A model of the ball lightning
V.K.Ignatovich
Laboratory of Neutron Physics, Joint Institute for Nuclear Research, 141980 Dubna
Moscow region, Russia

The ball lightning is supposed to be a shock wave of a point explosion frozen with electrostriction forces of the internal strong laser discharge. The life time of the ball with modest parameters is calculated.

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After many years I decided to rewrite the article about electromagnetic model of the ball lightning, which was first published in Russian in 1980 [1], and later in 1992 [2].

Electromagnetic models of ball lightning were considered, for instance, in papers [3]-[6] though in [7], it is said that they can not explain the observed parameters. Most of the authors treated the ball lightning as a plasma formation. In particular, in [4] the ball lightning was supposed to be a cavity with radiation locked by surrounding plasma.

One similar model was also considered in [1]. In that model the photons were contained in a sphere surrounded by a gas with anomalous dispersion at frequency of the stored radiation. There and in [2] it was also considered another model which can be visualized as a shock wave of a point explosion in the atmosphere. This shock wave can be stopped and frozen by a powerful laser discharge behind the shock wave front, if it arises in whispering gallery mode. The stopped shock represents a thin spherical shell filled with electromagnetic radiation. Radiation is retained in the shell because of the total internal reflection, and the shell is kept integer because of electrostriction forces and surface tension created by the radiation.

This model can explain both the high energy and the long life time of the ball lightning. More over it throws some light on the nature of other phenomena, such as hurricane and tornado and can be used in many other branches of physics. In [8] it is also described how such a model can explain unusual properties of the ball lightning, its ability to penetrate indoors through slits and windowpanes, and to move against wind.

To show how the model works we use an analogy with quantum mechanics of a particle. Since the stationary equation for electric field $E$ (for simplicity we use here the scalar approximation)

$$[\Delta + n^2 k_0^2]E(\mathbf{r}) = 0,$$

where $k_0 = \omega/c$, $\omega$ is the photon frequency, $c$ is the light speed in vacuum and $n$ is the index of refraction, is very similar to the Schrödinger equation

$$[\Delta + k^2 - u(\mathbf{r})]\psi(\mathbf{r}) = 0,$$
for particles [9]. Here $\psi$ is the wave function of a particle with wave number $k^2$, $u$ is a potential energy measured in units $\hbar^2/2m$ and $m$ is the mass of the particle.

To transform equation (1) to the form (2) it is necessary only to denote

$$u = (1 - n^2)k_0^2.$$  \hfill (3)

For the equation (2) it is known that, if $u$ is a potential well, the particle can have a bound state inside it. The same can be said about the photon. If the potential well is inside a matter, and the well depth increases with increase of the matter density, then the particle in a bound state in such a well is not only kept by the well but also compresses the matter. This is the analog of electrostriction forces.

Let us illustrate such an effect with the help of a slow neutron. It is known (see, for example [10]) that interaction of neutrons with nuclei creates neutron-matter interaction potential $u = 4\pi N_0 b$, where $N_0$ is the number of atoms in a unit volume, and $b$ is the coherent scattering amplitude. If $b > 0$ then $u > 0$ and matter repels the neutron. However if $b < 0$, the matter attracts the neutron. So a substance with a negative $u$ is a potential well for neutrons. But, and it is very important, because $u$ is proportional to atomic density, not only matter holds neutron in the well, but the neutron itself, if it is in a ground state, also holds the matter, i.e., it resists expansion of the matter.

Indeed, it is shown on fig.1 that, when the matter expands, the density $N_0$ and the well depth become lower. It leads to rise of the lowest energy level. Therefore expansion requires some work to increase the neutron energy.

\[
\begin{array}{c|c|c}
 U & U - dU & E + dE \\
 E & - & - \\
 r & - & - \\
 r + dr & - & - \\
\end{array}
\]

Fig.1

Figure 1: Illustration of striction of matter by a particle in the ground state in a well created by the matter.

It is easy to calculate the force with which the particle in the ground state compresses the matter. The equation for bound state energy can be
represented as
\[ \exp(2ikr)\rho^2(k) = 1, \tag{4} \]
where \( k = \sqrt{2mE/\hbar^2} \), \( m \) is the particle mass, and the bound state energy \( E > 0 \) is counted from the bottom of the well. This (4) is a condition for stationarity. It means, that if wave function of the particle in the well moving to the right has some value near the right wall, then after reflection with amplitude \( \rho(k) \) from that wall, propagation to the left, reflection with the same amplitude (we assume the well to be symmetric) from the left wall, and coming back to the right wall should have the same initial value of its wave function. Reflection amplitude is
\[ \rho(k) = \frac{k - ik'}{k + ik'} = \exp(-2i\varphi), \quad \varphi = \arccos(k/\sqrt{u}), \tag{5} \]
where \( k' = \sqrt{u - k^2} \) and \( u \propto 1/r \) determines the depth of the well. Equation (4) for the ground state is reduced to
\[ kr - \arccos(k/\sqrt{u}) = 0, \quad \cos(kr) = k/\sqrt{u}, \tag{6} \]
and it is a simple exercise to calculate the force \( dE/dr \) from this equation. The force is negative, which means compression of the substance.

Of course, the striction force created by a single particle is small. But if the number of particles large, the force can be also large. This may happen to be very important for, say, neutron stars. In a neutron star every neutron has a potential \( u = 4\pi N_0 b \), where \( N_0 \) here is the density of neutrons. The compression energy here is proportional to \( N_0^2 \). It is very high and may be even greater than the gravitational one. This question is considered in [12].

In the paper [6] the analogous interaction was considered for electrons. It is shown, that at some density of plasma exchange attractive interaction can become larger than direct Coulomb repulsion, and it leads to the coherent binding of plasma particles.

All these considerations can be applied also to \( \gamma \)-quanta. And it leads to our model for the ball lightning. Indeed \( n^2 = \epsilon = 1 + 4\pi N_0 \alpha \), where \( N_0 \) is the number of molecules in a unit volume and \( \alpha \) is polarizability of a molecule. So, the potential
\[ u = (1 - n^2)k_0^2 = -4\pi N_0 \alpha k_0^2, \tag{7} \]
is negative for positive \( \alpha \). It means that usually matter attracts photons, and this attraction is known as ”pondermotive force” and self focussing. But in analogy with quantum mechanics of a particle we can speak also about bound levels of photon, and such approach will give us a possibility to estimate the life time of the ball.
In the case of anomalous dispersion or at high frequencies the permittivity $\epsilon$ can be less then unity. In these cases $\alpha < 0$, and matter repels photons. This happens, for instance in the case of interaction with plasma.

Interaction of atoms and molecules with electromagnetic field is described by the expression

$$U_1 = -dE = -\alpha E^2 = -4\pi \alpha N_\gamma \hbar \omega,$$

where $d = \alpha E$ is the induced dipole moment (we suppose the absence of own dipole moment), $\alpha$ is the polarizability of the molecule and $N_\gamma$ is the number density of photons. This interaction shows, that the forces drawing matter inside the photon field is proportional to gradient of its density.

From quantum mechanics it follows [13, 14], that

$$\alpha = \frac{e^2}{2m_e} \sum_{k \neq 0} \frac{f_{0k}}{\omega_{0k}^2 - \omega^2 - i \omega \Gamma_k},$$

where $\omega$ is the incident photon frequency, $e$ is the electric charge, $m_e$ is the mass of the electron, $\omega_{kl} = \omega_k - \omega_l$ are the eigen frequencies defined by transition $k \rightarrow l$, $\omega_k$ are energy levels of the electron in the atom, $\Gamma_k$ is the width of the transition, $f_{lk}$ are oscillator strengths given by

$$f_{kl} = \frac{(2m_e/\hbar^2)\hbar \omega_{lk} |d_{kl}|^2},$$

and $d_{kl} = \langle k| r |l \rangle$ is a matrix element of dipole transition between states $|l\rangle$ and $|k \rangle$.

For $\omega < \omega_{01}$ an unexcited atom is pulled into regions of space with larger density of photons and, if it is excited, it will with the highest probability emit the same photon, which is already present in the field, because the matrix element of the transition is proportional to the square root of the total number of the photons present in the mode. So, the photons in the intense coherent field can not be scattered and atoms are stabilized [15]-[17].

This happens to neutral molecules. But electrons and ions are repelled by electromagnetic field, because their eigen frequencies in space can be put equal to zero. It means, that if ionization happened at the explosion then the light electrons will fly before the shock wave front, and after creation of thin spherical layer filled with intense electromagnetic field electrons remain to be separated from ions left behind the shock wave, and we obtain a charged spherical capacity.

Let us consider parameters of the ball lightning. We shall take radius and the energy of it to be given and equal to 10 cm and 10 kJ respectively and then estimate its life time.
In the spherical coordinate system the equation for radial part of the electromagnetic potential

\[ A(r) = \frac{R_L(r)}{r} P_L(\theta) \exp(i m \phi), \tag{11} \]

looks like

\[ \left[ \frac{d^2}{dr^2} + k^2 - u(r) - \frac{L(L+1)}{r^2} \right] R_L(r) = 0, \tag{12} \]

where \( L \) is an orbital momentum of photons, or (12) can be represented in the form

\[ \left[ \frac{d^2}{dr^2} + k^2 - V(r) \right] R_L(r) = 0, \tag{13} \]

with the effective potential \( V(r) = \frac{L(L+1)}{r^2} + u(r) \) Fig. 2. The potential is positive, and \( u \) represents a ”pocket” on a monotonously decreasing centrifugal potential curve. The photons have a metastable state in this pocket. Since the scattering is prohibited, the only way photons can leave this pocket is through tunnelling.

Let the wave length of trapped radiation be \( \lambda = 10^{-4} \) cm. Then, for sphere of radius \( r_0 = 10 \) cm we get \( L = kr/2\pi \approx 10^5 \). The life time \( T \) can be estimated by expression \( T = t_f/P \), where \( t_f \) is free flight time between two collisions with the shock wave front and \( P \) is the probability of tunnelling through the potential barrier. Since \( t_f < 10^{-10} \) s, the probability \( P \) must be very low. Let us find \( P \) with the usual quasiclassical approximation of quantum mechanics.

\[ P = \exp(-2\gamma), \quad \gamma = \int_R^{r_2} \sqrt{L^2/r^2 - k^2} \, dr. \tag{14} \]
The integration limits are determined by the relations
\[ (L/R)^2 = k^2 + |u|, \quad (L/r_2)^2 = k^2. \] (15)

At large \( L \) and small \( u \) the integral in (14) can be approximated by the expression
\[ \gamma = \int_0^{x_2} \sqrt{|u| - 2L^2x/R^3} \, dx = \frac{1}{3} \left( \frac{|u|}{k^2} \right)^{3/2} L, \] (16)

where \( x_2 = r_2 - R \). To get lifetime near \( 10^4 \) s it is necessary to have \( \gamma \approx 20 \) and for \( L = 10^5 \) the value of \( |u|/k^2 = \epsilon - 1 \) should be \( \approx 10^{-2} \). It gives the magnitude of the refraction index \( \epsilon \) inside the shock wave to be of the order of 1.007.

The angle \( \phi \) of total reflection is defined from \( \sin \phi = 1/n \). It shows that the width of the photon layer is
\[ d = r_0[1 - \sin \phi] \approx 0.01r_0 \approx 0.1 \text{ cm}. \] (17)

All these parameters are not extraordinary, so the life time of order 10 000 seconds seems to be quite achievable.

If total energy is concentrated in photons, then the layer must contain \( 10^{23} \) photons of energy 1 eV each. The density in the layer then is equal \( N_\gamma \approx 10^{27} \text{ m}^{-3} \). At such a density the surface tension is \( \sigma = (\epsilon - 1)\hbar \omega N_\gamma d \approx 10^3 \text{ J/m}^2 \). The surface tension is defined as \( \sigma = -dE_s/dS \), where \( E_s \) is the total energy of the layer, which here is \( E_s = (1 - \epsilon)\hbar \omega N_\gamma S d \), and \( S \) is the surface of the sphere.

Such a surface tension creates compressing pressure \( p = 2\sigma/r = 2 \cdot 10^4 \text{ J/m}^3 \). Since the normal atmospheric pressure is of the order \( 10^5 \text{ J/m}^3 \), the gas density inside the ball to withstand the compression should be only 20\% higher than outside pressure, or the temperature inside gas should be only 60 K higher than outside. Because of higher density inside the photon film, the ball is heavier than environment and falls down. If the gas density in the ball is lower than outside, its temperature must be higher, and the ball can be lighter than the air.

For photon frequencies very close to a resonance the magnitude of \( n^2 - 1 \) can be higher, and therefore the higher will be the surface tension and gas temperature inside the ball. Situation improves even more, if one takes into account the Lorenz-Lorentz correction.

The ball can be also a charged spherical capacitor, if during the point explosion a separation of charges takes place. For instance the light electrons will fly faster and become before the shock wave front, while heavier positive ions remain behind it. The energy of the capacitor depends on its charge. Let us suppose that the charge is equal to \( Q \). An outside electron is attracted
by the charge of ions with the force \( F_q = Ee = 9 \times 10^9 Qe/r_0^2 = Q \times 10^{-7} \) N. But the photons repel it. The interaction energy of an electron with the photon layer is

\[
u_e = \pi (e^2/mc^2) \lambda^2 \hbar \omega N_\gamma. \tag{18}\]

Let us show how to derive it with the help of the Dirac equation

\[
\left[ \gamma \left( p - \frac{e}{c} A \right) - mc \right] \psi = 0, \tag{19}
\]

where \( A \) is a four dimensional vector potential: \( A = (-A, \Phi) \), \( \Phi \) is its scalar and \( A \) is its three dimensional vector part.

We can transform (19) into equation of the second order multiplying it from the left by

\[
\left[ \gamma \left( p - \frac{e}{c} A \right) + mc \right],
\]

which gives

\[
\left[ \left( p - \frac{e}{c} A \right)^2 - m^2c^2 - \frac{e\hbar}{c} [\Sigma H - i\alpha E] \right] \psi = 0. \tag{21}
\]

We neglect \( \Phi \) and average of (21) over fast oscillations of the field. Then (21) is facilitated to

\[
\left[ p_0^2 - m^2c^2 - \hat{p}^2 - \left( \frac{e}{2c} \right)^2 A^2 \right] \psi = 0. \tag{22}
\]

We divide it by \( 2m \) (\( m \) is electron mass) and reduce (22) to nonrelativistic form

\[
\left[ -\frac{\hbar^2}{2m} k^2 + \frac{1}{2m} \hat{p}^2 + \frac{e^2}{4mc^2} A^2 \right] \psi = 0. \tag{23}
\]

Since \( E = dA/dt \), then we can substitute \( A^2 = (c/\nu)^2 E^2 = \lambda^2 E^2 \), and use the energy density in the form \( E^2/4\pi = \hbar \omega N_\gamma \). As a result we obtain

\[
\left[ -\frac{\hbar^2}{2m} \Delta + \pi \frac{e^2}{mc^2} \lambda^2 \hbar \omega N_\gamma - \frac{\hbar^2}{2m} k^2 \right] \psi = 0, \tag{24}
\]

where potential energy is given by (18).

The repulsive force is proportional to the gradient of \( N_\gamma \). The distribution of gamma quanta is determined by the Bessel function \( J_L(kr) \), so \( dJ_L(kr)/dr \approx (L/r_0)J_L(kr) \). It means that the force can be estimated as \( F_e = Lu_e/r_0 \), or \( F_e \approx 10^{-12} \) N. This force can withstand the attraction only if the charge is \( Q \leq 10 \) µCoul. So the total energy of the capacitor is of the order of 1 J, which is considerably smaller of the total energy. But this is true for a single electron. For a negative ion the potential (18) can be two order of magnitude higher, and it increases \( Q \) and its electrostatic energy.
To create the ball lightning it is necessary to make a point explosion inside a medium, where the shock wave makes excitation of atoms. Also it is possible to use an external pumping. The question is whether the laser discharge will have enough time to be developed.

To answer this question we compare the time of the light passage around the ball with that of the shock wave passage over the distance $d$. The first time is equal to $T_l = \frac{2\pi r_0}{c} \approx 10^{-9}$ s. The second one, $T_s$, is defined by an automodel solution [18]:

$$r = \left(t^2 W/\rho \right)^{1/5},$$

where $\rho$ is the density of the atmosphere and $W$ is the energy of the explosion. The speed of the front is equal to

$$v = dr/dt = \left(\frac{2}{5}\right)r/t = \left(\frac{2}{5}\right)\left(W/\rho \right)^{1/2}r^{-3/2}.\tag{26}$$

At $W = 10^4$ J, $\rho = 1$ kg/m$^3$, $r = 0, 1$ m the speed is $v \approx 10^3$ m/s. So the $T_s \approx 10^{-6}$ s. It shows that $T_l \ll T_s$, therefore the laser discharge has enough time to be developed.

It is not necessary that each point explosion will lead to a ball lightning. Probability of the ball creation is proportional to the probability of emitting a photon in the WGM mode.

It is interesting that external excited atoms incident on the ball may be reflected or deexcited. The last channel is the most probable. After deexcitation the photon layer pulls the atom inside it, so the ball moves in the direction of the positive gradient of the density of excited atoms and eats them up.

In fact, to have a stable ball it is not necessary that all photons inside the ball skin must be coherent. The coherence is necessary for fast process. If time scale is large enough we can obtain a similar object with incoherent radiation. It is possible that the origin of hurricanes and tornado can be related to similar processes (see also [19]).

Till now we considered the scalar case. A spherical solution for vector electromagnetic field [20] in a reference frame moving with a small velocity $k$ can be represented, for instance, in the form

$$E = C \exp(ikr - i\omega t) \times$$

$$\times \left[ \sqrt{\frac{l}{2l + 1}} Y_{l,l+1,M}(r')j_{l+1}(s|\mathbf{r} - k\mathbf{t}|) + \sqrt{\frac{l + 1}{2l + 1}} Y_{l,l-1,M}(r')j_{l-1}(s|\mathbf{r} - k\mathbf{t}|) \right],\tag{27}$$

similarly to nonspreading wave packet in quantum mechanics [21], where $Y_{j,l,M}(r')$ is a vector spherical harmonics, and $r' = (r - kt)/|\mathbf{r} - k\mathbf{t}|$. 
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