Influence of the Foam Morphology on the Mechanical Behavior of Flow-Through Foam Filters During Filtration Processes

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1. Introduction

Open-cell foams can be used as filters, for example, for the filtration of molten metals\cite{1,2} as catalysts\cite{3} for burners or carriers for biofilters. In all these and related applications, the foams are subjected to forces emanating from the fluid flowing through them. To ensure the integrity of the foams during the flow or filtration process an evaluation of the strength of such structures is necessary. Mechanical tests of foams under a load of fluids can be complicated or even impossible, for example, if the fluid is a very hot liquid metal. The macroscopic properties of foams are determined by the bulk material properties and by the topology and morphology of the porous structure.\cite{4,5} To assess the effective properties of foams often numerical models using representative volume elements (RVEs) are utilized. RVEs are usually spatial periodic structures often having special arrangements of pores derived from crystal structures. In Zhu et al.,\cite{6} an analytical model using tetraakidecahedrons (Kelvin cells) is employed to analyze the elastic properties of open-cell foams. Luxner et al.\cite{7} showed that small irregularities within such periodic structures can have quite a large influence on the effective elastic properties. Jang et al.\cite{8} investigated the compressive strength of metal foams, where buckling of the foam struts occurs for larger deformations. Buckling is not expected in ceramic foams with relative densities around 15% to 30% as they are considered in this contribution.

The generation of artificial models of foam structures with properties very similar to those of real foams is described in Abendroth et al.\cite{9} where Laguerre tessellations of periodic hard-sphere packings serve as initial topologies of soap foams. The surface energy of these foams is then relaxed utilizing Ken Brakke’s surface evolver.\cite{10} The foam skeleton (without foam faces) is mapped into a voxel grid, resulting in a strut length density field, which gets smoothed using a spatial gaussian filter. The width of the filter defined by its standard deviation influences the morphology of the filter. The outer surface of the artificial foam is finally defined by an isosurface (level-set) of the smoothed strut density field. The relative density can be controlled by the level value used for the isosurface. In Vecchio et al.,\cite{11} the strut system of the foams is obtained by binarization of CT images and subsequent smoothing procedures resulting in a segmentation, where each segment represents a foam pore. Further utilization of the already mentioned surface evolver improves the edge length and face area distributions toward realistic values of real foams.

The artificial or from CT-images derived geometric models can be meshed and analyzed using the finite element method (FEM).\cite{9,12,13} As fully resolved foam structures can have a very large number of pores or cells, the computational resources might not be sufficient to analyze them in a reasonable time. But the RVEs can be analyzed with reasonable numerical effort, even when the foam struts are meshed using 3D elements. Applying homogenization techniques, effective macroscopic properties can be derived from RVE simulations.

Despite the homogenized properties, there exist several analytical models, the most popular is probably the Deshpande–Fleck model\cite{14} describing the elastic-plastic behavior for an isotropic foam structure. Other models and the underlying theories are described by Ochtsner.\cite{13} Much work has been done to model and describes the damage and failure behavior of foam...

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structures, which is closely related to the description of the plastic behavior of foams. Here, often yield and failure surfaces\textsuperscript{[16–20]} are used to describe critical points in stress or strain space. Another application for foam RVEs is the simulation of fluid flow through it, as it happens in filtration of liquid metals or fluid flow through soils.\textsuperscript{[21]} Effective properties are also derived here, mainly the coefficients of the Darcy–Forchheimer law.\textsuperscript{[22]}

This study aims to provide a framework to assess the integrity of a certain flow-through foam structure. We concentrate on a quite specific setup to have a realistic scenario, which can be compared to some experimental results. But the general approach is not fixed to this specific problem and can be used also in a generalized manner.

It is assumed that a circular filter plate is located and fixed within a pipe, which is used to transport liquid metal from a tundish into a casting mold. The utilized filters are produced using the Schwartzwalder process.\textsuperscript{[23]} Further, this study focuses on the hydrodynamic contribution to the mechanical load while thermo-mechanical stresses are neglected. This condition is justified for filtration systems, which allow a preheating of the filter (\textsuperscript{[24–26]}) and where the melt flow rate is sufficiently high to ensure that heat losses to the surrounding have a negligible effect on the temperature distribution inside the filter. The questions to be answered are: Can the filter withstand the forces of the melt? How large are the occurring inelastic deformations after a certain time? Some parameters for this scenario are taken from publications\textsuperscript{[9,21,22,27–29]} of the collaborative research center CRC 920 (SFB 920).

In Section 2, the generation of geometric foam models is introduced, followed by Section 3 which explains the fluid and continuum mechanics involved in such a matter. Thereby a first-order homogenization procedure is utilized, where the macroscopic filter behavior is described by a polar orthotropic Mindlin plate and the (fluid-/solid-) mechanical properties of the foam are described in a smeared, averaged sense. With variations of morphological filter parameters and by application of a non-linear least square fit method an analytical model is derived, which can describe the deformation and strength of circular flow through foam structures, depending on a wide range of geometric, material, and process parameters. The model is applied to a set of different foam structures and the results are presented in Section 4. A conclusion and discussion section finalize this contribution.

2. Selected Foams

For the present study, 12 RVEs were created, using the modeling technique developed by Abendroth et al.\textsuperscript{[9]} The topology of the RVEs is derived from a Weaire–Phelan foam.\textsuperscript{[30]} Each RVE contains 8 pores. One full pore at the center, two half pores at each of the six sides, and one-eighth pore on each of the eight corners of the RVE. The RVEs are periodic in all three spatial directions. The relative density and the strut form factor are varied as \( \rho_{\text{rel}} = 15, 20, 25, 30\% \) and \( k = 0.5, 0.75, 1.0 \). As can be seen in Figure 1 an increasing relative density leads in general to thicker struts. The strut form factor \( k \) influences the thickness ratio between strut center and strut ends. Smaller values of \( k \) lead to more cylindrical struts, whereas larger values lead to struts, which are thinner at the center and thicker near the foam nodes, where always four struts meet.

3. Theory

3.1. Fluid Mechanics

The pressure gradient within a foam filter structure can be described by the Darcy–Forchheimer law\textsuperscript{[22]} as shown in Equation (1).

\[
\frac{dp}{dx} = \frac{\mu_f}{k_1} \cdot u_f + \frac{\rho_f}{k_2} \cdot u_f^2
\]

Therein, \( u_f \) is the fluid velocity, \( \mu_f \) and \( \rho_f \) are the dynamic viscosity and the density of the fluid. The parameters \( k_1 \) and \( k_2 \) depend on the geometrical properties of the foam. They are related to the dimensionless parameters \( k_1^* \) and \( k_2^* \) as shown in Equation (2), which only depend on the morphology of the foam.

\[
k_1 = k_1^* \cdot L^2; \quad k_2 = k_2^* \cdot L
\]

Thereby \( L \) is a reference length, which depends on the pore density \( \varphi \) as shown in\textsuperscript{[22]}

\[
L = \frac{1}{\sqrt{\varphi}}
\]

The porosity of foam is defined as

\[
\varepsilon = 1 - \frac{\rho}{\rho_0}
\]

and relative density of a foam is related to the porosity as

\[
\rho_{\text{rel}} = 1 - \varepsilon
\]

The two parameters \( k_1^* \) and \( k_2^* \) depend on the relative density \( \rho_{\text{rel}} \) and on a morphological factor \( k \), which describe the geometrical shape of the foam struts\textsuperscript{[9]}.

3.2. Small Strain Elasticity

In this article, only small strains are regarded. Hooke’s law for a macroscopic foam in general format relates the effective stress tensor \( \Sigma \) with the effective, elastic strain tensor \( F^\varepsilon \) using the effective fourth-order stiffness tensor \( C \) as shown in Equation (6).

\[
\Sigma = C \ast F^\varepsilon
\]

At the microscale, an isotropic material behavior at small strains is assumed

\[
\sigma_{ij} = \frac{E}{1 + \nu} \varepsilon_{ij} + \frac{\nu E}{(1 - 2\nu)(1 + \nu)} \frac{e^e_{ij}}{e^\varepsilon_{kj}} \delta_{ij}
\]

where \( \sigma \) and \( \varepsilon \) denote the local stress and strain tensors, respectively. The local elastic modulus and the Poisson’s ratio are denoted by \( E \) and \( \nu \).
3.3. Creep Deformation

At high temperatures, ceramics begin to creep. A general creep law for the rate of creep strain can be written as

$$\varepsilon_{cr} = F(\varepsilon_{cr}, \Theta, \ldots) \cdot \sigma_{eq}$$

(8)

The behavior of ceramics mainly falls into two distinct categories. Many ceramics exhibit a stress exponent $n$ close to $1$.11 A typical ceramic used for foam filters is aluminum bonded $\text{Al}_2\text{O}_3$–C. An investigation at a temperature of $\Theta \approx 1350^\circ\text{C}$ revealed a stress exponent $n = 1.06 \approx 1$.27 In addition, many ceramics have a stress exponent in the range between 3 and 5.31 In this study, only ceramics with $n \approx 1$ are regarded. It follows that the creep strain rate scales linearly with the stress. As it is hardly possible to include creep behavior in an analytical plate model, later FE plate models are used to calculate the creep deformation, and then a semi-analytical model for the maximum creep deformation is derived using a nonlinear least square method. The applied creep law in time hardening formulation reads

$$\varepsilon_{cr} = \frac{\sigma_{eq}}{a} \cdot t^m$$

(9)

where $a$ ([MPa$^{m+1}$]) and $m$ ([–]) are material parameters to be determined experimentally.
3.4. Effective Elastic Stiffness

The overall stiffness tensor \( C \) is investigated by condensing the stiffness matrix from a linear-elastic finite element foam model, using first-order homogenization. The macroscopic stress tensor is defined as

\[
\Sigma = \langle \sigma \rangle = \frac{1}{|V|} \int_V \sigma \, dV = \frac{1}{|V|} \int_{\partial V} \sigma \cdot n \, dS
\]  

(10)

To satisfy the condition that the averaged microscopic strain tensor is equal to the macroscopic strain tensor

\[
E = \langle \varepsilon \rangle \approx \frac{1}{|V|} \int_{\partial V} \varepsilon \cdot n \, dS
\]  

(11)

suitable boundary conditions are to be enforced. In this article, only periodic boundary conditions are considered, as they estimate the best overall response. The displacements of two periodic points can be formulated as

\[
u(x^+) = \nu(x^-) + E \cdot [x^+ - x^-]
\]  

(12)

The boundary of the microscopic model \( \partial V \) is split into two parts \( \partial V = \partial V^+ \cup \partial V^- \). Always two corresponding points on the boundary partitions \( x^+ \in \partial V^+ \) and \( x^- \in \partial V^- \) with oppositely oriented normal vectors \( n^- = -n^+ \) are linked together.

From the finite element model, the stiffness is condensed as

\[
[C] = \left[ \frac{d \Sigma}{d \varepsilon} \right] = \left[ \frac{\partial \Sigma}{\partial \varepsilon} \right] + \left[ \frac{\partial \Sigma}{\partial n} \right] \cdot \left[ \frac{d n}{d \varepsilon} \right]
\]  

(13)

Foams show in general an anisotropic behavior, which can in good estimation be viewed as orthotrophic, where the Voigt notation of Hooke's law is shown in Equation (14).

\[
\begin{bmatrix}
\Sigma_{11} \\
\Sigma_{22} \\
\Sigma_{33} \\
\Sigma_{12} \\
\Sigma_{13} \\
\Sigma_{23}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\gamma_{12} \\
\gamma_{13} \\
\gamma_{23}
\end{bmatrix} =
\begin{bmatrix}
E_{11} \\
E_{22} \\
E_{33} \\
2E_{12} \\
2E_{13} \\
2E_{23}
\end{bmatrix}
\]  

(14)

It shows that as a good estimation the following holds

\[
C_{11} \approx C_{22} \approx C_{33}, \quad C_{12} \approx C_{13} \approx C_{23} \quad \text{and} \quad C_{44} \approx C_{55} \approx C_{66}
\]  

(15)

which is also a special case of polar orthotropic, which will be important later when dealing with an orthotropic circular plate. For the filter problem plane stress conditions are considered as usual in plate bending theory. The condition \( \Sigma_{13} = \Sigma_{23} = 0 \) leads to \( E_{13} = E_{23} = 0 \) and finally with the condition \( \Sigma_{13} = 0 \) the stiffness is condensed as

\[
\begin{bmatrix}
\Sigma_{11} \\
\Sigma_{22} \\
\Sigma_{12} \\
\Sigma_{13}
\end{bmatrix} =
\begin{bmatrix}
[D_{11}] & [D_{12}] \\
[D_{12}] & [D_{22}]
\end{bmatrix}
\begin{bmatrix}
E_{11} \\
E_{22} \\
2E_{12} \\
E_{13}
\end{bmatrix}
\]  

(16)

with

\[
\Sigma_{33} = 0 = [D_{21}] \begin{bmatrix} E_{11} \\ E_{22} \\ 2E_{12} \\ E_{13} \end{bmatrix} + D_{22} \cdot E_{33} \rightarrow
\]

\[
E_{33} = -\frac{1}{D_{22}} [D_{21}] \begin{bmatrix} E_{11} \\ E_{22} \\ 2E_{12} \end{bmatrix}
\]  

(17)

leading to

\[
\begin{bmatrix}
\Sigma_{11} \\
\Sigma_{12} \\
\Sigma_{13}
\end{bmatrix} =
\begin{bmatrix}
[D_{11}] - \frac{1}{D_{22}} [D_{12}] [D_{21}] \\
[D_{12}] \\
[D_{21}]
\end{bmatrix}
\begin{bmatrix}
E_{11} \\
E_{22} \\
E_{13}
\end{bmatrix}
\]  

(18)

The condensed stiffness is summarized in Equation (19) and the fact was being used, that the stiffness is proportional to the Youngs modulus of the bulk material.

\[
E \cdot \begin{bmatrix}
\overline{\Sigma}_{11} \\
\overline{\Sigma}_{12} \\
\overline{\Sigma}_{13}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & 0 \\
C_{12} & C_{22} & 0 \\
0 & 0 & C_{33}
\end{bmatrix} - \frac{1}{C_{33}} \begin{bmatrix}
C_{13}^2 & C_{13}C_{23} & 0 \\
C_{13}C_{23} & C_{23}^2 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]  

(19)

The effective Poisson's ratio \( \nu_0 \) can be computed by

\[
\nu_0 = \frac{\overline{\Sigma}_{12}}{\overline{\Sigma}_{11}}
\]  

(20)

and the effective shear modulus is

\[
G_{tr} = E \cdot \overline{\Sigma}_{66}
\]  

(21)

3.5. Maximum Local Stresses

The maximum macroscopic stresses are only an average over the microscopic stress distribution. To evaluate the maximum local stresses, finite element models of the foams are used to evaluate the maximal microscopic von Mises stress. Since linear elasticity is considered, the superposition principle can be applied. From six linear independent simulations, where always only one of the effective stress components is non-zero, the local stress fields \( \sigma_i(x) \) are computed, where \( i \) denotes the Voigt index of the corresponding load case. Using the superposition principle, the local stress field can be computed using

\[
\sigma(x) = \sum_{i=1}^{6} \lambda_i \sigma_i(x)
\]  

(22)

Therein, \( \lambda_i = \Sigma_i / \Sigma_0 \) are dimensionless load factors, which define all components of the effective stress tensor and \( \Sigma_0 \) denotes a reference load, which is used for the specific non-zero
component for the six load cases. From (22) certain values used in the strength hypothesis can be computed, which can be compared to experimentally determined failure criteria. Here, we use the von Mises criterion

$$\sigma_{\text{VM}}(\mathbf{X}) = \max_{z \in \mathcal{B}} \left( \sqrt{3J_2(\mathbf{x}, \mathbf{X})} \right), \quad \text{with} \quad J_2 = \frac{1}{2} \mathbf{S} : \mathbf{S}$$

(23)

whereby \(\mathcal{B}\) is the domain of the microscopic foam model, \(\mathbf{x}\) the microscopic position vector and \(\mathbf{X}\) the macroscopic stress tensor, and \(\mathbf{S}\) its deviator.

### 3.6. Parameter Estimation

Since it is costly to generate foam models and evaluate the stiffness matrix, only 12 different foam morphologies are investigated as shown in Chapter 2. The foam parameters that influence the filter deflection are \(L\) (which does not depend on the foam morphology), \(k^r_1, k^r_2, \mathcal{Q}_{11}, \mathcal{Q}_{66},\) and \(n_0\). The so-called “Gibson–Ashby law” \(^3\)

$$\frac{\text{foam property}}{\text{bulk material property}} = \rho_{\text{rel}} = c_1 \cdot \rho_{\text{rel}}^{c_2}$$

(24)

states, that the overall property of foam is equal to the bulk material property times a constant \(c_1\) times the relative density \(\rho_{\text{rel}}\) to the power of another constant \(c_2\). In addition to the relative density \(\rho_{\text{rel}}\), also the strut shape factor \(k\) is varied. It happens, that the parameters \(\mathcal{Q}_{11}\) and \(\mathcal{Q}_{66}\) nearly show no influence of the factor \(k\). As the factors \(k^r_1\) and \(k^r_2\) are very weakly related to \(k\) this dependency is neglected.

To determine the constants \(c_1\) and \(c_2\), a non-linear least square method is deployed. It minimizes the sum

$$\min \sum_{i=1}^{12} \left( P_{\text{rel}} - c_1 \cdot \rho_{\text{rel}}^{c_2} \right)^2 = \min \left( P_{\text{rel}} - 2P_{\text{rel}} \cdot c_1 \cdot \rho_{\text{rel}}^{c_2} + c_1^2 \cdot \rho_{\text{rel}}^{2c_2} \right)$$

(25)

The problem is solved by deploying an iterative Gauss–Newton scheme. Since the effective Poisson’s ratio depends very strongly on \(k\), the influence cannot be neglected. Therefore, the following adaption to the Gibson–Ashby law is being made

$$\nu_{\text{rel}} = c_1 \cdot \rho_{\text{rel}}^{c_2} \cdot [1 + c_1 \cdot k]$$

(26)

### 3.7. Polar Orthotropic Mindlin Plate

The filter which is investigated in the present contribution, has a cylindrical shape, is integrated into a pipe and supported as shown in Figure 2. The fluid flowing through the filter exerts pressure on the foam struts. In this article, an analytical and fully homogenized solution method is chosen, as it is being best suited for parameter variations. The pressure on the foam struts can be approximated using the Darcy–Forchheimer Equation (1), which describes the pressure drop per unit length. The total pressure yielded by multiplication with the filter height. A constant temperature inside of the filter is assumed since the heat conduction of the ceramic filter can be neglected for the given problem.

Considering the shape of the filter, its material, and the mechanical loading, a plate model is best suited to describing its deformation behavior. The most common plate theory is the Kirchhoff theory. However, since the effect of transverse shear deformation is neglected in this theory, it is not well suited for thick plates. The most widely used displacement-based plate theory including shear effects was developed by Mindlin and is used here assuming polar orthotropic and a circular shape after Halil.\(^3\) The model is illustrated in Figure 3.

For an orthotropic first-order Mindlin plate the following kinematic assumptions\(^3\) are made

$$u_i(r, z) = z\psi(r); \quad u_0(r, z) = 0; \quad u_2(r, z) = w(r)$$

(27)

Through differentiation, the strains can be computed as

$$E_r = 2 \left( \frac{d\psi}{dr} \right); \quad E_\theta = z \frac{d\psi}{r} \quad \Gamma_{rz} = \frac{dw}{dr} + \psi$$

(28)

Halil shows, that a variational formulation leads to the ordinary system of differential equations\(^3\)

$$\kappa_s G_{rz} h \left( \frac{d^2\psi}{dr^2} + \frac{1}{r} \frac{dw}{dr} + \frac{dw}{r} + \psi \right) + p = 0$$

(29)

$$D_s \left( \frac{d^2\psi}{dr^2} + \frac{1}{r} \frac{dw}{dr} - b^2 \frac{\psi}{r} \right) - \kappa_s G_{rz} h \left( \frac{dw}{dr} + \psi \right) = 0$$

(30)

where \(b = \sqrt{\frac{D_s}{\kappa_s}}\) is an orthotropy coefficient, which is \(b \approx 1\) for the foams investigated and \(\kappa_s = 5/6\) is the shear correction factor, modifying the shear modulus. The plate bending stiffness is defined as

![Figure 3. First-order Mindlin plate model.](image-url)
The solution of the system of differential equations for \( w(r) \) and \( \psi(r) \) of the flow-through filter with the simply supported edge as shown in Figure 3 is after Halli[12]

\[
w(r) = \frac{p R^2}{4} \left[ \frac{R^2}{16 D_r} \left[ \frac{5 + \nu_0}{1 + \nu_0} - \frac{6 + 2\nu_0}{1 + \nu_0} \frac{r^2}{R^2} + \frac{\nu_0}{1 + \nu_0} \right] \right] + \frac{1}{\kappa_i G_{r_i} h_i} \left[ 1 - \left( \frac{r}{R} \right)^2 \right]
\]

(32)

and

\[
\psi(r) = \frac{p R^2 r}{16 D_r} \left[ \frac{3 + \nu_0}{1 + \nu_0} - \frac{r^2}{R^2} \right]
\]

(33)

Thereby, the pressure \( p \) is with the use of Equation (1) and (2)

\[
p = h \cdot \frac{dp}{dz} = h \left[ \frac{\mu_t}{k_1} \cdot u_r + \frac{\rho_t}{k_2} \cdot u_z \right] = h \left[ \frac{\mu_t}{k_1} \frac{L}{r} \cdot u_r + \frac{\rho_t}{k_2} \frac{L}{r} \cdot u_z \right]
\]

(34)

To characterize the problem in a nondimensional manner, a procedure known as Buckingham’s Π-Theorem[13] is adopted. The Reynolds number as the classical characteristic number of fluid mechanics is introduced similarly to[10] as

\[
Re = \frac{\rho_t u_r L}{\mu_t}
\]

(35)

Using the Reynolds number, \( p \) can be rewritten as

\[
p = \frac{\mu_t}{\rho_t L^2} h \cdot \frac{Re}{k_1} + \frac{Re}{k_2}
\]

(36)

To pursue the nondimensionalization, the dimensionless numbers \( \Pi_0, \Pi_1 \) and \( \Pi_2 \) are established as

\[
\Pi_0 = \frac{\mu_t^2}{\rho_t L^2}, \quad \Pi_1 = \frac{R}{R}, \quad \text{and} \quad \Pi_2 = \frac{h}{L}
\]

(37)

Thereby \( \Pi_0 \) is a mixed fluid/material property for a given fluid and bulk material and depends also on the characteristic foam length \( L \). \( \Pi_1 \) is the ratio of the filters height to its radius. For very low values of \( \Pi_1 \) the filter cannot be modeled as a plate anymore. For values between 1 and 3 a Mindlin model is appropriate as used in the present contribution and for very high values of \( \Pi_1 \) the Kirchhoff plate theory may be the more accurate choice. \( \Pi_2 \) is the ratio of the characteristic foam length to the height of the filter. High values of \( \Pi_2 \) indicate that the first-order homogenization is still valid as assumed here. For very low values of \( \Pi_2 \), second-order homogenization should be considered.

When the dimensionless coordinate \( \xi \) is established as \( \xi = \frac{r}{R} \), Equation (32) and (33) can be rewritten completely dimensionless as

\[
\frac{w(\xi)}{h} = \frac{\Pi_0 \Pi_1 \Pi_2}{4} \left[ \frac{Re}{k_1} + \frac{Re}{k_2} \right] \left[ \frac{3}{\overline{Q}_{11}} \left[ \frac{5 + \nu_0}{1 + \nu_0} + \frac{6 + 2\nu_0}{1 + \nu_0} \xi^2 + \frac{1 + \nu_0}{1 + \nu_0} \right] \right] + \frac{1}{\kappa_i G_{r_i} h_i} \left[ 1 - \left( \frac{\xi}{R} \right)^2 \right]
\]

(38)

and

\[
\psi(\xi) = \frac{3 \Pi_0 \Pi_1 \Pi_2}{4} \overline{Q}_{11} \left[ \frac{Re}{k_1} + \frac{Re}{k_2} \right] \left[ \frac{3 + \nu_0}{1 + \nu_0} - \xi^2 \right]
\]

(39)

The other occurring dimensionless parameters \( \overline{Q}_{11}, \overline{Q}_{66}, \) and \( \nu_0 \), as introduced previously, do entirely depend on the foams morphology.

To calculate the stress state, the strains are evaluated from Equation (28).

\[
E_1 = \frac{d \psi}{dr} = \frac{3 + \nu_0}{1 + \nu_0} - \frac{r^2}{R^2}
\]

(40)

\[
E_0 = \frac{\mu_t}{\rho_t} \left[ \frac{3 + \nu_0}{1 + \nu_0} - \frac{r^2}{R^2} \right]
\]

(41)

\[
\Gamma_{rz} = \frac{d \psi}{dr} + \psi = \frac{\mu_t}{\rho_t} \left[ \frac{3 + \nu_0}{1 + \nu_0} - \frac{r^2}{R^2} \right]
\]

(42)

We find the stress state by Hooke’s law and use \( \overline{Q}_{11} \approx \overline{Q}_{22} \) and \( \overline{Q}_{11}/\overline{Q}_{66} = \nu_0 \) as shown in Equation (43)-(45).

\[
\frac{\Sigma_r}{\overline{E}} = \left[ \overline{Q}_{11} \overline{E}_r + \overline{Q}_{12} \overline{E}_t \right]
\]

(43)

\[
\frac{\Sigma_t}{\overline{E}} = \left[ \frac{3}{4} \right] \left[ \Pi_0 \Pi_2 \left[ \frac{Re}{k_1} + \frac{Re}{k_2} \right] \left[ 3 + \nu_0 \right] \left[ 1 - \xi^2 \right] \right]
\]

(44)

\[
\frac{\Sigma_{rz}}{\overline{E}} = \left[ \overline{Q}_{11} \overline{E}_r + \overline{Q}_{22} \overline{E}_t \right]
\]

(45)

4. Results

4.1. Effective Elastic Constants

Figure 4 shows the three elastic constants depending on the geometrical and morphological parameters \( k \) and \( \rho_{rel} \). With increasing relative density \( \rho_{rel} \) more material is available to carry the loads and all stiffness constants increase. The strut form parameter \( k \) influences the ratio of the strut thickness at the center and thickness at the end of struts or at the foam nodes, which is also illustrated in Figure 1. As \( k \) increases, the minimum strut
thickness at the strut centers decreases, whereby the values of the effective stiffness tensor components are only affected weakly.

Table 1 shows the calculated values for \( \frac{\sqrt{c}}{C_{138}} \) used within the equations for the minimization of the least square problem in Section 3.6. The graphs in Figure 5–7 show the results of the least square analysis:

4.2. Elastic Filter Deflection

The maximal filter deflection occurs at the location \( \xi = 0 \) as can be seen from Equation (38)

\[
\frac{w(0)}{h} = \frac{\Pi_0 \Pi_1 \Pi_2}{4} \left[ \frac{Re}{k_1^2} + \frac{Re^2}{k_2^2} \right] \left[ \frac{3}{4} + \frac{5}{v_0} + \frac{1}{\Pi_0 k_s Q_{66}} \right]
\]  

(46)

To investigate the influence of the foam topology parameters, the relations gained from the least square analysis are inserted into Equation (46) resulting in

\[
\frac{w(0)}{h} = \frac{\Pi_0 \Pi_1 \Pi_2}{4} \left[ \frac{Re}{0.0012 \rho_{rel}} + \frac{Re^2}{0.08 \rho_{rel}^{1.46}} \right] 
\cdot \left[ \frac{0.84}{\rho_{rel}^{0.73}} \left[ 1 + \frac{4}{1 + 0.13 \rho_{rel}^{0.46}(1 - 0.19 \rho_{rel})} \right] + 3.16 \right]
\]

(47)

Finally \( \rho_{rel}^{1.73} \) is factored out, yielding

\[
\frac{w(0)}{h} = 9.90 \cdot \Pi_0 \Pi_1 \Pi_2 \left[ \frac{67.54 Re}{\rho_{rel}^{0.52}} + \frac{Re^2}{\rho_{rel}^{0.27}} \right] 
\cdot \left[ 0.26 \left[ 1 + \frac{4}{1 + 0.13 \rho_{rel}^{0.46}(1 - 0.19 \rho_{rel})} \right] + 1 \right]
\]

(48)

For an investigation on the influence of \( \rho_{rel} \) on the bending deflection, \( \frac{w(0)}{h} \) is plotted in Figure 8 for different Reynolds numbers \( Re \).

It shows that the influence of the strut shape factor \( k \) on the maximum filter deflection is almost negligible, because in good estimation \( k \) only influences \( \nu_{\theta} \). However, with an increasing relative density \( \rho_{rel} \) the deflection decreases. For example, at a Reynolds number of \( Re = 30000 \) and the given parameters, the maximum filter deflection drops by 17% when the relative density doubles from 15% to 30%.

4.3. Creep Filter Deflection

The foam deformation resulting from local creep mechanisms at the foam struts is a complex, irreversible, and time-dependent process. As analytical expressions for the filter deflection over time are favorable, for example, for dimensioning a filter that does not reach a critical deflection in a given time, finite element simulations with the Monolith FE2 package\(^{[34]}\) of a foam creep test are performed using the Creep law (9) at the microscale to get parameters for a homogenized
macroscopic creep law. Thereby an additive split of the strain rate is assumed as

$$\varepsilon_{ij} = \varepsilon_{ij}^{el} + \varepsilon_{ij}^{cr}$$  \hspace{1cm} (49)$$

The creep strain rate is presumed to be purely deviatoric

$$\varepsilon_{ij}^{cr} = \frac{3}{2} s_{ij} \sigma_{eq} \varepsilon^{cr}$$  \hspace{1cm} (50)$$

with

$$\sigma_{eq} = \sqrt[3]{2} s_{ijkl} s_{ijkl}$$  \hspace{1cm} (51)$$

The creep law is discretized using the Euler backward scheme. To describe the macroscopic behavior a creep law is formulated equivalent to Equation (9) as

$$\dot{E}_{cr} = \sum_{eq} A(\rho_{rel}) \cdot \rho_{rel}^m$$  \hspace{1cm} (52)$$

It is assumed in accordance with\(^\text{[27]}\) that the time exponent \(m\) of the macro- and microscopic creep law are equal and the parameter \(A\) is a function of the relative density. For \(A\) an ansatz of the “Gibson–Ashby”-law is being made as shown in Equation (53) which is equivalent to Equation (25).
Macroscopic creep curve, \( \rho_{\text{rel}} = 0.15, k = 0.75 \).

**Figure 9.** Macroscopic creep behavior a) Macroscopic creep curve, \( \rho_{\text{rel}} = 0.15, k = 0.75 \). b) Result of the least square analysis for \( A \).

\[
A(\rho_{\text{rel}}) = c_1 \cdot \rho_{\text{rel}}^2
\]  

(53)

Creep tests are simulated by setting a constant (macroscopic) load in one direction and unhindered expansion in the other directions. The results are macroscopic creep curves as shown exemplarily in Figure 9a. From these curves, the parameter \( A \) is extracted using Equation (52) and linear least square analyses. Figure 9a shows that the homogenized law describes the FE solution with only a very small deviation. The values \( A \) are now used to determine the parameters \( c_1 \) and \( c_2 \). The result is shown in Table 2 and illustrated in Figure 9b.

To get an analytical expression for the maximal filter deformation over time, a semi-analytical procedure is applied. From Equation (52) it can be seen that the creep deformation scales linearly with the equivalent stress \( \Sigma_{\text{eq}} \) and from Equation (43)-(45) it shows that there exists a constant pre-factor for all macroscopic stresses. We make the assumption, that the stress state remains nearly constant over time, which is a good estimation as the flow-through problem is stress controlled. Therefore, we make an assumption for the maximum filter deflection as shown in Equation (54).

\[
\frac{w(\xi = 0, t)}{h} = \frac{w_{\text{el}}(\xi = 0)}{h} + \frac{w_{\text{el}}(\xi = 0)}{h} + \int_0^1 \frac{1}{A} \cdot \Pi_0 \Pi_1 \Pi_2 [\frac{Re}{k_1} + \frac{Re^2}{k_2}] \cdot f(\Pi_1) \, d\theta
\]

\[
= \frac{w_{\text{el}}(\xi = 0)}{h} + \frac{w_{\text{el}}(\xi = 0)}{h} + \frac{m+1}{m+1} \frac{1}{A} \cdot \Pi_0 \Pi_1 \Pi_2 \left[ \frac{Re}{k_1} + \frac{Re^2}{k_2} \right] \cdot f(\Pi_1)
\]  

(54)

**Table 2.** Constants of the least square analysis for creep parameter \( A \).

| \( c_1 \) | \( c_2 \) |
|----------|----------|
| 0.9657   | 1.776    |

Thereby it is assumed that the Reynolds number \( Re \) does not change over time. A function \( f \) is introduced, which accounts for the geometry of the filter over the dimensionless ratio of radius to height \( \Pi_1 \). To get this function, FE simulations are conducted with axisymmetric plate elements with quadratic shape functions and 5 Gauss points over the thickness using the macroscopic material parameters as determined before. The finite element model is shown in Figure 10. To show that the ansatz for \( f \) is valid, Figure 11 left plots \( f \) as determined by the finite element simulations (for different \( \rho_{\text{rel}}, k \), etc.), showing that it is approximately only a function of \( \Pi_1 \).

For function \( f \) the ansatz

\[
f(\Pi_1) = c_1 \cdot \Pi_1^2
\]  

(55)

is being made, which leads to a nonlinear least square problem equivalent to Equation (25). The constants of the fitting are shown in Table 3. The curve is illustrated in Figure 11 on the right-hand side.

Together with Equation (54), the final result for the filter deflection over time is obtained

\[
\frac{w(\xi = 0, t)}{h} = \frac{w_{\text{el}}(\xi = 0)}{h} + \frac{m+1}{m+1} \frac{817.3 \text{MPa}}{A} \cdot \Pi_0 \Pi_1 \Pi_2 \left[ \frac{Re}{k_1} + \frac{Re^2}{k_2} \right] \cdot f(\Pi_1)
\]

\[
= \Pi_0 \Pi_1 \Pi_2 \left[ \frac{Re}{k_1} + \frac{Re^2}{k_2} \right] \cdot \left[ \frac{3}{16 \xi_{11}(1 + \epsilon_{11})} + \frac{1}{4 \Pi_k Q_{66}} + \frac{m+1}{m+1} \frac{817.3 \text{MPa}}{A} \right]
\]

\[
= \Pi_0 \Pi_1 \Pi_2 \left[ 67.5 \frac{Re^2}{\rho_{\text{rel}}^{0.57}} \left[ \frac{10.5}{\rho_{\text{rel}}^{0.27}} + \frac{9.88}{\Pi_k^{2.61}} + \frac{1.02 \times 10^6 \text{MPa}}{m+1} \cdot a \cdot \rho_{\text{rel}}^{0.09 \, m+1} \right] \right]
\]  

(56)
4.4. Maximal Local Stresses

As it was stated previously in Chapter 3.5, the maximum local stresses are a function of the macroscopic stress state Σ. In the particular flow through filter problem Σ is a function of ξ and z and assumed to be no function of the time t. The maximal macroscopic stresses occur at z = h/2, hence only the dependence on ξ is investigated. To find the location ξ of the maximal local von Mises stress, its distribution is plotted at z = h/2 for different values of Πi as shown in Figure 12. Typical values for Re = 30000, Π0 = 3.5 · 10⁻¹³, and Π2 = 7 are chosen only for illustration purposes. We see that Π1 has an impact on the shape of the stress distribution over ξ. For realistic values of Π1 (1.0 ≤ Π1 ≤ 3.0), the maximal Mises stress is in good estimation at the location ξ = 0. As we are only interested in the global maximum of the local von Mises stress, a parameter variation and least square analysis are only done for this location (Figure 13).

The stress state at ξ = 0 reads with the use of Equation (43)–(45)

\[
\frac{\Sigma(\xi = 0, z = h/2)}{E} = \Pi_0 \Pi_2 \Pi_2 \left[ \frac{Re}{k_1^2} + \frac{Re^2}{k_2^2} \right] \frac{3}{8} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 + \nu_0 \\ 1 + \nu_0 \\ 1 + \nu_0 \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ A_2 \end{bmatrix}
\]

(57)

The maximum local von Mises stress scales linearly with a given stress amplitude A. The prefactor in Equation (57) is known and the principal stress directions are independent of the parameters chosen. The (relative) von Mises stress can be calculated from the RVEs finite element models and is for a given stress direction a function of the relative density ρrel and the strut factor k and scales linearly with an amplitude. An ansatz is made in the form of

\[
\frac{\sigma_{\text{V},\text{M},\text{max}}}{E} = \frac{\sigma_{\text{V},\text{M}}(\Sigma(\xi = 0, z = h/2))}{E} = A(\Pi_0, \Pi_1, \Pi_2, Re, \rho_{\text{rel}}) \cdot f(\rho_{\text{rel}}, k)
\]

with

\[
f(\rho_{\text{rel}}, k) = c_1 \cdot \rho_{\text{rel}}^{c_2} \cdot k^{c_3}
\]

To determine the constant of the ansatz f, again a nonlinear least square analysis is used. The fitted constants are shown in Table 4.

The final result for the maximal local von Mises stress is gained with the use of Equation (57)–(59) as

\[
\frac{\sigma_{\text{V},\text{M},\text{max}}}{E} = \Pi_0 \Pi_2 \Pi_2 \left[ \frac{Re}{0.0012 \rho_{\text{rel}}^{1.365} + 0.088 \rho_{\text{rel}}^{1.459}} \right] \cdot 2.502 \cdot \rho_{\text{rel}}^{0.696} \cdot k^{-0.362}
\]

(60)

For typical values Π0 = 3.5 · 10⁻¹³, Π1 = 2.0, and Π2 = 7, the maximal local stress is plotted in Figure 14 as a function of the relative density ρrel and the strut shape factor k for different values of the Reynolds number Re. It can be seen that both an

![Figure 10. Finite element model using Abaqus SAX2 elements.](image)

![Figure 11. Function f(Π1) over time t, right: f(Π1) over Π1 with least square result.](image)

![Figure 12. Typical values for Re = 30000, Π0 = 3.5 · 10⁻¹³, and Π2 = 7 are chosen only for illustration purposes.](image)

Table 3. Constants of the least square analysis for function f.

| c1  | c2  |
|-----|-----|
| 817.3 MPa m⁻¹ | 3.0  |

The maximum local von Mises stress scales linearly with a given stress amplitude A. The prefactor in Equation (57) is known and the principal stress directions are independent of the parameters chosen. The (relative) von Mises stress can be calculated from the RVEs finite element models and is for a given stress direction a function of the relative density ρrel and the strut factor k and scales linearly with an amplitude. An ansatz is made in the form of

\[
\frac{\sigma_{\text{V},\text{M},\text{max}}}{E} = \frac{\sigma_{\text{V},\text{M}}(\Sigma(\xi = 0, z = h/2))}{E} = A(\Pi_0, \Pi_1, \Pi_2, Re, \rho_{\text{rel}}) \cdot f(\rho_{\text{rel}}, k)
\]

with

\[
f(\rho_{\text{rel}}, k) = c_1 \cdot \rho_{\text{rel}}^{c_2} \cdot k^{c_3}
\]

To determine the constant of the ansatz f, again a nonlinear least square analysis is used. The fitted constants are shown in Table 4.

The final result for the maximal local von Mises stress is gained with the use of Equation (57)–(59) as

\[
\frac{\sigma_{\text{V},\text{M},\text{max}}}{E} = \Pi_0 \Pi_2 \Pi_2 \left[ \frac{Re}{0.0012 \rho_{\text{rel}}^{1.365} + 0.088 \rho_{\text{rel}}^{1.459}} \right] \cdot 2.502 \cdot \rho_{\text{rel}}^{0.696} \cdot k^{-0.362}
\]

(60)

For typical values Π0 = 3.5 · 10⁻¹³, Π1 = 2.0, and Π2 = 7, the maximal local stress is plotted in Figure 14 as a function of the relative density ρrel and the strut shape factor k for different values of the Reynolds number Re. It can be seen that both an
increasing relative density $\rho_{\text{rel}}$ and increasing strut shape factor $k$ lead to a decreasing maximal local von Mises stress. At a Reynolds number of $Re = 3000$, we see a drop of the maximal stress by 22% going from a relative density $\rho_{\text{rel}}$ of 15% to 30% and by the same amount from a strut shape factor $k$ of 0.5 to 1.0.

### 4.5. Comparison between Different Solution Approaches

To compare different solution approaches additionally to the semi-analytically model and the plate-FE model, a solid model is investigated. The solid model is simulated with two different material models. In one simulation, a homogenized macroscopic material is adopted which uses the homogenized parameters determined in the previous chapters and in one simulation a concurrent FE$^2$ procedure is employed with aid of the MonolithFE$^2$ program. Figure 15 shows all models.
In Figure 16, the macroscopic creep curves of all 4 models are compared. The results show quite some differences but a qualitatively good overall agreement. The difference between the analytical and FE-plate simulation can be explained by a different plate formulation used by Abaqus which is less stiff than the Mindlin plate model. The difference between the plate and solid models can be explained, as in the solid models, a contact at the coating is incorporated, leading for example to contact nonlinearities and expected deviations from the kinematic assumptions of the plate model.

Figure 15c) shows the relative von Mises stress of the micro model at the underside of the filter at $\xi = 0$. It reveals maximal relative stress of $\approx 2.84\%$ in the middle of the foam struts. The analytical model (Equation (60)) predicts maximal relative stress $2.89\%$ which is very close to the result of the FE$^2$ simulation. Admittedly, the maximal stress of the solid model is located in the surrounding of the coating. It is obvious that no real results are expected at that location in the plate model.
4.6. Failure Surfaces

Following Storm et al.\(^{[35]}\) failure surfaces for certain local failure criteria can be computed. Since the foam RVEs used in this contribution show an orthotropic elastic behavior, a definition for the orientation of the RVEs is necessary. Here, three-unit vectors \(\mathbf{N}_i, i \in [1, 2, 3]\) with the properties \(\mathbf{N}_i \cdot \mathbf{N}_j = 0\) for \(i \neq j\) and \(\|\mathbf{N}_i\| = 1\) are defined, denoting the three principal axes of the RVE. A unit tensor

\[
\mathbf{\hat{n}} = \frac{I}{\sqrt{3}} \sin \alpha + \frac{n}{\sqrt{2}} \cos \alpha
\]

is defined. Therein, \(I\) is the second-order unit tensor and \(\mathbf{\hat{n}}\) a deviatoric tensor.

Figure 17. Strength of the foam structures for a) a uniaxial load, b) a biaxial load, c) a pure hydrostatic load, d) a deviatoric load with Lode angle \(\theta = 0\), and e) a deviatoric load with Lode angle \(\theta = \pi/6\).
\( \hat{\lambda} = \sum_{k=1}^{3} \varepsilon_k M_k \)

composed of the three tensor eigen bases \( M_k = N_k \otimes N_k \) and three eigen values \( \varepsilon_k \) defined by

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\end{bmatrix} = 
\begin{bmatrix}
\cos \theta - \frac{\sin \theta}{\sqrt{3}} \\
\frac{2 \sin \theta}{\sqrt{3}} \\
- \cos \theta - \frac{\sin \theta}{\sqrt{3}} \\
\end{bmatrix}
\]

depending on the Lode angle \( \theta \). The unit tensor \( n \) serves as stress direction and its six independent components as the values for \( \lambda_i \)

\( \rho_{\text{rel}} = 0.15, \ k = 0.50 \)

\( \rho_{\text{rel}} = 0.20, \ k = 0.50 \)

\( \rho_{\text{rel}} = 0.25, \ k = 0.50 \)

\( \rho_{\text{rel}} = 0.30, \ k = 0.50 \)

**Figure 18.** Failure surfaces for 4 different foams with a strut shape factor \( k = 0.50 \) using a von Mises criterion with \( \sigma_{\text{crit}} = 20 \) MPa. Left) Complete failure surface projected into principal stress space. Center) Meridian cross section cuts at different Lode angles \( \theta \). Right) Cross section at different angles \( \alpha \) projected on the \( \Pi \)-plane. The cross sections are also indicated in the 3D plots.
used in the superposition of the six independent load cases in Equation (22). The resulting local stress field \(\sigma(x)\) needs to be scaled by a scalar factor \(\lambda_{\text{crit}}\) such that the maximum of the local stress criterion becomes equal to a critical value \(\sigma_{\text{crit}}\), which can be obtained from experiments.

\[
\lambda_{\text{crit}} \max\left\{ \sigma_\nu \left[ \sigma(x) \right] \right\} = \lambda_{\text{crit}} \max\left\{ \sigma_\nu \left[ \sum_{i=1}^{6} \lambda_i \sigma_i(x) \right] \right\} = \sigma_{\text{crit}} \quad (64)
\]

The six components of a critical effective stress state are defined by

\[
\Sigma_{\text{crit}} = \lambda_{\text{crit}} \sigma_0 \quad (65)
\]

and can be easily computed for each combination of \(\alpha\) and \(\theta\). The nature of the curves in Figure 17 follows the extended Gibson–Ashby power law as defined by Equation (26) or (59), where an additional term is used to incorporate the strut form factor \(k\).

The graphs in Figure 18 are obtained by varying \(\alpha \in [-\pi/2, \pi/2]\) and \(\theta \in [0, 2\pi]\) in 49 equidistant steps.

5. Conclusion and Discussion

In the present contribution, the influence of the foam morphology, macroscopic dimensions, and material parameters on the mechanical behavior of a flow-through foam filter has been investigated. The foam material is assumed to show an elastic-creep behavior and small strains were considered.

Finite element models of foam RFVs with different morphology parameters are used to evaluate the overall material parameters and the interaction with a flow-through fluid, which leads to effective parameters for the Darcy–Forchheimer law. To get an analytical expression for these macroscopic parameters in dependence of the foam morphology parameters, least square analyses have been performed.

To describe the deformation of the filter, a Mindlin plate model is deployed. The resulting equations have been nondimensionalized with characteristic numbers, for example, the Reynolds number \(Re\). When examining the influence of different foam parameters on the maximum filter deflection it shows, that the influence of the foam strut shape factor is negligible and the relative density influences the deflection only weakly. This results from the fact that the stiffness is decreasing with the relative density but so does the pressure drop when all other parameters stay constant. To include creep deformation, a semi-analytical procedure for the maximal filter deflection is applied, where the creep deflection is calculated by an FE plate model and incorporated into the analytical model.

To investigate the integrity of the filter, the maximal local von Mises stress has been calculated, showing that its global maximum lies in the filter middle for common aspect ratios of radius to height. At this point, another least square analysis has been performed to get an analytical expression for the maximal stress in dependence of the foam topology, filter dimensions, and fluid parameters, which shows that with increasing relative density and strut shape factor the maximal occurring local von Mises stress drops considerably. To check the integrity of an arbitrary stress state, a method of calculating failure surfaces for a given permissible stress level is shown and the results have been visualized graphically.

The value of this study is not only to show the influence of the foam topology on the filter deformation local stresses, and so on but also how analytical methods can be combined with numerical homogenization methods to effectively calculate the mechanical response if one is mainly interested in the overall macroscopic behavior.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

Research data are not shared.

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