The role of field redefinition on renormalisability of a general $N = \frac{1}{2}$ supersymmetric gauge theories

A. F. Kord $^{1,2}$, M. Haddadi Moghaddam $^1$, N. Ghasempour $^1$

1: Department of Physics, Hakim Sabzevari University (HSU), P.O.Box 397, Sabzevar, Iran

2: School of Particles and Accelerators, Institute for Research in Fundamental Sciences(IPM), P.O. Box 19395-5531, Tehran, Iran

E-mail: afarzaneh@hsu.ac.ir

Abstract

We investigate some issues on renormalisability of non-anticommutative supersymmetric gauge theory related to field redefinitions. We study one loop corrections to $N = \frac{1}{2}$ supersymmetric $SU(N) \times U(1)$ gauge theory coupled to chiral matter in component formalism, and show the procedure which has been introduced for renormalisation is problematic because some terms which are needed for the renormalisability of theory are missed from the Lagrangian. In order to prove the theory is renormalisable, we redefine the gaugino and the auxiliary fields($\lambda, \bar{F}$), which result in a modified form of the Lagrangian in the component formalism. Then, we show the modified Lagrangian has extra terms which are necessary for renormalisability of non-anticommutative supersymmetric gauge field theories. Finally we prove $N = \frac{1}{2}$ supersymmetric gauge theory is renormalisable up to one loop corrections using standard method of renormalisation; besides, it is shown the effective action is gauge invariant.
1 Introduction

In recent years, deformed quantum field theories have received more attention due to their natural appearance in string theory. One type of deformation is space-time deformation in which the commutators of the space-time coordinates become non-zero, which results in non-commutative field theories. The non-commutativity leads to a nontrivial product of fields which is called the $\star$-product. Another type of deformation is noncommutativity in the Grassmann coordinates $\theta^\alpha$, leaving the anticommutators of $\bar{\theta}^{\dot{\alpha}}$ unchanged. It has been indicated that this superspace geometry can occur in string theory in the presence of a graviphoton background [1, 2]. Theories defined on non-anticommutative (NAC) superspace have been studied extensively during last ten years [1, 3, 4, 5, 6]. It is straightforward to construct a field theory over NAC superspace in terms of superfields with the star-product where the Lagrangian is deformed from the original theory by the non-anticommutativity parameter $\{\theta^\alpha, \theta^{\beta}\} = C^\alpha{}_{\beta}$ where $C$ is a nonzero constant.

During the last ten years, the renormalisability of the NAC field theories has been the subject of numerous research. NAC field theories are not power-counting renormalisable; however, it has been shown that they could be made renormalisable if some additional terms are added to the Lagrangian in order to absorb divergences to all orders [7]-[15]. The issues of renormalisability of NAC versions of the Wess-Zumino model and supersymmetric gauge theories have been studied. The renormalisability of NAC versions of the Wess-Zumino model has been discussed [7, 8], with explicit computations up to two loops [9]. The renormalisability of supersymmetric gauge field theories has been discussed in WZ gauge [13, 14]. Drawing on the component approach, authors [16, 17] have provided the most complete results for the one-loop quantum corrections of the deformed component theory. Working in components in the WZ gauge, they have argued that in order to restore gauge invariance, it is necessary to define one-loop divergent field redefinitions of the gaugino field($\lambda$) and the auxiliary field($\bar{F}$) (in the case of matter fields). It has been manifested, the one-loop divergences(1PI), with the relevant diagrams containing
only $C$-deformed vertex figures, cannot be cancelled by the Lagrangian since they contain contributions which do not appear in the original Lagrangian; in other words, the theory is nonrenormalisable. Their results imply that there are problems with the assumption of gauge invariance, which is required to rule out some classes of divergent structure in the NAC theory. In their findings, one can see that even at one loop divergent non-gauge-invariant terms are generated. In order to remove the non-gauge-invariant terms and restore gauge invariance at one loop they have introduced one loop divergent field redefinitions. They realised that by adding new deformation-parameter-dependent terms to the theory, the one-loop effective action can be renormalisable. However, these kinds of divergent field redefinitions are problematic because there is no theoretical justification or interpretation for the field redefinition, as mentioned by the authors [16, 17].

On the other hand, the authors in [19, 20, 21] started from the superspace formalism and discussed renormalisability and supergauge invariance. They have argued that suitable deformations of the classical actions are necessary in order to achieve renormalisability at one loop level [19]. Using the background field method, they have computed the effective action. They have proved that divergent field redefinitions are not required and the original effective action is not only gauge but also supergauge invariant up to one loop corrections. An important feature of their work is that although they obtain the effective action without any difficulty in the superspace formalism, they have found that some new terms have to be added to the superfield action due to the one loop divergent contribution for $C$ deformed section.

In this paper we investigate the renormalisability of $N = \frac{1}{2}$ supersymmetric gauge theory coupled to chiral matter and propose a modified classical action which is necessary in component formalism. First, we briefly review NAC supersymmetric gauge theories and their Lagrangian in the component formalism and also the field redefinitions which are described in refs [16]. Then we concentrate on the one-loop corrections of three and four-point functions (in the C-deformed sector) and show that anomalous terms appear in the 1PI functions which spoil the renormalisabil-
ity of the theory. In order to absorb these anomalous terms and renormalise the theory we suggest a new kind of field redefinition which results in a new form of the Lagrangian in the component formalism (though the form of the Lagrangian remains unchanged in the superspace formalism). Then, we investigate its effects on the renormalisability of the theory. We shall prove $N = \frac{1}{2}$ supersymmetric gauge theory is renormalisable at one loop level, using the standard method of renormalisation without any need for divergent field redefinitions. Our method confirms that the effective action is gauge invariant which is consistent with superspace formalism [19].

Working in the component case, we initially encounter some very important issues such as the field redefinition of the gaugino field $\lambda$ and auxiliary field $\bar{F}$ that Seiberg and other authors have introduced at the beginning of their extension of the standard theory. With these redefinitions some terms have been effectively removed from the Lagrangian; we reveal the necessity of restoring these hidden terms by new generalized redefinitions based on [1, 6]. Secondly, the effective action in the component case violates gauge invariance owing to some unusual terms—the so-called $Y$ terms. Nonetheless, we confirm a number of results in both of the works in [17, 19] relating to renormalisability of the theory and preservation of the algebraic structure of the star product.

2. $N = 1/2$ supersymmetric $U(N)$ gauge theory Lagrangian

In this section we review the classical form of the $N = \frac{1}{2}$ supersymmetric gauge theory Lagrangian. The $N = 1/2$ supersymmetric gauge theory Lagrangian was first introduced in Ref. [1, 6] for the gauge group $U(N)$. However, as it was noted in Refs. [16, 17], at the quantum level the $U(N)$ gauge invariance cannot be retained since the $SU(N)$ and $U(1)$ gauge couplings renormalise differently; and they have obliged to consider a modified $N = \frac{1}{2}$ invariant theory with the gauge
The $U(N)$ gauge invariant supersymmetric Lagrangian for NAC superspace formalism is as follows:

\[
L = \int d^2\theta d^2\bar{\theta} \Phi^{*} e^{V} \Phi + \frac{1}{16k^2} \left( \int d^2\theta tr W^{\alpha} * W_{\alpha} + \int d^2\bar{\theta} \bar{W}_{\dot{\alpha}} * \bar{W}^{\dot{\alpha}} \right),
\]

where $W_{\alpha}$ and $\bar{W}_{\dot{\alpha}}$ are chiral and antichiral field strengths. $V$, $\Phi$ and $\bar{\Phi}$ are vector, chiral and anti chiral superfield respectively.

When one discusses the non-anticommutative theory, he or she starts with the superspace formalism. In the superspace gauge transformation, the gauge parameter is a superfield. When one rewrites it into the component formalism, it is necessary to impose the Wess Zumino gauge. Using this gauge fixing, one obtains the component gauge transformation, which is smaller than the original superspace gauge transformation.

In Wess Zumino gauge, the vector superfield $V$ is presented as:

\[
V(y, \theta, \bar{\theta}) = -\bar{\theta} \sigma^{\mu} \theta v_{\mu}(y) + i\theta \theta \bar{\lambda}(y) - i\bar{\theta} \bar{\theta} \lambda(y) + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} (D - i\partial^{\nu}v^{\nu})(y),
\]

where we write $V = V^{A}R^{A}$ with $R^{A}$ being the generators of the gauge group $U(N)$. Performing a gauge transformation on the vector superfield in the Wess-Zumino gauge results in the gauge transformations of the component fields. They are given by:

\[
\delta^{*}_{\varphi} A_{\mu} = -2\partial_{\mu}\varphi + i[\varphi, A_{\mu}],
\]

\[
\delta^{*}_{\varphi} \lambda_{\alpha} = i[\varphi, \lambda_{\alpha}] + \frac{1}{4}(\epsilon C\sigma^{\mu})_{\alpha\dot{\alpha}}(-2\partial_{\mu}\varphi + \bar{\lambda}^{\dot{\alpha}}),
\]

\[
\delta^{*}_{\varphi} \bar{\lambda}_{\dot{\alpha}} = i[\varphi, \bar{\lambda}_{\dot{\alpha}}],
\]

\[
\delta^{*}_{\varphi} D = i[\varphi, D],
\]

where $\varphi$ is the gauge transformation parameter. These gauge transformations are not canonical because the transformation of $\lambda$ depends on the deformation parameter $C$. In order to obtain the canonical form of the gauge transformations, the
authors [1, 6] proposed the following \( \lambda \) redefinition:

\[
\lambda'_\alpha = \lambda_\alpha - \frac{1}{4}(\varepsilon C\sigma^\mu)_{\alpha\dot{\alpha}}\{A_\mu, \bar{\lambda}^{\dot{\alpha}}\},
\]  
(7)

so that its canonical gauge transformation is given by:

\[
\Rightarrow \delta^*_\varphi \lambda'_\alpha = i[\varphi, \lambda'_\alpha].
\]  
(8)

Then, the vector superfield is redefined as:

\[
V^A(y, \theta, \bar{\theta}) = -(\theta \sigma^\mu \bar{\theta})A^A_\mu(y) + i\theta \theta \bar{\theta} \bar{\lambda}^A(y) - i\theta \bar{\theta} \bar{\theta}\left(\lambda'^A_\alpha + \frac{1}{4}g^{AB\dot{C}}C^{\mu\nu} A^{B}_{\mu} \bar{\lambda}^{\dot{C}}\right) + \frac{1}{2}\theta \theta \bar{\theta}(D^A - i\partial^A_\mu A^A_\mu)(y)
\]  
(9)

Gauge transformations of the chiral and antichiral superfields have been studied in [1, 6]. The chiral and anti-chiral superfields are written as:

\[
\Phi(y, \theta) = \phi(y) + \sqrt{2}\theta \psi(y) + \theta \theta F(y)
\]  
(10)

\[
\bar{\Phi}(\bar{y}, \bar{\theta}) = \bar{\phi}(\bar{y}) + \sqrt{2}\bar{\theta} \bar{\psi}(\bar{y}) + \bar{\theta} \bar{\theta} \bar{F}(\bar{y})
\]  
(11)

In order to have canonical gauge transformations of the component fields, the \( \bar{F} \) component field should be redefined as:

\[
\bar{F}'(\bar{y}) = \bar{F}(\bar{y}) - iC^{\mu\nu} \partial_\mu (\bar{\phi} A_\nu)(\bar{y}) + \frac{1}{4}C^{\mu\nu} \bar{\phi} A_\mu A_\nu(\bar{y})
\]  
(12)

Then

\[
\Rightarrow \delta^*_\varphi \bar{F}' = -i\bar{F}' \varphi
\]  
(13)

Thus, the antichiral superfield is given by [6]:

\[
\bar{\Phi}(\bar{y}, \bar{\theta}) = \bar{\phi} + \sqrt{2}\bar{\theta} \bar{\psi} + \bar{\theta}\bar{\theta}\left(\bar{F}' + iC^{\mu\nu} \partial_\mu (\bar{\phi} A_\nu) - \frac{1}{4}C^{\mu\nu} \bar{\phi} A_\mu A_\nu\right)
\]  
(14)

Using the above field redefinitions and rescaling \( V \) and \( C^{\alpha\beta} \), the authors [6, 17] have suggested an \( N = \frac{1}{2} \) supersymmetric U(N) gauge theory action coupled to chiral matter. It is given by:

\[
S = \int d^4x \left[ Tr\{-\frac{1}{2}F^{\mu\nu}F_{\mu\nu} - 2i\bar{\lambda}\partial^\mu (D_\mu \lambda) + D^2\}\right]
\]
\[-2igC^{\mu\nu}Tr\{F_{\mu\nu}\bar{\lambda}\bar{\lambda}\} + g^2 |C|^2 Tr\{\bar{\lambda}\bar{\lambda}\}\]

\[
+ \left\{ \bar{F}^\tau F - i\bar{\psi}\sigma^\mu (D_\mu \psi) - D_\mu \bar{\phi}D^\mu \phi + g\bar{\phi}D\phi + i\sqrt{2}g(\bar{\phi}\bar{\lambda}'\psi - \bar{\psi}\bar{\lambda}\phi) + igC^{\mu\nu}\bar{\Phi}_{\mu\nu}F + \sqrt{2}gC^{\mu\nu}D_\mu \bar{\phi}\bar{\sigma}_\nu \psi + \frac{|C|^2}{4} g^2 \bar{\phi}\bar{\lambda}\bar{\lambda}F \\
+ (\phi \rightarrow \bar{\phi}, \psi \rightarrow \bar{\psi}, F \rightarrow \bar{F}, C^{\mu,\nu} \rightarrow -C^{\mu,\nu}) \right\},
\]

(15)

where in order to ensure anomaly cancellation, a multiplet \{\phi, \psi, F\} transforming according to the fundamental representation and, a multiplet \{\bar{\phi}, \bar{\psi}, \bar{F}\} transforming according to its conjugate are included in the Lagrangian. Moreover, there are

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu],
\]

\[
D_\mu \lambda' = \partial_\mu \lambda' + ig[A_\mu, \lambda'],
\]

\[
D_\mu \phi = \partial_\mu \phi + igA_\mu,
\]

(16)

and

\[
A_\mu = A_\mu^A R^A, \quad \lambda' = \lambda'^A R^A, \quad D = D^A R^A.
\]

(17)

Corresponding to any index \(a\) for SU(N), we introduce the index \(A = (0, a)\). Thus, \(A\) runs from 0 to \(N^2 - 1\), with \(R^A\) being the group matrices for U(N) in the fundamental representation. These satisfy

\[
[R^A, R^B] = if^{ABC} R^C, \quad \{R^A, R^B\} = d^{ABC} R^C,
\]

(18)

where \(f^{ABC}\) is completely antisymmetric, \(f^{abc}\) is the same as SU(N) and \(f^{0bc} = 0\), while \(d^{ABC}\) is completely symmetric; \(d^{abc}\) is the same as SU(N), \(d^{0bc} = \sqrt{2/N} \delta^{bc}\), \(d^{00c} = 0\) and \(d^{000} = \sqrt{2/N}\). In particular, \(R^0 = \sqrt{1/N}1\), and

\[
Tr\{R^A R^B\} = \frac{1}{2}\delta^{AB}
\]

(19)

\(C^{\mu\nu}\) is related to the non-anti-commutativity parameter \(C^{\alpha\beta}\) by:

\[
C^{\mu\nu} = C^{\alpha\beta} \epsilon_{\beta\gamma} \sigma^{\mu\nu}_{\alpha\gamma},
\]

(20)

and

\[
|C|^2 = C^{\mu\nu} C_{\mu\nu}.
\]

(21)
Besides, our conventions are consistent with ref [1].

It is easy to show there are some extra terms in the action when one uses the original definition of the gaugino field (λ) and the auxiliary field (F) instead of the field redefinitions (λ') and (F'). Thus, the problem of renormalisability of the theory is solved by these extra terms as it is shown in next section.

2.1 \( N = \frac{1}{2} \) supersymmetric \( SU(N) \times U(1) \) action

To ensure renormalisability it is necessary to decompose \( U(N) \) into \( SU(N) \times U(1) \) because the \( U(N) \) gauge symmetry is not preserved under renormalisation. In fact, the two gauge coupling constants renormalise differently [17, 19]. To obtain a renormalisable theory one must introduce different couplings for the \( SU(N) \) and \( U(1) \) parts of the gauge group and then the \( U(N) \) gauge-invariance is lost. Therefore, the authors of [17] have suggested an \( N = \frac{1}{2} \) supersymmetric \( SU(N) \times U(1) \) gauge theory coupled to chiral matter. Following their work, the action is given by:

\[
S = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu}^A F_{\mu\nu}^A - i \bar{\lambda}^A \tilde{\sigma}^\mu (D_\mu \lambda)^A + \frac{1}{2} D_\mu D^\mu A \\
- \frac{1}{2} i \gamma^{ABC} A^{ABC} C_{\mu\nu} F_{\mu\nu}^A \bar{\lambda}^B \bar{\lambda}^C + \frac{1}{8} |C|^2 d^{abc} d^{cde} g^2 (\bar{\lambda}^a \bar{\lambda}^b) (\bar{\lambda}^c \bar{\lambda}^d) \\
+ \frac{1}{4N} |C|^2 (\frac{g^2}{g_0})^2 (\bar{\lambda}^a \bar{\lambda}^a) (\bar{\lambda}^b \bar{\lambda}^b) \\
+ \left\{ \tilde{F}^A - i \tilde{\psi} \tilde{\sigma}^\mu D_\mu \psi - D_\mu \tilde{\Phi} D_\mu \phi + \tilde{\Phi} \tilde{D} \phi + i \sqrt{2} (\tilde{\phi} \tilde{\psi} - \tilde{\psi} \tilde{\phi}) \\
+ \sqrt{2} C_{\mu\nu} D_\mu \tilde{\Phi} \tilde{\sigma}_\nu \psi + i C_{\mu\nu} \tilde{\Phi} \tilde{F}_{\mu\nu} F + \frac{1}{4} |C|^2 \tilde{\phi} \tilde{\phi} \tilde{\phi} \tilde{\phi} \tilde{\phi} \\
+ (\phi \rightarrow \tilde{\phi}, \psi \rightarrow \tilde{\psi}, F \rightarrow \tilde{F}, C_{\mu\nu} \rightarrow -C_{\mu\nu}) \right\},
\]

(22)

where \( \hat{A}_\mu \) is defined by

\[
\hat{A}_\mu = \hat{A}_\mu^A R^A = g A^a_\mu R^a + g_0 A^0_\mu R^0,
\]

(23)

with similar definitions for \( \hat{\lambda}', \tilde{D} \) and \( \tilde{F}_{\mu\nu} \), and

\[
D_\mu \phi = (\partial_\mu + i \hat{A}_\mu) \phi.
\]

(24)

\( \gamma^{ABC} \) is given by:

\[
\gamma^{abc} = g, \quad \gamma^{a0b} = \gamma^{ab0} = \gamma^{000} = g_0, \quad \gamma^{0a0} = \frac{g^2}{g_0}
\]

(25)
Eq. (22) reduces to the original U(N) Lagrangian Eq. (15) derived from nonanticommuting superspace upon setting $g_0 = g$.

In order to investigate the renormalisability of the theory, one needs to compute the one-loop one-particle-irreducible (1PI) graph contributions. The one-loop graph corrections of $N = 1$ part of the theory are not affected by $C$-deformation. So the anomalous dimensions and gauge $\beta$-functions are as for $N = 1$. The 1PI graph corrections contributing to the new terms (those containing $C$) are calculated in ref [17] in the component formalism and we present them in the Appendix (We note that the one loop divergent contributions for the $C$ deformed sector have also been computed using the background field method in the superspace approach in ref [19]).

However, it is easy to see the component version of the theory could not be renormalisable because some anomaly terms which are proportional to $Y^{\mu\nu} = C^{\mu\rho}g_{\rho\lambda}(\bar{\sigma}^{\lambda\nu})$ appear in 1PI corrections, but there are no similar terms in the Lagrangian; therefore, these terms spoil the renormalisability of the theory. Moreover, the existence of these terms violates the gauge invariance of the effective action (albeit this issue does not happen in the effective action according to superspace formalism).

In order to solve these issues and renormalise the theory in component formalism, the authors of ref [17] have proposed a procedure of two steps. Firstly, they have modified the action and added new terms to the action. Their modified action is given by:

$$
S = \int d^4x \left[ -\frac{1}{4} F^{\mu\nu A} F_{\mu\nu}^A - i\bar{\lambda}^{A\sigma} (D_{\mu} \lambda^{\sigma})^A + \frac{1}{2} D^A D^A \\
- \frac{1}{2} i\gamma^{ABC} d^{ABC} C^{\mu\nu} F_{\mu\nu}^A \bar{\lambda}^{B\lambda} C^{C} + \frac{1}{8} \left| C \right|^2 d^{abc} d^{cde} g^{2} (\bar{\lambda}^{a\lambda}) (\bar{\lambda}^{c\lambda}) (\bar{\lambda}^{d}) \\
+ \frac{1}{4N} \left| C \right|^2 (\frac{g^{2}}{g_{0}}) (\bar{\lambda}^{a\lambda}) (\bar{\lambda}^{b} \bar{\lambda}) \\
+ \frac{1}{N} \partial_{\mu} g^{2} C^{2} (\bar{\lambda}^{a\lambda}) (\bar{\lambda}^{b} \bar{\lambda}) + \left\{ \bar{F}^{\mu} F - i\bar{\psi} \bar{\sigma}^{\mu} D_{\mu} \psi - D^{\mu} \frac{\bar{\phi}}{\phi} D_{\mu} \phi + \bar{\phi} D_{\phi} + i\sqrt{2} (\bar{\phi} \bar{\lambda}^0 \psi - \bar{\psi} \bar{\lambda}^0 \phi) \\
+ \sqrt{2} C^{\mu\nu} D_{\mu} \bar{\phi} \bar{\sigma}_{\nu} \psi + iC^{\mu\nu} \bar{\phi} F_{\mu\nu} F + \frac{1}{4} \left| C \right|^2 \frac{\bar{\phi}}{\phi} \bar{\lambda} \bar{\lambda} C^{F} \\
- \partial_{\mu} C^{\mu\nu} g \left( \sqrt{2} D_{\mu} \bar{\phi} \bar{\lambda}^{a} R^{a} \bar{\sigma}_{\nu} \psi + \sqrt{2} \bar{\phi} \bar{\lambda}^{a} R^{a} \bar{\sigma}_{\nu} D_{\mu} \psi + i\phi F^{a}_{\mu\nu} R^{a} F \right) \\
+ (\phi \rightarrow \bar{\phi}, \psi \rightarrow \bar{\psi}, F \rightarrow \bar{F}, C^{\mu\nu} \rightarrow -C^{\mu\nu}) \right\} ,
\right.
$$

(26)
where $\vartheta_1$ and $\vartheta_2$ are constants. These new terms (those are proportional to $\vartheta_1$ and $\vartheta_2$ constants) are separately invariant under $N = 1/2$ supersymmetry and must be included to obtain a renormalisable Lagrangian. However, these terms are not obtained from the original superspace action. In this case a similar feature also appeared in [19, 20, 21] where the superspace action had to be modified in order to get a renormalised theory. In their procedure, the renormalised couplings $\vartheta_1$ and $\vartheta_2$ have been set to zero for calculational simplicity. In other words, they have not contributed to 1PI corrections. Secondly, they have introduced divergent field redefinitions or, to put it another way, added non-linear terms to the bare action at the end of their calculations. This scenario has been used in several papers [18]. However, the scenario is problematic because of divergent field redefinitions which have no theoretical justification. The another problem is that the action is changed. Moreover, there is disagreement with the superfield formalism where divergent field redefinitions are not needed [19].

In next section we introduce field redefinitions which lead to a different classical action in the component formalism, then discuss the renormalisability of the theory, and prove the theory is renormalisable up to one loop corrections. Besides, we indicate divergent field redefinitions are not needed which is in agreement with the superfield formalism.

3 Generalized Wess Zumino gauge, Field redefinitions and Renormalisation

In this section we generalise Wess Zumino gauge, and introduce field redefinitions which modify the $N = \frac{1}{3}$ supersymmetric gauge theory action. Then we prove the modified action is renormalisable.

Discussing the non-anticommutative theory, one starts from the superspace formalism. In the superspace gauge transformation, the gauge parameter is a superfield. In order to obtain the $N = \frac{1}{2}$ supersymmetric gauge theory Lagrangian in the com-
ponent formalism, one should impose the Wess Zumino gauge. Using this gauge fixing, the component gauge transformation is obtained which is smaller than the original superspace gauge transformation. In order to obtain the canonical forms of the gauge transformations, Seiberg proposed to take the following Wess Zumino gauge [1].

\[ V^A(y, \theta, \bar{\theta}) = -\left( \theta \sigma^\mu \bar{\theta} \right) A^A_\mu(y) + i \bar{\theta} \sigma^\mu \bar{\lambda}^A(y) - i \bar{\theta} \bar{\theta} \left( \lambda^A_\alpha + \frac{1}{4} d^{ABC} C^{\mu\nu} \sigma_{\nu\alpha\dot{\alpha}} A^B_\mu \bar{\lambda}^C \right) + \frac{1}{2} \bar{\theta} \bar{\theta} \bar{\theta} \left( D^A - i \partial^\mu A^A_\mu \right)(y). \]  

(27)

What we do is to impose the generalised version of this Wess Zumino gauge fixing in the form:

\[ V = \cdots - i \bar{\theta} \bar{\theta} \left( \lambda^A_\alpha + \frac{1}{4} d^{ABC} C^{\mu\nu} \sigma_{\nu\alpha\dot{\alpha}} A^B_\mu \bar{\lambda}^C (1 + \kappa^{ABC}) \right) + \cdots \]  

(28)

For \( \kappa^{ABC} = 0 \), it reduces to the Seiberg’s case. For \( \kappa^{ABC} = -1 \), it reproduces the form of \( \lambda \) before the redefinition appearing in [1][6]. We postulate that the difference between the two gauge fixing is related by a certain superspace gauge transformation. Therefore, we believe that the parameter \( \kappa^{ABC} \) simply corresponds to a choice of gauge fixing. In the explicit calculation, we will see that \( \kappa^{ABC} \) is renormalised.

It means that we change the gauge fixing condition during the renormalisation. In the usual renormalisation method, we keep the gauge fixing condition. In this sense, our method does not look very natural conceptually although it does not necessarily mean that our computation is problematic. If we would like to keep the same Wess Zumino gauge, we can go back to the original gauge fixing after the 1-loop computation by putting the renormalised coupling \( \kappa^{ABC}_{\text{ren}} = 0 \) by hand in the renormalised Lagrangian.

In the same way we generalise the anti-chiral superfield as follow:

\[ \bar{\phi}(y, \bar{\theta}) = \bar{\phi} + \sqrt{2} \bar{\theta} \bar{\phi} + \bar{\theta} \bar{\phi} \left( \bar{F} + i C^{\mu\nu} \partial_\mu (\bar{\phi} A_\nu) - \frac{1}{4} C^{\mu\nu} \sigma_{\nu\alpha\dot{\alpha}} A^B_\mu \bar{\lambda}^C + \frac{1}{2} \bar{\theta} \bar{\theta} \right) - \frac{1}{8} h_{ABC} |C|^2 g_{B9} d^{ABC} R^{A_\mu \bar{\lambda}^B \bar{\lambda}^C} \]  

(29)
For $h_{ABC} = \varepsilon_A = \kappa_A = 0$, it reduces to the Eq. (14).

Using the generalised vector and anti-chiral superfield, Eqs. (28,29) result in field redefinitions $\lambda'$ and $\bar{F}'$ which are:

$$\lambda'^A \rightarrow \lambda^A - \frac{1}{4} \kappa^{ABC} d^{ABC} C_{\mu\nu} A^B_{\mu} \bar{\phi} \chi^C$$

$$\bar{F}' \rightarrow \bar{F} - i C_{\mu\nu} \kappa_A R^A \bar{\phi} \partial_{\mu} A^A_{\nu} + \frac{i}{8} \varepsilon_A g R^A f^{ABC} C_{\mu\nu} \bar{\phi} A^B_{\mu} A^C_{\nu}$$

$$+ \frac{1}{8} h_{ABC} |C|^2 g_{BC} d^{ABC} R^A \bar{\phi} \chi^B \chi^C.$$

These field redefinitions are similar to Eqs (7) and (12). According to the above equations, the WZ gauge has been parameterised by some new non zero parameters $\kappa^{ABC}$, $\kappa_A$ and $\varepsilon_A$. The field redefinitions in eqs. (30) lead to new gauge and SUSY transformations which are not canonical because the transformation of $\lambda$ and $\bar{F}$ depend on the NAC parameter $C$. The gauge transformations are given by:

$$\delta_{\varphi} A^A_{\mu} = -2 \partial_{\mu} \phi^A - f^{ABC} \phi B A^C_{\mu},$$

$$\delta_{\varphi} \tilde{A}^A_{\dot{\alpha}} = - f^{ABC} \phi B \chi^C,$$

$$\delta_{\varphi} \chi^A = - f^{ABC} \phi B - \frac{1}{2} \kappa^{ABC} d^{ABC} C_{\mu\nu} \sigma_{\mu\nu\dot{\alpha}} \partial_{\mu} \phi B \chi^C$$

$$\delta_{\varphi} D^A = - f^{ABC} \phi B D^C,$$

$$\delta_{\varphi} \phi = i \phi \phi,$$

$$\delta_{\varphi} \bar{\phi} = - i \bar{\phi} \phi,$$

$$\delta_{\varphi} \psi_{\dot{\alpha}} = i \phi \psi_{\dot{\alpha}},$$

$$\delta_{\varphi} \bar{\psi}_{\dot{\alpha}} = - i \bar{\psi}_{\dot{\alpha}} \phi$$

$$\delta_{\varphi} F = i \phi F,$$

$$\delta_{\varphi} \bar{F} = - i \bar{F} \phi - i C_{\mu\nu} \kappa_A R^A f^{ABC} \bar{\phi} \partial_{\mu} (\phi B A^C_{\nu})$$

$$+ \frac{i}{2} \varepsilon_A g R^A f^{ABC} C_{\mu\nu} \bar{\phi} (\partial_{\mu} \phi B) A^C_{\nu}$$

$$+ \frac{i}{4} \varepsilon_A g R^A f^{ABC} f^{BDE} C_{\mu\nu} \bar{\phi} D \phi B A^E A^C_{\nu}$$

$$+ \frac{1}{4} h_{ABC} |C|^2 g_{BC} d^{ABC} R^A \bar{\phi} \psi D \phi B \chi^E \chi^C.$$
Moreover, the $N = \frac{1}{2}$ SUSY transformations are given by:

$$\delta A^A = -i \bar{\lambda}^A \bar{\sigma}_\mu \epsilon ,$$

$$\delta \lambda^A = i \epsilon_\alpha D^A + (\sigma^{\mu \nu})_\alpha [F_\mu^A + \frac{1}{2} i C_\mu (\gamma^{ABC} + \frac{1}{2} \kappa^{ABC}) d^{ABC} \bar{\lambda} B \bar{\lambda} C],$$

$$\delta \bar{\lambda}^A = 0 \ , \ \delta D^A = - \epsilon \sigma^\mu D_\mu \bar{\lambda}^A ,$$

$$\delta \phi = \sqrt{2} \epsilon \psi \ , \ \delta \bar{\phi} = 0 ,$$

$$\delta \psi^\alpha = \sqrt{2} \epsilon^\alpha F \ , \ \delta \bar{\psi}^\alpha = -i \sqrt{2} (D_\mu \bar{\psi}) (\epsilon \sigma^\mu)_\alpha ,$$

$$\delta F = 0 ,$$

$$\delta \bar{F} = -i R^A \bar{\phi} \epsilon \lambda^A + \sqrt{2} \epsilon \sigma^\mu (\partial_\mu \bar{\psi} + ig R^A A^A_\mu \bar{\psi})$$

$$+ i (1 - \kappa_A) C^{\mu \nu} R^A \phi (\epsilon \sigma_\nu \partial_\mu \bar{\lambda}^A) - \frac{1}{4} (1 - \epsilon_A) g R^A f^{ABC} C^{\mu \nu} \bar{\phi} A^B_\mu (\epsilon \sigma_\nu \bar{\lambda}^C)(32)$$

### 3.1 The modified action

The field redefinitions Eqs. (30) lead to a modified NAC action in component formalism. Therefore, we should replace Eq. (22) by:

$$S = \int d^4 x \left[ - \frac{1}{4} F^{\mu \nu A} F_{\mu \nu} - i \bar{\lambda}^A \bar{\sigma}_\mu (D_\mu \lambda)^A + \frac{1}{2} D^A D^A ight.$$

$$- \frac{1}{2} i \gamma^{A B C} d^{A B C} C^{\mu \nu} F_{\mu \nu} \bar{\lambda}^B \bar{\lambda}^C + \left. \frac{1}{8} | C |^2 d^{a b c} d^{d e f} g^2 (\bar{\lambda}^a \bar{\lambda}^b) (\bar{\lambda}^c \bar{\lambda}^d) \right]$$

$$+ \frac{1}{4 N} | C |^2 (\frac{g^2}{d}) (\bar{\lambda}^a \bar{\lambda}^b) + \frac{1}{N} \partial_\mu g^2 \bar{C}^2 (\bar{\lambda}^a \bar{\lambda}^b) (\bar{\lambda}^c \bar{\lambda}^d)$$

$$+ \frac{i}{16} d^{A B C} \kappa^{A B C} C^{\mu \nu} (\partial_\mu A_\nu^A - \partial_\nu A_\mu^A) \bar{\lambda}^B \bar{\lambda}^C$$

$$+ \frac{i}{4} g \kappa^{E D B} d^{B D E} f^{A C E} C^{\mu \nu} A_\mu^C A_\nu^D \bar{\lambda}^A \bar{\lambda}^B$$

$$+ \frac{i}{16} g \kappa^{D B C} d^{A B C} A_\mu^A (\partial_\nu \bar{\lambda}^B Y^{\mu \nu} \bar{\lambda}^C - \bar{\lambda}^B Y^{\mu \nu} \partial_\nu \bar{\lambda}^C)$$

$$+ \frac{i}{4} g \kappa^{E D B} f^{A C E} d^{B D E} A_\mu^C \bar{\lambda}^A Y^{\mu \nu} \bar{\lambda}^B$$

$$+ \left[ \bar{F} \bar{F} - \bar{\psi} \bar{\sigma}_\mu D_\mu \psi - D_\mu \bar{\phi} D_\mu \phi + \bar{\phi} \bar{D} \phi + i \sqrt{2} (\bar{\phi} \lambda \psi - \bar{\psi} \lambda \phi) \right]$$

$$+ \sqrt{2} C^{\mu \nu} D_\mu \bar{\phi} \bar{\lambda} \bar{\sigma}_\nu \psi + i C^{\mu \nu} \bar{\phi} \bar{F}_{\mu \nu} \phi + \frac{1}{4} | C |^2 \bar{\phi} \lambda \bar{\lambda} \bar{C} \bar{F}$$

$$- \partial_\mu g^2 \left[ \sqrt{2} D_\mu \bar{\phi} \lambda \bar{\sigma}_\nu \psi + \sqrt{2} \bar{\phi} \lambda \bar{F}_{\nu} \bar{F}_{\mu} \phi \right]$$

$$+ i \sqrt{2} g \kappa^{A B C} d^{A B C} R^{\mu \nu} \bar{\phi} A^B_\mu \bar{\lambda}^C \bar{\sigma}_\nu \psi$$

$$- i C^{\mu \nu} \kappa_A R^{\phi}_A \phi \partial_\mu A_\nu^A + \frac{i}{8} \epsilon_A g R^A f^{ABC} C^{\mu \nu} \bar{\phi} A^B_\mu A_\nu^C$$

$$+ \frac{1}{8} \kappa_{A B C} | C |^2 g b g c d^{A B C} R^A \bar{\phi} \bar{\lambda}^B \bar{\lambda}^C \bar{F}$$
\[ + (\phi \rightarrow \tilde{\phi}, \psi \rightarrow \tilde{\psi}, F \rightarrow \tilde{F}, C^{\mu
u} \rightarrow -C^{\mu
u}) \}, \]  \hspace{1cm} (33)\]

where \( \kappa^{ABC}, \kappa_A \) and \( \varepsilon_A \) are some constants. Besides, because of the renormalisability of \( \text{NAC } SU(N) \times U(1) \) gauge theory, we require choosing

\[
\kappa^{ABC} = \xi \gamma^{BAC}a^A e^B \theta^C, \quad \kappa_A = (\zeta c_A + \eta(1 - c_A)) g_A
\]

\[
\kappa_a = \zeta g, \quad \kappa_0 = \eta g_0, \quad \varepsilon_A = \tau g_A c^A,
\]

where \( \xi, \zeta, \eta, \tau \) and \( h_{ABC} \) are some coefficients. Moreover, \( h_{ABC} \) depend on indices \( A, B, C \). We note that \( h_{a0b} = h_{ab0} \). In addition, \( g_a \equiv g, \ c^A = 1 - \delta^{A0} \), and \( d^A = 1 + \delta^{A0} \). We also have

\[
(Y^{\mu\nu})^{\alpha\beta} = \epsilon^{\alpha\beta} C^{\mu\nu} g_{\mu\lambda} (\bar{\sigma}^{\lambda\nu})^{\beta}_{\nu}.
\]

According to the above equations, the action has been parameterised by some new non zero parameters \( \kappa^{ABC}, \kappa_A \) and \( \varepsilon_A \). Such parameters have not been introduced in refs \[1, 17, 6\] because they have worked in spacial Wess Zumino gauge. However, the renormalisation procedure reveals the necessity for these non-zero couplings. We have realised that if these new coefficients are zero, then some terms are hidden in the classical action, meanwhile divergent contributions due to Feynman diagrams produce them.

It is straightforward to show that Eq. (33) is preserved under the gauge, and SUSY transformations of Eqs. (31,32).

### 3.2 Renormalisation of the \( SU(N) \times U(1) \) modified action

The divergences in one-loop diagrams should be cancelled by the one-loop divergences in bare action, obtained by replacing the fields and couplings in Eq. (33) with bare fields and couplings given by

\[
A^a_{B\mu} = Z^a_{A\mu} A^a_{\mu}, \quad A^0_{B\mu} = Z^0_{A\mu} A^0_{\mu}
\]

\[
\lambda^a_B = Z^a_{A\lambda} \lambda^a, \quad \lambda^0_B = Z^0_{A\lambda} \lambda^0, \quad \phi_B = Z^a_{\phi} \phi, \quad \psi_B = Z^a_{\psi} \psi, \quad g_B = Z^g g
\]

\[
C^{\mu\nu}_B = Z_C C^{\mu\nu}, \quad |C|_B = Z_{|C|^2} |C|_B
\]
\[ \xi_B = Z\xi, \quad \zeta_B = \frac{Z\zeta}{\xi}, \quad \eta_B = \frac{Z\eta}{\xi}, \]
\[ \tau_B = Z\tau, \quad \hat{h}_{(ABC)\text{Bare}} = Z_{h_{ABC}} h_{ABC} \]
\[ \vartheta_1B = Z_{\vartheta_1}, \quad \vartheta_2B = Z_{\vartheta_2}. \]  (36)

In Eq. (36) we have set the renormalised couplings \( \xi, \zeta, \eta, \zeta, \tau \) to zero for simplicity. The other renormalisation constants start with tree-level values of 1. Therefore, we have:

\[ \xi_B = Z(1)\xi, \quad \zeta_B = Z(1)\zeta, \quad \eta_B = Z(1)\eta, \]
\[ \tau_B = Z(1)\tau, \quad \hat{h}_{(ABC)\text{Bare}} = Z_{h_{ABC}}(1)h_{ABC}, \]
\[ \vartheta_1B = Z(1)\vartheta_1, \quad \vartheta_2B = Z(1)\vartheta_2. \]  (37)

The \( C \)-independent one-loop corrections are cancelled by the one-loop divergences in the \( C \)-independent part of the bare action. Thus, the renormalisation constants for the fields and for the gauge couplings \( g, g_0 \) are the same as in the ordinary \( N = 1 \) supersymmetric theory [13], and up to one loop corrections they are given by [22]:

\[ Z_\lambda = 1 - 2L(N + 1), \quad Z_{\lambda^0} = 1 - 2L, \]
\[ Z_A = 1 + 2L(N - 1), \quad Z_{A^\phi} = 1 - 2L, \]
\[ Z_g = 1 + L(1 - 3N), \quad Z_{g_0} = 1 + L, \]
\[ Z_\phi = 1, \quad Z_\psi = 1 - 2L\hat{C}_2, \]  (38)

where (using dimensional regularisation with \( d = 4 - \epsilon \))

\[ \hat{C}_2 = (N + \frac{1}{N}\Delta) \]  (39)

with

\[ \Delta = \left(\frac{g_0}{g}\right)^2 - 1 \]  (40)

Upon inserting Eq. (38) into Eq. (33) one could obtain the one-loop contributions from \( S_{\text{Bare}} \) as

\[ S_{\text{Bare}}^{(1)} = \int d^4x \left( (4NL + 2L)i g C^{\mu\nu} d^{abc} \partial_\mu \bar{A}_a^\nu \tilde{\lambda}^b \tilde{\lambda}^c + 4i L g_0 C^{\mu\nu} d^{ab0} \partial_\mu \bar{A}_a^\nu \tilde{\lambda}^b \tilde{\lambda}^0 \right) \]
\[+(8NL + 2L)g^2 C_{\mu
u} \delta^{abc} \partial_\mu A^0_\nu \lambda^b \lambda^0 + 2iLg_0 C_{\mu
u} \delta^{000} \partial_\mu A^0_\nu \lambda^0 \lambda^0 + 2Lg_0 C_{\mu
u} \delta^{000} \partial_\mu A^0_\nu \lambda^0 \lambda^0 \]
\[-(3NL + L)g^2 C_{\mu
u} \delta^{abc} f^{cde} A^c_\nu A^d_\mu \lambda^a \lambda^b \]
\[-(2NL + 2L)g g_0 C_{\mu
u} \delta^{abc} f^{cde} A^c_\nu A^d_\mu \bar{\lambda}^0 \bar{\lambda}^b \]
\[-\frac{1}{4} \left(\frac{g^2}{g_0}\right) L + \frac{1}{4NL} \left| C \right|^2 d^{abc} d^{cde} \bar{\lambda}^a \bar{\lambda}^b \left(\bar{\lambda}^c \bar{\lambda}^d\right) \]
\[+(\left(\frac{g^2}{g_0}\right) L + \frac{1}{2NL} \left| C \right|^2 \left(\bar{\lambda}^a \lambda^a\right) \left(\bar{\lambda}^b \lambda^b\right) \]
\[-\frac{i}{2} Z_{[C]}^{(1)} \frac{g}{g_0} C_{\mu
u} \delta^{abc} d^{abc} \bar{\lambda}^a \left(\bar{\lambda}^b \lambda^b\right) \]
\[+\sqrt{2} (-\bar{L} \bar{C}_2 - 4NL) g C_{\mu\nu} g C_{\mu\nu} \delta_{\mu} \bar{\sigma} \psi + \sqrt{2} (-\bar{L} \bar{C}_2) g_0 C_{\mu\nu} \partial_\mu \bar{\lambda}^0 R \bar{\sigma} \psi \]
\[+\sqrt{2} (6NL + \bar{L} \bar{C}_2) g^2 C_{\mu\nu} \delta A^\mu_\nu \bar{\lambda}^0 \lambda^0 \lambda^C \bar{\lambda}^C \]
\[+\sqrt{2} (4NL + L \bar{C}_2) g g_0 C_{\mu\nu} \delta A^\mu_\nu \bar{\lambda}^0 \lambda^0 \lambda^0 \lambda^C \bar{\lambda}^C \]
\[+\sqrt{2} \left(\frac{g^2}{g_0}\right) \left(\bar{L} \bar{C}_2\right) \frac{g}{g_0} C_{\mu\nu} \delta A^\mu_\nu \bar{\lambda}^0 \lambda^0 \lambda^0 \lambda^C \bar{\lambda}^C \]
\[+\left(-4NL\right) C_{\mu\nu} \delta R^a \bar{\sigma} \partial_\mu A^a_\nu F + i\left(4NL\right) g^2 C_{\mu\nu} \delta R^a f^{abc} A^b_\mu A^c_\nu F \]
\[+\left(-NL\right) g^2 \left| C \right|^2 d^{abc} R^a \bar{\lambda}^b \lambda^c F + \left(-NL\right) g_0 \left| C \right|^2 d^{abc} R^a \bar{\lambda}^b \lambda^c F \]
\[+\sqrt{2} \left(Z_{[C]}^{(1)} \right) C_{\mu\nu} D_{\mu} \delta \bar{\lambda} \bar{\sigma} \psi + Z_{[C]}^{(1)} C_{\mu\nu} \delta \bar{\lambda} \bar{\sigma} \psi F + \frac{1}{4} Z_{[C]}^{(1)} \left| C \right|^2 \delta \bar{\lambda} \bar{\sigma} \psi \bar{\lambda} \bar{\sigma} \psi \]
\[+\frac{i}{8} \left(Z_{[C]}^{(1)} \right) g d^{abc} C_{\mu\nu} \partial_\mu A^a_\nu \lambda^b \lambda^c - \frac{i}{8} \left(Z_{[C]}^{(1)} \right) g^2 f^{cde} d^{abc} C_{\mu\nu} A^c_\mu A^d_\nu \bar{\lambda}^a \bar{\lambda}^b \bar{\lambda}^c \]
\[+\frac{i}{4} \left(Z_{[C]}^{(1)} \right) \xi g d^{abc} \left(\partial_\mu \bar{\lambda}^b Y_{\mu\nu} \bar{\lambda}^c - \bar{\lambda}^b Y_{\mu\nu} \partial_\mu \bar{\lambda}^c\right) A^a_\mu \]
\[+\frac{i}{4} \left(Z_{[C]}^{(1)} \right) g^2 f^{abc} d^{cde} A^c_\mu A^d_\nu \lambda^a \lambda^b \lambda^c \]
\[+\frac{i}{2} \left(Z_{[C]}^{(1)} \right) g_0 d^{abc} \left(\partial_\mu \lambda^b Y_{\mu\nu} \lambda^0 - \lambda^b Y_{\mu\nu} \partial_\mu \lambda^0\right) A^a_\nu \]
\[-\frac{i}{2} \left(Z_{[C]}^{(1)} \right) g_0 f^{abc} d^{cde} C_{\mu\nu} A^c_\mu A^d_\nu \bar{\lambda}^0 \bar{\lambda}^b \bar{\lambda}^c \]
\[+\frac{i}{4} \left(Z_{[C]}^{(1)} \right) g^2 C_{\mu\nu} \delta A^b_\mu \bar{\lambda}^0 \bar{\lambda}^c \bar{\lambda} \lambda^c \bar{\lambda} \lambda^0 \lambda^C \bar{\lambda}^C \]
\[-\left(Z_{[C]}^{(1)} \right) C_{\mu\nu} g R^a \delta \partial_\mu A^a_\nu F - i\left(Z_{[C]}^{(1)} \right) C_{\mu\nu} g_0 R^0 \delta \partial_\mu A^a_\nu F \]
\[+\frac{i}{8} \left(Z_{[C]}^{(1)} \right) g^2 C_{\mu\nu} \delta R^a \bar{\sigma} \partial_\mu A^a_\nu F + \frac{1}{4} \left(Z_{[C]}^{(1)} \right) \left| C \right|^2 \delta \bar{\lambda} \bar{\sigma} \psi \bar{\lambda} \bar{\sigma} \psi \]
\[-\sqrt{2} \left(Z_{[C]}^{(1)} \right) g C_{\mu\nu} \delta \partial_\mu \bar{\lambda} \bar{\sigma} \psi \bar{\lambda} \bar{\sigma} \psi \]
\[-\sqrt{2} \left(Z_{[C]}^{(1)} \right) g C_{\mu\nu} \delta \bar{\lambda} \bar{\sigma} \psi \partial_\mu \bar{\lambda} \bar{\sigma} \psi - 2i \left(Z_{[C]}^{(1)} \right) g C_{\mu\nu} \delta \bar{\lambda} \bar{\sigma} \psi \partial_\mu A^a_\nu F \]
\[+i \left(Z_{[C]}^{(1)} \right) g^2 C_{\mu\nu} \delta R^a \bar{\sigma} \partial_\mu A^a_\nu \lambda^0 F + \frac{1}{N} \left(Z_{[C]}^{(1)} \right) g_0^2 \left| C \right|^2 \left(\bar{\lambda}^a \lambda^a\right) \left(\bar{\lambda}^0 \lambda^0\right) \]

(41)
The results $\Gamma_{i-1PI}^{(1)}$, $i = 1, \ldots, 8$ for the one loop divergences from the 1PI graphs coming from the $C$ dependent part of the $N = 1/2$ supersymmetric gauge theory coupled to matter are given in Appendix A[17]. We find that with

$$Z_C^{(1)} = Z_{|C|^2 = 0}^{(1)} = 0, \quad Z_\zeta^{(1)} = -2NL, \quad Z_{\phi_2}^{(1)} = -NL$$

$$Z_\zeta^{(1)} = -(5N + 2\hat{C}_2)L, \quad Z_\eta^{(1)} = 2\hat{C}_2L, \quad Z_\tau^{(1)} = -(18N + 8\hat{C}_2)L$$

$$Z_{h_{abc}}^{(1)} = -37N + 32\hat{C}_2L, \quad Z_{h_{a00}}^{(1)} = -2(N - \hat{C}_2)L$$

$$Z_{h_{0bc}}^{(1)} = \frac{3}{4}NL, \quad Z_{h_{000}}^{(1)} = 2L\hat{C}_2, \quad Z_{\vartheta_1}^{(1)} = -3NL,$$

they can be canceled by Eq. (41). In fact, we have

$$S_{Bare}^{(1)} + \sum_{i=1}^{8} \Gamma_{i-1PI}^{(1)} = \text{finite},$$

Our results indicate the theory is renormalisable in the usual procedure [23] without using one-loop divergent field redefinitions of the gaugino and the auxiliary fields($\lambda, \bar{F}$). However, it is necessary to include the terms involving $\vartheta_1, \vartheta_2$ in Eq. (33) since further divergent configurations arise at one-loop which are $N = 1/2$ supersymmetric. These terms are not in the original formulation of the theory though they are independently $N = 1/2$ supersymmetric. Therefore, one should modify the classical superspace Lagrangian Eq. (1) because these terms are not obtained from the original superfield action Eq. (1). This point is consistent with results [19, 20, 21]. They have modified the classical action Eq. (1) in order to have one-loop renormalisable theory.

In our work, we modify the classical action of Eq (26) in order to make the theory be renormalisable and gauge invariant. In the modified action Eq (33), there are extra terms which were absent in [17]. In our model, we have new renormalised couplings ($\xi, \zeta, \eta, \tau, h_{ABC}$) which start with tree-level values of zero for simplicity. In order to renormalise the theory we use the field redefinitions of the gaugino and the auxiliary fields($\lambda, \bar{F}$) in component formalism, but the classical superspace action is not modified. In fact we use a different Wess Zumino gauge in compare with that used in refs [1, 6]. Moreover, we obtain the same divergent contributions $Z_{\vartheta_1}, Z_{\vartheta_2}$ as those in Ref [17] which is a good check of our results.

17
In our work we use the following gaugino field redefinition

$$
\lambda'^A \rightarrow \lambda^A - \frac{1}{4} \kappa^{ABC} d^{ABC} C^\mu\nu A^B_\mu \sigma_\nu \bar{\lambda}^C
$$

(47)

or, we have:

$$
\delta \lambda^A = \lambda - \lambda' = \frac{1}{4} \kappa^{ABC} d^{ABC} C^\mu\nu A^B_\mu \sigma_\nu \bar{\lambda}^C = \frac{1}{4} \xi \gamma^{BAC} c^A c^B d^C d^{ABC} C^\mu\nu A^B_\mu \sigma_\nu \bar{\lambda}^C
$$

(48)

After the renormalisation, we have

$$
\delta \lambda^A = \frac{1}{2} N L \gamma^{BAC} c^A c^B d^C d^{ABC} C^\mu\nu A^B_\mu \sigma_\nu \bar{\lambda}^C
$$

(49)

which is similar to the nonlinear gaugino field redefinition introduced in ref [17].

Now we can give an interpretation of the nonlinear gaugino field redefinition in [17]. They have worked in Seiberg’s Wess Zumino gauge (\( \kappa^{ABC} = 0 \) in Seiberg’s parametrisation but this choice is not preserved in the renormalisation) which is not a suitable convention for the renormalisation, then they have been forced to use the nonlinear field redefinition. However, in order to renormalise the theory we use a generalized Wess Zumino gauge where our redefinition is associated with some parameters, so we do not need to use the nonlinear field redefinition. The same comparison between our auxiliary field (\( \bar{F} \)) redefinition the divergent auxiliary field redefinition of ref [17] can be used in order to interpret the nonlinear auxiliary field redefinition. In fact, they have used the nonlinear field redefinitions to absorb unusual divergent contributions which are produced by 1PI graphs, and have found that variation of \( \lambda \) and \( \bar{F} \) result in a change in the action; besides, adding these divergent contributions to the classical action the theory is renormalisable up to one loop corrections. In this sense, we would like to conclude that the divergent field redefinitions used in [17] were actually correct, although their interpretation was not clearly written.

We have obtained \( Z_{C^2} = |Z_C|^2 = 1 \) which means the non-anticommutative structure is preserved by the renormalisation despite the fact that the modified action (Eq. 41) has an explicit dependence on the NAC parameter. Therefore, the star product does not get deformed by quantum corrections which is consistent with
ref [17] for the case of the component formalism and ref [19] for the case of the superspace formalism.

The authors of Ref [19] have studied one-loop quantum properties of the deformed superspace theory and showed that the one-loop effective action could be renormalised if one modifies the NAC action in superspace formalism and our work should be compared to their work. Generally, we confirm their work although some of the details may differ. Working in superspace, in a background field approach, they have shown that new divergences were present which cannot be renormalised away. In order to make the theory be renormalisable they have modified the classical action from the start by adding new terms which allow for the cancellation of all the divergent terms at one loop. We have taken the same approach by adding new terms to the Lagrangian from start in the component formalism. We prove that subtraction of one loop divergences does not require nonlinear field redefinitions which is consistent with [19], and also the discussion is cleaner. The important point however was the check that indeed, even in the presence of NAC, the effective action is gauge invariant and therefore the safety of going to WZ gauge is ensured.

In this work we assume the renormalised couplings $\xi, \zeta, \eta, \tau, \vartheta_1, \vartheta_2$ and $h_{ABC}$ are set to zero. These assumptions simplify our calculations; in other words, we do not consider contributions from terms which are proportional to these couplings to the loop divergences. However, in a future extended work we will consider non-zero values for these renormalised couplings, and calculate their contributions to quantum corrections. In our previous paper [24], we have computed one-loop corrections which come from extra terms in $N = 1/2$ supersymmetric pure gauge theory. Moreover, Our results are consistent with ref [19] which used the superspace formalism. In both cases, in order to obtain a renormalisable Lagrangian it is vital to add some new terms to the original Lagrangian.
4 Conclusion

We have investigated the renormalisability of a general $N = \frac{1}{2}$ supersymmetric $SU(N) \times U(1)$ gauge theory coupled to chiral matter at one loop order. We have proved the theory is renormalisable up one loop order using the standard method of renormalisation by adding some extra terms which are generated by field redefinitions of the gaugino and the auxiliary fields ($\lambda, \bar{F}$), and some new terms which are put by hand to the original component Lagrangian. Moreover, we have shown the effective action is gauge invariant up to one-loop corrections.

We have indicated there is no need to employ divergent redefinitions of $\lambda$ and $\bar{F}$. We have used the $N = \frac{1}{2}$ gauge group $SU(N) \times U(1)$ because of the requirements of gauge invariance and renormalisability. As discussed in [7] the non-anticommutative $SU(N)$ gauge theory is not well-defined, and the non-anticommutative $U(N)$ gauge theory is not renormalisable [17, 19].

We have shown that the problem of the renormalisability of the non-anticommutative theory in the component formalism can be solved by field redefinitions. One of advantages of the field redefinition method is that it does not change the original Lagrangian in the superspace formalism; in other words, Eq (1) is preserved under field redefinitions and the theory is renormalised. We have proved that the complete divergent part of the effective action which come from $C$-deformed section is gauge invariant even though term by term these quantum corrections are not gauge invariant, and also arrived at the conclusion that there is no need to renormalise the non-anticommutativity parameter $C$, which is consistent with Ref [19].

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A Divergent contributions for 1PI graphs

The divergent contributions to the effective action from the $C$ dependent diagrams with one gauge, two gaugino lines ($A_\mu \bar{\lambda} \bar{\lambda}$) are given by:

\[
\Gamma^{(1)}_{1-1PI} = -\left( \frac{15}{4}NL + 2L \right) C^{\mu \nu} d^{abc} \delta_{a} \bar{A}^{b} \bar{\lambda}^{c} - 4iLg_{0} C^{\mu \nu} d^{abc} \partial_{\mu} A_{a}^{0} \bar{\lambda}^{b} \bar{\lambda}^{0}
\]

\[
- (8NL + 2L) i g^{2} C^{\mu \nu} d^{cde} \partial_{\mu} \bar{A}^{0} \bar{\lambda}^{b} \bar{\lambda}^{0} - 2iLg_{0} C^{\mu \nu} d^{000} \partial_{\mu} A_{a}^{0} \bar{\lambda}^{b} \bar{\lambda}^{0}
\]

\[
+ \frac{i}{2} NLg_{0} C^{\mu \nu} d^{abc} \bar{A}^{a}_{\mu} \bar{\lambda}^{b} \bar{\lambda}^{0} - NLg_{0} C^{\mu \nu} d^{000} \partial_{\mu} A_{a}^{0}
\]

\[
- 2iNLg_{0} d^{abc} \bar{\lambda}^{b} Y^{\mu \nu} \partial_{\mu} \bar{\lambda}^{0}
\]

The divergent contributions to the effective action from the $C$ dependent diagrams with two gauge and two gaugino lines ($A_\mu \bar{\lambda} \bar{\lambda}$) are:

\[
\Gamma^{(1)}_{2-1PI} = \left( \frac{11}{4}NL + L \right) g^{2} C^{\mu \nu} d^{abc} f^{cde} A^{c}_{\mu} A^{d}_{\nu} \bar{\lambda}^{a} \bar{\lambda}^{b}
\]

\[
+ \frac{1}{2} NLg^{2} f^{abc} d^{cde} A^{c}_{\mu} A^{d}_{\nu} Y^{\mu \nu} \bar{\lambda}^{b}
\]

\[
+ (NL + 2L) i g_{0} g^{d} d^{cde} C^{\mu \nu} A^{c}_{\mu} A^{d}_{\nu} \bar{\lambda}^{a} \bar{\lambda}^{0}
\]

The divergent contributions to the effective action from the $C$ dependent diagrams with four gaugino lines $(\bar{\lambda} \bar{\lambda})^{2}$ are:

\[
\Gamma^{(1)}_{3-1PI} = \left( \frac{5}{4}NL + \frac{1}{4}L \right) g^{2} | C |^{2} d^{abc} d^{cde} (\bar{\lambda}^{a} \bar{\lambda}^{b})(\bar{\lambda}^{c} \bar{\lambda}^{d})
\]

\[
+ (4 \frac{g^{2}}{g_{0}^{2}} L + \frac{1}{2N} L) | C |^{2} (\bar{\lambda}^{a} \bar{\lambda}^{b})(\bar{\lambda}^{c} \bar{\lambda}^{d})
\]

\[
+ 3NLg_{0}^{2} | C |^{2} (\bar{\lambda}^{a} \bar{\lambda}^{b})(\bar{\lambda}^{0} \bar{\lambda}^{0})
\]

The divergent contributions to the effective action from the $C$ dependent diagrams with one gaugino, one scalar and one chiral fermion line $(\bar{\phi} \bar{\lambda} \psi)$ are given by:

\[
\Gamma^{(1)}_{4-1PI} = \sqrt{2}(L \hat{C}_{2} + 3NL) g C^{\mu \nu} \partial_{\mu} \bar{\phi} \bar{\lambda}^{a} R^{a} \bar{\sigma}_{\nu} \psi
\]

\[
+ \sqrt{2}(L \hat{C}_{2}) g_{0} C^{\mu \nu} \partial_{\mu} \bar{\phi} \bar{\lambda}^{0} R^{0} \bar{\sigma}_{\nu} \psi
\]

\[
+ \sqrt{2}(-NL) g C^{\mu \nu} \bar{\phi} \bar{\lambda}^{a} R^{a} \bar{\sigma}_{\nu} \partial_{\mu} \psi
\]

The divergent contributions to the effective action from the $C$ dependent diagrams
with one gaugino, one scalar, one chiral fermion and one gauge line \((A_\mu \bar{\phi} \lambda \psi)\) are:

\[
\Gamma^{(1)}_{5-1PI} = i\sqrt{2}NLg^2C^{\mu\nu}A^b_\nu \bar{\phi} \lambda^a \sigma_\nu \psi \left[\frac{1}{2} d^{abc} R^c - 2i f^{abc} - 3d^{abc} R^c \right] \\
+i\sqrt{2}NLg g_0 C^{\mu\nu} A^0_\nu \bar{\phi} \lambda^b R^a R^b \sigma_\nu \psi [-4] \\
-i\sqrt{2}NLg^2 C^{\mu\nu} A^0_\nu \bar{\phi} \lambda^b R^a R^b \sigma_\nu \psi [\hat{C}_2] \\
-i\sqrt{2}NLg g_0 C^{\mu\nu} A^0_\nu \bar{\phi} \lambda^b R^a R^b \sigma_\nu \psi [\hat{C}_2] \\
-i\sqrt{2}NLg g_0 C^{\mu\nu} A^0_\nu \bar{\phi} \lambda^b R^a R^b \sigma_\nu \psi [\hat{C}_2] \\
-i\sqrt{2}NLg^2 C^{\mu\nu} A^0_\nu \bar{\phi} \lambda^0 (R^0)^2 \sigma_\nu \psi [\hat{C}_2] \\
\]

The divergent contributions to the effective action from the \(C\) dependent diagrams with one gauge, one scalar and one auxiliary line \((\bar{\phi}A_\mu F)\) are given by:

\[
\Gamma^{(1)}_{6-1PI} = i(-2L\hat{C}_2)gC^{\mu\nu} \bar{\phi} \partial_\mu A^a_\nu R^a F + i(-3NL)gC^{\mu\nu} \bar{\phi} \partial_\mu A^a_\nu R^a F \\
+i(-2L\hat{C}_2)g_0 C^{\mu\nu} \bar{\phi} \partial_\mu A^0_\nu R^0 F \\
\]

The divergent contributions to the effective action from the \(C\) dependent diagrams with two gaugino, one scalar and one auxiliary line are:

\[
\Gamma^{(1)}_{7-1PI} = i(L\hat{C}_2)g^2 C^{\mu\nu} \bar{\phi} R^a f^{abc} A^b_\mu A^c_\nu F \\
+i\left(-\frac{1}{4}NL\right)g^2 C^{\mu\nu} \bar{\phi} R^a f^{abc} A^b_\mu A^c_\nu F \\
\]

The divergent contributions to the effective action from the \(C\) dependent diagrams with two gaugino, one scalar and one auxiliary line \((\bar{\phi}\lambda\lambda F)\) are:

\[
\Gamma^{(1)}_{8-1PI} = \left\{ \begin{array}{l}
\frac{45}{8}NLg^2 \left[ C \right]^2 d^{abc} R^a \bar{\phi} \lambda^b \lambda^c F \\
+(-4L\hat{C}_2)g^2 \left[ C \right]^2 d^{abc} R^a \bar{\phi} \lambda^b \lambda^c F \\
+(5NL)g g_0 \left[ C \right]^2 d^{0bc} R^c \bar{\phi} \lambda^0 \lambda^b F \\
+(-4L\hat{C}_2)g g_0 \left[ C \right]^2 d^{0bc} R^c \bar{\phi} \lambda^0 \lambda^b F \\
+(\frac{1}{4}NL)g^2 \left[ C \right]^2 d^{0bc} R^0 \bar{\phi} \lambda^b \lambda^c F \\
+(-2L\hat{C}_2)(g_0)^2 \left[ C \right]^2 d^{000} R^0 \bar{\phi} \lambda^0 \lambda^0 F \\
\end{array} \right. \\
\]

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