Numerical analysis of extensional flow through the pharyngeal duct

M Preciado-Méndez¹, M Salinas-Vázquez², W Vicente³, E Brito-de la Fuente³, G Ascanio⁴

¹Programa de Maestría y Doctorado en Ingeniería, Universidad Nacional Autónoma de México, Ciudad Universitaria, México.
²Instituto de Ingeniería, Universidad Nacional Autónoma de México, Circuito Escolar
³Fresenius-Kabi Deutschland GmbH, Bad Homburg, Germany.
⁴Centro de Ciencias Aplicadas y Desarrollo Tecnológico, Universidad Nacional Autónoma de México, Circuito Exterior Ciudad Universitaria, México.

E-mail: mpreciadom@iingen.unam.mx

Abstract. The flow through the pharynx from the glossopalatal junction (GPJ) to the upper esophageal sphincter (UES) has been numerically investigated with a non-Newtonian fluid obeying the power-law with similar rheological indices to a contrast medium used in videofluoroscopy. For that purpose, a three-dimensional model of the transport of food bolus along the pharynx has been proposed using the immersed boundaries method, which allow representing the shape of the pharynx using Cartesian grids. The pharyngeal wall has been considered to be an elastic membrane. Flow fields in terms of the axial velocity, pressure, shear rate and strain rate were obtained. Results show that the highest velocity concentrates in the central stream as the fluid enters into the pharynx. In addition, as the flow quits the pharynx, a recirculation zone appears inside the cavity, resulting in low velocity zone, which increases with the coefficient of elasticity. A strong dependence on the coefficient of elasticity was observed on the pressure fields; so that as such a coefficient increases, the pressure in the pharyngeal wall will increase. It has been also observed that the bolus head travels faster than the bolus tail, which indicates that the bolus is not only subjected to shear but also to elongation. Results from this work can be further used for a rheological characterization (shear and extension) of oral nutritional supplements for patients suffering from swallowing disorders.

1. Introduction

1.1. Role of bolus rheology when swallowing

Extensional rheological properties play an important role in processes in which the fluid is subjected to highly decelerated or accelerated flows, particularly important extensional effects are found in flows through converging or diverging geometries [1]. Many real flows, including those which are biologically relevant, are complex and can include both shear and extensional components. Extensional flows can significantly stretch macromolecules, providing orders of magnitude increases in elastic forces and extensional viscosity [2].

Rheology, defined as the science of deformation and flow of matter, is by common consent a difficult subject [3]; when it comes to the oral behavior of fluid foods analysis, rheology has been estimated as a very important factor. The flow behavior and the rate of deformation of a fluid food inside the mouth are considered as play a critical role in sensory perception and oral transportation. The rheology of complex polymeric biological fluids is vital to the correct functioning of many processes in the body.
1.2. Swallowing as subject of study

Recently, the importance of the extensional rheological properties to study the dynamics of complex fluids has become an area of important research activity, nevertheless, little information is available due to generating and sustaining uniaxial elongational flow experimentally remains a challenge [4]. The knowledge of behaviour of fluids does apply to the knowledge of biological functions, such is the case of swallowing process, which is a neurological function wherein more than fifty pair of muscles are involved. The pharynx plays a crucial role in this complex process because of its dual function since it transports both air to the larynx and the bolus to the esophagus in respiration and swallowing, respectively. Hence, the airway protection needs to be guaranteed, it is for this reason that swallowing is carried out within a second and it cannot be stopped or suspended before its completion. Furthermore, carrying out the swallowing process requires the accomplishment of a series of process, namely, closure of the nasopharynx, tongue pressure, pharyngeal peristalsis, and opening of upper esophageal sphincter, among others.

Swallowing is divided into three major stages according to the anatomic location of the bolus, namely, oral, pharyngeal and esophageal. The former one is in turn subdivided into two sub phases, preparatory and transport phase.

The oral preparatory phase occurs when food enters into the mouth aimed to be transformed in an appropriate bolus for swallowing. Subsequently, the oral transport phase begins, here, the tongue exerts a pressure against the palate, forcing the bolus to move posteriorly toward the oropharynx. This stage has a quite high importance due to the lingual activity since it helps form and maintain the bolus and also, lingual force might be considered as the main driving force for bolus flow [5]; it should be noticed that it stimulates the oropharyngeal receptors and thus, triggers the pharyngeal swallow reflex. Once this stage has been completed, it follows the pharyngeal phase, here, the airway protection and the bolus transport through the pharynx towards the esophagus are carried out, for this reason, this phase is considered as the most important and a precise synchronized work of the epiglottis to fold over the glottis and stop liquid and food from entering the trachea and the pharynx are needed. The latter phase of swallowing begins when bolus passes though the upper esophageal sphincter (UES).

2. Governing equations

A non-Newtonian shear-thinning fluid was considered in this work because that is the type of fluid used for the assessment of swallowing process as well as for treating dysphagic patients. Meng et al [6] suggested that non-Newtonian of fluids would be safer to swallow since they would allow the neuromuscular system more time to shut off air passages and reduce the risk of aspiration, for this reason it is important to simulate pharyngeal swallows of non-Newtonian boluses. A density of 1800 kg/m3 and a dynamic viscosity following the Sisko-model were used.

The Navier-Stokes equations can be written as an extension of the following compressible equations [7].

\[
\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}_i}{\partial x_i} = \mathbf{S}_p
\]

(1)

where \(\mathbf{U}\) is a four-component vector defined by Eq. (2) and \(\mathbf{F}_i\) are the fluxes:

\[
\mathbf{U} = (\rho, \rho u, \rho v, \rho w)
\]

(2)

\[
\mathbf{F}_i = \begin{pmatrix}
\rho u_i u_1 + p \delta_{i1} - 2 \mu S_{i1} \\
\rho u_i u_2 + p \delta_{i2} - 2 \mu S_{i2} \\
\rho u_i u_3 + p \delta_{i3} - 2 \mu S_{i3}
\end{pmatrix}
\]

According to the equation of state for an artificial incompressible fluid the pressure is replaced in Navier-Stokes equation:

\[
p = \rho c^2
\]

(3)

where \(p\) is the pressure and \(\rho\) is the density considered; \(c\) is the sound speed of the fluid. The values for the coefficients for the Sisko-model, Eqs.(4) and (5), were obtained from rheological measurements and are shown in Table 1.
\[
\mu = \mu_\infty + k \dot{\gamma}^{(n-1)} \quad (4)
\]
\[
\mu = \mu_\infty + \begin{cases} 
\mu_{max} \dot{\gamma} < 0.1 \\
\mu_{min} \dot{\gamma} > 500 \\
0.1 \leq \dot{\gamma} \leq 500
\end{cases} \quad (5)
\]

Table 1. Parameters for Sisko-model

| Parameter | Value |
|-----------|-------|
| \(k\)     | 11.0 Pa\,s\(^n\) |
| \(n\)     | 0.32  |
| \(\mu_\infty\) | 0.42 Pa\,s |

\(S_F\) in Eq.(1) is the vector of source terms. For this case, the four terms in \(S_F\) are equal to:

\[
S_p = S_{pu} = S_{pv} = S_{pw} = 0 \quad (6)
\]

\(S_F = 0\)

Gravitational body forces were considered negligible, since bolus transport is due to the peristaltic movement [8].

2.1. Numerical resolution.

As the bolus passes through the pharynx, its cross section gets modified in an expedite form as a consequence of the peristaltic flow. Cook et al. [9] proposed aspatio-temporal shape of the pharynx, from the glossopalatal junction (GPJ) to the upper esophageal sphincter (UES), and that was the basis of this work.

To pursue with the numerical simulations, the governing equations had to be discretized in each node of the computational grid and for this reason, it was necessary to build numerically the components which allow to represent in an accurate way the physic phenomenon, namely, the pharynx geometry.

The immersion of the geometry of the pharynx, follows a process illustrated in Figure 1. The nodes that do not belong to the pharynx surface are blocked by imposing in them constant speeds or a value of zero, allowing the equations to be solved exclusively in the nodes inside the geometry, i.e. the fluid will flow only inside the pharynx. For those nodes remaining near the real body surface, within the flow area, velocity was interpolated at every time step in order to avoid stepped flow in nodes nearby the blocked areas [12]. Temporal functions (high-order polynomials) of the position of 21 reference points for 10 different times allowed obtaining the temporal evolution of the pharynx surface. From these functions, it is possible to know the approximate position of each point at every instant. Interpolations between these reference points were used to know the position of the remaining points of the mesh conforming the surface. Deriving over the time the functions corresponding to the change of position in each point on the surface of the pharynx leads to obtain the radial velocity. The temporal deformation occurring on the surface is considered to be produced only in the radial direction. Thus, the two-dimensional geometries used by Chang et al. [3] were converted into three-dimensional geometries, as shown in Fig. 2 and, which are used in this work.

The generalized coordinate system is solved by extension of the fully explicit McCormack scheme, second order in time and fourth order in space, developed by Gottlieb and Turkel [10]. The inlet and outlet boundary conditions used herein are those proposed by Poinsot and Lele [11].
3. Peristaltic flow and movable pharyngeal walls

As abovementioned, the peristaltic flow is responsible of the movement of the bolus through the pharynx. Fluid motion in elastic tubes has a high practical and theoretical relevance, however, not much results have been produced due to the complications on the calculation of simultaneous fluid-structure evolution \[13\]. In order to consider the walls of the pharynx to be movable, an axial velocity was fixed on the surface (peristaltic wave velocity, \( \approx 4.81 \, \text{cm/s} \)); also, a velocity normal to the wall and a wall pressure were fixed. The wall pressure was computed considering the pharyngeal wall like a non inertia membrane according to Predizzetti \[13\] from:

\[
p_w = \tau_n - \frac{2}{Re} \left( \frac{U'_x - R'U'_x}{1 + R'^2} + \frac{U'_r}{R} \right)
\]

where \( Re \) is the global Reynolds number, and \( U'_x \) is streamwise velocity and \( U'_r \) is the radial wall velocity both defined by \( U'_x = \frac{\partial x}{\partial t} \): and \( U'_r = \frac{\partial R}{\partial t} \), \( R \) is the local transversal radio of the pharynx.

The hydrodynamics of the gastrointestinal duct is influenced by its mechanical characteristics. In this work, we focused on the coefficient of elasticity, \( K \), defined by Eq. (8), hence, values of about \( 10^2 \) to \( 10^3 \, \text{Pa} \cdot \text{m} \) for \( K \) were taken. The membrane was considered isotropic, homogeneous, with constant wall width and a hyperelastic behavior. In Eq (7) \( \tau_n \) is the wall normal stress, which is function of the principal deformation ratios and of the coefficient of elasticity \( K \).

\[
K = hE
\]

4. Numerical method

The computational domain has as dimensions \( 5.0 \, \text{cm} \times 2.8 \, \text{cm} \times 2.8 \, \text{cm} \) in \( x, y \) and \( z \) directions respectively are: This domain was discretized through a uniform grid distribution employing \( 150 \times 109 \times 109 \) nodes; the grid spacing is \( 0.3 \times 0.25 \times 0.25 \, \text{mm} \) in \( x, y \), and \( z \) directions, respectively. According to Hasegawa et al. \[12\] the total simulation time was fixed in \( 1.04 \, \text{s} \) and divided into three stages: From \( 0–0.34 \, \text{s} \) the filling stage with only the entrance (GPJ) open; from \( 0.34–0.54 \, \text{s} \), the
intermediate stage, here both inlet and outlet are open and the emptying stage from 0.54–1.04 s where the entrance is already close and only the exit (UES) remains open.

4.1. Boundary and initial conditions
A bolus volume of 10 cm$^3$ was used, being this amount one of the most commonly used in experiments and also corresponding to a mouthful; also a preexisting volume inside the pharynx of 2.5 cm$^3$, these value is similar to the value used by Chang et al. [3]. The velocities were imposed at the inlet in order to assure the right volumetric flow in the pharynx. The oral preparatory phase is represented by a time of 0 s, where both the GPJ and the UES are closed and it is considered that all velocities are null in the entire domain.

The change of volume of the pharynx at every time instant, allowed obtaining the inlet and outlet volumetric flows. The flow rate of the inlet is obtained by the change of volume of the pharynx at the time step $\Delta t$:

$$\dot{V}_{in} = \left[ \frac{\Delta V_{pharynx}}{\Delta t} \right]$$  \hspace{1cm} (9)

the mean axial velocity at the inlet is given by:

$$U_{inlet} = \frac{\dot{V}_{in}}{A_{in}}$$  \hspace{1cm} (10)

where $A_{inlet}$ corresponds to the perpendicular area to the inlet flow.

In the intermediate stage, $t = 0.34$ s to 0.54 s, it is necessary to know the volumetric flow at one of the two boundaries, and for that, a small and constant inlet and outlet volumetric flow difference was considered.

The bolus has to be propelled from the oral cavity to the pharynx, the tongue is responsible for carrying out this task. Therefore, the pressure applied by the tongue had to be considered in the simulation and it was modeled by setting a membrane wall pressure, $P = P_{inlet}$, in the nodes close to the entrance, allowing the inlet boundary condition adapting to the pressures inside the pharynx. The membrane wall pressure at these points will be the maximum between $P_{inlet}$ and the value obtained locally with Eq. (10).

5. Results
Different cases were studied and the identical configuration for the temporal evolution of the pharynx was used in all studied cases. The value of the volume increased up to $V_{max} \approx 12.47$ cm$^3$ at $t = 0.423$ s. The value of 12.5 cm$^3$ was not reached (initial volume plus bolus volume), since the emptying process began at $t = 0.33$ s. Part of the bolus volume has been evacuated at $t = 0.423$ s. The coefficient of viscosity $K$ was varied and remained the same through the whole process; the values of $P_{inlet}$ were chosen of the maximums pressures obtained in each simulation. The values used for the most significant cases simulated are shown in

| Case | $P$ (Pa) | $K$ (Pa · m) |
|------|----------|--------------|
| A    | 2.0      | 25           |
| B    | 4.0      | 50           |
| C    | 6.0      | 75           |
| D    | 8.0      | 100          |

5.1. Velocity profiles
5.1.1. Filling stage $(0 \ s < t < 0.33 \ s; \ GPJ \ open \ and \ UES \ closed)$. Regarding to the velocity, simulations showed that all the four studied cases presented a same behaviour in the first stage. Thereby, rises of the velocity were observed at the axis when the fluid (bolus) entered the pharynx and it is accelerated until reaching the pharyngeal wall which provokes also an increase of pressure.
Fig. 3 shows some profiles for the velocity in Case B at this stage, the values were close to those found by Nguyen et al. [13] who found values for the velocity of the bolus head of 37.1 ± 1.1 cm/s using the technique of videofluoroscopy. It is important to point out that the flow is not necessarily axisymmetric even though the geometries were generated from revolution surfaces as can be seen from Figure 5 with an isosurface of velocity.

5.1.2. Intermediate stage (0.33 s < t < 0.54 s). It was observed in this stage that both the entrance (GPJ) and the exit (UES) are open but is the entrance that begins to close and the bolus continues its movement to the outlet. Values under 40 cm/s were observed for this case and those results are in good agreement with those reported by Hasegawa et al [12] who estimated the maximum velocity reached in the swallowing process of 50 cm/s.

5.1.3. Emptying stage (0.54 s < t < 1.04 s). At time t = 0.54 s the emptying phase begins and as the peristaltic contraction occurs, a recirculation zone (negative velocities) is generated and becomes larger near to the entrance where the GPJ is now closed. The zones with a highest shear rate were still found near the walls and in the recirculation zone. Recirculation zones with negative velocities were observed at the occluded zone and this was more relevant as the values coefficient of elasticity $K$ incremented.

**Figure 3.** Velocity profiles for Case B and D (right) and global velocities at the occluded zone (t = 0.7 s)
Figure 4. Shear rate (left) and viscosity (right) profiles at the occluded zone (t = 0.7 s)

Figure 5. Isosurface of velocity at t = 0.2s

Figure 6. Case 2, velocity profiles.

a) t = 0.06 s, b) t = 0.2 s, c) t = 0.34 y
d) t = 0.46 s

5.2. Pressure profiles

5.2.1. Filling stage. At this stage, it can be noticed that the process is highly influenced by the pressure exerted by the tongue to initiate the movement and also by the coefficient of elasticity. As the latter increased its value, so did the pressure.
5.2.2. Intermediate stage. Figure 8 shows the flow fields in terms of the pressure for Case B, the pressure in the pharynx reached values around 4 kPa which is the value estimated by Ferguson et al [14].

5.2.3. Emptying stage. The emptying process is mainly controlled by the volumetric flow, so that the elasticity level has a little influence. Under real conditions, changes in the coefficient of elasticity would create changes in the bolus clearance time.

5.3. Extensional viscosity and extensional rate calculation. The calculation of the nominal extensional strain rate was made by:

\[ \dot{\varepsilon} = \frac{\partial u}{\partial x} \]  

(11)

and the extensional viscosity was reckoned by the

\[ \eta_E(t, \dot{\varepsilon}) = \frac{\tau_{xx}(t, \dot{\varepsilon}) - \tau_{yy}(t, \dot{\varepsilon})}{\dot{\varepsilon}} \]  

(12)

Central lines where plotted in order to have a general idea of the rheological behavior of the fluid used in the simulations.

![Figure 7. Extensional viscosity and t the occluded zone (t = 0.7 s)
Figure 8. Pressure profiles for Case B at six different times (0.06, 0.2, 0.46, 0.54, 0.7 and 0.9 s)

Conclusion
A three-dimensional model of the transport of food bolus along the pharynx has been proposed using the immersed boundaries method, which allows representing the shape of the pharynx using Cartesian grids. The pharyngeal wall has been considered to be an elastic membrane, and its temporal shape has been taken from previous experimental works using three-dimensional interpolations. The effect of the tongue force acting on the bolus was incorporated as a pressure exerted by the pharyngeal wall close to the GPJ. The effect of the elastic coefficient on the flow behavior has been analyzed.

References
[1] P. J. Carreau, G. Ascanio, E. Brito-De La Fuente and P. A. Tanguy, "Orifice Flowmeter for Measuring Extensional Rheological Properties," The Canadian Journal of Chemical Engineering, vol. 80, pp. 1189-1196, 2002.
[2] S. J. Haward, J. A. Odel, M. Berry and T. Hall, "Extensional rheology of human saliva," Rheologica Acta, vol. 50, pp. 869-879, 2011.
[3] J. F. Steffe, It is important to point out that the flow is not necessarily, East Lansing: Freeman Press, 1996.
[4] P. K. Bhattacharjee, A. G. McDonnell, R. Prabhakar, L. Y. Yeo and J. Friend, "Extensional flow of low-viscosity fluids in capillary bridges formed by pulsed surface acoustic," New Journal of Physics, vol. 13, 2011.
[5] A. Okada, M. Honoma, S. Nomura and Y. Yamada, "Oral behavior from food intake until termila swallow.," Physiology and Behavior., vol. 90, pp. 172-179, 2007
[6] Y. Meng, M. A. Rao and K. Datta, "Computer simulation of the pharyngeal bolus transport of non-newtonian fluids.," Institution of Chemical Engineers Journals. Food and Bioproducts Processing., vol. 83(C4), pp. 297-305, 2005
[7] M. Salinas-Vázquez, M. de la Lama, W. Vicente and E. Martinez, "Large Eddy Simulation of a flow through circular tube bundle," Applied Mathematical Modelling, vol. 35 (9), pp. 4393-4406, 2011.
[8] J. B. Palmer, "Bolus aggregation in the oropharynx does not depend on gravity.," *Archives of Physical Medicine and Rehabilitation.*, vol. 79, pp. 691-696, 1998.

[9] I. Cook, W. J. Dods and R. O. Dantas et al, "Opening mechanisms of the human upper esophageal sphincter," *American Journal of Physiology.* vol. 257, pp. 748-759, 1989.

[10] D. Gotlieb and E. Turkel, "Dissipative two-four methods for time dependent problems," *Mathematics of computation.* vol. 30, pp. 703-723, 1976.

[11] T. Poinset and S. Lele, "Boundary conditions for Direct Simulations of Compressible Viscous Flows," *Journal of Computational Physics.* vol. 101, pp. 104-129, 1992.

[12] A. Hasegawa, A. Otoguro, H. Kumagai and F. Nakasawa, "Velocity of swallowed gel food in the pharynx by ultrasonic method," *Journal of the Japanese Society for Food Science and Technology.* vol. 52, pp. 441-447, 2005.

[13] H. Nguyen, J. Silny, D. Albers, Roeb E., Gartung C., G. Rau and S. Matern, "Dynamics of esophageal bolus transport in healthy subjects studied using multiple intraluminal inmedancometry," *American Journal of Physiology.* vol. 273, pp. G958-64, 1997.

[14] D. Ferguson, A. Shuttleworth and D. Whittaker, *Oral Bioscience,* Edingburgh: Churchil Livingstone., 1999.

[15] A. Chorin, "A Numerical Method for Solving Incompressible Viscous Flow Problems," *Journal of Computational Physics.* vol. 135, pp. 118-125, 1967.

[16] M. W. Chang, B. Rosendall and B. A. Finlayson, "Mathematical modeling of normal pharyngeal bolus transport: A preliminar study," *Journal of Rehabilitation Research and Development.* vol. 35, pp. 327-334, 1998.

[17] G. Pedrizetti, "Fluid flow in a tube with an elastic membrane insertion," *Journal of Fluid Mechanics.* vol. 375, pp. 39-64, 1998.

[18] W. Yang, T. Fung, K. Chian and C. Chong, "Finite element simulation of food transport through the esophageal body," *World Journal of Gastroenterology.* vol. 13, no. 9, pp. 1352-1359, 2007.