In-medium properties of SU(3) baryons

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Changes of baryon properties in nuclear matter are investigated within the framework of an in-medium modified SU(3) Skyrme model. Introducing the medium functionals in the SU(2) sector and considering the alteration of kaon properties in nuclear medium, we are able to examine the medium modification of the nucleon and hyperons. The functionals introduced in the SU(2) sector are related to ordinary nuclear matter properties near the saturation point. The results indicate that the changes of the baryon properties in the strange sector are strongly correlated with the in-medium properties of kaons.

Keywords: Skyrmions, nucleons, hyperons, nuclear matter
I. INTRODUCTION

Understanding how the hyperons undergo changes in nuclear matter is a very important issue in contemporary nuclear physics. In particular, it is of great interest to see how the hyperons are related to in-medium kaon properties at low densities and how they can be changed in higher densities that can be found in the interior of neutron stars \cite{1,2}. In the present contribution, we will discuss a recent work on the hyperon properties in nuclear matter, which was carried out in a simple but plausible framework of a chiral soliton approach to nonzero density phenomena in the SU(3) sector \cite{3}. Previously, a similar approach was developed in the non-strangeness sector to study various phenomena in medium (for example, see Ref. \cite{4} and references therein) and the results were in qualitative agreement in the SU(3) sector \cite{3}. Previous similar approach was carried out in a simple but plausible framework of a chiral soliton approach to nonzero density phenomena \cite{1,2}. In the present contribution, we will discuss a recent work on the hyperon properties in nuclear matter, at low densities and how they can be changed in higher densities that can be found in the interior of neutron stars \cite{1,2}. [\cite{3}] Here, the Skyrme parameter, the masses of the \( \pi \), \( \rho \) mesons are given respectively as \( m_\pi = 134.976 \text{MeV} \) and \( m_K = 495 \text{MeV} \), and the mass matrix of the pseudo-Nambu-Goldstone bosons \( M \) has the diagonal form \( M = (m_\pi^2, m_\rho^2, m_K^2) \). The density-dependent functions \( \alpha_i^4(\rho), \alpha_i^5(\rho), \alpha_i^6(\rho), \alpha_i^7(\rho) \) and \( \alpha_{iSB}(\rho) \) reflect the changes of the meson properties in nuclear medium. In an approximation of homogeneous infinite nuclear matter they are expressed in terms of the three linear density-dependent functions \( f_i(\rho) = 1 + C_i \rho \), \( i = 1, 2, 3 \). The numerical values of \( C_i \) are fixed to be \( C_1 = -0.279, C_2 = 0.737 \) and \( C_3 = 1.782 \), respectively. They reproduce very well the equations of state (EoS) for symmetric nuclear matter near the normal nuclear matter density \( \rho_0 \) and at higher densities that may exist in the interior of a neutron star. The medium modification of the kaon properties is achieved by considering the following scheme

\[ F_\pi m_K \rightarrow F_\pi^K m^*_K = F_\pi m_K (1 - C_1 \rho / \rho_0) \]

and can be explained in terms of the alteration of the kaon decay constant and/or of the kaon mass in nuclear environment.

The quantization of the model is performed by considering the time-dependent rigid rotation of a static soliton

\[ U(r, t) = A(t) U_0(r), A(t)^\dagger, \]

where \( U_0(r) \) denotes the static SU(3) chiral soliton with trivial embedding. The time-dependent rotational matrix \( A(t) \) is decomposed

\[ A(t) = \begin{pmatrix} A(t) & 0 \\ 0^\dagger & 1 \end{pmatrix} S(t), \]

in terms of the SU(2) isospin rotation \( A(t) = k_0(t) 1 + i \sum_{a=1}^{3} \tau_a k_a(t) \) and fluctuations into the strangeness sector given by the matrix \( S(t) = \exp \left\{ i \sum_{p=4}^{7} k_p \lambda_p \right\} \). Here \( \tau_{1,2,3} \) denote the Pauli matrices, whereas \( \lambda_p \) stand for the strange part of the SU(3) Gell-Mann matrices. The time-dependent functions \( k_a(t) \) \( (a = 0, 1, 2, \ldots, 7) \) represent arbitrary collective coordinates. The more details of the approach can be found in Ref. \cite{3}. 

II. THE MODEL

The Lagrangian of the present model written by the following form

\[ \mathcal{L} = -\frac{F_\pi^2}{16} \alpha_2^4(\rho) \text{Tr} L_0 L_0 + \frac{F_\pi^2}{16} \alpha_2^5(\rho) \text{Tr} L_i L_i - \frac{\alpha_4^4(\rho)}{16 \pi^2} \text{Tr} [L_0, L_i]^2 + \frac{\alpha_4^5(\rho)}{32 \pi^2} \text{Tr} [L_i, L_j]^2 + \frac{\alpha_4^7(\rho)}{16} \text{Tr} [L_i, L_j] + \mathcal{L}_{WZ}, \]

where \( L_\mu = U^\dagger \partial_\mu U \) and \( U(x, t) \) is a chiral field in SU(3). The Wess-Zumino term \[ \mathcal{L}_{WZ} \] in the Lagrangian constrains the soliton to be identified as a baryon and is expressed by a five-dimensional integral over a disk \( D \)

\[ S_{WZ} = -\frac{i N_c}{240 \pi^2} \int_D d^5 \xi \epsilon^{\mu \nu \alpha \beta \gamma} \text{Tr} (L_\mu L_\nu L_\alpha L_\beta L_\gamma). \]

Here \( \epsilon^{\mu \nu \alpha \beta \gamma} \) is the totally antisymmetric tensor defined as \( \epsilon^{01234} = 1 \) and \( N_c = 3 \) is the number of colors. The values of input parameters are defined in free space: \( F_\pi = 108.783 \text{MeV} \) denotes the pion decay constant, \( e = 4.854 \) represents the Skyrme parameter, the masses of the \( \pi \) and \( K \) mesons are given respectively as \( m_\pi = 134.976 \text{MeV} \) and \( m_K = 495 \text{MeV} \), and the mass matrix of the pseudo-Nambu-Goldstone bosons \( M \) has the diagonal form \( M = (m_\pi^2, m_\rho^2, m_K^2) \). The density-dependent functions \( \alpha_2^4(\rho), \alpha_2^5(\rho), \alpha_4^4(\rho), \alpha_4^5(\rho) \) and \( \alpha_{4SB}(\rho) \) reflect the changes of the meson properties in nuclear medium. In an approximation of homogeneous infinite nuclear matter they are expressed in terms of the three linear density-dependent functions \( f_i(\rho) = 1 + C_i \rho \), \( i = 1, 2, 3 \). The numerical values of \( C_i \) are fixed to be \( C_1 = -0.279, C_2 = 0.737 \) and \( C_3 = 1.782 \), respectively. They reproduce very well the equations of state (EoS) for symmetric nuclear matter near the normal nuclear matter density \( \rho_0 \) and at higher densities that may exist in the interior of a neutron star. The medium modification of the kaon properties is achieved by considering the following scheme

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III. RESULTS AND DISCUSSIONS

The results of this contribution are presented in the form of tables and figures. The table shows the values of various parameters and the figure illustrates the changes of the hyperon properties as a function of density. The numerical results are in good agreement with the expectations and the previous works. The study of the hyperon properties in nuclear medium is of great interest in contemporary nuclear physics.
III. RESULTS AND DISCUSSIONS

All model parameters in free space and in nuclear matter, except for the parameter $C$ in Eq. (3), are fixed in the SU(2) sector. The only remaining parameter $C$ could be fixed by data on kaon-nucleus scattering and kaonic atoms. However, in the present work we carry out a qualitative analysis of the effects in the baryonic sector due to the modification of the kaon properties in nuclear medium. Consequently, we discuss the density dependence of the mass splittings among the various baryon multiplet members. In our calculation, the parameter value $C = 0$ corresponds to the case when the properties of kaon will not change in nuclear matter whereas a nonzero value of the parameter $C \neq 0$ indicates that the mass and/or kaon dynamics is altered in a dense nuclear environment.

The results show that in general the masses of the baryon octet tend to decrease in nuclear matter. Only $\Sigma$ showed a different tendency if the parameter value is set to be $C = 0$. In the case of $C = 0$, $m_\Sigma$ also tends to decrease as the density of nuclear matter increases [3]. In comparison, the results from SU(3) chiral effective field theory [6] show that $m_\Lambda^*$ is decreased by about 17\% at normal nuclear matter density $\rho_0$. The $\Xi$ hyperon is behaved in a similar manner. At $\rho_0$ the change in the mass of $\Xi^*$ was about 6\% and 16\% for the corresponding parameter values $C = 0$ and $C = 0.2$, respectively. The masses of the baryon decuplet increase in general as $\rho$ increases. Changes are dramatic for $C = 0$ while for $C = 0.2$ they are less changeable.

We present the density dependence of the mass splittings among the multiplet members in Figs. 1 and 2. Figure 1 shows the density dependence of the mass splittings among the baryon octet members. The mass splittings in nuclear matter are normalized to the corresponding free space mass splittings. The left and right panels in the figure correspond to the results with $C = 0$ and $C = 0.2$, respectively.

![Density dependence of the mass splittings among the baryon octet members.](image1)

**FIG. 1.** (Color online.) Density dependence of the mass splittings among the baryon octet members. The mass splittings in nuclear matter are normalized to the corresponding free space mass splittings. The left and right panels in the figure correspond to the results with $C = 0$ and $C = 0.2$, respectively.

![Density dependence of the mass splittings among the baryon decuplet members.](image2)

**FIG. 2.** (Color online.) Density dependence of the mass splittings among the baryon decuplet members. Notations are the same as in Fig. 1.

shows the density dependence of the mass splittings among the baryon octet members while Fig. 2 depicts the results corresponding to the mass splittings among the decuplet members. All the mass splittings in nuclear matter are
normalized to the values of the corresponding ones in free space. The left and right panels in the figures illustrate the results with two different values of parameter $C$, respectively.

It is interesting to see that except $m^*_\Sigma - m^*_\Lambda$ all the mass splittings tend to decrease up to $(1.5-2)\rho_0$. This behavior can be explained in terms of the density-dependent functionals $\omega^*$ and $c^*$ entering into the mass formula (see Eq. (36) in Ref. [3]). The first functional describes the fluctuations in the strangeness direction and comes into play for the mass splitting formula between the same strangeness members while all other mass splittings presented in the figures depend linearly on $\omega^*$. This indicates that at large densities the fluctuations in strangeness direction gets weaker. From the figures one concludes also that at large densities SU(3) flavor symmetry tends to be restored.

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