An investigation on apportion of mathematical loss in transmission loss/cost allocation approach

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ABSTRACT

Cost allocation of highly non-linear transmission loss is complex and essential in competitive electricity market. In most of the existing transmission loss/cost allocation approaches, real power loss depends on selection of slack bus and hence the cost of transmission losses which are allocated to the generators and the loads also varies. In this paper, a complete analysis on the impact of slack bus selection on transmission loss allocation with and without mathematical loss is made. One of the existing approaches, proportional generation and proportional load (PGPL) method is taken to illustrate the impact. Mathematical loss is the loss without generation and load in the network and can be obtained from power flow solution by taking generation and load as zero. The cost incurred for this mathematical loss is allocated to the transmission lines while the cost of transmission loss due to bilateral contracts is allocated among the sources and the consumers. These loss/cost allocations with and without considering mathematical loss is shown using an IEEE 30 bus, 57 bus, 75 bus and 118 bus systems. The simulation results are obtained using MATLAB R2014a.

Keywords:
Bilateral contact
Electricity market
Mathematical loss
Slack bus selection
Transmission loss

INTRODUCTION

Competition is introduced in the electricity market mainly to reduce the price of electricity. It is the responsibility of the Independent System Operator (ISO) to fairly allocate the cost of transmission real power loss among the involved parties. Competitive electricity market can be of bilateral market, pool market or hybrid market. Most of the loss/cost allocation approaches are presented in the literature and it can be broadly classified into the following categories like proportional sharing (PS) methods, methods based on incremental transmission loss (ITL) coefficients. It is dependent on slack bus selection and results in over recovery of loss cost [5]. A priority list is prepared based on penalised quoted cost (PQC) for the sources to supply total system losses sequentially without considering reactive loads. This method uses ITL coefficients to compute penalty factor [6]. A modified PQC approach considering the reactive power demand of the loads is presented in [7]. Methods based on power or current injection is presented in [8-10]. Power injections are derived in terms of real and imaginary currents which are traced to find the contribution of each source to load [11]. Bailek's tracing method provides the details of contribution of each sources or load in power flowing through a particular line [12].

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Transmission loss allocation based on relative electrical distance (RED) is presented in [13] which determine the location of loads to the generators. Graph theory approach for loss allocation in open access and bilateral market is discussed in [14-16]. It calculates the contribution of source and load to the line flows. A comparative study on graph theory method with PS method, Zbus method and modified Zbus method is presented in [17]. An analysis on different loss allocation methods like pro-rata, marginal loss, proportional sharing, Zbus, modified Zbus, RED, PGPL methods are widely discussed in [18-20].

Loss allocation procedure with impact of circulating currents among the generators and mutual inductance between transmission lines is presented in [21] and [22] respectively. Abhyankar et. al. [23] presented a new loss allocation method based on point of connection where charges are decentralised. Kyung-II Min et. al. [24] developed a new algorithm where losses are allocated accurately to the respective buses based on path-integrals. This path results in increased accuracy considering the non-linearity of the system losses. A new tracing method combining hybrid artificial bee colony (ABC) algorithm and Support Vector Machine (SVM) to trace the active and reactive power is presented in [25].

A new methodology in [26] to dispatch reactive power and energy simultaneously in a multiobjective framework. It considers four objectives like maximization of social welfare, maximization of load served, minimization of loss and volatility stability enhancement index. Francesco Arci et al. presented a forecasting methodology using artificial neural networks for Irish electricity market. Results shows that artificial neural network based load forecasting is more accurate than other methods [27]. Adline K. Bikkeri et al. presented a study on effect of generating companies’ market power on unit commitment decision. This paper combines the bidding strategy and unit commitment problem using particle swarm optimization [28].

The proposed approach is applicable for normal operation of the power system. During congestion, the proposed approach is not fair and a separate procedure has to be followed like generation rescheduling to alleviate or reduce congestion in the grid. Moreover, congestion occurs for a shorter period of time when compared with normal operation. Therefore, the scope of the paper is restricted to normal operation of the grid.

In all the above discussed papers, the loss allocation problems are addressed without considering the mathematical loss. Literally, transmission real power losses are allocated to the sources and loads. Ideally, system loss exists only when generators and loads exist. But the transmission system shows system loss even without generation and load which is referred as mathematical loss and the respective cost should be allocated to the transmission sector. This mathematical loss is dependent on choice of slack bus. The impact of slack bus selection on transmission loss allocation (with and without mathematical loss) is illustrated using proportional generation and proportional load (PGPL) method. This fact can be applied to any of the loss/cost allocation approach. By this method, the associated cost of the transmission loss is allocated to the generators, loads and transmission sector.

2. PROPORTIONAL GENERATION AND PROPORTIONAL LOAD METHOD

In PGPL method, cost is allocated to the respective buses based on generator and demand loss factors. Generator loss factor and demand loss factor is proportional to its active power generation and load respectively [29]. In this method, reactive loads are neglected. In transmission loss/cost allocation process ‘N’ buses are involved with ‘NG’ generator buses, ‘ND’ load buses and ‘n’ dummy buses. The total active power loss (Ploss) is obtained from the solved power flow solution which is computed from the loss given in (1).

\[ P_{loss} = \sum_{i=1}^{NG} \sum_{j=1}^{NG} P_{Gi} B_{ij} P_{Gj} + \sum_{i=1}^{NG} B_{i0} P_{Gi} + B_{00} \]  

where, \( P_{Gi} \) - real power generation at bus ‘i’.

a. \( B_{ij}, B_{i0} & B_{00} \) - AC loss coefficients

b. The AC loss coefficients are computed from the results obtained from the load flow solution. Total cost for meeting the loss is calculated using incremental cost of loss using (2).

\[ TC_{loss} = P_{loss} \cdot \lambda \]  

where, \( \lambda \) - incremental cost of loss in $/MWhr,

The total cost of real power loss is allocated to the respective buses based on loss factors. The generator loss factor of generator ‘i’ is given in (3).

\[ LF_{Gi} = 0.5 \left( \frac{P_{Gi}}{\sum_{i=1}^{NG} P_{Gi}} \right) \]
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c. The demand loss factor of load 'j' is shown in (4).

\[ LF_{DJ} = 0.5 \left( \frac{PD_{j}}{\sum_{i=1}^{n} PD_{i}} \right) \]  

(4)

where, \( PD_{j} \) - Real power load at bus 'j'.

d. Cost allocation for 'NG' generators and 'ND' loads are given in (5) and (6) respectively.

\[ GC_{i} = LF_{Gi} \times TC_{loss} \]  

(5)

\[ LC_{j} = LF_{DJ} \times TC_{loss} \]  

(6)

In some cases, bus 'k' will have generation and load. In such case bus 'k' is considered as generator bus if generation is greater than demand or as load bus if it is vice-versa. Therefore, the real power generation and load of bus 'k' is given in (7) and (8) respectively.

\[ P_{Gk} = P_{Gk} - P_{Dk} \text{ if } P_{Gk} > P_{Dk} \]  

(7)

\[ P_{Dk} = P_{Dk} - P_{Gk} \text{ if } P_{Dk} > P_{Gk} \]  

(8)

If real power generation and load at bus 'k' are equal then the bus is treated as dummy bus without generation and load. PGPL method with the impact of reactive loads in active power loss allocation is addressed in [30].

3. RESULTS AND DISCUSSION

The impact of slack bus and mathematical loss on transmission loss allocation is demonstrated using an IEEE 30 bus system. It consists of 6 sources, 18 consumers and 6 dummy buses. The incremental or marginal cost of loss is assumed as 200 $/hr. The transaction data for IEEE 30 bus system is given in Table 1. The cost allocation to the sources and demands considering with and without mathematical loss is given in Table 3 and Table 2 respectively for different slack bus selection. The reactive power demand of the loads are neglected in real power loss/cost allocation. All node voltages are 1.0 p.u.

| Generator bus No. | Load bus No. | Real Power Transaction (MW) |
|-------------------|--------------|-----------------------------|
| 1                 | 7            | 22.8                        |
| 2                 | 3,4,12       | 2.4, 7.6, 30.0              |
| 5                 | 21,24        | 8.7,3.5                     |
| 8                 | 26,29,30     | 11.2,2.4,10.6               |
| 11                | 10,20,21     | 3.2,9.5,2.2                 |
| 13                | 14,15,16,17,18,19 | 6.2,8.2,3.5,9.0,5.8,17.5 |

Table 1. Transaction Data for IEEE 30 Bus System

The mathematical loss for this system is 0.0354 MW with bus 1 as slack bus which is 1.15% of total loss. This loss and its associated cost (0.0354 MW * 200 $/MWhr = 7.08 $/hr) have to be borne by the transmission sector. The real power loss varies from 3.089 MW to 3.175 MW with change of slack bus. The total variation is 0.086 MW which contributes to 2.78% of total loss. In terms of cost, the total variation is 17.2 $/hr. Therefore, the mathematical loss and slack bus selection has a significant impact on loss allocation. The cost allocated to the respective buses for both the cases are given in table 2 and table 3. The cost allocation for dummy buses at bus 6, bus 8, bus 22, bus 25, bus 27 and bus 28 are zero.

The real power loss, mathematical loss and their respective costs for IEEE 57, 75, and 118 bus system is given in Table 4 for all possible slack bus selection. For IEEE 57 bus system, the total real power loss significantly differs from 15.8234 MW to 46.309 MW. In this case, the loss increases thrice with change in slack bus. The loss is minimum for slack bus 12 and maximum with bus 6. Mathematical loss varies from 0.932 MW to 2.3297 MW. This contributes 2% to 5.03% of total loss.
### Table 2. Cost Allocation without Mathematical Loss for IEEE 30 Bus System

| Slack Bus | Bus 1 | Bus 2 | Bus 3 | Bus 4 | Bus 5 | Bus 6 | Bus 7 | Bus 8 | Bus 9 | Bus 10 | Bus 11 | Bus 12 | Bus 13 |
|-----------|------|------|------|------|------|------|------|------|------|-------|-------|-------|-------|
| Loss (MW) | 3.175| 3.169| 3.148| 3.104| 3.089| 3.124|
| Cost ($/MWh) | 635 | 633.8 | 629.6 | 620.8 | 617.8 | 624.8 |

### Table 3. Cost Allocation with Mathematical Loss for IEEE 30 Bus System

| Slack Bus | Bus 1 | Bus 2 | Bus 3 | Bus 4 | Bus 5 | Bus 6 | Bus 7 | Bus 8 | Bus 9 | Bus 10 | Bus 11 | Bus 12 | Bus 13 |
|-----------|------|------|------|------|------|------|------|------|------|-------|-------|-------|-------|
| Loss (MW) | 3.1397| 3.1347| 3.1127| 3.0687| 3.0537| 3.0687|
| Cost ($/MWh) | 627.94 | 626.74 | 622.54 | 613.74 | 610.74 | 617.74 |

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The real power loss considering with and without mathematical loss in transmission loss/cost is shown in Figure 1 for different slack bus selection.

![Figure 1. Transmission loss with and without mathematical loss for IEEE 30 bus system](image)

**The real power loss considering with and without mathematical loss for IEEE 57 bus, 75 bus and 118 bus system is shown in Figure 2, Figure 3 and Figure 4 respectively. For IEEE 75 bus system, the real power loss varies from 105 MW to 232 MW but variation of mathematical loss is 1 MW. The mathematical loss contributes to 9.8% of total loss. IEEE 75 bus system consists of 15 generators and slack bus selection.**

**Table 4. Real Power Loss for different IEEE Test Systems**

| System | Choice of Slack bus | Mathematical Loss (MW) | Cost borne by transmission sector ($/hr) | Real Power Loss in MW (with Mathematical loss) | Total Cost of Real Power Loss in $/hr (with Mathematical loss) |
|--------|---------------------|------------------------|------------------------------------------|-----------------------------------------------|---------------------------------------------------------------|
| 57     | Bus 1               | 0.932                  | 186.40                                   | 27.6875                                       | 5537.50                                                      |
|        | Bus 2               | 1.8706                 | 374.12                                   | 40.158                                        | 8031.60                                                      |
|        | Bus 3               | 2.3297                 | 465.94                                   | 36.0681                                       | 7213.62                                                      |
|        | Bus 6               | 1.9155                 | 383.10                                   | 46.309                                        | 9261.80                                                      |
|        | Bus 8               | 0.9365                 | 187.30                                   | 38.7136                                       | 7742.72                                                      |
|        | Bus 9               | 1.8146                 | 362.92                                   | 24.1088                                       | 4821.76                                                      |
|        | Bus 12              | 0.935                  | 187.00                                   | 15.8234                                       | 3164.68                                                      |
| 75     | Bus 1               | 23.2283                | 4645.660                                  | 172.8401                                      | 34568.02                                                     |
|        | Bus 2               | 23.6209                | 4724.180                                  | 232.3998                                      | 46479.96                                                     |
|        | Bus 3               | 23.2259                | 4645.180                                  | 218.0274                                      | 43605.48                                                     |
|        | Bus 9               | 23.0735                | 4614.700                                  | 178.4886                                      | 35697.72                                                     |
|        | Bus 10              | 22.8239                | 4564.780                                  | 175.2180                                      | 35043.60                                                     |
|        | Bus 12              | 23.0142                | 4602.840                                  | 177.0151                                      | 35403.02                                                     |
|        | Bus 13              | 23.2938                | 4658.760                                  | 183.0933                                      | 36618.66                                                     |
|        | Bus 14              | 22.7821                | 4556.420                                  | 105.1023                                      | 21020.46                                                     |
|        | Bus 12              | 9.0448                 | 1808.96                                   | 234.4094                                      | 46881.88                                                     |
|        | Bus 25              | 9.5464                 | 1909.28                                   | 287.5957                                      | 57519.14                                                     |
|        | Bus 26              | 8.7572                 | 1751.44                                   | 286.4532                                      | 57290.64                                                     |
|        | Bus 31              | 8.8027                 | 1760.54                                   | 289.9213                                      | 57984.26                                                     |
|        | Bus 46              | 8.7900                 | 1758.00                                   | 245.5988                                      | 49119.76                                                     |
|        | Bus 49              | 8.7425                 | 1748.50                                   | 170.4732                                      | 34094.64                                                     |
|        | Bus 54              | 8.8193                 | 1763.86                                   | 166.2434                                      | 33248.68                                                     |
|        | Bus 59              | 8.8833                 | 1776.66                                   | 165.8427                                      | 33168.54                                                     |
| 118    | Bus 61              | 8.7212                 | 1744.24                                   | 173.7503                                      | 34750.06                                                     |
|        | Bus 65              | 8.9616                 | 1792.32                                   | 168.6066                                      | 33721.32                                                     |
|        | Bus 66              | 8.7425                 | 1748.50                                   | 176.5316                                      | 35306.32                                                     |
|        | Bus 69              | 8.7468                 | 1749.36                                   | 168.6225                                      | 33724.50                                                     |
|        | Bus 80              | 8.7414                 | 1748.28                                   | 184.0204                                      | 36804.08                                                     |
|        | Bus 100             | 8.7737                 | 1754.74                                   | 256.9922                                      | 51398.44                                                     |
|        | Bus 103             | 8.7993                 | 1759.86                                   | 299.9042                                      | 59980.84                                                     |
is not feasible for all generators. The load flow results are presented for feasible set of slack bus. The generators with non-feasible solution of load flows are bus 4, bus 5, bus 6, bus 7, bus 8, bus 11 and bus 15.

In 118 bus system, the real power loss varies from 165.84 MW to 299.9 MW and mathematical loss varies around 1 MW. It consists of 19 generators and load flow solution is not feasible for three generators at bus 87, bus 89 and bus 111. In this case mathematical loss contributes to 3.18% of total loss. The main objective of this paper is to highlight the impact of slack bus selection and mathematical loss on transmission loss/cost allocation. Therefore, the cost allocation to the individual buses is not given for IEEE 57 bus, 75 bus and 118 bus systems.

Figure 2. Transmission loss with and without mathematical loss for IEEE 57 bus system

Figure 3. Transmission loss with and without mathematical loss for IEEE 75 bus system

The mathematical loss and real power loss for different IEEE test systems is shown in Figure 4. For all the systems, bus 1 is taken as slack bus to compute loss. The mathematical loss changes with system size.
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4. CONCLUSION

Cost of transmission losses are shared by generators and loads in deregulated electricity market while the transmission sector is uncharged. This paper proposes a methodology to involve the transmission sector in real power loss or cost allocation process with a study on impact of slack bus selection and mathematical loss. To demonstrate the results, PGPL method is considered. This study is carried out for sample four test systems like IEEE 30 bus, 57 bus, 75 bus and 118 bus system. The results show that the slack bus selection and mathematical loss has a significant impact on transmission loss or cost allocation. Moreover, the mathematical loss varies with system size and hence cannot be ignored.

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