Epistemological Obstacles on the Topic of Ratio and Proportion among Junior High School Students

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Abstract. This study intends to investigate how students’ understanding of ratio and proportion concept indicates epistemological obstacles. It was part of Didactical Design Research which was conducted to 72 students of 8th grade who ever learned about ratio and proportion. Data were collected through the students’ answers and interview in solving ratio and proportion problems. The results show that students’ conception, the ways of applying other mathematical concepts, the ways of using the rules, and variety of contexts are factors influencing epistemological obstacles in teaching and learning of ratio and proportion. These conditions can affect the students’ understanding of related mathematics topics. Based on analysis of the results, this study is expected to overcome or minimize the epistemological obstacles.

1. Introduction

One of the obstacles that the teachers need to consider in mathematics teaching and learning is epistemological obstacle. It has to be done for developing anticipative relationship between teacher and materials called didactical and pedagogical anticipation, besides didactical relationship and pedagogical relationship [1]. Epistemological obstacle is the obstacle in gaining new knowledge. The emergence of this obstacle was due to the limited of students’ understanding or knowledge in certain contexts [2, 3]. It is indicated by students’ difficulties to apply concept or knowledge to different contexts. This investigation is based on the ways of thinking and the ways of understanding [4]. Besides that, epistemological obstacle is also seen as an obstacle which is related to the aspects of understanding a concept or regarding knowledge in which there was a wrong way of understanding new knowledge [5]. As we know that understanding the concept is one of common goals in mathematics teaching and learning [6]. When the students can understand a concept well, they can work on many problems and develop other mathematical skills. It can be achieved by providing a variety of contexts [7] so that the students can develop their experience and knowledge.

Regarding the importance of conceptual understanding in mathematics teaching and learning, one of the topics that shows the emergence of epistemological obstacles is ratio and proportion. This is one of the topics which exists in curriculum in Indonesia [8] so that it is important to be mastered by students. This topic will be widely encountered and utilized by students when learning other mathematics topics, learning science or other disciplines, and solving problems in daily life [9, 10, 11, 12]. Understanding of the concept of ratio and proportion became a central to mathematics, as a basis of rational number operations, unit partitioning, and solving algebra as well as geometry problems [13]. The students need to understand about the representation of ratio as \( a:b \) where \( b \neq 0 \) [14] so that they can be trained to understand proportion. They also need to know about the relation of
proportion with the same product, “if \( \frac{a}{b} = \frac{c}{d} \), then \( ad = bc \), where \( a, b, c, \) and \( d \) are positive integers” [15] and to understand how the idea works. It leads to how the students develop their understanding creatively.

The facts show that student experiences based on previous didactical designs have an impact on the emergence of learning obstacles. Many studies found that the students do not understand basic concept of ratio and just can solve the problems which can be solved using calculations because their proportional reasoning is not developed [16, 17, 18, 12, 19]. In addition, most students have not been able to identify and distinguish the problems of direct proportion and inverse proportion because they do not understand the meaning of mathematical model for those kinds of proportion [20, 21, 19]. They also find the difficulties to solve the problems of ratio and proportion when dealing with different contexts [12, 22]. These facts surely affect not only students’ understanding in learning other mathematics topics, but also in learning science. The students are going to deal with intensive quantities, such as density, speed, and temperature, which combine direct and inverse proportionality [10].

Based on the results of studies that had been done related to identification of learning obstacles, this study basically found some learning obstacles, including epistemological obstacle. This paper is intended to investigate more about the factors influencing epistemological obstacles. The findings are expected not only to show an understanding of epistemological obstacles in the topic of ratio and proportion, but also to be the basis for developing didactical designs. This is important for teachers because the teachers play a very important role in designing didactical situations so that the teaching and learning process occurs within the students [1]. This role needs to be supported by the teacher’s knowledge of the things that might hinder the learning process, especially knowing the epistemological obstacles which are related to how students acquire the knowledge from the contexts. The teachers should be able to predict every student’s response. Thus, this study is expected to overcome or minimize the emergence of epistemological obstacles in teaching and learning of ratio and proportion.

2. Method

The study used an interpretive paradigm which was part of Didactical Design Research. This paradigm was used by researchers to view the world through the perceptions and experiences of participants [23]. It was used in this study to identify epistemological obstacles in the topic of ratio and proportion based on previous didactical design. It would be the first step in didactical design research which could be followed up on critical paradigm. It means that this study would lead to design innovations based on the learning obstacles identified.

This study was conducted to 72 students (two classes which consist of 35 and 37 students) of 8th grade in one of junior high school in Bandung, Indonesia. The participants were the students who had learned about ratio and proportion. This condition was selected because the researchers wanted to know students’ ability in understanding ratio, proportion, and its applications based on previous didactical design. Each student was given a test which was consisted of six problems related to ratio and proportion. The time provided was 80 minutes. After they did the test, the researchers observed the students’ answer and chose some students for an in-depth interview. It was needed to attain a deeper students’ understanding of the problems and to find out the epistemological obstacle they have got. Furthermore, the interview data was analyzed qualitatively.

3. Result and Discussion

Based on the analysis of students’ answers on the test given, the researchers found the epistemological obstacles which were identified from students’ strategies in solving the first, fifth, and sixth problems. The results show that students’ conception, the ways of applying other mathematical concepts, the ways of using the rules, and variety of contexts are factors influencing epistemological obstacles in teaching and learning of ratio and proportion.
3.1. Students' Conception
There are several conditions how students’ conceptions indicate the epistemological obstacles. It becomes one of the factors when the students already know what concept should be used, but there is a misunderstood part of the concept they have known. This can be depicted in students’ way in solving Problem 1 in figure 1.

Mrs. Fitri has two teams which are going to participate in science competition. Team A consists of 2 girls and 3 boys, while team B consists of 2 girls and 4 boys. Which team has more girls?

Figure 1. Problem 1 about comparing ratios.

The researchers found that the epistemological obstacle was identified both in the students’ correct answers and in the wrong ones. Figure 2 shows that the student has the correct answer, he uses fraction as the notation of ratio, but he just tries to guess using irrelevant information to answer the question. He has perceived that it has unlikely answer if both teams have same proportion of girls because there is another condition which is different. The difference affects his understanding to notice the quantities of boys in every team. This is related to students’ conception since they have understood which team has more girls section or part, but their conception is distracted with another condition. It can be denoted as an understanding in additive situation. It is also known from the interview. He cannot explain the relationship between the students’ number and proportion being asked. He also cannot explain why they choose the least students number to determine more girls proportion. It means that the students who were able to apply certain concepts to the problems did not necessarily have any epistemological obstacles [24]. Hence, it needs to be proven by giving the other contexts to the students.

Figure 2. Photograph of student’s answer in solving Problem 1.

The students’ conception is also depicted in student’s answer in figure 3. The student recognizes the way to show ratio in solving the problem, but she still notices the quantity of girls in each team. It makes her think that the answer is “equal”. She can basically apply the concept of ratio, yet she does not try to compare the two ratios obtained and just compares between the quantities only.

Figure 3. Photograph of student’s answer in solving Problem 1.

The students’ conception is also observed when the students solve the problem which is an application of proportion. Most of them have same conception to solve this following Problem 6.
The students explain that stretching is related to additive situation. They just perceive it as increasing the number of quantities as in figure 6. The student chooses photo D and E because if the length of photo E increases 5 cm, then the width also increases 5 cm. He does not think of multiplicative relationship at all.

The result of interview also indicates that the students focus on the increases in size in the given context. Although they realize stretching is related to multiples, they cannot apply it to this problem. They just know that it has to deal with integers, for instance stretching doubled of 5 cm × 10 cm will be 10 cm × 20 cm. This condition is based on the interview below.

**Student**: Since it is about stretching, it must have relationship with multiple, for instance, the reason why I choose photo D and E is...if this quantity (five) is multiplied by two, it must be 10 or 5+5=10, then another quantity (ten) must be added to five and the result is 15.

**Teacher**: So, this is about multiple or sum?

**Student**: So, this is about sum, I think.

**Teacher**: Sum? What do you think of multiple?

**Student**: If this is related to multiple, it should be 20 (stretching of width from 5 cm × 10 cm). So, this quantity of 5 cm becomes 10 cm, and 10 cm becomes 20 cm.

**Teacher**: So, you think that stretching is about multiple or sum?

**Student**: Sum.

**Teacher**: Are you sure?

**Student**: It can be multiple, and the sum can be too.

The interview above indicates a wrong conception in understanding the context involving scaling up. Teaching and learning of proportion leads to the applications which involve scaling up (stretching).
or scaling down (shrinking) [25], such as the context of map-making and scaling in similarity transformations of object in daily life or geometric shapes, but in fact their conception does not fully understand that it deals with multiplicative relationship. The students can imagine the contexts, but they never infer that a multiplicative relationship underlying the concept of ratio and proportion.

Based on the results of students’ answer, the teachers need to undertake to overcome and to anticipate the epistemological obstacles which are caused by students’ conception. They need to have a good grasp and understand the students’ thinking in teaching and learning of ratio and proportion. They have to understand the concept genuinely about quantities, multiplicative relationships between quantities, and the intertwinement between the other mathematical concepts. They need to provide the contexts involving situations in which multiplicative and additive relationships exist so that the students can identify mathematical relationships underlying the concept of ratio and proportion. They also need to give the contexts that lead the students to imagine the situations qualitatively. The proportion problems in the real contexts would provide an opportunity for students to be able to see kind of proportion in terms of qualitative way [26]. This opportunity will ease the students to identify the direct proportion and inverse proportion.

3.2. The Ways of Applying Other Mathematical Concepts

The ways students apply mathematical concepts that they have ever got before can also become a factor influencing epistemological obstacles. Mathematical concepts are essentially interrelated or not emerged partially [27], so the students must apply them properly. Based on the students’ answer, the students try to relate the concepts that have been obtained to solve the problem, but actually they do not have a good grasp of it.

![Figure 6](image.png)

**Figure 6.** Student 1 uses the concept of fraction in comparing proportion of girls in each team (left), while student 2 uses percentage (right).

Figure 6 shows that the students have the correct answer for Problem 1 (figure 1). The students try to use fraction as ratio, but they do not understand why they should use it. They do not understand how to apply the concept of ratio, fraction, or percentage to the problem. They just realize that when they find that kind of problem, it can be solved by the concept of fraction or percentage as revealed by the student in the following interview.

*Teacher:* Why do you use fraction?
*Student:* Because I just understand about it.

*Teacher:* Then why does it become $\frac{2}{3}$ or $\frac{2}{4}$ and what’s the purpose?
*Student:* Hmm... I don’t know, Ma’am. I just know this way.

The wrong ways of applying the concept can also be depicted in figure 7. Some students use that way and do not understand the problem that the question is about the proportion of girls in each team which leads them to compare between the ratio of team A and team B. They think of the problem in additive situation. They solve it by applying the concept of set and using Venn diagram. They set the number of girls as the intersection of sets. They basically answer directly that the answer is "equal". Furthermore, they prove it by using the concept of set. They do not realize the existence of the differences in the number of boys because they think that the question focuses on the number of girls so that they do not need to pay attention to the other conditions.
The way of applying mathematical concept to the problem is also shown by the students in solving Problem 6 (figure 4). The problem has provided all of information about quantities and the students just need to choose and prove their choice using the concept of proportion. It becomes kind of problem that students may rarely get in the class. Figure 8 shows that the student tries to find the meaning of stretching. The information about length and width denoted by \( p \times l \) makes her try to calculate the multiplication so that she finds an area of each photo. When she finds that an area of photo can be scaled up to another area in the choices given, she confidently deems that those photos must be the answer. Therefore, the student constructs a new understanding in the wrong way that stretching can be sought by finding the area.

![Figure 7. Photograph of student’s answer in solving Problem 1.](image)

![Figure 8. Photograph of student’s answer in solving Problem 6.](image)

Based on those ways above, the teachers need to anticipate it by providing the contexts that involve some related mathematical concepts to the topic being taught. They need to emphasize interrelated concepts and to provide students with the right understanding, including the use of concept of fraction, percentages, similarity, and algebra [28].

3.3. The Ways of Using the Rules
The ways of using the rules without understanding it also becomes a factor influencing epistemological obstacles. It can be shown in students’ way solving Problem 5.

**Daniel calculates that the distance on the map between two places is 7.2 cm. A map scale is 1 : 5,000,000. He usually went there by motorcycle from 7 a.m and would arrive at 4 p.m without calculating the interlude. How much speed does Daniel need to get there within 6 hours?**

![Figure 9. Problem 5 about direct proportion and inverse proportion.](image)
they misplace the quantities properly. They also seem to ignore whether the answer makes sense or not because they believe in the formula. It is what the student does in figure 10.

The other condition can be seen in figure 11. The student knows that the first step to do is finding the actual distance because the speed is related to it and time, but she does not understand what the speed formula is. This makes her speculate only with the information she obtains so she uses some number operations without understanding them.

![Figure 10. Photograph of student’s answer in solving Problem 5.](image)

![Figure 11. Photograph of student’s answer in solving Problem 5.](image)

Based on the students’ answer, the way students use the rules or formulas should be noticed by teachers in teaching and learning of ratio and proportion. The teachers need to provide the contexts that lead the students to develop new knowledge based on a formula generally. They do not need to give many formulas to students with the same scope. They just need to manage student’s learning to be active in constructing new knowledge from a concept so that the students do not rely on the existence of the formulas that the teachers give in the class.

### 3.4. Variety of Contexts

Variety of contexts becomes a factor influencing epistemological obstacle based on the students’ answer in solving Problem 1 (figure 1). Some students directly reveal that the answer is "equal", but they do not show it mathematically (figure 12). The fact is the students think that there is something wrong with the problem. They repeatedly ask the researcher "Is this problem wrong?" and do not realize about the situation of the problem at all. They are deceived by the word of “girls” and "more" in the question so that their understanding is still in additive situation. It shows about the lack of contexts since they are not used to face such problem.

![Diagram](image)
Figure 12. Photograph of student’s answer in solving Problem 1.

Similar condition was shown in figure 13 in which the student basically recognizes the concept of ratio, but he tends to notice only the quantities of one object. It can happen because he used to be faced by different quantities so that he never constructs his understanding of the existence of the same quantity with different total of objects.

![Figure 13. Photograph of student’s answer in solving Problem 1.](image)

Related to variety of contexts, it is also shown when the students solve Problem 6 (figure 4). Most students have no idea how to solve the problem. This is because they are basically just faced the contexts about scale factor which comprise three quantities and an unknown. Besides that, the contexts have already informed about the scale, but the students do not really understand the meaning of scale. This situation makes them hard to solve the problem which provides all of the quantities and requires only a proof by using the concept of proportion. It is known from the following interview.

Teacher : Do you know what topic this problem is about?
Student : Ehmm...Ratio and proportion.
Teacher : Ratio and proportion? How about if you solve this using that concept? Can you?
Student : I forget the formula.
Teacher : Do you think that it must use a formula?
Student : Not really, but I just forget the steps.

Based on the conditions above, the teachers should anticipate it by providing variety of contexts so that the students know several applications of ratio and proportion in daily life. The students should be able to apply their knowledge in various situations or contexts [29]. The teachers also need to vary the problem contexts which demand the students to prove the conditions. It means that the students are not always asked to find the unknown. It becomes a learning that will familiarize the students to be skilled in solving problems because they find creative ideas through exploration [19].

The factors influencing epistemological obstacle can be a basic understanding for teachers to design mathematics teaching and learning. Each factor identified becomes a consideration for finding a more effective teaching and learning design. The students’ difficulties and even misconceptions when solving the problems of ratio and proportion could be a starting point for designing effective teaching and learning of its topic [30]. The learning process provided is to emphasize the process of a person to be able to choose, maintain, and transform the information actively [31]. It will affect students’ thinking and understanding on the topic being learned and make the students' memory last long because of the experience they have got in the class.

4. Conclusion

The goals of this study were to investigate the epistemological obstacle on the topic of ratio and proportion and to reveal some factors influencing the epistemological obstacles. It is essential to find out how the students’ understanding of ratio and proportion concept can hinder students’ learning. The factors discovered in this study are students’ conception, the ways of applying other mathematical concepts, the ways of using the rules, and variety of contexts. Students’ conception becomes a factor influencing the epistemological obstacles when the students already know what concept should be used, but there is a misunderstood part of the concept they have known. The way of applying other mathematical concepts is also a factor when the students do not have a good grasp of the concepts used or they misunderstand how to apply mathematical concepts in the problems. Furthermore, the way of using rules becomes a factor when the students do not really understand the basic concepts of
the rules used. In addition, variety of contexts also becomes a factor when the students are always confronted with certain context themes so that they tend to memorize the contexts related to the topic being taught.

Each factor identified is expected to be a consideration for teachers in designing teaching and learning of ratio and proportion in the class. The teachers can provide the contexts that involve developing students’ understanding about ratio and proportion concept and about some mathematical concepts related to the topic. They also need to provide contexts which can urge the students to construct their new knowledge about the rules or formulas. The contexts should be varied in how ratio and proportion are applied in daily life. Thus, this study was expected to overcome or minimize the epistemological obstacle in teaching and learning of ratio and proportion in the class.

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