Spin transverse force and quantum transverse transport

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We present a brief review on spin transverse force, which exerts on the spin as the electron is moving in an electric field. This force, analogue to the Lorentz force on electron charge, is perpendicular to the electric field and spin current carried by the electron. The force stems from the spin-orbit coupling of electrons as a relativistic quantum effect, and could be used to understand the Zitterbewegung of electron wave packet and the quantum transverse transport of electron in a heuristic way.

I. INTRODUCTION

In electrodynamics it is well known that a magnetic field would exert a transverse force, i.e. the Lorentz force, on an charged particle if it is moving. This Lorentz force can lead to a lot of fundamental phenomena such as the Hall effect in solid.\(^1\) The interaction of the electron spin in the electromagnetic field behaves as if the spin is a gauge charge and the interaction is due to the SU(2) gauge field.\(^2\) Essentially the electron spin is an intrinsic quantum variable, not just a classical tiny magnetic moment. If we want to manipulate and control quantum spin states, a natural question raises: what type of force exerts on a moving spin in an electric field? In the recent work,\(^3\) it was found that an electric field exerts a transverse force on an electron spin if it is moving and the spin is projected along the electric field. The force stems from the spin-orbit coupling which can be derived from the Dirac equation for an electron in a potential in the non-relativistic limit or the Kane model with the \(k \cdot p\) coupling between the conduction band and valence band. It is heuristic to understand that the Zitterbewegung of electronic wave packet is driven by the spin transverse force on a moving spin. Based on the concept of spin force balance, a relation between spin current and spin polarization is given. The role of spin transverse force is also discussed in the anomalous Hall effect, the spin Hall effect and its reciprocal effect driven by the pure spin current in semiconductors. It was shown that the Kubo formula can be formalized in terms of the spin force. Some authors also discussed the spin force in the systems with Rashba coupling, and its relation with the spin transport by means of numerical simulation,\(^4\) the Boltzmann equations,\(^5\) and the continuity equation of the momentum current.\(^6\) The forces induced by the Yang-Mills filed such as the Rashba and Dresselhaus fields, the shear strain field acting on spin and spin current were also derived.\(^7\)

II. SPIN TRANSVERSE FORCE

We consider an electron in a confining potential \(V\) and a vector potential \(\mathbf{A}\) for a magnetic field, \(\mathbf{B} = \nabla \times \mathbf{A}\). In the relativistic quantum mechanics, the Dirac equation determines the behaviors of electron. In the nonrelativistic limit, neglecting the higher order terms of expansion, the Dirac equation is reduced to the Schrödinger equation by introducing the spin-orbit coupling and the Zeeman exchange coupling,

\[
H \approx \frac{(\mathbf{p} + \frac{A}{c})^2}{2m} + V_{\text{eff}} + \mu_B \sigma \cdot \mathbf{B} + \frac{\hbar}{4m^2c^2}(\sigma \times \nabla) \cdot \mathbf{V},
\]

where \(m\) and \(e\) are the electron mass and charge, respectively, \(c\) is the speed of light, \(\sigma\) are the Pauli matrices, \(\mu_B = e\hbar/2mc\) and \(V_{\text{eff}} = V + \frac{\hbar^2}{8mc^2} \nabla^2 V\). From the Heisenberg motion equation, we can obtain the quantum mechanical analogue of Newton’s second law, known as the Ehrenfest theorem,

\[
F = m \frac{d\mathbf{v}}{dt} = \frac{m}{i\hbar} [\mathbf{v}, H] = -\frac{m}{\hbar^2} [\mathbf{r}, H], H. \tag{2}
\]

Of course there is no concept of force in quantum mechanics, this is just an operator equation. The physical meaning of force is contained in the expectation value in a quantum state. The quantum mechanical version of the force \(F\) comprises three parts, i.e., Lorentz force \(F_h\), spin electromagnetic force \(F_g\) and spin transverse force \(F_f\). The form of Lorentz force \(F_h\), which has its counterpart in the classic limit of \(\hbar \to 0\), is

\[
F_h = \frac{e}{c} (\mathbf{v} \times \mathbf{B} - \mathbf{B} \times \mathbf{v})/2 - \nabla (V_{\text{eff}} + \mu_B \sigma \cdot \mathbf{B}), \tag{3}
\]

where \(\mathbf{B} = \mathbf{B} + \nabla \times \mathbf{A}\), with \(\mathbf{A} = \frac{\hbar}{4mc^2} \sigma \times \nabla V\). It indicates clearly that \(\mathbf{A}\) plays a role of a SU(2) gauge vector potential. Recently it is proposed that the force can generate a pure spin current if we assume \(\nabla B_z\) is a constant.\(^2\) The spin electromagnetic force \(F_g\) is written as

\[
F_g = \frac{\mu_B}{2mc^2} (\sigma \cdot \nabla V) - (\mathbf{B} \sigma \cdot \nabla)\mathbf{V}. \tag{4}
\]

It is non-zero only when the electric and magnetic fields coexist, as suggested by Anandan and others.\(^2\) The spin transverse force \(F_f\) is derived as

\[
F_f = \frac{\hbar}{8m^2c^4} (\sigma \cdot \nabla V) (\mathbf{v} \times \nabla V), \tag{5}
\]

which comes from the SU(2) gauge potential or spin-orbital coupling. As the force is related to the Planck
constant it has no counterpart in classic mechanics and is purely quantum mechanic effect. The spin transverse force for a single electron on a quantum state can be written in a compact form,
\[ \langle F_f \rangle = \frac{e^2 |E|}{4m^2 c^3} \hat{x} \times E, \] (6)
where the spin current is defined conventionally, \( j^F = \frac{1}{2} \langle \{ V, \sigma \cdot E \}/|E| \rangle \). The force is proportional to the square of electric field \( E \) and the spin current which polarization is projected along the field. It is important to note that an electron in a spin state perpendicular to the electric field will not experience any force. Comparing with a charged particle in a magnetic field, \( j_e \times B \), where \( j_e \) is a charge current density, the spin force is nonlinear to the electric field and depends on the spin state of electron.

III. APPLICATIONS OF SPIN TRANSVERSE FORCE

Zitterbewegung of wave package—The rapid oscillation of the electron wave packet is known in literatures as the Zitterbewegung of electron as a relativistic quantum mechanical effect, which is physically regarded as a result of admixture of the positron state in electron wave packet. However, this effect is usually expected to be measured in high energy physics, but fails to do experimentally. Note that the effective models describing the band structure of III-V semiconductors are similar to the Dirac equation such that some authors proposed that the experimental observation of Zitterbewegung may be more realistic in semiconductors with spin-orbit coupling rather than in high energy physics. Recently Schliemann et al proposed that Zitterbewegung can be observed in both n- and p-doped III-V zinc-blende semiconductor quantum wells. In the p-doped three-dimensional bulk semiconductors described by the Luttinger model will generate the Zitterbewegung as calculated by Jiang et al. The Zitterbewegung of electrons in three-dimensional bulk III-V semiconductors was also discussed by Zawadzki. Katsnelson analyzed the oscillatory motion of the electron related to the Zitterbewegung in graphene which is a gapless semiconductor with massless Dirac energy spectrum. Very recently, a unified treatment of Zitterbewegung for spintronic, graphene, and superconducting systems was presented. Readers are referred to see other recent worked devoted to Zitterbewegung phenomena in references.

In the recent work, a heuristic picture is given to understand that the Zitterbewegung of electronic wave packet is driven by the spin transverse force on a moving spin. Here we consider the motion of an electron confining in a two-dimensional plane subjected to a perpendicular electric field, without loss of generality, and assume that the spin-orbit coupling provides an effective magnetic field along the \( y \) direction. In this case, only the spin transverse force \( F_y \) exerts on the spin while the Lorentz force \( F_x \) and spin electromagnetic force \( F_y \) vanish. Because of the spin-orbit coupling the the electron spin precesses in the spin \( x-z \) plane. The spin \( \sigma_z(t) \) varies with time and the spin current is always along the \( x \) direction. As a result the spin transverse force is always perpendicular to the \( x \) direction. If the initial state is polarized along \( y \) direction the electron spin does not vary with time as it is an energy eigenstate of the system. In this case the spin current carried by the electron is always zero, as a result, the spin transverse force is also zero. If the initial state is along the spin \( z \) direction, the spin state will evolve with time as the the state is not the energy eigenstate. It can be understood that the spin precession makes the spin current whose polarization is projected along the electric field changes with time such that the spin force along the \( y \) direction also oscillates. This force will generate a non-zero velocity of electron oscillating along the \( y \) direction, and then the trajectory oscillates with the time.

Though the spin transverse force on a moving spin is very analogous to the Lorentz force on a moving charge, because of spin precession, its effect is completely different with the motion of a charged particle in a magnetic field, where the amplitude of the Lorentz force is constant and the charged particle will move in a circle. The Zitterbewegung of the electronic wave packet near the boundary will cause some edge effect as shown in recent numerical calculations. The edge effect is determined by the electron momentum. The smaller the momentum, the larger the edge effect.

Spin force balance—Intrinsic spin Hall effect has been received a great deal of attention recently since the works by Murakami et al. and Sinova et al. that a spin Hall current can be produced by an electric field in p-doped III-V semiconductors and the two-dimensional electron gas with Rashba coupling. Vertex correction and numerical calculations show that disorder can cancel spin Hall effect in the two-dimensional electron gas with Rashba coupling. On the other hand, if the exchange coupling is taken into account, spin Hall effect should survive. In fact, the result can be derived based on the concept of spin force balance. We consider an effective Hamiltonian for a two-dimensional ferromagnetic system with Rashba coupling, \( H = p^2/(2m^*) + \lambda (p_x \sigma_y - p_y \sigma_x) + h_0 \sigma_z \), where \( m^* \) is the effective mass of conduction electron and the exchange field \( h_0 \) due to the magnetic impurities. The spin-orbit coupling induces the spin transverse force \( F_y = -2m^* \lambda h_0 [\sigma_y \hat{y} - \sigma_x \hat{x}] \), and the spin electromagnetic force \( F_y = -2m^* \lambda h_0 [\sigma_x \hat{x} + \sigma_y \hat{y}] \). If the disorder potential \( V_{\text{disorder}} \) is taken into account, in a steady state, the spin force must reach at balance,
\[ \frac{1}{i\hbar} \left\langle \left[ \frac{\hat{c}}{c} \mathbf{A}, H + V_{\text{disorder}} \right] \right\rangle = \langle F_f + F_y \rangle = 0. \] (7)
This result is independent of the non-magnetic disorder and interaction because the spin gauge field commutes with non-magnetic potential \( V_{\text{disorder}} \). From the spin force balance we have a relation between spin cur-
rent and spin polarization, \( \langle J_z^x \rangle = \frac{\hbar m}{2e} \langle \sigma_y \rangle \), and \( \langle J_z^y \rangle = -\frac{\hbar m}{2e} \langle \sigma_x \rangle \). It is obvious that the spin Hall current vanishes in the case of \( h_0 = 0 \), while if \( h_0 \neq 0 \) spin Hall current should survive.

Anomalous Hall effect-The spin transverse force can be also regarded a driven force of an anomalous Hall effect. Here we give a clear picture for anomalous Hall effect in ferromagnetic metals and semiconductors. When an external electric field is applied along the \( x \) axis, it will circulate an electric current \( J_{\alpha x} \), and also a spin current \( J_{\alpha y} \) since the charge carriers are partially polarized. The spin-orbit coupling exerts a spin transverse force on the spin current, \( J_{\alpha y} \), and generate a drift velocity or the anomalous Hall current \( J_{\alpha y} \). From the Rashba coupling the spin polarization tends to be normal to the momentum or electric current. The electric current \( J_{\alpha x} \) along \( E \) induces a non-zero \( \langle \sigma_y \rangle \) and the anomalous Hall current \( J_{\alpha y} \) induces non-zero \( \langle \sigma_x \rangle \). These non-zero spin polarization maintains the balance of spin transverse force, and further a non-zero spin current in a steady state. Thus the anomalous electronic transverse transport is robust against the disorder in the ferromagnetic metals and semiconductors.

Spin resolved charge Hall effect-The spin transverse force can be applied understand the spin-resolved Hall effect. When a spin current is injected into 2DEG with the spin-orbit coupling, the spin-orbit coupling exerts the spin transverse force on the spin current, and drives electrons to form a charge Hall current perpendicular to the spin current. The injected spin current which can be generated in various ways, such as the ac-magnetic field, the spin force \( \nabla (\hat{\mu} \cdot \mathbf{B}) \) and circularly or linearly polarized light injection.

Spin transverse force and Kubo formula-The quantum transverse transport of electrons caused by a weak electric field \( \mathbf{E} \) can be calculated within the framework of linear response theory. Based on the concept of spin force, the Kubo formula of linear response theory can be re-formalized. We consider an effective Hamiltonian for electrons with spin \( 1/2 \), \( H = \frac{p^2}{2m^*} + \sum_{\alpha=x,y,z} d_\alpha(p) \sigma_\alpha \), where \( d_\alpha(p) \) are the the momentum-dependent coefficients which describes the spin-orbit interactions and exchange interaction. The spin force is derived as \( F_\alpha = \frac{2m^*}{\hbar} \sigma_\alpha \cdot \left( \frac{\partial \chi_{ij}}{\partial p_j} \times \mathbf{d} \right) \). In general, for any observable \( O \), its linear response to an external electric field \( \mathbf{E} \) has the formula, \( \langle O_i \rangle = -\chi_{ij} \mathbf{E} \), and

\[
\chi_{ij} = -\frac{e\hbar^2}{16m^*\Omega} \sum_p \left( f_{p,-} - f_{p,+} \right) \frac{d}{dE} \text{Tr} \left[ \mathbf{F}_i \mathbf{O}_j \right],
\]

where \( d = \sqrt{d_x^2 + d_y^2 + d_z^2} \) and \( f_{p,\pm} \) are the Dirac-Fermi distribution functions.

IV. CONCLUSION

In conclusion, compared with the Lorentz force brought by the magnetic field upon a charged particle, the spin-orbit coupling produces a spin transverse force on a moving electron spin. It has no classical counterpart as the coefficient is divided by \( h \), but it reflects the tendency of spin asymmetric scattering of a moving electron subject to the spin-orbit coupling. It stems from the spin-orbit coupling as a relativistic quantum mechanical effect. However, this quantity may provide an new way to understand the spin transport in semiconductors.

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