Magnetic field effects on natural convection flow of nanofluid in a rectangular cavity using the Lattice Boltzmann model

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Lattice Boltzmann method; Nanoparticle; MHD flow; Natural convection.

Abstract This work applied the Lattice Boltzmann Method (LBM) to investigate the effect of CuO nanoparticles on natural convection with magnetohydrodynamic (MHD) flow in a square cavity. The left and right vertical walls of the cavity were kept at constant temperatures, $T_h$ and $T_c$, respectively, with two insulated walls at the top and bottom. A uniform magnetic field was used in a horizontal direction. Results were carried out for different Hartmann numbers ranging from 0–100, Rayleigh numbers from $10^3$ –$10^5$ and the solid volume fraction from 0 to 0.05. Effects of the solid volume fraction and magnetic field on hydrodynamic and thermal characteristics were investigated and discussed. The averaged Nusselt numbers, on hot wall, streamlines, temperature contours, and the vertical component of velocity for different values of a solid volume fraction, Hartmann and Rayleigh numbers were illustrated. The results indicate that the averaged Nusselt number increases for nanofluids when increasing the solid volume fraction, while, in the presence of a high magnetic field, this effect decreases.

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1. Introduction

The problem of natural convection in a cavity has been a major topic of research, due to its occurrence in industrial and technological applications, such as crystal growth, electronic cooling, oil extraction, and solar collectors, etc. Hydro-magnetic flow and heat transfer have been considered by many researchers for different industrial applications [1–3]. Kanadaswamy et al. [4] numerically investigated flow and temperature fields in a square cavity with partially active vertical walls for a Prandtl number of 0.71. They found that heat transfer rate is maximum for middle–middle thermally active locations, while it is poor for top–bottom thermally active locations. Oztop et al. [5] analyzed magnetohydrodynamic buoyancy induced heat transfer and fluid flow in a non-isothermally heated square enclosure using the finite volume method. The bottom wall of the enclosure was heated and cooled, periodically, like a sinusoidal function, and the top wall was cooled isothermally. Their numerical results showed that heat transfer decreases by increasing the Hartmann number and amplitude of the sinusoidal function.

Iliuta and Arachi [6] investigated the magneto hydrodynamics of trickle bed reactors with an experimental simulation. They have illustrated that, in the presence of a magnetic field, when the Kelvin force density acts against the flow direction, the pressure drop is larger than that without a magnetic field. Rudraiah and Barron [7] numerically studied the natural convection of an electrically conducting fluid in a rectangular enclosure in the presence of a magnetic field. They have pointed out that the average Nusselt number decreases with an increase in the Hartmann number and the Nusselt number approaches one under a strong magnetic field. Robillard et al. [8], numerically and analytically, investigated the effect of an electromagnetic field on free convection in a vertical rectangular porous cavity saturated with an electrically conducting binary mixture. They have concluded that the flow is parallel in the core of the cavity at constant heat and mass fluxes. Pangrle et al. [9] performed experimental research, in which Magnetic Resonance Imaging (MRI) was used to measure, noninvasively, the steady, incompressible, laminar fluid flow in an inorganic porous tube and shell module. They used a porous tube module in closed end mode for Reynolds numbers of 100–200, based on the tube radius, to study the flow and heat transfer.
In nanofluid, Choi [10] was the first to coin the term “nanofluids” for this new class of fluids with superior thermal properties. Khanafer et al. [11] investigated heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids for a range of Grashof numbers and volume fractions. They found that the heat transfer across the enclosure increases with the volumetric fraction of the copper nanoparticles in water for different Grashof numbers. Recently, Kang et al. [12] demonstrated that a nanofluid consisting of silver nano-particles in DI-water (Ag Nanofluid) enhanced the thermal performance of a grooved heat pipe. Abu-Nada et al. [13] investigated the influences of nanoparticles on natural convection heat transfer enhancement in horizontal annuli with various nanoparticles and volume fractions. They reported an enhancement of heat transfer in horizontal annuli. Nguyen et al. [14] covered the fluid flow and heat transfer characteristics of nanofluids in forced and free convection flows, and their potential applications. Wang et al. [15] investigated the free convection heat transfer of Al2O3–water nanofluids in a horizontal and vertical rectangular enclosure. They reported that the ratio of heat transfer coefficient of nanofluids to that of base fluid decreases as the size of the nanoparticles increases. Jou and Tzeng [16] numerically attempted to simulate the natural convection in a rectangular cavity with two different aspect ratios for just Cu–water nanofluid. They simulated the flow field in aspect ratios where the convection heat transfer is more dominant than conduction heat transfer, but they did not report the details of heat transfer enhancement. Oztop and Abu-Nada [17] simulated the natural convection flow in a rectangular cavity by adding a heater at the right hand side of the cavity. Their findings show that the Cu–water mixture has better heat transfer enhancement compared to the Al2O3–water mixture.

Table 1: Thermo physical properties of different phases.

| Property     | Fluid phase (water) | Solid phase (CuO) |
|--------------|---------------------|-------------------|
| c (J/kg K)   | 4179                | 540               |
| ρ (Kg/m³)    | 997.1               | 6500              |
| k (W/mK)     | 0.613               | 18                |
| β × 10⁹ (K⁻¹)| 21                  | 0.85              |
| dₙ (nm)      | 0.384               | 29                |

The LB model used here is the same as that employed in [25,26]. The thermal LB model utilizes two distribution functions, f, g and B, for the flow, temperature and magnetic fields, respectively. It uses the modeling of the movement of fluid particles to capture macroscopic fluid quantities, such as...
velocity, pressure, temperature and the magnetic field. In this approach, the fluid domain is discretized to uniform Cartesian cells. Each cell holds a fixed number of distribution functions, which represent the number of fluid particles moving in these discrete directions. The D2Q9 model was used, and values of \( w_0 = 1/4 \) for \( |c_0| = 0 \) (for the static particle), \( w_{1-4} = 1/9 \) for \( |c_{1-4}| = 1 \) and \( w_{5-9} = 1/36 \) for \( |c_5-9| = \sqrt{2} \) are assigned in this model. Where \( w_i \) and \( c_i \) denote the equilibrium distribution weight and discrete velocity in the direction of \( \alpha \).

The density and distribution functions, i.e. \( f, g \) and \( B \), are calculated by solving the Lattice Boltzmann Equation (LBE), which is a special discretization of the kinetic Boltzmann equation. After introducing BGK approximation [27], the general form of the Lattice Boltzmann equation with external force is:

For the flow field:

\[
f_i(x + c_i \Delta t, t + \Delta t) = f_i(x, t) + \frac{\Delta t}{\tau_v} \left[ f_i^eq(x, t) - f_i(x, t) \right] + \Delta t c_i F_i.
\] (1)

For the temperature field:

\[
g_i(x + c_i \Delta t, t + \Delta t) = g_i(x, t) + \frac{\Delta t}{\tau_c} \left[ g_i^eq(x, t) - g_i(x, t) \right] \]

where \( \Delta t \) denotes the lattice time step, \( c_i \) is the discrete lattice velocity in direction \( i \), \( \tau_v \) is the external force in the direction of the lattice velocity, \( \tau_v \), and \( \tau_c \) denotes the lattice relaxation time for the flow and temperature fields. The kinetic viscosity, \( \nu \), and the thermal diffusivity, \( \alpha \), are defined in terms of their respective relaxation times, i.e. \( \nu = c^2 \tau_v (1 - 1/2) \) and \( \alpha = c^2 \tau_c (1 - 1/2) \), respectively. Note that limitation \( 0.5 < \tau \) should be satisfied for both relaxation times to ensure that viscosity and thermal diffusivity are positive. Furthermore, the local equilibrium distribution function determines the type of problem that needs to be solved. It also models the equilibrium distribution functions for flow and temperature fields, respectively. In this study, the density distribution function, \( f_i^eq \), was modified to consider the magnetic effect:

\[
f_i^{eq} = \frac{w_i \rho}{\tau_v} \left[ 1 + \frac{c_i \cdot u}{c_i^2} + \frac{1}{2} \left( \frac{c_i \cdot u}{c_i^2} \right)^2 - \frac{1}{2} \frac{u^2}{c_i^2} \right] + \frac{w_i}{2c_i^2} \left[ \frac{B^2}{2} - \left( c_i \cdot B \right)^2 \right].
\] (3)

\[
g_i^{eq} = \frac{w_i \rho}{\tau_c} \left[ 1 + \frac{c_i \cdot u}{c_i^2} \right].
\] (4)

where \( B \) is the magnetic field, \( w_i \) is the weighting factor, \( c_i \) is the speed of sound and defined by \( c_i = \frac{1}{\sqrt{\gamma}} \). Similar to the density equilibrium function \( f_i^{eq} \), for calculating the magnetic field, the magnetic equilibrium function is considered, as follows [24]:

\[
h_{ix}^{eq} = \lambda_i \left[ B_x + \frac{1}{c_i^2} \frac{\partial}{\partial x_i} \left( \rho \epsilon_{x_i} u_B - u_x u_B \right) \right],
\] (5)

\[
h_{iy}^{eq} = \lambda_i \left[ B_y + \frac{1}{c_i^2} \frac{\partial}{\partial y_i} \left( \rho \epsilon_{y_i} u_B - u_y u_B \right) \right],
\] (6)

where \( \lambda_i \) is the weighting factor of the magnetic field and defined in the fifth direction by Dellar [24]:

\[
\lambda_i = \begin{cases} 
1 & \text{for } i = 0 \\
\frac{1}{3} & \text{for } i = 1-4.
\end{cases}
\] (7)

For solving the velocity and magnetic field, the following equation must be considered [24]:

\[
h_i(x + c_i \Delta t, t + \Delta t) = h_i(x, t) + \frac{\Delta t}{\tau_m} \left[ h_i^{eq}(x, t) - h_i(x, t) \right].
\] (8)

Magnetic resistivity, like kinetic viscosity, \( \nu \), and thermal diffusivity, \( \alpha \), is defined in terms of its respective relaxation time, \( \eta = c^2 (\tau_m / 2) \). In order to incorporate buoyancy force in the model, the force term in Eq. (1) needs to be calculated, as below, in a vertical direction (\( y \)):

\[
F = 3w_g \rho \beta \theta.
\] (9)

For natural convection, the Boussinesq approximation is applied, and radiation heat transfer is negligible. To ensure that the code works in the near incompressible regime, the characteristic velocity of the flow for a natural, \( V_{natural} = \sqrt{\rho g c^2 (\Delta T)} \), regime must be small compared with the fluid speed of sound. In the present study, the characteristic velocity is selected as 0.1 of sound speed.

Finally, macroscopic variables are calculated using the following formula:

\[
\rho = \sum_i f_i, \quad \rho u = \sum_i c_i f_i, \quad T = \sum_i g_i,
\] (10)

\[
B_x = \sum_i h_x, \quad B_y = \sum_i h_y.
\]

2.2. The Lattice Boltzmann model for nanofluid

In order to simulate nanofluid by the Lattice Boltzmann method, because of nanoparticle potentials and other forces on the nanoparticles, nanofluid behaves differently from pure liquid. In the absence of nanoparticles in the enclosures, the governed equations are Eqs. (1)–(10). However, for modeling the nanofluid, because of changes in fluid thermal conductivity, density, heat capacitance and thermal expansion, some of the governed equations should change.

The thermal diffusivity is:

\[
\alpha = \frac{k_{nf}}{(\rho c_p)_{nf}}.
\] (11)

The effect of density at the reference temperature is:

\[
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s.
\] (12)

Moreover, the heat capacitance and thermal expansion of nanofluid are explained as [28]:

\[
(\rho c_p)_{nf} = (1 - \phi) (\rho c_p)_f + \phi (\rho c_p)_s,
\] (13)

\[
\beta_{nf} = (1 - \phi) \beta_f + \phi \beta_s.
\] (14)

The viscosity of nanofluid containing a dilute suspension of small rigid spherical particles is [14]:

\[
\mu_{nf} = \mu_f (1 - \phi)^{2.5}.
\] (15)

Chon et al. [23] introduced the effective thermal conductivity of two component entities of spherical-particle suspension as follows:

\[
\frac{k_{nf}}{k_f} = 1 + 64.7 \phi^{0.764} \left( \frac{d_i}{d_f} \right)^{0.369} \left( \frac{k_f}{k_s} \right)^{0.7476} Pr_i \text{Re}_i^{0.2221}.
\] (16)
where $Pr_T$ and $Re_T$ are:

$$Pr_T = \frac{\mu_f c_f}{\rho_f c_f}, \quad Re_T = \frac{\rho_f k_b T}{3 \pi \mu_f l_f^2},$$  \hspace{1cm} (17)

where $l_f$ is the mean path of the fluid particle that is given as 17 nm, similar to Saha et al.

$$Pr_T = \frac{3}{2} \frac{\mu_f l_f}{c^2} + 0.5 = \frac{3}{2} \frac{\mu_f l_f}{c^2} + 0.5.$$  \hspace{1cm} (18)

That $lbm$ subscript determines the lattice scale. This procedure is the same as Das et al. in the simulation of variable thermal conductivity in Lattice Boltzmann.

3. Results and discussions

In this study, physical properties are constant, except the density variation in the body force term of the momentum equation, which is satisfied by Boussinesq approximation. The effect of nanoparticle suspension in water, for $Ra = 10^3 - 10^5$, $Ha = 0-100$, and solid volume fraction 0%-5%, is studied. Water is the base fluid, with $Pr = 6.57$ at $22^\circ$C, and the fluid is a water-based nanofluid containing CuO nanoparticles.

To validate the numerical simulation, the results for natural convection flow in an enclosed cavity filled by pure fluid have been compared with those obtained by Jou and Tzeng [16]. This comparison reveals good agreement between results (Table 2). Figure 2 shows the effect of a transverse magnetic field on natural convection flow inside a rectangular enclosure, which is compared with the result of Rudraiah et al. [7]. In addition, this result shows good agreement with previous studies.

Figure 3 indicates the streamlines at $Ra = 10^4$ and $10^5$ for different volume fractions of CuO nanoparticles. This figure illustrates the effect of the volume fraction and Hartmann number on the flow field for $\varphi = 0$, 0.03 and 0.05, at $Ha = 0, 10$.
and 50. In the absence of a magnetic field at low Rayleigh number, because of the buoyancy effect, a main recirculation zone is created in the cavity, at the center of which is, approximately, the recirculation core. By increasing the Rayleigh number, the center of the created vortex divides into two intense vortexes. As the volume fraction increases, the intensity of streamlines increases, due to the high-energy transport through the flow because of the irregular motion of the ultra fine particles. This treatment becomes remarkably stronger by increasing the Ra number, as can be seen in Figure 3. By adding a magnetic field, the intensity of the recirculation decreases, and the vortex core stretches vertically. It can clearly be seen that the shape of the vortex changes from circular to elliptic. This phenomenon is because of the magnetic force, which is against the flow direction and which causes a considerable reduction in the intensity of streamlines. Also, by increasing the solid volume fraction, the trend is the same as cases in the absence of a magnetic field.

Figure 4 shows the effect of volume fraction and Ha number on the vertical component of velocity (LBM velocity) at the horizontal centerline. As the volume fraction increases, the irregular and random movement of particles increases the energy exchange rates in the fluid. Therefore, as shown in Figure 4, at a high volume fraction, high peaks of the vertical component occur. This increment causes thermal dissipation in the flow of nanofluid to be enhanced. However, the increase in Ha number dramatically decreases the amount of vertical velocity, as, for Ha ≥ 50, there is no change along the horizontal centerline.

Figure 5 shows the effect of the magnetic field and solid volume fraction on temperature contours at different Rayleigh numbers. As can be seen, buoyancy force increases by increasing the Rayleigh number, which causes the convective

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Table 2: The averaged Nusselt number on the left wall in comparison with the previous study at Kn = 10^{-4}.

| Ra   | Ref [16] | Present study | Ref [16] | Present study | Present study |
|------|----------|---------------|----------|---------------|---------------|
| Grid | 40 × 40  | 40 × 40       | 80 × 80  | 80 × 80       | 100 × 100     |
| 10^5 | 4.487    | 4.449         | 4.523    | 4.508         | 4.512         |
| 10^6 | 8.798    | 8.623         | 8.928    | 8.883         | 8.891         |
heat transfer to increase, in comparison with conduction, at $Ha = 0$. By immersing the magnetic field, flow field is affected and, subsequently, the dominant mechanism of heat transfer changes. The magnetic field has a negative effect on buoyancy force and decreases the flow motion. This reduction in velocity causes a decrease in the convection heat transfer. By increasing the magnetic force (higher $Ha$ number), conduction heat transfer becomes the dominate mechanism in heat transfer. By increasing the conduction effect, isothermal lines are parallel, as shown in Figure 5. At high Rayleigh numbers, this effect occurs at higher $Ha$ numbers. For example, at $Ra = 10^4$ and $Ha = 50$, isothermal lines are almost parallel, whereas this phenomenon, for $Ra = 10^5$, occurred at $Ha = 100$.

Figures 6 and 7 illustrate how the addition of nanoparticles influences the $Nu$ number distribution along the heated surface, at $Ra = 10^5$, and for different $Ha$ numbers. For $Ha = 0$, by comparing the value of the local Nusselt number with the case of pure fluid, it is clearly evident that increasing the volume fraction increases the $Nu$ number, particularly close to the bottom of the hot wall, while there is a slight increase at the top. For high $Ha$ numbers, it is obvious that the Nusselt number along the hot wall decreases, because, by increasing the $Ha$ number, the effect of convection reduces and the dominant heat transfer mechanism is conduction. Figure 7 shows this statement in more detail. In addition, for nanofluid, the Nusselt number differences between the top and bottom decrease, which causes a reduction in the effect of nanoparticles.

Figure 8 shows the average Nusselt number at $Ra = 10^5$ and for different volume fractions and $Ha$ number. From this figure, it can be found that, as the solid volume fraction increases from 0% to 5%, the $Nu$ number distribution along the heated surface increases by about 93.2%, 102.8%, 78.0%, and 70.7% for $Ha = 0, 10, 50$ and 100, respectively. As we can see, nanofluid has more effect for $Ha = 10$ than pure fluid and high $Ha$ number. Figure 9 shows the effect of nanofluid with a solid volume fraction of 5%, in comparison with pure fluid, on Nusselt numbers with different $Ra$ and $Ha$ numbers. It can be seen that percentages of increment in the Nusselt number increase at higher $Ra$ numbers, although, for $Ha = 100$, all reach the same point. In addition, maximum increment occurs at $Ra = 10^5$ and $Ha = 10$.

4. Conclusion

In this investigation, the effect of nanofluid on natural convection at a rectangular cavity in the presence of a magnetic field is numerically studied. The Lattice Boltzmann method was employed for the solution of the present problem. Some conclusions are summarized, as follows:

a. The Lattice Boltzmann method, based on a multi-distribution function, is a powerful approach for simulating nanofluid flow in the presence of a magnetic field. This method can
simulate the velocity, temperature and magnetic fields with second order accuracy.

b. The buoyancy force increases by increasing the Rayleigh number, which causes an increase in the Nu number. Also, increasing the volume fraction increases the Nu number, particularly close to the bottom of the hot wall.

c. The magnetic field reduces the circulation in the cavity. When the magnetic field becomes stronger, it causes the convection heat transfer to reduce and, subsequently, conduction heat transfer becomes dominant.

d. By adding nanoparticles to fluid, the average Nusselt number increases, and by increasing the volume fraction, this phenomenon becomes more sensible.

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