Experimental Quantum Error Rejection for Quantum Communication

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We report an experimental demonstration of a bit-flip error rejection protocol for error-reduced transfer of quantum information through a noisy quantum channel. In the experiment, an unknown state to be transmitted is encoded into a two-photon entangled state, which is then sent through an engineered noisy quantum channel. At the final stage, the unknown state is decoded by a parity measurement, successfully rejecting the erroneous transmission over the noisy quantum channel.

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A crucial step in the full realization of long-distance quantum communication is to overcome the problems caused by decoherence and exponential photon loss in the noisy quantum channel [1]. As a general solution, two distant parties could first share highly entangled photon pairs, the transmission of quantum states for various applications in quantum communication can then be achieved by using ancilla entanglement. As the quantum repeater [2], combining entanglement purification [3] and entanglement swapping [4], could provide an efficient way to generate highly entangled states between distant locations, many experimental efforts have been made to achieve entanglement swapping, entanglement purification and quantum memory [5, 6, 7, 8], and even the demonstration of a prototype of quantum relay [9, 10]. However, one still has a long way to go before the above techniques can be realistically applied to long-distance quantum communication.

Meanwhile, in the context of quantum error correction (QEC) the way to protect a fragile unknown quantum state is to encode the state into a multi-particle entangled state [11, 12, 13]. Then, the subsequent measurements, i.e. the so-called decoding processes, can find out and correct the error during the quantum operations. Several QEC protocols have been experimentally demonstrated in the NMR [14, 15] and ion-trap [16] systems. Although the QEC are primarily designed for large scale quantum computing, the similar idea was also inspired to implement error-free transfer of quantum information through a noisy quantum channel [17].

The main idea in the original scheme is to encode an unknown quantum state of single particle into a two-particle entangled state [17]. After the encoded state is transmitted over the noisy quantum channel, a parity check measurement [18] is sufficient to reject the transmission with bit-flip error. Such a scheme has the advantage of avoiding the difficult photon-photon controlled-NOT gates necessary for the usual QEC. Moreover, the error rejection scheme proposed promises additional benefit of high efficiency, compared to the QEC based on linear optics quantum logic operations [14]. This is because the crucial feed-forward operations in linear optics QEC will lead to very low efficiency. Although the original scheme is within the reach of the current technology as developed in the recent five-photon experiments [20, 21], it is not optimal in its use of ancilla entangled state because the encoding process is implemented via a Bell-state measurement.

Remarkably, it is found recently [22] that one pair of ancilla entangled state is sufficient to implement the two-photon coding through two quantum parity measurements. Thus, an elegant modification of the previous experiment on four-photon entanglement [23] would allow a full experimental realization of the error rejection code.

In this letter, we report an experimental realization of bit-flip error rejection for fault-tolerant transfer of quantum states through a noisy quantum channel. An unknown state to be transmitted is first encoded into a two-photon entangled state, which is then sent through an engineered noisy quantum channel. At the final stage, the unknown state is decoded by quantum parity measurement, successfully rejecting the erroneous transmission over the noisy quantum channel.

Let us first consider the scenario that Alice wants to send a single photon in an unknown polarization state
\[ \alpha |H\rangle_1 + \beta |V\rangle_1 \] to Bob through a noisy quantum channel. As shown in Fig. 1, instead of directly sending it to Bob, Alice can encode her unknown state onto a two-photon entangled state with an ancilla pair of entangled photons:

\[ |\varphi^+\rangle_{2\prime 3} = \frac{1}{\sqrt{2}} (|HH\rangle_{2\prime 3} + |VV\rangle_{2\prime 3}). \tag{1} \]

The photon in the unknown polarization state and one photon out of the ancilla entangled photon are superposed in a polarization beam splitting (PBS\(_1\)). Behind the PBS\(_1\), with a probability of 50% we obtain the renormalized state corresponding to the three-fold coincidence among modes 1\' , 2\' , and 3.

\[ |\psi\rangle_{1\prime 2\prime 3} = \alpha |HHH\rangle_{1\prime 2\prime 3} + \beta |VVV\rangle_{1\prime 2\prime 3}. \tag{2} \]

Conditional on detecting photon 1\' in the + state with a probability of 50%, where \(|\pm\rangle = \frac{1}{\sqrt{2}} (|H\rangle \pm |V\rangle)\), the remaining two photons will then be projected onto the following entangled state:

\[ |\psi\rangle_{2\prime 3} = \alpha |HH\rangle_{2\prime 3} + \beta |VV\rangle_{2\prime 3}. \tag{3} \]

Thus, through a quantum parity measurement between modes 1\' and 2\', a two-photon encoding operation can be realized.

After finishing the encoding process, Alice sends photons 2' and 3 to Bob through a noisy quantum channel and Bob will recombine the two photons at the PBS\(_2\) in order to identify and reject the erroneous transmission. If there is no error in the quantum channel, Bob will obtain the same quantum state as in (3) after PBS\(_2\). Projecting photon 2'' into the + state with a success probability of 50%, photon 3' will be left in the unknown state \(\alpha |H\rangle + \beta |V\rangle\). Through the decoding process, i.e. conditional on detecting in mode 2'' one and only one +-polarized photon, Bob can recover the state originally sent by Alice.

If a bit-flip error occurred for one of the two transmitted photons, the two photons will have different polarizations and exit the PBS\(_2\) in the same output arm. Therefore, no coincidence will be observed between modes 2'' and 3'. That is to say, the bit-flip error during the transmission of quantum states over the noisy channel has been simply rejected by the final quantum parity measurement. However, if both bit-flip errors occurred simultaneously for the two transmitted photons, Bob would finally obtain the polarization state of \(\alpha |V\rangle + \beta |H\rangle\) via the same quantum parity measurement for decoding operation and the error can not be effectively rejected.

Moreover, the detection of photon 1\' in the - state also leads to encoding of the initial quantum state in a two photon state, provided the associated phase flip is taken into account. Obviously, the same holds for the decoding at Bob’s: projection onto the - state is associated with a phase flip that can be compensated for.

The coding and decoding efficiency can thus be increased by a factor of two each.

Specifically, suppose that Alice would send photons to Bob in one of the three complementary bases of \(|H\rangle / |V\rangle\), \(|+\rangle / |−\rangle\) and \(|R\rangle / |L\rangle\) (where \(|R\rangle = \frac{1}{\sqrt{2}} (|H\rangle + i |V\rangle)\), and \(|L\rangle = \frac{1}{\sqrt{2}} (|H\rangle - i |V\rangle)\), and each qubit is directly sent through the noisy quantum channel with bit-flip error rate of \(E_0 = p\). The quantum bit error rate (QBER) after the decoding process 22 will be:

\[ E_1 = \frac{p^2}{(1 - p)^2 + p^2}, \tag{4} \]

for the polarization states of \(|H\rangle / |V\rangle\) and \(|R\rangle / |L\rangle\), and no error occurs for the \(|+\rangle / |−\rangle\). Therefore, the QBER of \(E_1\) will be lower, compared to the QBER of \(E_0\) for any \(p < 1/2\). For small \(p\), \(E_1\) is on the order of \(p^2\). The transmission fidelity can thus be greatly improved by using the quantum error rejection code.

Note that, conditional detection of photons in mode 1' implies that there is either zero or one photon in the mode 2'. But, as any further practical application of such a coding involves a final verification step, detecting a threefold coincidence makes sure that there will be exactly one photon in each of the modes 2' and 3. This feature allows us to perform various operations like, for example, the recombination of two photons at PBS\(_2\) before the final detection. This makes our encoding scheme significantly different from a previous two-photon encoding experiment [24], where there are certain probabilities of containing two photons in one of two encoding modes. Thus, the previous two-photon encoding experiment cannot be applied to the error-rejection code.

Moreover, we would like to emphasize that, compared to the two recent experiments on fault-tolerant quantum information transmission [25, 26], our protocol has two essential advantages. On the one hand, the work in [25] can only encode and send a known state instead of encoding and sending arbitrary unknown states required by many quantum communication protocols. On the other hand, the experiment in [26] can only filtrate half of the single phase-shift error. Thus, if the error rate of the channel is \(p\), after applying the error filtration method the remaining QBER is still larger than \(p/2\) even in the ideal case. Note that, the error filtration probability in [26] can be increased by coding a qubit in a larger number of time-bins, however, this would need much more resources. While our method can in principle reject any one bit-flip error with certainty as analyzed before. In fact, the ability to suppress the first order error (\(p\)) to the second order (\(p^2\)) is essential to overcome the channel noise in scalable quantum communication.

A schematic drawing of the experimental realization of the error rejection is shown in Fig. 2. A UV pulse (with a duration of 200fs, a repetition rate of 76MHz and a central wavelength of 394nm) passes through a BBO crystal.
the state $\phi^+$ twice to generate two entangled photon pairs 1, 4 and 2, 3 in the state $\phi^+$ [23]. The high quality of two-photon entanglement is confirmed by observing a visibility of $(94 \pm 1)\%$ in the $|+\rangle / |-\rangle$ basis. One quarter wave plate (QWP) and one polarizer (Pol.) in front of detector $D_4$ are used to perform the polarization projection measurement such that the input photon in mode 1 is prepared in the unknown state.

The two photons in modes 1 and 2 are steered to the PBS$_1$, where the path length of photon 1 have been adjusted by moving the delay mirror Delay 1 such that they arrive simultaneously. Conditional on detecting photon 1 in the $|+\rangle$ polarization, the unknown polarization state was encoded into the modes in 2' and 3. The encoded two-photon state is transmitted through the engineered quantum channel and then recombined at the PBS$_2$. Furthermore, the path length of photon 2 has been adjusted by moving the Delay 2 such that photons in modes 2' and 3 arrive at the PBS$_2$ simultaneously. Through the whole experiment, spectral filtering (with a FWHM 3nm, $F$ in Fig. 2) and fiber-coupled single-photon detectors have been used to ensure good spatial and temporal overlap between photons in modes 1 and 2, and photons in modes 2' and 3 [23].

To characterize the quality of the encoding and decoding process, we first measure the interference visibility at the PBS$_1$. Since photon pairs 1-4 and 2-3 are in the state $|\phi^+\rangle$, it is easy to see that the four-fold coincidence in 1', 2', 3 and 4 corresponds to a four-photon GHZ state $\frac{1}{\sqrt{2}}(|HHHH\rangle_{1'2'3'4}+|VVVV\rangle_{1'2'3'4})$ [23]. The four-photon entanglement visibility in the $|+\rangle / |-\rangle$ basis was observed to be $(83 \pm 3)\%$. Similarly, the four-photon entanglement visibility in modes 1', 2'', 3' and 4 is observed to be $(80 \pm 3)\%$, before introducing artificial channel noise. Note that, the visibility is obtained after compensating the birefringence effect of the PBSes [23].

In the experiment, the noisy quantum channels are simulated by one half wave plate (HWP) sandwiched with two QWPs. Each of two QWP is set at 90° such that the horizontal and vertical polarization will experience 90° phase shift after passing through the QWPs. By randomly setting the HWP axis to be oriented at $\pm \theta$ with respect to the horizontal direction, the noisy quantum channel can be engineered with a bit-flip error probability of $p = \sin^2 \theta(2\theta)$. In order to show that our experiment has successfully achieved the error rejection code, the quantum states to be transmitted in mode 1 are prepared along one of the three complementary bases of $|H\rangle / |V\rangle$, $|+\rangle / |-\rangle$, and $|R\rangle / |L\rangle$. The error rates in the engineered quantum channel can be varied by simultaneously changing the axis of each half-wave plate. Specifically, we vary the angle $\theta$ to achieve various error rates from 0 to 0.40 with an increment 0.05 in the quantum channel.

The experimental results of three different input states, after passing through the noisy quantum channel, are shown in Fig. 3. The triangle dots in Fig. 3, corresponding to the bit-flip error rates of single photons, were measured by directly sending the quantum state of photon 1 (after passing through a PBS and some wave-plates for state preparing) through the engineered quantum channel while with both PBS$_1$ and PBS$_2$ removed. These dots also shows the quality of the simulated error of the quantum noisy channel. The quadrangle dots show the final bit-flip error rates after performing error rejection operation with the help of PBS$_1$ and PBS$_2$. Fig. 3a,
3b and 3c shows the experimental results for the input states \(|V\rangle, |–\rangle, \text{ and } |L\rangle\), respectively. The other three input states have the similar results as the one with the same basis respectively. And Fig. 3d shows the average QBER calculated over all six input states.

In Fig. 3, one can clearly see that our error-rejection operation itself also introduces significant error rates, even with \(E_0 = 0\). Therefore, if the original \(E_0\) is comparable with the error rate caused by the experimental imperfection, no improvement will be gained after error-rejection. In the \(|H\rangle/|V\rangle\) experiment, the experimental error rate is about 5%. In both \(|+\rangle/|–\rangle\) and \(|R\rangle/|L\rangle\) experiments an experimental error rate of 10% is observed.

We notice that, whereas both the \(|+\rangle/|–\rangle\) and \(|R\rangle/|L\rangle\) experiments have roughly the same visibility, a better visibility is obtained in the \(|H\rangle/|V\rangle\) experiment. This is mainly due to our two-photon entanglement source, which has a better visibility in the \(|H\rangle/|V\rangle\) basis (97%) than in the \(|+\rangle/|–\rangle\) or \(|R\rangle/|L\rangle\) basis (94%). Moreover, it is partly due to the imperfect birefringent compensation at the PBS\(_1\) and PBS\(_2\) \(29\), which leads to a reduction of interference visibility, hence imperfect encoding and decoding process. Moreover, the imperfect encoded state passing through the noisy channel also leads that in the \(|+\rangle/|–\rangle\) basis the result becomes deteriorate as increasing of artificial noise.

From Fig. 3a and 3c, it is obvious that our error-rejection method can significantly reduce the bit-flip error as long as \(E_0\) is larger than the experimental error rates. However, although ideally in the \(|+\rangle/|–\rangle\) experiment no error should occur after the error-rejection operation, an error rate no less than 10% is observed, which is in accordance with the limited visibility of 80%.

Although our experimental results are imperfect, they are sufficient to show a proof of principle of a bit-flip error rejection protocol for error-reduced transfer of quantum information through a noisy quantum channel. Moreover, Fig. 3d shows that for a substantial region our experimental method does provide an improved QBER over the standard scheme in a six-state quantum key distribution (QKD). This implies, with further improvement, the error-rejection protocol may be used to improve the threshold of tolerable error rate over the quantum noisy channel in QKD \(30\).

Our experimental realization of bit-flip error rejection deserves some further comments. First, the same method can be applied to reject the phase-shift error because phase errors can be transformed into bit-flip errors by a 45° polarization rotation. In this way we can reject all the 1 bit phase-shift error instead of bit-flip error. Second, by encoding unknown states into higher multi-photon (\(N\)-photon) entanglement and performing multi-particle parity check measurement \(18\) either the higher order (up to \(N – 1\)) bit-flip error or phase-shift error can be rejected for more delicate quantum communication.

In summary, our experiment shows a proof of principle of a bit-flip error rejection protocol for error-reduced transfer of quantum information through a noisy quantum channel. Moreover, by further improvement of the quality of the resource for multi-photon entanglement, the method may also be used to enhance the bit error rate tolerance \(31 32\) over the noisy quantum channel and offer a novel way to achieve long-distance transmission of the fragile quantum states in the future QKD.

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