Solving fully fuzzy transportation problem using pentagonal fuzzy numbers

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Abstract. In this paper, we propose a simple approach for the solution of fuzzy transportation problem under fuzzy environment in which the transportation costs, supplies at sources and demands at destinations are represented by pentagonal fuzzy numbers. The fuzzy transportation problem is solved without converting to its equivalent crisp form using a robust ranking technique and a new fuzzy arithmetic on pentagonal fuzzy numbers. To illustrate the proposed approach a numerical example is provided.

1. Introduction
Transportation problem is an important network structured linear programming problem that arises in several contexts and received a great deal of attention in the literature. Transportation problem can be used for a wide variety of situations such as production, investment, plant location, inventory control, employment scheduling and many others. In general, transportation problems are solved with the assumptions that the transportation costs, supplies at sources and demands at destinations are specified in a precise way i.e., in crisp environment. However, in many cases the decision maker may not able to get precise values for the decision parameters for the transportation problem. If the nature of the information is vague, that is, if it has some lack of precision, then the corresponding coefficients or elements defining the problem can be formulated by means of fuzzy numbers and thus fuzzy transportation problems arise.

Several researchers have carried out investigations on fuzzy transportation problems (FTP). Lai and Hwang [8] developed transportation model that solving the problem when quantities are fuzzy and prices are crisp. Chanas and Kuchta[2] presented the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers and developed an algorithm for obtaining the optimal solution. Chanas and Kuchta presented an algorithm that solves the transportation problem with fuzzy supply and demand values and integrality condition imposed on the solution. Liu and Kao[11] provided a process to derive the fuzzy objective value of the fuzzy transportation problem, in
that the supply and demand quantities and the cost coefficients are fuzzy numbers basing on extension principle are analyzed the transportation problem with fuzzy supply values of deliverers and with fuzzy demand values of the receivers. Ahlatioglu, Sivri and Güzel [1] proposed a solution algorithm finding all fuzzy optimal solution of the transportation problem that the cost coefficients and the supply and demand quantities are fuzzy numbers. Gen [6] describes an implementation of genetic algorithm to solve Bicriteria Solid Transportation Problem. Pandian and Natarajan[13] introduced the new algorithm, zero point method to find the fuzzy optimal solution of fuzzy transportation problem. Parra [12] proposed a method for solving fuzzy transportation problem and also to find the possibility distribution of the objective value of the transportation problem provided all the inequality constraints are of ≤ types or ≥ types. Nagoor Gani and Abdul Razak [7] obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers using a parametric approach. Edward [5] proposed the simplex type algorithm for the solving the Fuzzy transportation problems. Lin. F.T[9] introduced a genetic algorithm solving the fuzzy relation with liner objective functions. Dinagar[4] proposed a new algorithm for finding a fuzzy optimal solution for fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers. In general, most of the existing techniques provide only crisp solutions for the fuzzy transportation problem. In this article a simple method for the solution of fuzzy transportation problem without converting them into classical transportation problem is proposed where all parameters are pentagonal fuzzy numbers.

The rest of this paper is organized as follows. In Section 2, some basic definition and arithmetic operations are reviewed. In Section 3, we attempt to introduce a formulation of fuzzy transportation problem with pentagonal fuzzy numbers. In Section 4, we propose a simple method for solving fuzzy transportation problems without converting to its equivalent crisp form using a robust ranking technique and a new fuzzy arithmetic on pentagonal fuzzy numbers. In Section 5, a numerical example is presented to illustrate the proposed method. Finally, conclusions are presented in Sect. 6.

2. Preliminaries

We review the fundamental notation of fuzzy set theory, initiated by Bellman and Zadeh.

Definition 2.1. A fuzzy set \( \tilde{A} \) defined on the set of real numbers \( \mathbb{R} \) is said to be a fuzzy number, if its membership function \( \mu_\tilde{A} : \mathbb{R} \to [0,1] \) has the following characteristics:

(i) \( \mu_\tilde{A} \) is convex, i.e., 
   \[ \mu_\tilde{A}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_\tilde{A}(x_1), \mu_\tilde{A}(x_2)\}, \lambda \in [0,1] \text{ and } \forall x_1, x_2 \in \mathbb{R}. \]

(ii) \( \mu_\tilde{A} \) is normal, i.e., there exists an \( x \in \mathbb{R} \) such that \( \mu_\tilde{A}(x) = 1 \).

(iii) \( \tilde{A} \) is upper semi-continuous.

(iv) \( \sup (\tilde{A}) \) is bounded in \( \mathbb{R} \).

Definition 2.2. A fuzzy number \( \tilde{A} \) on \( \mathbb{R} \) is said to be a pentagonal fuzzy number (PFN) or linear fuzzy number if its membership function \( \tilde{A} : \mathbb{R} \to [0,1] \) has the following characteristics:
where the middle point $r$ has the grade of membership 1 and $q, s$ has the grades $u_1, u_2$ respectively. Note that every PFN is associated with two weight $u_1$ and $u_2$.

From the above figure, it is clear that $\tilde{A}$ has a piecewise continuous graph of five points in its domain, forming a pentagonal shape. As chosen, the points in the domain have the ordering $p < q < r < s < t$; $p, q, r, s, t \in R$. We have to choose the value of membership function at $q, s$ in such a way that $u_1 \geq \frac{q-p}{r-p}$ and $u_2 \geq \frac{p-t}{r-t}$. Otherwise, the convexity properties of the fuzzy number fail for the pentagonal fuzzy number.

Remark 2.3. (i). The pentagonal fuzzy numbers $\tilde{A}$ becomes a triangular fuzzy number then $u_1 = u_2 = 0$ and $\tilde{A} = (p, q, r, s, t) \equiv (q, r, s)$.  
(ii). The pentagonal fuzzy numbers $\tilde{A}$ becomes a trapezoidal fuzzy number then $u_1 = u_2 = 1$ and $\tilde{A} = (p, q, r, s, t) \equiv (p, q, s, t)$.

2.4. Arithmetic Operations: Let $\tilde{A} = (p_1, q_1, r_1, s_1, t_1)$ and $\tilde{B} = (p_2, q_2, r_2, s_2, t_2)$ are two fuzzy numbers where $p_1 \leq q_1 \leq r_1 \leq s_1 \leq t_1$, similarly $p_2 \leq q_2 \leq r_2 \leq s_2 \leq t_2$ then the arithmetic operations are defined as:
(i). Addition :
\[ \tilde{A} + \tilde{B} = (p_1 + p_2, q_1 + q_2, r_1 + r_2, s_1 + s_2, t_1 + t_2). \]

(ii). Subtraction :
\[ \tilde{A} - \tilde{B} = (p_1 - t_2, q_1 - s_2, r_1 - r_2, s_1 - q_2, t_1 - t_2). \]

(iii). Multiplication :
\[ \tilde{A} \times \tilde{B} = \left( \frac{p_1 + p_2}{5}, \frac{q_1 + q_2}{5}, \frac{r_1 + r_2}{5}, \frac{s_1 + s_2}{5}, \frac{t_1 + t_2}{5} \right), \text{ where } \beta_b = (p_2 + q_2 + r_2 + s_2 + t_2). \]

(iv). Division :
\[ \tilde{A} \div \tilde{B} = \left( \frac{5p_1}{\beta_b}, \frac{5q_1}{\beta_b}, \frac{5r_1}{\beta_b}, \frac{5s_1}{\beta_b}, \frac{5t_1}{\beta_b} \right), \text{ if } \beta_b \neq 0 \text{ where } \beta_b = (p_2 + q_2 + r_2 + s_2 + t_2). \]

(v). Scalar Multiplication :
\[ k\tilde{A} = (k p, k q, k r, k s, k t) \text{, if } k > 0 \]
\[ (k t, k s, k r, k q, k p) \text{, if } k < 0 \]

2.5. Ranking Function: In this paper we proposed a new ranking technique. Let \( \tilde{A} = (p, q, r, s, t) \) be pentagon fuzzy numbers and it is divided into four parts which includes two triangles and two trapezoids, by using Robust ranking technique for triangles and trapezoidal fuzzy numbers here new ranking was introduced.
\[ R(\tilde{A}) = \int_0^1 0.5 (a^L_\alpha, a^U_\alpha) d\alpha. \]
\[ (i.e.) \quad R(\tilde{A}) = \int_0^1 0.5 [(q - p)\alpha + p, t + (t - s)\alpha] d\alpha \]

3. Fuzzy Transportation Problems
The Fuzzy Transportation problems deal with the Transportation of a single product from several sources to several sinks. In general, let there be \( m \) sources \( S_1, S_2, ..., S_m \) with \( \tilde{a}_i \) \( (i = 1, 2, ..., m) \) available supplies or capacity at each source \( i \), to be allocated among \( n \) destinations \( D_1, D_2, ..., D_n \) with \( \tilde{b}_j \) \( (j = 1, 2, ..., n) \) specified requirements at each destination \( j \). Let \( \tilde{c}_{ij} \) be the cost of shipping one from \( i \) to destination \( j \) for each route. Then, if \( \tilde{x}_{ij} \) be the units shipped per route from source \( i \) to destination \( j \), the problem is to determine the transportation schedule so as to minimize the total transportation cost satisfying the supply and demand condition.

The problem may as stated as follows:
\[ \text{minimize} \quad \tilde{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij} \]
subject to the constraints:
For a feasible solution to exist, it is necessary that supply equals total requirement,

\[ \sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j \]

The fuzzy transportation problem is explicitly represented by the following fuzzy transportation table:

|       | \( D_1 \) | \( D_2 \) | \( \ldots \) | \( D_m \) | Supply |
|-------|-----------|-----------|-------------|-----------|--------|
| \( S_1 \) | \( \tilde{c}_{11} \) | \( \tilde{c}_{12} \) | \( \ldots \) | \( \tilde{c}_{1n} \) | \( \tilde{a}_1 \) |
| \( S_2 \) | \( \tilde{c}_{21} \) | \( \tilde{c}_{22} \) | \( \ldots \) | \( \tilde{c}_{2n} \) | \( \tilde{a}_2 \) |
| \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) |
| \( S_m \) | \( \tilde{c}_{m1} \) | \( \tilde{c}_{m2} \) | \( \ldots \) | \( \tilde{c}_{mn} \) | \( \tilde{a}_m \) |
| Demand | \( \tilde{b}_1 \) | \( \tilde{b}_2 \) | \( \ldots \) | \( \tilde{b}_n \) |        |

4. Solution Procedure

Fuzzy transportation problems require two methods for its optimal basic feasible solution.

4.1. Fuzzy version of VAM method

Step 1: Calculate penalties by taking differences between the minimum and next to transportation cost in each row and each column.
Step 2: Circle the largest Row Difference and Column Difference. In the event of a tie, choose either.
Step 3: Allocate as much as possible in the lowest cost cell of the row (or column) having a circled Row (or Column) Difference.
Step 4: In the case the allocation is made fully to a row (or column), ignore that row (or column) for further consideration, by crossing it.
Step 5: Revise the differences again and cross out the earlier figures. Go to step 2.
Step 6: Continue the procedure until all rows and columns have been crossed out. i.e., distribution is complete.

4.2. Fuzzy version of MODI method

For checking optimality of the current basic feasible solution of the fuzzy transportation problem may be summarized in the following iterative procedure:
Step 1: Check the number of occupied cells. if these are less than \( m+n-1 \), there exists degeneracy and we introduced a very small positive assignment of \( \varepsilon \approx 0 \) in suitable independent positions, so that the number of occupied cells is exactly equal to \( m+n-1 \).
Step 2: For each occupied cell in the current solution, solve the system of equations \( u_i + v_j = c_{ij} \) starting initially with some \( u_i = 0 \) or \( v_j = 0 \) and entering successively the values of \( u_i \) and \( v_j \) in the transportation table margins.
Step 3: Compute the net evaluations \( z_{ij} = c_{ij} - (u_i + v_j) \) for all unoccupied basic cells and enter them in the upper right corners of the corresponding cells.
Step 4: Examine the sign of each $z_{ij} - c_{ij}$. If all $z_{ij} - c_{ij} \geq 0$, then the current basic feasible solution is on optimum one. If at least one $z_{ij} - c_{ij} \leq 0$, select the unoccupied cell, having the largest negative net evaluation enter the basis.

Step 5: Let the unoccupied cell $(r, s)$ enter the basis. Allocate an unknown quantity, say $\theta$, to the cell $(r, s)$. Identify a loop that starts and ends at the cell $(r, s)$ and connects some of the basic cells. Add and subtract $\theta$ interchangeably, to and from the basic cells of the loop, in such a way that the rim requirements remain satisfied.

Step 6: Assign the minimum value to $\theta$ in such a way that the value of one basic variable becomes zero and the other basic variables remain non-negative. The basic cell, whose allocation has been reduced to zero, leaves the basis.

Step 7: Return to Step 2 and repeat the process until an optimum basic feasible solution has been obtained.

5. Numerical Example
Consider a fully fuzzy Transportation problem in which supply at sources, demand at destinations and fuzzy unit transportation costs, etc…. are assumed to be pentagon fuzzy numbers.

| Destinations | $D_1$ | $D_2$ | $D_3$ | Fuzzy Supply |
|--------------|-------|-------|-------|-------------|
| Sources      |       |       |       |             |
| $O_1$        | (5,10,13,14,18) | (1,2,3,4,5) | (2,6,8,10,14) | (2,11,23,34,45) |
| $O_2$        | (3,4,5,6,7) | (1,5,6,7,11) | (1,4,5,9,16) | (10,47,52,65,76) |
| $O_3$        | (3,6,9,12,15) | (2,5,7,8,8) | (1,1,1,1) | (3,18,56,76,87) |
| Fuzzy Demand | (11,16,51,67,75) | (20,40,60,80,100) | (15,30,45,75,110) | |

The given fuzzy transportation problem is unbalanced. Now we convert the unbalanced fuzzy transportation problem into balanced fuzzy transportation.

| Destinations | $D_1$ | $D_2$ | $D_3$ | Fuzzy Supply |
|--------------|-------|-------|-------|-------------|
| Sources      |       |       |       |             |
| $O_1$        | (5,10,13,14,18) | (1,2,3,4,5) | (2,6,8,10,14) | (2,11,23,34,45) |
| $O_2$        | (3,4,5,6,7) | (1,5,6,7,11) | (1,4,5,9,16) | (10,47,52,65,76) |
| $O_3$        | (3,6,9,12,15) | (2,5,7,8,8) | (1,1,1,1) | (3,18,56,76,87) |
| $O_4$        | (0,0,0,0,0) | (0,0,0,0,0) | (0,0,0,0,0) | (-162,-89,25,146,270) |
| Fuzzy Demand | (11,16,51,67,75) | (20,40,60,80,100) | (15,30,45,75,110) | |

By Using VAM Method to obtain the IBFS as
The fuzzy initial basic feasible solution

\[
\begin{align*}
(1,2,3,4,5) & \cdot (2,11,23,34,45) + (3,4,5,6,7) & \cdot (-259,-130,26,156,237) + (1,5,6,7,11) & \cdot (-334,-166,37,241,407) + (1,4,5,9,16) & \cdot (-72,-46,-115,7107) \\
+ (1,1,1,1) & \cdot (3,18,56,76,87) + (0,0,0,0,0) & \cdot (-162,-89,25,146,270) \\
= & (133,331,404,498,724).
\end{align*}
\]

When applying fuzzy version of MODI method for optimal solution of the fuzzy transportation problem, it took two more iteration for improving the fuzzy initial basic feasible solution and the optimal solution is obtained in the third iteration.

The fuzzy optimal solution is given by

\[
\begin{align*}
\tilde{x}_{12} & = (2,11,23,34,45), \quad \tilde{x}_{21} = (-600,-322,51,405,686), \\
\tilde{x}_{22} & = (-367,-186,1,215,367), \quad \tilde{x}_{33} = (3,18,56,76,87), \quad \tilde{x}_{41} = (-269,-146,36,192,342), \\
\tilde{x}_{42} & = (-72,-46,-115,7107).
\end{align*}
\]

The corresponding fuzzy optimal cost Rs =

\[
\begin{align*}
(1,2,3,4,5) & \cdot (2,11,23,34,45) + (3,4,5,6,7) & \cdot (-600,-322,51,405,686) + (1,5,6,7,11) & \cdot (-367,-186,1,215,367) \\
+ (1,1,1,1) & \cdot (3,18,56,76,87) + (0,0,0,0,0) & \cdot (-269,-146,36,192,342) + (0,0,0,0,0) & \cdot (-72,-46,-115,7107) \\
= & (209,300,373,446,537).
\end{align*}
\]
6. Conclusion

In this paper, we have proposed a simple method for solving fully fuzzy transportation problems with pentagonal fuzzy numbers. A numerical example solved by using the proposed method without converting the given problem to crisp equivalent problem.

References

[1] Ahlatcioglu M, Sivri M, Güzel N, 2004 Transportation of the Fuzzy Amounts Using the Fuzzy Cost, Journal of Marmara for Pure and Applied Sciences, vol. 8, pp 139–155.

[2] Chanas S D, Kuchta D, 1996 A concept of the optimal solution of the transportation problem with fuzzy cost coefficients, Fuzzy Sets and Systems, vol. 82, pp 299- 305.

[3] Chanas S, Kolodziejczky W, Machaj A A, 1984 A fuzzy approach to the transportation problem, Fuzzy Sets and Systems, vol. 13, pp 211-221.

[4] Dinagar D S, Palanivel K, 2009 The transportation problem in fuzzy environment, International Journal of Algorithms, Computing and Mathematics, vol. 2, pp 65-71.

[5] Edward Samuel A, Raja P, 2017 Algorithmic Approach To Unbalanced Fuzzy Transportation Problem, International Journal of Pure and Applied Mathematics (IJPAM), vol. 5, pp 553-561.

[6] Gen M, Ida K, Li Y, 1994 Solving Bicriteria Solid Transportation Problem, Humans Information and Technology, IEEE International Conference, vol. 2, pp 1200-1207.

[7] Nagoor Gani Abdul Razak K A, 2006 Two stage fuzzy transportation problem, Journal of Physical Sciences, vol. 10, pp 63-69.

[8] Lai Y J, Hwang C L, 1992 “Fuzzy Mathematical Programming Methods and Application”, Springer, Berlin.

[9] Lin F T, 2009 Solving the Transportation Problem with Fuzzy Coefficients using Genetic Algorithms, Proceedings IEEE International Conference on Fuzzy Systems, pp 20-24.

[10] Liou T S, Wang M J, 1992 Ranking fuzzy number with integral values, Fuzzy Sets and Systems, vol. 50, pp 247-255.

[11] Liu S T, Kao C, 2004 Solving fuzzy transportation problems based on extension principle, European Journal of Operational Research, vol. 153, pp 661-674.

[12] Parra M A, Terol A B, Uria M V R, 1999 Solving the Multi-objective possibilistic linear programming problem, European Journal of Operational Research, vol. 117, pp 175-182.

[13] Pandian P, Natarajan G, 2010 A new algorithm for finding a fuzzy optimal solution for fuzzy Transportation problems, Applied Mathematical Sciences, vol. 4, pp 79-90.

[14] Zadeh L A, 1965 Fuzzy sets, Information and Control, vol. 8, pp 338-353.