The effects of Core-polarization on the form factors of $^{17}\text{O}$, $^{19}\text{F}$ and $^{48}\text{Ca}$

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Abstract: Inelastic electron scattering for selected nuclei $^{17}\text{O}$, $^{19}\text{F}$ and $^{48}\text{Ca}$ are investigated by considering the energies of higher configurations outside core $^{16}\text{O}$ and $^{40}\text{Ca}$ cores. Higher energy configurations, called core polarization effects. Bohr-Mottelson calculations were also conducted to study the form factors of the above-mentioned nuclei using Oxbash. The predicted form factors were compared to the experimental data available. The effect of higher excited configurations is found to be essential in the dynamic transfer dependence of the form factors and is remarkably well aligned with the measured data.

Keywords: Longitudinal and transverse form factors, p-shell, sd-shell, Core polarization effects, Oxbash

1. Introduction

Electron scattering from nuclei gives us an invaluable tool for testing the broad range of nuclear and nuclear characteristics, because it has proved to be one of the most effective ways to study the characteristic of atomic nuclei energy levels, this it has supplied a wealth of data for the excitation and collective states of single-particle states and the mapping of nuclear ground state charges and accurate transition states [1]. Because electrons are charged and light, they radiate during the scattering process by necessity. This is one of electron scattering’s technical difficulties. Quantum electrodynamics (QED) described this radiation and even the associated virtual electromagnetic effects. The produced virtual quantum of electromagnetic radiation interacts with the target in electron scattering experiments in which the momentum of the initial and final electron is defined, this quantum’s energies are determined by the electron energy transmission and the quantum momentum from the transmission momentum. The transferred momentum and energies of the exchanged virtual photon can also differ independent of one another in electron scattering experiments, as long as the virtual photon resembles space [2]. In the research of the $^{17}\text{O}$ magnetic form factor, core polarization impacts were taken into consideration by Arima et al. [3] in the study of the magnetic form factor of $^{17}\text{O}$. A microscopic model was suggested to include outside model space impacts [4] to include effects from outside the model space. In sd-shell nuclei, the Coulomb of factors for E4 transition The core polarization impacts were addressed using self-consistent Hartree-Fock + random phase approximation calculations, which provided a strong agreement with experimental data. Radhi et al. carried out unrestricted large model space configurations calculations on $^{19}\text{F}$ nucleus. The elastic and inelastic electron scattering on $^{19}\text{F}$ is studied by taking in to account all major shells s, p, sd and pf are with $(0−2)\hbar\omega$ truncations [5]. Coulomb form factors have been researched by R. A. Radhi for the single-particle quadrupole transitions of p-shell and sd-shell nuclei [6]. A microscopic calculation has been performed by F.A. Majeed [7] to study the C2 form factors of some selected states of $^{10}\text{B}$ where the results were in
excellent agreement with the experimental data. Majeed and Najim [8] calculated the longitudinal and transverse electron scattering factors for some of the selected positive parity and negative parity states for the nuclei \(^{7}\)Li, \(^{13}\)C, and \(^{17}\)O which they are lies in the in p- and sd shells, by considering the configurations of high energy outside the conventional model space of p- and sd shells. The C2 and C4 longitudinal from factors have been investigated by Majeed and Hussain for some nuclei lies in the fp-shell region, where the authors used different effective interactions to account for the CP effects [9]. Majeed and Obaid [10] have carried out a wide base shell model computation for studies of the \(^{20}\)Ne, \(^{22}\)Ne, and \(^{24}\)Mg nuclei nuclear structure.

This study aimed to investigate the effect of configuration with high energy on the estimation of the inelastic electron scattering form factors for \(^{17}\)O, \(^{19}\)F and \(^{48}\)Ca. The effect of the configurations with higher energy will be considered by a microscopic theory that allows the projection of particles from the closed core to the model space with \(4\hbar\omega\). The results will be compared with the available measured data.

2. Theory

The matrix element of the one-body is expressed as a linear combination of the single-particle matrix element for shell-model wave functions and final spin \(\nu\) [11].

\[
\langle J_f | \hat{F}_\nu | J_i \rangle = \Sigma_{j_f, j_i} \zeta^{\nu}_{j_f, j_i} \langle j_i | j_f \rangle \langle J_f | \hat{F}_\nu | J_i \rangle.
\]

(1)

Where the structure factor (one-body density matrix element) is located in range \(\zeta^{\nu}_{j_f, j_i}\), \(j_i\) and \(j_f\) denote the initial and final single-particle state, A reduced one-body operator's single matrix element \(\hat{F}\) can be expressed as the sum of three terms, one sd-shell or fp-shell part, two-core matrix-element (CP) polarization [12]

\[
\langle j_f | \hat{F}_\nu | j_i \rangle = \langle j_f | \hat{F}_\nu | j_i \rangle + \langle j_f | \hat{F}_\nu^{0} V_{res} | j_i \rangle + \langle j_f | V_{res}^{0} \hat{F}_\nu | j_i \rangle
\]

(2)

where the operator \(O\) projects are located outside the configuration sd-shell or fp-shell. The two core polarization conditions are detailed calculations indicated in Ref [9]. The remaining MSDI interaction is used as \(V_{res} A, B, C\) and \(T\) where \(T\) indicates the strength of the MSDI, where \(T\) shows the isospin \((0\ or\ 1)\). The parameters are set to \(A = A_1 = B = 25/\hbar\ MeV\) and \(C = 0\) [12] and are used for all states in this work in which \(A\) is the mass number. Ref. [7] uses this set of parameters. The intermediate one-particle-one-hole states are taken up to \(6\hbar\omega\) excitations, where \(^{16}\)O and \(^{46}\)Ca core excitations are taken into account. A harmonic oscillator is the single-particle wave functions. The longitudinal form factor of electron scattering for a particular multipolarity is expressed as \(\nu\) [12].

\[
|F_\nu(q)|^2 = \frac{1}{2j_f+1} \left(\frac{4\pi}{\kappa^2}\right) \langle j_f | \hat{F}_\nu | j_i \rangle^2 \frac{2J_f^2 f_{s,m}^2}{2j_f+1}.
\]

(3)

In which \(f_{s,m}\), the correction of the finite-nucleon size is and \(f_{s,m}\). The radiative width of transition is defined as

\[
\Gamma_\nu(f \rightarrow i) = 2Z^2 e^2 k |F_\nu(q = k)|^2 \frac{2j_i+1}{2j_f+1},
\]

(4)

where \(e^2 = 1.64 MeV\), \(f\) and \(k = \frac{E}{\hbar c}\).
3. Result and Discussion

A. $^{17}$O Nucleus ($0.870\text{MeV}\ J_f^T = \frac{1}{2}^+_\frac{1}{2}$), ($5.084\text{ MeV}\ J_f^T = \frac{3}{2}^+_\frac{1}{2}$)

This single neutron nucleus outside $^{16}$O core according to the conventional shell model is distributed over the $1d_{5/2}$, $2s_{1/2}$ and $1d_{3/2}$. The red dashed line gives the results obtained using the sd wave functions in all the proceeding diagrams. Figure 1 shows the form factor $C_2$; the calculations of the model space only were not able to account for the measured $C_2$ form factors and unable to correctly locate the diffraction minima. The outcomes are shown in the blue solid line. The one body density matrix (OBDM) elements were determined in the p-shell and their values are listed in Tables 1 and 2. The including of CP effects improve calculations and very clearly describes data at the first and second maximums as well as the minimum diffraction in its correct place. The transverse $M_3$ factor for the ($J_f^T = \frac{1}{2}^+_\frac{1}{2}$) is presented in Figure 2. CP effects were included to describe the form factor measured in alpha-moment transmission areas and to properly reproduce the place of the diffraction minimum. Our studies are comparable to the calculation from the Ref [11]. While the Fig is there. 3 The solid curves are the $C_2$ and $C_4$ multipole decomposition, which can very well reproduce the data in the range $q \approx 0.86 - 1.92\text{ fm}^{-1}$. The Bohr-Mottelson agrees with the first maxima and underestimate the second maxima for $C_2$ form factor, overestimate the $M_3$ transverse form factor and reasonably agrees with $C_2+C4$ from factor. The estimated form factors are in agreement with the previous study of Ref. [13].

| $J_i$ | $J_f$ | OBDM ($\Delta T=0$) | OBDM ($\Delta T=1$) |
|------|------|----------------------|----------------------|
| $1p_{1/2}$ | $1p_{1/2}$ | 1.51477 | 1.62347 |
| $1d_{5/2}$ | $1d_{5/2}$ | 0.24718 | 0.22482 |
| $2s_{1/2}$ | $2s_{1/2}$ | 0.0571 | 0.98713 |

Table (1): The values of the OBDM elements for the longitudinal $C_2$ and transverse $M_1$ transition of the state $\frac{1}{2}^+_\frac{1}{2}$ of $^{17}$O.

| $J_i$ | $J_f$ | OBDM ($\Delta T=0$) | OBDM ($\Delta T=1$) |
|------|------|----------------------|----------------------|
| $1p_{1/2}$ | $1p_{1/2}$ | 0.00341 | 0.00935 |
| $1d_{5/2}$ | $1d_{5/2}$ | 0.01748 | -0.0162 |
| $2s_{1/2}$ | $2s_{1/2}$ | -0.0191 | 0.01747 |

Table (2): The values of the OBDM elements for the longitudinal $C_2$ and $C_4$ transitions of the state $\frac{3}{2}^+_\frac{1}{2}$ of $^{17}$O.
FIG. 1: The Longitudinal C2 form factor for the $\frac{1}{2}^-$ state in $^{17}$O with and without CP effects. The experimental data are taken from [13].

FIG. 2: The Transverse M3 form factor for the $\frac{3}{2}^+$ state in $^{17}$O with and without CP effects. The experimental data are taken from [13].

FIG. 3: The Longitudinal C2+C4 form factor for the $\frac{3}{2}^+$ state in $^{17}$O with and without CP effects. The experimental data are taken from [13].
B. $^{19}$F Nucleus $(0.00 \text{ MeV } J_f^T = \frac{1+1}{2_1^1 2})$, $(1.554 \text{ MeV } J_f^T = \frac{3+1}{2_1^1 2})$

Figure 4 shows the compared Coulomb C0 form factor computed at the $E_x=0.00$ MeV for the $(J_f^T = \frac{1+1}{2_1^1 2})$, which provides good consistency with experimental data taken from [12] from every momentum transfer area calculated for the model space wave functions and inclusion of the CP effects. The one body density matrix (OBDM) elements were determined in the p-shell and their values are listed in Tables 3 and 4. The M1 magnetic form factors are shown in the figure. 5. The model space results are shown by (red dashed curve) and the Cp effects are compared with the experimental data (blue solid curve) [13]. The form factor does not describe in the data region $1 < q < 1.5 \text{ fm}^{-1}$. In Figure 6, we found the longitudinal C2 form factor for the $(J_f^T = \frac{3+1}{2_1^1 2})$ at $E_x=1.554 \text{ MeV}$ state under prediction of the data by approximately a factor of five at the first maximum and the inclusion of the CP improves the calculations and the formula factor for experimental values in all momentum transfer regions. The experimental data are extracted from [14]. The transverse M1 + E2 form factors in the same state in Figure 7 doesn’t significantly affect the calculation of form factor including core polarization. The Bohr-Mottelson calculations shown with green curve agrees reasonably well with the measured form factors for all the studied states of $^{19}$F except M1+E2 form which overshoots the experimental data.

Table (3): The values of the OBDM elements for the longitudinal C0 and transverse M1 transition of the ground state $\frac{1+1}{2_1^1 2}$ of $^{19}$F.

| $J_i$ | $J_f$ | OBDM ($\Delta T=0$) | OBDM ($\Delta T=1$) |
|-------|-------|----------------------|----------------------|
| 1d_3/2 | 1d_3/2 | 1.51477              | 1.62347              |
| 1d_3/2 | 1d_3/2 | 0.24718              | 0.22482              |
| 2s_1/2 | 2s_1/2 | 0.0571               | 0.98713              |

Table (4): The values of the OBDM elements for the longitudinal C2 transition of the state $\frac{3+1}{2_1^1 2}$ of $^{19}$F.

| $J_i$ | $J_f$ | OBDM ($\Delta T=0$) | OBDM ($\Delta T=1$) |
|-------|-------|----------------------|----------------------|
| 1d_3/2 | 1d_3/2 | -0.09847             | 0.05636              |
| 1d_3/2 | 1d_3/2 | -0.05358             | -0.00185             |
| 1d_3/2 | 1d_3/2 | 0.05279              | -0.02911             |
| 1d_3/2 | 1d_3/2 | 0.06106              | 0.02288              |
| 1d_3/2 | 2s_1/2 | 0.26754              | 0.01943              |
| 2s_1/2 | 1d_3/2 | -0.11861             | -0.00165             |
| 2s_1/2 | 2s_1/2 | -0.0003              | -0.01597             |
FIG. 4: The longitudinal C0 form factor for the $1/2^+_1$ state in $^19$F with and without CP effects. The experimental data are taken from [14].

FIG. 5: The transverse M1 form factor for the $1/2^+_1$ state in $^19$F with and without CP effects. The experimental data are taken from [15].

FIG. 6: The longitudinal C2 form factor for the $3/2^+_1$ state in $^19$F with and without CP effects. The experimental data are taken from [15].

FIG. 7: The transverse component M1+E2 form factor for the $3/2^+_1$ state in $^19$F with and without CP effects. The experimental data are taken from [15].
C. \(^{48}\text{Ca} \quad \text{Nucleus} \quad \begin{align*} & (0.00 \text{ MeV} J^\pi T = 0^+_1 \ 4), \quad (3.831 \text{ MeV} J^\pi T = 2^+_1 \ 4) \quad \text{and} \quad (4.503 \text{ MeV} J^\pi T = 4^+_1 \ 4) \end{align*} \)

The C0 charge form factor is the first property to be discussed where the charge distribution is presented in the momentum transmission scale. Figure 8 displays the electron scattering form factor of the Coulomb C0 as a function of momentum transfer \(q\). The one body density matrix (OBDM) elements were determined in the p-shell and their values are listed in Tables 5, 6 and 7. From Fig. 8, the theoretical and experimental curves coincide with both model space and CP effect at initial maximum, but the theoretical curves are quenched at the second and third maximum. The experimental data is extracted from [16]. For calculating the model of 1f2p shell in \(^{48}\text{Ca}\), the quadrupoles transition C2 form factors are shown in Figure 9 to the state of the first. The total form factors for these transitions are only caused by the core polarization because the model space neutrons have a very small contribution to Coulomb. The first lobe of the form factor has an excellent agreement where the data is reproduced correctly up to \(q = 1.5 \text{ fm}^{-1}\). For the second lobe, the estimated data is approximately two times higher, but a qualitative agreement is reached. The C4 transition form factor for \(^{48}\text{Ca}\) with excitation energy \(E_x=4.503 \text{ MeV}\) is shown in Fig. 10. The model space calculation is overestimated, and the inclusion of CP strengthens the calculations and raises the form factor to \(q=2.6 \text{ fm}^{-1}\). The form factor for C4 transition in \(^{48}\text{Ca}\) with excitation energy \(E_x=4.503 \text{ MeV}\) is displayed in Fig. 10. The model space calculation underestimates the experiment and the inclusion of the CP enhances the calculations and brings the form factor to the up to \(q=2.6 \text{ fm}^{-1}\). The value of higher \(q\) is underestimated. Bohr-Mottelson calculations agrees well for all the studied states of \(^{48}\text{Ca}\) as displyes in Figs. 8, 9 and 10. The experimental data are taken from Ref. [14].
Table (5): The values of the OBDM elements for the longitudinal C0 and transverse M1 transition of the ground state $^1_0$ of $^{48}$Ca.

| $J_i$ | $J_f$ | OBDM ($\Delta T=0$) | OBDM ($\Delta T=1$) |
|-------|-------|-----------------------|-----------------------|
| $1f_{7/2}$ | $1f_{7/2}$ | 0 | 2.79025 |
| $1f_{5/2}$ | $1f_{5/2}$ | 0 | 0.01313 |
| $2p_{3/2}$ | $2p_{3/2}$ | 0 | 0.03353 |
| $2p_{1/2}$ | $2p_{1/2}$ | 0 | 0.00619 |

Table (6): The values of the OBDM elements for the longitudinal C2 transition of the state $^2_1$ of $^{48}$Ca.

| $J_i$ | $J_f$ | OBDM ($\Delta T=0$) | OBDM ($\Delta T=1$) |
|-------|-------|-----------------------|-----------------------|
| $1f_{7/2}$ | $1f_{7/2}$ | 0 | 0.01792 |
| $1f_{5/2}$ | $1f_{5/2}$ | 0 | 0.03756 |
| $1f_{7/2}$ | $2p_{3/2}$ | 0 | 0.09361 |
| $1f_{5/2}$ | $1f_{7/2}$ | 0 | -0.10351 |
| $1f_{5/2}$ | $2p_{3/2}$ | 0 | -0.00309 |
| $1f_{7/2}$ | $2p_{1/2}$ | 0 | 0.00283 |
| $2p_{3/2}$ | $1f_{7/2}$ | 0 | 0.95681 |
| $2p_{3/2}$ | $1f_{5/2}$ | 0 | -0.00011 |
| $2p_{3/2}$ | $2p_{3/2}$ | 0 | 0.01009 |
| $2p_{3/2}$ | $2p_{1/2}$ | 0 | 0.00482 |
| $2p_{1/2}$ | $2p_{3/2}$ | 0 | 0.00357 |
| $2p_{1/2}$ | $2p_{1/2}$ | 0 | -0.01208 |

Table (7): The values of the OBDM elements for the longitudinal C4 transition of the state $^4_2$ of $^{48}$Ca.

| $J_i$ | $J_f$ | OBDM ($\Delta T=0$) | OBDM ($\Delta T=1$) |
|-------|-------|-----------------------|-----------------------|
| $1f_{7/2}$ | $1f_{7/2}$ | 0 | 0.00997 |
| $1f_{5/2}$ | $1f_{5/2}$ | 0 | 0.01901 |
| $1f_{7/2}$ | $2p_{3/2}$ | 0 | 0.02762 |
| $1f_{5/2}$ | $2p_{1/2}$ | 0 | 0.00717 |
| $1f_{5/2}$ | $1f_{5/2}$ | 0 | -0.14793 |
| $1f_{5/2}$ | $2p_{3/2}$ | 0 | 0.0026 |
| $1f_{5/2}$ | $2p_{1/2}$ | 0 | -0.00633 |
| $2p_{3/2}$ | $1f_{7/2}$ | 0 | 0.94276 |
| $2p_{3/2}$ | $1f_{5/2}$ | 0 | 0.00176 |
| $2p_{3/2}$ | $2p_{3/2}$ | 0 | -0.05457 |
FIG. 8: The longitudinal C0 form factor for the $0^+_4$ state in $^{48}$Ca with and without CP effects. The experimental data are taken from [16].

FIG. 9: The longitudinal C2 form factor for the $2^+_4$ state in $^{48}$Ca with and without CP effects. The experimental data are taken from [16].

FIG. 10: The longitudinal C4 form factor for the $4^+_4$ state in $^{48}$Ca with and without CP effects. The experimental data are taken from [16].
4. Conclusion
The form factors of inelastic electron scattering for different states in $^{17}$O, $^{19}$F and $^{40}$Ca nuclei have been investigated. The effect of configurations with higher energy outside the model space which is called the polarization of the core were considered in the calculations and shows major contribution to the calculations of the form factors. The choice of MSDI as effective residual interaction to mediate the calculations between initial and final states is found very adequate for the electron scattering calculations. The Bohr-Mottelson calculations are successful to study the form factors for the selected states of the studied nuclei. The calculations can be further extended in this mass areas to study more nuclei.

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