Mathematical model of a milking machine manipulator based on stepper motors

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Abstract. The automation of agriculture increases labour productivity and contributes to an increase in the output of agricultural products, an increase in their quality. These processes are closely related to the application of industrial production technology in agriculture, the improvement of planning and management. Also, the improvement of product quality and the growth of labour productivity are associated with the development of automated control systems that contribute to the autonomous operation of entire agricultural complexes. This paper deals with the development of a mathematical model of a milking installation based on stepper motors. The authors of the work recreated the operation of the milking installation, taking into account the operation of the kinematic scheme of the robot and the dynamics of the stepping motors, positioning of its working body near the tops of the teats of the udder of the animal, which are automatically recognized using image analysis from the 3D TOF camera. The uniqueness of this model lies in the fact that various stages of the operation of an autonomous milking installation are concentrated in one software product. This technology makes it possible to study the effect of each stage separately on the static positioning error of the manipulator working body. The model has been tested, an acceptable accuracy of the results of the model operation is achieved. This model can be recommended for use for engineering calculations and the development of control algorithms for the movement of the working body of the manipulator.

1. Introduction

A three-dimensional time-of-flight or 3D ToF camera is a type of non-scanning lidar (a device for detecting, identifying and ranging using light) that uses powerful optical pulses of several nanoseconds duration to capture depth information (usually at short distances) within the range of interest area.

Time-of-Flight cameras are most often used in the following ways: navigation in a confined space, gesture recognition, object scanning, object tracking, volume measurement, observation of the target area, and fast, accurate indication of the distance to the target, augmented reality, estimation of the size and shape of objects. So, the first smartphone to receive a ToF camera was the iPhone X, released in the fall of 2017. In it, it is a key element of the FaceID system that scans the user's face.

In 2015, a 3D image of a statue of Christ the Savior in Brazil was successfully created using a drone and a ToF camera. Before this, there was no exact model of this statue. ToF technology is used not only on drones and phones, but also in various fields such as logistics, surveillance and security, robotics, medicine, games, photography, filmmaking, auto industry, archeology, environmental projects, and farm automation. Within the framework of this article, the issue of using a 3D camera for detecting a biological image is considered [1].
An important part of the automatic milking system is the automatic teat recognition system. This problem cannot be effectively solved with the help of technical vision, since in this case, it is necessary to know for certain the necessary threshold value for preparing the original image in order to detect the desired object.

2. Materials and methods
The object of research in laboratory conditions was a model of the udder, printed on a 3D printer (Figure 1). The geometrical dimensions of the laboratory model are in good agreement with the real dimensions. Cylindrical rubber tips are used as nipples for the animal. On the one hand, rubber tips prevent damage to the working body of the manipulator when searching for an optimal control algorithm or errors in the operation of an automated control system. On the other hand, the rubber tips simulate real objects well on a tactile and physical level.

3. Description of the prototype model of the milking machine manipulator
Figure 1 shows a schematic view of a prototype milking machine manipulator designed for automatic detection of the tops of the udder teats and automatic positioning of the working body near them.

The prototype includes a 3D ToF camera, which is attached to the third link of the manipulator. The origin of the coordinate system of the experimental space passes near the first link of the manipulator. The model of the udder is mounted on a rigid base (stand), which has fixed geometric dimensions and shapes. In the width of the plane 0XZ, the stand does not have any requirements for the geometric size, since the angle of attack of the camera is located in a different plane (Fig. 1a). Therefore, in the plane 0YZ (Fig. 1b) to the width of the stand |AC| the following requirement is imposed: the width of the experimental stand must be greater than the aperture angle of the chamber β, which guarantees that the camera captures the studied static udder model for further analysis of the algorithm. This stand is located on the marked plane 0XY. On the stand, there are four distinct points A, B, C and D. The composition of the marked plane is such that when comparing any of the above points with the marked plane, it is easy to determine their coordinates (x, y). Knowing the coordinates of absolutely any two points of the stand allows determining the exact coordinates of each nipple during physical modelling, thanks to the rigid structure.
4. Description of the stepper motor model

In the mathematical description of models of stepper electric drives, many assumptions generally accepted in the theory of electrical machines are always accepted. These assumptions make it possible, instead of a real machine, to study the idealized one with sufficient accuracy within the framework of the conditions under consideration [2-4].

- The magnetic permeability of the stator and rotor cores is assumed to be equal to infinity. This circumstance makes it possible to truly unambiguously determine the field pattern from the current of each winding. Also, it makes it possible to use the superposition principle to determine the resulting field in the gaps with the simultaneous action of currents in all windings of an electric machine. This implies that the idealized machine is assumed to be saturated, and the relationship between the current of the electrical circuits and the flux linkage is assumed to be linear. Saturation is taken into account here only indirectly, depending on the choice of inductive parameters [5, 6].

- The distribution of fields’ distribution mutual inductions of the stator and rotor windings and self-induction fields of three-phase windings along the circumference of the electric machine should be taken as sinusoidal, with a spatial half-period that is equal to the pole division. It turns out that only the first harmonic in the indicated fields is taken into account and the influence of the jagged fields in the gap is not taken into account. These fields are caused by the gearing of the stator and rotor, and by the presence of higher subharmonics of the field caused by the corresponding harmonics of the magnetomotive force of the windings. The basis for this assumption is the ability of a three-phase winding to "filter out" higher field harmonics in the gaps [7]. In a normally designed machine, it is possible to obtain higher harmonics of the EMF, due to many higher harmonics of the field, of very,
very small amplitude. The magnetic fields of those higher harmonics of three-phase windings that induce the EMF of the fundamental frequency are referred to as the stray fields of these windings. This circumstance also implies a neglect of the participation of higher harmonics in the formation of the electromagnetic moment [8, 9].

The magnetic circuit and windings are assumed to be symmetrical, i.e. the magnetic circuit has the same shape at all-pole divisions, and within the pole division, and it is symmetrical with respect to the longitudinal and transverse axes [10]. This fact means that all phase windings have the same number of turns, active resistances and mutual displacement of magnetic axes, and damper windings distributed along the circumference of the rotor can be represented by concentrated short-circuited circuits in the longitudinal and transverse axes of the rotor. Hence it follows that it is enough to consider the processes in a two-pole machine since the physical processes taking place in it and a multi-pole machine are completely equivalent. Therefore, the idealized machine is assumed to be two-pole [11].

For a correct and unambiguous mathematical description of the processes occurring in stepper drives, it is necessary to specify a coordinate system, as well as positive directions of currents, rotor rotation and MDS vectors [12].

Figure 2 presents a structural diagram of a generalized model of a stepping electric drive, which was developed within the framework of this work, according to the introduced assumptions.

Figure 2. Stepper drive mathematical model designed in Matlab

Figure 3 presents the transient response of a stepper motor. These characteristics show the results of the stepper motor operation when positioning the motor shaft at a distance of one step.
5. Description of the refined mathematical model of the milking machine manipulator

Figure 4 shows a block diagram of the refined mathematical model of the milking machine.

Figure 3. Stepper motor transient response

The coordinates of the positioning point are fed to the input of the refined mathematical model of the milking machine, which is processed in the "Inverse kinematics problem" block. At the output, the angles of turns of each link are formed, which are fed to the input of mathematical models of stepper motors. Further, in the current article, an experiment will be carried out to test the refined model, which consists in positioning the udder near the top of the nipple in order to build the dynamic characteristics of the engines.
6. Results of approbation of the mathematical model

Figure 5 shows a graphical representation of the movement of the working body from the top of the nipple No. 1 to the top of the nipple No. 2.

![Graphical representation of the movement of the working body from the top of the nipple No. 1 to the top of the nipple No. 2.](image)

**Figure 5.** Moving the working body from the top of the nipple No. 1 to the top of the nipple No. 2

Table 1 shows the sequence of movement of the working body.

**Table 1.** Moving the working body to the nipple No. 2

| No. | Name of movement | Distance $S$, cm | Working body speed $v$, cm/s | Movement time $t$, s | Angle $\alpha$, degree | Angle $\beta$, degree |
|-----|------------------|-----------------|-----------------------------|---------------------|------------------------|------------------------|
| 1   | Movement along the 0Z axis | 10,00 | 3,9 | 2,56 | 0 | 0 |
| 2   | Movement in plane | 14,23 | 3,9 | 3,64 | -16,34 | -16,62 |
| 3   | Movement along the 0Z axis | 10,00 | 3,9 | 2,56 | 0 | 0 |

Figure 6 shows the telemetry of the engine operation in step 1, which is responsible for the movement of the working body along the 0Z axis (engine operation time $t = 2.56$ s).
7. Conclusions
During the experiment, it was found that the developed mathematical model reliably fulfils the trajectory of the manipulator working body on the boundary conditions. Therefore, this model can be recommended for engineering calculations and the development of optimal control algorithms for stepper motors.

It was clearly shown that when the stepper motor rotor is turned by one step, there is a certain inertia in the positioning system. This fact is because the rotor turns at a small angle, while the rotation speed is high enough, which leads to 12% overshoot in the positioning system. The developed mathematical
model makes it possible to estimate the time range of the installation when servicing one animal, which does not exceed 40 seconds.

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