In an introductory manner, the nature of dark energy is addressed, how it is observed and what further tests are needed to reconstruct its properties. Several theoretical approaches to dark energy will be discussed.

1 Plan of the Talk

- What observations and theoretical assumptions underly dark energy (DE)?
- If general relativity (GR) holds at all length scales, the most conservative assumption, then DE follows from the supernovae Type 1A (SNe1A) or, independently, from the Cosmic Microwave Background (CMB) combined with Large Scale Structure (LSS).
- Should we seriously query GR at large distance scales?

2 Einstein-Friedmann Equation

The Einstein equations relate geometry on the Left-Hand-Side (LHS) to the distribution of mass-energy on the Right-Hand-Side (RHS)

\[ G_{\mu\nu} = -8\pi G T_{\mu\nu} \]  

We hesitate to change the LHS but it is really checked with precision only at Solar System (SS) scales. At cosmological length scales, we may consider using a modification such as higher-dimensional gravity.

On the RHS, if we include only luminous and dark matter it is insufficient (keeping the LHS intact) and there is needed a further term which could be a cosmological constant or, more generally, dark energy.
3 Observational Issues

How can we constrain DE?

- Measurement of the expansion history $H(t)$
- The time-dependence of the equation of state $w(t)$
- Looking for any clustering property of DE. No evidence for this presently.
- How does DE couple to Dark Matter (DM)? This is related to the question of clustering.
- Local tests of GR and the equivalence principle, though the extrapolation from the SS to the Universe is some 13-15 orders of magnitude comparable to the extrapolation from the weak scale to the GUT scale in particle phenomenology. The usual prior is a desert hypothesis.

4 $\Lambda$ as DE: Why $10^{-122}$ (Planck Mass)$^4$?

We know from the Lamb Shift and Casimir Effect in quantum electrodynamics that vacuum fluctuations are real effects.

If we calculate the value of $\Lambda$, it will naively be ultra-violet (UV) quartically divergent. The most natural UV cut-off in GR is the Planck mass $\sim 10^{19} GeV$ whereupon

$$\Lambda \sim (10^{19} GeV)^4 = (10^{28} eV)^4 = 10^{112}(eV)^4$$

If we use, instead, the weak scale $\sim 100 GeV$ as our UV cut-off, we arrive at

$$\Lambda \sim (100 GeV)^4 = (10^{11} eV)^4 = 10^{44}(eV)^4$$

The observed value for $\Lambda$, by contrast, is approximately

$$\Lambda \sim (3 \times 10^{-3} eV)^4 \sim 10^{-10}(eV)^4$$

5 Coincidence Problem

As if the fine-tuning problem for $\Lambda$ were not enough, there is a second problem with $\Lambda$, the coincidence problem. Let us define $\Omega_\Lambda = \rho_\Lambda/\rho_C$ as the fraction of the critical density $\rho_C$.

The present value is $\Omega_\Lambda \sim 0.7$ but it scales, since $\rho_\Lambda$ is constant and assuming $\Omega_{TOT} = 1$, like $\rho_C^{-1} \sim (1 + Z)^{-3}$ so at a redshift $Z > 10$ it was $\Omega_\Lambda < 0.001$ while for a future redshift $Z < -0.9$ one has $\Omega_\Lambda > 0.999$.

If we plot $\Omega_\Lambda$ versus log $R$ over cosmic history from $-60 < \log_{10} R < +60$, it appears like a step function changing from zero to one abruptly around the present era. Even more dramatic is a plot of $d\Omega_\Lambda/dR$ which approximates a Dirac delta function and the coincidence problem is then why we live right in the middle of the spike of the delta function.

If the dark energy had appeared earlier it would have interfered with structure formation: if later, we would still be unaware of it.

6 The Quintessence Possibility

One parametrization of the dark energy can be made using a dynamical scalar field, now generically called quintessence.
6.1 Scaling potentials

Examples are:

\[ V \sim e^{-\lambda \Phi} \]  

(5)

as in [12],

\[ V \sim ((\Phi - A)^2 + C)e^{-\lambda \Phi} \]  

(6)

as in [3].

6.2 Tracker Potentials

Examples are

\[ V \sim \Phi^{-\alpha} \]  

(7)

as in [4],

\[ V \sim \exp \left( \frac{M}{Q} - 1 \right) \]  

(8)

as in [5].

6.3 Approaches to the Coincidence Problem

We may assume that our universe sees periodic epochs of acceleration with potential

\[ V \sim M^4 e^{-\lambda \Phi}(1 + A \sin a \Phi) \]  

(9)

Another possibility is that it is important that our epoch is close to the matter/radiation equality time. This may be incorporated by having a non-minimal coupling to matter to gravity or in a k-essence theory with a non-trivial kinetic term in the lagrangian.

7 Dark Energy with Equation of State \( w = p/\rho < -1 \)

Present data on SNe1A, CMB and LSS are consistent with \( w = -1 \) as for a cosmological constant.

Since the possibility that \( w < -1 \) is still allowed, I shall spend a disproportionate amount of time on it because, if it persisted, it could well signal new physics.

One interpretation of dark energy comes from string theory, closed strings on a toroidal cosmology. This leads generically to \( w < -1 \).

In general, without dark energy (as in most cosmology texts pre-1998), the destiny of the Universe was tied to geometry in a simple manner: the Universe will expand forever if it is open or flat; it will stop expanding and contract to a Big Crunch if it is closed.

With Dark Energy, this connection between geometry and destiny is lost and the future fate depends entirely on how the presently-dominant dark energy will evolve.

This question is studied in [13][14][15]. If \( w < -1 \) and is time-independent, the scale factor diverges at a finite future time - the Big Rip. Generally, this will be at least as far in the future as the Big Bang was in the past.

Such a cosmology may have philosophical appeal? There is more symmetry between past and future.

If one allows a time-dependent \( w(t) \), there are two other possible fates:

(i) An infinite-lifetime universe where dark energy dominates at all future times.
(ii) A disappearing dark energy where the Universe becomes (again) matter dominated. The case $w < -1$ gives rise to some exceptionally interesting puzzles for theoretical physics.

There is the question of violation of the weak energy condition universally assumed in general relativity. This means there are inertial frames where the energy density is negative signaling vacuum instability\[16\]17.

Let us make three assumptions, any or all of which may be incorrect, just so that we may say something more: that (i) There is a stable vacuum with $\Lambda = 0$; (ii) The dark energy decays to it by a 1st-order phase transition; (iii) There is some, albeit feeble, interaction between dark energy and the electromagnetic field.

Then one can use old arguments\[16\] to investigate nucleation. The result is that\[16\] even with the tiniest coupling of dark energy to the electromagnetic field the dark energy would have spontaneously decayed long ago unless the appropriate bubble radius is at least galactic in size.

In this model, because the energy density of the DE is so small compared to e.g. the energy density in a common macroscopic magnetic field of, say, 10T the 1st order phase transition can be adequately suppressed only by decoupling the DE completely from all but gravitational forces or by arguing that a collision would need to be between galaxies or larger objects to be effected. Certainly, no terrestrial experiment can be influenced: for one contrary suggestion of a Josephson junction experiment which might well be justified for other reasons, see e.g.\[19\].

Of course, this is only a toy model but the general conclusion is probably correct - that there can be no microscopic effect of the dark energy.

This makes the DE very difficult or impossible to investigate except through astronomical observations.

8 Dark Energy and Neutrinos

It has been pointed out by many theorists that the density of the dark energy $\sim (10^{-3} eV)^4$ is suggestive of the neutrino mass.

Very interesting attempts to strengthen such a connection have been made\[20\],\[21\]. Such mass-varying neutrino models seek to make a direct identification of the DE density with neutrino mass\[22\],\[23\] itself.

9 Precision Experiment

We know well of the precision experiments to test Newton’s Law of Gravity down to a distance of 100 microns and below.

One originator of such ideas suggests\[21\] a different precision test, of the Earth-Moon distance, to a similar accuracy of 100 microns, presumably the distance between the centers of mass. A particular modification of gravity\[25\] might have a tiny effect on our lunar system. Clearly if this experiment can be achieved, the present accuracy being at the level of centimeters, it would be an impressive achievement.

10 Conclusions: Observation and Theory

- The theoretical community has yet to come up with a definitive proposal to explain the dark energy.
- The nature of the dark energy is so profound for cosmology and particle physics that we desperately need more SNe1A observations from important proposed experiments e.g. SNAP (for which NASA funding has sadly been suspended for 5 years as a result of prioritizing sending humans to Mars!), as well as complementary observational constraints on the CMB from e.g. the Planck mission.
• The equation of state will be decisive. If $w=-1$, it’s a cosmological constant with its fine-tuning and coincidence problems. If $w > -1$ quintessence will receive a shot in the arm.

• If the data would settle down to a value $w < -1$ we could be at the dawn of a revolution in theory with general relativity at the largest distance scales called into question.

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References

1. C. Wetterich, Nucl. Phys. B302, 668 (1988).
2. P.G. Ferreira and M. Joyce, Phys. Rev. D58, 023503 (1998).
3. A. Albrecht and S. Skodis, Phys. Rev. Lett. 84, 2076 (2000); Phys. Rev. D66, 043523 (2002).
4. B. Ratra and P.J. E. Peebles, Phys. Rev. D37, 3406 (1988). P.J. E. Peebles and B. Ratra, Ap. J. 325, 217 (1988).
5. P.J. Steinhardt, L.-M. Wang and I. Zlatev, Phys. Rev. D59, 123504 (1999).
6. S. Dodelson, M. Kaplinghat and E. Stewart, Phys. Rev. Lett. 85, 5276 (2000).
7. R. Bean and J. Magueijo, Phys. Rev. D66, 063505 (2002).
8. F. Perrotta and C. Baccigalupi, Phys. Rev. D65, 23505 (2002).
9. C. Armendariz-Picon, V. Mukhanov and P.J. Steinhardt, Phys. Rev. Lett. 85, 4438 (2000).
10. A. Melchiorri, L. Mersini, C.J. Odman and M. Trodden, Phys. Rev. D68, 043509 (2003).
11. M. Bastero-Gil, P.H. Frampton and L. Mersini, Phys. Rev. D65, 106002 (2002).
12. P.H. Frampton, Phys. Lett. B555, 139 (2003) astro-ph/0209037
13. R. Kallosh and A. Linde, JCAP 0302: 002 (2003).
14. P.H. Frampton and T. Takahashi, Phys. Lett. B557, 135 (2003) astro-ph/0211544
15. P.H. Frampton and T. Takahashi, Astropart. Phys. (in press) astro-ph/0405333

J.L. Crooks, J.O. Dunn, P.H. Frampton, H.R. Norton and T. Takahashi, Astropart. Phys. 20, 361 (2003). astro-ph/0305495
17. R.R. Caldwell, Phys. Lett. B545, 23 (2002).
16. P.H. Frampton, Mod. Phys. Lett. A19, 801 (2004). hep-th/0302007
17. S.M. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D68, 023509 (2003). astro-ph/0311273
18. P.H. Frampton, Phys. Rev. Lett. 37, 1378 (1976); Phys. Rev. D15, 2922 (1977).
19. C. Beck and M.C. Mackey. astro-ph/0406504
20. P.Q. Hung, hep-ph/0010126. P.Q. Hung and H. Paes. astro-ph/0311131
21. R. Fardon, A.E. Nelson and N. Weiner. astro-ph/0309800
22. D.B. Kaplan, A.E. Nelson and N. Weiner. hep-ph/0401099
23. P.H. Frampton and P. Vogel, Physics Reports 82, 339 (1982).
24. P.H. Frampton. hep-ph/0403164. hep-ph/0310217. P.H. Frampton, S.T. Petcov and W. Rodejohann, Nucl. Phys. B687, 31 (2004). hep-ph/0401206. P.H. Frampton, S.L. Glashow and T. Yanagida, Phys. Lett. B548, 119 (2002). hep-ph/0208157. P.H. Frampton, M.C. Oh and T. Yoshikawa, Phys. Rev. D66, 033007 (2002). hep-ph/0204273
Phys. Rev. D65, 073014 (2002). [hep-ph/0110300]. P.H. Frampton, S.L. Glashow and D. Marfatia, Phys. Lett. B536, 79 (2002). [hep-ph/0201008]. P.H. Frampton and S.L. Glashow, Phys. Lett. B461, 85 (1999). [hep-ph/9906375].

24. G. Dvali, A. Gruzinov and M. Zaldarriaga, Phys. Rev. D68, 024012 (2003). [hep-ph/0212069].

25. G.R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B485, 208 (2000). [hep-th/0005016].