Radiative meson transitions in the quark model

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Abstract.
Radiative meson transitions in the light and heavy flavor sector are investigated in a potential quark model. Relativistic corrections and higher order diagrams are analyzed. The decay rates calculated with the relativistic corrections taken into account are in most of the cases in better agreement with the experimental values. Higher order diagrams seem to have larger a effect for the light quark decays.

1. Introduction
Meson and baryon radiative transitions deserve a lot of investigation since they are very easily produced and could help us understand theory of strong interactions. Since they belong to the non-perturbative regime of QCD they cannot be described from first principles. One of the theories which had number of successes in describing non perturbative part of QCD is the quark model. In particular, the quark model can describe the whole spectrum of meson masses. But one needs other observables in order to better understand strong interactions since very different potentials can lead to very similar mass spectra of mesons.

One of the options here is to study transitions between various mesons which are very sensitive to the inter-quark potential. In particular, electromagnetic transitions can provide significant help in testing various meson potentials and wave functions since the transition operator is very well known.

In dealing with radiative transitions some typical approximations are usually in use. Some of them are impulse approximation, dipole approximation for E1 transitions, long wave length approximation, non relativistic approximation. Also spherical harmonic oscillator (SHO) wave functions are widely used to represent the meson wave functions. And almost in all the cases the study of radiative transitions is performed only for the particular sector of meson spectra (for example only heavy or only light mesons) [1, 2, 3, 4].

Most of these approximations are taken from atomic and nuclear physics where they describe radiative transitions rather well. But when applied to mesons they are not always justified. For example, long wave length approximation is defined by condition \( k_\gamma R \ll 1 \) where \( k_\gamma \) is the photon momentum, and \( R \) is the size of the source. For the meson radiative transitions typically \( k_\gamma = 0.1 - 0.5 \text{GeV} \) and \( R = 0.5 - 1 \text{fm} = 2.5 - 5 \text{GeV}^{-1} \) so that the long wave length condition is not always true. Also the long wave length approximation leads to neglecting the recoil of the final meson, and in reality the momentum of the final meson is often comparable to its mass. That convinces us that not only recoil should not be neglected but the nonrelativistic approximation is not suited in this case. To preserve gauge invariance both transition operator and meson wave functions should be relativistic. Some attempts have been made to take the
relativistic effects in radiative meson transitions into account but other approximations have been made which can have a larger effect on the result [2, 5, 6].

The motivation for our work was to perform a detailed study of meson radiative transitions and investigate the effects of different approximations. We used wave functions calculated from the realistic potentials as well as SHO wave functions (for comparison). Relativistic corrections in the transition operator as well as in the wave functions have been taken into account. Higher order diagrams (beyond impulse approximation) have been analyzed. The decays rates have been calculated for all available experimental results from the Particle Data Group (Table 1).

| Process             | \( \gamma \text{(MeV)} \) | Gaussian nonrel | rel | Coulomb+linear nonrel | rel | Experiment |
|---------------------|-----------------------------|-----------------|-----|------------------------|-----|------------|
| \( J/\psi \rightarrow \gamma \eta_c \) | 115                         | 2.85            | 2.52| 2.82                   | 2.11| 1.13       |
| \( X_{C0} \rightarrow \gamma J/\psi \) | 303                         | 194             | 167 | 349                    | 276 | 165        |
| \( X_{C1} \rightarrow \gamma J/\psi \) | 389                         | 221             | 193 | 422                    | 325 | 291        |
| \( X_{C2} \rightarrow \gamma J/\psi \) | 430                         | 137             | 114 | 352                    | 260 | 389        |
| \( \Psi(2S) \rightarrow \gamma X_{C0} \) | 261                         | 29.1            | 22.1| 19.8                   | 11.5| 26.1       |
| \( \Psi(2S) \rightarrow \gamma X_{C1} \) | 171                         | 60.8            | 45.3| 39.6                   | 22.6| 25.2       |
| \( \Psi(2S) \rightarrow \gamma X_{C2} \) | 127                         | 76.0            | 57.4| 49.6                   | 29.1| 20.4       |
| \( h_c \rightarrow \gamma \eta_c \) | 496                         | 189             | 162 | 497                    | 363 |            |
| \( \Upsilon(2S) \rightarrow \gamma \chi_{b0}(1P) \) | 162                         | 0.80            | 0.76| 0.24                   | 0.15| 1.67       |
| \( \Upsilon(2S) \rightarrow \gamma \chi_{b1}(1P) \) | 130                         | 1.96            | 1.84| 0.60                   | 0.36| 2.99       |
| \( \Upsilon(2S) \rightarrow \gamma \chi_{b2}(1P) \) | 110                         | 2.74            | 2.60| 0.83                   | 0.53| 3.08       |
| \( \Upsilon(3S) \rightarrow \gamma \chi_{b0}(2P) \) | 122                         | 1.23            | 1.11| 0.45                   | 0.36| 1.42       |
| \( \Upsilon(3S) \rightarrow \gamma \chi_{b1}(2P) \) | 100                         | 3.01            | 2.74| 1.10                   | 0.82| 2.97       |
| \( \Upsilon(3S) \rightarrow \gamma \chi_{b2}(2P) \) | 86                          | 4.35            | 4.02| 1.61                   | 1.21| 3.00       |
| \( \rho^0 \rightarrow \gamma \pi^0 \) | 374                         | 50.5            | 20.5| 41.3                   | 13.0| 11.8       |
| \( \rho^+ \rightarrow \gamma \pi^\pm \) | 373                         | 50.4            | 20.5| 41.1                   | 13.0| 67.1       |
| \( \rho^- \rightarrow \gamma \pi^- \) | 191                         | 53.7            | 23.5| 39.9                   | 14.2| 56.7       |
| \( \omega \rightarrow \gamma \pi^0 \) | 380                         | 473             | 191.3| 387.1                 | 121.3| 73.4       |
| \( \omega \rightarrow \gamma \eta \) | 200                         | 6.70            | 3.13| 4.99                   | 1.77| 5.49       |
| \( \eta' \rightarrow \gamma \rho^0 \) | 168                         | 121             | 57.2| 91.0                   | 33.4| 59.6       |
| \( \eta' \rightarrow \gamma \omega \) | 159                         | 11.6           | 5.48| 8.63                   | 3.18| 6.12       |
| \( \phi \rightarrow \gamma \eta \) | 363                         | 43.4            | 32.8| 47.8                   | 32.8| 55.3       |

2. Calculations
Nonrelativistic meson wave functions have been calculated from the Schrodinger equation with the Hamiltonian:

\[ H_{NR} = \frac{p^2}{2m} + V(r) \]

\[ V(r) = -\frac{4}{3} \frac{\alpha}{r} + br \]
The electromagnetic transition operator in nonrelativistic case is:

\[ H_{em} = -\frac{e}{m} \mathbf{A} \cdot \mathbf{p} \]

and relativistic is:

\[ H_{em} = -e\alpha \cdot \mathbf{A} \]

which we can get from the relativistic Hamiltonian:

\[
H_{rel} = \int d\mathbf{x} \psi^\dagger (-i\alpha \cdot \nabla + \beta \mathbf{m})\psi \\
+ \frac{1}{2} \int d\mathbf{y}d\mathbf{y} \psi^\dagger (\mathbf{x}) \frac{\lambda^a}{2} \psi(\mathbf{x}) \mathbf{V}(\mathbf{x} - \mathbf{y}) \psi^\dagger (\mathbf{y}) \frac{\lambda^a}{2} \psi(\mathbf{y})
\]

In the simplest case of SHO wave functions and nonrelativistic transition operator decay rates for E1 and M1 transitions are:

\[
\Gamma(3S_1 \rightarrow 1S_1 \gamma) = \frac{1}{3} \alpha \left( \frac{Q_1}{m_1} + \frac{Q_2}{m_2} \right)^2 \frac{E_f k^3 e^{-k^2/8\beta^2}}{m_i}
\]

\[
\Gamma(3P_J \rightarrow 3S_1 \gamma) = \frac{2}{3} \alpha \left( \frac{Q_1}{m_1} + \frac{Q_2}{m_2} \right)^2 \frac{E_f k^3 e^{-k^2/8\beta^2}}{m_i}
\]

\[
\Gamma(2S_1 \rightarrow 3P_J \gamma) = (2J + 1) \frac{16}{81} \alpha \left( \frac{Q_1}{m_1} + \frac{Q_2}{m_2} \right)^2 \frac{E_f k^3 e^{-k^2/8\beta^2}}{m_i}
\]

For realistic wave functions and relativistic transition operator decay rates have been calculated numerically. Some of the results are presented in Table 1.

In order to consider higher order diagrams in the nonrelativistic case we have to include an additional term in Hamiltonian which describes the possibility of quark-antiquark pair appearing from the vacuum.

One way to do that is to take the nonrelativistic approximation of quark-antiquark pair term of a relativistic QCD Hamiltonian. In this case we get Cornell model term [5]:

\[
H_{pc} = \int d^3k d^3p d^3q \frac{\lambda^a\lambda^b}{4} K^{(0)}(q) b_k^\dagger (\sigma \cdot q) d_{-k-q}^\dagger b_p b_{p+q} + \text{other terms}
\]

In practice it is common to use phenomenological $^3P_0$ model to describe the possibility of quark-antiquark pair appearing nonrelativistically. The reason for that is that $^3P_0$ model describes hadron transitions with better agreement with experiment than Cornell model. The operator for $^3P_0$ model is [7]:

\[
H_{^3P_0} = \gamma \int d^3k \ b_k^\dagger (\sigma \cdot k) d_{-k}^\dagger + H.C.
\]

When we include $^3P_0$ term in the nonrelativistic Hamiltonian there are four possible higher order diagrams (Figure 1).

Amplitudes of higher order diagrams have to be summed over all possible intermediate states. For diagrams 3 and 4 these sums are equal to zero. For diagrams 1 and 2 they are not equal to zero and give significant contribution to the impulse approximation diagrams especially for light mesons.

A comparison of the higher order diagram amplitudes to the impulse approximation is presented in Table 2.
Figure 1. Second order diagrams.

Table 2. Higher order diagram amplitudes in comparison to impulse approximation. ($\text{GeV}^{-1/2}$)

|       | Impulse | Diagram1 | Diagram2 |
|-------|---------|----------|----------|
| $\rho \rightarrow \gamma\pi$ | 0.057 ±0.061 | ±0.005 |
| $J/\psi \rightarrow \gamma\eta_c$ | 0.032 ±0.038 | ±0.013 |

3. Summary
Different aspects of radiative meson transitions have been analyzed. Although for some of the transitions our formalism gives satisfactory agreement with experiment, it is obvious that some physics is still missing. The next step is to include relativistic effects in the wave functions and in the higher order diagrams.

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