Tyre Side-Slip Deformation of Dual-Steering Multi-Axle Vehicles Based on Steady-State Cornering

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Abstract. It is well-known that the tyre side-slip deformation is a static indeterminacy problem for the full-vehicle tyres. For the dual-steering multi-axle vehicles on steady-state cornering, it is a closed and dissipative system in a stable dynamic balance state. Therefore, the minimum energy dissipation rate principle is adopted to solve the above indeterminacy problem. Moreover, a new method is proposed to develop a formula for calculating the tyre side-slip angles of the vehicles with dual steering systems (DSS). The validity of the formula is examined using the formula from literature and simulation results of the reference model.

1. Introduction
In practise, multi-axle vehicles with a single steering system have some steering reliability problems. Concern for achieving better steering reliability performance has led to the use of a dual system with a hydraulic-assisting steering link mechanism system (HASLMS) for the front axles and an electrohydraulic active-steering system (EHASS) for the rear axles on these vehicles. Multi-axle vehicles such as the all-terrain cranes QAY1000 from XCMG and LTM 1400.7.1 from Liebher have become available. Figure 1 shows the chassis of the all-terrain crane LTM 1400.7.1.

![Figure 1. Chassis of all-terrain crane LTM 1400.7.1.](image-url)

The motion of a vehicle is governed by the forces generated between the tyre and the road. Therefore, tyre lateral force measurements are of interest to vehicle designers for stability control and ABS braking applications [1]. They are also essential to study vehicle dynamics, which have been examined by several researchers [2–4]. The lateral tyre force is generated as a function of tyre side-slip angle. However, the lateral force obtained is ultimately limited by the friction limit of the road. Because knowledge of the side-slip angle is important for vehicle stability control, real-time information about the side-slip angle is critical to the performance of integrated vehicle dynamics control (IVDC) strategies [4]. Owing to the difficulty of directly measuring the side-slip angle [6–7], its estimation accuracy becomes very important. Researchers have demonstrated that a combination of GPS and inertial sensors can provide accurate measurements of the slip angle [8–9]. However, until...
now, no attempt has been made with regard to side-slip-angle-estimation-based lateral dynamics control of multi-axle steering vehicles with double steering systems. This motivates the current research.

2. Steady-state cornering of multi-axle vehicles with DSS

Figure 2. Dynamic model of multi-axle vehicles with DSS and coordinate systems.

Figure 2 shows coordinate systems used to describe a multi-axle vehicle with side-slip angle $\beta$, vehicle velocity $v$, and yaw rate $r$ at the vehicle centre of gravity. $\beta$ and $r$ are positive in the counterclockwise direction, and $v$ is positive in the $x$ direction. Assuming that all of the left and right wheels are concentrated to the centre line of the vehicle, the model of the multi-axle vehicle becomes a simplified multi-axle bicycle one. The vehicle has two steering systems with HASLMS for axles $1$--$k$ and EHASS for axles $(k+1)$--$n$. The vehicle has interconnected hydropneumatic suspensions for supporting the vehicle body. Because the tyre side-slip angle and yaw rate at the centre of gravity are constant in steady-state cornering, $dv/dt$, $d\beta/dt$, and $dr/dt$ become zero. Moreover, the lateral force of the side wind is neglected.

Point $G$ is the vehicle centre of gravity, and a moving coordinate system $(x, y)$ is attached to the point. Point $O_f$ is the turning centre of the front wheels steered by HASLMS. $\Delta_0$ denotes the distance between the foot point of the point $O_f$ on the $x$-axis and the vehicle centre of gravity $G$. The numerical value of $\Delta_0$ is positive as point $O_f$ is in front of point $G$; Otherwise, it is negative. Point $O_r$ denotes the turning centre of the rear wheels steered by EHASS. $\Delta$ denotes the distance between the foot point of point $O_r$ on the $x$-axis and the vehicle centre of gravity $G$. The numerical value of $\Delta$ is positive as point $O_r$ is in front of point $G$ along the vehicle’s travelling direction; Otherwise, it is negative.

The numerical value of $\Delta$ is calculated according to the active steering control algorithm of the rear wheels. $l_i$ denotes the distance from the vehicle centre of gravity $G$ to the $i$th axle in the $x$-direction. $l_i$ is positive as the $i_{th}$ axle is in front of the vehicle centre of gravity $G$. In fact, the actual turning centre of all wheels is the point $O$ considering the tyre side-slip angles produced by the centrifugal force. $\lambda$ is the coordination-deformation coefficient of the tyres. The distances from points $O_f$, $O$, and $O_r$ to the $x$-axis are $R_f$, $R$, and $R_r$, respectively. $F_{xi}$ and $F_{yi}$ denote the $x$-component and $y$-component of the frictional forces $F_i$ acting on the $i_{th}$ wheel, respectively. The steering angle of each tyre is described by $\Delta_i$. The subscript $i$, ranging from 1 to $n$, indicates the position of each tyre, starting from the front to the rear. The side-slip angle of the $i_{th}$ tyre is $\alpha_i$.

Assuming the wheel steering angle is small, for steering angles of wheels steered by SLMSHA, the ratio $H_i$ of $\Delta_i$ to $\Delta_f$ can be expressed as
Because the distance between points Of and Or increases as the vehicle speed increases, the steering angles of wheels steered by EHASS should be calculated according to the turning centre Or. Therefore, for the steering angles of wheels steered by EHASS, the ratio \( K_i \) of \( \delta_1 \) to \( \delta_i \) is different than the previous ratio \( H_i \). This can be expressed as

\[
\frac{l_i - \Delta_0}{l_i - \Delta} = \frac{\tan \delta_i}{\tan \delta_1} \Rightarrow H_i = \frac{\delta_1}{\delta_i} \approx \frac{l_i - \Delta_0}{l_i - \Delta}
\]

(1)

3. Deformation of tyres with side slips of vehicles with DSS

It’s well known that the side-slip angles are highly significant in maintaining the stability of the vehicle motion. The deformation distribution of tyres with side slip of an entire vehicle may provide an important way to evaluate the motion stability of the vehicle. For multi-axle vehicles, however, the calculation of side-slip angles is an indeterminate problem. Here, a new method for calculating tyre side-slip angles is introduced in detail.

According to the definition of the minimum energy dissipation rate principle, for a closed and dissipative system in a stable dynamic balance state [10], the energy consumption rate is minimum corresponding to the constraints imposed on the system. The static indeterminate problem of the deformation of tyres with side slip is to be solved by inducing this principle. Therefore, the deformation energy consumption rate of tyres with side slip caused by a cornering motion can be written as

\[
P = \sum_{i=1}^{n} F_i \Delta v_i
\]

(3)

where the side-slip velocity of a tyre can be written as

\[
\Delta v_i = \frac{\nu}{\rho_i} \sin \alpha_i
\]

(4)

where \( \rho_i \) is the turning radius of the \( i \)th wheel, and \( \rho \) is that of the vehicle centre of gravity.

In the region where the side-slip angle \( \alpha_i \) is small, \( \sin \alpha_i \approx \alpha_i \), and the terms with an order of more than three of Taylor’s series of \( \alpha_i \) can be ignored. Therefore, the energy dissipation rate of the deformation of tyres with side slip can be written as

\[
P = \frac{\nu}{\rho} \sum_{i=1}^{n} C_i \alpha_i^2 \rho_i
\]

(5)

Equation (5) is an objective function. For the deformation of tyres with side slip to correspond to real conditions, the following constraints should be satisfied:

(1) The sum of all forces in the y direction should be zero, neglecting the longitudinal forces \( F_x \).

(2) The sum of the moments of all lateral forces around the vehicle centre of gravity G should be zero.

The constraint equations can be described as

\[
\phi_i = \sum_{i=1}^{n} C_i \alpha_i \cos \delta_i - \frac{mv^2}{\rho} \cos \beta = 0
\]

(6)
\[ \varphi_2 = \sum_{i=1}^{n} C_i l_i \alpha_i \cos \delta_i = 0 \]  
(7)

Adopting the method of Lagrange multipliers to solve the optimization problem, the optimization function can be written as

\[ L = \frac{v}{\rho} \sum_{i=1}^{n} C_i \rho \alpha_i^2 + \eta_1 \left( \sum_{i=1}^{n} C_i \alpha_i \cos \delta_i - \frac{mv^2}{\rho} \cos \beta \right) + \eta_2 \sum_{i=1}^{n} C_i l_i \alpha_i \cos \delta_i \]  
(8)

Take a partial derivation of the function \( L \) with respect to \( \alpha_i, \eta_1, \) and \( \eta_2, \) and make it equal to zero:

\[ \frac{\partial L}{\partial \alpha_i} = 2 \frac{v}{\rho} C_i \rho \alpha_i + \eta_1 C_i \alpha_i \cos \delta_i + \eta_2 C_i l_i \alpha_i \cos \delta_i = 0 \]  
(9)

\[ \frac{\partial L}{\partial \eta_1} = \sum_{i=1}^{n} C_i \alpha_i \cos \delta_i - \frac{mv^2}{\rho} \cos \beta = 0 \]  
(10)

\[ \frac{\partial L}{\partial \eta_2} = \sum_{i=1}^{n} C_i l_i \alpha_i \cos \delta_i = 0 \]  
(11)

From Equation (9), the side-slip angle of the \( i \)th tyre can be obtained:

\[ \alpha_i = -\frac{\rho \eta_1 C_i \cos \delta_i + \eta_2 C_i l_i \cos \delta_i}{2v} \]  
(12)

Substituting Equation (12) into (11):

\[ \eta_1 = -\frac{\eta_2 \sum_{i=1}^{n} C_i l_i^2 \cos \delta_i^2}{\rho_i \sum_{i=1}^{n} C_i l_i \cos \delta_i} \]  
(13)

Substituting Equation (12) and (13) into (10), and the following formula can be obtained:

\[ \eta_2 = \frac{2mv^3}{\rho^2} \frac{\cos \beta \sum_{i=1}^{n} C_i l_i \cos \delta_i^2}{\rho_i \sum_{i=1}^{n} \sum_{j=1}^{n} C_i C_j \cos \delta_i \cos \delta_j \cos \delta_i \cos \delta_j \frac{l_i}{l_i - l_j}} \]  
(14)

Substituting Equation (14) into (13):

\[ \eta_1 = -\frac{2mv^3}{\rho^2} \frac{\cos \beta \sum_{i=1}^{n} C_i l_i^2 \cos \delta_i^2}{\rho_i \sum_{i=1}^{n} \sum_{j=1}^{n} C_i C_j \cos \delta_i \cos \delta_j \cos \delta_i \cos \delta_j \frac{l_i}{l_i - l_j}} \]  
(15)

Substituting Equations (14) and (15) into (12):
\[ a_i = \frac{mv^2 \cos \beta \cos \delta_i}{\rho \rho_i} \sum_{j=1}^{n} C_j l_j (l_j - l_i) \cos^2 \delta_i + \rho \rho_j \sum_{k=1}^{n} \sum_{j=1}^{n} C_k C_j \cos^2 \delta_i \cos^2 \delta_j l_k (l_k - l_j) \]  \hspace{2cm} (16)

4. Validation of the method

4.1. Comparison with the model from literature

To validate the method, a sample of five-axle steering vehicles with HASLMS was used for a simulation analysis, and comparisons were conducted between the calculation results from the derived formula (16) and from the formula calculating the tyre side-slip angle in [11]. The formula for the tyre side-slip angle in [11] can be described as

\[ a_i = \frac{\sum_{j=1}^{n} C_j l_j^2 - l_i \sum_{j=1}^{n} C_j l_i}{\sum_{j=1}^{n} C_j \sum_{i=1}^{n} C_i l_i - \left( \sum_{i=1}^{n} C_i l_i \right)^2} \frac{mv^2}{2 \rho} \]  \hspace{2cm} (17)

If \( \rho_i \approx \rho \), \( \cos \beta \approx 1 \), and \( \cos \Delta_i \approx 1 \), Equation (16) is transformed into Equation (17). Therefore, the calculation accuracy of Equation (16) is higher than that of Equation (17), and Equation (16) is a more generalised form for tyre side-slip angles. For example, if \( n=2 \) in formula (17), it becomes the calculation formula for tyre side-slip angles \( \beta_f \) and \( \beta_r \) of two-axle vehicles, noting that \( l_1-l_2, C_1, C_2, l_f, \) and \( l_r \) here are the equivalents of \( l, 2K_f, 2K_r, l_f, \) and \( l_r \) on page 66 in [12], respectively. This further validates that the calculation formula for the tyre side-slip angle derived from the proposed method is correct in theory.

4.2. Comparison with the reference model

4.2.1 Setting of simulation parameters.

Comparative analyses of the tyre side-slip angles calculated with the two methods were performed using MATLAB software. The setting of the simulation parameters was as follows: \( l_1=4.07 \text{m}, l_2=1.32 \text{m}, l_3=-0.3 \text{m}, l_4=-1.92 \text{m}, l_5=-3.54, \) vehicle weight \( m=58000 \text{kg}, \) turning radius of vehicle gravity centre calculated by Equation (9), turning radius of every axle calculated by Equation (11), cornering stiffness \( C=612 \text{kN/rad} \) for all tyres, and \( \Delta = \Delta \phi =0 \) for the vehicle with HASLMS.

![Figure 3. Comparison of results of tyre side-slip angles.](image)

4.2.2 Analytic demonstration of simulation results by comparison.

In the computer simulation, a steady-state cornering scenario was considered. The vehicle speed accelerated from 0 m/s to 23 m/s at
a constant steering angle of 0.1 rad for the first wheel. The comparison results are shown in Figure 3. The results calculated using Equation (16) and those from Equation (17) are marked with mdet* and rdet*, respectively, with the asterisk * representing the serial number of axles. The maximum discrepancy between them was less than 0.01 rad because Equation (17) is just the simplified form of Equation (16) as previously mentioned. Hence, it can be thought that they are approximately equal, thereby proving the validity of the method.

5. Conclusions
The paper presents the deformation distribution of tyres with side slip for multi-axle vehicles with DSS. A new method was proposed based on the deformation energy optimization function related to all constrained tyres. A minimum energy dissipation rate principle and Lagrange multiplier method were introduced for the method. A formula for calculating the tyre side-slip angle was derived, and it has higher calculation accuracy than that of the model from literature.

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