Electroweak and QCD Corrections to $Z$ and $W$ pole observables in the SMEFT

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Abstract

We compute the next-to-leading order QCD and electroweak corrections to $Z$ and $W$ pole observables using the dimension-6 Standard Model effective field theory and present numerical results that can easily be included in global fitting programs. Limits on SMEFT coefficient functions are presented at leading order and at next-to-leading order under several assumptions.
I. INTRODUCTION

The LHC experiments provide strong evidence that the $SU(3) \times SU(2) \times U(1)$ Standard Model (SM) gauge theory describes physics at the electroweak scale[1]. To date, there is no evidence of new interactions or high mass particles. Taken together, these features suggest that the weak scale can be described by an effective field theory (SMEFT) having the SM as its low energy limit. The SMEFT is defined by an infinite tower of on-shell and higher operators, involving only the SM particles and assumes that the Higgs boson is part of an $SU(2)$ doublet[2]. The effects of the higher dimension operators are suppressed by powers of a high scale, $\Lambda$, and we assume that the most numerically relevant operators are those of dimension-6. All possible new physics phenomena are contained in the coefficient functions.

Numerous studies have been performed extracting limits on the coefficients of dimension-6 operators from global fits to Higgs measurements, vector boson pair production, electroweak measurements at the $Z$ and $W$ poles, top quark measurements, and low energy data[3–10]. Typically, these fits use the most accurately known SM predictions, while the SMEFT effects are treated at lowest order (LO). A program of calculations has begun to treat the SMEFT contributions at NLO, for both the QCD and electroweak (EW) contributions. The SMEFT QCD corrections to gauge boson pair production[11, 12] and top quark production and decay[13–16] are known. The electroweak SMEFT corrections to Higgs decays to $b\bar{b}$[17–19], $\gamma\gamma$[20–23], $Z\gamma$[24, 25], $ZZ$[24], and $WW$[20] have also been computed, along with partial corrections to the Drell Yan process[26]. The SMEFT NLO corrections to $Z$ pole decays at NLO are also only partially known[27–29].

In this work, we take a major step by computing the next-to-leading order (NLO) EW and QCD corrections in the SMEFT to $Z$ and $W$ pole observables. We assume flavor universality and use the Warsaw basis[30]. We are particularly interested in the numerical effects of the NLO corrections on the global fits. In Section II, we review the basics of the SMEFT theory and in Section III, we describe our NLO calculations. Our results are given in Section IV and Appendix A, which contains numerical expressions for the $Z$ and $W$ pole observables, as well as limits on the SMEFT coefficients at LO and NLO. Section V contains some conclusions and a discussion of the implications of our results for global fits.
\[ O_{H} (\bar{l} \gamma \mu l) (\bar{l} \gamma \mu l) \]  
\[ O_{\phi W B} (\phi^\dagger \tau^a \phi) W^a_{\mu \nu} B^{\mu \nu} \]  
\[ O_{\phi D} (\phi^\dagger D^\mu \phi) (\phi D^\mu \phi) \]  
\[ O_{\phi e} (\phi^\dagger D^\mu \phi) (\bar{q} \tau^a \gamma^\mu q) \]  
\[ O_{\phi u} (\phi^\dagger D^\mu \phi) (\bar{u} \gamma^\mu u) \]  
\[ O_{\phi d} (\phi^\dagger D^\mu \phi) (\bar{d} \gamma^\mu d) \]  
\[ O_{\phi q} (\phi^\dagger D^a \phi) (\bar{q} \tau^a \gamma^\mu q) \]  
\[ O_{\phi l} (\phi^\dagger D^a \phi) (\bar{l} \gamma^\mu l) \]

| \( O_{H} \) | \( (\bar{l} \gamma \mu l) (\bar{l} \gamma \mu l) \) | \( O_{\phi W B} \) | \( (\phi^\dagger \tau^a \phi) W^a_{\mu \nu} B^{\mu \nu} \) | \( O_{\phi D} \) | \( (\phi^\dagger D^\mu \phi) (\phi D^\mu \phi) \) |
| --- | --- | --- | --- | --- | --- |
| \( O_{\phi e} \) | \( (\phi^\dagger D^\mu \phi) (\bar{e} \gamma^\mu e) \) | \( O_{\phi u} \) | \( (\phi^\dagger D^\mu \phi) (\bar{u} \gamma^\mu u) \) | \( O_{\phi d} \) | \( (\phi^\dagger D^\mu \phi) (\bar{d} \gamma^\mu d) \) |
| \( O^{(3)}_{\phi q} \) | \( (\phi^\dagger i \hat{D}^a \phi) (\bar{q} \gamma^\mu \gamma^\mu q) \) | \( O^{(1)}_{\phi q} \) | \( (\phi^\dagger i \hat{D}^a \phi) (\bar{q} \gamma^\mu \gamma^\mu q) \) | \( O^{(3)}_{\phi l} \) | \( (\phi^\dagger i \hat{D}^a \phi) (\bar{l} \gamma^\mu \gamma^\mu l) \) |

**TABLE I**: Dimension-6 operators contributing to the \( Z \) and \( W \) pole observables of this study at tree level.

**II. SMEFT BASICS**

The SMEFT parameterizes new physics through an expansion in higher dimensional operators,

\[ \mathcal{L} = \mathcal{L}_{SM} + \sum_{k=5}^{\infty} \sum_{i=1}^{n} \frac{C_i^k}{\Lambda_k^4} O_i^k, \]

where the \( SU(3) \times SU(2)_L \times U(1)_Y \) invariant dimension-\( k \) operators are constructed from SM fields and all of the effects of the beyond the SM (BSM) physics reside in the coefficient functions, \( C_i^k \). We assume all coefficients are real and do not consider the effects of CP violation. We use the Warsaw basis [30] and at tree level (neglecting flavor) there are 10 dimension-6 operators contributing to the \( Z \) and \( W \) pole observables of our study. These operators are listed in Table I, where \( \phi \) is the \( SU(2)_L \) doublet, \( \tau^a \) are the Pauli matrices, \( D_\mu = \partial_\mu + i g_s T^A G^A_\mu + i g_2 \frac{\alpha_s}{2} W^a_\mu + i g_1 Y B_\mu \), \( q^T = (u_L, d_L) \), \( t^T = (v_L, e_L) \), \( W^a_{\mu \nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g_2 \epsilon^{abc} W^b_\mu W^c_\nu \), \( \phi^\dagger i \hat{D}^a \phi = i \phi^\dagger (D^a_\mu \phi) - i (D^a_\mu \phi)^\dagger \phi \), and \( \phi^\dagger i \hat{D}^a_\mu \phi = i \phi^\dagger \tau^a D^a_\mu \phi - i (D^a_\mu \phi)^\dagger \tau^a \phi \).

At NLO, there are 22 additional operators that contribute:

\[ O_{ed}, O_{ee}, O_{eu}, O_{td}, O_{te}, O^{(1)}_{iq}, O^{(3)}_{iq}, O_{\phi B}, O_{\phi W}, O_{\square}, \]
\[ O_{qe}, O_{uB}, O_{uW}, O_{W}, O^{(1)}_{qd}, O^{(3)}_{qq}, O^{(1)}_{qq}, O^{(1)}_{ud}, O_{uu}, O_{dd}. \]

Definitions for these operators can be found in Refs. [30?]. We use the Feynman rules in \( R_\xi \) gauge from Ref. [31].

The SMEFT interactions cause the gauge field kinetic energies to have non-canonical
normalizations and following Ref. [31], we define "barred" fields and couplings,

\[ \overline{W}_\mu \equiv (1 - C_\phi \mu^2 / \Lambda^2) W_\mu, \]
\[ \overline{B}_\mu \equiv (1 - C_\phi B^2 / \Lambda^2) B_\mu, \]
\[ \overline{g}_2 \equiv (1 + C_\phi W^2 / \Lambda^2) g_2, \]
\[ \overline{g}_1 \equiv (1 + C_\phi B^2 / \Lambda^2) g_1, \]

(3)
such that \( \overline{W}_\mu \overline{g}_2 = W_\mu g_2 \) and \( \overline{B}_\mu \overline{g}_1 = B_\mu g_1 \). The "barred" fields have canonically normalized kinetic energy interactions. The masses of the W and Z fields to \( \mathcal{O} \left( \frac{1}{\Lambda^2} \right) \) are [31, 32],

\[ M^2_W = \frac{\overline{g}^2_2 v^2}{4}, \]
\[ M^2_Z = \frac{(\overline{g}^2_1 + \overline{g}^2_2) v^2}{4} + \frac{v^4}{\Lambda^2} \left( \frac{1}{8} (\overline{g}^2_1 + \overline{g}^2_2) C_{\phi D} + \frac{1}{2} \overline{g}_1 \overline{g}_2 C_{\phi WB} \right). \]

(4)

Dimension-6 4-fermion operators give contributions to the decay of the \( \mu \), changing the relation between the vev, \( v \), and the Fermi constant \( G_\mu \),

\[ G_\mu \equiv \frac{1}{\sqrt{2}v^2} - \frac{1}{\sqrt{2}\Lambda^2} C_{\mu} + \frac{\sqrt{2}}{\Lambda^2} C_{\phi l}^{(3)}. \]

(5)

The tree level SMEFT couplings of fermions to the Z and W are given in terms of our input parameters \( (\alpha, M_Z, G_\mu) \),

\[ L \equiv 2M_Z \sqrt{2G_\mu} Z \left[ g_L^Z q + \delta g_L^Z q \right] \overline{q}_R \gamma_\mu q + 2M_Z \sqrt{\sqrt{2G_\mu} Z \left[ g_R^Z u + \delta g_R^Z u \right] \overline{u}_R \gamma_\mu u_R \]
\[ + 2M_Z \sqrt{2G_\mu} Z \left[ g_R^Z d + \delta g_R^Z d \right] \overline{d}_R \gamma_\mu d_R + 2M_Z \sqrt{\sqrt{2G_\mu} Z \left[ g_L^Z l + \delta g_L^Z l \right] \overline{l}_R \gamma_\mu l \]
\[ + 2M_Z \sqrt{\sqrt{2G_\mu} Z \left[ g_R^Z e + \delta g_R^Z e \right] \overline{e}_R \gamma_\mu e_R + 2M_Z \sqrt{\sqrt{2G_\mu} \left( \delta g_R^Z \nu \right) \overline{\nu}_R \gamma_\mu \nu_R \]
\[ + \frac{\overline{g}_2}{\sqrt{2}} \left[ W_\mu \left( 1 + \delta g_L^W q \right) \overline{n}_L \gamma_\mu d_L + \left( \delta g_R^W q \right) \overline{d}_R \gamma_\mu d_R \right] \]
\[ + W_\mu \left( 1 + \delta g_L^W l \right) \overline{\nu}_L \gamma_\mu e_L + \left( \delta g_R^W \nu \right) \overline{e}_R \gamma_\mu e_R \right] + h.c. \].

(6)

We assume all couplings are flavor independent and we neglect CKM mixing. The weak coupling in Eq. 6 is evaluated using the LO SM relation and Eq 7 serves as the definition of \( s^2_W \),

\[ \overline{g}_2^2 = 2\sqrt{2G_\mu} M^2_Z \left( 1 + \sqrt{1 - \frac{4\pi \alpha}{\sqrt{2G_\mu} M^2_Z}} \right). \]
\[ s^2_W \equiv \frac{4\pi \alpha}{\overline{g}_2^2}. \]

(7)
\[
\begin{array}{|c|c|}
\hline
\text{Warsaw Basis} & \\
\hline
\delta g_{L}^{Zu} & -\frac{v^2}{2\Lambda^2} \left( C_{\phi q}^{(1)} - C_{\phi q}^{(3)} \right) + \frac{1}{2} \delta g_{Z} + \frac{2}{3} \left( \delta s_{W}^2 - s_{W}^2 \delta g_{Z} \right) \\
\delta g_{L}^{Zd} & -\frac{v^2}{2\Lambda^2} \left( C_{\phi q}^{(1)} + C_{\phi q}^{(3)} \right) - \frac{1}{2} \delta g_{Z} - \frac{1}{3} \left( \delta s_{W}^2 - s_{W}^2 \delta g_{Z} \right) \\
\delta g_{L}^{Zu} & -\frac{v^2}{2\Lambda^2} \left( C_{\phi l}^{(1)} - C_{\phi l}^{(3)} \right) + \frac{1}{2} \delta g_{Z} \\
\delta g_{L}^{Ze} & -\frac{v^2}{2\Lambda^2} \left( C_{\phi l}^{(1)} + C_{\phi l}^{(3)} \right) - \frac{1}{2} \delta g_{Z} - \left( \delta s_{W}^2 - s_{W}^2 \delta g_{Z} \right) \\
\delta g_{R}^{Zu} & -\frac{v^2}{2\Lambda^2} C_{\phi u} + \frac{2}{3} \left( \delta s_{W}^2 - s_{W}^2 \delta g_{Z} \right) \\
\delta g_{R}^{Zd} & -\frac{v^2}{2\Lambda^2} C_{\phi d} - \frac{1}{3} \left( \delta s_{W}^2 - s_{W}^2 \delta g_{Z} \right) \\
\delta g_{R}^{Ze} & -\frac{v^2}{2\Lambda^2} C_{\phi e} - \left( \delta s_{W}^2 - s_{W}^2 \delta g_{Z} \right) \\
\delta g_{L}^{Wq} & \frac{v^2}{\Lambda^2} C_{\phi q}^{(3)} + \frac{2}{3} \delta s_{W}^2 + \delta s_{W}^2 \\
\delta g_{L}^{Wl} & \frac{v^2}{\Lambda^2} C_{\phi l}^{(3)} + \frac{2}{3} \delta s_{W}^2 + \delta s_{W}^2 \\
\delta g_{Z} & -\frac{v^2}{\Lambda^2} \left( \delta v + \frac{1}{4} C_{\phi D} \right) \\
\delta v & C_{\phi l}^{(3)} - \frac{1}{2} C_{\phi l} \\
\delta g_{W}^{2} & \frac{v^2}{\Lambda^2} - \frac{s_{W} c_{W}}{\Lambda^2 - s_{W}} \left[ 2 s_{W} c_{W} \left( \delta v + \frac{1}{4} C_{\phi D} \right) + C_{\phi WB} \right] \\
\hline
\end{array}
\]

TABLE II: Anomalous fermion couplings at LO in the Warsaw [30] basis.

Since we are working to \( O \left( \frac{v^2}{\Lambda^2} \right) \), we omit dipole type operators that do not interfere with the SM contributions to \( Z \) and \( W \) pole observables. Similarly, the contributions from right-handed \( W \) couplings and the right-handed \( Z \nu \nu \) interaction do not contribute to our study. The tree level couplings are,

\[
g_{R}^{Zf} = -s_{W}^2 Q_{f} \quad \text{and} \quad g_{L}^{Zf} = T_{3} f - s_{W}^2 Q_{f}
\]

with \( T_{3} f = \pm \frac{1}{2} \). \( SU(2) \) invariance implies,

\[
\delta g_{L}^{Wq} = \delta g_{L}^{Zu} - \delta g_{L}^{Zd} \quad \text{and} \quad \delta g_{L}^{Wl} = \delta g_{L}^{Zv} - \delta g_{L}^{Ze}.
\]

The SMEFT contributions to the effective couplings are listed in Table II[33].

III. \( W \) AND \( Z \) POLE OBSERVABLES TO NLO

The observables we consider are:

\[
M_{W}, \Gamma_{W}, \Gamma_{Z}, \sigma_{h}, R_{t}, A_{l, FB}, R_{b}, R_{c}, A_{FB,b}, A_{FB,c}, A_{b}, A_{c}, A_{l}.
\]
The SM results for these observables are quite precisely known, and as a by product of our study we recover the known NLO QCD and NLO EW results as a check of our calculation\cite{34}. The next-to-leading order contributions to $Z$ and $W$ pole observables require the calculation of one loop virtual diagrams in the SMEFT and in most cases, the contribution also of real photon and gluon emission diagrams. Since the SMEFT theory is renormalizable order by order in the ($v^2/\Lambda^2$) expansion, we retain only terms of $\mathcal{O}(v^2/\Lambda^2)$. The one-loop SMEFT calculations contain both tree level and one-loop contributions from the dimension-6 operators, along with the full electroweak and QCD one-loop SM amplitudes. Sample diagrams contributing to the $Z$ decay widths at NLO are shown in Fig. 1. For $W$ decays, there are also dipole-like $\gamma(g)WF$ contact interactions that we include. (The corresponding $\gamma(g)ZF$ operators first arise at dimension-8.) Since we concentrate on the $Z$ pole physics, we calculate the cross sections, $e^+e^- \rightarrow$ hadrons, using the narrow width approximation:

$$\sigma_{\text{had}}^0 = \sum_{f=u,d,s,c,b} \frac{12\pi}{M_Z^2} \frac{\Gamma_f}{\Gamma_Z^2}. \quad (11)$$

Corrections to this formula are of higher order and we do not include them \cite{35}. Non-resonant contributions, such as photon exchange, box diagrams and 4-fermions interactions \cite{33}, are also not included because they do not contribute to the observables on the $Z$ pole to $\mathcal{O}(1/\Lambda^2)$.

We employ a modified on-shell (OS) scheme, where the SM parameters are renormalized in the OS scheme. The effective field theory coefficients of the dimension-6 operators are treated as $\overline{MS}$ parameters and the poles of the one-loop coefficients $C_i$ are known from Refs. \cite{32,36,37},

$$C_i(\mu) = C_{0,i} - \frac{1}{2\hat{\epsilon}} \frac{1}{16\pi^2} \gamma_{ij} C_j, \quad (12)$$

where $\mu$ is the renormalization scale, $\gamma_{ij}$ is the one-loop anomalous dimension,

$$\hat{\epsilon} = \frac{dC_i}{d\mu} = \frac{1}{16\pi^2} \gamma_{ij} C_j, \quad (13)$$

and $\hat{\epsilon}^{-1} \equiv \epsilon^{-1} - \gamma_E + \log(4\pi)$.

The renormalized SM gauge boson masses are,

$$M_V^2 = M_{0,V}^2 - \Pi_{VV}(M_V^2), \quad (14)$$

where $\Pi_{VV}(M_V^2)$ is the one-loop correction to the 2-point function for $Z$ or $W$ computed on-shell and tree level quantities are denoted with the subscript 0 in this section. The gauge
FIG. 1: Sample electroweak diagrams contributing to $Z \to f\bar{f}$ at NLO in the SMEFT: (a) Tree level SMEFT diagram, (b) virtual SMEFT diagram, and (c) real photon emission in the SMEFT. The circles represent potential insertions of dimension-6 SMEFT operators.

Boson 2-point functions in the SMEFT can be found analytically in Refs. [38, 39]. The one-loop relation between the vacuum expectation value and the Fermi constant is,

$$G_\mu + \frac{C_{ll}}{\sqrt{2}\Lambda^2} - \sqrt{2}\frac{C_{\phi l}^{(3)}}{\Lambda^2} \equiv \frac{1}{\sqrt{2}v_0^2}(1 + \Delta r),$$

where $v_0$ is the unrenormalized minimum of the potential and $\Delta r$ is obtained from the one-loop corrections to $\mu$ decay. Complete analytic expressions for $\Delta r$ in both the SM and the SMEFT at dimension-6 are given in Ref. [24]. Finally, the on-shell renormalization of $\alpha$ is extracted from the renormalization of the $l\bar{l}\gamma$ vertex.

We obtain the relevant amplitudes for the virtual contributions using FeynArts [40] with a model file generated by FeynRules [41] and the Feynman rules of Ref. [31]. Then we use FeynCalc [42, 43] to manipulate and reduce the integrals and LoopTools [44] for the numerical evaluation.

The $Z$ decays to charged fermions receive contributions from one-loop virtual diagrams and from real photon emission that are separately IR divergent and we regulate these divergences with a photon mass. Since we only consider the inclusive quantities of Eq. 10, the photon mass dependence cancels after integration over the photon phase space and there
is no need for a photon energy cut. The complicated form of the SMEFT vertices makes
direct integration of the phase space difficult, so we use the method of Ref. [45], where the
integration over the photon phase space is replaced with a loop integration. This is possible
after we use the identity,
\begin{equation}
2i\pi \delta(p^2 - m^2) = \frac{1}{p^2 - m^2 + i0} - \frac{1}{p^2 - m^2 - i0}.
\end{equation}
(16)
After making this replacement, we treat the momenta of the outgoing particles as internal
loop momenta, the integration over the phase space becomes an integration over the loop
momenta and we can use the IBP relations to reduce the loop integrals to known master
integrals. In the case of $Z \to f\bar{f}\gamma$, the integrals are 2-point 2-loop integrals, for which a
generic basis of master integrals is known [46, 47] and the reduction can be done using FIRE
[48]. This is identical to the technique we applied in the calculation of the real contributions
to $H \to W^+W^-\gamma$ in Ref. [20].

IV. RESULTS

We take as our physical input parameters,
\begin{align*}
G_\mu &= 1.1663787(6) \times 10^{-5}\text{GeV}^{-2} \\
M_Z &= 91.1876 \pm .0021\text{GeV} \\
\frac{1}{\alpha} &= 137.035999139(31) \\
\Delta\alpha^{(5)}_{\text{had}} &= 0.02764 \pm 0.00009 \\
\alpha_s(M_Z) &= 0.1181 \pm 0.0011 \\
M_H &= 125.10 \pm 0.14 \text{GeV} \\
M_t &= 172.9 \pm 0.5 \text{GeV}.
\end{align*}

The lowest order SMEFT contributions to $Z$ pole observables, $O_i$ are well known. We
write the SMEFT predictions for the observables as,
\begin{align*}
O_i^{\text{SMEFT,LO}} &= O_i^{\text{SM}} + \delta O_i^{LO} \\
O_i^{\text{SMEFT,NLO}} &= O_i^{\text{SM}} + \delta O_i^{NLO},
\end{align*}
(17)
and we present our results numerically. In Table III, we summarize the current state of
the SM theory and the experimental results. The theory errors include the parametric
TABLE III: Experimental results and SM predictions for $W$ and $Z$ pole observables, assuming lepton universality. The theory includes the full set of 2-loop contributions for the $Z$ pole observables, along with higher order corrections when known. When not specified, the numbers are taken from Table 10.5 of the electroweak review of Ref. [55]. The theory predictions are computed using the formulae in the indicated references and our input parameters, and the theory errors include the parametric uncertainties on $M_t$ and $M_H$ [35], along with the estimated theory uncertainties described in the respective papers.

| Measurement      | Experiment          | ”Best” theory                                           |
|------------------|---------------------|---------------------------------------------------------|
| $\Gamma_Z$(GeV)  | 2.4952 ± 0.0023     | 2.4945 ± 0.0006 [35, 49, 50]                            |
| $\sigma_h$(nb)   | 41.540 ± 0.037      | 41.491 ± 0.008[35, 49, 50]                             |
| $R_l$            | 20.767 ± 0.025[51]  | 20.749 ± 0.009[35, 49, 50]                             |
| $R_b$            | 0.21629 ± 0.00066   | 0.21586 ± 0.0001[35, 49, 50]                           |
| $R_c$            | 0.1721 ± 0.0030     | 0.17221 ± 0.00005[35, 49, 50]                          |
| $A_l$            | 0.1465 ± 0.0033[51] | 0.1472 ± 0.0004 [35, 52]                               |
| $A_c$            | 0.670 ± 0.027       | 0.6679 ± 0.0002[35, 52]                                |
| $A_b$            | 0.923 ± 0.020       | 0.92699 ± 0.00006[35, 52, 53]                          |
| $A_{l,F,B}$      | 0.0171 ± 0.0010     | 0.0162 ± 0.0001 [35, 52]                               |
| $A_{b,F,B}$      | 0.0992 ± 0.0016     | 0.1023 ± 0.0003 [35, 52, 53]                           |
| $A_{c,F,B}$      | 0.0707 ± 0.0035     | 0.0737 ± 0.0003 [35, 52]                               |
| $A_l(SLD)$       | 0.1513 ± 0.0021[51] | 0.1472 ± 0.0004[35, 52]                                |
| $\sin^2 \theta_{l,eff}$ | 0.23179 ± 0.00035 [54] | 0.23150 ± 0.00006[35, 52, 53]                       |
| $M_W$(GeV)       | 80.379 ± 0.012 [55] | 80.359 ± 0.006[56, 57]                                 |
| $\Gamma_W$(GeV)  | 2.085 ± 0.042 [55]  | 2.0904 ± 0.0003[58]                                   |

uncertainties on $M_t$ and $M_H$ [35]. In evaluating $O^{SM}_i$ in Eq. 17, we always use the most accurately calculated value given in Table III.

We do not include the effective weak leptonic mixing angle in our fit since it can be
directly derived from other observables, but present it here for completeness.

\[
\delta \sin^2 \theta_{\ell, \text{eff}}^{\text{LO}} = \frac{v^2}{\Lambda^2} \left\{ -0.28785C_{\phi e} - 0.21215C_{\phi d}^{(1)} + 0.36851C_{\phi l}^{(3)} - 0.29033C_{\phi l} \\
+ 0.14517C_{\phi D} + 0.71015C_{\phi WB} \right\}
\]

\[
\delta \sin^2 \theta_{\ell, \text{eff}}^{\text{NLO}} = \frac{v^2}{\Lambda^2} \left\{ -0.2726C_{\phi e} - 0.23666C_{\phi d}^{(1)} + 0.42246C_{\phi l}^{(3)} - 0.31904C_{\phi l} \\
+ 0.16629C_{\phi D} + 0.77518C_{\phi WB} \\
-0.00036C_{\phi d} - 0.00100C_{ee} + 0.00677C_{ee} + 0.00161C_{\phi d} + 0.01033C_{\phi d}^{(1)} \\
-0.00871C_{\phi q}^{(3)} - 0.01424C_{\phi u} - 0.00028C_{\phi d} - 0.00064C_{\phi e} - 0.00401C_{\phi q}^{(1)} \\
-0.00106C_{\phi l q}^{(3)} + 0.00531C_{\phi u} + 0.00032C_{\phi B} + 0.00004C_{\phi W} + 0.00032C_{\phi W} \\
-0.00512C_{\phi e} + 0.01087C_{\phi B} + 0.00917C_{\phi W} + 0.00053C_{\phi W} \right\}.
\]

(18)

The NLO corrections to sin²θ_{\ell, \text{eff}} change the numerical effects of the coefficients appearing at tree level by \(\mathcal{O}(5 - 10\%)\), and introduce dependencies on other coefficients.

For the W mass and total width, we find the predictions,

\[
\delta M_W^{\text{LO}} = \frac{v^2}{\Lambda^2} \left\{ -29.827C_{\phi d}^{(3)} + 14.914C_{\phi l} - 27.691C_{\phi D} - 57.479C_{\phi WB} \right\} \text{GeV}
\]

\[
\delta M_W^{\text{NLO}} = \frac{v^2}{\Lambda^2} \left\{ -35.666C_{\phi d}^{(3)} + 17.243C_{\phi l} - 30.272C_{\phi D} - 64.019C_{\phi WB} \\
-0.137C_{\phi d} - 0.137C_{\phi e} - 0.166C_{\phi l}^{(1)} - 2.032C_{\phi q}^{(1)} + 1.409C_{\phi q}^{(3)} + 2.684C_{\phi u} \\
+0.438C_{\phi l q}^{(3)} - 0.027C_{\phi B} - 0.033C_{\phi W} - 0.035C_{\phi W} - 0.902C_{\phi u} - 0.239C_{\phi u} - 0.15C_{\phi W} \right\} \text{GeV}
\]

\[
\delta \Gamma_W^{\text{LO}} = \frac{v^2}{\Lambda^2} \left\{ -5.092C_{\phi d}^{(3)} + 2.784C_{\phi q}^{(3)} + 3.242C_{\phi l} - 2.143C_{\phi D} - 4.448C_{\phi WB} \right\} \text{GeV}
\]

\[
\delta \Gamma_W^{\text{NLO}} = \frac{v^2}{\Lambda^2} \left\{ -5.556C_{\phi d}^{(3)} + 2.996C_{\phi q}^{(3)} + 3.340C_{\phi l} - 2.260C_{\phi D} - 4.777C_{\phi WB} \\
-0.01C_{\phi d} - 0.01C_{\phi e} - 0.017C_{\phi l}^{(1)} - 0.153C_{\phi q}^{(1)} + 0.203C_{\phi u} + 0.048C_{\phi q}^{(3)} \\
-0.002C_{\phi B} - 0.003C_{\phi W} - 0.004C_{\phi W} - 0.03C_{\phi q}^{(1)} - 0.094C_{\phi q}^{(3)} \\
-0.068C_{\phi u} - 0.014C_{\phi W} - 0.13C_{\phi W} \right\} \text{GeV}.
\]

(19)

It is interesting to note that some of the contributions to the W mass and width change by more than 10% when going from LO to NLO in the SMEFT. The NLO SMEFT contributions to the other observables of Eq. 10 given in Appendix IV.
We fit to the experimental data given in Table III, (omitting $\sin^2 \theta_{\text{eff}}$ since it can be directly derived from other observables). The most accurate SM predictions are given in the right-hand column and we use these values in our fits, as opposed to the LO or NLO SM contributions directly calculated. The pole observables we consider are\[59–61\]:

\[
M_W, \Gamma_W, \Gamma_Z, \sigma_h, R_l, A_{l,FB}, R_b, R_c, A_{FB,b}, A_{FB,c}, A_b, A_c, A_l,
\]

where we assume lepton universality and the experimental correlations can be found in Ref.\[51\]. We include the measurements of $A_l$ from LEP and SLD as separate data points.

The $\chi^2$ is computed from,

\[
\chi^2 = \sum_{i,j} (O_{i}^{\text{exp}} - O_{i}^{\text{SMEFT}}) \sigma_{ij}^{-2} (O_{j}^{\text{exp}} - O_{j}^{\text{SMEFT}}).
\]

Using the LO SMEFT expressions for the observables and taking $\Lambda = 1\,\text{TeV}$, we find\(^1\),

\[
\chi^2_{LO} = \chi^2_{SM} + 32C_{\phi d} + 105C_{\phi e} - 445C_{\phi l}^{(1)} + 639C_{\phi l}^{(3)} - 49C_{\phi q}^{(1)} - 60C_{\phi q}^{(3)} - 11C_{\phi q} - 424C_{ll} + 491C_{\phi D} + 114C_{\phi WB} + \vec{C}_{LO}^{T} M_{LO} \vec{C}_{LO}
\]

where

\[
\vec{C}_{LO}^{T} = \left( C_{ll}, C_{\phi WB}, C_{\phi u}, C_{\phi q}^{(3)}, C_{\phi q}^{(1)}, C_{\phi l}^{(3)}, C_{\phi l}^{(1)}, C_{\phi e}, C_{\phi D}, C_{\phi d} \right)
\]

and we find $\chi^2_{SM} \sim 13.42$. The symmetric matrix $M_{LO}$ is,

\[
M_{LO} = \begin{pmatrix}
25279 & -108322 & 1799 & 14960 & 4513 & -71171 & 27975 & 16835 & -37889 & -831 \\
148456 & -851 & -11405 & -3882 & 165479 & -51962 & -55619 & 102746 & -629 \\
574 & 6873 & 1615 & -7314 & -6662 & 3620 & -899 & -697 \\
24474 & 13826 & -54867 & -45834 & 27540 & -7486 & -5161 \\
3097 & -15840 & -12754 & 8236 & -2257 & -1569 \\
70369 & 18870 & -56402 & 60803 & 4835 \\
58121 & -44390 & -13987 & 5382 \\
31734 & -8417 & -2193 \\
21176 & 415 \\
318
\end{pmatrix}
\]

\(^1\) Our results are consistent with SMEFT fits to purely LEP observables using slightly different sets of inputs\[9, 59–61\].
Using the NLO SMEFT expressions we find $\chi^2_{NLO}$, (for $\Lambda = 1 \, TeV$),

$$\chi^2_{NLO} = \chi^2_{SM} - 403 C_{ll} + 1070 C_{WB} - 53 C_{\phi u} - 93 C_{\phi q}^{(3)}$$

$$- 18 C_{\phi d}^{(1)} + 666 C_{\phi d}^{(3)} - 402 C_{\phi d}^{(1)} + 176 C_{\phi e} + 502 C_{\phi D} + 27 C_{\phi d}$$

$$- 1.48 C_{qq}^{(1)} + 0.55 C_{\phi \Box} + 0.62 C_{\phi W} + 0.48 C_{\phi B} + 6.55 C_{uW}$$

$$+ 15 C_{uB} + 0.23 C_{ed} + 0.063 C_{dd} + 0.56 C_{ee} + 1.40 C_{qq}^{(3)} + 2.38 C_{W}$$

$$+ 0.53 C_{uu} - 0.54 C_{ud}^{(1)} + 1.05 C_{uq}^{(1)} - 4.88 C_{lq}^{(3)} + 2.8 C_{qe} + 0.34 C_{qd}^{(1)}$$

$$+ 9.8 C_{lu} - 0.32 C_{le} - 0.49 C_{ld} - 3.8 C_{eu} - 7.5 C_{lq}^{(1)} + \bar{C}^T_{NLO} M_{NLO} \bar{C}_{NLO},$$

where,

$$\bar{C}^T_{NLO} = \left( C_{ll}, C_{\phi WB}, C_{\phi u}, C_{\phi q}^{(3)}, C_{\phi q}^{(1)}, C_{\phi d}^{(3)}, C_{\phi d}^{(1)}, C_{\phi e}, C_{\phi D}, C_{\phi d}, C_{ed}, C_{ee}, C_{eu}, C_{lu}, C_{ld}, C_{le}, C_{lq}^{(1)}, C_{lq}^{(3)}, C_{\phi B}, C_{\phi W}, C_{\phi \Box}, C_{ue}, C_{uB}, C_{uW}, C_{W}, C_{qd}^{(1)}, C_{qq}^{(3)}, C_{qq}^{(1)}, C_{uq}^{(1)}, C_{ud}^{(1)}, C_{uu}, C_{dd} \right),$$

where the numerical form of $M_{NLO}$ is given in the supplemental material. At NLO, the $\chi^2$ now depends on 32 coefficients, and the effects of the coefficients appearing at LO have shifted by $5 - 10\%$. The (relatively) large shift of the coefficients of $C_{\phi u}$ and $C_{\phi d}^{(1,3)}$ are due to the top quark loop.

To study the numerical importance of the NLO effects, we begin by keeping only one coefficient non-zero at a time. We find the 95% confidence level regions at LO and NLO shown in Tables IV and V. The largest effect of the NLO corrections is on the coefficient of $C_{\phi u}$. The relatively large allowed values for $C_{\phi d}$ are the result of the discrepancy in the measured value $A_{FB,b}$ from the SM prediction.

At lowest order, the $\chi^2_{LO}$ is sensitive to 8 combinations of operators, implying that there are 2 blind directions[61–63]. These 8 combinations can be thought of as the combinations of operators contributing to $\delta g_L^{Zu}, \delta g_L^{Zd}, \delta g_U^{Ze}, \delta g_U^{Z\nu}, \delta g_R^{Zu}, \delta g_R^{Zd}, \delta g_R^{Ze},$ and $M_W$. Because of the $SU(2)$ symmetry of Eq. 9, at LO there is no additional information from $\Gamma_W$. Since our study includes only 14 data points, we clearly cannot fit to all of the SMEFT coefficients appearing at one loop. At NLO, the fit is sensitive to only 10 combinations of operators. The additional information can be thought of as coming from $\delta g_L^{Zb}$ and $\Gamma_W$ where the top quark makes significant contributions. Since there are 32 coefficients that contribute to the NLO
fit to the electroweak observables, resolving these 22 blind directions requires input from other processes and/or assumptions about which operators can be safely neglected.

We chose to perform our fits setting \( C_{\phi e} = 0 \) and \( C_{(3)\phi q} = 0 \), along with setting all of the operators that first appear at NLO to 0. We then marginalize over the remaining operators to study the numerical impacts of the NLO contributions. These results are shown in Tab. VI. We see that the effects of the NLO corrections can be significant, although the numerical results are sensitive to which operators are set to 0. Our results suggest that including the NLO corrections in the global fits (where the complete set of operators can potentially be bounded) may be important.

As another way of examining the impact of the NLO contributions, we consider the oblique parameters. The tree level SMEFT contributions are,

\[
\alpha \Delta S = 4c_W s_W \frac{v^2}{Λ^2} C_{\phi WB} \\
\alpha \Delta T = -\frac{v^2}{2Λ^2} C_{\phi D}.
\]

For the NLO oblique parameter fit, we set all coefficients to 0, except \( C_{\phi WB} \) and \( C_{\phi D} \). The resulting limits are shown in Fig. 2, (where what we are really plotting are the limits on

| Coefficient | LO | NLO |
|-------------|-----|-----|
| \( C_{ul} \) | \([-0.0039, 0.021]\) | \([-0.0045, 0.019]\) |
| \( C_{\phi WB} \) | \([-0.0088, 0.0013]\) | \([-0.0080, 0.0016]\) |
| \( C_{\phi u} \) | \([-0.072, 0.091]\) | \([-0.035, 0.085]\) |
| \( C_{(3)\phi q} \) | \([-0.011, 0.014]\) | \([-0.010, 0.014]\) |
| \( C_{(1)\phi q} \) | \([-0.027, 0.043]\) | \([-0.031, 0.036]\) |
| \( C_{(3)\phi l} \) | \([-0.012, 0.0029]\) | \([-0.010, 0.0028]\) |
| \( C_{(1)\phi l} \) | \([-0.0043, 0.012]\) | \([-0.0047, 0.012]\) |
| \( C_{\phi e} \) | \([-0.013, 0.0094]\) | \([-0.013, 0.0080]\) |
| \( C_{\phi D} \) | \([-0.025, 0.0019]\) | \([-0.023, 0.0023]\) |
| \( C_{\phi d} \) | \([-0.16, 0.060]\) | \([-0.13, 0.064]\) |

TABLE IV: 95% confidence level allowed ranges for single parameter fit to coefficients contributing to the lowest order predictions. The scale Λ is taken to be 1 TeV.
| Coefficient | LO          | NLO         |
|-------------|-------------|-------------|
| $C_W$       | $[-4.8, 0.48]$ |              |
| $C_{uB}$    | $[-0.57, 0.11]$ |              |
| $C_{qq}^{(1)}$ | $[-0.93, 1.5]$ |              |
| $C_{\phi \Box}$ | $[-22, 1.9]$ |              |
| $C_{lu}$    | $[-0.49, 0.19]$ |              |
| $C_{le}$    | $[-5.3, 1.1]$ |              |
| $C_{ed}$    | $[-12, 6.7]$ |              |
| $C_{(1)ad}$ | $[-3.0, 5.6]$ |              |

| Coefficient | LO          | NLO         |
|-------------|-------------|-------------|
| $C_{uu}$    | $[-1.1, 0.99]$ | $[-0.78, 0.29]$ |
| $C_{(1)uq}$ | $[-2.2, 1.3]$ | $[-0.32, 0.29]$ |
| $C_{(3)qq}$ | $[-0.75, 0.48]$ |              |
| $C_{\phi W}$ | $[-17, 2.2]$ |              |
| $C_{(1)lu}$ | $[-0.32, 0.57]$ | $[-0.25, 0.66]$ |
| $C_{lt}$    | $[-3.8, 8.7]$ | $[-51, 26]$ |
| $C_{ce}$    | $[-3.9, 2.4]$ | $[-0.36, 0.58]$ |
| $C_{(1)tq}$ | $[-0.025, 0.12]$ | $[-0.039, 0.16]$ |
| $C_{\phi u}$ | $[-0.12, 0.37]$ | $[-0.21, 0.41]$ |
| $C_{(1)\phi}$ | $[-0.0086, 0.036]$ | $[-0.0072, 0.037]$ |
| $C_{lt}$    | $[-0.085, 0.035]$ | $[-0.088, 0.033]$ |
| $C_{(1)\phi u}$ | $[-0.060, 0.076]$ | $[-0.095, 0.076]$ |

TABLE V: 95% confidence level allowed ranges for single parameter fit to coefficients not contributing to the lowest order predictions. The scale $\Lambda$ is taken to be $1\,TeV$.

| Coefficient | LO          | NLO         |
|-------------|-------------|-------------|
| $C_{\phi D}$ | $[-0.034, 0.041]$ | $[-0.039, 0.051]$ |
| $C_{\phi W B}$ | $[-0.080, 0.0021]$ | $[-0.098, 0.012]$ |
| $C_{\phi d}$ | $[-0.81, -0.093]$ | $[-1.07, -0.03]$ |
| $C_{(3)\phi}$ | $[-0.025, 0.12]$ | $[-0.039, 0.16]$ |
| $C_{\phi u}$ | $[-0.12, 0.37]$ | $[-0.21, 0.41]$ |
| $C_{(1)\phi}$ | $[-0.0086, 0.036]$ | $[-0.0072, 0.037]$ |
| $C_{lt}$    | $[-0.085, 0.035]$ | $[-0.088, 0.033]$ |
| $C_{(1)\phi u}$ | $[-0.060, 0.076]$ | $[-0.095, 0.076]$ |

TABLE VI: 95% confidence level allowed ranges for fit to coefficients marginalizing over the other 7 operators we are considering. The coefficients of all operators not listed in the table are set to 0. The scale $\Lambda$ is taken to be $1\,TeV$.

The effect of the NLO SMEFT corrections is small. At NLO, new coefficients can influence the oblique parameters, and the complete one-loop SMEFT result is given in Ref. [38].

Our calculation includes only the resonant $Z$ and $W$ contributions to the precision elec-
FIG. 2: 95% CL limits from a 2 parameter fit to $C_{\phi WB}$ and $C_{\phi D}$, setting all other coefficients to 0. The scale $\Lambda = 1 \text{ TeV}$. The solid line is the result of the LO fit, while the dotted line is the NLO fit to the electroweak parameters of this study.

troweak observables. In the SMEFT, there are tree level non-resonant contributions due to 4-fermion operators that can complicate the experimental extraction of the widths from the data. The size of these effects has been estimated in Ref. [33]. For example,

$$\delta \Gamma(Z \rightarrow \text{hadrons}) \sim 0.6 \text{ MeV} C_{4f}(\frac{1 \text{ TeV}}{\Lambda})^2,$$

(28)

where $C_{4f}$ is a generic 4-fermion operator. These effects could potentially be similar in size to the electroweak corrections we have computed. There are also off-shell corrections proportional to $q^2/\Lambda^2$, where $q^2$ is the momentum running through the $Z$ boson propagator. The experimental cuts [51] were designed to extract predominantly the on-shell $Z$ events, so although the size of these effects is expected to be small a detailed theoretical study would be needed to quantify them.

V. CONCLUSIONS

We have computed the NLO electroweak and QCD corrections to the SMEFT predictions for the precision electroweak observables. Our results are presented in a numerical form that can easily be incorporated in the global fitting programs. We also present numerical results for the LO and NLO $\chi^2$ that can be customized for the reader’s use. Our studies suggest that the NLO SMEFT corrections may have a sizable effect on the global fits. Numerical
results for the SMEFT NLO expressions for the observables considered here, along with the $\chi^2_{LO}$, $\chi^2_{NLO}$ and the matrix $M_{NLO}$, are posted at https://quark.phy.bnl.gov/Digital_Data_Archive/dawson/ewpo_19.

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Appendix A: Observables to LO and NLO in the SMEFT

In this appendix, we report the contributions to the observables of Table II using the definitions of Eq. 17. The contributions to the $Z$ width are,

$$\delta \Gamma(Z \rightarrow \nu \nu)^{LO} = \frac{v^2}{\Lambda^2} \left\{ -0.3318C_{\phi l}^{(1)} + 0.1659C_{\phi l} - 0.0829C_{\phi D} \right\} \text{GeV}$$

$$\delta \Gamma(Z \rightarrow \nu \nu)^{NLO} = \frac{v^2}{\Lambda^2} \left\{ -0.3446C_{\phi l}^{(1)} + 0.1640C_{\phi l} - 0.0853C_{\phi D} - 0.0003C_{\phi d} - 0.0003C_{\phi e} \\
-0.0018C_{\phi q}^{(3)} - 0.0073C_{\phi q}^{(1)} + 0.0054C_{\phi q}^{(3)} + 0.0083C_{\phi u} - 0.0004\mathcal{C}_{ld} \\
-0.0004\mathcal{C}_{te} - 0.0061\mathcal{C}_{lq}^{(1)} - 0.0061\mathcal{C}_{lq}^{(3)} + 0.008\mathcal{C}_{lu} - 0.0002C_{\phi \square}^{(1)} - 0.0001\mathcal{C}_{\phi W} \\
+0.0063C_{\phi WB} + 0.0001\mathcal{C}_{uW} - 0.0001\mathcal{C}_{W} \right\} \text{GeV}$$

$$\delta \Gamma(Z \rightarrow l^+ l^-)^{LO} = \frac{v^2}{\Lambda^2} \left\{ -0.1408C_{\phi e} + 0.191C_{\phi l}^{(1)} - 0.037C_{\phi l}^{(3)} + 0.114C_{ll} - 0.057C_{\phi D} \\
-0.0713C_{\phi WB} \right\} \text{GeV}$$

$$\delta \Gamma(Z \rightarrow l^+ l^-)^{NLO} = \frac{v^2}{\Lambda^2} \left\{ -0.1596C_{\phi e} + 0.1834C_{\phi l}^{(1)} - 0.0221C_{\phi l}^{(3)} + 0.0985C_{ll} - 0.0508C_{\phi D} \\
-0.0349C_{\phi WB} - 0.0001C_{\phi W} - 0.0002C_{\phi d} - 0.0005C_{ee} + 0.0035C_{eu} \\
-0.0002C_{\phi d} - 0.0042C_{\phi q}^{(1)} + 0.0032C_{\phi q}^{(3)} + 0.0049C_{\phi u} + 0.0002C_{ld} \\
+0.0001C_{te} + 0.0034C_{lq}^{(1)} - 0.0031C_{lq}^{(3)} - 0.0045\mathcal{C}_{lu} - 0.0001C_{\phi \square}^{(1)} \\
-0.0027C_{qe} - 0.0007\mathcal{C}_{uB} - 0.0007\mathcal{C}_{uW} - 0.0001\mathcal{C}_{W} \right\} \text{GeV}$$
\[ \delta \Gamma(Z \rightarrow u\bar{u})^{LO} = \delta \Gamma(Z \rightarrow c\bar{c})^{LO} \]
\[ = \frac{v^2}{\Lambda^2} \left\{ -0.9261C_{\phi d}^{(3)} - 0.7138C_{\phi q}^{(1)} + 0.7138C_{\phi q}^{(3)} + 0.2815C_{\phi u} + 0.4631C_{\phi \ell} \\
-0.2315C_{\phi D} - 0.4093C_{\phi W B} \right\} \text{GeV} \]
\[ \delta \Gamma(Z \rightarrow u\bar{u})^{NLO} = \delta \Gamma(Z \rightarrow c\bar{c})^{NLO} \]
\[ = \frac{v^2}{\Lambda^2} \left\{ -0.9620C_{\phi d}^{(3)} - 0.7559C_{\phi q}^{(1)} + 0.7477C_{\phi q}^{(3)} + 0.3515C_{\phi u} + 0.4634C_{\phi \ell} \\
-0.2402C_{\phi D} - 0.4016C_{\phi W B} + 0.0004C_{\phi u} - 0.0013C_{\phi d} - 0.0013C_{\phi e} \\
-0.0022C_{\phi d}^{(1)} - 0.0009C_{\phi q}^{(1)} + 0.0099C_{\phi q}^{(3)} + 0.0004C_{\phi u} - 0.0002C_{\phi B} \\
-0.0003C_{\phi R} - 0.0004C_{\phi W} - 0.0009C_{\phi q}^{(1)} - 0.0009C_{\phi e} - 0.0448C_{\phi q}^{(1)} \\
-0.0629C_{\phi q}^{(3)} + 0.0223C_{\phi q}^{(1)} - 0.006C_{\phi B} + 0.0004C_{\phi u}^{(1)} - 0.0221C_{\phi u} \\
-0.0049C_{\phi W} - 0.0005C_{\phi W} \right\} \text{GeV} \]
\[ \delta \Gamma(Z \rightarrow d\bar{d})^{LO} = \delta \Gamma(Z \rightarrow s\bar{s})^{LO} \]
\[ = \frac{v^2}{\Lambda^2} \left\{ -0.1408C_{\phi d} - 1.0299C_{\phi q}^{(3)} + 0.8545C_{\phi q}^{(1)} + 0.8545C_{\phi q}^{(3)} + 0.5149C_{\phi \ell} \\
-0.2575C_{\phi D} - 0.3379C_{\phi W B} \right\} \text{GeV} \]
\[ \delta \Gamma(Z \rightarrow d\bar{d})^{NLO} = \delta \Gamma(Z \rightarrow s\bar{s})^{NLO} \]
\[ = \frac{v^2}{\Lambda^2} \left\{ -0.1659C_{\phi d} - 1.1057C_{\phi q}^{(3)} + 0.8818C_{\phi q}^{(1)} + 0.9150C_{\phi q}^{(3)} + 0.5329C_{\phi \ell} \\
-0.275C_{\phi D} - 0.3582C_{\phi W B} - 0.0004C_{\phi d} + 0.0002C_{\phi d} - 0.0013C_{\phi e} \\
-0.0024C_{\phi d}^{(1)} + 0.0255C_{\phi q} + 0.0002C_{\phi q} + 0.0011C_{\phi q} + 0.011C_{\phi q}^{(3)} \\
-0.0002C_{\phi B} - 0.0004C_{\phi R} - 0.0004C_{\phi W} - 0.0016C_{\phi q}^{(1)} \\
+0.0011C_{\phi q} + 0.0293C_{\phi q}^{(1)} - 0.0118C_{\phi q}^{(3)} + 0.0205C_{\phi q}^{(1)} \\
-0.0053C_{\phi B} + 0.0035C_{\phi d}^{(1)} - 0.0041C_{\phi W} - 0.0005C_{\phi W} \right\} \text{GeV} \]
\[ \delta \Gamma(Z \rightarrow b\bar{b})^{LO} = \frac{v^2}{\Lambda^2} \left\{ -0.1400C_{\phi d} - 1.0242C_{\phi q}^{(3)} + 0.8498C_{\phi q}^{(1)} + 0.8498C_{\phi q}^{(3)} + 0.5121C_{\phi \ell} \\
-0.2561C_{\phi D} - 0.3361C_{\phi W B} \right\} \text{GeV} \]
\[ \delta \Gamma(Z \to b\bar{b})^{NLO} = \frac{v^2}{\Lambda^2} \left\{ -0.1649 C_{\phi_d} - 1.0666 C_{\phi_d}^{(3)} + 0.8724 C_{\phi_d}^{(1)} + 0.8790 C_{\phi_d}^{(3)} + 0.5134 C_{\phi_d}^{(1)} \\
 -0.2659 C_{\phi_D} - 0.3413 C_{\phi_{WB}} - 0.0004 C_{\phi_{dd}} - 0.0002 C_{\phi_{ed}} - 0.0013 C_{\phi_{ee}} \\
 -0.0023 C_{\phi_l}^{(1)} + 0.0222 C_{\phi_u} - 0.0002 C_{\phi_{ld}} + 0.0011 C_{\phi_{iq}}^{(1)} + 0.0109 C_{\phi_{iq}}^{(3)} \\
 -0.0002 C_{\phi_B} - 0.0004 C_{\phi_{WB}} - 0.0016 C_{\phi_{qd}} + 0.0011 C_{\phi_{qe}} \\
 +0.0292 C_{\phi_{qq}}^{(1)} - 0.0117 C_{\phi_{qq}}^{(3)} - 0.0204 C_{\phi_{qq}}^{(1)} - 0.0069 C_{\phi_{uB}} + 0.0035 C_{\phi_{ud}}^{(1)} \\
 -0.0168 C_{\phi_{uW}} - 0.0027 C_{\phi_{W}} \right\} \text{GeV}. \]

The SMEFT contributions to the total Z width are,

\[ \delta \Gamma_{Z}^{LO} = \frac{v^2}{\Lambda^2} \left\{ -0.4223 C_{\phi_d} - 0.4223 C_{\phi_e} - 0.4223 C_{\phi_d}^{(1)} - 5.053 C_{\phi_d}^{(3)} + 1.1361 C_{\phi_d}^{(1)} \\
 +3.9911 C_{\phi_q}^{(3)} + 0.5631 C_{\phi_u} + 3.3106 C_{\phi_l} - 1.6553 C_{\phi_D} - 2.0463 C_{\phi_{WB}} \right\} \text{GeV} \]

\[ \delta \Gamma_{Z}^{NLO} = \frac{v^2}{\Lambda^2} \left\{ -0.5008 C_{\phi_d} - 0.4862 C_{\phi_e} - 0.4951 C_{\phi_d}^{(1)} - 5.2739 C_{\phi_d}^{(3)} + 1.0898 C_{\phi_d}^{(1)} \\
 +4.2302 C_{\phi_q}^{(3)} + 0.8157 C_{\phi_u} + 3.2934 C_{\phi_l} - 1.7061 C_{\phi_D} - 1.9465 C_{\phi_{WB}} \\
 -0.0013 C_{\phi_{dd}} - 0.0011 C_{\phi_{ed}} - 0.0016 C_{\phi_{ee}} + 0.0113 C_{\phi_{eu}} - 0.0011 C_{\phi_{ld}} - 0.0011 C_{\phi_{le}} \\
 -0.0065 C_{\phi_{lq}}^{(1)} + 0.025 C_{\phi_{lq}}^{(3)} + 0.0113 C_{\phi_{lu}} - 0.0014 C_{\phi_{B}} - 0.0028 C_{\phi_{WB}} - 0.0026 C_{\phi_{W}} \\
 -0.0065 C_{\phi_{qd}}^{(1)} - 0.0065 C_{\phi_{qe}} - 0.0017 C_{\phi_{qq}}^{(1)} - 0.161 C_{\phi_{qq}}^{(3)} - 0.0168 C_{\phi_{qq}}^{(1)} - 0.0318 C_{\phi_{uB}} \\
 +0.0113 C_{\phi_{ud}}^{(1)} - 0.0443 C_{\phi_{uW}} - 0.0365 C_{\phi_{uW}} - 0.0054 C_{\phi_{W}} \right\} \text{GeV}. \]

The ratios are defined to be

\[ R_t = \frac{\Sigma_q \Gamma(Z \to q\bar{q})}{\Gamma(Z \to ll)} \]

\[ R_c = \frac{\Gamma(Z \to u\pi)}{\Sigma_q \Gamma(Z \to q\bar{q})} \]

\[ R_b = \frac{\Gamma(Z \to d\bar{d})}{\Sigma_q \Gamma(Z \to q\bar{q})} \]
and the SMEFT contributions are,

\[
\delta R_{\ell}^{LO} = \frac{v^2}{\Lambda^2} \left\{ -4.978C_{\phi d} + 33.673C_{\phi e} - 45.688C_{\phi l}^{(1)} - 49.393C_{\phi l}^{(3)} + 13.39C_{\phi q}^{(1)} + 47.041C_{\phi q}^{(3)} \\
+ 6.637C_{\phi u} + 1.853C_{ll} - 0.926C_{\phi D} - 4.532C_{\phi WB} \right\}
\]

\[
\delta R_{\ell}^{NLO} = \frac{v^2}{\Lambda^2} \left\{ -5.8831C_{\phi d} + 39.214C_{\phi e} - 45.510C_{\phi l}^{(1)} - 56.368C_{\phi l}^{(3)} + 14.398C_{\phi q}^{(1)} + 49.304C_{\phi q}^{(3)} \\
+ 8.0458C_{\phi u} + 5.3901C_{ll} - 2.8457C_{\phi D} - 13.262C_{\phi WB} - 0.015C_{dd} + 0.038C_{dd} + 0.124C_{ee} \\
- 0.834C_{eu} - 0.063C_{ld} - 0.012C_{le} - 0.795C_{tq}^{(1)} + 1.362C_{tq}^{(3)} + 1.083C_{tu} - 0.004C_{\phi B} \\
- 0.002C_{\phi W} - 0.005C_{\phi W} - 0.077C_{qg}^{(1)} + 0.654C_{qe} - 0.020C_{qg}^{(1)} - 1.898C_{qg}^{(3)} - 0.198C_{qu} \\
- 0.168C_{uB} + 0.133C_{ud} - 0.522C_{uu} - 0.254C_{uW} - 0.037C_{W} \right\}
\]

\[
\delta R_{c}^{LO} = \frac{v^2}{\Lambda^2} \left\{ 0.0421C_{\phi d} - 0.0449C_{\phi e}^{(3)} - 0.5279C_{\phi q}^{(1)} + 0.0164C_{\phi q}^{(3)} + 0.1073C_{\phi u} \\
+ 0.0224C_{ll} - 0.0112C_{\phi D} - 0.0549C_{\phi WB} \right\}
\]

\[
\delta R_{c}^{NLO} = \frac{v^2}{\Lambda^2} \left\{ 0.0487C_{\phi d} - 0.0380C_{\phi e}^{(3)} - 0.5455C_{\phi q}^{(1)} + 0.0136C_{\phi q}^{(3)} + 0.1253C_{\phi u} \\
+ 0.0182C_{ll} - 0.0096C_{\phi D} - 0.0465C_{\phi WB} + 0.0001C_{dd} + 0.0001C_{dd} + 0.0001C_{eu} \\
- 0.0001C_{\phi e} - 0.0001C_{\phi l}^{(1)} + 0.0001C_{ld} - 0.0007C_{tq}^{(1)} + 0.0005C_{tq}^{(3)} + 0.0001C_{tu} \\
+ 0.0001C_{uq}^{(1)} - 0.0007C_{uq} - 0.0258C_{qq}^{(1)} - 0.0205C_{qq}^{(3)} + 0.0146C_{qq}^{(1)} - 0.0005C_{uB} \\
- 0.0009C_{ud}^{(1)} - 0.0084C_{uu} + 0.0006C_{uW} + 0.0002C_{W} \right\}
\]

\[
\delta R_{b}^{LO} = \frac{v^2}{\Lambda^2} \left\{ -0.0208C_{\phi d} + 0.02993C_{\phi e}^{(3)} + 0.3519C_{\phi q}^{(1)} - 0.01094C_{\phi q}^{(3)} \\
- 0.07156C_{\phi u} - 0.01497C_{ll} + 0.00748C_{\phi D} + 0.03661C_{\phi WB} \right\}
\]

\[
\delta R_{b}^{NLO} = \frac{v^2}{\Lambda^2} \left\{ -0.03276C_{\phi d} + 0.03271C_{\phi e}^{(3)} + 0.3618C_{\phi q}^{(1)} - 0.01678C_{\phi q}^{(3)} \\
- 0.08398C_{\phi u} - 0.01583C_{ll} + 0.00828C_{\phi D} + 0.03465C_{\phi WB} - 0.0008C_{dd} \\
- 0.00004C_{dd} - 0.0009C_{eu} + 0.00007C_{\phi e} + 0.00010C_{\phi l}^{(1)} - 0.00004C_{ld} \\
+ 0.00044C_{tq}^{(1)} - 0.00034C_{tq}^{(3)} - 0.00009C_{tu} + 0.00001C_{\phi B} + 0.00001C_{\phi W} \\
- 0.00008C_{qg}^{(1)} + 0.00044C_{qe} + 0.01717C_{qq}^{(1)} + 0.01367C_{qq}^{(3)} - 0.00972C_{qq}^{(1)} \\
- 0.00028C_{uB} + 0.00060C_{ud}^{(1)} + 0.00562C_{uu} - 0.00534C_{uW} - 0.00098C_{W} \right\}.
\]
\begin{align*}
\sigma_{h}^{\text{LO}} &= \frac{v^2}{\Lambda^2} \left\{ 4.05 C_{\phi d} - 55.524 C_{\phi e} + 109.235 C_{\phi l}^{(1)} + 32.796 C_{\phi l}^{(3)} - 10.896 C_{\phi q}^{(1)} - 38.278 C_{\phi q}^{(3)} - 5.4 C_{\phi u} + 4.319 C_{\phi l} - 2.16 C_{\phi D} - 10.565 C_{\phi WB} \right\} \text{ nb} \\
\sigma_{h}^{\text{NLO}} &= \frac{v^2}{\Lambda^2} \left\{ 4.656 C_{\phi d} - 62.835 C_{\phi e} + 106.805 C_{\phi q}^{(1)} + 40.603 C_{\phi q}^{(3)} - 11.515 C_{\phi q}^{(1)} - 38.995 C_{\phi u} - 1.176 C_{\phi l} + 0.738 C_{\phi D} + 3.178 C_{\phi WB} + 0.012 C_{dd} - 0.068 C_{ed} + 0.205 C_{ee} + 1.382 C_{en} + 0.142 C_{ld} + 0.063 C_{le} + 1.945 C_{lq}^{(1)} - 1.101 C_{lq}^{(3)} - 2.598 C_{lu} - 0.001 C_{\phi B} + 0.002 C_{\phi D} + 0.063 C_{qd}^{(1)} - 1.064 C_{qe} + 0.017 C_{qq}^{(1)} + 1.544 C_{qq}^{(3)} + 0.161 C_{uq}^{(1)} - 0.009 C_{uB} - 0.108 C_{uW} + 0.424 C_{uu} + 0.067 C_{uW} + 0.027 C_{W} \right\} \text{ nb}
\end{align*}

The asymmetries are defined as,
\begin{align*}
A_t &= \frac{\Gamma(Z \rightarrow e_L^+e_L^-) - \Gamma(Z \rightarrow e_R^+e_R^-)}{\Gamma(Z \rightarrow e^+e^-)} \\
A_c &= \frac{\Gamma(Z \rightarrow u_L\bar{u}_L) - \Gamma(Z \rightarrow u_R\bar{u}_R)}{\Gamma(Z \rightarrow u\bar{u})} \\
A_b &= \frac{\Gamma(Z \rightarrow d_L\bar{d}_L) - \Gamma(Z \rightarrow d_R\bar{d}_R)}{\Gamma(Z \rightarrow d\bar{d})},
\end{align*}
and the SMEFT contributions are,

\[
\delta A_{t}^{LO} = \frac{\nu^2}{A^2} \left\{ 2.1503\mathcal{C}_{\phi e} + 1.5848\mathcal{C}_{\phi d}^{(1)} - 2.7529\mathcal{C}_{\phi d}^{(3)} + 2.1689\mathcal{C}_{ll} - 1.0844\mathcal{C}_{\phi D} - 5.305\mathcal{C}_{\phi WB} \right\}
\]

\[
\delta A_{t}^{NLO} = \frac{\nu^2}{A^2} \left\{ +2.1666\mathcal{C}_{\phi e} + 1.8745\mathcal{C}_{\phi d}^{(1)} - 3.3587\mathcal{C}_{\phi d}^{(3)} + 2.5342\mathcal{C}_{ll} - 1.3250\mathcal{C}_{\phi D} - 6.1599\mathcal{C}_{\phi WB} \\
+0.0027\mathcal{C}_{ed} + 0.0076\mathcal{C}_{ee} - 0.0518\mathcal{C}_{eu} - 0.0128\mathcal{C}_{\phi d}^{(1)} - 0.0867\mathcal{C}_{\phi q}^{(1)} + 0.0719\mathcal{C}_{\phi q}^{(3)} \\
+0.1190\mathcal{C}_{\phi u} + 0.0021\mathcal{C}_{ld} + 0.0049\mathcal{C}_{le} + 0.0307\mathcal{C}_{lq}^{(1)} + 0.0098\mathcal{C}_{lq}^{(3)} - 0.0406\mathcal{C}_{lu} \\
-0.0026\mathcal{C}_{W} + 0.0004\mathcal{C}_{\phi DD} - 0.0026\mathcal{C}_{\phi W} + 0.0392\mathcal{C}_{qe} \\
-0.0866\mathcal{C}_{uB} - 0.0711\mathcal{C}_{dW} - 0.0046\mathcal{C}_{W} \right\}
\]

\[
\delta A_{c}^{LO} = \frac{\nu^2}{A^2} \left\{ -1.779\mathcal{C}_{\phi d}^{(3)} - 0.65\mathcal{C}_{\phi q}^{(1)} + 0.65\mathcal{C}_{\phi q}^{(3)} - 1.648\mathcal{C}_{\phi u} + 0.889\mathcal{C}_{ll} - 0.445\mathcal{C}_{\phi D} - 2.175\mathcal{C}_{\phi WB} \right\}
\]

\[
\delta A_{c}^{NLO} = \frac{\nu^2}{A^2} \left\{ -2.295\mathcal{C}_{\phi d}^{(3)} - 0.867\mathcal{C}_{\phi q}^{(1)} + 0.856\mathcal{C}_{\phi q}^{(3)} - 1.798\mathcal{C}_{\phi u} + 1.110\mathcal{C}_{ll} - 0.581\mathcal{C}_{\phi D} - 2.699\mathcal{C}_{\phi WB} \\
-0.002\mathcal{C}_{eu} - 0.006\mathcal{C}_{ed} - 0.006\mathcal{C}_{\phi e} - 0.007\mathcal{C}_{\phi q}^{(1)} - 0.001\mathcal{C}_{lq}^{(1)} + 0.025\mathcal{C}_{lq}^{(3)} - 0.002\mathcal{C}_{lu} \\
-0.001\mathcal{C}_{\phi B} - 0.001\mathcal{C}_{\phi W} - 0.001\mathcal{C}_{qd}^{(1)} - 0.001\mathcal{C}_{qe} - 0.045\mathcal{C}_{qq}^{(1)} - 0.063\mathcal{C}_{qq}^{(3)} - 0.014\mathcal{C}_{qu}^{(1)} \\
-0.037\mathcal{C}_{uB} - 0.002\mathcal{C}_{ud} + 0.13\mathcal{C}_{uu} - 0.03\mathcal{C}_{uW} - 0.002\mathcal{C}_{W} \right\}
\]

\[
\delta A_{b}^{LO} = \frac{\nu^2}{A^2} \left\{ 0.727\mathcal{C}_{\phi d} - 0.328\mathcal{C}_{\phi d}^{(3)} + 0.12\mathcal{C}_{\phi q}^{(1)} + 0.12\mathcal{C}_{\phi q}^{(3)} + 0.164\mathcal{C}_{ll} - 0.082\mathcal{C}_{\phi D} - 0.401\mathcal{C}_{\phi WB} \right\}
\]

\[
\delta A_{b}^{NLO} = \frac{\nu^2}{A^2} \left\{ +0.842\mathcal{C}_{\phi d} - 0.424\mathcal{C}_{\phi d}^{(3)} + 0.149\mathcal{C}_{\phi q}^{(1)} + 0.157\mathcal{C}_{\phi q}^{(3)} + 0.205\mathcal{C}_{ll} - 0.107\mathcal{C}_{\phi D} - 0.501\mathcal{C}_{\phi WB} \\
+0.002\mathcal{C}_{dd} + 0.001\mathcal{C}_{ed} - 0.001\mathcal{C}_{\phi e} - 0.001\mathcal{C}_{\phi d}^{(1)} + 0.009\mathcal{C}_{\phi q} + 0.001\mathcal{C}_{ld} + 0.005\mathcal{C}_{lq}^{(3)} \\
+0.014\mathcal{C}_{qd}^{(1)} + 0.005\mathcal{C}_{qq}^{(1)} - 0.092\mathcal{C}_{qq}^{(3)} - 0.003\mathcal{C}_{qu}^{(1)} \\
-0.007\mathcal{C}_{uB} - 0.018\mathcal{C}_{ud}^{(1)} - 0.007\mathcal{C}_{uW} - 0.001\mathcal{C}_{W} \right\}
\]

Finally, the forward backward asymmetries are defined as

\[
A_{FB,i} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B},
\]

where defining \(\theta\) to be the angle between the incoming \(l^-\) and the outgoing \(\bar{f}_i\), \(\sigma_F\) has \(\theta\)
between \((0, \frac{\pi}{2})\) and \(\sigma_B\) has \(\theta\) between \((\frac{\pi}{2}, \pi)\). The SMEFT results are,

\[
A_{FB,b}^{LO} = \frac{v^2}{\Lambda^2} \left\{ 0.9547 C_{\phi e} + 0.7037 C_{\phi l}^{(1)} - 1.2223 C_{\phi l}^{(3)} + 0.9630 C_{ll} - 0.4815 C_{D} - 2.3555 C_{WB} \right\}
\]

\[
A_{FB,b}^{NLO} = \frac{v^2}{\Lambda^2} \left\{ +0.4783 C_{\phi e} + 0.4138 C_{\phi l}^{(1)} - 0.7414 C_{\phi l}^{(3)} + 0.5594 C_{ll} - 0.2925 C_{D} - 1.3598 C_{WB} \\
+0.0006 C_{ed} + 0.0017 C_{ee} - 0.0114 C_{eu} - 0.0028 C_{old} - 0.0191 C_{eq} + 0.0159 C_{qf} + 0.0263 C_{fu} \\
+0.0005 C_{ld} + 0.0011 C_{le} + 0.0068 C_{lq}^{(1)} + 0.0022 C_{lq}^{(3)} - 0.009 C_{lu} - 0.0006 C_{fb} - 0.0001 C_{\phi D} \\
-0.0006 C_{W} + 0.0086 C_{qe} - 0.0191 C_{uB} - 0.0157 C_{uW} - 0.0010 C_{W} \right\}
\]

\[
A_{FB,c}^{LO} = \frac{v^2}{\Lambda^2} \left\{ 1.1785 C_{\phi e} + 0.8686 C_{\phi l}^{(1)} - 1.9036 C_{\phi l}^{(3)} - 0.1443 C_{\phi q}^{(1)} + 0.1443 C_{\phi q}^{(3)} \\
-0.3658 C_{\phi u} + 1.3861 C_{ll} - 0.693 C_{D} - 3.3903 C_{WB} \right\}
\]

\[
A_{FB,c}^{NLO} = \frac{v^2}{\Lambda^2} \left\{ +1.0846 C_{\phi e} + 0.9381 C_{\phi l}^{(1)} - 1.9356 C_{\phi l}^{(3)} - 0.1391 C_{\phi q}^{(1)} + 0.1305 C_{\phi q}^{(3)} \\
-0.1388 C_{\phi u} + 1.3918 C_{ll} - 0.7278 C_{D} - 3.3833 C_{WB} + 0.0014 C_{ed} + 0.0038 C_{ee} \\
-0.0262 C_{eu} - 0.007 C_{od} + 0.0011 C_{ld} + 0.0024 C_{le} + 0.0153 C_{lq}^{(1)} + 0.0076 C_{lq}^{(3)} \\
-0.0206 C_{lu} - 0.0014 C_{fb} - 0.0002 C_{\phi D} - 0.0195 C_{qe} \\
-0.0050 C_{qq}^{(1)} - 0.0070 C_{qq}^{(3)} - 0.0016 C_{qu}^{(1)} - 0.0475 C_{uB} \\
-0.0002 C_{ud}^{(1)} + 0.0143 C_{uu} - 0.0389 C_{uW} - 0.0025 C_{W} \right\}
\]

\[
A_{FB,b}^{LO} = \frac{v^2}{\Lambda^2} \left\{ 0.1615 C_{\phi d} + 1.5275 C_{\phi e} + 1.1258 C_{\phi l}^{(1)} - 2.0284 C_{\phi l}^{(3)} + 0.0266 C_{\phi q}^{(1)} \\
+0.0266 C_{\phi q}^{(3)} + 1.5771 C_{ll} - 0.7886 C_{D} - 3.8576 C_{WB} \right\}
\]

\[
A_{FB,b}^{NLO} = \frac{v^2}{\Lambda^2} \left\{ +0.0840 C_{\phi d} + 1.5062 C_{\phi e} + 1.3031 C_{\phi l}^{(1)} - 2.3819 C_{\phi l}^{(3)} - 0.0439 C_{\phi q}^{(1)} \\
+0.0673 C_{\phi q}^{(3)} + 1.7845 C_{ll} - 0.9331 C_{D} - 4.3379 C_{WB} + 0.0002 C_{dd} \\
+0.0020 C_{ed} + 0.0053 C_{ee} - 0.0360 C_{eu} + 0.0838 C_{\phi u} + 0.0016 C_{ld} + 0.0034 C_{le} \\
+0.0214 C_{lq}^{(1)} + 0.0073 C_{lq}^{(3)} - 0.0283 C_{lu} - 0.0018 C_{\phi B} - 0.0003 C_{\phi D} - 0.0018 C_{\phi W} \\
+0.0015 C_{qd}^{(1)} + 0.0273 C_{qe} + 0.0005 C_{qq}^{(1)} - 0.0002 C_{qq}^{(3)} - 0.0003 C_{qu}^{(1)} \\
-0.0610 C_{uB} - 0.0020 C_{ud}^{(1)} - 0.0502 C_{uW} - 0.0033 C_{W} \right\}.
\]
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