Establishment and verification of longitudinal aerodynamic model of tandem wing aircraft

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Abstract. This paper clarified the contribution of front wing and rear wing to the longitudinal aerodynamic derivative by analyzing the characteristics of tandem wing aircraft. By combining the structure and aerodynamic simulation data of the aircraft, the aerodynamic derivative can be directly solved through simplification of force without introducing the tail capacity $V_{th}$, and the longitudinal short-period (control) transfer function of the tandem wing aircraft can be obtained by substituting the aerodynamic model which is composed by aerodynamic derivative into the short period motion equation. This transfer function can be used to build the control system in Simulink. When the input signal is the same, the short-period response of the system has high goodness of fit with the value measured in flight test. Therefore, the longitudinal aerodynamic model of the aircraft is well verified, and it can be used for the design of the aircraft controller and its aerodynamic shape. The modeling method and the verification method are applicable to fixed-wing aircraft including tandem wing aircraft.

1. Introduction

Multidisciplinary cross-analysis and design no longer stay in the overall design stage of the aircraft, but also go deep into the control system design and implementation level. The new requirements, new layout, and new control of future aircraft will make the aerodynamics, structure, propulsion unit and flight control more closely coupled[1]. However, the aerodynamic derivative of the aircraft is the key parameter in the design of the dynamic characteristics of the aircraft as well as the basis for the analysis and optimization of the dynamic quality of the aircraft[2]. As the controlled object of the flight control system, the aerodynamic model of the aircraft composed by aerodynamic derivative is the basic input of the flight controller design.

The aerodynamic derivative is the derivative of the aerodynamic to motion state parameters. The aerodynamic derivative can be obtained by the combination of calculation and simulation (that is, according to the regular that the aerodynamic force of the aircraft varies with the state quantity); it can also be obtained by parameter identification (that is, processing a large amount of flight test data and solving by data fitting). For both methods, the former has the advantages of high speed and low cost. Therefore, it is adopted in this paper to obtain the longitudinal aerodynamic derivatives of tandem wing aircraft.
2. Solutions of longitudinal aerodynamic derivatives

At present, there is a method for calculating the aerodynamic derivative $M$ of a conventional layout aircraft. However, since the aerodynamic center of wing is close to the center of mass for the conventional layout aircraft, the wing has little effect on longitudinal control, which is usually not considered. However, for a tandem wing aircraft, the front wing ($w$) and the rear wing ($t$) are similar in area, then the aerodynamic center of front wing has a similar distance with that of the aerodynamic center of rear wing to the center of mass of the aircraft. Therefore, the effect of both wings must be considered.

2.1. Aerodynamic simulation and measurement

In this paper, the body coordinate system is used. The “front, right and bottom” of the aircraft respectively correspond to the positive direction of “X, Y, Z”, and the origin O coincides with the center of mass of the aircraft, as shown in Figure 1.

When the flight height is 200m and the speed is 25m/s, the surface pressure distribution situation for aircraft obtained by aerodynamic simulation is shown in Figure 2. The simulation results of the lift of the front and rear wings change with the angle of attack are shown in table 1.

![Figure 1. Coordinate system.](image1)

![Figure 2. Distribution of pressure.](image2)

Table 1. Data obtained from aerodynamic simulation.

| Angle of attack $\alpha$ (rad) | 0.000 | 0.035 | 0.070 | 0.105 | 0.140 | 0.175 |
|--------------------------------|-------|-------|-------|-------|-------|-------|
| Lift of front wings $L_w$ (N)  | -26.76| -34.04| -39.88| -44.50| -46.58| -47.41|
| Lift of rear wings $L_t$ (N)   | -6.33 | -8.39 | -10.45| -12.41| -14.02| -14.94|

After the data is fitted, it can be obtained that the slope of the front wing lift to the angle of attack $L_{\alpha w} = 141.81$ and the slope of the rear wing lift to the angle of attack $L_{\alpha t} = 54.93$.

Table 2 shows the measured physical quantities of the aircraft, include mass of aircraft $m$, moment of inertia around Y-axis $I_y$, coordinate of the aerodynamic center of front wing on X-axis $x_{aw}$, coordinate of the aerodynamic center of rear wing on X-axis $x_{at}$, aspect Ratio Of front wing ARw, area percentage of elevator $\tau$, area of front wing $S_w$, area of rear wing $S_t$. The position of the aerodynamic center is at a length of 1/4 chord from the leading edge of the wing, which is referenced by the thin airfoil theory[3].

Table 2. Physical parameters of the aircraft.

| $m$ (kg) | $I_y$ (kg·m²) | $x_{aw}$ (m) | $x_{at}$ (m) | ARw | $\tau$ | $S_w$ (m²) | $S_t$ (m²) |
|----------|---------------|--------------|--------------|------|-------|------------|------------|
| 4.3      | 0.1534        | 0.1055       | -0.4058      | 11   | 34.9% | 0.1960     | 0.0967     |

2.2. The solution of $M_\alpha$
\( M \) is the derivative of the pitch angular acceleration to the angle of attack \( \alpha \) of the aircraft. First, according to the coordinate system shown in figure 1, the lifting and resistance of the front and rear wings are simplified to the center of mass of the aircraft, as shown in Figure 3. The torque of front wing:

\[
M_{aw} = L_w \cdot \cos \alpha_F \cdot l_w + D_w \cdot \sin \alpha_F + L_w \cdot \sin \alpha_F \cdot z_a + D_w \cdot \cos \alpha_F \cdot z_a + M_{aw}
\]  

(1)

Assume that the angle of attack is small: \( \cos \alpha = 1 \), \( \sin \alpha = \alpha \), \( C_\alpha > C_D \), \( z_a = 0 \). It can be obtained by simplification: \( M_{aw} = M_{aw} + L_w l_w \) (\( M_{aw} \) is not relevant to the angle of attack[4]).

\[
M_{aw} = M_{aw} + \frac{1}{2} \rho v_w^2 S_w C_{L_{aw}} l_w = M_{aw} + Q_w S_w C_{L_{aw}} (\alpha_w - \alpha_{wo}) l_w
\]

(2)

In equation (2), \( Q_w = \frac{1}{2} \rho v_w^2 \), \( C_{L_{aw}} \) is the slope of the lift coefficient of the front wing, \( \alpha_{wo} \) is the zero-lift angle of the front wing.

In order to facilitate the derivation, equation (2) can be divided into two parts:

\[
M_{aw} = M_{aw} + Q_w S_w C_{L_{aw}} \alpha_w l_w
\]

(3)

In equation (3), \( M_{aw} = M_{aw} - Q \cdot S \cdot C_{L_{aw}} \alpha_{wo} l_w \), and is not relevant to angle of attack, \( \Delta \alpha_w = \Delta \alpha_F \). Therefore:

\[
M_{aw} = \frac{\partial M_{aw}}{\partial \alpha_F} / l_y = Q_w S_w C_{L_{aw}} l_w / l_y
\]

(4)

In the same way, the torque of rear wing can be obtained:

\[
M_{at} = M_{at} + Q \cdot S \cdot C_{L_{at}} (\alpha_t - \alpha_{t0}) l_t
\]

(5)

As shown in Figure 3, the angle of attack of rear wing \( \alpha_t = \alpha_F + i_t - \varepsilon_w \). \( \alpha_F \) is the angle of attack of the aircraft, \( i_t \) is the setting angle of rear wing, and \( \varepsilon_w \) is the angle of downwash of front wing airflow. Therefore:

\[
M_{at} = M_{at} + Q \cdot S \cdot C_{L_{at}} (\alpha_t - \alpha_{t0}) l_t + Q \cdot S \cdot C_{L_{at}} \alpha_F l_t - Q \cdot S \cdot C_{L_{at}} \varepsilon_w l_t
\]

(6)

There is one special case: for the elliptical ring-shaped wing, the angle of downwash remains unchanged along the span[5], that is \( \varepsilon_w = \frac{\varepsilon}{2v_w} = \frac{2C_{L_{aw}}}{\pi \alpha R_w} \). Though splitting the equation (6):

\[
M_{at} = M_{at} + Q \cdot S \cdot C_{L_{at}} (\alpha_t - \alpha_{t0}) l_t
\]

(7)

In equation (7), \( M_{at} = M_{at} + Q \cdot S \cdot C_{L_{at}} (\alpha_t - \alpha_{t0}) l_t \), which is not relevant to \( \alpha_t \), So:

\[
M_{at} = \frac{\partial M_{at}}{\partial \alpha_F} / l_y = Q \cdot S \cdot C_{L_{at}} l_t / l_y + Q \cdot S \cdot C_{L_{at}} l_t \frac{\partial \alpha_F}{\partial \alpha_t} / l_y
\]

(8)

The aerodynamic derivative of the aircraft:

\[
M_t = M_{aw} + M_{at} = Q_w S_w C_{L_{aw}} l_t / l_y + Q \cdot S \cdot C_{L_{at}} l_t / l_y + Q \cdot S \cdot C_{L_{at}} l_t \frac{\partial \alpha_F}{\partial \alpha_t} / l_y
\]

(9)

2.3. The solution of \( M_q \)

\( M_q \) is the derivative of the pitch angle acceleration of the aircraft to the pitch rate. When the airflow reaches the aerodynamic center of rear wing from the center of mass of the aircraft, the pitching torque variation contributed by the rear wing can be calculated:

\[
\Delta M_t = -\Delta L_t l_t = -Q \cdot S \cdot C_{L_{at}} \Delta \alpha_t l_t
\]

(10)

\( " - " \) indicates that the pitching torque provided by rear wing lift is negative.

By Substituting \( \Delta \alpha_t = \frac{q_{\phi t}}{v_w} \) into the equation (10), it can be obtained:

\[
\Delta M_t = -Q \cdot S \cdot C_{L_{at}} \frac{q_{\phi t}}{v_w} l_t
\]

(11)

\[
M_{q,t} = \frac{\partial \Delta M_t}{\partial q} / l_y = -Q \cdot S \cdot C_{L_{at}} \frac{1}{v_w} l_t / l_y
\]

(12)

In the same way, for front wing:

\[
\Delta M_w = \Delta L_w l_w = Q_w S_w C_{L_{aw}} \Delta \alpha_w l_w
\]

(13)
By Substituting $\Delta \alpha_w = -\frac{q_{lw}}{v_{\infty}}$ into the equation (13), it can be obtained: ("-" indicates that the angle of attack of the front wing changes before the airflow reaches the center of mass of the aircraft).

$$
\Delta M_w = -Q_w S_w C_{L_{aw}} \frac{q_{lw}}{v_{\infty}} l_w
$$

(14)

$$
M_{qw} = \frac{\partial \Delta M_w}{\partial q} = -Q_w S_w C_{L_{aw}} \frac{l_w}{v_{\infty}} l_w / I_y
$$

(15)

$$
M_t = M_{qw} + M_{qt} = -\left(Q_w S_w C_{L_{aw}} \frac{l_w}{v_{\infty}} l_w + Q_t S_t C_{L_{at}} \frac{l_t}{l_t} l_t \right) / I_y
$$

(16)

2.4. The solution of $M_\alpha$

$M_\alpha$ is the derivative of the pitch angle acceleration to the angle of attack of the aircraft. When the angle of attack of front wing changes, the angle of downwash $\varepsilon_w$ changes accordingly. According to Figure 3, the changes of $\varepsilon_w$ will change the angle of attack of rear wing, and then the lift of rear wing, thus the pitching torque will also change accordingly. The equation (17) and equation (18) can be derived from the Section 3.6.3 of reference[4]:

$$
\Delta M_t = -\Delta L_t l_t = -Q_t S_t C_{L_{at}} \Delta \alpha_t l_t
$$

(17)

$$
\Delta \alpha_t = \Delta \frac{d\alpha_t}{dt} = \Delta t \frac{d\varepsilon_w}{d\alpha} \Delta \alpha
$$

(18)

$\Delta t$ is the time required for the downwash airflow of front wing reaching the rear wing. $\Delta \alpha_t$ is the variation of downwash angle of front wing during $\Delta t$. As for tandem wings aircraft, $\Delta t = \frac{l_w}{v_{\infty}}$, so:

$$
M_\alpha = \frac{d\Delta M_t}{d\alpha} / I_y = -Q_t S_t C_{L_{at}} \frac{l_w + l_t}{v_{\infty}} \frac{d\varepsilon_w}{d\alpha} l_t / I_y
$$

(19)

2.5. Solutions of other aerodynamic derivatives

Derivative of the total lift of the aircraft $Z$ to the angle of attack $\alpha$:

$$
Z_\alpha = Z_{\alpha w} + Z_{\alpha t} = -\left(C_{L_{aw}} + C_{D_{ow}}\right) Q_w S_w / m - \left(C_{L_{at}} + C_{D_{ot}}\right) Q_t S_t / m
$$

(20)

Derivative of the pitch angle acceleration of the aircraft to the deflection angle of the elevator:

$$
M_{\delta_e} = -C_{L_{at}} \tau Q_t S_t l_t / I_y
$$

(21)

Derivative of The total lift of aircraft to the deflection angle of the elevator[6]:

$$
Z_{\delta_e} = Q_t S_t C_{L_{at}} \tau / m
$$

(22)

3. Longitudinal approximation algorithm

In this paper, due to the large dihedral angle of the rear wing, the interference airflow from front wing has little influence on the rear wing. After analyzing the streamline chart of the aerodynamic simulation, it is considered that the downwash angle of the airflow $\varepsilon_w$ of the front wing has little influence on the rear wing, which can be neglected. The dynamic pressure of rear wing is $Q_\alpha = Q_w = \frac{1}{2} \rho v_{\alpha}^2$, by substituting the simulation and measurement results into equations in section 2, it can be obtained:

| $M_\alpha$ | $M_q$ | $M_{\delta_e}$ | $Z_\alpha$ | $Z_{\delta_e}$ |
|-----------|-------|-------------|----------|----------|
| -382.50   | -48.68| -0.06       | -46.35   | -83.42   |
|           |       |             | 537.91   |          |

Ignoring the change of flight speed $u_0$ and the thrust $T$, the short-period longitudinal motion equation of the aircraft derived from the Small-Perturbation Theory is simplified as in equation (23):

$$
\begin{bmatrix}
\Delta \dot{\alpha} \\
\Delta \dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
\frac{Z_\alpha}{u_0} & 1 & \frac{Z_{\delta_e}}{u_0} \\
M_\alpha + M_\alpha Z_\alpha / u_0 & M_q + M_q \Delta \theta / \Delta \theta & M_{\delta_e} + M_{\delta_e} Z_{\delta_e} / u_0
\end{bmatrix}
\frac{[\Delta \theta]}{[\Delta \alpha]}
$$

(23)

Derived from equation (23), the transfer function of the change in pitch rate to the change in the angle of elevator is shown in equation (24):

$$
\frac{\Delta \theta(s)}{\Delta \alpha(s)} = \frac{(M_{\delta_e} + M_{\delta_e} Z_{\delta_e} / u_0) s + M_{\delta_e} Z_{\delta_e} / u_0 - M_{\delta_e} Z_{\delta_e} / u_0}{s^2 - (M_q + M_q \Delta \theta / \Delta \theta) s + Z_{\delta_e} M_q / u_0 - M_\alpha}
$$

(24)
By substituting the calculation result of the aerodynamic derivative into equation (24), the transfer function of the pitch rate to the angle of elevator can be obtained as equation (25):

$$\frac{\Delta q(s)}{\Delta \delta_e(s)} = \frac{-84.09s - 3958.51}{s^2 + 49.65s + 426.74}$$

(25)

4. Test and Verification

4.1. Verification method

Build the open-loop control system in Simulink according to the transfer function (the pitch rate to the angle of elevator) obtained in part 2. Set the control data (PitchCon) of the elevator recorded in the flight test as the input signal of the control system. Compare the curve of pitch rate (PitchRateSim) obtained by the simulation system with the curve of the PichtRate measured by the sensors in the flight test.

4.2. Verification process

The flight test was carried out by the aircraft (shown in Figure 1), and the record data conducted by the test aircraft was taken for verification. The total flight time is 77s, of which 70s is in manual mode, and 7s in stabilization mode.

Figure 4 shows the longitudinal state quantity generated by the Pixhawk autopilot. The rudder servo model is KINGMA CLS0512W, the pitch control data (PitchCon) is dimensionless, and the relationship between PitchCon value and the elevator deflection angle $\delta_e \text{ (rad)}$ can be shown as: $\delta_e = 0.5 \times \text{PitchCon(rad)}$.

In order to improve the verification accuracy of the longitudinal aerodynamic model, this paper excludes the take-off, stabilization flight and crash periods of the aircraft, and selects the period of zero roll control to avoid yaw and rolling state quantities from interfering with longitudinal analysis. It should be noted that: the model reads directly the control data (PitchCon) of the controller on the elevator (Pixhawk autopilot output ATTC.Pitch, PitchCon in the text), therefore, the controller does not affect the model verification regardless of whether the "manual" mode or the "stabilization" mode is selected. In addition, the data recorded during the stabilization is excluded since the state quantity of the longitudinal channel is too small and it is not conducive to model verification.
Figure 6. Pitch rate response curve of the model.

In view of the above considerations, the PitchRate data from 180s to 198.6s of the flight test is selected. The pitch rate response (PitchRateSim) data obtained by simulation during 180s to 198.6s is also selected. Figure 7 is a comparison of PitchRate and PitchRateSim, and the green chain line is the pitch control data (PitchCon).

Figure 7. Comparison of pitch angle rate obtained by simulation and measure.

4.3. Conclusion
It can be seen from Figure 7 that, at the same input, the pitch rate of the model is in good agreement with the pitch rate measured by the sensor in the flight test. Figure 8 shows that the differences fluctuate within a small range. The mean difference is 0.015 rad/s and the mean variance is 0.104. Regression analysis was performed in matlab, the regression coefficient of the model output to the measured data reached 0.93, and the R-squared of the model output with the measured data reached 0.860. The differences are caused by the following reasons: wind interference, sensor data is not filtered, and the inertia of the servo is ignored.
Figure 8. Difference between pitch angle rate obtained by simulation and measured.

Based on the physical parameters and aerodynamic simulation data of the three-dimensional model of the aircraft, the longitudinal aerodynamic derivative suitable for the tandem wing aircraft is obtained to form the longitudinal aerodynamic model of the aircraft. Under the same input signals, the aerodynamic model response and the flight test record data are in good agreement, which proves the accuracy of the aerodynamic model. The aerodynamic model established by this method can help the design of the aircraft controller in the middle and small angle of the attack flight envelope. The modeling method and the verification method are applicable to fixed-wing aircraft including tandem wing aircraft.

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