Non linear $\sigma$ models:
renormalisability versus geometry.\textsuperscript{*}

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Abstract

After some recalls on the standard (non)-linear $\sigma$ model, we discuss the interest of B.R.S.
symmetry in non-linear $\sigma$ models renormalisation. We also emphasise the importance of
a correct definition of a theory through physical constraints rather than as given by a
particular Lagrangian and discuss some ways to enlarge the notion of renormalisability.
1 Introduction

The subject of non-linear $\sigma$ models is relevant to this meeting for two reasons:

- Carlo Becchi has been involved in that field in several occasions. A tentative list could be:
  - Axial anomaly in a linear $\sigma$ model with fermions (1973) \cite{1},
  - Non-linear $\sigma$ model and the bosonic string (1986-1988) \cite{2},
  - Renormalisability and I.R. finiteness in non-linear $\sigma$ models on coset spaces (1987-1988) \cite{3},
  - Non-linear $\sigma$ model with Wess-Zumino term (1989) \cite{4},
  - Non-linear $\sigma$ model with (4,0) supersymmetry (1990) \cite{5}.

- In Paris, with Galliano Valent and François Delduc \cite{6}, between 1983 and 1987 we studied the quantisation of non-linear $\sigma$ models in 2 space-time dimensions for a large class of target spaces including $CP^1$, $CP^n$, the grassmannians, Kähler symmetric then Kähler homogeneous ones. After his sabbatical year in Paris in 1981, we kept contact with Carlo and his group and I had the chance to suggest a collaboration between our two groups, which led to the renormalisation of all coset-space models with the algebraic methods à la B.R.S. \cite{3}. My personal interest in that field was mostly its use as a toy model to discuss new theoretical ideas such as on shell renormalisation and the enlargement of the notion of what a renormalisable theory might be.

In the first part of this talk, I will recall some well known facts on non-linear $\sigma$ models whose physical interest goes back to current algebra, P.C.A.C., broken symmetries and Goldstone mechanism. Other fields such as statistical models in $2+\epsilon$ dimensions, $1/N$ non perturbative methods, mass gap computations or string vacuum description, will not be considered (see however the talk of François Delduc \cite{7}).

In a second part, I will explain the power of B.R.S. methods in that field, the possible “apparent” non-renormalisability and end with some open questions on “generalised renormalisability”.

2 The “old” $O(N+1)$ $\sigma$ model.

2.1 The linear $\sigma$ model.

In their famous paper, GellMann and Lévy \cite{8} present several Lagrangians ensuring a realisation of the Partially Conserved Axial Current (P.C.A.C.) observation; one of them was proposed by Schwinger \cite{9} and writes:

$$\mathcal{L} = (1/2)[(\partial_\mu \vec{\pi})^2 + (\partial_\mu \sigma)^2] - (1/2)m^2[\vec{\pi}^2 + \sigma^2] + (\lambda/4)[\vec{\pi}^2 + \sigma^2]^2 + \mathcal{L}_{\text{breaking}},$$

where the breaking of $O(N+1)$ to $O(N)$ is given by a non vanishing $\sigma$ field expectation value, resulting from a linear

$$\mathcal{L}_{\text{breaking}} = c \sigma.$$
The generators of the symmetry being respectively:

\[ W_{ij}^V \sim \int dx \left[ \pi_i \frac{\delta}{\delta \pi_j} - \pi_j \frac{\delta}{\delta \pi_i} \right] , \quad W_k^A \sim \int dx \left[ \sigma \frac{\delta}{\delta \pi_k} - \pi_k \frac{\delta}{\delta \sigma} \right] , \] (3)

the \( O(N+1) \) algebra is:

\[
\begin{align*}
[W_{ij}^V, W_{kl}^V] &= \delta_{jk} W_{il}^V - \delta_{il} W_{jk}^V - \delta_{ik} W_{jl}^V + \delta_{jl} W_{ik}^V , \\
[W_{ij}^V, W_k^A] &= \delta_{jk} W_i^A - \delta_{ik} W_j^A , \\
[W_i^A, W_j^A] &= -W_{ij}^V .
\end{align*}
\] (4)

In presence of the term (2), the “axial” current conservation is broken according to the strong P.C.A.C.- Adler condition:

\[
J_{\mu}^A i(x) = \sigma \partial_\mu \pi^i - \pi^i \partial_\mu \sigma , \quad \partial_\mu < J_{\mu}^A i(x) X > = -c < \pi^i(x) X > + \text{contact terms} . \] (5)

Note that, in a nice paper of 1973 [1], Carlo Becchi gave the first complete analysis of a linear \( \sigma \) model with fermions, defined by the requirement of the P.C.A.C.-Adler condition (3).

2.2 The non-linear \( S^N \) model.

In their paper, GellMann and Lévy also proposed another Lagrangian, taking into account the experimental absence of any particle \( \sigma \) similar to the pions; moreover, it also explains the smallness of the mass of the pions when compared to others hadrons. Thanks to the constraint

\[
\bar{\pi}^2 + \sigma^2 = f_\pi^2 ,
\]

the \( O(N+1) \) symmetry is now non-linearly realised [1]. The algebra (3) is left unchanged but the \( W_i^A \) generators are now non-linear ones thanks to the replacement \( \sigma = \sqrt{f_\pi^2 - \bar{\pi}^2} \). The pion field belongs to the symmetric space \( S^N = O(N+1)/O(N) \). The Lagrangian (3) now describes massless particles. The non zero mass of the physical pions - and the correlative non exact conservation of the axial current - will result either from an explicit breaking

\[
\Delta L = c \sqrt{f_\pi^2 - \bar{\pi}^2} ,
\]

or from a Nambu-Golstone mechanism, through the spontaneous breaking of the \( O(N+1) \) symmetry to the “vector” \( O(N) \) one. The “axial” symmetry is now non-linearly realised, according to:

\[
\delta_{\beta} \bar{\pi} = \bar{\beta} \sqrt{f_\pi^2 - \bar{\pi}^2} , \quad \delta_{\gamma} \delta_{\beta} \bar{\pi} = -\bar{\beta} (\bar{\gamma}, \bar{\pi}) , \] (6)

and the axial current is:

\[
J_{\mu}^A i(x) = \sqrt{f_\pi^2 - \bar{\pi}^2} \partial_\mu \pi^i - \pi^i \partial_\mu \sqrt{f_\pi^2 - \bar{\pi}^2} \quad \text{with} \quad \partial_\mu J_{\mu}^A i(x) = -c \pi^i(x) . \] (7)

At the end of the sixties, physicists use the geometry of coset spaces \( G/H \) to study various realisations of the chiral symmetry [ understood as being rather \( \frac{SU(2) \times SU(2)}{SU(2)} \) than \( \frac{O(4)}{O(3)} \) ]; later on, this geometrical approach was generalised, in the framework of 2-dimensional field theory, to an arbitrary riemannian manifold (see [1] for a review), as I recall in the next subsection.
2.3 The non-linear $\sigma$ model.

Given an arbitrary $N$-dimensional riemannian space, the lagrangian density is written:

$$\mathcal{L} = (ds)^2 = \frac{1}{2} \left[ \eta^{\mu\nu} g_{ij}(\phi) \partial_\mu \phi^i \partial_\nu \phi^j \right], \quad (8)$$

where

- the fields $\phi^i$ are considered as coordinates on the manifold (the target space),
- $g_{ij}(\phi)$ is a metric on the manifold,
- $\eta^{\mu\nu}$ is the space-time (inverse)metric.

When isometries are present - for instance when the manifold is a homogeneous one (coset space $G/H$) - they may be realised in various ways:

$$\left[ W_{i}^{G/H}, \phi^j \right] = -i f_{ij}(\phi); \quad (9)$$

$f_{ij}(\phi)$ is constrained by the Lie algebra structure and, thanks to the invariance of the distance, related to the target space metric. For example, in the $S^N$ case, the standard parametrisation gives:

$$f_{ij}(\vec{\pi}) = \delta_{ij} \frac{f_{\pi}^2 - \vec{\pi}^2}{f_{\pi}^2 - \vec{\pi}^2}, \quad g_{ij}(\vec{\pi}) = \delta_{ij} + \frac{\vec{\pi}_i \vec{\pi}_j}{f_{\pi}^2 - \vec{\pi}^2};$$

the stereographic parametrisation of Schwinger \[12\] gives:

$$f_{ij}(\vec{\phi}) = \frac{1}{2} \delta_{ij} (f_{\pi}^2 - \vec{\phi}^2) + \phi^i \phi^j, \quad g_{ij}(\vec{\phi}) = \frac{\delta_{ij}}{(f_{\pi}^2 + \vec{\phi}^2)^2};$$

note that the change of coordinates between these two parametrisations is non-linear:

$$\vec{\phi} = \frac{f_{\pi} \vec{\pi}}{f_{\pi}^2 + \sqrt{f_{\pi}^2 - \vec{\pi}^2}} \iff \vec{\pi} = \frac{2 f_{\pi}^2 \vec{\phi}}{(f_{\pi}^2 + \vec{\phi}^2)}.\quad (10)$$

Other choices have been considered, for example one where $\det |g_{ij}| = 1$ (Charap \[13\]), with the hope that they will lead, to all-order of perturbation theory, to finite S-matrix elements in 4 dimensions.

Let us end that Section by some comments.

2.4 Remarks and comments.

- As physics should be independent of the parametrisation, in those days some people consider the choice of parametrisation as a kind of gauge choice \[13\] : this idea was later on taken up in the framework of the use of B.R.S. symmetry in non-linear $\sigma$ models and non-linear field redefinitions \[14\].

Notice that in a recent work, Blasi et al. \[15\] prove the equivalence theorem - independence of S matrix elements under reparametrisations of the fields - by that very idea : the new terms in the Lagrangian density, corresponding to a change of gauge, are B.R.S. variation of something, and then unphysical ones.

1 Here we do not consider a Wess-Zumino term in the Lagrangian (in two space-time dimensions, anti-symmetric tensors $\epsilon^{\mu\nu}$ and $b_{ij}$ would respectively replace $\eta^{\mu\nu}$ and $g_{ij}$). Note that Carlo Becchi has also been involved in that situation (in a work with Olivier Piguet \[4\]).
A perturbative approach requires the choice of a special point “S” on the manifold $\mathcal{M}$ (e.g. the south pole for stereographic parametrisation). However, if the manifold is an homogeneous one, all points are equivalent. On the other hand, in the case of generalised non-linear $\sigma$ models defined on an arbitrary target space, this independence results from the existence at the quantum level of some compatibility conditions ensuring the existence of a globally defined metric on $\mathcal{M}$, as pointed out by Friedan [11] and thoroughly discussed by Carlo Becchi at the 1987 Ringsberg meeting [16].

Given some point “S” on $\mathcal{M}$, it would be simpler to have a covariant perturbative expansion : as first shown by Meetz, normal coordinates give such a geometric formulation [17]. At the one-loop order, the divergences are proportional to the Ricci tensor of the target space metric :

$$\Delta L \sim \delta^4(0) R_{\mu\nu} \partial_{\mu}\phi^i \partial_{\nu}\phi^j .$$

For any symmetric space the Ricci tensor is proportional to the metric tensor, then the corresponding models are one-loop renormalisable. Other situations, as well as an all-order analysis, will be discussed in the next Section.

I would like to emphasize that, in the sixties, physicists tried to take the P.C.A.C. - Adler condition (5) as the very definition of a right theory. They considered the Lagrangians (8) as effective lagrangians and agreed on the importance of the algebra of isometries - the Ward identities. For example, many efforts [18] were done to enforce P.C.A.C. in higher orders of perturbation theory (in 4 space-time dimensions).

It seems to me that after 1972 or so, there appeared some confusion : I feel that the successes of dimensional regularisation for gauge theories - with the kind of “magic” that accompanies its use - and the discovery of the “standard model” with its simple Lagrangian, somehow obscure the comprehension ; so, in the rest of this talk, I shall comment on the definition and renormalisation of non-linear $\sigma$ models, this time in two space-time dimensions where they are power counting renormalisable, the canonical dimension of the fields being zero.

3 From Ward identities to B.R.S. symmetry.

In the presence of non-linear transformations of the fields, the standard trick is to add to the effective action sources for these variations, as well as for their successive iterations. In doing so, one may need an infinite series of sources. This makes the quantum analysis a priori a tremendous task ; however, in some cases it may be carried out :

- when there exists a special parametrisation of the coset space such that the series of transformations closes in a finite number of steps. This is the case for $S^N$ in the standard parametrisation, as well as for the Grassmannians [19] ;

- for Kähler coset spaces, there is a $U(1)$ charge, which helps to control the infinite tower of transformations. In particular, our group in Paris was able to exhibit a parametrisation where the isometries are holomorphic :

$$\delta \phi^i = \epsilon^i + F_{jk}^i \epsilon^j \phi^k + G_{jk}^i \epsilon^j \phi^k , \quad \delta \phi^\bar{i} = \epsilon^\bar{i} + F_{\bar{j}\bar{k}}^\bar{i} \epsilon^{\bar{j}} \phi^{\bar{k}} + G_{\bar{j}\bar{k}}^\bar{i} \epsilon^{\bar{j}} \phi^{\bar{k}} ,$$

and so to give a complete all order analysis of the non-linear $\sigma$ models on homogeneous Kähler spaces [20].
But, if one recalls that the algebra involves commutators rather than products of transformations and that for coset spaces the commutator of two infinitesimal transformations gives another infinitesimal group transformation, it is tempting to promote the parameters of the transformations to anti-commuting ones (Fadeev-Popov constants). This idea was proposed by Alberto Blasi and Renzo Collina for the $S^N$ model [21], and we used it in our Genoa-Paris collaboration to obtain a complete algebraic proof of the renormalisability of any homogeneous non-linear $\sigma$ model [3]. Let us remark that:

- we replace Ward Identities by **B.R.S. invariance and Slavnov identities**. No parametrisation being now specified, it is not surprising that the field may be non-linearly renormalised 2. The B.R.S. invariant effective action writes:

$$\Gamma^{(0)} = S\Delta^{(-1)} + A^{(0)},$$

where $S$ is the Slavnov operator, $A^{(0)}$ the invariant action and $\Delta^{(-1)}$ an arbitrary local functional in the fields and their derivatives, constrained by power counting arguments and of Fadeev-Popov charge -1;

- this **first extension of the notion of renormalisability** (as an infinite number of renormalisation constants appear) was first exhibited by Piguet and Sibold in supersymmetric models [22] and widely discussed at the 1987 workshop in Ringsberg [14].

During this workshop, Breitenlohner and Maison exposed their efforts to quantise N=2 super Yang-Mills theory [23]: on the one hand, they came up against the difficulty of an infinite series of sources and realized that this B.R.S. idea will reveal itself very fruitful. On the other hand, they also came up against the difficulty of open algebras that occur in supersymmetric theories (the algebra only **closes on-shell** and, moreover, in their situation, *modulo* a gauge transformation). This problem of on-shell closed algebras was tackled by Kallosh [24], solved through the formalism of Balatin and Vilkovisky [25] and exemplified by Piguet and Sibold in their analysis of Wess-Zumino model without auxiliary fields [26].

This offers a **new success of B.R.S. formalism** as one may define a modified Slavnov operator and a modified effective action that take care of that non-closure of the algebra. The challenge of Breitenlohner and Maison was finally answered by Maggiore [27]; moreover, this method was also used in the study of $N = 2$ and $N = 4$ supersymmetric non-linear $\sigma$ models [28] and recently applied by Blasi and Maggiore to broken algebras [29].

4 On some apparent non-renormalisabilities.

In that Section, I want to illustrate on concrete examples, two problems that result from too naive a use of dimensional regularisation.

4.1 Non-linear renormalisation of the fields.

Take the $O(N)$ invariant Lagrangian:

$$\mathcal{L}_c = \frac{1}{2g^2} \left[ \frac{(\partial_\mu \vec{\varphi})^2}{(1 + \vec{\varphi}^2)^2} - m^2 \frac{\vec{\varphi}^2}{(1 + \vec{\varphi}^2)} \right].$$

\(^2\) Contrarily to the homogeneous Kähler case where in our parametrisation, using Ward identities, we prove that the field is not renormalised ($Z_\varphi = 1$)
Dimensional regularisation gives as 1-loop effective lagrangian:

\[ \mathcal{L}_c + \Delta \mathcal{L}_{\text{min.}} = \left[ 1 + \frac{\hbar g^2}{\pi \epsilon} \right] \mathcal{L} + \frac{(N-2)\hbar g^2}{4g^2} \frac{(\partial_\mu \phi)^2}{(1 + \phi^2)} - 2 \frac{(\bar{\phi} \partial_\mu \phi)^2}{(1 + \phi^2)^2} . \] (12)

Using standard arguments, one would conclude that for \( N \neq 2 \), the model is non-renormalisable. This would be rather unpleasant as the Lagrangian (11) is the ordinary \( S^N \) one in the stereographic parametrisation (subsect. 2.3). As shown in [30], this is a special case of on-shell renormalisability, as the S-matrix elements can be renormalised through a coupling and mass renormalisation:

\[ Z_{g^2} = 1 - \frac{(N - 1)\hbar g^2}{\pi \epsilon} , \quad Z_{m^2} = 1 - \frac{(N - 2)\hbar g^2}{2\pi \epsilon} . \]

Indeed, result (12) may be understood as a non-linear renormalisation of the fields according to:

\[ \bar{\phi}_0 = \phi \left[ 1 - \frac{(N - 2)\hbar g^2}{4\pi \epsilon} (1 + \phi^2) \right] . \] (13)

4.2 Unsufficient definition of the model.

Consider another \( O(N) \) invariant Lagrangian:

\[ \mathcal{L}_c = \frac{1}{2g^2} \frac{(\partial_\mu \phi)^2}{(1 - \phi^2)} - m^2 \phi^2 . \] (14)

A calculation with dimensional regularisation give as 1-loop effective lagrangian:

\[ \mathcal{L}_c + \Delta \mathcal{L}_{\text{min.}} = \frac{1}{2g^2} \frac{(\partial_\mu \phi)^2}{(1 - \phi^2)} - m^2 \phi^2 - \frac{1}{2\pi \epsilon} \frac{\hbar g^2}{(1 - \phi^2)} - \frac{m^2 \phi^2}{2g^2} - 2 \frac{\hbar g^2}{2\pi \epsilon} - \frac{(N - 2)\hbar g^2}{4g^2} \frac{(\partial_\mu \phi)^2}{(1 - \phi^2)} + m^2 \phi^2 + \frac{(\bar{\phi} \partial_\mu \phi)^2}{(1 - \phi^2)^2} . \] (15)

- For \( N = 2 \), the model appears to be 1-loop renormalisable with:

\[ Z_{\phi} = 1 - \frac{\hbar g^2}{2\pi \epsilon} , \quad Z_{m^2} = 1 + \frac{\hbar g^2}{\pi \epsilon} \quad \text{and} \quad Z_{g^2} = 1 , \]

which means a vanishing \( \beta \) function at that order [32]. This is fine, but, at 2-loop order, still in the minimal dimensional scheme, the calculation gives non-renormalisability. It is then tempting to try an \( \mathcal{O}(\hbar) \) additive modification of the Lagrangian : as shown in [33], renormalisability is recovered, but the candidate 1-loop Lagrangian is not completely determined : this signals an insufficient definition of the model !

- The solution: for \( N = 2 \), the classical action satisfies the factorisation properties of an integrable model (Complex Sine-Gordon [34]). The model should be defined by the non-production physical property, which means an infinite number of identities. With François Delduc, we prove that, enforcing one-loop factorisability, the \( \mathcal{O}(\hbar) \) finite counterterms are completely fixed and as an extra bonus we get the vanishing of the \( \beta \) function at 2-loops order [33].
– Comments:

* As the factorisation property is specific of 2-dimensional space-time, it is not surprising that dimensional regularisation leads to quantum corrections, often called “spurious anomalies”.

* In the eighties, many uncorrect claims were published - for example on a possible violation of the Adler-Bardeen theorem in Super-Yang-Mills. As shown in [35], most of them were due to the identification of (minimal) regularisation with a renormalisation scheme. The “magic” of minimal dimensional regularisation leads to a blind faith understanding of the Lagrangian as the theory, as opposed to people involved in soft pions and chirality business in the sixties.

This is the right moment to recall a nice expression of Carlo Becchi:

“The Lagrangian is an opinion”...

• For $N > 2$, the model is 1-loop non-renormalisable, even on-shell, as S-matrix elements cannot be made finite through some renormalisation of the parameters of the classical Lagrangian [14]. This should be related to the absence, in ordinary perturbation theory, of O(N) symmetric integrable models.

5 Concluding remarks: generalised renormalisability?

In the previous subsections, we discussed how the study of non-linear $\sigma$ models, taken as “toy” models, leads to “trivial” extensions of the notion of renormalisability: possible non-linear renormalisations of the fields and on-shell renormalisability. In these concluding Section, I comment on two other directions:

• Friedan’s approach [11]:
Friedan discusses renormalisation “in the space of metrics”, which, in the absence of isometries, a priori leads to some unpredictiveness as a theory with an arbitrary metric $g_{ij}[\phi]$ on the manifold $\mathcal{M}$ would involve an infinite number of parameters. If only a finite number of them are “physical” ones, the theory is acceptable and may be called a renormalisable theory. Of course, the important part lies in the definition of what are “physical” parameters. For instance, does a geometrical constraint (Ricci flatness, hyperkählerness,...) sufficiently reduce the space of parameters for a given manifold? A complete answer to that question is not known yet.

• The effective action point of view [36]:
The B.R.S. methods have been used in a series of papers [37] where it is claimed that the goal of the physicists of the sixties [18] - that is to say the validity, to all orders of perturbation theory, of the low energy theorems for non-linear $\sigma$ models, in $D = 4$ dimensions - has been reached. In another spirit, Gomis and Weinberg [36] note that, given a standard renormalisable theory, the integration over some massive fields leads to an Action which is power counting non-renormalisable. However, they argue that such a theory is physically predictive and then should be considered as “renormalisable” in an extended sense. Moreover they discuss the a priori constraints that should be imposed on the form of the bare action to make possible the absorption of any infinity coming from loop graphs by an allowed counterterm. Here also, more work is to be done.
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