A Trade Credit Inventory Model with Multivariate Demand for Non-Instantaneous Decaying products

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Abstract

The present study proposed a mathematical example using multivariate demand with non-instant decaying products. For any business organization, the carrying cost is an important term to find total inventory cost. Here we consider the numerical example to get the best optimum solution for understanding the behavior of inventory model. We also used sensitivity analysis to show the effect of variation in total profit per item with respect to changes in the other constraints to illustrate the model. The scenario of today's market is to encourage the retail dealers allowing them a delay in making the payments without them incurring any interest.

Keywords: Inventory, Multivariate Demand, Non-Instantaneous Deterioration, Ordering Cost, Trade Credit

1. Introduction

A large number of the researchers on the inventory models do not assume the trade credit with the inflation simultaneously. Trade credit and inflation play a crucial role in the optimum ordering policy inventory model with influence the demand of some items.

Proposed a mathematical model having stock dependent consumption rate with trade credit policy in payment conditions¹. Presented a mathematical model for obtaining the trade credit policy, optimum pricing and ordering policy¹. Design a model with constant deterioration and demand with increase exponentially due to inflation¹. Investigated a mathematical model with inventory level-dependent demand under the progressive payment scheme environment⁵. Discussed the inventory models which show the effect of inflation on two warehouse management⁶⁷. Various associated research reported in⁸⁻¹⁴. Presented a model with time dependent demand for decaying products using the Trade credit scheme condition¹⁵. Presented an inventory model with trade credit policy¹⁶. Discussed an optimal order policy with stock-level demand rate for non-instant diminishing products¹⁷. Developed an improved inventory model with stock-dependent demand¹⁸. Formulated a mathematical example with inflation and stock-level demand rate¹⁹⁻²¹. Developed a multi item model for diminishing products having expiry date under the shortages²². Discussed an optimum payment scheme for an inventory model with shortages and trade credit²³. Investigated a mathematical model under the inflation under partial backlogging²⁰⁻²⁴.

This paper investigates a trade credit inventory model with multivariate demand under the inflation condition. The main objective of this paper is to obtain the optimum restock procedure with multivariate demand under the inflation for non-instantaneous diminishing items. The maintenance and individuality of the finest solutions to the optimal circumstances are provided.
2. Notations and Assumptions

The inventory notations and assumptions of the proposed study are given below:

- The demand rate is \( D(t) = d_1 + d_2 t + d_3 Z(t) \), Where, \( d_1, d_2, d_3 \geq 0 \).
- Lead-time is assumed to be zero.
- The retailer can build up the returns and produce interest followed by the payment of customers in purchasing cost from the retailer in eagerness of the finishing of the permissible delay in payment time presented by the dealer.
- \( \theta \) rate of deterioration
- \( C_i \) cost of ordering per order
- \( C_s \) cost of carrying cost of inventory per item
- \( C_e \) cost of deterioration per item
- \( C_p \) cost of purchasing per item
- \( C_{sr} \) cost of sales revenue per item
- \( r \) discount rate, representing the time value of money
- \( i \) inflation rate
- \( R \) net discount rate of inflation; \( R = r - i \)
- \( M \) trade credit offered by supplier in months
- \( \gamma_1 \) interest charges per $ per month
- \( \gamma_2 \) interest earned per $ in stocks per month
- \( Z \) presents the non-decaying inventory level in duration \([0, t_1]\).
- \( Z_1 \) presents the inventory level in which demand and deterioration level goes to '0' in duration \([t_1, t_2]\).
- \( t_1 \) the time period in which product maintain their freshness.
- \( t_2 \) the time period in which the deterioration in the product takes place.
- TVP the total optimal function of profit per item.

3. Mathematical Model

We assumed a mathematical inventory model with multivariate demand which is used for non-instant decaying products under the trade credit policies. Depending upon this proposed inventory model the following equations describes the level of inventory (\( Z \)) at \( t \):

\[
\frac{dZ(t)}{dt} = -(d_1 + d_2 t + d_3 Z(t)) \quad t \in [0, t_1]
\]  

(1)

\[
\frac{dZ_1(t)}{dt} + \theta Z_2(t) = -(d_1 + d_2 t + d_3 Z(t)) \quad t \in [t_1, T]
\]  

(2)

Showing the boundary conditions \( Z(t_1) = W, Z(T) = 0 \), respectively, solving above equations (1) and (2), we get

\[
Z(t) = (W + \gamma_i) e^{-\theta t} - \gamma_1 - \frac{d_2}{d_3} t, \quad t \in [0, t_1],
\]  

(3)

\[
Z_1(t) = (\gamma_2 - t \gamma_3) + e^{d_3 t \gamma_1 t} (T_\gamma_3 - \gamma_2) \quad t \in [t_1, T],
\]  

(4)

Where \( \gamma_1 = \frac{d_1 d_3 - d_2}{d_3^2}, \gamma_2 = \frac{d_1 - d_2 (d_1 + d_2)}{(d_1 + d_2)^2}, \gamma_3 = \frac{d_1}{d_3 + \theta} \)

At \( t = t_1 \), using Eqns (3) and (4), we have considered \( Z_1(t_1) = Z(t_1) \)

\[
W = \frac{d_1}{d_3} t_1 + \gamma_1 - \gamma_2 - t_\gamma_3 e^{d_3 t} - \gamma_1 - e^{\theta t} (\gamma_2 - T \gamma_3)
\]  

(5)

Total profit for each sequence contains the subsequent components:

1. The ordering cost (CO) = \( C_1 \).
2. The carrying cost (CH)

\[
CH = C_1 \int_0^T e^{-\theta t} Z_1(t)dt + \int_0^T e^{-\theta t} Z(t)dt
\]  

(6)

\[
= \left[ \frac{W + \gamma_1}{R + d_3} + \left( e^{\theta t} - 1 \right) \left( \frac{dR \gamma_1 + d_1}{dR^2} \right) \right] + \frac{d_1}{dR} t e^{d_3 t} + \frac{\gamma_2}{R} \left( T e^{-\theta t} - t e^{-\theta t} \right) + \frac{\gamma_1}{R} \left( R e^{-\theta t} - t e^{-\theta t} \right)
\]  

(7)

3. The deterioration cost (CD) is

\[
CD = C_2 \int_0^T e^{-\theta t} Z_2(t)dt
\]

\[
= \frac{(\gamma_3 - \gamma_2 R)}{R} \left( e^{-\theta t} - e^{-\theta t} \right) + \frac{\gamma_2}{R} \left( T e^{-\theta t} - t e^{-\theta t} \right) + \frac{\gamma_1}{R} \left( R e^{-\theta t} - t e^{-\theta t} \right)
\]  

(8)

4. The purchasing cost (CP) is \( CP = C_3 x W \)

\[
= C_3 \left[ \frac{d_1}{d_3} t_1 + \gamma_1 - \gamma_2 - t_\gamma_3 e^{d_3 t} - \gamma_1 - e^{\theta t} (\gamma_2 - T \gamma_3) \right]
\]  

(9)

5. The sales revenue cost (CSR) is

\[
CSR = C_4 \int_0^T e^{-\theta t} (d_1 + d_2 t + d_3 z(t))dt
\]
In this paper we have considered permissible delay in payment in two periods: (on the basis of the length of T and M)

**Case-I (M∈[t, T])**, here the interest payable is

\[
IP_i = C_iZ_i \int M \int (t, d_i + d_i(t)) dt + \int M \int (t, d_i + d_i(t)) dt
\]

The interest earned is

\[
IE_i = C_iZ_i \int M \int \left[ \frac{d_i M_i}{2} + \frac{d_i M_i^3}{3} + \left( \frac{W + \gamma_i}{d_i} \right) \left( 1 - e^{\gamma_i(d_i + M_i) + 1} \right) - \frac{\gamma_i d_i^3}{2} - \frac{\gamma_i^3}{3} \right] dt
\]

Total profit function TVP₁ per unit time is

\[
TVP_i = \frac{1}{T} \left[ CSR - CO - CH - CD - CP - IP_i + IE_i \right]
\]

The total profit function TVP₁ is maximum if

\[
\frac{dTVP_i}{dt_i} = 0
\]

and \( \frac{dTVP_i}{dt_i} < 0 \)

**Case-II (M≥T)**, here in this period for any product interest charges are not reward, i.e.,

\[
IP_i = 0
\]

The interest earned is

\[
IE_i = C_iZ_i \int M \int \left[ t, (d_i + d_i(t)) dt + \int M \int (t, d_i + d_i(t)) dt + D(T) + M \right] dt
\]

The function of the total profit TVP₂ is

\[
TVP_i = \frac{1}{T} \left[ CSR - CO - CH - CD - CP - IP_i + IE_i \right]
\]

The total profit function TVP₂ is maximum if

\[
\frac{dTVP_i}{dt_i} = 0
\]

and \( \frac{dTVP_i}{dt_i} < 0 \)

### Solution Algorithm

Follow the given steps below to find the optimum solution:

**Step.1.** Input the constraints \( C_1, C_2, C_3, C_4, C_5, \theta, R, d_1, d_2, \ldots, M, Z, t, t_i \) values.

**Step.2. Case-I:** With the help of equation (14) determine the value of \( t_i \) and then compute the value of profit function TVP₁ using the equation (13).

**Case-II:** With the help of equation (15) determine the value of \( t_i \) and then compute the value of profit function TVP₂ using the equation (18).

**Step.3. Case-I:** Now if the value of \( t_i \) satisfies the condition \( \frac{dTVP_i}{dt_i} < 0 \) then the solution is optimal solution, if not move to step 1 and reset the constraints values.

**Case-II:** Now if the value of \( t_i \) satisfies the condition \( \frac{dTVP_i}{dt_i} < 0 \) then the solution is optimal solution, if not move to step 1 and reset the constraints values.

### 4. Numerical Example and Sensitivity Analysis

**Ex.1.** Consider \( C_1 = 100, C_2 = 0.40, C_3 = 0.05, C_4 = 40, C_5 = 75, Z = 0.1, Z = 0.08, d_1 = 200, d_2 = 0.5, d_3 = 0.2, \theta = 0.40, \)
and R=0.01. **Case-I** assume M=0.6 month, and **Case-II** assume M=1.5 month. From the Table 1.1, we monitor that (TVP) is maximum, if \( t_1=1/2, t_2=0.604 \) month, TVP \(*=3725.6 \) and optimal order quantity is \( W=383.543 \).

From the Table 1.2, we study that the profit (TVP) is maximum when \( t_1=1/2, t_2=0.2229, TVP_2*=5692.3 \) and optimal order quantity is \( W=251.0385 \).

## 5. Sensitivity Analysis

In the given inventory model the constraints are analyzed which shows that the total profit (TVP, and TVP) changes significantly with the change in the various constraint values as shown in the following cases:

**Table 1.** Sensitivity Analysis for Case I

| Parameter | change | \( t_1^* \) | \( W^* \) | TVP \(*^* \) |
|-----------|--------|-------------|-------------|-------------|
| \( C_1 \)  | -10%   | 0.602       | 382.799     | 3734.7      |
|           | 0%     | 0.604       | 383.543     | 3725.6      |
|           | +10%   | 0.606       | 384.286     | 3716.6      |
| \( C_2 \)  | -10%   | 0.605       | 384.034     | 3732.8      |
|           | 0%     | 0.604       | 383.543     | 3725.6      |
|           | +10%   | 0.603       | 383.055     | 3718.4      |
| \( C_3 \)  | -10%   | 0.6041      | 383.561     | 3725.7      |
|           | 0%     | 0.6041      | 383.543     | 3725.6      |
|           | +10%   | 0.6041      | 383.525     | 3725.5      |
| \( C_4 \)  | -10%   | 0.657       | 404.470     | 5130.1      |
|           | 0%     | 0.604       | 383.543     | 3725.6      |
|           | +10%   | 0.568       | 369.640     | 2329.3      |
| \( C_5 \)  | -10%   | 0.604       | 368.642     | 1940.6      |
|           | 0%     | 0.604       | 383.543     | 3725.6      |
|           | +10%   | 0.651       | 401.952     | 5518.4      |
| \( d_1 \)  | -10%   | 0.606       | 345.918     | 3345.5      |
|           | 0%     | 0.604       | 383.543     | 3725.6      |
|           | +10%   | 0.602       | 421.167     | 4105.8      |
| \( d_2 \)  | -10%   | 0.602       | 383.956     | 3724.1      |
|           | 0%     | 0.604       | 383.543     | 3725.6      |
|           | +10%   | 0.6040      | 383.530     | 3727.1      |
| \( d_3 \)  | -10%   | 0.661       | 394.763     | 4028.4      |
|           | 0%     | 0.604       | 383.543     | 3725.6      |
|           | +10%   | 0.555       | 373.958     | 3436.9      |
| \( \Theta \) | -10% | 0.606       | 384.338     | 3734.7      |
|           | 0%     | 0.604       | 383.543     | 3725.6      |
|           | +10%   | 0.602       | 382.755     | 3716.5      |

**Case-I:**

1. Rise in the demand rate \( (d_1) \), reduces the time \( t_2 \) and an increment in the order quantity \( (W) \) as well as total profit \( (TVP) \).
2. Rise in the demand rate \( (d_2) \), reduces the time \( t_2 \) as well as order quantity \( (W) \) and an increment in total profit \( (TVP) \).
3. Rise in the demand rate \( (d_3) \), also gives rise to time \( t_2 \), order quantity \( (W) \) and total profit \( (TVP) \).
4. Rises in the deterioration rate \( (\Theta) \), purchasing cost \( (C_1) \), deteriorating cost \( (C_2) \), carrying cost \( (C_3) \) increases and inflation rate \( (R) \), reduces time \( t_2 \), order quantity \( (W) \) and the total profit \( (TVP) \).
5. Rise in the ordering cost \( (C_4) \), gives increment in time \( t_2 \) as well as order quantity \( (W) \) on the other hand it reduces the total profit \( (TVP) \).
6. Rise in the sales revenue cost \( (C_5) \), gives increment in time \( t_2 \), order quantity \( (W) \) and the total profit \( (TVP) \).

**Table 1.** Sensitivity Analysis for Case 2

| Parameter | change | \( t_1^* \) | \( W^* \) | TVP \(*^* \) |
|-----------|--------|-------------|-------------|-------------|
| \( C_1 \)  | -10%   | 0.2202      | 250.216     | 5706.2      |
|           | 0%     | 0.2229      | 251.039     | 5692.3      |
|           | +10%   | 0.2255      | 251.858     | 5678.5      |
| \( C_2 \)  | -10%   | 0.2233      | 251.171     | 5697.9      |
|           | 0%     | 0.2229      | 251.039     | 5692.3      |
|           | +10%   | 0.2224      | 250.907     | 5686.8      |
| \( C_3 \)  | -10%   | 0.2229      | 251.043     | 5692.3      |
|           | 0%     | 0.2229      | 251.039     | 5692.3      |
|           | +10%   | 0.2229      | 251.034     | 5692.3      |
| \( C_4 \)  | -10%   | 0.1877      | 240.280     | 7085.2      |
|           | 0%     | 0.2229      | 251.039     | 5692.3      |
|           | +10%   | 0.2479      | 258.845     | 4305.8      |
| \( C_5 \)  | -10%   | 0.2522      | 260.195     | 3717.8      |
|           | 0%     | 0.2229      | 251.039     | 5692.3      |
|           | +10%   | 0.1889      | 240.655     | 7674.0      |
| \( d_1 \)  | -10%   | 0.2256      | 226.682     | 5111.4      |
|           | 0%     | 0.2229      | 251.039     | 5692.3      |
|           | +10%   | 0.2206      | 275.393     | 6273.3      |
| \( d_2 \)  | -10%   | 0.2231      | 251.111     | 5690.2      |
|           | 0%     | 0.2229      | 251.039     | 5692.3      |
|           | +10%   | 0.2227      | 250.966     | 5694.4      |
| \( d_3 \)  | -10%   | 0.2348      | 255.329     | 5690.2      |
|           | 0%     | 0.2229      | 251.039     | 5692.3      |
|           | +10%   | 0.2120      | 247.163     | 5694.4      |
| \( \Theta \) | -10% | 0.2334      | 248.918     | 5932.5      |
|           | 0%     | 0.2229      | 251.039     | 5692.3      |
|           | +10%   | 0.2106      | 252.201     | 5464.7      |
| \( R \)    | -10%   | 0.2240      | 251.386     | 5698.2      |
|           | 0%     | 0.2229      | 251.039     | 5692.3      |
|           | +10%   | 0.2218      | 250.693     | 5686.4      |
Case-II:
1. The order quantity ($W$) and total profit ($TVP_2$) rises and the time ($t_2$) reduces, if there is an increment in the demand rate ($d_1$).
2. The total profit ($TVP_2$) rises whereas the order quantity ($W$) and the time ($t_2$) reduces, if there is an increment in the demand rate ($d_1$).
3. The total profit ($TVP_2$) rises whereas the order quantity ($W$) and the time ($t_2$) reduces, if there is an increment in the demand rate ($d_1$).
4. The order quantity ($W$) rises whereas total profit ($TVP_2$) and time ($t_2$) reduce, if there is an increment in the deterioration rate ($\theta$).
5. The total profit ($TVP_2$) rises whereas the order quantity ($W$) and the time ($t_2$) reduce, if there is an increment in the sales revenue cost ($C_5$).
6. If there is an increment in purchasing cost ($C_4$) and ordering cost ($C_1$) then it will give rise to time $t_2$ and order quantity ($W$) whereas reduction in the total profit ($TVP_2$).
7. If there is an increment in carrying cost ($C_2$), deteriorating cost ($C_3$) and inflation rate ($R$), then it reduces the time $t_2$, order quantity ($W$) and total profit ($TVP_2$).

The graphs in (Figures 1.1 and 1.2) indicates the relation among profit functions ($TVP_1$* and $TVP_2$*) and time $t_1$* and $t_2$*.

**Figure 1.** 3D view of Total Profit $TVP_1$* v/s $t_2$* and $t_1$* values.

**Figure 2.** 3D view of Total Profit $TVP_2$ v/s $t_2$* and $t_1$* values.

6. **Conclusion**

This paper designs a mathematical model for the inventory system using, concept of effect of inflation and trade credit policy with the non-instant diminishing products which is very realistic in daily life environment. Finally, for the developed inventory model, numerical example, graphical representation of profit function with constraints and analyzed results are given to exemplify, the significant features of the results with various parameters. Some possible extension of this research paper, considering variable lead time, variable carrying cost, shortage, production model and two warehouses model etc.

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