Estimation of local temperature dependent heat transfer coefficient for dynamic thermal analysis of electronic circuits

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Abstract. The value of the surface heat transfer coefficient depends strongly on temperature. This phenomenon is one of the main sources of nonlinearities arising in the dynamic thermal simulation of electronic systems. This paper presents the problem of estimating the temperature dependence of this coefficient based on a practical example of a hybrid circuit. The estimation is carried out by the coupling of the forward Green’s function solver with the inverse solver using the modified Landweber iteration.

1. Introduction

The increased density of power dissipated in modern electronic systems forces engineers to seek for new ways of thermal analysis of such systems. The heat transfer processes are modelled by the partial differential heat equation [1]-[2]. In most cases, thermal simulations are carried out for the linear heat equation, where the thermal model parameters do not depend on temperature. On the other hand, many parameters appearing in the heat equation are inherently temperature dependent. Moreover, quite often the exact values of these parameters are extremely difficult to be determined based on some theoretical considerations, e.g. due to the complex problem geometry or because they are a function of multiple factors, such as the water temperature and flow rate, level of turbulence, etc. [3].

The research carried out over the past decades demonstrated clearly that the main source of the nonlinearities is the spatial and temporal variation of the heat transfer coefficient, which reflects the heat exchange with the ambient at the outer surfaces of electronic systems [4]. The usual engineering practice is to assume in thermal simulations some average value of the coefficient estimated from the measurements in the steady state. However, electronic circuits rarely operate in the steady state and proper simulation results can be obtained only by considering the local instantaneous values of the heat transfer coefficient which in turn are strongly temperature dependent and might vary for the same circuit in the considered temperature range by more than an order of magnitude. The considerations presented here are based on the temperature measurements of a hybrid module, whose operation was investigated in different environmental conditions, that is with the forced water and still air cooling. Additionally, the position of the entire assembly and the coolant temperature was varied during the measurements. The next section of this paper formulates rigorously the problem under consideration and presents the solution methods applied later on for the numerical experiments.

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2. Heat equation

This section, as mentioned previously, is devoted to the mathematical formulation of the particular problem of the estimation of temperature dependent heat transfer coefficient. Subsequently the direct and inverse problems are given. Finally, the solution algorithm for the inverse problem is proposed. All the presented here mathematical apparatus will be employed in the next section for the numerical experiments.

2.1. Direct problem

Let $\Omega$ be a domain in $\mathbb{R}^3$ representing an electronic structure. The temperature distribution $T$ in the structure can be described by the following equation [1]-[3]:

$$\rho(x)c_p(x)\frac{\partial}{\partial t}T(x, t) = \text{div}(\lambda(x)\nabla T(x, t)), \quad (1)$$

where $\rho$, $c_p$, and $\lambda$ are the material density, the specific heat and the thermal conductivity respectively. Additionally, the initial condition is

$$T(x, 0) = 0; \quad x \in \Omega;$$

and the boundary conditions are adiabatic ones on the lateral surfaces

$$\lambda \frac{\partial}{\partial n}T(x, t) = 0; \quad x \in \Omega_1;$$

on the top surface the mixed Neuman-Robin boundary conditions are prescribed

$$\lambda \frac{\partial}{\partial n}T(x, t) = -h_T(T(x, t))(T(x, t)) + q(x, t); \quad x \in U;$$

and finally on the bottom surface the pure Robin condition is imposed

$$\lambda \frac{\partial}{\partial n}T(x, t) = -h_B(T(x, t))T(x, t); \quad x \in B;$$

where $U$, $B$ are the top and the bottom surfaces where the heat transfer occurs. Here, $q$ is a given heat source strength and $h_T$ $h_B$ are the heat transfer coefficients on the top and the bottom surfaces respectively. The temperature $T$ is always considered as the difference to the ambient temperature.

The direct problem consists typically in computing the temperature distribution in a structure, given the problem geometry, the material properties, the heat transfer coefficient values and the heat source strength. Note that here the heat transfer coefficients $h$ depend on temperature, which makes the direct problem a nonlinear one. Throughout this paper the direct problem is solved employing the Green's function method [5] and the nonlinearity is treated by the fixed-point iteration [6].

2.2. Green's function solution

The solution of the direct problem in terms of the Green’s function can be formulated as follows. Let $h_{G, U}$ $h_{G, B}$ be some a-priori chosen constants related to the heat transfer at the top and bottom surfaces and $G(x, t; x', t')$ be the Green’s function associated to the heat equation (1) with adiabatic lateral boundary conditions and Robin type homogenous boundary conditions at the top and bottom:

$$\lambda \frac{\partial}{\partial n}T(x, t) = -h_{G, U}T(x, t) \text{ on } U$$

$$\lambda \frac{\partial}{\partial n}T(x, t) = -h_{G, B}T(x, t) \text{ on } B$$

For the considered three-dimensional rectangular geometry, the Green’s functions can be computed explicitly. Moreover, this approach can be generalized to multilayered structures [7] and the Green’s functions can be used to find an implicit expression for the temperature distribution $T$ [8]. Indeed, from the equation describing the heat transfer, temperature $T$ satisfies for all $x \in \Omega$ and $t \geq 0$
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\[ T(x, t) = \int_0^t \int_U G(x, t; x', t') \left[ q(x', t') - \left( h_U(T(x', t') - h_{G,U})T(x', t') \right) \right] dx' dt' + \\
+ \int_0^t \int_B G(x, t; x', t') (q(x', t') - (h_B(T(x', t')) - h_{G,B})T(x', t')) dx' dt' \]

Note that the above correction terms involving the constants \( h_{G,U}, h_{G,B} \) come from the arbitrary choice of the boundary conditions for the Green’s function. When the temperature observation point \( x \) is set on the top surface \( U \), it is possible to find on the bottom surface \( B \) a system of fixed point equations involving only the temperature values at these two surfaces. Then, the nonlinear problem can be solved by a fixed-point iteration choose the initial temperature \( T(x, 0) = 0 \) and computing the \( n+1 \)-th temperature update as

\[ T_{n+1}(x, t) = \int_0^t \int_U G(x, t; x', t') \left[ q(x', t') - \left( h_U(T_n(x', t') - h_{G,U})T_n(x', t') \right) \right] dx' dt' + \\
+ \int_0^t \int_B G(x, t; x', t') (q(x', t') - (h_B(T_n(x', t')) - h_{G,B})T_n(x', t')) dx' dt' \]

for \( x \in T \) and for \( x \in B \). When \( h_B \) and \( h_U \) are differentiable functions of temperature, whose values and derivatives are close to the a-priori assumed constants \( h_{G,B} \) and \( h_{G,U} \), this iteration yields a contraction and hence the procedure will converge to the unique fixed point, which determines the temperature field solving the nonlinear problem.

Note that here the iteration was defined in such a way that in each step the temperature is solved for the whole time interval. A more advanced algorithm was proposed in [9], where the iteration is done in small time intervals and then, after the convergence is reached, the time is increased and the fixed-point iteration is performed again. This solution is probably more accurate, but here the particular implementation was chosen because of the computational speed and simplicity.

2.3. Inverse problem

Considering the thermal models of electronic systems their geometry and the material thermo-physical properties are usually known. The only model parameter, whose value is not certain, is the temperature dependent heat transfer coefficient. Thus, this dependence should be determined based on the data available from experiment, which in turn leads to the inverse problem of identifying the heat transfer coefficient (in particular its dependence on temperature) on top and bottom surfaces \( h = (h_T, h_U) \) from temperature measurements available at the set of \( m \) points \( x_m = (x_1, x_m) \) located on the top surface. Thus, the data are given as

\[ y(t) = T(x_m, t) \]

In order to formulate the mathematical inverse problem in a rigorous way, the following parameter-to-data mapping is used

\[ F: h \rightarrow y \]

where \( h \) denotes the unknown heat transfer coefficients (two functions) and the operator \( F \) maps these parameters to the temperature field. Thus, an evaluation of the operator \( F(h) \) means computing the nonlinear direct problem stated above. The inverse problem can now be formulated as solving the operator equation \( F(h) = y \) for the heat transfer coefficient \( h \) given the temperature data \( y \).

The main difficulty in solving inverse problems consists in their ill-posedness. This means that the operator equation cannot be inverted in a stable way, i.e. \( F^{-1}(y) \) is not continuous. To circumvent this difficulty some kind of regularisation has to be applied. Then, instead of the original problem, some related, but stable neighbouring problem is solved [10]. There exists several choices to perform the regularization for nonlinear problems, including the best known Tikhonov regularization, which is based on the minimization of a functional. Here, an iterative Landweber-type method is proposed, which has the advantage of its computational simplicity because it requires only the Frechet-derivative of the parameter-to-data mapping and its adjoint to formulate the iterative regularization algorithm.
2.4. Frechet derivative

The Frechet derivative $F'[h]$ of the mapping $F(h)$ is the derivative with respect to the parameter $h$. Here this derivative is derived formally, because a more thorough derivation would require the choice of appropriate function spaces for the parameter $h$. For the derivation, the difference of two different temperatures $T_1$ and $T_2$, associated to two heat transfer coefficient values $h_1$ and $h_2$, is considered:

$$\Theta := \Theta_1 \Theta: F \Theta_2 \Theta' = \Theta_1 \Theta: F \Theta_2 \Theta'$$

Because of the linearity, the difference $U$ satisfies the same heat equation as previously given; only the boundary conditions on the top and bottom surfaces are respectively

$$\lambda \frac{\partial}{\partial n} \Theta(x,t) = - (h_{U,1}(T_1) T_1 - h_{U,2}(T_2) T_2); \quad x \in U \cup B;$$

and

$$\lambda \frac{\partial}{\partial n} \Theta(x,t) = - (h_{B,1}(T_1) T_1 - h_{B,2}(T_2) T_2); \quad x \in U \cup B.$$

Using the Taylor expansion it can be written for the top surface that

$$h_{U,1}(T_1) T_1 - h_{U,2}(T_2) T_2 = h_{U,1}(T_1) T_1 - h_{U,2}(T_1) T_1 + h_{U,2}(T_1) T_1 - h_{U,2}(T_2) T_2 =$$

$$= \Delta h_U(T_1) T_1 + f'(T_1)(T_1 - T_2) + o(||T_1 - T_2||)$$

where

$$\Delta h_U(x) = h_{U,1}(x) - h_{U,2}(x), \quad f(x) = h_{U,2}(x)$$

and consequently

$$f'(x) = h_{U,2}(x) x + h_{U,2}(x).$$

Thus, the formally linearized boundary conditions can be written as

$$\lambda \frac{\partial}{\partial n} \Theta(x,t) = - f'(T_1) \Theta - (\Delta h_U)(T_1) T_1$$

for the top surface and for the bottom one as

$$\lambda \frac{\partial}{\partial n} \Theta(x,t) = - f'(T_1) \Theta - (\Delta h_B)(T_1) T_1$$

Finally, with $h = (h_U, h_B)$ and $w = (w_U, w_B)$, the Frechet-derivative is the mapping

$$F'[h]: w \to \Theta(x_m, t)$$

where $\Theta$ solves the problem with the boundary condition $(w_U, w_B) = (\Delta h_U, \Delta h_B)$, and where $\Theta$ is the solution to the nonlinear problem with the heat transfer coefficient $h$.

2.5. Adjoint operator

For the Landweber iteration, as already mentioned, it is required also to compute the adjoint operator of the Frechet derivative $F'[h]$: i.e. the operator

$$(F'[h] w, z) = (F'[h]^* z, w).$$

This operator takes a function $z(t)$, which lives on the same space as the data do and maps it to the function space where the parameter $h$ lives. Let us consider an arbitrary function $z(t)$ and let $\Theta$ be the solution associated to $F'[h] w$. Moreover, consider the solution to the adjoint problem $W(X, t)$

$$-\rho(x) c_p(x) \frac{\partial}{\partial t} W(x, t) = div(\lambda(x) \nabla W(x, t))$$

with the end condition

$$W(t_{fin}, x) = 0$$
and the boundary conditions
\[ \lambda \frac{\partial}{\partial n} W(x, t) = 0; \quad x \in \Omega_i; \]
\[ \lambda \frac{\partial}{\partial n} W(x, t) = -(h_U(T) + h_B(T)) W + p(x, t) \]
where \( p(x, t) \) is a source of type
\[ p(x, t) = \sum_i \delta(x - x_i) z_i(t), \quad (3) \]
\( x_i \) are the measurement points and \( z_i \) are arbitrary functions.

Using the weak formulation and the integration by parts, the adjoint operator is defined implicitly
\[ (F^*[h](z_i), (w_U, w_B)) = \int_0^{T_{fin}} \int_0^T w_U(T) W(x, t) dx + \int_0^{T_{fin}} \int_0^T w_B(T) T(x, t) W(x, t) dx \quad (4) \]

When \( w_T \) is constructed out of discrete basis functions
\[ W_U = \sum_i w_i \phi_i(x), \]
then, given that \( z \) solves equation (4), the discrete adjoint operator requires the computation of \( W \) and maps it to the coefficients
\[ \int_0^{T_{fin}} \int_\Theta \phi_i(T(x, t)) W(x, t) dx dt \]

For piecewise constant basis functions \( \phi_i \), the integration is needed only over these \((x, t)\), where \( T \in \text{supp}(\phi) \). Similar reasoning can be carried out also for the bottom surface.

Finally, the adjoint is computed by solving an additional problem; this time with linear boundary conditions. This can be achieved employing the similar solution procedure as for the direct problem.

2.6. Regularization algorithm
The statement of the iterative regularization scheme used in the numerical simulations will be started with the presentation of the classical Landweber iteration. For the considered case, the unknown heat transfer coefficient values would be computed successively as
\[ h_{n+1} = h_n - \tau F^*[h_n]^*(F(h_n) - y), \quad \tau > 0, \]
where \( \tau \) is a sufficiently small step size. One step of the update requires the following operations:
1. computation of \( F(h_n) \) by solving the nonlinear forward solver;
2. subtraction of \( F(h_n) \) from the residual: \( z = F(h_n) - y \);
3. determination of the adjoint by solving \( W \) with the residual \( z \).

Thus, for the computation of the adjoint the equation for \( W \) has to be solved with the corresponding boundary conditions. This involves a problem with non-constant heat-transfer coefficients at the top and at the bottom. The Landweber iteration is a classical general purpose method iteration method for inverse problems, and it is based on a steepest descent method for the least squares functional. In the inverse heat transfer problems this method and similar ones have been proposed by Alifanov [11], who considered both steepest descent and conjugate gradient type methods. However, in order to reduce computational complexity, here we will employ a variant of this iteration method, which does not use the steepest descent direction, but only a descent direction. The main difference is, that we do not evaluate the adjoint at the current iterate \( h_n \), but at a constant heat transfer function \( h_0 \). This variant, proposed by Kuegler [12], is known as the derivative free Landweber iteration. The only difference to the original one is that instead of the adjoint problem one has to solve a simpler problem with the following boundary conditions at the top and bottom:
\[
\lambda \frac{\partial}{\partial n} W(x, t) = p(x, t) \\
\lambda \frac{\partial}{\partial n} W(x, t) = 0
\]

where \( p(x, t) \) is a source defined in equation (3), when the \( h \) terms are equalled to 0.

Because in the considered case only the Green’s functions for the constant \( h_G \) were accessible, this iteration scheme was modified further and the boundary condition for the adjoint problem were

\[
\lambda \frac{\partial}{\partial n} W(x, t) = 0 \quad x \in \Omega_1 \\
\lambda \frac{\partial}{\partial n} W(x, t) = -h_G \gamma W + p(x, t) \\
\lambda \frac{\partial}{\partial n} W(x, t) = -h_G \beta W
\]

This approach is the same as computing the adjoint \( F' \ast [h_G] \), where \( h_G \) are the constant heat transfer coefficients used for the Green’s functions. Thus, the proposed algorithm can be considered as a frozen Landweber iteration (i.e. the adjoint is always taken at \( h_G \)):

\[
h_{n+1} = h_n - \tau F' \ast [h_G](F(h_n) - y)
\]

where \( h_G \) are the constant heat transfer coefficients used in the Green’s functions. The convergence of this algorithm is an open question but, as will be demonstrated, it can provide useful results.

Another important aspect worth mentioning is that in general the iterative regularization methods do not converge for ill-posed problems, but they show only the semi-convergence, i.e. the iteration will diverge in the case of noisy data. Thus, the iteration must be stopped applying a suitable stopping rule depending on the amount of noise in the data. The heat transfer coefficient value \( h \) obtained at the final iteration \( n_{\text{stop}} \) is considered as the sought approximation to the unknown parameter. For the case of Landweber iteration, it was shown in [13] that the iteration scheme combined with an appropriate stopping rule converges to the true unknown parameter value when the noise level tends to 0.

3. Measurement and simulation
This section of the paper presents the numerical experiments consisting in the estimation of the heat transfer coefficient thermal dependence. The simulations are carried out for the real measurement data obtained from the recorded infrared images of the circuit. First the measurement setup is described. Then, the estimation results for different cooling conditions are presented and discussed.

3.1. Measurements
The infrared measurements were performed on a commercial power operational amplifier manufactured in the hybrid IMS technology. For the measurements the circuit was sprayed with black matt paint so as to assure uniform emissivity of the surface. The 4.2 cm x 5.3 cm circuit contained two transistors in its output stage as shown in the infrared image of the top surface in figure 1. One transistor, indicated by HS, served as the heat source. The infrared images were saved at the maximal camera speed of 10 seconds. The temperature values were read from the images at 9 points, marked by circles, and located in different parts of the circuit, which was done expressly to cover the widest possible range of temperatures.

![Figure 1. Exemplary infrared image.](image-url)
3.2. **Forced water cooling**

First, the forced water cooling heat transfer coefficient dependence on temperature was investigated. The circuit was attached to a heat sink, through which continuous water flow was forced. The constant water temperature was maintained by means of a thermostat. The measurements were repeated for the water temperatures of 20 °C and 80 °C and the power dissipation of 37.2 W. The recorded temperature rise values for the water temperature of 20 °C for 3 points marked as black dots in figure 1 are shown by solid lines in figure 2. For the higher water temperature the heat source temperature rise was 6 °C lower, which could be expected because of the improved overall cooling at the outer surfaces.

Next, the circuit was simulated using the simple two-layer thermal model consisting of the thin insulating dielectric and the aluminium base plate. The heat source was represented by a surface heat flux and the Robin boundary conditions were applied at the top at the bottom. The initial values of the heat transfer coefficient \( h_0 \) were set so as to obtain the correct temperature values at the coldest of the selected points. Then, the temperature dependence of the coefficient was retrieved applying the earlier described algorithm and forcing the monotonous rise of the coefficient value with temperature. The estimated coefficient values are shown in figure 3 and the circuit temperature simulation results obtained for these coefficient values are presented as the dashed lines together with the measurements. As can be seen, the heat transfer coefficient varies quite significantly with temperature. Unfortunately, the lack of data for the intermediate temperature values does not allow simultaneous estimation of the coefficient in the entire temperature range. When the simulated temperature values are concerned they match the experimental ones very well and the error in the steady state does not exceed 1 °C.

3.3. **Still air cooling**

The second part of the experiment was devoted to the estimation of the heat transfer coefficient in the case of still air cooling. Now, the circuit was firmly attached to a large multi-fin aluminium heat sink. The measurements were carried out for the horizontal and for the vertical position of the assembly. Obviously, such a change should influence the value of the heat transfer coefficient because when the radiator was put vertically, the hot air could be evacuated more efficiently by the gravitational forces. The measurement results, shown for the horizontal configuration in figure 4 with solid lines confirmed this fact since after the change the heat source steady state temperature decreased by 2 °C. For both cases, the estimated heat transfer coefficient dependence on temperature is presented in figure 5. Additionally, the simulated temperature values are compared with measurements in figure 4. As can be seen, this time again the proposed algorithm was capable of differentiating between the two cases. Unfortunately, the simulated temperature values match the experimental ones only near the steady state and are visibly lower in the early stages of the heating process, which is probably caused by too simplified thermal model used in the simulation.

![Figure 2. Simulation and measurement - forced water cooling 20°C.](image1.png)

![Figure 3. Estimated values of the heat transfer coefficient for different temperatures.](image2.png)
4. Conclusions

This paper presented a method to estimate the temperature dependence of the heat transfer coefficient. The simulations showed that the proposed algorithm was able to retrieve even very subtle differences, however its robustness is hard to judge. First of all, the estimate uncertainty can be improved using the detailed thermal model, but this would require rather the application of some numerical software as the forward solver. Further improvements would be also expected when the input data would be more accurate and cover the entire temperature range, but this would involve more sophisticated hardware.

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