Controller confidentiality for nonlinear systems under sensor attacks

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Abstract—Controller confidentiality under sensor attacks refers to whether the internal states of the controller can be estimated when the adversary knows the model of the plant and controller, while only having access to sensors, but not the actuators. We show that the controller’s state can be estimated accurately when the nonlinear closed-loop system is detectable. In the absence of detectability, controller confidentiality can still be breached with a periodic probing scheme via the sensors under a robust observability assumption, which allows for the controller’s state to be estimated with arbitrary accuracy during the probing period, and with bounded error during the non-probing period. Further, stealth can be maintained by choosing an appropriate probing duration. This study shows that the controller confidentiality for nonlinear systems can be breached by balancing the estimation precision and the stealthiness of the adversary.

I. INTRODUCTION

The cyber security of dynamical systems have gained traction in recent years as cyber-physical systems become increasingly interconnected, see [1] and [2] for a tutorial overview. While the connectivity improves performance and enhances the capabilities of cyber-physical systems, it also exposes vulnerabilities which can be exploited maliciously. The objective of the adversary is to gather data in order to launch an attack to disrupt operation, while avoiding detection by the system operator.

Although there are many vulnerable points in cyber-physical systems, the vulnerability of sensors has been widely studied thus far. In this setup, a subset of the sensor measurements can be read and manipulated by an adversary and various attack strategies have been investigated to avoid detection in works by [3], [4], [5] to name a few, and to then still provide good estimates of the system states in works by [6], [7], [8], [9] and more. Underlying the attack strategies mentioned earlier is the adversary’s knowledge of the controller’s state, which motivated a line of work investigating the confidentiality of control systems [10], [11], [12], [13]. In all these works, control systems with only linear dynamics is considered.

In this paper, we analyse the controller confidentiality of nonlinear systems. Precisely, we provide rigorous analysis on whether the states of the controller can be estimated when the adversary can read and manipulate the sensors. We consider plant and controllers models with a general nonlinear structure, which already has some inherent stability properties, as all well designed control systems possess. The adversary knows the plant and controller models and has access to the sensors, but not the actuators. We show that if the closed-loop system is detectable (assumed in [10]), then the adversary only needs to read the sensors and not manipulate them to reconstruct the controller’s state exactly. In the absence of closed-loop detectability, the adversary needs to manipulate the sensors, which we call the act of probing, such that the controller’s states can be estimated within a bounded margin of error. As the adversary now needs to probe the closed-loop system, this could raise alarms as anomaly detection schemes are typically employed in well designed control systems. In this scenario, we show that stealth can be maintained when the adversary employs a dual-mode probing scheme.

First, we assume that the closed-loop system is semiglobal asymptotically stable and has a robust observability property. With these assumptions, the estimation of the controller’s state (with bounded error) and stealth (semiglobal practical stability of the closed-loop system) can be achieved. To do so, the adversary probes the closed-loop system via the sensors periodically for a short period. During which, a fast estimator can reconstruct the controller’s state with desired precision. After which, the probing signal is turned off for a specified time interval to preserve the semiglobal practical stability of the closed loop system (maintain stealth), while still keeping the estimated controller’s state within a neighborhood of the true controller’s state. During the non-probing interval, the estimator is turned off and the estimate of the controller’s state is held until the end of the non-probing interval. This scheme is reminiscent of the time-sharing strategies in [14] and [15] for sampled-data output feedback for nonlinear systems and Wiener systems, respectively. This paper focuses only on confidentiality breaching strategies. Hence, future work will involve developing defense strategies which involve the introduction of uncertainties known to the system operator, but not known to the adversary, for example.

The proofs of the lemmas are provided in the full version of this paper [16].

II. PRELIMINARIES

Let \( \mathbb{R} = (-\infty, \infty), \mathbb{R}_{\geq 0} = [0, \infty), \mathbb{R}_{> 0} = (0, \infty) \). Let \( \mathbb{N}_{\geq 1} = \{i, i+1, i+2, \ldots\} \). A finite set of integers \( \{i, i+1, i+2, \ldots, i+k\} \) is denoted as \( \mathbb{N}_{[i,i+k]} \). The identity matrix of dimension \( n \) is denoted by \( I_{n} \). A diagonal matrix with matrices \( d_{i}, i \in [1,n] \) is denoted by \( \text{diag}(d_{1}, d_{2}, \ldots, d_{n}) \). Given a symmetric matrix \( P \), its maximum (minimum) eigenvalue is denoted by \( \lambda_{\text{max}}(P) \) \( (\lambda_{\text{min}}(P)) \). The infinity norm of a vector \( x \in \mathbb{R}^{n} \), is denoted \( |x| := \max_{i \in [1,n]} |x_{i}| \) and for a matrix \( A \in \mathbb{R}^{n \times n} \), \( |A| := \max_{i \in [1,n]} \sum_{j \in [1,n]} |a_{ij}| \), where
$a_{ij}$ is the row $i$-th and column $j$-th element of matrix $A$.

See [17] for the usual definitions of class $K$ and $KL$ functions.

III. MOTIVATION AND PROBLEM FORMULATION

A. Plant and controller models

We consider nonlinear systems of the form

$$\Sigma_p: \begin{align*}
\dot{x}_p &= f_p(x_p, u), \\
y &= h(x_p) + a,
\end{align*}
$$

where $x_p \in \mathbb{R}^{n_p}$ is the system’s state, $u \in \mathbb{R}^{n_u}$ is the input, $y \in \mathbb{R}^{n_y}$ is the output and $a : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_y}$ is an attack signal, respectively. The functions $f_p$ and $h$ are locally Lipschitz and $h$ is sufficiently smooth.

We consider controllers with a general nonlinear structure taking the following form

$$\Sigma_c: \begin{align*}
\dot{x}_c &= f_c(x_c, y), \\
u &= \kappa(x_c, y),
\end{align*}
$$

where $x_c \in \mathbb{R}^{n_c}$ is the controller’s state, the locally Lipschitz function $\kappa : \mathbb{R}^{n_c} \to \mathbb{R}^{n_x}$ is the control law and the function $f_c : \mathbb{R}^{n_y} \times \mathbb{R}^{n_c} \to \mathbb{R}^{n_x}$ is locally Lipschitz such that for all initial conditions $x(0) \in \mathbb{R}^{n_y}$ and $x_c(0) \in \mathbb{R}^{n_c}$, the trajectories of (1), (2) and (3) exist for all time $t \geq 0$.

The controller model in (3) captures both state and output feedback schemes. In the case where state feedback is employed to stabilise the plant (1), the plant output (2) is $h(x_p) = x_p$ (in the absence of sensor attacks) and the controller model in (3) becomes $f_c(x_c, y) = 0$, $x_c(0) = 0$ and $\kappa(x_c, y) = \kappa(0, h(x_p))$. When an output feedback stabilisation scheme is used, then the controller model (3) takes the role of a state observer of the plant (1) with $\kappa(x_c, y)$ being the control law.

In this paper, we focus on control schemes (3) which render the closed-loop system composed of (1), (2) and (3) semiglobally asymptotically stable in the absence of sensor attacks ($\alpha(t) = 0$, for all $t \geq 0$) as stated in the assumption below.

Assumption 1 (Closed-loop system is SG-AS): Let $\Sigma := (x_p, x_c)$. The closed loop system from (1), (2) and (3) with the following dynamics for all $t \geq 0$,

$$\dot{x} = \begin{pmatrix} f_p(x_p, \kappa(x_c, h(x_p))) \\ f_c(x_c, h(x_p)) \end{pmatrix} =: f(x),$$

with $\alpha(t) = 0$, is asymptotically stable, i.e., there exist a class $KL$ function $\beta_x$ such that

$$\|x(t)\| \leq \beta_x(\|x(0)\|, t), \forall t \geq 0.$$  \hfill (5)

When (5) holds for $\|x(0)\| \leq \Delta_x$, where $\Delta_x > 0$, we say that the closed-loop system (4) is semiglobally asymptotically stable (SG-AS).

Control schemes (3) for nonlinear systems which involve output feedback results in a closed-loop system that is SG-AS for certain classes of systems, see [18], [19], [20], [21], [22], for example. For linear plant and controllers, this problem is well studied and Assumption 1 holds thanks to the well-known separation principle which yields a controller (3) that results in a closed loop system that is globally exponentially stable.

B. Adversary model and objectives

We assume that the adversary has knowledge of the plant and controller models, but not their initial conditions $x(0)$ and $x_c(0)$. The adversary can manipulate the sensors $y$, but does not have access to the actuators $u$. Precisely, the adversary operates under the following conditions.

Assumption 2 (Adversary model):

1. The adversary can manipulate the sensor readings $h(x_p)$ via an attack signal $\alpha$, modelled by (2).
2. The adversary knows the functions $f_p$, $h$, $f_c$ and $\kappa$ from (1), (2), (3).
3. The adversary does not know the control input $u$, nor the initial state of the plant and controller models $x(0)$.

The objectives of the adversary are to obtain an estimate of the controller’s states $x_c$ under the operating conditions stated in Assumption 2 without letting the state of the closed loop system $x(t)$ become unbounded in finite time, in the sense that $\lim_{t \to T} |x(t)| = \infty$, for $T < \infty$. We state these two objectives precisely below.

Objective 1 (Estimation of the controller’s state $x_c$):

The estimate of the controller’s state (3) denoted by $\hat{x}_c$ converges to a neighbourhood of the controller’s state $x_c$.

Objective 2 (Maintaining stealth): The closed loop system (4) is semiglobally practically stable, i.e. for any $K_x \geq \Delta_x > 0$, the solution to the closed loop system (4) satisfies

$$|x(0)| \leq \Delta_x \implies |x(t)| \leq K_x, \forall t \geq 0.$$  \hfill (6)

When both of the aforementioned objectives are achieved, we say that the adversary has achieved stealthy estimation of the controller’s state. In other words, the confidentiality of the control system has been breached. Figure 1 illustrates the problem setup.

In the sections that follow, we describe how an adversary can achieve these goals. The stealthy estimation of the controller’s state $x_c$ can be realised without manipulating the sensor measurements when the closed-loop system (4) is detectable in Section IV by using the sensor measurements $y$ to probe the closed-loop system in a time-shared manner in Section V.

IV. CLOSED-LOOP SYSTEM (4) IS DETECTABLE

We first consider the case where the closed-loop system (4) is detectable, which is defined as follows.

Definition 1 (Detectability): The closed-loop system (4) is detectable if there exists a function $l : \mathbb{R}^{n_y} \to \mathbb{R}^{n_x + n_c}$. 

Fig. 1. Problem setup
with \( l(0) = 0 \), such that the estimate \( \hat{x} := (\hat{x}_p, \hat{x}_c) \) is the solution to the following system with dynamics given by
\[
\dot{x} = f(\hat{x}, y) + l(y - \hat{y}), \quad \hat{y} = h(\hat{x}_p),
\]
and the closed-loop system (4) satisfy
\[
|\dot{x}(t) - x(t)| \leq \beta_x(|\dot{x}(0) - x(0)|, t), \quad |x(t)| \leq \beta_x(|\dot{x}(0) - x(0)|, t), \quad \forall t \geq 0,
\]
for all \( t \geq 0 \), for all initial conditions \( \dot{x}(0), x(0) \in \mathbb{R}^{n_x+n_c} \), \( \beta_x \in \mathcal{KL} \) and \( \gamma_x \in \mathcal{K} \).

The function \( l(y - \hat{y}) \) is known as an output injection term and the system (7) whose solution \( \dot{x} \) provides the estimate of \( x \) is known in the literature as a nonlinear observer. According to Definition 1, observers (7) with property (8) are known as input-to-state (ISS) observers with respect to the attack signal \( a \). The design of observers (7) for detectable systems (4) according to Definition 1 is done for specific classes of systems, which exploits the inherent structure of the system, see [23] for an overview.

Hence, if the closed-loop system (4) is detectable, the adversary can estimate the controller’s state \( x_c \) (Objective 1) by only monitoring the sensor measurements \( y \) without manipulating them (i.e., \( a(t) = 0 \) for all \( t \geq 0 \)) and thereby remaining stealthy (Objective 2) under Assumption 1. We summarise this in Proposition 1 below.

Proposition 1: Consider the closed-loop system (4) and adversary model under Assumptions 1 and 2, respectively. If the closed-loop system (4) is detectable, then the adversary achieves Objectives 1 and 2 using (7) with \( a(t) = 0 \), for all \( t \geq 0 \), in the sense that
\[
|\dot{x}(t) - x(t)| \leq \beta_x(|\dot{x}(0) - x(0)|, t), \quad |x(t)| \leq \beta_x(|\dot{x}(0) - x(0)|, t), \quad \forall t \geq 0,
\]
for all initial conditions \( \dot{x}(0), x(0) \in \mathbb{R}^{n_x+n_c} \) satisfying \( |\dot{x}(0)| \leq \Delta_x \) and \( |x(0)| \leq \Delta_x \), and \( \beta_x \in \mathcal{KL} \) and \( \beta_x \in \mathcal{KL} \) comes from Definition 1 and Assumption 1, respectively. □

As seen in (9), the adversary performs better than the stated objectives by achieving asymptotic convergence of the estimates \( \dot{x}_c \) to the controller’s state \( x_c \) and the closed-loop system (4) preserves the inherent SG-AS property from Assumption 1.

This setup was studied in discrete-time for an LTI closed-loop system in [10] in the presence of Gaussian process and measurement noise where a time-varying Kalman filter is proposed as the optimal controller’s state estimator. Here, we do not consider the presence of noise, but leveraging noise to preserve the confidentiality of the controller will be an important future endeavour of this work.

The crucial assumption in [10] is the detectability of the closed-loop system. To the best of our knowledge, no results exist in the literature for when the closed-loop system (4) is not detectable according to Definition 1. Hence, the novelty of this work is in showing that controller confidentiality can be breached in the absence of closed-loop detectability. The rest of the paper is dedicated to this unexplored aspect.

V. CLOSED-LOOP SYSTEM (4) IS NOT DETECTABLE

When the closed-loop system (4) is NOT detectable, the stealthy estimation of the controller’s state \( x_c \) can be achieved through manipulating the sensor measurement \( y \) by way of the attack signal \( a \), modelled by (2). The compromised sensor \( y \) is used to probe the closed-loop system (4) periodically within the time interval \([kT, (k + 1)T]\), \( k \in \mathbb{N}_{\geq 0} \), for a short period of time \( t^* > 0 \) such that the controller’s state \( x_c \) can be estimated within some desired margin of error during the probing interval of \([kT, kT + t^*]\) and with bounded error for the remainder of the interval. The probing however, may lead to detection by the operator, and hence is only held sufficiently long, such that the closed-loop system (4) remains practically stable, i.e., stealth is maintained according to (6).

To this end, we require a modification of an observability notion first introduced in [14] where we need to apply an open-loop probing signal \( y^* \) for the closed-loop system (4) within a finite time interval such that its states can be estimated. Adopting the same terminology as in [14], such an observability notion is defined as follows:

Definition 2 (Robust observability): System (4), (2) is semiglobal q-robust observable (SGq-RO) for \( t \in [0, t^*] \), \( t^* > 0 \), if, for each \( \Delta_x \geq 0 \), there exist an integer \( q \in \mathbb{N}_{\geq 1} \), a C\(q+1\) function \( y^*: [0, t^*] \to \mathbb{R}^n_y \) and a function \( \Psi: \mathbb{R}^{2(q+1)n_y} 
\] with initial condition \( |x(0)| \leq \Delta_x \), the following is satisfied for \( t \in [0, t^*] \):
\[
\dot{x} = f(x, y^*) = \left( f_p(x_p, f_q(x_p, y^*)) \right), \quad y = h(x_p),
\]
• the solution \( x(t) \) to (11) exists,
• the function \( \Psi \) maps the measurement \( h(x_p) \) and the probing signal \( y^* \) as well as their derivatives to the solution \( x(t) \) as follows
\[
x(t) = \Psi(Y(t), Y^*(t)),
\]
where \( Y := (h(x_p), L_{f_p} h(x_p), \ldots, L_{f_p} h^{(q)}(x_p)) \) and \( Y^* := (y^*, y^*, \ldots, y^*(q)) \), where \( L_{f_p} h(q)(x_p) \) denotes the \( q \)-th time derivative of \( h(x_p) \),
• there exists \( \rho_\Psi \in \mathbb{K}_\infty \) such that
\[
\left| \Psi(\tilde{Y}, Y^*) - \Psi(Y, Y^*) \right| \leq \rho_\Psi \left| \tilde{Y} - Y \right|
\]

For examples of systems which are SGq-RO and on how to construct the probing signal \( y^* \), the reader is referred to the origin of this observability notion in [14]. A consequence of the SGq-RO property of system (4), (2) is that the sensor measurement \( h(x_p) \) needs to be sufficiently smooth. For the proposed adversarial scheme to work, we further require the following.

Assumption 3: Suppose that the closed-loop system (4) is SGq-RO. For a given \( t^* > 0 \), there exist compact sets \( H_q \) and \( H_{q+1} \) such that for all \( t \in [0, t^*] \),
\[
\left( L_{f_p} h(x_p(t)), \ldots, L_{f_p} h^{(q)}(x_p(t)) \right) \in H_q, \quad \text{and} \quad L_{f_p} h^{(q+1)}(x_p(t)) \in H_{q+1}.
\]
The proposed adversarial strategy which ensures that the adversary’s estimate of the controller state \( \hat{x}_c \) converges to a neighborhood of the controller’s state \( x_c \), and the closed-loop system (4) is semiglobal practical stable (maintaining stealth), takes the following form under the assumption that the closed-loop system (4) is \( \text{SGq-RO} \).

First, a probing duration \( t^* > 0 \) is chosen and then a suitable total duration \( T > t^* \) is selected. Then each time interval \([kT, (k + 1)T]\), for \( k \in \mathbb{N}_{\geq 0} \), is subdivided into a probing interval \( T_k := [kT, kT + t^*) \) and non-probing interval \( T_k := [kT + t^*, (k + 1)T) \). During the probing interval \( T_k \), the adversary probes the system for a short duration \( t^* > 0 \) by compromising the sensor measurements \( y \) via the attack signal \( a \). After which, the adversary stops probing to maintain stealth during \( T_k \) such that,

\[
y(t) = \begin{cases} y^*(t - kT), & t \in T_k, \\ h(x_p(t)), & t \in T_k, \end{cases}
\]

and the resulting closed-loop system is

\[
\dot{x}(t) = \begin{cases} f(x(t), y^*(t - kT)), & t \in T_k, \\ f(x(t), h(x_p(t))), & t \in T_k. \end{cases}
\]

By the probing procedure of (13), the controller’s state estimate \( \hat{x}_e \) is obtained via

\[
\dot{\hat{y}}(t) = \begin{cases} \hat{A} \hat{y}(t) + \theta \Delta \hat{H} \left( h(x_p(kT)) - \hat{C} \hat{y}(t) \right), & t \in T_k, \\ 0, & t \in T_k, \end{cases}
\]

where \( \hat{Y} \in \mathbb{R}^{(q+1)n_y}, \hat{C} = [I_{n_y}, 0_{a_q \times (q+1)n_y}] \), \( \hat{H} = [a_1 I_{n_y}, a_2 I_{n_y}, \ldots, a_q I_{n_y}]^T \) with \( a_i \) chosen such that the polynomial \( s^{q+1} + a_1 s^q + a_2 s^{q-1} + \cdots + a_{q+1} \) is Hurwitz, \( \Delta \hat{y} = \text{diag}(I_{n_y}, \theta I_{n_y}, \theta^2 I_{n_y}, \ldots, \theta^{q} I_{n_y}) \) with a constant tuning parameter \( \theta > 1 \), and \( \hat{A} = \begin{bmatrix} 0_{a_q \times n_y} & I_{n_y} \\ 0_{n_y \times n_y} & 0_{n_y \times a_q} \end{bmatrix} \). The initialisation of (15) is chosen to be

\[
\hat{y}(kT) \in \mathcal{Y}(h(x_p(kT))),
\]

where \( \mathcal{Y}(r) := \{ \hat{Y} \in \mathbb{R}^{(q+1)n_y} : |\hat{Y} - r| \leq \epsilon_y, \epsilon_y > 0 \} \).

The adversary can then obtain an estimate of the controller’s state as follows for all \( t \in T_k \cup T_k \),

\[
\hat{x}(t) = \begin{cases} \hat{x}_p(t), & t \in T_k, \\ \Psi \left( \hat{Y}(t), y^*(t - kT) \right), & t \in T_k, \end{cases}
\]

where \( q \) and \( \Psi \) come from the assumption that the closed-loop system is \( \text{SGq-RO} \) as defined in Definition 2. The probing duration \( t^* > 0 \) is dictated by the robustness of the closed-loop system (4) such that it is semiglobally practically stable (Objective 2: maintaining stealth).

VI. MAIN RESULT

The proposed probing scheme was inspired by the dual mode sampled-data output feedback control strategy in [14]. Consequently, elements of the proof of Theorem 1 follow that of [14] and [24] with some modifications as our resulting closed-loop system (4), (2) does not have a sample-and-hold input. The adversary’s estimate of the controller’s state does employ a sample-and-hold observer (15), (16), (17), but is not employed in the closed-loop system (4).

In the sequel, we will pave the way towards our main result (Theorem 1) in Section VI-C by addressing the fulfillment of Objective 1 and 2 in Sections VI-A and VI-B, respectively.

A. Objective 1: controller’s state estimation

Proposition 2: Given \( t^* > 0 \), consider the closed-loop system (14), (13) that is \( \text{SGq-RO} \) and satisfies Assumption 1 and 3, the adversary model under Assumption 2, the estimator (15), (16) and the controller’s state estimate (17). For all \( K_{\tilde{x}} > 0 \), there exist \( \theta \geq 1 \) and \( \sigma_{\tilde{x}} \in K_{\infty} \) such that for all \( k \in \mathbb{N}_{\geq 0} \) and \( t \in T_k \),

\[
|\hat{x}(kT + t^*) - x(kT + t^*)| \leq K_{\tilde{x}}, \quad |\hat{x}(t) - x(t)| \leq \sigma_{\tilde{x}}(\Delta e, k \theta^{q-1}),
\]

where \( \Delta e, k := \max\{|Y_a - Y_b|, Y_a, Y_b \in \mathcal{Y}(h(x_p(kT)))\} \). Further, suppose \( |x(kT + t^*)| \leq \Delta_{x} \). Then, there exists \( \sigma_{\tilde{x}} \in K_{\infty} \) such that

\[
|\hat{x}(t) - x(t)| \leq K_{\tilde{x}} + \Delta_{x} + \sigma_{\tilde{x}}(\Delta_{x}), \quad \forall t \in T_k,
\]

and for all \( \epsilon_{\tilde{x}} > 0 \), there exists \( T > 0 \) such that

\[
|\hat{x}(kT + 1)T - x((k + 1)T)| \leq K_{\tilde{x}} + \Delta_{x} + \epsilon_{\tilde{x}}.
\]
Remark 2: For subsequent non-probing intervals $T_k$, where $k \in \mathbb{N}_{\geq 0}$, the estimation error bounds in (23) and (24) become
\[ |\dot{x}(t) - x(t)| \leq K_{\dot{x}} + \Delta_x + \overline{\epsilon}_x \] \hspace{1cm} \forall t \in T_k, \tag{25}
and
\[ |\dot{x}(kT) - x(kT)| \leq K_{\dot{x}} + \Delta_x + \epsilon_x. \tag{26} \]
The periodic interval $T > 0$ is chosen a-priori based on the desirable margin of estimation error which can be made small up to $K_{\dot{x}} + \Delta_x$, which is the sum of the error margin at the end of the probing period and the size of the initial condition of the non-probed closed loop system (4), (2).

B. Objective 2: maintaining stealth

The analysis that allows the adversary to maintain stealth hinges on the fact that the uncompromised closed-loop system (4) is inherently semiglobal asymptotically stable (Assumption 1), which by application of a converse Lyapunov theorem (see [17, Theorem 4.14], for instance), there exists a $C^1$ closed-loop control Lyapunov function $V: \mathbb{R}^{n_t+n_e} \to \mathbb{R}_{\geq 0}$ such that there exist $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{K}_{\infty}$ where

\begin{align*}
(V1) & \quad \alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|), \\
(V2) & \quad (\nabla V(x), f(x, h(x_p))) \leq -\alpha_3(V(x)), \text{ for } x \in \mathcal{V}(R), \text{ with } R := \alpha_2(\Delta_x), \\
(V3) & \quad \text{where we define } \mathcal{V}(r, R) := \{x \in \mathbb{R}^{n_t+n_e} : r \leq V(x) \leq R\} \text{ and denote } \mathcal{V}(-\infty, R) \text{ by } \mathcal{V}(R).
\end{align*}

By straightforward application of [17, Lemma 4.4] and the comparison lemma [17, Lemma 3.4], a consequence of (C2) is stated below.

\begin{align*}
(C2') & \quad \text{there exist } \beta_V \in \mathcal{KL} \text{ such that } V(x(t)) \leq \beta_V(V(x(0)), t), \text{ for all } t \geq 0, \text{ for } x \in \mathcal{V}(R), \text{ with } R := \alpha_2(\Delta_x).
\end{align*}

We will use this closed-loop control Lyapunov function $V$ to show that the probed closed-loop system (14) is semiglobally practically stable. To do so, the given control Lyapunov function $V$ and the vector field $f$ of the probed closed-loop system (14) have to possess the following properties.

Assumption 4:

(V1) There exists $\rho \in \mathbb{K}_{\infty}$ such that $|V(x) - V(w)| \leq \rho(|x - w|)$ for all $x, w \in \mathcal{V}(R, R + R_m)$, for some $R_m > 0$.

(V2) There exists a constant $F^* > 0$ such that $|f(x, y^*)| \leq F^*$, for all $x \in \mathcal{V}(R + R_m)$.

(V3) There exists a constant $F > 0$ such that $|f(x, h(x_p))| \leq F$, for all $x \in \mathcal{V}(R + R_m)$.

We employ a key lemma in showing an $L_1$-type robustness with respect to additive disturbance for the first interval $[0, T]$. The results carry over to subsequent intervals $[kT, (k+1)T]$ which will be stated in a remark below. The proof of the lemma below is inspired by [24] and can be found in the Appendix.

Lemma 3: Consider the closed-loop system (4) under Assumption 1 and (V1) of Assumption 4. Given $T > 0$ and $0 < r < R$, consider
\[ \dot{x}(t) = f(x(t), h(x_p(t))) + d(t), \forall t \in [0, T], \tag{27} \]
and for all $x(0) \in \mathcal{V}(R)$. Let $\sigma \in [0, R - r)$. If $d(t)$ satisfies
\[ \max_{t \in [0, T]} \left| \int_0^T d(s)ds \right| \leq \rho^{-1}(\sigma)e^{-LT}, \tag{28} \]
where $\dot{L} := l_x + l_yh > 0$, where $l_x > 0$ and $l_y > 0$ are the Lipschitz constants of the function $f$ with respect to each of its arguments\(^1\), respectively, and $l_h > 0$ is the Lipschitz constant of the function $h$. Then the solution $x(t)$ to (27) exists and satisfies
\[ V(x(t)) \leq \beta_V(V(x(0), t) + \sigma, \forall t \in [0, T], \tag{29} \]
and for all $x(0) \in \mathcal{V}(R)$. \hfill \Box

Remark 3: The result of Lemma 3 is applicable to subsequent time intervals $[kT, (k+1)T]$, where (30) is replaced with
\[ V(x(t)) \leq \beta_V(V(x(kT), t - kT) + \sigma, \forall t \in [kT, (k+1)T], \tag{30} \]

Rewriting our probed closed-loop system (14) in perturbed form, we get the perturbed system (27) for $t \in T_k \cup T_k = [kT, (k+1)T]$, with
\[ d(t) = \begin{cases} f(x(t), y^*(t)) - f(x(t), h(x_p(t))), & t \in T_k, \\ 0, & t \in T_k. \end{cases} \tag{31} \]

To apply Lemma 3, we observe that
\[ \max_{t \in [kT, (k+1)T]} \int_{kT}^{(k+1)T} d(s)ds \leq \int_{kT}^{T + \sigma} f(x(s), y^*(s)) - f(x(s), h(x_p(s)))ds \leq (F^* + F)t^*, \tag{32} \]
where we got the last inequality using (V2) and (V3).

Therefore, with (33) in Proposition 3 below, the hypothesis of Lemma 3 is fulfilled. We can then apply Lemma 3 to prove the following proposition.

Proposition 3: Consider the probed closed-loop system (14) under Assumption 1, 2, 4. Given $T > 0$, suppose there exist $t^* < T$ and $r \in (0, R)$ satisfying
\[ r \leq \beta_V(R, t^*), \quad (F^* + F)t^* \leq \rho^{-1}(\sigma)e^{-LT}, \tag{33} \]
with $\sigma \in [0, R - r]$ and $\dot{L} > 0$ is as defined in Lemma 3. Then, the solution $x(t)$ to the probed closed-loop system (14) exists and satisfies the following for all $t \in [kT, (k+1)T], k \in \mathbb{N}_{\geq 0}$,
\[ V(x(t)) \leq \beta_V(V(x(kT)), t - kT + \sigma, V(x(kT + t^*)) \leq \sigma, \forall x(kT) \in \mathcal{V}(R). \tag{34} \]

C. Achieving Objective 1 and 2: stealthy estimation

We are now ready to state our main result.

Theorem 1: Suppose the closed-loop system (4) and (2) is SGqRO (Definition 2) and satisfies Assumptions 1, 3, and the adversarial model satisfies Assumption 2. Then, the adversary can employ the probing scheme of (13), (14),

\(^1\)Since the functions $f_p$, $f_c$ and $\kappa$ are all locally Lipschitz in their arguments, the function $f$ is also locally Lipschitz in its arguments.
(17), (15) initialised according to (16) to achieve stealthy estimation, if 
(i) Assumption 4 holds, and 
(ii) For any $\Delta x > 0$, $K_2 > 0$ and $\varepsilon_2 > 0$, there exist $t^* > 0$, $T > t^*$ and $r \in (0, R)$ satisfying (33) and Lemma 2 with $R := \alpha_1(\Delta_2)$, $\sigma_2(\varepsilon_2) \in [0, R-r)$, $\beta_V \in KL$ from (C2'), $\rho < K_{\infty}$, $F > 0$, $F^* > 0$ from Assumption 4.

Stealthy estimation is achieved in the sense that the objectives are fulfilled in the following manner:

- **Objective 1 (estimation of the controller’s state):** there exist $\theta > 1$, $\sigma_1, \sigma_2 \in K_{\infty}$ such that for all $t \in [kT, (k+1)T)$, $k \in \mathbb{N}_{\geq 0}$,
  $$|\hat{x}(t) - x(t)| \leq \max\{\sigma_2(\Delta_2)^{\theta-1}, K_2 + \Delta_2 + \sigma_2(\Delta_2)\},$$
  $$|\hat{x}(kT + t^*) - x(kT + t^*)| \leq K_2,$$
  $$|\hat{x}(kT) - x(kT)| \leq K_2 + \Delta_2 + \varepsilon_2,$$  
  (35)

  with $\Delta_2 := \max\{\hat{Y}_a - \bar{Y}_a : \hat{Y}_a, \bar{Y}_a \in \mathcal{C}(h(x_p(kT)))\}$.

- **Objective 2 (maintaining stealth):** there exists $K_2 = K_x(\Delta_2) > 0$ such that for all $t \in [kT, (k+1)T)$, $k \in \mathbb{N}_{\geq 0}$,
  $$|x(t)| \leq K_2.$$  
  (36)

**Proof:** Let $k \in \mathbb{N}_{\geq 0}$ and $t \in [kT, (k+1)T)$. We will employ Proposition 2 and 3. Since (33) and Lemma 2 hold, we first apply Proposition 3 to obtain (34). From the first inequality of (34) and using (C1), we obtain
  $$\alpha_1(|x(t)|) \leq \beta_V(R, 0) + R - r$$
  $$\Rightarrow |x(t)| \leq \alpha_1^{-1}(\beta_V(R, 0) + R).$$  
  (37)

Since $R := \alpha_3(\Delta_2)$, we achieve (36) with $K_2 := \alpha_1^{-1}(\beta_V(\alpha_1(\Delta_2), 0) + \alpha_1(\Delta_2))$.

Next, we see that condition requiring $|x(kT + t^*)| \leq \Delta_2$ in Proposition 2 is fulfilled with the second inequality of (34) using (C1). By applying Proposition 2, we obtain (35) as desired.

**VII. CONCLUSION AND FUTURE WORK**

We have shown that the confidentiality of the controller’s states can easily be breached stealthily when the closed-loop system is detectable. In this scenario, the adversary merely needs to gather measurement data from the sensors. In the absence of a detectable closed-loop system, but under a relaxed robust observability property, the adversary may employ a time-shared probing scheme by manipulating the sensor data to estimate the controller’s state within a desired margin of error during the probing period and with bounded error during the non-probing period. Additionally, stealth is maintained in the sense that the closed-loop system remain semiglobally practically stable. Future work includes devising defence mechanisms to obfuscate the adversary’s estimate of the controller’s state.

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