Correction factors for two new reference beta radiation fields

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Abstract

Several correction factors for two new reference beta-particle radiation fields complementing ISO 6980 have been measured and calculated for both primary beta dosimetry and the operational quantities in radiation protection. The following correction factors for primary beta dosimetry for the determination of the absorbed dose to tissue, \(D_t\), have been determined by calculations: \(k_{ba}\) for backscatter from the collecting electrode, \(k_{pe}\) for perturbation by the chamber’s side walls, \(k_{ih}\) for inhomogeneity inside the collecting volume, \(k_{Sta}\) for the stopping power ratio at different phantom depths, and \(k_{SA}\) for the use of the Spencer-Attix theory, while the following have been measured: \(k_{br}\) for the effect of bremsstrahlung and \(k_{abs}\) for variations in the attenuation and scattering of beta particles between the source and the collecting volume due to variations from reference conditions and for differences of the entrance window to a tissue-equivalent thickness of 0.07 mm. Furthermore, calculations were undertaken to determine the following correction factors to assess the operational quantities \(H_{p}(0.07)\), \(H_{p}(3)\), \(H'_{p}(0.07)\), and \(H'_{p}(3)\): \(k_{rod}(0.07)\) for the rod instead of the slab phantom, \(k_{cyl}(3)\) for the cylinder instead of the slab phantom, as well as \(k'_{(0.07)}\) and \(k'_{(3)}\) for the ICRU sphere instead of the slab phantom, while the correction factor for oblique radiation incidence has been measured. The newly determined correction factors were determined in the same way as those for several well-established beta-particle radiation fields described in two earlier publications by the author. Furthermore, they are ready to be implemented in an updated version of the ISO 6980 series and in the software of the Beta Secondary Standard BSS 2.

Keywords electron dosimetry, ionization chambers, measurement units and standards, computer modeling and simulation

(Some figures may appear in colour only in the online journal)

1. Introduction

Beta dosimetry in radiation protection is based on the ISO series ISO 6980 [1–3]. In ISO 6980–1 [1], the methods of production of the reference beta-particle radiation fields are described, in ISO 6980–2 [2], the primary dosimetry for the absorbed dose to tissue, \(D_t\), in a slab phantom is outlined, while in ISO 6980–3 [3], the determination of the operational quantities, \(H\), is described. The radiation fields described in ISO 6980 show a gap with respect to mean energy between radiation from \(^{85}\)Kr sources (0.24 MeV mean energy) and \(^{90}\)Sr/\(^{90}\)Y sources (0.8 MeV mean energy). Therefore, two new energy-reduced radiation fields from \(^{90}\)Sr/\(^{90}\)Y sources were suggested resulting in a fluence weighted mean energy of about 0.6 MeV [4–6]. They are defined at a distance of 20 cm from a \(^{90}\)Sr/\(^{90}\)Y source, i.e. a reference distance of \(y_0 = 20\) cm, behind an absorber made of polymethyl methacrylate (PMMA) of 3 mm or 4 mm thickness positioned at 4 cm distance from the source.

To determine \(D_t\) and \(H\), several correction factors are required. For the radiation qualities already contained in ISO 6980, these correction factors are given in part 2 and 3 of that standard for \(D_t\) and \(H\), respectively. Therefore, for the two new radiation fields all these correction factors were determined.
within this work in the same way as described in two earlier publications by the author [7, 8].

Section 2 gives a short overview of the basic relations in beta dosimetry, section 3 describes the determination of the correction factors for primary dosimetry, while section 4 lays down the methods to obtain the correction factors for the operational quantities.

2. Basic relations in beta dosimetry

2.1. Absorbed dose to tissue

The primary quantity in beta dosimetry is the reference beta-particle absorbed dose rate to tissue, \( D_{R, \beta} \) [2], i.e. the absorbed dose rate to tissue in a tissue equivalent slab phantom at 0.07 mm depth due to beta-particles. It is determined by measurement with extrapolation chambers using the following equation according to ISO 6980–2 [2]:

\[
D_{R, \beta} = \frac{W_0 / e}{q_{0,0} \cdot a} \cdot \left[ \frac{d}{dl} (k' \cdot I(l)) \right]_{l=0}
\]

with \( (W_0 / e) \), the quotient of the mean energy required to produce an ion pair in air under reference conditions and the elementary charge \( e \), with a recommended value of \((33.88 \pm 0.05) \text{ J/C}\),

\( q_{0,0} \), the density of air at the reference conditions of temperature, pressure and relative humidity,

\( a \), the effective area of the collecting electrode,

\( k' \), the product of the correction factors which are independent of the chamber depth,

\( k \), the product of the correction factors which vary with the chamber depth and \( \frac{d}{dl} \{ k' \cdot I(l) \} \) \( l=0 \), the limiting value of the slope of the corrected current versus chamber depth \( l \).

The factors varying with the chamber depth are:

- \( k_{\text{slab}} \), correction factor for variations in the attenuation of beta particles between the source and the collecting volume due to variations from reference conditions,
- \( k_{\text{att}} \), correction for the attenuation of beta particles in the collecting volume,
- \( k_{\text{ad}} \), correction factor for the variations of the air density in the collecting volume from reference conditions,
- \( k_{\text{bd}} \), correction factor for the radioactive decay of the beta-particle source,
- \( k_{\text{adf}} \), correction factor for the axial non-uniformity of the beta-particle field,
- \( k_{\text{f}} \), correction factor for the perturbation of the beta-particle flux density by the side walls of the extrapolation chamber, and
- \( k_{\text{ion}} \), correction factor for ionisation losses due to ionic recombination.

The factors independent of the chamber depth are:

- \( k_{\text{bs}} \), correction factor for the difference in backscatter between tissue and the material of the collecting electrode,
- \( k_{\text{fe}} \), correction factor for the effect of bremsstrahlung from the beta-particle source,
- \( k_{\text{el}} \), correction factor for the electrostatic attraction of the entrance window due to the collecting voltage,
- \( k_{\text{h}} \), correction factor for the effect of humidity of the air in the collecting volume on the average energy required to produce an ion pair,
- \( k_{\text{in}} \), correction factor for interface effects between the air in the collecting volume and the adjacent entrance window and collecting electrode, and
- \( k_{\text{r}} \), correction for the radial non-uniformity of the beam, i.e. perpendicular to the beam axis.

These correction factors and a few more will be treated in section 3 of this work.

For simplification, in the following equations the dose, \( D \), instead of the dose rate, \( D \), will be used.

From the reference beta-particle absorbed dose, \( D_{R, \beta} \), the reference absorbed dose (due to beta-particles and photons), \( D_R \), results according to

\[
D_R = D_{R, \beta} / k_{\text{bs}}
\]

2.2. Operational quantities \( H_p(0.07) \), \( H_p(3) \), \( H'(0.07) \), and \( H'(3) \)

As for betas and photons the quality factor is unity, \( Q = 1 \text{ Sv Gy}^{-1} \), and as \( D_R \) is valid for a reference depth of 0.07 mm in a tissue equivalent slab phantom, the operational quantity personal dose equivalent at 0.07 mm depth in a slab phantom, \( H_p(0.07) \), results to

\[
H_p(0.07) = D_R \cdot 1 \text{ Sv/Gy}
\]

For different source nuclides and geometries, \( source \), as well as for normal and oblique radiation incidence, \( H_p(0.07; source; \alpha) \), results from the conversion coefficient \( h_{p,D}(0.07; source; \alpha) \) to

\[
H_p(0.07; source; \alpha) = D_R \cdot h_{p,D}(0.07; source; \alpha)
\]

Ring dosemeters are calibrated and irradiated on a rod phantom. The correction factor \( k_{\text{rod}}(0.07; source; \alpha) \) leads to

\[
H_p(0.07; source; \alpha) = H_p(0.07; source; \alpha)_{\text{slab}} \cdot k_{\text{rod}}(0.07; source; \alpha) = \frac{D_R \cdot h_{p,D}(0.07; source; \alpha)}{1 + \frac{D_R}{D_{R, \beta}}}
\]

The personal dose equivalent at 3 mm depth in a slab phantom, \( H_p(3; source; \alpha) \), results from the conversion coefficient \( h_{p,D}(3; source; \alpha) \) to

\[
H_p(3; source; \alpha) = D_R \cdot h_{p,D}(3; source; \alpha)
\]

Eye lens dosemeters are calibrated and irradiated on a cylinder phantom. The correction factor \( k_{\text{cyl}}(3; source; \alpha) \) leads to

\[
H_p(3; source; \alpha) = H_p(3; source; \alpha)_{\text{slab}} \cdot k_{\text{cyl}}(3; source; \alpha) = \frac{D_R \cdot h_{p,D}(3; source; \alpha)}{1 + \frac{D_R}{D_{R, \beta}}}
\]
Area dosemeters are calibrated and irradiated free in air. The corresponding quantity directional dose equivalent at 0.07 mm depth, $H'(0.07)$, is defined in the ICRU sphere [9]. Thus, the correction factor $k'(0.07; source;\alpha)$ leads to

$$H'(0.07; source;\alpha) = H_p(0.07; source;\alpha) \cdot k'(0.07; source;\alpha) = D_R \cdot h_p(0.07; source;\alpha)$$ (8)

The area quantity to estimate the eye lens dose is $H'(3)$. The correction factor $k'(3; source;\alpha)$ leads to

$$H'(3; source;\alpha) = H_p(3; source;\alpha) \cdot k'(3; source;\alpha) = D_R \cdot h_p(3; source;\alpha)$$ (9)

The conversion coefficients and correction factors described in this subsection will be treated in section 4.

3. Correction factors for primary beta dosimetry to determine the absorbed dose to tissue, $D_t$

3.1. General remarks

The correction factors described in this section were determined for several other reference beta-particle radiation fields within an earlier work [8] based on their field characteristics supplied earlier [10]. Therefore, the corresponding descriptions are only rephrased briefly in this work. Regarding details as well as information on the radiation transport simulations performed, the reader is referred to my previous work [8]. For comparison, the figures shown in this work also contain the results obtained for the well-established sources [8] together with the results for the two new radiation fields. The tables in appendix A of this work only contain the results for the two new source types. The Beta Secondary Standard BSS 2 [11, 12] which is commercially available [13], can be used to realize all these radiation fields.

As in my previous work, all correction factors were determined for two primary extrapolation chambers used at PTB: the Böhm chamber [14] (Beta Primary Standard, BPS1) and a new extrapolation chamber (BPS2) [8]. Both BPS chambers were constructed at PTB and regarding their details such as dimension and material, the reader is referred to the given references [8, 14].

The use of the correction factors treated in this section is explained in section 2.1 of this work.

3.2. Correction factors adopted from earlier work

The following correction factors can be taken from ISO 6980–2:2004 [2]:

- correction for the electrostatic attraction of the entrance window, $k_{el}^p$
- correction for ionization losses due to recombination, $k_{ad}^p$
- correction for the effect of humidity of the air in the collecting volume on the average energy required to produce an ion pair, $k_{hu}^p$
- correction for interface effects between the air in the collecting volume and the adjacent entrance window and collecting electrode, $k_{in}^p$
- correction for radioactive decay of the beta-particle source, $k_{de}^p$

The following corrections factors can directly be taken from the previous work mentioned above [8]:

- correction for the air density in the collecting volume, $k_{ad}$
- correction for the source to chamber distance at different phantom depth, $k_{ph}$

Values for all these correction factors are compiled in the previous work [8].

3.3. Correction for backscatter from the collecting electrode, $k_{ba}^p$

To correct the backscatter of the chamber’s material to that of ICRU tissue (as the dose to ICRU tissue is to be determined) the following ratio is used:

$$k_{ba} = \frac{D_{tissue}}{D_{chamber}}$$ (10)

with $D_{tissue}$, the calculated dose in the active volume with a tissue back in the chamber and $D_{chamber}$, the calculated dose in the active volume with the real chamber material.

The values of $k_{ba}$ are independent of the chamber depth $l$ (250 $\mu$m up to 2500 $\mu$m). Therefore, figure 1 shows the calculated values (mean for ten chamber depths in steps of 250 $\mu$m) of $k_{ba}$ depending on the source, i.e. on the radiation’s field mean energy, and the angle of incidence, $\alpha$, for both BPS1 and BPS2; the corresponding data are listed in the tables A1 and A2 in appendix A. The values for the two new radiation fields fit perfectly into the former ones.

3.4. Correction for perturbation by the chamber’s side wall, $k_{pe}^p$

To correct the perturbation of the chamber’s side wall to that of air (as the collecting volume is made of air) the following ratio is used:

$$k_{pe} = \frac{D_{wallless \ chamber}}{D_{chamber}}$$ (11)

with $D_{wallless \ chamber}$, the calculated dose in the active volume with the chamber wall replaced by air and $D_{chamber}$, the calculated dose in the active volume with the real chamber material.

The resulting correction factor depends on the chamber depth $l$. The data points can be fitted by a 2nd order polynomial regression resulting for $l = 0$ $\mu$m in $k_{pe} = 1.0$, i.e. through the point (0/1):

$$k_{pe} = 1 + f_1 \cdot l + f_2 \cdot l^2$$ (12)

with $f_1$ and $f_2$, parameters obtained from a 2nd order polynomial regression through (0/1) and $l$, the chamber depth.
Figure 1. Calculated correction factor $k_{\text{ba}}$ for BPS1 (top) and BPS2 (bottom) depending on the source, i.e. on the radiation’s field mean energy and the angle of incidence, $\alpha$, see legend. The data points represent the mean for the ten chamber depths from $l = 250 \mu m$ … 2500 \mu m, while the uncertainty bars show the standard deviation of the values of the ten chamber depths.

The same equation and parameter names are used in ISO 6980-2 [2]. The parameters $f_7$ and $f_8$ obtained are listed for the two new radiation fields in tables A3 and A4 for BPS1 and BPS2, respectively, in appendix A. To get an overview of all sources, figure 2 shows $k_{\text{pe}}$ for the largest chamber depth, i.e. for $l = 2500 \mu m$, depending on the radiation’s field mean energy and the angle of incidence $\alpha$. For both BPS1 and BPS2, the behaviour of $k_{\text{pe}}$ is similar to larger effects for BPS1 which is plausible as the chamber diameters are 6 cm and 8 cm, respectively, for the BPS1 and BPS2.

3.5. Correction for inhomogeneity inside the collecting volume, $k_{\text{ih}}$

To correct the inhomogeneous dose within the finite active chamber volume to the dose in a small point, the following ratio is used:

$$k_{\text{ih}} = \frac{D_{\text{part of act. vol.}}}{D_{\text{total act. vol.}}}$$  \hspace{1cm} (13)

with $D_{\text{part of act. vol.}}$ the dose in the centre front part of the active volume of 0.5 cm diameter and 100 \mu m depth, and
Figure 2. Calculated correction factor $k_{pe}$ for BPS1 (top) and BPS2 (bottom) depending on the source, i.e. on the radiation’s field mean energy and the angle of incidence, $\alpha$, see legend, for a chamber depth of $l = 2500 \mu m$ (usually resulting in the largest deviation from unity). The uncertainty bars represent the uncertainty of the single simulations ($\leq 0.25\%$).

$D_{\text{total act. vol.}}$, the dose in the total active volume of 3 cm diameter and the whole chamber depth $l$.

$D_{\text{part of act. vol.}}$, was used as a representative for the dose in a single centre point at the front of the chamber.

The resulting correction factor linearly depends on the chamber depth $l$. Therefore, a linear regression was used to fit the values:

$$k_{ih} = b + m \cdot l$$  \hspace{1cm} (14)

with $b$ and $m$, parameters obtained from a linear regression and $l$, the chamber depth.

The parameters $b$ and $m$ obtained from the regression are listed for all sources in tables A5 and A6 for BPS1 and BPS2, respectively, in appendix A. To get an overview of all sources, figure 3 shows the mean of $k_{ih}$ for all chamber depths together with its largest value (which it always takes for the largest $l = 2500 \mu m$) and its smallest value (which it always takes for the smallest $l = 250 \mu m$) depending on the radiation’s field mean energy and the angle of incidence $\alpha$. For both BPS1 and BPS2, the behaviour of $k_{ih}$ is the same with larger values for BPS1 as already seen earlier [8].
3.6. Correction factor for the effect of bremsstrahlung from the beta-particle source, $k_{br}$

The contribution of photons to the total dose was measured by placing an absorber made of PMMA in front of the extrapolation chamber, sufficiently thick to stop all beta particles and by comparing the corresponding ionization current with the one at 0.07 mm tissue depth:

$$k_{br} = 1 - \left\{ \frac{I_{\text{phot}}}{I(0.07)} \right\}$$  \hspace{1cm} (15)

with $I(0.07)$, the ionization current interpolated to 0.07 mm tissue depth and

$I_{\text{phot}}$, the ionization current behind a thick PMMA absorber.

$I(0.07)$ is considered to represent the total dose at reference conditions, $D_R$, while $I_{\text{phot}}$ is interpreted as contribution due to photons, $D_{br} = (D_R - D_{R,\beta})$. In an earlier work their ratio, $\tau_{br} = D_{br}/D_R = I_{\text{phot}}/I(0.07)$, was given for several other radiation fields \[12\]. Table A7 in appendix A lists values of $k_{br}$ for the two new radiation fields.
3.7. Correction factor for variations in the attenuation and scattering of beta particles due to the source and the collecting volume due to variations from reference ambient conditions and for differences of the entrance window to a tissue-equivalent thickness of 0.07 mm, \( k_{abs} \)

The correction factor \( k_{abs} \) accounts for variations in the attenuation and scattering of beta particles between the source and the collecting volume due to variations from reference ambient conditions and for the difference of the entrance window to a tissue-equivalent thickness of 0.07 mm [2]. It is given by

\[
k_{abs} = \frac{T(d_0)}{\rho} = \frac{T(d_0)}{\rho_{a0}} \cdot \frac{\left( \eta_{m,t} \rho_{m,t} \right)}{\eta_d \rho_d} \cdot \frac{\left( \eta_{a,t} \rho_{a,t} \right)}{\eta_{a0} \rho_{a0}} \cdot \frac{d_0}{d_a}
\]

with \( d_0 \) the reference thickness of the entrance window of the extrapolation chamber, 0.07 mm or 3 mm ICRU tissue equivalent thickness, \( \rho_{a0} = 1.9794 \cdot 10^{-3} \text{ g cm}^{-3} \) the reference air density, \( \rho_d \) the air density during the measurement, \( \eta_{m,t} \rho_{m,t} \) the tissue-equivalent thickness of a window of medium \( m \), thickness \( d_m \) and density \( \rho_m \), and \( \eta_{a,t} \rho_{a,t} \) the tissue-equivalent thickness from the reference air path \( y_0 \) with \( y_0 = 20 \text{ cm} \) for the two new radiation fields.

The measured depth dose curves due to beta radiation, i.e. the transmission functions \( T(d) \), are adequately represented by functions of the form [12, 15, 16]

\[
T(d) = \sum_{i=0}^{l=8} \left( T_i \cdot \cos \left( i \cdot \arccos \left( X(d) \right) \right) - \tau_{br} \right) \frac{1}{1 - \tau_{br}}
\]

with \( X(d) = 2 \cdot \frac{\log_{10}(d + \delta) - \log_{10}(d_{\text{max}} + \delta)}{\log_{10}(d_{\text{max}} + \delta) - \log_{10}(d_{\text{min}} + \delta)} - 1 \) a variable transformation from \( d \) to \( X(d) \in [-1; 1] \) and \( T_i \) for \( i = 0 \ldots 8 \) nine parameters, and \( \tau_{br} \) the bremsstrahlung contribution. See table A7 in appendix A and section 3.6 for \( k_{abs} \) and further details.

Values for the parameters \( T_i \), \( i = 0 \ldots 8 \), as well as for \( d_{\text{min}} \), \( d_{\text{max}} \) and \( \delta \) for the two new radiation fields are shown in table A8 in appendix A. They were obtained as fits of measurements of transmission through polyethylene terephthalate (PET) foils and PMMA absorbers [15, 16]. The values for \( \alpha = 0^\circ \) are to be applied to extrapolation curve measurements at all angles of incidence, i.e. at \( \alpha = 0^\circ \) and at \( \alpha \neq 0^\circ \).

3.8. Correction factor for the stopping power ratio at different phantom depths, \( k_{Sta} \)

The stopping power ratio, \( s_{Sta} \), accounts for the fact that the measurement volume is filled with air, not tissue. The value of \( s_{Sta} \) is equal to the ratio of the energy transfer of electrons in tissue and air. It is not a constant as it depends on the energy \( E \) of the electrons which decreases continuously as they pass deeper into matter due to the energy loss of the electrons along their path. To obtain the stopping power ratios at different depths in a tissue equivalent slab phantom, \( d_a, s_{Sta}(d_a)_{BG} \), the spectral fluences at depth \( d_a \) were folded with the stopping power ratios of tissue and air for monoenergetic electrons. With that, the correction factor can be formulated [15]

\[
k_{Sta} = \frac{s_{Sta}(0)_{BG} + a \cdot (d_a/\mu m)^b}{s_{Sta}(0)_{BG}}
\]

with \( s_{Sta}(0)_{BG} \) the stopping power ratio due to Bragg-Gray at the phantom’s front; their values are given in table A9 in appendix A. The spectral electron fluences were taken from literature [6]. The parameters \( a \) and \( b \) were determined with the method described in literature [15] and are given in table A10 in appendix A.

3.9. Use of the Spencer-Attix theory via correction factor, \( k_{SA} \)

The Spencer-Attix cavity theory is considered to be more accurate than the Bragg-Gray theory as it accounts for the variation in the response measured as a function of cavity dimension \( l \), whereas the Bragg-Gray theory does not [17]. From the two new radiation fields, the spectral information is freely available [6]. From these data, corresponding Spencer-Attix stopping power ratios, \( s_{SA}(l)_{SA} \), depending on the chamber depth, \( l \), were calculated using the method described above [17], the results will be published in [18]. Their ratio to the Bragg-Gray stopping power ratio, \( s_{Sta}(0)_{BG} \), see table A9 in appendix A, can be interpreted as correction factor \( k_{SA} \) to apply the Spencer-Attix theory:

\[
k_{SA} = \frac{s_{SA}(l)_{SA}}{s_{Sta}(0)_{BG}}
\]

This ratio can be fitted to a 2nd order polynomial function depending on the chamber depth \( l \) of the form

\[
k_{SA} = c_0 + c_1 \cdot l + c_2 \cdot l^2
\]

with \( c_0, c_1 \) and \( c_2 \), parameters obtained from a 2nd order polynomial regression and \( l \), the chamber depth.

The parameters \( c_0, c_1 \) and \( c_2 \) obtained from the regression are listed for the two new radiation fields in table A11 in appendix A for the primary extrapolation chamber of PTB, i.e. a Böhm chamber (BPS1) [14].

4. Conversion coefficients and correction factors to determine the operational quantities, \( H \)

4.1. General remarks

The conversion coefficients and corrections factors described in this section were determined for several other reference beta-particle radiation fields (sources) earlier [7, 12]. Therefore, the corresponding descriptions are only rephrased briefly in this work. Regarding details as well as information on the measurements and on the radiation transport simulations performed, the reader is referred to the previous publications. For comparison, the figures shown in this section also contain the results obtained earlier for the well-established sources together with the results for the two new radiation fields. The tables in appendix B of this work contain the results for the two new radiation fields. The use of the conversion coefficients and
correction factors treated in this section is explained in section 2.2 of this work.

4.2. Conversion coefficient from \(D_R\) to \(H_p(0.07; source; \alpha)_{slab}\) for angle \(\alpha\) and source, \(h_{p,D}(0.07; source; \alpha)_{slab}\)

The conversion coefficient from \(D_R\) to \(H_p(0.07; source; \alpha)_{slab}\) was measured using the primary extrapolation chamber of PTB, i.e. a Böhm chamber (BPS1) \([14]\), for several angles of incidence \(\alpha\). The conversion coefficient is given by the ratio of the ionization current at \(\alpha \neq 0^\circ\) and at \(\alpha = 0^\circ\), both interpolated to 0.07 mm tissue equivalent depth:

\[
h_{p,D}(0.07; source; \alpha)_{slab} = I(0.07; source; \alpha) / I(0.07; source; 0^\circ)
\]  

(21)

The interpolation method of the measured depth dose curves to 0.07 mm is outlined in section 3.7. The results from the equation (21) are shown in figure 4 and listed in table B2 in appendix B. Figure 4 shows that the behaviour of the two new radiation fields fits perfectly into the well-established radiation fields, namely: the smaller the mean energy the less prominent the dose build-up effect and the steeper the drop of \(h_{p,D}(0.07; source; \alpha)_{slab}\) with increasing angle of incidence.

The use of \(h_{p,D}(0.07; source; \alpha)_{slab}\) to determine \(H_p(0.07; source; \alpha)_{slab}\) is outlined in equation (4) in section 2.2.

4.3. Conversion coefficient from \(D_R\) to \(H_p(3; source; \alpha)_{slab}\) for angle \(\alpha\) and source, \(h_{p,D}(3; source; \alpha)_{slab}\)

The conversion coefficient from \(D_R\) to \(H_p(3; source; \alpha)_{slab}\) was determined in the same way as described in the previous section 4.2 with the exception that the numerator of the ratio was the ionization current interpolated to 3 mm tissue equivalent depth:

\[
h_{p,D}(3; source; \alpha)_{slab} = I(3; source; \alpha) / I(0.07; source; 0^\circ)
\]  

(22)

The results from the equation (22) are shown in figures 5 and listed in table B2 in appendix B. They also fit perfectly to those of the well-established radiation fields, namely: the smaller the mean energy the smaller the values of \(h_{p,D}(3; source; \alpha)_{slab}\) and the steeper their decrease with increasing angle of incidence due to the stronger scattering and absorption of electrons in the 3 mm tissue material. Furthermore, no dose build-up effect is present as this is completed at depths smaller than 3 mm.

The use of \(h_{p,D}(3; source; \alpha)_{slab}\) to determine \(H_p(3; source; \alpha)_{slab}\) is outlined in equation (6) in section 2.2.

4.4. Correction factors to account for the phantom shape and depth, \(k_{rod}(0.07)\), \(k_{cyl}(3)\), \(k'(0.07)\), and \(k'(3)\)

The four correction factors were determined by simulations calculating the dose at 0.07 mm and 3 mm depth in a slab phantom made of ICRU tissue and comparison with the dose in 0.07 mm of an ICRU tissue rod phantom for \(k_{rod}(0.07)\), by comparison with the dose in 3 mm of an ICRU tissue cylinder phantom for \(k_{cyl}(3)\), and by comparison with the dose in 0.07 mm and 3 mm of an ICRU tissue sphere for \(k'(0.07)\) and \(k'(3)\), respectively \([7]\):

\[
k_{rod}(0.07; source; \alpha) = H_p(0.07; source; \alpha)_{rod} / H_p(0.07; source; \alpha)_{slab}
\]  

(23)
Figure 5. Measured values of the conversion coefficient from $D_R$ to $H_p(3; \text{source}; \alpha)_{\text{slab}}$ for angle $\alpha$ and source, for the slab phantom: $h_{p,R}(3; \text{source}; \alpha)_{\text{slab}}$. The statistical standard uncertainties are smaller than the symbols and therefore not shown.

Figure 6. Calculated correction factors $k_{\text{rod}}(\alpha)$ for the rod phantom (left) and $k'(0.07; \alpha)$ for the ICRU sphere (right). The ordinate of the left graph omits the range between 1.05 and 1.11 in order to have a larger scale for the remaining ranges. The uncertainty bars show the statistical standard uncertainties.
The use of $k_{\text{rod}}(0.07; \alpha)$ to determine $H_p(0.07; \alpha)_{\text{rod}}$ is outlined in equation (5), the use of $k_{\text{cyl}}(3; \alpha)$ to determine $H_p(3; \alpha)_{\text{cyl}}$ in equation (7), the use of $k'(0.07; \alpha)$ to determine $H'(0.07; \alpha)$ in equation (8) and finally the use of $k'(3; \alpha)$ to determine $H'(3; \alpha)$ in equation (9), all in section 2.2.

Finally, appendix C gives tables with combined conversion coefficients from the reference dose $D_R$ to the operational quantities $H_p(0.07; \alpha)_{\text{rod}}$, $H_p(3; \alpha)_{\text{cyl}}$, $H'(0.07; \alpha)$ and $H'(3; \alpha)$, i.e. $h_{p,D}(0.07; \alpha)_{\text{rod}}$, $h_{p,D}(3; \alpha)_{\text{cyl}}$, $h'_p(0.07; \alpha)$ and $h'_p(3; \alpha)$ in tables C1–C4, respectively. These values including their uncertainties were calculated according to the equations mentioned above.

5. Conclusions

For two new reference beta-particle radiation fields both a complete set of correction factors for primary beta dosimetry as well as conversion coefficients and correction factors for the determination of the operational quantities were determined, for both normal and oblique radiation incidence. With the data determined in this work it is possible to implement the two new radiation fields in the current revision of the ISO 6980 series [1–3] and especially the tables B1, B2 C1–C4, in ISO 6980–3 [3].

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Table A1. Calculated values of the backscatter factor $k_{bs}$ for BPS1 (PMMA back electrode): mean and standard deviation of the values of the ten chamber depths (from $l = 250$ µm up to $2500$ µm in steps of $250$ µm), see section 3.3

| source                  | $0^\circ$ | $15^\circ$ | $30^\circ$ | $45^\circ$ | $60^\circ$ | $60^\circ$ |
|-------------------------|-----------|------------|------------|------------|------------|------------|
| $^{90}$Sr + $^{90}$Y with 3 mm PMMA; $\gamma_0 = 20$ cm | 1.017     | 0.001     | 1.017     | 0.001     | 1.014     | 0.001     | 1.011     | 0.001     |
| $^{90}$Sr + $^{90}$Y with 4 mm PMMA; $\gamma_0 = 20$ cm | 1.017     | 0.001     | 1.018     | 0.002     | 1.017     | 0.002     | 1.015     | 0.002     | 1.012     | 0.001     |

Table A2. Calculated values of the backscatter factor $k_{bs}$ for BPS2 (PEEK back electrode): mean and standard deviation of the values of the ten chamber depths (from $l = 250$ µm up to $2500$ µm in steps of $250$ µm), see section 3.3

| source                  | $0^\circ$ | $15^\circ$ | $30^\circ$ | $45^\circ$ | $60^\circ$ | $60^\circ$ |
|-------------------------|-----------|------------|------------|------------|------------|------------|
| $^{90}$Sr + $^{90}$Y with 3 mm PMMA; $\gamma_0 = 20$ cm | 1.013     | 0.001     | 1.013     | 0.001     | 1.012     | 0.001     | 1.011     | 0.001     | 1.008     | 0.001     |
| $^{90}$Sr + $^{90}$Y with 4 mm PMMA; $\gamma_0 = 20$ cm | 1.013     | 0.001     | 1.012     | 0.001     | 1.011     | 0.001     | 1.011     | 0.001     | 1.009     | 0.001     |

Table A3. Values of the fit parameters $f_7$ and $f_8$ for the calculation of the perturbation factor, $k_{pe}$, for the BPS1, see equation (12) in section 3.4. The parameters were determined via 2nd order polynomial regression through $(0/1)$ of the calculated perturbation factor, $k_{pe}$.

| source                  | $f_7(0^\circ)/f_7(15^\circ)/f_7(30^\circ)/f_7(45^\circ)/f_7(60^\circ)$ (mm$^{-1}$/mm$^{-2}$/mm$^{-3}$/mm$^{-4}$/mm$^{-5}$) |
|-------------------------|---------------------------------------------------------------------------------------------------------------|
| $^{90}$Sr + $^{90}$Y with 3 mm PMMA; $\gamma_0 = 20$ cm | $-5.06/-0.217/-6.14/0.365/-6.68/1.19/-5.97/2.75/4.13/3.46$                                                                 |
| $^{90}$Sr + $^{90}$Y with 4 mm PMMA; $\gamma_0 = 20$ cm | $-4.61/0.0847/-7.05/1.42/-5.61/1.24/-4.92/3.06/3.60/4.68$                                                                 |

Note 1: The unit 1/(mm$^{-1}$) $\equiv 1/(mm^{-2}) \equiv 1/(m-1)$ while the unit 1/(mm$^{-2}$) $\equiv 1/(m-2)$.
Note 2: An entry of ‘-5.06’ means $f_7 = -5.06 \times 10^{-1} m^{-1} = 0.00506 m^{-1}$
Note 3: An entry of ‘-0.217’ means $f_8 = 2.17 \times 10^{-4} m^{-2} = -0.000217 m^{-2}$

Table A4. Values of the fit parameters $f_7$ and $f_8$ for the calculation of the perturbation factor, $k_{pe}$, for the BPS2, see equation (12) in section 3.4. The parameters were determined via 2nd order polynomial regression through $(0/1)$ of the calculated perturbation factor, $k_{pe}$.

| source                  | $f_7(0^\circ)/f_7(15^\circ)/f_7(30^\circ)/f_7(45^\circ)/f_7(60^\circ)$ (mm$^{-1}$/mm$^{-2}$/mm$^{-3}$/mm$^{-4}$/mm$^{-5}$) |
|-------------------------|---------------------------------------------------------------------------------------------------------------|
| $^{90}$Sr + $^{90}$Y with 3 mm PMMA; $\gamma_0 = 20$ cm | $-1.69/-0.806/-1.28/-0.762/-1.79/-0.457/-2.50/0.626/-1.36/2.12$                                                                 |
| $^{90}$Sr + $^{90}$Y with 4 mm PMMA; $\gamma_0 = 20$ cm | $-1.36/-0.539/-2.71/0.133/-1.94/0.0485/-1.93/0.671/-0.986/2.68$                                                                 |

Note 1: The unit 1/(mm$^{-1}$) $\equiv 1/(mm^{-2}) \equiv 1/(m-1)$ while the unit 1/(mm$^{-2}$) $\equiv 1/(m-2)$.
Note 2: An entry of ‘-1.69’ means $f_7 = -1.69 \times 10^{-1} m^{-1} = 0.00169 m^{-1}$
Note 3: An entry of ‘-0.806’ means $f_8 = -8.06 \times 10^{-4} m^{-2} = -0.000806 m^{-2}$

measurements with the primary extrapolation chamber and George Winterbottom for the very quick production of the 3 mm and 4 mm PMMA absorbers.

Appendix A. Tables with the calculated and measured correction factors

Tables A1 and A2 list the backscatter factors, $k_{bs}$, for BPS1 and BPS2, respectively, together with their uncertainties, tables A3 and A4 list the fit parameters $f_7$ and $f_8$ for the calculation of the perturbation factors, $k_{pe}$, and tables A5 and A6 list the inhomogeneity factors, $k_{in}$, for BPS1 and BPS2, respectively, together with their uncertainties. Furthermore, the correction factors for bremsstrahlung, $k_{br}$, table A7, the fit parameters for the transmission functions $T(d)$, table A8, the Bragg-Gray stopping power ratio at 0 µm phantom depth, table A9, the fit parameters to calculate $k_{SA}$ at different phantom depths, table A10 and the fit parameters to calculate $k_{SA}$ to use the Spencer-Attix theory, table A11, are given.
### Table A5. Values of the fit parameters $m$ and $b$ for the calculation of the inhomogeneity factor, $k_{ih}$, for the BPS1, see equation (14) in section 3.5. The parameters were determined via linear regression of the calculated inhomogeneity factor, $k_{ih}$.

| source | $m(0^\circ)/\text{y (mm}^{-1})$ | $m(15^\circ)/\text{y (mm}^{-1})$ | $m(30^\circ)/\text{y (mm}^{-1})$ | $m(45^\circ)/\text{y (mm}^{-1})$ | $m(60^\circ)/\text{y (mm}^{-1})$ |
|--------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $^{90}\text{Sr}+^{90}\text{Y}$ with 3 mm PMMA; $y_0 = 20$ cm | 0.999 | 4.15 | 0.997 | 4.12 | 0.993 | 4.69 | 0.993 | 3.79 |
| $^{90}\text{Sr}+^{90}\text{Y}$ with 4 mm PMMA; $y_0 = 20$ cm | 1.000 | 4.09 | 0.996 | 5.33 | 0.995 | 6.01 | 0.996 | 3.80 | 0.994 | 3.95 |

Note 1: The unit 1/(mm$^{-1}$) = 1/(m$^{-1}$).
Note 2: An entry of ‘4.15’ means $m = 4.15 	imes 10^{-3}$ mm$^{-1} = 0.0415$/mm

### Table A6. Values of the fit parameters $m$ and $b$ for the calculation of the inhomogeneity factor, $k_{ih}$, for the BPS2, see equation (14) in section 3.5. The parameters were determined via linear regression of the calculated inhomogeneity factor, $k_{ih}$.

| source | $m(0^\circ)/\text{y (mm}^{-1})$ | $m(15^\circ)/\text{y (mm}^{-1})$ | $m(30^\circ)/\text{y (mm}^{-1})$ | $m(45^\circ)/\text{y (mm}^{-1})$ | $m(60^\circ)/\text{y (mm}^{-1})$ |
|--------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $^{90}\text{Sr}+^{90}\text{Y}$ with 3 mm PMMA; $y_0 = 20$ cm | 0.996 | 4.35 | 0.999 | 2.56 | 0.995 | 3.43 | 0.995 | 1.80 | 0.994 | 0.20 |
| $^{90}\text{Sr}+^{90}\text{Y}$ with 4 mm PMMA; $y_0 = 20$ cm | 0.995 | 4.98 | 0.996 | 3.75 | 0.996 | 3.65 | 0.994 | 2.35 | 0.990 | 1.56 |

Note 1: The unit 1/(mm$^{-1}$) = 1/(m$^{-1}$).
Note 2: An entry of ‘4.35’ means $m = 4.35 	imes 10^{-3}$ mm$^{-1} = 0.00435$/mm

### Table A7. Values of the correction factor for bremsstrahlung, $k_{br}$, and its standard uncertainty, see section 3.6 for details.

| source | $k_{br}$ | $u(k_{br})$ |
|--------|----------|-------------|
| $^{90}\text{Sr}+^{90}\text{Y}$ with 3 mm PMMA; $y_0 = 20$ cm | 0.9987 | 0.07% |
| $^{90}\text{Sr}+^{90}\text{Y}$ with 4 mm PMMA; $y_0 = 20$ cm | 0.9958 | 0.21% |

### Table A8. Fit parameters for the transmission functions $T(d)$ for $\alpha = 0^\circ$, see equation (16), to calculate, $k_{abs}$, see equation (17) in section 3.7

| $T_0$ | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ | $T_6$ | $T_7$ | $T_8$ | $d_{min}$ | $d_{max}$ | $\delta$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----------|-----------|--------|
| $^{90}\text{Sr}+^{90}\text{Y}$ with 3 mm PMMA; $y_0 = 20$ cm | 0.45255 | -0.61321 | 0.0889 | 0.15182 | -0.08177 | -0.01019 | 0.00957 | 0.00774 | -0.00368 | 11408 | 1085 |
| $^{90}\text{Sr}+^{90}\text{Y}$ with 4 mm PMMA; $y_0 = 20$ cm | 0.42302 | -0.59573 | 0.13071 | 0.12922 | -0.10115 | 0.00524 | 0.01668 | 0.00629 | -0.00622 | 11403 | 860 |

### Table A9. Bragg-Gray stopping power ratio, $s_{\alpha}(0)_{BG}$, at 0 μm phantom depth for the two new radiation fields, see equation (18) in section 3.8 for details. The standard uncertainty is about 0.6%.

| source | $s_{\alpha}(0)_{BG}$ |
|--------|----------------------|
| $^{90}\text{Sr}+^{90}\text{Y}$ with 3 mm PMMA; $y_0 = 20$ cm | 1.11415 |
| $^{90}\text{Sr}+^{90}\text{Y}$ with 4 mm PMMA; $y_0 = 20$ cm | 1.11599 |

### Table A10. Fit parameters $a$ and $b$ to calculate $k_{Sta}$ at different phantom depths, see equation (18) in section 3.8 for details.

| $E_{\text{mean}}$/MeV | $^{90}\text{Sr}+^{90}\text{Y}$ with 3 mm PMMA; $y_0 = 20$ cm | $^{90}\text{Sr}+^{90}\text{Y}$ with 4 mm PMMA; $y_0 = 20$ cm |
|-----------------------|--------------------------------------------------|--------------------------------------------------|
| $a$ | 0.61 | 1.11110^{-4} | 1.14410^{-4} |
| $b$ | 0.54 | 0.516 | 0.49364 |
### Table A11. Fit parameters \( c_0 \), \( c_1 \) and \( c_2 \) to calculate \( k_{SA} \) to use the Spencer-Attix theory for the BPS1, see equation (20) in section 3.9. The parameters were determined via a 2nd order polynomial regression of the ratio of the calculated Spencer-Attix theory factor to the Bragg-Gray ratios, \( k_{SA} \), see equation (19) in section 3.9.

| source | \( c_0 \) | \( c_1/(\text{mm}^{-1} \cdot 10^{-3}) \) | \( c_2/(\text{mm}^{-2} \cdot 10^{-3}) \) |
|--------|----------|---------------------------------|----------------------------------|
| \(^{90}\text{Sr} + ^{90}\text{Y} \) with 3 mm PMMA; \( y_0 = 20 \) cm | 1.0078 | -3.14 | 0.707 |
| \(^{90}\text{Sr} + ^{90}\text{Y} \) with 4 mm PMMA; \( y_0 = 20 \) cm | 1.0079 | -3.75 | 0.978 |

Note 1: The unit 1/(\text{mm}^{-1} \cdot 10^{-3}) \equiv 1/(\text{m}^{-1}) while the unit 1/(\text{mm}^{-2} \cdot 10^{-3}) \equiv 1/(\text{m}^{-2}) .

Note 2: An entry of ‘-3.14’ means \( c_1 = -3.14 \times 10^{-3} \text{ mm}^{-1} = -0.00314/\text{mm} \)

Note 3: An entry of ‘0.707’ means \( c_2 = 0.707 \times 10^{-3} \text{ mm}^{-2} = 0.000707/\text{mm}^2 \)

### Appendix B. Tables with the conversion coefficients and correction factors to determine the operational quantities

Tables B1 and B2 list the conversion coefficients from \( D_R \) to \( H_p(0.07; \text{source}; \alpha)_{\text{lab}} \) and from \( D_R \) to \( H_p(3; \text{source}; \alpha)_{\text{lab}} \), while tables B3–B6 list the correction factors \( k_{rad}(0.07) \), \( k_{cyl}(3) \), \( k'(0.07) \) and \( k'(3) \), respectively. Details for the values are discussed in section 4.

#### Table B1. Measured values of the conversion coefficient from \( D_R \) to \( H_p(0.07; \text{source}; \alpha)_{\text{lab}} \) for angle \( \alpha \) and \( \text{source} \), for the slab phantom: \( h_{p,D}(0.07; \text{source}; \alpha)_{\text{lab}} \) in \( \text{Sv/Gy} \), see equation (21) in section 4.2 for its determination and equation (4) in section 2.2 for its application.

| source | \( 0^\circ \) | \( u(0^\circ) \) | \( 15^\circ \) | \( u(15^\circ) \) | \( 30^\circ \) | \( u(30^\circ) \) | \( 45^\circ \) | \( u(45^\circ) \) | \( 60^\circ \) | \( u(60^\circ) \) |
|--------|-------------|----------------|-------------|----------------|-------------|----------------|-------------|----------------|-------------|----------------|
| \(^{90}\text{Sr} + ^{90}\text{Y} \) with 3 mm PMMA; \( y_0 = 20 \) cm | 1.00 | 0.0% | 1.019 | 0.14% | 1.065 | 0.54% | 1.124 | 1.17% | 1.109 | 2.00% |
| \(^{90}\text{Sr} + ^{90}\text{Y} \) with 4 mm PMMA; \( y_0 = 20 \) cm | 1.00 | 0.0% | 1.015 | 0.14% | 1.056 | 0.54% | 1.099 | 1.17% | 1.054 | 2.00% |

#### Table B2. Measured values of the conversion coefficient from \( D_R \) to \( H_p(3; \text{source}; \alpha)_{\text{lab}} \) for angle \( \alpha \) and \( \text{source} \), for the slab phantom: \( h_{p,D}(3; \text{source}; \alpha)_{\text{lab}} \) in \( \text{Sv/Gy} \), see equation (22) in section 4.3 for its determination and equation (6) in section 2.2 for its application.

| source | \( 0^\circ \) | \( u(0^\circ) \) | \( 15^\circ \) | \( u(15^\circ) \) | \( 30^\circ \) | \( u(30^\circ) \) | \( 45^\circ \) | \( u(45^\circ) \) | \( 60^\circ \) | \( u(60^\circ) \) |
|--------|-------------|----------------|-------------|----------------|-------------|----------------|-------------|----------------|-------------|----------------|
| \(^{90}\text{Sr} + ^{90}\text{Y} \) with 3 mm PMMA; \( y_0 = 20 \) cm | 0.182 | 0.51% | 0.166 | 0.55% | 0.125 | 0.95% | 0.0743 | 1.83% | 0.0323 | 3.04% |
| \(^{90}\text{Sr} + ^{90}\text{Y} \) with 4 mm PMMA; \( y_0 = 20 \) cm | 0.0715 | 0.51% | 0.0649 | 0.55% | 0.0478 | 0.95% | 0.0266 | 1.83% | 0.0118 | 3.04% |

#### Table B3. Calculated values of the correction factor \( k_{rad}(0.07) \), see equation (23) in section 4.4 for its determination and equations (5) in section 2.2 for its application.

| source | \( 0^\circ \) | \( u(0^\circ) \) | \( 15^\circ \) | \( u(15^\circ) \) | \( 30^\circ \) | \( u(30^\circ) \) | \( 45^\circ \) | \( u(45^\circ) \) | \( 60^\circ \) | \( u(60^\circ) \) | \( 75^\circ \) | \( u(75^\circ) \) |
|--------|-------------|----------------|-------------|----------------|-------------|----------------|-------------|----------------|-------------|----------------|-------------|----------------|
| \(^{90}\text{Sr} + ^{90}\text{Y} \) with 3 mm PMMA; \( y_0 = 20 \) cm | 0.990 | 0.51% | 0.987 | 0.51% | 0.987 | 0.51% | 0.992 | 0.50% | 1.024 | 0.50% | 1.144 | 0.50% |
| \(^{90}\text{Sr} + ^{90}\text{Y} \) with 4 mm PMMA; \( y_0 = 20 \) cm | 0.990 | 0.51% | 0.988 | 0.51% | 0.989 | 0.51% | 0.994 | 0.50% | 1.023 | 0.50% | 1.127 | 0.50% |

#### Table B4. Calculated values of the correction factor \( k_{cyl}(3) \), see equation (24) in section 4.4 for its determination and equations (7) in section 2.2 for its application.

| source | \( 0^\circ \) | \( u(0^\circ) \) | \( 15^\circ \) | \( u(15^\circ) \) | \( 30^\circ \) | \( u(30^\circ) \) | \( 45^\circ \) | \( u(45^\circ) \) | \( 60^\circ \) | \( u(60^\circ) \) | \( 75^\circ \) | \( u(75^\circ) \) |
|--------|-------------|----------------|-------------|----------------|-------------|----------------|-------------|----------------|-------------|----------------|-------------|----------------|
| \(^{90}\text{Sr} + ^{90}\text{Y} \) with 3 mm PMMA; \( y_0 = 20 \) cm | 1.000 | 0.50% | 1.006 | 0.50% | 1.021 | 0.50% | 1.035 | 0.50% | 1.047 | 0.50% | 1.084 | 0.86% |
| \(^{90}\text{Sr} + ^{90}\text{Y} \) with 4 mm PMMA; \( y_0 = 20 \) cm | 1.000 | 0.50% | 1.009 | 0.50% | 1.010 | 0.50% | 1.024 | 0.61% | 1.038 | 0.85% | 1.054 | 1.30% |
Table B5. Calculated values of the correction factor $k'(0.07)$, see equation (25) in section 4.4 for its determination and equation (8) in section 2.2 for its application.

| source                  | 0°  | 15°  | 30°  | 45°  | 60°  | 75°  |
|-------------------------|-----|------|------|------|------|------|
| $^{90}\text{Sr} + ^{90}\text{Y}$ with 3 mm PMMA; $\gamma_0 = 20$ cm | 1.000 | 0.50% | 1.000 | 0.50% | 1.000 | 0.50% | 1.000 | 0.50% | 1.000 | 0.50% |
| $^{90}\text{Sr} + ^{90}\text{Y}$ with 4 mm PMMA; $\gamma_0 = 20$ cm | 1.000 | 0.50% | 1.000 | 0.50% | 1.000 | 0.50% | 1.000 | 0.50% | 1.000 | 0.50% |

Table B6. Calculated values of the correction factor $k'(3)$, see equation (26) in section 4.4 for its determination and equation (9) in section 2.2 for its application.

| source                  | 0°  | 15°  | 30°  | 45°  | 60°  | 75°  |
|-------------------------|-----|------|------|------|------|------|
| $^{90}\text{Sr} + ^{90}\text{Y}$ with 3 mm PMMA; $\gamma_0 = 20$ cm | 1.000 | 0.50% | 1.000 | 0.50% | 1.019 | 0.50% | 1.021 | 0.50% | 1.042 | 0.51% | 1.055 | 0.87% |
| $^{90}\text{Sr} + ^{90}\text{Y}$ with 4 mm PMMA; $\gamma_0 = 20$ cm | 1.000 | 0.50% | 1.000 | 0.50% | 1.008 | 0.50% | 1.014 | 0.61% | 1.024 | 0.85% | 1.031 | 1.30% |

Appendix C. Tables with further conversion coefficients to determine the operational quantities

Tables C1–C4 list the conversion coefficients from reference dose $D_R$ to the operational quantities $H_p(0.07; \alpha_{rod}$), $H_p(3; \alpha_{cyl}$), $H'_{07}(0.07; \alpha)$ and $H'(3; \alpha)$, respectively. Details for the values are discussed in section 4.4

Table C1. Conversion coefficients from $D_R$ to $H_p(0.07; \alpha_{rod}$) for angle $\alpha$ and source $\gamma$, for the rod phantom: $h_{p,R}(0.07; \alpha_{rod}$) in Sv/Gy.

| source                  | 0°  | 15°  | 30°  | 45°  | 60°  |
|-------------------------|-----|------|------|------|------|
| $^{90}\text{Sr} + ^{90}\text{Y}$ with 3 mm PMMA; $\gamma_0 = 20$ cm | 0.990 | 0.51% | 1.006 | 0.53% | 1.051 | 0.74% |
| $^{90}\text{Sr} + ^{90}\text{Y}$ with 4 mm PMMA; $\gamma_0 = 20$ cm | 0.990 | 0.51% | 1.003 | 0.52% | 1.045 | 0.74% |

Table C2. Conversion coefficients from $D_R$ to $H_p(3; \alpha_{cyl}$) for angle $\alpha$ and source $\gamma$, for the cylinder phantom: $h_{p,D}(3; \alpha_{cyl}$) in Sv/Gy.

| source                  | 0°  | 15°  | 30°  | 45°  | 60°  |
|-------------------------|-----|------|------|------|------|
| $^{90}\text{Sr} + ^{90}\text{Y}$ with 3 mm PMMA; $\gamma_0 = 20$ cm | 0.182 | 0.71% | 0.167 | 0.74% | 0.128 | 1.07% |
| $^{90}\text{Sr} + ^{90}\text{Y}$ with 4 mm PMMA; $\gamma_0 = 20$ cm | 0.0715 | 0.71% | 0.0655 | 0.74% | 0.0483 | 1.07% |

Table C3. Conversion coefficients from $D_R$ to $H'(0.07; \alpha)$ for angle $\alpha$ and source $\gamma$: $h'_{07}(0.07; \alpha)$ in Sv/Gy.

| source                  | 0°  | 15°  | 30°  | 45°  | 60°  |
|-------------------------|-----|------|------|------|------|
| $^{90}\text{Sr} + ^{90}\text{Y}$ with 3 mm PMMA; $\gamma_0 = 20$ cm | 1.000 | 0.50% | 1.019 | 0.52% | 1.065 | 0.74% |
| $^{90}\text{Sr} + ^{90}\text{Y}$ with 4 mm PMMA; $\gamma_0 = 20$ cm | 1.000 | 0.50% | 1.015 | 0.52% | 1.056 | 0.74% |

Table C4. Conversion coefficients from $D_R$ to $H'(3; \alpha)$ for angle $\alpha$ and source $\gamma$: $h'(3; \alpha)$ in Sv/Gy.

| source                  | 0°  | 15°  | 30°  | 45°  | 60°  |
|-------------------------|-----|------|------|------|------|
| $^{90}\text{Sr} + ^{90}\text{Y}$ with 3 mm PMMA; $\gamma_0 = 20$ cm | 0.182 | 0.71% | 0.167 | 0.74% | 0.127 | 1.07% |
| $^{90}\text{Sr} + ^{90}\text{Y}$ with 4 mm PMMA; $\gamma_0 = 20$ cm | 0.0715 | 0.71% | 0.0649 | 0.74% | 0.0482 | 1.07% |
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