Filamentary collapse flow in molecular clouds

Raúl Naranjo-Romero⋆, Enrique Vázquez-Semadeni†, Robert M. Loughnane‡

Instituto de Radioastronomía y Astrofísica,
Universidad Nacional Autónoma de México, Apdo. Postal 3-72, Morelia, Michoacán, 58089, México

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

We present idealized numerical simulations of prestellar gravitational collapse of a moderate initial filamentary perturbation with an additional central ellipsoidal enhancement (a core) considering a uniform, and a stratified background, the latter representing flattened clouds. Both simulations maintain the filamentary structure during the collapse, developing a hierarchical accretion flow from the cloud to the plane; from there to the filament, and from the filament to the core. The flow changes direction smoothly at every step of the hierarchy, with no density divergence nor a shock developing at the filament’s axis during the studied prestellar evolution. The flow drives accretion onto the central core and drains material from the filament, slowing down the growth of the latter. As a consequence, the ratio of the central density of the core to the filament density increases in time, diverging at the time of singularity formation in the core. The stratified simulation produces the best match for observed Plummer-like radial column density profiles of filaments, while the uniform simulation does not produce a flat central density profile. This result supports recent suggestions that MCs may be preferentially flattened structures. We examine the possibility that the filamentary flow might approach a quasi-stationary regime in which the radial accretion onto the filament is balanced by the longitudinal accretion onto the core. A simple argument suggests that such a stationary state may be an attractor for the system. Our simulations, do not attain this stationary stage, but appear to be approaching it during the prestellar stage.

Key words: Gravitation – hydrodynamics – ISM: structure – ISM: clouds – ISM: evolution – Stars: formation

1 INTRODUCTION

1.1 Molecular Cloud substructure and star formation

Although observations over many decades have shown that molecular clouds (MCs) are the nurseries of star formation, the process through which MCs evolve to form stars has undergone major transformations in recent years. In particular, the existence of filamentary structure in MCs has been known for several decades (e.g., Bally et al. 1987; Nagahama et al. 1998; Myers 2009). Improved observations, especially with the Herschel Observatory (see, e.g., the review by André et al. 2014, hereafter A14) and numerical simulations (e.g., Heitsch et al. 2009; Smith et al. 2011; Gómez & Vázquez-Semadeni 2014) have shown that there is a number of intermediate stages before the final formation of stars, identified as filaments, hubs and dense cores. In particular, as discussed by A14, the Herschel continuum observations and surveys have revealed that filamentary structures are ubiquitous in MCs, that they contain the majority of the prestellar cores in MCs, and that the formation of filaments appears to precede star formation. This multi-stage collapse process can be understood in terms of a global, hierarchical collapse (GHC), in which small-scale collapses occur within larger-scale ones and are shifted in time (collapses within collapses; Vázquez-Semadeni et al. 2009). Moreover, the larger-scale collapses produce filamentary structures because they occur in a nearly pressureless form, due to the large number of Jeans masses they contain. As a consequence, they collapse fastest along their shortest dimension, in which anisotropies are amplified, thus forming sheets and filaments (e.g., Lin et al. 1965; Gómez & Vázquez-Semadeni 2014). Hence, filaments correspond to the intermediate-scale collapse, between the cloud and the core scales. It is noteworthy that the latter authors argue that the collapse of clouds proceeds from the cloud at large to sheet-like objects, then to filaments, and finally to clumps, so it is possible that the filaments tend to be embedded in sheet-like clouds. A more detailed and comprehensive scenario of the GHC has recently been described by Vázquez-Semadeni et al. (2019) and put in the context of the lifecycle of Giant Molecular Clouds in the Galaxy by Chevance et al. (2020).

Observations from the Herschel Gould belt survey (e.g., Arzoumanian et al. 2011; Palmeirim et al. 2013) have also found that the filaments have radial column density pro-
files falling off as \( r^{-1.5} \) to \( r^{-2.5} \) at large radii, and flattened profiles at small radii, with typical widths \( \sim 0.1 \) pc, although the universality of this width was questioned by Panopoulou et al. (2017), who argued that it is an artifact of sampling a truncated power-law distribution with uncertainties.

Filaments appear to be highly dynamic entities. Rivera-Ingraham et al. (2017) have shown evidence that the filaments evolve from a subcritical (i.e., stable; Inutsuka & Miyama 1992) to a supercritical (i.e., unstable) regime by accretion of material from their environment according to recent observations of nearby (\( d < 500 \) pc) filaments (Rivera-Ingraham et al. 2016). They found that self-gravitating filaments in dense environments (\( A_v \sim 3, N_H^2 \sim 2.9 \times 10^{20} \) cm\(^{-2}\)) can become supercritical on timescales of \( \sim 1 \) Myr, and suggested that filaments evolve in tandem with their environment. Also, Arzoumanian et al. (2013) found that thermally subcritical filaments have transonic velocity dispersions independent of their column density, while thermally supercritical filaments have higher velocity dispersions scaling roughly as the square root of the column density. They suggest that the higher velocity dispersions of supercritical filaments may not directly arise from supersonic interstellar turbulence, but instead may be driven by gravitational contraction/accretion. Finally, a number of molecular-line studies of filaments have suggested that there is a net gas flow along the filaments, perhaps feeding the hubs and clumps as a consequence of the global collapse of the filaments (e.g., Schneider et al. 2010; Peretto et al. 2013; Kirk et al. 2013).

Many existing analytical models (e.g., Ostriker 1964; Inutsuka & Miyama 1992; Fischera & Martin 2012) consider hydrostatic equilibrium, while others (e.g., Heitsch 2013a,b; Hennebelle & André 2013), together with some numerical simulations (e.g., Clarke et al. 2016), do consider accretion. To our knowledge, however, the longitudinal flow along filaments, seen both in observations (Schneider et al. 2010; Peretto et al. 2013; Kirk et al. 2013) and reported in simulations of MC evolution (Gómez & Vázquez-Semadeni 2014) has not been discussed in analytical models or controlled numerical simulations, to investigate the essential characteristics of the flow regime.

In this work, we extend our previous study of the collapse of an idealized spherical core embedded in a uniform, unstable background medium (Naranjo-Romero et al. 2015, hereafter Paper I), by adding a filamentary perturbation that triggers non-spherical collapse motions. We investigate the fundamental underlying flow pattern in a filamentary structure that is part of the collapse flow from the clump scale (a few parsecs) down to the core scale (a few times \( 0.01 \) pc), and discuss whether or not this regime is capable of explaining some of the observed structural and kinematic features of MC filaments. In Sec. 2, we first present the numerical simulations, and in Sec. 3, we present the results concerning the flow pattern, the approximation to a stationary state, the evolution of structural features, and a comparison with observations. Finally, in Sec. 4, we discuss and summarize our main results.

2 THE SIMULATIONS

We use a spectral, fixed mesh numerical code (Léorat et al. 1990; Vázquez-Semadeni et al. 2010) to perform two numerical simulations of hierarchically collapsing density perturbations initially at rest. These simulations consist of a spherical density enhancement with a gaussian radial profile (“the core”) embedded in a cylindrical perturbation, also with a radial density profile (“the filament”), which in turn is immersed in a uniform density background (“the cloud”). One simulation includes a stratification representing the possibility that the clouds may be flattened (e.g., Beaumont & Williams 2010; Veena et al. 2017; Kusume et al. 2019). As in Paper I, we restrict our study to the prestellar stage of the evolution because our code does not include a prescription for the creation of sink particles.

For the density background (i.e., “the cloud”), we have considered two different configurations, corresponding to each of our two simulations. The first has a uniform density background, and therefore has axial symmetry, so it is labeled RunA. The second is stratified in the \( y \) direction, and is thus labeled RunS (see Fig. 1). RunS therefore represents the case of a filament embedded in a sheet-like cloud, which is the outcome of cloud formation by the convergence of oppositely-directed gas streams in the warm, diffuse atomic gas (e.g., Vázquez-Semadeni et al. 2006; Heitsch et al. 2008; Heitsch 2013b; Wareing et al. 2019). For convenience, in this run we will refer to the central plane perpendicular to the direction of stratification as the dense plane. We will discuss the variation of some physical quantities in three directions: one perpendicular to the dense plane, one parallel to it (i.e., on the plane), but perpendicular to the filament, and another one on the plane but running along the filament, to which we will refer as the longitudinal direction. In contrast, in RunA, since the “parallel” and “perpendicular” directions are undistinguishable, we will only refer to the longitudinal and perpendicular directions, with respect to the filament.

The various components (stratification, filament, and core) of the initial density field are set up as successive enhancements over a uniform density background with gaussian profiles in one, two or three directions, as illustrated schematically in Fig. 1. The stratification (in RunS only) is set up as a gaussian enhancement along the \( y \) direction only, peaking at half the length of the box in this direction, with a standard deviation of 1.38 pc and an amplitude of 50\% over the uniform background. The filamentary component is set up as a gaussian enhancement in the \( x \) and \( y \) directions with a standard deviation of 0.69 pc and amplitude of 50\% of the uniform background above its corresponding background (uniform cloud in RunA and the stratified background in RunS), centered at the middle point of the \((x, y)\) plane. Finally, the core was set up as a gaussian enhancement in all three directions, centered at the middle of the numerical box, with a
Figure 1. Illustration of the setup for the density field in the stratified simulation, RunS.

standard deviation of 0.50 pc, and an amplitude of 100% of the uniform background over the filamentary component.

After all components are added up, the whole density field is renormalized in order to recover the mean density of 100 cm$^{-3}$. Figure 2 shows the resulting initial density profiles along the various directions for the two simulations. For practical reference, their measured full widths at half maximum (FWHM) are also shown.

The gas is initially at rest, and no gravity-counteracting forces such as a magnetic field or small-scale turbulence are included, so gravitational contraction starts immediately. This setup represents the premise of the GHC scenario that substructures begin their local gravitational contraction when their masses become larger than the mean Jeans mass in their parent structure as a consequence of the large-scale contraction of the latter, which progressively reduces the global mean Jeans mass (Vázquez-Semadeni et al. 2019).

For the advanced stages of the evolution, to identify the boundary of each substructure, we have chosen to use a density threshold criterion over the density of the corresponding parent structure, given by

\[ \rho_{\text{bd}, \text{fil}} = \alpha_{\text{fil}} \langle \rho \rangle_{\text{box}} \]
\[ \rho_{\text{bd}, \text{core}} = \alpha_{\text{core}} \rho_{\text{ax}, l}, \]

with $\alpha_{\text{fil}} = 1.3$ and $\alpha_{\text{core}} = 9$. The value of $\alpha_{\text{fil}}$ was selected simply as a slight enhancement over the mean density of the numerical simulation. The value of $\alpha_{\text{core}}$ was selected so that the choice of core boundary allows our cores to match observational data (see Sec. 3.3). Note that $\rho_{\text{ax}, l}$ is the density at the axis of the filament but a distance $l$ away from the border of the computational box. The latter will be useful when measuring properties of the filament, away from the core.

Additionally, it is useful to identify two different regions along the filament, depending on whether they contain the core or not (see Fig. 3) for comparison with observed cores and filaments, respectively. The off-core region, roughly halfway between the center of the core and the border of the computational box, safely removes the central core while not being too strongly affected by periodic boundary effects. The other is the core region, it extends from the center of the filament to the boundary of the core.

To investigate the evolution of the filament-core system, we have chosen some selected timesteps in both runs (see Table 1).

### 3 RESULTS

In this section, we first describe the uniform-background, axisymmetric simulation (RunA), and then the stratified-background simulation (RunS). All figures are labeled with the type of background setup, the time in terms of the free-fall time, and the orientation along which we have calculated the plotted quantities for each panel.

#### 3.1 Overall evolution

Since the initial conditions we use are globally Jeans-unstable in both runs, the simulations begin to undergo gravitational contraction as soon as they start evolving. Nevertheless, it is important to note that the contraction never becomes completely radial, but instead maintains the filamentary structure throughout the (prestellar) evolution, developing a filamentary collapse flow directed toward the central core. This is in agreement with the large-scale numerical simulations of GMC formation evolution of Gómez & Vázquez-Semadeni (2014), in which the global and hierarchical gravitational contraction of the cloud generates filamentary structures onto which the rest of the cloud accretes.

Figure 4 shows cross sections at various times of the volume density field of both simulations on the planes passing through the center of the computational box (see labels), with the normalized velocity field overlaid. It is noteworthy that, due to the presence of the stratification, RunS produces a ribbon-like filament and, also the core is thinner in the direction perpendicular to the stratification. The ranges in the color scales correspond to the minimum and maximum values of the number density (in cm$^{-3}$) for the overall evolution on

| Run | Snapshot | Time (t$_{\text{ff}}$) | Time (Myr) |
|-----|----------|----------------------|------------|
| RunA | 1 | 0.03 | 0.11 |
| 8 | 0.27 | 0.89 |
| 16 | 0.53 | 1.77 |
| 24 | 0.79 | 2.65 |
| 32 | 1.05 | 3.53 |
| 33 | 1.09 | 3.64 |
| RunS | 1 | 0.03 | 0.11 |
| 8 | 0.26 | 0.88 |
| 9 | 0.30 | 1.00 |
| 16 | 0.53 | 1.77 |
| 24 | 0.79 | 2.65 |
| 25 | 0.82 | 2.76 |
| 31 | 1.02 | 3.42 |
| 32 | 1.05 | 3.53 |
Figure 2. Density profiles at the initial conditions along the three main directions, passing through the center of the computational box for the axisymmetric simulation (RunA, left panel) and the stratified one (RunS, right panel). The red lines represent the direction perpendicular to the dense plane; the blue lines, represent the direction parallel to (on) the plane (perpendicular to the filament), and the green lines, the longitudinal direction (along the filament). The numbers on the right-hand side of the panels correspond to the FWHM of the density peak for each of the profiles. Note that, in RunA, the parallel and perpendicular profiles overlap due to the axial symmetry.

Figure 3. Illustration of the selection of the core and non-core filament sections.

each run. The color bar also indicates the values of the density at the boundaries of the filament and the core (labeled $f$ and $c$), according to eqs. (1) and (2), with $\alpha_{fil} = 1.3$ and $\alpha_{core} = 9$. The horizontal lines define a slice across the filament at a fixed distance $L_{J,\text{init}}/2 \approx 1.12$ pc away from the boundary at the initial conditions, where we have computed various physical quantities (cf. Sec. 3.2).

In RunA (left panels), the position of the filament’s boundary, as defined by eq. (1), does not vary significantly throughout the evolution (except in the on-core region), growing from $\sim 0.73$ pc at the initial conditions to $\sim 1.03$ pc. A similar situation occurs for RunS in the direction perpendicular to the plane. On the other hand, in this run, in the parallel direction (on the central plane), the filament’s boundary steadily moves outwards until it reaches the box boundary. Concerning the core boundary, it is seen that in all cases it changes dramatically in shape, evolving from a nearly spherical shape to a highly elongated one.

The density field is almost uniform along the filament and away from the core, and in the background region away from the filament. Also, as suggested in the less idealized simulation of Gómez & Vázquez-Semadeni (2014), the velocity field smoothly changes direction as it approaches the filament, being mostly perpendicular to the filament at large distances from it, but becoming longitudinal in the filament’s central axis, pointing towards the core. It is important to note that no shocks develop during the prestellar evolution. In addition, the velocity field direction remains quite constant throughout the evolution, suggesting an approximately stationary flow. We return to this point below in Sec. 3.2.

Figure 5 shows the density profiles for the selected timesteps in both simulations, with RunA shown on the two top panels and RunS shown in the three bottom panels, respectively. The various lines shown correspond to the selected timesteps, listed in Table 1. The longitudinal slices (right top and bottom panels) confirm that the density in the filament, away from the core, increases steadily in time through accretion from the cloud, although in RunA the increase rate appears to decrease towards the final stages of the prestellar evolution, suggesting an approach to stationarity.

Also, note that for RunS, the density of the dense plane increases, even far from the filament (bottom left panel, far from the core), while the density of the box away from the plane decreases (middle bottom panel, far from the core). Finally, the density in the filament also increases (bottom right panel, far from the core). This indicates that there is accretion from the box onto the plane, from the plane onto the filament, and from the filament onto the core, thus constituting an extremely anisotropic and hierarchical accretion flow.

Figure 6 shows, in its top panels, the evolution of the filament’s mass. The bottom panels show the evolution of the linear mass density (often misleadingly referred to as the “line mass” in the literature) for the filament+core system (red
lines) and the filament alone (blue lines). The green lines in the bottom panels show the evolution using values for the core and filament density thresholds from the initial conditions.

Interestingly, the filament evolution seems to transition from a regime of increasing rates of mass and linear density growth to a regime of decreasing growth rates, again suggesting an approximation to a stationary regime. This seems to be more clear in the stratified case. However, full stationarity (i.e., constant mass) is clearly not fully reached during the prestellar phase investigated in this work). Also, note that this behaviour is not observed when we use the fixed boundaries from the initial conditions (green lines), and that in this case both runs behave remarkably similarly.

Figure 7 shows the radial profiles for the number density, column density, the radial and longitudinal velocities for RunA (left column) and for RunS, both in the direction parallel to the dense plane (middle column) and perpendicular to it (right column). The solid lines show the profiles at the center of the box (i.e., at the position of the core), while the dashed lines show the profiles midway between the core and the boundary; i.e., on the off-core region of the filament. In this figure, we use a logarithmic radial axis to emphasize the internal structure of both the core and the filament.

From the solid lines, we can see that the collapse in the on-core position proceeds from the outside-in (that is, with the velocity peak removed roughly one Jeans length from the center, Whitworth & Summers 1985; Keto & Caselli 2010), in a similar way as in the spherical case described in Paper I and other works (e.g., Whitworth & Summers 1985; Gómez et al. 2007; Gong & Ostriker 2009). Early in the evolution, at the on-core position (solid lines), the velocities are largest at large
Figure 5. Evolution of the density profile for RunA perpendicular to the filament (top left panel) and along the filament (top right panel), and for RunS, perpendicular to the filament on the dense plane (bottom right panel), perpendicular to the filament and to the dense plane (middle bottom panel) and along the filament (bottom right panel). The various colored lines in all panels correspond to the snapshots listed in Table 1.

Figure 6. Evolution of the mass $M$ (top panels) and linear density (mass per unit length, $M_{\text{line}}$; bottom panels) of the filaments, either identified according to Eqs. (1)-(2) (red and blue lines) or at the fixed boundaries from the initial conditions (green lines in the bottom panels). RunA is shown on the left panels, and RunS is shown on the right panels.
radial distances away from the core’s center, while the inner parts develop a velocity profile roughly linear with radius. At timestep 9, \( t \sim 0.3 \tau_\text{ff} \sim 1.0 \text{ Myr} \), a transonic point appears at \( \sim 0.5 \text{ pc} \) away from the center, that then splits into two points that move in opposite directions, i.e., one outwards and one inwards. At later times, these transonic points enclose a region of almost uniform supersonic inward velocity. The density profile resembles that of the Bonnor-Ebert sphere, being flat in the inner region where the velocity is uniform, while approaching an \( r^{-2} \) profile in the exterior parts.

Similarly, Figure 8 shows the radial profiles along the longitudinal direction along the filamentary structure in the last timestep of each simulation at the on-core position. The behaviour is quite similar to that of the core on top of the uniform density background described in Paper I, with the main difference that the flat inner part of the density and column density profiles for the core are more extended.

Figure 9 shows the radial column density profile of the filament and from Fig. 9 we plot the histogram of the density to continue growing. But then, the increase in the longitudinal direction along the filamentary structure in the last timestep of each simulation at the on-core position. The behaviour is quite similar to that of the core on top of the uniform density background described in Paper I, with the main difference that the flat inner part of the density and column density profiles for the core are more extended.

Table 2. Fitted parameters for the column density profile for RunA.

| Snapshot | \( R_{\text{flat}} \) (pc) | \( p \) | \( A_p \) |
|----------|-----------------|-----|-----|
| 0        | 0.81            | 1.20| 4.03|
| 16       | 0.61            | 1.23| 3.65|
| 33       | 0.18            | 1.32| 0.06|

at the end of the simulations. The parameters of the fitting are the radius of the central flat part of the filament’s radial profile, \( R_{\text{flat}} \), its power-law exponent \( p \), and \( A_p \) (see also Tables 2-3), which is a finite constant factor for \( p > 1 \). For comparison, it also shows the profile for an infinite, hydrostatic isothermal cylinder, for which \( p = 4 \) (Ostriker 1964)\(^2\), and the derived profile for observed filaments, for which \( p = 2 \) (Arzoumanian et al. 2011). Also shown is the profile of an observed filament from Arzoumanian et al. 2011, with the upper and lower errors denoted by the dotted lines. We can see that the fitted values of the slope (\( p = 1.33 \) for RunA and, \( p = 1.7 \) and \( p = 1.25 \) for RunS, in the perpendicular and parallel directions, respectively), are in general less than the observed slope. Although it is possible that the final slope is not attained until after the formation of a singularity (i.e., a protostar), which we do not consider here.

Figure 10 shows the evolution of \( R_{\text{flat}} \) obtained by fitting the column density profiles (see Figure 9) for both runs. As we can see from both panels, even though at early times the fitted values of \( R_{\text{flat}} \) are quite smaller than the typical values \( \sim 0.1 \text{ pc} \) reported from Herschel observations, at later times, this typical size scale is approached.

It is noteworthy that the evolution of \( R_{\text{flat}} \) in the perpendicular direction in RunS is very similar to the evolution in RunA. In RunA, \( R_{\text{flat}} \) decreases by a factor \( \sim 4.5 \) between the initial and final states, while for RunS it varies by a factor of \( \sim 2.9 \) and \( \sim 9.9 \) in the parallel and perpendicular directions, respectively. We speculate that, after singularity formation, \( R_{\text{flat}} \) may become approximately constant as a consequence of a possible approach to stationarity of the system (as long as the gas supply from the cloud remains, as we discuss in Sec. 3.2). This may explain the apparent observed “universality” of the filament widths (e.g., Arzoumanian et al. 2011; Palmeirim et al. 2013; André et al. 2014).

3.2 The approach to a stationary regime and mass flux in the filament

As discussed in Sec. 3.1 (cf. Fig. 4), the velocity field in the simulations tends to remain constant in space, first in direction and then in magnitude as time advances, suggesting an approach to a stationary regime. To further search for evidence of this, in Figure 11 we plot the histogram of the angles of the velocity vectors with respect to the horizontal axis in each of the columns of Fig. 4, at both the first snapshot after the start of the simulation (\( t = 0.03 \tau_\text{ff} \)) and at the final time of the evolution. In both runs, the frequency distribution of these angles is seen to remain almost constant, with its peak shifting only few degrees throughout the evolution\(^3\), reinforcing the view that the velocity field remains approximately stationary.

In fact, an approach to stationarity is expected. As can be seen from the bottom panels of Fig. 6 and from Fig. 7, both the filament’s linear density \( M_{\text{line}} \) and longitudinal velocity \( v_z \) increase monotonically over the entire prestellar evolutionary stage we simulate. This implies that the longitudinal mass accretion rate along the filament, \( \dot{M} = M_{\text{line}} v_z \), also increases.

Now, it appears safe to assume that the radial accretion rate from the cloud onto the filament, which feeds the filament, varies on longer timescales than the variation of the longitudinal accretion rate from the filament onto the core (which drains the filament), because the former evolves on the cloud free-fall time, which is longer because the cloud is at lower density. The increase in the filament’s linear mass density suggests that the radial accretion is initially larger than the longitudinal one, causing the filament’s linear density to continue growing. But then, the increase in the lon-

---

\(^2\) For an infinite isothermal filament in hydrostatic equilibrium, \( A_p = \pi/2 \) and, \( R_{\text{flat}} \) corresponds to the thermal Jeans length at the center of the filament.

\(^3\) Except at later steps of the RunS, for low degrees and in the perpendicular direction, when the filament is dense enough so the radial accretion is dominant.
Figure 7. Radial profiles of the volume density, column density and radial and longitudinal velocity measured in the on-core (solid lines) and off-core (dashed lines) positions for RunA (left column) and RunS, on the dense plane (center column) and perpendicular to it (right column), at $t = 0.00t_{\text{ff}}$ (top panels), $t = 0.53t_{\text{ff}}$ (middle panels) and $t \approx 1.02t_{\text{ff}}$ (bottom panels). Note that in the on-core position, the longitudinal (parallel to the filament) velocity is zero at all radii.

Figure 8. Longitudinal profiles of the volume density, column density, and longitudinal velocity towards the end of the evolution over the central axis of the filament for RunA (left panel) and RunS (right panel). Note that the radial velocity vanishes at the central axis at all positions along the filament’s length.
Figure 9. Radial column density profiles of the filament at the off-core position (black) close to the end of the simulation (RunA, top panel; RunS, bottom panels). The green crosses indicate the points used for computing the best-fit parameters using a Plummer-like function. For comparison, the plot shows the profile for a typical observed filament from Arzoumanian et al. 2011 (magenta continuous line) with the observed dispersion indicated by the magenta dotted curves. Also shown are Plummer profiles, given by eq. (3), with $p = 4$ (adequate for an infinite hydrostatic isothermal cylinder; Ostriker 1964), and with $p = 2$ (adequate for the filament sample of Arzoumanian et al. 2011).

Figure 10. Evolution of $R_{\text{flat}}$ for RunA (left panel) and RunS (right panel).
gitudinal accretion rate suggests that it should approach the radial rate. Conversely, assume that eventually the longitudinal rate becomes larger than the radial one. In this case, the filament would lose mass and decrease its linear density, decreasing the longitudinal accretion rate. Therefore, it appears that the longitudinal rate will tend to always balance the radial rate, as long as the variation in the latter occurs over significantly longer timescales than that of the former.

To test numerically the hypothesis of a stationary regime, we have selected a slice of the filament of thickness 10 pixels, at a distance $l = L_{\text{init}}/2$ from the boundary, to measure the mass per unit length (the linear density), as well as the longitudinal and radial mass fluxes in the filament. This choice for the slice was made so that the slice is sufficiently removed both from the core, since we are interested in the filament in this case, and from the boundary of the numerical box, where the longitudinal flow velocity is forced to remain at zero. These chosen slices for each case are shown by the black lines in Fig. 4.

Consider Fig. 12. If the flow approaches stationarity, then the mass flux into the slice, $\dot{M}_{\text{in}}$ (both through the top “lid” of the slice, in the direction toward the core, as well as in the radial direction, due to accretion from the cloud) must approach the flux out of the slice, $\dot{M}_{\text{out}}$, through its bottom lid. That is, we expect that the total mass change rate ratio,

$$\mu_{\text{tot}} \equiv \frac{\dot{M}_{\text{in}}}{\dot{M}_{\text{out}}} \sim 1,$$

where

$$\dot{M}_{\text{in}} = \int_A \rho \cdot \mathbf{v} \cdot dA + \int_B \rho \cdot \mathbf{v} \cdot dB,$$

$$\dot{M}_{\text{out}} = \int_C \rho \cdot \mathbf{v} \cdot dC,$$

\(\mathbf{v}\) is the velocity vector, and \(A\) is the area of the top lid, \(B\) is the perimetal area of the slice, and \(C\) is the area of the bottom lid.

Figure 13 shows the evolution of $\mu_{\text{tot}}$ through the slice for the two runs. As we can see, the mass flux ratio for the axisymmetric run has not yet reached unity by the time it stops, although $\mu_{\text{tot}}$ is clearly increasing and may eventually reach unity. For the stratified run, on the other hand, $\mu_{\text{tot}}$ does reach unity; although, somewhat surprisingly, it exceeds this value at the end of the simulation and likely would continue to grow. Although this behaviour may be transient, it is necessary to perform simulations that evolve past the time of formation of a singularity (a protostar), which, however, requires us to use a different numerical code. We therefore postpone this task to a future study.

Another relevant diagnostic is the ratio of the mass accretion rate onto the slice through the perimetal area, $\dot{M}_{\text{in,B}}$, to the total mass rate of change of the slice itself, $\dot{M}_{\text{slice}}$, given by

$$\dot{M}_{\text{in,B}} = \int_B \rho \cdot \mathbf{v} \cdot dB,$$

and

$$\dot{M}_{\text{slice}} = \dot{M}_{\text{in}} - \dot{M}_{\text{out}}.$$

We then expect that, at early times, most of the mass change in the slice is due to the perimetal accretion, and so the mass rate ratio

$$\mu_B \equiv \frac{\dot{M}_{\text{in,B}}}{\dot{M}_{\text{slice}}}$$

should be close to unity. On the other hand, at late times, if the mass of the slice tends to a constant (i.e., $\dot{M}_{\text{slice}} \to 0$), then this ratio should diverge, indicating that the mass accreted through the perimetal area plus the mass accreted through the top lid, is expelled through the bottom lid. Therefore the perimetal accretion no longer contributes to an increase in the slice’s mass.

Figure 14 shows the evolution of $\mu_B$ for the two simulations. For both, this ratio is indeed close to (although slightly larger than) unity at early stages. This means that the mass flux through the perimetal area is close to, but slightly larger than, the total mass increase rate of the slice. This in turn implies that, during the early stages, this perimetal mass flux is the main driver of the mass growth of the slice, although some of it is lost by evacuation from the bottom lid.

On the other hand, at later times, for RunS, the ratio begins to increase, indicating that the evacuation (through the bottom lid) increases in relation to the perimetal inflow rate, so that the mass of the slice begins to approach constancy (its mass rate of change decreases); i.e., the slice approaches stationarity. In fact, after $t \sim 0.7t_f$, $\mu_B$ increases strongly and then changes sign, indicating that the total mass change rate for the slice becomes negative; i.e., the slice begins to lose mass. Figure 15, however, shows the evolution of the mass of the slice (blue lines) for the two runs. Surprisingly, the slice mass is growing monotonically in the two runs, a fact that appears inconsistent with the conclusion from the mass change ratios that the slice begins to lose mass at $t \sim 0.93t_f$ in RunS.

This apparent inconsistency is resolved by noting that, so far, we have considered the boundary of the filament (and thus the perimetal area of the slice) using the definition given by eq. (1). As discussed in Sec. 3.1, with this definition, the boundary of the filament in RunS moves outwards as it evolves. Thus, the mass growth occurs because the filament radius increases, in spite of the fact that the net instantaneous mass flux across the filament’s boundaries is negative. This behaviour is confirmed by the red dashed lines in Figs. 13, 14, and 15, which show the evolution of the ratios of mass increase and the slice mass when the boundary of the filament is forced to remain fixed at its initial radius. In this case, it is seen in Figure 14 that the ratio of the perimetal accretion rate to the total mass rate of change, $\mu_B$, remains close to unity, and, while the mass still increases, it does so much more slowly than in the moving-boundary case (blue lines, labeled “inst”).

It is also worth noting in Figure 15 that, in the moving-boundary case the filament slice mass grows in two stages, first accelerating and then decelerating, in both simulations. Instead, when the filament boundary is defined to remain fixed at its initial position, the filament slice mass growth curve is always concave—i.e., always accelerating, at least during the prestellar stage considered in this work. This behaviour suggests that the more extended boundary represents a region where the approach to stationarity has advanced further than the region enclosed by the fixed boundary, which remains further inside the filament.
Filamentary collapse flow in molecular clouds

3.3 Comparison with observations of prestellar cores

As in Paper I (see Sec. 4.1 and Figure 3 there, for further details about the observed prestellar cores and the simulated core), Figure 16 shows a plot of $M_{\text{core}}/M_{\text{BE}}$ vs. $M_{\text{core}}$, where $M_{\text{core}}$ is the core mass and $M_{\text{BE}}$ is the Bonnor-Ebert mass for the mean density $\langle n \rangle$ and temperature $T$ of the core, given by

$$M_{\text{BE}} = 1.82 \left( \frac{\langle n \rangle}{10^4 \text{ cm}^{-3}} \right) \left( \frac{T}{10 \text{ K}} \right)^{3/2} M_\odot. \quad (9)$$

This diagram was studied by Lada et al. (2008), who concluded that in the Pipe molecular cloud, most low-mass cores are gravitationally unbound and confined by external pressure. Here, we have plotted the same sample of observed cores we considered in Paper I, coming from the Pipe (Rathborne et al. 2009) and from the Orion (Ikeda et al. 2007) clouds, as well as the evolutionary track in this diagram of our filament-embedded cores. Note that in the case of the cores from the Pipe, even though not all prestellar cores in the sample of Rathborne et al. (2009) are extracted from the main filamentary structure, it has been suggested that the molecular cloud is indeed formed by the collision of filaments (Frau et al. 2015). The Orion molecular cloud is more complex, but the sample includes prestellar cores from a filamentary structure in the north part of the cloud and also prestellar cores within the south region of more diffuse emission where no clear filamentary structure has been found (Ikeda et al. 2007). From Figure 16, we can see that the evolutionary track of our cores, with their boundaries defined by eq. (2) tracks almost exactly the locus of the observed prestellar cores.

The agreement between the pictures of formation of prestellar cores immersed in both a filamentary structure (this work) and in apparent isolation (Paper I), indeed suggests that the GHC scenario is a plausible mode for star formation from the large scale and down to the small scale (i.e., cloud-sheet-filament-core), where the main physical mechanisms are the gravitational focusing and the direct accretion from the different successive hierarchies in the sequence of collapse.

Note that we had to choose here a factor $\alpha_{\text{core}} = 9$ that is rather large compared to those (1.125, 1.2, and 1.5, respectively) utilized in Paper I, in which the core was placed directly over the background, with no intermediary filament. Lower choices of this factor resulted in the evolutionary tracks being displaced to higher masses in this diagram. This sug-

---

Figure 11. Histogram of the angle between the velocity vector and the horizontal axis of the plots in Fig. 4 axis for RunA (top panel, corresponding to the left column in Fig. 4) and RunS, both on the central dense plane (bottom left panel, corresponding to the middle column in Fig. 4) and perpendicular to it (bottom right panel, corresponding to the right column of Fig. 4) at the initial (blue) and final (red) timesteps. It can be seen that the orientation of the flow is almost invariant throughout the evolution.

Figure 12. Illustration of mass flux through the filament’s slice.

The agreement between the pictures of formation of prestellar cores immersed in both a filamentary structure (this work) and in apparent isolation (Paper I), indeed suggests that the GHC scenario is a plausible mode for star formation from the large scale and down to the small scale (i.e., cloud-sheet-filament-core), where the main physical mechanisms are the gravitational focusing and the direct accretion from the different successive hierarchies in the sequence of collapse.
Figure 13. Evolution of the mass flux ratio, $\mu_{\text{tot}}$, (eq.[4]) in the slice for Run A (left panel) and RunS (right panel). The blue solid lines indicate the ratio of the mass fluxes across the moving filament boundary defined by eq.(1). The red dashed lines indicate the fluxes calculated across a boundary fixed at its initial position.

Figure 14. Evolution of the ratio of perimetral accretion rate to total mass change rate, $\mu_B$, (eq.[8]) for the filament slice for RunA (left panel) and RunS (right panel). The blue solid lines indicate the ratio calculated for the mass fluxes across the moving filament boundary defined by eq.(1). The red dashed lines indicate the fluxes calculated across a boundary fixed at its initial position.

gests that the observational definition criterion for cores is somewhat arbitrary, as it often relies on the noise level of the observations (see, e.g., André et al. 2014), and moreover depends on the presence of an embedding filamentary structure for the cores. It remains to be tested whether this choice is consistent with regular observational core-definition procedures, a task that we defer to a future study.

4 SUMMARY AND DISCUSSION

The results from the previous section can be summarized as follows:

(i) We have numerically simulated the prestellar collapse of a filamentary perturbation containing a spherical central enhancement (a core) in its center. We considered two variants of this filamentary collapse, one with axial symmetry (RunA), and one with additional stratification in one of the directions perpendicular to the filament (RunS). The latter represents the case in which the filament is in turn embedded in a flattened cloud.

(ii) The presence of the filamentary perturbation changes the symmetry of the collapse flow which, away from the core, proceeds first toward the filament, and smoothly changes direction as it approaches the filament axis, to becoming longitudinal there, i.e., oriented towards the core. No shocks develop in the filament, in spite of the flow eventually becoming supersonic, in analogy with the prestellar collapse of a purely spherical core.

(iii) To measure properties of the filament and the core, and intending to represent common observational procedures, we defined their boundaries in terms of density enhancements above their respective parent structures (eqs.[1] and [2]). Thus defined, both the filament and the core evolve by grow-
Figure 15. Slice mass vs. time for RunA (left panel) and RunS (right panel). The blue solid lines indicate the mass contained within the moving filament boundary defined by eq. (1). The red dashed lines indicate the mass contained within a boundary fixed at its initial position.

Figure 16. Evolution in the diagram $M_{\text{core}}/M_{\text{BE}}$ vs. $M_{\text{core}}$ (cf., Lada et al. 2008) of the cores (immersed in the filaments) for RunA (left panel) and RunS (right panel). Also shown are the observed cores from Table 2 in (Rathborne et al. 2009), and Table 1 from (Ikeda et al. 2007). Each point has a vertical error bar that spans the values of the BE-mass at temperatures of 10 and 13 K (top and bottom respectively). The bottom panels show the same data as those in the top panel, but with a logarithmic vertical axis, to better appreciate the location of the low-mass cores.
The filament is never hydrostatic, and instead shows some signatures of approaching a stationary regime in which the material accreted through the boundary flows longitudinally toward the core. We suggested that such a stationary regime may be an attractor for the evolution of the filament, because, if the longitudinal “drainage” of material is lower than the peripheral accretion, the filament’s linear density must increase, increasing the longitudinal flux. Conversely, if the longitudinal flux exceeds the peripheral accretion, then the filament’s linear density must decrease, reducing the longitudinal flux.

(v) In both runs, we measured the evolution of various mass flux rates, as well as the total rate of mass change and their ratios in a slice on the filament, far from the core, to determine whether or not the slice mass approaches stationarity, and if so, how it is approached. The mass fluxes were measured across the perimetral boundary of the filament and along the filament. We found that the perimetral mass flux first causes an increase in the filament mass, but later the longitudinal flux tends to cancel this effect. The stationary state, however, is not fully attained during the prestellar stage we have considered in this paper, and may require consideration of the subsequent protostellar stages of evolution. We plan to address this possibility in future work.

(vi) We found that the radial column density profile in the filament away from the core can be fitted by a Plummer profile, and that the radius of the flat central part slowly decreases in time, approaching the “typical” observed values of order 0.1 pc at the end of the simulations, near the time of protostar formation in the core. The profile on the filament retains its flattened shape near the filament axis even at the time when the central density in the core diverges (the time of protostar formation). This behaviour is due to the fact that, while the mass is accumulated at the core, it just “traverses” the filament but does not accumulate there, thus never causing a divergence of the density in the filament axis. Therefore filaments simply act as intermediary “funnels” for the material to flow from the cloud to the core.

4.1 Comparison with previous work

Our results are consistent with those of simulations of the formation and evolution of turbulent molecular clouds (e.g., Heitsch et al. 2009; Smith et al. 2011; Gómez & Vázquez-Semadeni 2014), and reinforce the notion that the filaments constitute intermediate stages of the collapse. Our idealized setup, of a perfectly cylindrical geometry with an initially spherical central core, allows us to focus on the essential flow features of filament-core systems.

We find that, as in calculations of spherical collapse (e.g., Larson 1969; Penston 1969, see also Paper I), a shock does not develop anywhere in the system before the formation of a singularity (the time of protostar formation). Moreover, along the filament, the growth rate of the central density in the filament slows down, so that it is nowhere near developing a singularity at the time when this occurs in the core (the end time of our simulations). Indeed, the flow seems to approach a stationary regime where the mass flux across the perimetral boundary of the filament is drained into the core by a longitudinal mass flux.

The approached steady state may be considered the filamentary analogue of the similarity flow developing in spherical structures after the formation of the singularity in flows that do not start from a singular hydrostatic initial condition (the singular isothermal sphere, or SIS), but rather have been growing from before the formation of the singularity (Whitworth & Summers 1985). For these, Murray et al. (2017) recently showed that the density at a given radius approaches a constant, implying that the flow becomes stationary. In this spherical case, the mass is drained into the singularity (the central protostar), so that the latter increases its mass, but the gaseous core may maintain a stationary density and velocity configuration (neglecting the increase in the core mass). In the filament, our results suggest that the drainage into the core may allow the filament to also develop stationarity, with our simulations approaching, but not quite reaching, it. Moreover, in the case of our filament, because the center of the collapse flow (the center of the core) is far from the middle position on the filament in which we have measured the mass fluxes, the density at the filament axis remains finite, and allows the radial density and column density profiles to remain flat for a long time. Thus, although $R_{\text{flat}}$ does evolve in time during the prestellar stages, it does so rather slowly, and seems to approach a value consistent with those observed in Herschel filaments (Arzoumanian et al. 2011; Palmeirim et al. 2013).

ACKNOWLEDGEMENTS

R.N.-R. acknowledges financial support from PAPIIT grant IA103517 from DGAPA-UNAM to Bernardo Cervantes-Sodi. E.V.-S acknowledges CONACYT grant 255295.

REFERENCES

André P., Di Francesco J., Ward-Thompson D., Inutsuka S.-I., Pudritz R. E., Pineda J. E., 2014, Protostars and Planets VI, pp 27–51
Arzoumanian D., et al., 2011, A&A, 529, L6
Arzoumanian D., André P., Peretto N., Könyves V., 2013, A&A, 553, A119
Bally J., Langer W. D., Stark A. A., Wilson R. W., 1987, ApJ, 312, L45
Beaumont C. N., Williams J. P., 2010, ApJ, 709, 791
Chevance M., et al., 2020, Space Sci. Rev., 216, 50
Clarke S. D., Whitworth A. P., Hubber D. A., 2016, MNRAS, 458, 319
Fischera J., Martin P. G., 2012, A&A, 542, A77
Frau P., Girart J. M., Alves F. O., Franco G. A. P., Onishi T., Román-Zúñiga C. G., 2015, A&A, 574, L6
Gómez G. C., Vázquez-Semadeni E., 2014, ApJ, 791, 124
Gómez G. C., Vázquez-Semadeni E., Shadmehri M., Ballesteros-Paredes J., 2007, ApJ, 669, 1042
Gong H., Ostriker E. C., 2009, ApJ, 699, 230
Heitsch F., 2013a, ApJ, 769, 115
Heitsch F., 2013b, ApJ, 776, 62

\footnote{Even though we don’t impose a velocity field here}
Filamentary collapse flow in molecular clouds

Heitsch F., Hartmann L. W., Slyz A. D., Devriendt J. E. G., Burkert A., 2008, ApJ, 674, 316
Heitsch F., Ballesteros-Paredes J., Hartmann L., 2009, ApJ, 704, 1735

Hennebelle P., André P., 2013, A&A, 560, A68
Ikeda N., Sunada K., Kitamura Y., 2007, ApJ, 665, 1194

Imotsuka S.-I., Miyama S. M., 1992, ApJ, 388, 392
Keto E., Caselli P., 2010, MNRAS, 402, 1625
Kirk J. M., et al., 2013, MNRAS, 432, 1424
Kusune T., et al., 2019, PASJ, 71, 55

Lada C. J., Muench A. A., Rathborne J., Alves J. F., Lombardi M., 2006, ApJ, 672, 410
Larson R. B., 1969, MNRAS, 145, 271
Leorat J., Passot T., Pouquet A., 1990, MNRAS, 243, 293
Lin C. C., Mestel L., Shu F. H., 1965, ApJ, 142, 1431

Murray D. W., Chang P., Murray N. W., Pittman J., 2017, MNRAS, 465, 1316
Myers P. C., 2009, ApJ, 700, 1069

Nagahama T., Mizuno A., Ogawa H., Fukui Y., 1998, AJ, 116, 336
Naranjo-Romero R., Vázquez-Semadeni E., Loughnane R. M., 2015, ApJ, 814, 48

Ostriker J., 1964, ApJ, 140, 1056
Palmeirim P., et al., 2013, A&A, 550, A38
Panopoulou G. V., Psaradaki I., Skalidis R., Tassis K., Andrews J. J., 2017, MNRAS, 466, 2529

Penston M. V., 1969, MNRAS, 144, 425
Peretto N., et al., 2013, A&A, 555, A112

Rathborne J. M., Lada C. J., Muench A. A., Alves J. F., Kainulainen J., Lombardi M., 2009, ApJ, 699, 742
Rivera-Ingraham A., et al., 2016, A&A, 591, A90
Rivera-Ingraham A., et al., 2017, A&A, 601, A94

Schneider N., Csengeri T., Bontemps S., Motte F., Simon R., Hennebelle P., Federrath C., Klessen R., 2010, A&A, 520, A49

Smith R. J., Glover S. C. O., Bonnell I. A., Clark P. C., Klessen R. S., 2011, MNRAS, 411, 1354

Vázquez-Semadeni E., Ryu D., Passot T., González R. F., Gazol A., 2006, ApJ, 643, 245

Vázquez-Semadeni E., Gómez G. C., Jappsen A.-K., Ballesteros-Paredes J., Klessen R. S., 2009, ApJ, 707, 1023

Vázquez-Semadeni E., Colín P., Gómez G. C., Ballesteros-Paredes J., Watson A. W., 2010, ApJ, 715, 1302

Vázquez-Semadeni E., Palau A., Ballesteros-Paredes J., Gómez G. C., Zamora-Avilés M., 2019, MNRAS, 490, 3061

Veena V. S., Vig S., Tej A., Kantharia N. G., Ghosh S. K., 2017, MNRAS, 465, 4219

Wareing C. J., Falle S. A. E. G., Pittard J. M., 2019, MNRAS, 485, 4686

Whitworth A., Summers D., 1985, MNRAS, 214, 1

This paper has been typeset from a TeX/LaTeX file prepared by the author.
This figure "illustration_slice.png" is available in "png" format from:

http://arxiv.org/ps/2012.12819v1
This figure "illustration_substructures.png" is available in "png" format from:

http://arxiv.org/ps/2012.12819v1