Matter enhanced $\nu_\mu \to \nu_e$ signals using various Fermilab main injector beam configurations

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1 Introduction

In this report we estimate the sensitivity of experiments optimized for measuring $|U_{e3}|^2$ and the sign of $\Delta m^2_{23}$ in various Fermilab main injector beam configurations. We calculate the sensitivity to $\nu_\mu \to \nu_e$ appearance making the assumption of 3 neutrino generations, and that only solar and atmospheric neutrino oscillations signals are "real." We have implemented an exact calculation of oscillation probabilities in matter from Barger et.al. [1] for the FNAL-Soudan (732km), and hypothetical FNAL-BNL (1500km) and FNAL-SLAC (2900km) beamlines, as shown in figures 1, 2, and 3. Of course, these appearance probabilities must be folded into the neutrino flux spectrum produced by a particular Fermilab main injector neutrino beam configuration, as well as the cross-section in the far detector. For the neutrino flux spectrum we have used the GEANT simulation of the NuMI beamline [2], and for the neutrino cross-section we have used the Soudan 2 [3] event generator. For the purposes of illustration, we shall use the test point $\Delta m^2_{23} = 0.003eV^2$ (regular hierarchy), $|U_{e3}|^2 = 0.003$ (ie. $\theta_{13} = 3^\circ$), $U_{\mu 3} = U_{\tau 3} = \Delta m^2_{\text{solar}} = 0.00003eV^2$, $U_{e1} = U_{e2}$, and phase $\phi = 0$. Using these parameters, the numbers of $\nu_\mu \to \nu_e$ charged current interactions per kiloton-year (in steel) are as follows:

| Event rates for steel exposure | FNAL-Soudan | FNAL-BNL | FNAL-SLAC |
|-------------------------------|-------------|-----------|-----------|
| PH2 low $\nu_\mu \to \nu_e$ CC | 0.93 / kt-yr | 0.55 / kt-yr | 0.23 / kt-yr |
| PH2 medium $\nu_\mu \to \nu_e$ CC | 1.41 / kt-yr | 1.15 / kt-yr | 0.79 / kt-yr |
| PH2 high $\nu_\mu \to \nu_e$ CC | 0.98 / kt-yr | 0.95 / kt-yr | 0.87 / kt-yr |

Note that the peak energy values for the PH2 low, medium, and high energy beams are roughly 3, 6, and 12 GeV respectively. In this report we will consider two examples of detectors: a 10-kt “MINOS-like” detector, and a 10-kt “OPERA-like” detector optimized for $\nu_e$ appearance. We define “MINOS-like detector” in this report to be a 10-kt detector composed of alternating 2.54cm Fe plates and scintillator strips. We define the “OPERA-like” detector as follows.

In an earlier note, we have described how a hybrid emulsion detector (HED) can obtain low background measurements of $\nu_\mu \to \nu_\tau$ and $\nu_\mu \to \nu_e$ oscillation events in the NuMI beam [4]. We now consider a HED designed exclusively to measure $\nu_e$. The tracking requirements for $\nu_\tau$ appearance are more stringent than those for $\nu_e$ appearance, and thus we can significantly relax certain parameters of a HED optimized for $\nu_\tau$ [5] if we only care about identifying $\nu_e$. The main difference is that
we propose to use 5 mm Fe instead of 1 mm Pb as in OPERA [9]. Thus, for roughly the same emulsion cost as in OPERA (a 2-kt detector) we obtain a detector with 3.5 times greater mass.

The detector would be composed of 10 cm thick steel-emulsion stacks, separated by RPC planes. The RPC planes would be used for triggering and event location. Emulsion sheets would be sandwiched between 5 mm thick steel plates. If the planes have an area of 8x8 square meters, then the mass of one steel-emulsion target plane would be 50 tons. The entire detector would consist of 200 planes. Note that there is enough floor space behind the first two MINOS supermodules in the MINOS Soudan cavern for such a detector.

We now attempt to estimate how much a HED would cost. We use the cost of emulsion film from the OPERA proposal [9] (converted to US dollars). We estimate the cost of salary, wages, and institutional overhead to be the same fraction as in the MINOS TDR [4], i.e. 50% of total cost before contingency. We add a 50% contingency. This results in a total detector cost of roughly $18.4M per kt. The rough cost breakdown is shown in the following table:

| component          | cost per unit | # units | total cost |
|--------------------|---------------|---------|------------|
| emulsion film      | $150 / m^2    | 256,000 | $38.4M     |
| steel              | $1000 / ton   | 10,000  | $10.0M     |
| RPC+electronics    | $500 / m^2    | 25,600  | $12.8M     |
| wages, salary, overhead |           |         | $61.2M     |
| contingency        |               |         | $61.2M     |
| total              |               |         | $183.6M    |

We now turn to a comparison of MINOS-like and OPERA-like detector analyses.

2 MINOS-like data analysis

The MINOS-like detector analysis (taken from [7]) is based upon the full “official” GEANT simulation of the MINOS detector [8]. We use the case of PH2 medium to SLAC as an example. The basic strategy can be summarized as follows: 1) reject events with $P_\mu > 1$GeV, 2) fraction of energy in the highest energy cluster $E_{CLUST}/E_{TOT} > 0.7$ (see figure 4), 3) number of strips in the highest energy cluster $N_{STRIPS} >= 9$ (see figure 5), 4) neural net estimator consistent with $\nu_e$ (see figure 6), and 5) reject events in which the total energy ($E_{TOT}$) does not fall within some energy range optimized to $\Delta m^2$ (see figures 7 and 8). The numbers of events after each sequential cut are tabulated in table 1.

3 HED data analysis

The HED data analysis [6] is based upon simple gaussian smearing of individual Monte Carlo particle truth information. Of course, the smearing is appropriate to the HED detector under consideration. The basis for the inputs to the smearing come from the OPERA proposal [9] (which has an $X_0=0.18$ for each target plate), with the appropriate extrapolation to the thicker plates in the HED described in this report ($X_0=0.28$). The resolution on electromagnetic showers from track counting in the OPERA proposal is 20%/sqrt(E), so we use a gaussian smearing of 25%/sqrt(E) for electrons and gammas in our calculation. The resolution on charged particle momentum using the multiple Coulomb scattering method in the OPERA proposal is 16%, so we use a gaussian smearing of 20% to (non-electron) charged particles in our calculation. We use a 1 milliradian angular resolution for charged particle direction (assuming that we use the emulsion film described in the OPERA proposal) and we add in quadrature the angular smearing due to multiple coulomb scattering in the
target steel plate (using a randomly chosen scattering vertex position in the target plate).

We use the case of PH2 medium to SLAC as an example. The data reduction can be summarized as follows: 1) reject events with $P_\mu > 1 GeV$, 2) require the highest energy electromagnetic shower (either from an electron or prompt gamma) to be greater than some optimized threshold (see figure 6), 3) reject prompt gammas which do not convert before passing through the first emulsion sheet after the primary interaction vertex (this rejects 88% of prompt gammas), 4) require missing transverse momentum to be less than some optimized threshold (see figure 10), and 5) require the total visible energy to fall within an energy range optimized to $\Delta m^2$ (see figures 11 and 12). The numbers of events after each sequential cut are tabulated in table 2.

4 Results

We assume 40 kt-yr exposures of the detectors in Fermilab main injector beams whose fluxes have been upgraded by a factor of 4. We also assume that the systematic error is dominated by uncertainty in the number of background events, and that this error is 10%. With these assumptions, we may summarize the significance of the signal in units of $\sigma_{\text{stat}}$ and $\sigma_{\text{syst}}$ after all data reduction cuts in the following table:

| Significance of the signal after all cuts in units of $\sigma_{\text{stat}}$ ($\sigma_{\text{syst}}$) | MINOS-like detector | OPERA-like detector |
|---|---|---|
| | regular hierarchy, $\Delta m^2_{23} = 0.003 eV^2$, $|U_{e3}|^2 = 0.003$ | Soudan | BNL | SLAC | Soudan | BNL | SLAC |
| PH2 low | 1.5 (0.8) | 2.3 (2.1) | 1.4 (2.2) | 2.6 (2.2) | 3.2 (4.3) | 2.0 (5.2) |
| PH2 medium | 1.1 (0.4) | 1.9 (1.3) | 3.1 (2.6) | 1.4 (1.3) | 2.5 (4.2) | 4.5 (10.0) |
| PH2 high | 0.4 (0.1) | 0.7 (0.4) | 1.7 (0.8) | 1.1 (0.8) | 1.9 (2.6) | 3.3 (7.5) |

The predicted sensitivities to $|U_{e3}|^2$ as a function of $\Delta m^2_{23}$ are shown in figures 13 through 18. Two general conclusions may be made: 1) OPERA-like is better than MINOS-like, and 2) running “on the highest energy oscillation peak” is best. If we take these calculations at face value, then the best beam/site/detector setup for $\Delta m^2_{23} = 0.003 eV^2$ appears to be the PH2 medium beam pointed at SLAC/HED. (If $\Delta m^2_{23}$ is slightly higher or lower, then a slightly higher or lower energy beam is optimal.) The predicted sensitivity to $|U_{e3}|^2$ including statistical and systematic errors for a 40-kt exposure of HED at SLAC in PH2 medium (4x regular flux) is shown in figure 19. Similarly, the discovery potential for $|U_{e3}|^2$ and sign($\Delta m^2_{23}$) is shown in figure 20. Note that $\nu_\mu \rightarrow \nu_e$ appearance signals and the sign of $\Delta m^2_{23}$ can be obtained at the 3-sigma level for $\theta_{13} > 3$ degrees.

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Figure 1: Probability for $\nu_\mu \to \nu_e$ for FNAL-Soudan (L=732km), given $|U_{e3}|^2 = 0.003$ (ie. $\theta_{13} = 3^\circ$), $\Delta m_{23}^2 = 0.003 eV^2$, $\Delta m_{12}^2 = 0.00003 eV^2$, $U^2_{\mu 3} = U^2_{\tau 3}$, and $U^2_{e1} = U^2_{e2}$. The black lines are for neutrinos, and the red lines are for anti-neutrinos. The panels on the left side have regular mass hierarchy, and the panels on the right side have inverted mass hierarchy. The upper panels have zero phase, the middle panels have +90 degrees phase, and the lower panels have -90 degrees phase.
Figure 2: Probability for $\nu_\mu \rightarrow \nu_e$ for FNAL-BNL (L=1500km), given $|U_{e3}|^2 = 0.003$ (ie. $\theta_{13} = 3^\circ$), $\Delta m_{23}^2 = 0.003 eV^2$, $\Delta m_{12}^2 = 0.00003 eV^2$, $U_{\mu 3}^2 = U_{\tau 3}^2$, and $U_{e1}^2 = U_{e2}^2$. The black lines are for neutrinos, and the red lines are for anti-neutrinos. The panels on the left side have regular mass hierarchy, and the panels on the right side have inverted mass hierarchy. The upper panels have zero phase, the middle panels have +90 degrees phase, and the lower panels have -90 degrees phase.
Figure 3: Probability for $\nu_\mu \rightarrow \nu_e$ for FNAL-SLAC (L=2900km), given $|U_{e3}|^2 = 0.003$ (ie. $\theta_{13} = 3^\circ$), $\Delta m_{23}^2 = 0.003eV^2$, $\Delta m_{12}^2 = 0.00003eV^2$, $U_{\mu 3} = U_{\tau 3}$, and $U_{e1}^2 = U_{e2}^2$.

The black lines are for neutrinos, and the red lines are for anti-neutrinos. The panels on the left side have regular mass hierarchy, and the panels on the right side have inverted mass hierarchy. The upper panels have zero phase, the middle panels have +90 degrees phase, and the lower panels have -90 degrees phase.
FNAL-SLAC (2900 km) 40 kt-yr exposure of MINOS-like detector in PH2 medium (4x)

Table 1: Summary of data reduction for 40 kt-yr exposure of MINOS-like detector at SLAC in the PH2 medium beam (4x). An exact calculation of the oscillation probability in matter was used for $\Delta m^2_{23} = 0.003$ eV$^2$ (regular hierarchy), $|U_{e3}|^2 = 0.1$ (θ$^{13} = 3^\circ$), $U_{\mu3} = U_{\tau3}$, $\Delta m^2_{12} = 0.00033$ eV$^2$, $U_{\alpha}^2 = U_{\beta}^2$, and phase $\phi = 0$.

| Sequential cuts: | signal | background |
|------------------|--------|------------|
|                  | CC     | CC         | $E_\nu < 10$ GeV | $E_\nu > 10$ GeV |
| all events       | 126.4  | 130.4      | 4589.8            | 1325.7            | 3312.7            | 1279.3            |
| $P_\mu < 1$ GeV  | 126.4  | 130.4      | 374.9             | 1127.6            | 3312.7            | 1279.3            |
| $E_{CLUST}/E_{TOT} > 0.7$ | 59.3  | 68.2       | 18.8              | 278.2             | 753.1             | 203.0             |
| cluster $N_{STRIPS} \geq 9$ | 57.9  | 67.4       | 16.5              | 230.2             | 135.9             | 94.6              |
| Neural net $Y > 0$ | 52.0  | 60.7       | 8.8               | 173.6             | 86.3              | 71.6              |
| $400 pe < E_{TOT} < 800pe$ | 34.9  | 12.3       | 3.0               | 82.8              | 29.4              | 15.6              |
| Efficiency, bkgd fraction | 0.28  | 0.094      | 0.0007            | 0.062             | 0.009             | 0.012             |
in the PH2 medium beam (4x). An exact calculation of the oscillation probability in matter was used for $\Delta m^2_{32} = 0.003 eV^2$ (regular hierarchy), $|U_{e3}|^2 = 0.003$ (ie. $\theta_{13} = 3^\circ$), $U_{23} = U_{33} = 0.00003 eV^2$, $\Delta m^2_{12} = 0.00003 eV^2$, and phase $\phi = 0$.  

| Sequential cuts: | signal | background |
|------------------|--------|------------|
|                  | CC     | CC         | NC        |
|                  | $\nu_\mu \rightarrow \nu_e$ | $\nu_\mu \rightarrow \nu_\mu$ | $E_\nu < 10 GeV$ | $E_\nu > 10 GeV$ |
| all events       | 126.4  | 130.4      | 4589.8    | 1325.7  | 3312.7  | 1279.3  |
| $P_\mu < 1 GeV$  | 126.4  | 130.4      | 374.9     | 1127.6  | 3312.7  | 1279.3  |
| $E_{e,\gamma} > 5 GeV$ | 28.9   | 20.6       | 4.8       | 39.7    | 0.9     | 28.5    |
| $\gamma$ conversion | 28.9   | 20.6       | 0.6       | 39.7    | 0.1     | 3.4     |
| missing $PT < 400 MeV$ | 19.0   | 11.0       | 0.09      | 11.3    | 0       | 0       |
| $E_{vis} < 10 GeV$ | 17.9   | 6.6        | 0.002     | 6.6     | 0       | 0       |
| Efficiency, bkgd fraction | 0.14   | 0.051      | 0         | 0.005   | 0       | 0       |
Figure 4: PH2 medium/MINOS-like/SLAC event distributions of the fraction of total energy in the highest energy cluster ($E_{\text{CLUST}}/E_{\text{TOT}}$). We require $E_{\text{CLUST}}/E_{\text{TOT}} > 0.7$. The histograms are normalized to 4x 40 kt-yr data samples. An exact calculation of the oscillation probability in matter was used for $\Delta m^2_{23} = 0.003eV^2$ (regular mass hierarchy), $|U_{e3}|^2 = 0.003$ (ie. $\theta_{13} = 3^{\circ}$), $U_{\mu 3}^2 = U_{\tau 3}^2$, $U_{e1}^2 = U_{e2}^2$, $\Delta m^2_{12} = 0.00003eV^2$, and phase $\phi = 0$. 
Figure 5: PH2 medium/MINOS-like/SLAC event distributions for number of strips ($N_{STRIPS}$) in the highest energy cluster. We require $N_{STRIPS} >= 9$. The histograms are normalized to 4x 40 kt-yr data samples. An exact calculation of the oscillation probability in matter was used for $\Delta m^2_{23} = 0.003 eV^2$ (regular mass hierarchy), $|U_{e3}|^2 = 0.003$ (i.e. $\theta_{13} = 3^\circ$), $U_{\mu3}^2 = U_{\tau3}^2$, $U_{e1}^2 = U_{e2}^2$, $\Delta m^2_{12} = 0.00003 eV^2$, and phase $\phi = 0$. 
Figure 6: PH2 medium/MINOS-like/SLAC event distributions for the neural net estimator ($Y$). We require $Y > 0$. The histograms are normalized to 4x 40 kt-yr data samples. An exact calculation of the oscillation probability in matter was used for $\Delta m_{23}^2 = 0.003eV^2$ (regular mass hierarchy), $|U_{e3}|^2 = 0.003$ (i.e. $\theta_{13} = 3^o$), $U_{\mu3}^2 = U_{\tau3}^2$, $U_{e1}^2 = U_{e2}^2$, $\Delta m_{12}^2 = 0.00003eV^2$, and phase $\phi = 0$. 
Figure 7: PH2 medium/MINOS-like/SLAC event distributions for the total energy ($E_{TOT}$). We require 400 photoelectrons $< E_{TOT} < 800$ photoelectrons. The histograms are normalized to 4x 40 kt-yr data samples. An exact calculation of the oscillation probability in matter was used for $\Delta m^2_{23} = 0.003 eV^2$ (regular mass hierarchy), $|U_{e3}|^2 = 0.003$ (ie. $\theta_{13} = 3^\circ$), $U^2_{\mu3} = U^2_{\tau3}$, $U^2_{e1} = U^2_{e2}$, $\Delta m^2_{12} = 0.00003 eV^2$, and phase $\phi = 0$. 


Figure 8: PH2 medium/MINOS-like/SLAC total energy event distributions for background (black line) and background plus signal (red line). We require the total energy to be in the range between 400 and 800 photoelectrons (green region). The histograms are normalized to 4x 40 kt-yr data samples. An exact calculation of the oscillation probability in matter was used for $\Delta m_{23}^2 = 0.003 eV^2$ (regular mass hierarchy), $|U_{\nu_3}|^2 = 0.003$ (i.e. $\theta_{13} = 3^\circ$), $U_{\mu 3}^2 = U_{\tau 3}^2$, $U_{e 1}^2 = U_{e 2}^2$, $\Delta m_{12}^2 = 0.00003 eV^2$, and phase $\phi = 0$. 

Figure 9: PH2 medium/HED/SLAC event distributions for the maximum energy electromagnetic shower ($E_{e,\gamma}$). We have used a 25%/sqrt(E) electromagnetic shower energy resolution (from track counting). We require $E_{e,\gamma} > 5$ GeV. The histograms are normalized to 4x 40 kt-yr data samples. An exact calculation of the oscillation probability in matter was used for $\Delta m^2_{23} = 0.003 eV^2$ (regular mass hierarchy), $|U_{e3}|^2 = 0.003$ (ie. $\theta_{13} = 3^\circ$), $U^2_{\mu 3} = U^2_{\tau 3}$, $U^2_{e1} = U^2_{e2}$, $\Delta m^2_{12} = 0.00003 eV^2$, and phase $\phi = 0$. 


Figure 10: PH2 medium/HED/SLAC event distributions for the missing transverse momentum ($P_T$). We have used a 20% track momentum resolution (from the multiple scattering method) and 1 mrad track angular resolution. (Obviously, we have not used neutrinos, neutrons, or K-long in the $P_X, P_Y$ sums.) We have incorporated multiple Coulomb scattering; in order to minimizing the smearing due to multiple scattering we only use charged particles with momentum greater than 400 MeV in the sums. We require missing transverse momentum $P_T < 400$ MeV. The histograms are normalized to 4x 40 kt-yr data samples. An exact calculation of the oscillation probability in matter was used for $\Delta m^2_{32} = 0.003eV^2$ (regular mass hierarchy), $|U_{e3}|^2 = 0.003$ (ie. $\theta_{13} = 3'$), $U_{\mu 3}^2 = U_{\tau 3}^2$, $U_{e1}^2 = U_{e2}^2$, $\Delta m^2_{12} = 0.0003eV^2$, and phase $\phi = 0$. 

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Figure 11: PH2 medium/HED/SLAC event distributions for the visible energy ($E_{vis}$).
We have used a 20% track momentum resolution (from the multiple scattering method) and a 25%/sqrt(E) electromagnetic shower energy resolution (from track counting). We require $E_{vis} < 10$ GeV. The histograms are normalized to 4x 40 kt-yr data samples. An exact calculation of the oscillation probability in matter was used for $\Delta m_{23}^2 = 0.003 eV^2$ (regular mass hierarchy), $|U_{e3}|^2 = 0.003$ (ie. $\theta_{13} = 3^\circ$), $U_{\mu 3}^2 = U_{\tau 3}^2$, $U_{e1}^2 = U_{e2}^2$, $\Delta m_{12}^2 = 0.00003 eV^2$, and phase $\phi = 0$. 
Figure 12: PH2 medium/HED/SLAC total energy event distributions for background (black line) and background plus signal (red line). We require the total energy to be less than 10 GeV (green region). The histograms are normalized to 4x 40 kt-yr data samples. An exact calculation of the oscillation probability in matter was used for $\Delta m^2_{23} = 0.003 \text{eV}^2$ (regular mass hierarchy), $|U_{e3}|^2 = 0.003$ (ie. $\theta_{13} = 3^\circ$), $U_{\mu3}^2 = U_{\tau3}^2$, $U_{e1}^2 = U_{e2}^2$, $\Delta m^2_{12} = 0.00003 \text{eV}^2$, and phase $\phi = 0$. 
Figure 13: (4x) 40 kt-yr 90% confidence level limits on $|U_{e3}|^2$ with Fermilab main injector beam pointed at Soudan/MINOS-like. We have assumed a systematic error of 10% on the number of background events. An exact calculation of the oscillation probability in matter was used for regular mass hierarchy, $U_{\mu3}^2 = U_{\tau3}^2$, $U_{e1}^2 = U_{e2}^2$, $\Delta m_{12}^2 = 0.00003 eV^2$, and phase $\phi = 0$. 

$\Delta m_{23}^2 (eV^2)$ vs. $\log_{10}(|U_{e3}|^2)$
Figure 14: (4x) 40 kt-yr 90% confidence level limits on $|U_{e3}|^2$ with Fermilab main injector beam pointed at BNL/MINOS-like. We have assumed a systematic error of 10% on the number of background events. An exact calculation of the oscillation probability in matter was used for regular mass hierarchy, $U^2_{\mu 3} = U^2_{\tau 3}$, $U^2_{e1} = U^2_{e2}$, $\Delta m^2_{12} = 0.00003 eV^2$, and phase $\phi = 0$. 
Figure 15: (4x) 40 kt-yr 90% confidence level limits on $|U_{e3}|^2$ with Fermilab main injector beam pointed at SLAC/MINOS-like. We have assumed a systematic error of 10% on the number of background events. An exact calculation of the oscillation probability in matter was used for regular mass hierarchy, $U_{\mu 3}^2 = U_{\tau 3}^2, U_{e1}^2 = U_{e2}^2$, $\Delta m_{12}^2 = 0.00003 eV^2$, and phase $\phi = 0$. 
Figure 16: (4x) 40 kt-yr 90% confidence level limits on $|U_{e3}|^2$ with Fermilab main injector beam pointed at Soudan/HED. We have assumed a systematic error of 10% on the number of background events. An exact calculation of the oscillation probability in matter was used for regular mass hierarchy, $U_{\mu 3}^2 = U_{\tau 3}^2$, $U_{e1}^2 = U_{e2}^2$, $\Delta m_{12}^2 = 0.00003 eV^2$, and phase $\phi = 0$. 
Figure 17: (4x) 40 kt-yr 90% confidence level limits on $|U_{e3}|^2$ with Fermilab main injector beam pointed at BNL/HED. We have assumed a systematic error of 10% on the number of background events. An exact calculation of the oscillation probability in matter was used for regular mass hierarchy, $U^2_{\mu 3} = U^2_{\tau 3}$, $U^2_{e1} = U^2_{e2}$, $\Delta m^2_{12} = 0.00003eV^2$, and phase $\phi = 0$. 

$\log_{10}(|U_{e3}|^2)$ vs $\Delta m^2_{23}$ (eV$^2$)
Figure 18: (4x) 40 kt-yr 90% confidence level limits on $|U_{e3}|^2$ with Fermilab main injector beam pointed at SLAC/HED. We have assumed a systematic error of 10% on the number of background events. An exact calculation of the oscillation probability in matter was used for regular mass hierarchy, $U_{\mu 3} = U_{\tau 3}$, $U_{e1} = U_{e2}$, $\Delta m^2_{12} = 0.00003eV^2$, and phase $\phi = 0$. 

$90\%$ C.L. limits, regular hierarchy, $U_{\mu 3} = U_{\tau 3}$

- PH2 low SLAC/HED
- PH2 med SLAC/HED
- PH2 high SLAC/HED

$\Delta m^2_{23} (eV^2)$

$\log_{10}(|U_{e3}|^2)$
Figure 19: Predicted 90% confidence level limits on $|U_{e3}|^2$ with Fermilab main injector beam. We have assumed a systematic error of 10% on the number of background events. An exact calculation of the oscillation probability in matter was used for regular mass hierarchy, $U_{\mu 3} = U_{\tau 3}$, $U_{e1}^2 = U_{e2}^2$, $\Delta m_{12}^2 = 0.00003eV^2$, and phase $\phi = 0$. 
Figure 20: Discovery potential for $|U_{e3}|^2$ and sign($\Delta m_{23}^2$) with Fermilab main injector beam. We have assumed a systematic error of 10% on the number of background events. For sign($\Delta m_{23}^2$) we have assumed an additional sample of data with the same exposure except with an anti-neutrino beam. An exact calculation of the oscillation probability in matter was used for regular mass hierarchy, $U_{\mu 3}^2 = U_{\tau 3}^2$, $U_{e 1}^2 = U_{e 2}^2$, $\Delta m_{12}^2 = 0.00003 eV^2$, and phase $\phi = 0$. 
