Searching for Boosted Dark Matter mediated by a new Gauge Boson

Wonsub Cho, 1 Ki-Young Choi, 1‡ and Seong Moon Yoo 1†
1 Department of Physics, Sungkyunkwan University, 2066, Seobu-ro, Jangan-gu, Suwon-si, Gyeong Gi-do, 16419 Korea

We study the possibility to directly detect the boosted dark matter generated from the scatterings with high energetic cosmic particles such as protons and electrons. As a concrete example, we consider the sub-GeV dark matter mediated by a \( U(1)_D \) gauge boson which has mixing with \( U(1)_Y \) gauge boson in the standard model. The enhanced kinetic energy of the light dark matter from the collision with the cosmic rays can recoil the target nucleus and electron in the underground direct detection experiments transferring enough energy to them to be detectable. We show the impact of BDM with existing direct detection experiments as well as collider and beam-dump experiments.

PACS numbers:

I. INTRODUCTION

The nature of dark matter (DM) is one of the unsolved problems in the astro-particle physics that spans from the small scales of galaxy to the large scales of the Universe [1]. The underground direct-detection experiment is one of the ways to search for the non-gravitational nature of DM and the sensitivity of the elastic scattering cross section on the nucleus. Now goes down to \( \sigma_{\chi p} \gtrsim 4.1 \times 10^{-47} \text{cm}^2 \) at 30 GeV of DM mass [2]. The constraints on the scattering cross section of DM with electron is \( \sigma_{\chi e} \gtrsim 3 \times 10^{-38} \text{cm}^2 \) at 100 MeV [3, 4].

In these studies of the DM direct detection, the DMs are assumed to be non-relativistic with a Maxwell-Boltzmann distribution around the Milky Way galaxy with speed around \( 10^{-3} \text{c} \), with the speed of light \( c \). However recently it was noticed that the small amount of DMs in the Milky Way can be boosted due to the scatterings with high energetic cosmic rays (CRs) of nuclei [5, 6]. The boosted DM (BDM) can transfer large momentum to the target and make the recoil energy above the detector threshold even with the light DM. This was used to search for dark matter in simple models [7, 8].

In this paper, we apply this novel method to the light DM mediated by a new \( U(1)_D \) gauge boson which has a mixing with \( U(1)_Y \) in the Standard Model [9, 10], which is one of the simplest extension of the Standard Model (SM). In this model, the mixing connects the visible and hidden sector through the mediation of the gauge bosons and opens the portal to the DM in the hidden sector. Here the DMs can interact with both nuclei and electrons, and therefore it is necessary to consider both scatterings with nuclei and electrons in the BDM generation as well as in the direct detection. This gives different behavior and constraints compared to the previous analysis assuming a single kind of interaction. In this study, we give the realization of the up-scattered DM by cosmic rays of a vector-mediation [11] and complements the existing constraints on this model [12, 13].

In Sec. II we introduce the model we consider, and in Sec. III we summarize the generation of BDM and attenuation. In Sec. IV we show the results with constraints from BDM, and conclude in Sec. VI.

II. MODEL

We consider a model of Dirac fermion dark matter with a dark gauge symmetry \( U(1)_D \) which mediates the interaction between dark and SM sector through mixing with \( U(1)_Y \) in the Standard Model [14, 21, 22]. The Lagrangian is given by

\[
\mathcal{L}_{Z_d} = -\frac{1}{4} \hat{Z}_{d\mu\nu} \hat{Z}^{\mu\nu}_d + \frac{\sin \varepsilon}{2} \hat{B}_{\mu\nu} \hat{Z}^{\mu\nu}_d + \frac{1}{2} (m_{Z_d}^0)^2 \hat{Z}^{\mu}_d \hat{Z}_d\mu, \tag{1}
\]

where \( \hat{B}_{\mu\nu} \) and \( \hat{Z}_{d\mu\nu} \) are the field strengths of \( U(1)_Y \) in the SM and \( U(1)_D \) in the dark sector respectively, with a small mixing term parametrized by \( \sin \varepsilon \), and \( m_{Z_d} \) is the mass of dark gauge boson. Here we assume that the hidden sector gauge symmetry is spontaneously broken by additional Higgs so that the mass of hidden gauge boson \( Z_d \) is generated. The fermion dark matter \( \chi \) has gauge interaction with hidden gauge boson with gauge coupling \( g_d \) as

\[
\mathcal{L}_{int} = g_d \hat{Z}_{d\mu} \chi \gamma^\mu \chi. \tag{2}
\]

Below the electroweak symmetry breaking, the mass eigenstates (without hat) are related to the bare gauge fields (with hat) as

\[
\hat{A} = A_{SM} - c_w t_e s_X Z_{SM} + c_w t_e c_X Z_d, \\
\hat{Z} = (c_X + s_w t_e s_X) Z_{SM} + (s_X - s_w t_e c_X) Z_d, \\
\hat{Z}_d = -\frac{s_X}{c_X} Z_{SM} + \frac{c_X}{e_X} Z_d,
\]

with the mixing angle \( \theta_X \) given by

\[
\tan 2\theta_X = \frac{2(m_{Z_d}^0)^2 s_w t_e}{(m_{Z_d}^0)^2(1 - s_w^2 t_e^2) - (m_{Z_d}^0 c_w t_e)^2}. \tag{4}
\]
Here \( m_Z \) is the mass of Z-boson in the SM, and we use the abbreviations defined by \( s_W = \sin \theta_W \), \( c_W = \cos \theta_W \) with Weinberg mixing angle \( \theta_W \), and \( t_\varepsilon = \tan \varepsilon \), \( c_\varepsilon = \cos \varepsilon \), \( s_\varepsilon = \sin \varepsilon \), and similarly for \( c_X = \cos \theta_X \), and \( s_X = \sin \theta_X \).

In the SM, the gauge interaction for a fermion \( \psi \) with \( SU(2) \) charge \( q_\psi \) and electromagnetic charge \( e_\psi \) is given by

\[
L_{SM, int} = \bar{\psi} \gamma^\mu \psi \left\{ eQ \hat{A}_\mu + \frac{e}{s_W c_W} (T_3 - Q s_W^2) \hat{Z}_\mu \right\},
\]

where \( \psi = \nu_L, e_L, e_R \), etc and \( e = |e| \). In Appendix, we show the corresponding interaction Lagrangian between DM and proton, neutron, electron and neutrino, from which the elastic scattering cross sections are calculated.

For the scattering with nucleus, the cross section at finite momentum transfer is corrected with a form factor as given by

\[
\sigma_{XN}(s, q^2) = \sigma_{XN}(s) \times F^2(q^2),
\]

where \( q^2 = 2m_N T_N \) with the mass of the target \( m_N \) and recoil kinetic energy \( T_N \). Here we use the Helm form factor \( \Phi_D \) with

\[
F(q^2) = \frac{3}{2} \frac{j_1(q r_n)}{q r_n} e^{-q^2 s^2/2},
\]

where \( j_1 \) is the spherical Bessel function, \( s = 1 \) fm is the nuclear skin thickness, and \( r_n = (c^2 + \frac{5}{2} \pi^2 a^2 - 5 s^2)^{1/2} \) parametrizes the nuclear radius, with \( c = 1.23 A^{1/3} - 0.6 \) fm and \( a = 0.52 \) fm, and \( A \) is the mass number of the nucleus.

In Fig. 1 we show the total scattering cross sections as the initial kinetic energy of CRs of proton (blue), He (red), and electron (green), in the rest frame of DM with mass \( m_\chi = 10^{-3} \) GeV (Left) and 0.1 GeV (Right). Here we used the parameters \( m_{Z_d} = 0.03 \) GeV, \( \alpha_d = g_\chi^2/(4\pi) = 1 \), and \( \sin^2 \varepsilon = 10^{-7} \). We can see that the temperature dependence of the cross section varies for different mass parameters. When \( m_\chi < m_{Z_d} \) (Left), the cross section grows corresponding to the momentum transfer between \( m_\chi \) and \( m_{Z_d} \). However when \( m_\chi > m_{Z_d} \) (Right), the cross section decreases with \( T_i \) for CR proton and Helium, because momentum transfer is larger than the mass of \( m_{Z_d} \).

In Fig. 2, the contour plots of the scattering cross section of DM with cosmic ray proton are shown on the plane of \((m_\chi, \sin^2 \varepsilon)\) with \( m_{Z_d} = 0.03 \) GeV (Left) and on the plane of \((m_{Z_d}, \sin^2 \varepsilon)\) with \( m_\chi = 0.1 \) GeV (Right) with initial kinetic energy \( T_i = 1 \) GeV and \( \alpha_d = 1 \).

### III. BOOSTED DARK MATTER FROM SCATTERINGS WITH COSMIC RAYS

**Boosted DM** The DMs in the Galactic halo are scattered by the cosmic rays. In the initial rest frame of DM, the recoiled kinetic energy of DM after scattering \( T_\chi \) can be written as

\[
T_\chi = T_\chi^{\max} \frac{1 - \cos \theta}{2},
\]

\[
T_\chi^{\max} = \frac{T_i^2 + 2m_i T_i}{T_i + (m_\chi + m_i)^2/(2m_\chi)},
\]

where \( m_\chi \) and \( m_i \) are the mass of DM and the colliding CR particle, respectively, and \( \theta \) is the scattering angle in the center-of-mass frame between DM and CR particle. Here \( T_\chi^{\max} \) is the maximum kinetic energy that the DM can have after scattering. The momentum transfer in the collision can be written as \( Q^2 = 2m_i T_\chi \). In other way, the minimum kinetic energy of the cosmic particles to make DM with \( T_\chi \) is given by

\[
T_i^{\min} = \left( \frac{T_\chi}{2} - m_i \right) \left( 1 \pm \sqrt{1 + \frac{2T_\chi (m_i + m_\chi)^2}{m_\chi (2m_i - T_\chi)^2}} \right),
\]

where + for \( T_\chi > 2m_i \) and − for \( T_\chi < 2m_i \). When DM collides to the nuclei in the rest frame, \( i \) and \( \chi \) are interchanged in the above equations.
FIG. 2: The contour plot of the scattering cross section \( \log_{10}(\sigma / \text{cm}^2) \) of DM with CR proton on the plane of \((m_\chi, \sin^2 \varepsilon)\) with \(m_{Zd} = 0.03\text{ GeV}\) (Left) and \((m_{Zd}, \sin^2 \varepsilon)\) with \(m_\chi = 0.1\text{ GeV}\) (Right) with initial kinetic energy of proton \(T_i = 1\text{ GeV}\) and \(\alpha_d = 1\).

FIG. 3: Differential flux in terms of kinetic energy of CR proton, Helium, and electron [27].

To find the flux of BDM, we follow the method in Ref. [6]. The differential flux of BDM with the kinetic energy \(T_\chi\) is obtained by integrating the flux of DM after scattering with initial kinetic energy of cosmic particle \(T_i\) as

\[
\frac{d\Phi_\chi}{dT_\chi} = \sum_{i=p, He, e} \int_{T_i^{\text{min}}}^{\infty} dT_i \frac{d\Phi_i}{dT_i dT_\chi},
\]

\[
= \frac{\rho_{\chi}^{\text{local}}}{m_\chi} D_{\text{eff}} \sum_{i=p, He, e} \int_{T_i^{\text{min}}}^{\infty} dT_i \frac{d\sigma_{\chi i}(T_i)}{dT_\chi} \frac{d\Phi_{T_i}}{dT_i},
\]

where \(T_i^{\text{min}}\) is the minimum energy of cosmic rays to give DM kinetic energy \(T_\chi\) after collision. Here we summed over the contributions from each CR of proton, Helium, and electron. In the second line, the scattering cross section between DM and CR \(\sigma_{\chi i}\) is a function of \(T_i\). For the flux of cosmic particles, we use the interstellar spectrum of the high energy cosmic particles observed by Voyager 1 [27]. In Fig. 3 we show the flux of CRs we used, and assume that the CR flux is uniform in the DM halo.

In the second line, the effective distance \(D_{\text{eff}}\) is defined as

\[
D_{\text{eff}} = \left(\frac{\rho_{\chi}^{\text{local}}}{4\pi}\right)^{-1} \int d\Omega \int d\ell \rho_{\chi},
\]

where we used \(\rho_{\chi}^{\text{local}} = 0.3\text{ GeV/cm}^3\). Here as a representative value we use the effective distance \(D_{\text{eff}} = 1\text{ kpc}\).

In Fig. 4 we show the flux of the BDM generated from scatterings with proton (blue), He (orange), electron (green), and the total (black), for reference values of \(m_\chi = 0.1\text{ GeV}, m_{Zd} = 30\text{ MeV}, \alpha_d = 1,\) and \(\sin^2 \varepsilon = 10^{-7}\). For heavier DM with \(m_\chi = 0.1\text{ GeV}\) (Right), the proton and Helium dominates, however for the light DM with \(m_\chi = 1\text{ MeV}\) (Left), the electron scattering is comparable to those from proton and Helium. This can be easily understood from the Fig. 1. When the mass of DM is lowered, the number of DM increases, and the cross section to nuclei is however decreased at \(T_i \sim \text{ GeV}\), and they more or less compensate. However for electron CR, the cross section is almost the same, and thus the BDM flux increases for lighter DM. As can be seen from the Fig. 4 (Left) with \(m_\chi = 10^{-3}\text{ GeV}\), the contribution of the CR proton and Helium is dominant at \(T_\chi \lesssim 0.1\text{ GeV}\), while the electron contribution is larger at \(T_\chi \gtrsim 0.1\text{ GeV}\).

**Attenuation** When the DMs come through the Earth crust, they can interact with the medium and lose energy. This attenuation of kinetic energy could make DM...
undetectable because the DMs cannot reach the detector or the kinetic energy of DM become too small for the threshold in the direct detection. The energy loss of DM particles per depth that passing through the medium is

$$\frac{dT_X}{dz} = - \sum_N n_N \int_0^{T_r^{\text{max}}} \frac{d\sigma_{\chi N}(T_r)}{dT_r} T_r dT_r,$$

where $T_r$ is the energy lost by BDM in a collision with nucleus $N$. In a realistic model, the energy dependence of the cross section must be considered.

As a simple example, for a constant scattering cross section and for isotropic scattering $\frac{d\sigma_{\chi N}(T_r)}{dT_r} = \sigma_{\chi N}/T_r^{\text{max}}$, we can approximate Eq. (12) as,

$$\frac{dT}{dz} = -\frac{1}{2} \sum_N n_N \sigma_{\chi N} T^{\text{max}} \approx -\frac{1}{2m_{\chi} \ell} (T_X^2 + 2m_{\chi} T_X),$$

where $n_N$ is number density of $N$ nuclei, $T^{\text{max}}$ is maximum kinetic energy of recoiled nucleus. In the second equation, we used $T_X \ll m_N$, and

$$\ell^{-1} = \sum_N n_N \sigma_{\chi N} \frac{2m_N m_{\chi}}{(m_N + m_{\chi})^2},$$

where $N$ includes relevant nuclei in the medium. The Eq. (13) is solved as

$$T_X^z = 2m_{\chi} \left[ \frac{2m_{\chi}}{T_0} e^{z/\ell} - 1 \right]^{-1}.$$

After solving differential equation Eq. (13), we obtain the differential flux

$$\frac{d\Phi_X}{dT_X} = \left( \frac{dT_X}{dT_X} \right) \frac{d\Phi_X}{dT_X} = \frac{4m_{\chi}^2 e^{z/\ell}}{(2m_{\chi} + T_X^z - T_X^z e^{z/\ell})^2} \frac{d\Phi_X}{dT_X},$$

where $T_X^z$ is the kinetic energy of dark matter at depth $z$. For non-relativistic case, $m_{\chi} \gg T_X$, the flux is suppressed efficiently at depth $z$ by the exponential factor $e^{z/\ell}$ for the cross section larger than

$$\sigma_{\chi N} \sim 10^{-27} \text{cm}^2 \left( \frac{10^{23} \text{cm}^{-3}}{n_N} \right) \left( \frac{m_{\chi}}{10 \text{GeV}} \right) \left( \frac{1 \text{GeV}}{m_{\chi}} \right),$$

which depends inversely on $m_{\chi}$. For relativistic DM, $m_{\chi} \ll T_X$, Eq. (16) becomes

$$\frac{d\Phi_X}{dT_X} = \frac{1}{(1 - z T_X^z \sum_N n_N \sigma_{\chi N}/m_{\chi})^2} \frac{d\Phi_X}{dT_X},$$

and the attenuation happens at depth $z$ for the cross section larger than

$$\sigma_{\chi N} \sim 10^{-27} \text{cm}^2 \left( \frac{m_{\chi}}{10 \text{GeV}} \right) \left( \frac{1 \text{GeV}}{T_X} \right) \left( \frac{10^{23} \text{cm}^{-3}}{n_N} \right),$$

which is independent of $m_{\chi}$, however depends on $T_X^z$.

Since the elastic scatterings on nuclei dominates the attenuation compared to the electrons, we consider only the nuclei to determine the critical cross section for the attenuation.

IV. DIRECT DETECTION OF BOOSTED DM

The DMs that survived the attenuation of the Earth crust reach the underground detector and can scatter the nuclei or the electrons.

A. DM-nucleus interaction

The BDMs that reach down the Earth could collide with target nucleus inside in the detector. This time, the nucleus is at rest and the DM is moving, which is the opposite situation for upscattering DM by cosmic

FIG. 4: Flux of BDM around Earth generated from scatterings with proton (blue), He (orange), electron (green), and the total (black). Here we used $m_{\chi} = 10^{-3}$ GeV (Left), and 0.1 GeV (Right), with $m_{\chi} = 0.03$ GeV, $\alpha_d = 1$, and $\sin^2 \theta = 10^{-7}$.
similarly to Eq. (10) as rays. The differential rate per target nucleus is obtained similarly to Eq. (11) as

$$\frac{d\Gamma}{dT_N} = \int_{T_x(T_x^{\text{min}},z)}^{\infty} dT_x d\sigma_{\chi N} d\Phi_{\chi},$$

(20)

where the $T_x(T_x^{\text{min}},z)$ is kinetic energy of boosted DM particle outside Earth which gives the minimum kinetic energy of target nucleus $T_N$. Then we can calculate the count rate by integrating between the experimentally accessible recoil energies $T_N \in \{T_1, T_2\}$, and compare it with the observational constraint.

For the present bound from Xenon-1T, we use $T_1 = 4.9$ kV, $T_2 = 40.9$ kV, and require that $\Gamma_N < \Gamma_{\text{Xenon-1T}}^{\text{MIN}} \approx 10^{-34} /s$. For future prospect, we use factor 10 higher sensitivity with Xenon nT [28], and 500 to get to the neutrino floor.

**B. DM-electron interaction**

The BDM scatterings with electron can be probed if the recoil energy of the electron $T_e$ is large enough [5]. Using the results of Super-K with 161.9 kton yr [28], that is searching signals with $T_e > 100$ MeV, we apply the number of the events in the range $0.1 \text{ GeV} < T_e < 1.33 \text{ GeV}$ is smaller than 4042 for 2628.1 days of SK to put the constraint.

**C. Results**

In Fig. 5 we show the constraints on the parameters of $(m_\chi, \sin^2 \varepsilon)$ from BDM for the fixed values of $\alpha_d = 1$ and $m_{Z_d} = 30$ MeV. The red (blue) shaded region in the left top is disallowed from the direct detection of the BDM with nuclei (electrons) in the detector. The future prospects are shown with dashed (10 times) dotted lines (500 times). The constraints from other direct detection experiments are shown with thin colors: direct detection with nuclei at Xenon-1T (orange) [5], and direct detection with electrons at Super-K (green) [29].

The BDM constraints complements the other bounds of the direct detection with the non-relativistic DM. This new bound closes a small open spot at around $m_\chi = 0.1$ GeV and $\sin^2 \varepsilon = 3 \times 10^{-5}$ and exclude the region of $m_\chi < 4$ MeV and $\sin^2 \varepsilon \gtrsim 10^{-5}$ which is not probed by non-relativistic DM direct detection. However these white regions of left-bottom and right-top are also constrained when we include the bound from the beam-dump experiments and cosmological considerations.

In this realistic model of DM, the shape of the constraint is different from those where constant cross section was assumed [8], or that where a simple vector mediation model to the nucleon was used [3]. The $m_\chi$-dependence of the BDM constraint can be understood as follows.

First, the number of DM in the halo is inversely proportional to $m_\chi$. For the DM-nucleus direct detection, we need to have recoil energy of nucleus larger than keV. For $m_\chi \lesssim 10$ MeV, this is satisfied for the DM kinetic energy larger than around 10 MeV, at which the BDM flux is mainly from CR of proton and Helium as well as comparable contribution from electron. The energy transferred from CR proton to DM scales as $T_e \propto 2m_\chi T_p^2/m_p^2$, and the integral of the CR flux, which scales $\sim T_p^{-2.7}$, is proportional to $(T_e^{\text{min}})^{-1.7} \propto m_\chi^{-0.85}$. Therefore the event rate is proportional to $\Gamma \propto m_\chi^{-1} \varepsilon^4 m_\chi^{-0.85} = \varepsilon^4 m_\chi^{-0.15}$, which gives $\varepsilon^4 \propto m_\chi^{-0.075}$. For $m_\chi \gtrsim 10$ MeV, the CR
proton to DM scattering cross section becomes dependent on $\varepsilon^2 m_\chi^2$, and also the recoil energy of nucleus scales $T_N \approx \frac{2m_\chi T_1}{m_N}$, with $T_1 \approx 2m_\chi T_1^2/m_p^2$. Therefore $\Gamma \propto \varepsilon^4 m_\chi^{2.7}$, which gives $\varepsilon^2 \propto m_\chi^{-1.35}$. That explains the up and down of the BDM constraint (red) in Fig. 3.

For DM-electron direct detection in Super-K, it is necessary that the recoil energy of electron be larger than 100 MeV. For $m_\chi \lesssim 10$ MeV, the dominant contribution to BDM comes from CR electron, and for $m_\chi \gtrsim 10$ MeV it comes from CR proton/Helium. For low $m_\chi$ region, the event rate scales as $\Gamma \propto m_\chi^{-1} T_1^2$, resulting in $\varepsilon^2 \propto \frac{1}{m_\chi^{0.7}}$. For large $m_\chi$ region, $T_2 \approx 2m_\chi T_1^2/m_\chi^2$, and $\Gamma \propto \varepsilon^4 m_\chi^{-2.7}$. This gives $\varepsilon^2 \propto m_\chi^{1.35}$.

In Fig. 6 we show the constraints on the plane of $(m_{Zd}, \sin^2 \varepsilon)$ for $m_\chi = 100$ MeV and $a_d = 1$, with other direct detection bound (orange and green) as well as the constraints from collider [31] and beam-dump experiments [32] (grey). The present BDM constraint is already within the bounds of collider and in the future BDM may touch the unbounded region by them, though it is already ruled-out by the Xenon10 experiment.

V. ASTROPHysical CONSTRAINTs

The large kinetic mixing of the hidden gauge boson with SM may change the effective number of neutrinos, which represents the degrees of freedom of relativistic decoupled species. The current Planck observation gives lower bound on the allowed mass of hidden gauge boson around 8.5 MeV for the mixing parameter $\sin \varepsilon \gtrsim 10^{-9}$ [18].

The large annihilation of DMs in the early Universe also can affect the BBN and CMB [22, 33]. However this may be avoided for a specific models of dark matter such as asymmetric dark matter. This requires non-thermal production of dark matter, which is beyond of our simple model of kinetic mixing [34].

VI. CONCLUSION

We studied the impact of the boosted dark matter generated by scatterings of the high energy cosmic rays mediated by the $U(1)$ gauge kinetic mixing. The non-observation in the underground direct detection combined with the BDM constrains the light dark matter region, independently of the previous bounds of the direct detection as well as the collider and beam-dump experiments.

VII. APPENDIX

A. Kinematics

The differential cross section for elastic scattering of particle 1 and 2 is given by

$$\frac{d\sigma}{dt} = \left| \frac{M^2}{16\pi\lambda(s, m_1^2, m_2^2)} \right|^2,$$

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca = \left[ a - \sqrt{b + c} \right]^2 \left[ a - \sqrt{b - c} \right]^2.$

If particle 2 is at rest initially, the Mandelstam variables are given by

$$s = m_1^2 + m_2^2 + 2E_1 m_2,$$

$$t = 2m_2^2 - 2m_2 E_2 = -2m_2 T_2,$$

$$u = 2(m_1^2 + m_2^2) - s - t = M^2 - 2m_1 m_2 - 2m_2(T_1 - T_2),$$

where $M^2 = m_1^2 + m_2^2$, and

$$\lambda(s, m_1^2, m_2^2) = (s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)$$

$$= (2E_1 m_2 - 2m_1 m_2)(2E_1 m_2 + 2m_1 m_2)$$

$$= 4m_2^2(T_1^2 + 2m_1 T_1)$$

$$= 2s - m_2 \cdot T_2^{\text{max}}.$$
Therefore we can write Eq. 21 into

$$\frac{d\sigma}{dT_2} = -2m_2 \frac{d\sigma}{dt} = -\frac{|\mathcal{M}|^2}{16\pi T_2^{max}}.$$  

If $|\mathcal{M}|^2$ is constant, the total cross section becomes

$$\sigma_{tot} = \int_{-2m_2 T_2^{max}}^{0} \left( \frac{d\sigma}{dt} \right) dt = \frac{|\mathcal{M}|^2}{16\pi s}.$$  

### B. Scattering cross section of DM in the model of dark gauge boson

The Lagrangian we are using is written by

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{4} \hat{Z}_{d \mu \nu} \hat{Z}^{\mu \nu}_d + \frac{\sin \varepsilon}{2} \hat{B}_{\mu \nu} \hat{Z}^{\mu \nu}_d + \frac{1}{2} (m_{Z_d}^0)^2 \hat{Z}_d^{\mu} \hat{Z}_d^{\mu} + \mathcal{L}_{int},$$  

where $\hat{B}_{\mu \nu}$ and $\hat{Z}_{d \mu \nu}$ are the field strengths of $U(1)_Y$ in the SM and $U(1)_D$ in the dark sector respectively, with a small mixing term parametrized by $\sin \varepsilon$, and $m_{Z_d}$ is the mass of dark photon. The fermion dark matter $\chi$ has gauge interaction with hidden gauge boson with gauge coupling $g_d$ as

$$\mathcal{L}_{int} = g_d \chi \gamma^\mu \hat{Z}_{d \mu}.$$  

The mixing term between $\hat{B}$ and $\hat{Z}_d$ can be removed by the field redefinition,

$$\begin{bmatrix} B_\mu^0 \\ Z_{d \mu}^0 \end{bmatrix} = \begin{bmatrix} 1 - \sin \varepsilon \\ \cos \varepsilon \end{bmatrix} \begin{bmatrix} \hat{B}_\mu \\ \hat{Z}_{d \mu} \end{bmatrix}. $$  

The electroweak symmetry breaking generates mass to $\hat{Z}$ boson with massless $\hat{A}$, which are defined by

$$A_\mu = c_W \hat{B}_\mu + s_W \hat{W}_\mu^3, \quad Z_\mu = -s_W \hat{B}_\mu + c_W \hat{W}_\mu^3,$$

in terms of Weinberg mixing angle $\theta_W$ with $c_W \equiv \cos \theta_W$ and $s_W \equiv \sin \theta_W$. The mass term can be written in terms of $Z_0$ and $Z_0^3$ by

$$\frac{1}{2} (m_Z^0)^2 \hat{Z}_\mu \hat{Z}^\mu = \frac{1}{2} (m_Z^0)^2 (-s_W \hat{B}_\mu + c_W \hat{W}_\mu^3) (-s_W \hat{B}^\mu + c_W \hat{W}^3, \mu),$$

$$= \frac{1}{2} (m_Z^0)^2 Z_\mu^0 Z_{\mu}^{0,\mu} - \frac{(m_Z^0)^2}{2} s_W t_\varepsilon Z_\mu^0 Z_{\mu}^{0,\mu} + \frac{1}{2} (m_Z^0)^2 s_W t_\varepsilon Z_\mu^0 Z_{\mu}^{0,\mu}, $$

where

$$Z_\mu^0 = -s_W B_\mu^0 + c_W W_\mu^3, \quad A_\mu^0 = A_\mu.$$

Then the mass matrix in the basis of $(A_\mu^0, Z_\mu^0, Z_{d \mu}^0)$ is written as

$$M^2 = \begin{bmatrix} 1 & 0 & (m_Z^0)^2 s_W t_\varepsilon \\ 0 & -(m_Z^0)^2 s_W t_\varepsilon & -\frac{(m_Z^0)^2}{2} s_W t_\varepsilon \\ 0 & \frac{(m_Z^0)^2}{2} s_W t_\varepsilon & (m_Z^0)^2 c_W^2 t_\varepsilon + (m_Z^0)^2 s_W^2 t_\varepsilon \end{bmatrix}. $$
which can be diagonalized to find the mass eigenstates \((A_{SM}, Z_{SM}, Z_d)\)

\[
\begin{bmatrix}
A_{SM\mu} \\
Z_{SM\mu} \\
Z_{d\mu}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta_X & -\sin \theta_X \\
0 & \sin \theta_X & \cos \theta_X
\end{bmatrix}
\begin{bmatrix}
A_\mu^0 \\
Z_\mu^0 \\
Z_{d\mu}^0
\end{bmatrix},
\]

(34)

with the mixing angle \(\theta_X\) given by

\[
\tan 2\theta_X = \frac{2(m_{Z_d}^0)^2 s_W t_X}{(m_Z^0)^2 (1 - s_W^2 t_X^2) - (m_{Z_d}^0)^2 / c_v^2}.
\]

(35)

The mass eigenvalues for \(Z_{SM}\) and \(Z_d\) are \([20]\)

\[
\begin{align*}
m_{Z_{SM}}^2 &= (m_Z^0)^2 (1 + s_W t_X), \\
m_{Z_d}^2 &= \frac{(m_{Z_d}^0)^2}{c_v^2} \left(1 + s_W t_X\right)^{-1}.
\end{align*}
\]

(36)

In this paper, with small \(\varepsilon\), we can approximate \(m_{Z_{SM}} \simeq m_Z^0\) and \(m_{Z_d} \simeq m_{Z_d}^0\). By rearranging the above terms, we can find the relations between the mass eigenstates of the gauge bosons \((A_{SM}, Z_{SM}, Z_d)\) and the interaction eigenstates \((\hat{A}, \hat{Z}, \hat{Z}_d)\) as

\[
\begin{align*}
\hat{A} &= A_{SM} - c_W t_x s_X Z_{SM} + c_W t_x c_X Z_d, \\
\hat{Z} &= (c_X + s_W t_x s_X) Z_{SM} + (s_X - s_W t_x c_X) Z_d, \\
\hat{Z}_d &= -\frac{s_X}{c_v} Z_{SM} + \frac{c_X}{c_v} Z_d.
\end{align*}
\]

(37)

For the standard model, the gauge interaction for a fermion \(\psi\) with \(SU(2)\) charge \(T_3\) and EM charge \(Q\) is

\[
\mathcal{L}_{int} = \bar{\psi} \gamma^\mu \psi \left\{ e Q \hat{A}_\mu + \frac{e}{s_W c_\gamma} (T_3 - Q s_W^2) \hat{Z}_\mu \right\},
\]

(38)

where \(\psi = \nu_L, e_L, e_R, \) etc and \(e = |e|\). By using Eq. (37), we can find easily the interaction of SM particles to the mass eigenstates of the gauge bosons.

C. DM-electron scattering

The interaction Lagrangian of electron is given by

\[
\mathcal{L}_{int} = e \gamma^\mu e \left[ -e A_{SM\mu} + g_C Z_{SM\mu} + g_{Cd} Z_{d\mu} \right] + e \gamma^\mu \gamma^5 e \left[ g_A Z_{SM\mu} + g_{Ad} Z_{d\mu} \right],
\]

(39)

where

\[
\begin{align*}
g_C &= \frac{e}{4} \left[ c_X (3 \tan \theta_W - \cot \theta_W) + \frac{3 s_X t_x}{c_W} \right], \\
g_{Cd} &= \frac{e}{4} \left[ s_X (3 \tan \theta_W - \cot \theta_W) - \frac{3 c_X t_x}{c_W} \right], \\
g_A &= \frac{e}{4 c_W} \left[ \frac{c_X}{s_W} + s_X t_x \right], \\
g_{Ad} &= \frac{e}{4 c_W} \left[ \frac{s_X}{s_W} - c_X t_x \right].
\end{align*}
\]

(40)

Note that \(t_x \simeq s_W t_x (1 - m_{Z_d}^2 / m_Z^2)^{-1}\) for very small \(\varepsilon\) and \(\theta_X\), and thus \(g_{Cd}\) and \(g_{Ad}\) becomes

\[
\begin{align*}
g_{Cd} &\sim \frac{e m_{Z_d}^2}{4 m_Z^2 - m_{Z_d}^2} \frac{c_W^2 - 3 s_W^2}{c_W} \varepsilon, \\
g_{Ad} &\sim \frac{e m_{Z_d}^2}{4 m_Z^2 - m_{Z_d}^2} \frac{1}{c_W} \varepsilon.
\end{align*}
\]

(41)
The invariant matrix element $M$ is

$$iM = \bar{u}_{p_x}^s \left( ig_d \frac{sX}{\sqrt{1-\varepsilon^2}} \gamma^\mu \right) u_{k_x}^{k'} \left[ -i \left( \eta_{\mu\nu} - \frac{g_d q_{\nu_0}}{m_Z^2} \right) \frac{1}{q^2 - m_{Z_d}^2} \right] \bar{u}_{p_x}^{k'} \left( i\gamma^\nu (g_C + g_A \gamma^5) \right) u^{k'}_{k_x},$$

and the spin-averaged amplitude squared is

$$\overline{|M|^2} = \frac{2g_d^2}{1-\varepsilon^2} \left( \frac{sXg_C}{t - m_Z^2} - \frac{c_Xg_{Cd}}{t - m_{Z_d}^2} \right)^2 A(m_\chi, m_\nu) + \left( \frac{sXg_A}{t - m_Z^2} - \frac{c_Xg_{Ad}}{t - m_{Z_d}^2} \right)^2 B(m_\chi, m_\nu),$$

where

$$A(m_\chi, m_\nu) = 2tM^2 + (s - M^2)^2 + (u - M^2)^2,$n

$$B(m_\chi, m_\nu) = (s - M^2)^2 + (u - M^2)^2 + 2t(m_\chi^2 - m_\nu^2) - 8m_\chi^2 m_\nu^2,$$ (44)

with $M^2 = m_\chi^2 + m_\nu^2$.

For non-relativistic limit, $s \to (m_1 + m_2)^2$, $t \to 0$, and $u \to (m_1 - m_2)^2$, then $A(m_\chi, m_\nu) = 8m_\chi^2 m_\nu^2$, and $B(m_\chi, m_\nu) = 0$. In this limit, Eq. 22 becomes

$$t = -2m_2 T_2,$n

$$s - M^2 = 2m_1 m_2 + 2m_2 T_1,$$ (45)

$$u - M^2 = -2m_1 m_2 - 2m_2 (T_1 - T_2).$$

For the non-relativistic limit, $\overline{|M|^2}$ becomes

$$\overline{|M|^2} = \frac{16g_d^2 m_\chi^2 m_\nu^2}{1-\varepsilon^2} \left( \frac{sXg_C}{m_Z^2} - \frac{c_Xg_{Cd}}{m_{Z_d}^2} \right)^2,$$ (46)

and the scattering cross section is given by

$$\sigma_{\chi e}^{NR} = \frac{g_d^2 m_\chi^2 m_\nu^2}{\pi(1-\varepsilon^2)} \left( \frac{sXg_C}{m_Z^2} - \frac{c_Xg_{Cd}}{m_{Z_d}^2} \right)^2.$$ (47)

## D. DM-neutrino scattering

The interaction Lagrangian of neutrino is given by

$$L_{int} = \bar{\nu}_e \gamma^\mu (1 - \gamma^5) [g_A Z_{SM \mu} + g_A Z_{dR}] \nu_e.$$ (48)

With the invariant matrix element $M$ given by

$$iM = \bar{\chi}(p') \left( ig_d \frac{sX}{\sqrt{1-\varepsilon^2}} \gamma^\mu \right) \chi(p) \left[ -i \left( \eta_{\mu\nu} - \frac{g_d q_{\nu_0}}{m_Z^2} \right) \frac{1}{q^2 - m_{Z_d}^2} \right] \bar{\nu}_e (k') \left( -ig_A \gamma^\nu (1 - \gamma^5) \right) \nu_e (k),$$ (49)
the spin-averaged amplitude squared is obtained as

$$|\mathcal{M}|^2 = \frac{4g_2^2 A(m_\chi, 0)}{(1 - \varepsilon^2)} \left( \frac{s x g_A}{t - m_Z^2} - \frac{c x g_{Ad}}{(t - m_{Za}^2)} \right)^2. \tag{50}$$

E. DM-nucleus scattering

The interaction Lagrangian of the proton and neutron is given by

$$\mathcal{L}_{int} = \bar{p} \gamma^\mu p (e A_{SM\mu} - g_C Z_{SM\mu} - g_{Cd} Z_{d\mu}) + \bar{p} \gamma^\mu g_5 p (-g_A Z_{SM\mu} - g_{Ad} Z_{d\mu}),$$

and thus the interaction of the Nucleus with mass number $A$ and the number of proton $Z$ is

$$\mathcal{L}_{int} = \bar{N} \gamma^\mu N [Z e A_{SM\mu} - g_{NC} Z_{SM\mu} - g_{NCd} Z_{d\mu}] + \bar{N} \gamma^\mu g_5 N [-g_{NA} Z_{SM\mu} - g_{NAd} Z_{d\mu}],$$

$$g_{NC} = Z g_C + (A - Z) g_A, \quad g_{NCd} = Z g_{Cd} + (A - Z) g_{Ad}, \quad g_{NA} = (2Z - A) g_A, \quad g_{NAd} = (2Z - A) g_{Ad}.$$ \tag{53}

The invariant matrix element $\mathcal{M}$ is

$$i\mathcal{M} = \bar{u}^s p_x \left( i g_d \frac{s x}{\sqrt{1 - \varepsilon^2}} \gamma^\mu \right) u^r_k \left[ -i \left( \eta_{\mu\nu} - \frac{q_{\mu} q_{\nu}}{q^2} \right) \right] \bar{u}^r_{pN} \left( -i \gamma^\nu (g_{NC} + g_{NA} g_5) \right) u^r_k,$$

and the spin-averaged amplitude squared is

$$|\mathcal{M}|^2 = \frac{2g_2^2}{1 - \varepsilon^2} \left[ \left( \frac{s x g_{NC}}{t - m_Z^2} - \frac{c x g_{NCd}}{m_{Za}^2} \right) \right]^2 A(m_\chi, m_N) + \left( \frac{s x g_{NA}}{t - m_Z^2} - \frac{c x g_{Ad}}{m_{Za}^2} \right)^2 B(m_\chi, m_N). \tag{55}$$

For non-relativistic limit, it becomes

$$|\mathcal{M}|^2 = \frac{16g_2^2 m_\chi^2 m_N^2}{1 - \varepsilon^2} \left( \frac{s x g_{NC}}{m_Z^2} - \frac{c x g_{NCd}}{m_{Za}^2} \right)^2,$$

$$= \frac{16g_2^2 m_\chi^2 m_N^2}{1 - \varepsilon^2} \left( Z \left( \frac{s x g_C}{m_Z^2} - \frac{c x g_{Cd}}{m_{Za}^2} \right) + (A - Z) \left( \frac{s x g_A}{m_Z^2} - \frac{c x g_{Ad}}{m_{Za}^2} \right) \right)^2, \tag{56}$$

and the total scattering cross section becomes

$$\sigma_{XN}^{NR} = \frac{g_2^2 m_\chi^2 m_N^2}{\pi (1 - \varepsilon^2)} \left( Z \left( \frac{s x g_C}{m_Z^2} - \frac{c x g_{Cd}}{m_{Za}^2} \right) + (A - Z) \left( \frac{s x g_A}{m_Z^2} - \frac{c x g_{Ad}}{m_{Za}^2} \right) \right)^2. \tag{57}$$

Acknowledgments. The authors were supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (NRF-2019R1A2B5B01070181).

[1] A. Del Popolo, Int. J. Mod. Phys. D 23 (2014), 1430005 doi:10.1142/S0218271814300055 [arXiv:1305.0456 [astro-ph.CO]].
[2] E. Aprile et al. [XENON], Phys. Rev. Lett. 121 (2018) no.11, 111302 doi:10.1103/PhysRevLett.121.111302 [arXiv:1805.12562 [astro-ph.CO]].

[3] R. Agnese et al. [SuperCDMS], Phys. Rev. Lett. 121 (2018) no.5, 051301 doi:10.1103/PhysRevLett.121.051301 [arXiv:1804.10697 [hep-ex]].

[4] M. Crisler et al. [SENSEI], Phys. Rev. Lett. 121 (2018) no.6, 061803 doi:10.1103/PhysRevLett.121.061803 [arXiv:1804.00088 [hep-ex]].

[5] R. Essig, A. Manalaysay, J. Mardon, P. Sorensen and T. Volansky, Phys. Rev. Lett. 109 (2012), 021301 doi:10.1103/PhysRevLett.109.021301 [arXiv:1206.2644 [astro-ph.CO]].

[6] T. Bringmann and M. Pospelov, Phys. Rev. Lett. 122 (2019) no.17, 171801 doi:10.1103/PhysRevLett.122.171801 [arXiv:1810.10543 [hep-ph]].

[7] Y. Ema, F. Sala and R. Sato, Phys. Rev. Lett. 122 (2019) no.18, 181802 doi:10.1103/PhysRevLett.122.181802 [arXiv:1811.00520 [hep-ph]].

[8] C. Cappiello and J. F. Beacom, Phys. Rev. D 100, no. 10, 103011 (2019) doi:10.1103/PhysRevD.100.103011 [arXiv:1906.11283 [hep-ph]].