Neutrino mixing matrices with relatively large $\theta_{13}$ and with texture one-zero

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Abstract

The recent T2K, MINOS and Double Chooz oscillation data hint a relatively large $\theta_{13}$, which can be accommodated by some general modification of the Tribimaximal/Bimaximal/Democratic mixing matrices. Using such matrices we analyze several Majorana mass matrices with texture one-zero and show whether they satisfy normal or inverted mass hierarchy and phenomenologically viable or not.

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It is now well established by the recent neutrino oscillation experiments \cite{1} that neutrinos do have a tiny but finite nonzero mass. Since neutrinos are massive, there will be flavor mixing in the charged current interaction of the leptons and a leptonic mixing matrix will appear analogous to the CKM mixing matrix for the quarks. Thus, the three flavor eigenstates of neutrinos ($\nu_e$, $\nu_\mu$, $\nu_\tau$) are related to the corresponding mass eigenstates ($\nu_1$, $\nu_2$, $\nu_3$) by the unitary transformation

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
V_{e1} & V_{e2} & V_{e3} \\
V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\
V_{\tau 1} & V_{\tau 2} & V_{\tau 3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},
$$

(1)

where $V$ is the $3 \times 3$ unitary matrix known as PMNS matrix \cite{2}, which contains three mixing angles and three CP violating phases (one Dirac type and two Majorana type). In the standard parametrization \cite{3} the mixing matrix is described by three mixing angles $\theta_{12}$, $\theta_{23}$, $\theta_{13}$ and three CP-violating phases $\delta$, $\rho$, $\sigma$ as

$$
V =
\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\
s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23}
\end{pmatrix}
P_\nu,
$$

(2)

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$ and $P_\nu \equiv \{e^{ip}, e^{i\sigma}, 1\}$ is a diagonal matrix with CP violating Majorana phases $\rho$ and $\sigma$. The global analysis of the recent results of various neutrino oscillation experiments \cite{4} suggest the neutrino masses and mixing parameters at 1$\sigma$ (3$\sigma$) level to be

$$
\Delta m_{21}^2 = 7.59 \pm 0.20^{(+0.61)}_{(-0.69)} \times 10^{-5}\text{eV}^2,
$$

$$
\Delta m_{31}^2 =
\begin{cases}
+(2.46 \pm 0.12(\pm0.37)) \times 10^{-3}\text{ eV}^2 & \text{for normal hierarchy (NH)} \\
-(2.36 \pm 0.11(\pm0.37)) \times 10^{-3}\text{ eV}^2 & \text{for inverted hierarchy (IH)}
\end{cases},
$$

$$
\theta_{12} = 34.5 \pm 1.0(\pm3.2)^\circ, \quad \theta_{23} = 42.8^{+4.7}_{-2.9}(^{+10.7}_{-7.3})^\circ, \quad \theta_{13} = 5.1^{+3.0}_{-3.3}(\leq 12.0)^\circ,
$$

(3)

and as per the the latest T2K result \cite{5}, $\theta_{13}$ at 90% confidence level is found to be

$$
5.0^\circ \lesssim \theta_{13} \lesssim 16.0^\circ \quad (\text{NH}),
$$

$$
5.8^\circ \lesssim \theta_{13} \lesssim 17.8^\circ \quad (\text{IH}),
$$

(4)
for a vanishing Dirac CP-violating phase $\delta$. Moreover, the best-fit value of $\theta_{13}$ is found to be $\theta_{13} \simeq 9.7^\circ$ for NH and $\theta_{13} \simeq 11.0^\circ$ for IH. Soon after the T2K report, the MINOS Collaboration [6] has also released new data which indicate $\theta_{13} \neq 0^\circ$ at the 1.5$\sigma$ level. Furthermore, evidence for $\theta_{13} \neq 0^\circ$ at about 3$\sigma$ level has been obtained in a global analysis [7, 8]. The first result from Double Chooz experiment [9] reported at the LowNu conference as

$$\sin^2 2\theta_{13} = 0.085 \pm 0.029 \text{ (stat.)} \pm 0.042 \text{ (syst.)} ,$$  

which also hints towards a non-zero $\theta_{13}$. This issue has already been discussed in the literature by several groups [10–22]. If this mixing angle is confirmed to be not very small by new data from these two experiments and upcoming reactor neutrino experiments, it will provide an important constraint on theoretical model building for neutrino mixing.

In the absence of any convincing flavor theory several approaches have been proposed to study the flavor problems of massive neutrinos e.g., radiative mechanisms, texture zeros, flavor symmetries, seesaw mechanisms, extra dimensions etc. The neutrino mass matrices with texture-zeros are phenomenologically very useful as they allow the possibility of calculating the neutrino mass matrix $M_{\nu}$ from which both the neutrino mass spectrum and the flavor mixing pattern can be more or less predicted. Theoretically, various neutrino mixing patterns have been proposed using discrete flavor symmetries, e.g., $A_4$, $\mu - \tau$ symmetry etc. Among those there are three well established patterns which are of special interest: bi-maximal mixing pattern (BM) [23], tri-bimaximal mixing pattern (TB) [24] and democratic mixing pattern (DC) [25], whose explicit forms are given below

$$V_{TB} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{\sqrt{2}}{\sqrt{2}} \end{pmatrix}, \quad V_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix},$$

$$V_{DC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{pmatrix}. \quad (6)$$

Clearly, all these patterns suggest vanishing $\theta_{13}$ contradicting the recent observations of $\theta_{13}$ being considerably large. Thus, to accommodate large $\theta_{13}$ Refs. [10] and [11] have considered possible perturbations to the Democratic neutrino mixing pattern and to the tri-bimaximal mixing pattern respectively. In Ref. [26] it has been shown that if one assumes some
general modification of the neutrino BM/TBM/DC mixing pattern then it is possible to get appropriate neutrino mixing angles that may fit the T2K data. The possible modification [26] could be of the following forms

1. \( V_{PMNS} = V_\alpha \cdot V_{ij}, \)
2. \( V_{PMNS} = V_{ij} \cdot V_\alpha, \)
3. \( V_{PMNS} = V_\alpha \cdot V_{ij} \cdot V_{kl}, \)
4. \( V_{PMNS} = V_{ij} \cdot V_{kl} \cdot V_\alpha, \)

where \( \alpha = \text{TB, BM or DC} \) and \((ij), (kl) = (12), (13), (23)\) respectively. The perturbation mixing matrices \( V_{ij} \) are given by

\[
V_{12} = \begin{pmatrix}
\cos x & \sin x & 0 \\
-\sin x & \cos x & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad V_{23} = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos y & \sin y e^{i\delta} \\
0 & -\sin y e^{-i\delta} & \cos y
\end{pmatrix}
\]

\[
V_{13} = \begin{pmatrix}
\cos z & 0 & \sin z e^{i\delta} \\
0 & 1 & 0 \\
-\sin z e^{-i\delta} & 0 & \cos z
\end{pmatrix}
\]

However, this modification is not the only way to obtain the large reactor angle \( \theta_{13} \). It has been shown in Ref. [27] that a non-zero \( \theta_{13} \) can arise at the leading order from type-1 see-saw mechanism in the extension of TBM mixing matrix with partially constrained sequential dominance. Such partially constrained sequential dominance can be realized in the GUT models with a non-abelian discrete family symmetry, such as \( A_4 \), spontaneously broken by flavons with particular vacuum alignment. Recently, it has also been discussed in [28] that it could be possible to achieve large \( \theta_{13} \) through the deviation of from the exact TBM mixing, such as in a model with \( S_4 \) flavor symmetry.

In the present work, we study the phenomenological implications of the above mixing matrices with some texture one-zero structure in the neutrino mass matrices. Such type of neutrino mass matrices with one/two texture zeros or one/two vanishing minors with \( \mu - \tau \) symmetry are discussed in Ref. [29] where they have imposed additional constraint of TBM mixing.

It is well known that in the basis where the charged lepton mass matrix is diagonal, the
complex symmetric mass matrix $M_{\nu}$ can be diagonalized by unitary matrix $V$ as

$$M_{\nu} = V M_{\nu}^{\text{diag}} V^T.$$  \hfill (12)

where

$$M_{\nu}^{\text{diag}} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. $$

Thus, from (12) we have matrix elements of $M_{\nu}$ as

$$ (M_{\nu})_{11} = m_1 V_{11}^2 + m_2 V_{12}^2 + m_3 V_{13}^2, $$

$$ (M_{\nu})_{12} = m_1 V_{11} V_{21} + m_2 V_{12} V_{22} + m_3 V_{13} V_{23}, $$

$$ (M_{\nu})_{13} = m_1 V_{11} V_{31} + m_2 V_{12} V_{32} + m_3 V_{13} V_{33}, $$

$$ (M_{\nu})_{21} = m_1 V_{11} V_{21} + m_2 V_{12} V_{22} + m_3 V_{13} V_{23}, $$

$$ (M_{\nu})_{22} = m_1 V_{21}^2 + m_2 V_{22}^2 + m_3 V_{23}^2, $$

$$ (M_{\nu})_{23} = m_1 V_{21} V_{31} + m_2 V_{22} V_{32} + m_3 V_{23} V_{33}, $$

$$ (M_{\nu})_{31} = m_1 V_{11} V_{31} + m_2 V_{12} V_{32} + m_3 V_{13} V_{33}, $$

$$ (M_{\nu})_{32} = m_1 V_{21} V_{31} + m_2 V_{22} V_{32} + m_3 V_{23} V_{33}, $$

$$ (M_{\nu})_{33} = m_1 V_{31}^2 + m_2 V_{32}^2 + m_3 V_{33}^2. $$  \hfill (13)

The above mass matrix no longer respects $\mu - \tau$ symmetry, however it would satisfy the $\mu - \tau$ symmetry property in the limit $V_{2i} = V_{3i}$.

Now we apply the six possible texture one-zero condition to $M_{\nu}$. By doing so we can further filter which modifications to TB, BM, DC mixing patterns will give results comparable to experimental results. Furthermore we choose the CP violating phases $\delta$, $\rho$ and $\sigma$ to be zero for simplicity in our analysis.

We first consider the phenomenologically feasible perturbations to TB mixing pattern i.e., $V_{PMNS} = V_{TB} V_{23} V_{12}$, where $V_{TB}$ and $V_{23(12)}$ are given in (6) and (11). Comparing the resulting mixing matrix with its standard parametrization form (2), we obtain the mixing
angles as

\[
\tan \theta_{12} = \frac{\cos x \cos y + \sqrt{2} \sin x}{\sqrt{2} \cos x - \cos y \sin x},
\]

\[
\sin \theta_{13} = \frac{1}{\sqrt{3}} \sin y,
\]

\[
\tan \theta_{23} = \frac{\sqrt{3} \cos y + \sqrt{2} \sin y}{-\sqrt{3} \cos y + \sqrt{2} \sin y},
\]

(14)

By substituting the values of \( \theta_{23} \) and \( \theta_{12} \) we obtained the values of \( x \) and \( y \) as

\[
x = (33.2^{+1.4}_{-3.2})^\circ, \quad y = -(88.2^{+1.7}_{-3.1})^\circ.
\]

(15)

Using the above mixing matrix, the elements of neutrino mass matrix \( M_\nu \) can be obtained from (12). Now we consider the following texture one-zero pattern on the above \( M_\nu \) matrix elements, and see whether they are phenomenologically viable or not.

**Case 1:** \( (M_\nu)_{11} = 0 \)

By considering the (1,1) element of the neutrino mass matrix to be zero we obtain the condition that

\[
m_2 (\sqrt{3} \cos x \cos y + \sqrt{6} \sin x)^2 + m_1 (\sqrt{6} \cos x - \sqrt{3} \cos y \sin x)^2 + 3m_3 \sin^2 y = 0.
\]

(16)

From this equation it is not possible to infer any conclusion regarding the ratio of the neutrino masses as it involves mass terms of all three neutrinos. However, one can obtain the results by assuming the hierarchical nature of neutrino masses. For example for normal hierarchy where the neutrino masses follow the pattern \( m_1 < m_2 << m_3 \). Since the absolute scale of the neutrino mass is not precisely known, we assume the limit \( m_1 \to 0 \) for normal hierarchy. Similarly for inverted hierarchy which has the mass ordering as \( m_3 << m_1 < m_2 \), we assume the limit \( m_3 \to 0 \).

With these assumptions, thus, we find for Normal Hierarchy i.e., \( m_1 \to 0 \)

\[
\frac{m_2}{m_3} = \left| \frac{\sin^2 y}{\cos^2 x \cos^2 y + 2 \sin^2 x + \sqrt{2} \cos y \sin 2x} \right|,
\]

(17)

and for Inverted Hierarchy i.e., in the limit \( m_3 \to 0 \),

\[
\frac{m_1}{m_2} = \left| \frac{\cos^2 x \cos^2 y + 2 \sin^2 x + \sqrt{2} \cos y \sin 2x}{2 \cos^2 x - \sqrt{2} \sin 2x \cos y + \sin^2 x \cos^2 y} \right|.
\]

(18)
By substituting the values of $x$ and $y$, as given in Eq. (15) we obtain the ratio of the masses as and the variation of $m_2/m_3$ and $m_1/m_2$ with the perturbation parameter $x$ are shown in Figure-1. From the figure, one can conclude that

\[
\frac{m_2}{m_3} > 1 \quad \text{(for NH)},
\]
\[
\frac{m_1}{m_2} < 1 \quad \text{(for IH)}. \tag{19}
\]

Thus, the above analysis shows that the neutrino mass matrix having texture one-zero with vanishing $(M_\nu)_{11}$ will follow Inverted Mass Hierarchy pattern. Now since we know that texture-zero type with $(M_\nu)_{11}$ supports Inverted mass hierarchy pattern, it is possible to determine the absolute mass scale of the lightest neutrino i.e., $m_3$ from Eq. (16). Now writing Eq. (16) in the symbolic form

\[
am_1 + cm_3 = -bm_2, \tag{20}
\]

where $a$, $b$ and $c$ are the coefficients of $m_1$, $m_2$ and $m_3$ in (16) (e.g., $a = (\sqrt{6}\cos x - \sqrt{3}\cos y\sin x)^2$ and so on). As these masses follow IH pattern we can express $m_1$ and $m_2$ in terms of $m_3$ and the mass square differences $\Delta m_{21}^2$ and $\Delta m_{31}^2$ as

\[
m_1^2 = m_3^2 + \Delta m_{13}^2
\]
\[
m_2^2 = m_3^2 + \Delta m_{21}^2 + \Delta m_{13}^2. \tag{21}
\]

FIG. 1: Variation of the neutrino mass ratios $m_2/m_3$ with the perturbation parameter $x$ (left panel) and $m_1/m_2$ on the right panel.
Now squaring both sides of Eq. (20) and substituting the values of $m_1$, $m_1^2$ and $m_2^2$ we obtain

$$P m_3^2 + Q m_3 + R = 0, \quad (22)$$

with

$$P = a^2 + c^2 - b^2, \quad Q = 2ac\sqrt{\Delta m_{13}^2}, \quad R = a^2 \Delta m_{13}^2 - b^2(\Delta m_{13}^2 + \Delta m_{21}^2). \quad (23)$$

This is a quadratic equation in $m_3^2$ which can be solved numerically. Now using the allowed values of the mass square differences, in Figure-2, we show the variation of $m_3$ with the perturbation parameters $x$ and $y$. From the figure it can be seen that the mass scale which is $\sim O(10^{-2})$, decreases with the increase of $x$ and $y$.

![Graphs showing variation of $m_3$ with $x$ and $y$.](image)

**FIG. 2:** Variation of the lightest neutrino neutrino mass ($m_3$ for IH) with the perturbation parameter $x$ (left panel) and $y$ (right panel).

**Case 2:** ($M_{\nu}$)$_{23} = 0$

Now equating the (2,3) element of the neutrino mass matrix (12) to zero, we obtain the condition

$$m_2 \sin^2 x + \cos^2 y(-3m_3 + 2m_1 \sin^2 x) + \sqrt{2}(m_1 - m_2) \cos y \sin 2x$$

$$+ 2m_3 \sin^2 y - 3m_1 \sin^2 x \sin^2 y + \cos^2 x(m_1 + 2m_2 \cos^2 y - 3m_2 \sin^2 y) = 0. \quad (24)$$

Since the above equation also involves the mass terms of all the three neutrinos, it is not possible to infer any definite result regarding the ratio of neutrino masses. However proceeding as in the previous case, one can obtain in the $m_1 \to 0$ limiting case i.e., for normal
hierarchy
\[
m_2 = \left| \frac{2 \cos 2y + \cos^2 y}{\sin^2 x - \sqrt{2} \cos y \sin x + \cos^2 x(2 \cos 2y - \sin^2 y)} \right|.
\]  
(25)

Similarly taking the limit \( m_3 \to 0 \) for Inverted Hierarchy case we obtain the following mass ratio
\[
\frac{m_1}{m_2} = \left| \frac{\sin^2 x - \sqrt{2} \cos y \sin x + \cos^2 x(2 \cos 2y - \sin^2 y)}{\sin^2 x(\sin^2 y - 2 \cos 2y) - \sqrt{2} \cos y \sin x - \cos^2 x} \right|.
\]  
(26)

Now substituting the allowed ranges of \( x \) and \( y \) we obtain
\[
\frac{m_2}{m_3} > 1 \quad \text{(for NH)}
\]
\[
\frac{m_1}{m_2} < 1 \quad \text{(for IH)}.
\]  
(27)

Thus, the above results implied that there is no feasible solution or in other words the neutrino masses do not satisfy either normal or inverted hierarchy pattern.

**Case 3:** \((M_\nu)_{22} = 0\)

Taking the \((M_\nu)_{22}\) element to be zero we obtain
\[
m_1(\cos x + \sqrt{2} \sin x \cos y - \sqrt{3} \sin x \sin y)^2 + m_3 \left( \frac{\cos y}{\sqrt{2}} + \frac{\sin y}{\sqrt{3}} \right)^2
\]
\[
+ m_2(\sin x + \sqrt{3} \cos x \sin y - \sqrt{2} \cos x \cos y)^2 = 0.
\]  
(28)

Proceeding as in the previous cases we obtain
\[
\frac{m_2}{m_3} > 1 \quad \text{(for NH)}
\]
\[
\frac{m_1}{m_2} < 1 \quad \text{(for IH)}.
\]  
(29)

Thus, the above analysis implies that the neutrino mass structure will follow Inverted mass hierarchy.

**Case 4:** \((M_\nu)_{33} = 0\)

Similarly equating the \((M_\nu)_{33}\) element to zero we obtain
\[
m_1(\cos x + \sqrt{2} \sin x \cos y + \sqrt{3} \sin x \sin y)^2 + m_3(\sqrt{3} \cos y - \sqrt{2} \sin y)^2
\]
\[
+ m_2(\sin x - \sqrt{2} \cos x \cos y - \sqrt{3} \cos x \sin y)^2 = 0.
\]  
(30)

Now, substituting the allowed values of \( x \) and \( y \) we obtain
\[
\frac{m_2}{m_3} < 1
\]  
(31)
for normal hierarchy and
\[ \frac{m_1}{m_2} > 1, \]  
(32)
for Inverted Hierarchy \((m_3 \to 0)\). Thus, implying that the neutrino mass matrix having texture one-zero with vanishing \((M_\nu)_{33}\) obeys Normal Hierarchy.

Now since we know that texture-zero type with \((M_\nu)_{33}\) supports normal mass hierarchy pattern, it is possible to determine the absolute mass scale of the lightest neutrino i.e., \(m_1\) from Eq. (30). Proceeding as in the previous case and substituting \(m_2^2\) and \(m_3^2\) as
\[ m_2^2 = m_1^2 + \Delta m_{21}^2, \]
\[ m_3^2 = m_1^2 + \Delta m_{31}^2, \]  
(33)
we obtain the quadratic equation on \(m_1\)
\[ P_1 m_1^2 + Q_1 m_1 + R_1 = 0, \]  
(34)
with
\[ P_1 = a^2 + c^2 - b^2, \quad Q_1 = 2ac\sqrt{\Delta m_{31}^2}, \quad R_1 = c^2\Delta m_{31}^2 - b^2\Delta m_{21}^2. \]  
(35)
This is a quadratic equation in \(m_1^2\) which can be solved numerically. Now using the allowed values of the mass square differences, in Figure-3, we show the variation of \(m_1\) with the perturbation parameters \(x\) and \(y\). From the figure it can be seen that the mass scale which is \(\sim \mathcal{O}(10^{-2})\), increases with the increase of \(y\) and it is insensitive to the variation of \(x\).

**Case 5: \((M_\nu)_{12} = 0\)**

Similarly equating the \((M_\nu)_{12}\) element to zero we obtain
\[ m_1 (\cos y + \sin y - \sqrt{2} \cos x -)(\cos x + \sin x(\sqrt{2} \cos y - \sqrt{3} \sin y)) \]
\[ + m_2 (\cos x \cos y + \sqrt{2} \sin x)(- \sin x + \sqrt{3} \cos x \cos y - \sqrt{2} \cos x \sin y) \]
\[ + m_3 \sin y(\sqrt{3} \cos y + \sqrt{2} \sin y) = 0. \]  
(36)
Now, substituting the allowed values of \(x\) and \(y\) we obtain
\[ \frac{m_2}{m_3} > 1 \]  
(37)
for normal hierarchy and
\[ \frac{m_1}{m_2} < 1, \]  
(38)
FIG. 3: Variation of the lightest neutrino mass scale ($m_1$) with the perturbation parameter $x$ (left panel) and $y$ (right panel).

for Inverted Hierarchy ($m_3 \to 0$). Thus, implying that the neutrino mass matrix having texture one-zero with vanishing $(M_\nu)_{12}$ obeys Inverted Hierarchy.

**Case 6:** $(M_\nu)_{13} = 0$

Similarly equating the $(M_\nu)_{13}$ element to zero we obtain

$$m_1(\sqrt{2} \cos x - \sin x \cos y)(- \cos x - \sin x(\sqrt{2} \cos y + \sqrt{3} \sin y))$$

$$+ m_2(\cos x \cos y + \sqrt{2} \sin x)(- \sin x + \cos x(\sqrt{2} \cos y + \sqrt{3} \sin y))$$

$$+ m_3 \sin y(-\sqrt{3} \cos y + \sqrt{2} \sin y) = 0 . \quad (39)$$

Now, substituting the allowed values of $x$ and $y$ we obtain

$$\frac{m_2}{m_3} < 1 \quad (40)$$

for normal hierarchy and

$$\frac{m_1}{m_2} > 1, \quad (41)$$

for Inverted Hierarchy ($m_3 \to 0$). Thus, implying that the neutrino mass matrix having texture one-zero with vanishing $(M_\nu)_{13}$ obeys Normal Hierarchy.

Similarly, the above analysis is repeated for all the perturbations and the results are presented in Table-1. The patterns for which there does not exist any feasible solution for any texture one-zero cases are not listed in the Table.

Next we will consider the perturbation of the form $V_\alpha \cdot V_{ij}$. It has already been shown in Ref. [26] that perturbation of this form with $\alpha=$BM and DC are excluded because of the
 following reasons.

i. $V_{BM} \cdot V_{12}$ gives $\theta_{13} = 0$ and hence excluded by the recent MINOS and T2K results.

ii. $V_{BM} \cdot V_{23}$ gives $\theta_{12} > 45^\circ$ and hence excluded by solar neutrino data

iii. $V_{BM} \cdot V_{13}$ gives $\theta_{13} \in [26.7^\circ, 33.7^\circ]$ and hence excluded by the recent MINOS and T2K results.

iv. $V_{DC} \cdot V_{12}$ gives $\theta_{13} = 0$ and hence excluded by the recent MINOS and T2K results.

v. $V_{DC} \cdot V_{23}$ gives $\theta_{13} \in [26^\circ, 35^\circ]$ and hence excluded by the recent MINOS and T2K results.

vi. $V_{DC} \cdot V_{13}$ gives $\theta_{13} > 45^\circ$ and hence excluded.

For the perturbation of the type $V_{TB} \cdot V_{ij}$, we found that

i. For $ij = 12$, $\sin \theta_{13}$ turns out to be zero and hence not allowed by the current data.

ii. For $ij = 23$, we obtain the mixing angles by comparing both sides of the relation $V_{PMNS} = V_{TB} \cdot V_{ij}$ as

$$\tan \theta_{12} = \frac{\cos y}{\sqrt{2}}, \quad (42)$$

which gives the value of the perturbation angle $y = (13.6^{+7.0}_{-6.0})^\circ$. We find that for such a perturbation the only allowed possibility is $(M_\nu)_{33} = 0$ texture-zero, which supports normal hierarchy.

iii. For $ij = 13$, we obtain $z = (13.7^{+7.3}_{-6.2})^\circ$, for which there is no feasible solution that exists for any type of texture one-zero.

Similar analysis is performed for the perturbation of the form $V_{ij} \cdot V_\alpha$ and it is found that $\alpha = \text{BM}$ and $\text{DC}$ cases are excluded as in the previous case due to the following reasons.

i. $V_{12} \cdot V_{BM}$ gives $x = - (14.69^{+1.4}_{-1.3})^\circ$ and no feasible solution is found for such perturbation.

ii. $V_{23} \cdot V_{BM}$ gives $\theta_{13} = 0$ and hence excluded by the recent MINOS and T2K results.

iii. $V_{13} \cdot V_{BM}$ gives $z = (14.69^{+1.38}_{-1.33})^\circ$ and no feasible solution is found for such perturbation.

iv. $V_{12} \cdot V_{DC}$ gives $\theta_{13} \in [13.5^\circ, 45^\circ]$ and hence excluded by the recent MINOS and T2K results.

v. $V_{23} \cdot V_{DC}$ gives $\theta_{13} = 0$ and hence excluded by the recent MINOS and T2K results.

vi. $V_{13} \cdot V_{DC}$ gives $\theta_{23} > 54.7^\circ$ and hence excluded by the atmospheric neutrino data.

Furthermore, for $\alpha = \text{TB}$ among all possibilities, we observe that only $V_{23} \cdot V_{TB}$ has a feasible solution for $(M_\nu)_{11} = 0$ and it supports Inverted hierarchy.
TABLE I: The allowed mass matrix texture for different type of perturbations to BM/TBM/DC mixing matrices.

| Perturbation          | Angles (in degree) | Case-1 $(M_\nu)_{11} = 0$ | Case-2 $(M_\nu)_{23} = 0$ | Case-3 $(M_\nu)_{22} = 0$ | Case-4 $(M_\nu)_{33} = 0$ | Case-5 $(M_\nu)_{12} = 0$ | Case-6 $(M_\nu)_{13} = 0$ |
|-----------------------|--------------------|-----------------------------|----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 1 $V_{TB}.V_{23}.V_{12}$ | $x = 33.2_{-3.2}^{+1.4}$, $y = - (88.2_{-3.1}^{+1.7})$ | IH                          | No soln                    | IH                          | NH                          | IH                          | NH                          |
| 2 $V_{TB}.V_{13}.V_{12}$ | $x = - (53.7_{-2.1}^{+0.7})$, $z = 88.7_{-2.2}^{+0.7}$ | IH                          | NH                         | NH                          | IH                          | No Soln                     | NH                          |
| 3 $V_{TB}.V_{23}.V_{13}$ | $y = 18.3_{-5.7}^{+4.4}$, $z = 25.1_{-4.2}^{+2.0}$ | No soln                     | No soln                    | IH                          | NH                          | IH                          | NH                          |
| 4 $V_{23}.V_{13}.V_{TB}$ | $z = -1.1 \pm 1.4$, $y = 87.8_{-3.8}^{+2.1}$ | No soln                     | No soln                    | No soln                     | No soln                     | NH                          | NH                          |
| 5 $V_{23}.V_{12}.V_{TB}$ | $x = -1.1 \pm 1.4$, $y = 87.8_{-3.8}^{+2.1}$ | No soln                     | No soln                    | No soln                     | No soln                     | NH                          | NH                          |
| 6 $V_{BM}.V_{13}.V_{12}$ | $z = 3.1_{-5.4}^{+5.3}$, $x = 45.2_{-0.1}^{+1.3}$ | NH                          | IH                         | IH                          | IH                          | NH                          | No Soln                     |
| 7 $V_{BM}.V_{23}.V_{12}$ | $y = -3.1_{-5.3}^{+5.4}$, $x = 44.8_{-1.3}^{+0.1}$ | NH                          | IH                         | IH                          | IH                          | IH                          | IH                          |
| 8 $V_{23}.V_{12}.V_{BM}$ | $x = -10.5 \pm 1.0$, $y = -1.8_{-3.6}^{+3.8}$ | No soln                     | No soln                    | No soln                     | IH                          | NH                          | No Soln                     |
| 9 $V_{DC}.V_{13}.V_{12}$ | $x = 51.6_{-3.2}^{+3.5}$, $z = 16.6_{-5.2}^{+5.1}$ | NH                          | IH                         | IH                          | IH                          | NH                          | IH                          |
| 10 $V_{DC}.V_{23}.V_{13}$ | $y = 37.9_{-1.9}^{+1.8}$, $z = 12.7_{-1.7}^{+1.1}$ | IH                          | IH                         | IH                          | NH                          | IH                          | NH                          |
| 11 $V_{23}.V_{13}.V_{DC}$ | $y = -12.6 \pm 3.9$, $z = -12.8 \pm 1.2$ | No soln                     | No soln                    | No soln                     | No soln                     | No Soln                     | NH                          |
| 12 $V_{23}.V_{12}.V_{DC}$ | $x = - (17.8_{-1.6}^{+1.2})$, $y = - (10.6_{-3.5}^{+3.6})$ | NH                          | IH                         | IH                          | No soln                     | No Soln                     | NH                          |
| 13 $V_{13}.V_{12}.V_{DC}$ | $x = 34.96_{-6.17}^{+5.65}$, $z = -6.4 \pm 0.1$ | IH                          | NH                         | NH                          | No soln                     | No Soln                     | IH                          |
| Perturbation Scenarios | Angles (in degree) | Case-1 | Case-2 | Case-3 | Case-4 | Case-5 | Case-6 |
|-----------------------|-------------------|--------|--------|--------|--------|--------|--------|
| 14 $V_{TB}V_{23}$    | $y = 13.6^{+7.0}_{-6.0}$ | No Soln | No Soln | No Soln | NH     | No Soln | NH     |
| 15 $V_{23}.V_{TB}$   | $y = 13.6^{+7.0}_{-6.0}$ | IH     | No Soln | No Soln | No Soln | No Soln | No Soln |
| 16 $V_{TB}.V_{13}$   | $z = 13.7^{+7.3}_{-6.2}$ | No Soln | No Soln | No Soln | No Soln | No Soln | NH     |
| 17 $V_{12}.V_{BM}$   | $x = -(14.7^{+1.4}_{-1.3})$ | No Soln | No Soln | No Soln | No Soln | NH     | No Soln |
| 18 $V_{13}.V_{BM}$   | $z = 14.7^{+1.3}_{-1.4}$ | No Soln | No Soln | No Soln | No Soln | No Soln | NH     |

To summarize, motivated by the recent data from T2K and MINOS which show the evidence of a relatively large $\theta_{13}$ at $3\sigma$ level, we study the phenomenological implications of BM/TBM/DC mixing matrices with some modifications. It has been shown in Ref. [26] that such modified mixing matrices could accommodate the observed large $\theta_{13}$. Using such mixing matrices and assuming texture one-zero in the neutrino mass matrices we have shown that whether such mixing matrices satisfy normal or inverted mass hierarchy, or phenomenologically viable or not.

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