Phenomenological Gaussian screening in the nonextensive statistics approach to fully developed turbulence

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(Dated: 30 December 2002)

We propose a simple phenomenological modification, a Gaussian screening, of the probability distribution function which was obtained by Beck to explain experimentally measured distribution from fully developed fluid turbulence, within the framework of nonextensive statistical mechanics. The modified distribution provides a good fit to new experimental results on the acceleration of fluid particles reported by Crawford, Mordant, and Bodenschatz. Theoretical foundations of such a modification requires a separate study.

Recently, Beck have studied application of Tsallis formalism to turbulent flows and achieved a good agreement with experimental measurements. Remarkably, no fitting parameters have been used by Beck to reproduce histogram of the acceleration $a$ of a test particle advected by the turbulent flow, while the stretch exponential fit requires three free parameters for this purpose,

$$P(a) = C \exp \left[ a^2 / (1 + \frac{a^\beta}{\sigma} \gamma^2) \right], \quad (1)$$

where $\beta = 0.513$, $\gamma = 1.600$, $\sigma = 0.563$ and $C$ is a normalization constant.

The main idea underlying generalized statistical mechanics approach to turbulence is to introduce fluctuation of temperature or fluctuation of energy dissipation described by gamma distribution.

Crawford, Mordant, and Bodenschatz reported new experimental results, which are slightly different from that reported in earlier experiment, and pointed out that the Beck’s distribution do not correctly capture the tails of the experimental distribution function. Here, $q = 3/2$ (Tsallis entropic index), $\beta = 4$, and $C = 2/\pi$ is a normalization constant, all the parameter values are due the theory. An essential discrepancy is clearly seen from the contribution $a^4 P(a)$ to the fourth moment.

To achieve a better fit, we suggest the modified distribution,

$$P(a) = \frac{C \exp[-a^2/a_0^2]}{(1 + \frac{\beta(q-1)a^2}{\sigma^2})^{1/(q-1)}}, \quad (2)$$

which is obtained by a “Gaussian screening” of the Beck’s distribution (2). Again, $C$ is a normalization constant, $\beta = 4$, and $q = 3/2$, while $a_0$ is a our free parameter, which we use for a fitting.

Plots of the distribution (3) and resulting $a^4 P(a)$, for certain value of $a_0$, are shown in Fig. 1. One can observe a better fit of the experimental $P(a)$ (top panel, solid line) and much better agreement of the plot $a^4 P(a)$ (bottom panel, solid line) to the new experimental results presented by Crawford, Mordant, and Bodenschatz, as compared to that derived from Eq. (2) (dashed line).

We conclude by a few comments.

Despite we use only one parameter, $a_0$, to fit the experimental data with a good accuracy (values of the other parameters are due to Eq. (2)), it is required to derive
Eq. (3) with the help of the same approach as used by Beck in Ref. [1], to provide a self-consistent description. This would allow one to unravel physical picture lying behind the proposed phenomenological Gaussian screening in Eq. (3).

We note that the experimentally observed large $a$ asymptotics of $a^4 P(a)$ which is evidently not reproduced by Beck’s result may require some modification of the theoretical set up.

To this end, one may be interested in finding a more appropriate statistical distribution of the inverse temperature parameter, instead of gamma distribution, or in modifying of the nonlinear Langevin equation for velocity, used by Beck.

However, it should be noted that while the gamma distribution is justified from various aspects, see, e.g., Refs. [1, 7, 8], as it provides Tsallis distribution with associated Tsallis entropy, and is known to be in correspondence with a nonlinear Langevin equation for fluctuating temperature [2] within the framework of the Landau-Lifschitz theory of fluctuations, the use of some different distributions [8, 10], with large variance of the inverse temperature fluctuations, may send one out of the established anzatz.

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