Longitudinal profiles of Extensive Air Showers with inclusion of charm and bottom particles

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Abstract

Charm and bottom particles are rare in Extensive Air Showers but the effect of its presence can be radical in the development of the Extensive Air Showers (EAS). If such particles arise with a large fraction of the primary energy, they can reach large atmospheric depths, depositing its energy in deeper layers of the atmosphere. As a consequence, the EAS observables ($X_{\text{max}}$, $\text{RMS}$ and $N_{\text{max}}$) will be modified, as well as the shape of the longitudinal profile of the energy deposited in the atmosphere. In this paper, we will modify the CORSIKA Monte Carlo by the inclusion of charm and bottom production in the first interaction of the primary cosmic ray. Results for different selections of the typical $x_F$ values of the heavy particles and distinct production models will be presented.

1 Introduction

The description of the Extensive Air Showers (EASs) is fundamental for the Cosmic Ray Physics. Primary particles reach the Earth with energies up to $10^{20}$ eV. Such energies are well above those reached in colliders, and, therefore, the simulations of these EASs require an extrapolation of the known physics.

Very energetic charm and bottom heavy hadrons may be produced in the upper atmosphere when a primary cosmic ray or a leading hadron in an EASs collide with the air. Because of its short mean life, $\approx 10^{-12}$ s ($\approx 300 \mu m$), heavy hadrons decay before interacting. At $E \approx 10^7$ GeV heavy hadrons reach their critical energy and its decay probabilities decrease rapidly. Decay lengths grow to considerable values. At $E \approx 10^8$ GeV their decay length becomes of order $\approx 10$ km, implying that they tend to interact in the air instead of decaying. Since the inelasticity in these collisions is much smaller than the one in proton and pion collisions, keeping a higher fraction of their energy after each interaction, there could be rare events where a heavy hadron particle transports a significant amount of energy deep into the atmosphere, giving rise to additional contributions to the EAS development. Heavy particles can be produced at any stage of the EAS development, but it is mainly during the first interaction of the primary collision that they are produced with a significant fraction of primary energy.

The collisions of heavy hadrons with the air are very elastic. For example, a $D$ meson after a $10^9$ GeV collision could keep around 55% of the initial energy, whereas a $B$ meson will have typically 80% of the incident energy after colliding with an air nucleus. In contrast, the leading meson after a $10^9$ GeV pion collision would carry in average just 22% of the energy \cite{1}. If heavy hadrons are produced with a high fraction of the primary particle, they will interact rather than decaying. If several elastic interactions occur, these heavy hadrons could transport a significant amount of energy deep into the atmosphere and likely have observable effects on the EAS development.

The energy deposition of a heavy particle near of the ground would produce muons and other particles that could change significantly the EASs longitudinal profile seen in fluorescence telescopes.
(e.g. in the Pierre Auger Observatory \cite{2}) and/or the temporal distribution observed in the surface detectors. That will modify the EAS observables ($X_{\text{max}}$, $\text{RMS}$ and $N_{\text{max}}$), besides a considerable change of the EAS longitudinal profile shape.

In this work, we will use the CORSIKA \cite{3} (Cosmic Ray Simulations for Kaskade) Monte Carlo to simulate the evolution of EASs in the atmosphere. We will a modified code of CORSIKA, with charm and bottom production at the cosmic ray first interaction \cite{4} (hereafter denoted HQ CORSIKA). In the next section, we will explain how this version works. Moreover, the HQ CORSIKA predictions with those derived using the standard CORSIKA (original code - without charm particle production) (hereafter denoted STD CORSIKA). In both versions (HQ CORSIKA and STD CORSIKA) we will use the QGSJET01 MC to describe the high energy hadronic interactions and the FLUKA one for the description of the low energy hadronic interactions. We will demonstrate that the charm and bottom production rates are only relevant for the highest energies. The modified MC allows us to analyse the effects that the production and propagation of heavy hadrons has in the EAS development.

Our goal is to verify the impact on the EAS observables of different selections for the minimum value of the $x_F$ variable \footnote{Probability of production of a heavy hadron carrying values larger than a certain fraction of primary energy.} as well of different models to describe the heavy hadron production - CGC (Color Glass Condensate) and IQM (Intrinsic Quark Model).

## 2 Heavy particle simulation

As mentioned earlier, we have included in the CORSIKA MC the production of heavy hadron (mesons and/or baryons) in the first interaction of primary cosmic ray with an incident energy $> 3 \times 10^{17}$ eV. We consider that $\Lambda_s$, $\Xi_s$, $\Sigma_s$, $\Omega_s$, $D_s$ and $B_s$ are produced in this interaction. These particles are produced according to the probability for the quark charm to be present in different species of hadrons after $p-\text{Air}$ collision \cite{6}.

The CORSIKA MC doesn’t explicitly include the charm and bottom heavy interactions. It is necessary that in the extraction of the source code the option CHARM is “switch on”. Not all packages can handle production and propagation of heavy particles. In the CORSIKA, the DPMJET and QGSJET packages of high energy hadronic interactions consider the interaction of these particles. The $D^+_s$ (main source of tau leptons) particle (PDG code 431) is not included by the QGSJET01.

We assume that the heavy particles can be produced via two models:

- Color Glass Condensate - CGC.
- Intrinsic Quark Model - IQM.

The choice of the production model and the kind of heavy particle (charm or bottom) generated in the first interaction is made via CORSIKA INPUT - key COLLDR. From the first interaction, the package of hadronic interaction PYTHIA \cite{5} makes the decay and interaction of the heavy particles. The key PROPAQ determines whether the propagation of heavy particles will be handled by PYTHIA, or by standard routine (e.g. QGSJET01). Through key SIGMAQ, the cross sections for the charm and bottom mesons and baryons are determined \cite{4}.

### 2.1 Color Glass Condensate

In this model, a heavy flavor quark-antiquark pair is created through the fluctuation of a gluon in the projectile particle. Charmed and bottom hadrons are formed from the hadronization of these heavy quarks with sea quarks, in a mechanism called Uncorrelated Fragmentation. More information of the model in \cite{7} and \cite{8}.

When a proton of energy $E_p$ (in GeV) collides with a nucleus in the atmosphere, the probability to produce a heavy hadron carrying a fraction of energy is given by:

\[ \frac{1}{X_{\text{max}}} \]
• Above 5% \((x_F > 0.05)\):  
\[
P(E_p) = 0.00129672 - 0.0000974551 * \ln(E_p) + 0.000055122 * \ln(E_p)^2
\]  
(1)

• Above 1% \((x_F > 0.01)\):  
\[
P(E_p) = -0.0676118 + 0.00544162 * \ln(E_p) + 0.000166688 * \ln(E_p)^2
\]  
(2)

The energy distribution of charms produced has the general form [6]:  
\[
\frac{dP}{dx_F} = a \left(1 - x_F^2\right)^b \tag{3}
\]

where:

\[
a = 0.0094058 + (6.7535 \times 10^{-4}) * \ln(10 * E_p), \quad b = 8.9416 + (-0.02078) * \ln(10 * E_p), \quad c = 1.3578 + (0.01281) * \ln(10 * E_p)
\]  
(4)

2.2 Intrinsic Quark Model

At leading order in QCD, heavy quarks are produced by the processes \(qq \rightarrow QQ\) and \(gg \rightarrow QQ\). When these heavy quarks arise from fluctuation of the initial state, its wave function can be represented as a superposition of Fock state fluctuations:

\[
|h> = c_0|n_v> + c_1|n_vg> + c_2|n_vgq>| + c_3|n_vQQ> ...
\]  
(5)

where \(|n_v>\) is the hadron ground state, composed only by its valence quarks.

When the projectile scatters on the target the coherence of the Fock components is broken and the fluctuations can hadronize, either with sea quarks or with spectator valence quarks. The latter mechanism is called Coalescence. For instance, the production of \(\Lambda_c^+\) in \(p - N\) collisions comes from fluctuation of the Fock state of the proton to \(|uudc\rangle\) state. To obtain a \(\Lambda_c^-\) in the same collision a fluctuation to \(|uud\bar{c}d\bar{c}\rangle\) would be required. Thus, since the probability of a five quarks state is larger than that of a 9 quarks state, \(\Lambda_c^+\) production is favored over \(\Lambda_c^-\) in proton reactions. The co-moving heavy and valence quarks have the same rapidity in these states but the larger mass of the heavy quarks implies they carries most of the projectile momentum. Heavy hadrons form from these states can have a large longitudinal momentum and carry a large fraction of the primary energy, which is crucial for their propagation [4]. At the figure [1] we can see the differential energy fraction distribution for some charmed and bottom hadrons.

![Figure 1: Feynman - x distribution for some charmed (left) and bottom (right) hadrons derived using the Intrinsic Quark production model][4].
The probability to have a state $|uudc\bar{c}\rangle$ in the proton is \[9\]:

$$P(p \to uudc\bar{c}) \approx [m_p^2 - \sum_{i=1}^{5} m_{1i}^2/x_i]^{-2}$$  

(6)

where the transverse mass is $m_{1i}^2$ and we take $i = 4, 5$ for $c, \bar{c}$. A detailed description of this model can be found in \[9\] and \[10\].

### 3 Longitudinal Profiles

In this section we will analyse the longitudinal profiles of total energy deposited in the atmosphere, considering $\approx 400$ EAS. We will assume that the primary cosmic ray is $1 \times 10^{20}$ eV, the zenithal angle is $60^\circ$ and primary particle is a proton. Production of charm or bottom heavy particles can occur via two models - CGC and IQM. We will use two selections for the minimum of $x_F$ (via CGC) - $x_F > 0.01$ and $x_F > 0.05$. The comparisons will be made between CORSIKA HQ and STD CORSIKA. For the longitudinal profile of energy deposited, the curves will be separated according to the energy fraction ($F_E$) of the heavy particles produced in the first interaction - $F_E < 0.1$ and $F_E \geq 0.1$ for the CGC model and $F_E < 0.5$, $F_E \geq 0.5$ and $F_E \geq 0.8$ for the IQM production model.

For this analysis, we will restrict the heavy hadron production to $\Lambda_c^+, D^0, D^+, D_s^+, B^+, B^0$ and their anti-particles.\[^2\]

In Fig. 2 we show the evolution of production of secondary charms in the EAS. The charm particles are “written” as they are produced during the EAS development, from the first interaction of cosmic ray down to the sea level. We assume bins of 100 $g/cm^2$ of atmospheric depths and half decade of energy. The charm particles are dominantly produced with low energy (below $10^9$ GeV) and are produced between 100 and 400 $g/cm^2$. On the other hand, the energetic charms are produced in the first interactions, i.e. when the depth of the atmosphere is less than 200 $g/cm^2$. We have that $\approx 47$ is the average number of charm produced above $10^8$ GeV, which will decay and produce high energy $\mu$ and $\nu$ that reach the ground. Regarding the total number of charms produced in all bins (energy higher than $10^4$ GeV), we have an average of $\approx 1000$ charms, being $\approx 400$ $D^0$, $\approx 330$ $D^+$, $\approx 47$ $D_s$ and $\approx 240 \Lambda_c$. From this total we have that $\approx 4$ charm are produced with energy higher than $10^8$ GeV.

\[^2\]Not all high hadronic interactions packages can handle this kind of heavy particles. $D_s^+, B^+ B^0$ and their anti-particles for example is not considered by QGSJET01.
charms reach higher energies. For example, for $x_F > 0.01$ some particles reach $\approx 3 \times 10^{18}$ eV. For this energy the charm decay length becomes of order of $\approx 50$ km, what could make such particle reach the ground with reasonable energy. Comparing now different heavy production models, CGC and IQM, the secondary generated via Intrinsic Quark Model reaches a much larger energy than produced via Color Glass Condensate. Via IQM, we have secondary particles been produced at first interaction with larger fractions of primary energy. Such particles can carry almost all primary energy.

![Energy distribution of the charm hadrons generated at the first interaction of the HQ CORSIKA. The curves are separated according to $x_F$ values ($x_F > 0.01$ and $x_F > 0.05$) and the model used to describe the heavy quark production. We assume that the energy of the primary cosmic ray is $3 \times 10^{19}$ eV.](image)

Considering now the energy distribution of the charmed particles that hits the ground. Independently of the selection for the values of $x_F$ and the production model(CGC and IQM), the number of particles is negligible. Consequently, the most part of charm particles that are produced in the EAS decay or interact before hit the ground.

Muons are a key prediction in EAS simulations. Although the presence of heavy hadrons will not introduce significant differences in the total number of muons at the ground level, there are other observables that may be more sensitive to these heavy hadrons: Events with late energy deposition from the decay of a heavy meson or a $\tau$ lepton. The fraction of these events is low. Events with leptons of PeV energies, coming from charm decays.

![Longitudinal profiles for the total energy deposited in the atmosphere for charm production via CGC ($x_F > 0.01$) and $F_E \geq 0.1$.](image)

In the Figs. 4, 5, and 6 we present our predictions for the longitudinal profiles. The longitudinal profiles are separated according to the fraction of the energy of the heavy particles generated in the
first interaction, \( F_E < 0.1 \) and \( F_E \geq 0.1 \), which allows to highlight the profiles. Concerning to \( F_E < 0.1 \) \((x_F > 0.05)\), for both charm and bottom production (via CGC), the longitudinal profiles follow approximately the same behaviour in comparison with the standard profiles (STD CORSIKA).

Concerning to \( F_E \geq 0.1 \) \((x_F > 0.01, \text{via CGC})\), presented in the Fig. 4, they represent less than 6% of total\(^4\). Looking for the profiles with a fraction < 0.1 we can’t see significant changes in the profile. In fact, they follow the same general shape of profiles with no heavy hadron production (STD CORSIKA). If we look now at the profiles with larger fractions, the effect is more pronounced. We have a slight difference in the maximum of energy deposited, i.e, we have a shift to deeper layers of the atmosphere. These profiles have a smaller value for the peak of energy deposited.

![Figure 5: Longitudinal profiles of the total energy deposited in the atmosphere for charm production via IQM, \( F_E \geq 0.8 \).](image)

Once again we separated the profiles for \( F_E < 0.5 \), \( F_E \geq 0.5 \) and \( F_E \geq 0.8 \) for the IQM heavy production model, considering the charm and bottom production separately. For these cases, we have a large number of profiles with high energy fraction\(^5\). For the highlighted profiles with \( F_E \geq 0.5 \) we predict a non-negligible change of the longitudinal profiles, for both particles, charm and bottom. In particular, for \( F_E \geq 0.8 \), we have the most important results. In some profiles the maximum of total energy deposit is largely shifted to larger depths. We also see in some profiles a big change in the profile shape, i.e, the peak of energy deposited is much smaller\(^6\) and the profiles are quite elongated (See Figs.

\(^4\)The highest value of the fraction of primary energy reached is \( \approx 0.2 \) (CGC production model), i.e, \( \approx 6 \times 10^{18} \text{ eV} \).

\(^5\)As mentioned before, heavy hadrons produced via IQM model can have a large longitudinal momentum and carry a large fraction of the primary energy.

\(^6\)In some cases the peak of total energy deposited reaches just 100 \( \text{PeV/g/cm}^2 \). The standard profile reaches about 250 \( \text{PeV/g/cm}^2 \).
This happens because heavy hadrons produced via IQM at first interaction have high energy fractions, thus carrying a big amount of energy deep into the atmosphere. For profiles with $F_E \geq 0.8$, we see significant changes, mainly for bottom production. In particular, we can see some double core profiles.

4 Discussions

Our purpose in this analysis was to study how the presence of a heavy hadrons could modify the fundamental parameters of the cascade development, such as the shape of longitudinal profile, the number of particles reaching the ground, the position and deviation of the shower maximum. Heavy hadrons propagating with an energy above its critical value will travel long paths. In particular, showers with zenithal angle of 60 degrees have an atmosphere slant depth of $\approx 2100 \text{ g/cm}^2$. After several elastic interactions, we expect heavy particles to deposit its remaining energy deep in the atmosphere and some could reach the ground carrying substantial energy fraction. If the heavy hadron carries a significant fraction of the primary’s energy, we can expect a large impact on the EAS observables. As the heavy particle energy fraction increases, more accentuated these effects will be.

In what follows we will present our predictions for the average longitudinal profiles. In Fig. 7 we
can’t observe significant differences between the average profiles. Small changes of the observable occur only for $x_F > 0.01$ ($F_E \geq 0.1$). In this case the $X_{\text{max}}$ is more deeper and the $\text{RMS}$ larger in comparison with EAS simulated by the STD CORSIKA. We have a relative difference of 9% to $X_{\text{max}}$ and 28% to $\text{RMS}$. Comparing now the $x_F$ assumed, we have small differences in the shape of longitudinal profiles. For $x_F > 0.01$, the $X_{\text{max}}$ and $\text{RMS}$ is slightly larger.

Regarding the average profiles obtained using IQM production model, presented in Figs. 8 and 9, we can see significant changes in the longitudinal profiles. We have a deeper $X_{\text{max}}$ and a higher $\text{RMS}$ when the energy fraction of the heavy particles is increased. For charm production with $F_E \geq 0.8$ we predict the values of 898.8 g/cm$^2$ and 87.8 g/cm$^2$, respectively, for $X_{\text{max}}$ and $\text{RMS}$. For bottom production with $F_E \geq 0.8$ we obtain 972.9 g/cm$^2$ and 128.8 g/cm$^2$. We can also observe a significant change in the shape of the profile. Depositing less energy according to energy fraction. Bottom and charm particle interaction are more elastic than other particles, therefore charms and bottoms produced with high primary fraction will deposit energy more slowly in the atmosphere and can carry large energies deeper in the atmosphere. Such effect is larger in bottom particles. At higher fractions of energy ($F_E \geq 0.8$), the impact on the average longitudinal profile is larger. In the case of charm production we have a $X_{\text{max}}$ shifted to deeper layers of the atmosphere in relation to standard CORSIKA, with the relative difference being about 12%. For the $\text{RMS}$, we have a relative difference of 40%. Regarding bottom production we have a more radical effect, being 22% for $X_{\text{max}}$ (shift to deeper layers) and 100% to $\text{RMS}$. For the $\text{RMS}$, we have larger values, which means that the fluctuation of depth of the maximum of EAS ($X_{\text{max}}$) is larger. This happens because the heavy particle interaction is more elastic. All these EAS effects ($X_{\text{max}}$ shift, larger $\text{RMS}$ and more elongated shape of the EAS profiles) can change significantly the EAS longitudinal profile seen in fluorescence telescopes and/or the temporal distribution observed in the surface detectors. In fact, we could see some double core profiles. The global effect of all these changes in the longitudinal profile is the lower energy reconstructed of the EAS and higher uncertainties. For the energy of the primary cosmic rays considered ($\approx 10^{20}$ eV), the values of $X_{\text{max}}$ and $\text{RMS}$ found in experiments such as the Pierre Auger Observatory for example are respectively $760 \pm 70$ g/cm$^2$ and $\approx 26$ g/cm$^2$ [11]. The $X_{\text{max}}$ and $\text{RMS}$ are directly linked to the mass composition of the primary cosmic ray. The appearance of charm and bottom in the EAS makes it more difficult to make such a connection, because of the deeper $X_{\text{max}}$ and larger $\text{RMS}$.

![Figure 8: Longitudinal profiles of the energy deposited in the atmosphere considering HQ CORSIKA (IQM - charm production) and the STD CORSIKA. Predictions for $F_E < 0.5$, $F_E \geq 0.5$ and $F_E \geq 0.8$.](image)

In Figs. 8 and 9 we can also see a more elongated shape of the EAS, ie, a slower energy deposit. In these figures we analyze when the energy deposition in the shower is shifted to large depths. The

\[^{9}\text{Here it is taken into account that we have a mixture of several mass compositions (H, He, C, Fe, etc.)}\]
amplitude and position of EAS $X_{\text{max}}$ is affected. The number of particles at maximum decrease, while the number of particles that reach the ground increase. In Fig. 10 we show the ratio between the number of particles in the maximum of the shower and the number of particles at the ground level ($E_{\text{max}}/E_{\text{ground}}$) according to the fraction of energy carried by the heavy hadron. Such ratio is sensitive to the change in the profile’s shape. The EAS energy deposited amplitude decrease as the $X_{\text{max}}$ is shifted to higher depths. The effect is higher to bottom particle production because of its higher elasticity.

Figure 9: Longitudinal profiles of the energy deposited in the atmosphere considering the HQ CORSIKA (IQM - bottom production) and the STD CORSIKA. Predictions for $F_E < 0.5$, $F_E \geq 0.5$ and $F_E \geq 0.8$.

Figure 10: Ratio between the number of particles at EAS maximum and the number of particles at ground level according to fraction of energy carried by the heavy hadron. The red point it related to ratio predicted by the STD CORSIKA.

In Fig. 11 we present the average longitudinal profiles for the energy deposited by muons and neutrinos. The shape of curves is shifted to IQM bottom production, both for muons an neutrinos. Again, this effect is more significant when we consider higher energy primary fractions.
5 Conclusions

Regarding average longitudinal profile via CGC, we observe small changes in the $X_{\text{max}}$, $RMS$ and $N_{\text{max}}$ observables. We can see small effects only for individual profiles. The energy of the heavy secondaries produced via CGC reaches up to $\approx 3 \times 10^{18}$ eV, what could make such particles reach the ground with reasonable energy. However, the fraction of energy carried by the particles is very small to cause considerable effects in the EAS development.

Regarding average longitudinal profiles via IQM, the heavy particles reach higher energy fraction, causing larger changes in the observables. We observed a considerable change in $X_{\text{max}}$, $RMS$ and $N_{\text{max}}$. We highlight some longitudinal profiles with high larger modification in its shape. In fact we can observe some double core profiles. This behavior certainly will cause important effects in EAS detection. Charm and bottom particles are very rare in EAS, but we shown that its effects are radical in the EAS development. All these EAS effects ($X_{\text{max}}$ shifted, larger $RMS$ and more elongated shape of the EAS profiles) can change significantly the EASs longitudinal profiles seen in fluorescence telescopes and/or the temporal distribution observed in the surface detectors. The global effect of all these changes in longitudinal profile is a smaller value of the energy reconstructed of the EAS and higher uncertainties.

As for the energy of the primary cosmic rays in question ($\approx 10^{20}$ eV), the values of $X_{\text{max}}$ and $RMS$ found in experiments such as the Pierre Auger are smaller. The $X_{\text{max}}$ and $RMS$ are directly linked to the mass composition of the primary cosmic ray. The appearance of charms and bottoms in the EAS makes it more difficult to make such a connection, because of the deeper $X_{\text{max}}$ and larger $RMS$. The discussion of whether the detection of EAS with heavy particles is feasible in Fluorescence telescopes or Surface Detectors needs to be analyzed in more detail and will be done in a future publication.

The inclusion of heavy hadrons in CORSIKA opens the possibility to test new possibilities of theories
to heavy hadron production and propagation.

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