Numerical investigation of the liquid motion influence on the collapse of a cavitation bubble near a wall

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Abstract. Axially symmetric dynamics of a single cavitation bubble near a plane wall in the process of its expansion and collapse with transformation of the bubble into a toroidal one is considered. The influence of the liquid motion at the end of expansion on the following collapse is studied, depending on the initial distance between the bubble and the wall. The liquid is assumed ideal incompressible, its motion potential. The numerical technique is based on the stepping method of describing the bubble contour movement and the boundary element method. It is shown that with increasing the initial distance between the bubble and the wall, the differences in the shape of the bubble and the liquid pressure at the bubble collapse with and without allowing for the liquid motion at the beginning of collapse significantly decrease.

1. Introduction

Studying dynamics of bubbles near a wall is of importance for understanding features of cavitation damage. This topic is considered in many theoretical and experimental works devoted to investigating the dependence of the bubble dynamics on small spheroidal disturbances in the initial bubble shape [1], the gravity [2], the surface tension [3], etc. A numerical technique, proposed in [4] for the case of ideal incompressible liquid and based on the boundary element method, is now widely used. The technique was then modified for calculating the motion of a toroidal bubble resulting from the impact of a cumulative jet on a wall or the liquid layer between the bubble and the wall. A significant number of publications consider the dynamics of an initially small bubble, somewhat distant from the wall, with the initial internal pressure being appreciably higher than the pressure in the surrounding liquid [6-9]. The main attention is payed to the influence of the distance between the bubble and the wall on the shape of the bubble, the liquid pressure and velocity fields, and the pressure on the wall. In the present work, the abovementioned technique is used for studying the dependence of the bubble dynamics on the liquid motion realizing in the process of expansion and collapse of a bubble near a wall at the moment the bubble attains its maximum volume, depending on the initial distance between the bubble and the wall.

2. Problem statement and numerical technique

At the initial moment of time a small spherical bubble with the radius $R_0$ and the center located at the distance $d$ from a plane wall first expands under the action of the internal pressure $p_{b0}$ significantly exceeding the pressure of the liquid (water, the density $\rho=1000 \text{ kg/m}^3$) at large distance from the
bubble, and then collapses. In experiments and in practice, the shape of a bubble at the beginning of its collapse (i.e., at the end of its expansion) seems relatively easy registered, whereas more significant efforts are necessary to determine the liquid pressure and velocity fields in the neighborhood of the bubble. Moreover, during transition from the expansion to collapse the liquid velocity is relatively small. Taking those into account, the possibility of neglecting the liquid motion at the end of the bubble expansion to analyze the following collapse is studied in the present work. For this purpose, two cases of the bubble collapse are compared. In the first case, the bubble collapse is a natural continuation of the preceding expansion. In the second case, the bubble collapse is realized at the same bubble shape and pressure fields at its beginning but at the zero liquid velocity (at the motionless liquid).

A mathematical model is used, assuming that the liquid is incompressible, its motion is potential, the surface tension is small. The pressure in the bubble is taken uniform, changing according to the adiabatic law

\[ p_b = p_{b0} (V_0 / V)^\kappa, \]

where \( V_0, V \) are respectively the initial and current bubble volumes, \( \kappa \) is the ratio of the specific heats (\( \kappa = 1.33 \)).

A two-stage technique is applied, based on the stepping method of describing the bubble contour movement and the boundary element method [6, 9]. In the first stage, the motion of a simply connected bubble till the moment the cumulative jet arising on the bubble surface hits the liquid layer between the bubble and the wall, is calculated. In the second stage, the movement of a toroidal bubble resulted from the contact between the jet and the liquid layer, is determined. For providing the numerical stability, the both stages use a smoothing procedure for the functions describing the bubble contour and its liquid velocity potential by applying cubic spline [10]. The technique also includes an algorithm of computing the liquid pressure and velocity fields in the vicinity of the bubble. The liquid pressure is determined by the Bernoulli integral. In doing so, the boundary element method is also used for calculating the partial derivative of the liquid velocity potential with respect to time to meet the Laplace equation [11]. The liquid velocity components are calculated using analytic expressions obtained by differentiation of the general solution of the Laplace equation for the velocity potential presented as sum of potentials of the plain and double layers [12].

### 3. Results

The results are presented utilizing the following dimensionless quantities:

\[ p^* = p / p_{o0}, \quad v^* = v / (p_{o0} \rho)^{1/2}, \quad r^* = r / R_{\text{max}}, \quad z^* = z / R_{\text{max}}, \quad t^* = t / [R_{\text{max}} (p / p_{o0})^{1/2}], \quad \gamma = d / R_{\text{max}}, \]

where \( p \) is the pressure, \( v \) is the velocity modulus, \( r, z \) are the radial and axial coordinates of the cylindrical reference system with the origin on the wall surface and the \( z \)-axis passing through the bubble center orthogonally to the wall, \( t \) is the time, \( \rho \) is the density of the liquid, \( h \) is the thickness of the liquid layer between the bubble and the wall, \( R_{\text{max}} \) is the maximum radius attained by the bubble during its expansion in the unlimited liquid volume.

The results are analyzed for the following three values of the parameter \( \gamma \) characterizing the remoteness of the bubble from the wall:

\[ \gamma = 0.8, 1.2 \text{ и } 1.6 \]

at \( p_{o0}^* = 1, \ p_{b0}^* = 100, \ R_0^* = 0.16. \)

Figure 1 presents the shapes of the bubble, the liquid velocity and pressure fields, and the profiles of the velocity modulus along the contour of the bubble at the moment of its maximum expansion for the considered values of \( \gamma \). One can see that in all the cases, the pressure and velocity of the liquid motion in the neighborhood of the bubble are relatively small. As the distance from the bubble grows, the liquid pressure increases, whereas the velocity decreases. At \( \gamma = 0.8 \) the bubble is most pushed to the wall, its shape larger differs from the spherical one (in particular, the portion of its surface in the vicinity of the wall is nearly plane), the liquid velocity on the bubble surface is maximum, and its
distribution along the bubble surface is most non-uniform. With the growth of $\gamma$, the influence of the wall decreases, resulting in that the bubble at the moment of its maximum expansion becomes ever closer to the spherical one. At that, the liquid velocity on its surface as well as the non-uniformity of its distribution decrease.

**Figure 1.** The liquid velocity and pressure fields in the vicinity of the bubble for $\gamma=0.8$ (a), 1.2 (b), 1.6 (c) and the velocity modulus profiles along the contour of the bubble (d) at the moment of its maximum expansion ($s$ is the arc coordinate counted off from the upper point of the bubble contour, $s_{\text{cont}}$ is the arc coordinate of the lower point of the contour).

**Figure 2.** The bubble cavities (shaded) and the pressure contours in liquid at moment $t^*$ for $\gamma=0.8$, 1.2, 1.6 at the bubble collapse with (a) and without (b) allowing for the liquid motion at the beginning of the collapse.
During the following bubble collapse (with and without allowing for the liquid motion at the beginning of the collapse) a wall-directed cumulative jet is formed on the bubble surface part remote from the wall. At some moment of time $t^*_c$ it hits the bubble surface part close to the wall, turning the bubble into a toroidal one. Figure 2 characterizes the bubble shape and the liquid pressure field realizing at moment $t^*_c$ of the bubble collapse with and without the account of the liquid motion at the beginning of the collapse for the considered values of $\gamma$. One can see that for equal $\gamma$ the bubble shapes in the both cases are similar, whereas their volumes differ significantly. With neglecting the liquid motion at the beginning of the collapse, the bubbles are compressed stronger and the maximum liquid pressure realizing at the base of the jet appears appreciably higher.

Table 1. Characteristics of the bubble dynamics at moment $t^*_c$ in the two cases of the bubble collapse for the three values of $\gamma$.

| Collapse | $\gamma$ | $h^*$ | $v^*_c$ | $p^*_b$ | $p^*_00$ |
|----------|---------|------|--------|--------|--------|
| case 1   | 0.8     | 0.0176 | 7.50   | 0.86   | 0.86   |
|          | 1.2     | 0.1769 | 8.37   | 4.29   | 4.37   |
|          | 1.6     | 0.8112 | 9.54   | 16.57  | 9.59   |
| case 2   | 0.8     | 0.0431 | 10.66  | 5.08   | 5.08   |
|          | 1.2     | 0.4207 | 12.22  | 23.15  | 16.38  |
|          | 1.6     | 1.0184 | 13.99  | 56.79  | 20.20  |

Table 1 demonstrates the influence of parameter $\gamma$ on a number of characteristics of the bubble dynamics at moment $t^*_c$ of the bubble collapse with (case 1) and without (case 2) allowing for the liquid motion at the beginning of the collapse: the thickness of the liquid layer between the bubble and the wall $h^*$, the jet end velocity $v^*_c$, the internal bubble pressure $p^*_b$, and the pressure on the wall $p^*_{00}$ at the point of the symmetry axis $z$. One can see that neglecting the liquid motion at the beginning of the collapse leads to significantly growing the values of all the presented quantities. As the distance between the bubble and the wall increases (i.e., with the growth of $\gamma$), the values of the characteristics rise in the both cases, whereas the ratio of the corresponding values in the second case to those in the first case, except for the ratio for $v^*_c$, decrease. This is apparently explained by the fact that with increasing $\gamma$, the liquid velocity at the beginning of the bubble collapse decreases. (figure 1d).

Figure 3. Contours of the toroidal bubbles at the moment of attaining the maximum pressure on the wall at the bubble collapse with (solid lines) and without (dotted lines) allowing for the liquid motion at the beginning of the collapse for the three values of $\gamma$.

Figure 3 shows the influence of $\gamma$ on the difference in the toroidal bubble shape at the moment of attaining the maximum pressure on the wall during the bubble collapse with and without the account of the liquid motion at the beginning of the collapse. One can see that for corresponding values of $\gamma$,
some similarity of the bubble shape is observed. The bubble collapse also appears significantly deeper in the second case than in the first one. At that, the volume ratio is equal to 3.9 for γ=0.8 and 2.0 for γ=1.6.

Table 2. Moment of realization of the maximum pressure on the wall $t^*_T = t^* - t^*_c$ in the two cases of the bubble collapse for the three values of γ.

| Collapse | γ=0.8 | γ=1.2 | γ=1.6 |
|----------|-------|-------|-------|
| case 1   | 0.0154| 0.0103| 0.0087|
| case 2   | 0.0071| 0.0063| 0.0048|

Table 2 presents the dependence on γ of the time of attaining the maximum pressure on the wall $t^*_T = t^* - t^*_c$, counted off from the beginning of the toroidal phase of the bubble motion, in the two considered cases of the bubble collapse. One can see that in the first case, this time is larger than in the second case and in the both cases it decreases with rising γ.

Figure 4. The radial profiles of the pressure on the wall at the moments of attaining its maximum at the bubble collapse with (a) and without (b) allowing for the liquid motion at the beginning of the collapse for γ=0.8 (solid lines), 1.2 (dashed lines) and 1.6 (dotted lines).

Figure 4 shows the influence of γ on the radial profiles of the pressure on the wall at the moment of attaining its maximum value in the cases of the bubble collapse with and without allowing for the liquid motion at the beginning of the collapse. One can see that for all γ in the both cases the pressure maximum is realized on the axis of symmetry. In the both cases, the maximum appreciably lowers: from 50.0 (γ=0.8) to 30.6 (γ=1.6) in the first case and from 123.1 (γ=0.8) to 35.6 (γ=1.6) in the second one. As above, the difference between the maxima in the first and second cases decreases here with the growth of γ.

4. Conclusions

Numerical investigation of the influence of the liquid motion at the moment of maximum expansion of a bubble near a wall on its following collapse has been performed, depending on the initial distance between the bubble and the wall. The results show that the differences in the bubble dynamics with and without allowing for the liquid motion at the beginning of the bubble collapse appreciably decrease as the initial distance between the bubble and the wall grows. This is apparently explained by the fact that with rising the initial distance between the bubble and the wall, the velocity of the liquid motion in the neighborhood of the bubble at the moment of the maximum bubble expansion decreases.

Acknowledgments

This work was financially supported by the Russian Science Foundation (grant No. 17-11-01135).
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