Nuclear level densities and gamma-ray strength functions of $^{145,149,151}$Nd isotopes

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Abstract. The nuclear level densities and gamma-ray strength functions are the key elements for Hauser-Feshbach statistical model calculations to predict reaction cross sections which have many applications including astrophysics. The nuclear level densities and $\gamma$-ray strength functions have been determined for $^{145,149,151}$Nd isotopes below the neutron separation energies using the Oslo method with the $^{144,148,150}$Nd$(d,p)\gamma$ reactions. The results from the first measurements as well as planned experiments at OCL will be presented.

1. Introduction
The study of nuclear level densities has been gaining a growing interest in the nuclear science community in recent years. Microscopic description of heavy nuclei up to high excitation energy has become possible with the introduction of new theoretical approaches and fast computers [1].

Currently, experimental level densities and $\gamma$-ray strength functions play an important role for testing of nuclear structure models and they are also important tools for various nuclear applications such as for calculation of reaction cross sections applied to astrophysical nucleosynthesis. They are important ingredients in designing nuclear power plants and also used for calculations in transmutation of nuclear waste.

The nuclear physics group at the University of Oslo has developed a method [2, 3] to determine the level density and the $\gamma$-ray strength function ($\gamma$SF) simultaneously by using particle-$\gamma$ coincidences. In this work, the Oslo method is applied to extract the level densities and $\gamma$-ray strength functions of the $^{145,149,151}$Nd isotopes.

In the second section, the experimental procedure and the data analysis are described and in the third section the level densities and $\gamma$-ray strength functions extracted are presented.

2. Experimental Procedure and Data Analysis
Three experiments were performed at the MC-35 Scanditronix cyclotron of the Oslo Cyclotron Laboratory (OCL). The self-supporting $^{144}$Nd (97% enrichment and 2 mg/cm$^2$ thickness), $^{148}$Nd (95% enrichment and 1.9 mg/cm$^2$ thickness), and $^{150}$Nd (97% enrichment and 1.9 mg/cm$^2$ thickness) targets were bombarded with a 13.5-MeV deuteron beam, and $^{144}$Nd$(d,p)\gamma$,$^{145}$Nd, $^{148}$Nd$(d,p)\gamma$,$^{149}$Nd and
150Nd(d, p)151Nd reactions were studied. Particle-γ coincidences were measured with the SiRi particle telescope and the CACTUS γ-detector system [4, 5] which are shown in figures 1 and 2.

![Figure 1: SiRi particle-detector system. (a) The layout of one silicon chip with its eight ΔE detectors used for different reaction angles. (b) The whole detector system with the cables to read out the signal.](image1)

![Figure 2: Detector setup. (a) Positioning of the SiRi with respect to the target and beam (b) Complete setup of the CACTUS array.](image2)

In order to reduce the intense elastically scattered deuterons and to obtain a broad and higher spin distribution, the 64 SiRi telescopes are placed in backward direction and they cover eight angles relative to the beam axis. The front and back detectors have thicknesses of 130 μm and 1550 μm, respectively. The CACTUS array consists of 26 collimated 5” x 5” NaI(Tl) detectors with a total efficiency of 14.1% at Eγ = 1.33 MeV.

By using the reaction kinematics, the excitation energy of the compound nucleus is calculated from the measured ejectile energy. The particles and γ-rays are measured in coincidence, thus each γ-ray can be assigned to a certain initial excitation energy of the residual nucleus, by which the particle-γ ray coincidence matrix is obtained (Figure 3).

First main steps of the Oslo method are shown in Figure 4. After obtaining the raw matrix by sorting the data into a matrix of initial excitation energies E versus the NaI energy signal (a), this raw matrix is unfolded [6] by using the NaI response function for each excitation bin (b); and finally, by applying an iterative subtraction technique [7] to separate the distribution of the first generation γs from the total γ cascade, the first generation (primary) γ-ray matrix P(E, Eγ) is obtained (c).
Figure 3: (a) $\Delta E - E$ bananas for $^{144}\text{Nd}$ and (b) particle–$\gamma$ ray coincidence matrix for $^{145}\text{Nd}$.

Figure 4: Initial excitation energy $E$ versus $\gamma$-ray energy $E_\gamma$ from particle-$\gamma$ coincidences obtained via the $^{144}\text{Nd}(d,p\gamma)^{145}\text{Nd}$ reaction. Main steps are: (a) obtaining a raw $\gamma$-ray spectra, (b) unfolding the spectra by using the NaI response function, and finally (c) extracting the primary (or first generation) $\gamma$-ray spectra $P(E, E_\gamma)$ as a function of excitation energy.

According to the Brink hypothesis [8] the $\gamma$-ray transmission coefficient $\mathcal{T}$ is approximately independent of excitation energy, hence the first generation matrix $P(E, E_\gamma)$ can be factorised as follows:

$$P(E, E_\gamma) \sim \mathcal{T}(E_\gamma) \rho(E - E_\gamma),$$

where $\rho(E - E_\gamma)$ is the level density at the excitation energy after the first $\gamma$-ray is emitted. This allows us to extract the level density and the $\gamma$-ray transmission coefficient simultaneously.

3. Level Densities and $\gamma$-ray Strength Functions

The level density obtained by applying equation (1) to the first-generation matrix determines only the functional form of $\mathcal{T}$ and $\rho$. If one solution of $\mathcal{T}$ and $\rho$ is known, it is possible to construct infinitely many fits to the $P(E, E_\gamma)$ matrix by the following equations:

$$\tilde{\rho}(E - E_\gamma) = A \exp[\alpha(E - E_\gamma)] \rho(E - E_\gamma),$$

$$\tilde{\mathcal{T}}(E_\gamma) = B \exp(\alpha E_\gamma) \mathcal{T}(E_\gamma).$$

In order to normalize the two functions, one needs information to fix the $A, \alpha$, and $B$ parameters. For the level density, two fixed points are used to determine $A$ and $\alpha$. The procedure is shown in Figure 5 for $^{145}\text{Nd}$. The level density is normalized to the known discrete levels [9] at low excitation energy. At high excitation energy, we make use of the neutron resonance spacing $D_0$ at the neutron separation energy $S_n$. Further descriptions of the Oslo method are given in references [2, 3].
Figure 5: Normalization procedure for experimental level density. The level density of $^{145}$Nd is normalized to known discrete levels at low energies and to $(S_n)$ at the binding energy, which is calculated from neutron resonance spacing data.

The normalization procedure is performed by using the Constant Temperature (CT) formula at above $E = 2\Delta$ energy, where $\Delta$ is the pairing gap parameter. Here, the CT formula is as follows:

$$\rho_{CT}(E_x) = \frac{1}{T_{CT}} \exp \frac{E_x - E_0}{T_{CT}},$$

where the shift in excitation energy $E_0$ is given by:

$$E_0 = S_n - T_{CT} \ln[\rho(S_n)T_{CT}].$$

To normalize the $\gamma$-ray strength function, we also need the parameter $B$. We can determine this parameter by using the known average total radiation width $\langle \Gamma_\gamma \rangle$ at the neutron separation energy. By assuming that dipole transitions dominate the decay and equal parity holds for all excitation energies and spins, it can be found [2,10]:

$$\langle \Gamma_\gamma(S_n) \rangle = \frac{1}{2\pi} \frac{\rho(S_n)}{\rho(S_n - E_\gamma, I_f)} \sum_{I_f} dE_\gamma B T_{\gamma}(E_\gamma) \rho(S_n - E_\gamma, I_f).$$

Finally, as seen in figure 6, we obtain the level densities (left panel) and the $\gamma$-ray strength functions (right panel) for $^{145,149,151}$Nd nuclei.

The parameters used for the Oslo method are displayed in Table 1 for the $^{145,149,151}$Nd nuclei. The resonance spacing and total gamma width parameters are taken from RIPL-3 database [11]. The level density and back-shift parameters are taken from the systematic study of Egidy and Bucurescu [12]. The level density at $S_n$ is calculated by using the following formula:

$$\rho(S_n) = \frac{2\sigma^2}{D_0 (I + 1)} \frac{1}{\exp[-(I + 1/2)^2/2\sigma^2]} [i + \exp[-I^2/2\sigma^2]].$$

Here, $I$ is the spin of the target nucleus. For the spin distribution $g(E_x, I)$ Gilbert and Cameron’s work is used [13], and for the spin cut-off parameter $\sigma$, Egidy and Bucurescu’s rigid body moment of inertia approach [12] is used in the following formulas:

$$g(E_x, I) = \frac{2I + 1}{2\sigma^2(E_x)} \exp[-(I + 1/2)^2/2\sigma^2],$$

$$\sigma^2(E_x) = 0.0146A^{2/3} \frac{1 + \sqrt{4aU(E_x)}}{2a},$$
where, $A$ is the mass number of the nucleus, $a$ is the level density parameter, $U(E_x) = E_x - E_1$ is the intrinsic excitation energy and $E_1$ is the back-shift parameter.

Table 1: Parameters used to calculate level density of $^{145, 149, 151}$Nd.

| Nucleus | Neutron binding energy $(S_n)$ (MeV) | Level density parameter $(a)$ (MeV$^{-1}$) | Back-shift parameter $(E_1)$ (MeV) | Spin cut-off parameter $(\sigma)$ | Total Gamma width $(\Gamma)$ (meV) | Resonance spacing parameter $(D_0)$ (eV) | Level density at $S_n$ $(\rho(S_n))$ (MeV$^{-1}$) |
|---------|---------------------------------|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $^{145}$Nd | 5755 | 15.942 | -0.052 | 6.09 | 47 | 450 | 1.673E+05 |
| $^{149}$Nd | 5.039 | 17.867 | -0.446 | 5.97 | 40 | 155 | 4.662E+05 |
| $^{151}$Nd | 5.335 | 17.820 | -0.596 | 6.153 | 67 | 165 | 4.278E+05 |

Figure 6: Level densities of $^{145, 149, 151}$Nd nuclei (left panel) and $\gamma$-ray strength functions of the $^{145, 149, 151}$Nd nuclei (right panel).

Figure 7: The $\gamma$-ray strength function for $^{151}$Nd compared to models with systematic parameters.
As seen in figure 6 (left), level density for $^{145}\text{Nd}$ is lower than those for the other two nuclei. This is because, $^{145}\text{Nd}$ is closer to the N=82 shell closure than the other two nuclei. Also, when we plot the $\gamma$-ray strength functions (figure 6, right), we see that the scissors resonance around 2.5 – 3.5 MeV increases as we move from $^{145}\text{Nd}$ to $^{151}\text{Nd}$, which is due to the increasing deformation. For $^{145}\text{Nd}$, it seems that the missing scissors strength reveals a low-energy enhancement.

In figure 7, $\gamma$-ray strength function for $^{151}\text{Nd}$ is shown together with the $\gamma$-ray strength functions obtained by Vasiliev [14] and Carlos [15]. As seen, although there is a region with no data between ~6-8 MeV, our results are compatible with the measurements of Vasiliev. The $\gamma$-ray strength function used as a “background” to our data is described by the sum of three strength functions (purple) (see figure 7).

4. Conclusions

Nuclear level densities and $\gamma$-ray strength functions were obtained for $^{145,149,151}\text{Nd}$ isotopes by using the so-called Oslo method. Preliminary results show that scissors mode resonance strength increases with deformation.

For our future work, we plan to study the $^{144}\text{Nd}(d,d'\gamma)^{144}\text{Nd}$, $^{148}\text{Nd}(d,d'\gamma)^{148}\text{Nd}$, and $^{150}\text{Nd}(d,d'\gamma)^{150}\text{Nd}$ channels and to conduct $^{142,146}\text{Nd}(d,p\gamma)^{143,147}\text{Nd}$ experiments. Thus we will be able to study the level densities and $\gamma$-ray strength functions for a wide range of neodymium isotopes, i.e. $^{142-151}\text{Nd}$ isotopes, systematically in detail.

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