Comment on “Anomalous Conductance Distribution in Q1D Gold Wires: Possible Violation of the One-Parameter Scaling Hypothesis”.

Mohanty and Webb [1] claim that their data on conductance fluctuations in gold wires contradict the one-parameter scaling. We show that flaws in extracting values of the cumulants \( \langle g^n \rangle \) of the conductance distribution (for \( n = 3, 4 \)) invalidate all the conclusions made there. The actual values of \( \langle g^n \rangle \) determined by us from the published raw data contained in Ref. [1] are orders of magnitude smaller than those claimed. We show that a visible (but small) deviation of the distribution in [1] from the normal shape results from a systematic error due to the limited applicability of the ergodicity hypothesis. Thus, the data of [1] do not warrant any statement on the violation or validity of the one-parameter scaling.

1. **Comparing data with theory.** Table I below copies part of the Table II in [1] as cumulants extracted from magnetofingerprints \( \delta g(B) \) (in units of \( e^2/h \)) measured in 3 samples. Comparing [1] these cumulants with the scaling result \( \langle g^n \rangle \sim \langle g \rangle^{2^n} \) obtained [2] for phase coherent samples with \( L < L_c \) (like 1dA) leads to an apparent contradiction: not only do \( \langle g^n \rangle \) not decrease with \( n \) but \( \langle g^3 \rangle > \langle g^2 \rangle \) in long samples 1dC, 1dD. A contradiction with the central limiting theorem is even stronger: for such long wires (with \( L = NL_c, N \approx 5 \) as quoted in [1]) magnetofingerprint \( \delta g(B) \) arises from independent fluctuations \( \delta G(B) \) in \( N \) phase-coherent conductors connected in series, so that the main contribution to \( \langle g^n \rangle \) is of order \( \langle G^n \rangle / N^{2n-1} \). For sample 1dC, this would need \( \langle G^3 \rangle > 10^2 \) and \( \langle G^4 \rangle > 10^4 \), inconceivable after comparing to the data, short to a wire sample, 1dA.

2. **Status of the reported statistical data.** We have digitized the distribution functions for three samples from the postscript files for Figs. 1 and 2 of Ref. [1] and evaluated \( \langle g^n \rangle \) for \( n \leq 4 \). Following [1], we have excluded the tail affected by the weak anti-localization indicated by arrows in Fig. 2 of [1]. Our results are displayed in Table II. While \( \langle g \rangle \) and \( \langle g^2 \rangle \) agree with Ref. [1], the values of \( \langle g^3 \rangle \) and \( \langle g^4 \rangle \) are smaller by 4 to 7 orders of magnitude [4]. We have also evaluated the skewness \( \langle g^3 \rangle / \langle g^2 \rangle^{3/2} \) and the kurtosis, \( \langle g^4 \rangle / \langle g^2 \rangle^2 \) [5] and found them to be of the order of \( \langle g^3 \rangle \) and \( \langle g^4 \rangle \) in Table I. Thus, we speculate that the data in Table I are an artefact of a mere confusion in Ref. [1]: \( \langle g^4 \rangle \) is referred to on page 146601-2 as “the fourth cumulant (gggg) or kurtosis”.

3. **Limited applicability of the ergodicity hypothesis (EH).** Some of the values of \( \langle g^3, 4 \rangle \) of Table II remain large in comparison with the theory. We argue that the source of this discrepancy is not the violation of scaling but the limited applicability of EH (the assumption that one magnetofingerprint is sufficient to represent statistics of sample-to-sample conductance fluctuations).

The cumulants discussed in Ref. [1] are sample-specific quantities \( \langle g^n \rangle \) averaged over the magnetic field, \( B \). For instance, \( \langle g^4 \rangle_B = \langle g^4 \rangle_B - 3 \langle g^2 \rangle^2 \langle g \rangle + 2 \langle g \rangle^3 \) and \( \langle g^4 \rangle_B = \langle g^4 \rangle_B - 4 \langle g^2 \rangle_B \langle g \rangle_B + 12 \langle g^2 \rangle_B^2 - 3 \langle g^2 \rangle_B^2 - 6 \langle g \rangle_B^3 \), where \( \langle X \rangle_B = \int_{B_1}^{B_k} X(B) dB / B_0 \). We have estimated the r.m.s. value of these sample-to-sample fluctuations from the average \( \langle \langle g^4 \rangle_B \rangle \) evaluated using the standard combinatorial rules [3]: \( \langle \langle g^4 \rangle_B \rangle = \langle g^4 \rangle + (B_c / B_0) \langle g^2 \rangle n! \int_{-\infty}^{\infty} \left[ K(x) \right] dx + O \left(B_c^2 / B_0^2 \right) \). Here \( B_c \) and \( K(x) \) are defined via the conductance covariance as \( \langle g(B) g(B') \rangle = \langle g^2 \rangle K(B-B') / B_0 \). For a quasi-1D diffuse wire of width \( w \) and length \( L > L_c \), \( \sqrt{K/B_0} > w, K(x) = [1 + x^2]^{-1/2} \) and \( B_c \sim \Phi_0 / w L_c \), so that

\[
\langle \langle g^4 \rangle_B \rangle = (\alpha_n B_c / B_0) \langle g^2 \rangle + \langle g^4 \rangle,
\]

where \( \alpha_3 = 12, \alpha_4 = 12 \pi \). Although the limit \( B_0 \to \infty \) indeed yields the ergodicity \( \langle g^n \rangle_B = \langle g^n \rangle \), even for \( B_0 = 15T \) and \( B_c \sim 0.04T \) as in [1], the measured \( \langle g^2 \rangle_B \rangle \) is dominated by the \( n^2 / 2 \) power of the variance: \( \langle g^3 \rangle_B \sim \pm 1.6 \langle g^2 \rangle^3 / 2 \) and \( \langle g^4 \rangle_B \sim \pm 0.3 \langle g^2 \rangle^2 \). Numerically, this is comparable to the values in Table II. Therefore, when extracted from a small number of measured magnetofingerprints, only those values of the cumulants of conductance distributions exceeding the first term in the r.h.s. of Eq. (1) would signal a deviation from the one-parameter scaling. This is not the case in Ref. [1].

V.I. Fal’ko\(^1\), I.V. Lerner\(^2\), O. Tsyplyatyev\(^1\), I.L. Aleiner\(^3\)
\(^1\) Physics Department, Lancaster University, LA1 4YB, UK
\(^2\) School of Physics, Birmingham University, B15 2TT, UK
\(^3\) Physics Department, Columbia University, NY 10027

[1] P. Mohanty and R.A. Webb, Phys. Rev. Lett. 88, 146601 (2002).
[2] B.L. Altshuler, V.E. Kravtsov, and I.V. Lerner, Zh. Eksp. Teor. Fiz. 91, 2276 (1986).
[3] *Elementary Statistical Methods*, G.B. Wetherill, Methuen & Co Ltd, London 1967.
[4] A visual inspection of the data for sample 1dC in Fig. 2 of [1] shows that this range of conductance values obeys \( |g-(g)| < 0.2 \). As the 3\(^rd\) cumulant can be represented by \( \langle g^3 \rangle_B = \langle g-(g) \rangle^3 \), it cannot exceed 0.008 while its value in Table I is 20 times bigger than such an upper bound.
[5] O. Tsyplyatyev et al. Phys. Rev. B 68, 121301 (2003).
[6] B.L. Altshuler and D.E. Khmelnitskii, JETP Lett. 42, 359 (1986); P.A. Lee, A.D. Stone, and H. Fukuyama, Phys. Rev. B 35, 1039 (1987).