AN OPPENHEIM TYPE DETERMINANTAL INEQUALITY FOR THE KHATRI–RAO PRODUCT

YONGTAO LI AND LIHUA FENG

Abstract. The Khatri-Rao product is a generalization of the classical Hadamard product for block matrices. In this paper, we give an Oppenheim type determinantal inequality for the Khatri-Rao product of two block positive semidefinite matrices, and then we extend our result to multiple block matrices.

Mathematics subject classification (2020): 15A45, 15A60, 47B65.
Keywords and phrases: Khatri-Rao product, Hadamard product, Oppenheim’s inequality, Fischer’s inequality.

REFERENCES

[1] T. ANDO, Inequalities for M–matrices, Linear Multilinear Algebra 8 (1980) 291–316.
[2] S. CHEN, Some determinantal inequalities for Hadamard product of matrices, Linear Algebra Appl. 368 (2003) 99–106.
[3] S. CHEN, Inequalities for M–matrices and inverse M–matrices, Linear Algebra Appl. 426 (2007) 610–618.
[4] S. DONG, Q. LI, Extension of an Oppenheim type determinantal inequality for the block Hadamard product, Math. Inequal. Appl. 23 (2) (2020) 539–545.
[5] X. FU, Y. LIU, Some determinantal inequalities for Hadamard and Fan products of matrices, J. Inequal. Appl. (2016) 262–269.
[6] M. GÜNTER, L. KLOTZ, Schur’s theorem for a block Hadamard product, Linear Algebra Appl. 437 (2012) 948–956.
[7] R. A. HORN, C. R. JOHNSON, Matrix Analysis, 2nd ed., Cambridge University Press, 2013.
[8] C. G. KHATRI, C. R. RAO, Solutions to some functional equations and their applications to characterization of probability distributions, Sankhya, 30 (1968) 167–180.
[9] S. KIM, J. KIM, H. LEE, Oppenheim and Schur type inequalities for Khatri-Rao product of positive definite matrices, Kyungpook Math. J. 57 (2017) 641–649.
[10] J. LIU, L. ZHU, Some improvement of Oppenheim’s inequality for M–matrices, SIAM J. Matrix Anal. Appl. 18 (2) (1997) 305–311.
[11] S. LIU, Matrix results on the Khatri-Rao and Tracy-Singh products, Linear Algebra Appl. 289 (1999) 267–277.
[12] S. LIU, Several inequalities involving Khatri-Rao products of positive semidefinite matrices, Linear Algebra Appl. 354 (2002) 175–186.
[13] S. LIU, G. TRENKLER, Hadamard, Khatri-Rao, Kronecker and other matrix products, Int. J. Inf. Syst. Sci. 4 (2008) 160–177.
[14] M. LIN, An Oppenheim type inequality for a block Hadamard product, Linear Algebra Appl. 452 (2014) 1–6.
[15] A. OPPENHEIM, Inequalities connected with definite Hermitian forms, J. London Math. Soc. 5 (1930) 114–119.
[16] G. P. H. STYAN, Hadamard products and multivariate statistical analysis, Linear Algebra Appl. 6 (1973) 217–240.
[17] Z. YANG, J. LIU, Some results on Oppenheim’s inequalities for M–matrices, SIAM J. Matrix Anal. Appl. 21 (3) (2000) 904–912.
[18] X.-D. Zhang, The equality cases for the inequalities of Fischer, Oppenheim, and Ando for general $M$-matrices, SIAM J. Matrix Anal. Appl. 25 (3) (2004) 752–765.

[19] X.-D. Zhang, C.-X. Ding, The equality cases for the inequalities of Oppenheim and Schur for positive semi-definite matrices, Czechoslovak Math. J. 59 (2009) 197–206.

[20] F. Zhang, Matrix Theory: Basic Results and Techniques, 2nd ed., Springer, New York, 2011.