Do house price-earnings ratios in England and Wales follow a power law? An application of Lavalette’s law to district data

David Gray
University of Lincoln, Lincoln, UK

Abstract
This paper considers Lavalette’s function and its applicability to district house price-earnings ratios. Drawing on work in the urban scaling literature and Zipf’s law, in conjunction with finance theories of pricing and affordability, the paper considers how stable the distribution of ratios is over time, how robust the ranking order of ratios is in the face of variations in affordability over 2004–2019, and proffers an explanation for the shape and movement of the distribution. It draws on issues found in the economic growth literature where sigma-convergence is applied to spatial variables, and a narrowing of the distribution is said to indicate convergence. It proposes that, when plotted over time, the Lavalette exponent and Spearman’s correlation coefficient point to divergence and rank-order stability.

Keywords
House price-earnings ratio, England and Wales, Lavalette’s law

Introduction
Batty (2006) observes that objects and events scale with size. He highlights that most analyses that entail a power law to characterise that distribution do so for a single point in time. The city-size distribution characterised as Zipf’s law is an empirical regularity that predicts the size and number of population nodes are inversely related (Arshad et al., 2018). Batty (2006) rank clock reveals that although the distribution may be stable, the rank order of city populations is not.

In the economic growth literature, beta-convergence is said to exist when a region with a relatively low income per head experiences a faster growth rate than the average of the group of regions, and the gap between it and richer regions is reduced; low-income regions have a higher growth rate than high-income ones. An initial dispersion of regional values is maintained in rank order, but there is ‘catch-up’. Sigma-convergence is a narrowing in the spread of income levels from across geographic areas at time $t$ relative to time $t−p$. Where lower income regions catch-up with the
average at rates not related to distance to the average there will be rank instability, or intra-
distributional mobility (Sala-i-Martin, 1996). To assess this, Boyle and McCarthy (1997) introduce
the use of the trend in rank concordance, in conjunction with the trend in the coefficient of variation,
a measure commonly used for sigma-convergence.

The Organisation for Economic Co-operation and Development (2011) questions the meaning of
a ‘mean’ regional level. There is no clustering of regional levels around the ‘average’. As such,
converging on the mean is meaningless. Lavalette’s ranking power-law is a good candidate for
describing both long-tailed and non-long-tailed data (Chlebus and Divgi, 2007), when the Zipf
regularity is not expected and power laws can be applied. It can characterise much narrower spatial
distributions and has been applied to Italian regions and municipalities (Cerqueti and Ausloos,
2015a, 2015b). Particularly with policy initiatives around inequality, having alternative indicators of
spread not based on a mean could be useful. Indeed, Lavalette’s exponent and changes in the
exponent could provide useful insights into spatial inequalities.

This paper considers Lavalette’s function and its applicability to district house price-earnings
ratios (HPERs). Housing affordability is a topical local issue yet a gap in our knowledge is what
happens at the local level. One underutilised source of data provided by the Office for National
Statistics is district house price to earnings ratios in England and Wales. At the regional level,
Gregoriou et al. (2014) analyse the UK housing affordability picture over 1983–2009, averring there
is a dislocation between income and price. Is this the case across all districts?

The paper considers how stable the distribution of house price-earnings ratios is over time, how
robust the ranking order in ratios are in the face of variations in affordability, and proffers an
explanation for the shape and movement of the distribution. The robustness of the ranking order is
undertaken using a time-based Spearman’s rank correlation. The time-path is used to demonstrate
the degree intra-distributional mobility over the period. Moreover, it is concluded that, as it has
strong similarities with the time-path of the coefficient of variation, Lavalette’s exponent provides
an alternative measure of sigma-convergence. Drawing on work in the urban scaling literature and
Zipf’s law, in conjunction with finance theories of pricing and affordability, explanations for
stability of order and spread are proffered.

The paper is structured as follows. First, there is a brief discussion of house price deter-
determination and the distribution of affordability metrics. Second, Zipf’s law and central place theory
are outlined, and also the complication that administrative spaces injects. Next, rank preservation
and Batty’s clock are considered. This is followed by an outline of convergence in house prices
and the law of one price. There is a discussion of Spearman’s rank autocorrelation coefficient
under method. It is adapted to assess the degree of rank order instability. Pareto and Lavalette’s
exponents are also reviewed in method. The analysis shows that Lavalette’s exponent provides a
useful description of the distribution of data drawn from administrative units. The time-path of
the exponent reveals a pattern of sigma-convergence followed by divergence. Spearman’s au-
tocorrelation function reveals intra-distributional mobility, but too little to claim instability.
Rather than the general rise in the house price-earnings ratio implied by Gregoriou et al. (2014),
when ordered by rank, the resulting ‘gradient’ appears to steepen over the 15 years. An increase in
both Lavalette’s calibrating value and the exponent are consistent with a steepening of the
‘gradient’ of ratios. The steepening reveals at the bottom end districts are almost untouched by
liberal finance.

The distribution of local house prices

The Bank of England (2015) posits that rental yield (rent ÷ price) is a determinant of fundamental
value. Following this asset pricing model, beyond standard structural features such as size and
proximity to amenities, underlying a house price is the cost of capital. Miles and Monro (2019)
estimate that the sustained decline in real interest rates between 1985 and 2018 can account for all of
the doubling of house prices relative to incomes over that period.

Asset pricing models project that housing yields reflect expected rental income growth. With
anticipated city productivity growth (Coulson et al., 2013; Van Nieuwerburgh and Weill, 2010) or
population growth, rental yields are expected to have a greater locational value in the future, and so
current house prices will be higher.

A common thesis is that price rises since the 1980s have been related to more liberal
finance. Hay (2009) argues that, from the early 1990s, consumer-led growth strategies and the incentives around
allocating mortgage debt inflated house price in both the UK and Eire, propitiously.

A persistent decline in the risk-free rate of return and more accessible credit does not mean that all
HPERs should rise in the same proportion. Stein (1995) argues that, because of the inflated property
equity that is bestowed on existing owners, where the metrics are more generous, a price rise
accelerates [decelerates] more in a boom [bust] periods, which is more likely to be in high house
price area. In other words, high house prices and high house price-earnings ratios coincide with
greater volatility. This is linked to the financial accelerator (Aoki et al., 2004). With rising prices,
higher leveraged dwellings appear lower [not higher] risk to lenders over some of that rise.

In contrast, pessimistic expectations of future growth, locally, will raise agency costs to the lender
so that properties with the given rental stream would have lower prices. Glaeser and Gyourko (2005)
posit that, as properties are more readily available, a city with a declining population would ex-
perience falling local housing prices. Consistent with this, some US states have declining HPER
trends (e.g. Indiana, Mississippi and South Carolina) (Hu and Oxley, 2018).

The credit constraint that most buyers will face will be related, in part, to the lender’s view of risk
in lending. A hierarchy in HPERs could be viewed as reflecting risk-adjusted returns across space to
a dwelling purchase (Sinai, 2010). In this light, one could envisage the rank order of HPERs like the
term structure of interest rates. This implies that the ‘gradient’ of that HPER order will be steeper
[shallower] in a bubble [bust].

Urban systems and power laws

Mainstream urban economics posits that co-location generates increasing returns to scale in social,
household and business domains. The enhanced productivity that underpins higher wages should
induce city in-migration. Crowding costs that come with population density strongly feature
relatively expensive accommodation, acting as a counterweight to in-migration. A spatial general
equilibrium framework (Roback, 1982) posits that individual agents migrate within and between
population nodes to maximise their utility. The larger node may have a higher house price but not
necessarily a greater house price to earnings ratio for a standard dwelling.

An observed regularity and explained by central place theory (Hsu, 2012) is the urban size
structure characterised by Zipf’s law. A decreasing number of increasingly large nodes of activity
can be expressed as a non-linear relationship between the rank-size of cities and their corresponding
populations. Brakman et al. (2020: 304) summarise that empirically at the continental level, Zipf’s
law is not strictly adhere to. It is better supported at the national level but with a tendency towards a
more uneven distribution of city-sizes in the last 60 years. The distribution of city-sizes across
Germany and also within its regions has been shown to follow Zipf’s law (Giesen and Südekum,
2011).

Rozenfeld et al. (2011) show that the area of a city also follows a power law. The third leg of the
stool, population density, does not. Indirectly, they consider two distributions in ratio form revealing
a Zipf exponent of zero: the density of population is unrelated to the corresponding rank order. It
highlights that ratios should not necessarily have a power coefficient of unity.
As administrative territories may not necessarily reflect economic units, subdivisions may be arbitrary, and as some English and Welsh cities are divided into districts, this will inflate the number of tracts. Fontanelli et al., 2017 propose that the ‘split-merge’ process of district resizing can render initial power laws of cities into other kinds of probabilistic laws. The distributions in latter stages are represented well by the discrete generalised beta distribution (DGBD). A special case of a DGBD is Lavalette’s law. When examining Italian regions and municipalities’ aggregated tax income over the period 2007–2011, Cerqueti and Ausloos (2015a) find a Lavalette function is statistically appealing in describing a size-rank rule.

**Rank preservation**

A test of Zipf’s law is that population growth is randomly distributed across the urban size spectrum (Arshad et al., 2018) so that growth [shocks] should not alter the rank order in a general way. Batty (2006) rank clock reveals that the rank order of city populations is unstable. Duranton (2007) considers three groups of city population growth rates: fast, slow, and still. The fast entails rapid changes driven by the location of dynamic industries. The slow refers to movement up or down the urban hierarchy as they grow relative to other cities, and the still reflects the size distribution almost unmoved by the passage of time. This last group tends to be of very large cities.

For housing, shocks have differential effects. Following a positive house price increase, Bogin et al. (2017) find that, in large cities, prices are consistently higher than the ‘pre-acceleration’ level. In small cities, real price gains are not so clear. They conclude that the average growth acceleration rate is a signal of a permanent shift in a location’s economic fundamentals. Using a cost of capital framework, Himmelberg et al. (2005) argue that price sensitivity to changes in real interest rates favours higher price levels in rapidly growing cities. It is concluded that rank inconsistency or [rank-] shuffling in HPERs may favour more expensive districts, possibly in larger or growing cities.

**Convergence**

Beta-convergence has been applied to regional housing in the UK (inter alia Cook, 2012; Drake, 1995; Montagnoli and Nagayasu, 2015; Tsai, 2018). Tsai (2018) claims the beta-convergence process verifies the law of one price in a house price context. This ‘law’, which draws heavily on spatial arbitrage, implies that a common price results from relocating a good from the low to the high-priced sites, squeezing out differentials. In house prices, this relates to the buyer switching to searching in lower-priced locales.

Beta-convergence in a Sala-i-Martin (1996) sense is described as intra-distributional mobility. In effect, the initial distribution becomes less representative at some point. A gauge of this is the degree of commonality between a benchmark [initial] order and any subsequent one. Mobility could result from a deviation from, or retroversion to, an established order in an error correction sense. So, to be consistent with the growth literature’s view of change, the tendency for dissimilarity to increase with time would be more apposite. A converged group would be characterised by random changes only. Indeed, this could be a phase. As proposed by Quah (1996), a logical outcome of beta-convergence is that the faster growth of the catch-up group carries on, ‘leapfrogging’ formally richer states, generating beta-divergence. The time variability of the distribution or spread should be considered concurrently with the rank order (Gray, 2020).

**Method**

Zipf’s law is a special case of the general expression: $R = A P^{-\alpha}$ (or the equivalent Pareto-form $P = K R^{-\gamma}$), where $P$ is the population of the city, $R$ is the rank of the city (by size) and $A, K$ are scaling
constants. A version of the Pareto-form can be expressed as $P_R = P_1 R^{-q}$ (Arshad et al., 2018) where $(q)$ is referred to as the [Pareto] exponent, $P_R$ is the population of city ranked as $R$ and $P_1$ the calibrating value. For Zipf’s law to hold, $\alpha$, the Zipf exponent, should be $-1$. With $P_1$ as the population of the largest city and $-q = 1$ (so consistent with Zipf’s law), the second city would be expected to be half the size of the first and the third city, one third. The $10^{th}$ city is $1.5 \times$ the size of the $15^{th}$ city. However, with $-q$ of 0.2 the $10^{th}$ city is $1.5^{0.2} = 1.084 \times$ the size of the $15^{th}$ city, so a lower exponent is consistent with a more even distribution.

Lavalette’s ranking power-law, established by the French biophysicist Lavalette (1996), can be expressed as $P_R = P_1 [(N \times R) \div (N - R + 1)]^{-q}$ (2) where $N$ is the total number of cities. The formula describes very well a characteristic semi-logarithmic S-shape (Chlebus and Divgi, 2007). With $-q$ of 0.2 and $N = 338$, the Pareto-form (1) [Lavalette’s law (2)] predicts the $12^{th}$ city is around $1.072 [1.075] \times$ the size of the $17^{th}$ city. The $327^{th}$ centre is around $1.003 [1.110] \times$ the size of the $332^{th}$ centre, so the two have different lower tails, with (2) better able to capture those cases that are unusually poor.

Simulations show that a greater exponent $(q$ in absolute terms) corresponds with a larger cross-sectional coefficient of variation (CoV). The dispersion, as measured by the standard deviation, as well as the mean, increase linearly with the calibrating value. By implication, the CoV is independent of the calibrating value.

Changes that occurred in a hierarchy of district values can be discussed using the statistical distribution of rank changes or the distribution of differences. One such measure of change, a time-dependent of the calibrating value.

$N_i$ is the population of city ranked as $i$. The formula

$$P_R = P_1 \left( \frac{N \times R}{N - R + 1} \right)^{-q}$$

reflected the mean change in rank between pairs of periods over $N$ districts.

Where the rank order at time $t$ and $t+p$ is the same, there is no shuffling of rank, and $H(p) = 0$. $H$ will increase with $N$. To cater for this, a Spearman’s rank autocorrelation function is used so that the measure of rank stability becomes $AC(p) = 1 - \frac{6 \sum_{i=1}^{N} |R_{it} - R_{it+p}|^2}{N(N+1)(N-1)}$. A value of 1 implies rank preservation; $-1$, rank inversion; and 0, random co-ranking. The time lapse, $p$, can be increased progressively from $p = 1$ to $T-1$ which when plotted $AC(p)$ is similar to a rank autocorrelation function. As with Spearman’s statistic has the advantage in allowing negative values. For tests and confidence intervals (CI), Ruscio (2008) advocates using $\sigma_{AC(p)}^2 = \frac{1 + 0.5 \cdot AC(p)}{N-3}$ and the Fisher transform. Caggiano and Leonida (2009) use a parametric ACF approach to test for convergence based on the detrended output per capita. They interpret the ACF as revealing the transitional dynamics of the economy to its steady-state path: once the economy departs from this path, the time required for the ACF to go to zero is a measure of the time necessary for the economy to return to its long-run equilibrium.

Data

Regional and Local Authority District house price and earnings data are supplied by the UK’s Office for National Statistics (ONS). Earnings data are taken from the Annual Survey of Hours and Earnings. Annual estimates of gross earnings are based on the tax year in the reference year and relate to employees on adult rates of pay who have been in the same job for more than a year. Both workplace-based and residential district earnings are available. Work-based earnings are used to
capture the effects of commuting of workers across district boarders. House Price Statistics for Small Areas are drawn from the Land Registry, which provides a comprehensive record of property transactions in the UK. The ONS values are all actual prices, including those not involving a mortgage, and annual earnings, including those not engaged in house purchase. The median values capture the typical worker’s earnings and house price paid in a district.

England and Wales (E&W) is divided into 10 regions and, excluding the Isles of Scilly which has intermittent data, 338 districts. The median HPER of ‘all dwellings’ in a year is available for regions and districts from 2004 to 2019. This covers the run up to, and the recovery from, the financial crisis of 2007–2008. An English and Welsh Local Authority District has an average population of around 150,000 but the largest, Birmingham, has around 7 times that.

The average HPER for E&W for 2004 is reported in Table 1 as 6.529. The ratio is reasonably stable from 2004 to 2013, but at the end of the period, it increases by 1.1 ratio points. The range between the highest ratio region (mostly London) and the lowest (always North East) floats around 4 ratio points until 2010. After this, it increases by 0.8 ratio points per annum for 6 years. Interestingly, there is no drift in the minimum values over time, implying a broader range. What stands out is a hierarchy with a North < Midlands < South < London order.

The simple average of values across the 338 E&W districts for 2004 is reported in Table 2 as 6.96. The HPER for England and Wales is reasonably stable from 2004 to 2013, but at the end of the study, the ratio then increases by 1.6 ratio points. This pattern, but not the values, is reflected in Table 1. The average local HPERs are often inflated relative to the regional ones. Take 2018. The E&W average of 7.85 reported in Table 2 is smaller than the district mean [median] of 9.16 [8.75], consistent with a longer upper tail in the District distribution. However, the harmonic mean provides a very similar average value to the actual E&W one. The similarities between the district-based harmonic means and the corresponding regional ones are close for all bar London, the region where high ratios are most featured.

| Year | North | Midlands | South |
|------|-------|----------|-------|
|      | E&W  | North    | West  | Wales | East | West  | Midlands | East | Midlands | South | West  | East  | South  | London |
| 2004 | 6.529| 4.78     | 4.851 | 5.427 | 5.139| 6.02  | 5.945    | 7.729| 7.193    | 7.622 | 7.548 |
| 2005 | 6.738| 5.182    | 5.35  | 6.009 | 5.493| 6.201 | 6.071    | 7.987| 7.385    | 7.842 | 7.663 |
| 2006 | 6.960| 5.676    | 5.682 | 6.253 | 5.767| 6.364 | 6.151    | 8.022| 7.441    | 8.025 | 7.906 |
| 2007 | 7.174| 5.789    | 5.832 | 6.586 | 6.029| 6.477 | 6.435    | 8.295| 7.77     | 8.404 | 7.941 |
| 2008 | 6.897| 5.563    | 5.581 | 6.322 | 5.737| 6.08  | 6.102    | 8.035| 7.616    | 8.216 | 8.076 |
| 2009 | 6.347| 5.202    | 5.204 | 5.702 | 5.295| 5.841 | 5.419    | 7.239| 6.863    | 7.284 | 7.418 |
| 2010 | 6.842| 5.403    | 5.471 | 5.824 | 5.659| 6.189 | 5.805    | 7.76 | 7.447    | 8.108 | 8.238 |
| 2011 | 6.733| 5.116    | 5.276 | 5.578 | 5.39 | 5.928 | 5.708    | 7.611| 7.321    | 8.068 | 8.506 |
| 2012 | 6.759| 5.004    | 5.243 | 5.632 | 5.352| 5.901 | 5.578    | 7.587| 7.267    | 7.986 | 8.529 |
| 2013 | 6.736| 5.018    | 5.177 | 5.539 | 5.294| 5.843 | 5.618    | 7.525| 7.43     | 8.264 | 8.956 |
| 2014 | 6.954| 5.019    | 5.423 | 5.643 | 5.54 | 6.209 | 5.94     | 7.778| 7.831    | 8.564 | 10.076 |
| 2015 | 7.370| 5.149    | 5.552 | 5.74  | 5.676| 6.269 | 6.279    | 8.17 | 8.425    | 9.128 | 11.047 |
| 2016 | 7.590| 5.223    | 5.645 | 5.818 | 5.779| 6.375 | 6.487    | 8.532| 8.963    | 9.764 | 12.027 |
| 2017 | 7.775| 5.212    | 5.794 | 5.763 | 5.892| 6.633 | 6.839    | 8.85 | 9.664    | 10.253| 12.375 |
| 2018 | 7.854| 5.312    | 5.845 | 5.92  | 5.95 | 6.791 | 6.963    | 8.925| 9.776    | 10.373| 12.256 |
| 2019 | 7.705| 5.201    | 5.864 | 5.818 | 5.892| 6.833 | 6.857    | 8.795| 9.474    | 10.118| 12.054 |
Figure 1 displays district HPERs arranged in rank order for 2004 and 2018. HPER at the bottom end has a shorter tail than at the top. Both distributions are S-shape. They have a similar lowest value but the highest ratio in 2018 (the calibrating value, $P_1$) is $2.6 \times$ the one in 2004. The distribution steepened.

Results 1: Rank stability

Time-paths of Spearman’s coefficients for E&W and the regions are displayed in Figure 2. The path labelled E&W (right group, thick line) is of a rank autocorrelation function based on 2004, such that the value of $AC(14) = 0.88$. $AC(p)$ shows decreasing similarity with the 2004 order, but at a slow rate. Bifurcating the distribution, $AC(14)$ among the lower and upper half of the HPER distribution in Figure 1 are 0.84 and 0.45, respectively. This difference is significant ($z = 6.06 \ [0.000]$), supporting that view that change is linked with HPER level.

One might explore the time-paths to look for patterns. The $AC(p)$ paths in Figure 2 are traced in three regional groups for ease of interpretation. Most regions exhibit decreasing alignment as $p$ increases. However, in the post-2008 era, Wales, Yorks-Humberside and North West exhibit stable values. Indeed, in general, there is greater flux within the regions of the South, perhaps reflecting more rapid growth in ratios compared with elsewhere. There is no evidence of rank inversion in the Quah (1996) convergence–divergence sense.

Results 2: Exponents

Normally, power laws are expected when the rank-size plot shows an approximate linearity in the log–log representation. The log–log plots using expressions (1) and (2) reveal a concave and an almost linear pattern, respectively. Reported in Table 3, in 2004, the Pareto exponent for E&W is

| Year | Mean | St Dev | Median | H Mean | St Dev |
|------|------|--------|--------|--------|--------|
| 2004 | 6.960| 1.965  | 6.986  | 6.354  | 2.187  |
| 2005 | 7.249| 1.901  | 7.177  | 6.727  | 2.038  |
| 2006 | 7.438| 1.891  | 7.318  | 6.976  | 1.853  |
| 2007 | 7.751| 2.040  | 7.640  | 7.261  | 1.902  |
| 2008 | 7.573| 2.089  | 7.383  | 7.059  | 1.894  |
| 2009 | 6.842| 1.947  | 6.671  | 6.382  | 1.675  |
| 2010 | 7.336| 2.235  | 7.130  | 6.769  | 1.908  |
| 2011 | 7.272| 2.388  | 6.988  | 6.629  | 1.997  |
| 2012 | 7.219| 2.464  | 6.840  | 6.556  | 2.033  |
| 2013 | 7.328| 2.716  | 6.957  | 6.588  | 2.098  |
| 2014 | 7.822| 3.225  | 7.294  | 6.933  | 2.310  |
| 2015 | 8.291| 3.444  | 7.742  | 7.269  | 2.569  |
| 2016 | 8.728| 3.733  | 8.167  | 7.548  | 2.803  |
| 2017 | 9.054| 3.884  | 8.602  | 7.805  | 2.977  |
| 2018 | 9.156| 3.924  | 8.748  | 7.889  | 3.083  |
| 2019 | 8.993| 3.674  | 8.620  | 7.825  | 2.949  |

H Mean = Harmonic Mean.
St Dev = Standard Deviation.
estimated as $-0.261$ ($R^2 = 0.706$) with the corresponding Lavalette exponent as $-0.168$ ($R^2 = 0.972$). To assess the goodness of fit, the $R^2$ and Akaike’s information criterion coefficient are reported for both the Pareto and Lavalette functions. The latter is a superior fit for each of the 16 years. Bootstrap 95% confidence intervals indicate that the Lavalette exponents are not close to zero or unity. It is concluded that Lavalette’s function proffers a good representation of the HPERs between 2004 and 2019. As often found when Zipf’s power law is applied to various city populations, the primary node is found to be too large (Brakman et al., 2020: 328); this is found with Lavalette’s function and HPERs.
Table 3. Pareto and Lavalette exponents.

| Year | Lavalette | Pareto |
|------|-----------|--------|
|      | \( q \)  | \( -q \) | \( R^2 \) | AIC | MAX | \( q \) | \( R^2 \) | AIC |
| 2004 | 0.168     | 0.161 | 0.972 | -0.160 | 16.8 | 0.261 | 0.706 | 2.207 |
| 2005 | 0.153     | 0.148 | 0.981 | -0.753 | 17.1 | 0.242 | 0.738 | 1.893 |
| 2006 | 0.142     | 0.139 | 0.988 | -1.313 | 18.8 | 0.233 | 0.791 | 1.523 |
| 2007 | 0.143     | 0.139 | 0.980 | -0.798 | 22.4 | 0.237 | 0.803 | 1.479 |
| 2008 | 0.149     | 0.145 | 0.985 | -1.038 | 22.0 | 0.249 | 0.828 | 1.421 |
| 2009 | 0.148     | 0.144 | 0.983 | -0.911 | 22.6 | 0.251 | 0.848 | 1.278 |
| 2010 | 0.159     | 0.155 | 0.988 | -1.092 | 24.3 | 0.270 | 0.852 | 1.403 |
| 2011 | 0.171     | 0.167 | 0.988 | -0.950 | 24.9 | 0.291 | 0.859 | 1.493 |
| 2012 | 0.174     | 0.17 | 0.985 | -0.744 | 28.5 | 0.295 | 0.848 | 1.607 |
| 2013 | 0.183     | 0.178 | 0.987 | -0.764 | 30.6 | 0.315 | 0.878 | 1.484 |
| 2014 | 0.194     | 0.187 | 0.982 | -0.317 | 38.3 | 0.337 | 0.892 | 1.488 |
| 2015 | 0.203     | 0.198 | 0.985 | -0.407 | 37.9 | 0.351 | 0.880 | 1.680 |
| 2016 | 0.213     | 0.208 | 0.982 | -0.094 | 38.9 | 0.367 | 0.870 | 1.863 |
| 2017 | 0.216     | 0.211 | 0.984 | -0.182 | 41.0 | 0.369 | 0.858 | 1.972 |
| 2018 | 0.216     | 0.211 | 0.985 | -0.282 | 44.0 | 0.365 | 0.841 | 2.089 |
| 2019 | 0.209     | 0.205 | 0.988 | -0.590 | 39.6 | 0.354 | 0.843 | 2.007 |

1 Bootstraps confidence interval.

To illustrate the fit, Figure 1 shows the distributions generated for 2004 and 2018 using the estimated Lavalette exponents from Table 3 and the mean values in Table 1. The simulated values are a remarkably close fit for the realisations. As the goodness of fit measures are all over 84% (not reported), Lavalette’s function can also summarise the districts when in their regional groups as well.

Figure 3 shows the time-paths of the exponents in the three groups again. The S-shape for E&W (third set, thick line in Figure 3) highlights decreasing values up to the price bust of 2008, followed by increasing ones to 2018. The southern regions’ exponents appear to fan out or diverge. London’s divergent path to 2014 is followed by reversion to a more similar value to the rest. East of England lags behind its neighbour by 2 years. The North’s regional decays are strikingly similar. The initial 2004–2008 period entails a rapid decline in the exponents, which is followed by a stable phase. The Midlands’ coefficients are also stable over time.

Comment

Lavalette exponents and the cross-sectional CoVs are found to trace such similar time-paths for E&W and for the regions that reproducing both is unnecessary. The long tails at the top end render simple means of the district ratios to be biased upwards, so explaining why harmonic means are effective in aligning district and regions averages. One might expect long-tailed data to be a common phenomenon. For example, the Organisation for Economic Co-operation and Development (2011: 47) displays a distribution of net employment creation across the OECD (Figure 1.12) which has striking similarities with Figure 1.

The autocorrelation function shows decreasing similarity with the 2004 order. There is some reranking but despite the degree of general movement, there are strong similarities between the initial and terminal structures. This appears to be a function of the HPER level. Wales, Yorks-
Humberside and North West exhibit stable internal rankings, despite an increasing time lapse, and stable estimated $q$, akin to a [club-] converged group. These have lower HPERs.

At odds with Tsai (2018), the law of one price is not supported as a practical notion. With CoV being a standard measure of sigma-convergence, bifurcating the period into pre- and post-2008, in the first period, HPERs are found to converge, which is later trumped by greater divergence over 2009–2018. In the first period, the North saw unusually large spreads when the South experienced the obverse. Post-2008, regions outside of the South maintained stable spreads, whilst inside they were broadening. The E&W Lavalette exponents are high relative to most regions’ and do not trace a representative path. The broader distribution towards the end of the period appears to reflect distinct regional groups that are diverging, with the top half of the distribution featuring southern districts.

Over the 2004–2018 interval, there is a broadening of district spreads. This is evident from a similar minimum value but with a much higher maximum one in 2018 in Figure 1. Ignoring the tails, the distribution ‘gradient’ of the HPER in 2018 is steeper. Gregoriou et al. (2014) aver that there is a dislocation between income and price in UK regional housing affordability. This is not wholly supported at the district level. Rather, there is a similar finding to Van Nieuwerburgh and Weill, who reveal a steep rise in the dispersion of house prices across US regions during 1975-2007, using the (CoV) as a scale-neutral measure of dispersion. Here, there is a steepening in HPERs, captured by both the calibrating value and Lavalette’s $q$ increasing over the time interval. The steepening of the gradient and the difference in systematic changes between the lower and upper halves of the distribution or between the South regions and the rest support a Bogin et al. (2017)-type argument where fundamentals do not shift for those at the bottom in the same way as the top of the value hierarchy with a rise in credit.

**Conclusion**

This paper considers the evolution of district house price-earnings ratios over 2004–2019 as provided by the Office for National Statistics. They proffer a richer basis for affordability analysis than regional data. This paper finds the ratios can be summarised well by a Lavalette function. As it
caters for long tails, it proffers perhaps a more useful summary of spatial data than simple descriptive statistics where there is a tendency to maintain differentials.

Although there is a general rise in ratios, there is also a steepening of the ‘gradient’ of ratios. Explanations for both are argued to be linked to local risk assessment, credit liberalisation and real interest rates. There is an increase in both the calibrating value and the exponent over the period. The time-path of the Lavalette exponent is used to provide a measure of sigma-convergence–divergence, revealing that spreads expanded. Combined with a rising calibrating value, it is consistent with ratios rising at different rates. As such, assertions in the literature about the law of one price in housing are difficult to support.

Spearman’s rank autocorrelation coefficient is used to assess the stability of the rank order. Despite 15 years of random and systematic movements, and so intra-distributional mobility particularly at the top end of the distribution, rank instability is not generally found. The terminal and initial rank order are associated. The regions in the South and the top half of the distribution is subject to more change than elsewhere, posited to reflect lending practices and price accelerants (Aoki et al. 2004; Stein, 1995).

Overall, recognising persistence in spatial inequalities, Lavalette’s law and Spearman’s correlation coefficient offer simple scale-neutral additional measures for the analyses of inequalities, without presuming a clustering around an average.

**Declaration of conflicting interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

**Funding**

The author(s) received no financial support for the research, authorship, and/or publication of this article.

**ORCID iD**

David Gray [https://orcid.org/0000-0002-1715-5549](https://orcid.org/0000-0002-1715-5549)

**Notes**

1. 97% CI (0.84, 0.90) for (2004:2018) fits bootstraps CI at 95%.
2. CoVs for E&W are generated using both harmonic and non-harmonic means and standard deviations (not shown).

**References**

Aoki K, Proudm J and Vlieghe G (2004) House prices, consumption and monetary policy: a financial accelerator approach. *Journal of Financial Intermediation* 13: 414–435.

Arshad S, Hu S and Ashra B (2018) Zipf’s law and city size distribution: A survey of the literature and future research agenda. *Physica A: Statistical Mechanics and Its Applications* 492: 75–92.

Bank of England (2015) The Financial Policy Committee’s Powers over Housing Tools. Available at: [https://www.bankofengland.co.uk/-/media/boe/files/statement/2015/the-financial-policy-committees-powers-over-housing-tools.pdf?la=en&hash=824555A3E0E43904679511189F70706FC5EE5EA2](https://www.bankofengland.co.uk/-/media/boe/files/statement/2015/the-financial-policy-committees-powers-over-housing-tools.pdf?la=en&hash=824555A3E0E43904679511189F70706FC5EE5EA2) (accessed 1 February 2018).

Batty M (2006) Rank clocks. *Nature* 444: 592–596.

Bogin A, Doerner W and Larson W (2017) Local house price paths: accelerations, declines, and recoveries. *The Journal of Real Estate Finance and Economics* 58(2): 201–222.

Boyle G and McCarthy T (1997) A simple measure of beta-convergence. *Oxford Bulletin of Economics and Statistics* 59(2): 257–264.
Brakman S, Garretsen H and Van Marrewijk C (2020) An Introduction to Geographical and Urban Economics: A Spiky World. Cambridge, UK: Cambridge University Press.

Caggiano G and Leonida L (2009) International output convergence: evidence from an autocorrelation function approach. Journal of Applied Econometrics 24: 139–162.

Cerqueti R and Ausloos M (2015a) Evidence of economic regularities and disparities of Italian regions from aggregated tax income size data. Physica A: Statistical Mechanics and Its Applications 421(1): 187–207.

Cerqueti R and Ausloos M (2015b) Cross ranking of cities and regions: population versus income. Journal of Statistical Mechanics: Theory and Experiment 7: P07002.

Chlebus E and Divgi G (2007) A novel probability distribution for modeling internet traffic and its parameter estimation. IEEE Global Telecommunications Conference Global Telecommunications Conference, Washington, DC, 26–30 November 2007, IEEE GLOBECOM 2007 Proceedings; 4670-4675.

Cook S (2012) β-convergence and the cyclical dynamics of UK regional house prices. Urban Studies 49: 203–218.

Coulson NE, Liu C and Villupuram S (2013) Urban economic base as a catalyst for movements in real estate prices. Regional Science and Urban Economics 43(6): 1023–1040.

Drake L (1995) Testing for convergence between UK regional house prices. Applied Economics 29: 357–366.

Duranton G (2007) Urban evolutions: the fast, the slow, and the still. American Economic Review 97(1): 197–221.

Fontanelli O, Miramontes P, Cocho G, et al. (2017) Population Patterns in World’s Administrative Units. Royal Society Open Science, 4, 170281. Available at: http://dx.doi.org/10.1098/rsos.170281.

Giesen K and Südekum J (2011) Zipf’s law for cities in the regions and the country. Journal of Economic Geography 11(4): 667–686.

Glaeser E and Gyourko J (2005) Urban decline and durable housing. Journal of Political Economy 113(2): 345–375.

Gray D (2020) A simple measure of beta-convergence revisited. Urban Studies 58(12): 2569–2583.

Gregoriou A, Kontonikas A and Montagnoli A (2014) Aggregate and regional house price to earnings ratio dynamics in the UK. Urban Studies 51: 2916–2927.

Havlin S (1995) The distance between Zipf plots. Physica A: Statistical Mechanics and Its Applications 216: 148–150.

Hay C (2009) Good inflation, bad inflation: the housing boom, economic growth and the disaggregation of inflationary preferences in the UK and Ireland. The British Journal of Politics and International Relations 11(3): 461–478.

Himmelberg C, Mayer C and Sinai T (2005) Assessing high house prices: bubbles, fundamentals and mis-perceptions. Journal of Economic Perspectives 19(4): 67–92.

Hsu W-T (2012) Central place theory and city size distribution. The Economic Journal 122(563): 903–932.

Hu Y and Oxley L (2018) Bubbles in US regional house prices: evidence from house price-income ratios at the State level. Applied Economics 50(29): 3196–3229.

Lavalette D (1996) Facteur d’impact: impartialité ou impuissance? In: Internal Report INSERM U350 Institut Curie–Recherche, Bât. 112. Orsay, France: Centre Universitaire, 91405 Available at: http://www.curie.upsud.fr/U350/ (accessed November 1996).

Miles D and Monro V (2019) UK house prices and three decades of decline in the risk-free real interest rate. In: Staff Working Paper No. 837. London, UK: Bank of England.

Montagnoli A and Nagayasu J (2015) UK house price convergence clubs and spillovers. Journal of Housing Economics 30: 50–58.

Organisation for Economic Co-operation and Development (2011) Regional Outlook, Building Resilient Regions for Stronger Economies. Paris, France: OECD Publishing.

Quah D (1996) Empirics for economic growth and convergence. European Economic Review 40(6): 1353–1375.

Roback J (1982) Wages, rents, and the quality of life. Journal of Political Economy 90: 1257–1278, In press.
Rozenfeld H, Rybski D, Gabaix X, et al (2011) The area and population of cities: new insights from a different perspective on cities. The American Economic Review 101(5): 2205–2225.

Ruscio J (2008) Constructing confidence intervals for Spearman’s rank correlation with ordinal data: a simulation study comparing analytic and bootstrap methods. Journal of Modern Applied Statistical Methods 7(2): 416–434.

Sala-i-Martin X (1996) Regional cohesion: evidence and theories of regional growth and convergence. European Economic Review 40(6): 1325–1352.

Sinai T (2010) Feedback between real estate and urban economics. Journal of Regional Science 50(1): 423–448.

Stein J (1995) Prices and trading volume in the housing market: a model with downpayment effects. The Quarterly Journal of Economics 110: 379–405.

Tsai I-C (2018) House price convergence in Euro zone and non-Euro zone countries. Economic Systems 42(2): 269–281, In press.

Van Nieuwerburgh S and Weill P (2010) Why has house price dispersion gone up? The Review of Economic Studies 77(4): 1567–1606.