The heat balance model of walling constructions

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Abstract. The inverse problem of identifying the heat loss model in a building through building envelopes is considered. The linear equations of heat balance in heated rooms are used. These equations take into account the thermal conductivity through the walls and the glazing of the openings, two-way convective heat exchange of the room air with heating devices and enclosing structures, and convective heat dissipation into the environment. The results of model identification based on experimental measurements of temperature fields in the building envelope are presented.

1. Introduction

The theoretical assessment of the predicted temperature conditions of buildings is carried out during their design and is aimed at providing a comfortable temperature in the premises by proper selection of the heating system (HS).

Calculated mode of operation is winter, in which the outdoor temperature is taken in accordance with the standards for the region. Along with the temperature, the factors of humidity and wind speed, which affect the heat loss, are taken into account [1]. The most comprehensive review of studies on the economical heat supply of buildings is contained in [2].

Evaluation of the predicted temperature in the building is carried out with varying degrees of detail, depending on the purpose of such an assessment. A common approach to obtaining it is the heat balance of a building or individual premises [3-7]. The power released by the HS in the form of heat is transferred from the heating devices to the internal air environment, and then through the enclosing structures is discharged into the outside air. The temperature in the premises is constant, provided that the power transferred by the HS is equal to that allocated through the building envelope in the form of heat losses.

Changing temperature conditions changes the temperature of the coolant at the entrance to the heating system; when the ambient temperature decreases, the supplier increases the temperature in the main pipeline. Due to this, the air temperature in the room changes to a lesser extent, which allows the use of stationary models in the diagnosis of the heating systems state. Actual temperature field measurements are used to identify heat loss parameters.

2. Problem Statement

Temperature fields in a heated building are described by a stationary mathematical model. This model includes the equations of heat balance, thermal conductivity of walling, heat and mass transfer in the heating system and heat transfer from the elements of the heating system to the room, from room to
walling and from walling to the environment. The stationary model is chosen because the rate of the heating processes of the enclosing structures is slow and the change in the coolant temperature is insignificant depending on the ambient temperature, so that the temperature in the rooms remains fairly stable (its fluctuations are much less than the temperature changes of the environment). The non-stationarity of the building thermal regime, when the weather changes, is smoothed by the heating system. The interaction of heat transfer processes in a heated building can be represented as a diagram shown in Figure 1.

![Diagram of heat exchange processes](image)

**Figure 1.** Scheme of heat exchange processes.

In Figure 1, the numbers I, II, III and IV denote the processes. Let us consider these processes in more detail.

Process I - heat transfer from the heating element to the room air. Power from the heating element is calculated using the convective heat transfer model. This power is equal to the product of the heat flux density and the area of the heating element, and the heat flux density is proportional to the temperature difference between the heating element surface and the air temperature in the room. The heat transfer coefficient in the air is known for the regulatory conditions of heat transfer (free convection into the air) and is given in the technical specifications for each model of the heating element.

Process II - heat transfer from indoor air to the enclosing structure. The model of this process is adapted in the form of convective heat exchange: the density of the heat flux is proportional to the difference in air temperature in the room and the inner surface of the enclosing structures. The heat transfer coefficient from air to enclosing structures is taken from well-known reference data (in the range from 5 to 10 W/(m² K)).

Process III - the thermal conductivity of the enclosing structures material. This process is described by the stationary heat conduction equation. The thermal conductivity coefficient of the enclosing structures material adapted by the reference data, available for most building materials.

Process IV - heat transfer from the outer surface of the walls to the surrounding air. This process is described by a free convection model. The density of the heat flux is proportional to the temperature difference between the outer surface of the enclosing structures and the surrounding air. The convection coefficient depends on the type of building material and the state of the surface.

Thus, the heat balance conditions are expressed by the equality of the power of heat sources in the building and the power of heat loss.
Heat sources include elements of the heating system and additional sources: electrical equipment (including computers), kitchen equipment, electric heaters, etc.

Heat loss is the loss through enclosing structures (walls, door and window openings) and losses through ventilation and infiltration of cold air through the openings of the building.

Variable temperatures are used to describe heat transfer processes. In the material of enclosing structures, the main component of the heat flux is directed perpendicular to the plane. However, in areas adjacent to the slopes of window and doorways, the distribution of heat is spatial.

Therefore, the defining equation for the material is the Fourier law:

\[
q = -\lambda \nabla T,
\]

\(\lambda\) is thermal conductivity coefficient, W/(m\(\cdot\)K);
\(\nabla T\) is temperature gradient.

The differential equation of heat balance at a material point is:

\[
\text{div} \ q = 0.
\]

Equations (1) - (2) are solved with boundary conditions of the 3rd kind.

The thermal conductivity coefficients of building materials are known and are determined by reference data. However, heat losses through the enclosing structures are characterized by thermal resistance coefficients \(R\). \(R\) is the ratio of the temperature difference inside and outside the room to the heat flux density. These factors are interrelated; when the temperature changes only along the normal to the glazing plane we get:

\[
R = \frac{d}{\lambda},
\]

\(d\) is thickness of the structural element.

Heat losses through window openings are affected by the thermal conductivity and thickness of the material and the looseness of the connection between the glass and the frame. Therefore, the coefficient of thermal resistance should be adjusted by the amount of the amendment taking into account these factors. Also the heat conductivity coefficient is corrected. Violation of wall insulation, erosion of brickwork and other similar phenomena alter the thermal resistance of walls.

Finally, equation (1) is taken in the form:

\[
q = -k \cdot \lambda \nabla T,
\]

\(k\) is correction factor determined when setting up a mathematical model.

The discrete model of the thermal balance of the building envelope is based on the finite element method. The topological and geometric model of the building, which defines the size, shape and topology of the rooms, walls and openings, is of great importance.

In the topological and geometric model of the building, the following structural components are distinguished: walls, rooms and heating elements.

A wall (with windows and doors) is a set of spatial elements of a hexagonal shape. These elements are connected at common nodal points - the vertices of the elements, so that the temperatures at the coinciding nodes of neighboring elements are the same. This applies to both adjacent elements of the wall and glazing elements. The wall has an external and internal surface; each section of the inner surface of the wall refers to one room. The air temperature in the room is constant at all internal points. Temperatures in different rooms are different. There are heating devices in the room.

3. Numerical solution

The method of solving the problem of heat conduction is based on a finite element model. The task accepts elements of the wall, elements of windows, elements of doors (entrance and balcony), elements for specifying convective heat exchange: external convectors on the outer surface of the walls, double-sided convectors between the inner surface of the wall and indoor air, and convectors between the surface of heating devices and room air.

The problem of stationary conductive-convective heat transfer can be set in a variational form, as the task of minimizing the functional [1]:

\[
\text{min} \ \int_{\Omega} \left( \frac{1}{2} \rho \nabla T^2 + q \cdot \nabla T \cdot n \right) d\Omega + \int_{\Gamma} \left( h \left( T - T_0 \right) \right) d\Gamma.
\]
\[ \Phi(T) = \frac{1}{2} \iiint q(T) \nabla T \cdot \nabla T \, dV + \frac{1}{2} \int_{S_{\text{conv}}} h(T - T_\infty) \, dS - \int_{S_{\text{heat}}} q_n(T) \, dS. \]  

(5)

\( S_{\text{conv}} \) is surface area through which convective heat exchange takes place, \( S_{\text{heat}} \) is surface area heated by a constant heat flux density \( q_n \), \( T \) is unknown temperature, variable in volume and surface of the object.

The functional (5) is approximated by a linear combination of basic functions of coordinates. The condition of its minimum is transformed into a system of linear algebraic equations for the coefficients of the basic functions. When using the method of finite displacements in the Lagrangian variant, the coefficients are the nodal values of temperature. Basic functions correspond to the form of finite elements and the location of their nodes.

According to the form of the element, they can be divided into the following types:
- spatial hexagonal element of the enclosing structure, in which the temperature is variable over the entire volume of the element;
- spatial hexagonal element of the room, in which the temperature is constant throughout its volume;
- quadrilateral element of a one-sided convector, which is located on the surface of the hexagon element and is in contact with the ambient temperature;
- octagonal element of a two-sided convector, which is located on the surface of the hexagonal element and has additional nodes connected to the hexagonal element of the room.

Let us consider in detail each of these types of finite elements.

4. The finite element of the enclosing structure

Element hexagon shape is shown in Figure 2.

![Figure 2](image)

**Figure 2.** The hexagon element of the enclosing structure: a) dimensional coordinates; b) dimensionless canonical coordinates

The vertices (nodes) of the element are numbered as follows: the first four nodes are located on the near face counterclockwise; the following four nodes are located in the same order on the far edge. Let us apply the well-known technique of isoparametric mappings of the canonical element (Fig. 2, b) to the dimensional coordinates. We introduce the basis of interpolation of coordinates and desired temperatures; let us denote \( N_v^{(e)}(\xi, \eta, \zeta) \) by basic spline function equal to one in the \( v \)-th node of the element and to zero in its other nodes. The specific form of such functions is repeatedly given in the literature.
The mapping of the canonical element to the dimensional coordinates is:

\[
\begin{pmatrix}
x \\
y \\
z \\
\end{pmatrix} = \sum_{\nu} N_{\nu}^{(e)}(\xi, \eta, \zeta) \begin{pmatrix}
\xi \\
\eta \\
\zeta \\
\end{pmatrix},
\]

(6)

the coordinates in the left and right sides of (6) are variable values. The temperature field in the element is uniquely determined by the temperature values at eight nodal points:

\[
\{T\} = [T_1 \ T_2 \ \ldots \ T_8]^T.
\]

(7)

Between the nodal values of temperature (7) and the variable temperature at an arbitrary point, the following interpolation equality is held:

\[
T(\xi, \eta, \zeta) = \sum_{\nu} N_{\nu}^{(e)}(\xi, \eta, \zeta) \{T\}.
\]

(8)

Temperature gradient is expressed in terms of nodal temperature values:

\[
\nabla T(\xi, \eta, \zeta) = \sum_{\nu} \nabla N_{\nu}^{(e)}(\xi, \eta, \zeta) \{T\}.
\]

(9)

Matrix view of the Fourier thermal conductivity law:

\[
\begin{bmatrix}
q_x \\
q_y \\
q_z \\
\end{bmatrix} = \begin{bmatrix}
\lambda_x & 0 & 0 \\
0 & \lambda_y & 0 \\
0 & 0 & \lambda_z \\
\end{bmatrix} \nabla T,
\]

(10)

\(\lambda_x, \lambda_y, \lambda_z\) are the material thermal conductivity coefficients in the directions of the coordinates axes.

The difference in the thermal conductivity coefficients for most building materials is insignificant, but in the case of layered materials, the thermal conductivity coefficient in the normal direction can be significantly less than in the plane of the layers.

Substituting in (9) the expression of the temperature gradient through the nodal variables, we get:

\[
\{q\} = [D] \sum_{\nu} \nabla N_{\nu}^{(e)}(\xi, \eta, \zeta) \{T\} = [D][B(\xi, \eta, \zeta)]\{T\},
\]

(11)

\([D]\) is matrix of thermal conductivity coefficients for the material.

The final result is an expression of the thermal conductivity matrix of the element \([K]^{(e)}\):

\[
[K]^{(e)} = \int_B \left[ \frac{\partial B(\xi, \eta, \zeta)}{\partial \xi} [B(\xi, \eta, \zeta)] \right] dV,
\]

(12)

where the integration is performed on the volume element, the index "T" stands for matrix transposition.

The finite element of thermal conductivity matrix maps the column vector of nodal temperatures (7) to the column vector of nodal powers.

5. The finite element of a one-way convector

The element of a one-sided convector has a quadrangular shape (Fig. 3).

![Figure 3. Element of one-way convector.](image-url)
The temperature field is determined by four variables: the temperature values at the nodes common to the heated surface (the outer surface of the wall or the surface of the heating element): \( \{T\} = \{T_1, T_2, \ldots, T_4\}^T \). (13)

The set of basic interpolation functions contains four functions, according to the number of nodes depending on two coordinates. The temperature at an arbitrary point is expressed by the interpolation of nodal values:

\[
T(\xi, \eta, \xi) = \sum_i N^{(e)}(\xi, \eta) \cdot \{T\} = [N^{(e)}(\xi, \eta)] \{T\}. \tag{14}
\]

The interpolation matrix \([N^{(e)}(\xi, \eta)]\) contains a basic function in each column. The thermal conductivity matrix of the element is:

\[
[K]^{(e)} = \int [N^T(\xi, \eta)] [D][N(\xi, \eta)] dS,
\]

this matrix has a size of 1x1 and contains a heat transfer coefficient, and the integration is performed over the surface area of the convector element. Nodal powers (right parts) contain the following:

\[
[Q]^{(e)} = \int [N^T(\xi, \eta)] T^e dS. \tag{16}
\]

6. The structure of the thermal balance equations for rooms and enclosing structures

We introduce the following partition of the unknown variables of the heat balance problem.

In the first group of variables the room nodes temperature are combined. Let us denote this column vector by \( T^I \). The number of independent variables is equal to the number of rooms in the building. Unknown \( T^I \) enters the heat balance equations along with the known temperatures of the heating elements and the unknown temperatures of the inner surface of the walls.

The second group of variables \( T^{II} \) combines the temperatures of the inner surface of the wall. These unknowns enter into the heat balance equations along with the unknown room temperatures \( T^I \) and the unknown temperatures of the internal components of the enclosing structures.

In the third group of unknowns, \( T^{III} \), the temperatures of the external components of the enclosing structures in contact with the outside air through the elements (the convectors of the outer wall) are combined. These unknowns enter the heat balance equations along with a fixed outdoor temperature.

The fourth group of unknowns \( T^{IV} \) combines the temperatures of the internal components of the enclosing structures that are not located either on the outside or on the inside surface. These temperatures are included in the heat balance equations, which also contain the temperature of adjacent surface nodes of the enclosing structures.

The values fixed in these equations (the temperatures of the heating elements) are distinguished into the fifth group \( T^V \) (they will be variable in the problem of the heat balance of the heating system), and the outdoor temperatures are combined into a separate group \( T^{VI} \).

Two different variables enter the same equation if they are contained in variables of the same finite element. The fixed values will be transferred to the right side of the equations, and the variable variables will be grouped on the left side. The structure of the equations general system for the thermal balance of a building in a matrix form can be represented as follows:

\[
\begin{bmatrix}
K_{11} & K_{12} & 0 & 0 \\
K_{21} & K_{22} & 0 & 0 \\
0 & 0 & K_{33} & 0 \\
0 & 0 & 0 & K_{44}
\end{bmatrix}
\begin{bmatrix}
T^I \\
T^{II} \\
T^{III} \\
T^{IV}
\end{bmatrix}
= \begin{bmatrix}
Q_{11} & 0 \\
0 & 0 \\
0 & Q_{22} \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
T^I \\
T^{II} \\
T^{III} \\
T^{IV}
\end{bmatrix}. \tag{17}
\]

Matrices in the left and right sides are the block ones. Blocks \( K_{ij} \) are formed by assembling the finite elements from local matrices, blocks \( Q_{ij} \) are formed by assembling the right parts of convector elements from the matrices.
7. Calculation example

The above technique was used in the diagnosis of heat loss in a seven-story educational building. Figure 4 shows a fragment of the finite element model for the building facade wall. When identifying the model, the characteristics of the heating devices were set in accordance with the actually measured average temperatures of the radiators, and the variable coefficients were the effective thermal conductivities of the window openings and walls. These coefficients were determined from the condition of a deviations squares sum minimum for the calculated and measured indoor temperatures.

Figure 4. Fragment of the building facade wall with openings for windows and balconies.

Figure 5a shows the calculated temperatures in the rooms adjacent to the building’s facade, prior to setting up the model (at standard values of thermal conductivity coefficients), and in fig. 5, b - with the values of the coefficients found by identification. The room number is indicated at the top of the rectangle to indicate the room. The difference between the calculated and measured temperatures before setting up the model reaches 11°C, while after setting it does not exceed 1.5°C.

Figure 5. Calculated temperatures in rooms adjacent to the facade:
   a) before identification, b) after identification of the model.
Conclusions
The balanced model parameters identification by the measured actual temperatures makes it possible to determine the actual coefficients of heat transfer to the external environment from each room of the heated building. Model tuning can be performed according to measurements at different external temperatures and different modes of heat consumption.

The adjusted balanced model of the building’s heat loss allows predicting the change in the heat regime in the premises after the scheduled repair work (thermal insulation of the building envelope, repair of heating appliances) and determining rational options for the current repair with limited resources of the operating organization.

The system of equations (17) is a part of the heat balance equations of a heated building, since the right side contains variables, some of them are unknown. Temperature determination of heating elements is a separate subject of modeling.

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