Using $SU(3)$ Flavor to Constrain the CP Asymmetries in $B \to PP, VP, VV$ Decays involving $b \to s$ Transitions

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Abstract

We use the approximate $SU(3)$ flavor symmetry of the strong interaction to put bounds on the CP asymmetries in $b \to s$ decays of the $B_0^0$ and $B^+_u$ mesons. We extend the work of [1] to include all relevant $B \to PP$, $B \to VP$ and $B \to VV$ decays. We obtain the strongest constraints from current data, and provide a list of $SU(3)$ relations which can be used when future data is obtained.

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I. INTRODUCTION

This work concerns the CP asymmetries in two-body $b \to s$ decays of $B$ mesons:

$$A_f(t) \equiv \frac{\Gamma(B^0(t) \to f) - \Gamma(B^0(t) \to \bar{f})}{\Gamma(B^0(t) \to f) + \Gamma(B^0(t) \to \bar{f})}.$$  \hspace{1cm} (1)

For a final state $f$ which is a CP eigenstate, the CP asymmetry is time dependent,

$$A_f(t) = -C_f \cos(\Delta m_B t) + S_f \sin(\Delta m_B t),$$ \hspace{1cm} (2)

while for a flavor specific final state the CP asymmetry is time independent,

$$A_f(t) = A_f.$$ \hspace{1cm} (3)

In the standard model, the $b \to s$ transition amplitudes are naively dominated by a single weak phase. A contribution from a second weak phase is CKM suppressed by order $O(\lambda^2)$. Consequently, the corresponding CP asymmetries are naively expected to fulfill $-\eta_f S_f \approx S_{\psi K \bar{s}}$ and $C_f, A_f \approx 0$ up to order of a few percent. A deviation from those expectations can serve as a signal for new physics while an agreement with them would imply yet another success of the standard model.

The word “naively” is used here for a reason. While the CKM suppression factors are well known, the amplitude also depends on hadronic matrix elements for which there is no fundamental theory which is proven to a high level of precision. In order to estimate the allowed deviation of the CP asymmetries from the naive expectation within the SM, one needs to calculate these hadronic matrix elements (or at least, the ratios between them).

In this work we follow the method of [1] and use the approximate $SU(3)$ flavor symmetry of the strong interaction to bound the ratio between the relevant terms in the $b \to s$ decay amplitudes. We extend the analysis of [1] to include $b \to s$ decays in all two-body $B \to PP$, $B \to VP$ and $B \to VV$ decays. Here, $B$ stands for either $B^0_d$ or $B^+_s$, $P$ stands for the pseudoscalar meson nonet and $V$ stands for the vector meson nonet. The inclusion of $B \to VV$ modes requires some adjustment of the arguments in [1] so that they will apply to the case of non-single final state (similar in spirit to [2, 3]). The currently available experimental values of the CP asymmetries for these modes, taken from [4], are collected into table I ($B \to PP$), table II ($B \to VP$) and table III ($B \to VV$).

We do not consider here $b \to s$ decays of the $B^0_s$ meson although the same method applies here as well. We do include, however, $B^0_s$ decay modes in the theoretical decomposition to
| Mode             | $-\eta f S_f$ | $C_f, -A_f$ |
|------------------|---------------|-------------|
| $B^0_d \to \eta K_S$ | $-$           | $-$         |
| $B^0_d \to \eta' K_S$ | $0.50 \pm 0.09$ | $-0.07 \pm 0.07$ |
| $B^0_d \to \pi^0 K_S$ | $0.31 \pm 0.26$ | $-0.02 \pm 0.13$ |
| $B^0_d \to \pi^- K^+$ | $n/a$         | $0.115 \pm 0.018$ |
| $B^+_u \to \eta K^+$ | $n/a$         | $0.33 \pm 0.12$ |
| $B^+_u \to \eta' K^+$ | $n/a$         | $0.031 \pm 0.021$ |
| $B^+_u \to \pi^+ K^0$ | $n/a$         | $0.02 \pm 0.04$ |
| $B^+_u \to \pi^0 K^+$ | $n/a$         | $-0.04 \pm 0.04$ |

TABLE I: Measured CP asymmetries in $B \to PP$, $b \to s$ decays.

| Mode             | $-\eta f S_f$ | $C_f, -A_f$ |
|------------------|---------------|-------------|
| $B^0_d \to \phi K_S$ | $0.47 \pm 0.19$ | $-0.09 \pm 0.14$ |
| $B^0_d \to \omega K_S$ | $0.63 \pm 0.30$ | $-0.44 \pm 0.23$ |
| $B^0_d \to \rho^0 K_S$ | $-$           | $-$         |
| $B^0_d \to \rho^- K^+$ | $n/a$         | $-0.17^{+0.16}_{-0.15}$ |
| $B^0_d \to K^{*0} \eta$ | $n/a$         | $0.01 \pm 0.08$ |
| $B^0_d \to K^{*0} \eta'$ | $n/a$         | $-$         |
| $B^0_d \to K^{*0} \pi^0$ | $n/a$         | $0.01^{+0.26}_{-0.27}$ |
| $B^0_d \to K^{*-} \pi^-$ | $n/a$         | $0.05 \pm 0.14$ |
| $B^+_u \to \phi K^+$ | $n/a$         | $-0.037 \pm 0.050$ |
| $B^+_u \to \omega K^+$ | $n/a$         | $-0.02 \pm 0.07$ |
| $B^+_u \to \rho^+ K^0$ | $n/a$         | $-$         |
| $B^+_u \to \rho^0 K^+$ | $n/a$         | $-0.31^{+0.11}_{-0.12}$ |
| $B^+_u \to K^{*-} \eta$ | $n/a$         | $-0.03^{+0.10}_{-0.11}$ |
| $B^+_u \to K^{*-} \eta'$ | $n/a$         | $-$         |
| $B^+_u \to K^{*0} \pi^+$ | $n/a$         | $0.093 \pm 0.060$ |
| $B^+_u \to K^{*0} \pi^0$ | $n/a$         | $-0.04 \pm 0.29$ |

TABLE II: Measured CP asymmetries in $B \to VP$, $b \to s$ decays.
Table III: Measured CP asymmetries in $B\rightarrow VV$, $b\rightarrow s$ decays.

SU(3) invariant amplitude. We also comment on several one-to-one amplitude relations between specific $B_d^0$ and $B_s^0$ decays usually due to the U-spin subgroup of SU(3).

Using SU(3) has two weaknesses. First, this symmetry is broken by effects of order $m_s/\Lambda_{\chi}\sim 0.3$. The bounds we obtain can therefore be violated to this order. Second, our method often provides only a (conservative) upper bound on the deviation and not an estimate. The actual deviation can be substantially lower than our bounds. The advantage of our method, on the other hand, is its hadronic model independence. Thus, methods that use the approximate symmetries of the strong interaction [1, 2, 3, 5, 6, 7] are complementary to methods employing direct calculation of hadronic matrix elements within factorization related schemes [8, 9, 10, 11, 12].

The structure of this work is as follows: In section II we review the formalism relevant to the use of SU(3) in constraining the CP asymmetries in $b\rightarrow s$ transitions and comment on the strategy of this work. Section III is divided into three subsections dedicated to $B\rightarrow PP$ (subsection IIIA), $B\rightarrow VP$ (subsection IIIB) and $B\rightarrow VV$ (subsection IIIC) decays. For each decay mode we give the SU(3) relation which leads to the strongest constraint currently available. We also provide a list of additional relations for each mode and argue that, in the future, one of those relations is likely to provide the strongest bound. We conclude in section IV. For readers who are only interested in the current strongest constraints on the CP asymmetries, we summarized our results in this section. In addition, we give the details on how to derive the SU(3) reduced matrix elements relations in appendix A and provide
the resulting decomposition of all physical modes in three tables. We also quote the current available experimental branching ratios for all relevant modes in appendix [B]

II. FORMALISM

In this section we provide a short review of the formalism and introduce the notation relevant to the use of $SU(3)$ symmetry in $b \to s$ decays [1, 2]. We also provide further comments on several relevant issues.

We consider a $B \to f$ decay process involving a $b \to s$ transition, and write the decay amplitude as

$$A_f = \lambda_s^c a_f^c + \lambda_s^u a_f^u.$$  

(4)

We use the compact notation $\lambda_q^{q'} \equiv V_{qb}^* V_{qq'}$.

In the context of this work, writing the amplitude as in eq. (4) has two important merits: First, the parameters we use have definite behaviour under CP conjugation. In the conjugate decay process $\bar{B} \to \bar{f}$, the CKM factors get complex conjugated while the $a_{f}^q$ terms remain unchanged. The CP asymmetries can be therefore related in a simple way to those parameters. Second, in this way of writing we isolate the approximate $SU(3)$ invariant part of the amplitude, the strong factors $a_{f}^q$, from the explicitly $SU(3)$ breaking ones, the weak CKM factors.

The difficulty in interpreting the CP asymmetries lies in the fact that there is no fundamental theory for calculating the $a_{f}^q$ parameters. Those are essentially hadronic matrix elements which involve non-perturbative calculations of the strong interaction. The effect of these hadronic parameters on CP asymmetries, however, can be encoded, for each decay process, into a single object [1]

$$\xi_f \equiv \frac{|\lambda_u^s| a_f^u}{|\lambda_c^s| a_f^c}.$$  

(5)

For a final state $f$ which is a CP eigenstate, the observed CP asymmetries are given, to first order in $\xi_f$, by

$$-\eta_f S_f - \sin 2\beta = 2 \cos 2\beta \sin \gamma \Re \xi_f,$$  

(6)

$$C_f = -2 \sin \gamma \Im \xi_f.$$  

(7)

Combining the two relations (6) and (7) we have

$$\left(\frac{-\eta_f S_f - \sin 2\beta}{\cos 2\beta}\right)^2 + C_f^2 = 4 \sin^2 \gamma |\xi_f|^2.$$  

(8)
For a final state $f$ which is flavor specific, the CP asymmetry $-A_f$ is given by the same formula (7).

$SU(3)$ relations provide a way to constrain $|\xi_f|$ and therefore constrain the CP asymmetries. Owing to the approximate $SU(3)$ of the strong interaction, the $a_q^f$ parameters can be expressed using corresponding parameters of $b \to d$ decay processes,

$$a_q^f = \sum_{f'} X_{f'} b_{f'}^q ,$$

(9)

where the $b_{f'}^q$ enters the $B \to f'$ decay amplitude

$$A_{f'} = \lambda_c^d b_{f'}^c + \lambda_u^d b_{f'}^u .$$

(10)

Typically, for any mode $f$ there are many possible relations of the form (9). Finding the various coefficients $X_{f'}$ is the main topic of this work.

Once a relation of the form (9) is established, we can use the $X_{f'}$’s to define

$$\hat{\xi}_f \equiv \left| \frac{V_{us}}{V_{ud}} \right| \left| \sum_{f'} X_{f'} A_{f'} \right| \leq \left| \frac{V_{us}}{V_{ud}} \right| \frac{\sum_{f'} |X_{f'}| \sqrt{B(B \to f')}}{\sqrt{B(B \to f')}} .$$

(11)

If experimental data yield an upper bound on $\hat{\xi}_f$ in the range between $\lambda^2$ and 1 ($\hat{\xi}_f$ is positive by definition), a constraint on $|\xi_f|$ is implied:

$$|\xi_f| \leq \frac{\hat{\xi}_f + \lambda^2}{1 - \hat{\xi}_f} .$$

(12)

In fact, if we use the charge averaged branching ratios in (11), (and correspondingly replace the $|\sum_{f'} X_{f'} A_{f'}|$ with $\sqrt{\sum_{f'} |X_{f'} A_{f'}|^2}$ and the $|A_f|$ with $\sqrt{|A_f|^2 + |A_f|^2}$ in the definition for $\hat{\xi}$) the constraint on $|\xi_f|$ can be slightly stronger than (12), depending on the value of the weak phase $\gamma$ [2]. Keeping a conservative path, we take the worst case scenario and make no assumptions regarding $\gamma$.

Current experiments give no data on the relative phase of the various decay amplitudes. Since we wish to obtain an upper bound on $\xi_f$, we must make use of the triangle inequality and add the terms in (11) with an absolute value. While this method indeed guarantees that we obtain an upper bound (assuming $SU(3)$), it also has the potential of weakening this bound considerably.

Using this formalism, therefore, the best prospects for obtaining a strong constraint on the CP asymmetries lies in those amplitude relations which involve the smallest number
of modes. In this work we follow this logic and present relations with up to three modes involved. Currently those relations do give the strongest available constraints.

We stress that when we calculate $\xi_f$ from (11) we use either the experimental bound on the branching ratio $B(B \to f')$, if only a bound exists, or the central value, if an actual measurement exists. The fact that we use central values can, potentially, somewhat strengthen $\xi_f$. Being conservative in other respects we do not consider this to be a significant issue.

In $B \to VV$ decays, the measured CP asymmetry and branching ratios represent a sum over three possible final states with distinct orbital angular momentum configuration, namely $l = 0, 1$ or 2. Since all $B \to VV$ decays with $b \to s$ transitions are flavor specific, the only relevant CP asymmetry is $A_f$. As was explained in detail in [2, 3], an extension of the same method can be used to constrain the CP asymmetry obtained in the case of a sum over several final states.

Since here there are only three final states we write the expression explicitly. We consider the three amplitudes

$$A_{f; l} = \lambda^s_c a^l_{f; l} + \lambda^u a^u_{f; l},$$

with $l = 0, 1$ or 2. The modified parameter

$$\xi_f \equiv \frac{\left| \lambda^s_a \right| a^c_{f;0} a^u_{f;0} + a^{c*}_{f;1} a^u_{f;1} + a^{c*}_{f;2} a^u_{f;2} }{\left| \lambda^s_c \right| \left| a^c_{f;0} \right|^2 + \left| a^c_{f;1} \right|^2 + \left| a^c_{f;2} \right|^2},$$

enters the CP asymmetry in the same way as before:

$$A_f = 2 \sin \gamma I m \xi_f.$$  

As can be seen, when there is only a single final state, (14) reduces to (5).

Taking $B(B \to f^{(*)})$ in (11) to indicate a sum over the branching ratios of the three final states, we find that $|\xi_f|$ is constrained by the relation given in eq. (12). Thus the same formalism can be applied to the multiple final states $B \to VV$.

It is worthwhile noticing the special case in which there is a relation between a single $b \to s$ mode and a single $b \to d$ mode. Besides being a good candidates for giving the tightest bounds, if there is a measurement, such a relation may allow for actual extraction of the hadronic terms and their relative phase from three observables.

To be specific, consider that, for a CP eigenstates $f$, we have the relation

$$a^q_{f} = b^q_{f'},$$

(16)
If we measure the rate $\mathcal{B}(B \to f)$, and the two CP asymmetries $C_f$ and $S_f$, we can predict the rate $\mathcal{B}(B \to f')$ and the CP asymmetries $C_{f'}$ and $S_{f'}$ (given that CKM factors are known). For example, in many cases, the U-spin subgroup of SU(3) gives such a relation between $B_d^0$ and $B_s^0$ decay. When accurate experimental data will be available, this idea can be used to predict the expected decay rate for various $B_s^0$ decay mode, within the SU(3) symmetry approximation.

Our last comment for this section concerns the issue of $\eta - \eta'$ and $\phi - \omega$ mixing. In this work, as was done in [1], we do not assume SU(3) to be an approximate symmetry in the $\eta - \eta'$ and $\phi - \omega$ mixing. Instead, we use the phenomenological description of the mixing and apply SU(3) relation to each component individually (although, for $\eta - \eta'$ mixing the SU(3) breaking is small enough that one could just use SU(3) straightforwardly and identify $\eta = \eta_8$ and $\eta' = \eta_1$). In this way we are able to apply SU(3) symmetry to decay processes without assuming SU(3) symmetry in the masses.

While there is still room left for better theoretical understanding, we make in this work a distinction between the physics governing masses and mixing and the physics which governs decays. A large breaking effect in the masses does not imply by itself a similar breaking in decays. It is possible that, for some unknown reason, large SU(3) breakings in the decays do occur, but currently no data suggest this to be the case.

III. RESULTS

A good way to obtain the SU(3) relations in the form of (9) is to write all decay processes using SU(3) invariant matrix elements. In appendix A we give full details how this is done. We also provide the result of the calculation in the form of three tables which are relevant to any possible two-body decay of a $B$ meson into pseudo-scalars and vectors. The tables we obtain match those in [1] and are extended to include $B_s^0$ modes.

Using the tables it is next possible to find amplitude relations between physical modes. In what follows we do exactly that for all $b \to s$ process.

A. $B \to PP$ modes.

There are eight $b \to s$ decay modes involving final states with two nonet pseudo-scalars.
1. $B^0_d \to \eta K^0$ and $B^0_d \to \eta' K^0$

These two decays were specifically studied in [1]. We provide here an update of experimental data as well as additional comments.

We use $\eta - \eta'$ mixing of the form

$$\eta = \eta_1 \sin \theta_{\eta\eta'} + \eta_8 \cos \theta_{\eta\eta'} ,$$

$$\eta' = \eta_1 \cos \theta_{\eta\eta'} - \eta_8 \sin \theta_{\eta\eta'} .$$

We take $\theta_{\eta\eta'} = 20^\circ$ to be the mixing angle [13].

According to our framework, in order to derive the amplitude relations for $B^0_d \to \eta K^0$, we need to discuss separately $B^0_d \to \eta_1 K^0$ and $B^0_d \to \eta_8 K^0$. The $B^0_d \to \eta_1 K^0$ relations are simpler and can be obtained from table IV. There is a single relation involving just one amplitude

$$a^{q}_{B^0_d \to \eta_1 K^0} = b^{q}_{B^0_s \to \eta_1 K^0} .$$

There is one relation which involves two modes (neither involves $B^0_s$)

$$a^{q}_{B^0_d \to \eta_1 K^0} = \sqrt{\frac{3}{2}} b^{q}_{B^0_d \to \eta_1 \eta_8} - \sqrt{\frac{1}{2}} b^{q}_{B^0_d \to \eta_1 \pi^0} .$$

There are no relations involving three amplitudes or more.

The $B^0_d \to \eta_8 K^0$ relations are obtained using table IV. There are two relations involving just one amplitude. They both involve a $B^0_s$ decay:

$$a^{q}_{B^0_s \to \eta_8 K^0} = b^{q}_{B^0_s \to \eta_8 K^0} ,$$

$$a^{q}_{B^0_s \to \eta_8 K^0} = \sqrt{\frac{1}{3}} b^{q}_{B^0_s \to \eta_8 \pi^0} .$$

Combining relations (18) and (20) we get the U-spin relations which, for the physical $\eta$ and $\eta'$, imply

$$a^{q}_{B^0_d \to \eta K^0} = b^{q}_{B^0_s \to \eta K^0} ,$$

$$a^{q}_{B^0_d \to \eta' K^0} = b^{q}_{B^0_s \to \eta' K^0} .$$

We now demonstrate the power of such single amplitude relations. We consider the relation (23). Using the three measured observables \[4\], $S_{B^0_s \to \eta' K_S}$, $C_{B^0_s \to \eta' K_S}$ and $B (B^0_d \to \eta' K_S)$,
together with the CKM parameters [14], we solve for the hadronic part of the amplitude. We use the $SU(3)$ relation to calculate the expected values of the observables in the $B_s^0 \to \eta' K_S$ decay. We get

$$S_{B_s^0 \to \eta' K_S} \approx -0.46 \pm 0.29,$$  (24)
$$C_{B_s^0 \to \eta' K_S} \approx +0.18 \pm 0.11,$$  (25)
$$2 \mathcal{B}(B_s^0 \to \eta' K_S) \approx (29 \pm 18) \times 10^{-6}.$$  (26)

The resulting distribution is not normal. The factor of two in (26) is due to the $K$ mixing.

Going back to listing the relations, there are two relations involving two amplitudes. One of them involves $B_s^0$ and is of less interest here. The other is

$$a_{B_d^0 \to \eta K^0} = \frac{1}{\sqrt{6}} b_{B_d^0 \to K^+ K^-}^q - \frac{1}{\sqrt{3}} b_{B_d^0 \to \pi^0 \eta^0}^q .$$  (27)

Thus, the most “economical” relation is obtained from combining (19) and (27):

$$a_{B_d^0 \to \eta' K^0} = -\frac{s}{\sqrt{6}} b_{B_d^0 \to K^+ K^-}^q + \frac{s}{\sqrt{3}} b_{B_d^0 \to \pi^0 \eta^0}^q + \sqrt{3} c^2 s b_{B_d^0 \to \eta' \eta}^q - \sqrt{3} c^2 s b_{B_d^0 \to \eta' \eta'}^q ,$$  (28)
$$a_{B_d^0 \to \eta K^0} = \frac{c}{\sqrt{6}} b_{B_d^0 \to K^+ K^-}^q - \frac{c}{\sqrt{3}} b_{B_d^0 \to \pi^0 \eta^0}^q + \sqrt{3} c s^2 b_{B_d^0 \to \eta' \eta}^q - \sqrt{3} c s^2 b_{B_d^0 \to \eta' \eta'}^q ,$$  (29)

where we use $c \equiv \cos \theta_{\eta' \eta}$ and $s \equiv \sin \theta_{\eta' \eta}$.

Substituting the coefficient from the relations (28) in (11), and using the experimental branching ratios, we get

$$\hat{\xi}_{B_d^0 \to \eta' K^0} \leq 0.17 ,$$  (30)

which means using (12)

$$|\xi_{B_d^0 \to \eta' K^0}| \leq 0.26 .$$  (31)

A bound on $\hat{\xi}_{B_d^0 \to \eta K^0}$ cannot be obtained since the branching ratio for this mode is only bounded and not yet measured. We therefore have no knowledge on how small the denominator in (11) is. We can, however, use the relation (29) and write the bound on $\hat{\xi}_{B_d^0 \to \eta K^0}$ as a function of this branching ratio:

$$\hat{\xi}_{B_d^0 \to \eta K^0} \leq \frac{0.63}{\sqrt{10^6 \times \mathcal{B}(B_d^0 \to \eta K^0)}} .$$  (32)
We note that already at the current upper bound, $B(B_d^0 \to \eta K^0) \leq 1.9 \times 10^{-6}$, we get a rather weak bound on $\hat{\xi}_{B_d^0 \to \eta K^0}$.

Interestingly, $SU(3)$ relation can give some information on $B(B_d^0 \to \eta K^0)$. We write the following relation between $b \to s$ amplitudes

$$a_{B_d^0 \to \eta K^0} = \frac{1}{\sqrt{3}} a_{B_d^0 \to \pi^0 K^0}.$$  \hspace{1cm} (33)

Combined with $\eta - \eta'$ mixing we get the relation

$$a_{B_d^0 \to \eta K^0} = \tan \theta_{\eta\eta'} a_{B_d^0 \to \eta' K^0} + \frac{1}{\sqrt{3} \cos \theta_{\eta\eta'}} a_{B_d^0 \to \pi^0 K^0}.$$  \hspace{1cm} (34)

The complex phase between the two amplitudes on the RHS of eq. (34) is not determined by experimental data, and therefore no exact value for the $B_d^0 \to \eta K^0$ amplitude can be obtained. However, we can use the relation to obtain the bounds

$$0.66 \times 10^{-6} \lesssim B(B_d^0 \to \eta K^0) \lesssim 25 \times 10^{-6}.$$  \hspace{1cm} (35)

We therefore conclude that the branching ratio is expected to within a factor of 3 of the current experimental bound (see eq. (B1)).

There are nine relations involving $B_d^0 \to \eta_8 K^0$ with three other amplitudes, but two of them involve $B_s^0$ decays and we do not show them. We list the remaining seven relations and the implied constraints on $\hat{\xi}_{B_d^0 \to \eta'(\prime) K^0}$. The constraints are obtained by combining each relation with (19) and rotating to physical modes. We do not give here the explicit expression in terms of physical states which can be easily obtained by substitution. (One should take care, though, to properly normalize final states with two identical mesons. See appendix A for details.)

$$a_{B_d^0 \to \eta_8 K^0} = \frac{1}{\sqrt{6}} b_{B_d^0 \to K^+ K^-} + \frac{1}{\sqrt{3}} b_{B_d^0 \to \pi^+ \pi^-} - \frac{1}{\sqrt{6}} b_{B_d^0 \to \pi^+ \pi^-},$$

$$\implies \hat{\xi}_{B_d^0 \to \eta' K^0} \leq 0.18, \quad \hat{\xi}_{B_d^0 \to \eta K^0} \leq \frac{0.96}{\sqrt{10^6 \times B(B_d^0 \to \eta K^0)}},$$  \hspace{1cm} (36)

$$a_{B_d^0 \to \eta_8 K^0} = \sqrt{3} b_{B_d^0 \to \eta_8 \eta_8} - \sqrt{\frac{2}{3}} b_{B_d^0 \to \pi^0 K^0} - \frac{1}{\sqrt{6}} b_{B_d^0 \to K^+ K^-},$$

$$\implies \hat{\xi}_{B_d^0 \to \eta' K^0} \leq 0.17, \quad \hat{\xi}_{B_d^0 \to \eta K^0} \leq \frac{1.01}{\sqrt{10^6 \times B(B_d^0 \to \eta K^0)}},$$  \hspace{1cm} (37)
\[
a^{q}_{B_{d}^{0}\rightarrow\eta_{s}K^{0}} = \frac{1}{\sqrt{6}} B_{d}^{0}\rightarrow K^{0}\pi^{0} - \frac{1}{\sqrt{3}} b_{d}^{0}\rightarrow \pi^{0}\eta^{0} + \frac{1}{\sqrt{2}} b_{d}^{0}\rightarrow \eta_{s}\pi^{0} ,
\]
\[
\Rightarrow \hat{\xi}_{B_{d}^{0}\rightarrow \eta'K^{0}} \leq 0.18 , \quad \hat{\xi}_{B_{d}^{0}\rightarrow \eta_{s}K^{0}} \leq \frac{0.95}{\sqrt{10^{6} \times B(B_{d}^{0}\rightarrow \eta_{s}K^{0})}} , \quad (38)
\]

\[
a^{q}_{B_{d}^{0}\rightarrow \eta_{s}K^{0}} = - \frac{1}{\sqrt{6}} B_{d}^{0}\rightarrow K^{0}\pi^{0} - \frac{1}{2\sqrt{3}} b_{d}^{0}\rightarrow \pi^{0}\eta^{0} + \frac{\sqrt{2}}{3} b_{d}^{0}\rightarrow \eta_{s}\pi^{0} ,
\]
\[
\Rightarrow \hat{\xi}_{B_{d}^{0}\rightarrow \eta'K^{0}} \leq 0.17 , \quad \hat{\xi}_{B_{d}^{0}\rightarrow \eta_{s}K^{0}} \leq \frac{0.65}{\sqrt{10^{6} \times B(B_{d}^{0}\rightarrow \eta_{s}K^{0})}} , \quad (39)
\]

\[
a^{q}_{B_{d}^{0}\rightarrow \eta_{s}K^{0}} = - \sqrt{3} b_{d}^{0}\rightarrow K^{0}\pi^{0} - \frac{1}{2} b_{d}^{0}\rightarrow \pi^{0}\eta^{0} + \sqrt{3} b_{d}^{0}\rightarrow \eta_{s}\eta^{0} ,
\]
\[
\Rightarrow \hat{\xi}_{B_{d}^{0}\rightarrow \eta'K^{0}} \leq 0.17 , \quad \hat{\xi}_{B_{d}^{0}\rightarrow \eta_{s}K^{0}} \leq \frac{1.20}{\sqrt{10^{6} \times B(B_{d}^{0}\rightarrow \eta_{s}K^{0})}} , \quad (40)
\]

\[
a^{q}_{B_{d}^{0}\rightarrow \eta_{s}K^{0}} = - \sqrt{3} b_{d}^{0}\rightarrow K^{0}\pi^{0} + \sqrt{2} b_{d}^{0}\rightarrow \pi^{0}\eta^{0} + \sqrt{3} b_{d}^{0}\rightarrow \eta_{s}\eta^{0} ,
\]
\[
\Rightarrow \hat{\xi}_{B_{d}^{0}\rightarrow \eta'K^{0}} \leq 0.18 , \quad \hat{\xi}_{B_{d}^{0}\rightarrow \eta_{s}K^{0}} \leq \frac{1.45}{\sqrt{10^{6} \times B(B_{d}^{0}\rightarrow \eta_{s}K^{0})}} , \quad (41)
\]

\[
a^{q}_{B_{d}^{0}\rightarrow \eta_{s}K^{0}} = - \sqrt{3} b_{d}^{0}\rightarrow \pi^{0}\pi^{0} + \frac{1}{2\sqrt{2}} b_{d}^{0}\rightarrow \pi^{0}\eta^{0} + \frac{\sqrt{3}}{4} b_{d}^{0}\rightarrow \eta_{s}\eta^{0} ,
\]
\[
\Rightarrow \hat{\xi}_{B_{d}^{0}\rightarrow \eta'K^{0}} \leq 0.17 , \quad \hat{\xi}_{B_{d}^{0}\rightarrow \eta_{s}K^{0}} \leq \frac{0.67}{\sqrt{10^{6} \times B(B_{d}^{0}\rightarrow \eta_{s}K^{0})}} . \quad (42)
\]

As can be seen, there are several relations which give similar bounds on \( \hat{\xi} \). Typically, the value of \( \hat{\xi} \) for these relations differ in the next significant digits, which were rounded here. Relation (42) is the one which was used in [1]. Indeed, using the available experimental data at the time [1] was written, relation (42) gives a slightly stronger bound compared to the relation (27). We can see by comparing the bounds in (30) and (32) to the ones in the list above, that the naive expectation that “the fewest modes the better” is not unreasonable.

Using this way of presenting the relations, it is easy to foresee how future improvement in experimental data would impact the bound. In particular, there is yet room for considerable improvement of the bound if the branching ratios \( \mathcal{B}(B_{d}^{0}\rightarrow \eta_{s}) \), \( \mathcal{B}(B_{d}^{0}\rightarrow \eta'_{s}) \) and \( \mathcal{B}(B_{d}^{0}\rightarrow \eta'\eta') \) will have a stronger constraint. As a demonstration, if all three are below the \( 1 \times 10^{-6} \) level, this would imply \( \hat{\xi}_{B_{d}^{0}\rightarrow \eta'K^{0}} \leq 0.09. \)
2. \( B^+_u \rightarrow \eta K^+ \) and \( B^+_u \rightarrow \eta' K^+ \).

The results here update and extend the results in [1]. As in the previous section, a separate \( SU(3) \) analysis is required for \( B^+_u \rightarrow \eta_1 K^+ \) and \( B^+_u \rightarrow \eta_8 K^+ \). The former has a single relation:

\[
a^q_{B^+_u \rightarrow \eta_1 K^+} = b^q_{B^+_u \rightarrow \eta_1 \pi^+}. \quad (43)
\]

For the latter, there are no relations involving just one amplitude. The relations involving two modes are

\[
a^q_{B^+_u \rightarrow \eta_8 K^+} = \frac{1}{\sqrt{3}} b^q_{B^+_u \rightarrow \pi^0 \pi^+} - \frac{1}{\sqrt{6}} b^q_{B^+_u \rightarrow K^+ K^0},
\]

\[
\Rightarrow \quad \hat{\xi}_{B^+_u \rightarrow \eta' K^+} \leq 0.02, \quad \hat{\xi}_{B^+_u \rightarrow \eta K^+} \leq 0.24, \quad (44)
\]

\[
a^q_{B^+_u \rightarrow \eta_8 K^+} = b^q_{B^+_u \rightarrow \eta_8 \pi^+} - \sqrt{\frac{2}{3}} \frac{b^q_{B^+_u \rightarrow \pi^0 \pi^+}}{b^q_{B^+_u \rightarrow \eta_8 \pi^+}},
\]

\[
\Rightarrow \quad \hat{\xi}_{B^+_u \rightarrow \eta' K^+} \leq 0.03, \quad \hat{\xi}_{B^+_u \rightarrow \eta K^+} \leq 0.50, \quad (45)
\]

\[
a^q_{B^+_u \rightarrow \eta_8 K^+} = \frac{\sqrt{3}}{2} b^q_{B^+_u \rightarrow \pi^0 \pi^+} - \frac{b^q_{B^+_u \rightarrow \eta_8 \pi^+}}{\sqrt{6}} ,
\]

\[
\Rightarrow \quad \hat{\xi}_{B^+_u \rightarrow \eta' K^+} \leq 0.03, \quad \hat{\xi}_{B^+_u \rightarrow \eta K^+} \leq 0.43. \quad (46)
\]

Note that (46) is a linear combination of (44) and (45).

There are six relations involving three modes but only two of them do not involve \( B^0_s \):

\[
a^q_{B^+_u \rightarrow \eta_8 K^+} = \frac{1}{\sqrt{6}} b^q_{B^0_s \rightarrow \pi^+ \pi^-} - \frac{1}{\sqrt{3}} \frac{b^q_{B^0_s \rightarrow \pi^0 \pi^0}}{b^q_{B^+_u \rightarrow \eta_8 \pi^+}} - \frac{1}{\sqrt{6}} b^q_{B^+_u \rightarrow K^+ K^0},
\]

\[
\Rightarrow \quad \hat{\xi}_{B^+_u \rightarrow \eta' K^+} \leq 0.02, \quad \hat{\xi}_{B^+_u \rightarrow \eta K^+} \leq 0.28, \quad (47)
\]

\[
a^q_{B^+_u \rightarrow \eta_8 K^+} = \frac{1}{2} \frac{b^q_{B^0_s \rightarrow \pi^+ \pi^-}}{\sqrt{2}} - \frac{1}{2} \frac{b^q_{B^0_s \rightarrow \pi^0 \pi^0}}{b^q_{B^+_u \rightarrow \eta_8 \pi^+}} - \frac{1}{2} b^q_{B^+_u \rightarrow \eta_8 \pi^+},
\]

\[
\Rightarrow \quad \hat{\xi}_{B^+_u \rightarrow \eta' K^+} \leq 0.03, \quad \hat{\xi}_{B^+_u \rightarrow \eta K^+} \leq 0.48. \quad (48)
\]

As was the case in [1], the strongest constraint on \( \hat{\xi}_{B^+_u \rightarrow \eta' K^+} \) (and on \( \hat{\xi}_{B^+_u \rightarrow \eta K^+} \)) comes from the relation (44) (It is slightly stronger than (47)):

\[
|\xi_{B^+_u \rightarrow \eta' K^+}| \leq 0.05, \quad (49)
\]

\[
|\xi_{B^+_u \rightarrow \eta K^+}| \leq 0.38. \quad (50)
\]

Note that since \( \hat{\xi}_{B^+_u \rightarrow \eta' K^+} < \lambda^2 \) the upper bound on \( |\xi_{B^+_u \rightarrow \eta' K^+}| \) in (49), is simply \( \lambda^2 \).
3. $B_d^0 \to \pi^0 K^0$.

The amplitude of the $B_d^0 \to \pi^0 K^0$ decay is related by $SU(3)$ to the amplitude of the $b \to s$ transition $B_s^0 \to \eta_s K^0$

$$a_{B_d^0 \to \pi^0 K^0}^q = \sqrt{3} a_{B_s^0 \to \eta_s K^0}^q .$$

(51)

As a consequence, the $SU(3)$ relations can be read off relations (20), (21), (27) and (36)–(42). For example, a one amplitude relation is read off (21):

$$a_{B_d^0 \to \pi^0 K^0}^q = b_{B_s^0 \to \pi^0 K^0}^q .$$

(52)

Using the experimental values of $S_{B_d^0 \to \pi^0 K^0}$, $C_{B_d^0 \to \pi^0 K^0}$ and $B(B_d^0 \to \pi^0 K^0)$ we can solve for the hadronic parameters and obtain an estimate for the $B_s^0 \to \pi^0 K_S$ observables:

$$S_{B_s^0 \to \pi^0 K_S} \approx -0.67 \pm 0.27 ,$$

(53)

$$C_{B_s^0 \to \pi^0 K_S} \approx +0.12 \pm 0.12 ,$$

(54)

$$2 B(B_s^0 \to \pi^0 K_S) \approx (11 \pm 12) \times 10^{-6} .$$

(55)

The single relation involving two amplitudes without $B_s^0$ decay is read off (21):

$$a_{B_d^0 \to \pi^0 K^0}^q = \frac{1}{\sqrt{2}} b_{B_d^0 \to K^+ K^-}^q - b_{B_d^0 \to \pi^0 \pi^0}^q .$$

(56)

This relation gives the strongest constraint

$$\hat{\xi}_{B_d^0 \to \pi^0 K^0} \leq 0.09 ,$$

(57)

which results with

$$|\xi_{B_d^0 \to \pi^0 K^0}| \leq 0.15 .$$

(58)

The other relations give far weaker constraints.

4. $B_d^0 \to \pi^- K^+$

A single amplitude relation due to U-spin involves a $B_s^0$ mode:

$$a_{B_d^0 \to \pi^- K^+}^q = b_{B_s^0 \to \pi^+ K^-}^q .$$

(59)
Three amplitude relations involve two modes, but only one does not involve a $B_s^0$ mode:

\[ a^q_{B_d^0 \rightarrow \pi^- K^+} = b^q_{B_d^0 \rightarrow \pi^+ \pi^-} - b^q_{B_d^0 \rightarrow K^+ K^-} , \quad \Rightarrow \quad \hat{\xi}_{B_d^0 \rightarrow \pi^- K^+} \leq 0.12 , \quad (60) \]

giving

\[ |\xi_{B_d^0 \rightarrow \pi^- K^+}| \leq 0.19 . \quad (61) \]

There are six relations involving three amplitudes, but only two do not involve a $B_s^0$ mode:

\[ a^q_{B_d^0 \rightarrow \pi^- K^+} = \sqrt{2} b^q_{B_d^0 \rightarrow \pi^0 \pi^-} - b^q_{B_d^0 \rightarrow K^- K^+} + \sqrt{2} b^q_{B_u^+ \rightarrow \pi^0 \pi^+} , \quad \Rightarrow \quad \hat{\xi}_{B_d^0 \rightarrow \pi^- K^+} \leq 0.27 , \quad (62) \]

\[ a^q_{B_d^0 \rightarrow \pi^- K^+} = b^q_{B_d^0 \rightarrow \pi^- \pi^+} - b^q_{B_d^0 \rightarrow K^0 K^+} - \sqrt{3} b^q_{B_u^+ \rightarrow \eta \pi^0} , \quad \Rightarrow \quad \hat{\xi}_{B_d^0 \rightarrow \pi^- K^+} \leq 0.35 . \quad (63) \]

Clearly (and reasonably), the relation (60) gives the strongest bound.

5. $B_u^+ \rightarrow \pi^+ K^0$

A single one amplitude relation due to the U-spin subgroup gives, as expected, the strongest constraint

\[ a^q_{B_u^+ \rightarrow \pi^+ K^0} = b^q_{B_u^+ \rightarrow K^0 K^+} , \quad \Rightarrow \quad \hat{\xi}_{B_u^+ \rightarrow \pi^+ K^0} \leq 0.05 , \quad (64) \]

which leads to

\[ |\xi_{B_u^+ \rightarrow \pi^+ K^0}| \leq 0.05 . \quad (65) \]

In principle, whenever a single amplitude is involved in the calculation of $\hat{\xi}_f$, a lower bound on $|\xi_f|$ can be placed as well as an upper bound. However, since here $\hat{\xi}_{B_u^+ \rightarrow \pi^+ K^0} \sim \lambda^2$ the lower bound is actually zero.

A weaker bound is obtained from the single two amplitudes relation

\[ a^q_{B_u^+ \rightarrow \pi^+ K^0} = \sqrt{\frac{3}{2}} b^q_{B_u^+ \rightarrow \eta \pi^+} - \sqrt{\frac{1}{2}} b^q_{B_u^+ \rightarrow \pi^0 \pi^+} , \quad \Rightarrow \quad \hat{\xi}_{B_u^+ \rightarrow \pi^+ K^0} \leq 0.21 . \quad (66) \]
Three relations involve three amplitudes but only one does not involve a $B_s^0$ mode

$$a^q_{B_u^+ \to \pi^0 K^0} = \sqrt{\frac{3}{2}} b^q_{B_u^+ \to \eta_{8} \pi^+} + \sqrt{\frac{1}{2}} b^q_{B_d^0 \to \pi^0 \pi^0} - \frac{1}{2} b^q_{B_d^0 \to \pi^- \pi^+} ,$$

$$\implies \hat{\xi}_{B_u^+ \to \pi^0 K^0} \leq 0.23 . \quad (67)$$

This relation gives a still weaker bound.

6. $B_u^+ \to \pi^0 K^+$

Three amplitude relations involve two modes

$$a^q_{B_u^+ \to \pi^0 K^+} = b^q_{B_u^+ \to \pi^0 \pi^+} + \sqrt{\frac{1}{2}} b^q_{B_u^+ \to \pi^0 \pi^+} , \quad \implies \hat{\xi}_{B_u^+ \to \pi^0 K^+} \leq 0.20 , \quad (68)$$

$$a^q_{B_u^+ \to \pi^0 K^+} = \sqrt{3} b^q_{B_u^+ \to \eta_{8} \pi^+} - \sqrt{\frac{1}{2}} b^q_{B_u^+ \to \pi^0 \pi^+} , \quad \implies \hat{\xi}_{B_u^+ \to \pi^0 K^+} \leq 0.32 , \quad (69)$$

$$a^q_{B_u^+ \to \pi^0 K^+} = \frac{1}{2} b^q_{B_u^+ \to \pi^0 \pi^+} + \sqrt{\frac{3}{4}} b^q_{B_u^+ \to \eta_{8} \pi^+} , \quad \implies \hat{\xi}_{B_u^+ \to \pi^0 K^+} \leq 0.21 . \quad (70)$$

The relation (70) is a linear combination of (68) and (69). Two out of six relations involving three amplitudes do not involve $B_s^0$ modes

$$a^q_{B_u^+ \to \pi^0 K^+} = \sqrt{\frac{1}{2}} b^q_{B_d^0 \to \pi^- \pi^+} - b^q_{B_d^0 \to \pi^0 \pi^0} + \sqrt{\frac{1}{2}} b^q_{B_u^+ \to \pi^0 \pi^+} ,$$

$$\implies \hat{\xi}_{B_u^+ \to \pi^0 K^+} \leq 0.23 , \quad (71)$$

$$a^q_{B_u^+ \to \pi^0 K^+} = \sqrt{\frac{1}{8}} b^q_{B_d^0 \to \pi^- \pi^+} - \frac{1}{2} b^q_{B_d^0 \to \pi^0 \pi^0} + \sqrt{\frac{3}{4}} b^q_{B_u^+ \to \eta_{8} \pi^+} ,$$

$$\implies \hat{\xi}_{B_u^+ \to \pi^0 K^+} \leq 0.23 . \quad (72)$$

The strongest bound (68) implies

$$|\xi_{B_u^+ \to \pi^0 K^+}| \leq 0.31 . \quad (73)$$
B. $B \to VP$ modes.

The are sixteen $b \to s$ decay processes in which one final meson belongs to the vector nonet and the other belongs to the pseudo-scalar nonet. The method of calculation is generally similar to the $B \to PP$ case but using the appropriate tables of reduced matrix elements (with table IV taking the role of table V). We consider the relevant amplitude relations and the resulting constraints below

1. $B \to \phi K^0$ and $B \to \omega K^0$

The $\omega - \phi$ mixing is given by \[\omega = \sqrt{\frac{2}{3}} \phi_1 + \sqrt{\frac{1}{3}} \phi_8 , \tag{74}\]
\[\phi = \sqrt{\frac{1}{3}} \phi_1 - \sqrt{\frac{2}{3}} \phi_8 . \]

We need to discuss both $B \to \phi_1 K^0$ and $B \to \phi_8 K^0$. Since, similarly to $\eta_1$, $\phi_1$ is a singlet of $SU(3)$, the same $SU(3)$ relations obtained from table IV hold. We can therefore simply adapt the single relation (19):
\[a_{qB_0}^{d\to \phi_1 K^0} = \sqrt{\frac{3}{2}} b_{qB_0}^{d\to \phi_1 \eta_8} - \sqrt{\frac{1}{2}} b_{qB_0}^{d\to \phi_1 \pi^0} . \tag{75}\]

Turning now to the $B_0^d \to \phi_8 K^0$ mode, the single relation involving two amplitudes and five out of the six relations involving three amplitudes, involve $B_0^s$ modes. The remaining single amplitude with three amplitudes which does not involve $B_0^s$ modes is
\[a_{qB_0}^{d\to \phi_8 K^0} = \sqrt{\frac{3}{2}} b_{qB_0}^{d\to \phi_8 \eta_8} - \sqrt{\frac{3}{2}} b_{qB_0}^{d\to \phi_8 \pi^0} - \sqrt{\frac{1}{2}} b_{qB_0}^{d\to \phi_8 \pi^0} . \tag{76}\]

Combining (75) and (76) we obtain
\[a_{qB_0}^{d\to \phi K^0} = b_{qB_0}^{d\to K^0 K^0} - \sqrt{\frac{3}{2}} b_{qB_0}^{d\to \phi_8 \eta_8} + \sqrt{\frac{3}{2}} \cos \theta_{\eta \eta'} b_{\phi \eta} - \sqrt{\frac{3}{2}} \sin \theta_{\eta \eta'} b_{\phi \eta'} , \tag{77}\]
\[a_{qB_0}^{d\to \omega K^0} = -\sqrt{\frac{1}{2}} b_{qB_0}^{d\to K^+ K^-} - \sqrt{\frac{3}{2}} b_{qB_0}^{d\to \omega \pi^0} + \sqrt{\frac{3}{2}} \cos \theta_{\eta \eta'} b_{\omega \eta} - \sqrt{\frac{3}{2}} \sin \theta_{\eta \eta'} b_{\omega \eta'} . \tag{78}\]

Unfortunately, a constraint on $B(B_0^d \to K^0 K^0)$ is currently not available. As a consequence, a constraint on $|\xi_{B_0^d \to \phi K^0}|$ and $|\xi_{B_0^d \to \phi K^0}|$ cannot be obtained. In fact, it can be shown that any amplitude relation for $B_0^d \to \phi_8 K^0$ must involve at least one of the modes $B_0^d \to K^{*+} K^-$,
\(B^0_d \to K^*-K^+,\; B^0_d \to \overline{K}^{*0}K^0\) or \(B^0_d \to K^{*0}\overline{K}^0\). As long as the branching ratios for these modes remain unknown, no bound can be obtained.

We can, however, write \(\hat{\xi}_{B^0_d \to \phi K^0}\) and \(\hat{\xi}_{B^0_d \to \omega K^0}\) as a function of the missing branching ratio. We get

\[
\hat{\xi}_{B^0_d \to \phi K^0} \leq 0.21 + 0.08 \sqrt{10^6 \times \mathcal{B}(B^0_d \to \overline{K}^{*0}K^0)},
\]

\[
\hat{\xi}_{B^0_d \to \omega K^0} \leq 0.31 + 0.07 \sqrt{10^6 \times \mathcal{B}(B^0_d \to \overline{K}^{*0}K^0)}.
\] (79) (80)

2. \(B^+_u \to \phi K^+\) and \(B^+_u \to \omega K^+\)

The relation for \(B^+_u \to \phi_1 K^+\) can be read off (43). There is a single amplitude relation

\[
a^q_{B^+_u \to \phi_1 K^+} = b^q_{B^+_u \to \rho K^+}.
\] (81)

For \(B^+_u \to \phi_8 K^+\) we find three relations involving two amplitudes:

\[
a^q_{B^+_u \to \phi_8 K^+} = \sqrt{\frac{1}{3}} b^q_{B^+_u \to \rho K^+} - \sqrt{\frac{1}{6}} b^q_{B^+_u \to \overline{K}^{*0}K^+},
\]

\[
\Rightarrow \; \hat{\xi}_{B^+_u \to \phi K^+} \leq 0.26, \; \hat{\xi}_{B^+_u \to \omega K^+} \leq 0.31,
\] (82)

\[
a^q_{B^+_u \to \phi_8 K^+} = b^q_{B^+_u \to \phi_8 K^+} - \sqrt{\frac{3}{2}} b^q_{B^+_u \to \overline{K}^{*0}K^+},
\]

\[
\Rightarrow \; \hat{\xi}_{B^+_u \to \phi K^+} \leq 0.22, \; \hat{\xi}_{B^+_u \to \omega K^+} \leq 0.36,
\] (83)

\[
a^q_{B^+_u \to \phi_8 K^+} = \sqrt{\frac{3}{4}} b^q_{B^+_u \to \rho K^+} - \frac{1}{2} b^q_{B^+_u \to \phi_8 K^+},
\]

\[
\Rightarrow \; \hat{\xi}_{B^+_u \to \phi K^+} \leq 0.29, \; \hat{\xi}_{B^+_u \to \omega K^+} \leq 0.28,
\] (84)

The relation (84) is a linear combination of (82) and (83). Those relations are similar to relations (44)–(46).

As was the case in [1], relation (83) gives the strongest bound on \(|\xi_{B^+_u \to \phi K^+}|\) which currently implies

\[
|\xi_{B^+_u \to \phi K^+}| \leq 0.34.
\] (85)

Relation (84) is the one that gives the strongest bound on \(|\xi_{B^+_u \to \omega K^+}|\) which implies

\[
|\xi_{B^+_u \to \omega K^+}| \leq 0.46.
\] (86)

There are no amplitude relations involving three modes.
3. $B_d^0 \to \rho^0 K^0$

The single relation that involves two amplitudes, involves $B_s^0$ modes. Out of the six amplitude relation that involves three amplitude, only one does not involve $B_s^0$ modes:

$$a_{B_d^0 \to \rho^0 K^0}^q = \sqrt{\frac{1}{2} b_{B_d^0 \to K^*0}^q} + \sqrt{\frac{3}{2} b_{B_d^0 \to \rho^0 \eta_8}^q} - \sqrt{\frac{1}{2} b_{B_d^0 \to \rho^0 \pi^0}^q}. \quad (87)$$

Similarly to $B_d^0 \to \phi K^0$ and $B_d^0 \to \omega K^0$, the fact that the branching ratio $B(B_d^0 \to K^*0 K^0)$ has not been measured yet, prevents a bound from being obtained. At least one of the four modes $B_d^0 \to K^{*+} K^-, B_d^0 \to K^{*-} K^+, B_d^0 \to K^{*0} K^0$ or $B_d^0 \to K^{*0} \overline{K}^0$, must be measured in order for a bound to be placed.

We can write the resulting constraint as a function of the missing branching ratio:

$$\hat{\xi}_{B_d^0 \to \rho^0 K^0} \leq 0.32 + 0.07 \sqrt{10^6 \times B(B_d^0 \to K^*0 K^0)}. \quad (88)$$

4. $B_d^0 \to \rho^- K^+$

We note first a single amplitude U-spin relation

$$a_{B_d^0 \to \rho^- K^+}^q = b_{B_d^0 \to K^{*-} \pi^+}^q. \quad (89)$$

On a more practical ground, we find a single amplitude relation involving two modes

$$a_{B_d^0 \to \rho^- K^+}^q = b_{B_d^0 \to K^{*-} \pi^+}^q - b_{B_d^0 \to K^{*+} K^+}^q. \quad (90)$$

$B(B_d^0 \to K^{*-} K^+)$ is needed in order to place a bound from eq. (90), or, more generally, at least one of the four modes $B_d^0 \to K^{*+} K^-, B_d^0 \to K^{*-} K^+, B_d^0 \to K^{*0} K^0$ or $B_d^0 \to K^{*0} \overline{K}^0$ is needed.

As a function of the missing branching ratio we get

$$\hat{\xi}_{B_d^0 \to \rho^- K^+} \leq 0.34 + 0.07 \sqrt{10^6 \times B(B_d^0 \to K^{*-} K^+)}. \quad (91)$$

5. $B_u^+ \to \rho^0 K^+$

Three amplitude relations involve two modes

$$a_{B_u^+ \to \rho^0 K^+}^q = \sqrt{\frac{1}{2} b_{B_u^+ \to K^*0 K^+}^q} + b_{B_u^+ \to \rho^0 \pi^+}^q, \quad \implies \hat{\xi}_{B_u^+ \to \rho^0 K^+} \leq 0.49, \quad (92)$$
\[ a_{B_u^+ \to \rho^0 K^+}^q = -\sqrt{\frac{1}{2}} b_{B_u^+ \to K^0 K^+}^q + \sqrt{3} b_{B_u^+ \to \phi K^+}^q , \quad \Rightarrow \quad \hat{\xi}_{B_u^+ \to \rho^0 K^+} \leq 0.55 , \quad (93) \]

\[ a_{B_u^+ \to \rho^0 K^+}^q = \frac{1}{2} b_{B_u^+ \to \rho^0 \pi^+}^q + \sqrt{\frac{3}{4}} b_{B_u^+ \to \phi \pi^+}^q , \quad \Rightarrow \quad \hat{\xi}_{B_u^+ \to \rho^0 K^+} \leq 0.34 . \quad (94) \]

Relation (94) is a linear combination of (92) and (93). The strongest bound (94) implies

\[ |\hat{\xi}_{B_u^+ \to \rho^0 K^+}| \leq 0.61 . \quad (95) \]

6. \( B_u^+ \to \rho^+ K^0 \)

The branching ratio \( \mathcal{B}(B_u^+ \to \rho^+ K^0) \) has currently only an upper bound. As a consequence, no bound on \( |\hat{\xi}_{B_u^+ \to \rho^+ K^0}| \) can be placed. We list here the amplitude relations that will become useful once this branching ratio is measured.

A single one amplitude relation is

\[ a_{B_u^+ \to \rho^+ K^0}^q = b_{B_u^+ \to \rho^0 K^0}^q . \quad (96) \]

The branching ratio \( \mathcal{B}(B_u^+ \to K^{*0} K^{0}) \) has not been measured yet.

There is a single relation involving two modes:

\[ a_{B_u^+ \to \rho^+ K^0}^q = \sqrt{\frac{3}{2}} b_{B_u^+ \to \rho^0 \eta^8}^q - \sqrt{\frac{1}{2}} b_{B_u^+ \to \rho^0 \pi^0}^q , \quad \Rightarrow \quad \hat{\xi}_{B_u^+ \to \rho^0 K^0} \leq \frac{1.66}{\sqrt{10^6 \times \mathcal{B}(B_u^+ \to \rho^+ K^0)}} . \quad (97) \]

There are no amplitude relations involving three modes.

Considering the upper bound on \( \mathcal{B}(B_d^0 \to \rho^+ K^0) \), a strong bound can still result.

7. \( B_d^0 \to K^{*0} \eta \) and \( B_d^0 \to K^{*0} \eta' \)

The branching ratio \( \mathcal{B}(B_d^0 \to K^{*0} \eta') \) has currently only been bounded. As a consequence, a bound on \( |\hat{\xi}_{B_d^0 \to K^{*0} \eta'}| \) cannot be placed.

The relation for \( B_d^0 \to K^{*0} \eta_1 \) can be read off (19):

\[ a_{B_d^0 \to K^{*0} \eta_1}^q = \sqrt{\frac{3}{2}} b_{B_d^0 \to \phi \eta_1}^q - \sqrt{\frac{1}{2}} b_{B_d^0 \to \rho \eta_1}^q . \quad (98) \]
A single relation for \( B_d^0 \to K^{*0}\eta_8 \) involving two amplitudes, and five out of six relation involving three amplitudes, involve \( B_s^0 \) modes. The one additional relation involving three amplitude is

\[
a_{B_d^0 \to K^{*0}\eta_8}^q = \sqrt{\frac{3}{2}} b_{B_d^0 \to \phi_8\eta_8}^q - \sqrt{\frac{3}{2}} b_{B_d^0 \to K^{*0}K^0}^q - \sqrt{\frac{1}{2}} b_{B_d^0 \to \rho^0\eta_8}^q .
\]  

(99)

Since \( B(B_d^0 \to K^{*0}K^0) \) has not been constrained yet, no bound on \( |\xi_{B_d^0 \to K^{*0}\eta}| \) and \( |\xi_{B_d^0 \to K^{*0}\eta'}| \) can be currently obtained. At least one of the four \( B_d^0 \to K^{*}K \) modes is required for a bound.

We can write the resulting constraint as a function of the missing branching ratio. Since \( B(B_d^0 \to K^{*0}\eta') \) has not been measured we only have

\[
\hat{\xi}_{B_d^0 \to K^{*0}\eta} \leq 0.14 + 0.06 \sqrt{10^6 \times B(B_d^0 \to K^{*0}K^0)} .
\]  

(100)

8. \( B_u^+ \to K^{*+}\eta \) and \( B_u^+ \to K^{*+}\eta' \)

\( B(B_u^+ \to K^{*+}\eta') \) has not been measured yet and therefore \( |\xi_{B_u^+ \to K^{*+}\eta'}| \) cannot be bounded. The single relation for \( B_u^+ \to K^{*+}\eta_1 \) can be read off (103):

\[
a_{B_u^+ \to K^{*+}\eta_1}^q = b_{B_u^+ \to \rho^+\eta_1}^q .
\]  

(101)

For \( B_u^+ \to K^{*+}\eta_8 \) the relations are equivalent to (102)–(104):

\[
a_{B_u^+ \to K^{*+}\eta_8}^q = \frac{1}{\sqrt{3}} b_{B_u^+ \to \rho^+\pi^0}^q - \frac{1}{\sqrt{6}} b_{B_u^0 \to K^{*+}K^0}^q ,
\]  

(102)

\[
a_{B_u^+ \to K^{*+}\eta_8}^q = b_{B_u^+ \to \rho^+\eta_8}^q - \sqrt{\frac{2}{3}} b_{B_u^0 \to K^{*+}K^0}^q ,
\]  

(103)

\[
a_{B_u^+ \to K^{*+}\eta_8}^q = \sqrt{\frac{3}{4}} b_{B_u^+ \to \rho^+\pi^0}^q - \frac{1}{2} b_{B_u^+ \to \rho^+\eta_8}^q .
\]  

(104)

Relation (104) is a linear combination of (102) and (103).

Since \( B(B_u^+ \to K^{*+}K^0) \) has not been bounded yet, the only useful relation currently is (104). We get

\[
\hat{\xi}_{B_u^+ \to K^{*+}\eta} \leq 0.26 , \quad \hat{\xi}_{B_u^+ \to K^{*+}\eta'} \leq \frac{1.37}{\sqrt{10^6 \times B(B_u^+ \to K^{*+}\eta')}} ,
\]  

(105)

which gives

\[
|\xi_{B_u^+ \to K^{*+}\eta}| \leq 0.42 .
\]  

(106)

At the current upper bound of \( B(B_u^+ \to K^{*+}\eta') \) the constraint on \( \hat{\xi}_{B_u^+ \to K^{*+}\eta'} \) is already not very strong.
9. \( B_d^0 \rightarrow K^{*0}\pi^0 \)

There is one amplitude relation involving two modes and six amplitude relations involving three modes. The only relation, however, which does not involve \( B_s^0 \) modes is

\[
a_{B_d^0 \rightarrow K^{*0}\pi^0}^q = \sqrt{\frac{1}{2}} b_{B_d^0 \rightarrow K^{*0}K^0}^q - \sqrt{\frac{1}{2}} b_{\rho^0\pi^0}^q + \sqrt{\frac{3}{2}} b_{B_d^0 \rightarrow \phi\pi^0}^q .
\] (107)

Once more, since \( \mathcal{B}(B_d^0 \rightarrow K^{*0}K^0) \) is not bounded, no bound on \( |\xi_{B_d^0 \rightarrow K^{*0}K^0}| \) is attained. Again, at least one of the four \( B_d^0 \rightarrow K^*K \) already mentioned must be bounded in order for \( |\xi_{B_d^0 \rightarrow K^{*0}K^0}| \) to be bounded.

As a function of the missing branching ratio we can write

\[
\hat{\xi}_{B_d^0 \rightarrow K^{*0}\pi^0} \leq 0.46 + 0.12 \sqrt{10^6 \times \mathcal{B}(B_d^0 \rightarrow K^{*0}K^0)} ,
\] (108)

which seems to give a rather weak bound in any case.

10. \( B_d^0 \rightarrow K^{*+}\pi^- \)

We first note a U-spin relation involving one amplitude (with a \( B_s^0 \) mode):

\[
a_{B_d^0 \rightarrow K^{*+}\pi^-}^q = b_{B_d^0 \rightarrow \rho^+K^-}^q .
\] (109)

There is a single relation involving two amplitudes

\[
a_{B_d^0 \rightarrow K^{*+}\pi^-}^q = b_{B_d^0 \rightarrow \rho^+\pi^-}^q - b_{B_d^0 \rightarrow K^{*+}K^-}^q .
\] (110)

There is no relation involving three modes (or four). At least one of the four \( B_d^0 \rightarrow K^*K \) is needed. Since \( \mathcal{B}(B_d^0 \rightarrow K^{*+}\pi^-) \) is not bounded, \( |\xi_{B_d^0 \rightarrow K^{*+}\pi^-}| \) is not bounded currently.

As a function of the missing branching ratio we can write

\[
\hat{\xi}_{B_d^0 \rightarrow K^{*+}\pi^-} \leq 0.32 + 0.06 \sqrt{10^6 \times \mathcal{B}(B_d^0 \rightarrow K^{*+}K^-)} .
\] (111)
11. $B^+_u \to K^{*+}\pi^0$

Three amplitude relations involve two modes (similar to (92)–(94)):

\[ a^q_{B^+_u \to K^{*+}\pi^0} = \sqrt{\frac{1}{2}} b^q_{B^+_u \to K^{*+}\pi^0} + b^q_{B^+_u \to \rho^+\pi^0} , \quad (112) \]

\[ a^q_{B^+_u \to K^{*+}\pi^0} = -\sqrt{\frac{1}{2}} b^q_{B^+_u \to K^{*+}\pi^0} + \sqrt{3} b^q_{B^+_u \to \rho^+\eta_8} , \quad (113) \]

\[ a^q_{B^+_u \to K^{*+}\pi^0} = \sqrt{\frac{3}{4}} b^q_{B^+_u \to \rho^+\eta} + \sqrt{\frac{3}{4}} b^q_{B^+_u \to \rho^+\eta} . \quad (114) \]

Relation (114) is a linear combination of (112) and (113).

Since $B(B^+_u \to K^{*+}\overline{K}^0)$ has not been bounded yet a bound comes only from (114):

\[ \hat{\xi}_{B^+_u \to K^{*+}\pi^0} \leq 0.45 , \quad (115) \]

which implies

\[ |\xi_{B^+_u \to K^{*+}\pi^0}| \leq 0.91 . \quad (116) \]

12. $B^+_u \to K^{*0}\pi^+$

A single U-spin relation involving one amplitude gives

\[ a^q_{B^+_u \to K^{*0}\pi^+} = b^q_{B^+_u \to K^{*0}\pi^+} , \quad \implies \quad \hat{\xi}_{B^+_u \to K^{*0}\pi^+} \leq 0.16 . \quad (117) \]

A single relation involving two amplitudes is

\[ a^q_{B^+_u \to K^{*0}\pi^+} = \sqrt{\frac{3}{2}} b^q_{B^+_u \to \phi\pi^+} - \sqrt{\frac{1}{2}} b^q_{B^+_u \to \rho\pi^+} , \quad \implies \quad \hat{\xi}_{B^+_u \to K^{*0}\pi^+} \leq 0.30 . \quad (118) \]

There are no other relations.

As expected, the strongest bound is obtained from (117)

\[ |\xi_{B^+_u \to K^{*0}\pi^+}| \leq 0.25 . \quad (119) \]

C. $B \to V V$ modes

The relations for $b \to s$ transitions in $B \to VV$ decays can be read off the $B \to PP$ relations by a direct substitution. The only additional difference is that the $\eta-\eta'$ mixing should be replaced by the $\omega-\phi$ mixing with a different mixing angle.

Currently, only four $b \to s B \to VV$ are measured (the other four are only bounded). We present below the details.
1. $B_d^0 \to \phi K^*0$ and $B_d^0 \to \omega K^*0$

The relevant relations are those that correspond to the relations in section III A 1. However, the relation equivalent to (19) involves the mode $B_d^0 \to \phi_1 \phi_8$ which, by mixing, involves $B(B_d^0 \to \omega \omega), B(B_d^0 \to \phi \omega)$ and $B(B_d^0 \to \phi \phi)$. Only the latter have been currently bounded and therefore no bounds on $|\xi_{B_d^0 \to \phi K^*0}|$ or $|\xi_{B_d^0 \to \omega K^*0}|$ result.

We can express the constraint as a function of the missing branching ratios. Since only $B(B_d^0 \to \phi K^*0)$ is currently measured, we only consider this mode. The most useful relation is the one corresponding to relations (42) which gives:

$$\hat{\xi}_{B_d^0 \to \phi K^*0} \leq 0.15 + 0.03 \sqrt{10^6 \times B(B_d^0 \to \omega \omega)} ,$$

(120)

2. $B_u^+ \to \phi K^{*+}$ and $B_u^+ \to \omega K^{*0}$

The relevant relations are those corresponding to the ones in section III A 2. Due to the different mixing, the strongest bounds on $|\xi_{B_u^+ \to \phi K^{*+}}|$ and $|\xi_{B_u^+ \to \omega K^{*+}}|$ both come from the relation which corresponds to (43). Since $B(B_u^+ \to \omega K^{*+})$ has only been bounded, we get

$$\hat{\xi}_{B_u^+ \to \omega K^{*+}} \leq \frac{1.53}{\sqrt{10^6 \times B(B_u^+ \to \omega K^{*+})}} ,$$

(121)

Already at the current upper bound of $B(B_u^+ \to \omega K^{*+})$ a useful bound on $|\xi_{B_u^+ \to \omega K^{*+}}|$ does not result.

For $B_u^+ \to \phi K^{*+}$ we get

$$\hat{\xi}_{B_u^+ \to \phi K^{*+}} \leq 0.41 ,$$

(122)

which leads to

$$|\xi_{B_u^+ \to \phi K^{*+}}| \leq 0.78 .$$

(123)

3. $B_d^0 \to \rho^0 K^{*0}$

The relevant relation correspond to the ones in section III A 3.

$$a^q_{B_d^0 \to \rho K^{*0}} = \sqrt{3} b^q_{B_d^0 \to \phi K^{*0}}$$

(124)

the actual relation can be therefore read off the ones in section III A 1.
Since $\mathcal{B}(B^0_d \to \rho^0 K^{*0})$ is not measured yet, we cannot give an explicit bound here. Had the branching ratio been measured, the strongest bound would come from the relation corresponding to (38):

$$
\hat{\xi}_{B^0_d \rightarrow \rho^0 K^{*0}} \leq \frac{2.04}{\sqrt{10^6 \times \mathcal{B}(B^0_d \to \rho^0 K^{*0})}}.
$$

(125)

Already at the current upper bound of $\mathcal{B}(B^0_d \to \rho^0 K^{*0})$ a useful bound does not result.

4. $B^0_d \to \rho^- K^{*+}$

The relevant relations correspond to those in section III A 4. Again, since $\mathcal{B}(B^0_d \to \rho^- K^{*+})$ is not measured yet, no bound on $|\xi_{B^0_d \rightarrow \rho^- K^{*+}}|$ can be attained. Had it been measured, the strongest bound would have come from a relation corresponding to (63):

$$
\hat{\xi}_{B^0_d \rightarrow \rho^- K^{*+}} \leq \frac{3.68}{\sqrt{10^6 \times \mathcal{B}(B^0_d \to \rho^- K^{*+})}}.
$$

(126)

5. $B^+_u \to \rho^+ K^{*0}$

The relevant relations are those corresponding to the ones in section III A 3. The strongest bound comes from a relation corresponding to (64):

$$
|\xi_{B^+_u \rightarrow \rho^+ K^{*0}}| \leq 1.44.
$$

(127)

6. $B^+_u \to \rho^0 K^{*+}$

The relevant relations corresponds to those in section III A 6. The strongest bound comes from the relation corresponding to (70) which gives

$$
|\xi_{B^+_u \rightarrow \rho^0 K^{*+}}| \leq 1.02.
$$

(128)

IV. SUMMARY

We summarize the constrains on the various $|\xi_{B \to f}|$’s obtained in section III. For $B \to PP$ modes, the only mode which is not measured yet is $B^0_d \to \eta K^0$. For the other modes we get
the following bounds

\[ |\xi_{B_0^d \to \eta^0 K^0}| \leq 0.26 , \tag{30} \]
\[ |\xi_{B_0^d \to \pi^0 K^0}| \leq 0.15 , \tag{38} \]
\[ |\xi_{B_0^d \to \pi^- K^+}| \leq 0.19 , \tag{51} \]
\[ |\xi_{B_0^d \to \eta' K^+}| \leq 0.05 , \tag{59} \]
\[ |\xi_{B_0^d \to \eta' K^+}| \leq 0.38 , \tag{60} \]
\[ |\xi_{B_0^d \to \pi^+ K^0}| \leq 0.05 , \tag{65} \]
\[ |\xi_{B_0^d \to \pi^0 K^+}| \leq 0.31 . \tag{78} \]

For \( B \to VP \) modes we have three modes which have not been measured yet and have only an upper bound: \( B_0^d \to K^* \eta' \), \( B_u^+ \to \rho^+ K^0 \) and \( B_u^+ \to K^*+ \eta' \). The \( B_0^d \to VP \) modes are unbounded due to the lack of bounds on various \( B_0^d \to K^* K \) modes. We therefore write \( \hat{\xi} \) for these modes as a function of the missing branching ratios. We get the following results:

\[ \hat{\xi}_{B_0^d \to \phi K^0} \leq 0.21 + 0.08 \sqrt{10^6 \times \mathcal{B}(B_0^d \to \overline{K}^0 K^0)} , \tag{79} \]
\[ \hat{\xi}_{B_0^d \to \omega K^0} \leq 0.31 + 0.07 \sqrt{10^6 \times \mathcal{B}(B_0^d \to \overline{K}^0 K^0)} , \tag{80} \]
\[ \hat{\xi}_{B_0^d \to \rho^0 K^0} \leq 0.32 + 0.07 \sqrt{10^6 \times \mathcal{B}(B_0^d \to \overline{K}^0 K^0)} , \tag{88} \]
\[ \hat{\xi}_{B_0^d \to \rho^- K^+} \leq 0.34 + 0.07 \sqrt{10^6 \times \mathcal{B}(B_0^d \to K^*^- K^+)} , \tag{91} \]
\[ \hat{\xi}_{B_0^d \to \phi K^0} \leq 0.14 + 0.06 \sqrt{10^6 \times \mathcal{B}(B_0^d \to K^* K^0)} , \tag{100} \]
\[ \hat{\xi}_{B_0^d \to \rho^0 K^0} \leq 0.46 + 0.12 \sqrt{10^6 \times \mathcal{B}(B_0^d \to K^* K^0)} , \tag{108} \]
\[ \hat{\xi}_{B_0^d \to \rho^- K^+} \leq 0.32 + 0.06 \sqrt{10^6 \times \mathcal{B}(B_0^d \to K^*+ K^-)} , \tag{111} \]

\[ |\xi_{B_u^+ \to \phi K^+}| \leq 0.34 , \tag{S5} \]
\[ |\xi_{B_u^+ \to \omega K^+}| \leq 0.46 , \tag{S6} \]
\[ |\xi_{B_u^+ \to \rho^0 K^+}| \leq 0.61 , \tag{95} \]
\[ |\xi_{B_u^+ \to \rho^- K^+}| \leq 0.42 , \tag{105} \]
\[ |\xi_{B_u^+ \to K^*+ \eta}| \leq 0.91 , \tag{116} \]
\[ |\xi_{B_u^+ \to K^*+ K^0}| \leq 0.25 . \tag{119} \]
For $B \to VV$ modes there are four modes which are still to be measured. Those are $B^0_d \to \omega K^{*0}$, $B^0_d \to \rho^0 K^{*0}$, $B^0_d \to \rho^- K^{*+}$ and $B^+_u \to \omega K^{*+}$. For the other modes we get

$$\hat{\xi}_{B^0_d \to \phi K^{*0}} \leq 0.15 + 0.03 \sqrt{10^6 \times B(B^0_d \to \omega \omega)},$$

$$|\xi_{B^+_u \to \phi K^{*+}}| \leq 0.78,$$

$$|\xi_{B^+_u \to \rho^+ K^{*0}}| \leq 1.44,$$

$$|\xi_{B^+_u \to \rho^0 K^{*+}}| \leq 1.02,$$

All our constraints on the CP asymmetries in $b \to s$ transitions agree quite well with the observed CP asymmetries. Some constraints cannot be currently obtained. First, there are all the $b \to s$ modes which have not been measured yet and are only bounded. Obviously, a measurement of those is also needed if CP asymmetry is to be measured. Second, all the $B^0_d \to VP$ modes require a bound on some $B^0_d \to K^*K$ branching ratio. The most useful $K^*K$ mode is different for each $B^0_d \to VP$ mode. Third, the CP asymmetry of $B^0_d \to \phi K^{*0}$ (and $B^0_d \to \omega K^{*0}$) require a bound on the branching ratios of $B^0_d \to \omega \omega$.

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**APPENDIX A: OBTAINING SU(3) RELATIONS**

A simple way to obtain all $SU(3)$ relations is to use tensor methods. In this appendix we give the calculation details. The results agree with the tables in [1] with the addition of $B_s$ modes which we include here.

We write the singlet and octet pseudo-scalar and vector mesons as $SU(3)$ tensors of the appropriate rank

$$P_1 = \eta_1,$$

$$(P_8)^i_j = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & K^0 \\ K^- & \frac{1}{\sqrt{2}} K^0 & -\sqrt{\frac{2}{3}} \eta_8 \end{pmatrix},$$
\[ V_1 = \phi_1 , \]  

\[(V_8)_j^i = \begin{pmatrix} \frac{1}{\sqrt{2}} \rho^0 + \frac{1}{\sqrt{6}} \phi_8 & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}} \rho^0 + \frac{1}{\sqrt{6}} \phi_8 & K^{*0} \\ K^{*-} & \overline{K^{*0}} & -\sqrt{\frac{3}{2}} \phi_8 \end{pmatrix}, \]  

The \( B \) mesons form a triplet of \( SU(3) \):

\[(B_3)_i = \begin{pmatrix} B_u^- \\ B_d^0 \\ B_s^0 \end{pmatrix} . \]  

We combine the \( b \to d \) and \( b \to s \) Hamiltonian operators into three rank 3 tensors \([15, 16]\):

\[ ((H_3^3)_k^j)^i = \begin{pmatrix} \lambda^d_q \\ \lambda^d_q \\ \lambda^s_q \end{pmatrix} \]

\[ ((H_3^2)_k^j)^i = \begin{pmatrix} \lambda^d_q \\ \lambda^d_q \\ \lambda^s_q \end{pmatrix} \]

\[ ((H_3^1)_k^j)^i = \begin{pmatrix} \lambda^d_q \\ \lambda^d_q \\ \lambda^s_q \end{pmatrix} \]

\[ ((H_1^1)_k^j)^i = \begin{pmatrix} \lambda^d_q \\ \lambda^d_q \\ \lambda^s_q \end{pmatrix} \]

where \( \lambda_q^d = V_{qb}^* V_{qq'} \) and \( q \) can be either \( u \) or \( c \).

The effective Hamiltonian is obtained by contracting in all possible ways the Hamiltonian operators with the meson tensors. We next proceed and show how this is done for the relevant final states.

1. \( B \to P_1 P_8, P_1 V_8, V_1 P_8 \) and \( V_1 V_8 \)

In all the cases of \( B \to P_1 P_8, P_1 V_8, V_1 P_8 \) and \( V_1 V_8 \), the combination of meson representation is always an octet. The effective Hamiltonian is therefore obtained by considering the
following contractions

\[ \mathcal{H}_{\text{eff}} = A_{q}^{8} (B_{3})_{i} (H_{15}^{q})_{ij} (S_{1})_{k} (M_{8})_{j}^{k} \]
\[ + A_{8}^{q} (B_{3})_{i} (H_{6}^{q})_{ij} (S_{1})_{k} (M_{8})_{j}^{k} \]
\[ + A_{8}^{q} (B_{3})_{i} (H_{3}^{q})_{ji} (S_{1})_{k} (M_{8})_{j}^{k} , \]

where both \( S \) and \( M \) can stand for either \( P \) or \( V \) and it is understood that \( q \) is summed over \( q = u, c \). The coefficients \( A_{3}^{q}, A_{6}^{q} \) and \( A_{15}^{q} \) are the reduced matrix elements and can have, as far as the \( SU(3) \) analysis is concerned, arbitrary values.

We point out, for clarity, that there are other ways to contract the indices in (A9). For example we can change the \( i \) and \( j \) indices in \( H_{6}^{q} \) and \( H_{15}^{q} \). However, one can easily check that all other possible contractions give the same coefficients as the three that already appear in (A9) and therefore contribute to physical processes in the same way. There is, therefore, no need to consider them.

Expanding the effective Hamiltonian (A9) we arrange the resulting numerical factors in Table IV. For concreteness we list the \( B \to P_{1} P_{8} \) case, but the three other cases can be obtained by simple substitution. We do not list the CKM factors in Table IV since they can be easily understood. For example, the \( B_{u}^{+} \to \eta_{1} K^{+} \) decay amplitude is obtained from the effective Hamiltonian term

\[ B_{u}^{+} \eta_{1} K^{+} \left[ \lambda_{c}^{s} (3 A_{15}^{c} + A_{6}^{c} + A_{3}^{c}) + \lambda_{u}^{s} (3 A_{15}^{u} + A_{6}^{u} + A_{3}^{u}) \right] . \]
There are six ways (i.e. six representations) to combine two octets into \( SU(3) \) invariant tensors:

\[
(T^{MN}_1) = (M_8)_m^n (N_8)_m^n ,
\]

(\text{A11})

\[
(T^{sN}_{8s})_j^i = \frac{1}{2} \left( (M_8)_m^n (N_8)_j^m + (N_8)_m^n (M_8)_j^m \right) - \frac{1}{3} (T^{MN}_1) \delta_j^i ,
\]

(\text{A12})

\[
(T^{aN}_{8a})_j^i = \frac{1}{2} \left( (M_8)_m^n (N_8)_j^m - (N_8)_m^n (M_8)_j^m \right) ,
\]

(\text{A13})

\[
(T^{MN}_{10})_{ijk} = \frac{1}{6} \left( (M_8)_i^m (N_8)_j^k \varepsilon_{kmm} + (M_8)_j^m (N_8)_k^n \varepsilon_{imm} + (M_8)_k^m (N_8)_i^n \varepsilon_{jmn} \right.
\]

\[
+ (M_8)_k^m (N_8)_j^n \varepsilon_{imm} + (M_8)_j^m (N_8)_k^n \varepsilon_{kmm} + (M_8)_i^m (N_8)_k^n \varepsilon_{jmn} \right) ,
\]

(\text{A14})

\[
(T^{MN}_{10})_{ijk} = \frac{1}{6} \left( (M_8)_i^m (N_8)_j^k \varepsilon_{kmm} + (M_8)_j^m (N_8)_k^n \varepsilon_{imm} + (M_8)_k^m (N_8)_i^n \varepsilon_{jmn} \right.
\]

\[
+ (M_8)_k^m (N_8)_j^n \varepsilon_{imm} + (M_8)_j^m (N_8)_k^n \varepsilon_{kmm} + (M_8)_i^m (N_8)_k^n \varepsilon_{jmn} \right) ,
\]

(\text{A15})

\[
(T^{MN}_{27})_{kl}^i = \frac{1}{4} \left( (M_8)_k^i (N_8)_l^j + (M_8)_l^i (N_8)_k^j \right)
\]

\[
- \frac{1}{10} \left( \delta_i^l (T^{MN}_1)_l^i + \delta_i^j (T^{MN}_1)_k^j + \delta_i^k (T^{MN}_1)_l^k + \delta_i^l (T^{MN}_1)_j^l \right) \delta_i^j (T^{MN}_1)_k^j \delta_i^k (T^{MN}_1)_l^k
\]

\[
- \frac{1}{24} \left( \delta_i^l \delta_i^j + \delta_i^j \delta_i^k \right) (T^{MN}_1) .
\]

(\text{A16})

Here both \( M \) and \( N \) stand for either \( P \) or \( V \).

When \( M = N \), as in the \( B \rightarrow P_8 P_8 \) and \( B \rightarrow V_8 V_8 \) case, the only non zero tensors are \((T^{MM}_1), (T^{sN}_{8s}) \) and \((T^{MN}_{27}) \). Contacting them with the Hamiltonian operators and the \( B \) triplet we write

\[
\mathcal{H}_{\text{eff}} = C^{q15}_{27} (B_3)_i (H^{q}_{15})_{jk} (T^{MM}_1)_l^i (T^{MM}_8)_j^k + C^{q15}_{88} (B_3)_i (H^{q}_{15})_{l}^i (T^{MM}_8)_j^k
\]

\[
+ C^{q9}_{88} (B_3)_i (H^{q}_{9})_{l}^i (T^{MM}_8)_j^k + C^{q9}_{1} (B_3)_i (H^{q}_{9})_{l}^i (T^{MM}_1)_j^k .
\]

(\text{A17})

Again, \( M \) stands for either \( P \) or \( V \), and the index \( q \) is summed over \( q = u, c \). As before, there are other ways to contract the indices, which are equivalent to the ones we present and therefore redundant.

The numerical factors are given in table \[\text{V} \]. We write the mesons for the \( B \rightarrow P_8 P_8 \) case while the \( B \rightarrow V_8 V_8 \) case is obtained by simple substitution. One should note that if the
amplitudes are to be related to physical decay rates in a consistent way, an additional factor of $\sqrt{2}$ needs to be introduced by hand for final states which contain two identical mesons, such as $\pi^0\pi^0$ or $\eta_8\eta_8$, due to the different phase space. Such a factor was introduced in the table.

| $B_d \to K^0\eta_8$ | $C^q_{27}$ | $C^q_{15}$ | $C^q_{8s}$ | $C^q_{8s}$ | $C^q_{3s}$ | $C^q_{3s}$ |
|---------------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $B_d \to K^0\pi^0$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ |
| $B_d \to K^+\pi^-$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ |
| $B_u \to K^+\pi^0$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ |
| $B_u \to K^+\eta_8$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ |
| $B_u \to K^0\pi^+$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ |
| $B_d \to K^0\bar{K}^0$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ |
| $B_d \to K^-K^+$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ |
| $B_d \to \pi^0\pi^0$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ |
| $B_d \to \eta_8\pi^0$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ |
| $B_d \to \eta_8\eta_8$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ |
| $B_d \to \pi^+\pi^+$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ | $\frac{15}{3}$ |

**TABLE V**: $SU(3)$ decomposition of $B \to M_8N_8$. 

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3. $B \to P_3 V_8$.

Setting $M = P$ and $N = V$, all six invariant tensors $\{A11\}-\{A16\}$ are now non-zero. We write the fully contracted Hamiltonian by adding the necessary terms to $\{A17\}$

$$
H_{\text{eff}} = E_{15}^{q15} (B_3)_i (H_1^{q15})_i^j (T_{27}^{PV})_j^k + E_{8a}^{q8} (B_3)_i (H_6^{q8})_i^j (T_{8a}^{PV})_j^k
$$

$$
+ E_{15}^{q15} (B_3)_i (H_1^{q15})_i^j (T_{27}^{PV})_j^k + E_{15}^{q15} (B_3)_i (H_1^{q15})_i^j (T_{27}^{PV})_j^k
$$

$$
+ E_{10}^{q10} (B_3)_i (H_6^{q10})_i^j (T_{10}^{PV})_j^k (\varepsilon)_m^k_\mathrm{nm} + E_{8a}^{q8} (B_3)_i (H_6^{q8})_i^j (T_{8a}^{PV})_j^k
$$

$$
+ E_{10}^{q10} (B_3)_i (H_6^{q10})_i^j (T_{10}^{PV})_j^k (\varepsilon)_m^k_\mathrm{nm} + E_{8a}^{q8} (B_3)_i (H_6^{q8})_i^j (T_{8a}^{PV})_j^k
$$

$$
+ E_{10}^{q10} (B_3)_i (H_6^{q10})_i^j (T_{10}^{PV})_j^k (\varepsilon)_m^k_\mathrm{nm} + E_{8a}^{q8} (B_3)_i (H_6^{q8})_i^j (T_{8a}^{PV})_j^k
$$

(A18)

Again $q$ is summed over $q = u, c$ and we do not write other possible contractions which contribute in the same way. The numerical factors are given in table VI.

**APPENDIX B: EXPERIMENTAL DATA**

We collect in this appendix the current experimental data [4]. All branching ratios are given in units of $10^{-6}$. Since we only list $B_u^+$ and $B_d^0$ branching ratios, the identity of the decaying meson is self evident.

1. $B \to PP, b \to s$ modes

\[
\mathcal{B}(\eta K^0) < 1.9 , \quad \text{(B1)}
\]
\[
\mathcal{B}(\eta' K^0) = 63.2 \pm 3.3 , \quad \text{(B2)}
\]
\[
\mathcal{B}(\pi^0 K^0) = 11.5 \pm 1.0 , \quad \text{(B3)}
\]
\[
\mathcal{B}(\pi^- K^+) = 18.9 \pm 0.7 , \quad \text{(B4)}
\]
\[
\mathcal{B}(\eta K^+) = 2.5 \pm 0.3 , \quad \text{(B5)}
\]
\[
\mathcal{B}(\eta' K^+) = 69.4 \pm 2.7 , \quad \text{(B6)}
\]
\[
\mathcal{B}(\pi^0 K^+) = 12.1 \pm 0.8 , \quad \text{(B7)}
\]
\[
\mathcal{B}(\pi^+ K^0) = 24.1 \pm 1.3 . \quad \text{(B8)}
\]
\[
\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
& E^{15}_{7} & E^{15}_{8} & E^{17}_{8} & E^{7}_{10} & E^{9}_{13} & E^{15}_{17} & E^{15}_{80} & E^{8}_{6} & E^{9}_{3} \\
\hline
B_{s}^{0} \rightarrow K^{0} \phi_{8} & 2 \sqrt{\frac{3}{2}} & 1 & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 & -4 \sqrt{\frac{1}{2}} & 0 & -\sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\
B_{d}^{0} \rightarrow K^{0} \rho^{0} & 6 \sqrt{\frac{3}{10}} & 1 & -\frac{1}{2 \sqrt{3}} & -\frac{1}{2 \sqrt{3}} & 0 & -4 \sqrt{\frac{2}{3}} & 4 \sqrt{\frac{3}{2}} & -1 & 2 \sqrt{\frac{3}{2}} \\
B_{s}^{0} \rightarrow \eta_{8} K^{*+} & 2 \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 & 4 \sqrt{\frac{2}{3}} & 0 & 0 & -\sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\
B_{d}^{0} \rightarrow \pi^{0} K^{*0} & 6 \sqrt{\frac{3}{10}} & -\frac{1}{\sqrt{2}} & -\frac{1}{2 \sqrt{3}} & -\frac{1}{2 \sqrt{3}} & 0 & 4 \sqrt{\frac{2}{3}} & 4 \sqrt{\frac{3}{2}} & -1 & 2 \sqrt{\frac{3}{2}} \\
B_{s}^{0} \rightarrow \pi^{-} K^{++} & 2 & -\frac{1}{\sqrt{2}} & -\frac{1}{2} & 0 & -\frac{1}{2} & 1 & 0 & -\frac{1}{2} & -\frac{1}{2} \\
B_{d}^{0} \rightarrow \rho^{-} K^{+} & -\frac{5}{3} & -1 & \frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\
B_{s}^{0} \rightarrow \pi^{-} K^{++} & 8 \sqrt{\frac{2}{3}} & 0 & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 0 & 0 & 4 \sqrt{\frac{2}{3}} & 3 \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{2}} \\
B_{d}^{0} \rightarrow \eta_{8} K^{*+} & 4 \sqrt{\frac{2}{3}} & 0 & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 & 0 & 0 & -\sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\
B_{s}^{0} \rightarrow \pi^{+} K^{*0} & -\frac{3}{5} & 0 & -\frac{1}{4} & -\frac{1}{4} & 0 & 0 & -4 \sqrt{\frac{2}{3}} & -\frac{5}{2} & -\frac{1}{2} \\
B_{d}^{0} \rightarrow K^{+} \rho^{0} & 8 \sqrt{\frac{2}{3}} & 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 3 \sqrt{\frac{2}{3}} & 3 \sqrt{\frac{2}{3}} \\
B_{s}^{0} \rightarrow K^{+} \phi_{8} & 4 \sqrt{\frac{2}{3}} & 0 & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 & 0 & 0 & -\sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\
B_{d}^{0} \rightarrow K^{0} \rho^{0} & -\frac{5}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\
B_{s}^{0} \rightarrow \pi^{0} \phi_{8} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
B_{d}^{0} \rightarrow \eta_{8} \phi_{8} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
B_{s}^{0} \rightarrow \pi^{+} \rho^{-} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
B_{d}^{0} \rightarrow \pi^{+} \phi_{8} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
B_{s}^{0} \rightarrow K^{0} \rho^{0} & -\frac{5}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
B_{d}^{0} \rightarrow K^{+} \phi_{8} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
B_{s}^{0} \rightarrow \pi^{0} \phi_{8} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
B_{d}^{0} \rightarrow \eta_{8} \phi_{8} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
B_{s}^{0} \rightarrow \pi^{+} \rho^{-} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
B_{d}^{0} \rightarrow \pi^{+} \phi_{8} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
B_{s}^{0} \rightarrow K^{0} \rho^{0} & -\frac{5}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
B_{d}^{0} \rightarrow K^{+} \phi_{8} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
B_{s}^{0} \rightarrow \pi^{0} \phi_{8} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
B_{d}^{0} \rightarrow \eta_{8} \phi_{8} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
B_{s}^{0} \rightarrow \pi^{+} \rho^{-} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
B_{d}^{0} \rightarrow \pi^{+} \phi_{8} & -\frac{1}{3} & 0 & -\frac{1}{\sqrt{3}} & 0 & \frac{3}{\sqrt{3}} & -\frac{1}{3} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
\hline
\end{array}
\]

**Table VI: SU(3) decomposition of \( B \rightarrow P V \).**
2. \( B \to PP, \ b \to d \) modes

\[
\mathcal{B}(\eta \eta) < 2.0 , \quad (B9)
\]
\[
\mathcal{B}(\eta' \eta') < 10 , \quad (B10)
\]
\[
\mathcal{B}(\eta' \eta) < 4.6 , \quad (B11)
\]
\[
\mathcal{B}(\pi^0 \pi^0) = 1.45 \pm 0.29 , \quad (B12)
\]
\[
\mathcal{B}(\pi^+ \pi^-) = 5.0 \pm 0.4 , \quad (B13)
\]
\[
\mathcal{B}(\eta \pi^0) < 2.5 , \quad (B14)
\]
\[
\mathcal{B}(\eta' \pi^0) < 3.7 , \quad (B15)
\]
\[
\mathcal{B}(K^0 \overline{K^0}) = 0.96^{+0.25}_{-0.24} , \quad (B16)
\]
\[
\mathcal{B}(K^+ K^-) = 0.05^{+0.10}_{-0.09} , \quad (B17)
\]
\[
\mathcal{B}(\eta \pi^+) = 4.3 \pm 0.4 , \quad (B18)
\]
\[
\mathcal{B}(\eta' \pi^+) = 2.53^{+0.59}_{-0.50} , \quad (B19)
\]
\[
\mathcal{B}(\pi^0 \pi^+) = 5.5 \pm 0.6 , \quad (B20)
\]
\[
\mathcal{B}(\overline{K^0} K^+) = 1.2 \pm 0.3 . \quad (B21)
\]

3. \( B \to VP, \ b \to s \) modes

\[
\mathcal{B}(\phi K^0) = 8.3^{+1.2}_{-1.0} , \quad (B22)
\]
\[
\mathcal{B}(\omega K^0) = 4.7 \pm 0.6 , \quad (B23)
\]
\[
\mathcal{B}(\rho^0 K^0) = 5.1 \pm 1.6 , \quad (B24)
\]
\[
\mathcal{B}(\rho^- K^+) = 9.9^{+1.6}_{-1.5} , \quad (B25)
\]
\[
\mathcal{B}(K^0 \eta) = 18.7 \pm 1.7 , \quad (B26)
\]
\[
\mathcal{B}(K^0 \eta') < 7.6 , \quad (B27)
\]
\[
\mathcal{B}(K^{*0} \pi^0) = 1.7 \pm 0.8 , \quad (B28)
\]
\[
\mathcal{B}(K^{*+} \pi^-) = 11.7^{+1.5}_{-1.4} \, . \quad (B29)
\]
\[ \mathcal{B}(\phi K^+) = 9.03^{+0.65}_{-0.63} , \]  
(\text{B30})

\[ \mathcal{B}(\omega K^+) = 6.5 \pm 0.6 , \]  
(\text{B31})

\[ \mathcal{B}(\rho^0 K^+) = 4.23^{+0.56}_{-0.57} , \]  
(\text{B32})

\[ \mathcal{B}(\rho^+ K^0) < 48 , \]  
(\text{B33})

\[ \mathcal{B}(K^{*-} \eta) = 24.3^{+3.0}_{-2.9} , \]  
(\text{B34})

\[ \mathcal{B}(K^{*-} \eta') < 14 , \]  
(\text{B35})

\[ \mathcal{B}(K^{*-} \pi^0) = 6.9 \pm 2.3 , \]  
(\text{B36})

\[ \mathcal{B}(K^{*-0} \pi^+) = 10.8 \pm 0.8 . \]  
(\text{B37})

4. \textit{B} \rightarrow VP, \textit{b} \rightarrow \textit{d} \textit{modes}

\[ \mathcal{B}(\phi \eta) < 1.0 , \]  
(\text{B38})

\[ \mathcal{B}(\phi \eta') < 4.5 , \]  
(\text{B39})

\[ \mathcal{B}(\phi \pi^0) < 1.0 , \]  
(\text{B40})

\[ \mathcal{B}(\omega \eta) < 1.9 , \]  
(\text{B41})

\[ \mathcal{B}(\omega \eta') < 2.8 , \]  
(\text{B42})

\[ \mathcal{B}(\omega \pi^0) < 1.2 , \]  
(\text{B43})

\[ \mathcal{B}(\rho^0 \eta) < 1.5 , \]  
(\text{B44})

\[ \mathcal{B}(\rho^0 \eta') < 4.3 , \]  
(\text{B45})

\[ \mathcal{B}(\rho^0 \pi^0) = 1.83^{+0.56}_{-0.55} , \]  
(\text{B46})

\[ \mathcal{B}(\rho^+ \pi^-) = 24.0 \pm 2.5 , \]  
(\text{B47})

\[ \mathcal{B}(\rho^- \pi^+) = 24.0 \pm 2.5 , \]  
(\text{B48})

\[ \mathcal{B}(K^{*-0} K^0) < \text{Not measured yet} , \]  
(\text{B49})

\[ \mathcal{B}(\bar{K}^{*-0} K^0) < \text{Not measured yet} , \]  
(\text{B50})

\[ \mathcal{B}(K^{*-} K^-) < \text{Not measured yet} , \]  
(\text{B51})

\[ \mathcal{B}(K^{*-} K^+) < \text{Not measured yet} , \]  
(\text{B52})
\[ B(\phi\pi^+) < 0.41 , \quad \text{(B53)} \]
\[ B(\omega\pi^+) = 6.6 \pm 0.6 , \quad \text{(B54)} \]
\[ B(\rho^0\pi^+) = 8.7^{+1.0}_{-1.1} , \quad \text{(B55)} \]
\[ B(\rho^+\eta) = 8.1^{+1.7}_{-1.5} , \quad \text{(B56)} \]
\[ B(\rho^+\eta') < 22 , \quad \text{(B57)} \]
\[ B(\rho^+\pi^0) = 10.8^{+1.4}_{-1.5} , \quad \text{(B58)} \]
\[ B(K^{*0}K^+) < 5.3 , \quad \text{(B59)} \]
\[ B(K^{*+}\overline{K^0}) = \text{Not measured yet} . \quad \text{(B60)} \]

5. \( B \to VV, \ b \to s \) modes

\[ B(\phi K^{*0}) = 9.5 \pm 0.9 , \quad \text{(B61)} \]
\[ B(\omega K^{*0}) < 6.0 , \quad \text{(B62)} \]
\[ B(\rho^0 K^{*0}) < 2.6 , \quad \text{(B63)} \]
\[ B(\rho^- K^{*+}) < 24 , \quad \text{(B64)} \]

\[ B(\phi K^{*+}) = 9.7 \pm 1.5 , \quad \text{(B65)} \]
\[ B(\omega K^{*+}) < 7.4 , \quad \text{(B66)} \]
\[ B(\rho^0 K^{*+}) = 10.6^{+3.8}_{-3.5} , \quad \text{(B67)} \]
\[ B(\rho^+ K^{*0}) = 10.6 \pm 1.9 . \quad \text{(B68)} \]
6. \( B \to VV, \ b \to d \) modes

\[
\begin{align*}
B(\phi \phi) &< 1.5, \quad (B69) \\
B(\omega \omega) &< \text{Not measured yet}, \quad (B70) \\
B(\phi \omega) &< \text{Not measured yet}, \quad (B71) \\
B(\rho^0 \rho^0) &< 1.1, \quad (B72) \\
B(\rho^+ \rho^-) &< 26.2^{+3.6}_{-3.7}, \quad (B73) \\
B(\phi \rho^0) &< 13, \quad (B74) \\
B(\omega \rho^0) &< 3.3, \quad (B75) \\
B(K^{*0} K^{*0}) &< 22, \quad (B76) \\
B(K^{*+} K^{*-}) &< 141, \quad (B77)
\end{align*}
\]

\[
\begin{align*}
B(\phi \rho^+) &< 16, \quad (B78) \\
B(\omega \rho^+) &< 12.6^{+4.0}_{-3.7}, \quad (B79) \\
B(\rho^0 \rho^+) &< 26.4^{+6.1}_{-6.4}, \quad (B80) \\
B(\overline{K^{*0}} K^{*+}) &< 71. \quad (B81)
\end{align*}
\]

[1] Y. Grossman, Z. Ligeti, Y. Nir and H. Quinn, Phys. Rev. D 68, 015004 (2003) [arXiv:hep-ph/0303171].
[2] G. Engelhard, Y. Nir and G. Raz, arXiv:hep-ph/0505194.
[3] G. Engelhard and G. Raz, arXiv:hep-ph/0508046.
[4] The Heavy Flavor Averaging Group, [http://www.slac.stanford.edu/xorg/hfag/](http://www.slac.stanford.edu/xorg/hfag/).
[5] C. W. Chiang, M. Gronau, Z. Luo, J. L. Rosner and D. A. Suprun, Phys. Rev. D 69, 034001 (2004) [arXiv:hep-ph/0307395].
[6] M. Gronau, Y. Grossman and J. L. Rosner, Phys. Lett. B 579, 331 (2004) [arXiv:hep-ph/0310020].
[7] M. Gronau, J. L. Rosner and J. Zupan, Phys. Lett. B 596, 107 (2004) [arXiv:hep-ph/0403287].
[8] A. Ali, G. Kramer and C. D. Lu, Phys. Rev. D 58, 094009 (1998) arXiv:hep-ph/9804363.

[9] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 606, 245 (2001) arXiv:hep-ph/0104110.

[10] M. Beneke and M. Neubert, Nucl. Phys. B 675, 333 (2003) arXiv:hep-ph/0308039.

[11] G. Buchalla, G. Hiller, Y. Nir and G. Raz, arXiv:hep-ph/0503151.

[12] M. Beneke, arXiv:hep-ph/0505075.

[13] S. Eidelman et al. [Particle Data Group], Phys. Lett. B 592, 1 (2004).

[14] J. Charles et al. [CKMfitter Group], arXiv:hep-ph/0406184.

[15] A. S. Dighe, Phys. Rev. D 54, 2067 (1996) arXiv:hep-ph/9509287.

[16] M. J. Savage and M. B. Wise, Phys. Rev. D 39, 3346 (1989) [Erratum-ibid. D 40, 3127 (1989)].