Beyond mean-field description of Gamow-Teller resonances and $\beta$-decay

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Abstract. $\beta$-decay half-lives set the time scale of the rapid neutron capture process, and are therefore essential for understanding the origin of heavy elements in the universe. The random-phase approximation (RPA) based on Skyrme energy density functionals is widely used to calculate the properties of Gamow-Teller (GT) transitions, which play a dominant role in $\beta$-decay half-lives. However, the RPA model has its limitations in reproducing the resonance width and often overestimates $\beta$-decay half-lives. To overcome these problems, effects beyond mean-field can be included on top of the RPA model. In particular, this can be obtained by taking into account the particle-vibration coupling (PVC). Within the RPA+PVC model, we successfully reproduce the experimental GT resonance width and $\beta$-decay half-lives in magic nuclei. We then extend the formalism to superfluid nuclei and apply it to the GT resonance in $^{120}$Sn, obtaining a good reproduction of the experimental strength distribution. The effect of isoscalar pairing is also discussed.
1. Introduction

The Gamow-Teller (GT) resonance is a prominent collective mode with spin-isospin character [1]. In the GT excitation, the orbital angular momentum of the nucleus is not changed, but the spin and isospin are changed by one unit. GT transitions can be studied in the laboratory by \((p,n)\) [2] or \((^3\text{He},t)\) [3] reactions. The low-energy GT transitions lying below the \(Q_\beta\) value also represent the dominant transitions in \(\beta\)-decay. \(\beta\)-decay plays an important role in the rapid neutron capture process (\(r\)-process), which is responsible for the production of heavy elements in the universe.

Therefore, much efforts have been made to investigate \(\beta\)-decay half-lives of neutron-rich nuclei. Experimentally, the study of half-lives of neutron-rich nuclei has made much progress due to the development of radioactive beam facilities. Theoretically, self-consistent QRPA model based on Skyrme [4] or relativistic density functionals [5, 6] have been extensively applied in the investigation of \(\beta\)-decay half-lives.

However, the accurate description of \(\beta\)-decay half-lives remains a hard problem. The QRPA model tends to overestimate the half-lives [4, 6]. This problem could be cured to a good extent by including the isoscalar proton-neutron pairing with a free pairing strength parameter. By adjusting this parameter, the half-lives could be reproduced for some open-shell nuclei. However, this does not help at all for closed shell nuclei, like \(^{78}\text{Ni}\) and \(^{132}\text{Sn}\) [4, 6–8]. Therefore, there must exist other correlations, other than proton-neutron pairing, able to reduce the half-lives. For instance, half-lives can be reduced to some extent by including the effects of the tensor force [9]. However, also the inclusion of tensor terms requires more parameters in the Hamiltonian.

It is well known that the RPA model only considers 1-particle 1-hole (1p-1h) configurations in the model space, so it is interesting to check whether the calculation of lifetime can be improved by including correlations beyond the RPA configuration space (1p-1h states), and considering more complicated states of 2p-2h character. This is also necessary for the description of the experimental spreading width. A direct approach of this kind is represented by the second RPA [10, 11]. However, the associated configuration space, which includes the 2p-2h states, is too large for practical calculations in heavy nuclei. We will instead consider the effects of the particle-vibration coupling (PVC), including 1p-1h states coupled with collective phonons [12]. The self-consistent RPA+PVC model has been developed based on both relativistic and Skyrme density functionals [13–16].

In this paper, we will first present the RPA+PVC model based on Skyrme density functional and its successful application to the GT resonance and to \(\beta\)-decay half-lives in magic nuclei. Then we report on recent progress concerning the extension of RPA+PVC model to the quasiparticle case, i.e., the QRPA + QPVC model, and present some results of GT resonance in superfluid nuclei, including also the effects of isoscalar pairing.

2. RPA+PVC model and its applications

In this section, we will first introduce briefly the formulas of RPA+PVC model, and then we will present the results of GT resonance in \(^{208}\text{Pb}\) and \(\beta\)-decay in closed shell nuclei.

2.1. Formulas of RPA+PVC model

The RPA+PVC equation reads

\[
\left( \mathcal{D} + \mathcal{A}_1(\omega) \right) \begin{pmatrix} \mathcal{A}_2(\omega) \\ -\mathcal{A}_3(\omega) \end{pmatrix} \begin{pmatrix} F^{(\nu)} \\ F^{(\nu)} \end{pmatrix} = \left( \Omega_\nu - \frac{\Gamma_\nu}{2} \right) \begin{pmatrix} F^{(\nu)} \\ F^{(\nu)} \end{pmatrix},
\]

We write the RPA+PVC equation in the basis of RPA eigenstates, so \(\mathcal{D}\) is a diagonal matrix containing the positive RPA eigenvalues. The \(\mathcal{A}_i\) matrices are associated with the coupling to
the doorway states. The expressions of \( A_i \) is also written in the RPA basis \(| n \rangle \),
\[
(A_1)_{mn} = \sum_{p, p', h'} W^1_{p p' h'}(\omega) X^{(m)}_{p h} X^{(n)}_{p' h'} + W^1_{p p' h'}(-\omega) Y^{(m)}_{p h} Y^{(n)}_{p' h'},
\]
(2)
\[
(A_2)_{mn} = \sum_{p, p', h'} W^2_{p p' h'}(\omega) X^{(m)}_{p h} Y^{(n)}_{p' h'} + W^2_{p p' h'}(-\omega) Y^{(m)}_{p h} X^{(n)}_{p' h'},
\]
(3)
\[
(A_3)_{mn} = \sum_{p, p', h'} W^3_{p p' h'}(\omega) Y^{(m)}_{p h} X^{(n)}_{p' h'} + W^3_{p p' h'}(-\omega) X^{(m)}_{p h} Y^{(n)}_{p' h'},
\]
(4)
\[
(A_4)_{mn} = \sum_{p, p', h'} W^4_{p p' h'}(\omega) Y^{(m)}_{p h} Y^{(n)}_{p' h'} + W^4_{p p' h'}(-\omega) X^{(m)}_{p h} X^{(n)}_{p' h'},
\]
(5)
where \( W^\dagger \) reads
\[
W^\dagger_{p p' h'}(\omega) = \sum_N \frac{\langle p|V|N \rangle \langle N|V|p' h' \rangle}{\omega - \omega_N}.
\]
(6)
The matrix elements \( W^\dagger_{p p' h'} \) are given by the sum of the four following terms,
\[
W^\dagger_{1p p' h'} = \delta_{hh'}\delta_{jj'}\sum_{p', nL} \frac{1}{\omega - (\omega_{nL} + \epsilon_{p'} - \epsilon_h) + i\Delta} \langle p|V||p' nL \rangle \langle p'|V||p'' nL \rangle,
\]
\[
W^\dagger_{2p p' h'} = \delta_{pp'}\delta_{jj''}\sum_{h', nL} \frac{1}{\omega - (\omega_{nL} - \epsilon_{h'} + \epsilon_p) + i\Delta} \langle h''|V||h, nL \rangle \langle h''|V||h' nL \rangle,
\]
\[
W^\dagger_{3p p' h'} = \sum_{nL} \frac{(-)^{j_p - j_h + J + L}}{\omega - (\omega_{nL} + \epsilon_p - \epsilon_h) + i\Delta} \left\{ \frac{J_p}{J_h} \frac{J_h}{J_p'} \right\} \langle p|V||p nL \rangle \langle h'|V||h nL \rangle,
\]
\[
W^\dagger_{4p p' h'} = \sum_{nL} \frac{(-)^{j_p - j_h + J + L}}{\omega - (\omega_{nL} + \epsilon_p - \epsilon_h) + i\Delta} \left\{ \frac{J_p}{J_h} \frac{J_h}{J_p'} \right\} \langle p|V||p' nL \rangle \langle h|V||h' nL \rangle.
\]
(7)
In the above formulas, \( p \) and \( h \) label particle and hole states, respectively. The corresponding angular momentum and single-particle energies are given respectively by \( j_p, j_h \) and \( \epsilon_p, \epsilon_h \). \( \omega_{nL} \) is a shorthand notation for \( 2j_p + 1 \), while \( \omega_{nL} \) denotes the energy of the phonon state \(| n L \rangle \). The averaging parameter \( \Delta \) is introduced to avoid singularities in the denominator of Eq. (7).

The GT strength associated with RPA+PVC, is given by
\[
S(\omega) = -\frac{1}{\pi} \text{Im} \sum_{\nu} \langle 0|\hat{O}_{GT \pm} |\nu \rangle^2 \frac{1}{\omega - \Omega_{\nu} + i(\frac{\Gamma_{\nu}}{2} + \Delta)},
\]
(8)
where the GT operator is \( \hat{O}_{GT \pm} = \sum_{i} \sigma(i) r_{\pm}(i) \). In our calculation, we will only focus on the GT\(^-\) excitations. More details can be found in Ref. [15, 16].

### 2.2. GT resonance and \( \beta \)-decay in magic nuclei

Fig. 1 shows the Gamow-Teller strength distributions for \(^{208}\text{Pb}\) calculated by RPA and RPA+PVC model with the interaction SkM*, and compared with experimental data [17]. The RPA model could only give discrete peaks, and overestimates the centroid energy of GT resonance. With the inclusion of PVC, the peak energy of GTR is shifted downwards by around 1 MeV, in better agreement with experimental data. At the same time, a spreading width is also developed, and the RPA+PVC result reproduces the experimental lineshape quite well.

We further check if RPA+PVC model could help in reducing the \( \beta \)-decay half-lives. So we present the \( \beta \)-decay half-lives of \(^{132}\text{Sn}\), \(^{68}\text{Ni}\), \(^{34}\text{Si}\), and \(^{78}\text{Ni}\), calculated by RPA and RPA+PVC models.
approaches with the interaction SkM*, respectively, in comparison with experimental values [18]. The arrow denotes an infinite half-life. The RPA results overestimate the half-lives systematically. In particular, the RPA model predicts an infinite half-life for $^{132}$Sn. With the inclusion of PVC, the half-lives get reduced systematically, and become in reasonable agreement with experimental data. For more results and discussions on half-lives, one can refer to Ref. [19].

In summary, the RPA+PVC model with interaction SkM* gives a good description both of the spreading width and of $\beta$-decay half-lives in magic nuclei.

3. QRPA+QPVC model and its applications
In this section, we will first introduce briefly the formulas of QRPA+QPVC model, and then we will present the results of GT resonance in the superfluid nucleus $^{120}$Sn.

3.1. Formulas of QRPA+QPVC model
In the case of QRPA+QPVC model, the QRPA+QPVC equation and the $A$ matrices are the same as Eq. (1) and Eq. (2) in the RPA+PVC model, except that the particle-hole configuration $ab$ is replaced by two-quasiparticle configuration $ab$. For nuclei not far from the stability line, the BCS quasi-particle states represent a convenient and accurate approximation to the HFB states. So we obtain the following expression for the spreading matrix elements.

\[
W_{lab,a'b'}^{ij} = \delta_{bb'}\delta_{ja,j'd} \frac{1}{2} \sum_{a'',nL} \frac{\langle a|V|a'',nL\rangle\langle a''|V|a',nL\rangle}{E - [\omega_{nL} + E_{a'} + E_b \pm (\lambda_n - \lambda_p)] + i\Delta},
\]

\[
W_{lab,a'b'}^{ij} = \delta_{aa''}\delta_{j'a',j''} \frac{1}{2} \sum_{b'',nL} \frac{\langle b|V|b'',nL\rangle\langle b''|V|b',nL\rangle}{E - [\omega_{nL} + E_{b'} + E_a \pm (\lambda_n - \lambda_p)] + i\Delta},
\]

\[
W_{3lab,a'b'}^{ij} = (-)^{j_a+j_b} \frac{1}{2} \sum_{j'''} \frac{\langle a'|V|a',nL\rangle\langle b|V|b',nL\rangle}{E - [\omega_{nL} + E_{a'} + E_{b'} \pm (\lambda_n - \lambda_p)] + i\Delta},
\]

\[
W_{4lab,a'b'}^{ij} = (-)^{j_a+j_b} \frac{1}{2} \sum_{j'''} \frac{\langle a'|V|a',nL\rangle\langle b|V|b',nL\rangle}{E - [\omega_{nL} + E_{a'} + E_{b'} \pm (\lambda_n - \lambda_p)] + i\Delta}.
\]
where $E_a$ is the BCS quasi-particle energy, and $\lambda_{n(p)}$ is the chemical potential for neutron (proton). The reduced matrix element has the following form

$$\langle a||V||a''', nL \rangle = \frac{\hat{L}}{\sqrt{1 + \delta_{cd}}} \sum_{cd} \tilde{V}(cdLa''', a) X_{cd}^{nL} + (-1)^{j_a - j_a'''} + L \tilde{V}(cdLa''', a''') Y_{cd}^{nL},$$

where

$$\tilde{V}(cdLa''', a) = V_{a'''}_{c'}_{u'}_{d'}^{Lpp}(u_a u_{a'''} v_{c'} v_{d'} - v_a v_{a'''} u_{c'} u_{d'} + (-1)^{j_e - j_d + L} V_{a'''}_{u'}_{d'}_{c'}^{Lpp}(u_a u_{a'''} v_{c'} v_{d'} - v_a v_{a'''} u_{c'} u_{d'}),$$

and

$$\tilde{V}(cdLa'; a''') = V_{a'''_{c'}_{u'}_{d'}^{Lpp}}(u_a u_{a'''} v_{c'} v_{d'} - v_a v_{a'''} u_{c'} u_{d'}) + V_{a'''}_{c'}_{u'}_{d'}^{Lpp}(u_a u_{a'''} v_{c'} v_{d'} - v_a v_{a'''} u_{c'} u_{d'}) (-1)^{j_e - j_d + L}$$

and

$$\tilde{V}(cdLa; a''') = V_{a'''}_{c'}_{u'}_{d'}^{Lpp}(u_a u_{a'''} v_{c'} v_{d'} - v_a v_{a'''} u_{c'} u_{d'}).$$

The above matrix elements $V$ are calculated in the canonical basis. $v_a$ is the square root of the occupation probability for the state $a$ in the canonical basis, and $u_a = \sqrt{1 - v_a^2}$. The ph and pp interaction will take the same form as that used for non-charge-exchange QRPA calculation. For more details, see Ref. [20].

3.2. GT resonance in $^{120}$Sn

In the QRPA calculation, besides the isovector $T = 1$ pairing both in the ground state and in the residual interaction, the isoscalar $T = 0$ pairing is also included in the residual interaction in the QRPA calculation. We adopt a density-dependent, zero-range surface pairing force in both $T = 1$ and $T = 0$ channel. We multiply the pairing strength of $T = 0$ channel by a constant factor $f$, and compare the results obtained with the same strength used for $T = 1$ pairing ($f = 1$) or neglecting $T = 0$ pairing ($f = 0$).

![Figure 3. GT strength distributions of $^{120}$Sn calculated by QRPA and QRPA+QPVC model using the interaction SkM* with isoscalar pairing ($f = 1$) [panel (a)] and without isoscalar pairing ($f = 0$) [panel (b)], and compared with experimental data [21].](image-url)

We take $^{120}$Sn as an example, and plot the GT strength distributions calculated by QRPA and QRPA+QPVC model using the interaction SkM*. The calculated results with and without isoscalar pairing are shown in panel (a) and (b) respectively. The experimental strength distribution is estimated from the cross section of (p,n) reaction in Ref. [21]. In this work, besides the cross section $\sigma_0^{(0^+)}$, the unit cross section $\sigma = 2.78 \pm 0.16$ mb/sr was also determined. We can
obtain an approximate value for the B(GT) strength, using the relation $\sigma(0^o) = \hat{\sigma}_F(q, \omega) B(GT)$, and assuming the factor $F(q, \omega)$, that gives the dependence on momentum and energy transfer of cross section, to be constant and equal to 1. Comparing panel (a) and (b), we can see that with the inclusion of the attractive isoscalar pairing the strength in the low-energy region increases; and the strength is redistributed in favour of the lower ones of the two peaks in GR region in the QRPA level. For the QRPA+QPVC results, the profile of the strength function in the giant resonance region is similar in the $f = 0$ and $f = 1$ cases, although the strength of the peaks in the low-energy region is increased and the strength of the highest peak is decreased with the inclusion of isoscalar pairing. With the inclusion of QPVC effect, the spreading width and lineshape of the giant resonance region are very well reproduced. However, the low-lying strength is still overestimated, even neglecting isoscalar pairing.

4. Conclusions
We have reported recent progress made on the study of GT resonance and $\beta$-decay within a theoretical framework beyond mean-field. Firstly, the self-consistent RPA+PVC model based on Skyrme density functional has been presented and applied to the study of GT strength distribution and $\beta$-decay half-lives in magic nuclei. The model reproduces well the spreading width of GTR in $^{208}$Pb with the Skyrme interaction SkM*. $\beta$-decay half-lives for $^{132}$Sn, $^{68,78}$Ni and $^{34}$Si are also well described using the same Skyrme interaction. Then we have presented the extension of the model to the calculation of superfluid nuclei, i.e., the self-consistent QRPA+QPVC model, and its application to the GT strength distribution in $^{120}$Sn. The QPVC also produces a large spreading width and the model reproduces the experimental data of GTR region, using the same Skyrme interaction SkM*. The effect of isoscalar pairing has also been discussed.

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