Spin current shot noise as a probe of interactions in mesoscopic systems

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It is shown that the spin resolved current shot noise can probe attractive or repulsive interactions in mesoscopic systems. This is illustrated in two physical situations: i) a normal-superconducting junction where the spin current noise is found to be zero, and ii) a single electron transistor (SET), where the spin current noise is found to be Poissonian. Repulsive interactions may also lead to weak attractive correlations (bunching of opposite spins) in conditions far from equilibrium. Spin current shot noise can also be used to measure the spin relaxation time $T_1$, and a set-up is proposed in a quantum dot geometry.

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Non-equilibrium noise plays a key role in mesoscopic physics [1]. Low-temperature correlations of the time fluctuations of the electronic current indeed give unique information about the charge and the statistics of quasiparticles. For non-interacting electrons, the scattering approach is very powerful for a variety of systems [2,3].

The reduction of shot noise from the Schottky value originates from the Pauli exclusion principle which forbids two wavepackets with the same quantum numbers to be superimposed [4]. On the other hand, Coulomb interactions also act in correlating wavepackets, and noise is indeed more sensitive to interactions than the conductance. Coulomb interactions may decrease or increase noise correlations, depending on the physical regimes [2,3].

Full counting statistics are also promising as a probe of interactions [5]. Yet, in a given mesoscopic structure, the effects on the shot noise of Fermi statistics and of interactions are intimately mixed. In contrast, we propose in this Letter that spin-resolved shot noise can unambiguously probe the effects of electronic interactions. The basic idea is that the Pauli principle acts only on electrons with the same spin. Therefore currents wavepackets carried by quasiparticles with opposite spins are only correlated by the interactions.

Spin-resolved shot noise has received very little attention up to now, contrarily to the total current shot noise. For instance, spin shot noise was recently considered in absence of charge current [6]. On the other hand, the effect of a spin-polarized current on charge and spin noise was investigated, with complex behaviours due to spin accumulation [6]. Noise is also an efficient probe for testing quantum correlations in two-electron spin-entangled states [1-4] or electron spin teleportation [8]. In contrast, we consider here simple and general mesoscopic structures in which the average current is not spin-polarized, but where the currents carried by quasiparticles with different spins can be separately measured. A possible set-up for this purpose will be described at the end of this Letter. To clarify our statement, let us first consider a general mesoscopic device made of a normal metal with non-interacting electrons, non magnetic terminals $i, j, ...$, and one channel for simplicity (generalization is obvious). In absence of magnetic fields and spin scattering of any kind, the scattering matrix is diagonal in the spin variable and spin-independent, $s_{\sigma\sigma'}^{ij} = \delta_{\sigma\sigma'} s_{ij}$. This trivially leads to spin-independent averaged currents $\langle I_{ij}^\sigma \rangle = \langle I_{ij}^{-\sigma}\rangle$. In a similar way, spin-resolved noise, defined as $S_{\sigma\sigma'}^{ij}(t-t') = \frac{1}{2} (\Delta I_{ij}^\sigma(t) \Delta I_{ij}^{-\sigma}(t') + \Delta I_{ij}^{-\sigma}(t') \Delta I_{ij}^\sigma(t))$, where $\Delta I_{ij}^\sigma(t) = I_{ij}^\sigma(t) - \langle I_{ij}^\sigma(t) \rangle$, can be evaluated. One easily finds that at any frequency the noise power between terminals $i$ and $j$ is diagonal in the spin variables, $S_{\sigma\sigma}^{ij}(\omega) = \delta_{\sigma\sigma} S_{ij}(\omega)$. Thus, choosing an arbitrary spin axis $z$, the total (charge) current noise $S_{ij}^{ch} = S_{ij}^{\uparrow\uparrow} + S_{ij}^{\downarrow\downarrow} + S_{ij}^{\uparrow\downarrow} + S_{ij}^{\downarrow\uparrow}$ and the spin current noise $S_{ij}^{sp} = S_{ij}^{\uparrow\uparrow} + S_{ij}^{\downarrow\downarrow} - S_{ij}^{\uparrow\downarrow} - S_{ij}^{\downarrow\uparrow}$, defined as the correlation of the spin currents $I_{ij}^{\sigma}(t) = I_{ij}^{\uparrow}(t) - I_{ij}^{\downarrow}(t)$, are strictly equal. On the contrary, in presence of Coulomb interactions, one expects that $S_{ij}^{\uparrow\uparrow} = S_{ij}^{\downarrow\downarrow}$, or equivalently $S_{ij}^{sp} \neq 0$. This can happen for instance if the scattering matrix couples carriers with opposite spins, as Andreev scattering at a normal- superconductor (NS) interface, or in quantum dots in presence of strong Coulomb repulsion. The sign of the correlation $S_{ij}^{\uparrow\downarrow}$ (bunching or antibunching) is of special interest.

Let us first consider a NS junction, where $S$ is a singlet superconductor and $N$ a normal metal. The scattering matrix coupling electron (e) and holes (h) quasiparticles in the metal is composed of spin-conserving normal elements $s_{ee}^{\sigma\sigma'}$, $s_{hh}^{\sigma\sigma'}$, and Andreev elements $s_{eh}^{\sigma\sigma'}$, $s_{he}^{\sigma\sigma'}$ coupling opposite spins. The calculation of the total zero-frequency noise $S_{ij}^{ch} = \sum_{\sigma\sigma'} S_{\sigma\sigma'}^{ij}$, using the unitarity of the scattering matrix, reduces at zero temperature to the well-known result $S_{ij}^{ch} = \frac{2e^2}{h} Tr [s_{he}^{\dagger} s_{he}(1 - s_{he}^{\dagger} s_{he})]$, where the trace is made on the channel indexes [10].

We remark here that it is easy to calculate the spin-resolved correlations $S_{\sigma\sigma}$ and $S_{-\sigma\sigma}$, and to check that they are exactly equal. The result of this observation is that for a NS junction, at $T = 0$, the spin current shot noise is strictly zero, $S_{sp} = 0$. The current correlation be-
This implies that the transitions from one level of energy \( E_x \), \( x \) to reservoir \( L \) do not depend on the charge state, depending on whether the level is occupied by zero, one or two electrons (Fig. 1). These states will be indexed as \( N = 0, N = 1 \) (with spins \( \uparrow, \downarrow \)) and \( N = 2 \). Let us denote as \( U(N) \) the Coulomb energy for the state \( N \), \( \Delta E_{L,R}^+(N) = E_0 - \mu_{L,R} + U(N + 1) - U(N) \) the energy to add an electron to state \( N \) from leads \( L, R \), and \( \Delta E_{L,R}^-(N) = -E_0 + \mu_{L,R} + U(N - 1) - U(N) \) the energy to remove an electron from state \( N \) towards \( L, R \). Let us further assume that \( \Delta E_{L,R}^+(0), \Delta E_{L,R}^-(1) \ll -k_B T \). This implies that the transitions from \( N = 0 \) to \( N = 1 \) involve electrons coming only from \( L \), and the transitions from \( N = 1 \) to \( 0 \) involve electrons going only into \( R \). Let us allow the Coulomb energy to vary and consider the possibility of transitions from \( N = 1 \) to \( 2 \), only from \( L \), e.g. \( \Delta E_{R}^-(2) \ll -k_B T \). Yet, \( \Delta E_{L}^+(1) \) can take any value. This describes the following situation: if \( \Delta E_{L}^+(1) \gg k_B T \), the transition to state \( N = 2 \) is forbidden and one has the same SET case with only two charge states \( N = 0, 1 \), in the resonant regime at low temperature (Fig. 1a). If on the contrary \( \Delta E_{L}^+(1) \ll -k_B T \), then the three charge states \( 0, 1, 2 \) are involved in the charge transport (Fig. 1b). The physical situation under consideration corresponds for instance to fixing the gate voltage such as \( U(1) = U(0) \), and varying the ratio between \( k_B T \) and the Coulomb excess energy \( U(2) - U(1) \).

Let us write the master equation describing this system. Assuming a constant density of states in the reservoirs and defining \( x \) as the Fermi function \( x = 1 + \exp(\beta \Delta E_{L}(1))^{-1} \), the non-zero transition rates are \( \Gamma_{L,R}^x(1) = \Gamma_{L,R}, \Gamma_{L,R}^x(2) = \Gamma_{L,R}^x \) (1 – \( x \)\( \Gamma_{L,R}^x \)). Then the populations \( p_0, p_1, p_2 \) verify

\[
\dot{p}_0 = -2\Gamma_L p_0 + \Gamma_R (p_1 + p_2) \\
\dot{p}_1 = -(\Gamma_R + x\Gamma_L) p_1 + \Gamma_L p_0 + (1 - x)\Gamma_L + \Gamma_R p_2 \\
\dot{p}_2 = -2((1 - x)\Gamma_L + \Gamma_R) p_2 + x\Gamma_L (p_1 + p_2) \\
\]

Let us first consider the limit \( x = 1 \). Then the transition rates from \( L \) or into \( R \) do not depend on the charge state, which means that this limit is equivalent to that of a resonant state without Coulomb charging energy. The solution of Eqs. factorizes in this case, e.g. \( p(n_\uparrow, n_\downarrow) = p(n_\uparrow) p(n_\downarrow) \), so that for each spin component, the probabilities \( p(0) \) and \( p(1) \) of empty and occupied states verify \( \dot{p}(0) = -\dot{p}(1) = -\Gamma_L p(0) + \Gamma_R p(1) \). From this one derives the average current \( \langle I \rangle = 2e\Gamma_L \frac{\Gamma_R}{(2\Gamma_L + \Gamma_R)^2} \) and the zero-frequency shot noise \( S_{ij}^{\sigma \sigma} (\omega = 0) = 2e^2 \langle I \rangle (1 - \langle I \rangle (2\Gamma_L + \Gamma_R)^2) \), independently of the couple of junctions \( i, j \) = \( L, R \). Moreover, it is simple to check that spin \( \uparrow \) and \( \downarrow \) currents are uncorrelated, thus \( S_{ij}^{\uparrow \uparrow} = S_{ij}^{\downarrow \downarrow} = 0 \), or equivalently \( S^{sp} = S^{ch} \), as can also been derived by the scattering method in the quantum coherent regime. We thus have another example of the general behavior for uncorrelated transport.

Let us now consider the SET case \( x = 0 \), where charge transport is maximally correlated. The charge noise is given by the expression \( S_{ij}^{\sigma \sigma} (\omega = 0) = 2e^2 \langle I \rangle (1 - \langle I \rangle (2\Gamma_L + \Gamma_R)^2) \). Apart from an effective doubling of the rate \( \Gamma_L \), this result is qualitatively similar to that obtained without interactions. Therefore the charge noise is not the best possible probe of interactions. We show that, on the contrary, the behaviour of the spin noise is completely different. Indeed, using the method by Korotkov \[21\], we find that

\[
S_{ij}^{\sigma \sigma} = e\langle I \rangle (1 - \frac{2\Gamma_L \Gamma_R}{(2\Gamma_L + \Gamma_R)^2}) \quad S_{ij}^{\sigma \sigma} = -e\langle I \rangle \frac{2\Gamma_L \Gamma_R}{(2\Gamma_L + \Gamma_R)^2} \quad S^{sp} = 2e\langle I \rangle 
\]
The striking result (Eq. (2)) for $S^{sp}$ resembles a Poisson result and corresponds to maximal fluctuations. The correlations between currents of opposite spins are negative, like a partition noise. Yet spin-up and spin-down channels are separated energetically rather than spatially, and wavepackets with up or down spins exclude each other because of interactions rather than statistics.

The above result, obtained at zero temperature with perfect spin coherence inside the island, can be interpreted in the following way: electrons come from reservoir $L$ with random spins. Even though the average spin current is zero, each junction is sequentially crossed due to Coulomb repulsion by elementary wavepackets with well-defined but uncorrelated spins. This implies very short time correlations (on the scale of tunneling through one of the barriers) therefore the spin current exhibits Poisson statistics. On the contrary, charge current wavepackets are correlated on times $\sim h/\Gamma$, leading to the reduction as compared to the Poisson value. Notice that Eqs. (2) is a consequence of the restriction to two charge states: as can be easily checked, the analysis of the SET involving only $N = 1$ and 2 states (instead of 0, 1) yields exactly the same result.

The general solution of Eqs. (1) offers an interpolation between the uncorrelated and the maximally correlated regimes. We find that the average current is given by $\langle I \rangle = e \frac{2\Gamma_L \Gamma_R}{(\Gamma_R + \Gamma_L)(\Gamma_R + \gamma x \Gamma_L)}$. The spin current noise components $S^{sp}_{ij}$ $(i,j = L,R)$ can also be calculated and do not depend on the couple $(i,j)$ of junctions chosen. The expression for the spin noise is

$$S^{sp}_{ij} = 2e\langle I \rangle \left(1 - \frac{2x\Gamma_L \Gamma_R}{(\Gamma_R + \Gamma_L)(\Gamma_R + x\Gamma_L)}\right)$$

The expression for the total (charge) noise $S^{ch}$ is too lengthy to be written here. Figs. 2, 3 shows the variation with $x$ of the charge and spin current noise. The spin noise maximum for $x = 0$, decreases monotonously and merges the charge noise at $x = 1$. The role of the asymmetry of the junctions is very striking. First, if $\Gamma_R > \Gamma_L$, we find that $S^{sp}$ is always larger than $S^{ch}$ (Fig. 2), like in the ideal SET ($x = 0$). On the other hand, if $\Gamma_R < \Gamma_L$, $S^{sp}$ happens to be smaller than $S^{ch}$ for $x > x_c \sim \Gamma_R/\Gamma_L$ (Fig. 3). This implies that $S^{\uparrow \uparrow} > 0$, contrarily to the naive expectation for repulsive interactions. This unexpected behavior can be explained as follows: if $\Gamma_R < \Gamma_L$, the low charge states are unfavored and the high ones favored, despite of Coulomb repulsion: for $x > x_c$, $p(2)$ becomes larger than $p(0)$. When the SET occasionally reaches the state $N = 0$, a first transition leads to state 1, but then the most probable transition is to state 2 since $\Gamma^+_R(1) = x\Gamma_L > \Gamma^-_R(1) = \Gamma_R$: two electrons enter the dot successively, with opposite spins, leading to a certain degree of bunching. Here the anomaly is due to a kind of “population inversion” (the most energetic state is favoured), manifesting a strong departure from equilibrium (Fig. 3). Yet, the effect is rather weak, less than 10%, contrarily to the NS junctions where attractive correlations are 100%.

Let us now consider how the above results are modified by spin relaxation, due for instance to spin-orbit scattering. Let us simply focus on the SET with two charge states 0, 1 and introduce a spin-flip rate $T^{-1}_f = \gamma_{sf}$. The master equations will be written in this case

$$\dot{p}_0 = -2\Gamma_L p_0 + \Gamma_R (p_\uparrow + p_\downarrow)$$
$$\dot{p}_\uparrow = -\Gamma_R p_\uparrow + \Gamma_L p_0 + \frac{1}{2} \gamma_{sf} (p_\downarrow - p_\uparrow)$$
$$\dot{p}_\downarrow = -\Gamma_R p_\downarrow + \Gamma_L p_0 + \frac{1}{2} \gamma_{sf} (p_\uparrow - p_\downarrow)$$

The introduction of spin relaxation obviously changes neither the average current nor the charge shot noise. On the contrary, the spin shot noise is altered. For instance, opposite spin noise correlations between junctions $L$ and $R$ become $S^{\sigma - \sigma}_{LL,R} = -e \langle I \rangle \left(\frac{2\Gamma_L \Gamma_R}{(2\Gamma_L + \gamma x \Gamma_L)^2} - \frac{\gamma_{sf}}{2(\Gamma_R + \gamma_{sf})}\right)$, and can even become positive. This results in a spin noise

$$S^{sp}_{LL} = 2e\langle I \rangle \Gamma_R \frac{\Gamma_R}{\Gamma_R + \gamma_{sf}}$$

FIG. 2: Spin shot noise and charge shot noise in the SET, as a function of $x$ (see text): $x = 0$ denotes the maximal correlation, $x = 1$ the uncorrelated case. $\Gamma_R = 2\Gamma_L$: antibunching of opposite spins.

FIG. 3: Same as Fig. 2, $\Gamma_R = 0.2\Gamma_L$: bunching of opposite spins for $x > x_c$. The inset shows the probabilities of states $N = 0, 1, 2$ and the population inversion at large $x$. 

\[ \text{FIG. 2: Spin shot noise and charge shot noise in the SET, as a function of } x \text{ (see text): } x = 0 \text{ denotes the maximal correlation, } x = 1 \text{ the uncorrelated case. } \Gamma_R = 2\Gamma_L: \text{ antibunching of opposite spins.} \]

\[ \text{FIG. 3: Same as Fig. 2, } \Gamma_R = 0.2\Gamma_L: \text{ bunching of opposite spins for } x > x_c. \text{ The inset shows the probabilities of states } N = 0, 1, 2 \text{ and the population inversion at large } x. \]
The reduction of the spin noise $S_{LR}^{SP}$ from the "Poisson" value is a fingerprint of spin relaxation. Remarkably enough, the spin noise on junction $L$ is not affected, since the spins of entering wavepackets are uncorrelated whatever happens in the island. While transient current measurements have allowed to measure $T_1$ in presence of Zeeman splitting [25], our result suggests an alternative method which does not require a magnetic field. Notice that noise was recently proposed to test spin flip in absence of Coulomb repulsion [26].

Let us now propose a set-up for the measurement of spin current correlations in a single-electron transistor. One may consider a four-terminal configuration [26], where the two left terminals $L_1$, $L_2$ are ferromagnetic metals with opposite spin polarizations, having the same chemical potential $\mu_L$ (Fig. 4). Similarly, terminals $R_1$ and $R_2$ have opposite polarizations, respectively parallel to those of $L_1$, $L_2$, and the same chemical potential $\mu_R$. If the junction parameters are the same for $L_1$, $L_2$ on one hand, and for $R_1$ and $R_2$ on the other hand, then the net current flowing through the SET is not spin polarized. Yet, it is possible to measure separately the spin current components in each of the four terminals, e. g. measure the noise correlations $S_{LL_1L_2}$, $S_{LL_1R_1}$, $S_{LL_2R_2}$, etc... If each terminal generates a fully spin-polarized current, the analysis of this set-up can be mapped onto the above model, and the previous results hold. In the more realistic case where polarization is not perfect, the above measurement would yield a mixing of the spin noise with the charge noise. If those are sufficiently different (strong repulsive correlations), they could still be distinguished, which allows to probe the Coulomb correlations by the method of spin current noise.

In summary, we have proposed to probe the attractive or repulsive correlations induced by interactions by measuring the noise correlations of the spin components of the current. This requires not to break the spin symmetry in the device, e.g. the total current is not spin-polarized. We have illustrated this trend on two simple and classical mesoscopic devices. First, a NS junction shows opposite spin bunching due to attractive correlations. Second, a SET in the sequential regime shows in general repulsive correlations (antibunching), but those can be weakly attractive far from equilibrium. Extensions to other regimes or multiple dot systems is quite promising.

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