We provide general arguments regarding the connection between low-energy theories (gravity and quantum field theory) and a hypothetical fundamental theory of quantum gravity, under the assumptions of (i) validity of the holographic bound and (ii) preservation of unitary evolution at the level of the fundamental theory. In particular, the appeal to the holographic bound imposed on generic physical systems by the Bekenstein-Hawking entropy implies that both classical geometry and quantum fields propagating on it should be regarded as phenomena emergent from the dynamics of the fundamental theory. The reshuffling of the fundamental degrees of freedom during the unitary evolution then leads to an entanglement between geometry and quantum fields. The consequences of such scenario are considered in the context of black hole evaporation and the related information-loss issue: we provide a simplistic toy model in which an average loss of information is obtained as a consequence of the geometry-field entanglement.

I. INTRODUCTION

In this paper we shall combine two general considerations regarding quantum theories of gravity and study their implications for black hole evaporation. The first consideration is that, if the Bekenstein upper bound on the entropy of any physical system is correct, there probably exist more fundamental entities than the ones we deem to be elementary \[1–3\]. The second consideration is that, at our energy scales, these fundamental entities must organize themselves as quantum fields acting on classical spacetimes (our best understanding of the low-energy physics) by “making” both the quantum fields and the classical geometry.

The consequence of such assumptions is then the following: since the bound is reached only when the black hole is formed, and it is an upper bound, then and only then all the degrees of freedom of the fundamental entities have been excited. Hence one can think of the black hole state as a gas-like high-energy “phase”, with few (one in the simplest case) macroscopic parameters characterizing all the microscopic states. When the energy is lowered (that is after the evaporation of the black hole) these fundamental entities have at their disposal a large amount of different, nonequivalent re-arrangements. These are their physically nonequivalent low-energy “phases”, that can only be described as specific quantum fields acting on specific geometries. Similar lessons can be learned from ordinary states of matter, made of ordinary particles. On the other hand there is a crucial difference with the usual phases of matter, namely the fact that we introduce here a democracy between spacetime and fields/particles which, in this view, both emerge from one underlying dynamics. We call this view the “quasiparticle” picture, as we shall explain below.

With this in mind, it is clearly virtually impossible that after the black hole evaporation we can retrieve the very same “phase” we had before the black hole was formed. Hence, the information associated to the quantum fields in the “phase” before the formation of the black hole is, in general, only partially recovered in the “phase” after the black hole has evaporated; the information loss is due to the entanglement between the fields and the geometry. This is how we intend to address the information paradox.

The structure of the paper is as follows. In Sec. II we explain more in detail the quasiparticle picture and its role in the emergence of both quantum fields and classical geometry. In Sec. III we briefly present the construction of the Page curve as well as analogous concepts and antagonist views. In order to demonstrate possible implications of our picture, in Sec. IV we shall construct a toy model of black hole evaporation that exhibits partial loss of information and, hence, leads to a modification of the Page curve.

II. QUASIPARTICLE PICTURE

As widely known, the entropy \(S\) of any physical system contained in a volume \(V\), including the volume itself, is supposed to be bounded from above by the value of the Bekenstein-Hawking entropy associated to a black hole whose event horizon coincides with the boundary of \(V\) \[4\]

\[
S \leq S_{\text{BH}} = \frac{1}{4} \frac{\partial V}{r_p^2},
\]

where \(r_p = \sqrt{\hbar G/c^3} \sim 1.6 \times 10^{-35}\) m. The generality of the original bounds for ordinary matter (i.e., when gravity is not included) posited by \[4\] is the subject of intense investigation and debates \[5\]. Nonetheless, it is widely accepted that for black holes the upper bound is saturated.

By the number of degrees of freedom \(N\) of a quantum physical system we mean the number of bits of information necessary to describe the generic state of the system. In other words, \(N\) is the logarithm of \(\mathcal{N}\), the dimension of the Hilbert space of the quantum system. In the extreme case of a black hole \(\mathcal{N} = e^{3N_{\text{BH}}}\). Hence, formula

\[\text{arXiv:1704.00345v1 [gr-qc] 2 Apr 2017}\]
This failure leads to the existence of different, unitarily
systems of finite degrees of freedom and trivial topology \[31\].
Rem \[29, 30\] that holds only for quantum mechanical sys-
manifestation of the failure of the Stone-von Neumann theo-
degrees of freedom and/or to the nontrivial topology of the
understood in QFT as due to the infinite number of de-
tum mechanical sectors or “phases”. This complexity is
can have a rich structure \[27\] with nonequivalent quan-
it is well recognized by now that the quantum vacuum
quantum vacuum at a continuous relativistic regime \[26\] (e.g., in condensed matter). Indeed,
(hence deemed to be fundamental) and in its nonrela-
tion between the actual fundamental degrees of freedom.
On the one hand, the idea that gravity is an emergent
phenomenon arising from more fundamental degrees of
freedom is not new and goes back to Shakarov \[8, 9\].
On the other hand, emergent, nonequivalent descrip-
tions of the same underlying dynamics are a built-in char-
acteristic of QFT \[24\], both in its relativistic regime \[25\]
(hence deemed to be fundamental) and in its nonrela-
tivistic regime \[29\] (e.g., in condensed matter). Indeed,
it is well recognized by now that the quantum vacuum
can have a rich structure \[27\] with nonequivalent quan-
tum mechanical sectors or “phases”. This complexity is
understood in QFT as due to the infinite number of de-
grees of freedom and/or to the nontrivial topology of the
system, such as the presence of topological defects \[28\].
Remains that the so-called “Page curve” \[45\] which describes the complete information retrieval in the Hawking radiation at the final stage of the black hole evaporation. In our picture, however, the probability that after the complete evaporation the fundamental degrees of freedom reorganize just like before the collapse leading to black hole, is inversely proportional to the number of possible nonequivalent rearrangements of the fundamental degrees of freedom. Therefore, even if one demands the dynamics of the fundamental degrees of freedom to be unitary, as we shall do, one expects that the entanglement between the geometry and the quantum fields due to the reshuffling of fundamental degrees of freedom could lead to an effective loss of information in the Hawking radiation.

The loss of information, in the sense of evolution of a
pure state into a mixed state, can have two causes. The
The first one is that the laws of quantum theory are indeed violated in some regimes. The second one is that only some subsystem of the universe is accessible, hence there will always be a residual entanglement of the subsystem with the inaccessible parts. In our picture we do not consider the first possibility, rather we suggest that part of the total system is always hidden: this produces entanglement between emergent fields and geometry and leads to an effective loss of information on the field side.

Let us conclude this introduction with a schematic summary of the logic behind the model we propose.

i) We interpret the upper bound as indication of the existence of finite number of fundamental degrees of freedom, fully excited (saturated bound) only for a black hole. To access these fundamental degrees of freedom, one would need resolutions of order (which might not be possible at all, as suggested, e.g., in ).

ii) Everything we see at our low-energy scale (low-resolution) is classical spacetimes and quantum fields. Both emerge from the properties of and the interactions between fundamental degrees of freedom.

iii) Since the bound is not reached at our energies, the particles we call elementary are in fact emergent quasiparticles.

iv) Being discrete entities, the fundamental degrees of freedom must arrange into discrete structures. Their nature (symmetries, type of interaction, etc.) is not specified here, as we shall make model-independent considerations. In other words, we do not construct here a specific model of quantum gravity, but try to convey general considerations.

v) There are, in general, different configurations of the fundamental degrees of freedom which give rise to the same classical geometry. These configurations yield different numbers of degrees of freedom for the fields. Thus, even if the geometries before the formation of the black hole and after its evaporation are the same, the emerging quantum fields will be, in general, different (i.e., live in different Hilbert spaces).

vi) Even though, for simplicity, we assume unitary evolution on the fundamental level, the rearrangement of the fundamental degrees of freedom during the evaporation process leads to an entanglement between the emerging geometry and the emerging fields, thereby producing a loss of information on the field side.

III. PAGE CURVE

Our primary motivation is to address the black hole information paradox, i.e., the problem of the apparent loss of information during the process of a black hole evaporation. There are many proposals how to resolve this paradox. There are arguments that in the presence of gravity, and especially in the presence of a black hole, we have to expect some modifications of the quantum theory and, perhaps, deviations from unitary evolution at the fundamental level. From this perspective, there is no paradox in losing information during the formation of a singularity and the subsequent evaporation of the back hole, because the underlying theory does not require the information conservation.

On the other hand, it has been advocated by and that the evolution is always unitary and the information loss is prohibited. These arguments rely on the holographic principle, string theory models of black hole evaporation and the paradigm of the black hole complementarity. Another confirmation of information conservation has been provided by , who employed the quantum perturbations of the event horizon and the AdS/CFT correspondence in order to argue that information can, in fact, escape from the black hole.

There have been also arguments that it is impossible to reconcile the unitary evolution, the principle of equivalence and the low energy effective quantum field theory. These arguments have been embodied in the controversial “firewall paradox” introduced in . Very recently, it was proposed in how to avoid the firewall paradox by appropriate identification of the antipodal points of the event horizon.

A more conservative approach to the problem has been adopted by Page and it is based on purely quantum mechanical considerations. Following , one considers the splitting of a Hilbert space into a bipartite system, , where superscripts and indicate the dimension of corresponding Hilbert space, so that . Next, one chooses an arbitrary fixed state and a random unitary matrix ; then is a random state in . To such state we associate the density matrix , by tracing out the subsystem , and the corresponding entanglement entropy . Averaging through we get the average entanglement entropy of the subsystem ,

\[
S_{m,n} = \langle S_{m,n}(U) \rangle_{\text{average through } U},
\]

and the average information contained in ,

\[
I_{m,n} = \ln m - S_{m,n}.
\]

For mathematical details of this construction see the original paper and also . Page conjectured – and it was later proved in – that the average information is

\[
I_{m,n} = \ln m + \frac{m - 1}{2n} - \sum_{k=n+1}^{m} \frac{1}{k}, \quad \text{for } m < n.
\]

These results are applied to the black hole evaporation problem in . It is assumed that the evolution of the collapsing matter to produce a black hole and the subsequent evaporation of that black hole is a unitary process,
and hence there exists a $S$-matrix relating the initial collapsing matter to the final state when black hole is fully evaporated and only the Hawking radiation remains. The Hilbert space of the Hawking radiation is factorized into a product as before, where the subsystem $A$ now corresponds to the states under the horizon and the subsystem $B$ corresponds to photons already emitted from the black hole.

When the black hole is formed, there is no Hawking radiation outside and, hence, $n = 1$ and $m = \dim H$. Thus, by assumption, the entanglement entropy is trivially zero. As the black hole evaporates, dimension $n$ increases and $m$ decreases, while $mn$ is kept constant. Since the emitted photons are entangled with the particles under the horizon, entanglement entropy increases. At some stage of the evaporation (approximately half time of evaporation process) the information stored below the horizon starts to leak from the black hole, decreasing the entanglement entropy. Finally, when the black hole fully evaporates, $m = 1$ and $n = \dim H$ and the entanglement entropy returns to zero. The process is shown in Fig. 1.

A natural generalization of the Page analysis is to consider tripartite system instead of a bipartite one. In [58], the authors investigate the possibility that the particles emitted by a black hole are transformed either into the Hawking radiation or into another form of matter, which can be, e.g., a remnant. In this case, even when the black hole is fully evaporated, there can still exist entanglement between these two forms of matter. Hence, the Hawking radiation does not contain the full information and it is not in a pure state (at least on average).

In the aforementioned works, no analysis of the interaction between the matter fields and the space-time geometry has been given, nor even addressed. However, in order to consider the full black hole evaporation process, one certainly cannot treat matter as the test field on a given, say Schwarzschild, background. It is the system gravity+matter which evolves unitarily, not just the matter field. Therefore, we propose the possibility that at the end of the evaporation the Hawking radiation is not in a pure state because of its entanglement with the geometry itself and it is a purpose of this paper to clarify this statement.

Thus, in a sense, we proceed analogously to Page, and it is important to stress the points where we differ. First, we interpret both gravity and fields as emergent phenomena. Thus, similarly to Page, we consider what we call fundamental Hilbert space $H$, but we do not split it into a direct product of the two spaces, because we claim that on the fundamental level there is no distinction between the field and the geometry at all. Instead, we introduce effective Hilbert spaces representing the states of the geometry and of the fields, and mappings which extract the geometrical/matter content from the states of $H$. Second, as a consequence, for the effective loss of the information we do not require the presence of a third, unknown kind of matter like in [58], because it is the entanglement of the field with the geometry which implies this effective loss. Nonetheless, our description allows an arbitrary number of different fields, hence includes the possibility for such unknown kind of matter.

On the other hand, our approach is similar to that of Page in that we do not specify any particular microscopic dynamics. We merely provide a kinematical framework which allows us to estimate the entanglement entropy.

IV. MODEL OF BLACK HOLE EVAPORATION

Our goal in this section is to construct a simple kinematical model which mimics the evaporation of the black hole, keeping in mind the illustrated conceptual framework for which both geometry and quantum fields are emergent phenomena. We consider the following idealized scenario:

1. Initially, there is a quantum field (in an almost flat space) which collapses and eventually forms a black hole of mass $M_0$.
2. The black hole starts to evaporate in a discrete way; for simplicity we assume that each emitted quantum of the field has the same energy $\varepsilon$, so that $M_0 = N_G \varepsilon$ for some integer $N_G$.
3. At the end of the evaporation, the space becomes almost flat again and the field is in excited state with $N_G$ quanta.

We assume:

1. There exists a fundamental Hilbert space $H$. That is the Hilbert space of the fundamental degrees of freedom of the total system, i.e., black hole, radiation and space outside the black hole. Since here we focus on a finite region accessible to a generic observer and big enough to contain the black hole
at initial time and the emitted radiation at a later
time, \( H \) here is finite-dimensional;

2. For a specific observer at low-energy scale, the
states of \( H \) appear as classical spatial geometry and
quantum fields propagating on it.

3. There are states in \( H \) which represent the same clas-
cical geometry but are microscopically different.

4. In general, there is exchange of the number of de-
grees of freedom between the fields and geometry.

In this model, we introduce a space of classical geomet-
ries representing spatial slices of space-time containing
a black hole of a given mass \( M^{(a)} = a \varepsilon \). That is, we
introduce an orthonormal set of the states
\[
|g^{(a)}\rangle, \quad a = 0, 1, \ldots, N_G - 1,
\]
where \( N_G \) is therefore the number of geometries allowed
in our model. For convenience, we introduce the Hilbert space of classical geometries \( H_G \) as the linear span of the states (5) and define the “mass operator” \( M \) by
\[
M|g^{(a)}\rangle = M^{(a)}|g^{(a)}\rangle \equiv \varepsilon a |g^{(a)}\rangle. \tag{6}
\]
An operator of this kind should represent the possibility of measuring geometric properties of the space, such as the three-dimensional metric, as seen by a specific observer. The assumption that the geometry of the space is a result of some coarse-graining procedure associated with a specific observer means there is some mapping \( P_G : H \rightarrow H_G \) which assigns to a microscopic state in \( H \) corresponding classical geometry or an appropriate superposition of such geometries.

Similarly, we shall assume the existence of some mapping \( P_F \) which extracts the “field content” of a state in \( H \). Then, \( H_F \) can be, e.g., an appropriate Hilbert (Fock) space representing the states of the fields; concrete definitions will depend on the particular theory of quantum gravity. Schematically, the states of the fundamental Hilbert space \( H \) can be interpreted as states with some classical geometry via the mapping \( P_G \), and with some state of the quantum field via the mapping \( P_F \):

After introducing these mappings, one can label the states in \( H \) by the values of the coarse-grained quantities, i.e., \( |\psi\rangle = |g^{(a)}\rangle, |\phi\rangle \).

For simplicity we assume that any state of \( H \) can be inter-
preted in such a way, although in reality this is much
more complicated: classical geometries are expected to
be very special superpositions of basis states with no
classical analogues. Since we are not building a spec-
cific model of quantum gravity, we ignore this complica-
tion. On the other hand, one can argue that among
the states corresponding to definite classical geometries
one can choose a subset of (sufficiently distinct) states
which are approximately orthogonal and consider only
a subspace of \( H \) generated by this (approximately) or-
thonormal set.

In Page’s picture described in the Sec. [III], he considers
splitting of the Hilbert space representing the states of
the field into “inside” and “outside” part with respect to
the horizon of the black hole. In our model we wish to im-
plement the idea that the geometry and its fundamental
degrees of freedom must be brought into the picture, so
that one should split the fundamental space \( H \) into a di-
rect product of “geometrical” and “field” part. However,
for our argument it is essential to entertain the possi-
bility that the distribution of the microscopic degrees of
freedom between the geometry and the fields is not fixed
and can change during the evolution of the system.

The following simple model will serve just as a useful
visualization and to provide a terminology convenient for
the subsequent construction. However, the construction
itself does not rely on such visualization. Let there be
a certain number \( N \) of fundamental degrees of freedom
in the sense explained in the Introduction. The states of
each fundamental degree of freedom form a \( d \)-dimensional
Hilbert space, so that the Hilbert space of all fundamen-
tal degrees of freedom has dimension \( d^N \). Now, the states
of the fundamental degrees of freedom give rise to the no-
tions of spatial geometry (distance, topology, dimension)
and of quantum fields. We think of the set of all funda-
mental degrees of freedom as distributed among the ver-
tices of a graph and their links. More specifically, suppose
that the fact that there is some geometrical relation (e.g.,
the distance) between two vertices can be represented as
a link between corresponding vertices of the graph and a
quantitative measure of such relation is represented by a
weight (or a set of weights) of the link. Thus, one could
interpret the geometry as encoded in the states of all links
in the graph. However, in order to keep the total degrees of
freedom constant, the vertices have to “offer” some of
their degrees of freedom to form the Hilbert space \( H_G \)
corresponding to the states of the links which are inter-
preted as the state of the geometry. Then, the remaining
fundamental degrees of freedom can be represented as
excitation states of the vertices of the graph and they
form a Hilbert space \( H_F \) whose elements are interpreted
as the states of the emergent field. So, the state of the
entire graph is an element of the Hilbert space \( H_G \otimes H_F \)
of dimension \( d^N \). The point is that it is the topology
of the graph (by which we mean simply a specific distribu-
tion of the links, ignoring their weights and the states of
the vertices) which dictates how the available fundamen-
tal degrees of freedom are distributed between the fields
and the geometry. During a standard, “nonviolent” evolu-
tion, we might expect that the topology of the lattice
does not change, but as the black hole and singularity
form, significant changes of the topology happen, imply-
ing both topological and causal changes in the emergent
spatial geometry and possible deviations from standard
QFT on curved space-time in the following sense: the change of the
topology of the lattice means reshuffling of the fundamental
degrees of freedom between the geometry and the fields, so that the structure of the new graph
is \(H_G' \otimes H_F\); the fields now live in a Hilbert space \(H_F'\) of
different dimension than \(H_F\). In this case we have to expect the deviations from the unitary evolution on the
effective field side, although the underlying evolution of microsopic degrees of freedom is purely unitary.

We do not stick to this oversimplified picture in which the
weights of the links are related directly to the metric and the states of the vertices are related directly to
the states of the fields. We shall, however, stick to the idea that there are several ways how the fundamental
degrees of freedom are reshuffled between the fields and the geometry and, in addition, there might exist different
microscopic configurations which, on the effective level, give rise to the same coarse-grained geometry. On the
microscopic configurations which, on the effective level,
give rise to the same coarse-grained geometry. On the
microscopic configurations which, on the effective level,
give rise to the same coarse-grained geometry. On the
effective level it is impossible to distinguish between two
such microscopic configurations but, microscopically, the
two configurations differ by the number of degrees of free-
dom available for the fields. That is, the fields in the two
cases are elements of different Hilbert spaces and, hence, the
resulting field cannot be in a pure state.

A. Toy model

Hence, starting from the fundamental Hilbert space \(H\), we assume it can be split into a direct sum of the
subspaces \(T_{(i)}\),

\[
H = \bigoplus_{i=1}^{N_T} T_{(i)}, \quad \dim H = N_T N,
\]

where each \(T_{(i)}\) has a fixed dimension \(N\) and consists of states with some specific distribution of the degrees of freedom between the geometry and the fields; in the language of the simplistic “graph model”, \(T_{(i)}\) is a set of states for one specific choice of the topology of the graph
and hence we shall refer to \(T_{(i)}\) as the set of the states with specific topology; \(N_T\) is then the number of different
topologies. By assumption, each \(T_{(i)}\) has a structure

\[
T_{(i)} = H_{G(i)}^p \otimes H_{F(i)}^q, \quad p_i q_i = N,
\]

where \(H_{G(i)}^p\) (\(H_{F(i)}^q\)) is a Hilbert space of dimension \(p\) (\(q\)) representing possible microscopic states of the geometry (fields).

A general state \(|\psi\rangle \in H\) admits the expansion adapted to the splitting of \(H\) which is in the form

\[
|\psi\rangle = \bigoplus_{i=1}^{N_T} \sum_{q=0}^{q_i-1} \sum_{t=0}^{t_i} c_{t(i)} |I_t(i)\rangle \otimes |n_{i}\rangle,
\]

where vectors \(|I_t\rangle\) and \(|n_{i}\rangle\) form a basis of spaces \(H_{G(i)}^p\) and \(H_{F(i)}^q\), respectively.

Let us denote by \(P_{(i)} : H \mapsto T_{(i)}\) a projector onto the
subspace \(T_{(i)}\). Then, the squared norm of the state
\(P_{(i)}|\psi\rangle\) is the probability \(p_{(i)}\) of finding the system in the state with the topology \(T_{(i)}\),

\[
p_{(i)} = \|P_{(i)}|\psi\rangle\|^2.
\]

In general, state in \(T_{(i)}\) is a state with the entanglement between the geometry and the field in the sense that its
decomposition reads

\[
P_{(i)}|\psi\rangle = \sum_{I,n} c_{t(i)}|I_t(i)\rangle \otimes |n_{i}\rangle.
\]

Associated density matrix representing the state of the field is

\[
\rho_{(i)} = \text{Tr}_{H_G^p} |\psi\rangle \langle \psi|_i,
\]

where we first define the normalized state

\[
|\psi\rangle_i = \frac{1}{p_{(i)}} P_{(i)}|\psi\rangle
\]

and then trace over the degrees of freedom of the gravitational field. Corresponding entanglement entropy will be denoted by

\[
S_{(i)} = - \text{Tr}_{H_G^p} \rho_{(i)} \ln \rho_{(i)};
\]

\(S_{(i)}\) is the entanglement entropy between the geometry and the fields for a given topology of the lattice. Since
for the observer it is impossible to distinguish between different topologies of the lattice, expected value of the
entanglement between the fields and the geometrical degrees of freedom will be

\[
\langle S \rangle = \sum_{i} p_{(i)} S_{(i)}.
\]

In our toy model we shall assume that only two topologies are possible, i.e., \(N_T = 2\), and that both topologies
admit the same family of classical geometries, i.e., we assume

\[
P_G(T_{(1)}) = P_G(T_{(2)}) = H_G.
\]

Let us fix the number of degrees of freedom for each type of the lattice to \(N = 1500\) and let us set

\[
T_{(1)} = H_G^{300} \otimes H_F^{50}, \quad p_1 q_1 = 30 \times 50,\]

\[
T_{(2)} = H_G^{60} \otimes H_F^{25}, \quad p_2 q_2 = 60 \times 25;
\]

then we have \(\dim H = 3000\). Finally, assume that the maximal mass \(M_0\) of the black hole is split into \(N_{G}\) quanta. That is, for a black hole of mass \(M^{(a)} = a \varepsilon\) there is exactly one state in \(H_G^a\) such that the mapping \(P_G\) maps it to a state \(|\gamma^{(a)}\rangle\), while in \(H_F^a\) there are two
such states. Hence, the subspaces \(T_{(i)}\) are generated by the following bases:

\[
T_{(1)} = \text{span} \{|(a)\rangle \otimes |n\rangle\},
\]

\[
T_{(2)} = \text{span} \{|(a), 1\rangle \otimes |n\rangle, |(a), 2\rangle \otimes |n\rangle\},
\]
where
\[ P_G : |(a)\rangle \otimes |n\rangle \in H_G^{(30)} \otimes H_F^{(50)} \rightarrow |g^{(a)}\rangle \in H_G, \]
\[ : |(a), 1\rangle \otimes |n\rangle \in H_G^{(60)} \otimes H_F^{(25)} \rightarrow |g^{(a)}\rangle \in H_G, \]
\[ : |(a), 2\rangle \otimes |n\rangle \in H_G^{(60)} \otimes H_F^{(25)} \rightarrow |g^{(a)}\rangle \in H_G. \]  
(19)

We shall interpret elements \(|n\rangle \in H^g_F\) as the states of the quantum field with \(n\) particles.

A general state in \(H\) can be now written in the form
\[
|\psi\rangle = \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_1-1} \alpha_{an} |(a)\rangle \otimes |n\rangle +
+ \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_2-1} (\beta_{an} |(a), 1\rangle \otimes |n\rangle + \gamma_{an} |(a), 2\rangle \otimes |n\rangle),
\]  
(20)

and the expected value of the mass of the black hole is
\[
\langle M \rangle = \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_1-1} M^{(a)} |\alpha_{an}|^2 +
+ \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_2-1} M^{(a)} (|\beta_{an}|^2 + |\gamma_{an}|^2), \]  
(21)

and similarly for the expected number of particles \(\langle n \rangle\).

The probabilities for finding the lattice in the state with the topology \(T_1\) and \(T_2\), respectively, are
\[
P_1 = \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_1-1} |\alpha_{an}|^2, \]  
(22)
\[
P_2 = \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_2-1} (|\beta_{an}|^2 + |\gamma_{an}|^2). \]  
(23)

During the evolution (first formation of the black hole, then evaporation) the system evolves continuously and unitarily in \(H\). We start the analysis at the point where the black hole just formed and the field outside the black hole is in the ground state, i.e., in the state with zero particles. Then the evaporation starts which we mimic by prescribing the expected values of mass \(\langle M \rangle\) and the expected value of number of particles \(\langle n \rangle\). Since we do not know the underlying microscopic dynamics, we shall consider, similarly to Page, all states which are compatible with these expectation values.

Let us find a convenient parametrization of such states. First, since the general state \(|\psi\rangle\) must be normalized,
\[
\sum_{a=0}^{N_G-1} \sum_{n=0}^{q_1-1} |\alpha_{an}|^2 + \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_2-1} (|\beta_{an}|^2 + |\gamma_{an}|^2) = 1
\]  
(24)

we can set
\[
P_1 = \sum_{n=0}^{q_1-1} |\alpha_{an}|^2 = \cos^2 \theta, \]
\[
P_2 = \sum_{n=0}^{q_2-1} (|\beta_{an}|^2 + |\gamma_{an}|^2) = \sin^2 \theta, \]  
(25)

where \(\theta \in (0, \pi/2)\) without the loss of generality. Let us parametrize the coefficients by
\[
|\alpha_{an}| = \mu_{an} \cos \theta, \]
\[
|\beta_{an}| = \nu_{an} \sin \theta \cos \phi, \]
\[
|\gamma_{an}| = \lambda_{an} \sin \theta \sin \phi, \]  
(26)

where \(\phi \in (0, \pi/2)\). This is analogous to introducing the Hopf coordinates on the sphere \(S^n\). This parametrization implies
\[
\sum_{a=0}^{N_G-1} \sum_{n=0}^{q_1-1} \mu_{an}^2 = \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_2-1} \nu_{an}^2 = \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_2-1} \lambda_{an}^2 = 1. \]  
(27)

We also define
\[
\mu_M = \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_1-1} M^{(a)} \mu_{an}, \quad \mu_N = \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_2-1} n \mu_{an}^2 \]  
(28)

and similarly for \(\nu_M, \nu_N, \lambda_M, \lambda_N\). In this notation, the expected value of the mass of the black hole and the expected number of particles are given by
\[
\langle M \rangle = \mu_M \cos^2 \theta + \sin^2 \theta (\nu_M \cos^2 \phi + \lambda_M \sin^2 \phi),
\]
\[
\langle n \rangle = \mu_N \cos^2 \theta + \sin^2 \theta (\nu_N \cos^2 \phi + \lambda_N \sin^2 \phi). \]  
(29)

A generic state \(|\psi\rangle\) now acquires the form
\[
|\psi\rangle = \cos \theta \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_1-1} \mu_{an} e^{i \chi_{an}^\alpha} |(a)\rangle \otimes |n\rangle +
+ \sin \theta \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_2-1} \left( \nu_{an} e^{i \chi_{an}^\beta} \cos \phi |(a), 1\rangle \otimes |n\rangle +
+ \lambda_{an} e^{i \chi_{an}^\gamma} \sin \phi |(a), 2\rangle \otimes |n\rangle \right). \]  
(30)

The normalized projections of \(|\psi\rangle\) onto \(T_1\) and \(T_2\) correspond to the first and the second sum in \(\text{(30)}\), respectively, with factors \(\sin \theta \) and \(\cos \theta\) omitted:
\[
|\psi\rangle_1 = \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_1-1} \mu_{an} e^{i \chi_{an}^\alpha} |(a)\rangle \otimes |n\rangle, \]  
(31a)
\[
|\psi\rangle_2 = \cos \phi \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_2-1} \nu_{an} e^{i \chi_{an}^\beta} |(a), 1\rangle \otimes |n\rangle +
+ \sin \phi \sum_{a=0}^{N_G-1} \sum_{n=0}^{q_2-1} \lambda_{an} e^{i \chi_{an}^\gamma} |(a), 2\rangle \otimes |n\rangle. \]  
(31b)

Corresponding density matrices are
\[
\rho_{(i)} = \sum_{m,n=0}^{q_1-1} c_{nm}^{(i)} |n\rangle \langle m|, \quad i = 1, 2, \]  
(32)
Hence, we generate two random unit 3-vectors 
\[ \chi_{an}^x = e^{i\chi_{an}^x} \]
and 
\[ \chi_{an}^y = e^{i\chi_{an}^y} \]
normally with uniform distribution on the interval \((0, 2\pi)\). In this way we generate a random state yielding the prescribed expectation values \(\langle M \rangle\) and \(\langle n \rangle\); the entropy is then calculated by means of Eqs. (14) and (33). Then we calculate the average value of entanglement entropy from 5000 runs of this procedure.

Now consider the scenario of evaporation of the black hole. At the beginning, let the black hole have its maximal mass \(M_0 = (N_G - 1)\varepsilon\), where we choose \(N_G = 30\) and let there be vacuum outside the black hole. That does not necessarily mean that the Hilbert space for the field outside the black hole has dimension 1 (as it is in Page’s case); indeed, in our model we have chosen the dimension to be either 50 or 25, depending on the topology of the lattice. However, since there is only one vacuum state \(|0\rangle \in H_{\text{BH}}^{50}\) and only one vacuum state \(|0\rangle \in H_{\text{BH}}^{25}\) in both topologies, the field is disentangled from the geometry. This can be also seen from Eq. (31) which shows that for \(\langle n \rangle = 0\) we have
\[ \mu_N = \mu_N = \lambda_N = 0 \]
which, by (28), implies
\[ \mu_{an} = 0 \quad \text{for} \quad n > 0, \]
and similarly for \(\nu_{an}\) and \(\lambda_{an}\). Requirement \(\mu_M = M_0\) then implies that the only nonzero \(\mu_{an}\) is
\[ \mu_{(N_G-1),0} = 1. \]
Then both states \(|\psi_1\rangle\) and \(|\psi_2\rangle\) in (31) are unentangled and we have \(\langle S \rangle = 0\). Hence, our starting point coincides with the starting point of Page: expected entanglement vanishes at the beginning of the evaporation.

Now we assume that black hole starts to evaporate. We assume continuous unitary evolution of the state in \(\mathcal{H}\) but take the “snapshots” of the system when the expected values are
\[ \langle M \rangle = (N_G - 1 - k)\varepsilon, \quad \langle n \rangle = k, \]
where \(k\) acquires discrete values
\[ k = 0, 1, \ldots N_G - 1; \]
\(k = 0\) corresponds to black hole of maximal mass \(M_0 = M_0(N_G - 1)\varepsilon\) and vacuum outside the black hole; in \(k\)-th step, black hole already emitted \(k\) quanta of the field, so its mass decreased by the value \(k\varepsilon\), while the field is in the state with \(k\) quanta outside; black hole is fully evaporated for \(k = N_G - 1\) and the field is in the state with \(N_G - 1\) particles.

Notice that at the end of the evacuation, the state \(|\psi_1\rangle\) is disentangled again, but the state \(|\psi_2\rangle\) remains entangled. Hence, the expected value of the entanglement entropy decreases but remains nonzero.

In Fig. 2 we show the entanglement entropy as the function of the discrete parameter \(k\). Although this graph
V. CONCLUSIONS

The fact that the number of degrees of freedom that determine the state of a system in a compact volume \( V \) is bounded from above by the Bekenstein-Hawking entropy of a black hole with horizon area \( \partial V \), leads us to entertain the possibility that such fundamental degrees of freedom should describe the state of both fields and geometry contained in said volume. The immediate consequence of such statement is that fields and geometry should be regarded as emergent phenomena at ordinary energy scales. In this picture the particles of the Standard Model are regarded as analogous to quasiparticles, arising together with the classical background geometry from the interactions between the fundamental degrees of freedom. We have not provided here a framework for such dynamical emergence (see [12–15, 17–23] for some examples along these lines); however we assume that the unitary evolution – which is a central requirement of the emergent quantum theory – is preserved down to the fundamental level. The unitary evolution inevitably leads to a reshuffling of the fundamental degrees of freedom and this is reflected on the emergent level as an entanglement between quantum fields and geometry. In order to investigate the consequences of such scenario, we provided here a kinematical framework that allows us to address the longstanding information-loss paradox in the context of black hole evaporation. Through a simple toy model of evaporation it is shown how the entanglement between fields and geometry can lead, after the evaporation is completed, to an average loss of the initial information. We claim that such modification of the original Page curve should be regarded as a common feature of any theory of quantum gravity in which both the space-time geometry and the quantum fields propagating on it are emergent features of an underlying fundamental and unitary theory.

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\[ S_{\text{final}} \]

starts at the point \((M_0, 0)\) which corresponds to the same origin of the Page curve in Fig. 1 at the final stage of the evaporation the entanglement entropy does not go to zero; at this point we differ from the prediction of the Page curve. It is clear that allowing for more microscopic realizations of the same effective geometry, i.e., more topologies, would in general increase the final deviation of \( \langle S \rangle \) from the pure state value.

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In fact, the concepts of elementary and collective excitations are interchangeable in theories where electromagnetic duality is at play [59, 60]. Actually, inspired by that fact that different arrangements of the carbon atoms can give rise to the same emergent spacetime geometry, in our model we take into account the possibility that the same emergent geometry can be realized through different arrangements of the fundamental degrees of freedom. These microscopic arrangements are indistinguishable at our (low) energy level.

In fact, in the case of graphene, geometries can indeed be seen as emergent [61, 62].