Theory of the Breakdown of the Quantum Hall Effect

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Breakdown of the Quantum Hall Effect at high values of injected current is explained as a consequence of an abrupt formation of a metallic “river” percolating from one edge of the sample to the other. Such river is formed when lakes of compressible liquid, where the long-range disorder potential is screened, get connected with each other due to the strong electric field. Our theory predicts critical currents consistent with experiment values and explains various features of the breakdown.

Since the discovery of the Quantum Hall Effect (QHE) great variety of phenomena associated with the behavior of the two-dimensional electron gas (2DEG) in the strong magnetic field has been studied both experimentally and theoretically. In this paper we discuss the breakdown of the QHE at high injected currents. First observed by Ebert as a sudden onset of the dissipation when the injected current exceeds some critical value, the breakdown of the integer QHE (IQHE) has been intensively investigated since then. While the absence of the dissipation ($\sigma_{xx} = 0$) and the quantization of the transverse conductivity $\sigma_{xy}$ in the IQHE regime at low temperatures are well understood, the breakdown of the dissipationless regime has not been given a clear explanation. Existing theories of this effect include production of hot electrons, inter-Landau level transitions in the high local electric field (due to tunnelling or emission of phonons), increase in the number of the delocalized states in the Landau level. None of these produced results consistent with experiments: except for the hot-electron picture, all others predict values of the critical current density that are two orders of magnitude higher than those observed experimentally. Also, neither of them can explain hysteresis and localized nature of the breakdown phenomenon, nor the existence of the transient switching and broadband noise before the breakdown, as well as peculiar steps in the magnetic field dependence in the regime of critical current. Besides, all these theories need to use some artificial assumptions and are too complicated to be correct.

Experiments on the breakdown of the QHE are performed on GaAs heterostructures or on high-quality MOSFET devices both characterized by the presence of long-range fluctuations of the disorder potential. In the GaAs systems the disorder potential due to the remote dopants has predominantly long wave-length fluctuations ($\lambda > d > l_H$, where $\lambda$ is the wavelength of the fluctuations, $d$ is the spacer thickness, and $l_H$ is the magnetic length). Long-range potential fluctuations are also present in high-mobility MOSFET devices though their nature is not that evident. When the magnetic field and the density of the 2DEG correspond to a filling factor close to an integer number, these fluctuations are not screened by the electrons in most parts of the system. The region with completely filled Landau level percolates through the sample, leading to the QHE (at the end of this paper we discuss what happens when the filling factor is far from an integer). In the percolating incompressible region electric fields are typically about the same as those of the bare disorder potential. This leads to: $E_{inc} \approx \sqrt{<E_{bare}^2>} = \sqrt{n_0/2\pi e^2/\epsilon_0 d}$. For a typical clean sample this is $E_{inc} \approx 0.1\hbar \omega_c/\ell_H$, where $\omega_c$ is the cyclotron frequency. There are, however, isolated areas where the long wave-length fluctuations of the disorder potential are screened. The temperatures in the experiments on the breakdown ($T \sim 1$ K) are high compared to the energy scales of both the shorter wave-length fluctuations (that are left unscreened) and of the residual inter-electron interaction. Under such conditions a compressible liquid fills these isolated areas of screened potential which, therefore, behave like metallic lakes. In each of those lakes, all states of the highest available Landau level are partially occupied, and the screened potential fluctuates around the Fermi level $\epsilon_F$ with an amplitude of the order of $T$ which is much smaller than the amplitude of the potential fluctuations in the incompressible region. Below we consider the potential in the compressible lakes to be flat. The main idea of our theory is that at high enough currents the insulating region separating two lakes, suddenly breaks down due to the high electric field; this leads to the connection of the lakes. When such connected lakes form a metallic river percolating from one edge of the sample to the other, abrupt onset of the dissipative regime is observed.

In general, the distance between two adjacent lakes can be large, and there can be several fluctuations of the potential in the incompressible region separating two lakes. We will see, however, from the results of the numerical simulations that the lakes closest to each other get connected first. Let us, therefore, consider a simple model in which two metallic lakes are separated by a narrow insulating region with one parabolic potential fluctuation characterized by the root mean square electric field $E = E_{inc} \approx 0.1\hbar \omega_c/\ell_H$. In equilibrium, all electrons in the lakes have energies...
equal to $\epsilon_F$ (up to $T$), and all electrons in the incompressible region separating them have energies below the Fermi energy. Electrons cannot move from the lake into the incompressible region because no states are available there (except on the next Landau level, which is much higher in energy). They do not flow into the lake either, because the energy of any available state $\epsilon_F$ is higher than that of any state in the completely filled region. Injection of the current into the 2DEG leads to the appearance of the Hall electric field throughout the sample. Assume that the direction of the electric field coincides with the direction from one lake to the other. In the insulating region this external field can be considered to be uniform on the scale of the distance between the lakes. In the lakes the charges are redistributed to screen the external field. Now, however, the two lakes are at different potentials (Fig. 1a). One can easily show that when the external field $E_0 = \sqrt{3} E_{\text{inc}}$, the minimum of the parabolic potential fluctuation shifts to the boundary of the metallic lake, so that the potential monotonically drops from one lake towards the other. At this point, electrons can freely move from the incompressible region into the lake, as their energies are higher than those of the partially filled states in the lake. In this simplified picture, $E_0$ is a critical field at which the two lakes get connected. Possible existence of several fluctuations in the separating region does not change the picture. Lakes get connected when the potential drops monotonically between them, i.e. when the last (the fastest) fluctuation is smoothed out by the electric field. The estimate for the critical field remains basically the same because the probability for the electric field of the disorder potential to be larger than $0.2\hbar\omega_c/l_H$ is exponentially small.

The above value of the electric field necessary to connect the lakes does not need to be equal to the critical average electric field $E_c = V_H/W$, where $V_H$ is the measured Hall voltage, and $W$ is the width of the sample. It is easy to see that the electric field in the narrow region between the two lakes can be much larger than the average Hall electric field: if the size of the separating region is smaller than the size of the lakes, the equipotential lines are highly squeezed between the two lakes. Therefore, an average electric field much smaller than $E_0$ is needed to connect the lakes. When the two lakes merge, the electric field is expelled from the former incompressible region now covered with compressible liquid. Therefore, the electric field becomes stronger outside the newly formed large lake, facilitating further connections as the injected current is increased. One would expect that after the metallic region acquires some critical length in the direction of the Hall electric field upon the increase of the injected current, the process of further connection must take an avalanche form. Finally, when the compressible liquid forms a river flowing from one edge to the other, dissipation drastically increases. It is important to notice that just before the formation of the metallic river, the longitudinal conductivity is still determined by the variable-range hopping; also, the effective size of the insulating system is still quite large at this moment, as there are many lakes on the way of the future river that are not yet connected. This is the reason why the longitudinal conductivity jumps several orders of magnitude at the critical current: the mechanism of the conductivity abruptly switches from hopping to the metallic one.

![FIG. 1. a. Distribution of the potential in the two adjacent islands and between them before breakdown; b. and c. Charge distribution from numerical simulations for the 2.5 µm $\times$ 2.5 µm sample with $d = 25$ nm, $l_H = 10$ nm, $\nu = 1.93$, $T = 0.5$ K. White areas are the $\nu = 2$ incompressible liquid, grey areas the $\nu < 2$ partially filled compressible liquid. b. at zero electric field, c. at $E = 500$ V/cm](image)

To see how the breakdown occurs in real systems we performed numerical simulations (the details will be published elsewhere). In equilibrium, the free energy of the interacting 2DEG in the strong magnetic field and in the smooth disorder potential was minimized with respect to the occupation numbers at finite temperature. The disorder potential was created by a randomly chosen distribution of charged impurities in a plane a distance $d$ away from the 2DEG. Coulomb interaction between electrons was taken into account in the mean-field approximation. Fig. 1b shows the charge distribution for a clean sample at $\nu = 1.93$. Lakes of compressible liquid ($\nu < 2$, grey areas) are immersed into...
the percolating incompressible region \((\nu = 2, \text{ white areas})\). The effect of the Hall current shows up in the appearance of the external electric field which is taken to be uniform before the response of the charges in the system. Relaxation current is allowed to flow only in the compressible regions. The charge density of the same system but with external field of 500 V/cm is shown on Fig. 1c. Evidently, this field is already higher than the critical one: a metallic river is well established. This is in good agreement with experimental data.

The fact that breakdown is localized in space directly follows from our theory: a metallic river is very narrow at the breakdown. Connection of the lakes at fields below the critical one causes charge redistribution and energy relaxation. The amount of dissipated energy varies with the size of the connecting lakes. This energy relaxation shows up as a broadband noise. The transient switching is the result of the formation and immediate disconnection of the river at the fields just below critical. This happens since after the river is formed, relaxation is available all over the sample width. The system can then relax to a new steady state with incompressible region still percolating. Our numerical simulations show such connection/disconnection processes. As in any dielectric breakdown, hysteresis is the result of the irreversible change of the system properties (namely the pattern of rivers and lakes) after the current was allowed to flow through the system.

Finally, the peculiar steps observed in the magnetic field dependence of the longitudinal voltage in the critical current regime show the evidence of the opening of new metallic channels when the system is moved away from the center of the plateau. In equilibrium, as the difference of the filling factor from the integer becomes larger, the size of the lakes of compressible liquid increases while their separation becomes smaller. Therefore, in the critical regime the number of the percolating narrow metallic channels also grows, each giving its own discrete contribution in the voltage drop. The question of what happens if at certain filling factor compressible region percolates even in equilibrium, will be addressed in another paper. Here, however, we want to mention that even at zero temperature in the mean-field approximation and with only long-range disorder, the relative width of the QH plateau is less than 0.4, and can be very small for clean samples. While the residual electron-electron interaction leads to the fractional QHE (FQHE) in very clean samples, the systems not exhibiting FQHE will still show step-like transition between QH plateaux. Such behavior can be observed only at very low temperatures when the fluctuations of the screened long-range potential in the compressible region \((\approx T)\) become comparable with the amplitude of the short wave-length \((< l_H)\) disorder potential fluctuations which, though being exponentially small, remain unscreened. At such low temperatures the properties of the system are determined by the short-range random potential.

In conclusion, we propose a theory of the breakdown of the QHE based on the existence of the compressible regions in the inhomogeneous 2DEG. The predictions of the theory agree with experimental data.

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