On the number of $e$-folds in the Jordan and Einstein frames

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Abstract. We investigate how inflationary predictions are affected by the difference in the number of $e$-folds between the Jordan and Einstein frames. We study several test models in relation to a Jordan frame defined by the common Higgs-inflation-like non-minimal coupling to gravity and consider two different formulations of gravity: metric and Palatini. We find that the difference is quite contained in case of a metric Jordan frame while can be quite remarkable in case of a Palatini Jordan frame. We also discuss a way to overcome the discrepancy by introducing a frame invariant physical distance and a consequently frame invariant number of $e$-folds.

Keywords: Inflation, frame equivalence, Palatini, non-minimal coupling
1 Introduction

Several observations of the cosmic microwave background radiation (CMB) indicate that at large scales the Universe is flat and homogeneous. These properties can be explained by assuming an accelerated expansion during the very early Universe [1–4]. Such an inflationary era is also able to generate and preserve the primordial inhomogeneities which generated the subsequent large-scale structure that we observe. In its simplest version, inflation is usually formulated by adding to the Einstein-Hilbert action one scalar field, the inflaton, whose energy density drives the near-exponential expansion.

Recently, the BICEP/Keck Array experiment [5] has reduced even more the available parameter space, disfavoring several inflationary models. On the other hand, two of the most popular models, namely Starobinsky [1] and non-minimal Higgs inflation (e.g. [6]) still lie in the allowed region. Both of these two models share the fact that they are initially formulated in the Jordan frame, where gravity is non-minimal, and then usually moved to the Einstein frame where the inflaton is minimally coupled to gravity and inflationary computations are simpler. Since it has been proven that cosmological perturbations are invariant under frame transformations (e.g. [7, 8]), it is possible to consistently work in either the Jordan or the Einstein frame. On the other hand the equivalence of the Einstein and Jordan frames (specially at the quantum level) is still a debated question (e.g. [9–34].) An important issue comes from the definition of the number of e-folds (e.g. [28, 35] and refs therein), which is a quantity that measures the expansion of the Universe and, in its current definition, a clearly frame-dependent quantity. Using the formalism of [8], it can be proven [28] that both the Jordan frame number of e-folds and the Einstein frame one can be written in terms of invariant quantities and that the difference is an invariant itself. Therefore the issue becomes a choice between two different invariants. However the ambiguity still remains.

Another issue that arises in the context of non-minimally coupled theories concerns the choice of the dynamical degrees of freedom. In the metric formulation, the metric is the only dynamical degree of freedom and the connection chosen to be the Levi-Civita one. On the contrary, in the Palatini formulation, the metric and the connection are treated as independent variables with their corresponding equations of motion. When the action is linear in the curvature scalar, the two formalisms lead to equivalent theories, otherwise the theories are completely different [36] and lead to different phenomenological results, as recently investigated in (e.g. [37–80]).
This article is organized as follows. In section 2 we set the theoretical framework and introduce the difference between the metric and Palatini formulation and the difference between the Jordan and Einstein frame number of e-folds. In section 3 we test numerically the issue of the number of e-folds for several inflationary models in relation to a Jordan frame defined by the common Higgs-inflation-like non-minimal coupling to gravity. In section 4 we discuss a way to overcome the discrepancy by introducing a frame invariant physical distance and a consequently frame invariant number of e-folds. Finally, in section 5 we present our conclusions.

2 Theoretical framework

We start by assuming the following Jordan frame action

\[ S = \int d^4x \sqrt{-g} \left( -\frac{M_P^2}{2} f(\phi) R^J(\Gamma) + \frac{(\partial \phi)^2}{2} - V(\phi) \right), \tag{2.1} \]

where \( M_P \) is the reduced Planck mass, \( R^J \) is the Jordan frame Ricci scalar constructed from a connection \( \Gamma \), \( f(\phi) \) is the non-minimal coupling to gravity and \( V(\phi) \) is the potential of the inflaton scalar. It is possible to perform the inflationary analysis in the Jordan frame, however, since cosmological perturbations are invariant under frame transformations (e.g [7, 8]), it is usually convenient to move the problem to the Einstein frame and perform the analysis there. Such a frame is obtained via the Weyl transformation

\[ g_{\mu\nu}^E = f(\phi) g_{\mu\nu}^J, \tag{2.2} \]

that leads to the Einstein frame action

\[ S = \int d^4x \sqrt{-g^E} \left( -\frac{M_P^2}{2} R^E + \frac{(\partial \chi)^2}{2} - U(\chi) \right), \tag{2.3} \]

where the scalar potential \( U(\chi) \) is given by

\[ U(\chi) = \frac{V(\phi(\chi))}{f^2(\phi(\chi))}. \tag{2.4} \]

The canonically normalized field \( \chi \) depends on the function \( f(\phi) \) and on the assumed gravity formulation. In the common metric formulation we have

\[ \frac{\partial \chi}{\partial \phi} = \sqrt{\frac{3}{2} \left( \frac{M_P}{f} \frac{\partial f}{\partial \phi} \right)^2 + \frac{1}{f}}, \tag{2.5} \]

where the first term comes from the transformation of the Jordan frame Ricci scalar and the second from the rescaling of the Jordan frame scalar field kinetic term. On the other hand, in the Palatini formulation [36], the field redefinition is generated only by the rescaling of the inflaton kinetic term i.e.

\[ \frac{\partial \chi}{\partial \phi} = \sqrt{\Gamma}, \tag{2.6} \]

where no additional term comes from the Jordan frame Ricci scalar. Assuming slow-roll, the inflationary dynamics can described in the Einstein frame by the potential slow-roll parameters

\[ \epsilon = \frac{1}{2} M_P^2 \left( \frac{1}{U} \frac{dU}{d\chi} \right)^2, \quad \eta = M_P^2 \frac{1}{U} \frac{d^2U}{d\chi^2}. \tag{2.7} \]
Inflation takes place when $\epsilon \ll 1$. The consequent expansion of the Universe is measured in number of $e$-folds

$$N^E_e = \frac{1}{M^2_P} \int_{\chi_f}^{\chi_N} d\chi \left( \frac{dU}{d\chi} \right)^{-1},$$

(2.8)

where the field value at the end of inflation, $\chi_f$, is determined by $\epsilon(\chi_f) = 1$. The field value $\chi_N$ at the time a given scale left the horizon is given by the corresponding $N_e$. Other two important observables, i.e. the scalar spectral index and the tensor-to-scalar ratio are respectively written in terms of the slow-roll parameters as

$$n_s \simeq 1 + 2\eta - 6\epsilon$$

(2.9)

$$r \simeq 16\epsilon.$$  

(2.10)

To reproduce the correct amplitude for the curvature power spectrum, the potential has to satisfy \[81\]

$$\ln \left( 10^{10} A_s \right) = 3.044 \pm 0.014 ,$$

(2.11)

where

$$A_s = \frac{1}{24\pi^2 M^4_P} \frac{U(\chi_N)}{\epsilon(\chi_N)}.$$  

(2.12)

This constraint is commonly used to fix the normalization of the inflaton potential. When an exact solution for the inverse field redefinition $\phi(\chi)$ (and the Einstein frame potential $U(\chi)$) is not possible, all the phenomenological parameters can be still derived using $\phi$ as computational variable, the chain rule $\frac{\partial}{\partial \chi} = \frac{\partial \phi}{\partial \phi} \frac{\partial \phi}{\partial \chi}$ and eq. (2.5) or (2.6).

Even though we presented only the Einstein frame’s equations, $r$, $n_s$ and $A_s$ can be unequivocally defined also in the Jordan frame and the original frame of definition should not affect the phenomenological results (e.g \[7, 8\]). On the other hand, there is a subtle issue regarding the evaluation of the number of $e$-folds and consequently $\chi_N$ and $\phi_N$. Assuming a FRW metric in the Jordan frame, the corresponding line element is

$$ds^2_J = dt^2 - a_J^2(t)dx^2$$

(2.13)

where $a_J$ is the scale factor of the Jordan frame metric $g^J_{\mu\nu}$. Therefore we can define the number of $e$-folds in the Jordan frame as

$$N^J_e = \ln \left[ \frac{a_J(t_f)}{a_J(t_N)} \right] ,$$

(2.14)

where $t_f$ is the time when inflation ends (i.e. $\phi = \phi_f$ and $\chi = \chi_f$) and $t_N$ is the time when $\phi = \phi_N$ and $\chi = \chi_N$. Analogously, the Einstein frame line element coming from the scaling in eq. (2.2) is

$$ds^2_E = dt^2 - a_E^2(t)dx^2$$

(2.15)

where we have defined

$$dt_E = \sqrt{f} dt , \quad a_E(t) = \sqrt{f} a_J(t) ,$$

(2.16)

and the corresponding number of $e$-folds in the Einstein frame is

$$N^E_e = \ln \left[ \frac{a_E(t_f)}{a_E(t_N)} \right] .$$

(2.17)
By using eq. (2.16) and a bit of algebra we obtain

\[ N_e^E = \ln \left[ \frac{a_E(t_f)}{a_E(t_N)} \right] = \ln \left[ \sqrt{\frac{f(\phi_f)}{f(\phi_i)} a_J(t_f)} \right] = \ln \left[ \frac{a_J(t_f)}{a_J(t_N)} \right] + \ln \left[ \sqrt{\frac{f(\phi_f)}{f(\phi_i)}} \right] = N_e^J + \frac{1}{2} \ln \left[ \frac{f(\phi_f)}{f(\phi_i)} \right] \]

(2.18)
i.e. a mismatch between the two number of e-folds. This has a tremendous impact in the determination of \( \chi_N \) and \( \phi_N \) and therefore in the computation of the observables \( r, n_s \) and \( A_s \). Using the formalism of [8], it can be proven [28] that both \( N_e^J \) and \( N_e^E \) can be written in terms of invariant quantities and that the difference \( \Delta N_e = N_e^E - N_e^J \) is an invariant itself. Therefore the issue is actual about a choice between two invariants. For practical purposes, a solution is found by invoking the slow-roll approximation, treating \( \Delta N_e \) as subdominant under slow-roll and therefore ignoring it. Therefore full unequivocal invariance is restored as an approximated result under slow-roll with \( N_e^E \simeq N_e^J \). However, first of all, given the increase in precision of observational data, such a difference might play a key role in the actual compatibility of inflationary predictions with current and future data. Moreover, from a theoretical point of view, this does not solve the issue of finding an unequivocal definition for the number of e-folds and undermines all the idea behind frame equivalence.

3 Numerical examples

In this section we study the impact of the different definitions of the number of e-folds with some numerical examples. It is customary to set the theory in the Jordan frame and then move it to the corresponding Einstein one. In our case we act in the opposite way. We first set the theory in the Einstein frame and then we move it to corresponding Jordan frame, according to the gravity formulation under consideration, metric or Palatini. Therefore, we first consider the following Einstein frame inflaton potentials:

\[
U(\chi) = \Lambda^4 \left( \frac{\chi}{M_P} \right)^n \quad \text{(power-law potential)}, \tag{3.1}
\]

\[
U(\chi) = \Lambda^4 \left[ 1 + \cos \left( \frac{\chi}{\mu} \right) \right] \quad \text{(natural inflation)}, \tag{3.2}
\]

\[
U(\chi) = \Lambda^4 \left[ 1 - \left( \frac{\chi}{\mu} \right)^4 \right] \quad \text{(quartic hilltop inflation)}, \tag{3.3}
\]

\[
U(\chi) = \Lambda^4 \left[ \tanh^2 \left( \frac{\chi}{\sqrt{6} \alpha M_P} \right) \right] \quad \text{(\( \alpha \) attractor)}, \tag{3.4}
\]

and compute for each one of them inflationary predictions for \( N_e^E \in [50, 60] \). Then we compute the same prediction for \( N_e^J \in [50, 60] \) with the Jordan frame corresponding to the popular non-minimal coupling

\[ f(\phi) = 1 + \xi \left( \frac{\phi}{M_P} \right)^2. \tag{3.5} \]

We consider two different Jordan frames, one corresponding to a metric formulation of gravity (see eq. (2.5)) and one to the Palatini one (see eq. (2.6)). The results for \( r \) vs. \( n_s \) are presented in Figs. 1-4 for 50 (dashed line) and 60 (continuous line) e-folds, computed in the
Figure 1. $r$ vs. $n_s$ for the power-law potential in eq. (3.1) with $-2 < \log_{10}(n) < 2$ computed in the Einstein frame (red), in the metric Jordan frame (purple) and in the Palatini Jordan frame (green). Dashed (continuous) line represents $N_e = 50$ (60). The gray areas represent the $1, 2\sigma$ allowed regions from the latest combination of Planck, BICEP/Keck and BAO data [5].

Figure 2. $r$ vs. $n_s$ for the natural inflation potential in eq. (3.2) with $0 < \log_{10}(\mu/M_p) < 2.5$ computed in the Einstein frame (red), in the metric Jordan frame (purple) and in the Palatini Jordan frame (green). Dashed (continuous) line represents $N_e = 50$ (60). The gray areas represent the $1, 2\sigma$ allowed regions from the latest combination of Planck, BICEP/Keck and BAO data [5]. On the left panels we show the results for $\xi = 1$ and on the right ones for $\xi = 1000$. In Fig. 1 we have the results for the power-law potential in eq. (3.1) with $-2 < \log_{10}(n) < 2$. In Fig. 2 we have the results for the natural inflation potential in eq. (3.2) with $0 < \log_{10}(\mu/M_p) < 2.5$. In Fig. 3 we have the results for the quartic hilltop potential in eq. (3.3) with $-2 < \log_{10}(\mu/M_p) < 2$. In Fig. 4 we have the results for the $\alpha$-attractor T-model in eq. (3.4) with $-6 < \log_{10}(\alpha) < 4$. In all the cases we
Figure 3. \( r \) vs. \( n_s \) for the quartic hilltop potential in eq. (3.3) with \(-2 < \log_{10}(\mu/M_p) < 2\) computed in the Einstein frame (red), in the metric Jordan frame (purple) and in the Palatini Jordan frame (green). Dashed (continuous) line represents \( N_e = 50 \) (60). The gray areas represent the 1,2\( \sigma \) allowed regions from the latest combination of Planck, BICEP/Keck and BAO data [5].

Figure 4. \( r \) vs. \( n_s \) for the alpha attractor potential in eq. (3.4) with \(-6 < \log_{10}(\alpha) < 4\) computed in the Einstein frame (red), in the metric Jordan frame (purple) and in the Palatini Jordan frame (green). Dashed (continuous) line represents \( N_e = 50 \) (60). The gray areas represent the 1,2\( \sigma \) allowed regions from the latest combination of Planck, BICEP/Keck and BAO data [5].

can see that the difference in the results increases with \( \xi \) increasing. However the increase in the metric Jordan frame is more or less contained: between \( \xi = 1 \) and \( \xi = 1000 \) the difference is not that remarkable but still visible in the plots. On the other hand, the increase of the discrepancy from \( \xi = 1 \) to \( \xi = 1000 \) in the Palatini Jordan frame is dramatic. While for \( \xi = 1 \) the predictions in the Palatini Jordan frame are usually only slightly altered, for \( \xi = 1000 \) they are completely changed. In the power law case, Fig. 1, from being ruled out in all the regions of the parameters space because of a too large \( r \) or \( n_s \), we pass to a strong reduction of \( r \) and \( n_s \) values (even reaching the core of 1\( \sigma \) region) and being allowed for very small
values of $n$. In the natural inflation case, Fig. 2, from being ruled out in all the region of the parameters space because of a too large $r$ or too small $n_s$ for $\xi = 1$, we again completely invert the situation for $\xi = 1000$ with $r$ and $n_s$ values even in the core of $1\sigma$ region for $5.18 < \mu/M_p < 5.82$. In the quartic hilltop inflation case, Fig. 3, for $\xi = 1$ the allowed region is $6.1 < \mu/M_p^2 < 35.5$ with $r \sim 10^{[-3,-2]}$. Instead for $\xi = 1000$ it is $0.68 < \mu/M_p^2 < 2.85$ with $r \sim 10^{[-6,-5]}$. In the $\alpha$-attractor case, Fig. 4, for $\xi = 1$ the discrepancy is visible only for $r \sim 10^{[-3,-2]}$ and most of the parameters space is in the allowed region. Instead for $\xi = 1000$, both $r$ and $n_s$ values decrease sensibly and a lot of the parameters space is out of the allowed region.

4 A possible solution

The number of $e$-folds describes how much the Universe expanded because of inflation and it is usually defined directly from the scale factor of the metric:

$$N_e = \ln \left[ \frac{a(t_f)}{a(t_N)} \right]. \quad (4.1)$$

An alternative but more operative definition would involve instead the physical distance between two fixed points in the comoving frame measured at the times $t_N$ and $t_f$

$$N_e = \ln \left[ \frac{\Delta r(t_f)}{\Delta r(t_N)} \right] = \ln \left[ \frac{a(t_f)\Delta x}{a(t_N)\Delta x} \right] \equiv \ln \left[ \frac{a(t_f)}{a(t_N)} \right] \quad (4.2)$$

where $\Delta r$ is the physical distance and $\Delta x$ is the comoving distance. The definitions (4.1) and (4.2) are exactly equivalent and from both we can immediately see the issue with frame invariance. The metric (and therefore the scale factor) and absolute physical distances are not frame invariant and therefore neither is the number of $e$-folds. On the other hand, operatively, we do not measure absolute distances, but only relative ones i.e. we need a unit of measurement. Therefore, we introduce the concept of normalized distance i.e. the ratio of the distance over the fundamental reference length, the Planck length of the corresponding frame: $\ell_P^i$, where $i$ labels the frame. Hence, we can define the normalized physical distance as

$$\Delta \bar{r} = \frac{\Delta r_i}{\ell_P^i} = \frac{a(t)\Delta x}{\ell_P^i} \quad (4.3)$$

where $i$ labels the frame. It is easy to verify that such a definition is actually frame invariant. In the Einstein frame we have

$$\Delta \bar{r} = \frac{\Delta r_E}{\ell_P^E} = a(t)E \Delta x M_P \quad (4.4)$$

while in the Jordan frame we have

$$\Delta \bar{r} = \frac{\Delta r_J}{\ell_P^J} = a(t)J \Delta x \sqrt{f(\phi)M_P} = a(t)E \Delta x M_P \quad (4.5)$$

where we used eq. (2.16). The Jordan frame normalized distance in eq. (4.5) is identical to the Einstein frame one in eq. (4.4). Therefore we can define an invariant number of $e$-folds

$$\bar{N}_e = \ln \left[ \frac{\Delta \bar{r}(t_f)}{\Delta \bar{r}(t_N)} \right] \quad (4.6)$$
We can easily check that such a definition recovers the standard Einstein frame definition in both the Jordan and Einstein frame

\[
\bar{N}_e = \ln \left[ \frac{\Delta r(t_f)}{\Delta r(t_N)} \right] = \ln \left[ \frac{\Delta r_J(t_f)/\ell_P}{\Delta r_J(t_N)/\ell_P} \right] = \ln \left[ \frac{a(t_f)E \Delta x M_P}{a(t_N)E \Delta x M_P} \right] = \ln \left[ \frac{a(t_f)E}{a(t_N)E} \right] = \ln \left[ \frac{\Delta r_P}{\Delta r_J(t_N)/\ell_P} \right] = \ln \left[ \frac{a(t_f)E \Delta x M_P}{a(t_N)E \Delta x M_P} \right] = \ln \left[ \frac{a(t_f)E}{a(t_N)E} \right]
\]

(4.7)

(4.8)

solving the discrepancy between the old standard definitions of the number of \(e\)-folds in different frames.

5 Conclusions

In this article we investigated how inflationary predictions are affected by the difference in the number of \(e\)-folds between the Jordan and Einstein frames. We studied as test examples four well-known models: monomial inflation, natural inflation, quartic hilltop inflation and \(\alpha\)-attractors. We considered a Jordan frame defined by the popular Higgs-inflation-like non-minimal coupling to gravity and two different formulations of gravity: metric and Palatini. We found that for all tested models, the difference between the predictions for an Einstein frame number of \(e\)-folds and a metric Jordan frame one remains quite contained regardless of the size of the non-minimal coupling to gravity, but still appreciable in the \(r\) vs. \(n_s\) plot. On the other hand the difference between the results for an Einstein frame number of \(e\)-folds and a Palatini Jordan frame one becomes gigantic in case of a relatively big non-minimal coupling to gravity. This might have a tremendous impact in ruling in or out inflationary models, specially in light of the precision of the forthcoming experiments (e.g. Simons Observatory [82], PICO [83], CMB-S4 [84] and LITEBIRD [85]).

Finally, we discuss a way to overcome the discrepancy by introducing the concept of renormalized distance, i.e. the ratio of the distance to the Planck length in the corresponding frame. We prove that the renormalized physical distance is frame invariant. In particular it can be used to define an invariant number of \(e\)-folds as the ratio of the renormalized physical distance (between two fixed points in the comoving frame) evaluated at end of inflation to the same distance at the time when a given scale leaves the horizon. If accepted, this new definition, would put an end to the ambiguity of inflationary computations between the Jordan and Einstein frame.

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