Leptogenesis and the Violation of Lepton Number and CP at Low Energies

John Ellis\textsuperscript{1} and Martti Raidal\textsuperscript{1,2}

\textsuperscript{1}Theory Division, CERN, CH-1211 Geneva 23, Switzerland
\textsuperscript{2}National Institute of Chemical Physics and Biophysics, Tallinn 10143, Estonia

Abstract

In the context of the minimal supersymmetric seesaw model, we study the implications of the current neutrino data for thermal leptogenesis, $\beta\beta_0\nu$ decay, and leptonic flavour- and CP-violating low-energy observables. We express the heavy singlet-neutrino Dirac Yukawa couplings $(Y_\nu)_{ij}$ and Majorana masses $M_{N_i}$ in terms of the light-neutrino observables and an auxiliary hermitian matrix $H$, which enables us to scan systematically over the allowed parameter space. If the lightest heavy neutrino $N_1$ decays induce the baryon asymmetry, there are correlations between the $M_{N_1}$, the lightest active neutrino mass and the primordial lepton asymmetry $\epsilon_1$ on the one hand, and the $\beta\beta_0\nu$ decay parameter $m_{ee}$ on the other hand. However, leptogenesis is insensitive to the neutrino oscillation phase. We find lower bounds $M_{N_1} \gtrsim 10^{10}$ GeV for the normal light-neutrino mass hierarchy, and $M_{N_1} \gtrsim 10^{11}$ GeV for the inverted mass hierarchy, respectively, indicating a potentially serious conflict with the gravitino problem. Depending on $M_{N_1}$, we find upper (upper and lower bounds) on the lightest active neutrino mass for the normal (inverted) mass hierarchy, and a lower bound on $m_{ee}$ even for the normal mass ordering. The low-energy lepton-flavour- and CP-violating observables induced by renormalization are almost independent of leptogenesis. The electron electric dipole moment may be close to the present bound, reaching $d_e \sim 10^{-27}$ to $10^{-28}$ e cm in our numerical examples, while $d_\mu$ may reach $d_\mu \sim 10^{-25}$ e cm.

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1 Introduction

The only convincing experimental evidence for physics beyond the Standard Model, so far, is provided by neutrino oscillations [1, 2], which are generally interpreted as evidence for non-zero neutrino masses. The smallness of neutrino masses is explained naturally via the seesaw mechanism [3] with heavy singlet Majorana neutrinos. Leptogenesis scenarios envisage that CP violation in their out-of-equilibrium decays may induce a non-zero lepton asymmetry, which may be converted into the observed baryon asymmetry of the Universe via electroweak sphaleron processes [4]. If, in addition, there is low-energy supersymmetry as motivated by the hierarchy problem, which is exemplified by the vastly different mass scales involved in the seesaw mechanism, low-energy processes violating charged-lepton flavour and CP may be observable, induced by the neutrino Dirac Yukawa interactions via renormalization below the heavy-singlet mass scale [5, 6]. In this scheme, the only source of many observables - neutrino oscillations, $\beta\beta_0\nu$ decay, the baryon asymmetry of the Universe, and flavour-violating decays of charged leptons and their electric dipole moments (EDMs) - are the Dirac Yukawa couplings $(Y_\nu)_{ij}$ and the Majorana masses $M_{N_i}$ of the three heavy singlet neutrinos.

Today, most of our knowledge of $Y_\nu$ and $M_{N_i}$ comes from the light-neutrino mass and mixing parameters measured in oscillation experiments, with an additional constraint from searches for $\beta\beta_0\nu$ decays [7]. The oscillation data are converging towards unique solutions for each of the atmospheric and solar neutrino anomalies [8], which represents a major breakthrough in neutrino physics. On the other hand, the determination of the baryon asymmetry of the Universe has significantly improved in recent years [9], and will improve still further with further astrophysical and cosmological observations, such as those of the MAP and Planck satellites. In addition, one expects significant improvements in the future experiments searching for $\beta\beta_0\nu$ decay [10], lepton-flavour-violating (LFV) processes [11, 12, 13] and electric dipole moments (EDMs) [14, 15, 16]. These prospects motivate us to perform a comprehensive study of these observables in the minimal seesaw model.

Any study of leptogenesis, neutrino masses, LFV processes and the EDMs of charged leptons faces the generic difficulty of relating the experimental information on light neutrino masses and mixings with other observables. If one takes a top-down approach and fixes $Y_\nu$ and $M_{N_i}$ by some theoretical or phenomenological argument [16], such as GUT relations, $U(1)$ or non-Abelian flavour models, phenomenological textures, democratic principles, arguments of minimal fine-tuning, etc., one can study the pattern of typical predictions of the model considered, but cannot perform a comprehensive phenomenological study of the interesting
observables. Even correct numerical consistency with light-neutrino data is a difficult task in this approach, since it may involve fine-tunings and must be checked \textit{a posteriori}.

These problems can be solved in a bottom-up approach \cite{17} to neutrino observables. Parametrizing \((Y_\nu)_{ij}\) and \(M_N\) in terms of the light neutrino mass matrix \(M_\nu\) and an auxiliary Hermitian matrix \(H\), as in \cite{18}, compatibility with the light-neutrino data is automatic, because \(M_\nu\) is an input. In addition, since the Hermitian matrix \(H\) has a physical interpretation as \(H = Y_\nu^\dagger D Y_\nu\), where \(D\) is a real diagonal matrix, one has also control over the renormalization-induced LFV decays. For every \((Y_\nu)_{ij}\) and \(M_N\) generated in this way, one can therefore calculate exactly the weighted light neutrino mass \(m_{ee}\) measured in \(\beta\beta_{0\nu}\) decay \cite{19}, rates for LFV decays \cite{3, 20, 21}, and EDMs of charged leptons \cite{22, 23}. Moreover, one can also calculate consistently the leptogenesis CP asymmetries \(\epsilon_i\) \cite{24, 25}, and, assuming the standard thermal leptogenesis scenario, also the washout parameters \(\kappa_i\) \cite{26, 27, 28}. Hence one can study the correlations between all these parameters and their dependences on the light and heavy neutrino masses. To our knowledge, there has so far been no study of leptogenesis in which all the \(\epsilon_i, \kappa_i, m_{\nu_1}\) and \(M_N\) are treated simultaneously as dynamical variables determined in consistency with the oscillation data.

In this paper we perform such a comprehensive phenomenological study of three classes of leptonic observables in the minimal supersymmetric seesaw model: the effective light-neutrino parameters in \(M_\nu\), the baryon asymmetry of the Universe generated via thermal leptogenesis, and the renormalization induced LFV processes and EDMs of charged leptons. Assuming the large-mixing-angle (LMA) solution to the solar neutrino anomaly, we parametrize \(Y_\nu\) and \(M_N\) in terms of \(M_\nu\) and \(H\). In our previous paper \cite{18} we considered only the case \(H = Y_\nu^\dagger D Y_\nu\) where \(D_{ii} = \ln(M_{\text{GUT}}/M_{N_i})\). Here we also consider a different form for \(D\) \cite{17}, which yields maximal EDMs for the charged leptons. We assume the standard thermal leptogenesis scenario in which the observed baryon asymmetry of the Universe is generated only by the decays of the lightest singlet neutrino \(N_1\). The wash-out parameter \(\kappa_1\) is calculated by solving numerically the analytical Boltzmann equations of \cite{29}, and correcting by a constant factor in order to be consistent with the exact results in \cite{28}.

We scan randomly over the input parameters: the lightest neutrino mass \(m_{\nu_1}\) (or \(m_{\nu_3}\) for the inverted hierarchy of light-neutrino masses), the two Majorana phases in \(M_\nu\), and the entries of the parameter matrix \(H\). We require the Yukawa couplings \(Y_\nu\) to remain perturbative until the GUT scale, we require the generated baryon asymmetry to be consistent with

\footnote{This assumption is not valid in the case of highly-degenerate heavy neutrinos, but still holds well in the case of moderate degeneracy \cite{25}. As the \(M_N\) are output parameters in our approach, it is not suitable for systematic studies of very degenerate heavy neutrino phenomena.}
observation, and we also require consistency with all the bounds on LFV processes.

We find interesting correlations between the neutrino observables and leptogenesis parameters, whilst the LFV processes and EDMs are almost independent of the leptogenesis constraints. The influence of the neutrino oscillation CP phase $\delta$ on leptogenesis is negligible: the existence of a baryon asymmetry does not require it to be non-vanishing. The experimental bound $Y_B \gtrsim 3 \times 10^{-11}$ on the baryon-to-entropy density ratio $Y_B$ implies the lower bounds $M_{N_1} \gtrsim 10^{10}$ GeV and $M_{N_1} \gtrsim 10^{11}$ GeV for the normally and inversely ordered light neutrino masses, respectively. These bounds put the findings of [30] on rigorous numerical ground, and indicate a serious potential conflict with the gravitino problem in supergravity models [31]. Leptogenesis also implies non-trivial bounds on the mass of lightest light neutrino. For the normal mass ordering, there is an $M_{N_1}$-dependent upper bound on $m_{\nu_1}$, whilst for the inverted hierarchy there are both upper and lower bounds on $m_{\nu_3}$. Successful leptogenesis with $m_{\nu_1} \gtrsim 0.1$ eV could be allowed for $M_{N_1} \gtrsim 10^{12}$ GeV. There is also an $M_{N_1}$-dependent lower bound on $m_{ee}$ for normally-ordered light neutrinos, implying its possible measurability in future experiments. On the other hand, $m_{ee}$ has a preferred value determined by $\Delta m^2_{atm}$ even in the case of the inverted mass hierarchy. It tends to be below $\mathcal{O}(10^{-1})$ eV, making improbable the discovery of $\beta\beta_0\nu$ decay in current experiments.

The rates of LFV processes and EDMs depend also on the soft supersymmetry-breaking parameters, which we fix by choosing one of the post-LEP benchmark points [32]. We find that $Br(\tau \to \mu(e)\gamma)$ and $Br(\mu \to e\gamma)$ may easily saturate their present lower bounds, and that the EDMs of electron and muon may reach $d_e \sim 10^{-27} - 10^{-28}$ e cm and $d_{\mu} \sim 10^{-25}$ e cm, respectively, in our random samples. We stress in particular that the electron EDM $d_e$ may be less than an order of magnitude from the present experimental bound $d_e \lesssim 1.6 \times 10^{-27}$ e cm [38] and offers a sensitive probe of the supersymmetric seesaw model. We find also some correlation between LFV $\tau$ decay rates and the lower bound on $M_{N_1}$.

The paper is organized as follows. In Section 2 we classify the observables and discuss our parametrization. Section 3 contains phenomenological studies, and finally our conclusions are presented in Section 4.

2 Parametrization of Neutrino Observables

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2These conclusions on neutrino parameters and leptogenesis are valid also in non-supersymmetric models.
2.1 Observables and Physical Parameters

The leptonic superpotential of the minimal supersymmetric model which implements the seesaw mechanism is

\[ W = N_i^c(Y_{\nu})_{ij}L_jH_2 - E_i^c(Y_e)_{ij}L_jH_1 + \frac{1}{2}N_i^c(M_N)_{ij}N_j^c + \mu H_2H_1, \]

(1)

where the indices \(i, j\) run over three generations and \(M_N\) is the heavy singlet-neutrino mass matrix. One can always work in a basis where the charged leptons and the heavy neutrinos both have real and diagonal mass matrices:

\[ (Y_e)_{ij} = Y^D_{e_i} \delta_{ij}, \quad (M_N)_{ij} = M_N \delta_{ij}. \]

(2)

The matrix \(Y_{\nu}\) contains six physical phases and can be parametrised as

\[ (Y_{\nu})_{ij} = Z^*_i Y^D_{\nu_k} X^*_k, \]

(3)

where \(X\) is the analogue of the quark CKM matrix in the lepton sector and has only one physical phase, and \(Z \equiv P_1 \bar{Z} P_2\), where \(\bar{Z}\) is a CKM-type matrix with three real mixing angles and one physical phase, and \(P_{1,2} \equiv \text{diag}(e^{i\theta_1}, e^{i\theta_2}, 1)\). This implies that we have 15 physical parameters in the Yukawa coupling \(Y_{\nu}\), which together with the 3 unknown heavy masses \(M_N\), make a total of 18 parameters in the minimal seesaw model\(^3\).

These 18 unknown neutrino parameters give rise to three classes of physical observables, as presented diagrammatically in Fig.1:

(i) Low-energy effective neutrino masses arising from the seesaw mechanism:

\[ \mathcal{M}_\nu = Y^T_{\nu} (M_N)^{-1} Y_{\nu} v^2 \sin^2 \beta. \]

(4)

The effective light-neutrino mass matrix \(\mathcal{M}_\nu\) is symmetric and can be diagonalized by a unitary matrix \(U\) as follows:

\[ U^T \mathcal{M}_\nu U = \mathcal{M}^D_\nu. \]

(5)

By a field redefinition, one can rewrite \(U \equiv VP_0\), where \(P_0 \equiv \text{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)\) and \(V\) is the MNS matrix. Therefore, all the low-energy neutrino observables such as neutrino oscillations, \(\beta\beta_{0v}\) decay, etc., depend on the 9 effective parameters in \(\mathcal{M}_\nu\), which are functions of all the 18 parameters in \(\mathcal{M}_\nu\). Whilst neutrino oscillations measure the

\(^3\)The parameter counting is identical in models with and without supersymmetry.
mass-squared differences of neutrinos and their mixing angles, $\beta\beta_{0\nu}$ decay measures one particular combination of their masses and mixing matrix elements,

$$|m_{ee}| \equiv \left| \sum_i U_{ei}^* m_{\nu_i} U_{ie}^\dagger \right|,$$  \hspace{1cm} (6)

which involves also the Majorana phases. The NMS mixing phase $\delta$ can in principle be measured in neutrino oscillations with neutrino factory experiments, but measurements of the Majorana phases are less straightforward [19, 34, 35].

(ii) The idea of baryogenesis via leptogenesis is first to produce a lepton asymmetry, whose ratio to the entropy density we denote by $Y_L$, via the out-of-equilibrium decays of the heavy neutrinos $N_i$. This asymmetry is converted into the baryon asymmetry of the universe via $(B + L)$-violating electroweak sphaleron processes:

$$Y_B = \frac{C}{C - 1} Y_L.$$  \hspace{1cm} (7)
where \( C = 8/23 \) in the minimal supersymmetric extension of the Standard Model (MSSM). The generated lepton asymmetry depends on the initial neutrino-plus-sneutrino density, the CP asymmetries in neutrino decays, and on the washout effects. The CP asymmetry produced in out-of-equilibrium decays of \( N_i \) is given by [24]

\[
\epsilon_i = -\frac{1}{8\pi} \sum_l \frac{\text{Im} \left( (Y_{\nu}Y_{\nu}^\dagger)_{il} (Y_{\nu}^\dagger Y_{\nu})_{il} \right)}{\sum_j |(Y_{\nu}^\dagger Y_{\nu})_{ij}|^2} \sqrt{x_l} \left[ \log(1 + 1/x_l) + \frac{2}{(x_l - 1)} \right],
\]

(8)

where \( x_l \equiv (M_{N_l}/M_{N_1})^2 \). It is clear from (8) that the generated asymmetry depends only on the 9 parameters (including 3 phases) in

\[
Y_{\nu}^\dagger Y_{\nu} = P_1^\dagger Z^\dagger (Y_{\nu}^D)^\dagger Z P_1
\]

(9)

and on the heavy neutrino masses.

In the case of non-degenerate heavy neutrinos, the lepton asymmetry is, to a good approximation, generated only via the decays of the lightest heavy neutrino \( N_1 \) (and the corresponding sneutrinos), because the very rapid washout processes mediated by \( N_1 \) erase whatever asymmetry had been produced previously in \( N_2,3 \) decays [26, 25]. In this case one has a simple relation

\[
Y_L = Y_{N_1}^{eq}(0) \epsilon_1 \kappa_1,
\]

(10)

where \( Y_{N_1}^{eq}(0) \) is the initial thermal equilibrium density of the lightest neutrinos and \( \kappa_1 \) is the washout parameter. The factors multiplying \( \epsilon_1 \), in (10) depend on the cosmological scenario. In the case of the standard leptogenesis scenario with thermally produced heavy neutrinos one has in thermal equilibrium the initial condition \( Y_{N_1}^{eq}(0) \approx 1/(2g_*) \), where \( g_* \sim 230 \) is the number of effective degrees of freedom in the MSSM. The washout parameter \( \kappa_1 \) can be precisely calculated by solving the set of Boltzmann equation for \( Y_L \) and for the neutrino density \( Y_{N_1} \),

\[
\begin{align*}
\frac{dY_{N_1}}{dz} + \gamma_{N_1}(z) &= 0, \\
\frac{dY_L}{dz} + \gamma_L(z)Y_L + S_L(z) &= 0,
\end{align*}
\]

(11)

where \( Y_L, Y_{N_1} \) and the factors \( \gamma_{N_1}(z), \gamma_L(z) \) and \( S_L(z) \) depend on the temperature, which is parametrized by \( z = M_1/T \). In this scenario, \( \gamma_{N_1}(z), S_L(z) \) and \( \gamma_L(z) \) are
worked out in great detail \cite{26,28} in the Standard Model, for which analytical approximations exist \cite{29}, and also in the supersymmetric framework \cite{27}. In general, the generated asymmetry depends on four parameters: the CP asymmetry $\epsilon_1$, the heavy neutrino mass $M_{N_1}$, and the lightest light neutrino mass, which determines the overall light-neutrino mass scale. The effective mass parameter

$$\tilde{m}_1 \equiv \left( Y_\nu Y_\nu^\dagger \right)_{11} \frac{v^2 \sin^2 \beta}{M_{N_1}}$$

is also used in discussions of leptogenesis: it depends on the 10 parameters introduced above. Fixing the baryon asymmetry to agree with observation implies a relation between $\epsilon_1$ and $\kappa_1$ via (10). Therefore, only the relations between $\epsilon_1$ and the masses $M_{N_1}$ and $m_{\nu_1}$ (or $m_{\nu_3}$ for the inverted mass hierarchy) are physically relevant.

(iii) Renormalization of soft supersymmetry-breaking parameters due to the presence of $Y_\nu$ above the heavy-neutrino decoupling scales induces low-energy processes such as the charged-lepton decays $\mu \to e\gamma$, $\mu \to eee$, $\tau \to l\gamma$, $\tau \to 3l$, and (in the presence of CP violation) EDMs for the electron and muon. In a leading-logarithmic approximation, renormalization modifies the left-sleppton mass matrix $m_\tilde{L}$ and trilinear soft supersymmetry-breaking $A_e$ terms according to \cite{6}

$$\begin{align*}
(\delta m_\tilde{L}^2)_{ij} & \simeq -\frac{1}{8\pi^2}(3m_0^2 + A_0^2)(Y^\dagger LY)_{ij}, \\
(\delta A_e)_{ij} & \simeq -\frac{1}{8\pi^2}A_0 Y_{ei}(Y^\dagger LY)_{ij},
\end{align*}$$

which are proportional to

$$Y^\dagger LY = X Y^D P_2 \overline{Z}^T L \overline{Z} P_2 X^* Y^D X^\dagger,$$

where $L$ is a diagonal matrix

$$L_{ij} = \ln(M_{GUT}/M_{N_i})\delta_{ij}. \tag{15}$$

We note that the CP-violating observables at low energies depend on the leptogenesis phase in $\overline{Z}$. The expression (14) contains 16 neutrino parameters altogether, but in completely different combinations from the seesaw mass matrix (4). Only the two Majorana phases in $P_1$ cancel out in (14). If the heavy neutrinos are exactly degenerate in mass, all the renormalization-induced observables are proportional to $Y^\dagger Y = X(Y_v^D)^2 X^\dagger$, but this is not a good approximation, in general.
The CP-violating observables in LFV processes all depend on a single CP invariant, which is related to \( H = Y^\dagger \nu L \nu \) by \( J = \text{Im} H_{12} H_{23} H_{31} \) \[36\]. This influences slepton physics at colliders and also determines the T-odd asymmetry in \( \mu \rightarrow 3e \) \[37\]. The dominant contribution to the lepton EDMs arises from threshold corrections to the trilinear coupling \( A_e \) due to the non-degeneracy of heavy neutrino masses. Diagonal phases in \( A_e \) are proportional to \[23, 18\]

\[
\text{Im}[X_j, X_k]_{ii} \log M_{N_k}/M_{N_j} \neq 0, \tag{16}
\]

where \((X_k)_{ij} = (Y^\nu(M_{N_k}))_{kj}(Y^\nu(M_{N_k}))_{kj}\). This depends non-trivially on the CP-violating phases, including the two Majorana phases in \( M^\nu \) and two phases in \( H \) that are irrelevant for LFV.

### 2.2 Phenomenological Parametrization of \( Y_\nu \) and \( M_{N_i} \)

None of the three classes of observables discussed in the previous subsection, by itself, allows one to measure all the parameters in the neutrino superpotential \([1]\). Moreover, at the moment the only parameters known experimentally are the 2 effective light neutrino mass differences and 2 mixing angles, and the baryon asymmetry of the Universe, whose interpretation requires some cosmological inputs. A central issue for comprehensive studies of the neutrino sector is how to parametrize \( Y_\nu \) and \( M_{N_i} \) in such a way that the effective neutrino parameters measured in the oscillation experiments are incorporated automatically, and all other phenomenological constraints are also satisfied.

One may attempt to fix \( Y_\nu \) and \( M_{N_i} \) in \([1]\) using model predictions or some other principle. In that case, however, satisfying the measured effective neutrino masses and mixings in \([1]\) is a non-trivial task. One can study the patterns of typical predictions for LFV processes, EDMs and the baryon asymmetry in any specific model, but not make comprehensive numerical studies which cover all the allowed parameter space.

However, as discussed in \([17]\), in the supersymmetric seesaw model the low-energy degrees of freedom may in principle be used to reconstruct the high-energy neutrino parameters. In \([18]\) we presented a parametrization of \( Y_\nu \) and \( M_{N_i} \) in terms of the light-neutrino mass matrix \( M^\nu \) and an auxiliary Hermitian matrix \( H \),

\[
H = Y^\dagger \nu D Y_\nu, \tag{17}
\]

where the diagonal matrix \( D \) was chosen in \([18]\) to be \( D_{ij} = \ln(M_{GUT}/M_{N_i})\delta_{ij} \), motivated by \([14]\). In this case, the parameter matrix \( H \) is directly related to the solutions
of the renormalization-group equations (RGEs) for the soft supersymmetry-breaking slepton masses, according to (13), and allows us to control the rates for LFV processes. Conversely, if any LFV process is observed, one can use its value to parametrize the heavy neutrino masses and couplings.

In principle the diagonal matrix $D$ can be an arbitrary real matrix. In order to study the maximal range for the charged lepton EDMs in the supersymmetric seesaw model, we also study in this paper the parametrization with $D = 1$, which is the case discussed in [17]. According to (13), the EDMs are not proportional to $H$ in either case. However, the choice $D = 1$ departs from (16) more than the choice $D = L$. As some of the entries of $H$ must be suppressed in order to satisfy the stringent experimental upper bound on $\mu \rightarrow e\gamma$, one expects the EDMs to get somewhat larger values if $D = 1$, and this is indeed the case.

We now present the details of the parametrization. Starting with (17) and using the parametrization [21]:

$$Y_\nu = \frac{\sqrt{M_N R \sqrt{\mathcal{M}^D_{\nu}} U^\dagger}}{v \sin \beta},$$

one can recast (17) into a form

$$H' = R'^\dagger \sqrt{M_N} R',$$

where $\sqrt{M_N}$ is a diagonal matrix

$$(\sqrt{M_N})_{ii} = D_{ii} M_{Ni},$$

and

$$H' = \sqrt{M^{D}_{\nu}}^{-1} U^\dagger H U \sqrt{M^{D}_{\nu}}^{-1} v^2 \sin^2 \beta.$$  \hspace{1cm} (21)

If (19) can be solved, i.e., if the matrix $H'$ can be diagonalized with an orthogonal matrix $R'$, then one can solve the heavy neutrino masses from (20) and calculate the neutrino Dirac Yukawa couplings from (18). Schematically,

$$(\mathcal{M}_\nu, H) \longrightarrow (\mathcal{M}_\nu, \sqrt{M_N}, R') \longrightarrow (Y_\nu, M_{Ni}),$$

where the quantities $\sqrt{M_N}$ and $R'$ are calculated in the intermediate step and do not have any independent physical meaning. Thus, one has converted the 9 low-energy effective neutrino parameters and the 9 free parameters in the Hermitian parameter matrix $H$ into the 18 physical neutrino parameters in $Y_\nu$ and $M_{Ni}$.
The advantages of this parametrization have already been mentioned: it allows one to control the rates for LFV processes and to scan efficiently over the allowed parameter space at the same time. The disadvantage of the parametrization (22) is that it is not continuous, because for some choice of the parameter $H$ there may not exist a matrix $R'$ that diagonalizes $H'$. However, in the case of multi-dimensional parameter spaces such as the one we study, scanning randomly over the allowed parameters is the most powerful tool, and in practice this disadvantage does not hinder such a phenomenological study.

3 Phenomenological Analysis

Using the parametrization developed in the previous section, we now perform a comprehensive phenomenological study of all three types of leptonic observables in the minimal supersymmetric seesaw model.

We fix the known light neutrino parameters by $\Delta m_{32}^2 = 3 \times 10^{-3}$ eV$^2$, $\Delta m_{21}^2 = 4.5 \times 10^{-5}$ eV$^2$, $\tan^2 \theta_{23} = 1$ and $\tan^2 \theta_{12} = 0.4$, corresponding to the LMA solution for the solar neutrino anomaly. Since the experimental constraint on the angle $\theta_{13}$ is quite stringent, $\sin^2 2\theta_{13} \lesssim 0.1$ \cite{38, 39}, our results depend only weakly on its actual value. We fix $\sin \theta_{13} = 0.1$ and study two cases with the limiting values of the neutrino mixing phase $\delta = \pi/2$ and $\delta = 0$. We consider both normally and inversely ordered light neutrino masses, since neutrino oscillations do not discriminate between them at present\footnote{Future neutrinoless double beta decay experiments could resolve this ambiguity \cite{19}.}

We assume the standard thermal leptogenesis scenario in which the baryon asymmetry originates only from the lightest heavy neutrino decays, as described by (10). In our calculations, we solve (11) numerically using the approximate analytical expressions for the thermally-averaged interactions given in \cite{29}. We start at $T \gg M_{N_1}$ with the initial conditions $Y_{N_1} = Y_{N_1}^{eq}$, and $Y_L = 0$. As is appropriate for supersymmetric models, we concentrate on the low-$M_{N_1}$ regime, $M_{N_1} \lesssim 10^{13}$ GeV, where the approximate solutions for $\kappa_1$ differ from the exact ones \cite{28} by just a constant factor. We correct our output by this factor to be consistent with \cite{28}. These results for the washout parameter $\kappa_1$ were derived in the context of non-supersymmetric seesaw models, but are expected to be a good approximation also in supersymmetric models, especially for the low $M_{N_1}$ and the moderate values of $\tan \beta$ that we consider in this work. Because the heavy neutrinos thermalize very fast, our results are actually independent of the initial conditions in this low-$M_{N_1}$ regime. Here the wash-out parameter $\kappa_1$ depends practically only on the effective parameter $\tilde{m}_1$ (with a small dependence on $M_{N_1}$). Therefore, with high accuracy, the observed baryon asymmetry implies via (10)
that \( \epsilon_1 \) and \( \tilde{m}_1 \) have a one-to-one correspondence.

In our subsequent analysis, we require the induced baryon asymmetry be in the range

\[
3 \times 10^{-11} \lesssim Y_B \lesssim 9 \times 10^{-11},
\]

and study its implications on the light and heavy neutrino masses, the CP asymmetry \( \epsilon_1 \), the \( \beta\beta_{\nu_0} \) decay parameter \( m_{ee} \), the LFV decays and the EDMs of the charged leptons \( e \) and \( \mu \).

As was shown in [18], the stringent experimental limit on \( Br(\mu \to e\gamma) \) implies the phenomenological constraints \( H_{12} \ll O(1) \) and \( H_{13}H_{32} \ll O(1) \) on elements of our parameter matrix \( H \). Therefore we study two different textures of the matrix \( H \):

\[
H_1 = \begin{pmatrix}
a & 0 & 0 \\
0 & b & d \\
0 & d^\dagger & c
\end{pmatrix}
\]

and

\[
H_2 = \begin{pmatrix}
a & 0 & d \\
0 & b & 0 \\
d^\dagger & 0 & c
\end{pmatrix},
\]

where \( a, b, c \) are real and \( d \) is a complex number. The texture \( H_1 \) suppresses \( \tau \to e\gamma \) and \( d_e \) while \( \tau \to \mu\gamma \) and \( d_\mu \) can be large, and \textit{vice versa} for \( H_2 \), since these processes are sensitive to \( H_{13} \) and \( H_{23} \), respectively. We consider two forms of the matrix \( H \) ([17]), namely those with \( D = L \) and \( D = 1 \). For the textures \( H_1, H_2 \), the former suppresses \( \mu \to e\gamma \) very efficiently and all the parameters \( a, b, c, |d| \) can be simultaneously of order unity. However, for the \( D = 1 \) case one of \( a, b, c \) must necessarily be very small in order to keep \( \mu \to e\gamma \) below the present experimental bound.

In our model, the rates for the LFV processes and EDMs depend on the soft supersymmetry-breaking parameters. In the following numerical calculations we fix them at the GUT scale to coincide with one of the post-LEP benchmark points [32] \( m_{1/2} = 300 \text{ GeV}, m_0 = 100 \text{ GeV}, A_0 = -300 \text{ GeV}, \tan \beta = 10 \) and \( \text{sign}(\mu) = +1 \). This choice ensures that all other phenomenological constraints, such as those on the lightest Higgs boson mass, supersymmetric contributions to \( b \to s\gamma \) and \( g_\mu - 2 \), cosmological arguments, etc., are satisfied within the necessary accuracy.

As the input parameters we thus have the lightest neutrino mass \( m_{\nu_1} \) (or \( m_{\nu_3} \) for inversely-ordered neutrinos), the two low-scale Majorana phases \( \phi_{1,2} \), and the entries \( a, b, c, d \) of the parameter matrices \( H_1 \) and \( H_2 \). We generate these input parameters randomly in the ranges
$(10^{-4} - 1) \text{ eV for } m_{\nu_1} \text{ (or } m_{\nu_3}), \ (0 - 2\pi) \text{ for } \phi_{1,2}, \text{ and } (10^{-7} - 10) \text{ for } a, b, c, |d|. \text{ The distribution for each of them is flat on a logarithmic scale. We require that } Y_\nu \text{ remains perturbative up to } M_{\text{GUT}} \text{ and impose the present constraints on all the LFV processes.}

### 3.1 Normally-Ordered Light Neutrinos

First we study the implications on leptogenesis parameters, $\beta\beta_{\nu_0}$ decay, and LFV decays and charged lepton EDMs in the case of normally-ordered light neutrino masses and mixings. Apart from the EDMs, the two parametrizations with $D = L$ and $D = 1$ give practically indistinguishable results. Therefore, we present the $D = 1$ results only for the EDMs and the LFV decays, while all the other plots present results for the parametrization with $D = L$, as in [18].

![Figure 2: Scatter plot of the CP-violating asymmetry $\epsilon_1$ and the lightest heavy neutrino mass $M_{N_1}$ for the two extreme choices of the MNS phase $\delta = \pi/2$ and $\delta = 0$, assuming normally-ordered light neutrinos and the texture $H_1$. The baryon asymmetry is required to be in the range $[23]$.](image)

We present in Fig. 2 scatter plots for the CP-violating asymmetry $\epsilon_1$ and the lightest heavy neutrino mass $M_{N_1}$, for two extreme values of the MNS phase: $\delta = \pi/2$ and $\delta = 0$, assuming the texture $H_1$. Here we also introduce a colour code to study the distribution of points with different $M_{N_1}$ in the following plots. The points within a factor of five from the lower bound on $M_{N_1}$ are black, while the points with larger $M_{N_1}$ are grey (green).
Fig. 2 shows immediately that there is no distinction between the plots for $\delta = \pi/2$ and for $\delta = 0$. In special cases, there have been studies whether the observed non-zero baryon asymmetry can be related to the NMS phase $\delta$ [10]. Our results imply that this is not possible in general, and that successful leptogenesis does not require a non-zero value for $\delta$.

We also see immediately in Fig. 2 that there is an $M_{N_1}$-dependent upper bound [30] on the cosmological CP-violating asymmetry $\epsilon_1$, and there is a strong lower bound $M_{N_1} > \sim 10^{10}$ GeV on the $N_1$ mass. This indicates a potential serious conflict with conventional supersymmetric cosmology which requires an upper bound on the reheating temperature of the Universe after inflation, derived from avoiding gravitino overproduction [31].

In [30], the analytical bound

$$|\epsilon_1| \lesssim \frac{3}{\delta \pi v^2 \sin^2 \beta} (m_{\nu_3} - m_{\nu_1}),$$

(26)

was found in the limit of very hierarchical heavy neutrinos, $M_{N_1} \ll M_{N_2} \ll M_{N_3}$. Our results improve this bound, by including the best available numerical results for the washout parameter $\kappa_1$ and allowing for moderately degenerate heavy neutrinos. We have compared the bound (26) with our numerical calculations. It is well satisfied for the low-$M_{N_1}$ points in Fig. 2 but can be violated by a large factor for points with high $M_{N_1}$ and high $\epsilon_1$. In these cases, the heavy neutrinos are not hierarchical in mass and $\epsilon_1$ is therefore enhanced. As we see in the next figures, for these points the light neutrino masses can also be moderately degenerate.

To study correlations between the leptogenesis parameters, we give in Fig. 3 scatter plots of the CP-violating asymmetry $\epsilon_1$ and the effective mass parameter $\tilde{m}_1$ versus the lightest neutrino mass $m_{\nu_1}$ for the texture $H_1$. The shape of the both plots is the same, verifying that, for fixed $Y_B$, the parameters $\epsilon_1$ and $\tilde{m}_1$ are not independent parameters in our scenario, as discussed above. There is no lower bound on $m_{\nu_1}$ in this scenario, and the upper bound on $m_{\nu_1}$ depends on $M_{N_1}$, as indicated by the distribution of colours. The allowed band for $\epsilon_1$ depends only weakly on $m_{\nu_1}$ for $m_{\nu_1} \lesssim 10^{(-1-2)}$ eV, but the dependence becomes strong for larger $m_{\nu_1}$. While degenerate light neutrino masses are disfavoured by our results, confirming the claims in [28, 41], light neutrino masses $m_{\nu_1} \sim O(0.1)$ eV are still perfectly consistent with leptogenesis. Also, notice that for small $m_{\nu_1}$ the lower bound on $\tilde{m}_1$ is much stronger than the limit $\tilde{m}_1 > m_{\nu_1}$ derived in [28, 11].

To study the possible implications of the light neutrino masses and leptogenesis for $\beta\beta_0\nu$ decay, the LFV processes and EDMs in the texture $H_1$, we give in Fig. 4 a scatter plot of the $M_{N_1}$ against the $\beta\beta_0\nu$ parameter $m_{ee}$, and the muon electric dipole moment $d_\mu$ versus the branching ratio for $\tau \to \mu\gamma$. Surprisingly, there are both lower and upper limits on $m_{ee}$,
Figure 3: Scatter plot of the CP-violating asymmetry $\epsilon_1$ and the effective mass parameter $\tilde{m}_1$ versus the lightest neutrino mass $m_{\nu_1}$ for the normal mass hierarchy and the texture $H_1$.

Figure 4: Scatter plots of $M_{N_1}$ and the $\beta\beta_{0\nu}$ parameter $m_{ee}$, and of the muon electric dipole moment $d_\mu$ and the branching ratio for $\tau \rightarrow \mu \gamma$ decay, for the normal neutrino mass ordering and the texture $H_1$. 

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which depend on $M_{N_1}$. For $M_{N_1} \lesssim 10^{11}$ GeV we get $10^{-3} \lesssim m_{ee} \lesssim 10^{-2}$ eV.

There is no correlation between leptogenesis and the renormalization-induced LFV decays and EDMs, as seen in Fig. 4. With the chosen soft supersymmetry-breaking parameters, $Br(\tau \to \mu \gamma)$ can attain the present experimental bound in our random sample, and the muon EDM may reach $d_\mu \sim 10^{-25}$ e cm. The former may be observable at the LHC and in B-factory experiments, which may reach sensitivities $Br(\tau \to \mu \gamma) \sim 10^{-8-9}$, and the latter in experiments at the front end of a neutrino factory, which may be able to reach $d_\mu \sim 5 \times 10^{-26}$ e cm. The texture $H_1$ suppresses $Br(\tau \to e \gamma)$ and $d_e$ below observable limits.

![Figure 5: Scatter plots of $M_{N_1}$ versus the $\beta\beta_{0\nu}$ parameter $m_{ee}$, and of the electron electric dipole moment $d_e$ versus the branching ratio for $\tau \to e \gamma$, for the normal neutrino mass ordering and the texture $H_2$.](image)

We have performed a similar analysis for the texture $H_2$. The behaviour of the leptogenesis parameters is almost the same as for the texture $H_1$, already shown in Figs. 3 and 4, so we do not present further plots here. The most important differences from $H_1$ can be seen, however, in Fig. 5, where we present scatter plots of $M_{N_1}$ versus the $\beta\beta_{0\nu}$ parameter $m_{ee}$, and of the electron electric dipole moment $d_e$ versus the branching ratio for $\tau \to e \gamma$. Whilst the lower bound on $M_{N_1}$ is the same as in the previous case, the distribution of points clearly favours large values of $M_{N_1}$. This is a result of a mismatch between the structure of $H_2$ and the light neutrino mass hierarchy $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$. Because $H_{11}, H_{13} \neq 0$ in $H_2$, the
Yukawa couplings for the first generation tend to be larger than in the case of the texture $H_1$. As the lightest $N_1$ tends to be related to the lightest light neutrino mass $m_{\nu_1}$, the seesaw mechanism implies that larger $M_{N_1}$ are usually needed. Fig. 5 indicates that this mismatch can be compensated with a tuning of the input parameters, resulting in a relatively small number of points at low $M_{N_1}$.

For the texture $H_2$, $Br(\tau \rightarrow e\gamma)$ and the electron EDM can be large, as seen in Fig. 5, whilst $Br(\tau \rightarrow \mu\gamma)$ and $d_\mu$ are suppressed below the observable ranges. We find that $Br(\tau \rightarrow e\gamma)$ can be of the same order of magnitude as $Br(\tau \rightarrow \mu\gamma)$, shown in Fig. 4. Importantly, the electron EDM may exceed $d_e \sim 10^{-28}$ e cm in our numerical examples. This is just one order of magnitude below the present bound $d_e \lesssim 1.6 \times 10^{-27}$ e cm [33]. As we have not made special attempts to maximize it, it might even reach larger values with special values of the soft supersymmetry-breaking parameters.

We see that there are no black points in the right-hand panel of Fig. 5 corresponding to the facts that $Br(\tau \rightarrow e\gamma)$ and $d_e$ are very much suppressed. This implies a correlation between the lower bound on $M_{N_1}$ and the rates of the FLV processes and EDMs. If $Br(\tau \rightarrow e\gamma)$ were to be found at the LHC while $Br(\tau \rightarrow \mu\gamma)$ were not, the lower bound on $M_{N_1}$ from leptogenesis would be above $10^{11}$ GeV.

### 3.2 Inversely-Ordered Light Neutrinos

The inverse ordering of light neutrino masses, $m_{\nu_2} > m_{\nu_1} > m_{\nu_3}$, is still a viable option, since present neutrino oscillation experiments are not sensitive to the sign of $\Delta m^2_{32}$. Therefore, it is interesting to study whether this case can be discriminated from the normal hierarchy of light neutrino masses, using leptogenesis, LFV and CP-violating processes.

In the inverted light-neutrino mass hierarchy, the correlations between $\epsilon_1$ and the heavy and light neutrino masses are practically the same for both textures $H_{1,2}$, and it is sufficient to present results for just one of them. Since $d_\mu$ is suppressed in $H_1$ by two orders of magnitude compared with the normal hierarchy, due to smaller third-generation Yukawa couplings, we choose $H_2$ for presentation.

We present in Fig. 8 scatter plots for the lightest heavy neutrino mass $M_{N_1}$ and the CP-violating asymmetry $\epsilon_1$ versus the lightest light neutrino mass $m_{\nu_3}$. As expected, the lower bound on $M_{N_1}$ is higher than in the normally-ordered case, and is $M_{N_1} \gtrsim 10^{11}$ GeV. This result follows from the seesaw mechanism, since the Yukawa couplings for the first two generations must be larger. Somewhat surprisingly, leptogenesis implies also a lower bound on the lightest neutrino mass $m_{\nu_3}$. Therefore the light neutrinos tend to be degenerate if
Figure 6: Scatter plots of the lightest heavy neutrino mass $M_{N_1}$ and of the CP-violating asymmetry $\epsilon_1$ versus $m_{\nu_3}$, for inversely-ordered light neutrino masses and the texture $H_2$.

Figure 7: Scatter plots of $M_{N_1}$ versus $m_{ee}$ and of $d_e$ versus $\text{Br}(\tau \rightarrow e\gamma)$, for the inversely-ordered light neutrino masses and the texture $H_2$. 
their masses are inversely ordered. The CP-violating asymmetry $\epsilon_1$ can be larger than in the previous case, implying stronger washout effects and somewhat larger light neutrino masses, as seen in Fig. 3.

In Fig. 7 we give scatter plots of $M_{N_1}$ versus $m_{ee}$ and of $d_e$ versus $Br(\tau \to e\gamma)$ for the same texture as previously. The sharp lower bound on $m_{ee}$ is the consequence of the inverted mass ordering. There is a preferred region for $m_{ee}$ determined by $\Delta m^2_{\text{atm}}$, and relatively few points extend above $m_{ee} \gtrsim \mathcal{O}(0.1)\text{ eV}$. Therefore, even for the inverted mass hierarchy, observation of $\beta\beta_0\nu$ decay in current experiments is improbable. Again, $d_e$ and $Br(\tau \to e\gamma)$ reach the same values as in the case of normally-ordered neutrinos, and no strong correlation between leptogenesis, $d_e$ and $Br(\tau \to e\gamma)$ is present.

4 Discussion and Conclusions

In the context of the minimal supersymmetric seesaw model, we have studied relations between the light and heavy neutrino masses, thermal leptogenesis, and LFV decays and EDMs of charged leptons, scanning over the phenomenologically-allowed parameter space as suggested in [18]. There are lower bounds $M_{N_1} \gtrsim 10^{10}\text{ GeV}$ and $M_{N_1} \gtrsim 10^{11}\text{ GeV}$ for the normal and inverse hierarchies of light neutrino masses, respectively. These bounds are in potential conflict with the gravitino problem in supersymmetric cosmology if the gravitino mass is below TeV.

In the thermal leptogenesis scenario, one can avoid these bounds by fine-tuning model parameters, namely the Yukawa couplings $Y_\nu$ and the heavy neutrino masses $M_N$. It is well known that the CP-violating asymmetry may be enhanced if the heavy neutrinos are degenerate in mass [25]. This may allow one to lower [25, 13] the lightest heavy neutrino mass $M_{N_1}$, and so avoid the bounds on the reheating temperature [31]. Degenerate heavy neutrinos are also consistent with the LMA solution to the solar neutrino problem [44]. Because our parametrization gives $Y_\nu$ and $M_N$ as an output, such fine tunings cannot be studied by our random scan over the parameter space, and would require a different approach.

Another way out of the problem would be leptogenesis with non-thermally produced heavy neutrinos [13, 41]. In such a case, the reheating temperature of the Universe does not limit leptogenesis. However, they do not have the same predictivity, and the implications of the observed $Y_B$ on neutrino parameters are lost. It is also possible that gravitino is the lightest supersymmetric particle, in which case [43] the upper bound on the reheating temperature is $10^{11}\text{ GeV}$ and thermal leptogenesis is possible.

We have found interesting correlations between the heavy and light neutrino mass param-
eters. For normally-ordered masses, there is an $M_{N_1}$-dependent upper bound on $m_{\nu_1}$, whilst for the inverted hierarchy there are both upper and lower bounds on $m_{\nu_3}$. Successful leptogenesis with $m_{\nu_1} \gtrsim 0.1$ eV is allowed for $M_{N_1} \gtrsim 10^{12}$ GeV. There is also an $M_{N_1}$-dependent lower bound on $m_{ee}$ for normally-ordered light neutrinos, implying its possible testability in future experiments. On the other hand, $m_{ee}$ has a preferred value determined by $\Delta m^2_{\text{atm}}$ even in the case of the inverted mass hierarchy. It tends to be below $\mathcal{O}(10^{-1})$ eV, making the discovery of $\beta\beta_0\nu$ decay in current experiments improbable.

There is no correlation between leptogenesis, and the LFV decays and EDMs of charged leptons. The branching ratios for $\tau \rightarrow \mu(e)\gamma$ and $\mu \rightarrow e\gamma$ may saturate their present lower bounds, and the EDMs of electron and muon reach $d_e \sim 10^{-27-28}$ e cm and $d_\mu \sim 10^{-25}$ e cm in our random samples. There is some correlation between $M_{N_1}$ and the LFV $\tau$ decays for normally-ordered light neutrino masses: observation of $Br(\tau \rightarrow e\gamma)$ would require $M_{N_1}$ to be an order of magnitude above the lower bound.

The type of parametrization discussed in [17] and applied in [18] and in this work is a useful tool for studying the large parameter space of the minimal supersymmetric seesaw model. In the future, it may help attempts to devise an experimental strategy for determining systematically all the 18 parameters in this sector. It would also be interesting to combine this approach with models aiming at predictions for some (all) of the seesaw parameters. As we have shown in this paper, this type of parameterization can teach us some salutary lessons about leptogenesis and its relations to other observables.

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