The ‘Higgs’ amplitude mode at the two-dimensional superfluid/Mott insulator transition

Manuel Endres1, Takeshi Fukuhara1, David Pekker2, Marc Cheneau1, Peter Schauß1, Christian Gross1, Eugene Demler3, Stefan Kühn4 & Immanuel Bloch1,5

Spontaneous symmetry breaking plays a key role in our understanding of nature. In relativistic quantum field theory, a broken continuous symmetry leads to the emergence of two types of fundamental excitation: massless Nambu–Goldstone modes and a massive ‘Higgs’ amplitude mode. An excitation of Higgs type is of crucial importance in the standard model of elementary particle physics1, and also appears as a fundamental collective mode in quantum many-body systems2. Whether such a mode exists in low-dimensional systems as a resonance-like feature, or whether it becomes overdamped through coupling to Nambu–Goldstone modes, has been a subject of debate2–9. Here we experimentally find and study a Higgs mode in a two-dimensional neutral superfluid close to a quantum phase transition into a Mott insulating phase. We unambiguously identify the mode by observing the expected reduction in frequency of the onset of spectral response when approaching the transition point. In this regime, our system is described by an effective relativistic field theory with a two-component quantum field2–8, which constitutes a minimal model for spontaneous breaking of a continuous symmetry. Additionally, all microscopic parameters of our system are known from first principles and the resolution of our measurement allows us to detect excited states of the many-body system at the level of individual quasi-particles. This allows for an in-depth study of Higgs excitations that also addresses the consequences of the reduced dimensionality and confinement of the system. Our work constitutes a step towards exploring emergent relativistic models with ultracold atomic gases.

Higgs modes are amplitude oscillations of a quantum field and appear as collective excitations in quantum many-body systems as a consequence of spontaneous breaking of a continuous symmetry. Close to a quantum critical point, the low-energy physics of such systems is in many cases captured by an effective Lorentz-invariant critical theory2. The minimal version of such a theory describes the dynamics of a complex order parameter \( \Psi = |\Psi| e^{i\phi} \) near a quantum phase transition between an ordered (\( |\Psi| > 0 \)) and a disordered phase (\( |\Psi| = 0 \)). Within the ordered phase, the classical energy density has a ‘Mexican hat’ shape (Fig. 1a) and the order parameter takes on a non-zero value in the minimum of this potential. Its phase, \( \phi \), thereby acquires a definite value through spontaneous breaking of the rotational symmetry (that is, U(1) symmetry). Expanding the field around the symmetry-broken ground state leads to two types of mode: a Nambu–Goldstone mode and a Higgs mode. These modes are related to phase and amplitude variations of \( \Psi \), respectively (Fig. 1a). In contrast to the phase mode, the amplitude mode has a finite excitation gap (that is, a finite mass), which is expected to show a characteristic softening when approaching the disordered phase (Fig. 1a). The sketched minimal model of an order parameter with \( N = 2 \) components belongs to a class of O(\( N \)) relativistic field theories, which are essential for the study of quantum phase transitions2.

Despite the fundamental nature of the amplitude mode, a full theoretical understanding of it has not yet been achieved. In particular, the decay of the amplitude mode into lower-lying phase modes, especially in two dimensions, has led to considerable theoretical interest. Specifically, it has been discussed whether a resonance-like feature persists or the decay results in a low-frequency divergence2–8.

The earliest experimental evidence for a Higgs mode stems from the observation of an unexpected peak in Raman scattering in a superconducting charge density wave compound10, which was later interpreted as a signal of an amplitude mode11. Further examples of experiments in solid-state systems can be found in ref. 6. None of these experiments have studied the mode spectrum across a quantum phase transition, except for neutron scattering experiments on quantum antiferromagnets11. In contrast to the work presented here, a resonance-like response of an amplitude mode is expected in these systems, because the phase transition occurs in three dimensions.

Ultracold bosonic atoms in optical lattices offer unique possibilities to study quantum phase transitions in a system with reduced dimensionality11. These systems are nearly ideal realizations of the Bose–Hubbard model, which is parameterized by a tunnelling amplitude \( J \) and an on-site interaction energy \( U \) (Methods). The coupling parameter \( f = J/U \) is easily tunable via the lattice depth, and the dimensionality of the system can be reduced by suppressing hopping in a certain direction12. At a critical coupling \( f_c \) and commensurate filling, the system undergoes a quantum phase transition from a superfluid (ordered) to a Mott insulating (disordered) phase13, which is described by an O(2) relativistic field theory13. A number of theoretical works have studied the Higgs mode in this system13–15. In particular, it has been argued that a modulation of the lattice depth can reveal a Higgs mode even in a two-dimensional system6–8.

Previous experiments using a lattice modulation amplitude of 20% were unable to identify the gapped amplitude mode6,16, most likely owing to the strong drive. A recent theoretical analysis of experiments using Bragg scattering in three-dimensional superfluids interpreted parts of the measured spectrum to be the result of nonlinear coupling to a short-wavelength amplitude mode17. Here we experimentally study the long-wavelength and low-energy response, which is described by a relativistic field theory at the quantum critical point.

Our experiment began with the preparation of a two-dimensional, degenerate gas of \(^{87}\text{Rb} \) atoms in a single antinode of an optical standing wave18. To realize different couplings \( J \), we loaded the two-dimensional gas into a square optical lattice with variable depth \( V_0 \) (Fig. 1b). For our trapping parameters and atom numbers (Methods), the density in the centre of the trap is typically one atom per lattice site. We then modulated the lattice depth with an amplitude of 3% at variable frequencies \( \nu_{\text{mod}} \). The modulation time \( T_{\text{mod}} \) was set to 20 oscillation cycles (\( T_{\text{mod}} = 20/\nu_{\text{mod}} \)), thus avoiding an unwanted enhanced response at higher frequencies present in experiments with a fixed modulation time18,21. We allowed for an additional hold time such that the sum of modulation and hold time was constant at 200 ms. To quantify the response, we adiabatically increased the lattice depth to reach the atomic limit (\( j \approx 0 \)) and measured the temperature of the
The results for selected lattice depths $V_0$ are shown in Fig. 2b. We observe a gapped response with an asymmetric overall shape that will be analysed in the following paragraphs. Notably, the maximum observed temperature after modulation is well below the ‘melting’ temperature for a Mott insulator in the atomic limit, $T_{\text{mol}} \approx 0.2U/k_B$ ($k_B$, Boltzmann’s constant), demonstrating that our experiments probe the quantum gas in the degenerate regime. To obtain numerical values for the onset of spectral response, we fitted each spectrum with an error function centred at a frequency $v_0$ (Fig. 2b, black lines). With $j$ approaching $j_c$, the shift of the gap to lower frequencies is already visible in the raw data (Fig. 2b) and becomes even more apparent for the fitted gap $v_0$ as a function of $j/j_c$ (Fig. 2a, filled circles). The $v_0$ values are in quantitative agreement with a prediction for the Higgs gap $v_{\text{SF}}$ at commensurate filling (solid line):

$$h v_{\text{SF}} / U = \left[ \frac{3}{\sqrt{2} - 4} (1 + j/j_c) \right]^{1/2} (j (j/c) - 1)^{1/2}$$

Here $h$ denotes Planck’s constant. This value is based on an analysis of variations around a mean-field state through the manuscript, we have rescaled $j_c$ in the theoretical calculations to match the value $j_c \approx 0.06$ obtained from quantum Monte Carlo simulations.

The sharpness of the spectral onset can be quantified by the width of the fitted error function, which is shown as vertical dashed lines in Fig. 2a. Approaching the critical point, the spectral onset becomes sharper, and the width normalized to the centre frequency $v_0$ remains constant (Supplementary Fig. 3). The constancy of this index indicates that the width of the spectral onset scales with the distance to the critical point in the same way as the gap frequency.

We observe similar gapped responses in the Mott insulating regime (Supplementary Information and Fig. 5a), with the gap closing continuously when approaching the critical point (Fig. 2a, open circles). We interpret this as a result of combined particle and hole excitations with a frequency given by the Mott excitation gap that closes at the transition point. The fitted gaps are consistent with the Mott gap $v_{\text{MIG}} / U = \left[ 1 + (12\sqrt{2} - 17) j/j_c \right]^{1/2} (1 - j/j_c)^{1/2}$ where $v_{\text{MIG}}$ is the Mott gap as predicted by mean-field theory (Fig. 2a, dashed line).

The observed softening of the onset of spectral response in the superfluid regime has led to an identification of the experimental signal with a response from collective excitations of Higgs type. To gain further insight into the full in-trap response, we calculated the eigenspectrum of the system in a Gutzwiller approach (Methods and Supplementary Information). The result is a series of discrete eigenfrequencies (Fig. 3a), and the corresponding eigenmodes show in-trap superfluid density distributions, which are reminiscent of the vibrational modes of a drum (Fig. 3b). The frequency of the lowest-lying amplitude-like eigenmode $v_{0,C}$ closely follows the long-wavelength prediction for homogeneous commensurate filling $v_{\text{SF}}$ over a wide range of couplings $j/j_c$ until the response rounds off in the vicinity of the critical point due to the finite size of the system (Fig. 3c). Fitting the low-frequency edge of the experimental data can be interpreted as extracting the frequency of this mode, which explains the good quantitative agreement with the prediction for the homogeneous commensurate filling in Fig. 2a. Modes at different frequencies from the lowest-lying amplitude-like mode broaden the spectrum only above the onset of spectral response.

An eigenmode analysis, however, does not yield any information about the finite spectral width of the modes, which stems from the interaction between amplitude and phase excitations. We will consider the question of the spectral width by analysing the low-, intermediate- and high-frequency parts of the response separately. We begin by examining the low-frequency part of the response, which is expected to be governed by a process coupling a virtually excited amplitude mode to a pair of phase modes with opposite momenta. As a result, the response of a strongly interacting, two-dimensional superfluid is
Figure 2 | Softening of the Higgs mode. a, The fitted gap values $\nu_0/U$ (circles) show a characteristic softening close to the critical point in quantitative agreement with analytic predictions for the Higgs and the Mott gap (solid line and dashed line, respectively; see text). Horizontal and vertical error bars denote the experimental uncertainty of the lattice depths and the fit error for the centre frequency of the error function, respectively (Methods). Vertical dashed lines denote the widths of the fitted error function and characterize the sharpness of the spectral onset. The blue shading highlights the superfluid region. b, Temperature response to lattice modulation (circles and connecting blue line) and fit with an error function (solid black line) for the three different points labelled in a. As the coupling $j$ approaches the critical value $j_c$, the change in the gap values to lower frequencies is clearly visible (from panel 1 to panel 3). Vertical dashed lines mark the frequency $\nu_0/U$ corresponding to the on-site interaction. Each data point results from an average of the temperatures over $\sim50$ experimental runs. Error bars, s.e.m.

Figure 3 | Theory of in-trap response. a, A diagonalization of the trapped system in a Gutzwiller approximation shows a discrete spectrum of amplitude-like eigenmodes. Shown on the vertical axis is the strength of the response to a modulation of $j$. Eigenmodes of phase type are not shown (Methods) and $Y_{0,G}$ denotes the gap as calculated in the Gutzwiller approximation, a.u., arbitrary units. b, In-trap superfluid density distribution for the four amplitude modes with the lowest frequencies, as labelled in a. In contrast to the superfluid density, the total density of the system stays almost constant (not shown). c, Discrete amplitude mode spectrum for various couplings $j/j_c$. Each red circle corresponds to a single eigenmode, with the intensity of the colour being proportional to the line strength. The gap frequency of the lowest-lying mode follows the prediction for commensurate filling (solid line; same as in Fig. 2a) until a rounding off takes place close to the critical point due to the finite size of the system. d, Comparison of the experimental response at $V_0 = 9.5E_r$ (blue circles and connecting blue line; error bars, s.e.m.) with a $2 \times 2$ cluster mean-field simulation (grey line and shaded area) and a heuristic model (dashed line; for details see text and Methods). The simulation was done for $V_0 = 9.5E_r$ (grey line) and for $V_0 = (1 \pm 0.02) \times 9.5E_r$ (shaded grey area), to account for the experimental uncertainty in the lattice depth, and predicts the energy absorption per particle $AE$.

Figure 4 | Scaling of the low-frequency response. The low-frequency response in the superfluid regime shows a scaling compatible with the prediction $(1 - j/j_c)^{-2/3}$ (Methods). Shown is the temperature response rescaled with $(1 - j/j_c)^2$ for $V_0 = 10E_r$ (grey), $9.5E_r$ (black), $9E_r$ (green), $8.5E_r$ (blue) and $8E_r$ (red) as a function of the modulation frequency. The black line is a fit of the form $a \nu^b$ with a fitted exponent $b = 2.9(5)$. The inset shows the same data points without rescaling, for comparison. Error bars, s.e.m.
approximately valid in our case because the frequency spacing between different phase modes is much smaller than the typical gap to the lowest amplitude mode. The model yields quantitative agreement with the low- to intermediate-frequency experimental data for a range of couplings (Fig. 3d, dashed black line, and Supplementary Information), where a relativistic field theoretical treatment of this type is applicable. Furthermore, the response at frequencies more than twice that at the absorption edge remains slightly underestimated.

Part of this high-frequency response might stem from the excitation of several amplitude modes or combinations of amplitude and phase modes, which cannot be described with the Gutzwiller approximation used so far and is only partly captured in the field theoretical treatment. Therefore, we performed a dynamical simulation based on a $2 \times 2$ cluster variational wavefunction, which captures the excitation of multiple modes as well as intermode coupling, at least at high momenta. The result is compared with experimental data in Fig. 3d and shows good overall agreement (also compare Fig. 5a with Fig. 5b near the critical point). Notably, the simulation predicts the low-frequency edge, the overall width and the absolute strength of the experimental critical point. Notably, the simulation predicts the low-frequency edge, the overall width and the absolute strength of the experimental critical point. Notably, the simulation predicts the low-frequency edge, the overall width and the absolute strength of the experimental critical point. Notably, the simulation predicts the low-frequency edge, the overall width and the absolute strength of the experimental critical point. Notably, the simulation predicts the low-frequency edge, the overall width and the absolute strength of the experimental critical point.

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Figure 5 | Response between the strongly interacting limit and the weakly interacting limit. a, Change in temperature $\Delta T$ as a function of $j/j_c$ and the modulation frequency $\nu_{\text{mod}}$ in units of $U$. A pronounced feature close to $j/j_c = 1$ directly shows the existence of the gap and its softening, which is also observed in units of $f$ (Supplementary Fig. 8). As the weakly interacting limit (higher values of $j/j_c$) is approached, the response broadens and vanishes. b, Simulation using a variational $2 \times 2$ cluster wavefunction predicting the energy absorption per particle $AE$ for the same parameter range. The simulation shows agreement with the experimental data near the critical point in both the softening of the response and the overall width of the absorption band. However, the simulation does not fully reproduce the vanishing of the response at higher $j/j_c$ values. A splitting in the excitation structure at $j/j_c = 3$ is visible, which might also be present in the experimental data. A low-frequency-charge associated with density oscillations at the edges of the trap due to the excitation of phase-like modes is clearly seen in the simulations. This feature occurs below the lowest measured frequency in the experiment and thus is not visible in a (except in the vicinity of the critical point, where the lowest modulation frequencies are close to this feature). Black solid lines show the mean-field predictions as plotted in Fig. 2a.

$2 \times 2$ cluster treatment still cannot fully capture the broadening of the modes due to coupling to low-energy phase modes.

Our analysis so far has shown the existence of an amplitude mode in the Bose–Hubbard model close to the critical point ($j/j_c \approx 1$), where the low-energy description of the system is approximately Lorentz invariant. In the weakly interacting limit ($j/j_c \gg 1$), however, the low-energy description (Gross–Pitaevskii theory) becomes effectively Galilean invariant, which precludes the existence of such a mode. To probe the evolution of the amplitude mode response when approaching the weakly interacting limit, we extended our measurements to higher values of the coupling $j$. The results are shown in Fig. 5a as a density plot, where a pronounced signal for $j/j_c \leq 3$ directly shows the softening of the mode close to the critical point. As the weakly interacting limit is approached with higher $j/j_c$ values, the response gradually broadens and finally disappears. Despite earlier theoretical treatments of the system in this regime\(^{27,28}\), a prediction of the disappearance of the response is still lacking. Also, results of the $2 \times 2$ cluster variational wavefunction approximation could only partly capture this effect (Fig. 5b).

In conclusion, we have identified and studied long-wavelength Higgs modes in a neutral, two-dimensional superfluid close to the quantum phase transition to a Mott insulating state. This was made possible by recent advances in the high-resolution imaging of single atoms in optical lattices\(^{29,30}\), leading to a new level of precision for the spectroscopy of ultracold quantum gases. The obtained spectra show softening at the quantum phase transition and are consistent with the generic $^3$ low-frequency scaling for a rotationally invariant coupling to the order parameter in a two-dimensional, strongly interacting superfluid. Furthermore, our results require the development of a quantitative theory valid between the strongly and the weakly interacting regimes capable of predicting the observed disappearance of the response. Our data also call for a first-principles treatment of the discrete nature of Higgs modes in a confined system. In this regard, we note an interesting connection to particle physics, where the spectrum of the Higgs boson in theories with compact extra dimensions\(^{30}\) may acquire a similar discrete spectrum.

**METHODS SUMMARY**

The preparation of the two-dimensional, degenerate gas in a single antinode of a vertical optical standing wave is described in ref. 23. The lattice constant for the vertical and both horizontal optical lattices was $a_{\text{lat}} = 532$ nm and the trapping frequencies for the two-dimensional system were typically $60$ Hz. We calibrated the lattice depths by performing amplitude modulation spectroscopy\(^{23}\). The Bose–Hubbard parameters $J$ and $U$ were calculated from the lattice depths by a numerical band-structure calculation\(^{13}\). Our systems contained an atom number of $190(36)$ (where the value in parentheses denotes the standard deviation), resulting in a central density of close to one atom per lattice site. The data point in Fig. 2a at $j/j_c = 1.1$ was measured with slightly different parameters ($T_{\text{tot}} = 300$ ms and $T_{\text{mod}} = 15/T_{\text{mod}}$) where $T_{\text{tot}}$ is the sum of modulation time $T_{\text{mod}}$ and additional hold time (see Fig. 1b) and is excluded from Figs 4 and 5a. We fitted the temperature response with

$$T = T_0 + \frac{\Delta T}{2} \left\{ \frac{1}{\sqrt{3}} \left( (\nu_{\text{mod}} - \nu_0) + 1 \right) \right\}$$

where erf($x$) denotes the error function. The fitting parameters were the temperature offset $T_0$, the temperature increase $\Delta T$, the width $\sigma$, and the centre frequency $\nu_0$. For the plot in Fig. 4, we chose frequencies in a span from $\nu_0 + 1.5\sigma$ to $\nu_0 + 0.5\sigma$, with $\nu_0$ and $\sigma$ taken from the error-function fit of the individual responses. To perform the eigenmode analysis in Fig. 3a–c, we first calculated the ground-state wavefunction in a Gutzwiller approximation\(^{16,22}\) and then linearized the equations of motion. The resulting eigenvalue problem was solved by a Bogoliubov transformation. Furthermore, we separated the Bose–Hubbard Hamiltonian into a time-independent part and a time-dependent part that describes the lattice modulation: $H_{\text{BH}} = H_0 + \sin(\omega t)H'$. The line strengths in Fig. 3a, c were calculated using Fermi’s golden rule and are proportional to the square of the matrix element of $H'$. Further details, as well as descriptions of the low-frequency scaling, the heuristic model and the cluster wavefunction approach, can be found in Methods.
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Full Methods and any associated references are available in the online version of the paper at www.nature.com/nature.

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METHODS

Experimental details. The preparation of the two-dimensional, degenerate gas is described in ref. 23. During the experiment, the gas was held in a single antinode of a vertical optical standing wave with a depth of \(20(2)E_r\), where \(E_r\) denotes the lattice recoil energy (\(E_r = h^2 / 8ma^2\)) with \(m\) the atomic mass of \(^87\)Rb. The lattice constant for the vertical and both horizontal optical lattices was \(a_{\text{lat}} = 532\) nm and the trapping frequencies for the two-dimensional system were typically 60 Hz. The ramp for lattice loading and the ramp to the atomic limit were \(s^\prime\) shaped with total durations of 120 and 75 ms, respectively. Our systems had an atom number of 190(36) (where the value in parentheses denotes the standard deviation), resulting in a central density of close to one atom per lattice site. The data point in Fig. 2a at \(f^t/\hbar = 1.1\) was measured with slightly different parameters (\(T_{\text{mod}} = 300\) ms and \(T_{\text{mod}} = 15\) ms). The uncertainties in the coupling analysis, we used the Gutzwiller trial wavefunction\(^{16,22}\) in the entire trap. Next we linearized the equations of motion, which were obtained by minimizing the effective action \(\mathcal{S}_{\text{eff}}(\Psi_1)\) around the stationary solution. The resulting eigenvalue problem was solved by a Bogoliubov transformation \(M_{kk}'\) that relates Bogoliubov creation operators \(\hat{f}_k^\dagger\) (and annihilation operators \(\hat{f}_k\)) to the small fluctuations \(\delta\hat{a}_k\) and \(\delta\hat{\phi}_k\) around the stationary solution

\[
\mathcal{J}_k = \sum_i \left( M_{kl} \hat{f}_{k,1}^\dagger + M_{kl} \hat{f}_{l,1}^\dagger \right)
\]

where \(k\) is the eigenmode index. We can identify the modes as amplitude-like or phase-like using a measure of ‘amplitude’

\[
A = \sum_{\ell} |M_{kk}|^2
\]

which is positive for amplitude-like modes and negative for phase-like modes (Supplementary Information).

To describe lattice modulation spectroscopy, we separated the Bose–Hubbard Hamiltonian into a time-independent part that describes the system with no modulation and a time-dependent part that describes the lattice modulation: \(H_{\text{mod}} = H_0 + \sin(\omega t)U\). The rate of excitation of the 5th amplitude or phase mode is given by Fermi’s golden rule: \(\mathcal{F}(\omega) = \delta(\omega - \omega_0) |\langle \psi_{\text{mod}} || \mathcal{J}_5 \rangle|^2\). The line strengths plotted in Fig. 3a are proportional to \(S_i = |\langle \psi_{\text{mod}} || \mathcal{J}_i \rangle|^2\). The line strengths decrease with increasing frequency, because higher-energy modes show short-wavelength spatial variations and do not efficiently couple to lattice modulation.

Heuristic model. We constructed a heuristic model (dashed line in Fig. 3d and Supplementary Information), which combines the frequencies and line strengths of our Gutzwiller calculation with the shape of the response calculated by field theoretical methods. For a given \(j\) value, the Gutzwiller approach yields a series of amplitude-like normal modes with frequencies \(\nu_j\) and corresponding line strengths \(S_j\). The heuristic model consists of summing up a response function \(F(v, v_{\text{mod}})\) for each of these frequencies weighted with the corresponding line strengths. A calculation based on a large-N expansion of a two-dimensional O(N) field theory\(^{16}\) yielded a scalar response function for the homogeneous and commensurate system of the form

\[
F(v, v_{\text{mod}}) \propto \frac{v_{\text{mod}}}{(v_{\text{mod}} - v)^2 + 4\nu_j^2 v_{\text{mod}}^2}
\]

A parameterization of the \(N = 2\) case of the model can be found in ref. 7 and yields \(b_0/\hbar = 1/8\). Assuming this response function at each individual normal mode (and measuring all frequencies in units of \(\hbar b_0\)) results in the final model function

\[
F_h(v_{\text{mod}}) = A_1 + A_2 \sum_j \frac{S_j v_{\text{mod}}}{(v_{\text{mod}} - \nu_j)^2 + 4\nu_j^2 v_{\text{mod}}^2}
\]

with \(\gamma = 1/8\) and fit parameters \(A_1\) and \(A_2\).

Dynamical evolution: \(2 \times 2\) cluster wavefunctions. We performed a study of the dynamical evolution of the system using \(2 \times 2\) cluster variational wavefunctions

\[
|\psi_{\text{mod}}\rangle = \prod_i \left( |a_i(t)\rangle + b_i(t) |1\rangle + c_i(t) |0\rangle \right)
\]

where \(a_i(t), b_i(t), c_i(t)\) are the variational parameters. We restrict the maximum occupation number per site to two. To initialize the dynamics, we obtained the initial trial wavefunction, corresponding to the state of the system before modulation spectroscopy begins, by minimizing \(|\langle \psi_{\text{mod}} || H(\Psi_1) \rangle|\). Next we dynamically evolved the trial wavefunction during the modulation drive, the hold time and the ramp to the atomic limit (Fig. 1b). Finally we measured the total energy absorption per particle \(\Delta E\) of the resulting state (in units of the on-site interaction \(U\) in the atomic limit).