A simplified theory of "stickiness" due to electroadhesion between rough surfaces

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Abstract

Building on theories of Persson, we derive a simpler theory for electroadhesion between rough surfaces using BAM (Bearing Area Model) of Ciavarella, or previous ideas by Persson and Tosatti. Rather surprisingly, in terms of stickiness, we obtain very simple and similar results for pure power law power spectrum density (PSD), confirming stickiness to be mainly dependent on macroscopic quantities. We define a new dimensionless parameter for electroadhesive stickiness.

Keywords:
Electroadhesion, JKR model, DMT model, soft matter, roughness models.

1. Introduction

Applied electric voltage between two rough solids leads to accumulation of charges of opposite sign on the surfaces, which obviously results in an electrostatic attraction, which adds to the external repulsive loads (omitting for simplicity the presence of van der Vaals independent forces). Also, this increases the area of contact between the solids, and therefore the friction force if solids are sliding, with particular reference to touch screen applications (Vardar et al., 2017). Persson (2018) has extended his well known theory of contact mechanics to the case of electroadhesion, and developed a general mean-field theory in two limiting cases: (i) when an electric insulating film separate two conducting bodies and (ii) two electric conducting solids, which results in contact resistance and in voltage drop $V$ over a narrow region at the interface, using the theorem by Barber (2013), that contact resistance
is proportional to the mechanical contact stiffness. Simplified results were obtained also by Popov & Hess (2018).

As suggested in a very interesting recent paper by Dalvi et al. (2019), surface topography can easily have more than seven orders of magnitude of almost power law spectrum, including down to the Ångström-scale (see Fig.S2 in Dalvi et al. (2019) which gives the 2D isotropic PSD). The very interesting recent paper by Dalvi et al. (2019) reports extensive adhesion measurements and corroborate ideas originally suggested by Persson & Tosatti (2001) and reworked in terms of stickiness criteria by Ciavarella (2019). We shall extend this to electroadhesion, including the stickiness criteria, and we shall find surprisingly universal results, despite the very different origin of the various proposals we compare.

2. Electro adhesion

Consider an elastic solid with surface roughness above a rigid solid with a flat surface. Both solids are conducting materials with insulating surface layers of thickness \( d_1 \) and \( d_2 \) and dielectric constants \( \varepsilon_1 \) and \( \varepsilon_2 \). An electric voltage difference \( V \) occurs between the two conducting solids. The interfacial separation \( u = u(x) \) depends on the lateral coordinate \( x = (x, y) \).

![Fig.2- Basic geometry of the problem, an elastic solid with surface roughness above a rigid solid with a flat surface. Both solids are conducting materials with insulating surface layers of thickness \( d_1 \) and \( d_2 \) and dielectric constants \( \varepsilon_1 \) and \( \varepsilon_2 \). An electric voltage difference \( V \) occurs between the two conducting solids. The interfacial separation \( u = u(x) \) depends on the lateral coordinate \( x = (x, y) \).](image-url)

With roughness, Persson finds using his contact mechanics theory an equation which gives the real contact area and the nominal pressure as a
function of voltage. From his Fig.1 we see that the area becomes highly non linear with voltage and also pressure.

Additionally, this “real contact” area remains highly ill-defined, depends critically on the truncation of the PSD spectrum of roughness.

From eqt.just above (2) in Persson (2018), we have for the case of insulating solids

\[ \sigma_{zz}(x) = \frac{\varepsilon_0}{2} \left( \frac{V}{u(x) + h_0} \right)^2 \] (1)

where

\[ h_0 = \frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \] (2)

is a fixed quantity. For the case of insulating solids, hence, we have an adhesive term \( p_{ad} = \langle \sigma_{zz}(x) \rangle \) and being \( p_{rep} \) the repulsive pressure, the final external pressure results in (notice that it is negative when tensile),

\[ \sigma = p_{rep} - p_{ad} = p_{rep} - \frac{\varepsilon_0}{2} V^2 \int_0^\infty \left( \frac{1}{u(x) + h_0} \right)^2 P(p, u) \, du \] (3)

where \( P(p, u) \) is the distribution of interfacial separations computed by Persson theory in Almquist et al. (2011).

In what follows, we shall take a typical power law PSD

\[ C(q) = Z q^{-2(1+H)} \] (4)

for \( q > q_0 = \frac{2\pi}{\lambda_L} \), where \( \lambda_L \) is the longest wavelength in the roughness spectrum, \( H \) is the Hurst exponent (equal to \( 3 - D \) where \( D \) is the fractal dimension of the surface), and specifically, \( Z = \frac{H}{2\pi} \left( \frac{h_0}{q_0} \right)^2 \left( \frac{1}{q_0} \right)^{-2(1+H)} \) where \( h_0^2 = 2h_{rms}^2 \). Here, \( h_{rms} \) is the rms amplitude of roughness.

3. Idea 1 – BAM method

Instead of integrating the full distribution, we can use the idea of BAM (Ciavarella, 2018), which gives a very simple estimate of adhesion of hard solids with rough surfaces based on a bearing area model. Suppose we neglect van der Waals adhesion, we just replace the true expression of the stress as a function of separation [1] with a Maugis-Dugdale equivalent for which the tensile stress is defined as a function of gap \( u \) as
\( \sigma_{ad}(u) = \sigma_0, \quad u \leq h_0 \)
\( \sigma_{ad}(u) = 0, \quad u > h_0 \) \hspace{1cm} (5)

Upon integration for nominally flat surfaces,
\[
\Delta \gamma = \frac{\varepsilon_0 V^2}{2} \int_0^\infty \left( \frac{1}{u(x) + h_0} \right)^2 du = \frac{\varepsilon_0}{2h_0} V^2 \hspace{1cm} (6)
\]
and for this to be equal to the BAM corresponding integral \( \Delta \gamma = \int_0^{h_0} \sigma_0 du = \sigma_0 h_0 \), we need to have
\[
\sigma_0 = \frac{\varepsilon_0}{2h_0^2} V^2 \hspace{1cm} (7)
\]
after which we can use BAM (Ciavarella, 2018).

For a Gaussian nominally flat surface, this results in
\[
\frac{A_{ad}}{A_0} = \frac{1}{2} \left[ \text{Erfc} \left( \frac{\bar{u} - \varepsilon}{\sqrt{2}h_{rms}} \right) - \text{Erfc} \left( \frac{\bar{u}}{\sqrt{2}h_{rms}} \right) \right] \hspace{1cm} (8)
\]
where \( \bar{u} \) is the mean separation of the surfaces, \( h_{rms} \) is rms amplitude of roughness. The total force is obtained by superposition of the repulsive pressure at indentation \( \Delta \) which is easily obtained with Persson’s theory (Persson, 2007) which, for the simplest power law PSD, and \( D \approx 2.2 \) gives
\[
\frac{p_{rep}(\pi)}{E^*} \simeq q_0 h_{rms} \exp \left( \frac{-\bar{u}}{\gamma h_{rms}} \right) \hspace{1cm} (9)
\]
where \( \gamma \approx 0.5 \) is a corrective factor. Therefore, summing up repulsive and attractive \( (\sigma_0 A_{ad}(\pi)) \) contributions, BAM gives
\[
\frac{\sigma(\bar{u})}{\sigma_0} \simeq q_0 h_{rms} \frac{E^*}{\sigma_0} \exp \left( \frac{-\bar{u}}{\gamma h_{rms}} \right) - \frac{1}{2} \left[ \text{Erfc} \left( \frac{\bar{u} - \varepsilon}{\sqrt{2}h_{rms}} \right) - \text{Erfc} \left( \frac{\bar{u}}{\sqrt{2}h_{rms}} \right) \right] \hspace{1cm} (10)
\]
which obviously results in a pull off finding the minimum as a function of \( \pi \).

Notice that
\[
\beta = \frac{E^*}{\sigma_0} = \frac{2E^* h_0^2}{\varepsilon_0 V^2} \hspace{1cm} (11)
\]
for low voltage goes to infinity whereas for very high voltage goes to zero, suggesting the material encounters a rigid wall. Notice that in the standard case of van der Waals forces, \( \frac{E^*}{\sigma_0} \approx \frac{E^* \gamma}{\Delta \gamma/\varepsilon} = \frac{\lambda}{l_a} \) where \( l_a = \Delta \gamma/E^* \) defines a
characteristic adhesion length (for Lennard Jones with crystals of the same material is \( l_a \approx 0.05 \epsilon \) or \( \frac{\epsilon}{l_a} \approx 20 \)). In that case, \( \sigma_0 = \frac{\Delta \gamma}{\epsilon} = \frac{l_a E^*}{\epsilon} = 0.05E^* \), namely the theoretical strength. In the electroadhesion case, \( \beta = \frac{2E^* h_0^2}{\epsilon_0 V^2} \) replaces \( \frac{\epsilon}{l_a} \) and depending on the voltage, can span very many orders of magnitude, probably more than the van der Waals case.

In the generic case, when both van der Waals adhesion and electroadhesion are at play, one would need to sum their effects.

Fig. 2 shows the abrupt decay in pull-off values for constant \( \beta = 20 \), stickiness is defined (for example) when \( -\sigma_{\text{min}}/\sigma_0 = 10^{-8} \) finding this by numerical routines. Moreover, Fig. 3 shows the effect of increasing \( \beta \), that is to decrease the voltage, which clearly reduces the tensile tractions.

Fig. 2- Curves of decay of pull-off normalized pressure \(-\sigma_{\text{min}}/\sigma_0\) as a function of normalized rms roughness amplitude \( h_{\text{rms}}/h_0 \) for constant voltage parameter \( \beta = \frac{E^*}{\sigma_0} = \frac{2E^* h_0^2}{\epsilon_0 V^2} = 20 \). Here, the reference long wavelength cutoff \( \lambda_{L0} = \frac{\eta \sigma_0}{2\pi} = 2048 \) and the curves shift to the right with increasing \( \lambda_L/\lambda_{L0} = 10^0, 10^1...10^5 \).
Fig. 3 - Curves of decay of pull-off normalized pressure $-\sigma_{\text{min}}/\sigma_0$ as a function of normalized rms roughness amplitude $h_{\text{rms}}/h_0$ for constant voltage parameter $\beta = \frac{E^*}{\sigma_0} = \frac{2E^*h_0^2}{\varepsilon_0V^2 \lambda_L^3} = 10, \ldots, 10^5$. Here, the reference long wavelength cutoff $\lambda_L = \frac{q_0}{2\pi} = 2048h_0$

3.1. Area - load

Persson’s theory (2018) has a prediction for the proportion of actual contact at a given nominal pressure which reads

$$\frac{A_{\text{rep}}}{A_0} = \text{erf}\left(\frac{\sqrt{\pi} \cdot p_{\text{rep}}}{2 \sigma_{\text{rough}}^2}\right)$$

(12)

where $\sigma_{\text{rough}} = E^*h_{\text{rms}}^2/2$ where $h_{\text{rms}}'$ is the rms slope of the surface, and $p_{\text{rep}}$ can be estimate as a function of $u$ from (9). This could be used to estimate the friction load as proportional to the repulsive contact area.

3.2. Stickiness criterion

The results show that the pull-off traction is principally determined by $h_{\text{rms}}, q_0$ and upon increasing the "magnification" of the surface, $\zeta = q_1/q_0$, converges rapidly, as in the adhesionless load-separation relation (9).
Summarizing, we can obtain similarly to the recent paper by Ciavarella (2019) for normal van der Waals adhesion, that for stickiness
\[ h_{rms} < \left( \frac{0.6 \sigma_0 h_0}{E^* \lambda L} \right)^{0.5} = 0.775 \sqrt{\frac{h_0 \lambda L}{\beta}} \]  
(13)
where we recognize that the dimensionless parameter previously defined \( \beta \) is an "electroadhesive" stickiness.

4. Idea 2 - Energy method

Alternatively, we could use the energy balance in the Persson-Tosatti’s theory for van der Waals adhesion. Persson & Tosatti (2001) argue with an energy balance between the state of full contact and that of complete loss of contact that the effective energy available at pull-off with a rough interface is
\[ \Delta \gamma_{eff} = \frac{A}{A_0} \Delta \gamma - \frac{U_{el}}{A_0} \]  
(14)
where \( A \) is an area in full contact, increased with respect to the nominal one \( A_0 \), because of an effect of roughness-induced increase of contact area, \( \frac{A}{A_0} > 1 \). Also, \( U_{el} \) is the elastic strain energy stored in the halfspace having roughness with isotropic power spectrum \( C(q) \) when this is squeezed flat
\[ \frac{U_{el} (\zeta)}{A_0} = \frac{\pi E^*}{2} \int_{q_0}^{q_1} q^2 C(q) dq = E^* l (\zeta) \]  
(15)
where we have integrated over wavevectors in the range \( q_0, q_1 \), and \( E^* = E/(1 - \nu^2) \) is the plane strain elastic modulus, where \( \nu \) is Poisson’s ratio. We have introduced in (15) a length scale \( l (\zeta) \) where \( \zeta = q_1/q_0 \) is the so-called "magnification".

With electroadhesion, we could sum the \( \Delta \gamma = \frac{\varepsilon_0}{2h_0} V^2 \) term, resulting in (let us remove the van der Waals contribution)
\[ \Delta \gamma_{eff} = \frac{\varepsilon_0}{2h_0} V^2 - \frac{U_{el}}{A_0} \]  
(16)

\[ ^{\text{Notice we use the original Persson’s convention and notation for } C(q) \text{ and not Dalvi et al. (2019) which is } C_{iso} (q) = 4\pi^2 C(q).} \]
We can then obtain a new ”Persson-Tosatti” stickiness criterion, by imposing $\Delta \gamma_{eff} = 0$ in (16) obtaining in terms of roughness amplitude, and using results in Ciavarella (2019), the condition (using $\beta$ from (11)) is obtained
\[
h_{rms} < \sqrt{\frac{h_0 \lambda_L}{\beta} \frac{2H - 1}{\pi H}}
\] (17)
which for $H = 0.8$ (the most typical Hurst exponent, see Persson, 2014), becomes
\[
h_{rms} < 0.5 \sqrt{\frac{h_0 \lambda_L}{\beta}}
\] (18)
which compares well with the other criterion obtained with BAM, except the threshold is 0.5 instead of 0.775.

We can rewrite the criteria in the form
\[
\frac{h_{rms}}{h_0} < 0.5 \sqrt{\frac{1}{\beta} \frac{\lambda_L}{h_0}} ; \quad \text{Persson-Tosatti} \quad (19)
\]
\[
\frac{h_{rms}}{h_0} < 0.775 \sqrt{\frac{1}{\beta} \frac{\lambda_L}{h_0}} ; \quad \text{BAM} \quad (20)
\]
and a comparison is shown in Fig.4, where Persson-Tosatti is reported in blue solid line, BAM as black solid line. Clearly, considering the two criteria have so different origin, it is remarkable that they give so qualitatively close results. All results are for $H = 0.8$ ($D = 2.2$), which is the most typical fractal dimension (Persson, 2014).
Fig. 4. A comparison of the three derived stickiness criteria: Persson-Tosatti (blue line) (19), BAM (black solid line) (20), in terms of the rms amplitude of roughness. All results are for $H = 0.8$ ($D = 2.2$), which is the most typical fractal dimension (Persson, 2014).

5. Conducting solids in the contact area

Most solids have a non-zero electric conductivity and we need to take into account that an electric current will flow through the asperity contact regions when an electric voltage is applied between the solids: also, the voltage drop $V$ over the contacting interface will depend on the electric conductivities 1 and 2 of the solids and on the contact resistance and Persson (2018, eqt.16) finds approximately

$$\Delta V = \frac{V}{1 + 4 \frac{d_0}{\sigma_{rms}} p_{rep}}$$

(21)

where

$$d_0 = \kappa \left( \frac{d_1}{\kappa_1} + \frac{d_2}{\kappa_2} \right)$$

(22)

and $\kappa_i$ is the electric conductivity of solid $i$ and $\kappa$ is the effective electric conductivity of the two solids. Hence, when the repulsive pressure increases, we have much less voltage drop available. Substituting this result in the previous results, we have a complete picture.
6. Conclusions

We have derived a simplified theory for electroadhesion, stemming from Persson (2018), for power law PSD spectrum, based on BAM, and we have also derived corresponding a stickiness criterion, which we further compared with an alternative derivation based on energy balance, stemming from Persson-Tosatti’s (2001), ideas. We have defined a new dimensionless parameter for electroadhesive stickiness.

7. References

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