Structure functions at small $x_{Bj}$
in a Euclidean field theory approach

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Abstract

The small-$x_{Bj}$ limit of deep inelastic scattering is related to the high-energy limit of the forward Compton amplitude in a familiar way. We show that the analytic continuation of this amplitude in the energy variable is calculable from a matrix element in Euclidean field theory. This matrix element can be written as a Euclidean functional integral in an effective field theory. Its effective Lagrangian has a simple expression in terms of the original Lagrangian. The functional integral expression obtained can, at least in principle, be evaluated using genuinely non-perturbative methods, e.g., on the lattice. Thus, a fundamentally new approach to the long-standing problem of structure functions at very small $x_{Bj}$ seems possible. We give arguments that the limit $x_{Bj} \to 0$ corresponds to a critical point of the effective field theory where the correlation length becomes infinite in one direction.

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1 Introduction

The behaviour of the structure functions of deep inelastic lepton-nucleon scattering (DIS) at small values of the Bjorken variable $x_{\text{Bj}}$ has been a topic of intense experimental and theoretical research in recent years. The strong rise of the structure functions for $x_{\text{Bj}} \to 0$ observed at HERA [1] opened the way for a more detailed experimental study of the small-$x_{\text{Bj}}$ limit of QCD. Today, many theoretical models try to explain the behaviour of the structure functions in this limit.

In a very pragmatic perturbative approach, the well-known DGLAP dynamics of the $Q^2$ evolution of structure functions [2] is taken seriously in the small-$x_{\text{Bj}}$ region (cf., for example, the parametrizations of [3]). Given a sufficiently small $Q^2$ starting scale of the perturbative evolution, the small-$x_{\text{Bj}}$ rise of structure functions can even be described on the basis of valence-like parton distributions (see [4] and refs. therein). In this case, all of the small-$x_{\text{Bj}}$ rise is produced by perturbative QCD.

If, in a more modest approach, a constant behaviour or a soft growth for $x_{\text{Bj}} \to 0$ is ascribed to the input distributions, the DGLAP evolution allows for a good fit to the data even in the case of a larger $Q^2$ starting scale. This observation underlies successful analyses based on the idea of double asymptotic scaling [5]. It is also at the heart of a recent combined analysis of diffractive and inclusive structure functions [6], where a universal logarithmic growth of the input distributions was assumed.

An obvious problem of the above, purely DGLAP-based approaches is the presence of large $\ln(1/x_{\text{Bj}})$ corrections, which can spoil the usefulness of the perturbation series as an asymptotic expansion. Using the BFKL resummation method [7] (see [8] for reviews and further references), all leading $\ln(1/x_{\text{Bj}})$ contributions can be included in the structure function analysis (see, e.g., [9]). It is, at present, not completely clear whether the predicted very strong power-like growth is borne out by HERA data. Also, no final conclusions can yet be made concerning the effect of the recently obtained next-to-leading order results in the BFKL framework (see [10] and refs. therein) on structure function analyses. Different methods of dealing with the apparent large size of the corrections have been suggested [11].

Unfortunately, because of the problem of infrared diffusion [12], it is not clear that the perturbative resummation of $\ln(1/x_{\text{Bj}})$ terms at any given order in $\alpha_s$ can reveal the asymptotic behaviour. One possibility to go beyond the above resummation schemes is the construction of a high-energy effective action for reggeons (see [13] and refs. therein). Another possibility is the derivation of a small-$x_{\text{Bj}}$ evolution equation in the framework of the semiclassical approach. This method goes back to the treatment of hadron-hadron scattering in the functional integral approach using the eikonal approximation introduced in [14]. For recent results concerning the energy dependence of cross sections in this framework see, e.g., [15].

A very different point of view is advertised in [16], where, in addition to the familiar soft pomeron, a phenomenological hard pomeron is introduced to describe small-$x_{\text{Bj}}$ structure functions. A further analysis [17], motivated by the successful phenomenology of [16], concludes that the appearance of new small-$x_{\text{Bj}}$ singularities due to $Q^2$ evolution


is inconsistent with the analyticity principle governing Regge theory and underlying the approach of [16]. Note also the successful analysis of various experimental cross sections reported in [18], which combines ideas of [14] and [16].

In spite of the large amount of work invested and the good description of data achieved in most of the above approaches, our fundamental understanding of the small-$x_{\text{Bj}}$ asymptotics in QCD is still not satisfactory. Therefore, we propose yet another method to achieve this goal. In this article, we will derive a connection between the small-$x_{\text{Bj}}$ behaviour of the structure functions of DIS and the behaviour of correlation functions in a certain effective field theory, which we consider both in Minkowski and Euclidean space-time. The analytic continuation from Minkowski to Euclidean space-time to be discussed below is similar in spirit to the one described in [19] for the high-energy hadron-hadron scattering amplitudes. However, the details are quite different and we will comment on the connection between the two approaches.

For simplicity, we study in this article a scalar model field theory where scalar currents replace the electromagnetic currents of real life, i.e., QCD. However, it will become clear that our methods allow a straightforward generalization to QCD.

Our article is organized as follows. In Sect. 2 we define the model and discuss the properties of our scalar analogue of the virtual Compton amplitude. In Sect. 3 we introduce a class of effective Hamiltonians depending on a parameter $r$. We also make a rotation from Minkowski to Euclidean space-time and show that the high-$Q^2$, $x_{\text{Bj}} \to 0$ limit corresponds to $r \to 0$. In Sect. 4 the amplitude is written as a functional integral with $r$-dependent action. In Sect. 5 the formalism is applied to a simple model of free fields. We show that – at least in the free-field limit – we get a diverging correlation length in our effective theory for $r \to 0$, i.e., a critical phenomenon. Finally, two alternative continuations from Minkowski to Euclidean space-time are outlined in Sect. 6. Section 7 contains our conclusions.

2 The model and the scalar analogue of the virtual Compton amplitude

We consider a model with one real scalar field $\phi(x)$ and the classical Lagrangian

$$\mathcal{L}(x) = \frac{1}{2} \partial_{\mu} \phi(x) \partial^{\mu} \phi(x) - \frac{m^2}{2} \phi(x)^2 - \frac{\lambda}{4!} \phi(x)^4, \quad (2.1)$$

where $m$ and $\lambda$ are the unrenormalized mass and coupling parameter, respectively. The momentum canonically conjugate to $\phi(x)$ is

$$\pi(x) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}(x)} = \dot{\phi}(x). \quad (2.2)$$

The Hamiltonian and the third component $P^3$ of the momentum operator read

$$H = \int_{x^0 = \text{const.}} d^3 x \left\{ \frac{1}{2} \Pi(x)^2 + \frac{1}{2} \left( \nabla \Phi(x) \right)^2 + \frac{m^2}{2} \Phi(x)^2 + \frac{\lambda}{4!} \Phi(x)^4 \right\}, \quad (2.3)$$
\[ P^3 = -\frac{1}{2} \int_{x^0=\text{const.}} d^3\vec{x} \left( \Pi(x) \frac{\partial \Phi(x)}{\partial x^3} + \frac{\partial \Phi(x)}{\partial x^3} \Pi(x) \right). \]  \hspace{1cm} (2.4)

In Eqs. (2.3) and (2.4) no vacuum expectation values are subtracted. Note that we use lower- and upper-case letters for the classical field variables and the corresponding operators respectively.

We assume that the theory has physical states with reasonable spectrum\(^1\) etc. We will call ‘proton’ the particle of lowest mass \(M\). We introduce now an ‘electromagnetic’ coupling of our field to a scalar ‘photon’ with field \(A(x)\):

\[ \mathcal{L}'(x) = -e J(x) A(x) \quad \text{with} \quad J(x) = \Phi(x)^2. \]  \hspace{1cm} (2.5)

This scalar current is our analogue of the electromagnetic current of QCD. In the following we will study DIS, i.e., the cross section for the absorption of the scalar ‘photon’ on the scalar ‘proton’ (Fig. 1).

\[ X(p) \]

\[ q \quad X(p') \]

\[ p \]

**Figure 1**: Diagram for the absorption of a scalar photon on the scalar proton.

We define the structure function for this reaction as

\[ W(\nu, Q^2) = \sum_X \frac{1}{2} (2\pi)^3 \delta(4)(p' - p - q) \langle P(p)|J(0)|X(p')\rangle \langle X(p')|J(0)|P(p)\rangle, \]  \hspace{1cm} (2.6)

where \(Q^2 = -q^2 > 0\) and \(\nu = pq/M > 0\). \hspace{1cm} (2.7)

All the notation is standard. The conventions for the metric, normalization of states etc. follow [20].

The object to study from a theoretical point of view is the virtual Compton amplitude. To be precise, we will study the amplitude corresponding to the retarded commutator (see, e.g., [21]):

\[ T_r(\nu, Q^2) = \frac{i}{2\pi} \int d^4x \, e^{i q \cdot x} \theta(x_0) \langle P(p)|[J(x), J(0)]|P(p)\rangle. \]  \hspace{1cm} (2.8)

Here and in the remainder of the paper, we always assume that the connected part of the relevant matrix element is taken, \(\langle \cdots \rangle = \langle \cdots \rangle_c\). It is easy to see that

\[ \text{Im} T_r(\nu + i\varepsilon, Q^2) = \text{sgn}(\nu) W(|\nu|, Q^2) \]  \hspace{1cm} (2.9)

for \(\nu\) real and \(\varepsilon \to 0+\).

\(^1\) Of course we know that the theory defined by Eq. (2.1) is trivial if the cutoff goes to infinity in four dimensions. The purist may always think of a theory with a finite cutoff.
We consider now Eq. (2.8) in the proton rest frame, where we can set
\[ p = (M, \vec{0}) \quad , \quad q = (\nu, \bar{e}_3 \sqrt{\nu^2 + Q^2}) \quad (2.10) \]
with \( \bar{e}_j \) \((j = 1, 2, 3)\) the spatial cartesian unit vectors. Rotational invariance gives
\[ \langle P(p) | J(\vec{x}, x^0) J(0) | P(p) \rangle = \langle P(p) | J(\pm |\vec{x}|\bar{e}_3, x^0) J(0) | P(p) \rangle . \quad (2.11) \]
Inserting this in Eq. (2.8) and using Eq. (2.10) and the fact that the commutator in Eq. (2.8) vanishes outside the light cone, we find
\[ T_r(\nu, Q^2) = -\frac{1}{\sqrt{\nu^2 + Q^2}} \int_0^\infty dx^0 \int_{-x^0}^{x^0} dx^3 x^3 e^{ix^0 \nu - ix^3 \sqrt{\nu^2 + Q^2}} \langle P(p) | [J(x^3 \bar{e}_3, x^0), J(0)] | P(p) \rangle . \quad (2.12) \]
In Eq. (2.12) we have a convenient starting point for the analytic continuation of \( T_r(\nu, Q^2) \) into the upper half of the complex \( \nu \) plane, keeping always \( Q^2 \) fixed. In fact, it is easy to see that the integral in Eq. (2.12) defines an analytic function for \( \text{Im} \nu > 0 \) (cf. Appendix A). Using standard methods, one finds then that \( T_r(\nu, Q^2) \) can be extended to a function analytic in the whole \( \nu \) plane except for poles and cuts on the real axis for \( |\nu| \geq Q^2/2M \) (cf. Fig. 2).

\textbf{Figure 2}: The analyticity structure of the virtual Compton amplitude \( T_r(\nu, Q^2) \) in the \( \nu \) plane. Singularities can only occur for real \( \nu \), \( |\nu| \geq Q^2/2M \). The integral in Eq. (2.12) is a representation of \( T_r(\nu, Q^2) \) for \( \text{Im} \nu > 0 \).

We are interested in the behaviour of \( T_r(\nu, Q^2) \) for \( \nu \to \infty \) on the real axis. Instead of studying this directly, we will study \( T_r(\nu, Q^2) \) on the imaginary axis, i.e., for \( \nu = i\eta, \eta \to +\infty \). With standard assumptions for the high-momentum behaviour of the amplitudes of QFT (cf. Appendix A), we can apply the Phragmén-Lindelöf theorem (see, e.g., [22]), which ensures that the asymptotic behaviour on the real and imaginary axes are related by analytic continuation. Our object of study is thus:
\[ T_r(i\eta, Q^2) = \frac{i}{\sqrt{\eta^2 - Q^2}} \int_0^\infty dx^0 \int_{-x^0}^{x^0} dx^3 x^3 e^{-x^0 \eta + x^3 \sqrt{\eta^2 - Q^2}} \langle P(p) | [J(x^3 \bar{e}_3, x^0), J(0)] | P(p) \rangle . \quad (2.13) \]
It turns out to be convenient to generalize this amplitude slightly. Let \( \mu \) with \( \text{Im} \mu \geq 0 \)
be a complex parameter with dimension of mass and define:

\[ T_{+}(i\eta, Q^2, \mu) = \frac{2i}{\sqrt{\eta^2 - Q^2}} \int_{0}^{\infty} dx \int_{-x^0}^{x^0} dx^3 x^3 e^{x^3(i\mu - \eta) + x^3\sqrt{\eta^2 - Q^2}} \langle P(p)|J(x^3\hat{e}_3, x^0)J(0)|P(p)\rangle. \]  

(2.14)

We have

\[ T_{r}(i\eta, Q^2) = \text{Re} T_{+}(i\eta, Q^2, 0), \]  

(2.15)

which is obvious for \( \eta > Q \) and requires a little thought for \( 0 < \eta \leq Q \) (cf. Appendix A).

In the next section we will represent \( T_{+}(i\eta, Q^2, \mu) \) for \( \text{Re} \mu < -M \) as a correlation function in an effective Euclidean field theory. The amplitude \( T_{r}(i\eta, Q^2) \) is then obtained from Eq. (2.15) after analytic continuation of \( T_{+}(i\eta, Q^2, 0) \) to \( \mu = 0 \). We will also write \( T_{+}(i\eta, Q^2, 0) \) as a correlation function in the corresponding effective theory in Minkowski space-time.

### 3 From Minkowski to Euclidean space

Let us introduce new coordinates in Eq. (2.14):

\[ x^0 = \xi \cosh \chi, \quad x^3 = \xi \sinh \chi \]  

(3.1)

with

\[ 0 \leq \xi < \infty, \quad -\infty < \chi < \infty. \]  

(3.2)

In this section we always consider \( \eta > Q \). Then we insert a sum over a complete set of states and perform the \( \xi \) integration:

\[ T_{+}(i\eta, Q^2, \mu) = \sum_{\chi} \frac{2i}{\sqrt{\eta^2 - Q^2}} \int_{-\infty}^{\infty} d\chi \sinh \chi \int_{0}^{\infty} d\xi \xi^2 \times e^{-\xi \left[(\eta + i(p^0 - M - \mu)) \cosh \chi - (\sqrt{\eta^2 - Q^2} + ip^3) \sinh \chi\right]} \langle P(p)|J(0)|X(p')\rangle \langle X(p')|J(0)|P(p)\rangle. \]  

(3.3)

The convergence of the \( \xi \) integration in Eq. (3.3) is guaranteed since

\[ \eta \cosh \chi - \sqrt{\eta^2 - Q^2} \sinh \chi > 0, \]  

(3.4)

and we get

\[ T_{+}(i\eta, Q^2, \mu) = \sum_{\chi} \frac{2i}{\sqrt{\eta^2 - Q^2}} \int_{-\infty}^{\infty} d\chi \sinh \chi \times 2 \left[(\eta + i(p^0 - M - \mu)) \cosh \chi - \left(\sqrt{\eta^2 - Q^2} + ip^3\right) \sinh \chi\right]^{-3} \times \langle P(p)|J(0)|X(p')\rangle \langle X(p')|J(0)|P(p)\rangle. \]  

(3.5)

Since \( p^0 \geq |p^3| \), we have also

\[ (p^0 - M - \text{Re} \mu) \cosh \chi - p^3 \sinh \chi > 0 \]  

(3.6)
for \( \text{Re} \mu < -M \). This allows us to write Eq. (3.5) again as an integral over a parameter \( \xi \):

\[
T_+(i\eta, Q^2, \mu) = \sum_X \frac{-2}{\sqrt{\eta^2 - Q^2}} \int_{-\infty}^{\infty} d\chi \sinh \chi \int_0^{\infty} d\xi^2 \\
\times e^{-\xi \left[ (\mu^0 - \mu - i\eta) \cosh \chi - (\mu^3 - i\sqrt{\eta^2 - Q^2}) \sinh \chi \right]} \\
\times \langle P(p)|J(0)|X(p')\rangle \langle X(p')|J(0)|P(p) \rangle \\
= \frac{-2}{\sqrt{\eta^2 - Q^2}} \int_{-\infty}^{\infty} d\chi \sinh \chi \int_0^{\infty} d\xi^2 \\
\times e^{\xi \left[ (\mu + i\eta) \cosh \chi - \xi \sqrt{\eta^2 - Q^2} \sinh \chi \right]} \mathcal{M}_E,
\]

where

\[
\mathcal{M}_E = \langle P(p)| e^{\xi (H \cosh \chi - P^3 \sinh \chi)} J(0) e^{-\xi (H \cosh \chi - P^3 \sinh \chi)} J(0)|P(p) \rangle.
\]

Effectively we have shifted the \( \xi \)-integration in Eq. (3.3) from the real to the imaginary axis, \( \xi \to -i\xi \). In Eq. (3.8) \( H \) and \( P^3 \) denote the Hamilton operator and the third component of the momentum operator of the original theory, Eqs. (2.3) and (2.4), respectively.

The matrix element \( \mathcal{M}_E \) can now be interpreted as one of a Euclidean field theory. For this we set

\[
y^0 = \xi \cosh \chi, \quad y^3 = \xi \sinh \chi, \quad y_{\pm} = y^0 \pm y^3 = \xi e^{\pm \chi}
\]

and

\[
r = \frac{2y_-}{y_+ + y_-} = 1 - \frac{y^3}{y^0}, \quad \eta_{\pm} = \eta \pm \sqrt{\eta^2 - Q^2}.
\]

We have

\[
y_+ \geq 0, \quad y_- \geq 0, \quad 0 \leq r \leq 2.
\]

Furthermore we define an effective, \( r \)-dependent Hamiltonian:

\[
H_{eff}(r) = H - (1 - r)P^3.
\]

With this we can write

\[
\mathcal{M}_E \equiv \mathcal{M}_E(y_0, r) = \langle P(p)| e^{y^0 H_{eff}(r)} J(0) e^{-y^0 H_{eff}(r)} J(0)|P(p) \rangle
\]

and

\[
T_+(i\eta, Q^2, \mu) = \frac{1}{2\sqrt{\eta^2 - Q^2}} \int_0^{\infty} dy_+ \int_0^{\infty} dy_- (y_+ - y_-) e^{\frac{i}{2}y_+(\mu + \eta_-) + \frac{i}{2}y_-(\mu + \eta_+)} \mathcal{M}_E(y_0^0, r)
\]

\[
= \frac{-2}{\sqrt{\eta^2 - Q^2}} \int_0^{\infty} dy_0^0(y_0^0)^2 \int_0^2 dr (1 - r) e^{y^0[\mu + i(1 - \frac{r}{2})\eta_- + i\frac{r}{2}\eta_+]} \mathcal{M}_E(y_0^0, r).
\]
We see from Eq. (3.10) that for \( \eta \to \infty \), \( Q^2 \) fixed, we have \( \eta_+ \sim 2\eta, \eta_- \sim Q^2/(2\eta) \). Then the oscillating term \( \exp(iy^0 r\eta_+/2) \) restricts the \( r \)-integration to
\[
r \lesssim \frac{1}{\eta y^0},
\]
(3.15)
Thus, except for the small-\( y^0 \) range, \( 0 \leq y^0 \lesssim 1/\eta \), the behaviour of the matrix element \( M_E(y_0, r) \) for \( r \to 0 \) will be essential. Going from \( r = 1 \) to \( r = 0 \) in Eq. (3.12) corresponds, of course, to going from the ordinary Hamiltonian \( H \) to the light-cone Hamiltonian \( H - P^3 \). In conventional light-cone calculations the theory is quantized directly on a light-like subspace (for reviews and further refs. see [23]). However, we always stay away from the light cone by having \( r \neq 0 \). For studies concerning the possibility to approach the light cone continuously in the Hamiltonian method, we refer to [24]. In Sect. 4 below we will use the Lagrangian method and for us the light-like limit concerns only the endpoint \( r = 0 \) of the \( r \)-integration in Eq. (3.14).

Whereas for \( r \in (0, 2) \), the effective Hamiltonian \( H_{eff}(r) \) has an energy gap, this gap vanishes for \( r = 0 \) and 2. We have
\[
\min \{ \langle X|H_{eff}(r)|X \rangle - E_0 \} = \sqrt{r(2-r) M},
\]
(3.16)
where \( E_0 \) is the vacuum energy:
\[
E_0 = \langle 0|H_{eff}(r)|0 \rangle = \langle 0|H|0 \rangle,
\]
(3.17)
and the minimum is taken over all normalized states \( |X \rangle \) orthogonal to \( |0 \rangle \). Equation (3.16) is already indicative of a large correlation length and a critical phenomenon for \( r = 0 \) and we will give more arguments for this in Sect. 5.

Instead of representing \( T_+(i\eta, Q^2, \mu) \) as an integral over a Euclidean matrix element \( M_E(y_0, r) \) in Eq. (3.13), we can also represent it as an integral over the corresponding Minkowskian matrix element \( M_E(y_0, r) = M_M(-iy^0, r) \):
\[
T_+(i\eta, Q^2, \mu) = \frac{2i}{\sqrt{\eta^2 - Q^2}} \int_0^\infty dx^0(x^0)^2 \int_0^2 dr (1-r) e^{-x^0[-i\mu+(1-\frac{r}{2})\eta_-+\frac{1}{2}r\eta_+] M_M(x^0, r)}.
\]
(3.18)
with
\[
M_M(x^0, r) = \langle P(p) | e^{+ix^0H_{eff}(r)} J(0) e^{-ix^0H_{eff}(r)} J(0) | P(p) \rangle.
\]
(3.19)
In Eq. (3.18) there is no problem when setting \( \mu = 0 \).

4 Functional integral representation

In this section we will rewrite the matrix element of Eq. (3.13) in terms of a path integral, using standard procedures as in the conventional case \( H_{eff} = H \), i.e., \( r = 1 \). We assume for simplicity that the states \( |P(p)\rangle \) have the quantum numbers of the \( \Phi \) field, i.e., that
the $\Phi$ field is an interpolating field for these states. Then, according to the basic principles of the LSZ reduction formalism, we have to construct the operators

$$A(p, x^0) = i \int_{x^0=\text{const.}} d^3 \vec{x} e^{ipx} \frac{\partial}{\partial x^0} \Phi(x). \quad (4.1)$$

We have then, in the weak sense,

$$\lim_{x^0 \to \pm \infty} \frac{1}{\sqrt{Z}} A^\dagger(p, x^0)|0\rangle = |P(p)_{\text{out}}\rangle = |P(p)\rangle,$$  

(4.2)

where $Z$ is the wave function renormalization constant.

The Euclidean version of Eq. (4.2) for the proton with zero three-momentum reads:

$$\frac{1}{\sqrt{Z}} \lim_{t \to \infty} \left( e^{-Ht} A^\dagger(0)|0\rangle \right) e^{(M+E_0)t} = |P(p)\rangle,$$  

(4.3)

where we set $A(0) \equiv A(p,0)$ with $p = (M, \vec{0})$. In terms of the field operator and its conjugate canonical momentum, we have

$$A(0) = \int_{x^0=0} d^3 \vec{x} (i\Pi(x) + M\Phi(x)),$$  

(4.4)

$$A^\dagger(0) = \int_{x^0=0} d^3 \vec{x} (-i\Pi(x) + M\Phi(x)).$$  

(4.5)

Before rewriting the matrix element of Eq. (3.13) in terms of a path integral, let us recall the corresponding procedure in the conventional case $H_{\text{eff}} = H$, i.e., $r = 1$. Using Eq. (4.3) we have

$$\mathcal{M}_E(\tau, 1) = \frac{1}{Z} \lim_{\tau_f \to -\infty} \lim_{\tau_i \to +\infty} e^{(\tau_f - \tau_i)(M+E_0)} (0|A(0) e^{-(\tau_f - \tau)H} J(0) e^{-(\tau - 0)H} J(0) e^{-(0 - \tau_i)H} A^\dagger(0)|0).$$  

(4.6)

Now the standard procedure is to introduce alternating intermediate states which are eigenstates of the field operators $\Pi$ and $\Phi$ with definite classical field eigenvalues $\pi$ and $\phi$ on a sufficiently dense grid in Euclidean time. The $\pi$ integrations can then be carried out explicitly, leaving one with a product of $\phi$ integrations. In the limit where the time slices become arbitrarily thin, this defines the Euclidean path integral.

In realizing this procedure, all operators in Eq. (1.6) are replaced by functionals of $\pi$ and $\phi$. For $H$ and $J$, the relevant expressions are given in Eqs. (2.3) and (2.5), for the annihilation and creation operator $A$, $A^\dagger$ in Eqs. (4.4) and (4.5).

The standard procedure outlined above gives the result

$$\mathcal{M}_E(\tau, 1) = \frac{1}{Z} \lim_{\tau_f \to +\infty} \lim_{\tau_i \to -\infty} e^{(\tau_f - \tau_i)M} Z^{-1} \int D\phi a(\tau_f) j(\tau) j(0)a^\dagger(\tau_i) e^{-\int d^4x \mathcal{L}_E}, \quad (4.7)$$

(4.7)

$$Z = \int D\phi e^{-\int d^4x \mathcal{L}_E}, \quad (4.8)$$

(4.8)
with the Euclidean Lagrangian
\[
\mathcal{L}_E = -i\pi(x)\dot{\phi}(x) + \mathcal{H} \\
= -i\pi(x)\dot{\phi}(x) + \frac{1}{2}\pi(x)^2 + \frac{1}{2}(\nabla\phi(x))^2 + \frac{m^2}{2}\phi(x)^2 + \frac{\lambda}{4!}\phi(x)^4.
\] (4.9)

Here the canonical momentum \(\pi\) entering Eq. (4.7) via \(a, a^\dagger\) and \(\mathcal{L}_E\) has to be understood as a function of \(\phi\). In the Euclidean theory, the relation between \(\phi\) and \(\pi\) is defined by
\[
i\dot{\phi} = \frac{\partial \mathcal{H}}{\partial \pi} = \pi.
\] (4.10)

In this way, Eq. (4.7) and the expressions for \(a\) and \(\mathcal{L}_E\) follow directly from Eq. (4.6) and the explicit form of the Hamiltonian density \(\mathcal{H}\).

The result at \(r \neq 1\) can be obtained by following the standard procedure for converting a matrix element into a functional integral expression outlined above. It is easy to guess the correct answer by observing that the effective Hamiltonian density is defined by the substitution
\[
\mathcal{H} \rightarrow \mathcal{H}_{eff} = \mathcal{H} + (1-r)\pi \partial_3 \phi.
\] (4.11)

Thus, the expression for \(\pi\) in terms of \(\phi\) following from Eq. (4.10) now reads
\[
i\dot{\phi} = \frac{\partial \mathcal{H}_{eff}}{\partial \pi} = \pi + (1-r)\partial_3 \phi.
\] (4.12)

This is to be used when expressing \(a, a^\dagger\) and the new effective Lagrangian \(\mathcal{L}_{E, eff}\) in terms of \(\phi\) in Eq. (4.7).

We summarize the results of this calculation, some details of which are given in Appendix B: the Euclidean matrix element of Eq. (3.13) can be calculated as
\[
\mathcal{M}_E(\tau, r) = \frac{1}{Z} \lim_{\tau_j \rightarrow -\infty} \int D\phi a(\tau_f) j(\tau) \mathcal{Z}^{-1} \mathcal{E}_{E, eff} e^{-\int d^4 x \mathcal{L}_{E, eff}}.
\] (4.13)

\[
\mathcal{Z} = \int D\phi e^{-\int d^4 x \mathcal{L}_{E, eff}},
\] (4.14)

with
\[
\mathcal{L}_{E, eff} = \frac{1}{2}\left(\partial_0 \phi(x) + i(1-r)\partial_3 \phi(x)\right)^2 + \frac{1}{2}(\nabla\phi(x))^2 + \frac{m^2}{2}\phi(x)^2 + \frac{\lambda}{4!}\phi(x)^4.
\] (4.15)

and
\[
a(\tau_f) = \int_{x^0=\tau_f} d^3 x \left( -\partial_0 \phi(x) - i(1-r)\partial_3 \phi(x) + M\phi(x) \right),
\] (4.16)
\[
a^\dagger(\tau_i) = \int_{x^0=\tau_i} d^3 x \left( \partial_0 \phi(x) + i(1-r)\partial_3 \phi(x) + M\phi(x) \right),
\] (4.17)
\[
j(\tau) = \phi^2(\vec{0}, \tau).
\] (4.18)

Note that in the Euclidean path integral, Eq. (4.13), the functionals \(a(\tau)\) and \(a^\dagger(\tau)\) are not complex conjugate to each other and \(\mathcal{L}_{E, eff}\) is not a real function for \(r \neq 1\).
The matrix element Eq. (4.13) is related to the high-energy limit of the structure function $W$ via Eqs. (2.9), (2.15) and (3.14). The above Euclidean functional integral expression for $\mathcal{M}_E$ is one of the central results of this paper. We propose to evaluate it using genuinely non-perturbative methods, e.g., lattice Monte Carlo simulations.

Similarly, the Minkowskian matrix element of Eq. (3.19) can be expressed in terms of an appropriate Minkowskian functional integral (cf. Appendix B).

5 Free-field example

To illustrate the meaning of the formalism developed, let us consider the simple example of free fields, i.e., the case $\lambda = 0$.

Using Eqs. (4.6), (4.16) and (4.17), the matrix element is expressed as

$$\mathcal{M}_E(\tau, r) = \frac{1}{Z} \lim_{\tau_i \to -\infty} \lim_{\tau_f \to +\infty} e^{(\tau_f - \tau_i)M} \int d^3 \vec{x}_f \, d^3 \vec{x}_i \left( -\frac{\partial}{\partial \tau_f} + M \right) \left( \frac{\partial}{\partial \tau_i} + M \right) G_{\phi j j \phi}(x_f, x, 0, x_i),$$

where

$$x_i = (\vec{x}_i, \tau_i), \quad x_f = (\vec{x}_f, \tau_f), \quad x = (\vec{0}, \tau),$$

and $G_{\phi j j \phi}$ is the connected Green function of two fields $\phi$ and two currents $j$ illustrated in Fig. 3. The terms with $x^3$ derivative disappear because of the $x^3$ integration.

Figure 3: The Euclidean Green function required for the calculation of $\mathcal{M}_E$.

In the free-field case, Fig. 3 contains only the two diagrams shown in Fig. 4, where the propagator following from the free part of the Euclidean Lagrangian, Eq. (4.15), has to be used. It reads

$$G(x) = \int \frac{d^4 k}{(2\pi)^4} e^{i k x} \frac{1}{k^T A_r k + m^2} = \frac{m}{4\pi^2 \sqrt{x^T A_r^{-1} x}} K_1 \left( m \sqrt{x^T A_r^{-1} x} \right),$$

where

$$A_r = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 - (1 - r)^2 & i(1 - r) \\ 0 & 0 & i(1 - r) & 1 \end{pmatrix}$$

with $x = \begin{pmatrix} x^1 \\ x^2 \\ x^3 \\ \tau \end{pmatrix}$, $k = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix}$.

$K_1$ is the modified Bessel function, and $m = M$ in our free-field model. It is easy to see that $x^T A_r^{-1} x \simeq 2r\tau^2$ for $x = (\vec{0}, \tau)$ and $r \to 0$. According to Eq. (5.3), this means that
the correlation length in the Euclidean time direction diverges and we are dealing with a critical phenomenon.

To study in general the behaviour of $G(x)$ for $x \to \infty$ we set

$$x = R \hat{x}, \quad R = |x| = \sqrt{x_1^2 + x_2^2 + x_3^2 + \tau^2} \quad (5.5)$$

and consider the limit $R \to \infty$ for fixed unit vector $\hat{x}$. We get from Eq. (5.3):

$$G(x) \to \frac{1}{4\pi^2} \sqrt{\frac{\pi}{2}} M^2 \left( M^2 R^2 \hat{x}^T A_r^{-1} \hat{x} \right)^{-3/4} e^{-MR\sqrt{\hat{x}^T A_r^{-1} \hat{x}}} \quad \text{for} \quad R \to \infty. \quad (5.6)$$

Thus the decay of $|G(x)|$ as $R \to \infty$ is governed by $\exp \left[ -MR \text{Re} \sqrt{\hat{x}^T A_r^{-1} \hat{x}} \right]$. This suggests to define a correlation length depending on $\hat{x}$ as

$$\xi(\hat{x}) = \left[ M \text{Re} \sqrt{\hat{x}^T A_r^{-1} \hat{x}} \right]^{-1}. \quad (5.7)$$

We have, in the Euclidean time direction,

$$\xi(\vec{0}, 1) = \left[ M \sqrt{r(2-r)} \right]^{-1}, \quad (5.8)$$

and, in the space directions,

$$\xi(\vec{e}_j, 0) = M^{-1}. \quad (5.9)$$

Thus for $r \to 0$ the correlation in this free-field model becomes of long range only in the time direction. There $\xi(\vec{0}, 1)$ diverges as $1/\sqrt{r}$, i.e., in a manner characteristic of mean-field theories for critical phenomena in statistical mechanics, identifying $r$ with $T - T_c$, the deviation of the temperature from the critical value.

We also note the behaviour of $G(x)$, Eq. (5.3), for $R \to 0$:

$$G(x) \to \frac{1}{4\pi^2} \left( R^2 \hat{x}^T A_r^{-1} \hat{x} \right)^{-1} \quad \text{for} \quad R \to 0. \quad (5.10)$$

In the Euclidean time direction this can be written as

$$G(\vec{0}, \tau) \to \frac{M^2}{4\pi^2} \left( \frac{\xi(\vec{0}, 1)}{\tau} \right)^2 \quad \text{for} \quad \tau \ll \xi(\vec{0}, 1). \quad (5.11)$$

This means that for times much smaller than the correlation length the Green function in the free theory has a simple power behaviour, again similar to what is found in the study of critical phenomena.
Now we describe the calculation of $T_+$ in Eq. (3.14) in the free-field case. Changing the integration variables in Eq. (3.14) from $y_\pm$ to $y^0$ and $y^3$, using the expression of Eq. (5.1) for $M_E$, and calculating the Green function $G_{\phi j\phi}$ according to Fig. 4, the amplitude $T_+$ reads

$$T_+(i\eta, Q^2, \mu) = \frac{-1}{\pi^2 \sqrt{\eta^2 - Q^2}} \int_0^\infty dy^0 \int_{-y^3}^{y^3} y^3 e^{\frac{i}{2}[(y^0+y^3)\mu + i\eta_-(\mu + i\eta_+)]} \left[ e^{My^0} + e^{-My^0} \right] \frac{M}{\sqrt{(y^0)^2 - (y^3)^2}} K_1 \left( M \sqrt{(y^0)^2 - (y^3)^2} \right).$$

(5.12)

Applying the identity

$$-\frac{y^3 M}{\sqrt{(y^0)^2 - (y^3)^2}} K_1 \left( M \sqrt{(y^0)^2 - (y^3)^2} \right) = \frac{\partial}{\partial y^3} K_0 \left( M \sqrt{(y^0)^2 - (y^3)^2} \right),$$

(5.13)

integrating by parts, and returning to the variables $y_\pm$, one finds

$$T_+(i\eta, Q^2, \mu) = \frac{-i}{4\pi^2} \int_0^\infty dy_+ \int_0^\infty dy_- e^{\frac{i}{2}[(y_+(\mu + i\eta_-) + y_-(\mu + i\eta_+)]} \left[ e^{M(y_+ + y_-)/2} + e^{-M(y_+ + y_-)/2} \right] K_0 \left( M \sqrt{y_+ y_-} \right).$$

(5.14)

For sufficiently negative Re$\nu$, the integrals can be evaluated using the relations 6.631.3, 9.224 and 6.224.1 of [25]. The result is

$$T_+(i\eta, Q^2, \mu) = -\frac{i}{2\pi^2} \left\{ f(M) + f(-M) \right\}$$

(5.15)

with

$$f(M) = \frac{1}{\mu^2 + 2M\mu - Q^2 + 2i\eta(M + \mu)} \ln \left[ \frac{1}{M^2} \left\{ (M + \mu)^2 - Q^2 + 2i\eta(M + \mu) \right\} \right].$$

(5.16)

When this expression is analytically continued to $\mu = 0$, a careful treatment of the imaginary part of the logarithms is essential. The singularities of $T_+(i\eta, Q^2, \mu)$ in the complex $\mu$ plane are shown in Fig. 5. The continuation in $\mu$ from Re$\mu < -M$ to $\mu = 0$ has to be done along the real axis staying above the left hand singularities.

From Eq. (2.15) one finds the behaviour of the retarded amplitude on the imaginary axis in the $\nu$ plane:

$$T_+(i\eta, Q^2) = \text{Re} T_+(i\eta, Q^2, 0) = \frac{1}{2\pi} \left( \frac{1}{Q^2 - 2i\eta M} + \frac{1}{Q^2 + 2i\eta M} \right).$$

(5.17)

The corresponding behaviour on the real axis follows from the analytic continuation $i\eta \to \nu$ for $\nu$ real:

$$T_+(\nu, Q^2) = \frac{1}{2\pi} \left( \frac{1}{Q^2 - 2\nu M - i\varepsilon} + \frac{1}{Q^2 + 2\nu M + i\varepsilon} \right).$$

(5.18)
Figure 5: The singularities of $T_+(i\eta, Q^2, \mu)$ in the complex $\mu$ plane. There are poles at $\pm M - i\tilde{\eta}_+, \pm M - i\tilde{\eta}_-$ with $\tilde{\eta}_\pm = \eta \pm \sqrt{\eta^2 - Q^2 - M^2}$ and logarithmic branch cuts starting at $\pm M - i\eta_+$ and $\pm M - i\eta_-$. Here we assume $\eta^2 > Q^2 + M^2$.

This is in agreement with what one would have found by simply adding the two diagrams for the forward scattering of a scalar proton and a scalar photon in the original Minkowskian theory.

We will close this section with some speculative remarks. Looking at Eq. (3.14) we see that for $\eta \to \infty$ the typical integration range over $\mathcal{M}_E(y^0, r)$ is

$$0 \leq r \lesssim \frac{1}{\eta y^0},$$

$$0 \leq y^0 \lesssim \frac{2\eta}{Q^2} \equiv y^0_m.$$  

Let us insert the mean value of $y^0$ of Eq. (3.14) into Eq. (5.19) to get as typical value for $r$:

$$\bar{r} = \frac{Q^2}{\eta^2}.$$  

Then we have two relevant scales for the $y^0$ integration in Eq. (3.14): $y^0_m$ and the length scale of the decay of $\mathcal{M}_E(y^0, \bar{r})$. Also in the general case of the theory with interactions this should be governed by a correlation length $\xi(\bar{r})$ similar to $\xi^{(0)}(\bar{r}) \equiv \xi(\vec{0}, 1)$, Eq. (5.8), in the free-field case. Clearly the behaviour of the amplitude $T_+$ will depend crucially on whether $y^0_m$ is smaller or bigger than $\xi(\bar{r})$.

In the free-field case we have

$$y^0_m \lesssim \xi^{(0)}(\bar{r}) \quad \text{for} \quad Q^2 \geq 8M^2.$$  

Here for large $Q^2$ and $\eta \to \infty$ the integration in Eq. (3.14) probes only the region of $\mathcal{M}_E(y^0, r)$ where $y^0$ is smaller than the correlation length and thus where Eq. (5.11) applies. We can speculate that in the theory with interaction we should again be able to distinguish two regimes, 

$$y^0_m \lesssim \xi(\bar{r}),$$  

where $\bar{r} = \frac{Q^2}{\eta^2}$.  

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where, from our experience with critical phenomena, we would expect $\xi(\bar{r})$ to satisfy a scaling law for $\bar{r} \to 0$,

$$\xi(\bar{r}) \simeq \frac{c}{M}(\bar{r})^{-\rho}, \quad c = \text{const.}, \quad \rho > 0,$$

(5.24)

but not with the mean field exponent $\rho = 1/2$ as in Eq. (5.8).

For $0 < \rho < 1$ the dividing line between the two regimes of Eq. (5.23) would then be roughly:

$$\eta = \frac{2M}{c} \left( \frac{Qc}{2M} \right)^{\frac{2-2\rho}{2\rho}}$$

(5.25)

and it is tempting to speculate that this could correspond to the dividing line between hard [$y_m^0 < \xi(\bar{r})$] and soft [$y_m^0 > \xi(\bar{r})$] pomeron regimes. But much more work is needed before such a conjecture can be substantiated.

Let us, nevertheless, assume for the moment that such a conjecture is true and apply Eq. (5.25) also in the physical region, replacing $\eta$ by $\nu$. Then we should find a dividing line between hard and soft pomeron effects in the structure functions in the $x_{\text{Bj}}$ versus $Q^2$ plane:

$$Q^2 = \frac{4M^2}{c^2}(cx_{\text{Bj}})^{2-\frac{1}{\rho}}.$$  

(5.26)

This is illustrated in Fig. 6. Thus, taking the limit $x_{\text{Bj}} \rightarrow 0$ at fixed $Q^2 > 0$ we would finally always go from the hard to the soft regime for $0 < \rho < 1/2$ and vice versa for $1/2 < \rho < 1$.

![Figure 6](image-url)  

**Figure 6**: Sketch of the regions of soft and hard pomeron effects in the $x_{\text{Bj}}$–$Q^2$ plane using Eq. (5.26).

6 **Alternative analytic continuations to Euclidean space**

It is clear that the form of the analytic continuation from the Minkowski to the Euclidean theory discussed in the previous sections is by no means unique. Other possible forms of analytic continuations from Minkowski to Euclidean space can be derived: in this section
we discuss two of these alternative Euclidean formulations and compare them with the one of Sect. 3.

Working in the rest frame of the proton, consider the quantity

\[ T(q^0, \vec{q}) = \frac{i}{2\pi} \int d^4x \theta(x^0)e^{iqx} \langle P(p)|J(x)J(0)|P(p) \rangle \]  

(6.1)
as an analytic function of \( q^0 \) with the spatial part \( \vec{q} \) fixed and real. Using standard methods, one finds

\[ T(q^0, \vec{q}) = -\sum_X \frac{(2\pi)^2 \delta^{(3)}(\vec{q} - \vec{p}')}{q^0 + M - p'^0} |\langle P(p)|J(0)|X(p') \rangle|^2, \]  

(6.2)

which reveals the cut along the positive real axis. The structure function of Eq. (2.6) is related to \( T \) by

\[ \text{Im}[T(q^0 + i\varepsilon, \vec{q})] = W(\nu, Q^2), \]  

(6.3)

where \( \varepsilon \to 0^+ \). Now, consider \( T(q^0, \vec{q}) \) on the positive imaginary axis, i.e., for \( q^0 = i\eta \) with \( \eta > 0 \). Using the identity

\[ \frac{1}{i\eta + M - p'^0} = -\int_{0}^{+\infty} dx_4 e^{i(x_4 + M - p'^0)x_4} \]  

(6.4)

(where the problematic lowest value of \( p'^0, p'^0 = M \), can be treated by giving \( \eta \) a small positive imaginary part), the following expression for \( T \) is obtained from Eq. (6.2):

\[ T(i\eta, \vec{q}) = \frac{1}{2\pi} \int d^3\vec{x} \int_{0}^{+\infty} dx_4 e^{i(x_4 + M - p'^0)x_4} \langle P(p)|J_E(\vec{x}, x_4)J_E(0)|P(p) \rangle. \]  

(6.5)

Here \( J_E(\vec{x}, x_4) \) is the current of the conventional Euclidean theory:

\[ J_E(\vec{x}, x_4) = e^{Hx_4}J(0, \vec{x})e^{-Hx_4}. \]  

(6.6)

Using the methods of Sect. 3 [cf. Eqs. (2.11) and (2.12)], invariance under spatial rotations can be employed to further simplify the above expression for \( T \).

To summarize, the amplitude \( T \) on the real axis, which determines the physical structure function, can be obtained from the one on the imaginary axis by analytic continuation in \( q^0: (q^0 = i\eta) \to (q^0 = \nu) \). According to Eq. (6.3), the latter one can be calculated from a conventional Euclidean matrix element.

This approach is simpler than the one discussed in the previous sections in that the Euclidean theory is based on the usual Hamiltonian \( H \) and not on the more complicated effective Hamiltonian of Eq. (3.12). However, in order to derive the large-\( \nu \) behaviour of the structure function \( W(\nu, Q^2) \) at fixed \( Q^2 \), one needs the analytic continuation of \( T(q^0, \vec{q}) \) in \( q^0 \) for every value of \( \vec{q} \). Only at this point can one study the asymptotic limit \( \nu \to \infty \), which involves both \( q^0 \to \infty \) and \( |\vec{q}| \to \infty \) in the physical region. This analytic continuation in \( q^0 \) is expected to be more troublesome than the \( \mu \) continuation of Sect. 3, which is decoupled from the high-energy limit.
We consider now a third possible form of analytic continuation from Minkowski to Euclidean space, which is close in spirit to [19]. The basic object is still the virtual Compton amplitude Eq. (2.8), which we write now as:

\[ T_\nu(\nu, Q^2) = \frac{1}{2} (\tilde{T}_+(q) + \tilde{T}_+^*(-q^*) ) \]  

(6.7)

where

\[ \tilde{T}_+(q) = \frac{i}{\pi} \int d^4x \theta(x^0)\theta((x^0)^2 - (x^3)^2) e^{ix\theta} \langle P(p)|J(x)J(0)|P(p) \rangle. \]  

(6.8)

Here we work in the rest system of the proton and choose now \( q \) in the form

\[ q = (\nu, \vec{e}_\perp \sqrt{\nu^2 + Q^2}, 0), \]  

(6.9)

with \( \vec{e}_\perp \) a unit vector in the \((q^1, q^2)\) plane. With the parametrization \( x = (\xi \cosh \chi, \vec{x}_\perp, \xi \sinh \chi) \), we find

\[ \tilde{T}_+(q) = \frac{i}{\pi} \int d^2\vec{x}_\perp e^{-i\vec{e}_\perp \cdot \vec{x}_\perp \sqrt{\nu^2 + Q^2}} \int_{-\infty}^{+\infty} d\chi \int_{-\infty}^{+\infty} d\xi \xi e^{i\nu\xi \cosh \chi} \langle P(p)|J(x)J(0)|P(p) \rangle. \]  

(6.10)

We will show how to compute this quantity and its \( \nu \) derivatives at \( \nu = 0 \) using an analytic continuation from Minkowski to Euclidean space. First of all, we write

\[ \tilde{T}_+(q)|_{\nu=0} = \int d^2\vec{x}_\perp e^{-i\vec{q}_\perp \cdot \vec{x}_\perp} \int_{-\infty}^{+\infty} d\chi A^{(1)}(\chi, \vec{x}_\perp), \]  

(6.11)

where \( \vec{q}_\perp = Q\vec{e}_\perp \) and the function \( A^{(1)} \) is defined by

\[ A^{(1)}(\chi, \vec{x}_\perp) = \frac{i}{\pi} \int_{0}^{+\infty} d\xi \langle P(p)|J(x)J(0)|P(p) \rangle \]

\[ = \frac{-i}{\pi} \sum_{X} \langle P(p)|J(0, \vec{x}_\perp, 0)|X(p')\rangle \langle X(p')|J(0)|P(p) \rangle \]

\[ \times \frac{1}{[(p^0 - M \cosh \chi - p^3 \sinh \chi - i\varepsilon)^2]}, \]  

(6.12)

where finally the limit \( \varepsilon \to 0^+ \) is taken. The singularity structure in the complex \( \chi \) plane is such that the function can be continued to the imaginary axis, \( \chi \to i\theta \) where, however, the \( \theta \) range is restricted by \( \theta \in (-\pi/2, \pi/2) \). In this region, \( A^{(1)} \) is related to a Euclidean matrix element,

\[ A^{(1)}(i\theta, \vec{x}_\perp) = \frac{-i}{\pi} \int_{0}^{\infty} d\xi \langle P(p)|J_E(x_E)J_E(0)|P(p) \rangle \]

\[ = \frac{-i}{\pi} \sum_{X} \langle P(p)|J(0, \vec{x}_\perp, 0)|X(p')\rangle \langle X(p')|J(0)|P(p) \rangle \]

\[ \times \frac{1}{[(p^0 - M \cos \theta - ip^3 \sin \theta - i\varepsilon)^2]}, \]  

(6.13)

where \( x_E = (\vec{x}_\perp, \xi \sin \theta, \xi \cos \theta) \) is a Euclidean four-vector and the Euclidean current \( J_E(x_E) \) is defined by Eq. (5.6).
More generally, the expression for the \( n \)-th derivative of \( \tilde{T}_+ \) with respect to \( \nu \), evaluated at \( \nu = 0 \), reads

\[
\partial_\nu^n \tilde{T}_+(q) |_{\nu=0} = \sum_{l=0}^{n} \int d^2 \vec{x}_\perp e^{-i \vec{q}_\perp \cdot \vec{x}_\perp} C^{n}_l (\vec{q}_\perp \cdot \vec{x}_\perp, Q^2) \int_{-\infty}^{+\infty} d\chi \left( \cosh\chi \right)^l A^{(l+1)}(\chi, \vec{x}_\perp),
\] (6.14)

with explicitly calculable coefficients \( C^{n}_l \) and functions \( A^{(l)} \) defined by

\[
A^{(l)}(\chi, \vec{x}_\perp) = \frac{i}{\pi} \int_0^{\infty} d\xi \xi^l \left( \langle P(p) | J(x) J(0) | P(p) \rangle \right)
\]

\[
= \frac{(-i)^{(l+1)}}{\pi} \sum_\chi \langle P(p) | J(0, \vec{x}_\perp, 0) | X(p') \rangle \langle X(p') | J(0) | P(p) \rangle
\]

\[
\times \frac{1}{[(p^0 - M) \cosh \chi - p^3 \sinh \chi - i\varepsilon]^{l+1}}.
\] (6.15)

As in the case \( l = 1 \), the values of this function for real \( \chi \) can be obtained from

\[
A^{(l)}(i\theta, \vec{x}_\perp) = \frac{(-i)^l}{\pi} \int_0^{\infty} d\xi \xi^l \left( \langle P(p) | J_E(x_E) J_E(0) | P(p) \rangle \right)
\]

\[
= \frac{(-i)^{(l+1)}}{\pi} \sum_\chi \langle P(p) | J(0, \vec{x}_\perp, 0) | X(p') \rangle \langle X(p') | J(0) | P(p) \rangle
\]

\[
\times \frac{1}{[(p^0 - M) \cos \theta - ip^3 \sin \theta - i\varepsilon]^{l+1}}.
\] (6.16)

by the analytic continuation \( i\theta \rightarrow \chi \).

To summarize, one first computes \( A^{(l)}(i\theta, \vec{x}_\perp) \) for \( l = 1, \ldots, n+1 \) and \( \theta \in (-\pi/2, \pi/2) \) in the Euclidean field theory (for example, on the lattice). Then, the value of \( \partial_\nu^n \tilde{T}_+(q) |_{\nu=0} \) can be obtained from Eq. \( \text{(6.14)} \) using the analytic continuations of the \( A^{(l)} \) to the real axis. Thus, using Eq. \( \text{(6.14)} \) and a Taylor expansion of \( \tilde{T}_+ \) around \( \nu = 0 \), the behaviour of \( T_r(\nu, Q^2) \) at small \( \nu \) (corresponding to large \( x_{\text{Bj}} \)) can be studied. The values of \( \partial_\nu^n T_r(\nu, Q^2) |_{\nu=0} \) can then be related to integrals of the structure function \( W(\nu, Q^2) \) using dispersion relations in \( \nu \). Assuming for simplicity no subtractions this reads:

\[
T_r(\nu, Q^2) = \frac{1}{\pi} \int_{Q^2/2M}^{\infty} d\nu' W(\nu', Q^2) \left[ \frac{1}{\nu' - \nu - i\varepsilon} + \frac{1}{\nu' + \nu + i\varepsilon} \right],
\] (6.17)

\[
\partial_\nu^n T_r(\nu, Q^2) |_{\nu=0} = \frac{n!}{\pi} \int_{Q^2/2M}^{\infty} d\nu' W(\nu', Q^2) [1 + (-1)^n], \quad (n = 0, 1, 2, \ldots).
\] (6.18)

Again, the advantage with respect to the method of Sect. \( \text{[3]} \) is the use of the conventional Hamiltonian in the Euclidean theory. The disadvantage is the limited sensitivity to the region of small \( x_{\text{Bj}} \).

### 7 Conclusions

In this paper, a fundamentally new approach to the long-standing problem of small-\( x_{\text{Bj}} \) structure functions in DIS has been developed. It is based on the well-known relation...
of the small-$x_{\text{Bj}}$ limit of DIS and the high-energy limit of forward virtual Compton scattering. Instead of taking the energy of the forward virtual Compton amplitude to infinity along the real axis, we propose to consider the limit of large imaginary energy. According to the theorems of Phragmén and Lindelöf type, the asymptotic behaviour in both directions is related by analytic continuation.

A slight generalization of the above virtual Compton amplitude with imaginary energy, which contains an additional mass variable $\mu$ for convergence, can be written as an integral over a matrix element in an effective Euclidean field theory. Using standard methods, this matrix element is expressed in terms of a Euclidean functional integral with a simple effective Lagrangian, which is given explicitly. The essential proposal of our paper is to attempt an evaluation of this functional integral using genuinely non-perturbative methods, e.g., lattice Monte-Carlo simulations. Finally, an analytic continuation to $\mu = 0$ and to the real axis in the complex energy plane should allow the extraction of the desired asymptotic small-$x_{\text{Bj}}$ limit of DIS structure functions.

To illustrate how our approach works, we have explicitly performed all necessary steps in a simple model with free scalar fields. In this case, the Euclidean path integral is given by two tree-level Feynman diagrams and the subsequent analytic continuations in $\mu$ and the energy variable can be carried out explicitly. As expected, the final result for the structure function is in agreement with a direct diagrammatic calculation in Minkowski space.

We have shown that in the free-field case the high-energy limit of the virtual Compton amplitude is governed by the behaviour of the effective Euclidean theory near the critical point $r = 0$, where $r$ is our parameter defined in Eq. (3.10). We have then assumed that also in the effective theory with interaction a critical point occurs at $r = 0$ and discussed the possible consequences of this for the hard and soft pomeron regimes of the structure function.

We do not claim that we have found the optimal method for an analytic continuation of high-energy amplitudes. In fact, in Sect. 6 of this paper we introduce two alternative possibilities. They have the advantage that the Euclidean field theory obtained is based on the conventional Hamiltonian as opposed to the effective Hamiltonian required in the original approach. However, in both cases it appears to be more difficult to recover the small-$x_{\text{Bj}}$ limit of structure functions from the final result of the calculation.

A lot of work remains to be done before phenomenologically relevant information can be extracted from the approach suggested. Note first that all of the discussion in the present paper is based on a model with a scalar ‘photon’ coupled to scalar partons. This has to be extended to the realistic case of a vector photon and QCD. However, we do not expect any fundamental problems with this generalization.

Furthermore, the feasibility of a lattice calculation or any alternative non-perturbative treatment of our Euclidean path integral expression has to be evaluated. Note that our effective Lagrangian contains an imaginary part, which, even though it does not raise any fundamental problems, may lead to technical difficulties on the lattice. Note also that the subsequent analytic continuation back to the physical region could prove highly non-trivial.
On a more fundamental level, one has to ask whether better methods for the treatment of the small-$x_{\text{Bj}}$ limit of DIS in a Euclidean theory exist. We have attempted to address this question in our cursory discussion of alternative methods of analytic continuation. However, we are not able to give a general answer at present.

Given the above reservations, the importance of our results lies in their potential to relate small-$x_{\text{Bj}}$ cross sections in DIS to quantities derivable from an effective Euclidean field theory. One can then hope to understand the limit $x_{\text{Bj}} \to 0$ in terms of the critical behaviour in this Euclidean theory using all the tools which have been developed in statistical mechanics for the study of criticality. Also one can hope to calculate the relevant quantities on the lattice. This goal is fundamental since lattice calculations remain practically the only non-perturbative method in QFT that can be derived strictly from first principles. Thus, it is certainly worthwhile to continue the exploration of possibilities to obtain high-energy or small-$x_{\text{Bj}}$ cross sections from Euclidean quantities.

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Appendix

A Considerations concerning the analytic continuation of amplitudes in the $\nu$ plane

Here we will first discuss the analytic continuation of $T_r(\nu, Q^2)$, Eq. (2.12), from real values of $\nu$ into the half plane $\text{Im} \nu > 0$, where we have

$$\text{Im} \nu > \text{Im} \sqrt{\nu^2 + Q^2} \geq 0. \quad (A.1)$$

Proof: the second inequality is obvious. The first inequality holds for imaginary $\nu$. By continuity, the inequality can only be violated if there exists some $\nu$ in the upper half plane such that

$$\text{Im} \nu = \text{Im} \sqrt{\nu^2 + Q^2}.$$  \hspace{1cm} (A.2)

Thus, for some $a \in \mathbb{R}$,

$$\sqrt{\nu^2 + Q^2} = \nu + a. \quad (A.3)$$

Squaring this equation, one finds

$$Q^2 = 2\nu a + a^2, \quad (A.4)$$

which can not be fulfilled for non-zero $Q$ and $\text{Im} \nu > 0$. Therefore, the first inequality in Eq. (A.1) holds everywhere in the upper half plane and our proof is complete.

With Eq. (A.1) we find that the factor $\exp(ix^0\nu - ix^3\sqrt{\nu^2 + Q^2})$ in Eq. (2.12) decreases exponentially for $\text{Im} \nu > 0$ as $x^0 \to \infty$ with $|x^3|/x^0 \leq 1$. Thus the integral in
Eq. (2.12) should be well convergent and define an analytic function in \( \nu \) for \( \text{Im} \nu > 0 \). There is still the possibility of \( T_r(\nu, Q^2) \) having a cut on the imaginary \( \nu \) axis for \( 0 \leq \text{Im} \nu \leq Q \) due to the explicit factor \( \sqrt{\nu^2 + Q^2} \) in Eq. (2.12). But using Eq. (2.11) it is easy to see that the values of \( T_r(\nu, Q^2) \) when approaching the imaginary axis from both sides are equal. Then the “edge of the wedge” theorem (cf., e.g., [26]) guarantees that \( T_r(\nu, Q^2) \) is also analytic for \( 0 < \text{Im} \nu < Q \). The point \( \nu = iQ \) could still be an isolated singularity but this is incompatible with the square root in Eq. (2.12). This concludes our discussion of the analyticity of \( T_r(\nu, Q^2) \) for \( \text{Im} \nu > 0 \).

The discussion of the analyticity of \( T_+(\nu, Q^2, \mu) \) defined as in Eq. (2.14) but with \( i\eta \) replaced by \( \nu \) is analogous. The proof of Eq. (2.15) for \( 0 < \eta < Q \) is straightforward using Eq. (2.11).

In the remainder of this appendix we will discuss the connection between the limits \( \nu \to \infty \) either on the real or the positive imaginary axis. We will outline one possibility of specifying the requirements for guaranteeing that the analytic continuation of the asymptotic behaviour of the amplitude from the real to the imaginary axis is possible.

The retarded amplitude \( T_r(\nu, Q^2) \) is an analytic function of \( \nu \) with cuts along the positive and negative real axis (Fig. 2). Let us assume that there exists a function \( f(\nu) \) analytic for \( \text{Im} \nu > 0 \) such that \( f(\nu) \neq 0 \) for \( \text{Im} \nu \geq 0 \), and

\[
R(\nu) \equiv \frac{T_r(\nu, Q^2)}{f(\nu)}
\]

is a bounded function, \( |R(\nu)| \leq \text{const.} \) for \( \text{Im} \nu \geq 0 \). (Since \( Q^2 \) is kept fixed we suppress this argument in \( R(\nu) \) and \( f(\nu) \).) Let us furthermore assume that

\[
\lim_{\eta \to \pm \infty} R(\eta) = C_\pm .
\]

Then the Phragmén-Lindelöf theorem (see theorem 5.64 of [22]) states that \( C_+ = C_- \) and that \( R(\nu) \) approaches the same limit \( C_+ \) along any ray \( \nu = \eta \exp(i\phi) \), \( 0 \leq \phi \leq \pi \), \( \phi = \text{const.} \), \( \eta \to +\infty \). Thus we have also on the imaginary axis

\[
\lim_{\eta \to +\infty} R(i\eta) = C_+ .
\]

Suppose now that from the study of \( T_r(\nu, Q^2) \), e.g., by lattice calculations, we can deduce the asymptotic behaviour for \( \eta \to \infty \), construct a suitable function \( f(i\eta) \) and deduce the value of the constant \( C_+ [\text{Eq. (A.7)}] \) with \( C_+ \neq 0 \).

Typically one would try to fit lattice data to functions reflecting our general expectations concerning the high-energy behaviour of amplitudes in QFT like \( f(\nu) = (\nu - \nu_0)^\alpha \) or \( f(\nu) = (\nu - \nu_0)^\alpha \ln(\nu - \nu_0) \). Here \( \alpha \) should be taken as a real constant and \( \nu_0 \) as a constant with \( \text{Im} \nu_0 < 0 \). With the assumptions specified in Eqs. (A.5) and (A.6) we get for the physical amplitude on the real axis:

\[
T_r(\nu, Q^2) \longrightarrow C_+ f(\nu) \quad \text{for} \quad \nu \to \infty .
\]

To summarize: if we assume that the limit of \( T_r(\nu, Q^2) \) for \( \nu \to \infty \), \( 0 \leq \text{arg} \nu \leq \pi \) is governed by a suitable analytic function \( f(\nu) \), then this function can be determined from the study of \( T_r(\nu, Q^2) \) on the positive imaginary axis.
We have only outlined here the simplest assumptions making the analytic continuation of the high-energy limits on the real and imaginary axes possible. No doubt, using the methods of [27] one can relax these assumptions and still get useful relations between the two limits.

B The functional integrals for $\mathcal{M}_E$ and $\mathcal{M}_M$

In this appendix we give the details of the derivation of Eq. (1.13). We start with Eq. (B.13) and using Eq. (4.3) rewrite it as

$$\mathcal{M}_E(\tau, r) = \frac{1}{Z} \lim_{\tau_i \to -\infty} \lim_{\tau_f \to +\infty} e^{(\tau_f - \tau_i)M} \langle 0 | e^{-(\tau_f' - \tau_i')H_{eff}(r)} A(0) e^{-(\tau_f - \tau)H_{eff}(r)} \rangle \times J(0) e^{-(\tau - 0)H_{eff}(r)} \langle J(0) e^{-(0 - \tau_i)H_{eff}(r)} A'(0) e^{-(\tau - \tau_i')H_{eff}(r)} | 0 \rangle e^{-(\tau_f' - \tau_i')}E_0.$$  \hspace{1cm} (B.1)

Here we have introduced further times $\tau_f', \tau_i'$ with $\tau_f < \tau_f' < \tau_i < \tau_f < \tau_f'$ and the limits in Eq. (B.1) are to be taken in the order indicated there.

Let $|\phi\rangle$ be eigenstates of the field operator $\Phi(x)$ at $x^0 = 0$:

$$\Phi(\vec{x}, 0) |\phi\rangle = \phi(\vec{x}) |\phi\rangle,$$  \hspace{1cm} (B.2)

where $\phi(\vec{x})$ are classical functions. We have

$$\int D\phi |\phi\rangle \langle \phi | = 1.$$  \hspace{1cm} (B.3)

The basic relation of the path integral formalism [28] discussed in many textbooks (see, e.g., [29]) is in our case

$$\langle \phi^{(2)} | e^{-\Delta \tau H_{eff}(r)} | \phi^{(1)} \rangle = \int D\pi^{(2)} e^{i \int d^3\vec{x} \left[ \pi^{(2)}(\vec{x})(\phi^{(2)}(\vec{x}) - \phi^{(1)}(\vec{x})) + i\Delta \tau H_r \left( \pi^{(2)}(\vec{x}), \phi^{(2)}(\vec{x}) \right) \right]} + \mathcal{O}(\Delta \tau^2).$$  \hspace{1cm} (B.4)

where the measure includes a factor $1/(2\pi)$ for each $\pi^{(2)}(\vec{x})$ integration. Here $\pi^{(2)}(\vec{x})$ is the classical momentum field and $H_r$ the classical Hamilton density corresponding to $H_{eff}(r)$:

$$H_r(\pi, \phi) = \frac{1}{2} \pi(\vec{x})^2 + (1 - r)\pi(\vec{x})\partial_3 \phi(\vec{x}) + \frac{1}{2} \left( \nabla \phi(\vec{x}) \right)^2 + \frac{1}{2} m^2 \phi(\vec{x})^2 + \frac{\lambda}{4!} \phi(\vec{x})^4.$$  \hspace{1cm} (B.5)

Next we choose a grid on the time interval $[\tau_i', \tau_f']$: $\tau_i' \equiv \tau^{(0)} < \tau^{(1)} < \tau^{(2)} \ldots < \tau^{(N)} \equiv \tau_f'$, where also $\tau_i, 0, \tau, \tau_f$ should occur as intermediate points $\tau^{(j)}$. We discuss the factor $e^{-(\tau_f' - \tau_i')}E_0$ which we rewrite using Eqs. (B.3) and (B.4) as:

$$e^{-(\tau_f' - \tau_i')}E_0 = \langle 0 | e^{-(\tau_f' - \tau_i')H_{eff}(r)} | 0 \rangle \hspace{1cm} (B.6)$$

$$= \langle 0 | e^{-(\tau^{(N)} - \tau^{(N-1)})H_{eff}(r)} \ldots e^{-(\tau^{(1)} - \tau^{(0)})H_{eff}(r)} | 0 \rangle$$

$$= \int \prod_{j=0}^N D\phi^{(j)} \langle 0 | \phi^{(N)} \rangle \langle \phi^{(N)} | e^{-(\tau^{(N)} - \tau^{(N-1)})H_{eff}(r)} | \phi^{(N-1)} \rangle$$

$$\ldots \langle \phi^{(1)} | e^{-(\tau^{(1)} - \tau^{(0)})H_{eff}(r)} | \phi^{(0)} \rangle \langle \phi^{(0)} | 0 \rangle,$$
\[ e^{-(\tau_f' - \tau_i')} E_0 = \lim_{N \to \infty} \int_0^N D\phi^{(j)} \prod_{j=1}^N D\pi^{(j)} \langle 0 | \phi^{(N)} \rangle \] (B.7)

\[ \times \prod_{j=1}^N e^{\int d^3 x [\pi(\vec{x}, \tau^{(j)}) (\pi(\vec{x}, \tau^{(j)} - \phi(\vec{x}, \tau^{(j-1)}))] + i(\tau^{(j)} - \tau^{(j-1)}) \partial_\tau (\pi(\vec{x}, \tau^{(j)}), \phi(\vec{x}, \tau^{(j)}))] \langle \phi(0) | 0 \rangle . \]

Here we set \( \phi^{(j)}(\vec{x}) \equiv \phi(\vec{x}, \tau^{(j)}) \) and \( \pi^{(j)}(\vec{x}) \equiv \pi(\vec{x}, \tau^{(j)}) \).

Now the integration over the \( \pi \) fields in Eq. (B.7) can be performed since the integral is of Gaussian type. In the limit \( N \to \infty, \Delta \tau \to 0 \) we get

\[ e^{-(\tau_f' - \tau_i')} E_0 = C \int D\phi e^{-\int_{t_i}^{t_f'} d\tau \int d^3 \vec{x} L_{E, \text{eff}}(\vec{x}, \tau)} , \] (B.8)

where \( L_{E, \text{eff}} \) is given in Eq. (4.13) and \( C \) is a constant (which is infinite). Performing the same steps for the matrix element in the numerator in Eq. (3.11) and taking the limit \( \tau_f' \to -\infty, \tau_i' \to +\infty \) we get the expression for \( M_E(\tau, r) \) of Eq. (4.13). If we perform the analogous steps for the Minkowskian matrix element, Eq. (3.19), we get:

\[ M_M(t, r) = \frac{1}{Z} \lim_{t_f \to -\infty, t_i \to +\infty} e^{i(t_f - t_i)M} Z^{-1} \int D\phi a(t_f) j(t) j(0) a^\dagger(t_i) e^{i \int d^4 x L_{M, \text{eff}}} , \] (B.9)

\[ Z = \int D\phi e^{i \int d^4 x L_{M, \text{eff}}} , \] (B.10)

with

\[ L_{M, \text{eff}} = \frac{1}{2} \left( \partial_\tau \phi(x) - (1 - r) \partial_3 \phi(x) \right)^2 - \frac{1}{2} \left( \nabla \phi(x) \right)^2 - \frac{m^2}{2} \phi(x)^2 - \frac{\lambda}{4!} \phi(x)^4 \] (B.11)

and

\[ a(t_f) = \int_{x^\phi = t_f} d^3 \vec{x} \left( i \partial_\phi \phi(x) - i(1 - r) \partial_3 \phi(x) + M \phi(x) \right) , \] (B.12)

\[ a^\dagger(t_i) = \int_{x^\phi = t_i} d^3 \vec{x} \left( - i \partial_\phi \phi(x) + i(1 - r) \partial_3 \phi(x) + M \phi(x) \right) . \] (B.13)

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