Ballistic Target Signal Separation Based on Differential Evolution Algorithm

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Abstract. During the course of the ballistic target's mid-flight, it is very important to accurately identify the target. Separating the target micro-Doppler curve is the key to accurate identification. Aiming at this problem, this paper proposes and improves a differential evolution algorithm to separate the micro-Doppler curves of the scattering points. According to the results of the simulation experiment, the signal is separated well, which verifies the effectiveness of the proposed algorithm.

1. Introduction
In order to cope with the threat of ballistic missiles, all countries are rushing to study the construction of ballistic missile defense systems[1]. Ballistic target recognition technology is one of the core technical problems that need to be solved in ballistic missile defense. However, in recent years, the penetration technology of ballistic missiles has been greatly developed. With the increasing maturity of sub-guided multi-warhead and missile decoy technology[2], ballistic target recognition technology based on traditional feature quantities has been unable to adapt to modern high-tech warfare. Demand. In 2000, Professor V. C. Chen extended the micro-movement to the field of radar observation, and pointed out that the micro-movement feature of the target is the inherent attribute of the target[3]. Because this characteristic is difficult to be imitated, and different warheads and decoy motion forms have obvious differences, this characteristic can be used to identify ballistic missiles.

Since precession will produce micro-Doppler modulation of radar echo, the micro-Doppler of radar echo can be analyzed, and then the characteristic parameters of the target can be obtained for identification[4][7]. In order to study the characteristics of the target, it is necessary to separate the echo signals of the group targets, and extract the micro-motion signals of the sub-targets separately. Literature [8] uses the segmented Viterbi algorithm to separate the compensated time-frequency curve. Literature [9] combined the Viterbi algorithm with adaptive field of view cluster matching, and obtained the optimal matching path of the target micro-Doppler curve, which realized signal separation. Literature [10] uses the EMD algorithm to decompose the micro-Doppler characteristics of the aircraft to achieve target resolution. Literature [11] uses the estimated period to segment the echo signal, and uses the support domain obtained from the strong energy region of the signal to perform time-frequency joint filtering on the echo, thereby separating different echoes. Literature [12] proposed a method of using sliding windows to separate the fretting curves, but this method did not mention the separation of ballistic group targets with different motion laws. Literature [13] proposed a learning algorithm using C-means clustering to improve the ICA mixing matrix, and then using sparse decomposition to separate the source signal, but this method relies on the accurate estimation of the mixing matrix. When the mixing matrix is inaccurate, the separation algorithm will not be able to Play its due role.
Based on the above-mentioned research status, this paper proposes the use of differential evolution algorithm to achieve the purpose of signal separation by seeking the optimal solution. Aiming at the problems of traditional differential evolution algorithm which is easy to fall into local optimum and premature convergence, its related strategies are improved, and the JADE algorithm is proposed. According to the results of the simulation experiment, the signal separation is realized. It lays a good foundation for extracting target feature parameters in the next step.

2. Precession model of ballistic target
The center of mass of the ballistic target is \( O \). As shown in Figure 1, the reference coordinate system \( O-XYZ \) is established with \( O \) as the origin. The ballistic target performs conical rotation on the \( Z \) axis, and the conical rotation angular velocity is \( \omega_c \); spins on its own symmetry axis, and the spin angular velocity is \( \omega_s \). The angle between the cone axis and the spin axis, that is, the precession angle is \( \omega \). At the beginning, the azimuth and elevation angles of the radar line of sight \( LOS \) in the reference coordinate system \( O-XYZ \) are \( \alpha \) and \( \beta \) respectively. The body coordinate system of the ballistic target is \( Oxyz \), rotate \( \phi_e \) around the \( z \) axis, rotate \( \theta_e \) around the \( x \) axis, and rotate \( \varphi_e \) around the \( y \) axis to get the reference coordinate system. The distance from the center of mass \( O \) of the target to the vertex \( A \) of the cone is \( l_1 \), and the distance from the center of the bottom surface of the cone is \( l_2 \).

![Figure 1 Scattering model of precession cone ballistic target](image)

In the body coordinate system \( Oxyz \), the initial position of any scattering point on the cone target is \( \mathbf{r}_0 = (r_{0x}, r_{0y}, r_{0z})^T \), then the position vector of the scattering point in the reference coordinate system at time \( t \) is \(^{[14]}\)

\[
\mathbf{r} = T_c T_s R_{int} \mathbf{r}_0
\]

In the formula: \( T_c \) is the spin matrix of the ballistic target, \( T_s \) is the cone spin matrix of the ballistic target, and \( R_{int} \) is the Euler rotation matrix, which is determined by the initial Euler angle \( (\phi_e, \theta_e, \varphi_e) \).

According to the literature \(^{[15]}\) and Rodrigues formula \(^{[16]}\), the expressions of \( T_c \), \( T_s \) and \( R_{int} \) matrix can be obtained as follows

\[
T_c = I + \tilde{\omega}_c \sin(\Omega, t) + \tilde{\omega}_c^2 (1 - \cos(\Omega, t))
\]

\[
T_s = I + \tilde{\omega}_s \sin(\Omega, t) + \tilde{\omega}_s^2 (1 - \cos(\Omega, t))
\]
In the formula: \( I \) is the identity matrix, \( \Omega = \| \omega_c \|, \Omega_s = \| \omega_s \| \), where \( \omega_c = (\omega_{cX}, \omega_{cY}, \omega_{cZ})^T \), \( \omega_s = (\omega_{sX}, \omega_{sY}, \omega_{sZ})^T \), \( \omega_{c}^{i} = \omega_{c} / \Omega_{c}, \omega_{s}^{i} = \omega_{s} / \Omega_{s}, \omega_{c}^{\hat{}} \) and \( \omega_{s}^{\hat{}} \) are the skew symmetric matrices of \( \omega_{c} \) and \( \omega_{s} \), respectively. Construct the following oblique symmetric matrix[17]

\[
\begin{bmatrix}
\cos \theta_{c} & -\sin \theta_{c} & 0 \\
\sin \theta_{c} & \cos \theta_{c} & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi_{c} & -\sin \phi_{c} \\
0 & \sin \phi_{c} & \cos \phi_{c}
\end{bmatrix}
\begin{bmatrix}
\cos \phi_{c} & -\sin \phi_{c} & 0 \\
\sin \phi_{c} & \cos \phi_{c} & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  

(4)

In the formula: \( \Omega = \| \omega_c \|, \Omega_s = \| \omega_s \| \), where \( \omega_c = (\omega_{cX}, \omega_{cY}, \omega_{cZ})^T \), \( \omega_s = (\omega_{sX}, \omega_{sY}, \omega_{sZ})^T \), \( \omega_{c}^{i} = \omega_{c} / \Omega_{c}, \omega_{s}^{i} = \omega_{s} / \Omega_{s}, \omega_{c}^{\hat{}} \) and \( \omega_{s}^{\hat{}} \) are the skew symmetric matrices of \( \omega_{c} \) and \( \omega_{s} \), respectively. Construct the following oblique symmetric matrix[17]

\[
\begin{bmatrix}
0 & -\omega_{cZ} & \omega_{cY} \\
\omega_{cZ} & 0 & -\omega_{cX} \\
-\omega_{cY} & \omega_{cX} & 0
\end{bmatrix}
\begin{bmatrix}
0 & -\omega_{sZ} & \omega_{sY} \\
\omega_{sZ} & 0 & -\omega_{sX} \\
-\omega_{sY} & \omega_{sX} & 0
\end{bmatrix}
\]  

(5)

Therefore, the distance between the scattering point and the radar at time \( t \) is

\[
R(t) = \| R_0 + \mathbf{r}_0 \| = n_{LOS}^T (R_0 + T_s R_{\text{rot}} r_0)
\]  

(6)

In the formula: \( R_0 \) is the position vector of the warhead's rotation center from the radar, and \( n_{LOS} \) is the direction of the radar's line of sight.

According to formula \( R(t) = \| R_0 + \mathbf{r}_0 \| = n_{LOS}^T (R_0 + T_s R_{\text{rot}} r_0) \) (6), the echo signal of the target can be obtained, and the fast Fourier transform is used, and then the envelope is obliquely removed, and a high-resolution range image can be obtained[17]

\[
S_{f}(f, t_m) = \sum_{i=1}^{L} \sigma_{T_{p}} \sin c(T_{p} (f + \frac{2\mu}{c} R_{\lambda})) \exp (-j \frac{4\pi f}{c} R_{\lambda})
\]  

(7)

In the formula: \( T_{p} \) is the pulse width, \( \mu \) is the frequency modulation, \( f_{c} \) is the radar carrier frequency, and \( R_{\lambda} \) is the radial distance between the scattering center and the rotation center.

3. Micro-motion information signal separation

3.1 Introduction and principle of differential evolution algorithm

Differential Evolution (DE) is a heuristic random search algorithm based on group differences. This algorithm is proposed by R.Storn and K.Price to solve Chebyshev polynomials, and is mainly used to solve real number optimization problems. This algorithm is a kind of group-based adaptive global optimization algorithm, which is a kind of evolutionary algorithm. Because of its simple structure, easy implementation, fast convergence, and strong robustness, it is widely used in data mining and pattern recognition, Digital filter design, artificial neural network, electromagnetics and other fields[18].

The DE algorithm uses floating-point vectors for encoding to generate population individuals. In the process of DE algorithm optimization, firstly, two individuals are selected from the parent individuals to perform vector difference to generate a difference vector. Secondly, another individual is selected and the difference vector is summed to generate an experimental individual. Then, the parent individual is compared with the difference vector. Corresponding experimental individuals perform crossover operations to generate new offspring individuals. Finally, a selection operation is performed between the parent individual and the offspring individual, and the individuals that meet the requirements are saved to the next generation group. Figure 2 shows the flow chart of the DE algorithm.

![Figure 2 Flow chart of DE algorithm](image-url)
3.2 Differential evolution algorithm process\(^{[19]}\)

Suppose the optimization model is as follows:

$$\min f(x_1, x_2, ..., x_D)$$

$$s.t. \quad x^L_j \leq x_j \leq x^U_j, \quad j = 1, 2, ..., D$$

(8)

In the formula: \(D\) is the dimension of the solution space, and \(x^L_j\) and \(x^U_j\) respectively represent the upper and lower bounds of the value range of the \(j\)-th component \(x_j\).

3.2.1 Initial population

The initial population is:

$$\{x_i(0) | x^L_j \leq x_{ij}(0) \leq x^U_j, i = 1, 2, ..., NP; j = 1, 2, ..., D\}$$

(9)

Various groups of individuals are randomly generated by the following formula:

$$x_{ij}(0) = x^L_j + x^U_j - 2 \times rand(0, 1) \times (x^U_j - x^L_j)$$

(10)

In the formula: \(x_i(0)\) represents the \(i\)-th individual of the 0th generation in the population, and \(x_{ij}(0)\) represents the \(j\)-th component of the \(i\)-th individual of the 0th generation. \(NP\) represents the population size, and \(rand(0, 1)\) represents a random number uniformly distributed in the (0,1) interval.

3.2.2 Mutations

The most significant difference between differential evolution algorithm and genetic algorithm is that the individual variation of DE is realized through differential strategy. The commonly used differential strategies are as follows:

$$v_i(g + 1) = x_i(g) + \lambda \times (x_{i'}(g) - x_{i''}(g))$$

$$i \neq i' \neq i''$$

(11)

In the formula: \(\lambda\) is the scaling factor, and \(x_i(g)\) is the \(g\)-th individual in the \(i\)-generation population.

That is, by randomly selecting two different individuals in the population, the vector difference is scaled and then the vector is synthesized with the individual to be mutated.

In the evolution process, the validity of the newly generated solution must be guaranteed, so it must be judged whether the generated solution meets the boundary conditions, if not, it needs to be regenerated (the generation scheme is the same as the initial population).

Generation \(g\) population:

$$\{x_i(g) | x^L_j \leq x_{ij}(g) \leq x^U_j, i = 1, 2, ..., NP; j = 1, 2, ..., D\}$$

(12)

Intermediate after mutation-Generation \(g + 1\) population:

$$\{x_i(g + 1) | x^L_j \leq x_{ij}(g + 1) \leq x^U_j, i = 1, 2, ..., NP; j = 1, 2, ..., D\}$$

(13)

3.2.3 Cross

Use the generation \(g\) population and its variant intermediates to cross:

$$u_{ij}(g + 1) = \begin{cases} v_{ij}(g + 1), \quad \text{rand}(0,1) \leq CR \| j = j_{rand} \\ x_{ij}(g + 1), \quad \text{otherwise} \end{cases}$$

(14)

In the formula: \(CR\) is the crossover probability, \(j_{rand}\) is the random integer of [1,2,\ldots,D].

3.2.4 Select

The greedy algorithm is used to select the next-generation population individuals:
\[ x_i(g+1) = \begin{cases} u_i(g+1), & f(u_i(g+1)) \leq f(x_i(g)) \\ x_i(g), & \text{otherwise} \end{cases} \]  

(15)

3.3 Improvement of DE algorithm

For the DE algorithm, as the number of iterations increases, the difference between individuals will gradually decrease, and the convergence speed will also decrease, which will make the DE algorithm easy to fall into local optimal and premature convergence. Therefore, it is necessary to seek various improvements on the original classic DE algorithm to improve the DE algorithm’s optimization ability, convergence speed, and overcome premature convergence. Therefore, the JADE algorithm is proposed.

According to the dimension \( D \) of the problem, we use a \( D \)-dimensional vector to represent a result. It should be noted that each dimension should have its own upper and lower bounds. Pack a fixed number (\( NP \)) of results to form a set.

The JADE algorithm is an improvement of the DE algorithm, so the basic logic remains unchanged. The main differences are as follows:

1. A new mutation strategy is proposed. Its strategy is shown in the following formula

\[ v_i(g+1) = x_i(g) + F_i\cdot(x_i(g) - x_j(g)) + F_i\cdot(x_i(g) - x_k(g)) \]

\[ i \neq r_1 \neq r_2 \neq r_3 \]

In the formula: \( x_{best}(g) \) is to directly select the best result in this set

2. \( F \)'s self-adaptation. The \( F_i \) of each result in each generation is a random number conforming to the Cauchy distribution \( C(\mu_F,0.1) \).

\[ \mu_F \text{ proportional parameter is 0.1, and conforms to the self-update formula } \]

\[ \mu_F = (1-c)\cdot\mu_F + c\cdot\text{mean}_F(S_F) \]

In the formula: The initial value of \( \mu_F \) is 0.5, and \( c \) is a constant. Generally speaking, \( 1/c \in [5,20] \), \( \text{mean}_F(S_F) = \frac{\sum_{i \in S_F} F_i^2}{\sum_{i \in S_F} F_i} \), and \( S_F \) are the set of \( F \) each time the result of the offspring replaces the result of the parent.

3. Adaptive of \( CR \). The \( CR \) of each result in each generation is a random number conforming to the normal distribution \( N(\mu_{CR},0.1) \) and the size is constrained within \([0,1]\), where \( \mu_{CR} \) follows the self-renewing formula\(^{[20]}\)

\[ \mu_{CR} = (1-c)\cdot\mu_{CR} + c\cdot\text{mean}_F(S_{CR}) \]

In the formula: The initial value of \( \mu_{CR} \) is 0.5, and \( c \) is a constant. Generally speaking, \( 1/c \in [5,20] \) and \( \text{mean}_F \) refer to the calculation of the arithmetic mean, and \( S_{CR} \) is the set of \( CR \) each time the result of the offspring replaces the result of the parent.

4. Simulation

The distance \( l_1 = 2m \) from the center of mass of the cone target to the vertex, the distance \( l_2 = 0.6m \) from the center of the bottom surface, and the radius \( r = 1m \) of the bottom surface. The cone spin frequency \( \omega_c = 4\pi \) and spin frequency \( \omega_s = 5\pi \) of the target. The precession angle is \( \theta = \pi/18 \), and the initial Euler angle is \( (\phi_0, \theta_0, \psi_0) = (\pi, \pi/18, \pi) \). The signal transmitted by the radar is a chirp signal with a bandwidth of 3GHz, a pulse width of \( T_p = 40\mu s \), a signal carrier frequency of \( f = 10GHz \), and a pulse repetition frequency of \( PRF = 1000Hz \). The radar is located at the reference coordinate system \( (100,300,-500)Km \), and the line of sight of the radar is \( \mathbf{n} = (1/\sqrt{35},-3/\sqrt{35},5/\sqrt{35}) \).

According to the distance formula and related parameters obtained in the previous section, the cone target movement is simulated, and the radar echo with a duration of \( 2s \) is received together, and the
target micro-Doppler curve as shown in Figure 3 is obtained.

Figure 4 simulates the JADE algorithm to separate the obtained cone target micro-Doppler signal. According to Figure 4, it can be seen that the micro-Doppler curves of the scattering points are better separated.

5. In conclusion
The radar micro-Doppler curve of the ballistic target is superimposed by multiple scattering points. For the follow-up related research, the micro-Doppler curve of the scattering points is separated first. This paper proposes the DE algorithm, and improves the JADE algorithm according to its related shortcomings, which realizes the separation of signals. The simulation experiment verifies the effectiveness of the algorithm and realizes the signal separation better.

In practice, the scattering point characteristics of the target are more complicated. At the same time, during the flight of the target, there are a lot of warhead fragments and decoys along with the flight of the target. Therefore, in the next step, we can study the micro-motion characteristics of the group target in the case of complex scattering points.

At the same time, after signal separation of ballistic targets, the next step is to perform feature extraction and recognition of the target. Here, by deriving the mathematical formula of the relevant parameters, the neural network algorithm is introduced in order to achieve the purpose of feature extraction and classification and recognition of the target.

References
[1] Liang, L. (2018) Development Trend of Intercontinental Ballistic Missile Penetration Technology. Winged Missiles Journal, 8: 55-57+63.
[2] Menq, J., Tuan, P., Liu, T. (2007) Discrete Markov ballistic missile defense system modeling. European Journal of Operational Research, 178: 560–578.
[3] Chen, V.C. (2000) Analysis of Radar Micro-Doppler Signature with Time-Frequency Transform. In: Proc. IEEE Workshop on Statistical Signal and Array Processing. Pocono Manor, PA, USA. pp. 463-466.
[4] Peng, L., Sun, J.P., Wang, J., et al. (2012) Micro-Motion Parameter Estimation of Free Rigid Targets Based on Radar Micro-Doppler. IEEE Transactions on Geoscience and Remote Sensing, 50: 3776-3786.
[5] Wang, J., Lei, P., Sun, J.P., et al. (2014) Spectral characteristics of Mixed Micro-Doppler time-frequency data sequences in Micro-Motion and Inertial Parameter Estimation of Radar Targets. IET Radar, Sonar & Navigation, 8: 275-281.
[6] Chen, V.C. (2014) Advances in Application of Radar Micro-Doppler Signatures. In: Proc. of the 2014 IEEE Inf. on Antenna Measurements & Application(CAMA). Antibes, France. pp. 1-4.
[7] Wang, Z.Y., Zhang, X.G., Bai, Y.C. (2014) Precession and Structural Parameter Estimation of
Cone-shaped Target Based on Micro-Doppler. Journal of Nanjing University (Natural Sciences), 50: 148-153.

[8] Han, L.X., Tian, B., Feng, C.Q., et al. (2019) Translation Compensation and Resolution of Ballistic Target with Precession. Journal of Beijing University of Aeronatics and Astronautics, 45: 1459-1466.

[9] Li, J.Q., Feng, C.Q., Zhang, D. (2015) Multi-target Separation and Extraction Based on Adaptive Vision Cluster Matching. Systems Engineering and Electronics, 37: 1974-1979.

[10] Li, Y., Ren, D.L., Zhang, C., et al. (2018) Aircraft Micro-Doppler feature analysis based on empirical mode decomposition. Radar & ECM, 38: 37-40+48.

[11] He, S.S., Zhao, H.N., Zhang, Y.S. (2015) Signal Separation for Target Group in Midcourse Based on Time-frequency Filtering. Journal of Radars, 4: 545-551.

[12] Zhao, M.M., Zhang, Q., Chen, Y.J., et al. (2015) A Sliding Window Tracking Algorithm for Distinguishing Space Group Targets. Journal of Astronautics, 36: 1187-1194.

[13] Chen, X.J., Cheng, H., Tang, B. (2010) Underdetermined Blind Radar Signal Separation Based on ICA. Journal of Electronics & Information Technology, 32: 919-924.

[14] Lei, T., Liu, J.M., Xu, F.P., et al. (2012) A New Procession Signature Extraction Method of Ballistic Target Based on Range-profile. Journal of Signal Processing, 28: 73-79.

[15] Hu, X.W., Tong, N.N., He, X.Y., et al. (2016) Three-dimensional Imaging of Precession Targets with Unsymmetrical Appendixes Based on Micro-motion. Systems Engineering and Electronics, 38: 501-505.

[16] Yao, H.W., Wei, X.Z., Xu, S.K., et al. (2012) Micro-Motion Characteristics of Non-Ideal Scattering Centers of Midcourse Targets with Precession. Acta Electronica Sinica, 40: 1844-1851.

[17] Zhang, Q., Luo, Y. (2013) Micro-Doppler Model of Target in Narrowband Radar. In: Mao, J.Q. (Eds.), Micro-Doppler Effect of Radar Targets. National Defense Industry Press, Beijing. 22-46.

[18] Min, T., Yang, S. (2021) Differential Evolution Algorithm Based on Low-Density Individual in Neighborhood for Multimodal Optimization Problem. Computer Systems & Applications, 30: 117-125.

[19] Niu, Y.F., Ma, J. (2021) Bayesian Network Structure Learning Method Based on Differential Evolution Strategy. Computer Simulation, 38: 242-246+255.

[20] Wang, B., Shen, L.K., Wang, X., et al. (2021) Research on energy efficiency optimal control strategy of parallel pump set based on improved DE algorithm. Transducer and Microsystem Technologies, 40: 133-139+143.