1. Introduction

Steel plate products have variety of applications such as ships, bridges, architectures, public constructions and mining tools. The quality improvement of heavy steel plate is now of great interest for the safety and reliability purpose. In this respect, the microstructure control realised by optimising the alloying elements has acquired a long term research interests as well as developing the heat treatment process such as quenching, tempering, normalising and annealing. More recent achievement of this kind includes Thermo-Mechanical-Control-Process (TMCP) technique which enables finer microstructures.

An Online Rolling Model for Plate Mill Using Parallel Computation

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A new online rolling model of the draft schedule setup for a plate mill has been developed. This model comprises plate temperature, rolling force function and flow stress calculations and their coupling for the roll separating force estimation. The roll separating force calculation is also used when the work roll gap control is made realising a precise plate thickness control for each rolling pass, which is often referred to as an adaptive control. The temperature model and roll separating force model, as well as its inverse calculation (calculate entry thickness from exit thickness and given roll separating force), are involved in the draft schedule setup calculations. Plate rolling is carried out according to the setup calculation results and thus the product plate quality is largely attributable to the setup calculation preciseness. In this model, a one dimensional finite element model is employed to the temperature calculation that enables a precise temperature control which is necessary for the Controlled Rolling (CR) technology. Another development includes the rolling force function model; a new mathematical model which takes the peening effect into account, derived from the three-dimensional rigid-plastic finite element calculations. Finally, a flow stress model is developed taking into account the metallurgical nature such as work hardening, recrystallization and recovery. The coupling of these models allows to a physical based precise model without unnecessary artificial fitting parameters.

In addition, for eliminating the convergence loop, an attempt has been made introducing a multi thread computing using General Purpose computing on Graphic Processing Unit (GPGPU). Thanks to this parallel computing technique, the computational time was remarkably reduced. The model was installed in a process computer and some trial rolling tests were conducted.

KEY WORDS: plate mill; rolling schedule; draft setup model; flow stress; rolling function; controlled rolling; parallel computing; GPGPU.

leads to a decrease in productivity or a yield loss. For this reason, a precise control model is necessary for the stable production and therefore many attempts have been carried out dedicating the draft setup schedule including temperature and roll separating force models.2)

In this research, an adaptive control model as well as the setup model is developed and their application test results are obtained. The model comprises temperature, rolling function and flow stress (deformation resistance) model reflecting the rolling conditions and metallurgical nature.

The difficulty underlying in the setup model for heavy plate rolling is that the rolling pass number is not a priori fixed and hence the schedule has a high degree of freedom as contrasted with hot strip rolling mills where the rolling pass number always equals to the finishing rolling stand number. This has caused the excessive computing time in the process line computer for heavy plate mills. Therefore, in this research, a parallelisation algorithm for reducing computational time is developed taking advantage of efficient parallelisation using General Purpose computing on Graphic Processing Unit (GPGPU). The modified computing flow using GPGPU and the typical application results will be discussed.
2. Online Model

2.1. Model Description

The draft setup calculation defines the total rolling pass number and each reduction amount as well as the temperature of each pass for the plate quality control requirement. In addition to the setup calculation, before each rolling pass, recalculation of roll separating force for the next pass for optimising roll-gap is conducted reflecting the actual rolling condition, such as time elapse since the start of rolling and application of the descaling water blow. This recalculation is often called adaptive control calculation.

Both the rolling setup model and the adaptive control model contain a function of the roll separating force estimation. To estimate the roll separating force, the temperature distribution along the thickness direction of a plate is calculated. Then, the flow stress (deformation resistance) is estimated based on the temperature calculation results. The flow stress depends on the microstructure of a plate in the rolling process as a function of time, temperature, rolling condition and alloying elements.

However, the hardness reduces with time after hot rolling and static recovery and the static recrystallization. Therefore, these two (flow stress and hardness) additional force by, for example, friction between work roll and rolled material. Therefore, these two (flow stress and rolling function) models are necessarily implemented in an integrated roll separating force calculations.

Again, the particular difficulty in the plate rolling setup model is that the total rolling pass number is not fixed. Thereby, the minimum possible rolling pass number should be chosen from unlimited possibilities of rolling schedule under the load, torque and shape control (constant crown ratio) restrictions. This requires excessive computing time for optimisation, which is reduced in this study by parallel computation using GPGPU.

The details of each model will be presented as follows.

2.2. Temperature

Temperature calculation model is a 1-dimensional x (in thick) finite element model. The one-dimensional heat conduction equation is:

\[ \rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + L, \] .......................... (1)

where, \( T \) is temperature, \( \rho \) is density, \( c \) is heat capacity, \( k \) is thermal conductivity and \( L \) is heat generation per unit second. The finite-element discrete equation is given by:

\[
\left( \int_{V} k (\nabla T)^{T} [N]^{T} [N] dV + \int_{S} h [N]^{T} [N] dS \right) [T] + \int_{V} \rho c [N]^{T} [N] dV = \int_{S} h [N] T_{0} dS - \int_{S} [N] \dot{q} dV = 0, \] .......................... (2)

where \( V \) is a volume, \( S \) is a surface of boundary, \( h \) is heat transfer coefficient, \( T_{0} \) is atmosphere temperature and \( N \) is a one dimensional shape function presented as follows:

\[ [N] = \frac{1}{2} \left[ 1 - \xi, 1 + \xi \right]. \] .......................... (3)

Here, normalised position in an element takes \(-1 \leq \xi \leq 1\). Thus, if one defines

\[ [H] = \int_{V} k (\nabla T)^{T} [N]^{T} [N] dV + \int_{S} h [N]^{T} [N] dS, \]
\[ [P] = \int_{V} \rho c [N]^{T} [N] dV, \]
\[ [Q] = \int_{V} [N] \dot{q} dV, \]
\[ [f] = \int_{S} h [N] T_{0} dS, \]

and linear approximation in a time increment \( \Delta t \):

\[ [T] = \frac{[T]_{t-\Delta t} + [T]_{t}}{\Delta t}, \]

the implicit form for the finite element equation is given as:

\[ [H] \frac{+ [P]}{\Delta t} [T]_{t} = \frac{[P]}{\Delta t} [T]_{t-\Delta t} + [Q] + [f]. \] .......................... (6)

The analytical solution of Eq. (4) can be given as:

\[ [H] = \sum_{e} \left\{ \frac{k}{l} \left[ 1 - \frac{1}{l} \right] + \mu h \left[ 1 \left[ 0 \ 0 \right] + vb \left[ 0 \ 1 \right] \right]\right\}, \]
\[ [P] = \sum_{e} \frac{\rho c}{6} \ell \left[ 2 \ 1 \right], \]
\[ [Q] = \sum_{e} \ell \left[ 1 \right], \]
\[ [f] = \sum_{e} \left[ \mu h T_{0} \left[ 1 \right] + vb T_{0} \left[ 0 \right] \right]. \]

where \( e \) is an element number, \( \ell \) is the one dimensional element length and \( \mu \) and \( v \) take 1 at boundary of each element and otherwise take 0. The boundary condition can be heat transfer by radiation, convection, descaling water, heat conduction to work roll and heat generation by friction. These boundary conditions are treated as an equivalent heat transfer coefficient.

2.3. Flow Stress

A quantitative formulation of flow stress is made here by considering the microstructure evolution in the hot plate rolling process. The flow stress is simply estimated by dislocation density using Bailey-Hirsch equation as follows:

\[ \sigma = \alpha \cdot G \cdot \| \rho \| \left( \rho_{0} - \rho_{e} \right)^{n}, \]

where \( \rho_{0} \) and \( \rho_{e} \) are current and initial dislocation density, \( G \) is stiffness, \( b \) is Burger’s vector and \( \alpha \) is a fitting parameter. The dislocation density is calculated throughout the rolling process as a function of time, temperature, rolling condition and alloying elements.

The dislocation density increases during rolling reflecting work hardening and, in the inter-pass, decreases due to the static recovery and the static recrystallization.5–8)

First of all, following equation describes the static recovery phenomenon; decreasing the dislocation density \( \rho_{e} \) with time \( t \), such that:
\[ \rho_s = (\rho_{0i-1} - \rho_s) \exp(-d \cdot t) + \rho_s \] .......................... (9)

where \( \rho_{0i-1} \) is the average dislocation density of previous pass and before initiating the static recovery, \( \rho_s \) is the saturation dislocation density and \( d \) is the parameter

\[ d = d_0 \exp(C \cdot NbT) \cdot D_{rec}^{m} \cdot \exp(Q_d / RT_0) \] .......................... (10)

where \( d \) is a variable for recovery rate, \( NbT \) is Nb content, \( D_{rec} \) is average grain size of previous pass, \( R \) is gas constant and \( d_0, C, m_d, Q_d \) are the constant parameters.

Next, the static recrystallization initiates. Using 50% recrystallizing time\(^5\) tos, the recrystallization fraction \( X \) and average grain size of both recrystallized \( D_{rec} \) and overall grain size \( D'_r \) are:

\[ X = 1 - \exp(-\beta (t / tos)^3), \]
\[ D_{rec} = g_0 \cdot D_{0i-1}^{m} \cdot \varepsilon_{0i-1} / X^{\beta}, \] .......................... (11)
\[ D'_r = D_{rec} \cdot X + D_{0i-1}(1 - X). \]

The rate form of Eq. (11) is better in obtaining precise recrystallization fraction as the temperature changes each moment and thus the tos is varying as a function of temperature, such that

\[ dX = 2\beta (t / tos)^3 \cdot \exp(-\beta (t / tos)^3) \cdot dt. \] .......................... (12)

The integration of Eq. (12) with time gives the static recrystallization fraction by considering temperature change in the inter-pass cooling process.

Analogous to the grain size estimation, the average dislocation density after recrystallization \( \rho' \) is:

\[ \rho' = X \rho_0 + (1 - X) \rho_s, \] .......................... (13)

where \( \varepsilon_{0i-1} \) is the strain accumulation before recrystallization and \( \beta, g_0, m_d, n_g, l_g \) are the parameters.

For plate rolling, the reduction is not so large and thus it is unlike to take place dynamic recrystallization. Therefore, dynamic recrystallization is neglected here and only work hardening and dynamic recovery are taken into account.

In this case, the dislocation density \( \rho_{d} \) during rolling is:

\[ \rho_{d} = \beta c \cdot (1 - \exp(-c \varepsilon_{a} / \dot{\varepsilon})) + \rho_0. \] .......................... (14)

Here, work hardening and dynamic recovery are described respectively as:

\[ b = b_0 \exp(A \cdot NbT) \cdot D_{rec}^{m} \cdot \dot{\varepsilon}^{n} \cdot \exp(Q_d / RT_0), \] .......................... (15)

\[ c = c_0 \exp(B \cdot NbT) \cdot D_{rec}^{m} \cdot \dot{\varepsilon}^{n} \cdot \exp(Q_d / RT_0), \] .......................... (16)

where \( D_{rec} \) is an average recrystallized grain size (micrometer), \( \dot{\varepsilon} \) is the strain rate and \( T_0 \) is the average temperature of rolled material, \( b_0, c_0, A, B, m_d, n_g, n_t, Q_d \) and \( \dot{\varepsilon} \) are the parameters.

As for strain accumulation \( \varepsilon_{a} \), the residual strain \( \varepsilon_{r} \) should be considered for the multiple rolling process. The residual strain \( \varepsilon_{r} \) takes the form:

\[ \varepsilon_{r} = (-\dot{\varepsilon} / c) \ln(1 - (\rho' - \rho_0) \varepsilon / \beta c), \] .......................... (17)

The prominent achievement by this model is that one can predict proper flow stress considering metallurgical nature including pinning effect by microalloy elements (Nb). Indeed, the recent plate rolling uses the microalloy for producing heavy plates with finer grains. Other efective microalloys like Ti are also considered being treated as equivalent Nb contents. The values of above-mentioned parameters are listed in Table 1.\(^5\)\(^-\)\(^9\)

These parameters can be identified by, for example, uniaxial compression tests as is explained in the references.\(^5\)\(^-\)\(^8\)\) Firstly, \( c \) in Eq. (14) is identified analytically which will be used to fit the parameters appear in Eq. (16) to the experimental results. Once \( c \) is obtained, \( \alpha \) in Eq. (8) and \( b \) in Eq. (14) and related parameters in Eq. (15) are calculated.

2.4. Rolling Force Function

The mechanistic nature of rolling is not as simple as uniaxial compression or plane strain compression because of the deformation heterogeneity appears in the roll byte. This means that the roll separating force is not a simple product of flow stress and contact area of work roll and rolled material. Reflecting the complexity of deformation of rolling, attempts have been reported to compensate the difference between plane strain compression and rolling. The parameter compensates the difference is called rolling function. Orowan has suggested the simplified rolling model considering non-uniform deformation\(^10\) and then Bland & Ford\(^11\) and Sims\(^3\) prescribed the analytical solution of Orowan’s model. These models are widely used in the roll separating force prediction in hot strip finishing mills or in cold rolling mills. However, it was also Orowan’s indication,\(^13\) the model sometimes fails in a precision especially when a plate

| Table 1. Parameters for flow stress.\(^5\)\(^-\)\(^9\) |
|---------------------------------|-----------------|
| Parameter symbols | Values |
| \( \alpha \) | 1.83 (\(\cdot\)) |
| \( \rho_0 \) | 1.0 × 10^{-3} (mm^{-2}) |
| \( d_0 \) | 1.06 × 10^{-9} (s^{-1}) |
| \( C \) | -85.6 (mass\%^{-1}) |
| \( m_d \) | -0.9 (\(\cdot\)) |
| \( Q_d \) | -1.8 × 10^{3} (J{\cdot}K^{-1}{\cdot}mol^{-1}) |
| \( R \) | 8.31 (m^{2}{\cdot}kg{\cdot}s^{-2}{\cdot}K^{-1}{\cdot}mol^{-1}) |
| \( \beta \) | 0.693 (\(\cdot\)) |
| \( g_0 \) | 5.17 (\(\mu\)m) |
| \( m_g \) | 0.29 (\(\cdot\)) |
| \( n_g \) | -0.75 (\(\cdot\)) |
| \( \dot{\varepsilon}_g \) | 0.29 (\(\cdot\)) |
| \( b_0 \) | 1.33 × 10^{7} (mm^{-2}) |
| \( A \) | 0.92 (mass\%^{-1}) |
| \( m_t \) | -0.207 (\(\cdot\)) |
| \( n_t \) | 0.105 (\(\cdot\)) |
| \( Q_t \) | 3.41 × 10^{7} (J{\cdot}K^{-1}{\cdot}mol^{-1}) |
| \( c_0 \) | 1.44 × 10^{7} (s^{-1}) |
| \( B \) | -4.3 (mass\%^{-1}) |
| \( m_c \) | -0.182 (\(\cdot\)) |
| \( n_c \) | 1.02 (\(\cdot\)) |
| \( Q_c \) | -1.82 × 10^{4} (J{\cdot}K^{-1}{\cdot}mol^{-1}) |
is relatively thick. In this case, the actual roll separating force is much greater than that of the model prediction. This phenomenon is known to take place when shape factor (ratio of contact length and average plate thickness) is smaller than a certain value (typically 1) and is called peening effect.\(^{14}\) The peening effect is thus considered in this study in developing a new rolling force model.

The authors used three-dimensional Finite Element model\(^{15}\) to simulate a rolling condition and developed a rolling force function which best fits the calculated roll separating force, such that:

\[
Q_a = a + b \cdot \Gamma^c + c / \Gamma^g , \;
\text{ .......... (18)}
\]

where \(Q\) is a rolling force function, \(\Gamma\) is a shape factor, \(a, b, c\) and \(g\) are the constants while \(g_a\) is the function of angle of contact \(\phi\). The obtained rolling function draws a typical curve as demonstrated in Fig. 1. The schematic illustration of angle of contact \(\phi\) is also shown in Fig. 1. The third term of Eq. (18) is an effect of peening. The values of each parameter appear in Eq. (18) is demonstrated in Table 2.

3. Algorithm

Using the previous discussion results, the roll separating force \(P\) (N) can be written as:

\[
P = k_m \cdot Q \cdot l_d \cdot W , \;
\text{ .......... (19)}
\]

where \(k_m\) is a 2-dimensional deformation resistance (MPa), \(W\) and \(l_d\) are plate width (mm) and contact length (mm) respectively.

The contact length \(l_d\) is a function of the work roll flattening, and the work roll flattening is the function of roll separating force, and so the contact length. Therefore, to solve the right roll separating force, an iterative computation is employed. The roll separating force prediction sequence is roughly summarised in Fig. 2. This roll separating force model is used for the setup model as well as for the adaptive control defining the roll gap of the subsequent rolling pass. The work roll flattening is calculated according to the Hitchcock model:

\[
R' = \left(1 + \frac{C_0}{\Delta h} P\right) R, \; C_0 = \frac{16(1 - \nu^2)}{\pi E} , \;
\text{ .......... (20)}
\]

where \(R'\) is a radius of flattened roll, \(R\) is an initial roll radius, \(\Delta h\) is the reduction, \(\nu\) is Poisson’s ratio and \(E\) is Young’s modulus\(^{16,17}\) at ambient temperature, i.e. \(\nu = 0.3, \; E = 205\) (GPa).

The setup model is consisted of trial schedule assembling and verifying process. In the trial schedule assembling, we

### Table 2. Parameters for rolling function.

| Parameter symbols | Values |
|-------------------|--------|
| \(a\)              | 0.602  |
| \(b\)              | 0.230  |
| \(c\)              | 0.209  |
| \(g_a\)            | −0.032+1.12 |
| \(g_b\)            | 1.176  |

![Fig. 1. Relation between shape factor and rolling force function.](image1)

![Fig. 2. Flow of rolling separating force calculation.](image2)

![Fig. 3. Convergence flow of entry thickness calculation.](image3)
start with the production plate thickness and conduct an inverse calculation up to slab thickness: calculation of an entry thickness derived from exit thickness, roll separating force, torque and constant crown ratio requirements. At the first iteration, the flow stress is empirically defined. When preliminary draft schedule is obtained, the precise calculation of temperature, flow stress is then conducted. A convergence flow of the iterative scheme is shown in Fig. 3. These trial and verification calculations are carried out repeatedly until the final convergence is achieved.

A typical calculated temperature and dislocation density evolution is shown in Fig. 4. Note that CR start and CR end in Fig. 4 refers to the inter-pass time during which air cooling or water cooling is carried out to achieve the target temperature, typically austenite recrystallization temperature. It is evident that the strain induced by previous rolling is disappeared during long time hold of temperature controlling process, which results in an abrupt decrease in roll separating force.

4. Setup Model on Parallel Computation Using GPGPU

4.1. Computation Time

The physical model was applied to a process computer of a plate mill. The first attempt of the setup model application revealed that the computing time for a setup calculation of a slab was approximately 90.4 s, which exceeds the process limitation of 2.0 s. A comparison of calculation time of equivalent setup models between hot strip rolling and plate rolling is summarised in Table 3. It is clear that the iteration of optimising total rolling pass number in plate rolling is the most time-consuming scheme.

Thus, an assessment of profiled computing time module by module was conducted yielding an apparent result of the major time-consuming agent. Comparing to the CR calculation (air and water cooling during interval of time between rolling) and schedule verification (recalculation and check if all the calculated parameters are compatible with the rolling mill hard and temperature and plate quality are within the requirement), Fig. 5 reveals that the inverse convergence calculation (from delivery thickness and roll separating force to entry thickness) was the majority of CPU consumption (about 95%). To tackle this problem, a parallel computation system using GPGPU is developed. The detailed description of computer hardware with GPGPU is demonstrated in Table 4.

4.2. Parallel Computing on GPGPU

In the trial schedule assembling of the first version of rolling setup model provides an iterative solution for calculating entry thickness. The iteration scheme, however, was found to repeat more than 30 times resulting in the excessive computation time that cannot be used online. The parallelisation of this flow to eliminate the convergence iteration is thus attempted using GPGPU (the parallel computing eliminates all the sequential steps to find \( h_{in} \) shown in Fig. 3).
From the maximum afforded rolling load, the maximum reduction in a rolling pass is set to be 40 mm and hence the first attempt of parallel computing is every 0.02 mm from delivery thickness to delivery + 40 mm with 2,000 threads. Every entry thickness trial calculation result is compared with the prescribed roll separating force and one of the best results will be chosen. Note that the maximum possible threads available for parallel computing are about 5 millions. However, the 2,000 parallel computing gives enough preciseness for the entry thickness calculation and too much parallelisation make the program code complex and may cause excessive data transmission time. In this case, the convergence loop, which potentially counts more than 50 times, is reduced to a single calculation. The overall computation time for the setup calculation has thus achieved then requirement of the online use which is less than 2 seconds.

5. Trial Rolling Application Results

To verify the physical model performance, the adaptive control calculation results of roll separating force are confronted with the measured ones. As is already mentioned above (and in Fig. 4), the roll separating force prediction even after CR with intermediate cooling is in good agreement with measured load. It is because the model traces dislocation density evolution during intermediate cooling process. The dislocation (strain) accumulation and the recovery (including static recrystallization) phenomena are in Fig. 6 exhibiting the effect of intermediate cooling.

The next application example is a comparison of roll separating force among the setup model, the adaptive control model and measured values. While all the measurable conditions, such as rolling speed, descaling and intermediate cooling conditions, are available for the adaptive control calculation, they are unknown variables for the setup calculation. It is because the adaptive calculation is conducted after each rolling pass so the variables are “experienced” ones, whilst the setup calculation is completed before the first rolling pass and thus the variables are “predicted” ones.

Table 4. Details of computer system used in the research.

| Component                        | Specification                                                                 |
|----------------------------------|-------------------------------------------------------------------------------|
| OS                               | CentOS release 6.5 (Final) 2.6.32-431.23.3.el6.x86_64                        |
| CPU                              | Intel(R) Xeon(R) CPU E5-2637 v3 @ 3.50 GHz                                   |
| GPU                              | NVIDIA Tesla K40c                                                            |
| CUDA Driver Version/Runtime Version | 7.5/6.5                                                                 |
| CUDA Capability Major/Minor version number | 3.5                                                                 |
| Total amount of global memory    | 11 520 MBytes (12 079 136 768 bytes)                                        |
| (15) Multiprocessors, (192) CUDA Cores/MP | 2 880 CUDA Cores                                                        |
| GPU Clock rate                   | 745 MHz (0.75 GHz)                                                           |
| Memory Clock rate                | 3 004 Mhz                                                                   |
| Maximum number of threads per multiprocessor | 2 048                                                               |
| Maximum number of threads per block | 1 024                                                                      |

Fig. 6. Evolution of temperature and dislocation density in conjunction with thickness change by rolling.

(a) Temperature

(b) Roll separating force

Fig. 7. Comparison of temperature and roll separating force among setup, adaptive control model and measurement.

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This makes the setup model even more difficult to achieve the precise prediction than the adaptive model. Figure 7 exhibits a comparison of roll separating force and temperature between prediction by setup and adaptive control and measured ones. It is clear that the adaptive control prediction agrees very well with the measured values. On the other hand, the setup model predicts a bit higher temperature than the measured one, resulting in the lower value prediction of the roll separating force. The reason why the setup model predicts higher temperature is that the descaling was used more than the setup initial guess resulting in the discrepancy between the setup and the adaptive control predictions. Despite the small discrepancies, the setup model and the adaptive control model predictions are found to have enough accuracy.

6. Conclusions

New online rolling adaptive control and setup models for plate rolling have been developed. These models consider the microstructure evolution from the reheating furnace to the finishing mill. The microstructure model enables to predict the precise flow stress change during hot rolling process as well as the intermediate cooling process.

For the setup model, entry thickness is calculated inversely from delivery thickness, maximum roll separating force and torque. The preliminary version of setup model uses an iterative scheme to solve it, which appears to take excessive computation time. Hence, a parallel computation using GPGPU was applied to eliminate the iteration resulting in the calculation time to be within the industrial requirements (2 seconds for each slab).

Though the trial tests, the model reveals that the assembled rolling schedule is enough accurate yielding a precise dimension of a plate.

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