Some remarks on Bianchi type-II, VIII and IX models

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Within the scope of anisotropic non-diagonal Bianchi type-II, VIII and IX spacetime it is shown that the off-diagonal components of the corresponding metric impose severe restrictions on the components of the energy momentum tensor in general. If the energy momentum tensor is considered to be diagonal one, the spacetime, expect a partial case of BII, becomes locally rotationally symmetric.

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I. INTRODUCTION

Spatially homogeneous and anisotropic cosmological models play a significant role in the description of the large scale behavior of the Universe. In search of a realistic picture of the early Universe such models have been widely studied within a framework of General Relativity. In this note we confine our study within the scope of Bianchi type-II (BII), type-VIII (BVIII) and type-IX (BIX) space-times, which has recently been studied by a number of authors. Christodoulakis et al [1] investigated the set of spacetime general coordinate transformations which leave the line element of a generic Bianchi-type geometry quasi form invariant; Einsteins field equations for stationary BII models with a perfect fluid source were investigated by Nilsson and Uggla [2]; Ram and Singh [3] presented analytical solutions of the Einstein-Maxwell equations for cosmological models of locally rotationally symmetric (LRS) Bianchi type-II, VIII and IX; two-fluid BII cosmological models were studied by Pant and Oli [4]; a BII cosmological model with constant deceleration parameter was considered by Singh and Kumar [5]; Belinchon [6, 7] studied a massive cosmic string within the scope of a BII model, while LRS BII cosmological models in the presence of a massive cosmic string and varying cosmological constant were studied by Pradhan et. al. [8], Kumar [9] and Yadav et. al. [10], respectively. Other recent work includes exact solutions for BII cosmological model in the Jordan Brans-Dicke scalar-tensor theory of gravitation [11], study of a BII Lyttleton-Bondi Universe [12], and determination of an anisotropic BII cosmological model in the presence of source-free electromagnetic fields in Lyra’s manifold [13]. A general scheme for the derivation of exact solution of Einstein equations corresponding to perfect fluid plus radiation was outlined in [14] within the scope of a BIX spacetime. Bianchi type-VIII and IX models in the Lyttleton-Bondi universe was studied by Shanthi and Rao [15]. Bianchi type-IX cosmic strings within the scope of scalar-tensor theory was studied in [16]. Spatially homogeneous cosmological models within the framework of BII, BVIII and BIX are considered in [17]. A Bianchi type-IX viscous cosmological model in general relativity was studied by Bali and Yadav [18]. In a recent paper [19] within the scope of a BII model, we have shown that the choice of metric and energy momentum tensors are mutually related and off-diagonal components of Einstein tensor and energy momentum tensor impose some restrictions on metric or material fields. Though for the isotropic distribution of matter, the off-diagonal components of Einstein’s system of equations for

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II. BIANCHI SPACETIME: GENERAL REMARKS

We consider the anisotropic non-diagonal Bianchi spacetime in the form

\[ ds^2 = dt^2 - a_t^2(t)dx_1^2 - [h^2(x_3)a_t^2(t) + f^2(x_3)a_2^2(t)]dx_2^2 - a_3^2(t)dx_3^2 + 2a_1^2(t)h(x_3)dx_1dx_2, \]  

(2.1)

with \( a_1, a_2, a_3 \) being the functions of \( t \) and \( h, f \) functions of \( x_3 \) only. Defining

\[ \delta = -\frac{1}{f} \frac{\partial^2 f}{\partial x_3^2} \]  

(2.2)

from (2.1) we find BII, BVIII and BIX models, respectively as follows:

\[ \delta = 0, \quad \text{corresponds to BII model}, \]  

(2.3a)

\[ \delta = -1, \quad \text{corresponds to BVIII model}, \]  

(2.3b)

\[ \delta = 1, \quad \text{corresponds to BIX model}. \]  

(2.3c)

Let us write the non-zero components of Einstein tensor explicitly corresponding to (2.1)

\[ G_1 = \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} - \frac{3}{4} \frac{a_1^2 \dot{h}^2}{a_3 a_2^2} - \frac{f''}{a_3^2} - \frac{1}{2} \frac{a_1^2 h}{f^2 a_3^3 a_2} \left( h'' - \frac{h' f'}{f} \right), \]  

(2.4a)

\[ G_2 = \frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} + \frac{1}{4} \frac{a_1^2 \dot{h}^2}{a_2^2 a_3^2} + \frac{1}{2} \frac{a_1^2 h}{f^2 a_3^3 a_2} \left( h'' - \frac{h' f'}{f} \right), \]  

(2.4b)

\[ G_3 = \frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{1}{4} \frac{a_1^2 \dot{h}^2}{a_2^2 a_3^2}, \]  

(2.4c)

\[ G_0 = \frac{\dot{a}_1}{a_1} \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} + \frac{1}{4} \frac{a_1^2 \dot{h}^2}{a_2^2 a_3^2} - \frac{f''}{f a_3^3}, \]  

(2.4d)

\[ G_3 = \frac{f'}{f} \left( \frac{\dot{a}_3}{a_3} - \frac{\dot{a}_2}{a_2} \right), \]  

(2.4e)

\[ G_3 = -\frac{f'}{f a_3^3} \left( \frac{\dot{a}_3}{a_3} - \frac{\dot{a}_2}{a_2} \right), \]  

(2.4f)

\[ G_1 = -\frac{1}{2} \frac{a_1^2}{f^2 a_3^3 a_2^3} \left( h'' - \frac{h' f'}{f} \right), \]  

(2.4g)

\[ G_2 = \frac{h \frac{\dot{a}_1}{a_1} - h \frac{\dot{a}_2}{a_2} + h \frac{\dot{a}_3}{a_3} \frac{\dot{a}_1}{a_1} - h \frac{\dot{a}_2}{a_2} \frac{\dot{a}_3}{a_3} + \frac{a_1^2 h \dot{h}^2}{a_2^2 a_3^2} + \frac{h f''}{a_3^3 f} - \frac{1}{2} \frac{a_1^2 h^2}{f a_3^3 a_2^3} \left( h'' - \frac{h' f'}{f} \right) \]  

(2.4h)
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It can be verified that
\[ G_2^1 = h(G_2^0 - G_1^1) + (h^2 + f^2 a^2 / a_1^2) G_1^2. \]  
(2.5)

It should be noted that in cosmology researchers usually choose the Einstein equations in the form
\[ G_{\mu}^\nu = -\kappa T_{\mu}^\nu. \]  
(2.6)

It means that relation analogous to (2.5) should be held for the components of energy momentum tensor as well, i.e.,
\[ T_2^1 = h(T_2^0 - T_1^1) + (h^2 + f^2 a^2 / a_1^2) T_1^2. \]  
(2.7)

Considering the energy momentum tensor in diagonal form is a common feature in cosmology. In connection with that we assume
\[ T_{\mu}^\nu = \{ T_0^0, T_1^1, T_2^2, T_3^3 \}. \]  
(2.8)

Under this assumption, from (2.7) immediately follows
\[ T_1^1 = T_2^2, \]  
(2.9)

i.e., if the energy momentum tensor of the material field possesses only non-zero diagonal components, the off-diagonal components of the Einstein tensor corresponding to the metric (2.1) leads to (2.9).

On account of (2.8), (2.9) and (2.4) the Einstein system of equations (2.6) corresponding to (2.1) now can be written as

\[
\begin{align*}
\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_2 \dot{a}_3}{a_2 a_3} - \frac{3 \; a_2^2 h^2}{4 f^2 a_2^2 a_3^2} + \frac{\delta}{a_3^2} &= -\kappa T_1^1, \\
\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3 \dot{a}_1}{a_3 a_1} + \frac{a_2^2 h^2}{4 f^2 a_2^2 a_3^2} &= -\kappa T_1^1, \\
\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{a_3^2 h^2}{4 f^2 a_3^2 a_2^2} &= -\kappa T_3^3, \\
\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1 \dot{a}_3}{a_1 a_3} - \frac{a_1^2 h^2}{4 f^2 a_1^2 a_3^2} + \frac{\delta}{a_3^2} &= -\kappa T_0^0, \\
\frac{f'}{f} \left( \frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3} \right) &= 0, \\
h'' - \frac{h' f'}{f} &= 0, \\
\frac{\ddot{a}_1}{a_1} - \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} - \frac{\ddot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{a_1^2 h^2}{f^2 a_1^2 a_3^2} - \frac{\delta}{a_3^2} &= 0.
\end{align*}
\]  
(2.10a-e)

Here we take into account (2.10f) to rewrite (2.10a), (2.10b) and (2.10g). It should be noted that under the condition (2.9) the Eq. (2.10g) is identically fulfilled, hence can be omitted.

From (2.4g) we obtain
\[ \frac{h''}{h'} = \frac{f'}{f}, \]  
(2.11)

with the solution
\[ h' = f, \]  
(2.12)

where the constant is taken to be unity.
So finally we can rewrite the system (2.10) as follows:

\[
\begin{align*}
\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_2}{a_2} \frac{\dot{a}_3}{a_3} - \frac{3}{4} \frac{a_1^2}{a_2 a_3} + \delta &= -\kappa T^1_1, \\
\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3} \frac{\dot{a}_1}{a_1} - \frac{1}{4} \frac{a_1^2}{a_2^2 a_3^2} + \delta &= -\kappa T^3_3, \\
\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{1}{4} \frac{a_1^2}{a_2^2 a_3^2} + \delta &= -\kappa T^0_0, \\
f'' - \frac{\ddot{a}_2}{a_2} - \frac{\ddot{a}_3}{a_3} &= 0.
\end{align*}
\] (2.13)

In what follows, we consider some concrete form of Bianchi metrics. In doing so we first solve the equation (2.2), i.e.,

\[
\frac{\partial^2 f}{\partial x_3^2} + \delta f = 0,
\] (2.14)

for different values of \(\delta\).

### III. BIANCHI SPACETIME: CONCRETE EXAMPLES

In this section we study the properties system given in the foregoing section for the concrete choice of \(\delta\).

#### A. BII metric

The metric (2.1) gives rise to BII space time if \(\delta = 0\),

\[
\frac{\partial^2 f}{\partial x_3^2} = 0.
\] (3.1)

Eqs. (3.1) allows two cases:

**Case I** As a partial solution we may consider the case when \(f' = 0\). For simplicity we may write \(f = 1\). Then from (2.12) we obtain \(h = x_3\) setting the integration constant to be trivial. In this case from (2.10c) one finds that the model allows a general case with \(a_2 \neq a_3\).

The corresponding metric takes the form \([8, 19]\)

\[
ds^2 = dt^2 - a_1^2 dx_1^2 - [x_3^2 a_1^2 + a_2^2] dx_2^2 - a_3^2 dx_3^2 + 2a_1^2 x_3 dx_1 dx_2.
\] (3.2)
The Einstein system in this case reads

\[
\begin{align*}
\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} + \frac{\ddot{a}_2}{a_2} \frac{\dot{a}_3}{a_3} - \frac{3}{4} \frac{a_1^2}{a_2^2a_3^3} &= -\kappa T_1^1, \\
\frac{\dot{a}_3}{a_3} + \frac{\dot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} \frac{\dot{a}_1}{a_1} + \frac{1}{4} \frac{a_2^2}{a_2^2a_3^3} &= -\kappa T_1^1, \\
\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\ddot{a}_1}{a_1} \frac{\dot{a}_2}{a_2} + \frac{1}{4} \frac{a_3^2}{a_2^2a_3^3} &= -\kappa T_3^3, \\
\frac{\ddot{a}_1 a_2}{a_1 a_2} + \frac{\ddot{a}_2 a_3}{a_2 a_3} + \frac{\ddot{a}_3 a_1}{a_3 a_1} - \frac{1}{4} \frac{a_1^2}{a_2^2a_3^3} &= -\kappa T_0^0.
\end{align*}
\]  

(3.3a) (3.3b) (3.3c) (3.3d)

Thus we see that in the case considered the off-diagonal Eqn. (2.10g) can be ignored thanks to (2.9). And it is a must condition. In this case anisotropy in the system can be introduced only along \(x_3\) axis, i.e., magnetic field, cosmic string etc. should be directed along \(x_3\) axis.

**Case II** In this case from (3.1) one finds \(f' = \text{const.} = 1\) (say), which leads to \(f = x_3\) and \(h = x_3^2/2\). In this case Eq. (2.13e) leads to

\[
\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3} = 0.
\]  

(3.4)

Taking into account that

\[
\frac{\ddot{a}_2}{a_2} = \frac{d}{dt} \left( \frac{\dot{a}_2}{a_2} \right) + \left( \frac{\dot{a}_2}{a_2} \right)^2 \frac{d}{dt} \left( \frac{\dot{a}_3}{a_3} \right) + \left( \frac{\dot{a}_3}{a_3} \right)^2 = \frac{\ddot{a}_3}{a_3},
\]  

(3.5)

subtraction of (2.13c) from (2.13b) leads to

\[
T_1^1 = T_3^3.
\]  

(3.6)

It means, in case of \(f' \neq 0\) the model is locally rotationally symmetric and the matter distribution is isotropic, i.e., \(T_1^1 = T_2^2 = T_3^3\).

The corresponding metric takes the form [17]

\[
ds^2 = dt^2 - a_1^2 dx_1^2 - \left[ \frac{1}{4} x_3^4 a_1^2 + x_3^2 a_3^2 \right] dx_2^2 - a_2^2 dx_3^2 + a_1^2 x_3^2 dx_1 dx_2,
\]  

(3.7)

The Einstein system in this case reads

\[
\begin{align*}
2 \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_2}{a_2}^2 - \frac{3}{4} \frac{a_1^2}{a_2^2} &= -\kappa p, \\
\frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1} \frac{\dot{a}_2}{a_2} + \frac{1}{4} \frac{a_3^2}{a_2^2} &= -\kappa p, \\
2 \frac{\ddot{a}_1}{a_1} \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_2}{a_2}^2 - \frac{1}{4} \frac{a_1^2}{a_2^2} &= -\kappa \epsilon,
\end{align*}
\]  

(3.8a) (3.8b) (3.8c)

where we define \(T_1^1 = T_2^2 = T_3^3 = -p\) and \(T_0^0 = \epsilon\). Thus we see that the model Universe can be described by these three equations if and only if the pressure distribution is isotropic.
B. BVIII metric

The metric (2.1) gives rise to BVIII space time if \( \delta = -1 \),

\[
\frac{\partial^2 f}{\partial x_3^2} - f = 0.
\]  

(3.9)

One of the solutions to (3.9) is \( f = \sinh(x_3) \) with \( h = \cosh(x_3) \). As it was proved for BII model with \( f' \neq 0 \), the BVIII is also locally rotationally symmetric and its matter distribution is isotropic, i.e., \( T_1^1 = T_2^2 = T_3^3 \).

The corresponding metric takes the form [17]

\[
ds^2 = dt^2 - a_1^2 dx_1^2 - [\cosh^2(x_3)a_1^2 + \sinh^2(x_3)a_2^2]dx_2^2 - a_3^2 dx_3^2 + 2a_1^2 \cosh(x_3)dx_1dx_2,
\]  

(3.10)

The Einstein system in this case reads

\[
2 \ddot{a}_2 + \frac{\dot{a}_2^2}{a_2} - \frac{3 a_1^2}{4 a_2^4} - \frac{1}{a_2^2} = \kappa p,
\]  

(3.11a)

\[
\frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_2}{a_1} + \frac{\dot{a}_2}{a_1 a_2} + \frac{1}{4 a_2^4} = \kappa p,
\]  

(3.11b)

\[
2 \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2^2}{a_2} - \frac{1}{4 a_2^4} = -\kappa \dot{e}.
\]  

(3.11c)

Thus we see that for the system to be consistent the pressure distribution should be isotropic.

C. BIX metric

The metric (2.1) gives rise to BIX space time if \( \delta = 1 \),

\[
\frac{\partial^2 f}{\partial x_3^2} + f = 0.
\]  

(3.12)

One of the solutions to (3.12) is \( f = \sin(x_3) \) with \( h = \cos(x_3) \). It can be emphasized that like BVIII the BIX is also locally rotationally symmetric and its matter distribution is isotropic, i.e., \( T_1^1 = T_2^2 = T_3^3 \).

The corresponding metric takes the form [16–18]

\[
ds^2 = dt^2 - a_1^2 dx_1^2 - [\cos^2(x_3)a_1^2 + \sin^2(x_3)a_2^2]dx_2^2 - a_3^2 dx_3^2 + 2a_1^2 \cos(x_3)dx_1dx_2,
\]  

(3.13)

The Einstein system in this case reads

\[
2 \ddot{a}_2 + \frac{\dot{a}_2^2}{a_2} - \frac{3 a_1^2}{4 a_2^4} + \frac{1}{a_2^2} = \kappa p,
\]  

(3.14a)

\[
\frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_2}{a_1} + \frac{\dot{a}_2}{a_1 a_2} + \frac{1}{4 a_2^4} = \kappa p,
\]  

(3.14b)

\[
2 \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2^2}{a_2} - \frac{1}{4 a_2^4} = -\kappa \dot{e}.
\]  

(3.14c)

As in previous cases, consideration of (3.14) imposes the isotropic distribution of matter, otherwise the system becomes inconsistent.
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IV. CONCLUSION

Within the scope of anisotropic non-diagonal Bianchi type-II, VIII and IX spacetime we study the role of the off-diagonal components of the corresponding metric that it plays on the choice of the energy momentum tensor in general. It is shown that if the energy momentum tensor of the source field possesses only non-zero diagonal components:

\[ T_{\mu}^{\nu} = \{ T_0^0, T_i^i, T_j^j, T_k^k \} \] (4.1)

and the space-time is given by the metric

\[ ds^2 = dt^2 - a_i^2(t)dx_i^2 - [h^2(x_k) a_i^2(t) + f^2(x_k) a_j^2(t)] dx_j^2 - a_k^2(t) dx_k^2 + 2a_i^2(t) h(x_k) dx_i dx_j, \] (4.2)

with \( a_i, a_j, a_k \) being the functions of \( t \) and \( h, f \) functions of \( x_k \) only, the off-diagonal components of the Einstein tensor immediately lead to

\[ T_i^i = T_j^j. \] (4.3)

In case of \( f' = 0 \), which is a partial case of BII space-time, the matter distribution may be anisotropic with magnetic field, cosmic string etc. directed along \( x_k \) axis. In case of \( f' \neq 0 \), the metric functions \( a_j \) and \( a_k \) are linearly dependent which gives rise to locally rotationally symmetric (LRS) Bianchi models. Moreover, in this case the off-diagonal components of Einstein tensor leads to the isotropic distribution of matter, i.e., the components of the energy momentum tensor must satisfy

\[ T_i^i = T_j^j = T_k^k. \] (4.4)

If the energy momentum tensor possesses non-zero off-diagonal components, for the system to be consistent, the relation

\[ T_j^i = h(T_j^j - T_i^i) + (h^2 + f^2 a_j^2/ a_i^2) T_j^i, \] (4.5)

must hold.

In this note we give some general remarks on the choice of energy momentum tensor. We plan to study the evolution of the Universe given by Bianchi models filled with spinor, scalar and other fields in near future.

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