Recent Developments on Representation of Experimental Data by Non-polynomial Curve

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Abstract

Recently some studies have been made on representing numerical data on a pair of variables by some standard non-polynomial curves namely exponential curve, modified exponential curve, logistic curve, Makeham’s curve etc. in connection with the development of some formula/method, more convenient than the existing ones, of interpolation. This paper is based on a brief review on the recent developments of the methods of representing numerical data on a pair of variables by these non-polynomial curves along with their application in real data.

Keywords: Pair of variables, numerical data, mathematical representation, non-polynomial curve

1 Introduction

Observations or data, collected from experiment or survey, normally suffer from various types of errors/causes which can be broadly divided into two types namely (1) Assignable error/cause that is avoidable/controllable & (2) Chance error/cause that is unavoidable / uncontrollable [Chakrabarty, 2014a,b,c,d,e,f,g,h, 2015a,b,c,d,e,f]. Even if all the assignable causes of error are controlled or eliminated, observations still do not become free from error. Each of them still suffers from some error which occurs due to some unknown and unintentional cause that is nothing but the chance cause. Consequently the findings obtained by analyzing the observations which are free from the assignable errors are also subject to errors due to the presence of chance error in the observations. Determination of constant(s) associated to mathematical model(s), in different situations, based on the observations is also subject to error due to the same reason.

A number of mathematical models have been identified for describing the association of chance error(s) in determining constant(s) in some distinct situations where observations/data are of measurement type [Chakrabarty, 2014g, 2016e,j, 2017c].

There are two broad aspects of statistical determination of parameters involved in the respective models describing the dependence of the dependent variable on the independent variable(s). One of them is based on the basic philosophy behind statistics [Chakrabarty, 2018g,h, 2019b], which consists of determining the parameter(s) from numerical data compromising with some degree of error in findings. Several

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statistical methods have already been developed for determination (estimation in statistical literature) of such parameter(s) which are available in the standard literatures in statistics. However, existing statistical methods of estimation cannot normally yield error free estimate(s) of parameters [Chakrabarty, 2014g, 2016c,j]. The same fact happens in the case of some recently developed methods of estimation [Chakrabarty, 2011, 2014a, 2016c,d,j; Chakrabarty and Dutta, 2007; Chakrabarty and Rahman, 2007, 2008]. Recently, some studies have been made on attempting of determining error free estimates of such parameter(s) [Chakrabarty, 2014b,f,g, 2015c,d,f, 2017b,c, 2018c, 2019a,d, 2020d] along with attempting of application in the case of real data [Chakrabarty, 2015b, 2016a, 2019h,i] . These studies have been done on the basis of various measures of average namely the three Pythagorean means (namely arithmetic mean, geometric mean & harmonic mean), median, generalized mean and others [Chakrabarty, 2016g, 2017c, 2018a,c,d,e,f, 2019c,e,f,g, 2020a,b,c,d,e, 2021a,b,c,d,e,f,g,h].

Standard methods of statistical representation of numerical data like least squares method, orthogonal polynomial method etc. are available in the standard literature of statistics. Some studies, extension in nature, have recently been done on statistical representation of numerical data by some special mathematical curves namely linear curve, quadratic curve, exponential curve etc. [Chakrabarty, 2011, 2014a, 2016c,d, 2017a; Chakrabarty and Dutta, 2007; Chakrabarty and Rahman, 2007, 2008; Rahman and Chakrabarty, 2009, 2011, 2015a,b,c,d,e,f].

Later on, some studies have been made on the representation of numerical data on a pair of variables by suitable mathematical equation/mathematical curve. In those studies [Chakrabarty, 2016a,b,f,g,h,i, 2017d,e, 2018b; Das and Chakrabarty, 2016a,b,c,d,e,f, 2017a,b,c,d, 2020] , some formulas/methods have been developed for representing numerical data on a pair of variables by polynomial curve in connection with the development of some more convenient formula/method of interpolation which is a technique of estimating approximately the value of the dependent variable corresponding to a value of the independent variable lying between its two extreme values on the basis of the given values of the independent and the dependent variables [Bathe and Wilson, 1976; Hummel, 1947; Wisniewski, 1930]. The formulas developed are based on usual algebraic operation, forward difference operation, backward difference operation, divided difference operation, backward divided difference operation, difference and ratio operation and backward difference & ratio operation [Bathe and Wilson, 1976; Conte and de Boor, 1980; Dokken and Lyche, 1979; Gerald and Patrick, 1994; Jordan and Jordán, 1965; Lee, 1989; Vertesi, 1990; Whittaker and Robinson, 1967], while the methods developed are based on matrix inversion by Cayley-Hamilton Theorem, matrix inversion by Gauss Jordan method and matrix inversion by elementary column transformation. Recently some studies have been made on representing numerical data on a pair of variables by some standard non-polynomial curves namely exponential curve, modified exponential curve, logistic curve, Makeham's curve etc. in connection with the development of some formula/method, more convenient than the existing ones, of interpolation. This paper is based on a brief review on the recent developments of the methods of representing numerical data on a pair of variables by these non-polynomial curves along with their application in real data.

2 Method of Representation of Numerical Data

Methods of representation of numerical data on a pair of variables by some standard non-polynomial curves, as mentioned above, have been discussed below:
2.1 Exponential Curve

The Exponential curve is of the form

\[ y = ab^x \] (1)

Eq. 1 implies,

\[ \log y = \log a + x \log b \] (2)

Since there are two parameters in the exponential curve, two equations are necessary for determining the values of the parameters and accordingly two sets of values the pair of variables are necessary.

Let \( y_0, y_1 \) be the values of \( y \) corresponding to the values \( x_0, x_1, \) of \( x \) respectively. Then the points \((x_0, y_0)\) and \((x_1, y_1)\) lie on the Eq. 1 and hence on Eq. 2.

Accordingly,

\[ \log y_0 = \log a + x_0 \log b \]
\[ \log y_1 = \log a + x_1 \log b \]

From which one can obtain that

\[ b = \frac{\log y_1 - \log y_0}{x_1 - x_0} = \text{antilog} \left( \frac{\Delta \log y_0}{\Delta x_0} \right) \] (3)

\[ a = \text{antilog} \left( \log y_0 - x_0 \left( \frac{\Delta y_0}{\Delta x_0} \right) \right) = \text{antilog} \left[ \log y_1 - x_1 \left( \frac{\Delta y_0}{\Delta x_0} \right) \right] \] (4)

2.2 Modified Exponential Curve

The modified exponential curve is of the form

\[ y = a + bc^x \] (5)

where, \( a, b \) and \( c \) are parameters.

Since there are three parameters in the modified exponential curve, three equations are necessary for determining the values of the parameters and accordingly three sets of values the pair of variables are necessary.

Let \( y_0, y_1, y_2 \) be the values of \( y \) corresponding to the values \( x_0, x_1, x_2 \) of \( x \) respectively so that three points \((x_0, y_0), (x_1, y_1)\) and \((x_2, y_2)\) lie on the Eq. 1. Then,

\[ y_0 = a + bc^{x_0}, \ y_1 = a + bc^{x_1}, \ y_2 = a + bc^{x_2} \]

i.e., \( \Delta y_0 = b(c^{x_1} - c^{x_0}) \) \& \( \Delta y_1 = b(c^{x_2} - c^{x_1}) \)

If \( x_0, x_1, x_2 \) are equally spaced then,

\[ x_1 - x_0 = x_2 - x_1 = h, \]
\[ i.e., \ x_1 = x_0 + h, \ \& \ x_2 = x_0 + 2h \]

This means,

\[ y_0 = a + bc^{x_0}, \ y_1 = a + bc^{x_0+h}, \ \& \ y_2 = a + bc^{x_0+2h} \]

Accordingly,

\[ \Delta y_0 = bc^{x_0}(c^h - 1) \ \& \ \Delta y_1 = bc^{x_1}(c^h - 1) \]

which implies,

\[ \frac{\Delta y_1}{\Delta y_0} = c^h \]

i.e.,

\[ c = \text{antilog} \left[ \frac{1}{h} \log \left( \frac{\Delta y_1}{\Delta y_0} \right) \right] \] (6)
and consequently,

\[ b = \frac{\Delta y_0}{e^{x_0}(c^h - 1)} = \frac{\Delta y_1}{e^{x_1}(c^h - 1)} \]

(7)

where \( c \) is given by the Eq. 6.

\[ a = y_0 - bc^{x_0} = y_1 - bc^{x_1} = y_2 - bc^{x_2} \]

(8)

where \( b \) and \( c \) are given by the Eq. 6 and Eq. 7 respectively.

2.3 Logistic Curve

The logistic curve is the form

\[ y = \frac{A}{B + C^x} \]

(9)

where, \( A, B \) and \( C \) are parameters.

In this case also there are three parameters so that three equations are necessary for determining the values of the parameters and accordingly three sets of values the pair of variables are necessary.

As earlier, let \( y_0, y_1, y_2 \) be the values of \( y \) corresponding to the values \( x_0, x_1, x_2 \), of \( x \) respectively so that three points \((x_0, y_0), (x_1, y_1)\) and \((x_2, y_2)\) lie on the curve described by Eq. 9. Then

\[ y_0 = \frac{A}{B + C^{x_0}}, \quad y_1 = \frac{A}{B + C^{x_1}} \quad \& \quad y_2 = \frac{A}{B + C^{x_2}}, \]

so that

\[ \frac{1}{y_0} = \frac{B}{A} + \frac{1}{A}c^{x_0}, \quad \frac{1}{y_1} = \frac{B}{A} + \frac{1}{A}c^{x_1} \quad \& \quad \frac{1}{y_2} = \frac{B}{A} + \frac{1}{A}c^{x_2} \]

i.e.,

\[ \Delta \left( \frac{1}{y_0} \right) = \frac{1}{A} (c^{x_1} - c^{x_0}) \quad \& \quad \Delta \left( \frac{1}{y_1} \right) = \frac{1}{A} (c^{x_2} - c^{x_1}) \]

If \( x_0, x_1, x_2 \) are equally spaced then

\[ x_1 - x_0 = x_2 - x_1 = h \]

i.e., \( x_1 = x_0 + h \) \& \( x_2 = x_0 + 2h \)

this means,

\[ y_0 = \frac{A}{B + C^{x_0}}, \quad y_1 = \frac{A}{B + C^{x_0+h}} \quad \& \quad y_2 = \frac{A}{B + C^{x_0+2h}}, \]

i.e., \( \frac{1}{y_0} = \frac{B + C^{x_0}}{A}, \quad \frac{1}{y_1} = \frac{B + C^{x_0+h}}{A} \quad \& \quad \frac{1}{y_2} = \frac{B + C^{x_0+2h}}{A} \)

Accordingly,

\[ \Delta \left( \frac{1}{y_0} \right) = \frac{1}{A} C^{x_0}(c^h - 1) \]

\& \[ \Delta \left( \frac{1}{y_1} \right) = \frac{1}{A} C^{x_1}(c^h - 1) = \frac{1}{A} C^{x_0+h}(c^h - 1) = \frac{c^h}{A} C^{x_0}(c^h - 1) \]

which implies,

\[ \frac{\Delta \frac{1}{y_1}}{\Delta \frac{1}{y_0}} = c^h \]
\[ C = \text{antilog} \left[ \frac{1}{h} \log \left( \frac{\Delta \left( \frac{1}{y_0} \right)}{\Delta \left( \frac{1}{y_1} \right)} \right) \right] \]  

(10)

and consequently,

\[ A = \frac{c_0^2 (c^h - 1)}{\Delta \left( \frac{1}{y_0} \right)} = \frac{c_1^2 (c^h - 1)}{\Delta \left( \frac{1}{y_1} \right)} \]

(11)

where \( C \) is given by Eq. 10.

\[ B = \frac{A}{y_0} - c^x = \frac{A}{y_1} - c^{x_1} = \frac{A}{y_2} - c^{x_2} \]

(12)

where \( A \) and \( C \) are given by Eq. (11) and Eq. 10.

### 2.4 Makeham’s Curve

The Makeham’s curve is of the form

\[ y = ab^x e^{dx} \]

(13)

where \( a, b, c \) and \( d \) are parameters.

In this case there are four parameters so that four equations are necessary for determining the values of the parameters and accordingly four sets of values the pair of variables are necessary.

As earlier, let \( y_0, y_1, y_2, y_3 \) be the values of \( y \) corresponding to the values \( x_0, x_1, x_2, x_2 \) of \( x \) so that the four points \((x_0, y_0), (x_1, y_1), (x_2, y_2), (x_3, y_3)\) lie on the curve describe by Eq. 13.

Then,

\[ y_0 = ab^{x_0} e^{dx_0}, y_1 = ab^{x_1} e^{dx_1}, y_2 = ab^{x_2} e^{dx_2}, y_3 = ab^{x_3} e^{dx_3} \]

i.e.,

\[
\log y_0 = \log a + x_0 \log b + d^{x_0} \log c \\
\log y_1 = \log a + x_1 \log b + d^{x_1} \log c \\
\log y_2 = \log a + x_2 \log b + d^{x_2} \log c \\
\log y_3 = \log a + x_3 \log b + d^{x_3} \log c \\
\log y_4 = \log a + x_4 \log b + d^{x_4} \log c
\]

If \( x_0, x_1, x_2, x_3 \) are equally spaced then,

\[ x_1 - x_0 = x_2 - x_1 = x_3 - x_2 = h \]

i.e., \( x_1 = x_0 + h, x_2 = x_0 + 2h, x_3 = x_0 + 3h, \)

so that

\[
\Delta \log y_0 = h \log b + (d^h - 1) \log cd^{x_0} \\
\Delta \log y_1 = h \log b + (d^h - 1) \log cd^{x_1} \\
\Delta \log y_2 = h \log b + (d^h - 1) \log cd^{x_2}
\]

This implies,

\[
\frac{\Delta^2 \log y_1}{\Delta^2 \log y_0} = \frac{d^{x_1}}{d^{x_0}} = \frac{d^{x_0+h} - d^{x_0}}{d^{x_0}} = \frac{d^{x_0} d^h}{d^{x_0}}
\]

If \( h = 1, \)

\[ d = \frac{\Delta^2 \log y_1}{\Delta^2 \log y_0} \]

(14)
Table 1: Total Population of India

| Year | 1951 | 1961 | 1971 | 1981 | 1991 | 2011 |
|------|------|------|------|------|------|------|
| x    | 0    | 1    | 2    | 3    | 4    | 5    |
| Number of Person | 361088090 | 439234771 | 548159652 | 683329097 | 846302688 | 1210193422 |

Also,
\[ \Delta^2 \log y_0 = (d-1)^2 \log c \]

Which implies,
\[ c = \text{antilog} \left( \frac{\Delta^2 \log y_0}{(d-1)^2} \right) \quad \text{if} \quad x_0 = 0 \quad (15) \]

and consequently,
\[ b = \text{antilog} \left( \Delta \log y_0 - \frac{\Delta^2 \log y_0}{(d-1)} \right) \quad (16) \]
\[ a = \text{antilog} \left( \Delta \log y_0 - \frac{\Delta^2 \log y_0}{(d-1)^2} \right) \quad (17) \]

3 Application to Numerical Data

The Table 1 shows the data on total population of India corresponding to the years from 1951 onwards:

3.1 Representation by Exponential Curve

Let us first represent the total populations corresponding to the years 1951 and 1961 by the exponential curve described by Eq. 1

After computations of the values of the parameters \( a \) and \( b \) (which have been found to be 1.21641999047 and 361088090 respectively), the form of the exponential curve that can represent the total populations corresponding to the years 1951 and 1961 has been found as
\[ y = 361088090 \times 1.21641999047^x \]

The forms of exponential curves, obtained, for representing the total population of India corresponding to the other pairs of years have been obtained have been shown in the Table 2

3.2 Representation by Modified Exponential Curve

In order to represent the total populations corresponding to the years 1951, 1961 and 1971 by modified exponential curve let us construct the Table 3 After computations it is obtained that
\[ c = 1.3938516595, \quad b = 198416533.52 \quad & \quad a = 162671556.48 \]

Therefore, the modified exponential curve that can represent the given data is
\[ y = 162671556.48 + 198416533.52 \times 1.3938516595^x \]

Similarly, the modified exponential curves representing the total populations corresponding to the other years obtained have been shown in Table 4
Table 2: Exponential curve representing total population of India

| Years (x=0,1) | Curve representing total population |
|---------------|-----------------------------------|
| 1951, 1961    | \( y = 361088090 \times 1.21641999047^x \) |
| 1961, 1971    | \( y = 439234771 \times 1.247987837465^x \) |
| 1971, 1981    | \( y = 548159652 \times 1.246587731342^x \) |
| 1981, 1991    | \( y = 683329097 \times 1.238499416627^x \) |
| 1991, 2001    | \( y = 846302688 \times 1.213531826807^x \) |
| 2001, 2011    | \( y = 1027015247 \times 1.178359742501^x \) |
| 1951, 1971    | \( y = 361088090 \times 1.51807353367^x \) |
| 1961, 1981    | \( y = 439234771 \times 1.555726327049^x \) |
| 1971, 1991    | \( y = 548159652 \times 1.543898178043^x \) |
| 1981, 2001    | \( y = 683329097 \times 1.502958459560^x \) |
| 1991, 2001    | \( y = 846302688 \times 1.429977050953^x \) |
| 1951, 1981    | \( y = 361088090 \times 1.892416603937^x \) |
| 1961, 1991    | \( y = 439234771 \times 1.926766148483^x \) |
| 1971, 2001    | \( y = 548159652 \times 1.873569576405^x \) |
| 1981, 2001    | \( y = 683329097 \times 1.771025743397^x \) |
| 1951, 1991    | \( y = 361088090 \times 2.34375685999^x \) |
| 1961, 2001    | \( y = 439234771 \times 2.338192043999^x \) |
| 1971, 2011    | \( y = 548159652 \times 2.20778963611^x \) |
| 1951, 2001    | \( y = 361088090 \times 2.84422543900^x \) |
| 1961, 2011    | \( y = 439234771 \times 2.755231374885^x \) |
| 1951, 2011    | \( y = 361088090 \times 3.351518522806^x \) |

### 3.3 Representation by Logistic Curve

In order to represent the total populations corresponding to the years 1951, 1961 1971 by logistic curve let us construct the Table 5

After computations it is obtained that

\[
C = 0.9181690619, \quad A = 166079821.3578811825 \quad \text{and} \quad B = -0.5400573269
\]

Therefore, the logistic curve that can represent the given data is

\[
y = 166079821.3578811825/(-0.5400573269 + 0.9181690619^x)
\]

Similarly, the modified exponential curves representing the total populations corresponding to the other years obtained have been shown in the Table 6

### 3.4 Representation by Makeham’s Curve

In order to represent the total populations corresponding to the years 1951, 1961, 1971 and 1981 by Makeham’s curve let us construct the Table 7 After computations it is obtained that

\[
d = -0.043813525170, \quad c = 1.023793397619 \quad \text{and} \quad a = 352696247.934125408029
\]
Table 3:

| Year | $x$ | Number population | $f(x) = y_i$ | $\Delta y_i$ | $\frac{\Delta y_i}{\Delta y_0}$ | $\log \left( \frac{\Delta y_i}{\Delta y_0} \right)$ |
|------|-----|-------------------|-------------|-------------|-----------------|-------------------------------|
| 1951 | 0   | 361088090         | 78146681    | 1.393851659547 | 0.332070893151 |
| 1961 | 1   | 439234771         | 108924881   |             |                  |                               |
| 1971 | 2   | 548159652         |             |             |                  |                               |

Table 4: Modified exponential curve representing total population of India

| Years $x = (0, 1, 2)$ | Curve Representing total population |
|-----------------------|-----------------------------------|
| 1951, 1961, 1971      | $y = 162671556.48 + 198416533.52 \times 1.3938516595^x$ |
| 1961, 1971, 1981      | $y = -12844741.85 + 452079512.85 \times 1.2409418651^x$ |
| 1971, 1981, 1991      | $y = -108964607.87 + 657124259.87 \times 1.2056984549^x$ |
| 1981, 1991, 2001      | $y = -813961579.52 + 1497290676.52 \times 1.1088456594^x$ |
| 1991, 2001, 2011      | $y = -12398675095.11 + 13244977783.11 \times 1.0136438552^x$ |
| 1951, 1981, 2011      | $y = -146377372.04 + 507465462.04 \times 1.6350008643^x$ |

Table 5:

| Year | $X$ | $\frac{1}{y_i}$ | $\Delta \frac{1}{y_i}$ | $\frac{\Delta \frac{1}{y_i}}{\Delta y_i}$ |
|------|-----|-----------------|-------------------|----------------------------------|
| 1951 | 0   | 0.166079821     | 0.000000002769407321 | 0.9181690619                  |
| 1961 | 1   | 0.000000002276686878 |             | 0.00000000942720533            |
| 1971 | 2   | 0.000000001824286384 |             | 0.000000001824286384           |

Table 6: Logistic curve representing total population of India

| Years $x = (0, 1, 2)$ | Curve Representing total population |
|-----------------------|-----------------------------------|
| 1951, 1961, 1971      | $y = \frac{166079821.3578811825}{-0.5403573269 + (0.9181690619)^x}$ |
| 1961, 1971, 1981      | $y = \frac{0.0132632202 + (0.7976607733)^x}{607054365.009564391}$ |
| 1971, 1981, 1991      | $y = \frac{0.1074037156 + (0.7800442825)^x}{990490540.3659358186}$ |
| 1981, 1991, 2001      | $y = \frac{0.361708115514 + (0.7377715206)^x}{1400316634.9560702305}$ |
| 1991, 2001, 2011      | $y = \frac{0.6546289697 + (0.7088342327)^x}{292165885.5970385244}$ |
| 1951, 1981, 2011      | $y = \frac{0.0860648533 + (0.4878388268)^x}{591080500.0657298}$ |
Table 7:

| Year | x_i | y_i | log y_i | ∆ log y_i | ∆² log y_i |
|------|-----|-----|---------|-----------|------------|
| 1951 | 0   | 361088090 | 19.704632503150 | 0.195912110819 | 0.02562041346 |
| 1961 | 1   | 439234771  | 19.900544613969  | 0.221532524279 | −0.00112252063 |
| 1971 | 2   | 548159652  | 20.122077138248  | 0.220410003649 | 0.02562041346 |
| 1981 | 3   | 683329097  | 20.342487141897  | 0.220410003649 | 0.02562041346 |

Table 8: Makeham’s curve representing total population of India

| Years | x = (0, 1, 2, 3) | Curve Representing total population |
|-------|-----------------|-----------------------------------|
| 1951, 1961, 1971, 1981 | y = 352696247.934125408029 × (1.246646468467)^x × (1.023793397619)^{(−0.043813525170)^x} |
| 1961, 1971, 1981, 1991 | y = 439256180.12431468776914 × (1.24827978453195169)^x × (0.99995239519317406)^x5.799006765234579297866 |
| 1971, 1981, 1991, 2001 | y = 548947760.20731398904366 × (1.25040580623912214)^x × (0.9985643293142933046)^x5.128382992022534292060511 |
| 1981, 1991, 2001, 2011 | y = 757632444.355018001412 × (1.29660574080506142197)^x × (0.9019269199614684234)^x1.4414320808258161559716 |
| 1951, 1971, 1991, 2011 | y = 360890060.48806901244 × (1.52270198184210846967)^x × (1.00054872531447159209)^x−4.544806655016345495885 |

Thus the Makeham’s curve satisfying the data is

\[ y = (1.246646468467)^x (1.023793397619)^{−0.043813525170}^x \]

Makeham’s curves to the other years obtained have been shown in the following Table 8.

4 Conclusion

The methods described above can be used to represent a given set of numerical data on a pair of variables by non-polynomial curves namely exponential curve, modified exponential curve, logistic curve and Makeham’s curve respectively. Each of the four curves, studied here, that represents a given set of numerical data can be used for interpolation at any position of the independent variable lying within its two extreme values. They can also be suitably applied in inverse interpolation also. A set of numerical data on a pair of variables can be represented by each of the four curves namely exponential curve, modified exponential curve, logistic curve and Makeham’s curve, applying the corresponding methods described above, only when the values of the independent variable are equally spaced. In the situation where the values of the independent variable are not equally spaced, the methods composed here fail to do so.

References

Bathe, K.-J. and Wilson, E. L. (1976). Numerical methods in finite element analysis. Prentice Hall.
Chakrabarty, D. (2011). Finite difference calculus: Method of determining least squares estimates. *Aryabhata Journal of Mathematics & Informatics, 3*(2):363–373.

Chakrabarty, D. (2014a). Curve fitting: Step-wise least squares method. *Aryabhata Journal of Mathematics & Informatics, 6*(1):15–24.

Chakrabarty, D. (2014b). Determination of parameter from observations composed of itself and errors. *International Journal of Engineering Science and Innovative Technology, 3*(2):304–311.

Chakrabarty, D. (2014c). Natural interval of monthly extreme temperature in the context of assam. *Journal of Chemical, Biological and Physical Sciences, 4*(3):2424–2433.

Chakrabarty, D. (2014d). Natural limits of annual total rainfall in the context of india. *International Journal of Agricultural and Statistical Sciences, 10*(1):105–109.

Chakrabarty, D. (2014e). Observation composed of a parameter and chance error : an analytical method of determining the parameter. *International Journal of Electronics and Applied Research, 1*(2):20–38. https://doi.org/10.33665/IJEAR.2014.v01i02.001.

Chakrabarty, D. (2014f). Observation composed of a parameter and chance error : an analytical method of determining the parameter. *International Journal of Electronics and Applied Research, 1*(2):20–38. https://doi.org/10.33665/IJEAR.2014.v01i02.001.

Chakrabarty, D. (2014g). Observation consisting of parameter and error: Determination of parameter. Paper presented in National Seminar on Advances in Electronics and Allied Science & Technology, 2014, held in Gauhati University, India, Abstract ID: CMAST-NaSAEAST-1401(Inv).

Chakrabarty, D. (2014h). Temperature in assam: Natural extreme value. *Journal of Chemical, Biological and Physical Sciences, 4*(2):1479–1488.

Chakrabarty, D. (2015a). Central tendency of annual extremum of surface air temperature at guwahati. *Journal of Chemical, Biological and Physical Sciences. Sec. C, 5*(3):2863 – 2877.

Chakrabarty, D. (2015b). Central tendency of annual extremum of surface air temperature at guwahati based on midrange and median. *Journal of Chemical, Biological and Physical Sciences. Sec. D, 5*(3):3193 – 3204.

Chakrabarty, D. (2015c). A method of finding true value of parameter from observation containing itself and chance error. *Indian Journal of Scientific Research and Technology, 3*(4):14–21.

Chakrabarty, D. (2015d). Observation composed of a parameter and chance error : an analytical method of determining the parameter. *International Journal of Electronics and Applied Research, 2*(1):35 – 47. https://doi.org/10.33665/IJEAR.2015.v02i01.001.

Chakrabarty, D. (2015e). Observation composed of a parameter and chance error : an analytical method of determining the parameter. *International Journal of Electronics and Applied Research, 2*(2):29 – 45. https://doi.org/10.33665/IJEAR.2015.v02i01.001.

Chakrabarty, D. (2015f). Observation consisting of parameter and error: Determination of parameter. In *Proceedings of the World Congress on Engineering*, volume 2, pages 680–684.

Chakrabarty, D. (2016a). Confidence interval of annual extremum of ambient air temperature at guwahati. *Journal of Chemical, Biological and Physical Sciences, 6*(1):192 – 203.
Chakrabarty, D. (2016b). Difference and ratio operators: Representation of numerical data on a pair of variables by a polynomial curve. *Journal of Environmental Science, Computer Science and Engineering & Technology, Section C*, 5(4):549–560.

Chakrabarty, D. (2016c). Elimination-minimization principle: Fitting of exponential curve to numerical data. *International Journal of Advanced Research in Science, Engineering and Technology*, 3(6):2256–2264.

Chakrabarty, D. (2016d). Elimination-minimization principle: Fitting of polynomial curve to numerical data. *International Journal of Advanced Research in Science, Engineering and Technology*, 3(5):2067–2078.

Chakrabarty, D. (2016e). Impact of error contained in observed data on theoretical model: study of some important situations. *International Journal of Advanced Research in Science, Engineering and Technology*, 3(1):1255–1265.

Chakrabarty, D. (2016f). Interpolation: One method of representation of numerical data on a pair of variables by a polynomial curve expressed in the simplest form. *Journal of Environmental Science, Computer Science and Engineering & Technology*, 5(3):405–418.

Chakrabarty, D. (2016g). Pythagorean mean: concept behind the averages and lot of measures of characteristics of data. Paper presented in National Seminar on Advances in Electronics and Allied Science & Technology, 2016, held in Gauhati University, India. Abstract ID: CMAST-NaSAEAST-1601(Inv).

Chakrabarty, D. (2016h). Recent developments on representation of numerical data by a polynomial curve. *International Journal of Electronics and Applied Research*, 3(2):125–158. https://doi.org/10.33665/IJEAR.2016.v03i01.001.

Chakrabarty, D. (2016i). Representation of numerical data on a pair of variables by a polynomial curve expressed in the simplest form. *International Journal of Electronics and Applied Research*, 3(1):26–39. https://doi.org/10.33665/IJEAR.2016.v03i01.001.

Chakrabarty, D. (2016j). Theoretical model and model satisfied by observed data: One pair of related variables. *International Journal of Advanced Research in Science, Engineering and Technology*, 3(2):1527–1534.

Chakrabarty, D. (2017a). Elimination-minimization principle: Fitting of gompertz curve to numerical data. *International Journal of Advanced Research in Science, Engineering and Technology*, 4(1):3180–3189.

Chakrabarty, D. (2017b). Numerical data containing one parameter and chance error: evaluation of the parameter by convergence of statistic. *International Journal of Electronics and Applied Research*, 4(2):59–73. https://doi.org/10.33665/IJEAR.2017.v04i02.001.

Chakrabarty, D. (2017c). Observations containing single parameter and random errors: One method of evaluation of the parameter. *Journal of Environmental Science, Computer Science and Engineering & Technology, Section C*, 6(4):432–449.

Chakrabarty, D. (2017d). Representation of numerical data by some special mathematical curves. *International Journal of Electronics and Applied Research*, 4(1):52–74. https://doi.org/10.33665/IJEAR.2017.v04i01.001.

Chakrabarty, D. (2017e). Some forms of interpolation formula based on divided difference. *Journal of Environmental Science, Computer Science and Engineering & Technology, Section C*, 6(2):199–211.
Chakrabarty, D. (2018a). Derivation of some formulations of average from one technique of construction of mean. *American Journal of Mathematical and Computational Sciences*, 3(3):62–68.

Chakrabarty, D. (2018b). Finite difference and ratio operations: Representation of numerical data on a pair of variables by a polynomial curve. *International Journal of Electronics and Applied Research*, 5(2):76–91. https://doi.org/10.33665/IJEAR.2018.v05i02.001.

Chakrabarty, D. (2018c). General technique of defining average. Paper presented in National Seminar on Advances in Electronics and Allied Science & Technology, 2018, held in Gauhati University, India, Abstract ID: CMAST-NaSAEAST-1801(Inv).

Chakrabarty, D. (2018d). Generalized $f_g$ mean: derivation of various formulations of average. *American Journal of Computation, Communication and Control*, 5(3):101–108.

Chakrabarty, D. (2018e). Observed data containing one parameter and chance error: evaluation of the parameter applying pythagorean mean. *International Journal of Electronics and Applied Research*, 5(1):32–45. https://doi.org/10.33665/IJEAR.2018.v05i01.001.

Chakrabarty, D. (2018f). One generalized definition of average: derivation of formulations of various means. *Journal of Environmental Science, Computer Science and Engineering & Technology*, 7(3):212–225.

Chakrabarty, D. (2018g). Statistics and bioscience: association in research. *Significances of Bioengineering & Biosciences*, 2(5):001–007.

Chakrabarty, D. (2018h). Understanding the space of research. *Biostatistics and Biometrics International Journal*, 8(3):104–109.

Chakrabarty, D. (2019a). Arithmetic-geometric mean: Evaluation of parameter from observed data containing itself and random error. *International Journal of Electronics and Applied Research*, 6(2):98–111. https://doi.org/10.33665/IJEAR.2018.v06i01.001.

Chakrabarty, D. (2019b). Association of statistics with biostatistics research. *Biometrics & Biostatistics International Journal*, 8(3):104–109.

Chakrabarty, D. (2019c). A general method of defining average of function of a set of values. *Aryabhatta Journal of Mathematics & Informatics*, 11(2):269–284.

Chakrabarty, D. (2019d). Observed data containing one parameter and chance error: probabilistic evaluation of parameter by pythagorean mean. *International Journal of Electronics and Applied Research*, 6(1):24–40. https://doi.org/10.33665/IJEAR.2018.v06i01.001.

Chakrabarty, D. (2019e). One definition of generalized $f_g$ mean: derivation of various formulations of average. *Journal of Environmental Science, Computer Science and Engineering & Technology, Section-C*, 8(2):51–66.

Chakrabarty, D. (2019f). One general method of defining average: derivation of definitions formulations of various means. *Journal of Environmental Science, Computer Science and Engineering & Technology, Section-C*, 8(4):327–338.

Chakrabarty, D. (2019g). Pythagorean geometric mean: measure of relative change in a group of variables. Abstract ID: CMAST-NaSAEAST-1902(Inv), Paper presented in National Seminar on Advances in Electronics and Allied Science & Technology, 2019, held in Gauhati University, India.
Chakrabarty, D. (2019h). Significance of change in ambient air temperature in the context of India. *Journal of Chemical, Biological and Physical Sciences*, 9(4):251 – 261. https://doi.org/10.33665/IJEAR.2018.v06i01.001.

Chakrabarty, D. (2019i). Significance of change of rainfall: Confidence interval of annual total rainfall. *Journal of Chemical, Biological and Physical Sciences, Section-C*, 9(3):2249–1929.

Chakrabarty, D. (2020a). Arithmetic-harmonic mean : evaluation of parameter from observed data containing itself and random error. *International Journal of Electronics and Applied Research*, 7(1):29–45. https://doi.org/10.33665/IJEAR.2020.v07i01.001.

Chakrabarty, D. (2020b). Central tendency of annual extremum of surface air temperature at guwahati by aghm. *International Journal of Advanced Research in Science, Engineering and Technology*, 7(12):16088–16098.

Chakrabarty, D. (2020c). Definition/formulation of average from first principle. *Journal of Environmental Science, Computer Science and Engineering & Technology, Section - C*, 9(2):151–163.

Chakrabarty, D. (2020d). AGM: a technique of determining the value of parameter from observed data containing itself and random error. *Journal of Environmental Science, Computer Science and Engineering & Technology, Section - C*, 9(3):473–486.

Chakrabarty, D. (2020e). AHM: a measure of the value of parameter $\mu$ of the model $x = \mu + e$. *International Journal of Advanced Research in Science, Engineering and Technology*, 7(10):15268–15276.

Chakrabarty, D. (2021a). Aghm as a tool of evaluating the parameter from observed data containing itself and random error. *International Journal of Electronics and Applied Research*, 7(2):05 – 23. https://doi.org/10.33665/IJEAR.2020.v07i02.001.

Chakrabarty, D. (2021b). Arithmetic-Harmonic Mean : A measure of central tendency of ratio-type data. *International Journal of Advanced Research in Science, Engineering and Technology*, 8(5):17324–17333.

Chakrabarty, D. (2021c). Comparison of measures of parameter of the model $X = \mu + e$ based on pythagorean means. *International Journal of Advanced Research in Science, Engineering and Technology*, 8(3):16948–16956.

Chakrabarty, D. (2021d). Four formulations of average derived from pythagorean means. *International Journal of Mathematics Trends and Technology*, 67(6):97–118.

Chakrabarty, D. (2021e). Model describing central tendency of data, international journal of advanced research in science. *International Journal of Advanced Research in Science, Engineering and Technology*, 8(9):18193–18201.

Chakrabarty, D. (2021f). Recent development on general method of defining average : a brief outline. *International Journal of Advanced Research in Science, Engineering and Technology*, 8(8):17947–17955.

Chakrabarty, D. (2021g). AGM, AHM, GHM & AGHM : evaluation of parameter $\mu$ of the model $x = \mu + e$. *International Journal of Advanced Research in Science, Engineering and Technology*, 8(2):16691–16699.

Chakrabarty, D. (2021h). AHM as a measure of central tendency of sex ratio. *Biometrics & Biostatistics International Journal*, 10(2):50–57. DOI: 10.15406/bbij.2021.10.00330.
Chakrabarty, D. and Dutta, K. (2007). A method of fitting exponential curve to population of Indian. *International Journal of Agricultural and Statistical Science*, 3(1):109–113.

Chakrabarty, D. and Rahman, A. (2007). Exponential curve: Estimation using the just preceding observation in fitted curve. *International Journal of Agricultural and Statistical Science*, 3(2):381–386.

Chakrabarty, D. and Rahman, A. (2008). Gompertz curve: Estimation using the just preceding observation in fitted curve. *International Journal of Agricultural and Statistical Science*, 4(2):421–424.

Conte, S. and de Boor, C. (1980). *Elementary Numerical Analysis, 3rd Ed*. McGraw-Hill, New York, USA.

Das, B. and Chakrabarty, D. (2016a). Inversion of matrix by elementary transformation: Representation of numerical data by a polynomial curve. *Journal of Mathematics and Systems Sciences*, 12:27 – 32.

Das, B. and Chakrabarty, D. (2016b). Lagrange’s interpolation formula: Representation of numerical data by a polynomial curve. *International Journal of Mathematics Trends and Technology*, 34(2):64–72.

Das, B. and Chakrabarty, D. (2016c). Matrix inversion: Representation of numerical data by a polynomial curve. *Aryabhatta Journal of Mathematics & Informatics*, 8(2):267–276.

Das, B. and Chakrabarty, D. (2016d). Newton’s backward interpolation: Representation of numerical data by a polynomial curve. *International Journal of Applied Research*, 2(10):513–517.

Das, B. and Chakrabarty, D. (2016e). Newton’s divided difference interpolation formula: Representation of numerical data by a polynomial curve. *International Journal of Mathematics Trends and Technology*, 35(3):197–203.

Das, B. and Chakrabarty, D. (2016f). Newton’s forward interpolation: Representation of numerical data by a polynomial curve. *International Journal of Statistics and Applied Mathematics*, 1(2):36–41.

Das, B. and Chakrabarty, D. (2017a). Backward divided difference: Representation of numerical data by a polynomial curve. *International Journal of Statistics and Applied Mathematics*, 2(2):1–6.

Das, B. and Chakrabarty, D. (2017b). Inversion of matrix by elementary column transformation: Representation of numerical data by a polynomial curve. *International Journal of Mathematics Trends and Technology*, 42(1):45–49.

Das, B. and Chakrabarty, D. (2017c). Representation of numerical data by exponential curve. *Aryabhatta Journal of Mathematics & Informatics*, 9(1):157–162.

Das, B. and Chakrabarty, D. (2017d). Representation of numerical data by modified exponential curve. *Journal of Mathematics and Systems Sciences*, 13(1):1–6.

Das, B. and Chakrabarty, D. (2020). Recent developments on representation of experimental data by polynomial curve. *International Journal of Electronics and Applied Research*, 7(1):46–76. https://doi.org/10.33665/IJEAR.2014.v07i01.003.

Dokken, T. and Lyche, T. (1979). A divided difference formula for the error in hermite interpolation.

Gerald, C. F. and Patrick, O. (1994). Wheatly. applied numerical analysis.

Hummel, P. (1947). A note on interpolation. *The American Mathematical Monthly*, 54(4):218–219.

Jordan, C. and Jordán, K. (1965). *Calculus of finite differences*, volume 33. American Mathematical Soc.
Lee, E. (1989). A remark on divided differences. *The American Mathematical Monthly*, 96(7):618–622.

Rahman, A. and Chakrabarty, D. (2009). Linear curve: a simpler method of obtaining least squares estimates of parameters. *International Journal of Agricultural and Statistical Science*, 5(2):415–424.

Rahman, A. and Chakrabarty, D. (2011). General linear curve: a simpler method of obtaining least squares estimates of parameters. *International Journal of Agricultural and Statistical Science*, 7(11):429–440.

Rahman, A. and Chakrabarty, D. (2015a). Basian-markovian principle in fitting of linear curve. *The International Journal Of Engineering And Science*, 4(6):31–43.

Rahman, A. and Chakrabarty, D. (2015b). Basian-markovian principle in fitting of quadratic curve. *International Research Journal of Natural and Applied Sciences*, 2(6):186–210.

Rahman, A. and Chakrabarty, D. (2015c). Elimination of parameters and principle of least squares: Fitting of linear curve to average maximum temperature data in the context of assam. *Aryabhatta Journal of Mathematics & Informatics*, 7(1):23–28.

Rahman, A. and Chakrabarty, D. (2015d). Elimination of parameters and principle of least squares: Fitting of linear curve to average minimum temperature data in the context of assam. *International Journal of Engineering Sciences & Research Technology*, 4(2):255–259.

Rahman, A. and Chakrabarty, D. (2015e). Method of least squares in reverse order: Fitting of linear curve to average maximum temperature data at guwahati and tezpur. *International Journal in Physical & Applied Sciences*, 2(9):24–38.

Rahman, A. and Chakrabarty, D. (2015f). Method of least squares in reverse order: Fitting of linear curve to average minimum temperature data at guwahati and tezpur. *Aryabhatta Journal of Mathematics & Informatics*, 7(2):305–312.

Vertesi, P. (1990). A contrast of some runge-kutta formula pairs. *SIAM Journal on Numerical Analysis*, 27(5):1332–1344.

Whittaker, E. T. and Robinson, G. (1967). *The Calculus of Observations: A Treatise on Numerical Mathematics*, 4th Edition, chapter The Gregory-Newton Formula of Interpolation and An Alternative Form of the Gregory-Newton Formula.

Wisniewski, J. K. (1930). Note on interpolation. *Journal of the American Statistical Association*, 25(170):203–205.