Light Propagating in a Born–Infeld Background as Seen by an Accelerated Observer

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The propagation of light in the Born–Infeld (BI) background as seen by an accelerated observer is examined. In a Born–Infeld electromagnetic field, light trajectories are governed by the null geodesics of the effective optical metric. The accelerated observer is in a Rindler frame, a situation that, according to the Einstein equivalence principle, is equivalent to being in a uniform gravitational field. The phase velocity of light propagating through a purely magnetic or electric BI background is determined as measured by the Rindler observer. The BI field and the acceleration of the frame have opposite effects on the propagating light; while the intense electric or magnetic BI background decreases the velocity of light, in the accelerated frame light may exceed its velocity in vacuum. Light propagating parallel or transversal to the acceleration direction of the Rindler frame is considered. The redshift of light pulses sent from one Rindler observer to another in the BI background is also calculated.

1. Introduction

In the presence of intense electromagnetic fields, quantum electrodynamics (QED) predicts that vacuum has the properties of a material medium as a consequence of self-interactions in the electromagnetic field. Nonlinear electromagnetic (NLEM) effects such as light–light interaction or pair production from vacuum excited by an electric field then arise. These effects become significant when the electromagnetic field strengths approach $F_{\mu \nu} \approx m_e^2 c^2/\hbar e \approx 10^{18} \text{ V m}^{-1}$ or $B_\mu \approx 10^7 \text{ Tesla}$; $B_\mu$ represents the field at which the cyclotron energy equals $m_e c^2$, and defines the field scale at which the impact of the external field on quantum processes becomes significant.

These NLEM interactions can be described in an effective way by Lagrangians $\mathcal{L}_{\text{NLED}}(F, G)$ that depend nonlinearly on the two Lorentz and gauge invariants of the Faraday tensor $F_{\mu \nu}$, $F = F^{\mu \nu} F_{\mu \nu} = 2(B^2 - E^2)$, and $G = F^{\mu \nu} F_{\mu \nu} = -4 \mathbf{B} \cdot \mathbf{E}$. Several theories exist to describe such NLEM proposals, but two stand out: the Euler–Heisenberg (EH) and the Born–Infeld (BI) theories. The EH theory was derived from QED principles by W. Heisenberg and H. Euler in 1936;[1] for a discussion on the history of the Euler–Heisenberg approach see ref. [2], and a pedagogical review can be found in ref. [3]. By treating the vacuum as a medium, EH effective action predicts rates of nonlinear light interaction processes since it takes into account vacuum polarization to one loop and is valid for electromagnetic fields that change slowly compared to the inverse electron mass.

The other relevant NLED theory was developed by M. Born and L. Infeld. Born and Infeld (1934)[4] presented a theory with nonlinear corrections to Maxwell electrodynamics but from a classical perspective. The aim was to contribute to discussion on the nature of the electromagnetic mass of charged particles; at that time opinion was divided between as to whether the mass of a charged particle is a manifestation of the electromagnetic field or if field and particle are two separate entities. A second aim was to solve the singularity of the field and energy of a point charge at its position, proposing the existence of a maximum attainable electromagnetic field, given by the BI parameter $b$, with a magnitude $b = e/r_0^2 = 10^{18} \text{ V m}^{-1}$, where $r_0$ is the classical electron radius. This is a classical theory that effectively models vacuum polarization as a material medium, in this sense resembling EH theory. Another interesting feature is that it presents neither birefringence nor shock waves. The BI Lagrangian is given by

$$\mathcal{L}_{\text{BI}}(F, G) = -4b^2 \left( 1 - \sqrt{1 + \frac{F}{2b^2} - \frac{G^2}{16b^4}} \right)$$

(1)

The linear electromagnetic Maxwell theory is recovered in the limit that $b \to \infty$, then $\mathcal{L}_{\text{Maxwell}}(F) = F$.

Efforts are currently in progress for measuring some of the NLED effects, a few of which are listed below. Light-by-light interactions can be studied using heavy-ion collisions. The electromagnetic (EM) field strengths produced, for example by a Pb nucleus, are up to $10^{25} \text{ V m}^{-1}$. These intense EM fields can be treated as a beam of quasi-real photons, and light-by-light scattering has been measured in Pb + Pb collisions at the large Hadron Collider.[5] Other experimental evidence includes the measurement of photon splitting in strong magnetic fields,[6] the search for vacuum polarization with laser beams crossing magnetic fields, and the detection of vacuum birefringence with intense laser pulses.[7] QED vacuum nonlinearity has also been detected using waveguides.[8] Vacuum pair production, known as the Sauter–Schwinger effect,[9] was predicted in the EH 1936
paper, however, the necessary electric field strengths, corresponding to a critical laser intensity of about \( I_p = 4.3 \times 10^{20} \text{W cm}^{-2} \)\(^{[10]} \) have not yet been reached experimentally. Strong magnetic fields are also of interest in astrophysics; as neutron stars can possess magnetic fields in the range of \( 10^6 - 10^{15} \text{T} \), processes such as photon splitting and pair conversion are expected to occur in their vicinity\(^{[11,12]} \).

On the other hand, it is well known that an electromagnetic wave traveling through intense EM fields has a reduced phase velocity due to vacuum polarization. This subject has been addressed since the 1970s in the literature\(^{[13,14]} \) as well as recently; see for instance\(^{[15,16]} \) for the EH theory, and ref.\(^{[17]} \) for wave propagation in a BI background. In ref.\(^{[18]} \), the use of a Michelson–Morley interferometer is proposed for measuring the changes in the phase velocity due to NLEM theories, specifically the Born–Infeld theory. They concluded that for a BI parameter of the order \( b \approx 10^{20} \text{V m}^{-1} \), the intensity of the background fields in question will not be able to be experimentally attained in the near future. Instead, using realistic intensities of background fields \( (B \approx 1 \text{T}) \), interferometric experiments could place bounds on the BI parameter \( b \) to an order of \( 10^{14} - 10^{15} \text{V m}^{-1} \).

In this paper, we examine the propagation of an electromagnetic wave in the BI-NLED background as seen by an accelerated observer. This setup is of interest because, according to the equivalence principle, an accelerated frame is equivalent to a uniform gravitational field, and we are always under the influence of such a gravitational environment. A light ray moving in such an accelerated frame will modify its velocity and pulses will be redshifted. We determine the phase velocity of light, its expression showing the interplay of the magnetic (electric) background and the acceleration of the frame. In the limit of zero acceleration, we recover the wave propagating in the BI background, and in the absence of BI field, we recover the Rindler propagation and the corresponding frequency shifts.

The paper is organized as follows: In the next section we present the effective optical metric, whose null geodesics are the light trajectories, as well as the phase velocity for a BI electromagnetic background. In Section 3 we present the Rindler spacetime and the coordinate transformation that connects the Minkowski metric with the accelerated frame. In Section 4 we derive the phase velocity of light propagating through a purely magnetic BI background from the point of view of the accelerated observer (uniform gravitational field). In Section 5 we analyze the phase velocity of light propagating in a purely electric BI background from the point of view of the Rindler observer. In both cases we consider light propagating parallel and perpendicular to the accelerated frame. The limits of zero acceleration and vanishing BI electromagnetic field are also presented. Section 6 is devoted to determining the redshift of the propagating light, and how the presence of the BI electromagnetic field affects the frequency shift. Finally, the conclusions are presented in Section 7.

### 2. Effective Optical Metric and Phase Velocity of Light in a BI Background

It is well known that intense EM fields, where the Maxwell theory is no longer valid, can resemble a curved spacetime, in the sense that light trajectories are not straight lines but undergo deflection. Deviations from the straight trajectories in vacuum are described in NLED by the null trajectories of an effective optical metric. The effective optical metric is obtained from the analysis of the propagation of discontinuities or perturbations of the EM field\(^{[19-21]} \) that is, according to this formalism the EM fields of the propagating wave are much smaller than the background fields. The effective optical metric approach turns out to be equivalent to the soft photon approximation. Splitting the total electromagnetic field into a background field \( F_{\mu \nu} \) and a propagating photon \( f_{\mu \nu} \), and keeping the linear approximation with respect to \( f_{\mu \nu} \) in the equations of motion, leads to an eigenvalue equation for the propagating modes\(^{[15,22]} \).

Considering that \( k_j \) is a null vector normal to the characteristic surface of the wave, the effective optical metric \( g_{ij}^{\text{eff}} \) is given by

\[
g_{ij}^{(\text{eff})} k_i k_j = 0, \quad i, j = 1, 2 \tag{2}
\]

where the (i) superscript corresponds to the two metrics that can arise in NLED, when the phenomenon of birefringence occurs. The equations of the propagation of the field discontinuities in nonlinear electrodynamics characterized by a Lagrangian \( \mathcal{L}(F, G) \) are derived by analyzing the propagation of linear waves associated with the discontinuity of the field in the limit of geometrical optics\(^{[21]} \) from the pair of coupled equations

\[
\zeta k^2 = \frac{4}{L_p} F^{\mu \nu} F_{\mu \nu} k_i k_j (L_{FG} \zeta + L_{GG} \zeta^*) - G_{\mu \nu} k^2 (L_{FG} \zeta + L_{GG} \zeta^*) \tag{3}
\]

\[
\zeta^* k^2 = \frac{4}{L_p} F^{\mu \nu} F_{\mu \nu} k_i k_j (L_{FG} \zeta^* + L_{GG} \zeta) - G_{\mu \nu} k^2 (L_{FG} \zeta^* + L_{GG} \zeta) + 2 \frac{F}{L_p} k^2 (L_{FG} \zeta + L_{GG} \zeta^*) \tag{4}
\]

where \( k^2 = n^{\mu \nu} k_\mu k_\nu \). Equations (3) and (4) were also presented in ref.\(^{[20]} \). For a NLED Lagrangian if we turn off either the electric or the magnetic field, that is, \( E = 0 \) or \( B = 0 \), then \( G = 4 \) \( E \cdot B = 0 \). If additionally the Lagrangian is such that \( \mathcal{L}_{FG} = 0 \), then the effective metrics in Equations (3) and (4) become

\[
k^2 = \frac{4}{L_p} F^{\mu \nu} F_{\mu \nu} k_i k_j L_{FF} \rightarrow k_i k_j (L_{FG} n^{\mu \nu} - 4 L_{FF} F_{\mu \nu} F^{\mu \nu}) = 0 \tag{5}
\]

\[
k^2 = \frac{4}{L_p} F^{\mu \nu} F_{\mu \nu} k_i k_j L_{GG} + 2 \frac{F}{L_p} k^2 L_{FG} \rightarrow k_i k_j ((\mathcal{L}_F - 2 L_{GG} F) n^{\mu \nu} - 4 L_{GG} F_{\mu \nu} F^{\mu \nu}) = 0 \tag{6}
\]

defining the effective metrics for a Lagrangian \( \mathcal{L} = \mathcal{L}(F) \) as

\[
g_{\mu \nu}^{(\text{eff})} = (\mathcal{L}_F - 2 L_{GG} F) n^{\mu \nu} - 4 L_{GG} F_{\mu \nu} F^{\mu \nu} \tag{7}
\]

\[
g_{\mu \nu}^{(\text{eff})} = \mathcal{L}_F n^{\mu \nu} - 4 L_{FF} F_{\mu \nu} F^{\mu \nu} \tag{8}
\]

where the subscript in the Lagrangian indicates derivative respect to that invariant, \( \mathcal{L}_x = d\mathcal{L}/dX \), and \( n^{\mu \nu} = \text{diag}[+1, -1, -1, -1] \) is the Minkowski metric. In the Maxwell case \( \mathcal{L} = F \), \( \mathcal{L}_F = 1 \), \( L_{FF} = 0 \) and \( L_{GG} = 0 \), then both effective metrics become conformal to the Minkowski metric, \( g_{\mu \nu}^{(\text{eff})} = g_{\mu \nu}^{(\text{Mink})} = n^{\mu \nu} \), and the null geodesics coincide with the light trajectories in the Minkowski spacetime.
See ref. [23] for a study on the Fresnel equation in nonlinear electrodynamics and ref. [24] for a classification of the effective metrics. The Born–Infeld non linear theory is characterized by the non existence of birefringence, meaning that the two metrics in Equations (7) and (8) become conformal, \( g^{\mu \nu}_{\text{eff}} = \Omega^2 g^{\mu \nu}_{\text{Mink}} \). We know that a conformal factor does not alter null geodesics. Therefore, in what follows we shall omit the superscript \((i)\), \( i = 1, 2 \).

For the BI case, the effective optical metric is given by

\[
g^{\mu \nu}_{\text{eff}} = \left( \frac{b^2 + F}{2} \right) \eta^{\mu \nu} + F^{\mu \nu} \delta^{\mu \nu}
\]

where the EM invariant \( F = 2(B^2 - E^2) \).

For a light ray propagating along the \( j \)-direction, \( j = x, y, z \), with wave frequency \( \omega \) and wave number \( k^j = k \), its phase velocity can be obtained from Equation (2), that is a dispersion relation, as

\[
g^{\mu \nu}_{\text{eff}} \omega^2 + 2g^{\mu j}_{\text{eff}} \omega k^j + g^{ij}_{\text{eff}} k^i k^j = 0
\]

\[
\nu_{\text{ph}} = \frac{\omega}{k} = \frac{\omega}{b_{\text{eff}}} \pm \sqrt{\left( \frac{b_{\text{eff}}}{a_{\text{eff}}} \right)^2 - \left( \frac{g^{ij}_{\text{eff}}}{b_{\text{eff}}} \right)^2}, \quad j = x, y, z
\]

Since our goal is to study the propagation of light as seen by an accelerated observer, for completeness, we introduce the accelerated frame or Rindler spacetime and some of its relevant features in the next section.

3. The Accelerated Frame or the Rindler Spacetime

We are interested in the characteristics of BI electromagnetic fields as seen by observers in a uniform gravitational field. According to the Einstein equivalence principle (EEP), a gravitational field can be (locally) modeled by an accelerated frame. An example of a "fake" gravity or accelerated frame is the Rindler space, which we briefly address in this section.

Let us consider an accelerated observer with constant acceleration \( a = \sqrt{a_i a_i} \) in the \( z \)-direction, and proper time \( \tau \). The Minkowski coordinates \((t, z)\) are related to the accelerated observer by

\[
t = \frac{1}{a} \text{Sh}(a \tau); \quad z = \frac{1}{a} \text{Ch}(a \tau)
\]

where we denote \( \cosh(x) = \text{Ch}(x) \) and \( \sinh(x) = \text{Sh}(x) \). The line element in Minkowski coordinates is related to the Rindler space by ref. [25]

\[
ds^2 = dt^2 - dx^2 - dy^2 - dz^2 = (1 + aZ)^2 dT^2 - dZ^2 - dx^2 - dy^2
\]

From this expression we see that the coordinate transformation does not cover the entire Minkowski spacetime. At \( Z = \frac{1}{a} \) the metric becomes degenerated, and therefore the range of the coordinates must be \( Z \in (-\infty, 0) \), \( T \in (-\infty, \infty) \). These coordinates cover only a part of the Minkowski spacetime, denoted as region I in Figure 1, with the different regions separated by event horizons located at \( z = \pm t \).

Equation (12) represents a hyperbolic curve in Minkowski spacetime, with the semimajor axis at \( 1/a \),

\[
z^2 - t^2 = \frac{1}{a^2}
\]

The accelerated observer’s trajectory is a hyperbola as shown in Figure 1. It starts at \( z \to \infty \), then slows down as it approaches the origin, returns at a finite distance from the origin and speeds up as it approaches \( z \to \infty \). Light rays are shown traveling along 45° null lines. The observer can “send signals” toward the upper quadrant II, as represented by the cone A, but the observer in quadrant I cannot receive signals from the upper quadrant II. The observer in the Rindler frame cannot access any signal beyond the horizon, located at \( z = \pm t \), but can receive signals from the inferior quadrant IV, as illustrated with the light cone D; however, A cannot send signals toward D [23].

To determine what the accelerated observer sees, we need to carry out a coordinate transformation from the Minkowski frame, which can also be termed the lab frame, to the accelerated observer’s frame. In the accelerated frame we will describe the phenomena from the point of view of a privileged observer at \( Z = 0 \) with proper time \( \tau = T \), but the general transformation from Minkowski to a Rindler observer located at \( Z_i \), with acceleration \( a_i \), is given by refs. [28, 29]

\[
t_i = \frac{1}{a_i} + Z_i \text{Sh}(a_i T_i); \quad z_i = \left( \frac{1}{a_i} + Z_i \right) \text{Ch}(a_i T_i)
\]

Note that taking \( Z = 0 \) and \( T = \tau \) results in Equation (12).

Other Rindler observers (other hyperbolas) have different accelerations \( a_i \), and different \( z_i \) (lab coordinates). In the Rindler frame, they have different \( Z_i \), but they all have the same velocity when their world lines intersect a line of simultaneity, as shown in Figure 2, where the lines of simultaneity are the dot-
In Rindler space—other at rest, then it is said that they form a rigid lattice of the 4-velocity parallel to each other, and they measure each frame so that, along the line of simultaneity, both frames have accelerated frame there is a momentarily co-moving inertial theydointheRindlerframe.

in the Lab frame, these observers do not form a rigid lattice, as does not imply that all the observers have the same acceleration; the Rindler space as a “Uniformly accelerated reference frame” different proper acceleration in the Lab frame. Then referring to $v$ the coordinate velocity of a Rindler observer is

$$v = \frac{\partial x^i}{\partial \tau}$$

For the null trajectories, $d\tau^2 = 0$

$$d\tau^2 = g_{TT} dT^2 + g_{ij} dx^i dx^j = 0$$

and the phase velocity for a light ray traveling in the $z$-direction in the Rindler frame is

$$v_{ph} = \frac{dZ}{d\tau} = \sqrt{-\frac{g_{TT}}{g_{zz}}} = (1 + aZ)$$

If we interpret $d\tau/dT$ as the rate of flow of time\cite{28} in the Rindler space, then the speed of light equals the rate of flow of time. The velocity now depends on the position of the observer; it is zero at the horizon $Z = -\frac{1}{a}$ and when $Z = 0$ the velocity is that in vacuum, $c = 1$. The position $Z = 0$ corresponds to the “principal observer”. From Equation (19) we see that an observer at $Z > 0$, measures a light velocity greater than $c = 1$. This result is explained using the global line of simultaneity that is rotating in the Lab frame, like a radar. When it intersects the worldlines it moves faster for an observer at higher $z$ (Lab frame) than for an observer closer to the origin. The interpretation is that time flows faster for higher $z$ because the proper time of the observers at greater $z$ is aging faster than for observers closer to the origin. Light therefore covers more distance per unit of time as $z$ increases. However, each accelerated observer measures a local speed of light being $c = 1$, therefore any of them could be chosen as the “master” or “principal” observer.

We can visualize the trajectory of light in Rindler spacetime integrating Equation (19).

$$Z = \frac{1}{a} \left[ -1 + (1 + aZ_0) \exp a(T - T_0) \right]$$

When $Z \to -\frac{1}{a}$, the velocity tends to zero, as expected. Then, the measurement of light velocity must be done locally, that is, at $Z = 0$, the position of the “master” observer.

Figure 2. The worldlines of two observers A and B in the Minkowski spacetime are shown as the gray and black thick lines in the figure to the left. The dotted lines represent the lines of simultaneity in the Rindler frame. To the left are illustrated the observers in the Minkowski spacetime, and to the right are the trajectories in the Rindler frame. We are considering A as the principal observer, whose trajectory is a vertical straight line (fixed $Z$); the dot at $(Z = -\frac{1}{a}, T = 0)$ is the Lab frame; the observer B is represented by the black thick line; the blue curve corresponds to a third Rindler observer, located between $Z = 0$ and $Z = 1/a_A$. 

The proper position of each observer is $Z_i = 0$ and \{ $t = Sh(a_i T)/a_i, Ch(a_i T)/a_i$\}, resulting in each observer having a different proper acceleration in the Lab frame. Then referring to the Rindler space as a “Uniformly accelerated reference frame” does not imply that all the observers have the same acceleration; in the Lab frame, these observers do not form a rigid lattice, as they do in the Rindler frame.

To define simultaneity, we consider that for each uniformly accelerated frame there is a momentarily co-moving inertial frame so that, along the line of simultaneity, both frames have the 4-velocity parallel to each other, and they measure each other at rest, then it is said that they form a rigid lattice of observers who all agree on simultaneity\cite{28}. In Rindler spacetime the lines that define the simultaneity are $T = n = constant$ (shown in Figure 2), and correspond to $t = z Th(an)$ in Minkowski spacetime.

Let $\tau_A$ be the proper time of the principal observer, which dictates what is measured by the other observer’s clocks. When $\tau_A = 0$ all the observers coincide with the $z$-axis: $T = 0, t = 0$. Let us distinguish the proper time $\tau$ and the coordinate time $T$. The Rindler line element is given by

$$ds^2 = g_{TT} dT^2 + g_{ij} dx^i dx^j, \quad i, j = \{X, Y, Z\}$$

The differential proper time $d\tau$ and the spatial line element $dl^2 = g_{ij} dx^i dx^j$, are such that $dl^2 = -d\tau^2 \mid_{\tau = 0}$ and the coordinate velocity of a Rindler observer is

$$v^2 = -\frac{g_{ij} dx^i dx^j}{d\tau^2}$$

where $v^2$ is non negative since we are taking the Minkowski metric as $\eta_{\mu\nu} = \text{diag}[+1, -1, -1, -1]$. 

$\eta_{\mu\nu}$

$\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$

$\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$

$\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}$
In the following two sections, Sections 4 and 5, we address the propagation of light in purely magnetic and purely electric BI backgrounds, respectively, as seen in the Rindler frame.

4. Light Propagating through a Born–Infeld Magnetic Background in the Rindler Space

In this section, we determine the phase velocity of light rays propagating in a purely magnetic BI background as seen by an accelerated observer. According to the EEP, an accelerated frame is equivalent to a uniform gravitational field; therefore, our treatment describes the propagation of light under the influence of a very intense BI electric or magnetic background, as seen by an observer in a uniform gravitational field.

Light trajectories through an intense magnetic field are the null geodesics of the effective optical metric $\tilde{g}_{\mu\nu}^{\text{eff}}$, Equation (9). We begin with the effective optical metric corresponding to a BI uniform magnetic field $B$ and then transform it into the Rindler accelerated frame.

Considering that the nonvanishing electromagnetic tensor components of the magnetic background are $F_{\mu\nu} = -B_y$, the transformed effective optical metric, obtained from $\tilde{g}_{\mu\nu}^{\text{eff}} = R_{\mu\nu} g_{\alpha\alpha}^{\text{eff}} R^{\alpha\beta}$, is

\[
\tilde{g}_{\mu\nu}^{\text{Rindler}} = \begin{pmatrix}
\frac{(aZ + 1)^2}{aZ + 1} & -\frac{aZ + 1}{aZ + 1} & \frac{aZ + 1}{aZ + 1} & 0 \\
-\frac{aZ + 1}{aZ + 1} & B_y Sh(aT) & B_x Sh(aT) & -(b^2 + B_y^2) \\
-\frac{aZ + 1}{aZ + 1} & B_x Sh(aT) & B_y Sh(aT) & -(b^2 + B_x^2) \\
0 & -(b^2 + B_y^2) & -(b^2 + B_x^2) & aT \\
\end{pmatrix}
\]

where the superscript “$R$” denotes the effective optical metric $\tilde{g}_{\mu\nu}^{\text{Rindler}}$ in the Rindler frame and $B^2 = B_x^2 + B_y^2$. In the case of $a = 0$, the effective optical metric Equation (21) is recovered. As a consequence of the transformation to the magnetic background, electric components arise in a similar way to a Lorentz boost. The nonvanishing electromagnetic tensor components are now $F^x_{\alpha\beta} = \frac{B_y Sh(aT)}{1 + az}$, $F^y_{\alpha\beta} = \frac{B_x Sh(aT)}{1 + az}$, $F^z_{\alpha\beta} = B_x Ch(aT)$, $F^{\text{av}}_{\alpha\beta} = -B_y Ch(aT)$, $F^{\text{av}}_{\alpha\beta} = B_x Ch(aT)$. The invariants are preserved, so $F = 2B^2$, $G = 0$.

The corresponding covariant effective optical metric is

\[
\tilde{g}_{\mu\nu}^{\text{eff,R}} = \begin{pmatrix}
(aZ + 1)[b^2 - B_y^2 Sh^2(aT)] & B_x B_y Sh(aT) & B_y B_x Sh(aT) & -\frac{1}{2} B_y^2 Sh(2aT) \\
B_x B_y Sh(aT) & \frac{b^2 + B_y^2}{aZ + 1} & \frac{b^2 + B_y^2}{aZ + 1} & -\frac{B_y^2}{aZ + 1} Sh(aT) \\
B_y B_x Sh(aT) & \frac{b^2 + B_y^2}{aZ + 1} & \frac{b^2 + B_y^2}{aZ + 1} & -\frac{B_y^2}{aZ + 1} Sh(aT) \\
-\frac{1}{2} b^2 Sh(2aT) & \frac{b^2 + B_y^2}{aZ + 1} & \frac{b^2 + B_y^2}{aZ + 1} & \frac{b^2 + B_y^2}{aZ + 1} \\
\end{pmatrix}
\]

$B = B_x \hat{x} + B_y \hat{y} = B \sin \theta \hat{x} + B \cos \theta \hat{z}$. From Equation (11), the phase velocity $v_{\phi\beta}$ of light propagating through a BI magnetic background, along the $\hat{z}$-direction, with wave frequency $\omega$ and wave number $k^2 = k^2$, is

\[
v_{\phi\beta} = 1 - \frac{B^2}{b^2 + B^2}
\]

where $B^2 = B_x^2 + B_y^2$. Then the effect of the BI magnetic background is slowing down the phase velocity, unless the magnetic component transversal to the propagating light is zero, in which case the phase velocity is that of light in vacuum. This case was studied in ref. [17].

To determine the phase velocity measured by a Rindler observer, the effective optical metric Equation (21) is transformed to the Rindler frame. The transformation matrix to transform to a Rindler frame with acceleration $\ddot{a} = a \hat{z}$ is given by

\[
R_{\mu\nu} = \frac{\partial x^\mu}{\partial X^\nu} = \begin{pmatrix}
1 & (1 + aZ) Ch(aT) & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
(1 + aZ) Sh(aT) & 0 & 0 & Ch(aT) \\
\end{pmatrix}
\]

The wave number $k^2 = k^2$, is

\[
\nu_{\phi\beta} = 1 - \frac{B^2}{b^2 + B^2}
\]
conventionally three different propagating directions of the wave: \( \pm \hat{z}, -\hat{z} \), and \( +\hat{x} \); the setting is shown in Figure 3. We analyze the corresponding phase velocities in the next subsections.

### 4.1. Magnetic BI Background: Light Propagating in the \( \pm \hat{z} \) Direction

For a wave traveling in the \( \pm \hat{z} \) direction: \( \mu = (\omega, 0, 0, \mp k) \) the phase velocity, from Equation (11), is given by

\[
\frac{v^\mu_{\text{ph}} \cdot \hat{z}}{1 + aZ} = \frac{\omega}{k} = \frac{\pi B_y Sh(aT)Ch(aT) + (b^2 + B^2)(b^2 + B^2)}{(b^2 + B^2 + B_\perp^2 Sh^2(aT))}
\]

(26)

where the \( \pi \) sign corresponds to the waves propagating in \( +\hat{z} \) and in \( -\hat{z} \) directions, respectively, and \( B^2 = B_x^2 + B_y^2 \). From this expression we can see that \( v^\mu_{\text{ph}} \) depends on \( aT \) and \( aZ \), that is, the effect of increasing time is equivalent to increasing the acceleration of the frame, recalling that the Rindler observer will remain at a fixed \( Z \). In addition, the acceleration affects the phase velocity only if there is a magnetic component that is transversal to the acceleration (in this case \( B_\perp \)). In the limit, \( a \to 0 \), the expression for the pure BI magnetic field Equation (22) is recovered. Note that the only difference between the wave propagating along \( +\hat{z} \) and \( -\hat{z} \) is the sign in the first term in Equation (26), which indicates that the Rindler frame distinguishes the travel direction of light. Moreover, it implies that the phase velocity for the wave along \( -\hat{z} \) will be greater than for the one traveling in the opposite direction.

Expanding \( v^\mu_{\text{ph}} \) Equation (26) in powers of \( b^2/b^2 \) and keeping terms of order \( O(1/b^2) \), that is, neglecting terms of order \( O(1/b^4) \) and higher, we arrive at

\[
\frac{v^\mu_{\text{ph}} \cdot \hat{z}}{1 + aZ} = 1 - \frac{b^2}{2b^2} \left( 1 - 2Sh(aT)Ch(aT) \right) \]

(27)

This approximation is still valid for very strong fields. If we consider, for instance, that \( B^2/b^2 \approx 10^{-2} \), then the BI field \( B \) is of the order \( 10^{19} \) V m\(^{-1} \), that is ten times the critical Schwinger field \( B_{cr} \approx 10^9 \) Tesla. In the limit, \( a \to 0 \), the expression for the pure BI magnetic field Equation (22) is recovered, up to terms in \( O(1/b^2) \). Notice that even if \( B \to b \), the phase velocity reaches a minimum of half the velocity of light in vacuum. The linear limit (no BI field) is obtained with \( b \to \infty \) and then \( (v^\mu_{\text{ph}})^2 = (1 + aZ)^2 \) is recovered.

In Equation (27), the term that depends on \( aT \) increases monotonically \((Sh(aT)Sh(aT) + Ch(aT))\), for the wave traveling in \( +\hat{z} \), while for the wave traveling in \( -\hat{z} \), the corresponding term, \( Sh(aT)[Sh(aT) - Ch(aT)] \), decreases monotonically up to \(-1/2\). The effect of the accelerating frame in one case \(( +\hat{z}) \) therefore contributes to slowing down the wave and in the other \((-\hat{z}) \) it increases the phase velocity. In the latter case the Rindler effect is opposite to the BI effect. Therefore, for a Rindler observer accelerated in \( +\hat{z} \) direction the phase velocity of the wave propagating in the \( +\hat{z} \) direction is smaller than that of the wave along \(-\hat{z} \). Intuitively, this corresponds to traveling in the same direction of the wave in the first case, and in the opposite direction to the wave propagation in the second case. This effect is superposed with the BI slowing down of the wave and the result is illustrated in Figures 4 and 5. As the intensity of the background field increases the light velocity diminishes, and by turning off the BI background field the light velocity in vacuum is recovered.

Contrary to what happens for the wave propagating in \( +\hat{z} \) direction, for the wave propagating in \(-\hat{z} \) direction the pace of slowing down decreases as \( aT \) increases. In the limit of long times or large acceleration, \( aT \to \infty \), the BI effect of slowing down is lost and the Rindler phase velocity \( (v^\mu_{\text{ph}})^2 = (1 + aZ)^2 \) is recovered.

### 4.2. Magnetic BI Background: Light Propagating in the \( +\hat{x} \) Direction

For a wave moving in the \( \hat{x} \) direction with wave number \( \kappa = (\omega, -k_y, 0, 0) \), the magnetic background component along \( \hat{x} \), \( B_x \), is parallel to the propagating ray and perpendicular to the Rindler acceleration, while \( B_\perp \) is perpendicular to the light direction. The phase velocity, from Equation (11), is given by

\[
\frac{v^\kappa_{\text{ph}} \cdot \hat{x}}{1 + aZ} = \frac{B_y Sh(aT) + (b^2 + B^2)(b^2 + B_\perp^2 Ch^2(aT))}{b^2 + B^2 + B_\perp^2 Sh^2(aT)}
\]

(28)

In this case the effect of turning off the \( B_x \) component eliminates the terms depending on \( aT \), since the only significative magnetic field is the one transversal to the acceleration of the frame.

Making \( a = 0 \) we obtain

\[
\frac{v^\kappa_{\text{ph}} \cdot \hat{x}}{1 + aZ} = \sqrt{1 - \frac{B_\perp^2}{b^2 + B^2}}
\]

(29)

that corresponds to the pure BI effect of slowing down \( v^\mu_{\text{ph}} \). Expanding \( v^\kappa_{\text{ph}} \) in powers of \( b^2/b^2 \) and keeping terms of order...
Figure 4. The plots for the squared phase velocity of a light ray propagating in \( z \)-direction through a BI magnetic background as seen by an observer with acceleration \( a \) in the \( \hat{z} \)-direction. The plots are for different values of \( aT \); as \( aT \) increases, the velocity slows down more rapidly. The intensities of the perpendicular magnetic components are varied as shown and \( B^2 = B_x^2 + B_z^2 \).

Figure 5. Phase velocities for a wave moving in the \( -\hat{z} \) direction for different values of \( aT \); \( v_{\text{ph}} \) is smaller as the magnetic field increases. As \( aT \) increases, the velocity slows down at a slower pace. The intensities of the perpendicular magnetic components are varied as shown and \( B^2 = B_x^2 + B_z^2 \).

\[ O(1/b^4) \] that is, neglecting terms of order \( O(1/b^4) \) and higher, we arrive at

\[
\frac{v_{\text{ph}}^{R\hat{x}}}{(1 + aZ)} = 1 - \frac{1}{2b^2} [B_x - B_z \text{Sh}(aT)]^2 \tag{30}
\]

The linear limit, \( b \to \infty \), gives the Rindler velocity, \( (v_{\text{ph}}^{R\hat{x}})^2 = (1 + aZ)^2 \). In this case the acceleration of the frame gives an effect opposite to the BI slowing down but only for a certain range of \( (aT) \). The minimum value of Equation (30) occurs for

\[
aT = \text{ArcSinh} \left( \frac{B_x}{B_z} \right) \tag{31}
\]

For \( aT \) larger than the one of the minimum, \( aT > \text{ArcSinh} \left( \frac{B_x}{B_z} \right) \), \( v_{\text{ph}}^{R\hat{x}} \) decreases such that in the limit of large \( aT \), \( v_{\text{ph}}^{R\hat{x}} \) approaches zero.

Making zero the component parallel to the acceleration, \( (B_z = 0) \), the phase velocity becomes

\[
\frac{v_{\text{ph}}^{R\hat{x}}}{(1 + aZ)} = 1 - \frac{B_x^2}{2b^2} \text{Sh}^2 (aT) \tag{32}
\]

while if the magnetic component that is perpendicular to the acceleration vanishes \( (B_x = 0) \)

\[
\frac{v_{\text{ph}}^{R\hat{x}}}{(1 + aZ)} = 1 - \frac{B_z^2}{2b^2} \tag{33}
\]

Then if we aim to diminish the phase velocity by the maximum amount, the most effective way would be to turn off the component along the Rindler acceleration, \( B_z = 0 \), since \( \text{Sh}(aT) \geq 0 \). \( \forall aT \). Figure 6 shows the phase velocity of the wave traveling in \( \hat{x} \) direction for different values of \( aT \).
Figure 6. Phase velocities for the wave moving in the $\hat{x}$-direction, for different values of $aT$. In the plot to the left, for $aT = 0.5$, as $B_x$ increases the phase velocity tends to the velocity of light in vacuum. To the right, for $aT = 1$, even for large field components, $(B_x/B)^2 = 0.9$ the phase velocity does not approach the one in vacuum.

Figure 7. The phase velocities for the three directions of propagation of the light wave in the Rindler frame are shown. The Rindler acceleration is $a\hat{z}$. For all three directions $v_{ph}^0$ decreases as $B/b$ increases and $aT = 0.3$. The smallest velocity is for the wave moving in the $+\hat{z}$ direction. In this plot the parallel and the transversal magnetic field components have the same intensity $B_x = B_z = B/\sqrt{2}$.

Figure 8. The squared modulus of phase velocities as a function of $aT$ are shown, for a fixed magnetic background, $(B/b)^2 = 0.2$. The magnetic field components are $B_x = B_z = B/\sqrt{2}$. The black thick line represents the phase velocity of the light ray in the presence of the BI magnetic background when $aT = 0$. For the waves moving in the $+\hat{z}$ and $+\hat{x}$ directions, $v_{ph}^0$ decreases up to a minimum and then increases reaching the velocity in vacuum. This corresponds to the Rindler observer's view, which initially is behind the light ray, reaches it, and then moves away from it. Since the accelerated frame is no longer inertial, there is no problem with surpassing the light velocity in vacuum. Light in the $-\hat{z}$ direction reaches $c = 1$ increasing monotonically.

In Figure 7 we compare the phase velocities for the three different directions of light rays. For all three directions of the light ray, $v_{ph}^0$ decreases as $B$ increases for a fixed ($aT$). The smallest velocity occurs for the wave moving in the $+\hat{z}$ direction.

Figure 8 shows the phase velocities of the light rays propagating along the three considered directions as a function of $aT$, for fixed $(B/b)^2 = 0.2$. The parallel and transversal magnetic field components have the same intensities $B_x = B_z = B/\sqrt{2}$.

The increase or decrease in phase velocity depends on the values of $aT$. For smaller values of $aT$, the velocity is smaller for the wave moving along $+\hat{z}$ and closer to $c = 1$ for the waves traveling in the $-\hat{z}$ and $+\hat{x}$ directions. For values of $aT > 2$, $v_{ph}^0$ is smaller for the wave moving along $+\hat{x}$ and approaches $c = 1$ for the waves traveling in the $\pm\hat{z}$ directions. As $aT$ increases the BI effect of slowing down the phase velocity is canceled for the waves traveling in the $\pm\hat{z}$ directions. Recall that these are the velocities as measured by the accelerated observer. This observer moves with respect to the light ray, first toward it, then reaching it, and moving away. This is possible since the frame is no longer an inertial one. The trajectories of light rays in $\pm\hat{z}$ directions are shown in Figure 9.

From the previous analysis we can summarize the behavior of the phase velocity of a light ray propagating through the BI magnetic field as seen by an observer with acceleration $\vec{a} = a\hat{z}$: the phase velocity depends on $aT$ such that the effect of increasing the acceleration is the same as that of time elapsing. For waves moving in the $+\hat{z}$ and $+\hat{x}$ directions, $v_{ph}^0$ decreases to a minimum and then increases to reach the velocity in vacuum, while for light in the $-\hat{z}$ direction it reaches $c = 1$ departing from the Rindler $v_{ph}^0$ and increasing monotonically. The change in direction of light
Finally moves away from the light ray (the accelerated observer sees that the light ray changes direction). If the magnetic field is in the same direction as the Rindler acceleration, then there is no effect on the phase velocity. For a fixed BI magnetic field, the smallest velocity is reached faster for the wave moving in the +z direction. Figures 7 and 8 illustrate these behaviors.

The question of whether the values of the BI field are reachable in experiments is discussed in ref. [18]. It is not clear whether it would be possible to build such an accelerated frame, or if it would be possible to maintain the acceleration for long enough to observe the predicted changes in the phase velocities.

5. Light Propagating through a BI Electric Background in the Rindler Space

The propagation of an EM wave through an intense uniform electric field is of interest, since the production of vacuum electron–positron pairs has been predicted.[10] This has not yet been measured; however, it may be feasible in the near future, due to the high power reached recently by lasers.[31, 32] Considering that the nonvanishing electromagnetic tensor components of a purely uniform electric background are \( F^{\mu\nu} = E_{\mu} F^{\nu} = E_{\nu} F^{\mu} = E_{\mu} \), and the electromagnetic invariant \( F = -2E^{2} \) \( (B = 0) \), the expression for the effective optical metric from Equation (9) is

\[
\gamma_{\text{eff}}^{\mu\nu} = \begin{pmatrix}
0 & 0 & 0 \\
-2F_{\mu}E_{\nu} & 0 & 0 \\
0 & 0 & -2F_{\mu}E_{\nu}
\end{pmatrix}
\]

From Equation (11) the phase velocity of light propagating through a uniform BI electric is determined as

\[
\nu_{\text{ph}}^{2} = 1 - \frac{E_{\mu}^{2}}{b^{2}}
\]

with \( E_{\mu} \) being the electric field component perpendicular to the acceleration direction. Note that in the absence of the perpendicular component, the phase velocity is that in vacuum. This expression was also derived in ref. [17]. According to Equation (35) in principle, the wave could reach zero velocity; however, taking the average in polarization and the electric field components, the minimum accessible phase velocity is \( < \nu_{\text{ph}} > \approx 1/3 \). [33]

Transforming the effective metric for the electric BI background to the Rindler frame with Equation (23), we obtain

\[
\gamma_{\text{eff},R}^{\mu\nu} = \begin{pmatrix}
\frac{E_{\mu}E_{\nu}(|aT|+b^{2})}{aZ+1} & \frac{E_{\mu}E_{\nu}(|aT|)}{aZ+1} & -\frac{E_{\mu}E_{\nu}(|aT|)Ch(aT)}{aZ+1} \\
\frac{E_{\mu}E_{\nu}(|aT|)}{aZ+1} & \frac{E_{\mu}E_{\nu}(|aT|)}{aZ+1} & -\frac{E_{\mu}E_{\nu}(|aT|)Ch(aT)}{aZ+1} \\
-\frac{E_{\mu}E_{\nu}(|aT|)Ch(aT)}{aZ+1} & -\frac{E_{\mu}E_{\nu}(|aT|)Ch(aT)}{aZ+1} & E_{\mu}E_{\nu}(|aT|)Ch^{2}(aT) - b^{2}
\end{pmatrix}
\]

As previously, we consider that the frame acceleration is \( \ddot{a} = a\ddot{z} \) and that, with no loss of generality, the uniform BI field is located in the XZ plane, \( \vec{B} = E_{\nu}\hat{x} + E_{z}\hat{z} \). The phase velocity of the wave moving in the \( \pm \hat{z} \) and \( \pm \hat{x} \) directions is then obtained from Equation (11) and analyzed in the next two subsections.

5.1. Electric BI Background: Light Propagating in the \( \pm \hat{z} \) Direction

For the wave moving in the \( \pm \hat{z} \) direction, and considering the position of the principal observer at \( Z = 0 \), the phase velocity resulting from Equation (11) is given by

\[
\nu_{\text{ph}}^{\pm \hat{z}} = \frac{\mp E_{\mu}E_{\nu}(|aT|)Ch(aT) + \sqrt{b^{2} + E_{\mu}^{2}Ch^{2}(aT)}}{b^{2} + E_{\mu}^{2}Ch^{2}(aT)}
\]
The term under the square root sign is used to recover the phase velocity in absence of acceleration. The difference in the phase velocity between the wave propagating in $+\hat{z}$ direction and the one along $-\hat{z}$ is then only the sign in the first term. The consequence of this difference is that the magnitude of the phase velocity for the wave in the $+\hat{z}$ direction is smaller than the one traveling in the opposite direction. There are several features shared with the magnetic case: $v_{\text{ph}}^R$ depends on $aT$ and $aZ$, that is, the effect of increasing time $T$ or spatial coordinate $Z$ is equivalent to increasing the acceleration of the frame. In addition, the acceleration affects the phase velocity only if there is an electric component that is transversal to the acceleration (in this case $E_x$); that is, the acceleration of the frame is affected only by the electric component that is transversal to $\vec{a} = a\hat{z}$. Expanding $v_{\text{ph}}^R$ in powers of $E^2/b^2$ and keeping terms of order $O(1/b^4)$, that is, neglecting terms of order $O(1/b^6)$ and higher, we arrive at

$$v_{\text{ph}}^R \pm \frac{1}{2b^2} \left\{ 1 + 2Sh(aT) [Sh(aT) \pm Ch(aT)] \right\}$$

(38)

Comparing the previous equation with the corresponding equation for the magnetic BI background, Equation (27), we notice that the expression is the same apart from changing the magnetic to the electric component, $B_y \rightarrow E_x$. This means that the application of a magnetic uniform background produces the same effect as the application of an electric background, as long as the transversal magnetic and electric components have the same magnitude. Consequently, the limiting cases are very similar to the ones for the magnetic background: in the limit $a \rightarrow 0$, the expression for the pure BI electric field Equation (35) is recovered. The linear limit (no BI field) is obtained with $b \rightarrow \infty$ and then $v_{\text{ph}}^{B_x} = 1$ is recovered ($Z = 0$). In the limit of long times or large acceleration ($aT \rightarrow \infty$) the BI effect of slowing down the phase velocity is lost, and the Rindler phase velocity $v_{\text{ph}}^{\text{RT}} = (1 + aZ)^2$ is recovered. In this case, we omit the figures for $v_{\text{ph}}^R$ for the wave propagating along $+\hat{z}$ due to redundancy.

5.2. Electric BI Background: Light Propagating in the $+\hat{x}$ Direction

For the wave moving in the $+\hat{x}$ direction, the phase velocity is

$$v_{\text{ph}}^{\text{ES}} = \frac{E_y E_x Sh(aT) + \sqrt{b^2 (E_y^2 Sh^2(aT) + b^2 - E_x^2)}}{E_x^2 Sh^2(aT) + b^2}$$

(39)

Recall that $E_x$ is perpendicular with respect to the acceleration of the Rindler frame, $\vec{a} = a\hat{z}$. In this case both components of the BI electric field, parallel and perpendicular to the acceleration, play a role. In the case when $E_z = 0$ there is no effect of the acceleration of the frame, and the wave slows down due to the pure BI effect

$$v_{\text{ph}}^{\text{ES}} = \sqrt{1 - \frac{E_z^2}{b^2}} \approx 1 - \frac{E_z^2}{2b^2}$$

(40)

Note that even if $E_z = b$, the light velocity does not vanish but there is a lower bound of $v_{\text{ph}}^{\text{min}} = 1/2$. Expanding $v_{\text{ph}}^R$ in powers of $E^2/b^2$ and keeping terms of order $O(1/b^4)$, that is, neglecting terms of order $O(1/b^6)$ and higher, we arrive at

$$v_{\text{ph}}^{\text{ES}} = 1 - \frac{1}{2b^2} [E_x - E_y Sh(aT)]^2$$

(41)

Note that this is the same expression as Equation (30) with $B_x \rightarrow E$. Then, as in the $+\hat{z}$ directions, the limiting cases are very similar to those for the magnetic background, so we omit the figures for the wave propagating along $\hat{x}$ and the comparison between the considered directions.

For the light propagating along $+\hat{z}$ direction, $v_{\text{ph}}^R$ diminishes monotonically in an interval of $(aT)$ until reaching a minimum that is arbitrarily close to zero. For $(aT)$ larger than a certain $aT_c$ given by

$$aT_c = \frac{1}{2} \text{ArcSh} \left( \sqrt{\frac{b^2 - E_x^2}{E_x^2}} \right)$$

(42)

$v_{\text{ph}}^R$ changes its propagation direction and increases to reach the light velocity in vacuum. For the wave in the $+\hat{x}$ direction, the behavior is qualitatively similar than that for the wave along $+\hat{z}$. As $aT$ increases the wave slows down monotonically until reaching a minimum that is arbitrarily close to zero, then increases till reaching $v_{\text{ph}}^R = 1$; the slowing down is maximized for $E_x = 0$. For the wave moving in the $-\hat{z}$ direction, as $aT$ increases the phase velocity reaches that in vacuum. Recall that these are the velocities as measured by the accelerated observer, and the relative directions change as $aT$ increases. Initially, the observer chases the light ray, and as it approaches the wave, the velocity of the wave seems to decrease, and eventually reaches zero (the moment the observer reaches the wave). The relative direction then changes, since subsequently, the observer moves away from the wave, and the velocity of the wave starts to increase. The plot shows the modulus squared of the phase velocity. Since the accelerated frame is no longer inertial, no special relativity velocity invariance is expected.

If the electric field component that is perpendicular to the acceleration vanishes, then there is no effect of either the acceleration or the BI field on the phase velocity, and its value in vacuum.

5.3. The Phase Velocities for Strong Fields

The series expansion of the phase velocities up to terms of order $B^2/b^2$ gives the same behavior for BI magnetic and electric backgrounds. However, it is worth (briefly) discussing the behavior of the phase velocities considering the exact expressions, Equations (37) and (39). For strong fields, that is, when the fields approach the maximum attainable electromagnetic field, differences arise and the electric and the magnetic backgrounds can be distinguished.

Figure 10 plots the exact expressions for the phase velocities, Equations (37) and (39), for waves moving in the $\pm\hat{z}$ and $\hat{x}$ directions.

In Figure 10a, for waves moving in the $\pm\hat{z}$ direction, the lowest phase velocity is reached by the wave in an electric background. The wave moving in the $\hat{x}$ direction reaches lower values in both
background fields. The behavior of the wave moving in the $\hat{z}$ direction is similar to the one in $-\hat{z}$ but it reaches slightly higher values in both background fields.

Note in Figure 10 that for the range $\frac{\text{Field}_b^2}{B^2} < 0.2$ it is not possible to distinguish between the magnetic and electric background, as discussed previously.

For completeness, in Figure 11 we compare the behavior of the phase velocities as a function of $aT$ for fixed electric and magnetic BI background. Comparing Figure 10a,b, note that the behavior is similar for both background fields. For waves moving in the $\pm \hat{z}$ directions, as $aT$ is higher the phase velocity tends to the velocity of light in vacuum. The phase velocity tends to zero as $aT$ increase for the wave moving in $\hat{x}$ direction.

According to the EEP, an accelerated frame is equivalent to a gravitational field, and it is therefore expected that light pulses sent from one Rindler observer to another will modify their frequency, resembling what happens in the presence of a gravitational field. This is indeed what happens, see refs. [28, 33–35].

In this section, we determine how the redshift due to the acceleration of the frame is affected by the addition of a BI magnetic background.

The redshift, denoted as $z_R$, can be written in terms of the intervals in proper time of emission $\Delta \tau_e$ and reception $\Delta \tau_r$ of two light rays with wavelength $\lambda$ and frequency $f$ as

$$z_R + 1 = \frac{\lambda_r}{\lambda_e} = \frac{f_e}{f_r} = \frac{\Delta \tau_r}{\Delta \tau_e} > 1 \quad (43)$$

where the subscripts $e$ and $r$ refer to the emitter and receiver, respectively. Therefore, to determine the redshift in the Rindler frame we first need to calculate the proper time intervals elapsed between the two sent signals, $\Delta \tau_e$, and the interval elapsed between the reception of the same two pulses, $\Delta \tau_r$, as measured.
Figure 11. a–c) The exact expressions, Equations (37) and (39), are shown as a function of $aT$, for a fixed BI background. The components of the background fields are of the same magnitude, $B_x = B_z$, $E_x = E_z$, and $B^2/b^2 = 0.2$, $E^2/b^2 = 0.2$; the behavior of the phase velocity is similar in both backgrounds.

by the receiver. These intervals will be different if a redshift occurs, as illustrated in Figure 12. Figure 13 shows the intervals in the world lines of the Rindler observers in the presence of a BI magnetic background.

When A sends a light pulse, the light trajectory is from the event of emission $A_1$ to the reception $B_1$. When B receives the signal, its velocity with respect to the Lab frame is higher than the velocity of A at the moment of emission. An analogous situation exist for the second signal, in such a way that the proper time intervals of reception and emission, respectively, are given by,

$$\Delta \tau_r = \tau_B^2 - \tau_B^1, \quad \Delta \tau_e = \tau_A^2 - \tau_A^1$$

(44)

The $(z, t)$ coordinates in the Lab frame in terms of the proper time $\tau_i$ and the acceleration $a_i$ of each observer are

$$z_i = \frac{1}{a_i} Ch(a_i \tau_i), \quad t_i = \frac{1}{a_i} Sh(a_i \tau_i), \quad i = A, B$$

(45)

As each observer is considered the principal observer, their proper positions are $Z_i = 0$.

To determine the redshift we restrict ourselves to a wave moving in the (+$\hat{z}$) direction in a magnetic BI background located in the plane $XZ$, with the Rindler acceleration being $\vec{a} = a \hat{z}$. The
light trajectory, in Minkowski coordinates \((t, z)\), is calculated by integrating the phase velocity in Equation (22). \(v_{\text{ph}} = \beta = \frac{dt}{dz} = \sqrt{1 - \frac{B^2}{\nu^2 + B^2}}\)

\[
z - z_0 = \sqrt{1 - \frac{B^2}{\nu^2 + B^2}} (t - t_0) = \beta (t - t_0)
\]

(46)

where the phase velocity is denoted by \(\beta\). When \(\beta \rightarrow 1\) (zero BI field) the trajectory is that of light in vacuum; this case is examined in ref. [28].

Considering that the first light pulse is emitted by \(A\) at the initial coordinates \(z_0 = z_1\) and \(t_0 = t_1\), the trajectory of the light pulse is \(z - z_1 = \beta (t - t_1)\), which in Rindler coordinates, Equation (45), becomes

\[
z - \frac{1}{a_A} \text{Ch}(a_A \tau_{A1}) = \beta t - \frac{1}{a_A} \text{Sh}(a_A \tau_{A1})
\]

(47)

This light ray intersects the world line of \(B\) at \((z_1, t_1)\) when the proper time of \(B\) is \(\tau_{B1}\). Since \(t_1 = \text{Sh}(a_B \tau_{B1})/a_B\), then knowing \(t_1\) we can determine \(\tau_{B1}\). Solving for \(t_1\) implies solving the system of equations consisting of the hyperbola equation for \(B\) and the light trajectory equation, that is, solving the system

\[
z_1^2 = \frac{1}{a_B^2} + t_1^2
\]

(48)

\[
z_1 - z_1 = \beta (t_1 - t_1)
\]

(49)

From these equations is obtained a quadratic equation for \(t_1\) with solution

\[
a_A a_B t_1 = \frac{1}{(\beta^2 - 1)} \left[ \beta a_B X_1 \pm \sqrt{a_B^2 X_1^2 + a_A^2 (\beta^2 - 1)} \right]
\]

(50)

where \(X_1 = a_A (\beta t_1 - z_1)\). In terms of the Rindler coordinates for the emitter \(A\), this is

\[
x_1 = a_A (\beta t_1 - z_1) = [\beta \text{Sh}(a_A \tau_{A1}) - \text{Ch}(a_A \tau_{A1})]
\]

(51)

Using \(t_j\) in Rindler coordinates, \(t_j = \text{Sh}(a_B \tau_{B1})/a_B\), we obtain

\[
\text{Sh}(a_B \tau_{B1}) = \frac{1}{a_B (\beta^2 - 1)} \left( \beta a_B x_1 \pm \sqrt{a_B^2 x_1^2 + a_A^2 (\beta^2 - 1)} \right)
\]

(52)

Following the same procedure for the second light ray, we obtain

\[
\text{Sh}(a_B \tau_{B2}) = \frac{1}{a_B (\beta^2 - 1)} \left( \beta a_B x_2 \pm \sqrt{a_B^2 x_2^2 + a_A^2 (\beta^2 - 1)} \right)
\]

(53)

where we have defined \(f\) and \(g\) as the right-hand sides of the previous equations. After determining \(\Delta \tau_j = \tau_{B2} - \tau_{B1}\) and \(\Delta \tau_j = \tau_{A1} - \tau_{A2}\) from the previous expressions, we can measure the redshift of the two light pulses propagating through a BI magnetic background in the Rindler frame.

From Equations (53) and (52) the proper time interval of the reception of the pulses is

\[
a_B (\tau_{B2} - \tau_{B1}) = \text{ArcSh} f - \text{ArcSh} g = \log \frac{\sqrt{f^2 + 1} + f}{\sqrt{g^2 + 1} + g}
\]

(54)

Expanding the result for “small” intensities of the field, that is, neglecting terms of order \((B/\nu)^2\) and higher

\[
\tau_{B2} - \tau_{B1} \approx \frac{a_A}{a_B} (\tau_{A2} - \tau_{A1}) + \left( \frac{a_A^2 - a_B^2}{2a_B^2} \right) \left( e^{2a_B \tau_{A2}} - e^{2a_B \tau_{A1}} \right) \frac{B^2}{4a_B^2}
\]

(55)

In this equation the first term corresponds to the redshift due to the acceleration of the observer’s frame, while the second term is the contribution due to the presence of the BI electromagnetic background, which clearly depends on the magnetic BI component that is transversal to the acceleration of the Rindler frame. We can obtain a simpler expression approximating for small proper times; in this case we approximate the exponentials as \(e^x \approx 1 + x\), neglecting terms \((a_A \tau_A)^2\) and higher

\[
\tau_{B2} - \tau_{B1} \approx \frac{a_A}{a_B} (\tau_{A2} - \tau_{A1}) \left( 1 + \frac{a_A^2 - a_B^2}{2a_B^2} \frac{B^2}{b^2} \right)
\]

(56)

\[
\Delta \tau_B \approx \frac{a_A}{a_B} \Delta \tau_A \left( 1 + \frac{a_A^2 - a_B^2}{2a_B^2} \frac{B^2}{b^2} \right)
\]

(57)
To obtain Equation (57) in terms of the frequency we note that if \( A \) sends a pulse with a proper frequency \( f_A \), \( A \) will measure the number of waves per unit of their proper time, while \( B \) receives and measures a proper frequency \( f_B \). Since the number of light pulses is the same

\[
 f_A \Delta \tau_A = f_B \Delta \tau_B \quad \text{(58)}
\]

then

\[
 \frac{f_A}{f_B} = \frac{\Delta \tau_A}{\Delta \tau_B} \approx \frac{a_A}{a_B} \left( 1 + \frac{a_B^2 - a_A^2}{2a_A^2} \frac{B_x^2}{b^2} \right) \quad \text{(59)}
\]

Since the proper accelerations are \( a_A/a_B > 1 \), and the BI term is always positive, then for the frequencies it is the case that \( \Delta \tau_A/\Delta \tau_B > 1 \), and in terms of the redshift parameter

\[
 z_a = \frac{a_A}{a_B} \left( 1 + \frac{a_B^2 - a_A^2}{2a_A^2} \frac{B_x^2}{b^2} \right) - 1 \quad \text{(60)}
\]

Analyzing the redshift expression, we see that there is a loss of energy by the light pulse. This loss is composed of two terms: the first part is used in overcoming the gravitational field (Rindler acceleration), as expected, and the second part is due to the effect of the BI field, that is, the pulse has to spend additional energy while traveling through the magnetic background, resulting in a larger redshift. Figure 14 shows two examples.

Another way of writing the redshift is in terms of the position of the observers. Considering the coordinate transformation to \((\bar{T}, \bar{Z})\) coordinates

\[
 \bar{Z} = Z + \frac{1}{a} \quad \bar{T} = T \quad \bar{X} = X \quad \bar{Y} = Y
\]

\[
 B_x = B_y
\]

Figure 14. The redshift in Equation (60) is plotted with respect to the intensity of the BI magnetic background, for two different proper acceleration differences between the emitter (A) and the receiver (B). As the difference becomes smaller, the resulting redshift is also smaller. The dashed lines represent the pure Rindler effect, that is, the one corresponding to a vanishing BI magnetic background, while the continuous lines account for the total redshift. The difference between the two lines (continuous minus dashed) corresponds to the BI redshift. In the plot the magnetic components are of the same magnitude, \( B_x = B_y \).

Figure 15. The redshift in Equation (64) is plotted with respect to the proper coordinates of the receiver \( Z_B \), for different values of the intensity of the BI field. The redshift increases as the intensity of the field increases. In this plot \( Z_a = 0.1 \). The redshift is larger for greater distances between the receiver and the emitter.

7. Conclusions

We have considered an electromagnetic wave, or light ray, propagating through an intense uniform Born–Infeld (BI) background, and have determined the phase velocities measured by an accelerated observer. For the accelerated frame we have considered a Rindler spacetime. This situation also models an environment with a uniform gravitational field, according to the Einstein equivalence principle. The phase velocities are determined from the effective optical metric, which is a curved spacetime produced by the presence of the intense BI magnetic or electric field.
Using the NLED effective optical metric approach\(^{[19,20]}\) and then applying a Rindler transformation, we obtain the phase velocity of the propagating wave from the null geodesics of the transformed effective optical metric. Our treatment is valid for very strong fields; if we consider, for instance, that \(B^2/b^2 \approx 10^{-2}\), then the BI field is of the order of \(10^{19}\) V m\(^{-1}\) that is ten times the critical Schwinger field or \(B_\text{c} \approx 10^9\) Tesla. We first considered a uniform BI magnetic background and then a purely electric background, and three different directions of the propagating wave, with the setting shown in Figure 3.

For the BI magnetic background, the phase velocity of the propagating light slows significantly for a wave moving in the same direction as the Rindler acceleration, diminishing as \(B\) grows; conversely, the phase velocity increases for a wave moving in the directions that are opposite and transversal to the Rindler acceleration, which we have considered as being \(a\hat{a}\). The phase velocities depend on \(aT\), such that the effect of increasing the acceleration is the same as that of time elapsing. If the magnetic field component transversal to the acceleration vanishes, then there is no effect on the phase velocity of either the acceleration or the BI field, and its value is that in vacuum.

For fixed values of the BI magnetic field, the phase velocity of waves moving in the \(+\hat{z}\) and \(+\hat{x}\) directions decreases to zero and then increases again as \(aT\) increases, while the phase velocity of waves propagating in the \(-\hat{z}\) direction approaches the one in vacuum as \(aT\) increases. In Figures 7 and 8, these behaviors are shown for the squared phase velocities. Recall that these are the velocities as measured by the accelerated observer, and the relative directions change as \(aT\) increases. Initially, the observer chases the light ray, and as it approaches the wave, the wave’s velocity seems to decrease. Eventually, it is zero (the moment the observer reaches the wave), and then the relative direction changes as the observer moves away from the wave and the wave’s velocity starts to increase. Since the accelerated frame is no longer inertial, no special relativity velocity invariance is expected. We also addressed the situation of a propagating wave through an intense electric background field. The effect of the BI electric field (decreasing the phase velocity) is quantitatively very similar to the one of the BI magnetic background, and, in the approximation taken up to \(B^2/b^2\) terms, the expressions for the phase velocities are the same apart from changing \(B_i \leftrightarrow E_i\). For \(E_z = 0\) the decrease in velocity is maximized. The wave traveling in \(-\hat{z}\) is not affected by the BI field and as \(aT\) increases the phase velocity reaches that in vacuum. If the electric field component that is perpendicular to the acceleration vanishes, then there is no effect of either the acceleration or the BI field on the phase velocity, and its value is that in vacuum. For strong fields, when \(B\) approaches the maximum attainable BI field \(b\), the behavior of the phase velocities is quantitatively different depending on whether the background is electric or magnetic. The most effective for slowing down the phase velocity is the electric background, for all travel directions of the light rays.

Finally, we analyzed the redshift of a light pulse sent from one Rindler observer and received by another when the light pulses travel through the BI magnetic background. From the trajectory of the pulse and the hyperbola worldline of the emitter and receiver, we determined the proper time intervals elapsing between the emission of the two pulses and then the proper time interval of the reception. Using these intervals we calculated the redshift. In the approximation of small fields (that as we commented above, are still very intense) and small intervals of time, we then found the expression for the redshift. Two contributions can be distinguished, one due to the acceleration of the frame and the other produced by the presence of the BI magnetic background, resulting in a larger total redshift.

In summary, we have analyzed the phase velocity of light propagating under the effect of a very intense magnetic or electric BI field as measured by an accelerated (Rindler) observer. According to the EEP, the situation is equivalent to measurements by an observer under the influence of a uniform gravitational field.

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### Conflict of Interest

The authors declare no conflict of interest.

### Data Availability Statement

This manuscript has no associated data or the data will not be deposited. [Authors’ comment: It is a theoretical work and the figures were generated analytically.]

### Keywords

Born–Infeld electrodynamics, light propagation, nonlinear electromagnetic field, non inertial frames, Rindler spacetime

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