Reconciling Hubble Constant Discrepancy from Holographic Dark Energy

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Holographic dark energy (HDE) models the vacuum energy in a cosmic IR region whose total energy saturates the limit of collapsing into a black hole. HDE predicts that the dark energy equation of the state (EoS) transiting from greater than \(-1\) regime to less than \(-1\), accelerating the Universe slower at early stage and faster at late stage. We show that this model provides a natural reconciliation of the Hubble constant \((H_0)\) discrepancy between CMB measurement and local measurements. With Planck+BAO data, we fit HDE model’s \(H_0\) as 71.54 ± 1.78 km s\(^{-1}\)Mpc\(^{-1}\), consistent with local \(H_0\) measurements by LMC Cepheid Standards \([1]\) (R19) at 1.4σ level. Combining Planck+BAO+R19, we find \(c = 0.51 ± 0.02\) and \(H_0 = 73.12 ± 1.14\) km s\(^{-1}\)Mpc\(^{-1}\), which fits cosmological data at all redshifts. Future CMB and large-scale structure surveys will further probe this scenario.

Introduction. — The cosmological observations derived from “Early” and “Late” Universe tend to prefer different values of the Hubble constant, leading to the discrepancy between the two types of measurements. The Planck measurement of the cosmic microwave background (CMB) constrained the Hubble constant to 1% precision, \(H_0 = 67.4 ± 0.5\) km s\(^{-1}\)Mpc\(^{-1}\) \([2]\), whereas local measurements, such as the SH0ES measurement of Cepheids data obtained from Hubble Space Telescope (HST) provides \(H_0 = 74.03 ± 1.42\) km s\(^{-1}\)Mpc\(^{-1}\) (denoted by R19 hereafter) \([1]\). In addition, replacing the Cepheids with the oxygen-rich Miras discovered in NGC4258, Ref. \([3]\) measured the Hubble constant \(H_0 = 73.3 ± 3.9\) km s\(^{-1}\)Mpc\(^{-1}\). Using the geometric distance to the megamaser-hosting galaxies CGCG 074-064 and NGC 4258, Ref. \([4]\) gives \(H_0 = 73.9 ± 3.0\) km s\(^{-1}\)Mpc\(^{-1}\). In a complementary probe of using gravitationally lensed quasars with measured time delays in a flat ΛCDM cosmology, the H0LiCOW team found \(H_0 = 73.3^{+1.7}_{-1.8}\) km s\(^{-1}\)Mpc\(^{-1}\) \([5]\) and more recently 82.4\(^{+8.3}_{-8.1}\) \([6]\). A combination of different local measurements yields \(H_0 = 73.3 ± 0.8\) km s\(^{-1}\)Mpc\(^{-1}\), which is 6.1σ discrepant from the aforementioned Planck result \([7]\).

Various theories have been proposed to resolve this discrepancy, mainly from two prospects \([8]\): (i) Modifying the early-universe physics to shrink down sound horizon at drag epoch \(r_{\text{drag}}\) \([9]\), such as including interactions in-between neutrinos to make them free-streaming later. (ii) Modifying dark energy evolution, by considering dark section interactions \([10]\) and early dark energy component \([11,12]\). In this work, we adopt the later approach, by using the holographic dark energy (HDE) to reconcile the \(H_0\) discrepancy. We demonstrate that with one more parameter \(c \simeq 0.5\) of the HDE model, its equation of state (EoS) transits from \(w > -1\) to \(w < -1\), which can naturally explain the \(H_0\) discrepancy between CMB and local measurements.

Holographic Dark Energy Model. — Inspired by the Bekenstein upper bound of black hole entropy in an effectve field theory, Cohen et al. \([13]\) suggested that, in quantum field theory a short distance ultraviolet (UV) cutoff is connected to a long distance infrared (IR) cutoff due to the limit set by the black hole formation, i.e., the maximum total energy set by the UV cutoff in a region of size \(L\) should not exceed the mass of a black hole with the same size, namely, \(L^2\rho_{\Lambda} \lesssim M_{\text{Pl}}^2\), and thus the energy density \(\rho_{\Lambda} \lesssim M_{\text{Pl}}^2L^{-2}\), where \(M_{\text{Pl}} = (8\pi G)^{-1/2}\) is the reduced Planck mass. Li \([14]\) subsequently proposed that, to make the largest \(L\) saturating the above inequality, the energy density of this holographic dark energy should be

\[ \rho_{\text{de}} = 3c^2M_{\text{Pl}}^2L^{-2} \]  \(\text{(1)}\)

where \(c\) is a constant coefficient. Ref. \([14]\) also found that, only if the IR cutoff \(L\) is taken as the future event horizon of the Universe, \(L = R_{\text{eh}} = a\int_0^\infty dt'/a(t')\), the dark energy can provide repulsive force and thus explain the cosmic acceleration.

We combine Eq.(1) with the energy-momentum conservation and Friedmann equation,

\[ \dot{\rho}_{\text{de}} + 3H(1 + w_{\text{de}}(z))\rho_{\text{de}} = 0, \]  \(\text{(2a)}\)

\[ 3M_{\text{Pl}}^2H^2 = \sum_j \rho_j, \]  \(\text{(2b)}\)

where \(w_{\text{de}}(z)\) is the EoS parameter of the HDE and \(H\) is the Hubble parameter. In Eq. (2b), the sum of energy densities includes matter (\(\rho_m\)), radiation (\(\rho_r\)), and dark energy (\(\rho_{\text{de}}\)). Among these, \(\rho_r = \Omega_r\rho_c(1+z)^4\) is fixed
by the observed CMB temperature. From Eq. (2), we can derive the following differential equations governing the dynamics of background expansion,

\[
\frac{d\rho_{de}}{dt} = -2H\rho_{de} \left(1 - \frac{\rho_{de}^\frac{4}{3}}{\sqrt{3}cM_{Pl}H}\right), \tag{3a}
\]

\[
\frac{dH}{dt} = \frac{1}{6M_{Pl}^2H} \sum_j \rho_j, \tag{3b}
\]

and the EoS of HDE,

\[
w_{de}(z) = -\frac{1}{3} - \frac{\rho_{de}^\frac{4}{3}}{3\sqrt{3}cM_{Pl}H}. \tag{4}
\]

In comparison with the “vanilla” ΛCDM model, the HDE model has an extra free parameter \(c\) as in Eq. (1), which controls the behaviour of HDE. We numerically solve the background evolution of the Universe through Eqs. (3a) and (3b), and compute the EoS of HDE (4), which is depicted as the blue solid curve in Fig. 1 for the case \(c = 0.5\). It shows that the HDE has \(w_{de}\) greater than \(-1\) (corresponding to Einstein’s cosmological constant \(Λ\)) at early epoch \((z > 1)\), transits and goes below \(-1\) at later epoch \((z < 1)\). This suggests that, comparing to Λ, the repulsive force in HDE model (quantified as the pressure \(P = w_{de}ρ\)) was weaker at earlier epoch than the present epoch. Hence, it causes the Universe to have smaller acceleration earlier on, and faster acceleration at later stage, but can still keep the total angular diameter distance to the last-scattering surface unchanged. This is exactly what is needed to resolve the \(H_0\) tension, because the present-day expansion rate \(H_0\) measured by local measurements is larger than that of CMB measured, whereas the angular diameter distance to the last-scattering surface needs to be fixed due to high-precision CMB measurement. As an analogue, a Marathon runner can run slower at early period, but accelerate at later stage to keep the total time and distance unchanged.

To explore this “\(w\)-transition” behaviour, we seek two parametrized models of dynamical dark energy with 2 and 4 more parameters than ΛCDM, which can mimic the behaviour of HDE.

**Two Parametrized Models.** — In general, if a dark energy model with EoS \(w_{de} > -1\) at early epoch and transits to \(w_{de} < -1\) at late epoch, it has the potential to imitate HDE. We first propose a “TransDE” parametrization with 4 free parameters \((w_1, w_2, z_t, Δz)\),

\[
w(x = \ln(1+z)) = w_1 + \frac{w_2}{2} \left(1 + \tanh \frac{x-x_t}{Δx}\right), \tag{5}
\]

where \(x_t = \ln(1+z_t)\) and \(Δx = Δz/(1+z_t)\) determine the redshift \(z_t\) and width \(Δz\) of transition in \(x\)-function. The \((w_1 + w_2)\) and \(w_1\) control the asymptotic behaviour of EoS at the infinite past \((z \to -∞)\) and infinite future \((z \to -1)\). We substitute this EoS into Eq. (2a) and obtain an analytical solution for the dark energy density,

\[
ρ_{de} = \frac{ρ^0_{de}}{\cosh(\frac{x}{Δx})} \exp \left[3(1+w_1 + \frac{w_2}{2})\right]
\]

\[+ \frac{3}{2} w_2 Δx \ln \left(\cosh \frac{x-x_t}{Δx}\right), \tag{6}
\]

where \(ρ^0_{de} = Ω_{de}ρ_{cr}\) is the present-day dark energy density, and \(ρ_{cr} = 3H_0^2M_{Pl}^{-2}\) is the critical energy density.

The other model is the famous Chevallier-Polarski-Linder (CPL) parametrization, \(w(a) = w_0 + w_a(1 - a)\), which behaves like the TransDE model at high-\(z\), but the difference is non-negligible if a rapid transition of EoS happens at low-\(z\) [15]. The energy density of CPL dark energy is

\[
ρ_{de} = ρ^0_{de} \exp \{-3[w_1(1-a) + (1+w_0 + w_a)\ln a]\}. \tag{7}
\]

We fit the HDE model with \(c = 0.5\) by using TransDE and CPL model respectively in Fig. 1. Due to extra free parameters, both TransDE and CPL model can mimic the HDE model, with the minimum deviations found by the global optimizer PyGMO [16]. As will be shown below, the HDE model is the most economical model to resolve the \(H_0\) discrepancy.

**Data Analysis.** — We combine R19 data (local measurement), Type-Ia supernovae “Pantheon” dataset (median redshift), and BAO and Planck CMB data (high redshifts) in our model fitting.

R19 is the measurement of \(H_0\) from Large Magellanic Cloud Cepheid Standards by Riess et al. [1], which gives

![FIG. 1: Fitting HDE with TransDE and CPL dark energy. The blue solid curve shows the EoS of HDE with \(c = 0.5\) and the other physical parameters fixed to the typical Planck best-fit values [2]. The orange dashed and green dash-dotted curves are the best-fit \(w(z)\) of the CPL dark energy and the TransDE, respectively, in the redshift range \(z \in (0, 10^3)\).](image-url)
$H_0 = 74.03 \pm 1.42 \text{ km s}^{-1} \text{ Mpc}^{-1}$, deviating from Planck measurement at 4.4σ level. Pantheon is a new set of light-curve samples, which gives a total of 1048 supernovae spanning the redshift range $0.01 < z < 2.3$ [17]. In this work, we do not use the entire Pantheon data in our analysis, but only use 837 SN sub-samples in the range $z \geq 0.2$ to the Planck+BAO constraints. The reason is that at low-redshift, Type-Ia SN luminosity distance $d_L \approx cz/H_0$, which gives a model-independent measurement of $H_0$. The Pantheon low-z data prefer a lower value of $H_0$ [17], making it directly inconsistent with R19 result regardless any cosmological model assumed. Hence, we only use $z \geq 0.2$ Pantheon data in our fitting, which are consistent with Planck+BAO12+R19 as shown in the model comparison of Table II.

We use the final full-mission baseline Planck likelihood data (the 2018 release), which includes the low-ℓ temperature likelihood (Commander), low-ℓ EE likelihood (SimAll), high-ℓ TT, TE and EE likelihood (Plik) [13], and the additional CMB lensing likelihood [19]. In the following, “Planck” denotes the combination of the aforementioned Planck data.

The BAO data includes the “consensus” SDSS/DR12 data [20], the 6dF [21] data and MGS [22] BAO data. Table I enumerates the effective redshift for each measurement, ranging from 0.106 to 0.61. $r_d$ is the sound horizon at drag epoch and $D_M$ is the comoving angular diameter distance. $D_V$ is related to the angular diameter distance $D_A$ and the Hubble parameter $H(z)$ through $D_V = (cD_A^2(1+z)^3/H(z))^{1/3}$. The 6dF and MGS data give the measurement of $r_d/D_V$ at redshift $z_{\text{eff}} = 0.106$ and the measurement of $D_V/r_d$ at redshift $z_{\text{eff}} = 0.15$, respectively. BOSS DR12 data include $D_M r_{\text{fid,d}}/r_d$ and $H r_{\text{fid,d}}/r_{\text{fid,d}}$ at redshift $z_{\text{eff}} = \{0.38, 0.51, 0.61\}$, where $r_{\text{fid,d}} = 147.78 \text{ Mpc}$ is a fiducial sound horizon. Since DR12 data are correlated between different redshifts, we include all their covariance matrix in our CosmoMC likelihood package.

Besides, SDSS quasar data and the combination of Lyman-α auto-correlation and Quasar-Lyman-α cross-correlation data have put BAO constraints at redshift $z > 2$ [23–25]. But unlike galaxy BAO measurement, Lyα measurements require a number of additional assumptions of the models of quasar continuum spectra and absorption line system, which are more complicated than galaxy BAO measurements. Thus, we do not include the Lyα BAO in the parameter constraints, but only plot them in Fig. 3 ($z \geq 2$) for visual comparison. Hence BAO12 represents 6dF, MGS, and SDSS/DR12 data.

**FIG. 2:** Marginalized constraints on the Hubble constant $H_0$ versus the HDE parameter $c$, at 68% C.L. (contours with dark colors) and 95% C.L. (contours with light colors). The combinations of three datasets are shown in the legend.

**Results and Discussions.** — We modify the Boltzmann CAMB code [26] to embed the HDE and TransDE models into the background expansion of the Universe, and use public code CosmoMC (version of July 2019) to explore the parameter space with Markov Chains Monte Carlo (MCMC) technique [27].

Figure 2 presents the marginalized 2D contour of the HDE parameter $c$ versus $H_0$. The Planck-only constraint on $H_0$ is relatively weak, but including the BAO12 and BAO12+R19 data tighten up the bounds. In Fig. 3, we plot the evolution of Hubble parameter $H(z)$ as a function of the redshift within range $z \in [0, 20]$ for the HDE (blue) and $\Lambda$CDM model (grey) under $1\sigma$ and $2\sigma$.
FIG. 3: Planck+BAO12+R19 constraints on the Hubble parameter for HDE (blue) and ΛCDM model (grey). The dark (light dark) colored stripes present the 68% (95%) limits, and the black solid curve in the center corresponds to the mean value. The orange dots with 68% error bars are the (marginalized) measurements. From left to right, the first point is R19, the next 3 points are BAO DR12 constraints listed in Table I and the last 3 points are the eBOSS DR14 QSO, BOSS DR12 Lyα and BOSS DR12 QSOxLyα, which are listed in table 1 of Ref. [28]. The BAO data at $z \gtrsim 1$ cannot help distinguishing the two models because they are close to each other.

FIG. 4: Marginalized distributions of Hubble constant $H_0$. The dashed (solid) curves show the Planck+BAO12 (Planck+BAO12+R19) constraints. The vertical shaded bands are allowed by the R19 measurements at 1σ (dark grey) and 2σ (light grey) confidence levels.

CMB and local measurements.

We finally stress that, other than the CPL and TransDE parametrization, the HDE model is based on the physically well-motivated holographic principle which connects the total energy of vacuum state to black hole mass in the IR limit. It naturally provides a dynamical dark energy with negative pressure which is less significant at early time, and becomes more significant at late time. We have demonstrated that this attractive HDE model can successfully resolve the $H_0$ discrepancy between the CMB and local $H_0$ measurements. Future measurements will improve the constraints and further discriminate the HDE model from the benchmark ΛCDM universe.

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\[ H(z)/(1 + z)^{3/2} \text{ (km s}^{-1}\text{Mpc}^{-1}) \]

\[ 0.01 \leq z \leq 1.0 \]

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