A rational analytical approach for buckling analysis of orthotropic double-nanoplate-systems

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Abstract. A novel analytical Hamiltonian-based approach is proposed for buckling analysis of orthotropic double-nanoplate-systems (DNPSs) under uniaxially compression embedded in an elastic medium. In the Hamiltonian system, the governing equations for in-phase and out-of-phase buckling are established in a unified form based on Eringen’s nonlocal plate theory. The buckling analysis of the orthotropic DNPS is reduced to an eigenproblem in the symplectic space. Analytical buckling equations and buckling mode shape functions can be obtained by the symplectic eigensolutions and boundary conditions simultaneously. Comparison studies demonstrate the accuracy and efficiency of the proposed method. Key influencing factors which may benefit the design of complex 3D mesostructures are studied in detail. Some new results are given also.

1. Introduction

Nano-/Micro-electro-mechanical systems (NEMS/MEMS) have increased considerable attention among the experimental and theoretical research communities due to their wide ranging applications. However, most of current NEMS/MEMS devices are limited to 2D geometries such as nanobeam-based or nanoplate-based structures [1, 2]. These devices usually operate in a largely simple manner and are unavailable when 3D motions are required. In recent years, a rapidly expanding field of research in materials science is fabricating 3D mesostructures with the aid of buckling of nanostructures [3]. Some 3D mesostructures have been successfully assembled by the compressive buckling of monolayer nanoplates [4]. Similar to the monolayer nanoplate, double-nanoplate-systems (DNPSs) may find applications in forming more complex multi-layer 3D mesostructures. Therefore, investigating the stability of DNPSs is important to the technique of mechanically guided self-assembly.

At nanometer scale, the long-range interatomic and intermolecular cohesive forces become prominent [5, 6], the scale free classical continuum elasticity theory cannot accurately predict the mechanical behaviors of nanostructures. Therefore, many researchers resorted to nonlocal elasticity theory to understand the relevant behaviors. One of the most famous size-dependent continuum theories is Eringen’s nonlocal theory which captures the small size effect by assuming that the stress at
a point as a function of the strains at all the other points [7, 8]. In the framework of Eringen’s nonlocal elasticity theory, numerous works have been reported on the stability analysis of nanoplate-like structures [9, 10]. Compared to the study on the buckling of monolayer nanoplate, the stability of DNPSs or multi-layered nanoplate systems was seldom mentioned. Only a few literatures were reported on the buckling analysis of DNPSs [11, 12], orthotropic DNPSs [13-16], viscoelasticity DNPSs [17], magneto-electro-elastic DNPSs [18], multi-layered nanoplate system [19-21]. Although many analytical solutions were found in the aforementioned literatures, most of them were derived by the Navier solution (only for fully simply supported DNPSs). In the assembly of 3D nanostructures, the nanoplate should have at least two free edges. Therefore, understanding the DNPSs with non-Navier type boundary conditions will provide significant guidance to the design of forming 3D mesostructures.

This paper presents a new analytical approach for buckling of DNPSs with Levy-type boundary conditions by reforming the governing equations into the Hamiltonian description [22, 23]. The determination of critical buckling loads and corresponding buckling modes is reduced to solving an eigenproblem in the symplectic space. Key influencing factors are investigated through a comprehensive parametric study. Numerical solutions for buckling of orthotropic DNPSs can be used for the future application in the assembly of complex 3D nanostructures.

2. Nonlocal orthotropic DNPS model

Consider an orthotropic DNPS embedded in an elastic medium, as shown in Fig. 1. The two nanoplates are uniaxially compressed by force \(N\). For mathematical modeling, the surrounding elastic medium is represented by a continuously distributed Winkler’s spring system; the interaction between the two nanoplates is also assumed as the Winkler’s springs, which may be used to substitute the Van der Waals (vdW) force, electrostatic force, capillary force, Casimir force and others. The stiffness coefficients of the elastic medium and interaction force are taken as \(K\) and \(k\). The two nanoplates are assumed to be made of the same material and with the same dimensions.

Figure 1. A uniaxially compressed orthotropic DNPS embedded in an elastic medium.

According to Eringen’s nonlocal theory, the constitutive equation for the orthotropic DNPS can be expressed as

\[
L \begin{bmatrix} \dot{\sigma}_{xx}^{(i)} \\ \dot{\sigma}_{yy}^{(i)} \\ \dot{\sigma}_{xy}^{(i)} \end{bmatrix} = \frac{12}{\hbar^3} \begin{bmatrix} D_x & D_{xy} & 0 \\ D_{xy} & D_y & 0 \\ 0 & 0 & 2D_z \end{bmatrix} \begin{bmatrix} \varepsilon_{xx}^{(i)} \\ \varepsilon_{yy}^{(i)} \\ \varepsilon_{xy}^{(i)} \end{bmatrix}
\]

where \(\sigma\) and \(\varepsilon\) are the stress tensor and strain tensor, respectively; \(\varepsilon_{xx}^{(i)} = -z\tilde{\sigma}^2w(\xi)/\tilde{\sigma}^2\), \(\varepsilon_{yy}^{(i)} = -z\tilde{\sigma}^2w(\eta)/\tilde{\sigma}^2\), \(\varepsilon_{xy}^{(i)} = -2z\tilde{\sigma}^2w(\xi)/\tilde{\sigma}^2\tilde{\sigma}\eta\); \(D_x = E_x\hbar^2/[12(1-\nu_x\nu_y)]\), \(D_y = E_y\hbar^2/[12(1-\nu_x\nu_y)]\), \(D_{xy} = E_{xy}\hbar^3/[12(1-\nu_x\nu_y)]\),
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By using Eq. (2), the governing equation governing the buckling of orthotropic DNPSs under in-plane compressive force is expressed as [12]

\[
\frac{\partial^2 M_x^{(i)}}{\partial x^2} + 2 \frac{\partial^2 M_y^{(i)}}{\partial x \partial y} + \frac{\partial^2 M_y^{(i)}}{\partial y^2} + L \left( N_y \frac{\partial^2 w^{(i)}}{\partial y^2} - P^{(i)} \right) = 0
\]

where \( P^{(i)} = -K_w \left( w^{(i)} - w^{(j)} \right) - K_w w^{(j)} \) ( \( j = 2, 1 \) ) are the pressures exerted on each nanoplate induced by the elastic medium and interaction force. It is noted that Eq. (3) is represented by classical variables.

By means of variable substitution, the buckling of orthotropic DNPSs can be divided into two subproblems, i.e., in-phase sequence (\( \omega_{ip} = w^{(i)} + w^{(2)} \)) and out-of-phase sequence (\( \omega_{op} = w^{(i)} - w^{(2)} \)) [11, 12]. Substituting the assumptions into Eq. (3), we have

\[
\frac{\partial^2 M_x^{(i)}}{\partial x^2} + 2 \frac{\partial^2 M_y^{(i)}}{\partial x \partial y} + \frac{\partial^2 M_y^{(i)}}{\partial y^2} + L \left( N_y \frac{\partial^2 w^{(i)}}{\partial y^2} - K \right) w_i = 0,
\]

where \( K = \phi K_w + K_c; \ \phi = 0, \ w = \omega_{ip}; \ \phi = 2, \ w = \omega_{op} \).

The Levy-type boundary conditions are selected and are mathematically expressed as

1. S: \( w^{(i)} = 0, \ M_x^{(i)} = 0 \) at \( x = 0, a; \ w^{(i)} = 0, \ M_y^{(i)} = 0 \) at \( y = 0, b; \)
2. C: \( w^{(i)} = 0, \ \theta_x^{(i)} = 0 \) at \( x = 0, a; \ w^{(i)} = 0, \ \theta_y^{(i)} = 0 \) at \( y = 0, b; \)
3. F: \( M_x^{(i)} = 0, \ V_x^{(i)} = 0 \) at \( x = 0, a; \ M_y^{(i)} = 0, \ V_y^{(i)} = 0 \) at \( y = 0, b; \)

Here, S, C, and F represent the simply supported, clamped, free edges, respectively; \( \theta_x^{(i)} = -\partial w^{(i)} / \partial x \) and \( \theta_y^{(i)} = -\partial w^{(i)} / \partial y \) are the angles of rotation; \( V_x^{(i)} = \partial M_x^{(i)} / \partial x + 2 \partial M_y^{(i)} / \partial y - N_x \theta_x^{(i)} \) and \( V_y^{(i)} = \partial M_y^{(i)} / \partial y + 2 \partial M_y^{(i)} / \partial x - N_y \theta_y^{(i)} \) are the classical equivalent shear forces.

3. Governing equations in the Hamiltonian system

To establish the Hamiltonian system of the buckled DNPS, a total unknown vector [23, 24] is defined as \( \Psi = \begin{bmatrix} w, \theta_x, V, M_x \end{bmatrix} \), where \( V = V_x - \left( N_x \partial^2 \tilde{w} / \partial y^2 - K \tilde{w} \right) \tilde{\xi}^2 \) is the generalized shear force. By using total unknown vector, the governing equations (4) can be rewritten in a matrix form, i.e.,

\[
\Psi' = H \Psi
\]

where \( (\cdot)' = (\cdot) / \partial x \), \( L_y = 1 - \xi^2 \partial^2 / \partial y^2 \); \( H \) is the Hamiltonian operator matrix as presented as follows,
Eqs. (6) are the governing equations in the Hamiltonian system. Noting that $H$ does not contain any derivative respect to $x$, the method of separation of variables can be employed to find the solution to Eq. (6). Let $v_j(x, y) = \Psi_j(y) e^{\mu_j x}$, the eigenequation is

$$H v_j = \mu_j v_j$$

(7)

where $\mu_j$ and $v_j$ are eigenpairs. The characteristic equation of Eq. (7) is expressed as

$$d_1 \lambda^4 + d_2 \lambda^2 + d_3 = 0$$

(8)

whose roots are

$$\lambda_{1,2} = \pm \sqrt{-d_2 - \sqrt{d_2^2 - 4d_1d_2}} / 2d_1, \quad \lambda_{3,4} = \pm \sqrt{-d_2 + \sqrt{d_2^2 - 4d_1d_2}} / 2d_1$$

(9)

where $d_1 = D_y + \xi^2$, $d_2 = 2D_x \mu^2 - N_y \left(1 - \xi^2 \mu^2\right) - K \xi^2$, $d_3 = D_x \mu^4 + K \left(1 - \xi^2 \mu^2\right)$.

$D_y = D_{yy} + 2D_x$. It can be proved that the eigensolutions exist only when $\lambda_{1,2} \neq 0$, $\lambda_{3,4} \neq 0$ and $\lambda_{1,2} \neq \lambda_{3,4}$[24]. Therefore, the general solution of the eigensolutions can be expressed as

$$\Psi = \begin{bmatrix} A_1 \sinh(\lambda_{1} y) + A_2 \cosh(\lambda_{1} y) + A_3 \sinh(\lambda_{3} y) + A_4 \cosh(\lambda_{3} y) \\ B_1 \sinh(\lambda_{1} y) + B_2 \cosh(\lambda_{1} y) + B_3 \sinh(\lambda_{3} y) + B_4 \cosh(\lambda_{3} y) \\ C_1 \sinh(\lambda_{1} y) + C_2 \cosh(\lambda_{1} y) + C_3 \sinh(\lambda_{3} y) + C_4 \cosh(\lambda_{3} y) \\ D_1 \sinh(\lambda_{1} y) + D_2 \cosh(\lambda_{1} y) + D_3 \sinh(\lambda_{3} y) + D_4 \cosh(\lambda_{3} y) \end{bmatrix}$$

(10)

where $A_i$, $B_i$, $C_i$, $D_i$ are the undetermined coefficients; and they satisfy the following relations

$$B_i = -\mu A_i \quad (i = 1, 2, 3, 4),$$

(11a)

$$C_i = -\mu \left[ D_x \mu^2 + \left( N_y \xi^2 + 4D_x + D_{yy} \right) \lambda_i^2 - K \xi^2 \right] A_i \quad (i = 1, 2)$$

(11b)

$$C_i = -\mu \left[ D_x \mu^2 + \left( N_y \xi^2 + 4D_x + D_{yy} \right) \lambda_i^2 - K \xi^2 \right] A_i \quad (i = 3, 4)$$

(11c)

$$D_i = -\left( D_x \mu^2 + D_{yy} \lambda_i^2 \right) A_i \quad (i = 1, 2)$$

$$D_i = -\left( D_x \mu^2 + D_{yy} \lambda_i^2 \right) A_i \quad (i = 3, 4)$$

(11d)

Substituting solution (10) into the boundary condition (5a), we have

$$Z \eta = 0$$

(12)
where \( \mathbf{Z} = \begin{bmatrix} \mathbf{Z}_1 & \mathbf{Z}_2 \\ \mathbf{Z}_3 & \mathbf{Z}_4 \end{bmatrix} \), \( \mathbf{Z}_1 = \begin{bmatrix} 0 & 1 \\ \sinh(b \lambda_1) & \cosh(b \lambda_1) \end{bmatrix} \), \( \mathbf{Z}_2 = \begin{bmatrix} 0 & 1 \\ \sinh(b \lambda_2) & \cosh(b \lambda_2) \end{bmatrix} \).

\( \mathbf{Z}_3 = -Y_3 \mathbf{Z}_1 \), \( \mathbf{Z}_4 = -Y_4 \mathbf{Z}_2 \), \( Y_3 = D_{xx} \mu^2 + D_{xy} \lambda_3^2 \), \( Y_4 = D_{yy} \mu^2 + D_{yx} \lambda_3^2 \), and \( \eta = \{ A_1, A_2, A_3, A_4 \}^T \) is composed of independent undetermined coefficients. According to the non-trivial condition of Eq. (12), the determinant of \( \mathbf{Z} \) must be zero, i.e.,

\[
D_\mu^2 (\lambda_1^2 - \lambda_2^2)^2 \sinh(b \lambda_1) \sinh(b \lambda_2) = 0. \tag{13}
\]

whose roots are

\[
\lambda_1 = \pm \frac{in \pi}{b} \quad \text{and} \quad \lambda_3 = \pm \frac{in \pi}{b} \quad (n = 1, 2, 3, \ldots). \tag{14}
\]

Substituting Eq. (14) into Eq. (8), the symplectic eigenvalues are achieved, i.e.,

\[
\mu_{1,2} = \pm \frac{1}{2D_s} \left[ d_4 + \sqrt{d_4^2 - 4D_s \left( d_5 + d_5 \xi \left( \frac{n \pi}{b} \right)^2 + D_y \left( \frac{n \pi}{b} \right)^4 \right)} \right], \tag{15a}
\]

\[
\mu_{3,4} = \pm \frac{1}{2D_s} \left[ d_4 - \sqrt{d_4^2 - 4D_s \left( d_5 + d_5 \xi \left( \frac{n \pi}{b} \right)^2 + D_y \left( \frac{n \pi}{b} \right)^4 \right)} \right] \tag{15b}
\]

where \( d_4 = 2D_s \left( \frac{n \pi}{b} \right)^2 + d_5 \xi^2 \), \( d_5 = K + N_y \left( \frac{n \pi}{b} \right)^2 \).

The symplectic eigensolution (10) can be rewritten as

\[
\begin{align*}
\psi^{(1)}_n &= \left\{ 1, \mu, d_6, -D_s \mu^2 + D_{xy} \left( \frac{n \pi}{b} \right)^2 \right\} \sin \left( \frac{n \pi}{b} y \right), \\
\psi^{(2)}_n &= \left\{ 1, \mu, -d_6, -D_s \mu^2 + D_{xy} \left( \frac{n \pi}{b} \right)^2 \right\} \sin \left( \frac{n \pi}{b} y \right), \\
\psi^{(3)}_n &= \left\{ 1, -\bar{\mu}, d_7, -D_s \bar{\mu}^2 + D_{xy} \left( \frac{n \pi}{b} \right)^2 \right\} \sin \left( \frac{n \pi}{b} y \right), \\
\psi^{(4)}_n &= \left\{ 1, -\bar{\mu}, -d_7, -D_s \bar{\mu}^2 + D_{xy} \left( \frac{n \pi}{b} \right)^2 \right\} \sin \left( \frac{n \pi}{b} y \right)
\end{align*} \tag{16}
\]

where \( \mu = \mu_1 \), \( \bar{\mu} = \mu_3 \), \( d_6 = -\mu \left[ D_s \mu^2 - \left( N_{xx} \xi^2 + 4D_x + D_{xy} \right) \left( \frac{n \pi}{b} \right)^2 - K \xi^2 \right] \), \( d_7 = -\bar{\mu} \left[ D_s \bar{\mu}^2 - \left( N_{xx} \xi^2 + 4D_x + D_{xy} \right) \left( \frac{n \pi}{b} \right)^2 - K \xi^2 \right] \). It should be mentioned that the first component of \( \psi_n(y) e^{ix \xi} \) is just the buckling mode shape functions of the orthotropic DNPS.

4. Buckling equations of orthotropic DNPS

In this section, the buckling equations of the orthotropic DNPS with all six possible boundary conditions at \( x = 0, a \) will be obtained.

(i) SS

Substituting Eq. (16) into Eq. (5a) yields

\[
f_1(N_y) = 0 \tag{17}
\]
where \( f_1(N_y) = \sinh(a \mu) \sinh(a \bar{\mu}) \). The corresponding buckling load is
\[
N_y = -\frac{b^2}{n^2 \pi^2} \left[ \frac{D_y \left( \frac{m \pi}{a} \right)^4 + 2D_y \left( \frac{n \pi}{b} \right)^2 + D_y \left( \frac{n \pi}{b} \right)^4}{1 + \varepsilon^2 \left( \frac{m \pi}{a} \right)^2 + \varepsilon^2 \left( \frac{n \pi}{b} \right)^2} \right]
\]
(18)
where \( m \) stands for the half wave number in \( x \)-direction \( (m = 1, 2, 3, \cdots) \).

The buckling equations for the other boundary conditions can be derived in a similar manner.

(ii) SC
\[
\mu f_2(N_y) - \bar{\mu} f_3(N_y) = 0;
\]
(19)
(iii) CC
\[
2\mu \bar{\mu} \left[ f_1(N_y) - 1 \right] - (\mu^2 + \bar{\mu}^2) f_1(N_y) = 0;
\]
(20)
(iv) SF
\[
\alpha_1 \beta_2 \mu f_2(N_y) - \alpha_1 \beta_2 \bar{\mu} f_3(N_y) = 0;
\]
(21)
(v) CF
\[
\mu \bar{\mu} (\alpha_1 \beta_2 + \alpha_2 \beta_1) - \mu \bar{\mu} f_4(N_y) (\alpha_1 \beta_2 + \alpha_2 \beta_1) + \left[ \mu^2 \bar{\mu} \beta_2 + \bar{\mu}^2 \alpha_2 \beta_2 \right] f_1(N_y) = 0;
\]
(22)
(vi) FF
\[
2\mu \bar{\mu} \alpha_1 \beta_2 \beta_2 \left[ 1 - f_4(N_y) \right] + \left[ \mu^2 \bar{\mu} \beta_1 + \bar{\mu}^2 \alpha_1 \beta_1 \right] f_1(N_y) = 0.
\]
(23)

where
\[
\alpha_1 = -D_y \mu^2 + D_{xy} (n \pi / b)^2, \quad \alpha_2 = -D_y \bar{\mu}^2 + D_{xy} (n \pi / b)^2
\]
\[
\beta_1 = D_y \mu^2 - (4D_y + D_{xy}) (n \pi / b)^2, \quad \beta_2 = D_y \bar{\mu}^2 - (4D_y + D_{xy}) (n \pi / b)^2 - N_y.
\]

5. Numerical examples
In this section, the critical buckling loads are computed by the symplectic method. The accuracy and efficiency of the proposed method are checked by comparing with the existing benchmarks in 5.1. The parametric studies of small size effect, interaction effect, elastic medium and aspect ratio on the buckling of DNPSs is performed in 5.2. To simplify the manipulation, the non-dimensional parameters are used here, i.e., \( \bar{K}_w = K_w b^4 / D_y, \quad \bar{K}_e = K_e b^4 / D_y \quad \bar{N}_y = N_y b^2 / (D_y \pi^2) \), \( \bar{\xi} = \xi / b \).

5.1. Verification
At first, a single-layered graphene sheet with computation parameters in Table 1 [25] is taken into consideration to verify the symplectic method. The critical buckling load for this case is 1.6389 nN/nm. The results are compared well with 1.96 nN/nm the result of Xiang and Shen [26] using the molecular dynamics (MD) simulation. Subsequently, the critical buckling loads of a Levy-type isotropic square nanoplate with \( v_1 = v_2 = 0.25, \quad \bar{\xi} = 0.05 \) are computed by the proposed method. Table 2 tabulates the comparison of the non-dimensional critical buckling loads to evaluated the efficient of the symplectic method. It is clear that the computed results are in excellent agreement with the results of Sarrami-Foroushani and Azhari [27]. At last, a further comparison study is carried out by considering an isotropic DNPS with \( a/b = 1, \quad \bar{\xi} = 0.1 \). The non-dimensional critical buckling loads for various interaction coefficients are summarized in Table 3. Due to the assumption of in-phase buckling, the DNPS can be treated as a monolayer nanoplate and the results for \( K = 0 \) are the critical buckling loads.
for in-phase sequence. It is also found that, the present results are in a good agreement with Murmu et al. [11]. From the above three comparisons, it is concluded that the symplectic method is appropriated for the buckling analysis of the DNPS and could provide highly accurate numerical results.

**Table 1.** Computation parameters (300K) of a zigzag graphene sheet [25].

| chirality |  $E_x$ (TPa) |  $E_y$ (TPa) |  $G$ (TPa) |  $a$ (nm) |  $b$ (nm) |  $H$ (nm) |  $\xi$ (nm) |  $\nu$ |
|-----------|-------------|-------------|----------|---------|---------|---------|---------|------|
| Zigzag    | 1.987       | 1.974       | 0.857    | 3.799   | 3.659   | 0.154   | 0.22    | 0.205 |

**Table 2.** Non-dimensional critical buckling loads of a Levy-type isotropic square nanoplate.

| type | SS | SC | CC | SF | CF | FF |
|------|----|----|----|----|----|----|
| Present | 3.8119 | 5.4528 | 6.8206 | 1.4002 | 1.6599 | 0.9477 |
| Ref. [27] | 3.8120 | 5.4537 | 6.8252 | 1.3901 | 1.6430 | 0.9445 |

**Table 3.** Non-dimensional critical buckling loads of a SSSS DNPS with various $\overline{K}_w$.

| $\overline{K}_w$ | 0 | 20 | 60 | 100 | 200 |
|------------------|---|----|----|-----|-----|
| In-phase buckling | Present | 3.3406 | 3.7512 | 4.5725 | 5.3938 | 7.4470 |
| Ref. [11] | 3.3406 | 3.7512 | 4.5725 | 5.3938 | 7.4470 |

5.2. Benchmark solution and parametric study

In this example, an orthotropic DNPS with $E_x=1765$ Gpa, $E_y=1588$ Gpa, $G = 678.85$ Gpa, $\nu_x = 0.3$, $\nu_y = 0.27$, $a = b = 10$ nm, $\overline{K}_w = 60$, $\overline{K}_w = 100$ is considered here.

To demonstrate the small size effect on the buckling of the orthotropic DNPS, the non-dimensional critical buckling loads for all six possible boundary conditions are tabulated in Table 4 with $\xi = 0, 0.5, 1, 1.5, 2$ nm. For a specific boundary condition, the critical buckling loads for in-phase or out-of-phase buckling show decreasing trends with the increasing nonlocal parameter. The observations are in accordance with those found in the study of monolayer nanoplates [28, 29]. Furthermore, for a fixed boundary condition and nonlocal parameter, the results of out-of-phase buckling are much greater than those of in-phase buckling. It illustrates that the in-phase buckling occurs more easily than the out-of-phase buckling. In other words, the design of forming 3D mesostructures should provide an appropriate driving force to achieve the out-of-phase sequence.

The effects of the interaction between the two nanoplates and the surrounding elastic medium on the buckling of the orthotropic DNPS are presented in Table 5. The computation parameters are the same as those in Table 4. The boundary condition is selected as SFSF. Since the in-phase buckling is independent on $K_x$, only the results for $K_x = 0$ are tabulated in Table 5. From the tabular data, it is found that the critical buckling loads for the out-of-phase buckling show an increasing trend with the increase of $K_x$. In another word, the stronger interaction leads to a greater driving force for inducing out-of-phase buckling of the DNPS. It is also found that the critical buckling loads for in-phase and out-of-phase buckling monotonously increase as $K_x$ increases. It indicates that the surrounding elastic medium would prevent buckling of the DNPS. All these phenomena imply that the coupling effect and surrounding elastic medium of the DNPS have significant influences on the driving force in fabricating 3D mesostructures.

The effect of aspect ratios on the critical buckling loads is investigated by considering the same DNPS as that in Table 5. The variations of non-dimensional critical buckling load versus the aspect ratio for $a/b = 0.25, 0.5, 0.75, 1.0, 1.5, 2.0, 3.0$ are shown in Fig.2 with $\xi = 0, 0.5, 1.0$ nm. In general, the curves of critical buckling loads show decreasing trends as the aspect ratio increases. It is
interesting to find that the critical buckling loads for in-phase sequence could be greater than those for out-of-phase sequence when the aspect ratio is smaller than a specific value (about 1), e.g., the black line is above the green and orange lines. However, the critical buckling loads for out-of-phase sequence are always greater than those of in-phase sequence when the aspect ratio is greater than that specific value (about 1). It indicates that the aspect ratio is also a key influencing factor for the buckling of the orthotropic NDPS. The out-of-phase buckling may be induced by a small driving force when a suitable aspect ratio is selected.

In addition, to get a better understanding of the buckling of DNPSs, the first six buckling shape modes for in-phase and out-of-phase sequences of a SFSF orthotropic DNPS are plotted in Figs. 3 and 4, respectively. It is apparent that all the buckling shape modes strictly satisfy the given boundary conditions.

### Table 4. Non-dimensional critical buckling loads of a Levy-type DNPS for various nonlocal parameters $\xi$.

| $\xi$ (nm) | SS    | SC    | CC    | SF    | CF    | FF    |
|------------|-------|-------|-------|-------|-------|-------|
|            | In-phase buckling |       |       |       |       |       |
| 0          | 4.8802 | 6.6141 | 7.4927 | 2.3347 | 2.5897 | 1.8880 |
| 0.5        | 4.6990 | 5.9430 | 6.6734 | 2.3043 | 2.5552 | 1.8702 |
| 1          | 4.1421 | 4.5149 | 5.0452 | 2.2207 | 2.4600 | 1.8209 |
| 1.5        | 3.0064 | 3.2578 | 3.4845 | 2.1029 | 2.3250 | 1.7503 |
| 2          | 2.1680 | 2.2183 | 2.2446 | 1.8253 | 1.8775 | 1.6701 |
|            | Out-of-phase buckling |       |       |       |       |       |
| 0          | 6.1121 | 6.9661 | 7.8007 | 3.5666 | 3.8217 | 3.1199 |
| 0.5        | 5.7301 | 6.2510 | 6.9814 | 3.5362 | 3.7871 | 3.1021 |
| 1          | 4.4500 | 4.8229 | 5.3532 | 3.4425 | 3.5288 | 3.0528 |
| 1.5        | 3.3143 | 3.4991 | 3.6214 | 2.7014 | 2.7693 | 2.4951 |
| 2          | 2.2642 | 2.2720 | 2.2755 | 2.1333 | 2.1585 | 1.9922 |

### Table 5. Non-dimensional critical buckling loads of a SFSF DNPS for various $K_w$ and $K_e$.

| $K_w$ | $K_e$ | $\xi$ (nm) | 0   | 0.5  | 1   | 1.5  | 2   |
|-------|-------|------------|-----|------|-----|------|-----|
|       |       | In-phase buckling |     |      |     |      |     |
| 0     | 0     | 0.8614     | 0.8436 | 0.7943 | 0.7236 | 0.6435 |
| 25    | 1.1181 | 1.0033     | 1.0509 | 0.9830 | 0.9002 |
| 50    | 1.3747 | 1.3569     | 1.3076 | 1.2370 | 1.1568 |
| 100   | 1.8880 | 1.8702     | 1.8209 | 1.7503 | 1.6701 |
|       |       | Out-of-phase buckling |     |      |     |      |     |
| 20    | 0     | 1.2721     | 1.2542 | 1.2049 | 1.1343 | 1.0541 |
| 25    | 1.5287 | 1.5109     | 1.4616 | 1.3910 | 1.3108 |
| 50    | 1.7854 | 1.7675     | 1.7182 | 1.6476 | 1.5674 |
| 100   | 2.2987 | 2.2808     | 2.2315 | 2.1609 | 1.7869 |
| 40    | 0     | 1.6827     | 1.6649 | 1.6156 | 1.5450 | 1.4648 |
| 25    | 1.9393 | 1.9215     | 1.8722 | 1.8016 | 1.6971 |
| 50    | 2.1960 | 2.1782     | 2.1289 | 2.0583 | 1.7612 |
| 100   | 2.7093 | 2.6915     | 2.6422 | 2.3924 | 1.8896 |
Figure 2. Variation of non-dimensional critical buckling loads versus the aspect ratio for different nonlocal parameters.

Figure 3. First six buckling mode shapes for in-phase sequences of a SFSF DNPS: (a) 1st mode (1,1); (b) 2nd mode (1,2); (c) 3rd mode (2,1); (d) 4th mode (2,2).

Figure 4. First six buckling mode shapes for out-of-phase sequences of a SFSF DNPS: (a) 1st mode (1,1); (b) 2nd mode (1,2); (c) 3rd mode (2,1); (d) 4th mode (2,2).

6. Conclusions
Axial elastic instability study of orthotropic DNPSs embedded in the elastic medium is carried out in the framework of Hamiltonian mechanics. Eringen’s nonlocal elasticity plate theory is employed to take the small size effects of the DNPS into consideration. By using a change of variable, the buckling of the DNPS can be divided into two types, in-phase sequence and out-of-phase sequence. The governing equations for the two type buckling modes are formulated in a unified form by introducing a
total unknown vector composed of the displacement, angle of rotation, generalized shear force and bending moment. The high-order governing differential equation in the classical Lagrangian system is changed into a set of ordinary differential equations in the Hamiltonian system. Therefore, the stability analysis of the orthotropic DNPS is regarded as the determination of symplectic eigenvalues and eigensolutions by using the method of separation of variables. Highly accurate critical buckling loads and exact buckling shape modes for all six possible boundary conditions are presented in the numerical examples. The parametric studies of the nonlocal parameter, stiffness coefficients of the interaction effect and elastic medium, aspect ratio are presented in detail.

Acknowledgments
In this research work, the supports of Shenzhen Science and Technology Funding Fundamental Research Program (No. JCYJ20170413141248626); Dalian Innovation Foundation of Science and Technology (No. 2018J11CY005); the National Natural Science Foundation of China (No. 11672054); the Key Program of Natural Science Foundation of Liaoning Province of China (No. 20170540186); High Level Talents Support Plan of Dalian of China (No. 2017RQ111) and the Fundamental Research Funds for the Central Universities (No. DUT17LK57) are gratefully acknowledged.

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