Ductile barrier deformation mechanism

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Abstract. Impact loading of a barrier from ductile material is considered. It is postulated that deformation process can be modeled two independent interfaced tasks: deformation of a barrier as plate of final thickness from elastic and viscous-plastic material the hard puncher and evaluation of energy loss of the puncher on material deformation under it and overcoming force of front resistance. It is shown that on the one hand, losses of energy of the puncher on barrier material deformation by longitudinal waves of tension both on formation, and on a stopper cut, and with another are most significant, process of promotion of the created stopper is longest and determines generally time of breaking through the barrier.

1. Introduction
Impact interaction of rods with different types of barriers and the description of destruction of the last is one of difficult and relevant tasks of deformable solid mechanics. There is a set of mathematical models which describe deformation and destruction of different types of barriers. As a rule, their essential shortcoming is the complexity of the implementation demanding considerable expenses of time, existence of initial experimental data about process of interaction of the studied puncher with this barrier and acceptances of various hypotheses and considerable assumptions. In the paper [1] destruction of a fragile barrier which material works within elastic deformations is considered and collapses practically without residual deformations. It is not possible to extend this model to a ductile barrier as its destruction is followed by considerable plastic deformations.
In this paper the calculation model of deformation parameters allowing to build the movement law of the puncher in a ductile barrier at almost acceptable engineering level is implemented. Such factors as longitudinal shear of material under the puncher and possible formation of a stopper, radial pressure in a barrier and friction on a side contact surface of the puncher, longitudinal waves of tension in barrier materials and the puncher, and a possibility of their destructions with a splitting off are considered. When modeling the principle of superposition of the listed factors is used [2].

2. Problem definition
It is postulated that deformation process can be modelled two independent interfaced tasks, and the speed losses of the puncher defined in them are summarized.
The first task describes deformation of a barrier as plates of final thickness from elastic and viscous-plastic material by the rigid puncher. On the basis of the solution of wave tasks the possibility of formation of a stopper and its promotion for a back surface of a barrier is estimated.
The second task estimates speed losses of the puncher on material deformation under it and overcoming force of front resistance without taking note of the processes proceeding on a side cylindrical surface of radius $R_0$ of material under the puncher. It is modelled by inelastic impact of two rods, one of which is a puncher and has length $L$, the second describes behavior of material of a real target under it and has length $H$. 
3. Theory

At the solution of the first task, it is considered that the puncher and a part of material of a plate under it move as a uniform body. In a barrier out of moving area radial longitudinal shear waves and radial waves of pressure extend. The shear and radial stresses arising at the same time has axial symmetry, are constant on barrier thickness and are the only components of a stress tensor which cannot be neglected (Fig. 1).

![Figure 1. Modelling of the first task describes deformation of a barrier in the form of plates of final thickness from elastic and viscoplastic material the rigid puncher](image)

Radial waves of longitudinal shear in the elastic and viscous-plastic plate with a hole are generated on a hole surface with radius $R_0$ by instantly enclosed impulses of stresses of longitudinal shear. The decision is passed in the cylindrical system of coordinates $R\theta Z$ which axis $Z$ coincides with a hole axis. Then the stress-strain state of material plate is defined by the components of stress tensor $\sigma_{RZ}$ and deformation tensor $\varepsilon_{RZ}$, which are not depending on coordinates $\theta$ and $Z$, and the being functions only of the coordinate $R$ and time $t$. And other components of stress tensor and deformation tensor are equal to zero. These assumptions reduce deformation process to distribution in a plate of radial waves of longitudinal shear. The system of the equations describing a task includes, as well as in the previous case, the equation of the movement, a condition of continuity and the constitutive equation [3]:

$$
\begin{align*}
\rho \frac{\partial V(R,t)}{\partial t} &= \frac{\partial \sigma_{RZ}(R,t)}{\partial R} + \frac{\sigma_{RZ}(R,t)}{R} \\
\frac{\partial \varepsilon_{RZ}(R,t)}{\partial t} &= \frac{\partial V(R,t)}{\partial R} \\
\frac{\partial \varepsilon_{RZ}(R,t)}{\partial R} &= \frac{1}{G} \frac{\partial \sigma_{RZ}(R,t)}{\partial t} + \Phi(\sigma_{RZ}, \varepsilon_{RZ})
\end{align*}
$$

(1)

where $G$ is the shear module; $\Phi(\sigma_{RZ}, \varepsilon_{RZ})$ is the function approximating material properties [4]. Initial conditions correspond to not stress and not deformed condition of material plate:

$$
\sigma_{RZ}(R, 0) = \varepsilon_{RZ}(R, 0) = V(R, 0) = 0.
$$

For the solution of a system (1) the method of characteristics was used. The solution of a task on longitudinal shear waves in a plate with a hole is provided in work [5]. As boundary conditions the movement equation of the «the puncher – barrier material under the puncher» systems which the movement resistance is formed by stresses of longitudinal shear is accepted $\sigma_{RZ}(R_0, t)$:

$$
(M + m) \frac{dV_1(R_0,t)}{dt} = -2\pi R_0 H \sigma_{RZ}(R_0, t)
$$

(2)

where $M$ and $m$ are the mass of the puncher and material barrier under it, respectively; $V_1(R_0, t)$ is longitudinal speed on a boundary cylindrical surface; $R_0$ is radius of midlength section of the puncher; $H$ is a barrier thickness.

If tension $\tau(R_0, t) = \sigma_{RZ}(R_0, t)$ exceeds the dynamic strength of a barrier material at shear in the timepoint $t = t^*$ equal to time of a double run of a longitudinal compression wave in barrier material,
then the stopper cut is possible. The mechanism of formation of a stopper is in detail considered in paper [6]. After a cut of «stopper» the kinetic energy of the puncher will be spent only for overcoming friction forces on the side surface of contact arising because of radial expansion of stopper material. In this case the equation (2) takes a form:

\[
(M + m) \frac{dV_1(R_0, t)}{dt} = -2\pi R_0 \tau_{fr}(R_0, t)(H - \int_0^t V_1(t)dt)
\]

(3)

where \(\tau_{fr}(R_0, t)\) is intensity of friction forces on a side surface of a stopper:

\[
\tau_{fr}(R_0, t) = f(t)\sigma R(R_0, t).
\]

where \(f(t)\) is friction coefficient on a side contact surface which depends on the relative speed of the friction surfaces; \(\sigma R(R_0, t)\) is radial pressure on a boundary surface in timepoint \(t > t^*\). It is defined from the solution of a wave task on radial pressure waves.

On a hole surface \(R = R_0\) in timepoint \(t = 0\) radial pressure \(P(t)\) is suddenly put, where \(P(0) \neq 0\). It is considered that the stress-strain state of the plate material is defined by components of stress tensor \(\sigma_{RR}\) and \(\sigma_{\nu\nu}\) (further are designated respectively \(\sigma_R\) and \(\sigma_{\nu}\)) and deformation tensor \(\varepsilon_{RR}, \varepsilon_{\nu\nu}\) and \(\varepsilon_{ZZ}\) (further \(\varepsilon_R, \varepsilon_{\nu}\) and \(\varepsilon_Z\) respectively), the functions which are not depending on coordinates \(\theta\) and \(Z\) and being only of the coordinate \(R\) and time \(t\). And other components of stress tensor and deformation tensor are equal to zero [7].

The movement equation of a ring element of plate mass is written so [8]:

\[
\rho \frac{\partial V_R}{\partial t} = \frac{\partial \sigma_R}{\partial R} + \frac{\sigma_R - \sigma_{\nu}}{R}
\]

(4)

where \(\rho\) is the mass density of plate material; \(V_R\) is radial speed of material particles.

Deformations \(\varepsilon_R\) and \(\varepsilon_{\nu}\) and speed \(V_R\) are connected among themselves by the known conditions of compatibility:

\[
\frac{\partial \varepsilon_R}{\partial t} = \frac{\partial V_R}{\partial R}
\]

(5)

As the constitutive equations for pipe material the generalized Hooke's law is used.

As boundary conditions is accepted:

\[
\varepsilon_Z(R_0, t) = \varepsilon_0,
\]

where \(\varepsilon_0\) is initial value of axial deformation: \(\varepsilon_0 = V_0/\alpha\); \(\alpha\) is speed of longitudinal elastic waves in barrier material.

At this modeling of the perforation the distribution speeds of the puncher, which received by means of the equations (2) and (3), reflects losses of speed of the puncher in real process, which are spent for overcoming shear stresses in a barrier and formation of a stopper and also friction forces on a side surface of the created stopper at its promotion for a back surface.

In the solution of the second task (Fig. 2) it is considered that in both rods’ longitudinal waves of stretching-compression stress extend [9], and required change of a contact surface speed \(V_2(0, t)\) will be defined from the solution of a task on longitudinal waves distribution of normal compression stress. The task solution on propagation of longitudinal waves of normal compression stress is under construction within a hypothesis of flat sections and the stress-strain state of the rod material completely is described by components of stress tensor \(\sigma_{ZZ}\) (further \(\sigma\)) and deformation tensor \(\varepsilon_{ZZ}\) (further \(\varepsilon\)). The system of the equations describing a task includes the movement equation, a condition of continuity and the constitutive equation [10]:
\[
\begin{aligned}
\frac{\partial \sigma(z,t)}{\partial z} & = \rho \frac{\partial V(z,t)}{\partial t} \\
\frac{\partial V(z,t)}{\partial z} & = \frac{\partial \varepsilon(z,t)}{\partial t} \\
E \frac{\partial \varepsilon(z,t)}{\partial z} & = \frac{\partial \sigma(z,t)}{\partial t} + E \Phi(\sigma, \varepsilon)
\end{aligned}
\]

where $\rho$ is rod material density; $V(z,t)$ is axial speed of rod particles; $\Phi(\sigma, \varepsilon)$ is the function approximating material properties of a rod [4].

In initial timepoint material of a rod is not stress, is not deformed and is not mobile:

$$\sigma(Z,0) = \varepsilon(Z,0) = V(Z,0) = 0,$$

As a boundary condition the identity of the stress-strain state on contact border of two inelastic rods is used.

**Figure 2.** Modelling of the second task is considered that in both inelastic rods’ longitudinal waves of stretching–compression stress extend

It should be noted that at the solution of this task change of lengths of the puncher and a stopper (material under the puncher) because of their partial destruction with a splitting off is possible. This problem is in detail discussed in paper [6], and its account does not represent basic complexity.

Now the true speed of the puncher in the course of penetration is defined so:

$$V(t) = V_1(t) - V_2(t),$$

and time a barrier perforation $t_{br}$ is defined from the solution of the equation:

$$B = \int_0^{t_{br}} V(t) dt$$

where $B = H$ - in case of lack of destructions and of a splitting off of a barrier; $B = H - l_{br}$ - at a splitting off of a part of the puncher length $l_{br}$.

4. **Discussion of results**

Results of numerical calculations at the following basic data are given below: $H = 15 \text{ mm}; \ R_0 = 2.35 \text{ mm}; \ L = 100 \text{ mm}$. Barrier material is aluminum; the puncher's material is steel 9260 (USA) [2].
Figure 3. Losses of puncher energy on deformation of a barrier under it and on overcoming force of front resistance:
- Curve 1 - change of puncher speed is carried out of the solution of the second task;
- Curve 2 - the change of puncher speed received from the solution of the first task;
- Curve 3 - the true change of puncher speed in a barrier

Fig. 3 illustrates change of puncher speed ($V_0 = 1000$ m/s). The curve 1 gives an idea of losses of puncher energy on deformation of a barrier under it and on overcoming force of front resistance and is carried out of the solution of the second task. The curve 2 shows the change of puncher speed received from the solution of the first task. True change of puncher speed in a barrier is illustrated by a curve 3. The interface point on it corresponds to the moment of stopper formation. The area under this curve limited at the left to ordinate axis, and on the right $t = t_{br}$ corresponds to thickness of a barrier $H$.

Figure 4. Speed change of the puncher and appropriate perforation times at various impact speeds:
- Curve 1 - impact speed is 1500 m/s, penetration time is 15 $\mu$s;
- Curve 2 - impact speed is 1350 m/s, penetration time is 21 $\mu$s;
- Curve 3 - impact speed is 1000 m/s, penetration time is 30 $\mu$s.

In Fig. 4 dependences of speed change of the puncher and perforation times corresponding to them are shown at various initial speeds of impact. The dotted line in Fig. 4 shows that at this time there was a penetration over the barrier, and the puncher is behind the back side of the barrier (flies further).
The data analysis on Fig. 4 shows, that if speed impact is reduced in 1.5 times, the penetration time increases twice. At the initial speed impact of 1500 m/s, the puncher's speed after penetration decreases to 1000 m/s; at the initial impact speed of 1000 m/s, the puncher's speed after penetration decreases to 450 m/s.

5. Conclusions

The mathematical model is carried out and the method of evaluation of the getting action of the puncher which are based on use of superpositions of solutions of a number of one-dimensional wave tasks is realized, and allow to construct the law of speed change of the puncher in a barrier and to define time a perforation.

The carried-out calculations showed that, on the one hand, losses of energy of the puncher on barrier material deformation by longitudinal stress waves and on formation and a cut of a stopper, and on the other hand are most significant, process of promotion of the created stopper is longest and defines, generally time a perforation.

6. References

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