General Analysis of Inflation in the Jordan frame Supergravity

Kazunori Nakayama\textsuperscript{(a)} and Fuminobu Takahashi\textsuperscript{(b)}

\textsuperscript{(a)} Theory Center, KEK, 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan
\textsuperscript{(b)} Institute for the Physics and Mathematics of the Universe, University of Tokyo, Chiba 277-8583, Japan

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Abstract

We study various inflation models in the Jordan frame supergravity with a logarithmic Kähler potential. We find that, in a class of inflation models containing an additional singlet in the superpotential, three types of inflation can be realized: the Higgs-type inflation, power-law inflation, and chaotic inflation with/without a running kinetic term. The former two are possible if the holomorphic function dominates over the non-holomorphic one in the frame function, while the chaotic inflation occurs when both are comparable. Interestingly, the fractional-power potential can be realized by the running kinetic term. We also discuss the implication for the Higgs inflation in supergravity.
I. INTRODUCTION

The inflation is strongly motivated by the recent WMAP results [1]. However, it is a non-trivial task to construct a successful inflation model, partly because the properties of the inflaton are poorly known.

Recently, a new class of inflation models was proposed by one of the authors (FT) [2], in which the kinetic term grows as the inflaton field, making the effective potential flat [3, 4]. This model naturally fits with a high-scale inflation model such as chaotic inflation [5], in which the inflaton moves over a Planck scale or even larger within the last 50 or 60 e-foldings [6]. This is because the precise form of the kinetic term may well change after the inflaton travels such a long distance. In some cases, the change could be so rapid, that it significantly affects the inflaton dynamics. We named such model as running kinetic (RK) inflation. In order to realize a chaotic inflation in supergravity, some sort of shift symmetry is necessary. One way to implement the RK inflation model in supergravity is to impose a shift symmetry on a composite field:

$$\phi^n \rightarrow \phi^n + \alpha,$$

where $\alpha \in \mathbb{R}$ is a transformation parameter, $n$ is a positive integer, and we adopt the Planck unit in which $M_P = 2.4 \times 10^{18}$ GeV is set to be unity. If $n = 1$, this symmetry is reduced to that considered in Ref. [7]. Interestingly, the power of the inflaton potential generically changes in the RK inflation models, which makes it possible to realize chaotic inflation with e.g. a linear and fractional-power potential [2]. The phenomenological aspects of the RK inflation was studied in detail in Ref. [8], and the idea led to a new Higgs chaotic inflation in supergravity [9].

Another way to obtain a flat potential is to introduce a non-minimal coupling to gravity [10–14]. This idea has recently attracted much attention since the proposal of the standard model (SM) Higgs inflation [15]. There are studies on the Higgs inflation in supergravity with the same spirit [16–20]. In the Jordan frame supergravity, the non-minimal coupling to gravity is represented by a holomorphic function $J(z)$ and a generic non-holomorphic

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1 The shift symmetry is sufficient but not necessary for having the RK inflation, and a more general form of the Kähler potential leads to the RK inflaiton.
function $g(z, \bar{z})$ in the frame function $\Omega^2(z, \bar{z})$:

$$\frac{1}{\sqrt{-g}} L_{\text{grav}} = \frac{1}{2} \Omega^2(z, \bar{z}) R + \cdots,$$

$$\Omega^2(z, \bar{z}) = 1 - \frac{1}{3} \left( g(z, \bar{z}) + J(z) + \bar{J}(\bar{z}) \right),$$

where $R$ denotes a curvature scalar, $z$ and $\bar{z}$ are complex scalar fields, and $g(z, \bar{z})$ and $J(z)$ are non-holomorphic and holomorphic functions, respectively. If $g(z, \bar{z}) = |z|^2$ and $J(z) = 0$, $z$ has a canonical kinetic term with a conformal coupling to gravity. The frame function is related to the Kähler potential as

$$K(z, \bar{z}) = -3 \log \Omega^2(z, \bar{z}).$$

In Ref. [18], they studied various inflation models and one of them is such that $g(z, \bar{z}) = |z|^2$ and $J(z) = \frac{3\chi}{4} z^2$ with $\chi = \pm 2/3$, which exhibits an (accidental) shift symmetry on $z^2$.

One of the purposes of this letter is to investigate what kind of inflation models are possible in the Jordan frame supergravity with a logarithmic Kähler potential. In particular, we would like to clarify the relation among the RK inflation, inflation with non-minimal coupling to gravity, and the chaotic inflation with an accidental shift symmetry. Also, the analysis on the RK inflation was performed with a polynomial Kähler potential so far, and it is a non-trivial question whether the RK inflation occurs with a logarithmic Kähler potential.

In this letter we study inflation in the Jordan frame supergravity with a logarithmic Kähler potential, which contains general holomorphic and non-holomorphic functions. For superpotential, we introduce an additional singlet $X$ to have a successful chaotic inflation [7]. Focusing on a large-field inflation model, we find that three types of inflation are possible in this framework, namely, the Higgs-type inflation, the power-law inflation [23], and the chaotic inflation with/without a running kinetic term. It is interesting that all the three inflation models can be consistent with the current CMB observations. In the next section we will study the inflation models with a logarithmic Kähler potential, and Sec. 3 is devoted to discussion and conclusions.

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Footnote 2: In Ref. [21], a shift symmetry on $H_u$ and $H_d$ is introduced to fortify the form of the Kähler potential. However, the resultant potential is a quartic power of the inflaton, which is severely constrained by the WMAP observation [1]. In fact, the model of Ref. [21] is similar to the early work on the inflation using the MSSM flat direction [22].
II. ANALYSIS

Let us consider the frame function in the following form,

\[ \Omega^2 = 1 - \frac{1}{3} \left( g(\phi, \bar{\phi}) + |X|^2 + \zeta|X|^4 + J(\phi) + \bar{J}(\bar{\phi}) \right), \]

where \( \phi \) is the inflaton, and \( X \) is a singlet field. The superpotential is given by

\[ W = \lambda X \phi^m, \]

where \( \lambda \) is a coupling constant, and \( m \) is a positive integer. Using the phase degree of freedom of \( X \), we take \( \lambda \) to be real and positive. The presence of \( X \) is essential for constructing chaotic inflation in supergravity, and it makes the form of the scalar potential simple. Since the \( X \) can be stabilized at the origin by the quartic coupling in \( \Omega \), we will set \( \langle X \rangle = 0 \) in the following analysis.

We consider the following three cases:

1. \( g(\phi, \bar{\phi}) \gg |J(\phi)| \),

2. \( g(\phi, \bar{\phi}) \ll |J(\phi)| \),

3. \( g(\phi, \bar{\phi}) \sim |J(\phi)| \),

where the inequalities are estimated during inflation. To simplify the analysis, we focus on the case that \( g \) and \( J \) can be approximated as a power of \( \phi \) and \( \bar{\phi} \) during the relevant epoch of the inflation:

\[ g(\phi, \bar{\phi}) \approx |\phi|^2 + a|\phi|^{2\ell}, \]

\[ J(\phi) \approx b \phi^n, \]

where \( a \) and \( b \) are real and complex parameters, respectively, and \( \ell > 1 \) and \( n \) are positive integers. Here we allow the non-holomorphic function \( g(\phi, \bar{\phi}) \) to take a general form, because the kinetic term could change during inflation especially if the inflaton travels a large distance. Note that the inflation is still canonically normalized about the origin. We will set \( b \) to be real and positive by re-defining the phase of \( \phi \) without loss of generality. In a more
general case, there could be other scheme that is not described by our analysis. As a first step, however, the above three cases cover reasonably large portion of the possible inflation models.

Before going to the analysis on each case, we here show the Kähler metric and the scalar potential for the inflaton:

\[
\mathcal{L} = K_{\phi\bar{\phi}} \partial\phi \partial\bar{\phi} - V(\phi, \phi^\dagger), \tag{9}
\]

with

\[
K_{\phi\bar{\phi}} = \frac{1}{\Omega^4} \left( 1 - \frac{1}{3} a(\ell - 1)^2 |\phi|^{2\ell} + a\ell^2 |\phi|^{2\ell - 2} + \frac{1}{3} b(n - 1)(\phi^n + \phi^\dagger n) 
+ \frac{1}{3} b n^2 |\phi|^{2n - 2} - \frac{1}{3} ab\ell(\ell - n)|\phi|^{2\ell - 2}(\phi^n + \phi^\dagger n) \right), \tag{10}
\]

\[
V(\phi, \phi^\dagger) = \frac{\lambda^2 |\phi|^{2n}}{\Omega^4}, \tag{11}
\]

\[
\Omega^2 = 1 - \frac{1}{3} \left( |\phi|^2 + a|\phi|^{2\ell} + b\phi^n + b\phi^\dagger n \right), \tag{12}
\]

where we set \(\langle X \rangle = 0\). Here and in what follows we adopt the Einstein frame.

\section{A case of \(g(\phi, \bar{\phi}) \gg |J(\phi)|\)}

First let us consider the case of \(g(\phi, \bar{\phi}) \gg |J(\phi)|\). In this limit, we can set \(b = 0\). As we will see below, the inflation does not take place in this case. The Kähler metric and the frame function are given by

\[
K_{\phi\bar{\phi}} = \frac{1}{\Omega^4} \left( 1 - \frac{1}{3} a(\ell - 1)^2 |\phi|^{2\ell} + a\ell^2 |\phi|^{2\ell - 2} \right), \tag{13}
\]

\[
\Omega^2 = 1 - \frac{1}{3} |\phi|^2 + a|\phi|^{2\ell}. \tag{14}
\]

For \(a > 0\), it is clear from the expression of \(\Omega^2\), \(\phi\) cannot take a value much larger than \(O(1)\), since otherwise \(\Omega^2\) becomes negative and unphysical. We can easily see that both the Kähler metric and the potential \(V(\phi, \phi^\dagger)\) diverge where \(\Omega^2\) vanishes. (The numerator of the Kähler metric does not vanish at this point). Let us estimate the effective potential in terms of a canonically normalized field near the point where \(\Omega^2 = 0\), in case of \(a = 0\). The situation is similar (actually even worse) in the case of \(a > 0\). Let us define \(\varphi \equiv |\phi|\). The
canonically normalized field $\hat{\varphi}$ is related to $\varphi$ as
\begin{equation}
\hat{\varphi} = \int \sqrt{2K_{\phi\bar{\phi}}} \, d\varphi \tag{15}
\end{equation}
\begin{equation}
\approx \sqrt{\frac{3}{2}} \log \left( \frac{2\sqrt{3}}{\sqrt{3} - \varphi} \right) \quad \text{as} \quad \varphi \to \sqrt{3}. \tag{16}
\end{equation}
As $\varphi$ approaches $\sqrt{3}$, the canonically normalized field $\hat{\varphi}$ goes to infinity. At a sufficiently large $\hat{\varphi}$, the potential is approximated with
\begin{equation}
V(\hat{\varphi}) \approx \frac{3^m \lambda^2}{16} \exp \left( \sqrt{\frac{8}{3} \hat{\varphi}} \right). \tag{17}
\end{equation}
Thus, the effective potential is an exponentially growing function and the inflation does not occur.

If $a$ is negative and sufficiently large, $\Omega^2$ does not vanish at a large value of $\varphi$. However, in this case, the Kähler metric necessarily vanishes at a finite value of $\varphi \gtrsim O(1)$, and the inflaton will be strongly coupled near the point. The inflation does not occur in this case, either.

The fact that the inflation does not occur in this case strongly motivates us to introduce a holomorphic function $J(\phi)$, which should play an important role for the inflation. Interestingly, two different types of inflation are possible depending on the relative size of the holomorphic and non-holomorphic functions.

**B. A case of $g(\phi, \bar{\phi}) \ll |J(\phi)|$**

Secondly we consider a case that the holomorphic function $J(\phi)$ dominates over the non-holomorphic function $g(\phi, \bar{\phi})$. This is the case that the non-minimal coupling to gravity plays an important role, and the Higgs inflation in the next-to-minimal supersymmetric standard model (NMSSM) falls in this category. For simplicity we set $a = 0$, and the situation is qualitatively similar in the case of $a \neq 0$. The Kähler metric and the frame function are given by
\begin{equation}
K_{\phi\bar{\phi}} = \frac{1}{\Omega^4} \left( 1 + \frac{1}{3} b(n - 1)(\phi^n + \phi^{\dagger n}) + \frac{1}{3} b^2 n^2 |\phi|^{2n-2} \right), \tag{18}
\end{equation}
\begin{equation}
\Omega^2 = 1 - \frac{1}{3} \left( |\phi|^2 + b\phi^n + b\phi^{\dagger n} \right). \tag{19}
\end{equation}
Note that, in contrast to the previous case, there are directions in the field space of $\phi$ such that both $\Omega^2$ and Kähler metric neither vanish nor diverge at finite values of $\phi$. Since the $\Omega^2$ appears in the denominator of the potential $V$ in Eq. (11), the phase of $\phi$ is stabilized so that $\phi^n + \phi^\dagger$ takes the minimal value, for sufficiently large $\phi$. (Remember that we set $b > 0$). To see this let us decompose $\phi = \varphi e^{i\theta}$. Then the frame function is given by

$$\Omega^2 = 1 - \frac{1}{3} \varphi^2 - \frac{2b}{3} \varphi^n \cos n\theta,$$

and therefore the potential is minimized at

$$\theta_{\text{min}} = \frac{\pi}{n} (2k + 1) \quad (21)$$

with $k = 0, \cdots n - 1$. Along the inflation trajectory given by (21), the radial component $\varphi$ can take a super-Planckian value. This is because, when the holomorphic function is large enough, there is an approximate shift symmetry on $J(\phi) = b\phi^n$ in the frame function (19), namely,

$$\phi^n \to \phi^n + i\alpha,$$

which is equivalent to (11). It is remarkable that a shift symmetry on a composite field $\phi^n$ appears in the limit that the holomorphic function becomes large, namely, the non-minimal coupling to gravity gets large. Indeed, the inflationary trajectory (21) coincides with that of the RK inflation considered in Ref. [8]. However, the form of the kinetic term is not same because of the logarithmic Kähler potential. (If the last term in the numerator in Eq. (18) dominated and if $\Omega^2$ were a constant, the kinetic term would grow at large $\phi$, leading to the RK inflation.) As we will see shortly, the RK inflation is realized when the holomorphic function is comparable to the non-holomorphic one.

Let us comment on the lower bound on $b$. For $\Omega^2$ not to vanish along the trajectory (21), $b$ must be larger than $b_c$ given by

$$b > b_c \equiv \begin{cases} \frac{(n - 2)^{\frac{n-2}{2}}}{3^{\frac{n-2}{2}} n^2} & \text{for } n > 2 \\ \frac{1}{2} & \text{for } n = 2 \end{cases} \quad (23)$$

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4 The mass of the phase is of $O(H)$ for $n = O(1)$. However, it becomes light for $n \gtrsim O(10)$, which may result in the isocurvature perturbation or non-Gaussianity.
If this inequality is met, the Kähler metric does not diverge at a finite value of \( \varphi \). Furthermore, in order for the scalar potential not to have a local maximum (and minimum), \( b \) must be larger than \( b'_c \),

\[
b > b'_c \equiv \frac{(m - 2)\bar{\varphi}}{m - 2}b_c,
\]

where we have assumed \( m > n \). For \( m > n \geq 2 \), \( b'_c \) is greater than or equal to \( b_c \). As we will see below, if \( m < n \), the potential has a local maximum and exhibits runaway behavior. If \( m = n \geq 3 \), there is always local maximum for any \( b > b_c \). If \( m = n = 2 \), there is no local maximum for any \( b > b_c \). We assume (23) is satisfied in the following.

Using (21), we obtain the Lagrangian,

\[
L = \frac{1}{n} - \frac{1}{2b} (n - 1)\varphi^n + \frac{b^2}{3} n^2 \varphi^{2n - 2} (\partial \varphi)^2 - \frac{\lambda^2 \varphi^{2m}}{(1 - \frac{1}{3} \varphi^2 + \frac{2b}{3} \varphi^n)^2}
\]

Let us consider the limit \( \varphi \gg b^{-1/(n-2)} \), in which case the above Lagrangian is simplified as

\[
L \approx \frac{3n^2}{4} \varphi^{-2} \partial \varphi^2 - \frac{9\lambda^2}{4b^2} \varphi^{2(m-n)} \left(1 - \frac{1}{2b} \varphi^{2-n} + \frac{3}{2b} \varphi^{-n}\right)^{-2}.
\]

The canonically normalized inflaton \( \hat{\varphi} \) is related to \( \varphi \) as

\[
\hat{\varphi} \approx \sqrt{\frac{3}{2}} n \ln(\varphi),
\]

and the scalar potential in terms of \( \hat{\varphi} \) is given by

\[
V(\hat{\varphi}) \approx \frac{9\lambda^2}{4b^2} \hat{\varphi}^{2n} \left(1 - \frac{1}{2b} e^{\alpha^{2-n} \hat{\varphi}} + \frac{3}{2b} e^{-\alpha \hat{\varphi}}\right)^{-2},
\]

where we defined \( \alpha \equiv \sqrt{2/3} \). As we have mentioned, the potential \( V \) exhibits runaway behavior for \( m < n \) as well as \( m = n \geq 3 \), while the potential is an exponentially growing function for \( m > n \). If \( m = n = 2 \), the scalar potential asymptotically approaches a constant value and the tilt of the potential is exponentially suppressed. The last case corresponds to the Higgs inflation [15–20], and it was extensively studied in the literatures, and so, we do not repeat the analysis here.

Let us consider the case of \( m > n \). In this case the scalar potential grows exponentially, and so, one might think that no inflation occurs in this case. However, the inflation does
occur if the coefficient in the exponent is small enough. This is actually the power-law inflation with a positive exponent. The slow-roll parameters $\varepsilon$ and $\eta$ are given by

$$\varepsilon = \frac{4}{3} \left(\frac{m-n}{n}\right)^2, \quad \eta = \frac{8}{3} \left(\frac{m-n}{n}\right)^2.$$  

(29)

Therefore the inflation occurs if $(m-n)/n \ll 1$. Note that this is not a severe tuning of parameters; $(m-n)/n \sim 0.1$ is sufficient. Such a choice of $m$ and $n$ can be justified for a certain choice of discrete and $U(1)_R$ symmetries. The inflaton field is related to the e-folding number $N$ as

$$\dot{\phi} \simeq \sqrt{\frac{8}{3}} \frac{m-n}{n} N.$$  

(30)

Interestingly, the tensor-to-scalar ratio $r$ and the scalar spectral index do not depend on the e-folding number, and they are determined by $m$ and $n$:

$$n_s \simeq 1 - \frac{8}{3} \left(\frac{m-n}{n}\right)^2, \quad r \simeq \frac{64}{3} \left(\frac{m-n}{n}\right)^2,$$  

(31)

and

$$1 - n_s = \frac{r}{8}.$$  

(32)

We note that the power-law inflation ends when $\varphi \sim b^{-1/(n-2)}$. If $1 > b > b'$, a chaotic inflation with a potential $\propto \varphi^{2m}$ occurs after the power-law inflation. Since $n_s$ and $r$ do not depend on the duration of the power-law inflation, the above prediction is not changed in this case, if the e-folds of the chaotic inflation is smaller than 50.

Thus, the inflation model considered here can be either the power-law inflation ($0 < (m-n)/n \ll 1$) or the Higgs-type inflation ($m = n = 2$).

### C. A case of $g(\phi, \bar{\phi}) \sim |J(\phi)|$

Thirdly we consider a case of $g(\phi, \bar{\phi}) \sim |J(\phi)|$. For the equality to hold for a reasonably large field space, we take (i) $a = 0$ and $b = 1/2$ for $n = 2$, or (ii) $|a| \sim b$ and $2\ell = n$ for $n > 2$. As we shall see, depending on the value of $a$, there appears an approximate flat direction corresponding to a shift symmetry on $\phi^\ell$, which results in the RK inflation.

First consider the case (i) ($n = 2$). If we take $a = 0$ and $b \to 1/2$, we find that there
appears an accidental shift symmetry $\phi \to \phi + i\alpha$, noting that

$$K_{\phi\bar{\phi}} = \frac{1}{\Omega^4} \left( 1 + \frac{1}{6}(\phi + \phi^\dagger)^2 + \frac{1}{3} \left( b - \frac{1}{2} \right) \left( \phi^2 + \phi^\dagger_2 + (4b + 2)|\phi|^2 \right) \right),$$

(33)

$$\Omega^2 = 1 - \frac{1}{6} (\phi + \phi^\dagger)^2 - \frac{1}{3} \left( b - \frac{1}{2} \right) (\phi^2 + \phi^\dagger_2).$$

(34)

By minimizing the potential about $\theta = \theta_{\text{min}}$ we find that both the Kähler metric and frame function become unity: $\Omega^2 = 1$ and $K_{\phi\bar{\phi}} = 1$. Thus the resulting scalar potential is simply given by $V = \lambda^2 |\phi|^{2m}$ and chaotic inflation occurs. This case was noted in Ref. [18], which also considered inflation models with a more generic superpotential. As the value of $b$ increases, the shift symmetry $\phi^2 \to \phi^2 + i\alpha$ appears and the theory approaches to that studied in Sec. II B. In particular, for $m = 2$, the potential becomes flat and the Higgs-type inflation occurs for sufficiently large field value. If $b - 1/2 \lesssim 3/(4mN)$, the last $N$ e-folds is in the chaotic inflation regime. Remember that, for a sufficiently large $b$, the potential exhibits a runaway behavior for $m < 2$ and the potential becomes too steep for the inflation to occur at large field value for $m > 2$.

Next, let us consider the case (ii) $(n > 2)$. In this case, the Kähler metric and frame function are given by

$$K_{\phi\bar{\phi}} = \frac{1}{\Omega^4} \left( 1 - \frac{1}{3}\ell(\ell - 1)|\phi|^{2\ell} + a\ell^2|\phi|^{2\ell-2} + \frac{1}{3}b(2\ell - 1)(\phi^{2\ell} + \phi^{\dagger 2\ell}) \\
+ \frac{4}{3}b\ell^3|\phi|^{4\ell-2} + \frac{1}{3}ab\ell^2|\phi|^{2\ell-2}(\phi^{2\ell} + \phi^{\dagger 2\ell}) \right),$$

(35)

$$\Omega^2 = 1 - \frac{1}{3} \left( |\phi|^2 + a|\phi|^{2\ell} + b(\phi^{2\ell} + \phi^{\dagger 2\ell}) \right).$$

(36)

In the limit $b \gg |a|$, this approaches to the case of Sec. II B and the theory has a shift symmetry $\phi^{2\ell} \to \phi^{2\ell} + i\alpha$. There is another interesting limit $b = \pm 2a$, where this has an accidental shift symmetry, noting that the frame function is rewritten as

$$\Omega^2 = 1 - \frac{1}{3} \left( |\phi|^2 + \frac{2b + a}{4}(\phi^\ell + \phi^{\dagger \ell})^2 + \frac{2b - a}{4}(\phi^\ell - \phi^{\dagger \ell})^2 \right).$$

(37)

Thus there appears an approximate shift symmetry, $\phi^\ell \to \phi^\ell + i\alpha$ for $b = a/2$, and $\phi^\ell \to \phi^\ell + \alpha$ for $b = -a/2$. The latter case leads to a negative kinetic term for a large field value. Therefore, we consider the case $b \simeq a/2$ in the following. Writing $\phi$ as $\varphi e^{i\theta}$, the frame function is given by

$$\Omega^2 = 1 - \frac{1}{3}\varphi^2 - \frac{1}{3}\varphi^n (a + 2b \cos n\theta).$$

(38)
The scalar potential is minimized at $\theta = \theta_{\text{min}}$ given in (21) independently of the sign of $a$ as long as the region connected to the origin without singularities are concerned. Then the frame function and Kähler metric, along the direction of $\theta = \theta_{\text{min}}$, are given by

$$K_{\phi\bar{\phi}} = \frac{1}{\Omega^4} \left( 1 + a\ell^2 \varphi^{2\ell - 2} - \frac{1}{3} \left( a\ell^2 + (2b-a)(2\ell-1) \right) \varphi^{2\ell} + \frac{2}{3} b\ell^2 (2b-a) \varphi^{4\ell - 2} \right),$$  \hspace{1cm} (39)

$$\Omega^2 = 1 - \frac{1}{3} \varphi^2 + \frac{1}{3} (2b-a) \varphi^n.$$  \hspace{1cm} (40)

First, setting $a = 2b$, we find that the frame function is simply reduced to $\Omega^2 = 1 - \frac{1}{3} \varphi^2$. Although the potential diverges at $\varphi = \sqrt{3}$, a sufficient amount of inflation still takes place for $\varphi < \sqrt{3}$ as is shown in the following. The kinetic term in the Lagrangian takes the following form,

$$\mathcal{L}_K = (1 + 2b\ell^2 \varphi^{2\ell - 2}) (\partial \varphi)^2,$$  \hspace{1cm} (41)

in the limit $\varphi \ll \sqrt{3}$. The canonically normalized field at large field value is given by

$$\hat{\varphi} = 2\sqrt{b} \varphi^\ell \quad \text{for} \quad 2b\ell^2 \varphi^{2\ell - 2} > 1.$$  \hspace{1cm} (42)

In the opposite limit $2b\ell^2 \varphi^{2\ell - 2} < 1$, the canonically normalized field is $\tilde{\varphi} = \sqrt{2} \varphi$. Thus the scalar potential changes its form as

$$V = \lambda^2 \left( \frac{1}{2\sqrt{b}} \right)^{2m/\ell} \hat{\varphi}^{2m/\ell} \quad \text{for} \quad 2b\ell^2 \varphi^{2\ell - 2} > 1,$$  \hspace{1cm} (43)

and

$$V = \frac{\lambda^2}{2^m \hat{\varphi}^{2m}} \quad \text{for} \quad 2b\ell^2 \varphi^{2\ell - 2} < 1.$$  \hspace{1cm} (44)

This is nothing but the RK inflation model found in Refs. [2, 8, 9]. One of the features of the RK inflation is that the power of the potential becomes smaller at a large field value. In the present case, the power of the potential during inflation is $2m/\ell$. In particular, a fractional power is possible in the RK inflation. Inflation ends at $\hat{\varphi} \sim 1$ and the field value corresponding to the e-folding number $N$ is $\hat{\varphi}_N = \sqrt{4mN/\ell}$. The corresponding field value of $\varphi$ is given by $\varphi_N \sim (mN/b\ell)^{1/(2\ell)}$. This must satisfy the constraint $\varphi_N < \sqrt{3}$ for the above analysis to be valid. This translates into the bound on $b$ as

$$b > \frac{mN}{3\ell}.$$  \hspace{1cm} (45)

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\[6\] Inflation ends at the large field regime if $b > 2^{\ell - 2}\ell^{-2\ell}$. Otherwise, the last stage of the inflation may be described in the small field regime. In this case we expect a running of the scalar spectral index at the scale corresponding to the transition from the large to small field regime.
The inflaton dynamics and the corresponding thermal history after inflation in the RK inflation model have been studied in detail in [8] and not repeated here. We only show the spectral index and the tensor to scalar ratio,

\[ n_s \simeq 1 - \left( 1 + \frac{m}{\ell} \right) \frac{1}{N}, \quad r \simeq \frac{8m}{\ell N}. \]  

(46)

In the above analysis we assumed \( 2b = a \). Let us see how the dynamics is affected if this equality is violated. One can show that if the following condition is satisfied,

\[ b - \frac{a}{2} > b_c \equiv \frac{(n - 2)^{\frac{2}{n - 2}}}{3^{\frac{n - 2}{2}} n^{\frac{n}{2}}}, \]  

(47)

the scalar potential does not diverge along the direction \( \theta = \theta_{\text{min}} \). There is a local maximum of the potential along \( \theta = \theta_{\text{min}} \), which may be an obstacle to the inflation. The condition that the local maximum disappears is written as

\[ b - \frac{a}{2} > b'_c = \frac{(m - 2)^{\frac{2}{m - 2}}}{m^{\frac{m - 2}{2}} (m - n)} b_c. \]  

(48)

One can show that \( b'_c \geq b_c \) for \( 2 \leq n < m \). The dynamics of the RK inflation is not much affected as long as \( |b - a/2| \lesssim \varphi_N^{-2e} \sim b\ell/(mN) \). Otherwise, if \( b \) is sufficiently large, higher order terms in the frame function and Kähler metric becomes important, and the theory approaches to that studied in Sec. IIIB.

To summarize, the RK inflation is realized when \( |b - a/2| \lesssim b\ell/(mN) \), and the Higgs-type or power-law inflation is realized when \( b - a/2 \gg b'_c \) for certain choices of \( m \) and \( n \). In the case of \( b - a/2 \gg b'_c \) but \( |b - a/2| \lesssim b\ell/(mN) \), the power-law inflation is followed by the RK inflation for the last \( N \) e-foldings.

III. DISCUSSION AND CONCLUSIONS

We have studied the inflation models in Jordan frame supergravity with a logarithmic Kähler potential, and found that the three types of inflation are possible: the Higgs-type, the power-law and the RK inflation, depending on the relative importance of the holomorphic and non-holomorphic functions in the frame function. More precisely speaking, when the holomorphic function is important, the potential exhibits runaway behavior for \( m < n \) and \( m = n \geq 3 \), and the inflation does not occur. The Higgs-type inflation occurs if \( m = n = 2 \), and the power-law inflation takes place if \( 0 < (m - n)/n \ll 1 \). We have pointed out that, in
In this case, there is a shift symmetry on a holomorphic function, which is basically equivalent to (1) considered in the RK inflation. Although the dynamics is not same because of the logarithmic form of the Kähler potential, it is remarkable that the inflationary path is same as that considered in Ref. [8]. On the other hand, if the non-holomorphic function and the holomorphic one are comparable to each other, there appears another shift symmetry. In the case of \( n = 2 \), this leads to a usual chaotic inflation, while the RK inflation is realized for \( n = 2\ell > 2 \). Interestingly, a fractional power potential is possible for the RK inflation due to the running kinetic term. In particular, we have shown that the same dynamics considered in Refs. [2, 8, 9] is realized with the logarithmic Kähler potential.

The relation of the inflation models is schematically shown in Fig. 1. In the left panel corresponding to the case of \( n = 2 \), the Higgs-type inflation is possible for sufficiently large \( b \) (blue triangle), while the chaotic inflation with the potential \( \propto \varphi^{2m} \) occurs for \( b = 1/2 \) and \( a \approx 0 \) (green circle) because there appears a shift symmetry. We note that, if \( b \) is sufficiently large, the Higgs-type inflation is possible for any \( \ell \) and \( a \). In the right panel corresponding to \( n > 2 \), the power-law inflation occurs for large \( b \) (blue triangle), while there appears an approximate shift symmetry along \( b = 2a \), leading to the RK inflation with with potential \( \propto \varphi^{2m/\ell} \) if \( n = 2\ell \). Interestingly, in the overlapping region, the power-law inflation takes place, subsequently followed by the RK inflation at smaller field values, as in Refs. [2, 8]. Note that, if \( n \neq 2\ell \) and \( n > 2 \), only the power-law inflation is possible for sufficiently large \( b \).

Lastly we briefly mention the implication for the Higgs inflation in supergravity. In NMSSM, there is an interaction of the Higgs fields and an additional singlet \( S \),

\[
W = \lambda S H_u H_d,
\]

which is same as (6) with \( m = 2 \), noting that \( H_u H_d \) can be described as \( \phi^2 \) along the D-flat direction. The interaction generates a quartic coupling about the origin. There are two issues in using the Higgs fields as the inflaton, if we extrapolate the quartic potential up to a large field value. First, the chaotic inflation with a quartic potential is excluded by the WMAP observation. Second, the coupling needed for obtaining the correct magnitude of the density perturbation is very small, \( \lambda \sim 10^{-6} \). As to the first issue, we need to somehow make the potential flatter at a large field value in order to realize a Higgs inflation that is consistent with observation. As far as we know, there are two ways; one is to introduce a
non-minimal coupling to gravity, and the other is to consider a running kinetic term [9]. As is well known, the former leads to a potential given by a constant plus an exponentially suppressed tilt, while the latter enables e.g. quadratic or even fractional-power potentials. These two possibilities predict different tensor-to-scalar ratio $r$, and the RK Higgs inflation tends to predict a larger $r$ within the reach of the Planck satellite [24]. Concerning the second issue on the small coupling, one needs either a large non-minimal coupling to gravity or a small $\lambda$ in the former case [19]. On the other hand, $\lambda$ can be as large as $O(0.1)$ without generating a large $\mu$ term in the RK Higgs inflation. This is because the kinetic term after inflation is different from that during inflation. Applying our analysis in this letter to the case of the Higgs inflation, we can see that the two possibilities, the non-minimally coupled Higgs inflation and the RK Higgs inflation, are related to each other. Their difference arises from the relative size of the holomorphic and non-holomorphic functions. Note that the power-law inflation does not take place because $1 < (m - n)/n \ll 1$ cannot be satisfied for $m = 2$. 

FIG. 1: The type of inflation models realized in supergravity with the logarithmic Kähler potential. Here $a$ and $b$ denote the coefficients of the non-holomorphic and holomorphic functions, respectively. See the text for details. Scales of both axes are arbitrary, and so, the area of each region is not necessarily proportional to the likelihood.
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