Network equivalent by vector fitting-based rational approximation in wind power integrated power systems

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Abstract: Considering the electromagnetic transient (EMT) simulation for a large power system is complicated and impractical, a feasible solution is to model the area not of interest (external system) as a frequency-dependent network equivalent circuit by vector fitting (VF)-based rational approximation methods. However, when the external system is wind power integrated, errors may be generated from the equivalent network by traditional approximation methods. A proposed algorithm in which the wind turbines or dynamic loads in the external system are reduced at its working points of steady-state firstly and the results from VF method are passivity enforced can improve the accuracy of network equivalent both in frequency and time domain. A case study is considered to evaluate the performance of the proposed algorithm and the simulation results prove the validity of the algorithm for the good accuracy and less time consuming in simulation.

1 Introduction

Electromagnetic transient (EMT) simulation is an important method to study the micro- or milli-second quick dynamics in power systems. As more and more renewable power generators such as wind turbine generators (WTG) or dynamic loads are integrated into power systems or increasing power electronic interface devices are introduced, which will desire for intensive EMT calculation due to the dynamics of power electronic controller considered. Thus, the requirement to speed up the computation process via some advanced techniques such as EMT rational approximation in wind power integrated external system becomes very urgent when the large-scale electrical power system is analysed by EMT calculation [1–3].

Vector fitting (VF) has attracted wide attention to the frequency-dependent network equivalent (FDNE) problem due to its robustness and stability [4–9] in which the external system is considered as a black-box. Current studies about VF-based network equivalent mostly focus on the detailed process about identifying the element parameters of a linear RLC circuit whose admittance-frequency characteristic is similar to the given external network [3, 10–12]. However, it is worthy to be noticed that there may exist some complex non-linear elements such as WTGs which cause error in the sampling of the frequence response in the external system.

Therefore, a reduction procedure of the external system should be added before the network equivalent to simplify the WTGs and dynamic loads as linear models, which may improve the accuracy of the equivalent. In this paper, a WTG modelled as an asynchronous machine of which the active power and mechanical torque are given is considered in the system. So a modified two-step VF-based FDNE method with the simplification of the wind power integrated external system with satisfied stability and passivity is proposed.

The paper is organised as follows. In Section 2, the principle of the network equivalent is introduced, whose detailed procedure is described in Section 3. In Section 4, the effect of the number of poles (NP) of admittance rational functions is described. In Section 5, the proposed algorithm is applied to a case study and its performance is investigated and analysed. Finally, Section 6 summarises this paper.

2 Principle of network equivalent

To reduce the computational burden of the full system, an effective way is to reserve the detailed EMT model over a specific frequency range of the interested region, and the rest of the system can be reduced by equivalent models, which can be constituted in frequency domain using a rational approximation of the network admittance matrix obtained by VF method or in time domain requiring the complete representation of the system in electromagnetic transients program.

2.1 Network reduction

In the study, the whole system is divided into two parts:

(i) the study system with detailed models
(ii) the external system as shown in Fig. 1a, which will be modelled as a FDNE network shown in Fig. 1b [13–16].

According to the system in Fig. 1, for the purpose of network equivalent, only the boundary nodes and external system nodes are considered. Suppose the node admittance matrix, the node injection current vector and the node voltage of the external network are denoted as \(\mathbf{Y}(s), \mathbf{I}(s)\) and \(\mathbf{U}(s)\), respectively. The relationship among them can be written as the following formulas:

\[
\mathbf{I}(s) = \mathbf{Y}(s)\mathbf{U}(s)
\]  

\[
\begin{bmatrix}
\mathbf{I}_b(s) \\
\mathbf{I}_i(s)
\end{bmatrix} =
\begin{bmatrix}
\mathbf{Y}_{bb}(s) & \mathbf{Y}_{bi}(s) \\
\mathbf{Y}_{ib}(s) & \mathbf{Y}_{ii}(s)
\end{bmatrix}
\begin{bmatrix}
\mathbf{U}_b(s) \\
\mathbf{U}_i(s)
\end{bmatrix}
\]  

where the variable subscript ‘\(b\)’ represents all boundary buses and ‘\(i\)’ represents the internal buses of the external network. \(\mathbf{Y}_{bb}(s), \mathbf{Y}_{bi}(s), \mathbf{Y}_{ib}(s)\) and \(\mathbf{Y}_{ii}(s)\) are defined in (2).

2.1.1 Passive network reduction: In (1) and (2), since the injected current \(\mathbf{I}_i(s)\) into any internal buses of the passive network can be regarded as zero, all the loads in the network are considered as constant impedance models and can be merged into \(\mathbf{Y}(s)\). Since only the port buses are to be retained in the study, internal buses in (2) can be eliminated using

\[
\mathbf{U}_i(s) = -\mathbf{Y}_{ii}^{-1}(s)\mathbf{Y}_{bi}(s)\mathbf{U}_b(s)
\]  

\[
\mathbf{I}_i(s) = (\mathbf{Y}_{bb}(s) - \mathbf{Y}_{bb}(s)\mathbf{Y}_{ii}^{-1}(s)\mathbf{Y}_{ib}(s))\mathbf{U}_b(s)
\]

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2.2 VF-based FDNE modelling

In FDNE, the original external system is reduced by an equivalent circuit with similar frequency response in a certain band. In this circuit, a group of RLC parallel branches and a current source at fundamental frequency are used, as illustrated in Fig. 1.

The main idea of VF-based FDNE is to find a rational admittance matrix $Y_{a0}(s)$ which is to be fitted with the $Y_i(s)$ in s-domain whose elements is formulated as (9) and can be transferred to the corresponding equivalent circuit, as illustrated in Fig. 2

$$Y_{i,j}(s) \approx Y_{a0,i,j} = \sum_{k=1}^{NP} \frac{c_k'}{s - a_k'} + d' + h' \cdot s \quad (9)$$

where NP is the order of system equivalent and the $Y_{i,j}(s), Y_{a0,i,j}(s)$ represents each element of the matrix $Y_i(s), Y_{a0}(s)$, respectively.

Fig. 2a shows the topology of a multi-terminal VF-based FDNE network. The admittance blocks $y_j(s)$ are calculated from each element of $Y_{a0}(s)$ by (10) and (11) which can be written as polynomials as (12) and realised by Fig. 2b [16]. $I_{cs}(s)$ and $I_{cs,b}(s)$ represent the $i$th and $j$th element of $I_c(s)$, respectively

$$y_j(s) = \sum_{i=1}^{NP} Y_{i,j} \quad (10)$$

$$y_j(s)|_{s = j} = - Y_{a0,i,j} \quad (11)$$

$$y_j(s) = \sum_{k=1}^{NP} c_k - a_k + d + h \cdot s \quad (12)$$

where $n$ is the number of terminals of the FDNE network. The parameters of elements in Fig. 2b are calculated by (13)–(20) [3, 4, 7]. $a_k, a_k', (c_k, c_k')$ represents the real and imaginary parts of the $a_k(c_k)$ in (12), respectively, when it is complex

$$C_n = h \quad (13)$$

$$R_n = d \quad (14)$$

$$R_{a_k} = a_k / c_k \quad (15)$$

$$L_{ck} = 1 / c_k \quad (16)$$

$$L_k = 1 / 2 c_k \quad (17)$$

$$R_k = (- 2 a_k + 2 (c_k a_k + c_k') L_{ck}) L_{ck} \quad (18)$$

$$1 / C_{ck} = (a_k^2 + a_k') + 2 (c_k a_k + c_k') R_k L_{ck} \quad (19)$$

$$G_{ck} = - 2 (c_k a_k + c_k') C_{ck} L_{ck} \quad (20)$$

Moreover, models obtained from VF directly are often non-passive, which means the equivalent model may generate energy. So the passivity verification of the equivalent network has also been a research interest recently [3, 9, 10]. The admittance of the equivalent circuit at each frequency is calculated to check whether the eigenvalues of the real part of the fitted admittance matrix $Y_{a0}(s)$ is less than 0, until the regulation (21) is conformed

$$\text{Re}(\text{real}(Y_{a0}(s))) > 0 \quad (21)$$

3 Two-step equivalent algorithm

The proposed two-step equivalent algorithm partitions the whole process of VF-based rational approximation into two main steps as shown in Fig. 3, which will be specifically described in 3.1 and 3.2
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represent driving point and transfer admittance, which are also the
circuited currents from each terminal should be obtained. However,
to several kHz) after all the voltage/current sources are short/open-
voltage source with varying frequency in steps (usually from 10 Hz
constant impedance models.

Fig. 3 Flowchart of the proposed algorithm

3.1 Reduction of the network

Consider there is a WTG modelled as an asynchronous machine in
the external system. To determine the value of \( I_{\text{ref}}(s) \), the short-
circuited currents from each terminal should be obtained. However,
the short circuit current will be dropped to zero at around 0.15 s. A
solution is to reduce the WTG with an equivalent voltage source in
series with an impedance with the same output active power before
the measurement. The power flow and voltage distribution of the
study system are approximate before and after the reduction.

3.2 FDNE modelling

Some details about the forming of the FDNE model are provided as
follows.

(i) The frequency responses of \( Y_i \) can be measured from the ports
in the condition that the external system is energised with constant
voltage source with varying frequency in steps (usually from 10 Hz
to several kHz) after all the voltage/current sources are short/open-
circuited, respectively [17]. For \( n \)-port passive networks, only \( n \)
driving point admittances and \( n(n - 1)/2 \) transfer admittances need
to be considered for its symmetry [11]. Each element of \( Y_i(s) \) can
be measured according to (22) and (23), where \( Y_{ri}(s) \) and \( Y_{ij}(s) \)
represent driving point and transfer admittance, which are also the
diagonal and non-diagonal elements of the \( Y_i(s) \), respectively

\[
Y_{ri}(s) = \frac{I_i(s)}{U_i(s)} \quad Y_{ij}(s) = \frac{I_j(s)}{U_i(s)} \quad Y_{ji}(s) = \frac{I_j(s)}{U_j(s)}
\]

(ii) Determine the NP of the equivalent circuit, whose influence on
the equivalent accuracy will be analysed in Section 4.

(iii) Identify parameters in \( Y_{\text{ref}}(s) \) by VF method [3].

(iv) Obtain the FDNE model as Fig. 2.

4 Effect of number of poles

The complexity of the equivalent circuit can be directly determined
by the NP in (9), which will affect the speed and the accuracy of
the EMT simulation. For the network in which WTGs and dynamic
loads are approximated, the NP of the rational admittance
diagonal and non-diagonal elements of the

\[
\begin{align*}
Y_{ri}(s) &= \frac{I_i(s)}{U_i(s)} \bigg|_{U_j=0,i \neq j} \\
Y_{ij}(s) &= \frac{I_j(s)}{U_i(s)} \bigg|_{U_j=0,i \neq j} \\
Y_{ji}(s) &= \frac{I_j(s)}{U_j(s)}
\end{align*}
\]

where \( n \) is the number of terminals of the network and \( N_p \) is
the number of the data points of the given frequency response data [3].
It is obvious that the more the \( E_{\text{rms}} \) value closer to zero the better
fitting effect or equivalent effect is obtained. The RMS error for
VF method is usually monotonically decreasing with the increasing
of NP. It is worth noting that a high-order equivalent circuit may be
needed for a complex external system, e.g. in case that many
distributed parameter transmission lines exist. For instance, the
order of each block of the equivalent network for a 7-bus with five
transmission lines subsystem can be more than 30 for precise
accuracy of the frequency responses in the band range 10–1000 Hz
(see Fig. 4), which may lead to the simulation process of the
reduced model being quite slow and violate the original purpose of
the network equivalent.

As can be seen in Fig. 4, the equivalent RMS error in frequency
domain can be gradually decreased as the NP is increased.
However, so many initial poles (e.g. NP > 20) may incur high
calculational burden of EMT simulation. So NP setting needs
careful treatment to achieve trade off between the algorithm
performance and its computation time.

5 Case study

5.1 Test system

The method is implemented in a 29-bus power system shown in
Fig. 5 which can be found in Simulink demo. The whole system
consists of 29 buses and 7 generators, in which a WTG modelled as
an asynchronous machine with negative given torque is located in
the QUE-Load subsystem. Besides, a comprehensive load network
which contains an induction motor is located at bus ‘MTL7’.

Define the single-port subsystem of QUE Load and Wind
Generation as the external system in which the WTG is reduced by
a voltage source with the same active power output, and the
remaining part of the grid is treated as study system. Then the short

circuit current from the port of the external system is measured
which is set as the value of the parallel injection current source in
the FDNE model.

5.2 Equivalent result

The parameters of the parallel current source and the VF-based
FDNE network are shown in Tables 1 and 2, respectively

5.3 Equivalent validation in frequency domain

The RMS error for the external system’s admittance rational
approximation with respect to the NP is shown in Fig. 6. It can be

\[
E_{\text{rms}} = \frac{\sqrt{\sum (Y_{ij} - Y_{\text{ref},ij})^2}}{N_p}
\]
seen that the rational function fits the admittance frequency response quite well when NP = 5, so the external network can be reduced as a sixth-order FDNE model (including the shunt capacitance) after the voltage sources inside is short circuited. The frequency responses of the original network and its FDNE model are shown in Fig. 7, which has a satisfactory equivalent accuracy.

5.4 Equivalent validation in time domain

A three-phase fault is set on Bus ‘QUE7’, which is located on the boundary of the external system, at 0.1 s for a duration of 0.05 s to assess the performance of the proposed equivalent algorithm under transient conditions. Figs. 8a–c show the voltage on bus ‘QUE7’ and the current & active power flowing from the port of the

Table 1  Circuit parameters for the parallel current source

| Branch no. | Peak amplitude, A | Phase, deg | Frequency, Hz |
|------------|-----------------|------------|--------------|
| 1          | 48.11           | −99.4072   | 60           |

Table 2  Circuit parameters for the FDNE circuit model

| Branch no. | R, Ω | L, mH | C, μF | G, S |
|------------|------|-------|-------|------|
| 2          | —    | —     |       | —    |
| 3          | 1.03 × 10^5 | —    | —     | —    |
| 4          | 1.60 × 10^3  | 1.37 × 10^5 | —    | —    |
| 5          | 2.41 × 10^3  | 3.10 × 10^4  | —    | —    |
| 6          | 9.24 × 10^1   | 3.45 × 10^1   | —    | —    |
| 7          | −5.99 × 10^7  | 1.15 × 10^6   | 3.2 × 10^{-7} | 1.67 × 10^{-8} |

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external system, respectively, and from the results it can be seen that the original system and the reduced model are in quite close agreement in time-domain response both in steady state and transient state. In Figs. 8a–b, the error of the reduced model is within 1% in magnitude and the phases of the two waves are exactly the same. The error in Fig. 8c is also within 5%. An oscillation occurs after the fault happens which is raised by the induction motor located in the MTL-Load subsystem. Moreover, the initial power flow and the voltage distribution at each bus in the study system are also quite close between the two system models, and the reduced model can save around 20% of the simulation time of the original system model.

6 Conclusion

A two-step equivalent method for finding the FDNE of a power system with WTGs integrated for accelerating the process of the EMT simulation has been proposed in this paper. Modifications have been made on that the WTGs in the external system are simplified as equivalent voltage sources, and the dynamic loads can be equivalent by constant impedance load models before the scanning of the network’s frequency response. Taking the case of 29-bus system with wind power integrated from Simulink Demo for instance, the simulation results both in frequency and time domains validate that the reduced model has the same behaviour as the original power system. The advantage of this method is that a stable and passive FDNE model can be obtained and the VF-based rational approximation can be well applied more widely to the occasions when the power system is wind power integrated.

7 References

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Fig. 6 RMS error with respect to the NP for the case study

Fig. 7 Comparison of the resultant admittance
(a) Magnitude of admittance w.r.t. frequency, (b) Phase angle of admittance w.r.t. frequency

Fig. 8 Comparison between the original model and the reduced model in time-domain response
(a) Voltage of bus ‘QUE7’ (phase A), (b) Current flowing from the external system (phase A), (c) Active power from the external system
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