Research on Calculation Model of Horizontal Wellbore Shape with Non-uniform In-situ Stress

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Abstract: In horizontal Wells, non-uniform ground stress is the root cause of irregular borehole shape. The failure of caliper instrument in horizontal section leads to the failure of accurate evaluation of borehole shape, which makes it difficult to calculate the frictional resistance of tubing string in the later stage. This paper studies the borehole shape calculation model from the following two aspects: (1) in the aspect of diameter expansion caused by shaft wall collapse. By establishing coulomb-Mohr criterion in cylindrical coordinate system, the well wall collapse instability is judged, and the hole diameter expansion is evaluated by establishing the calculation model of the instability range. (2) Shrinkage caused by stress concentration. Firstly, the boundary position of formation deformation is determined. Secondly, the calculation model of borehole deformation is established by analyzing the stress characteristics of formation, which reflects the time effect of borehole deformation. Therefore, the establishment of borehole shape calculation model of horizontal well provides a basis for calculating the friction between the pipe string and the horizontal section.

1. Introduction
After a borehole is formed through drilling in the strata, the supporting effect of the drilled strata on the wellbore is substituted by the pressure inside the drilling fluid columns, and consequently, the balance of the original underground stress system is broken. The stress concentration phenomenon occurs to rocks around the borehole, the strata rocks undergo unavoidable deformation due to the stress change, and more seriously, the well borehole may collapse. Without any doubt, both will aggravate the irregularity of the borehole shape, but as the borehole diameter measuring instrument cannot be stretched into the horizontal segment in the horizontal well, it is impossible to accurately evaluate the borehole shape. By establishing a calculation model, this paper provides a theoretical basis for calculating the borehole shape.

2. Model research on rock stress distribution
The deep strata bear the actions of triaxial ground stress [1-3], vertical principal stress σv, maximum horizontal ground stress σH and minimum horizontal ground stress σh, the horizontal borehole passes through the stratigraphic section 2 and forms an α angle (namely borehole azimuth angle) with the direction of minimum horizontal ground stress, the borehole is filled with drilling fluid and the liquid fluid pressure is Pw. At the time, the calculation model of the rock stress distribution around the...
borehole is as below:

\[
\sigma_r = \frac{r^2}{r^2} P_w + \frac{\sigma_H}{2} \sin^2 \alpha + \frac{\sigma_H}{2} \cos^2 \alpha + \frac{\sigma_z}{2} \left(1 - \frac{r^2}{r^2}ight) + \frac{\sigma_r}{2} \sin^2 \alpha - \sigma_r \cos^2 \alpha \left(1 + \frac{3r^4}{r^4} - \frac{4r^2}{r^2}\right) \cos 2\theta
\]  

\[
\sigma_\theta = -\frac{r^2}{r^2} P_w + \frac{\sigma_H}{2} \sin^2 \alpha + \frac{\sigma_H}{2} \cos^2 \alpha + \frac{\sigma_z}{2} \left(1 - \frac{r^2}{r^2}\right) - \frac{\sigma_r}{2} \sin^2 \alpha - \sigma_r \cos^2 \alpha \left(1 + \frac{3r^4}{r^4}\right) \cos 2\theta
\]  

\[
\sigma_z = \left(\sigma_H \cos^2 \alpha + \sigma_z \sin^2 \alpha\right) - 2\mu \left[\sigma_r - \sigma_r \sin^2 \alpha - \sigma_r \cos^2 \alpha\right] \left(\frac{r_w}{r}\right)^2 \cos 2\theta
\]

Where \(\sigma_r\) is radial stress, MPa; \(\sigma_\theta\) is circumferential stress, MPa; \(\sigma_z\) is vertical stress, MPa; \(\tau_{r\theta}, \tau_{rz}\) and \(\tau_{zr}\) are shear stresses, MPa; \(r\) is the distance from wellbore to the borehole axis, m; \(r_w\) is borehole diameter, m; \(\theta\) is circumferential angle, °.

Fig. 1 Stress analysis diagram of rocks around the well borehole at azimuth angle \(\alpha\)

3. Analysis of horizontal wellbore collapse failure [4-8]

After the borehole is formed, the pressure of drilling fluid columns substitutes the supporting effect of the drilled strata on the wellbore. Under low fluid column pressure inside the well, the stress borne by rocks around the wellbore exceeds the rock strength, a shear failure is generated and wellbore collapse is caused, and moreover, the borehole diameter is enlarged due to the collapse, and the irregularity degree of the borehole shape is elevated to a certain extent. The wellbore collapse failure is judged using Coulomb-Mohr criterion.

Meanwhile, the scope of wellbore collapse instability can be obtained:

\[
r_i = \sqrt{-S - \frac{S^2 - 4RT}{2R}}
\]

Where:

\[
R = M \left(1 - P\right) - N \cos 2\theta \left(1 + P\right) - 2CQ
\]

\[
S = \sigma_\theta \left[M \left(1 + P\right) - W \left(1 + P\right) + 4NP \cos 2\theta\right]
\]

\[
T = -3C \sigma_r \cos 2\theta \left(1 + P\right)
\]

\[
M = \frac{\sigma_r + \sigma_H \sin^2 \alpha + \sigma_z \cos^2 \alpha}{2}
\]

\[
N = \frac{\sigma_r - \sigma_H \sin^2 \alpha - \sigma_z \cos^2 \alpha}{2}
\]

\[
P = \frac{1 + \sin \phi}{1 - \sin \phi}, Q = \frac{\cos \phi}{1 - \sin \phi}, W = P_c
\]

4. Calculation model research on horizontal wellbore deformation

Following the formation of the borehole, the original stress state of the strata rocks within a certain scope of the wellbore is destroyed, so as to form a ground stress disturbed zone, where rock
deformation takes place unavoidably due to the stress change. The sum of rock deformations at various points constitutes the radius shrinkage amount in the direction of one circumferential angle, and this lays a foundation for accurate calculation of the borehole shape.

4.1. Determination method of boundary location for strata deformation after borehole formation

According to the rock stress distribution model around the borehole at different azimuth angles, when \( r \to \infty \), the strata at this position will keep its original ground stress state while not being influenced by the stress concentration, and then the original ground stress distribution model can be expressed as below:

\[
\begin{align*}
\sigma_{\theta} &= \frac{\sigma_{\alpha} \sin^2 \alpha + \sigma_{\alpha} \cos^2 \alpha + \sigma_{\text{a}}}{2} + \frac{\sigma_{\text{a}} - \sigma_{\alpha} \sin^2 \alpha - \sigma_{\alpha} \cos^2 \alpha}{2} \cos 2\theta \\
\sigma_{\phi} &= \frac{\sigma_{\alpha} \sin^2 \alpha + \sigma_{\alpha} \cos^2 \alpha + \sigma_{\text{a}}}{2} - \frac{\sigma_{\text{a}} - \sigma_{\alpha} \sin^2 \alpha - \sigma_{\alpha} \cos^2 \alpha}{2} \cos 2\theta \\
\sigma_{\text{y}} &= \sigma_{\alpha} \cos^2 \alpha + \sigma_{\text{a}} \sin^2 \alpha
\end{align*}
\]

The original ground stress distribution model combined, when \( r = N r_w \), \( N = 1, 2, 3, \ldots \), the distance from the borehole center will become farther as \( N \) increases, and the stress concentration parameter in the rock stress model satisfy the following equation:

\[
\frac{r^2}{r_w^2} \to 0, \quad 1 - \frac{r^2}{r_w^2} \to 1, \quad 1 + \frac{3r^4}{r_w^4} - \frac{4r^2}{r_w^2} \to 1
\]

Eq. (7) indicates that the influence of stress concentration becomes smaller and smaller. When this influence is weakened to a certain extent, the strain of the strata rocks at this position, which is defined as strata deformation boundary \( r_b \), can be neglected. Under \( r \leq r_b \), the strata within this scope undergo major deformation; When \( r > r_b \), it can be deemed that the stress borne by the strata within this scope is original ground stress.

4.2. Analysis of strata stress and deformation characteristics

When it comes to strata deformation [6-7], which is different from the deformation of ground material, as the scope of strata rocks is quite large, the load applied to the wellbore can be transferred outside the thick-walled strata only after a long time of wellbore stress change, and thus the strata present the following stress and deformation characteristics:

(1) Rock deformation on wellbore can be triggered by the wellbore stress change. Afterwards, the rock deformation breaks the original stress balance of surrounding strata and changes the surrounding stresses, the latter of which will lead to strain and stress change of farther surrounding rocks. As a result, the stress change and strain range are enlarged as seen in the dotted line in Fig. 2; (2) As the time passes by, the radius of dotted line continues to increase, so does the scope of deformation and accumulative deformation amount; (3) Both stress and deformation in the dotted line are already changed, but the stress and size outside the dotted line remain unchanged. When the dotted line reaches outside (deformation boundary position \( r_b \)), both stress and size of the whole thick-walled strata are changed while the strata beyond radius \( r_b \) do not experience deformation as they are not disturbed.
4.3. Calculation model research on borehole deformation

When the calculation model of borehole deformation is established, the following hypotheses are made first: (1) The strata are of solid petrographic composition with fixed shape; (2) The elastic modulus, strength and Poisson’s ratio are consistent in the whole strata; (3) The elasticity modulus, strength and Poisson’s ratio before and after (after the borehole is formed) strata drilling are unchanged; (4) The strata deformation follows generalized Hooke Law; (5) The deformation amount in the direction of well track is 0 under strata deformation.

The basic vertical, circumferential and radial stress-strain relations can be expressed as follows:

\[
\varepsilon_z = \frac{\sigma_z - \mu(\sigma_x + \sigma_y)}{E}, \quad \varepsilon_\theta = \frac{\sigma_\theta - \mu(\sigma_x + \sigma_y)}{E}, \quad \varepsilon_r = \frac{\sigma_r - \mu(\sigma_x + \sigma_y)}{E}
\]  

(8)

Where \( \varepsilon_z \) is vertical strain, dimensionless; \( \varepsilon_\theta \) denotes circumferential strain, dimensionless; \( \varepsilon_r \) is radial strain, dimensionless; \( \mu \) represents Poisson’s ratio of strata rocks, dimensionless; \( E \) is elasticity modulus of strata rocks, MPa.

According to the basic hypotheses, the strata deformation amount in the direction of well track is 0, then the vertical stress-strain relational expression is solved as 0, and meanwhile, the following can be obtained by combining the relational expression of radial strain:

\[
\varepsilon_r = \frac{\sigma_r - \mu(\sigma_x + \sigma_y)}{E} = \frac{\sigma_r(1-\mu^2)-\mu(1+\mu)\sigma_\theta}{E}
\]  

(9)

The calculation model (1) of radial stress and calculation model (2) of circumferential stress are respectively substituted into Eq. (9). Within the scope of strata deformation \( r_w\sim r_b \), the deformation amount of the whole borehole can be obtained by taking the integral of radial strain, and because the borehole deformation amount \( \Delta r \) is opposite to the strain symbol, its calculation model is written into the following form:

\[
\Delta r = -\int_{r_w}^{r_b} \varepsilon_r dr = -\frac{1-\mu^2}{E} \left[ \left( r_w - \frac{r_w^2}{r_6} \right) P_w + \sigma_\mu \sin^2 \alpha + \sigma_\alpha \cos^2 \alpha + \sigma \left( \frac{r_w^2}{r_b^2} - 2r_w \right) \right] + \\
\frac{1}{2} \left[ \sigma_r - \sigma_\theta \sin^2 \alpha - \sigma_\alpha \cos^2 \alpha \left( \frac{r_w^4}{r_b^2} - \frac{r_w^4}{r_6^2} + 4r_w^2 \right) \cos 2\theta \right] + \\
\frac{\mu(1+\mu)}{E} \left[ \left( r_w - \frac{r_w^2}{r_6} \right) P_w + \sigma_\mu \sin^2 \alpha + \sigma_\alpha \cos^2 \alpha + \sigma \left( \frac{r_w^2}{r_b^2} - 2r_w \right) \right] + \\
\frac{1}{2} \left[ \sigma_r - \sigma_\theta \sin^2 \alpha - \sigma_\alpha \cos^2 \alpha \left( \frac{r_w^4}{r_b^2} - \frac{r_w^4}{r_6^2} \right) \cos 2\theta \right]
\]  

(10)

5. Calculation model research on horizontal well borehole shape

Through the above analysis, the horizontal well borehole shape is formed under joint actions of borehole radius expansion, which is induced by wellbore collapse, and borehole shrinkage, which is
caused by strata deformation in the disturbed zone. Given this, the calculation model of borehole shape is established by keeping to the following ideas: (1) Judgment of wellbore collapse failure; (2) Under wellbore collapse, the scope of wellbore collapse \( r_t \) is determined according to the calculation model of collapse scope; (3) The borehole shrinkage \( \Delta r \) is calculated using the calculation model of borehole deformation; (4) The final borehole shape is algebraic sum of \( r_t \) and \( \Delta r \). The calculation model of borehole shape is presented as below:

\[
 r = r_t + \Delta r
\]  

If wellbore collapse does not occur, \( r_t = r_w \).

6. Calculated example and analysis

Based on the basic data of different parameters in Tab. 1, the borehole shapes under different azimuth angles (\( \alpha = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ \) and \( 90^\circ \)) are calculated and analyzed.

| Name                        | Symbol | Value | Unit   |
|-----------------------------|--------|-------|--------|
| Vertical depth              | \( H \) | 3,500 | m      |
| Borehole radius             | \( r_w \) | 0.1   | m      |
| Minimum horizontal principal stress | \( \sigma_h \) | 60   | MPa    |
| Maximum horizontal principal stress | \( \sigma_H \) | 80   | MPa    |
| Vertical principal stress   | \( \sigma_v \) | 90   | MPa    |
| Rock elasticity modulus     | \( E \) | 25,000 | MPa    |
| Drilling fluid density      | \( \rho_0 \) | 1.25 | g/cm³  |
| Rock Poisson’s ratio        | \( \mu \) | 0.3   | Dimensionless |
| Rock cohesion               | \( C \) | 20    | MPa    |
| Rock internal friction angle| \( \phi \) | 30    | °      |

The final horizontal well borehole shape can be acquired according to the calculation model (12) as shown in Fig. 3.

Fig. 3 The changing rule of wellbore shape with circumferential Angle

As shown in Fig. 3, under nonuniform ground stress condition, the borehole shape is irregular at different azimuth angles, and the stronger the nonuniformity degree of ground stress, the higher the borehole irregularity; The borehole shape is influenced by two factors, that is, borehole radius expansion caused by wellbore collapse and borehole shrinkage induced by stress concentration, where wellbore collapse is the primary factor resulting in borehole irregularity at circumferential 90° and 270° directions of the borehole section. However, at circumferential 0° and 180° directions of the borehole section, the borehole shrinkage arising out of stress concentration is the principal factor giving rise to borehole irregularity.

7. Example verification

A horizontal well is taken for example, where the well depth, vertical depth and scope of depth of the
horizontal segment are 3,270 m, 2,430.25 m and 2,150 m-3,270 m, respectively, and the horizontal segment is in the same direction of the minimum horizontal principal stress. The ground stresses in three directions at the vertical depth, that is, vertical principal stress, maximum horizontal principal stress and minimum horizontal principal stress, are 62.49 MPa, 55.54 MPa and 41.66 MPa, respectively, the drill size is 165.1 mm, and the density of the used drilling fluid is 1.25g/cm3. 10% of borehole radius expansion rate taken into consideration, the borehole diameter data of horizontal segment measured in the practical drilling process are presented in Fig. 4.

![Fig.4 Borehole radius change curve of horizontal segment](image)

In Fig. 4, the average borehole radius is 170.64 mm. Through the calculation method in the calculated example, it can be confirmed that the wellbore does not undergo collapse instability, and the deformation boundary position under this stress action is r=65rw. The borehole shape is decided by radius shrinkage induced by stress concentration at the time, the minimum radius shrinkage at different circumferential angles on the borehole cross section of the horizontal well is 0.012 m (circumferential angles 0°and 180°), and the corresponding maximum borehole radius is 0.1576 m. The coincidence rate of the result calculated through the theoretical model with the measured data reaches as high as 92.36%.

8. Conclusions

1) The horizontal well borehole shape under nonuniform ground stress condition depends on both radius expansion due to wellbore collapse and radius shrinkage caused by stress concentration, and the higher the nonuniformity degree of ground stress, the more irregular the borehole shape;

2) As the cumulative sum of rock stresses in the whole ground stress disturbed zone, borehole shrinkage reflects the stress and strain propagation characteristics in infinitely great strata after drilling, and meanwhile, it is also manifested that borehole shrinkage is of time effect;

3) In the direction perpendicular to the maximum stress borne by the borehole, the radius expansion triggered by wellbore collapse is the main reason for irregular borehole shape, but in the same direction of the maximum stress carried by the borehole, the radius shrinkage caused by stress concentration takes the major responsibility for the irregularity of borehole shape; The wellbore collapse can be prevented by properly elevating the pressure of drilling fluid columns inside the well, so as to guarantee regular borehole shape to a certain degree.

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