Comptonization of cosmic microwave background photons in dwarf spheroidal galaxies

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ABSTRACT

We present theoretical modelling of the electron distribution produced by annihilating neutralino dark matter in dwarf spheroidal galaxies (dSphs). In particular, we follow up the idea of Colafrancesco and find that such electrons distort the cosmic microwave background (CMB) by the Sunyaev–Zeldovich (SZ) effect. For an assumed neutralino mass of 10 GeV and beam size of 1 arcsec, the SZ temperature decrement is of the order of nano-Kelvin for dSph models with a soft core. By contrast, it is of the order of micro-Kelvin for the strongly cusped dSph models favoured by some cosmological simulations. Although this is out of reach of current instruments, it may well be detectable by future mm telescopes, such as the Atacama Large Millimetre Array. We also show that the upscattered CMB photons have energies within reach of upcoming X-ray observatories, but that the flux of such photons is too small to be detectable now. None the less, we conclude that searching for the dark matter induced SZ effect is a promising way of constraining the dark distribution in dSphs, especially if the particles are light.

Key words: galaxies: dwarf – intergalactic medium – cosmic microwave background – dark matter – X-rays: galaxies.

1 INTRODUCTION

Dwarf spheroidal galaxies (dSphs) are important probes of dark matter. They are among the highest mass-to-light systems known, and the dynamics of their sparse stellar populations are governed by the dominant dark matter distribution. In addition, no emission has been detected from dSphs in wavebands other than the optical, indicating a lack of internal dust or gas (Fomalont & Geldzahler 1979; Bonanos et al. 2004).

Here, we consider the distortion of the cosmic microwave background (CMB) by the non-thermal population of secondary electrons generated by dark matter annihilation. This is an example of the Sunyaev–Zeldovich (SZ) effect (Zeldovich & Sunyaev 1969). Since we are dealing with electrons produced from dark matter annihilation, we write the distortion as the dSZ effect. Ensslin & Kaiser (2000) and Colafrancesco, Marchegiani & Palladino (2003) calculated the signal expected from the SZ effect of a relativistic plasma, while Colafrancesco (2004) determined the dSZ effect in galaxy clusters.

As first pointed out in Colafrancesco (2004), dSphs are attractive targets because they have very high mass-to-light ratios and because they have few contaminants. In particular, they have little or no internal magnetic field, so it is not possible for synchrotron emission (from annihilation electrons or otherwise) to contaminate the dSZ effect. Since dSphs are believed to be almost devoid of interstellar gas, other mechanisms such as H i or CO line emission will not be important. CMB distortions are therefore a rather clean method to detect the annihilation signature. We therefore use existing models of the dark matter distribution in dSphs, and a possible form of the energy spectrum of electrons produced by dark matter annihilation, to derive the temperature change in the CMB. We focus on dark matter models explicitly constrained by observations, rather than simulations.

For our predictions, we assume that the cold dark matter particle is the lightest supersymmetric particle, the neutralino (Jungman, Kamionkowski & Griest 1996). Current limits on the neutralino mass \(M_\chi\) and centre-of-mass velocity-averaged cross-section \(\langle \sigma V \rangle_\chi\) have been reviewed recently by Bertone, Hooper & Silk (2005). Based on these results, and considerations of current or upcoming experiments, we investigate the parameters \(\langle \sigma V \rangle_\chi = 10^{-26}\, \text{cm}^3\, \text{s}^{-1}\) and \(M_\chi = 10\, \text{GeV}\). Such low-mass particles may provide a sizeable contribution to the matter density in the Universe (Bottino, Fornengo & Scopel 2003), and hence are worthy of consideration. However, some dark matter candidates—such as the neutralino in the most commonly studied minimal
supersymmetric models or the lightest Kaluza–Klein particle—must be more massive than this. The value assumed for \((\sigma V)_A\) is consistent with the expected relic density in a Universe with \(\Omega_m = 0.3\) and \(H_0 = 70\, \text{km s}^{-1}\, \text{Mpc}^{-1}\) (Colafrancesco & Mele 2001). We derive the dependency of the dSZ signal on \((\sigma V)_A\) and \(M_{\chi}\), and show that the brightness temperature decrement \(\Delta T_B \propto (\sigma V)_A M_{\chi}^{-2}\).

We initially calculate the expected signal for this optimistic choice of dark matter parameters.

The format of the paper is as follows. In Section 2, we discuss our dSph dark halo models, based on current work in the literature. We then discuss possible products of dark matter particle annihilation in Section 3. Observational consequences of such annihilation events are presented in Section 4. We conclude in Section 5.

2 Dwarf Spheroidal Models

With spherical symmetry of the dark halo assumed, we use the results of Evans, Ferrer & Sarkar (2004), who fit observational data on the Draco dSph (currently orbiting the Milky Way) from Wilkinson et al. (2004) to two sets of models via the Jeans equation (e.g. Binney & Tremaine 1987).

2.1 Cusped halo models

Cusped halo models (CHMs) are favoured by numerical simulations, as reported by Navarro, Frenk & White (1997) (NFW) and Moore et al. (1998). Using arguments based on the survivability of cusps, as reported by Navarro, Frenk & White (1997) (NFW) and Cusped halo models (CHMs) are favoured by numerical simulation (e.g. Binney & Tremaine 1987).

Wilkinson et al. (2004) point towards a milder density slope of \(\gamma \simeq 1.1\). However, we retain the Moore profile here as the models satisfy the observational constraints from stellar radial velocities. In addition, this profile allows an upper limit of the magnitude of the dSZ effect, enabling a broad yet well-motivated parameter space to be investigated.

Table 2. List of CPL model parameters for the Draco dSph. Values for \(r_1\) in parentheses are for NFW models of the Milky Way, as opposed to isothermal power-law models which are without brackets.

| \(\alpha\) | \(v_{\text{th}}\)/km s\(^{-1}\) | \(r_c\)/kpc | \(r_1\)/kpc | \(M(r_1)/10^8 M_\odot\) |
|---|---|---|---|---|
| 0.2 | 24.7 | 0.25 | 6.2 (1.3) | 4.6 |
| 0 | 22.9 | 0.23 | 7.8 (1.4) | 9.5 |
| −0.2 | 20.9 | 0.21 | 10.1 (1.6) | 22.43 |

Typically, \(r_{\text{min}} \sim 10^{13} \text{m}\), and for such a small value, it is unlikely that tidal forces (from M31 or the Milky Way) could disrupt the central cusp. It should be noted that the Moore profile \((\gamma = 1.5)\) represents an extreme case for the inner slope of the cusp. Recent numerical simulations (e.g. Diemand, Moore & Stadel 2004; Navarro et al. 2004) point towards a milder density slope of \(\gamma \simeq 1.1\). However, we retain the Moore profile here as the models satisfy the observational constraints from stellar radial velocities. In addition, this profile allows an upper limit of the magnitude of the dSZ effect, enabling a broad yet well-motivated parameter space to be investigated.

2.2 Cored power-law models

The second family of dark halo profiles studied is the cored power-law (CPL) models (Evans 1994). Again, these satisfy the Draco velocity dispersion observations, and take the form

\[ n(\hat{\rho}) = n_0 a(\hat{\rho}), \]

where \(n_0 = A r_c^{-3} / M_{\chi}\), and \(a(\hat{\rho}) = \hat{\rho}^{-\gamma} (1 + \hat{\rho})^{-\alpha/2}\), with \(\hat{\rho} = r / r_c\). Table 1 gives the slope \(\alpha\), the core radius \(r_c\), the tidal radius \(r_t\), and the overall normalization \(A\). The model is truncated at \(r_t\), whose value depends on the Milky Way dark halo model.

To address the problem of the divergent central density in this model, we use the arguments of Blasi & Colafrancesco (1999) and Tyler (2002). The density profile is truncated at a radius \(r_{\text{min}}\), where the dark matter annihilation rate matches the collapse time-scale of the cusp. With this assumption, a small constant density core is created with radius \(r_{\text{min}}\)

\[ r_{\text{min}} = r_c (\sigma V)_A^{1/2} \left( \frac{n_{\text{min}}}{4\pi G M_{\chi}} \right)^{1/4}, \]

where \(n_{\text{min}}\) is the number density at the location of the constant density region. At small radii, equation (1) reduces to \(n(\hat{\rho}) \simeq (A M_{\chi}^{-1} r_c^{-3} \hat{\rho}^{-\gamma})\), and hence we find

\[ r_{\text{min}}^{1+\alpha/4} = r_c (\sigma V)_A^{1/2} \left( \frac{A r_c^{-3}}{4\pi G M_{\chi}} \right)^{1/4}. \]

Table 1. List of CHM model parameters for the Draco dSph. Values for \(r_1\) in parentheses are for NFW models of the Milky Way, as opposed to isothermal power-law models which are without brackets.

| \(\gamma\) | \(A \times 10^5 M_\odot\) | \(r_c\)/kpc | \(r_1\)/kpc | \(M(r_1)/10^8 M_\odot\) |
|---|---|---|---|---|
| 0.5 | 2.3 | 0.32 | 6.6 (1.5) | 5.5 |
| 1.0 | 3.3 | 0.62 | 7.0 (1.6) | 6.6 |
| 1.5 | 2.9 | 1.0 | 6.5 (1.5) | 5.5 |

Energy losses are efficient in the dSph halo, and dark matter annihilation from infalling material continuously refills the electron spectrum. In a first approximation, it seems reasonable to neglect diffusive effects, as dark matter annihilation replenishes the electron spectrum efficiently, at least in the central parts of the dSph.

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Furthermore, we consider the dark matter to maintain a constant radial profile on the time-scale of dark matter annihilation. This means that any time-variation of the electron distribution is negligible. The source function of electrons, $Q_{\epsilon}(E, r)$, generated in this fashion therefore obeys the stationary diffusion-loss equation (see e.g. Longair 1994)

$$\frac{\partial}{\partial E} \left[ \frac{d\epsilon_{\nu}(E, r)}{dE} \right] = -Q_{\epsilon}(E, r),$$

(8)

which we integrate to obtain the annihilation-produced electron energy distribution $d\epsilon_{\nu}(E, r)/dE$:

$$\frac{d\epsilon_{\nu}(E, r)}{dE} = - \frac{1}{b(E)} \int Q_{\epsilon}(E, r) dE.$$

(9)

The source function itself results from annihilation products of neutralino collisions. In this calculation, we follow the work of Tyler (2002), who used the Hill (1983) formula to determine the densities of particles produced by dark matter annihilation. Quark pairs and their subsequent fragmentation lead to pions as the main annihilation products. The primary decay particles are neutral pions, which decay to gamma rays, and charged pions, which decay as

$$\pi^+ \rightarrow \mu^+ \nu_\mu \quad \text{and} \quad \pi^- \rightarrow \mu^- \bar{\nu}_\mu.$$

(10)

The muons then decay to electrons via

$$\mu^+ \rightarrow e^+ \bar{\nu}_e \quad \text{and} \quad \mu^- \rightarrow e^- \nu_e.$$

(11)

The number spectrum of electrons from a single $\chi \chi$ annihilation is then given by

$$d\epsilon_{\nu} = \int_{E_{\mu}}^{E_{\nu}/2} W_{\mu} \frac{dN^{(\mu)}_{\bar{\nu}}}{dE_{\bar{\nu}}} dE_{\bar{\nu}} dE_{\mu},$$

(12)

where $W_{\mu} \equiv (m_\mu/m_\chi)^2$, and the charged pion multiplicity per annihilation event is

$$W_{\pi} = \frac{4}{3} \frac{15}{16M_\chi^2} \frac{E_{\mu}}{M_\chi} \left[ 1 - \frac{E_{\mu}}{M_\chi} \right]^2,$$

(13)

where the factor of $4/3$ accounts for the fact that annihilation electrons are only produced by charged pions, and that quarks (which eventually decay to charged pions) are produced in pairs. The number spectrum of muons produced per charged pion decay is

$$dN^{(\mu)}_{\bar{\nu}} = \frac{1}{E_{\mu}} \frac{m_\mu^2}{m_\pi^2 - m_\mu^2},$$

(14)

and

$$dN^{(\mu)}_{\bar{\nu}} = \frac{2}{E_{\mu}} \left[ \frac{5}{6} - \frac{3}{2} \left( \frac{E_{\mu}}{E_{\nu}} \right)^2 + \frac{2}{3} \left( \frac{E_{\mu}}{E_{\nu}} \right)^3 \right],$$

(15)

is the number spectrum of electrons per muon decay. After some algebra, with the above forms for the decay product energy spectrum, equation (12) has an analytic solution:

$$d\epsilon_{\nu} = \frac{15}{8M_\chi^2} \frac{m_\mu^2}{m_\pi^2 - m_\mu^2}$$

$$\times \left[ c_1 z^{-3/2} + c_2 z^{-1/2} + c_3 + c_4 z^{1/2} + c_5 z^2 + c_6 z^3 \right].$$

(16)

in units of GeV$^{-1}$, where $z = E_{\nu}/M_\chi$. The coefficients $c_i$ are (0.1039, −1.2218, 2.4800, −1.5406, 0.2205, −0.04197).

Scaling this expression, we arrive at the source function

$$Q_{\epsilon}(E, r) = \frac{4}{3} \frac{15}{8} \frac{m_\mu^2}{m_\pi^2 - m_\mu^2} \frac{M_\chi}{9} \left( \frac{n_\chi}{\text{cm}^{-3}} \right)^2 \left( \frac{\langle \sigma V \rangle_A}{10^{-26} \text{cm}^3 \text{s}^{-1}} \right)$$

$$\times \left[ c_1 z^{-3/2} + c_2 z^{-1/2} + c_3 + c_4 z^{1/2} + c_5 z^2 + c_6 z^3 \right].$$

(17)

where the appropriate units are GeV$^{-1}$ cm$^{-3}$ s$^{-1}$. This needs to be multiplied by a further factor of 2 to account for the contributions of electrons and positrons. Using this expression, and equation (9), we arrive at the equilibrium electron energy distribution

$$d\epsilon_{\nu} = 4.6848 \times 10^{-9} \left( \frac{M_\chi}{\text{GeV}} \right)^{-2} \left( \frac{n_\chi}{\text{cm}^{-3}} \right)^2 \left( \frac{\langle \sigma V \rangle_A}{10^{-26} \text{cm}^3 \text{s}^{-1}} \right)$$

$$\times \left[ d_1 z^{-5/2} + d_2 z^{-3/2} + d_3 z^{-1} + d_4 z^{1/2} + d_5 z^2 + d_6 z^3 \right].$$

(18)

in units of GeV$^{-1}$ cm$^{-3}$. In this expression, the radial dependence of $d\epsilon_{\nu}/dE_{\nu}$ arises in the $n_\chi$ factors. The new coefficients $d_i$ are (0.2078, 2.4436, −2.4800, 1.0271, −0.07333, 0.01049). Equations (17) and (18) are displayed in Figs 1 and 2, respectively.

For completeness, in Fig. 1 we also display two other possible source functions presented in Colafrancesco & Mele (2001). These source functions arise from fermion-dominated annihilation in the first instance (denoted by CM $\chi \chi \rightarrow ff$), and gauge boson dominated annihilation (CM $\chi \chi \rightarrow WW$) in the second. At the crucial low-energy end of the spectrum, the three separate source functions exhibit amplitudes within 1.5 orders of magnitude, and similar power-law slopes. This implies that our calculations may increase or decrease by approximately one order of magnitude in either

![Figure 1](https://academic.oup.com/mnras/article-abstract/368/2/659/984824/2)

**Figure 1.** Electron source function, as described by equation (17) (denoted by Hill), assuming $M_\chi = 100$ GeV, $\langle \sigma V \rangle_A = 10^{-26}$ cm$^3$ s$^{-1}$ and $n_\chi = 1$ cm$^{-3}$. Also shown are the source functions described in Appendix A of Colafrancesco & Mele (2001), i.e. electrons produced in fermion-dominated annihilation $\chi \chi \rightarrow ff$, and gauge boson dominated annihilation, $\chi \chi \rightarrow WW$. 

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direction, depending on the precise details of the source function. As the results of Colafrancesco & Mele (2001) were derived in an independent manner to those of Hill (1983), our results may also hold for neutralino compositions other than those considered here. The crucial quantity for the magnitude of the dSZ effect is the lower limit of electron energy, which can increase rapidly for any power-law source function \( Q_e \sim z^{-\beta} \), where \( \beta > 0 \) is a generic slope parameter. It is highly unlikely on energetic grounds that electrons produced by annihilating dark matter can have \( Q_e \) rising with \( z \). Although more complicated forms for \( Q_e \) from general electron sources are possible, such as a double power law (Colafrancesco & Mele 2001), these again have negative slopes, and therefore it is the lowest electron energies that are most significant.

From equation (18), we may calculate the number density of electrons in the halo:

\[
 n_e = \int_{0.01M_J}^{M_J} \frac{dE_e}{dE_e} \, dE_e. \tag{19}
\]

The limits are chosen to be \( 0.01M_J < E_e < M_J \), based on the analysis of Kamionkowskii & Turner (1991), who derived the positron source function in a similar manner to that described above. We will see that the upper limit is somewhat irrelevant, as the source function falls rapidly to zero as the electron energy approaches the neutralino rest mass energy.

Evaluating this leads to

\[
 n_e = 8.09 \times 10^{-7} \left( \frac{n_e}{\text{cm}^{-3}} \right) \left( \frac{\langle \sigma V \rangle}{10^{-26} \text{cm}^3 \text{s}^{-1}} \right) \left( \frac{M_J}{\text{GeV}} \right)^{-1}, \tag{20}
\]

in units of \( \text{cm}^{-3} \).

We now represent equation (18) in terms of its momentum spectrum, as this quantity will be used later to calculate the frequency spectrum of upscattered CMB photons. Specifically, we write

\[
 \frac{dn_e}{dp} = n_e(r) f_e(p), \tag{21}
\]

where \( p = \beta_e \gamma_e \) is the normalized electron momentum, and \( f_e(p) \) has the property

\[
 \int_0^\infty dp f_e(p) = 1. \tag{22}
\]

In the case of dark matter annihilation, \( \beta_e = 1.0 \) to a very good approximation. Furthermore, \( \gamma_e = E_e/m_e c^2 \), which will be in the range \( 19.6M_J < \gamma_e < 1960M_J \), corresponding to energies in the range \( 0.01 \sim 1.0 M_J/\text{GeV} \). Equation (21) can be written explicitly by retaining the power-law terms in equation (18) and their associated ‘weights’ \( d_i \), and introducing a normalizing factor \( A = 1/172.77 \).

The momentum spectrum is then written as

\[
 f_e(p) = A \sum_{i=1,6} d_i \left( \frac{m_e}{M_J} \right)^{1-\alpha_i} p^{-\alpha_i}, \tag{23}
\]

where \( \alpha_i = (2.5, 1.5, 1.0, 0.5, -1.0, -2.0) \). Finally then, we have

\[
 \frac{dn_e}{dp} = n_e(r) A \sum_{i=1,6} d_i \left( \frac{m_e}{M_J} \right)^{1-\alpha_i} p^{-\alpha_i}. \tag{24}
\]

4 OBSERVABLES

We now calculate the magnitude of dSZ radio emission, and the flux of upscattered CMB photons, using the model parameters for Draco as described in Section 2. The computations use an approximate method based on the work of Ensslin & Kaiser (2000). This holds to first order in the electron optical depth, neglects multiple scatterings and is valid for a single electron population only. Throughout, we employ units where \( \langle \sigma V \rangle_n \) is measured in \( 10^{-26} \text{cm}^3 \text{s}^{-1} \), and \( M_J \) in GeV. Distances such as \( r_h \) are taken in kpc, and \( n_e,0 \) is in \( \text{cm}^{-3} \).

4.1 The SZ effect

The SZ effect is caused by inverse Compton scattering of CMB photons off energetic electrons. On average, the photons gain energy, shifting their spectrum to higher frequencies and causing a distortion in the CMB radiation field. This is commonly characterized by the Compton \( y \)-parameter, the line-of-sight integral of gas pressure through the electron cloud:

\[
 y = \int \frac{P_e(r)}{m_e c^2} \, dl. \tag{25}
\]

We may also compute the integrated \( y \)-parameter, \( \bar{Y} \), over the solid angle \( \Omega \) of the dSph:

\[
 \bar{Y} = \int \frac{\sigma T}{m_e c^2} \int P_e \, dl \, d\Omega. \tag{26}
\]

An integral over solid angle can be written as \( d\Omega = dA/r_h^2 \), hence we can equivalently write

\[
 \bar{Y} = \int \frac{\sigma T}{m_e c^2 r_h^2} \int P_e \, dV, \tag{27}
\]

where \( r_h \) is the heliocentric distance to the dSph (assumed to be 80 kpc for Draco). For the ultrarelativistic gas considered here, where \( E_e \sim \text{GeV} \), we apply the relationship between pressure and energy density \( P_e = \epsilon/3 \). The (dominant) kinetic energy density \( \epsilon \) is obtained from (Ensslin & Kaiser 2000)

\[
 \epsilon = n_e(r) \int_{0.01M_J/m_e}^{M_J/m_e} dp f_e(p)(\sqrt{1 + p^2} - 1)m_e c^2, \tag{28}
\]
where again the upper limit is not too important, as the source function falls rapidly to zero as the electron energy approaches the neutralino rest mass energy. We already showed that \( p > 19.5695 M_\gamma \), and so even for a low neutralino mass of 10 GeV, \( p^2 \gg 1 \). In this case, we can simplify equation (28) to

\[
\epsilon = n_e(r) m_e c^2 \int_{0.01 M_\gamma/m_e}^{M_\gamma/m_e} dp f_e(p) p.
\]

(29)

The pressure is then evaluated using equations (20), (23) and (28) leading to

\[
P_e = 1.585 \times 10^{-15} n_e^2 (\sigma V) d J m^{-3}.
\]

(30)

Note that the neutralino mass is present only in the number density term here.

We may now calculate the integrated \( Y \)-parameter, using this result and equation (27), for each dSph model in Section 2:

\[
Y = 3.974 \times 10^{-5} r_i^{-2} a^2 (\sigma V) \int_{0}^{r_i/2} F^2 d^2(\hat{r}) d\phi
\]

(31)

where \( r_i \) and \( a(\hat{r}) \) should be replaced with \( r_c \) and \( b(\hat{r}) \) for the CPL models.

To derive temperature shifts, we will work in terms of the mean Compton parameter averaged over the dSph, i.e.

\[
\bar{y} = \frac{Y}{\Omega}
\]

(32)

where \( \Omega \) is the angular extent over which the \( Y \)-parameter is averaged in equation (26). Once converted to temperature units, \( \bar{y} \) measures the temperature decrement inside a telescope beam of angular size \( \Omega \). We choose \( \Omega \) to take three values, first to match the angular size of the whole dSph, secondly within a 1-arcmin beam and finally within a 1-arcsec beam.

For the assumed neutralino mass, annihilation electrons are always ultrarelativistic. Since we expect these particles to have energies of the order of a few GeV, the effect of such a non-thermal electron population is to completely remove photons from the spectral range of the CMB. The problem is one of electron number density: a low neutralino mass increases the number density of electrons as \( M_\gamma \), which raises the scattering probability accordingly. It is clear that the relativistic SZ signature really measures the electron number in the dSph. Since there is a correspondence between the number of electrons and the number of annihilating neutralinos, \( Y \) is a direct measure of the dSph mass.

4.1.1 The intensity shift

The fractional change in the CMB intensity field is

\[
\delta i(x) = \frac{\Delta I(x)}{I_0},
\]

(33)

where \( x = h v/k_B T_{\text{CMB}} \) is the dimensionless frequency, and \( I_0 = 2(2\pi)^3\hbar^2/\hbar c^2 \), contributions to such a distortion are written as a product of a spectral function, \( g(x) \), and the Compton parameter \( y \). The product \( g(x) \) usually refers to the thermal electron population in clusters of galaxies; for the relativistic gas considered here, we write \( g(x) \) to make the distinction explicit. The fractional distortion averaged over the whole dSph is therefore written as (Raphaeli 1995)

\[
\delta i(x) = \frac{\Delta I(x)}{I_0} = \bar{y} g(x) \bar{y}
\]

(34)

There are two contributions to \( g(x) \). First, photons are removed from the infinitesimal frequency band \( x, x + \Delta x \) by collisions with the ultrarelativistic electrons. This contribution is written as \(-i(x)\).

The second effect is the photons scattered into this band from lower frequencies, which is written as \( j(x) \). Scattered CMB photons have their frequency increased (on average) by a factor of \( 4\gamma^2/3 - 1/3 \), so CMB photons are upscattered to the X-ray regime. Therefore, we are interested in two distinct frequency bands—near \( x \sim 2.5 \) in the radio corresponding to \( i(x) \), and the band close to \( x \sim 10^6 \) in X-rays, described by \( j(x) \).

Essentially no photons are scattered into the radio frequency band under consideration from lower frequencies. Photons are simply removed from the spectrum, causing a decrease in the number of photons, and thus a corresponding decrease in the specific intensity. The factor \( i(x) \) therefore has the spectral form of the CMB \( i(x) = x^3/(e^x - 1) \), which is maximal at \( x = 2.82 \).

The spectral factor \( g(x) \) is thus given by (Ensslin & Kaiser 2000)

\[
g(x) = [j(x) - i(x)] \frac{m_e c^2}{k_B T_e}.
\]

(35)

We define the pseudo-temperature \( k_B T_e \), as the ratio of the gas pressure \( P_e \) to the electron number density \( n_e \). For a thermal electron distribution, this is equal to the thermodynamic temperature. In this case, we have

\[
k_B T_e = \frac{P_e}{n_e} = 0.0122426 \left( \frac{M_\gamma}{\text{GeV}} \right) \text{GeV}.
\]

(36)

Using equations (34) and (35), the fractional distortion in the CMB at radio wavelengths, integrated over the dSph, is therefore

\[
\delta i(x) = -\bar{y} \left( \frac{m_e c^2}{k_B T_e} \right) j(x).
\]

(37)

The approximation of dropping \( j(x) \) is valid provided \( x < 10 \) (Ensslin & Kaiser 2000).

4.1.2 The temperature shift

The above result can equally be expressed as a temperature shift. Expressing the result in this manner has the advantage of a direct comparison to typical CMB telescope noise temperatures.

\[
\Delta T = \left| \frac{\partial I(x)}{\partial T} \right| \Delta I
\]

(38)

gives the expected temperature shift in the CMB, where \( I(x) = I_0 i(x) \). The partial derivative is

\[
\frac{\partial I(x)}{\partial T} = I_0 \frac{x^e e^t}{(e^x - 1)^2}
\]

(39)

and in conjunction with equations (33), (37) and (38), we have

\[
\Delta T(x) = -\frac{e^x - 1}{x} \frac{m_e c^2}{k_B T_e} \bar{y} T_{\text{CMB}}.
\]

(40)

Table 3 shows the results for the fiducial case of \( M_\gamma = 50 \) GeV and \( (\sigma V)_A = 10^{-26} \text{cm}^3 \text{s}^{-1} \), assuming a heliocentric distance \( r_q = 80 \) kpc for Draco and with a beam size that matches the angular size of the dSph. We assume an observing frequency of 35 GHz [\( x = 0.616 \), close to Band 1 of the forthcoming Atacama Large Millimetre Array (ALMA)]. Such \( y \) values are prohibitively low for current or near-future experiments—for example, the upcoming South Pole Telescope (Ruhl et al. 2004) will only achieve noise temperatures of \( \sim\mu \text{K} \) on arcmin scales. However, noise temperatures of the order of \( \sim 10^{-9} \) K may well be reached by a future generation of radio/sub-mm telescopes, such as ALMA. Finally, Tables 4–6 show the same quantities, but for beam sizes of 1 arcmin.
of Colafrancesco (2004) on clusters of galaxies. Although clusters
1 and 0.1 arcsec, encompassing a range of target specifications for

density is at least a factor of 10 lower than in the dSph models
considered here. Since the electron density is proportional to $n_e^2$, we
therefore have a relative increase in dSZ pressure of at least a factor
of 100 in dSphs. Using equation (25), it is also clear that the integral
along the line of sight will be proportional to the scalelength of the

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**Table 3.** List of $\gamma$-parameters, fractional intensity shifts $\delta_i$, and temperature decrements $\Delta T$ for both sets of dSph models, assuming $M_\gamma = 10 \text{ GeV}$ and $(\sigma v)_\gamma = 10^{-26} \text{ cm}^3 \text{ s}^{-1}$. All quantities are averaged over the angular extent of the dSph. Bracketed values correspond to the different Milky Way models as described in Table 1. A heliocentric distance to Draco of 80 kpc is assumed. For the intensity and temperature shifts, we use a frequency of $\nu = 0.616$, i.e. 35 GHz.

| $\gamma$ | $\bar{\gamma}/10^{-11}$ | $\bar{\delta_i}/10^{-13}$ | $\Delta T/10^{-13} K$ |
|----------|-------------------------|--------------------------|-----------------------|
| 0.5      | 7.019 (1.338 $\times 10^5$) | 8.026 (1.530 $\times 10^5$) | -5.973 (-1.103 $\times 10^5$) |
| 1.0      | 7.095 (1.336 $\times 10^5$) | 8.114 (1.528 $\times 10^5$) | -6.038 (-1.137 $\times 10^5$) |
| 1.5      | 4.514 $\times 10^2$ (9.365 $\times 10^5$) | 5.162 $\times 10^3$ (1.071 $\times 10^5$) | -3.841 $\times 10^3$ (-7.970 $\times 10^5$) |

| $\alpha$ | $\bar{\gamma}/10^{-11}$ | $\bar{\delta_i}/10^{-13}$ | $\Delta T/10^{-13} K$ |
|----------|-------------------------|--------------------------|-----------------------|
| 0.2      | 4.045 (9.040 $\times 10^3$) | 4.626 (1.034 $\times 10^5$) | -3.442 (-7.693 $\times 10^3$) |
| 0.0      | 2.970 (8.599 $\times 10^3$) | 3.396 (9.833 $\times 10^3$) | -2.527 (-7.318 $\times 10^3$) |
| -0.2     | 2.250 (7.586 $\times 10^3$) | 2.573 (8.676 $\times 10^3$) | -1.915 (-6.456 $\times 10^3$) |

**Table 4.** List of $\gamma$-parameters for 1-arcmin beam, fractional intensity shifts $\delta_i$, and temperature decrements $\Delta T$ for both sets of dSph models. The remaining dSph parameters are as recorded in Table 3. We assume $M_\gamma = 10 \text{ GeV}$ and $(\sigma v)_\gamma = 10^{-26} \text{ cm}^3 \text{ s}^{-1}$. For the intensity and temperature shifts, we use a frequency of $\nu = 0.616$, i.e. 35 GHz.

| $\gamma$ | $\bar{\gamma}/10^{-7}$ | $\bar{\delta_i}/10^{-10}$ | $\Delta T/10^{-9} K$ |
|----------|-------------------------|--------------------------|-----------------------|
| 0.5      | 3.209 (3.208) | 3.670 (3.590) | -2.731 (-2.671) |
| 1.0      | 9.619 (9.582) | 1.100 $\times 10^1$ (1.096 $\times 10^1$) | -8.186 (-8.155) |
| 1.5      | 5.119 $\times 10^2$ (5.499 $\times 10^2$) | -5.854 $\times 10^5$ (6.289 $\times 10^5$) | -4.356 $\times 10^5$ (-4.680 $\times 10^5$) |

| $\alpha$ | $\bar{\gamma}/10^{-8}$ | $\bar{\delta_i}/10^{-10}$ | $\Delta T/10^{-10} K$ |
|----------|-------------------------|--------------------------|-----------------------|
| 0.2      | 4.399 (4.397) | 5.031 (5.029) | -3.744 (-3.742) |
| 0.0      | 4.473 (4.470) | 5.115 (5.112) | -3.806 (-3.804) |
| -0.2     | 4.418 (4.414) | 5.053 (5.048) | -3.760 (-3.757) |

**Table 5.** The same as Table 4, but for a 1-arcsec beam.

| $\gamma$ | $\bar{\gamma}/10^{-6}$ | $\bar{\delta_i}/10^{-9}$ | $\Delta T/10^{-9} K$ |
|----------|-------------------------|--------------------------|-----------------------|
| 0.5      | 1.066 (1.057) | 1.219 (1.209) | -9.075 (-8.999) |
| 1.0      | 8.102 $\times 10^4$ (7.251 $\times 10^4$) | 9.266 $\times 10^4$ (8.292 $\times 10^4$) | -6.895 $\times 10^4$ (-6.171 $\times 10^4$) |
| 1.5      | 1.356 $\times 10^5$ (1.466 $\times 10^5$) | 1.551 $\times 10^5$ (1.677 $\times 10^5$) | -1.154 $\times 10^5$ (-1.248 $\times 10^5$) |

| $\alpha$ | $\bar{\gamma}/10^{-8}$ | $\bar{\delta_i}/10^{-11}$ | $\Delta T/10^{-10} K$ |
|----------|-------------------------|--------------------------|-----------------------|
| 0.2      | 4.528 (4.471) | 5.178 (5.113) | -3.853 (-3.805) |
| 0.0      | 4.624 (4.548) | 5.288 (5.201) | -3.935 (-3.871) |
| -0.2     | 4.599 (4.497) | 5.259 (5.143) | -3.914 (-3.827) |

**Table 6.** The same as Table 3, but for a 0.1-arcsec beam.

| $\gamma$ | $\bar{\gamma}/10^{-6}$ | $\bar{\delta_i}/10^{-9}$ | $\Delta T/10^{-8} K$ |
|----------|-------------------------|--------------------------|-----------------------|
| 0.5      | 1.636 (1.515) | 1.871 (1.732) | -1.392 (-1.289) |
| 1.0      | 1.544 $\times 10^3$ (9.999 $\times 10^2$) | 1.766 $\times 10^3$ (1.143 $\times 10^3$) | -1.314 $\times 10^3$ (-8.505 $\times 10^2$) |
| 1.5      | 7.893 $\times 10^6$ (9.879 $\times 10^6$) | 9.026 $\times 10^6$ (1.130 $\times 10^6$) | -6.717 $\times 10^6$ (-8.407 $\times 10^6$) |

| $\alpha$ | $\bar{\gamma}/10^{-8}$ | $\bar{\delta_i}/10^{-10}$ | $\Delta T/10^{-10} K$ |
|----------|-------------------------|--------------------------|-----------------------|
| 0.2      | 5.200 (4.605) | 5.946 (5.266) | -4.249 (-3.919) |
| 0.0      | 5.440 (4.694) | 6.221 (5.368) | -4.630 (-3.995) |
| -0.2     | 5.601 (4.665) | 6.405 (5.335) | -4.767 (-3.970) |
Comptonization of CMB photons in dwarf spheroidal galaxies

4.2 Upscattered photons

We now explicitly consider the spectrum and flux of upscattered CMB photons, adopting the first-order, approximate approach described in Ensslin & Kaiser (2000).

4.2.1 Frequency spectrum

The scattered spectrum of photons, \( j(x) \), can be expressed as

\[
j(x) = \int_0^\infty dp f_e(p) P(t) \frac{1}{x/t}.
\]

where the photon redistribution function \( P(t) \) gives the probability of a photon being scattered to a frequency \( t \) times greater than its original frequency. If the electron momentum spectrum is \( f_e(p) dp \), the photon redistribution function is

\[
P(t) = \int_0^\infty dp f_e(p) P(t; p).
\]

In this expression, \( P(t; p) \) is the redistribution function for a monoenergetic electron distribution. This has an analytic form

\[
P(t; p) = \frac{3}{32 p^3 t^3} \left[ 1 + 10 + 8 p^2 + 4 p^4 t + t^2 \right] + \frac{3(1 + t)}{8 p^5} \left[ \frac{3 + 3 p^2 + p^4}{\sqrt{1 + p^2}} - \frac{3 + 2 p^2}{2p} (2 \text{arcsinh}(p) + |\ln(t)|) \right],
\]

with the condition that \( P(t; p_1, p_2) = 0 \) if \( |\ln(t)| > 2 \text{arcsinh}(p_2) \). We may apply this formalism to the normalized electron momentum spectrum in equation (23). For such high electron energies, it is more convenient to express equation (43) in terms of the logarithmic frequency shift \( s = \ln(t) \), in which case

\[
P(s; p) = P(e^s; p) e^s ds.
\]

Figure 3. Planck temperature sensitivity for 24 h and 1-yr ALMA observation.

Figure 4. Photon redistribution function, for the momentum spectrum equation (24), assuming \( M_x = 10 \text{ GeV} \).
4.2.2 X-ray flux

The X-ray intensity produced by the upscattered photons can be calculated from equation (34). In this instance, we replace $\tilde{g}(x)$ by $j(x)$, and then the X-ray flux density is

$$F_{\text{X-ray}}(x) = I_0 j(x) Y,$$  \hspace{1cm} (45)

where we integrate over the solid angle of the whole dSph.

We consider the X-ray emission integrated over a uniform efficiency energy band 0.1–10 keV (corresponding to $x_1 = 4.247 \times 10^3$ and $x_2 = 4.247 \times 10^5$), as an approximation to the current X-ray satellites Chandra and XMM–Newton. Dividing equation (45) by $x$ and integrating over the bandpass yield the X-ray photon flux

$$F_{\text{TOT}}(x_1, x_2) = \frac{I_0 \tilde{Y}}{kT_{\text{CMB}}} \int_{x_1}^{x_2} \frac{j(x)}{x} \, dx.$$  \hspace{1cm} (46)

For $M_\chi = 10 \text{ GeV}$ and $\langle \sigma V \rangle_\Lambda = 10^{-26} \text{ cm}^3 \text{s}^{-1}$, the integral above evaluates to

$$\int_{x_1}^{x_2} \frac{j(x)}{x} \, dx = 0.414.$$  \hspace{1cm} (47)

The results for each of the models considered previously are listed in Table 7. The X-ray fluxes are all of the order of $10^{-12} \text{ cm}^{-2} \text{s}^{-1}$, above current estimates of the gamma-ray flux from direct annihilation in X-rays near $10^5$, with X-ray satellites.

5 CONCLUSIONS

In the above analysis, we have shown the following.

(i) The SZ effect caused by secondary electrons produced from dark matter annihilation in dwarf galaxies (the dSZ effect) proposed by Colafrancesco (2004) could be measurable. The Comptonization parameters averaged over the angular size of the dSph are $\bar{y} \sim 10^{-11}$ for low neutralino masses of $10 \text{ GeV}$ and $\langle \sigma V \rangle_\Lambda = 10^{-26} \text{ cm}^{-1} \text{s}^{-1}$. The temperature decrement for an assumed beam size of 1 arcsec is of the order of milli-Kelvin for extremely cusped dSph halo models and a few tenths of a nano-Kelvin for cored models. This may provide a definitive test between these competing hypotheses, if the signal for cusped model is large enough to be detectable by future radio telescopes. This result holds before the noisy effects of primestellar CMB, radio point sources, and SZ from clusters, have been taken into account. This, however, is only a concern for cored models, in which the signal comes from the bulk of the dSph. Even then, dSPhs are clean and uncontaminated objects, devoid of magnetic fields and gas, and mostly free from point sources such as supernova remnants. For cusped models, most of the signal comes from the very centre, and so contaminating point sources are not a worry.

(ii) Our main aim here has been to demonstrate the feasibility of measuring the dSZ effect. We caution that our calculations make use of an approximate, single-scattering formalism that holds good for low electron optical depth. We may therefore have underestimated the size of the effect in the very innermost regions of cusped dSph models. Further numerical treatments, for example using the methods of Colafrancesco et al. (2003), in the vicinity of dark matter spikes are desirable.

(iii) Upscattered CMB photons lie in the X-ray band, with the emission peak near $x = 2.5 \times 10^5$ for the neutralino mass considered here. Their integrated fluxes are $\sim 10^{-12} \text{ cm}^{-2} \text{s}^{-1}$, comparable in size to that from gamma-rays produced by direct annihilation channels. However, next generation X-ray satellites such as Generation-X (Windhorst et al. 2006), with collecting areas of $\sim 1000 \text{ cm}^2$, may detect such a signal.

(iv) Our assessment of the importance of the dSZ effect is quite optimistic. If dark haloes are strongly cusped, then we conclude that the dSZ effect may be measurable in the near-future by telescopes such as ALMA. However, if dark haloes are only weakly cusped, or if the dark matter particles are heavy ($\gtrsim 50 \text{ GeV}$), then even the most generous integration times with ALMA may not yield a positive detection. None the less, it is worth bearing in mind that here are some circumstances in which a larger effect may be produced. First, the recently discovered very dark dSph Ursa Major (Willman et al. 2005; Kleya et al. 2006) may be the first representative of the

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**Table 7.** List of integrated X-ray fluxes for Draco, in the energy band 0.1–10 keV, for $M_\chi = 10 \text{ GeV}$ and $\langle \sigma V \rangle_\Lambda = 10^{-26} \text{ cm}^{-1} \text{s}^{-1}$.

| $\gamma$   | $F_{\text{TOT}}/10^{-12} \text{ cm}^{-2} \text{s}^{-1}$ |
|------------|------------------------------------------------------|
| 0.5        | 2.604 (2.564)                                        |
| 1.0        | 2.962 (2.914)                                        |
| 1.5        | 1.625 $\times 10^4$ (1.795 $\times 10^4$)            |
| $\alpha$   | $F_{\text{TOT}}/10^{-12} \text{ cm}^{-2} \text{s}^{-1}$ |
| 0.2        | 1.325 (1.301)                                        |
| 0.0        | 1.539 (1.436)                                        |
| $-0.2$     | 1.955 (1.654)                                        |

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missing dark satellites predicted by numerical simulations (Moore et al. 1998). In this case, there may be undetected, very dark dSphs much closer to us than Draco, which is beneficial as the flux received obviously varies like the inverse square of distance. Secondly, our calculations apply only to the case of the neutralino dark matter candidate. There are other possibilities, including light (1–5 MeV) scalar dark matter (Boehm et al. 2004; Hooper et al. 2004) and Kaluza–Klein dark matter (Bertone et al. 2005), whose induced dSZ signals could well be of interest.

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