Abstract

We present a class of interacting nonlocal quantum field theories, in which the CPT invariance is violated while the Lorentz invariance is present. This result rules out a previous claim in the literature that the CPT violation implies the violation of Lorentz invariance. Furthermore, there exists the reciprocal of this theorem, namely that the violation of Lorentz invariance does not lead to the CPT violation, provided that the residual symmetry of Lorentz invariance admits the proper representation theory for the particles. The latter occurs in the case of quantum field theories on a noncommutative space-time, which in place of the broken Lorentz symmetry possesses the twisted Poincaré invariance. With such a CPT-violating interaction and the addition of a C-violating (e.g., electroweak) interaction, the quantum corrections due to the combined interactions could lead to different properties for the particle and antiparticle, including their masses.
1 Introduction

Lorentz symmetry and the $CPT$ invariance are two of the most fundamental symmetries of Nature, whose violation has not yet been observed. While the Lorentz invariance is a continuous symmetry of space-time, the $CPT$ involves the discrete space- and time-inversions, $P$, $T$, and the charge conjugation operation on the fields, $C$. Although the individual symmetries, $C$, $P$ and $T$ have been observed to be violated in various interactions, their combined product, $CPT$, remarkably remains still as an exact symmetry. The first proof of $CPT$ theorem was given by Lüders and Pauli [1,2] based on the Hamiltonian formulation of quantum field theory, which involves locality of the interaction, Lorentz invariance and Hermiticity of the Hamiltonian. Later on the theorem was proven by Jost [3] (see also [4–6]) within the axiomatic formulation of quantum field theory without reference to any specific form of interaction. This proof of $CPT$ theorem relaxes the requirement of locality or ”local commutativity” condition to the so-called ”weak local commutativity”. Lorentz symmetry has been an essential ingredient of the proof, both in the Hamiltonian and in the axiomatic proofs.

A simple phenomenological classification of possible $C$, $P$, $T$, $CP$, $PT$, $TC$ and $CPT$-violating effects is presented in [7]. For consequences of $CPT$ and their experimental tests, as well as some theoretical considerations on the possibilities of violation of Lorentz invariance and $CPT$ in the known interactions, we refer to [8–13] and references therein.

It is important to clarify the relation between the $CPT$ and Lorentz invariance and in particular to see whether the violation of any of them implies the violation of the other. This issue has recently become a topical one due to the growing phenomenological importance of $CPT$ violating scenarios, namely in neutrino physics as well as its cosmological and astrophysical consequences. Indeed, the relation between the $CPT$ and Lorentz invariance has acquired a prominent place in nowadays particle physics with the attempts of explaining in a unified manner the contradictory results, ”anomalies”, in the interpretation of various neutrino physics experiments, without enlarging the neutrino sector. The idea was first suggested by Murayama and Yanagida [14] in the form of different masses for neutrino and antineutrino, based on phenomenological considerations. This proposal was formalized as a $CPT$-violating quantum field theory with a mass difference between neutrino and antineutrino in [15] (see also [16]). The issue was taken up in relation with the Lorentz symmetry by Greenberg [17], the conclusion of Greenberg’s analysis being that $CPT$ violation implies violation of Lorentz invariance. This result was given as a ”theorem”, the dispute on the validity of which is the subject of this Letter.

We should emphasize that a theorem which states that $CPT$ violation implies violation of Lorentz invariance has to be explicit, first of all about what is meant by the charge conjugation in a Lorentz violating theory. Is the violation complete or is any subgroup of Lorentz symmetry
left, which should have the needed spin-representations to which the particles are assigned? Does the corresponding theory which violates both \textit{CPT} and Lorentz invariance contain fields with a plausible description in terms of equations of motion?

2 \textit{CPT}-violating free field model

A free field model in which particle and antiparticle have different masses was proposed in \cite{15}. Although the model was hoped to be Lorentz-invariant, a closer examination \cite{17} showed that it is not – the propagator is not Lorentz covariant, unless the masses of particle and antiparticle coincide. The model is also nonlocal and acausal: the \(\Delta(x,y)\)-function, i.e. the commutator of two fields, does not vanish for space-like separation, unless the two masses are the same, thus violating the Lorentz invariance. This was considered in \cite{17} as supporting a general "theorem" that interacting fields that violate \textit{CPT} symmetry necessarily violate Lorentz invariance.

We would like to point out that the model taken in \cite{17} is utmost pathological and can not be considered as a quantum field theory. There, the claim was that the model represents a free complex scalar field, quantized in such a way that the mass of the antiparticle differs from that of the particle. However, there is no definite equation of motion that this "field" satisfies, and no quantization procedure that would support the claim that the mode expansion with different masses for "particle" and "antiparticle" really represents a free quantized field. Also, two such "free fields" separated by a space-like distance do not commute, i.e. the theory is acausal at the free level without invoking interaction.

Moreover, by requiring that the classical symmetries and in particular the global \(U(1)\) symmetry for a free complex scalar field, i.e. the conservation of electric charge, be preserved at the quantum level, one can show that using the expansion for a free "field" as proposed in \cite{17}, would bring it back uniquely to the usual field expansion in terms of creation and annihilation operators with \(m = \bar{m}\) – otherwise, the electric charge is not conserved.

Furthermore, in a quantum field theory with acausal free fields, as taken in \cite{17}, observables, which are functions of those fields, do not commute when separated by space-like distances. This, according to Pauli’s proof of the spin-statistics theorem, implies that there is no spin-statistics relation already for \textit{the free fields}. Thus, one has no rule whether to apply commutation or anticommutation relations in quantizing the fields. But the worst is that in such a model, where Lorentz invariance is violated by the free fields, there is \textit{no concept of spin} to start with altogether.
3 CPT-violating but Lorentz-invariant nonlocal model

Here, as an example, we propose a model which preserves Lorentz invariance while breaking the CPT symmetry through a (nonlocal) interaction. The latter attitude is taken as responsible for the violations of a symmetry, based on our experience that all the discrete, C, P and T invariances, as well as other symmetries, are broken in our description of Nature by means of interaction. We also know that nonlocal field theories appear, in general, as effective field theories of a larger theory.

Consider a field theory with the nonlocal interaction Hamiltonian of the type

$$
\mathcal{H}_{int}(x) = \lambda \int d^4 y \, \phi^*(x)\phi(x)\phi^*(x)\theta(x_0 - y_0)\theta((x - y)^2)\phi(y) + h.c.,
$$

(3.1)

where $\lambda$ is a coupling constant with dimension appropriate for the Hamiltonian density, $\phi(x)$ is a Lorentz-scalar field in the interaction picture and $\theta$ is the Heaviside step function, with values 0 or 1, for its negative and positive argument, respectively. The combination $\theta(x_0 - y_0)\theta((x - y)^2)$ in (3.1) ensures the Lorentz invariance, i.e. invariance under the proper orthochronous Lorentz transformations, since the order of the times $x_0$ and $y_0$ remains unchanged for time-like intervals, while for space-like distances the interaction vanishes. Also, the same combination makes the nonlocal interaction causal at the tree level, which dictates that there is no interaction when the fields are separated by space-like distances and thus there is a maximum speed of $c = 1$ for the propagation of information.

On the other hand, it is clear that C and P invariance are trivially satisfied in (3.1), while T invariance is broken due to the presence of $\theta(x_0 - y_0)$ in the integrand.

One can always insert into the Hamiltonian (3.1), without changing its symmetry properties, a weight function or form-factor $F((x - y)^2)$, for instance of a Gaussian type:

$$
F = \exp \left( -\frac{(x - y)^2}{l^2} \right),
$$

(3.2)

with $l$ being a nonlocality length in the considered theory. Such a weight function would smear out the interaction and would guarantee the desired behaviour of the integrand in (3.1); in the limit of fundamental length $l \to 0$ in (3.2), the Hamiltonian (3.1) would correspond to a local, CPT- and Lorentz-invariant theory. A weight function such as (3.2) would make the acausality of the model (see the next section) restricted only to very small distances, of the order of $l$. The latter could be looked upon as being a characteristic parameter relating the effective field theory to its parent one, for instance the radius of a compactified dimension when the parent theory is a higher-dimensional one. Furthermore, with such a weight function, the interaction vanishes at infinite $(x - y)^2$ separations and thus one can envisage the existence of in- and out-fields.
There exists a whole class of such CPT-violating, Lorentz-invariant field theories involving different, scalar, spinor or higher-spin interacting fields. Typical simplest examples are:

\[ H_{\text{int}}(x) = \lambda \int d^4y \, \phi_1^*(x) \phi_1(x) \theta(x_0 - y_0) \theta((x - y)^2) \phi_2(y) + \text{h.c.} \tag{3.3} \]

\[ H_{\text{int}}(x) = \lambda \int d^4y \, \bar{\psi}(x) \psi(x) \theta(x_0 - y_0) \theta((x - y)^2) \phi(y) + \text{h.c.} \tag{3.4} \]

\[ H_{\text{int}}(x) = \lambda \int d^4y \, \phi(x) \theta(x_0 - y_0) \theta((x - y)^2) \phi^2(y) + \text{h.c.} \tag{3.5} \]

4 Quantum theory of such nonlocal interactions

The S-matrix in the interaction picture is obtained as solution of the Lorentz-covariant Tomonaga-Schwinger equation [18,19] (see also [20,21]):

\[ i \frac{\delta}{\delta \sigma(x)} \Psi[\sigma] = H_{\text{int}}(x) \Psi[\sigma], \tag{4.1} \]

with \( \sigma \) a space-like hypersurface, and the boundary condition:

\[ \Psi[\sigma_0] = \Psi, \tag{4.2} \]

where \( H_{\text{int}} \) is for instance the Hamiltonian (3.3) with the fields in the interaction picture. Then Eq. (4.1) with the boundary condition (4.2) represent a well-posed Cauchy problem.

The existence of a unique solution for the Tomonaga-Schwinger equation is ensured if the integrability condition

\[ \frac{\delta^2 \Psi[\sigma]}{\delta \sigma(x) \delta \sigma(x') - \delta^2 \Psi[\sigma]} = 0, \tag{4.3} \]

with \( x \) and \( x' \) on the surface \( \sigma \), is satisfied. The integrability condition (4.3), inserted into (4.1), requires that the commutator of the interaction Hamiltonian densities vanishes at space-like separation:

\[ [H_{\text{int}}(x), H_{\text{int}}(y)] = 0, \quad \text{for } (x - y)^2 < 0. \tag{4.4} \]

Since in the interaction picture the field operators satisfy free-field equations, they automatically satisfy Lorentz-invariant commutation rules. The Lorentz-invariant commutation relations are such that (4.4) is fulfilled only when \( x \) and \( y \) are space-like separated, \( (x - y)^2 < 0 \), i.e. when \( \sigma \) is a space-like surface. As a result, the integrability condition (4.4) is equivalent to the micro-causality condition for local relativistic QFT. When the surfaces \( \sigma \) are hyperplanes of constant time, the Tomonaga-Schwinger equation reduce to the single-time Schrödinger equation.

Inserting, e.g., the expression (3.5) into (4.4), we have:

\[ [H_{\text{int}}(x), H_{\text{int}}(y)] = \lambda^2 \int d^4a \, d^4b \, \theta((x - a)^2) \theta(x^0 - a^0) \theta((y - b)^2) \theta(y^0 - b^0) \]
×[\phi(x)\phi^2(a) + h.c., \phi(y)\phi^2(b) + h.c.].  

(4.5)

The commutator on the r.h.s. will open up into a sum of products of field at the points \(x, y, a, b\), multiplied by commutators of free fields like \([\phi(x),\phi(y)],[\phi(x),\phi(b)],[\phi(a),\phi(y)],[\phi(a),\phi(b)]\). In order for the commutator (4.5) to vanish, all the coefficients of the products of fields in the expansion have to vanish, since the fields at different space-time points are independent. Clearly, the terms with the coefficient \(\Delta(x - y) = [\phi(x),\phi(y)]\) vanish for \((x - y)^2 < 0\). However, the commutator (4.5) does not vanish for \((x - y)^2 < 0\). In order to show this, it is enough to show that one independent product of fields has nonzero coefficient.

Let us consider the products which contain the fields \(\phi(x),\phi(y),\phi(a),\phi(b)\). A straightforward calculation shows that the terms containing these fields are:

\[
\int d^4a \, d^4b \, \theta((x - a)^2)\theta(x^0 - a^0)\theta((y - b)^2)\theta(y^0 - b^0)2\Delta(a - b)\{\phi(a),\phi(b)\}\phi(x)\phi(y) + h.c. \tag{4.6}
\]

A closer study of the expression (4.6) shows that it does not vanish at space-like distances between \(x\) and \(y\) and thus the causality condition (4.4) is not satisfied.

This, in turn, implies that the field operators in the Heisenberg picture, \(\Phi_H(x)\) and \(\Phi_H(y)\), do not satisfy the locality condition

\[
[\Phi_H(x),\Phi_H(y)] = 0, \quad \text{for } (x - y)^2 < 0, \tag{4.7}
\]

when the quantum corrections are taken into account. This is in accord with the requirement of locality condition (4.7) for the validity of CPT theorem both in the Hamiltonian proof \([1, 2]\) and as well in the axiomatic one \([3]-[6]\), taking into account that there is no example of a QFT, which satisfies the weak local commutativity condition (WLC) but not the local commutativity (LC). For general considerations on the causality and unitarity properties of nonlocal relativistic quantum field theories, we refer to \([22, 23]\) and references therein.

Instead of the description in terms of \(\mathcal{H}_{int}\) and the interaction picture as done above, one can also consider a whole class of CPT-violating, Lorentz-invariant nonlocal quantum field theories described by their actions or Lagrangians. An example, analogous to (3.5) is given by the following action:

\[
S = \int d^4x \left( \frac{1}{2} \partial_\mu \Phi^\mu_H(x) \partial_\mu \Phi_H(x) - \frac{1}{2} m^2 \Phi^2_H(x) - \lambda \int d^4 y \left( \Phi_H(x)\theta(x_0 - y_0)\theta((x - y)^2)\Phi^2_H(y) + h.c. \right) \right), \tag{4.8}
\]

with the corresponding field equation given by

\[
(\Box + m^2)\Phi_H(x) = -\lambda \int d^4 y \theta(x_0 - y_0)\theta((x - y)^2) \left( \Phi^2_H(y) + 2\Phi_H(x)\Phi_H(y) + h.c. \right). \tag{4.9}
\]

Analogous to (3.1)-(3.4), nonlocal actions can be written down in a similar way.
We recall that the relation between the action (or Lagrangian) and the Hamiltonian in a nonlocal relativistic field theory is not so straightforward as in the case of local field theories. For instance, from the action (4.8) does not follow the Hamiltonian given by (3.5) and one should adopt instead a more involved prescription (see, e.g., [22, 23]). The quantum treatment of such theories as well should be performed through the use of Yang-Feldman equation [24] with the fields, denoted by $\Phi_H(x)$, in the Heisenberg picture.

With such a $CPT$-violating interaction as in (3.1)-(3.5) or (4.8), and the addition of a $C$-violating (e.g., electroweak) interaction, the quantum corrections due to the combined interactions could lead to different properties for the particle and antiparticle, including their masses.

5 Lorentz-invariance violating but $CPT$-invariant quantum field theories: Reciprocal theorem

During the last decade, we have learned that the violation of Lorentz invariance does not necessarily lead to the violation of the $CPT$ theorem. The example comes from the quantum field theory on noncommutative space-time (NC QFT) with the canonical, Heisenberg-like, commutation relations for coordinate operators:

$$[x^\mu, x^\nu] = i\theta^{\mu\nu},$$

(5.1)

with $\theta^{\mu\nu}$ an antisymmetric constant matrix [25].

In this case, by the nature of the above noncommutativity parameter $\theta^{\mu\nu}$ being a constant but not a tensor, Lorentz invariance is broken, but not the $CPT$ symmetry [26][29]. Translational invariance is valid. In addition to the Lorentz invariance violation, such NC QFTs are nonlocal in the noncommuting coordinates. However, the Lorentz symmetry violation is of a very particular form, and invariance under the stability group of the matrix $\theta^{\mu\nu}$ is preserved under the so-called residual symmetry $O(1,1) \times SO(2)$ [30]. This reduced symmetry is enough to prove the $CPT$ theorem only for the scalar fields (for which the $C$ operation is a simple Hermitian conjugation) on the noncommutative space-time (5.1) [27]. A full proof of the $CPT$ theorem in Lorentz-violating noncommutative quantum field theory, however, could be achieved [28] only by using the twisted Poincaré symmetry [31,32] which these theories possess. The twisted Poincaré invariance is a deformation of the Poincaré symmetry, considered as a Hopf algebra, a concept coming from the theory of quantum groups [33], as compared with the Lie algebra. The irreducible representations of twisted Poincaré are identical to those of the usual Poincaré algebra, i.e. labeled by the mass and spin of the particles [31,32]. Therefore, the meaning of the charge conjugation has survived intact in the noncommutative quantum field theories. While parity and time reversal symmetries can be defined with any concept of space and time, the notion of charge conjugation has meaning...
only in the framework of Lorentz symmetry. Antiparticles are a consequence of special relativity. Particle and antiparticle are in the same irreducible representation of the Poincaré group. The CPT theorem is thus strongly connected to the Poincaré group representations, and not so much to the Lorentz symmetry, as the validity of the CPT theorem in the noncommutative space-time shows.

There are other examples of Lorentz-invariance violating but CPT-invariant theories, as in the extensions of the Standard Model given in [10] or with aether compactification [34]. However, in such cases the Lorentz invariance broken theory does not in general admit the usual representation content for the particles, unless the breaking of Lorentz invariance is made to be a spontaneous one.

6 Conclusion

We have presented a whole class of interacting nonlocal quantum field theories, such as the ones in (3.1)-(3.5) or (4.8), which violate CPT invariance while being Lorentz-invariant. This result invalidates a general claim made previously [17], that "CPT violation implies violation of Lorentz invariance". With such a CPT-violating interaction as in (3.1)-(3.5) or (4.8), and the addition of a C-violating (e.g., electroweak) interaction, the quantum corrections due to the combined interactions could lead to different properties for the particle and antiparticle, including their masses. Furthermore, there exists the reciprocal of this theorem, namely that the violation of Lorentz invariance does not necessarily lead to CPT violation.

Acknowledgements

We are grateful to Luis Álvarez-Gaumé, whose belief in the result of this Letter encouraged us to pursue the study. We are indebted to José Gracia-Bondía and Peter Prešnajder for useful discussions and for their invaluable contribution to this work. We thank Samoil Bilenky for many clarifying discussions on neutrino-antineutrino data in connection with the CPT violation. A.D.D. and V.A.N. would like to thank the RF Ministry for Science and Education for partial support under the Contract 02.740.11.5158. A.T. acknowledges the support of the Academy of Finland under the Project Nos. 136539 and 140886, and the support for this research of the Vilho, Yrjö and Kalle Väisälä Foundation, Finland.
References

[1] G. Lüders, *On the Equivalence of Invariance under Time-Reversal and under Particle-Antiparticle Conjugation for Relativistic Field Theories*, Det. Kong. Danske Videnskabernes Selskab, Mat.-fys. Medd. 28 (5) (1954).

[2] W. Pauli, *Niels Bohr and the Development of Physics*, W. Pauli (ed.), Pergamon Press, New York, 1955.

[3] R. Jost, *A remark on the C.T.P. theorem*, Helv. Phys. Acta 30 (1957) 409.

[4] R. Jost, in *Theoretical Physics in the Twentieth Century*, Interscience, New York, 1960.

[5] R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and All That*, Benjamin, New York, 1964.

[6] N. N. Bogoliubov, A. A. Logunov and I. T. Todorov, *Introduction to Axiomatic Quantum Field Theory*, Benjamin, Reading, 1975.

[7] L. B. Okun, *C, P, T are broken. Why not CPT?*, Talk given at 14th Rencontres de Blois: Matter-Anti-matter Asymmetry, Chateau de Blois, France, 17-22 Jun 2002 [arXiv:hep-ph/0210052].

[8] S. R. Coleman and S. L. Glashow, *High-Energy Tests of Lorentz Invariance*, Phys. Rev. D 59 (1999) 116008 [arXiv:hep-ph/9812418].

[9] J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, *CPT violation in string modified quantum mechanics and the neutral kaon system*, Int. J. Mod. Phys. A 11 (1996) 1489 [arXiv:hep-th/9212057]; J. R. Ellis, J. L. Lopez, N. E. Mavromatos and D. V. Nanopoulos, *Precision tests of CPT symmetry and quantum mechanics in the neutral kaon system*, Phys. Rev. D 53 (1996) 3846 [arXiv:hep-ph/9505340]; N. E. Mavromatos, *CPT Violation and Decoherence in Quantum Gravity*, J. Phys. Conf. Ser. 171 (2009) 012007 [arXiv:0904.0606 [hep-ph]].

[10] D. Colladay and V. A. Kostelecky, *CPT violation and the standard model*, Phys. Rev. D 55 (1997) 6760 [arXiv:hep-ph/9703464]; *Lorentz-violating extension of the standard model*, Phys. Rev. D 58 (1998) 116002 [arXiv:hep-ph/9809521]; V. A. Kostelecky and R. Lehnert, *Stability, causality, and Lorentz and CPT violation*, Phys. Rev. D 63 (2001) 065008 [arXiv:hep-th/0012060].
[11] V. A. Kostelecky, ed., *CPT and Lorentz Symmetry I-IV*, World Scientific, Singapore, 1999-2008; R. Bluhm, *Overview of the SME: Implications and phenomenology of Lorentz violation*, Lect. Notes Phys. **702** (2006) 191 [arXiv:hep-ph/0506054].

[12] A. G. Cohen and S. L. Glashow, *Very special relativity*, Phys. Rev. Lett. **97** (2006) 021601 [arXiv:hep-ph/0601236].

[13] Ralf Lehnert, *Tests of Lorentz symmetry*, Presented at 5th Patras Workshop on Axions, WIMPs and WISPs, Durham, England, United Kingdom, 13-17 Jul 2009. Published in "Durham 2009, Patras Workshop on Axions, WIMPs and WISPs" 171-174 [arXiv:1008.1746 [hep-ph]].

[14] H. Murayama and T. Yanagida, *LSND, SN1987A, and CPT violation*, Phys. Lett. **B 520** (2001) 263 [arXiv:hep-ph/0010178].

[15] G. Barenboim, L. Borissov, J. D. Lykken and A. Y. Smirnov, *Neutrinos as the messengers of CPT violation*, JHEP **0210** (2002) 001 [arXiv:hep-ph/0108199]; G. Barenboim, L. Borissov and J. Lykken, *Neutrinos that violate CPT, and the experiments that love them*, Phys. Lett. **B 534** (2002) 106 [arXiv:hep-ph/0201080].

[16] S. M. Bilenky, M. Freund, M. Lindner, T. Ohlsson and W. Winter, *Tests of CPT invariance at neutrino factories*, Phys. Rev. D **65** (2002) 073024 [arXiv: hep-ph/0112226].

[17] O. W. Greenberg, *CPT violation implies violation of Lorentz invariance*, Phys. Rev. Lett. **89** (2002) 231602 [arXiv:hep-ph/0201258].

[18] S. Tomonaga, *On a Relativistically Invariant Formulation of the Quantum Theory of Wave Fields*, Bull. IPCR (Rikeniko) **22** (1943) 545 (in Japanese); Prog. Theor. Phys. **1** (1946) 27.

[19] J. Schwinger, *Quantum Electrodynamics. I. A Covariant Formulation*, Phys. Rev. **74** (1948) 1439.

[20] S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory*, Row, Peterson and Company, 1961.

[21] K. Nishijima, *Fields and Particles*, W. A. Benjamin, 1969.

[22] R. Marnelius, *Action principle and nonlocal field theories*, Phys. Rev. D **8** (1973) 2472.

[23] R. Marnelius, *Can The S Matrix Be Defined in Relativistic Quantum Field Theories With Nonlocal Interaction?*, Phys. Rev. D **10** (1974) 3411.
[24] C. N. Yang and D. Feldman, *The S Matrix in The Heisenberg Representation*, Phys. Rev. 79 (1950) 972.

[25] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, JHEP 9909 (1999) 032 [arXiv:hep-th/9908142].

[26] M. Chaichian, K. Nishijima and A. Tureanu, *Spin statistics and CPT theorems in noncommutative field theory*, Phys. Lett. B 568 (2003) 146 [arXiv:hep-th/0209006].

[27] L. Álvarez-Gaumé and M. A. Vázquez-Mozo, *General properties of noncommutative field theories*, Nucl. Phys. B 668 (2003) 293 [arXiv:hep-th/0305093].

[28] M. Chaichian, M. N. Mnatsakanova, K. Nishijima, A. Tureanu and Yu. S. Vernov, *Towards an axiomatic formulation of noncommutative quantum field theory*, J. Math. Phys. 52 (2011) 032303 [arXiv:hep-th/0402212].

[29] M. M. Sheikh-Jabbari, *C, P, and T invariance of noncommutative gauge theories*, Phys. Rev. Lett. 84 (2000) 5265 [arXiv:hep-th/0001167].

[30] L. Alvarez-Gaume, J. L. F. Barbon and R. Zwicky, *Remarks on time-space noncommutative field theories*, JHEP 0105 (2001) 057 [arXiv:hep-th/0103069].

[31] M. Chaichian, P. P. Kulish, K. Nishijima and A. Tureanu, *On a Lorentz-invariant interpretation of noncommutative space-time and its implications on noncommutative QFT*, Phys. Lett. B 604 (2004) 98 [arXiv:hep-th/0408069].

[32] M. Chaichian, P. Prešnajder and A. Tureanu, *New concept of relativistic invariance in NC space-time: Twisted Poincaré symmetry and its implications*, Phys. Rev. Lett. 94 (2005) 151602 [arXiv:hep-th/0409096v1].

[33] M. Chaichian and A. P. Demichev, *Introduction to Quantum Groups*, World Scientific, Singapore, 1996.

[34] S. M. Carroll and H. Tam, *Aether Compactification*, Phys. Rev. D 78 (2008) 044047 [arXiv:0802.0521 [hep-ph]].