Gauge Dependence of Gravitational Waves Generated from Scalar Perturbations

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Abstract

A tensor-type cosmological perturbation, defined as a transverse and traceless spatial fluctuation, is often interpreted as gravitational waves. While decoupled from the scalar-type perturbations in linear order, the tensor perturbations can be sourced from the scalar-type in nonlinear order. The tensor perturbations generated by the quadratic combination of a linear scalar-type cosmological perturbation are widely studied in the literature, but all previous studies are based on a zero-shear gauge without proper justification. Here, we show that, being second order in perturbation, such an induced tensor perturbation is generically gauge dependent. In particular, the gravitational wave power spectrum depends on the hypersurface (temporal gauge) condition taken for the linear scalar perturbation. We further show that, during the matter-dominated era, the induced tensor modes dominate over the linearly evolved primordial gravitational wave amplitude for $k > 10^{-2} [\mathrm{h}/\mathrm{Mpc}]$ even for the gauge that gives the lowest induced tensor modes with the optimistic choice of primordial gravitational waves $(r = 0.1)$. The induced tensor modes, therefore, must be modeled correctly specific to the observational strategy for the measurement of primordial gravitational waves from large-scale structure via, for example, the parity-odd mode of weak gravitational lensing, or clustering fossils.

Key words: cosmology: theory – gravitational waves – large-scale structure of universe

1. Introduction

Since the beginning of cosmological perturbation theory (Lifshitz 1946), it has been well known that linear-order relativistic perturbations can be decoupled into the scalar-, vector-, and tensor-type perturbations and the tensor-type perturbations are gauge invariant in the (spatially homogeneous and isotropic) Friedmann background world model. The gauge dependence, especially the temporal gauge (hypersurface or slicing) dependence, of the scalar-type perturbations is also well known in the literature (Bardeen 1980). It was Bradeen who suggested a practical strategy of utilizing the gauge dependence as an advantage in analyzing the perturbations. He wrote, “The moral is that one should work in the gauge that is mathematically most convenient for the problem at hand” (Bardeen 1988).

A natural question arises: among all possible gauge conditions, which one is more relevant for interpreting the physical world? The answer to this question does not depend on the mathematical structure of the gauge: neither on the gauge invariance of the variable in the chosen gauge, nor on the explicit gauge invariance of the combination of perturbation variables. As a matter of fact, all perturbation variables without the gauge-mode ambiguity are gauge invariant in the sense that their values evaluated in any other gauge remain the same. This is the case for all perturbation variables in the several fundamental gauges introduced in Bardeen (1980, 1988) except for the synchronous gauge. For these gauges, each perturbation variable uniquely corresponds to a gauge-invariant combination of perturbation variables. Again, according to Bardeen, “While a useful tool, gauge invariance in itself does not remove all ambiguity in physical interpretation,” and “Many gauge-invariant combinations of these scalars can be constructed, but for the most part they have no physical meaning independent of a particular time gauge, or hypersurface condition” (Bardeen 1988). For a given variable, say density perturbation or velocity perturbation, we can, in fact, construct the variable with infinitely different gauge conditions, and all of them correspond to the gauge-invariant combinations; this is because the constant-time hypersurface can be deformed in a continuous manner. These statements are also true to the nonlinear order in cosmological perturbation theory (Noh & Hwang 2004; Hwang & Noh 2013a). It is, therefore, safe to treat a variable evaluated in different gauges as entirely different variables. Finally, according to Bardeen, “Gauge-invariant variables give mathematically unambiguous ways of comparing results obtained in different gauges, but their physical interpretation is not necessarily straightforward, in that it is usually tied to a particular way of slicing the spacetime into hypersurfaces. I know of no way to characterize completely the deviations from homogeneity and isotropy independent of the slicing into spacelike hypersurfaces” (Bardeen 1988).

Instead, which gauge-invariant variable corresponds to the one that we measure from observation depends on the nature of the observation. That is, a specification of observation must tell us which gauge-invariant variable or combination of them is the right one for that particular observation (assuming, of course, that perturbation theory in the Friedmann world model handles the observed phenomena). We discuss this issue further in Section 6.

To the nonlinear order, the three types of perturbations couple to each other on the equation level and the decomposition itself becomes ambiguous. It is because we can introduce many different ways of decomposing the perturbation to scalar-, vector-, and tensor-types (Hwang & Noh 2013a); hereafter, we simply call it the scalar perturbation or scalar mode, etc. Naturally, from the second order, even the tensor perturbation becomes gauge dependent.
In particular, the second-order tensor perturbations generated from the quadratic combinations of linear scalar perturbations (induced tensor perturbations) must depend on the gauge condition, as the linear scalar perturbations depend on the choice of the constant-time hypersurface. There are studies of such induced tensor modes in the literature (Mollerach et al. 2004; Ananda et al. 2007; Baumann et al. 2007; Arroja et al. 2009; Assadullahi & Wands 2009, 2010; Jedamzik et al. 2010; Alabidi et al. 2013; Saga et al. 2015), but all of these studies have been based on one particular gauge condition, the zero-shear gauge in our terminology (Harrison 1967). Note that, with no entirely clear reason, in the literature, this gauge condition is often termed as the longitudinal, Newtonian, conformal-Newtonian, or Poisson gauge, etc (Bertschinger 1995). The zero-shear gauge causes the scalar part of the shear of the normal frame vector to vanish; if we ignore the vector and tensor part, this statement is valid for fully nonlinear orders in perturbation, see Equations (B7) and (C7) in Hwang & Noh (2013a).

In this work, we explicitly show the gauge dependence of the power spectrum of the induced tensor perturbations. The main results are summarized in Equation (61) in a unified form, and shown in Figure 1. This paper is organized as following. In Section 2, we introduce our notations and basic equations. In Section 3, we use the nonlinear gauge transformation to show the gauge dependence of the tensor modes to the second order and relations of the tensor-mode solutions among different gauge conditions, see Equations (42) and (58). In Section 4, we present the tensor power spectrum in several gauge conditions in a unified form. We discuss the implication of our result to the future observations in Section 6. We set $c = 1$.

2. Equations

As the metric convention, we have

$$
\begin{align*}
\text{d}s^2 &= -a^2(1 + 2\Lambda)\text{d}\eta^2 - 2a^2B_i\text{d}\eta\text{d}x^i + a^2(\gamma_{ij} + 2C_{ij})\text{d}x^i\text{d}x^j,
\end{align*}
$$

(1)

where the spatial indices of $B_i$ and $C_{ij}$ are raised and lowered using the background metric of the comoving coordinate $\gamma_{ij}$; for a spatially flat background with $K = 0$ we have $\gamma_{ij} = \delta_{ij}$; $i$, $j$, $k$,... are the three-dimensional spatial indices and $a$, $b$, $c$,... are spacetime indices. We decompose the spatial vector $B_i$ and the spatial tensor $C_{ij}$ into the scalar, vector, and tensor perturbations as (York 1973)

$$
\begin{align*}
A &\equiv \alpha, \quad B_i \equiv \beta_i + B_i^{(v)}, \quad C_{ij} \equiv \varphi\gamma_{ij} + \gamma_{ij} + C_{ij}^{(v)} + h_{ij},
\end{align*}
$$

(2)

where $B_i^{(v)} \equiv 0 \equiv C_{ij}^{(v)}$ (transverse vector) and $h_{ij} \equiv 0 \equiv h^t_i$ (transverse-traceless tensor); a vertical bar indicates a covariant derivative based on $\gamma_{ij}$ as the metric; like $B_i$ and $C_{ij}$, indices of $B_i^{(v)}$, $C_{ij}^{(v)}$, and $h_{ij}$ are raised and lowered using $\gamma_{ij}$ as the metric; we define $A_{ij} \equiv \frac{1}{2}(\gamma_{ij} + A_{ij})$. We set $\chi \equiv a(\beta + a\gamma)$; an overdot indicates time derivative based on cosmic time $t$ (defined with $dt \equiv a\text{d}\eta$). For the energy-momentum tensor, we have

$$
\begin{align*}
\bar{T}_{ab} &= \bar{\mu}\bar{u}_a\bar{u}_b + \bar{p}(\bar{\bar{g}}_{ab} + \bar{u}_a\bar{u}_b) + \bar{\pi}_{ab},
\end{align*}
$$

(3)

where $\bar{\mu}$, $\bar{\rho}$, $\bar{\varphi}$, and $\bar{\pi}_{ab}$ are the energy density, the pressure, the fluid four-vector, and the anisotropic stress, respectively (Ellis 1971, 1973; Ehlers 1993). We decompose them as

$$
\begin{align*}
\bar{\mu} &\equiv \mu + \delta\mu, \quad \bar{\rho} \equiv p + \delta p, \quad \bar{\varphi} \equiv a\Gamma v_i, \\
v_i &\equiv -\nu_i + \nu_i^{(v)}, \quad \Gamma \equiv -\tilde{n}_i\tilde{\omega}^i = \frac{1}{\sqrt{1 - \alpha^2\tilde{R}^2}v_i v_j}, \\
\bar{\pi}_{ij} &\equiv a^2\Pi_{ij}, \quad \Pi_{ij} \equiv \frac{1}{a^2}\left(\Pi_{ij} - \frac{1}{3}\gamma_{ij}\Delta\Pi\right) + \frac{1}{a}\Pi_{ij}^{(v)} + \Pi_{ij}^{(v)}, \quad \Theta \equiv \frac{1}{3}\gamma_{ij}\Pi_{ij}.
\end{align*}
$$

(4)

where $\tilde{n}_i$ is the normal four-vector and $\Gamma$ is the Lorentz factor. Indices of $\nu_i$, $\nu_i^{(v)}$, $\Pi_{ij}$, $\Pi_{ij}^{(v)}$, and $\Pi_{ij}^{(v)}$ are raised and lowered using $\gamma_{ij}$ as the metric. We have $\nu_i^{(v)} = 0 \equiv \Pi_{ij}^{(v)}$ and $\Pi_{ij}^{(v)} = 0 \equiv \Pi_{ij}^{(v)}$; tildes indicate the covariant quantities. We added the $\Pi_{ij}^t$ term in the decomposition of $\Pi_{ij}$ as we have $\Pi_{ij}^t = 0$ to the nonlinear order. $\tilde{R}_{ij}$ is the inverse metric of the ADM (Arnowitt-Deser-Misner) metric $\tilde{g}_{ij}$ defined as $\tilde{g}_{ij} \equiv \tilde{g}_{ij}$. For $\gamma \equiv 0 \equiv C_{ij}^{(v)}$ and $h_{ij} \equiv 0$, we can derive an explicit form of the inverse metric and we have (Hwang & Noh 2013a; Hwang et al. 2016)

$$
\begin{align*}
\tilde{g}_{ij} &= a^2(1 + 2\varphi)\gamma_{ij}, \\
\tilde{h}_{ij} &= \frac{1}{a^2(1 + 2\varphi)}\gamma_{ij}, \\
\Gamma &= \frac{1}{\sqrt{1 - \frac{\nu^2}{1 + 2\varphi}}}.
\end{align*}
$$

(5)

Setting $\gamma = 0 \equiv C_{ij}^{(v)}$ corresponds to taking the spatial gauge condition without losing any generality and without missing any physics: according to Bardeen “Since the background 3-space is homogeneous and isotropic, the perturbations in all physical quantities must in fact be gauge invariant under purely spatial gauge transformations.” (Bardeen 1988); this statement is true to fully nonlinear order (Noh & Hwang 2004; Hwang & Noh 2013a).

The tracefree ADM propagation equation can be written as

$$
\begin{align*}
\frac{1}{a^2}\left(\nabla\nabla - \frac{1}{3}\gamma_{ij}\Delta\right)\left[\frac{1}{a}(a\chi) - \alpha - \varphi - 8\pi\Gamma\Pi_{ij}\right] \\
+ \frac{1}{a}\nabla_0\left\{\frac{1}{a^2}\left[a^2(B_{ij}^{(v)} + aC_{ij}^{(v)}) - 8\pi\Gamma\Pi_{ij}^{(v)}\right]\right\} \\
+ \tilde{h}_{ij} + 3H\tilde{h}_{ij} - \frac{\Delta - 2K}{a^2}\tilde{h}_{ij} - 8\pi\Gamma\Pi_{ij}^{(v)} = n_{ij},
\end{align*}
$$

(6)

where $n_{ij}$ indicates pure nonlinear parts, see Equation (109) of Hwang & Noh (2007); in our notation, we absorb the $\Pi_{ij}^t$-part to the right-hand side, thus $n_{ij}$ is tracefree. The tensor part of Equation (6) becomes

$$
\tilde{h}_{ij} + 3H\tilde{h}_{ij} - \frac{\Delta - 2K}{a^2}\tilde{h}_{ij} - 8\pi\Gamma\Pi_{ij}^{(v)} = s_{ij},
$$

(7)
with \( s_{ij} \) being the transverse-tracefree projection of \( n_{ij} \):

\[
s_{ij} = \mathcal{P}_{ij}^{\perp} n_{\ell \ell} = n_{ij} - \frac{1}{3} \gamma_{ij} n_{\ell \ell}^{\ell} + \frac{1}{2} \left( \nabla n_{j} - \frac{1}{3} \gamma_{ij} \Delta \right) \times (\Delta + 3K)^{-1} \left[ n_{k}^{k} - 3\Delta^{-1} (n_{\ell}^{\perp})_{\ell} \right] - 2 \nabla_{i} (\Delta + 2K)^{-1} \left[ n_{j}^{k} - n_{j}^{\perp} \right] \Delta^{-1} (n_{\ell}^{\perp})_{\ell},
\]

where \( \mathcal{P}_{ij}^{\perp} \) is a transverse-tracefree projection operator on a symmetric spatial tensor. For the spatially fully nonlinear order in perturbation.

For later convenience, here we summarize the basic set of linear-order scalar perturbation equations, see Equations (95)-(101) in Hwang & Noh (2007):

\[
\kappa = 3H \alpha - 3 \varphi - \frac{\Delta}{a^2} \chi, \quad \text{(11)}
\]

\[
4\pi G \delta \varphi + H \kappa + \frac{\Delta + 3K}{a^2} \varphi = 0, \quad \text{(12)}
\]

\[
\kappa + \frac{\Delta + 3K}{a^2} \chi - 12\pi G (\mu + p) v_{\nu} = 0, \quad \text{(13)}
\]

\[
\kappa + 2H \kappa - 4\pi G (\delta \varphi + 3 \varphi) + \left( 3H + \frac{\Delta}{a^2} \right) \alpha = 0, \quad \text{(14)}
\]

\[
\chi + H \chi - \varphi - \alpha - 8\pi G \Pi = 0, \quad \text{(15)}
\]

\[
\kappa + 3H (\delta \mu + \delta \varphi) - (\mu + p) \left( \kappa + 3H \alpha + \frac{\Delta}{a^2} \right) v_{\nu} = 0, \quad \text{(16)}
\]

\[
\frac{[a^4 (\mu + p) v]}{a^4 (\mu + p)} \frac{1}{a} - \frac{1}{a^2} \varphi - \varphi = 0.
\]

\[
\kappa \text{ is a perturbed part of the trace of extrinsic curvature.}
\]

### 2.1. Zero-shear Gauge

In the zero-shear gauge, we set \( \beta \equiv 0 \equiv \gamma \), thus \( \chi = 0 \). For \( \Pi = 0 \), we have \( \alpha = -\varphi \) to the linear order, and Equation (10) gives

\[
n_{ij} \chi = -\frac{1}{a^2} \left[ 4\varphi \varphi_{ij} + 2 \varphi_{i} \varphi_{j} - \frac{1}{3} \gamma_{ij} (4\varphi \Delta \varphi + 2 \varphi^{k} \varphi_{k}) \right] + 8\pi G (\mu + p) v_{i} v_{j}.
\]

\[
\kappa_{ij} = \frac{1}{a^2} \left( \varphi_{ij} + 3\varphi \delta \varphi_{ij} - \varphi_{i} \varphi_{j} \right) + 8\pi G (\mu + p) v_{i} v_{j}.
\]

These equations are valid for general \( K \).

In the matter-dominated era, the growing solutions to the linear order for \( K = 0 = \Lambda \) are (Hwang 1994):

\[
\varphi = \frac{3}{5} C, \quad \alpha = -\frac{3}{5} C, \quad \kappa = -\frac{9}{5} HC,
\]

\[
v_{\nu} = -\frac{2}{5 aH} C, \quad \delta_{\nu} = \frac{6}{5} \left( 1 - \frac{1}{3 \frac{1}{a^2} H^2} \right) C.
\]

#### 2.2. Comoving Gauge

In the comoving gauge, we set \( \nu \equiv 0 \equiv \gamma \). In the zero-pressure case with \( \Pi = 0 \) and \( K = 0 \), to the linear order, from Equations (11) and (13), we have

\[
\alpha = 0, \quad \varphi = 0, \quad \frac{1}{a} (a \chi) = \varphi, \quad \kappa = -\frac{\Delta}{a^2} \chi.
\]

Using this, Equation (10) gives

\[
n_{ij} = \frac{1}{a^2} (\kappa \chi_{ij} - 2 \varphi \varphi_{ij} - \varphi_{i} \varphi_{j}) + \left[ \frac{1}{a^2} \chi^{k}_{i} \chi_{\ell k} - \frac{1}{a^2} \chi^{k}_{i} \chi_{\ell k} \right] + \frac{1}{a^2} \chi^{k}_{i} \chi_{\ell k}. \quad \text{(22)}
\]

where the sub-index \( v \) indicates the comoving gauge. In this gauge, we have \( \alpha = \alpha_{v}, \varphi = \varphi_{v}, \chi = \chi_{v}, \) and \( \kappa = \kappa_{v} \).

In the matter-dominated era, the growing solutions to the linear order for \( K = 0 = \Lambda \) are (Hwang 1994):

\[
\varphi = C, \quad \alpha_{v} = 0, \quad \chi_{v} = \frac{2}{5} \frac{1}{H} C, \quad \kappa_{v} = -\frac{2}{5} \frac{\Delta}{a^2 H^2} C. \quad \text{(23)}
\]
The normalization of the growing solution is based on the conserved nature of a variable $\gamma \equiv C$.

3. Gauge Issue

We consider the gauge transformation $\tilde{x}^a = x^a + \xi^a(x^i)$. To the linear order, using $\xi^{(v)} = \xi^{\alpha}_{(v)}$ with $\xi^{\alpha}_{(v)}(0) \equiv 0$, we have (see Equation (250) in Noh & Hwang 2004):

\[
\tilde{\alpha} = \alpha - \frac{1}{a} (a\xi^0)' \gamma, \quad \tilde{\beta} = \beta - \xi^0 + \left(\frac{1}{a} \xi^i\right) \gamma, \\
\tilde{B}^{(v)}_i = B^{(v)}_i + \xi^{(v)}_i, \quad \tilde{\gamma} = \gamma - \frac{1}{a} \xi^0, \quad \tilde{\chi} = \chi - a \xi^0, \quad \tilde{\kappa} = \kappa + \left(3H + \frac{\Delta}{a^2}\right)a\xi^0, \\
\tilde{\varphi} = \varphi - \frac{a'}{a} \xi^0, \quad \tilde{v} = v - \xi^0 - \frac{\mu}{\varphi} \xi^0 = \tilde{\varphi} + 3(1 + w) \frac{a'}{a} \xi^0, \\
\tilde{\dot{h}}_{ij} = h_{ij} + \mathcal{P}_{ij} \xi^{(v)}.
\]

(24)

where the prime indicates a time derivative based on $\eta$ and $x^0 = \eta; w \equiv p/\mu$. From these, we have constructed gauge-invariant combinations

\[
\chi_v \equiv \chi - av, \quad \chi_e \equiv \chi - \frac{1}{H} \varphi, \quad \chi_\kappa \equiv \chi + \frac{\kappa}{3H + \frac{\Delta}{a^2}}, \\
\chi_{\xi} \equiv \chi + \frac{\xi}{3(1 + w)H},
\]

(25)

which correspond to $\chi$ (the scalar part of the shear of the normal frame vector) in, respectively, the comoving gauge ($\chi_v$), the uniform curvature gauge ($\chi_e$), the uniform expansion gauge ($\chi_\kappa$), and the uniform density gauge ($\chi_{\xi}$), and

\[
\varphi_{ij} \equiv \varphi - aHv, \quad \varphi_{\xi} \equiv \varphi - H\chi,
\]

(26)

are the scalar metric (curvature) perturbation $\varphi$ in the comoving gauge ($\varphi_v$) and the zero-shear gauge ($\varphi_{\xi}$).

To the linear order, we fix the spatial (including the scalar and vector) gauge by conditions (Bardeen 1988)

\[
\gamma \equiv 0 \equiv C^{(v)}_i.
\]

(27)

Under these gauge conditions, the spatial gauge degrees of freedom are fixed completely with $\xi = 0 = \xi^{(v)}$, thus $\xi_i = 0$, to the linear order. The second-order gauge transformation is given from Equation (231) in Noh & Hwang (2004) as

\[
\tilde{C}^{(v)}_i = C^{(v)}_i - \frac{a'}{a} \xi^0 \gamma_{ij} - \xi^{(v)}_{(p)} - \left(C^{(v)}_j + 2 \frac{a'}{a} C^{(v)}_y\right) \xi^0 \\
- \frac{1}{2} \xi^0 \xi^{(v)}_{(p)} + \xi^0 \left[\frac{a'}{a} \xi^0 + \frac{1}{2} \left(\frac{a'}{a} + \frac{a''}{a^2}\right) \xi^0 \right] \gamma_{ij} \\
\equiv C^{(v)}_i - \frac{a'}{a} \xi^0 \gamma_{ij} - \xi^{(v)}_{(p)} + C^{(v)}_{ij},
\]

(28)

where $C^{(v)}_i$ indicates pure quadratic parts of the gauge transformation property of $C^{(v)}_i$. From this, using the decomposition in Equation (2), we can show

\[
\tilde{\varphi} = \varphi - \frac{a'}{a} \xi^0 + \frac{1}{2} \left(\Delta + 3K\right)^{-1} \left[\left(\Delta + 2K\right)C^{(v)}_y - C^{(v)}_{ij} \left|\mu\right|\right],
\]

(29)

\[
\tilde{\gamma} = \gamma - \frac{1}{a} \xi^0 - \frac{1}{2} \left(\Delta + 3K\right)^{-1} [C^{(v)}_{ik} - \Delta^{-1} (C^{(v)}_{ik} \left|\mu\right|)],
\]

(30)

\[
\tilde{C}^{(v)}_i = C^{(v)}_i - \xi^{(v)} + 2 \left(\Delta + 2K\right)^{-1} [C^{(v)}_{ik} - \Delta^{-1} (C^{(v)}_{ik} \left|\mu\right|)],
\]

(31)

\[
\tilde{h}_{ij} = h_{ij} + P_{ij} \xi^{(v)}.
\]

(32)

To the second order, we can continue taking the spatial and rotational gauge by the same conditions in Equation (27). These are possible by suitable choices of $\xi$ and $\xi^{(v)}$ using Equations (30) and (31); i.e., the spatial gauge conditions to the second order determine $\xi$ and $\xi^{(v)}$ to the second order as

\[
\xi = - \frac{a}{2} \left(\Delta + 3K\right)^{-1} [C^{(v)}_{ik} - 3 \Delta^{-1} (C^{(v)}_{ik} \left|\mu\right|)],
\]

\[
\xi^{(v)}_i = 2 \left(\Delta + 2K\right)^{-1} [C^{(v)}_{ik} - \nabla_i \Delta^{-1} (C^{(v)}_{ik} \left|\mu\right|)].
\]

(33)

Notice that even in the case of vanishing vector perturbation, we should not ignore $\xi^{(v)}_{ij}$ to the second order. By taking conditions in Equation (27), we have

\[
C^{(v)}_i = \left(\frac{1}{a} \chi_{(v)} + \Psi^{(v)}_{(v)} \right) \xi^0 - \frac{1}{2} \Psi^{(v)}_{(v)} \xi^0 - \left(h^{(v)}_{ij} + 2 \frac{a'}{a} h_{ij}\right) \xi^0 + \xi^0 \left[- \varphi' - \frac{2}{a} \varphi + \frac{a'}{a} \xi^0 + \frac{1}{2} \left(\frac{a''}{a} + \frac{a'^2}{a^2}\right) \xi^0 \right] \gamma_{ij},
\]

(34)

where

\[
\chi \equiv a (\beta + \gamma'), \quad \Psi^{(v)}_{(v)} \equiv B^{(v)}_i + C^{(v)}_{ij},
\]

(35)

are spatially gauge-invariant combinations to the linear order. We can show that the $\gamma_{ij}$ part in $C^{(v)}_i$ does not affect the tensor-mode gauge transformation in Equation (32).

Now, we consider pure scalar perturbation to the linear order. Ignoring the $\gamma_{ij}$ part that does not contribute to the tensor modes, we have

\[
C^{(v)}_{ij}^{(tensor)} = \frac{1}{a} \chi_{(v)} \xi^0 - \frac{1}{2} \xi^0 \xi^0.
\]

(36)

Using Equations (24)–(26) and (36), we can construct a set of variables $C^{(v)}_{ij}$ such that the gauge transformation is given with $C^{(v)}_{ij}^{(tensor)}$,

\[
\tilde{C}^{(v)}_{ij} = C^{(v)}_{ij} - C^{(v)}_{ij}^{(tensor)}.
\]

(37)
where $x = \chi, \, v, \, \varphi, \, \kappa, \, \text{and} \, \delta$. The explicit expressions for $C_{ij}$ that we consider here are given as

$$C_{ij}^{\varphi} \equiv -\frac{1}{2a^2} \varphi_{i,\ell} \varphi_{j,\ell}, \quad C_{ij}^{\chi} \equiv \frac{1}{2a^2} \chi_{i,\ell} \chi_{j,\ell},$$

$$C_{ij}^{\kappa} \equiv -\frac{1}{a^2} \chi_{i,\ell} \chi_{j,\ell} - \frac{1}{2a^2} \chi_{i,\ell} \chi_{j,\ell} + \frac{1}{a^2} \chi_{i,\ell} \chi_{j,\ell},$$

$$C_{ij}^{\delta} \equiv -\frac{1}{a^2} \chi_{i,\ell} \chi_{j,\ell} - \frac{1}{2a^2} \chi_{i,\ell} \chi_{j,\ell} - \frac{1}{a^2} \chi_{i,\ell} \chi_{j,\ell} + \frac{1}{a^2} \chi_{i,\ell} \chi_{j,\ell} - \frac{1}{a^2} \chi_{i,\ell} \chi_{j,\ell}.$$

(38)

Note that, unlike the gauge-invariant variables $\chi$, $\varphi$, and $\kappa$ that we defined earlier, $C_{ij}^{\varepsilon}$ is not a gauge-invariant notation. With these new variables, we can express

$$C_{ij} - C_{ij}^{\varphi} = -\frac{1}{a^2} \chi_{i,\ell} \chi_{j,\ell},$$

$$C_{ij} - C_{ij}^{\chi} = -\frac{1}{a^2} \chi_{i,\ell} \chi_{j,\ell},$$

$$C_{ij} - C_{ij}^{\delta} = -\frac{1}{a^2} \chi_{i,\ell} \chi_{j,\ell} - \frac{1}{a^2} \chi_{i,\ell} \chi_{j,\ell}.$$

(39)

Using these notations, we can construct a unified form of the explicit gauge-invariant combination $h_{ij}$ as

$$h_{ij} = h_{ij} + C_{ij}^{\varphi} = \frac{1}{a^2} \chi_{i,\ell} \chi_{j,\ell} - \frac{1}{a^2} \chi_{i,\ell} \chi_{j,\ell} - \frac{1}{a^2} \chi_{i,\ell} \chi_{j,\ell} - \frac{1}{a^2} \chi_{i,\ell} \chi_{j,\ell}.$$

(40)

(41)

Complete sets of solutions for $\chi_\ell$ in all fundamental gauges are presented in the tables of Hwang (1994) for a pressureless medium and tables of Hwang (1993) for an ideal fluid medium.

4. Fourier Analysis

We consider a spatially flat background, where the plane wave solutions are eigenfunctions of the Laplacian operator. It is, then, convenient to work in the Fourier space. We introduce a Fourier decomposition of a tensor perturbation $h_{ij}(x, \ell)$ as

$$h_{ij}(x, \ell, t) = \frac{1}{(2\pi)^3} \int d^3k e^{-i k \cdot x} [h(k, \ell, t) e_{ij}(k) + \overline{h}(k, \ell) \overline{e}_{ij}(k)],$$

(43)

where

$$e_{ij}(k) \equiv \frac{1}{\sqrt{2}} [\varepsilon_i(k) e_j(k) - \varepsilon_j(k) e_i(k)],$$

$$\overline{e}_{ij}(k) \equiv \frac{1}{\sqrt{2}} [\varepsilon_i(k) e_j(k) + \varepsilon_j(k) e_i(k)],$$

(44)

are polarization bases for spin-2 fields; we construct the bases from two transverse unit vectors $e_i$ and $\varepsilon_i$ (satisfying $|e_i| = 1 \equiv |\varepsilon_i|$ and $e^i e_i = \varepsilon^i \varepsilon_i = 0$). Note that the two polarization bases are orthogonal $(e^{(\ell)}(k) \varepsilon_{ij}(k) = 0)$ and normalized as $e^{(\ell)}(k) e_{ij}(k) = e^{(\ell)}(k) \varepsilon_{ij}(k) = 1$. These two bases are sometimes called $e^\pm = \sqrt{2} e_i$ and $\varepsilon^\pm = \sqrt{2} \varepsilon_i$ in the literature; see, for example, Dai et al. (2012). By using the orthogonality of the polarization bases, we have

$$h(k, \ell) = e^{(\ell)}(k) \int d^3x e^{-i k \cdot x} h(x, \ell) = \overline{h}(k, \ell) \int d^3x e^{-i k \cdot x} \overline{h}(x, \ell).$$

(45)

For the spatially flat case ($K = 0$) and in the absence of the genuine tensor-origin contribution to anisotropic stress $\Pi_{ij}^{(0)}$, which is the case for the standard cosmological models, the gravitational wave equation in Equation (7) becomes

$$\left(\partial_i^2 + 3H \partial_i + \frac{k^2}{a^2}\right) h(k, \ell) = e^{(\ell)}(k) \int d^3x e^{-i k \cdot x} s_{ij}(x, \ell) = e^{(\ell)}(k) \int d^3x e^{-i k \cdot x} b_{ij}(x, \ell),$$

(46)

with $s_{ij}$ given in terms of $n_{ij}$ in Equation (9). The other polarization mode $\overline{h}(k, \ell)$ obeys the equation parallel to Equation (46) with $\mathbf{s}(k, \ell)$ defined with $\varepsilon_{ij}$ instead of $e_{ij}$.

In the parity-preserving universe, the two polarization modes $h(k, \ell)$ and $\overline{h}(k, \ell)$ must have exactly the same statistical properties. We, therefore, shall focus only on $h(k, \ell)$ in what follows. The effect of $\overline{h}(k, \ell)$ will be taken into account by simply adding the same contribution at the end of the calculation.

We transform Equation (46) by introducing the new variable $\nu = \alpha h$ and using the conformal-time derivative (denoted by prime) as

$$v''(k, \eta) + \frac{k^2}{a^2} v'(\eta) = a(\eta) s(k, \eta).$$

(47)

The solution is then given by

$$v(k, \eta) = \int_0^\eta a(\eta') s(k, \eta') d\eta',$$

(48)

by using the Green's function

$$g(k, \eta, \eta') = \frac{v_1(\eta) v_2(\eta') - v_1(\eta') v_2(\eta)}{v_1(\eta) v_2(\eta) - v_1(\eta') v_2(\eta')},$$

(49)
for $\eta \geq \bar{\eta}$ and 0 otherwise, because the scalar source at time $\bar{\eta}$, $s(k, \bar{\eta})$, only affects the gravitational waves at later times $\eta \geq \bar{\eta}$. Here, $v_1$ and $v_2$ are two linearly independent solutions for the homogeneous part of Equation (47).

In the flat, matter-dominated universe ($K = 0 = \Lambda$), we have $a \propto \eta^2$, and the solutions to the linear-order gravitational waves are (Lifshitz 1946; Weinberg 1972)

$$v = \frac{a}{h} \propto x j_i(x),$$
$$x y_i(x) = -\cos x + \frac{\sin x}{x}, \quad -\sin x - \frac{\cos x}{x}, \quad (50)$$

where $x \equiv k \eta$. The tensor amplitude $v$ is gauge invariant in the linear order: that is, it is independent of the gauge condition taken for the scalar perturbation. The induced tensor amplitudes appear from the second order as Equation (48), where the Green’s function is given as

$$g_\kappa(\eta, \bar{\eta}) = \frac{\bar{x} x}{k} \left[j_1(\bar{x}) y_i(x) - j_i(x) y_1(\bar{x})\right]. \quad (51)$$

The induced tensor amplitude depends on the temporal gauge chosen for the scalar perturbations.

In this section, we shall present explicit expressions for the induced tensor amplitude in various different temporal gauge conditions. In particular, we shall calculate the Fourier space expression for the source term $s_\kappa(k, \eta)$ with a temporal gauge condition denoted by the subscript $c$. We start from the zero-shear gauge and find $s_\kappa(k, t)$ in the matter-dominated universe, and generalize to the other gauges by using the gauge transformation that we have presented in Section 3.

### 4.1. Zero-shear Gauge

In the zero-shear gauge, using Equations (18) and (46), we can show

$$\left(\frac{\partial^2}{\partial t^2} + 3H \partial_t + \frac{k^2}{a^2}\right)h_\kappa(k, t) = \frac{1}{(2\pi)^3} \int d^3q [e^{ij}(k)q]_\kappa \varphi_i(k - q, t) + 8\pi G(\mu + p)\varphi_i(k, t) \varphi_\kappa(k - q, t) \equiv \frac{1}{a^2} s_\kappa(k, t). \quad (52)$$

In the matter-dominated era, using Equation (20), we have

$$s_\kappa(k) = \frac{6}{5} \frac{1}{(2\pi)^3} \int d^3q [e^{ij}(k)q]_\kappa \varphi_i(k - q, t) C(q) C(k - q). \quad (53)$$

The general solution to the second order is

$$h_\kappa(k, \eta) = \frac{s_\kappa(k)}{k^2} + \frac{1}{a} [c_1 x j_1(x) + c_2 x y_1(x)]. \quad (54)$$

Imposing the initial condition $h_\kappa = 0 = h_\kappa'$ at $\eta = 0$ we have $c_2 = 0$ and (Mollerach et al. 2004)

$$h_\kappa(k, \eta) = \frac{s_\kappa(k)}{k^2} \left(1 + \frac{3}{\eta} \frac{x \cos x - \sin x}{x^3}\right) \equiv \frac{s_\kappa(k)}{k^2} g(k). \quad (55)$$

For $x \ll 1$, we have $g = \frac{1}{10} x^2$ thus $h_\kappa = \frac{1}{10} s_\kappa h_\kappa$. For $x \gg 1$, we have $g = 1$ thus $h_\kappa = s_\kappa / k^2$.

### 4.2. Unified Expression in Other Gauges

Solutions in other gauge conditions simply follow from the one in the zero-shear gauge, as Equation (42) gives

$$h_\kappa(k, t) = h_\kappa(k, t) - \frac{1}{2a^2} \frac{1}{2(2\pi)^3} \int d^3q \times [e^{ij}(k)q]_\kappa \varphi_i(q, t) \varphi_\kappa(k - q, t). \quad (56)$$

The linear solutions in the matter-dominated era for $K = 0 = \Lambda$ are (see Table 1 of Hwang 1994)

$$\chi_0 = \frac{2}{5} \frac{1}{H} C, \quad \chi_\varphi = \frac{3}{5} \frac{1}{H} C, \quad \chi_\kappa = -\frac{9}{5} \frac{H}{3 + \frac{1}{a^2}} C, \quad \chi_\delta = \frac{2}{15} \frac{1}{H} \left[3 - \frac{\Delta}{a^2 H^2}\right] C, \quad (57)$$

which are $\chi$ value evaluated in, respectively, the comoving gauge $x = v$, the uniform curvature gauge $x = \varphi$, the uniform expansion gauge $x = \kappa$, and the uniform density gauge $x = \delta$. In the case of the comoving gauge ($x = v$), we can check that the solution of $h_\kappa$ derived from the gauge transformation in Equation (56) coincides with the solution directly derived from Equations (7), (9), and (22).

From Equations (53), (55), and (56), we have the unified expression for the pure second-order contribution,

$$h_\kappa(k, \eta) = \frac{6}{5} \frac{1}{k^2} \frac{1}{(2\pi)^3} \int d^3q [e^{ij}(k)q]_\kappa \varphi_i(k - q, t) \times C(q) C(k - q) W_\kappa(k, q, \eta), \quad (58)$$

where

$$W_\kappa = g(k)\varphi, \quad W_\kappa = g(k) - \frac{1}{15} \frac{k^2}{a^2 H^2},$$
$$W_\varphi = g(k) - \frac{3}{20} \frac{k^2}{a^2 H^2},$$
$$W_\kappa = g(k) - \frac{1}{15} \frac{k^2}{a^2 H^2} \left(1 + \frac{2}{9} \frac{q^2}{a^2 H^2}\right)^{-1} \left(1 + \frac{2}{9} \frac{k - q^2}{a^2 H^2}\right)^{-1},$$
$$W_\kappa = g(k) - \frac{1}{15} \frac{k^2}{a^2 H^2} \left(1 + \frac{1}{3} \frac{q^2}{a^2 H^2}\right) \left(1 + \frac{1}{3} \frac{k - q^2}{a^2 H^2}\right). \quad (59)$$

We have $\frac{1}{a^2 H^2} = \frac{1}{2} \eta$. For $x \ll 1$, we have $g = \frac{1}{10} x^2$ thus $h_\kappa \propto h_\kappa$. Thus, we have $h_\kappa = 0 = h_\kappa'$ at $\eta = 0$. For $x \gg 1$, we have $g = 1$ thus $h_\kappa \propto h_\kappa$ except for $x = \kappa$.

### 4.3. Power Spectrum: Unified Expression

Using the definition of power spectra

$$\langle C(k) C(k') \rangle \equiv (2\pi)^3 \delta^D(k + k') P_\kappa(k),$$
$$\langle h_\kappa(k, \eta) h_\kappa(k', \eta) \rangle \equiv (2\pi)^3 \delta^D(k + k') \frac{1}{2} P_h(k, \eta), \quad (60)$$
we have the expression for the induced tensor power spectrum as

\[
P_{\text{hs}}(k, \eta) = \frac{144}{25} \frac{1}{k^4} \frac{1}{(2\pi)^3} \int d^3q [e^{i\mathbf{q}\cdot\mathbf{r}}(k)q] \epsilon_{\text{tensor}}(k, \eta, \mathbf{q}, \mathbf{r})^2 \times P_C(q)P_C(|k - q|) W_s^2(k, \eta, \mathbf{q}, \mathbf{r}). \tag{61}
\]

Note that the factor 1/2 in Equation (60) accounts for the two polarization modes whose power spectrum must be equal. Here, we assume that primordial curvature perturbations follow Gaussian statistics. Note that, although it is of the same order, the cross-term multiplying linear-order and third-order scalar perturbations is not present because there is no linear-order scalar contribution to the induced tensor mode in the standard Friedmann–Robertson–Walker world models.

5. Spectrum of Induced Gravitational Waves

We calculate the spectrum of induced gravitational waves in the standard ΛCDM world model adopting the best-fitting cosmological parameters (maximum likelihood values in the table entitled "base_plikHM_TTTEEE_lowTEB_lensing_post_BAO_H080p6_JLA") from Planck 2015 (Planck Collaboration et al. 2015): \( \Omega_b h^2 = 0.022307, \Omega_c h^2 = 0.11865, \Omega_{\Lambda} h^2 = 0.00638, \) \( \Omega_{\Lambda} = 0.69179, \) with a current Hubble expansion rate of \( H_0 = 67.78 \text{ km/s/Mpc}. \) Primordial scalar power spectrum amplitude and spectral index are, respectively, \( n_s = 2.147 \times 10^{-9} \) and \( n_s = 0.9672 \) that yield the normalization of the matter power spectrum at the present time as \( \sigma_8 = 0.8166 \).

From the Einstein equation in the comoving gauge, we find that

\[
C_+(k) = \frac{5}{2} \Omega_m a^{-3} H^2 \epsilon_+(k), \tag{62}
\]

which relates the power spectrum of \( C \) to the usual matter linear power spectrum in the comoving gauge \( P_L(k) \) as

\[
P_C(k) = \frac{25}{4} \Omega_m \left( \frac{a^{-3} H^2}{k^2} \right)^2 P_L(k). \tag{63}
\]

We have calculated the power spectrum of induced tensor perturbations in Figure 1, as the gravitational wave energy density parameter per logarithmic interval

\[
\Omega_{GW}(k) \equiv \frac{1}{12 H^2} \frac{k^3 P_h(k)}{2 \pi^2} \approx \frac{1}{12 H^2} \frac{k^5 P_{\text{hs}}(k)}{2 \pi^2}. \tag{64}
\]

This follows from the 00-component of the energy-momentum tensor of the gravitational waves with \( \rho_{GW} \propto (h_s^s)^2 \) (Watanabe & Komatsu 2006). The second approximated is accurate in subhorizon scales. We show both wavenumber (\( k, \) along the top x-axis) and frequency (\( f = kc/2\pi, \) along the bottom x-axis). Figure 1 shows the induced tensor perturbations calculated in zero shear (ZS) gauge and uniform expansion (UE) gauge (dark blue), comoving gauge (Co, dark red), and uniform curvature gauge (UC, dark green) at four different redshifts (\( z = 6, z = 5, z = 2, \) and \( z = 0 \) from top, left to bottom, right). We do not present the power spectrum for the pathological uniform density gauge, because the integrand in this gauge blows up on small-scales (for larger \( k \)). We plot the result down to \( k = 100 / h/Mpc, \) just for the presentation purpose. Of course, the second-order perturbation theory, breaks down well before \( k \approx 1 / h/Mpc \) even for the highest redshift (\( z = 6 \)) shown here; see, for example, Jeong & Komatsu (2006).

First of all, we note the gauge dependence of the power spectra of induced tensor perturbations. While zero shear gauge and uniform expansion gauge show the same power spectrum, the power spectra calculated from comoving gauge and uniform curvature gauge are very different. As the induced tensor modes result from the nonlinear interactions, the power on large scales is suppressed and scales as \( h_s(k) \propto k^2 \) in the \( k \to 0 \) limit for all cases. Even on these near-horizon scales, however, amplitude is different for all cases.

To facilitate the comparison, we have also shown the power spectrum of linear tensor perturbations with \( r = 0.1 \) with (red solid) and without (black dotted) the damping due to free-streaming neutrinos after neutrino decoupling epoch (\( T \approx 1.5 \text{ MeV}; \) Weinberg 2004). Here, we adopt the damping factor calculation of Watanabe & Komatsu (2006) that the primordial linear gravitational waves \( h_{\text{prim}} \) are damped by 80.313% for the modes that are re-entering the horizon during the radiation-dominated epoch. For larger-scale modes, we estimate the damping factor by linearly re-scaling the small-scale damping factors with the neutrino fraction at the time of the horizon-crossing. That is, the damping factor is applied as \( h^1(\nu_\text{free})/h^1(\nu_\text{free} = 1) \) \( \approx (1 - 0.48582 \Omega_\nu (\eta - 1)^{k^{-1}}) \), where \( h^1(\nu_\text{free})/h^1(\nu_\text{free} = 1) \).

At higher redshift (\( z = 6 \) and \( z = 5 \)), the induced tensor power spectrum is much bigger compared to the linear power spectrum for \( k \gtrsim 10^{-2} / h/Mpc, \) even for the lowest case for the zero-shear gauge or uniform expansion gauge. While the amplitude of primordial gravitational waves decays as a linear theory tensor mode (without the linear source), in the matter-dominated epoch, the amplitude of induced tensor modes stays constant because the induced tensor mode is proportional to the gravitational potential perturbation, which stays constant. For lower redshift (\( z = 2 \) and \( z = 0 \), we observe the competition between the cosmological redshift of the linear tensor mode and the damping of gravitational potential in the presence of the cosmological constant (\( \Lambda \)). Note that for the lower redshift, we estimate the induced tensor modes by simply using the result in the matter-dominated era (Equations (58)–(59)) and re-scale the gravitational potential power spectrum Equation (63) with the linear growth factor.

Although the result for the zero-shear gauge has been reported in previous studies (Mollerach et al. 2004; Amendola et al. 2007; Baumann et al. 2007; Arroja et al. 2009; Assadullahi & Wands 2009, 2010; Jedamzik et al. 2010; Alabidi et al. 2013; Saga et al. 2015), the gauge dependence as well as total domination of induced tensor modes over primordial gravitational waves signature is the new result in this work. From these figures, it is clear that the induced tensor-mode contribution must be understood properly in conjunction with the exact observable that is being considered; that is the only way to remove the ambiguity due to gauge choice, and, therefore, to extract a truly primordial gravitational wave signature from the large-scale structure observables.
The primordial signature for inter-galactic scales is better than the zero-shear gauge and uniform expansion gauge properly reproduce the Newtonian velocity and gravitational potential (Hwang & Noh 1999a). The comoving gauge is a curious case. In the zero-pressure limit, the equations for density and velocity exactly coincide with the Newtonian equations for the density, velocity, gravitational potential (Hwang & Noh 1999a).

6. Discussion

We have presented the leading order induced tensor power spectrum generated by the quadratic combination of linear scalar perturbation in the matter-dominated era. The tensor power spectrum depends on the slicing conditions taken for the linear scalar perturbation. The results are summarized in Equations (58) and (61) for the solutions and the power spectra, respectively, in unified forms, and in Figure 1.

First of all, we emphasize again that the tensor power spectrum is gauge dependent because it naturally has to be to the nonlinear order. Comparing the induced tensor power spectrum with the linearly evolved spectrum of the primordial gravitational waves (1GW) with (thin, red) and without (dashed, black) the damping due to free-streaming neutrinos. Results from the four redshifts ($z = 0, 2, 5, 6$), and $z = 0$, from top left to bottom right, are shown to highlight the time evolution. The induced tensor modes completely dominate over the primordial signature for $k \gtrsim 10^{-2} \text{[h/Mpc]}$ for comoving gauge and uniform curvature gauge, while the induced tensor modes from zero shear gauge and uniform expansion gauge show moderate excess between $10^{-2} \text{[h/Mpc]}$ and $1 \text{[h/Mpc]}$. Searching for the signatures of primordial gravitational waves, therefore, must take into account the detailed study of the induced tensor modes including their gauge dependence.

Then, an important question arises: which gauge is the right choice for the observation of induced gravitational waves? As mentioned in Section 1, in order to properly address the question, one has to specify the observational strategy. First, the frequency range that we have considered here is too low (naturally, of the order of a Hubble timescale, $f \sim 10^{-19} - 10^{-16}$ Hz) to be detected from the direct detection methods using interferometers such as LIGO or LISA, or the pulsar timing array. The large-scale structure of the galaxy distribution offers futuristic, but compelling methods of detecting tensor modes by, for example, the parity-odd (B-mode) part of the gravitational weak lensing (Schmidt & Jeong 2012a), galaxy clustering (Jeong & Schmidt 2012; Chisari et al. 2014;
Schmidt et al. 2014, cosmic ruler (Schmidt & Jeong 2012b), as well as clustering fossils (Jeong & Kamionkowski 2012). Because these observables measure the tensor part of the metric perturbations on scales that we are considering here, and blind about the origin of the tensor perturbation, we need to understand the induced tensor perturbation properly in order to pin down the signatures from primordial gravitational waves. As mentioned earlier, the proper choice of gauge is subject to the exact way that the tensor perturbations are measured from each observable.

This was true even for the linear-order density and velocity power spectra. Because the behavior of the density perturbation depends on the gauge (Lifshitz 1946; Bardeen 1980), its power spectrum should depend on the gauge as well: in many (but not all) fundamental gauge conditions used in the literature the behavior of density perturbation happens to coincide far inside the horizon (Bardeen 1980). The issue has been resolved in the density perturbation case by addressing the strategy of measuring the density power spectrum: by observing the photons traveled from galaxies (Yoo et al. 2009; Yoo 2010; Bonvin & Durrer 2011; Challinor & Lewis 2011; Jeong et al. 2012; Yoo 2014; Jeong & Schmidt 2015). In the case of the cosmic microwave background temperature anisotropy power spectrum, the observational strategy of measuring temperature “difference” between different angular directions in the sky makes the observed quantities naturally gauge invariant (Abbott & Wise 1984; Abbott & Schaeffer 1986; Hwang & Noh 1999b).

In this work, we have only clarified the gauge dependence of the second-order tensor perturbation power spectrum generated by linear scalar perturbation. The issue of which one variable, or a combination of variables, is the right choice for the observed power spectrum is left for future investigation.

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