Modeling of Spectral Occupation through Time Series

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Abstract

Spectral occupation modeling is a key aspect for an adequate prediction of spectral opportunities, which results in a more efficient use of the radio spectrum. The objective of this work is to present a comparative evaluation of the performance of four time series models: AR, MA, ARMA and SARIMA, in the modeling of spectral occupation for cognitive radio networks. The results achieved show that the MA model presents the best relationship between a high precision in the modeling of spectral occupation and a low level of computational cost.

Keywords: Time series, spectral occupation, wireless networks, mathematical models.

I. INTRODUCTION

Cognitive radio networks (CRN) allow a more efficient use of radio spectrum through dynamic spectrum allocation (DSA) in wireless communication networks [1]. This is materialized through the opportunistic use of the licensed spectrum by cognitive radio users or secondary users (SU). Achieving an adequate selection of the available frequencies or spectral opportunities is possible from an adequate modeling of the spectral occupation pattern of the frequency band of interest [2], [3].

Making an adequate selection of spectral opportunities by secondary users is key to obtaining excellent performance in quality of service parameters in SU communications. To achieve the above, it is necessary to have an adjusted modeling of the spectral occupation in the corresponding frequency band, however, obtaining high levels of precision in the respective modeling may imply a high level of processing, which ultimately affects the levels of delay in the transmission of data by the SU. The problem then consists in being able to obtain an adequate level of precision in the modeling of the spectral occupation with a low level of computational processing.

One of the models with an excellent trade-off between precision and processing levels is the time series. The three fundamental models based on time series are: Autoregressive (AR), Moving Averages (MA) and Autoregressive of Moving Averages (ARMA). These time series have shown excellent performance in various publications in the current literature [4].

In accordance with the above, the objective of this article is to present a comparative evaluation between four time series models, in the modeling of spectral occupation and the prediction of spectral opportunities for wireless networks: Autoregressive (AR), Moving Averages (MA), Autoregressive of Moving Averages (ARMA) and Integrated Autoregressive of Seasonal Moving Averages (SARIMA), given the seasonal behavior of spectral occupation in wireless networks. The performance evaluation of the time series models will be contrasted from six evaluation metrics: total handoff, handoff with interference, anticipated handoff, perfect handoff, average bandwidth and average delay.

II. METHODOLOGY

Despite the fact that there are currently several prediction models, it was decided to select the time series, due to their good results evidenced in publications and their low computational level, given that they are linear models [4]. Time series are ideal models for correlated series such as mobile network traffic [4]. In order to analyze the performance of different types of time series, the AR, MA, ARMA and SARIMA models were selected to make predictions about the behavior of the PU, as they are the models with the best relationship between performance and computational cost.

To develop the four time series models, the Box-Jenkins methodology [5] was followed, as it is the most widely used and recognized. This methodology consists of building a time series model in four stages: (1) Identification, (2) Estimation of parameters, (3) Verification of the model, and (4) Forecasting the model [5]. The four iterative steps of the Box-Jenkins methodology are described below [6], [7].

Step 1: Identification. This step focuses on the selection of parameters d, D, p, P, q and Q. The order number can be identified by looking at the autocorrelation diagram (ACF) and the partial autocorrelation diagram (PACF).

Step 2: Estimation of parameters. Historical data is used to estimate the parameters of the model identified in step 1.

Step 3: Verification of the model. Through a correlation test between the residuals of the estimated model, it is verified
whether this is correct, if it is, proceed to step 4, otherwise, return to step 1.

Step 4: Model forecast. The model verified in step 3 is used to forecast future values.

Given that there is an AR, MA, ARMA and SARIMA model, for each of the 461 observed Wi-Fi channels, and each of these has the ability to be updated, there is no point in presenting such models. However, Equations (1) to (6) describe the general mathematical models of the AR, MA and ARMA time series.

II.I AR Model
The AR model is based on the observations of the time series. AR (p) indicates the current value of the series, which depends on the past p values, where p establishes the number of lags necessary to make the predictions. The order of p is given by Eq. (1).

\[ X_t = \phi_0 + \phi_1 X_{t-1} + \ldots + \phi_p X_{t-p} + \epsilon_t \]  

(1)

II.II MA Model
MA model applies to function to smooth the original time series through average elements subset; this model assumes linearity and the current value of series is given by the smoothing function. Order q MA is given by Equation (2).

\[ X_t = \theta_0 - \theta_1 \epsilon_{t-1} - \ldots - \theta_q \epsilon_{t-q} - \epsilon_t \]  

(2)

II.III ARMA Model
The ARMA model is the hybrid between the AR and MA models, and is given by Eq. (3).

\[ X_t = \phi_0 + \phi_1 X_{t-1} + \ldots + \phi_p X_{t-p} + \epsilon_t + \theta_0 - \theta_1 \epsilon_{t-1} - \ldots - \theta_q \epsilon_{t-q} - \epsilon_t + \theta_0 \epsilon_{t-1} + \ldots + \theta_q \epsilon_{t-q} \epsilon_t \]  

(3)

II.IV SARIMA Model
In general, if a time series presents a potential seasonality indexed by s, then it is advantageous to use a seasonal ARIMA model (p, d, q) (P, D, Q) s, where d is the level of non-seasonal differentiation, p is the autoregressive non-seasonal order (AR), q is the non-seasonal moving average (MA) order, P is the number of seasonal autoregressive terms, D is the number of seasonal differences, and Q is the number of seasonal moving averages. Box and Jenkins' seasonal autoregressive integrated moving average model [8] is presented in Eq. (4) [9].

\[ \phi_p (B) \phi_p (B^s) \epsilon_t = \theta_q (B) \theta_q (B^s) \epsilon_t \]  

(4)

Where B is the backward shift operator, xt is the observed load time series at time t, et is the identical, normally distributed independent error (random shock) in period t, \( \epsilon_t \) and \( \epsilon_t^{(s)} \) are the seasonal ones, AR (p) and MA (q) are the operators, respectively, which are defined in Eq. (5) and (6).

\[ \phi_p (B^s) = 1 - \phi_1 B^s - \phi_2 B^{2s} - \ldots - \phi_p B^{ps} \]  

(5)

\[ \theta_q (B^s) = 1 - \theta_1 B^s - \theta_2 B^{2s} - \ldots - \theta_q B^{qs} \]  

(6)

Where \( \phi_1, \phi_2, \ldots, \phi_p \) are parameters of the seasonal model AR (p), \( \theta_1, \theta_2, \ldots, \theta_q \) are the parameters of the seasonal model MA (q).

II.V Evaluation Matrices
To evaluate the performance of the selected time series models, six evaluation metrics described in Table 1 are used.

| Name                               | Description                                                                 | Type   |
|------------------------------------|-----------------------------------------------------------------------------|--------|
| Total number of handoffs           | Corresponds to the total number of handoffs during the 9 minutes of the transmission. | Cost   |
| Interference handoff number        | Corresponds to the number of reactive handoffs made after the arrival of the PU, during the 9 minutes of transmission. | Cost   |
| Number of anticipated handoffs     | Corresponds to the number of anticipated handoffs, made well in advance of the arrival of the PU, during the 9 minutes of transmission. | Cost   |
| Perfect handoff number             | It corresponds to the number of perfect handoffs, which were made exactly before the arrival of the PU, during the 9 minutes of the transmission. | Benefit|
| Average bandwidth                  | It is the average bandwidth of the communication during the 9 minutes of the transmission of the SU. | Benefit|
To evaluate the performance of each handoff, a simulation environment progressively reconstructs the behavior of the spectrum occupation using the traces of data captured in the Wi-Fi frequency band. This allows to accurately evaluate the behavior of the PU and also, to evaluate and validate the performance of each handoff. The spectral occupation data correspond to a one-week observation captured in the city of Bogotá, Colombia [10].

### III. RESULTS

Figures 1 to 6 present the performance of each of the metrics described in Table 1, for each of the time series models: AR, MA, ARMA and SARIMA.

The figures present the results obtained during a nine-minute transmission in the Wi-Fi frequency band, with a high level of traffic, that is, for few spectral opportunities.

Figure 1 shows the total number of handoffs, here the ARMA model has the worst performance with 7% more handoff than the other models, which have a very similar behavior.

Figure 2 presents the number of handoffs with interference being the MA model, the one that shows the best performance with 21% less handoff with interference than the ARMA model that has the highest level of interference.

![Fig. 1. Total number of handoffs](image1)

![Fig. 2. Interference handoff number](image2)
Fig. 3. Number of anticipated handoffs

Fig. 4. Perfect handoff number

Fig. 5. Average bandwidth
Figure 6 describes the average delay. Figure 3 describes the number of anticipated handoffs, here the algorithm with the worst performance is SARIMA, while the others exhibit very similar behavior.

Figure 4 shows the number of perfect handoffs, with the ARMA model having the highest number of perfect handoffs, closely followed by the AR and MA algorithms.

Figures 5 and 6 describe the average bandwidth and average delay, respectively. In both, the SARIMA model shows the best performance, however, its poor performance in the other metrics does not make it a viable candidate, also taking into account that it is the model with the highest level of computational cost. The other time series models show similar behavior, making the MA model the best candidate for modeling spectral occupancy in Wi-Fi wireless networks with high traffic levels.

IV. CONCLUSIONS
This document highlights the importance of measuring not only the interference generated based on predictive capacity, but also presents the possibility of validating its effectiveness given the characteristics of traffic and QoS for communication. Data processing for prediction plays an important role; therefore, data smoothing techniques need to be established to insert a higher correlation that improves prediction. Similarly, traffic behavior affects the quality of the prediction, especially if it is of the ON-OFF type.

V. ACKNOWLEDGMENTS
The authors wish to thank the Center for Research and Scientific Development of the Universidad Distrital Francisco Jose de Caldas for the support and financing of this research.

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