JET ACCELERATION BY TANGLED MAGNETIC FIELDS

SEBASTIAN HEINZ\(^1\) AND MITCHELL C. BEGELMAN\(^2,3\)

Joint Institute for Laboratory Astrophysics, University of Colorado and National Institute for Standards and Technology, Boulder, Colorado 80309-0440

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ABSTRACT

We explore the possibility that extragalactic radio jets might be accelerated by highly disorganized magnetic fields that are strong enough to dominate the dynamics until the terminal Lorentz factor is reached. Following the twin-exhaust model by Blandford & Rees, the collimation under this scenario is provided by the stratified thermal pressure from an external medium. The acceleration efficiency then depends on the pressure gradient of that medium. In order for this mechanism to work there must be continuous collimation of the magnetic field, changing the magnetic equation of state away from pure flux freezing (otherwise, conversion of Poynting flux to kinetic energy flux is suppressed). This is a complementary approach to models in which the plasma is accelerated by large-scale ordered fields. We include a simple prescription for magnetic dissipation, which leads to trade-offs among conversion of magnetic energy into bulk kinetic energy, random particle energy, and radiation. We present analytic dynamical solutions of such jets, assess the effects of radiation drag, and comment on observational issues, such as the predicted polarization and synchrotron brightness. Finally, we try to make the connection to observed radio galaxies and \(\gamma\)-ray bursts.

Subject headings: acceleration of particles — galaxies: jets — MHD

1. INTRODUCTION

Many extragalactic radio jets move with bulk Lorentz factors \(\Gamma > 10\), as evidenced by very short variability timescales, superluminal proper motion of jet features, and dilated particle lifetimes, yet the process that actually accelerates the material is not well known, and neither is the mechanism that collimates the outflow.

One of the first serious models for the large-scale dynamics of extragalactic radio jets is the “twin exhaust” model (Blandford & Rees 1974, hereafter BR74). In this model, relativistic particle pressure provides the bulk acceleration via conversion of internal to kinetic energy. The collimation comes from confinement by an external medium, the pressure of which is very likely stratified in the gravitational field of the central black hole and the host galaxy and cluster (either in a hydrostatic equilibrium configuration or as a wind). Pressure balance then dictates the evolution of the jet’s Lorentz factor \(\Gamma\) and the jet diameter \(R\). However, cooling processes for particles with highly relativistic random motions (necessary to produce outflows with large bulk Lorentz factors \(\Gamma\)) are very efficient, thus competing with and probably disabling bulk acceleration.

The problem of radiative losses is similarly limiting in the case of radiatively accelerated jets (Phinney 1982). Furthermore, these jets rely on the presence of a strong pointlike radiation source (the terminal Lorentz factor is limited by the solid angle subtended by the radiation source due to relativistic aberration). The same inverse Compton (IC) scattering effects that lead to the bulk acceleration lead to even stronger radiative losses. In fact, radiation drag can actually decelerate the jet if the external radiation field is isotropic, which leads to both random energy losses and kinetic energy losses by the same process.

Radiative losses are much less limiting if the energy is stored in the magnetic field. Organized magnetic fields have been suggested to provide both acceleration and collimation (e.g., Blandford & Payne 1982; Li, Chiuhe, & Begelman 1992); however, such solutions, in which the collimation is provided by toroidal field, seem to be hampered by instabilities (Begelman 1998). Furthermore, it is difficult to achieve collimation on the basis of magnetic tension alone without contribution from an external pressure source (Begelman 1995, hereafter B95). Polarization measurements show that the magnetic field in jets is probably not well organized—the polarization is generally well below the maximal value of \(~70\%\); only in knots does the polarization tend toward this value (note, however, that interpretations of polarization measurements are often ambiguous, since different field geometries can sometimes lead to the same net polarization). This argues for the presence of largely unorganized, chaotic fields, which could easily account for the high polarization measured in the knots if they are interpreted as shocks, compressing the field in the shock plane (Laing 1980; B95). This goes hand in hand with the fact that the field produced in the disk by dynamo processes is expected to be highly chaotic. Since the conditions in the jet will likely be controlled by disk physics, we should expect the same statement to be true for the magnetic field in the jet at least close to the disk. These arguments led us to investigate the dynamics of jets containing large amounts of such disorganized magnetic fields.

The rate of acceleration in a jet propelled by internal (isotropic) particle pressure in an external pressure gradient is limited to \(\Gamma \propto p_{\text{ext}}^{-1/4}\) (BR74), which means that the acceleration to bulk Lorentz factors of \(\Gamma \sim 10–100\) would occur over length scales \(\gtrsim 1000r_d\) for external pressure gradients \(p \propto z^{-2}\). One might think that an anisotropic pressure in the form of chaotic magnetic fields could increase the rate at which the jet is accelerated if the excess momentum flux is oriented along the direction of the jet. We will show that

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\(^{1}\) heinzs@rocinante.Colorado.edu.

\(^{2}\) mitch@jila.Colorado.edu.

\(^{3}\) Also at Institute for Theoretical Physics, University of California, Santa Barbara.

\(^{4}\) Also at Department of Astrophysical and Planetary Sciences, University of Colorado, Boulder.
under a given set of simple assumptions the rate of acceleration is actually the same as in the classic case considered by BR74, i.e., \( \Gamma \propto p_{\text{ext}}^{-1/4} \).

It is unlikely that the magnetic field evolves without some form of dissipation, especially if it is highly unorganized (reconnection is a diffusive process, so strong gradients in the field, as are present if the field is highly tangled on small scales, will likely lead to increased dissipation). These loss processes can compete with the efficiency of bulk acceleration by removing energy from the flow reservoir. We will investigate the effects such a trade-off might have on the dynamics and appearance of jets.

We will lay out the simplifying assumptions going into this model in \( \S \) 2. Section 3 contains a brief discussion of allowed solutions and the sonic transition of such solutions. In \( \S \) 4 we will discuss self-similar and asymptotic solutions and present some full analytic solutions, including a simple estimate of the effects of radiation drag. Section 5 contains a discussion of the aforementioned trade-off between dissipation and acceleration, some predicted observational consequences, and an outlook on applications of this model to radio galaxies and \( \gamma \)-ray bursts. Finally, we will summarize our results in \( \S \) 6.

2. THE MODEL

The model we are employing here is closely related to (and an extension of) the “twin-exhaust” model put forward by BR74. We adopt a similar scenario under which the jet is launched into a stratified external medium. In the case we are considering, turbulent and highly disordered magnetic fields dominate the internal energetics of the jet.

We have tried to illustrate the overall picture we are employing in Figure 1. The interstellar magnetic field is advected inward by the accretion disk. Turbulent shear then amplifies the field and tangles it up (dynamo action). This effect will grow stronger with decreasing distance to the black hole. Eventually, regions of very high field strength will develop. Because of their buoyancy they will accelerate away from the black hole, forming an initial outflow. This outflow is then collimated by the pressure of the external medium. The jet channel is constrained by pressure balance; i.e., the jet will expand or contract in such a way that an equilibrium solution is set up for which the flow is stationary. As the flow expands, we assume that microinstabilities and turbulence constantly rearrange the field. As in the pure particle pressure case, the flow can go through a critical point, where the radius \( R \) has a minimum and beyond which the flow will become self-similar (if the external pressure itself behaves self-similarly with distance to the black hole) before the rest mass energy starts dominating the inertia, at which point the jet will reach a terminal Lorentz factor \( \Gamma_c \). Along the way, the field might dissipate energy via reconnection-like processes and radiation drag might alter the dynamics.

We assume that no energy or particles are exchanged between jet and environment, except for radiative losses. However, the momentum discharge (the jet thrust \( Q \)) need not be conserved along the jet. By assuming that quantities like \( \rho_v, U_i' \equiv \langle B_i^2 \rangle / 8\pi \) (where a prime denotes that the quantity is measured in the comoving frame) do not vary significantly across the jet, we simplify the analysis to a quasi–one-dimensional solution. We ignore effects of shear at the jet boundaries. We assume throughout most of the paper that the advected matter is cold (i.e., enthalpy density

\[ h' \approx n'm_{\text{particle}} c^2 \]. We are looking for stationary flow along the jet (i.e., far from the terminal shock), enabling us to drop time derivatives. Finally, to make this quasi–one-dimensional treatment possible, we will need to make the assumption that the jet is narrow, which in our case implies that the opening angle is small compared to the beaming angle, i.e., \( dR/dz \ll 1/\Gamma \). As we will later show, this also implies that the jet is in causal contact with its environment (as required by the assumption that the jet is in pressure equilibrium with the surrounding medium).

2.1. Treatment of the Magnetic Field

We use cylindrical coordinates \((r, \phi, z)\) with the \( z \)-axis oriented along the jet axis. The flow velocity is not aligned with the \( z \)-axis for \( r > 0 \) (the jet expands). We assume that the sideways velocity is small compared to \( v_z \) but non-vanishing, i.e., that the flow is well collimated. The magnetic field is expressed in a different basis, since the standard basis vectors \( e_r \) and \( e_z \) are not orthogonal in the comoving frame for \( r \neq 0 \). One axis of this new basis is aligned with the local velocity vector, \( e_{||} \). The second unit vector \( e_{\phi} \) is coincident with the \( \phi \) unit vector of the lab coordinate system. The third unit vector \( e_{\alpha} \) is obtained from the cross product of
the other two. In the comoving frame we have
\[ B = B^\perp e_\perp + B^\parallel e_\parallel. \]  
(1)
As mentioned before, the one-dimensional approximation is possible only if the opening angle is small compared to the beaming angle. This is because the Lorentz factor will not be nearly uniform across the jet otherwise. The assumption that \( dR/dz < 1/\Gamma \) simplifies the equations of motion significantly.

Following B95, who investigated similar jets in the non-relativistic limit, we assume that the magnetic field is highly disorganized. In the comoving frame, averages over the individual components and cross terms vanish while the energy density in the individual components is not zero:
\[ \langle B_i B_j \rangle = 0, \quad \text{for } i \neq j, \quad \langle B_i^2 \rangle \equiv 8\pi U_i' \neq 0. \]  
(2)
Lorentz transformation of the field to the lab frame (and to the cylindrical coordinate system aligned with the jet axis) yields
\[ B = \left( \frac{v_z}{v} \Gamma B^\perp + \frac{v_r}{v} B^\parallel \right) \hat{e}_r + \Gamma B^\perp \hat{e}_\phi + \left( \frac{v_z}{v} B^\parallel - \frac{v_r}{v} \Gamma B^\perp \right) \hat{e}_z. \]  
(3)
Some of the components are now correlated. The electric field in the lab frame is
\[ E = v_z \Gamma B^\perp \hat{e}_r - \sqrt{v_r^2 + v_z^2} \Gamma B^\perp \hat{e}_\phi - v_r \Gamma B^\perp \hat{e}_z. \]  
(4)

2.1.1. Magnetic Equation of State

Without the presence of turbulent rearrangement of the field, flux freezing would govern the behavior of the individual components. If we assume the presence of turbulent mixing between the different field components, we might expect the field to follow a modified evolution according to
\[ dB^i = \sum_j x_{ij} \frac{\partial B^j}{\partial \Gamma v} \Bigg|_{tt} \ d(\Gamma v) + \sum_j \beta_{ij} \frac{\partial B^j}{\partial R} \Bigg|_{tt} \ dR, \]  
(5)
where the subscript “ff” denotes the value the derivative would take under flux freezing and \( x_{ij} \) and \( \beta_{ij} \) are arbitrary mixing coefficients. Based on this picture we therefore choose the following convenient ad hoc parameterization of the field evolution with Lorentz factor \( \Gamma \) and jet radius \( R \), including rearrangement:
\[ B^\perp \propto (\Gamma v)^{-2 + \mu_1} R^{-2 + \mu_2}, \]
\[ B^\parallel \propto (\Gamma v)^{-1} R^{-4 + \mu_4}. \]  
(6)
This is the magnetic equation of state we use. In the case of pure flux freezing, \( \mu_i = 0 \) for all \( i \). In the case of a completely isotropic field we have \( \mu_1 = \frac{2}{3}, \mu_2 = -\frac{2}{3}, \mu_3 = -\frac{2}{3}, \mu_4 = \frac{1}{2} \). Note that this prescription is still fully general [until we make some limiting assumptions about the \( \mu_i(r, z) \)]. Since the rearrangement process mixes the perpendicular and parallel components of the field, we would expect that the field behavior is changed from flux freezing in such a way that the coefficients \( \mu_i \) are bracketed by the values they take in the case of flux freezing. Since the case of a purely isotropic field must be included in our analysis, it is clear that this condition requires that \( \mu_2 < 0 < \mu_1, \mu_3 < 0 < \mu_2 \).

We define two quantities to characterize the anisotropy of the magnetic pressure:
\[ \zeta \equiv \frac{U'_\perp - U'_\parallel}{U'_\perp + U'_\parallel} \quad \text{and} \quad \delta \equiv \frac{U'_\parallel - U'_\perp}{U'_\perp + U'_\parallel}, \]  
(7)
where \( U'_\parallel \equiv U'_\perp + U'_\parallel \). Thus, the magnetic field is purely perpendicular for \( \zeta = -1 \) and purely parallel for \( \zeta = 1 \). The perpendicular component is purely radial for \( \delta = -1 \) and purely toroidal for \( \delta = 0 \). It is obvious from equation (6) that \( \delta \) is constant for any combination of parameters, since \( U'_\perp \propto U'_\parallel \) by assumption.

While this parameterization alone is rather unrestrictive, we can limit it to a one parameter family by assuming that the \( \mu_i \) are constants under any possible variation of \( \Gamma \) and \( R \), and that the rearrangement process does not change the total comoving energy density in the magnetic field. (Otherwise the same process would have to act as an energy sink, since we assume that the magnetic field is the dominant term in the internal energy budget. We would therefore be dealing with a dissipative process, which we will address in § 2.1.2.) We can solve for \( \mu_i \) in terms of \( \zeta \) by fixing either \( \Gamma \) or \( R \) and demanding that the total energy density \( U' \equiv \sum U_i' \) behave the same as it would following flux freezing:
\[ dU' = U'_\parallel [(\mu_1 - 2)d(\Gamma v) + (\mu_2 - 2)dR] \]
\[ + U'_\parallel [(\mu_3 \delta d(\Gamma v) + (\mu_4 - 4)dR] \]
\[ = -2U'_\parallel [d(\Gamma v) + dR] - 4U'_\parallel dR \]  
(8)
for arbitrary \( d(\Gamma v) \) and \( dR \). Constancy of any of the \( \mu_i \) then implies constancy of \( \zeta \), and substitution of \( \zeta \) from equation (7) yields
\[ U'_i \propto (\Gamma v)^{1/3} R^{-3/4}; \]
\[ \mu_1 = 1 + \zeta, \quad \mu_2 = -1 - \zeta, \]
\[ \mu_3 = \zeta - 1, \quad \mu_4 = 1 - \zeta \]  
(9)
which includes the isotropic case, where the magnetic field behaves like a relativistic gas, for which \( \zeta = -\frac{1}{2}, \delta = 0 \).

It turns out that one can find special analytic solutions with constant \( \mu_i \) that satisfy equation (8) without the requirement that \( \zeta \) be constant (see § 4). For these solutions the rearrangement process conserves the comoving magnetic energy density only under the variations in \( \Gamma \) and \( R \) allowed by the Bernoulli equation [i.e., \( d(\Gamma v) \) and \( dR \) in eq. (8) are not arbitrary]. The only condition on the parameters \( \mu_i \) for such a solution is that \( \mu_i/\mu_2 = \mu_3/\mu_4 \). These solutions are limited to the self-similar range, where the jet is dominated by magnetic pressure. Once they approach the terminal phase (i.e., \( \rho' \gtrsim U' \)), the parameters \( \mu_i \) must vary with \( z \). For the rest of the paper we will assume that equation (9) holds unless indicated otherwise.

2.1.2. Dissipation of Magnetic Energy

It is unlikely that the tangled magnetic field evolves without any dissipation of its energy (e.g., via reconnection). We thus include a simple, ad hoc prescription of magnetic energy losses. We base our parameterization on the idea that the magnetic field is always in a nearly force-free equilibrium. However, it is impossible to maintain perfect force-free conditions everywhere and as the jet expands in either
direction, the field responds by rearrangement between the different components (eq. [9]) and by dissipation of some of its energy. We therefore assume that the dissipation rate is roughly proportional to the divergence of the velocity in the comoving frame:

\[
\left( \frac{\partial U'_i}{\partial t} \right)_{\text{diss}} \approx -\Lambda U'_i \ln (\Gamma v R^2) \]

or, in the lab frame,

\[
\left( \frac{\partial U'_i}{\partial z} \right)_{\text{diss}} \approx -\Lambda U'_i \left( \frac{\partial}{\partial z} \ln (\Gamma v R^2) \right). \tag{11}\]

This Ansatz can easily be generalized to different \( \Lambda_i \) for different field components (e.g., if the Alfvén velocity factors into \( \Lambda \)). For a more realistic dissipation model see the Appendix.

We will assume that the dissipated energy goes into isotropic particle pressure, which is then either (1) radiated away immediately as isotropic radiation in the comoving frame or (2) accumulated until the jet reaches a state of equipartition between particle pressure and magnetic field.

### 2.2. Equations of Motion

We write the relativistic continuity equation as

\[
\rho' v_z \Gamma R^2 = \text{const}. \tag{12}\]

The energy and momentum equations are given by

\[
T_{00}' = 0 \quad (T \text{ is the stress-energy tensor, separable into a matter and an electromagnetic part}). \]

In the absence of gravity, this reduces to \( T_{00}' = 0 \), which will be sufficient for the analysis through most of this paper since most of the acceleration will likely take place at distance \( z \gg r_g \equiv GM/c^2 \), where \( M \) is the mass of the central black hole. It turns out, however, that gravity is important in discussing the critical points of the jet, in which case we approximate the covariant derivative by a Newtonian potential \((- \Gamma_{00}' = \Gamma_{33}' = \Gamma_{00} = r_g/z^2)\). We will comment on the accuracy of this approximation in § 3.1.

Since there is no energy exchange between the jet and the environment, using the expression for the electromagnetic field measured in the lab frame from § 2.1 we can write the energy equation as \( T_{00}' = 0 \) (neglecting gravity). We integrate the equation over a cross-sectional volume of the jet and convert it to a surface integral using Gauss’s law. The contribution from the side wall is zero, giving

\[
\Gamma^2(\rho' c^2 + 4p')v_z \pi R^2 + \frac{1}{4\pi} v_z \Gamma^2(B'_\phi^2 + B'_\alpha^2)nR^2 \equiv L = \text{const}. \tag{13}\]

(where the \( B'_i^2 \) are now averaged quantities). Dividing equation (12) into equation (13) gives the relativistic Bernoulli equation. In the more general case including radiative losses and gravity we have

\[
\frac{d}{dz} \Gamma^2 v_z R^2 \left( \rho' c^2 + 4p' + 2 \frac{B'^2}{8\pi} \right) + 2 \frac{r_g}{z^2} \Gamma^2 v_z R^2 (\rho' c^2 + 4p' + 2U'_\perp) + S_{\text{rad}} = 0, \tag{14}\]

where \( S_{\text{rad}} \) is the energy lost to radiation leaving the jet. We have to make some assumption about the form of \( S_{\text{rad}} \), i.e., the amount of energy radiated away [cases (a) and (b) from § 2.1.2].

The \( z \)-momentum flux \( Q \) can be calculated in much the same way (integrating \( T_{33}' \) across a jet cross section). Since the jet can exchange \( z \)-momentum with the environment, the momentum discharge need not be conserved, however. Dropping terms of order \( v'_z \), the integration yields

\[
Q \equiv \int dT_{33}' \approx \Gamma^2 v^2 \pi R^2 (\rho' + 4p'/c^2) + \pi R^2 p' + \pi R^2 \Gamma^2(1 + v^2)U'_\perp - U'_\parallel. \tag{15}\]

The sideways momentum equation is given by

\[
T_{13}' - v_z T_{33}' = 0. \quad \text{The condition that the solution be stationary \cite[i.e., \( \partial R(z)/\partial t = 0 \)]{2} gives the pressure balance condition between the jet and its environment. We assume that the internal structure of the field adjusts to maintain the given cross section. Since we assume that \( v_z < v_{t,0} \), the internal variation will be sufficiently small, \( \partial/\partial t \sim (v_z/v_{t,0})\partial/\partial z \), to justify the assumption of uniformity (note: this assumption is satisfied only if the jet is in causal contact). We are thus interested only in the pressure balance condition at the jet walls, \( r = R \), which gives

\[
p_{\text{ext}} = U'_\phi + U'_\perp - U'_\alpha + p'. \tag{16}\]

Note that \( U'_\alpha = 0 \) directly at the jet boundaries, since the magnetic field is assumed not to penetrate the contact discontinuity. However, since interior pressure balance demands that \( U'_\phi + U'_\perp - U'_\alpha + p' \) be constant, we can set \( U'_\phi - U'_\alpha = \text{const.} \) and substitute it for \( U'_\phi \) at \( r = R \), which gives equation (16).

### 3. DYNAMICAL SOLUTIONS

Before we start analyzing the equations presented above, it is worth noting that in the case of a cold \( (p' = 0) \) jet and a magnetic field following pure flux freezing \( (\mu_t = \mu_\alpha = 0) \), the only possible solution to equation (13) far away from the core \cite[i.e., \( z > r_g \) is \( \Gamma(z) = \text{const.} ; \ i.e., \ the \ jet \ expands \ sideways \ to \ satisfy \ pressure \ balance, \ without \ accelerating. \ This \ is \ because \ both \ the \ kinetic \ energy \ flux \ and \ the \ Poynting \ flux \ do \ not \ vary \ with \ R, \ while \ they \ do \ vary \ with \ v, \ so \ that \ equation \ (13) \ becomes \ an \ equation \ of \ \Gamma \ only. \ Thus, \ fixing \ the \ total \ jet \ power \ L \ fixes \ \Gamma. \ While \ a \ scenario \ like \ this \ might \ explain \ the \ coasting \ phase \ of \ the \ jet \ (where \ no \ more \ acceleration \ occurs), \ it \ cannot \ account \ for \ the \ initial \ bulk \ acceleration \ we \ are \ looking \ for. \n
Note that this is different than the case of anisotropic, relativistic particle pressure in the absence of isotropization (i.e., simply under adiabatic behavior of the individual components). In that case, the components scale like \( p_\phi \propto (\Gamma v)^{-2}R^{-2} \), \( p_\alpha \propto p_\phi \propto (\Gamma v)^{-1}R^{-3} \). We might expect a behavior like this for a relativistic turbulent pressure term. The sideways pressure is simply \( p_r \). At relativistic speeds, the solution approaches the one found by BR74, \( \Gamma \propto R \propto p^{-1/4} \). Thus, unlike in the magnetic case described in this paper, it is generally possible to accelerate a jet with anisotropic particle pressure without making any arbitrary assumptions about the randomization process. This is simply because only the perpendicular component of the field, \( U'_\perp \), contributes to the Poynting flux, while all components of the pressure enter equation (13), which introduces a dependence on \( R \), making a solution \( \Gamma \neq \text{const.} \)
possible. For a magnetically dominated solution to exist, on the other hand, we need a field rearrangement process at work, such as was described in § 2.1.1. However, even under such favorable conditions, a proper, accelerating solution is not always guaranteed.

3.1. Critical Points

Since the jet will likely be injected with subrelativistic speed, the question arises as to where the jet crosses possible critical points and at what velocity. If the jet is injected at large distances from the central black hole, we can neglect gravity; if it is injected close to the hole we will have to include at least a phenomenological gravity term.

As a first step, we will set $M = 0$ artificially (still assuming the presence of an external pressure gradient) and neglect dissipation ($S_{\text{rad}} = 0$). Equation (14) gives

$$\Gamma^2[(p'c^2 + 4p') + 2(1 + \zeta)U_{z*}] \frac{dv}{v dz} + [4(2 - \gamma_{\text{ad}})p' - 2(1 + \zeta)U_{z*}] \frac{dR}{dz} = 0,$$

where $\gamma_{\text{ad}}$ is the adiabatic index of the particles, $\gamma_{\text{ad}} = d\ln p'/d\ln \rho'$. This equation has a critical point when the expression in square brackets vanishes. At such a point, the jet cross section must satisfy $dR/dz = 0$; i.e., the jet must go through a nozzle, the position and cross section of which are determined by the dynamics of the flow. Following the notation of BR74, the velocity at which that happens is

$$c_* = \sqrt{\frac{4p'_{*}(\gamma_{\text{ad}} - 1) - 2U'_{z*} \zeta}{\rho'_{*} c^2 + 4p'_{*} + 2U'_{z*}}},$$

where the subscript asterisk indicates that the quantity is evaluated at the critical point $z_*$. Since for a magnetically dominated jet limit $c_* = 0$, the critical point exists only for $\zeta < 0$. For a purely isotropic field, where $\zeta = -\frac{1}{3}$, $c_*$ reduces to the speed of a relativistic gas with $\gamma_{\text{ad}} = \frac{4}{3}$.

Locally we can always write $p_{\text{ext}} \propto z^{-\zeta}$, thus we define

$$\zeta \equiv -d\ln p_{\text{ext}}/d\ln z.$$

We can then substitute the pressure balance condition (16) into equation (17) in the limit $p'c^2 + 4p' \ll U'$ and eliminate $R$, which yields

$$\Gamma^2[(3 + \zeta)v^2 + (1 + 3\zeta)] \frac{dv}{v dz} = (1 + \zeta) \frac{z}{z_*}.$$

This equation also has a critical point with a critical speed of

$$c_* \equiv \sqrt{\frac{1 + 3\zeta}{-3 - \zeta}}.$$ (21)

Unlike equation (17), solutions cannot cross this critical point, since there $dv/dz \to \infty$ (but see § 3.2).

We expect $dp_{\text{ext}}/dz < 0$, so solutions always accelerate (decelerate) for $v > c_*$ ($v < c_*$). Since $c_*$ exists only for $\zeta < -\frac{1}{3}$, solutions with $\zeta > -\frac{1}{3}$ always accelerate. In that case equation (17) implies that for $v > c_*$ ($v < c_*$) the jet is expanding (contracting) in the $r$-direction. Since $c_*$ exists only for $\zeta < 0$, solutions with $\zeta > 0$ always expand sideways.

If, on the other hand, $\zeta < -\frac{1}{3}$, two branches of solutions exist: (1) solutions that are injected with $v > c_*$, which always accelerate and go through a nozzle at $v = c_*$, and (2) solutions that are injected with $v < c_*$, which always decelerate. Thus, at sufficiently large distances from the core for gravity to be negligible (see § 3.2), highly anisotropic solutions with $\zeta \approx -1$ have to be injected at relativistic velocities to be accelerating, since $c_* \to 1$ as $\zeta \to -1$. This corresponds to the right branch of the dashed solutions plotted in Figure 2 (which includes the effects of gravity; see § 3.2).

It is instructive to look at the case of pure anisotropic relativistic particle pressure again. We define the pressure anisotropy as $\zeta_p \equiv (p_\parallel - p_\perp)/(p_\parallel + p_\perp)$. If we fix $\zeta$ by some rearrangement process as we did for the magnetic field in § 2.1.1 (which might occur, for example, if there is a coupling between magnetic field and turbulent pressure as suggested by B95), the behavior is very similar in the sense that accelerating solutions for $\zeta_p > -\frac{1}{3}$ (i.e., for $p_\parallel > 2p_\perp$) have to be injected at supercritical velocity $v > c_{\text{crit}} = [(1 + 3\zeta_p/\gamma_{\text{ad}})]^{1/2}$.

If we simply let the components of $p$ evolve adiabatically (without rearrangement), we arrive at a different critical velocity, $c_{\text{crit}} = [(5 + 7\zeta_p)/(5 + 5\zeta_p)]^{1/2}$ (note that $\zeta$ is not constant in this case), which exists only for $\zeta_p > -\frac{1}{3}$. The solution once again has a nozzle at $c_{\text{crit}} = [(2 + 2\zeta_p)/(3 + \zeta_p)]^{1/2}$. Since $c_{\text{crit}} > c_*$ for $\zeta_p > -\frac{1}{3}$ and since $dR/dz < 0$ for $v < c_*$, solutions injected with $v > c_*$ must accelerate to satisfy pressure balance, which means that $\zeta_p$ increases with $z$, reducing $c_{\text{crit}}$ and thus making the flow more supercritical (i.e., once above the critical point, the solution moves away from it).

3.2. The Effects of Gravity on the Sonic Transition

As seen in the previous section, a solution for $\zeta > -\frac{1}{3}$ that starts out with $v < c_*$ will always decelerate in the absence of gravity. However, as is the case in the solar wind, gravity can actually help a flow go through a critical point. We thus consider $M > 0$ in this section.

We can go through the same arguments as in § 3.1. The critical speeds are still given by equations (18) and (21), but now the critical conditions are different. At $c_*$, equation (14) gives

$$\Gamma^2[(3 + \zeta)v^2 + (1 + 3\zeta)] \frac{dv}{v dz} = (1 + \zeta) \frac{z}{z_*},$$

$$2\Gamma^2[(3 + \zeta)v^2 + (1 + 3\zeta)] \frac{dv}{v dz} = (1 + \zeta) \frac{z}{z_*} - (3 + \zeta) \frac{2\zeta_p}{z_*}.$$ (23)
If \( v \neq c_t \) at \( z_t \), the solution must follow \( dv/dz = 0 \) at that point. This is true for all \( \zeta \). Since that is the only zero of equation (23), we can therefore conclude that solutions accelerating at any \( z > z_t \) will accelerate for all \( z > z_t \).

Solving the equations for \( dR/dz \) instead gives

\[
\left[ (\zeta^2 - 1) - (3 + \zeta)(v^2 + \zeta) \right] \frac{dR}{R} dz = - \left(\frac{v^2 + \zeta}{z} + (\zeta - 1)\frac{2r_g}{z^2} \right),
\]

which also has a critical point at \( c_t \). For \( v = c_t \), the right-hand side of this equation vanishes only at \( z_t \). In that case, \( dR/dz \) remains finite. For all other solutions (i.e., if \( z \neq z_t \) when \( v = c_t \)), we must have singularities in both \( dv/dz \) and \( dR/dz \). The singularity in \( dv/dz \) is evident from Figure 2 and from equation (23); pressure balance then requires that \( dR/dz \) have a singularity of opposite sign, since \( dp_{ext}/dz \) is assumed to be finite; i.e., \( p_{ext} \) is continuous.

We have numerically integrated equation (23) for two representative cases \( \zeta = -0.9 \) and \( \zeta = 0, \zeta = 2 \) and plotted them in Figure 2. Solutions are qualitatively different for \( \zeta < -\frac{1}{2} \) and \( \zeta > -\frac{1}{2} \).

1. For \( \zeta < -\frac{1}{2} \), there is one accelerating transonic solution, given by the condition in equation (24), shown in the upper panel of Figure 2 as a thick solid black curve. This is also the only solution accelerating for all \( z \). As in the case of a regular adiabatic wind (Parker 1958), there also exists a decelerating transonic solution. Regions where solutions contract in the \( r \)-direction (i.e., where \( dR/dz < 0 \)) are shown as hatched areas. There are four more branches of solutions. Two branches are double-valued (shown as dashed curves in Fig. 2). The left branch of those solutions can be rejected since those solutions exist only for \( z < z_t \). For an accelerating solution on the right branch to exist, it must be injected with \( v > c_t \). This corresponds to the solutions discussed in § 3.1 for which gravity can be neglected.

The remaining two branches are solutions that are always sub- or supercritical (thin solid curves in Fig. 2). The subcritical solutions decelerate for large \( z \) and always stay subrelativistic. They are uninteresting as possible candidates for relativistic jets. The supersonic solutions decelerate for \( z < z_t \) and accelerate for \( z > z_t \). These solutions correspond to the supersonic solutions mentioned in § 3.1. They always expand in the sideways direction. As we let \( \zeta \to -1, \ z_t \to \infty \). This is not necessarily an indication that no solution is possible for \( \zeta \approx -1 \), since for those cases the critical speed is very close to 1, thus the solution can attain large \( \Gamma \). Furthermore, as we saw above, the solution is expanding even before it goes through \( z_t \). We can thus have a regular (though subcritical) accelerating jet even for \( \zeta \approx -1 \).

2. For \( \zeta > -\frac{1}{2} \), there is only one branch of solutions, all of which start out decelerating, shown in the bottom panel of Figure 2. As the solutions reach \( z_t \), they begin to accelerate and behave the same way as described in § 3.1. Since \( z_t \) moves inward for increasing \( \zeta \), this is no handicap. For \( \zeta > -\frac{1}{2} \) we have \( z_t < 8r_g/\zeta \) from equation (24), which, for reasonable values of \( \zeta \), is well in the regime where relativistic corrections become important and inside the region where we expect the injection to occur. All of these solutions have positive sideways expansion \( dR/dz > 0 \) for all \( z \).

Now solutions can cross the critical point \( c_t \), since the right-hand side of equation (23) vanishes at

\[
z_t = \left( \frac{3 + \zeta}{1 + \zeta} \right) \frac{2r_g}{\zeta}.
\]

(24)
The transonic solution for \( \zeta < -\frac{1}{2} \) has some additionally nice features. Since we know \( z_+ \), we can relate the jet cross section to the total jet power \( L_+ \) at \( z_+ \). Assuming that the jet is still magnetically dominated at \( z_+ \), the kinetic luminosity of the jet is

\[
L_+ = \pi R_+^2 \Gamma_+^2 \frac{3}{2} U_+^3. \tag{26}
\]

The external pressure at \( z_+ \) is \( p_{\text{ext}} \), so

\[
R_+ = \sqrt{\frac{L_+ \delta (1 + \zeta) / (1 - \zeta) [1 + \zeta]}{\pi p_{\text{ext}} \sqrt{-\zeta}}}. \tag{27}
\]

While we do not know \( p_{\text{ext}} \), for most parameter choices \( L_+ \) is very nearly equal to \( L_{\infty} \), which we have a reasonably good handle on from an observational point of view. Furthermore, we can estimate the jet width at observable distances and scale the solution back to \( z_+ \), which gives us an estimate of \( p_+ \) and thus \( U_+ \). This in turn will allow us to determine the original matter loading of the jet from estimates of the terminal Lorentz factor \( \Gamma_{\infty} \).

4. Solutions in the self-similar regime and asymptotic solutions

For an already-relativistic jet in the “self-similar” range \( z_+ \ll z \ll z_{\text{inertia}} \) (where \( z_{\text{inertia}} \) is the location where \( \rho c^2 = 2U_+^2 \)), a self-similar solution can be found. Equations (9) and (13) give \( \Gamma_{\infty} \propto R^{-\mu_2/\mu_1} \), and with equation (9) we have \( \Gamma_{\infty} \propto R \).

Under the conditions of equation (9), the pressure balance condition gives

\[
\Gamma \propto R \propto p_{\text{ext}}^{-1/4}, \tag{28}
\]

the same as in the case of isotropic particle pressure considered by BR74. For future reference we define the acceleration efficiency

\[
\eta \equiv -\frac{d \ln \Gamma}{d \ln p_{\text{ext}}}; \tag{29}
\]

thus for this simple case \( \eta = \frac{1}{4} \).

If we adopt the less limiting restriction \( \mu_1/\mu_2 = \mu_3/\mu_4 \) (see § 2.1.1) instead of equation (9), we can find power-law solutions in three limiting cases:

1. For \( \delta = 0 \), the solution is given by \( \Gamma_{\infty} \propto R^{-\mu_2/\mu_1} \propto p_{\text{ext}}^{2/4\mu_1} \), which can be very efficient for \( \mu_1 \ll -\mu_2 \) as pointed out in § 2.1.1, one would generally expect that \( \mu_2 < 0 < \mu_1 \);

2. For \( \zeta \approx -1 \), the solution is given by \( \Gamma_{\infty} \propto R^{-\mu_2/\mu_1} \propto p_{\text{ext}}^{1/(2\mu_1/\mu_2 - 2)} \), which has a limiting efficiency of \( \eta \leq \frac{1}{2} \); and

3. For \( \zeta \approx 1 \) the solution is approximately the same as case 1.

If \( \delta > 0 \), the solution will in general approach solutions 2 or 3 (i.e., \( \zeta \rightarrow \pm 1 \)). For \( \delta < 0 \) it is possible that the solution approaches a finite terminal Lorentz factor and zero opening angle if radial tension cancels the pressure due to \( U_\parallel \) and \( U_\perp \). Figure 3 shows the different regimes. Note once again that these solutions exist only for \( \Gamma > 1 \) and \( \rho c^2 \ll 2U_+^2 \). For all other cases the coefficients \( \mu_1, \mu_2, \mu_3, \) and \( \mu_4 \) are not constant. Note that we would generally expect that \( \mu_1 \approx -\mu_2 \), since otherwise the rearrangement process would be acting preferentially for changes in geometry in one specific direction, which seems arbitrary. Thus, these results reduce to the well known \( \eta \approx \frac{1}{4} \).

We can look for solutions in the presence of dissipation of magnetic energy. We now have to consider equation (14). We assume that the energy goes completely into relativistic particles; thus energy conservation implies

\[
\frac{d p'}{dz}_{\text{dissipation}} = -\frac{1}{3} \frac{d U'}{dz}_{\text{dissipation}}. \tag{30}
\]

The particle energy can subsequently be radiated away as isotropic radiation. As long as \( p' \ll U'_\perp \), the radiative case is no different from the nonradiative case, since the adiabatic term in the particle pressure does not contribute to the dynamics.

A power-law–type solution is once again possible only if the enthalpy is negligible compared to the magnetic energy density (see § 4.2). In that case the solution in the self-similar range is given by \( \Gamma_{\infty} \propto p_{\text{ext}}^\eta \) with \( \eta = 1/[4 + 6\alpha(3 + 5\zeta)/(3 - 3\zeta^2 - 2\Lambda - 6\zeta) \leq 1 \), with \( \eta = 1 \) at \( \zeta = -1 \). We have plotted \( \eta \) as a function of \( \zeta \) and \( \Lambda \) in Figure 4. For \( \zeta < -\frac{1}{2} \) the efficiency is larger than in the case without dissipation; for \( \zeta > -\frac{1}{2} \) it is smaller. The limiting efficiency that can be achieved in such a jet under the condition that \( d R/dz > 0 \) is given by \( \eta \leq \frac{1}{2} \), as \( \Lambda \rightarrow 0 \). This happens because the dissipative process can convert energy in the parallel field component \( U_\parallel \) (which does not enter eq. [13]) into particle pressure, which must be taken into account in the energy balance. Also shown are areas in \( \zeta \)-\( \Lambda \) space where the conditions \( d R/dz > 0, d R/dz > 0 \) are not satisfied.

4.1. Opening Angles and Causal Contact

Since a jet is generally defined as a collimated outflow, we can ask under which conditions the solutions from above are actually collimated. The collimation condition \( d R/dz < 1 \) translates to a pressure gradient...
No. 1, 2000 JET ACCELERATION BY TANGLED MAGNETIC FIELDS 111

\[ -d \ln p_{\text{ex}}/d \ln z = \xi < 4 \]

as long as the jet is magnetically dominated, the same as in the particle pressure-dominated case. The presence of dissipation can alter this value. Generally, the collimation is increased by dissipation since the sideways pressure is reduced; thus the jet does not need to expand as much. In the coasting phase, where the jet is no longer accelerating, this condition changes to \( \xi < 3 + \xi < 4 \).

Given the solutions from above, we can investigate the ratio of the opening angle \( \alpha_{\theta} \) to the beaming angle \( \alpha_{b} \sim \Gamma^{-1} \). In the absence of dissipation we can write

\[ \alpha_{\theta}/\alpha_{b} = \Gamma \frac{dR}{dz} \sim z^{\xi/2 - 1}, \]

independent of \( \zeta \). Thus, for steep pressure gradients \( \xi > 2 \), the opening angle will grow faster than the beaming angle and will thus always become larger even if it starts out being smaller. For shallow pressure gradients \( \xi < 2 \), the situation is reversed; i.e., the beaming angle will eventually become larger than the opening angle. The presence of dissipation changes this behavior qualitatively: the ratio \( \alpha_{\theta}/\alpha_{b} \) now depends on both \( \Lambda \) and \( \zeta \), as illustrated in Figure 4.

This has consequences for the appearance of the jet, since the effective beaming angle is given by the larger of the two angles. Under the assumption that the jet is always collimated, the opening angle in the coasting phase will always become smaller than the beaming angle, since the jet does not accelerate anymore. This could have important consequences for the morphology of superluminal sources: if the beaming angle were smaller than the opening angle, one might expect to see larger jet misalignments, or lose the jet morphology altogether. The appearance would become sensitive to the emissivity and local Lorentz factor as a function of position across the jet cross section.

Jets that expand too fast will eventually lose causal contact with their environment. This happens when the Alfvén crossing time of the jet becomes larger than the expansion time (in the comoving frame), i.e., when

\[ \tau_{\Lambda} = \frac{R}{v_{\text{Alfvén}}} \approx \frac{R}{c} > \tau_{\text{exp}} = \frac{p'}{\Gamma v dp'/dz} \approx \frac{z}{\Gamma c \zeta} \]

(32)

(where we approximated \( v_{\text{Alfvén}} \sim c \)) or

\[ R > \frac{z}{\Gamma c \zeta}. \]

(33)

This corresponds (up to the factor \( \xi \)) to the criterion when \( \alpha_{\theta} > \alpha_{b} \). Thus, for \( \xi > 2 \) (in the absence of dissipation) the jet will eventually lose causal contact with its surroundings (see Figure 4 for values of \( \zeta \) and \( \xi \) where this is the case). As mentioned in § 2, a quasi-one-dimensional treatment is no longer possible since the internal pressure balance is now regulated by shocks traveling inward from the jet walls. After the jet reaches the terminal phase, it will regain causal contact, since the opening angle will continually decrease (assuming the jet is still collimated).

4.2. Equipartition

Constant pumping of magnetic energy into particle pressure can lead to equipartition between \( U' \) and \( p \). We can use the self-similar solution to estimate \( \Gamma_{\text{eq}} \) where the accumulated particle pressure surpasses the magnetic energy density (including effects of adiabatic cooling on the accumulated particle pressure, where we assume that it behaves as a relativistic gas, which gives an upper limit on the resulting pressure). Thus, for \( \Gamma_{\text{eq}} \approx \Gamma_{\text{eq}} \) the solution might be altered. For some parameter values \( p \) never reaches the level of \( U' \). In that case we estimate the asymptotic ratio \( p'/U_{\text{eq}} \). Figure 5 shows the results of those estimates. For large \( \zeta \), equipartition can be reached quickly; thus, unless the energy going into particles is subsequently radiated away, the assumption that the particle pressure be negligible compared to the magnetic field energy density will be violated beyond \( \Gamma_{\text{eq}} \).
We define the energy distribution function as
\[
f(\gamma) \equiv F_0 \gamma^{-s}, \quad \int_{\gamma_1}^{\gamma_2} f(\gamma) d\gamma = n',
\]
where \(\gamma\) is the Lorentz factor according to a particle’s random motion, measured in the comoving frame, and \(\gamma_1 \ll \gamma_2\) are the lower and upper spectral cutoffs. Since we assume that the magnetic field is dominating the internal energy budget, synchrotron losses can be very strong, provided the particle energy spectrum is flat enough so that most of the energy is at high particle energies (i.e., \(s < 2\)). In that case we can expect most of the energy to be radiated away immediately and the corresponding electrons will lose most of the inertia, thus the dissipated energy will not lead to a buildup of particle pressure. If, however, synchrotron losses are weak compared to dissipation (e.g., if the spectrum is too steep, or if synchrotron self-absorption traps most of the radiation to inhibit cooling), the effects of particle pressure can become important, as demonstrated in Figure 5. For a discussion of the observational effects of the different radiative scenarios see § 5.1.

4.3. Full Solutions

We can solve the full equation (14) in the regime \(\Gamma \gg 1\), i.e., for relativistic jets. As mentioned before, the pressure balance condition leads to a simple algebraic equation in \(\Gamma\) and \(R\). In the absence of dissipation and gravity, equation (13) is in fact another algebraic equation relating \(\Gamma\) and \(R\); thus, the two equations can be solved for \(\Gamma(p_{\text{ext}})\) using a numerical root finder. Apart from reproducing the scaling behaviors established in § 3, this will enable us to determine the terminal Lorentz factors and the length scales over which the transitions between different dynamical phases occur. Furthermore, we can use the full dynamical model to investigate the evolution of such observational quantities as polarization and synchrotron brightness.

In the absence of dissipation, the terminal Lorentz factor that can be reached with such a jet is simply
\[
\Gamma_\infty \equiv \lim_{p_{\text{ext}} \to 0} \Gamma = \Gamma_0 \left( \frac{\rho_0 c^2 + 2 U_{L,0} + 4 p_0}{\rho_0 c^2} \right),
\]
where the subscripts “0” denote quantities evaluated at some arbitrary upstream point.

This simple solution is no longer possible in the presence of dissipation, which introduces a sink term into equation (13). As a result, we have to use equation (14) instead. Once the energy has been converted into particle pressure, it can be radiated away as isotropic radiation, which will not affect the dynamics of the jet any further (assuming that \(p\) is dynamically unimportant). If the energy is stored as particle pressure, the pressure could eventually become dynamically important (see § 4.2). Until that happens, though, the two solutions are identical. The terminal Lorentz factor is always reduced (see § 5.1), but the acceleration efficiency can be increased for \(\xi < -\frac{3}{2}\) (see § 4). We have plotted the solution for the radiative case (the one case solvable analytically) in Figure 6.

4.4. Radiation Drag

The presence of ultrahigh-energy particles in active galactic nucleus (AGN) jets suggests that inverse Compton (IC) enhanced radiation drag might be dynamically important. While O’Dell (1981) initially suggested that pair jets might be accelerated by the Compton rocket effect, Phinney (1982) showed that it is hard to accelerate a plasma beyond fairly modest Lorentz factors by radiation pressure without a continuous source of particle acceleration to offset the strong IC cooling of the plasma. Furthermore, if the radiation source is not pointlike, the terminal Lorentz factor is limited by the solid angle \(\Omega\) the radiation source subtends. On the other hand, radiation drag can hamper the bulk acceleration of plasmas containing relativistic particles in the presence of a radiation field, if those particles are continuously reheated to overcome the IC losses. The dissipation mechanism discussed above could provide such reheating. Here we will consider the effect of radiation drag in the simplest possible prescription.

We assume that the IC cooling and the dissipational heating time scales are short compared to the adiabatic timescale. If this is not satisfied, the influence of radiation drag will be reduced. We can then expect dissipational heating to nearly balance IC losses in a near equilibrium situation. Thus the amount of IC drag is controlled by how much dissipation there is. For this approximation to be valid, IC losses must dominate the loss processes of the particles; i.e., the radiation energy density \(U'_{\text{rad}}\) in the comoving frame must be large compared to the magnetic field energy density \(U'\) (for large enough \(\Gamma\) this is always going to be the case, since the external field will be Doppler boosted). Finally, we assume that \(\langle B^2 \gamma^2 \rangle \gg 1\), where \(\gamma\) is the particle Lorentz factor in the comoving frame. This sets an upper limit of
\[
U'_{\text{rad}} \ll 6 \times 10^3 \text{ ergs cm}^{-3}
\]
\[
\times \left( \frac{\Gamma U'}{\rho c^2} \frac{d \ln \Gamma R^2}{d \ln z} \frac{10^{14} \text{ cm}}{z} \right)
\]
on the comoving radiation energy density (otherwise IC cooling would have lowered the upper spectral cutoff to \(\gamma_2 \sim 1\)). These assumptions allow us to eliminate \(U'_{\text{rad}}\) from
the equations, since the drag term and the cooling term are both proportional to $U_{\text{rad}}$. In a sense, then, we are presenting an upper limit on the importance of IC radiation drag over large length scales. It has to be kept in mind, though, that drag can be much more important in nonstationary situations (like, for example, in shocks), which are beyond the scope of this paper.

We assume the jet is moving through a radiation field that is locally isotropic in the lab frame. Following the Phinney (1982) treatment, we can calculate the loss rate and the force density due to IC scattering in the comoving frame and then transform back to lab frame to find the additional term for equation (13). We find that radiation drag always decreases both the acceleration efficiency $\eta$ and the terminal Lorentz factor $\Gamma_{\infty}$ by moderate amounts. It does not, however, introduce qualitatively new features. To demonstrate this, we have plotted a solution including radiation drag for otherwise identical parameters in Figure 6.

5. DISCUSSION

5.1. Trade-off between Dissipation and Acceleration and Synchrotron Brightness

In the following section we will investigate the observational effects of the jets we have introduced in this paper. A highly dissipative jet will radiate away a large fraction of its internal energy along the way before reaching the terminal Lorentz factor $\Gamma_{\infty}$, while a nondissipative jet will convert all its internal energy into kinetic energy flux. Since the jet will ultimately terminate and reconvert its kinetic energy flux into random particle energy when it slams into the surrounding medium, the ratio of kinetic luminosity (which could be estimated based on the energy input into the lobes, based on the source size and its age) to the radiative luminosity $L_{\text{diss}}(z)$ (i.e., the integrated luminosity of the jet before reaching the terminal shock) should give us some indication of the importance of dissipation.

We have already seen in § 4.3 that the presence of dissipation can lower $\Gamma_{\infty}$, thus lowering the kinetic energy flux at the terminal shock, $L_{\infty}$ (dominated by cold kinetic energy flux), and the produced hot-spot luminosity. A given fraction $b$ of the terminal luminosity $bL_{\infty}$ will be radiated away, which can be estimated from the hot-spot and cocoon luminosity (calculating $b$ is, of course, a highly nontrivial matter), giving us a handle on $\Lambda$. We have plotted the ratio

$$e \equiv \frac{L_{\text{diss}}}{L_{\infty}}$$

in Figure 7. To make that plot, we chose parameters such that in the absence of dissipation the jet would reach $\Gamma_{\infty}(\Lambda = 0) = 2U_{\infty}/(\rho_0 c^2) = 100$. If we had chosen a larger (smaller) value of this parameter, the lines in the plot would move down (up), since $\Gamma_{\infty}$ depends nonlinearly on both $\Lambda$ and $2U_{\infty}/(\rho_0 c^2)$, so this plot is only a representative one of a family of similar plots for different $\Gamma_{\infty}(\Lambda = 0)$.

Another question is what the spatial distribution of the jet emission is, since there are several competing effects: the optical depth to self absorption (and inverse Compton upscattering), Doppler boosting and opening angle (the larger of which will determine the opening angle of the cone into which most of the radiation goes), and of course the dissipative power of the jet itself [depending on $\Lambda$ and $\Gamma(z)$, $R(z)$]. This question can be asked with respect to the frequency integrated brightness $I$ or the spectral brightness $I_{\nu}$. If we assume that all the dissipated energy is radiated away on the spot, we can calculate the local dissipation rate, which must then be equal to the local frequency integrated jet emissivity $j'$ in the comoving frame. This assumption depends on the injected particle energy spectrum. If the spectrum is flatter than $s = 2$, most of the energy is in the high-energy particles. In that case, synchrotron cooling can be efficient enough for our assumption of on the spot radiation to be effective. If, on the other hand, $s > 2$, most of the energy is in the low-energy particles, synchrotron radiation will not be efficient (unless $\gamma_1$ is very high, in which case the injected spectrum would rapidly cool to a quasi-monoenergetic distribution), and the jet will accumulate particle pressure or radiate by other means (note that Compton cooling would be equally insufficient to balance heating in this case).

To investigate the first case we will set $s < 2$. Given a viewing angle we can then determine the observed total intensity $I$, given by

$$I \propto \left(\frac{dU'}{dz}\right)_{\text{diss}} D^2 \frac{R}{\sin \theta},$$

where $\theta$ is the angle between line of sight and jet axis and $D \equiv [\Gamma(1 - v \cos \theta)]^{-1}$ is the Doppler factor. This expression takes relativistic beaming and the relativistic corrections to foreshortening into account. One might expect that the integrated brightness peaks at a certain distance from the core, since $D$ is strongly peaked at $\Gamma \sim 1/\theta$. However, in our prescription the dissipation drops off too fast for this effect to be important. The main difference in the brightness evolution is that a dissipative jet has a different efficiency $\eta$ and generally expands less rapidly in the sideways direction (and will reach a smaller terminal Lorentz factor $\Gamma_{\infty}$). This effect will become important once the jet has reached $\Gamma_{\infty}$ and only for $\Lambda$ large enough to significantly alter the dynamics ($\Lambda \gtrsim 0.1$). We have plotted $I$ as a function of $z$ for...
different values of $\Lambda, \zeta = 0$, $U_0 = 20\rho_0 c^2$, and a viewing angle of $\theta = 10^\circ$ in Figure 8, arbitrarily normalized to $I(\Lambda = 0.01)$ to increase dynamic range (the brightness decreases by many orders of magnitude along the jet). As is obvious from the plot, for small $\Lambda$ only the overall normalization of $I$ varies with $\Lambda$, whereas for large enough $\Lambda$ the brightness distribution itself changes shape because of the altered dynamics. Also shown are the Doppler factors $D$ for the different parameters, which are primarily responsible for the different shapes.

The situation can become more difficult in the opposite case, i.e., if most of the radiation is trapped (e.g., by synchrotron self-absorption, which implies a steep spectrum, $s > 3$) or if the deposited energy is simply not efficiently radiated (e.g., if $2 < s < 3$). In that case the emission of dissipated energy could be delayed, leading to a relative brightness peak downstream. To briefly investigate this possibility, we assume the latter case, i.e., $2 < s < 3$, which is not an unreasonable choice for AGNs (see § 5.3, for example). We assume that the energy flux $\nu F_\nu$ peaks at high energies $\nu_p$: either at the spectral break of $\lambda \nu \sim \frac{1}{2}$ expected in a scenario in which high-energy particles are constantly reinjected, where the spectral index $\alpha$ is given by

$$\alpha = \frac{d \ln I}{d \ln \nu},$$

or at the spectral cutoff produced by synchrotron and IC cooling (in the absence of a strong break). The position of $\nu_p$ depends on adiabatic effects, radiative cooling, and heating due to dissipation. The spectrum will be self-absorbed at low frequencies, which generally leads to an observed spectral index of $\alpha \sim -\frac{2}{3}$ (for an exact treatment of the spectral shape at the self absorption turnover, see De Kool, Begelman, & Sikora 1989). If we take $2 < s < 3$ or $\frac{1}{2} < \alpha < 1$, the self absorbed part contributes a negligible fraction to the total brightness and most of the energy is emitted at the high end of the spectrum.

In this case it is impossible to calculate the brightness analytically as a function only of $\Gamma, R$, and $z$. Rather, one can numerically integrate the evolution equation of the peak frequency under adiabatic cooling, dissipative heating [we assume a self-similar transfer of energy from magnetic field to the particles such that $(dU/dz)_{\text{diss}} \propto \gamma (dU'/dz)_{\text{diss}}$, and synchrotron cooling on the basis of the solution $\Gamma(z)$ given above. We can estimate the run of $I$ by scaling it with the brightness at $\nu_p$, taking account of relativistic beaming and aberration. If we choose to normalize the intensity curves as we did in the previous case, we can get around fixing the absolute normalization of $U'$, since it enters only linearly into the intensity and will thus cancel out upon normalization. We used $U'_{\nu_0} = 20\rho_0 c^2$, along with $s = \frac{3}{2}$ and plotted the frequency integrated intensity in Figure 9 with otherwise the same parameter values as in Figure 8, once again normalized to $I$ for $\Lambda = 0.01$. Note that for large values of $\Lambda$ our assumption that the jet be magnetically dominated and that particle pressure be negligible can break down (see Fig. 5), so curves with high $\Lambda$ are to be taken with a grain of salt. Nevertheless, it is interesting to note that the intensity drops less rapidly with $z$ for larger values of $\Lambda$, corresponding to the delayed emission mentioned above. The shapes of these curves depend only weakly on $s$. The different bends in the curves stem from the evolution of the Doppler factor (shown in Fig. 8), and the evolution of the peak frequency. The slopes of the curves are produced mostly by the evolution of the lower cutoff frequency $\gamma_s$ (see eq. [34]), which enters the expression for the synchrotron brightness through the power-law normalization of the particle distribution, and by the evolution of Lorentz factor and jet radius (entering through the particle density and the integration of the emissivity across the jet).

**Fig. 8.** Frequency integrated brightness for a radiative jet (i.e., all the dissipated energy is radiated away on the spot) as a function of $z$ for different values of $\Lambda, \zeta = 0, \xi = 2, U_0/\rho_0 = 20$, and $\theta = 10^\circ$, arbitrarily normalized to the intensity curve for $\Lambda = 0.01$ to increase the contrast (thick black curves). Also shown are the corresponding Doppler factors $D$ as thin gray curves. The shapes of the individual intensity curves are mainly determined by the variation of $D$. Note that a significant observable effect is achievable only for $\Lambda \lesssim 0.1$.

**Fig. 9.** Frequency-integrated brightness for a marginally non-radiative jet (only a dynamically small amount of the dissipated energy is radiated away on the spot) as a function of $z$ for different values of $\Lambda, \zeta = 0, U_0/\rho_0 c^2 = 20, \xi = 2$, and $\theta = 10^\circ$, arbitrarily normalized to the intensity curve for $\Lambda = 0.01$ to increase the contrast (the absolute intensity drops by many orders of magnitude). Note that these curves do not depend on the absolute value of the magnetic field, which cancels out because of the normalization.
5.2. Polarization

While the degree of polarization is highest for homogeneous magnetic fields, jets with tangled or disorganized field can exhibit a net polarization if there is a net anisotropy in the field (Laing 1980). The measured polarization will depend not only on $\zeta$, but also on the viewing angle $\theta$ and the bulk Lorentz factor $\Gamma$.

The polarization of radiation from a power-law distribution of electrons with index $s$ in a region of homogeneous field is given by

$$\Pi = \frac{I_{\perp} - I_{\parallel}}{I_{\perp} + I_{\parallel}} = \frac{s + 1}{s + 3},$$

(40)

where $I$ is the intensity at a given optically thin wavelength. To calculate the integrated polarization, we average across the jet. To do this we decompose the radiation into polarization along the jet axis and perpendicular to it. Furthermore, we assume that field directions are distributed among all solid angles and introduce a weighting function that distributes the field orientations to the required anisotropy,

$$w(\delta) = \sin \delta k$$

(41)

where $\delta$ is the angle between the field and the $z$-axis and $k$ is determined by the anisotropy. We can solve for $k$ under the condition that $\langle B_z^2 \rangle = (1 + \zeta)/(1 - \zeta)\langle B_z^2 \rangle$:

$$k = \frac{1 + 3\zeta}{1 + \zeta}.$$  

(42)

We correct the viewing angle for relativistic aberration, which has a significant impact on the observed polarization, since the average polarization will go to zero for a jet seen head on. Furthermore, the angle between the line of sight and the magnetic field is important in determining the relative brightness of a region. Figure 10 shows the predicted polarization for the cases shown in Figure 6, a spectral index of $\alpha = 0.5$, and a viewing angle of $\theta = 10^\circ$. Since the anisotropy of the jet is fixed, the variation in the polarization $\Pi(\theta)$ is solely caused by changes in the viewing angle due to relativistic aberration. Generally, the polarization will be perpendicular to the jet axis if $\zeta < -\frac{1}{2}$ and parallel if $\zeta > -\frac{1}{3}$. As long as equation (9) holds, an extremum in $\Pi(\theta)$ will be present, and the location of that extremum should indicate the position where $\Gamma = 1/\sin \theta$, i.e., where the viewing angle corrected for aberration is $\theta^* = 90^\circ$. Note that this polarization is averaged across the jet. In order to compare these predictions to actual measurements, a relatively small correction for the emission weighted averaging across the jet at different angles must be made. The qualitative predictions of this section should be unaffected by that caveat.

5.3. Applications

Finally, we sketch out some applications for this model. The obvious candidates for jet models are radio galaxies and all related jet-powered AGNs. The best studied example is M87, since it is the closest unobscured source (though by no means a particularly powerful one). The central black hole has a relatively well determined mass of $M \approx 2.4 \times 10^9 M_\odot$ or $r_s \approx 3.5 \times 10^{14}$ cm. The kinetic power of M87 is estimated to be of order $L_{\text{kin}} \approx 10^{43} - 44$ ergs s$^{-1}$ (Reynolds et al. 1996; Bicknell & Begelman 1996).

Taking the new proper motion measurements based on Hubble Space Telescope data (Biretta, Sparks, & Macchetto 1999) at face value, the terminal bulk Lorentz factor likely falls into the range $6 < \Gamma_\infty < 10$ with a viewing angle of $\theta \sim 20^\circ$. (Note that VLA measurement show slower proper motions [Biretta, Zhou, & Owen 1995] and the relationship between the observed pattern speeds and $\Gamma$ is not known. There are, however, no direct measurements of bulk motions, so for lack of better knowledge we will use the larger values obtained from the optical data.) The average polarization at kiloparsec distances (where the jet has likely reached terminal velocity) is roughly $\Pi \sim 10^\%$ parallel to the jet axis, which corresponds to $\zeta \sim 0.3$ with the numbers given above.

Reynolds et al. (1996) showed that the jet probably consists of pair plasma rather than ionized gas, so the jet might actually still be accelerating (if indeed a large fraction of the particles has $\gamma > 10$). Note, however, that the presence of shocks, clearly visible as knots in all wavelengths, calls for a more sophisticated model that takes time-dependence and MHD instability effects into account. For a discussion of the nature of the shocks observed in M87, see Bicknell & Begelman (1996). Furthermore, there is evidence that the jet is not magnetically dominated on large scales ($z > 100$ pc) and that the magnetic field is actually somewhat below equipartition (Heinz & Begelman 1997), which means in this context that the magnetic field must have been dissipated nonradiatively, accounting for the particle pressure at larger distances. The observed spectral index at optically thin wavelengths is $\alpha \sim 0.6$, which is at least in the correct regime for synchrotron radiation to be inefficient at radiating away the dissipated energy (see § 5.1).

At a viewing angle of $20^\circ$ the jet radius at knot A ($z \sim 3$ kpc) is $R \sim 35$ pc with an approximate opening angle of $\alpha_0 \sim 0.7$, much smaller than the beaming angle of $\alpha_0 \lesssim 6^\circ$; thus the jet is narrow, at least at VLA resolution. Note that entrainment of ambient material may be important before the jet reaches knot A (Bicknell & Begelman 1996). Using knot D as a reference point does not change this analysis significantly, however. There are no good estimates of the pressure gradient in the innermost regions of the M87 X-ray
atmosphere (this situation should change, though, with the launch of Chandra). The pressure at large distances is approximately $p_{\text{ISM}} \sim 10^{-16}$ dyn cm$^{-2}$, but Bicknell and Begelman (1996) argue that the pressure in the radio lobes is significantly higher than this, $p_{\text{ext}} \sim 1.5 \times 10^{-9}$ dyn cm$^{-2}$. For lack of better information we will assume the latter value and a smooth pressure gradient with $\xi = \text{const.}$ The radiative luminosity of the jet itself is $L_{\text{rad}} \gtrsim 3 \times 10^{44}$ ergs s$^{-1}$ (Reynolds et al. 1996), which argues for $\Lambda \gtrsim 4 \times 10^{-3}$. If (as we speculated above) much of the particle pressure at large distances was indeed produced by dissipation, $\Lambda$ could be much larger.

The jet probably reaches terminal $\Gamma_{\infty} \sim 10$ somewhere between VLBI scales and VLA scales, $2 \times 10^{3} r_{g} \sim 0.2$ pc $< z_{\infty} < 200$ pc $< 2 \times 10^{6} r_{g}$. For $\Lambda$ small enough to be dynamically unimportant we can assume that $\eta \sim \frac{1}{2}$ in the self-similar regime. Arbitrarily setting $z_{0} \sim 10 r_{g}$ gives $0.8 \lesssim \xi \lesssim 1.74$. The central pressure $p_{0}$ is then $2 \times 10^{-4}$ dyn cm$^{-2} < p_{0} < 200$ dyn cm$^{-2}$, which requires an rms magnetic field of $0.09$ G $< B_{0} < 90$ G for pressure balance (using $\xi \sim 0.3$). The initial jet width strongly depends on $\xi$: $8 r_{g} < R_{0} < 9000 r_{g}$ for an assumed $z_{0} = 10 r_{g}$. The latter value is unrealistic (and inconsistent with the limits put on the jet width by VLBI observations; Junor & Biretta 1995) since most of the energy output of the disk into the jet will be provided close to the black hole. It is therefore most reasonable to assume that $z_{\infty} \sim 2 \times 10^{4} r_{g}$ and $\xi \sim 1.7$. The total jet power implied by the numbers given is of order $L \sim 2 \times 10^{44}$ erg s$^{-1}$, consistent with the estimate by Bicknell & Begelman (1996). Overall it seems that this model is consistent with the observed properties of M87 to first order if we adopt a pressure gradient following $\xi \sim 1.7$.

The total isotropic energy output of $E \gtrsim 10^{44}$ ergs of GRB 990123 (Bloom et al. 1999) argues strongly in favor of nonisotropic gamma-ray burst (GRB) scenarios, so jet models explaining the apparently beamed nature of these sources (e.g., Meszáros & Rees 1997; Sari, Piran, & Halpern 1999) enjoy newly enhanced popularity. One of the standard scenarios for the energy sources of GRBs is a massive accretion event (either a neutron star–neutron star merger or the accretion of a neutron star by a black hole; Narayan, Paczyński, & Piran 1992), which leads to the formation of a disk after tidal disruption of one of the objects. Since neutron stars already display large magnetic fields, one might expect that they be amplified by the interaction with the jet. We estimate the impact of dissipation of magnetic energy on the dynamics of the flow by considering a simple, phenomenological prescription of the loss process. The presence of dissipation lowers the terminal Lorentz factor $\Gamma_{\infty}$ and generally changes the rate at which the jet is accelerated (the latter effect is noticeable only if the dissipation rate is comparable to the adiabatic expansion rate). We also include a simple prescription of the effects of magnetic energy. We find that radiation drag always lowers the efficiency $\eta$ and $\Gamma_{\infty}$. The amount of radiation drag in our model is controlled by the amount of dissipation replenishing the high-energy particle pool but seems to be dynamically unimportant.

We calculate the frequency integrated surface brightness for the extreme cases where all or very little of the dissipated energy is radiated away on the spot and find that, while the brightness drops off very rapidly in all considered cases because of the expansion of the jet, values of the dissipation efficiency $\Lambda \gtrsim 0.05$ can have a significant impact on the intensity as a function of $z$. In the marginally nonradiative case the buildup of particle energy leads to a slower decline in intensity with $z$ for larger $\Lambda$. Finally, we apply this model
to the prototypical radio galaxy M87 and find that it is consistent with the observed properties.

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APPENDIX

A MORE REALISTIC DISSIPATION LAW

The dissipation law we assumed in § 2.1.2 was only one of many plausible ad hoc models. Since (for finite conductivity, i.e., beyond the limit of perfect MHD) reconnection will occur whenever there is field reversal on sufficiently small scales (which would certainly be the case in a highly tangled geometry), we would expect dissipation to occur even if there were no change in field geometry due to expansion or acceleration. In that case we might expect that the dissipation timescale would be proportional to the time it takes a disturbance to travel a given characteristic length (e.g., the jet width) in the comoving frame, i.e.,

$$\frac{dU_i}{dz} \bigg|_{\text{diss}} \sim \Lambda U_i \frac{v_{\text{Alfven}}}{\Gamma R},$$

where the parameter $\Lambda$ absorbs the effects of resistivity and all the unknown physics of the reconnection process. Once again, it is straightforward to generalize to the case of different $\Lambda_i$ for different components of the field.

It is possible to solve the set of equations in the magnetically dominated case (i.e., setting $\rho' + 4p' = 0$), under the assumption that $\zeta = \text{const}$. and, in the relativistic limit, $\Gamma \gg 1$. In this limit $v_{\text{Alfven}} = 1$, which simplifies the treatment significantly. We assume that the dissipated energy is radiated away immediately, which leads to a modified equation (14):

$$\frac{\Lambda}{\Gamma R} \left( \frac{2 + 6\zeta}{3 - 3\zeta} + (2 + 2\zeta) \frac{d\Gamma}{\Gamma dz} - (2 + 2\zeta) \frac{dR}{R dz} \right) = 0. \quad (A2)$$

The pressure balance equation (16) is also modified:

$$\frac{(\zeta - 1)}{14} \frac{d\Gamma}{dz} - (3 + \zeta) \frac{dR}{R dz} - \frac{\Lambda}{\Gamma R} = - \frac{\zeta}{z}, \quad (A3)$$

where we used $U' \propto p_{\text{ext}}$ and equation (19). This set of equations can be solved to give

$$\Gamma(z) = \Gamma_0 (z/z_0)^{\zeta/4} \{1 - A[(z/z_0)^{1-\zeta/2} - 1]\}^{(3 + 5\zeta)(1/4 + 4\zeta)} \quad (A4)$$

with

$$A \equiv \frac{\Lambda}{\Gamma_0 R_0} \frac{1}{(1 - \zeta/2)(1 + 3\zeta)}. \quad (A5)$$

Subscripts “0” denote quantities evaluated at some arbitrary upstream point $z_0$. In the limit of $\Lambda \to 0$ this solution appropriately reduces to the result without dissipation, i.e., $\Gamma \propto z^{\zeta/4}$. The solution can essentially take on three different behaviors. If $\zeta > 2$, the solution will asymptotically approach $\Gamma \propto z^{\zeta/4}$, i.e., $\eta = \frac{1}{2}$. If $\zeta < 2$, on the other hand, two different scenarios can occur: the flow can either approach a self-similar behavior with $\eta \neq \frac{1}{2}$ (but still constant) in the limit of $z \gg z_0$, or, if $A > 0$ (i.e., $\zeta > -\frac{1}{2}$), the flow can actually stall, i.e., $\Gamma \to 0$ for $z \to z_0[[1 - \zeta/2(1 + 3\zeta)/A - 1]^{1/(1-\zeta/2)}].$ In this limit, the magnetic field dissipates away too quickly to satisfy pressure balance and the jet must contract and decelerate to increase its internal pressure, thereby increasing its dissipation rate, which leads to a runaway process. Eventually, the massless approximation will break down, in which case the Alfvén velocity will drop, lowering the dissipation rate (furthermore, the particle pressure will gain in importance, eventually stabilizing the jet against external pressure, in which case the jet would behave as described by BR74). This process would produce an observable hot spot (and possibly a shock) at a fixed distance.

REFERENCES

Begelman, M. C. 1995, Proc. Natl. Acad. Sci., 92, 11442 (B95)
Bicknell, G. V., & Begelman, M. C. 1996, ApJ, 467, 597
Biretta, J. A., Sparks, W. B., & Macchetto, F. 1999, ApJ, 520, 621
Biretta, J. A., Zhou, F., & Owen, F. N. 1995, ApJ, 447, 582
Blandford, R. D., & Payne, D. G. 1982, MNRAS, 199, 883
Blandford, R. D., & Rees, M. J. 1974, MNRAS, 169, 395 (BR74)
Bloom, J. S., et al. 1999, ApJ, 518, L1
Boswell, W., & Biretta, J. A. 1995, AJ, 109, 500
Daugherty, R. A. 1980, MNRAS, 193, 439
De Kool, M., Begelman, M. C., & Sikora, M. 1989, ApJ, 337, 66
Heinz, S., & Begelman, M. C. 1997, ApJ, 490, 653
Junor, W., & Biretta, J. A. 1993, AJ, 105, 500
Laing, R. A. 1980, MNRAS, 193, 439
Li, Z. Y., Chiueh, T., & Begelman, M. C. 1992, ApJ, 394, 459
Meszaros, P., & Rees, M. J. 1997, ApJ, 482, L29
Narayan, R., Paczyński, B., & Piran, T. 1992, ApJ, 515, L69
O'Dell, S. L. 1981, ApJ, 243, L147
Paczynski, B. 1998, ApJ, 494, L45
Parker, E. N. 1958, ApJ, 128, 664
Phinney, E. S. 1982, MNRAS, 198, 1109
Reynolds, C. S., Fabian, A. C., Celotti, A., & Rees, M. J. 1996, MNRAS, 283, 873
Sari, R., Piran, T., & Halpern, J. P. 1999, ApJ, 519, L17
Thompson, C. 1994, MNRAS, 270, 480
Woosley, S. E. 1993, ApJ, 405, 273