Structure and dynamics of a rotating superfluid Bose-Fermi mixture

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(Dated: July 29, 2014)

We investigate the structure and dynamics of a rotating superfluid Bose-Fermi mixture (SBFM) made of superfluid bosons and two-component (spin up and down) superfluid fermions. A ground-state phase diagram for the nonrotating case of a SBFM with specific parameters is given. We show that, depending on the boson-boson (BB), fermion-fermion (FF) and boson-fermion (BF) interactions, a nonrotating SBFM can exhibit different phase structures, such as mixed phases, layer separated phases, and inlaid separated phases. When the BB interaction is fixed, the ground-state configuration of a nonrotating SBFM is mainly determined by the BF interaction. For the rotating case of a SBFM with a sufficiently large rotation frequency, the system supports a mixed phase and three typical layer separated phases (one with a visible vortex lattice in the bosonic superfluid, one with a visible vortex lattice in the fermionic superfluid, and another one with a visible vortex lattice in both the bosonic and fermionic superfluids). In addition, we find that different separated phases at the initial time may lead to almost the same equilibrium structures. Furthermore, the visible vortex formation in the fermionic superfluid exhibits a remarkable hysteresis effect during the dynamical evolution of a rotating SBFM, which is evidently different from the case of rotating two-component Bose-Einstein condensates.

PACS numbers: 67.85.Pq, 67.85.Lm, 03.75.Kk, 03.75.Lm

I. INTRODUCTION

Superfluidity plays a key role in many fields of physics, such as liquid Helium\textsuperscript{1}, ultracold atomic gases\textsuperscript{2, 3}, quantum magnets\textsuperscript{4}, and astrophysics\textsuperscript{5}. In particular, superfluid Bose-Fermi mixtures (SBFMs) in ultracold atomic gases have attracted considerable interest recently. It is well known that for liquid Helium the superfluidity of bosonic \textsuperscript{4}He and fermionic \textsuperscript{3}He can be achieved separately. However, the simultaneous superfluidity for the two isotopes is remarkably prevented by the strong interactions between the two species in spite of such a superfluid \textsuperscript{4}He-\textsuperscript{3}He mixture being a long-sought object\textsuperscript{6}. By contrast, a superfluid Bose-Fermi mixture (SBFM) in ultracold atomic gases can be realized by using the combination of the Feshbach resonance and radio-frequency techniques. Most recently, a SBFM consisting of condensed \textsuperscript{7}Li bosons and ultracold \textsuperscript{6}Li fermions in two-spin states has been produced by Salomon’s group\textsuperscript{7}. The experimental breakthrough provides new opportunities to study the intriguing properties of SBFMs inaccessible in a Bose-Einstein condensate (BEC) with arbitrary spin\textsuperscript{3, 9}, pure bosonic superfluid mixtures and pure fermionic superfluid mixtures. As a matter of fact, many novel physical characters have been predicted theoretically in SBFMs, including the saturation effect of nonlinear interaction in the strong-coupling unitarity limit\textsuperscript{10}, localized Bose-Fermi bright soliton\textsuperscript{11}, possibility of simulating dense quantum chromodynamics\textsuperscript{12}, phase transition overlapping with phase separation\textsuperscript{13}, and the Faraday pattern generation\textsuperscript{14}. All existing studies of the SBFMs refer to the nonrotating case. Considering that one of the most striking hallmarks of a superfluid is its response to rotation, in this paper we investigate the combined effects of rotation and nonlinear interatomic interaction on the exact two-dimensional (2D) topological structure and dynamics of a rotating SBFM, focusing on the superfluid \textsuperscript{7}Li-\textsuperscript{6}Li mixture case as the prototype. Here we present a phenomenological dissipation model\textsuperscript{15} combining with the unitary Schrödinger (US) equation for an interacting SBFM\textsuperscript{10} to describe the dynamics of a rotating SBFM. First, a ground-state phase diagram for a nonrotating SBFM with specific parameters is given. We find that the ground-state structure of a nonrotating SBFM is mainly determined by the ratio of the boson-fermion (BF) scattering length to the boson-boson (BB) scattering length. Secondly, it is shown that the rotating SBFM with a sufficiently large rotation frequency can display four typical steady structures. One is a mixed phase for attractive BF interaction below the threshold for collapse, where the density peak of the BEC in the trap center is surrounded by a triangular vortex lattice, and the fermionic superfluid keeps unchanged basically except for several ghost vortices lying on its outskirts. The other three are layer separated phases for repulsive BF interaction, where the BEC is repelled to the outskirts of the fermionic superfluid and there exist visible vortices in the BEC or the fermionic superfluid or both the bosonic and fermionic superfluids. Thirdly, we find that different separated phases at the initial time may result in almost the same steady structures. Finally, we show

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that the generation of visible vortices in the fermionic superfluid displays an evident hysteresis effect during the time evolution of a rotating SBFM. Due to the Cooper pair entity of fermionic superfluid and the presence of BF and fermion-fermion (FF) interactions, it is demonstrated that the topological structure and the dynamics of a rotating SBFM are evidently different from the usual cases of rotating two-component Bose-Einstein condensates (BECs).

The paper is organized as follows. In Sec. II, we describe a phenomenological model for a rotating SBFM. In Sec. III, we study the equilibrium structure of the rotating SBFM. A phase diagram for a nonrotating SBFM is given, and the typical phase structures of the rotating SBFMs with various parameter values are analyzed. In Sec. IV, we discuss the dynamics of vortex formation in a rotating SBFM. The conclusion is outlined in the last section.

II. MODEL

We consider a superfluid Bose-Fermi mixture composed of superfluid bosons and two-component superfluid fermions with equal populations of spin up and down, corresponding to the most favorable condition for Cooper pairing. The superfluid Bose-Fermi gas is confined in a harmonic trap rotating around z axis with angular velocity ω. In the present work we use a combined phenomenological model based on a phenomenological dissipation model \[15\] and a Galilei-invariant nonlinear unitarity model \[10\] to investigate the dynamics of the vortex formation and the structure of the equilibrium state of a rotating SBFM. In this phenomenological model, the order parameter \(\Psi_b\) of a bosonic superfluid (i.e., a Bose-Einstein condensate) and the order parameter \(\Psi_p\) of a fermionic superfluid \[10\] obey the coupled equations in the rotating frame

\[
(i - \gamma)\frac{\partial \Psi_b}{\partial t} = \left[ -\frac{\hbar^2 \nabla^2}{2m_b} + U_b + \mu_b(n_b, a_b) - \Omega L_z \right] \Psi_b + G_{bp} |\Psi_p|^2 \Psi_b, \\
(i - \gamma)\frac{\partial \Psi_p}{\partial t} = \left[ -\frac{\hbar^2 \nabla^2}{2m_p} + U_p + \mu_p(n_p, a_f) - \Omega L_z \right] \Psi_p + G_{bp} |\Psi_b|^2 \Psi_p,
\]

where for simplicity we assume that the degree of dissipation \(\gamma\) of the fermionic superfluid is the same as that of the BEC. \(m_b\) is the mass of a bosonic atom, \(m_p = 2m_f\) is the mass of a Cooper pair with \(m_f\) being the mass of a single fermion, and \(L_z = i\hbar(y\partial_x - x\partial_y)\) denotes the \(z\) component of the angular-momentum operator.

\(U_b = m_b(\omega_{b1}^2 + \omega_{b2}^2 x^2) / 2\) is the external trapping potential for the bosons, \(U_p = m_p(\omega_{p1}^2 + \omega_{p2}^2 x^2) / 2\) is the trapping potential for the fermionic superfluid, \(\omega_b(\omega_f)\) and \(\omega_{b1}(\omega_{f1})\) are the radial trap frequency and the axial trap frequency for the bosons (fermions), respectively. \(G_{bp} = 4\pi \hbar^2 a_{bf} / m_f\) is the BF \(s\)-wave scattering length, and \(m_{bf} = m_m m_f / (m_m + m_f)\) is the BF reduced mass. The bulk chemical potential of the BEC is given by

\[
\mu_b(n_b, a_b) = \frac{\hbar^2}{m_b} n_b^{2/3} f(n_b^{1/3} a_b),
\]

where

\[
f(x) = \frac{4\pi(x + \alpha x^{5/2})}{1 + \lambda x^{3/2} + \beta x^{5/2}},
\]

\(n_b\) is the local density of bosons, and \(a_b\) is the BB \(s\)-wave scattering length. Here we choose \((\alpha - \lambda) = 32 / (3\sqrt{\pi})\), \(\beta = 4\pi \alpha / \eta\) with \(\eta = 22.22\), \(\alpha = 32\xi / (3\sqrt{\pi})\), and \(\lambda = 32(\xi - 1) / (3\sqrt{\pi})\) with \(\xi = 1.1\). \[10\] The bulk chemical potential of the Fermi superfluid is expressed as \[10\] \[16\] \[17\]

\[
\mu_p(n_p, a_f) = \frac{2\hbar^2}{m_p}(6\pi^2 n_p^{2/3} g(2^{1/3} n_p^{1/3} a_f),
\]

where

\[
g(x) = 1 + \frac{\delta x}{1 - k x},
\]

\(n_p = n_f / 2\) is the local density of Cooper pairs with \(n_f\) being the total local density of fermions, \(a_f\) denotes the attractive FF scattering length, \(\delta = 20\pi / (3\pi^2)^{2/3}\), and \(k = \delta(1 - \zeta) / \zeta\) with \(\zeta = 0.44\). The choices of the parameters in Eqs. \[4\] and \[6\] are consistent with the unitarity and the Lee-Yang-Huang limits \[10\] \[18\] \[19\] as well as the relevant results of the Monte Carlo calculations \[20\] \[22\].

This phenomenological model is a variation of that in \[23\] \[24\] and a generalization of that of a rotating single-component BEC \[19\], and it has good predictive power \[25\]. The largest merit of the phenomenological dissipation model is that by virtue of this model one can not only obtain the steady states of a rotating system but also reveal the dynamics of vortex formation. In addition, the present model is valid from weak-coupling to unitarity for both bosons and fermions \[10\].

Next, we consider the two-dimensional problem by assuming the translation invariance along the \(z\) axis (i.e., \(\omega_{zb} = \omega_z = 0\)), reducing the order parameters as \(\psi_i(\vec{r}, t) = \psi_i(x, y, t) / \sqrt{R_z} (i = b, p)\) with the typical size \(R_z\) along the \(z\) axis. After a straight-forward calculation, we obtain 2D coupled equations,

\[
(i - \gamma)\frac{\partial \psi_b}{\partial t} = \left[ -\frac{\hbar^2}{2m_b} (\nabla_x^2 + \nabla_y^2) + F(\psi_b) - \Omega L_z \right] \psi_b + \frac{m_b \omega_{b1}}{2} (x^2 + y^2) + \frac{G_{bp}}{R_z} |\psi_p|^2 \psi_b,
\]

\[
(i - \gamma)\frac{\partial \psi_p}{\partial t} = \left[ -\frac{\hbar^2}{2m_p} (\nabla_x^2 + \nabla_y^2) + J(\psi_p) - \Omega L_z \right] \psi_p + \frac{m_p \omega_{p1}}{2} (x^2 + y^2) + \frac{G_{bp}}{R_z} |\psi_b|^2 \psi_p,
\]
where
\[
F(\psi_b) = \frac{4\pi \hbar^2}{m_b} \left( a_b R_{z}^{-1} |\psi_b|^2 + \alpha a_b^{5/2} R_{z}^{-3/2} |\psi_b|^3 \right).
\]
and
\[
J(\psi_p) = \frac{2\hbar^2}{m_p} (6\pi^2)^{2/3} |\psi_p|^{4/3} \left( 1 + \frac{21/3 \delta a_f |\psi_p|^{2/3}}{1 - 21/3 k a_f |\psi_p|^{2/3}} \right).
\]

The initial order parameters are normalized as \( N_i = \int \psi_i(x,y,t=0) dxdy \) with \( N_b \) being the initial number of bosons and \( N_p \) the initial number of Cooper pairs \( (N_b = N_f/2 \) and \( N_f \) is the total number of fermions). By introducing the notations \( \omega_{bf} = (\omega_b + \omega_f)/2 \), \( d_0 = \sqrt{\hbar/(2m_b \omega_{bf})} \), \( x_0 = x/d_0 \), \( y_0 = y/d_0 \), \( t_0 = t/\omega_{bf} \) and \( \Omega_0 = \Omega/\omega_{bf} \), and replacing the wave functions as \( \psi_i \rightarrow \sqrt{\Omega_0} \psi_i/d_0 \), we obtained the rescaled dimensionless 2D equations for the mixture
\[
(i - \gamma) \frac{\partial \psi_b}{\partial t} = \left[ -\frac{m_b}{m_b} (\nabla^2_x + \nabla^2_y) + C_1 |\psi_p|^2 - \Omega L_z \right] \psi_b
+ \frac{4\pi \hbar}{m_b \omega_{bf}} \left( A_1 |\psi_p|^2 + \alpha B_1 |\psi_b|^{4/3} \right) + \frac{m_b \omega_{bf}^2}{4m_b \omega_{bf}^2} (x^2 + y^2) \psi_b,
\]
\[
(i - \gamma) \frac{\partial \psi_p}{\partial t} = \left[ -\frac{m_p}{m_p} (\nabla^2_x + \nabla^2_y) + C_2 |\psi_b|^2 - \Omega L_z \right] \psi_p
+ A_2 |\psi_p|^{4/3} \left( 1 + \frac{\delta B_2 |\psi_p|^{2/3}}{1 - k B_2 |\psi_p|^{2/3}} \right) + \frac{m_p \omega_{bf}^2}{2m_p \omega_{bf}^2} (x^2 + y^2) \psi_p,
\]

where the number subscript 0 is omitted for simplicity. The corresponding coefficients are \( A_1 = a_b N_b d_0^{-2} R_z^{-1} \), \( B_1 = a_b^{5/2} N_b^{-3/2} a_0^{-3} R_z^{-3/2} \), \( C_1 = 8 \pi a_b N_p R_z^{-1} \), \( D_1 = \alpha a_b^{5/2} N_b^{-1/2} a_0^{-1} R_z^{-1/2} \), \( E_1 = \alpha a_b^{3/2} N_b^{1/2} a_0^{-3/2} R_z^{-5/6} \), \( A_2 = (2\hbar/m_p \omega_{bf}^2) (6\pi^2 N_p d_0^{-2})^{2/3} \), \( B_2 = (2N_p d_0^{-2})^{1/3} a_f \), and \( C_2 = 8 \pi a_b N_b R_z^{-1} \).

In the following, we numerically solve the 2D coupled equations \[ \text{(11)} \] and \[ \text{(12)} \] which require an enormous computation effort. The initial-state order parameter \( \psi_b(x,y,t=0) = 0 \) and \( \psi_p(x,y,t=0) = 0 \) of the system can be obtained by the imaginary-time propagation method \[ \text{(24,28)} \] based on the Peaceman-Rachford method \[ \text{(29,30)} \]. Here we consider a superfluid \(^7\text{Li}^{6}\text{Li} \) mixture in harmonic traps satisfying \( U_b(x,y) = m_b \omega_x^2 (x^2 + y^2) = U_f(x,y) = m_f \omega_f^2 (x^2 + y^2) \) with \( m_b \) being the mass of \(^7\text{Li} \) atom and \( m_f \) the mass of \(^6\text{Li} \) atom. The parameters are chosen as \( \omega_x = 2\pi \times 100 \text{ Hz} \), \( R_z = 10 \mu \text{ m} \), and \( \gamma = 0.03 \) corresponding to a temperature of about 0.1\text{\( T_c \)} \[ \text{(31)} \]. Recently, a mixture of \(^7\text{Li} \) superfluid and \(^6\text{Li} \) superfluid has been realized by Salomon’s group \[ \text{(3)} \]. Thus

the above assumption is valid and feasible, and the relevant results can be tested under the current experimental conditions.

III. STEADY STATE CONFIGURATION OF A ROTATING SBFM

For convenience, we introduce two relative interaction strengths, \( R_1 = a_{bf}/a_b \) and \( R_2 = a_f/a_b \). The FF attractive scattering length \( a_f \) can be varied from zero to negative infinity, and the absolute value of the BF scattering length \( |a_{bf}| \) is assumed to be not too large (otherwise, it requires special attention \[ \text{(10)} \]).

Figure 1 shows the ground state phase diagram of a static superfluid \(^7\text{Li}^{6}\text{Li} \) mixture with fixed BB repulsive interaction. The corresponding parameters are \( a_f = 50 \text{ nm} \), \( N_b = 1000 \), and \( N_p = 100 \). There exist four possible phases depending on the values of \( R_1 \) and \( R_2 \), and the typical density profiles corresponding to phases II-IV are displayed in Fig. 2. In Fig. 1, region I denotes the system collapse regime. For a sufficiently strong attractive BF interaction (here \( R_1 < -0.5 \), i.e. \( a_{bf} < -25 \text{ nm} \)), the system can undergoes a simultaneous collapse of the density profile of the BEC and that of the fermionic superfluid. Physically, the critical value of BF scattering length is governed by the balance between the kinetic energy of bosons and Cooper pairs and the mutual attractive BF interaction. When the BF attraction becomes sufficiently strong, it cannot be stabilized by the kinetic energy anymore. Therefore the mixture lowers its energy via increasing the densities of bosons and Cooper pairs, and finally the bosonic superfluid or the fermionic one or both the bosonic and fermionic superfluids collapse simultaneously due to instability. Region II represents a miscible phase in which the density pro-
FIG. 2: (color online) Density profiles (a)-(c) correspond to phases II-IV in Fig.1, respectively. Here 1 labels the bosonic superfluid and 2 denotes the fermionic one of the system. (a) \( R_1 = -0.4, R_2 = -20 \), (b) \( R_1 = 1, R_2 = -20 \), and (c) \( R_1 = 6, R_2 = -20 \). The other parameters are the same as those in Fig.1. The darker color area indicates the lower density. \( x \) and \( y \) are in units of \( d_0 \).

The ground-state structures of SBFMs are different from those of degenerate boson-fermion mixtures \([32, 36]\). In the latter case, there is no s-wave interaction between identical fermions in the spin-polarized state due to the Pauli exclusion principle. Depending on the BB and BF interactions, the density profile of a degenerate boson-fermion mixture may display a core-shell-shaped separated phase, where the Fermi gas forms a shell around a core inside the BEC, or even both \([32, 36]\). However, our simulation shows that in a wide range of parameter values the SBFM supports neither a separated phase of fermions constituting a shell around the BEC nor a staggered separated phase of fermions becoming both a shell around and a core inside the BEC \([32, 36]\). Hence, the trap center is occupied by a density peak of bosons due to the attractive BF interaction. However, no any visible vortex can generated in the density of the superfluid fermions. Since these phase defects are invisible in the in situ density profile of fermionic superfluid and contribute to neither the angular momentum nor the energy of the SBFM, there are referred to as ghost vortices \([15, 22, 25, 30]\). As shown in Figs. 3(a1) and 3(a3), when the rotating SBFM with an initial state of \( R_1 = -0.4 \) and \( R_2 = -20 \) reaches an equilibrium state one can see that there are several visible vortices \([15, 22]\) constituting a triangular lattice in the periphery of the bosonic superfluid. In the meantime, the trap center is occupied by a density peak of bosons due to the attractive BF interaction. However, no any visible vortex can generated in the density of the fermionic superfluid even for large rotation frequency of \( \Omega = 0.94 \) [see Fig. 3(a2)]. From Fig. 3(a4), there exist some phase defects that are located on the outskirts of the SBFM. Since these phase defects are invisible in the in situ density profile of fermionic superfluid and contribute to neither the angular momentum nor the energy of the SBFM, there are referred to as ghost vortices \([15, 22, 25, 30]\). For the case of \( R_1 = 1 \) and \( R_2 = -20 \), there is a large circular density hole of bosons in the trap center which looks like a giant vortex.
(a multiquantized vortex) and is surrounded by an expected visible vortex lattice [Fig. 3(b1)]. In the phase profile displayed in Fig. 3(b3), we find that there are three phase defects in the trap center which are close to each other. The three singly quantized phase defects show that the circular density hole is not a giant vortex. Actually, they are known as singly quantized hidden vortices because they carry significant angular momentum, though they are invisible in the in situ density profile of the BEC. Only after including the hidden vortices can the Feynman rule be satisfied. As seen in Figs. 3(b2) and 3(b4), the fermionic superfluid with ghost vortices distributing on the outskirts of the cloud is completely pulled inside the trap center (i.e., the region of the large density hole of the BEC) due to the competition between the BF repulsion and the rotation repulsion, which indicates a fully separated phase of the rotating SBFM.

With the further increase of the BF repulsive interaction, a nonrotating SBFM will develop into an inlaid separated configuration, where the superfluid fermions lie on the outer edges of the bosons via the form of two fragments [Figs. 2(c1) and 2(c2)]. Counterintuitively, we find that the steady structure of a rotating SBFM with \( R_1 = 6 \) and \( R_2 = -20 \) is similar to that with \( R_1 = 1 \) and \( R_2 = -20 \) as shown in Figs. 3(c1)-3(c4). Here the fermionic superfluid is separately repelled to the trap center region rather than the outskirts of BEC, which is quite different from the nonrotating case. In the presence of dissipation, the steady visible vortex lattice in the BEC is mainly formed by the competition between the rotating driving and the BB repulsive interaction. The BF repulsion and the centrifugal force acting on the bosons tend to push the fermionic superfluid toward the outside or the inside. Considering the well coherence of fermionic superfluid with FF attractive interaction, it evidently prefers to occupy the trap center, especially in the presence of central density hole for the bosons, because the corresponding potential energy for the superfluid fermions is relatively small such that the system energy of the SBFM reaches the minimum.

In Figs. 4(a1) and 4(a2), we show the density profiles of the BEC and the fermionic superfluid in a static SBFM with \( N_b = 10^3 \), \( N_p = 10^3 \), \( a_b = 5 \) nm, \( R_1 = 6 \) and \( R_2 = -20 \). Compared with the layer separated phase in Figs. 2(a1) and 2(a2), here the torus density profile of the BEC becomes thinner while the circular density distribution of the fermionic superfluid gets larger. The steady structure at \( t = 500 \) of the SBFM rotating with \( \Omega = 0.92 \) is displayed in Figs. 4(b1) and 4(b2), and the corresponding phase profiles are given in Figs. 4(c1) and 4(c2). From Figs. 4(b1) and 4(c1), we can see that there is no visible vortices but there exist some hidden vortices in the BEC, where the trap center is occupied by multiple close singly quantized hidden vortices. Here the central blurred region in the phase profile is mainly resulted from the inevitable numerical errors or fluctuations in the numerical computations. In contrast to Fig. 3, a triangular visible vortex lattice made of six visible vortices forms in the rotating fermionic superfluid as shown in Figs. 4(b2) and 4(c2). Although there is attractive FF interaction between two fermions in a Cooper pair, the fermionic superfluid can display a weak effective repulsive interaction between the Cooper pairs under the appropriate parameters according to Eqs. [2], [3], [10] and [12], which is consistent with the relevant theoretical prediction. This point may explain why visible vortices can be generated in the rotating fermionic superfluid.

IV. DYNAMICS OF VORTEX FORMATION IN A ROTATING SBFM

In order to reveal the vortex formation process in the rotating superfluid mixture of bosons and Cooper pairs, we consider a bigger superfluid Bose-Fermi system which
requires more computation effort. In Figs. 5(a1) and 5(a2), we present the ground-state structure of a static SBFM, where the parameters are \( N_b = 10^4, N_p = 10^5, a_b = 50 \text{ nm}, R_1 = 6, \) and \( R_2 = -20. \) The steady structure at \( t = 500 \) of the SBFM rotating with \( \Omega = 0.96 \) is shown in Figs. 5(b1) and 5(b2), and the corresponding phase profiles are displayed in Figs. 5(c1) and 5(c2). The initial state of the SBFM at \( t = 0 \) has a shell-shaped separated structure with fermionic superfluid being surrounded by the bosonic superfluid, which is similar to that in Figs. 2(b1) and 2(b2). In the presence of dissipation, a steady visible vortex lattice forms eventually in the outer of the superfluid BEC [Fig. 5(b1)], where the energy of the rotating SBFM reaches the minimum in the rotating frame. This point is similar to the case of Fig. 3(b1). The large density hole of the bosons corresponds to twenty two singly quantized hidden vortices indicated by the phase profile in Fig. 5(c1). The central blurred region in the phase profile is mainly caused by the inevitable numerical errors or fluctuations in the numerical computations. In contrast to the case of \( N_b = 1000 \) and \( N_p = 100 \) (Fig. 2 and Fig. 3), seven evident visible vortices are generated in the fermionic superfluid [see Figs. 5(b2) and 5(c2)].

The dynamical evolution of the rotating SBFM is illustrated in Fig. 6, where the top two rows denote the time evolutions of the density profiles of the BEC (row 1) and fermionic superfluid (row 2), while the bottom two rows represent those of the phase profiles of the BEC (row 3) and fermionic superfluid (row 4). The evolution times are \( t = 11.5 \) (left), \( t = 20 \) (middle), and \( t = 60 \) (right), respectively. Initially, the superfluid densities \( |\psi_b(x, y, t = 0)|^2 \) and \( |\psi_p(x, y, t = 0)|^2 \) in a stationary isotropic harmonic trap are shown in Figs. 5(a1) and 5(a2). With the development of time, the boundary surface of the bosonic superfluid undergo complex turbulent oscillation and many ghost vortices appear at the outskirts of the BEC, which can be seen in Figs. 6(a1) and 6(a3). In contrast, the fermionic superfluid remains basically unchanged as shown in Figs. 6(a2) and 6(a4). Essentially, these ghost vortices are generated by collective excitations through the nonlinear atomic interactions and the Landau instability associated with the negative excitation frequency \( \omega_b = 2, 4 \) because the rotating harmonic trap is isotropic and has rotation symmetry. This characteristic is evident especially for the component of fermionic superfluid [see Figs. 6(a2) and Figs. 6(a4)-6(c4)]. Thus here the formation mechanism of topological defects is different from the case of a rotating anisotropic harmonic potential, where ghost vortices are mainly resulted from the dynamical instability through the rapid modulation of trapping anisotropy \( \omega_b = 2, 4 \). The angular momentum is transferred into the SBFM via the excitations of surface modes or the generation of visible vortices and hidden vortices. With the further time evolution, some ghost vortices penetrate into the BEC and become visible vortices or hidden vortices due to the Landau instability [see Figs. 6(b1)-6(c1) and Figs. 6(b3)-6(c3)], where the visible vortices arrange themselves irregularly. When the rotating SBFM reaches an equilibrium state the visible vortices form a triangular lattice such that the energy of the system approaches the minimum in the rotating frame. Compared with the...
rotating BEC, the vortex (including ghost vortex and visible vortex) formation in the rotating fermionic superfluid exhibits an evident hysteresis effect as shown in Fig. 6 and Fig. 5.

The structure and dynamics of a rotating SBFM are remarkably different from those of a rotating two-component BECs. First, in the latter case the vortices in the two BECs are always generated simultaneously \([49, 51, 52]\), while here the vortex formation in the superfluid fermions is far later than that in the BEC due to the presence of Cooper pair with attractive FF interaction. Secondly, in the presence of dissipation there are triangular visible vortex lattices and singly quantized hidden vortices formed in a rotating SBFM. In our simulation, we did not observe the generation of vortex sheet or square vortex lattice occurring in a rotating two-component BEC \([49, 51, 52]\). These structures as well as the giant vortex are suppressed by the dissipation term in our simulation.

V. CONCLUSION

We have studied the structure and dynamics of a rotating superfluid Bose-Fermi mixture. For specific parameters, we give a ground-state phase diagram for a nonrotating SBFM, where different phase structures (including mixed phase, layer separated phase, and inlaid separated phase) can be formed. It is shown that the ratio of BF interaction to BB interaction plays a key role in determining the ground-state structure of a nonrotating SBFM. For fixed BB and BF s-wave scattering lengths, the variation of FF (spin up and down) s-wave scattering length does not influence the phase diagram. Depending on the choice of parameters, the rotating SBFM with a sufficiently large angular velocity can display four typical steady structures. For attractive BF interaction below the threshold for collapse, the equilibrium structure of the rotating SBFM is a mixed phase, where the density peak of the BEC in the trap center is surrounded by a triangular vortex lattice, and the fermionic superfluid keeps unchanged basically except for several ghost vortices lying on the outskirts of the superfluid Fermi gas. For repulsive BF interaction, the steady structure of the rotating SBFM is a layer separated phase, where the BEC is expelled to the outskirts of the fermionic superfluid and there is a visible vortex lattice in the BEC or the fermionic superfluid or both the bosonic and fermionic superfluids. In the bosonic superfluid, the region of the central density hole is occupied by multiple close singly quantized hidden vortices rather than a giant vortex. In addition, we show that different separated phases at the initial time may result in almost the same steady structures. In particular, we find that the generation of visible vortices in the fermionic superfluid exhibits an evident hysteresis effect during the time evolution of a rotating SBFM. We expect that our findings can be observed and tested in future experiments. In the mean time, the present investigation provides a new way to test further the validity of the unitarity model \([10]\) for a SBFM.

Acknowledgments

We thank Biao Wu, Yongping Zhang, Li Mao and Yong Xu for helpful discussions. L.W. acknowledges the research group of Prof. Chuanwei Zhang in The University of Texas at Dallas, where part of the computations were carried out. This work was supported by the NSFC under Grants No. 11047033 and No. 11304270, and the Ph.D. foundation of Yanshan University under Grant No. B846.
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