Rydberg-interaction gates via adiabatic passage and phase control of driving fields

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In this paper, we propose two theoretical schemes for implementation of quantum phase gates by engineering the phase sensitive dark state of two atoms subjected to Rydberg-Rydberg interaction. Combining the conventional adiabatic techniques and newly developed approaches of phase control, a feasible proposal for implementation of a geometric phase gate is presented firstly, where the conditional phase shift (Berry phase) is achieved by adiabatically and cyclically changing the parameters of the driving fields. Here, we find that the geometric phase acquired is related to the way how the relative phase is modulated. In the second scheme, the system Hamiltonian is adiabatically changed in a noncyclic manner, so that the acquired conditional phase is not a Berry phase. A detailed analysis of the experimental feasibility and the effect of decoherence is also given. The proposed schemes provide new perspectives for adiabatic manipulation of interacting Rydberg systems with tailored phase modulation.

I. INTRODUCTION

Neutral atoms in highly excited and long-lived Rydberg states are considered as the ideal architecture for quantum information processing since it provides strongly interatomic interaction on demand, while keeps interacting with the environment weakly [1,2]. There have been numerous proposals to use Rydberg-Rydberg interactions for implementation of quantum logic gates [3–9], quantum error correction [10,11], quantum algorithms [12–14] and quantum repeater [15–18]. By following the pioneering works proposed by Jaksch et al. [19] and Lukin et al. [20], promising schemes for realizing two-qubit controlled-Z and controlled-NOT gates that rely on dynamical control of dipolar coupling and intrinsic Förster interaction have been widely studied in both the Rydberg blockade [5,21–24] and antiblockade regimes [25,26]. Therein, the validity of the gate operations is predominantly determined by the detailed laser parameters as well as the Rydberg interaction strength. Experimental demonstrations in producing quantum entanglement of few Rydberg atoms [27,28] and two-qubit logic operations [24] have recently made great progress by addressing the system’s evolutionary dynamics, however, the fidelity achieved to date is significantly limited by the imprecise control of experimental parameters.

The requirement of precise control of coherent dynamics can be avoided by using the adiabatic techniques, such as stimulated Raman adiabatic passage (STIRAP) and adiabatic rapid passage (ARP), where the sensitivity to imprecise Rabi control and other experimental perturbations is strongly suppressed [29]. The theoretical proposals based on the STIRAP and the ARP have been proposed for coherent population transfer [30–33], preparation of entangled states [34–36], and implementation of quantum logic gates [37–39] with Rydberg atoms, which exhibit robustness properties against moderate fluctuations of experimental parameters. Furthermore, the adiabatic technique alternatively provides a chance for geometric manipulation of Rydberg systems [35–37], which is naturally robust against certain control errors [39,40] and is a promising approach for implementation of a built-in fault-tolerant two-qubit logic gate.

Here, we put forward two new schemes for implementing quantum phase gates via adiabatic passage and phase control of the driving fields. The first scheme is based on the geometric manipulation of the system’s Hamiltonian in the parameter space. In contrast to the previously similar approach [35], the geometric phase acquired here is not due to the variance of the phase difference of the control pulses, and is alternatively accumulated by changing the phases of the driving fields in step and keeping the phase difference null. Remarkably, we find that the geometric phase acquired is strongly dependent on the way how the relative phase is modulated. In the second scheme, neither is the conditional phase shift of dynamical origin since the qubit system evolves in the dark state space, nor is it a Berry adiabatic phase as the system Hamiltonian is not cyclically changed. The conditional phase arises from the adiabatic manipulation of the dark state with staircase phase control. The experimental feasibility, gate fidelity and decoherence effect for the proposed schemes are carefully studied.

This paper is organized as follows. In Sec. II, we propose the level addressing scheme for two neutral atoms interacting via the Rydberg-Rydberg interaction and examine the role of the phases of driving fields in adiabatic control. In Sec. III, the schemes for implementing conditional phase gates based on Berry phase and non-Berry adiabatic phase are presented. In Sec. IV, we provide a detailed discussion about the experimental feasibility of the two schemes. In Sec. V, the effect of atomic spontaneous emission and interatomic force on the gate fidelity is studied. The conclusion appears in Sec. VI.
II. DARK STATE OF TWO INTERACTING RYDBERG ATOMS

We first introduce the schematic description of the system. Consider a pair of identical three-level atoms with a ground state \( |1\rangle \), an intermediate state \( |2\rangle \), and a highly excited Rydberg state \( |3\rangle \), see Fig. 1(a), which are trapped in optical tweezers or optical lattices. Two excitation lasers of optical frequencies resonantly drive the atomic transitions \( |1\rangle \leftrightarrow |2\rangle \) and \( |2\rangle \leftrightarrow |3\rangle \) with the Rabi frequencies \( \Omega_p \) and \( \Omega_\mu \) are set at frequencies \( \hbar \omega_p \) of optical frequencies resonantly drive the interacting Rydberg atoms with \( \hbar \) being the reduced Planck constant. The ground state \( |0\rangle \) is the auxiliary qubit state. \( V_{22} \), \( V_{23} \), and \( V_{33} \) are energy shifts induced by the Rydberg-Rydberg interaction.

In terms of the symmetric two-atomic basis states spanned by \( \{|\phi_j\rangle\}, \ j=1,\ldots, 6 \), with
\[
\begin{align*}
|\phi_1\rangle &= |1\rangle|1\rangle, \\
|\phi_2\rangle &= \frac{1}{\sqrt{2}}(|1\rangle|2\rangle + |2\rangle|1\rangle), \\
|\phi_3\rangle &= \frac{1}{\sqrt{2}}(|1\rangle|3\rangle + |3\rangle|1\rangle), \\
|\phi_4\rangle &= |2\rangle|2\rangle, \\
|\phi_5\rangle &= \frac{1}{\sqrt{2}}(|2\rangle|3\rangle + |3\rangle|2\rangle), \\
|\phi_6\rangle &= |3\rangle|3\rangle,
\end{align*}
\]
\( \mathcal{H}_R \) can be rewritten as
\[
\mathcal{H}_R = \begin{bmatrix}
0 & \sqrt{2}\Omega_\mu & 0 & 0 & 0 & 0 \\
\sqrt{2}\Omega_p & 0 & \Omega_\mu & \sqrt{2}\Omega_\mu & 0 & 0 \\
0 & \Omega_\mu & 0 & 0 & \Omega_\mu & 0 \\
0 & 0 & \sqrt{2}\Omega_p & 0 & \sqrt{2}\Omega_\mu & 0 \\
0 & 0 & \Omega_\mu & \sqrt{2}\Omega_\mu & 0 & \sqrt{2}\Omega_\mu \\
0 & 0 & 0 & 0 & 0 & V_{33}
\end{bmatrix}.
\] (3)

There exists a nondegenerate eigenspace and a unique dark state for the Hamiltonian \( \mathcal{H}_R \), which is given by
\[
|d_2(t)\rangle \propto (|\Omega_\mu|^2 - |\Omega_p|^2)|\phi_1\rangle + \Omega_p^*|\phi_4\rangle - \sqrt{2}\Omega_\mu\Omega_p|\phi_3\rangle.
\] (4)

Expressing the relative strength of the two Rabi frequencies \( \Omega_p, \Omega_\mu \) as \( \tan\theta = |\Omega_p|/|\Omega_\mu| \) and keeping their phases nonvanishing, the Eq. (4) after normalization is rewritten as follows:
\[
|d_2(t)\rangle = \mathcal{N}^{-1} \left[ (\cos^2\theta - \sin^2\theta)|\phi_1\rangle + \sin^2\theta e^{i2\phi_\mu}|\phi_4\rangle - \sqrt{2}\sin\theta e^{-i(\phi_\mu - \phi_\phi)}|\phi_3\rangle \right],
\] (5)

where
\[
\begin{align*}
\cos\theta &= \frac{|\Omega_\mu|}{\sqrt{|\Omega_\mu|^2 + |\Omega_p|^2}}, \\
\sin\theta &= \frac{|\Omega_p|}{\sqrt{|\Omega_\mu|^2 + |\Omega_p|^2}},
\end{align*}
\]
and
\[
\mathcal{N} = \sqrt{\cos^4\theta + 2\sin^4\theta}.
\]

The Eq. (5) has a similar form to the dark state firstly studied by Möller et al. \cite{85}, where a time-dependent relative phase \( \phi_\mu(t) \equiv \phi_\mu - \phi_\phi \) is found to be relevant for acquisition of geometric phases, except that the Eq. (5) contains an additional exponential factor \( e^{i2\phi_\mu} \) for \( |\phi_4\rangle \). It implies that by setting \( \phi_\mu = 0 \) or \( \phi_\phi = \pi/2 \), the two-atom system will transfer to the anti-symmetric superposition state \( |EPR\rangle_{as} = (|\phi_1\rangle - |\phi_4\rangle)/\sqrt{2} \) or the symmetric superposition state \( |EPR\rangle_{s} = (|\phi_1\rangle + |\phi_4\rangle)/\sqrt{2} \).
by adiabatically following the dark state with $\theta$ changing from 0 to $\pi/2$ (see later discussion for detail). This is numerically confirmed by examining the probability for detecting the two-atomic states $|EPR\rangle_{as}$ after the applied STIRAP pulse sequence [see Fig. 1(b)], which is a sinusoidal function of $\phi_\rho$, exhibiting wavelike interference fringes, as shown in Fig. 2. The co-existence of the phase factors $\alpha$, $e^{-i\phi_\rho}$ and $e^{i2\phi_\rho}$ in the superposition coefficients for the three components $|\phi_1\rangle$, $|\phi_3\rangle$ and $|\phi_4\rangle$, respectively, can significantly modify the geometric phases acquired during adiabatic evolution assisted by phase control and can find special use for construction of quantum logic gates.

The level configuration [as in Fig. 1(a)] including a single Rydberg state suffers from an irreversible spontaneous decay since the optically excited intermediate state $|2\rangle$ has a short lifetime, therefore adiabatic manipulation of the (unstable) dark state becomes not experimentally feasible (see section V for further discussion). To avoid the defect, we then consider atoms with two ground hyperfine states $|0\rangle$ and $|1\rangle$ and two Rydberg states $|2\rangle$ and $|3\rangle$, see Fig. 1(c). The atomic transition $|1\rangle \leftrightarrow |2\rangle$ is resonantly excited by a single-photon field $\Omega_\rho$ and the transition $|2\rangle \leftrightarrow |3\rangle$ is driven by a microwave field $\Omega_\mu$. The auxiliary level $|0\rangle$ is introduced as a qubit information for the later discussed gate protocols. While both atoms are excited to the Rydberg states, two relevant interparticle interactions are involved, i.e. the van der Waals (vdW) interaction $V_{22}$ ($V_{33}$) between the states $|2\rangle$ ($|3\rangle$) and the exchange dipole-dipole interaction (DDI) $V_{23}$ between an atom in $|2\rangle$ and another in $|3\rangle$. When including the Rydberg-Rydberg interaction, the two-atom Hamiltonian governing the temporal evolution of the compound system takes the form

$$H'_{R} = \mathcal{H}_1 \otimes I_2 + I_1 \otimes \mathcal{H}_2 + V_{33}|3\rangle_1|3\rangle_2|2\rangle_1|2\rangle_2 + V_{23}(|2\rangle_1|3\rangle_2|2\rangle_2|1\rangle_1 + |2\rangle_1|2\rangle_2|1\rangle_1|3\rangle_2) \tag{6}$$

If the two atoms only weakly interact with each other while they are in the state $|2\rangle$ (e.g., due to a dispersive Förster process) such that the vdW shift $V_{22}$ becomes negligible comparing with other Rydberg interaction energies $V_{33}$, $V_{23}$, i.e. $V_{22} \ll V_{23}, V_{33}$, then the Hamiltonian $H'_{R}$ with $V_{22} \to 0$ has one dark state, which is exactly given by Eq. (5). The interatomic DDI $V_{23}$ does not shift the zero eigenenergy and change the form of the dark state. Therefore, the single-Rydberg-level effects with respect to $|d_2\rangle$ hold true for the multiple-Rydberg-level model as long as the adiabatic condition is well guaranteed, and adiabatic control of the dark state becomes more feasible for long radiative lifetime of the Rydberg levels.

In another parameter regime where the interaction between the Rydberg states $|3\rangle$ is sufficiently weak comparing with the vdW shift $V_{22}$ and the DDI strength $V_{23}$, i.e. $V_{33} \ll V_{22}, V_{23}$, by setting $V_{33} = 0$ we again find a dark state for $H'_{R}$, but with a different form

$$d_2(t) = \cos^2\theta e^{i\phi_\rho} |\phi_1\rangle + \sin^2\theta |\phi_0\rangle - \sqrt{2}\sin\theta \cos\theta e^{i\phi_\rho} |\phi_3\rangle, \tag{7}$$

which can be exactly expressed as the direct product of the dark states for the single-atom Hamiltonian $H_i$ ($i = 1, 2$), i.e. $|d_2(t)\rangle = |d_1(t)\rangle_1 \otimes |d_1(t)\rangle_2$, with

$$|d_1(t)\rangle_1 = \cos\theta e^{i\phi_\rho} |1\rangle_1 - \sin\theta |3\rangle_1. \tag{8}$$

In this case, the relative phase $\phi_\rho$ is the only degree of freedom for phase modulation during the system’s adiabatic evolution along $|d_2(t)\rangle$.

If we further assume that $V_{22} = V_{33} = 0$ but with $V_{23} \neq 0$, the zero-energy eigenstate for the two-atom Hamiltonian $H'_{R}$ can then be written as the superposition of the degenerate dark states $|d_2(t)\rangle$ and $|d_2'(t)\rangle$. A finite Rydberg interaction strength $V_{22}$ or $V_{33}$ between the states $|2\rangle$ or $|3\rangle$ results in the removal of the degeneracy, which cannot occur with only the DDI due to the missing component $|\phi_0\rangle$.

Suppose that the Hamiltonian $H'_{R}(t)$ is time dependent through the set of parameters $R(t) = (\theta(t), \phi_\rho(t), \phi_\mu(t))$ and the interacting two-atom system is initially in the ground eigenstate $|g(R(0))\rangle$ of the instantaneous $H'_{R}(t = 0)$. If $R(t) = (\theta(t), \phi_\rho(t), \phi_\mu(t))$ is modulated under the condition

$$|\langle e(t) | dH'_{R} / dt | g(t) \rangle | \ll |E_e - E_g|^2$$

Figure 2. (color online) (a) $\phi_\rho$-dependent wavelike interference fringes in the probabilities of $|EPR\rangle_{as}$ (solid blue) and $|EPR\rangle_{as}$ (dashed red). (b), (c) Numerically calculated time-dependencies of the phases (divided by $\pi$) of the states $|\phi_1\rangle$ (solid blue) and $|\phi_4\rangle$ (dashed red) for $\phi_\rho = 0$ and $\phi_\rho = \pi/2$, respectively. The two-atom system is initially in the state $|\phi_1\rangle$ and adiabatically evolves along $|d_2(t)\rangle$. The Rabi frequencies are modeled by sine-function pulses $\Omega_p(t) = \Omega \sin(\pi t/\tau) e^{i\phi_\rho}$, $\Omega_\mu(t) = \Omega \cos(\pi t/\tau) e^{i\phi_\rho}$ with $0 \leq t \leq \tau$. We fix units of $\Omega = 1$, and set $\Omega \pi/2 = 6$, $V_{23}/\Omega = 1.1$, and $V_{33}/\Omega = 0.9$. 

|Population $|B\rangle \& |D\rangle v.s. p/\pi$ phase $C_1$, $C_2$ time | 0 0.5 1 1.5 20 | 0.2 | 0.4 | 0.6 | 0.8 | 1 | 0 2 4 60 | 0.5 | 1 | 0 2 4 60 | 0.5 | 1 |

| Phase of $|B\rangle$ states | Phase of $|D\rangle$ states | $\Omega t/2\pi$ | $\Omega t/2\pi$ |
such that the Hamiltonian is adiabatically changed along a closed curve $C$ in the parameter space (i.e. $\mathbf{R}(T) = \mathbf{R}(0)$), where $|\psi\rangle$ is any one of the instantaneous excited state, then the system will keep in the ground state and acquire a purely geometric phase $\varphi_g$ in additional to the usual dynamical phase $\varphi_d$:

$$|g(\mathbf{R}(T))\rangle = \exp\{i[\varphi_g(T) + \varphi_d(T)]\}|g(\mathbf{R}(0))\rangle,$$

(9)

where

$$\varphi_g = i \oint_C d\mathbf{R} \cdot (\langle g(\mathbf{R}(t))|\nabla_\mathbf{R}|g(\mathbf{R}(t))\rangle)$$

(10)

and

$$\varphi_d(T) = - \int_0^T E_d(\mathbf{R}(t)) dt,$$

(11)

which is vanished for a dark state $|g(t)\rangle = |d(\mathbf{R}(t))\rangle$ with zero eigenenergy $E_g = 0$.

III. SCHEMES FOR IMPLEMENTING CONTROLLED-Z GATES VIA ADIABATIC PASSAGE

We encode qubit information on the ground state $|1\rangle$ and the auxiliary level $|0\rangle$ that is uncoupled from any pulse sequences of the control field. Thus, the computational basis states are given by $\{|0\rangle_1|0\rangle_2, |0\rangle_1|1\rangle_2, |1\rangle_1|0\rangle_2, |1\rangle_1|1\rangle_2\}$. The controlled-Z gate is implemented by applying a counterintuitive pulse sequence and by modulating the phases of the control fields.

Scheme 1. Geometric phase gate with the phases of the Rabi frequencies varying in step (i.e. $\phi \rightarrow \text{const.}$). It has been realized that the dark states $|3\rangle$ and $|8\rangle$ under adiabatic evolution can acquire the geometric phases $\varphi_2 = 2 \int \sin^2\theta \cos^2\theta (\cos^2\theta + 2 \sin^2\theta)^{-1} d\phi_\tau$ and $\varphi_1 = \int \sin^2\theta d\phi_\tau$, respectively, for a nonvanishing and time-dependent relative phase $\phi_\tau(t)$ [35]. In contrast, we find that the two-atom dark state $|5\rangle$ can acquire a Berry phase even though the relative phase is kept invariant.

Suppose the two-atom system is initially in $|d_2(0)\rangle = |1\rangle_1|1\rangle_2$ (i.e. $\cos \theta = 1$) and the phases of the driving fields are $\phi_\rho(0) = 0$, $\phi_\mu(0) = 0$ without loss of generality. The time-dependent amplitudes of the Rabi frequencies are chosen as $(0 \leq t \leq 2\tau)$

$$|\Omega_\rho(t)| = \Omega \sin\left(\frac{\pi}{2\tau} t\right), \ |\Omega_\mu(t)| = \Omega \cos\left(\frac{\pi}{2\tau} t\right),$$

(12)

which corresponds to $\theta(t)$ varying from 0 to $\pi/2$ and the corresponding reverse process. The phases $\phi_{\rho,\mu}(t)$ are synchronized with each other in real time and have a simply linear time dependence $\phi_{\rho,\mu}(t) = \pi t/\tau$. Therefore, the system makes a cyclic evolution with starting point and ending point $\theta = 0$, see temporal evolution of

the probability amplitudes and the phases of the relevant states as shown in Fig. 3. The geometric phase $\varphi'_2$ (i.e. the Berry phase) accumulated during the adiabatic process can be calculated by using the standard formula Eq. (10). Since $\phi_\tau$ remains zero at any time, the relevant parameter space reduces to $\mathbf{R}(t) = (\theta(t), \phi_\rho(t))$. Thus, we have

$$\varphi'_2 = - \oint_C \frac{2 \sin^4 \theta}{\cos^3 \theta + 2 \sin^4 \theta} d\phi_\rho = - \oint_C \frac{4 \sin^3 \theta}{\cos \theta + 2 \sin^4 \theta} d\theta$$

(13)

for $d\phi_\rho(t)/d\theta(t) = 2$ taken in our example. Apart from that, while the system is initially in the state $|0\rangle_1|0\rangle_2$, $|0\rangle_1|1\rangle_2$ or $|1\rangle_1|0\rangle_2$, no geometric phases can be acquired during the cyclic evolution. The sudden increase of the phases of $|0\rangle_1|1\rangle_2$ ($|1\rangle_1|0\rangle_2$) around $t = \tau$ is due to imperfect state transfer and is automatically eliminated at the end of the pulse sequence. Thus, we have successfully implemented a controlled phase gate based on the
conditionally geometric phase shift:

\[
|0\rangle_1|0\rangle_2 \rightarrow |0\rangle_1|0\rangle_2, \quad |0\rangle_1|1\rangle_2 \rightarrow |0\rangle_1|1\rangle_2,
\]
\[
|1\rangle_1|0\rangle_2 \rightarrow |1\rangle_1|0\rangle_2, \quad |1\rangle_1|1\rangle_2 \rightarrow e^{i\phi_r}|1\rangle_1|1\rangle_2.
\] (14)

Since we have guaranteed \(\phi_r = \text{const.}\) during the adiabatic evolution, the required solid angle for obtaining the geometric phase \(\phi_2''\) is induced by the concurrency control of \(\phi_p\) and \(\phi_\mu\), where an additional reference oscillator should be included. While for the system initially being in the two-atom dark state \(|d_2(0)\rangle\) under the condition \(V_{33} \ll V_{22}, V_{23}\), no geometric phases can be acquired since \(\phi_r\) is invariant.

However, if \(\phi_\mu\) becomes time variant, note that the phases of the Rabi frequencies may be modulated in two fashions, leading to different geometric phase shift for the state \(|d_2\rangle\). First, \(\phi_\mu = \text{const.}\) and \(\phi_r(t) = \phi_p(t) - \phi_\mu\) is time dependent via \(\phi_\mu(t)\). In this case, the geometric phase acquired is exactly given by \(\varphi_2\). While for the other case where \(\phi_\mu = \text{const.}\) and \(\phi_r(t) = \phi_p - \phi_\mu(t)\) is determined by \(\phi_p(t)\), the geometric phase acquired is then alternatively given by

\[
\varphi_2'' = -\frac{1}{2\sqrt{C}} \sqrt{\frac{2\sin^2\theta}{\cos^4\theta + 2\sin^4\theta}} d\phi_p
\]
\[
= -\frac{1}{2\sqrt{C}} \sqrt{\frac{4\sin^2\theta}{\cos^4\theta + 2\sin^4\theta}} d\theta.
\] (15)

Thus, we find three different ways of phase control for geometrically manipulating the interacting two-atom system. A comparison of the acquired geometric phases for the three cases is shown in Fig. 4, from which one can easily read out \(\varphi_2\), \(\varphi_2'\) and \(\varphi_2''\) by \(4\theta_m f(\theta)\), with \(\theta_m\) and \(f(\theta)\) being the given \(\theta\) with respect to the preset dark state \(d_2(\theta_m, \phi_p, \phi_\mu)\) and the \(\theta\) average of the curves on the plots (indicated by dash lines for scheme 1), respectively.

Scheme 2. Gate based on non-Berry adiabatic phase arisen from staircase phase control. The operation procedure is generally divided into two steps during the time interval \(0 \leq t \leq 2\tau\), in which the time-dependent amplitudes of the Rabi frequencies again vary according to Eq. (12), and the phases of the driving fields follow

\[
\phi_p(t) = \text{const.}, \quad \phi_\mu(t) = \frac{\pi}{2} \Theta(t - \tau),
\] (16)

with \(\Theta(x)\) being the unit step function. Note that the relative phase is changed only at the end of the first half of the pulse sequence (\(t = \tau\)) without the limit of adiabaticity, and the system Hamiltonian is not changed along a closed curve in the parameter space \(R(t) = (\theta(t), \phi_\mu(t))\). Therefore, it is fundamentally different from the geometric operation (leading to the Berry phase) proposed by Møller et al. [35], where the relative phase should be adiabatically modulated when the applied pulses overlap, and the initial and the final Hamiltonian of the evolutionary system should remain the same (i.e. \(R(2\tau) = R(0)\)). The idea of realizing a phase gate through adia-
batic manipulation of the dark state with staircase phase control was firstly studied in ion traps [41].

In the first step (0 ≤ t ≤ τ), the phase factors φρ, φμ are set to be equal so that φρ = 0, e.g. φρ = φμ = 0 for simplicity. θ is adiabatically increased from 0 to π/2 by adjusting the relative intensity of the coupling fields as in Eq. (12). For V_{22} ≪ V_{23},V_{33}, the temporal evolution of the basis states will follow the dark states [Eqs. (5) and (6)] throughout the procedure, leading to the transformations

\[
|0\rangle_1|0\rangle_2 → |0\rangle_1|0\rangle_2, \quad |0\rangle_1|1\rangle_2 → -|0\rangle_1|3\rangle_2,
\]

\[
|1\rangle_1|0\rangle_2 → -|3\rangle_1|0\rangle_2, \quad |1\rangle_1|1\rangle_2 → \frac{1}{\sqrt{2}}(|\phi_1⟩ + |\phi_4⟩).
\]

(17)

In the second step (τ ≤ t ≤ 2τ), θ is tuned adiabatically from π/2 back to 0 but with φρ = 0 and φμ = π/2 (i.e. φμ = π/2), which gives rise to

\[
|0\rangle_1|0\rangle_2 → |0\rangle_1|0\rangle_2, \quad -|0\rangle_1|3\rangle_2 → e^{iπ/2}|0\rangle_1|1\rangle_2,
\]

\[
-|3\rangle_1|0\rangle_2 → e^{iπ/2}|1\rangle_1|0\rangle_2, \quad \frac{1}{\sqrt{2}}(|\phi_1⟩ + |\phi_4⟩) → |1\rangle_1|1\rangle_2.
\]

(18)

Since the two processes are highly adiabatic, the population of the basis states return to the initial state after the counterintuitive pulse sequence. It is interesting to see that the basis states |0⟩_1|0⟩_2 and |1⟩_1|0⟩_2 finally acquire an additional phase factor e^{iφng} with φng = π/2, which does not exist for |0⟩_1|0⟩_2 and |1⟩_1|1⟩_2 (see Fig. 5). Because the dark states are the eigenstates of \(\mathcal{H}_i (i = 1, 2)\) with zero eigenvalues, φng has no dynamic origin. On the other hand, here the Hamiltonian is not required to make a cyclic evolution in the parameter space as for the accumulation of the adiabatic Berry phase.

Finally, by applying single-qubit operations \(|1⟩_1|2⟩ \rightarrow e^{iπ/2}|1⟩_1|2⟩\) to both atoms, we recover the familiar controlled-Z gate

\[
|0⟩_1|0⟩_2 → |0⟩_1|0⟩_2, \quad |0⟩_1|1⟩_2 → |0⟩_1|1⟩_2,
\]

\[
|1⟩_1|0⟩_2 → |1⟩_1|0⟩_2, \quad |1⟩_1|1⟩_2 → -|1⟩_1|1⟩_2,
\]

(19)

which can be easily transformed to a controlled-NOT gate by using two additional π/2 pulses rotating the target qubit around the y-axis in the opposite directions. Note that for \(V_{33} ≪ V_{22},V_{23}\), repeating the operation procedure above will lead to the transformation for the basis states: \(|0⟩_1|1⟩_2 \rightarrow e^{iπ/2}|0⟩_1|1⟩_2, |1⟩_1|0⟩_2 \rightarrow e^{iπ/2}|1⟩_1|0⟩_2\) and \(e^{iπ}|1⟩_1|1⟩_2\), which is impossible to become a universal binary gate under any local operations.

From a comparison between the two schemes we can see that the non-Berry phase gate via the staircase phase control is built on a completely different mechanism in contrast to the normal dynamical and geometric phase gates: the qubit system does not undergo any dynamical phase shift since it works in the zero-energy eigenspace; the Hamiltonian is not changed along a closed curve in the parameter space; precisely adiabatic modulation of the phases of the driving fields and adiabatic control of the population transfer at the same time is unnecessary, thus, the errors in obtaining the required geometric solid angle are avoided and the operation procedure is simplified.

IV. PHYSICAL REALIZATION: ASYMMETRIC RYDBERG COUPLING

In the context of Rydberg experiments, the strongly asymmetric coupling condition \(V_{22} ≪ V_{23},V_{33}\) can be found, for example, by mapping the Rydberg states to \(|2⟩ = 40p_{3/2}, m = 1/2\) and \(|3⟩ = 41s_{1/2}, m = 1/2\) of Rubidium atoms separated at an interatomic distance \(R\) of several micrometers. In this case, the blockade interaction between the states \(|2⟩\) and \(|3⟩\) is an exchange process of resonant dipole nature (\(\sim n^4/R^3\) with \(n\) being the principal quantum number), where the zero-interaction angle can be avoided either by using a spatial light modulator to create the preset trap pattern or by applying a weak external magnetic field \((B = 10^{-7} \text{ T})\) to couple the atomic Zeeman states of different magnetic quantum numbers. The anisotropic interaction between states \(|2⟩\) and isotropic interaction between states \(|3⟩\) are both induced by the Förster process, where the two-atomic interaction potential can transit from the dipole-dipole to the van der Waals limit (\(\sim n^{11}/R^6\)), depending on the interatomic distance [42]. It is therefore possible to restrict our consideration to the asymmetric coupling regime, which represents the dominant interaction mechanism at the atomic separation of interest. For \(R = 3\mu m\), the interaction strengths \(V_{23}\) and \(V_{22}\) can respectively vary from 5 MHz to 20 MHz, and from 0.02 MHz to 0.1 MHz by adjusting the angle between the dipoles, and the interaction strength \(V_{33}\) approximates \(2\pi \times 3.7 \text{ MHz} [43]\).

On the other hand, the excitation of Rydberg \(p\)-states from ground \(s\)-states in a single photon transition has recently become feasible due to the availability of ultraviolet (UV) laser sources, which results in much larger Rabi frequency \(\Omega\) (scaling as \(\Omega \sim n^{-3/2}\)) compared to a three-photon excitation process [44]. In addition, the optical excitation of a Rydberg state followed by a microwave-driven coupling between two neighboring Rydberg levels has been experimentally demonstrated as well, where the Rabi frequency of the Rydberg-Rydberg transition can reach several tens of MHz by increasing the intensity of the microwave field [45]. Thus, it becomes very promising to implement the proposed schemes with asymmetric Rydberg-Rydberg interaction by integrating the current experimental techniques [46].

To evaluate the performance of the controlled-Z gate, we use the fidelity \(F = |\text{Tr}\sqrt{\rho_{\text{tar}}\sigma(2\pi)\sqrt{\rho_{\text{tar}}}^2}|\) to measure the desired output \(\rho_{\text{tar}}\) given an input of all the logi-
bust against the variation of the Rydberg interactions however, note that the non-Berry operation is more ro-
abatic transfer towards the nonzero-energy eigenstates), will lead to reduction of the gate fidelity (due to nonadi-
abatic implementation of the Berry-phase-based controlled-Z gate as well. To gain the insight, we have
to consider. In this case, the nonadiabatic transi-
tions, docoherence induced by atomic spontaneous emission and atomic motion. In Fig. 6, we have shown the fidelity of the controlled-Z gate [ Eq. (14) and Eq. (19) ] under the condition of asymmetric Rydberg coupling $V_{22} \ll V_{23}, V_{33}$ in the coherent regime. Considering the dynamically perturbative effect of $V_{22}$ (i.e. $V_{22}\tau \ll 1$), the fidelity reaches its optimum at $V_{33} = 0.7$, $V_{23} = 1$ and $V_{33} = 0.9$, $V_{23} = 1$ for gates based on the Berry and the non-Berry adiabatic phase, respectively, where the interaction strengths $V_{23}, V_{33}$ are of comparable magnitude with the maximum of the Rabi frequencies $\Omega$, corresponding to the intermediate coupling regime $V_{23}, V_{33} \sim \Omega$. Further increasing $V_{23}$ or $V_{33}$ will lead to reduction of the gate fidelity (due to nonadi-
abatic transfer towards the nonzero-energy eigenstates), however, note that the non-Berry operation is more ro-
Assistant_1
Figure 8. (color online) Fidelities of the gates based on Berry phase (a) and non-Berry adiabatic phase (b) for different atomic spontaneous emission rates as functions of the Rydberg interaction $V_{33}$ for $(V_{22}, V_{23})/\Omega = (0.005, 1)$ (solid) and $V_{22} = V_{23} = 0$ (dash). We here set $\gamma_2 = \gamma_3 = \gamma$.

the dark state $|d^2_2(t)\rangle$, where the phase difference $\phi_\tau$ of the control fields becomes the only relevant phase factor for the modulation process. For the operation to obtain Berry phases, the system acquires no geometric phase during the cyclic evolution since $\phi_\tau$ is kept invariant [see Fig. 7(a-b)]. Alternatively, for the operation to obtain non-Berry adiabatic phases, the rise up of $\phi_\tau$ at $t = \tau$ introduces phase factors $e^{i2\phi_\tau} = e^{i\pi}$ and $e^{i\phi_\tau} = e^{i\pi/2}$ to the basis states $|1\rangle_1|1\rangle_2$ and $|1\rangle_1|0\rangle_2$ (or $|0\rangle_1|1\rangle_2$), respectively, which are irrelevant to a binary gate. While for a finite $V_{33}$, the instantaneous ground state of $H'_k$ evolves from the bare $|1\rangle_1|1\rangle_2$ state into a “dressed” state with some admixture of $|3\rangle_1|3\rangle_2$, which additionally supplements a dynamical phase $\varphi_d \approx \int \sin^2\theta V_{33} dt$ to $|1\rangle_1|1\rangle_2$ [see Fig. 7(c-d)]. Therefore, the implementation of a controlled-Z gate via the completely dynamical control is still available for $\varphi_d = \pi$ for both cases, and is sensitive to the fluctuation of Rydberg interactions nevertheless.

Figure 9. (color online) Level configuration. Two hyperfine ground states $|1\rangle$, $|2\rangle$ are coupled via the Raman process with the effective Rabi frequency $\Omega_\mu$. The Rydberg state $|3\rangle$ is excited via a single photon transition of the Rabi frequency $\Omega_\nu$.

V. THE EFFECT OF SPONTANEOUS EMISSION AND INTERATOMIC FORCE

The two atoms excited to Rydberg states are subjected to decoherence due to atomic spontaneous emission and interatomic force. The dissipative dynamics can be calculated by the Lindblad master equation for the density operator $\rho$ of the two-atom system,

$$\dot{\rho}(t) = -i[H_{R}, \rho(t)] + \sum_{i=1}^{2} \sum_{k=1}^{2} \mathcal{L}[A_{i,k}]\rho(t), \quad (20)$$

where $\mathcal{L}[A_{i,k}]\rho = A_{i,k}\rho A_{i,k}^\dagger - \frac{1}{2} \{A_{i,k}^\dagger A_{i,k}, \rho\}$. $A_{i,1} = \sqrt{\gamma_2}\, |1\rangle_2\langle i|_i$ and $A_{i,2} = \sqrt{\gamma_3}\, |2\rangle_2\langle i|_i$ with $\gamma_2$ and $\gamma_3$ being the spontaneous decay rates for the transition channels $|2\rangle_i \rightarrow |1\rangle_i$ and $|3\rangle_i \rightarrow |2\rangle_i$ respectively. In Fig. 8 we show overlap (fidelity) between the realistic density matrix $\rho(2\tau)$ at the end of the pulse sequences from Eq. (20) and the ideal result $\rho_{\text{tar}}$ (for $\gamma_2 = \gamma_3 = 0$), for the system initially in $|\psi_0\rangle$. Since the lifetime of the Rydberg states $|2\rangle$ ($|3\rangle$) with principal quantum number $n = 40$ or 41 is around 60\,µs, thus the decay rates are taken as $\gamma_2 = \gamma_3 = 10^{-4}\Omega$, $10^{-3}\Omega$, corresponding to the peak Rabi frequency $\Omega/2\pi = 20\,\text{MHz}, 2\,\text{MHz}$, respectively. For the former case, the fidelities of the gates relying on the Berry phase and non-Berry adiabatic phase are 0.983 and 0.992, respectively; while for the latter case, fidelity of better than 0.92 is still achievable for both schemes. On the other hand, the interatomic force (induced by double excitation of Rydberg states) during the gate operation can couple the internal degree of freedom to the externally atomic motion. Its perturbative effect on the gate fidelity can be estimated by $\sim \frac{3\hbar\lambda_0 V_{33}}{R_{\text{tr}}}(1 - e^{-i\omega_0\tau})$ to the first order, with $\omega_0$ the trapping frequency and $\lambda_0$ the wavelength of trapping light [7]. Thus, one can enlarge the Rabi frequency $\Omega$ to reduce the gate duration $\tau$ or alternatively use an optical lattice (instead of an optical tweezer trap) with higher trapping frequency to trap the atoms such that the motional effect can be reasonably ignored.

As mentioned before, the controlled-Z gate can be implemented as well with the atomic level scheme involving...
only a single Rydberg state. In this case, the microwave control becomes unnecessary. However, for the usual Rydberg EIT configuration (i.e., a cascaded three-level system), the populated intermediate state $|2\rangle$ (such as $6p_{1/2}$ for Rb and $7p_{1/2}$ for Cs atoms) is an excited state with strong spontaneous emission rate, which will irreversibly deteriorate the coherent population transfer and then the gate fidelity. However, the obstacle can be overcome by using the single-photon excitation scheme for the ground-Rydberg transition and by mapping $|1\rangle$ and $|2\rangle$ to the atomic hyperfine states, which can couple to each other via two-photon Raman processes (see Fig. 3). Therefore, the fidelity can be further improved by selecting a Rydberg state with larger principal quantum number and longer lifetime. For example, in a 300K environment, the Cs Rydberg states $|90p\rangle$, $|95p\rangle$ have the lifetimes 361µs and 406µs, respectively. Moreover, double excitation of Rydberg states is avoided, and thus the effect of the interatomic force that may entangle their motional degree of freedom can be neglected.

**VI. CONCLUSION**

In conclusion, we have shown that the Rydberg-Rydberg interaction between two highly excited atoms can be exploited for implementing a reliable controlled-Z gate via adiabatic passage and tailored phase modulation. The newly developed addressing schemes drive the system Hamiltonian to change in a cyclic or a non-cyclic manner, giving rise to a Berry phase or a non-Berry adiabatic phase for implementation of conditional phase gates. In the former case, the geometric phase is acquired through concurrent control of the phases of the driving fields, and can be alternatively obtained via modulation of the relative phase in two different ways, while for the latter the requirement of adiabatic phase control becomes unnecessary and therefore the experimental complexity can be significantly released. We also pointed out that the implementation of the schemes with multilevel atomic configuration involving a unique Rydberg state might be more promising for experimental demonstration. We note that our adiabatic Rydberg gates may not replace the conventional approaches with fast dynamical control, however, the merits of the adiabatic technique itself and the new addressing schemes of phase modulation found here will provide new perspectives for adiabatic manipulation of interacting Rydberg systems.

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