A New Method for Spinning Projectile Aerodynamic Estimation: Extreme Learning Machine Optimized by Cuckoo search algorithm

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Abstract: Aiming at the problem of aerodynamic parameter identification of spinning projectile, a Cuckoo search algorithm for extreme learning machine algorithm is proposed in this paper. The algorithm uses Cuckoo search algorithm to optimize the hidden layer weight and threshold of the extreme learning machine, so as to avoid the problem of unstable identification results caused by the random weight and threshold of the traditional extreme learning machine. The simulation results show that the algorithm can effectively identify the aerodynamic parameters of projectile, with high identification accuracy and fast convergence speed. The proposed algorithm is practical in engineer applications.

1. Introduction

Accurate acquisition the aerodynamic parameters of the uncontrolled projectile is the key to reduce the dispersion of impact point and improve the strike accuracy [1]. Generally, the ways to obtain the aerodynamic parameters of projectile include theoretical calculation method, wind tunnel blowing method and actual flight data identification method. The first two methods usually have large errors, but the aerodynamic parameters identified through the actual flight test can truly reflect the flight state of the projectile and have high accuracy [2]. Projectile aerodynamic parameter identification technology is an important branch in the field of aircraft aerodynamic parameter identification. Studying the method and theory of projectile aerodynamic parameter identification can more effectively understand the motion state of projectile [3], which has important theoretical research and engineering application values. In the past decades, many scholars have been studying aerodynamic identification, such as Warner and Norton, which have laid a theoretical foundation for the research of aerodynamic parameter identification [4]. With the rapid development of modern statistical theory, control theory and computer technology, new methods and technologies of parameter identification are also developed [5]. At present, the more mature parameter identification methods include least square method [6-8], maximum likelihood method [9-12], Kalman filtering method [13-14] and intelligent optimization algorithm [15-16], etc. Extreme learning machine (ELM) [17] is an algorithm for training single hidden layer feed-forward neural networks (SLFNS) proposed by Huang guangbin et al. Compared with other traditional feed-forward neural network (FNN) learning algorithms, ELM has the advantages of good global generalization ability, high efficiency and less parameters adjustment. It has been widely used in cloud computing, data visualization and random projection [18-20]. However, ELM randomly produces the weight and threshold of hidden layer, which leads to the unstable prediction effect when solving the prediction problem. The cuckoo search algorithm (CS) [21] has a simple structure, few parameters, an
excellent search path and strong optimization ability. Therefore, CS optimized ELM algorithm (CS-ELM) is proposed in this paper and uses CS to optimize the input weights and hidden layer thresholds of ELM. The algorithm is applied to the aerodynamic parameter identification problem of an uncontrolled high-spin projectile.

2. Modeling

In this paper, the modified particle trajectory model (4D model for short) is used as the theoretical model of parameter identification. The 4D model is established in the ground coordinate system, and its specific expression is as follows,

\[
\frac{dV_x}{dt} = \frac{1}{2m} \rho S(C_{00} + C_{r2} \cdot \alpha_r^2) V_y (V_z - W_z) + \frac{1}{2m} \rho S C_{y} V_y^2 \alpha_y + \frac{1}{2m} \rho S \rho C_{y} [\alpha_y (V_z - W_z) - \alpha_x V_x] \quad (1)
\]

\[
\frac{dV_y}{dt} = \frac{1}{2m} \rho S(C_{00} + C_{r2} \cdot \alpha_r^2) V_x (V_z - W_z) + \frac{1}{2m} \rho S C_{r} V_x^2 \alpha_x + \frac{1}{2m} \rho S \rho C_{r} [\alpha_x (V_z - W_z) - \alpha_y V_y] - g \quad (2)
\]

\[
\frac{dV_z}{dt} = -\frac{1}{2m} \rho S(C_{00} + C_{r2} \cdot \alpha_r^2) V_x V_y (V_z - W_z) + \frac{1}{2m} \rho S C_{r} V_x V_y^2 \alpha_x + \frac{1}{2m} \rho S \rho C_{r} [\alpha_x V_x - \alpha_y (V_x - W_x)] \quad (3)
\]

\[
\frac{dx}{dt} = V_x \quad (4)
\]

\[
\frac{dy}{dt} = V_y \quad (5)
\]

\[
\frac{dz}{dt} = V_z \quad (6)
\]

\[
\frac{d\gamma}{dt} = V_x' \quad (7)
\]

The velocity of the projectile relative to the air is

\[
V_r = \sqrt{(V_x - W_x)^2 + V_y^2 + (V_z - W_z)^2} \quad (9)
\]

The direct calculation formula of dynamic balance angle \(\alpha_e\) is:

\[
\alpha_{ex} = \frac{x_1 a_g (x_2 V_x V_e - x_2 V_r)}{(x_1^3 + x_2^2 x_1 V_r^2)} \quad (10)
\]

\[
\alpha_{ey} = \frac{-x_1 x_2 a_g (V_{xx}^2 + V_{rr}^2)}{(x_1^3 + x_2^2 x_1 V_r^2)} \quad (11)
\]

\[
\alpha_{ez} = \frac{x_1 a_g (x_2 V_{xx} + x_2 V_{yy} V_{rr})}{(x_1^3 + x_2^2 x_1 V_r^2)} \quad (12)
\]

\[
\alpha_e = \sqrt{\alpha_{ex}^2 + \alpha_{ey}^2 + \alpha_{ez}^2} \quad (13)
\]

Where,

\[
x_1 = 1 - (a_a - a_b) b_2 \dot{V}_r V_r^2 \quad (14)
\]

\[
x_2 = (a_a - a_b) b_2 V_r^2 \quad (15)
\]

\[
b_2 = \frac{1}{2m} \rho S C_{y} \quad (16)
\]


\[ b_z = \frac{1}{2m} \rho S d C'_z \]  

(17)

\[ a_u = \frac{2ml d \gamma m'_y}{(\rho Slm'_z C'_y V'_r^2 + \rho Sld^2 C'_z m'_y \gamma^2 V'_r^2)} \]  

(18)

\[ a_b = \frac{2C \gamma C'_y}{\rho Slm'_z C'_y V'_r^4 + \rho Sld^2 C'_z m'_y \gamma^2 V'_r^2} \]  

(19)

\[ V_{rx} = V_x - W_x \]  

(20)

\[ V_{ry} = V_y \]  

(21)

\[ V_{rz} = V_z - W_z \]  

(22)

Where, \( V_x \), \( V_y \), \( V_z \) is the velocity component of the projectile in the \( x \), \( y \) and \( z \) axes respectively; \( x \), \( y \) and \( z \) are the position of the projectile; \( \gamma \) is the roll angle; \( \dot{\gamma} \) is the roll angle rate; \( m_0 \) is the mass of projectile; \( d \) is the projectile reference diameter; \( l \) is the length of projectile; \( g \) is the gravitational acceleration; \( \rho \) is the air density; \( S \) is the projectile reference area; \( C_{xz0} \) is the zero lift drag coefficient; \( C_{xz} \) is the induced drag coefficient; \( V_r \) is the projectile velocity relative to the air; \( C'_y \) is the derivative of the lift coefficient; \( C''_y \) is the derivative of the derivative of the Markov force coefficient; \( C \) is the polar axial moment of inertia; \( m'_{xz} \) is the derivative of the axial damping moment coefficient; \( W_x \), \( W_y \) is the wind velocity on \( x \) and \( y \) direction; \( m''_x \) is the derivative of the derivative of the Markov moment coefficient; \( m''_z \) is the derivative of the static moment coefficient.

3. The optimization method for ELM using CS algorithm

Using the CS algorithm to optimize the ELM to solve the problem of projectile aerodynamic parameter identification is a comprehensive application of multiple algorithms. The CS algorithm is responsible for optimizing the input weights and thresholds of hidden layer neurons that generate extreme learning. On this basis, The ELM realizes projectile aerodynamic parameter identification. The Cs algorithm and ELM will be described in this section as prior knowledges.

3.1 ELM

ELM is derived from single hidden layer feed-forward neural networks (SLFNs). Different from traditional SLFNs, ELM randomly generates the connection weight and threshold of hidden layer neurons. The randomly generated weights and thresholds are independent of each other and do not need iterative adjustment. Therefore, ELM can be regarded as generalized SLFNs, and its basic structure is shown in Figure 1.

![Fig 1. ELM Structure](image)
Given N training samples \((x_i, t_i) \in \mathbb{R}^n \times \mathbb{R}^m\), where \(x_i\) is the \(n\)-dimensional input vector, \(t_i\) is the \(m\)-dimensional objective vector. For the ELM with \(\tilde{N}\) hidden layer neurons and with the activation function \(G(a_i, b_i, x_j)\), the output is

\[
f_{\tilde{N}}(x_j) = \sum_{i=1}^{\tilde{N}} \beta_i G(a_i, b_i, x_j) = t_j, \quad j = 1, \ldots, N, \tag{23}
\]

Where, \((a_i, b_i)\) is the weight and threshold of the hidden layer respectively; \(\beta_i\) is the connection weight vector connecting the \(i\)-th hidden layer neuron and the output neuron. Equation (23) can be simplified to matrix form:

\[
H^\top \beta = T \tag{24}
\]

Where,

\[
H(a_1, \ldots, a_{N_{\tilde{N}}}, b_1, \ldots, b_{N_{\tilde{N}}}, x_1, \ldots, x_N) = \begin{bmatrix}
G(a_1, b_1, x_1) & \cdots & G(a_{N_{\tilde{N}}}, b_{N_{\tilde{N}}}, x_1) \\
\vdots & \ddots & \vdots \\
G(a_1, b_1, x_{N_{\tilde{N}}}) & \cdots & G(a_{N_{\tilde{N}}}, b_{N_{\tilde{N}}}, x_{N_{\tilde{N}}})
\end{bmatrix}_{N \times N_{\tilde{N}}}
\]

\[
\beta = \begin{bmatrix}
\beta_1^T \\
\vdots \\
\beta_{N_{\tilde{N}}}^T
\end{bmatrix}_{N_{\tilde{N}} \times m} T = \begin{bmatrix}
t_1^T \\
\vdots \\
t_{N_{\tilde{N}}}^T
\end{bmatrix}_{N \times m} \tag{25}
\]

\(H\) is the hidden layer output matrix of the ELM. Once the training set is given and the weights and thresholds \((a_i, b_i)\) of hidden layer neurons are randomly generated, the forward calculation can quickly solve \(H\). Now, ELM can be regarded as a linear system, and the connection weight is output based on the least square criterion \(\beta\) It can be calculated by the following formula:

\[
\hat{\beta} = H^+ T \tag{27}
\]

Where, \(H^+\) is the Moore Penrose generalized inverse matrix of \(H\).

### 3.2 CS algorithm

The CS algorithm is inspired by the parasitic reproduction of cuckoos and is based on the Levi flight model. Similar to other swarm intelligence algorithms, the location information of each bird's nest in this algorithm represents a feasible solution. After the initial population is randomly generated, iterate continuously according to the position update formula to generate a new population. When the optimal selection judgment is made, if the convergence condition is met, the iteration stops and the optimal solution is output.

For a \(d\)-dimensional optimization problem, the individual coding form of each cuckoo is:

\[
x = [x_1, x_2, \ldots, x_d]^T \tag{28}
\]

The CS algorithm uses the Levy Flight operator to update the position, and the update formula is as follows:

\[
X_{i}^{k+1} = X_i^k + \alpha \oplus L(\lambda) \quad i = 1, 2, \ldots, N_{\text{pop}}, k = 1, 2, \ldots, k_{\text{max}} \tag{29}
\]

Where \(X_i^k\) is the \(i\)-th cuckoo individual in the \(k\)-th iteration, \(\alpha\) is the step factor, \(\oplus\) is the dot multiplication operation, \(k\) is the current iteration number, \(k_{\text{max}}\) is the maximum number of iterations of the algorithm, \(L\) is the search step length, which obeys the Levy distribution, that is:

\[
L(\lambda) \sim u = t^{-\lambda}, \quad 1 < \lambda \leq 3 \tag{30}
\]

The complete Levy Flight operator is as follows:
\[ X_i^{k+1} = X_i^k + \alpha \frac{u}{|v|^{1/\beta}} (X_i^k - X_{\text{best}}^k), \quad \beta \in [1, 2] \]  

\[ X_{\text{best}}^k \text{ is the optimal individual in the } k\text{-th generation of cuckoo population, } \beta \text{ is a constant, both } u \text{ and } v \text{ obey the normal distribution, satisfying } u \sim N(0, \sigma_u^2), v \sim N(0, \sigma_v^2), \text{ where:} \]

\[ \sigma_u^2 = \left\{ \frac{\Gamma(1 + \beta) \sin(\pi \beta / 2)}{\Gamma[(1 + \beta) / 2] \beta 2^{(\beta-1)/2}} \right\}^{1/\beta}, \quad \sigma_v^2 = 1 \]  

### 3.3 Identification model based on CS-ELM

This paper proposes a projectile aerodynamic parameter identification model based on ELM as shown in Figure 1. On this basis, the CS algorithm is used to optimize the weights and thresholds of the ELM network model. The model adopts a hierarchical structure to extract features layer by layer from the input bottom data. The model framework of projectile aerodynamic parameter identification shown in Figure 2 includes a network input layer, a hidden layer and an output layer.

![Figure 2. CS-ELM flow chart](image-url)
The input layer, as the input interface of the model, is mainly responsible for preprocessing the original data to form the matrix data that the model can perform efficient calculation and batch processing. The input nodes included are the speed data, position data and the angular velocity of the projectile during the actual flight of the projectile. In particular, the input layer also needs to perform min-max normalization processing on the above data, map all data to the [0,1] interval in order to eliminate the influence of different dimensions on data analysis, and improve the identification accuracy and convergence speed of the model. The specific formula of min-max normalization is as follows

$$x_i^* = \frac{x_i - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}}$$

(33)

Where, $x_i$ is original input data; $x_{\text{min}}$ is the minimum value of input data; $x_{\text{max}}$ is the maximum value of input data; $x_i^*$ is the normalized input data.

ELM is a single hidden layer structure. The hidden layer is the core processing part of the overall model. Its design mainly includes the selection of the activation function and the selection of the number of neurons in the hidden layer. The activation function can introduce non-linear characteristics into the network and improve the expression ability of the model. Since ELM does not need to pass the gradient back, there is no gradient disappearance problem when the Sigmoid function is selected as the activation function. Reasonably setting the number of neurons in the hidden layer can effectively improve the identification accuracy of the model. If the number of nodes is too small, the model accuracy is low; if the number of nodes is too much, the training time is too long, and the final identification accuracy may also decrease due to overfitting. Based on the empirical formula $L = \sqrt{m + n + a}$ (a usually takes a constant of 1 ~ 10), the number of neurons in the hidden layer is continuously optimized.

The output layer is used to output the parameters of the projectile to be identified. The aerodynamic parameters to be identified in this paper are: Zero lift drag coefficient $C_{x0}$, Induced drag coefficient $C_{x2}$, the derivative of lift coefficient $C'_y$, the derivative of the derivative of the Markov force coefficient $C''_z$, the derivative of the axial damping moment coefficient $m_{xz}$, the derivative of the derivative of the Markov moment coefficient $m'_z$, the derivative of the static moment coefficient $m''_z$.

4. Identification results

Figure 3~9 shows the results of aerodynamic parameter identification of a certain type of an uncontrolled high-spin projectile. In the figures, the abscissa is the Mach number, the ordinate is the parameter to be identified, the black curve is the actual observation curve, and the red and blue curves are the identification result curves of ELM and CS-ELM, respectively. The fit of the CS-ELM identification result curve is better than the ELM identification result curve, especially when the nonlinear aerodynamic parameters $C_{x2}$, $C'_y$ and $m'_z$ are identified, the ELM identification result curve varies with the Mach number increasingly, there is a noticeable turbulence.
Table 1 shows the model structure, identification success rate and identification time of the projectile aerodynamic parameters identified by ELM and CS-ELM. Using the SC algorithm to optimize the ELM can effectively improve the identification success rate and simplify the model structure; although the weight and threshold of ELM generated by SC algorithm iterative optimization will increase the identification time, compared with the basic ELM, the identification time only increases by 0.34 s.

| Algorithm | Model Structure | Success Rate | Time/s |
|-----------|----------------|--------------|--------|
| ELM       | 5-72-7         | 76%          | 7.68   |
| SC-ELM    | 5-60-7         | 100%         | 8.02   |

(Note: The recognition success rate and recognition time refer to the average value after 100 independent experiments)
5. Conclusion
This paper combines the Cuckoo Search algorithm with the Extreme Learning Machine, and proposes a CS-ELM algorithm. The algorithm uses the CS algorithm to optimize the input weights and the threshold of hidden layer neurons to realize the aerodynamic parameter identification of the projectile. Based on the results of the simulation experiment, the following conclusions are drawn:

(1) The CS-ELM algorithm can effectively identify the aerodynamic parameters of a certain type of high-rotation uncontrolled projectile, and the accuracy of the aerodynamic parameters obtained by this method can meet the actual engineering needs.

(2) Using the CS algorithm to optimize the weight and threshold of the ELM can effectively improve the identification success rate and accuracy, simplify the model structure; but the iterative optimization process will cause the identification time to be slightly higher than that of the basic ELM.

References
[1] Yan Zhanggen, Qi Zaiang. Shooting table technology. Beijing: National Defense Industry Press, 2000.
[2] Fresconi, Frank, Harkins, Tom. “Experimental Flight Characterization of Asymmetric and Maneuvering Projectiles from Elevated Gun Firings.” Journal of Spacecraft and Rockets, 2012.
[3] Burchett, B.T., and Costello, M., “Model Predictive Lateral Pulse Jet Control of an Atmospheric projectile,” Journal of Guidance, Control, and Dynamics, 2002.
[4] Warner E P, Norton F H. Preliminary Report on Free Flight Tests. Technical Report Archive & Image Library, 1920.
[5] Wang Chao, Zhang Shengxiu, Zheng Jianfei, et al. Adaptive control of aircraft against saturation based on aerodynamic characteristics identification. Acta Aeronautica Sinica, 2013.
[6] Morelli E A, Klein V. Accuracy of aerodynamic model parameters estimated from flight test data. Journal of Guidance, Control, and Dynamics, 1997.
[7] Kamali C, Pashikar A A, Raol J R. Evaluation of Recursive Least Squares algorithm for parameter estimation in aircraft real time applications. Aerospace Science & Technology, 2011.
[8] Morelli, E. A., “Global nonlinear aerodynamic modeling using multivariate orthogonal functions,” Journal of Aircraft, 1995.
[9] Xia Zhixun, Sheng xiangrao, Tang Guojin. Identification of aerodynamic parameters of axisymmetric aircraft by maximum likelihood method. Acta Aeronautica Sinica, 1998.
[10] Shantha K N, Janardhana R N. Estimation of stability and control derivatives of light canard research aircraft from flight data. Defence Science Journal, 2004, 54(3):277-292.
[11] Carnduff S, Cooke A. Application of aerodynamic model structure determination to UAV data. Aeronautical Journal, 2011.
[12] Zhang Tianjiao, Qian Weiqi, et al. Research on aerodynamic parameter identification technology in wind tunnel free-flight test based on Maximum Likelihood Estimation. Journal of Experiments in Fluid Mechanics, 2015.
[13] Jieliang S, Yan S, Qing L, et al. Calculation and Identification of the Aerodynamic Parameter for Small-Scaled Fixed-Wing UAVs. Sensors, 2018.
[14] SEO,G.,KIM,Y.and SADERLA,S. Kalman filter based online system identification of fixed-wing aircraft in upset condition, Journal of Aerospace Science Technology 2019.
[15] JIAN Zhaosheng, AI Jianliang. Application of Differential Evolution Algorithm for Aerodynamic Parameter Identification. Journal of Fudan University (Natural Science), 2017.
[16] Mohamad A, Karimi J, Naderi A. Dynamic aerodynamic parameter estimation using a dynamic particle swarm optimization algorithm for rolling airframes. Journal of the Brazilian Society of Mechanical Sciences and Engineering, 2020.
[17] Huang G B, Zhu Q Y, Siew C K. Extreme learning machine: Theory and applications. Neurocomputing, 2006.
[18] Lin jiuran, Yin jianpin, et al. Secure outsourcing of extreme learning machine in cloud computing. Computer Engineering & Science, 2015.
[19] A A kusok, Baek S , Miche Y , et al. ELMVIS+: Fast nonlinear visualization technique based on cosine distance and extreme learning machines. Neurocomputing, 2016.
[20] Chuangquan, Chen, Chi-Man, et al. Efficient extreme learning machine via very sparse random projection. Soft Computing A Fusion of Foundations Methodologies & Applications, 2018.
[21] Shokri-Ghaleh H , Alfi A . Optimal synchronization of teleoperation systems via cuckoo optimization algorithm[J]. Nonlinear Dynamics, 2014, 78(4):2359-2376.