Influence of Adoptment of Geocentric 3D Coordinate System on Chinese Existent Maps

CHEN Junyong  DANG Yamin

ABSTRACT If a geocentric 3D coordinate system is adopted in China to replace 2D non-geocentric coordinate system, the coordinates of the surface points will be changed accordingly. The influences on the current maps of China, especially the topographic maps, are discussed due to the replacement of the coordinate systems. Taking the replacement of Xi'an 80 coordinate system by GRS80 for a numerical example, this paper analyzes the changes of latitude, longitude, Gauss plane coordinates of the surface points, as well as the orientation and the length changes between the points on the map, including the changes of map border lines and sheet corner points.

KEY WORDS replacement of coordinate system; map; geocentric 3D coordinate system

CLC NUMBER P226.3

Introduction

If a geocentric 3D coordinate system is adopted in China, the coordinates for the surface points in the new coordinate system will be different from those in the original system, i.e. the coordinate of those points will change due to the coordinate system replacement. The influence on the geodetic coordinates and Gauss plane coordinates of surface points in Chinese territory, as well as those of map sheet corner points will be discussed in this paper, for example, as the Xi'an 80 geodetic coordinates system is replaced by the Geodetic Reference System 1980 (GRS80) (1). Besides, the orientation and the length changes of the lines on the map, including the changes of map border lines are also described. Thereafter it is possible to estimate the influence on the existent maps of China caused by the replacement of the mentioned coordinate system in China.

1 Changes of geodetic longitude and latitude due to the replacement of coordinate system

If the Xi'an 80 coordinate system is replaced by GRS80 geocentric 3D coordinate system, the change range of geodetic latitude for surface points in China is from $-1.6^\circ$ to $+0.7^\circ$. The rule for the absolute values of latitude changes generally is from small to large, when the location of surface points moves either northward (i.e. the geodetic latitude values of surface points from small to large) or westward (i.e. the geodetic longitude values of surface points from large to small). The average absolute value of the geodetic latitude changes in China caused by the replacement of the coordinate system is $0.57^\circ$. The geodetic latitude changes of surface points in China territory due to the replacement of Xi'an 80 coordinate system by GRS80 are shown in Table 1.
Table 1 Change of geodetic latitude of the surface points in China territory caused by the replacement of Xi'an 80 coordinate system by GRS80 coordinate system

| Longitude L/(°) | 72 | 78 | 84 | 90 | 96 | 102 | 108 | 114 | 120 | 126 | 132 |
|----------------|----|----|----|----|----|-----|-----|-----|-----|-----|-----|
| dBGeodetic/(°) | -1.6 | -1.3 | -1.2 | -1.0 | -0.7 | -0.5 | -0.2 | -0.1 | +0.1 | +0.4 | +0.7 |

Note: The longitude L is according to that of 1 : 1 M scale map sheet corners. dB is the average value of geodetic latitude changes along the corresponding longitude of the map border line.

Due to the replacement of the coordinate system, the range of geodetic longitude change for surface points in China territory is from -6.4° to -3.0°. The rule for the absolute values of longitude change generally is from small to large, when the location of surface points moves either eastward (i.e., the geodetic longitude values of surface points from small to large) or northward (i.e., the geodetic latitude values of surface points from small to large). The average absolute value of the geodetic longitude changes in China caused by the replacement of the coordinate system is 4.4°. The geodetic longitude change of surface points in China due to the replacement of Xi'an 80 coordinate system by GRS80 is shown in Table 2.

Table 2 Change of geodetic longitude of the surface points in China territory caused by the replacement of Xi'an 80 coordinate system by GRS80 coordinate system

| Longitude L/(°) | 72 | 78 | 84 | 90 | 96 | 102 | 108 | 114 | 120 | 126 | 132 |
|----------------|----|----|----|----|----|-----|-----|-----|-----|-----|-----|
| dL/(°)         | -3.1 | -3.4 | -3.8 | -4.1 | -4.3 | -4.5 | -4.5 | -4.5 | -4.7 | -4.9 | -5.3 |

Note: The longitude L is according to that of 1 : 1 M scale map sheet corners in China; dL is the average value of geodetic longitude changes along the corresponding longitude of the map border line.

Beijing 54 coordinate system is still employed by many Chinese maps. In case that this coordinate system is replaced by GRS80 geocentric 3D coordinate system, then the changes of geodetic latitude and longitude of surface points in Chinese territory, according to the estimation data in Reference [2], are as described below. The change of geodetic latitude is in the range from -1.5° to +3.0°, and the average absolute value of the changes is 1.3°. The change of geodetic longitude is in the range from -4.0° to +6.0°, and the average absolute value of the change is 2.1°.

2 Changes of Gauss plane coordinates due to the replacement of coordinate system

If the Xi'an 80 coordinate system is replaced by GRS80 geocentric 3D coordinate system, the plane coordinate of surface points in Gauss projection will be different from those before the replacement of the coordinate system. The changes of Gauss plane ordinate value X for surface points in Chinese territory caused by the replacement of the coordinate system will be discussed firstly.

The change range of Gauss plane ordinate values X for surface points in Chinese territory, of course including the corresponding map sheet corner points, is from -48.6 m to 22.7 m, due to the replacement of coordinate system. The average absolute value of the changes for X is 16.31 m, and the rule for the change of X value is similar to that of the geodetic latitude mentioned above.

The change range of Gauss plane ordinate values X (in 1:50 000 and 1:10 000 scale maps) for the surface points in China territory is shown in Table 3.

The change range of Gauss plane abscissa values Y for surface points in Chinese territory, of

Table 3 Change of Gauss plane ordinate values X of the surface points in China territory caused by the replacement of Xi'an 80 coordinate system by GRS80 coordinate system

| Longitude L/(°) | 72 | 78 | 84 | 90 | 96 | 102 | 108 | 114 | 120 | 126 | 132 |
|----------------|----|----|----|----|----|-----|-----|-----|-----|-----|-----|
| dX/m           | -45.3 | -37.7 | -35.5 | -29.7 | -19.9 | -14.3 | -5.3 | -2.7 | +3.5 | +16.4 | +20.4 |
| 1 : 50 000/mm  | -0.9 | -0.8 | -0.7 | -0.6 | -0.4 | -0.3 | -0.1 | -0.0 | +0.0 | +0.2 | +0.4 |
| 1 : 10 000/mm  | -4.5 | -3.8 | -3.6 | -3.0 | -2.0 | -1.4 | -0.5 | -0.3 | +0.4 | +1.0 | +2.0 |

Note: The longitude L is according to that of 1:1 M scale map sheet corners in China; dX is the average value of the change of Gauss plane ordinate values X along the corresponding longitude of the map border line.
course including the corresponding map sheet corner points, is from −115.7 m to −75.7 m, due to the replacement of coordinate system. The average absolute value of the abscissa Y values changes is 106.9 m. The rule for the absolute change value of Y is from small to large gradually, when the location of surface point is from west to east (i.e., its longitude value from small to large), and it seems that there is no close relationship between the change value of Y and the geodetic latitude of the studied surface point when the coordinate replacement happen.

The change of Gauss plane abscissa values Y (in 1:50 000 and 1:10 000 scale maps) for the surface points in China territory is shown in Table 1.

| Longitude L(°) | 72  | 78  | 84  | 90  | 96  | 102 | 108 | 114 | 120 | 126 | 132 |
|---------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| dY m          | −75.8 | 81.1 | −92.3 | −94.0 | −101.8 | −109.3 | −112.7 | −114.8 | −115.7 | −115.2 | −113.7 |
| 1:50 000 mm   | −1.5 | −1.7 | −1.8 | 2.0 | −2.1 | −2.2 | −2.3 | −2.3 | −2.3 | −2.3 | −2.3 |
| 1:10 000 mm   | −7.6 | −8.4 | −9.2 | −9.9 | −10.3 | −10.9 | −11.3 | −11.5 | −11.6 | 11.5 | −11.4 |

Note: The longitude L is according to that of 1:1 M scale map sheet corners in China, dY is the average value of the change of Gauss plane abscissa values Y along the corresponding longitude of the map border line.

### 3 Orientation changes of the map border lines due to the replacement of coordinate system

The orientation and length of a line between any two points on a map will change due to the replacement of the coordinate system. Now the map border line can be taken as an example to explain the change. As the longitude and latitude of the four map sheet corner points (as well as their Gauss plane coordinates) in a same map sheet are different from each other, so the changes of those coordinates mentioned above are different from each other after the replacement of the coordinate system. Hence, the orientation and length of a map border lines will change as they are constructed by the map sheet corner points. The changes for the orientation of a map border line in east-west direction will be discussed firstly, then turn to those in south-north direction.

Now taking the 1:1 M scale map as an example, before the replacement of the coordinate system, the east and the west map sheet corner points are located on a east-west direction map border line, and the map border line should be along a parallel of the earth, i.e., the latitudes for the east and the west map sheet corner points are the same in a same map sheet, of course their longitudes are different and the difference usually is 6° for a Chinese 1:1 M scale map sheet. After the replacement of the coordinate system, the latitudes of the east and west map sheet corner points mentioned above will be changed into different latitude values. Then an east-west direction map sheet border line in a map sheet will not be constructed with a parallel of the same value. Then the orientation of the latter map border line will be changed in comparison with that of the original east-west map border line. Supposing that the geodetic coordinates of the east and the west corner point of a map sheet on an earth-west direction map border line are \( B^0 \), \( L^0 \) and \( B^0 \), \( L^0 + 6^\circ \), respectively, and \( \Delta dB \) and \( \Delta dX \) are the latitude and Gauss plane ordinate differences between east and west map sheet corner points of the same east-west map sheet border line (1:1 M scale) in Chinese territory after the replacement of the coordinate system, then we have

\[
\Delta dB = dB_{B, L+6^\circ} - dB_{B, L^0}, \\
\Delta dX = dX_{B, L+6^\circ} - dX_{B, L^0}.
\]

Here, \( dB_{B, L+6^\circ} \), \( dX_{B, L+6^\circ} \), \( dB_{B, L^0} \), and \( dX_{B, L^0} \) are the changed values of latitude and Gauss plane ordinate for the east and west sheet corner point, respectively, after the replacement of the coordinate system. Now the map border line in east-west direction in a same map sheet will not be constructed by the same parallel, and it will be a line connected by east and west map sheet corner points but with different latitudes. Therefore there is an orientation change of an east-west border line of a map sheet as compared
The differences $\delta dB$ and $\delta dX$ can be taken as the indexes to estimate the magnitude for the orientation change of the east-west sheet border line. If $\delta dB$ and $\delta dX$ are positive, it says that the latitude value (also the value of Gauss plane ordinate) of east point of the map border line is larger than that of west point after the replacement of the coordinate system. It also means that the east-west map border line after the replacement of the coordinate system rotates anti-clockwise in respect to the original map border line, and it shows that the former becomes inclination northward in comparison with the original parallel, and if $\delta dB$ and $\delta dX$ are minus, the situation will be vice versa. If the Xi'an 80 coordinate system is replaced by GRS80, the change range for $\delta dB$ in Chinese territory will be $+0.1''$ to $+0.3''$, and its average value is $+0.21''$; for $\delta dX$, $+2.5$ m to $+10.2$ m, and its average value is $+5.9$ m.

The orientation change values, $\delta dB$ and $\delta dX$, of the east-west map sheet border line in 1:1 M scale map of Chinese territory after the replacement of coordinate system are shown in Table 5.

| Latitude $B$ (°) | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 | 52 |
|------------------|----|----|----|----|----|----|----|----|----|----|
| $\delta dB$ (°)  | 0.11 | 0.13 | 0.16 | 0.17 | 0.19 | 0.21 | 0.23 | 0.25 | 0.28 | 0.31 |
| $\delta dX$/m    | +2.5 | +3.2 | +3.9 | +4.4 | +5.1 | +5.8 | +6.7 | +7.7 | +8.8 | +10.2 |

Note: The latitude $B$ is according to that of 1:1 M scale map sheet corners in China; $\delta dB$ and $\delta dX$ are the average values of the orientation changes of east-west map sheet border line in 1:1 M scale map of Chinese territory.

According to Table 5, it can be seen that map border line in east-west direction will rotate anti-clockwise (northward) after Xi'an 80 coordinate system is replaced by GRS80.

Now the orientation change for the map border line in south-north direction (i.e. along the longitude line of the earth) after the replacement of the coordinate system will be discussed thereafter. Still taking the 1:1 M scale map as an example, before the replacement of the coordinate system the longitudes of the south and the north corner point of the map sheet on a south-north direction map sheet border line (i.e. the map border line along the parallel of the earth) are the same in a map sheet, and their latitudes are certainly different (the difference is usually 4° for Chinese 1:1 M scale map) in the same map sheet.

After the replacement of the coordinate system, the longitudes of the same two map sheet corner points mentioned above will change into different values. Consequently, a south-north direction map border line will not be constructed with the same longitude. It is to say that in the same map at 1:1 M scale the map border line will be connected by the two map sheet corner points with different longitude values, and then the orientation of latter map sheet border line will change in comparison with that of the original south-north map sheet border line. Supposing that geodetic coordinates of the north and the south corner points of the map sheet on a north-south direction map border line are $B^\circ$, $L^\circ$ and $B'$, $L'$, respectively, and $\delta dL$ and $\delta dY$ are the differences for longitude and Gauss plane abscissa between north and south sheet corner points of the same north-south map border line (1:1 M scale) in Chinese territory after the replacement of coordinate system, then

$$\delta dL = dL_{(B',L')} - dL_{(B,L)};$$
$$\delta dY = dY_{(B',L')} - dY_{(B,L)}.$$  

Here, $dL_{(B',L')}$, $dY_{(B',L')}$ and $dL_{(B,L)}$, $dY_{(B,L)}$ are the changed values of longitude and Gauss plane abscissa of the north and south map sheet corner point, respectively, after the replacement of coordinate system. $\delta dL$ and $\delta dY$ can be taken as the orientation change magnitude of south-north map border line. If $\delta dL$ and $\delta dY$ are positive, it says that the longitude value (also the value of Gauss plane abscissa) of east point of the map sheet border line is larger than that of west point after the replacement of coordinate system.
If the Xi'an 80 coordinate system is replaced by GRS80, the change range for $\Delta dL$ in Chinese territory will be from $-0.17'$ to $-0.39'$, and its average value is $-0.24'$; for $\Delta dY$ in 1:1 M scale map within Chinese territory is very small, and the orientation change of the south-north map border line almost cannot be manifested in those maps after the replacement of the Xi'an coordinate system by GRS80.

The orientation change of south-north map border line of 1:1 M scale map caused by the replacement is shown in Table 6.

| Orientation change of south-north map border line of 1:1 M scale map caused by the replacement of Xi'an 80 coordinate system by GRS80 coordinate system |
|---|---|---|---|---|---|---|---|---|
| Longitude ($L_i$) | 72 | 78 | 84 | 90 | 96 | 102 | 108 | 114 | 120 | 126 | 132 |
| $\Delta dL/(')$ | -0.17 | -0.18 | -0.22 | -0.23 | -0.23 | -0.21 | -0.21 | -0.24 | -0.28 | -0.39 |
| $\Delta dY/m$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

Note: The longitude $L_i$ is according to that of 1:1 M scale map sheet corners in China; $\Delta dL$ and $\Delta dY$ are the average values of the orientation changes for south-north map sheet border line in 1:1 M scale map in Chinese territory.

From Table 5 and Table 6, it can be seen that all the values of $\Delta dX$ and $\Delta dY$ is smaller than 0.1 mm in the 1:1 M scale maps in Chinese territory. On the basis of the further research of the similar situation existing in all the maps whose scales are larger than 1:1 M, it can be found that if the Xi'an 80 coordinate system is replaced by GRS80, the orientation changes of the map border lines or the lines connected by any two points on the maps at various scales in Chinese territory can be ignored.

### 4 Length changes of the map border lines due to the replacement of coordinate system

Still taking 1:1 M scale map as an example, before the replacement of the coordinate system, the longitude difference is 6' for the east and west map corner points of a map border line in east-west direction (along latitude circle). After the replacement of coordinate system, there are different longitude changes for the two corner points, then the longitude difference for the east and west map sheet corner points along the map border line will change and not be 6'. The projection length of the map border line will also change and not be the original length corresponding to the longitude difference 6'. Let $dL$ and $dY$ be obtained from the following formulae.

$$dL = dL_{(B,L_i + 6')} - dL_{(B,L)}$$
$$dY = dY_{(B,L_i + 6')} - dY_{(B,L)}$$

Here, $dL_{(B,L_i + 6')}$ and $dY_{(B,L_i + 6')}$ are the change values of coordinate for east map sheet corner points (their geodetic coordinates are $B$, $L_i + 6'$ before the coordinate system replacement); $dL_{(B,L)}$ and $dY_{(B,L)}$ are the change values of coordinate for west map sheet corner points (their geodetic coordinates are $B$, $L_i$ before the coordinate system replacement).

If $dL$ and $dY$ are positive, it means that after the replacement of the coordinate system the length of east-west map border line becomes longer than the original one, and if minus, it becomes shorter. If the Xi'an 80 coordinate system is replaced by GRS80, the change range for $dL$ in 1:1 M scale map in Chinese territory will be $-0.2''$ to $+0.1''$, and its weight average value is $-0.13''$. The change range for $dY$ will be $-5.0$ m to $+1.0$ m, and its weight average value is $-4.20$ m. The length change of east-west map border line of 1:1 M scale map caused by the replacement of Xi'an 80 coordinate system by GRS80 coordinate system is shown in Table 7.

| Length change of east-west map border line of 1:1 M scale map caused by the replacement of Xi'an 80 coordinate system by GRS80 coordinate system |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Latitude ($B'$) | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 | 52 |
| $dL/(')$ | -0.05 | -0.05 | -0.07 | -0.14 | -0.15 | -0.18 | -0.16 | -0.13 | -0.13 | -0.05 |
| $dY/m$ | -1.50 | -1.50 | -2.25 | -4.01 | -4.00 | -4.52 | -3.96 | -3.43 | -2.87 | +1.04 |

Note: The latitude $B'$ is according to that of 1:1 M scale map sheet corners in China; $dL$ and $dY$ are the averages values of the length changes for east-west map sheet border line in 1:1 M scale map in Chinese territory.
Similar to the length change of east-west map border line mentioned above, the length in 1:1 M scale map for the latitude difference or projected length between south and north map sheet corners will not be the original length corresponding to the latitude difference 4 in Gauss projection. \( \Delta dB \) and \( \Delta dX \) listed in Table 8 are the differences given by the following formulae.

\[ \begin{align*}
\Delta dB &= dL_{(\gamma'-\gamma, \lambda')} - dL_{(\gamma, \lambda)}, \\
\Delta dX &= dX_{(\gamma'-\gamma, \lambda')} - dX_{(\gamma, \lambda)}
\end{align*} \]

Here, \( dL_{(\gamma'-\gamma, \lambda')} \) and \( dX_{(\gamma'-\gamma, \lambda')} \) are the coordinate changes for the north sheet corner point (its geodetic coordinate before the coordinate replacement is \( \gamma_{17}, \lambda_1 \)) of the south-north map border line, and \( dL_{(\gamma, \lambda)} \) and \( dX_{(\gamma, \lambda)} \) are the coordinate changes for the south sheet corner point (its geodetic coordinate before the coordinate replacement is \( \gamma_2, \lambda_2 \)) of the south-north map border line.

If \( \Delta dB \) and \( \Delta dX \) are positive, it means that after the replacement of coordinate system the length of south-north map border line becomes longer than the original one, and if minus, it becomes shorter. If the Xi’an 80 coordinate system is replaced by GRS80, the change range for \( \Delta dB \) in 1:1 M scale map in Chinese territory will be \(-0.16"\) to \(+0.02"\), and its average value is \(-0.10"\). The change range for \( \Delta dX \), will be \(-6.6 \) m to \(+1.6 \) m, and its average value is \(-2.20 \) m. The length change, \( \Delta dB \) and \( \Delta dX \), of south-north map border line of 1:1 M scale map caused by the replacement of Xian 80 coordinate system by GRS80 coordinate system is shown in Table 8.

### Table 8 Length change of south-north map border line of 1:1M scale map caused by the replacement of Xi’an 80 coordinate system by GRS80 geocentric 3D coordinate system

| Longitude L/(\degree) | 72 | 78 | 84 | 90 | 96 | 102 | 108 | 114 | 120 | 126 | 132 |
|----------------------|----|----|----|----|----|-----|-----|-----|-----|-----|-----|
| \( \Delta dB/" \)   | -0.16 | -0.15 | -0.14 | -0.12 | -0.10 | -0.08 | -0.06 | -0.04 | -0.01 | -0.01 | +0.02 |
| \( \Delta dX/m \)    | -6.56 | -5.84 | -5.26 | -4.52 | -3.56 | -2.75 | -1.81 | -1.03 | -0.21 | +0.62 | +1.55 |

Note: The longitude L is according to that of 1:1 M scale map sheet corners in China; \( \Delta dB \) and \( \Delta dX \) are the average value of the length change for south-north map sheet border line along longitude in 1:1 M scale map in Chinese territory.

In general, the length change, \( \Delta dB \) and \( \Delta dY \), of map border lines either in south-north direction or in east-west direction is usually shorter than 0.1 mm and hence they can be ignored on the map. On the basis of the relevant research, the similar situation also exists on those maps, whose scales are larger than 1:1 M. It is to say that if the Xi’an coordinate system is replaced by GRS80, the length changes of map border line in the maps at various scales in Chinese territory can be neglected.

### 5 Conclusions

1) If the geocentric three-dimension coordinate system is adopted in China, i.e., if Xi’an coordinate system, which is used in China now, is replaced by GRS80 geocentric three-dimension coordinate system, the geodetic latitude and longitude of surface points in Chinese territory will change. The ranges of the change is \(-1.6"\) to \(+0.7"\) and \(-6.4"\) to \(-3.0"\), respectively, their average absolute values are \(0.57"\) and \(4.4"\), respectively.

2) As the replacement between the two coordinate systems, the change ranges for Gauss ordinate \( X \) and abscissa \( Y \) values are \(-48.6\) m to \(+22.7\) m and \(-115.7\) m to \(-75.7\) m, respectively, their average absolute values are \(16.31\) m and \(106.9\) m, respectively. Therefore, the geographic location change of map sheet corners in 1:50 000 and larger scale map are beyond the mapping accuracy, then those corners should be retagged in the original map, and the map border lines should be revised accordingly.

3) As a result between the replacement of the two coordinate systems, the orientation changes will happen for the east-west and south-north map border lines of the existent maps. The range of orientation change for the east map sheet corner in respect to the west map sheet corner (along the east-west map border line of 1:1 M scale map in Chinese territory) is \( +2.5 \) m to \(+2.5\) m (Continued on Page 103)