Walking in the third millennium.

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(Dated: September 11, 2006)

Based on recent progress in the study of strong dynamics in conformal field theories, I construct a simple, one family model of dynamical electro-weak symmetry breaking. Non-perturbative effects can be computed systematically. The model does not suffer from the well known, parametrically big, phenomenological difficulties of traditional technicolor models, is compatible with all present experimental data and is testable at the LHC.

PACS numbers: 11.10.Kk, 12.15.Lk, 12.60.Nz

INTRODUCTION

Models of dynamical electro-weak symmetry breaking suffer from well-known parametrically big phenomenological problems. A possible solution consists of assuming that the dynamics be very different from QCD, with an energy range of quasi-conformal behavior at strong coupling near the confinement scale and the presence of large anomalous dimensions (walking). This provides a parametric enhancement of fermion masses, and might soften the problems with precision electro-weak data. But the lack of a reliable computational tool for the non-perturbative effects has been a strong limitation on quantitative studies.

Some peculiar four-dimensional strongly interacting field theories, based on $SU(N_T)$ gauge theories at large-$N_T$, admit a dual description in terms of a weakly coupled gravity theory in a higher dimensional space-time. This is the conjectured AdS-CFT correspondence. It is the ideal tool for the description of walking technicolor and allows to address the long-standing problem of estimating the magnitude of non-perturbative effects on precision electro-weak parameters.

In this letter, I propose a very simple model of walking technicolor, formulated in the language of the AdS-CFT correspondence, in which non-perturbative effects are calculable. The model is not complete. It lacks a dynamical explanation for the hierarchical pattern of fermion masses. It also does not provide a UV-complete four-dimensional Lagrangian valid at energies above the scale at which conformal symmetry is lost.

This simple one-family model illustrates how a modern approach to the physics of strong-interacting, conformal field theories, leads very naturally to the solution of long-standing problems of model-building, and allows to construct realistic, calculable and testable models of walking technicolor. Fine-tuning problems with the $S$ and $T$ parameters and the top mass $m_t$ are easily softened by one order of magnitude. Higher-order operators are suppressed by the UV cut-off at which conformal symmetry is lost, estimated to be in the $5 - 10$ TeV range. The bounds from precision electro-weak physics can be satisfied even in the large-$N_T$ regime, in contrast to what suggested by perturbative estimates. The low-energy phenomenology is the one of the standard model (SM) with a heavy, broad, strongly interacting and model-dependent scalar Higgs. The model is testable: a set of four spin-1 excited states have degenerate masses in the $2 - 4$ TeV range, with strong couplings to the SM currents, and hence unsuppressed production and decay rates at the LHC.

THE MODEL.

The energy window above the electro-weak scale is described by a slice of $AdS_5$, a five-dimensional space-time containing a gravity background with the metric:

$$ds^2 = \left(\frac{L}{z}\right)^2 \left(\eta_{\mu\nu}dx^{\mu}dx^{\nu} - dz^2\right),$$

where $x^\mu$ are four-dimensional coordinates, $\eta_{\mu\nu}$ the Minkoski metric with signature $(+,−,−,−)$, and $z$ is the extra (warped) dimension. $L$ is the $AdS_5$ curvature. Conformal symmetry is broken by the boundaries $L_0 < z < L_1$, with $L_0 > L$. $L_0$ and $L_1$ correspond to the UV and IR cut-offs of the conformal theory. At energies above $1/L_0$ the four-dimensional strong-interacting sector is no longer conformal, and the gauge coupling of the dual $SU(N_T)$ theory runs toward asymptotic freedom. The SM with a heavy, strongly interacting, Higgs boson is recovered below $1/L_1$.

The field content in the bulk of the five-dimensional model consists of a complex scalar $\Phi$ transforming as a $(2,1/2)$ of the gauged $SU(2)_L \times U(1)_Y$. The generators of $SU(2)_L$ are $T_i = \tau_i/2$ with $\tau_i$ the Pauli matrices. The bulk action for $\Phi$ and the gauge bosons $W = W_i T_i$ and $B$ is

$$S_5 = \int d^4x \int_{L_0}^{L_1} dz \sqrt{G} \left[ (G^{MN}(D_M\Phi)^\dagger D_N\Phi - M^2|\Phi|^2) \left( -\frac{1}{2} \text{Tr} (W_M W_R S) - \frac{1}{4} B_{MN} B_{RS} \right) G^{MR} G^{NS} \right].$$

This is the conjectured AdS-CFT correspondence.
with the boundary terms given by
\[ S_4 = \int d^4x \int_{L_0}^{L_1} dz \sqrt{g} \left[ \delta(z - L_0) g^{\mu\nu} G_{\mu\nu} \right] 
\]
\[ \left( -\frac{1}{2} D \text{Tr} [W_{\mu
u} W_{\rho\sigma}] - \frac{1}{4} D B_{\mu
u} B_{\rho\sigma} \right) 
\]
\[ -\delta(z - L_i) 2\lambda_i \left( |\Phi|^2 - \frac{v^2}{2} \right)^2 \]
where the \( D_M \Phi \) is the covariant derivative and \( i = 0,1 \).

in the action, \( M^2 = -4/L^2 \) is a bulk mass term for the scalar, and \( g \) and \( g' \) are the (dimensionful) gauge couplings in five-dimensions.

Electro-weak symmetry breaking is induced by the VEV of \( \Phi \):
\[ \langle \Phi \rangle = \frac{v(z)}{\sqrt{2}} \left( \begin{array}{c} 0 \\ 1 \end{array} \right) . \]
In the \( \lambda_i \to +\infty \) limit, the bulk equations admit the solution
\[ v(z) = \frac{v_1}{L_1^2} z = \frac{v_0}{L_0^2} z^2 . \]
With this choice symmetry-breaking is triggered by a chiral condensate of dimension \( d = 2 \), i.e. a bilinear fermionic condensate with anomalous dimension \( \gamma = 1 \).

In the following, I focus on the spin-1 sector of the model in unitary gauge. The complete Lagrangian, including the appropriate gauge-fixing terms, is discussed in [3].

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**ELECTRO-WEAK PHENOMENOLOGY.**

I define:
\[ V^M = \frac{g' W_3^M + g B^M}{\sqrt{g^2 + g'^2}} , \]
\[ A^M = \frac{g W_3^M - g' B^M}{\sqrt{g^2 + g'^2}} , \]
so that the massless mode of \( V^\mu \) is the photon, and the lightest mode of \( A^\mu \) is the \( Z \) boson.

After Fourier transformation in the four-dimensional Minkoski coordinates:
\[ A^\mu(q,z) \equiv A^\mu(q) v_Z(z,q) , \]
and analogous for \( W_{1,2} \) and \( V \), where \( q = \sqrt{q^2} \) is the four-dimensional momentum. The bulk equations are:
\[ \partial_z \frac{L}{z} \partial_z v_i - \mu_i^4 L z v_i = -q^2 \frac{L}{z} v_i , \]
where \( i = v, Z, W \), with \( \mu_v = 0 \), \( \mu_W^4 = 1/4 g^2 v_0^2 / L^2 \) and \( \mu_Z^2 = 1/4 (g^2 + g'^2) v_0^2 / L^2 \).

The bulk equations can be solved exactly. The matrix of the polarizations \( \pi_i(q^2) \) of the SM gauge bosons can be written in terms of the action evaluated at the UV boundary:

\[ \pi_+ / N^2 = Dq^2 + \frac{\partial_z v_W}{v_W} (q^2, L_0) , \]
\[ \pi_{BB} / N^2 = Dq^2 + \frac{g^2}{g^2 + g'^2} \frac{\partial_z v_W}{v_W} (q^2, L_0) + \frac{g'^2}{g^2 + g'^2} \frac{\partial_z v_Z}{v_Z} (q^2, L_0) , \]
\[ \pi_{WB} / N^2 = \frac{g g'}{g^2 + g'^2} \left( \frac{\partial_z v_W}{v_W} (q^2, L_0) - \frac{\partial_z v_Z}{v_Z} (q^2, L_0) \right) , \]
\[ \pi_{WW} / N^2 = Dq^2 + \frac{g'^2}{g^2 + g'^2} \frac{\partial_z v_W}{v_W} (q^2, L_0) + \frac{g^2}{g^2 + g'^2} \frac{\partial_z v_Z}{v_Z} (q^2, L_0) , \]

where \( N \) is chosen to produce canonical kinetic terms in the limit in which the heavy resonances decouple. The precision electro-weak parameters are defined as
\[ S = \frac{g_4}{g_4^{(i)}} \pi_{WB}(0) , \]
\[ T \equiv \frac{1}{M_W^2} \left( \pi_{WW}(0) - \pi_{+}(0) \right) , \]
where \( g_4^{(i)} \) are the (dimensionless) gauge couplings of the SM in four-dimensions, and where \( \pi' \equiv d\pi / dq^2 \).

Taking the limit \( L_0 \to L \), and expanding for small \( L_0 \to 0 \), from [3]:

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\[
\frac{\partial v_e}{v_e}(q^2, L_0) = q^2 L_0 \left( \frac{\pi V_0(q L_1)}{2 J_0(q L_1)} - \left( \gamma_E + \ln \frac{q L_0}{2} \right) \right),
\]
\[
\frac{\partial v_Z}{v_Z}(q^2, L_0) = L_0 \left\{ \mu_Z^2 - q^2 \left[ \gamma_E + \ln(\mu_Z L_0) + \frac{1}{2} \ln \left( -\frac{q^2}{4 \mu_Z^2} \right) - \frac{c_2}{2 c_1} \Gamma \left( -\frac{q^2}{4 \mu_Z^2} \right) \right] \right\},
\]

where, after imposing Neumann boundary conditions in the IR,
\[
c_1 = 2L \left( -1 + \frac{q^2}{4 \mu_Z^2}, \mu_Z^2 L_1^2 \right) + L \left( \frac{q^2}{4 \mu_Z^2}, -1, \mu_Z^2 L_1^2 \right),
\]
\[
c_2 = -U \left( -\frac{q^2}{4 \mu_Z^2}, 0, \mu_Z^2 L_1^2 \right) + \frac{q^2}{2 \mu_Z^2} U \left( 1 - \frac{q^2}{4 \mu_Z^2}, 1, \mu_Z^2 L_1^2 \right),
\]

and where \( \partial v_W/v_W = \partial v_Z/v_Z(\mu_Z \to \mu_W) \).

The localized counterterm
\[
D = L_0 \left( \ln \frac{L_0}{L_1} + \frac{1}{\varepsilon^2} \right)
\]
cancels the logarithmic divergences, and for \( \mathcal{N}^2 = \varepsilon^2/L_0 \) all the dependence on \( L_0 \) disappears (at leading order in \( L_0 \)), the limit \( L_0 \to 0 \) can be taken, and the model is renormalized, with finite SM couplings.

Expanding for \( \mu_Z^2 L_1^2 \ll 1 \),
\[
\hat{S} = \varepsilon \frac{1}{2} \mu_W^2 L_1^2,
\]
\[
\hat{T} = \frac{\varepsilon}{M_W^2} \left( \mu_W^2 \tanh \frac{\mu_W^2 L_1^2}{2} - \frac{\mu_W^2}{\mu_Z^2} \tanh \frac{\mu_Z^2 L_1^2}{2} \right)
\]
\[
\simeq \frac{\varepsilon}{M_W^2} \frac{\mu_W^4 L_1^6}{24} (\mu_Z - \mu_W),
\]
with \( \varepsilon \simeq 2.7 \).

The spin-1 spectrum consists of the massless photon, the \( W \) and \( Z \) gauge bosons, and towers of their excited states. The four towers are essentially degenerate: the splitting between the first excited state of \( W \) and \( Z \) is parametrically smaller than their mass, for which all the mass is \( M_{\rho} = k / L_1 \) of the first excited state of the photon, the lightest techni-p resonance, where \( k \in \{2.4, 4.7\} \) grows with \( \varepsilon \). The mass of the \( W \) gauge boson is approximately given by
\[
M_W^2 \simeq \varepsilon^2 \left( \mu_W^2 \tanh \frac{\mu_W^2 L_1^2}{2} \right) \simeq \frac{1}{2} \mu_W^2 \mu_Z^2 L_1^2,
\]
while \( M_Z^2 \simeq (g^2 + g'^2)/g^2 M_W^2 \). Equivalently,
\[
M_W^2 = \frac{1}{8} \varepsilon^2 g^2 \eta^2 \left( \frac{L_0}{L_1} \right)^2 = \frac{1}{4} g^2 \eta^2,
\]
where \( \eta \) is related to the Fermi decay constant \( G_F \) by
\[
\eta^2 = L_0 \left( \frac{L_0}{L_1} \right)^2 = \frac{1}{\sqrt{2} G_F} \simeq (246 \text{ GeV})^2.
\]

Substituting in the precision parameters yields:
\[
\hat{S} \simeq \frac{1}{6} M_W^2 L_1^2 = \frac{k^2 M_W^2}{\varepsilon^2 M_{\rho}^2},
\]
\[
\hat{T} = \frac{M_Z^2 - M_W^2}{6 \varepsilon^2} L_1^2 = \frac{k^2 M_Z^2 - M_W^2}{6 \varepsilon^2 M_{\rho}^2}.
\]

I take as indicative of the experimentally allowed ranges (at the 3\( \sigma \) level):
\[
\hat{S}_{\text{exp}} = (-0.9 \pm 3.9) \times 10^{-3},
\]
\[
\hat{T}_{\text{exp}} = (2.0 \pm 3.0) \times 10^{-3},
\]
from [2]. These bounds are extrapolated to the case of a Higgs boson with mass of 800 GeV. The comparison has to be done with some caution. The one-loop level SM analysis used in the extraction of the bounds is not reliable for a heavy, strongly coupled Higgs, the mass of which is not controllable in this model.

All SM fields are fundamental fields with no TC interactions, hence localized on the UV-boundary. With usual assignments to the representations of the SM gauge group, the mass of the top originates after symmetry breaking from the (dimensionful) Yukawa coupling \( - \delta (z - L_0) y_t \tilde{q}_L \Phi u_R \) where \( \Phi = i \tau_2 \Phi \) and where for simplicity \( L = L_0 \). The renormalized top-Yukawa in four dimensions \( y_t \equiv y_t L_0 / L_1 \sqrt{L} \) reproduces the experimental value \( m_t \sim 175 \text{ GeV} \) for \( y_t \sim 1 / \sqrt{2} \).

At finite \( L_0 \sim L \), this requires:
\[
\frac{y_t}{\sqrt{L}} = \frac{L_1}{\sqrt{2} L_0}.
\]

**NATURALNESS AND EXPERIMENTAL BOUNDS.**

As long as \( g^2 / L \) is small the (tree-level) computations in the five-dimensional gravity background should provide a good estimate of the non-perturbative effects in the dual four-dimensional conformal gauge theory. This
is the large-$N_T$ limit. Since $g_\rho^2 = \varepsilon^2 g^2 / L$ is the SM $SU(2)_L$ gauge coupling, parametrically small values for $\varepsilon$ imply parametrically big uncertainties. Hence, I assume $\varepsilon > 1/2$ ($g_\rho / \sqrt{L} < 1.3$). Numerically, $k(\varepsilon = 1/2) \simeq 2.8$. For these values of $\varepsilon$, the bounds on $\tilde{S}$ are automatically satisfied if the bounds on $\tilde{S}$ are.

The indicative bound on $\tilde{S}$ implies:

$$\frac{1}{L_1} > \frac{M_W}{\sqrt{\varepsilon S_{\max}}} = 890 \text{ GeV}.$$  \hspace{1cm} (32)

The techni-$\rho$ mesons have mass $M_\rho \simeq 2.5$ TeV. There are four of them, for all practical purposes degenerate in mass. The next excited states have masses bigger than 5 TeV. With small values of $g_\rho / \sqrt{L}$, the couplings of the lightest techni-$\rho$’s to the SM currents are not suppressed, and hence they should be observable at the LHC.

In order to gauge the amount of fine-tuning that might be hidden in the definition of the renormalized couplings, it is convenient to look at the physical quantities at finite $L_0$ and $L$. The bound on $\tilde{S}$ is independent of $g$ and $L_0$. From the expression for $\eta$ it translates into:

$$v_1^2 L_0^2 = \varepsilon^2 L_1^2 < \left( \frac{1}{3.6} \right)^2.$$ \hspace{1cm} (33)

The natural expectation for $v_1$ is

$$v_1 \simeq \frac{2.4}{g L_1},$$ \hspace{1cm} (34)

leading to the bound:

$$\frac{L L_0}{g^2 L_1^2} \left( \frac{1}{6} \right)^2.$$ \hspace{1cm} (35)

Before commenting on this bound, Eq. 33 deserves to be explained. $L_1$ is the confinement scale, at which the symmetry-breaking condensate of the $SU(N_T)$ theory $v_1$ forms. $v_1$ has dimensions 3/2, which on dimensional grounds explains the extra factor of $g$, since these are the only two parameters relevant on the IR-brane. The numerical coefficient is deduced from the real-world QCD relation $\sqrt{2} g_\rho f_\pi = M_\rho$, with $g_\rho = g / \sqrt{L}$ and $M_\rho = 2.4 / L_1$. The value of $g_\rho \simeq 6$ is extracted from the first coefficient of the large-$q^2$ expansion of the vector-vector correlator: $L / g^2 = N_c / 12 \pi^2$. Eq. 33 should not be taken too literally. It only gives a size of the natural expectation for the value of the symmetry-breaking condensate evaluated at the confinement scale.

Eq. 33 is the main result of this paper. At large-$N_T$ (small $g$), the bound on $\tilde{S}$ is satisfied thanks to the parametric scale separation between $L_0$ and $L_1$, which provides a suppression mechanism to compensate for the smallness of $g_\rho^2 / L$. For $g^2 / L \sim 1$, this implies $1 / L_0 \sim 6 / L_1 \sim 5.3$ TeV, high enough to provide a sufficient suppression of higher order operators, and enough to suppress FCNC transitions in a complete ETC model $\mathcal{E}$. The value of the top Yukawa is in this case somewhat large $y_t / \sqrt{L} \sim 4$, but still natural. From a phenomenological perspective, walking makes the axial and vectorial contributions to $\tilde{S}$ cancel each other, so that the bounds are satisfied even if the spin-1 resonances have unsuppressed couplings to the SM currents.

This has to be contrasted with what happens in a small-$N_T$ QCD-like technicolor theory, in which there is no substantial cancellation between axial and vectorial contribution to $\tilde{S}$, and the bounds are satisfied because a large value of $g^2 / L$ parametrically suppresses the techni-$\rho$ decay constants. From the perspective of the searches at the LHC, the scenario described here is very appealing. A more quantitative study is needed, but going to large-$N_T$ renders the production and decay rates bigger than in a QCD-like model, improving on the feasibility of the search itself.

The perturbative estimate of $\tilde{S}$ completely misses the $(L_0 / L_1)^2$ suppression factor coming from walking. The limit case $g / \sqrt{L} \sim 1.3$ corresponds roughly to a $SU(N_T)$ theory with critical number $N_d \sim 2N_T$ of techni-fermions transforming on the fundamental of both $SU(2)_L$ and $SU(N_T)$ for $N_T \sim 8$. In this case, the perturbative estimate is:

$$\tilde{S}_p = \frac{\alpha}{4 \sin^2 \theta_W} \frac{N_d N_T}{6 \pi} \sim 0.06,$$ \hspace{1cm} (36)

compared with $\tilde{S} \simeq 0.003$, obtained for $1 / L_0 \sim 5$ TeV.

I am grateful to T. Appelquist for encouraging this study and for useful discussions. This work is partially supported by the DOE grant DE-FG02-96ER40956.

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