One-loop divergences in the Galileon model

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Abstract. The investigation of UV divergences is a relevant step in better understanding of a new theory. In this work the one-loop divergences in the free field sector are obtained for the popular Galileons model. The calculations are performed by the generalized Schwinger-DeWitt technique and also by means of Feynman diagrams. The first method can be directly generalized to curved space, but here we deal only with the flat-space limit. We show that the UV completion of the theory includes the $\pi\Box^2\pi$ term. According to our previous analysis in the case of quantum gravity, this means that the theory can be modified to become superrenormalizable, but then its physical spectrum includes two massive ghosts and one massive scalar with positive kinetic energy. The effective approach in this theory can be perfectly successful, exactly as in the higher derivative quantum gravity, and in this case the non-renormalization theorem for Galileons remains valid in the low-energy region.

MSC: 81T15, 81T18, 83D05

PACS: 11.10.Gh, 04.50.Kd

1 Introduction

The Galileons are qualitatively new models of scalar field with numerous applications. Originally Galileons were introduced in the context of Dvali-Gabadadze-Porrati model of gravity [1] and attracted a great deal of interest (see, e.g., [2, 3, 4] and further references therein). One of the standard motivations to consider Galileons is that they are assumed to possess some unusual renormalization properties [5, 6], especially when treated in the effective quantum field theory framework [7, 8, 9].

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In short, Galileons are second derivative theories with much more derivatives in the action. This means that the kinetic term of such a model has only two derivatives, while the interacting terms have much more derivatives. As a result, the tree-level propagator of the theory is free from higher derivative ghosts and, at the same time, the possible divergences have so many derivatives that the terms which are present in the classical action, never get renormalized \[5, 6\]. In this respect, the Galileon model is interesting to compare with the fourth derivative quantum gravity (HDQG) theory \[10\] (see also \[11\] for detailed introduction) which was further generalized in \[12\]. The common point between the two theories is the presence of higher derivatives. In case of HDQG this makes the theory renormalizable \[10\] or even superrenormalizable \[12\]. However, there is a price to pay: the particle spectrum of the theory includes higher derivative ghost \[10\] or ghosts \[12\]. At the same time, Galileon model is very much nonrenormalizable, in a sense that there are many possible divergences, however all of the possible logarithmically divergent terms have much more derivatives that the initial classical action, as a result the low-energy sector of the theory is free from strong UV quantum corrections.

The described scheme of constructing a theory which is not affected, in the UV limit, by quantum corrections, looks very attractive, but there is one important point to verify. If there are higher derivative divergences in the propagator sector, then the UV completion of the theory actually has massive ghost. Eliminating this ghost from the spectrum leads to the possible unitarity breaking, exactly as in the HDQG. And, exactly as in the HDQG, we can try to formulate the theory in such a way that the ghost is not generated at relatively low energies. So, the program of exploring Galileons at quantum level should be supplemented by direct calculation of the quantum contributions to the scalar field propagator. Such a calculation, at the one-loop level, is the subject of the present paper.

In general, the status of the quantum theory essentially depends on the structure of UV divergences in the free field sector. It may happen that the quantum theory is non-renormalizable, or it can be completed to become renormalizable or even superrenormalizable, like the HDQG model of \[12\]. In the last case the one-loop contribution may be the only one which is relevant. In what follows we shall figure out what is the situation, namely whether the Galileons theory is non-renormalizable, renormalizable or superrenormalizable and whether it can be unitary in a strong sense or only as an effective field theory.

The paper is organized as follows. In Sect. 2 we present a very brief description of the classical action of the model. In Sect. 3 the derivation of one-loop divergences is performed by means on the generalized Schwinger-DeWitt technique \[13\] with expansions originally developed in \[14\]. The main advantage of this technique is that it can be also applied to the similar derivation in curved space, however in the present Letter we restrict
our attention only by the flat-space limit, which is indeed sufficient to answer the questions formulated above. In Sect. 4, which was written for better control and for illustrative purposes, we show how the same result can be obtained by means of a more conventional Feynman diagrams technique. In Sect. 5 we present some additional discussions of the result and draw our conclusions.

2 Brief description of the model

In a four dimensional space-time there are only five Lagrangians with single scalar field \( \pi \), which are invariant (up to a total derivative) under the following transformation

\[
\pi \rightarrow \pi + b_\mu x^\mu + c,
\]

where \( c \) and \( b_\mu \) are constants. The transformation (1) is called Galilean transformation and the field \( \pi \) is called Galileon. These five Lagrangians can be represented by the following structures:

\[
\begin{align*}
L_1 &= \pi, \\
L_2 &= \frac{1}{2} \partial_\mu \pi \partial^\mu \pi, \\
L_3 &= \partial_\mu \pi \partial^\mu \pi \Box \pi, \\
L_4 &= \frac{1}{2} \partial_\mu \pi \partial^\mu \pi (\Box \pi)^2 - \partial_\mu \pi \partial^\mu \partial^\nu \pi \partial_\nu \pi \Box \pi - \frac{1}{2} \partial_\mu \partial_\nu \pi \partial^\mu \partial^\nu \pi \partial_\rho \pi \partial^\rho \pi \\
&\quad + \partial_\mu \pi \partial^\mu \partial^\nu \pi \partial_\nu \partial_\rho \partial_\nu \pi \partial^\rho \pi, \\
L_5 &= \frac{1}{6} \partial_\mu \pi \partial^\mu \pi \Box \pi (\Box \pi)^3 - \frac{1}{2} \partial_\mu \pi \partial^\mu \partial^\nu \pi \partial_\nu \pi \Box \pi - \frac{1}{2} \partial_\mu \partial_\nu \pi \partial^\mu \partial^\nu \pi \partial_\rho \pi \partial^\rho \pi \Box \pi \\
&\quad + \partial_\mu \pi \partial^\mu \partial^\nu \pi \partial_\nu \partial_\rho \pi \partial^\rho \pi \Box \pi + \frac{1}{3} \partial_\mu \partial^\nu \pi \partial_\nu \partial^\rho \pi \partial_\rho \pi \partial^\nu \pi \partial^\nu \pi \partial^\rho \pi \partial^\rho \pi, \\
&\quad + \frac{1}{2} \partial_\mu \partial_\nu \pi \partial^\mu \partial^\nu \pi \partial_\nu \partial^\rho \pi \partial_\rho \pi \partial^\rho \pi \partial^\nu \pi \partial^\nu \pi \partial^\rho \pi \partial^\rho \pi \partial^\rho \pi.
\end{align*}
\]

The full Lagrangian for the field \( \pi \) is a linear combination of the above Lagrangians

\[
L_\pi = \sum_{i=1}^{5} c_i L_i,
\]

where \( c_i \)'s are generic coefficients.

3 Calculation of the one-loop counterterms

In this section we shall present the details of the calculation of the one-loop counterterms of the theory (3). For the purpose of calculating the divergences we shall apply the
background field method (see, e.g., Chapter 2 of [11] for introduction) and the generalized Schwinger-DeWitt technique [13]. Let us start with the usual splitting of the fields into background and quantum part

$$\pi \rightarrow \pi' = \pi + \sigma.$$  \hfill (4)

The one-loop effective action is given by the expression

$$\Gamma^{(1)} = \frac{i}{2} \text{Tr} \ln \hat{H},$$  \hfill (5)

where $\hat{H}$ is the bilinear form of the action given by Lagrangian (3). Substituting (4) in (3) one can find the bilinear form of the action

$$S^{(2)} = \frac{-1}{2} \int d^4x \sigma \hat{H} \sigma,$$  \hfill (6)

where $\hat{H}$ has a form

$$\hat{H} = \Box + \hat{P}_1 + \hat{P}_2 + \cdots,$$  \hfill (7)

here $\hat{P}_1 \sim O(\pi)$, $\hat{P}_2 \sim O(\pi^2)$ and so on. Let us note that the scheme of calculation which we are going to perform is designed to get the correction to the propagator, therefore we do not need to take into account the terms beyond $\hat{P}_2$ and the square of the $\hat{P}_1$-term in this and further expansions.

In order to calculate the divergent part of the one-loop effective action (5), in the second order in $\pi$, one can perform the expansion, which is similar to the one which was previously used in [14] (described also in details in Chapter 8 of the book [11]),

$$\text{Tr} \ln \hat{H} = \text{Tr} \ln (\Box + \hat{P}_1 + \hat{P}_2 + \cdots) = \text{Tr} \ln \Box + \text{Tr} \ln \left(1 + \hat{P}_1 \frac{1}{\Box} + \hat{P}_2 \frac{1}{\Box} + \cdots\right) = \text{Tr} \ln \Box + \text{Tr} \left(\hat{P}_1 \frac{1}{\Box} + \hat{P}_2 \frac{1}{\Box} - \frac{1}{2} \hat{P}_1 \frac{1}{\Box} \hat{P}_1 \frac{1}{\Box}\right) + \cdots$$  \hfill (8)

The omitted terms are $O(\pi^3)$ and hence (as we have already mentioned above) they are actually irrelevant for our purposes. In order to reduce the amount of calculations we shall consider only the particular case of the flat background space-time. Then the only type of universal trace that does not vanish has the form

$$\text{Tr} \partial_{\mu_1} \cdots \partial_{\mu_{2n-4}} \frac{1}{\Box^n} \bigg|_{\text{div}} = -\frac{2i}{\epsilon} \int d^4x \frac{g^{(n-2)}_{\mu_1 \cdots \mu_{2n-4}}}{2^{n-2} (n-1)!},$$  \hfill (9)
with \( n \geq 2 \) and
\[
g_{\mu_1 \cdots \mu_{2n-4}}^{(n-2)} = g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} \cdots g_{\mu_{2n-3} \mu_{2n-4}} + \text{all permutations}.
\]

By using the formula (9) in (8) we can see that the trace of the \( \hat{P}_2(1/\Box) \)-term corresponds to \( n = 1 \) in Eq. (9) and hence it is finite in dimensional regularization. Therefore, only the last term of equation (8) gives contribution to the divergences. Then,
\[
\lim \ln \hat{H} \bigg|_{\text{div}} = -\frac{1}{2} \text{Tr} \hat{P}_1 \frac{1}{\Box} \hat{P}_1 \frac{1}{\Box} \bigg|_{\text{div}}.
\]
To calculate (11) we need an explicit form of \( \hat{P}_1 \), namely
\[
\hat{P}_1 = \hat{U}^{\mu \nu} \partial_\mu \partial_\nu, \quad \text{where} \quad \hat{U}^{\mu \nu} = 4c_3 \left[ (\Box \pi) g^{\mu \nu} - (\partial^\mu \partial^\nu \pi) \right].
\]

The commutations which enable one to reduce the problem to the universal trace (9) are performed as follows:
\[
\lim \text{Tr} \hat{P}_1 \frac{1}{\Box} \hat{P}_1 \frac{1}{\Box} \bigg|_{\text{div}} = \text{Tr} \hat{U}^{\mu \nu} \partial_\mu \partial_\nu \frac{1}{\Box} \bigg|_{\text{div}} = \text{Tr} \hat{U}^{\alpha \beta} \partial_\alpha \partial_\beta \left\{ \hat{U}^{\mu \nu} \partial_\mu \partial_\nu \frac{1}{\Box^2} + \left[ \frac{1}{\Box} , \hat{U}^{\mu \nu} \right] \partial_\mu \partial_\nu \frac{1}{\Box} \right\} \bigg|_{\text{div}}.
\]
Furthermore, the last commutator can be transformed as
\[
\left[ \frac{1}{\Box} , \hat{U}^{\mu \nu} \right] \partial_\mu \partial_\nu \frac{1}{\Box} = - (\Box \hat{U}^{\mu \nu}) \partial_\mu \partial_\nu \frac{1}{\Box^3} + (\Box^2 \hat{U}^{\mu \nu}) \partial_\mu \partial_\nu \frac{1}{\Box^4}
+ 4 (\partial^\lambda \Box \hat{U}^{\mu \nu}) \partial_\lambda \partial_\mu \partial_\nu \frac{1}{\Box^4} - 12 (\partial^\lambda \partial^\sigma \Box \hat{U}^{\mu \nu}) \partial_\lambda \partial_\mu \partial_\nu \frac{1}{\Box^4} - 2 (\partial^\lambda \hat{U}^{\mu \nu}) \partial_\lambda \partial_\mu \partial_\nu \frac{1}{\Box^4}
+ 4 (\partial^\lambda \partial^\sigma \hat{U}^{\mu \nu}) \partial_\lambda \partial_\mu \partial_\nu \partial_\rho \partial_\mu \partial_\nu \frac{1}{\Box^5} - 8 (\partial^\rho \partial^\sigma \partial^\lambda \hat{U}^{\mu \nu}) \partial_\rho \partial_\lambda \partial_\mu \partial_\nu \frac{1}{\Box^5}
+ 16 (\partial^\rho \partial^\sigma \partial^\lambda \partial^\tau \hat{U}^{\mu \nu}) \partial_\rho \partial_\lambda \partial_\mu \partial_\tau \partial_\mu \partial_\nu \frac{1}{\Box^6} + O \left( \frac{1}{\Box^7} \right).
\]
The terms with background dimension (we assume the reader is familiar with the terminology of [13]) of more than \( 1/l^4 \) can be safely omitted here because they do not contribute to divergences. Substituting the equation (11) into (13) we finally get, after some tedious calculations, the following expression:
\[
\lim \text{Tr} \hat{P}_1 \frac{1}{\Box} \hat{P}_1 \frac{1}{\Box} \bigg|_{\text{div}} = \text{Tr} \left\{ - \hat{U}^{\alpha \beta} (\partial_\alpha \partial_\beta \Box \hat{U}^{\mu \nu}) \partial_\mu \partial_\nu \frac{1}{\Box^3} + \hat{U}^{\alpha \beta} (\Box^2 \hat{U}^{\mu \nu}) \partial_\alpha \partial_\beta \partial_\mu \partial_\nu \frac{1}{\Box^4}
+ 8 \hat{U}^{\alpha \beta} (\partial_\alpha \partial^\lambda \Box \hat{U}^{\mu \nu}) \partial_\beta \partial_\lambda \partial_\mu \partial_\nu \frac{1}{\Box^4} - 12 \hat{U}^{\alpha \beta} (\partial^\lambda \partial^\sigma \Box \hat{U}^{\mu \nu}) \partial_\alpha \partial_\beta \partial_\lambda \partial_\mu \partial_\nu \frac{1}{\Box^4}
+ 4 \hat{U}^{\alpha \beta} (\partial_\alpha \partial_\beta \partial^\lambda \partial^\tau \hat{U}^{\mu \nu}) \partial_\lambda \partial_\tau \partial_\mu \partial_\nu \frac{1}{\Box^4} - 16 \hat{U}^{\alpha \beta} (\partial_\alpha \partial^\rho \partial^\lambda \partial^\sigma \hat{U}^{\mu \nu}) \partial_\beta \partial_\rho \partial_\lambda \partial_\sigma \partial_\mu \partial_\nu \frac{1}{\Box^5}
+ 16 \hat{U}^{\alpha \beta} (\partial^\rho \partial^\sigma \partial^\lambda \partial^\tau \hat{U}^{\mu \nu}) \partial_\alpha \partial_\beta \partial_\rho \partial_\lambda \partial_\tau \partial_\mu \partial_\nu \frac{1}{\Box^6} \right\} \bigg|_{\text{div}}.
\]
Now we are in position to use the universal trace \([9]\). In this way, after certain work, we obtain the following traces \([\text{here } \epsilon = (4\pi)^2 (n-4) \text{ in dimensional regularization}]\):

\[
- \quad \text{Tr} \left[ \hat{U}^\alpha \beta \left( \partial_\alpha \partial_\beta \Box \hat{U}^{\mu \nu} \right) \partial_\mu \partial_\nu \frac{1}{\Box^3} \right]_{\text{div}} = \frac{i}{2\epsilon} \int d^4 x \ \hat{U}^\alpha \beta \left( \partial_\alpha \partial_\beta \Box \hat{U}^{\mu}_{\mu} \right),
\]

\[
\text{Tr} \left[ \hat{U}^\alpha \beta (\Box^2 \hat{U}^{\mu \nu}) \partial_\alpha \partial_\beta \partial_\mu \partial_\nu \frac{1}{\Box^4} \right]_{\text{div}} = - \frac{i}{12\epsilon} \int d^4 x \ \left\{ 2 \hat{U}^{\mu \nu} (\Box^2 \hat{U}^{\mu \nu}) + \hat{U}^\alpha (\Box^2 \hat{U}^{\mu}_{\mu}) \right\},
\]

\[
8 \ \text{Tr} \left[ \hat{U}^\alpha \beta (\partial_\alpha \partial_\beta \Box \hat{U}^{\mu \nu}) \partial_\mu \partial_\nu \frac{1}{\Box^4} \right]_{\text{div}} = - \frac{2i}{3\epsilon} \int d^4 x \ \left\{ 2 \hat{U}^{\nu \alpha} (\partial_\alpha \partial_\nu \Box \hat{U}^{\mu \nu}) + \hat{U}^\alpha (\Box^2 \hat{U}^{\mu}_{\mu}) \right\},
\]

\[
4 \ \text{Tr} \left[ \hat{U}^\alpha \beta (\partial_\alpha \partial_\beta \partial_\mu \Box \hat{U}^{\mu \nu}) \partial_\nu \partial_\lambda \partial_\tau \partial_\nu \frac{1}{\Box^4} \right]_{\text{div}} = - \frac{i}{3\epsilon} \int d^4 x \ \left\{ 2 \hat{U}^{\nu \beta} (\partial_\nu \partial_\beta \partial_\mu \Box \hat{U}^{\mu \nu}) + \hat{U}^\alpha (\Box^2 \hat{U}^{\mu}_{\mu}) \right\},
\]

\[
- 12 \ \text{Tr} \left[ \hat{U}^\alpha \beta (\partial_\alpha \partial_\beta \Box \Box \hat{U}^{\mu \nu}) \partial_\nu \partial_\lambda \partial_\tau \partial_\nu \partial_\mu \partial_\nu \frac{1}{\Box^6} \right]_{\text{div}} = \frac{i}{4\epsilon} \int d^4 x \ \left\{ \hat{U}^{\nu \alpha} (\partial_\nu \partial_\mu \Box \hat{U}^{\mu}_{\mu}) + \frac{1}{2} \hat{U}^\alpha (\Box^2 \hat{U}^{\mu}_{\mu}) + 4 \hat{U}^\nu (\partial_\nu \Box \hat{U}^{\mu \nu}) + \hat{U}^{\nu \nu} (\Box^2 \hat{U}^{\mu \nu}) + \hat{U}^\nu (\Box \Box \hat{U}^{\mu \nu}) \right\},
\]

\[
- 16 \ \text{Tr} \left[ \hat{U}^\alpha \beta (\partial_\alpha \partial_\beta \partial_\mu \partial_\tau \Box \hat{U}^{\mu \nu}) \partial_\nu \partial_\lambda \partial_\nu \partial_\mu \partial_\nu \partial_\tau \frac{1}{\Box^6} \right]_{\text{div}} = \frac{i}{2\epsilon} \int d^4 x \ \left\{ \hat{U}^{\nu \beta} (\partial_\nu \partial_\mu \Box \hat{U}^{\mu \nu}) + 2 \hat{U}^\alpha (\Box^2 \hat{U}^{\mu}_{\mu}) + 2 \hat{U}^\alpha (\partial_\lambda \partial_\nu \Box \hat{U}^{\mu \nu}) \right\},
\]

and

\[
16 \ \text{Tr} \left[ \hat{U}^\alpha \beta (\partial_\mu \partial_\nu \partial_\alpha \partial_\beta \Box \hat{U}^{\mu \nu}) \partial_\nu \partial_\mu \partial_\alpha \partial_\beta \partial_\nu \partial_\tau \partial_\nu \partial_\mu \partial_\nu \partial_\tau \frac{1}{\Box^6} \right]_{\text{div}} = \frac{i}{5\epsilon} \int d^4 x \ \left\{ \frac{1}{4} \hat{U}^\alpha (\Box^2 \hat{U}^{\mu}_{\mu}) + \hat{U}^\alpha (\partial_\nu \partial_\nu \Box \hat{U}^{\mu \nu}) + \hat{U}^\alpha (\partial_\alpha \partial_\beta \Box \hat{U}^{\mu}_{\mu}) \right. \\
\left. + 2 \hat{U}^\mu (\partial_\alpha \partial_\beta \partial_\mu \partial_\nu \Box \hat{U}^{\mu \nu}) + 4 \hat{U}^{\nu \nu} (\partial_\alpha \partial_\mu \Box \hat{U}^{\mu \nu}) + \frac{1}{2} \hat{U}^{\nu \nu} (\Box^2 \hat{U}^{\mu \nu}) \right\}. \quad (15)
\]

By using the traces listed above, Eq. \([14]\) can be reduced to

\[
\begin{align*}
\text{Tr} \left[ \hat{P}_1 \hat{P}_1 \right] & = \frac{2i}{\epsilon} \int d^4 x \ \left\{ \frac{1}{40} \hat{U}^{\alpha \beta} (\partial_\alpha \partial_\beta \Box \hat{U}^{\mu}_{\mu}) - \frac{1}{120} \hat{U}^{\mu \nu} (\Box^2 \hat{U}^{\mu \nu}) \\
- \frac{1}{240} \hat{U}^{\alpha} (\Box^2 \hat{U}^{\mu}_{\mu}) - \frac{1}{30} \hat{U}^{\alpha \beta} (\partial_\alpha \partial_\beta \partial_\mu \partial_\nu \Box \hat{U}^{\mu \nu}) - \frac{1}{15} \hat{U}^{\nu \nu} (\partial_\alpha \partial_\nu \Box \hat{U}^{\mu \nu}) + \frac{1}{40} \hat{U}^{\alpha} (\partial_\mu \partial_\nu \Box \hat{U}^{\mu \nu}) \right\}.
\end{align*}
\]
Replacing, term by term, the explicit form of operator $\tilde{U}^{\mu\nu}$ given by Eq. (12) into Eq. (16), after some algebra we arrive at

$$\text{Tr} \left. \hat{P}_1 \frac{1}{\Box} \hat{P}_1 \frac{1}{\Box} \right|_{\text{div}} = -\frac{2i}{\varepsilon} \frac{c_3^2}{c_3^2} \int d^4x \, \pi \Box^4 \pi .$$

(17)

Finally, from this expression and the equations (5), (11) we obtain the result for the one-loop divergences of effective action in the free field sector,

$$\Gamma^{(1)}_{\text{div}} = -\frac{c_3^2}{2} \int d^4x \, \pi \Box^4 \pi .$$

(18)

The expression (18) is providing us some relevant information about the quantum properties of the Galileons theory. Let us present it in the systematic form.

- The UV logarithmic divergences of the theory require $\pi \Box^4 \pi$-type counterterms. This means that the consistency of the theory requires that the same term is included into the classical action of the theory. If we do not include such a term, it will emerge with infinite coefficient anyway and then no control over this term via the effective approach will be possible.

- According to the analysis of the similar gravitational theory in [12], the theory with an extra $\pi \Box^4 \pi$-type classical term has, typically, two massive ghosts (excitations with negative kinetic energy), one massive scalar degree of freedom and, of course, the “original” massless scalar mode. This situation means that the propagator of the field $\pi$ has the general structure

$$G(p) = \frac{1}{p^2} - \frac{A_1}{p^2 + m_1^2} + \frac{A_2}{p^2 + m_2^2} - \frac{A_3}{p^2 + m_3^2} ,$$

(19)

$$A_{1,2,3} > 0 , \quad m_3 > m_2 > m_1 > 0 .$$

Here we assumed Euclidean signature and that there are no tachyons in the spectrum. The last can be always provided by adjusting the coefficients of subleading $\pi \Box^2 \pi$-type and $\pi \Box^3 \pi$-type terms. Let us note that similar, curvature-dependent, terms are likely to be requested also by the divergences in curved space-time. On the other hand, all considerations presented below should be valid also in the presence of tachyons.

- According to the analysis of the similar gravitational theory in [12], the theory with an extra $\pi \Box^4 \pi$-type classical term is superrenormalizable, for any choice of the coefficients $c_k$ in the interacting sector of (2). This means, in our case, that the divergences can emerge only at the one-loop level. Starting from the second loop only one-loop sub-diagrams can be divergent. This feature does not depend on the flatness of space-time and will definitely hold also in curved space-time case (see [11] for introduction to the general theory of renormalization in curved space-time).
• The effective approach in the quantum Galileon theory is perfectly possible if we put a sufficiently small parameter in front of the classical $\int \pi \Box^4 \pi$-term in the action. Such parameter should have the form of $M^{-6}$ and, therefore, the choice of a small coefficient of the higher derivative term means we choose a huge mass parameter $M$. The masses of both massive ghosts $m_1, m_3$ and of the massive scalar with positive kinetic energy, $m_2$, will be of the same order of magnitude as $M$ (see [12] for details). If we consider classical or quantum phenomena at the energies much smaller than $M$, the massive modes of the scalar do not become active and the conclusions of [5, 6] and [7, 8] remain correct, including the non-renormalization theorem. In the effective framework the “correct” quantum result is supposed to be the one we have derived above, and not the one of the complete theory with the $\int \pi \Box^4 \pi$-term. Finally, this means that (1) is the low-energy symmetry, so the fact that it is violated by the one-loop divergence (18) is irrelevant in this framework.

4 One-loop divergences from Feynman diagrams

In the previous Section we calculated the one-loop logarithmic divergences by means of the generalized Schwinger-DeWitt technique. This approach has an advantage because it is relatively easy to generalize the results for a curved space-time. However, since in practise we are dealing only with flat space-time limit, here we perform an equivalent calculations by a more traditional Feynman diagrams-based calculation. The purpose of this section is to have an extra control of the result and also for the illustrative reasons. Since the theory looks unusual, this consideration may be instructive. Without going into full details, we will also provide a comparison between diagrams and universal traces, considered in the previous Section.

Let us consider the diagrams which contribute to the two-point function of the Galileon field at the one-loop level. The first set of diagrams which are all generated by the Lagrangian $\mathcal{L}_3$ is shown in Fig. 1.

![Figure 1](image)

**Figure 1.** The first set of diagrams coming from the $\mathcal{L}_3$-term vertex, which contribute to the propagator of the Galileon field. Here primes indicate derivatives.

We will be interested in the behavior in the large momentum regime. The integral
associated with the first diagram from Fig. 1 is

\[ \Pi_1(p) = c_3^2 \int \frac{d^4q}{(2\pi)^4} \frac{p^4 q_\mu q_\nu (q^\mu - p^\mu)(q^\nu - p^\nu)}{q^2(q - p)^2}. \]  

(20)

Expanding the denominator of equation (20), we found at large momentum scale the relation (in this equation we omit all tensorial indices and coefficients, and are only interested in power counting related to the divergences and powers of momentum \( p \))

\[ \Pi_1(p) \xrightarrow{q \to \infty} c_3^2 \int_0^\infty dq \left( \cdots + p^6 q + p^7 + \frac{p^8}{q} + \cdots \right). \]

(21)

One can easily see that this diagram contains ultraviolet logarithmic divergences and the power of \( p \) corresponding to this divergence is eight, which fits the \( \Box^4 \)-term obtained in the previous Section.

The others diagrams of our interest are the ones shown in Fig. 2

![Diagram 1](image1.png) (Figure 2). The second set of (tadpole-like) diagram provided by \( \mathcal{L}_3 \), which contribute to the two-point function.

and also the graphs shown in Fig. 3,

![Diagram 2](image2.png) (Figure 3). The diagrams generated by lagrangian \( \mathcal{L}_4 \) which may contribute for propagator of Galileon Model.

In fact, the diagrams from Figures 2 and 3 do not contribute to the divergences in the case of Galileon theory. The reason is that these diagrams include derivatives of the propagator in a single space-time point, and hence vanish.

Using this simple analysis of the diagrams one can explain qualitatively the contributions for the divergences for each term in the expansion for effective action \( (8) \). First let us consider the term \( \text{Tr} \hat{P}_1 \frac{\Box}{\Box} \). Since \( \hat{P}_1 \) is \( \mathcal{O}(\pi^3) \), it contains only contributions of the Lagrangian \( \mathcal{L}_3 \) and is proportional to \( c_3 \). Then, \( \text{Tr} \hat{P}_1 \frac{\Box}{\Box} \) is also proportional to \( c_3 \) and we can see that this term does not contribute to divergences because it corresponds to
Feynman diagrams which are proportional to $c_3$ and are zero after we use Wick’s theorem, since $\mathcal{L}_3$ has an odd number of fields.

The next term is $\text{Tr} \, \hat{\mathcal{P}}_2 \frac{1}{\gamma}$. Remember that $\hat{\mathcal{P}}_2$ is $\mathcal{O}(\pi^4)$ and is proportional to $c_4$, then the contribution of this term are given by tadpoles diagrams of Fig. 3. As we have mentioned above, this diagrams do not contribute, hence we can see why this term makes no contribution to the counterterms. Finally, the last trace is $\text{Tr} \, \hat{\mathcal{P}}_1 \frac{1}{\gamma} \hat{\mathcal{P}}_1 \frac{1}{\gamma}$. This term is proportional to $c_3^2$, its contribution to the logarithmic divergences is different from zero and is given by diagrams of Fig. 1.

The considerations presented above enable us to write the expression for the divergent part of the two-point function as presented in Fig. 4.

$$G^{(1-\text{loop})}_{\text{div}}(x, y) = \frac{1}{2} (\alpha_1 \times \begin{array}{c} x \end{array} \begin{array}{c} y \\ y \end{array} + \alpha_2 \times \begin{array}{c} x \end{array} \begin{array}{c} y \\ y \end{array} + \alpha_3 \times \begin{array}{c} x \end{array} \begin{array}{c} y \\ y \end{array} + \alpha_4 \times \begin{array}{c} x \end{array} \begin{array}{c} y \\ y \end{array}) \bigg|_{\text{div}} = x \begin{array}{c} y \\ y \end{array}$$

**Figure 4.** Diagrammatic representation of the Green function. $\alpha_{1,2,3,4}$ are combinatorial coefficients.

The definition of the full polarization operator is given in Fig. 5.

$$\begin{array}{c} \bullet \\ \bullet \end{array} = \Pi(p) = \sum_{i=1}^{4} \frac{1}{2} \alpha_i \Pi_i(p)$$

**Figure 5.** Full polarization operator.

with

$$\Pi_1(p) = c_3^2 \int \frac{d^4q}{(2\pi)^4} \frac{p^\mu q_\nu (q^\mu - p^\mu)(q^\nu - p^\nu)}{q^2(q-p)^2},$$

$$\Pi_2(p) = c_3^2 \int \frac{d^4q}{(2\pi)^4} \frac{p^2 q^2 q_\mu p_\nu (q^\mu - p^\mu)(q^\nu - p^\nu)}{q^2(q-p)^2},$$

$$\Pi_3(p) = c_3^2 \int \frac{d^4q}{(2\pi)^4} \frac{p_\mu p_\nu q^2 (p^\nu - q^\nu)(p_\alpha - q_\alpha)(p^\alpha - q^\alpha)}{q^2(q-p)^2}$$

and

$$\Pi_4(p) = c_3^2 \int \frac{d^4q}{(2\pi)^4} \frac{p_\mu p_\nu q^4(p^\mu - q^\mu)(p^\nu - q^\nu)}{q^2(q-p)^2}.$$
To evaluate these integrals we used dimensional regularization, reformulating the theory in the space-time of $2\omega$ complex dimensions, where the integrals are convergent. Using the formulas of the Appendix, we found the result

\begin{align}
\Pi_1(p, \omega) &= \frac{i c_3^2}{4} p^8 I_1, \\
\Pi_i(p, \omega) &= 0, \quad i = 2, 3, 4.
\end{align}

The integral $I_1$ is defined in the Appendix. To find the divergent part of the polarization operator we consider the limit $\omega \to 2$. The divergent part is given by the pole of $\Gamma(2 - \omega)$ in $I_1$. We find

\begin{equation}
I_1\bigg|_{\text{div}} = -\frac{2}{\epsilon},
\end{equation}

where $\epsilon = (4\pi)^2(n - 4)$. Taking into account the combinatorial coefficient $\alpha_1 = 4$, we arrive at

\begin{equation}
\Pi_{\text{div}}(p, \omega) = \frac{i c_3^2}{2} p^8 I_1^{\text{div}} = -\frac{i c_3^2}{\epsilon} p^8.
\end{equation}

The last part is to calculate the divergent part of Effective Action in the coordinate representation. For this end one can use the Dyson’s formula

\begin{equation}
G^{-1} = G_0^{-1} - \Pi(p).
\end{equation}

Then the divergent part of Effective Action can be written as

\begin{equation}
\frac{\delta^2 \Gamma^{(1)}_{\text{div}}}{\delta \pi \delta \pi} = -i \Pi_{\text{div}}(\Box) = -\frac{c_3^2}{\epsilon} \Box^4.
\end{equation}

The equation (30) can be easily solved and we arrive at

\begin{equation}
\Gamma^{(1)}_{\text{div}} = -\frac{c_3^2}{2\epsilon} \int d^4x \pi \Box^4 \pi,
\end{equation}

that is exactly the result obtained in previous section.

5 Concluding discussions

We have developed the background field method and calculated the one-loop divergences for the Galileon model. It turns out that the UV completion of the theory includes higher derivative sectors, as it was indeed anticipated in [5, 6]. An interesting new aspect is that this UV completion leads to the superrenormalizable quantum theory, where only
the one-loop contribution to the effective action is divergent and everything beyond the one-loop order is finite.

There is an interesting similarity between the quantum Galileons model with this higher derivative completion and the higher derivative quantum gravity (HDQG). In fact, the unique conceptual difference is that the Galileon model with an extra $\int \pi \Box^4 \pi$-term is strongly superrenormalizable, while HDQG admits different levels of renormalizable and superrenormalizable theories. At the same time, the status of ghosts in these two theories is very close. In case of HDQG the Planck mass $M_P$ plays the role of the massive parameter $M$ which was discussed at the end of the section 3. In both cases one can provide the absence of ghosts at the tree level for sufficiently low energies.

One simple test of the last statement has been applied recently in the cosmological framework [18]. It was shown that the physically relevant cosmological solutions in the higher derivative gravity theory (even with complicated semiclassical corrections) are stable with respect to graviton perturbations (gravitational waves). Definitely, this output is expected to hold only until we do not start to deal with the perturbations with the initial amplitude of the Planck order of magnitude. However, after the Universe passed through its initial Planck-scale epoch, such violent perturbations are never generated, and therefore the theory is safe at the classical level. Probably, this should mean that the quantum theory is also free of the ghost problem at the tree level. Of course, this is not an obvious statement, because it is not really clear how the most relevant classical solutions (such as cosmological one, for example) of the gravitational theory can be reproduced via the linearized gravity approach. At the same time, the stability of the cosmological solution [18] is definitely more fundamental issue than our skills in linearizing gravity, so we can definitely say that we have a strong positive arguments in favor of higher derivative theories.

Let us, finally, discuss some practical lessons which we can learn from the analogy with the HDQG case. As far as the main applications of Galileons is related to cosmology, it would be definitely interesting to consider the stability of the classical cosmological solutions in the presence of higher derivative terms which are a necessary UV completion of the theory. For this end one has to complete the $\int \pi \Box^4 \pi$-term derived here by the corresponding curvature-dependent terms. Regardless of most of the relevant information for such a cosmological application can be perfectly well obtained from power counting arguments, it would be anyway reasonable to generalize our calculation of the one-loop divergences to the curved space case.

---

3There are many other interesting proposals towards the solution of the ghost issue in HDQG [19, 20, 21, 22, 23] which can be also productive. In any case it is important to care about higher derivative terms in gravity, since they are requested by consistency of the quantum theory of matter fields [11, 24].
6 An extra note on more general scalar-tensor theories

It is possible, in principle, to generalize the results considered above to the more complicated and general scalar-tensor model with second-order field equations. Such a theory has been recently considered in [17] (see further references therein). The action of this general model does not satisfy the symmetry (1) and this opens the way for infinitely many new terms in the Lagrangian. The form of the curved-space Lagrangian is

\[ \mathcal{L} = \sum_{i=2}^{5} \mathcal{L}_i, \]

\[ \mathcal{L}_2 = K(\phi, X), \quad \text{where} \quad X = -\frac{1}{2} \partial_{\mu}\phi \partial^{\mu}\phi, \]

\[ \mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \]

\[ \mathcal{L}_4 = G_4(\phi, X)R + G_{4, X} [(\Box\phi)^2 - (\nabla_{\mu}\nabla_{\nu}\phi)(\nabla^{\mu}\nabla^{\nu}\phi)] \]

\[ \mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu} - \frac{1}{6} G_{5, X} [(\Box\phi)^2 - 3(\Box\phi)(\nabla_{\mu}\nabla_{\nu}\phi)(\nabla^{\mu}\nabla^{\nu}\phi) + 2(\Box\phi)(\nabla^{\mu}\nabla_{\alpha}\phi)(\nabla^{\alpha}\nabla_{\beta}\phi)(\nabla^{\beta}\nabla_{\mu}\phi)], \]

where \( G_k(\phi, X) \), with \( k = 3, 4, 5 \), are arbitrary functions and \( G_{k, X}(\phi, X) \) are the corresponding derivatives with respect to \( X \).

The question in which we are interested in is whether and how the result (18) gets modified in the more general model (32). In order to address this issue, it is sufficient to perform the analysis of the power counting. Hence, our consideration of this model will be brief, so that we leave the details of the performed analysis as exercise for an interested reader and give only the main result.

As far as we are interested only in the counterterms which contribute to the propagator of \( \phi \) and do not intend to quantize metric, the modifications which come from the curvature-dependent terms in (32) are irrelevant. Furthermore, according to our previous analysis, only the contributions to the vertices with three legs are significant, while the vertices with four and more legs play no role. Taking these two observations into account, one can easily see that the result strongly depends on the presence of a constant term in the function \( G_{5, X} \).

Let us assume that \( G_5(\phi, X) \) can be expanded as

\[ G_5(\phi, X) = G_{50}(\phi) + G_{51}(\phi)X + G_{52}(\phi)X^2 + \ldots, \]

where

\[ G_{51}(\phi)X = G_{510} + G_{511}\phi + G_{512}\phi^2 + G_{513}\phi^3 + \ldots. \]  

It is easy to see that if the coefficient \( G_{510} \) in (33) is zero, then the result (18) does not change (except the coefficient, of course). However, in case \( G_{510} \neq 0 \) the consideration
of the superficial degree of divergences of the relevant diagram shows that the leading counterterm will be different from \((18)\). In this case we can expect the counterterms of the form

\[
\Gamma_{\text{div}}^{(1)} \sim \int d^4 x \, \phi (\Box^6 + \ldots) \phi ,
\]

where the dots indicate the presence of possible terms with lower powers of \(\Box\). The higher order in derivatives in \((34)\) compared to \((18)\) is because the \(G_{510}\)-term in \((32)\) leads to the vertices with three legs and five derivatives, which are not present in the Galileon case \((2)\). Qualitatively, the consequences of the higher derivative terms remain the same as it was discussed above. The theory with the corresponding UV completion would be superrenormalizable, and possesses a (larger, in this case) set of ghosts and massive scalar states with positive kinetic energy.

**Appendix. Massless integrals**

To calculate the integrals of Feynman diagrams from Sect. 4, we need the divergent parts of some massless integrals, which are given below. The basic formulas \((35)-(39)\) can be found in \([15]\) and other integrals can be obtained by the method explained in \([16]\). All integrals are defined over Euclidian space and must be understood though the prescription for massless case explained in \([15]\).

\[
\int \frac{d^{2\omega} q}{(2\pi)^{2\omega} q^2 (q-p)^2} = I_1 ,
\]

\[
\int \frac{d^{2\omega} q q\mu}{(2\pi)^{2\omega} q^2 (q-p)^2} = p\mu I_1 ,
\]

\[
\int \frac{d^{2\omega} q q\mu q\nu}{(2\pi)^{2\omega} q^2 (q-p)^2} = \delta_{\mu\nu} I_3 + p\mu p\nu I_4 ,
\]

\[
\int \frac{d^{2\omega} q q\mu q\nu q\alpha}{(2\pi)^{2\omega} q^2 (q-p)^2} = p\mu p\alpha I_5 + E_{\mu\nu\alpha} I_6 ,
\]

\[
\int \frac{d^{2\omega} q q\mu q\nu q\alpha q\beta}{(2\pi)^{2\omega} q^2 (q-p)^2} = p\mu p\alpha p\beta I_7 + G_{\mu\nu\alpha\beta} I_8 + H_{\mu\nu\alpha\beta} I_9 ,
\]

\[
\int \frac{d^{2\omega} q q\mu q\nu q\alpha q\beta q\rho}{(2\pi)^{2\omega} q^2 (q-p)^2} = p\mu p\nu p\alpha p\beta p\rho I_{10} + K_{\mu\nu\alpha\beta\rho} I_{11} + L_{\mu\nu\alpha\beta\rho} I_{12} ,
\]
\[
\int \frac{d^{2\omega} q q_{\mu} q_{\nu} q_{\alpha} q_{\beta} q_{\rho} q_{\omega}}{(2\pi)^{2\omega} q^2 (q - p)^2} = p_{\mu} p_{\nu} p_{\alpha} p_{\beta} p_{\rho} p_{\omega} I_{13} + R_{\mu \nu \alpha \beta \rho \omega} I_{14} + S_{\mu \nu \alpha \beta \rho \omega} I_{15} \\
+ T_{\mu \nu \alpha \beta \rho \omega} I_{16},
\]

where

\[
E_{\mu \nu \alpha} = \delta_{\mu \nu} p_{\alpha} + \text{all permutations},
\]

\[
G_{\mu \nu \alpha \beta} = \delta_{\mu \nu} p_{\alpha} p_{\beta} + \text{all permutations},
\]

\[
H_{\mu \nu \alpha \beta} = \delta_{\mu \nu} \delta_{\alpha \beta} + \text{all permutations},
\]

\[
K_{\mu \nu \alpha \beta \rho} = \delta_{\mu \nu} p_{\alpha} p_{\beta} p_{\rho} + \text{all permutations},
\]

\[
L_{\mu \nu \alpha \beta \rho} = \delta_{\mu \nu} \delta_{\alpha \beta} p_{\rho} + \text{all permutations},
\]

\[
R_{\mu \nu \alpha \beta \rho \omega} = \delta_{\mu \nu} p_{\alpha} p_{\beta} p_{\rho} p_{\omega} + \text{all permutations},
\]

\[
S_{\mu \nu \alpha \beta \rho \omega} = \delta_{\mu \nu} \delta_{\alpha \beta} p_{\rho} p_{\omega} + \text{all permutations},
\]

\[
T_{\mu \nu \alpha \beta \rho \omega} = \delta_{\mu \nu} \delta_{\alpha \beta} \delta_{\rho \omega} + \text{all permutations}.
\]

The integrals \( I_2, \ldots, I_{16} \) can be expressed in terms of the basic integral \( I_1 \),

\[
I_1 \equiv \frac{1}{(4\pi)^{\omega} \Gamma(2-\omega) \Gamma(\omega-1) \Gamma(\omega-1) p^{2(\omega-2)}}
\]

and are given by the expressions

\[
I_2 = \frac{1}{2} I_1,
\]

\[
I_3 = \frac{-p^2}{4(2\omega - 1)} I_1,
\]

\[
I_4 = \frac{\omega}{2(2\omega - 1)} I_1,
\]

\[
I_5 = \frac{(\omega + 1)}{4(2\omega - 1)} I_1.
\]
\[ I_6 = \frac{-p^2}{8(2\omega - 1)} I_1, \quad (55) \]

\[ I_7 = \frac{(\omega + 1)(\omega + 2)}{4(4\omega^2 - 1)} I_1, \quad (56) \]

\[ I_8 = \frac{-(\omega + 1)p^2}{8(4\omega^2 - 1)} I_1, \quad (57) \]

\[ I_9 = \frac{p^4}{16(4\omega^2 - 1)} I_1, \quad (58) \]

\[ I_{10} = \frac{(\omega + 3)(\omega + 2)}{8(4\omega^2 - 1)} I_1, \quad (59) \]

\[ I_{11} = \frac{-(\omega + 2)p^2}{16(4\omega^2 - 1)} I_1, \quad (60) \]

\[ I_{12} = \frac{p^4}{32(4\omega^2 - 1)} I_1, \quad (61) \]

\[ I_{13} = \frac{(\omega + 4)(\omega + 3)(\omega + 2)}{8(2\omega + 3)(4\omega^2 - 1)} I_1, \quad (62) \]

\[ I_{14} = \frac{-(\omega + 3)(\omega + 2)p^2}{16(2\omega + 3)(4\omega^2 - 1)} I_1, \quad (63) \]

\[ I_{15} = \frac{(\omega + 2)p^4}{32(2\omega + 3)(4\omega^2 - 1)} I_1, \quad (64) \]

\[ I_{16} = \frac{-p^6}{64(2\omega + 3)(4\omega^2 - 1)} I_1. \quad (65) \]

**Acknowledgments**

T.P.N. thanks FAPEMIG for supporting his Ms. project. I.Sh. is grateful to CNPq, CAPES, FAPEMIG and ICTP for partial support of his work.
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