Heavy Flavor Suppression, Flow and Azimuthal Correlation: Boltzmann vs Langevin

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Abstract. The propagation of heavy flavor through the quark gluon plasma has been treated commonly within the framework of Langevin dynamics, i.e. assuming the heavy flavor momentum transfer is much smaller than the light one. On the other hand a similar suppression factor $R_{AA}$ has been observed experimentally for light and heavy flavors. We present a thorough comparison in terms of nuclear suppression, $R_{AA}$, elliptic flow, $v_2$, and $c\bar{c}$ back to back correlation between the Langevin equation and the full collisional Boltzmann collision integral within the framework of Boltzmann transport equation. We have shown that the Langevin dynamics overestimates the interaction and even for a fixed $R_{AA}$ the full two-body collision integral shows that the elliptic flow is larger with respect to that predicted by a Langevin dynamics. Furthermore we have found that Boltzmann approach gives rise to a larger spreading of $c\bar{c}$ correlation in comparison with the Langevin approach.

1. Introduction
The experimental efforts at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies are aimed at creating and characterizing the properties of the quark gluon plasma (QGP). The heavy flavors, namely, charm and bottom quarks (c and b) are playing a crucial role to serve this purpose because of two-fold reason: the first, typical of particles physics, is that the mass $M_Q \gg \Lambda_{QCD}$ which makes possible the evaluation of cross section and $p_T$ spectra within next-to-next-to-lead order (NLLO) [1, 2] in a perturbative QCD (pQCD) scheme; the second, more inherent to plasma physics is that $M_Q \gg T$ and therefore the thermal production in the QGP is expected to be negligible because it is suppressed approximately by a $\sim e^{-M/T}$ term. Hence for Heavy Quarks (HQ) one has a nearly exact flavor conservation during the evolution of the plasma in both the partonic and hadronic stages. Hence, heavy flavors are the witness of the entire phase-space evolution of the QGP and can be used as a probe to characterize the properties of the QGP. The most common tool to study heavy flavors propagation in QGP is based on the Fokker-Planck approach [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] that can be obtained from the Boltzmann equation in the limit of small angle approximation or in other words in the limit in which the momenta transfered in the collisions between the particles of the bulk and the heavy quarks are small compared to the total momentum of the heavy quarks. The Fokker Planck approximation is expected to be asymptotically valid for $m/T \to \infty$. We are presenting in this proceeding a first study on the validity of such an approximation for charm and
bottom quarks. The proceeding is organized as follows. In section 2 we discuss the Boltzmann and the Fokker Planck approach which are used to describe the propagation of heavy quark through the QGP. In section 3 we discuss the evolution of the HQ spectra in a static medium. In section 4 the results for $c\bar{c}$ correlation are presented. In section 5 we compare the results we get for $R_{AA}$ and $v_2$ of HQ with both the two approaches when realistic simulations of ultra relativistic heavy ion collisions are performed and finally section 6 contains the conclusions.

2. Boltzmann vs Fokker-Planck dynamics

The propagation of HQ in QGP has been quite often described by the Fokker-Planck equation \[\text{[3, 4, 5, 6, 7, 14, 8, 10, 16, 11, 12, 13, 15, 17, 18, 19]}\] in order to calculate the nuclear suppression factor. Such a scheme has been very widely employed \[\text{[5, 6, 7, 8]}\] that also the Boltzmann equation reduces to a Fokker-Planck equation \[\text{[4]}\], which constitutes a significant simplification of in medium dynamics. Such a scheme has been very widely employed \[\text{[3, 4, 5, 6]}\]. The main reason is that it was believed that their motion can be encased into a Brownian motion due to their large masses that should generically lead to collisions sufficiently forward peaked and/or with small momentum transfer. Under such constraints it is known that also the Boltzmann equation reduces to a Fokker-Planck equation \[\text{[4]}\], which constitutes a significant simplification of in medium dynamics. Such a scheme has been very widely employed \[\text{[5, 6, 7, 8]}\] in order to calculate the nuclear suppression factor \[\text{[36, 37, 38]}\] of in medium dynamics. Such a scheme has been very widely employed \[\text{[3, 4]}\] and the elliptic flow \[\text{[36]}\] that have been observed for the non-photonic single electron spectra that are related to the decay of charm and bottom mesons. Along with the Fokker-Planck approach in some works a description of HQ within a relativistic Boltzmann transport approach has been developed \[\text{[9, 22, 34, 24, 25, 35]}\]. The Boltzmann equation for the HQ distribution function can be written in a compact form as:

\[
p^\mu \partial_\mu f_Q(x, p) = C[f_Q](x, p) \tag{1}
\]

where \(C[f_Q](x, p)\) is the relativistic Boltzmann-like collision integral in which the phase-space distribution function of the bulk medium appears as an integrated quantity, see for example Refs \[\text{[21, 32]}\]. This equation describes the evolution of the heavy quarks distribution function \(f_Q(x, p)\). The collision integral can be written in a simplified form \[\text{[3, 4]}\] in the following way:

\[
C[f_Q](x, p) = \int d^3 k [\omega(p + k, k)f_Q(x, p + k) - \omega(p, k)f_Q(x, p)] \tag{2}
\]

where \(\omega(p, k)\) express the collision rate of heavy quark per unit of momentum phase space which changes the heavy quark momentum from \(p\) to \(p - k\). The first term in the integrand is the gain of probability through collisions and the second term represent the net loss out of that momentum space volume. In this work we have not considered radiative processes therefore the heavy quarks interact with the medium by mean of two-body collisions regulated by the scattering matrix of the process \(g + Q \rightarrow g + Q\), therefore defining the relative velocity between the two colliding particles as \(v_{\text{rel}}\) the transition rate can be written as:

\[
\omega(p, k) = \int \frac{d^3 q}{(2\pi)^3} g_g(x, p)v_{\text{rel}} \frac{d\sigma_{g+Q\rightarrow g+Q}}{d\Omega} \tag{3}
\]

where \(\sigma_{g+Q\rightarrow g+Q}\) is related to the scattering matrix \(|M_{gQ}|^2\) as indicated below

\[
v_{\text{rel}} \frac{d\sigma_{g+Q\rightarrow g+Q}}{d\Omega} = \frac{1}{d_e} \frac{1}{4E_g E_q} \frac{|M_{gQ}|^2}{16\pi^2 E_{p-k} E_{q+k}} \delta^0(E_p + E_q - E_{p-k} - E_{q+k}) \tag{4}
\]

The explicit expression of the scattering matrix is indicated in this reference \[\text{[4]}\]. We point out that the scattering matrix is the real kernel of the dynamical evolution for both the Boltzmann approach and the Fokker-Planck one. Of course all the calculations discussed in this proceeding will originate from the same scattering matrix for both cases.
The Boltzmann equation is solved numerically dividing the space into a three-dimensional lattice and using the test particle method to sample the distributions functions. The collision integral is solved by means of a stochastic algorithm in which whether a collision happen or not is sampled stochastically comparing the collision probability related to two body collisions between HQ and gluons \((P_{22} = v_{rel} g_{g+Q-g+Q} \Delta t/\Delta x}\) with a random number extracted between 0 and 1 \([20, 30, 21, 33]\). The code has been widely tested and in particular it has been checked that the correct collision rate is reproduced and that given an initial non-equilibrium distribution the dynamical evolution brings the system towards equilibrium described by a Boltzmann-Juttner distribution. We have performed these tests as a function of cross section, temperature and mass of the particles, including non-elastic collisions \([31]\).

The non-linear integro-differential Boltzmann equation can be simplified employing the Landau approximation whose physical relevance can be associated to the dominance of soft scatterings with small momentum transfer \(|\mathbf{k}|\) respect to the particle momentum \(p\). This simplification is obtained expanding \(\omega(p + k, k)f(x, p + k)\) around the momentum transfer, \(k\),

\[
\omega(p + k, k)f_Q(x, p + k) \approx \omega(p, k)f(x, p) + k \frac{\partial}{\partial p} (\omega f) + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j} (\omega f) \tag{5}
\]

Inserting Eq.(5) into the Boltzmann collision integral, indicated in Eq.(2), one obtains the Fokker Planck Equation:

\[
\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i} \left[ A_i(p)f + \frac{\partial}{\partial p_j} [B_{ij}(p)] \right] \tag{6}
\]

by simply defining \(A_i = \int d^3k w(p, k)k_i = A(p)p_i\) and \(B_{ij} = \int d^3k w(p, k)k_i k_j\) that are directly related to the so called drag and diffusion coefficient. The Fokker-Planck equation can be solved by a stochastic differential equation i.e the Langevin equation that can be written as \([3, 6, 14]\):

\[
\begin{align*}
    dx_i &= \frac{p_i}{E} dt, \\
    dp_i &= -A p_i dt + (\sqrt{2B_0 P^\perp_{ij}} + \sqrt{2B_1 P^\parallel_{ij}}) \rho_j \sqrt{dt} 
\end{align*} \tag{7}
\]

where \(dx_i\) and \(dp_i\) are the coordinates and momenta changes in each time step \(dt\). \(A\) is the drag force and \(B\) the longitudinal and transverse diffusions, \(\rho_j\) is a stochastic Gaussian distributed variable and

\[
P^\perp_{ij} = \delta_{ij} - \frac{p_ip_j}{p^2}, P^\parallel_{ij} = \frac{p_ip_j}{p^2}. \tag{8}
\]

are the transverse and longitudinal tensor projectors. We will employ the common assumption, \(B_0 = B_1 = D\) \([6, 7, 14, 8, 9, 11, 15]\). To achieve the equilibrium distribution \(f_{eq} = e^{-E/T}\) with \(E = \sqrt{p^2 + m^2}\) as the final distribution one need to adjust the drag coefficient \(A\) in accordance with the Einstein relation \([26]\) (see also \([27]\))

\[
A(p) = \frac{D(p)}{ET} - \frac{D'(p)}{p}. \tag{9}
\]

3. Heavy quark diffusion in momentum space in a static medium

To investigate the similarities and the differences in between the Langevin and Boltzmann approach we have studied the heavy quark momentum evolution considering the initial charm and bottom quark distribution as a delta distribution at \(p = 10\) GeV. We have solved both the Langevin and Boltzmann equation in a box where the bulk consists of only gluon at equilibrium at \(T = 400\) MeV. The elastic collisions of heavy quarks with gluon has been considered within the
pQCD framework. In particular we have evaluated the drag and the diffusion coefficients, which are the basic ingredients required to solve the Langevin equations, using the scattering matrix elements shown in reference [4] and with the same scattering matrix we have also evaluated the total cross section that is required in order to solve the Boltzmann collision integral. In this work we have used $\alpha_s = 0.35$. The momentum evolution of the charm and bottom quarks within the Langevin dynamics are displayed respectively in Fig. 1 (left) and Fig. 2 (left). One can notice that both the charm and bottom quarks spectra assumes a gaussian shape as expected by construction. In fact it is well known the Langevin dynamics consists of a shift of the average momenta with a fluctuation around it that includes also the possibility to gain energy for the HQ as one can observe from the tail of the momentum distribution that overshoots the initial momentum $p = 10\text{ GeV}$ at $t = 2\text{ fm}/c$, black solid line in Fig.1 (left).

![Figure 1](image1.png)

**Figure 1.** Evolution of the momentum distribution for charm quarks within Langevin dynamics (left) and Boltzmann equation (right) considering the initial momentum distribution of the charm quarks as a delta distribution at $p=10$ GeV.

![Figure 2](image2.png)

**Figure 2.** Evolution of the momentum distribution for bottom quarks within Langevin dynamics (left panel) and the Boltzmann (right panel) considering the initial momentum distribution of the bottom quark as a delta distribution at $p=10$ GeV.

In Fig. 1 (right) we depict the momentum distribution for charm quark within the Boltzmann equation. From this figures it appears a very different evolution of the charm quark momentum distribution which does not have a Gaussian shape and already at $t = 2\text{ fm}/c$ has a very different spread in momentum with a larger contribution from processes where the charm quark can
gain energy and a long tail at low momenta. Such a tail indicates that charms during their propagation thorough the medium can lose a quite large amount of energy. The fact that charm spectra at different time does not show a gaussian shape essentially indicates that for a particle with \( M \sim \langle p \rangle \sim 3T \), as it is for the charm at a temperature \( T = 0.4 \text{ GeV} \), the momentum evolution is not of Brownian type. For the bottom quarks, shown in Fig. 2, the momentum evolution gives a much better agreement between the Boltzmann and the Langevin evolution because \( M_{\text{bottom}}/T \simeq 10 \).

4. Azimuthal correlations of heavy quarks
In the initial hard scatterings heavy quark pairs are produced back-to-back at leading order due to momentum conservation and this lead to a back-to-back correlation in azimuthal angle. These produced heavy quarks pairs move in opposite direction and hence suffer different interactions depending if they propagate towards the inner part of the bulk or towards the outer. This would drastically alter their initial back-to-back correlation [28, 29]. We have studied the azimuthal correlation of the \( c \bar{c} \) pairs within both the Boltzmann and Langevin dynamics in a box. We initialize the \( c \bar{c} \) pairs at the boundary of the box with position \( x=z=0 \) and \( y=-2.5 \text{ fm} \), and having momentum \( p_x = p_z = 0 \) and \( p_y = 10 \text{ GeV} \). Hence, the initial azimuthal correlation will be a delta around \( \Delta \phi = 0 \). As a consequence of the interaction with the thermal bath this initial azimuthal correlation will broaden around \( \Delta \phi = 0 \). We have studied the final azimuthal correlation for charm quarks within the momentum cut \( 3 < p < 4 \text{ GeV} \).

![Figure 3](image1.png)

**Figure 3.** Left: Azimuthal correlation of \( c \bar{c} \) within Langevin dynamics at different times. ; Right: Azimuthal correlation of \( c \bar{c} \) within Boltzmann equation at different times.

In Fig. 3 (left) the azimuthal correlation of \( c \bar{c} \) within Langevin dynamics has been displayed at different times while in Fig. 3 (right) the azimuthal correlation of \( c \bar{c} \) within Boltzmann equation has been shown. A larger spreading of momentum distribution in case of Boltzmann equation give rise a larger spreading of \( c \bar{c} \) correlation in comparison with the Langevin dynamics.

5. Heavy flavor suppression and flow
We have compared the Boltzmann and Langevin approach also for realistics simulations of heavy ion collisions. We have carried out simulations of \( Au + Au \) collisions at \( \sqrt{s} = 200 \text{GeV} \) for \( 20 - 30\% \) centrality. The initial conditions for the bulk in coordinate space are given by the standard Glauber model while in momenta space we employ a Boltzmann-Juttner distributions up to a transverse momentum \( p_T = 2 \text{ GeV} \) and a minijet distributions for higher momenta as calculated by pQCD at Next to leading order. For the initial distributions of charm quarks in coordinate space we use \( f_{c=0} = 4.1 \times 10^2/(0.7 + 0.09p)^{15.44} \) while for bottoms quarks we use \( f_{b=0} = 1/(57.74 + p^2)^{5.04} \). In coordinate space the HQ are distributed according to the number

![Figure 4](image2.png)
of binary collisions $N_{\text{coll}}$. Moreover the bulk that we obtained using the transport approach is then used as background for the Langevin evolution of HQ. This is a novelty of our work since usually the background for the Langevin is given by hydrodynamic simulations.

Our purpose is to perform the comparison between the Langevin and Boltzmann transport equations for different momentum transfer scenario that can be directly related to the angular distribution of scattering matrix or cross section. This can be achieved by using three different values of the Debye screening masses ($m_D$) needed to shield the divergence associated with the t-channel of the scattering matrix. We have chosen three values for $m_D$: $m_D = 0.4$ GeV which implies a very forward peaked cross section and thus a small average momentum transferred; $m_D = 0.83$ GeV and $m_D = 1.6$ GeV which implies an almost isotropic cross section and thus a quite large average momentum transferred.

![Figure 4](image1.png)

**Figure 4.** $R_{AA}$ (left) and $v_2$ (right) as a function of $p_T$ at $m_D=0.4$ GeV within both Langevin (LV) and Boltzmann (BM) approach.

![Figure 5](image2.png)

**Figure 5.** $R_{AA}$ (left) and $v_2$ (right) as a function of $p_T$ at $m_D=0.83$ GeV within both Langevin (LV) and Boltzmann (BM) approach.

In the present work we have tried to reproduce the same $R_{AA}$ in both Langevin and Boltzmann approach and we have studied the relation with the corresponding $v_2$ for all the three $m_D$ case.
Figure 6. $R_{AA}$ (left) and $v_2$ (right) as a function of $p_T$ at $m_D=1.6$ GeV within both Langevin (LV) and Boltzmann (BM) approach.

In Fig. 4 we have displayed the $R_{AA}$ (left) and $v_2$ (right) at $m_D=0.4$ GeV within both Langevin and Boltzmann approach. As expected, in this near forward peaked angular distribution of the cross section case the Langevin dynamics is quite close to the Boltzmann one (for details we refer to our earlier work [24]). In this case to reproduce the same $R_{AA}$ we need to reduce the interaction from the Langevin side by 30% (or equivalently we need only the 70% of LV interaction). In Fig. 5 we have displayed the results same as Fig 4 with $m_D=0.83$ GeV. In this case to have the same $R_{AA}$ (within LV and BM), we need to reduce the interaction from the Langevin side by almost 50%. In this case Boltzmann dynamics generate a larger $v_2$ in comparison with the Langevin dynamics. In Fig. 6 we have displayed the $R_{AA}$ (left) and $v_2$ (right) at $m_D=1.6$ GeV within both Langevin and Boltzmann approach. In this near isotropic angular distribution of the cross section case, we need to reduce the interaction by 60% from the Langevin side to get the same $R_{AA}$ as of Boltzmann. In this case ever reducing the Langevin interaction to have the same $R_{AA}$ obtained with the Boltzmann we get in the first case a smaller elliptic flow and this indicates that the Boltzmann approach is more efficient in the generation of the $v_2$.

6. Conclusions
In this proceeding we have presented a study of the approximations involved by Langevin equation making a direct comparison with the full collisional integral within the framework of Boltzmann transport equation in a box where the bulk consists of only gluon at $T=0.4$ GeV as well as for a realistic simulations of heavy ion collisions. We have found that the Langevin approach is a good approximation for bottom quark whereas for charm quark Langevin approach deviates from Boltzmann approach and the difference is larger for larger $m_D$. It has also been found that to get a similar suppression factor for both the approach we need to reduce the interaction of the Langevin approach. For the same $R_{AA}$ Boltzmann dynamics produced larger $v_2$, which may be helpful for a simultaneous reproduction of $R_{AA}$ and $v_2$ for the same set of inputs. Moreover we have found that the Boltzmann approach gives rise to a larger spreading of $c\bar{c}$ correlation in comparison with the Langevin approach.

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