Further study on $5q$ configuration states in the chiral SU(3) quark model †

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Abstract

The structure of the $5q$ configuration states with strangeness $S = +1$ is further studied in the chiral SU(3) quark model based on our previous work. We calculate the energies of fifteen low configurations of the $5q$ system, four lowest configurations of $J^\pi = \frac{1}{2}^-$ with $4q$ partition $[4]_{orb}(0s^4)[31]\sigma_f$, four of $J^\pi = \frac{1}{2}^+$ with $4q$ partition $[31]_{orb}(0s^30p)[4]\sigma_f$ and seven of $J^\pi = \frac{1}{2}^+$ with $4q$ partition $[4]_{orb}(0s^30p)[31]\sigma_f$. Some modifications are made in this further study, i.e., the orbital wave function is extended as an expansion of 4 different size harmonic oscillator forms; three various forms (quadratic, linear and error function form) of the color confinement potential are considered; the states with $4q$ partition $[4]_{orb}(0s^30p)[31]\sigma_f$ are added, which are unnegligible in the $J^\pi = \frac{1}{2}^+$ case and were not considered in our previous paper, further the mixing between configurations $[31]_{orb}(0s^30p)[4]\sigma_f$ and $[4]_{orb}(0s^30p)[31]\sigma_f$ is also investigated. The results show that the $T = 0$ state is still always the lowest one for both $J^\pi = \frac{1}{2}^-$ and $J^\pi = \frac{1}{2}^+$ states, and $J^\pi = \frac{1}{2}^-, T = 0$ state is always lower than that of $J^\pi = \frac{1}{2}^+$. All of these modifications can only offer several tens to hundred MeV effect, and the theoretical value of the lowest state is still about 245 MeV higher than the experimental mass of $\Theta^+$. It seems to be difficult to get the calculated mass close to the observed one with the reasonable parameters in the framework of the chiral SU(3) quark model when the model space is chosen as a $5q$ cluster.

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1 Introduction

Last year some Labs. reported that they observed a new resonance state $\Theta$ with positive strangeness $S = +1$ [1, 2]. The mass of this $\Theta$ particle is around $M_\Theta = 1540$ MeV and the upper limit of the width is about $\Gamma_\Theta < 25$ MeV. People suggested this exotic particle as a pentaquark state because its strangeness quantum number is $+1$. Although there are also several negative reports from some other Labs. [2, 3], it has motivated an enormous amount of experimental and theoretical studies in the baryon physics area, because if the existence of this exotic particle can be confirmed, it will be the first multi-quark state people discovered. Actually there have been a lots of theoretical works on the study of the pentaquark baryons with various quark models, e.g. [4, 5, 6, 7, 8], and other approaches [9, 10, 11, 12], but its structure is still a challenging problem.

It is well known that the non-perturbative quantum chromodynamics (NPQCD) effect is very important in the light quark system, but up to now there is no serious approach to really solve the NPQCD problem. In this sense, people still need QCD-inspired models to help in the low energy region. The constituent quark model [13, 14] is quite successful in explaining the baryon spectrum, especially when the non-perturbative QCD effect is considered by introducing the chiral field coupling [14, 15], the Roper resonance and the low excited states of $\Lambda$ can be explained simultaneously. At the same time, the chiral SU(3) quark model can reasonably reproduce the binding energy of deuteron, the nucleon-nucleon ($NN$) and the kaon-nucleon ($KN$) scattering phase shifts of different partial waves, and the hyperon-nucleon ($YN$) cross sections by the resonating group method (RGM) calculations [16, 17, 18, 19]. Inspired by these achievements, we try to extend this model to study the 5 quark system.

In our previous work [7], we calculated the energies of eight low configurations of the $5q$ system, four lowest configurations of $J^\pi = \frac{1}{2}^-$ with 4$q$ partition $[4]_{\text{orb}}(0s^4)[31]^\sigma f$ and four of $J^\pi = \frac{1}{2}^+$ with 4$q$ partition $[31]_{\text{orb}}(0s^30p)[4]^\sigma f$. But the results of the adiabatic approximation calculation show that the mass of the lowest $5q$ cluster state is about $150 - 300$ MeV higher than the observed one of the $\Theta^+$ particle when the model parameters are taken in the reasonable region.

The purpose of this paper is to do a further study on the $5q$ system with strangeness $S = +1$ based on Ref.[7]. Some modifications are made: (1) The orbital wave function is extended as an expansion of the harmonic oscillator form with 4 different
sizes $b_i(i = 1 - 4)$ to improve the adiabatic approximation. (2) Three various forms (quadratic, linear and error function form) of the color confinement potential are considered to examine the effects from the different confinement potential. (3) Seven low lying $J^\pi = \frac{1}{2}^\pm$ states of partition $[4]_{orb}(0s^30p)[31]^{nf}$ are added and the mixing between configurations $[31]_{orb}(0s^30p)[4]^{nf}$ and $[4]_{orb}(0s^30p)[31]^{nf}$ is also studied. Meanwhile, the parameters are chosen three different groups, one is fitted by the $NN$ and $YN$ scattering experimental data [17, 18] and the other two are fitted by the $KN$ scattering phase shifts [19]. The results show that with various groups of parameters and different color confinement potentials, the $T = 0$ state is still always the lowest one for both $J^\pi = \frac{1}{2}^-$ and $J^\pi = \frac{1}{2}^+$ states, and $J^\pi = \frac{1}{2}^-$, $T = 0$ state is always lower than that of $J^\pi = \frac{1}{2}^+$. In addition, the modification of the adiabatic approximation and the confinement potential of error function form can improve the calculated energy by several tens MeV to the lowest state, respectively. When the parameters are taken for the case by fitting the $KN$ phase shifts, the energies of the system are much higher than those for the case by fitting $NN$ and $YN$ data, because the $S$ wave interaction between $K$ and $N$ is repulsive. Some of the $J^\pi = \frac{1}{2}^+$ states with 4q partition $[4]_{orb}(0s^30p)[31]^{nf}$ are lower than those with partition of $[31]_{orb}(0s^30p)[4]^{nf}$ and the mixing between these two configurations can make the results 40 – 90 MeV lower. As a consequence, all of these modifications can offer several tens to hundred MeV effect, and the calculated energy of the lowest state is still about 245 MeV higher than the experimental mass of $\Theta$. It seems that when the model space is chosen as 5q cluster, it is difficult to get the calculated mass close to the observed one by using the chiral quark model with the reasonable parameters.

The paper is arranged as follows. The theoretical framework of the chiral SU(3) quark model and the determination of parameters are briefly introduced in Section 2. The calculation results of different confinement potentials with three groups of parameters are listed and discussed in Section 3. Finally conclusions are drawn in Section 4.

2 Theoretical framework

2.1 The model

As mentioned in Ref [7], the Hamiltonian of the 5q system is written as
\[ H = \sum_i T_i - T_G + \sum_{i<j=1}^4 V_{ij} + \sum_{i=1}^4 V_{i5}, \]  
(1)

where \( \sum_i T_i - T_G \) is the kinetic energy of the system, \( V_{ij}(i, j = 1 - 4) \) and \( V_{i5}(i = 1 - 4) \) represent the interactions between quark-quark \((q - q)\) and quark-antiquark \((q - \bar{q})\) respectively. In the chiral \( SU(3) \) quark model, the quark-quark interaction includes three parts: color confinement potential \( V_{ij}^{conf} \), one gluon exchange (OGE) interaction \( V_{ij}^{OGE} \) and chiral field coupling induced interaction \( V_{ij}^{ch} \),

\[ V_{ij} = V_{ij}^{conf} + V_{ij}^{OGE} + V_{ij}^{ch}, \]  
(2)

where the confinement potential \( V_{ij}^{conf} \), which provides the non-perturbative QCD effect in the long distance, is taken as three different forms in this work, i.e. quadratic, linear, and error function form. And the expression of \( V_{ij}^{OGE} \) is

\[ V_{ij}^{OGE} = \frac{1}{4g_i g_j} \left( \lambda_i^c \cdot \lambda_j^c \right) \left\{ \frac{1}{r_{ij}} - \frac{\pi}{2} \delta (\vec{r}_{ij}) \left( \frac{1}{m_{q_i}^2} + \frac{1}{m_{q_j}^2} + \frac{3}{4} \frac{1}{m_{q_i} m_{q_j}} (\vec{\sigma}_i \cdot \vec{\sigma}_j) \right) \right\} + V_{ij}^{OGE\ tensor} + V_{ij}^{OGE\ \bar{\ell} \cdot \bar{s}}, \]  
(3)

which governs the short-range perturbative QCD behavior. \( V_{ij}^{ch} \) represents the interactions from chiral field couplings and describes the nonperturbative QCD effect of the low-momentum medium-distance range, which can be derived from the chiral-quark coupling interaction Lagrangian

\[ \mathcal{L}_I^{ch} = -g_{ch} F(q^2) \bar{\psi} \left( \sum_{a=0}^8 \sigma_a \lambda_a + i \sum_{a=0}^8 \pi_a \lambda_a \gamma_5 \right) \psi, \]  
(4)

where \( \lambda_0 \) is a unitary matrix, \( \sigma_0, \ldots, \sigma_8 \) are the scalar nonet fields and \( \pi_0, \ldots, \pi_8 \) the pseudoscalar nonet fields. \( \mathcal{L}_I^{ch} \) is invariant under the infinitesimal chiral \( SU(3)_L \times SU(3)_R \) transformation. Thus \( V_{ij}^{ch} \) can be expressed as

\[ V_{ij}^{ch} = \sum_{a=0}^8 V_{sa}(\vec{r}_{ij}) + \sum_{a=0}^8 V_{psa}(\vec{r}_{ij}). \]  
(5)

The expressions of \( V_{sa}(\vec{r}_{ij}) \) and \( V_{psa}(\vec{r}_{ij}) \) can be found in Refs [17, 18].
The interaction between $q$ and $\bar{q}$ includes two parts: direct interaction and annhilation part,

$$V_{q\bar{q}} = V_{q\bar{q}}^{\text{dir}} + V_{q\bar{q}}^{\text{ann}},$$

(6)

$$V_{q\bar{q}}^{\text{dir}} = V_{q\bar{q}}^{\text{conf}} + V_{q\bar{q}}^{\text{OGE}} + V_{q\bar{q}}^{\text{ch}},$$

(7)

with

$$V_{q\bar{q}}^{\text{ch}}(\vec{r}) = \sum_i (-1)^{G_i} V_{q\bar{q}}^{\text{ch},i}(\vec{r}).$$

(8)

Here $(-1)^{G_i}$ describes the $G$ parity of the $i$th meson. For the $\Theta$ particle case without vector meson exchanges, $q\bar{q}$ can only annihilation into a $K$ meson, thus $V_{q\bar{q}}^{\text{ann}}$ can be expressed as

$$V_{q\bar{q}}^{\text{ann}} = V_{q\bar{q}}^{K,\text{ann}},$$

(9)

with

$$V_{q\bar{q}}^{K,\text{ann}} = C_{q\bar{q}}^{K,\text{ann}} \left(1 - \vec{\sigma}_q \cdot \vec{\sigma}_{\bar{q}}\right)_{\text{spin}} \left(2 + 3\lambda_q \cdot \lambda_{\bar{q}}^*\right)_{\text{color}} \left(\frac{19}{9} + \frac{1}{6} \lambda_q \cdot \lambda_{\bar{q}}^*\right)_{\text{flavor}} \delta(\vec{r}_q - \vec{r}_{\bar{q}}),$$

(10)

where we treat $C_{q\bar{q}}^{K,\text{ann}}$ as a parameter and adjust it to fit the mass of the $K$ meson.

In this work, calculations are carried on with three groups of parameters. First, we take the parameters which can reasonably reproduce the experimental data of $NN$ and $NY$ scattering (case I) [17, 18]. By some special constraints, the model parameters are fixed in the following way: the chiral coupling constant $g_{ch}$ is fixed by

$$\frac{g_{ch}^2}{4\pi} = \left(\frac{3}{5}\right)^2 \frac{g_{NN,\pi}^2}{4\pi} \frac{m_u^2}{M_N^2},$$

(11)

with $g_{NN,\pi}^2/4\pi = 13.67$ taken as the experimental value. The mass of the mesons are also adopted the experimental values, except for the $\sigma$ meson, whose mass is treated as an adjustable parameter. $\eta, \eta'$ mesons are mixed by $\eta_0, \eta_8$,

$$\eta' = \eta_8 \sin \theta^{PS} + \eta_0 \cos \theta^{PS},$$
The one gluon exchange coupling constants \( g_u \) and \( g_s \) can be determined by the mass splits between \( N, \Delta \) and \( \Sigma, \Lambda \) respectively. The confinement strengths \( a_{\text{uu}}^c, a_{\text{us}}^c, \) and \( a_{\text{ss}}^c \) are fixed by the stability conditions of \( N, \Lambda, \) and \( \Xi, \) and the zero point energies \( a_{\text{uu}}^0, a_{\text{us}}^0, \) and \( a_{\text{ss}}^0 \) by fitting the masses of \( N, \Sigma, \) and \( \Xi + \Omega, \) respectively.

Another two groups of parameters are fitted by \( KN \) scattering [19], in which the scalar meson mixing between the flavor singlet and octet mesons is considered, i.e. \( \sigma \) and \( \epsilon \) mesons are mixed by \( \sigma_0 \) and \( \sigma_8, \)

\[
\sigma = \sigma_8 \sin \theta_S + \sigma_0 \cos \theta_S,
\]

\[
\epsilon = \sigma_8 \cos \theta_S - \sigma_0 \sin \theta_S.
\]

The mixing angle \( \theta_S \) is an open problem because the structure of the \( \sigma \) meson is unclear and controversial. We adopt two possible values by which we can get reasonable \( KN \) phase shifts, one is ideally mixing \( \theta_S = 35.264^\circ \) (case II), which means that \( \sigma \) only acts on the \( u(d) \) quark, and \( \epsilon \) on the \( s \) quark, the other is \( \theta_S = -18^\circ \) (case III), which is provided by Dai and Wu based on their recent investigation [20].

The three sets of model parameters are tabulated in Table 1, the first column (case I) is for the case fitted by \( NN \) and \( YN \) scattering, and the second (case II) and third (case III) columns for the case fitted by \( KN \) scattering.

### 2.2 The configurations

For the 5\( q \) system with \( S = +1 \), there are a lot of different configurations. In the actually calculating process, we have to choose some lower configurations. Using this model, we considered 15 states of the 5\( q \) system with strangeness \( S = +1 \): four lowest \( J^\pi = \frac{1}{2}^− \) states with \( 4q \) partition \([4]_{\text{orb}}(0s)^4\)

\[
[4]_{\text{orb}}[31]_{ls=01}^{s}\sigma_f, LST = 0\frac{1}{2}0, J^\pi = \frac{1}{2}^-,
\]

\[
[4]_{\text{orb}}[31]_{ls=10}^{s}\sigma_f, LST = 0\frac{1}{2}1, J^\pi = \frac{1}{2}^-,
\]

\[
[4]_{\text{orb}}[31]_{ls=11}^{s}\sigma_f, LST = 0\frac{1}{2}1, J^\pi = \frac{1}{2}^-,
\]

\[
[4]_{\text{orb}}[31]_{ls=21}^{s}\sigma_f, LST = 0\frac{1}{2}2, J^\pi = \frac{1}{2}^-,
\]

and four configurations of \( J^\pi = \frac{1}{2}^+ \) with \([31]_{\text{orb}}(0s)^3(0p)\)

\[
[31]_{\text{orb}}[4]_{ls=00}^{s}\sigma_f, LST = 1\frac{1}{2}0, J^\pi = \frac{1}{2}^+,
\]

\[
[31]_{\text{orb}}[4]_{ls=11}^{s}\sigma_f, LST = 1\frac{1}{2}1, J^\pi = \frac{1}{2}^+,
\]
Table 1: Model parameters. The meson masses and the cut-off masses: $m_{\sigma'} = 980$ MeV, $m_\kappa = 980$ MeV, $m_\epsilon = 980$ MeV, $m_\pi = 138$ MeV, $m_K = 495$ MeV, $m_\eta = 549$ MeV, $m_{\eta'} = 957$ MeV, $\Lambda = 1100$ MeV.

| For $NN,YN$ cases (I) | For $KN$ case | $\theta^S = 35.264^\circ (II)$ | $\theta^S = -18^\circ (III)$ |
|------------------------|----------------|-------------------------------|-------------------------------|
| $b_u$ (fm)             | 0.5            | 0.5                           | 0.5                           |
| $m_u$ (MeV)            | 313            | 313                           | 313                           |
| $m_s$ (MeV)            | 470            | 470                           | 470                           |
| $g_u$                  | 0.886          | 0.886                         | 0.886                         |
| $g_s$                  | 0.755          | 0.755                         | 0.755                         |
| $m_\sigma$ (MeV)      | 595            | 675                           | 675                           |
| $\ast a^c_{uu}$ (MeV/fm$^2$) | 48.1        | 52.4                          | 55.2                          |
| $\ast a^c_{us}$ (MeV/fm$^2$) | 60.7        | 72.3                          | 68.4                          |
| $\ast a^0_{uu}$ (MeV) | -43.6          | -50.4                         | -55.1                         |
| $\ast a^0_{us}$ (MeV)  | -38.3          | -54.2                         | -48.7                         |
| $\diamond a^c_{uu}$ (MeV/fm) | 86.6        | 95.1                          | 100.6                         |
| $\diamond a^c_{us}$ (MeV/fm) | 101.0        | 121.7                         | 114.7                         |
| $\diamond a^0_{uu}$ (MeV) | -75.6        | -86.0                         | -93.1                         |
| $\diamond a^0_{us}$ (MeV) | -72.3        | -96.5                         | -88.2                         |
| $\ast a^c_{uu}$ (MeV)  | 194.4          | 213.3                         | 225.7                         |
| $\ast a^c_{us}$ (MeV)  | 218.2          | 262.9                         | 247.8                         |
| $\ast a^0_{uu}$ (MeV)  | -87.4          | -99.0                         | -106.8                        |
| $\ast a^0_{us}$ (MeV)  | -82.8          | -109.1                        | -100.0                        |

Here $a^c_{uu}$, $a^c_{us}$, $a^0_{uu}$, $a^0_{us}$ with symbol ‘$\ast$’ are for the quadratic form color confinement, those with ‘$\diamond$’ are for the linear form confinement, and those with ‘$\star$’ are for the error function form confinement.
Here the symbols \([f]_{\text{orb}}\) and \([f']_{\sigma f}\) are the partitions of orbital space and flavor-spin space respectively; \([4]_{\text{orb}}\) represents the total symmetric state in the orbital space, where four quarks are all in \((0s)\) state; and \([31]_{\text{orb}}\) is the orbital space partition of \((0s)^3(0p)\). Symbol \(t\) and \(s\) denote the isospin and spin of the four quark part; after coupling with the fifth quark \(\bar{s}\), the total orbital angular momentum, spin and isospin of the five quark system are expressed as \(LST\). The color part is \((10)_c\) of \(4q\) and \((01)_c\) of the anti-quark \(\bar{s}\) respectively, here we omitted them in the expressions.

Meanwhile, there are some other \(J^\pi = \frac{1}{2}^+\) states (we did not consider them in our previous work [7]) in which the \(\bar{s}\) can be in the \((0p)\) state. The wave functions of these states should be orthogonal to the excited states of the center of mass motion. The expression of the wave function with various \(ts\) and \(LST\), \(\Psi_{ts}^{LST}(5q)\), is given as following

\[
\Psi_{ts}^{LST}(5q) = \sqrt{\frac{m_s}{4m_u + m_s}} \left( \Psi_{4q}((0s)^3(0p))[4]_{\text{orb}}[31]_{ts}^{\sigma f}[211]_{c}\Phi_{0s}(\bar{s}) \right)_{LST}
- \sqrt{\frac{4m_u}{4m_u + m_s}} \left( \Psi_{4q}(0s^4)[4]_{\text{orb}}[31]_{ts}^{\sigma f}[211]_{c}\Phi_{0p}(\bar{s}) \right)_{LST}.
\]

Where in the first term, the \(4q\) orbital wave function is \((0s)^3(0p)\) with partition \([4]_{\text{orb}}\) and \(\bar{s}\) is in \((0s)\), while in the second term, the \(4q\) orbital wave function is \((0s)^4\) with partition \([4]_{\text{orb}}\) and \(\bar{s}\) is in \((0p)\). \([31]_{ts}^{\sigma f}\) is the \(4q\) partition of spin-flavor space. The total orbital angular momentum, spin and isospin of the five quark system are \(LST\). We considered seven low states of \(\Psi_{ts}^{LST}(5q)\), they are

\[
[4]_{\text{orb}}[31]_{ts=01}^{\sigma f}\bar{s}, LST = 1\frac{3}{2}0, J^\pi = \frac{1}{2}^+,
[4]_{\text{orb}}[31]_{ts=10}^{\sigma f}\bar{s}, LST = 1\frac{3}{2}1, J^\pi = \frac{1}{2}^+,
[4]_{\text{orb}}[31]_{ts=11}^{\sigma f}\bar{s}, LST = 1\frac{1}{2}1, J^\pi = \frac{1}{2}^+,
[4]_{\text{orb}}[31]_{ts=21}^{\sigma f}\bar{s}, LST = 1\frac{1}{2}2, J^\pi = \frac{1}{2}^+,
[4]_{\text{orb}}[31]_{ts=01}^{\sigma f}\bar{s}, LST = 1\frac{3}{2}0, J^\pi = \frac{1}{2}^+,
[4]_{\text{orb}}[31]_{ts=11}^{\sigma f}\bar{s}, LST = 1\frac{3}{2}1, J^\pi = \frac{1}{2}^+,
[4]_{\text{orb}}[31]_{ts=21}^{\sigma f}\bar{s}, LST = 1\frac{3}{2}2, J^\pi = \frac{1}{2}^+.
\]

In order to improve the adiabatic approximation, in this work the so-called "breath model" is taken to carry on the energy calculation. The trail wave function can be written as an expansion of the \(5q\) states with several different harmonic oscillator frequency \(\omega_i\),
\[ \Psi_{5q} = \sum_i^n \alpha_i \Phi_{5q}(b_i). \]  

(15)

Where \((b_i)^2 = \frac{1}{m_{q_i}}\). Using the fractional parentage coefficient (f.p.) technique, we can easily write down the wave functions of the above 15 states and by solving the Schrödinger equation the energies of these states are obtained.

### 3 Results and discussions

We calculate energies of fifteen low configurations, four lowest configurations of \(J^\pi = \frac{1}{2}^-\) with 4q partition \([4]_{\text{orb}}(0s^4)[31]^{\sigma f}\), four of \(J^\pi = \frac{1}{2}^+\) with 4q partition \([31]_{\text{orb}}(0s^30p)[4]^{\sigma f}\) and seven low configurations of \(J^\pi = \frac{1}{2}^+\) with 4q partition \([4]_{\text{orb}}(0s^30p)[31]^{\sigma f}\). Comparing with our previous work [7], some modifications are made in this work. The trial wave function is taken as an expansion of harmonic oscillator wave functions with four different size parameter \(b_i (i = 1 - 4)\) to improve the adiabatic approximation. Three different forms (quadratic, linear and error function form) of the confinement potential are considered to study the effects from various confinement potentials. More low lying states of \(J^\pi = \frac{1}{2}^-\) with 4q partition \([4]_{\text{orb}}(0s^40p)\) and the mixing between some configurations are considered. At the same time, parameters are chosen three different groups as listed in Table 1. The results are given in Table 2. In this table, case I means the parameters are fitted by NN and YN scattering, while case II by KN scattering with \(\theta^S = 35^\circ\) and case III by KN scattering with \(\theta^S = -18^\circ\). And \(r^2, r\) and erf represent the confinement potential is adopted as quadratic, linear and error function form respectively.

From Table 2, one can see that: (1) the isoscalar state \(T = 0\) is always the lowest state both in \(J^\pi = \frac{1}{2}^-\) and in \(J^\pi = \frac{1}{2}^+\) cases, and \([4]_{\text{orb}}[31]^{\sigma f}_{ls=01\bar{s}}, LST = 0\frac{1}{2}0, J^\pi = \frac{1}{2}^-\) is the lowest one among all the states. For two sets of \(J^\pi = \frac{1}{2}^+\) states, some configurations with 4q partition \([4]_{\text{orb}}, especially ([4]_{\text{orb}}[31]^{\sigma f}_{ls=01\bar{s}}, LST = 1\frac{1}{2}0, J^\pi = \frac{1}{2}^+),\) are about \(100 - 200\) MeV lower than those of \([31]_{\text{orb}}\) in various cases. It means that in \(J^\pi = \frac{1}{2}^+\) state, the configurations with 4q partition \([4]_{\text{orb}}\) can not be neglected. (2) The effect of various confinement potentials is about several tens MeV. The energies obtained from the error function confinement potential are the lowest, and the influence is larger to the high energy configurations. (3) The energies of case I are lower than those of the other two cases, this is obvious, because in the NN case the S wave interaction is
Table 2: Energies (in MeV) of fifteen configurations with 4 different size $b_i (i = 1 - 4)$.

| configurations | case I | case II | case III |
|----------------|--------|---------|----------|
| $J^\pi = \frac{1}{2}^-$ | $r^2$ | $r$ | $erf$ | $r^2$ | $r$ | $erf$ | $r^2$ | $r$ | $erf$ |
| $(\[4\]_{orb}[31]^\sigma_f[^{00}_{ls=01}\bar{s}])_{\frac{1}{2}0}$ | 1799 | 1790 | 1785 | 1901 | 1890 | 1884 | 1897 | 1886 | 1880 |
| $(\[4\]_{orb}[31]^\sigma_f[^{10}_{ls=10}\bar{s}])_{\frac{1}{2}1}$ | 2085 | 2061 | 2044 | 2155 | 2127 | 2107 | 2156 | 2128 | 2108 |
| $(\[4\]_{orb}[31]^\sigma_f[^{11}_{ls=11}\bar{s}])_{\frac{1}{2}1}$ | 2150 | 2120 | 2098 | 2219 | 2184 | 2159 | 2220 | 2185 | 2160 |
| $(\[4\]_{orb}[31]^\sigma_f[^{21}_{ls=21}\bar{s}])_{\frac{1}{2}2}$ | 2353 | 2294 | 2251 | 2417 | 2352 | 2305 | 2420 | 2355 | 2308 |
| $J^\pi = \frac{1}{2}^+$ | $r^2$ | $r$ | $erf$ | $r^2$ | $r$ | $erf$ | $r^2$ | $r$ | $erf$ |
| $(\[31\]_{orb}[4]^\sigma_f[^{00}_{ls=00}\bar{s}])_{\frac{1}{2}0}$ | 2153 | 2129 | 2114 | 2221 | 2193 | 2174 | 2206 | 2179 | 2162 |
| $(\[31\]_{orb}[4]^\sigma_f[^{11}_{ls=11}\bar{s}])_{\frac{1}{2}1}$ | 2257 | 2221 | 2198 | 2323 | 2282 | 2255 | 2308 | 2269 | 2243 |
| $(\[31\]_{orb}[4]^\sigma_f[^{22}_{ls=22}\bar{s}])_{\frac{1}{2}2}$ | 2337 | 2291 | 2257 | 2409 | 2354 | 2317 | 2391 | 2339 | 2304 |
| $J^\pi = \frac{1}{2}^-$ | $r^2$ | $r$ | $erf$ | $r^2$ | $r$ | $erf$ | $r^2$ | $r$ | $erf$ |
| $(\[4\]_{orb}[31]^\sigma_f[^{00}_{ls=00}\bar{s}])_{\frac{1}{2}0}$ | 2071 | 1958 | 1917 | 2110 | 1975 | 1924 | 2123 | 1994 | 1945 |
| $(\[4\]_{orb}[31]^\sigma_f[^{10}_{ls=10}\bar{s}])_{\frac{1}{2}1}$ | 2239 | 2105 | 2048 | 2267 | 2110 | 2043 | 2283 | 2132 | 2066 |
| $(\[4\]_{orb}[31]^\sigma_f[^{11}_{ls=11}\bar{s}])_{\frac{1}{2}1}$ | 2294 | 2152 | 2089 | 2322 | 2157 | 2083 | 2337 | 2178 | 2107 |
| $(\[4\]_{orb}[31]^\sigma_f[^{21}_{ls=21}\bar{s}])_{\frac{1}{2}2}$ | 2458 | 2285 | 2198 | 2485 | 2289 | 2192 | 2501 | 2311 | 2216 |
| $(\[4\]_{orb}[31]^\sigma_f[^{01}_{ls=01}\bar{s}])_{\frac{1}{2}0}$ | 2275 | 2138 | 2077 | 2300 | 2140 | 2070 | 2317 | 2162 | 2094 |
| $(\[4\]_{orb}[31]^\sigma_f[^{11}_{ls=11}\bar{s}])_{\frac{1}{2}1}$ | 2270 | 2132 | 2071 | 2298 | 2137 | 2066 | 2313 | 2158 | 2090 |
| $(\[4\]_{orb}[31]^\sigma_f[^{21}_{ls=21}\bar{s}])_{\frac{1}{2}2}$ | 2253 | 2200 | 2127 | 2383 | 2205 | 2133 | 2398 | 2227 | 2146 |

Configurations are express as: $(\[f\]_{orb}[f']^\sigma_f[^{ls}_{ls}\bar{s}])_{LST}$, in which $LST$ are the total orbital angular momentum, spin and isospin of the five quark system.
Table 3: Comparison between the energies with 1 \( b \) (adiabatic approximation) and 4 \( b \) of three lowest configurations of \( J^\pi = \frac{1}{2}^- \) and \( J^\pi = \frac{1}{2}^+ \) by taking the parameters of case I, which are fitted by the \( NN \) and \( YN \) scattering experimental data.

|                      | \( r^2 \) | \( r \) | \( erf \) | \( r^2 \) | \( r \) | \( erf \) |
|----------------------|---------|-------|----------|---------|-------|----------|
|                      | 1 \( b \)* (MeV) | 4 \( b \) (MeV) |
| \([4]_{orb}[31]_{ls=01}^{af} S_{0\frac{1}{2}}\) \( J^\pi = \frac{1}{2}^- \) | 1821   | 1820   | 1820    | 1799   | 1790   | 1785     |
| \([31]_{orb}[4]_{ls=00}^{af} S_{1\frac{1}{2}}\) \( J^\pi = \frac{1}{2}^+ \) | 2162   | 2145   | 2135    | 2153   | 2129   | 2114     |
| \([4]_{orb}[31]_{ls=01}^{af} S_{1\frac{1}{2}}\) \( J^\pi = \frac{1}{2}^- \) | 2079   | 1973   | 1936    | 2071   | 1958   | 1917     |

* Here the energies with 1\( b \) are a little bit higher than those in Ref. [7]. This is because that the annihilation to \( K^* \) is not considered in our present calculation, but it is involved in Ref. [7].

Table 4: Comparison between the energies without and with configuration mixing between two \( J^\pi = \frac{1}{2}^+ \) states by taking the parameters of case I.

|                      | without mixing | with mixing |
|----------------------|----------------|-------------|
|                      | \( r^2 \) (MeV) | \( r \) (MeV) | \( erf \) |
|                      | \( r^2 \) (MeV) | \( r \) (MeV) | \( erf \) |
| \( LST = 1{\frac{1}{2}} \) \( 1 \) | 2203 | 2152 | 2089 | 2165 | 2054 | 2018 |
| \( LST = 1{\frac{1}{2}} \) \( 2 \) | 2294 | 2173 | 2154 | 2331 | 2271 | 2225 |
| \( LST = 1{\frac{3}{2}} \) \( 1 \) | 2257 | 2132 | 2071 | 2220 | 2082 | 2032 |
| \( LST = 1{\frac{3}{2}} \) \( 2 \) | 2270 | 2221 | 2198 | 2306 | 2271 | 2237 |

In the column with the configuration mixing, the third row gives two groups of results with considering the mixing between \([31]_{orb}[4]_{ls=11}^{af} S_{1\frac{1}{2}}\) \( 1 \) and \([4]_{orb}[31]_{ls=11}^{af} S_{1\frac{1}{2}}\) \( 1 \), and the forth row gives those with considering the mixing between \([31]_{orb}[4]_{ls=11}^{af} S_{1\frac{1}{2}}\) \( 1 \) and \([4]_{orb}[31]_{ls=11}^{af} S_{1\frac{1}{2}}\) \( 1 \).
Table 5: Comparison between the energies with $b_u = 0.5$ fm and $b_u = 0.6$ fm

| $r^2$ | $r$ | $erf$ | $r^2$ | $r$ | $erf$ |
|-------|-----|-------|-------|-----|-------|
| $b_u = 0.5$ fm (MeV) | $b_u = 0.6$ fm (MeV) |
| ([4]$_{orb}$[31]$^f_{ts=01\bar{s}}$)$_{0\frac{1}{2}}$; $J^\pi = \frac{1}{2}^-$ | 1799 | 1790 | 1785 | 1668 | 1666 | 1665 |
| ([31]$_{orb}$[4]$^f_{ts=00\bar{s}}$)$_{1\frac{1}{2}0}$; $J^\pi = \frac{1}{2}^+$ | 2153 | 2129 | 2114 | 1976 | 1968 | 1963 |
| ([4]$_{orb}$[31]$^f_{ts=01\bar{s}}$)$_{1\frac{1}{2}0}$; $J^\pi = \frac{1}{2}^+$ | 2071 | 1958 | 1917 | 1887 | 1878 | 1875 |

attraction, while in the $KN$ case it is repulsive.

The comparison between the results of adiabatic approximation (the wave function is taken as harmonic oscillator function with 1$b$) and those of the "breath model" (the wave function is treated as an expansion of the harmonic oscillator functions with 4 different frequency $\omega$) is shown in Table 3. Here the parameters are taken as case I, which is fitted by $NN$ and $YN$ scattering data. The results show that this modification can only reduce the energies about 10–30 MeV for different configurations. This means that even the trial wave function is modified, the energy of the 5$q$ system can not be improved a lot.

In the case of $J^\pi = \frac{1}{2}^+$, ([31]$_{orb}$[4]$^f_{ts=11\bar{s}}$)$_{1\frac{1}{2}}$ and ([4]$_{orb}$[31]$^f_{ts=11\bar{s}}$)$_{1\frac{1}{2}1}$ as well as ([31]$_{orb}$[4]$^f_{ts=11\bar{s}}$)$_{1\frac{1}{2}1}$ and ([4]$_{orb}$[31]$^f_{ts=11\bar{s}}$)$_{1\frac{1}{2}1}$ have the same quantum numbers. The configuration mixing has to be considered for these two cases. The results comparing with those without configuration mixing are given in Table 4. From this table, one can see that the configurations mixing effect is not very small, it can make energies about 40–90 MeV lower.

We also try to adjust the size parameter $b_u$ to be larger to see the influence. As an example, the results of lower configurations with $b_u = 0.6$ fm in the chiral SU(3) quark model are given compared with the former results with $b_u = 0.5$ fm in Table 5. In this case, the energies of all states become smaller, caused by the kinetic energy of the system is reduced for larger $b_u$. But the lowest energy 1665 MeV is still about 125 MeV higher than the experimental mass of the observed $\Theta$.

In addition, we notice that recently Sachiko Takeuchi et al. [8] investigated $uudd\bar{s}$ pentaquarks by employing their quark models with the meson exchange and the effective gluon exchange as $qq$ and $q\bar{q}$ interactions, and dynamically solved the system
by taking two quarks as a diquark-like $qq$ correlation. Their calculated value of the lowest configuration was $1947 \pm 2144$ MeV and the low mass close to the observed one could not be obtained, which was similar to our results. The main difference is that in Ref. [8] there were parameter sets where the mass of the lowest positive-parity states became lower than that of the negative-parity states. However, in our work, where the model parameters are fitted by the $NN,YN$ and $KN$ scattering phase shifts and the $q\bar{q}$ $s$-channel interactions are fitted by the mass of the kaon meson, the results show that the $J^\pi = \frac{1}{2}^-$, $T = 0$ state is always the lowest one. As discussed in our previous work [7], if we omitted the interactions between $4q$ and $\bar{q}$, then the state with positive parity can be lower than that with negative parity, in agreement with what is claimed by Takeuchi and Shimizu [8], and Stancu and Riska [4]. This means that how to treat the annihilation interactions reasonably is very important in the calculation.

Meanwhile, it is important to investigate the narrow width as well as the low mass of the $\Theta$ particle. But in our present work, we concentrate our study on the masses of some low-lying $5q$ configurations with various quantum numbers. Since the coupling between $5q$ configuration states and continuum baryon-meson states such as $KN$ is not included, we can only get some qualitative information about the width from the wave functions. For example, by using the group theory method[21] the lowest state of $J^\pi = \frac{1}{2}^-$ and $T = 0$, $\Psi([4]_{orb}(0s)^4[31]_{ts=01}\bar{s}(0s))_{LST=040,(00)c}$, can be expanded as:

$$\Psi([4]_{orb}(0s)^4[31]_{ts=01}\bar{s}(0s))_{LST=040,(00)c} =$$

$$-\frac{1}{2} \left( \Phi_{123}(s=\frac{1}{2},t=\frac{1}{2})(11)_c \right) \left( \Phi_{45}(s=0)(11)_c \right)$$

$$-\frac{1}{2} \sqrt{\frac{1}{3}} \left( \Phi_{123}(s=\frac{1}{2},t=\frac{1}{2})(11)_c \right) \left( \Phi_{45}(s=1)(11)_c \right)$$

$$+\sqrt{\frac{1}{3}} \left( \Phi_{123}(s=\frac{1}{2},t=\frac{1}{2})(11)_c \right) \left( \Phi_{45}(s=1)(11)_c \right)$$

$$+\frac{1}{2} \left( \Phi_{123}(s=\frac{1}{2},t=\frac{1}{2})(00)_c \right) \left( \Phi_{45}(s=0)(00)_c \right)$$

$$+\frac{1}{2} \sqrt{\frac{1}{3}} \left( \Phi_{123}(s=\frac{1}{2},t=\frac{1}{2})(00)_c \right) \left( \Phi_{45}(s=1)(00)_c \right),$$

where the wave function is antisymmetrized and all terms are orthogonal and linearly independent each other. In the above expression (Eq.(16)), the fourth term is the component of $S$-wave $KN$, whose probability is $(\frac{1}{2})^2 = \frac{1}{4}$. Such a large (25%) $KN$ component would consequently make the width of this state to be quite large. In this
sense the $5q$ cluster $\frac{1}{2}^+$ state can be expected to have a smaller width. However in our present work, when the model parameters are fitted by the $NN,YN$ and $KN$ scattering phase shifts, the results show that the masses of the positive-parity states are higher than those of the negative-parity states, also much higher than the experimental value. In this framework, it is difficult to understand both the low mass and the narrow width of the $\Theta$. But up to now, the existence of the $\Theta$ particle is still a controversial problem, and some high-statistics experimental collaborations showed the negative results. From our results, we can just say that if the $\Theta$ particle do exist, it can not be explained as a five-constituent-quark cluster, and its structure should be understood from other mechanism.

4 Conclusions

The structures of $5q$ cluster states with $S = +1$ are further studied based on the $5q$ cluster configurations studied in the SU(3) chiral quark model. Fifteen low configurations and the mixing between some configurations are considered. In the calculation, the trial wave function is taken as an expansion of harmonic oscillator wave functions with four different size parameters. The effect of various color confinement potential is examined by taken three different forms (quadratic, linear and error function forms). With various groups of parameters and different color confinement potentials, the isoscalar state $T = 0$ is always the lowest state both in $J^{\pi} = \frac{1}{2}^-$ and in $J^{\pi} = \frac{1}{2}^+$ cases. And $J^{\pi} = \frac{1}{2}^-$, $T = 0$ is the lowest one among all of these configurations. The modification of the adiabatic approximation and the confinement potential of error function form can improve the calculated energy by several tens MeV to the lowest state, respectively. Since the $S$ wave interaction between $K$ and $N$ is repulsive, when the parameters are taken for the case by fitting the $KN$ phase shifts, the energies of the system are much higher than those for the case by fitting $NN$ and $YN$ data. The configurations mixing of $J^{\pi} = \frac{1}{2}^+$ states can make the calculated mass $40 - 90$ MeV shifted. If we adjust the size parameter larger, $b_u = 0.6$ fm, the energy of the lowest configuration will be 1665 MeV. All of these modifications can only offer several tens to hundred MeV improvement, and the calculated value of the lowest state is still about 125 MeV higher than the experimental mass of $\Theta$. It seems that when the model space is chosen as $5q$ cluster, it is difficult to get the calculated mass close to the observed one in the framework of the chiral quark model with the reasonable parameters. Though
the existence of the \( \Theta \) particle is still an open problem, this work just presents that it can not be regarded as a five-constituent-quark cluster, if it do exist, its structure should be explained by other mechanism.

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