A technique has been developed for reducing the material intensity of highly stressed tail sections of launch vehicles, taking into account strength and stability constraints as well as technological requirements. A cylindrical longitudinally and transversely ribbed waffle-grid (lattice) shell with rectangular holes is taken as the design scheme of the tail section, with its lower end being fastened at locations of support brackets, and the upper one being loaded with longitudinal compressive forces, evenly distributed along the contour, due to the action of the weight of higher-located structure elements. The optimization algorithm is based on the principle of ensuring discrete uniform strength of individual elements (substructures). The structural geometric dimensions of cross-sections of a standard tail section and the stiffness parameters of longitudinal and transverse load-bearing frames, the wall thicknesses of shell elements, the dimensions of lattice shells, etc., are selected from the requirements of stress-strength reliability: constraints on the limiting values of equivalent stresses (strength conditions), compressive stresses of the local and general buckling, and a number of design and technological requirements. The direct calculation of the tail section and the search for its variable geometric parameters are proposed to be performed using an interactive numerical-analytical (finite element method – engineering analysis) algorithm. The initial calculation of the static stress-strain state of the lattice-reinforced tail section was carried out by the finite element method, which is implemented in the NASTRAN package. To discretize the shell and its ribbing, flat finite elements were used. In the process of the finite-element numerical modeling of the tail section state, the reliability of the obtained results of calculating the equivalent stresses was analyzed by studying the convergence of the results of calculations on a series of meshes with different refinement. Results of the application of the developed technique to reduce the mass of the standard tail section of the Antares launch vehicle are presented.

Keywords: launch vehicle, tail section, material consumption, stress-strain state.
Thus, the tail section has a substantially inhomogeneous structure and is under conditions of intense uneven loading, which entails the same significant unevenness of its stress-strain state (SSS), which is characterized by the presence of both overstressed sections and sections with a sufficiently low level of forces and stresses [6].

It is obvious that the development of an approach to ensure the equalization of the SSS parameters through a more rational use of the capabilities of the tail section material to reduce its material intensity is a very pressing application problem.

Formulation of the Tail-Section Weight-Optimization Problem

A cylindrical longitudinally and transversely ribbed lattice shell with rectangular holes is taken as the design scheme of the tail section, with its lower end being fastened at locations of support brackets, and the upper one being loaded with longitudinal compressive forces, evenly distributed along the contour, due to the action of the weight of higher-located structural elements.

The structural geometric dimensions of cross-sections of a standard tail section and the stiffness parameters of longitudinal and transverse load-bearing frames, the wall thicknesses of shell elements, the dimensions of lattice shells, etc., are selected from the requirements of stress-strength reliability: constraints on the limiting values of equivalent stresses (strength conditions), compressive stresses of the local and general buckling, and a number of design and technological requirements.

It is proposed to reduce the material intensity of such a lattice shell structure by optimizing the incrementally-variable, in the circumferential and longitudinal directions, geometric dimensions of the sections $j (j=1, 2, ..., J)$ of its constituent substructures of the tail section load-bearing frame:

$$V = \sum_{j=1}^{J} V_j \to \min .$$  \hspace{1cm} (1)

An operable (admissible) load-bearing frame will be the one for which, for all $J$ of its constituent substructures, $N$ conditions of stress-strength reliability are fulfilled in the form

$$g_{j,n}(\delta) \leq 0, n = 1,2,...,N ,$$  \hspace{1cm} (2)

where $\delta = (\delta_1, \delta_2,...,\delta_s)$ is the variation vector of technologically admissible geometrical and physical structural parameters

$$\delta^n_s \leq \delta_s \leq \delta^n_s, s = 1,2,...,S .$$  \hspace{1cm} (3)

The requirement that the strength conditions for structural elements is satisfied for, in the general case, several possible $t$ types of loading can be formulated as constraints on the limiting values of equivalent stresses in the form

$$\max_{t} \sigma^{(j,t)}_{\text{eq}} \leq [\sigma]_{j,t} ,$$  \hspace{1cm} (4)

where $\max_{t} \sigma^{(j,t)}_{\text{eq}}$ is the maximum equivalent stress for all points of the $j$-th substructure, which can be determined according to one of the accepted strength theories, at the $t$-th loading option, and $[\sigma]_{j,t}$ is their allowable stress for the material of the $j$-th substructure (element) at the $t$-th option of effect, respectively.

The total number of constraints (4) will be equal to the product of the number of substructures (selected according to some principle of a set of structural elements) by the total number of options for external effects.

In the case when the design calculation is carried out for one type of loading, constraints (4) take the form

$$\sigma^j_{\text{eq}} \leq [\sigma]_j , j = 1,2,...,J .$$  \hspace{1cm} (5)

Similarly represented are the dependencies for critical compressive stresses, which determine the possible buckling of individual substructures of system [8–10]

$$\sigma^{i,j}_{\text{cr}} \leq [\sigma]_i , i = 1,2,...,I, I \leq J, i \in (j = 1, J) .$$  \hspace{1cm} (6)

The limit case of conditions (5), (6) leads to the creation of a discretely equal stress structure, all substructures of which have the same stress-strength reliability [11].

Constraints (5), (6) can be represented in dimensionless form

$$q'(\delta) = f'(\delta) - 1 \leq 0, e \in 1,2,...,I + J .$$  \hspace{1cm} (7)
If for the $j$-th element in (7) $q^e(\delta) \leq 0$, then the corresponding substructure of the deformable system is considered underloaded, and when $q^e(\delta) > 0$, overloaded. Thus, $q^e(\delta)$ characterizes the level of uneven loading of the $j$-th substructure.

To check the consistency of the mathematical model of the problem in form (1–7), a numerical calculation of the entire system was carried out using the finite element method. The results obtained were used to calculate the objective function and the values of constraints for some initial (with regular parameters) standard design option.

Such an (initial) calculation of the static SSS of the lattice-reinforced tail section is implemented in the NASTRAN package [6, 7]. Flat finite elements were used to discretize the shell and its ribbing. In the process of the finite-element numerical modeling of the tail section state, the reliability of the obtained results of calculating the equivalent stresses was analyzed by studying the convergence of the results of calculations on a series of meshes with different refinement. If the results of calculations of equivalent stresses on these meshes turned out to be close, then the convergence and reliability of the calculation were considered achieved. With the selected initial values of standard design parameters, the tail section was in the elastic region with the given safety factor $f=1.22$.

Based on the results of the finite element calculation and analysis (7), a significant non-uniformity of the SSS of the standard tail section body was established, which served as the basis for determining the non-uniformity coefficient which was used later in the design optimization procedure.

The purpose of this study is to use this unevenness to reduce the weight of the tail section by controlling the geometry of the reinforcement, the size of cells and the thickness of the shell.

**Construction of an Algorithm for Reducing the Tail-Section Material Intensity**

The choice of the optimization method and its efficiency substantially depend [12] on the number and nature of variable parameters and constraints, as well as on the time required to carry out one such direct calculation of the system being optimized using the available technical means of computer implementation. Therefore, for structures with a large number of heterogeneous substructures and, as a consequence, a significant number of variable parameters and constraints, the problem becomes rather cumbersome for the successful application of well-known mathematical optimization methods [9], [12 – 14].

In the case under consideration, the tail section as a substantially inhomogeneous reinforced shell structure, which requires that in the process of numerical investigation the mesh be condensed in places of stress concentration (which entails an increase in the number of stresses and dimension of the stiffness matrix, the use of finite elements with different properties and other techniques of reliable finite element analysis [6, 15]), the direct calculation algorithm turns out to be quite costly in terms of preparing the initial data and implementation time. In addition, the nature of the change in the sought-for variable parameters is different, since they include both the variable geometric dimensions of the reinforcement sections and wall thicknesses of shell elements, and integral components (number of reinforcement elements, etc.).

One of the serious difficulties in constructing an effective algorithm for optimizing design parameters in this case is also the need to rebuild the finite element mesh for the direct calculation of the subsequent iteration, which is due to the change in the structure topology at each step of the iterative optimization process owing to varying geometrical dimensions. This leads to the need to develop additional fragments of special software to automate the restructuring of the finite element mesh in connection with the change in the input data at each subsequent step or to the use of interactive design methods, which are rather labor-intensive with a large number of iterations.

As a result, the use of traditional mathematical optimization methods [12, 13] to reduce the material intensity of the tail section in the case under consideration is very problematic.

In this article, the solution to the arising problem of weight optimization is proposed to be carried out using the principle of discrete uniform strength [11] in combination with an interactive numerical-analytical algorithm for choosing variable parameters and analyzing the SSS of the structure. When solving the problem of optimizing parameters, the system under consideration, with account taken of the results of calculating the initial (standard) version of the composite structure, is conventionally divided into $J$ substructures: individual reinforcement elements or a set of rods, plates, panels, shells, etc., whose geometric dimensions determine the consumption of construction material.
To divide the structure into $J$ zones, a certain number of $Z$ levels of stresses ($z \leq J$) are set according to the principle

$$
\left(\sigma_{\text{max}}^i, \sigma_{\text{cr}}^i\right) \in \left[\sigma_{\min}^i, (m-1)\left|\sigma_{\max}^i-\sigma_{\min}^i\right|/Z, \sigma_{\min}^i+m\left|\sigma_{\max}^i-\sigma_{\min}^i\right|/Z\right], \quad m = 1, 2, \ldots, Z,
$$

where $\left|\sigma_{\max}^i\right|$ and $\left|\sigma_{\min}^i\right|$ are the maximum and minimum stress values on the entire structure of the tail section.

In this case, $Z = 1$ corresponds to the generally accepted design calculation. For the case when $Z \to \infty$, it is possible to create an equally stressed structure, with all the points of each of its elements having stresses that are equal to the allowable ones [16].

If the volume of material is assumed to be linearly dependent on the geometric control parameters of the section $\delta_s, (s = 1, S, S \geq J)$, the objective function (1) will be linear

$$
V(\bar{\delta}) = \sum_{s=1}^{S} c_s \delta_s, c_s > 0, \delta_s > 0.
$$

Hence, it is obvious that in the presence of constraints in the form (3), (5), (6), the solution to the problem of finding the minimum of function (9) will be on the boundary of one of the regions determined by the conditions of strength, stability or technological requirements, that is, we can assume that the optimal solution will be the envelope of these constraints.

In discretely equal stress structures [11, 16], the material is used most efficiently, since there are no zones with overestimated or underestimated stresses, with account taken of a given safety factor. Therefore, in the limit of the iterative optimization algorithm, it is possible to require the fulfillment of constraints (5), (6) in the form of equal stress conditions for each $j$-th substructure

$$
\sigma_{\text{max}}^j(\bar{\delta}) = [\sigma_j], \sigma_{\text{cr}}^j(\bar{\delta}) = [\sigma_k]^p, \quad j = 1, J, i = 1, I, I \leq J.
$$

An exception may be substructures for which the structural or technological requirements for the dimensions of the sections of elements overlap the stiffness parameters selected from the conditions of stress-strength reliability (10).

The essence of the proposed design algorithm for a discretely equal-strength structure is the rational redistribution of the material of the structure in such a way that if a certain zone is overstressed, then the size of the sections (stiffness parameters) of its elements increases to such a level that the excess stress is removed. If the zone is not loaded, then the variable stiffness parameters of its elements decrease.

After setting a certain initial $k=0$ approximation for the variable stiffness parameters $\delta_s^0, s = 1, 2, \ldots, S$ of elements of the standard structure of the tail section, the stresses $\sigma_{\max}^j$ in its constituent elements are determined from the results of direct calculation. This makes it possible to determine $\delta_j^1$ of the first approximation and then $\Delta \bar{\delta} = \bar{\delta} - \bar{\delta}^0$ using the methods of approximate engineering design calculation [8, 9, 17].

Continuing the recalculation of the structure in this way, one can similarly calculate $\Delta \bar{\delta}^k = \bar{\delta}^k - \bar{\delta}^{k-1}$, where $\bar{\delta}^k$ is the value of the change in the variable stiffness parameters for the next two search steps.

As a rule [11, 16], the iterative process of designing a discretely-equal strength structure makes significant changes to its configuration only in a few initial cycles, and then the difference between two successive approximations becomes rather small.

Therefore, the stress $\sigma_j^k$ at the $k$-th iteration, up to second-order small numbers, can be obtained from their values $\sigma_j^{k-1}$ at the previous iteration, using the linear continuation [16]

$$
\sigma_j^k = \sigma_j^{k-1} + \sum_{i=1}^{N} \frac{\partial \sigma_j^{(k-1), l}}{\partial \delta_j} \Delta \bar{\delta}_j^{k-1},
$$
where $\frac{\partial \sigma_{j}^{k+1}}{\partial \delta_{i}^{k}}$ is calculated as the change in stresses between two $(k-1, k)$ subsequent iterations, and $\sigma_{j}^{k}, \sigma_{j}^{k-1}$ are the maximum stress values for all points of each substructure at two subsequent iterations.

Expression (11) can be written in matrix form

$$\bar{\sigma}^{k} = \bar{\sigma}^{k-1} + A(\bar{\delta}^{k-1})\Delta \bar{\delta}^{k-1},$$

where $\bar{\sigma}^{k}, \bar{\sigma}^{k-1}, \Delta \bar{\delta}^{k-1}$ are the corresponding column vectors, and the matrix $A$ has the form

$$A = \begin{bmatrix} \frac{\partial \sigma_1}{\partial \delta_1} & \frac{\partial \sigma_1}{\partial \delta_2} & \cdots & \frac{\partial \sigma_1}{\partial \delta_j} \\ \frac{\partial \sigma_2}{\partial \delta_1} & \frac{\partial \sigma_2}{\partial \delta_2} & \cdots & \frac{\partial \sigma_2}{\partial \delta_j} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \sigma_j}{\partial \delta_1} & \frac{\partial \sigma_j}{\partial \delta_2} & \cdots & \frac{\partial \sigma_j}{\partial \delta_j} \end{bmatrix}.$$

It should be noted that in statically determinate systems, internal forces do not depend on the stiffness of neighboring elements, since they are determined only from the equilibrium conditions. This implies that for such systems all the matrix $A$ components lying outside the main diagonal will be equal to zero and the iterative process in this case can be represented as

$$\sigma_{j}^{k+1}(\bar{\delta}^{k}) = \sigma_{j}^{k} + \frac{\partial \sigma_j}{\partial \delta_j} \Delta \delta_j.$$

In statically indeterminate systems, the matrix $A$ components lying outside the main diagonal, due to the presence of strain compatibility conditions, express the magnitude of the influence of the change in the $i$-th elements of neighboring substructures on the change in stresses in elements of the $j$-th substructure.

As indicated in [11, 16] and verified in this work, on the basis of a systemic numerical experiment for the problem, in particular, of the optimal distribution of material in the zone of the edge effect of a cylindrical reservoir under the action of hydrostatic pressure, the matrix elements lying outside the main diagonals turn out to be significantly smaller than the values for elements of the main diagonal.

Therefore, based on the physical concepts of stress distribution in composite structures, as well as the results of the above-mentioned numerical experiment, this paper assumes that it is possible, when constructing an iterative numerical-analytical algorithm for designing a discrete equal-strength structure, to neglect, between two sequential finite element calculations, the components lying outside the main diagonal of the matrix $A$.

It should be noted that, on the one hand, the accepted assumption makes it possible to significantly reduce the computational costs of finding an optimal project due to the possibility of using “internal” iterations in form (11), and on the other hand, it does not affect the accuracy of determining the variable parameters as a whole, since the stress state of the structure at separate (reference) steps of the search iterative algorithm is recalculated using a sufficiently reliable finite element analysis.

At the same time, it is assumed that, at known stress values $\sigma_{j}^{k}$, engineering (analytical) methods make it possible to determine the stiffness parameters $\delta_{j}^{k}$ of the corresponding element, and expression (12) can be approximately represented in the following finite-dimensional form:

$$\bar{\sigma}_{j}^{k+1} = \bar{\sigma}_{j}^{k} + \gamma \Delta \bar{\sigma}_{j}^{k} \Delta \bar{\delta}_{j}^{k}.$$

Here, $\Delta \bar{\delta}_{j}^{k} = \bar{\delta}_{j}^{k} - \bar{\delta}_{j}^{k-1}$,

$$\Delta \bar{\sigma}_{j}^{k} = (\bar{\sigma}_{j}^{k} - \bar{\sigma}_{j}^{k-1})/|\sigma|.$$

$\bar{\delta}_{j}^{k}, \bar{\sigma}_{j}^{k-1}$ are the variable stiffness parameters of each $j$-th substructure, which are determined using the methods of engineering stress-strength analysis by the known stresses (between the main finite element calculations at $(k-1)$-th and $k$-th steps of the process being optimized); $\gamma$ is the relaxation factor whose essence is...
DYNAMICS AND STRENGTH OF MACHINES

to smooth out the influence of a possible sharp change in the stiffness of the \( j \)-th element for two successive steps of the search for stress in individual structural elements [13].

Thus, the presence of the multiplier \( 0 < \gamma \leq 1 \) prevents the formation of a yawing iterative process (solution "overshoot"). As a result, it often turns out to be possible to significantly reduce the required number of cycles of the iterative finite element calculation of a structure [12, 13].

The essence of the hybrid numerical-analytical approach used in this paper is that between the "main" iterations, due to the sufficiently costly finite element calculation of the entire tail section, successive approximations of the search for rational parameters of individual substructures are performed using (11), (13) and methods of engineering design calculation [2, 3, 8, 10, 17].

The overall strategy for lightweight construction is as follows. Based on the results of the initial finite element analysis of the standard design and further determination of variation factors \( q^* (\vec{\delta}) \) (7), the structure surface is conventionally divided into the "main" zones, in which the coefficients of uneven loading are positive, and the "lightweight" zones where the coefficients of uneven loading turn out to be negative.

The "main" zone is characterized by a significant value of stresses, which is why they are enhanced by increasing the ribbing and thickness of the shell, based on the dependencies of engineering analysis. In the "lightweight" zone the stresses are small. Therefore, the ribbing and thickness of the shell can be reduced in accordance with (13), since in this case (14) is the corresponding component \( \bar{\Delta \sigma^k} < 0 \) and the structure can thus be lightened.

The parameters of reinforcement for each zone of the surface of the lower and upper shells were determined from the condition for ensuring their stability as they were determined for regular lattice shells with a given coefficient of uneven loading. In this case, the choice of the ribbing and thickness of the shell itself with the purpose of lightening the LV tail section was carried out on the basis of analytical formulas of engineering analysis, with account taken of (12), (13) and the results of a preliminary finite element calculation. The use of such a hybrid approach in the form of a sequence of steps "finite element calculation – engineering analysis" in the process of determining the optimal parameters of the structure significantly simplifies the optimization procedure.

The parameters of the lattice shell structure were determined from conditions (5, 6, 10) to ensure its strength in the form [2, 3, 8]

\[
T_{\text{разр}} = \min \left( \frac{T_{\text{кр},0}}{T_{\text{кр},m}} \right) \geq \tilde{T}_{\text{экв}},
\]

where \( T_{\text{кр},0} \) and \( T_{\text{кр},m} \) are critical loads of its general and local buckling. To calculate them, we used well-proven approximate formulas whose structure is presented in [2, 3, 8], and their application was tested on the basis of an analysis of a large number of results of experimental studies carried out at the Yuzhnoye Design Bureau.

To determine \( T_{\text{кр},0} \), we used the dependence

\[
T_{\text{кр},0} = 2\pi kE\varphi \delta_{\text{экв},k} \sqrt{\frac{E_\sigma}{E_\tau}} \delta_{\text{экв},k} \cos 2\alpha / kn,
\]

where \( k = 0.732 \left[ 1 + \exp \left( -\frac{1}{k_n^2} \right) \right] \left[ 0.1 + 20 \frac{\delta_{\text{экв},k}}{R} \right] \); \( k_n \) is the coefficient of the overall stability of the shell under axial compression (\( k_n \geq 0.2 \)); \( \varphi = \sqrt{E_\sigma E_\tau} \) is the plasticity function; \( E_\tau \) is the tangent modulus of the diagram \( \sigma_\tau (\epsilon_\tau) \) of material; \( E_\sigma \) is the secant modulus of the material loading diagram; \( \delta \) is the shell thickness; \( \delta_{\text{экв},k} = \delta + \frac{F_{\text{кр}}}{B} \) is the effective thickness of the structure along each stringer; \( F_{\text{кр}} \) is the cross-sectional area of a stringer; \( B \) is the distance between stringers; \( \sigma_{\text{экв},k} = \sqrt{\frac{12(1-\mu^2)}{A}} I_{\text{экв}} \) is the equivalent thickness of the frame; \( I_{\text{экв}} \) is the moment of inertia of the cross-section of the frame with the attached skin; \( A \) is the distance between frames; \( \mu \) is Poisson’s ratio; \( \epsilon_\tau \) is the stress intensity; \( \epsilon_u \) is the intensity of strains.

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In practical calculations to determine critical loads, instead of the \( \sigma \varepsilon \) curve, the \( \sigma - \varepsilon \) curve of material tension can be used.

The critical loads of local buckling \( T_{kr,m} \) of a lattice cylindrical shell are calculated by the formula

\[
T_{kr,m} = \frac{2\pi k_n R E \phi \delta_{np,m}}{k_n} \left( \frac{\delta}{B - \delta_{pn} - r} \right)^2,
\]

where \( k_n = 6.0 \) is the coefficient of local buckling of the shell without ribbing during axial compression; \( r \) is the radius of the transition from the rib to the cell sheet; \( \delta_{pn} \) is the stringer thickness.

Analytical formulas (15), (16) were further used to determine the parameters of the load-bearing frame and the shell thickness.

After choosing the optimal lightweight lattice structure, a finite element calculation of the stress-strain state of the entire structure was carried out with verification of all strength conditions and design requirements, which made it possible to more accurately assess the strength of the optimal lightweight structure and serve, if necessary, as the initial data for the next stage of design calculation.

Although the correct proof of the optimality of the resulting design with this approach remains open (see [16]), its use allows a step-by-step reduction in material intensity of a rather complex design of the LV tail section.

Numerical Results of Finite Element Modeling of the Optimal Tail Section

Based on the approach proposed above, the middle surface of the Antares tail section shell, consisting of the upper and lower shells, was divided into "main" zones and "lightweight" zones. In total, in accordance with (8), seven stress levels of such zones were considered, as a result of which \( J=16 \) irregularity zones appeared in the tail section. In the "main" zones, the reinforcement of the shell did not increase, since the equivalent stresses in these zones were less than the yield strength of the material. In the "lightweight" zone, the parameters of the ribbing and thickness of the shell were reduced using relations (12, 13, 15, 16). Already in the first approximation (finite element calculation – engineering analysis), the weight of the lightweight tail section turned out to be 18\% less than that of the original tail-section structure.

The results of the verification calculation of the field of equivalent stresses of the modernized tail section, carried out in the NASTRAN software package, are shown in Fig. 2, which corresponds to the data in [6, 7].

Thus, the application of the proposed approach already at the first step made it possible to significantly reduce the weight of the tail section. The prototype of the lightweight tail compartment, manufactured according to the results of rational design, was subjected to static tests, the results of which [18, 19] indicate the correspondence of the calculated and experimental data.

Incomplete equalization of the stress state (Fig. 2) is explained by the need to fulfill the design and technological requirements (3), in the action zones of which the equivalent stresses are forced to be lower than the allowable ones.

The existing stress unevenness in the resulting project also indicates the possibility of further reducing the material intensity of the tail section by increasing the number \( Z \) (decreasing the size) of the zones under consideration, since within each of them optimization is carried out according to the maximum equivalent stress for the entire zone.

The rational redistribution of the tail section material was further carried out by continuing the iterative process of finding the optimal parameters based on the results of subsequent finite element calculations.
After five steps of the main iterative process, the material savings were about 23%. At the same time, the difference in the calculated parameters of the stress-strain state of the tail section decreased (which indicates the convergence of the process) and turned out to be within the accuracy specified in the calculation.

However, it should be noted that due to the increase in the degree of variability of the stiffness parameters, the implementation of these promising possibilities is limited by the complication of technology and, as a consequence, the increase in the cost of manufacturing the section.

Conclusions

A method has been developed for reducing the material intensity of highly-stressed LV tail sections, with account taken of strength and stability constraints as well as technological requirements, using the principle of ensuring discrete uniform strength of an inhomogeneous (stepwise variable stiffness) lattice cylindrical shell with edged rectangular holes reinforced by a cross-sectional load-bearing frame.

To select the variable geometric parameters and implement the direct calculation of the tail section, an original interactive numerical-analytical algorithm is proposed, which can significantly reduce the number of rather labor-intensive finite element calculations.

The developed approach to optimizing the load-bearing frame of lattice-reinforced LV tail sections has been introduced into the design practice of the Yuzhnoye Design Bureau. The proposed approach was used to modernize the tail section of the Antares LV. A significantly lightweight lattice structure with a variable load-bearing frame and shell thickness was obtained.

The developed approach can also be used to optimize other structural elements of launch vehicles (tanks, ribbed sections) and other structures used in various fields of mechanical engineering.

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Методика зниження матеріаломісткості хвостових відсіків ракет-носіїв

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Розроблено методику зниження матеріаломісткості високоопріоруджених хвостових відсіків ракет-носіїв з урахуванням обмежень міцності, стійкості і технологічних вимог. Як розрахункова схема хвостового відсіку приймається оребрена в поздовжньому і поперечному напрямках вафельна циліндрична оболонка з прямоугольними отворами, нижній торець якої закріплені в місцях розташування опорних кронштейнів, а верхній навантажений рівномірно розподіленими по контуру поздовжніми тискуваннями зусиллями від дії ваги розташованих вище елементів конструкції. Алгоритм оптимізації побудований за принципом забезпечення дискретної рівномірності орієнтування елементів (підкінструкції). Конструктивні геометричні розміри перерізів штатного хвостового відсіку і жорсткості параметри поздовжнього і поперечного силових наборів, повністю співтік оболонкових елементів, розміри вафельних обичайок та ін., вибираються з вимог міцності і стійкості, вимог стійкості відсіку від розрахованих напружень, вимог стійкості від розрахованих напружень у зоні розташування елементів конструкції, з вимогів технологічності структурних елементів конструкції. Для вирішення задачі побудований алгоритм, який використовується в пакеті NASTRAN. В результаті виконання алгоритму отримані результати розрахунку еквівалентних напружень шляхом дослідження процесів збіжності результатів розрахунку на серії стонкі з різними розташуваннями елементів конструкції. Наведена методика дозволяє знизити матеріаломісткість хвостового відсіку ракет-носіїв з альтернативну."
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