The advantage of the method of hierarchy analysis, the statistical methods of decision support

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Abstract. The Work is devoted to the analysis of the effectiveness of the use of statistical methods and MAI with the support of decision making aimed at configuring the security system of an economic object. On practical examples the insufficiency of statistical methods of decision support is shown. It is theoretically and practically demonstrated that the method of hierarchy analysis allows simplifying the process of making complex decisions based on diverse and unstructured information on economic objects. It is defined that the correct statement of a problem in the process of application of MAI, a task of initial, boundary conditions, selection of experts and a choice of criteria is of great importance. The conclusion is made about the advantages of MAI over other methods.

1. Introduction
Until now, one of the most common approaches to possible solutions in our time remains brainstorm the problem. Such a process is rarely requiring a lot of time, taking into account various factors and the end result can be influenced by one person. Often, the person who makes the decisions, choosing one of the many possible options, guided by intuition based on their knowledge. As a result, it may be that important factors were omitted. Decisions will be uncertain, will not be effective and the information is not complete. When there is a question of choice model and security configuration is crucial identity and profession of the people involved in the decision. Finance insists on minimizing costs, Head of IT department will propose to charge all the computer system, and the head of the department of protection – install iron doors, grilles and put guards around every corner. Head, in the decision, should take into account all the views, but it is necessary to rely on the calculations, confidently proving the benefits and effectiveness of a choice. Let us use some mathematical approaches to decision– making on the choice of safety system of the entity.

2. Statistical models
Statistical methods are often used when it is necessary to justify the choice or decision [1– 2]. They are quite a lot, but despite the ease of use they have some significant drawbacks. [3] For example, you must select the fire safety system in a supermarket. Let us consider three different systems, A1, A2, A3, and each in three configurations – maximum, medium and minimum, B1, B2, B3.

Supposethat:

- P– the probability of failure of the system and make a matrix of failures
Table 1. Matrix of failures.

|       | AT 2 | AT 3 |
|-------|------|------|
| A1    | 0.1  | 0.2  | 0.3  |
| A2    | 0.2  | 0.3  | 0.4  |
| A3    | 0.3  | 0.4  | 0.5  |

Then the probability of operation of the system \( P \).

Get the income matrix \( Pr = (1 - P) \times D \), the probability of the system operation multiplied by the maximum losses \( D \) – from damages, where \( D = 90\% \);

Table 2. Income matrix.

|       | AT 2 | AT 3 |
|-------|------|------|
| A1    | 81   | 72   | 63   |
| A2    | 72   | 63   | 56   |
| A3    | 63   | 56   | 45   |

risk matrix \( R = P \times D \), the probability of failure of the system multiplied by the maximum loss.

Table 3. Risk matrix.

|       | AT 2 | AT 3 |
|-------|------|------|
| A1    | 9    | 18   | 27   |
| A2    | 18   | 27   | 36   |
| A3    | 27   | 36   | 45   |

Now we apply the standard model of decision – making under uncertainty.
Maximax model. It is believed that the events will develop in the most favorable scenario: choose the most out of the matrix income.

\[
K (A, B) (Pr) = \max_i \max_j a_{ij} = 81 \quad (A1)
\]

(1)

Wald model. Selected the best of the worst options for the implementation of the matrix income.

\[
K (A, B) (Pr) = \max_i \min_j a_{ij} = 63 \quad (A1)
\]

(2)

Savage Model. Selects the smallest value of maximum risk.

\[
K (A, B) (R) = \min_i \max_j a_{ij} = 9 \quad (A1)
\]

(3)

Hurwitz model. We find the average between the maximum and the minimum income in each row and choose the maximum value.

\[
K (A, B) (Pr) = \max_i (a \times \min_j a_{ij} + (1 - a) \times \max_j a_{ij}) = 72 \quad (A1)
\]

(4)

We see that all the statistical model chosen A1 system, but to determine the preferred configuration, it is necessary to conduct additional studies of the brackets arguments were many and very important criteria, such as the cost of creation and maintenance, etc. Therefore, these models can be used only as additional arguments.
3. The method of analysis of hierarchies

We will not dwell on the description of the MAI, the literature on this topic in the public domain abounds. [4-11] Let us only on the basic concepts and methodology.

The first task is to describe the decision-making model, obtaining the mutual influence of these criteria and factors for the weights of different solutions.

The next task – the formalization of data included in our model.

The third task – to connect the data model and to prioritize possible (considered) the decisions and assess the impact of the factors taken into account.

As a result, we can mathematically have justified to take the "A" solution, ensuring its qualitative superiority over the "B".

"MAI – mathematical tool systematic approach to decision-making problems. MAI does not prescribe to the decision maker (DM), any "right" decisions and allows him to interactively find such an option (alternative), which is most consistent with its understanding of the nature of the problem and requirements to its decision"[12].

We get a graph (chart) in formulating the problem of decision-making and use of the MAI, which consists of the following units:

- Group for alternative solutions,
- Home Solutions ranking criteria (credit ratings)
- a set of factors affecting the rating assigned,
- various directional links, showing the effect on decision-making, communication criteria and factors to each other.

All this can significantly increase the validity of the decision.

Widespread method is due to its versatility and "omnivorous." It can be used in decision-making in a variety of cases in all areas of activity. It allows you to formalize and use the data, parameters and criteria that cannot be applied to other models of decision-making. The main problem with the use of the MAI is big enough subjectivity of estimates produced by the user on their own or received from experts. Real means to get rid of this subjectivity is averaging estimates of factors and criteria of several experts.

When solving a simple problem MAI known list of possible solutions: P1, P2, P3, ... Pk, and is known for a list of criteria that influence the decision taken: K1, K2, K3, ... Kn. We know the characteristics of each of the criteria and how this or that criterion influences the choice of solutions. The goal – to choose the optimal solution.

First, you need to rank criteria, place them on the importance of the degree of influence on the decision to determine for themselves their importance.

We make up the matrix of pairwise comparisons between the criteria is the next step. It turns back symmetric matrix (nxn). By TL Saaty 'Matrix elements a(i, j) is the intensity of display element hierarchy i relative hierarchy item j, estimated by the intensity scale of 1 to 9, where the evaluation has the following meanings:

- equal importance - 1;
- moderate superiority -3;
- a significant advantage - 5;
- strong superiority - 7;
- very strong superiority - 9;

even estimates put in intermediate cases: 2, 4, 6, 8 (e.g., 4 – between moderate and considerable superiority) "[4].

In fact, hierarchy’s intensity scale can be any, from the simplest (1,2,3), to an arbitrarily complex. It is chosen (or created) under the influence of the complexity of the relationship between the criteria themselves and their degree of influence on the decision.

In the process of filling the matrix and intercomparisons criteria the user answers to the following questions:

Which of the two criteria is more important (greater effect) to make a decision? How?
What criteria is more important (or desirable) for the user? How? The probability of which criteria above? How?

In response to these (or similar) issues and applying a scale intensity we form a matrix as follows: If element i important element j, then the field (i,j), corresponding to line i and column j, filled integer from Intensity Scale, and the field (j,i), corresponding to row j and column i, is filled with a fractional number, a unit where the numerator and the denominator a (i,j). Criteria matrix affix descending importance.

The following parameters are calculated after the construction of the matrix:

– geometric mean in each row

\[ a_i = \sqrt[n]{n_i 1 \times n_i 2 \times \ldots \times n_i j} \]  

(5)

where \( i \) varies from 1 to \( n \);

– the sum of the geometric means

\[ S = \sum_{i=1}^{n} a_i \]  

(6)

– the elements of the normalized vector priorities in each row

\[ NV Pi = a_i / \sum_{i=1}^{n} a_i \]  

(7)

NVPi element is a measure of the importance (significance) \( i \)-th criterion \( K \). To check the coherence of the CWP expect:

– eigenvalue

\[ \lambda_{\text{max}} = \sum_{i=1}^{n} (\sum_{j=1}^{n} (aij \times NV Pi)) \]  

(8)

– index matching

\[ IP = (\lambda_{\text{max}} - n) / (n - 1); \]  

(9)

– the harmonization

\[ OS = IS / PSS; \]  

(10)

MSS in this case – an indicator of the consistency of the random, tabular value, Table 1.

| \( n \) | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|-------|---|---|---|---|---|---|---|----|----|----|
| PSS   | 0.58 | 0.90 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 | 1.51 | 1.48 |

Table 4. PSS – indicator random consistency.

It is generally accepted that our criteria (evaluation) are well matched, if the OS is in the range from 0% to 10% (0 – 0.10), just consistent, if the OS is in the range of 10% to 15% (0.10 – 0.15), and weakly coordinated in the range from 15% to 20% (0.15 – 0.2). If the operating system is more than 20%, it is necessary to review all the peer reviews and make the matrix again.
The next step for each criterion \( K_i \) held pairwise comparison solutions \( P_j \) where \( j \) from 1 to \( k \), \( pij \) priorities are determined and filled in \( n \) matrices (\( k \times k \)). For each of the obtained matrix was determined by the parameters of the formulas (5), (6) and 7).

The main criteria (priorities) are determined from the data obtained for each considered solution:

\[
K (P_j) = (11) \sum_{j=1}^{k} (pij * NVPi)
\]

That decision, for which \( K (P_j) = \text{max} \) is the optimum (best).

Now it is necessary to calculate and ensure the accuracy of the results.

Generalized matching index is calculated as

\[
OIS = (12) \sum_{j=1}^{k} (ISj * NVPi)
\]

The generalized ratio matching – \( CAB = OIS / OPSS \) where OPSS defined as MSS, from the table.

The reliability of the solution is also determined, as well as for the OS.

More details about the MAI and its practical applications can be found in the following references to [4–11].

### 4. Practical application (example)

Consider, for example, a shop. You must select one of three security systems – A1, A2, A3, the following criteria:

- \( K_1 \) – cost of the system;
- \( K_2 \) – fire protection;
- \( K_3 \) – theft protection of goods in the sales area;
- \( K_4 \) – protection against theft of cash from the safe.

Security systems have the following:

- A1 – inexpensive sprinkler system does not protect against theft;
- A2 – anti-theft systems with fire component;
- A3 – dear ant trauma system, not having the fire component.

To apply MAI first rank the most important criteria:

- \#1 (most important) \( K_2 \) – fire protection (as in case of fire you can lose all the property and possible loss of life, the probability of occurrence, subject to building regulations operation is small);
- \#2 \( K_1 \) – cost of the system (since the cost of loss prevention can be less than the cost of creating and maintaining system performance);
- \#3 \( K_3 \) – theft protection of goods in the sales area (the losses are relatively small, but constant, the probability is very high);
- \#4 \( K_4 \) – protection against theft of cash from the safe (as the least likely and the most poor-selling risk).

The matrix composition on the basis of expert assessments:

|      | K2 | K1 | K3 | K4 |
|------|----|----|----|----|
| K2   | one| 3  | five| four|
| K1   | 1/3| one| 3  | 2  |
| K3   | 1/5| 1/3| one| four|
| K4   | 1/4| 1/2| 1/4| one|

According to formulas (5 – 10) was calculated values:
\[ a_i = (2.78 \ 1.19 \ 0.72 \ 0.42); \]

\[ S = 5.11; \]

\[ NVP_i = (0.544 \ 0.233 \ 0.141 \ 0.082) \]

To test the add up the values obtained, the amount is equal to one, there is no error.

To check the coherence of the CWP expect:

- eigenvalue \( \lambda_{\text{max}} = 4.30; \)
- index matching IC = 0.101;
- the harmonization of the OS = 0.112 or 11.2%, so our criteria for consistent evaluation and matched correctly.

Next to each criterion \( K_i \) build the matrix \( A_i \times A_j \) and expect all the same options as above. Received four sets of matrices and calculations will not give. Putting there sults into the final table:

|       | K1     | K2     | K3     | K4     |
|-------|--------|--------|--------|--------|
| A1    | 0.197  | 0.726  | 0.089  | 0.091  |
| A2    | 0.657  | 0.208  | 0.323  | 0.182  |
| A3    | 0.146  | 0.067  | 0.588  | 0.727  |

By the formula (7), forward final normalized vector priorities for each selection:

\[ NVP (A_i) = 0.296; \ 0.466; \ 0.238. \]

From the analysis of NVP it can be seen that the solution A2 in terms of our model has the highest priority. To check the relevance and coherence of the final count index matching IMS = 0.03, and the final (combined) the harmonization of the ITS = 0.052 or 5.2%, indicating that all the results are good agreement and have practical application.

5. Conclusion
Decision support programs have proliferated around the world and are used in the economies of different countries. The main advantage of the analytic hierarchy process is the high versatility – the method can be used to solve a variety of tasks. For example, in the execution of scientific research, marketing, personnel management, production, accounting and strategic planning of the enterprise to its development, management and resource allocation.

The disadvantage of the analytic hierarchy process is the need for large amounts of information from experts. Method to the greatest extent for those cases where the bulk of the data is based on the preferences of the decision maker in the process of choosing the best variant of the decision of a variety of alternatives.

Of great importance is the correct formulation of the problem in the process of applying analytic hierarchy process, the task of initial conditions, boundary conditions, the selection of experts and the selection criteria.

From the foregoing, it can be concluded that the hierarchy analysis method will simplify the process of enterprises make complex decisions based on diverse and unstructured information. And information systems based on this method, make it possible to avoid mistakes and to make decision-making easier and more effective.
References

[1] Shiryaev 2014 Probabilistic and statistical methods in decision theory EBook ISBN: 978– 5– 4439 – 0247 – 0 p 145

[2] Rogov S F 2013 Mathematical methods in decision theory (Publisher: Sputnik) p 148

[3] Chernyak V Z, Ėriashvili N D and Barikaev E N 2015 Management of business risks to the economic security of the system Theoretical aspect: monograph UNITY– DANA; Law and Law p 159

[4] Saaty T 1993 Decision Making Method of analysis of hierarchies (Moscow "Radio and Communication) p 278

[5] Podinovskii O V 2010 Method of analysis of hierarchies as the adoption of multi– criteria decision support method. Higher School of Economics, Information technology modeling and management 1 (60) pp 71 – 80

[6] Seredenko N N 2011 Development Analytic Hierarchy Process (AHP), the HSE, Open formation 2– 1 pp 39– 48

[7] Mukhametzyanov I Z 2017 Fuzzy inference and fuzzy analytic hierarchy process in decision support systems: an application to the assessment of the reliability of technical systems Cybernetics and programming 2 pp 59– 77

[8] Podinovskii O V 2010 Method of analysis of hierarchies as the adoption of multi– criteria decision support method Higher School of Economics, Information technology modeling and management 1 (60) pp 71– 80

[9] Seredenko N N 2011 Development Analytic Hierarchy Process (AHP), the HSE, Open formation 2– 1 pp 39– 48

[10] Podinovskii V V and Podinovskii O V 2012 Esche just about incorrect analytic hierarchy process, management problems 4 pp 75– 78

[11] Tsibizova T Y Karpunin A A 2015 Application of the analytic hierarchy process in the evaluation of quality management processes Modern problems of science and education 2 pp 47– 58