The calculation of the masses of the lightest nucleon resonances using lattice QCD is surveyed. Recent results for the mass of the first radial excitation of the nucleon, the Roper resonance, are reviewed and the interpretation in terms of models of hadronic resonances, such as the quark model and hadronic molecules, discussed. The talk concludes with an outline of prospects for future calculations.

1. Introduction

The calculation of the light hadron spectrum has historically been the benchmark calculation of lattice QCD, but the predictive focus has been on the phase structure of QCD and on weak matrix elements. There is now increasing interest given to the study of hadronic structure, both through the measurement of form factors and structure functions, and through the determination of the hadron spectrum, and in particular the nucleon resonance spectrum. In this talk, I will review recent lattice results, emphasising what lattice QCD measurements can tell us about the threshold resonances.

I will begin by briefly outlining the theoretical and computational issues in the determination of the nucleon spectrum. I will then review recent lattice results, before addressing the question of what they can tell us about the nature of the observed threshold states, in particular by emphasising the importance of continuing the studies to physical values of the light-quark masses, and by the pursuing the study of “molecular” states. I will conclude by discussing prospects for future calculations.

2. Nucleon Spectrum Cookbook

The recipe for determining the mass of a low-lying state from Euclidean lattice QCD is straightforward: choose an operator $O_N$ having a large
overlap with the state, and form the time-sliced correlator $C(t)$:

$$C(t) = \sum_{\vec{x}} \langle 0 | O_N(\vec{x}, t)O_N(0) | 0 \rangle$$

where $M$ is the mass of the lightest particle in that channel. In practice, there are systematic uncertainties that have to be accounted for: finite-size and discretisation effects, the extrapolation to physical values of the light quark masses, and, until recently, the systematic uncertainty due to the use of the quenched approximation. For the light-hadron spectrum, there have been many precise calculations of the lowest-lying states both in the quenched approximation, and in “full” QCD; in the former case, the measured spectrum agrees with experiment to around 10% in most channels.

The calculation of the nucleon resonance spectrum has particular delicacies. Firstly, the cubic symmetry group of the lattice limits our ability to construct interpolating operators of a specific $J^{PC}$; only the lightest states for $J = 1/2, 3/2$ and $5/2$ can be delineated unambiguously from a single correlator. Secondly, the propagation of heavier states is subject to worsening signal-to-noise ratios. Finally, the determination of the higher resonances within a channel requires the determination of a matrix of correlators.

There have been several studies of the excited nucleon spectrum, focusing on the ground states of both parities for spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$. All of these studies have also extracted a radial excitation in the $J^P = \frac{1}{2}^+$ channel by employing two interpolating operators:

$$N_1 = (uC\gamma_5d)u$$
$$N_2 = (uC\gamma_5d)u.$$ (3)

These both have an overlap onto $\frac{1}{2}^+$ states, but $N_2$ vanishes in the non-relativistic limit and is expected to couple predominantly to the radial excitation of the nucleon; a sample calculation is shown in Figure 1. All the calculations share the feature that the ordering of the states is broadly in accord with quark-model expectations, with, for example, the “Roper” in excess of 2 GeV, and no evidence for a light $\Lambda(1405)^-$. 
3. Light quarks, and more of them.

The preceding calculations have several limitations. Firstly, they were all obtained in the quenched approximation to QCD. Secondly, they employed quarks with masses around that of the strange quark. Finally, they used a limited basis of operators, and in particular operators that would be expected to couple primarily to three-quark states.

The elimination of the quenched approximation to QCD imposes the greatest computational demands. It is thus crucial to extract the maximum physical information; the use of Bayesian statistics with suitable priors provides a possible means of so doing. An application of this method to the study of the nucleon correlator, albeit for quark masses around the $s$-quark mass, reveals the same quark-model-like ordering of states observed in the quenched approximation.

The discovery of a lattice fermion action possessing an exact chiral symmetry has opened the prospect for calculations at physical light-quark masses, though at considerably computational cost than with “traditional” Wilson fermions. A recent calculation using the overlap realisation of this action has allowed pion masses as low as 180 MeV to be attained, enabling the exploration of the region in which the effects of the pion cloud can emerge. Bayesian statistics are used, and a fit to the nucleon correlator reveals!
Figure 2. Masses of the ground and first-excited states of positive and negative parity, obtained from a Bayesian fit to $N_{17}$.

The authors are taking care to investigate the possible sources of systematic uncertainties in their calculation. Most notably, even in the quenched approximation to QCD, both the radial excitation of the nucleon and its parity partner can “decay” for suitably light pion masses. In the scalar-meson sector, this is manifest through non-unitary behaviour of the scalar propagator, which can be understood within quenched chiral-perturbation theory. The final interpretation of the results in the nucleon sector must await this analysis.

If the experimentally observed spectrum proves to defy interpretation from these calculations, is it possible that we can interpret the anomalously light components of the spectrum, such as the Roper and $\Lambda(1405)^-$, in terms of molecular or multiquark states? All calculations so far have used local, three-quark interpolating operators, and are therefore sensitive to states having broadly that structure. Therefore I will conclude this talk with the issue of how we might observe molecular states.

Typically the binding of hadronic “molecules” is small on the scale of QCD, of the order of a few MeV, and furthermore the states are large on the scale of the box sizes in current lattice calculations. The most extensively
studied multiquark state has been the H dibaryon, a proposed six-quark state and the lightest possible spin-0 state with strangeness -2. This was originally computed in the bag model to be $O(100)$ MeV below the $\Lambda\Lambda$ threshold. This relatively large predicted binding energy has encouraged several lattice studies over the past 15 years\textsuperscript{9,10,11,12}, but with conclusions complicated by the need to extrapolate to infinite volume. The most recent study provides no evidence for binding in the infinite-volume limit\textsuperscript{12}.

A very fruitful arena in which to explore binding of molecular states in QCD is the heavy-quark sector, and in particular the $BB$ system\textsuperscript{13}. By using static heavy quarks, the two “atoms” are fixed in space, and an adiabatic potential can be defined between them which can then be probed; here there is evidence of a binding potential in some channels\textsuperscript{14,15}, but models are required to extend the analysis to physical quark masses.

An elegant way of exploring the issues of scattering lengths and hadronic interactions is provided by examining the volume dependence of the two-particle spectrum\textsuperscript{16}. The method typically requires the computation of all-to-all propagators. Whilst such calculations are computationally very expensive for QCD, the advent of terascale computing resources should enable the tackling of these challenging problems over the next few years.

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