Machine Wear Calculation Module in Computer-Aided Design Systems

A Dykha, R Sorokaty, O Pasichnyk, P Yaroshenko, and T Skrypnyk

Khmelnytsky national university, 11, Institutskaya str., Khmelnytsky, 29016, Ukraine
Sumy National Agrarian University, 160, H. Kondratieva str., Sumy, 40021, Ukraine
E-mail: tribosenator@gmail.com

Abstract. It has been established that computer-aided design systems are currently an effective tool in engineering practice. Based on the methods of computer modeling, it is possible to assess the performance of any technical system in a given period of time. The operability of almost all technical systems is associated with the analysis of the process of wear of its elements. For the numerical modeling of wear processes, an iterative approach is proposed, which involves consideration of a number of discrete states through which the tribosystem passes during its operation. To characterize tribosystems, the method uses a unified mathematical apparatus to simulate the wear of various friction units. This takes into account a wide range of operating factors that limit the physical characteristics of wear processes and their changes in the process of functioning. An algorithm of interaction of the module for calculating the wear of an element with a computer-aided design system and an example of its implementation are presented.

1. Introduction
Design automation is a complex discipline, the components of which are a lot of modern information technologies. This determines its special place in the range of CAD information technologies - complexes as the main automation tool for the design of complex modern software systems. Currently, CAD - complexes have become an effective tool in engineering. They allow computer simulation methods to evaluate the performance of a technical system in a given period of time. The adequacy of the models used in CAD complexes depends on taking into account the physical processes that determine the behavior of technical systems. In this regard, it is necessary to improve the elements of information technology. In particular, the mathematical and algorithmic support of CAD are the systems that take into account the influence of physical processes in mathematical models. One of the most important and influencing behavior of technical systems is the wear processes of elements. Therefore, the main direction of improving the software systems at the present stage is the development of calculation methods for the analysis of wear processes.

2. Background and problem definition
Recently, in the calculation modules of CAD - systems, attempts have been made to take into account the influence of wear on the behavior of technical systems by simulating and analyzing the physical processes occurring in the wear process.

In [1-5], the authors solve the problem by dividing the complex problem of assessing the wear of friction units into a number of problems. These problems can be solved by means of CAD-complexes...
with the subsequent use of calculation results to assess the wear of friction pairs.

In some CAD systems for modeling wear processes, the following algorithm is implemented [1, 2]:

- in the CAD solver - ABAQUS systems, the contact geometric parameters were determined;
- the results of the calculation analysis were transferred to an external software application, where the depreciation was calculated;
- the results of the calculation of wear were transferred to the preprocessor to build a new finite element model and carry out VAT calculations taking into account changes in geometry due to wear;
- above procedures were repeated until the set value of the wear of the friction pair elements was reached.

The main disadvantage of this algorithm is the constant restructuring of the entire finite element grid, which leads to a significant increase in the calculation time. A wear calculation module was included in the system based on the use of the Archard model for steady abrasive wear [6–9]. This approach has the following features. If the geometry of the friction pair changes, after a wear cycle, there is no need to rebuild the entire finite element grid. It is enough to change the coordinates of only the nodes of the elements that were in the contact zone and have wear. This will significantly reduce the time of calculation analysis. The models used to describe the wear processes in CAD systems should be invariant to the methods of obtaining the initial data for determining the parameters of the models. It is impossible to be based only on theoretical models, since in tribology a significant part of the data was obtained experimentally. The fact that the wear process is a random process is also ignored. In addition to the main prevailing factors, a large number of random factors affect wear. Another approach for taking into account physical processes that occur during wear in models was implemented by MSC Software, namely the use of integrated modules in a CAD system. This system developers have included a module for calculating wear using several deterministic wear models [10-12]. A similar architectural solution was implemented by the developers of the CAD-system ANSYS.

The combined effect of these factors at the physical level has not been fully studied either theoretically or experimentally. The influence of factors that are not taken into account in wear models leads to the fact that the wear process behaves as a random process. For an adequate description of the wear process, the function of the mathematical expectation of wear over time and the dispersion of values relative to this function at each moment of time should be determined. The analysis allows us to formulate the requirements for the mathematical support of information technology for systems of the tribological sector. Namely:

- The software should be based on the principles of spatio-temporal discretization, and be able to interact with other calculation modules of the CAD system.
- Mathematical models should be based on the use of a unified mathematical apparatus and methodological approach to describe the behavior of various types of technical tribosystems.
- Models should describe the wear process as a non-stationary random process.
- The aim of this work is to improve the mathematical and algorithmic support of information technology of CAD-systems of tribological direction, taking into account these requirements.

3. Improving the mathematical support of information technology CAD-systems of tribological direction

It has been proposed to use a numerical method for modeling the behavior of tribosystems, the method of triboelements (MTE), as the basic method for the mathematical support of information technology for CAD systems of the tribological sector [13]. This method most fully takes into account the basic requirements for the software systems. It can be implemented in their architecture as a separate calculation module or integrated directly into one of the existing modules. The triboelement method is
based on an iterative approach, which involves considering a number of discrete states through which the tribosystem passes in the process of functioning. This method (like most modeling methods in SAE modules) is based on spatio-temporal discretization. This allows you to organize the interaction of the calculation modules at the level of the source data and the calculation results. To describe the behavior of tribosystems, the method uses a unified mathematical apparatus and a methodological approach to simulate the wear of various types of friction units. The method takes into account a wide range of factors affecting the course of wear processes and their changes in the process of functioning. The method takes into account that the wear process at the macro level is an evolutionary, non-stationary random process. The determination of the parameters of a random process is based on the correspondence between the parameters of the mathematical model and the physical characteristics of the wear process. The method for determining the model parameters is invariant to methods for obtaining the physical characteristics of the process. To describe the wear of elements of friction pairs, arrays of probability vectors of wear values of discrete points of the surface, called "triboelements", are used. Triboelements are modeled by non-stationary random processes of the Markov type. The change in the size of the bodies in the direction perpendicular to the friction surface is estimated using the mathematical expectation of the probability of the presence of triboelements in a certain state. The shape of the worn surface is determined using the cubic spline approximation of the mathematical expectations of wear at the points of location of the triboelements, which ensures a minimum of potential energy of the wear surface. In accordance with the method, wear is described by a random Markov process with discrete time and states [13]. At time \( t = 1 \) the probabilities of finding triboelements are defined as the product of the vector of initial states and the transition probability matrix (TPM) \( [W_{ij}] \):

\[
[p_i(t=1)] = [p_i(t=0)] [W_{ij}], \quad i, j = 1, 2, ..., K_C,
\]

where \( [p_j(t=0)] \) is the vector of initial states; \( [p_j(t = 1)] \) is the vector of unconditional probabilities of finding triboelements in \( j \) states at time \( t = 1 \); \( [W_{ij}] \) is the transition probability matrix.

The components of the initial state vector \( [p_i(t=0)] \) are determined from the assumption that at the initial moment of time the triboelement was in the first state:

\[
[p_i(t=0)] = [1, 0, 0, ..., 0]
\]

If more detailed information is known about the probabilities of finding the triboelements at the initial moment of time (for example, dispersion of the initial parameters of the elements of the tribosystem), the vector of initial states may have a different form.

Probabilities of the state of triboelements at time \( t > 1 \) defined as a product \( [p_j(t-1)] \) vector of unconditional probabilities at the moment \( t - 1 \) on the transition probability matrix, defining behavior of a triboelement at a time \( t \):

\[
[p_i(t)] = [p_i(t-1)] [W_{ij}], \quad i, j = 1, 2, ..., K_C
\]

To describe the behavior of triboelements, a matrix of transition probabilities with single jumps up and the presence of an absorbing state is used:

\[
[W_{ij}] = \begin{bmatrix}
w_{11}(t) & w_{12}(t) & 0 & 0 & ... & 0 \\
0 & w_{22}(t) & w_{23}(t) & 0 & ... & 0 \\
... & ... & ... & ... & ... & ... \\
0 & 0 & 0 & 0 & ... & 1
\end{bmatrix}
\]

The components of the matrix \( w_{ij}(t) \) are defined as follows:

\[
w_{ij}(t) \approx \lambda_j(t) \Delta t,
\]
where $\lambda_i(t) = V_i(t)/h$; is the wear flow rate; $\Delta t$ is the time step; $h = \varepsilon/(K_C - 1)$ is the value determined from the condition of ordinary wear flow; $V_i(t)$ is the wear rate at time $t$.

It should be noted that the wear rate function is invariant with respect to the methods of its preparation. As a function, one can use both the approximation functions of empirical data and the dependences obtained by calculation methods. Through the function of the wear rate, the main characteristic of the wear flow, represented in the form of a Markov chain, is determined as the intensity of the wear flow $\lambda_i(t)$, which determines the transitions of a system from state to state.

The wear flow rate can be determined by the calculation method \[9\], based on the thermokinetic theory of fracture.

$$
\lambda_i(t) = \frac{1}{\ln \theta} \exp \left( - \frac{\sum_{n(x,y,z,t) \in \Delta \lambda_i} \left( \sigma_n(x,y,z,t) \cdot \frac{M_{k,u}}{b_w} \left( \frac{U_0}{RT} \right)^{\pm \Delta G} \right)}{1} \right)
$$

where $\sigma_n(x,y,z,t)$ is the contact pressure and temperature, respectively, at the point with coordinates $(x,y,z)$ at the time $t$; $f_d$ is the dynamic coefficient of friction; $f_s$ is the static coefficient of friction; $v_{sk}$ is the relative sliding speed at the point of contact; $\beta$ is the coefficient; $L_S$ is the sliding friction path; $M$ is the molar volume; $h_w$ is the wear layer thickness; $U_0$ is the activation energy of the leading mechanism of destruction; $\Delta G$ is the coefficient taking into account the influence of the external environment; $R$ is gas constant.

Dependence of wear flow rate $\lambda_i(t)$ can be used to simulate tribosystems with a wide range of input parameters.

The value of wear is determined through the mathematical average $\overline{m}_t$:

$$
\overline{m}_t = \sum_{i=1}^{K_c} i \pi_t(i)
$$

where $\overline{m}_t = \sum_{i=1}^{K_c} \pi_t(i)$, $i = 1, 2, ..., K_C$; $\pi_t(i)$ is the vector of unconditional probabilities of triboelement states.

Topological equations describing the relationships in the composition of the simulated system are represented by a cubic spline approximation of the mathematical expectations of wear. This provides a minimum of potential energy of the wear surface:

$$
Z_{k,u}(l) = -\frac{1}{6 \delta_k} \left[ M_{k,u} \left( l_{k,u+1} - l_{k,u} \right)^3 + M_{k,u} \left( 1 - l_{k,u} \right)^3 + \frac{1}{6 \delta_k} \left( Z_{k,u} - M_{k,u} \delta_k^2 \right) \left( l_{k,u+1} - l_{k,u} \right)^3 + \frac{1}{6 \delta_k} \left( Z_{k,u} - M_{k,u} \delta_k^2 \right) \left( l_{k,u+1} - l_{k,u} \right)^3 \right],
$$

where $d_{k,u} = l_{k,u+1} - l_{k,u}$; $k = 1, 2, ..., K$; $u = 1, 2, ..., U$; $K$, $U$ is the number of triboelements with wear in the radial and axial directions.

$M_{k,u}$ are determined from systems of linear equations for each $k$:

$$
d_{k,u} M_{k,u} + 2 \left( d_{k,u} + d_{k,u+1} \right) M_{k,u+1} + d_{k,u+1} M_{k,u+2} = 6 \left( \frac{Z_{k,u+1} - Z_{k,u}}{d_{k,u+1}} - \frac{Z_{k,u} - Z_{k,u+2}}{d_{k,u}} \right)
$$

Cubic spline function of the dependence of wear values is:

$$
Z_{r+1}(l_r; \phi) = -\frac{1}{6 \delta_l} \left[ M_r \left( \phi_r + \phi_r - \phi_r \right)^3 + M_{r+1} \left( \phi_r + \phi_r - \phi_r \right)^3 + \frac{1}{6 \delta_l} \left( Z_{r+1} - M_{r+1} \delta_l^2 \right) \left( \phi_r + \phi_r - \phi_r \right)^3 + \frac{1}{6 \delta_l} \left( Z_{r+1} - M_{r+1} \delta_l^2 \right) \left( \phi_r + \phi_r - \phi_r \right)^3 \right],
$$
where \( d_r = \phi_{r+1} - \phi_r \); \( r = 1, 2, ..., K_f \).

\( M_r \) is determined from a system of linear equations:

\[
d_r M_r + 2(d_r + d_{r+1}) M_{r+1} + d_{r+1} M_{r+2} = 6 \left( \frac{Z_{r+2} - Z_r}{d_{r+1}} - \frac{Z_{r+1} - Z_r}{d_r} \right)
\]

(11)

### 4. Generalized Interaction Algorithm for the Wear Calculation Module with CAD Systems

To the effectiveness of the calculation method will depend on how much the basic requirements for the mathematical support of CAD/CAE systems and the architectural features of the software package for its implementation are taken into account. To consider the influence of wear processes on the behavior of the technical system in information technology, it is necessary to ensure the interaction of the calculation modules of the software complex, taking into account changes in the input parameters at each step of the iterative process. For example, the initial data for determining the parameters of the wear model are the results of the analysis of the stress-strain state of the elements of the friction pair.

On the other hand, the stress-strain state depends on geometric parameters that change due to wear of the elements. In addition, the dynamic processes occurring in the system influence the stress-strain state. The dynamic behavior of the system determines, among other things, the wear of elements of friction pairs. In connection with the above, a generalized algorithm for the interaction of the module for calculating wear by the triboelement method with the calculation complex can be represented as follows.

1. Construction of a geometric model of a technical system using a CAD complex preprocessor, including tribopairs.
2. Construction by means of CAD-complex of the calculated finite-element model of the system and tribopairs.
3. Saving the set of coordinates of the nodes of the finite elements of the tribological conjugation located on the wear surface.
4. Determination of the stress-strain state of the elements of the technical system (including tribological conjugation elements) using the solver CAD-complex, taking into account a given set of input parameters.
5. Determination of the parameters of the wear model using the obtained results of the solution of p. 4 in the wear calculation module. The parameters are the values of the components of the transition probability matrix for each node of the finite element that has come into contact.
6. In the module for calculating wear, the mathematical expectation of the amount of wear is determined for each node of the final element located on the wear surface and which has come into contact. Saving the obtained vectors of unconditional wear probabilities for finite element nodes.
7. Determination of the amount of wear and the set of new coordinates of the nodes of the finite elements located on the wear surface.
8. By means of the CAD complex, the set of coordinates of the nodes of finite elements located on the wear surface and having wear is adjusted (taking into account the results obtained (p. 7)).
9. The cyclic repetition of p. p. 4-8 until the specified boundary operating time of the technical system or the specified parameters of the state of the system.

### 5. Approbation of the Solutions

The proposed solutions for improving the mathematical support of information technology, the interaction algorithms of the wear calculation module with the elements of a CAD complex have been tested. Using this method, new results were obtained by calculation, which received experimental confirmation. The method of triboelements was verified by comparison with empirical and calculated data available in the literature. For various types and conditions of functioning of tribosystems, sufficient accuracy and efficiency of the method was obtained in solving tribotechnical problems [13 - 16].
To test the mathematical models of the method, software was developed to calculate the wear of various types of friction units (Figure 1, 2). The software was implemented both in the form of autonomous modules (Figure 1), and in the form of computational modules integrated into the CAD complex (Figure 2).

![Figure 1](image1.jpg)

**Figure. 1.** The results of the calculations of the autonomous module.

When implementing integrated calculation modules, a generalized algorithm for the interaction of the wear calculation module with CAD systems was implemented and tested.

![Figure 2](image2.jpg)

**Figure. 2.** The results of the calculations of the integrated module.

The testing of the mathematical support of information technology was carried out both for normal operating conditions of technical systems [13 - 15], and for systems operating in extreme conditions [16]. Mathematical models have shown sufficient accuracy in assessing the wear of rifled small arms trunks.

6. **Conclusion**

Based on the analysis, it is proposed to use the triboelement method as the basic method of mathematical support of information technology for CAD-systems of the tribological sector. This method most fully takes into account the basic requirements for the CAD systems of mathematical software and can be implemented in software systems. The proposed solutions to improve the mathematical support of information technology, the interaction algorithms of the wear calculation
module with the elements of the CAD complex are quite approved. The proposed generalized algorithm for the interaction of the wear calculation module with CAD - systems has been implemented and tested by implementing the software of integrated calculation modules.

References

[1] Xue Y, Chen J, Guo S 2018 Finite element simulation and experimental test of the wear behavior for self-lubricating spherical plain bearings. Friction Vol 6, 297
[2] Ali Rezaeia, Wim Van Paepegema, Patrick De Baets, Wouter Ost 2012 Adaptive finite element simulation of wear evolution in radial sliding bearings. Wear. Vol 296 91-20, 660
[3] Shaoyu Zhu, Jun Sun, Biao Li, Xiaoyong Zhao, Guixiang Zhu 2019 Stochastic models for turbulent lubrication of bearing with rough surfaces. Tribology International, Vol 136, 224
[4] Bhatt C D, Nadarajan M, Balaji R 2020 Isukapalli roihith, Ashish Selokar. Leaf spring model for heavy load vehicle using solid works and ANSYS analysis, Materials Today https://doi.org/10.1016/j.matpr.2020.08.3
[5] Abdullah O I, Schlattmann J 2016 Temperature analysis of a pin-on-disc tribology test using experimental and numerical approaches. Friction 4, 135 https://doi.org/10.1007/s40544-016-0110-1
[6] Khan Z A, Chacko V & Nazir H 2017 A review of friction models in interacting joints for durability design. Friction 5, 1. https://doi.org/10.1007/s40544-017-0143-0
[7] Etsion I 2013 Modeling of surface texturing in hydrodynamic lubrication. Friction Vol 1, 195 https://doi.org/10.1007/s40544-014-0018-y
[8] Shuhui Cui, Le Gu, Liqin Wang, Bo Xu 2018 Chuanwei Zhang. Numerical analysis on the dynamic contact behavior of hydrodynamic journal bearings during start-up. Tribology International, Vol 121, 260
[9] Mohanavel R, Iyankumar M, Sundar P, Kiran Kumar L, Pugazhendhi 2020 Modelling and finite element analysis of anti-roll bar using ANSYS software, Materials Today: Proceedings https://doi.org/10.1016/j.matpr.2020
[10] Xu Y, Jackson R L 2019 Boundary element method (BEM) applied to the rough surface contact vs. BEM in computational mechanics. Friction Vol 7, 359 https://doi.org/10.1007/s40544-018-0229-3
[11] Vasil’ev Y N, Fugol’ V A 2010 Mathematical model of friction and wear of polycrystalline solids. J. Fric. Wear 31, 89. https://doi.org/10.3103/S1068366610020017
[12] Sorokaty R V, Pisarenko V G, Dyka M A 2013 Analysis of Wear Surface Geometry Formation in Plain Bearings with Misaligned Shaft and Bush Axes, J. Friction and Wear. Vol 34, 274 doi: 10.3103/S1068366613040119
[13] Dmitry A D 2018 Prediction the wear of sliding bearings. International Journal of Engineering & Technology, Vol 7(2.23), 4 doi: https://doi.org/10.14419/ij.et.v7i2.23.11872
[14] Dyka A, Sorokaty R, Makovkin O, Babak O 2017 Calculation-experimental modeling of wear of cylindrical sliding bearings. Eastern-European Journal of Enterprise Technologies Vol 5(1), 51 doi: 10.15587/1729-4061.2017.109638
[15] Sorokaty R V, Dyka A V 2015 Analysis of Processes of Tribodamages under the Conditions of High-Speed Friction. J. Friction and Wear, Vol 36, 422 doi: 10.3103/S106836661505013X
[16] Sorokaty R, Chernets M, Dyka A, Mikosyanchyk O 2019 Phenomenological Model of Accumulation of Fatigue Tribological Damage in the Surface Layer of Materials Mechanisms and Machine Science, Vol 73, 3761 https://link.springer.com/chapter/10.1007%2F978-3-030-20131-9_371