Impact of Lorentz Violation on Cosmology

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Abstract

We discuss the impact of Lorentz violation on the cosmology. Firstly, we show that the Lorentz violation affects the dynamics of the chaotic inflatonary model and gives rise to an interesting feature. Secondly, we propose the Lorentz violating DGP brane models where the Lorentz violating terms on the brane accelerate the current universe. We conjecture that the ghost disappears in the Lorentz violating DGP models.

1 Introduction

Various observations suggest the existence of two accelerating stages in the universe, namely, the past inflationary universe and the current acceleration of the universe. In cosmology, therefore, how to accelerate the universe in the past and present is a big issue. Many attempts to resolve this issue have been performed and failed. Given the difficulty of the problem, it would be useful to go back to the basic point and reexamine it. Here, we consider the possibility to break the Lorentz symmetry.

Typically, the Lorentz violation yields the preferred frame. In the case of the standard model of particles, there are strong constraints on the existence of the preferred frame. In contrast, there is no reason to refuse the preferred frame in cosmology. Rather, there is a natural preferred frame defined by the cosmic microwave background radiation (CMB). Therefore, there is room to consider the gravitational theory which allows the preferred frame.

Now, we present our model with which we discuss the cosmological acceleration problems. Suppose that the Lorentz symmetry is spontaneously broken by getting the expectation values of a vector field $u^\mu$ as $\langle u^\mu u_\mu \rangle = -1$. We do not notice the existence of this field because the frame determined by this field coincides with the CMB frame. However, in the inhomogeneous universe, both frames can fluctuate independently. Hence, we can regard the spatial hypersurface of our universe as a kind of membrane characterized by the extrinsic curvature $K_{ij}$ with a time like vector field $u^\mu$. Based on this observation, we propose the model

\begin{equation}
S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} R + \beta_1(\phi) K_{ij} K^{ij} - \beta_2(\phi) K^2 - \gamma_1(\phi) \nabla^\mu u^\nu \nabla_\mu u_\nu - \gamma_2(\phi) \nabla^\mu u^\nu \nabla_\nu u_\mu - \gamma_3(\phi) (\nabla_\mu u^\mu)^2 - \gamma_4(\phi) u^\mu u^\nu \nabla_\mu u^\alpha \nabla_\nu u_\alpha + \lambda (u^\mu u_\mu + 1) - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right],
\end{equation}

where $g_{\mu\nu}$, $R$, and $\lambda$ are the 4-dimensional metric, the scalar curvature, and a Lagrange multiplier, respectively. Here, we also considered the scalar field $\phi$ with the potential $V$ and it couples to other terms with the coupling function $\beta_i(\phi)$ and $\gamma_i(\phi)$. This is a generalization of the Einstein-Ether gravity \cite{1}. Since $\beta_i, \gamma_i$ at present can be different from those in the very early universe, we do not have any constraint on these parameters in the inflationary stage. Of course, ultimately, they have to approach the observationally allowed values at present. Here, the Lorentz symmetry is violated both spontaneously and explicitly.

The purpose of this paper is to discuss the impact of the Lorentz violation both on the inflationary scenario and the current acceleration. In the former case, the Lorentz violation merely modifies the scenario. In the latter case, however, the impact of Lorentz violation could be appreciable. The inclusion of Lorentz violation may give rise to a resolution of the ghost problem in the DGP brane model \cite{2}. 

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2 Impact of Lorentz Violation on Inflationary Scenario

Now, let us consider the chaotic inflationary scenario and clarify to what extent the Lorentz violation affects the inflationary scenario [3]. We take the model

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} R - \beta(\phi)K^2 - \frac{1}{2} \left( \nabla \phi \right)^2 - V(\phi) \right]. \]

Let us consider the homogeneous and isotropic spacetime

\[ ds^2 = -dt^2 + e^{2a(t)}\delta_{ij}dx^idx^j. \]

The scale of the universe is determined by \( \alpha \). Now, let us deduce the equations of motion. First, we define the dimensionless derivative \( Q' \) by

\[ \dot{Q} = \frac{dQ}{d\alpha} \frac{d\alpha}{dt} \equiv Q' \frac{d\alpha}{dt}. \]

Then, the equations of motion are

\[ \left( 1 + \frac{1}{8\pi G\beta} \right) H^2 = \frac{1}{3} \left[ \frac{1}{2} \frac{H^2}{\beta} + \frac{V}{\beta} \right] \]

\[ \left( 1 + \frac{1}{8\pi G\beta} \right) \frac{H'}{H} + \frac{1}{2} \frac{\phi'^2}{\beta} + \frac{\beta'}{\beta} = 0 \]

\[ \phi'' + \frac{H'}{H} \phi' + 3\phi' + \frac{V,\phi}{H^2} + 3\beta,\phi = 0, \]

where \( \beta,\phi \) denotes the derivative with respect to \( \phi \). We have taken \( H = \dot{\alpha} \) as an independent variable. As is usual with gravity, these three equations are not independent. Usually, the second one is regarded as a redundant equation.

The above equation changes its property at the critical value \( \phi_c \) defined by

\[ 8\pi G\beta(\phi_c) = 1. \]

When we consider the inflationary scenario, we usually require the enough e-folding number, say \( N = 70 \). Let \( \phi_i \) be the corresponding initial value of the scalar field. If \( \phi_c > \phi_i \), the effect of Lorentz violation on the inflationary scenario would be negligible. However, if \( \phi_c < \phi_i \), the standard scenario should be modified. It depends on the models. To make the discussion more specific, we choose the model \( \beta = \xi \phi^2 \), \( V = \frac{1}{2}m^2\phi^2 \), where \( \xi \) and \( m \) are parameters. For this model, we have \( \phi_c = \frac{M_{pl}}{\sqrt{8\pi \xi}} \). As \( \phi_i \sim 3M_{pl} \) approximately in the standard case, the condition \( \phi_i > \phi_c \) implies the criterion \( \xi > 1/(72\pi) \sim 1/226 \) for the Lorentz violation to be relevant to the inflation. For other models, the similar criterion can be easily obtained.

Now, we suppose the Lorentz violation is relevant and analyze the two regimes separately.

For a sufficiently larger value of \( \phi \), both the coupling function \( \beta \) and the potential function \( V \) are important in the model (2). During this period, the effect of Lorentz violation on the inflaton dynamics must be large. In the Lorentz violating regime, \( 8\pi G\beta \gg 1 \), we have

\[ H'^2 + \frac{1}{2} \frac{H^2}{\beta} + \frac{V}{\beta} = 0 \]

\[ \phi'' + \frac{H'}{H} \phi' + 3\phi' + \frac{V,\phi}{H^2} + 3\beta,\phi = 0. \]

To have the inflation, we impose the condition \( H^2 \phi'^2 \ll V \) as the slow roll condition. Consequently, Eq.(7) is reduced to

\[ H^2 = \frac{1}{3\beta} V. \]

Using Eq.(10), the slow roll condition can be written as \( \phi'^2 \ll \beta \). Now, we also impose the condition \( H'/H \ll 1 \) as the quasi-de Sitter condition. Then, Eq.(8) gives us the condition \( \beta' \ll \beta \). We also require the standard condition \( \phi'' \ll \phi' \). Thus, we have the slow roll equations (10) and

\[ \phi' + \frac{V,\phi}{3H^2} + \beta,\phi = 0. \]
For our example, we can easily solve Eqs. (10) and (11) as \( \phi(\alpha) = \phi_i e^{-4\xi \alpha} \). For this solution to satisfy slow roll conditions, we need \( \xi < 1/16 \). Thus, we have the range \( 1/226 < \xi < 1/16 \) of the parameter for which the Lorentz violating inflation is relevant. Note that, in our model, the Hubble parameter (10) becomes constant

\[
H^2 = \frac{m^2}{6\xi},
\]
even though the inflaton is rolling down the potential. This is a consequence of Lorentz violation.

After the inflaton crosses the critical value \( \phi_c \), the dynamics is governed entirely by the potential \( V \).

In the standard slow roll regime \( 8\pi G \beta \ll 1 \), the evolution of the inflaton can be solved as

\[
\phi^2(\alpha) = \phi_c^2 - \frac{\alpha}{2\pi G},
\]
The scale factor can also be obtained as \( a(t) = \exp \left[ 2\pi G (\phi_c^2 - \phi^2(t)) \right] \). The standard inflation stage ends and the reheating commences when the slow roll conditions violate.

Now it is easy to calculate e-folding number. Let \( \phi_i \) be the value of the scalar field corresponding to the e-folding number \( N = 70 \). The total e-folding number reads

\[
N = \frac{1}{4\xi} \log \frac{\phi_i}{\phi_c} + 2\pi G \left( \phi_c^2 - \phi_i^2 \right),
\]
where \( \phi_c \sim 0.3M_{pl} \) is the value of scalar field at the end of inflation. Note that the first term arises from the Lorentz violating stage. As an example, let us take the value \( \xi = 10^{-2} \). Then, \( \phi_c \sim 2M_{pl} \). The contribution to the e-folding number from the inflation end is negligible. Therefore, we get \( \phi_i \sim 12M_{pl} \).

In this simple example, the coupling to the Lorentz violating sector disappears after the reheating. Hence, the subsequent homogeneous dynamics of the universe is the same as that of Lorentz invariant theory of gravity. However, it is possible to add some constants to \( \beta \), which are consistent with the current experiments. In that case, the effect of the Lorentz violation is still relevant to the subsequent history.

The tensor part of perturbations can be described by

\[
ds^2 = -dt^2 + a^2(t) \left( \delta_{ij} + h_{ij}(t, x^i) \right) dx^i dx^j,
\]
where the perturbation satisfy \( h_{ii} = h_{ij,j} = 0 \). The quadratic part of the action is given by

\[
S = \int d^4x \frac{a^3}{16\pi G} \left[ \frac{1}{4} h_{ij} h^{ij} - \frac{1}{4a^2} h_{ij,k} h^{ij,k} \right].
\]
In the case of chaotic inflation model, the Hubble parameter is constant (12) during Lorentz violating stage. The spectrum is completely flat although the inflaton is rolling down the potential. This is a clear prediction of the Lorentz violating chaotic inflation.

### 3 Impact of Lorentz Violation on Current Acceleration

To solve the current acceleration problem is much more difficult than the past one. The most interesting proposal is the DGP model [2]. However, it suffers from the ghost. Hence, it is not a stable model. Here, we would like to argue the Lorentz violation may resolve the instability problem.

Our basic observation is that the Lorentz violating term itself can accelerate the universe. Let us consider the simplest braneworld model:

\[
S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-G} R - \int d^4x \sqrt{-g} \beta K^2,
\]
where \( G \) is the determinant of the 5-dimensional metric and \( \kappa_5 \) is the 5-dimensional gravitational coupling constant. Here, we have assumed the scalar field is stabilized at present. Let us assume the \( Z_2 \) symmetry. Then the junction condition \( K_{\mu\nu} - g_{\mu\nu} K = \kappa_5^2 T_{\mu\nu} \) gives the effective Friedman equation:

\[
\pm H = \frac{\kappa_5^2}{6} [3\beta H^2 - \rho].
\]
If we take the positive sign, in the late time, we have the de Sitter spacetime with

$$H = \frac{2}{\kappa_5^2 \beta}.$$  (19)

Thus, the late time accelerating universe can be realized due to the Lorentz violation.

More generally, we propose the following model

$$S = \frac{1}{2\kappa_5^2} \int d^5 x \sqrt{-GR}$$
$$+ \int d^4 x \sqrt{-g} \left[ \frac{1}{16\pi G} R + \beta_1 K^{ij} K_{ij} - \beta_2 K^2 - \gamma_1 \nabla^\mu u^\nu \nabla_\mu u_\nu$$
$$- \gamma_2 \nabla^\mu u^\nu \nabla_\nu u_\mu - \gamma_3 (\nabla^\mu u^\nu)^2 - \gamma_4 u^\mu u^\nu \nabla_\mu u_\alpha \nabla_\nu u_\alpha + \lambda (u^\mu u_\mu + 1) \right].$$  (20)

It should be stressed that any term on the brane can generate the current acceleration. As the tensor structure in the action is very different from the original DGP model, we can expect the above action contains no ghost. In fact, we have many parameters here, hence there is a chance to remove the ghost from our models. Thus, we expect that some of the above Lorentz violating brane models does not contain any ghost. If this is so, one can say the Lorentz violation explains the current stage acceleration of the universe.

4 Conclusion

We have discussed the impact of Lorentz violation on cosmology. In the first place, we found that the Lorentz violating inflation shows an interesting feature. In the second place, we proposed a Lorentz violating DGP model which have a possibility to avoid the ghost problem.

It would be interesting to study the evolution of fluctuations completely. If the vector modes of perturbations can survive till the last scattering surface, they leave the remnant of the Lorentz violation on the CMB polarization spectrum. It is also intriguing to seek for a relation to the large scale anomaly discovered in CMB by WMAP. The calculation of the curvature perturbation is much more complicated. However, it must reveal more interesting phenomena due to Lorentz violating inflation. The tensor-scalar ratio of the power spectrum would be also interesting. These are now under investigation.

More importantly, we need to show the stability of Lorentz violating DGP model. If the Lorentz violation kills the ghost, this is a great progress in the cosmology. Even in case that it turns out that all of the models are unstable, it makes our understanding of the ghost issue profound.

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