Correlated Isocurvature Fluctuation of Quintessence and Curvaton Scenario

TAKEO MOROI\(^1\) and TOMO TAKAHASHI\(^2\)

\(^1\)Department of Physics, Tohoku University, Sendai 980-8578, Japan
\(^2\)Institute for Cosmic Ray Research, University of Tokyo, Kashiwa 277-8582, Japan

ABSTRACT

We consider cosmic microwave background (CMB) anisotropy in models with quintessence taking into account of isocurvature fluctuation in the quintessence. It is shown that, if the primordial fluctuation of the quintessence has a correlation with the adiabatic density fluctuations, CMB angular power spectrum \(C_l\) at low multipoles can be suppressed. Possible scenario of generating correlated mixture of the quintessence and adiabatic fluctuations is also discussed.

1. Introduction

Current cosmological observations suggest that the present universe is dominated by an enigmatic component called “dark energy.” Although many candidates for dark energy have been proposed so far, a slowly evolving scalar field, dubbed as “quintessence” is an interesting possibility. Since the quintessence field is a scalar field, its amplitude may fluctuate which can become a new source of the cosmic density fluctuations and affects the CMB angular power spectrum. In this talk, we discuss the CMB angular power spectrum in models with quintessence, paying particular attention to effects of the correlated isocurvature fluctuation between quintessence and adiabatic fluctuations [1].

2. Framework

Here we present the framework of our study. Although various models of quintessence have been proposed, we adopt a simple approximation for the quintessence potential of a quadratic form with a constant term\(^*\) :

\[
V(Q) = V_0 + \frac{1}{2} m_Q^2 Q^2.
\]  

(1)

In our study, we consider \(m_Q\) comparable to (or smaller than) the present Hubble parameter. In addition, \(V_0\) is assumed to be of the order of the present critical density or smaller. With such a small value of \(m_Q\) (and \(V_0\)), slow-roll condition for \(Q\) is satisfied until very recently and energy fraction of the quintessence becomes sizable only at the very recent epoch. Since the quintessence is a dynamical scalar field, its amplitude may fluctuate which can become a new source of the cosmic density fluctuations and affects the CMB angular power spectrum.

Evolution and effects of the primordial fluctuation of \(Q\) have been studied for the case where the primordial fluctuation of the quintessence is not correlated with the adiabatic fluctuations [2].

\(^*\)Notice that this potential is a good approximation for some quintessence models like the cosine-type one [2].
fluctuations. Fluctuation of the quintessence field can be, however, correlated with the adiabatic fluctuations. If some correlation exists, effects of the primordial fluctuation of $Q$ on the CMB angular power spectrum are expected to be different from those in the uncorrelated case. Hereafter, we discuss the effects of quintessence fluctuation for the case where the correlation between the fluctuation of $Q$ and the adiabatic fluctuations exists.

In order to parameterize the relative size of the primordial quintessence fluctuation and the adiabatic fluctuations, we define

$$r_Q \equiv \frac{\delta Q_{\text{init}}}{M_* \Psi_{\text{RD}}}.$$

Here, $\delta Q_{\text{init}}$ is the primordial fluctuation of $Q$, $\Psi$ denotes the fluctuation of the $(0, 0)$ component of the metric in the Newtonian gauge: $g_{00} = a^2(1 + 2\Psi)$ with $a$ being the scale factor, and $M_*$ is the reduced Planck scale. In addition, $\Psi_{\text{RD}}$ is the metric perturbation related to the adiabatic density fluctuation in the radiation-dominated epoch. We assume that $\Psi_{\text{RD}}$ is (almost) scale-invariant. If we calculate the CMB angular power spectrum with non-vanishing values of $\delta Q_{\text{init}}$ and $\Psi_{\text{RD}}$, we obtain

$$C_l = C_l^{(\text{adi})} + C_l^{(\text{corr})} + C_l^{(\text{uncorr})}.$$  

Here, $C_l^{(\text{adi})}$ is the result with purely adiabatic density fluctuations. $C_l^{(\text{uncorr})}$ is the CMB angular power spectrum purely generated from $\delta Q_{\text{init}}$, while $C_l^{(\text{corr})}$ parameterizes the effects of correlation. When $\delta Q_{\text{init}}$ and $\Psi_{\text{RD}}$ are uncorrelated, $C_l^{(\text{corr})} = 0$. Furthermore, $C_l^{(\text{uncorr})}$ is increased at low multipoles. As a result, the total CMB angular power spectrum may be significantly enhanced at low multipoles when no correlation is assumed. If $\delta Q_{\text{init}}$ and $\Psi_{\text{RD}}$ have correlation, $C_l^{(\text{corr})}$ plays important roles. In Fig. we show the CMB angular power spectrum for the case where $\delta Q_{\text{init}}$ and $\Psi_{\text{RD}}$ are fully correlated. As one can see, in this case, sizable suppression of the low multipoles is possible compared to the $\Lambda$CDM model. Notice that, in the uncorrelated case, such a suppression of $C_l$ at the low multipoles cannot be realized.

3. A possible model

Here we present a possible scenario which generates the correlated fluctuations. We use the fact that fluctuations of two scalar fields can be correlated if they have a mixing during the inflation. We consider the case with two scalar fields, $Q$ and $\phi$. $Q$ and $\phi$ are defined as mass eigenstates in the present universe. In the early universe (i.e., for example, during inflation), however, Hubble-induced interaction may cause a mixing between $Q$ and $\phi$ and hence the mass eigenstates may be linear combinations of them. We denote the mass eigenstates as $\xi$ and $\eta$, and define the mixing angle $\theta$ as

$$\left( \begin{array}{c} \xi \\ \eta \end{array} \right) = \left( \begin{array}{cc} \cos \theta(t) & -\sin \theta(t) \\ \sin \theta(t) & \cos \theta(t) \end{array} \right) \left( \begin{array}{c} Q \\ \phi \end{array} \right).$$

Strictly speaking, the following expression is valid only for the case where $\delta Q_{\text{init}}$ and $\Psi_{\text{RD}}$ are fully correlated. For the case where $\delta Q_{\text{init}}$ and $\Psi_{\text{RD}}$ are uncorrelated, for example, it should be understood as $r_Q = \sqrt{\langle \delta Q_{\text{init}}^2 \rangle / M_* \langle \Psi_{\text{RD}}^2 \rangle}$. 

\[\text{\cite{extension}}\]
In our model, $\theta(t)$ varies from non-vanishing value during inflation $\theta_{\text{inf}}$ to the present value $\theta_{\text{now}} = 0$. If the mass of $\eta$ is large during inflation while that of $\xi$ is negligible, only the $\xi$ field acquires the quantum fluctuation as $\delta \xi_{\text{inf}} = H_{\text{inf}}/2\pi, \delta \eta_{\text{inf}} = 0$, where $H_{\text{inf}}$ is the expansion rate during the inflation. Assuming that $\theta$ rapidly changes from $\theta_{\text{inf}}$ to 0 at the end of inflation, primordial fluctuation of $Q$ and $\phi$ are given by $\delta Q_{\text{init}} = \delta \xi_{\text{inf}} \cos \theta_{\text{inf}}, \delta \phi_{\text{init}} = \delta \xi_{\text{inf}} \sin \theta_{\text{inf}}$, then correlated fluctuations are generated in $Q$ and $\phi$.

The above situation may be realized if the potential of the scalar fields is of the form

$$V = V_0 + \frac{1}{2} m_Q^2 Q^2 + \frac{1}{2} m_\phi^2 \phi^2 + V_{\text{Hubble}},$$

(5)

where $V_{\text{Hubble}} = H^2_{\text{vac}} (Q \sin \theta_{\text{inf}} + \phi \cos \theta_{\text{inf}})^2$. Here, $V_{\text{Hubble}}$ is the Hubble-induced interaction which is effective only during the inflation with $H_{\text{vac}}$ being the Hubble parameter induced by the “vacuum energy”; $H_{\text{vac}} = H_{\text{inf}}$ and $H_{\text{vac}} \simeq 0$ for during and after the inflation, respectively. With this potential, one of the mass eigenstates $\eta \simeq \phi + \theta_{\text{inf}} Q$ acquires an effective mass comparable to the expansion rate and its quantum fluctuation during inflation becomes negligibly small. Other mass eigenstate $\xi \simeq Q - \theta_{\text{inf}} \phi$, on the contrary, stays almost massless and it acquires the quantum fluctuation.

If the decay rate of the inflaton field is larger than $m_\phi$, slow roll condition is satisfied for $\phi$ at the time of the inflaton decay. In this case, $\delta \phi_{\text{init}}$ may become the dominant source of the adiabatic fluctuations. In order to generate the adiabatic fluctuations from the fluctuation of $\phi$, we can use the curvaton mechanism [6] where the primordial fluctuation of the curvaton becomes the dominant source of the adiabatic fluctuations\(^*\). Indeed, if the

\(^*\)Here, we do not identify $\phi$ as inflaton; in our model, $\phi$ should acquire large effective mass during
energy density of $\phi$ once dominates the universe, $\phi$ plays the role of curvaton and the metric perturbation in the radiation dominated epoch is given by $\Psi_{\text{RD}} = -(4/9)(\delta \phi_{\text{init}}/\phi_{\text{init}})$ where $\phi_{\text{init}}$ is the initial amplitude of $\phi$ determined during the inflation \[7\]. As a result, correlated mixture of adiabatic and quintessence fluctuations is generated. In this model, the $r_Q$ parameter is estimated as

$$r_Q = \frac{1}{2\pi} \frac{H_{\text{inf}}}{M_\ast \Psi_{\text{RD}}} \cos \theta_{\text{inf}}.$$  \hfill (6)

Notice that, using $\Psi_{\text{RD}} \sim O(10^{-5})$ and the upper bound $H_{\text{inf}}/M_\ast \lesssim 7 \times 10^{-5}$ \[8\], $r_Q \lesssim 1$ in this simple model. Larger value of $r_Q$ is, however, possible if we extend the model. For example, if the coefficient of the kinetic term of $Q$ varies after the inflation, value of $\delta Q$ (for the canonically normalized field) also changes \[1\].

4. Summary

We have seen that the CMB angular power spectrum at small $l$ can be suppressed if the primordial fluctuation of the quintessence has correlation with the adiabatic density fluctuations\(^d\). We have also pointed out that such a correlation may be generated during inflation if the quintessence field has some mixing with other scalar field which is responsible for generating the adiabatic density fluctuations.

Acknowledgments: T.T. would like to thank the Japan Society for Promotion of Science for financial support. This work was partially supported by the Grand-in-Aid of the Ministry of Education, Science, Sports and Culture of Japan No. 15540247 (T.M.).

5. References

[1] T. Moroi and T. Takahashi, Phys. Rev. Lett. 92, 091301 (2004)
[2] J. E. Kim, JHEP 9905 (1999) 022.
[3] L. R. Abramo and F. Finelli, Phys. Rev. D 64 (2001) 083513.
[4] M. Kawasaki, T. Moroi and T. Takahashi, Phys. Rev. D 64 (2001) 083009; Phys. Lett. B 533 (2002) 294.
[5] G. Hinshaw et al., Astrophys. J. Suppl. 148, 135 (2003).
[6] K. Enqvist and M. S. Sloth, Nucl. Phys. B 626 (2002) 395; D. H. Lyth and D. Wands, Phys. Lett. B 524 (2002) 5; T. Moroi and T. Takahashi, Phys. Lett. B 522 (2001) 215 [Erratum-ibid. B 539 (2002) 303].
[7] T. Moroi and T. Takahashi, in Ref. \[6\]; Phys. Rev. D 66 (2002) 063501.
[8] H. V. Peiris et al., Astrophys. J. Suppl. 148 (2003) 213; V. Barger, H. S. Lee and D. Marfatia, Phys. Lett. B 565 (2003) 33; S. M. Leach and A. R. Liddle, Phys. Rev. D 68, 123508 (2003)
[9] C. Gordon and W. Hu, arXiv:astro-ph/0406496

\(^d\)For subsequent work along this line, see Ref. \[9\].