A MODEL FOR THE FORMATION AND EVOLUTION OF COSMOLOGICAL HALOS

Marcelo A. Alvarez,1,2 Paul R. Shapiro,1 and Hugo Martel1

1. INTRODUCTION

Adaptive SPH and N-body simulations were carried out to study the collapse and evolution of dark matter halos that result from the gravitational instability and fragmentation of cosmological pancakes. Such halos resemble those formed by hierarchical clustering from realistic initial conditions in a CDM universe and, therefore, serve as a convenient test-bed model for studying halo dynamics. Our halos are in approximate virial equilibrium and roughly isothermal, as in CDM simulations. Their density profiles agree quite well with the fit to N-body results for CDM halos by Navarro, Frenk, & White (1997; NFW).

This test-bed model enables us to study the evolution of individual halos. The masses of our halos evolve in three stages: an initial collapse involving rapid mass assembly, an intermediate stage of continuous infall, and a final stage in which infall tapers off as a result of finite mass supply. In the intermediate stage, halo mass grows at the rate expected for self-similar spherical infall, with \( M(a) \propto a \). After the end of initial collapse at \( a = a_c \), the concentration parameter grows linearly with the cosmic scale factor \( a_c(a) \) just after virialization is about 1.35, a value close to that of the N-body results for CDM halos, as predicted by the truncated isothermal sphere model (TIS) (Shapiro, Iliev, & Raga 1999) and consistent with the value expected for a virialized halo in which mass infall contributes an effective surface pressure. Thereafter, the virial ratio evolves towards the value expected for an isolated halo, \( 2T/W \) \( \propto 1 \), as the mass infall rate declines. This mass accretion history and evolution of concentration parameter are very similar to those reported recently in N-body simulations of CDM analyzed by following the evolution of individual halos. We therefore conclude that the fundamental properties of halo collapse, virialization, structure, and evolution are generic to the formation of cosmological halos by gravitational instability and are not limited to hierarchical collapse scenarios or even to Gaussian-random-noise initial conditions.

2. HALO FORMATION VIA PANCAKE INSTABILITY

Test-bed Model: Cosmological pancakes – modelled as single plane-wave density fluctuations – are gravitationally unstable; density perturbations transverse to the direction of pancake collapse cause the pancake to fragment (Valinia et al. 1997). When a pancake is perturbed by two transverse density modes with wavevectors in the plane of pancake collapse, a quasi-spherical halo forms at the intersection of two filaments in the pancake plane. This halo closely resembles those formed by hierarchical clustering from initial conditions in a CDM universe.

ASPH/P3M Simulations: Two simulations were run of the formation of a dark matter halo by pancake instability for use as a test-bed model for halo formation, with \( 64^3 \) DM particles, both with and without \( 64^3 \) gas particles. The primary pancake collapses (i.e. first forms accretion shocks and caustics) at a scale factor \( a_c \). By \( a/a_c = 3 \), a well-defined pancake-filament-halo structure appears (Fig. 1).

3. HALO PROFILES & EQUILIBRIUM

Density Profile: By \( a/a_c = 3 \), the DM halo (simulated with and without gas) has a spherically-averaged density profile close to the fit by NFW to N-body simulations of halos formed by hierarchical clustering in CDM (Fig. 2).
Fig. 2. Density profiles: spherically-averaged simulation of DM no gas (dots), best-fit NFW profiles (solid curves), at $a/a_c = 3, 5, 7, 10$; fractional deviations from NFW, $(\rho_{NFW} - \rho)/\rho_{NFW}$, above.

**Jeans Equilibrium & Anisotropy:** After $a/a_c = 3$, the halo is close to equilibrium, according to the Jeans equation in spherical symmetry, and is close to isothermal. Hereafter, we shall consider the halo to form at $a_0 = 3a_c$. The velocity distribution is somewhat more anisotropic than found in simulations of CDM, $0.2 < \beta < 0.8$, whereas the CDM simulations give $0.0 < \beta_{CDM} < 0.6$, where $\beta = 1 - (\langle v_r^2 \rangle / (2\langle v^2 \rangle))$.

**4. EVOLUTION**

**Concentration Parameter:** The concentration parameter of the best-fitting NFW density profile at each epoch evolves linearly with scale factor (Fig. 3). For $a > a_0$, after the halo formation epoch, we find $c \simeq 4(a/a_0)$, almost identical to that reported by Wechsler et al. (2002) for N-body simulations of CDM halos.

**Mass Growth Rate:** For $2 < a/a_c < 3$, $M_{200}$ (mass within $r_{200}$) grows rapidly, while for $3 < a/a_c < 7$, $M_{200} \propto a$, consistent with self-similar spherical infall (Bertschinger 1985). For $a/a_c > 7$, growth flattens due to finite mass supply. This mass history closely resembles that for CDM halos found by Wechsler et al. (2002) (Fig. 4).

**Self-Similar Infall:** The radial velocity profile, mass, and radius are consistent with self-similar infall for $3 < a/a_c < 7$, with $\lambda_{200}/\lambda_c \simeq 0.8$, where $\lambda_{200} = r_{200}/r_a$, $r_a$ is the time-varying turnaround radius, and $\lambda_c$ is the radius of the outermost caustic in the self-similar solution.

**Virial Ratio:** The virial ratio $2T/|W|$ just after virialization is $\sim 1.35$, close to that of the N-body results for CDM halos, as predicted by the TIS model and consistent with the value expected for a virialized halo in which mass infall contributes an effective surface pressure. Thereafter, the virial ratio evolves towards the value expected for an isolated halo, $2T/|W| \sim 1$, as the mass infall rate declines.

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Fig. 3. Concentration parameter vs. scale factor.

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