Measurement-based quantum computation using two-component BECs

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Abstract
Measurement-based quantum computation (MBQC) using two-component Bose–Einstein condensates (BECs) is proposed in this paper. An arbitrary state for one logical qubit is obtained by three-body measurement. Furthermore, a method is proposed for implementing controlled-Z gates for logical qubits in a graph state using BEC qubits and controlled-Z gates for BEC-type encoding. Results showed that the state after a measurement depends on the number of particles. These results pave the way for a novel quantum computing process based on particle control.

1. Introduction
Bose–Einstein condensation (BEC) is a phenomenon unique to many-body systems. It has been experimentally and theoretically attracting attention since it was discovered in a cold atomic system in 1995 [1, 2]. It is an outstanding challenge in the application of quantum technologies, such as quantum computation, quantum simulation [3], and quantum metrology [4]. Thus, quantum computing using two-component Bose–Einstein condensates (BECs) was proposed [5]. The authors of [5] used two modes of the hyperfine state for quantum computing. Quantum computing was implemented by BEC encoding using an appropriate time evolution. In addition, entangled states between two BECs were considered in [6], and the two types of CZ gates proposed in this paper may be realized by similar methods. Promising systems for quantum computing using BEC are optical lattices [1, 2], atom chips [7], and yttrium iron garnet (YIG) [8].

Preparing many-body systems is referred to as graph states and performing quantum tasks by measuring the particle angle is referred to as measurement-based quantum computation (MBQC) [9, 10]. Graph states are useful quantum states for quantum computation. There are many studies on the quantum computation of graph states, hypergraph states [11] using general controlled-Z gates over two qubits, and qudit graph states [12–14] using d-dimensional Hilbert spaces. Conventional quantum computers, such as the circuit-type quantum computer, perform unitary operations for the qubit. While MBQC prepares the graph state by quantum operation, one can break the state by measurement to perform quantum calculations. This distinction between quantum operations and classical measurement operations has produced many new results. To date, research related to MBQC includes matrix product state representation [15–17], correlation space [18, 19], valence bond solids states [20], and projected entangled pair states [21–23]. The graph state has been experimentally realized in optical lattice systems [24]. It should be noted whether it will be realized in atom chip and YIG systems as well.

In recent years, applications of quantum many-body systems using atom chips [7] or optical lattices [25] and YIG hybrid systems [26] have been shown to be remarkable resources for quantum computing. Many qubit systems, including superconducting quantum computers, operate at very low temperatures. Therefore, energy issues are substantial. The Diamond NV Center [27], optical quantum computers [28], silicon-based quantum computers [29], and YIG [8] systems are promising quantum computers that operate at room temperature.

In this paper, MBQC using BECs is proposed. In constructing the theory, many-body effects must be introduced to MBQC. This was realized by incorporating creation and annihilation operators for MBQC. Thus, many-body systems could be manipulated in the MBQC. The formulation incorporates genuine many-body
effects. Additionally, two types of controlled-Z gates are defined. By one gate, quantum computing by manipulating the Bose particle numbers can be achieved. That is, particle number quantum computing is realized via creation and annihilation operators. One promising system for performing MBQC using BECs is the YIG system, which can generate BECs at room temperature. This study, considering the YIG system, is useful for the implementation of quantum computers running at room temperature.

2. Preliminaries

2.1. MBQC and graph states

In the MBQC, when preparing graph states, $|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$ was put on a vertex, and a controlled-Z (CZ) gate was implemented between two vertices to create entanglement. Graph states were as follows.

$$ |G\rangle \equiv \left( \prod_{e \in E} CZ_e \right)|+\rangle^{|V|}, $$

where $CZ_e$ affects two vertices belonging to edge $e$, and $V$ is the vertex. Measurements of the qubit $Z$, $X$ basis change the graph state structure. Edge addition and deletion can be performed by the CZ gate.

Any qubit states can be calculated using graph states simply by local measurement. Consider an explicit two-qubit case as an example. The prepared graph states are

$$ |G\rangle = |0\rangle_1 |+\rangle_2 + |1\rangle_1 |-\rangle_2. $$

Then qubit one was measured in $|0\rangle_1 \pm e^{i\theta}|1\rangle_1$ basis, and if projected into $|0\rangle + e^{i\theta}|1\rangle$ basis, the following is obtained

$$ |+\rangle_2 + e^{-i\theta}|-\rangle_2 = He^{i\theta Z/2}|+\rangle_2, $$

where $H$ is the Hadamard gate. Any one-qubit unitary operator can be realized with $H$ and $e^{i\theta Z/2}$.

In addition, two-qubit gates can be implemented. The CZ gate can be implemented by applying it beforehand. One measurement left four qubits on the X basis since the measurement and the CZ gate can be replaced (figure 1).

2.2. Two-component BECs

Quantum computation using BECs was proposed [5]. BEC quantum computation denotes two levels of the hyperfine state via creation operators $a^\dagger$ and $b^\dagger$. Arbitrary BEC qubit states are represented by

$$ |\alpha, \beta\rangle \equiv \frac{1}{\sqrt{N!}}(\alpha a^\dagger + \beta b^\dagger)^N|\text{vac}\rangle. $$

Here $a^\dagger$ and $b^\dagger$ are creation Bose operators, obeying commutation relations $[a, a^\dagger] = [b, b^\dagger] = 1$. $\alpha$ and $\beta$ are arbitrary complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$ and $|\text{vac}\rangle$ in a vacuum state. The boson number $N = a^\dagger a + b^\dagger b$ is taken to be the conserved quantity. Then, to manipulate two-component BECs, the Stokes operator is introduced.
Those operators satisfy the commutation relation with same spin-1/2 particles,
\[ [S^i, S^j] = 2\epsilon_{ijk} S^k \]
where \( \epsilon_{ijk} \) is the Levi-Civita antisymmetric tensor \( i, j, k = x, y, z \). An entangled state generated by \( S^i_z S^j_z \) interaction is generated by
\[
|\psi\rangle = \frac{1}{\sqrt{2^N}} \sum_{|k\rangle} \left( e^{-i(2k_1-N_1)t} S^i_0 |k\rangle \right) |k_2\rangle_2,
\]
where \( k \) is a Fock state defined by
\[
|k\rangle = \frac{(a^+)^k(b^+)^{N-k}}{\sqrt{k!(N-k)!}}|\text{vac}\rangle.
\]

Another important operator is a BEC-type CZ gate, which is operating to two BECs, and described by Hamiltonian time evolution
\[
H = -S^x_1 S^x_2 + N S^2_1 - N S^2_2 + N^2,
\]
which affects two BECs and for a time evolution \( t = \pi/4N \). This gives
\[
\frac{1}{\sqrt{2^N}} \sum_{|k\rangle} \left( \frac{N}{k_1} \right) |k\rangle |\text{vac}\rangle |k_2\rangle_2.
\]

In quantum computing using BECs, Schwinger’s construction can be used to represent SU(2) with spin multiplicity 1. This is a harmonic oscillator in a high-energy excited state that may provide quantum parallelism (figure 2).

3. MBQC with two-component BECs

3.1. Formalism

BECs graph states are proposed. The BEC qubit \(|\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle\) was put in the vertex, instead of \( |+\rangle \). The CZ gate was implemented between two connected BEC qubits, as usual for MBQC. Let us define two types of BEC CZ gates. Those that are Hamiltonian are defined by
\[
H_1 = -S^x_1 S^x_2 + N S^2_1 - N S^2_2 + N^2 \quad \text{(10)}
\]
\[
H_2 = S^x_1 S^x_2 + N S^2_1 + N S^2_2 + N^2. \quad \text{(11)}
\]

In this paper, the time evolution \( t = \pi/4 \) is taken. A Hamiltonian (10) time evolution acts on two BEC qubits as
\[
\exp(-iH_1 t) |\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle_1 |\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle_2,
\]
\[
= \frac{1}{\sqrt{2^N}} \sum_{|k\rangle} \left( \frac{N}{k_1} \right) |k\rangle_1 |\frac{1}{\sqrt{2}}, e^{-ik} \rangle_2.
\]

Figure 2. Representation of the Fock state. Schwinger’s construction of SU(2) is represented with spin multiplicity 1.
This type of Hamiltonian time evolution is referred to as a right-hand component phase shift CZ gate (r-CZ). Conversely, a left-hand component phase shift CZ gate (l-CZ) Hamiltonian (11) can be defined. This acts between two BEC qubit as

\[
\exp(-iH_2t)\left|\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right>_1\left|\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right>_2
\]

\[
= \frac{1}{\sqrt{2^N}} \sum_{k_1=0}^{N} \left(N\right)_{k_1} \left(\frac{e^{-ik_1}}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)_1|k_2\rangle_2
\]

\[
= \frac{1}{\sqrt{2^N}} \sum_{k_1=0}^{N} \left(N\right)_{k_1} |k_1\rangle e^{-ik_1}\left|\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right>_2. \tag{13}
\]

The above two CZ gates are described by a similar formalism. However, there are different calculation results after the measurement steps. Hamiltonian (10) results in any N-dependent logical qubit state. Conversely, (11) is independent. Next, graph states are constructed using BECs.

\[
|G\rangle = \prod_{e \in \mathcal{E}} \exp(-iH_2t)_e \left|\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right>_1^{[V]} \tag{14},
\]

where, the r-CZ gate Hamiltonian was used, and CZ_e = exp(-iH_2t), affects two vertices belonging to an edge e. After discussion, binomial coefficient factors were neglected. Indeed, that is the only difference between odd and even binomial coefficients. For an explicit example of two BEC cases, and after summing over k, the BEC graph states are written as

\[
|G\rangle = |e\rangle_1\left|\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right>_2 + |o\rangle_1\left|\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right>_2. \tag{15}
\]

A total even number ket |e\rangle is defined, along with a total odd number ket |o\rangle

\[
|e\rangle = \sum_{k=0}^{k_{\text{even}}} \sqrt{N} C_k |k\rangle
\]

\[
|o\rangle = \sum_{k=1}^{k_{\text{odd}}} \sqrt{N} C_k |k\rangle. \tag{16}
\]

Equation (15) is a simple extension of equation (2). Indeed, one can confirm by replacing |0\rangle \rightarrow |e\rangle, |1\rangle \rightarrow |o\rangle, \left|\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle \rightarrow \left|\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle, \text{ and } \left|\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\rangle \rightarrow \left|\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\rangle\)
3.2. Three body measurement

Three-body measurement was demonstrated (figure 3). Any rotation Bloch sphere can be implemented on a logical qubit in this procedure. Three-body measurement consists of the following eight steps [protocol1].

First, the r-CZ gate is implemented between BECs1 and BECs2. Then, the r-CZ gate is implemented between BECs2 and BECs3. The system becomes

\[ |G\rangle = \sum_{k_2=0}^{N} e^{-ik_2} \sum_{k_1=0}^{N} \left| k_2 \right\rangle_{1} \left| k_2 \right\rangle_{2} |k_3\rangle_3, \]  

where

\[ |k, e^{-ik}\rangle \equiv \frac{(a^\dagger)^k (b^\dagger e^{-iNk})^N-k}{\sqrt{k!(N-k)!}} |\text{vac}\rangle. \]

A summation is taken over \( k_3 \) in equation (17). Thus the following is obtained

\[ \sum_{k=0}^{N} \left( e^{-ik} \left| \frac{1}{\sqrt{2}} e^{-ik} \right\rangle \right)_1 \left| -k_2 \right\rangle_2 + \left| \frac{1}{\sqrt{2}} e^{-ik} \right\rangle \right)_1 \left| k_2 \right\rangle_2 \]

The next step is odd–even measurement. BECs3 is measured by \( |\text{e}\rangle \pm |\text{o}\rangle \) basis. If \( |\text{e}\rangle + e^{i\theta} |\text{o}\rangle \) basis is projected, the following is obtained

\[ \left( 1 + e^{-i(\phi+N\pi)} \right) \left| \frac{1}{\sqrt{2}} e^{-i\phi} \right\rangle \right)_1 + e^{-i\phi} \left( 1 - e^{-i(\phi+N\pi)} \right) \left| \frac{1}{\sqrt{2}} e^{-i\phi} \right\rangle \right)_1 \]

Next, the BEC-type Hadamard gate is implemented on BECs1.

\[ e^{-i3\pi/4} \left| \frac{1}{\sqrt{2}} e^{-i\phi} \right\rangle \right)_1 = \left| 1, 0 \right\rangle, \]

\[ e^{-i3\pi/4} \left| -1, 0 \right\rangle = \left| 0, 1 \right\rangle. \]

Finally, the BECs1 (N+1 dimensional Hilbert space) is projected into a logical qubit (2-dimensional Hilbert space),

\[ |\tilde{0}\rangle \equiv \left| 1, 0 \right\rangle, \quad |\tilde{1}\rangle \equiv \left| 0, 1 \right\rangle. \]

Thus, any single-qubit state can be implemented after three-body measurement. An outcome state depends on N. This result implies that the outcome state is determined beforehand in the prepared boson number. After all steps are implemented, the following outcome states are obtained, ignoring the overall phase.

\[ \left( \cos \frac{\phi + N\pi}{2} \left| \tilde{0}\right\rangle + e^{-i\phi} \sin \frac{\phi + N\pi}{2} \left| \tilde{1}\right\rangle \right). \]

When the orthogonal bases of BECs3 and BECs2 are measured, if \( |\text{e}\rangle_3 - e^{i\phi} |\text{o}\rangle_3 \) and \( |\text{e}\rangle_2 - e^{i\theta} |\text{o}\rangle_2 \) are projected, the following is obtained

\[ \left( i \sin \frac{\phi + N\pi}{2} \left| \tilde{0}\right\rangle - e^{-i\phi} \cos \frac{\phi + N\pi}{2} \left| \tilde{1}\right\rangle \right). \]
The state can be specified in advance and depends only on the number of particles initially.

3.3. Measurement and controlled-Z gate

Using similar procedures for MBQC, the CZ gate can be implemented for MBQC using BECs (figure 4). The measurement and the CZ gates are commuted. Thus, the CZ gates can be implemented beforehand.

\[
\cos \left( \frac{N\pi}{2} |\bar{0}\rangle_1 + i \sin \left( \frac{N\pi}{2} |\bar{1}\rangle_1 \right). \tag{26}
\]

The state can be specified in advance and depends only on the number of particles initially.

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**Figure 4.** The black circle denotes BECs before summing over \(k\). The white circle denotes executed BECs summing over \(k\). A blue line represents the l-CZ, and a red line represents the r-CZ.

**Figure 5.** A logical-type CZ gate can be implemented by measuring the two white circles in the center of the \(|e\rangle \pm |o\rangle\) basis. The blue line connects the two white circles and the red line connects the white circle and the black circle.

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**protocol2: State controlling by removing middle BEC particles**

1. Implement CZ gate between BECs2 and BECs3.
2. Sum over BECs3.
3. Measure by \(|e\rangle \pm e^{i\theta}|o\rangle\) basis at BECs3
4. Remove the particles in BECs2
5. Implement CZ gate between BECs1 and BECs2.
6. Sum over BECs2.
7. Measure by \(|e\rangle \pm e^{i\theta}|o\rangle\) basis at BECs2
8. Implement BEC-type Hadamard gate on BECs1.
9. Projection into logical qubit on BECs1.
A logical CZ gate (figure 5) can be implemented. First, the four-BEC qubit graph state is

\[ |G\rangle = \sum_{k_0=0}^{N} \sum_{k_1=0}^{N} \left( \frac{1}{\sqrt{2}} e^{-i \pi k_1} |e\rangle + \frac{1}{\sqrt{2}} e^{-i \pi k_2} |o\rangle \right) |e\rangle^{k_1} |o\rangle^{k_2} |k_0 \rangle \]

where

\[ |e\rangle = (a^\dagger b |0\rangle)^{k_1} (a^\dagger c |0\rangle)^{k_2} |\text{vac}\rangle \]

Then, after taking the sum of \( k_2 \) and \( k_3 \), the two BECs were measured in the middle of the \( |e\rangle \pm |o\rangle \) basis. The following was obtained.

\[ |G\rangle = |0\rangle |0\rangle + (-1)^{s_1} |0\rangle |\bar{1}\rangle + (-1)^{s_2} (-1)^{s_1} |\bar{1}\rangle |0\rangle \]

where \( s_1 \) and \( s_2 \) are measurement results. It is a logical-type CZ gate when projected into a logical qubit if

\[ s_1 = s_2 = 0. \]

The logical-type CZ gate implemented in the BEC graph state for logical qubits is shown in figure 6.

A general graph state of the MBQC using BECs, which connected the three-body measurement part and the CZ gate part, is shown in figure 7.

3.4. Systems with different numbers of particles

Let us consider a system with a different number of particles. First, a different number of CZ gates is suggested. Two BEC CZ gates are defined by

\[
H_1' = -N_i S_i^z + \mathcal{N}_i S_i^+ + \mathcal{N}_i S_i^- + \mathcal{N}_i N_2 \\
H_2' = S_i^z S_j^z + \mathcal{N}_i S_i^+ + \mathcal{N}_i S_j^- + \mathcal{N}_i N_2,
\]

where \( \mathcal{N}_i = a_i^\dagger a_i + b_i^\dagger b_i + \mathcal{N}_i \) and \( \mathcal{N}_i = a_j^\dagger a_j + b_j^\dagger b_j + \mathcal{N}_i \) are also defined. The Hamiltonians are asymmetric concerning BECs1 and BECs2. Using the same procedures, the three-body measurement is conducted. After the Hamiltonian time evolution, the following outcome states are obtained.
The proposed encoding method uses the same method as creation and annihilation operator formalisms of BECs. The BECs graph state was also determined depending on the middle BEC qubit particle number. Consider protocol2, which is changed from protocol1 considered in subsection III.D. In this protocol, the r-CZ gate is operated between BECs1 and BECs2 after the additional CZ gate operates on the graph state after measurement, the outcome state can be controlled depending on the middle BEC qubit particle number. In the middle of the measurement, the task of removing the particles from the central BECs is considered. If the loss is estimated to be low. Moreover, there is research on the detection of a single magnon coupling YIG magnons and photons in a cavity QED (quantum electrodynamics) system [31].

There are possible errors, and this section will discuss how to correct them. First, the following errors in which the state of particles simultaneously transfers to other states are considered to be unlikely to occur due to the coherence property of the multiparticle system. \(|e\rangle \rightarrow |o\rangle, |o\rangle \rightarrow |e\rangle\) and \(|\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle \rightarrow |\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle\). In fact, to the best of the authors’ knowledge, no such errors have been reported. In two-component BECs systems, phase shift error was theoretically and experimentally pointed out as a possible error [32] when a calculation step, such as particle collision, was inserted. The method for detecting phase errors has been studied [32], and uses a Ramsey interferometer in the atom chip system. Phase error correction is achieved by applying appropriate time evolution using Stokes operators on the system. The BECs unitary gate \(\exp (-iS^z\delta\phi)\) can be used to rotate the system in the XY plane in the opposite phase to the phase error, where \(\delta\phi\) is a phase error rate.

3.5. State controlling by removing middle BEC particles

In the middle of the measurement, the task of removing the particles from the central BECs is considered. If the additional CZ gate operates on the graph state after measurement, the outcome state can be controlled depending on the middle BEC qubit particle number. Consider protocol2, which is changed from protocol1 considered in subsection III.D. In this protocol, the r-CZ gate is operated between BECs1 and BECs2 after the BECs2 particles are removed. By this operation, the output BECs1 state changed according to the number of removed particles.

\[
|e\rangle_{2,3} + e^{i\theta}|o\rangle_{2,3} \\
\rightarrow \left( \cos \frac{\phi + N_2\pi}{2} |0\rangle_1 + e^{-i\theta} \sin \frac{\phi + N_2\pi}{2} |1\rangle_1 \right)
\]

(32)

\[
|e\rangle_{2,3} - e^{i\theta}|o\rangle_{2,3} \\
\rightarrow \left( i \sin \frac{\phi + N_2\pi}{2} |0\rangle_1 - e^{-i\theta} \cos \frac{\phi + N_2\pi}{2} |1\rangle_1 \right).
\]

(33)

where \(N_2\) is a middle Bose particle number in the three-body measurement, and (in \( |e\rangle_{2,3} + e^{i\theta}|o\rangle_{2,3}\)) index 2, 3 are adjacent measurements for BECs2 and BECs3. The phase of cos and sin can be changed by controlling the number of middle particles. For example, evolve \(N_2 \rightarrow N_2 - 1\)

\[
(32) \rightarrow \left( \sin \frac{\phi + N_2\pi}{2} |0\rangle_1 - e^{-i\theta} \cos \frac{\phi + N_2\pi}{2} |1\rangle_1 \right)
\]

(34)

\[
(33) \rightarrow \left( -i \cos \frac{\phi + N_2\pi}{2} |0\rangle_1 - e^{-i\theta} \sin \frac{\phi + N_2\pi}{2} |1\rangle_1 \right).
\]

3.6. Decoherence and error correction

In the two-component BECs system, the main factors of decoherence are particle loss and dephasing. These two species were showed not to scale to the number of particles by [5], unless it involved quantum calculation steps. The proposed encoding method uses the same method as [5], and it is predicted that particle loss and dephasing are not scaling to \(N\). In addition, in optical lattice systems and atom chip systems, BEC is kept in space, so the particle loss from the trap cannot be ignored. However, in the YIG system, BEC occurs in solids, so the particle loss is estimated to be low. Moreover, there is research on the detection of a single magnon [30], and it is possible to realize high manipulation of particles. Furthermore, there is high controllability that can be realized by coupling YIG magnons and photons in a cavity QED (quantum electrodynamics) system [31].

There are possible errors, and this section will discuss how to correct them. First, the following errors in which the state of particles simultaneously transfers to other states are considered to be unlikely to occur due to the coherence property of the multiparticle system. \(|e\rangle \rightarrow |o\rangle, |o\rangle \rightarrow |e\rangle\) and \(|\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle \rightarrow |\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle\). In fact, to the best of the authors’ knowledge, no such errors have been reported. In two-component BECs systems, phase shift error was theoretically and experimentally pointed out as a possible error [32] when a calculation step, such as particle collision, was inserted. The method for detecting phase errors has been studied [32], and uses a Ramsey interferometer in the atom chip system. Phase error correction is achieved by applying appropriate time evolution using Stokes operators on the system. The BECs unitary gate \(\exp (-iS^z\delta\phi)\) can be used to rotate the system in the XY plane in the opposite phase to the phase error, where \(\delta\phi\) is a phase error rate.

4. Conclusions

In this paper, we constructed MBQC using BECs by introducing the effect of many-body systems through the creation and annihilation operator formalisms of BECs. The BECs graph state was also defined and takes the BECs system time evolution at \(t = \pi/4\). As a result, the maximum entangled state between logical qubits was obtained. When performing quantum calculations, there are two types of CZ gates (r-CZ and l-CZ). The r-CZ is formulated by a right-hand component phase shift of the CZ gate. This type of CZ gate generates a particle number factor in the middle of the calculation step. In addition, three-body measurement was considered, which showed that any one logical qubit state is obtained by consuming two BEC qubits. Moreover, the BEC-type CZ gate and the logical-type CZ gate can be implemented in a similar way to that of standard graph states because the measurement and the CZ gate in BECs are commuted. The general graph state was constructed by combining the three-body measurement and the CZ gate part.

However, there were some difficulties with implementation. One such difficulty was how to measure the mixed odd and even number states. In the Wiggner tomography method [33, 34], a parity operator \(P = e^{i\theta_b\phi}\) may be used to measure the interference effect for odd/even measurements. The parity operator is defined by
the Stokes operator $H = \mathcal{N} - S^2 = b^\dagger b$, and time evolution of $t = \pi/2$. The parity operator acts on $P(\text{even}) = |\text{even}\rangle$ and $P(\text{odd}) = -|\text{odd}\rangle$. Another difficulty is that the final state of the three-body measurement is the Schrödinger cat state. Thus, it is necessary to take measures immediately when the final state is reached.

A brief comparison was made to other quantum computers operating at room temperature, especially the Diamond NV Center [27], which is attracting attention from the viewpoint of coherence operability by microwaves as well as a long coherence time. Unlike superconducting quantum computers, the Diamond NV Center operates at room temperature. In recent years, the YIG system in which BEC occurs at room temperature has been discovered [8]. Thus, the Diamond NV Center and the YIG system are compared from the viewpoint of coupling with other systems. Coupling is also important from the perspective of scalable quantum computer construction. In cavity QED, there is a method to couple qubits in the Diamond NV Center with photons; however, coupling is weak. There are also problems with scalability. In the YIG system, the coupling of magnon and photon as well as magnon and superconducting qubits have been studied. In addition, it can cause strong coupling.

Finally, it is noted that the key to the realization of the proposed theory in a laboratory will depend on the ability to manipulate the particle dissipation of the system. Recent experiments on optical lattices are attempting to control dissipation in the cold atomic system as inelastic collisions and light scattering of particles [35, 36]. That has the potential to be promising realization milestones for this study.

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Data availability statement

No new data were created or analysed in this study.

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