A recent derivation of the interpolation between the free energy and conformal anomaly for free fields on spheres is generalised to hemispheres with Neumann (N) and Dirichlet (D) conditions at the rim for GJMS scalar fields. It is shown that the N minus D interpolation is minus a quarter of that for a higher derivative fermion on the spherical rim. In particular, since, for ordinary bosons $k = 1$, the related fermion is irregular propagating according to a second order (pseudo) operator. ($2k$ is the derivative order in the equation of motion.)

It is suggested that the relation has a role to play in the Type–B AdS/CFT mismatch.

The DN boundary value problem is enlarged upon in the context of Branson and Gover’s construction of the D to N operator. Contact is made with a result of Park and Wojciechowski which is then related to a duality relation of Barvinsky and Nesterov.
1. Introduction

In [1], a particular analytic continuation of the spectral ζ–function on spheres was used to obtain the (known) dimensional interpolation between the free energy (odd dimensions) and the conformal anomaly (even dimensions). In the present brief note I give a technical extension by replacing the sphere with a hemisphere, and imposing boundary conditions at the rim. I make a speculative remark on the Type–B mismatch.

The analysis is then extended to a discussion of the boundary value problem and the related Dirichlet to Neumann operator.

In a number of earlier calculations, it was found expedient to consider the spectra on the full sphere as the union of those on the hemisphere, with appropriate boundary conditions e.g. Neumann\(^2\) (N) and Dirichlet (D) for scalar fields. The particular works, [2] and [3], contain the spectral information and some basic calculations that I will draw upon to avoid too much repetition. The essential facts are contained in [4] and I can be brief.

2. The hemisphere ζ–functions and an interpolation

The relevant ζ–function is shown in [3] to be,

$$\zeta(s, a, \alpha) = \sum_{m=0}^{\infty} \frac{1}{((a + m.1_d)^2 - \alpha^2)^s},$$

where \(m\) and \(1_d = (1, 1, \ldots, 1)\) are integer \(d\)–vectors. If \(\alpha = 1/2\), the quantities in brackets are the eigenvalues of the conformally invariant Penrose–Yamabe Laplacian. The \(\square^k\) GJMS ζ–function involves a finite product on \(\alpha\) of the summand.

The parameter \(a\) equals \((d - 1)/2 \equiv a_N\) for Neumann, and \((d + 1)/2 \equiv a_D\) for Dirichlet conditions.

It is convenient to define the combinations,

$$\zeta_{(rot)}(s, d, \alpha) \equiv \zeta(s, a_N, \alpha) + \zeta(s, a_D, \alpha)$$
$$\zeta_{(diff)}(s, d, \alpha) \equiv \zeta(s, a_N, \alpha) - \zeta(s, a_D, \alpha).$$

\(\zeta_{(rot)}(s)\) is the ζ–function for the full (rotationally invariant) sphere which has already been treated in [1] and will not be considered further here. The difference,

\(^2\) These are sufficient as the boundary has zero extrinsic curvature.
\( \zeta_{\text{diff}}(s) \), can be regarded as a boundary effect (section 3). Some specific values were early given in [5,6] but this is not my primary aim here which is to obtain an interpolation generalisation of the discussion of the boundary \( F \)-theorem, [4].

Following Candelas and Weinberg, [7], and Minakshisudaram, [8], I employ the Bessel function form,

\[
\zeta(s, a, \alpha) = \frac{\pi}{\Gamma(s)} \int_0^\infty \frac{d\tau}{(1 - \exp(-\tau))^d} \left( \frac{\tau}{2\alpha} \right)^{s-1/2} \mathcal{I}_{s-1/2}(\alpha \tau),
\]

for both terms in (2). \( \mathcal{I} \) is the second kind modified Bessel function.

Putting in the values for \( a_N \) and \( a_D \),

\[
\zeta_{\text{diff}}(s, d, \alpha) = \frac{2^{2-d} \sqrt{\pi}}{\Gamma(s)} \int_0^\infty \frac{dz}{\sinh^{d-1} z} \left( \frac{z}{\alpha} \right)^{s-1/2} \mathcal{I}_{s-1/2}(2\alpha z) \equiv \frac{\mathcal{I}(s, \alpha)}{\Gamma(s)}. \tag{4}
\]

In order to continue in \( s \), \( z \) is extended into the complex plane and the parity properties of the integrand,

\[ f(e^{\pm\pi i} z) = e^{\pm\pi i \sigma} f(z), \quad \sigma = 2s - d, \]

give the continuations with two, ultimately equivalent contours, \( \mathcal{C}_\pm \),

\[
\mathcal{I}_\pm(s, \alpha) = \frac{2^{2-d} \sqrt{\pi}}{1 + e^{\pm\pi i \sigma}} \int_{\mathcal{C}_\pm} dz \frac{1}{\sinh^{d-1} z} \left( \frac{z}{\alpha} \right)^{s-1/2} \mathcal{I}_{s-1/2}(2\alpha z), \tag{5}
\]

where \( z = x + iy \), and the contours, \( \mathcal{C}_\pm \), run from \(-\infty \pm iy_0\) to \( \infty \pm iy_0 \) with \( 0 < y_0 < \pi \).

As described in the cited references, the free energy difference is given by

\[
F_{\text{diff}}(s, \alpha) = -\mathcal{I}_\pm(0, \alpha)/2 = -\frac{2^{1-d}}{1 + e^{\pm\pi i \sigma}} \int_{\mathcal{C}_\pm} dz \frac{\cosh 2\alpha z}{z \sinh^{d-1} z}.
\]

At this point, I extend the theory immediately to GJMS higher derivatives by summing over \( \alpha = \alpha_j = (j + 1/2), \ j = 0 \to k - 1 \) to produce,

\[
F_{\text{diff}}(k) = -\frac{2^{1-d}}{1 + e^{\pm\pi i \sigma}} \int_{\mathcal{C}_\pm} dz \frac{\sinh 2kz}{z \sinh^d z}. \tag{6}
\]
In order to find an interpolation, it is possible to follow the route in [1] but much easier to note that (6) is (minus) a quarter of the free energy for \( k \)-fermions on the \((d - 1)\)-sphere (the hemisphere boundary), and so the interpolating expression of the generalised hemisphere GJMS free energy difference can be taken at once from [1], where specific forms for \( \tilde{F}_f \) are given, the relation being,

\[
\tilde{F}_{(diff)}(d, k) = -\frac{1}{4} \tilde{F}_f(d - 1, k),
\]

which interpolates between the free energy (difference), \((-1)^{(d/2)}F_{(diff)}\), for even \( d \) and the conformal anomaly\(^3\) (difference), \((-1)^{(d+1)/2}\zeta_{(diff)}(0)/2\pi\), for odd.

Equation (7) relates GJMS scalar fields, on the left, and higher derivative spinors on the right in a holographic way and is the calculational conclusion of this note. A possible application is outlined in section 4.

Various explicit expressions for \( F_{(rot)}, F_{(diff)}, \text{etc.} \) can be found in [1,2,9].

A simple, more direct eigenvalue justification of (7) is given in the next section.

3. Eigenvalue expression for \( \zeta_{(diff)} \)

By trivial termwise cancellation,

\[
\zeta_{(diff)}(s) \equiv \zeta(s, a_N, \alpha) - \zeta(s, a_D, \alpha)
= \sum_{m=0}^{\infty} \frac{1}{(((d - 1)/2 + m.1_d)^2 - \alpha^2)^s}
- \sum_{m_1=1, m=0}^{\infty} \frac{1}{(((d - 1)/2 + m_1 + m.1_{d-1})^2 - \alpha^2)^s}
= \sum_{m=0}^{\infty} \frac{1}{(((d - 1)/2 + m.1_{d-1})^2 - \alpha^2)^s}
\]

which equals the \( \zeta \)-function for the spin–half operator \((\nabla/|\nabla|)(|\nabla|^2 - \alpha^2)\), per component, on the \((d - 1)\)-sphere. The conformal \( d = 2 \) case was noted in [5].

This elementary eigenvalue argument shows that, at least for spheres, the appearance of a spin–half quantity is somewhat of a kinematic coincidence.

\(^3\) For higher derivative fields, the actual scaling anomaly is \( k\zeta(0) \).
4. Boundary considerations

A boundary free energy associated with the $d$-hemisphere can be defined by

$$F_{\partial} = F_{(N,D)\text{-hemisphere}} - \frac{1}{2} F_{\text{sphere}}$$

$$= \pm \frac{1}{2} F_{(\text{diff})},$$

(cf e.g. Gaiotto, [10]).

The results above, show that this hemisphere boundary free energy has a specific boundary spectral meaning as one quarter of the free energy of higher derivative spinors on the rim.

5. Numerology and a suggestion on the Type–B mismatch

A comment of a speculative nature now follows.

The calculations of Günyadin, Skvortsov and Tran, [11] and Giombi, Klebanov and Tan, [12], give values for the $(d + 1)$–AdS free energy, summed over spins with various spectra (‘A’, ‘B’ and ‘C’). In odd dimension $d$, for non–minimal Type-B, the results fall into the pattern, $\tilde{F}_f(d,1)/4$, yielding the mismatch. (See also [13]). From (7) this is seen to equal $-(-1)^{(d+1)/2} F_{(\text{diff})}(d + 1, 1)$, on returning to the ordinary free energy difference.

I now note that the regularised volume of an even dimensional hyperboloid is the same as that of a hemisphere, up to the sign factor $(-1)^{d/2}$. This suggests that the hemisphere quantity, $-(-1)^{d/2} F_{(\text{diff})}(d + 1, 1)$, can be converted into an AdS integral giving the difference in free energy for two ordinary (rescaled) scalar fields having different AdS boundary conditions. If so, then the bulk spectrum might be accordingly modified to produce a vanishing total and eliminate the mismatch. Whether this spectrum is associated with any CFT would have to be investigated.

If this mechanism proves workable (but see note below), the irregular fermion seems to play only an intermediate, bookkeeping role.

I finally note that the ratio of volumes of the hyperboloid and hemisphere differ also by a factor of $(-1)^{(d+1)/2}/2\pi$ in odd dimensions which is in accordance with the relation of the conformal anomaly to the generalised free energy, $\tilde{F}$.

*Added note:*

It seems that the calculation here is nothing more than a compact (i.e. spherical) version of the AdS double trace computation (e.g. [14]). Hence the relation (7) is entirely equivalent to the results in [1] and a resolution of the mismatch is no closer.
6. Boundary value problem

Irrespective of any direct AdS/CFT relevance, it is instructive to put the analysis into a wider boundary value problem context such as that provided by Branson and Gover, [15], who discuss a conformally invariant, higher derivative operator, $\Box_{2k}$,\(^4\) of some generality, where $k$ is an integer. The idea is to generalise to $\Box_{2k}$ the usual Dirichlet and Neumann boundary value problems, in particular the D to N map.

The upshot is that a set of $k$ conformally invariant differential boundary operators, $\delta'_m$, ($m = \{m_j\}, m_j \in \mathbb{Z}$), can be defined such that the associated boundary value problems are both elliptic and self-adjoint.

Two such sets, suggested by the special case of powers of the Laplacian (the highest order term in $\Box_{2k}$) are the *iterated Dirichlet set*,

$$m = m_D = (0, 2, \ldots, 2k - 2)$$

and the *iterated Neumann set*

$$m = m_N = (1, 3, \ldots, 2k - 1).$$

The derivative order of the $\delta'_m$ is $m_j$. Their construction is quite involved in detail but the essential point is that the leading term is

$$\delta'_m \sim (\partial_n)^m,$$

in terms of the (inward) normal derivative.

Green’s (second) identity then reads

$$\int_{\mathcal{M}} (u \Box_{2k} v - v \Box_{2k} u) = \int_{\partial \mathcal{M}} \sum_{i=0}^{k-1} \left[ (\delta'_i u) \delta'_{2k-1-i} v - (\delta'_i v) \delta'_{2k-1-i} u \right] \quad (9)$$

(A boundary divergence has been thrown away.)

Introducing now the $B$ Green function in $\mathcal{M}$, i.e. the operator inverse with boundary conditions on $\partial \mathcal{M}$ satisfying\(^5\)

$$\Box_{2k} G^B_{2k} = 1_{\mathcal{M}}$$

\(^4\) I have doubled their index $k$ in order to accord with the notation in [\[\].

\(^5\) I assume no zero modes. Also note that $\Box$ is a positive operator.
with
\[ B : \delta'_i G_{2k}^B = 0. \quad i \in m_B. \]

In the usual way, setting \( v = G_{2k}^B \) in (9), gives,
\[ u = \int_{\partial M} \sum_{i \in m_B} sg(i) \left( \delta'_i u \right) \delta'_{2k-1-i} G_{2k}^B \]
(10)
where
\[ sg(i) = 1, \quad 0 \leq i \leq k - 1 \]
\[ = -1, \quad k \leq i \leq 2k - 1. \]

(10) ‘solves’ the boundary value problem \( B \) when the data \( \delta'_{m_B} u \) are specified, \( G_{2k}^B \) is a ‘bulk–to–boundary’ Green function.

For definiteness, I write out the \( k = 2 \) (Paneitz) case for N and D separately,
\[ u = \int_{\partial M} \left( u \partial^3_n + \partial_n u \partial^2_n \right) G_{4,N}^D - \int_{\partial M} \left( \partial^2_n u \partial_n + \partial^3_n u \right) G_{4,N}^D \]
\[ u_N = \int_{\partial M} \left( \partial_n u \partial^2_n G_{4}^N - \partial^3_n u G_{4}^N \right) \]
\[ u_D = \int_{\partial M} \left( u \partial^2_n G_{4}^D - \partial^3_n u \partial_n G_{4}^D \right) \]
(11)
u\(_N\) solves the Neumann problem with boundary data \( [\partial_n u, \partial^3_n u] \) and \( u_D \) likewise for the Dirichlet data, \( [u, \partial^2_n u] \).

The model example of the hemisphere is treated in [15]. Then \( \delta'_m \) can be replaced by \( (\partial_n)^m \) where \( m \in m \). The N and D boundary conditions can be accommodated in the usual ways, either spectrally through the mode parity at the boundary, or via the image (better, pre–image) construction of the Green functions. Either way shows that odd order normal derivatives vanish for N and even ones for D.

Branson and Gover’s quest is for a (non–local) operator on the boundary that converts D data to N data, (and the inverse). Again, the general construction is involved, but on the hemisphere explicit expressions are found.

Branson and Gover construct D to N operators, \( P' \), which relate boundary data. On the hemisphere
\[ P'_{k,m,j,2k-1-m_j} \sim A_{2k-1-2m_j} = B \prod_{h=1}^{k-1-m_j} (B^2 - h^2), \quad m_j < k - 1/2 \]
\[ = \left[ B \prod_{h=1}^{-k+1+m_j} (B^2 - h^2) \right]^{-1}, \quad m_j > k - 1/2 \]
(12)
derived group theoretically as a representation intertwinor.

For the Paneitz case, \( k = 2 \) and \( m_0 = 0, m_2 = 2 \) for D and \( m_1 = 1, m_3 = 3 \) for N.

Spelling out the operators

\[
P_{2,0,3} \sim A_3 = B(B^2 - 1), \quad P_{2,1,2} \sim A_1 = B \]
\[
P_{2,2,1} \sim A_{-1} = B^{-1}, \quad P_{2,3,0} \sim A_{-3} = (B^2 - 1)^{-1}B^{-1},
\]

and the \( D \to N \) action,

\[
\begin{pmatrix} A_3 & 0 \\ 0 & A_{-1} \end{pmatrix} \begin{pmatrix} \partial_0^2 u \\ \partial_n^2 u \end{pmatrix} = \begin{pmatrix} \partial_3^3 u \\ \partial_n^3 u \end{pmatrix}.
\]

To give one more example, for \( k = 3 \), one has \( m_0 = 0, m_2 = 2, m_4 = 4 \) for D and \( m_1 = 1, m_3 = 3, m_5 = 5 \) for N. Then,

\[
P_{3,0,5} \sim A_5 = B(B^2 - 1)(B^2 - 4), \quad P_{3,1,4} \sim A_3 = B(B^2 - 1) \quad P_{3,2,3} \sim A_1 = B \]
\[
P_{3,3,2} \sim A_{-1} = A_{-1}^{-1}, \quad P_{3,4,1} \sim A_{-3} = A_{-3}^{-1}, \quad P_{3,5,0} \sim A_{-5} = A_{-5}^{-1}
\]

and

\[
\begin{pmatrix} A_5 & 0 & 0 \\ 0 & A_1 & 0 \\ 0 & 0 & A_{-3} \end{pmatrix} \begin{pmatrix} \partial_0^2 u \\ \partial_n^2 u \\ \partial_3^3 u \end{pmatrix} = \begin{pmatrix} \partial_5^5 u \\ \partial_n^5 u \end{pmatrix}.
\]

The pseudo–operator \( B = \sqrt{Y + 1/4} \) in terms of the standard conformally covariant Yamabe–Penrose operator, \( Y \), on the sphere, and the \( A_{2k'} \) are the GJMS operators, \( \Box_{2k'} \), of odd order in (12) but now defined generally by the Branson intertwinor form,

\[
A_{2k'} = \frac{\Gamma(B + k' + 1/2)}{\Gamma(B - k' + 1/2)}.
\]  

(13)

In the general case, I write the action of the \( N \) to \( D \) operator, \( A \), as

\[
A^{(2k)} \beta_D = \beta_N
\]

where \( \beta_D \) and \( \beta_N \) are boundary data vectors of dimension \( k \) whose components can be ordered such that \( A \) has the structure,

\[
A^{(2k)} = \text{diag}(A_{2k-1}, A_{3-2k}, \ldots, A_{(1)^{k+1}})
\]

noting that \( A_{-2k'} = A_{2k'}^{-1} \).
An associated spectral quantity is,

$$\log \det A^{(2k)} = \log \det A_{2k-1} - \log \det A_{2k-3} + \ldots - (-1)^k \log \det A_1.$$  

which can be telescoped after remarking that

$$\log \det A_{2k-1} = \frac{1}{2} \log \det (D_{2k} D_{2k-2}), \tag{14}$$

where $D_{2k}$ is the spherical Dirac GJMS operator, defined by (13) with $B = |\nabla|$. This leads to

$$\log \det A^{(2k)} = \frac{1}{2} \log \det D_{2k}. \tag{15}$$

for integral $k$. (The right-hand side is actually defined for all $k$.)

A comparison of this with (7) reveals that the right-hand sides are the same (up to a factor of 2). Hence one has the relation,

$$\log \Det_N \Box_{2k} - \log \Det_D \Box_{2k} = \log \det A^{(2k)}, \tag{16}$$

between the bulk free energies and the logdet of the total D to N operator on the boundary. The notation is that $\Det_B$ is taken in the bulk Hilbert space, with boundary condition $B$, and $\det$ in the boundary Hilbert space. An alternative form is $\Det_G^{B} = -\Det_B \Box_{2k}$.

For comparison, a known result is (e.g. [16,17]),

$$\frac{\Det \Delta_N}{\Det \Delta_D} = \det \mathcal{N} \tag{17}$$

where $\Delta$ is a Laplace (2nd order) type operator and $\mathcal{N}$ is the corresponding D to N operator on the boundary of a Riemannian bulk.

For the model hemisphere, the higher derivative relation (16) has been demonstrated here by explicit calculation of each side separately, assuming the form of the D to N operator, which was found by an independent, intertwinor argument.

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6 This recursion was obtained in [9] from contour integral expressions for the logdets, valid for any $k$, and a trigonometric identity. The spin factors have been removed.

7 The general derivations of this eminently reasonable formula would seem to be unreasonably involved.
7. The D to N mapping

In this section I derive some basic relations involving the classic D to N operator in as elementary (i.e. non rigorous) a way as possible. This will still allow me to make some comments connecting to other work.

For ease I consider the standard D,N problems in some Riemannian manifold with a Laplace–type operator.

For the D case, take given boundary data, $\phi$, and a harmonic function, $u$, in the bulk, to be found such that $\phi = u|_{\partial\mathcal{M}}$. The solution is written formally as $u = \mathcal{E}\phi$. The D to N mapping, $\mathcal{N}$, is defined by,

$$\mathcal{N}\phi := -\partial_n(\mathcal{E}\phi), \quad \partial_n = n.\nabla|_{\partial\mathcal{M}},$$

in terms of the inward normal derivative at the boundary. $\mathcal{N}$ is known to be an elliptic boundary (pseudo)–operator and maps D boundary data to N boundary data. The sign is so as to make $\mathcal{N}$ positive.

Consider now the particular N problem having $\partial_n(\mathcal{E}\phi)$ as data. Introducing the N Green function, the solution is,

$$u_N = -\int_{\partial\mathcal{M}} G^N \partial_n(\mathcal{E}\phi)$$

and, by construction and uniqueness, this must coincide with the original D problem i.e. that with solution $\mathcal{E}\phi$ viz.

$$u_D = \int_{\partial\mathcal{M}} G^D \partial_n \phi.$$

Setting $u_D = u_N$ leads to,

$$\int_{\partial\mathcal{M}} G^D \partial_n \phi = -\int_{\partial\mathcal{M}} G^N \partial_n(\mathcal{E}\phi) = \int_{\partial\mathcal{M}} G^N \mathcal{N}\phi = \int_{\partial\mathcal{M}} G^N \mathcal{N}\phi,$$

since $\mathcal{N}$ is self–adjoint on the boundary. Cancelling the arbitrary $\phi$ gives the bulk–to–boundary statement,

$$G^D \partial_n = G^N \mathcal{N}.$$

Taking the bulk coordinate (the one on the left of $G$) to the boundary, the left–hand side becomes the unit operator (delta function) on the boundary (so that $u \to \phi$) and we discover that (working in the boundary Hilbert space)

$$\mathcal{N}^{-1} = G^N,$$

8 I choose the basic N condition. For conformal situations Robin are required but would make the formulae less neat.
in terms of the boundary-to-boundary N Green function. In this way, contact is made with the work of Barvinsky and Nesterov, [18], whose eqns. (1.9) and (1.10) show, in the light of (22), that the D to N operator, \( N \), is the kernel (‘propagation operator’) in the action induced on the boundary by integrating out the bulk fields. (See also Barvinsky, [19].)

Furthermore, using path-integrals, [18] derives a one-loop relation, (1.11), that, when combined with (22), is identical to (17). This could be considered a physicist’s simpler derivation of this mathematical formula.

An alternative expression for \( N \) follows by acting on (21) with \( -\partial_n \) and noting that the strictly boundary-to-boundary quantity \( -\partial_n G^N \) is the boundary delta function so giving,

\[
N = -\partial_n G^D \partial_n.
\]

(See [18].)

For the special case of the hemisphere, the higher derivative equivalent is that the induced effective boundary field theory consists of a set of \( k \) scalars propagating according to the non-local operators \( A^{(2k)} \) with alternating statistics. At the one-loop level, this system collapses spectrally to a single higher derivative, non-local spinor.

8. Comments

The analysis in section 6 applies, as it stands, only for \( k \) an integer when the bulk is local and the boundary non-local. However, the one-loop equivalences can be continued to real \( k \), as the interpolations and numerics verify. A particular case is \( k \) a half-integer, when, this time, the bulk theory is non-local and the boundary non-local. This raises the question of what is the corresponding bulk boundary value problem that would yield a D to N map. The problem centres on the construction of a relevant Green identity.

Similar boundary value problems, though usually on a domain, such as a ball, in \( \mathbb{R}^n \), have been the subject of intensive consideration in pure and applied mathematics over the years with the non-local operator being the fractional Laplacian, \( \Delta^a \), a normally restricted to lie between 0 and 1.

There seems to be no consensus on the ‘correct’ approach. There are several, non-equivalent, definitions of the fractional Laplacian on bounded domains, dependant on practical applications, frequently probabilistic. The most popular is the ‘spectral’ definition which is actually that universally employed in field theory as
being the most natural (obvious) and amounts to taking the operator (13), (for example) considered as a function, $A_{2k}(\Delta)$, of the Laplacian to equal $A_{2k}(\Delta_B)$, i.e. the boundary condition, $B$, is applied to the Laplacian in the usual way and thence to $A_{2k}$.

The works of Grubb, [20,21] contain sufficient history from which to trace these developments.

I have concentrated on the effective action but the conformal anomaly can also be discussed. In addition, the the fact that hemisphere is in the same conformal class as the Euclidean ball can be employed to gain further information and insight.

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