Vibration Analysis of Roots Rotor System Considering Bearing Clearance

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Abstract. In this paper, the dynamic model of rotor-bearing-gear-cavity system has been established. The dynamic model of bearing has been established by using Hertz contact theory. And the rotating shaft is regarded as the elastic Euler beam. And the rotation response is approximated by the low order modal superposition. The modal truncation method is used to list the differential equation of motion of shaft. The calculation results of the Jeffcott rotor model are compared with the methods described in this paper to verify the accuracy of the new model. The influence of modal intercepting order on the calculation results is analyzed. The appropriate modal intercept order is selected. The influence of the bearing clearance on the shaft response is analyzed by making the waterfall of the rotor response under different bearing clearance. Which provides the theoretical basis for bearing selection and structural improvement.

Preface

The Roots vacuum pump is a volumetric vacuum pump with double rotor [1]. And is a very common device in the factory that uses only mechanical means to obtain a higher vacuum. The meshing theory of the rotor and the design of the rotor profile have been studied, and great progress has been made. But the influence of the bearing parameters on the vibration response of the rotor system is neglected. Bearing the rotor is an important part of supporting the rotor, its parameters will greatly affect the rotor vibration response. Therefore, the design of the bearing parameters is important for reducing the vibration and noise of the Roots pump.

In order to study the effect of bearing parameters on the vibration response of the rotor system, an accurate rotor system model is established, and the effect of the bearing clearance on the rotor response is studied.

Figure 1. Coupling Dynamic Model of Roots Pump.
Dynamic Model of Roots Vacuum Pump

Rotary Shaft Dynamics Model Based On Equal Section Free Beam

Assume that the shaft is symmetrical. The gyro effect is not considered. As shown in Figure 2, it is rotational axis model based on Euler free beam with constant cross section. The coordinate’s o-xyz are located at the leftmost position of the shaft and are consolidated with the shaft. The transverse axis of the axis in the oyz plane and the oxz plane is considered, and the axial displacement is ignored [2].

![Figure 2. Rotational Axis Model Based on Euler Free Beam with Constant Cross Section.](image)

(1) Transverse bending vibration in the oxz plane

The \( x_j(z,t) \) is vibration displacement of the shaft. The elastic modulus of the shaft is \( E \), the moment of inertia of the shaft section is \( I \), the density is \( \rho \), and the cross-sectional area is \( A \). The vibration differential equation of the shaft is:

\[
E I \frac{\partial^4 x_j(z,t)}{\partial z^4} + \rho A \frac{\partial^2 x_j(z,t)}{\partial t^2} = F_{shL}(z-L) + F_{shR}(z) + F_{sp}\delta(z-L/2)
\]

Where, \( F_{sp} = m\omega^2 \cos(\omega t) \) is the rotor eccentric force, \( F_{shL}, F_{shR} \) are bearing forces.

As Eq. (1), this type of equation cannot be directly solved. So the equation should be converted. According to the vibration theory of the beam, the above equation can be converted into a second order differential equation by means of superposition principle. Because the first few modes of the beam can be used to approximate the actual state of motion of the beam, we choose the modal truncation theory. We choose several low-order modes (\( N \)) to approximate the vibration response of the free beam. Therefore, the free beam orthogonal function system is introduced \( X_n \) (\( n=1\sim N \)).

\[
X_1 = 1
\]

\[
X_2 = \sqrt{3}(1-2z/L)
\]

\[
X_n = (\cosh \beta_m z + \cos \beta_m z) - C_m (\sinh \beta_m z + \sin \beta_m z), m > 2
\]

Where, \( C_m \) and \( \beta_m \) is constant. The values of \( C_m, \beta_m \) are shown in Table 1

| \( m \) | 1 | 2 | 3 | 4 | 5 | \( \geq 6 \) |
|---|---|---|---|---|---|---|
| \( C_m \) | - | - | 0.982502 | 1.000777 | 0.999966 | 1.000000 |
| \( \beta_m L \) | 0 | 0 | 4.73004 | 7.853200 | 10.995600 | (m-\( \pi \))/2 |

The transverse bending vibration of the axis in the oxz plane can be approximated as:

\[
x_j(z,t) = \sum_{n=1}^{NM} X_n(z)T_n(t)
\]
Substituting Eq. (3) into Eq. (1) and multiplying $X_p(z)$ on both sides of the equation. In the equation $p=1,2,3...NM$. Where, $z$ is integrated over the full length of the shaft. The differential equation is obtained by using the modal orthogonality and the $\delta$ function of the property.

$$\rho aT_{ws}(t) \int_0^L X_n^2(z)dz + EI T_{ws}(t) \int_0^L X_n^2(z)dz \frac{d^4X_n(z)}{dz^4}dz = F_{shx}X_n(L) + F_{shx}X_n(0) + F_{yp}X_n(L/2) \quad (4)$$

The following formula is obtained from the orthogonality condition of the beam

$$\int_0^L X_n^2(z)dz = L$$
$$\int_0^L X_n^2(z)\frac{d^4X_n(z)}{dz^4}dz = 1 \beta_n^4 \quad (5)$$

Thus equation (4) can be reduced to:

$$\tilde{T}_{ws}(t) + \frac{E I \beta_n^4}{\rho A} T_{ws}(t) = \frac{F_{shx}}{\rho AL} X_n(0) + \frac{F_{shx}}{\rho AL} X_n(L) + \frac{F_{yp}}{\rho AL} X_n(L/2) \quad (6)$$

Where $n = 1 \sim N$. Eq. (6) is the second order ordinary differential equation of the axis in the oxz plane.

(2) Transverse bending vibration in the oyz plane
Assuming that the transverse bending vibration displacement variable of the axis in the oyz plane is $y_n(z,t)$. The vibration differential equation of the shaft is obtained

$$EI \frac{d^4y_n(z,t)}{dz^4} + \rho A \frac{d^2y_n(z,t)}{dt^2} = F_{shx}\delta(z) + F_{shx}\delta(z-L) + F_{yp}\delta(z-L/2) \quad (7)$$

Among them, $F_{yp}=meo \cdot \cos(\omega t)$ is the rotor eccentric force, $F_{yp}=meo \cdot \sin(\omega t), F_{yp}, F_{yp}$, are bearing forces.

In the same way as above, the free beam orthogonal function is introduced. $Y_n(n=1 \sim N)$. The transverse bending vibration of the axis in the oyz plane can be approximated as

$$y_n(z,t) = \sum_{n=1}^{NM} Y_n(z)T_{ym}(t) \quad (8)$$

In the oyz plane, the second-order ordinary differential equations ($n = 1 \sim N$) of the axis model in the mode coordinate are as follows:

$$\tilde{T}_{ym}(t) + \frac{E I \beta_n^4}{\rho A} T_{ym}(t) = \frac{F_{shx}}{\rho AL} Y_n(0) + \frac{F_{shx}}{\rho AL} Y_n(L) + \frac{F_{yp}}{\rho AL} Y_n(L/2) + \frac{\rho}{L} \int_0^L Y_n(z)dz \quad (9)$$

**Rolling Bearing Dynamic Model**

Figure 3 shows the rolling bearing model. Assuming that there is no sliding between the bearing ball and the inner and outer rings of the bearing, only the pure rolling is connected. The balls are equidistantly arranged in the cage[4].
Assume $v_{\text{out}}$ is the linear velocity at the position where the bearing roller is in contact with the bearing outer ring. The $v_{\text{in}}$ is the linear velocity at the position where the bearing roller is in contact with the bearing inner ring. $w_{\text{outer}}$ is the rotation angle of the bearing outer ring. $w_{\text{inner}}$ is the rotation angle of the bearing inner ring. When the bearing roller rotates about the axis, it forms an annular raceway. $R$ is the outer radius of the ring raceway. And $r$ is the inner radius of the ring raceway. We can get that $v_{\text{out}} = w_{\text{outer}} \times R$ and $v_{\text{in}} = w_{\text{inner}} \times r$. The linear speed of the cage (The linear velocity at the center of the ball) is:

$$v_{\text{cage}} = \frac{(v_{\text{out}} + v_{\text{in}})}{2}$$  \hspace{1cm} (10)

As the outer ring is fixed, we can get $v_{\text{out}} = 0$. The angular velocity of the retainer is:

$$\omega_{\text{cage}} = \frac{v_{\text{cage}}}{(R+r)/2} = \frac{(\omega_{\text{outer}} \times r)/2}{(R+r)/2} = \frac{\omega_{\text{outer}} \times r}{R+r}$$  \hspace{1cm} (11)

Since the inner ring is synchronized with the axis rotation and the outer ring is fixed to the cavity, it can be found that $\omega_{\text{outer}} = \omega_{\text{rotor}}$, $\omega_{\text{outer}} = 0$. The number of balls in the bearing is $N_b$. The frequency of the ball rolling is:

$$\omega_{\text{c}} = \omega_{\text{cage}} \times N_b = \omega \times \left( \frac{r}{R+r} \times N_b \right)$$  \hspace{1cm} (12)

Assume that the angular position of the jth ball is $\theta_j$. We can get:

$$\theta_j = \omega_{\text{cage}} \times t + \frac{2\pi}{N_b} (j-1)$$  \hspace{1cm} (13)

Where $j = 1,2,3,\ldots,N_b$.

Assuming that the displacements of the centers of the inner rings in the x and y directions are expressed as x and y, respectively. At the same time, assuming that the bearing clearance is $r_0$. The amount of the normal contact deformation between the jth ball and the raceway is:

$$\delta_j = x \cos \theta_j + y \sin \theta_j - r_0$$  \hspace{1cm} (14)

The contact pressure between the jth ball and the raceway can be obtained according to the Hertz theory.
\[ F_j = C_b \delta_j^2 \cdot H(\delta_j) = C_b (x \cos \theta_j + y \sin \theta_j - e_0)^2 \cdot H(x \cos \theta_j + y \sin \theta_j - e_0) \] (15)

Where \( H(\delta_j) \) is the helix function and \( C_b \) is the Hertz contact stiffness.

The component of \( F_j \) in the x and y directions is:

\[ F_{jx} = F_j \cos \theta_j \]
\[ F_{jy} = F_j \sin \theta_j \] (16)

The bearing force produced by the rolling bearing:

\[ F_x = \sum_{j=1}^{N_b} F_{jx} = \sum_{j=1}^{N_b} F_j \cos \theta_j \]
\[ F_y = \sum_{j=1}^{N_b} F_{jy} = \sum_{j=1}^{N_b} F_j \sin \theta_j \] (17)

So we can get bearing force:

1. \( x = x_r (L, t) - x_{ad}, y = y_r (L, t) - y_{ad}, F_{ad1} = F_x, F_{ad2} = F_y, \)
2. \( x = x_r(0, t) - x_{ad}, y = y_r(0, t) - y_{ad}, F_{std1} = F_x, F_{std2} = F_y. \)

Where \( x_r (L, t) \) is the displacement of the axis in the x direction at the length \( L \). The \( F_{ad1}, F_{ad2}, F_{std1}, F_{std2} \) are the forces of the left and right bearings, respectively. And \( x_{ad}, y_{ad}, y_{ad1}, y_{ad2} \) are the displacement of the left and right bearing outer rings, respectively.

Table 2. Symbol meaning and calculation parameter.

| Parameter name         | Value   |
|------------------------|---------|
| Bearing outer diameter | 85mm    |
| Bearing inner diameter | 45mm    |
| The number of rollers  | 10      |
| Roller diameter        | 10mm    |

This type of Roots vacuum pump uses deep groove ball bearings. The calculation parameters of the bearing are shown in Table 2.

The Vibration Equation of The Rest of the Parts

The rest of the parts are treated as quality points. Using Newton’s law, the vibration equation is derived.

\[ m_{xL} \ddot{x}_{xL} + k_{xL} (x_{xL} - x_{ad}) + c_{xL} (x_{xL} - x_{ad}) + F_{xL} = 0 \]
\[ m_{yL} \ddot{y}_{yL} + k_{yL} (y_{yL} - y_{ad}) + c_{yL} (y_{yL} - y_{ad}) + F_{yL} = -m_{xL} g \]
\[ m_{xR} \ddot{x}_{xR} + k_{xR} (x_{xR} - x_{ad}) + c_{xR} (x_{xR} - x_{ad}) + F_{xR} = 0 \]
\[ m_{yR} \ddot{y}_{yR} + k_{yR} (y_{yR} - y_{ad}) + c_{yR} (y_{yR} - y_{ad}) + F_{yR} = -m_{xR} g \]
\[ m_{xL} \ddot{x}_{xL} + k_{xL} (x_{xL} - x_{ad}) + c_{xL} (x_{xL} - x_{ad}) + F_{xL} = 0 \]
\[ m_{yL} \ddot{y}_{yL} + k_{yL} (y_{yL} - y_{ad}) + c_{yL} (y_{yL} - y_{ad}) + F_{yL} = -m_{xL} g \]
\[ m_{xR} \ddot{x}_{xR} + k_{xR} (x_{xR} - x_{ad}) + c_{xR} (x_{xR} - x_{ad}) + F_{xR} = 0 \]
\[ m_{yR} \ddot{y}_{yR} + k_{yR} (y_{yR} - y_{ad}) + c_{yR} (y_{yR} - y_{ad}) + F_{yR} = -m_{xR} g \]
\[ m_{xL} \ddot{x}_{xL} + k_{xL} (x_{xL} - x_{ad}) + c_{xL} (x_{xL} - x_{ad}) + F_{xL} = 0 \]
\[ m_{yL} \ddot{y}_{yL} + k_{yL} (y_{yL} - y_{ad}) + c_{yL} (y_{yL} - y_{ad}) + F_{yL} = -m_{xL} g \]
\[ m_{xR} \ddot{x}_{xR} + k_{xR} (x_{xR} - x_{ad}) + c_{xR} (x_{xR} - x_{ad}) + F_{xR} = 0 \]
\[ m_{yR} \ddot{y}_{yR} + k_{yR} (y_{yR} - y_{ad}) + c_{yR} (y_{yR} - y_{ad}) + F_{yR} = -m_{xR} g \]
\[ m_{xL} \ddot{x}_{xL} + k_{xL} (x_{xL} - x_{ad}) + c_{xL} (x_{xL} - x_{ad}) + F_{xL} = 0 \]
\[ m_{yL} \ddot{y}_{yL} + k_{yL} (y_{yL} - y_{ad}) + c_{yL} (y_{yL} - y_{ad}) + F_{yL} = -m_{xL} g \]
\[ m_{xR} \ddot{x}_{xR} + k_{xR} (x_{xR} - x_{ad}) + c_{xR} (x_{xR} - x_{ad}) + F_{xR} = 0 \]
\[ m_{yR} \ddot{y}_{yR} + k_{yR} (y_{yR} - y_{ad}) + c_{yR} (y_{yR} - y_{ad}) + F_{yR} = -m_{xR} g \]
Calculation and Testing of Models

To verify the correctness of the methods described in this article. The Jeffcott rotor model and the rotor model described in this paper are respectively solved and the response results are compared. The calculation parameters are shown in Table 3. We make the speed equal to 5Hz.

![Graphs of vibration response](image)

(a) The vibration response of the rotor in the x direction  
(b) The vibration response of the rotor in the y direction  
(c) Frequency diagram of rotor vibration

Figure 4. The vibration response of the rotor is ignored when the shaft quality is neglected.

![Graphs of vibration response](image)

(a) The vibration response of the rotor in the x direction  
(b) The vibration response of the rotor in the y direction  
(c) Frequency diagram of rotor vibration

Figure 5. The vibration response of the rotor when the rotor is regarded as the elastic body Euler beam.

It can be seen that the method of treating the axis as an elastomeric Euler beam is substantially the same as the simulation result obtained by the method mentioned in this paper. And it is consistent with the actual situation. It is proved that the method of treating the rotating shaft as the elastic body Euler beam is accurate and feasible. And the results obtained by the method described in this article are more accurate.
Table 3. Symbol meaning and calculation parameter.

| Parameter                                      | symbol | Value     |
|------------------------------------------------|--------|-----------|
| Cavity (stator) quality                        | $c_m$  | 50.0 [kg] |
| Rotor quality                                  | $m_r$  | 14.059 [kg] |
| Rolling bearing outer ring quality              | $m_{wr}$ | 2.0 [kg] |
| bearing block quality                          | $m_{bl}$ | 10.0 [kg] |
| modulus of elasticity                          | $E$    | 2.07x10^9 [Pa] |
| section inertia                                | $I$    | 5.2x10^4 [m^4] |
| The length of the shaft                        | $L$    | 0.45 [m] |
| The product of the axis density and the cross-sectional area | $\rho A$ | 1.543 x 10^3 [kg/m] |
| The stiffness between the rotating shaft and the disc | $k_r$  | 10^4 [N/m] |
| Damping between rotating shaft and disc         | $c_r$  | 1000 [N·s/m] |
| Stiffness between bearing outer ring and bearing block | $k_i$  | 2.5x10^7 [N/m] |
| The squeeze film damping between the bearing outer ring and bearing block | $c_i$  | 1050 [N·s/m] |
| Stiffness between the cavity and the bearing block | $k_i$  | 2.5x10^7 [N/m] |
| Damping between the cavity and the bearing block | $c_i$  | 2100 [N·s/m] |
| Stiffness between cavity and foundation        | $k_r$  | 2.5x10^7 [N/m] |
| The damping between the cavity and the foundation | $c_r$  | 2100 [N·s/m] |
| The eccentricity of the rotor                   | $e$    | 0.01 [mm] |

Study on the Relationship between Bearing Parameters and Shaft System Vibration Response

Selection of Modal Interception Order

![Figure 6. Effect of NM on Rotor Response.](attachment:image)

(a) Mean square value of displacement      (b) Mean square velocity

It can be seen from Fig. 6 that the root mean square of the displacement tends to stabilize with the increase of the modal intercept order (NM). When NM $\geq 3$, the calculation results have been very accurate. In order to balance the accuracy of computing and the relationship between resources, we select NM equal to 5 to solve the response.

Influence of Bearing Clearance on Rotor Response

The calculation results show that when the bearing clearance is 5µm, the rotor critical speed is about 90Hz. When the bearing clearance increases to 40µm, the first order critical speed is reduced to about 70Hz, which is equivalent to the support stiffness is reduced. In addition, it can be seen in the figure, there are natural frequency about 57Hz. It can be seen, when the bearing clearance is too large,
the system will appear resonance phenomenon. Therefore, the bearing clearance should be minimized in order to reduce the rotor vibration response.

![Graphs showing displacement of waterfall in x direction of rotor for different bearing clearances](image)

(a) bearing clearance 5µm  
(b) bearing clearance 40µm

Figure 7. Displacement of waterfall in x direction of rotor.

**Conclusion**

(1) The dynamic model of Roots pump rotor system is established, The shaft is considered as an elastomer Euler beam; The roller bearing model was established by using the Hertz contact theory; The equations are solved and compared with the results obtained by the Jeffcott rotor model. The method proposed in this paper is verified to be accurate and feasible.

(2) The effect of modal intercept order on the accuracy of calculation is considered. The rotor response is calculated when the bearing clearance is 5um and 40um. From the result we know that changing the bearing clearance is equivalent to reducing the support stiffness. In addition, as can be seen from Figure 7 (b), when the bearing clearance is too large, the system will resonate. Therefore, the bearing clearance should be minimized to reduce the rotor vibration response.

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