Using Back-Scattered Laser Beams to Detect CP Violation in the Neutral Higgs Sector

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Abstract

We demonstrate that the ability to polarize the photons produced by back-scattering laser beams at a TeV scale linear $e^+e^-$ collider could make it possible to determine whether or not a neutral Higgs boson produced in photon-photon collisions is a CP eigenstate. The relative utility of different types of polarization is discussed. Asymmetries that are only non-zero if the Higgs boson is a CP mixture are defined, and their magnitudes illustrated for a two-doublet Higgs model with CP-violating neutral sector.

⋆ Work supported, in part, by the Department of Energy.
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An important issue for understanding electroweak symmetry breaking and physics beyond the Standard Model (SM) is whether or not there is CP violation in the Higgs sector. For instance, both spontaneous and explicit CP violation are certainly possible in the context of a general two-Higgs-doublet model (2HDM), whereas CP violation is not possible at tree-level for the specific 2HDM of the Minimal Supersymmetric Model (MSSM) or its simplest extensions involving additional singlet scalar Higgs fields.\[1] The sensitivities of a variety of experimental observables to CP violation in the neutral sector have been examined in the literature, ranging from neutron\[2−4] and electron\[8−11] electric dipole moments (for a general review and more complete references, see Ref. [12]) to asymmetries in top quark decay and production.\[13,14] However, CP violation in these situations arises via loop graphs involving the neutral Higgs bosons. Numerically, current EDM experiments are not sufficiently sensitive to constrain CP violation in the neutral Higgs sector, although future results could begin to impose restrictions. Even if a non-zero EDM is experimentally observed its interpretation will be uncertain since other types of new physics could also be involved. Of course, detection of CP violation in top quark decays and production must await the large top production rates of the SSC and LHC.

Even after a Higgs boson has been directly observed, it may be difficult to determine whether or not it is a pure CP eigenstate. In a CP-conserving 2HDM, there are three neutral Higgs bosons: two CP-even scalars, \( h^0 \) and \( H^0 \) \((m_{h^0} \leq m_{H^0})\) and one CP-odd scalar, \( A^0 \). If the 2HDM is CP-violating there will simply be three mixed states, \( \phi_i \) \(i=1,3\). In principle, the presence of CP violation can be detected through the existence and/or strength of various couplings. For instance, in the CP-conserving case, at tree-level the \( A^0 \) is predicted to have no \( WW, ZZ \) couplings while the \( h^0 \) and \( H^0 \) together should saturate the \( WW, ZZ \) couplings. In the CP-violating case, all of the \( \phi_i \) would have \( WW, ZZ \) couplings at tree-level. But, even if three \( \phi_i \) are observed to have \( WW, ZZ \) couplings, it would not be clear whether this was due to CP violation in a 2HDM or to the existence of more than two doublets. A better possibility is to note\[15\] that CP violation at tree-level would be required if the couplings \( ZZ\phi_1, ZZ\phi_2 \) and \( Z\phi_1\phi_2 \) are all non-zero. To completely avoid contamination from C-violating one-loop diagrams, three or more neutral Higgs bosons must be detected. Non-zero values for all three of the couplings \( Z\phi_1\phi_2, Z\phi_1\phi_3 \) and \( Z\phi_2\phi_3 \) are only possible if CP violation is present. Finally, we note that the use of correlations between the decay planes of the decay products of the \( WW \) or \( ZZ \) vector boson pairs, in order to analyze the CP properties of the decaying Higgs boson,\[16\] will not be useful in the most probable case where the CP-even component of a mixed-CP \( \phi \) state accounts for essentially all of the \( WW, ZZ \) coupling strength.

We wish to contrast the above rather significant difficulties with the situation that arises in collisions of polarized photons. High luminosity for such collisions is possible using back-scattered laser beams at a TeV scale linear \( e^+e^- \) collider.\[17,18\] Detection of the SM Higgs boson and of the neutral Higgs bosons of the MSSM using back-scattered laser beam photons was first studied in Ref. [19]. More detailed Monte Carlo results have appeared in Ref. [20]. These studies show that
expected luminosities are adequate to make large numbers of Higgs bosons via such $\gamma\gamma$ collisions, and that backgrounds to their detection in $q\bar{q}$ and $ZZ$ decay channels are not serious. In $q\bar{q}$ channels, $\gamma\gamma \rightarrow q\bar{q}$ production can be suppressed in the $m_{\text{Higgs}} \gg 2m_q$ limit by appropriate choices for the helicities of the incoming photons. The $ZZ$ channel in which one of the $Z$’s decays to $l^+l^-$ is virtually background free, and observation of even a handful of events would constitute an adequate signal. In this paper, we demonstrate that the ability to control the polarizations of back-scattered photons provides a powerful means for exposing the CP properties of any single neutral Higgs boson that can be produced with reasonable rate. In particular, we find that there are three polarization asymmetries which are only non-zero if the Higgs boson is not a pure CP eigenstate, and which could well be large enough to be measurable. We will compute the maximal effects achievable in a general 2HDM while maintaining a sizeable production rate.

The basic physics behind our techniques is well-known.\textsuperscript{[1]} A CP-even scalar couples to $\gamma_1 \gamma_2$ via $F_{\mu\nu}F^{\mu\nu}$, yielding (in the center of mass of the two photons) a coupling strength proportional to $e \cdot \tilde{e}$, while a CP-odd scalar couples via $F_{\mu\nu}\tilde{F}^{\mu\nu}$, implying a coupling proportional to $(e \times \tilde{e})_z$. \textsuperscript{[Quantities without (with) a tilde belong to $\gamma_1$ ($\gamma_2$).]} In the helicity basis, we employ conventions such that (for $\gamma_1$ moving in the $+z$ direction and $\gamma_2$ moving in the $-z$ direction)

$$e_\pm = \mp \frac{1}{\sqrt{2}}(0, 1, \pm i, 0), \quad \tilde{e}_\pm = \mp \frac{1}{\sqrt{2}}(0, -1, \pm i, 0).$$

For these choices we find ($\lambda, \tilde{\lambda} = \pm 1$):

$$e \cdot \tilde{e} = -\frac{1}{2}(1 + \lambda\tilde{\lambda}), \quad (e \times \tilde{e})_z = \frac{1}{2}i\lambda(1 + \lambda\tilde{\lambda}).$$

We write the general amplitude for a mixed-CP state $\phi$ to couple to $\gamma\gamma$ as $M = e \cdot \tilde{e} + (e \times \tilde{e})_z o$, where $e$ ($o$) represents the CP-even (-odd) coupling strength. Then using Eq. (2) the helicity amplitude squares and interferences of interest are:

$$|M_{++}|^2 + |M_{--}|^2 = 2(|e|^2 + |o|^2), \quad 2\text{Re}(M^*_{--}M_{++}) = 2(|e|^2 - |o|^2),$$

$$|M_{++}|^2 - |M_{--}|^2 = -4\text{Im}(eo^*), \quad 2\text{Im}(M^*_{--}M_{++}) = -4\text{Re}(eo^*).$$

It is useful to define the three ratios:

$$A_1 \equiv \frac{|M_{++}|^2 - |M_{--}|^2}{|M_{++}|^2 + |M_{--}|^2}, \quad A_2 \equiv \frac{2\text{Im}(M^*_{--}M_{++})}{|M_{++}|^2 + |M_{--}|^2}, \quad A_3 \equiv \frac{2\text{Re}(M^*_{--}M_{++})}{|M_{++}|^2 + |M_{--}|^2}. \quad (4)$$

Note that $A_1 \neq 0$, $A_2 \neq 0$, and $|A_3| < 1$ only if \textit{both} the even and odd CP couplings $e$ and $o$ are present. For a CP-even (-odd) eigenstate $\mathcal{A}_1 = \mathcal{A}_2 = 0$ and $\mathcal{A}_3 = +1 (-1)$. In the following we demonstrate how to probe these three ratios, and we will compute their magnitudes in a general CP-violating 2HDM.
The event rate for $\gamma\gamma$ production of any final state can be written in the form

$$dN = dL_{\gamma\gamma} \sum_{i,j=0}^{3} \langle \zeta_i \tilde{\zeta}_j \rangle d\sigma_{ij},$$

(5)

where $dL_{\gamma\gamma}$ is the luminosity for two-photon collisions, the $\zeta_i$ ($\tilde{\zeta}_j$) are the Stokes polarization parameters (with $\zeta_0 = \tilde{\zeta}_0 \equiv 1$) for $\gamma_1$ ($\gamma_2$), and $\sigma_{ij}$ are the corresponding cross sections. $\zeta_2$ and $\tilde{\zeta}_2$ are the mean helicities of the two photons, while $l = \sqrt{\zeta_1^2 + \zeta_3^2}$ and $\tilde{l} = \sqrt{\tilde{\zeta}_1^2 + \tilde{\zeta}_3^2}$ are their degrees of linear polarization. $dL_{\gamma\gamma}$ and $\langle \zeta_i \tilde{\zeta}_j \rangle$ are obtained as functions of the $\gamma\gamma$ center-of-mass energy, $W$, by averaging over collisions, including a convolution over the energy spectra for the colliding photons. $^*$ Expressing the $\sigma_{ij}$ in terms of the helicity amplitudes, we obtain in the case of Higgs boson production:

$$dN = dL_{\gamma\gamma} d\Gamma \frac{1}{4} (|M_{++}|^2 + |M_{--}|^2) \left\{ (1 + \langle \zeta_2 \tilde{\zeta}_2 \rangle) + (\langle \zeta_2 \rangle + \langle \tilde{\zeta}_2 \rangle) A_1 + (\langle \zeta_3 \tilde{\zeta}_1 \rangle + \langle \zeta_1 \tilde{\zeta}_3 \rangle) A_2 + (\langle \zeta_3 \tilde{\zeta}_3 \rangle - \langle \zeta_1 \tilde{\zeta}_1 \rangle) A_3 \right\},$$

(6)

where $d\Gamma$ represents an appropriate element of the final state phase space, including an initial state flux factor.

The behaviors of $dL_{\gamma\gamma}$ and the $\langle \zeta_i \tilde{\zeta}_j \rangle$ as functions of $W$ are crucial to our considerations. Using standard results for Compton scattering, it is shown in Ref. [18] that both quantities depend sensitively upon the polarizations of the incoming electron and laser beams. In order to understand these convolution-weighted quantities, it will be useful to first discuss the energy spectrum and Stokes parameters for an individual back-scattered photon, obtained after integrating only over the azimuthal angles for its emission. These are determined by four functions. In particular, the energy spectrum for $\gamma_1$ is directly proportional to a function $C_{00}$, while its Stokes parameters take the form $\zeta_i = C_{i0}/C_{00}$, where

$$C_{00} = \frac{1}{1 - y} + 1 - y - 4r(1 - r) - 2\lambda_e P_c x (2r - 1)(2 - y)$$

$$C_{20} = 2\lambda_e x [1 + (1 - y)(2r - 1)^2] - P_c (2r - 1) \left[ \frac{1}{1 - y} + 1 - y \right]$$

$$C_{30} = 2r^2 P_t \sin 2\kappa, \quad C_{30} = 2r^2 P_t \cos 2\kappa.$$  

In Eq. (7) $P_c$ ($P_t$) is the degree of circular (transverse) polarization of the initial

* We note that the definition of $\langle \ldots \rangle$ employed here is not the same as that of Ref. [18], Eq. (29), which does not include a convolution at fixed $W$. Our definition of $\langle \ldots \rangle$ is that implicitly employed in Figs. 13-15 of Ref. [20].
laser photon ($P^2_c + P^2_t \leq 1$), $\kappa$ is the azimuthal angle of the direction of its maximum linear polarization, and $\lambda_e$ is the mean helicity of the electron off of which the photon is scattered; note that any sensitivity to the transverse polarization of the initial electron has vanished after the azimuthal emission angle integration. The quantities $r$, $x$ and $y$ are defined by

$$
r = \frac{y}{x(1 - y)}, \quad y = \frac{\omega}{E}, \quad x = \frac{4E\omega_0}{m_e^2} \approx 15.3 \left( \frac{E}{\text{TeV}} \right) \left( \frac{\omega_0}{eV} \right),$$

where $\omega$ ($\omega_0$) is the final (initial) photon energy, and $E$ is the initial electron energy. The maximum value of $y$ is $y_{max} = x/(1 + x)$, in which limit $r = 1$. All these same quantities as related to the second back-scattered photon will be denoted with a tilde.

Given that the mass of the Higgs boson is not likely to be near to the $e^+e^-$ center-of-mass energy, a flat luminosity spectrum as a function of $W$ is best for Higgs boson searches. This means that a flat energy spectrum as a function of $y$ is preferred for the individual photons, and to study the CP properties of the Higgs boson large values of the $\zeta_i$ ($i = 1, 2, 3$) are required. To simultaneously achieve both, it turns out that the two extreme choices of full circular and full transverse polarization for the initial laser photon are most useful. Consider first $2\lambda_c P_c = \pm 1$, i.e. full circular polarization for the initial laser photon and maximal average helicity for the incoming electron. For $2\lambda_c P_c = -1$, $C_{00}$ (and, consequently, the photon spectrum) is peaked as a function of $y$ just below $y_{max}$, whereas for $2\lambda_c P_c = +1$ one finds a rather flat (and, hence, more desirable) spectrum over a broad range of $y$ falling sharply to 0 as $y \to y_{max}$. The behaviors of the $\zeta_i$ follow from Eq. (7). For $P_c = \pm 1$, $P_t$ must be zero and only $\zeta_2$ (and $\zeta_0 \equiv 1$) can be non-zero. The choice of $|2\lambda_c P_c| = 1$ allows $\zeta_2$ to be maximal; one finds $\zeta_2 \to +P_c, -P_c$ for $y \to 0, y_{max}$, respectively. In the case of $2\lambda_c P_c = +1$ (preferred for Higgs studies), $\zeta_2 \sim +P_t$ over almost the entire $y$ range; only very near to $y = y_{max}$ does $\zeta_2$ change sign and approach $-P_c$. This is highly desirable behavior for isolating $\mathcal{A}_1$. In contrast, in the case of $2\lambda_c P_c = -1$, associated with a peaked energy spectrum, $\zeta_2$ slowly switches sign in the middle of the allowed $y$ range. Thus, we have a very fortunate conspiracy in which the $2\lambda_c P_c = +1$ choice which yields the best photon energy spectrum for study of a Higgs boson, also yields nearly 100% circular polarization for the back-scattered photon’s polarization for most $y$ values.

The other extreme choice for the incoming laser beam polarization is to take $P_c = 0, P_t = 1$. In this case, $C_{00}$ is independent of $\lambda_c$ and varies slowly as a function of $y$ over the entire range $y = 0$ to $y = y_{max}$. Eqs. (6) and (7) make it apparent that large $l$ will be required in order to measure $\mathcal{A}_2$ and $\mathcal{A}_3$. For $P_t = 1$, the linear polarization, $l = \sqrt{\zeta_1^2 + \zeta_3^2}$, vanishes at $y = 0$ ($l \approx y^2/x^2$) and is rather small until $y \gtrsim y_{max}/2$; as $y \to y_{max}$, $l$ approaches a maximum of $l_{max} = 2/[(1+x)+(1+x)^{-1}]$. Obviously, to maximize $l$ it would be highly advantageous to have a machine design with as small a value of $x$ as possible. It is also useful to note that if $|2\lambda_c| = 1$ then $\zeta_2$ goes from 0 at $y = 0$ to a maximum of $|\zeta_2|_{max} = \sqrt{1-l^2}$ at $y = y_{max}$. 


For typical values of \( x \) (e.g. of order 2 – 4) \( |\zeta_2|_{\text{max}} \) can be sufficiently large that it would be useful in suppressing \( q\bar{q} \) backgrounds.

Of course, to isolate \( \mathcal{A}_1, \mathcal{A}_2 \) and \( \mathcal{A}_3 \), it is necessary to consider the polarizations of both of the back-scattered photons. When good circular polarization for both laser beams is available, \( \mathcal{A}_1 \) would be most easily isolated by making the wide-spectrum choices of \( 2\lambda_e P_c = +1 \) and \( 2\lambda_e \bar{P}_c = +1 \). From Eq. (6), we see that the term proportional to \( \mathcal{A}_1 \) changes sign if we reverse the sign of all of the helicities of the incoming electrons and laser beams — \( \lambda_e, \lambda_e, P_c \) and \( \bar{P}_c \) — thereby keeping \( 2\lambda_e P_c \) and \( 2\lambda_e \bar{P}_c \) fixed at +1 while reversing the sign of both \( \zeta_2 \) and \( \tilde{\zeta}_2 \). To determine \( \mathcal{A}_2 \) and \( \mathcal{A}_3 \) we would take \( P_t = 1 \) and \( \bar{P}_t = 1 \) and note that the coefficient of \( \mathcal{A}_2 \) (\( \mathcal{A}_3 \)) is proportional to \( \langle \tilde{l}\tilde{l} \rangle \sin 2(\kappa + \kappa) \) (\( \langle \tilde{l}\tilde{l} \rangle \cos 2(\kappa + \kappa) \)). \( \mathcal{A}_2 \) could thus be isolated by taking the difference of cross sections for \( \kappa + \kappa = +\pi/4 \) and \(-\pi/4 \), while the difference of cross sections for \( \kappa + \kappa = 0 \) and \( \pi/2 \) would determine \( \mathcal{A}_3 \).

A convenient explicit form for the number of Higgs boson events is obtained by normalizing to the two-photon decay width of the Higgs boson obtained after summing over final state photon polarizations. From Refs. [19] and [20] the number of Higgs bosons, \( N_\phi \), produced after averaging over colliding photon polarizations is:

\[
N_\phi = \frac{dL_{\gamma\gamma}}{dW} \bigg|_{W=m_\phi} \frac{4\pi^2\Gamma(\phi \to \gamma\gamma)}{m_\phi^2} \left( \frac{E_{ee}}{\text{GeV}} \right)^{-1} \left( \frac{\Gamma(\phi \to \gamma\gamma)}{\text{KeV}} \right) \left( \frac{m_\phi}{\text{GeV}} \right)^{-2} F(m_\phi),
\]

where \( F(W) = (E_{ee}/L_{ee})dL_{\gamma\gamma}/dW \) is a slowly varying function whose value depends upon details of the machine design, but is \( \mathcal{O}(1) \). In Eq. (9), \( E_{ee} \) and \( L_{ee} \) are the \( e^+e^- \) machine energy and integrated luminosity. For the case of interest, where some of the Stokes parameters have non-zero average values, Eq. (9) is modified by the curly bracket appearing in Eq. (6), with \( \zeta_i \) and \( \tilde{\zeta}_i \) replaced by \( \langle \zeta_i \rangle \) and \( \langle \tilde{\zeta}_i \rangle \), etc. All such averages depend upon the \( \gamma\gamma \) invariant mass, \( W \). For instance, for \( \mathcal{A}_1 = \mathcal{A}_2 = 0 \), and \( P_t = \bar{P}_t = 0 \), the expressions for \( N_\phi \) in Eq. (9) are multiplied by \( (1 + \langle \zeta_2\tilde{\zeta}_2 \rangle(m_\phi)) \).

Typical behaviors of \( F(W) \) and \( \langle \zeta_2\tilde{\zeta}_2 \rangle(W) \) are amply illustrated in Ref. [20] for the case of \( 2\lambda_e P_c \simeq +1 \) (i.e. \( P_t = 0 \)) upon which we shall focus. The most important points to note are the following.

(a) A broad spectrum for \( F(W) \), advantageous for Higgs studies, can be achieved using \( 2\lambda_e P_c \) and \( 2\lambda_e \bar{P}_c \) both as close to +1 as possible.

(b) For \( 2\lambda_e P_c \sim +1 \), \( 2\lambda_e \bar{P}_c \sim +1 \) and \( P_c \bar{P}_c \sim +1 \), \( \langle \zeta_2\tilde{\zeta}_2 \rangle(W) \) is near to +1 for \( W \) up to 50%–70% of \( E_{ee} \). This means that for \( m_\phi \lesssim 70\% E_{ee} \), Higgs boson
Reduced couplings for the charged leptons follow those for $bF$ charges, and leptons, we illustrate by displaying the expressions for fermion. Although our computations include fermion loops from all quarks and $\tau$'s derive (at the one-loop triangle diagram level) from $i = \text{charged fermion}$. Although our computations include fermion loops from all quarks and leptons, we illustrate by displaying the expressions for $e$ and $o$ keeping only $b$, $t$, $W$, and $H^+$ triangles. In this case, we have

\begin{align}
e &= N_c e_t \tau_1 F_{1/2}^s (\tau_1) + N_c e_b \tau_5 F_{1/2}^s (\tau_5) + s_{W+} W^{-} F_{1} (\tau_{W^+}) + s_{H^+} H^{-} F_{0} (\tau_{H^+}) , \\
o &= N_c e_t \tau_1 P_{1/2}^p (\tau_1) + N_c e_b \tau_5 P_{1/2}^p (\tau_5) , (12)
\end{align}

where $N_c = 3$ for quarks, $e_t = 2/3$ and $e_b = -1/3$ are the $t$ and $b$ fractional charges, and $\tau_i \equiv 4m_i^2/m_\phi^2$. The $F_i$'s are those defined in Appendix C of Ref. [1]. We remind the reader that for large $\tau$, $F_1 (\tau) \rightarrow 7$, $F_0 (\tau) \rightarrow -1/3$, $F_{1/2}^s (\tau) \rightarrow -4/3$, and $F_{1/2}^p (\tau) \rightarrow -2$; note in particular the small size of $F_0 (\tau)$ in this limit. The reduced CP-even (scalar, $s$) and CP-odd (pseudoscalar, $p$) couplings are given by

\begin{align}s_{\tau f} &= \frac{u_2}{\sin \beta} , \\
p_{\tau f} &= -u_3 \cot \beta , \\
s_{W^+ W^-} &= \frac{u_1}{\cos \beta} , \\
p_{W^+ W^-} &= -u_3 \tan \beta , (13)
\end{align}

Reduced couplings for the charged leptons follow those for $b\bar{b}$, while in Eq. (12) one

\[ \Gamma(\phi \rightarrow \gamma\gamma) = \frac{\alpha^2 g^2 m_\phi^2}{1024\pi^3 m_W^2} \left( |e|^2 + |o|^2 \right) , \]
would have $N_c = 1$ and charge $-1$. In the above, the $u_i$ specify the eigenstate $\phi$ in the $\Phi_i$ basis of Ref. [3] (see Ref. [13] for more details). In a 2HDM, $\sum_i u_i^2 = 1$, but they are otherwise unconstrained. Results for the SM Higgs boson correspond to taking $u_1 = \cos \beta$, $u_2 = \sin \beta$, and $u_3 = 0$. More generally, for a CP-even eigenstate we would have $u_3 = 0$, while for a CP-odd eigenstate $|u_3| = 1$. We note that the branching ratios for the $\phi$ to decay to $b\bar{b}, t\bar{t}, W^+W^-, ZZ$, etc. are determined using these same reduced couplings by appropriately weighting the results for CP-even and CP-odd scalars as given in Appendix B of Ref. [1]. In Eq. (13) we have not given an expression for $s_{H^+H^-}$. The most general possibility is rather complicated, and there is a great deal of freedom in its magnitude. However, it is proportional to $m_W^2/m_{H^+}^2$ and will not be large if $m_{H^+}$ is large, unless the mass of one of the other neutral Higgs bosons is much larger than $m_{H^+}$. Because of the very uncertain value of $s_{H^+H^-}$, the reasonable probability that it will be small, and the fact that it enters with a relatively small coefficient from $F_0(\tau_H)$, we neglect the charged Higgs loop in the numerical estimates to be given later.

Of course, it is also necessary to understand the structure of possible backgrounds to our Higgs boson signal. Consider the case of the $b\bar{b}$ background with $m_\phi \gg 2m_b$. In this limit, the amplitudes $M_{++}$ and $M_{--}$ for $\gamma\gamma \rightarrow b\bar{b}$ may be neglected. Further, parity invariance implies that $M_{-+} = M_{+-}$. We then obtain as the form of the background corresponding to that of Eq. (6) for the Higgs boson:

$$dN = dL_{\gamma\gamma} d\Gamma_{1/2} |M_{+-}|^2 \left(1 - \langle \zeta_2 \tilde{\zeta}_2 \rangle + \langle \zeta_3 \tilde{\zeta}_3 \rangle + \langle \zeta_1 \tilde{\zeta}_1 \rangle \right), \quad (14)$$

Note that if the colliding photons have perfect circular polarization with $\zeta_2 = \tilde{\zeta}_2 = \pm 1$ (in which case $\zeta_1 = \zeta_3 = \tilde{\zeta}_1 = \tilde{\zeta}_3 = 0$), then this background from $b\bar{b}$ production can be essentially eliminated.\footnote{Even if perfect polarization cannot be achieved, $\gamma\gamma \rightarrow b\bar{b}$ can be dramatically suppressed by requiring that the $b$ and $\bar{b}$ not emerge close to the beam direction.} Most importantly for determining the CP properties of the $\phi$, we note that in Eq. (14) there are no terms containing the same $\zeta/\tilde{\zeta}$-dependent factors that multiply $A_1, A_2$ and $A_3$ in Eq. (6). This remains true even if $M_{++}$ and $M_{--}$ cannot be neglected (as, in particular, in the case of the $t\bar{t}$ background). Thus, for example, the background cancels when isolating $A_1$ by simultaneously changing the sign of both $\zeta_2$ and $\tilde{\zeta}_2$.

We will not attempt to study the backgrounds in detail here. Instead, we will use the results of Refs. [19] and [20] to estimate that a Higgs boson could be seen if its polarization-averaged event rate is such that there are at least 80 $b\bar{b}$ decays, or 80 $t\bar{t}$ decays, or 20 clean $ZZ$ (with one $Z \rightarrow l^+l^-$) decays. Of course, a more detailed Monte Carlo would be required to determine accurately the minimal signal as a function of $m_\phi$ that would be required in the various channels. For instance, for $b\bar{b}$ invariant masses below about 50 GeV, 80 Higgs events in the $b\bar{b}$ channel would
not be adequate. In addition, the number of background events depends strongly on the polarization mode employed. As we have already noted, the $b\bar{b}$ background can be greatly suppressed if $2\lambda_e P_c = 2\tilde{\lambda}_e \tilde{P}_c \sim +1$ and $m_\phi$ is below about 70% of $E_{ee}$ (so that $\langle \zeta_2 \tilde{\zeta}_2 \rangle(m_\phi) \sim 1$). This is certainly the appropriate configuration for measuring $A_1$. On the other hand, the maximal achievable $\langle \zeta_2 \tilde{\zeta}_2 \rangle$ for $P_c = \tilde{P}_c = 0$ (so as to maximize $P_\ell$ and $\tilde{P}_\ell$) would not be very near 1 and full suppression of the $b\bar{b}$ background would not be possible when probing $A_2$ and $A_3$.

For our illustrative results, we will adopt a machine energy of $E_{ee} = 0.5$ TeV, an integrated luminosity of $L_{ee} = 20$ fb$^{-1}$, and use the rough value of $F(W) \approx 1$ in Eq. (9) in computing event rates. We will search for the values of $|A_1|$, $|A_2|$, and $|A_3|$ that maximize the observability of CP violation, subject to the minimum event number requirements stated above. This search is performed separately for each of the observables by randomly scanning over all allowed $u_i$ values. We will present our results as functions of $m_\phi$ for various values of $m_t$ and $\tan \beta$. We will not consider $m_\phi$ smaller than 60 GeV. For such low $m_\phi$, not only would the $b\bar{b}$ background be large (as noted above), but also it turns out that existing experimental constraints from $Z$ decays at LEP require that the $\phi W^+W^-$ coupling must be so suppressed that a significant number of $\phi$ cannot be made in $\gamma\gamma$ collisions.$^{\dagger}$

Regarding the statistical significance, $N_{SD}$, of event number differences deriving from the $A_{1,2,3}$, it will be useful to define two standardized scenarios, the first appropriate for measuring $A_1$ and the second for measuring $A_2$ and $A_3$. Suppose the number of events of a given type for unpolarized photons is $N$. For measuring $A_1$ we take $\langle \zeta_2 \rangle \simeq \langle \tilde{\zeta}_2 \rangle = \pm 1$ (this requires $P_c \tilde{P}_c \simeq 1$ and $m_\phi \lesssim 70\%E_{ee}$) and find $2N(1 \pm A_1)$ events. The statistical significance of the asymmetry fluctuation is then $N_{1SD}^1 \equiv \sqrt{2N}/|A_1|$. For measuring $A_2$ and $A_3$, we take $P_\ell \simeq \tilde{P}_\ell \simeq 1$ and adopt the “typical” values of $\langle l \rangle \approx \langle \tilde{l} \rangle \approx 0.4$ (not attained until $m_\phi \gtrsim 60\%E_{ee}$ according to our estimates) and $\langle \zeta_2 \tilde{\zeta}_2 \rangle = 0.5$ (we assume that $2\lambda_e \simeq 2\tilde{\lambda}_e \simeq 1$ so that this latter average is large, in order to suppress $q\bar{q}$ backgrounds, when $l$ and $\tilde{l}$ are large — see earlier discussion). The number of events obtained (after averaging over $2\lambda_e = 2\tilde{\lambda}_e = \pm 1$ for $\kappa + \tilde{\kappa} = +\pi/4, -\pi/4$ (for $A_2$) or for $\kappa + \tilde{\kappa} = 0, \pi/2$ (for $A_3$) is $N[(1 + \langle \zeta_2 \tilde{\zeta}_2 \rangle) \pm \langle \tilde{l} \rangle] A_{2,3}$. Using the typical values stated above we obtain an approximate statistical significance for the event number fluctuation of $N_{SD}^2 \equiv 0.13\sqrt{N}/|A_2|$, in the case of $A_2$. For $A_3$, recall that the signal for CP violation is $|A_3| < 1$. Thus, what matters in this case is whether the magnitude of fluctuation predicted for $|A_3| = 1$ could be distinguished from that associated with some smaller value of $|A_3|$. The statistical significance that we shall associate with this difference will be $N_{SD}^3 \equiv 0.13\sqrt{N}(1 - |A_3|)$.

$^{\dagger}$ Recall that the $W$ loop is the most important contributor to the $\phi\gamma\gamma$ coupling.
Results for $\tan \beta = 2$ and $m_t = 150$ GeV are presented in Figs. 1 and 2. In Fig. 1 we plot the values of $|A_1|$, $|A_2|$ and $1 - |A_3|$ which yield the maximal statistical significance ($N_{SD}^1$, $N_{SD}^2$, or $N_{SD}^3$, as defined above) for detection of each of these three observables. In all cases, we impose the minimal event number requirements stated earlier. In Fig. 2 we plot the corresponding $N_{SD}$ values themselves. The best statistical significances (as plotted) are achieved using the $b\bar{b}$ channel for $m_\phi < 2m_Z$, using the $ZZ$ channel for $2m_Z < m_\phi < 2m_t$, and using the $t\bar{t}$ channel for $m_\phi > 2m_t$. In all cases, the best $u_i$ choices are such that $u_3$ is large, thereby guaranteeing a significant contribution to the CP-odd amplitude, $o$. Also, as discussed in more detail below, in any particular region of $m_\phi$, the $u_i$ that maximize the $N_{SD}$'s have certain preferred relative signs. Otherwise, the $u_i$ choices which yield the extrema plotted are unnoteworthy. Fine tuning is not required to obtain $N_{SD}$ values close to those illustrated. We have also verified that fine-tuning of the parameters of the 2HDM potential (see Ref. [3], Eq. (55)) is not necessary to attain these $u_i$ choices. This remains true even if we demand that $\phi$ is the Higgs eigenstate of lowest mass and that the contribution of the $H^+$ loop to the $\gamma\gamma\phi$ coupling is negligible. Of course, if $\phi$ is not the lightest eigenstate, then decays
Figure 2: The maximum statistical significances $N_{SD}^1$, $N_{SD}^2$, and $N_{SD}^3$ for observing $|A_1|$ (———), $|A_2|$ (− − − −), and $(1 - |A_3|)$ (· · · · · ·), respectively (for the standard scenarios defined in the text), as a function of $m_\phi$. We have taken $\tan \beta = 2$ and $m_t = 150$ GeV. Extrema are obtained for 150,000 random choices of the $u_i$ subject to the requirement that there be at least 80 events in the $b\bar{b}$ decay channel of the $\phi$, or 20 events in the $ZZ$ (one $Z \to t^+t^-$) channel, or 80 events in the $t\bar{t}$ channel when the colliding photon polarizations are averaged over.

such as $\phi \to \phi'\phi''$, $\phi \to \phi'Z$, etc. become possible, and can easily be dominant. Since the CP asymmetries we consider are defined by the initial photon polarizations, presumably these alternative final states would also allow measurement of the associated production rate differences.

According to Fig. 2, at least one of the asymmetries could be observable throughout almost the entire $m_\phi$ range studied. However, we must keep in mind that the $N_{SD}$ values presented, based on the polarization averages assumed for the standard scenarios, are overestimated in certain mass regions. In particular, in the case of $A_1$, for $m_\phi > 2m_t$ $\langle \zeta_2 \rangle \simeq \langle \tilde{\zeta}_2 \rangle = \pm 1$ may not be achievable for $E_{ee} = 0.5$ TeV, since $m_\phi > 70\% E_{ee}$ for much of this range. In the case of $A_2$ and $A_3$, the large linear polarizations characterizing our standard $A_2, A_3$ scenario can only be realized for $m_\phi \gtrsim 60\% E_{ee}$. However, in practice this does not appear to be an important restriction, since viable statistical significances for detection of CP violation via these latter two asymmetries are mostly attained for $m_\phi > 2m_t$, in any case. Thus, determination of the CP properties of the $\phi$ via $A_2$ and $A_3$ will be confined to the $m_\phi > 2m_t$ region.

Let us now discuss the asymmetry observables corresponding to these maxi-
mal $N_{SD}$ values, see Fig. 1. We focus first on $A_1$. $|A_1|$ is generally small and decreases with increasing $m_\phi$ for $m_\phi < 2m_W$. This is because the only significant imaginary contribution to the sum of loop amplitudes comes from the $b$ loop, and this imaginary part declines in magnitude as $\tau_b \ln(4/\tau_b)$. In this region, maximal statistical significance is achieved for values of $u_3 \gtrsim 0.6$, and if $u_1$ and $u_2$ have the same sign. The latter implies that the $\phi WW$ coupling is not small, thereby keeping the basic production rate significant (recall that the $W$ loop generally dominates). Once $m_\phi$ is above the $W^+W^-$ threshold a much larger imaginary part for the CP-even amplitude, $e$, is possible. In order to maximize interference, preferred values of $u_3$ remain large. Further, $u_1$ and $u_2$ continue to have the same sign; this allows for a large $W$ loop imaginary part, large basic production cross section, and large $\phi \to ZZ$ branching ratio. Not surprisingly, the best signal is in the $ZZ$ channel for $2m_Z < m_\phi < 2m_t$. As $m_\phi$ passes beyond $2m_t$, for this case where $m_t$ is significantly larger than $m_W$ it is easier to achieve a large $t$-loop imaginary part than $W$-loop imaginary part. The best statistical significance is achieved if $u_1$ and $u_2$ have opposite signs, so as to suppress the $\phi WW$ coupling. This has a twofold effect: the imaginary part of the $t$ loop is enhanced relative to the overall amplitude magnitude (enhancing the level of interference) and $\phi \to t\bar{t}$ decays can be dominant, thereby allowing for the most significant signal to be found in the $t\bar{t}$ channel.

For this same case of $m_t = 150$ GeV and $\tan \beta = 2$, $A_2$ and $A_3$ are less promising for CP studies, and the reasons behind their behaviors are more complex. We discuss only the $m_\phi > 2m_t$ region, where reasonably large linear polarization is most likely to be achieved. Here, the best $N^2_{SD}$ and $N^3_{SD}$ values are attained using the $t\bar{t}$ decay channel. $u_1$ and $u_2$ are chosen to have appropriately balanced opposite signs (see the form for $s_{W^+W^-}$ in Eq. (13)) such as to (simultaneously) severely suppress the $W$ triangle loop contribution to $e$ and the decays $\phi \to WW, ZZ$. For moderate values of $m_t$, the CP-even and CP-odd amplitudes can then be made comparable because of the dominance of the $t$ loop and $|A_2|$ and $1 - |A_3|$ take on values near unity. Values of $N^2_{SD}$ and $N^3_{SD}$ above 1 are generally possible. Overall, however, the suppression arising from the small value of $\langle |l\bar{l}| \rangle \sim 0.16$ that is present $ab initio$ implies that large statistical significances for these asymmetries would require much larger luminosity. For this reason, the remainder of our discussion will focus on $A_1$.

The achievable statistical significances for $A_1$ (as well as the other asymmetries) depend upon $\tan \beta$ and $m_t$. In Fig. 3 results for the maximal achievable statistical significance (subject to minimum event number requirements) for observation of $A_1$ are given for a variety of other $\tan \beta, m_t$ choices. The corresponding values of $|A_1|$ are given in Fig. 4. We briefly explain the principle features of these plots. First, consider keeping $m_t$ fixed at 150 GeV and increasing $\tan \beta$ to 10. Recall that large $\tan \beta$ suppresses the $t\bar{t}$ coupling and enhances the $b\bar{b}$ coupling (see Eq. (13)). For small $m_\phi$, $|A_1|$ is enhanced compared to small $\tan \beta$ since the (only) imaginary components of $e$ and $o$ come from the $b$ loop and are now much larger. Meanwhile, a large event rate can be maintained through the $W$ loop contribution to $e$. However, for $2m_t \gtrsim m_\phi \gtrsim 2m_W$, even though $\text{Im } e$ can be large due to the $W$ loop, $o$ came
Figure 3: The maximum statistical significances for observing $|A_1|, N_{1SD}^1$ (as defined in the text), as a function of $m_\phi$. Various values for $m_t$ and tan $\beta$ are considered: tan $\beta = 2, m_t = 200$ GeV (———), tan $\beta = 2, m_t = 100$ GeV (−−−−), tan $\beta = 10, m_t = 200$ GeV (⋯⋯⋯⋯⋯⋯), tan $\beta = 10, m_t = 150$ GeV (⋯⋯⋯⋯⋯⋯), and tan $\beta = 10, m_t = 100$ GeV (⋯⋯⋯⋯⋯⋯). Extrema are obtained as described for Fig 2. Curves terminate when the minimal event number requirements can no longer be met.

mainly from the $t$ loop at moderate tan $\beta$, which loop is now severely suppressed by the large tan $\beta$ value. For $m_\phi > 2m_t$, $\tau_W$ is getting small and the $W$ loop contribution to event rate cannot be kept large. Since the $t$ loop contribution to the $\phi$ production rate and $B(\phi \to t\bar{t})$ are also both suppressed, it becomes impossible to maintain minimal event rates in either the $ZZ$ or $t\bar{t}$ channels. Thus, detection of $A_1$ for $m_\phi \gtrsim 2m_W$ becomes difficult at large tan $\beta$.

Keeping tan $\beta = 2$ and increasing $m_t$ from 150 GeV to 200 GeV simply moves the $m_\phi = 2m_t$ threshold (beyond which the $t$ loop and $t\bar{t}$ channel dominate $A_1$ and $\phi$ decays, respectively) to a higher value. The systematics behind the behaviors of $N_{SD}^1$ and $|A_1|$ are very much as described above for tan $\beta = 2$ and $m_t = 150$ GeV. Lowering $m_t$ to 100 GeV at tan $\beta = 2$ means that for $m_\phi \gtrsim 200$ GeV the $e$ and $\phi$ amplitudes can both have large imaginary parts. Thus, for $m_\phi \sim 200$ GeV both $N_{SD}^1$ and the corresponding $|A_1|$ are much larger than for $m_t = 150$ GeV. However, since both $\tau_W$ and $\tau_t$ become small as $m_\phi$ increases further, the $W$- and $t$-loop contributions to $e$ and $\phi$ (and event rates) fall, and both $N_{SD}^1$ and $|A_1|$ decline rapidly. Beyond a certain point ($m_\phi \sim 275$ GeV) there are simply not enough $ZZ$
Figure 4: The values for $|A_1|$ which yield the largest $N_{SD}^1$ values as a function of $m_\phi$. Various values of $m_t$ and $m_\phi$ are illustrated: $\tan \beta = 2, m_t = 200$ GeV (———), $\tan \beta = 2, m_t = 100$ GeV (− − − −), $\tan \beta = 10, m_t = 200$ GeV (· · − · ·), $\tan \beta = 10, m_t = 150$ GeV (· − · −), and $\tan \beta = 10, m_t = 100$ GeV (· · · · · ·). Extrema are obtained as described for Fig. 2. Curves terminate when the minimal event number requirements can no longer be met.

In summary, of the three possible CP-sensitive polarization asymmetries, $A_1$ provides the best opportunities for studying the CP properties of a neutral Higgs boson produced in $\gamma\gamma$ collisions of polarized back-scattered laser photons. A non-zero value for $A_1$ requires that the $\phi\gamma\gamma$ coupling have an imaginary part, as well as both CP-even and CP-odd contributions. For a mixed-CP Higgs boson with $m_\phi \lesssim 2m_W$, measurement of $A_1$ will be easiest if $\tan \beta$ is large since the $b$ loop, which makes the only large contribution to the imaginary part for such $m_\phi$ values, will be enhanced. For $m_\phi > 2m_W$, the required imaginary part is dominated by the $W$ loop (or $t$-loop if $m_\phi$ is also $> 2m_t$). Large $\tan \beta$ makes detection of $A_1$ in this region more difficult since the dominant CP-odd contribution derives from the $t$ loop, which will be suppressed. Nonetheless, it is clear from our analysis that collisions of polarized back-scattered laser photons will provide a significant opportunity for determining the CP properties of any neutral Higgs boson that can be produced with reasonable event rate. Certainly, a substantial effort should be made to design a machine with maximal polarization for the incoming electrons and laser beams and the highest possible luminosity.
Acknowledgements

We are grateful to D. Borden, D. Caldwell, and T. Barklow for discussions regarding the degree of polarization achievable for the backscattered photons. One of us (JFG) would like to thank the CERN theory group for support during the initial stages of this work. BG would like to thank U.C. Davis for support during the course of this research.

REFERENCES

1. For a review of Higgs bosons, see J.F. Gunion, H.E. Haber, G. Kane, S. Dawson, The Higgs Hunters Guide, Addison Wesley (1990).
2. S. Weinberg, Phys. Rev. Lett. 63 (1989) 2333.
3. S. Weinberg, Phys. Rev. D42 (1990) 860.
4. J.F. Gunion and D. Wyler, Phys. Lett. B248 (1990) 170.
5. C.Q. Geng and J.N. Ng, Phys. Rev. D42 (1990) 1509.
6. A. De Rujula, M.B. Gavela, O. Pene and F.J. Vegas, Phys. Lett. B245 (1990) 690.
7. I. Bigi and N.G. Uraltsev, Nucl. Phys. B353 (1991) 321.
8. S.M. Barr and A. Zee, Phys. Rev. Lett. 65 (1990) 21.
9. J.F. Gunion and R. Vega, Phys. Lett. B251 (1990) 157.
10. R.G. Leigh, S. Paban, and R.-M. Xu, Nucl. Phys. B352 (1991) 45.
11. D. Chang, W.-Y. Keung, and T.C. Yuan, Phys. Rev. D43 (1991) 14.
12. W. Bernreuther and M. Suzuki, Rev. Mod. Phys. 63 (1991) 313.
13. B. Grzadkowski and J.F. Gunion, preprint UCD-92-7 (1992), to be published in Phys. Lett. B.
14. C. Schmidt and M. Peskin, preprint SLAC-PUB-5788 (1992).
15. A. Mendez and A. Pomarol, Phys. Lett. B272 (1991) 313.
16. C.A. Nelson, Phys. Rev. D30 (1984) 1937 (E: D32 (1985) 1848); J.R. Dell’Aquila and C.A. Nelson, Phys. Rev. D33 (1986) 80,93; Nucl. Phys. B320 (1989) 86.
17. H.F. Ginzburg, G.L. Kotkin, V.G. Serbo, and V.I. Telnov, Nucl. Inst. and Meth. 205 (1983) 47.
18. H.F. Ginzburg, G.L. Kotkin, S.L. Panfil, V.G. Serbo, and V.I. Telnov, Nucl. Inst. and Meth. 219 (1984) 5.
19. J.F. Gunion and H.E. Haber, preprint UCD-90-25 (September, 1990); to appear in the Proceedings of the 1990 DPF Summer Study on High Energy Physics, Snowmass, July 1990.
20. D.L. Borden, D.A. Bauer, and D.O. Caldwell, preprint SLAC-PUB-5715 (January 1992).