SA-DPSO-based Weapon-target Assignment for Cooperative Air Defense with Anti-aircraft Gun-Missile Weapons

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Abstract. For the weapon-target assignment problem of anti-aircraft weapon formation air defense, a weapon-target assignment model was developed for anti-aircraft gun-missile weapon formation according to the different fire characteristics of anti-aircraft missile and anti-aircraft gun system. The model gave sufficient consideration to the damage zone of anti-aircraft gun and missile as well as the weapon-target join point, and the assignment result allows anti-aircraft gun-missile weapons to fire missiles and anti-aircraft guns at the right moment. On this basis, the method of Simulated Annealing-Discrete Particle Swarm Optimization (SA-DPSO) was proposed to solve the weapon-target assignment for formation anti-aircraft. The simulation result showed good rate of convergence and global searching ability of the algorithm.

1. Introduction

Anti-aircraft gun-missile weapons are anti-aircraft weapons formed by anti-aircraft missiles and small-caliber anti-aircraft guns through combining the public searching, tracking and fire control systems. They combine the advantages of anti-aircraft missiles in high firing accuracy and long firing range and of anti-aircraft guns in quick response, intensive fire and high damage probability at short range in a complementary manner so as to achieve the best defense effect[1-2]. The maturing method and theory of weapon-target assignment based on a single anti-aircraft gun-missile weapon mainly deals with the problem of when to fire missiles and when to fire anti-aircraft guns. Nowadays, air raids tend to apply multi-batch, all-range, multi-sortie and other continuous attack patterns. Ground anti-aircraft forces are often formed by multiple missile-gun weapons to attack several incoming targets through information sharing and mutual collaboration. How to make accurate and efficient weapon-target assignment plans for anti-aircraft gun-missile weapon groups has become a difficult problem to be solved in modern anti-aircraft operation[3-6].

This paper developed a mathematic model of weapon-target assignment and proposed relevant optimization plans, gave the weapon-target mode for anti-aircraft gun-missile weapons and to bring anti-aircraft gun-missile system into full play. According to the different fire characteristics of missiles and anti-aircraft guns, the model took account of anti-aircraft gun and missile’s weapon-target join point and damage boundary, to clarify the firing of missiles or anti-aircraft guns with anti-aircraft gun-missile weapons. Simulated Annealing-Discrete Particle Swarm Optimization (SA-DPSO) algorithm
was proposed to solve the weapon-target assignment problem and to improve the efficiency of the algorithm in searching globally optimal solutions.

2. Damage probability of anti-aircraft gun-missile weapons

There are both head-on attack area and tail-on attack area for missiles while guns only have attack head-on attack area. Figure 1 below shows the probability curve of missiles and anti-aircraft guns damaging targets.

Figure 1. Damage probability of anti-aircraft guns vs missiles

As shown in Figure 1, there is a weapon-target join point in the damage probability curve of missiles and anti-aircraft guns, which, as the main condition for weapon-target transition, ensures the maximum damage probability of incoming air targets. When targets are farther away than the weapon-target join distance, the efficiency of target inception by missiles is higher. In such case, missiles are preferred for target interception. Otherwise, anti-aircraft guns are preferred. When targets overfly, missiles are used for interception.

3. Weapon-target assignment model

We assume that an anti-aircraft gun-missile weapon formation has $m$ anti-aircraft gun-missile weapons and $n$ faces incoming targets. $p_{ij}$ is the probability of the $i_{th}$ anti-aircraft gun-missile weapon $i_{th}$ damaging the $j_{th}$ target. Let $x_{ij}$ be decision variable.

\[
\begin{align*}
    x_{ij} &= 1 & \text{if } \text{anti-aircraft gun-missile weapon } i \text{damages target } j \\
    x_{ij} &= 0 & \text{if } \text{anti-aircraft gun-missile weapon } i \text{does not damage target } j
\end{align*}
\]

Then, the weapon-target assignment model of anti-aircraft gun-missile weapon formation is[8]

\[
E = \max \sum_{j=1}^{n} W_j [1 - \prod_{i=1}^{m} (1 - p_{ij})^{x_{ij}}]
\]

\[s.t.\]

\[
\begin{align*}
    p_{ij} &= 1 - \prod_{i=1}^{m} (1 - p_{ij})^{x_{ij}} \geq P_{\text{max}} \\
    p_{ij} &= P_{V, \text{missile}}, \quad \text{if } i_{th} \text{ anti-aircraft gun-missile weapon attacks } j_{th} \text{ target with a missile} \\
    p_{ij} &= P_{V, \text{gun}}, \quad \text{if } i_{th} \text{ anti-aircraft gun-missile weapon attacks } j_{th} \text{ target with fire guns} \\
    p_{ij} &= 0, \quad \text{if } j_{th} \text{ target does not enter the attack range of } i_{th} \text{ anti-aircraft gun-missile weapon}
\end{align*}
\]

\[
\sum_{j=1}^{n} x_{ij} \leq 1 \quad \sum_{i=1}^{m} x_{ij} \leq 4
\]
Where: \( E \) is the objective function value; \( W_{j} \) is the threat degree of \( j_{th} \) target; \( P_{\min} \) is the damage probability threshold of the \( j_{th} \) target. The assignment model was analyzed as below:

1) If the joint damage probability of some target \( P_{j} \) is lower than the damage probability threshold, the assignment of the target is considered to be null.

2) An anti-aircraft gun-missile weapon can intercept at most one target each time.

3) The missile and anti-aircraft gun of an anti-aircraft gun-missile weapon cannot fire targets at the same time.

4) To avoid excessive fire resources, no more than 4 anti-aircraft gun-missile weapons are assigned to one target.

Subject to the damage probability threshold, the model produced plans of anti-aircraft gun-missile weapon assignment with maximum damage probability by comparing the damage probability of anti-aircraft gun-missile firing targets. It turned out to be an integral nonlinear programming problem.

The advantage of DPSO algorithm lies in the fact that it reserves the operation mode of continuation PSO algorithm and it is thus possible to utilize the superiority of DPSO algorithm in simplified and less time-consuming vector calculation. However, this algorithm has disadvantages of easily falling into premature convergence and local optimum. With the action of individual extremum and global extremum, particles lose diversity very soon and the algorithm loses global searching ability. For such disadvantages, the concept of simulation annealing (SA) was introduced into DPSO[9-10]. SA algorithm was utilized to temporarily accept the properties of some inferior solutions under certain probability control to improve the basic DPSO. SA searching was done around the initial solution generated by DPSO for the purpose of focused searching within a small range and avoiding premature and enhancing algorithm convergence. Based on this, SA and DPSO are combined here to solve weapon-target assignment problem. A coding method based on real numbers was used in which particle position represented one assignment plan and particles totaled \( R \), thus the position vector

\[
X = \{X_{1}, X_{2}, \ldots, X_{m}\} \text{ of the } c_{th} \text{ particle.}
\]

\[
X_{i} = \begin{cases} \text{jth anti-aircraft gun - missile weapon is assigned to } j\text{th target} & i = 1, 2, \ldots, m \\ \text{0} & j = 1, 2, \ldots, n \end{cases}
\]

The specific process of SA-DPSO hybrid optimization algorithm is as follows:

1) DPSO parameters are initiated. Inertia weight \( \omega \), learning factors \( c_{1}, c_{2} \), and group size \( R \) are determined, and the maximum number of iterations to \( k_{\text{max}} \) is set;

2) The population of \( R \) particles is randomly generated, i.e., \( R \) initial populations \( X_{r} \) and \( R \) initial velocity \( v_{r} \), \( r = 1, 2, \ldots, R \), \( k = 1, 2, \ldots, k_{\text{max}} \) are randomly generated;

3) Each particle’s fitness value \( G_{r} \) is calculated and \( G_{r} \) is compared with individual extremum \( p_{r} \) to update \( p_{r} \) with the optimal one;

4) Each particle’s individual extremum \( p_{r} \) is compared with global extremum \( p_{g} \) to update \( p_{g} \) with the optimal one;

5) If the termination condition is satisfied, the process is complete; otherwise, 6) is performed;

6) Each particle’s flight velocity \( v_{r}^{k+1} \) and position \( X_{r}^{k+1} \) are updated and limited within \( (v_{\text{min}}, v_{\text{max}}) \) and \( (X_{\text{min}}, X_{\text{max}}) \) respectively;

7) SA algorithm is performed;

Step 1: SA parameters are initiated by setting the initial temperature \( T \) and the number of iterations \( L \) of each \( T \) value;

Step 2: Steps 3-6 are performed for \( p = 1, 2, \ldots, L \);

Step 3: New solution \( x_{r}^{p} \) is produced;

Step 4: \( e(r) = G_{r} - G_{r}^{p} \) is calculated, where \( G_{r}^{p} \) is the fitness function of the new solution;

Step 5: If \( e(r) < 0 \), \( x_{r}^{p+1} = x_{r}^{p} \) is accepted; otherwise, \( x_{r}^{p+1} = x_{r}^{p} \) is accepted with probability \( \exp(-e(r)/T) \);

Step 6: If the termination condition is satisfied, the current solution is output as the optimal solution and the process is complete; otherwise, the process proceeds to 8);
8) The temperature is lowered at annealing temperature convergence rate \( \alpha \), i.e., \( T = \alpha T \); if \( T \geq 0 \), turn to 3); otherwise, the process is completed.

4. Simulated analysis

Assume the number of anti-aircraft gun-missile weapons \( m \) is equal to 8, and the number of incoming targets \( n \) is equal to 6. Table 1 shows target’s motion parameters and threat degree. To facilitate online calculation, the damage probability of anti-aircraft gun-missile weapons was obtained from the interpolation of damage probability curve in Figure 1. The maximum range of missiles is 10km and the weapon-target join point is at 2.2km. Inertia weight \( \omega \) was reduced from the maximum weight factor \( \omega_{\text{max}} \) to the minimum weight factor \( \omega_{\text{min}} \), that is

\[
\omega = \omega_{\text{max}} - k \frac{\omega_{\text{max}} - \omega_{\text{min}}}{k_{\text{max}}}
\]

Where: \( \omega_{\text{max}} = 1.2 \), \( \omega_{\text{min}} = 0.4 \), \( k \) and \( k_{\text{max}} \) are the current number of iterations and the maximum number of iterations respectively, \( c_1 = 1.15 \), \( c_2 = 1.15 \), \( k_{\text{max}} = 120 \) the number of particles \( R = 60 \), \( P_{\text{min}} = 0.3 \), the initial temperature in SA algorithm is 10,000, temperature cooling coefficient \( c_2 \), and SA number of iterations.

Table 1. Target’s threat degree and motion parameters

| Batch No. | Velocity (m/s) | Height (m) | Shortcut flight course (m) | Initial azimuth (°) | Initial distance (m) | Target threat |
|-----------|---------------|-----------|---------------------------|-------------------|----------------------|---------------|
| 1         | 250           | 500       | 100                       | 40                | 15,000               | 0.29          |
| 2         | 200           | 300       | 150                       | 100               | 15,000               | 0.21          |
| 3         | 220           | 400       | 120                       | 160               | 15,000               | 0.15          |
| 4         | 190           | 600       | -100                      | 220               | 15,000               | 0.10          |
| 5         | 230           | 200       | -130                      | 280               | 15,000               | 0.19          |
| 6         | 170           | 330       | -110                      | 340               | 15,000               | 0.06          |

Table 2. The results of weapon-target assignment of anti-aircraft gun-missile.

| Time (s) | Weapon 1 | Weapon 2 | Weapon 3 | Weapon 4 | Weapon 5 | Weapon 6 | Weapon 7 | Weapon 8 | Efficiency value E |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|--------------------|
| 0        | 2(M₄)    | 5(M₄)    | 1(M₄)    | 3(M₄)    | 4(M₄)    | 2(M₄)    | 5(M₄)    | 1(M₄)    | 0.7835             |
| 20       | 3(M₄)    | 2(M₄)    | 3(M₄)    | 1(M₄)    | 5(M₄)    | 2(M₄)    | 5(M₄)    | 1(M₄)    | 0.5676             |
| 40       | 5(M₄)    | 4(M₄)    | 3(M₄)    | 2(M₄)    | 5(M₄)    | 2(M₄)    | 1(M₄)    | 5(M₄)    | 0.6978             |
| 60       | 2(G)     | 5(G)     | 1(M₄)    | 4(M₄)    | 2(G)     | 3(G)     | 3(G)     | 6(M₄)    | 0.5876             |
| 80       | 2(M₆)    | 5(G)     | 3(M₆)    | 2(M₆)    | 4(M₆)    | 5(M₆)    | 1(M₆)    | 1(M₆)    | 0.6679             |
| 100      | 5(M₆)    | 2(M₆)    | 4(M₆)    | 3(M₆)    | 3(M₆)    | 2(M₆)    | 4(M₆)    | 5(M₆)    | 0.5639             |

As the assignment results in Table 2 shows, while the damage probability threshold was satisfied, missiles were preferred for attacking targets with greater threat degree, for example, assigning at least two anti-aircraft gun-missile weapons each time within the first 40s to attack targets; to avoid excessive fire resources, there was no case in the assignment results that more than three anti-aircraft gun-missile weapons were used to attack one target; with anti-aircraft gun-missile weapon-target join point taken into account in the assignment results, when targets entered the gun firing zone of anti-aircraft gun-missile weapons, they were assigned to guns (from 60s to 80s); when targets overflew, anti-aircraft gun-missile weapons were assigned to use missiles to attack the targets.
To verify the performance of the proposed algorithms, the proposed SA-DPSO algorithm, basic DPSO algorithm and genetic algorithm were used to calculate the weapon-target assignment results at 40s. Figure 2 shows the change of objective function values of the 3 algorithms.

![Figure 2. Comparison of different algorithms](image)

It is observed from Figure 2 that SA-PDSO algorithm is able to find the globally optimal solution at 31st iteration with high rate of convergence. Basic DPSO algorithm and genetic algorithm have such low rate of convergence that convergence is achieved after about 70th iteration and the optimum is not found; rather, they easily fall into local optimum. This explains that the proposed SA-PDSO algorithm is rapid and effective.

5. Conclusion

This paper proposed a mathematic model of weapon-target assignment of anti-aircraft gun-missile weapon formation, which, for the reasonable use of missiles or anti-aircraft guns to attack targets, took into consideration of each anti-aircraft gun and missile weapon’s weapon-target join point and damage boundary. SA-DPSO algorithm was proposed to solve weapon-target assignment problem. The simulation result showed the weapon-target assignment results of the model were reasonable and effective to give full play to the overall effectiveness of anti-aircraft gun-missile weapon formation. The proposed SA-DPSO algorithm is rapid and effective to significantly improve the efficiency of searching globally optimal solutions.

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