Some Statistical Properties of a Weighted Distribution and Its Application to Waiting Time Data

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Authors’ contributions

This work was carried out in collaboration between both authors. BPS designed the problem and performed the statistical analysis. UDD derived the statistical properties and wrote the first draft of the manuscript. Both authors read and approved the final manuscript.

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Abstract

The weighted distribution is mainly used in various real life fields such as ecology, reliability engineering, medical science etc. Some statistical properties are derived such as moment generating function, characteristic function, cumulative generating function, the hazard function, Renyi entropy, Cumulative residual entropy. For parameter estimation maximum likelihood estimation method is used. The considered weighted probability distribution is applied to two real data sets of waiting time to examine the suitability and applicability.

Keywords: MGF; MRLF; Entropy; MLE; Waiting Time; Fecundability.

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1 Introduction

In the recent past, various statistical distribution have been introduced for lifetime data which reveal decreasing or increasing failure rate function. Fisher [1] introduced the concept of weighted distributions. Patil and Rao [2] discussed the weighted distribution and size-biased sampling with applications to wildlife populations. Piegorsch et al. [3] give more detail about the applications and examples of weighted distribution. Gupta and Kundu [4] used the idea of Azzalini [5] and suggested a new class of weighted exponential distribution which is defined as follows: A random variable $X$ is said to have a weighted exponential distribution with the shape parameter $\alpha$ and scale parameter $\lambda$ if it has the following probability density function (PDF) as

$$f(x) = \left(\frac{1+w}{w}\right) \lambda e^{-\lambda x}[1 - e^{-\lambda x}] ; \quad \alpha > 0, \lambda > 0, x > 0$$

In recent time, the weighted exponential distribution has been rigorously explored in the area of probability distribution theory. Oguntunde [6] and [7] also generalized the weighted exponential distribution using the exponentiated family of distributions to propose the exponentiated weighted exponential distribution. Bashir and Naqvi [8] proposed a model named as weighted exponential distribution (WED) and discussed its properties. Rather and Subramanian [9] introduced Length biased erlang truncated exponential distribution and discussed its properties, current year Rather and Özel [10] proposed the weighted power lindley distribution and with properties and applications. Some other weighted distributions have also been discussed in the literature, for example, the weighted inverted exponential distribution, Hussain [11], the weighted weibull distribution, Mahdy [12] and Shahbaz and Nadeem [13], the weighted multivariate normal distribution, Kim [14], the weighted inverse weibull distribution, Kersey [15], a weighted three parameter weibull distribution Essam et al. [16]. In this paper we study a new weighted exponential distribution following the concept of previous research article Patil and Rao [17].

2 Weighted Distribution

According to Patil and Rao [17], if $f(x; \theta)$ be the probability distribution function of random variable $X$ and the unknown parameter $\theta$ the weighted distribution is defined as;

$$f(x; \theta) = \frac{w(x)f^*(x; \theta)}{E[w(x)]} ; \quad x \in \mathbb{R}, \theta > 0$$

where $w(x)$ is the weight function, and $f^*(x; \theta)$ is the base line distribution. Bashir and Naqvi [8] proposed a distribution where he has taken weight function as $w(x) = e^{wx}$, and exponential distribution with parameter $\lambda$ as the base line distribution which is defined as

$$f^*(x; \lambda) = \lambda e^{-\lambda x} ; \quad x > 0$$

Therefore the pdf of the weighted exponential distribution (WED) is given by

$$f(x, \lambda) = (\lambda - w)e^{-(\lambda - w)x} ; \quad x > 0, \lambda > 0, \lambda > w, 0 < \lambda < 1 \quad (2.1)$$

In this paper, for comparison with the above weighted exponential distribution (WED), the weight function is taken as $w(x) = x ; x > 0$, and the exponential distribution with parameter $\theta$ as the base line distribution thus the pdf of the weighted exponential distribution is given by the equation (2.2), in fact this is length biased exponential distribution (LBED). This distribution is also known as Gamma$(2, \theta)$.

$$f(x; \theta) = x\theta^2e^{-\theta x} ; \quad x > 0, \theta > 0 \quad (2.2)$$
Fig. 1. Probability density function of LBED

Fig. 1 shows us the probability density function of LBED for different values of $\theta$. As $\theta$ is increasing the shape of the distribution become more peaked and positively skewed. The cdf of the LBED is obtained as:

$$F(x) = \int_0^x f(t)dt = \int_0^x t^2 e^{-\theta t} dt = 1 - (1 + \theta x)e^{-\theta x}$$

Fig. 2. Cumulative distribution function of LBED

Fig. 2 indicates the shape of cumulative density function for different values of $\theta$.

3 Properties of LBED

The mean of the length biased exponential distribution is obtained as

$$E(x) = \int_0^\infty xf(x)dx = \frac{2}{\theta}$$

(3.1)
The Harmonic mean is given in the equation number (3.2)

\[
\frac{1}{H} = E\left(\frac{1}{x}\right) = \int_0^\infty \frac{x^2e^{-\theta x}}{x} dx = \theta^2 \int_0^\infty e^{-\theta x} dx = \theta
\]

Thus the geometric mean of the length biased exponential distribution is given by the equation (3.3)

\[
GM = \sqrt[\lambda HQ]{AM \times HM} = \sqrt{\frac{2}{\theta} \times \frac{1}{\theta}} = \frac{\sqrt{2}}{\theta}
\]

The moment generating function of the distribution is given as

\[
M_x(t) = E(e^{tx}) = \int_0^\infty e^{tx} f(x) dx = \int_0^\infty e^{tx} x^2 e^{-\theta x} dx
\]

\[
= \theta^2 \int_0^\infty x^{-1} e^{\left(-\frac{(t-1)x}{\theta}\right)} dx = \theta^2 \frac{\Gamma(2)}{(\theta - t)^2} = \frac{\theta^2}{(\theta - t)^2} - \left(1 - \frac{t}{\theta}\right)^{-2}
\]

\[
= 1 + 2\frac{t}{\theta} + 3\left(\frac{t}{\theta}\right)^2 + 4\left(\frac{t}{\theta}\right)^3 + 4\left(\frac{t}{\theta}\right)^4 + \ldots\quad \text{(3.4)}
\]

\(\mu_r = \text{coefficients of } \frac{t^r}{r!} \text{ in the expansion of } M_x(t)\)

The first four raw moments of the LBED are given bellow

\[
E(x) = \mu_1 = \frac{2}{\theta} \quad ; \quad \theta > 0
\]
\[
E(x^2) = \mu_2 = \frac{6}{\theta^2} \quad ; \quad \theta > 0
\]
\[
E(x^3) = \mu_3 = \frac{24}{\theta^3} \quad ; \quad \theta > 0
\]
\[
E(x^4) = \mu_4 = \frac{120}{\theta^4} \quad ; \quad \theta > 0
\]

Variance is obtained as

\[
V(x) = E(x^2) - (E(x))^2 = \frac{6}{\theta^2} - \frac{4}{\theta^2} - \frac{2}{\theta^2}
\]

Thus the standard deviation (SD) is obtained as

\[
\sigma = \frac{\sqrt{2}}{\theta} \quad ; \quad \theta > 0
\]

It is very interesting that the standard deviation (SD) and geometric mean of the distribution is exactly same. The higher order moment is defined as

\[
\mu_3 = \frac{12}{\theta^3} \quad ; \quad \theta > 0 \quad \mu_4 = \frac{24}{\theta^4} \quad ; \quad \theta > 0
\]
The median of this distribution is given by equation number (3.6)

\[ \int_{0}^{m} f(x)dx = \frac{1}{2} ; \int_{0}^{m} \theta^2 xe^{-\theta x} dx = \frac{1}{2} \]

i.e. \((1 + \theta m)e^{-\theta m} = \frac{1}{2}\) i.e. \(e^{\theta m} - 2\theta m - 2 = 0\) \hspace{1cm} (3.6)

We can compute \(m\) by iteration method.

The skewness and kurtosis of the LBED are obtained as

\[ \beta_1 = 18 \quad \text{and} \quad \gamma_1 = \sqrt{18} = 3\sqrt{2} \]
\[ \beta_2 = 6 \quad \text{and} \quad \gamma_2 = \beta_2 - 3 = 3 \]

Since skewness and kurtosis of the distribution are independent of parameter. Therefore it can be seen that the distribution is positively skewed and leptokurtic. As same as above the characteristic function is defined as follows

\[ \Phi_x(t) = E(e^{itx}) = \int_{0}^{\infty} e^{itx} f(x)dx = \left( 1 - \frac{it}{\theta} \right)^{-2} \] \hspace{1cm} (3.7)

The cumulative generating function is defined as

\[ \kappa_x(t) = \log M_x(t) = 2 \log \left( \frac{\theta}{\theta - t} \right) \] \hspace{1cm} (3.8)

The survival function of the LBED is

\[ S(x) = e^{-\theta x} + x\theta e^{-\theta x} ; \quad x > 0, \theta > 0 \] \hspace{1cm} (3.9)

The plot of survival function is shown in Fig. (3) for different values of \(\theta\).

![Fig. 3. Survival Function of LBED](image)

Now the hazard function of the LBED is obtained as

\[ h(x) = \frac{f(x)}{S(x)} = \frac{f(x)}{1 - F(x)} \]
\[ = \frac{x\theta^2 e^{-\theta x}}{e^{-\theta x} + x\theta e^{-\theta x}} = \frac{x\theta^2}{1 + x\theta} \] \hspace{1cm} (3.10)

The hazard function of the LBED is increasing however exponential distribution has constant hazard. Plot of hazard functions of LBED for parameter \(\theta\) are displayed in the Fig. (4) below. As the value of parameter is increasing the hazard is also increasing.
Now from equation (3.10) we have

$$h(x) = \frac{x^2}{1 + x^2} = \frac{\theta}{1 + \frac{1}{\theta}} \quad (3.11)$$

Since $x > 0$ ; $\theta > 0$ then $\frac{\theta}{1 + \frac{1}{\theta}}$ is an increasing hazard function and intercepts on origin.

for $x \to 0, h(x) = 0$ and $x \to \infty, h(x) = \theta$

The cumulative hazard rate of the LBED is

$$H(t) = \int_0^t h(x) \, dx = \int_0^t \frac{x^2}{1 + x^2} \, dx = \int_0^t \left( 1 - \frac{1}{1 + x} \right) \, dx = (\theta x - \log(1 + x)) \bigg|_0^t = \theta t - \log(1 + t\theta) \quad ; \theta > 0, t > 0 \quad (3.12)$$

The mean residual life function at the time point $t$ is obtained as

$$m(t) = \frac{1}{1 - F(t)} \int_t^\infty (1 - F(x)) \, dx = \frac{1}{1 - F(t)} \int_t^\infty \left( e^{-\theta x} + \theta xe^{-\theta x} \right) \, dx$$

$$= \frac{1}{1 - F(t)} \left[ \int_t^\infty e^{-\theta x} \, dx + \frac{1}{\theta} \int_t^\infty \theta xe^{-\theta x} \, d(\theta x) \right] = \frac{1}{1 - F(t)} \left[ \frac{e^{-\theta t} - e^{-\theta(\theta t + 1)}}{-\theta} + \frac{e^{-\theta(\theta t + 1)}}{\theta} \right]$$

$$= \frac{e^{-\theta t}(\theta t + 2)}{\theta e^{-\theta t}(\theta t + 1)} = \frac{\theta t + 2}{\theta(\theta t + 1)} \quad (3.13)$$

Putting $t = 0$ we get the mean of this distribution as $m(0) = \frac{2}{\theta}$
3.1 Rényi Entropy

An entropy is a measure of uncertainty; Rényi [18] gave an expression of the entropy which is defined by.

\[
e(\eta) = \frac{1}{1 - \eta} \log \left[ \int_{0}^{\infty} f^\eta(x)dx \right]
= \frac{1}{1 - \eta} \log \left[ \int_{0}^{\infty} (x^\eta e^{-\theta x})^\eta e^{-\theta x} d\theta \right] = \frac{1}{1 - \eta} \log \theta^{\eta+1} [\Gamma(\eta+1)/(\eta\theta)]^\eta
= \frac{1}{1 - \eta} \log \left( \theta^{\eta+1} \Gamma(\eta+1)/\eta^{\eta+1} \right)
\]

(3.14)

3.2 Cumulative Residual Entropy

The cumulative residual entropy is defined by

\[
\Upsilon_{CR} = -\int_{R} S(x) \log S(x)dx
\]

Where \( S(x) = e^{-\theta x} (1 + \theta x) \) is the survival function of the LBED. Thus the cumulative entropy is obtained as

\[
\Upsilon_{CR} = -\int_{0}^{\infty} e^{-\theta x} (1 + \theta x) \log \left( e^{-\theta x} (1 + \theta x) \right) dx = -\int_{0}^{\infty} [-\theta x + \log(1 + \theta x)] dx
= \frac{\Gamma(2) + \Gamma(3) - \frac{3}{\theta} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{\theta^n} \log(1 + \theta x)] dx
= \frac{3}{\theta} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{\theta^n} n + (n + 1)! \frac{n!}{\theta^n} + \frac{3}{\theta} \sum_{n=1}^{\infty} (-1)^{n+1} (n + 2)(n - 1)! \theta^n
\]
3.3 Shannon Entropy

For LBED, the Shannon entropy is defined in equation no (3.15)

$$E (- \log f(x)) = - \int_{0}^{\infty} (\log f(x)) f(x)dx$$

$$= - \int_{0}^{\infty} \log \left( x \theta^2 e^{-\theta x} \right) x \theta^2 e^{-\theta x} dx$$

$$= - \int_{0}^{\infty} (\log x + 2 \log \theta - \theta x) x \theta^2 e^{-\theta x} dx$$

$$= -2 \log \theta \int_{0}^{\infty} \theta xe^{-\theta x} d\theta + \int_{0}^{\infty} (\theta x)^2 e^{-\theta x} d\theta - \theta^2 \int_{0}^{\infty} \log \left( xe^{-\theta x} \right) dx$$

$$= -2 \log \theta \Gamma(2) + \Gamma(3) - \theta^2 \int_{0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{2n-1}}{n} \Gamma(k+2)$$

$$= 2(1 - \log \theta) - \theta^2 \sum_{n=0}^{\infty} \frac{(-1)^{2n-1}}{n} \sum_{k=0}^{\infty} \frac{\Gamma(k+2)}{\theta^k}$$

$$= 2 \log e - \sum_{n=0}^{\infty} \frac{(-1)^{2n-1}}{n} \sum_{k=0}^{\infty} \frac{(k+1)!}{\theta^k} (3.15)$$

4 Maximum Likelihood Estimation (MLE)

The maximum likelihood estimation of LBED defined as:

$$L(\theta; x_1, x_2, x_3, ..., x_n) = \prod_{i=1}^{n} f(x_i)$$

here the independent observations are $x_1, x_2, x_3, ..., x_n$, then the likelihood function of the LBED is

$$L(\theta; x_1, x_2, x_3, ..., x_n) = \prod_{i=1}^{n} x_i \theta^2 e^{-\theta x_i} = \theta^{2n} \prod_{i=1}^{n} x_i e^{-\theta \sum_{i=1}^{n} x_i}$$
Taking log both side and differentiate with respect to $\theta$ partially we get,
\[
\log L = 2n \log \theta + \sum_{i=1}^{n} \log x_i - \theta \sum_{i=1}^{n} x_i
\]
\[
\frac{\partial}{\partial \theta} \log L = \frac{2n}{\theta} - \frac{n}{\theta} \sum_{i=1}^{n} x_i = 0
\]
i.e. \( \hat{\theta} = \frac{2}{\bar{x}} \)
\[
\frac{\partial^2}{\partial \theta^2} \log L = -\frac{2n}{\bar{x}^2} \]
\[
\left. \frac{\partial^2}{\partial \theta^2} \log L \right|_{\hat{\theta} = \frac{2}{\bar{x}}} = -\frac{n (\bar{x})^2}{2} < 0
\]

Hence \( \hat{\theta} = \frac{2}{\bar{x}} \) is the mle of the distribution.

5 Application

To evaluate the flexibility of LBED over the WED previously proposed by Bashir and Naqvi [8], we use the waiting time (measured in min) bank customers before service is being rendered, this data set previously used by Ghitany et.al [19]. Actually model proposed by Bashir and Naqvi [8] has two parameters and both parameters are depending on each other, the for given a particular estimate of one parameter we have an estimate of another parameter but the fitting will be same for different estimate of the parameters. LBED has only one parameter and simple and has explanation in terms of statistical constants. The tables in Appendix provides fitting of the LBED relating to waiting time of 100 bank customers and waiting time to the first conception. The mean and variance of the first data set are 9.79 and 52.47 respectively. The second data set on the waiting time to the first conception, taken from NFHS-IV, [20] for Varanasi District of Uttar Pradesh, India. We have considered those females whose age at marriage is more than or equal to 16 years and marital duration is more than 10 years at the time of survey, for the analysis of waiting time to first conception. The mean and variance is 23.64 months and 306.65 months square respectively. Data on waiting time to conception is not available in NFHS-4 because it is not observed directly thus we subtract 9 months from the data on first birth interval available in the surveys for obtaining waiting time to conception.

6 Conclusion

Various properties of the LBED are discussed in this paper, reliability measures of the LBED also derived. LBED is increasing failure rate and this distribution also a positively skewed distribution. The MLE of this distribution is derived and it is closed form also it is unbiased. This distribution is applied on waiting time data of 100 bank customers and it is proved that this distribution is more tractable than the existing new WED proposed by Bashir and Naqvi [8] also they have taken the value of $w$ as 0.5 which seems to be not justified and the range of $w$ must be as $0 < w < 1$. Further the fitting of data and estimates as well as p-value are fairly good for LBED than the earlier distribution given by Bashir and Naqvi [8]. LBED is showing good fit for the data with chi-square value=0.48465 and p-value=0.922. Further Table 2 shows that suitability of LBED on the waiting time to first conception to the currently married female in Varanasi district. The chi-square value=3.81 and p-value=0.70 shows the excellent fitting of the distribution. Inverse of the
expectation of \( x \) is some time defined as fecundability which is known as the chance of conception in a lunar month and here the second data set is in months thus the value of the parameter \( \theta = 0.085 \) is able to provide the estimate of fecundability. It is worthwhile to mention here that the parameter \( \theta \) is two times of the inverse of the expectation of \( x \) hence the estimate of fecundability is 0.0425 and the yearly chance of conception is 12 times the fecundability i.e. 0.51. This also indicates according to the NFHS-IV data, [20] female in Varanasi district take on an average 2 years to get first conception.

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**Competing Interests**

Authors have declared that, no competing interest exists.

**References**

[1] Fisher RA. The effect of methods of ascertainment upon the estimation of frequencies. Annals of Eugenics. 1934;6(1):13-25.

[2] Patil GP, Rao CR. Weighted distributions and size-biased sampling with applications to wildlife populations and human families. Biometrics. 1978;34(2):179-189.

[3] Piegorsch, Walter W. Encyclopedia of environmetrics. Chichester: John; 2002.

[4] Gupta RD, Kundu D. A new class of weighted exponential distributions. Statistics. 2009;43(6):621-634.

[5] Azzalini A. A class of distributions which includes the normal ones. Scandinavian journal of statistics. 1985;12(2):171-178.

[6] Oguntunde P. On the exponentiated weighted exponential distribution and its basic statistical properties. Applied Science Reports. 2015;10(3):160-167.

[7] Oguntunde P, Adejumo A. The generalized inverted generalized exponential distribution with an application to a censored data. Journal of Statistics Applications & Probability. 2015;4(2):223-230.

[8] Bashir S, Naqvi IB. A new weighted exponential distribution and its applications on waiting time data. International Journal of Scientific and Research Publications. 2016;6(7):698-702.

[9] Rather AA, Subramanian C. The length-biased erlang-truncated exponential distribution with life time data. Journal of Information and Computational Science. 2019;9(8):340-355.

[10] Rather AA, Özel G. The weighted power Lindley distribution with applications on the life time data. Pakistan Journal of Statistics and Operation Research. 2020;16(2):225-237.

[11] Hussian M. A weighted inverted exponential distribution. International Journal of Advanced Statistics and Probability. 2013;1(3):142-150.

[12] Mahdy M. A class of weighted weibull distributions and its properties. Studies in Mathematical Sciences. 2013;6(1):35-45.

[13] Shahbaz S, Shahbaz MQ, Butt NS. A class of weighted weibull distribution. Pakistan Journal of Statistics and Operation Research. 2010;6(1):53-59.

[14] Kim HJ. A class of weighted multivariate normal distributions and its properties. Journal of Multivariate Analysis. 2008;99(8):1758-1771.
[15] Kersey JX. Weighted inverse weibull and beta-inverse weibull distribution. Electronic Theses and Dissertations. 2010;661.

[16] Essam AA, Mohamed AH. A weighted three-parameter weibull distribution. J. Applied Sci. Res. 2013;9:6627-6635.

[17] Patil GP, Rao CR. The weighted distribution: A survey of their applications. In Applications of Statistics, Krishnaiah PR (Ed.), North Holland Publishing Company. 1977;383-405.

[18] Rényi A. On measures of entropy and information. In Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, Volume 1: Contributions to the Theory of Statistics. The Regents of the University of California; 1961.

[19] Ghitany ME, Atieh B, Nadarajah S. Lindley distribution and its application. Mathematics and Computers in Simulation. 2008;78(4):493-506.

[20] IIPS. National family health survey (NFHS-4). 2015:16. International Institute for Population Sciences (IIPS), Mumbai, India; 2017.
## Appendix

### Table 1. Goodness of fit for LBED and WED on waiting time of 100 bank customers

| Waiting time (in minutes) | Observed frequencies | Expected frequencies of LBED | Expected frequencies of WED [8] |
|---------------------------|----------------------|------------------------------|---------------------------------|
| 0-5                       | 30                   | 27.22                        | 40.25                           |
| 5-15                      | 32                   | 33.34                        | 24.05                           |
| 10-15                     | 19                   | 20.47                        | 14.37                           |
| 15-20                     | 10                   | 10.42                        | 8.586                           |
| 20-25                     | 5                    | 4.85                         | 5.13                            |
| 25-30                     | 1                    | 2.14                         | 3.065                           |
| 30-35                     | 2                    | 0.91                         | 1.831                           |
| 35-40                     | 1                    | 0.64                         | 1.095                           |
| Total                     | 100                  | 100.00                       | 98.34                           |

Estimates of parameter: $\theta = 0.2044$, $\lambda = 0.60$, $w = 0.5$

Chi-square value: 0.48465, 7.628597

Degree of freedom: 3

p-value: 0.922, 0.054347

### Table 2. Goodness of fit for LBED on the waiting time to first conception for females (NFHS-IV data)

| Waiting time to first conception (in months) | Observed number of females | Expected number of females |
|---------------------------------------------|----------------------------|----------------------------|
| 0-3                                         | 12                         | 12.21                      |
| 3-15                                        | 165                        | 150.06                     |
| 15-27                                       | 125                        | 135.90                     |
| 27-39                                       | 72                         | 78.77                      |
| 39-51                                       | 40                         | 39.24                      |
| 51-63                                       | 17                         | 18.09                      |
| 63-75                                       | 10                         | 7.96                       |
| 75-87                                       | 4                          | 3.39                       |
| 87-99                                       | 1                          | 1.41                       |
| 99-111                                      | 2                          | 0.97                       |
| Total                                       | 448                        | 448.00                     |

Estimates of parameter: $\theta = 0.085$

Chi-square value: 3.51

Degree of freedom: 6

p-value: 0.70

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