Chapter

Fatigue Limit Reliability Analysis for Notched Material with Some Kinds of Dense Inhomogeneities Using Fracture Mechanics

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Abstract

This study proposes a quantitative method for predicting fatigue limit reliability of a notched metal containing inhomogeneities. Since the fatigue fracture origin of the notched metal cannot be determined in advance because of stress nonuniformity, randomly distributed particles, and scatter of a matrix, it is difficult to predict the fatigue limit. The present method utilizes a stress-strength model incorporating the “statistical hardness characteristics of a matrix under small indentation loads” and the “statistical hardness characteristics required for non-propagation of fatigue cracks from microstructural defects”. The notch root is subdivided into small elements to eliminate the stress nonuniformity. The fatigue limit reliability is predicted by unifying the survival rates of the elements obtained by the stress-strength model according to the weakest link model. The method is applied to notched specimens of aluminum cast alloy JIS AC4B-T6 containing eutectic Si, Fe compounds and porosity. The fatigue strength reliability at $10^7$ cycles, which corresponds to the fatigue limit reliability, is predicted. The fatigue limits of notch root radius $\rho = 2, 1, 0.3, \text{ and } 0.1 \text{ mm}$ are obtained by rotating-bending fatigue tests. It is shown that the fatigue limits predicted by the present method are in good agreement with the experimental ones.

Keywords: metal fatigue, fatigue limit reliability, notch effect, aluminum cast alloy, inhomogeneity

1. Introduction

Aluminum cast alloys are widely applied, for example, in motor vehicles, ships, aircraft, machines, and structures, owing to the high cast ability and high specific strength [1–3]. They can be improved so as to meet specific mechanical properties by tuning the casting method, the alloying elements, and the cooling and heat treatment conditions [4–6]. Generally, precipitation hardening, also called age-hardening, is used to strengthen the aluminum cast alloys, which brings the dense precipitate of particles such as eutectic Si. The precipitations form fine microstructures such as dendrites, which significantly improve the mechanical properties.
However, the resultant stress concentrations by the precipitations further to fatigue fracture unfortunately [7–9]. Moreover, the possibility of the fatigue fracture increases more and more if microstructural flaws such as porosity are created in the casting process [10–15]. Because the precipitate particles and the microstructural defects are unique, the fatigue strength of the aluminum cast alloys is obliged to treat statistically.

Statistical fatigue test methods [16, 17] are standardized to determine the reliability of the fatigue strength. However, because they require many fatigue tests, it is time-consuming to determine the fatigue strength reliability at $10^7$ stress cycles. Moreover, because the weakest region which controls the fatigue strength of the specimen is not known, the present materials cannot be improved rationally. Hence, a faster, rational method for quantitatively and nondestructively predicting the effect of inhomogeneities on fatigue strength is necessary for safe and reliable machine designs and for economical and quick material developments.

Several methods for predicting the fatigue strength at $10^7$ stress cycles, which are equivalent to the statistically determined fatigue limit of aluminum cast alloys, have been proposed [18–24]. Through a series of stress analyses and fatigue experiments, Murakami et al. [18–20] clarified the non-propagation limit of a fatigue crack initiated by a microstructural defect and proposed a simple formula for predicting the fatigue limit of a plain specimen containing defects [18–20]. The non-propagation limit of a fatigue crack initiated by microstructural defect is determined by the defect size and mechanical characteristics of the matrix near the defect. The maximum defect, which is often estimated by extreme statistics, is therefore assumed to be the origin of the fatigue fracture. Most of the methods are based on the assumption that fatigue fracture begins at the maximum defects, and they often do not consider the interference effects of inhomogeneities and the scatter of the hardness of the matrix [25]. Because aluminum cast alloys have much higher densities of inhomogeneities, it is presumed that the interference effect is not negligible and the maximum inhomogeneity is not in the severest mechanical state necessarily. Additionally, in the case of a notched specimen, the stress varies significantly. The most severe mechanical defect should be used for prediction, even if it is not maximal. Generally, the fatigue limit of a notched specimen of a homogeneous metal in which microstructural defect is not the origin of the fatigue fracture consists of the microcrack and macrocrack non-propagation limits [26–32]. This fact is widely used in predicting fatigue limit. However, since microstructural defects act as crack initiation sites, the fatigue limit of an inhomogeneous metal also cannot be predicted by these two types of crack non-propagation limits.

In this study, a quantitative method for predicting the fatigue limit reliability of a notched metal containing inhomogeneous particles is proposed. The present method is also based on the stress-strength model and is applied to notched specimens of an Al-Si-Cu alloy (JIS AC4B). The inhomogeneous particle in the alloy comprises eutectic Si and Fe compounds and porosity in the matrix. Rotating-bending fatigue tests are performed on the notched specimens of AC4B-T6 by changing notch root radius variously. The validity of the present method is examined by comparing its numerical prediction with experimental results.

2. Crack non-propagation limits for predicting fatigue limit of notched specimen

Generally, when fatigue tests are performed on a notched specimen by changing the notch root radius $\rho$ for a given notch depth $t$, the typical relationship between the fatigue limit $\sigma_{w}$ and $\rho$ is as shown in Figure 1; here, $\sigma_{w0}$ is the fatigue limit of the
plain specimen, $\sigma_{w_1}$ is the microcrack non-propagation limit, $\sigma_{w_2}$ is the macrocrack non-propagation limit, and $\rho_0$ is a material property known as the branch point, the critical value of which determines whether the non-propagating crack exists along the notch root [26, 27]. If the notch is sufficiently deep, $\rho_0$ is constant [27].

If $\rho > \rho_0$, $\sigma_{w_1}$ is the fatigue limit [33]. $\sigma_{w_1}$ can be predicted from the mechanical characteristics of the microstructure. Conversely, if $\rho \leq \rho_0$, $\sigma_{w_2}$ is the fatigue limit [33]. $\sigma_{w_2}$ is constant and independent of $\rho$. This means that the $\sigma_{w_2}$ is equal to the fatigue limit of the cracked specimen as $\rho \to 0$. That is, the notch can be assumed to be a crack and $\sigma_{w_2}$ can be predicted by the fracture mechanics.

In the case of metals containing microstructural defects, the non-propagation limit of the fatigue crack that originates from the microstructural defect may be the fatigue limit. Because the defect is categorized as a macrocrack, the low macrocrack
non-propagation limit is differentiated from $\sigma_{w2}$. The threshold stress intensity factor range $\Delta K_{th}$ determines whether the fatigue crack originating from the macrocrack is arrested. The value of $\Delta K_{th}$ is an indication of the dependency of the different crack lengths [20]. In this study, a crack for which $\Delta K_{th}$ is constant irrespective of its length, and which exhibits the small-scale yielding (SSY), is defined as a long macrocrack. Conversely, a crack for which $\Delta K_{th}$ is dependent on the length, and which exhibits the large-scale yielding (LSY), is defined as a small macrocrack [33, 34]. The three following types of crack non-propagation limits are introduced and defined to predict the fatigue limit of a notched specimen of aluminum cast alloy [35]:

| Limit | Description |
|-------|-------------|
| $\sigma_{w1}$ | This is the non-propagation limit of a microcrack that is initiated by repeated irreversible plastic strains in a homogeneous notch stress field without microstructural and structural stress concentrations. |
| $\sigma_{wd}$ | This is the non-propagation limit of a three-dimensional fatigue crack that originates from microstructural defects such as nonmetallic inclusions and pits in a homogeneous notch stress field without other microstructural and structural stress concentrations. |
| $\sigma_{w2}$ | This is the non-propagation limit of structural long macrocracks such as deep notches with $\rho < \rho_0$. |

**Figure 2** is a schematic illustration of the relationships between $\rho$ and each of $\sigma_{w1}$, $\sigma_{wd}$, and $\sigma_{w2}$. Further, $\rho_0$ and $\rho_d$ are, respectively, the branch point and limit notch root radius, which determines whether the fatigue limit is affected by the microstructural defects. $\sigma_{w1}$ and $\sigma_{wd}$ decrease as $\rho$ decreases, whereas $\sigma_{w2}$ attains a constant value and becomes independent of $\rho$. If $\rho \geq \rho_d$, $\sigma_{wd}$ is equal to the fatigue limit $\sigma_w$. If $\rho_0 < \rho < \rho_d$, $\sigma_{w1}$ is equal to $\sigma_w$. If $\rho \leq \rho_0$, $\sigma_{w1}$ and $\sigma_{wd}$ are cut off by $\sigma_{w2}$, and $\sigma_{w2}$ is equal to $\sigma_w$.

Figure 2. Schematic illustration of fatigue limit of a notched structure with defects.
Because the hardness is locally scattered and numerous defects are distributed through the material, the microcrack and defect that determine the fatigue fracture cannot be determined in advance. In this situation, the probabilities of the arrest of the microcrack and the fatigue crack originating from the defect are, respectively, determined by the statistical characteristics of the hardness and the statistical characteristics of the defect. That is, $\sigma_{w1}$ and $\sigma_{wd}$ are, respectively, described by probability distributions.

### 3. Method for predicting fatigue limit reliability of notched metal containing inhomogeneous particles

| Nomenclature | Description |
|--------------|-------------|
| $A_j$ | size of $j$th surface element |
| $A_j^*$ | size of $j$th region where the relative first principal stress corresponds to $\sigma_{1j}$ |
| $A_{opc}, A_{opb}, A_{opc}$ | region required for the non-propagation of fatigue crack |
| $\sqrt{\text{area}_p}$ | size of surface defect |
| $\sqrt{\text{area}_{p1}}$ | lower limit size of small surface crack |
| $\sqrt{\text{area}_g}$ | size of internal defect |
| $F, F_P, F_R$ | geometric correction factor |
| $F_{n_o}$ | fatigue limit reliability of notched specimen |
| $f_{HVM}$, $f^*_HVM$, $f^*_HVM$ | $H_{VM}$ distribution |
| $f_{\chi^2}$ | $\chi^2$ distribution |
| $g_{P}, g_{R}, g_{S}$ | limit hardness |
| $\gamma_w$ | stress relaxation effect |
| $H_V, H_{VM}$ | Vickers hardness |
| $K_{IP}, K_{IR}, K_{IP R}$ | stress intensity factor |
| $K_i$ | stress concentration factor |
| $\Delta K_w$ | threshold stress intensity factor range |
| $\Delta K_{w LL}$ | lower limit value of $\Delta K_w$ |
| $\Delta K_{w UL}$ | upper limit value of $\Delta K_w$ |
| $M_0(\sqrt{\text{area}_{p0}})$ | the number of surface cracks with $\sqrt{\text{area}_p} \geq \sqrt{\text{area}_{p0}}$ in a unit area |
| $M_{V0}(R_0)$ | the number of particles with $R \geq R_0$ in a unit volume |
| $M_k$ | types of inhomogeneous particles |
| $N_{V0}$ | the number of particles in a unit volume |
| $n_S$ | the number of surface elements |
| $n_V$ | the number of solid elements |
| $P, P_R$ | indentation load |
| $P_V(R_0)$ | existence probability of particles with $R \geq R_0$ |
| $R_i$ | limit size of small interior crack |
| $S_{n_o}$ | survival rate of notched specimen |
| $S_{s_{w1}}$ | survival rate of surface element with microcracks |
This section presents a method for predicting the fatigue limit reliability of a notched specimen with stress concentration factor $K_t$, notch depth $t$, and notch root radius $\rho$ under zero mean stress. The control volume is actually divided into surface and solid elements so that the stresses applied to the elements can be assumed to be constant. The fatigue strengths of all the elements are then stochastically evaluated by the stress-strength model on the mesoscale. The fatigue limit reliability is also predicted by assembling the fatigue strengths using the weakest link model [25].

3.1 Stress relaxation effect of interference of inhomogeneous particles

Figure 3 is a schematic illustration of the analytical model of a metal containing inhomogeneous particles. The metal is approximated by a cubic lattice model to determine the stress relaxation effect of the interference of the particles [36].

3.1.1 Statistical characteristics of inhomogeneous particles

The probability of existence of such particles is given by the following equation [37]:

![Figure 3. Approximate model of metal with inhomogeneous particles.](image-url)
Here, \( \nu \) and \( \lambda \) are material constants, \( R_0 \) is the particle radius, and \( P_V(R_0) \) is the probability of the existence of particles with radii greater than \( R_0 \).

The total number of particles in a unit volume is denoted by \( N_{V0} \). The average number of particles with radii greater than \( R_0 \) in a unit volume, \( M_{V0}(R_0) \), is given by the following equation \[37\]:

\[
M_{V0}(R_0) = N_{V0} \cdot P_V(R_0).
\] (2)

A particle cross-sectioned by the specimen surface is projected onto a plane perpendicular to the first principal stress. The projected area is then modified as shown in Figure 4 by considering the mechanics. The modified area is denoted by \( A_{P0} \). The average number of cross-sectioned particles with areas larger than \( \sqrt{A_{P0}} \) in a unit area, \( M_{S0}(\sqrt{A_{P0}}) \), is given by the following equation \[12, 24\]:

\[
M_{S0}(\sqrt{A_{P0}}) = \lambda N_{V0} \int_0^1 \frac{t}{\sqrt{1-t^2}} \left\{ \Gamma \left( 1 + \frac{1}{\nu}, \left( \frac{\sqrt{A_{P0}}}{\lambda \theta^+} \right) ^\nu \right) 
+ \Gamma \left( 1 + \frac{1}{\nu}, \left( \frac{\sqrt{A_{P0}}}{\lambda \theta^-} \right) ^\nu \right) \right\} dt,
\] (3)

\[
\theta^+ = \frac{\pi}{2} + 2\sqrt{1-t^2},
\] (4)

\[
\theta^- = \sin^{-1} t - t \sqrt{1-t^2}.
\] (5)

Here, \( \Gamma \) is a gamma function of the second kind.

### 3.1.2 Average radius and distance

The average particle radius is evaluated by the following equation:

\[
R_m = \int_0^\infty R \frac{dP_V}{dR} dR = \lambda \Gamma \left( 1 + \frac{1}{\nu} \right).
\] (6)

![Figure 4. Spheroidal particle cut by surface.](image-url)
If $N_{V_0}$ particles are regularly arranged in a unit volume as shown in Figure 3, the average distance between the particles is evaluated by the following equation:

$$p_m = \left( \frac{1}{N_{V_0}} \right)^{1/3}$$

(7)

### 3.1.3 Stress relaxation effect

Nisitani [38] proposed a method for approximately solving the interference problem of notches by superposing simple basic solutions to satisfy the equilibrium conditions at the stress concentration point.

When the uniform tensile stress at infinity, $\sigma_z = 1$, is applied to an infinite body, it is supposed that a stress field composed of $\sigma_z = Z_m$ and $\sigma_x = \sigma_y = \sigma_z = T_m$ is formed around the particle. $T_m$ and $Z_m$ are set to satisfy the equilibrium condition at point $(0, 0, R_m)$. Because the stress acting on a single particle in the $z$-direction, $T_m + Z_m$, is composed of $\sigma_z = 1$ and the stresses due to the other particles, the stress equilibrium condition in the $z$-direction is as follows:

$$T_m + Z_m = 1 + \sum_{(i, j, k) \neq (0, 0, 0)} \sum_{i, j, k = -\infty}^{\infty} \sigma_{zm}(T_m, Z_m)|_{x_{i,j,k}} = -ip_m$$

$$y_{i,j,k} = -jp_m$$

$$z_{i,j,k} = R_m - kp_m$$

(8)

Here, $\sigma_{zm}(T_m, Z_m)$ is the stress in the $z$-direction at $(0, 0, R_m)$ produced by the spherical particle located at $(ip_m, jp_m, kp_m)$ in the infinite body under $\sigma_z = Z_m$ and $\sigma_x = \sigma_y = \sigma_z = T_m$.

The stress equilibrium condition in the $y$-direction is also given by

$$T_m = \sum_{(i, j, k) \neq (0, 0, 0)} \sum_{i, j, k = -\infty}^{\infty} \sigma_{ym}(T_m, Z_m)|_{x_{i,j,k}} = -ip_m$$

$$y_{i,j,k} = -jp_m$$

$$z_{i,j,k} = R_m - kp_m$$

(9)

Here, $\sigma_{ym}(T_m, Z_m)$ is the stress in the $y$-direction at $(0, 0, R_m)$ produced by the spherical particle located at $(ip_m, jp_m, kp_m)$ in the infinite body under $\sigma_z = Z_m$ and $\sigma_x = \sigma_y = \sigma_z = T_m$.

$T_m$ and $Z_m$ are obtained by solving the simultaneous linear Eqs. (8) and (9). In this study, the stress relaxation effect $\gamma_m$ of the interference of the particles is assumed to be

$$\gamma_m = T_m + Z_m$$

(10)

### 3.1.4 Characteristics of elastic stress field near notch root

If the notch is sufficiently deep, a unique stress field determined by the maximum stress and $\rho$ is formed near the notch root [27]. The first principal stress normalized by the maximum stress at the notch root is denoted by $\sigma_1^*$. Figure 5 shows a contour map of $\sigma_1^* = [0.4, 1]$ near the notch root in a semi-infinite plate under tensile stress [35]. The value of $\sigma_1^*$ is independent of the notch shape [27]. If the notch root is divided as shown in Figure 5, the length of the notch edge and size of $j$-th region in which the relative first principal stress is $\sigma_{1j}^*$ are denoted by $l_j^*$ and $A_j^*$ ($j = 1, \cdots$), respectively. The values of $l_j^*$ and $A_j^*$ are given in Table 1 [35].

To predict the fatigue limit reliability, the control volume is set at the notch root and divided into surface and solid elements. The sizes of the solid and surface
elements are denoted by \( V_j \) and \( A_j \) \((j = 1, \ldots)\), respectively. In the case of a typical notch, as in the notched specimen, the control volume can be divided as shown in Figure 5. For example, when a circular bar with a circumferential notch is divided, \( V_j \) and \( A_j \) are approximated as follows [35]:

\[
V_j \approx A_j^* \times \text{average diameter of } j\text{th region} \times \pi,
\]

\[
A_j \approx l_j^* \times \text{average diameter of } j\text{th region} \times \pi.
\]

3.2 Fatigue survival rate of surface element containing microcracks

3.2.1 Statistical characteristics of Vickers hardness

The authors proposed a virtual small cell model for predicting the statistical characteristics of the Vickers hardness in a small region [25, 35]. If the population of the virtual small cells is described by an arbitrary distribution of the mean \( \mu \) and variance \( s^2 \), the statistical characteristics of the Vickers hardness can be described by the normal distribution of the mean \( \mu \) and the variance \( s^2 / n_c \), based on the central limit theory, where \( n_c \) is the number of virtual small cells in the indentation area.

If \( m \) Vickers hardness values are measured in this way using an indentation load \( P \), their statistical characteristics are described by the following normal distribution [25, 35]:

\[
\begin{array}{c|cccccc}
 j & 1 & 2 & 3 & 4 & 5 & 6 \\
 \sigma_{ij}^* & 0.95 - 0.9 & 0.9 - 0.8 & 0.8 - 0.7 & 0.7 - 0.6 & 0.6 - 0.5 \\
 l_j^* / \rho & 0.463 & 0.216 & 0.361 & 0.358 & 0.402 & 0.496 \\
 A_j^* / \rho & 0.0083 & 0.0181 & 0.0703 & 0.144 & 0.296 & 0.657 \\
\end{array}
\]

Table 1. Area of isostress near the notch root.
\[
\begin{align*}
\mathcal{H}_{VM}(H_{VM}) &= \frac{1}{\sqrt{2\pi s^2_{VM}}} \exp \left\{ -\frac{(H_{VM} - \mu_{VM})^2}{2s^2_{VM}} \right\}. 
\end{align*}
\] (13)

Here, \( H_{VM} \) is the Vickers hardness of the matrix that does not contain microstructural defects, \( \mu_{VM} \) is the sample mean, and \( s^2_{VM} \) is the sample variance.

Based on the central limit theorem, the relationship between the sample mean \( \mu_{VM} \) and the population mean \( \mu_{VM0} \) is
\[
\mu_{VM} = \mu_{VM0}. 
\]

Further, the relationship between the sample variance \( s^2_{VM} \) and the population variance \( s^2_{VM0} \) is described by the \( \chi^2 \) distribution with the freedom degree of \( n = m_1 - 1 \) [25, 35]:
\[
\begin{align*}
\chi^2 &= \frac{1}{2\Gamma(n/2)} \left( \frac{\chi^2}{2} \right)^{\frac{n}{2} - 1} \exp \left( -\frac{\chi^2}{2} \right), \\
\Gamma(t) &= \int_0^\infty \chi^{t-1}e^{-\chi}d\chi, \quad \chi^2 = m_1 \frac{s^2_{VM}}{s^2_{VM0}}. 
\end{align*}
\] (14)

3.2.2 Fatigue survival rate of surface element containing microcracks

The microcrack non-propagation limit \( \sigma_{w0} \) is determined by the average characteristics of the material properties around the microcrack. \( \sigma_{w0} \) can be empirically predicted by the following equations [20, 26]:
\[
\begin{align*}
\sigma_{w0} |_{\sigma_m=0} &= 1.6 H_{VM}, \\
\sigma_{w1} |_{\sigma_m=0} &= \frac{\sigma_{w0} |_{\sigma_m=0}}{K_t \sqrt{1 + 4.5\varepsilon_0 |_{\sigma_m=0} / \rho}} = f(H_{VM}, \rho, \sigma_m = 0, K_t). 
\end{align*}
\] (16), (17)

(\( \sigma_{w0}, \sigma_{w1}, \) and \( \sigma_m \) are in MPa, \( H_{VM} \) is in kgf/mm\(^2\), and \( \rho \) and \( \varepsilon_0 \) are in mm.)

If the stress relaxation effect \( \gamma_m \) is considered, \( \gamma_m \) \( \sigma_{1,j} / K_t \) is applied to \( j \)-th surface element. Because fatigue fracture occurs when \( \gamma_m \sigma_{1,j} / K_t \) is greater than \( \sigma_{w1} \), the limit hardness \( g_S(\sigma_{1,j}) \) that determines the occurrence is given by the following equation:
\[
\begin{align*}
g_S &= f^{-1}(H_{VM}, \rho, \sigma_m = 0, K_t) |_{\sigma_m=\gamma_m \sigma_{1,j} / K_t}, 
\end{align*}
\] (18)

Here, \( f^{-1} \) is a function obtained by solving Eq. (18) on \( H_{VM} \).

It is supposed that \( A_{NPC}S \) exhibits the microcrack non-propagation limit characteristics in Eqs. (16) and (17). If \( H_{VM} \) of the matrix is greater than \( g_S \) in \( A_{NPC}S \), fatigue fracture will not occur in \( A_{NPC}S \). Therefore, the probability \( S_{\sigma_{w0}0,j} \) that fatigue fracture does not occur in \( A_{NPC}S \) below \( \sigma_{1,j} \) is given by [25, 35]
\[
S_{\sigma_{w0}0,j} = \mathcal{F}_{H_{VM},S}(h_{VM}) \int_{h_{VM}} dh_{VM}. 
\] (19)

Here, \( \mathcal{F}_{H_{VM},S} \) is the normal distribution of \( \mu_{H_{VM},S} \) and \( s^2_{H_{VM},S} \).

The fatigue survival rate \( S_{\sigma_{w1}j} \) of \( j \)-th surface element containing microcracks is given by the following equation [25, 35]:
\[
S_{\sigma_{w1}j} = \int_0^\infty f_{\chi} \cdot (S_{\sigma_{w1}0,j})^A_j d\chi^2. 
\] (20)
If fatigue fracture does not occur in all the surface elements, the notched specimen would not be broken by the microcrack. Therefore, the fatigue survival rate of a surface element containing microcracks, \( S_{\sigma_1} \), is obtained by multiplying the fatigue survival rates of all the surface elements as follows [25, 35]:

\[
S_{\sigma_1} = \prod_{j=1}^{n_S} S_{\sigma_1,j}.
\]  

Here, \( n_S \) is the number of surface elements.

### 3.3 Fatigue survival rate of surface element containing microstructural defects

The authors [25] proposed a method for predicting the reliability of the small macrocrack non-propagation limit for a nonzero stress gradient using the “statistical hardness characteristics of a matrix under small indentation loads” and the “statistical hardness characteristics required for non-propagation of fatigue cracks originating from microstructural defects in a material” [25]. The stress relaxation effect was introduced into the method to make it applicable to a metal containing dense inhomogeneous particles.

\( \sigma_{wd} \) is divided into two crack non-propagation limits, namely, the non-propagation limit \( \sigma_{wdI} \) of the small crack originating from the interior defect and the non-propagation limit \( \sigma_{wds} \) of the small crack originating from the surface defect.

#### 3.3.1 Fatigue survival rate of solid element containing interior microstructural defects

Because the fatigue crack that originates from a defect propagates on the plane perpendicular to the first principal radial stress, a spherical particle of radius \( R \) is projected onto this plane and assumed to be a penny-shaped crack. If the projected area is denoted by \( \text{area}_R \), its square root is related to \( R \) as follows:

\[
\sqrt{\text{area}_R} = \sqrt{\pi}R.\tag{22}
\]

The stress intensity factor \( K_{IR} \) of the small interior crack is given by [20, 35]

\[
K_{IR} = 0.5 F_R \gamma_m \sigma_{1,j} \sqrt{\pi \sqrt{\text{area}_R}}, \tag{23}
\]

\[
F_R = \frac{4}{\pi^{3/4}} \left\{ 1 - \left( \frac{2}{\sqrt{\pi}} - \frac{4}{3 \pi} \frac{\sqrt{\text{area}_R}}{\rho} \right) \right\}. \tag{24}
\]

Moreover, the threshold stress intensity factor range \( \Delta K_w \) of the small surface defect of size \( \sqrt{\text{area}_P} \) in the metal with Vickers hardness \( H_{VM} \) is given by the following equation [33, 34]:

\[
\Delta K_w|_{\sigma_m=0} = \frac{2\alpha\beta \sqrt{\text{area}_P}^{1/3}}{\ln (2\beta/H_{VM} + 1)}, \tag{25}
\]

\( \alpha = 3.3 \times 10^{-3} \) and \( \beta = 120 \).

(\( \Delta K_w \) is in MPa\( \sqrt{\text{m}} \), \( H_{VM} \) is in kgf/mm\(^2 \), and \( \sqrt{\text{area}_P} \) is in \( \mu \text{m} \)).

The limit hardness that determines whether the fatigue crack originating from the interior microstructural crack is arrested, \( g_R(\sigma_{1,j}, \sqrt{\text{area}_R}) \), is given based on the relationship \( \sqrt{\text{area}_R} = 1.7 \sqrt{\text{area}_P} \) by the following equation [25, 35]:

\[
\]
\[
g_R = 240 / \left\{ \exp \left( \frac{1.56 \times 240}{F_{R \gamma_m} \sigma_{1,j} \sqrt{\text{area}_R}} \right) - 1 \right\}
\]  
(26)

(\sigma_{1,j} \text{ is in MPa, } g_R \text{ is in kgf/mm}^2, \text{ and } \sqrt{\text{area}_R} \text{ is in \(\mu\)m.)}

The relationship between \(\mu_{HVMR}\) and \(\mu_{HVM0}\) can be expressed as follows [25, 35]:

\[
\mu_{HVMR} = \mu_{HVM0} = \mu_{HVM1}.
\]  
(27)

Moreover, the relationship between \(s_{HVMR}^2\) and \(s_{HVM0}^2\) can be expressed as follows [25, 35]:

\[
A_{\text{npc R}} \cdot s_{HVMR}^2 = A_{HVM0} \cdot s_{HVM0}^2.
\]  
(28)

Here, \(A_{HVM0} = P / \mu_{HVM0}\), \(A_{\text{npc R}} = P_R / g_R\), and \(P_R\) are the loads used to create the indentation for obtaining the Vickers hardness \(g_R\) and the indentation area \(A_{\text{npc R}}\).

The fatigue survival rate of \(j\)-th solid element containing interior defects, \(S_{m}^{(m)}_{\sigma_{\text{int}},j}\), is given by the following equation [25, 35]:

\[
S_{m}^{(m)}_{\sigma_{\text{int}},j} = \int_{0}^{\infty} \int_{0}^{\infty} \left\{ - \int_{0}^{R} \left( \int_{0}^{p} p_{(m)}^{(m)} dM_{V,j}^{(m)} dR_{\text{VHM}} dh_{\text{vm}} \right) dR \right\} dR \; dR^2.
\]  
(29)

If the fatigue fracture does not occur in all the solid elements, the notched specimen would not be broken by the small interior defect. Therefore, the fatigue survival rate \(S_{m}^{(m)}_{\sigma_{\text{int}}}\) of a solid element containing interior microstructural defects is obtained by multiplying the fatigue survival rates of all the solid elements as follows [25, 35]:

\[
S_{m}^{(m)}_{\sigma_{\text{int}}} = \prod_{j=1}^{n_V} S_{m}^{(m)}_{\sigma_{\text{int}},j}.
\]  
(30)

Here, \(n_V\) is the number of solid elements.

### 3.3.2 Fatigue survival rate of surface element containing surface microstructural cracks

The stress intensity factor \(K_{IP}\) of a small surface crack of size \(\sqrt{\text{area}_P}\) is given by the following equation [20, 35]:

\[
K_{IP} = 0.65 \cdot R g_m \gamma_{\text{m}} \sigma_{1,j} \sqrt{\text{area}_P},
\]  
\[
F_R = 0.968 - 1.021 \cdot \frac{\sqrt{\text{area}_P}}{\rho}.
\]  
(31)

Further, the limit hardness \(g_P(\sigma_{1,j}, \sqrt{\text{area}_P})\) that determines whether the small surface crack is arrested is given by the following equation [25, 35]:

\[
g_P = 240 / \left\{ \exp \left( \frac{1.43 \times 240}{F_{R \gamma_m} \sigma_{1,j} \sqrt{\text{area}_P}} \right) - 1 \right\}.
\]  
(33)

(\sigma_{1,j} \text{ is in MPa, } g_P \text{ is in kgf/mm}^2, \text{ and } \sqrt{\text{area}_P} \text{ is in \(\mu\)m.)}

The fatigue survival rate of \(j\)-th surface element containing surface microstructural cracks, \(S_{m}^{(m)}_{\sigma_{\text{int}},j}\), is given by
\[ S_{\sigma_{\text{ads}}(m)} = \prod_{j=1}^{n} S_{\sigma_{\text{ads}}(m)} \]  

(35)

3.3.3 Fatigue survival rate of element with microstructural defects

The fatigue survival rate \( S_{\sigma_{\text{ads}}}^{(m)} \) of an element containing microstructural defects is obtained by multiplying \( S_{\sigma_{\text{ads}}}^{(m)} \) and \( S_{\sigma_{\text{ads}}}^{(m)} \) as follows [25, 35]:

\[ S_{\sigma_{\text{ads}}}^{(m)} = S_{\sigma_{\text{ads}}}^{(m)} \times S_{\sigma_{\text{ads}}}^{(m)} \]  

(36)

Because the material contains \( M_d \) types of inhomogeneous particles, the fatigue survival rate \( S_{\sigma_{\text{ads}}} \) is given by the following equation:

\[ S_{\sigma_{\text{ads}}} = \prod_{m=1}^{M_d} S_{\sigma_{\text{ads}}}^{(m)} \]  

(37)

3.4 Prediction of long macrocrack non-propagation limit \( \sigma_{w2} \)

\( \sigma_{w2} \) of the notched specimen with \( \rho \leq \rho_0 \) is equal to the fatigue limit of the cracked specimen obtained by \( \rho \rightarrow 0 \). \( \Delta K_{wUL} \) is the upper limit of \( \Delta K_w \) and is constant regardless of the crack length. \( \sigma_{w2}|_{\sigma_{w}=0} \) can be obtained as follows [35]:

\[ \sigma_{w2}|_{\sigma_{w}=0} = \frac{\Delta K_{wUL}|_{\sigma_{w}=0}}{2F \sqrt{\pi t}} \]  

(38)

3.5 Prediction of fatigue limit reliability

The probability that fatigue fracture is caused by microcracks or microstructural defects is obtained by the complementary event defined by the product of \( S_{\sigma_{\text{ads}}} \) and \( S_{\sigma_{\text{ads}}} \). Because the fatigue limit of a notched specimen, \( \sigma_{w} \), cannot be lower than \( \sigma_{w2} \), and its fatigue limit reliability \( F_{\sigma_{w}} \) is obtained as follows [35]:

\[ F_{\sigma_{w}} = \begin{cases} 0 & (\sigma_{w} \leq \sigma_{w2}) \\ 1 - S_{\sigma_{\text{ads}}} \times S_{\sigma_{\text{ads}}} & (\sigma_{w} > \sigma_{w2}) \end{cases} \]  

(39)

4. Fatigue experiment

4.1 Material, shape of specimen, and experimental procedure

The material used for the experiment was Al-Si-Cu alloy (JIS AC4B). The age-hardened aluminum cast alloy is identified as AC4B-T6. Table 2 shows its mechanical properties.
Figure 6 shows the configurations of the specimens. The notch depth $t$ and opening angle $\theta$ were set at 0.5 mm and 60°, respectively. The notch root radii $\rho$ were set at 2, 1, 0.3, and 0.1 mm, respectively. All the specimens were machined; polished with fine emery paper, alumina (3 $\mu$m), and diamond paste (1 $\mu$m); and also chemically polished. Rotating-bending fatigue tests were carried out according to JIS Z2274 and the earlier studies [27–31] under stress amplitude $\sigma_a = 60–110$ MPa and frequency $f = 50$ Hz. An Ono-type rotating-bending fatigue machine of a capacity 15 Nm was used for the tests. The nominal stress $\sigma_n$ used for the analyses of the experimental results was the stress at the minimum cross section, where the diameter $d$ was 5 mm. The fatigue life $N_f$ was defined as the total number of stress cycles to failure.

4.2 Experimental results

Figure 7 shows $S-N$ curves obtained from the results of the tests. Figure 8 shows optical micrographs of a specimen when $\rho = 0.1$ mm for $\sigma_w = 90$ MPa. Fatigue limit $\sigma_w = 105$ when $\rho = 2$ mm. $\sigma_w = 95$ when $\rho = 1$ and 0.3 mm. $\sigma_w = 90$ when $\rho = 0.1$ mm. Since the non-propagating macrocrack was not observed at the fatigue
limit when \( \rho = 1 \) and 2 mm, it can be said that the microcrack non-propagating limit \( \sigma_{w1} \) or the small macrocrack non-propagating limit \( \sigma_{wd} \) appears as the fatigue limit. When \( \rho = 0.1 \) mm for \( \sigma_w = 90 \) MPa and \( \rho = 0.3 \) mm for \( \sigma_w = 95 \) MPa, the non-propagating macrocracks were observed along the notch root. Therefore, the long macrocrack non-propagating limit \( \sigma_{w2} = 90 \) MPa.

5. Examination of validity of prediction method

5.1 Notch sensitivity to crack initiation limit in age-hardened aluminum alloy

Figure 9 shows the relationship between \( K_t \sigma_{w1}/\sigma_{w0} \) and \( \rho \) using early fatigue data of previous studies [28–30]. \( \epsilon_0 \) values of the curves are shown in Figure 10. When \( \epsilon_0 \) values were approximated with the lines by the least squares method, the following equation was obtained:
\[ \varepsilon_0|_{\sigma_w=0} = 5.0 \times 10^{-4} H_B - 0.0164, \quad (40) \]
\[ \rho \geq 0.5, \quad 97 \leq H_B \leq 207. \]

(\(\varepsilon_0\) and \(\rho\) are in mm, and \(H_B\) is in kgf/mm\(^2\).

Once \(\sigma_w\) has been predicted, \(H_{VM}\) can be used instead of \(H_B\).

---

**Figure 9.**
Relation between \(K_t\sigma_w/\sigma_m\) and \(1/\rho\).

**Figure 10.**
Relation between \(\varepsilon_0\) and \(H_B\).
5.2 $\Delta K_{wUL}$ of age-hardened aluminum alloy

Figure 11 shows the values of $\Delta K_{wUL}$ obtained from the early fatigue data of $\sigma_{w2}$ for different Al-Si-X alloys, where X is a transition element [28, 29, 31]. An approximation of $\Delta K_{wUL}$ obtained from the lines by the least squares method was used to derive the following equation:

$$\Delta K_{wUL}|_{\sigma_{w}=0} = \Delta K_{wLL} + 0.03 H_B,$$

$$40 \leq H_B \leq 100.$$  \hspace{1cm} (41)

($\Delta K_{wUL}$ and $\Delta K_{wLL}$ are in MPa$\sqrt{m}$, and $H_B$ is in kgf/mm$^2$.)

Here, $\Delta K_{wLL}$ is the lower limit of $\Delta K_{w}$ and $\Delta K_{wLL} = 0.5$ MPa$\sqrt{m}$ for the Al-Si-X alloys.

5.3 Evaluation of statistical characteristics of inhomogeneous particles

The present aluminum cast alloy AC4B-T6 contains three main types of inhomogeneous particles, namely, eutectic Si and Fe compounds and porosity. Surrounding an irregular cross section with a smooth convex curve as shown in Figure 12, the area is defined as $area_A$. The values of $r$ are obtained from $area_A$ by the following equation [12, 37]:

$$r = \frac{\sqrt{area_A}}{\sqrt{\pi}}.$$  \hspace{1cm} (42)

Figure 13 shows the measured $M_{A0}$ values of eutectic Si and Fe compounds and porosity. The relationship between $M_{V0}$ and $M_{A0}$ is as follows [12, 37]:

\[ R = -1 \]

- 6061-T6 (Nisitani and Kawagoishi, 1985)
- 61S-T6 (Nisitani and Goto, 1976)
- 61S-T6

\[ (\text{Kawagoishi et al., 1990}) \]

\[ \Delta AC4CH-F \]
\[ \Delta AC4CH-T4 \]
\[ \Delta AC4CH-T6 \]

![Figure 11](image)

Relation between $\Delta K_{wUL}$ and $H_B$. 

17
Here, $M_{A0}(r_0)$ is the number of cross-sectional particles in a unit area for which $r \geq r_0$ on a unit area. Considering the assumption that $M_{V0}(R_0)$ is given by Eqs. (1) and (2), the asymptotic characteristics of Eq. (42) are expressed by the following equation [12, 37]:

$$M_{A0}(r_0) = 2 \int_{r_0}^{\infty} \sqrt{R^2 - r^2} \frac{dM_{V0}}{dR} dR. \quad (43)$$

The line of Eq. (44) is drawn to best fit the $M_{A0}$ values obtained by Eq. (43) to determine the values of $N_{V0}$, $\nu$, and $\lambda$.

**Figure 14** shows the values of $M_{V0}$ for porosity, Fe compounds, and eutectic Si. **Figure 15** shows the values of $M_{S0}$ for the porosity, Fe compounds, and eutectic Si.

**Figure 12.**
Definition of area $A$.

**Figure 13.**
$M_{A0}$ of porosity and eutectic Si and Fe compounds.
5.4 Evaluation of statistical characteristics of Vickers hardness of matrix without inhomogeneous particles

In this study, the Vickers hardness was measured at the position of 2.5–3.0 mm from the center on the circular cross section obtained by cutting the specimen grip under indentation load $P = 29.4$ mN in consideration with the stress distribution at

![Graph](image1)

**Figure 14.**
Relation between $M_{V0}$ and $R$.

![Graph](image2)

**Figure 15.**
Relation between $M_{S0}$ and $\sqrt{\text{area}_P}$.

5.4 Evaluation of statistical characteristics of Vickers hardness of matrix without inhomogeneous particles

In this study, the Vickers hardness was measured at the position of 2.5–3.0 mm from the center on the circular cross section obtained by cutting the specimen grip under indentation load $P = 29.4$ mN in consideration with the stress distribution at
the notch root. Figure 16 shows the results plotted on a normal probability paper. The sample mean $\mu_{HVM}$ and sample variance $\sigma^2_{HVM}$ were 91.8 kgf/mm$^2$ and 486.0 (kgf/mm$^2$)$^2$, respectively.

5.5 Evaluation of $\gamma_m$ value

Because the values of $R_m/p_m$ for the eutectic Si were much greater than those of the Fe compounds and porosity, as shown in Table 3, $\gamma_m$ was calculated using only the eutectic Si. The eutectic Si was assumed to be a rigid body [7], and the following values were used for the calculation: $E_M = 68$ GPa, $E_I = 105$ GPa, and $\nu_M = \nu_I = 0.3$ [35]. Using $R_m/p_m = 0.192$, $\gamma_m$ was determined to be 1.055.

5.6 Evaluation of $A_{npc\ S}$, $A_{npc\ R}$, and $A_{npc\ P}$ values

5.6.1 Evaluation of $A_{npc\ S}$ value

Because a microcrack often grows radially, it is approximated by the semielliptical crack shown in Figure 17. $A_{npc\ S}$ in Eq. (20) can also be roughly evaluated. The approximation of the microcrack by the semielliptical macrocrack is such that the crack non-propagation
limits are equal. If the macrocrack is located near the notch root of radius $\rho$ and it is sufficiently small, its non-propagation limit $\sigma_{\text{wdS}}$ is given by the following equation:

$$\sigma_{\text{wdS}} = \frac{1.43(H_{\text{VM}} + 120)}{F_p \sqrt{\text{area}_p}^{1/6}},$$  \hspace{1cm} (45)

$$F_p = 0.968 - 1.021 \times 10^{-3} \frac{\sqrt{\text{area}_p}}{\rho}.$$  \hspace{1cm} (46)

($\sigma_{\text{wdS}}$ is in MPa, $H_{\text{VM}}$ is in kgf/mm$^2$, $\sqrt{\text{area}_p}$ is in $\mu$m, and $\rho$ is in mm.)

Conversely, when the macrocrack is sufficiently large, $\Delta K_{w}$ is greater than $\Delta K_{wUL}$. The macrocrack is thus treated as being large, and its non-propagation limit is categorized as $\sigma_{w2}$, which is given by the following equation:

$$\sigma_{w2} = \frac{13.0(H_{\text{VM}} + 16.7)}{F_p \sqrt{\text{area}_p}^{1/2}}.$$  \hspace{1cm} (47)

($\sigma_{w2}$ is in MPa, $H_{\text{VM}}$ is in kgf/mm$^2$, and $\sqrt{\text{area}_p}$: in $\mu$m.)

$F_p$ is approximated to be 1. Because the average Vickers hardness of the matrix of the present AC4B-T6 is about 91.8 kgf/mm$^2$, the microcrack non-propagation limit $\sigma_{w0}$ was estimated to be 160 MPa using Eq. (17). The non-propagating crack length of the present AC4B-T6 for $\rho = 20$ mm was about 60 $\mu$m when $\sigma_n = 120$ MPa. From these experimental results, the $l_{\text{npc}0}$ of the present study was assumed to be 70 $\mu$m.

$l_{\text{npc}}$ was set to achieve $b/l = 0.4$. Using $c = 2.5$ mm, $l_{\text{npc}0} = 70$ $\mu$m is equivalent to $\sqrt{\text{area}_{\text{npc}0}}$ of 39.2 $\mu$m. Figure 18 shows the relationship between $l_{\text{npc}}$ and $1/\rho$.

5.6.2 Evaluation of $A_{\text{npc} R}$ and $A_{\text{npc P}}$ values

$A_{\text{npc} R}$ is a function of $P_R$ and $g_R$; $A_{\text{npc P}}$ is a function of $P_P$ and $g_P$. Because $g_R$ and $g_P$ can be calculated using Eqs. (26) and (33), respectively, only $P_R$ and $P_P$ need to

Figure 17.
Schematic illustration of the microcrack.
be further examined. In this study, it is assumed that $P_P = 0.3$ kgf. Considering the difficulty in evaluating $P_R$, it is also assumed that $P_R = P_P$.

5.7 Comparison and examination of predicted and experimental results

The fatigue limit reliability of the notched specimen shown in Figure 6 was predicted by the present method. The region in which the first principal stress is within the range of $\sigma_1^* = \sigma_1/\sigma_{\text{max}} = [0.95, 1]$ at the center of the specimen was adopted as the control volume. In this case, the region was ring-like.

When $H_B = 152$ kgf/mm$^2$ is used, the $\Delta K_{w\text{UL}} = 5.06$ MPa is predicted from Eq. (41). Because the value of $\xi$ for the present specimen is 0.167 (i.e., using $d = 5$ mm and $t = 0.5$ mm, as in Section 4.1), $F$ is 0.754 [39]. In this case, the predicted value of $\sigma_{w2}$ is 84.7 MPa. Considering that the experimentally determined value of $\sigma_{w2}$ is 90 MPa, the prediction is confirmed to be good.

Figure 19 shows the fatigue limit reliability $F_{\sigma_{w}}$. The thick solid line represents the case of $\rho = 2$ mm, whereas the thin solid line represents the case of $\rho = 0.3$ mm. The value of $F_{\sigma_{w}}$ for $\rho = 0.3$ mm suddenly changes from 0 to 1 when $\sigma_{w} = 84.7$ MPa, which is due to $\sigma_{w1}$ and $\sigma_{w2}$ being cut off by $\sigma_{w2}$. In other words, the inhomogeneous particles have almost no effect on the fatigue limit reliability in terms of initiating a fatigue crack. Instead, the eutectic Si actually strengthens the matrix.

Figure 20 shows the relationship between $\sigma_{w}$ and $1/\rho$. The solid line represents 50% reliability, the broken line represents 90% reliability, the single-dotted chain line represents 99% reliability, and the open marks represent the experimental results. Because the fatigue limit obtained by the ordinary fatigue test is equivalent to 50% fatigue limit reliability, the solid line agrees well with the open marks. The little differences between the open marks of $\rho = 0.3$ and 0.1 mm and the solid line can be attributed to the fact that $\Delta K_{w\text{UL}}$ of the present AC4B-T6 was unknown and...
the corresponding value for the of Al-Si-X alloy was used for predicting $\sigma_{w2}$. It is expected that an even better prediction accuracy would be achieved by using the true $\Delta K_{UL}$. Nevertheless, $\sigma_w$ was well predicted, which validated the proposed method for notched AC4B-T6 specimens.
6. Conclusions

This study proposed a nondestructive method for predicting the fatigue limit reliability of notched specimens of a metal containing inhomogeneous particles densely. The method was applied to aluminum cast alloy JIS-AC4B-T6. Rotating-bending fatigue tests were performed on the notched specimens of AC4B-T6 with notch root radius \( \rho = 2, 1, 0.3, \) and 0.1 in order to examine the validity of the present method. Since the non-propagating macrocracks were observed along the notch root, the long macrocrack non-propagating limit \( \sigma_{w2} \) appears as the fatigue limit when \( \rho = 0.1 \) and 0.3 mm. On the other hand, since the non-propagating macrocrack was not observed when \( \rho = 1 \) and 2 mm, it can be said that the microcrack non-propagating limit \( \sigma_{w1} \) or the small macrocrack non-propagating limit \( \sigma_{wd} \) appears as the fatigue limit. The fatigue limits predicted by the present method were in good agreement with the experimental ones.

The method is not only convenient for use in predicting fatigue strength reliability for the reliable design of machine and structures, but it is also time effective and can be applied to the economic development of materials.

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