Abstract

We discuss the transport of matter waves in low-dimensional waveguides. Due to scattering from uncontrollable noise fields, the spatial coherence gets reduced and eventually lost. We develop a description of this decoherence process in terms of transport equations for the atomic Wigner function. We outline its derivation and discuss the special case of white noise where an analytical solution can be found.

Introduction

We discuss in this contribution the transport of atomic matter waves in a low-dimensional waveguide. Such structures may be created close to solid substrates using electro-magnetic fields: the magnetic field of a current-carrying wire combined with a homogeneous bias field, e.g., gives rise to a linear waveguide [1, 2, 3]. Planar waveguides may be constructed with repulsive magnetic [4] or optical [5] fields that ‘coat’ the substrate surface. The atomic motion is characterised by bound vibrations in the ‘transverse’ direction(s) and an essentially free motion in the ‘longitudinal’ direction(s) along the waveguide axis (plane), respectively. Although direct contact with the substrate is avoided by the shielding potential, the atoms feel its presence through enhanced electromagnetic field fluctuations that ‘leak’ out of the thermal solid, typically held at room temperature. We have shown elsewhere that these thermal near fields are characterised by a fluctuation spectrum exceeding by orders of magnitude the usual blackbody radiation [6, 7, 8, 9]. The scattering of the atoms off the near field fluctuations occurs at a rate that may be calculated using Fermi’s Golden Rule. The consequences of multiple scattering is conveniently described by a transport equation that combines in a self-consistent way both ballistic motion and scattering.
The purpose of this contribution is to outline a derivation of this transport equation. The status of this equation is similar to that of the quantum-optical master equations allowing to describe the evolution of the reduced density matrix of an atomic system, on a time scale large compared to the correlation time of the reservoir the system is coupled to, typically the vacuum radiation field. In the case of transport in waveguides, we face both temporal and spatial dynamics and therefore restrict our attention to scales large compared to the correlation time and length of a fluctuating noise potential. Our analysis uses a multiple scale expansion adapted from \[10\]. Similar to the quantum-optical case, we make an expansion in the perturbing potential to second order. In the resulting transport equation, the noise is thus characterised by its second-order correlation functions or, equivalently, its spectral density. In the case of white noise, the transport equation can be explicitly solved. We have shown elsewhere \[8\] that this approximation holds quite well for thermal near field fluctuations. For technical noise, it also holds when the noise spectrum is flat on a frequency scale roughly set by the ‘longitudinal’ temperature of the atoms in the waveguide. The explicit solution yields an estimate for the spatial coherence of the guided matter waves as a function of time. The paper concludes with some remarks on the limits of validity of the present transport theory. It cannot describe, e.g., Anderson localisation in one dimension \[11\] because on the coarser spatial scale of the transport equation, the scattering from the noise field is assumed to take place locally; interferences between different scattering sequences are not taken into account. Decoherence in ‘curved’ or ‘split’ waveguides also needs a refined theory because of the cross-coupling between the transverse and longitudinal degrees of freedom, the former being ‘frozen out’ in our framework.

1 Statistical matter wave optics

The simplest model for atom transport in a low-dimensional waveguide is based on the Schrödinger equation

\[ \text{i} \hbar \partial_t \psi(x, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi + V(x, t) \psi \]  

The coordinate \( x \) describes the motion in the free waveguide directions. The transverse motion is ‘frozen out’ by assuming that the atom is cooled to the transverse ground state. Atom-atom interactions are neglected, too. \( V(x, t) \) is the noise potential: for a magnetic waveguide, e.g., it is given by

\[ V(x, t) = \langle s | \mu \cdot B(x, t) | s \rangle, \]  

where \( |s\rangle \) is the trapped internal state of the atom (we neglect spin-changing processes), and \( B(x, t) \) is the thermal magnetic field. The noise potential is a
statistical quantity with zero mean and second-order correlation function

\[ C_V(s, \tau) = \langle V(x+s, t+\tau) V(x, t) \rangle, \tag{3} \]

where the average is taken over the realisations of the noise potential. We assume a statistically homogeneous noise, the correlation function being independent of \( x \) and \( t \). As a function of the separation \( s \), thermal magnetic fields are correlated on a length scale \( l_c \) given approximately by the distance \( d \) between the waveguide axis and the solid substrate \[8\]. This estimate is valid as long as the wavelength \( 2\pi c/\omega \) corresponding to the noise frequency \( \omega \) is large compared to \( d \): for micrometre-sized waveguide structures, this means frequencies below the optical range. The relevant frequencies of the noise will be identified below and turn out to be much smaller than this.

The coherence properties of the guided matter waves are characterised by the noise-averaged coherence function (the time dependence is suppressed for clarity)

\[ \rho(x; s) = \langle \psi^*(x - \frac{1}{2}s) \psi(x + \frac{1}{2}s) \rangle. \tag{4} \]

In complete analogy to quantum-optical master equations, this coherence function may be regarded as the reduced density matrix of the atomic ensemble, when the degrees of freedom of the noise are traced over. The Wigner function gives a convenient representation of the coherence function:

\[ W(x, p) = \int \frac{d^D s}{(2\pi \hbar)^D} e^{-ip \cdot s/\hbar} \rho(x; s), \tag{5} \]

where \( D \) is the waveguide dimension. This representation allows to make a link to classical kinetic theory: \( W(x, p) \) may be viewed as a quasi-probability in phase space. For example, the spatial density \( n(x) \) and the current density \( j(x) \) of the atoms are given by

\[ n(x) = \int d^D p W(x, p) \tag{6} \]
\[ j(x) = \int d^D p \frac{p}{m} W(x, p) \tag{7} \]

We also obtain information about the spatial coherence: the spatially averaged coherence function \( \Gamma(s, t) \), for example, is related to the Wigner function by

\[ \Gamma(s, t) \equiv \int d^D x \rho(x; s, t) \]
\[ = \int d^D x d^D p e^{ip \cdot s/\hbar} W(x, p, t) \tag{9} \]

In the next section, we outline a derivation of a closed equation for the Wigner function in terms of the noise correlation function.
2 Transport equation

Details of the derivation of the transport equation may be found in the appendix A. We quote here only the main assumptions underlying the theory.

(i) The noise potential is supposed to be weak so that a perturbative analysis is possible. As in quantum-optical master equations, a closed equation is found when the expansion is pushed to second order in the perturbation.

(ii) The scale $l_c$ over which the noise is spatially correlated is assumed to be small compared to the characteristic scale of variation of the Wigner function. This implies a separation of the dynamics on short and large spatial scales, the dynamics on the large scale being ‘enslaved’ by certain averages over the short scale. Similarly, we assume that the potential fluctuates rapidly on the time scale for the evolution of the Wigner function. These assumptions correspond to the Markov approximation of quantum optics, where the master equation is valid on a coarse-grained time scale.

The derivation of the master equation is based on a multiple scale expansion. Functions $f(x)$ of the spatial coordinate are thus written in the form

$$ f(x) = f(X, \xi) $$

where $X$ gives the ‘slow’ variation and the dimensionless variable $\xi = x/l_c$ gives the ‘rapid’ variation on the scale of the noise correlation length $l_c$. Spatial gradients are thus expanded using

$$ \nabla_x = \nabla_X + \frac{1}{l_c} \nabla_\xi $$

By construction, the first term is much smaller than the second one. Finally, the Wigner function is expanded as

$$ W(x, p, t) = W_0(X, p, t) + \eta^{1/2} W_1(X, \xi, p, t) + O(\eta) $$

where $\eta \ll 1$ is the ratio between the correlation length $l_c$ and a ‘macroscopic’ scale on which the coordinate $X$ varies. The expansion allows to prove self-consistently that the zeroth order approximation $W_0$ does not depend on the short scale $\xi$, and to fix the exponent $1/2$ for the first order correction.

The resulting transport equation specifies the evolution of the Wigner function $W_0$. Dropping the subscript 0, it reads

$$ \left( \partial_t + \frac{p}{m} \cdot \nabla_x \right) W(x, p) = $$

$$ \int d^d p' S_V(p' - p, E_{p'} - E_p) \left[ W(x, p') - W(x, p) \right], $$

4
where \( S_V \), the spectral density of the noise, is essentially the spatial and time Fourier transform of the noise correlation function

\[
S_V(q, \Delta E) = \frac{1}{\hbar^2} \int \frac{d^D s}{(2\pi\hbar)^D} C_V(s, \tau) e^{-i(q \cdot s - \Delta E \tau)/\hbar}.
\]  

(14)

The left hand side of the transport equation gives the free ballistic motion of the atoms in the waveguide. If an external force were applied, an additional term \( F \cdot \nabla p \) would appear. The right hand side describes the scattering from the noise potential. \( E_p = p^2 / 2m \) is the de Broglie dispersion relation for matter waves. We observe that scattering processes \( p \to p' \) occur at a rate given by the noise spectrum at the Bohr frequency \( (E_p - E_{p'})/\hbar \). If the potential noise is static (as would be the case for a ‘rough potential’), then its spectral density is proportional to \( \delta(\Delta E) \), and energy is conserved. If we are interested in the scattering between guided momentum states, then the initial and final energies \( E_p, E_{p'} \) are typically of the order of the (longitudinal) temperature \( kT \) of the ensemble. The relevant frequencies in the noise spectral density are thus comparable to \( kT/\hbar \).

3 Results

3.1 White noise

White noise is characterised by a constant spectral density, i.e., the noise spectrum \( S_V(q, \Delta E) \) is independent of \( \Delta E \). Equivalently, the noise correlation is \( \delta \)-correlated in time:

\[
C_V(s, \tau) = B_V(s) \delta(\tau).
\]  

(15)

The integration over the momentum \( p' \) in (13) is now not restricted by energy conservation, and the right hand side of the transport equation becomes a convolution. One therefore obtains a simple solution using Fourier transforms. Denoting \( k \) (dimension: wavevector) and \( s \) (dim.: length) the Fourier variables conjugate to \( x \) and \( p \), we find the equation

\[
\left( \partial_t + \frac{\hbar k}{m} \cdot \nabla_s \right) \tilde{W}(k, s) = -\gamma(s) \tilde{W}(k, s).
\]  

(16)

where we have introduced the rate

\[
\gamma(s) = \frac{1}{\hbar^2} (B_V(0) - B_V(s)).
\]  

(17)

Eq.(16) is easily solved using the method of characteristics, using \( s - \hbar kt/m \) as a new variable. One finds

\[
\tilde{W}(k, s; t) = \tilde{W}_i(k, s - \hbar kt/m) \times 
\times \exp \left[ - \int_0^t dt' \gamma(s - \hbar kt'/m) \right],
\]  

(18)
where \( \tilde{W}(k, s) \) is the Wigner function at \( t = 0 \).

We observe in particular that the spatially averaged coherence function \( \mathcal{G}(s; t) \) shows an exponential decay as time increases:

\[
\Gamma(s; t) = \Gamma_i(s) \exp \left[ -\gamma(s) t \right].
\]

We can thus give a physical meaning to the quantity \( \gamma(s) \): it is the rate at which two points in the matter wave field, that are separated by a distance \( s \), lose their mutual coherence. This rate saturates to \( \gamma = \gamma(\infty) = B_V(0)/\hbar^2 \) for distances \( s \gg l_c \) large compared to the correlation length of the noise field (the correlation \( B_V(s) \) then vanishes). This saturation has been discussed, e.g., in [12]. As shown in [9], the rate \( \gamma \) is equal to the total scattering rate from the noise potential, as obtained from Fermi's Golden Rule. For distances smaller than \( l_c \), the decoherence rate \( \gamma(s) \) decreases since the two points of the matter wave field 'see' essentially the same noise potential. The exact solution (19) thus implies that after a time of the order of the scattering time \( 1/\gamma \), the spatial coherence of the atomic ensemble has been reduced to the correlation length \( l_c \). The estimates given in [9] imply a time scale of the order of a fraction of a second for waveguides at a micrometre distance from a (bulk) metallic substrate. Significant improvements can be made using thin metallic layers or wires, nonconducting materials or by mounting the waveguide at a larger distance from the substrate [9].

At timescales longer than the scattering time \( 1/\gamma \), the spatial coherence length of the atoms decreases more slowly, approximately as \( l_c/\sqrt{\gamma t} \) [9]. This is due to a diffusive increase of the width of the atomic momentum distribution, with a diffusion constant of the order of \( D = \hbar^2 \gamma/l_c^2 \). This constant is in agreement with a random walk in momentum space: for each scattering time \( 1/\gamma \), the atoms absorb a momentum \( q_c = \hbar/l_c \) from the noise potential. The momentum step \( q_c \) follows from the fact that the noise potential is smooth on scales smaller than \( l_c \), its Fourier transform therefore contains momenta up to \( \hbar/l_c \).

### 3.2 Fokker-Planck equation

The momentum diffusion estimate given above can also be retrieved from the transport equation, making an expansion of the Wigner distribution as a function of momentum. We assume that the typical momentum transfer \( q_c \) absorbed from the noise is small compared to the scale of variation of the Wigner distribution, and expand the latter to second order. This manipulation casts the transport equation into a Fokker-Planck form

\[
\left( \frac{\partial}{\partial t} + \frac{p}{m} \cdot \nabla_x + F_{dr}(p) \cdot \nabla_p \right) W(x, p) =
\]

\[
\sum_{ij} D_{ij}(p) \frac{\partial^2}{\partial p_i \partial p_j} W(x, p),
\]

where \( D_{ij}(p) \) is the diffusion tensor.
where the drift force and the diffusion coefficient are given by

\[ F_{\text{dr}}(p) = -\int d^Dq q S_V(q, E_{p+q} - E_p) \]

(21)

\[ D_{ij}(p) = \int d^Dq q_i q_j S_V(q, E_{p+q} - E_p). \]

(22)

In the special case of white noise, the \( p \)-dependence of these quantities drops out. Also the drift force is then zero because the noise correlation function is real and the spectrum \( S_V(q) \) even in \( q \). Since \( q_c \) gives the width of the spectrum, the diffusion coefficient turns out to be of order \( q_c^2 \gamma \), as estimated before.

Casting the transport equation into Fokker-Planck form, one can easily take into account the scattering from the noise field in (classical) Monte Carlo simulations of the atomic motion: one simply has to add a random force whose correlation is given by the diffusion coefficient.

We note, however, that the Fokker-Planck equation cannot capture the initial stage of the decoherence process, starting from a wave field that is coherent over distances larger than the correlation length \( l_c \). Indeed, it may be shown (neglecting the \( p \cdot \nabla_x \) term and the drift force, assuming an isotropic diffusion tensor for simplicity) that (20) yields a spatially averaged coherence function

\[ \Gamma_{FP}(s, t) = \Gamma_i(s) \exp \left[ -Ds^2t/h^2 \right] \]

(23)

This result implies a decoherence rate proportional to \( s^2 \) without saturation. It is hence valid only at large times (compared to the scattering time \( 1/\gamma \)) where the exponentials in both solutions (19, 23) are essentially zero for \( s \geq l_c \).

4 Concluding remarks

We have given an outline of a transport theory for dilute atomic gases trapped in low-dimensional waveguides. This theory allows to follow the evolution of the atomic phase-space distribution (more precisely, the atomic, noise-averaged Wigner function) when the atoms are subject to a noise potential with fluctuations in space and time. The spatial coherence of the gas can be tracked over temporal and spatial scales larger than the correlation scale of the noise, in a manner similar to the master equations of quantum optics. We have given explicit results in the case of white noise, highlighting spatial decoherence and momentum diffusion.

The transport equation has to be taken with care for strong noise potentials because its derivation is based on second-order perturbation theory. It is certainly not valid when the ‘mean free path’ ~ \( \bar{v}/\gamma \) (\( \bar{v} \) is a typical velocity of the gas) is smaller than the noise correlation length \( l_c \) because then the Wigner distribution changes significantly over a small spatial scale. (In technical terms, the approximation of a local scattering kernel in (13) is no longer appropriate.)
Also, the theory cannot describe Anderson localisation in 1D waveguides with static noise \[11\]. This can be seen by working out the scattering kernel with \( S_V(q, \Delta E) = S_V(q) \delta(\Delta E) \):

\[
2m \int dp' S_V(p' - p) \delta(p'^2 - p^2) [W(x, p') - W(x, p)] = \frac{mS_V(2p)}{p} [W(x, -p) - W(x, p)].
\] (24)

We find a divergence of the scattering rate at \( p \to 0 \) since the spectrum \( S_V(2p) \) is finite in this limit. The one-dimensional, static case therefore merits further investigation. We also mention that it has been found recently that Anderson localisation is destroyed when time-dependent fluctuations are superimposed on the static disorder \[13, 14\]. In this context, transport (or master) equations similar to our approach have been used.

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A Multiple scale derivation of the transport equation

The Schrödinger equation (1) gives the following equation for the Wigner function

\[
(\partial_t + p \cdot \nabla_x) W(x, p) = -i \frac{\hbar}{\hbar} \int d^Dq \hat{V}(q, t) e^{iq \cdot x} \left[ W(x, p + \frac{1}{2}q) - W(x, p - \frac{1}{2}q) \right]
\] (25)

where \( \hat{V}(q, t) \) is the spatial Fourier transform of the noise potential. Since this potential is assumed weak and varies on a scale given by the correlation length \( l_c \), we introduce the following scaling

\[
\hat{V}(q, u, t) = \int d^Dx e^{-iq \cdot u / l_c} V(x, t) = l_c^{\beta} \eta^\beta \hat{V}(u, t)
\] (26)

where \( q_c = \hbar / l_c \) is the typical momentum width of \( \hat{V}(q, t) \) and \( u \) is a dimensionless vector. The parameter \( \eta \) is given by the ratio between the small scale \( l_c \) and the ‘macroscopic’ scale of the position distribution, the (positive) exponent \( \beta \) remains to be determined. We assume \( \eta \ll 1 \) and make the multiple scale expansion (12) for the Wigner function. Using the expansion (11) for the spatial gradient, we get

\[
\left[ \partial_t + \frac{p}{m} \cdot \left( \nabla_x + \frac{1}{l_c} \nabla_\xi \right) \right] (W_0 + \eta^\alpha W_1) =
\] (27)
\[-\frac{i\eta^\beta}{ql_c} \int \frac{d^D u}{(2\pi)^D} \hat{V}(u, t) e^{iu \cdot \frac{x}{l_c}} [W(x, p + q_c u/2) - W(x, p - q_c u/2)]\]

We now take the limit \( \eta \to 0, l_c \to 0 \) at fixed \( q_c \). The most divergent term on the left hand side is the one with \((1/l_c) \nabla_\xi W_0\). It could only be balanced with a term on the right hand side involving \( W_0 \), but due to the small factor \( \eta^\beta \), this term cannot have the same order of magnitude. We must therefore require that \((1/l_c) \nabla_\xi W_0\) vanishes individually: the zeroth order Wigner function is independent of the short scale variable \( \xi \).

The next terms on the left hand side contain \((\eta^\alpha/l_c) \nabla_\xi W_1\) and \( \nabla_X W_0 \), while on the right hand side the leading order is \((\eta^\beta/l_c) W_0\). We look for a connection between \( W_0 \) and \( W_1 \), and therefore, the left hand \( W_1 \) term must be more divergent than the \( W_0 \) term. This is the case if \( \eta^\alpha \mathcal{O}(1/l_c) \gg \mathcal{O}(1/X) \sim \eta \mathcal{O}(1/l_c) \). We thus conclude that \( \alpha < 1 \).

Comparing powers of \( \eta \) on the left and right hand side, we find \( \alpha = \beta \), since the vector \( u \) and the scaled distance \( \xi \) are of order unity. Therefore we get the equation

\[
\left( l_c \partial_t + \frac{p}{m} \cdot \nabla_X \right) W_1(X, \xi, p) = \tag{28}
-\frac{i}{q_c} \int \frac{d^D u}{(2\pi)^D} \hat{V}(u, t) e^{iu \cdot \xi} [W_0(X, p + q_c u/2) - W_0(X, p - q_c u/2)]
\]

In the exponential, only the short length scale \( \xi = x/l_c \) occurs. We thus find that the large scale variable \( X \) is a parameter in this equation, and get a solution via Fourier transforms with respect to \( \xi \) and \( t \). In the spirit of the Markov approximation, we take the slowly varying \( W_0 \) (as a function of time) out of the time integral

\[
\int_{-\infty}^{\infty} dt \, e^{i\omega t} \hat{V}(u, t) W_0(\ldots, t) \approx W_0(\ldots, t) \hat{V}[u, \omega] \tag{29}
\]

where \( \hat{V}[u, \omega] \) denotes the double space and time Fourier transform of the potential. We note \( \kappa, \omega \) the conjugate variables for the spatial Fourier transform and find the following solution for the first order Wigner function

\[
W_1(X, \xi, p) = \tag{30}
-\frac{i}{q_c} \int \frac{d\omega}{2\pi} \int \frac{d^D \kappa}{(2\pi)^D} \frac{e^{i\kappa \cdot \xi - i\omega t} \hat{V}[\kappa, \omega]}{i\kappa \cdot p/m - il_c \omega + 0} (W_0(X, p + q_c \kappa/2) - W_0(X, p - q_c \kappa/2))
\]

The \(+0\) prescription in the denominator is related to causality: it ensures that the poles in the complex \( \omega \)-plane are moved into the lower half plane, avoiding a blow-up of \( W_1 \).

This result will be inserted into the next order equation that also links \( W_0 \) to \( W_1 \):

\[
\left( \partial_t + \frac{p}{m} \cdot \nabla_X \right) W_0 =
\]
\[-\frac{i\eta^{2\alpha}}{q_{\text{cl}}}(2\pi)^D \int \frac{d^Du}{D} \hat{V}(u, t) e^{iu\cdot\xi} [W_1(X, \xi, p + q_cu/2) - W_1(X, \xi, u - q_cu/2)]\]

Note that this equation is scaled consistently if \(O(1/X) \sim \eta^{2\alpha}O(1/l_c) = \eta^{2\alpha-1}O(1/X)\). This determines the exponent \(\alpha = \frac{1}{2}\). The result is an equation for \(W_0\) only. We take the statistical average and make the factorisation

\[
\langle \hat{V}(u, t) \hat{V}[\kappa, \omega] W_0(X, p) \rangle = \langle \hat{V}(u, t) \hat{V}[\kappa, \omega] \rangle W_0(X, p).
\]

This may be justified heuristically as follows: it seems reasonable that the statistical average can also be performed via ‘spatial coarse graining’, i.e., taking an average over the small-scale fluctuations of the medium. This is precisely the picture behind transport theory: the individual scattering events are not resolved but only the behaviour of the matter wave on larger scales. The lowest order Wigner function \(W_0\) may be taken out of the coarse grain average because it does not depend on the short scale \(\xi\) by construction.

Finally, we introduce the spectral density \(\hat{S}(u, \omega)\) of the (scaled) noise potential

\[
\langle \hat{V}(u, t) \hat{V}[\kappa, \omega] \rangle = (2\pi)^D \hat{S}(u, \omega) e^{iut} \delta(u + \kappa)
\]

This allows to perform the integration over \(\kappa\) when (30) is inserted into (31). The result still contains a frequency integral where denominators of the following form appear

\[
\frac{1}{i(u/m) \cdot (p + q_cu/2) - il_c\omega + 0} = \frac{-iq_c}{E_{p+q_cu} - E_p - \hbar\omega - i0}
\]

A second term contains the sign-reversed energy difference. These denominators ensure that the kinetic energy change occurring in the scattering is compensated by a ‘quantum’ \(\hbar\omega\) from the noise potential.

We write the denominators (33) as a \(\delta\)-function plus a principal part. For the classical noise potential considered here, the power spectrum \(\hat{S}(u, \omega)\) is even in \(\omega\), so that the \(\delta\)-functions combine and the principal parts drop out. We finally get

\[
\left(\partial_t + \frac{p}{m} \cdot \nabla_X\right) W_0 =
\]

\[
\eta \frac{\hbar^3}{\hbar^2} \int \frac{d^D u}{(2\pi)^D} \hat{S}(u, \Delta E/\hbar) [W_0(X, p + q_cu) - W_0(X, p)]
\]

where \(\Delta E = E_{p+q_cu} - E_p\). It is easily checked that this is the transport equation (13), taking into account the relation between the scaled and non-scaled noise spectra

\[
\frac{\eta l^3}{\hbar^2} \hat{S}_V(u, \Delta E/\hbar) = S_V(q_cu, \Delta E)
\]

that follows from (14) and (20).
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