Parametric amplification versus collisions:
an illustrative application

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We investigate the time dependence of the quantum statistical model with $\lambda \phi^4/4!$ self-interaction and consider the resulting induced particle number density. For a special example in which the classical approximation exhibits parametric resonance, the effects of the back reaction and especially of collisions, treated in a perturbative way, are analysed.

I. INTRODUCTION

A widely used method for the investigation of the non-equilibrium evolution of quantum fields is the decomposition of the field into its mean value (or zero mode) and fluctuations, and the solution of the resulting coupled equations of motion. While the mean field rolls down some potential and/or oscillates around its minimum, its energy is transferred into the modes, a process resulting in particle production. This method has been applied to models of preheating in inflationary cosmology [1–5], to several examples of non-equilibrium dynamics of phase transitions [6,7] in the early universe and heavy ion collisions, to the calculation of the formation of DCCs [8], and also to more exotic processes like the decay [9] of parity odd metastable states in hot QCD [10].

In general, the coupled non-linear equations of motion which describe the real time evolution of the quantum fields cannot be solved exactly. The natural first step, then, is a classical approximation, in which the resulting mode equation exhibits parametric resonance [11] for certain parameter values, leading to a huge amplification of particle production. This approximation is applicable for the early stages of many processes, when the number of particles produced is not too large.

In the next step, the back reaction of the created particles has to be included. This is usually done with a term that changes the effective mass in the time-dependent oscillator equation. Most existing calculations make use of the Hartree-Fock- (or large N-) approximation, in which the coupled equations for the mean field and the fluctuations are solved self-consistently. If and how fast the inclusion of the back reaction influences or even destroys the parametric resonance, depends strongly upon the model under investigation. For example, in certain models of preheating with small coupling, the back reaction is not very destructive [2–5], whereas in the model for the decay of parity odd metastable states [9], particle production is strongly suppressed as compared to the classical case. Heuristically, this could be explained with the growing effective mass of the particles which makes them harder to be produced. Eventually, the Hartree-Fock approximation fails when the fluctuations grow comparable to the value of the mean field.

When the density of the particles created becomes large, the effects of collisions like scattering off or into the resonant modes, additional particle creation, dissipation to modes with higher energies and maybe thermalization, are expected to become important. Parametric resonance amplifies only certain low momentum modes, producing highly non-thermal states with the energy concentrated in the infrared. In order to approach thermal equilibrium, energy must be scattered into higher momentum modes. These effects are not taken into account by the Hartree-Fock approximation, which corresponds to a truncation at the level of two-point functions. An understanding of these processes which are expected to terminate the parametric resonance and should eventually lead to equilibration are crucial not only for the scenario of inflationary cosmology (for a review see [3]), but also for a deeper understanding of the different stages in heavy ion collisions [2].

In the present work, we consider a field theory model with $\lambda \phi^4$ interaction and propose to include collision terms represented perturbatively by the sunset diagrams of $O(\lambda^2)$, but expressed by the full two-point Green function in order to have an approximation applicable far from equilibrium. For the emergence of parametric resonance [1], typical parameters in the equation of motion must be periodically varying in time (e.g. the frequency in a harmonic oscillator equation), but the number of spatial dimensions is of no importance. Therefore, as a first step we consider the case of zero space dimensions, effectively a quantum statistical model, which simplifies the calculations considerably.

After introducing the set of equations in section II, their solutions and the application to the preheating model of [3] are studied in section III. It is shown how the parametric instability of the classical approximation is destroyed when the collision term is switched on, i.e. when additional transfer of energy between the zero mode and the fluctuations is introduced.
II. QUANTUM STATISTICAL MODEL WITH COLLISION TERMS

We consider a real one component scalar field with $\phi^4$ coupling, but for simplicity with zero spatial dimensions. Following the standard method as described, e.g., in [3–8,12], the field operator is decomposed into its zero mode $\varphi \equiv \langle \hat{\varphi}(t) \rangle$ and fluctuations $\chi$,

$$\hat{\varphi}(t) = \varphi(t) + \chi(t)$$

with $\langle \chi(t) \rangle = 0$. The full coupled, nonlinear equations of motion read

$$\ddot{\varphi}(t) + m^2 \varphi(t) = -\frac{\lambda}{3!} \varphi^3(t) - \frac{\lambda}{2} \varphi(t) \langle \dot{\chi}^2(t) \rangle - \frac{\lambda}{3!} \langle \dot{\chi}^3(t) \rangle$$

for the zero mode and

$$\ddot{\chi}(t) + m^2 \chi(t) = -\frac{\lambda}{2} \varphi^2(t) \chi(t) - \frac{\lambda}{2} \left( \chi^2(t) - \langle \chi^2(t) \rangle \right) \varphi(t) - \frac{\lambda}{3!} \left( \chi^3(t) - \langle \chi^3(t) \rangle \right)$$

for the fluctuations, which is an operator equation in $\chi$. In the following, we will make use of the mode functions $\chi(t)$, which are introduced in the same way as in the usual Fourier decomposition of the Heisenberg operators in field theory:

$$\chi(t) = \frac{1}{\sqrt{2\omega_0}} \left( a(t) + a^\dagger(t) \right)$$

$$= \frac{1}{\sqrt{2\omega_0}} \left( a \chi(t) + a^\dagger \chi^*(t) \right)$$

where $\omega_0 \equiv \omega(t_0)$ is defined below in (15). The time-independent creation and annihilation operators $a \equiv a(t_0), a^\dagger \equiv a^\dagger(t_0)$ act on the initial Fock vacuum state. The density matrix at $t_0$ is chosen to fulfill $\langle a \rangle = \langle a^\dagger \rangle = 0, \langle aa \rangle = \langle a^\dagger a^\dagger \rangle = 0$ etc., except for $\langle aa^\dagger \rangle$ and $\langle a^\dagger a \rangle$ which are assumed to be nonzero.

The first step towards the solution of the equations of motion is the classical approximation, which is linear in the mode functions and does not include any back reaction effects of the particles created:

$$\ddot{\varphi} + m^2 \varphi(t) + \frac{\lambda}{3!} \varphi^3(t) = 0$$

$$\ddot{\chi} + \left( m^2 + \frac{\lambda}{2} \varphi^2(t) \right) \chi(t) = 0.$$  

(For the solution, see section III.) In this case, the induced particle number density, which is defined as

$$n(t) = \langle a^\dagger(t) a(t) \rangle$$

$$= \frac{\text{Tr} a^\dagger(t) a(t) \rho(t_0)}{\text{Tr} \rho(t_0)}$$

$$= \frac{\text{Tr} a^\dagger(t_0) a(t_0) \rho(t)}{\text{Tr} \rho(t_0)},$$

can be written in terms of the mode functions as

$$n(t) = \frac{1}{4} \left( |\chi(t)|^2 + \frac{|\dot{\chi}(t)|^2}{\omega_0^2} \right) - \frac{1}{2},$$

just as for the harmonic oscillator, with the initial conditions $\chi(t_0) = 1$ and $\dot{\chi}(t_0) = -i\omega(t_0)$ consistent with $n(t_0) \equiv 0$.

A consistent method to include back reaction effects of the fluctuations on the mean field and on themselves is the Hartree-Fock approximation $\dot{\chi}^3 \rightarrow 3(\dot{\chi}^2)\dot{\chi}$, leading to $\langle \dot{\chi}^3 \rangle = 0$. The equations of motion are

$$\ddot{\varphi}(t) + m^2 \varphi(t) + \frac{\lambda}{3!} \varphi^3(t) = -\frac{\lambda}{2} \langle \dot{\chi}^2 \rangle \varphi(t)$$

$$\ddot{\chi}(t) + m^2 \chi(t) + \frac{\lambda}{2} \varphi^2(t) \chi(t) = -\frac{\lambda}{2} \langle \dot{\chi}^2 \rangle \chi(t)$$

(13)
with the back reaction term
\[ \langle \hat{\chi}^2 \rangle = \frac{1}{2\omega_0}|\chi(t)|^2. \] (14)

In this approximation, the Hamiltonian is still that of an oscillator, but with the time dependent frequency
\[ \omega^2(t) = m^2 + \frac{\lambda}{2}\omega^2(t) + \frac{\lambda}{2}\langle \hat{\chi}^2 \rangle. \] (15)

The definition of the particle number in this approximation can be kept as before (for a discussion of the time-dependent oscillator see, e.g., \[13\]), and the order in \( \lambda \) is the same.

Although the mode functions will be used below for the numerical calculations, we also need their relation to the Green functions to establish the equations of motion with collision terms. The Green functions are defined as (see, e.g., \[14\])
\[ G^>(t, t') = \langle \hat{\chi}(t)\hat{\chi}(t') \rangle \] (16)
and can be written in terms of the mode functions as
\[ G^>(t, t') = \frac{1}{2\omega_0}[(1 + n(t_0))\chi(t)\chi^*(t') + n(t_0)\chi^*(t)\chi(t')]. \] (17)

With \( n(t_0) = 0 \), this simplifies to
\[ G^>(t, t') = \frac{1}{2\omega_0}\chi(t)\chi^*(t'), \] (18)
and \( G^<(t, t') = G^>(t', t) \). Because we use the mode functions for further calculations, we effectively only need the equation of motion for the equal-time Green functions
\[ \left[ \partial_t^2 + m^2 + \Sigma^>(t) \right] G^>(t, t')|_{t'=t} = -i \int_{t_0}^{t} dt'' \Sigma^>(t, t'') G^>(t'', t')|_{t'=t} \] \[ -i \int_{t'}^{t_0} dt'' \Sigma^<(t, t'') G^<(t'', t')|_{t'=t}, \] (19)
which is the Schwinger-Dyson equation (see, e.g., \[14\]) in the limit of equal times \( t' = t \) (for the notation, see \[15\]). The integration contour is shown in fig.1.

FIG. 1. The Schwinger-Keldysh closed-time-path contour for the case \( t' = t \).
The tadpole self-energy

\[ \Sigma^\delta(t) = \frac{\lambda}{2} G^> (t, t) + \frac{\lambda}{2} \varphi^2(t) \quad (20) \]

is the part which already appeared in the Hartree-Fock approximation (12), (13). In order to close the system of equations by including effects due to collisions, we approximate the self-energy \( \Sigma^> \) in eq. (19) by the two-point function (18). In the presence of the condensate field \( \varphi(t) \), the sunset-type approximation (cf. [15])

\[ \Sigma^>(t, t'') = -\frac{\lambda^2}{6} [G^>(t, t'')]^3 \quad (21) \]

is generalized to (see fig.2)

\[ \Sigma^>(t, t'') = -\frac{\lambda^2}{6} [G^>(t, t'')]^3 - \frac{\lambda^2}{2} \varphi(t) [G^>(t, t'')] \varphi(t'') - \frac{\lambda^2}{4} \varphi^2(t) [G^>(t, t'')] \varphi^2(t''). \quad (22) \]

![Graphical representation of the rhs of the Schwinger-Dyson equation for the Green function \( G(t, t') \) (solid curve). The dashed curve denotes the mean field. The statistical factors (cf. [16]) are given.](image)

Expressing the Green functions in terms of the mode functions, we obtain the equation of motion up to \( O(\lambda^2) \):

\[ \left[ \ddot{\chi}(t) + \left( m^2 + \frac{\lambda}{2} \varphi^2(t) + \frac{\lambda}{4\omega_0} |\chi(t)|^2 \right) \chi(t) \right] \chi^*(t) = -\frac{\lambda^2}{24\omega^3_0} \int_{t_0}^{t} dt' \text{Im} \left[ (\chi(t)\chi^*(t'))^4 \right] \]

\[ -\frac{\lambda^2}{4\omega^3_0} \int_{t_0}^{t} dt' \text{Im} \left[ \varphi(t)\varphi(t') (\chi(t)\chi^*(t'))^3 \right] \]

\[ -\frac{\lambda^2}{4\omega^3_0} \int_{t_0}^{t} dt' \text{Im} \left[ \varphi^2(t)\varphi^2(t') (\chi(t)\chi^*(t'))^2 \right], \quad (23) \]

where

\[ \omega^2_0 = m^2 + \frac{\lambda}{2} \varphi^2(t_0) + \frac{\lambda}{4\omega_0}. \quad (24) \]

Correspondingly, the equation of motion for the mean field is (see fig.3)

\[ \left[ \partial^2_t + m^2 + \frac{\lambda}{6} \varphi^2(t) + \frac{\lambda}{2} G(t, t) \right] \varphi(t) = -i \int_{t_0}^{t} dt' \left( \hat{\Sigma}^>(t, t') - \hat{\Sigma}^<(t, t') \right) \varphi(t') \]

\[ = -i\lambda^2 \int_{t_0}^{t} dt' \varphi(t') \left[ \frac{1}{6} [G^>(t, t')]^3 - [G^<(t, t')]^3 \right] \]

\[ + \frac{1}{4} \varphi(t) [G^>(t, t')]^2 - [G^<(t, t')]^2 \varphi(t') \]

\[ + \frac{1}{12} \varphi^2(t) (G^>(t, t') - G^<(t, t')) \varphi^2(t') \];

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and the initial conditions are chosen as \( \varphi(t_0) = \varphi_0 \) and \( \dot{\varphi}(t_0) = 0 \). Inserting the mode functions, eq. (25) reduces to

\[
\dot{\varphi}(t) + \left( m^2 + \frac{\lambda}{6} \varphi^2(t) + \frac{\lambda}{4\omega_0} |\chi(t)|^2 \right) \varphi(t) = -\frac{\lambda^2}{24\omega_0^3} \int_{t_0}^{t} dt' \text{Im} \left[ \varphi(t') (\chi(t)\chi^* (t'))^3 \right] - \frac{\lambda^2}{8\omega_0^3} \int_{t_0}^{t} dt' \text{Im} \left[ \varphi(t') \varphi^2 (t') (\chi(t)\chi^* (t'))^2 \right] - \frac{\lambda^2}{12\omega_0} \int_{t_0}^{t} dt' \text{Im} \left[ \varphi^2 (t') \varphi^3 (t') (\chi(t)\chi^* (t')) \right],
\]

where it should be noted that \( \varphi(t) \) is a real function.

\[ \text{FIG. 3. Graphical representation of the r.h.s of the Schwinger-Dyson equation for the mean field (dashed curve).} \]

The particle number including collisions is calculated in general from

\[ \dot{n}(t) = i[H, n(t)]. \]

Following the derivation given in \[17,18\], this reads for a scalar field for the contributions due to collisions

\[ \dot{n}(t)_{\text{coll}} = -\frac{\lambda}{6\omega_0} \frac{\partial}{\partial t'} \langle (\varphi(t))^3 \varphi(t') \rangle |_{t' = t} \]

where \( t \) and \( t' \) are on the contour of fig. 1. Keeping terms of \( O(\lambda^2) \), we write

\[ \dot{n}(t)_{\text{coll}} = -i \int_{t_0}^{t} dt'' \Sigma^< (t, t'') \frac{\partial}{\partial t''} G^< (t'', t') + \int_{t_0}^{t} dt'' \Sigma^< (t, t'') \frac{\partial}{\partial t''} G^> (t'', t') |_{t' = t}, \]

where \( \Sigma \) is given by the graphs of fig. 2. With \( G \) expressed in terms of the mode functions, the result is, after integrating and adding the contribution \[13\] without collisions,

\[ n(t) = \frac{1}{4} \left( |\chi(t)|^2 + \frac{|\chi'(t)|^2}{\omega_0^2} \right) - \frac{1}{2} \]

\[ -\frac{\lambda^2}{48\omega_0^3} \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \text{Im} \left[ \chi'(t') \chi^3(t') (\chi^* (t''))^3 \right] \]

\[ -\frac{\lambda^2}{8\omega_0^3} \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \text{Im} \left[ \varphi(t') \varphi(t'') \chi'(t') (\chi^* (t''))^2 \right] \]

\[ -\frac{\lambda^2}{12\omega_0} \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \text{Im} \left[ \varphi^2(t') \varphi(t'') \chi'(t') (\chi^* (t'')) \right], \]

\[ +O(\lambda^3). \]

In the case of vanishing condensate field, i.e. \( \varphi = 0 \), but with collisions, the particle number in leading order of \( \lambda \) is

\[ n_{\text{coll}}(t) = \frac{\lambda^2}{48\omega_0^3} \sin^2(2\omega_0(t - t_0)) \]

\[ -\frac{\lambda^2}{8\omega_0^3} \frac{\sin^2(2\omega_0(t - t_0))}{8\omega_0^3}, \]

which is easy to find after approximating the mode functions in terms of free fields, \( \chi(t) = \exp(-i\omega_0 t) \). This corresponds to the expression (3.21) in \[18\] for the \( d = 0 \) case, neglecting \( O(\lambda^3) \) contributions. Here, we discuss \( n(t) \) of eq. (28) without expanding the mode functions in terms of the free field solution, but instead determine \( \chi(t) \) from the self-consistent set of coupled equations (23) and (24).
III. RESULTS FOR MODELS WITH COLLISION TERM

In order to study the effects of the collision term, we solve the coupled equations of motion numerically for the three steps of approximation.

As a first example, we consider the set of parameters $m^2 = 1.6$, $\lambda = 0.1$, and $\varphi_0 = 7.75$, and the other initial values for $\varphi$ and $\chi$ are as given as in section II. (This choice will be justified below.) The results for the classical approximation eqs.(6) and (7) (solid curve), including back reaction (dashed curve) and additionally including the collision term (dot-dashed curve), are shown in fig.4. The growth of the induced particle number density does not exhibit parametric resonance for this set of parameters, but it is sufficient to illustrate the dramatic effect of the collision term: The inclusion of the back reaction (eqs.(12),(13)) already suppresses this growth, and the additional inclusion of the collision term (eqs.(23),(26)) destroys it completely.

![Figure 4](image1.png)

**FIG. 4.** Particle number density for $m^2 = 1.6$, $\lambda = 0.1$; for the classical approximation (solid curve), including back reaction (dashed curve), and additionally including the collision term (dot-dashed curve).

![Figure 5](image2.png)

**FIG. 5.** Zero mode for $m^2 = 1.6$, $\lambda = 0.1$; for the three cases as in fig.4.
As the main example, we apply our approximation to a special preheating model of chaotic inflation treated in [5], where the development of parametric resonance does not depend on the expansion of the universe. The equation for the zero mode

$$\varphi''(x) + \varphi^3(x) = 0,$$

written for the conformal field $\varphi = a \phi$ ($a$ is the cosmological scale factor) and the dimensionless time $x = \sqrt{\lambda \varphi_0 \eta}$ ($\eta$ being the conformal time) has the solution

$$\varphi(x) = \varphi_0 \cn \left( x, \frac{1}{\sqrt{2}} \right),$$

a Jacobian cosine function, leading to a Lamé equation for the fluctuations of $\varphi$,

$$\varphi''_{\kappa}(x) + \left( \kappa^2 + 3 \left( \frac{\varphi(x)}{\varphi_0} \right)^2 \right) \varphi_{\kappa}(x) = 0,$$

where the comoving momentum $k$ appears in $\kappa^2 = k^2 / \lambda \varphi_0^2$. The Lamé equation is known to have unstable solutions of the form $\varphi_{\kappa}(x) = P(x) \exp(\mu_{\kappa} x)$, where $P(x)$ is a periodic function, for certain values of $\kappa$ which form the so called instability bands. The Floquet exponent $\mu_{\kappa}$ characterizes the growth of the solutions and of the particle number density $\ln n_{\kappa} \approx 2 \mu_{\kappa} x$, and is real for the instability bands. In this special case, the instability band is given by $3/2 < \kappa^2 < \sqrt{3}$ with its maximum value $\mu_{\kappa,\text{max}} \simeq 0.03598$ at $\kappa_{\text{max}}^2 \simeq 1.615$ [5]. As discussed in [19], the inclusion of different “back reaction”/rescattering effects will change the equations of motion, the frequency of oscillation and the effective masses. The instability bands are restructured, and scattering may lead to additional particle production and removal from the resonance.

To investigate effects of this kind with our approximation, we first have to introduce the dimensionless time variable $\tau = \sqrt{\lambda/6} \varphi_0 t$. Comparing our set of eqs. (6), (7) with the corresponding equations of [5] leads to the condition

$$\frac{6m^2}{\lambda \varphi_0^2} = \kappa^2,$$

i.e. the momentum $\kappa$, which does not appear in our equations because we consider zero spatial dimensions, is “replaced” by the mass. Moreover, we have to neglect the mass term in the zero mode equations (6), (12), and (26).

We choose the initial value $\varphi_0 = \sqrt{6/\lambda}$, which has the advantage that $\tau = t$, i.e. our time scale is the same as in [5]. Choosing $\lambda = 0.1$ and considering the special value $\kappa_{\text{max}}^2 = 1.615$ fixes $m^2$ for the mode equations. (This set of parameters corresponds to those of the first example.) Fig.6 shows the results for the particle number density, where the classical result (solid curve) reproduces that of fig.3 in [5]. The solid curve in fig.7 corresponds to the solution of eq.(33). The parametric amplification of the classical approximation is suppressed only weakly by the back reaction. In contrast, the resonance is completely destroyed by the collision term given in eqs.(23) and (26).

FIG. 6. Particle number density for $m^2 = 0, \lambda = 0.1$ and $\kappa^2 = 1.615$; for the classical approximation (solid curve), including back reaction (dashed curve), and additionally including the collision term (dot-dashed curve).
FIG. 7. Zero mode for $m^2 = 0$, $\lambda = 0.1$ and $\kappa^2 = 1.615$; for the three cases as in fig.6.

IV. CONCLUSION

The essential feature of this model for the inclusion of collisions is that the two-point functions are not expressed in terms of planar waves, i.e. $\chi(t)$ is not perturbatively expanded in terms of $\lambda$. This is crucial for obtaining parametric resonance, which leads to amplified particle production, in the classical equations and even when the back reaction is included. With the inclusion of collisions, the parametric amplification disappears, at least within our treatment. But, unfortunately, our approximation breaks down already for small times, $t_f \approx 11$ in this case (see figs.6 and 7). Decreasing the coupling $\lambda$, without leaving the resonance band of the classical solution, only shifts the instability of the approximation scheme towards larger times $\tilde{t}_f = \sqrt{\lambda/\tilde{\lambda}} t_f$. There is strong evidence that the effective potential becomes concave for $t \geq t_f$, as can be seen from the evolution of the zero mode (fig.7), which does not only grow larger than its initial value, but even seems to tend to infinity. Contrary to studies of thermalization (see, e.g., [2]), we are not able to see the long-time behaviour, although the model includes (memory) terms which are non-local in time. A definite statement about equilibration, i.e. about particle number densities including momentum dependence, would additionally require the inclusion of $d > 0$ spatial dimensions.

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