Deciphering the Seesaw Nature of Neutrino Mass from Unitarity Violation

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Abstract

If neutrino masses are obtained via the canonical seesaw mechanism, based on an underlying $2 \times 2$ mass matrix, unitarity violation of the neutrino mixing matrix is unavoidable, but its effect is extremely small. On the other hand, in the inverse (and linear) seesaw mechanisms, based on an underlying $3 \times 3$ mass matrix, it can be significant and possibly observable. This $3 \times 3$ matrix is examined in more detail, and a new variation (the lopsided seesaw) is proposed which has features of both mechanisms. A concrete example based on $U(1)_N$ is discussed.
In the famous canonical seesaw mechanism, the standard model (SM) of particle interactions is implemented with a heavy singlet “right-handed” neutrino $N_R$ per family, so that the otherwise massless left-handed neutrino $\nu_L$ gets a mass from diagonalizing the $2 \times 2$ mass matrix spanning $(\bar{\nu}_L, N_R)$:

$$M_{\nu,N} = \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix},$$  \hspace{1cm} (1)

resulting in

$$m_\nu \simeq -\frac{m_D^2}{m_N},$$  \hspace{1cm} (2)

with mixing between $\nu_L$ and $N_R$ given by

$$\tan \theta_1 \simeq m_D \frac{1}{m_N} \simeq \sqrt{|m_\nu/m_N|}. \hspace{1cm} (3)$$

Since $N_R$ does not have gauge interactions, the $3 \times 3$ mixing matrix linking the 3 neutrinos to the 3 charged leptons cannot be exactly unitary. However, for $m_\nu \sim 1$ eV and $m_N \sim 1$ TeV, this violation of unitarity is of order $10^{-6}$, which is much too small to be observed. Note that lepton-number conservation is recovered in the limit $m_N \rightarrow \infty$. [If neutrinos obtain Majorana masses directly through a Higgs triplet, thus doing without $N_R$, then there is no violation of unitarity to begin with.]

Consider now the idea of the inverse seesaw mechanism: a situation is established where $m_\nu = 0$ because of a symmetry, which is then broken by a small mass $[1, 2, 3, 4, 5, 6, 7]$. In contrast to the canonical seesaw mechanism, lepton-number conservation is recovered here in the limit this small mass goes to zero. The prototype model is to add a singlet Dirac fermion $N$, i.e. both $N_R$ and $N_L$, with lepton number $L = 1$ per family to the SM. The $3 \times 3$ mass matrix spanning $(\bar{\nu}_L, N_R, \bar{N}_L)$ is then given by

$$M_{\nu,N} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & \epsilon_R & m_N \\ 0 & m_N & \epsilon_L \end{pmatrix},$$  \hspace{1cm} (4)
where $\epsilon_{L,R}$ are lepton-number violating Majorana mass terms. This is a natural extension of the famous seesaw $2 \times 2$ mass matrix of Eq. (1), but it also has a clear symmetry interpretation, i.e. $\epsilon_{L,R}$ may be naturally small because their absence would correspond to the exact conservation of lepton number. [A linear combination of $\nu_L$ and $N_L$ would combine with $N_R$ to form a Dirac fermion, whereas its orthogonal combination would remain massless.]

Using $\epsilon_{L,R} << m_D, m_N$, the smallest mass eigenvalue of Eq. (4) is then

$$m_\nu \simeq \frac{m_D^2 \epsilon_L}{m_N^2},$$

(5)

with mixing between $\nu_L$ and $N_L$ given by

$$\tan \theta_2 \simeq \frac{m_D}{m_N} \simeq \sqrt{|m_\nu/\epsilon_L|}.$$  (6)

Note first that the mixing between $\nu_L$ and $N_R$ remains negligible, i.e. $m_D \epsilon_L/m_N^2 \simeq m_\nu/m_D$. More importantly, note that $m_N$ in Eq. (3) is replaced by $\epsilon_L$ in Eq. (6). This means that $\theta_2$ is not constrained in the same way as $\theta_1$, and it can be bigger by orders of magnitude.

For example, let $m_\nu \sim 1$ eV and $\epsilon_L \sim 10$ keV, which is compatible with $m_D \sim 10$ GeV and $m_N \sim 1$ TeV, then the mixing is of order $10^{-2}$. This dramatic change in possible unitarity violation means that it may be observable in future neutrino experiments [8, 9, 10, 11, 12, 13, 14, 15, 16]. If confirmed, it will be a big boost for the idea of the inverse seesaw.

If $\nu_e$ or $\nu_\mu$ mixes significantly with singlets, then the effective $G_F$ for $\mu \rightarrow e\nu_\mu \bar{\nu}_e$ would have to be redefined, and many precision electroweak measurements would be affected. Thus only $\nu_\tau$ mixing is likely to be significant, affecting the unitarity of the third row of the neutrino mixing matrix linking $\nu_{e,\mu,\tau}$ to the mass eigenstates $\nu_{1,2,3}$.

In Eq. (4), if $\epsilon_R = 0$ or relatively small, but $\epsilon_L$, renamed $m_L$, is very large, with $\epsilon_R, m_D << m_N^2/m_L$, then this is called the double seesaw. First, $N_R$ gets a medium large Majorana mass from $N_L$, i.e. $-m_N^2/m_L$. For example, $m_N \sim 10^9$ GeV, $m_L \sim 10^{15}$ GeV, then this seesaw mass is $\sim 1$ TeV. Second, $\nu_L$ gets a small Majorana mass from $N_R$, i.e. $m_D^2 m_L/m_N^2$. This
idea is often used in models of grand unification. In this case, the mixing between $\nu_L$ and $N_R$ is the same as in the canonical seesaw, and that between $\nu_L$ and $N_L$ is further suppressed.

A more recent proposal \cite{17} is the linear seesaw, i.e.

$$
\mathcal{M}_{\nu,N} = \begin{pmatrix}
0 & m_D & m_2 \\
m_D & 0 & m_N \\
m_2 & m_N & 0
\end{pmatrix},
$$

(7)

so that

$$
m_\nu \simeq -\frac{2m_2m_D}{m_N}.
$$

(8)

However, since both $N_R$ and $\bar{N}_L$ are singlets, they may be redefined by a rotation so that $m_2 = 0$. Let $m_2/m_D = \tan \theta$, and $c = \cos \theta$, $s = \sin \theta$, $c_2 = \cos 2\theta$, $s_2 = \sin 2\theta$, then

$$
\begin{pmatrix}
1 & 0 & 0 \\
0 & c & s \\
0 & -s & c
\end{pmatrix}
\begin{pmatrix}
0 & m_D & m_2 \\
m_D & 0 & m_N \\
m_2 & m_N & 0
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & c & -s \\
0 & s & c
\end{pmatrix} =
\begin{pmatrix}
0 & m_D/c & 0 \\
0 & m_Ns_2 & m_Nc_2 \\
0 & m_Nc_2 & -m_Ns_2
\end{pmatrix},
$$

(9)

which is the same as Eq. (4), resulting in

$$
m_\nu \simeq (m_D^2/m_N^2)(-2m_Nm_2/m_D) = -2m_2m_D/m_N,
$$

(10)

which is identical to Eq. (8) as expected. On the other hand, if there is a symmetry beyond that of the SM which enforces Eq. (7), then it may be considered on its own.

There is however another interesting variation (the lopsided seesaw), which has not been discussed before. Let $\epsilon_{L,R}$ be renamed $m_{L,R}$, i.e.

$$
\mathcal{M}_{\nu,N} = \begin{pmatrix}
0 & m_D & 0 \\
m_D & m_R & m_N \\
0 & m_N & m_L
\end{pmatrix},
$$

(11)

and do away with the notion of lepton number, then for

$$
m_D << m_R, \quad \frac{m_N^2}{m_R} << m_L << m_N << m_R,
$$

(12)

the neutrino mass is again given by $m_\nu \simeq -m_D^2/m_R$ as in the canonical seesaw, and the mixing of $\nu_L$ and $N_R$ is again $m_D/m_R$, but now the mixing of $\nu_L$ and $N_L$ is $m_Nm_D/m_Lm_R$ <
$m_D/m_N$ which can be significant. If $m_L$ is small enough, $N_L$ should be considered a sterile neutrino. In that case, the violation of unitarity may show up also in the first two rows of the neutrino mixing matrix. As an example, let $m_D \sim 1$ GeV, $m_R \sim 10^9$ GeV, $m_N \sim 10$ GeV, $m_L \sim 1$ keV, then $m_{\nu} \sim 1$ eV, and the mixing between $\nu_L$ and $N_L$ is $10^{-2}$.

If $m_L = 0$ and $m_R$ is very large, it appears from Eq. (7) at first sight that there are two seesaw masses, i.e. $-m_D^2/m_R$ and $-m_N^2/m_R$. However, since the determinant of $\mathcal{M}_{\nu,N}$ is zero in this case, the linear combination $(m_N \nu_L - m_D N_L)/\sqrt{m_N^2 + m_D^2}$ remains massless. If $m_L \neq 0$, i.e. the seesaw is lopsided, then there is no zero eigenvalue.

The neutrino mass matrices of Eqs. (4) and (7) are also very suited for gauge extensions of the SM, such as $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$, where $U(1)_X$ is orthogonal to $U(1)_Y$, an example of which is a linear combination of $U(1)_X$ and $U(1)_Y$ in $E_6$ models. As a concrete example, consider the lopsided seesaw and $U(1)_N$ of $E_6$ [18, 19, 20, 21, 22]. Under the maximum subgroup $SU(3)_C \times SU(3)_L \times SU(3)_R$ of $E_6$, the charges of its fundamental $27$ representation under $U(1)_N$ are given by $Q_N = 6Y_L + T_3R - 9Y_R$. Hence $(u, d), u^c, e^c$ have $Q_N = 1$, $(\nu, e), d^c$ have $Q_N = 2$, and $N^c$ has $Q_N = 0$. To allow for quark and lepton masses, two Higgs scalar doublets

$$\Phi_1 = (\phi^0_1, \phi^-_1) \sim (1, 2, -1/2, -3), \quad \Phi_2 = (\phi^+_1, \phi^0_1) \sim (1, 2, 1/2, -2) \quad (13)$$

under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N$ are needed, with the Yukawa interactions

$$(u\phi^0_2 - d\phi^+_2)u^c, \quad (d\phi^0_1 - u\phi^-_1)d^c, \quad (e\phi^0_1 - \nu\phi^-_1)e^c, \quad (\nu\phi^0_2 - e\phi^+_2)N^c. \quad (14)$$

In the $27$ of $E_6$, there is another fermion singlet $S$ which has $Q_N = 5$. Consider now the breaking of $U(1)_N$ by the Higgs scalars $\chi_1 \sim -5$ and $\chi_2 \sim 10$. The most general Higgs potential is given by

$$V_{\chi} = \sum_i \mu_i^2 \chi_i \chi_i + \frac{1}{2} \sum_{i,j} \lambda_{ij}(\chi^+_i \chi_i)(\chi^+_j \chi_j) + [\mu_{12} \chi_1 \chi_2 + H.c.], \quad (15)$$
where $\lambda_{12} = \lambda_{21}$. Let $\langle \chi_i \rangle = u_i$, then the conditions for $V_\chi$ to be at its minimum are

\begin{align}
u_1 [\mu_1^2 + \lambda_{11} u_1^2 + \lambda_{12} u_2^2 + 2 \lambda_{11} u_1 u_2 & = 0, \quad (16) \\
u_2 [\mu_2^2 + \lambda_{12} u_1^2 + \lambda_{22} u_2^2 + \lambda_{12} u_1 u_2 & = 0. \quad (17) \\A natural solution exists \cite{23, 24, 25}, such that $u_2 << u_1$, i.e.

$\nu_1 \sim -\frac{\mu_1^2}{\lambda_{11}}, \quad \nu_2 \sim -\frac{\mu_1^2 u_1^2}{\mu_2^2 + \lambda_{12} u_1^2}$. \quad (18)$

If $\mu_{12} = 0$, there would be an extra global U(1) symmetry in $V_\chi$. Hence a small $\mu_{12}$ is natural, and $u_2 << u_1$ may be maintained. The $3 \times 3$ mass matrix spanning $(\nu, N^c, S)$ is then of the form desired, with $m_R$ an invariant mass, $m_N$ coming from $\chi_1$ and $m_L$ coming from $\chi_2$.

To summarize, it has been shown in this paper that a $3 \times 3$ realization of the seesaw mechanism has three distinguishable scenarios, resulting in the inverse (or linear) seesaw, the double seesaw, and the lopsided seesaw. The last is a new proposal and may naturally be implemented in the $U(1)_N$ extension of the SM.

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