About ten stars orbit eclipsing binary XZ Andromedae

Lauri Jetsu
Department of Physics, P.O. Box 64, FI-00014, University of Helsinki, Finland;
email: lauri.jetsu@helsinki.fi

June 2, 2020

Abstract

A third body in an eclipsing binary system causes regular periodic changes in the observed (O) minus the computed (C) eclipse epochs. Fourth bodies have rarely been detected from the O-C data. We apply the new Discrete Chi-square Method (DCM) to the O-C data of the eclipsing binary XZ Andromedae. These data contain the periodic signatures of at least ten wide orbit stars (WOSs). Their orbital periods are between 1.6 and 9.7 years. Since no changes have been observed in the eclipses of XZ Andromedae during the past 127 years, the orbits of all these WOSs are most probably co-planar. We give detailed instructions of how the professional and the amateur astronomers can easily repeat all stages of our DCM analysis with an ordinary PC, as well as apply this method to the O-C data of other eclipsing binaries.

Key words: methods: data analysis - methods: numerical - methods: statistical - binaries: eclipsing

1 Introduction

Naked eye observations of Algol’s eclipses have been recorded into the Ancient Egyptian Calendar of Lucky and unlucky days (Jetsu and Porceddu, 2015; Jetsu et al., 2013; Porceddu et al., 2018, 2008). Today, eclipsing binary (EB) observations have become routine for the professional and the amateur astronomers. On November 29th 1890, Dr. Raymond S. Dugan recorded the first primary eclipse epoch of XZ Andromedae. The last epoch in our XZ And eclipse data is from December 24th, 2017. The primary (A4 IV-V, 3.2m⊙, 2.4R⊙) and the secondary (G IV, 1.3m⊙, 2.6R⊙) of this binary orbit each other during Par (2452500.0) = 357 days (Demircan et al., 1995).

Periodic long-term changes are sometimes observed O-C data of EBs. A third or a fourth body can cause such periodicity (e.g. Jetsu, 2020b), but there are also other alternatives, like a magnetic activity cycle (e.g. Applegate, 1992) or an apsidal motion (e.g. Borkovits et al., 2005). Such long-term changes have also been observed in XZ And (Chaplin, 2019; Demircan et al., 1995; Manzoori, 2016). When Hajdu et al. (2019) studied O-C data of 80 000 EBs, they detected only four EB candidates that may have a fourth body. Our preliminary analysis of XZ And with the new Discrete Chi-Square Method (DCM) already confirmed the presence of a third and a fourth body (Jetsu, 2020a). Here, we show that DCM can detect the periodic O-C signals of at least ten bodies in this system. Our appendix gives detailed instructions for repeating every stage of our DCM analysis.¹

2 Data

In September 2019, we retrieved the observed (O) minus the computed (C) primary eclipse epochs of XZ And from the Lichtenknecker-Database of the BAV. These data had been computed from the ephemeris

\[ \text{HJD} = 2452500.5129 + 1.35730911 \times E \]  

(1)

All original O-C data are shown in Fig. 1. We reject eight secondary minima and three primary minimum outliers (Table A2). The n = 1094 Heliocentric Julian Day ti = t⊙,i data

\[ y_i = y_{\text{HJD}}(t_{\odot,i}) = O - C \]  

(2)

are our first analysed sample (Table A3: hereafter HJD-data).

The yearly distribution of data is given in Table A4. Most of these observations have been made between August and January. The regular lack of observations between February and July is repeated in the data for 127 years. This “12-month window” can mislead DCM period analysis. In our second analysed sample, we create random time points

\[ t_i^* = t_{\odot,i} + \Delta t_i \]  

(3)

where t⊙,i time points are from Table A3. The random shifts Δti are uniformly distributed between -365.25/2 and +365.25/2. These new t∗i time points are rearranged into increasing order. No changes are made to yi and σi values of Table A3. We analyse one such arbitrary sample of t∗i, yi and σi data (Table A5: hereafter D-data). We will show that the Δti random shifts do not mislead the detection of longer periods, but they do help us in the identification of possibly spurious periods (Table A10). For example, the largest possible random shift of ±1384 is only about ±6% for Pmin = 3000d, which is our lower limit for the tested longer periods of XZ And.

¹ DCM program code and all other necessary files are freely available in the Zenodo database: doi 10.5281/zenodo.3871549. All files, variables and other code related items are printed in magenta colour.
In our third sample, we transform Table A3 data to

$$y_{\text{ID}}(t_{\odot,i}) = y_{\text{HJD}}(t_{\odot,i}) - \delta t_i,$$

where \(\delta t_i = t_{\odot,i} - t_{\oplus,i}\), and \(t_{\odot}\) and \(t_{\oplus}\) are the epochs when the same eclipse of XZ And is observed in the Sun and on the Earth (Table A6: hereafter JD-data). No changes are made to \(t_i\) and \(\sigma_i\) values of Table A3. We use XZ And coordinates \(\alpha = 01^h 53^m 48.876\) and \(\delta = +41^o 51' 24.97''\) in the transformation of Eq. 4. This transformation superimposes the artificial “1'-signal”, the Earth’s motion around the Sun, into these data. We use this 1'-signal for checking the reliability of our period analysis. For XZ And, this effect is about ±7.4 minutes = ±0.0051 days.

Since the errors \(\sigma\) of the data are unknown, we use the following arbitrary relative weights for different observations

- \(w_1 = 3\) for “C” \((n = 55)\) and “E” \((n = 29)\)
- \(w_2 = 2\) for “V” \((n = 939)\) and “F” \((n = 61)\)
- \(w_3 = 1\) for “P” \((n = 7)\) and “?” \((n = 3)\),

where the BAV observation systems in German language are “C = CCD”, “E = Fotometer”, “V = Visuel”, “F = Fotoserie”, “P = Platten” and “?” = Unbekannt”. First, we fix the errors for the most accurate “C” and “E” observations to \(\sigma_1 = 0.0001\). Then, the \(w = \sigma^{-2}\) relation gives the errors \(\sigma_2 = \sqrt{2\sigma_1^2/3} = 0.00122\) for the “V” and “F” observations, as well as \(\sigma_3 = \sqrt{2\sigma_1^2/3} = 0.00173\) for the “P” and “?” observations. The numerical values of these errors do not influence our results, because we use the same weight for every observation. These errors are used only to give the correct DCM code analysis format for our three files of data.

3 Method

The data are \(y_i = y(t_i) \pm \sigma_i\), where \(t_i\) are the observing times and \(\sigma_i\) are the errors \((i = 1, 2, ..., n)\). The time span is \(\Delta T = t_n - t_1\). We apply DCM to these data. This method can detect many signals superimposed on arbitrary trends. Detailed instructions for using the DCM python code were already given in Jetsu (2020a, Appendix). Here, we also provide all necessary information for reproducing our DCM analysis of XZ And data.

DCM model is a sum of a periodic function \(h(t)\) and an aperiodic function \(p(t)\)

$$g(t) = g(t, K_1, K_2, K_3) = h(t) + p(t),$$

where

$$h(t) = h(t, K_1, K_2) = \sum_{i=1}^{K_1} h_i(t)$$

and

$$h_i(t) = \sum_{j=1}^{K_2} B_{i,j} \cos(2\pi j f_i t) + C_{i,j} \sin(2\pi j f_i t).$$
\[ p(t) = p(t, K_3) = \sum_{k=0}^{K_3} p_k(t) \]  
\[ p_k(t) = M_k \left( \frac{2t}{\Delta T} \right)^k. \]

The periodic \( h(t) \) function is a sum of \( K_1 \) harmonic \( f_i \) frequency signals \( h_i(t) \). The order of these signals is \( K_2 \). The trend \( p(t) \) is an aperiodic \( K_3 \) order polynomial. The \( g(t) \) model has

\[ p = K_1 \times (2K_2 + 1) + K_3 + 1 \]

free parameters. We use the abbreviation “model\(K_1,K_2,K_3\)” for a model having orders \( K_1, K_2 \) and \( K_3 \). DCM determines the \( h_i(t) \) signal parameters

\[ P_i = 1/f_i = \text{Period} \]
\[ A_i = \text{Peak to peak amplitude} \]
\[ t_{i,\text{min},1} = \text{Deeper primary minimum epoch} \]
\[ t_{i,\text{min},2} = \text{Secondary minimum epoch (if present)} \]
\[ t_{i,\text{max},1} = \text{Higher primary maximum epoch} \]
\[ t_{i,\text{max},2} = \text{Secondary maximum epoch (if present)}, \]

and the \( M_k \) parameters of the \( p(t) \) trend.

We compute the DCM test statistic \( z \) from the sum of squared residuals \( R \), because the errors for the data are unknown (Jetsu, 2020a, Eqs. 9 and 11). The \( F = F_R \) test statistic gives the Fisher-test critical levels \( Q_F \) (Jetsu, 2020a, Eqs. 13). When we compare a simple and a complex model, the latter is a better model for the data if

\[ Q_F < \gamma_F = 0.001, \]

where \( \gamma_F = 0.001 \) is the pre-assigned significance level (Jetsu, 2020a, Eq. 14).

Our notations for the two signatures of unstable models are

\* = Dispersing amplitudes  
\† = Intersecting frequencies

The former can occur without the latter, but not vice versa (Jetsu, 2020a, Sect. 4.3.). The periods that are clearly too large are also denoted with \( \dagger \). We use the notation

\* = Failed model

when \* and \( \dagger \) both occur, or the model must be rejected with the criterion of Eq. 11.

4 Search for long periods

We first search for longer periods between \( P_{\text{min}} = 3000^d \) and \( P_{\text{max}} = 100 \ 000^d \), because the \( 1^y \)-window and \( 1^y \)-signal can mislead the detection of periods below \( P_{\text{min}} \). Later, we will also search for periods shorter than \( P_{\text{min}} \) (Sect. 5). Note that \( P_{\text{max}} > \Delta T = 46 \ 111^d \), because we will show that DCM can detect periods longer than the time span of data. We search for periodicity in three different samples, where the misleading \( 1^y \)-window or \( 1^y \)-signal is present or absent. These are

HJD-data: \( 1^y \)-window = Yes, \( 1^y \)-signal = No  
D-data: \( 1^y \)-window = No, \( 1^y \)-signal = No  
JD-data: \( 1^y \)-window = Yes, \( 1^y \)-signal = Yes

We analyse the original HJD-data first, because the \( 1^y \)-signal does not contaminate these data. The one signal model periods \( P = 58237^d \) and \( 13148^d \) are different (Table A7: Models \( M=1 \) and \( 2 \)). Fisher-test reveals that the last quadratic \( K_3 = 2 \) trend model\(1,2,3\) is certainly a better model for the data than the linear \( K_3 = 1 \) trend model\(1,1,1\) (Table A7: \( M=1, \dagger, F = 149, Q_F < 10^{-16} \)). Both two signal models \( M=3 \) and \( 4 \) give the period \( P \approx 13300^d \), but the \( P_2 \) periods of these models are different. Also for these two signal models, the \( K_3 = 2 \) quadratic trend model\(2,1,2\) is better than the linear \( K_3 = 1 \) trend model\(2,1,1\) (Table A7: \( M=3, \dagger, F = 390, Q_F < 10^{-16} \)). The same \( \approx 13300^d \) period is also present in the three signal model\(3,1,1\) and model\(3,1,2\), but the other two periods of these models differ (Table A7: \( M=5 \) and \( 6 \)). Model \( M=5 \) fails \( (*) \), because the two largest periods have dispersing amplitudes \( (*) \) and the largest one is unrealistic \( (*) \). This linear trend three signal model \( M=5 \) has \( p = 11 \) free parameters. Yet, the simple quadratic trend two signal model \( M=4 \), having only \( p = 8 \) free parameters, is certainly a better model for the data (Table A7: \( M=4, \dagger, F = 0, Q_F = 1.0 \)). This is very strong evidence for the \( K_3 = 2 \) quadratic \( p(t) \) trend in these data. The linear trend four signal model \( M=7 \) fails \( (*) \). All \( \dagger \) arrows point towards model \( M=8 \) in the second last column of Table A7. This four signal model\(4,1,2\) is certainly the best one of all eight compared models, because it beats the other seven models by an absolute certainty of \( Q_F < 10^{-16} \). The periodograms for this model\(4,1,2\), and the model itself, are shown in Figs. 2 and 3. The transparent diamonds denoting the red \( z_1 \) and the blue \( z_2 \) periodogram minima for the two weaker periodicities are certainly real. When all four periodograms are plotted in the same scale, these two minima appear to be shallow only because the two larger amplitude periodic signals clearly dominate in this model \( M=8 \). These two high amplitude signals have a much bigger impact on the squared sum of residuals \( R \) than the two low amplitude signals. When the tested frequencies approach zero, the yellow \( z_4 \) periodogram turns upwards in the lower panel of Fig. 2. Thus, DCM can confirm that none of the periods longer than \( \Delta T \) fit to these data. The level of residuals is stable, but some regular patterns indicate that there are more than four signals in these data (Fig. 3: blue dots). Each \( h_j(t_i) \) signal

\[ y_{i,j} = y_i - [g(t_i) - h_j(t_i)] \]  

is also shown in Fig. 4. The main results in Table A7 are

R1. These data contain a quadratic \( p(t) \) trend \( K_3 = 2 \).

R2. The periods and amplitudes of all quadratic trend \( K_3 = 2 \) models are consistent (Table A7: \( M=2, 4, 6 \) and \( 8 \)). When we detect a new signal, we re-detect exactly the same old signal periods and signal amplitudes.

R3. There are at least four signals, because model\(4,1,2\) beats the other models with an absolute certainty of \( Q_F < 10^{-16} \).
Figure 2: Periodograms for four signal model4,1,2 of Table A7 (M=8). Colours are red ($z_1$), blue ($z_2$), green ($z_3$) and yellow ($z_4$) (Jetsu, 2020a, Eq. 17). Open diamonds denote best frequencies.

Figure 3: Data and model M=8 of Table A7. (a) Data (black dots) and $p(t)$ trend (dotted black line). (b) Data minus $p(t)$ trend (black dots), $g(t)$ minus $p(t)$ (black line), $g_1(t)$ signal (red line), $g_2(t)$ signal (blue line), $g_3(t)$ signal (green line) and $g_4(t)$ signal (yellow line). Residuals (blue dots) are offset to -0.012 (dotted blue line).
Figure 4: Four signals in original data. Signals $y_{i,j}$ (Eq. 12) for model M=8 of Table A7. Each signal is plotted as a function of time ($t$) and phase ($\phi$).

Figure 5: Four signal residuals (Table A9: M=8), otherwise as in Fig. 3.
Figure 6: Four signals in four signal residuals. Signals $y_{i,j}$ (Eq. 12) for model M=8 in Table A9, otherwise as in Fig. 4.

Figure 7: Failed three signal model for original data (Table A9: M=5), otherwise as in Fig. 3.
Figure 8: Failed model for original data. Signals $y_{i,j}$ (Eq. 12) for model M=5 in Table A7. Otherwise as in Fig. 4.

Figure 9: $Y^1$-signal in JD-data. Signal $y_{i,j}$ (Eq. 12) for model M=9 in Table A15, otherwise as in Fig. 9.

We use the above R1, R2 and R3 abbreviations for these particular results. The quadratic $K_3 = 2$ trend is hereafter used in all models for the original data (R1 result).

It takes several days for an ordinary PC to compute the four signal model $4_1$ and its 20 bootstrap rounds (Figs. 2, 3 and 4). The five signal model computation would take months. We solve this problem with Test $3+3+3+3$ and Test $4+4+4$ approach. First, the original data are analysed with the quadratic trend model. Then, we analyse the three or four signal residuals with the constant trend model (e.g. Fig. 5). This gives us the six or eight signal residuals, which are analysed with the constant trend model. We end this process at $3+3+3+3$ or $4+4+4$ = 12 signals. The results for our three samples are given in Tables A8 and A9 (HJD-data), Tables A10 and A11 (D-data), and Tables A12 and A13 (JD-data). All these results are compared in Table A14.

Test $3+3+3+3$ results for HJD-data, D-data and JD-data agree in Table A14. The same applies to Test $4+4+4$ results. In all the above mentioned tables, we use the symbol

\[ \hat{\Psi}^9 \] = Weakest of first nine detected signals

to denote the periodicity that has the lowest amplitude of all nine first detected signals. We choose these $\hat{\Psi}^9$ periods from models M=9 (Test $3+3+3+3$) and M=12 (Test $4+4+4$). If these $\hat{\Psi}^9$ periods were ignored in Table A14, the results would be the same for the eight best Test $3+3+3+3$ and Test $4+4+4$ periods. The main results in Table A14 are

R4 These data contain at least seven or eight signals.

R5 Test $3+3+3+3$ and Test $4+4+4$ give the same results for the eight best periods in all three samples.

R6 There is no $Y^1$-window and no $Y^1$-signal bias, because all three samples give the same results.

We use the words “at least” in R4 result, because HJD-data or D-data may contain even more periodicities. For example, Fisher-test does not require the rejection of the ninth signal in HJD-data (Tables A8 and A9: M=9). Another uncertainty is connected to the failed (*) model M=6 in Tables A8, A9, A10 and A11. The two periods above and below 4500$^4$ show amplitude dispersion (*) and intersecting frequencies (!). DCM does not tend to find too many signals, but it may find too few (see Jetsu, 2020a, Sect. 4.4). However, we cannot confirm, if these two periods above and below 4500$^4$ represent two separate signals.

The results for the four signal residuals are shown in Figs. 5 and 6. These two figures “speak for themselves”, because the scatter of the new more accurate data is extremely small. It seems as if these black dots of new data were glued on the $h_i(t)$ curves. The scatter definitely increases for the older less accurate data. The eight signal residuals (blue dots) still show some regularity, but only for the more accurate new data. One failed model is also shown in Figs. 7 and 8. Note that the solution for $p(t)$ (black dotted line) makes no sense, and nor does the solution for $h_3(t)$ (green continuous line).
5 Search for short periods

The eight best Test$^{4+4+4}$ periods are the same for HJD-data, D-data and JD-data (R5 result). Therefore, we search for shorter periods in the residuals of the eight signal models M=8 of these three samples. The tested DCM period range is between $P_{\text{min}} = 300^d$ and $P_{\text{max}} = 3000^d$. Shorter $P_{\text{min}}$ values would only lead to the “detection” of spurious signals, like 1/2, 2/3, 3/4 x 1$^d$-window or 1$^d$-signal. The short period search results are given in Table A15.

The first period $P_1 = 365.46 \pm 78^d$ in HJD-data comes from the 1$^d$-signal period, although this signal should have been removed from these Heliocentric Julian Days. While the 1$^d$-window may cause this periodicity, it may also be present, because the units of some epochs are Julian Days, i.e. the $\delta t_i$ correction of Eq. 4 has not been applied. The amplitude $A_2 = 0.40013$ of the latter second period $P_2 = 616.46^d$ signal is approximately equal to the amplitudes of weakest nine first detected long period signals ($^{+8}_{-3}$) in Table A14. Hence, this second signal may represent a real periodicity. The third or the fourth short periodicity alternatives fail (Table A15: M=3 and 4 have $^{+8}_{-3}$).

Fisher-test indicates that there may be even three short period signals in the eight signal residuals of D-data (Table A15: models M=5-7). The first one, $P_1 = 364.49 \pm 33^d$, is again the 1$^d$-signal. This time the 1$^d$-window can not explain this periodicity, because this window is removed from D-data (Eq. 3). The only realistic cause for this periodicity is the wrong Julian Day units of some epochs. There can not be only a few such wrong epochs, because the $\delta t_i$ random shifts of Eq. 3 would otherwise eliminate this 1$^d$-signal. The second $P_2 = 593.41 \pm 26^d$ period is equal to the $P_2$ period already detected from HJD-data. It also has the same amplitude $A_2 = 0.40013$. The third $P_3 = 2244^d \pm 9^d$ signal is even stronger, $A_3 = 0.40016$. There is no fourth signal, because model M=8 fails ($^{+8}_{-3}$).

Only one period dominates the eight signal residuals of JD-data: 1$^d$-signal (Table A15: other models M=10-12 have $^{+8}_{-3}$). This regular artificial signal is shown in Fig. 9. DCM detects an extremely accurate value for this period, $P_1 = 365.34 \pm 0.08^d$. The amplitude $A_1 = 0.40110 \pm 0.0004$ of this 1$^d$-signal agrees perfectly with the expected superimposed signal amplitude $\pm 0.40051 \pm 0.00102$ (Eq. 4). Our unambiguous re-detection of this superimposed 1$^d$-signal is possible only if all earlier eight long period signals detected from JD-data are also real. Even a single wrong long period $h_i(t)$ signal could weaken or erase this 1$^d$-signal, let alone many wrong long period $h_i(t)$ signals or a wrong $p(t)$ trend. The unexpected 1$^d$-signal signatures in HJD-data and D-data also support this same detection of eight real long periods, because some epochs probably have the wrong units, Julian Days. The main results in Table A15 are

R7. The unambiguous detection of the artificially superimposed 1$^d$-signal in JD-data strongly supports the idea that the eight detected long periods of XZ And are real.

R8. Signatures of $\sim 600^d$ signal are detected in all three samples. D-data 2244$^d$ signal may also be a real periodicity.

6 Discussion

6.1 Period analysis

While it is not possible to determine the exact number of stars in XZ And, there are definitely many. For example, the current O-C data can not confirm if the two signals below and above 4500$^d$ represent one or two periodicities/stars (e.g. Table A8: M=6). Another example is the short 2244$^d$ period detected in D-data, which is not detected in HJD-data or JD-data (Table A15: M=7). This detection could be explained with 1$^d$-window, which has been removed only from the D-data. There are also Fisher-test cases that resemble the children’s game stone-paper-scissors, where stone wins scissors, paper wins stone, and scissors win paper (e.g. Table A15: models M=1-4 and M=9-12). The results for these tests also depend on the chosen pre-assigned critical level $\gamma_F$ in the criterion of Eq. 11. In all long period searches, the $Q_F$ critical levels are extremely significant for seven, eight or even nine first detected signals. Then this significance drops abruptly for the next signals. Exactly the same results were obtained for simulated data by Jetsu (2020a, their Table 4). DCM may find too few signals, especially if the $p(t)$ trend is wrong. However, DCM tends not to detect too many signals, because such models fail ($^{+8}_{-3}$).

DCM analysis does not require access to super computers. The results for Test$^{3+3+3+3}$ and Test$^{4+4+4}$ confirm that the computation capacity of small PCs is sufficient for detecting many signals in the O-C data of EBs. The computations for the bootstrap error estimates of three and four signal models take a long time. The best alternative is to run only a few bootstrap rounds first, and test many bootstrap rounds only if the preliminary analysis results make sense. Note that the bootstrap never gives exactly the same error estimates because it uses random samples of residuals.

We do not re-discuss our main results R1 - R8. The fact that DCM re-detects the artificial 1$^d$-signal in JD-data proves beyond any reasonable doubt that the other detected signals represent real periodicities. We succeed in this after removing a quadratic $p(t)$ trend and eight $h_i(t)$ signals.

The model used by Hajdu et al. (2019, their Eq. 11) was the sum of a parabola and a sinusoid. Their detection rate of fourth bodies in EBs was only 0.00005. Jetsu (2019) has already shown that the one-dimensional period finding methods, like the power spectrum method (Lomb, 1976; Scargle, 1982; Zechezmeister and Kürster, 2009), give spurious results if the data contains many signals. This applies also to the one-dimensional periodic model applied by Hajdu et al. (2019), and explains their low detection rate of fourth bodies. At the same time, their low detection rate merely highlights the potential of DCM.

Jetsu (2020a, Sect. 6.) already discussed the problems arising from the use of $K_2 = 2$ double wave models. All third bodies do not necessarily induce purely sinusoidal O-C changes. Therefore, the temptation for using $K_2 = 2$ harmonics arises. This approach opens up a real Pandora’s box, because two stars having periods $P_1$ and $P_2$ induce a synodic period $(P_1^{-1} - P_2^{-1})^{-1}$. These synodic periods are repeated
Table 1: Masses $m_3$ from $p_3$ and $a$ of Eq. 13. Eight periods are from long search (Long) and two periods from short search (Short). We refer to these stars as S$_1$, ..., S$_{10}$. Masses are computed for $i = 90^\circ$, $60^\circ$ and $30^\circ$ ($m_3=90^\circ$, $m_3=60^\circ$, $m_3=30^\circ$).

| Search | Star | $d$ | $y$ | $p_3$ | $a$ | $m_3=90^\circ$ | $m_3=60^\circ$ | $m_3=30^\circ$ |
|--------|------|-----|-----|-------|-----|----------------|----------------|----------------|
| Long S$_1$ | 33507 | 91.7 | 0.04300 | 1.16 | 1.38 | 2.74 |
| Long S$_2$ | 13537 | 37.1 | 0.02840 | 1.45 | 1.73 | 3.56 |
| Long S$_3$ | 7754 | 21.2 | 0.00310 | 0.20 | 0.23 | 0.40 |
| Long S$_4$ | 6346 | 17.4 | 0.00275 | 0.20 | 0.23 | 0.41 |
| Long S$_5$ | 4723 | 12.9 | 0.00200 | 0.18 | 0.20 | 0.36 |
| Long S$_6$ | 4320 | 11.8 | 0.00250 | 0.24 | 0.27 | 0.49 |
| Long S$_7$ | 3732 | 10.2 | 0.00130 | 0.13 | 0.15 | 0.27 |
| Long S$_8$ | 3019 | 8.3 | 0.00120 | 0.14 | 0.16 | 0.29 |
| Short S$_9$ | 2244 | 6.1 | 0.00080 | 0.11 | 0.13 | 0.23 |
| Short S$_{10}$ | 592 | 1.6 | 0.00060 | 0.21 | 0.25 | 0.44 |

throughout the whole data, and DCM certainly detects them. If we had used the $K_2 = 2$ option in our analysis, the interactions between the signals of all stars in XZ And would have caused an incredible mess.

6.2 Astrophysics

Activity cycles are quasi-periodic, and never give regular long-term residuals. Therefore, the Applegate (1992) mechanism cannot explain the numerous O-C periods of XZ And. An apsidal motion can cause only one period. If the light-time effect (LTE) of a third body causes these periodic O-C changes, the mass function fulfills

$$f(m_3) = \frac{(m_3 \sin i)^3}{m_1(1 + \frac{q}{m_3})^2} = \frac{(173.15 a)^3}{p_3^2},$$

where $i$ is the inclination of the orbital plane of the third body, $m_1$ is the mass of primary $[m_\odot]$, $q = m_2/m_1$ is the dimensionless mass ratio of secondary and primary, and $m_3$ is the mass of the third body $[m_\odot]$, $a = A/2$ is half of the peak the peak amplitude of O-C modulation caused by the third body $[d]$, and $p_3$ is the period of the modulations caused by the third body $[y]$ (Borkovits and Hegedüs, 1996; Tannveer, 2015; Yang et al., 2016). We use the masses $m_1 = 3.2 m_\odot$ and $m_2 = 1.3 m_\odot$ (Demircan et al., 1995, A4 IV + G5 IV). The long period search $p$ and $a$ values are those detected from HJD-data (Table A9: M=4 and 8). The respective two short period values are detected from HJD-data and D-data (Table A15: M=4 and 7). We compute the $m_3$ values for inclinations $i = 90^\circ$, $60^\circ$ and $30^\circ$ (Table 1). In the $i = 90^\circ$ alternative, the mass $1.45 m_\odot$ of star S$_2$ exceeds the $m_2 = 1.3 m_\odot$ mass of the secondary. The $1.16 m_\odot$ mass of S$_1$ star is just below this limit. If the remaining eight less massive stars were in the main sequence, they would all belong to spectral type M. The $m_1 = 3.2 m_\odot$ primary would dominate the luminosity of such a system of twelve stars. If the inclination were $i = 60^\circ$, the S$_1$ and S$_2$ star masses would exceed the secondary $m_2$ mass, but the $m_1$ primary would still dominate the luminosity of the whole system. The $i = 30^\circ$ alternative can be ruled out, because the S$_2$ star would be more massive and brighter than the primary, and this practically constant short-term radial velocity star would have been noticed long ago. The orbital period $37.81$ of this S$_2$ star has also been detected in several previous studies (Table 2).

The quadratic polynomial trend is

$$p(t) = M_0 + M_1 c t + M_2 c^2 t^2,$$

where $c = 2/\Delta T$. The time derivative is

$$\frac{dp(t)}{dt} = M_1 c + 2M_2 c^2 t.$$

We use the values $M_1 = -0.627 \pm 0.005$ and $M_2 = 0.082 \pm 0.002$ of the four signal model for the original HJD-data (Table A16: M=4). The $p(t)$ changes caused by parameter $M_1 c$ can be eliminated by computing the O-C values with a new constant period $P' = P - (M_1)c/P = 1.43573458$. Hence, the only real period change is $\Delta P = 2M_2 c^2 = 4.11 \times 10^{-10} = 3.6 \times 10^{-9}$ during $P = 1.435730911$. The period increases because $M_2 > 0$. The long-term increase rate is

$$\frac{\Delta P}{P} = 2M_2 c^2 = (3.04 \pm 0.07) \times 10^{-10},$$

which is about five times less than the $\Delta P/P = 1.45 \times 10^{-9}$ estimate of XZ And obtained by Manzoori (2016). The two and the three signal model results would have been nearly the same, but the one signal model $M_1$ and $M_2$ values would have given a completely wrong result (Table A16). We do not derive any mass transfer estimate for XZ And (e.g. Manzoori, 2016, their Eq. 8). This estimate would not be correct for a multiple star system, where the numerous WOSs can perturb the central EB, e.g. through the Kozai effect (Kozai, 1962), or the combination of Kozai cycle and tidal friction effects (Fabrycky and Tremaine, 2007).

Our solar system is stable because the orbital planes of planets are nearly co-planar, the biggest exception being the smallest planet Mercury with an inclination of seven degrees. The single stars, the multiple star systems and the planets form in co-planar protostellar disks (e.g. Watkins et al., 1998). If the orbital plane of a third body is not co-planar with, or perpendicular to, the orbital plane of the central EB, periodic long-term perturbations change the orbital plane of the central EB, and the eclipses may no longer occur (Soderhjelm, 1992).
7 Conclusions

Hajdu et al. (2019) detected only four candidates possibly having a fourth body in their O-C data of 80 000 eclipsing binaries. The probability for detecting a fourth body from their O-C data was only 0.00005. Here, we apply the new Discrete Chi-Square Method (DCM) to the O-C data of the eclipsing binary XZ And, and detect signatures of at least ten wide orbit stars (WOSs) orbiting the central EB. These WOSs have orbital periods between 1.76 and 91.77. Two WOSs are certainly more massive than the Sun. The orbits of all these WOSs are most probably co-planar with, or perpendicular to, the orbital plane of central EB, because no changes have been observed in the eclipses of XZ And in over a century. Considering the number of new companions detected in XZ And, it is actually a more interesting multiple star system than Algol itself (Jetsu, 2020b, “only” two new companions Algol D and Algol E). Our results for XZ And and Algol confirm that many EBs have unknown companions, which can be easily detected with our DCM. This abstract mathematical method is not designed for analysing any particular phenomena of the real physical world. The O-C data of EBs just happen to be one particular type of suitable data. We sincerely hope that these results would ignite a “renaissance” in the O-C data studies of other EBs. Valuable data have been patiently collected by the professional and the amateur astronomers since the well-known amateur astronomer Sir John Goodricke re-detected Algol’s periodic eclipses (Goodricke, 1783).

Acknowledgements. This work has made use of NASA’s Astrophysics Data System (ADS) services. We retrieved the O-C data of XZ And from the Lichtenknecker Database of the BAV.

References

Applegate, J. H. (1992). A mechanism for orbital period modulation in close binaries. ApJ, 385:621–629.

Baron, F., Monnier, J. D., Pedretti, E., Zhao, M., Schaefer, G., Parks, R., Che, X., Thureau, N., ten Brummelaar, T. A., McAlister, H. A., Ridgway, S. T., Farrington, C., Sturmann, J., Sturmann, L., and Turner, N. (2012). Imaging the Algol Triple System in the H Band with the CHARA Interferometer. ApJ, 752(1):20.

Borkovits, T., Forgács-Dajka, E., and Regály, Z. (2005). The combined effect of the perturbations of a third star and the tidally forced apsidal motion on the O–C curve of eccentric binaries, volume 333 of Astronomical Society of the Pacific Conference Series, page 128.

Borkovits, T. and Hegedues, T. (1996). On the invisible components of some eclipsing binaries. A&A, 120:63–75.

Chaplin, G. B. (2019). Medium-term Variation in Times of Minimum of Algol-type Binaries: XZ And, RZ Cas, U Cep, TW Dra, U Sge. Journal of the American Association of Variable Star Observers (JAAVSO), 47(2):222.

Demircan, O., Akalin, A., Selam, S., Derman, E., and Mueyesseroglu, Z. (1995). A period study of XZ Andromedae. A&AS, 114:167.

Fabrycky, D. and Tremaine, S. (2007). Shrinking Binary and Planetary Orbits by Kozai Cycles with Tidal Friction. ApJ, 669(2):1298–1315.

Goodricke, J. (1783). A Series of Observations on, and a Discovery of, the Period of the Variation of the Light of the Bright Star in the Head of Medusa, Called Algol. In a Letter from John Goodricke, Esq. to the Rev. Anthony Shepherd, D. D. F. R. S. and Plumian Professor at Cambridge. Philosophical Transactions of the Royal Society of London Series I, 73:474–482.

Hajdu, T., Borkovits, T., Forgács-Dajka, E., Sztakoviczs, J., Marschalkó, G., and Kutrovátsz, G. (2019). Eclipse timing variation analysis of OGLE-IV eclipsing binaries towards the Galactic Bulge - I. Hierarchical triple system candidates. MNRAS, 485(2):2562–2572.

Jetsu, L. (2019). Real light curves of FK Comae Berenices: Farewell flip-flop. arXiv e-prints, page arXiv:1808.02221.

Jetsu, L. (2020a). Discrete Chi-square Method for Detecting Many Signals. The Open Journal of Astrophysics, 3(1):4.

Jetsu, L. (2020b). Say hello to Algol D and Algol E. arXiv e-prints, page arXiv:2005.13360.

Jetsu, L. and Porceddu, S. (2015). Shifting Milestones of Natural Sciences: The Ancient Egyptian Discovery of Algol’s Period Confirmed. PLoS ONE, 10(12):e0144140.

Jetsu, L., Porceddu, S., Lyytinen, J., Kajatkari, P., Lehtinen, J., Markkanen, T., and Toivari-Viitala, J. (2013). Did the Ancient Egyptians Record the Period of the Eclipse Binary Algol - The Raging One? ApJ, 773:1.

Kozai, Y. (1962). Secular perturbations of asteroids with high inclination and eccentricity. AJ, 67:591–598.

Lomb, N. R. (1976). Least-squares frequency analysis of unequally spaced data. Ap&SS, 39:447–462.

Manzoori, D. (2016). XZ And a semidetached asynchronous binary system. Astronomy Letters, 42(5):329–338.
Porceddu, S., Jetsu, L., Markkanen, T., Lyytinen, J., Ka- jatkari, P., Lehtinen, J., and Toivari-Viitala, J. (2018). Al- gal as horus in the cairo calendar: The means and the mo- tives of the observations. Open Astronomy, 27:232–263.

Porceddu, S., Jetsu, L., Markkanen, T., and Toivari-Viitala, J. (2008). Evidence of periodicity in ancient egyptian calen- dars of lucky and unlucky days. Cambridge Archaeological Journal, 18(03):327.

Scargle, J. D. (1982). Studies in astronomical time series analysis. II - Statistical aspects of spectral analysis of unevenly spaced data. ApJ, 263:835–853.

Soderhjelm, S. (1975). The three-body problem and eclipsing binaries. Application to Algol and lambda Tauri. A&AJ, 42:229–236.

Tanrıver, M. (2015). Cyclic period changes and the light-time effect in eclipsing binaries: A low-mass companion around the system VV Ursae Majoris. NewA, 36:56–63.

Watkins, S. J., Bhattal, A. S., Boffin, H. M. J., Francis, N., and Whitworth, A. P. (1998). Numerical simulations of protostellar encounters - II. Coplanar disc-disc encounters. MNRAS, 300(4):1205–1213.

Yang, Y., Li, K., Li, Q., and Dai, H. (2016). Photometric Studies of Two Neglected Eclipsing Binaries AX Cassiopeia and V1107 Cassiopeia with Possibly Additional Companions. PASP, 128(962):044201.

Zechmeister, M. and Kürster, M. (2009). The general- ised Lomb-Scargle periodogram. A new formalism for the floating-mean and Keplerian periodograms. A&A, 496:577–584.

A Reproducing our results

In this appendix, we give all necessary information for reproduc- ing the results of our DCM period analysis. DCM manual manual.pdf, and all other required analysis files, can be copied from the Zenodo database. Copy the analysis files to the same folder in your computer. Do not edit these analysis files.

The DCM analysis program file is

dcm.py

The three different file1 data files are

hjdXZAnd.dat = HJD-data = Table A3
dXZAnd.dat = D-data = Table A5
jdXZAnd.dat = JD-data = Table A6

There are 98 control files, which are specified in the last columns of Tables A8, A9, A10, A11, A12, A13 and A15. All our results can reproduced by using these control files. For example, the two linux commands reproduce model1,1,1 period analysis results given in the first line of Table A7 (M=1). In other words, the control file dcm.dat is an exact copy of hjd14R111s.dat. This control file specifies what kind of a DCM analysis the program dcm.py should perform. The meaning of all control file dcm.dat parameters is explained in the manual manual.pdf.

The naming conventions of our control files are given in Ta- ble A1. These names are formed by using a sequence N1 + N2 + N3 + N4 + N5. The first N1 notation hjd in the file name hjd14R111s.dat means that we analyse HJD-data. The N2 notation 14 means that we search for small frequencies (long period search between 3000 and 100 000 days). All figures and files produced by dcm.py begin with the tag hjd14R111s, e.g. the periodogram figure hjd14R111sz.eps or the result file hjd14R111sParams.dat.

DCM analysis program dcm.py can be applied to the O-C data of any other star, if the format for the data file file1 and the control file dcm.dat are the same as in this paper.

| Table A1: dcm.dat control file name notations. |
| Meaning | N1 | N2 | N3 | N4 | N5 |
|---------|----|----|----|----|----|
| hjd     | hjd |   |    |    |    |
| jd      | d   |   |    |    |    |
| jd      | JD  |   |    |    |    |
| 14      | 14  |   |    |    |    |
| 58      | 58  |   |    |    |    |
| 912     | 912 |   |    |    |    |
| 13      | 13  |   |    |    |    |
| 46      | 46  |   |    |    |    |
| 79      | 79  |   |    |    |    |
| 1012    | 1012|   |    |    |    |
| R       | R   |   |    |    |    |
| 111     | 111 |   |    |    |    |
| 112     | 112 |   |    |    |    |
| 211     | 211 |   |    |    |    |
| 212     | 212 |   |    |    |    |
| 311     | 311 |   |    |    |    |
| 312     | 312 |   |    |    |    |
| 411     | 411 |   |    |    |    |
| 412     | 412 |   |    |    |    |
| 110     | 110 |   |    |    |    |
| 210     | 210 |   |    |    |    |
| 310     | 310 |   |    |    |    |
| 410     | 410 |   |    |    |    |
| s       | s   |   |    |    |    |
| 1       | 1   |   |    |    |    |

cp hjd14R111s.dat dcm.dat

dcm.py

python dcm.py
Table A2: Rejected data. Date, UT and rejection criterion

| Date       | UT   | Criterion       |
|------------|------|-----------------|
| 06.10.1923 | 21:36| Secondary minimum |
| 11.11.1949 | 05:47| Secondary minimum |
| 17.09.1965 | 01:13| Secondary minimum |
| 25.11.1975 | 17:56| Secondary minimum |
| 26.12.1996 | 00:04| Secondary minimum |
| 03.01.2008 | 18:51| Secondary minimum |
| 21.12.2009 | 19:03| Secondary minimum |
| 14.08.2010 | 23:02| Secondary minimum |
| 18.09.1923 | 17:02| Outlier         |
| 01.10.1923 | 18:57| Outlier         |
| 02.12.1923 | 19:00| Outlier         |

Table A3: O-C data in file *hjdXZAnd.dat*. Columns are primary eclipse epoch in the Sun ($t_\odot$) and observed minus computed eclipse epoch of Eq. 2 ($y_{HJD} \pm \sigma_{y_{HJD}}$). Only three first values of all $n = 1091$ values are shown.

| $t_\odot$ | $y_{HJD}$ | $\sigma_{y_{HJD}}$ |
|-----------|-----------|---------------------|
| [HJD]     | [d]       | [d]                 |
| 2411700.59000 | 0.789000 | 0.000173            |
| 2412080.62900 | 0.781000 | 0.000173            |
| 2412711.78000 | 0.784000 | 0.000173            |
| ...       | ...       | ...                 |

Table A4: Yearly distribution of data

| Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 98  | 62  | 25  | 5   | 4   | 11  | 56  | 142 | 220 | 197 | 153 | 121 |

Table A5: O-C data in file *dXZAnd.dat*. Time points computed from Eq. 3, otherwise as in Table A3.

| 2411655.32320 | 0.789000 | 0.000173 |
| 2411965.26421 | 0.781000 | 0.000173 |
| 2412702.44481 | 0.784000 | 0.000173 |
| ...           | ...       | ...       |

Table A6: O-C data in file *jdXZAnd.dat*. Columns are primary eclipse epoch on the Earth ($t_\oplus$) and observed minus computed eclipse epoch of Eq. 4 ($y_{JD} \pm \sigma_{y_{JD}}$), otherwise as in Table A3.

| $t_\oplus$ | $y_{JD}$ | $\sigma_{y_{JD}}$ |
|------------|---------|-------------------|
| [JD]       | [d]     | [d]               |
| 2411700.59000 | 0.793510 | 0.000173          |
| 2412080.62900 | 0.784770 | 0.000173          |
| 2412711.78000 | 0.786750 | 0.000173          |
| ...       | ...       | ...               |
Table A7: Long period search between 3000^d and 10000^d for HJD-data. Col 1. Model number M. Col 2. \(model_{K_1,K_2,K_3}\), \(P\) = number of free parameters and \(R = \) sum of squared residuals. Cols 3-6. Fisher period analysis results: Detected periods \(P_1,...,P_4\) and amplitudes \(A_1,...,A_4\). Cols 7-13. Fisher test results: "\(\uparrow\)" \(\equiv\) complex model above is better than left side simple model, "\(=\)" \(\equiv\) left side simple model is better than complex model above, \(F =\) Fisher test statistic and \(Q_F = \) critical level. Col 14. Figure numbers and control file .

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
\text{M} & \text{Model} & \text{Period} & \text{Analysis} & \text{Parameter} & \text{P} & \text{Fisher-test} & \text{Residuals} & \text{Fisher-test} & \text{Residuals} & \text{Fisher-test} & \text{Residuals} & \text{Fisher-test} \\
\hline
1 & \text{model}_{1,1,1} & 1.911 & - & 0.0572 & F < 0.0001 & Q_F < 10^{-16} & Q_F < 10^{-16} & Q_F < 10^{-16} & Q_F < 10^{-16} \\
\hline
\text{Model number} & \text{Parameter} & \text{Fisher-test} & \text{Residuals} & \text{Fisher-test} & \text{Residuals} & \text{Fisher-test} & \text{Residuals} & \text{Fisher-test} & \text{Residuals} & \text{Fisher-test} \\
\hline
2 & \text{model}_{2,1,2} & 1.592 & - & 0.0528 & F < 0.0001 & Q_F < 10^{-16} & Q_F < 10^{-16} & Q_F < 10^{-16} & Q_F < 10^{-16} \\
\hline
3 & \text{model}_{3,1,3} & 1.235 & - & 0.0552 & F < 0.0001 & Q_F < 10^{-16} & Q_F < 10^{-16} & Q_F < 10^{-16} & Q_F < 10^{-16} \\
\hline
\end{array}
\]

Table A8: Long period search between 3000^d and 10000^d. Test^3+3^3+3 for HJD-data. Notation \(\uparrow\)^{\#}\(\uparrow\) is explained in Sect 4, otherwise as in Table A7.
Table A9: Long period search between 3000^d and 100000^d. Test 4+4+4 for HJD-data, otherwise as in Table A7.

| Model | Period analysis | Original data (Table A3 = file1 = hjd2536.dat) | Fisher-test | model 1-1-2 | model 1-1-2 | model 1-1-2 |
|-------|-----------------|--------------------------------------------------|-------------|-------------|-------------|-------------|
|       | PK [d]          | PK [log]                                          | PK [d]      | Fisher-test | Fisher-test | Fisher-test |
| 1     | model 1-1-2     | 131482109                                        | -           | F=5157      | Q p<10^-16  | Q p<10^-16  |
| p=8   | -               | 0.009290001                                      | -           | F=2907      | Q p<10^-16  | Q p<10^-16  |
| R=0   | 363566           | -                                                 | -           | F=2228      | Q p<10^-16  | Q p<10^-16  |

| 2     | model 1-1-2     | 131482109                                        | -           | F=5157      | Q p<10^-16  | Q p<10^-16  |
| p=9   | -               | 0.058290004                                      | -           | F=44.0      | Q p<10^-16  | Q p<10^-16  |
| R=0   | 02354            | -                                                 | -           | F=50.9      | Q p<10^-16  | Q p<10^-16  |

| 3     | model 1-1-2     | 314212213                                        | -           | F=5157      | Q p<10^-16  | Q p<10^-16  |
| p=12  | -               | 0.094290004                                      | -           | F=44.0      | Q p<10^-16  | Q p<10^-16  |
| R=0   | 006434           | -                                                 | -           | F=50.9      | Q p<10^-16  | Q p<10^-16  |

| 4     | model 1-1-2     | 314212213                                        | -           | F=5157      | Q p<10^-16  | Q p<10^-16  |
| p=15  | -               | 0.094290004                                      | -           | F=44.0      | Q p<10^-16  | Q p<10^-16  |
| R=0   | 006434           | -                                                 | -           | F=50.9      | Q p<10^-16  | Q p<10^-16  |

| M     | Period analysis | Original data (Table A3 = file1 = hjd2536.dat) | Fisher-test | model 1-1-2 | model 1-1-2 | model 1-1-2 |
|-------|-----------------|--------------------------------------------------|-------------|-------------|-------------|-------------|
|       | PK [d]          | PK [log]                                          | PK [d]      | Fisher-test | Fisher-test | Fisher-test |
| 5     | model 1-1-2     | 422517                                            | -           | F=25.5      | Q p=1.6 x 10^-11 | Q p<10^-16 |
| p=4   | -               | 0.094290004                                      | -           | F=21.9      | Q p<10^-16  | Q p<10^-16  |
| R=0   | 006434           | -                                                 | -           | F=21.3      | Q p<10^-16  | Q p<10^-16  |

| 6     | model 1-1-2     | 430312213                                        | -           | F=25.5      | Q p=1.6 x 10^-11 | Q p<10^-16 |
| p=7   | -               | 0.01290101†                                      | -           | F=17.1      | Q p=7.0 x 10^-11 | Q p<10^-16 |
| R=0   | 006434           | -                                                 | -           | F=18.0      | Q p<10^-16  | Q p<10^-16  |

| 7     | model 1-1-2     | 391121238                                        | -           | F=25.5      | Q p=1.6 x 10^-11 | Q p<10^-16 |
| p=10  | -               | 0.062890003                                      | -           | F=18.0      | Q p=7.0 x 10^-11 | Q p<10^-16 |
| R=0   | 006434           | -                                                 | -           | F=18.0      | Q p<10^-16  | Q p<10^-16  |

| 8     | model 1-1-2     | 301921238                                        | -           | F=25.5      | Q p=1.6 x 10^-11 | Q p<10^-16 |
| p=13  | -               | 0.094290004                                      | -           | F=18.0      | Q p=7.0 x 10^-11 | Q p<10^-16 |
| R=0   | 006434           | -                                                 | -           | F=18.0      | Q p<10^-16  | Q p<10^-16  |

| M     | Period analysis | Original data (Table A3 = file1 = hjd2536.dat) | Fisher-test | model 1-1-2 | model 1-1-2 | model 1-1-2 |
|-------|-----------------|--------------------------------------------------|-------------|-------------|-------------|-------------|
|       | PK [d]          | PK [log]                                          | PK [d]      | Fisher-test | Fisher-test | Fisher-test |
| 9     | model 1-1-2     | 560521238                                        | -           | F=3.3       | Q p<0.0013  | Q p<0.0014  |
| p=4   | -               | 0.062890003                                      | -           | F=4.5       | Q p<0.0013  | Q p<0.0014  |
| R=0   | 006434           | -                                                 | -           | F=3.6       | Q p<0.0013  | Q p<0.0014  |

| 10    | model 1-1-2     | 560521238                                        | -           | F=3.3       | Q p<0.0013  | Q p<0.0014  |
| p=8   | -               | 0.062890003                                      | -           | F=4.5       | Q p<0.0013  | Q p<0.0014  |
| R=0   | 006434           | -                                                 | -           | F=3.6       | Q p<0.0013  | Q p<0.0014  |

| 11    | model 1-1-2     | 580921235                                        | -           | F=3.3       | Q p<0.0013  | Q p<0.0014  |
| p=10  | -               | 0.094290004                                      | -           | F=4.5       | Q p<0.0013  | Q p<0.0014  |
| R=0   | 006434           | -                                                 | -           | F=3.6       | Q p<0.0013  | Q p<0.0014  |

| 12    | model 1-1-2     | 554731238                                        | -           | F=3.3       | Q p<0.0013  | Q p<0.0014  |
| p=13  | -               | 0.062890003                                      | -           | F=4.5       | Q p<0.0013  | Q p<0.0014  |
| R=0   | 006434           | -                                                 | -           | F=3.6       | Q p<0.0013  | Q p<0.0014  |
Table A10: Long period search between 3000\textsuperscript{d} and 100000\textsuperscript{d}. Test\textsuperscript{3+3+3+3} for D-data, otherwise as in Table A7

| M  | Model     | Period analysis (Table A8 = Initial data) | Fisher-test |
|----|-----------|------------------------------------------|-------------|
|    |           | Original data | Fishertest  |              |
| 1  | model1\_1 |               |             |              |
|    | p=6 0.050500002 | - | - | F=4794 | QF\textless 10\textsuperscript{-16} |
|    | R=0 2107 |               |             |              |
| 2  | model2\_1 |               |             |              |
|    | p=6 0.057950003 | - | - | F=38.1 | QF\textless 10\textsuperscript{-16} |
|    | R=0 2528 |               |             |              |
| 3  | model3\_1 |               |             |              |
|    | p=6 0.004310004 | - | - | F=263 | QF\textless 10\textsuperscript{-16} |
|    | R=0 2286 |               |             |              |

Residuals of three signals (d13R312s.dat)

| M  | Model     | Period analysis | Fisher-test |
|----|-----------|-----------------|-------------|
| 1  | model1\_1 |               |             |              |
|    | p=4 0.003920003 | - | - | F=14.1 | QF=4.9 \times 10\textsuperscript{-9} |
|    | R=0 0076 |               |             |              |
| 2  | model2\_1 |               |             |              |
|    | p=7 0.002420003 | - | - | F=19.2 | QF\textless 10\textsuperscript{-16} |
|    | R=0 02009 |               |             |              |
| 3  | model3\_1 |               |             |              |
|    | p=10 0.0120 0.02\textsuperscript{+} | - | - | F=472.0 | QF\textless 10\textsuperscript{-16} |
|    | R=0 01899 |               |             |              |

Residuals of six signals (d46R210s.dat)

| M  | Model     | Period analysis | Fisher-test |
|----|-----------|-----------------|-------------|
| 1  | model1\_1 |               |             |              |
|    | p=4 0.002250004 | - | - | F=11.0 | QF=8.8 |
|    | R=0 01836 |               |             |              |
| 2  | model2\_1 |               |             |              |
|    | p=7 0.002420005 | - | - | F=4.4 | QF=4.9 \times 10\textsuperscript{-9} |
|    | R=0 01782 |               |             |              |
| 3  | model3\_1 |               |             |              |
|    | p=10 0.002420002 | - | - | F=4.9 | QF=4.9 \times 10\textsuperscript{-9} |
|    | R=0 01751 |               |             |              |

Residuals of nine signals (d13R312s.dat)

| M  | Model     | Period analysis | Fisher-test |
|----|-----------|-----------------|-------------|
| 1  | model1\_1 |               |             |              |
|    | p=4 0.001320003 | - | - | F=5.2 | QF=3.3 |
|    | R=0 01730 |               |             |              |
| 2  | model2\_1 |               |             |              |
|    | p=7 0.002420004 | - | - | F=3.3 | QF=3.3 |
|    | R=0 01715 |               |             |              |
| 3  | model3\_1 |               |             |              |
|    | p=10 0.0220 0.02\textsuperscript{+} | - | - | F=3.3 | QF=3.3 |
|    | R=0 01699 |               |             |              |
Table A11: Long period search between 3000$^d$ and 100000$^d$. Test$^{4+4+4}$ for D-data. otherwise as in Table A7.

| Model | Period analysis | Fisher-test | Original data (Table A5 = file4110.dat) |
|-------|-----------------|-------------|---------------------------------------|
|       | $T_1$, $A_1$ | $T_2$, $A_2$ | $T_3$, $A_3$ | $T_4$, $A_4$ | model1,2 | model3,4 | model5,6 | model7,8 | model9,10 |
| 1     | model1,2,3   | 0.059500 | 0.059500 | 0.059500 | 0.059500 | F=4794 | F=2663 | F=2626 | F=1196 | F=2626 |
|       | p=0.059500 | 0.059500 | 0.059500 | 0.059500 | QF<10$^{-16}$ | QF<10$^{-16}$ | QF<10$^{-16}$ | QF<10$^{-16}$ | QF<10$^{-16}$ |
| 2     | model1,2,3   | 0.059500 | 0.059500 | 0.059500 | 0.059500 | F=38.1 | F=46.0 | F=46.0 | F=46.0 | F=46.0 |
|       | p=0.059500 | 0.059500 | 0.059500 | 0.059500 | QF<10$^{-16}$ | QF<10$^{-16}$ | QF<10$^{-16}$ | QF<10$^{-16}$ | QF<10$^{-16}$ |
| 3     | model1,2,3   | 0.059500 | 0.059500 | 0.059500 | 0.059500 | F=14.0 | F=13.2 | F=13.2 | F=13.2 | F=13.2 |
|       | p=0.059500 | 0.059500 | 0.059500 | 0.059500 | QF<10$^{-16}$ | QF<10$^{-16}$ | QF<10$^{-16}$ | QF<10$^{-16}$ | QF<10$^{-16}$ |
| 4     | model1,2,3   | 0.059500 | 0.059500 | 0.059500 | 0.059500 | F=14.0 | F=13.2 | F=13.2 | F=13.2 | F=13.2 |
|       | p=0.059500 | 0.059500 | 0.059500 | 0.059500 | QF<10$^{-16}$ | QF<10$^{-16}$ | QF<10$^{-16}$ | QF<10$^{-16}$ | QF<10$^{-16}$ |

Note: $T_i$ and $A_i$ represent the period and amplitude for each signal, respectively. The Fisher-test results are given for each model, with significance levels indicating whether the signal is statistically different from zero.
Table A12: Long period search between 3000\textsuperscript{d} and 100000\textsuperscript{d}. Test\textsuperscript{3+3+3+3} for JD-data, otherwise as in Table A7.

| M | Model | Period analysis | Original data (Table A6 = jd\textsuperscript{12}d.dat) | Fisher-test |
|---|-------|----------------|------------------------------------------------------|-------------|
| 1 | model\textsubscript{1,1,0} | 4.84e-211 | 6.50e-201 | F=113.05, P<10\textsuperscript{-16} |
| R=0.3605 | | | | |
| 2 | model\textsubscript{2,1,0} | 4.84e-211 | 6.50e-201 | F=113.05, P<10\textsuperscript{-16} |
| R=0.3605 | | | | |
| 3 | model\textsubscript{3,1,0} | 4.84e-211 | 6.50e-201 | F=113.05, P<10\textsuperscript{-16} |
| R=0.3605 | | | | |

Residues of three signals (jd\textsuperscript{13}d.dat)

| M | Model | Period analysis | Original data (jd\textsuperscript{13}d.dat) | Fisher-test |
|---|-------|----------------|------------------------------------------------------|-------------|
| 1 | model\textsubscript{1,0,0} | 4.84e-211 | 6.50e-201 | F=113.05, P<10\textsuperscript{-16} |
| R=0.3605 | | | | |
| 2 | model\textsubscript{2,0,0} | 4.84e-211 | 6.50e-201 | F=113.05, P<10\textsuperscript{-16} |
| R=0.3605 | | | | |
| 3 | model\textsubscript{3,0,0} | 4.84e-211 | 6.50e-201 | F=113.05, P<10\textsuperscript{-16} |
| R=0.3605 | | | | |

Residues of six signals (jd\textsuperscript{14}d.dat)

| M | Model | Period analysis | Original data (jd\textsuperscript{14}d.dat) | Fisher-test |
|---|-------|----------------|------------------------------------------------------|-------------|
| 1 | model\textsubscript{1,0,0} | 4.84e-211 | 6.50e-201 | F=113.05, P<10\textsuperscript{-16} |
| R=0.3605 | | | | |
| 2 | model\textsubscript{2,0,0} | 4.84e-211 | 6.50e-201 | F=113.05, P<10\textsuperscript{-16} |
| R=0.3605 | | | | |
| 3 | model\textsubscript{3,0,0} | 4.84e-211 | 6.50e-201 | F=113.05, P<10\textsuperscript{-16} |
| R=0.3605 | | | | |

Residues of nine signals (jd\textsuperscript{15}d.dat)

| M | Model | Period analysis | Original data (jd\textsuperscript{15}d.dat) | Fisher-test |
|---|-------|----------------|------------------------------------------------------|-------------|
| 1 | model\textsubscript{1,0,0} | 4.84e-211 | 6.50e-201 | F=113.05, P<10\textsuperscript{-16} |
| R=0.3605 | | | | |
| 2 | model\textsubscript{2,0,0} | 4.84e-211 | 6.50e-201 | F=113.05, P<10\textsuperscript{-16} |
| R=0.3605 | | | | |
| 3 | model\textsubscript{3,0,0} | 4.84e-211 | 6.50e-201 | F=113.05, P<10\textsuperscript{-16} |
| R=0.3605 | | | | |
### Table A1: Comparison of photometric results

| Model | HJD-data | D-data | JD-data | Period analysis | Fisher-test | QF |
|-------|----------|--------|---------|-----------------|-------------|----|
| Model 1 | -        | -      | -       |                  |             |    |
| Model 2 | -        | -      | -       |                  |             |    |
| Model 3 | -        | -      | -       |                  |             |    |
| Model 4 | -        | -      | -       |                  |             |    |

For JD-data, differences in Table A1.

| Model | HJD-data | D-data | JD-data | Period analysis | Fisher-test | QF |
|-------|----------|--------|---------|-----------------|-------------|----|
| Model 1 | -        | -      | -       |                  |             |    |
| Model 2 | -        | -      | -       |                  |             |    |
| Model 3 | -        | -      | -       |                  |             |    |
| Model 4 | -        | -      | -       |                  |             |    |

For JD-data, differences in Table A1.

| Model | HJD-data | D-data | JD-data | Period analysis | Fisher-test | QF |
|-------|----------|--------|---------|-----------------|-------------|----|
| Model 1 | -        | -      | -       |                  |             |    |
| Model 2 | -        | -      | -       |                  |             |    |
| Model 3 | -        | -      | -       |                  |             |    |
| Model 4 | -        | -      | -       |                  |             |    |

For JD-data, differences in Table A1.
Table A15: Short period search between 300$^d$ and 3000$^d$. Fours best periods for the eight signal residuals (HJD-data: M=1-4, D-data: M=5-8, JD-data: M=9-12), otherwise as in Table A7.

| M  | Model | Period analysis | Fisher-test |
|----|-------|----------------|-------------|
|    |       | $P_{dK2-A}$ | $P_{dK2-A2}$ | $P_{dK3-A}$ | $P_{dK4-A}$ | model2,1,0 | model3,1,0 | model4,1,0 |
| 1  | model1,1,0 | 641.2 ± 11 | 0.0013 ± 0.0002 | - | - | $F = 4.8$ | $Q_F = 0.0026$ | $Q_F = 5.6 \times 10^{-7}$ | dcm.dat = hjd14R111.dat |
|    | p = 7 | 0.0013 ± 0.0002 | - | - | $F = 4.2$ | $Q_F = 0.0038$ | $Q_F = 5.6 \times 10^{-7}$ | dcm.dat = hjd14R210.dat |
|    | $R = 0.01462$ | - | - | - | - | - | - | - |
| 2  | model2,1,0 | 365.5 ± 10 | 616.4 ± 14 | - | - | $F = 3.6$ | $Q_F = 0.014$ | $Q_F = 1.6 \times 10^{-5}$ | dcm.dat = hjd14R210.dat |
|    | p = 7 | 0.0018 ± 0.0005 | 0.0013 ± 0.0004 | - | - | $F = 3.5$ | $Q_F = 0.014$ | $Q_F = 1.6 \times 10^{-5}$ | dcm.dat = hjd14R210.dat |
|    | $R = 0.01443$ | - | - | - | - | - | - | - |
| 3  | model3,1,0 | 314.4 ± 13 | 365.4 ± 98 | 617.2 ± 19 | - | - | - | - | dcm.dat = hjd14R210.dat |
|    | p = 10 | 0.0010 ± 0.0002 | 0.0013 ± 0.0005 | 0.0013 ± 0.0002 | - | - | - | - | dcm.dat = hjd14R210.dat |
|    | $R = 0.01429$ | - | - | - | - | - | - | - |
| 4  | model4,1,0 | 365.6 ± 9.8 | 365.4 ± 19 | 591.6 ± 187 | 616.4 ± 21 | - | - | - | dcm.dat = hjd14R210.dat |
|    | p = 13 | 0.0018 ± 0.0004 | 0.0012 ± 0.0003 | 0.0012 ± 0.0003$^*$ | 0.0012 ± 0.0003$^*$ | - | - | - | dcm.dat = hjd14R210.dat |
|    | $R = 0.01401$ | - | - | - | - | - | - | - |

| M  | Model | Period analysis | Fisher-test |
|----|-------|----------------|-------------|
|    |       | $P_{dK2-A}$ | $P_{dK2-A2}$ | $P_{dK3-A}$ | $P_{dK4-A}$ | model2,1,0 | model3,1,0 | model4,1,0 |
| 5  | model1,1,0 | 364.9 ± 0.3 | 0.0022 ± 0.0003 | - | - | - | - | - | dcm.dat = hjd14R111.dat |
|    | p = 4 | 0.0022 ± 0.0003 | - | - | - | - | - | - | dcm.dat = hjd14R111.dat |
|    | $R = 0.01635$ | - | - | - | - | - | - | - |
| 6  | model2,1,0 | 364.9 ± 0.3 | 593.1 ± 26 | - | - | - | - | - | dcm.dat = hjd14R111.dat |
|    | p = 7 | 0.0022 ± 0.0003 | 0.0013 ± 0.0002 | - | - | - | - | - | dcm.dat = hjd14R111.dat |
|    | $R = 0.01612$ | - | - | - | - | - | - | - |
| 7  | model3,1,0 | 364.9 ± 34 | 593.1 ± 26 | 2244 ± 49 | - | - | - | - | dcm.dat = hjd14R111.dat |
|    | p = 10 | 0.0023 ± 0.0003 | 0.0013 ± 0.0003 | 0.0013 ± 0.0003 | - | - | - | - | dcm.dat = hjd14R111.dat |
|    | $R = 0.01575$ | - | - | - | - | - | - | - |
|    | model4,1,0 | 364.9 ± 22 | 593.1 ± 30 | 2244 ± 372 | 7714 ± 1367 | - | - | - | dcm.dat = hjd14R111.dat |
|    | $p = 13$ | 0.0023 ± 0.0003 | 0.0012 ± 0.0003 | 0.0012 ± 0.0004$^*$ | 0.0010 ± 0.0003$^*$ | - | - | - | dcm.dat = hjd14R111.dat |
|    | $R = 0.01565$ | - | - | - | - | - | - | - |

| M  | Model | Period analysis | Fisher-test |
|----|-------|----------------|-------------|
|    |       | $P_{dK2-A}$ | $P_{dK2-A2}$ | $P_{dK3-A}$ | $P_{dK4-A}$ | model2,1,0 | model3,1,0 | model4,1,0 |
| 9  | model1,1,0 | 365.3 ± 0.08 | 0.0110 ± 0.0004 | - | - | - | - | - | dcm.dat = hjd14R111.dat |
|    | p = 4 | 0.0110 ± 0.0004 | - | - | - | - | - | - | dcm.dat = hjd14R111.dat |
|    | $R = 0.01485$ | - | - | - | - | - | - | - |
| 10 | model2,1,0 | 365.3 ± 0.08 | 365.4 ± 0.6 | - | - | - | - | - | dcm.dat = hjd14R111.dat |
|    | p = 7 | 0.0010 ± 0.0004$^*$ | 0.0010 ± 0.0004$^*$ | - | - | - | - | - | dcm.dat = hjd14R111.dat |
|    | $R = 0.01474$ | - | - | - | - | - | - | - |
| 11 | model3,1,0 | 365.3 ± 1.3 | 365.4 ± 30 | 593.3 ± 24 | - | - | - | - | dcm.dat = hjd14R111.dat |
|    | p = 10 | 0.0010 ± 0.0004$^*$ | 0.0010 ± 0.0004$^*$ | 0.0012 ± 0.0004 | - | - | - | - | dcm.dat = hjd14R111.dat |
|    | $R = 0.01453$ | - | - | - | - | - | - | - |
| 12 | model4,1,0 | 365.3 ± 1.3 | 365.5 ± 38 | 595.7 ± 26 | 5612 ± 844 | - | - | - | dcm.dat = hjd14R111.dat |
|    | $p = 13$ | 0.0010 ± 0.0004$^*$ | 0.0012 ± 0.0002 | 0.0012 ± 0.0002 | 0.0013 ± 0.0002 | - | - | - | dcm.dat = hjd14R111.dat |
|    | $R = 0.01431$ | - | - | - | - | - | - | - |
Table A16: $M_1$ and $M_2$ coefficients of $p(t)$.

| Model                  | $M_1$       | $M_2$       |
|------------------------|-------------|-------------|
| Table A9: $M=1$        | $-0.930 \pm 0.007$ | $0.209 \pm 0.003$ |
| Table A9: $M=2$        | $-0.646 \pm 0.006$ | $0.089 \pm 0.002$ |
| Table A9: $M=3$        | $-0.635 \pm 0.006$ | $0.085 \pm 0.002$ |
| Table A9: $M=4$        | $-0.627 \pm 0.005$ | $0.082 \pm 0.002$ |