The time-reversal odd side of a jet

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1. Introduction

With the advent of the Electron Ion Colliders, such as the EicC [1] and the EIC [2], the studies of the femtoscale structure of the nucleon are entering a new era, which will portray the full three-dimensional (3D) image of the nucleon with unprecedented precisions. To effectively decipher the mass [3], spin and flavor information from the experimental data, one requires the global analyses of the unpolarized and polarized parton distribution functions (PDFs) and their 3D counterparts such as the transverse momentum distributions (TMDs). The global analyses will simultaneously utilize all possible hard probes including inclusive cross sections, semi-inclusive hadron productions and jets, from all available programs such as the COMPASS, HERMES and the Relativistic Heavy Ion Collider (RHIC). Each probe has its own advantage and complements to the others. The idea of the jets, which originally emerged as a primary technique for studying the strong interactions, now further demonstrates its power in probing the fundamental properties of the nucleon and the nuclear medium through the measurements at the LHC and RHIC [4–8]. The high luminosity of the future EIC considerably boosts the studies of the jets and the jet substructures for exploring the nucleon spin structures and great progress has been made. The jets have been showed to have the ability to access the unpolarized TMDs as well as the transversely polarized Sivers functions [9–19]. The jet structure studies, like the soft-drop grooming [20], hadron in jets [21,22], the perturbative interference effect [23] as well as newly developed jet recombination and construction schemes [15,24,25], such as the “winner-take-all” (WTA) scheme [24,25], further enrich the content of the jet probes.

However, compared with the conventional semi-inclusive deep inelastic scattering (DIS) process, the jet probe of the nucleon structures faces two fundamental difficulties:

(1) It is hard to achieve the quark flavor separation.
(2) It only couples to very limited nucleon spin structures, due to the unpolarized and time-reversal even nature of a jet.

Recently proposed jet charge resolves the first issue [26], while the second remains challenging and limits the power of the jet probe. Specifically, all the time-reversal odd (T-odd) proton distributions, such as the transversity which is crucial for extracting the least known nucleon tensor charge [27–31], are regarded as unreachable directly by the jets. See Ref. [16] for recent efforts.

In this work, contrary to the conventional wisdom, we notice that the jets can acquire the T-odd contributions due to its non-perturbative ingredients, especially in the O(1 GeV) region where the interesting nuclear Femtography is performed [32]. The basic idea roots in the fact that the final state interactions induce an asymmetry for the partons with different spin orientations S [33–35] that initiate the jets. Similar to the Collins effect [36] in semi-inclusive DIS, the asymmetry can be observed as long as the jet axis is not aligned with the fragmenting par-

https://doi.org/10.1016/j.fjfm.2021.11.039
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ton momentum $P_{\alpha}$. This is the generic case for the WTA jets as sketched in Fig. 1 as well as the groomed jets. The situation also holds for the standard jets due to the soft radiations outside the jet cone [37]. The small imbalance between the jet axis and the parton momentum direction in the $-Q^2 (1 \text{ GeV})$ region could thus probe the asymmetry.

The T-odd jet couples directly to the T-odd proton distributions, therefore immediately opens up many unique opportunities for probing the nucleon intrinsic spin dynamics using jets, which were thought to be impossible or difficult. As concrete examples, we demonstrate the feasibility and advantage of T-odd jets to the proton transversity in transversely polarized DIS by extending the factorization of the conventional unpolarized jet production to the T-odd case. We also show the same T-odd jets can be constrained directly through the azimuthal angle asymmetry in the $e^+ e^-$ annihilation.

2. QCD factorization and the time-reversal odd jet function

To introduce the T-odd jet function, we consider jet production in the collision between an unpolarized electron $e$ and a transversely polarized proton $p$ with spin $s_T$ and momentum $P$, i.e., $P_\perp (P, s_T) \rightarrow \omega J (P_J) + X$, where we assume $P^\perp = \frac{Q}{2}(1, 0, 0, 1) = \frac{Q}{2} e^w$. We construct the jet out of the WTA scheme [24,25] with radius $R$. We emphasize that the use of the T-odd jet is for illustration. Our conclusion holds generically for any production. We are particularly interested in the case that $q_T \ll Q$, especially for $q_T \sim \Lambda_{QCD}$, to probe the intrinsic dynamics of the proton. Here $q_T = q - \frac{q}{2} e^w$, $q$ is the transverse momentum of the virtual photon in the frame that the jet axis defines the conjugate direction $\hat{n} = (1, 0, 0, 1)$. Here $q$ denotes the virtual photon momenta. In the frame that the virtual photon and the proton develop no transverse momenta (the $T - p$ frame), $q_T = -\frac{Q^2}{2e^w}$ up to $1/Q^2$ corrections where $z = n \cdot P_J / Q = P_T / Q$. Here we introduce the notation that for an arbitrary vector $\nu$, $\nu \cdot v \equiv v^\nu$ while $\hat{n} \cdot e^w \equiv e^w$.

We follow Refs. [24,25] to derive the factorization theorem for the cross section using Soft Collinear Effective Theory (SCET) [38–41]. In the small $q_T$ limit, the $n$ and $\hat{n}$ collinear sectors can be factorized and the cross section can be written as [24]:

$$\sigma = \frac{1}{2\pi} \int [d^4p'] \frac{e^{2q}L_{p'}}{Q^2} \int \frac{d^2x_T e^{x^T x_T} \langle P', s_T \bar{\epsilon}_a(x', x_T)|J & 1$
jet algorithm dependence will be included in the perturbative coefficient. As a consequence, the T-odd $J_T^p$ shares the comparable predictive power as $J_T$ in the small $k_T$ region relevant for the nucleon TMD and spin studies.

Besides, the flexibility of choosing the jet recombination schemes and hence the jet axis, allows us to adjust the sensitivity to different non-perturbative contributions. This extra control could provide the opportunity to “film” the QCD non-perturbative dynamics, if one manages to continuously change the axis from one to another.

The T-odd jets open up many new opportunities for probing the proton intrinsic dynamics, which were thought to be unreachable by using jets. Below we highlight some of the possible applications to the nucleon structure studies.

3. Transversely polarized deep-inelastic-scattering

From Eq. 4 to Eq. 5 and compare with the hadron production [42], one can immediately realize that the non-vanishing T-odd component allows the jet to probe the proton T-odd distributions. For example, the nucleon transversity $h_1$ can be accessed through the azimuthal distribution:

$$A(ζ, y, φ_j, P_{j⊥}) = 1 + ε|s_j| \sin(φ_j + φ_p) \frac{F_{UT}}{F_{UU}}$$

where $F_{UT}$ is the conventional unpolarized jet production which can be found in Ref. [24] and the azimuthal dependent part:

$$F_{UT} = \sum_1^2 \int \frac{d^3b}{4π^2} \frac{\delta(1 - b)}{b_{1J}} \frac{P_{j⊥}}{P_{j⊥}} \frac{h_1(ζ, b) δ_{1J} J_1^T(b)}$$

where $h_1(ζ, b)$ and $J_1^T(b)$ are the Fourier transformation of the proton transversity $h_1(ζ, p_T)$ and the T-odd jet function $J_1^T(k_T)$, respectively.

The azimuthal distribution is measured in the $γ - p$ frame, where the kinematics involved are illustrated in Fig. 2 with:

$$ε = \frac{1 - y - \frac{1}{2} \frac{y^2}{1 - y + \frac{1}{2} y^2 + \frac{1}{2} y^2}}{1 - \frac{y}{2} \frac{y^2}{1 - y + \frac{1}{2} y^2 + \frac{1}{2} y^2}}$$

and $y = \frac{2E_1}{M}$ with $M$ the proton mass.

Now we see clearly that the transversity can be measured via the jet probe which was thought to be impossible. Compared with measuring the final state hadron Collins effect, we see from Eq. 7 that the T-odd WTA jet probe of the incoming proton transversity reduces the non-perturbative degrees of freedom, which allows for a cleaner extraction of the transversity at the future EIC and EicC.

To access this cross section, we construct jet using the WTA scheme and perform the measurements in the $γ - p$ frame. We restrict us to the region where $P_{j⊥} \sim Λ_{QCD} \ll Q$.

One can avoid the possible cancellation between different quark flavors to further enhance the asymmetry by measuring the jet charge $Q_J$ [26]. The brief idea is to weigh the relative contributions of different quark flavors to the jet by selecting the appropriate jet charge region. For instance, it is found that a fraction of $r_{ud}^p = 52\%$ of the $u(\bar{d})$ quark initiating jets will contribute to $Q_J > 0.25$ while only $r_{d(\bar{d})}^p = 15\%$ of the $d(\bar{d})$-quark jets contribute to this jet charge region [26]. For $Q_J < -0.25$, the fractions change to $r_{ud}^p = 15\%$ and $r_{d(\bar{d})}^p = 52\%$. Therefore the $d(\bar{d})$ quark contribution will be dramatically suppressed when $Q_J > 0.25$ and enhanced in $Q_J < -0.25$.

A simulation of the azimuthal distribution generated by the transversity and the T-odd jet at the EIC is shown in Fig. 3 using PYTHIA8 [45] with the STRINGSPINNER plugin [46]. The events are generated in the lab frame while the asymmetry is measured in the $γ - p$ frame. The simulation manifests the testable effects of the T-odd jets. Notice that WTA scheme is applied in the simulation. We also tested the E-scheme by performing naive summing over hadrons to define the jet axis, which leads to negligible azimuthal asymmetry as can be understood by the sum rule of Collins effect [47]. Details on this simulation will be given in future publications.

4. Anisotropy in $e^+e^−$ annihilation

The same T-odd jet function will also arise in the back-to-back di-jet production in $e^+e^−$ annihilation, where the T-odd jets generate the angular anisotropy. Such asymmetry allows a direct and clean study of the T-odd jet via analyses of the BaBar or BELLE data.

Here again, we stick to the WTA jets with $q_T < \sqrt{2}R$, where $\sqrt{2}R$ is the center of mass energy and $q_T = -P_{j⊥}$. The kinematics that we considered are depicted in Fig. 4.

The azimuthal asymmetry takes the general form [48]:

$$R_{1T} = \cos(\phi_1 - \phi_2) \frac{F_T(q_T)}{F_U(q_T)}$$

where $F_T$ is the contribution from conventional jet, while $F_U$ from the T-odd component, respectively. The $F_U$ is known and found to be [24]:

$$F_U^{+} = \sum_{p} \sum_{q} \frac{d^3p}{2\pi^2} \frac{d^3q}{2\pi^2} \epsilon(p) \epsilon(q) g_{pq}$$

$$F_U^{−} = \sum_{p} \sum_{q} \frac{d^3p}{2\pi^2} \frac{d^3q}{2\pi^2} \epsilon(p) \epsilon(q) g_{pq}$$

where $g_{pq}$ is the planar Dirac distribution.

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where $g_{pq}$ is the planar Dirac distribution.
\[ F_T = q_T \sum_q \int \frac{d^4b}{(2\pi)^4} J_0(q_T b) J^q(b) \tilde{F}^q(b) \]  

where \( J_0(q_T b) \) is the Bessel function. The newly identified \( F_T \) is given by:

\[ F_T = q_T \sum_q \int \frac{d^4b}{(2\pi)^4} \left( \frac{2}{q_T} \frac{q_T^2}{q_T^2 + k^2} \right) J_0(q_T b) J^q(b) \tilde{F}^q(b) \]

Here in Eq. (10), we have:

\[ J^q(b) \tilde{F}^q(b) = e^{-\sum_{\text{pert}} S_{\text{pert}}^q} \left( 1 + \mathcal{O}(\alpha_s) \right) \]

where the \( \mathcal{O}(\alpha_s) \) corrections can be extracted from Ref. [24]. The \( S_{\text{pert}} \) is the perturbative Sudakov factor:

\[ S_{\text{pert}} = \int \frac{d^2 \mu^2}{\mu^2} \left( A \log \frac{\mu^2}{\mu^2} + B \right) \]

where the anomalous dimension \( A \) and \( B \) can be found in Ref. [49]. The scale \( \mu_0 = c_0/b_s \), where \( b_s = b/\sqrt{1 + b^2/b_{\text{max}}} \) with \( c_0 \approx 1.22 \) and \( b_{\text{max}} = 1.5 \text{ GeV}^{-1} \). The \( b_s \)-prescription introduces the non-perturbative Sudakov factor \( S_{\text{NP}}^q \), which can be parameterized as \( S_{\text{NP}}^q = g_s \ln(b_s/b_H) \ln(\sqrt{7} Q_0) + 2g_s b_s^2 \) with \( g_s = 0.84 \), \( \xi_0 = 0.042 \text{ GeV}^{-1} \) and \( Q_0 = 2.4 \text{ GeV}^2 \).

The T-odd jet function can be determined by the operator product expansion following the similar strategy in Ref. [44], which we left for future work. The similarity between the T-odd jet function and the Collins function [36,48] suggests the form of its contribution to be:

\[ \partial \phi \partial \phi J^q b^q J^{\mu} J^{\nu} = e^{-S_{\text{pert}} - S_{\text{NP}}^q} \frac{b^q b^q}{4} N^q_{\mu\nu}(b) \]

where \( S_{\text{NP}}^q \) is the non-perturbative Sudakov factor while \( N^q_{\mu\nu}(b) \) is the non-perturbative normalization which determines the sign and the magnitude of the T-odd jet function. Both \( S_{\text{NP}}^q \) and \( N^q_{\mu\nu}(b) \) could be constrained by experimental analyses. Here for simplicity we parameterize \( S_{\text{NP}}^q = g_s \ln(b_s/b_H) \ln(\sqrt{7} Q_0) + 2g_s b_s^2 \) with \( g_s = 0.0236 \text{ GeV}^{-1} \) and the rest are the same as \( S_{\text{NP}}^q \).

In this example, to further enhance the sensitivity, we again demand the jet charge \( Q_j > 0.25 \) for one of the jets while \( Q_j < -0.25 \) for the other. The only change in the factorization is to make the replacement \( J^q(b) \tilde{F}^q(b) \rightarrow J^q(b) \tilde{F}^q(b) \phi_c^q \) in Eq. (10) with \( \phi_c \). In Eq. (12), we parameterize \( N^q_{\mu\nu} = -0.3 r_s^q \) and \( N^q_{\nu\mu} = 0.1 r_s^q \) for \( Q_j > 0.25 \) and \( N^q_{\mu\nu} = 0.1 r_s^q \) and \( N^q_{\nu\mu} = -0.3 r_s^q \) for \( Q_j < -0.25 \). The values of the \( r_s \)s are given previously thanks to the universality.

In Fig. 5, we present a prediction for the azimuthal asymmetry \( R = 2 \int d \cos \theta \frac{2 \cos(2\phi_f)}{R_1^{1/2}} \) with \( \sqrt{s} = \sqrt{110} \text{ GeV} \).

We can see a non-vanishing azimuthal asymmetry induced by the T-odd jet. The actual magnitude and shape of this asymmetry should be determined by upcoming experimental data analyses. Similar azimuthal anisotropy can also be shown to exist in the dijet production in both pp and heavy-ion collisions, whose studies will be investigated in the future.

5. Conclusion

In this work, we re-examine the factorization theorem of the jet production for nucleon spin studies. Unlike the jet physics at the LHC, for nucleon studies, the focused kinematic region is much lower \( \sim \mathcal{O}(1 \text{ GeV}) \), and thus the jets involved will be inevitably sensitive to the non-perturbative effects. This allows the existence of a T-odd component. We found that the T-odd contribution, which was thought to vanish, gives rise to novel jet phenomena and endows the jets the ability to probe many nucleon spin structures which were thought to be inaccessible by jets. As direct demonstrations, we show the proton transversity can couple to the T-odd jet function in the \( q_T \) distribution in transversely polarized DIS. We also show the same T-odd jet function can be directly measured via the induced anisotropy in the \( e^+e^- \) annihilation at Belle and BaBar.

We emphasize that the T-odd feature of jets is not limited to the WTA jets discussed currently, but holds for generic jet production and could exist in many jet shape observables in the non-perturbative regime. Furthermore, different jet schemes and/or event shapes weighs differently the non-perturbative soft and collinear contributions, therefore will provide more comprehensive insights into the non-perturbative dynamics. We thus expect the T-odd jets identified in this work to maximize the outreach of the jet probes and enrich the inputs to the global analyses and complement the conventional hadron probes of the spin structures at current and future facilities.

Declaration of competing interest

The authors declare that they have no conflicts of interest in this work.

Acknowledgments

We thank Zhongbo Kang, Jian-Wei Qiu and Yuxiang Zhao for comments on the manuscript. This work is supported by the National Natural Science Foundation of China under Grants No. 12022512, 12035007 and 12175016 and by the Guangdong Major Project of Basic and Applied Basic Research No. 2020B0301030008.

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