Counting Statistics of Single-Photon Avalanche Diodes

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Abstract—This work presents stochastic approaches to modeling the counting behavior of single-photon avalanche diodes (SPADs). We present both analytical expressions and simulation algorithms predicting the distribution of the number of detections in a finite time window. We also present formulas for the mean detection rate. The approaches cover recovery time, afterpulsing, and twilight pulsing. We experimentally compare the theoretical predictions to measured data using commercially available silicon SPADs. Their total variation distances range from $10^{-5}$ to $10^{-2}$.

Index Terms—Single-photon avalanche diode (SPAD), afterpulsing, counting statistics, detection rate

I. SINGLE-PHOTON AVALANCHE DIODES

SINGLE-PHOTON avalanche diodes (SPADs) are the most affordable and widespread technology for detecting photons in the field of quantum optics [1], [2]. The most commonly used materials are silicon for the visible spectrum and InGaAs/InP for the infrared [3]. SPADs are used either individually for single-photon detection, or integrated – for photon-number resolution [4], [5], communications [6], and imaging applications [7]. In particular, multi-pixel SPADs offer new ways to measure the quantum statistics of light [8]–[10]. Multiplexed designs also offer ways of improving the dynamic range of single-photon detection [11], [12].

The properties of single SPADs that are most relevant in photon counting applications are detection efficiency, recovery time, dark counts, afterpulsing, and reset effects [1]. Efficiency is the probability of registering a detection if a single photon is incident on the SPAD [13], [14]. Dark counts are background detections occurring due to spontaneous thermal excitations or tunneling of charge carriers inside the SPAD [15], [16]. After each detection, the SPAD is quenched and cannot detect any more photons for a certain amount of time (dead time). During a brief transition period just after dead time, the detections exhibit different conditions including an extra delay due to bias voltage rise time (twilight pulses) [13], [17]. The SPAD therefore exhibits an effective dead time called recovery time, which is the minimum delay between successive detections. Twilight pulses arrive just after recovery time and their probability of occurrence is proportional to incident illumination during the rising bias.

Afterpulsing is a spontaneous detection triggered by a released carrier that was trapped in a deep energy level during a previous avalanche [18]. If the avalanche charge remains stable, afterpulses occur with approximately constant probability and exhibit a certain temporal distribution that depends on the impurities in the SPAD material [19]. This behavior has been treated using various models in the past [20].

These effects are a consequence of complex physical processes taking place in the SPAD semiconductor structure and its driving circuitry. Published SPAD models are mainly concerned with a detailed physical description of such processes and simulating the equivalent circuit, which is crucial for designing the SPAD quenching unit [16], [21]. Our aim is to describe the SPAD counting operation from a user’s perspective; that is model the detection times with respect to incident illumination.

II. THE CONTRIBUTION OF THIS PAPER

We present a counting model of the SPAD that determines the number of detections in a time window under continuous-wave illumination [22]. The model takes into account all above-stated SPAD imperfections. Additionally, it produces new mean-rate correction formulas. The model is based on simulating a self-exciting point process. However, suitable approximations are used to derive explicit expressions.

The proposed counting model generalizes known approaches that only consider dead time [23], [24] by incorporating afterpulsing and twilight pulsing. Afterpulsing is also generalized [25], [26], being treated as a translated point process [27]. A Monte-Carlo simulation algorithm is provided that accurately reproduces desired counting statistics under stated assumptions [28], [29].

The proposed model considers an arbitrary temporal distribution of afterpulses that can be obtained from experimental data; it also considers twilight pulsing and treats properly the interaction of radiation-induced detections and afterpulses. It assumes a physical model of a continuum of carrier traps [30] with an arbitrary life-time distribution. The model also gives an iterative detection rate formula that accurately incorporates all the discussed phenomena [22]. Additionally, simplified models of afterpulsing are used to derive explicit relations for counting probabilities and mean detection rates without the need for simulation. These models were already used to verify arbitrarily generated photon statistics in a previous work [31].

III. POINT PROCESS FORMULATION

To describe the counting statistics fully, we make a number of considerations that result in a formulation of a generalized self-exciting point process. This process can be simulated numerically to obtain the counting statistics, and its stationary intensity can be used to calculate the mean event rate.
The basis of the model is a homogeneous Poisson point process with a constant intensity $\mu$ that is modified by considering basic detector imperfections: recovery time $\tau_R$, twilight pulses and afterpulses. The detection efficiency $\eta$ and the dark count rate $\mu_0$ are included in $\mu = \eta \Phi + \mu_0$, where $\Phi$ is the incident photon flux. For more details on the mathematical treatment of point process models here, please refer to Appendix A.

Let us first discuss afterpulsing. Conventionally, it is formulated as a single event possibly triggered by each detection with a certain probability, and having a certain temporal probability distribution. The idea is that during each avalanche, there is a chance that a charge carrier gets caught in a deep-level trap and is subsequently released as the level exponentially decays, triggering another avalanche. Previous works suggested that there are multiple traps with different lifetimes and attempted to model the afterpulse distribution using various mixtures of exponentials, discrete or continuous [20], [30], [32].

In this work, we are going to assume a high number of traps, which is supported by the estimated junction volume $\sim 10^{13} \text{ m}^3$ and trap concentration $\sim 10^{17} \text{ m}^{-3}$ [33]. Each type of trap has its own exponential decay rate $\gamma$, its occurrence density in the material and a probability of being populated. These traps work independently [26], which means that the set of populated decay rates $\gamma_i$ after each detection is an inhomogeneous Poisson process with intensity $\rho(\gamma)$. Subsequently, the occurrence of afterpulses in time $t$ becomes a translated point process with intensity $\rho'(t) = \int \gamma \exp(-\gamma t) \rho(\gamma) dy$. This process is considered unaffected by detection avalanches [26] and is therefore superimposed on the counting process of the detector. Because of recovery time, we only consider the offset intensity $v(t) = \rho'(t + \tau_R)$. In the typical limit of the mean number of afterpulses $\langle n_{AP} \rangle = \int v(t) dt \ll 1$, afterpulsing approaches the model mentioned earlier – a random choice with a probability $\langle n_{AP} \rangle$ and a temporal probability density function (PDF) $p_{AP}(t) = v(t)/\langle n_{AP} \rangle$.

Next, we discuss twilight pulses. All carriers accumulated during the SPAD reset (rising bias voltage) are registered approximately at one point just after recovery time [17]. Due to the reset time being brief, the probability of such an event is approximately proportional to the incident intensity; $p_T \approx \alpha \mu$. This phenomenon has been called twilight pulsing and has mostly been treated as a linear contribution to afterpulsing.

Finally, we consider that during a recovery time $\tau_R$ after each detection, no further events can take place. This enables us to formulate the overall temporal point process, which incorporates everything except twilight pulsing (see also Fig. 1). The process intensity of the $n$-th detection given the history $\{t_i\}$ is

$$\lambda_n(t_n|\{t_i\}) = \begin{cases} 0 & t_n \leq t_{n-1} + \tau_R \\ \mu + \sum_{i=1}^{n-1} v(t_n - t_i - \tau_R) & t_n > t_{n-1} + \tau_R \end{cases}$$

(1)

This process takes place provided that a twilight pulse did not occur at time $t_{n-1} + \tau_R$ with a probability $p_T$. Taking both possibilities into account, the overall PDF of the $n$-th detection time given $\{t_i\}$ is

$$p_n(t_n|\{t_i\}) = p_T \delta(t_n - t_{n-1} - \tau_R) + (1 - p_T) \lambda_n(t_n) e^{-\int_{t_{n-1} + \tau_R}^{t_n} \lambda(t) dt}$$

(2)

for $t_n \geq t_{n-1} + \tau_R$ and $\delta$ being the Dirac delta function. This process can be numerically simulated using a Monte Carlo approach that is shown in a later section.

The mean detection rate of such a process can be calculated more efficiently, which is important for practical purposes, such as inferring the real rates $\Phi$ or $\mu$ from the observed detection rate $\mu_{det} = \lim_{n \to 0} n/t_n$. The idea is to find a stationary intensity $\lambda(t)$ and use it to calculate the detection rate.

Let us parameterize the $t_n$ of the $n$-th detection with respect to the most recent recovery: $\Delta t_n = t_n - t_{n-1} + \tau_R$, $\Delta t_n \in [0, \infty)$. The intensity $\lambda_n$ averaged over its history can be expressed recurrently with $\lambda_1(\Delta t_1) = \mu$ and, using (2),

$$\lambda_n(\Delta t_n) = \int_0^{\infty} \lambda_{n-1}(\Delta t_{n-1} + \tau_R + \Delta t_n) p_{n-1}(\Delta t_{n-1}) d\Delta t_{n-1} + v(\Delta t_n).$$

(3)
mean the already existing intensity $\lambda_{n-1}$ is displaced by the delay of the previous detection $\Delta t_{n-1}$, averaged, and afterpulsing $\nu$ is added. The stationary condition is $\bar{\lambda} = \lambda_n = \lambda_{n-1}$.
If we consider only the extra addition $f(\Delta t)$ to the constant intensity, $f(\Delta t) = \bar{\lambda}(\Delta t) - \mu$, and substitute in (3), we obtain an integral equation for $f$

$$f(\Delta t) = (1 - p_T) \int_0^{\infty} f(t + \tau_R + \Delta t)(\mu + f(t))e^{-\mu t - F(t)}dt + p_T f(\Delta t + \tau_R) + \nu(\Delta t)$$

with $F(t) := \int_0^t f(t')$, using the parameters $\mu$, $\tau_R$, $p_T$, and $\nu(t)$. A solution can be found efficiently using an iterative approach, starting with $f_1(t) = 0$ and substituting the left side in the right side repeatedly. Arriving at the solution $\bar{\lambda}(\Delta t)$, it can be substituted in (2) instead of $\lambda_n$ to get the stationary PDF

$$\bar{\lambda}(\Delta t) = p_T \delta(\Delta t) + (1 - p_T) \bar{\lambda}(\Delta t) \exp \left(- \int_0^{\Delta t} \bar{\lambda}(t)dt \right).$$

Assuming ergodicity, the mean detection rate is then obtained by averaging,

$$\bar{\mu}_{det} = \left(\langle \bar{\lambda}(\Delta t) \rangle_T + \tau_R \right)^{-1}.$$

Numerically, the cross-correlation in equation (4) can be efficiently evaluated using Fast Fourier Transform. This enables the relation (6) to be numerically inverted with respect to $\mu$, which provides detection rate correction using parameters that can be experimentally obtained with time-resolved detection techniques. For details on implementation, see Appendix B and the CodeOcean capsule [22].

IV. APPROXIMATE FORMULATIONS

We are going to introduce approximations that allow for the counting model to be described analytically. The first step is adopting an inter-arrival approach, where the whole process is described by a single PDF $p(\Delta t)$. Afterpulsing is considered to be a simple discrete-choice process with a probability $p_a$ and a temporal PDF $p_{AP}(\Delta t)$. This approach neglects the excitation of multiple afterpulses and their persistence over subsequent avalanches. The probabilistic mixture of all processes leads to

$$p(\Delta t) = p_T \delta(\Delta t) + (1 - p_T) p_a (p_{AP}(\Delta t) - P_{AP}(\Delta t)\mu) e^{-\mu \Delta t}$$

$$+ (1 - p_T) p_a e^{-\mu \Delta t},$$

where $P_{AP}(\Delta t) := \int_0^{\Delta t} p_{AP}(\tau)dt$. By averaging, we get

$$\bar{\mu}^{(1)}_{det} = \left[ \frac{1}{\mu} (1 - p_T) \left( 1 - p_a \int_0^{\infty} p_{AP}(\tau) e^{-\mu \tau} d\tau \right) + \tau_R \right]^{-1}.$$

Here the main element is the integral Laplace transform that eliminates those afterpulses that have not triggered before the next Poissonian event.

The next approximation is to neglect the PDF $p_{AP}(\Delta t)$ and consider all afterpulses and twilight pulses to arrive together at $\Delta t = 0$ with probability $\bar{p}_a$. This final simplification allows us to treat the whole problem analytically and derive expressions for the counting statistics. The mean detection rate (8) becomes

$$\bar{\mu}^{(2)}_{det} = \frac{\mu}{1 - \bar{p}_a + \mu \tau_R}.$$

If $\bar{p}_a$ is constant or increases with the first power of $\mu$, this formula can be directly inverted. All of the above models of the mean detection rate are compared in Fig. 2 along with the conventionally used corrections for dead time $\mu_{det}^{dead} = \mu/(1 + \mu \tau_R)$ and afterpulses $\mu_{det}^{AP} = (1 - \bar{p}_a)\mu$ [25].

The model (7) becomes simplified,

$$p_{amp}(\Delta t) = \bar{p}_a \delta(\Delta t) + (1 - \bar{p}_a) e^{-\mu \Delta t},$$

and can be used to calculate the PDF of the $n^{th}$ detection in a row, and eventually the counting statistics inside a time window $T$. Careful discussion of recovery time is needed to obtain mathematically correct results. The step-by-step derivation is presented in Appendix C. The resulting probability $P_n$ of $n$ detections occurring within a time $T$ is

$$P_0(T) = \frac{1 - \bar{p}_a}{1 - \bar{p}_a + \mu \tau_R} e^{-M_0},$$
$$P_{0 < n < N}(T) = \frac{1}{1 - \bar{p}_a + \mu \tau_R} \sum_{k=0}^{n} \binom{n}{k} \bar{p}_a^{n-k} (1 - \bar{p}_a)^k \left[ (k + 1 - \bar{p}_a) Q_{k+1}^{n+1} - M_{n+1} Q_k^{n+1} - \left( 2k + (k - 1) / n (1 - \bar{p}_a) \right) Q_{k+1}^n + (\bar{p}_a k / n + 2 M_n) Q_k^n + k Q_{k+1}^{n-1} - (k / n + M_{n-1}) Q_{k+1}^{n-1} \right],$$
$$P_N(T) = \frac{1}{1 - \bar{p}_a + \mu \tau_R} \left\{-M_0 + \sum_{k=0}^{N} \binom{N}{k} \bar{p}_a^{N-k} (1 - \bar{p}_a)^k \left[ (\bar{p}_a k / N + 2 M_N) Q_k^N - \left( 2k + (k - 1) / N (1 - \bar{p}_a) \right) Q_{k+1}^N + k Q_{k+1}^{N-1} - (k / N + M_{N-1}) Q_{k+1}^{N-1} \right] \right\} + N + 1,$n+1, \quad \text{and}
$$P_{N+1}(T) = \frac{1}{1 - \bar{p}_a + \mu \tau_R} \left\{M_0 + \sum_{k=0}^{N+1} \binom{N+1}{k} \bar{p}_a^{N+1-k} (1 - \bar{p}_a)^k \left[ k Q_{k+1}^N - \left( k / (N+1) + M_N \right) Q_k^N \right] \right\} - N,$$
where the terms $M$ and $Q$ are defined as

$$M_n = \mu(T - n\tau_R),$$  
$$Q_k^n := Q_k(M_n) = \begin{cases} 0 & \text{if } k = 0, \\ e^{-M_n} \sum_{i=0}^{k-1} M_n^i / i! & \text{if } k \geq 1. \end{cases}$$ \hspace{1cm} (15) \hspace{1cm} (16)

The number of detections where the analytical expression changes is $N = \lfloor T/\tau_R \rfloor$.

In the limit of $p_{\text{twilight}} \to 0$, or $p_{\text{twilight}} \equiv 0$ if one postulates $0^0 := 1$, the relations are reduced to the form published by Müller for a dead-time-only process (equations (32) in ref. [23]).

The relations (11) to (14) are an exact model of the point process defined by eq. (10) and by the recovery time $\tau_R$. As the definition (10) is rather simple, the model can be conveniently verified using a Monte Carlo simulation. Any of the more complex models above cannot be expressed explicitly; the only approach then is a numerical simulation.

V. NUMERICAL SIMULATION

Here we introduce an algorithm that generates the process defined by (1) to simulate the SPAD operation. Its output is a histogram of the number of detections.

In the following pseudo-code, a uniform number between zero and one is returned by $\text{random}(0,1)$, the mean incident rate $\mu$ is $\text{rate}$, the afterpulse mean $\langle n_{\text{AP}} \rangle$ is $\text{AP\_MEAN}$, and the twilight pulse probability $p_{\text{twilight}}$ is $\text{p\_twilight}$. Because afterpulsing is a point process governed by $\nu(t)$, the number of afterpulses $n_{\text{AP}}$ is Poissonian. The temporal distribution of afterpulses $p_{\text{AP}}(t)$ is governed by the function $\text{AP}(\cdot)$. Each afterpulse arrival time is stored in the queue $\text{AP\_queue}$. The width $T$ of the detection window is $\text{WINDOW}$.

One cycle of the simulation updates the variable $\text{time}$ to the time of the next detection event. The detection event counter is incremented and if needed, the count histogram is updated. A single loop cycle is

```plaintext
while (time > \text{WINDOW})
    \text{time} -= \text{WINDOW}
    for (\text{time\_AP} \in \text{AP\_queue})
        \text{RemoveFromQueue(\text{time\_AP})}
        \text{AddToQueue(\text{time\_AP})}
    \text{incrementHistogram(\text{detectionEvents})}
    \text{detectionEvents} += 1
end
```

The above loop cycle provides one sample of a detection event. It is written to simulate the process established by (1), but it can be simplified or expanded depending on how complex the detection model is. It can be conveniently run in multiple threads. An important technical note is that within one loop, each instance of a random number $\text{random}(0,1)$ should come from a separate pseudo-random-number generator. In our implementation, we found that if this condition is not met, the insufficiency in randomness is statistically observable in the simulation.

The number of runs for verifying of relations (11) to (14) was $10^{11}$. The same number of runs was used to verify the precision of the iterative mean-detection-rate formula (6) and it was found accurate within the statistical precision $\sigma = 3 \times 10^{-6} \mu$.

VI. SPAD MEASUREMENTS: RESULTS AND DISCUSSION

We compared our predictions with the counting statistics of three different actively quenched silicon SPAD modules made by two manufacturers (Excelitas SPCM 3605H and 3432H, Laser Components Count 20C). First, afterpulsing was measured in a separate experiment. Each SPAD was subjected to a pulsed signal coming from an attenuated gain-switched VCSEL diode (850 nm). Using a 81-ps time-tagging module (Qutools qutau), start-stop histograms were recorded for each gated optical pulse. The pulse frequency was 478 kHz, so afterpulsing was recorded within a 2-µs range. This way, we established $\langle n_{\text{AP}} \rangle$ and the histograms served as numerical inputs of $\nu(t)$ or $\text{p\_AP}(t)$ for the simulations.

Then, we subjected the SPADs to continuous-wave input signals of different orders of magnitude (LED filtered at 810 nm). For the counting statistics, we chose the time window to be 10 µs in order to cover low and high mean numbers as well as saturation. The measurement time was 1 hour for each signal. Detector output was recorded by a 156-ps time-tagging module (UQDevices Logic16). Recovery time was established directly from inter-arrival histograms. We found out that it is not constant and apparently increases for higher rates (see Appendix D). Twilight pulse probabilities were estimated from the twilight peaks in the histograms and for each detector, the constant $\alpha$ was established by a linear fit. Again, for higher rates, nonlinear behavior was observed for some detectors. Both of these irregularities can be explained by a different thermal equilibrium in the SPAD circuit, which slightly modifies the resetting of the bias current and therefore affects twilight pulse tardiness, efficiency, and effective reset time. For the detector 3605H, we also had to revise the value of $\langle n_{\text{AP}} \rangle$, which was originally 0.0141 in the pulsed measurement, but 0.0171 in the continuous measurement.
The results are shown in Fig. 3. Some of the data approach the model within statistical tolerance, but most exhibit systematic errors. The differences between measured and predicted distributions are expressed by their total variation distance – the maximum difference between probabilities of any two sets of results. These are usually in the order of $\sim 10^{-4}$, with the exception of $8 \times 10^{-3}$ in the case of saturated Count SPAD, which seems to produce narrower statistics than expected.

At this level of precision, several factors play a role. First, there are small fluctuations and drifts in detection efficiency and/or SLED intensity that affect the results of 1-hour-long integration. Moreover, inter-arrival histograms reveal that afterpulsing/twilight contributions sometimes do not behave as predicted. This could be due to SPAD temperature changing with count rate, which affects afterpulsing characteristics [34], [35]. In the inter-arrival time distributions, there are convex deviations from the negative-exponential region beyond the range of afterpulses that cannot be explained by relatively slow fluctuations, as those would result in a concave mixture.

Overall, some of these systematic effects could be compensated and corrected *ad hoc* by adjusting the parameters individually for each dataset or using parts of the inter-arrival histograms themselves as an input for the simulations. This would, however, be difficult to justify without additional independent measurements that would elaborate on the existing detection model. This is why the systematic errors were left uncorrected. Only the intensity $\mu$ was calculated individually as a free parameter to match each mean detection rate.
The approximated afterpulsing probability was chosen \( \tilde{p}_a = p_a + p_T - p_0 p_T \), where \( p_a = 1 - e^{-\langle N(t) \rangle} \) and \( p_T = \alpha \mu \). Fig. 3 also shows that often, the approximation error of (10) relative to (1) is much lower than other systematic errors. The results for the SPAD CD3605H counting statistics and mean detection rates (Figs. 2 and 3) indicate that the relative error made by the approximation alone is \( \lesssim 10^{-3} \).

VII. CONCLUSION

The formulation of a point process detection model allowed us to propose methods of mean-rate correction and counting statistics calculation that treat the detection aftereffects in new detail. The presented calculations can be evaluated both theoretically and using experimental data, such as numerical afterpulsing distributions. A multi-threadable algorithm was proposed that simulates the SPAD counting process. The presented methods can be used with any model of afterpulsing traps [20]. Approximations were shown that simplify the model down to explicit formulas.

Our results offer more accurate mean-rate corrections that are important for everyday rate estimation. Among applications that rely on predictable detector response are transmission measurements [36], single-photon imaging [37], or verification of Born’s rule [38]. The counting model improves the current treatments of SPAD counting statistics and its applications, such as estimating afterpulsing from a variance-to-mean ratio [29]. Counting statistics also offers a new way of characterizing SPAD non-Markovian phenomena [26], [39], as counting statistics is particularly affected by cumulative effects that cannot be fully distinguished in a start-stop histogram.

APPENDIX A

POINT PROCESSES

A. Preliminaries

This section provides some background and more detailed discussion of point processes and their use in the main text. A one-dimensional point process on \( t \in \mathbb{R} \) is a random process with a realization in a form of a set of points \( \{t_i\} \). It is usually described by an intensity function \( \lambda(t) \geq 0 \) that represents the average density of events. The intensity can be either explicitly given or it can depend on the particular realization \( \{t_i\} \) (self-exciting process). The process can be defined in term of survival probability (no events happening) between points \( t_1 \) and \( t_2 \). If we denote \( \Lambda(t_1, t_2) \) as the released \( \int_{t_1}^{t_2} \lambda(t)dt \), the survival probability is

\[
P_S(t_1, t_2) = e^{-\Lambda(t_1, t_2)}. \tag{17}
\]

An infinitesimal interpretation of this is a series of narrow regions \((t, t + dt)\), each having an independent probability of one event occurring equal to \( \lambda(t)dt \). The negative exponential is then a result of Euler’s limit.

The survival probability directly leads to calculating the inter-arrival probability. We wish to calculate the probability density function (PDF) \( p(t) \) of the first detection since \( t_0 \). First, we take the portion of events where no detection happened up to \( t \), which is \( P_S(t_0, t) \). Then, we wish to choose the subset where at least one detection happened during time \( dt \). So, we subtract the complement \(- P_S(t_0, t + dt) \). The resulting probability can then be converted to PDF by \( dt \to 0 \),

\[
p(t) = \frac{-\partial P_S(t_0, t)}{\partial t} = \lambda(t)e^{-\Lambda(t_0, t)}. \tag{18}
\]

The ideal detection of a constant flux of photons represents the simplest case, where \( \lambda(t) = \mu = \text{const} \). The result is a well-known homogeneous Poisson process with negative-exponentially distributed inter-arrival time and a Poisson distributions of the number of events in a finite time window.

B. Afterpulsing

Now let us explicitly discuss why afterpulsing is a point process under the assumption of a continuum of trap levels. We begin with a finite set of independent traps with decay rates \( \{\gamma_i\} \), each having a probability to be excited \( P_i \). Every trap, if excited, decays exponentially with time \( \Delta t \), and the released carrier triggers the next detection. So the survival probability for one trap is a combination of not being excited or decaying later than \( \Delta t \).

\[
P_{S,i}(\Delta t) = 1 - P_i \left(1 - e^{-\gamma_i \Delta t}\right). \tag{19}
\]

The total survival probability is simply a product \( P_S = \prod_i P_{S,i} \).

If we assume a continuum of traps with a certain excitation PDF \( \rho(\gamma) \), we get \( P_i = \rho(\gamma_i)d\gamma \) and

\[
P_S(\Delta t) = \lim_{dy \to 0} \prod_i \left[1 - \rho(\gamma_i)d\gamma \left(1 - e^{-\gamma_i \Delta t}\right)\right] 
= \exp\left(-\int_{\gamma} \rho(\gamma) \left(1 - e^{-\gamma \Delta t}\right) d\gamma \right) 
= \exp\left(-\int_0^{\Delta t} \rho(\gamma) e^{-\gamma \Delta t} d\gamma dr \right). \tag{22}
\]

We can see that (22) has a form of a temporal point process (17), where \( \Lambda(\Delta t) = \int_0^{\Delta t} \rho'(t) dt \) and \( \rho'(t) \) denotes \( \int \rho(\gamma) e^{-\gamma t} d\gamma \). This means that each afterpulse excitation is also a point process with intensity \( \rho'(t) \). Because only afterpulses at time \( t > \tau_R \) take place, we consider the intensity \( \nu(t) = \rho'(t + \tau_R) \). The number of excited afterpulses is a Poisson-distributed variable with the mean value \( \langle n_{AP} \rangle = \int_0^\infty \nu(t), \) typically in the order of \( 10^{-2} \) at most.

The afterpulsing intensity can be directly measured using a pulsed-excitation scheme and timing the detections in between the pulses. (A normalized) start-stop histogram of inter-arrival times samples the PDF

\[
p_{\text{hist}}(\Delta t) = \frac{1}{1 - e^{-\langle n_{AP} \rangle}} \nu(\Delta t) \exp\left(-\int_0^{\Delta t} \nu(t') dt' \right). \tag{23}
\]

The limit \( \langle n_{AP} \rangle \ll 1 \) then leads to \( p_{\text{hist}}(\Delta t) \approx \nu(\Delta t)/\langle n_{AP} \rangle \), which is often sufficient.

APPENDIX B

MEAN-RATE CORRECTION

Here we cover the practical implementation of the mean detection rate calculation (6). An example code is published on CodeOcean [22]. The most essential part is obtaining the afterpulsing intensity \( \nu(t) \), usually in the form of a histogram \( \{H_k\} \).
If the histogram bin width is \( \delta t \), then \( H_k = \int_{(k-1)\delta t}^{k\delta t} \nu(t) \, dt \). We denote the corresponding values of \( t_k := (k-1)\delta t \). The key step is calculating the discrete form of the cross-correlation in (4). We use the correlation theorem for fast Fourier transforms \( \mathcal{F} \) of vectors \( \mathbf{x}, \mathbf{y} \), each having \( N \) elements,

\[
\mathcal{F}^{-1} \{ \mathcal{F}(\mathbf{x}) \mathcal{F}(\mathbf{y})^\star \}_k = \sum_{i=0}^{N-1} y_i^\star \cdot x_{(i+k) \mod N}. \tag{24}
\]

To avoid the cyclical index wrap-around in \( \mathbf{x} \), both vectors’ lengths can be doubled by zeroing to each other.

Let us work with the vector \( \{ f_k \}_{k=1}^N \). The index offset representing recovery time would be \( n_R = \tau_R / \delta t \) (rounded). Then, let

\[
\mathbf{x} := \{ f_{i+n_R} \}_{i=1}^{2N}, \tag{25}
\]

\[
\mathbf{y} := \{ \mu \cdot \delta t + f_i \exp(-\mu t_i - \sum_{j=1}^i f_j) \}_{i=1}^{2N}, \tag{26}
\]

where we consider each out-of-bound value to be zero – namely, \( t_i, f_i := 0 \) \( \forall i > N \). Then, a single iteration step is

\[
f_{k+1}^\text{next} = (1 - p_T) \mathcal{F}^{-1} \left[ \mathcal{F}(\mathbf{x}) \mathcal{F}(\mathbf{y})^\star \right]_k + p_T f_k + n_R + H_k \tag{27}
\]

with the initial vector being \( f_0 \equiv 0 \).

The next step is averaging over \( \overline{\nu}(\Delta t) \) given in (5), where \( \overline{\lambda}_k = f_k + \mu \). Averaging beyond the value \( T_N \) of \( \overline{\lambda} \) needs to be written analytically. We can conveniently use the definition (26) with the resulting vector \( f \) and write

\[
(\Delta t)_{\overline{\nu}} = (1 - p_T) \left( \sum_i \tau_i y_i + \left( \frac{1}{\mu} + T_N \right) e^{-\Sigma_i f_i} \right), \tag{28}
\]

\[
\mu_{\text{det}} = (\Delta t)_{\overline{\nu}} + \tau_R \tag{29}
\]

The main caveats here are sampling and the number of iterations. The bin width \( \delta t \) needs to be short enough to neglect rounding errors in \( n_R \) and \( T_N \) needs to be long enough for \( f \) to approach zero at the end, which is mainly determined by \( H \). It is possible to work with a smaller bin width than the one given by the afterpulsing histogram, but then \( H \) needs to be interlaced by zeroes.

**Appendix C**

**Derivation of the Analytical Counting Model**

Here we derive the equations (11) to (14) in the main text. The process is given by the probability density function (PDF) of the time \( t \) between the end of detector recovery and the next detection, where \( \tau_R \) is a constant recovery time,

\[
p_{\text{inter}}(t) = \overline{\nu}_a \delta(t) + (1 - \overline{\nu}_a) e^{-\mu t}, \quad t \geq 0. \tag{30}
\]

The parameter \( \mu \) is the constant temporal density, \( \overline{\nu}_a \) is the afterpulse probability, and \( \delta(t) \) is the Dirac delta distribution. Let us work with the temporal PDFs of the \( 1^\text{st}, 2^\text{nd}, \ldots, n^\text{th} \) detection. First, let us consider the case when the detector is free (not recovering) at time zero. The probability of no detection up to time \( t \) is \( p^\text{free}_0(t) = \exp(-\mu t) \). The PDF of the first detection is simply \( p^\text{free}_1(t_1) = \mu \exp(-\mu t_1) \). Then, recovery time follows, so that the time of the second detection \( t_2 \geq t_1 + \tau_R \).

Of the second detection integrates over all possible times \( t_1 \) of the first detection:

\[
p^\text{free}_2(t_2) = \int_0^{\infty} p^\text{free}_1(t_1) p_{\text{inter}}(t_2 - (t_1 + \tau_R)) \, \, dt_1, \quad t_2 \geq \tau_R. \tag{31}
\]

By extension, the PDF of each detection is always a convolution of the PDF of the previous detection and PDF of the interarrival time,

\[
p^\text{free}_n(t) = \int_{(n-2)\tau_R}^{n\tau_R} p^\text{free}_{n-1}(t') p_{\text{inter}}(t - t' - \tau_R) \, \, dt', \tag{32}
\]

\[
p^\text{free}_n(t) = \mu e^{-\mu t} p^\text{free}_{n-1}(t) \left( 1 - \overline{\nu}_a \right) \tag{33}
\]

\[
\times \sum_{k=0}^{n-1} \binom{n-1}{k} \overline{\nu}_a^{n-1-k} \left( 1 - \overline{\nu}_a \right)^k \frac{[t - (n-1)\tau_R]^k}{k!},
\]

where \( t \geq (n-1)\tau_R \). Now, let us consider the probability of \( n \) detections in a time window between zero and \( T \). This means that the \( n^\text{th} \) detection happens at time \( t < T \) and no more detections happen afterwards. This must be split into two cases. In the first case, the \( n^\text{th} \) recovery time goes beyond the time window, \( t + \tau_R > T \). Then, no further detections inside the interval can take place. In the other case, if \( t \leq T - \tau_R \), then the probability of no further detections occurring is the product of no afterpulsing and no detections afterwards, which is equal to \( (1 - \overline{\nu}_a) \exp(-\mu t - \tau_R - t) \). Both cases are possible if \( n \leq [T/\tau_R] - 1 \). If we denote the maximum amount of recoveries that fit inside the detection window \( N := [T/\tau_R] \), then the maximum amount of detections is \( N + 1 \). Considering the time requirements of both cases, the probability of \( n \) detections is

\[
p^\text{free}_{n \leq N}(T) = (1 - \overline{\nu}_a) \int_{(n-1)\tau_R}^{T-\tau_R} p^\text{free}_{n-1}(t) e^{-\mu(T-\tau_R-t)} \, \, dt
\]

\[
+ \int_{N\tau_R}^{T} p^\text{free}_n(t) \, \, dt,
\]

\[
p^\text{free}_{N+1}(T) = \int_{N\tau_R}^{T} p^\text{free}_n(t) \, \, dt. \tag{35}
\]

Let us now abbreviate \( M_n := \mu(T - n\tau_R) \), which could be interpreted as an ideal mean number of detections in a time window reduced by \( n \) recovery times. Also, let \( Q_k(x) = \exp(-x) \sum_{m=0}^{k-1} \frac{x^m}{m!} \) be the regularized upper incomplete Gamma function, which in this special case of \( k \in \mathbb{N} \) represents the probability of a Poissonian variable with mean \( x \) to be less than \( k \) (note that \( Q_0(x) = 0 \)). Using this notation, let us substitute (33) into (34) and (35) to obtain

\[
p^\text{free}_0(T) = e^{-M_0}, \tag{36}
\]

\[
p^\text{free}_{1 \leq n \leq N}(T) = \sum_{k=0}^{n-1} \binom{n-1}{k} \overline{\nu}_a^{n-1-k} (1 - \overline{\nu}_a)^k \left[ Q_{k+1}(M_n) - Q_{k+1}(M_{n-1}) + (1 - \overline{\nu}_a) \frac{M_{n+k+1}}{(k+1)!} e^{-M_n} \right], \tag{37}
\]

\[
p^\text{free}_{N+1}(T) = 1 - \sum_{k=0}^{N} \binom{N}{k} \overline{\nu}_a^{N-k} (1 - \overline{\nu}_a)^k Q_{k+1}(M_N), \tag{38}
\]

where the terms \( Q_k(M_n) \) can be viewed with regard to the interpretations mentioned above. These equations give a
counting model assuming the detector is free at the beginning. However, if the detector is recovering at \( t = 0 \) and keeps inactive for a certain initial time \( \tau_i < \tau_R \), then the initial detection has the PDF

\[
p_{1}^{\text{rec}}(t, \tau_i) = \bar{p}_a \delta(t - \tau_i) + (1 - \bar{p}_a)e^{-\mu(t - \tau_i)}, \quad t \geq \tau_i. \tag{39}
\]

Like before, multiple convolutions result in the \( n \)th detection PDF

\[
p_{n}^{\text{rec}}(t, \tau_i) = \bar{p}_a^n \delta(t - (n - 1)\tau_R - \tau_i) + e^{-\mu(t - (n - 1)\tau_R - \tau_i)}
\times \sum_{k=1}^{n} \binom{n}{k} \bar{p}_a^{n-k} (1 - \bar{p}_a)^k \mu^k \left[ \frac{(T - (n - 1)\tau_R - \tau_i)^k}{(k-1)!} \right],
\tag{40}
\]

where \( t \geq (n - 1)\tau_R + \tau_i \). By integration analogous to (34), the probability of \( n \) detections in a time window \( T \) then is

\[
P_{n}^{\text{rec}}(T, \tau_i) = (1 - \bar{p}_a)e^{-\mu(T - \tau_i)}, \quad n \leq N, \tag{41}
\]

\[
P_{n+1}^{\text{rec}}(T, \tau_i) = (1 - \bar{p}_a)\bar{p}_a^n e^{-\mu(T - n\tau_R - \tau_i)} + (1 - \bar{p}_a)e^{-\mu(T - n\tau_R - \tau_i)}
\times \sum_{k=1}^{n} \binom{n}{k} \bar{p}_a^{n-k} (1 - \bar{p}_a)^k
\times \frac{(T - n\tau_R - \tau_i)^k}{k!}
\times \left[ Q_k(\mu(T - n\tau_R - \tau_i)) - Q_k(\mu(T - (n - 1)\tau_R - \tau_i)) \right], \tag{42}
\]

which is similar to (37), except for the first term and the \( \tau_i \) contribution. The first term is kept separate intentionally for consistent analytic integration in the subsequent step.

For the remaining cases of \( n \geq N \), the initial time \( \tau_i \) determines whether the \( N \)th recovery time can possibly be inside the detection window or not. The border value is \( \bar{\tau}_i = T - N\tau_R \). Therefore we need to split the two cases, while for \( n = N+1 \) the final recovery time always goes beyond the detection window and no more detections are possible.

\[
P_{N}^{\text{rec}}(T, \tau_i < \bar{\tau}_i) = \left. P_{n}^{\text{rec}} \right|_{n=N}(T, \tau_i), \tag{43}
\]

\[
P_{N}^{\text{rec}}(T, \tau_i > \bar{\tau}_i) = \int_{N\tau_R + \tau_i}^{T} P_{N}^{\text{rec}}(t, \tau_i) \, dt, \tag{44}
\]

\[
P_{N+1}^{\text{rec}}(T, \tau_i < \bar{\tau}_i) = \int_{N\tau_R + \tau_i}^{T} P_{N+1}^{\text{rec}}(t, \tau_i) \, dt, \tag{45}
\]

\[
P_{N+1}^{\text{rec}}(T, \tau_i > \bar{\tau}_i) = 0. \tag{46}
\]

Now we have obtained both distributions \( P_{n}^{\text{free}}(T) \) and \( P_{n}^{\text{rec}}(T, \tau_i) \) separately, where the distinction is the state of the detector at the beginning of the time window. We need to combine these cases by determining their statistical representation in a long measurement. Let us note that detection intervals are periodically distributed with a fixed length \( T \), while detections follow the probabilistic point process (30). So, in a long measurement, these two become uncorrelated and one can assume that the distribution of window beginnings with respect to detection events is completely random. Therefore, the proportion of “free” windows to “rec” windows is equal to

\[
P_n(T) = \frac{\langle \tau \rangle - \tau_R}{\langle \tau \rangle + \tau_R} P_n^{\text{free}}(T) + \frac{1}{\tau_R} \int_{0}^{\tau_R} P_n^{\text{rec}}(T, \tau_i) \, d\tau_i, \tag{47}
\]

This mixture needs to be evaluated separately for the cases of \( n = 0, n < N, n = N \), and \( n = N + 1 \), because the probability distributions differ and the integration over \( \tau_i \) needs to be split to accommodate the piecewise definitions (43) to (46). After integration, renumbering of the summation indices and using the property \( Q_0(x) = 0 \), we obtain the equations (11) to (14) in the main text.

**Appendix D Recovery time**

Here we show how the SPADs exhibit changes in recovery time. Fig. 4 shows the histograms of delays between two successive detections. The histograms are scaled so that the twilight/afterpulsing peak locations can be distinguished. The peaks mark the earliest detections and determine recovery time \( \tau_R \). Generally, \( \tau_R \) increases with rate and the changes become significant as the detectors start being saturated.
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REFERENCES

[1] A. Migdall, S. V. Polyakov, J. Fan, and J. C. Bienfang, *Single-Photon Generation and Detection*. Academic Press, Dec 2013.
[2] C. J. Chunnilall, I. P. Degiovanni, S. Kück, I. Müller, and A. G. Sinclair, “Metrology of single-photon sources and detectors: a review,” *Optical Engineering*, vol. 53, no. 8, p. 081910, Jul 2014.
[3] G. S. Buller, R. E. Warburton, S. Pellegrini, J. S. Ng, J. P. R. David, L. J. J. Tan, A. B. Krysa, and S. Cova, “Single-photon avalanche diode detectors for quantum key distribution,” *IET Optoelectron.*, vol. 1, no. 6, pp. 249–254, Dec 2007.
[4] P. Eraerds, M. Legrè, A. Rochas, H. Zbinden, and N. Gisin, “SiPM for fast photon-counting and multiphoton detection,” *Optics Express*, vol. 15, no. 22, pp. 14539–14549, Oct 2007.
[5] G. Chesi, L. Malinverno, A. Allevi, R. Santoro, M. Caccia, A. Martemiyanov, and M. Bondani, “Optimizing Silicon photomultipliers for Quantum Optics,” *Sci. Rep.*, vol. 9, no. 7433, pp. 1–12, May 2019.
[6] L. Zhang, D. Chitinis, H. Chun, S. Rajbhandari, G. Faulkner, D. O’Brien, and S. Collins, “A Comparison of APD- and SPAD-Based Receivers for Visible Light Communications,” *J. Lightwave Technol.*, vol. 36, no. 12, pp. 2435–2442, Feb 2018.
[7] C. Bruschi, H. Homuelle, I. M. Antolovic, S. Burri, and E. Charbon, “Single-photon avalanche diode imagers in biophotonics: review and outlook,” *Light Sci. Appl.*, vol. 8, no. 87, pp. 1–28, Sep 2019.
[8] D. A. Kalashnikov, S.-H. Tan, T. Sh. Ishkakov, M. V. Chekhova, and L. A. Krivitsky, “Measurement of two-mode squeezing with photon number resolving multipixel detectors,” *Opt. Lett.*, vol. 37, no. 14, pp. 2829–2831, Jul 2012.
[9] G. Chesi, L. Malinverno, A. Allevi, R. Santoro, M. Caccia, and M. Bondani, “Measuring nonclassicality with silicon photomultipliers,” *Opt. Lett.*, vol. 44, no. 13, pp. 3092–3094, Mar 2019.
[10] G. Lubin, R. Tennie, I. Michel Antolovic, E. Charbon, C. Bruschi, and D. Oron, “Quantum correlation measurement with single photon avalanche diode arrays,” *Opt. Express*, vol. 27, no. 23, pp. 32863–32882, Nov 2019.
[11] I. M. Antolovic, C. Bruschi, and E. Charbon, “Dynamic range extension for photon counting arrays,” *Opt. Express*, vol. 26, no. 17, pp. 22234–22248, Aug 2018.
[12] G. Brida, I. P. Degiovanni, F. Picentini, V. Schettini, S. V. Polyakov, and A. Migdall, “Scalable multiplexed detector system for high-rate telecom-band single-photon detection,” *Rev. Sci. Instrum.*, vol. 80, no. 11, p. 116103, Nov 2009.
[13] S. V. Polyakov and A. L. Migdall, “High accuracy verification of a correlated-photon-based method for determining photon-counting detection efficiency,” *Optics Express*, vol. 15, no. 4, pp. 1390–1407, Feb 2007.
[14] L. Cohen, Y. Pilnyak, D. Istrati, N. M. Studer, J. P. Dowling, and H. S. Eisenberg, “Absolute calibration of single-photon and multiplexed photon-number-resolving detectors,” *Physical Review A*, vol. 98, no. 1, p. 013811, Jul 2018.
[15] S. Cova, M. Ghioni, A. Lacaita, C. Samori, and F. Zampa, “Avalanche photodiodes and quenching circuits for single-photon detection.” *Applied Optics*, vol. 35, no. 12, pp. 1956–1976, Apr 1996.
[16] Z. Cheng, X. Zheng, D. Palubiak, M. J. Deen, and H. Peng, “A Comprehensive and Accurate Analytical SPAD Model for Circuit Simulation,” *IEEE Trans. Electron Devices*, vol. 63, no. 5, pp. 1940–1948, Mar 2016.
[17] M. Ware, A. Migdall, J. C. Bienfang, and S. V. Polyakov, “Calibrating photon-counting detectors to high accuracy: background and deadtime issues,” *Journal of Modern Optics*, vol. 54, no. 2-3, pp. 361–372, Jan 2007.
[18] S. Cova, A. Lacaita, and G. Ripamonti, “Trapping phenomena in avalanche photodiodes on nanosecond scale,” *IEEE Electron Device Letters*, vol. 12, no. 12, pp. 685–687, Dec 1991.
[19] G. Humer, M. Peev, C. Schaeff, S. Ramelow, M. Stipčević, and R. Ursin, “A simple and robust method for estimating afterpulsing in single photon detectors,” *Journal of Lightwave Technology*, vol. 33, no. 14, pp. 3088–3107, Jul 2015.
[20] A. W. Ziarkash, S. K. Joshi, M. Stipčević, and R. Ursin, “Comparative study of afterpulsing behavior and models in single photon counting avalanche photodiode detectors,” *Scientific Reports*, vol. 8, no. 1, p. 5076, Mar 2018.
[21] F. Zampa, A. Tosi, A. D. Mora, and S. Tisa, “SPICE modeling of single photon avalanche diodes,” *Sens. Actuators, A*, vol. 153, no. 2, pp. 197–204, Aug 2009.
[22] I. Straka, “SPAD counting model,” 2020. [Online]. Available: https://codeocean.com/capsule/8487128, https://github.com/ivo-s/SPAD-counting-model
[23] J. W. Müller, “Dead-time problems,” *Nuclear Instruments and Methods*, vol. 112, no. 1, pp. 47–57, Sep 1973.
[24] J. Rapp, Y. Ma, R. M. A. Dawson, and V. K. Goyal, “Dead Time Compensation for High-Flux Ranging,” *IEEE Trans. Signal Process.*, vol. 67, no. 13, pp. 3471–3486, May 2019.
[25] V. Kornilov, “Effects of dead time and afterpulses in photon detector on measured statistics of stochastic radiation,” *Journal of the Optical Society of America A*, vol. 31, no. 1, pp. 7–15, Jan 2014.
[26] F.-X. Wang, W. Chen, Y.-P. Li, D.-Y. He, C. Wang, Y.-G. Han, S. Wang, Z.-Q. Yin, and Z.-F. Han, “Non-Markovian property of afterpulsing effect in single-photon avalanche photodiodes,” *Journal of Lightwave Technology*, vol. 34, no. 15, pp. 3610–3615, Aug 2016.
[27] D. L. Snyder and M. I. Miller, *Random Point Processes in Time and Space*. Springer-Verlag New York, 1991.
[28] M. Stipčević and D. J. Gauthier, “Precise Monte Carlo simulation of single-photon detectors,” Advanced Photon Counting Techniques VII, vol. 8727, p. 87270K, May 2013.

[29] B.-W. Tzou, J.-Y. Wu, Y.-S. Lee, and S.-D. Lin, “Method to evaluate afterpulsing probability in single-photon avalanche diodes,” Optics Letters, vol. 40, no. 16, pp. 3774–3777, Aug 2015.

[30] D. B. Horoshko, V. N. Chizhevsky, and S. Y. Kilin, “Afterpulsing model based on the quasi-continuous distribution of deep levels in single-photon avalanche diodes,” Journal of Modern Optics, vol. 64, no. 2, pp. 191–195, Jan 2017.

[31] I. Straka, J. Mika, and M. Ježek, “Generator of arbitrary classical photon statistics,” Optics Express, vol. 26, no. 7, pp. 8998–9010, Apr 2018.

[32] M. A. Itzler, X. Jiang, and M. Entwistle, “Power law temporal dependence of InGaAs/InP SPAD afterpulsing,” Journal of Modern Optics, vol. 59, no. 17, pp. 1472–1480, Oct 2012.

[33] M. Ghioni, A. Gulinatti, I. Rech, P. Maccagnani, and S. Cova, “Large-area low-jitter silicon single photon avalanche diodes,” Quantum Sensing and Nanophotonic Devices V, vol. 6900, p. 69001D, Feb 2008.

[34] M. Stipčević, D. Wang, and R. Ursin, “Characterization of a commercially available large area, high detection efficiency single-photon avalanche diode,” Journal of Lightwave Technology, vol. 31, no. 23, pp. 3591–3596, Oct 2013.

[35] M. Anti, A. Tosi, F. Acerbi, and F. Zappa, “Modeling of afterpulsing in single-photon avalanche diodes,” Physics and Simulation of Optoelectronic Devices XIX, vol. 7933, p. 79331R, Feb 2011.

[36] J. Sabines-Chesterking, R. Whittaker, S. K. Joshi, P. M. Birchall, P. A. Moreau, A. McMillan, H. V. Cable, J. L. O’Brien, J. G. Rarity, and J. C. F. Matthews, “Sub-Shot-Noise Transmission Measurement Enabled by Active Feed-Forward of Heralded Single Photons,” Phys. Rev. Appl., vol. 8, no. 1, p. 014016, Jul 2017.

[37] A. Ingle, A. Velten, and M. Gupta, “High flux passive imaging with single-photon sensors,” in 2019 IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), June 2019, pp. 6753–6762.

[38] T. Kauten, R. Keil, T. Kaufmann, B. Pressl, Č. Brukner, and G. Weihs, “Obtaining tight bounds on higher-order interferences with a 5-path interferometer,” New J. Phys., vol. 19, no. 3, p. 033017, Mar 2017.

[39] M. A. Wayne, J. C. Bienfang, and S. V. Polyakov, “Simple autocorrelation method for thoroughly characterizing single-photon detectors,” Optics Express, vol. 25, no. 17, p. 20352, 2017.