A method of determining monitoring indexes for spatial deformation of super high arch dam based on extreme value theory

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Abstract: In the long service of arch dam, deformation is one of the main performance parameters which can directly reflect the working behavior of the dam. In order to effectively monitor the safety of dam operation, it is necessary to determine the monitoring control value of deformation. The traditional dam deformation monitoring control values only consider the single point, ignoring the spatial correlation between the measurement points. In order to overcome this limitation, this paper focuses on the spatial distribution characteristics of deformation measuring points of the arch dam, then by building dam deformation field and using the Projection Pursuit Analysis (PPA) project high-dimensional data to low dimensional space. On this basis, the POT model proposed to determine the monitoring control values. The PPA-POT model is applied to practical engineering, and the feasibility and applicability of the method in this paper are tested by comparing with the traditional methods, so as to provide a new analysis technique for the long-term service status safety evaluation of arch dams.

1. Introduction
The long-term deformation characteristic of concrete dam is an important representation of engineering safety, which can be effectively monitored and judged the operation behavior of the dam by the combination of instrument monitoring and numerical calculation[1]-[2]. Therefore, for a long time, the method of monitoring and evaluating the safety of the dam by monitoring the control value can provide more effective technical support for the safe operation of the project. The commonly methods to estimate the control value of monitoring effect quantity include mathematical statistics method and structural analysis method[3]-[4], among which it is usually convenient to use mathematical statistics method to determine the dam deformation monitoring control value for long-term monitoring data sequence, and it is widely used in practical projects.

In spite of this, dams are often equipped with multiple measurement points for monitoring, but the conventional dam monitoring control values are mostly based on a single measurement point, ignoring the spatial correlation between the measurement points. Therefore, this paper firstly constructs the spatial deformation field of super high arch dam, and then uses the Projection Pursuit Analysis to project the deformation field data into the low-dimensional space. Furthermore, the threshold is determined according to the graph method of Hill, and the POT model theory is applied to formulate the safety monitoring control value of the spatial deformation field of the dam body.
2. Model construction and weight determination method of super high arch dam space deformation field

2.1. Construction of super high arch dam space deformation field

In practice, a large number of different monitoring instruments are often arranged in different parts of the arch dam to monitor the working status of the dam. Therefore, it is necessary to comprehensively consider the correlation between different spatial monitoring quantities, so as to dig out the spatial characteristics and development rules of dam deformation from the data.

The displacement vector field generated by arch dam under external load can be decomposed into three directions: radial displacement $\delta_r$, tangential displacement $\delta_t$ and lead-straight displacement $\delta_u$, namely:

$$\mathbf{\delta} = \delta_r (x, y, z) \mathbf{i} + \delta_t (x, y, z) \mathbf{j} + \delta_u (x, y, z) \mathbf{k}$$  \hspace{1cm} (1)

Where $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ represent unit vectors in radial, tangential, and lead-straight directions, respectively. All radial displacement measuring points can be expressed as $\{\delta_{ij}|i = 1,2,3\ldots; j = 1,2,3\ldots,n\}$, where $\delta_{ij}$ is the $j$th measurement value of the $i$th measurement point.

On this basis, the spatial deformation values are transformed into synthetic deformation $\mathbf{\delta}'_j$ by using the Projection Pursuit Analysis:

$$\mathbf{\delta}'_j = w_i \hat{\delta}_{ij} \hspace{1cm} (j = 1,2,3\ldots,T)$$  \hspace{1cm} (2)

2.2. The weight of measuring points is determined by Projection Pursuit Analysis

The Projection Pursuit Analysis can project the high-dimensional monitoring data into the low-dimensional space, and the low-dimensional data is a projection vector that can reflect the characteristics of the original high-dimensional data by studying and analyzing the original high-dimensional data.

The measured value sequence after $T$ dimension normalization of radial displacement $\delta_{ij}$ is synthesized into the comprehensive projection value in the direction of projection $l = \{l_1, l_2, l_3, \ldots, l_T\}$:

$$T(i) = \sum_{j=1}^{T} l_j \hat{\delta}_{ij} \hspace{1cm} i = 1,2,3\ldots,n$$  \hspace{1cm} (3)

Where $l$ is the unit length vector, namely: $\sum_{j=1}^{T} l_j^2 (j) = 1$.

In this paper, the real coding based acceleration genetic algorithm (RAGA)[5] was used to optimize the projection index function, the optimal projection direction can be obtained, denoted as $l^*$. According to equation (3), the projection value of the sample point can be obtained. After processing the projection value, the weight value of each deformation measuring point can be obtained:

$$w_i = \frac{T^*(i)}{\sum_{i=1}^{n} T^*(i)} \hspace{1cm} i = 1,2,3\ldots,n$$  \hspace{1cm} (4)

Where, $w_i$ is the weight value of the $i$th measuring point; $T^*(i)$ is the optimal projection value of the measured value at the measuring point $i$. 


3. The monitoring and control values of spatial deformation field are determined Based on extremum theory

If the POT model is used to determine the dam safety monitoring control value, the size of the threshold \( u \) should be determined first, and then the sample distribution of the over threshold value should be studied. The parameter estimation methods mainly include graph method of Hill, average excess function method and peak value method. In this paper, the threshold value is determined according to graph method of Hill.

Let \( \{x_1, x_2, x_3, \cdots, x_n\} \) be \( n \) independent positive random variables with the same distribution, and its reverse order statistic is \( x_{n,1} \geq x_{n-1,1} \geq x_{n-2,1} \geq \cdots \geq x_{1,1} \), and \( H_{k,n} \) is the Hill estimation of the sample sequence:

\[
H_{k,n} = \frac{1}{k} \sum_{j=1}^{k} j \left( \log x_{n-j+1,n} - \log x_{n-j,n} \right) \quad 1 \leq k \leq n-1
\]

The curve formed by the set of points \( \{(k,1/H_{k,n}), 1 \leq k \leq n-1\} \) is called the Hill diagram. By selecting the starting point of a relatively stable region on the curve, the threshold \( u \) is the value corresponding to the stable starting point, and then the maximum logarithmic likelihood function is used to solve the parameters \( \sigma \) and \( \varepsilon \). After determining the parameters \( u, \varepsilon, \sigma \), the distribution function of the threshold value can be determined. According to the level of the dam, the failure probability can be determined to solve the monitoring index at a certain point. The failure probability function is:

\[
P(x < x_m) = P_a = \int_{x_0}^{x_m} f(x)dx
\]

Where \( f(x) \) is the probability density function. Generally \( P_a \) takes 1%~5%, so the estimated value \( \delta_m \) is:

\[
\delta_m = \begin{cases} 
  u + \frac{\sigma}{\varepsilon} \left( \frac{n}{P_a} - 1 \right) & (\varepsilon \neq 0) \\
  u - \sigma \ln \left( \frac{n}{P_a} \right) & (\varepsilon = 0)
\end{cases}
\]

4. Engineering example

A hydropower station is a water conservancy project with integrated benefits of power generation, flood control, irrigation and water transportation in the reservoir area. The water-retaining structure is a super high arch dam with the height of 294.5m, crest elevation of 1245m, lowest foundation elevation of 953m, crest length of 992.74m and base width of 69.49m. The layout of measuring points is shown in figure 1. In order to monitor the deformation of arch dam crest, plumb line measuring points (located at sections 4, 9, 15, 19, 22, 25, 29, 35 and 41 respectively) on the dam crest were selected. In this paper, 2191 radial displacement data (in total) from the above measurement points on January 1, 2013 solstice, December 31, 2018 were selected as samples, and the gross error was removed to perform linear interpolation on the vacancy value.
4.1. The weight of each measuring point in the space deformation field was determined

The projection weight \( w_i \) is obtained by reducing the dimension of the multidimensional data in the spatial deformation field through the Projection Pursuit Analysis, as shown in table 1. According to equation (3), the comprehensive displacement of the spatial deformation field can be obtained from the data of the arch ring plumb line measurement points on the dam crest, as shown in figure 2, indicating that the process line of the comprehensive displacement can well reflect the radial displacement of the entire arch ring.

| measuring point | C4-A04-PL-01 | C4-A09-PL-01 | C4-A15-PL-01 | C4-A19-PL-01 | C4-A22-PL-01 |
|-----------------|--------------|--------------|--------------|--------------|--------------|
| weight \( w_i \) | 0.1102       | 0.1217       | 0.1164       | 0.1160       | 0.1134       |

| measuring point | C4-A25-PL-01 | C4-A29-PL-01 | C4-A35-PL-01 | C4-A41-PL-01 |
|-----------------|--------------|--------------|--------------|--------------|
| weight \( w_i \) | 0.1204       | 0.1123       | 0.1136       | 0.0760       |

4.2. Formulation of deformation monitoring indicators

The above spatial deformation field's comprehensive displacement sequence was analyzed, and Q-Q map method was used to carry out thick tail test on the displacement historical monitoring data. The results are shown in figure 3. As can be seen from figure 3, the middle part of the displacement monitoring sequence is approximately linear and slightly concave, with the lower end warped to the left and the upper end bent downward, which conforms to the distribution characteristics of the thick tail and meets the prerequisite for the establishment of the extreme value theory POT model. The threshold was determined by using the graph method of Hill, and graph was drawn as figure 4. The threshold was
determined through the graph change process. According to figure 4, the Hill graph tends to be stable when the number of threshold values is 85. Based on the above description, the calculation parameters of POT model are listed, as shown in table 2.

![Figure 3](image3.png)  Q-Q chart of thick tail inspection of monitoring data.  

![Figure 4](image4.png)  Hill graph of monitoring data.  

| parameter | \( n \) | \( n_t \) | \( \mu \) | \( \xi \) | \( \sigma \) |
|-----------|------|------|------|------|------|
| numerical value | 2191 | 85 | 71.3 | -0.630 | 4.718 |

the engineering level for accident probability 1% combined with big I type engineering. The space deformation monitoring control value of the super high arch dam can be obtained by calculating the parameters of equation (7), which is \( \Theta_m = 76.86 \text{mm} \).

5. Conclusion
In this paper, according to the spatial structure correlation characteristics of the super high arch dam space measurement points, the weight is determined according to the inherent laws and characteristics of the data, and the dam space deformation field is constructed. Then the method of determining the monitoring control value of super high arch dam space deformation based on the extreme value theory is proposed. As the application of engineering example shows that the data distribution of the over threshold sequence can well reflect the characteristics of the dam to resist the load, the monitoring control value based on the over threshold sample is appropriate. Compared with the traditional method, it can effectively reflect the state of multi-point spatial deformation field, and the proposed monitoring control value can fully measure the overall safety status of the dam.

Acknowledgments
This work was supported by National Natural Science Foundation of China (Grant Nos. 51739003, 51909173), Free exploration project of Hohai University (No.B200201058), Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions (YS11001), Basic Research Project Funded of National Key Laboratory(20195025912). The authors sincerely appreciate the help of Hohai University and the support of the above funds.

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