On the finiteness of the BRS modulo–$d$
cocycles

Olivier Piguet and Silvio P. Sorella
Dép. de Physique Théorique, Université de Genève
CH-1211 Genève 4, Switzerland

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Ladders of field polynomial differential forms obeying systems of descent equations and corresponding to observables and anomalies of gauge theories are renormalized. They obey renormalized descent equations. Moreover they are shown to have vanishing anomalous dimensions. As an application a simple proof of the nonrenormalization theorem for the nonabelian gauge anomaly is given.
1 Introduction

A whole class of observables and anomalies in gauge theories are built by solving systems of descent equations [1] for sequences – "ladders" – of classical field polynomials. This amounts to studying the BRS modulo-d cohomolgy in the space of local field polynomials [2, 3]. The nontrivial solutions of the descent equations are uniquely specified by invariant ghost monomials of the general form Tr $c^n$, $n$ odd, where $c$ is the usual Faddeev-Popov ghost belonging to the Lie algebra of the gauge group.

The aim of the present paper is twofold. First, show the perturbative existence and uniqueness of quantum insertions, for any given classical ladder, which fulfil the quantum version of the descent equations. Second, prove that these quantum insertions are "ultraviolet finite", i.e. that they are characterized by vanishing anomalous dimensions [4, 5].

The proof of ultraviolet finiteness will be given in a particular gauge, namely the Landau gauge [6]. However, thanks to the gauge invariance of the ghost cocycle Tr $c^n$ and by means of the extended BRS technique [7], its validity persists in a generic covariant gauge, as shown for a particular case in [5]. The choice of the Landau gauge is the natural one for studying the ultraviolet properties of such ghost monomials. Indeed, as shown in [6], the Landau gauge is characterized by the "ghost equation" which controls the dependence of the theory on the ghost field $c$. It is this equation which implies the vanishing of the anomalous dimension of $c$ and also of all cocycles Tr $c^n$.

These results were already proved, by using Feynman graph considerations, in the case of the ladder related to the cocycle Tr $c^3$: this was a basic ingredient in the proof of the nonrenormalization theorem of the U(1) anomaly [5] in renormalizable nonabelian gauge theories.

We will give here the proof referring to the ladder $n = 5$ for simplicity, but in a way suitable for generalization to any $n$. This case is of particular importance for gauge theories in 4-dimensional space-time due to the relation of this ladder with the nonabelian gauge anomaly. As an application of our results we present here a completely algebraic proof of the nonrenormalization theorem [8, 9, 10] for the latter anomaly.

The paper is organized as follows. In Section 2 we introduce the cocycle Tr $c^5$ in the framework of a general renormalizable gauge theory, and we write down all the functional identities characterizing the properties of the model in the classical approximation. Section 3 is devoted to the renormalization of that cocycle. The ultraviolet finiteness of the renormalized Tr $c^5$ is proved in Section 4 by showing that it obeys a Callan-Symanzik equation without anomalous dimension. Section 5 extends the proof to the whole renormalized $n = 5$ ladder, and in particular to the anomaly
insertion. In Section 6 we present the algebraic proof of the nonrenormalization theorem. A useful proposition on local cohomology is given in the Appendix.

# 2 The $c^{(5)}$ cocycle

## 2.1 The functional identities

We consider here a massless gauge theory in 4-dimensional space-time. The gauge group is a compact Lie group $G$, assumed to be simple. Its generators obey the commutation rules

$$[T_a, T_b] = i f_{abc} T_c .$$

(2.1)

The gauge field $A^a_\mu$ as well as the Lagrange multiplier field $b^a$ and the ghost fields $c^a, \bar{c}^a$ belong to the adjoint representation, whereas the matter fields, collectively denoted by $\phi$, belong to some finite unitary representation of $G$ where the generators will also be denoted by $T_a$. [1]

The BRS transformations are:

$$sA_\mu = -\partial_\mu c + i[A_\mu, c] = -D_\mu c ,$$

$$sc = -\frac{i}{2} \{c, c\} = -ic^2 ,$$

$$s\bar{c} = b ,$$

$$s\phi = -ic\phi .$$

(2.4)

They are nilpotent.

The BRS-invariant gauge fixed classical action in the Landau gauge reads

$$\Sigma_{\text{Landau}} = \int d^4 x \left( -\frac{1}{4g^2} F_{\mu \nu}^a F_{\mu \nu}^a + \mathcal{L}_{\text{matter}} (\phi, D_\mu \phi) + b_\alpha \partial^\mu A_\mu^a + \bar{c}_\alpha \partial^\mu D_\mu c^a \right) .$$

(2.5)

We don’t specify the part of the gauge invariant action which depends on the matter field and on its covariant derivative $D_\mu \phi = (\partial_\mu - iA_\mu)\phi$, only restricted by the usual power-counting renormalizibility condition.

Let us define

$$c^{(5)} = \frac{1}{40} d_{abc} f_{bde} f_{cfg} c^a c^d c^e c^f c^g .$$

(2.6)

1 Fields $\varphi$ in the adjoint representation will often be written as matrices:

$$\varphi = T_a \varphi^a .$$

(2.2)

The Killing form is taken to be $\delta_{ab}$, i.e. the structure constants obey

$$f_{abc} f^{bcd} = \delta^d_a .$$

(2.3)
with $d_{abc}$ the completely symmetric invariant tensor of rank 3. This is a BRS cocycle (s-invariant but not of the form $s(\cdot \cdot \cdot)$). The purpose of this and the following section is to show the finiteness of $c^{(5)}$ with the help of the ghost equation governing the dependence of the theory on the ghost $c$. In order to control the renormalization properties of $c^{(5)}$ let us couple it to an external field $\rho$. It will turn out to be useful to couple also certain $c$-monomials of degree 4, 3 and 2 to external fields $\eta$, $\omega$, $\tau$ and $\sigma$. Coupling moreover the BRS variations of $A_{\mu}$ and $\phi$ to the external fields $\Omega^{\mu}$ and $Y$, we write the external field dependent part of the classical action as:

$$\Sigma_{\text{ext}} = \int d^4x \left( \Omega_{\mu}^a s A_{\mu}^a + Y s \phi + \rho c^{(5)} + \frac{1}{8} \eta^{ab} f_{acd} f_{be} f e^c d e^f + \frac{1}{2} (\omega^{ab} + \tau^{ab}) c_{a} f b c d e^f + \frac{1}{2} \sigma^{ab} c_a c_b \right).$$

with

$$\begin{align*}
\eta^{ab} &= \eta^{ba}, \\
\omega^{ab} &= \omega^{ba}, \\
\tau^{ab} &= -\tau^{ba}, \\
\sigma^{ab} &= -\sigma^{ba}.
\end{align*}$$

Dimensions and ghost numbers of the different fields are given in Table 1.

| $A_{\mu}$ | $c$ | $\bar{c}$ | $b$ | $\Omega^{\mu}$ | $\rho$ | $\eta$ | $\omega$ | $\tau$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|
| $d$ | 1 | 0 | 2 | 2 | 3 | 4 | 4 | 4 | 4 |
| $g$ | 0 | 1 | -1 | 0 | -1 | -5 | -4 | -3 | -3 |

Table 1: Dimensions $d$ and ghost numbers $g$.

BRS invariance of the external field part of the action together with nilpotence is achieved by demanding the following transformation rules ($\Omega^{\mu}$ and $Y$ being kept invariant as usual):

$$\begin{align*}
s\rho &= 0, \\
s\eta^{ab} &= 2 \omega^{ab}, \\
s\omega^{ab} &= 0, \\
s\tau^{ab} &= \sigma^{ab}, \\
s\sigma^{ab} &= 0.
\end{align*}$$

Invariance of the total classical action

$$\Sigma(A, c, \bar{c}, b, \Omega, Y, \rho, \eta, \omega, \tau, \sigma) = \Sigma_{\text{Landau}} + \Sigma_{\text{ext}}$$

(2.10)
under the BRS transformations (2.4) and (2.9) is expressed by the Slavnov identity:

$$\mathcal{S}(\Sigma) = \int d^4x \left( \frac{\delta \Sigma}{\delta \Omega_a^\mu} \delta A_a^\mu + \frac{1}{2} f^{abc} \frac{\delta \Sigma}{\delta \sigma^{bc}} \delta c^a + \frac{\delta \Sigma}{\delta Y} \frac{\delta \Sigma}{\delta \phi} + b^a \frac{\delta \Sigma}{\delta \bar{c}^a} + \omega^{ab} \frac{\delta \Sigma}{\delta \eta^{ab}} \right) = 0 .$$  

The action is invariant under the rigid transformations $\delta^\text{rig}_a$ of the group $G$:

$$\mathcal{R}^\text{rig}_a \Sigma = \sum_{\text{all fields } \varphi} \int d^4x \, \delta^\text{rig}_a \varphi \frac{\delta \Sigma}{\delta \varphi} = 0 .$$  

The Landau gauge condition is expressed by the equation

$$\frac{\delta \Sigma}{\delta b_a} = \partial^\mu A_a^\mu .$$  

The antighost equation

$$\mathcal{G}_a \Sigma = \frac{\delta \Sigma}{\delta \bar{c}^a} + \partial^\mu \frac{\delta \Sigma}{\delta \Omega^a_\mu} = 0$$  

follows from the Slavnov identity and the gauge condition.

The ghost equation, usually valid in the Landau gauge \[^\text{[5]}\], extends to the present case as:

$$\mathcal{G}_a \Sigma = \Delta_a ,$$  

where

$$\mathcal{G}_a = \int d^4x \left( \frac{\delta}{\delta \bar{c}^a} + f^{abc} \frac{\delta}{\delta b_c} + \frac{1}{2} \rho d^{abc} \frac{\delta}{\delta \eta_{bc}} ight.$$  

$$+ \frac{1}{2} f^{abc} \eta^{bc} \left( \frac{\delta}{\delta \omega^{dc}} + \frac{\delta}{\delta \tau^{dc}} \right)$$  

$$+ \frac{1}{2} \left( (\omega_{ab} + \tau_{ab}) f^{bcd} + 2(\omega^{cb} + \tau^{cb}) f^{ad} \right) \frac{\delta}{\delta \sigma^{cd}} \left) \right) ,$$  

and

$$\Delta_a = \int d^4x \left( f^{abc} \Omega^{\mu}_a A^\mu_c + \sigma_{abc} c^b + iYT_a \phi \right) .$$  

Finally it is easy to see, using the Jacobi identity obeyed by the structure constants, that the following "$\tau$-equation" holds:

$$\mathcal{T}_a \Sigma = f^{abc} \Delta_{bc} = 0 .$$  

\[^2\]The functional derivative of a functional $\mathcal{F}$ with respect to a symmetric or antisymmetric tensor field $t_{ab}$ is defined through the variation formula

$$\delta \mathcal{F} = \int d^4x \frac{1}{2} \delta t_{ab} \frac{\delta \mathcal{F}}{\delta t_{ab}} .$$  

4
The "linearized" Slavnov operator

\[ S_\Sigma = \int d^4x \left( \frac{\delta \Sigma}{\delta \Omega^a} \frac{\delta}{\delta A^a} + \frac{\delta \Sigma}{\delta A^a} \frac{\delta}{\delta \Omega^a} + \frac{1}{2} f^{abc} \frac{\delta \Sigma}{\delta \sigma^{bc}} \frac{\delta}{\delta c^a} + \frac{1}{2} f^{abc} \frac{\delta \Sigma}{\delta c^a} \frac{\delta}{\delta \sigma^{bc}} \right) \]

is nilpotent if the functional \( \Sigma \) is a solution, not only of the Slavnov identity (2.12), but also of the \( \tau \)-equation (2.19):

\[ S(\Sigma) = 0 \quad \Rightarrow \quad S^2 \Sigma = 0 . \]  

### 2.2 The functional algebra

Since BRS nilpotency holds only up to the \( \tau \)-equation (2.19), our first task in the next section will be to prove the validity of the latter to all orders. We thus will be interested, in the following, to the space of functionals \( \gamma \) restricted by:

\[ T_a \gamma = 0 \]  

In this space the functional operators introduced above obey a nonlinear algebra whose relevant part reads:

\begin{enumerate}
  \item \( S_\gamma S(\gamma) = 0 \),
  \item \( G_a S(\gamma) + S_\gamma (G_a \gamma - \Delta_a) = R^a_{\text{rig}} \gamma \),
  \item \( \frac{\delta}{\delta b^a} S(\gamma) - S_\gamma \left( \frac{\delta \gamma}{\delta b^a} - \partial A^a \right) = \bar{G}_a \gamma \),
  \item \( G_a S(\gamma) + S_\gamma \bar{G}_a = 0 \),
  \item \( T_a S(\gamma) = 0 \),
  \item \( \frac{\delta}{\delta b^a} (G_b \gamma - \Delta_b) - G_b \left( \frac{\delta \gamma}{\delta b^a} - \partial A^a \right) = 0 \),
  \item \( G_a \bar{G}_b \gamma + \bar{G}_b (G_a \gamma - \Delta_a) = f_{abc} \left( \frac{\delta \gamma}{\delta b^c} - \partial A^c \right) \),
  \item \( T_a (G_b \gamma - \Delta_b) = 0 \),
  \item \( \{ G_a, G_b \} = 0 \).
\end{enumerate}

### 3 Renormalization

We want to show that the functional identities of the preceding section hold to all orders for the vertex functional \( \Gamma \) – which coincides with the classical action \( \Sigma \) (2.10) in the tree graph approximation.
Let us consider generic classical identities
\[ \mathcal{F}_i \Sigma = 0 \ . \] (3.1)

According to the quantum action principle \[11, 12\], the possible breakings of such identities by the radiative corrections are given at their lowest nonvanishing loop order by local field functionals \( \Delta_i \) with quantum numbers and dimensions prescribed by those of the corresponding functional operators:
\[ \mathcal{F}_i \Gamma = \Delta_i + O(\hbar \Delta) \ . \] (3.2)

The functional operator algebra implies a set of consistency conditions on the \( \Delta \)'s which one has to solve. If the general solution has the form
\[ \Delta_i = \mathcal{F}_i \hat{\Delta} \quad \forall i \ , \] (3.3)
then \( \hat{\Delta} \) can be absorbed as a counterterm in the action, and the identities (3.1) are renormalizable.

The gauge fixing condition (2.14) and the antighost equation (2.15) are known to be renormalizable \[13\] and thus will be assumed to hold:
\[ \frac{\delta \Gamma}{\delta b^a} = \partial A_a \ , \quad \bar{G}_a \Gamma = 0 \ . \] (3.4)

The same can be assumed for the rigid invariance \[14\] (see (2.13)):
\[ \mathcal{R}^{\text{rig}}_a \Gamma = 0 \] (3.5)

### 3.1 The \( \tau \)-equation

The most general quantum breaking of the \( \tau \)-equation (2.19) – of dimension 0 and ghost number 3 – has the form
\[ Q_a = t_{abcd} c^b c^c c^d \ . \] (3.6)

Using (2.3) we can write it as
\[ Q_a = \frac{1}{2} \mathcal{T}_a \int d^4 x \ f^{def} \tau_{ef} t_{dghl} c^g c^h c^l \ . \] (3.7)

Hence the \( \tau \)-equation
\[ \mathcal{T}_a \Gamma = 0 \ . \] (3.8)

can be assumed to hold to all orders. As a consequence we shall be able to exploit the functional algebra displayed at the end of the preceding section.

\(^3\)In (3.1) we have omitted a possible classical breaking – i.e. a breaking linear in the quantum fields like in the ghost equation (2.16). If the operator \( \mathcal{F}_i \) is nonlinear, it is its linearized form (cf. (2.20)) which appears in (3.3).
3.2 The ghost equation

To discuss the renormalization of the ghost equation (2.16) let us write it as:

\[ G_a \Gamma = \Delta_a + \Xi_a , \quad (3.9) \]

where \( \Xi_a \) represents the possible breaking induced by the radiative corrections. \( \Xi_a \) is an integrated local functional with dimensions four and ghost number \(-1\) which, according to the nonlinear algebra (2.23), satisfies the conditions:

\[ \frac{\delta \Xi_a}{\delta b^r} = 0 , \quad G_c \Xi_a = 0 , \quad (3.10) \]

\[ \mathcal{R}^a_{\text{rig}} \Xi_b = -f_{abc} \Xi^c , \quad \mathcal{T}_a \Xi_b = 0 , \quad (3.11) \]

and

\[ G_a \Xi_b + G_b \Xi_a = 0 . \quad (3.12) \]

Equation (3.10) implies that \( \Xi_a \) is \( b \)-independent and depends on the fields \( \bar{c} \) and \( \Omega^\mu \) only through the combination

\[ \gamma_\mu = \partial_\mu \bar{c} + \Omega_\mu . \quad (3.13) \]

It follows then that \( \Xi_a \) can be parametrized as

\[ \Xi_a = \int d^4x \left( t_{abcde} \rho^{bc} c^d c^e + m_{abcde} \eta^{bc} c^d c^e f + r_{abcde} \omega^{bc} c^d c^e 
+ n_{abcde} r^{bc} c^d c^e + q_{abcde} \sigma^{bc} c^d + p_{abcde} \gamma^{b\mu} A^c_{\mu} + v Y T_a \phi \right) , \quad (3.14) \]

where, from (3.11), \((t, m, r, n, q, p)\) are invariant tensors in the adjoint representation, and

\[ \int m_{abcde} = 0 . \quad (3.15) \]

Since

\[ \int d^4x \ p_{abc} \gamma^{b\mu} A^c_{\mu} = G_a \int d^4x \ c^m p_{mbc} \gamma^{b\mu} A^c_{\mu} , \quad (3.16) \]

and

\[ \int d^4x \ Y T_a \phi = G_a \int d^4x \ c^m Y T_m \phi , \quad (3.17) \]

it follows that the nontrivial part of \( \Xi_a \) reduces to

\[ \Xi_a = \int d^4x \left( t_{abcde} \rho^{bc} c^d c^e + m_{abcde} \eta^{bc} c^d c^e f + r_{abcde} \omega^{bc} c^d c^e 
+ n_{abcde} r^{bc} c^d c^e + q_{abcde} \sigma^{bc} c^d + p_{abcde} \gamma^{b\mu} A^c_{\mu} \right) , \quad (3.18) \]

i.e. \( \Xi_a \) depends only on the variables \((c, \rho, \eta, \omega, r, \sigma)\) and does not contain any space-time derivative. To study the condition (3.12) on the local functional space (3.18)
we introduce a dimensionless space-time independent parameter $\xi^a$ with zero ghost number and we consider the operator
\[
D = \xi^a G_a ,
\] (3.19)
which, due to the relation $(ix)$ of equation (2.23), turns out to be nilpotent:
\[
D^2 = 0 .
\] (3.20)
The introduction of the operator $D$ allows to transform the equation (3.12) into a cohomology problem. Indeed, it is easy to check that if the general solution of the equation
\[
D X = 0 ,
\] (3.21)
where $X$ is an integrated local functional in the variable $(\xi, c, \rho, \eta, \omega, \tau, \sigma)$ with dimensions 4 and ghost number $-1$, is of the form
\[
X = D \hat{X} ,
\] (3.22)
then the general solution of equation (3.12) reads:
\[
\Xi_a = G_a \hat{\Xi} ,
\] (3.23)
which implies the absence of anomalies for the ghost equation (2.16). The most general form for the $X$-space is given by
\[
X = \int d^4x \left( L_{abcd}(\xi) \rho c^a c^b c^c c^d + R_{abdef}(\xi) \eta^{ab} c^e c^d c^e + S_{abdef}(\xi) \omega^{ab} c^e c^d + U_{abdef}(\xi) \tau^{ab} c^e c^d + V_{ab}(\xi) \sigma^{ab} c^d \right) ,
\] (3.24)
where the invariant tensors $L, R, S, U, V$ are power series in $\xi$. To characterize the cohomology of $D$ we introduce the filtering operator
\[
N = \xi^a \frac{\partial}{\partial \xi^a} + \int d^4x \left( c^a \frac{\delta}{\delta c^a} + \rho \frac{\delta}{\delta \rho} + \frac{1}{2} \left( \eta^{ab} \frac{\delta}{\delta \eta^{ab}} + \omega^{ab} \frac{\delta}{\delta \omega^{ab}} \right) + \frac{1}{2} \left( \tau^{ab} \frac{\delta}{\delta \tau^{ab}} + \sigma^{ab} \frac{\delta}{\delta \sigma^{ab}} \right) \right) ,
\] (3.25)
according to which, the operator $D$ decomposes as
\[
D = D^0 + D^1 ,
\] (3.26)
where
\[
D^0 = \int d^4x \ \xi^a \frac{\delta}{\delta c^a} , \quad D^0 D^0 = 0 .
\] (3.27)
It is now apparent that, due to the absence of space-time derivatives in the general expression (3.24), the cohomology of $D^0$ vanishes and, since the cohomology of $D$ is isomorphic to a subspace of the cohomology of $D^0$ [13], it follows that also $D$ has trivial cohomology on $X$. This proves that the ghost equation (2.16) can be implemented to all orders of perturbation theory.
3.3 The Slavnov identity

Having renormalized the $\tau$ and the ghost equations, let us discuss the quantum extension of the Slavnov identity (2.12). From the nonlinear algebra (2.23) one has that the breaking of the Slavnov identity

$$S(\Gamma) = \bar{h}^n \Delta + O(\bar{h}^{n+1}) ,$$

has to satisfy the conditions:

$$\frac{\delta \Delta}{\delta b^\mu} = 0 , \quad G_c \Delta = 0 ,$$

$$R_a^\mu \Delta = 0 , \quad T_a \Delta = 0 , \quad G_a \Delta = 0 ,$$

and

$$S_\Sigma \Delta = 0 ,$$

where $S_\Sigma$ is the linearized operator defined in (2.20) and $\Delta$ is an integrated local polynomial in the fields with ghost-number 1 and dimension four. The index $n$ appearing in (3.28) denotes the lowest nonvanishing order for $\Delta$.

As in the previous section, the conditions (3.29) imply that $\Delta$ is $b$-independent and that the fields $\bar{c}$ and $\Omega_{\mu}$ enter only in the combination $\gamma_{\mu}$ (3.13) so that $\Delta$ can be parametrized as:

$$\Delta = A(c, A_\mu, \phi) + \int d^4x \left( M_{abcdef} \rho c^a b^c d^e f^g + R_{abcdefg} \eta^{ab} c^a b^e f^g + S_{abcdef} \omega^{ab} c^a b^e f^g + U_{abcdef} \tau^{ab} c^a b^e f^g + \mathcal{V}_{abcdef} \sigma^{ab} c^a b^e f^g + L_{abcdef} \gamma^{ab} \gamma_{\mu} A_\mu^d + \alpha f_{abc} c^a b^d \gamma_{\mu} + \beta d_{abc} c^a b^e \partial_{\mu} \gamma_{\mu} + \lambda f_{abc} c^a b^e c^b Y T_c \phi \right) ,$$

where $M$, $R$, $S$, $U$, $V$ and $L$ are invariant tensors in the adjoint representation and $A(c, A_\mu, \phi)$ depends only on the ghost $c$, on the gauge field $A_\mu$ and on the matter fields $\phi$. It is easy to see that the ghost condition in (3.30) implies that

$$\alpha = \beta = \lambda = 0 ,$$

and

$$\int d^4x \frac{\delta A}{\delta c^a} = 0 ,$$

from which it follows that $A$ depends only on the space-time derivatives of $c$.

Finally, the Slavnov condition (3.31) reads:

$$\int d^4x \text{Tr} \left( - (D_\mu c) \frac{\delta}{\delta A_\mu} - i c^2 \frac{\delta}{\delta c} - ic\phi \frac{\delta}{\delta \phi} \right) A = 0 .$$
The most general solution of \((3.36)\) has been given in \[14\] and coincides, modulo a \(\mathcal{S}_\Sigma\) coboundary, with the well known expression for the gauge anomaly \[16\]:

\[
\mathcal{A} = \varepsilon^{\mu\nu\rho\sigma} \int d^4x \partial_\mu c^a \left( d_{abc} \partial_\nu A^b_\rho A^c_\sigma - \frac{D_{abcd}}{12} A^b_\nu A^c_\rho A^d_\sigma \right), \tag{3.37}
\]

with

\[
D_{abcd} = d_{ab}^n f_{ncd} + d_{ac}^n f_{ndb} + d_{ad}^n f_{nbc}, \tag{3.38}
\]

To summarize this section one has that the vertex functional \(\Gamma\)

\[
\Gamma = \Sigma + O(\bar{h}), \tag{3.39}
\]

obeys:

i) the gauge-fixing condition and the antighost equation

\[
\frac{\delta \Gamma}{\delta b_a} = \partial A^a, \quad \bar{G}_a \Gamma = 0, \tag{3.40}
\]

ii) the rigid gauge invariance and the \(\tau\) equation

\[
\mathcal{R}^{\text{rig}}_a \Gamma = 0, \quad \mathcal{T}_a \Gamma = 0, \tag{3.41}
\]

iii) the ghost equation

\[
\mathcal{G}_a \Gamma = \Delta_a, \tag{3.42}
\]

iv) the anomalous Slavnov identity

\[
\mathcal{S}(\Gamma) = \bar{h}^n r \mathcal{A} + O(\bar{h}^{n+1}), \tag{3.43}
\]

where \(r\) is a coefficient directly computable in terms of Feynman diagrams. Moreover, as we shall see in the next sections, the coefficient \(r\) turns out to satisfy a nonrenormalization theorem \[8, 9, 10\].

### 4 The Callan-Symanzik equation.

This section is devoted to the derivation of the Callan-Symanzik equation obeyed by the vertex functional \(\Gamma\).

Equations \((3.40) - (3.43)\) show that, besides the gauge fixing condition, the antighost equation and the \(\tau\) equation, the functional \(\Gamma\) obeys the ghost equation
This equation governs the dependence of $\Gamma$ on the ghost field $c$ and will impose quite strong constraints on the Callan-Symanzik equation; it will imply the absence of anomalous dimensions for the ghost and for the cocycle $c^{(5)}$ of equation (2.6).

Moreover, the presence of the gauge anomaly at the order $\hbar^n$ in the Slavnov identity (3.43) does not allow for a Callan-Symanzik equation which is invariant to all orders of perturbation theory: one expects a Slavnov invariant Callan-Symanzik equation up to the order $\hbar^{n-1}$.

However, it will be shown that the Callan-Symanzik equation will extend to the order $\hbar^n$, which is the order of the gauge anomaly; this property will be crucial for the nonrenormalization theorem of the anomaly coefficient $r$ in (3.43) [8, 9, 10].

To characterize the scaling properties of the model we look for a basis of insertions which are invariant under the set of equations satisfied by the functional $\Gamma$: i.e. we look at the most general integrated local polynomial in the fields $\hat{\Sigma}_{loc}$ with dimension four and zero ghost number which satisfies the stability conditions [6, 13]:

$$
\frac{\delta \hat{\Sigma}_{loc}}{\delta \psi^c} = 0, \quad \check{G}_c \hat{\Sigma}_{loc} = 0,
$$

(4.1)

$$
R^\text{rig}_a \hat{\Sigma}_{loc} = 0, \quad T_a \hat{\Sigma}_{loc} = 0, \quad \check{G}_a \hat{\Sigma}_{loc} = 0,
$$

(4.2)

and

$$
S_{\Sigma} \hat{\Sigma}_{loc} = 0.
$$

(4.3)

Proceeding as in the previous sections, it is not difficult to show that the most general expression for $\hat{\Sigma}_{loc}$ reads:

$$
\hat{\Sigma}_{loc} = -\frac{z_g}{4g^2} \int d^4x \, F^{a\mu}_a F^a_{\mu\nu} + S_{\Sigma} \int d^4x \left( z_A \gamma^a A_{a\mu} - z_\phi Y \phi \right) + \sum_i z_i \hat{\Sigma}_i(\phi),
$$

(4.4)

where $z_g$, $z_A$, $z_\phi$ and $z_i$ are arbitrary parameters and the $\hat{\Sigma}_i$ are all possible invariant matter self interactions. Expression (4.4) represents the most general invariant counterterm. Note that there is no term in the external fields $\rho$, $\eta$, $\omega$, $\tau$ and $\sigma$: this is due to the last condition (4.2). Thus one sees that the conditions (3.40) – (3.43) determine the model up to a renormalization of the gauge coupling constant $g$ (given by $z_g$), of the matter self couplings (given by $z_i$) and to a redefinition of the gauge field amplitude ($z_A$) and of the matter fields ($z_\phi$).

Expression (4.4) can be rewritten as

$$
\hat{\Sigma}_{loc} = -z_g g^2 \frac{\partial \Sigma}{\partial g^2} + z_A \check{N}_A \Sigma + z_\phi \check{N}_\phi \Sigma + \sum_i z_i \lambda_i \frac{\partial \Sigma}{\partial \lambda_i},
$$

(4.5)

where $\Sigma$ is the classical action (2.10) and $\check{N}_A$, $\check{N}_\phi$ are the Slavnov invariant counting operators defined by

$$
\check{N}_A = \int d^4x \left( A^{a\mu} \frac{\delta}{\delta A_{a\mu}} - b^a \frac{\delta}{\delta b^a} - c^a \frac{\delta}{\delta c^a} - \Omega^{a\mu} \frac{\delta}{\delta \Omega^{a\mu}} \right),
$$

(4.6)
\[ N_{\phi} = \int d^4x \left( \phi \frac{\delta}{\delta \phi} - Y \frac{\delta}{\delta Y} \right). \] 

and \( \lambda_i \) are the matter self-coupling constants.

It is apparent from (4.3) that the set
\[
\left( \frac{\partial \Gamma}{\partial g}, \frac{\partial \Gamma}{\partial \lambda_i}, \mathcal{N}_A \Gamma, \mathcal{N}_{\phi} \Gamma \right),
\]
can be taken as a basis for the insertions of dimensions four and ghost number zero which are Slavnov invariant and which are compatible with the gauge fixing condition and the antighost equation (3.41) as well as with the \( \tau \) and the ghost equations (3.41) – (3.42).

The insertion defined by \( \frac{\partial \Gamma}{\partial \mu} \), where \( \mu \) is the normalization point, obeys to the same conditions. Expanding it in the basis (4.8) we get then the Callan-Symanzik equation
\[
C \Gamma = \mu \frac{\partial}{\partial \mu} + \bar{h} \beta_g \frac{\partial}{\partial g} + \bar{h} \sum_i \beta_i \frac{\partial}{\partial \lambda_i} + \bar{h} \gamma_A \mathcal{N}_A + \bar{h} \gamma_{\phi} \mathcal{N}_{\phi}) \Gamma
\]
\[
\quad = \bar{h}^n \Delta^n_c + O(\bar{h}^{n+1}) ,
\]
where \( \Delta^n_c \) is an integrated local polynomial corresponding to the lack of the Slavnov invariance at the order \( \bar{h}^n \) according to equation (3.43).

Notice that the validity of the ghost equation (3.42) has led to the absence of anomalous dimensions for the ghost field \( c \).

From the algebraic relation
\[
C S(\Gamma) = S_c C \Gamma ,
\]

it follows that, applying the Callan-Symanzik operator to the anomalous Slavnov identity (3.43) and retaining the lowest order in \( \bar{h} \), one gets the equation:
\[
S_{\Sigma} \Delta^n_c = \mu \frac{\partial r}{\partial \mu} A ,
\]
which, due to the fact that \( r \) is a dimensionless coefficient, implies that
\[
S_{\Sigma} \Delta^n_c = 0 .
\]

This condition tells us that the local breaking \( \Delta^n_c \), in spite of the presence of the gauge anomaly, is Slavnov invariant and that it can be expanded in the basis (4.8). This allows the extension of the Callan-Symanzik equation in a Slavnov invariant way up to the order \( \bar{h}^n \), i.e.:
\[
C \Gamma = \bar{h}^{n+1} \Delta_c^{n+1} + O(\bar{h}^{n+2}) ,
\]
with $\Delta_c^{n+1}$ local.

Deriving this equation with respect to the source $\rho(x)$ we get

$$C[c^{(5)} \cdot \Gamma] = O(\hbar^{n+1}) ,$$

(4.14)

which expresses the finiteness, i.e. the vanishing of its anomalous dimension, of the ghost cocycle $\mathcal{G}$, whose quantum extension is defined by

$$[c^{(5)} \cdot \Gamma] = \frac{\delta \Gamma}{\delta \rho} .$$

(4.15)

5 The descent equations

The purpose of this section is to show that the gauge anomaly operator (3.37), here written in terms of differential forms

$$A(A, c) = \int \omega_{4}^{1} ,$$

$$\omega_{4}^{1} = \frac{1}{2} \text{Tr} (dc(AdA + dAA - iA^{3})) ,$$

(5.4)

can be renormalized in such a way that it obeys a Callan-Symanzik equation with vanishing anomalous dimension. This will follow from the vanishing of the anomalous dimension of the cocycle insertion (4.15) proved in Section 4.

Throughout this section we shall assume the validity of the gauge condition and antighost equation (3.40), and of the rigid invariance. This means that we shall deal with functionals of $A, \phi, c, \gamma$ (see (3.13)), $Y$ and $\sigma$ which is the external field coupled to the BRS variation of $c$, see Eq. (5.11) below.

---

4The symbole $\text{Tr}$ means the normalized trace

$$\text{Tr} (T_a T_b) = \delta_{ab} .$$

(5.1)

The symmetric tensor $d_{abc}$ in (2.3) is defined by

$$d_{abc} = \frac{1}{2} \text{Tr} (T_a \{T_b, T_c\}) .$$

(5.2)

$\omega_{q}^{p}$ indicates a $p$-form of ghost number $q$, the ghost number of $c$ being equal to 1. The gauge 1-form is $A = A_\mu dx^\mu$, its BRS transformation reads

$$sA = dc - i[c, A] .$$

(5.3)

By convention $d\omega = dx^\mu \partial_\mu \omega$ for any form $\omega$, and $s$ anticommutes with the exterior derivative $d$. Brackets $[,]$ denote graded commutators.
5.1 Renormalization of the anomaly operator

The starting point is the relation of the ghost cocycle (2.6) to the gauge anomaly through the descent equations [1]

\[ s\omega^q_{0-q} + d\omega^{q+1}_{4-q} = 0 \quad , \quad q = 1, \cdots, 4 \ , \]
\[ s\omega^5_0 = 0 \ . \]

where

\[ \omega^2_3 = - \text{Tr} \ ( (dc)^2 A) \ , \]
\[ \omega^3_2 = \text{Tr} \ ( (dc)^2 c) \ , \]
\[ \omega^4_1 = - \frac{i}{2} \text{Tr} \ (dc c^3) \ , \]
\[ \omega^5_0 = - \frac{1}{10} \text{Tr} \ (c^5) . \]

(5.5)

We have rewritten \( c^{(5)} \) in the matrix notation as

\[ c^{(5)} = \omega^5_0 . \]

(5.6)

What we need is a ladder of operator insertions \( \{ \Omega^q_{5-q} \} \) which is a quantum extension of the classical ladder \( \{ \omega^q_{5-q} \} \) and obeys the quantum descent equations – valid to the same order as the Slavnov identity (3.43) since the nilpotency of \( S_\Gamma \) is broken by the anomaly:

\[ S_\Gamma \Omega^q_{5-q} + d\Omega^{q+1}_{4-q} = O(h^n) \ , \quad q = 1, \cdots, 4 \ , \]
\[ S_\Gamma \Omega^5_0 = O(h^n) . \]

(5.8)

The renormalized anomaly operator insertion will be defined as

\[ A_R = \int \Omega^1_4 \]

(5.9)

and by

\[ \Omega^5_0 = [c^{(5)} \cdot \Gamma] \]

(5.10)

which is the finite insertion constructed in Section [1].

In order to construct the renormalized insertions \( \Omega^q_{5-q} \) we introduce external fields \( u_{q-1} \), with \( u_{q-1} \) a \((q - 1)\)-form of ghost number \(-q\), singlet of the gauge group [3]. We therefore start now with the classical action

\[ \Sigma = \Sigma_{\text{Landau}} + \int d^4x \left( \text{Tr} (\gamma^\mu sA_\mu + Y s\phi + \sigma sc) \\
+ \sum_{q=1}^5 u_{q-1} \omega^q_{5-q} \right) , \]

(5.11)

with \( \Sigma_{\text{Landau}} \) given by (2.5). We have discarded the external fields used in Section [4] for controlling the finiteness of \( c^{(5)} \), but we have introduced the external field \( \sigma \)
coupled to the BRS variation of $c$. This action is BRS-invariant, the external fields $u$ transforming as:

\[ su_0^{-1} = 0 , \]
\[ su_q^{-1} = -du_{q-2}^{-1} , \quad q = 2, \ldots, 5 . \]  

(5.12)

The classical Slavnov identity now reads

\[
\mathcal{S}(\Sigma) = \int d^4 x \left( \text{Tr} \left( \frac{\delta \Sigma}{\delta \Omega^\mu} \frac{\delta \Sigma}{\delta A_\mu} + \frac{\delta \Sigma}{\delta \sigma} \frac{\delta c}{\delta c} + b \frac{\delta \Sigma}{\delta c} \right) + \frac{\delta \Sigma}{\delta Y} \frac{\delta \phi}{\delta \phi} + \sum_{q=2}^5 su_{q-1}^{-1} \frac{\delta \Sigma}{\delta u_{q-1}^{-1}} \right) = 0 ,
\]

(5.13)

with a corresponding nilpotent linearized form (cf. (2.20)). The rigid Ward identity (2.13), the gauge condition (2.14) and the antighost equation (2.15) are left unchanged and will again be assumed to hold to all orders for the vertex functional $\Gamma$.

The construction of a ladder of insertions fulfilling the quantum descent equations (5.8) will be achieved by showing the validity of the Slavnov identity (5.13) (anomalous at the order $n$):

\[
\mathcal{S}(\Gamma) = h^n r A + O(h^{n+1}) .
\]

(5.14)

We have thus to show the triviality of the cohomology of $\mathcal{S}_\Sigma$ in the sector of ghost number 1 for the $u$-dependent integrated local functionals

\[
\Delta = \int \chi_4^1 .
\]

(5.15)

The condition $\mathcal{S}_\Sigma \Delta = 0$ and the triviality [17, 3] of the cohomology of $d$ mean that the 4-form $\chi_4^1$ belongs to a ladder of forms obeying descent equations similar to (5.3):

\[
\mathcal{S}_\Sigma \chi_{5-q}^q + d \chi_{4-q}^{q+1} = 0 , \quad q = 1, \cdots, 4 ,
\]
\[
\mathcal{S}_\Sigma \chi_0^5 = 0 .
\]

(5.16)

We have first to solve the last of these equations, which is a problem of local cohomology (see Def. A.2 in Appendix A). Then we have to solve the remaining equations, for increasing form degree, which again is a problem of local cohomology.

In the absence of the external fields $u$ the local cohomology depends only on the invariants $\text{Tr} c^n$ and on those made with the Yang-Mills curvature $F$ and its derivatives [15]. This uniquely leads to the ladder (5.3) and to the chiral anomaly (5.4).

It is shown in the Appendix that the dependence of the cohomology on the external fields $u$ occurs only through the zero-form $u_0^{-1}$. The most general $u$-dependent
expression for $\chi_0^5$ in the local cohomology would be the superposition of the monomials $u_0^{-1} \text{Tr} c^6$ and $u_0^{-1} (\text{Tr} c^3)^2$, but they vanish due to the cyclicity of the trace and the anticommutativity of the ghost fields. This proves the absence of $u$-dependent anomalies, hence of any obstruction to the construction of the renormalized ladder (5.8).

### 5.2 Callan-Symanzik equation for the anomaly

In order to prove the vanishing of the anomalous dimension for the renormalized anomaly operator defined in the preceding subsection (see (5.9)), we have to repeat in presence of the $u$-fields the construction of the Callan-Symanzik equation performed in Section 4, and to use the fact that the $c^{(5)}$ insertion (5.10) has no anomalous dimension.

The basis of classical invariant insertions used in Eq. (4.4) must be completed by adding to it all BRS invariants of the form

$$\Delta = \int \chi_4^0 ,$$

(5.17)

where $\chi_4^0$ is a 4-form of zero ghost number, depending on the $u$’s. Like in the case Eqs. (5.13) and (5.16), the latter belongs to a ladder obeying the descent equations:

$$S_\Sigma \chi_{4-q}^q + d\chi_{3-q}^{q+1} = 0 , \quad q = 0, \cdots, 3 ,$$

$$S_\Sigma \chi_4^0 = 0 .$$

(5.18)

The most general nontrivial expression for $\chi_4^0$ depending only on $u_0^{-1}$ and on $c$ according to the proposition of Appendix A, is $u_0^{-1} \text{Tr} c^5$. It leads to the expression, unique modulo $S_\Sigma$:

$$\int \chi_4^0 = \int \sum_{q=1}^5 u_{q-1}^{-q} \omega_q^{q} = N_u \Sigma ,$$

(5.19)

with the $u$-counting operator defined by:

$$N_u = \int \sum_{q=1}^5 u_{q-1}^{-q} \frac{\delta}{\delta u_{q-1}} .$$

(5.20)

The second equality in (5.19) follows from the observation that its left-hand side consists just of the $u$-dependent terms of the action (5.11). The most general form for the classical invariant $\Delta$ thus is a linear superposition of (5.19) and of the $S_\Sigma$-variation of an arbitrary local field polynomial. The quantum basis (4.8) can accordingly be completed by adding to it the insertion $N_u \Gamma$ and the $S_\Gamma$-variation of an arbitrary insertion of ghost number $-1$. This implies that the Callan-Symanzik equation in presence of the fields $u$ will take the form

$$C_{(u)} \Gamma = (C + \hbar \gamma_u N_u) \Gamma = \hbar S_\Gamma [\hat{\Delta} \cdot \Gamma] + O(\hbar^{n+1}) ,$$

(5.21)
where $\hat{\Delta}$ is some $u$-dependent insertion of ghost number $-1$; $C$ is the Callan-Symanzik operator defined in (4.9). The fact that the anomaly starts to produce effects only at the order $n + 1$ follows from the same argument used in Section 4.

Differentiation of (5.21) with respect to $u^{-5}_{-4}$ and use of the anticommutativity of this derivative with the linearized Slavnov operator $S_{\Gamma}$, yields the Callan-Symanzik equation for the insertion of the cocycle $c^{(5)}$ with anomalous dimension $\gamma_u$:

$$
(C + h\gamma_u)[c^{(5)} \cdot \Gamma] = -hS_{\Gamma}\left(\frac{\delta}{\delta u^{-5}_{-4}}[\hat{\Delta} \cdot \Gamma]\right) + O(h^{n+1}) = O(h^{n+1}).
$$

The last equality follows from the fact that the most general expression for the $u^{-5}_{-4}$-dependent part of $\hat{\Delta}$ reads $u^{-5}_{-4}\text{Tr}c^4$, which identically vanishes. But since the insertion $c^{(5)}$ was shown to have zero anomalous dimension, we conclude that

$$
\gamma_u = 0.
$$

(5.23)

Differentiating now (5.21) with respect to $u^{-1}_{-0}$ and integrating over space-time we get the Callan-Symanzik equation

$$
CA_R = -hS_{\Gamma}\int \frac{\delta}{\delta u^{-1}_{-0}}[\hat{\Delta} \cdot \Gamma] + O(h^{n+1}),
$$

(5.24)

for the anomaly insertion, without anomalous dimension as announced, up to an irrelevant $S_{\Gamma}$-variation following from the right-hand side of (5.21) and from the anticommutativity of $\int \delta/\delta u^{-1}_{-0}$ with $S_{\Gamma}$.

6 The nonrenormalization theorem of the gauge anomaly

As an application of the finiteness properties displayed by the quantum insertions $\{\Omega_{5-q}\}$ of (5.8), let us discuss the nonrenormalization theorem of the gauge anomaly.

Following [10], we can extend the anomalous Slavnov identity (3.43) to the order $h^{n+1}$ as:

$$
S(\Gamma) = h^n r A_R + h^{n+1} B + O(h^{n+2}),
$$

(6.1)

where $A_R$ is the renormalized anomaly operator insertion defined in (5.9) and $B$ is an integrated local functional of ultraviolet dimension four and ghost number one.

Applying the Callan-Symanzik operator to both sides of equation (6.1) and using the Callan-Symanzik equation without anomalous dimension (5.24) for $A_R$ one gets, to lowest order (i.e. order $n + 1$) in $h$, the equation:

$$
\left(\beta_{\mu}^{(1)} \frac{\partial r}{\partial g} + \sum_i \beta_{\lambda_i}^{(1)} \frac{\partial r}{\partial \lambda_i}\right) A = \mu \frac{\partial B}{\partial \mu} + S_{\Gamma}\hat{\Delta},
$$

(6.2)
where $A$ is the gauge anomaly expression (5.4), $\beta_{g}^{(1)}$ and $\beta_{i}^{(1)}$ are respectively the one-loop beta functions for the gauge and the self matter couplings and $\bar{\Delta}$ is a local integrated functional.

Taking into account that $B$ is homogeneous of degree zero in the mass parameters [10], i.e.:
\[ \mu \frac{\partial B}{\partial \mu} = 0 , \] (6.3)
and that the gauge anomaly $A$ cannot be written as a local $S_{\Sigma}$-variation one has the condition:
\[ \beta_{g}^{(1)} \frac{\partial r}{\partial g} + \sum_{i} \beta_{i}^{(1)} \frac{\partial r}{\partial \lambda_{i}} = 0 , \] (6.4)
which, in the generic case $\beta_{g}^{(1)} \neq 0$ and $\beta_{i}^{(1)} \neq 0$, implies [10] that if the coefficient $r$ vanishes at one loop order it will vanish to all orders.

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A Local cohomology for complete ladder fields

Let $\mathcal{V}$ be the space of nonintegrated local field functionals, the fields being defined on some differential manifold, and let us assume a coboundary operator $\delta$ acting on $\mathcal{V}$, nilpotent and anticommuting with the exterior derivative $d$.

**Definition A.1** A "complete ladder field" on a $D$-dimensional differentiable manifold is a set $\mathcal{L}^{(Q)} = \{u_{p}^{Q-p} | p = 0, \cdots, D\}$ of $D + 1$ differentiable forms, where $Q$ is some fixed algebraic integer. The coboundary operator $\delta$ acts on the $u$’s as:
\[ \delta u_{0}^{Q} = 0 , \]
\[ \delta u_{p}^{Q-p} = -du_{p-1}^{Q-p+1} , \quad p = 1, \cdots, D . \] (A.1)

**Definition A.2** The "local cohomology" space is the set of equivalence classes modulo $\delta$ of solutions $\mathcal{Q} \in \mathcal{V}$ of the equation
\[ \delta \mathcal{Q} = 0 . \] (A.2)

---

\[^{5}u_{p}^{Q}\] is a form of degree $p$ and ghost number $q$.  

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Proposition A.1 The local cohomology space depends on the complete ladder field $\mathcal{L}^Q$ only through its element $u^Q_0$, not derivated.

The proof is a slight generalization of the one given in Ref. [3]. The space of local functionals we have to consider consists of all the polynomials in the symmetric and antisymmetric derivatives of the form components $u^Q_{[\mu_1\cdots\mu_p]}$:

\[
S^{Q-p}_{\mu_1\cdots\mu_p,\nu_1\cdots\nu_r} = \partial_{\nu_1} \cdots \partial_{(\nu_r} u^Q_{\mu_1\cdots\mu_p)} , \quad p = 1, \cdots D; \ r \geq 0 ,
A^{Q-p}_{\mu_1\cdots\mu_p,\nu_1\cdots\nu_r} = \partial_{\nu_1} \cdots \partial_{(\nu_r} u^Q_{[\mu_1\cdots\mu_p)} , \quad p = 1, \cdots D; \ r \geq 0 ,
A^{Q}_{\nu_1\cdots\nu_r} = \partial_{\nu_1} \cdots \partial_{\nu_r} u^Q , \quad r \geq 1 ,
\]

(A.3)

The action of $\delta$ on these derivatives reads

\[
\delta S^{Q-p}_{\mu_1\cdots\mu_p,\nu_1\cdots\nu_r} = A^{Q-p+1}_{\mu_1\cdots\mu_{p+1},\nu_1\cdots\nu_r} , \quad p = 1, \cdots D; \ r \geq 0 ,
\delta A^{Q-p}_{\mu_1\cdots\mu_p,\nu_1\cdots\nu_r} = 0 , \quad p = 1, \cdots D; \ r \geq 0 ,
\delta u^Q = 0 .
\]

(A.4)

One sees that all fields and their derivatives – except the undifferentiated 0-form $u^Q$ – are displayed in BRS doublets, i.e. in couples $(U, V = \delta U)$. It is known [3, 18] that the cohomology cannot depend on such couples. This ends the proof of the proposition.

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