Outlier Eliminating Method of Gyro Array Data in Dynamic and Non-linear Condition

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Abstract. Outliers in gyro array system can affect the accuracy and even cause the filtering unstable. Simply diagnosing outliers as failures may lead gyro combination to switch frequently. The current methods have bad performance in the non-linear condition and the patch-type outliers. A method of outlier elimination for gyro array is proposed for the questions above. The method constructs parity vectors by singular value decomposition method to solve the detection function values of gyro array. The relationship between the detection function value and the outlier vector is deduced. The detection function value is checked by the Grubbs rule to mark out the outliers. The simulation results show that the method can eliminate outliers in both static and dynamic non-linear situation, and effectively improve the accuracy of that field.

1. Introduction

MEMS redundant inertial measurement unit (RIMU) is composed of MEMS accelerometers, MEMS gyros and integrated circuit. Through the data fusion algorithm to obtain the final measurement results, the measurement accuracy and stability of MEMS RIMU are improved. Researchers have done a lot of work in this area [1~3]. The measurement accuracy and stability of MEMS IMU are poor, where the outlier problem is serious [4, 5]. The outliers in the measurement data include isolated outliers and outlier patches [6]. The outliers in gyro measurement data will lead to serious pathological faults and divergence of filtering algorithms. But regarding outlier as faults directly may lead to the loss of available measurement information, high false alarm rate, frequent switching of gyro combination mode and breakpoint of system output data.

At present, the common methods to deal these problems are divided into the following three categories [7]: First, detecting outliers according to the statistical characteristics of measurement data [6]; the second is to detect outliers by information processing and feature extraction methods [8, 9]; and the third method is according to the statistical characteristics of error [10, 11]. The first type does not use all the information of the observation value, and the samples are small because of the real-time requirements of aircraft. So these methods are not reliable, and the false alarm rate is relative high. The second type is more complex, and the computation is large, which is not suitable for the real-time online work. The third method may have problems of miss-eliminating and false alarm, because most of the filtering has a bad accuracy in the estimation of nonlinear conditions.

In this paper, the singular value decomposition method is used to monitor the measurement data of the gyros in a MEMS RIMU. The detection function is presented. The new statistics is designed to detect the outliers with small sample. The problems of traditional outlier eliminating algorithm in the dynamic nonlinear situation are improved.
2. Detection Function Based on SVD

2.1. Mathematical Model of Gyro Array

The observation equation of a gyro array system, which composed of n gyros under normal working condition, is given as follow [12]:

\[ Z = Hx + \varepsilon \] (1)

Among them, Z is the n-dimensional system measurement value vector, \( H_{n \times 3} \) is the gyro array geometric configuration matrix, \( \varepsilon \) is the n-dimensional White noise vector, which statistical characteristics meet:

\[ E[\varepsilon] = 0, E[\varepsilon\varepsilon^T] = \sigma^2 I_n \] (2)

When there are outliers in the measurement data of the gyro array, the measurement equation can be summed up as

\[ Z = Hx + b + \varepsilon = Hx + b_\varepsilon \] (3)

The \( b \) is an n-dimensional outlier vector. If there are outliers in a gyro, the corresponding element in \( b \) is not zero, and the remaining elements are zero. The gyro configuration matrix \( H \) is related to the mounting position and angle of each gyro in the system.

2.2. Detection Function

In the SVD method, the geometric configuration matrix of gyro array is decomposed first.

\[ H = U_H \begin{pmatrix} S_H & 0 \\ 0 & 0 \end{pmatrix} L_H^T \] (4)

\[ U_H = [U_1 ; U_2] \] (5)

\[ V = U_2^T U_2^T \] (6)

The parity vector is calculated by the measurement data vector \( Z \) and matrix \( V \) as follows:

\[ p = VZ \] (7)

The detection function and isolation function are given as follow:

\[ F_D = p^T p \] (8)

\[ F_i^j = (p^T V_i)^2 / V_i^T V_i \] (9)
In the equation (9), the $V_i$ is the $i$-th column of matrix $V$, $F_j^i$ is corresponding to the $i$-th gyro in the array. The gyro corresponding to largest $F_j^i$ is most likely to have outliers at this moment. Form equation (3), (7) and (8), the equation (10) can be presented as follow:

$$F_D = [V(Hx + b_x)]^T [V(Hx + b_x)]$$

The decoupling matrix satisfies equations of equivalent space, so the detection function $F_D$ can be expressed as:

$$F_D = b_e^T b_e$$

From equation (11), the detection function value is only related to the system noise and the fault vectors. In practical application, the geometric configuration matrix $H$ will not change after calibrating. Even in the nonlinear case, the error output value of filter is not able to affect the detection function. This method will not introduce other errors, so the credibility is higher.

3. The Method and Criteria of Outlier Detection

3.1. Grubbs Test

The detection function values described above are subject to $\chi^2$ distribution when the noise statistical characteristics keep stable. But in the actual process, the statistical characteristics of noise will not remain constant. Once the noise changes for any reason, it will lead to an increase in false alarm rate or miss eliminating rate. At present, the commonly used methods contain Romanovsky test, Nair rule, Dixon test, Chauvenet test, Grubbs test and $3\sigma$ test. Among them, Nair rule is optimal, but it needs accurate error distribution model, which is not suitable for inertial navigation system. The $3\sigma$ test and Dixon test require large sample data, and may miss a part of outliers. The Grubbs test requires less sample data and has better detection performance under the condition of unknown error, which is easy to engineering practice.

Therefore, we select the Grubbs rule to test each gyro signal [13]. The Grubbs test assumes that the working environment of the system keeps constant in a sufficiently small duration, so that its error level remains unchanged. In the case of a small number of samples, the error values do not obey the normal distribution. Therefore, this paper proposes the new statistics $g_i$ by using the Grubbs rule:

$$g_i = \frac{(y_i - \bar{y})}{s}$$

Among the equation (12), the $s$ is given by:

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2}$$

After determining the false alarm rate $\alpha$ and sample quantity $n$, the threshold value $G(\alpha,n)$ can be determined by querying the table of Grubbs test. When $g_i > G(\alpha,n)$, the point could be diagnosed as the outlier, otherwise it would be a normal measurement data.
3.2. Outlier Detection Method in Gyro Array

In this paper, the value of detection function is calculated by SVD method. The false alarm rate $\alpha$ and sampling period $N$ are determined according to the requirements of system performance. Each sampling period contains $n$ sampling points, where $N$ generally takes between the 30–50 [13]. When the measurement data of $i$-th point obtained, the $g_i$ can be calculated as follow:

$$g_i = \frac{(y_i - \bar{y})}{\sqrt{\frac{1}{n-1} \sum_{k=i-n+1}^{i} (y_k - \bar{y})^2}}$$

(14)

$$y_i = \sqrt{F_{Di}}$$

(15)

4. Simulation Experiment

4.1. Outlier Detection in Static Situation

In the static situation, the gyro data generator is built by Simulink as Figure 1 shows. There are 4 gyros which are all sensitive to the angular velocity of the same direction; the sampling frequency is 0.5% Hz; the sampling time is 1000 s; the frequency of the pulse generator is 0.1 Hz; and the pulse duration is 0.5% of the pulse period. The output of a single gyro is shown in Figure 2.

![Simulink Model of Gyro Signal Generator](image-url)

**Figure 1.** Simulink Model of Gyro Signal Generator
Figure 2. Simulation Gyro Output Signal

From Figure 2, there is a certain amount of noise in each four groups of signals, and there are many obvious wild value points too. The Kalman filter is used for data fusion to calculate system output signal, and the result is shown in Figure 3.

Figure 3. Gyro Fusion Signal with Outliers

According to Figure 3, after the fusion through Kalman filter, the signal error is obviously reduced, but there are still many obvious outlier points in the fused signal. It can be seen that the data fusion algorithm by traditional Kalman filter cannot eliminate the influence caused by the outliers.

In contrast, the outliers of four sets of raw data are eliminated by the new method in this paper. The diagnosis period is 50; the false alarm rate is 99.5%; and the detection threshold is given by
The marked outlier points are replaced by the corresponding points of fitting curves. The fusion results are shown in Figure 4. From Figure 4, the gyro array fusion data has less outlier points after eliminating, which greatly reduces the influence of the anomaly value on the overall accuracy of the system.

\[ G_f(\alpha, n) = 1.496 \]

**Figure 4.** Comparison between Fusion Signal with Outliers and without Outliers

### 4.2. Outlier Detection in Dynamic Nonlinear Condition

In the case of dynamic and nonlinear simulation experiment, the actual static signal of the gyro is taken as the residual value of the dynamic simulation signal. The static signal plus the sinusoidal simulation signal to use as experimental data. The sampling time is 200 s; the sampling frequency is 100 Hz; the sinusoidal signal amplitude is 10 o/s; and the frequency is 0.1 rad/s, where the gyro configuration is same as the static situation. To verify the detection ability of the new method for outlier patches, an outlier pulse is injected every 10 s. The output signal and the detection function value are shown in Fig. 5.

**Figure 5.** Gyro Output Signal and Detection Function Value
The outlier points are marked by the new method and replaced by corresponding points from cubic spline curves. The signal after outlier eliminating is shown in Fig. 6. It can be seen from Fig. 6 that the method have a good performance at the position where the outlier point appears, whether it is an isolated outlier or an outlier patch.

![Comparison of the Signal before and After the Outliers Eliminating](image)

**Figure 6.** Comparison of the Signal before and After the Outliers Eliminating

5. Conclusion
In this paper, a SVD based method is proposed for the outlier problem in gyro array. This method greatly eliminates the effect of outliers on accuracy and stability of MEMS IMU under the premise of real-time requirement. The simulation experiments show that the proposed method can deal with the outliers in both static and dynamic nonlinear conditions, and the effect of isolated outliers and spotted outliers are handled well.

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