Research on Physical Laws Behind Irreversible Cartesian Diver

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Abstract. The float-sink is an instrument used to demonstrate the pressure transmission of liquids, and it is a vivid proof of Archimedes's principle and Pascal's principle. When pressure is applied to the container in which the sinker is located, the sinker sinks; when the pressure is released, the sinker floats up. This phenomenon qualitatively proves Archimedes's law. But when the sinker sinks to a certain critical depth, even if the pressure is released, the sinker will no longer float up. This phenomenon is the so-called "irreversible sinker". This article mainly analyzes the irreversible principle and influencing factors when the float-sink reaches the critical depth and explores the influence of different influencing factors on the critical depth through mathematical analysis and experimental methods. Finally, it is concluded that the critical depth when the float is irreversible is related to factors such as the mass of the float, the volume of gas in the float and the height of the water in the container, and the critical depth has a linear relationship with the mass of the float.

1. Introduction

1.1. Background

The floating sink is an instrument used to demonstrate the buoyancy of liquids, the compressibility of gases, and the transfer of pressure to fluids. It was created by French scientist Descartes. There are two types of floats, which are closed floats and open floats. The short float is a closed soft vial, and the available float is a rigid vial with a hole at the bottom. Take the open float sink as an example. According to Pascal’s law, the pressure is transferred to the water when the air is compressed. Gravity increases, greater than the buoyancy it receives, it sinks. When the hand leaves the rubber membrane, the volume of air on the water surface in the cylinder increases, and the pressure decreases. The compressed air in the float sink presses the water out. At this time, the gravity of the float sink is less than the buoyancy it receives, so it floats upward.

When the pressure of the float sink reaches a certain depth, after releasing it, the float will no longer automatically float up but will continue to sink. Why doesn't the floater sink? What is the reason? According to specific experiments, there should be a critical depth of sinking. When the sinking position is above this depth, the sinker will automatically float up. When the sinking position is greater than this depth, the sinker will no longer automatically float up, but continue to sink. At the same time, the air volume in the sinker should be related to the critical depth. If so, what should be the relationship? The above phenomenon cannot be quantified through ordinary experiments.
1.2. Research status
At present, scholars at home and abroad have conducted extensive research on the irreversible phenomenon of floats and sinks. Roberto De Luca and Salvatore Ganci mainly studied the pressure and temperature changes of the float sink during the complete sinking experiment and deduced the critical depth formula \( h_c = \frac{V_e}{\rho g V_a} p_a \) [1]. Liu Bingsheng mainly studied the factors that caused the sinking and floating of floats and sinks and deduced the formula \( x_0 = \frac{p_0 v_0}{g} - H \) [2]. Chang Jian mainly proved the existence of the equilibrium position of the float-sink. He provided a theoretical basis for the lower pressure on the upper part of the float-sink at a deeper level [3-5].

1.3. Research content
In this study, the stress state of the float-sink in the water was analyzed, and the critical depth of the float-sink was determined according to the condition of the force balance of the float in the water. If the float is above the critical depth, the float will automatically float up after the external force is released; if the float is below the critical depth, the float will automatically sink even after the external force is released.

In the study, a specific theoretical analysis of two different types of floats and sinks was carried out to explore which factors (the total mass of the floats and the volume of air in the floats and sinks in the initial state) of the floats and sinks under normal conditions will determine the critical depth of the floats and sinks. And quantitatively study the relationship between the critical depth and parameters of the float-sink so as to get the application conditions of the float-sink. Then through experiments to further discover and demonstrate the floating and sinking phenomenon, through the physical principles to reveal and verify the impact of these influencing factors on the critical depth.

2. Methods

2.1. Floats and sinkers and experiments on their ups and downs
Using simple experimental equipment, make an open float that can hang heavy objects, and verify the irreversible phenomenon of the float through experiments: when it reaches a certain depth, remove the external force, the float will automatically rise; when it reaches a certain depth After the external force is removed, the float will continue to sink and will no longer float.

During the experiment, record the position of the critical depth, and use the scatter plot to summarize the law.

2.2. Floats and sinkers and their physics laws
Determine the factors that affect the critical depth by analyzing the forces at different positions of the floats and sinks in the water and related theoretical derivations, and analyze how these factors cause the changes in the critical depth, so as to reveal and verify the ups and downs obtained from the floats and sinks experiments law.

Whether an object can float or sink in water is mainly due to the relationship between the buoyancy and gravity of the object. If the buoyancy is greater than the gravity, the object is in a floating state; if the buoyancy is equal to the buoyancy, the object is in a floating or suspended state; if the buoyancy is less than the gravity, the object is in a sinking state. In response to this phenomenon, we will explore the effects of different influencing factors on the irreversible phenomenon and critical depth of floats and sinks through a combination of experiment and theory and derive the essential empirical formula of depth.

3. Shortcomings and prospects
There are two types of floats, which are closed floats and open floats. A fast float is a soft balloon or a plastic bag with a heavy object hanging under it, filled with air, as shown in Figure 1(a); an open float is a hard plastic or glass bottle with a piece hanging under it. Heavy objects contain part of water and air, and there is a small hole at the bottom through which water can enter and drain, as shown in Figure 1(b).
When the closed float sink is fully immersed in water, if the water discharged by the air-bag is heavier than the weight of the rock, the float sink will float up. The float will cause the air-bag to expand and increase, and the speed of floating will increase until it floats to the surface. At this time, the buoyancy is equal to gravity; if the weight of the water discharged by the air-bag is less than the weight of the rock, the float sink will sink, and the sinking will cause the air-bag to shrink and shrink, and the sinking speed will also increase until it sinks to the bottom. Therefore, the enclosed floats and sinks have only two states in the water, namely floating on the water surface and sinking in the bottom.

When the open float sink is placed in the water, water will enter the float sink through the small holes. As the water intake increases, the buoyancy of the float sink becomes smaller and smaller. If the suspended heavy object is too heavy, the interior of the float will be filled with water, and the float will sink until it sinks to the bottom; if the suspended heavy object is light, the water and gas in the float will reach a state of equilibrium. At this time, buoyancy is equal to gravity. Therefore, there are only two states of open floats in the water, namely floating on the water surface and sinking in the bottom.

For the above two different types of floats and sinks, the following numerical analysis is done:

### 3.1. Theoretical Analysis on the Ups and Downs of Enclosed Floats and Sinks

#### 3.1.1. Conformity and meaning used in the theoretical analysis process

The symbol description of the enclosed float is shown in Table 1.

| Symbol | Meaning |
|--------|---------|
| $m$    | Total mass of floats and sinks |
| $V_0$  | The volume of gas in the float (inflation under standard atmospheric pressure) |
| $V_1$  | Block volume |
| $V$    | Gas volume after pressure change (dynamic change) |
| $\rho$ | Density of water ($10^3$ kg/m$^3$) |
| $p_0$  | Standard atmospheric pressure ($1.01 \times 10^5$ Pa) |
| $h_c$  | Critical depth |

#### 3.1.2. Theoretical analysis process

Assumption: The gas in the float meets ideal gas conditions.

When the float sinks into the water, the resultant force is:
\[ F = mg - \rho g(V + V_1) \]  \hspace{1cm} (1)

Since there is almost no change in temperature during the process of putting into water, the process of putting into water can be regarded as isothermal compression, according to \( pV = nRT \), where \( T \) is a constant, so \( pV \) is a constant. So when the balloon is placed in the water:

\[ p_0V_0 = (p_0 + \rho gh)V \]  \hspace{1cm} (2)

\[ V = \frac{p_0V_0}{p_0 + \rho gh} \]  \hspace{1cm} (3)

Substituting into the above formula, we get

\[ F = mg - \rho g \left( \frac{p_0V_0}{p_0 + \rho gh} + V_1 \right) = mg - \rho gV_1 - \rho gV_0 \frac{p_0}{p_0 + \rho gh} \]  \hspace{1cm} (4)

Let \( F = 0 \)

\[ mg - \rho gV_1 - \rho gV_0 \frac{p_0}{p_0 + \rho gh_c} = 0 \]  \hspace{1cm} (5)

\[ p_0 + \rho gh_c = \frac{\rho gV_0p_0}{mg - \rho gV_1} \]  \hspace{1cm} (6)

The critical depth is:

\[ h_c = \frac{p_0V_0}{mg - \rho gV_1} - \frac{p_0}{\rho g} = \frac{p_0}{\rho g} \left( \frac{\rho V_0}{m - \rho V_1} - 1 \right) \]  \hspace{1cm} (7)

The basic condition for the establishment of the above formula is: \( \rho V_0 > m - \rho V_1 > 0 \), otherwise \( h_c \) is a negative value, which does not match the actual situation.

\[ F = mg - \rho gV_1 - \rho gV_0 \frac{p_0}{p_0 + \rho gh} \]  \hspace{1cm} (8)

It can be seen from the above formula that, except for the variable depth \( h \), the rest are all fixed values. So the curve of \( F \) is roughly as shown in Figure 2:

![Figure 2](image)

**Figure 2.** The law of change of the resultant force \( F \).

When \( F > 0 \), the floater floats up; when \( F < 0 \), the floater sinks continuously.

\[ h_c = \frac{p_0}{\rho g} \left( \frac{\rho V_0}{m - \rho V_1} - 1 \right) \]  \hspace{1cm} (9)

It can be seen from the above formula that increasing the counterweight \( m \) can reduce the critical depth \( h_c \). Therefore, we stipulate the “weight sensitivity” (defined as the partial derivative of the mass of the float-sink to the critical depth) as:
\[
\left| \frac{\partial m}{\partial h_c} \right| = \frac{\rho^2 g p_0 V_0}{(p_0 + \rho g h_c)^2}
\]  

(10)

It reflects the sensitivity of the critical depth to this change when the counterweight \( m \) changes. Subsequent experiments can quantitatively increase the counterweight to study the influence of the counterweight \( m \) on the critical depth \( h_c \).

Suppose the mass and volume of the original float and sink are \( m_s \) and \( V_s \), the mass and volume of each counterweight block are \( m_2 \) and \( V_2 \), the number of counterweight blocks is \( k \), and the critical depth expression is:

\[
h_c = \frac{p_0}{\rho g} \left( \frac{\rho V_0}{(m_s + km_2) - \rho(V_s + kV_2)} - 1 \right) = \frac{p_0}{\rho g} \left( \frac{\rho V_0}{(m_s - \rho V_s) + k(m_2 - \rho V_2)} - 1 \right)
\]  

(11)

Define the depth coefficient \( H = \frac{p_0}{p_0 + \rho g h_c} \), so

\[
H = \frac{p_0}{p_0 + \rho g h_c} = \left( \frac{m_s - V_s}{V_0} \right) + k \left( \frac{m_2 - V_2}{V_0} \right)
\]  

(12)

It can be seen that the depth coefficient \( H \) is proportional to the number of counterweight blocks \( k \), and the resultant force \( F \) is:

\[
F = mg - \rho g V_1 - \rho g V_0 H
\]  

(13)

\[ \text{Table 2. Three Scheme comparing.} \]

| Symbol  | Meaning                                      |
|---------|----------------------------------------------|
| \( m_t \) | Total mass of floats and sinks               |
| \( V_t \) | Total volume of floats and sinks             |
| \( m_w \) | The mass of the water in the float           |
| \( V_w \) | The volume of water in the float             |
| \( m_p \) | The mass of floats                           |
| \( V_p \) | The volume of the float (ignored as 0)       |
| \( m_z \) | The mass of the suspended weight             |
| \( V_z \) | The volume of the suspended weight (ignored as 0) |
| \( V_{\text{ext}} \) | The volume on the surface of the float     |
| \( V_c \) | The volume below the surface of the float   |
| \( V_0 \) | The volume of gas in the float (initial state) |
| \( V_a \) | Gas volume in floats and sinks (dynamic change) |
| \( \rho \) | Density of water \( (10^3 \text{kg/m}^3) \) |
| \( p_0 \) | Standard atmospheric pressure \( (1.01 \times 10^5 \text{Pa}) \) |
| \( h_c \) | Critical depth                                |

\[ \text{3.2. Theoretical Analysis on the Ups and Downs of Open Floats and Sinks} \]

3.2.1. Conformity and meaning used in the theoretical analysis process

The symbol description of the open float is shown in Table 2.

\[ \text{3.2.2. Theoretical analysis process} \]

Assumption: The gas in the float meets ideal gas conditions.

Volume equation:

\[
V_t = V_{\text{ext}} + V_c + V_z = V_0 + V_w + V_p + V_z
\]  

(14)

When the float is floating on the water:

\[
F_{\text{float}} = G
\]  

(15)

\[
\rho g (V_t - V_{\text{ext}}) = (m_p + m_w + m_z)g
\]  

(16)
When the float is completely immersed in water:

\[ F = (m_p + m_z)g - \rho g V_a \]  

If there is a critical depth, then \( F = 0 \), so:

\[ (m_p + m_z)g - \rho g V_a = 0 \]  

According to \( pV = nRT \), \( T \) is a constant, so \( pV \) is a constant. Ignore the bottle height, so when the balloon sinks into the water:

\[ p_0 V_0 = (p_0 + \rho g h) V_a \]
\[ V_a = \frac{p_0 V_o}{p_0 + \rho g h} \]

So

\[ (m_p + m_z)g = \rho g \frac{p_0 V_o}{p_0 + \rho g h} \]

Therefore, the critical depth \( h_c \) is:

\[ h_c = \frac{p_0 V_o}{(m_p + m_z)g} - \frac{p_0}{\rho g} \]

The basic condition for the establishment of the above formula is: \( \rho V_0 > m_p + m_z > 0 \), otherwise, \( h_c \) is a negative value, which is inconsistent with reality.

4. Experimental design

4.1. Experiment equipment

The experimental equipment is as follows:

1. Measuring cylinder (500ml);
2. 100ml syringe;
3. Plastic tube;
4. High-precision electronic scale;
5. Float and sinker (lightweight glass medicine bottle);
6. Glass rod;
7. Water;
8. Straightedge;
9. Dropper.

4.2. Experiment program

The experimental program is as follows:

1. Use a reagent dropper and two paper clips (plus a part of the other paper clip) together, and the first test is whether it can float in a 200mm high glass. Then put the made float in a graduated cylinder and press it down with a plastic rod to test whether it will automatically sink when it reaches a certain depth. If so, judge the approximate depth and start the experiment; each case needs to be repeated once.
2. Experimental steps:
   1. Place the manufactured float and sink lightly on a high-precision electronic scale to weigh and record the corresponding data.
2) According to the depth of the preliminary test, add an appropriate amount of water to the measuring cylinder to ensure that the float sink can sink to the bottom.
3) Put the suction hose into the water in the graduated cylinder and connect it with the syringe. Use the syringe to slowly draw out the water in the graduated cylinder.
4) Carefully observe the state of the sinker during the slow pumping process until the sinker moves slightly (and then continues to float), and record the corresponding scale.
5) Repeat the test two more times to get three data.
6) Separate the floats and sinks, wipe off the water and weigh them separately.
7) To change the quality of floats and sinks, repeat steps 1-6.

4.3 Improvement of experimental measures
In the float-sink experiment, three methods were used to pressurize the gas in the measuring cylinder, namely: pressing method, drainage method and exhaust method. However, the use of the pressing method will have the problem of greater difficulty in force control, and it is easy to leak air and water when the measuring cylinder is sealed by the drainage method. In the end, we use the syringe exhaust method to pressurize the gas in the graduated cylinder, which is also easy to control.

4.4 Experimental data and its analysis
Since the wall thickness and volume of floats and sinks cannot be ignored, the critical height under test conditions is calculated according to the following formula:

\[ V_t = V_w + V_a + V_p \]  
\[ V_a = \frac{p_v \rho_0}{p_0 + \rho g h} \]  
\[ V_w = V_t - V_p - \frac{p_v \rho_0}{p_0 + \rho g h} = V_t - \frac{m_p}{\rho} \]  
\[ h_c = \frac{p_v \rho_0}{m_d g - \rho g V_p} - \frac{p_0}{\rho g} \] (28)

The data directly recorded in the experiment are shown in Table 3.

| No. | \( m_p \) (g) | \( m_t \) (g) | \( m_w \) (g) | \( V_p \) (ml) | \( V_t \) (ml) | \( V_w \) (ml) | \( V_0 \) (ml) | \( h_c \) (cm) Calculated | \( h_c \) (cm) Measured |
|-----|--------------|--------------|--------------|--------------|--------------|--------------|--------------|-----------------|-----------------|
| 1   | 8.75         | 16.11        | 7.36         | 3.74         | 16.5         | 7.36         | 5.40         | 6.5             | 5.805           |
| 2   | 8.75         | 16.05        | 7.30         | 3.74         | 16.5         | 7.30         | 5.46         | 6.8             | 5.929           |
| 3   | 8.75         | 16.00        | 7.25         | 3.74         | 16.5         | 7.25         | 5.51         | 7.4             | 6.032           |
| 4   | 8.75         | 15.98        | 7.23         | 3.74         | 16.5         | 7.23         | 5.53         | 7.4             | 6.073           |
| 5   | 8.75         | 15.94        | 7.19         | 3.74         | 16.5         | 7.19         | 5.57         | 7.9             | 6.156           |
| 6   | 7.68         | 14.97        | 7.29         | 3.12         | 15.0         | 7.29         | 4.59         | 5.0             | 5.068           |
| 7   | 7.68         | 14.91        | 7.23         | 3.12         | 15.0         | 7.23         | 4.65         | 5.5             | 5.204           |
| 8   | 7.68         | 14.85        | 7.17         | 3.12         | 15.0         | 7.17         | 4.71         | 5.9             | 5.340           |
| 9   | 7.68         | 14.82        | 7.14         | 3.12         | 15.0         | 7.14         | 4.74         | 5.9             | 5.408           |
| 10  | 7.68         | 14.77        | 7.09         | 3.12         | 15.0         | 7.09         | 4.79         | 6.1             | 5.521           |

The following scatter plot and its linear regression are only intuitive data relationships that illustrate the correlation between critical depth and influencing factors.

(1) The relationship between \( h_c \) and \( V_0 \) is shown in Figures 3 and 4. The two have an obvious positive correlation (when \( m_p = 8.75 \)g, \( R^2 = 0.976 \); when \( m_p = 7.68 \)g, \( R^2 = 0.9414 \)).
Figure 3. Scatter plot of critical depth when $m_p = 8.75g$.

Figure 4. Scatter plot of critical depth when $m_p = 7.68g$.

(2) The relationship between $h_c$ and $m_t$ is shown in Figures 5 and 6, and the two have an obvious negative correlation (when $m_p = 8.75g$, $R^2 = 0.9768$; when $m_p = 7.68g$, $R^2 = 0.9414$).
5. Experimental conclusion and summary

It can be found through experiments that the critical depth of the float is mainly related to the height of the water in the container, the mass of the float and the volume of air in the float. When the height of the water in the container is less than the critical depth, the float will return to its original position after decompression, even if the float touches the easy bottom. The critical depth of floats and sinks has a linear relationship with its mass, and the relationship is $h_c = \frac{\rho_0 V_0}{(m_p + m_z)g} - \frac{\rho_0}{\rho g}$, which has also been verified during the experiment. The air volume in the float will also affect the irreversible phenomenon of the float. Since the condition of $\rho V_0 > m_p + m_z > 0$ exists, if $\rho V_0 \leq m_p + m_z$, $h_c$ will no longer exist, and the irreversible phenomenon of floats and sinks will not occur.

Through experiments and theoretical analysis, we have a further understanding of the irreversible phenomenon of floats and sinks. Using the conclusion of the critical depth of floats and sinks, practical objects such as vehicles that sail or dive into water, indicator lights floating in the water, floats used for fishing, markers used for water warnings, etc. can be made according to requirements.

6. Shortcomings and prospects

In this study, there are mainly the following shortcomings:

1. Due to the limitation of the experimental conditions, only the open-type float-sink test has been completed this time, and there is no corresponding closed-type test data.

2. In the analysis process, the quality of the float and sink itself and the assumption that the water is in a static state are ignored, and the influence of other conditions is not considered.

In future research, we will consider the influence of other conditions in work, life and other scenarios to make the theoretical derivation more suitable for practical application scenarios.

7. References

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