Research on repeated loading and sinkage characteristics of soil by numerical analysis

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Abstract. The research on repeated loading and sinkage characteristics of soil can calculate the sinkage of wheel and track in driving process more precisely, so it can help calculate the vibration characteristics and driving resistance of vehicle more accurately. This paper analysis the soil repeated loading and sinkage characteristics based on finite element method. The results shows that the unloading and reloading process of soil can be assumed as a straight line with slope of k on load-sinkage curve, here k is called rebound coefficient. The research conclusion can provide reference to the vehicle terramachanics and other research involved soil deformation.

1. Introduction
Construction machinery and vehicles exert vertical load to ground when travel on it and cause sinkage [1]. The relation between sinkage and vertical load usually is established by plate sinking test [2]. The power function model and hyperbolic model are widely used now to describe the relationship between load and sinkage of soil under vertical load. Though these models all consider the sinkage characteristics of soil under uniaxial load and don not consider the unloading and reloading soil sinkage characteristics [3-8].

For wheeled vehicle travels on the soft soil, the front wheel pressed over a certain area, the soil under this certain area would have an unloading process, then the rear wheel get through this area again and this is a reloading process to the soil of this certain area. Also to the tracked vehicle travels on the soft soil, for a piece of track, when it touch down the ground, the loading wheel will rolling it from top to bottom, then the soil under this track will have a unloading and reloading process[9]. Therefore, based on the finite element theory, this paper did a research on repeated loading and sinkage characteristics of soil.

2. Constitutive relation of soil
Soil’s stress-strain character is a typical nonlinear elastoplastic behavior, so the major work of analysis deformation law of soil by finite element method is determining the soil constitutive relation.

2.1. Elastic behavior of soil
The generalized Hooke’s law can describe the elastic behavior of soil here [10], and this involve two parameters elastic module $E$ and Poisson ratio $v$. 
2.2. Plastic behaviour of soil

Incremental theory of plasticity is the major theory now to describe the plastic behavior of soil. Incremental theory of plasticity is based on plastic postulate, yield criterion, hardening law and flow rule as it three characteristics, for the plastic behavior of soil, also need to involve failure criterion [11-15]. The deformation characteristics of soil are different from other metals and materials, such as shear militancy and compression yield characteristics and so on.

The modified Drucker–Prager/cap plasticity model has been widely used in finite element analysis programs for a variety of geotechnical engineering applications. The cap model is appropriate to soil behavior because it is capable of considering the effect of stress history, stress path, militancy, and the effect of the intermediate principal stress. So here use modified Drucker–Prager/cap plasticity model to describe the plastic behavior of soil.

2.2.1. Yield criterion. The process by which the soil material enters the plastic state from the elastic state is called yield. The initial yield is the boundary between elastic strain and plastic strain and it does not represent soil damage. When the soil material enter initial yield, with the increase of strain and stress, the yield stress may continue increase and appear hardening phenomenon. Yield surfaces of the modified cap model in the $p-t$ plane as shown in Figure 1.

![Figure 1. Yield surface of modified Drucker-Prager cap model](image)

The cap yield surface given as

$$F_c = \sqrt{(p - p_a)^2 + \left(\frac{Rt}{1 + \alpha - \alpha / \cos \beta}\right)^2} - R(d + p_a \tan \beta) = 0$$

(1)

Where R is a material parameter that controls the shape of the cap and $\alpha$ is a small number (typically, 0.01 to 0.05) used to define a smooth transition surface between the shear failure surface and the cap surface. The cap yield surface can reflect the yield of soil due to isotropic compression, and this is very different from other metals.

2.2.2. Failure criterion. Failure criterion is the criterion to judge whether the soil is damaged or not, it is a certain stress combination based on the basic principle of the strengthen theory. For the failure of soil, Mohr-Coulomb criterion consider the influence of friction components and is considered the most suitable to soil damage conditions, so it is widely used in geotechnical materials. The failure criterion of modified cap model is same as the Mohr-Coulomb criterion, in the $p-t$ plane as a straight line as shown in Figure 1.

The modified cap model failure surface is given by
$$F_s = t - p \tan \beta - d = 0$$  \hspace{1cm} (2)

Where $\beta$ is the soil’s angle of friction and $d$ is its cohesion in the $p-t$ plane.

2.2.3. Flow rule. Flow rule is used to determine the direction of plastic strain increment of soil when soil enter plastic deformation process, that is the proportion of each component. Flow rule has two form, associated and no associated. Associated flow rule assume the flow potential same as yield surface. For the modified cap model failure surface and the transition yield surface, a no associated flow is assumed: The shape of the flow potential in the $p-t$ plane is different from the yield surface as shown in Figure 2.

![Figure 2. Flow potential of modified Drucker-Prager cap model](image)

In the cap region the elliptical flow potential surface is given as

$$G_c = \sqrt{\left( p - p_a \right)^2 + \left( \frac{Rt}{1+\alpha - \alpha / \cos \beta} \right)^2}$$ \hspace{1cm} (3)

The elliptical flow potential surface portion in the modified cap failure and transition regions is given as

$$G_s = \sqrt{\left( (p - p_a) \tan \beta \right)^2 + \left( \frac{t}{1+\alpha - \alpha / \cos \beta} \right)^2}$$ \hspace{1cm} (4)

2.2.4. Hardening law. $p_a$ is an evolution parameter that controls the hardening–softening behavior as a function of the volumetric plastic strain. Each $p_a$ corresponding a yield surface. And $p_a$ is the function of volumetric plastic strain, so the change of volumetric plastic strain will control the hardening law of soil.

2.3. Determination of constitutive model parameters
The parameters involved in this model are elastic module $E$, Poisson ratio $\nu$, angle of friction $\beta$ and cohesion $d$ in the $p-t$ plane. These parameters all can be get by triaxial test of soil. Here choose the rolling dirt road as the paper research object as shown in Figure 3.
Through triaxial test of soil finally got the constitutive model parameters are that elastic module $E$ is 20.2Mpa, Poisson ratio $\nu$ is 0.32, and cohesion $c$ 6.38Kpa angle of friction $\varphi$ 27.33° in $\sigma - \tau$ plane.

For triaxial stress conditions the Mohr–Coulomb parameters cohesion $c$ and angle of friction $\varphi$ in $\sigma - \tau$ plane can be converted to Drucker–Prager parameters angle of friction $\beta$ and cohesion $d$ in the $p - t$ plane as follows:

$$\tan\beta = \frac{6\sin\varphi}{3 - \sin\varphi}$$  \hspace{1cm} (5)

$$d = \frac{18c \cos\varphi}{3 - \sin\varphi}$$  \hspace{1cm} (6)

So got the cohesion $d$ is 40.15Kpa and angle of friction $\beta$ is 47.31° in $p - t$ plane.

3. Repeated loading and sinkage of soil

First study the sinkage characteristics of soil under vertical uniaxial load, then study the sinkage characteristics of soil under vertical unloading and reloading condition, finally through the comparison get the sinkage characteristics of soil.

3.1. Load-deformation relationship of soil under uniaxial loading

Load-deformation relationship of soil under uniaxial load usually can be determined by plate sinking test and that is press a plate representing the ground area of a tire or track into soil with uniformly distributed load, and get the relationship between sinkage $z$ and pressure $p$.

By numerical simulation, pressed a square plate $0.2m \times 0.2m$ into the soil at a uniform speed, and finally got the relationship between sinkage $z$ and pressure $p$ as shown in figure 4.
From the figure 5, we can see that sinkage curve has gone through three stages. (1): linear deformation stage: the relationship between load and sinkage is linear in this stage and the sinkage major caused by soil compaction. (2): local shear failure stage: in this stage with the load increase, the relationship between load and sinkage is gradually down warping curve linear, in this stage besides the soil compaction there has some area in soil bring into plastic deformation because the shear stress reached the shear strength in these area. (3) Completely failure stage: when the load continue increase to a critical value, soil sinkage rapid increase and at this time, the soil under the plate already be destroyed and cannot continue bear more load and the critical value is called ultimate bearing capacity.

3.2. Load-deformation relationship of soil under cyclic loading
When did the plate sinking test, take load as a control variable, when the load increase to 600Kpa unloading all, then reloading to 900Kpa and unloading all again, finally reloading to the soil failure. Through the numerical simulation, that got the relationship between sinkage z and pressure p as shown in figure 5.

![Figure 5. Soil sinkage curve under cycle loading](image)

From the figure 6, we can find that when unloading the load to zero that the sinkage decrease a lot but not to zero, that mean the soil emerge the residual plastic strain. In the reloading process, when the load pressure less than the unload point, the sinkage of soil will has the similar behavior as the unload process, when the load pressure big than the unload point, the soil sinkage curve will follow the first loading curve.

Load-sinkage curve of unloading and reloading process form a hysteresis loop, and this represents some energy dissipation in this process. Unloading and reloading process of soil can be assumed as a straight line with slope of \( k \) on load-sinkage curve, here \( k \) is called rebound coefficient.

From the figure 6, we can also conclude that if all wheels of a vehicle are subject to the same load \( p \), only the first wheel sink to the depth \( z \) and the rest wheels will roll in the same rut without obvious deeper sinkage.

4. Conclusion
Unloading and reloading process of soil can be assumed as a straight line with slope of \( k \) on load-sinkage curve, here \( k \) is called rebound coefficient.

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