Assessing Pandemic Uncertainty on Conditions of Vaccination and Self-isolation

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Abstract—A share of predictable information about the forthcoming state of a three-sided coin (susceptible—infected—immune) with the regular and random transition times between states is used for assessing the degree of pandemic uncertainty in our model. Unreliable and unsafe vaccines (that do not guarantee absolute immunity) as well as long self-isolation making transition times random increase the degree of pandemic uncertainty, worsening the damaging impact for both society and the economy.

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\[\text{In memory of Professor Gennady Leonov (1947–2018), a gentle man.}\]

1. INTRODUCTION

The coronavirus leaking from a lab caught the world by surprise [1–4]. Uncertainty about:

- the continuing spread of the increasing number of different strains of the SARS-CoV-2 virus, as the pandemic progresses [5, 6],
- a wide range of COVID-19 symptoms reported by patients—from mild symptoms to severe illness—that may, or may not appear 2–14 days after exposure to the virus [7],
- the COVID-19 tests results that may be falsely positive or negative, or may show an abnormality that doesn’t matter [8],
- the treatments for COVID-19 that currently have limited evidence of efficacy [9],
- the efficiency and possible side effects [10, 11] of the different brands of vaccines [12], as the vaccine war unfolds [13, 14],
- the effectiveness of self-quarantine [15–17], stay-at-home lockdowns [18], with physical distancing in public spaces, curfews, self-isolation of elderly people [19], and other country-based pandemic mitigation measures [16, 19, 20],

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make people fear for their lives and cautious about their spending. By the end of April, 2020 more than 20.5 mln jobs have been lost in the U.S. since the start of the pandemic [21].

Sweeping self-quarantine was imposed as a measure for slowing down the spread of COVID-19 and preventing the public health infrastructure from collapsing as hospitals overflow. A major obstacle to compliance for household quarantine determining the public compliance rates was individual concern about loss of income [22]. When lost wages compensation was assumed, the reported compliance rate jumped to 94%, but the compliance rate dropped to less than 57% when compensation was removed [22]. Therefore, self-quarantine did not eliminate the pandemic waves [23, 24] but rather stretched them over indefinite time (i.e., until the coronavirus-related restrictions are dropped). Moreover, many self-quarantined individuals practiced self-medication and self-reliance in preventing or relieving minor symptoms or conditions [25] and did not report any suspicious physical symptoms during self-isolation. Many patients were not admitted to hospitals keeping their normal activities at home. Further consequences of lockdown included people dying from other diseases because they were unable to access urgent care, individuals with mental-health issues [19], victims of domestic violence, and people suffering from intensifying poverty [21]. The pandemics led to a dramatic loss of human life worldwide, devastated the national systems of public health, ruined food systems and the world of work [26].

In our work, we gauge the pandemic uncertainty by using a biased “three-sided coin” model (see Fig. 1a and 1b), a discrete-time Markov chain with three states: “susceptible”, “infected”, and “immune” (see Sec. 2 for the details). The probability of transition to the state of being immune describes the overall effectiveness of COVID-19 vaccines in real-world conditions (of multiple virus traits and possible side effects). The amount of information revealed by the Markov chain at each step is the sum of predictable and unpredictable information components [27–30] (see Sec. 3 for the details). While the predictable information component quantifies the apparent uncertainty, that can be resolved from the previous history and the present state of the system, the unpredictable component represents the true uncertainty, i.e., the amount of information about the forthcoming state of a patient (i.e., diagnosis) that cannot be predicted anyway (see Sec. 3.3.1). We further assess the pandemic uncertainty on condition of prolonged self-isolation by assuming the random transition times between states of the three-sided coin (see Sec. 3.3.2). The results of our study on assessing pandemic uncertainty are summarized in Sec. 4. The major conclusions of our work are given in the last section (Sec. 5): it seems that unreliable and unsafe vaccines (that do not guarantee absolute immunity) and indefinitely long self-isolation increase the degree of pandemic uncertainty, worsening the damaging impact for both society and the economy.

The proposed methodology can be used for assessing uncertainty (at least qualitatively) in complex real-world multi-state systems.

2. MODEL: A BIASED THREE-SIDED COIN

For quantifying uncertainty related to the problem, we use a biased three-state coin model shown in Fig. 1 (a and b). A personal health record, \( \{X_t : t \in \mathbb{Z}_0 \} \); \( \Pr (X_{t+1}|X_t) \geq 0 \), in our model is generated.
by a discrete-time Markov chain describing transitions between three following states: ‘susceptible’ (S), ‘infected’ (I), and ‘immune’ (J). Transitions between these states might occur at discrete (regular) units of time.

In the symmetric case shown in Fig. 1, a, each of three possible state is repeated with the same probability 0 ≤ p ≤ 1, and every transition between the states occurs with the equal probability \( (1 - p)/2 \). In the uneven case (see Fig. 1b), the state of being immune (presumably after vaccination), \( J \), recurs with another probability, 0 ≤ q ≤ 1, quantifying the degree of safety and efficiency of vaccination (i.e., a perfect vaccine corresponds to \( q \to 1 \), and \( q \to 0 \) in the opposite case). When \( p \to 1 \), a patient is assumed to be dead if her current state \( X_t = J \), but safe and healthy provided \( X_t = S \).

The stochastic symmetric transition matrix defining the Markov chain shown on the diagram Fig. 1, a is given by

\[
T(p) = \begin{pmatrix}
p & (1 - p)/2 & (1 - p)/2 \\
(1 - p)/2 & p & (1 - p)/2 \\
(1 - p)/2 & (1 - p)/2 & p
\end{pmatrix},
\]

and the biased stochastic transition matrix defining the Markov chain on condition of vaccination (shown in Fig. 1b) is the following:

\[
T(p, q) = \begin{pmatrix}
p & (1 - p)/2 & (1 - p)/2 \\
(1 - p)/2 & p & (1 - p)/2 \\
(1 - q)/2 & (1 - q)/2 & q
\end{pmatrix}.
\]

To assess pandemic uncertainty on condition of self-isolation, we use the same models as shown in Fig. 1a and 1b (1), (2) with random transition times between states (see Sec. 3.3.2 for the details).

3. METHODS: ENTROPY DECOMPOSITION INTO INFORMATION COMPONENTS UNDER REGULAR AND RANDOM TRANSITION TIMES

The amount of information released at every flipping of a three-sided coin (see Fig. 1, a and b) is given by the following entropy function

\[
H(x) = - \sum_{k=1}^{3} \pi_k(x) \log_3 \pi_k(x), \quad x \equiv \{p, q\}, \quad 0 \cdot \log_3 0 \equiv 0,
\]

in which \( \pi_k \geq 0 \) is the density of the \( k \)-state in the Markov chains (1), (2).

For the symmetric Markov chain shown in Fig. 1, a, the density of states is uniform, i.e., \( \pi_S = \pi_I = \pi_J = 1/3 \), so that for any value of the state repetition probability 0 ≤ p ≤ 1 the amount of entropy (3) in the three-sided coin (1) is the same: \( H(p) = 1 \) trit that is equal to \( \log_3 3 (\approx 1.585) \) bits [31]. As the number of states in the chain naturally determines the base of logarithm in the entropy definition, trit is the most convenient unit to measure information in the problem in question. In the uneven model shown in Fig. 1b, the densities of states are unequal,

\[
\pi_S = \pi_I = \frac{q - 1}{p + 2q - 3}, \quad \pi_J = \frac{p - 1}{p + 2q - 3},
\]

and therefore the total average amount of information released at every transition is a function of both repetition probabilities, \( p \) and \( q \):

\[
H(p, q) = \frac{-2q + 2}{(p - 3 + 2q)} \log_3 \left( \frac{q - 1}{p - 3 + 2q} \right) + \frac{-p + 1}{(p - 3 + 2q)} \log_3 \left( \frac{p - 1}{p - 3 + 2q} \right).
\]

The profile of entropy function (4) is shown in Fig. 2: entropy attains the maximum value \( H(p, p) = 1 \) trit for the symmetric case, \( p = q \), and the minimum value \( H(p, q) = 0 \) for an absolutely immune patient \( q = 1 \), i.e., when there is no uncertainty about her state. If \( q < 1 \) but \( p = 1 \), there is however some uncertainty about whether the patient is dead \( (X_{t>0} = I \) indefinitely) or healthy \( (X_{t>0} = S \) indefinitely). In the latter case, \( H(1, q) = \log_3 2 \approx 0.6309 \) trit.
3.1. Entropy Decomposition Into Conditional Entropies

The entropy function (3) allows for the following decomposition involving the conditional entropies [29, 30]:

\[
H(X_t) = H(X_t) + H(X_{t+1}|X_t) + H(X_t|X_{t-1}) + H(X_{t+1}|X_{t-1})
\]

\[
= \frac{E(X_t)}{H(X_t|X_{t+1};X_{t-1})} + \frac{I(X_t;X_{t+1}|X_{t-1})}{H(X_t|X_{t+1};X_{t-1})} + (H(X_{t+1}|X_t) + H(X_t|X_{t-1}) - H(X_{t+1}|X_{t-1}))
\]

\[
H(X_t|X_{t+1};X_{t-1})
\]

where the excess entropy \(E(X_t)\) [29, 30, 32–34] quantifies the apparent uncertainty of the diagnosis (i.e., the forthcoming state of the Markov chain shown in Fig. 1) that can be resolved by studying the entire history of states observed in the past; the mutual information [29, 30, 32, 35] between the present and future states of the chain conditioned on its past state, \(I(X_t;X_{t+1}|X_{t-1})\), measures the efficacy of forecasting the future state of the Markov chain from the present state alone; finally, the latter term known as ephemeral entropy of the present state \(X_t\) conditional on the future and past states of the chain [29, 30, 33], \(H(X_t|X_{t+1};X_{t-1})\), assessing the amount of true uncertainty about the forthcoming state of the chain \(\{X_t\}\) that can neither be inferred from the past history, nor have any repercussion for the future states of patient.

The first two quantities together, \(E(X_t)\) and \(I(X_t;X_{t+1}|X_{t-1})\), taken together amount to the predictable information component \(P(X_t)\) of the entropy (3) that might be resolved from the sequence of states already known (i.e., the present and past states of the Markov chain). The latter quantity, \(U(X_t)\) \(= H(X_t|X_{t+1};X_{t-1})\), assesses the amount of unpredictable information related to the Markov chain, and, accordingly (5)

\[
H(X_t) = P(X_t) + U(X_t).
\]

Given a finite state Markov chain transition matrix,

\[
T_{ij} = \Pr(X_{t+1} = j|X_t = i) \geq 0, \quad i, j = \{1, \ldots, N\}, \quad t \geq 0,
\]

describing the transition probabilities between \(N\) states of the chain, the unpredictable information component in (6) is then given by [29, 30]

\[
U(X_t) = -\sum_{i,j=1}^{3} \pi_i T_{ij}^2 \log_3 T_{ij}^2,
\]
and the predictable information component,

\[ P(X_t) = H(X_t) - U(X_t) = -\sum_{i=1}^{3} \pi_i \left( \log_3 \pi_i - \sum_{j=1}^{3} T_{ij}^2 \log_3 T_{ij}^2 \right). \]  

(8)

The latter information component comprises of the excess entropy [29, 30], viz.,

\[ E(X_t) = -\sum_{i=1}^{3} \pi_i \left( \log_3 \pi_i - \sum_{j=1}^{2} T_{ij} \log_3 T_{ij} \right), \]  

(9)

and the mutual information between the present state and the future state conditioned on the past state [29, 30], i.e.,

\[ I(X_t; X_{t+1} | X_t) = \sum_{i,j=1}^{3} \pi_i \left( T_{ij} \log_3 T_{ij} - T_{ij}^3 \log_3 T_{ij}^3 \right). \]  

(10)

### 3.2. Random Transition Times Under Self-Quarantine

We model the pandemic uncertainty on condition of self-isolation by assuming that transition times in the Markov chain models (shown in Fig. 1) may be random. The motivation behind this choice is that self-quarantine did not eliminate the pandemic waves [23, 24] but rather extended them indefinitely (at least, until the moment when lockdown restrictions were eliminated).

While in quarantine, some patients were strictly complying with the self-isolation rules and, perhaps, made virtually no transitions (staying presumably healthy), others failed to do so [22]. Furthermore, they often practiced self-medication and self-reliance in preventing or relieving minor symptoms or virus-related conditions\(^1\), and often did not report any physical symptoms experienced during the quarantine period. We assume that while under self-isolation individuals might experience random number of transitions between the states in the models shown in Fig. 1, a and b, and defined by (1), (2).

To take into account the degree of randomness in transition times between states on condition of self-isolation, we use the fractional Markov chain \( T_\varepsilon \), with the fractionality parameter \( 0 \leq \varepsilon \leq 1 \), a convergent infinite power series of binomial type of a Markov chain transition matrix \( T \), as defined by us in [29], viz.,

\[ T_\varepsilon \equiv -\sum_{k=1}^{\infty} \frac{\Gamma(k-1+\varepsilon)}{\Gamma(k+1)\Gamma(-1+\varepsilon)} T^k, \]  

(11)

as following from the fractional differencing of non-integer order [36, 37] introduced in relation to discrete-time stochastic processes in [36–44]. The coefficients in the binomial type series (11) sum to one, and their magnitude decays rapidly with \( k \), for any value of the fractionality parameter \( \varepsilon \). The transition matrix of the fractional Markov chain defined by (11) coincides with the initial transition matrix \( T \) as \( \varepsilon \to 0 \), but includes substantial contributions from all powers of the transition matrix provided \( \varepsilon \to 1 \), as the variance of the distribution of coefficients in the series (11) diverges as \( \varepsilon \to 1 \). Therefore, the fractional order parameter \( \varepsilon > 0 \) can be considered as a degree of transition uncertainty on condition of self-isolation. The transition matrix for the fractional chain corresponding to the symmetric transition matrix (1) for \( 0 \leq \varepsilon \leq 1 \) is given by

\[ T_\varepsilon(p) = \frac{1}{\gamma^\varepsilon} \begin{pmatrix} \gamma^\varepsilon - 1 & 1 - p/2 & 1 - p/2 \\ 1 - p/2 & \gamma^\varepsilon - 1 & 1 - p/2 \\ 1 - p/2 & 1 - p/2 & \gamma^\varepsilon - 1 \end{pmatrix}, \quad \gamma \equiv \frac{3(1-p)}{2}. \]  

(12)

\(^1\)It was observed that all the surveyed drugs (acetaminophen, ibuprofen, azithromycin, penicillin, antiretrovirals and hydroxychloroquine) were consumed for various symptoms including: fever, fatigue, cough, sneezing, muscle pain, nasal congestion, sore throat, headache and breathing difficulty [25].
The latter fractional transition matrix equals the initial transition matrix defined by (1) for $\varepsilon = 0$, viz., $T_{\varepsilon=0}(p) = T(p)$ but all transition and state probabilities in (12) uniformly equal to $1/3$ (i.e., the density of states in the symmetric model Fig. 1, a) when $\varepsilon = 1$, viz.,

$$T_{\varepsilon=1} = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}. \quad (13)$$

The fractional transition matrix corresponding to the biased Markov chain (2) describing the diagnostic model on condition of vaccination (see Fig. 1b) is given by

$$T_{\varepsilon}(p, q) = \begin{pmatrix} 1 - \frac{1}{4}p\delta_\varepsilon & \frac{1}{4}p\sigma_\varepsilon & \frac{1}{2}p\chi^{-\varepsilon} \\ \frac{1}{4}q\sigma_\varepsilon & 1 - \frac{1}{4}p\delta_\varepsilon & \frac{1}{2}q\chi^{-\varepsilon} \\ \frac{1}{2}\chi^{-\varepsilon} & \frac{1}{2}\chi^{-\varepsilon} & 1 - (1 - q)\chi^{-\varepsilon} \end{pmatrix}, \quad (14)$$

where we have introduced the following auxiliary variables:

$$\chi \equiv (3 - p - 2q)/2, \quad \delta_\varepsilon \equiv \gamma^{-\varepsilon} + \chi^{-\varepsilon}, \quad \sigma_\varepsilon \equiv 3\gamma^{-\varepsilon} - \chi^{-\varepsilon}, \quad (15)$$

and $\gamma$ is defined in (12).

Similarly, the fractional transition matrix (14) coincides with the initial matrix $T(p, q)$ defined by (2) for $\varepsilon = 0$. When $\varepsilon = 1$, the rows of fractional transition matrix (14) are the same as the density of states in the biased model with vaccination (2), viz.,

$$T_{\varepsilon=1}(p, q) = \frac{1}{p - 3 + 2q} \begin{pmatrix} q - 1 & q - 1 & p - 1 \\ q - 1 & q - 1 & p - 1 \\ q - 1 & q - 1 & p - 1 \end{pmatrix}. \quad (16)$$

Thus, the minimal value of the fractional order parameter ($\varepsilon = 0$) in the model (11) may be attributed to a synchronous case of regular pace of time in the diagnostic models (Sec. 2)—with no self-isolation. The maximum value $\varepsilon = 1$ corresponds to the opposite situation of indefinitely long self-isolation when individuals may experience arbitrary number of transitions, and their actual state is foretasted by averaging over all possible scenarios, as recovering the density of states in the models presented in Fig. 1.

The predictable and unpredictable information components defined by (7)–(10) can be calculated for the fractional transition matrix (11), for any value of the fractional order parameter $0 \leq \varepsilon \leq 1$, in the same way as it may be done for the usual transition matrices.

### 4. Results: Exacerbating Uncertainty with Vaccination and Self-Quarantine Restrictions

We discuss the symmetric model shown in Fig. 1, a first. All information quantities in this model are the functions of a single probability $p$ quantifying the chance a state is repeated. For the symmetric Markov chain, the densities of all states are equal ($\pi = 1/3$), so that $H(p) = -\log_3 1/3 = 1$ trit of information released at each step of the chain, uniformly, for all $0 \leq p \leq 1$ (see Fig. 1a). Accordingly (5), the entropy function (7)–(10) comprises of three conditional information components ($E(X_t)$, $U(X_t)$, and $I(X_t; X_{t+1}\mid X_{t-1})$) that depend upon the repetition probability $p$, indeed (see Fig. 3a). These quantities are calculated as the functions of $p$, using (9) and (10), respectively.

The component responsible for unpredictable information ($U(p)$) attains maximum when the three-sided coin is fair, i.e., $p = 1/3$ (Fig. 3a)—no prediction about the forthcoming state of individual (diagnosis) is possible in such a case. The amount of predictable information (measured in trits) ($E(p) + I(p)$) in the symmetric model (1) as a function of $p$ is shown in Fig. 3b by the bold curve. For comparison, we have also shown the amount of predictable information as a function of the state.
repetition probability $p$ for a two-sided coin [29] (measured in bits) by the dotted curve in Fig. 3b.

The two-sided coin is defined by the following stochastic transition matrix $\begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix}$, where the states, ‘heads’ (“0”) or ‘tails’ (“1”), repeat themselves with the probability $0 \leq p \leq 1$. While a two-sided coin is fair at $p = 0$ (i.e., its forthcoming state is unpredictable when $p = 0$), a three-sided coin paradigmatic for our model is fair at $p = 1/3$ when the amount of predictable information about the forthcoming state of the chain turns to zero. The amount of predictable information attains maximum $P(p) = 1$ trit, as $p \to 1$. In contrast to the two-sided coin (where the simple alternation of states at $p = 0$ allows to predict the future state of the coin reliably), the amount of predictable information in the three-sided coin is inferior to one trit (since the uncertainty between two symmetric states—susceptible and infected—remains unresolved). The bold curve representing the amount of predictable information in the three-sided coin looks shifted in comparison to the symmetric profile of the curve for a two-sided coin.

We keep on comparing the three-sided and two-sided coin models also for the uneven case (2) shown in Fig. 1b. In Fig. 3, a, we presented the amount of predictable information in this model on condition of vaccination (with the probability of staying immune $0 \leq q \leq 1$). The amount of predictable information about the forthcoming state attains minimum for a symmetric fair coin, $p = q = 1/3$, and grows as $p \to 1$. However, the maximum of predictable information (of 1 trit) can be attained as $p \to 1$ only when $q \to 1$, i.e., in the case when vaccination guarantees immunity.

In Fig. 3b, we have shown the amount of predictable information in the model of the biased two-sided coin where ‘heads’ and ‘tails’ repeat themselves with the probabilities $0 \leq p \leq 1$ and $0 \leq q \leq 1$, respectively, $\begin{pmatrix} p & 1-p \\ 1-p & q \end{pmatrix}$, as a counter-part of the uneven three-sided coin (2). The two-sided coin is fair (i.e., its forthcoming state is unpredictable) whenever $p = q = 1/2$. The amount of predictable information shown in Fig. 3b is symmetric with respect to the axis $p = q$ and attains maximum as $p, q \to 0$, or $p, q \to 1$, simultaneously (Fig. 3b).

To elucidate the role of $0 \leq q \leq 1$ (the probability of staying immune) for pandemic uncertainty, in Fig. 5 we juxtapose a contour plot representing the amount of predictable information $P(p, q)$ in the model (2) and a plot of the two-dimensional vector field $(P(p, q), U(p, q))$, in which the components are the amount of predictable information (in horizontal direction) and unpredictable information $U(p, q)$ (in vertical direction) evaluated at the point $(p, q)$.
Fig. 4. (a) The amount of predictable information \( P(p,q) \) for a three-sided coin on condition of vaccination, with the probability of staying immune \( q \) and the probability of staying susceptible/infected—\( p \). (b) The amount of predictable information \( P(p,q) \) in the biased two-sided coin where 'heads' and 'tails' repeat themselves with the probabilities \( 0 \leq p \leq 1 \) and \( 0 \leq q \leq 1 \), respectively.

Fig. 5. The contour plot representing the amount of predictable information \( P(p,q) \) in the model (2) along with the two-dimensional vector field plot \( \langle P(p,q), U(p,q) \rangle \) indicating the role of immunity probability \( q \) for pandemic uncertainty in the model with vaccination.

Fig. 6. The amount of predictable information \( P(p) \) for the different values of the parameter \( 0 \leq \varepsilon < 1 \) in the symmetric three-sided coin model with random transition times between states (12).

It is worth mentioning that as the probability of staying immune grows \( q > 0 \), but until vaccination does not guarantee absolute immunity \( (q < 1) \), the vertical component (i.e., unpredictable information) drives the vector field \( \langle P(p,q), U(p,q) \rangle \) in the vertical direction, as increasing the degree of uncertainty of the forthcoming state (although the situation improves when \( p, q \rightarrow 1 \) simultaneously).
We now discuss the model with random transition times between states (11) that we use to model the effect of self-isolation on the degree of pandemic uncertainty. We consider the symmetric model (12) first. In Fig. 6, we have shown the amount of predictable information $P(p)$ for the different values of the fractional order parameter $0 \leq \varepsilon < 1$ in the three-sided coin model with random transition times between states (12).

For $\varepsilon = 0$, the model corresponds to the case of regular transition times, and $P(p)$ is the same as shown in Fig. 3, b by the solid curve. As $\varepsilon \to 1$, the amount of predictable information tends to zero for a wide range of values of the state repetition probability $p$. Literally speaking, the fractional three-sided coin is fair for any value of $p$, and therefore diagnostic of patient’s state on condition of indefinitely long self-isolation is as reliable as fortune telling by coin flipping.

Finally, in Fig. 6, we have shown the amount of predictable information $P(p, q)$ in the biased model with vaccination (14) for the consequent values of the fractional order parameter $0 \leq \varepsilon < 1$. Similarly to the symmetric model (12), the amount of $P(p, q)$ fades out to zero, as $\varepsilon \to 1$. Again, a reliable diagnostic of patient’s state is improbable on condition of indefinitely-long self-isolation.

5. CONCLUSION

Uncertainty is the major damaging factor of the continuing coronavirus pandemics. While risk is measurable and can be managed, uncertainty deals with outcomes we cannot predict or never saw coming [45] that makes it impossible to weigh costs and benefits of an economic shutdown and other possible responses to the pandemic.

In our work, we have proposed and studied two simple three-sided coin tossing models with regular and random transition times between three states, susceptible - infected - immune, to evaluate the degree of pandemic uncertainty on conditions of vaccination and self-isolation. The major conclusions of our model are the following:

1. If the transition rates between the states of the model uniformly equal $1/3$, a specific diagnostic, (i.e., prediction of patient’s actual state) is impossible, as the three-sided coins become fair.

2. If vaccines do not guarantee absolute virus immunity, their overall effect may worsen pandemic uncertainty.
3. Long self-isolation dramatically reduces the degree of predictability of patient’s actual state as well.

Vaccination with unreliable vaccines and indefinitely long self-isolation, increasing the degree of pandemic uncertainty, worsen the damaging impact for both society and the economy, indeed. The information decomposition technique developed in our paper may be used for the qualitative evaluation of the degree of uncertainty in complex real-world systems.

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