Constraining EDM and MDM lepton dimension five interactions in the electroweak sector

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We investigate dimension five Lorentz-violating nonminimal interactions in the electroweak sector, in connection with the possible generation of electric dipole moment (EDM), weak electric dipole moment (WEDM), magnetic dipole moment (MDM) and weak magnetic dipole moment (WMDM) for leptons. These couplings are composed of the physical fields and LV tensors of ranks ranging from 1 to 4. The CPT-odd couplings do not generate EDM behavior and do not provide the correct MDM signature, while the CPT-even ones yield EDM and MDM behavior, being subject to improved constraining. Tau lepton experimental data is used to constrain the WEDM and WMDM couplings to the level of $10^{-11}$ (GeV)$^{-1}$, whereas electron MDM and EDM data is employed to improve constraints to the level of $10^{-17}$ (GeV)$^{-1}$ and $10^{-11}$ (GeV)$^{-1}$, respectively.

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I. INTRODUCTION

Electric dipole moment (EDM) physics is a broad field of investigation [1–4] deeply connected with precise experiments and physics beyond the Standard Model (SM) [5]. EDM has as signature the violation of parity ($P$) and time reversal ($T$) symmetries, while preserving charge conjugation ($C$) and the $CPT$ symmetry. In the relativistic context, the electric dipole moment, $d = g(q/2m_S)S$, yields the interaction of $d(S \cdot E)$, with $d$ being the EDM modulus, $E$ being the electric field, and $S$ being the Dirac spin operator. The EDM Lagrangian is represented by the dimension five term $-d(\langle \bar{\psi} \sigma_{\mu\nu} \gamma_5 F^{\mu\nu} \psi \rangle)$, where $\psi$ is a Dirac spinor. It is important to mention that the EDM structure is only generated by radioactive corrections at four-loop order [4, 6], so that its magnitude is of about $d_e \approx 10^{-38} e \cdot cm$ in the SM framework. Analogously, the magnetic dipole moment, $\mu = g(q/2m_S)$, provides the relativistic magnetic interaction, $\mu(S \cdot B)$ [6, 7], whose Lagrangian representation, $\mu(\bar{\psi} \gamma_{\mu} F^{\mu\nu} \psi)$, appears in the SM framework at 1-loop order.

Each order of magnitude improvement in the EDM experiments leads to strong phenomenological consequences on a diversity of CP-violating theories. EDM measurements have been progressively improved [8], reaching the level of $10^{-29} e \cdot cm$ for the electron EDM [9, 10], and $10^{-30} e \cdot cm$ for the $^{199}$Hg nuclear EDM [11]. The gap of seven orders of magnitude still remaining between the experimental data and the theoretical evaluations for the electron EDM allows for new CP mechanisms, besides the usual CP violation sources already embedded in the SM. These sources may be relevant for explaining the observed baryon asymmetry of the universe, an issue possibly connected with axions and the strong CP problem [12]. EDM physics may also be related to Lorentz-violating theories, investigated in the broader framework of the Standard Model extension (SME), developed by Colladay and Kostelecký [13]. The SME incorporates dimension four and dimension three LV terms in all sectors of the Standard Model, including fermions [14–16], photons [17–20], photon-fermion interactions [21, 22], and electroweak (EW) processes [23–25]. Beyond the minimal SME, there are nonminimal extensions encompassing couplings with higher-order derivatives [26] and higher-dimensional operators [27–29].

Lorentz violation can work as a source of CP violation and EDM generation via radiative corrections [30], or even at tree level via dimension five nonminimal (NM) couplings [31, 32]. Dimension-five nonminimal couplings have been proposed as nonusual QED interactions between fermions and photons, yielding EDM Lagrangians pieces as $\lambda_1(\bar{\psi}K_F)_{\mu\nu\alpha\beta}\Gamma^{\mu\nu}\Gamma_{\alpha\beta}\psi$, $\lambda_2(\bar{\psi}T_{\mu\alpha}\Gamma^{\mu\nu}\Gamma_{\alpha\beta}\psi$, where $(K_F)_{\mu\nu\alpha\beta}$ and $T_{\mu\alpha}$ are $CPT$-even LV tensors, with $\Gamma_{\mu\nu} = \sigma_{\mu\nu}$ or $\sigma_{\mu\nu}\gamma_5$ [32]. Electron EDM experimental data has yielded upper bounds as tight as $10^{-25} (eV)^{-1}$ on the magnitude of these couplings. Considering the Schiff screening theorem [33], anisotropic electrostatic interactions were taken into account in order to engender LV corrections on the nuclear EDM and Schiff moment [34]. Recently, general dimension six nonminimal fermion-fermion couplings were proposed [35] and constrained at the level of $10^{-15}$ (GeV)$^{-2}$ by EDM data [36], considering these couplings as electron-nucleon P-odd and T-odd atomic interactions. LV contributions to MDM physics were also examined [37, 38], being constrained by precise experimental data.
If the Standard Model is addressed as a low-energy effective theory, it becomes worthy to consider higher dimensional terms in the Lagrangian. Extensions of the electroweak model containing higher dimension terms (mainly dimension six) have been analyzed as effective theories since the eighties [40]. Lists of dimension six EW and strong couplings have been presented and updated [41], so as to involve top quark physics and interactions with the Higgs [42]. CP-violating couplings in the Higgs sector, which comprise CP-violating interactions to quarks and tau lepton, are also represented by dimension six operators. Such couplings can generate EDM, providing an effective route of constraining [43]. Some of the best bounds on the anomalous CP-violating Higgs interactions come from EDM measurements. A plethora of dimension six terms yielding electroweak baryogenesis and CP violation has been considered in connection with the baryon asymmetry of the universe [44]. The role of EDM physics in electroweak interactions and electroweak baryogenesis has been a topical issue in the latest years [45, 46].

Dimension five nonminimal couplings in the Glashow-Salam-Weinberg (GSW) electroweak model have also been proposed in connection with CPT and Lorentz symmetry violation [47, 48]. Such couplings have been constrained by weak decay data at the level of $10^{-5} \text{(GeV)}^{-1}$. The repercussions of MDM and EDM physics on such nonminimal couplings has not been examined yet, and can be used to improve constraining on these couplings. In the electroweak sector, the weak magnetic moment (WMDM) and weak electric dipole interaction (WEDM) involve interaction with the Z boson field, being given by the effective Lagrangian [49, 50]:

$$\mathcal{L}_{EW} = \frac{1}{\sin 2\theta W} \left[ \frac{\alpha_w}{2m_t} \sigma^{\mu\nu} Z_{\mu\nu} + id_w \sigma^{\mu\nu} \tilde{f} Z_{\mu\nu} \right] \psi,$$

where $\alpha_w$ and $d_w$ represent the WMDM and WEDM magnitudes, $\theta$ is the Weinberg angle and $Z_{\mu\nu}$ is the $U(1)$ boson field strength. Experimental limits for tau lepton WMDM and WEDM are presented in Ref. [49]: $\alpha_w < 1 \times 10^{-3}$ and $d_w < 10^{-17} \text{e} \cdot \text{cm}$.

In this work, we analyze a few dimension five LV couplings in the GSW electroweak model concerning the possibility of generating EDM, WEDM, MDM, WMDM for leptons. While we propose CPT-odd and CPT-even couplings, only the latter ones generate EDM or MDM behavior. Using tau WEDM and WMDM experimental data, some couplings are constrained to the level of $10^{-5} \text{(GeV)}^{-1}$, while the electron EDM and MDM yield upper bounds to the level of $10^{-17} \text{(GeV)}^{-1}$ and $10^{-11} \text{(GeV)}^{-1}$, respectively.

II. THE GLASHOW-SALAM-WEINBERG ELECTROWEAK MODEL

In the GSW model, the left-handed leptons are disposed in isodoublets $(T = 1/2; T_3 = \pm 1/2)$, while the right-handed leptons are represented by isosinglets $(T = 0)$ under the $SU(2)$ group,

$$L_L = \begin{bmatrix} \psi_\mu \\ \psi_\tau \end{bmatrix}_L = \frac{1 - \gamma_5}{2} \begin{bmatrix} \psi_\mu \\ \psi_\tau \end{bmatrix},$$

$$R_i = (\psi_i)_R = \frac{1 + \gamma_5}{2} \psi_i,$$

with the generators, $T = (T_1, T_2, T_3)$, fulfilling the relation, $[T_i, T_j] = i\varepsilon_{ijk}T_k$. The GSW Lagrangian is

$$\mathcal{L} = \bar{L}_i \gamma^\mu i D_\mu L_i + \bar{R}_i \gamma^\mu i D_\mu R_i - \frac{1}{4} \mathcal{W}_{\mu\nu} \cdot \mathcal{W}^{\mu\nu} - \frac{1}{4} B_{\mu
u} B^{\mu\nu},$$

where the field strengths for the $U(1)$ and $SU(2)$ gauge fields, $B_{\mu\nu}$ and $\mathcal{W}_{\mu\nu}$, are $B_{\mu\nu} = \partial_\mu B_{\nu} - \partial_\nu B_{\mu}$ and $\mathcal{W}_{\mu\nu} = \partial_\mu \mathcal{W}_{\nu} - \partial_\nu \mathcal{W}_{\mu} + g (\mathcal{W}_\mu \times \mathcal{W}_\nu)$. Knowing that the $U(1)$ field is a combination of the electromagnetic and the boson $Z$ field, $B_\mu = \cos \theta A_\mu - \sin \theta Z_\mu$, one has

$$B_{\mu\nu} = (\cos \theta) F_{\mu\nu} - (\sin \theta) Z_{\mu\nu},$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$. The usual covariant derivative is

$$D_\mu = \partial_\mu - ig (T \cdot \mathcal{W}) - i\frac{g'}{2} Y B_\mu,$$

where $Y$ is the $U(1)$ generator. We have $Y_L = -1$ for $(\varepsilon_L, \mu_L, \tau_L)$ and $Y_R = -2$ for $(\varepsilon_R, \mu_R, \tau_R)$. Replacing the covariant derivative in the Lagrangian (4), we obtain

$$\mathcal{L} = i\bar{L}_i \gamma^\mu \partial_\mu L_i + i\bar{R}_i \gamma^\mu \partial_\mu R_i + \mathcal{L}_{int}^{(l)},$$

with the interaction piece being

$$\mathcal{L}_{int}^{(l)} = g (\bar{L}_i \gamma^\mu T L_i) \mathcal{W}_\mu - \left[ \frac{g'}{2} (\bar{L}_i \gamma^\mu L_i) + g' (\bar{R}_i \gamma^\mu R_i) \right] B_\mu.$$

III. CPT-ODD DIMENSION FIVE NONMINIMAL LV ELECTROWEAK COUPLING

We investigate some CPT-odd nonminimal couplings. They do not generate EDM nor possess the correct MDM signature under CPT operators. This can be argued by analyzing rank-1 or rank-3 nonminimal couplings, which are the simplest ones to be proposed.
A. Rank-1 CPT-odd NMC

Rank-1 CPT-odd and dimension five nonminimal coupling in the EW sector were proposed in Ref. [47], as

\[
\mathcal{L}_{\text{int}} = g'_{1} \left( L_{i} \gamma^{\mu} B_{\mu\nu} C_{i}^{\nu} L_{i} \right) + 2g'_{2} \left( R_{i} \gamma^{\mu} B_{\mu\nu} C_{i}^{\nu} R_{i} \right),
\]

where \( C_{i}^{\nu} \) is a fixed LV background, and \( Y = -1 \) and \( Y = -2 \) for left-handed and right-handed fermions, respectively. Using Eq. (2) and (3), we obtain the Lagrangian:

\[
\mathcal{L}_{\text{int}} = \frac{1}{2} g'_{1} \bar{\psi}_{i} \gamma^{\mu} \left( 1 - \gamma_{5} \right) \psi_{i} B_{\mu\nu} C_{i}^{\nu} + \frac{1}{2} g'_{2} \bar{\psi}_{i} \gamma^{\mu} \left( 3 + \gamma_{5} \right) \psi_{i} B_{\mu\nu} C_{i}^{\nu}.
\]

Here, it is important to note that the \( \gamma_{5} \) operator changes the behavior of the coupling in relation to the \( C, P \) and \( T \) operators. Thus, it is suitable to rewrite Lagrangian (11) in terms of two vector backgrounds, \( C_{i}^{\nu} \) and \( C_{i}^{A} \):

\[
\mathcal{L}_{(1)}^{(odd)} = \frac{1}{2} g'_{1} \bar{\psi}_{i} \gamma^{\mu} \psi_{i} B_{\mu\nu} C_{i}^{\nu} - \frac{1}{2} g'_{1} \bar{\psi}_{i} \gamma^{\mu} \gamma_{5} \gamma_{5} \psi_{i} B_{\mu\nu} C_{i}^{\nu} + \frac{3}{2} g'_{1} \bar{\psi}_{i} \gamma^{\mu} \psi_{i} B_{\mu\nu} C_{i}^{\nu} + \frac{1}{2} g'_{2} \bar{\psi}_{i} \gamma^{\mu} \gamma_{5} \psi_{i} B_{\mu\nu} C_{i}^{A}.
\]

For purpose of better investigation, one explicitly analyzes the lepton (\( l \)) content of Eq. (12), writing

\[
\mathcal{L}_{(1)}^{(odd)} = \frac{1}{2} g'_{1} \left[ 3 \bar{\psi}_{i} \gamma^{0} B_{0\mu} C_{i}^{\mu} \psi_{i} + \bar{\psi}_{i} \gamma^{0} \gamma_{5} B_{0\mu} C_{i}^{\mu} \psi_{i} \right] + \frac{3}{2} \bar{\psi}_{i} \gamma^{0} B_{i} C_{i}^{0} \psi_{i} + \frac{3}{2} \bar{\psi}_{i} \gamma^{0} \gamma_{5} B_{0\mu} C_{i}^{\mu} \psi_{i}.
\]

Lepton Lagrangian term \( \bar{\psi}_{i} \gamma^{0} \gamma_{5} \psi_{i} B_{i} \), \(\psi_{i} C_{i}^{0} A_{i}^{0} \) is the unique one compatible with the EDM signature, as shown in Table (1), since it contains the pieces

\[
\left( \bar{\psi}_{i} \gamma^{0} \Sigma_{i}^{A} \psi_{i} \right) B_{0\mu} C_{i}^{0},
\]

where \( B_{0\mu} = E^{i} + \bar{E}^{i} \) could yield electric and electroweak EDM, with \( E^{i} \) representing the weak electric field, as shown in (19). This same analysis holds equally for the neutrino terms in Lagrangian (11), where the EDM-like term is \( \left( \bar{\psi}_{\nu} \gamma^{0} \Sigma_{\nu} \psi_{\nu} \right) B_{0\mu} C_{i}^{0} \). The presence of the \( \gamma^{0} \) factor prevents the EDM behavior, since it avoids the resemblance to the EDM Lagrangian \( \left( \bar{\psi}_{i} \gamma^{0} E^{i} \psi_{i} \right) \). The \( \gamma^{0} \) factor disappears in the corresponding Hamiltonian form, yielding the relativistic interaction pieces, \( \Sigma^{i} E^{i}, \Sigma^{j} E^{j}, \)

which are undetectable (at first order) in accordance with the Schiff’s theorem [4, 33]. This occurs with the Hamiltonian interactions stemming from Eq. (14),

\[
\psi_{i} \Sigma^{i} E^{i} C_{0} \psi_{i}, \quad \psi_{i} \Sigma^{i} Z_{0} C_{0} \psi_{i},
\]

implying absence of EDM physics. Here, we have used

\[
\sigma^{0j} = i \alpha^{j}, \quad \sigma^{ij} = \epsilon_{ijk} \Sigma^{k},
\]

\[
\alpha_{i}^{k} \gamma_{5} = \Sigma^{k}, \quad \alpha_{i}^{l} = \Sigma^{j} \gamma_{5}, \quad F_{ij} = E^{j}, \quad F_{mn} = \epsilon_{mnp} B_{p}, \quad Z_{ij} = E^{j}, \quad Z_{mn} = \epsilon_{mnp} B_{p}.
\]

We can also investigate MDM behavior for leptons and neutrinos. The closest MDM term in Lagrangian (12) is

\[
\bar{\psi}_{i} \gamma^{0} \gamma_{5} B_{ij} C_{i}^{A} \psi_{i},
\]

in a non convoluted way (it couples the spin to a “rotated” LV background structure, \( C_{i}^{A} \)). As shown in Table (1), the coefficient \( C_{i}^{A} \) has not the exact signature of a MDM interaction, due the presence of \( \gamma^{0} \). Thus, it does not generate MDM or WMDM. For experimental purposes, as this tensor background has no diagonal components, \( C_{i} = 0 \), the contributions (21) could only be probed with a magnetic field orthogonal to the spin, as discussed in Refs. [32, 34]. The same conclusions hold for the neutrino counterparts.

B. Rank-3 CPT-odd NMC

We now examine a rank-3 dimension five nonminimal coupling in Lagrangian of the GSW model. It can be written as

\[
\mathcal{L}_{(3)}^{(odd)} = -g'_{3} \bar{Y}_{L} L_{i} \left( \gamma^{0} B_{\mu\nu} H_{\mu\nu} \right) L_{i} - g'_{3} \bar{Y}_{R} R_{i} \left( \gamma^{0} B_{\mu\nu} H_{\mu\nu} \right) R_{i},
\]

where \( H_{\mu\nu} \) is the LV background tensor, with the supposed symmetry \( H_{\mu\nu} = -H_{\mu\nu}, \) and \( Y_{L} = -1, Y_{R} = -2, \) so that

\[
\mathcal{L}_{(3)}^{(odd)} = g'_{3} \bar{L}_{i} \left( \gamma^{0} B_{\mu\nu} H_{\mu\nu} \right) L_{i} + g'_{3} \bar{R}_{i} \left( \gamma^{0} B_{\mu\nu} H_{\mu\nu} \right) R_{i},
\]

This EW Lagrangian can be written in terms of the lepton and neutrino pieces, \( \mathcal{L}_{(3)}^{(odd)} = \mathcal{L}_{(3)}^{(odd)} + \mathcal{L}_{(3)}^{(odd)}, \) given as

\[
\mathcal{L}_{(3)\mu}^{(odd)} = g'_{3} \bar{\psi}_{i} \left[ 3 \gamma^{0} \gamma_{5} B_{\mu\nu} H_{\mu\nu} \right] \psi_{i},
\]

\[
\mathcal{L}_{(3)\nu}^{(odd)} = g'_{3} \bar{\psi}_{i} \left[ \gamma^{0} \gamma_{5} B_{\mu\nu} H_{\mu\nu} - \gamma^{0} \gamma_{5} B_{\mu\nu} H_{\mu\nu} \right] \psi_{i},
\]

which are undetectable (at first order) in accordance with the Schiff’s theorem [4, 33]. This occurs with the Hamiltonian interactions stemming from Eq. (14),

\[
\psi_{i} \Sigma^{i} E^{i} C_{0} \psi_{i}, \quad \psi_{i} \Sigma^{i} Z_{0} C_{0} \psi_{i},
\]

implying absence of EDM physics. Here, we have used

\[
\sigma^{0j} = i \alpha^{j}, \quad \sigma^{ij} = \epsilon_{ijk} \Sigma^{k},
\]

\[
\alpha_{i}^{k} \gamma_{5} = \Sigma^{k}, \quad \alpha_{i}^{l} = \Sigma^{j} \gamma_{5}, \quad F_{ij} = E^{j}, \quad F_{mn} = \epsilon_{mnp} B_{p}, \quad Z_{ij} = E^{j}, \quad Z_{mn} = \epsilon_{mnp} B_{p}.
\]
TABLE II: Classification under $C, P, T$ for the CPT-odd rank-3 nonminimal couplings of Lagrangian (26).

| Coupling | $g_3^2 H_{00\bar{0}}$ | $g_4^2 H_{00\bar{1}}$ | $g_4^2 H_{01\bar{0}}$ | $g_4^2 H_{01\bar{1}}$ |
|----------|----------------------|----------------------|----------------------|----------------------|
| P        | $+$                   | $+$                   | $+$                   | $+$                   |
| C        | $-$                   | $+$                   | $+$                   | $+$                   |
| T        | $+$                   | $+$                   | $+$                   | $+$                   |

where we have introduced the rank-3 background $(H_A)_{\mu\alpha\beta}$ for the coupling involving $\gamma_5$, as we have done in Eq. (12). In order to verify the possibility of EDM generation for leptons, we investigate the tensor structure of lepton Lagrangian (24) that can expressed as

$$L^\text{(odd)}_{(3)} = g_2^2 \left[ \bar{\psi}_l \gamma^0 B_{00\bar{0}} \psi_l + \bar{\psi}_l \gamma^0 B_{00\bar{1}} \psi_l + \bar{\psi}_l \gamma^0 B_{01\bar{0}} \psi_l + \bar{\psi}_l \gamma^0 B_{01\bar{1}} \psi_l \right] \tag{26}$$

In Eq. (26), we see that the term, $\bar{\psi}_l \gamma^0 \gamma_5 B_{0\bar{0}} (H_A)_{\mu\bar{0}\bar{0}} \psi_l$, is the unique that has EDM signature, as shown in Table II. This piece can be written as

$$\bar{\psi}_l \gamma^0 \gamma_5 \Sigma_i B_{0\bar{0}} (H_A)_{\mu\bar{0}\bar{0}} \psi_l, \tag{27}$$

in which $B_{0\bar{0}}$ contains the electric and weak electric counterparts. Analogously to the rank-1 CPT-odd NM coupling, the presence of the $\gamma^0$ avoids the EDM behavior. As it occurs for the rank-1 case, the Table (II) shows that the couplings of Lagrangian (26) do not possess MDM behavior.

There are another possibilities of writing (hermitian) rank-3 nonminimal couplings. An example is

$$L^\text{(odd)}_{(3)i} = g_3^2 \bar{\psi}_i \gamma^\alpha B_{\bar{\alpha} \nu} \psi_l H_{\nu\alpha\beta} + g_3^2 \bar{\psi}_i \gamma^\alpha B_{\bar{\alpha} \nu} - \gamma^\beta B_{\bar{\alpha} \nu} \gamma_5 \psi_l (H_A)_{\mu\alpha\beta}. \tag{28}$$

These couplings do not generate EDM behavior, do not possess MDM correct signature, and will be no longer examined.

IV. CPT-EVEN DIMENSION FIVE NONMINIMAL LV ELECTROWEAK COUPLINGS

In this section, we analyze CPT-even dimension five nonminimal couplings composed of rank-2 and rank-4 tensors, which generate EDM and MDM behavior.

A. Rank–2 nonminimal coupling

The EDM Lagrangian terms should have the form presented in Eq. (1). Initially, the idea could be to propose a form written in terms of a covariant derivative into the interaction Lagrangian (9). In the hermitian form, we first propose a non axial (without $\gamma_5$) modified covariant derivative,

$$D_\mu = D_\mu - \frac{i}{2} \lambda_1 \left( T_{\mu\nu} B^{\nu\beta} - T^{\beta\nu} B_{\mu\nu} \right) \gamma_5, \tag{29}$$

based on the pattern first analyzed in Ref. [32]. Replacing this covariant derivative in the EW quiral Lagrangian structure for left-handed leptons, $\bar{L} \gamma^\mu i D_\mu L$, we obtain

$$\mathcal{L} = \bar{L} \gamma^\mu i \left[ -\frac{i}{2} \lambda_1 \left( T_{\mu\nu} B^{\nu\beta} - T^{\beta\nu} B_{\mu\nu} \right) \gamma_5 \right] L, \tag{30}$$

Using the identity, $\gamma^\mu \gamma_5 = (\delta^\mu_\beta - i \sigma^{\mu\beta})$, it becomes

$$\mathcal{L} = -i \lambda_1 \bar{L} \left[ \sigma^{\mu\beta} \left( T_{\mu\nu} B^{\nu\beta} \right) \right] L, \tag{31}$$

where it was neglected a term of the form $\bar{L} \left[ \left( T_{\beta\mu} B^{\mu\beta} \right) \right] L$, since it does not contain any gamma matrices nor spin components. Now it is necessary to remark that this nonminimal coupling is not properly communicated to the Lagrangian pieces of leptons and neutrinos. Indeed, we notice that

$$\left( \frac{1 \pm \gamma_5}{2} \right) X \left( \frac{1 \mp \gamma_5}{2} \right) = 0, \tag{32}$$

if the operator $X$ contains an even number of gamma matrices, which includes $X = \sigma^{\mu\beta}$ as a special case. Otherwise, if the operator $X$ possesses an odd number of gamma matrices, the quantity in Eq. (32) is not null, in principle. Thus, Lagrangian (31) yields a null contribution; the same holds for the right-handed fermions:

$$\bar{L} \left[ \sigma^{\mu\beta} \left( T_{\mu\nu} B^{\nu\beta} \right) \right] L = \bar{R} \left[ \sigma^{\mu\beta} \left( T_{\mu\nu} B^{\nu\beta} \right) \right] R = 0. \tag{33}$$

In order to circumvent this difficulty, we can propose $U(1)$ CPT-even NM couplings directly on the neutrino and lepton Lagrangian spinors:

$$L_{(2)\mu}^{\text{(even)}} = \lambda_{\mu} \bar{\psi}_l \left[ \sigma^{\mu\beta} T_{\mu\nu} B^{\nu\beta} - i \sigma^{\mu\beta} \gamma_5 R_{\mu\nu} B^{\nu\beta} \right] \psi_l, \tag{34}$$

$$L_{(2)\nu}^{\text{(even)}} = \lambda_{\nu} \bar{\psi}_l \left[ \sigma^{\mu\beta} T_{\mu\nu} B^{\nu\beta} - i \sigma^{\mu\beta} \gamma_5 R_{\mu\nu} B^{\nu\beta} \right] \psi_l. \tag{35}$$

where the imaginary factor was introduced with the matrix $\gamma_5$ in order to assure hermiticity. The leptons NM couplings in Eq. (34) exhibit a “non axial” (without $\gamma_5$) and an “axial” (with $\gamma_5$) interaction piece:

$$L_{(2)\mu(T)}^{\text{(even)}} = \lambda_{\mu} \bar{\psi}_l \left( \sigma^{\mu\beta} T_{\mu\nu} B^{\nu\beta} \right) \psi_l, \tag{36}$$

$$L_{(2)\mu(A)}^{\text{(even)}} = i \lambda_{\mu} \bar{\psi}_l \left( \sigma^{\mu\beta} \gamma_5 R_{\mu\nu} B^{\nu\beta} \right) \psi_l. \tag{37}$$
where the label \((T)\) refers to the tensor \(T_{\mu\nu}\) and the label \((A)\) refers to the “axial” tensor \(\gamma_5 R_{\mu\nu}\) coupling. Such couplings are represented by two distinct tensors, \(T_{\mu\nu}\) and \(R_{\mu\nu}\), to stress that the interactions with and without \(\gamma_5\) are physically different.

The lepton first piece can be explicitly written as

\[
\mathcal{L}^l_{(T)EDM} = \lambda_l \cos \theta \tilde{\psi}_i \left[ \lambda_l \epsilon_{\alpha l \epsilon_{a l}} T_{00} B^\alpha \right] \psi_i - i \lambda_l \epsilon_{\alpha l \epsilon_{a l}} T_{0 a} \alpha T_{00} B^\alpha \\
+ \lambda_l T_{i i} \Sigma^k B^k - \lambda_l T_{i k} \Sigma^k B^k \psi_i - \sin \theta \tilde{\psi}_i \left[ i \lambda_l \epsilon_{\alpha l \epsilon_{a l}} T_{00} B^\alpha \right] \psi_i + i \lambda_l \epsilon_{\alpha l \epsilon_{a l}} T_{0 a} \alpha T_{00} B^\alpha \\
+ \lambda_l T_{j i} \Sigma^k B^k + \lambda_l T_{i k} \Sigma^k \tilde{B}^k - \lambda_l T_{i k} \Sigma^k \tilde{B}^k \psi_i, \tag{38}
\]

where we have used the conventions \((16), (17), (18)\) and \((19)\). Such an expression provides “rotated” EDM and weak EDM contributions:

\[
\mathcal{L}^l_{(T)EDM} = \lambda_l \cos \theta \tilde{\psi}_i \left[ T_{j k} \Sigma^k E^j \right] \psi_i, \tag{39}
\]
\[
\mathcal{L}^l_{(W)EDM} = - \lambda_l \sin \theta \tilde{\psi}_i \left[ T_{j k} \Sigma^k \tilde{E}^j \right] \psi_i, \tag{40}
\]

having as counterpart the following Hamiltonian contributions:

\[
\mathcal{H}^l_{EDM} = - \lambda_l \cos \theta \tilde{\psi}_i \left[ T_{j k} \Sigma^k E^j \right] \psi_i, \tag{41}
\]
\[
\mathcal{H}^l_{WEDM} = \lambda_l \sin \theta \tilde{\psi}_i \left[ T_{j k} \Sigma^k \tilde{E}^j \right] \psi_i, \tag{42}
\]

with the \(\gamma^0\) factor circumventing the Schiff theorem \([33]\) and assuring the effective EDM character. The EDM signature is also revealed by the behavior of these couplings under CPT operations, as shown in Table (IV). Here, \(T_{j k}\) is a “rotated” background redefined as

\[
T_{j k} = \epsilon_{j k} T_{00}, \tag{43}
\]

that allows to write the interactions in a more direct way.

In expression \((38)\), we also identify MDM and weak MDM (WMDM) interactions for leptons associated with the Lagrangian terms:

\[
\mathcal{L}^l_{(T)(MDM)} = \lambda_l \left( \cos \theta \right) \tilde{\psi}_i \left[ T \Sigma^k B^k \right] \psi_i - \tilde{\psi}_i \left[ T \Sigma^k B^k \right] \psi_i, \tag{44}
\]
\[
\mathcal{L}^l_{(W)(MDM)} = \lambda_l \left( \sin \theta \right) \tilde{\psi}_i \left[ T \Sigma^k \tilde{B}^k \right] \psi_i + \tilde{\psi}_i \left[ T \Sigma^k \tilde{B}^k \right] \psi_i, \tag{45}
\]

where \(T = T_{i i} = T_{00}\) is the trace of space sector of the tensor \(T_{\mu\nu}\). Analogously, we can perform the same analysis for the second lepton piece \((37)\), whose tensor structure is

\[
\mathcal{L}^l_{(A)} = \cos \theta \tilde{\psi}_i \left[ \lambda_l R_{00} \Sigma^i E^i + \lambda_l R_{i k} \Sigma^i B^k - \lambda_l R_{i j} \Sigma^i \tilde{E}^j \right] \psi_i - i \lambda_l \epsilon_{i j k} \epsilon_{i j k} R_{i k} \Sigma^i B^k \\
- i \lambda_l \epsilon_{i j k} \epsilon_{i j k} R_{i j} \Sigma^i \tilde{E}^j \psi_i + \sin \theta \tilde{\psi}_i \left[ \lambda_l R_{00} \Sigma^i \tilde{E}^j + \lambda_l R_{i k} \Sigma^i \tilde{B}^k - \lambda_l R_{i j} \Sigma^i \tilde{E}^j \right] \psi_i - i \lambda_l \epsilon_{i j k} \epsilon_{i j k} R_{i j} \Sigma^i \tilde{B}^k \\
- i \lambda_l \epsilon_{i j k} \epsilon_{i j k} R_{i j} \Sigma^i \tilde{B}^k \psi_i, \tag{46}
\]

| Coupling | \(\lambda_l T\) | \(\lambda_l R_{00}\) | \(\lambda_l R_{i j}\) |
|----------|----------------|----------------|----------------|
| P        | +              | -              | -              |
| C        | +              | +              | +              |
| T        | +              | -              | -              |

| Coupling | \(\lambda_l T\) | \(\lambda_l R_{00}\) | \(\lambda_l R_{i j}\) | \(\lambda_l R_{i j}\) |
|----------|----------------|----------------|----------------|----------------|
| P        | +              | -              | -              | +              |
| C        | +              | +              | +              | +              |
| T        | +              | -              | -              | -              |

TABLE III: EDM, WEDM, MDM and WMDM contributions to the hamiltonian of the lepton nonminimal coupling \((34)\).

where we have used the relations \((16), (17), (18), (19)\), and \(R_{j k} = \epsilon_{j k} R_{00}\). Such an expression provides two direct Lorentz-violating EDM contributions for leptons:

\[
\mathcal{L}^l_{(A)(EDM)} = \lambda_l \cos \theta \tilde{\psi}_i \left( R_{00} \Sigma^i E^i \right) \psi_i - \tilde{\psi}_i \left( R_{00} \Sigma^i E^i \right) \psi_i \tag{47}
\]

and two direct Lorentz-violating weak EDM (weak dipole moment) pieces:

\[
\mathcal{L}^l_{(A)(WEDM)} = \lambda_l \sin \theta \left[ - \tilde{\psi}_i \left( R_{00} \Sigma^i \tilde{E}^i \right) \psi_i + \tilde{\psi}_i \left( R_{00} \Sigma^i \tilde{E}^i \right) \psi_i \right]. \tag{48}
\]

There are rotated lepton MDM and weak MDM contributions as well:

\[
\mathcal{L}^l_{(A)(MDM)} = \lambda_l \cos \theta \tilde{\psi}_i \left( R_{i k} \Sigma^i B^k \right) \psi_i, \tag{49}
\]
\[
\mathcal{L}^l_{(A)(WMDM)} = \lambda_l \sin \theta \tilde{\psi}_i \left( R_{i k} \Sigma^i \tilde{B}^k \right) \psi_i. \tag{50}
\]

All these terms are shown in Table (III), which contains the EDM, WEDM, MDM and WMDM contributions to the Hamiltonian of the lepton NM coupling in Eq. \((34)\).

The tau lepton data can be used to constrain the lepton weak EDM and weak MDM couplings of Table (III). Using the upper bound for the tau lepton WEDM \([49]\), the element \((40)\) leads to \(|\lambda_l \left( \sin \theta \right) T_{j k}| < 1.2 \times 10^{-17} \text{e} \cdot \text{cm}\), that is

\[
|\lambda_l T_{j k}| < 1 \times 10^{-4} \text{(GeV)}^{-1}, \tag{51}
\]
where we used $\sin \theta = 0.48$. Having in mind the definition $T_{ij} = T^{0a}_{ei} a_{ij}$, obviously the tensor has no isotropic component, $T_{ii} = 0$. The component $T_{jk}$ to be constrained depends on the direction of the electric field. In an apparatus the electric field points along the z-axis, the components to be restrained are $T_{13}, T_{23}$. The constraining procedure can be applied on the other WEDM pieces of Eq. (48), implying the upper bounds:

$$|\lambda_e R_{00}| < 1 \times 10^{-4} \text{(GeV)}^{-1},$$

$$|\lambda_e R_{ij}| < 1 \times 10^{-4} \text{(GeV)}^{-1}. \quad (52)$$

Tan WMDM experimental upper bounds [49] can also be used to constrain the tensor components of Table (III), $\alpha_w < 1 \times 10^{-3}$, or

$$\frac{e}{2m_e} \frac{\alpha_w}{\sin 2\theta} < 3 \times 10^{-5} \text{(GeV)}^{-1}, \quad (54)$$

which is the factor that bounds the WDM coefficients of Lagrangian (38) and (46). For the isotropic component, we write $\sin \theta |\lambda_e T_i| < 3 \times 10^{-5} \text{(GeV)}^{-1}$, or

$$|\lambda_e T_i| < 6 \times 10^{-5} \text{(GeV)}^{-1}. \quad (55)$$

The same holds for $|\lambda_e T_{ij}|$ and $|\lambda_e R_{ij}|$, as it appears in Table (VI).

In order to constrain the EDM couplings, we should use the electron EDM measurements, which represent the smallest EDM limit ever established, $d_e < 1.1 \times 10^{-31} e \cdot m$ [10]. For the isotropic component, $|\lambda_e (\cos \theta) R_{00}| < 1 \times 10^{-31} e \cdot m$,

$$|\lambda_e R_{00}| < 5 \times 10^{-17} \text{(GeV)}^{-1}, \quad (56)$$

where we have used $\cos \theta = 0.88$, the same holding for the other components $|\lambda_e T_{jk}|, |\lambda_e R_{ij}|$.

Concerning the MDM interaction,

$$\mathcal{L} = \bar{\psi} \left[ g \frac{e}{2m} \sigma^{\mu\nu} F_{\mu\nu} \right] \psi, \quad (57)$$

we can use the electron data to constrain it. The electron’s magnetic moment is $\mu = -g \mu_B S$, with $\mu_B = e/2m$ being the Bohr magneton and $g = 2(1 + a)$ being the gyromagnetic factor, with $a = \alpha/2\pi \approx 0.00116$ representing the deviation from the usual case, $g = 2$. The magnetic interaction is $H' = -\mu_B g (S \cdot B)$. Precise measurements reveal that the experimental imprecision on the electron EDM is at the level of 2.8 parts in $10^{13} [39]$, that is, $\Delta a \leq 2.8 \times 10^{-13}$. This value represents the window for new contributions that stem from dimension five terms, in such a way that $\lambda_e T \cos \theta < \mu_B \Delta a = 2.4 \times 10^{-20} \text{(eV)}^{-1}$, implying

$$\lambda_e T < 3 \times 10^{-11} \text{(GeV)}^{-1}. \quad (58)$$

We thus observe that the electron EDM data imply better couplings than the e-EDM data by a factor $10^3$, while the tau-WEDM and WMDM imply constraints of similar magnitude, which is explained by the large tau mass, which yields a much smaller Bohr magneton. The upper bounds obtained are presented in Tables (V) and (VI).

The same kind of analysis holds for the neutrino NM coupling contained in Lagrangian term (35), which can be analogously separated into two pieces,

$$\mathcal{L}_T^\nu = \lambda_\nu \bar{\nu}_{\nu} \sigma^{\mu\nu} \left( T^{\mu\nu}_{\nu} B^\nu_{\nu} \right) \psi_{\nu}, \quad (59)$$

$$\mathcal{L}_A^\nu = -i \lambda_\nu \bar{\nu}_{\nu} \left( \sigma^{\mu\nu} \gamma_5 R_{\mu\nu} B^\nu_{\nu} \right) \psi_{\nu}. \quad (60)$$

Due to the similar structure between the lepton and neutrino NM couplings in Eqs. (34) and (35), the EDM, WEDM, MDM and WMDM Lagrangian contributions for neutrinos are, in principle, the same ones of Table (III), only by replacing $\psi_i \rightarrow \nu_{\nu}$ and $\bar{\psi}_i \rightarrow \bar{\nu}_{\nu}$.

### B. Rank–4 dimension five nonminimal LV electroweak couplings

In this section, we introduce, directly on the GSW model Lagrangian, the rank–4 dimension five nonminimal LV couplings:

$$\mathcal{L}^{(even)}_{(4)} = \frac{\lambda_{ijkl}}{2} \bar{\psi}_i \left[ \sigma^{\mu\nu} K_{\mu\nu\alpha\beta} B^{\alpha\beta} + i \sigma^{\mu\nu} \gamma_5 \tilde{K}_{\mu\nu\alpha\beta} B^{\alpha\beta} \right] \psi_j$$

$$+ \frac{\lambda_{ijkl}}{2} \bar{\psi}_{\nu i} \left[ \sigma^{\mu\nu} K_{\mu\nu\alpha\beta} B^{\alpha\beta} + i \sigma^{\mu\nu} \gamma_5 \tilde{K}_{\mu\nu\alpha\beta} B^{\alpha\beta} \right] \psi_{\nu j}, \quad (61)$$

$\mathcal{L}^{(even)}_{(4)}$
where the rank-4 background tensors $K_{\mu\nu\alpha\beta}$, $\tilde{K}_{\mu\nu\alpha\beta}$ are antisymmetric in the two pairs:

$$K_{\mu\nu\alpha\beta} = -K_{\nu\mu\alpha\beta}, \quad (63)$$

$$\tilde{K}_{\mu\nu\alpha\beta} = -K_{\nu\mu\beta\alpha}. \quad (64)$$

Supposing $T_{\nu\beta} = (K)^\alpha_{\nu\alpha\beta}$ and $R_{\nu\beta} = (\tilde{K})^\alpha_{\nu\alpha\beta}$, one can propose the prescription,

$$(K)_{\mu\nu\alpha\beta} = \frac{1}{2} (g_{\mu\alpha} T_{\nu\beta} - g_{\mu\beta} T_{\nu\alpha} + g_{\nu\beta} T_{\mu\alpha} - g_{\nu\alpha} T_{\mu\beta}), \quad (65)$$

$$(\tilde{K})_{\mu\nu\alpha\beta} = \frac{1}{2} (g_{\mu\alpha} R_{\nu\beta} - g_{\mu\beta} R_{\nu\alpha} + g_{\nu\beta} R_{\mu\alpha} - g_{\nu\alpha} R_{\mu\beta}), \quad (66)$$

where the tensors $T_{\nu\beta}, R_{\nu\beta}$ are now symmetric and traceless. Replacing such a prescription in the lepton sector of the Lagrangian (62), we obtain:

$$L^{(even)}_{(4)} = \frac{\lambda_l}{2} \bar{\psi}_l [\sigma^{\mu\nu} T_{\nu\beta} B^\beta_\alpha + i \sigma^{\alpha\nu} g_{\beta\alpha} R_{\nu\beta} B^\beta_\alpha] \psi_l. \quad (67)$$

These couplings recover the ones involving ranking-2 tensors, already presented. Thus, if the rank-4 tensor is written as shown in expression (65), the upper bounds found in the last section hold equivalently for some components of $(K)_{\mu\nu\alpha\beta}$. For instance, $T_{00} = - (K)_{00\alpha\beta}$ and $T_{ij} = (K)_{00\alpha\beta} - (K)_{0\alpha\beta}$, so that the WEDM upper limits (52) and (53) are read as:

$$|\lambda_\tau (K)_{00\alpha}| < 1 \times 10^{-4} \text{ (GeV)}^{-1}, \quad (68)$$

$$\lambda_\tau |(K)_{0\alpha\beta} - (K)_{0\alpha\beta}| < 1 \times 10^{-4} \text{ (GeV)}^{-1}. \quad (69)$$

V. CONCLUSION AND FINAL REMARKS

We analyzed dimension five LV nonminimal couplings in the EW sector. The CPT-odd ones are not effective in generating EDM or MDM contributions, both in the rank-1 and rank-3 forms. Such impossibility is confirmed by the EDM-incompatible signature under C,P and T operators, as shown in Table (I). We also examined CPT-even nonminimal electroweak couplings, which generate tree level EDM, MDM, WEDM and WMDM contributions. We firstly have introduced rank-2 dimension five nonminimal couplings directly in the GSW model Lagrangian, using two rank-2 background tensors, $T_{\mu\nu}$ and $R_{\mu\nu}$, as presented in Lagrangians (36) and (37). We have identified the coefficients that generate EDM, MDM, WEDM and WMDM lepton contribution to the Hamiltonian. Then, we used experimental data of tau lepton to constrain the WEDM and WMDM couplings to the level of $10^{-4} \text{ (GeV)}^{-1}$, and electron MDM and EDM data to constrain EDM and MDM couplings to the level of $10^{-17} \text{ (GeV)}^{-1}$ and $10^{-11} \text{ (GeV)}^{-1}$, respectively. These upper bounds are shown in Tables (V) and (VI). We have also proposed CPT-even nonminimal EW couplings involving a rank-4 background tensor, $K_{\mu\nu\alpha\beta}$, coupled to the $U(1)$ field strength and the leptons' (neutrinos') spinors. Using a suitable relation, Eq. (66), we showed that some rank-4 couplings become equivalent to rank-2 couplings. Thus, the rank-4 nonminimal couplings that generate EDM, MDM, WEDM and WMDM are bounded to the same level of constraining presented in Tables (V) and (VI).

VI. ACKNOWLEDGEMENTS

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