One-loop predictions
for the pion VFF
in resonance chiral theory

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[Forthcoming; in collaboration with A. Pich and I. Rosell]
Outline:

- Framework for the $\pi\pi$-VFF:
  - Resonance chiral theory and $1/N_c$ expansion
- $\text{Im} F(s)$ at $s \to \infty$
- Full $F(s)$ at $s \to \infty$
- $F(s)$ at low energies
- Numerical estimates and conclusions
One-loop predictions for the $\pi\pi$-VFF in $R\chi T$

$\pi\pi$-VFF,

$R\chi T$

and the $1/N_C$ expansion
The amplitude:
\( \pi\pi \) vector form-factor

Hadronic interaction theory?
Resonance loops?

\[
\langle \pi^+\pi^- | \frac{1}{2} \bar{u} \gamma^\mu u - \frac{1}{2} \bar{d} \gamma^\mu d | 0 \rangle = (p_{\pi^+} - p_{\pi^-})^\mu F(q^2)
\]

with \( q = p_{\pi^+} + p_{\pi^-} \)

- Very good measurements
- Well dominated by the vector

\( \rightarrow \) Test for our hadronic theory
Ingredients of a chiral theory for resonances (RχT)

- Large $N_c \rightarrow U(n_f)$ multiplets
- $1/N_c$ suppression of meson loops
- Goldstones from $S\chi SB (\pi, K, \eta_8, \eta_1)$
- SRA: First resonance multiplets $(V, A, S, P)$
- Chiral symmetry invariance
- Just $O(p^2)$ operators $\Leftrightarrow$ High energy conditions $[\text{Trnka,SC'09}]$
  $\Leftrightarrow$ Field Redefinitions $[\text{Xiao,SC'07}]$
No restriction on the number of R fields: $\mathcal{L}_{R \times T} = \mathcal{L}_{GB} + \sum_R \mathcal{L}_R + \sum_{R,R'} \mathcal{L}_{RR'} + \sum_{R,R',R''} \mathcal{L}_{RR'R''} + \ldots$

The only relevant ones at large $N_C$ (tree-level LO):

$$\mathcal{L}_{GB}^\pi = \frac{F^2}{4} \langle u^\mu u_\mu \rangle + \chi^+$$

[Weinberg’79]

[ Gasser & Leutwyler’84]

[ Gasser & Leutwyler’85]

$$\mathcal{L}_V = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f^{\mu\nu}_+ \rangle + \frac{i G_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle$$

$$\mathcal{L}_A = \frac{F_A}{2\sqrt{2}} \langle A_{\mu\nu} f^{\mu\nu}_- \rangle$$

$$\mathcal{L}_S = c_d \langle S u_\mu u^\mu \rangle$$

[ Ecker et al.’89]
\[ \mathcal{L}_{VA} = i\lambda_2^{VA} \langle [V_{\mu}, A_{\nu\alpha}] h_{\mu}^{\alpha} \rangle + i\lambda_3^{VA} \langle [\nabla_{\mu} V_{\nu\alpha}, A_{\nu\alpha}] u_{\alpha} \rangle 
+ i\lambda_4^{VA} \langle [\nabla_{\alpha} V_{\mu\nu}, A_{\alpha\nu}] u_{\mu} \rangle + i\lambda_5^{VA} \langle [\nabla_{\alpha} V_{\mu\nu}, A_{\mu\nu}] u_{\alpha} \rangle \]

\[ \mathcal{L}_{SA} = \lambda_1^{SA} \langle \{\nabla_{\mu} S, A_{\mu\nu}\} u_{\nu} \rangle \]

\[ \mathcal{L}_{PV} = i\lambda_1^{PV} \langle [\nabla_{\mu} P, V_{\mu\nu}] u_{\nu} \rangle \]

\[ \mathcal{L}_{SP} = \lambda_1^{SP} \langle u_{\alpha} \{\nabla_{\alpha} S, P\} \rangle \]

\[ \sum_{R,R'} \mathcal{L}_{RR'} + \sum_{R,R',R''} \mathcal{L}_{RR'R''} + ... \] 

\[ \rightarrow \text{We will neglect RR'} \text{ absorptive cuts} \]
We must build the $R\chi_T$ that best mimics QCD

- **AT LONG-DISTANCE**: Chiral symmetry invariance
  Ensures the right low-energy QCD structure ($\chi_T$), even at the loop level!
  
  [Catà, Peris’02]
  [Harada, Yamawaki’03]
  [Rosell, Pich, SC’04 ’06 ’08] …

- **AT SHORT-DISTANCE**: Demand to the theory the high-energy power behaviour prescribed by QCD
  
  [Bordsky, Lepage’79]

One-loop predictions for the $\pi\pi$-VFF in $R\chi_T$
One-loop predictions for the $\pi\pi$-VFF in $R\chi T$

$F(q^2) = 1 + \frac{F_V G_V}{F^2} \frac{q^2}{M_V^2 - q^2}$

$F_V G_V = F^2$

• This can be also understood from a Padé app. Point of view:

  ⇒ [0/1] Pade Type ($M_V$ fixed)  [Masujan,Peris,SC’08]

• It leads to the low-energy large-$N_C$ expression

  $\mathcal{F}(s) = 1 + \frac{2L_9 s}{F^2} + \mathcal{O}(s^2)$

with the LEC prediction $L_9^{N_C \to \infty} = \frac{F_V G_V}{2M_V^2} = \frac{F^2}{2M_V^2} \simeq 6.8 \cdot 10^{-3}$  [Ecker et al.’89]

To be compared to the experimental numbers for $10^3 \cdot L_9(M_\rho)$:

- $6.9 \pm 0.7$  [O($p^4$)$\chi$PT; Gasser,Leutwyler’85],
- $7.04 \pm 0.23$  [$\tau$-$R\chi$T; Pich,SC’03],
- $6.54 \pm 0.15$  [O($p^4$) $\tau$-SR; Gonzalez-Alonso et al.’09]
Developing $R\chi T$ beyond large $N_C$

One Loop Diagrams $\rightarrow$ NLO Contributions

However, Loops=UV Divergences!!

New NLO pieces (NLO couplings)?

Removable through EoM
and with proper short-distance

$\delta \theta, \delta z, \delta z', \delta z''$

$[\text{Ecker et al.'89}, \ldots]$

$[\text{Rosell, Pich, SC'04, '06}]$

$[\text{Catà, Peris'02}]$

$[\text{Rosell et al.'05}]$

$[\text{Rosell Pich, SC'04}]$

$[\text{Xiao, SC'07}]$

$[\text{Ruiz-Femenia et al.'05,'06}]$

$[\text{SC'07 } ]$

$[\text{Xiao, SC'07 } ]$

$[\text{Trnka, SC'09} ]$
One-loop predictions for the $\pi\pi$-VFF in $R\chi T$

UV divergences $\rightarrow$ NLO lagrangian

- Only Goldstones counterterms
  \[ \mathcal{L}_{\text{NLO}}^{\text{GB}} = -i \tilde{L}_9 \left( f_+^{\mu \nu} u_\mu u_\nu \right) \]

- Counterterms with resonance fields
  \[ \mathcal{L}_{\text{NLO}}^{\text{V}} = X_Z \left\{ V_{\lambda \nu} \nabla^\lambda \nabla_\rho \nabla^2 V^{\rho \nu} \right\} \]
  \[ + X_F \left\{ V_{\mu \nu} \nabla^2 f_+^{\mu \nu} \right\} \]
  \[ + 2i X_G \left\{ V_{\mu \nu} \nabla^2 [u_\mu u_\nu] \right\} \]
However,

**Redundant operators** (Proportional to EoM) \[ \text{[Rosell,Pich,SC'}04] \]

\[
\nabla^\mu u_\mu = \frac{i}{2} \chi^- + ...
\]

\[
\nabla^\mu \nabla_\rho V^{\rho\nu} - \nabla^\nu \nabla_\rho V^{\rho\mu} = -M_V^2 V^{\mu\nu} - \frac{F_V}{\sqrt{2}} f_{\mu\nu}^+ - \frac{iG_V}{\sqrt{2}} [u^\mu, u^\nu] + ...
\]

\[ \mathcal{I}_V^{\text{NLO}} \text{ operators removable through meson field redefinitions } \xi \]

Instead of the original set of couplings, the amplitude depends only on effective combinations

\[
\begin{align*}
X_{Z,F,G} & \xrightarrow{\xi} 0, \\
\tilde{L}_9 & \xrightarrow{\xi} \tilde{L}_9 + (\sqrt{2}X_F G_V + 2\sqrt{2} F_V X_G - X_Z F_V G_V),
\end{align*}
\]

\[
\begin{align*}
F_V & \xrightarrow{\xi} F_V + (2X_Z F_V M_V^2 - 2\sqrt{2}X_F M_V^2), \\
G_V & \xrightarrow{\xi} G_V + (2X_Z G_V M_V^2 - 4\sqrt{2}X_G M_V^2), \\
M_V^2 & \xrightarrow{\xi} M_V^2 + 2X_Z M_V^4.
\end{align*}
\]
•VFF up to NLO in \( 1/N_C \)

\[
F(s) = 1 + \frac{F_V G_V}{F^2} \frac{s}{M_V^2 - s} \; \text{LO tree-level}
\]

\[
+ \frac{2 \tilde{L}_9 s}{F^2} - 2 X_Z \frac{F_V G_V}{F^2} \frac{s^3}{(M_V^2 - s)^2} - 4 \sqrt{2} \frac{F_V X_G}{F^2} \frac{s^2}{M_V^2 - s} - 2 \sqrt{2} \frac{X_F G_V}{F^2} \frac{s^2}{M_V^2 - s} \; \text{NLO tree-level}
\]

\[
+ F(s)_{1 \text{PI}} + \frac{F_V G_V}{F^2} \frac{s \Sigma(s)}{(M_V^2 - s)^2} + \frac{F_V \Gamma(s)}{F^2} \frac{s}{M_V^2 - s} + \frac{\Phi(s) G_V}{F^2} \frac{s}{M_V^2 - s} \; \text{1-loop}
\]

\[ [ F_V, G_V, X_Z, X_F \ldots \; \text{renormalize every single Vertex-Function} ] \]

•VFF up to NLO in \( 1/N_C \) (AFTER EoM SIMPLIFICATION)

\[
F(s) = 1 + \frac{F_V^{\text{eff}} G_V^{\text{eff}}}{F^2} \frac{s}{M_V^2 - s} + \frac{2 \tilde{L}_9^{\text{eff}} s}{F^2} + F(s)^{1-\text{loop}}
\]

**MEANING:** \( F_V G_V, M_V^2 \) and \( L_9 \) are able to make \( F(s) \) finite

•From now on, we will always refer to the simplified lagrangian \( \Rightarrow \) "eff" superscript assumed

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**J. J. Sanz Cillero**

*One-loop predictions for the \( \pi\pi \)-VFF in \( R\chi T \)*
Step 1) VFF spectral function:
\[ \text{Im} F(s) \text{ at } s \to \infty \]
• The full VFF must the behaviour $F(s)\to 0$ when $s\to \infty$

• Similarly, for its spectral function $\text{Im}F(s)\to 0$ when $s\to \infty$

• We will demand this behaviour for every two-meson cut

$$\text{Im}F(s)|_{m_1,m_2}\to 0 \quad \text{when } s\to \infty$$

• The spectral function shows the generic form:

$$\text{Im}F(s) = s\left(\alpha_1^{(p)} + \alpha_1^{(\ell)} \ln \frac{-s}{M^2}\right) + \left(\alpha_0^{(p)} + \alpha_0^{(\ell)} \ln \frac{-s}{M^2}\right) + \ldots$$

which requires the constraints

$$\alpha_1^{(p)} = \alpha_1^{(\ell)} = \alpha_0^{(p)} = \alpha_0^{(\ell)} = 0$$

[ Notice that at large $N_c$, $\alpha_k^{(p)}=\alpha_k^{(\ell)}=0$ trivially ]
One-loop predictions for the $\pi\pi$-VFF in $R\chi T$

$\pi\pi$ cut:

$$F_V G_V = F^2 \text{ [Ecker et al.'89]}$$

$$3G_V^2 + 2c_d^2 = F^2 \text{ [Guo,Zheng,SC'07]}$$

$$\left(0 \leq c_d^2 \leq F^2/2, \quad 0 \leq G_V^2 \leq F^2/3\right)$$

(Everything fixed in terms of $M_V,M_S,G_V$)

$P_\pi$ cut:

$$\lambda_{PV}^P = 0 \quad \Rightarrow \quad F(s)|_{P_\pi} = 0 \text{ trivially}$$
Complicate system $\Rightarrow$ Various solutions (6)

\(\Rightarrow (1^{\text{st}}) \text{ Trivial solution: } \lambda_3^{VA} - 2\lambda_2^{VA} = 0\)

\[2\lambda_2^{VA} - 2\lambda_3^{VA} + \lambda_4^{VA} + 2\lambda_5^{VA} = 0\]

\[\lambda_1^{SA} = 0\]

\[\text{Im}\mathcal{F}(s)|_{A\pi} = 0\]

\(\Rightarrow (2^{\text{nd}}) A\pi-\text{VFF constraint solution}\)

\[\lambda_3^{VA} - 2\lambda_2^{VA} = 0\]

\[2\lambda_2^{VA} - 2\lambda_3^{VA} + \lambda_4^{VA} + 2\lambda_5^{VA} = \frac{F_A}{F_V}\]

\[\lambda_1^{SA} = -\frac{F_AG_V (M_A^2 - 4M_V^2)}{3\sqrt{2}M_A^2 c_d F_V}\]

\[\text{Im}\mathcal{F}(s)|_{A\pi} \neq 0\]
Step 2) 

Full VFF asymptotic behaviour: 

\( F(s) \) at \( s \to \infty \)
After constraining \( \text{ImF(s)} \), the VFF has the structure:

\[
\mathcal{F}(s) = 1 + \frac{F_V G_V}{F^2} \frac{s}{M_V^2 - s} + \frac{2\tilde{L}_9 s}{F^2} + \mathcal{F}(s)^{1-\text{loop}}
\]

[Unique decomposition from dispersion relations] \[\text{Pich,Rosell,SC'08}\]

with the 1-loop structure \( \mathcal{F}(s)^{1-\text{loop}} = \overline{\mathcal{F}}(s)^{1-\text{loop}} + \frac{2s}{F^2} \hat{\delta}_1 + \hat{\delta}_0 \frac{s}{M_V^2 - s} + \hat{\delta}_{-1} \frac{s}{(M_V^2 - s)^2} \)

At high-energies, this results in:

\[
\mathcal{F}(s) = 1 + \left( \frac{F_V G_V}{F^2} + \hat{\delta}_0 \right) \frac{s}{M_V^2 - s} + \frac{2s}{F^2} \left( \tilde{L}_9 + \hat{\delta}_1 \right) + \overline{\mathcal{F}}(s)^{1-\text{loop}}
\]

At high-energies, this results

\[
\overline{\mathcal{F}}(s)^{1-\text{loop}} \overset{s \rightarrow \infty}{=} \hat{\delta}_0 + \mathcal{O}\left( s^{-1} \right)
\]

\[
\mathcal{F}(s) = \frac{2s}{F^2} \left( \tilde{L}_9 + \hat{\delta}_1 \right) + \left( 1 - \frac{F_V G_V}{F^2} - \hat{\delta}_0 + \hat{\delta}_0 \right) + \mathcal{O}\left( \frac{1}{s} \right)
\]
This leads to the NLO constraints:

\[
\tilde{L}_9 + \hat{\delta}_1 = 0
\]
\[
\frac{F_V G_V}{F^2} + \hat{\delta}_0 = 1 + \delta_0
\]

[Not really physical here; renorm.-scheme choice]

to be compared with their large-$N_C$ values

\[
\tilde{L}_9 = 0, \quad \frac{F_V G_V}{F^2} = 1
\]
• **\( \pi\pi \) contribution:**

After using the high-energy \( \text{Im} F(s)|_{\pi\pi} \) constraints \( \Rightarrow \delta_{0}^{\pi\pi} \approx 0.23 \)

\[ \Rightarrow \frac{F^2}{2M_V^2} \delta_{0}^{\pi\pi} \approx 1.5 \cdot 10^{-3} \]

• **\( P\pi \) contribution:**

From the \( \text{Im} F(s)|_{P\pi} \) constraints \( \Rightarrow \delta_{0}^{P\pi} = 0 \)

• **\( A\pi \) contribution:**

After using the high-energy \( \text{Im} F(s)|_{A\pi} \) constraints

\( \Rightarrow \) Complicate expression, but \( \Rightarrow \delta_{0}^{A\pi} \approx 0.14 \)

\[ \Rightarrow \frac{F^2}{2M_V^2} \delta_{0}^{A\pi} \approx 1.0 \cdot 10^{-3} \]
Low-energy expansion
At $s \rightarrow 0$, the RChT expression shows the structure,

\[
\mathcal{F}(s) = 1 + \frac{2s}{F^2} \left[ \tilde{L}_9 + \hat{\delta}_1 \right] + \frac{F^2}{2M_V^2} \left( \frac{F_V G_V}{F^2} + \hat{\delta}_0 \right) + \xi_{L_9} \]

\[
+ \frac{s}{F^2} \frac{G_9}{16\pi^2} \left( \frac{5}{3} - \ln \frac{-s}{\mu^2} \right) + \mathcal{O}(s^2)
\]

with the log coefficient $G_9 = \Gamma_9 = \frac{1}{4}$, matching ChPT.

This has the same form as ChPT,

\[
\mathcal{F}(s) = 1 + \frac{2L_9(\mu_\chi) s}{F^2} + \frac{s}{F^2} \frac{\Gamma_9}{16\pi^2} \left( \frac{5}{3} - \ln \frac{-s}{\mu_\chi^2} \right) + \mathcal{O}(s^2)
\]
• This leads to the prediction for the ChPT LECs,

\[
L_9(\mu_\chi) = \frac{F^2}{2M_V^2} \left( \frac{F_V G_V}{F^2} + \hat{\delta}_0 \right) + (\tilde{L}_9 + \hat{\delta}_1) + \xi_{L_9} + \frac{\Gamma_9}{32\pi^2} \ln \frac{\mu^2}{\mu_\chi^2}
\]

For instance, with only the $\pi\pi$ loops considered,

\[
\xi_{L_9} = \frac{c_3^2 \log \left( \frac{M^2}{\mu^2} \right)}{64\pi^2 F^2} - \frac{11c_3^2}{384\pi^2 F^2} + \frac{F_V G_V^3 \log \left( \frac{M^2}{\mu^2} \right)}{64\pi^2 F^4} - \frac{5F_V G_V^3}{192\pi^2 F^4} + \frac{G_V^2 \log \left( \frac{M_V^2}{\mu^2} \right)}{128\pi^2 F^2} + \frac{25G_V^2}{768\pi^2 F^2}
\]

• Using the high-energy constraints up to NLO \( \Rightarrow \)

\[
L_9(\mu) = \frac{F^2}{2M_V^2} \left( 1 + \hat{\delta}_0 \right) + \xi_{L_9}
\]

**TO NOTICE:** Exact recovery of the $\mu$ running dependence
• $\pi\pi$ contribution:

After using the high-energy $\text{Im}F(s)|_{\pi\pi}$ constraints $\Rightarrow \xi_{L9}^{\pi\pi} \approx -1.6 \cdot 10^{-3}$

• $P\pi$ contribution:

From the $\text{Im}F(s)|_{P\pi}$ constraints $\Rightarrow \xi_{L9}^{P\pi} = 0$

• $A\pi$ contribution:

After using the high-energy $\text{Im}F(s)|_{A\pi}$ constraints

$\Rightarrow$ Complicate expression, but $\xi_{L9}^{A\pi} \approx -0.1 \cdot 10^{-3}$
Numerical determinations

(PRELIMINARY)
• Inputs: $M_V = 0.76 \pm 0.02$ GeV, $M_S = 0.98 \pm 1.2$ GeV, $F = 89$ MeV, $G_V = F/\sqrt{3} = 40$ MeV,

$[A \pi$ channel $] M_A = 1.23 \pm 1.00$ GeV, $F_A = 123 \pm 89$ MeV

• At LO in $1/N_C$:

$L_9^{N_C \rightarrow \infty} = 6.7 \cdot 10^{-3}$

• NLO with $\pi\pi$:

$L_9(\mu_0) = (6.6 \pm 0.4) \cdot 10^{-3}$, with $\mu_0 = 0.77$ GeV

( =NLO with $\pi\pi + P_\pi$ )

• NLO with $\pi\pi + P_\pi + A_\pi$:

$L_9(\mu_0) = (7.5 \pm 0.5) \cdot 10^{-3}$, with $\mu_0 = 0.77$ GeV
Conclusions

and

PROSPECTS
• Clear theoretical organization of the NLO computation:
  - Loop contribution
  - Relevant physical couplings

• **Successful test** of the $1/N_C$ expansion in RChT

• Very slight problem with the $A_\pi$ channel ("high" range of $L_9^{\text{exp}}$ values)

• **Next step:**
  - Detailed uncertainty estimate
  - Extraction of the VFF $O(p^6)$ LECs
  - Analysis of experimental VFF data
One-loop predictions for the $\pi\pi$-VFF in $R\chi T$
\[ \frac{\mathcal{F}(t)}{t} = \frac{D(t)}{(M_V^2 - t)^2}, \]

\[ \frac{\mathcal{F}(s)}{s} = \frac{1}{2\pi i} \int \frac{\mathcal{F}(t)}{t(t - s)} \, dt. \]

\[ \frac{1}{s} \mathcal{F}(s) = \frac{1}{s} + \sum_{m_1, m_2} \frac{1}{s} \mathcal{F}(s)|_{m_1, m_2} - \frac{\text{Re} D'(M_V^2)}{M_V^2 - s} + \frac{\text{Re} D(M_V^2)}{(M_V^2 - s)^2}, \]

\[ \mathcal{F}(s)|_{m_1, m_2} = \lim_{\epsilon \to 0} \left[ \frac{s}{\pi} \int_0^{M_V^2 - \epsilon} \frac{\text{Im} \mathcal{F}(t)|_{m_1, m_2}}{t(t - s)} \, dt + \frac{s}{\pi} \int_{M_V^2 + \epsilon}^{\infty} \frac{\text{Im} \mathcal{F}(t)|_{m_1, m_2}}{t(t - s)} \, dt \right. \]

\[ \left. - \frac{2s}{\pi \epsilon} \lim_{t \to M_V^2} \left\{ (M_V^2 - t)^2 \frac{\text{Im} \mathcal{F}(t)|_{m_1, m_2}}{t(t - s)} \right\} \right]. \]

\[ \mathcal{F}(t) = 1 + \sum_{m_1, m_2} \mathcal{F}(t)|_{m_1, m_2} - \frac{s \text{Re} D'(M_V^2)}{M_V^2 - t} \]

\[ = 1 + \frac{F_V G_V}{F^2} \frac{s}{M_V^2 - s} + \sum_{m_1, m_2} \mathcal{F}(t)|_{m_1, m_2}, \]
• What about extra tadpole terms?

They all are real rational functions of the form of the $L_9$ (local) and $F_V G_V$ (pole) terms, i.e.,

$$\mathcal{F}(s)^\text{tad.} = \frac{2s}{F^2} \delta_1^{\text{tad.}} - \frac{\delta_0^{\text{tad.}}}{M_V^2 - s}$$

⇒ HOWEVER,

These couplings (or their combination with possible tadpole) will be fully fixed later through high-energy constraints

• Likewise, we will consider the on-shell mass scheme $\Rightarrow M_V=770$ MeV
• **Inputs:** \( M_V = 0.77 \text{ GeV} \), \( M_S = 0.98 \text{ GeV} \), \( M_A = 0.95 \pm 1.3 \text{ GeV} \)

\( F = 89 \text{ MeV} \), \( G_V = 45 \pm F/\sqrt{3} \)

• At LO in \( 1/N_C \):

\[
L_9^\infty = 6.7 \cdot 10^{-3}
\]

• NLO with \( \pi\pi \):

\[
L_9(\mu_0) = 6.6 \cdot 10^{-3}, \quad \text{with} \ \mu_0 = 0.77 \text{ GeV}
\]

( =NLO with \( \pi\pi + P\pi \) )

• NLO with \( \pi\pi + P\pi + A\pi \):

\[
L_9(\mu_0) = 7.5 \cdot 10^{-3}, \quad \text{with} \ \mu_0 = 0.77 \text{ GeV}
\]
• $\pi\pi$ contribution:

After using the high-energy $\text{Im}F(s)|_{\pi\pi}$ constraints \( \Rightarrow \delta_1^{\pi\pi} = \delta_0^{\pi\pi} = 0 \)

• $P_\pi$ contribution:

Similarly, from the $\text{Im}F(s)|_{P\pi}$ constraints \( \Rightarrow \delta_1^{P\pi} = \delta_0^{P\pi} = 0 \)

• $A_\pi$ contribution:

After using the high-energy $\text{Im}F(s)|_{A\pi}$ constraints

\( \Rightarrow \text{Complicate expression, but } \delta_1^{A\pi}, \delta_0^{\pi\pi} \neq 0 \)
Vector Form Factor to $A_\pi$ (Figure D.2)

$$
\langle A_{I=1}^0(p_A, \varepsilon)\pi^-(p_\pi)|\bar{d}\gamma^\mu u|0\rangle = \frac{i\sqrt{2}}{M_A} \left\{ \left(q\varepsilon^* p_A^\mu - q p_\pi \varepsilon^{*\mu} \right) F_{A\pi}^v(q^2) \\
+ \left(q\varepsilon^* p_\pi^\mu - q p_\pi \varepsilon^{*\mu} \right) G_{A\pi}^v(q^2) \right\},
$$

$$
F_{A\pi}^v(q^2) = \frac{F_A}{F} + \frac{F_V}{F} \frac{M_A^2 - q^2}{M_V^2 - q^2} \left[ - 2\lambda_2^V - 2\lambda_3^V - \lambda_4^V - 2\lambda_5^V \right],
$$

$$
G_{A\pi}^v(q^2) = \frac{2F_V}{F} \frac{M_A^2}{M_V^2 - q^2} \left[ - 2\lambda_2^V + \lambda_3^V \right],
$$
One-loop predictions for the $\pi\pi$-VFF in $\chi T$