A lattice test of strong coupling behaviour in QCD at finite temperature

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ABSTRACT: We propose a set of lattice measurements which could test whether the deconfined, quark–gluon plasma, phase of QCD shows strong coupling aspects at temperatures a few times the critical temperature for deconfinement, in the region where the conformal anomaly becomes unimportant. The measurements refer to twist–two operators which are not protected by symmetries and which in a strong–coupling scenario would develop large, negative, anomalous dimensions, resulting in a strong suppression of the respective lattice expectation values in the continuum limit. Special emphasis is put on the respective operator with lowest spin (the spin–2 operator orthogonal to the energy–momentum tensor within the renormalization flow) and on the case of quenched QCD, where this operator is known for arbitrary values of the coupling: this is the quark energy–momentum tensor. The proposed lattice measurements could also test whether the plasma constituents are pointlike (as expected at weak coupling), or not.
1. Introduction

The heavy ion experiments at RHIC have given two rather surprising and important results, namely the medium effects known as elliptic flow and jet quenching turned out to be much larger than simple expectations based on perturbative QCD. This has led to the picture that the deconfined matter produced at RHIC is a nearly perfect fluid, so like a strongly coupled plasma (see, e.g., the review papers [1, 2] and references therein). The coupling constant $\alpha_s = g^2/4\pi$ in QCD can never become large, because of asymptotic freedom, but it can be of order one at scales of order $\Lambda_{\text{QCD}}$, and this might lead to an effectively strong-coupling behaviour. It is notoriously difficult to do reliable estimates in QCD when $\alpha_s \simeq 1$, so it has become common practice to look to the strongly coupled $\mathcal{N} = 4$ supersymmetric Yang–Mills (SYM) theory for guidance as to general properties of strongly coupled field theories at finite temperature (see the review papers [3, 4, 5] for details and more references). Since conformal symmetry is an essential property of $\mathcal{N} = 4$ SYM, this theory is probably not a good model for the dynamics in QCD in the vicinity of the deconfinement phase transition, where the conformal anomaly associated with the running of the coupling in QCD is known to be important. But lattice studies [6] show that the relative conformal anomaly $(\epsilon - 3p)/\epsilon$ ($\epsilon$ is the energy density and $p$ is the pressure) decreases very fast with increasing $T$ above $T_c$ and becomes unimportant (smaller than 10%) for temperatures $T \gtrsim 2T_c \simeq 400$ MeV. Hence, there is a hope that, within the intermediate range of temperatures at $2T_c \lesssim T \lesssim 5T_c$, which is the relevant range for the phenomenology of heavy ion collisions at RHIC and LHC, the dynamics in QCD may be at least qualitatively understood by analogy with $\mathcal{N} = 4$ SYM theory at strong coupling.

One result suggested by SYM theory is that all the leading twist operators that occur in the operator product expansion of deep inelastic scattering, with the exception of the energy–momentum tensor, should have vanishing expectation values in a strongly coupled plasma. For the $\mathcal{N} = 4$ SYM plasma, whose strong-coupling limit can be studied via the gauge/gravity duality [7, 8, 9], this result follows from the fact that only protected operators — those whose anomalous dimension is zero because of a symmetry, or conservation law —
do not acquire large negative anomalous dimensions \([7, 8, 9, 10, 11, 12]\). This result is in fact natural in any field theory whose coupling is large. As one measures this theory at smaller and smaller space–time scales, one uncovers more and more strong evolution \(\text{(branching)}\) of the quanta of the theory \([13, 14, 15, 16]\). The smallness of the higher dimensional operators in the leading–twist series just represents the fact that the higher energy–moments of “bare” quanta are naturally small at strong coupling, because the energy has been shared among many quanta via the branching process. At finite temperature, it is natural to assume that the branchings have taken place between the temperature scale \(T\) and the “hard” resolution scale \(Q\), with \(Q \gg T\), at which the operator is evaluated.

It is important to emphasize that the renormalization flow of the operators and, in particular, their anomalous dimensions are determined by the vacuum properties of the theory — the temperature enters only as the natural scale at which this flow should begin (and which therefore controls the early running of the coupling in a theory like QCD). In particular, at weak coupling, the anomalous dimensions are computable in the zero–temperature perturbative expansion, which is a series in powers of \(\alpha_s\). This should be contrasted with the calculation of thermal expectation values, so like the pressure, whose perturbative expansion is quite subtle already at weak coupling, and in particular non–analytic in \(\alpha_s\), because of infrared problems associated with the thermal Bose–Einstein distribution \([17, 18, 19]\). Thus a non–perturbative study of the renormalization flow of the \(\text{(unprotected)}\) leading twist operators would allow one to distinguish between genuine strong–coupling effects and the failure of the perturbation theory due to medium effects, which occurs already at weak coupling. This would avoid the ambiguity inherent in the present lattice studies of the QCD thermodynamics \([1]\), whose results cannot be accommodated by a strict perturbative expansion, yet they appear to be consistent, for temperatures \(T \gtrsim 2.5T_c\), with the predictions of the HTL–resummed perturbation theory \([21, 21, 22, 23]\), and not too far away from the strong–coupling limit of \(\mathcal{N} = 4\) SYM \([24, 25]\).

In particular, recent lattice calculations \([26]\) of the fluctuations of the electric charge, baryonic number, and strangeness in the quark–gluon plasma appear to be remarkably close to the respective results of HTL–resummed perturbation theory \([27]\) (and also to the ideal gas limit) already for \(T \gtrsim 1.5T_c\), thus strongly supporting a quasiparticle picture of the weak coupling type. The fact that the approach towards the ideal gas limit when increasing \(T\) above \(T_c\) appears to be faster for the quark susceptibilities than for the pressure or energy density, is perhaps to be attributed to the fact that the conformal anomaly, which is so important for the thermodynamics in the vicinity of \(T_c\), is less important for fermionic observables, like the above susceptibilities.

Now, there is \textit{a priori} no contradiction in having a quasiparticle picture also at strong coupling, as shown by the fact that the entropy density of the \(\mathcal{N} = 4\) SYM plasma in the strong–coupling limit is close to the respective value at zero coupling. However if the effective coupling is large, one expects the quasiparticles to be highly composite, without a pointlike core carrying a significant fraction of the quasiparticle energy. This is illustrated by recent calculations of deep inelastic scattering (DIS) off the strongly coupled \(\mathcal{N} = 4\) SYM plasma, which show that, when this plasma is measured on a hard resolution scale \(Q \gg T\), one finds only low–energy constituents, with energy fractions \(x \lesssim T/Q \ll 1\) \([13, 16]\). The higher \(Q\) is,
the smaller are the energy fractions, meaning that there are no pointlike constituents.

How to determine what is the corresponding picture for the quark–gluon plasma? Of course, one cannot literally perform a deep inelastic scattering on the QCD matter produced at RHIC to test whether or not there are pointlike constituents having energy of order of $T$ (in the rest frame of the plasma). The only experimental evidence on this comes from the phenomenology of “jet quenching”, which within perturbative QCD at least, is the process responsible for both energy loss and transverse momentum broadening of a hard probe propagating through the medium. The relevant transport coefficient $\hat{q}$ (the “jet–quenching parameter”) is given by [28]

$$\hat{q} = \frac{4 \pi^2 \alpha_s N_c}{N_c^2 - 1} \frac{dxG(x, Q^2)}{dV}, \quad (1.1)$$

where $dxG(x, Q^2)/dV$ is the number of gluons per unit volume in the plasma measured on the relevant energy ($x$) and virtuality ($Q^2$) resolution scales. It is generally assumed that $Q$ is of the order of the saturation momentum of the plasma, since this is the typical momentum of the gluons exchanged between the jet and the medium. Weak–coupling estimates of $\hat{q}$ using ideal gas formulas for the density of the plasma constituents at the scale $T$ together with perturbative evolution to the hard scale $Q$ give $\hat{q} \simeq (0.5 \div 1)$ GeV$^2$/fm, while phenomenology [29, 30] rather suggests that $\hat{q}$ should be somehow larger, between 5 and 15 GeV$^2$/fm. This difference supports the picture of strong evolution in the plasma, and hence of strong coupling [31]. One should nevertheless keep in mind that this phenomenology is quite difficult and not devoid of ambiguities: strong assumptions are necessary in order to compute $\hat{q}$, and also to extract its value from the RHIC data (see, e.g., the discussion in [32]).

In view of the experimental difficulties, it is natural to ask whether lattice gauge theory can illuminate this question. Computing the DIS structure functions on the lattice is in principle possible: via the operator product expansion and for sufficiently high–$Q^2$, the moments of the structure functions can be related to expectation values of operators with spin $n$ and (classical) dimension $n + 2$ — the leading twist operators — which form an infinite series (only the even values of $n$ being relevant for DIS). Given the space–like kinematics of the DIS process, these operator expectation values are effectively Euclidean, and thus can be evaluated on the lattice. In order to reconstruct the structure functions from their moments, one would need to measure a large number (in principle, infinite) of the latter, which is practically tedious, if not impossible. Indeed, operators with spin $n = 4, 6, 8, \ldots$ involve too many derivatives to be accurately evaluated in lattice QCD, although some attempts were done in that sense, for the case of the proton structure functions (see, e.g., [33]).

However, in order to answer the limited questions that we address here, such a full reconstruction of the plasma structure functions is actually not needed. What we instead propose is to measure the expectation value of the unique leading–twist operator with $n = 2$ which is not protected by symmetries, and thus check whether the corresponding result is rapidly vanishing when approaching the continuum limit — as expected for a strong–coupling dynamics —, or rather it is slowly evolving away from the respective ideal gas expectations — as it should be the case at weak coupling.
Specifically, consider the two leading–twist operators with \( n = 2 \) in QCD, that is
\[
\mathcal{O}^{\mu \nu}_f \equiv \bar{q} \gamma^\mu i D^\nu q - \text{(trace)},
\]
and, respectively,
\[
\mathcal{O}^{\mu \nu}_g \equiv -F^{\mu \alpha}_a F^{\nu \alpha}_a + \frac{1}{4} g^{\mu \nu} F^{\alpha \beta}_a F_{\alpha \beta}.
\]
(It is understood that the fermionic operator \( \mathcal{O}^{\mu \nu}_f \) involves a sum over quark flavors and a symmetrization of the Lorentz indices, and we neglect the masses of the quarks.) These two operators are well defined only with a renormalization prescription, and thus implicitly depend upon the resolution scale \( Q^2 \). Since they have the same quantum numbers, they mix with each other under the renormalization flow. The following linear combination yields the total energy–momentum tensor,
\[
T^{\mu \nu} = \mathcal{O}^{\mu \nu}_f + \mathcal{O}^{\mu \nu}_g,
\]
which is a conserved quantity, and thus is insensitive to quantum evolution (it does not depend upon \( Q^2 \)). Clearly, this operator cannot be used to test whether the plasma has pointlike constituents, or not. Within perturbation theory, it is always possible to construct the linear combination of \( \mathcal{O}^{\mu \nu}_f \) and \( \mathcal{O}^{\mu \nu}_g \) which is orthogonal to \( T^{\mu \nu} \) within the renormalization flow and therefore vanishes in the continuum limit \( Q^2 \to \infty \) (the respective anomalous dimension being negative). This is the operator whose expectation value we would like to measure on the lattice. But if the coupling is strong, we do not know how to explicitly construct this orthogonal combination.

Fortunately, there is a simpler version of the theory where the identification of this operator becomes possible for any value of the coupling: this is *quenched* QCD. Loosely speaking, this is the theory obtained from QCD after removing all the quark loops. On the lattice, this is non–perturbatively defined by removing the fermionic determinant from the QCD action. Note that the quark fields are still present in this theory, but only as external probes. In particular, it makes sense to evaluate the fermionic operator (1.2) in quenched QCD: at finite temperature, this amounts to computing the Matsubara Dirac propagator in the background of the thermal fluctuations of the gauge fields. Such a calculation effectively resums all the respective Feynman graphs of QCD, except for those involving quark loops. For instance, if the coupling is weak (i.e., for high enough temperatures) and for a given resolution scale \( Q^2 \) which is not too hard, the expectation value \( \langle \mathcal{O}^{\mu \nu}_f(Q^2) \rangle_T \) should be close to the respective value for an ideal fermionic gas, as given by the Fermi–Dirac distribution. Moreover, when \( Q \sim T \), the temperature \( T \) is the only scale in the problem, so by dimensional arguments we expect
\[
\langle \mathcal{O}^{\mu \nu}_f(Q^2 \sim T^2) \rangle_T \propto T^4 \quad \text{for any value of the coupling},
\]
(We implicitly assume here that the temperature is sufficiently high for the QCD trace anomaly to become unimportant; in practice, \( T \gtrsim 2T_c \).) On the other hand, in the continuum limit \( Q^2 \to \infty \) at fixed \( T \), the above expectation value must vanish:
\[
\langle \mathcal{O}^{\mu \nu}_f(Q^2 \to \infty) \rangle_T \to 0 \quad \text{(fixed } T\text{)}. \tag{1.6}
\]
Eq. (1.6) will be derived in Sect. 4, but it is easy to see how it comes about. The quark can emit gluons — the more so, the harder the scale at which one probes its substructure. But the emitted gluons, as well as those from the thermal bath, are not allowed to emit quark–antiquark pairs. Hence, when the system is probed on a sufficiently hard scale, most of the total energy appears in the gluon fields. We thus see that, within quenched QCD, $O_f^{\mu\nu}$ is the $n = 2$ operator orthogonal to the (total) energy–momentum tensor. In the continuum limit, the latter reduces to its gluonic component: $T^{\mu\nu} \rightarrow O_g^{\mu\nu}(Q^2)$ as $Q^2 \rightarrow \infty$.

Whereas Eq. (1.6) holds for any value of the coupling, the rapidity of the evolution with increasing $Q^2$ — i.e., the rate at which $\langle O_f^{\mu\nu} \rangle_T$ approaches to zero — depends upon the strength of the interactions. For a weak coupling, this evolution would be quite slow; using lowest-order perturbative QCD, we shall estimate in Sect. 4 that for a temperature $T \simeq 3T_c$ and an inverse lattice spacing $a^{-1} \equiv Q \simeq 4$ GeV, the deviation of $\langle O_f^{00} \rangle_T$ from the corresponding ideal gas value should not exceed 30%. On the other hand, if the evolution is more like at strong coupling, and if the measurements of $\hat{q}$ are indicative of what should be expected in the strongly-coupled QCD plasma, one would expect $\langle O_f^{00} \rangle_T$ to be reduced by a factor of 5 or more. Of course, all the conclusions that could be drawn in this way would strictly apply to quenched QCD alone. However, we expect real (unquenched) QCD to behave similarly (within the same range of temperatures), because the asymptotic freedom property of QCD is driven by gluon dynamics.

2. Leading–twist operators: from weak to strong coupling

Although the main emphasis in this paper is not on the process of deep inelastic scattering by itself, but rather on the lattice evaluation of specific, low spin, leading–twist operators, it is nevertheless natural to introduce these operators in the context of DIS and thus summarize some of their properties to be used later on.

Within QCD, there are two infinite sequences of leading–twist operators: the fermionic ones,

$$ O_f^{(n)\mu_1...\mu_n} \equiv \bar{q} \gamma^\mu_1 (iD^\mu_2) \cdots (iD^\mu_n) q - \text{(traces)}, \quad (2.1) $$

(the curly brackets around Minkowski indices mean symmetrization), and the gluonic ones,

$$ O_g^{(n)\mu_1...\mu_n} \equiv -\frac{1}{2} F^{\mu_1\nu} (iD^\mu_2) \cdots (iD^\mu_{n-1}) F^\mu_{n-1} \nu - \text{(traces)}, \quad (2.2) $$

where a trace over color indices is implicit. Such operators have spin $n$, classical dimension $d = n + 2$, and hence twist $t = d - n = 2$. For $n = 2$, we recover the operators expressing the energy–momentum tensor for quarks and gluons, respectively, cf. Eqs. (1.2)–(1.3). For even values of $n$, $n = 2, 4, 6, \ldots$, these operators enter the OPE of the current–current correlator which determines the cross–section for the standard DIS process, as mediated by the exchange of a space–like photon. More precisely, if the expectation values of the operators are evaluated directly at the resolution scale $Q^2$ for DIS (the virtuality of the space–like photon), then the OPE involves only the quark operators (2.1). In practice, however, it is convenient to evaluate the operators at some fixed renormalization scale $\mu$, or at some intrinsic physical scale —
say, the temperature for the case of DIS off the quark–gluon plasma. In such a case, the operators at the DIS scale $Q^2$ are obtained by following the renormalization flow from the original scale $\mu^2$ (or $T^2$), and under this flow, the fermionic and gluonic operators having the same spin mix with each other.

It is generally stated that the leading–twist operators dominate the OPE for DIS for sufficiently high $Q^2$. This is strictly true only so long as the coupling is not too strong, as shown by the example of \( \mathcal{N} = 4 \) SYM theory, where explicit calculations were also possible at strong coupling. To illustrate this, consider the leading–twist contributions to the moments of the DIS structure function $F_2(x, Q^2)$, which can be expressed as (we follow the conventions in Ref. 34)

$$\int_0^1 dx \, x^{n-2} F_2(x, Q^2) \simeq A_f^{(n)}(Q^2),$$

(2.3)

where the approximate equality sign means that in the r.h.s. we have kept only the twist–2 contribution. The quantity $A_f^{(n)}(Q^2)$ is the expectation value of the spin–$n$ fermionic operator evaluated at the resolution scale $Q^2$ and with all the kinematical factors (responsible for the Minkowski tensor structure and for the actual dimension of the operator) stripped off. For instance, if $F_2$ refers to a proton with 4–momentum $P^\mu$, then

$$\langle P | O_f^{(n)}(Q^2) | P \rangle = A_f^{(n)}(Q^2) 2P^{\mu_1} \cdots P^{\mu_n} - \text{(traces)},$$

(2.4)

whereas for a plasma at temperature $T$:

$$\langle O_f^{(n)}(Q^2) \rangle_T = A_f^{(n)}(Q^2) T^n 2n^{\mu_1} \cdots n^{\mu_n} - \text{(traces)},$$

(2.5)

where $n^\mu$ is the four–velocity of the plasma, with $n^\mu = (1, 0, 0, 0)$ in the rest frame of the plasma. Note that $A_f^{(n)}$ and $F_2$ are dimensionless in the case of the proton, but they have mass dimension two in the case of the plasma; this difference is related to the normalization of the proton wavefunction. The Bjorken $x$ variable is defined as $x = Q^2 / 2(P \cdot q)$ for DIS off the proton and, respectively, $x = Q^2 / 2T(n \cdot q)$ for DIS off the plasma; here, $q^\mu$ is the 4–momentum of the virtual photon, with $q^\mu q_\mu = -Q^2$.

Let us assume that the operator $O_f^{(n)}$ is normalized at the scale $\mu_0$ and ignore the issue of operator mixing for the time being (we shall return to this issue in Sect. 3). The corresponding operator at a different resolution scale $Q$ is obtained by solving the renormalization group equation (henceforth the Minkowski indices will be kept implicit)

$$\mu^2 \frac{d}{d\mu^2} O_f^{(n)} = \gamma_f^{(n)} O_f^{(n)} \implies O_f^{(n)}(Q^2) = \exp \left\{ \int_{\mu_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \gamma_f^{(n)}(\mu^2) \right\} O_f^{(n)}(\mu_0^2),$$

(2.6)

where $\gamma_f^{(n)}$ is the corresponding anomalous dimension, which in QCD depends upon the scale because of the running of the coupling. Clearly, a similar evolution equation applies for the expectation value $A_f^{(n)}(Q^2)$ of the above operator. It turns out that the anomalous dimensions
are always negative, with the notable exception of the energy–momentum tensor (1.4), for which $\gamma = 0$. Hence, $A^{(n)}_{f}(Q^2) \to 0$ as $Q^2 \to \infty$ for any $n \geq 4$, whereas for $n = 2$ we have

$$\int_{0}^{1} dx F_{2}(x,Q^2) \to \text{const. \ as \ } Q^2 \to \infty,$$

which is simply the statement of energy–momentum conservation. These results imply that, when increasing $Q^2$, $F_{2}(x,Q^2)$ is increasing at small $x$, but decreasing at large $x$: the evolution acts to decrease the average value of the energy fraction of the partons in the wavefunction. This should be expected given the physical picture of the evolution in terms of parton branching, as described in the Introduction. The rate of the evolution towards zero for the unprotected operators, and also the weight of the small–$x$ partons in the sum–rule (2.7), are however quite different at weak and respectively strong coupling, as we now explain.

Consider weak coupling first. To lowest order in perturbative QCD, the anomalous dimensions are obtained as (for a generic leading–twist operator $O$)

$$\gamma_{O}(\mu^2) = -a_{O} \frac{\alpha_s(\mu^2)}{4\pi} = - \frac{a_{O}}{b_{0}\ln(\mu^2/\Lambda_{QCD}^2)},$$

where $a_{O}$ is a positive number and in writing the second equality we have used the one–loop expression for the QCD running coupling, with $b_{0} = (11N_{c} - 2N_{f})/3$. Then Eq. (2.6) implies

$$O^{(n)}_{f}(Q^2) = \left[\frac{\ln(\mu_0^2/\Lambda_{QCD}^2)}{\ln(Q^2/\Lambda_{QCD}^2)}\right]^{a_{(n)}_{f}/b_{0}} O^{(n)}_{f}(\mu_0^2),$$

which shows that the approach towards zero with increasing $Q^2$ is merely logarithmic. Still at weak coupling, consider the case of a conformal field theory, so like $\mathcal{N} = 4$ SYM, where the coupling $\alpha = g^2/4\pi$ is fixed; then, $\gamma_{O} = -\alpha(\alpha/4\pi)$ is a fixed number of $O(\alpha)$, and

$$O^{(n)}_{f}(Q^2) = \left[\frac{\mu_0^2}{Q^2}\right]^{a_{(n)}_{f}/4\pi} O^{(n)}_{f}(\mu_0^2),$$

so that the evolution is typically faster than in QCD, since it is not slowed down by the decrease of the coupling with increasing $Q^2$. But for both QCD and $\mathcal{N} = 4$ SYM, the anomalous dimensions are small $\sim O(g^2)$ at weak coupling, so the leading–twist operators dominate indeed the moments of the DIS structure functions at high $Q^2$: the corresponding contributions from higher–twist operators are suppressed by inverse powers of $Q^2$ with exponents of $O(1)$.

Consider also the energy–momentum sum–rule (2.7): although $F_{2}(x,Q^2)$ does rise at small $x$, as expected, pQCD predicts that this rise is rather mild, so that the integral in Eq. (2.7) is dominated by rather large values of $x$, of order 0.1. This is confirmed by the experimental data at HERA, which can be parameterized by a law $F_{2}(x,Q^2) \sim 1/x^{\lambda(Q^2)}$ where the effective exponent $\lambda(Q^2)$ rises slowly with $Q^2$, but it remains relatively small: $\lambda(Q^2) \approx 0.15 \div 0.3$. This expresses the fact that, at weak coupling, the branching proceeds via bremsstrahlung and favors the emission of small–$x$ gluons, whose number grows very fast,
but which carry only a tiny fraction of the energy of their parent partons. Accordingly, most of the total energy remains in the “valence” degrees of freedom at large $x$. Since this is true for arbitrarily high $Q^2$, it is clear that these valence constituents can be viewed as pointlike.

What is the corresponding situation at strong coupling? Since the respective results are not known for QCD, we focus on the $\mathcal{N} = 4$ SYM theory, whose strong–coupling limit has been addressed via the gauge/string duality. By “strong coupling”, we more precisely mean here the limit in which the gauge coupling is weak, $g^2 \ll 1$, but the number of colors is sufficiently large, $N_c \gg 1$, for the ’t Hooft coupling to be large: $\lambda \equiv g^2 N_c \gg 1$. (Recall that the ‘t Hooft coupling is the relevant coupling for perturbation theory at large $N_c$.) Via the AdS/CFT correspondence, the (gluonic) leading–twist operators are mapped onto excited string states — closed strings which rotate in the $\text{AdS}_5$ space–time geometry. By computing the energy spectrum for such states, one can deduce the quantum dimensions $\Delta(n) = n + 2 - 2\gamma(n)$ of the dual operators $O^{(n)}$, and thus extract their anomalous dimensions $\gamma(n)$. One has thus found \[10, 11, 12\]

$$\gamma^{(n)} \simeq -\frac{n}{2\lambda} \lambda^{1/4} \quad \text{for} \quad 1 \ll n \ll \sqrt{\lambda}, \quad (2.11)$$

and, respectively,

$$\gamma^{(n)} \simeq -\frac{\sqrt{\lambda}}{2\pi} \ln \frac{n}{\sqrt{\lambda}} \quad \text{for} \quad n \gg \sqrt{\lambda}. \quad (2.12)$$

That is, the anomalous dimensions are again negative (except, of course, for the protected energy–momentum tensor), and moreover they are extremely large: of $\mathcal{O}(\lambda^{1/4})$ for the operators with lower spin. Via Eq. (2.6), this implies that all the leading–twist operators with the exception of $T^{\mu \nu}$ are strongly suppressed at high $Q^2$, and hence they become irrelevant for DIS: the respective structure functions are rather controlled by $T^{\mu \nu}$ together with protected higher–twist operators which have zero anomalous dimensions.

The fact that the anomalous dimensions are so large at strong coupling means that the branching process is very fast and, as a result of it, all partons have fallen at small values of $x$. This is further confirmed by the fact that the anomalous dimensions (2.11)–(2.12) rise with $n$, showing (via the moments (2.3)) that the support of the structure function is now concentrated at small values of $x$.

As an illustration of the situation at strong coupling, let us recall the results for DIS off the $\mathcal{N} = 4$ SYM plasma in the strong–coupling limit $\lambda \to \infty$ (or $N_c \to \infty$). In that limit, the anomalous dimensions for the non–protected leading–twist operators become infinite, while the higher–twist protected operators cannot contribute to the DIS cross–sections because of the energy–momentum conservation. Accordingly, the explicit calculation in Ref. \[15\] finds that there is no power–like tail in $F_2(x, Q^2)$ at high $Q^2$. More interestingly, it also finds that there is an exponential tail,

$$F_2(x, Q^2) \sim x N_c^2 Q^2 \exp \left\{ -c \left( Q/Q_s(x) \right) \right\} \quad \text{for} \quad Q \gg Q_s(x) = \frac{T}{x} \quad (2.13)$$

($c$ is a number), which reflects a tunneling process, reminiscent of the Schwinger mechanism: the highly–virtual ($Q \gg Q_s(x)$) space–like current can decay into charged partons via a
tunnel effect induced by a uniform force \( \sim T^2 \), which represents the action of the plasma on the dipole fluctuations of the current in this large–\( N_c \) limit.

The exponential in Eq. (2.13) can be alternatively rewritten as \( \exp \{-c(x/x_s(Q))\} \) with \( x_s(Q) = T/Q \), showing that, for fixed \( Q \gg T \), the DIS structure function is essentially zero for any \( x \) larger than \( x_s(Q) \ll 1 \). This reflects the fact that, via successive branchings, all partons have fallen at small values of \( x \). And, indeed, for sufficiently small values \( x \ll x_s(Q) \) (or, equivalently, for low enough virtualities \( Q \ll Q_s(x) \) at a given \( x \)), the exponential suppression goes away, and one finds \[ F_2(x, Q^2) \sim x N_c^2 Q^2 \left( \frac{T}{x Q} \right)^{2/3} \quad \text{for} \quad Q \ll Q_s(x). \] (2.14)

These estimates are such that the energy–momentum sum–rule (2.7) is saturated by the partons along the saturation line, i.e., those having \( x \simeq x_s(Q) \):

\[
T^2 \int_0^1 dx \, F_2(x, Q^2) \simeq T^2 \, x_s F_2(x_s, Q^2) \sim N_c^2 T^4.
\]
(2.15)

As also emphasized above, this sum–rule reproduces the right order of magnitude for the energy density of the strongly–coupled plasma: \( \langle T^{00} \rangle_T \sim N_c^2 T^4 \). One can similarly check that the higher moments with \( n \geq 4 \) are power suppressed at high \( Q^2 \):

\[
\int_0^1 dx \, x^{n-2} F_2(x, Q^2) \sim x_s^2 N_c^2 Q^2 \sim N_c^2 Q^2 \left( \frac{T}{Q} \right)^n.
\]
(2.16)

### 3. Evolution of \( n = 2 \) operators in QCD for a generic coupling

In this section we describe the evolution of the \( n = 2 \) leading–twist operators. We focus on the respective flavor–singlet operators, of which there are two: the quark \((O^{\mu \nu}_f)\) and gluon \((O^{\mu \nu}_g)\) energy–momentum tensors displayed in Eqs. (1.2)–(1.3). Our emphasis will be on the mixing between these two operators under quantum evolution, leading to two orthogonal eigen–operators: one which is \textit{a priori} known for any value of the coupling, since this is protected by energy–momentum conservation and hence it is scale–independent — this is, of course, the total energy–momentum tensor, \( T^{\mu \nu} = O^{\mu \nu}_f + O^{\mu \nu}_g \), and the other one which is not protected and hence it depends upon the renormalization scale \( Q^2 \). The latter, that we shall denote as \( \Theta^{\mu \nu}(Q^2) \), is explicitly known in QCD only for sufficiently high \( Q^2 \), where perturbation theory can be used to compute the matrix of anomalous dimensions (see e.g. Ch. 18 in [34]). Here, we are rather interested in the situation at generic, and relatively strong, coupling, so our subsequent developments will be necessarily formal and incomplete: we shall try and use physical constraints and guidance from \( \mathcal{N} = 4 \) SYM theory in such a way to characterize the mixing matrix and the structure of \( \Theta^{\mu \nu} \) as well as we can without performing explicit calculations in QCD.

In full generality, the relevant renormalization group equations can be written in matrix form as (from now on we shall omit the Lorentz indices)

\[
\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} O_g \\ O_f \end{pmatrix} = \begin{pmatrix} \gamma_{gg} & \gamma_{gf} \\ \gamma_{fg} & \gamma_{ff} \end{pmatrix} \begin{pmatrix} O_g \\ O_f \end{pmatrix},
\]
(3.1)
which features the $2 \times 2$ anomalous dimension matrix $\gamma(\mu^2)$ of the $n = 2$ leading-twist operators. To lowest order in perturbation theory, the scale dependence of $\gamma$, as encoded in the running coupling $\alpha_s(\mu^2)$, factorizes out from the matrix structure. In that case it is convenient to pursue the analysis by diagonalizing the $\gamma$ matrix, since the corresponding eigenvectors are scale-independent (see e.g. Ref. [34]). However, such a simplification does not occur for the general case at hand. It is then preferable to consider the formal solution to Eq. (3.1), as obtained by integrating this equation from the conventional renormalization scale $\mu^2_0$ to the physically interesting scale $Q^2$. The solution reads

$$\begin{pmatrix} O_g(Q^2) \\ O_f(Q^2) \end{pmatrix} = \begin{pmatrix} M_{gg} & M_{gf} \\ M_{fg} & M_{ff} \end{pmatrix} \begin{pmatrix} O_g(\mu^2_0) \\ O_f(\mu^2_0) \end{pmatrix},$$

(3.2)

where the evolution matrix $M = (M_{ij})$, with $i, j = g$ or $f$, can be compactly written as

$$M(Q^2, \mu^2_0) = P \exp \left\{ \int_{\mu^2_0}^{Q^2} \frac{d\mu^2}{\mu^2} \gamma(\mu^2) \right\},$$

(3.3)

where the symbol $P$ in the r.h.s. indicates the $\mu^2$–ordering of the product of matrices in the series expansion of the exponential.

We shall now argue that, still in full generality, the matrix $M$ has only two independent components. This follows from energy–momentum conservation: the requirement that $T = O_f + O_g$ be scale-independent, that is,

$$O_g(Q^2) + O_f(Q^2) = O_g(\mu^2_0) + O_f(\mu^2_0),$$

(3.4)

implies two constraints on the components of the matrix $M$ (since the operators at the original scale $\mu^2_0$ should be viewed as two independent quantities), leading to

$$M_{fg} = 1 - M_{gg}, \quad M_{gf} = 1 - M_{ff},$$

(3.5)

and therefore

$$M = \begin{pmatrix} M_{gg} & 1 - M_{ff} \\ 1 - M_{gg} & M_{ff} \end{pmatrix}.$$  

(3.6)

So far we have not taken into account the fact that the coupling could be strong. Recall that our objective is to give a test of the idea that QCD is strongly coupled at a scale which is a few times the critical temperature for deconfinement $T_c$. So, let us assume that the coupling is strong at the scale $\mu_0$ at which one starts the evolution. (This is the scale to be identified with the temperature $T$ when the evolution takes place in the quark–gluon plasma phase.) Then we expect $M$ to have an eigenvalue which is extremely small, nearly zero, corresponding to the fact that the “unprotected” operator $\Theta$ has a large and negative anomalous dimension,

\footnote{There are of course similar constraints on the anomalous dimension matrix, which there imply $\gamma_{fg} = -\gamma_{gf}$ and $\gamma_{gf} = -\gamma_{ff}$. This means that, at any scale $\mu^2$, the matrix $\gamma(\mu^2)$ has the left eigenvector $(1, 1)$ with eigenvalue $\gamma_T = 0$. But the other eigenvector, orthogonal to $T$, is generally scale–dependent.}
which is exponentiated by the evolution. Let us give a more formal argument in that sense: to that aim, we divide the logarithmic phase–space for the evolution \( \ln(Q^2/\mu_0^2) \) into a large number \( N \) of small steps with width \( \epsilon = (1/N) \ln(Q^2/\mu_0^2) \), in such a way as to ensure that, within each interval, the anomalous dimension matrix is essentially constant. Then we can break the \( \mu^2 \)-ordered exponential in Eq. (3.3) into a product of \( N \) ordinary exponentials:

\[
M(Q^2, \mu_0^2) = e^{\gamma_i N} e^{\gamma_{N-1}} \cdots e^{\gamma_1},
\]

where, of course, the quantities \( \gamma_i \equiv \gamma(\mu_i^2) \) are \( 2 \times 2 \) matrices. The determinant \( \det M \) is equal to the product of the determinants of the \( N \) matrices in the r.h.s. For any such a matrix, we can diagonalize \( \gamma_i \) locally at \( \mu_i^2 \): \( \gamma_i = h_i \text{diag}(\gamma_T, \gamma_{\Theta(i)}) h_i^{-1} \), where \( \gamma_T = 0 \) (this is the anomalous dimension of the protected operator \( T \)), whereas \( \gamma_{\Theta(i)} \) is strictly negative (this is the anomalous dimension of the unprotected operator \( \Theta(\mu^2) \) at \( \mu^2 = \mu_i^2 \)). Then, clearly

\[
\det e^{\gamma_i} = e^{\gamma_{\Theta(i)} T} \implies \det M = \exp \left\{ \int_{\mu_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \gamma_{\Theta}(\mu^2) \right\},
\]

where the integrand in the exponent is negative at any \( \mu^2 \). Now, let us assume that for \( \mu^2 \) close to the lower limit \( \mu_0^2 \) (or anywhere else along the way from \( \mu_0^2 \) to \( Q^2 \)), the anomalous dimension is extremely large, so like at strong coupling: this implies that \( \det M \approx 0 \), as anticipated. By imposing \( \det M = 0 \) in Eq. (3.4), one finds \( M_{ff} = 1 - M_{gg} \). We thus finally deduce the following, particularly simple, expression for the evolution matrix (with \( m \equiv M_{gg} \))

\[
M = \begin{pmatrix} m & m \\ 1 - m & 1 - m \end{pmatrix},
\]

valid when the evolution takes place at least partially in a region in \( \mu^2 \) where the coupling is strong. Using this form for \( M \) in Eq. (3.2), one finds

\[
\mathcal{O}_g(Q^2) = m T, \quad \mathcal{O}_f(Q^2) = (1 - m) T,
\]

which allows us to identify the \( n = 2 \) operator orthogonal to \( T \), i.e., the one which has evolved essentially down to zero on the resolution scale \( Q^2 \):

\[
\Theta(Q^2) \equiv (1 - m) \mathcal{O}_g(Q^2) - m \mathcal{O}_f(Q^2) = 0.
\]

If the quantity \( m = m(Q^2, \mu_0^2) \) were known theoretically, then Eq. (3.11) would be a prediction that could be tested in lattice gauge theory. Unfortunately, we do not know how to determine this quantity within the scenario that QCD is strongly coupled at low scales.

What we do know, however, is that \( m \) should be independent of the precise value of the scale \( \mu_0^2 \) at which one starts the evolution: indeed, \( m \) is rather determined by the largest value of \( \mu^2 \) at which the anomalous dimension \( \gamma_{\Theta}(\mu^2) \) is still large. To see this, let us introduce an intermediate scale \( \mu_1^2 \), with \( \mu_0^2 < \mu_1^2 < Q^2 \), and thus write \( M = M_1 M_2 \), with

\[
M_1 = P \exp \left\{ \int_{\mu_0^2}^{\mu_1^2} \frac{d\mu^2}{\mu^2} \gamma(\mu^2) \right\}, \quad \text{and} \quad M_2 = P \exp \left\{ \int_{\mu_1^2}^{Q^2} \frac{d\mu^2}{\mu^2} \gamma(\mu^2) \right\}.
\]
Now, assume that $\mu_1$ is such that the coupling is still strong in its neighborhood, so that $M_1$ has the structure shown in Eq. (3.9) with $m \to m_1$. Then one can easily check that $M = M_1$, that is $m = m_1$, and this even for a matrix $M_2$ which has the most general possible structure, as shown in Eq. (3.3). (But of course in QCD we would also expect $M_2$ to be of the simpler form (3.9), since if the coupling is strong at some scale $\mu_1$, it is still strong at the softer scale $\mu_0 < \mu_1$.) Hence, if $\mu_S$ is the largest value at which the coupling is still effectively strong, then we have $m(Q^2, \mu_0^2) = m(Q^2, \mu_S^2)$ for any $\mu_0 < \mu_S$.

Finally, one may worry that in QCD anomalous dimensions are scheme dependent and that there is no meaning to say that $\gamma$ is large. However, when $Q^2$ is large, the operators on the left hand side of Eq. (3.2) have very little scheme dependence because $\alpha_s(Q^2)$ becomes small at large $Q^2$. The scheme dependence refers merely to the ability to transfer contributions between the evolution matrix $M(Q^2, \mu_0^2)$ and the operators $O_g(\mu_0^2)$ and $O_f(\mu_0^2)$ at the original scale. If QCD behaves like a strongly coupled field theory, then the operators $O_g(Q^2)$ and $O_f(Q^2)$ at the final scale are expressible in terms of the (protected) energy–momentum tensor, as shown in Eq. (3.10). We have modeled our discussion to resemble the situation in $\mathcal{N} = 4$ SYM theory (where there is no scheme dependence, because of the conformal symmetry), but we recognize that in QCD one could choose schemes in which a condition like Eq. (3.11) — i.e., the vanishing of $\Theta$ at the scale $Q^2$ — does not follow from the evolution (i.e., from the particular structure (3.9) of the evolution matrix $M$), but rather from the fact that a relation between $O_g$ and $O_f$ similar to (3.11) holds already at the original scale $\mu_0^2$.

4. Evolution of $n = 2$ operators in quenched QCD

Because we are unable to specify a definite value for the quantity $m$ in Eqs. (3.3) and (3.11), it is difficult to devise a test of strong coupling behaviour in terms of $n = 2$ leading–twist operators using lattice gauge theory for full (unquenched) QCD. However, experience with lattice calculations shows that there is generally not a large difference between quenched and unquenched QCD. Thus if full QCD is effectively a strongly coupled theory in the soft momentum region, one would naturally expect the same to be true for quenched QCD. As mentioned in the Introduction, quenched QCD consists in ignoring the quark loops, so the matrix element $\gamma_{fg}$ of the anomalous dimension matrix in Eq. (3.1) must vanish. (Recall that this element describes a transition from gluon to quark fields.) Since $\gamma_{gg} = -\gamma_{fg}$ by energy–momentum conservation, and similarly $\gamma_{ff} = -\gamma_{gf}$, we deduce that the $\gamma$ matrix has a very simple structure in quenched QCD:

$$
\gamma(\mu^2) = \begin{pmatrix}
0 & -\gamma_{ff} \\
0 & \gamma_{ff}
\end{pmatrix}
$$

(4.1)

This structure is already telling us that the operator $\Theta$ orthogonal to the energy–momentum tensor $T = O_f + O_g$ is simply the quark operator $O_f$. (Indeed, the matrix (4.1) has the left eigenvector $(0,1)$ with eigenvalue $\gamma_{ff} < 0$.) It is furthermore clear that the rest of the discussion of the renormalization group evolution for $n = 2$ goes exactly like in the previous section, so in particular Eqs. (3.3) and (3.11) are still true, but now with $m = 1$ (since gluon fields cannot change into fermions). Once again, Eq. (3.11) with $m = 1$ confirms that $\Theta = O_f$. 

\[\]
Thus, within quenched QCD, a strong–coupling scenario predicts $O_f(Q^2) \simeq 0$ for a sufficiently hard scale $Q^2$. At finite temperature, this in turn implies that the average value of the energy carried by a bare quark (one which is measured on a hard resolution scale $Q^2 \gg T^2$) which is in equilibrium with a strongly–coupled thermal bath of gluons is very small,

$$\langle \bar{q} \gamma_0 i D_0 q (Q^2) \rangle_T \simeq 0,$$  

and in particular much smaller than the corresponding ideal–gas value (the Stefan–Boltzmann law for a gas of free, massless, quarks):

$$\langle \bar{q} \gamma_0 i D_0 q \rangle_T^{(0)} = N_f N_c \frac{7 \pi^2}{60} T^4.$$

By contrast, in a weak coupling scenario, the corresponding lattice result should be rather close to the above ideal gas value, and slowly depart from it with decreasing lattice spacing $a = 1/Q$. One can easily evaluate the leading order perturbative corrections to (4.3), and thus get a better estimate for what should be the result at weak coupling: using

$$\gamma_{ff} = - - a_{ff} \frac{\alpha_s(\mu^2)}{4\pi}, \quad a_{ff} = \frac{8}{3} C_F,$$

one finds (cf. Eq. (2.9) with $a_f^{(n)} \rightarrow a_{ff}$ and $b_0 = 11N_c/3$)

$$\langle O_f(Q^2) \rangle \langle O_f(\mu_0^2) \rangle = \left[ \frac{\ln(\mu_0^2/\Lambda_{QCD}^2)}{\ln(Q^2/\Lambda_{QCD}^2)} \right]^{8C_F/3b_0}.$$

For example, for $Q = 4$ GeV, $\Lambda_{QCD} = 200$ MeV, and $\mu_0 = 3T_c \simeq 600$ MeV, one finds that the perturbative evolution reduces the ideal–gas result (4.3) by about 30%.

What could be the corresponding suppression in a strong–coupling scenario? It is of course very difficult to answer this question given our impossibility to perform calculations in QCD at strong coupling. But if the experimental results at RHIC for the jet quenching parameter $\hat{q}$ [29, 30] — which, we recall, suggest an enhancement by roughly a factor of 5 with respect to the respective weak–coupling estimate — are indeed indicative of the strength of the quantum evolution in the QCD plasma, then one might expect a similarly strong reduction, by a factor of 5 or more, for the quark energy density in quenched QCD. That such an expectation is not totally unreasonable (within that strong–coupling scenario) can be also viewed via the following argument:

Although there is no good reason to believe that the strong–coupling, large–$n$, estimates for the anomalous dimensions in $N = 4$ SYM theory, cf. Eq. (2.11), could be applied to the QCD problem at hand, let us nevertheless do so, by lack of a better argument. Previous studies in the literature, concerning the comparison between $N = 4$ SYM and thermal QCD in the temperature range of interest, suggest that a reasonable value for the QCD ’t Hooft coupling to be used in this context is $\lambda_{QCD} \simeq 5.5$ [33, 23]. (For instance, this is close to the naive estimate $\lambda_{QCD} = 3g^2$, with the 2–loop QCD running coupling $g^2(\bar{\mu})$ evaluated at the scale $\bar{\mu} = 2\pi T$.) Via Eq. (2.11), this yields (for $n = 2$) an anomalous dimension $|\gamma_{ff}| \sim 1.$
Assume now that there exists a window for strong–coupling dynamics, within which \( O_f(\mu^2) \) evolves according to Eq. (2.10). Then

\[
\frac{\langle O_f(\mu^2) \rangle}{\langle O_f(\mu_0^2) \rangle} \sim \left( \frac{\mu_0^2}{\mu^2} \right)^{|g| f f} \sim \frac{\mu_0^2}{\mu^2},
\]

(4.6)

whereas the subsequent evolution from \( \mu^2 \) to the harder scale \( Q^2 \) takes place at weak coupling, and hence it is much slower, cf. Eq. (4.5). Taking \( \mu_0 = 3T_c \simeq 600 \) MeV once again, it is clear that a reduction by a factor of 5 or larger is achieved as soon as \( \mu \gtrsim 2\mu_0 \sim 1.2 \) GeV, that is, even if the strong–coupling dynamics holds only in a rather narrow window. The current lattice QCD results for the QCD pressure or energy density show a rather smooth behaviour for temperatures \( T > 3T_c \), with almost no variation from \( 3T_c \) up to \( 6T_c \); hence, if it so happens that QCD is (effectively) strongly–coupled at the scale \( 3T_c \), there is no reason why this should not remain true until the slightly harder scale of \( 6T_c \).

To summarize, a lattice calculation for quenched QCD finding a result close to (4.3) would show that the “quasiparticles” of quenched QCD are close to being pointlike and that the theory is weakly coupled. On the other hand, a much smaller result, cf. (4.2), would be compelling evidence for an effectively strongly–coupled theory, with quasiparticles (if they exist) highly composite.

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