Structure of Cosmological CP Violation via Neutrino Seesaw

V. Barger, Duane A. Dicus, Hong-Jian He, Tianjun Li

1. Introduction

From Big Bang Nucleosynthesis (BBN) the observed abundance of deuterium determines the baryon asymmetry in the universe to be $\eta_B^{\text{BBN}} = (n_B - n_\gamma)/n_\gamma = (6.1^{+0.7}_{-0.5}) \times 10^{-10}$, where $n_\gamma$ is the current photon number density [1]. The precision of this asymmetry has been improved through measurements of the power spectrum of the cosmic microwave background radiation (CMB). From WMAP (Wilkinson Microwave Anisotropy Probe) data [2] the baryon density ($\Omega_B h^2 = 0.0224 \pm 0.0009$) is inferred, which corresponds to

$$\eta_B^{\text{CMB}} = (6.5^{+0.4}_{-0.3}) \times 10^{-10}. \quad (1)$$

The standard model (SM) of particle physics fails to explain the observed baryon asymmetry because the Higgs mass bound from searches at the CERN Large Electron Positron (LEP) collider precludes a sufficiently strong first-order electroweak phase transition [3]. Moreover, only a limited parameter range in the supersymmetric extension of the SM (MSSM) is still viable for baryogenesis.

Alternatively, leptogenesis [4] provides a very attractive mechanism for realistic baryogenesis. This possibility is especially well motivated by the discovery of low energy neutrino oscillations in the past five years (cf. Refs. [5, 6] for recent reviews). The atmospheric neutrino data [7] indicate $\nu_\mu \leftrightarrow \nu_\tau$ oscillations with nearly maximal mixing $\theta_{13} \approx 45^\circ$ at the scale of mass-squared-difference $\Delta_m \approx 2 \times 10^{-3}$ eV$^2$, while the solar and KamLAND data [8] single out $\nu_e \leftrightarrow \nu_\mu$ oscillations with large mixing angle (LMA) solution of $\theta_{12} \approx 32^\circ$ and the scale of mass-squared-difference $\Delta_s \approx 7 \times 10^{-5}$ eV$^2$. The CHOOZ and Palo Verde long baseline reactor experiments (in combination with the mass range of the atmospheric oscillation data) constrain the $\nu_e \rightarrow \nu_\tau$ transition to be fairly small, $\theta_x \lesssim 13^\circ$ (sin$\theta_x \lesssim 0.22$) at 95% C.L. [9], which bounds Dirac CP-violation. The seesaw mechanism [10] provides a most natural explanation for the tiny masses of active neutrinos, and the predicted Majorana nature induces two independent Majorana phases at low energy, relevant to neutrinoless double-beta ($0\nu\beta\beta$) decay experiments [11]. In addition, the seesaw mechanism further allows the natural realization of successful leptogenesis, with the light neutrino mass range compatible with the current low energy oscillation data [12].

A crucial ingredient for leptogenesis is the generation of lepton asymmetry via CP-violations in the neutrino seesaw mechanism [4]. Given the excitement from the various current and upcoming neutrino experiments (especially Superbeams and next generation $0\nu\beta\beta$ decay experiments), an intriguing possibility is that the baryon asymmetry of our universe may be connected to the low energy neutrino CP-violation phenomenon and thus highly testable. We note that a seesaw sector with $k$ right-handed neutrinos will contain $3(k - 1)$ independent CP-phases ($k = 1, 2, 3$). The required CP-violation for leptogenesis can naturally arise from the CP-phases in the seesaw sector, provided $k \geq 2$. The Minimal Neutrino Seesaw Models (MNSMs) for leptogenesis contain two heavy Majorana neutrinos ($N_1, N_2$). Although a link between high energy leptogenesis and low energy neutrino CP-violations is not guaranteed in general [13], in this Letter we will quantitatively derive a minimal criterion for the MNSMs under which the high energy leptogenesis CP-asymmetry can be completely reconstructed from the low energy neutrino CP-violations. We reveal and analyze two general classes of such MNSMs, and derive the predictions for the allowed ranges of low energy neutrino CP-observables that can generate the observed baryon asymmetry. Finally the extension of our reconstruction formalism to analyzing the leptogenesis in the three neutrino ($N_1, N_2, N_3$) seesaw is discussed.

2. Leptogenesis CP Asymmetry in the Minimal Neutrino Seesaw

A lepton asymmetry can be dynamically generated
in an expanding universe, provided that all three of Sakharov’s conditions[14] are realized: (i) the heavy Majorana neutrinos $N_j$ decay into a lepton-Higgs pair $\ell H$ and its CP-conjugate pair $\bar{\ell}H^*$ with different partial widths, thereby violating lepton number; (ii) CP-violation arises from phases in the seesaw sector; (iii) cosmological expansion causes departure from the thermal equilibrium. As the universe cools down, the $N_j$’s drop out of equilibrium and their decays generate a CP asymmetry,

$$\epsilon_j = \frac{\Gamma[N_j \rightarrow \ell H] - \Gamma[N_j \rightarrow \bar{\ell}H^*]}{\Gamma[N_j \rightarrow \ell H] + \Gamma[N_j \rightarrow \bar{\ell}H^*]}.$$  

This leads to a lepton asymmetry,

$$Y_L = \frac{n_L - n_{\bar{\ell}}} s = \sum_j \frac{\epsilon_j \kappa_j}{g_{s_j}},$$  

where $g_{s_j}$ represents the relativistic degrees of freedom which contribute to the entropy $s$ at a temperature of the order of the lightest heavy Majorana neutrino mass $M_j$; in the SM and MSSM, for instance, $g_{s_1} = 106.75$ and $g_{s_1} = 228.75$, respectively[15]. [Here and below we use SM (MSSM) to denote the usual SM (MSSM) with an addition of the neutrino seesaw sector.] In (3), $\kappa_j$ describes the wash-out of the asymmetry $\epsilon_j$ due to the various lepton-number-violating processes. The wash-out factor $\kappa_j$ can be computed from the full Boltzmann equations. Analytically $\kappa_1$ can be approximated as[15]

$$\kappa_1 \approx 0.3 \left[ \frac{10^{-3} \text{eV}}{m_1} \right]^{3/8},$$

for $m_1 = (10^{-2}-10^{-3})\text{eV}$, where $m_1 = (m_D m_D)^{11}/M_1$ [cf. (6)-(7) below for definitions of $m_D$ and $M_1$]. More accurate empirical formulas for $\kappa_1$ fitted to the exact solution were also derived[16]. The nonperturbative sphaleron interactions violate $B + L$ but preserve $B - L$ [17], so that the lepton asymmetry $Y_L$ is partially transmitted to a baryon asymmetry $Y_B$[18]

$$Y_B = \frac{n_B - n_{\bar{b}}}{s} = \frac{\xi}{1 + \xi} Y_L = \frac{\xi}{(\xi - 1)} \sum_j \frac{\epsilon_j \kappa_j}{g_{s_j}},$$  

where $Y_B \approx \eta_B/7.04$. The parameter $\xi$ depends on the other processes in equilibrium; it is expressed as $\xi = (8N_F + 4N_H)/(22N_F + 13N_H)$ with $N_F$ the number of fermion generations and $N_H$ the number of Higgs doublets. Thus, given the particle contents of the SM (MSSM), we have $\xi = 28/79$ (8/23).

The minimal seesaw Lagrangian can be written as

$$L_{ss} = -\bar{\ell}LM_L\ell_R - \bar{\nu}_D m_D \nu_D + \frac{1}{2} \bar{N}^C M \bar{N} \bar{N} + \text{h.c.},$$  

where the light neutrinos $\nu_L = (\nu_e, \nu_\mu, \nu_\tau)^T$ are in the flavor eigenbasis, while the charged leptons $\ell = (e, \mu, \tau)^T$ and heavy Majorana neutrinos $N = (N_1, N_2)^T$ are in their mass eigenbasis so the mass matrices $M_L = \text{diag}(m_e, m_\mu, m_\tau)$ and $M_R = \text{diag}(M_1, M_2)$ are diagonal and real. The $3 \times 2$ Dirac mass matrix $m_D$ contains all 3 independent CP-phases in the seesaw sector, and can be generally written as

$$m_D = \begin{pmatrix} a & a' \\ b & b' \\ c & c' \end{pmatrix} = \begin{pmatrix} \zeta_1 \bar{a} & \zeta_2 \bar{a} \\ \zeta_1 \bar{b} & \zeta_2 \bar{b} \\ \zeta_1 \bar{c} & \zeta_2 \bar{c} \end{pmatrix},$$

where $\zeta_1 \equiv \sqrt{m_0 M_1}$ and $\zeta_2 \equiv \sqrt{m_0 M_2}$ with $m_0 \equiv \sqrt{\Delta \alpha}$. In (7) we have defined, for convenience, the 6 dimensionless parameters, $(\bar{a}, \bar{b}, \bar{c}) \equiv (a, b, c)/\sqrt{m_0 M_1}$ and $(\bar{a}', \bar{b}', \bar{c}') \equiv (a', b', c')/\sqrt{m_0 M_2}$, which contain all 3 independent CP-phases. Integrating out the heavy neutrinos from the seesaw Lagrangian (6) we arrive at the symmetric $3 \times 3$ Majorana mass matrix for active neutrinos

$$M_\nu \approx m_D M_R^{-1} m_D^T = \begin{pmatrix} \bar{a}^2 + \bar{a}'^2 & \bar{a} \bar{b} + \bar{a}' \bar{b}' & \bar{a} \bar{c} + \bar{a}' \bar{c}' \\ \bar{b} \bar{a}' + \bar{b}' \bar{a} & \bar{b}^2 + \bar{b}'^2 & \bar{b} \bar{c}' + \bar{b}' \bar{c} \\ \bar{c} \bar{a}' + \bar{c}' \bar{a} & \bar{c}' \bar{b} + \bar{c} \bar{b}' & \bar{c} \bar{c}' + \bar{c}' \bar{c}' \end{pmatrix}.$$  

Note that $M_\nu$ can be completely expressed in terms of 6 dimensionless parameters $(\bar{a}, \bar{b}, \bar{c})$ and $(\bar{a}', \bar{b}', \bar{c}')$, up to an overall mass scale parameter $m_0 = \sqrt{\Delta \alpha}$.

We can readily verify that det($M_\nu$) = 0, so one of the three neutrino mass eigenvalues ($m_1, m_2, m_3$) must be zero. Therefore the minimal seesaw predicts a hierarchical neutrino mass spectrum and excludes a nearly degenerate mass spectrum. The hierarchical mass spectra include the Normal Hierarchy (NH) and Inverted Hierarchy (IH),

$$\text{NH: } 0 = m_1 < m_2 \ll m_3,$$

$$\text{IH: } m_1 \gtrsim m_2 \gg m_3 = 0,$$  

where for NH we have $m_2 = m_0 \sqrt{r}$, $m_3 = m_0 \sqrt{1 + r}$, and for IH we have $m_1 = m_0 \sqrt{1 + r}$, $m_2 = m_0$. Here $r$ is defined as the ratio $r \equiv \Delta \alpha/\Delta \alpha$, which is constrained by the oscillation data to the range $1.9 \times 10^{-2} \lesssim r \lesssim 7.4 \times 10^{-2}$ (95% C.L.), with a central value $r \approx 0.036$.

Now considering thermal leptogenesis with $M_1 \ll M_2$ for simplicity, only the decays of the lightest right-handed neutrino $N_1$ are relevant for generating the final baryon asymmetry $Y_B$ in (5) because the asymmetry caused by $N_2$ decays is washed out by the lepton-number-violating processes involving $N_1$’s that are more abundant at the high temperature $T \sim M_2 \gg M_1$. But when temperature $T \sim M_1 \ll M_2$ is reached, the heavier $N_2$’s have already decayed and the asymmetry from $N_1$ decays is preserved. Thus, under this mechanism, only the term $\epsilon_1 \kappa_1 / g_{s_1}$ in (5) will contribute. We explicitly derive the
CP-asymmetry parameter $\epsilon_1$ to be

$$
\epsilon_1 = \frac{m_0 M_2}{8 \pi} \left( \frac{\sqrt{2}}{v \sin \beta} \right)^2 F \left( \frac{M_2}{M_1} \right) \tau_1,
$$

$$
\overline{\tau}_1 = \frac{3 m \left[ (m_D^d m_D^d)_{12} \right]_{12}}{(m_0 M_2) (m_D^d m_D^d)_{11}} = \frac{3 m \left( \pi \pi' + \overline{\pi} \overline{\pi'} \right)^2}{|\pi|^2 + |\overline{\pi}|^2 + |\pi'|^2},
$$

where $\sin \beta = 1(\leq 1)$ for SM (MSSM), and the function $F(x)$ is given by [12]

$$
F(x) = \begin{cases} 
1 - (1 + x^2) \ln \left( \frac{1 + x^2}{x^2} \right) + \frac{1}{1 - x^2}, & (\text{SM}), \\
- \ln \left( \frac{1 + x^2}{x^2} \right) + \frac{2}{1 - x^2}, & (\text{MSSM}).
\end{cases}
$$

The CP-asymmetry parameter $\tau_1$ in (10) is solely expressed as a function of the six dimensionless parameters $(\pi, \overline{\pi}, \pi', \overline{\pi'}, \mathbf{a}, \mathbf{c})$ introduced for the Dirac mass matrix $m_D$ in Eq. (7). For the case $M_1 \gg M_2$, the relevant CP-asymmetry is $\epsilon_2$ from $N_2$ decays and can be independently analyzed in the same way as the reconstruction analysis of $\epsilon_1$ below.

3. Structure of Cosmological CP Violation from Low Energy Reconstruction

**Reconstructing the 3 × 3 Neutrino Mass Matrix**

Consider a generic $3 \times 3$ symmetric Majorana mass matrix $\mathcal{M}_\nu$ for 3 light flavor-neutrinos ($\nu_e, \nu_\mu, \nu_\tau$),

$$
\mathcal{M}_\nu = \begin{pmatrix} 
m_{ee} & m_{e\mu} & m_{e\tau} \\
m_{\mu\mu} & m_{\mu\tau} \\
m_{\tau\tau}
\end{pmatrix} \equiv m_0 \begin{pmatrix} 
A & C & D \\
B & E \\
F
\end{pmatrix}. \tag{12}
$$

The mass matrix (12) contains nine independent real parameters, which can be equivalently chosen as three mass eigenvalues ($m_1, m_2, m_3$) $\geq 0$, three mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$), and three CP-violation phases ($\delta, \phi, \phi'$) with $\delta$ the usual Dirac phase and ($\phi, \phi'$) the Majorana phases (irrelevant to neutrino oscillations). For MNSMs we have det($\mathcal{M}_\nu$) = 0 so that there are only two nonzero mass-eigenvalues [cf. (9)]. The neutrino mixing matrix [19] is $V = U \overline{U}'$, which diagonalizes $\mathcal{M}_\nu$ via $V^T \mathcal{M}_\nu V = \mathcal{M}_\nu^{\text{diag}}$, and contains six parameters (three rotation angles and three phases). The unitary matrix $V$ can be decomposed into a matrix $U$ (à la Cabibbo-Kobayashi-Maskawa) and a diagonal matrix $U'$ with two independent Majorana phases, i.e.,

$$
U = \begin{pmatrix} 
c_c c_x & -s_c s_x & -s_x e^{i\delta} \\
-c_c c_a + s_c s_a e^{-i\delta} & c_c c_a + s_c s_a s_x e^{-i\delta} & -s_c c_a e^{-i\delta} \\
-s_s s_a + c_s c_a s_x e^{-i\delta} & s_s s_a s_x e^{-i\delta} & s_s c_a e^{-i\delta}
\end{pmatrix} \tag{13}
$$

where for convenience we have introduced the notation $(\theta_s, \theta_a, \theta_x) \equiv (\theta_{12}, \theta_{23}, \theta_{13})$, with $(s, a, c) \equiv (\sin \theta, \cos \theta)$ for $\alpha = s, a, x$. The matrix $U'$ can be parametrized as, $U' = \text{diag}(e^{-i\delta/2}, e^{-i\phi/2}, 1)$ for NH schemes and $U' = \text{diag}(1, e^{-i\phi/2}, e^{-i\phi'/2})$ for IH schemes. [Our convention of $V$ is related to that of particle data book via $(\theta_{s,a,x}, \delta, \phi, \phi') \rightarrow -(\theta_{s,a,x}, \delta, \phi, \phi'')$.]

Because of the mass spectrum of the minimal seesaw in Eq. (9), we see that in the above parametrization of $U'$ only a single Majorana phase $\phi$ is physically relevant. From the mass diagonalization, we can reconstruct the neutrino mass matrix $\mathcal{M}_\nu$ via the relation

$$
\mathcal{M}_\nu = V^* \mathcal{M}_\nu^{\text{diag}} V', \tag{14}
$$

which gives

$$
\begin{align*}
m_{ee} &= c_s^2 m_1^1 + s_s^2 m_2^1 + \bar{p}^2 m_3^1, \\
m_{\mu\mu} &= (c_s c_a + p c_s s_a) m_1^1 + (c_s c_a + p s_s s_a) m_2^1 + s_s^2 c_a^2 m_3^1, \\
m_{\tau\tau} &= (c_s s_a + p c_s a_s) m_1^1 + (c_s s_a - p s_s a_s) m_2^1 + c_a^2 c_s^2 m_3^1, \\
m_{ee} &= c_x (c_s c_a (m_1^1 - m_2^1) - p s_s s_a (c_a^2 m_1^1 + s_s^2 m_2^1)) + \bar{p} s_s s_a m_3^1, \\
m_{\mu\tau} &= (c_s s_a - p c_s a_s) (c_s c_a - p s_s a_s) m_1^1 + (c_s c_a + p s_s s_a) m_2^1 - s_s s_a c_a^2 m_3^1. \tag{15}
\end{align*}
$$

For NH schemes $(m_1^1, m_2^1, m_3^1) \equiv (0, m_q, m_3) = m_0(\sqrt{1+r}, \sqrt{1+r})$, for IH schemes $(m_1^1, m_2^1, m_3^1) \equiv (m_1, m_2, 0) = m_0(\sqrt{1+r}, q, 0)$, and the two relevant CP-phase factors are $(p, q) = (\bar{p}^2, q) = (e^{i\delta}, e^{i\phi})$.

Unlike most other bottom-up approaches, our reconstruction formalism solely relies on the low energy neutrino observables without any extra ansatz about the specific forms of the neutrino mass matrix (12), and is thus completely general for a given seesaw sector.

**Reconstruction Theorem in Minimal Neutrino Seesaw**

From (8) and (12), we derive the reconstruction equation:

$$
\begin{pmatrix} 
\overline{\mathbf{c}}^2 + \overline{\mathbf{a}}^2 & \overline{\mathbf{c}} \overline{\mathbf{b}} & \overline{\mathbf{c}} \overline{\mathbf{e}} + \overline{\mathbf{c}} \overline{\mathbf{c}} \\
\overline{\mathbf{b}} \overline{\mathbf{b}} & \overline{\mathbf{b}} \overline{\mathbf{a}} \overline{\mathbf{c}} + \overline{\mathbf{b}} \overline{\mathbf{b}} & \overline{\mathbf{b}} \overline{\mathbf{b}} \\
\overline{\mathbf{e}} \overline{\mathbf{e}} + \overline{\mathbf{c}} \overline{\mathbf{c}} & \overline{\mathbf{e}} \overline{\mathbf{e}} + \overline{\mathbf{c}} \overline{\mathbf{c}} & \overline{\mathbf{a}} \overline{\mathbf{a}}
\end{pmatrix} = \begin{pmatrix} 
A & C & D \\
B & E \\
F
\end{pmatrix}. \tag{16}
$$

This equation contains 6 (complex) conditions among which 5 are independent and the sixth condition is redundant because of det($\mathcal{M}_\nu$) = 0 in the minimal seesaw. Accordingly, the property det($\mathcal{M}_\nu$) = 0 also makes one of the 6 elements $(A, B, C, D, E, F)$ to be dependent. Therefore we can analytically solve (reconstruct) 5 out of
the 6 dimensionless seesaw parameters \((\pi, \pi', \tilde{b}, \tilde{b}', \tilde{c}, \tilde{c}')\) of \(m_D\) in terms of the low energy neutrino observables contained in \((A, B, C, D, E, F)\). For \(A \neq 0\), we can exactly resolve (16) in terms of \(\pi\) or \(\pi'\),

\[
\pi' = \tilde{s}_a \sqrt{A - \pi'^2}, \quad \text{or,} \quad \pi = \tilde{s}_a \sqrt{A - \pi'^2},
\]

\[
\tilde{b} = \frac{1}{A} \left[ \pi C - \tilde{s}_b \pi' \sqrt{AB - C^2} \right],
\]

\[
\tilde{b}' = \frac{1}{A} \left[ \pi' C + \tilde{s}_b \pi \sqrt{AB - C^2} \right],
\]

\[
\pi = \frac{1}{A} \left[ \pi D - \tilde{s}_c \pi' \sqrt{AF - D^2} \right],
\]

\[
\pi' = \frac{1}{A} \left[ \pi' D + \tilde{s}_c \pi \sqrt{AF - D^2} \right],
\]

\[
E = \frac{1}{A} \left[ CD + \tilde{s}_b \tilde{s}_c \sqrt{(AB - C^2)(AF - D^2)} \right],
\]

where \((\tilde{s}_b, \tilde{s}_b', \tilde{s}_c, \tilde{s}_c') = \pm 1\). We see that with a single input of either \(\pi\) or \(\pi'\) the other 5 dimensionless parameters in \(m_D\) are completely determined from the low energy neutrino observables contained in \((A, B, C, D, E, F)\); the last equation is a consistency condition due to \(\text{det}(\mathbb{M}_e) = 0\) that will also fix the sign convention of \(\tilde{s}_b, \tilde{s}_c\) from the reconstructed 3 \(
\times\) 3 low energy neutrino mass matrix \(\mathbb{M}_e\) in (12) and (15). Noting that the reconstruction equation (16) is invariant under the joint transformations \(\pi \leftrightarrow \tilde{b}\), \(\pi' \leftrightarrow \tilde{b}'\), \(A \leftrightarrow B\), \(D \leftrightarrow E\), we can readily resolve (16) in terms of \(\tilde{b}\) or \(\tilde{b}'\), for \(B \neq 0\). Similarly, using the invariance of (16) under the exchanges \(\pi \leftrightarrow \pi', \pi' \leftrightarrow \pi\), \(A \leftrightarrow F\), \(C \leftrightarrow E\), we can reexpress the solutions in terms of \(\pi\) or \(\pi'\) instead, for \(F \neq 0\), and so on. The exact solution (17) explicitly shows that from the low energy neutrino observables in \(\mathbb{M}_e\), we can reconstruct 5 out of the 6 dimensionless parameters in (7) for \(m_D\). In the minimal seesaw \(m_D\) contains a maximal 6 \(\mathbb{C}\P\) -phases among which only 3 combinations are independent, and one independent combination gives the required \(\mathbb{C}\P\) -phase for leptogenesis. Hence, we can formulate a Reconstruction Theorem for leptogenesis:

**In the minimal neutrino seesaw models (MMNSMs) with the leptons and heavy Majorana neutrinos in their mass-eigenbasis, five out of six dimensionless parameters \((\pi, \pi', \tilde{b}, \tilde{b}', \tilde{c}, \tilde{c}')\) in \(m_D\) can be completely reconstructed from the low energy neutrino observables. A complete reconstruction can be realized in a class of MMNSMs where one of the six entries in \(m_D\) is not independent. The simplest such MMNSMs can be classified into Type-I and Type-II, where Type-I has 1-texture-zero and Type-II has 1-equality between two entries of \(m_D\), Consequently, for Type-I and -II, all \(\mathbb{C}\P\)-phases in \(m_D\) are fully reconstructable from low energy, so that a unique link is realized between the high energy leptogenesis \(\mathbb{C}\P\)-asymmetry and the low energy neutrino \(\mathbb{C}\P\)-violation observables.**

In fact the requirement of 1-texture-zero or 1-equality is the **Minimal Criterion** for a complete reconstruction of the seesaw sector (up to two mass parameters \(M_{1,2}\) of \(N_1\) and \(N_2\)). We note that in the above reconstruction the \(\mathbb{C}\P\)-phase distribution in all 6 entries of \(m_D\) is also fully fixed, and there is no more freedom for rephasing of lepton-doublets in the Yukawa interaction or Dirac mass term because we have made use of the rephasing of all three lepton-doublets to derive the standard parametrization of the MNSP mixing matrix \(V = U U'\) [19] in the leptonic charged currents such that only 3 independent \(\mathbb{C}\P\)-phases \((\phi, \phi', \phi'')\) enter our low energy formulation (where one of the two Majorana phases \((\phi, \phi')\) becomes irrelevant in the MNSMs). A unique feature of our reconstruction formalism is that it does not introduce any extra unphysical \(\mathbb{C}\P\)-phase beyond the standard MNSP mixing matrix \(V\) [19], so it has great physical transparency and technical simplicity.

Based upon the above reconstruction formulation, we can further define and classify the Most Minimal Neutrino Seesaw Models (MMNSMs) in which \(m_D\) contains 2-texture-zeros or 2-equalities or 1-texture-zero plus 1-equality, so that a single \(\mathbb{C}\P\)-phase alone generates all \(\mathbb{C}\P\)-violations at both high and low energies. (Our classification of the MMNSMs include the recent Frampton-Glashow-Yanagida leptogenesis scheme [20] as one sub-class.) A complete exploration of the MMNSMs will be given elsewhere [21].

### Classifying Minimal Schemes for Leptogenesis

According to the Reconstruction Theorem above, we explicitly classify our Type-I and Type-II MNSMs. For Type-I class of MNSMs, there are six 1-texture-zero schemes for \(m_D\), in total,

\[
m_D = \begin{pmatrix}
a & 0 \\
b & b' \\
c & b' \\
0 & a' \\
b' & 0 \\
c' & 0 
\end{pmatrix}, \quad \begin{pmatrix}
a & a' \\
b' & b' \\
c & c' \\
0 & a' \\
b' & 0 \\
c' & 0 
\end{pmatrix}, \quad \begin{pmatrix}
a & a' \\
b & b' \\
c & c' \\
0 & a' \\
b' & 0 \\
c' & 0 
\end{pmatrix}, \quad \begin{pmatrix}
a & a' \\
b + a' & b' \\
c & c' \\
0 & a' \\
b' & 0 \\
c' & 0 
\end{pmatrix}, \quad \begin{pmatrix}
a & a' \\
b & b' \\
c & c' \\
0 & a' \\
b' & 0 \\
c' & 0 
\end{pmatrix}
\]

which we label as Type-Ia through Type-If. For the Type-II MNSMs, there are 15 schemes depending on how the equality is chosen, which can be horizontal (such as \(a = a'\)), or vertical (such as \(b = c\)), or crossing (such as \(b = c'\)). The Horizontal Equalities (HE) in \(m_D\) are invariant under rephasing and are thus not affected when we fix the low energy parametrization of MNSP matrix in the leptonic charged currents. The Vertical Equalities (VE) or Crossing Equalities (CE) in \(m_D\) arising from an underlying theory could be changed by the rephasing so they are less appealing in comparison with the horizontal equalities. In the following we focus on the analysis of Type-II schemes with horizontal equality. There are only 3 such Type-II-HE schemes,

\[
m_{II} = \begin{pmatrix}
a \pm a \\
b & b' \\
c & c' \\
0 & a' \\
b' & 0 \\
c' & 0 
\end{pmatrix}, \quad \begin{pmatrix}
a & a' \\
b & b' \\
c & c' \\
0 & a' \\
b' & 0 \\
c' & 0 
\end{pmatrix}, \quad \begin{pmatrix}
a & a' \\
b & b' \\
c & c' \\
0 & a' \\
b' & 0 \\
c' & 0 
\end{pmatrix}
\]
where in each scheme we can have a variation scheme by simply flipping the sign of the equality.

4. Reconstruction Analysis for Leptogenesis

We start by proving a general theorem about the structure of the leptogenesis in the two-heavy-neutrino seesaw schemes. The theorem will establish the connection between a nonzero leptogenesis (due to $\epsilon_1 \neq 0$) and a nonzero solar mass-squared-difference (due to $\Delta_a \neq 0$). The Part-I of the theorem states that for any two-heavy-neutrino seesaw with IH, nonzero leptogenesis can be realized only if the low energy observable $\Delta_s$ is not zero (because $\epsilon_1 \to 0$ as $r = \Delta_s/\Delta_a \to 0$). To prove this, we derive, for any two-heavy-neutrino seesaw under $r \to 0$,

$$\begin{align*}
\left[ \vec{\pi} \vec{\varpi} + \vec{b} \vec{b} + \vec{c} \vec{c} \right]_{\text{IH}} &= \frac{c^2_c}{|A|^2} \left[ (\vec{\pi}^2 - \vec{\varpi}^2) + s_{\vartheta} s_{\delta_a} (|_{\varpi}^2 + |_{\varpi}^2) (q^2 - q^{\dagger 2}) \right] \\
&= \text{Imaginary, } \Rightarrow \epsilon_1 = 0 ,
\end{align*}$$

(20)

where $|A| \neq 0$ is ensured by the $\nu$-oscillation data.

We next consider the NH scenario. Using our reconstruction solution (17) and its variations, and defining $\Sigma = \left[ \vec{\pi} \vec{\varpi} + \vec{b} \vec{b} + \vec{c} \vec{c} \right]_{\text{NH}}$, we deduce, for $r \to 0$,

$$\Sigma = \frac{\vec{\pi} \vec{\varpi}}{c^2_a c^2_x} = \frac{\vec{b} \vec{b}}{s^2_a c^2_x} = \frac{\vec{c} \vec{c}}{s^2_x} ,$$

(21)

where the last equality is derived for $s_x \neq 0$. [The combined limit $(r, s_x) \to 0$ will force $(\vec{\pi}, \vec{\varpi}) \to 0$ and a vanishing solar angle $\theta_x \to 0$, but this is not our concern since we examine $r \to 0$ as an independent limit with all mixing angles confined to their physical ranges.] Eq. (21) shows that for any $m_D$ with 1-texture-zero or 1-horizontal-equality, we have, under the $r \to 0$ limit, $\Sigma = 0$ or $\Sigma = \text{Real}$, which means $\epsilon_1 = 0$. Hence, the Part-II of our theorem states: For NH, any viable 2-heavy-neutrino seesaw scheme with 1-texture-zero or 1-horizontal-equality in $m_D$ can have nonzero leptogenesis only if the low energy observable $\Delta_s$ is not zero ($r \neq 0$).

4.1. Analysis for Type-I Minimal Schemes

We next turn to explicit analysis of the Type-Ia scheme in (18) with $\alpha' = 0$. From (17) the complete reconstruction solution is given by

$$\begin{align*}
\vec{a} &= s_a \sqrt{A} , \quad \vec{a}^\dagger = 0 , \\
\vec{b} &= \frac{c_a}{\sqrt{A}} , \quad \vec{b}^\dagger = s_a s_b \sqrt{\frac{A B - C^2}{A}} , \\
\vec{c} &= \frac{D}{\sqrt{A}} , \quad \vec{c}^\dagger = s_a c_b \sqrt{\frac{A F - D^2}{A}} .
\end{align*}$$

(22)

We thus derive the CP-asymmetry $\bar{\epsilon}_1$ from (10),

$$\bar{\epsilon}_1 = \frac{3m}{|A| \left[ (|A|^2 + |C|^2 + |D|^2) \right]} \left[ (C^* \sqrt{A B - C^2} + s_b s_c D^* \sqrt{A F - D^2}) \right] ,$$

(23)

which is expressed solely in terms of the low energy neutrino observables [cf. Eqs. (12) and (15)].

- Type-I Schemes with Inverted Hierarchy

For the IH scenario, we derive for Type-Ia ($\alpha' = 0$),

$$\bar{\epsilon}_1 = \frac{s^2_a c^2_x \sqrt{1 + r (1 + r c^2_x)}}{c^2_a \left[ 1 + 2 s^2_a c^2_x \sin (\cos \phi - 1) \right]^{1/2}} \left(1 + \sqrt{1 + r} \right)^{1/2} ,$$

(24)

Strikingly, $\bar{\epsilon}_1$ depends on Majorana phase $\phi$ only; it is independent of $\theta_x$ and Dirac phase $\delta$.

Type-Id ($\alpha = 0$) is very similar to Type-Ia; $\bar{\epsilon}_1$ is given by (24) with the factor $\sqrt{1 + r (1 + r c^2_x)}$ replaced by $-1$. We have also derived the expressions for other Type-I schemes, which take simple analytical forms under the expansion in $\lambda \equiv \sqrt{r} s_x$.

For Type-Ib ($\beta' = 0$) and Type-Ie ($b = 0$), we find

$$\bar{\epsilon}_1 = \pm \frac{r s_a c_s}{c_a} \left[ s_a c_{c_s} \sin \phi + 2 s_a s_x z_1 \right] + O(\lambda^4) ,$$

(25)

$$z_1 = \frac{c^2_x \sin (\delta - \phi) + s^2_x \sin (\delta + \phi)}{c^2_s} ,$$

which are numerically very accurate. In (25) the overall $+(-)$ sign corresponds to Type-Ib (Ie) scheme.

For Type-Ic ($c' = 0$) and Type-II ($c = 0$), $\bar{\epsilon}_1$ is given by (25) with the replacements, $s_a \leftrightarrow c_a$ and $s_x \to -s_x$. Interestingly the CP-asymmetry $\bar{\epsilon}_1$ is independent of $\theta_x$ and the Dirac phase $\delta$ (Type-Ia and -Id), or depends on them only at higher orders. This means that in the Type-I schemes with inverted hierarchy, leptogenesis is controlled by a single CP-phase $\phi$ which also appears in the low energy $0\nu\beta\beta$ decay observable,

$$|\mathcal{M}_{ee}| = m_0 c^2 \left[ (c^2_x \sqrt{1 + r} + s^2_x) 2 - \sqrt{1 + r} \sin^2 2\theta_x \sin^2 \phi \right] .$$

(26)

Hence the Type-I schemes with IH predict an unique link between high energy and low energy CP-violations via a single phase $\phi$. But, given the anticipated future uncertainty in the nuclear matrix elements [11, 22], the small size of the predicted $|\mathcal{M}_{ee}|$, and the expected uncertainty in the eventual cosmological determination of the sum of neutrino masses for the hierarchical neutrino spectrum [23], it is hard to probe the Majorana phase $\phi$ [24]. Our study of the low energy test of leptogenesis provides a motivation for greatly improving the $|\mathcal{M}_{ee}|$ determination; such efforts are currently underway [22].

- Type-I Schemes with Normal Hierarchy

For NH scenario, we derive for Type-Ia,
\[ \tau_1 = \frac{s_\beta^2 c_\beta^2 \sqrt{r(1+r) \sin(\phi + 2\delta)}}{[r(1-c_\beta^2 c_\beta^2) + s_\beta^2]} \tau_2 \]
\[ = \frac{s_\beta^2 c_\beta^2 \sin(\phi + 2\delta)}{[s_\beta^2 + c_\beta^2 + \cos(\phi + 2\delta)s_\beta^2/\sqrt{r}]} + O(\lambda^3), \]
\[ \tau_2 = \left[ (c_\beta^2 s_\beta^2 \sqrt{r - s_\beta^2} \sqrt{1 + r})^2 + 2\sqrt{r(1+r)}s_\beta^2 c_\beta^2 \cos(\phi + 2\delta) - 1 \right]^{1/2}, \]
where \( \lambda = O(\sqrt{r}, s_\beta) \). Type-Id is again very similar to Type-Ia except that in (27) the denominator of \( \tau_1 \) will have its first brackets [\( \left[ \cdots \right] \) replaced by \(-[s_\beta^2 + c_\beta^2 s_\beta^2] \). Unlike the IH case, the \( \tau_1 \) is now controlled by a single phase-combination \( \phi + 2\delta \) from low energy. Interestingly, we find that the \( 0\nu\beta\beta \) decay observable now becomes

\[ |\mathcal{M}_{\nu}\rangle = m_0 [s_\beta^2 \sqrt{r} + s_\beta^2 \cos(\phi + 2\delta) + O(\lambda^3)] \, . \] (28)
which depends on the same phase-combination \( \phi + 2\delta \). Furthermore, the Dirac phase \( \delta \) can be measured at low energy in long-baseline neutrino oscillation experiments via the leptonic Jarlskog invariant [25],

\[ J = \frac{1}{8} \sin \theta s \sin \theta d s_\beta \cos \phi + \sin \delta. \] (29)
The results of \( \bar{\epsilon}_1 \) in other Type-I schemes are not so simple for the normal hierarchy. For Type-Ib, we find

\[ \tau_1 = \sqrt{c_\beta s_\beta} \left[ c_\beta s_\beta \sin \phi + 2s_\beta s_\beta s_\beta \sin(\phi + \delta) \right] \frac{[s_\beta^2 + c_\beta^2 s_\beta^2 \cos \phi]}{s_\beta^2 + \sqrt{c_\beta^2 s_\beta^2 \cos \phi}} + O(\lambda^3), \] (30)
while for Type-Ie, \( \tau_1 \) is given by

\[ -s_\beta^2 c_\beta^2 \left[ c_\beta s_\beta \sin \phi + 2s_\beta s_\beta s_\beta \sin(\phi + \delta) \right] \frac{[c_\beta^2 + s_\beta^2 s_\beta^2 + 2s_\beta s_\beta c_\beta s_\beta \cos \delta]}{[c_\beta^2 + \sqrt{c_\beta^2 s_\beta^2 \cos \phi}]} + O(\lambda^2). \] (31)
For \( c' = 0 \) (Type-Ic) or \( c = 0 \) (Type-IIf), \( \tau_1 \) is given by (30) or (31) with the replacements \( s_\beta \rightarrow c_\beta \) and \( s_\beta \rightarrow -s_\beta \).

We may mention that the phase convention below (15) is not unique. If we assign the independent Majorana phase instead to the mass \( m'_3 \) (NH), or \( m'_1 \) (IH), the results for \( \tau_1 \) would change only by \( \phi \rightarrow -\phi \) everywhere.

Finally we analyze the predictions in the Type-Ia schemes [cf. Fig. 1]. For the IH scenario, we first show the asymmetry \( \eta_B \) versus the low energy Majorana CP-phase \( \phi \), where we set \( x = M_2/M_1 = 10 \). The result is very insensitive to \( x \) for \( x \geq 3 \), because in the SM and MSSM, \( x_F(x) \) equals \(-\frac{2}{3}\) and \(-3\), to \( 7\% \) (2\%) accuracy or better for \( x \geq 3 \) (\( x \geq 5 \)). Fig. 1(a) shows that for the best fit values of \( \nu \)-oscillation parameters the CMB limit (1) constrains \( M_1 \geq 10^{13} \text{GeV} \) for IH, and for \( M_2 \geq 1.3 \times 10^{13} \text{GeV} \) the phase \( \phi \) is confined to two narrow regions. In Fig. 1(b), we plot the physical mass \( M_1 \) vs. the low energy observable \( |\mathcal{M}_{\nu}\rangle \) of \( 0\nu\beta\beta \) decay which are both controlled by the same CP-phase \( \phi \). Since \( \epsilon_1 \) is sensitive to \((r, m_0)\), we have scanned the 95\% C.L. range of \((\Delta s, \Delta a)\) (and also \( \theta_s \)) in addition to varying \( \phi \) and found that \( M_1 \) is always above \( 4 \times 10^{12} \text{GeV} \), but has little chance to go above \( 2 \times 10^{14} \text{GeV} \). Future improved precision of \((\Delta s, \Delta a)\) will strengthen the bound.

For the NH scenario, we first plot \( \eta_B \) as a function of \( s_\beta \) by varying the CP-phase angle \( \phi + 2\delta \) [cf. Fig. 1(c)]. Then, in Fig. 1(d) we analyze the correlation between the scale \( M_1 \) and the low energy CP-observable \( J \) in (29) by varying both phases \( (\phi, \delta) \in (0, 2\pi) \). We have scanned the 95\% C.L. ranges of all \( \nu \)-oscillation parameters, together with \( \eta_B \). Distinctly, we find \( M_1 \geq 3 \times 10^{10} \text{GeV} \), and for \(|J| \geq 0.01\), \( M_1 \) is mainly below \( 10^{12} \text{GeV} \).

4.2. Analysis for Type-II Minimal Schemes

Next we turn to the analysis of the Type-II schemes with horizontal equality. We consider the Type-II-Hea Schemes with the equality \( a = a' \). From (17) we thus derive a complete reconstruction solution,

\[ \bar{\tau} = \bar{s}_a \sqrt{A}_{\omega_2}, \quad \bar{\tau}' = \bar{s}_a \sqrt{A}_{\omega_1}, \]
\[ \bar{b} = \bar{s}_a \sqrt{A}_{\omega_2} \left[ C - \bar{s}_\nu \rho^{-1} \sqrt{AB - C^2} \right], \]
\[ \bar{b}' = \bar{s}_a \sqrt{A}_{\omega_1} \left[ C + \bar{s}_\nu \rho \sqrt{AB - C^2} \right], \]
\[ \bar{c} = \bar{s}_a \sqrt{A}_{\omega_2} \left[ D - \bar{s}_\nu \rho^{-1} \sqrt{AD - D^2} \right], \]
\[ \bar{c}' = \bar{s}_a \sqrt{A}_{\omega_1} \left[ D + \bar{s}_\nu \rho \sqrt{AD - D^2} \right], \]

where we have introduced the notation \( \rho \equiv \sqrt{M_2/M_1} \), \( \omega_1 = \sqrt{1 + \rho^2} \), and \( \omega_2 = \sqrt{1 - \rho^{-2}} \). The CP-asymmetry observable \( \tau_1 \) is given by

\[ \tau_1 = \frac{2(\rho + \rho^{-1})}{\omega_1^2 |A|} \bar{s}_\nu X + (\rho - \rho^{-1})Y_1 \bar{Y}_2, \] (33)

where

\[ X = X_1 - X_2, \]
\[ = |A|^2 + |C|^2 + |D|^2 - |AB - C^2| - |AF - D^2|, \]
\[ Y \equiv Y_1 + iY_2, \]
\[ = C^* \sqrt{AB - C^2} + \bar{s}_\nu \bar{s}_\nu D^* \sqrt{AF - D^2}, \] (34)

and \( \bar{s}_\nu \bar{s}_\nu = +(-) \) for IH (NH) Schemes.

- **Type-II Schemes with Inverted Hierarchy**

For the IH scenario, we derive the CP-asymmetry of leptogenesis for Type-II-Hea Schemes,

\[ \tau_1 = \frac{2s_\beta c_\beta \left[ \bar{s}_\nu (c_\beta^2 - s_\beta^2) + (\rho - \rho^{-1})s_\beta c_\beta \cos \phi \right]}{\left( \rho + \rho^{-1} \right) \left[ 1 - 4s_\beta^2 c_\beta^2 \sin^2 \phi \right]^{1/2}} \, \left( r \sin \frac{\phi}{2} \right), \] (35)
where we have expanded the exact formula up to $O(r)$ and ignored small sub-leading terms of $O(r^2)$. Again we see that for Type-II-HEa Schemes with IH, the CP-asymmetry $\tau_1$ only contains the Majorana phase $\phi$; it is independent of $s_x$ and the Dirac phase $\delta$. A new feature is the nontrivial dependence on $\rho = \sqrt{M_2/M_1}$ for $\rho \neq 1$.

**Type-II Schemes with Normal Hierarchy**

For the NH scenario, we derive the CP-asymmetry of leptogenesis for Type-II-HEa Schemes,

$$\tau_1 = \frac{-\bar{s}_\nu s_x r_1^{1/4} + (\rho - \rho^{-1}) s_x \cos \frac{\phi + 2\delta}{2}}{\rho(r s_x^2 + s_x^2) + \rho^{-1} s_x^2 \sqrt{1 - \bar{s}_\nu^2} r_1^{1/4} s_x \cos \frac{\phi + 2\delta}{2}} \times 2 s_x \sin \frac{\phi + 2\delta}{2} + O(\lambda^2).$$

A similar feature to Type-Ia with NH [cf. (27)] is that the CP-asymmetry $\tau_1$ in (36) also solely depends on the phase-combination $\phi + 2\delta$.

We have analyzed the prediction of baryon asymmetry $\eta_B$ in Type-II-HEa schemes. For the IH scenario, we first plot the asymmetry $\eta_B$ versus the low energy Majorana phase $\phi$ in Fig. 2(a). Unlike the Type-I schemes [cf. Eqs. (23)-(24)], we find $\eta_B$ to be very sensitive to the mass ratio $\rho^2 = M_2/M_1$ and the sign $\bar{s}_\nu$, according to Eqs. (33) and (35). We show, in Fig. 2(b), the correlation between the leptogenesis scale $M_1$ and the $0\nu\beta\beta$ decay observable $|\mathcal{M}_{ee}|$ where we have varied the mass ratio $M_2/M_1 = 5 - 100$ in addition to scanning all $\nu$-oscillation parameters within 95% ranges. We find that $M_1$ is always above $4 \times 10^{12}$ GeV and largely falls below $10^{14}$ GeV.

For the NH scenario, Fig. 2(c) plots $\eta_B$ versus $\sin \theta$, with different inputs of the ratio $M_2/M_1$ and sign $\bar{s}_\nu$, which can be compared with the Type-Ia in Fig. 1(c). Fig. 2(d) demonstrates the correlation between the mass $M_1$ and the $J$ parameter by scanning $M_2/M_1 \in (5, 100)$ and all other observables within 95% C.L. ranges. Unlike Fig. 1(d), the distribution of $M_1$ has no clear peak around

![Diagram](image-url)
The conversion factor close to the SM value of $\tan \beta$ of the MSSM but is nearly 1 for $\tan \beta$. The neutrino mixing angles and phases are generally small [26], and can be directly incorporated in our reconstruction analysis.

5. Summary and Extension

In this Letter we have presented a general formalism to quantitatively reconstruct leptogenesis for the observed matter-antimatter asymmetry (1) in the minimal neutrino seesaw. We have systematically classified and analyzed such minimal seesaw schemes in which the required high energy CP-asymmetry for leptogenesis is uniquely linked to the low energy CP-phases in the physical MNSP matrix. Imposing the cosmological bound (1) and the existing neutrino oscillation data, we have analyzed the constraints and correlations of the leptogenesis scale $M_1$ versus the low energy neutrino observables $|\mathcal{M}_{ee}|$ (for $0\nu\beta\beta$ decays) and $J$-invariant (for long baseline $\nu$-oscillations). The minimal schemes of Type-I (with one-equality) and Type-II (with one-texture-zero) result in distinctive predictions for leptogenesis and its link to the low energy CP-violation observables (cf. Figs. 1-2).

We can readily extend our reconstruction formalism to the general three-heavy-neutrino seesaw where $\mathcal{N} = (N_1, N_2, N_3)^T$ and $M_R = \text{diag}(M_1, M_2, M_3)$ in their mass-eigenbasis. Consequently the Dirac mass matrix $m_D$ becomes a $3 \times 3$ matrix,
\[ m_D = \begin{pmatrix} a & a' & a'' \\ b & b' & b'' \\ c & c' & c'' \end{pmatrix} = \begin{pmatrix} \zeta \pi & \zeta_\pi & \zeta_\pi' \\ \zeta_\pi & \zeta_\pi & \zeta_\pi' \\ \zeta_\pi & \zeta_\pi & \zeta_\pi' \end{pmatrix}, \] (37)

where \( \zeta_j = \sqrt{m_0 M_j} \) (\( j = 1, 2, 3 \)), and \( m_D \) contains 3 extra independent CP-phases in \( (\pi', \bar{\nu}', \nu') \). A general classification of minimal schemes for (37) and a complete reconstruction of leptogenesis will be given elsewhere. Here we comment on the thermal leptogenesis with a hierarchical mass-spectrum of \( \nu \) such as \( M_1 \ll M_2 \ll M_3 \), which is sometimes called light sequential dominance [27].

With this mass hierarchy, leptogenesis arises from \( \epsilon_1 \) via the \( N_1 \) decays, which is extended from (10) as

\[ \epsilon_1 = \frac{m_0 M_1}{8\pi} \left( \frac{\sqrt{2}}{v \sin \beta} \right)^2 [x_{21} F(x_{21}) \tau_{12} + x_{31} F(x_{31}) \tau_{13}], \]

\[ \tau_{13} = \frac{\Im m \left( (m_D^* m)_{13} \right)^2}{(m_D^* m)_{11}} = \frac{\Im m \left( (\pi \bar{\nu}') + (\bar{\nu} \nu') + (\nu \bar{\nu}') \right)^2}{|\pi|^2 + |\bar{\nu}|^2 + |\nu|^2}, \] (38)

where the formula for \( \tau_{12} \) takes the same form as \( \tau_1 \) in (10), and \( (x_{21}, x_{31}) \equiv (M_2/M_1, M_3/M_1) \). The entries in the Dirac mass matrix \( m_D \) are generally of order \( O(\nu) \), except for the possible texture zeros. Thus the mass hierarchy \( M_1 \ll M_2 \ll M_3 \) implies

\[ \frac{\tau_{13}}{\tau_{12}} = \frac{O(\pi', \bar{\nu}', \nu')^2}{O(\pi', \bar{\nu}', \nu')^2} = \frac{O(M_2/M_3)}{1}. \] (39)

Hence, with the above mass hierarchy, the CP-asymmetry \( \epsilon_1 \) is dominated by \( \tau_{12} \) (contained in the 2-neutrino seesaw), with the \( \epsilon_1 \) term suppressed by the factor \( M_2/M_3 \ll 1 \). Furthermore, the modifications to the reconstruction solutions of \( (\pi, \bar{\nu}, \nu) \) and \( (\pi', \bar{\nu}', \nu') \) relative to a minimal \( (N_1, N_2) \)-seesaw are also suppressed by \( M_2/M_3 \ll 1 \). Thus with \( M_1 \ll M_2 \ll M_3 \), leptogenesis effectively reduces to that of the minimal seesaw in Sec. 2-4,

\[ \epsilon_1 = \frac{m_0 M_2}{8\pi} \left( \frac{\sqrt{2}}{v \sin \beta} \right)^2 F(\frac{M_2}{M_1}) \tau_{12} \left[ 1 + O\left( \frac{M_2}{M_3} \right) \right], \] (40)

which allows the direct application of our Minimal Seesaw Schemes (Sec. 2-4) to the 3-neutrino seesaw at the leading order of the \( M_2/M_3 \) expansion.

Acknowledgments. This work was supported by the DOE under grants DE-FG02-95ER40896 and DE-FG03-93ER40757, by the NSF under grant PHY-0070928, and by the Wisconsin Alumni Research Foundation.

[1] E.g., V. Barger, J. Kneller, H.-S. Lee, D. Marfatia, and G. Steigmann, Phys. Lett. B266, 8 (2003) and references therein.
[2] D. N. Spergel, et al., astro-ph/0302209.
[3] A. Riotto, et al., Ann. Rev. Nucl. Part. Sci. 49, 35 (1999).
[4] M. Fukugita and T. Yanagida, Phys. Lett. B174, 45 (1986).
[5] V. Barger, D. Marfatia, K. Whisnant, hep-ph/0308123; and references therein.
[6] S. Pakvasa and J. W. Valle, hep-ph/0301061; M. C. Gonzalez-Garcia, Y. Nir, Rev. Mod. Phys. 75, 345 (2003).
[7] S. Fukuda, et al., [Super-Kaminokande Collaboration], Phys. Rev. Lett. 86, 5656 (2001); Phys. Lett. B539, 179 (2002); and hep-ex/0309011.
[8] S. N. Ahmed, et al., [SNO Collaboration], nucl-ex/0309004, and Q. R. Ahmad, et al., Phys. Rev. Lett. 89, 011301, 011302 (2002); K. Eguchi, et al., [KamLAND Collaboration], Phys. Rev. Lett. 90, 021802 (2003).
[9] M. Apollonio et al., [CHOOZ Collaboration], Eur. Phys. J. C27, 331 (2003); F. Boehm et al., [Palo Verde Collaboration], Phys. Rev. Lett. 84, 3764 (2000).
[10] T. Yanagida, in Unified Theories, eds. O. Sawada, et al., Feb., 1979; M. Gell-Mann, P. Ramond, R. Slansky, in Sanibel Symposium, CALT-68-709, Feb., 1979, and in Supergravity, eds. D. Freedman, et al. (Amsterdam, 1979); S. L. Glashow, in Quarks and Leptons, Cargese, eds. M. Levy, et al., pp.707, (Plenum, NY, 1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).
[11] S. R. Elliot and P. Vogel, Ann. Rev. Nucl. Phys. Sci. 52, 115 (2002); and references therein.
[12] E.g., for reviews, W. Buchmiller, hep-ph/0204288, hep-ph/0306047, and references therein.
[13] E.g., M. N. Rebelo, Phys. Rev. D67, 013008 (2003).
[14] A. D. Sakharov, JETP Lett. 5, 24 (1967).
[15] E. W. Kolb and M. S. Turner, The Early Universe, Addison-Wesley Pub. (1990); J. A. Harvey and M. S. Turner, Phys. Rev. D42, 3344 (1990); H. B. Nielsen and Y. Takanishi, Phys. Lett. B507, 241 (2001).
[16] M. Hirsch and S. F. King, Phys. Rev. D64, 113005 (2001); P. DiBari, hep-ph/0211175; G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, hep-ph/0310123.
[17] G. ’t Hooft, Phys. Rev. Lett. 37, 8 (1976).
[18] J. A. Harvey, M. S. Turner, Phys. Rev. D42, 3344 (1990).
[19] B. Pontecorvo, Sov. Phys. JETP 6, 420 (1958); Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. 28, 870 (1962).
[20] P. H. Frampton, S. L. Glashow, T. Yanagida, Phys. Lett. B548, 119 (2002); S. L. Glashow, hep-ph/0306100; M. Raidal and A. Strumia, Phys. Lett. B553, 72 (2003).
[21] V. Barger, D. A. Dicus, H.-J. He, T. Li, to appear.
[22] V. A. Rodin, A. Faessler, F. Simkovic, and P. Vogel, Phys. Rev. C68, 044302 (2003); S. R. Elliott and P. Vogel, private communications; A. S. Barash, [NEMO Collaboration], Phys. Atom. Nucl. 66, 458 (2003).
[23] E.g., S. Hannestad, Phys. Rev. D67, 085017 (2003).
[24] V. Barger, S.L. Glashow, P. Langacker, and D. Marfatia, Phys. Lett. B540, 247 (2002) hep-ph/0205290.
[25] L. L. Chau and W. Y. Keung, Phys. Rev. Lett. 53, 1802 (1984); C. Jarlskog, Phys. Rev. Lett. 55, 1039 (1985).
[26] E.g., S. Antusch, J. Kersten, M. Lindner, and M. Ratz, hep-ph/0305723; M. Ratz, private communication.
[27] E.g., S. F. King, JHEP 0209, 011 (2002).