Spontaneous R-parity breaking, Left-Right Symmetry and Consistent Cosmology with Transitory Domain Walls

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Abstract

Domain wall formation is quite generic in spontaneous Left-Right parity (D-parity) breaking models. Since they are in conflict with cosmology, we need some mechanisms to remove them. Planck scale suppressed effects have been considered to be quite successful for this purpose. We study this possibility in Minimal Supersymmetric Left-Right (SUSYLR) model originally proposed by Kuchimanchi et al [1] where both D-parity and R-parity \( R_p = (-1)^{3(B-L)+2s} \) are spontaneously broken. We find that Planck scale suppressed terms allowed for the specific particle content of this model can successfully remove the domain walls provided the D-parity breaking scale is relatively low \( (\leq 10^5 - 10^7 \text{GeV}) \). However, demanding this theory to be part of a grand unified theory such as \( SO(10) \) forces the D-parity breaking scale to be very high \( (\geq 10^{14} \text{GeV}) \) and hence is in conflict with the constraint from domain wall removal. We also find another class of R-parity violating SUSYLR models where both these constraints can be simultaneously satisfied.

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I. INTRODUCTION

Left-Right Symmetric Models (LRSM) provide a framework within which spontaneous parity breaking as well as tiny neutrino masses can be successfully implemented without reference to very high scale physics such as grand unification. Incorporating Supersymmetry (SUSY) into it comes with other advantages like providing a solution to the gauge hierarchy problem, and providing a Cold Dark Matter candidate which is the lightest supersymmetric particle (LSP). In Minimal Supersymmetric Standard Model (MSSM), the stability of LSP is guaranteed by R-parity, defined as $R_p = (-1)^{3(B-L)+2S}$ where $S$ is the spin of the particle. This is a discrete symmetry put by hand in MSSM to keep the baryon number (B) and lepton number (L) violating terms away from the superpotential. In generic implementations of Left-Right symmetry, R-parity is a part of the gauge symmetry and hence not ad-hoc like in the MSSM. In one class of models, spontaneous parity breaking is achieved without breaking R-parity. This was not possible in minimal supersymmetric left right (SUSYLR) models where the only way to break parity is to consider spontaneous R-parity violation. In minimal SUSYLR model parity, $SU(2)_R$ gauge symmetry as well as R-parity break simultaneously by the vacuum expectation value of right handed sneutrino.

Spontaneous breaking of exact discrete symmetries like parity (which we shall denote as D-parity hereafter), as well as R-parity have got cosmological implications since they lead to frustrated phase transitions leaving behind a network of domain walls (DW). These domain walls, if not removed will be in conflict with the observed Universe. It was pointed out that Planck scale suppressed non-renormalizable operators can be a source of domain wall instability. Supersymmetry dictates the structure of these non-renormalizable terms but also gives rise to the gravitino overabundance problem. Incorporating all these restrictions, the constraint on the D-parity breaking scale in R-parity conserving SUSYLR models has been discussed. Here we extend the analysis to a more general class of models where both R-parity and D-parity break spontaneously. It should be mentioned that the formation of domain walls is not generic in all Left-Right models. Models where D-parity and $SU(2)_R$ gauge symmetry are broken at two different stages do not suffer from this problem. In these models, the vacuum expectation value (vev) of a parity odd singlet field breaks the D-parity first and $SU(2)_R$ gauge symmetry gets broken at a later
stage by either Higgs triplets and Higgs doublets.

This paper is organized as follows. In section II we briefly review the domain wall dynamics. In section III we discuss minimal SUSYLR model with Higgs triplets, constraints on the symmetry breaking scale from successful removal of domain wall as well as gauge coupling unification and in section IV we do this same analysis for minimal SUSYLR model with Higgs doublets. We summarize our results in section V.

II. DOMAIN WALL DYNAMICS

Discrete symmetries and their spontaneous breaking are both common instances and desirable in model building. The spontaneous breaking of such discrete symmetries gives rise to a network of domain walls leaving the accompanying phase transition frustrated \cite{15, 16}. The danger of a frustrated phase transition can therefore be evaded if a small explicit breaking of discrete symmetry can be introduced.

Due to the smallness of such discrete symmetry breaking, the resulting domain walls may be relatively long lived and can dominate the Universe for a long time. Since this will be in conflict with the observed Universe, these domain walls need to disappear at a very high energy scale (at least before Big Bang Nucleosynthesis). Keeping this in mind, we summarize the three cases of domain wall dynamics discussed in \cite{19}, one of which originates in radiation dominated (RD) Universe and destabilized also within the radiation dominated Universe. This scenario was originally proposed by Kibble \cite{15} and Vilenkin \cite{26}. The second scenario was essentially proposed in \cite{27}, which consists of the walls originating in a radiation dominated phase, subsequent to which the Universe enters a matter dominated (MD) phase, either due to substantial production of heavy unwanted relics such as moduli, or simply due to a coherent oscillating scalar field. The third one is a variant of the MD model in which the domain walls dominate the Universe for a considerable epoch giving rise to a mild inflationary behavior or weak inflation (WI) \cite{28, 29}. In all these cases the domain walls disappear before they come to dominate the energy density of the Universe.

When a scalar field $\phi$ acquires a vev at a scale $M_R$ at some critical temperature $T_c$, a phase transition occurs leading to the formation of domain walls. The energy density trapped per unit area of such a wall is $\sigma \sim M_R^3$. The dynamics of the walls are determined by two quantities, force due to tension $f_T \sim \sigma/R$ and force due to friction $f_F \sim \beta T^4$ where
$R$ is the average scale of radius of curvature prevailing in the wall complex, $\beta$ is the speed at which the domain wall is navigating through the medium and $T$ is the temperature. The epoch at which these two forces balance each other sets the time scale $t_R \sim R/\beta$. Putting all these together leads to the scaling law for the growth of the scale $R(t)$:

$$R(t) \approx (G\sigma)^{1/2}t^{3/2}$$  \hspace{1cm} (1)

The energy density of the domain walls goes as $\rho_W \sim (\sigma R^2/R^3) \sim (\sigma/Gt^3)^{1/2}$. In a radiation dominated era this $\rho_W$ is comparable to the energy density of the Universe [$\rho \sim 1/(Gt^2)$] around time $t_0 \sim 1/(G\sigma)$.

The pressure difference arising from small asymmetry on the two sides of the wall competes with the two forces $f_F \sim 1/(Gt^2)$ and $f_T \sim (\sigma/(Gt^3))^{1/2}$ discussed above. For $\delta \rho$ to exceed either of these two quantities before $t_0 \sim 1/(G\sigma)$

$$\delta \rho \geq G\sigma^2 \approx \frac{M_R^6}{M_{Pl}^2} \sim M_R^4 \left(\frac{M_R}{M_{Pl}}\right)^2$$  \hspace{1cm} (2)

Similar analysis in the matter dominated era, originally considered in [27] begins with the assumption that the initially formed wall complex in a phase transition is expected to rapidly relax to a few walls per horizon volume at an epoch characterized by Hubble parameter value $H_i$. Thus the initial energy density of the wall complex is $\rho_W^{in} \sim \sigma H_i$. This epoch onward the energy density of the Universe is assumed to be dominated by heavy relics or an oscillating modulus field and in both the cases the scale factor grows as $a(t) \propto t^{2/3}$. The energy density scales as $\rho_{mod} \sim \rho_{mod}^{in}/(a(t))^3$. If the domain wall (DW) complex remains frustrated, i.e. its energy density contribution $\rho_{DW} \propto 1/a(t)$, the Hubble parameter at the epoch of equality of DW contribution with that of the rest of the matter is given by 

$$H_{eq} \sim \sigma^{3/4}H_i^{1/4}M_{Pl}^{-3/2}$$  \hspace{1cm} (3)

Assuming that the domain walls start decaying as soon as they dominate the energy density of the Universe, which corresponds to a temperature $T_D$ such that $H_{eq}^2 \sim GT_D^4$, the above equation gives

$$T_D^4 \sim \sigma^{3/2}H_i^{1/2}M_{Pl}^{-1}$$  \hspace{1cm} (4)

Under the assumption that the domain walls are formed at $T \sim \sigma^{1/3}$

$$H_i^2 = \frac{8\pi}{3} G\sigma^{4/3} \sim \frac{\sigma^{4/3}}{M_{Pl}^2}$$  \hspace{1cm} (5)
Now from Eq. (4)

\[ T^4_D \sim \frac{\sigma^{11/6}}{M_{Pl}^{3/2}} \sim \frac{M_R^{11/2}}{M_{Pl}^{3/2}} \sim M_R^4 \left( \frac{M_R}{M_{Pl}} \right)^{3/2} \]  

(6)

Demanding \( \delta \rho > T^4_D \) leads to

\[ \delta \rho > M_R^4 \left( \frac{M_R}{M_{Pl}} \right)^{3/2} \]  

(7)

The third possibility is the walls dominating the energy density of the Universe for a limited epoch which leads to a mild inflation. This possibility was considered in \[28, 29\]. As discussed in \[19\], the evolution of energy density of such walls can be expressed as

\[ \rho_{DW}(t_d) \sim \rho_{DW}(t_{eq}) \left( \frac{a_{eq}}{a_d} \right) \]  

(8)

where \( a_{eq}(a_d) \) is the scale factor at which domain walls start dominating (decaying) and \( t_{eq}(t_d) \) is the corresponding time. If the epoch of domain wall decay is characterized by temperature \( T_D \), then \( \rho_{DW} \sim T^4_D \) and the above equation gives

\[ T^4_D = \rho_{DW}(t_{eq}) \left( \frac{a_{eq}}{a_d} \right) \]  

(9)

In the matter dominated era the energy density of the moduli fields scale as

\[ \rho^d_{\text{mod}} \sim \rho^d_{\text{mod}} \left( \frac{a_{eq}}{a_d} \right)^3 \]  

(10)

Using this in equation (9) gives

\[ \rho^d_{\text{mod}} \sim \frac{T^4_D}{H^4_{eq}} \]  

(11)

Domain walls start dominating the Universe after the time of equality, \( \rho_{DW}(t_d) > \rho^d_{\text{mod}} \). So the pressure difference across the walls when they start decaying is given by

\[ \delta \rho \geq \frac{T^4_D G^2}{H^4_{eq}} \]  

(12)

where \( H^2_{eq} \sim G \rho_{DW}(t_{eq}) \). Replacing the value of \( H_{eq} \) from equation (3), the pressure difference becomes

\[ \delta \rho \geq M^4_R \frac{T^4_D M^3_{Pl}}{M^4_R} \]  

(13)

Unlike the previous two cases RD and MD, here it will not be possible to estimate \( T_D \) in terms of other mass scales and we will keep it as undetermined.
III. MINIMAL SUPERSYMMETRIC LEFT-RIGHT MODEL (MSLRM) WITH HIGGS TRIPLETS

We consider the minimal SUSYLR model of Kuchimanchi et al \cite{1} in this section. Although the minimality of the Higgs content is an attractive feature of this model, the authors concluded that D-parity can be spontaneously broken only at the expense of breaking R-parity spontaneously at the same energy scale by the vev of right handed sneutrino. Since the R-parity violation (RPV) is in the leptonic sector only, the dangerous proton decay problem can be evaded in this model. The models where left handed sneutrino vev gives rise to RPV are strongly disfavored by electroweak precision measurements \cite{30, 31}. However there is no such strict constraints on models where right handed sneutrino vev gives rise to RPV provided the extra gauge boson masses lie above the allowed lower bounds \cite{32, 33}. Here we find another constraint on this RPV scale from domain wall removal as well as gauge coupling unification.

The matter content of this model is

\[
Q = \begin{pmatrix} u \\ d \end{pmatrix} \sim (3, 2, 1, \frac{1}{3}), \quad Q_c = \begin{pmatrix} d_c \\ u_c \end{pmatrix} \sim (3^*, 1, 2, -\frac{1}{3}),
\]

\[
L = \begin{pmatrix} \nu \\ e \end{pmatrix} \sim (1, 2, 1, -1), \quad L_c = \begin{pmatrix} \nu_c \\ e_c \end{pmatrix} \sim (1, 1, 2, 1)
\]

The Higgs sector of this minimal consists of the Higgs bidoublets and Higgs triplets

\[
\Phi_1 = \begin{pmatrix} \phi_{11}^0 & \phi_{11}^+ \\ \phi_{12}^- & \phi_{12}^0 \end{pmatrix} \sim (1, 2, 2, 0), \quad \Phi_2 = \begin{pmatrix} \phi_{21}^0 & \phi_{21}^+ \\ \phi_{22}^- & \phi_{22}^0 \end{pmatrix} \sim (1, 2, 2, 0),
\]

\[
\Delta = \begin{pmatrix} \delta_L^+ / \sqrt{2} & \delta_L^{++} \\ \delta_L^0 & -\delta_L^+ / \sqrt{2} \end{pmatrix} \sim (1, 3, 1, 2), \quad \bar{\Delta} = \begin{pmatrix} \Delta_L^- \sqrt{2} & \Delta_L^0 \\ \Delta_L^- & -\Delta_L^- / \sqrt{2} \end{pmatrix} \sim (1, 3, 1, -2),
\]

\[
\Delta_c = \begin{pmatrix} \delta_R^+ / \sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+ / \sqrt{2} \end{pmatrix} \sim (1, 1, 3, -2), \quad \bar{\Delta}_c = \begin{pmatrix} \Delta_R^- / \sqrt{2} & \Delta_R^0 \\ \Delta_R^- & -\Delta_R^- / \sqrt{2} \end{pmatrix} \sim (1, 1, 3, 2)
\]

The renormalizable superpotential is

\[
W_{ren} = h^{lij}_i L^T \tau_2 \Phi_i \tau_2 L_c + h^{l}_{ij} Q^T \tau_2 \Phi_i \tau_2 Q_c + if L^T \tau_2 \Delta L + if L_c^T \tau_2 \Delta_c L_c
\]  

\[+ m_\Delta \text{Tr} \Delta \bar{\Delta} + m_\Delta \text{Tr} \Delta_c \bar{\Delta}_c + \mu_{ij} \text{Tr} \tau_2 \Phi_i^T \tau_2 \Phi_j \]
where \( h^{(i)}_{q,l} = h^{(i)\dagger}_{q,l}, \mu_{ij} = \mu_{ji}^{*} \) and \( f, h \) are symmetric matrices. It has been shown that with this minimal field content it is not possible to break the D-parity spontaneously. Adding a parity odd singlet also does not improve the situation. The authors showed that the D-parity breaking vacua in this case also give rise to the breaking of electromagnetic charge.

The authors [1] proposed an alternative scenario where it was shown that by allowing a non-zero vev for right handed sneutrino, \( \tilde{\nu}_c \) it is possible to get D-parity breaking minima which preserve electromagnetic charge. However, the vev of sneutrino which has odd \( U(1)_{B-L} \) charge also gives rise to spontaneous R-parity violation. Here we follow the approximations adopted by the authors to find a region in parameter space of the coupling constants giving rise to the desired minima. The first approximation is the one where they choose the parameter space, such that \( g^2 \) and \( g'^2 \) are smaller than the constants \( h^2 \) and \( f^2 \). With this approximation the \( D \)-terms become weaker than the trilinear terms that contain the triplet scalars and the sleptons. The second approximation is made in order to maintain the hierarchy between the electroweak scale and the parity breaking scale, i.e. \( \langle L_c \rangle, \langle \Delta_c \rangle \gg \langle \Phi \rangle \). With these approximations the scalar potential can now be written as,

\[
V = m^2_t (\tilde{L}^c \tilde{L} + \tilde{L}^c \tilde{L}_c) + M^2_2 Tr (\Delta \Delta^\dagger + \Delta_c \Delta_c^\dagger) + M^2_2 Tr (\bar{\Delta} \bar{\Delta}^\dagger + \bar{\Delta}_c \bar{\Delta}_c^\dagger) + |h|^2 \tilde{L}^c \tilde{L} \tilde{L} \tilde{L}
+ |f|^2 [ (\tilde{L}^c \tilde{L})^2 + (\tilde{L}^c \bar{L}_c)^2 ] + 4 |f|^2 [ (\tilde{L}^c \tau_2 \Delta^c)^2 + (\bar{L}^c \tau_2 \Delta_c)^2 ] + M'^2 Tr (\Delta \bar{\Delta} + \Delta_c \bar{\Delta}_c + h.c.)
+ [\tilde{L}^c \tau_2 (iv \Delta + iM^* f \bar{\Delta}^c) \tilde{L} + \tilde{L}^c \tau_2 (iv \Delta_c + iM^* F \bar{\Delta}_c) \tilde{L}_c + h.c.] \tag{16}
\]

Consider the case where the Right handed fields getting a non-zero vev, at the same time the vev for the Left handed fields is zero. So,

\[
\langle L \rangle = 0, \quad \langle \Delta \rangle = \langle \bar{\Delta} \rangle = 0
\]

\[
\langle L_c \rangle = \begin{pmatrix} l_c \\ 0 \end{pmatrix}, \quad \langle \Delta_c \rangle = \begin{pmatrix} 0 & 0 \\ d_c & 0 \end{pmatrix}, \quad \langle \bar{\Delta}_c \rangle = \begin{pmatrix} 0 & d_c \\ 0 & 0 \end{pmatrix}
\]

Now putting these vev’s in the scalar potential Eq.(16) gives rise to

\[
V = m^2_t (l_c^2) + M^2_2 (\bar{d}_c^2) + f^2 (l_c^4) + 4 f^2 l_c^2 d_c^2 + v l_c^2 d_c^2 + f M l_c^2 \bar{d}_c + c.c \tag{17}
\]

Minimising the above potential the authors get the solution for the parity breaking minima
as
\[ d_c = -\frac{v}{4f^2}, \quad \bar{d}_c = \frac{f M_R^2}{M_2^2} \]
\[ l_c^2 = \frac{(v^2 - 4f^2 m^2) M_2^2}{8f^4(M_2^2 - M_2^2)} \] (18)

Since the original theory is Left-Right symmetric there exists an equivalent minima corresponding to \( d_c \rightarrow d, \bar{d}_c \rightarrow \bar{d}, l_c \rightarrow l \). The degeneracy of these two equivalent vacua leads to the unavoidable consequences of formation of domain walls. For a successful phase transition accompanying the symmetry breaking, these domain walls should be unstable. We follow the idea\[17, 18\] where it is argued that Planck scale suppressed non-renormalizable operators can potentially solve the domain wall problem.

A. Constraints on \( M_R \) from domain wall removal

We adopt the technique developed in\[19\] to find the operators suppressed by Planck scale. And we find the constrains on the symmetry breaking scale from cosmological considerations.

We now find the \( 1/M_{Pl} \) terms in the effective potential by expanding the Kähler potential and superpotential in powers of \( 1/M_{Pl} \). We include the terms containing \( \Delta(\bar{\Delta}), \Delta_c(\bar{\Delta}_c), L(L_c) \) in the expansion. The Kähler potential in this model upto \( 1/M_{Pl} \) is
\[ K = \text{Tr}[\Delta \Delta^\dagger + \bar{\Delta} \bar{\Delta}^\dagger] + \text{Tr}[\Delta_c \Delta_c^\dagger + \bar{\Delta}_c \bar{\Delta}_c^\dagger] + \frac{c_L}{M_{Pl}} (L^T \tau_2 \Delta L + L^T \tau_2 \bar{\Delta} L + h.c.) \]
\[ + \frac{c_R}{M_{Pl}} (L_c^T \tau_2 \Delta_c L_c + L_c^T \tau_2 \bar{\Delta}_c L_c + h.c.) \] (19)

The superpotential upto the powers of \( 1/M_{Pl} \) is
\[ W = W_{ren} + \frac{a_L}{2 M_{Pl}} (\text{Tr}[\Delta \bar{\Delta}])^2 + \frac{a_R}{2 M_{Pl}} (\text{Tr}[\Delta_c \bar{\Delta}_c])^2 + \frac{b_L}{M_{Pl}} \text{Tr}[\Delta^2] \text{Tr}[\bar{\Delta}^2] + \frac{b_R}{M_{Pl}} \text{Tr}[\Delta_c^2] \text{Tr}[\bar{\Delta}_c^2] \]
\[ + \frac{f_1}{M_{Pl}} (\text{Tr}[\Delta \bar{\Delta}]) (\text{Tr}[\Delta_c \bar{\Delta}_c]) + \frac{f_2}{M_{Pl}} \text{Tr}[\Delta^2] \text{Tr}[\bar{\Delta}^2] + \frac{f_3}{M_{Pl}} \text{Tr}[\Delta_c^2] \text{Tr}[\bar{\Delta}_c^2] + \frac{f_4}{M_{Pl}} (L^T)^2 (L_c^T L_c) \]

Assuming a phase where only right type fields get non-zero vev and left type fields get zero vev, the scalar potential upto the leading term in \( 1/M_{Pl} \) becomes,
\[ V_{eff}^R \sim \frac{2fa_R}{M_{Pl}} l_c^2 d_c^2 + \frac{2a_R m_c}{M_{Pl}} \bar{d}_c^2 d_c \] (20)
Using Eq. (18) in the above equation we get

\[ V_{eff}^R \sim -\frac{f a_R}{2M_{Pl}} \left( \frac{M^2}{M^4_2} + \frac{M^4}{M^6_2} \right) v l c^6 \] (21)

Similarly assuming non-zero vev for left type fields only and not for right type fields the effective potential becomes,

\[ V_{eff}^L \sim -\frac{f a_L}{2M_{Pl}} \left( \frac{M^2}{M^4_2} + \frac{M^4}{M^6_2} \right) v l c^6 \] (22)

If the scale of parity breaking is \( M_R \) then \( l_c = M_R \). In this case where we consider the equal chance for left and right type fields getting a vev, then \( l = M_R \). So the effective energy difference arising from the operators is given by,

\[ \delta \rho \sim \frac{f (a_L - a_R)}{2M_{Pl}} \left( \frac{M^2}{M^4_2} + \frac{M^4}{M^6_2} \right) v M^6_R \] (23)

Now we shall compare this \( \delta \rho \) with the case in a matter dominated era where we have calculated the energy density for the domain wall to decay. Before going further we make some approximations. The constants \( v \) and \( M \) in Eq.(23) are the coefficients appearing in the trilinear terms in Eq.(16). These trilinear terms are the soft terms. So the coefficients are of electroweak scale, \( M_{ew} \). \( M^2_1 \) and \( M^2_2 \) appear in the mass terms for \( \Delta(\Delta_c) \) and \( \bar{\Delta}(\bar{\Delta}_c) \). Since these triplet fields get their vev’s at a scale \( M_R \), the scales \( M^2_1 \) and \( M^2_2 \) should be of order \( M_R \). But the scale of \( M_R \) is higher than the electroweak scale, hence we find \( M^2_2 > M^2 \). So

\[ \frac{M^2}{M^4_2} > \frac{M^4}{M^6_2} \] (24)

So the dominant term in Eq.(23) is,

\[ \delta \rho \sim \frac{f (a_L - a_R)}{2M_{Pl}} \frac{M^3_{ew} M^6_R}{M^4_R} \] (25)

Now by comparing,

\[ \frac{f (a_L - a_R)}{2M_{Pl}} M^3_{ew} M^2_R > M^4_R \left( \frac{M_R}{M_{Pl}} \right)^{3/2} \] (26)

Putting the electroweak scale as \( M_{ew} \sim 10^3 \text{GeV} \) and taking \( f \) as \( O(1) \) we get the constraint on \( (a_L - a_R) \) as

\[ (a_L - a_R) > 10^{-5} \left( \frac{M_R}{10^4 \text{GeV}} \right)^{7/2} \] (27)

Comparing the obtained \( \delta \rho \) with the case in a radiation dominated era we get,

\[ \frac{f (a_L - a_R)}{2M_{Pl}} M^3_{ew} M^2_R > M^4_R \left( \frac{M_R}{M_{Pl}} \right)^2 \] (28)
Proceeding as above we get the constraint on \((a_L - a_R)\) as

\[
(a_L - a_R) > 10^{-4} \left( \frac{M_R}{10^9 \text{GeV}} \right)^4
\]  

(29)

Taking the dimensionless parameters \(a_L, a_R\) to be of order one, the equation\(^\text{27}\) gives an upper bound on the scale \(M_R\) in a matter dominated era

\[
M_R < 2.7 \times 10^5 \text{GeV}
\]  

(30)

Similarly during the radiation dominated era, the equation\(^\text{29}\) gives an upper bound on \(M_R\)

\[
M_R < 1 \times 10^7 \text{GeV}
\]  

(31)

Allowing further fine tuning between \(a_L\) and \(a_R\) will make this bound even more strict. However as we will see below, such a low intermediate D-parity breaking scale is not favored from successful gauge coupling unification point of view which makes this model less attractive.

Comparing the obtained \(\delta \rho\) with the weak inflation case we have

\[
\frac{f(a_L - a_R)}{2M_{Pl}} M_{ew}^3 M_R^2 \geq M_R^4 T_D^{12} M_{Pl}^{15} \]

(32)

Taking the dimensionless coefficients to be of order one, we arrive at the following bound on \(M_R\)

\[
M_R \geq 1.4 \times 10^5 T_D^{12/13}
\]  

(33)

Thus for \(T_D\) of the order of electroweak scale, \(M_R\) remains just below the gravitino bound. However, if \(T_D > 1.44 \times 10^4 \text{ GeV}\), then the \(M_R\) is forced to be higher that \(10^9 \text{ GeV}\) which, as noted in \([19]\), can be problematic if the reheating temperature after the domain wall disappearance is comparable to the temperature scale of original phase transition. In that case, the Universe would reheat to a temperature higher than \(10^9 \text{ GeV}\) giving rise to gravitino overabundance.

### B. Constraints on \(M_R\) from Unification

Successful gauge coupling unification at scale \(M_G > 10^{16} \text{ GeV}\) puts tight constraints on the intermediate symmetry breaking scales. Assuming the Higgs triplets to be as heavy as the the scale \(M_R\), we get a lower bound on the scale \(M_R\) to be of the order of \(10^{14} \text{ GeV}\). A lower value of \(M_R\) will make the couplings of \(U(1)_{B-L}\) and \(SU(2)_{L,R}\) meet before the allowed
unification scale from proton decay constraints. Although for the minimal particle content, the $SU(3)_c$ couplings do not meet the other two at one point, we can always take into account of some additional fields in a Grand Unified Theory (GUT) like $SO(10)$ which survive the GUT symmetry breaking and can be as light as the scale $M_R$. Here we consider three pairs of extra heavy colored superfields $\chi(3, 1, 1, -\frac{2}{3}), \bar{\chi}(\bar{3}, 1, 1, \frac{2}{3})$ which can be naturally fitted within $SO(10)$ GUT theory inside the representations $\mathbf{120}, \mathbf{126}$. The resulting unification is shown in fig. 1. Thus the lower limit on $M_R$ from unification is in conflict with the bounds from domain wall removal (30), (31).

IV. MINIMAL SUPERSYMMETRIC LEFT-RIGHT MODEL WITH HIGGS DOUBLETS

Spontaneous R-parity breaking can be achieved even without giving vev to the sneutrino fields. If the $U(1)_{B-L}$ symmetry is broken by a Higgs field which has odd $B-L$ charge then R-parity is spontaneously broken. We call this model as Minimal Higgs Doublet (MHD)
The scalar potential is \( V \) follows (2)

where the numbers in brackets correspond to the quantum numbers corresponding to Model. The minimal such model \([25, 34]\) has the following particle content

\[
\begin{align*}
L(2, 1, -1), & \quad L_c(1, 2, 1), \quad S(1, 1, 0), \quad Q(2, 1, \frac{1}{3}), \quad Q_c(1, 2, -\frac{1}{3}) \\
H &= \left( \begin{array}{c}
H_L^+ \\
H_0^L/\sqrt{2}
\end{array} \right) \sim (2, 1, 1), \quad H_c = \left( \begin{array}{c}
H_R^+ \\
H_0^R/\sqrt{2}
\end{array} \right) \sim (1, 2, -1), \\
\bar{H} &= \left( \begin{array}{c}
\bar{h}_L^+ /\sqrt{2} \\
\bar{h}_L^-
\end{array} \right) \sim (2, 1, -1), \quad \bar{H}_c = \left( \begin{array}{c}
\bar{h}_R^+ /\sqrt{2} \\
\bar{h}_R^-
\end{array} \right) \sim (1, 2, 1), \\
\Phi_1(2, 2, 0), & \quad \Phi_2(2, 2, 0)
\end{align*}
\]

where the numbers in brackets correspond to the quantum numbers corresponding to \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \). The symmetry breaking pattern is

\[
SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{\langle H, H_c \rangle} SU(2)_L \times U(1)_Y \xrightarrow{\langle \Phi \rangle} U(1)_{em}
\]

(34)

The renormalizable superpotential relevant for the spontaneous parity violation is given as follows

\[
W_{ren} = h_1^{(i)} L^T \tau_2 \Phi_1 \tau_2 L_c + h_q^{(i)} Q^T \tau_2 \Phi_1 \tau_2 Q_c \\
+ \mu_{ij} \text{Tr} \tau_2 \Phi_i^T \tau_2 \Phi_j + f_1 (H^T \Phi_1 H_c + \bar{H}^T \Phi_1 \bar{H}_c) + m_h H^T \tau_2 \bar{H} + m_h H_c^T \tau_2 \bar{H}_c
\]

(35)

The scalar potential is \( V = V_F + V_D + V_{soft} \) where \( V_F = |F_i|^2, F_i = -\frac{\partial W}{\partial \Phi^i} \) is the F-term scalar potential, \( V_D = D^a D^a/2, D^a = -g(\phi_i^a T_{ij}^a \phi_j) \) is the D-term of the scalar potential and \( V_{soft} \) is the soft supersymmetry breaking scalar potential. We introduce the soft SUSY breaking terms to check if they alter relations between various mass scales in the model. The soft SUSY breaking superpotential in this case is given by

\[
V_{soft} = m_H^2 H^\dagger H + m_{H_c}^2 \bar{H}^\dagger \bar{H} + m_{H_c}^2 H^\dagger H_c + m_{H_c}^2 \bar{H}^\dagger \bar{H}_c + m_{\Phi_1}^2 \Phi_1^\dagger \Phi_1 \\
+ m_{\Phi_2}^2 \Phi_2^\dagger \Phi_2 + (B_1 H^\dagger \tau_2 \bar{H} + B_2 H_c^\dagger \tau_2 \bar{H}_c + B \mu_{ij} \text{Tr} [\tau_2 \Phi_i \tau_2 \Phi_j] + h.c.) \\
+(A_1 H^\dagger \Phi_1 H_c + A_2 \bar{H}^\dagger \Phi_1 \bar{H}_c + h.c.)
\]

(36)

where all the parameters \( m_H, m_{11}, m_{22}, B, A \) are of the order of SUSY breaking scale \( M_{susy} \sim \) TeV. We denote the vev of the neutral components of \( \Phi_1, \Phi_2, H_L, \bar{H}_L, H_R, \bar{H}_R \) as \( \langle (\Phi_1)_{11} \rangle = v_1, \langle (\Phi_2)_{22} \rangle = v_2, \langle H_L, \bar{H}_L \rangle = v_L, \langle H_R, \bar{H}_R \rangle = v_R \) Minimizing the potential with respect to \( v_L, v_R \), we get the relations

\[
\frac{\partial V}{\partial v_L} = \frac{1}{2} (4v_R^2 + v_1^2 + v_2^2) f_1^2 + 2v_R f_1 (m_h (v_1 + v_2) + 2v_1 (\mu_{11} + \mu_{12}) \\
2v_2 (\mu_{12} + \mu_{22})) + 4m_{H_R}^2 v_L - 2 v_L m_{H}^2 + 2 v_L B_1 + A_1 v_1 v_R = 0
\]

(37)
FIG. 2: Gauge coupling unification in minimal SUSYLR model with Higgs doublets, $M_{\text{susy}} = 1$ TeV, $M_R = 10^{12}$ GeV, $M_{\text{GUT}} = 10^{16.4}$ GeV

\[
\frac{\partial V}{\partial v_R} = \frac{1}{2} \left( v_R^2 (4 v_L^2 + v_1^2 + v_2^2) f_1^2 + 2 v_L f_1 (m_h (v_1 + v_2) + 2 v_1 (\mu_{11} + \mu_{12})) 
- 2 v_2 (\mu_{12} + \mu_{22}) + 4 m_R^2 v_R - 2 v_R m_h^2 + 2 v_L B_2 + A_1 v_1 v_L \right) = 0
\]  

(38)

From the above two equations we arrive at

\[
\frac{v_R}{v_L} \frac{\partial V}{\partial v_L} - v_L \frac{\partial V}{\partial v_R} = \frac{(v_R^2 - v_L^2)}{2} (4 v_L v_R f_1^2 + 2 f_1 (m_h (v_1 + v_2)) 
- 2 v_1 (\mu_{11} + \mu_{12}) + 2 v_2 (\mu_{22} + \mu_{12}) + A_1 v_1) = 0
\]  

(39)

Assuming $m_h, v_R \gg v_1, v_2, \mu_{ij} \sim M_{\text{EW}}$ the above relation gives the parity breaking solution $(v_L \neq v_R)$

\[
v_L \sim -\frac{m_h (v_1 + v_2)}{2 v_R f_1}
\]

From the above relations we can show that parity is broken spontaneously to give rise to the parity violating standard model. Also there is a seesaw between $v_L$ and $v_R$ from the above equation which can give rise to tiny neutrino masses $[25]$. 

13
A. Constraints on $M_R$ from domain wall removal

Similar to the previous section, here also we find the $1/M_p$ terms in the effective potential by expanding the Kähler potential and superpotential in powers of $1/M_p$. The superpotential upto the powers of $1/M_p$ is

$$W = W_{ren} + \frac{a_L}{2M_{Pl}}(H^T \tau_2 H)^2 + \frac{a_R}{2M_{Pl}}(H^c_T \tau_2 H_c)^2 + \frac{b_L}{M_{Pl}}(H^T H)(\bar{H}^2 \bar{H}) + \frac{b_R}{M_{Pl}}(H^c_T H_c)(\bar{H}^2 \bar{H}_c)$$

$$+ \frac{f_2}{M_{Pl}}(H^T \tau_2 \bar{H})(H^c_T \tau_2 \bar{H}_c) + \frac{f_3}{M_{Pl}}(H^T H)(H^c_T H_c) + \frac{f_4}{M_{Pl}}(\bar{H}^T \bar{H})(\bar{H}^c_T \bar{H}_c)$$

(40)

The Kähler potential in this model upto $1/M_p$ is

$$K = H^\dagger H + \bar{H}^\dagger \bar{H} + H^\dagger_c H_c + \bar{H}^\dagger_c \bar{H}_c$$

Assuming a phase where only right type fields get non-zero vev and left type fields get zero vev, the scalar potential upto the leading term in $1/M_p$ becomes

$$V_{eff}^R \sim (a_R + 2b_R)m_h \frac{v_R^4}{M_{Pl}}$$

(41)

Similarly for the phase where only left type fields get non-zero vev

$$V_{eff}^L \sim (a_L + 2b_L)m_h \frac{v_L^4}{M_{Pl}}$$

(42)

Taking $m_h \sim M_R$ the effective energy difference can now be calculated as

$$\delta \rho \sim [(a_R + 2b_R) - (a_L + 2b_L)] \frac{M_R^5}{M_{Pl}}$$

(43)

Thus for the matter dominated era we have

$$[(a_R + 2b_R) - (a_L + 2b_L)] > \frac{M_R^{1/2}}{M_{Pl}^{1/2}}$$

(44)

And for the radiation dominated era

$$[(a_R + 2b_R) - (a_L + 2b_L)] > \frac{M_R}{M_{Pl}}$$

(45)

Taking the various dimensionless parameters to be of order one, we get the same upper bound on $M_R$ in both the above cases

$$M_R < M_{Pl} = 1 \times 10^{19}\text{GeV}$$

(46)
Similarly, in the weak inflation scenario we have

\[
[(a_R + 2b_R) - (a_L + 2b_L)] \frac{M_R^2}{M_{Pl}} \geq M_R^4 \frac{T_D^{12} M_{Pl}^3}{M_R^{15}}
\]  

(47)

Assuming the dimensionless coefficients to be order one, this leads to

\[
M_R \geq 5.6 \times 10^4 T_D^{3/4}
\]  

(48)

Here \(T_D\) can be as high as \(5 \times 10^5\) GeV for \(M_R\) to remain below the gravitino bound. As noted in the previous section, \(M_R > 10^9\) GeV can lead to gravitino overabundance if the reheat temperature after the wall disappearance is same as the temperature of the original phase transition.

Thus the scale \(M_R\) is less restrictive in this model compared to the SUSYLR model with Higgs triplets. As we will see below, one can have successful gauge coupling unification in this model for \(M_R \geq 10^{12}\) GeV. Due to the possibility of successful removal of domain walls as well as successful gauge coupling unification, this model is the preferred one over the model with Higgs triplets.

**B. Constraints on \(M_R\) from Unification**

Similar to the minimal SUSYLR model with Higgs triplets, here also the intermediate symmetry breaking scales will be constrained by demanding successful gauge coupling unification at a very high scale \(M_G(> 10^{16}\text{GeV})\). Similar to the previous case, here also the couplings of \(U(1)_{B-L}\) and \(SU(2)_{L,R}\) meet much before the allowed Unification scale if the intermediate symmetry breaking scale \(M_R\) is lower than a certain value. For the minimal SUSYLR model with Higgs doublets, this lower bound on \(M_R\) is found to be of the order of \(10^{12}\) GeV. We also consider two additional heavy colored superfields so that the \(SU(3)_c\) coupling meet the other two couplings at one point. They are denoted as \(\chi(3, 1, 1, -\frac{2}{3}), \bar{\chi}(\bar{3}, 1, 1, \frac{2}{3})\) and can be accommodated within \(SO(10)\) GUT theory in the representations \(120, 126\). Here we assume that the structure of the GUT theory is such that these fields survive the symmetry breaking and can be as light as the \(SU(2)_R\) breaking scale. The resulting gauge coupling unification as shown in the figure.\(^2\)
V. RESULTS AND CONCLUSION

We have discussed the issue of domain wall formation due to the spontaneous breaking of D-parity in two different versions of supersymmetric left right models: one with Higgs triplets having \( B - L \) charge \( \pm 2 \) and the other with Higgs doublets having \( B - L \) charge \( \pm 1 \). Since stable domain walls are in conflict with cosmology, we consider the effects of Planck scale suppressed operators in destabilizing them. We consider the evolution and decay of domain walls in two different epochs: radiation dominated as well as matter dominated. We find that successful removal of domain walls put rather strict constraints on the D-parity breaking scale \( M_R \) in the model with Higgs triplets. The model with Higgs doublets is far less restrictive on the other hand.

| Model     | Gauge Coupling Unification | DW removal during MD era | DW removal during RD era | DW removal including WI |
|-----------|-----------------------------|--------------------------|--------------------------|--------------------------|
| MSLRM     | \( \geq 10^{14} \)          | \( < 2.7 \times 10^5 \)  | \( < 10^7 \)             | \( \geq 1.4 \times 10^5 T_D^{12/13} \) |
| MHD       | \( \geq 10^{12} \)          | \( < M_{Pl} \)           | \( < M_{Pl} \)           | \( \geq 5.6 \times 10^4 T_D^{3/4} \) |
| BDM       | \( \geq 1.5 \times 10^3 \)  | None                     | None                     | None                     |

| Model     | Gauge Coupling Unification | DW removal during MD era | DW removal during RD era | DW removal including WI |
|-----------|-----------------------------|--------------------------|--------------------------|--------------------------|
| ABMRS     | \( \geq 10^{15-16} \)       | \( < 10^7 \)             | \( < 10^{11} \)          | \( \geq 8.6 \times 10^4 T_D^{4/5} \) |
| BM        | \( \geq 10^{14} \)          | None                     | None                     | None                     |
| Bitriplet | \( \geq 5 \times 10^{12} \) | None                     | None                     | None                     |

We also find the constraint on the D-parity breaking scale in these models by demanding gauge coupling unification at a scale \( M_G > 10^{16} \) GeV. We use one-loop beta functions for
both the models and take into account of some heavy colored superfields to make the $SU(3)_c$ coupling meet the other two exactly at one point. The results are shown in table (I). We also mention the model by Bhupal Dev and Mohapatra (BDM) \[23\] in the table where the scale of $SU(2)_R \times U(1)_{B-L}$ symmetry breaking to $U(1)_Y$ denoted by $M_R$ can be as low as few TeV. However there is no constraint on the scale $M_R$ from domain wall disappearance due to the existence of a parity odd singlet which, after acquiring a vev breaks the degeneracy between two possible vacua.

In table (II) we summarize the results of similar analysis obtained for R-parity conserving SUSYLR models in some of our earlier works. The bounds from domain wall disappearance in Aulakh-Bajc-Melfo-Rasin-Senjanovic (ABMRS) \[11, 12\] model and Babu-Mohapatra (BM) \[13\] model were discussed in \[19\]. The bitriplet model \[14\] does not suffer from domain wall problem due to the existence of a parity odd singlet as pointed out in the introduction. The bounds on $M_R$ from gauge coupling unification in such models were discussed in \[24, 35\].

To summarize the result of this paper, it is shown that both domain wall removal and unification constraints can be satisfied in the Minimal Higgs doublet model or the MHD model whereas it is not possible to have successful removal of domain walls and gauge coupling unification together in the model with Higgs triplets (MSLRM).

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