Active Photon Regeneration for ALPS II

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ALPS II, the Any Light Particle Search, is a second-generation Light Shining through a Wall experiment that hunts for axion-like particles. It uses two optical cavities; one on each side of the wall, to first generate light particles from a very strong intra-cavity optical field and then turn these particles back into photons in the second cavity called the regeneration cavity. ALPS II will either detect axion-like particles or provide an upper limit on their coupling strength to two photons of \( g_{\alpha\gamma\gamma} \lesssim 2 \times 10^{-11} \text{GeV}^{-1} \). The experiment is currently transitioning from the design and construction phase to the commissioning phase with science runs expected to start in 2021. One of the challenges of ALPS II is that one of the cavities will have to track the length of the other cavity at the pm level. This paper discusses the possibility of replacing the regeneration cavity with an active regeneration system which promises a similar signal to noise ratio. For ALPS II, this is a risk reduction activity. However, the regeneration cavity in ALPS is fundamentally very similar to signal recycling in interferometric gravitational wave detectors (LIGO, VIRGO, GEO, and KAGRA) and the basic idea might very well be applicable there as well.

I. INTRODUCTION

Light shining through walls (LSW) experiments [1, 2] are a class of purely laboratory based searches for beyond standard model particles such as axions or axion-like particles, their scalar equivalents or hidden sector photons. These are just a few of the particles that have been invented to explain the currently unexplainable such as dark matter, the CP problem in QCD [3–5] or subtle observations like TeV transparency [6] or discrepancies in stellar cooling rates [7]. These searches explore predicted but extremely weak interactions with the electromagnetic field which allows photons to pass through light tight walls by turning into such a particle in front of the wall and back into a photon behind the wall. In the following, we will refer to all these particles as axion-like particles.

The leading experiment of this type will soon be ALPS II which is currently under construction at DESY in Germany [8]. It uses one 120 m long optical cavity to enhance the number of photons that can be turned into axion-like particles before the wall and a second identical cavity to amplify the regenerated field on the dark side of the wall. A sophisticated length and alignment sensing and control scheme has been developed to ensure that both cavities are simultaneously resonant at the same optical frequency [9].

This letter is about a second potentially easier way to build up the regenerated field which would not require the use of high speed position actuators to maintain resonance between the cavities, instead it relies on the electronic or active regeneration of the field. The idea is similar to the ones presented in [10, 11] for haloscope axion searches such as ADMX, HAYSTACK and ABRA-

CADABRA [12,14] except that it is applied here to an optical field instead of an RF fields.

II. REGENERATION CAVITY IN ALPS

Figure 1 shows the key components of the ALPS experiment. A strong laser field is injected into the production cavity. The cavity internal field builds up inside a 110 m long string of HERA dipole magnets. Inside the magnets, a small fraction of the photons turn into axion-like particles which travel through a light-tight wall into a second magnet string where they turn back into photons. ALPS uses a second optical cavity, the regeneration cavity, to build up the regenerated field and increase the regeneration rate. This last amplification is the topic of this letter.

The build-up inside the regeneration cavity is mathematically described by a set of well known equations:

\[
E(t) = \rho_1 \rho_2 e^{i\phi_{RT}} E(t - 2L/c) + E_{\text{sig}}(t)
\]

where \( E(t) \) is the field at time \( t \) measured at some reference plane inside the cavity, \( \rho_1 \) and \( \rho_2 \) are the amplitude reflectivities of the two mirrors and \( \phi_{RT} \) is the roundtrip phase. The signal field \( E_{\text{sig}} \) is generated by the axion field \( a \) and the B-field \( \vec{B} \)

\[
E_{\text{sig}} \propto g_{\alpha\gamma\gamma} a\vec{B}
\]

inside the magnet string. The details of this process are not relevant for this paper except that this is a coherent process in the sense that the generated E-field is monochromatic with a stable phase evolution that is evolving ideally with a constant rate of \( \omega_{\gamma} \), the angular frequency of the strong laser field that is injected into the production cavity.

This leads to the well known sum of the cavity internal...
field which can be solved using the geometric series:

$$E_C = \sum_{n=0}^{\infty} (\rho_1 \rho_2 e^{i \phi_{RT}})^n E_{\text{sig}}(t) = \frac{1}{1 - \rho_1 \rho_2 e^{i \phi_{RT}}} E_{\text{sig}}$$

This field still needs to leave the cavity through one of the mirrors before it can be detected:

$$E_{\text{det}} = i \tau_1 \frac{1}{1 - \rho_1 \rho_2 e^{i \phi_{RT}}} E_{\text{sig}}$$

where $\tau_1 = \sqrt{1 - \rho_1^2}$ is the amplitude mirror transmissivity of the output mirror; note that we use the Siegman convention of a $\pi/2$ phase shift in transmission and positive amplitude reflectivities for beam splitters independent of the side of reflection. In the resonant case, $\phi_{RT} = N \times 2\pi$, this turns into a signal gain which is proportional to $1/\tau_1$ when expressed as a field amplitude. This field can then be detected by beating it against a local oscillator; this technique is usually named heterodyne sensing (HET) \cite{9}, or using a photon counter such as a transition edge sensor (TES) \cite{8}. The here discussed system is a variation of the HET detection scheme; it is not compatible with TES.

ALPS II requires that the length of the production cavity tracks the length of the regeneration cavity at the pm level. For the HET system this is achieved by locking a high power laser (HPL) to the production cavity and a local oscillator laser (LO) to the regeneration cavity. A reference laser (RL) is then phase locked to the regeneration cavity transmitted field. These loops are high gain, high bandwidth feedback systems which are fairly standard in optical experiments. Lastly, the transmitted field of the production cavity is phase locked to the reference laser to maintain the dual resonance condition and ensure that the production cavity tracks all length changes of the regeneration cavity. This requires to actuate a 2” mirror with high gain and bandwidth, a rather difficult task, or very sophisticated (and expensive) suspension systems. Note that the entire system could be reversed such that the regeneration cavity tracks the length changes of the production cavity. In any case, one of the cavity mirrors requires actuation to control its position and match the lengths of the two cavities. The here presented idea would remove the regeneration cavity and replace it with the here described active regeneration system.

### A. Active Regeneration in ALPS

The idea is depicted in figure \ref{fig:ALPS}. We replace the cavity end mirror with an amplitude modulator which is driven by the power fluctuations measured by the depicted detector. The local oscillator field $E_0 e^{i \phi_{LO}}$ from the LO is injected through the amplitude modulator, a Faraday rotator and a half wave plate before it enters the regeneration area. The beam then reflects at the flat RC mirror and on the return path it is superimposed with the generated signal field with angular frequency $\omega_0 + \Omega$. This net field passes again through the half wave plate and the Faraday rotator before it is separated at the polarizer from the ingoing beam. The beat signal between the injected field and the signal field is detected by the detector and used to amplitude modulate the LO. The amplitude of the beat signal is also the science signal of ALPS and is integrated to measure the energy at the signal frequency $\Omega$.

The field on the photo detector $E_{PD}$ is:

$$E_{PD} = E_0 \left[ 1 + g P_{\text{sig}} \cos(\Omega (t - 2\tau) + \phi_{RF}) \right] e^{-i 2 \omega_0 \tau} + E_{\text{sig}} e^{i (\omega_0 + \Omega) (t - \tau)}$$

where $P_{\text{sig}}$ is the amplitude of the photo detector signal at frequency $\Omega$, delayed in time by the propagation through the regeneration region of time $2\tau = 2L/c$. $g$ is a gain factor. The power on the PD at frequency $\Omega$ is then:

$$P_{\text{sig}} e^{i \Omega t} = 2 E_0^2 g P_{\text{sig}} e^{i (\Omega (t - 2\tau) + \phi_{RF})} + E_0 E_{\text{sig}} e^{i \Omega (t - \tau)}$$

where $\phi_{RF}$ is an additional phase shift applied to the RF signal. Note that $P_{\text{sig}}$ is a complex number and the magnitude gives the amplitude at frequency $\Omega$. This leads to the well known infinite sum or geometric series:

$$P_{\text{sig}} e^{i \Omega t} = E_0 E_{\text{sig}} e^{i \Omega t} \sum_{n=0}^{\infty} r^n e^{-i \phi_{RT} n}$$

$$= \frac{E_0 E_{\text{sig}}}{1 - r e^{-i \phi_{RT}}}$$

where we use $r = 2E_0^2 g$ and the roundtrip phase $\phi_{RT} = \Omega \tau - \phi_{RF}$. As always, the convergence requires that the roundtrip gain $r < 1$.\[FIG. 1. ALPS II is the leading light shining through a wall experiment. A high power laser beam is injected into the production cavity which is placed inside a string of twelve HERA magnets. The generated axions pass through a wall and regenerate identical photons in the regeneration cavity which are then detected by the detector.\]
The response of this active feedback system is similar to the response of the regeneration cavity where the product of the amplitude reflectivities of the mirrors $\rho_1 \rho_2$ is replaced by $r = 2E_0^2g$ which will need to be carefully controlled. The system still requires that the round trip phase $\phi_{RT} = N \cdot 2\pi$ is tightly controlled although this will be achieved using the locking scheme described later. However, it does not longer require to move a mirror around.

This signal is then demodulated with $cos \Omega t$ and integrated over time $T$:

$$S = \frac{\sqrt{|$LO$|^2|\eta|^2}}{1 - r} T$$

where we expressed both field amplitudes by their equivalent photon rates. The integration time $T$ is expected to be on the order of a few weeks.

If we equate $r$ with the equivalent mirror reflectivities of the setup with the regeneration cavity, the signal is actually larger by the inverse amplitude transmissivity of the output coupler. One interpretation of this result is that the measurement here is actually done within the newly formed cavity.

**B. Random noise in active regeneration systems**

An amplification of the signal is only useful if it increases the signal to noise ratio. Our limiting noise source is shot noise which is caused by the incoherent vacuum fluctuations present in any eigenmode of the electromagnetic field. These fluctuations cause phase and amplitude modulations of the electro-magnetic field [15]:

$$E_{PD} = E_0 e^{i\omega_0 t} + v_{AM} \cos (\Omega t + \phi_{AM}) + iv_{PM} \cos (\Omega t + \phi_{PM})$$

where the phases $\phi_{AM}$ and $\phi_{PM}$ are random values between 0 and $2\pi$ while

$$<v_{AM}> = \frac{1}{\sqrt{2}} = <v_{PM}>$$

when we measure the amplitude of the electric field in square root of the photon rate. The measured power at some frequency $\Omega$ is then:

$$P_{SN} e^{i\Omega t} = r P_{SN} e^{i(\Omega(t-2\tau)+\phi_{RT})} + E_0 v_{AM}(t) e^{i(\Omega t + \phi_{RT})}$$

This can also be written as an infinite sum:

$$P_{SN} e^{i\Omega t} = E_0 e^{i\Omega t} \sum_{n=0}^{\infty} r^n v_{AM}(t-2n\tau) e^{i\phi_{AM}(t-2n\tau)}$$

where we assume $\phi_{RT} = N \cdot 2\pi$. Rewriting this as an inphase and a quadrature component:

$$P_{SN} e^{i\Omega t} = I_{SN} \cos \Omega t - Q_{SN} \sin \Omega t$$

with

$$I_{SN} = E_0 \sum_{n=0}^{\infty} r^n v_{AM}(t-2n\tau) \cos (\phi_{RT})$$

$$Q_{SN} = E_0 \sum_{n=0}^{\infty} r^n v_{AM}(t-2n\tau) \sin (\phi_{RT})$$

The average amplitudes vanish:

$$\langle I_{SN} \rangle = E_0 \sum_{n=0}^{\infty} r^n \langle v_{AM}(t-2n\tau) \rangle \langle \cos (\phi_{RT}) \rangle = 0$$

$$\langle Q_{SN} \rangle = E_0 \sum_{n=0}^{\infty} r^n \langle v_{AM}(t-2n\tau) \rangle \langle \sin (\phi_{RT}) \rangle = 0$$

as the average over the trigonometric functions vanish or, more fundamentally, the vacuum fluctuations have an average field of zero as all phases are equally likely.

The variance of the inphase component can be calculated via:

$$\langle I_{SN}^2 \rangle = E_0^2 \left( \sum_{n=0}^{\infty} r^n v_{AM}(t-n\tau) \cos (\phi_{RT}) \right) \times \left( \sum_{m=0}^{\infty} r^m v_{AM}(t-m2L/c) \cos (\phi_{RT}) \right)$$

FIG. 2. In the active photon regeneration system, the second cavity mirror at the far end is replaced by a system that injects an amplitude modulated field into the regeneration cavity region.
which generates terms proportional to
\[ r^{n+m} \langle v_{AM}(t-2n\tau)v_{AM}(t-2m\tau) \times \cos(\zeta(t-2n\tau))\cos(\zeta(t-2m\tau)) \rangle = \delta_{nm} \frac{r^{2n}}{2} \]

which vanishes for uncorrelated noise such as shot noise except when \( n = m \). The variances of the inphase and quadrature component are therefore:
\[ \langle I_{SN}^2 \rangle = E_0^2 \sum_{n=0}^{\infty} r^{2n} \frac{1}{2} = \frac{1}{2} E_0^2 \frac{1}{1-r^2} = \frac{1}{2} \frac{n_{LO}}{1-r^2} = \langle Q_{SN}^2 \rangle \]

The noise is also integrated over time. However, as the noise is uncorrelated, it’s variance will increase linear with time and the standard deviation or the relevant noise increases with \( \sqrt{T} \). The signal to noise ratio as a function of time is then:
\[ SNR(T) = \frac{\sqrt{n_{LO}^2} \sqrt{T}}{\frac{1}{2} \sqrt{\frac{n_{LO}}{1-r^2} \sqrt{T}}} = \sqrt{\frac{n_s T}{1-r^2}} \]

Note that \( n_s \) is the regenerated photon rate without feedback the same way \( E_S \) was the signal amplitude without feedback. This is again identical to the regeneration cavity case if \( r \) is equated with the mirror reflectivities \( \rho_1 \rho_2 \).

C. Implementation issues

As discussed before, active regeneration does not require to actuate any cavity mirror. Instead, an ALPS detector which implements this idea, the production cavity would act as a master oscillator or frequency reference. Similar to the current system, the HPL would be locked to the production cavity. The beat signal between the PC transmitted light and the RL would be used to phase lock the RL with a known offset frequency. Next, the LO would be offset phase locked to the RL using a leakage field through the return mirror. This system of high gain and high bandwidth feedback loops would allow tight frequency control without the need of a length actuator. The signal frequency is again the sum (or difference) frequency between the two offset frequencies used in the offset phase lock loops [7, 9].

The requirements and challenges associated with the phase stability of the signal are identical to the current HET design with the exception that potential changes of the free spectral range of the regeneration cavity are not longer an issue. However, the one technical issue that needs to be solved is the gain control which ensures that the roundtrip gain \( r \) stays just below unity to have ample gain but not generate a signal out of noise.

III. SUMMARY

The active photon regeneration system replaces the optical feedback within the regeneration cavity with an active feedback system in which the electrical signal itself is used to amplitude modulate the local oscillator field. This modulated field is injected into the regeneration area where it is superimposed with the signal field. The amplitude of the signal builds up faster than uncorrelated noise. The (idealized) signal to noise ratio of the active regeneration system is essentially identical to the signal to noise ratio of the regeneration cavity. This shouldn’t be a surprise as this is in principle nothing else than an autocorrelation technique which allows to recover periodic signals in uncorrelated noise.

The advantages of this approach compared to the regeneration cavity in ALPS II still need to be analyzed and will likely only play out if the simultaneous locking of the two recycling cavities turns out to be challenging and when techniques to control the gain in the active scheme are available.

The same principle is also applicable to RF detection schemes used for example in ADMX; papers from that community actually inspired this paper. Another area where optical feedback is used to amplify a signal field are interferometric gravitational wave detectors such as LIGO, VIRGO, GEO, and KAGRA. In that case, gravitational waves modulate the phase of a carrier field in each of the interferometer arms. This generates signal sidebands which leave the interferometer through the dark port. The signal recycling mirror sends this field back into the interferometer to either build up the signal at specific frequencies furthermore or to extract the signal from the arm cavities; the former is known as signal recycling [16], the latter as resonant sideband extraction [17]. Active regeneration might also replace the signal recycling cavity. This would allow to apply a frequency dependent feedback gain and tailor and therefore improve the sensitivity of these detectors [18]. Furthermore, it would allow to change the return phase and even the gain dynamically with high bandwidth and no vacuum incursion. Applying active recycling to LIGO-like detectors is currently under investigation.

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