Quantum properties of a non-Abelian gauge invariant action with a mass parameter

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We continue the study of a local, gauge invariant Yang-Mills action containing a mass parameter, which we constructed in a previous paper starting from the nonlocal gauge invariant mass dimension two operator \(F_{\mu\nu}(D^2)^{-1}F_{\mu\nu}\). We return briefly to the renormalizability of the model, which can be proven to all orders of perturbation theory by embedding it in a more general model with a larger symmetry content. We point out the existence of a nilpotent BRST symmetry. Although our action contains extra (anti)commuting tensor fields and coupling constants, we prove that our model in the limit of vanishing mass is equivalent with ordinary massless Yang-Mills theories. The full theory is renormalized explicitly at two loops in the \(\overline{\text{MS}}\) scheme and all the renormalization group functions are presented. We end with some comments on the potential relevance of this gauge model for the issue of a dynamical gluon mass generation.

I. INTRODUCTION.

Yang-Mills gauge theories, with quantum chromodynamics (QCD) modeling the strong interaction between elementary particles as one of the key examples, are quite well understood at very high energies. In this energy region, asymptotic freedom \([1, 2, 3, 4]\) sets in, which in turn ensures that the coupling constant \(g^2\) is small enough to make a perturbative expansion in powers of \(g^2\) possible. The elementary QCD excitations are the gluons and quarks.

Our current understanding of non-Abelian gauge theories is still incomplete in the infrared region. At lower energies, the interaction grows stronger, preventing the use of standard perturbation theory to obtain relatively acceptable results. Nonperturbative aspects of the theory come into play. The most notable, yet to be rigourously proven nonperturbative phenomenon, is the fact that the elementary gluon and quark excitations no longer belong to the physical spectrum, being \textit{confined} into colorless states such as glueballs, mesons and baryons.

A widely used strategy to parametrize certain nonperturbative effects of the theory amounts to the introduction of so called condensates, which are the expectation values of certain operators in the vacuum. Furthermore, one can employ the operator product expansion (OPE) (viz. short distance expansion) which can be applied to \textit{local} operators, in order to relate the associated condensates to nonperturbative power corrections which, in turn, give additional information next to the perturbatively calculable contributions.

As we are considering a gauge theory, these condensates should be gauge invariant if they are to enter physical observables. This puts rather strong restrictions on the possible condensates, the ones with lowest dimensionality are the dimension three quark condensate \(\langle \bar{\psi} \psi \rangle\) and the dimension four gluon condensate \(\langle F_{\mu\nu}^2 \rangle\). There is a variety of methods to obtain estimates of...
these condensates, such as the phenomenological approach based on the SVZ sum rules, for a recent overview, the use of lattice methods, as well as the use of instanton calculus.

Gauge condensates are necessarily nonperturbative in nature, as gauge theories do not contain a mass term in the action due to the requirement of gauge invariance. However, through nonperturbative effects, a nontrivial value for e.g. $\langle F_{\mu\nu}^2 \rangle$ can arise.

In the case where the dimension two gluon condensate $\langle A_{\mu}^2 \rangle$ has received much attention in the Landau gauge over the past few years. An OPE argument based on lattice simulations has provided evidence that this condensate could account for quadratic power corrections of the form $\sim 1/\mu^2$, reported in the running of the coupling constant as well as in the gluon propagator.

This nonvanishing condensate $\langle A_{\mu}^2 \rangle$ gives rise to a dynamically generated gluon mass. The appearance of mass parameters in the gluon two point function is a common feature of the expressions employed to fit the numerical data obtained from lattice simulations.

Let us mention that a gluon mass has been found to be useful also in the phenomenological context.

The local operator $A_{\mu}^2$ in the Landau gauge has witnessed a renewed interest due to the recent works, as the quantity

$$\langle A_{\mu\nu}^2 \rangle \equiv \min_{U \in SU(N)} \frac{1}{V T} \int d^4 x \left( \langle (A_{\mu}^2) \rangle \right),$$

which is gauge invariant due to the minimization along the gauge orbits, could be physically relevant. In fact, as shown in the case of compact three-dimensional QED, the quantity $\langle A_{\mu\nu}^2 \rangle$ seems to be useful in order to detect the presence of nontrivial field configurations like monopoles. One should notice that the operator $A_{\mu\nu}^2$ is highly nonlocal and therefore it falls beyond the standard OPE realm that refers to local operators. One can show that $A_{\mu\nu}^2$ can be written as an infinite series of nonlocal terms, see and references therein, namely

$$A_{\mu\nu}^2 = \frac{1}{2} \int d^4 x \left[ A_{\mu}^a \left( \delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{D^2} \right) A_{\nu}^a - g f^{abc} \left( \frac{\partial_\nu}{D^2} \partial_\mu A^a \right) \left( \frac{1}{D^2} \partial_\nu A^b \right) A_{\nu}^c \right] + O(A^4).$$

However, in the Landau gauge, $\partial_\mu A_\nu = 0$, all nonlocal terms of expression drop out, so that $A_{\mu\nu}^2$ reduces to the local operator $A_{\mu\nu}^2$, hence the interest in the Landau gauge and its dimension two gluon condensate. However, a complication, as already outlined in our previous paper, is that the explicit determination of the absolute minimum of $A_{\mu\nu}^2$ along its gauge orbit, and moreover of its vacuum expectation value, is a very delicate issue intimately related to the problem of the Gribov copies. We refer to for some more explanation and the original references concerning this point.

Nevertheless, some nontrivial results were proven concerning the operator $A_{\mu\nu}^2$. In particular, we mention its multiplicative renormalizability to all orders of perturbation theory, in addition to an interesting and numerically verified relation concerning its anomalous dimension. An effective potential approach consistent with the renormalization group requirements include possible effects of Gribov copies.

A somewhat weak point about the operator $A_{\mu\nu}^2$ is that it is unclear how to deal with it in gauges other than the Landau gauge. Till now, it seems hopeless to prove its renormalizability out of the Landau gauge. In fact, at the classical level, adding to the Yang-Mills action is equivalent to add the so-called Stueckelberg action, which is known to be not renormalizable. We refer, once more, to for details and references.

In recent years, some progress has also been made in the potential relevance of dimension two condensates beyond the Landau gauge. We were able to prove the renormalizability of certain local operators like: in the linear covariant gauges, in the nonlinear Curci-Ferrari gauges, and in the maximal Abelian gauges. A renormalizable effective potential for these operators has been constructed, giving rise to a nontrivial value for the corresponding condensates, and a dynamical gluon mass parameter emerged in each of these gauges. There also have been attempts to include possible effects of Gribov copies. Unfortunately, the amount of numerical data available from lattice simulations is rather scarce in the aforementioned gauges. Nevertheless, let us mention that a dynamical gluon mass in the maximal Abelian gauge has been reported in. In the Coulomb gauge too, the relevance of a dimension two condensate has been touched upon in the past.

Although the renormalizability of the foregoing dimension two condensates is a nontrivial and important fact in its own right, their lack of gauge invariance is a less welcome feature. Moreover, at present, it is yet an open question whether these operators might be related in some way to a gauge invariant gluon mass.

Many aspects of the dimension two condensates and of the related issue of dynamical mass generation in Yang-Mills theories need further understanding. An important step forwards would be a gauge invariant mechanism behind a dynamical mass, without giving up the important renormalization aspects of quantum field theory.

We set a first step in this direction in our previous paper. As a local gauge invariant operator of dimension two does not exist, and since locality of the action is almost indispensable to prove renormalizability to all orders and to have a consistent...
calculational framework at hand, we could look for a nonlocal operator that is localizable by introducing an additional set of fields. As pointed out in [40], this task looks extremely difficult for the operator $A^2_{\text{mun}}$ if we reckon that an infinite series of nonlocal terms is required, as displayed in (4). Instead, we turned our attention to the gauge invariant operator

$$F \frac{1}{D^2} F \equiv \frac{1}{VT} \int d^4x F^a_{\mu \nu} \left[ (D^2)^{-1} \right]^{ab} F^b_{\mu \nu},$$

where $D^2 = D_\mu D^\mu$ is the covariant Laplacian, $D_\mu$ being the adjoint covariant derivative. The operator (3) already appeared in relation to gluon mass generation in three-dimensional Yang-Mills theories [56].

The usefulness of the operator (3) relies on the fact that, when it is added to the usual Yang-Mills Lagrangian by means of $-\frac{1}{4} m^2 F \frac{1}{DF} F$, the resulting action can be easily cast into a local form by introducing a finite set of auxiliary fields [40]. Starting from that particular localized action, we succeeded in constructing a gauge invariant classical action $S_{el}$ containing the mass parameter $m$, enjoying renormalizability. This action $S_{el}$ was identified to be

$$S_{\text{phys}} = S_{el} + S_{gf},$$

$$S_{el} = \int d^4x \left[ \frac{1}{4} F^a_{\mu \nu} F^a_{\mu \nu} + \frac{im}{4} (B - \overline{B})^a_{\mu \nu} F^a_{\mu \nu} + \frac{1}{4} (\overline{D}^a_{\mu \nu} D^{ab}_{\mu \nu} B^b_{\mu \nu} - \overline{D}^a_{\mu \nu} D^{ab}_{\mu \nu} D^b_{\mu \nu} G^c_{\mu \nu} - \frac{3}{32} m^2 \lambda_1 (\overline{B}^a_{\mu \nu} B^a_{\mu \nu} G^c_{\mu \nu}) + \frac{3 m^2 \lambda_1}{16} (\overline{B}^a_{\mu \nu} B^a_{\mu \nu} G^c_{\mu \nu} - \overline{G}^a_{\mu \nu} G^b_{\mu \nu}) + \frac{\gamma^{abcd}}{16} (\overline{B}^a_{\mu \nu} B^b_{\mu \nu} G^c_{\mu \nu} - \overline{G}^a_{\mu \nu} G^b_{\mu \nu}) \right],$$

$$S_{gf} = \int d^4x \left( \frac{\alpha}{2} b^a b^a + b^a d_\mu a^a_{\mu} + \partial^\nu a^a_{\mu \nu} - \frac{\gamma^{abcd}}{16} (\overline{B}^a_{\mu \nu} B^b_{\mu \nu} G^c_{\mu \nu} - \overline{G}^a_{\mu \nu} G^b_{\mu \nu}) \right).$$

We notice that we had to introduce a new quartic tensor coupling $\lambda_{abcd}$, as well as two new mass couplings $\lambda_1$ and $\lambda_3$. The renormalizability was proven to all orders in the class of linear covariant gauges, implemented through $S_{gf}$, via the algebraic renormalization formalism [57]. Without the new couplings, i.e. when $\lambda_1 \equiv 0$, $\lambda_3 \equiv 0$, $\lambda_{abcd} \equiv 0$, the previous action would not be renormalizable.

In this paper, we present further results concerning the action (5) obtained in [40]. In section II, we provide a short summary of the construction of the model (5) and we present a detailed discussion of the renormalization of the tensor coupling $\lambda_{abcd}$ not given in [40]. We draw attention to the existence of an extended version of the usual nilpotent BRST symmetry for the model (5). We introduce a kind of supersymmetry between the novel fields $\{B^a_{\mu \nu}, G^c_{\mu \nu}, \overline{D}^a_{\mu \nu}, \overline{B}^a_{\mu \nu} \}$ which is enjoyed by the massless version, $m = 0$, of the action (4). In section III, we discuss the explicit renormalization of several quantities. The fields and the mass $m$ are renormalized to two loop order in the \(\overline{\text{MS}}\) scheme. The $\lambda_{abcd}$-function of the tensor coupling $\lambda_{abcd}$ is determined at one loop, by means of which it shall also become clear that radiative corrections (re)introduce anyhow the quartic interaction in the novel fields in the action (4). These fields are thus more than simple auxiliary fields, which appear at most quadratically. A few internal checks on the results are included, such as the explicit gauge parameter independence of the anomalous dimension of $m$. It is also found that the original Yang-Mills quantities renormalize identically as when the usual Yang-Mills action would have been used. This is indicative of the fact that the massless version of (4) might be equivalent to Yang-Mills theory, quantized in the same gauge. This is a nontrivial statement, due to the presence of the term proportional to the tensor coupling $\lambda_{abcd}$. In section IV, we use the aforementioned supersymmetry to actually prove the equivalence between the massless version of (4) and Yang-Mills theories. In the concluding section V, we put forward a few suggestions that might be useful for future research directions.

**II. SURVEY OF THE CONSTRUCTION OF THE MODEL AND ITS RENORMALIZABILITY.**

In this section we present a short summary of how we came to the construction of our model (5) in [40]. We shall also point out a few properties of the corresponding action not explicitly mentioned in [40].

**A. The model at the classical level.**

We start from the Yang-Mills action $S_{YM}$ supplemented with a gauge invariant although nonlocal mass operator

$$S_{YM} + S_O,$$

with the usual Yang-Mills action defined by

$$S_{YM} = \frac{1}{4} \int d^4x F^a_{\mu \nu} F^a_{\mu \nu},$$
and with
\[ S_0 = -\frac{m^2}{4} \int d^4x F_{\mu\nu}^a \left( (D^2)^{-1} \right)^{ab} F_{\mu\nu}^b . \]  

The field strength is given by
\[ F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c , \]
and the adjoint covariant derivative by
\[ D_\mu^{ab} = \partial_\mu S^{ab} - g f^{abc} A_\mu^c . \]

In order to have a consistent calculational framework at the perturbative level, we need a local action. To our knowledge, it is unknown how to treat an action like (7), such as proving its renormalizability to all orders of perturbation theory. This is due to the presence of the nonlocal term (9). As we have discussed in [40], the action (7) can be localized by introducing a pair of complex bosonic antisymmetric tensor fields, \((B_{\mu\nu}^a, \overline{B}_{\mu\nu}^a)\), and a pair of complex anticommuting antisymmetric tensor fields, \((\overline{G}_{\mu\nu}^a, G_{\mu\nu}^a)\), belonging to the adjoint representation, according to which
\[ e^{-S_0} = \int D\overline{B}DBD\overline{G}DG \exp \left[ - \left( \frac{1}{4} \int d^4x \overline{B}_{\mu\nu}^a D_\sigma^{ab} \overline{B}_{\rho\sigma} B_{\mu\rho}^b - \frac{1}{4} \int d^4x \overline{G}_{\mu\nu}^{ab} D_\sigma^{bc} G_{\rho\sigma}^{bc} G_{\mu\rho}^{ce} + \frac{im}{4} \int d^4x \overline{B}_{\mu\nu}^a F_{\mu\nu}^a \right) \right] . \]  

It is worth mentioning the special limit \( m \equiv 0 \), in which case we have in fact introduced nothing more than a rather complicated unity written as [83]
\[ \int D\overline{B}DBD\overline{G}DG \exp \left[ - \left( \frac{1}{4} \int d^4x \overline{B}_{\mu\nu}^a D_\sigma^{ab} \overline{B}_{\rho\sigma} B_{\mu\rho}^b - \frac{1}{4} \int d^4x \overline{G}_{\mu\nu}^{ab} D_\sigma^{bc} G_{\rho\sigma}^{bc} G_{\mu\rho}^{ce} \right) \right] = 1 . \]  

Hence, we have obtained a local, classical and gauge invariant action
\[ S = S_{YM} + S_{BG} + S_m , \]
where
\[ S_{BG} = \frac{1}{4} \int d^4x \left( \overline{B}_{\mu\nu}^a D_\sigma^{ab} \overline{B}_{\rho\sigma} B_{\mu\rho}^b - \overline{G}_{\mu\nu}^{ab} D_\sigma^{bc} G_{\rho\sigma}^{bc} G_{\mu\rho}^{ce} \right) , \]
\[ S_m = \frac{im}{4} \int d^4x \overline{B}_{\mu\nu}^a F_{\mu\nu}^a . \]

The gauge transformations are given by
\[ \delta A_\mu^a = \delta \omega_\mu^b , \]
\[ \delta B_{\mu\nu}^a = g f^{abc} \omega_\mu^b B_{\nu}^c , \]
\[ \delta \overline{B}_{\mu\nu}^a = g f^{abc} \omega_\mu^b \overline{B}_{\nu}^c , \]
\[ \delta G_{\mu\nu}^a = g f^{abc} \omega_\mu^b G_{\nu}^c , \]
\[ \delta \overline{G}_{\mu\nu}^a = g f^{abc} \omega_\mu^b \overline{G}_{\nu}^c , \]

with \( \omega^a \) parametrizing an arbitrary infinitesimal gauge transformation, so that
\[ \delta S = \delta (S_{YM} + S_{BG} + S_m) = 0 . \]

**B. The model at the quantum level.**

Evidently, the construction of the classical action (14) is only a first step. We still need to investigate if this action can be renormalized when the quantum corrections are included. This highly nontrivial task was treated at length in [40] to which we refer the interested reader for background information. Nevertheless, we shall take some time to explain the main idea as well as to present a detailed analysis of the quantum corrections affecting the quartic tensor coupling \( \lambda^{abcd} \).

As the quantization of a locally invariant gauge model requires the fixing of the gauge freedom, we shall employ the linear covariant gauge fixing from now on, as it was done in [40], and which is imposed via [6].
To actually discuss the renormalizability of \([14]\), we found it useful to embed it into a more general class of models described by the action

\[
\Sigma = S_{YM} + S_{gf} + \int d^4x \left( \mathcal{B}_i^a D_{\mu i}^a b_{\mu i} - \mathcal{G}_i^a D_{\mu i}^a G_{\mu i}^a \right) + \int d^4x \left( (\mathcal{V}_{\mu \nu} G_i^a + \mathcal{V}_{\mu \nu} B_i^a - \mathcal{V}_{\mu \nu} B_i^a + U_{\mu \nu} \mathcal{G}_i^a) F_{\mu \nu}^a + \chi_1 (\mathcal{V}_{\mu \nu} \partial^2 V_{\mu \nu} - \mathcal{U}_{\mu \nu} \partial^2 U_{\mu \nu}) \right) + \int d^4x \chi_2 (\mathcal{V}_{\mu \nu} \partial^2 V_{\mu \nu} - \mathcal{U}_{\mu \nu} \partial^2 U_{\mu \nu}) - \int d^4x \zeta (\mathcal{V}_{\mu \nu} U_{\mu \nu} \mathcal{U}_{\mu \nu} U_{\mu \nu} \mathcal{U}_{\mu \nu}) + \mathcal{V}_{\mu \nu} \mathcal{V}_{\mu \nu} \mathcal{V}_{\mu \nu} \mathcal{V}_{\mu \nu} \mathcal{V}_{\mu \nu}, \quad (18)
\]

where use has been made of a composite Lorentz index \(i \equiv (\mu, \nu)\), \(i = 1 \ldots 6\), corresponding to a global \(U(6)\) symmetry \([40]\) of the action \([13]\). The quantities \(V_{\mu \nu}, \mathcal{V}_{\mu \nu}, U_{\mu \nu}\) and \(\mathcal{U}_{\mu \nu}\) are local sources. The identification between objects carrying indices \(i\) and \((\mu, \nu)\) is determined through

\[
(B_i^a, B_i^a, G_i^a, G_i^a) = \frac{1}{2} \left( B_i^a, B_i^a, G_i^a, G_i^a \right), \quad (19)
\]

The free parameters \(\chi_1, \chi_2\) and \(\zeta\) are needed for renormalizability purposes. As far as we are considering Green functions of elementary fields, their role is irrelevant as they multiply terms which are polynomial in the external sources.

The reason for introducing the action \([13]\) is that for \(m = 0\), the action \([14]\) enjoys a few global symmetries which are lost for \(m \neq 0\), whereas the action \([13]\) also enjoys such symmetries when the global symmetry transformations are suitably extended to the sources. We refer to \([40]\) for the details. Evidently, the general action \([13]\) must possess the action \([14]\) we are interested in as a special case. The reader can check that the connection is made by considering the “physical” limit

\[
\lim_{\text{phys}} V_{\sigma \rho \mu \nu} = \lim_{\text{phys}} V_{\sigma \rho \mu \nu} = \frac{-i m}{2} \left( \delta_{\sigma \mu} \delta_{\rho \nu} - \delta_{\sigma \nu} \delta_{\rho \mu} \right), \quad (20)
\]

i.e.

\[
\lim_{\text{phys}} \Sigma = S. \quad (21)
\]

In \([40]\), it was shown that the action \([13]\) obeys a large set of Ward identities. We shall not list them here, but mention that the action \(\Sigma\) is invariant under a nilpotent BRST transformation \(s\), acting on the fields as

\[
\begin{align*}
\delta A \mu^a &= -D_{\mu i}^a b_i^a, \\
\delta c^a &= g_{fabc} c^b c^c, \\
\delta B_{\mu \nu}^a &= g_{fabc} B_{\mu \nu}^a + G_{\mu \nu}^a, \\
\delta G_{\mu \nu}^a &= g_{fabc} G_{\mu \nu}^a, \\
\delta B_{\mu \nu}^a &= g_{fabc} B_{\mu \nu}^a + G_{\mu \nu}^a, \\
\delta G_{\mu \nu}^a &= g_{fabc} G_{\mu \nu}^a, \\
\delta c^a &= b^a, \\
\delta b^a &= 0,
\end{align*}
\]

and on the sources as

\[
\begin{align*}
\delta V_{\mu \nu} &= U_{\mu \nu}, \\
\delta U_{\mu \nu} &= 0, \\
\delta U_{\mu \nu} &= V_{\mu \nu}, \\
\delta V_{\mu \nu} &= 0.
\end{align*}
\]

\[
\begin{align*}
\delta A \mu^a &= -D_{\mu i}^a b_i^a, \\
\delta c^a &= g_{fabc} c^b c^c, \\
\delta B_{\mu \nu}^a &= g_{fabc} B_{\mu \nu}^a + G_{\mu \nu}^a, \\
\delta G_{\mu \nu}^a &= g_{fabc} G_{\mu \nu}^a, \\
\delta b^a &= 0,
\end{align*}
\]

and on the sources as

\[
\begin{align*}
\delta V_{\mu \nu} &= U_{\mu \nu}, \\
\delta U_{\mu \nu} &= 0, \\
\delta U_{\mu \nu} &= V_{\mu \nu}, \\
\delta V_{\mu \nu} &= 0.
\end{align*}
\]

\[
\begin{align*}
\delta A \mu^a &= -D_{\mu i}^a b_i^a, \\
\delta c^a &= g_{fabc} c^b c^c, \\
\delta B_{\mu \nu}^a &= g_{fabc} B_{\mu \nu}^a + G_{\mu \nu}^a, \\
\delta G_{\mu \nu}^a &= g_{fabc} G_{\mu \nu}^a, \\
\delta b^a &= 0,
\end{align*}
\]

and on the sources as

\[
\begin{align*}
\delta V_{\mu \nu} &= U_{\mu \nu}, \\
\delta U_{\mu \nu} &= 0, \\
\delta U_{\mu \nu} &= V_{\mu \nu}, \\
\delta V_{\mu \nu} &= 0.
\end{align*}
\]
perturbation theory in the linear covariant gauge, imposed via case of the more general renormalizable action (25), since

\[ 6 \]

obtain the main outcome of the paper [40], which is the physical action generalized Jacobi identity not fully covered in [40].

The quantities \( \lambda_1 \) and \( \lambda_3 \) are independent scalar couplings, whereas \( \lambda^{abcd} \) is an invariant rank 4 tensor coupling, obeying the generalized Jacobi identity

\[ f^{man} \lambda^{abcd} + f^{mnb} \lambda^{amcd} + f^{mnc} \lambda^{abmd} + f^{mnd} \lambda^{abcm} = 0, \]

and subject to the following symmetry constraints

\[ \lambda^{abcd} = \lambda^{cdab}, \]
\[ \lambda^{abcd} = - \lambda^{bacd}, \]

which can be read off from the vertex that \( \lambda^{abcd} \) multiplies. When we specify the action (25) to the physical values (20), we obtain the main outcome of the paper [40], which is the physical action \( S_{\text{phys}} \) given in (4), that is renormalizable to all orders of perturbation theory in the linear covariant gauge, imposed via \( S_{\text{gen}} \). The renormalizability is of course ensured as [4] is a special case of the more general renormalizable action (25), since

\[ \lim_{\text{phys}} S_{\text{gen}} = S_{\text{phys}}. \]

We notice that the couplings \( \lambda_1 \) and \( \lambda_3 \) are in fact part of the mass matrix of the fields \( \{ B^{a}_{\mu \nu}, B^{i}_{\mu \nu}, G^{a}_{\mu \nu}, G^{i}_{\mu \nu} \} \).

We end this subsection by mentioning that the classical action \( S_{\text{cl}} \) is also invariant with respect to the gauge transformations (16), since the terms \( \propto \{ \lambda_1, \lambda_3, \lambda^{abcd} \} \) are separately gauge invariant.

C. The renormalization of the tensor coupling \( \lambda^{abcd} \).

As already mentioned, this section is devoted to providing further details of the renormalization of the tensor coupling \( \lambda^{abcd} \), not fully covered in [40].

The term we are interested in at the level of the bare [86] action is given by

\[ \int d^{4}x \left[ \frac{\lambda^{abcd}}{16} \left( B^{a}_{\mu \nu} B^{b}_{\mu \nu} - G^{a}_{\mu \nu} G^{b}_{\mu \nu} \right) \right], \]

where, in the notation of [40]

\[ \{ B, B, G, G \} \equiv \left[ 1 + \eta \left( a_3 + \frac{1}{2} a_4 \right) \right] \{ B, B, G, G \}, \]

where \( a_3, a_4 \) are arbitrary coefficients and \( \eta \) stands for a perturbative expansion parameter [40].
The most general counterterm corresponding to the renormalization of the 4-point vertex \( (\mathcal{F}_i B_i - \mathcal{G}_i G_i) (\mathcal{F}_j B_j - \mathcal{G}_j G_j) \) turns out to be given by

\[
(4a_3 + \tilde{a}_6) \frac{M_{abcd}^{\text{abed}}}{16} (\mathcal{F}_i B_i - \mathcal{G}_i G_i) (\mathcal{F}_j B_j - \mathcal{G}_j G_j) ,
\]

where \( \tilde{a}_6 \) is a free coefficient and \( M_{abcd}^{\text{abed}} \) is an arbitrary invariant rank 4 tensor, composed of all the other available tensors (such as \( \lambda_{abcd}, \delta_{ab}, \rho^{abc} \) and invariant objects constructed from these and \( T \)). By the Ward identities, it is nevertheless restricted by

\[
M_{abcd}^{\text{abed}} = M_{cdab}^{\text{abed}},
\]

\[
M_{abcd}^{\text{abed}} = M_{bacd}^{\text{abed}},
\]

which are of course the same symmetry constraints as those for \( \lambda_{abcd} \), see (28). Also the Jacobi identity (27) applies to \( M_{abcd}^{\text{abed}} \).

The counterterm is thus not necessarily directly proportional to the original tensor \( \lambda_{abcd} \). This has a simple diagrammatical explanation, as diagrams contributing to the 4-point interaction \( \propto (\mathcal{F}_i B_i - \mathcal{G}_i G_i) (\mathcal{F}_j B_j - \mathcal{G}_j G_j) \) can be constructed with the other available interactions. This also means that, even if \( \lambda_{abcd} = 0 \), radiative corrections shall reintroduce this 4-point interaction. This shall become more clear in section III, where the explicit results are discussed.

We can thus decompose the bare tensor coupling \( \lambda_{oabcd} \) as

\[
\lambda_{oabcd} = Z \lambda_{abcd} + Z_{aabcd} ,
\]

where \( Z \) and \( Z_{abcd} \) contain the counterterm information, more precisely \( Z \) contains the counterterm information directly proportional to \( \lambda_{abcd} \), while \( Z_{abcd} \) contains, so to say, all the rest. Evidently, \( Z_{abcd} \) will obey analogous constraints as given in (29) or (33). The tensor \( M_{abcd}^{\text{abed}} \) can be decomposed similarly into

\[
M_{abcd}^{\text{abed}} = \lambda_{abcd} + \frac{M_{abcd}^{\text{abed}} - \lambda_{abcd}}{\eta (\tilde{a}_6 - 2a_4)} ,
\]

In the previous paper [40], we erroneously omitted the \( \mathcal{N}_{abcd} \) part. Using (32), (34) and (35) allows for a simple identification, being

\[
Z = 1 + \eta (\tilde{a}_6 - 2a_4) ,
\]

\[
Z_{abcd} = \eta (4a_3 + \tilde{a}_6) \lambda_{abcd} .
\]

Consequently, the model is still renormalizable to all orders, although \( \lambda_{abcd} \) is not multiplicatively renormalizable in the naive sense. The situation can be directly compared with Higgs inspired models like the Coleman-Weinberg action \[60, 61\], in the sense that also there a similar mixing occurs between the different couplings, in casu the gauge coupling \( e^2 \) and the Higgs coupling \( \lambda \). This is nicely reflected in the \( \beta \)-functions for the couplings, which are series in both \( e^2 \) and \( \lambda \). It is even so that setting the Higgs coupling \( \lambda \equiv 0 \) does not make \( \beta_\lambda(e^2, \lambda) \) vanish. See [61] for the three loop expressions. As we shall discuss later in this paper, the \( \beta_{abcd} \)-function of the tensor coupling \( \lambda_{abcd} \) will be influenced by the gauge coupling \( g^2 \). Vice versa, one might expect that \( \lambda_{abcd} \) could enter, in a suitable colorless combination, the \( B_{ab} \)-function for \( g^2 \). This is however not the case. We shall present the general argument behind this in section IV. The \( B_{ab} \)-function remains thus identical to the well known \( \beta(g^2) \)-function of Yang-Mills theory.

Let us end this subsection by mentioning that the method of using the extended action (25), which is a generalization of another action like [41] and which exhibits a larger set of Ward identities, turns out to be a powerful tool in order to establish renormalizability to all orders. This is reminiscent of Zwanziger’s approach to prove the renormalizability of a local action describing the restriction to the first Gribov horizon [58, 52].

D. A few words about the BRST symmetry and a kind of supersymmetry.

Let us now return for a moment to the action \( S_{\text{phys}} \) given in (4). As it is a gauge fixed action, we expect that it should have a nilpotent BRST symmetry. However, one shall easily recognize that the BRST transformation \( s \) as defined in (22) no longer constitutes a symmetry of the action (4). This is due to the fact that setting the sources to their physical values (20) breaks the BRST \( s \) as the transformations (23) are incompatible with the desired physical values (20).
Let us take a closer look at the breaking of this BRST transformation $s$. Let us define another transformation $\tilde{s}$ at the level of the fields by

\[
\tilde{s}A_a^\mu = -D_a^{\mu b}c^b, \\
\tilde{s}c^a = \frac{g}{2}f^{abc}c^b c^c, \\
\tilde{s}B_{\mu \nu}^a = g f^{abc}c^b B_{\mu \nu}^c, \\
\tilde{s}\bar{B}_{\mu \nu} = g f^{abc}c^b \bar{B}_{\mu \nu}^c, \\
\tilde{s}G_{\mu \nu}^a = g f^{abc}c^b G_{\mu \nu}^c, \\
\tilde{s}\bar{G}_{\mu \nu} = g f^{abc}c^b \bar{G}_{\mu \nu}^c, \\
\tilde{s}c^a = b^a, \\
\tilde{s}b^a = 0.
\]  

(37)

A little algebra yields

\[
\tilde{s}S_{\text{phys}} = 0, \\
\tilde{s}^2 = 0.
\]  

(38)  

(39)

Hence, the action $S_{\text{phys}}$ is invariant with respect to a nilpotent BRST transformation $\tilde{s}$. We obtained thus a gauge field theory, described by the action $S_{\text{phys}}$, containing a mass term, and which has the property of being renormalizable, while nevertheless a nilpotent BRST transformation expressing the gauge invariance after gauge fixing exists simultaneously. It is clear that $\tilde{s}$ stands for the usual BRST transformation, well known from literature, on the original Yang-Mills fields, whereas the gauge fixing part $S_{gf}$ given in (6) can be written as a $\tilde{s}$-variation, ensuring that the gauge invariant physical operators shall not depend on the choice of the gauge parameter [57].

We can relate $\tilde{s}$ and $s$. Let us start from the original localized action (14) and let us set $m \equiv 0$. Then it enjoys a nilpotent "supersymmetry" between the auxiliary tensor fields \{\(B_{\mu \nu}^a, \bar{B}_{\mu \nu}^a, G_{\mu \nu}^a, \bar{G}_{\mu \nu}^a\}\}, more precisely if we define the (anticommuting) transformation $\delta_s$ as

\[
\delta_s B_{\mu \nu}^a = G_{\mu \nu}^a, \\
\delta_s G_{\mu \nu}^a = 0, \\
\delta_s \bar{G}_{\mu \nu} = 0, \\
\delta_s B_{\mu \nu}^a = 0, \\
\delta_s \Psi = 0 \text{ for all other fields } \Psi,
\]  

(40)

then one can check that

\[
\delta_s^2 = 0, \\
\delta_s (S_{\text{loc}} = 0) = 0.
\]  

(41)

Let us mention for further use that, $\delta_s$ being a nilpotent operator, it possesses its own cohomology, which is easily identified with polynomials in the original Yang-Mills fields \{\(A_a^\mu, b^a, c^a, \tau^a\)\}. The auxiliary tensor fields, \{\(B_{\mu \nu}^a, \bar{B}_{\mu \nu}^a, G_{\mu \nu}^a, \bar{G}_{\mu \nu}^a\)\}, do not belong to the cohomology of $\delta_s$, as a consequence of the fact that they form pairs of doublets [57].

Taking a closer look upon eqns. (22), (37) and (40), one immediately verifies that

\[
s = \tilde{s} + \delta_s.
\]  

(42)

When $m \neq 0$, the action (14) is no longer $\delta_s$-invariant. Nevertheless, this $\delta_s$-symmetry can be kept if the more general action (18) is employed, when we extend the $\delta_s$-invariance to the introduced sources [57] as

\[
\delta_s U_{\mu \nu} = V_{\mu \nu}, \\
\delta_s V_{\mu \nu} = 0, \\
\delta_s U_{\mu \nu} = 0, \\
\delta_s V_{\mu \nu} = 0.
\]  

(43)

Eventually, the most general and renormalizable action (25) turns out to be compatible with the $\delta_s$-invariance too, as it should
be. This is obvious if we recognize that we can write

\[ S_{\text{gen}} = S_{\text{YM}} + S_{gf} + \delta_1 \int d^4 x \tilde{G}_i \delta^{ab} \delta_1 B_i^a \]

\[ + \delta_2 \int d^4 x \left[ (V_{\mu\nu} \tilde{G}_i - \overline{U}_{\mu\nu} B_i^a) F_{\mu\nu} + \chi \overline{U}_{\mu\nu} \partial^2 V_{\mu\nu} \right] + \chi_2 \overline{U}_{\mu\nu} \partial_\alpha V_{\nu\alpha} - \xi (\overline{U}_{\mu\nu} V_{\mu\nu} V_{j\alpha\beta} V_{j\alpha\beta} - \overline{U}_{\mu\nu} V_{\mu\nu} \overline{U}_{j\alpha\beta} U_{j\alpha\beta}) \]

\[ + \delta_2 \int d^4 x \left[ \lambda_1 (B_i^a \tilde{G}_i^b (V_{\mu\nu} U_{\mu\nu} - \overline{U}_{\mu\nu} U_{\mu\nu})) + \lambda_2 (B_i^a \tilde{G}_i^b (B_i^a V_{\mu\nu} - \overline{G}_i^a U_{\mu\nu}))) \right.

\[ - \frac{1}{2} \lambda_3 (B_i^a \overline{U}_{\mu\nu} (G_i^a U_{\mu\nu} + B_i^a V_{\mu\nu})) + \frac{1}{2} \lambda_3 (\overline{G}_i^a U_{\mu\nu} (\overline{G}_i^a U_{\mu\nu} - B_i^a V_{\mu\nu})) \right], \]

(44)

and invoke the nilpotency of \( \delta_i \).

What happens when we return to the physical action \( \tilde{S} \)? Clearly, the \( \delta_i \)-invariance is broken as \( \lambda_0 \) and \( \lambda_2 \) are incompatible. The presence of the mass \( m \) thus breaks the supersymmetry \( \delta_i \). As a consequence, by keeping \( \lambda_0 \) in mind for the fields, the BRST transformation \( s \) is lost too. Fortunately, we recover another BRST invariance \( \tilde{s} \), for the physical action. We shall come back to the relevance and use of the \( \delta_i \)-supersymmetry in section IV.

E. Intermediate conclusion.

The classical gauge invariant action \( S_{\text{cl}} \) can be quantized in the linear covariant gauges, whereby a nilpotent BRST symmetry and renormalizability to all orders of perturbation theory are present. The most famous gauge models exhibiting renormalizability with the possibility of massive gauge bosons are of course those based on the Higgs mechanism, which is related to a spontaneous gauge symmetry breaking [64, 65, 66, 67, 68]. Few other Yang-Mills models exhibiting mass terms for the gauge bosons exist. We mention those based on the Stueckelberg formalism, which give rise to a nonpolynomial action in the extra Stueckelberg fields. However, these models lack renormalizability [44, 45]. Other models are based on the works [63, 64] by Curci and Ferrari. Although the resulting models are renormalizable, they do not have a classical gauge invariant counterpart, since the mass terms that are allowed/needed by renormalizability are not gauge invariant terms. Typically, the mass term is of the form \( \frac{1}{2} m^2 (A^{a\mu} A^{a}_\mu + \alpha F^{a\mu \nu} F^{a}_\mu \nu) \), where \( \alpha \) is the gauge parameter [68]. Next to the Curci-Ferrari gauges, the special case of the Landau gauge, corresponding to taking the limit \( \alpha = 0 \) for the Curci-Ferrari gauge parameter, and the maximal Abelian gauges can also be used to build up such renormalizable massive models. Unfortunately, these models have the problem of being not unitary [63, 70], a fact related to the lack of a nilpotent BRST transformation [69]. Nevertheless, in the past few years a lot of interest arose in these dimension two operators from the viewpoint of massless Yang-Mills theories quantized in a specific gauge. As these operators turn out to be renormalizable to all orders of perturbation theory in the specific gauge chosen [15, 47, 50], a consistent framework can be constructed to investigate the condensation of these renormalizable albeit non gauge invariant operators [16, 17, 18, 21, 46, 49, 50]. This has resulted in a dynamical mass generating mechanism in gauge fixed Yang-Mills theories [11, 16, 17, 19, 21, 46, 49, 50].

III. TWO LOOP CALCULATIONS.

We now detail the actual computation of the two loop anomalous dimension of the fields and the one loop \( \beta \)-function of the tensor coupling \( \lambda^{abcd} \). In order to deduce the renormalization group functions, there are two possible ways to proceed. One is to regard the extra gluon mass operator as part of the free Lagrangian and work with completely massive gluon and localizing ghosts throughout. It transpires that this would be extremely tedious for various reasons. First, although the propagators will be massive there will be a 2-point mixing between the gluon and localizing ghosts leading to a mixed propagator. Whilst it is possible to handle such a situation, as has recently been achieved in a similar localization [71], it requires a significantly large number of Feynman diagrams to perform the full renormalization. Moreover, one needs to develop an algorithm to systematically integrate massive Feynman diagrams where the masses are in principle all divergent. Although algorithms have been developed for similar but simpler renormalizations, we do not pursue this avenue here mainly because the extra effort for this route is not necessary given that there is a simpler alternative. This is to regard the mass operator as an insertion and split the Lagrangian into a free piece involving massless fields with the remainder being transported to the interaction Lagrangian. Hence to renormalize the operator will involve its insertion into a massless Green function, after the fields and couplings have been renormalized in the massless Lagrangian. This is possible since it has been demonstrated that the ultraviolet structure of the renormalization
constants remain unchanged in $\overline{\text{MS}}$ whether the gluon mass operator is present or not \cite{40}. Moreover, given that the massless field approach is simpler and more attractive, we can use the \textsc{Mincer} algorithm to perform the actual computations. This algorithm, \cite{72}, written in the symbolic manipulation language \textsc{FORM}, \cite{73,74}, is devised to extract the divergences from massless $2$-point functions. Therefore, it is ideally suited to deduce the anomalous dimensions of the fields. Hence we note that for the computations the propagators of the massless fields in an arbitrary linear covariant gauge are, \cite{40},

\[
\langle A_{\mu}^a(p)A_{\nu}^b(-p) \rangle = - \frac{\delta^{ab}}{p^2} \left[ \delta_{\mu\nu} - (1 - \alpha) \frac{p_{\mu}p_{\nu}}{p^2} \right],
\]

\[
\langle c^a(p)c^b(-p) \rangle = \frac{\delta^{ab}}{p^2}, \quad \langle \psi(p)\bar{\psi}(-p) \rangle = \frac{\eta}{p^2},
\]

\[
\langle B_{\mu\nu}^a(p)\bar{B}_{\alpha\beta}^{\alpha\beta}(-p) \rangle = - \frac{\delta^{ab}}{2p^2} \left[ \delta_{\sigma\rho}\delta_{\nu\sigma} - \delta_{\nu\rho}\delta_{\sigma\sigma} \right],
\]

\[
\langle G_{\mu\nu}^a(p)G_{\alpha\beta}^{\alpha\beta}(-p) \rangle = - \frac{\delta^{ab}}{2p^2} \left[ \delta_{\sigma\rho}\delta_{\nu\sigma} - \delta_{\nu\rho}\delta_{\sigma\sigma} \right],
\]

(45)

where $p$ is the momentum. Using \textsc{QGraf}, \cite{75}, to generate the two loop Feynman diagrams we have first checked that the \textit{same} two loop anomalous dimensions emerge for the gluon, Faddeev-Popov ghost and quarks in an arbitrary linear covariant gauge as when the extra localizing ghosts are absent. This is primarily due to the fact that the $2$-point functions of the fields do not involve any extra ghosts except within diagrams. Then they appear with equal and opposite signs due to the anticommutativity of the $G_{\mu\nu}$ ghosts and hence cancel. This observation shall be given an explicit proof in section IV. We thus note that the expressions obtained for the renormalization group functions are the same as the two loop $\overline{\text{MS}}$ results of \cite{3,4,76,77}. For the localizing ghosts there is the added feature that the properties of the $\lambda^{abcd}$ couplings have to be used, as specified in (27) and (28). We have implemented these properties in a \textsc{FORM} module. However, we note that in the renormalization of both localizing ghosts, we have assumed that

\[
\lambda^{acde}\lambda^{bcde} = \frac{1}{N_A} \delta^{ab}\lambda^{pqrs}\lambda^{pqrs}, \quad \lambda^{acde}\lambda^{bdce} = \frac{1}{N_A} \delta^{ab}\lambda^{pqrs}\lambda^{pqrs},
\]

(46)

which follows from the fact that there is only one rank two invariant tensor in a \textit{classical} Lie group. If this is not satisfied then one would require a $2$-point counterterm involving the $\lambda^{abcd}$ couplings which was not evident in the algebraic renormalization technology which established the renormalizability of the localized operator. Hence, at two loops in $\overline{\text{MS}}$ we find that

\[
\gamma_{\mu}(a, \lambda) = \frac{\gamma_{C}(a, \lambda)}{N_A} = (2\alpha - 3) + \left(\frac{\alpha^2}{4} + 2\alpha - \frac{61}{6}\right) C_A^2 + \frac{10}{3} T_f N_f \alpha^2 + \frac{1}{128N_A} \lambda^{abcd}\lambda^{acbd},
\]

(47)

where $N_A$ is the dimension of the adjoint representation of the colour group, $\alpha = g^2/(16\pi^2)$ and we have also absorbed a factor of $1/(4\pi)$ into $\lambda^{abcd}$ here and in later anomalous dimensions. These anomalous dimensions are consistent with the general observation that these fields must have the \textit{same} renormalization constants, in full agreement with the output of the Ward identities \cite{40}. In order to verify that (47) is in fact correct, we have renormalized the $3$-point gluon $B_{\mu\nu}^a$ vertex. Since the coupling constant renormalization is unaffected by the extra localizing ghosts (and we have checked this explicitly by renormalizing the gluon quark vertex), then we can check that the \textit{same} gauge parameter independent coupling constant renormalization constant emerges from gluon $B_{\mu\nu}^a$ vertex. Computing the $7$ one loop and $166$ two loop Feynman diagrams it is reassuring to record that the vertex is finite with the already determined two loop $\overline{\text{MS}}$ field and coupling constant renormalization constants. Prior to considering the operator itself, we need to determine the one loop $\beta$-function for the $\lambda^{abcd}$ couplings. As this is present in a quartic interaction it means that to deduce its renormalization constant, we need to consider a $4$-point function. However, in such a situation the \textsc{Mincer} algorithm is not applicable since two external momenta have to be nullified and this will lead to spurious infrared infinities which could potentially corrupt the renormalization constant. Therefore, for this renormalization only, we have resorted to using a temporary mass regularization introduced into the computation using the algorithm of \cite{78} and implemented in \textsc{FORM}. Consequently, we find the gauge parameter independent renormalization

\[
\lambda_{\rho}^{abcd} = \lambda^{abcd} - \frac{1}{8} \left[ \lambda^{abpq}\lambda^{cdpq} + \lambda^{abpq}\lambda^{cdpq} + \lambda^{apcq}\lambda^{bpdq} + \lambda^{apdq}\lambda^{bpqc} \right] - 6C_A \lambda^{abcd} a + 4C_A f^{abp} f^{cdp} a^2 + 8C_A f^{abp} f^{bcp} a^2 + 48d^{abcd}_A a^2 \frac{1}{\varepsilon},
\]

(48)

from both the $\lambda^{abcd} B_{\mu\nu}^a B_{\alpha\beta}^{\alpha\beta} B_{\gamma\delta}^{\gamma\delta} G_{\epsilon\zeta}^{\epsilon\zeta}$ and $\lambda^{abcd} B_{\mu\nu}^a B_{\alpha\beta}^{\alpha\beta} C_{\rho\sigma}^{\rho\sigma} G_{\epsilon\zeta}^{\epsilon\zeta}$ vertices where $d^{abcd}_A$ is the totally symmetric rank four tensor defined by

\[
d_f^{abcd} = \text{Tr} \left( T_f^{a} T_f^{b} T_f^{c} T_f^{d} \right),
\]

(49)
with $T_A^a$ denoting the group generator in the adjoint representation. Dimensional regularization in $d = 4 - 2\epsilon$ dimensions is used throughout this paper. Producing the same expression for both these 4-point functions, aside from the gauge independence, is a strong check on their correctness as well as the correct implementation of the group theory. Unlike for the gauge coupling and its $\beta$-function, the $\lambda^{abcd}$ $\beta$-function contains terms also involving the gauge coupling $g^2$ at one loop. Hence, to one loop $\beta^{abcd}_\lambda(a, \lambda)$ is given by

$$
\beta^{abcd}_\lambda(a, \lambda) = \left[ \frac{1}{4} (\lambda^{abpq} \epsilon^{cdpq} + \lambda^{apbq} \epsilon^{cdpq} + \lambda^{aqpc} \epsilon^{bdpq} + \lambda^{aqpd} \epsilon^{bcpq}) \\
- 12C_A \lambda^{abcd} a + 8C_A f^{abpq} f^{cdpq} a^2 + 16C_A f^{adpq} f^{bcpq} a^2 + 96\epsilon \lambda^{abcd} a^2 \right],
$$

such that in $d$ dimensions

$$
\mu \frac{\partial}{\partial \mu} \lambda^{abcd} = -2\epsilon \lambda^{abcd} + \beta^{abcd}.
$$

Another useful check is the observation that $\beta^{abcd}$ enjoys the same symmetry properties as the tensor $\lambda^{abcd}$, summarized in (28).

It is worth noticing that $\lambda^{abcd} = 0$ is not a fixed point due to the extra $\lambda^{abcd}$-independent terms. Put another way, if we had not included the $\lambda^{abcd}$-interaction term in the original Lagrangian, then such a term would be generated at one loop through quantum corrections, meaning that in this case there would have been a breakdown of renormalizability. Further, we note that with the presence of the extra couplings, the two loop term of this $\beta$-function is actually scheme dependent.

Finally, we turn to the two loop renormalization of the localized operator itself, or equivalently of the mass $m$. The operator can be read off from (15) and is given by

$$
O = (B^\nu_{\mu \nu} - B^\mu_{\nu \nu}) F^a_{\mu \nu}.
$$

To do this we extend the one loop calculation, (40), by again inserting this operator into a $A_{\mu}^a B_{\nu \sigma}^b$ 2-point function and deducing the appropriate renormalization constant $Z_O$, defined by

$$
O_o = Z_O O.
$$

One significant advantage of the massless field approach is that there is no mixing of this dimension three operator into the various lower dimension two operators, which was evident in the algebraic renormalization analysis, and would complicate this aspect of the two loop renormalization. In other words following the path of using massive propagators would have required us to address this mixing issue. Hence, from the 5 one loop and 131 two loop Feynman diagrams, we find the MS renormalization constant

$$
Z_O = 1 + \left[ \frac{2}{3} T_F N_f - \frac{11}{6} C_A \right] \frac{a}{\epsilon} \\
+ \left[ \frac{121}{24} C_A^2 + \frac{2}{3} T_F N_f^2 - \frac{11}{3} T_F N_f C_A \right] \frac{a^2}{\epsilon^2} + \left[ \left( \frac{1}{3} T_F N_f C_A - \frac{77}{48} C_A^2 + T_F N_f C_F \right) \frac{a^2}{\epsilon^2} + \frac{1}{512 N_A} \lambda^{abcd} \lambda^{abcd} - \frac{1}{32 N_A} f^{abe} f^{cde} \lambda^{adbc} a \right] \frac{1}{\epsilon},
$$

and therefore,

$$
\gamma_O(a, \lambda) = -2 \left[ \frac{2}{3} T_F N_f - \frac{11}{6} C_A \right] a - \left( \frac{4}{3} T_F N_f C_A + 4 T_F N_f C_F - \frac{77}{12} C_A^2 \right) \frac{a^2}{\epsilon^2} + \frac{1}{8 N_A} f^{abe} f^{cde} \lambda^{adbc} a - \frac{1}{128 N_A} \lambda^{abcd} \lambda^{abcd},
$$

as the two loop MS anomalous dimension, which is defined as (40)

$$
\gamma_O(a, \lambda) = \mu \frac{\partial}{\partial \mu} \ln Z_O.
$$

As at one loop it is independent of the gauge parameter, as expected from the fact that the operator is gauge invariant. Also, the two loop correction depends on the $\lambda^{abcd}$ couplings as well as the gauge coupling, as expected from our earlier arguments. We end this section by mentioning that a factor of $(-2)$ was erroneously omitted in the one loop anomalous dimension $\gamma_O(a)$ in eq.(6.9) of (40).
IV. EQUIVALENCE BETWEEN THE MASSLESS THEORY AND USUAL YANG-MILLS THEORY.

In this section, we shall discuss the usefulness of the $\delta_s$-supersymmetry, defined by \(40\), and show that Green functions which are built from the original Yang-Mills fields \(\{A_\mu^a, c^a, \bar{c}^a, b^a\}\) are independent from the tensor coupling $\lambda^{abcd}$, in the massless version of the physical model \(43\).

Next to this result, we shall also prove the stronger result that any Green function constructed from the original Yang-Mills fields \(\{A_\mu^a, c^a, \bar{c}^a, b^a\}\), evaluated with respect to the massless version of our action, gives the same result as if the Green function would be evaluated with the original Yang-Mills action.

A. The massless case $m \equiv 0$.

As we have already noticed in \(13\), the case corresponded originally to the introduction of a unity into the usual Yang-Mills action. Evidently, we expect that the model obtained with $m \equiv 0$ would be exactly equivalent to ordinary Yang-Mills theory. Nevertheless, this statement is a little less clear if we take a look at the massless action

\[
S^\text{phys}_{\mu=0} = S_{\text{YM}} + S_{gf} + \int d^4x \left[ \frac{1}{4} \left( T^a_{\mu\nu} D^b_{\sigma} D^c_{\sigma} B^{\nu b} - \bar{c}^a_{\sigma} D^b_{\sigma} G^c_{\nu} \right) + \frac{\lambda^{abcd}}{16} \left( T^a_{\mu\nu} B^b_{\nu\mu} - \bar{c}^a_{\mu} G^b_{\nu} \right) \left( T^c_{\rho\sigma} B^d_{\rho\sigma} - \bar{G}^d_{\rho\sigma} G^c_{\rho\sigma} \right) \right],
\]

(57)

which is obtained from \(43\). The reader shall notice that the quartic interaction \(\propto \lambda^{abcd}\) is anyhow generated, making the path integration over the tensor fields no longer an exactly calculable Gaussian integral. Hence, we could worry about the fact the tensor coupling $\lambda^{abcd}$ might enter the expressions for the Yang-Mills Green functions, which are those built out of the fields $A_\mu^a$, $c^a$, $\bar{c}^a$ and $b^a$, when the partition function corresponding to the action \(57\) would be used.

We recall here that the action \(57\) is invariant under the supersymmetry \(40\). This has its consequences for the Green functions. Let us explore this now. Firstly, we consider a generic $n$-point function built up only from the original fields \(\{A_\mu^a, c^a, \bar{c}^a, b^a\}\). More precisely, we set

\[
G_n(x_1, \ldots, x_n) = \left< G_n(x_1, \ldots, x_n) \right>_{\text{phys}}^\text{equiv}
= \int D\Phi G_n(x_1, \ldots, x_n) e^{-S^\text{phys}_{\mu=0}},
\]

(58)

with

\[
G_n(x_1, \ldots, x_n) = A(x_1) \ldots A(x_n) \bar{c}(x_1) \bar{c}(x_2) \ldots b(x_{k+1}) \ldots b(x_n),
\]

(59)

and we introduced the shorthand notation $\Phi$ denoting all the fields. We notice that $\delta_s G_n = 0$ whereas $G_n \neq \delta_s (\ldots)$, i.e. any functional of the form \(59\) belongs to the $\delta_s$-cohomology.

We are interested in the dependence of $G_n$ on $\lambda^{abcd}$. A small computation leads to

\[
\frac{\partial G_n}{\partial \lambda^{abcd}} = -\frac{1}{16} \int D\Phi G_n(x_1, \ldots, x_n) \int d^4x \left[ \left( T^a_{\mu\nu} B^b_{\nu\mu} - \bar{c}^a_{\nu} G^b_{\mu} \right) \left( T^c_{\rho\sigma} B^d_{\rho\sigma} - \bar{G}^d_{\rho\sigma} G^c_{\rho\sigma} \right) \right] e^{-S^\text{phys}_{\mu=0}}
= -\frac{1}{16} \int D\Phi \delta_s \left( G_n(x_1, \ldots, x_n) \int d^4x \left[ \left( T^a_{\mu\nu} B^b_{\nu\mu} - \bar{c}^a_{\nu} G^b_{\mu} \right) \left( T^c_{\rho\sigma} B^d_{\rho\sigma} - \bar{G}^d_{\rho\sigma} G^c_{\rho\sigma} \right) \right] \right) e^{-S^\text{phys}_{\mu=0}}
= -\frac{1}{16} \left< \delta_s \left( G_n(x_1, \ldots, x_n) \int d^4x \left[ \left( T^a_{\mu\nu} B^b_{\nu\mu} - \bar{c}^a_{\nu} G^b_{\mu} \right) \left( T^c_{\rho\sigma} B^d_{\rho\sigma} - \bar{G}^d_{\rho\sigma} G^c_{\rho\sigma} \right) \right] \right) \right>_{\text{phys}}^\text{equiv}
= 0.
\]

(60)

The last line of \(60\) is based on the fact that $\delta_s$ annihilates the vacuum as it generates a symmetry of the model.

We have thus shown that all original Yang-Mills Green functions will be independent of the tensor coupling $\lambda^{abcd}$. These are of course the most interesting Green functions, gauge variant (like the gluon propagator) or invariant (the physically relevant Green functions). The previous result does not mean that we can simply set $\lambda^{abcd} \equiv 0$ and completely forget about the quartic interaction \(\propto \left( T^a_{\mu\nu} B^b_{\nu\mu} - \bar{c}^a_{\nu} G^b_{\mu} \right) \left( T^c_{\rho\sigma} B^d_{\rho\sigma} - \bar{G}^d_{\rho\sigma} G^c_{\rho\sigma} \right)\). There is a slight complication, as quantum corrections reintroduce the quartic interaction \(\propto \lambda^{abcd}\) even when we set $\lambda^{abcd} = 0 \ [91]$. 

### B. Exact equivalence between the massless action and the Yang-Mills action.

In this subsection, we shall use once more the nilpotent $\delta$ symmetry to actually prove that

$$\langle G_n(x_1, \ldots, x_n) \rangle_{S_{YM}+S_\delta} = \langle G_n(x_1, \ldots, x_n) \rangle_{S_{phys}} ,$$  \hspace{1cm} (61)

meaning that the expectation value of any Yang-Mills Green function, constructed from the fields $\{A^a_{\mu}, e^a, \mathcal{T}^a, b^a\}$ and calculated with the original (gauge fixed) Yang-Mills action $S_{YM} + S_\delta$, is identical to the one calculated with the massless action $S_{phys}$, where it is tacitly assumed that the gauge freedom of both actions has been fixed by the same gauge fixing.

In this context, let us also mention that the physical content of the massless theory described by $S_{phys}$ will not depend on the extra tensor fields, as $(B_{\mu\nu}^a, G_{\mu\nu}^a)$ and $(\mathcal{B}_{\mu\nu}, \mathcal{G}_{\mu\nu})$ are both $\delta$-doublets, and hence any physical operator shall certainly not depend on these fields. Physical operators belong to the $\delta$-cohomology, which is independent of $\delta$-doublets\(^{57}\). More precisely, Yang-Mills theory and the massless action\(^{57}\) will have identical physical operators, which belong to the BRST $\delta$-cohomology.

Next to this, we also know that the extra fields of the massless model shall decouple from the physical spectrum as they belong to the trivial part of the additional $\delta$-cohomology.

Let us now prove the statement (61). We shall first prove the following.

**Theorem I** Let $S_0$ be an action constructed from a set of fields $\phi_i$ that enjoys a symmetry generated by a nilpotent operator $\delta$. Consider a second action $S_1 = S_0 + \Delta S$, whereby $\Delta S$ is constructed from the fields $\phi_i$, and an extra set $(\phi_k, \overline{\phi}_k)$ whereby $\phi_k$ and $\overline{\phi}_k$ are $\delta$-doublets, such that $S_1$ also enjoys the symmetry generated by $\delta$. We assume that the renormalizability of $S_0$ and $S_1$ has been established.

The physical operators of $S_0$ and $S_1$ both belong to the $\delta$-cohomology, whereas the extra fields $(\phi_k, \overline{\phi}_k)$ do not. This is due to the fact that these fields give rise to pairs of $\delta$-doublets, thus having nonvanishing cohomology. The difference $\Delta S = S_1 - S_0$ is then also necessarily $\delta$-exact. Let $\mathcal{H}$ be an operator belonging to the $\delta$-cohomology. Then we can write

$$\langle \mathcal{H} \rangle_{S_1} = \int d\phi_i d\phi_k \mathcal{H} e^{-S_0 - \Delta S} .$$  \hspace{1cm} (62)

We define a new action

$$S_\kappa = S_0 + \kappa \Delta S ,$$  \hspace{1cm} (63)

where $\kappa$ is a global\(^{92}\) parameter put in front of the action-part $\Delta S$. If we multiply the counterterm part, corresponding to $\Delta S$, which belongs to a trivial part of the $\delta$-cohomology, by $\kappa$, then the renormalizability will be maintained, without the need of introducing a counterterm for $\kappa$, so that $\kappa_0 \equiv \kappa$. The parameter $\kappa$ can be used to switch on/off the difference $\Delta S$.

More precisely, we can interpolate continuously between $S_0$ and $S_1$. Using this, it is not difficult to show that

$$\frac{\partial}{\partial \kappa} \langle \mathcal{H} \rangle_{S_\kappa} = 0 ,$$  \hspace{1cm} (64)

due to the $\delta$-exactness of $\Delta S$ and $\delta$-closedness of $\mathcal{H}$. As a consequence

$$\langle \mathcal{H} \rangle_{S_0} = \langle \mathcal{H} \rangle_{S_1} .$$  \hspace{1cm} (65)

A related theorem is the following.

**Theorem II** Let $S_0$ be an action constructed from a set of fields $\phi_i$. Consider a second action $S_1 = S_0 + \Delta S$, whereby $\Delta S$ is constructed from the fields $\phi_i$, and an extra set $(\phi_k, \overline{\phi}_k)$ whereby $\phi_k$ and $\overline{\phi}_k$ are $\delta$-doublets, such that $S_1$ enjoys the symmetry generated by the nilpotent operator $\delta$. We trivially extend the action of $\delta$ on the fields $\phi_i$ as $\delta \phi_i = 0$, so that of course $\delta S_0 = 0$. We assume that the renormalizability of $S_0$ and $S_1$ has been established.

The difference $\Delta S = S_1 - S_0$ is necessarily $\delta$-exact. Moreover, the physical operators of $S_1$ must belong to the $\delta$-cohomology, which is independent of the $\delta$-doublets $(\phi_k, \overline{\phi}_k)$. The operators constructed from the fields $\phi_i$, i.e. all operators of the model $S_0$, certainly belong to the $\delta$-cohomology of $S_1$. Let $\mathcal{K}$ be such an operator. A completely similar argument as used in Theorem I allows to conclude that

$$\langle \mathcal{K} \rangle_{S_0} = \langle \mathcal{K} \rangle_{S_1} .$$  \hspace{1cm} (66)
Let us now comment on the usefulness of the previous theorems. Theorem II is applicable to the Yang-Mills action \( S_{YM} + S_{ef} \) and the massless action \( S_{phys}^{m=0} \), where \( \delta_\ell \) is the nilpotent symmetry generator of \( S_{phys}^{m=0} \), trivially acting on \( S_{YM} + S_{ef} \). Said otherwise, we have just proven that the massless physical model \( (52) \) is equivalent with Yang-Mills, in the sense that the Green functions of the original Yang-Mills theory remain unchanged when evaluated with the action \( (52) \).

An important corollary of the previous result is that the running of the gauge coupling \( g^2 \) will be dictated by the usual \( \beta(g^2) \)-function known from common literature, as the renormalization factor for \( g^2 \) can be extracted from original Yang-Mills \( n \)-point Green functions. This result was confirmed in section III, as well as the fact that the other Yang-Mills renormalization group functions remain unaltered, again in agreement with Theorem II.

C. The massive case \( m \neq 0 \).

It might be clear that the \( \delta_\ell \)-supersymmetry was the key tool to prove the \( \lambda^{abcd} \) independence in the massless case, as well as the equivalence with Yang-Mills theory.

Evidently, we are more interested in the case that the mass \( m \) is present. The question arises what we may say in this case, as the supersymmetry is now explicitly broken, i.e.

\[
\delta_\ell S_{phys} \neq 0,
\]

with the massive physical action given in \( (4) \). The role of the tensor fields \( \{ B^a_{\mu
u}, F^a_{\mu
u}, G^a_{\mu
u}, \tau^a_{\mu
u} \} \) remains an open question, as they do not longer constitute a pair of doublets. But, we repeat, gauge or BRST invariance is kept, at both the classical and quantum level.

V. CONCLUSION AND OUTLOOK.

Returning to the rationale behind the construction of the action \( (4) \), we recall that is was based on the localization procedure, given in \( (2) \), of the nonlocal operator coupled to the Yang-Mills action, as displayed in \( (7) \). Clearly, we can return from the local physical action to the original nonlocal one only if the extra couplings \( \lambda_1, \lambda_3 \) and \( \lambda^{abcd} \) would be zero. As already explained, even if we set these couplings equal to zero from the beginning, quantum corrections will reintroduce their corresponding interactions.

Moreover, as the massive physical action is not \( \delta_\ell \)-supersymmetric, we cannot simply say that we can choose the extra couplings freely. Trying to get as close as possible to the original localized version \( (15) \) of the nonlocal action \( (7) \) where only one coupling constant is present, we can imagine taking the tensor coupling \( \lambda^{abcd} \) to be of the form

\[
\lambda^{abcd} = \ell_0^{abcd} g^2 + \ell_1^{abcd} g^4 + \ldots,
\]

i.e. we make it a series in the gauge coupling \( g^2 \), where the coefficients \( \ell_0^{abcd} \) are constant color tensors with the appropriate symmetry properties as \( (27) \) and \( (28) \). This already avoids the introduction of an independent coupling. We have temporarily reintroduced the Planck constant \( \hbar \) to make clear the distinction between classical and quantum effects. At the classical level, we could try to set \( \ell_0^{abcd} = 0 \) in order to kill the quartic interaction. Doing so, we would at least keep the classical equivalence between the local and nonlocal actions \( (4) \) and \( (7) \) by employing the classical equations of motion. Unfortunately, this is not possible. We should assure that \( (53) \) is consistent with the quantum model, meaning that we should assure the consistency with the renormalization group equations. Taking the quantum effects into account, next to the divergent contributions canceled by the available counterterms, there will be also quantum corrections to the quartic interaction which are finite but nonvanishing in \( \hbar g^2 \). We can fix the classical (tree level) value \( \ell_0^{abcd} \) by demanding that the proposal \( (68) \) is consistent with the renormalization group function \( (51) \), and likewise for the higher order coefficients \( \ell_i^{abcd} \), i.e. we should solve

\[
\frac{\partial}{\partial g^2} \lambda^{abcd}(g^2) = \beta(g^2), \quad \frac{\partial}{\partial g^2} \lambda^{abcd}(g^2) = \beta^{abcd}(g^2),
\]

order by order, with \( \lambda^{abcd}(g^2) \) defined as in \( (68) \). By using the renormalization group function \( (50) \) it is apparent that \( \ell_0^{abcd} = 0 \) is not a solution.

It is interesting to notice that the classical action should already contain the classical quartic coupling \( \lambda_0^{abcd} = \ell_0^{abcd} g^2 \) in order to allow for a consistent extension of the model at the quantum level. This classical value is in fact dictated by one loop quantum effects, as it is clear from the lowest order term of \( (52) \).

A completely similar approach could be used for the mass couplings \( \lambda_1 \) and \( \lambda_3 \). We could eliminate the extra couplings in favour of the gauge coupling, without making any sacrifice with respect to the renormalization group equations. Of course, this is a nontrivial task, as it should be checked whether for instance \( (52) \) possesses meaningful solution(s), as the coefficient tensors \( \ell_i^{abcd} \) should be at least realvalued, whereas the uniqueness of the solution might be not evident.
The aforementioned procedure is not new, as we became aware of works like [81, 82] and references therein, where the reduction of couplings and its use were already studied.

In this paper, the role of the extra couplings was not the primary motivation. The main purpose of this paper was to establish a further study of the gauge model (4) itself. We have reported on a few properties. We mentioned the existence of a nilpotent BRST symmetry of the physical action (4) and we have commented on the fact that the physical action can be embedded into a more general action that possesses an interesting supersymmetry amongst the new fields. We briefly returned to the renormalizability, in particular on the algebraic renormalization of the extra tensor coupling $\lambda^{abcd}$ where we drew an analogy with, for instance the Higgs inspired Coleman-Weinberg model. The physical action itself only enjoys the supersymmetry in the massless limit, in which case we were able to prove the equivalence with ordinary Yang-Mills theory by using an argument based on the cohomology of that supersymmetry. We also presented explicit results concerning the renormalization group functions of the model evaluated in the $\overline{\text{MS}}$ scheme: the anomalous dimensions of the original Yang-Mills fields and parameters were calculated to two loop order and turned out to be identical to the ones calculated in massless Yang-Mills, supplemented with the same gauge fixing, in agreement with the general argument that both models are equivalent in the massless limit. The one-loop anomalous dimension of the tensor coupling $\lambda^{abcd}$ has also been evaluated, giving explicit evidence that $\lambda^{abcd} = 0$ is not a fixed point of the model, and finally also the two loop anomalous dimensions of the auxiliary tensor fields as well as of the gauge invariant operator $O = (B - \overline{B})F$, given in [52], or equivalently of the mass $m$, have been calculated.

An interesting issue to focus on in the future would be the possible effects on the gluon Green functions arising from the massive physical model (4), to find out whether mass effects might occur in the gluon sector as, for instance, in the gluon propagator.

Returning to the necessity of introducing extra couplings, since we cannot keep the equivalence between actions (7) and (4) due to these couplings, the relation with the nonlocal gauge invariant operator $F = \frac{1}{2} F_{\mu\nu} F^{\mu\nu}$ has become obscured. However, this is not an unexpected feature, as this operator is highly nonlocal, due to the presence of the inverse of the covariant Laplacian. Nevertheless, we believe that the final model (4) is certainly relevant per se. To some extent, it provides a new example of a renormalizable massive model for Yang-Mills theories, which is gauge invariant at the classical level and when quantized it enjoys a nilpotent BRST symmetry.

There are several other remaining questions concerning our model. At the perturbative level for example, it could be investigated which (asymptotic) states belong to a physical subspace of the model, and in addition one should find out whether this physical subspace can be endowed with a positive norm, which would imply unitarity. Although the resolution of this topic is under study, it is worth remarking that the nilpotency of the BRST operator might be useful in this context.

Of course, at the nonperturbative level, not much can be said at the current time. The model is still asymptotically free, implying that at low energies nonperturbative effects, such as confinement, could set in. One could search for indications of confinement, similarly as it is done for usual Yang-Mills gauge theories. An example of such an indication is the violation of the spectral positivity, see e.g. [21, 82, 83]. Proving and understanding the possible confinement mechanism in our model is probably as difficult as for usual Yang-Mills gauge theories.

Finally, it would be interesting to find out whether this model might be generated dynamically. A possibility would be to start from the massless version of the action (4), which was written down in [52]. After all, it is equivalent with massless Yang-Mills, and we can try to investigate whether a nonperturbative dynamically generated term $m(B - \overline{B})F$ might emerge, which in turn could have influence on the gluon Green functions.

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[86] Bare quantities are denoted with a subscript “o”.

[87] In fact $\delta \equiv s, \bar{s} \equiv 0$ for the set of sources $\{U_{\mu\nu}, \overline{U}_{\mu\nu}, V_{\mu\nu}, \overline{V}_{\mu\nu}\}$. These sources are gauge (BRST $\bar{s}$) singlets.

[88] When $m \equiv 0$, these models are massless Yang-Mills theories fixed in the Curci-Ferrari gauge with $\alpha$ the associated gauge parameter.

[89] There is a BRST symmetry, but it is not nilpotent.

[90] We shall not consider matter (spinor) fields in this section, although the derived results remain valid.

[91] This could also be reinterpreted by stating that the bare coupling $\lambda_{ab0d}$ is not proportional to the renormalized coupling $\lambda_{abcd}$.

[92] It is tacitly assumed here that $\Delta S$ is, disjoint from $S_0$, invariant with respect to other possible symmetries. If not, we can evidently not introduce such a global factor $\kappa$. 

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