Convection of Viscoplastic Fluid in Square Cavity Heated from the Sidewalls

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Abstract. The paper is devoted to the numerical study of two-dimensional convective motion of a viscoplastic fluid in a closed region under lateral heating. The problem is solved numerically using the ANSYS Fluent software. The Bulkeley-Herschel model describing viscoplastic behavior was chosen as a non-Newtonian fluid model. Under certain rheological parameters, this model is reduced to a Newtonian fluid model, the behavior of which is also investigated as a limit case. Based on the results of the calculations, the dependence of the maximum value of the stream function in the cavity on the Rayleigh number is plotted and the evolution of the unyielded zones with an increase in the Rayleigh number is traced. The threshold value of the Rayleigh number, at which a sharp growth of the flow intensity is observed, has been determined. The calculated threshold value of the Rayleigh number for the Bulkeley-Herschel liquid was found to be very close to the threshold value of the Rayleigh number at which the flow of the Bingham liquid arises, which was obtained analytically.

1. Introduction

The present study is devoted to the convection of a non-Newtonian fluid in the square cavity heated from the sidewalls. Non-Newtonian fluids are actively used in chemical, oil refining, automotive and aviation industries. Engine oils used in car engines and aircraft engine gearboxes have non-Newtonian fluid properties, forming a thin layer on the walls of the unit, reducing friction between the parts. Research on the flow of non-Newtonian fluids are important from the scientific and engineering points of view.

There are many papers in the literature devoted to investigation of flows and heat and mass transfer in non-Newtonian fluids, for example, the paper [1] investigates heat transfer when non-Newtonian fluids flow in pipes of different diameters. Analysis of fluid behavior showed that temperature distribution for all variants of pipe geometry is similar. In the center of the pipe the temperature is low, on the pipe walls - high. Galullina et al. [2] investigated heat transfer during non-Newtonian fluids flow in heat exchanger channels. It was found that non-Newtonian fluid in a convergent-divergent channel is cooled more effectively than in a direct channel. According to the authors, this effect is explained by the presence of vortices in non-Newtonian fluid in the convergent-divergent channel, which contribute to more rapid heat transfer inside the fluid.

Convection of non-Newtonian fluids in closed cavities was first studied in [3, 4]. In [4] within the framework of the regularized Williamson rheological model, which allows to carry out calculations in a unified way in the whole cavity, the convection of a viscoplastic fluid under heating from the sidewalls was studied. It was found that, unlike the case of Newtonian fluid, at small values of Rayleigh number only weak convective flow is observed, and when some value of Rayleigh number Ra* is reached, sharp growth of the flow intensity is observed. The structure of unyielded zones at different Rayleigh numbers has been determined. A variational principle is formulated with the help of which the threshold value of the Rayleigh number Ra*, at which convection of Bingham fluid arises in a square cavity heated from the sidewalls, is found.
2. Statement of the problem

Consider the two-dimensional problem of convective flow of a viscoplastic fluid in a square cavity heated from the sidewalls in the gravity field (figure 1).

![Figure 1. Geometry of the computational domain](image)

Let us restrict ourselves to consideration of two-dimensional flow. To describe thermal buoyancy convection we use the Boussinesq approximation [5]:

\[\frac{\partial \rho}{\partial t} + \rho \frac{\partial \mathbf{u}}{\partial x} + \rho \mathbf{v} \frac{\partial \mathbf{u}}{\partial y} = - \frac{\partial p}{\partial x} + \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right) \]

\[\frac{\partial \rho \mathbf{v}}{\partial t} + \rho \mathbf{u} \frac{\partial \rho \mathbf{v}}{\partial x} + \rho \mathbf{v} \frac{\partial \rho \mathbf{v}}{\partial y} = - \frac{\partial p}{\partial y} + \left( \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \right) + \rho g \beta (T + T_c) \]

\[\frac{\partial \mathbf{u}}{\partial x} + \frac{\partial \mathbf{v}}{\partial y} = 0 \]

\[\rho C_p \frac{\partial T}{\partial t} + \rho u C_p \frac{\partial T}{\partial x} + \rho v C_p \frac{\partial T}{\partial y} = \kappa \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \]

Here, \( \mathbf{u} \) is x-component of velocity, \( \mathbf{v} \) is y-component of velocity, \( \rho \) is the density, \( p \) is the pressure, \( g \) is the gravitational acceleration, \( \beta \) is the coefficient of thermal expansion, \( C_p \) is the specific heat capacity, \( T \) is the temperature, \( \kappa \) is the thermal conductivity, \( \tau_{xx}, \tau_{xy}, \tau_{yy} \) are the components of the viscous stress tensor.

To describe the viscoplastic behavior of the fluid, the Bulkeley-Herschel model is used [6]:

\[\bar{\tau} = \mu_0 \bar{\gamma}, \text{при} \gamma < \gamma_0 \]

\[\bar{\tau} = \left( \frac{\tau_0}{\gamma} + K \cdot \bar{\gamma}^{n-1} \right) \bar{\gamma}, \text{при} \gamma > \gamma_0 \]

\[\gamma_0 = \frac{\tau_0}{\mu_0} \]

\[\gamma = \left[ \frac{1}{2} \kappa \tau \bar{\gamma} \right]^{\frac{1}{2}} \]

where \( \tau_0 \) is the yield stress, \( \gamma \) is the shear rate, \( K \) is the consistency coefficient, \( n \) is the nonlinearity index, \( \mu_0 \) is the yield viscosity. At very low shear rates \( \gamma < \gamma_0 \), the fluid viscosity \( \mu_0 \) is very high.

The rheological curve of the Bulkeley-Herschel fluid is shown in figure 2 for the case \( n=1 \). In this case, the Bulkeley-Herschel model corresponds to a piecewise linear rheological curve, and at \( \gamma_0 \to 0 \), it is transformed into the Bingham model.
Table 1 shows possible variants of rheological models obtained at different values of the rheological parameters of the Bulkley-Herschel model.

Table 1. Rheological models obtained at different values of the rheological parameters of the Bulkley-Herschel model

| Liquid          | τ₀ | K   | N   |
|-----------------|----|-----|-----|
| Newtonian       | = 0 | Newtonian viscosity  |
| Power-law       | = 0 | consistency coefficient  |
| Dilatant        | = 0 | consistency coefficient  |
| Casson Model    | > 0 | consistency coefficient  |
| Bingham Model   | > 0 | plastic viscosity  |

At the vertical rigid boundaries of the cavity, constant different temperatures were set. The horizontal boundaries were considered as rigid, with no-slip condition and the temperature changes according to a linear law.

\[ x = 0: u = 0, T = T_c \]  
\[ x = L: u = 0, T = T_u \]  
\[ T_u > T_c \]  
\[ y = 0: v = 0, T = \frac{T_u - T_c}{L} x + T_c \]  
\[ y = H: v = 0, T = \frac{T_u - T_c}{L} x + T_c \]

3. Numerical method

The problem was solved numerically using the ANSYS Fluent software. To solve the system of equations in Fluent, the First Order Upwind discretization scheme was selected - the discretization scheme of the first order of accuracy. The time step was chosen to be 0.1s.

The mesh for the computational domain was uniform, it was constructed using the ICEM CFD mesh builder with the size of 100x100 cells, the cell size - 0.1 mm.

The calculations were performed at fixed values of the parameters \( \rho, g, \beta, C_p, k, \mu_0, \tau_0, n \):  
\( \rho = 998 \text{ kg/m}^3; \ g = 9.81 \text{ m/s}^2; \ \beta = 0.0015 \text{ K}^{-1}; \ C_p = 4183 \text{ J/kg} \cdot \text{K}; \ k = 0.599 \text{ W/m} \cdot \text{K}; \ \mu_0 = 10 \text{ kg/m} \cdot \text{s}; \ \tau_0 = 0.000285 \text{ N/m}^2; \ n = 1.0; \ K = 0.01 \). The parameter \( \gamma_0 \) at these parameter values turns out to be 0.0000285 1/s. The Rayleigh number was varied by varying the temperature difference between the vertical boundaries.

Based on the results of the calculations, the dependences of the maximum value of the stream function in the cavity on the Rayleigh number and the fields of the stream function, temperature, and components of the viscous stress tensor were determined. The obtained fields of the components of the
viscous stress tensor were used to determine the unyielded zones. As in [3], the criterion for determining these zones was the condition:

\[ \sqrt{T_2} < \tau_0, T_2 = \frac{1}{2} \tau_{ij} \tau_{ij}, \]

where \( T_2 \) is the second invariant of the viscous stress tensor.

4. Results and discussion

Calculations were carried out for the Rayleigh numbers in the range \( Ra = 0 - 20000 \).

Calculations showed that at all Rayleigh numbers in investigated range in the cavity there is single-vortex flow (figure 3) directed clockwise.

Figure 3. Typical stream function field

Figure 4 shows the fields of the square root of the second invariant of the viscous stress tensor for value \( Ra = 6500 \). As calculations have shown, at these values of Rayleigh number very weak convective flow is observed, the temperature field is close to heat-conductive. As can be seen, the value in the entire cavity is below the yield stress. In the limit case of Bingham fluid this means complete absence of flow, in the case of Bulkley-Herschel fluid there is a flow, but its intensity is very small, the values of \( \gamma \) are smaller \( \gamma_0 \) (see, figure 2). At \( Ra = 6500 \) the situation is similar, only in narrow zones near the centers of the boundaries does the value exceed the yield stress.

At Rayleigh number approximately equal to 6750, there is a sharp growth of the flow intensity (figure 5), which leads to distortion of the temperature field. With further increase of Ra, the
maximum value of the stream function in the cavity grows with the increase of the Rayleigh number according to the law close to linear, which corresponds to the results of [3].

In [3] the variational principle was formulated and with its help the formula for the threshold Rayleigh number as a function of the yield stress for the Bingham fluid $\text{Ra}^* = 32\tau_0$ was obtained. For the rheological and geometrical parameters chosen in the present study, this formula gives the threshold value of the Rayleigh number $\text{Ra}^* \approx 6355$. The calculations carried out in the present study for the Bulkley-Herschel fluid give a threshold value of the Rayleigh number $\text{Ra}^*$ approximately equal to 6750, which is very close to the value obtained in [3].

![Figure 5. Dependence of the maximum value of the stream function on the Rayleigh number](image)

Figure 5 shows the fields of the maximum value of the stream function in the cavity for Rayleigh numbers equal to 6750 and 15000. As can be seen, at $\text{Ra} = 6750$ (figure 6a), a narrow annular zone of yielded flow appears near the boundaries of the cavity, in which $\sqrt{T_2} > \tau_0$ (shaded in red). The entire central cavity is occupied by a unyielded zone in which $\sqrt{T_2} < \tau_0$, in the corners of the cavity there are “stagnant” zones.

As the Rayleigh number increases, the area of the unyielded zone decreases. At $\text{Ra} = 15000$, it, similarly to the results of [3], takes the form of “cross” (figure 6b). At $\text{Ra} = 20000$ the unyielded zone in the center disappears, there are only “stagnant” zones in the corners of the cavity.

![Figure 6. The field of the square root of the second invariant of the viscous stress tensor at](image)

Figure 6. The field of the maximum value of the stream function at $\text{Ra} = 6750$, (b) $\text{Ra} = 15000$
5. Conclusion

The convective flow of a viscoplastic fluid in a square cavity under lateral heating is considered. The fluid behavior was described using the Bulkley-Herschel model.

For different values of the Rayleigh number, the fields of the stream function, temperature and the square root of the second invariant of the viscous stress tensor were obtained. The numerical data on the evolution of the unyielded zones with increasing Rayleigh number are close to those obtained in [3] in the framework of the Williamson model.

The dependence of the maximum value of the stream function in the cavity on the Rayleigh number was plotted. The threshold value of Rayleigh number at which sharp growth of motion intensity is observed has been determined. The obtained in calculations for the Bulkley-Herschel liquid threshold value of Rayleigh number turned out to be very close to the threshold value of Rayleigh number, at which the motion of the Bingham fluid appears, obtained analytically in [3]. Thus, numerical simulation using selected values of rheological parameters of the Bulkley-Herschel liquid allows to simulate accurately enough the occurrence and development of viscoplastic convection of the Bingham liquid in a closed cavity.

References

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