Quasinormal modes of bumblebee wormhole

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Abstract
In this work, we calculate the quasinormal frequencies from a bumblebee traversable wormhole. The bumblebee wormhole model is based on the bumblebee gravity, which exhibits a spontaneous Lorentz symmetry breaking. Supporting by the Lorentz violation parameter \( \lambda \), this model allows for the fulfillment of the flare-out and energy conditions, granted non-exotic matter to the wormhole. We analyze the parameters of the bumblebee wormhole in order to obtain a Regge–Wheeler’s equation with a bell-shaped potential. We obtain the quasinormal modes (QNMs) via the WKB approximation method for both scalar and gravitational perturbations. The time domain has decreasing oscillation (damping) profiles for the bumblebee wormhole.

Keywords: gravitational waves, wormholes, quasi-normal modes, Regge–Wheeler’s potential

1. Introduction

Wormholes are solutions of the Einstein equation denoted by tunnels that connect two different regions of spacetime [1, 2]. The first conception of such structures comes from 1935 by the Einstein–Rosen bridge [1], being the term wormhole later adopted by Misner and Wheeler in 1957 [2]. Unlike black holes, wormholes have no horizon, and it can be traversable depending on some conditions at the wormhole throat [3–6].

In a very recent work, Bueno et al [7] present wormholes as a model for a class of exotic compact objects (ECOs). As a matter of fact, in the context of gravitational waves and its detections [8], wormholes would be distinguished from black holes due to its features of ECOs, which can exhibit two bumps, instead of the usual single bump characteristic of black holes [7]. On the other hand, traversable wormholes violate the null convergence condition (NCC) close at the throat, which leads to a violation of the null energy condition (NEC).
Hence, in the context of the usual formulation of Einstein–Hilbert general relativity, traversable wormholes need an exotic matter source. However, recently some approaches were proposed in order to avoid such exotic matter by the modification of the gravity, namely, with the wormhole in a Born–Infeld gravity, in the \( f(R) \) theories, in the framework of Gauss–Bonnet and considering the wormhole in the bumblebee gravity scenario. The so-called bumblebee model was developed from the string theory, which a spontaneous Lorentz symmetry breaking (LSB) was verified. Such violations imply in the conception of the so-called standard model extension (SME). Recently, a Schwarzschild-like solution on a bumblebee gravity was proposed in, where the Lorentz violation parameter is upper-bounded by the Shapiro’s time delay of light. Moreover, a traversable wormhole solution in the framework of the bumblebee gravity was proposed by Övgün et al. In this bumblebee wormhole, the Lorentz violation parameter can support a normal matter source. At the limit of vanishing LSB, the deflection of light in this model becomes the same of the Ellis wormhole.

In this letter, we calculate the quasinormal modes (QNMs) and time domain profiles for scalar and gravitational perturbations in the Övgün bumblebee wormhole. We obtain a range over the Lorentz violation parameter that satisfies to all flare-out and energy conditions. Furthermore, we also choose suitable LSB parameter that performs quasinormal modes with a decreasing time-domain profile. It is important to highlight that the computation of QNMs for wormholes was featured. On the other hand, the quasinormal modes can distinguish black holes from wormholes, discussed by Konoplya and Zhidenko. It is worthwhile mentioning that Myrzakulov et al. shows an updated discussion about wormholes and the galaxies rotation curves. Others important features in this theme are present into Einstein-power-Maxwell black hole, charged BTZ-like wormholes and cloud of strings.

The paper is organized as follows. In section 2, the Övgün bumblebee wormhole is reviewed. In section 3, the energy conditions and the flare-out conditions are analyzed by the LSB parameter. Where we obtain a non-exotic wormhole. In section 3.1, we introduce the change of variable to obtain the Regge–Wheeler equation. In section 4, we describe the Regge–Wheeler equation for the scalar and tensorial perturbations in the bumblebee wormhole, where we made a suitable choice of parameters to obtain an exact bell-shaped Regge–Wheeler potential. In section 4.1, the quasinormal frequencies were computed by the third-order WKB method and the damping time-domain profile was evaluated for both scalar and tensorial perturbations. In section 5, we present our last discussions and summarize our results.

### 2. Bumblebee wormhole

In this section, we review the exact solution of bumblebee wormhole presented in. Let us start with the following bumblebee action

\[
S_B = \int dx^4 \sqrt{-\tilde{g}} \left[ \frac{\kappa}{2} \frac{R}{2\kappa} + \frac{\kappa}{2} \xi B^\mu B^\nu R_{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - V(B_{\mu} B^\mu \pm b^2) \right] + \int dx^4 \mathcal{L}_m,
\]

where \( B_{\mu} \) represents the bumblebee vector field, the \( B_{\mu\nu} = \partial_{[\mu} B_{\nu]} \) is the bumblebee field strength, and the \( \xi \) is the non-minimal curvature coupling constant. For the vacuum solutions \( V(B_{\mu} B^\mu \pm b^2) = 0 \), we have that \( b^2 = \pm B^\mu B_\mu = \pm b^\mu b_\mu \) is the non-null vector.
norm associated to the vacuum expectation value \( \langle B^\mu \rangle = b^\mu \) [15, 18]. The scalar curvature is denoted by \( R \), \( g \) is the metric determinant and \( \kappa \) the gravitational constant.

The energy–momentum tensor is modified by the bumblebee field in the following form [15, 19]:

\[
R_{\mu\nu} - \kappa G \left[ T^M_{\mu\nu} + T^B_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left( T^M + T^B \right) \right] = 0,
\]

(2)

where \( T^M = g^{\mu\nu} T^M_{\mu\nu} \) and the bumblebee energy–momentum tensor \( T^B_{\mu\nu} \) reads

\[
T^B_{\mu\nu} = -B_{\mu\alpha} B^\alpha_{\nu} - \frac{1}{4} B_{\alpha\beta} B^\alpha B^\beta g_{\mu\nu} - 2\nabla^\alpha B_{\mu\nu} B^\alpha + \frac{1}{2} g_{\mu\nu} (B_\alpha B^\alpha)
\]

\[
+ \frac{\lambda}{2} \nabla_\alpha \nabla_\beta (B^\alpha B^\beta).
\]

(3)

The modified Einstein equation in equation (2) with the energy–momentum tensor in equation (3) can be explicit as

\[
E^{\text{einstein}}_{\mu\nu} = R_{\mu\nu} - \kappa \left( T^M_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^M \right) - \kappa T^B_{\mu\nu} - 2\kappa g_{\mu\nu} V
\]

\[
+ \kappa B_{\mu\alpha} B^\alpha g_{\mu\nu} V - \frac{\lambda}{4} g_{\mu\nu} \nabla^2 (B_\alpha B^\alpha)
\]

\[
- \frac{\lambda}{2} g_{\mu\nu} \nabla_\alpha \nabla_\beta (B^\alpha B^\beta) = 0.
\]

(4)

At this point, the authors of [15] choose a static and spherically symmetric traversable wormhole solution in the following form [5, 15]

\[
ds^2 = e^{2\Lambda} dt^2 - \frac{dr^2}{1 - \frac{b(r)}{r}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2,
\]

(5)

where the red-shift function is made null \( (\Lambda = 0) \) and the bumblebee vector \( b_\mu \) is set to be correlated to the wormhole shape function \( b(r) \) as following [15]

\[
b_\mu = \left( 0, \sqrt{\frac{\alpha}{1 - \frac{b(r)}{r}}}, 0, 0 \right),
\]

(6)

where \( \alpha \) is a positive constant associated with the Lorentz violation term.

Besides, following the [15], the isotropic energy–momentum tensor can be decomposed as a perfect fluid \( (T''^\mu_\nu)^M = (\rho, -P, -P, -P) \) where \( P = w\rho \),

(7)

assuming \( \rho \geq 0 \). The \(-\frac{1}{3} < w \leq 1\) is a dimensionless constant responsible to hold the energy conditions.

Substituting the wormhole metric equation (5) into Einstein equation (4), we have the following new Einstein equations with the \( b(r) \) and \( w \) dependence

\[
E^{\text{einstein}}_\nu = -\kappa \rho r^3 (1 + 3w) + \lambda \dot{b}(r) - \lambda b(r) = 0,
\]

(8)
\[ E^\text{einstein}_{\theta\theta} = \kappa \rho \rho^3 (w - 1) + (2 + 3 \lambda) \dot{r} b - (2 + 3 \lambda) b(r) = 0, \]  
(9)

\[ E^\text{einstein}_{\theta\theta} = \kappa \rho \rho^3 (w - 1) + \rho \dot{b}(r) + (2 \lambda + 1) b(r) - 2 \lambda r = 0, \]  
(10)

where \( \lambda = \alpha \xi \) is defined as the Lorentz symmetry breaking (LSB) parameter and the dot denotes derivative concerning to coordinate \( r \).

The energy density \( \rho \) can be obtained from the system of equations (8)–(10). By solving directly \( \rho \) from equation (8) we obtain:

\[ \rho = \frac{\lambda (\dot{r} b(r) - b(r))}{\kappa r^3 (1 + 3w)}. \]  
(11)

Moreover, \( b(r) \) can be found by multiplying \((w - 1)\) by equation (8) and summing with \((1 + 3w)\) multiplied by equation (10). Additionally, the condition at wormhole throat \( b(r_0) = r_0 \) leads to the solutions

\[ b(r) = \frac{\lambda r}{\lambda + 1} + \frac{r_0}{\lambda + 1} \left( \frac{r_0}{r} \right)^\gamma, \]  
(12)

where \( \gamma(w, \lambda) = \frac{\lambda (5w + 3) + 3w + 1}{\lambda (w - 1) + 3w + 1} \).

The derivative \( \dot{b}(r) \) can be obtained as

\[ \dot{b}(r) = \frac{\lambda}{\lambda + 1} \left[ 1 - \frac{\gamma}{\lambda} \left( \frac{r_0}{r} \right)^{\gamma + 1} \right]. \]  
(13)

Finally, from equations (12) and (13), the energy-density of equation (11) can be found in the form

\[ \rho(r) = -\frac{2 \lambda r_0^{\gamma + 1}}{\kappa (\lambda (w - 1) + 3w + 1)} r^{-(\gamma + 3)}. \]  
(14)

In next section, we analyze the parameters \( \lambda \) and \( w \) in order to obtain a special case of the Övgün wormhole, where all energy conditions and flare-out holds. Moreover, we will also set these parameters in order to obtain a Regge–Wheeler potential with a bell-shaped curve.

### 3. Energy conditions and flare-out

In this section, we analyze the conditions imposed in the wormhole. We summarize the conditions in table 1. The NEC is the Null energy condition, WEC is the Weak energy conditions, SEC is the Strong energy condition, DEC is the Dominant energy conditions, and FOC is the flare-out condition.

In order to bound the \( w \) parameter, lets us assume that \( \rho > 0 \). Once that \( P = w \rho \) the DEC holds for \( P > |w| \) so \(-1 \leq w \leq 1\). On the other hand, the SEC is verified for \((1 + 3w) \rho > 0 \) so \( w \geq -\frac{1}{3} \). The NEC is expressed as \((1 + w) \rho > 0 \), so \( w \geq -1 \). Hence in order to obey NEC, SEC and DEC, the \( w \) parameter is must be such that

\[ \frac{\lambda (5w + 3) + 3w + 1}{\lambda (w - 1) + 3w + 1} \]  

**Table 1. Energy and flare-out condition.**

| Condition | NEC | WEC | SEC | DEC | FOC |
|-----------|-----|-----|-----|-----|-----|
| \( P + \rho \geq 0 \) | \( \rho > 0 \) and NEC | \( \rho + 3P \geq 0 \) and NEC | \( \rho \geq |P| \) | \( b(r) < 1 \) |
\[-\frac{1}{3} \leq w \leq 1. \tag{15}\]

From now on, we set \(\kappa = r_0 = 1\). The energy (14) is positive when \(-\frac{2\lambda}{(\gamma-1)\gamma+\lambda+1}\) > 0 and it vanishes when \(r \to \infty\) for \(\gamma > -3\). The region where the conditions are valid is shown in figure 1.

On the other hand, the flare-out condition (FOC) is necessary to maintain the structure of the wormhole traversable \([3, 5, 15, 29]\). The FOC is written as

\[b(r) - r \leq 0 \quad \text{and} \quad r \dot{b}(r) - b(r) < 0 \Rightarrow \dot{b}(r) < 1. \tag{16}\]

Considering the equation (13), the FOC and the vanishing of energy (14) when \(r \to \infty\), we plot in figure 2 the regions where the FOC and energy conditions are valid.

### 3.1. Tortoise coordinates and the energy and flare-out conditions

In section 4, we will obtain the Regge–Wheeler equation for the Övgün bumblebee wormhole. For this goal, a transformation to the radial variable \(r\) into a new variable \(x\) (tortoise coordinate) is required. This transformation, for \(\Lambda = 0\), is given by following the integral: \([30, 31]\)

\[x = \int dr \frac{1}{\sqrt{1 - \frac{b(r)}{r}}}. \tag{17}\]

To obtain an analytical function into \(x\) coordinate, we choose \(w = -1\) and \(r_0 = 1\), which transform the shape-function of equation (12) into

\[b(r) = \alpha r + \frac{\beta}{r}, \tag{18}\]

where \(\alpha = \frac{\lambda}{\lambda+1}\) and \(\beta = \frac{1}{\lambda+1} = (1 - \alpha)\).

Hence, by solving equation (17) with the of equation (18), the tortoise coordinate can be expressed as

\[x = \beta^{-1}\sqrt{\beta (r^2 - 1)} \quad \Rightarrow \quad r = \sqrt{\beta x^2 + 1}. \tag{19}\]

For this particular choice of \(w = -1\), the SEC is violated. However, the energy of equation (14) becomes \(\rho(r, \lambda) = \frac{\lambda}{\lambda+1}r^{-4}\) which is positive when \(\lambda > 0\) or \(\lambda < -1\). So the DEC, NEC and WEC still hold. The FOC obtained by \(\dot{b}(r)\) of equation (18) reads \(\frac{\lambda}{(\lambda+1)} - \frac{\beta^2}{(\lambda+1)} < 1\), which it is valid for \(\lambda > -1\). The figure 3 shows the attendance of DEC, NEC, WEC and FOC for \(w = -1\) and \(\lambda > 0\).

Furthermore, in the section 4 we need to set a value for the \(\lambda\) to compute the quasinormal modes. For a qualitative analysis, let us choose \(\lambda = 1\). The figure 4 shows the energy density \(\rho(r)\), the shape-function \(b(r)\) and its derivative \(\dot{b}(r)\) with \(w = -1\) and \(\lambda = 1\) fixed. Therefore, we note that, except the SEC, all other conditions above mentioned are satisfied.

### 4. Regge–Wheeler equation and quasinormal modes

In this section, we obtain the wave equation for the bumblebee wormhole. Let us consider an external perturbation, ignoring the back-reaction, and following all detailed linearization
procedure for the scalar and the axial gravitational perturbations of the references [30–34]. By considering a stationary solutions in the form $\Psi(x,t) = \phi(x)e^{-i\omega t}$, a simplified version of the so-called Regge–Wheeler equation can be represented by [27, 35, 36]

$$\left(\frac{d^2}{dx^2} + \omega^2 - V(r,l)\right)\phi(x) = 0,$$

where $\phi(x)$ is the wave function (for the scalar or gravitational perturbations) with $x$ tortoise coordinate (19), $\omega$ is the frequency and $V(r,l)$ is the Regge–Wheeler potential where $l$ is the azimuthal quantum number, related to the angular momentum.

For the scalar perturbations, the potential $V(r,l)$ reads [30, 33]

$$V_s(r,l) = e^{2\Lambda} \left[ \frac{l(l+1)}{r^2} - \frac{\dot{b}r - b}{2r^3} + \frac{1}{r} \left( 1 - \frac{b}{r} \right) \dot{\Lambda} \right].$$

\textbf{Figure 1.} Representation of the region where the energy conditions are satisfied. All conditions hold for the dark region into interval $-\frac{1}{3} < w \leq 1$. (the vertical dashed line denotes $w = -\frac{1}{3}$). In the grey region, only the WEC is secured.
Since that $\Lambda = 0$, by substituting the $b(r)$ (18) into potential (21), the $V_s(r, l) = \alpha r^2 + \beta r^4$ into $x$ coordinate (19) becomes

$$V_s(x, l) = \frac{l(l+1)}{r^2} + \left( \frac{1}{\Lambda + 1} \right) \left( \frac{x^2}{\Lambda + 1} + 1 \right)^{-2}.$$  \hspace{1cm} (22)

Similarly, for the axial gravitational perturbations, the potential $V(r, l)$ reads [32, 33, 36–38]

$$V_g(r, l) = e^{2\Lambda} \left[ \frac{l(l+1)}{r^2} + \frac{\hat{b} r - 5b}{2r^3} + \frac{1}{r} \left( 1 - \frac{b}{r} \right) \Lambda \right].$$  \hspace{1cm} (23)

With $b(r)$ given by equation (18), the $V_g(r, l) = \frac{l(l+1) - 2\alpha}{r^2} - \frac{3\beta}{r^2}$ into $x$ coordinate (19) becomes
\[ V_g(x, l) = \left( l(l + 1) - 2 \frac{\lambda}{\lambda + 1} \right) \left( \frac{x^2}{\lambda + 1} + 1 \right)^{-1} - \frac{3}{\lambda + 1} \left( \frac{x^2}{\lambda + 1} + 1 \right)^{-2}. \]  

(24)

The figure 5 shows the plots of the scalar potential of equation (22) and gravitational potential of equation (24). Note that both potentials are symmetric bell-shaped potentials centered at the origin. The increasing of the angular momentum increases the peaks of potentials. For the scalar perturbation, all potentials are repulsive. However, for the tensorial perturbation the first two values of $l$ lead to attractive potentials. The height of peaks changes the behavior of quasinormal modes, as discussed in the next subsection.

### 4.1. Quasinormal modes and the time-domain

To compute the quasinormal modes (QNMs) of Regge–Wheeler equation (20) we apply the semianalytic method of the third-order WKB approximation presented in [28]. This method requires a positive bell-shaped potential, but reliable frequencies are obtained only for $n < l$ [39]. Hence we impose $l \geq 4$ for the computation of first four modes ($n = 0, 1, 2, 3$) for the scalar and the tensorial perturbations.

Briefly, the QNMs can be found from the formula [28, 34, 39]

\[ \frac{i(\omega^2 - V_0)}{\sqrt{-2V_0}} = \sum_{i=2}^{6} \Lambda_i + n + \frac{1}{2}, \]

(25)

where $\dot{V}_0$ is the second derivative of the potential on the maximum $x_0$, $\Lambda_i$ are constant coefficients, and $n$ denotes the number of modes.

As a result, the quasinormal modes for the scalar perturbations are written in table 2. All frequencies found have negative imaginary part, denoting damping oscillations [21, 22, 34].

Similarly, the quasinormal modes for the gravitational perturbations are written in table 3. We start with $l = 4$, to guarantee positive potentials and $n < l$. Again, all frequencies have negative imaginary part.
Figure 5. The scalar (left) and gravitational (right) potentials of Regge–Wheeler equation for some $l$ parameters and $\lambda = 1$.

Table 2. QNMs for the scalar perturbations (22) with $w = -1$, $\kappa = r_0 = \lambda = 1$ and some $l$.

| $\omega_n$ | $l = 4$ | $l = 5$ | $l = 6$ |
|-----------|---------|---------|---------|
| $n = 0$   | $\pm 4.499 \, 130 - 0.355 \, 065 \, i$ | $\pm 5.499 \, 520 - 0.354 \, 591 \, i$ | $\pm 6.499 \, 710 - 0.354 \, 307 \, i$ |
| $n = 1$   | $\pm 4.441 \, 050 - 1.068 \, 920 \, i$ | $\pm 5.452 \, 660 - 1.066 \, 160 \, i$ | $\pm 6.460 \, 380 - 1.064 \, 590 \, i$ |
| $n = 2$   | $\pm 4.327 \, 290 - 1.793 \, 450 \, i$ | $\pm 5.360 \, 180 - 1.784 \, 700 \, i$ | $\pm 6.382 \, 470 - 1.779 \, 780 \, i$ |
| $n = 3$   | $\pm 4.162 \, 200 - 2.534 \, 190 \, i$ | $\pm 5.224 \, 440 - 2.514 \, 160 \, i$ | $\pm 6.267 \, 380 - 2.502 \, 820 \, i$ |

Table 3. QNMs for the gravitational perturbations (24) with $w = -1$, $\kappa = r_0 = \lambda = 1$ and some $l$.

| $\omega_n$ | $l = 4$ | $l = 5$ | $l = 6$ |
|-----------|---------|---------|---------|
| $n = 0$   | $\pm 4.156 \, 050 - 0.335 \, 541 \, i$ | $\pm 5.221 \, 450 - 0.341 \, 988 \, i$ | $\pm 6.265 \, 710 - 0.345 \, 476 \, i$ |
| $n = 1$   | $\pm 4.101 \, 510 - 1.010 \, 760 \, i$ | $\pm 5.175 \, 990 - 1.028 \, 450 \, i$ | $\pm 6.227 \, 100 - 1.038 \, 130 \, i$ |
| $n = 2$   | $\pm 3.995 \, 510 - 1.697 \, 660 \, i$ | $\pm 5.086 \, 450 - 1.722 \, 130 \, i$ | $\pm 6.150 \, 640 - 1.735 \, 770 \, i$ |
| $n = 3$   | $\pm 3.843 \, 440 - 2.401 \, 700 \, i$ | $\pm 4.955 \, 440 - 2.427 \, 030 \, i$ | $\pm 6.037 \, 830 - 2.441 \, 370 \, i$ |

Figure 6. Plot of QNMs for scalar (left) and gravitational (right) perturbations with $\lambda = 1$. 

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Moreover, figure 6 shows the points of QNMs tables 2 and 3. Note that all modes have smooth curves where the increase of \( l \) increases the modulus of the real part of each mode. The imaginary part changes slight with \( l \) but increases with \( n \). Also, the increasing of the number of modes for \( n > l \) can leads to not reliable results via the WKB method, requiring a numerical approach.

Besides, in figure 7, the time-domain of bumblebee wormhole is evaluated by the Gundlach’s method [40]. Note that both perturbations exhibit damping profiles, being the decreasing of scalar modes slower than the gravitational modes. Moreover, the \( l \) parameter faster the decay of these solutions.

5. Conclusions

In this work, we study the bumblebee wormhole. This scenario has a Lorentz violation parameter \( \lambda \), which allows for the preservation of energy conditions, leading to a wormhole generated by non-exotic matter [15]. We study the possible choices of the parameters \( \lambda \) and \( w \) (associated to the relation \( P = w \rho \)) that satisfies the flare-out and the energy conditions, as shown in figure 1. In order to achieve an analytical and simplified tortoise coordinates transformation of equation (19), we renounce the SEC condition. However, all the other ones, namely, NEC, WEC, DEC, and FOC remain valid.

Moreover, the scalar and tensorial perturbation of the bumblebee wormhole were obtained. We use the general expressions for the scalar and gravitational potential. Both the potentials admit positive bell-shaped (see figure 5). Hence we evaluate the quasinormal frequencies for both perturbations using the WKB method. The QNMs are reliable for \( n < l \) (as denoted in tables 2 and 3), and exhibits smooth curves (see figure 6). Also, we compute the time-domain profile for both perturbations in figure 7, from where we note that all QNMs studied perform damping oscillation profiles.

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