Fluid flow and convection heat transfer in concentric and eccentric cylindrical annuli of different radii ratios for Taylor-Couette-Poiseuille flow

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Abstract
This paper presents the results of fluid flow and convection heat transfer in concentric and eccentric annuli between two cylinders using a three-dimensional computational fluid dynamics model. Effects of rotational speed (n = 0, 150, 300, and 400 rpm) and eccentricity (e = 0, 0.15, 0.3, 0.45, and 0.6) on axial and tangential velocity distribution, pressure drop and forced convection heat transfer are investigated for radii ratios (h) of 0.2, 0.4, 0.6, and 0.8, Reynold’s number $2.0 \times 10^3$–$1.236 \times 10^5$, Taylor number $1.47 \times 10^6$–$1.6 \times 10^{10}$, and Prandtl number $3.71$–$6.94$. The parameters cover many applications, including rotary heat exchangers, mixers, agitators, etc. Nusselt numbers and friction factors for stationary and rotated concentric and eccentric annuli are correlated with four dimensionless numbers. The results revealed that when the speed of the inner cylinder increases from 0 to 400 rpm, the friction factor increases by 7.7%–103% for concentric and 8.2%–148% for eccentric annuli, whereas Nusselt number enhances by 37%–333% for concentric and 44%–340% for eccentric annuli. The radius ratio has a substantial effect on the heat transfer and pressure drop in annuli. The eccentricity enhances the heat transfer up to 12%, whereas its effect on the friction factor is not monotonic as it depends on Reynolds number, radii ratios, and rotational speed.

Keywords
Concentric and eccentric annuli, Taylor-Couette-Poiseuille flow, heat transfer, pressure drop, inner cylinder rotation, radius ratio

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Introduction
Fluid flow and heat transfer in the annular space between two circular cylinders, where the inner cylinder is rotating and the outer cylinder is stationary, is encountered in many types of equipment and engineering applications. A narrow annular space exists in the clearance between the stator and rotor of an electric motor or gas turbine or the peripheral clearance of twin-screw pumps and many other applications. Also, the wide annular space takes place in numerous industrial applications, including rotating heat exchangers, mixers, agitators, around nuclear fuel rods in reactors, and wells drilling (the annulus between a borehole and a drill pipe).¹

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The fluid flow in the annular space between two cylinders may be classified into a Couette flow, Taylor-Couette flow, Poiseuille flow, and Taylor-Couette-Poiseuille flow. The viscous flow between two cylinders where either one is rotating is termed a Couette flow, whereas for Taylor-Couette flow, the outer cylinder is stationary and the inner cylinder is rotating, without axial flow. In Poiseuille flow, both cylinders are stationary with the imposed axial flow, whereas for Taylor-Couette-Poiseuille flow, the outer cylinder is stationary, and the inner cylinder is rotating with the imposed axial flow. In addition, other applications may include imposed flow in the radial direction.

The annular space between the two cylinders may be a concentric or an eccentric one. The two cylinders have a common center for a concentric annulus, or one of the centers is shifted from the other by a distance $e$ to form an eccentric annulus. Figure 1 shows a section of an eccentric annulus where the eccentricity of the inner cylinder splits the annulus into a large space ($s = P_1$) and a small space ($s = P_2$); for concentric cylinders $P_1$ and $P_2$ are equal. The thermal boundary conditions of two walls in concentric or eccentric annular duct for laminar and turbulent flow were classified into four types.

The application of Taylor-Couette flow without imposed axial flow may be encountered in journal bearings. The present work focuses on heat exchangers applications, which include Poiseuille flow for stationary heat exchangers or Taylor-Couette-Poiseuille flow for rotating heat exchangers. Due to the importance and wide applications of the topic, a plethora of work is available on the literature, particularly for Taylor-Couette flow. A comprehensive literature review of the fluid flow work was reported by Tagg, Childs and Long, and Nakashima et al. The review of fluid and heat transfer work was reported by Fénot et al. and Togun et al. Table 1 lists a few recent papers on the topic of Taylor-Couette flow, where the aspect ratio (pipe length to hydraulic diameter) and rotational speed of the inner cylinder are the driving force for flow, heat, and absolute instability in such applications. The listed work includes concentric and eccentric annuli with stationary or rotating inner cylinders.

On the other hand, Table 2 presents a comprehensive review of fluid flow and heat transfer in Taylor-Couette-Poiseuille flow, where the axial flow is encountered in such applications. The listed work is ordered from recent to early papers within three categories, namely fluid flow work, forced convection heat transfer work for concentric, and eccentric annuli. The fluid flow work, reported in Table 2, indicated that the topic receives a major consideration in the literature. It includes laminar and turbulent flow,
stationary, and rotating inner cylinder, and concentric and eccentric annuli with various radii ratios.

Table 2 indicated that most of the work on forced convection heat transfer in concentric annuli was performed for a single radius ratio under stationary or rotating conditions. The exception to that is the work of Sreenivasulu and Prasad\(^45\) and Coney and El-Shaarawi\(^48\) that covers three radii ratios under stationary conditions in turbulent and laminar flow regions, respectively. Also, Poncet et al.\(^44\) covered two radii ratios (0.506 and 0.961) under rotating conditions, but for low Reynolds number range (1200, 2500, and 7500). Therefore, the effect of rotation on the annular space of different radii ratios is lacks in the literature. This is of

| No | Reference | Approach | Radius ratio | Speed (rpm) | Eccentricity | Re |
|----|-----------|----------|--------------|-------------|--------------|----|
| 1  | Bicalho et al.\(^22\) | E/N | 0.5 | 0, 200, 400 | 0.23, 0.46 | 40–1578 |
| 2  | Hamd\(^23\) | N | 0.5 | 0–250 | 0.2, 0.4 | 200 |
| 3  | Kelessidis et al.\(^24\) | E | 0.57 | 0 | 0, 1.0 | <10629 × 10^4 |
| 4  | Ninokata et al.\(^25\) | DNS | 0.1, 0.5 | 0 | 0.5 | 4000 |
| 5  | Chung and Sung\(^26\) | LES | 0.5 | 0, 270, 1080 | 0 | 8900 |
| 6  | Kaneda et al.\(^27\) | N | 0.001–0.99 | 0 | 0 | 10^2–10^6 |
| 7  | Dumont et al.\(^28\) | E | 0.615 | 30–600 | 0 | <4 |
| 8  | Escudier et al.\(^29\) | E/N | 0.2, 0.5, 0.8 | 0–400 | 0.2, 0.3, 0.4, 0.5, 0.6, 0.7 | 105, 115, 120 |
| 9  | Moser et al.\(^30\) | E | 0.5 | 15 | 0 | 0–20 |
| 10 | Wereley and Lueptow\(^31\) | E | 0.83 | 10–20 | 0 | 0–25 |
| 11 | Fang et al.\(^32\) | N | 0.2–0.8 | 0 | 0.2–0.8 | 6–15 |
| 12 | Nouri and Whitelaw\(^33\) | E | 0.5 | 0, 300 | 0 | 9000 |
| 13 | Torii and Yang\(^34\) | N | 0.2, 0.5, 0.8 | 0 | 0–0.8 | 10^4–10^5 |
| 14 | Velusamy and Garg\(^35\) | N | 0.1–0.8 | 0 | 0.2–0.9 | 6–15 |
| 15 | Nouri et al.\(^36\) | E | 0.5 | 0 | 0.5, 1.0 | 8.9 × 10^3–2.6 × 10^4 |
| 16 | Rehme\(^37\) | E | 0.02–0.1 | 0 | 0 | 2 × 10^4–2 × 10^5 |
| 17 | Jonsson and Sparrow\(^38\) | E | 0.28–0.75 | 0 | 0–1 | 1.8 × 10^4–1.8 × 10^5 |
| 18 | Barrow et al.\(^39\) | E | 0.25–0.6 | 0 | 0–1 | Turbulent (NA) |
| 19 | Chithrakumar et al.\(^40\) | E/N | 0.624 | 50–440 | 0 | 0–6760 |
| 20 | Yassin et al.\(^41\) | E | 0.333 | 0–400 | 0 | 3 × 10^9–9 × 10^9 |
| 21 | Lancial et al.\(^42\) | E | 0.922 | 0–1500 | 0 | 0–1,425 × 10^4 |
| 22 | Abou-Ziyan et al.\(^1\) | E | 0.333 | 0–400 | 0 | 8.1 × 10^4–1.8 × 10^5 |
| 23 | Aubert et al.\(^43\) | E | 0.888 | 200, 300 | 0 | 11,200 |
| 24 | Poncet et al.\(^44\) | N | 0.506, 0.961 | 0–2000 | 0 | 1200, 2500, 7500 |
| 25 | Sreenivasulu and Prasad\(^45\) | E | 0.5, 0.56, 0.67 | 0 | 0 | 2 × 10^4–2 × 10^5 |
| 26 | Lu and Wang\(^46\) | E | 0.8 | 0 | 0 | 10–30,000 |
| 27 | Jeng et al.\(^47\) | E | 0.8955 | 0–1028 | 0 | 30–1200 |
| 28 | Coney and El-Shaarawi\(^48\) | N | 0.1, 0.5, 0.9 | 0 | 0 | Laminar (NA) |
| 29 | Nouri-Borujerd and Nakhch\(^49\) | E | 0.928 | 0–1980 | 0.928 | 4.6 × 10^3–1.23 × 10^4 |
| 30 | Ali et al.\(^50\) | N | 0.333 | 0–500 | 0.2, 0.3, 0.4 | 5 × 10^5–5 × 10^5 |
| 31 | Teimouri et al.\(^51\) | N | 0.384 | NA | 0.6 | 275 |
| 32 | Chauhan et al.\(^52\) | N | 0.5–0.95 | 0 | 0.0–0.7 | 10^5–10^6 |
| 33 | Piot and Taveularis\(^53\) | E | 0.28 | 0 | 0.5–0.8 | 400–1200 |
| 34 | Nikitin et al.\(^54\) | N | 0.6, 0.7, 0.8 | 0 | 0.5, 1 | 8000 |
| 35 | Manglik and Fang\(^55\) | N | 0.25–0.75 | 0 | 0.0–0.6 | 6–15 |
| 36 | Ogino et al.\(^56\) | E | 0.3–0.48 | 0 | 0.25–0.75 | 5 × 10^3–4 × 10^4 |
| 37 | Feldman et al.\(^57\) | N | 0.5, 0.9 | 0 | 0.1, 0.4 | 2.5–50 |
| 38 | Trombetta\(^58\) | N | 0.5, 0.9 | 0 | 0.1–0.8 | 52–66 |
| 39 | Present \(^\text{work (2020)}\) | N (CFD) | 0.2, 0.4, 0.6, 0.8 | 0–400 | 0, 0.15, 0.3, 0.45, 0.6 | 2 × 10^3–1.23 × 10^5 |

DNS: direct normal simulation; LES: large eddy simulation; CFD: computational fluid dynamics.

\(^*\)E/N indicates paper approach Experimental/Numerical.
particular interest in many industrial applications such as agitators, mixers, and rotary heat exchangers.

Most of the reported work on forced convection heat transfer in eccentric annuli (Table 2) is conducted for stationary inner cylinders and a wide range of eccentricity (0–1) and laminar and turbulent flow conditions. Only the work presented in was performed under rotational conditions for a very narrow annulus with a radius ratio of 0.928 and an eccentricity of 0.928 or for a fixed annulus with a radius ratio of 0.333 and eccentricity ranges from 0 to 0.4. Therefore, the effect of eccentricity on various radii ratios under rotational conditions was not investigated. The interaction between the radius ratio and eccentricity on heat transfer and fluid flow in applications involved small, medium, and large radii ratios, and eccentricity needs to be explored.

The earlier discussion revealed that the previous work did not consider the effect of radius ratio, particularly for a rotating annular space on fluid flow and convection heat transfer. Also, the effect of the eccentricity of a rotating annulus of various radii ratios on heat transfer was not examined in the open literature as most of the investigated work for eccentric annuli was conducted under either a stationary inner cylinder or limited Reynolds numbers at a single radius ratio.

Therefore, the aim of the present work is to investigate the effect of radius ratio, eccentricity, and inner pipe rotation speed on fluid flow and heat transfer in wide, medium, and narrow annular gaps to cover various applications, including rotary heat exchangers. Concentric and eccentric annular space of radii ratios of 0.2, 0.4, 0.6, and 0.8 are investigated for eccentricity of 0, 0.15, 0.30, 0.45, and 0.60 and rotational speed of 0, 150, 300, and 400 rpm. The simulated data of Nusselt number and friction factor are correlated with Reynolds number from about 2000 to 123,600, Taylor’s number of 0, 0.15, 0.30, 0.40, and 0.60 and radial number from 3.71 to 6.94, and radius ratio from 0.2 to 0.8. The work is conducted using 3D-CFD simulations with a k-ε turbulence model.

**Numerical modeling and simulation**

This section describes the modeling and simulation of fluid flow and heat transfer of water flow through the annular space between two circular cylinders where the inner cylinder is assumed to be at constant surface temperature and the outer cylinder is adiabatic. The inner cylinder is either stationary or rotating, and the outer one is stationary. Fluid flow and heat transfer in the concentric or eccentric annulus can be described mathematically using the three-dimensional governing equations of mass, momentum, and energy. Several three-dimensional test cases have been simulated using ANSYS- Fluent version 18.2. In the case of incompressible flow under steady-state conditions, the equations solved in each computational cell are Navier-Stokes equations. The continuity equation (conservation of mass) in cylindrical coordinates is given in equation (1).

\[
\frac{1}{r} \frac{\partial}{\partial r} (ru_r) + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0
\]

(1)

Where \(u_r\), \(u_\theta\), and \(u_z\) are the velocity vector components in the cylindrical coordinates \(r\), \(\theta\), and \(z\). Conservation of momentum equation (equation of motion) in \(r\), \(\theta\), and \(z\) directions is expressed by equations (2)–(4), respectively.

\[
\rho \left( \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + u_\theta \frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} \right) = -\frac{1}{r} \frac{\partial p}{\partial r} + \rho g_r
\]

\[
+ \mu \left[ \frac{1}{r} \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2} \right]
\]

(2)

\[
\rho \left( \frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + u_\theta \frac{\partial u_\theta}{\partial \theta} + u_z \frac{\partial u_\theta}{\partial z} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_\theta
\]

\[
+ \mu \left[ \frac{1}{r} \frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial z^2} - \frac{u_\theta}{r^2} \right]
\]

(3)

\[
\rho \left( \frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + u_\theta \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} \right) = -\frac{1}{r} \frac{\partial p}{\partial z} + \rho g_z
\]

\[
+ \mu \left[ \frac{1}{r^2} \frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right]
\]

(4)

Where \(\rho\) and \(\mu\) are the fluid density and dynamic viscosity, \(p\) is the pressure, and \(g_r\) is the body force in the \(r\), \(\theta\), and \(z\) directions. Conservation of energy equation is given in equation (5):

\[
u_r \frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + u_\theta \frac{\partial T}{\partial \theta} + u_z \frac{\partial T}{\partial z} = \alpha \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right] + S_T
\]

(5)

Where \(T\) is the temperature, \(\alpha\) is the thermal diffusivity \((\alpha = k/c_p)\), \(k\) is the thermal conductivity of element, \(c_p\) is the specific heat, and \(S_T\) is the source term, which may be due to electrical current flow, chemical reaction, etc.

ANSYS FLUENT Workbench Release 18.2 is set to solve three-dimensional turbulent fluid flow and heat transfer in the concentric and eccentric horizontal annulus. The computational scheme used by ANSYS FLUENT Workbench is based on the finite volume discretization method in which the flow equations are integrated over each control volume.

Turbulence is accounted for by time-averaging the equations (1–5) to produce the Reynolds Averaged...
Navier-Stokes (RANS) equations, and to solve for the additional terms generated using this process; a turbulence model has to be implemented.

**Computational domain and boundary conditions**

As shown in Figure 2, the physical model consists of double cylinders with an inner cylinder of 50 mm diameter and a 2000 mm length at a constant surface temperature of 85°C. The outer cylinder is adiabatic with variable diameters to study the effect of radii ratios on both fluid flow and heat transfer. A downstream section of 1000 mm is added to the original domain length to eliminate the reversed backflow, which results from flow rotation in the low-pressure zone adjacent to the exit.

The working fluid used is water with an inlet temperature of 20°C and a variable inlet water velocity from 0.15 to 0.6 m/s to investigate the effect of Reynolds number on the fluid flow and heat transfer characteristics in the annular space. The outlet boundary conditions are pressure outlet with zero-gauge pressure. The outer cylinder is stationary with the no-slip surface condition. Reynolds averaged Navier-Stokes equations were solved using the realizable \( k \)–\( \varepsilon \) turbulence model with enhanced wall treatment while the pressure gradient effect is taken into consideration. The pressure velocity coupling was the semi-implicit method for pressure-linked equations (SIMPLE). The residuals were set to \( 10^{-5} \) for all transport equations to achieve convergence.

**Turbulence model selection**

Turbulence is a very complicated physical process to represent mathematically and consequently difficult to be accurately modeled. However, there are three common approaches to predict turbulent flow; Direct Numerical Simulation (DNS), Large Eddy Simulation (LES), and Reynolds-Averaged Navier-Stokes (RANS) equation simulation with an appropriate turbulence model. The RANS approach has become widely used in modeling fluid flow in the annulus due to its limited requirements of computer resources. Therefore, in this study, the RANS approach is adopted, and various turbulence models have been tested to predict the turbulence effects of the water flow. Among the tested models, the realizable \( k \)–\( \varepsilon \) turbulence model performed better than the other models in predicting more accurate results than the reported results in. 50

**Grid size study**

ANSYS meshing is used for mesh generation using the multi-zone method. Figure 3 shows a section of the mesh in the annular space. The mesh is created using edge sizing and the multi-zone method. The accuracy of the solution of the three-dimensional governing equations depends on the number and the size of the cells of the generated mesh. Therefore, a grid independence study was conducted on the three-dimensional geometries to determine the best mesh spacing. The output from the program is the outlet water temperature and the pressure drop across the test section. Table 3 lists the results of outlet temperature, pressure drop, and Nusselt number for three grid sizes 204,771, 418,500,
and 671,660. The comparison shows that the mesh size of 418,500 obtains results with 0.01%, 0.29%, and 0.45% difference from the finest mesh for outlet temperature, pressure drop, and Nusselt number, respectively, with a reasonable computational time.

**Model validation**

The predictions of the CFD model are validated against results reported by Ali et al.⁵⁰ for a concentric annulus with a radius ratio of 0.33. A reasonable agreement is obtained for the outlet temperature and pressure drop. The validation is conducted at rotational speeds of 0, 100, 300, 400, and 500 rpm at two axial water velocities of 0.05 and 0.20 m/s. Figure 4 shows the comparison between the results of the present work and those reported by Ali et al.⁵⁰ for the main program output. The percentage difference ranges from 0.06% to 0.67% for the outlet temperature and 0.4% to 11.52% for pressure drop.

**Data analysis and reduction**

As stated before, the inputs to the CFD program are the inlet water velocity ($U_b$) and temperature ($T_i$) with inner cylinder surface temperature ($T_s$) of 85°C. The output of the CFD simulation is the outlet water temperature ($T_o$), inlet pressure ($p_i$), and outlet pressure ($p_o$). Those data are used to calculate the heat transfer rate ($Q$) using equations (6). The mass flow rate of water ($m$) is calculated using equation (7). $A_c$ is the cross-sectional area of the annular space as given in equation (8), where $d_i$ and $d_o$ are the diameters of the inner and outer cylinders, respectively.

$$Q = m c_p (T_o - T_i)$$

$$m = \rho U_b A_c$$

$$A_c = \frac{\pi (d_o^2 - d_i^2)}{4}$$

The convection heat transfer coefficient ($h$) is calculated using equation (9), where $A_s$ is the surface area of heat exchange, and $L$ is the test section length. The logarithmic mean temperature difference (LMTD) along the test section is calculated using equation (11).

$$h = \frac{Q}{A_s L \text{LMTD}}$$

$$A_s = \pi d_i L$$

**Table 3.** Effect of cell size on the predicted results.

| Grid size | Parameters | % Difference |
|-----------|------------|--------------|
|           | $T_o$ (°C) | $\Delta P$ (Pa) | Nu | $T_o$ | $\Delta P$ | Nu |
| 204,771   | 26.540     | 3.42         | 83.58 | 0.075 | 0.87 | 0.96 |
| 418,500   | 26.557     | 3.45         | 84.38 | 0.01  | 0.29 | 0.45 |
| 671,660   | 26.560     | 3.46         | 84.76 | 0.00  | 0.00 | 0.00 |

**Figure 4.** Validation of the numerical results: (a) Outlet temperature and (b) pressure drop.
LMTD = \frac{T_o - T_i}{\ln\left(\frac{T_o - T_i}{T_i - T_e}\right)} \quad (11)

On the other hand, the pressure drop (\Delta p) is the difference between inlet and outlet pressure along the test section (equation (12)). Also, the pumping power (PP) required to drive the water through the annular space is calculated using equation (13).

\[ \Delta p = p_o - p_i \quad (12) \]

\[ PP = \Delta p U_b A_c \quad (13) \]

The convection heat transfer coefficient, pressure drop, and the water velocity are presented in dimensionless forms as Nusselt number (Nu), friction factor (f), and Reynolds number (Re) as defined in equations (14–16), respectively. Nu, f, and Re are based on the hydraulic diameter (D_h) that characterize the geometry of the annular channel (equation (17)). Also, dimensionless Prandtl number (Pr) accounts for the water properties, as defined in equation (18).

\[ Nu = \frac{h D_h}{k} \quad (14) \]

\[ f = \frac{2 \Delta p D_h}{\rho U_b^2 L} \quad (15) \]

\[ Re = \frac{\rho U_b D_h}{\mu} \quad (16) \]

\[ D_h = d_o - d_i \quad (17) \]

\[ Pr = \frac{c_p \mu}{k} \quad (18) \]

Taylor’s number (Ta) is a dimensionless quantity that characterizes the importance of centrifugal forces or so-called inertial forces due to rotation of fluid about an axis relative to viscous forces. There are various definitions of Taylor’s number in literature, which are not all equal. The most commonly used are Taylor’s number or the rotational Reynolds number (Re_t), as defined in equations (19–20)⁸ Taylor’s number incorporates the annulus radius or diameter ratio.⁹ It should be noted that some authors used the square root of Ta instead of Ta. The angular velocity (\omega) is defined by equation (21).

\[ Ta = \left(\frac{\omega d_i}{\nu}\right)^2 \left(\frac{d_o - d_i}{8(d_o + d_i)}\right)^3 \quad (19) \]

\[ Re_t = \frac{\omega d_i (d_o - d_i)}{4 \nu} \quad (20) \]

\[ \omega = 2 \pi n / 60 \quad (21) \]

Finally, the best-operating conditions are that lead to the highest ratio (R) of the heat transfer rate (Q) to the theoretical pumping power (PP) as defined by the following equation:

\[ R = \frac{Q}{PP} = \frac{Q}{\Delta p U_b A_c} = \frac{\rho c_p (T_o - T_i)}{\Delta p} \quad (22) \]

**Results and discussion**

Effects of rotation, eccentricity, and radius ratio on axial and tangential velocity distributions, friction factors, and Nusselt numbers in an annular channel between two concentric and eccentric cylinders are discussed in this section. The reported results cover concentric and eccentric annuli with radii ratios of 0.2, 0.4, 0.6, and 0.8 under stationary and rotating (n = 150, 300, and 400 rpm) inner cylinder with constant surface temperature and adiabatic stationary outer cylinder. The results cover Reynolds number from $2.0 \times 10^3$ to $1.236 \times 10^5$, Taylor number from $1.47 \times 10^6$ to $1.6 \times 10^{10}$, Prandtl number from 3.71 to 6.94, radius ratio (η) from 0.2 to 0.8, and eccentricity (ε) from 0 to 0.6.

**Axial and tangential velocity distribution**

**Concentric annuli.** The results of the concentric annular gap are reported for a fully developed turbulent flow (at $z = 1.9$ m and Re > 2000) under stationary (Ta = 0) and rotating conditions (Ta between $2.35 \times 10^7$ and $3.6 \times 10^{10}$). The simulations are carried out for wide to narrow annuli with radii ratios (η = Ri/Ro) from 0.2 to 0.8 with an eccentricity of the inner cylinder (ε = e/Ro) from 0 to 0.6, where e is the distance between the centers of the inner and outer cylinders, Ri the outer radius of the inner cylinder and Ro the inner radius of the outer cylinder, refer to Figure 1.

Taylor-Couette-Poiseuille flow in concentric annuli is affected by many factors such as the flow rate through the annulus, the radius ratio, and the rotational speed of the inner cylinder. Figure 5(a) to (d) shows the normalized axial (upper figures) and tangential (lower figures) velocity profiles in concentric annuli (ε = 0) of radii ratios of 0.2, 0.4, 0.6, and 0.8, respectively, at bulk fluid velocity of 0.35 m/s. The outer cylinder is stationary with an adiabatic surface, while the inner cylinder is isothermal and either stationary (n = 0) or rotating (n = 150, 300, and 400 rpm). Both axial (u) and tangential (w) velocities are normalized concerning the bulk fluid velocity ($U_b = 0.35$ m/s). The x-axis (r/s) is normalized concerning the distance between the inner and outer cylinders (s), with 0.0 at the outer cylinder surface and 1.0 at the inner cylinder surface.

The axial velocity distribution for a small radius ratio (wide annular space) shows a non-symmetric profile, particularly for the stationary inner cylinder as the maximum velocity occurs near the inner cylinder.
surface. For the stationary inner cylinder with the largest radius ratio (0.8), the velocity distribution is approximately symmetric, and the maximum axial velocity is near $r/s = 0.5$. This is mainly because as the radius ratio increases, the hydraulic diameter, and $Re$ decrease. Consequently, the velocity profile becomes less turbulent. On the other hand, the axial velocity distribution for the rotational cases is more symmetric than the stationary cases, with the maximum velocity takes place around $r/s = 0.5$. Thus, the axial velocities near the rotating inner cylinder surface are greater than those under stationary conditions, particularly for small radii ratios. The maximum axial velocities are about $1.12U_b$ for $h = 0.6$ and $1.18U_b$ for the largest radius ratio ($\eta = 0.8$), as shown in Figure 5.

The tangential velocity results mainly from the rotation of the inner cylinder. Thus, the tangential velocity ($v_r$) for the case of the stationary inner cylinder is zero and overlaps with the $x$-axis in Figure 5. Figure 5(a) to (d) (lower frames) show that the normalized tangential velocity is maximum at the rotating inner cylinder and zero at the outer stationary cylinder due to the no-slip condition in which the fluid velocity at all fluid-solid boundaries is equal to that of the solid boundary. Also, Figure 5(a) to (d) show that the tangential velocity increases with the rotational speed for all radii ratios. Also, the effect of the radius ratio on the tangential velocity is observed by comparing Figure 5(a) to (d). As the radius ratio increases, the annulus becomes smaller, and the effect of rotation on the tangential velocity becomes clearer. At the radius ratio of 0.8, the mean tangential velocity is greater than the axial velocity. Normalized tangential velocity reaches about $3U_b$ in the annular space for the concentric case, near the wall of the inner cylinder. This suggests a considerable effect of rotation in small annuli than in large annuli (small radii ratios).

Figure 6(a) to (d) show the axial (upper) and tangential (lower) velocity contours of the concentric annulus of radii ratios of 0.2, 0.4, 0.6, and 0.8, respectively, at the bulk fluid velocity of 0.35 m/s and 400 rpm. The contours are taken in the fully developed region at $z = 1.9$ m. These contours are corresponding to the cases represented by the red dashed curves in Figure 5(a) to (d). The contours of both axial and tangential velocities show no angular distribution for concentric annuli.
annuli. It is clear that the maximum velocity represented by a narrow ring (shown in dark red) occurs in the middle of the annulus of $\eta = 0.8$ (Figure 6(d)). The ring, representing the maximum axial velocity, is getting wider as $\eta$ decreases indicating the trend of higher turbulent flow as Re increases due to the hydraulic diameter increase. Also, the ring moves toward the inner cylinder as $\eta$ decreases for the wider annulus. It is also shown that the axial velocity is zero at the walls of the inner and outer cylinders (circles shown in dark blue color).

On the other hand, the development of the tangential velocity profile as the radius ratio increases from 0.2 to 0.8 is depicted in the lower part of Figure 6(a) to (d). It is clear that the tangential velocity is zero at the outer cylinder surface and maximum at the rotating inner cylinder surface. As the radius ratio increases from 0.2 to 0.8, the magnitude of the tangential velocity becomes higher, and the dark blue ring representing the zero tangential velocity becomes thinner. This is observed by comparing the development of the tangential velocity contours in Figure 6(a) to (d).

Similar trends to those shown in Figures 5 and 6 are observed for the other tested bulk fluid velocities (0.05–0.60 m/s) for the normalized axial velocity. However, the effect of rotation on the tangential velocity decreases as the bulk fluid velocity is increased. This is confirmed by the fact that, at 400 rpm, the maximum normalized tangential velocity reaches about 1.74, 2.97, and 6.97 for fluid bulk velocities of 0.6, 0.35, and 0.15 m/s, respectively.

**Eccentric annuli.** Figure 1 shows a section of an eccentric annulus where the eccentricity of the inner cylinder splits the annulus into a large space ($s = P1$) and a small space ($s = P2$). The results are reported for spaces P1 and P2 for a fully developed turbulent flow (at $z = 1.9$ m and $Re > 2000$) under stationary ($Ta = 0$) and rotating inner cylinder. The normalized axial and tangential velocity profiles are shown in Figures 7 and 8, respectively, in the eccentric annulus of spaces P1 and P2. The velocity profiles are presented at the largest tested eccentricity ($\varepsilon = 0.6$) for various radii ratios of 0.2, 0.4, 0.6, and 0.8, under stationary ($n = 0$) and rotation ($n = 150$, 300, and 400 rpm) conditions. Figures 7 and 8 are made of the same ordinate to ease visual comparison between velocity distribution in the annuli of various radii ratios for the large (P1, in the upper part of Figures 5 and 6) and small (P2, in the lower part of Figures 5 and 6) spaces.

In general, the axial and tangential velocity profiles at $\eta = 0.2$ (Figures 7(a) and 8(a)) in the large space (P1)
are somewhat similar to those of the concentric case (Figure 5(a)). This indicates a small effect of the eccentricity on the velocity profiles for a small radius ratio due to the low sensitivity of the large annular space to the eccentricity variation. On the other hand, the effect of eccentricity is more evident on the axial and tangential velocity distribution in the small space (P2) as presented in Figures 7(a) and 8(a) (lower frames). The axial velocity for the stationary inner cylinder, in the small space (P2), decreases in comparison with the concentric case, as can be seen in Figure 7(a). This is mainly because a small percentage of water flows in the small space (small flow area) due to a larger resistance to flow than that for the large space (large flow area).

Additionally, both axial and tangential velocities in the small space (P2) increase as the rotational speed increases. This is mainly because as the eccentricity increases, the space P2 becomes smaller and hence more sensitive to the rotation of the inner cylinder for both axial and tangential velocities. This is more obvious for greater radii ratios (η = 0.4–0.8) as the axial velocities under rotation conditions are higher than those under stationary conditions for the small eccentric space (P2) and lowers for the large eccentric space (Figure 7(b)–(d)). This pattern becomes clearer as the radius ratio increases. The velocity profile in the small eccentric space (P2) is symmetric only for η = 0.8. Also, the maximum axial velocity shifts near to the inner cylinder when it is stationary and to the outer cylinder when the inner cylinder is rotating.

Figure 8 shows that the tangential velocity profiles in the large eccentric space (P1) are similar to those of concentric annuli with η = 0.2–0.4 (Figure 5(a) and (b)), whereas those in the small eccentric space (P2) are comparable to those of concentric annuli with η = 0.6–0.8 (Figure 5(c) and (d)). The effect of eccentricity on
the tangential velocity in the small eccentric space is more significant than those in the case of the concentric annulus (Figure 5).

The effect of the radius ratio on the axial and tangential velocity contours in the eccentric annuli is shown in Figure 9(a) to (d) for radii ratios of 0.2, 0.4, 0.6, and 0.8, respectively. The velocity contours are taken in the fully developed region at $z = 1.9$ m for 4009 rpm. These contours are corresponding to the red dashed curves in Figures 7 and 8. Due to the combined effect of the inner cylinder rotation and eccentricity, the contours show an angular velocity distribution for both axial and tangential velocity. These angular variations of the axial and tangential velocity contours are different for each radius ratio. Accordingly, the maximum axial velocity in the large space (P1) takes place in the center of the eccentric annulus of the case of $\eta = 0.2$ in the wider ring, as shown in Figure 9(a). But the maximum axial velocity becomes thinner and shifts angularly in the cases of larger $\eta = 0.4$–0.8.

On the other hand, the tangential velocity contours (Figure 9) show lower values in the large eccentric space (P1) compared to those in the small eccentric space (P2). This is mainly because the rotation has a greater effect on the small space (P2) than the large space (P1). The tangential velocity contours have larger variations and magnitude for the larger radii ratios than for the small ones.

Figure 10(a) to (d) show the effect of rotation at 0, 150, 300, and 400 rpm, respectively, on the axial and tangential velocity contours for a radius ratio of 0.8 and eccentricity of 0.6. The velocity contours in Figure 10 are corresponding to the four cases presented in Figures 7(d) and 8(d). Figure 10(a) shows that for stationary annulus, the maximum axial velocity occurs at the large space (P1), but by increasing the rotational speed, the maximum velocity shifts angularly near the small space (P2). This is because the effect of rotation on the small space (P2) becomes more influential as the inner cylinder speed increases. On the other hand, the
Figure 9. Axial (upper) and tangential (lower) velocity contours of eccentric annulus ($e = 0.6$) of various radii ratios at bulk velocity, $U_b = 0.35 \text{ m/s}$ and 400 rpm: (a) $\eta = 0.2$, (b) $\eta = 0.4$, (c) $\eta = 0.6$, and (d) $\eta = 0.8$.

Figure 10. Axial (upper) and tangential (lower) velocity contours in eccentric annulus ($e = 0.6$), $\eta = 0.8$ and $U_b = 0.35 \text{ m/s}$: (a) 0 rpm, (b) 150 rpm, (c) 300 rpm, and (d) 400 rpm.
tangential velocity contours, in Figure 10(a) to (d), show that the velocity is zero for the stationary inner pipe. The tangential velocity increases as the rotational speed is increased, particularly in the small space (P2). It decreases from about 1 m/s at the rotating surface of the inner cylinder to zero at the surface of the stationary outer cylinder.

**Pressure drop and friction factors**

The pressure drop along the annular test section of the constant length of 2 m is predicted using the simulation program. The pressure drop increases as the water inlet velocity or the rotational speed of the inner cylinder is increased. The ratio of pressure drop in a wide annulus of radius ratio of 0.2 to a narrow one of radius ratio of 0.8 is listed in Table 4, for concentric and eccentric ($e = 0.6$) cases at rotational speeds 0, 150, 300, and 400 rpm. These ratios of pressure drop range from 0.019 for low to 0.042 for large inlet water velocity. Also, the rotational speed and eccentricity have a substantial effect on the ratio of pressure drop.

The pressure drop may be presented in the dimensionless form of friction factor, $f$, as defined by equation (15). Figure 11(a) to (d) show the effect of rotational speed on the friction factor of concentric ($e = 0$) and eccentric ($e = 0.6$) annuli for radius ratios 0.2, 0.4, 0.6, and 0.8, respectively, for the stationary and rotating inner cylinder.

In general, the friction factor decreases as the Reynolds number increases for the concentric and eccentric annuli (Figure 11). This is a typical relationship between friction factor and Reynolds number. It is also clear that the friction factor increases as the radius ratio increases for both concentric and eccentric annuli (Figure 11). This is mainly because as the radius ratio increases, the flow area decreases and, consequently, the resistance to flow increases. Also, the quantitative effect of rotational speed on the friction factor for concentric and eccentric annuli can be observed in Table 5. The ratio ($f_{rot}/f_{st}$) indicates that the friction factor is more pronounced for large radius ratio, low Reynolds number, and higher speed. As the rotational speed of the inner cylinder increases from 0 to 400 rpm, the friction factor increases by 7.7%–40.4% for a radius ratio of 0.2 and from 21.3% to 98.3% for a radius ratio of 0.8. The effect of rotation on the friction factor in eccentric annuli is larger than that in concentric annuli. As the speed increases from 0 to 400 rpm in the eccentric annulus ($e = 0.6$), the friction factor increases by 8.2%–46.6% for a radius ratio of 0.2 and 42.7%–148% for a radius ratio of 0.8 (Table 5).

In conclusion, the friction factor increases with increasing the rotational speed, or the radius ratio, where the effect is more pronounced at the smallest annulus space ($e = 0.8$) as the resistance to flow increases in the narrow annulus and the rotation makes the fluid more turbulent. Consequently, the pressure drop increases as the flow rate, rotational speed, and radius ratio increase, while the effect of rotation is more evident in the narrow annulus of high radius ratio ($e = 0.8$).

The effect of eccentricity on the friction factor for the stationary ($n = 0$ rpm) and rotating ($n = 400$ rpm) inner cylinder is presented in Table 6 for radii ratios 0.2, 0.4, 0.6, and 0.8. In a large stationary annulus (at $n = 0$ rpm), as the eccentricity increases from 0 to 0.6, the friction factor increases by less than 1% at low Reynolds number ($3.14 \times 10^5$) and decreases by about 2% at the largest Reynolds number ($1.236 \times 10^5$, Table 6). In intermediate stationary annulus (at $e = 0.4$), the friction factor increases with the eccentricity for all Reynolds numbers by 1%–2%. For a small stationary annulus (at $e = 0.6$ and $n = 0$ rpm), the friction factor decreases for all Re by 1%–17% (Table 6). Therefore, the effect of eccentricity on friction factor for stationary annuli depends on the radius ratio and Reynolds number.

Also, Table 6 presents the effect of eccentricity for rotational inner cylinder ($n = 400$) with radius ratios of 0.2, 0.4, 0.6, and 0.8, respectively, for eccentricity of 0, 0.15, 0.30, 0.45, and 0.60. In general, the friction factor increases as the eccentricity is increased for all Reynolds number by 1%–6%, except for wide annulus ($e = 0.2$) at high Re ($1.236 \times 10^5$), where the friction factor shows a slight decrease by about 2% with the eccentricity increase (Table 6). Also, the effect of

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**Table 4. Ratio of DP$_{0.2}$/DP$_{0.8}$ under different rotational speed for concentric and eccentric ($e = 0.6$) annuli.**

| V (m/s) | Concentric annuli (n, rpm) | Eccentric annuli, $e=0.6$ (n, rpm) |
|--------|---------------------------|-------------------------------|
|        | 0 | 150 | 300 | 400 | 0 | 150 | 300 | 400 |
| 0.15   | 0.026 | 0.024 | 0.020 | 0.019 | 0.032 | 0.023 | 0.020 | 0.019 |
| 0.25   | 0.031 | 0.029 | 0.025 | 0.023 | 0.036 | 0.028 | 0.025 | 0.023 |
| 0.35   | 0.033 | 0.032 | 0.029 | 0.027 | 0.039 | 0.032 | 0.028 | 0.027 |
| 0.45   | 0.035 | 0.034 | 0.032 | 0.030 | 0.041 | 0.034 | 0.031 | 0.029 |
| 0.60   | 0.037 | 0.037 | 0.035 | 0.033 | 0.042 | 0.037 | 0.034 | 0.032 |
Figure 11. Effect of speed on the friction factor of annuli at various radii ratios: (a) $h = 0.2$, (b) $h = 0.4$, (c) $h = 0.6$, and (d) $h = 0.8$. 

Concentric annulus ($\varepsilon=0$) (a) 0 rpm, (b) 150 rpm, (c) 300 rpm, (d) 400 rpm  
Eccentric annulus ($\varepsilon=0.6$)
Table 5. Ratio of $f_{rot}/f_{st}$ under different rotational speed for concentric and eccentric ($e = 0.6$) annuli.

| $\eta$ | Re $\times 10^4$ | Concentric annuli (n, rpm) | Eccentric annuli (n, rpm) |
|--------|------------------|-----------------------------|---------------------------|
|        |                  | 150 | 300 | 400 |                  | 150 | 300 | 400 |
| 0.2    | 3.139            | 1.137 | 1.304 | 1.404 | 1.144 | 1.323 | 1.446 |
|        | 12.36            | 1.016 | 1.051 | 1.077 | 1.025 | 1.067 | 1.082 |
| 0.4    | 1.177            | 1.234 | 1.581 | 1.822 | 1.262 | 1.712 | 1.919 |
|        | 4.636            | 1.036 | 1.104 | 1.157 | 1.028 | 1.101 | 1.170 |
| 0.6    | 0.523            | 1.283 | 1.715 | 2.028 | 1.395 | 1.886 | 2.259 |
|        | 2.060            | 1.039 | 1.128 | 1.204 | 1.063 | 1.191 | 1.311 |
| 0.8    | 0.196            | 1.275 | 1.699 | 1.983 | 1.599 | 2.140 | 2.480 |
|        | 0.773            | 1.037 | 1.131 | 1.213 | 1.172 | 1.323 | 1.427 |

Table 6. Ratio of $f_{ecc}/f_{con}$ under different eccentricity for stationary and rotating (n = 400) annuli.

| $\eta$ | Re $\times 10^4$ | Stationary annuli (n = 0 rpm) | Rotating annuli (n = 400 rpm) |
|--------|------------------|-------------------------------|-------------------------------|
|        |                  | 0.15 | 0.30 | 0.45 | 0.60 | 0.15 | 0.30 | 0.45 | 0.60 |
| 0.2    | 3.139            | 0.99 | 0.99 | 1.01 | 1.01 | 1.00 | 1.01 | 1.03 | 1.04 |
|        | 12.36            | 0.98 | 0.97 | 0.98 | 0.98 | 0.99 | 0.98 | 0.99 | 0.98 |
| 0.4    | 1.177            | 1.00 | 1.01 | 1.01 | 1.01 | 1.00 | 1.01 | 1.03 | 1.06 |
|        | 4.636            | 1.00 | 1.01 | 1.01 | 1.02 | 1.00 | 1.01 | 1.02 | 1.03 |
| 0.6    | 0.523            | 1.00 | 0.99 | 0.98 | 0.95 | 1.00 | 1.02 | 1.04 | 1.06 |
|        | 2.060            | 1.00 | 1.00 | 0.99 | 0.97 | 1.00 | 1.02 | 1.03 | 1.06 |
| 0.8    | 0.196            | 0.99 | 0.96 | 0.90 | 0.83 | 1.00 | 1.01 | 1.02 | 1.04 |
|        | 0.773            | 0.99 | 0.97 | 0.93 | 0.87 | 1.00 | 1.00 | 1.01 | 1.02 |

Table 7. Ratio of $h_{0.2}/h_{0.8}$ under different rotational speed for concentric and eccentric ($e = 0.6$) annuli.

| $V$ (m/s) | Concentric annuli (n, rpm) | Eccentric annuli, $e = 0.6$ (n, rpm) |
|-----------|-----------------------------|--------------------------------------|
|           | 0 | 150 | 300 | 400 | 0 | 150 | 300 | 400 |
| 0.15      | 0.933 | 1.322 | 1.552 | 1.712 | 0.982 | 1.299 | 1.478 | 1.581 |
| 0.25      | 0.949 | 1.186 | 1.301 | 1.443 | 1.005 | 1.197 | 1.333 | 1.391 |
| 0.35      | 0.967 | 1.114 | 1.267 | 1.341 | 1.022 | 1.146 | 1.266 | 1.319 |
| 0.45      | 0.993 | 1.068 | 1.209 | 1.276 | 1.020 | 1.108 | 1.215 | 1.271 |
| 0.60      | 0.982 | 1.029 | 1.142 | 1.204 | 1.016 | 1.082 | 1.167 | 1.216 |

Table 8. Ratio of $Nu_{rot}/Nu_{st}$ under different rotational speed for concentric and eccentric ($e = 0.6$) annuli.

| $\varepsilon$ | Re $\times 10^4$ | Concentric annuli (n, rpm) | Eccentric annuli (n, rpm) |
|---------------|------------------|-----------------------------|---------------------------|
|               |                  | 150 | 300 | 400 |                  | 150 | 300 | 400 |
| 0.2           | 3.139            | 2.11 | 3.43 | 4.33 | 2.22 | 3.54 | 4.30 |
|               | 12.36            | 1.12 | 1.44 | 1.68 | 1.14 | 1.46 | 1.70 |
| 0.4           | 1.177            | 2.17 | 3.45 | 4.30 | 1.99 | 3.32 | 4.10 |
|               | 4.636            | 1.10 | 1.37 | 1.61 | 1.12 | 1.38 | 1.62 |
| 0.6           | 0.523            | 1.98 | 3.14 | 3.88 | 1.84 | 3.15 | 4.02 |
|               | 2.060            | 1.14 | 1.44 | 1.68 | 1.08 | 1.37 | 1.61 |
| 0.8           | 0.196            | 1.49 | 2.06 | 2.36 | 1.67 | 2.35 | 2.67 |
|               | 0.773            | 1.07 | 1.24 | 1.37 | 1.07 | 1.27 | 1.42 |
Figure 12. Effect of speed on $Nu$ at various radii ratios: (a) $h = 0.2$, (b) $h = 0.4$, (c) $h = 0.6$, and (d) $h = 0.8$. 
eccentricity becomes less significant at high Re compared to that at low Re.

In conclusion, the effect of eccentricity on the friction factor is not monotonic for the different conditions. The trend is different for stationary and rotating conditions, small and large radii ratios, or low and high Reynolds numbers.

**Convection heat transfer and Nusselt numbers**

As discussed in section 3, the convection heat transfer coefficient \( (h) \) is evaluated at rotational speeds of the inner cylinder of 0, 150, 300, and 400 rpm and eccentricity of 0, 0.15, 0.30, 0.45, and 0.60 for radii ratios of 0.2, 0.4, 0.6, and 0.8. The effect of rotation on the ratio \( h_0.2/h_0.8 \) at various inlet water velocities is given in Table 7. It is clear that the ratio \( h_0.2/h_0.8 \) in a stationary concentric annulus is less than 1, particularly at a low water velocity. The rotation enhances \( h \) in a narrow annulus (\( \eta = 0.8 \)) more than in a wide annulus (\( \eta = 0.2 \)) to increase the ratio \( h_0.2/h_0.8 \) to become larger than 1. Also, the effect of rotation is more influential in the concentric annulus at a low water velocity. The effect of rotation on the ratio \( h_0.2/h_0.8 \) for the eccentric annulus is larger than that for the concentric annulus at the largest water velocity.

The results of heat transfer are presented in the form of Nu in Figure 12 for concentric (left column) and eccentric (\( \epsilon = 0.6 \), right column). Also, the quantitative effect of rotational speed on Nusselt number of concentric and eccentric annuli can be observed in Table 8. In the case of concentric cylinders at a radius ratio of 0.2, Nu increases by 111%, 243%, and 333% at Re of \( 3.14 \times 10^4 \) and by 12%, 44%, and 68% at Re of \( 1.236 \times 10^5 \) when the inner cylinder rotational speed increases from 0 to 400 rpm, respectively, as presented in Figure 12(a). The comparison between the augmentations in Nu indicates that Nu enhances as the rotation speed increases. Also, the effect of rotation on Nu enhancement decreases as Re increase. This can be noted in Figure 12(a) as the distances between the curves of different rotation decreases as Re increases. This is mainly because as axial Re increases, the flow turbulence increase, and the effect of rotational speed on the rate of heat transfer decreases.

The results in the case of concentric cylinders with a radius ratio of 0.6 (Figure 12(c)) indicates that at Re = \( 5.23 \times 10^3 \), Nu increases by 98%, 214%, and 288% as the rotational speed increases from 0 to 150, 300, and 400 rpm, respectively, as listed in Table 8. Comparing Nu of the concentric case in Figure 12(a) to (d) indicates that Nu in the wide annulus (with a low radius ratio) is greater than Nu in the small annulus (with a high radius ratio). Also, the effect of rotational speeds reduces as Re increases, which can be seen in Figure 12(a) to (d) by the narrower gap between curves at large Re compared to the wider gap at low Re.

Figure 12(a) to (d) (right column) show the effect of rotational speed on Nu for radii ratios of 0.2, 0.4, 0.6, and 0.8, respectively, for the annuli with an eccentricity of 0.6. Comparing Nusselt numbers for concentric and eccentric annuli, in Table 8, indicates that the rotational speed enhances Nu for eccentric annuli in a similar way to that reported for concentric annuli. Also, Nu for eccentric annulus of 0.6 is greater than Nu for concentric annulus of the same radius ratio. It can be concluded that the rotational speed has a positive effect on the convective heat transfer at all radii ratios for concentric and eccentric annuli.

Table 9 presents the effect of eccentricity (\( \epsilon = 0.15, 0.30, 0.45, \) and 0.60) on Nu of stationary and rotating (\( n = 400 \text{ rpm} \)) annuli for radii ratios 0.2, 0.4, 0.6, and 0.8, respectively. For stationary annulus, Nu enhances as the eccentricity of the inner cylinder increases from 0 to 0.6, particularly for intermediate radii ratios (0.4 and 0.6), with the largest Nu occurs at the eccentricity of 0.6. Except for the radius ratio of 0.8 (narrow annulus), where Nu decreases by 1%–4% at the eccentricity of 0.6. Also, the effect of eccentricity on Nu, for stationary annuli, is more pronounced for small Reynolds numbers (Table 9). This is due to the increase in the

| \( \eta \) | Re \( \times 10^4 \) | Stationary annuli \((n=0 \text{ rpm})\) | Rotating annuli \((n=400 \text{ rpm})\) |
|---|---|---|---|
| | 0.15 | 0.30 | 0.45 | 0.60 | 0.15 | 0.30 | 0.45 | 0.60 |
| 0.2 | 3.139 | 1.00 | 1.00 | 1.02 | 1.01 | 1.01 | 1.02 | 1.02 | 1.05 |
| 0.4 | 12.36 | 1.01 | 1.01 | 1.03 | 1.02 | 1.01 | 1.01 | 1.03 | 1.04 |
| 0.6 | 1.177 | 1.00 | 1.02 | 1.04 | 1.08 | 1.00 | 1.00 | 1.01 | 1.05 |
| 0.8 | 4.636 | 1.00 | 1.01 | 1.02 | 1.04 | 1.00 | 1.00 | 1.01 | 1.03 |
| 0.196 | 2.060 | 1.00 | 1.02 | 1.05 | 1.07 | 1.00 | 1.01 | 1.01 | 1.03 |
| 0.773 | 0.096 | 1.01 | 1.00 | 1.00 | 1.05 | 1.01 | 1.03 | 1.05 | 1.09 |
| 0.0 | 1.00 | 1.00 | 1.00 | 1.00 | 1.01 | 1.01 | 1.03 | 1.09 | 1.03 |
turbulence of the fluid, and hence the heat transfer and Nu due to the eccentricity increases, particularly for large eccentricity values.

On the other hand, Nu enhances up to 9% and 12% for radii ratios of 0.6 and 0.8, respectively, at an eccentricity of 0.6 when the inner cylinder rotates at 400 rpm, particularly at a low Reynolds number (Table 9). Comparing ($N_{ucc}/N_{con}$) for stationary and rotating annulus (Table 9) indicates that the effect of eccentricity ($\varepsilon = 0.60$) on Nu is more significant in the case of rotation ($n = 400$ rpm) compared to that of stationary. It can be stated that the combined effects of eccentricity and rotation enhance Nu significantly, particularly for radii ratios of 0.6 and 0.8 at low Re. Therefore, Nu increases by increasing both rotation and eccentricity of the inner cylinder up to 12%, in cases of large radii ratios due to the enhancement in the fluid and thermal boundary layers in narrow gaps.

**Correlations of Nusselt number and friction factor**

Nusselt number data points, in concentric and eccentric annuli for stationary and rotating cases, are correlated in terms of Reynolds number, Taylor number, Prandtl number, and radii ratios. The data of concentric and eccentric stationary annuli are fitted by a single equation with a correlation coefficient of 0.999 and 94 degrees of freedom, as given in equation (23). Also, the data for concentric and eccentric rotating cases are fitted by equation (24) with 294 degrees of freedom and correlation factors of 0.994. Figure 13(a) shows the predicted Nu using equations (23) and (24) for stationary and rotating cases versus the simulated Nu. The data are well scattered around the plotted 1:1 dashed line indicating the good accuracy of predicting Nu using the stated equations.

\[
N_{\text{st}} = 0.0244 \, Re^{0.834} \, Pr^{0.4} \, \eta^{-0.15} \tag{23}
\]

\[
N_{\text{rot}} = 0.126 \, Re^{0.19} \, Pr^{0.4} \, Ta^{0.276} \, \eta^{-0.115} \tag{24}
\]

The above equations are valid for Re between 2000 and 123,618, Pr between 3.71 and 6.94, Ta of zero and between $1.47 \times 10^{6}$ and $1.60 \times 10^{10}$, and $\eta$ between 0.2 and 0.8. The constant and exponents of Re and Pr in equation (23) are similar to those of the classical correlations of the turbulent flow inside cylinders or tubes (Sider-Tata correlation). The exponent of the radius ratio indicates that Nu for the smallest radius ratio (0.2) is higher than those of the largest radii ratios (0.8) by about 23.1% for the same Re and Pr numbers.

On the other hand, equation (24) indicates a substantial effect of rotation on the heat transfer data. This is indicated by the influential exponent of Ta and the considerable reduction in Re exponent from 0.834 for stationary annulus to 0.19 for rotating annulus. The effect of the radius ratio on Nu for the rotating case is lower than that for the stationary case, as indicated by the smaller exponent, which is lower than the exponent for stationary cases. In the rotating case, Nu for the smallest radius ratio is larger than that for the largest radius ratio by about 17.3%, for the same Re and Pr.

The 400 data points of the friction factor are correlated in a similar way to that described for the correlation of Nu. It was possible to correlate the friction factor of stationary concentric and eccentric annuli by one equation with a correlation factor of 0.927 and 94 degrees of freedom as given in equation (25). Similarly, the friction data for rotating concentric and eccentric annuli are correlated by equation (26). Equation (26) has a correlation factor of 0.953 and 288 degrees of freedom. Figure 13(b) shows the relation between the predicted and simulated friction factors. The friction

![Figure 13. Predicted versus simulated results for stationary and rotating data: (a) Nusselt number, (b) Friction factor.](image)
factor data for stationary and rotating eccentric annuli are fitted better than those of the rotating concentric annuli, which show relatively more scattered data around the plotted 1:1 line.

\[
\begin{align*}
    f_{st} &= 2.894 \frac{Re^{-0.42}}{Pr^{-0.44}} \\
    f_{rot} &= 10.62 \frac{Re^{-0.7}}{Pr Ta^{-0.089}} \eta^{-0.36}
\end{align*}
\]

Equations (25) and (26) are valid for the same ranges of variables specified for equations (23) and (24). Equations (25) and (26) indicated that the exponent for Re, in the case of stationary annuli, is noticeably different from the exponent for the rotating annuli. This is mainly due to the significant effect of Ta. Also, the effect of the radii ratios is considerable for either stationary or rotating cases.

Finally, it should be stated that due to the small and non-monotonic effect of the eccentricity on the Nusselt number and friction factor data under the investigated conditions, it was not practical to incorporate the eccentricity in equations (23)–(26). However, the newly developed correlations (equations (23)–(26)) incorporate the effect of radius ratio on stationary and rotating annuli, which was not correlated before.

**Summary and conclusion**

The investigation of fluid flow and heat transfer in concentric and eccentric annuli with a stationary adiabatic outer cylinder and a stationary or rotating isothermal inner cylinder is conducted for rotational speeds 0, 150, 300, and 400 rpm, radii ratios 0.2, 0.4, 0.6, and 0.8 with eccentricities 0.0, 0.15, 0.30, 0.45, and 0.6. The effects of radius ratio, rotational speed, and eccentricity are reported for Re, Ta, and Pr ranges of \(2.0 \times 10^5–1.236 \times 10^5\), \(1.47 \times 10^6–1.6 \times 10^6\), and \(3.71–6.94\), respectively. Based on the reported results, the following conclusions are drawn:

- Correlations for heat transfer and pressure drop, using 400 data points for each, are developed in the form of Nu and f as a function of Re, Ta, Pr, and radii ratios (\(\eta\)). Two correlations are developed for Nu and f under stationary and rotating conditions, with \(R^2\) greater than 0.99 and 0.92, respectively. The correlations facilitate the design of rotating and stationary heat exchangers and other applications based on the annular space between fixed and rotating cylinders.
- The heat transfer and pressure drop in cylindrical annuli are largely affected by the rotational speed, followed by the radius ratio, while the effect of eccentricity is small in the investigated range of 0–0.6.
- The rotational speed of the inner cylinder enhances the convective heat transfer for concentric and eccentric annuli, particularly at low Reynolds numbers. As the rotational speed increases from 0 to 400 rpm, Nu increases by 333% at Re of \(3.14 \times 10^4\) and by 68% at Re of \(1.236 \times 10^5\), for a radius ratio of 0.2.
- The radius ratio has a considerable effect on pressure drop and heat transfer. The pressure drop at a large radius ratio (narrow annuli) is 23.8–52.6 times that at a small radius ratio for concentric and eccentric stationary or rotating annuli.
- The effect of eccentricity on Nu for rotating annuli is larger than that for stationary annuli; as the eccentricity increases from 0 to 0.60, Nu enhances up to 8% for stationary annuli and 12% for rotating annuli at a speed of 400 rpm and a radius ratio of 0.60.
- As the rotational speed of the inner pipe increases from 0 to 400 rpm, the pressure drop increases by 40.4%, and 98.3%, in wide and narrow annular gaps, respectively, for low Reynolds number and by 7.6% and 21.3% for high Reynolds number.
- The friction factor increases with increasing the rotational speed or the radius ratio. However, the effect of eccentricity on the friction factor is not monotonic.
- For concentric annuli, the axial velocity profile is not symmetric for a small radius ratio (wide annular space) under stationary conditions and becomes relatively symmetric for large radii ratios or under rotation conditions.
- The effect of rotation is more important in small annuli (large radii ratios) than that in large annuli (small radii ratios).
- The effect of eccentricity on the axial and tangential velocity profiles is distinct for large radii ratios and a rotating inner cylinder.
- As the eccentricity splits the annulus into large and small spaces, the effect of eccentricity on the velocity profiles is more pronounced in the small annular space for the small radius ratio.
- The combined effect of eccentricity and rotation of the inner cylinder causes obvious angular distribution in the axial velocity contours, which is not exit for concentric annuli.

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**Appendix**

**Notation**

- $A_c$ : annulus flow cross section area (m²)
- $A_s$ : surface area of heat transfer (m²)
- $C$ : specific heat of air (J/kgK)
- $d_i$ : diameter of inner pipe (m)
- $d_o$ : diameter of outer pipe (m)
- $D_h$ : hydraulic diameter (m)
- $e$ : eccentric distance between the two pipes centers
- $f$ : friction factor (-)
- $h$ : convection heat transfer coefficient (W/m²K)
- $k$ : thermal conductivity (W/mK)
- $L$ : effective length of test section
- $m$ : water flow rate (kg/s)
- $n$ : rotational speed of inner cylinder (rpm)
- $Nu$ : Nusselt number (-)
$p$ \quad \text{pressure (Pa)}

$p_i$ \quad \text{inlet pressure (Pa)}

$p_o$ \quad \text{outlet pressure (Pa)}

$PP$ \quad \text{pumping power (W)}

$Pr$ \quad \text{Prandtl number (-)}

$Q$ \quad \text{heat transfer rate (W)}

$r$ \quad \text{r-coordinate}

$R$ \quad \text{Ratio of heat transfer to pumping power (-)}

$R_i$ \quad \text{radius of inner pipe (m)}

$R_o$ \quad \text{radius of outer pipe (m)}

$Re$ \quad \text{Reynolds number based on the free-stream conditions and hydraulic diameter}

$S_T$ \quad \text{source term}

$t$ \quad \text{time (s)}

$T$ \quad \text{temperature (K)}

$T_i$ \quad \text{water inlet temperature (K)}

$T_o$ \quad \text{water outlet temperature (K)}

$T_s$ \quad \text{surface temperature of the inner pipe (K)}

$Ta$ \quad \text{Taylor number (-)}

$u$ \quad \text{axial water velocity (m/s)}

$u_r$ \quad \text{r-velocity (m/s)}

$u_\theta$ \quad \text{$\theta$-velocity (m/s)}

$u_z$ \quad \text{z-velocity (m/s)}

$U_b$ \quad \text{bulk water velocity (m/s)}

$w$ \quad \text{tangential velocity (m/s)}

$z$ \quad \text{z-coordinate}

Greek symbol

$\alpha$ \quad \text{thermal diffusivity (m$^2$/s)}

$\Delta p$ \quad \text{pressure drop (Pa)}

$\varepsilon$ \quad \text{eccentricity (-)}

$\eta$ \quad \text{radius ratio} = \frac{R_i}{R_o}

$\theta$ \quad \text{\text{\theta-coordinate}}

$\mu$ \quad \text{dynamic viscosity (Pa.s)}

$\nu$ \quad \text{Kinematic viscosity (m$^2$/s)}

$\rho$ \quad \text{fluid density (kg/m$^3$)}

$\omega$ \quad \text{Angular velocity (rad/s)}

Abbreviations

ANSYS \quad \text{Analysis system}

CFD \quad \text{Computational fluid dynamics}

DNS \quad \text{Direct numerical simulation}

LES \quad \text{Large Eddy simulation}

RANS \quad \text{Reynolds averaged Navier-stokes}

SST \quad \text{Shear stress transport}