VERY SMALL STRANGELETS

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ABSTRACT

We study the stability of small strangelets by employing a simple model of strange matter as a gas of non-interacting fermions confined in a bag. We solve the Dirac equation and populate the energy levels of the bag one quark at a time. Our results show that for system parameters such that strange matter is unbound in bulk, there may still exist strangelets with $A < 100$ that are stable and/or metastable. The lifetime of these strangelets may be too small to detect in current accelerator experiments, however.
I. INTRODUCTION

With the advent of heavy ion colliders, it will soon be possible to search for stable or metastable lumps of quark matter with $S^* \sim A \sim 10 - 30$. The possible stability of strange quark matter (“strange matter”) in bulk was pointed out by Witten in 1984, and since then, there have been numerous attempts to predict the properties of strange matter in bulk and in finite lumps (“strangelets”). These studies generally only apply to baryon numbers much larger than those accessible in heavy ion colliders. Our intention in this letter is to present some qualitative information on very small strangelets obtained from a very elementary model. Our model includes only quark kinetic energy, the Pauli principle and confinement. It cannot tell us anything about important issues like the overall energy scale, or equivalently the bulk stability, of strange matter. It does, however, illustrate potentially interesting effects such as shell closures, “loading” and unloading of strangeness and isolated “islands of stability” in the $(S, A)$ plane. None of the details of our predictions should be taken very seriously; they would undoubtedly be changed in more sophisticated models. Nevertheless, the types of phenomena which occur in our model may well persist in others.

No one knows how to model quark matter in QCD accurately. Lattice simulations are as yet unable to cope with systems at non-zero chemical potential. Models of bulk strange matter have confined quarks in a bag and included residual gluon interactions perturbatively. Surface effects were included for large strangelets by including surface modifications of the quark density of states as well as Coulomb effects. The resulting Thomas-Fermi like model is only valid for strangelets with radii very large compared to the natural length scale of the

*For the sake of simplicity, throughout this Letter we assign the strange quark a strangeness of $+1$. 
system, $B^{-1/4} \sim 1 - 2$ fm. For typical strange matter densities, a strangelet with baryon number $A \sim 200$ has a radius of only $5 - 6$ fm, so only for such a large baryon number does the model of Ref. [6] become reliable. We model a small strangelet as a gas of non-interacting fermions confined in a bag. Rather than approximate the density of states, we instead fill the bag energy levels sequentially, obeying the exclusion principle, minimizing the energy (for each $A$) and adjusting the bag radius so the quark pressure balances the vacuum pressure, $B$. The free parameters we use are the energy per baryon in bulk, $\epsilon_b$, the mass of the strange quark $m_s$, and of course, the baryon number, $A$. We ignore residual perturbative QCD interactions following Ref. [6] where it was argued (on the basis of Ref. [5]) that the effect of the interactions can be largely absorbed into a redefinition of the overall energy scale parameterized by $\epsilon_b$. We also ignore Coulomb corrections. This should be a good approximation because $Z$ is very small for small $A$, $Z \ll A$. Both of these effects should be included in future calculations.

Both the quantum numbers and energetics of small strangelets show characteristic regularities reminiscent of atomic physics. We see that “shell” effects are extremely important when filling a bag with quarks: the rate of change of the energy per baryon with $A$ changes dramatically near shell closures and leads to enhanced stability. We find that there exist small regions of $A$ in which strangelets are stable even for system parameters such that strange matter is not bound in bulk. We also observe that the strangeness of the most stable strangelet is an erratic function of $A$. Strange and non-strange quark energy levels cross as a function of the bag radius $R$. When, as happens, a non-strange level dives below a strange level, the strange level “unloads” into the non-strange one, dropping strangeness by as much as $\Delta S = -6$ from one value of $A$ to the next. This phenomenon is similar to the filling and emptying of inner $d$-orbitals in the periodic table. Finally, we find that the spatial distribution of strangeness
is not uniform throughout a strangelet. Because they are less relativistic, strange quarks are concentrated in the interior and depleted on the surface. This phenomenon is related to the quark mass dependence of the surface tension in strange matter.

In our model, we describe the system as a Fermi gas under constant pressure. First, we fix the strange quark mass and the bag radius, \( R \). After determining the energy eigenvalues for the quarks, we fill the bag and vary its radius until the quark pressure at the surface equals the vacuum pressure, \( B \). This is equivalent to minimizing the total energy at constant pressure, \( B \). Values of \( B \) were taken from fits to bulk strange matter as described in Ref. [6].

There have been previous attempts to use similar models to study very small strangelets. In Ref. [7], the strangelets were constructed by filling the bag energy levels with the strangeness ratio held fixed. In the published work, \( S/A \) was fixed at 0.7 which is far from the optimal value for most \( A \). Fixing the strangeness ratio artificially prevents the system from finding a minimum energy configuration. Thus, the model of Ref. [7] is not adequate for studies of stability.

More recently, Madsen\(^8\) attempted to study very small strangelets using the asymptotic expansion of the density of states including the curvature term in order to achieve higher accuracy. In fact, we find that there is no region in \( A \) in which the curvature term gives a useful correction to the density of states. At large \( A \), the curvature term is negligible. At small \( A \), where it would be expected to be important, other even lower-order corrections are even more important. This is clearly displayed in Fig. 1 where we plot the integral of the density of states, \( N(k) \), for a light quark as a function of \( k = \sqrt{E^2 - m^2} \) in a rigid spherical cavity. For comparison we plot the first two terms in an asymptotic expansion for \( N(k) \) for large systems, obtained by integrating the density of states, \( \rho(k) \),

\[
\rho(k) = \frac{g}{2\pi^2} \left[ k^2 V - \frac{\pi k S}{4} \left( 1 - \frac{2}{\pi} \tan^{-1} \frac{k}{m} \right) + \ldots \right],
\]

(1)
where $V$ and $S$ are the volume and surface area of the bag, respectively. The first omitted term in Eq. (1) is proportional to the surface integral of the average curvature 
\[
\oint d^2 s \frac{1}{2} (1/R_1 + 1/R_2),
\]
where $R_1$ and $R_2$ are the principal curvatures at each point) and suppressed by $O(1/k^2)$ relative to the volume term. At low $k$, $N(k)$ is a noisy function of $k$ reflecting the details of the eigenvalue spectrum of the Dirac operator in a cavity. At large $k$, $N(k)$ is well-approximated by the integral of Eq. (1). There does not seem to be a significant intermediate region in which a $O(1/k^2)$ (curvature) correction is significant. We conclude that the asymptotic expansion of the density of states is not useful to study very small strangelets.

II. STRANGELETS AS NON-INTERACTING DIRAC FERMIONS

Since we are dealing with light quarks confined to a small bag, we must of course consider them as relativistic particles. We write down the Dirac equation and the appropriate boundary condition,
\[
(\bar{\alpha} \cdot \vec{p} + \beta m)\Psi = E\Psi , \quad \text{for } r < R , \quad \tag{2}
\]
\[
i\hat{r} \cdot \vec{\gamma} \Psi = \Psi , \quad \text{for } r = R . \quad \tag{3}
\]
This boundary condition ensures that there is no probability flux leaving the bag ($\vec{j} \cdot \hat{r} = 0$). Note that the wavefunction and the density, $\Psi^\dagger \Psi$, need not go to zero on the boundary, whereas for the non-relativistic case, the wave function must vanish at the boundary. This implies that the more massive, hence less relativistic, strange quarks will tend to shy away from the boundary of the bag.

Once the Dirac equation is solved with this boundary condition and geometry, we obtain expressions for the eigenfunctions and transcendental equations for the eigenvalues. We take
the energy, momentum, and mass to be $\omega, k, \text{ and } m$ respectively. We define $\alpha = \omega R, x = k R, \text{ and } \lambda = m R$, so that $\alpha^2 = x^2 + \lambda^2$. Thus, the eigenvalue equation reads

$$\sqrt{\alpha + \lambda} f_\kappa = -\sqrt{\alpha - \lambda} f_{\kappa-1}$$

(4)

where $f$ is the spherical Bessel function regular at the origin, and $\kappa = -\ell - 1$ for $j = \ell + 1/2$ and $\kappa = \ell$ for $j = \ell - 1/2$. Using this definition of $\kappa$, we can re-express the angular momentum/color degeneracy, $3(2j + 1)$ as $6|\kappa|$. The normalized wave function is

$$\Psi = \sqrt{\frac{x^2}{R^3 (2\alpha(\alpha + \kappa) + \lambda)f_\kappa^2(x)}} \left( \frac{if_\kappa \left( \frac{x R}{x} \right) \phi_{jm}^\ell}{\alpha + \lambda f_{\kappa-1} \left( \frac{x R}{x} \right) \vec{\sigma} \cdot \hat{r} \phi_{jm}^\ell} \right). \quad (5)$$

The eigenvalues are a function of the product $mR$, the mass of the particle times the radius of the bag. For the up and down quarks, the mass is taken to be zero.

As described in Ref. [6], we determine the bag constant and an estimate of the radius of the bag as a function of $\epsilon_b$, $m_s$, and $A$ by studying the bulk limit, $A \rightarrow \infty$. In bulk equilibrium, the Fermi seas for the three quark species must have the same Fermi energy or chemical potential: $\mu = \mu_{\text{up}} = \mu_{\text{down}} = \mu_{\text{strange}}$. $\mu$ is the change in total energy due to the addition of a quark. We obtain the number of particles/volume $n_a$ for $a = u, d, \text{ and } s$ by integrating the $k^2V$ term in the density of states, Eq. (1). For the massless $u$ and $d$ quarks, $n_{u,d} = (\mu^3/\pi^2)$ for the strange quark, $n_s = (\mu^3 \cos^3 \theta/\pi^2)$, where $\sin \theta = (m/\mu)$. In bulk, the baryon number is $A = (1/3) \sum_a n_a V$. The total energy of the bag is $E = \sum_a \mu_a n_a V$. By using these two expressions, we find that $\epsilon_b = 3\mu$. As noted in Ref. [6], the surface tension is positive. Thus, to minimize energy, the shape of the strangelet will be spherical. By inverting the relation between $A$ and $n$, we obtain a first estimate of the radius of the bag as a function of $\epsilon_b, m_s$ and $A$.

$$R = \left( \frac{9\pi A}{4} \left( 2 + \left[ \frac{\sqrt{\epsilon_b^2 - 9m^2}}{\epsilon_b} \right]^3 \right)^{-1} \right)^{1/3} \frac{3}{\epsilon_b}.$$

(6)
This is the radius of a lump of strange matter in which all surface effects are ignored and is used as a first approximation to the actual radius that will balance the quark pressure against the vacuum pressure. The equilibrium condition on the volume gives us the equation for $B$, $B = -\sum \Omega_a \equiv \text{quark pressure}$. Using the equations for $\Omega_a$ from Ref. [6], we get

$$B = \frac{\epsilon_b}{3} + \frac{1}{4\pi^2} \left[ \frac{\epsilon_b}{3} \sqrt{\left(\frac{\epsilon_b}{3}\right)^2 - m^2} \left(\left(\frac{\epsilon_b}{3}\right)^2 - \frac{5}{2} m^2\right) + \frac{3}{2} m^4 \ln \left(\frac{\epsilon_b}{3} + \sqrt{\left(\frac{\epsilon_b}{3}\right)^2 - m^2}\right) \right].$$

(7)

Once the quark mass has been chosen and a first approximation to the radius has been determined, we calculate the energy levels by solving the transcendental equation numerically. We then adjust the radius, and recalculate the energy levels, until the total energy is minimized. Once a strangelet is thus created, we can read off its energy per baryon, strangeness, and radius directly.

We have performed a variety of checks on this calculation. First, we have calculated the number of states with “momentum” less than $k$ ($k = \sqrt{\omega^2 - m^2}$). This function, $N(k)$, should be approximated by the integral of the asymptotic expansion of Eq. (1) for large $k$. This check is shown in Fig. 1 where it is clear that our model reproduces the asymptotic result and the surface correction. Second, we have checked that the energy per baryon and strangeness per baryon also converge to the bulk values as $A \to \infty$. These checks reassure us that our calculation has been performed correctly.

We now turn to issues of stability and composition of strangelets. If a strangelet is not in flavor equilibrium, it can decay via weak semileptonic decays, weak radiative decays, and electron capture all of which do not change baryon number. Other modes of decay such as fission, alpha decay, weak and strong neutron decays, and strong $\Lambda$, $\Sigma$, $\Xi$, $\Omega$ decays reduce
baryon number by one or more units. Our strangelets are already in equilibrium at a given $A$ and thus are only subject to alpha and strong or weak neutron decays, strange baryon decays, and fission. We check the stability of our strangelets against alpha decay, fission, and the other baryon decays noted.

A strong neutron decay will occur if the difference in energy between two strangelets of the same strangeness but $\Delta A = -1$ is greater than $m_n$. Similarly, a weak neutron decay is possible if the energy difference between two strangelets of $\Delta S = -1$ and $\Delta A = -1$ is greater than $m_n$. For the $\Lambda$, $\Sigma$, $\Xi$, $\Omega$ decays, we have $\Delta A = -1$ and $\Delta S = -1, -1, -2, -3$, respectively. We calculated these energies and discovered where stable regions exist within our model.

III. RESULTS

For small $A$, the dynamics are as follows. Given the choice between massive and massless particles, we opt to fill the bag with less energetic massless particles first. We continue to add massless particles until we build up a large enough Fermi sea so that it becomes energetically favorable to add a strange quark to the system. Soon, it again becomes favorable to add non-strange quarks to the system. One might expect that strange and non-strange levels will fill in an alternating sequence. However, Fig. 2a shows that this is not the case. This is because the massive quark energy levels change at a different rate with respect to the radius than the massless quark energy levels do. Energy levels can cross, and strange levels that have been filled may suddenly empty out into nonstrange levels. This can be seen on the data for $\epsilon_b = 950$ MeV, $m_s = 150$ MeV, where a level crossing occurs at $A = 30$ and the strangelets become stable until the next nonstrange level begins to fill at $A = 36$ (see Fig. 2a).
Our results indicate that within our model, there exist stable and metastable strangelets for various system parameter values. We generated strangelets of baryon numbers $1 - 100$ for various values of $m_s$ and $\epsilon_b$. We find that for $\epsilon_b < 930$ MeV, which is the value of $\epsilon$ for $^{56}_{26}$Fe, there exist many stable strangelets. The stability of strangelets with several choices of $\epsilon_b$ and $m_s$ is displayed in Fig. 3. Those species noted in the figure are stable against single baryon emission. Whenever the slope of the $\epsilon(A)$ curve is negative enough, the decrease in energy due to emission of a particle is not enough to offset the increase in energy due to the slope. Thus, for small $A$, it is frequently energetically unfavorable for a strangelet to decay via emission of a baryon. Many of the smaller strangelets, however, are subject to fissioning into several $\Lambda$ hyperons and a nucleus, or simply dissolving into $\Lambda$ hyperons and neutrons. This mode is a strong decay, but its rate will be suppressed by several orders of magnitude due to the unlikeness of the quarks simultaneously arranging themselves into the decay products. The suppression is difficult to estimate, however, because we are dealing with a collective, many-particle effect. The astute reader will observe that some quasistable species occur in regions where the slope of $\epsilon(A)$ is positive (see, for example, Fig. 3d near $A = 60$). In this region the strangeness charge between most stable species with $A$ and $-1$ is $\Delta s = -3$ (see Fig. 2a) requiring $\Omega^{-}$ emission which is energetically forbidden. Neutron emission requires $\partial \epsilon/\partial A$ at fixed strangeness to be positive. In the region of concern, direct calculation shows $\partial E/\partial A|_{s}$ to be negative.

Another interesting effect that can be seen is the phenomenon of shell closures. The first level to fill is a $1s_{1/2}$ level where $\kappa = -1$. This level may hold six quarks of each flavor. At every occurrence of a shell closure, the $\epsilon(A)$ curve takes a noticeable dip. This generates a large slope for $\epsilon(A)$ and thus, stable regions in the neighborhood of a closed shell. This is
similar to atomic physics where shell closures produce more stable, less chemically reactive elements. Shell closures can be seen at \( A = 4, 6, 14, 18, 22, \ldots \) (see Fig. 3). These particular values correspond to a non-strange \( s_{1/2} \)-shell, a strange \( s_{1/2} \)-shell, a non-strange \( p_{3/2} \)-shell, a non-strange \( p_{1/2} \)-shell, and a strange \( p_{3/2} \)-shell, respectively. The locations of these shell closures are a function of the self-consistent “potential,” in which the quarks are bound — in this case, the bag. Therefore, the precise values should not be taken too seriously.

The most surprising results are uncovered when we examine values of \( \epsilon_b > 930 \text{ MeV} \). Specifically, looking at \( \epsilon_b = 950, 970 \text{ MeV}, m_s = 150 \text{ MeV} \), we see that there still exist islands of stability against single baryon decay (see Fig. 3). This is interesting because the failure of terrestrial searches to find stable strange matter suggests that strange matter in bulk may well be unstable.\(^9\) Our results indicate that even though this may be the case, there is still a chance of detecting small strangelets in the laboratory provided the strong decay into light nuclei and several hyperons or complete dissolution does not proceed too rapidly to allow the produced strangelets to reach the detector before decaying. These islands of stability persist until \( \epsilon_b \sim 1000 \text{ MeV}, m_s = 150 \text{ MeV} \).

The charge systematics of light strangelets are important for experimenters. In bulk, we expect roughly equal numbers of \( u-, d- \) and \( s \)-quarks, thus \( Z/A \ll 1 \). Even for nuclei, where \( Z \sim A \), Coulomb effects are not important for small \( A \). For small strangelets we ignore them. The possible charges of small strangelets are determined by which shells are filled, and which one is currently filling. Figure 2b shows that the allowed charges for strangelets as a function of baryon number is a complex function that reflects the nature of the shell filling process. Throughout our region of interest, the charge remains relatively small (and occasionally negative) in comparison to \( A \), so we are justified in neglecting the Coulomb energy contribution.
We also plotted the spatial density for the quarks in the strangelets. As is guaranteed by the Dirac equation, the heavier, less relativistic, strange quarks in fact have a distribution that is concentrated closer to the center of the strangelet than the up and down quarks (see Fig. 4). This is a reflection of the boundary condition imposed. By requiring that no probability flux leave the bag rather than requiring that $\Psi = 0$, a relativistic quark may have a non-zero density, $\Psi^\dagger \Psi$, at the boundary. As the mass of the particle increases, we approach the non-relativistic limit where the boundary condition becomes $\Psi = 0$. Thus, strange (heavier) quarks are depleted near the surface.

We have shown that the energetics associated with shell closures are likely to be important in the study of very small strangelets. Our admittedly crude method brings out this aspect of the system that is not seen when the smoothed density of states is employed. Our results are consistent with those obtained for large $A$. We therefore conclude that metastable strange matter may be found in small lumps. The suppressed strong decay into a nucleus (or many neutrons) and $\Lambda$ hyperons, might render it difficult to detect, however. One characteristic that would identify a strangelet is its unusual charge/mass ratio. The charge is typically small since flavor equilibrium favors charge neutrality even for relatively small $A$. 

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Fig. 1: The scaled number of states as a function of $k$ compared with the asymptotic expansion including the surface correction. Here, $m_s = 150$ MeV, $R = 1$, $g = 6$.

Fig. 2a: The strangeness as a function of $A$ for the most stable species, illustrating the “unloading” of strange quarks into non-strange energy levels as strange and non-strange energy levels cross. Here $\epsilon_b = 950$ MeV, $m_s = 150$ MeV.

Fig. 2b: The allowed range of charges for strangelets as a function of $A$.

Fig. 3: Energy per baryon as a function of $A$ for various choices of $\epsilon_b$ and $m_s$, including some for which bulk strange matter is unstable.

Fig. 4: The ratio of radial strangeness density to radial total matter density for $A = 14, 36, 100$, showing the depletion of strangeness near the boundary of the bag. Here $\epsilon_b = 950$ MeV, $m_s = 150$ MeV.