Robust Planning for High-Tech Supply Chain Networks under Uncertainty

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Abstract

The decision making on production and distribution for high-tech companies is one of most important work in terms of right time and accuracy as well as total cost. This study proposes an efficient robust production and distribution planning methodology to cope with demand uncertainty inherent in supply chain networks of high-tech manufacturing. Computational experiments show that the performance of the proposed method is superior to that of deterministic approach using various scenarios.

Keywords: High-Tech Industries, Uncertainty, Production, Distribution, Supply Chain Networks, Robustness.

1. Introduction

Generally, a manufacturing industry that requires large quantities of parts such as semiconductors and electronic components has a supply chain network based on more than two regionally dispersed suppliers, production bases, and vendors. In other words, the multi-site procurement-production-distribution system (MPPDS) is a supply chain network that sends materials acquired from many raw material suppliers to production bases and delivers finished products to distributors.

The main characteristic of the MPPDS supply chain network addressed in this study is that MPPDS is a two-stage hybrid production-distribution chain. In general, the MPPDS hybrid production-distribution chain includes the delivery of upstream FAB (fabrication), back-end process (final option assembly), and downstream distribution (Figure 1). In a semiconductor industry, wafer fabrication in the production-distribution chain is accomplished in the upstream FAB stage, while finished products are acquired after final assembly in the back-end process stage. The FAB stage mainly follows a push-type make-to-stock method to improve the resource utilisation in response to relatively longer production lead time and customer needs for quick delivery. On the other hand, the back-end process production stage at the downstream level follows a pull-type make-to-order strategy because of the high variability of orders and customer needs for various specifications. Moreover, unique characteristics of the MPPDS production-distribution chain, which make production and distribution plans more difficult, include an alternative BOM (bill of material) based on quality specifications, volatilities of price and demand, distribution among multiple facilities, etc.¹–³. The MPPDS chain requires alternative BOM to manufacture finished products by using the main parts supplied by subcontractors or other parts with different characteristics. However, alternative BOM is a factor that causes various product groups and product structures and increases the complexity of production and distribution control (PDC). In fact, customisation and quality specification classes of products are considered a main cause for high volatility in consumer goods manufacture as well as semiconductor production.

The volatility inherent in a supply chain network, as considered in this study, is a distinct feature that cannot be found in previous studies about the traditional PDC problem. Although the traditional PDC problem focuses

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on the optimisation of the product distribution network by either the minimization of total cost or only by demand volatility, they do not take account of the price uncertainty that exists in the MPPDS supply network.

This study suggests a methodology for an effective production and distribution plan in semiconductor MPPDS with various inherent volatilities to solve the problem. Firstly, the study defines a model that reflects realistic constraint conditions such as the cost of plan changes and production capacity restrictions while developing a stochastic mixed integer linear programming model using a variability-reflected scenario. In addition, the study evaluates the validity of models and shows the significance of the suggested algorithm through experiments conducted in various scenarios.

2. PDC Algorithm Considering Volatility

Lead time in the MPPDS production-distribution chain depends largely on production stages (up and downstream) and distribution methods. For example, FAB production stage in the upstream requires a relatively longer lead time than assembly and distribution stage in the downstream. As MPPDS demonstrates big changes for lead time in production and distribution stages and, as mentioned earlier, may include demand volatility and price uncertainty depending on product specifications, it is necessary to develop a methodology for making effective PDC decisions in the production-distribution chain.

This study introduces two types of models: one is a stochastic model relating to various scenarios (possible situations) and the other is a deterministic model relating to re-plan costs. The models include two phases. Firstly, a decision made in the first stage of a stochastic model regarding various scenarios is relevant to the establishment of FAB production decision making, based on expected demand and price information. Therefore, robust decisions in the first phase should be developed by considering all kinds of possible and realistic scenarios for planning, so that they can be adopted for any future scenario. In the second phase, when uncertainty of demand and price disappears as actual customer orders are obtained after a certain amount of time, decisions related to assembly and distribution in the downstream stage are readjusted to improve profits. At that time, decision variables set in the first phase are fixed and cannot be changed, while variables included in the second phase can be re-planned within the realized scenario. Further, diverse part specifications that are produced as a push-type in the upstream manufacturing process of the first phase should be taken into account as alternative BOM in the second phase.

The first phase of a deterministic model relating to re-plan costs deducts decisions for both upstream and downstream stages based on the optimised current information judged by a company. In the second phase, re-plans should be established based on changes of information that have occurred until the moment at which final decisions should be made in the downstream stage. In this sense, the second phase includes making a decision whether re-plans are to be established, by comparing costs related to the re-plan with the deterministic model in the first phase.

2.1 Stochastic Model

Stochastic programming or the stochastic mixed-integer linear programming (MILP) model is developed in order to design a mathematical model and set decisions regarding the volatility characteristics in the MPPDS production-distribution chain in the semiconductor industry. The purpose of model development is to discover the production and distribution policy regarding volatility. A brief introduction to the PDC model of the MPPDS production-distribution chain, the object of this study, is described below:

- Capacity restrictions on upstream production, downstream assembly, and distribution
- Order backlogs are not allowed and penalties are imposed on unfulfilled demand
- Probability distribution is provided based on historical data on demand and price

Figure 1. Configuration of supply chain networks.
Demand can be fulfilled alternatively in the downward direction towards depreciation. That is, customer orders can be delivered with the original specification levels or upper-class levels.

This study aims to design a model that seeks a robust solution to the PDC problem including volatility and uncertainty of price and demand by using possible scenario sets \(\Omega = \{1, \ldots, S\}\) for a certain amount \(S\). In the process of solving the PDC problem, a deterministic model that does not consider demand volatility and price uncertainty is used. By ignoring the uncertainty, the PDC problem can be designed as a model that has only one scenario (e.g. \(S = 1\)). This model is a PDC-D (PDC-deterministic) model that is currently employed in the MPPDS supply network of high-tech manufacturing companies, including semiconductor manufacturing firms. The PDC-D model should be developed to design a robust optimisation (RO)-type model by using scenarios that assume the existence of uncertainty. For this goal, prototypes of PDC-D models can be arranged as described below. However, the PDC-D model should be designed under the assumption that a certain scenario \(S\) has already been chosen as the model is deterministic.

### 2.2 Stochastic Models

**Indices**
- \(i = 1, 2, \ldots, N\): Product group,
- \(j = 1, 2, \ldots, M\): Assembly manufacturing facility,
- \(k = 1, 2, \ldots, K\): Quality specification level of FAB product (wafer),
- \(l = 1, 2, \ldots, L\): Quality specification level of finished product,
- \(s = 1, 2, \ldots, S\): Possible scenario in future,
- \(t = 1, 2, \ldots, T\): Planning horizon,

**Parameters**

**In terms of whole model**
- \(P_{it}^l\): Average sale price for finished product group \(i\) with quality specification level \(l\) during period \(t\),
- \(D_{it}^l\): Forecasted demand of finished product group \(i\) with quality specification level \(l\) during period \(t\),
- \(d_{it}^l\): Fulfilled demand quantity of finished product group \(i\) with quality specification level \(l\) during period \(t\),

**In terms of cost**
- \(hc_{it}^l\): Unit inventory holding cost of wafer product group \(i\) with quality specification level \(l\),
- \(hpi_{it}^l\): Unit inventory holding cost of finished product group \(i\) with quality specification level \(l\),
- \(ta_{ij}^l\): Unit shipping cost for air transportation of wafer product group \(i\) to facility \(j\),
- \(ts_{ij}^l\): Unit shipping cost for sea transportation of wafer product group \(i\) to facility \(j\),
- \(\pi_i^l\): Unit opportunity cost for finished product group \(i\) with quality specification level \(l\),

**In terms of inventory and production**
- \(lp_{ij}^l\): Inventory quantity of wafer (upstream stage) product group \(i\) with quality specification level \(l\) during period \(t\),
- \(lmc_{ij}^l\): Inventory quantity of wafer product group \(i\) with quality specification level \(l\) in facility \(j\) during period \(t\),
- \(impi_{ij}^l\): Inventory quantity of finished product group \(i\) with quality specification level \(l\) in facility \(j\) during period \(t\),
- \(yp_{ij}^l\): Production yield of wafer product group \(i\) with quality specification level \(l\) during period \(t\),
- \(ym_{ij}^l\): Production yield of finished product with quality specification level \(l\) to be assembled by wafer product group \(i\) with quality specification level \(l\) in facility \(j\), where \(\sum_k yp_{ij}^k \leq 1, \forall i\),

**In terms of capacity**
- \(cp_{i}^l\): Production capacity of wafer product group \(i\),
- \(cm_{ij}^l\): Production capacity of finished product group \(i\) in facility \(j\),
- \(ca_{ij}^l\): Air transportation capacity when shipping wafer product group \(i\) to facility \(j\),

**Decision variables**

1\(^{st}\) stage robust decision variables
- \(PQ_{it}^l\): Production quantity of wafer product group \(i\) during period \(t\),
- \(S_{it}^l\): Sea transportation quantity when shipping wafer product group \(i\) with quality specification level \(k\) to facility \(j\) during period \(t\),

2\(^{nd}\) stage decision variables
- \(AQ_{ij}^k\): Assembly quantity of wafer product group \(i\) with quality specification level \(k\) in facility \(j\) during period \(t\),
- \(A_{ij}^k\): Air transportation quantity when shipping wafer product group \(i\) with quality specification level \(k\) to facility \(j\) during period \(t\),
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\textbf{PDC-D}

Maximize \( R \)

Subject to

\[ R = \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{t=1}^{T} p_{ij}^{t} d_{ij}^{t} - \sum_{i=1}^{N} \sum_{t=1}^{T} h_{ij}^{t} I_{ij}^{t} \]
\[ - \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{t=1}^{T} c_{ij}^{t} A_{ij}^{t} - \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{t=1}^{T} s_{ij}^{t} S_{ij}^{t} \]
\[ \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{t=1}^{T} \pi^{t} (D_{ij}^{t} - d_{ij}^{t}) \]

(1)

\[
I_{ij}^{t} = I_{ij}^{t-1} + y_{ij}^{t} P_{ij}^{t} - \sum_{k} A_{ij}^{t} - S_{ij}^{t}, \forall i, k, t 
\]

(2)

\[
I_{ij}^{t} = I_{ij}^{t-1} + A_{ij}^{t} + S_{ij}^{t} - A_{ij}^{t}, \forall i, j, k, t 
\]

(3)

\[
I_{ij}^{t} = \max(I_{ij}^{t-1} + \sum_{k} y_{ij}^{t} A_{ij}^{k} - d_{ij}^{t}, \forall i, j, l, t 
\]

(4)

\[
D_{ij}^{t} \geq \sum_{k} d_{ij}^{k}, \forall i, l, t 
\]

(5)

\[
\sum_{k} A_{ij}^{k} + \sum_{l} S_{ij}^{l} \leq I_{ij}^{t-1} + y_{ij}^{t} P_{ij}^{t}, \forall i, k, t 
\]

(6)

\[
d_{ij}^{t} \leq \max(I_{ij}^{t-1} + \sum_{k} y_{ij}^{t} A_{ij}^{k}, \forall i, j, k, t 
\]

(7)

\[
P_{ij}^{t} \leq c_{p}^{t}, \forall i 
\]

(8)

\[
\sum_{k} A_{ij}^{k} \leq c_{m}^{t}, \forall i, j, t 
\]

(9)

\[
\sum_{k} A_{ij}^{k} \leq c_{a}^{t}, \forall i, j, k, l, t 
\]

(10)

\[
P_{ij}^{t} \geq 0; A_{ij}^{t} \geq 0; S_{ij}^{t} \geq 0, \forall i, j, k, t 
\]

(11)

To briefly explain the model described above, \( R \) in the first formula shows gross profits regarding sales revenue, inventory holding, distribution, and stock-out costs. Constraints (2)–(4) show inventory balance equations, while constraints (5)–(7) set both upper and lower limits for the volume of order transportation. Constraints (8)–(10) are formulated to reflect capacity restrictions on production facilities and distribution methods (sea or air). The RO model is a method used to set a plan stochastically when many possible scenarios exist owing to demand volatility and price uncertainty. According to a recent study, the RO model is a methodology that increases the robustness of decision making while decreasing the variance of objective function values. It is especially known for calculating relatively reasonable results when uncertainty about critical issues exists at decision point.

When the probability mass function is given for each possible scenario of discrete random variable and \( w' (w' > 0) \) is occurrence probability of scenario \( s \), \( \sum_{s \in \Omega} w' = 1 \). This study suggests PDC-MEV (PDC-max expected value) for the RO model. The purpose of the PDC-MEV model is to find robust decision variables to maximize expected profits under various types of possible scenarios. The PDC-MEV model adjusts the objective function of a basic PDC-D type designed to be deterministic and reflects the scenarios set. The basic framework of a PDC-MEV model is described below.

\textbf{PDC-MEV}

Maximize \( E[R'] = \sum_{s \in \Omega} w'R' \)

Subject to

\[
R' = \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{t=1}^{T} p_{ij}^{t} d_{ij}^{t} - \sum_{i=1}^{N} \sum_{t=1}^{T} h_{ij}^{t} I_{ij}^{t} 
\]
\[ - \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{t=1}^{T} c_{ij}^{t} A_{ij}^{t} - \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{t=1}^{T} s_{ij}^{t} S_{ij}^{t} \]
\[ \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{t=1}^{T} \pi^{t} (D_{ij}^{t} - d_{ij}^{t}) \]

(12)

\[
A'X' = B', \forall s 
\]

(13)

\[
X' \geq 0; s = 1, 2, ..., S 
\]

(14)

In the PDC-MEV model above, \( R' \), \( p_{ij}^{t} \), \( D_{ij}^{t} \), \( d_{ij}^{t} \), \( I_{ij}^{t} \), \( A_{ij}^{t} \), \( S_{ij}^{t} \), \( Imc_{ij}^{t} \), and \( Imp_{ij}^{t} \) are parameters in a certain scenario \( s \). To make the explanation simple, constraints (2)–(10) of the PDC-D model are replaced for the expression of \( A'X' = B' \). However, those constraints should reflect every scenario (\( s \in \Omega \)). Moreover, the decision variable \( x \) can be expressed as \( x = (x^{1st}, x^{2nd}) \) within the two-stage PDC system framework of the study. In addition, \( x^{1st} \) and \( x^{2nd} \) each mean the amount of common part production and sea transportation of the first phase (upstream stage) and the amount of finished goods assembly production and air transportation of the second phase (downstream stage). Therefore, the first phase finds a robust solution regarding all scenarios by using the PDC-MEV model and the second phase seeks a \( x^{2nd} \) solution without uncertainty after fixing a \( x^{1st} \) variable under realised scenarios with the lapse of time.

\textbf{2.3 Deterministic Model}

The first phase of the deterministic model with re-plan costs, PDC-RPC, draws both upstream and downstream
decisions based on the most accurate information at that time. For the second phase, re-plans are established based on changes of information that have occurred until the time when final decisions are supposed to be made in the downstream stage. The second stage decides whether the re-plan is to be established by comparing gross profits ($R$) with all the costs related to the re-plan with the deterministic model in the first phase. Digitization of re-plan related costs makes it possible to calculate the cost of changing plans (CP) by separating a frozen period (FP) and re-planning period (RP) in a planning period ($T$). If $CF$ is a cost (constant) that occurs when a plan should be inevitably changed in a frozen period, a re-planning cost is defined as a formula (15), while a cost function followed by plan change is expressed as shown in Figure 2.

$$
CP_{t} = \begin{cases} 
CF, & \text{for } FP \geq t \\
CF - \frac{(t - FP)CF}{(T - FP - RP)}, & \text{for } FP < t \leq T - RP \\
0, & \text{for } T - RP < t 
\end{cases}
$$

(15)

3. Results and Analysis

This study designs an experiment to evaluate the performance of the two robust algorithms, which are the stochastic model using PDC-MEV model and the deterministic model with PDC-RPC model, and seeks an opportunity for it to be applied in reality. The design of the experiment can be summarized as shown in Table 1. According to Table 1, the semiconductor MPPDS supply network, which is the target of this experiment, consists of three product groups, three manufacturing facilities, and three quality levels. The parameters for input data such as inventory holding cost, opportunity cost, transport cost, capacity of production and transportation, etc. are created from fixed uniform distributions. Further, $CF$, a cost constant used in the PDC-RPC model, is set to be 20% of gross profits ($R$) obtained from the PDC-D model. A planning time in the second phase is made to occur randomly and is located in each FP, semi-frozen period, and RP sections with the probabilities of 0.1, 0.7, and 0.2, respectively.

3.1 Results Analysis

This section analyses the performance of the algorithms for the PDC-MEV stochastic model and the PDC-RPC deterministic model by using the experimental data

| Table 1. Experimental data |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| Product group | High quality | Medium quality | Low quality |
| Inventory holding cost for wafer | $k = 1,2,3$ | U[6.8] | U[5.7] | U[4.6] |
| k = 1.2 | U[8.10] | U[7.9] | U[6.8] |
| k = 3 | U[7.9] | U[6.8] | U[5.7] |
| k = 1 | U[200,250] | U[150,200] | U[100,150] |
| k = 2 | U[300,350] | U[250,300] | U[200,250] |
| k = 3 | U[200,250] | U[150,200] | U[100,150] |
| Opportunity cost | $k = 1,2,3$ | U[200,250] | U[150,200] | U[100,150] |
| Production capacity in upstream | $k = 1,2,3$ | U[3000,2500] | U[3000,2500] | U[3500,4000] |
| Assembly capacity in Downstream | $k = 1,2,3$ | U[4000,5000] | U[4000,5000] | U[4500,5500] |
| Air transportation capacity | $k = 1.2$ | U[400,500] | U[400,500] | U[450,550] |
| Air transportation capacity | $k = 3$ | U[300,400] | U[300,400] | U[300,400] |
| Air transportation cost | $k = 1,2,3$ | U[100,150] | U[200,250] | U[100,150] |
described in Tables 1 and 2. In addition, objective function values from each algorithm are compared with those from the PDC-D model. As a measure for relative comparison, relative error (RE) is used in the experiment.

\[
RE = \frac{S_{\text{proposed}} - S_D}{S_D}
\]

(16)

Where,

- \(S_D\): objective function value of the PDP-D model
- \(S_{\text{proposed}}\): objective function value acquired from the PDP-MEV stochastic model or the PDC-RPC deterministic model.

As a result, robustness, the performance measure, becomes more powerful as \(RE\) values increase. Unlike the PDC-MEV and PDC-RPC models, which include two steps, the PDC-D model determines all parameters including demand and price information during the planning stage so that changes do not occur once decisions are made in the first phase. PDC-D model is supposed to follow only a normal case scenario with medium quality in Tables 1 and 2.

Figure 3 shows the experimental results of the three scenarios (optimistic, normal, and pessimistic). For every case, both the PDC-MEV stochastic model and the PDC-RPC deterministic model show superior functions than that of the PDC-D model. In particular, the PDC-MEV model shows profits about 60% higher than those of the PDC-RPC model in all three scenarios. For each model, the optimistic and pessimistic scenarios show a better result than the normal one. This means that the two algorithms suggested in the study show more robustness in optimistic/pessimistic situations that have higher volatility than in the normal scenario. This experiment has proven that designing a model that reflects volatility is effective for a semiconductor MPPDS supply network operated by a two-stage system.

### Table 2. Data generation criteria for input data with uncertainty

| Quality group | Optimistic case | Normal case | Pessimistic case |
|---------------|----------------|-------------|------------------|
| Demand        |                |             |                  |
| Mean (\(\mu\)) | High           | U[1300,1400] | U[1100,1200] | U[900,1000] |
|               | Medium         | U[1500,1600] | U[1300,1400] | U[1100,1200] |
|               | Low            | U[1200,1100] | U[1000,900]  | U[800,700]   |
| Standard deviation (\(\sigma\)) | High         | U[1400,1500] | U[1200,1300] | U[1000,1100] |
| Price         | Medium         | U[1200,1100] | U[1000,900]  | U[800,700]   |
| Mean (\(\mu\)) | Low            | U[1000,900]  | U[800,700]   | U[600,500]   |
| Standard deviation (\(\sigma\)) | U[80,110]    | U[70,100]   | U[60,90]     |

![Figure 3. Performance results (RE) for three situations.](image-url)
4. Conclusion

This study aimed to solve a decision-making problem of a semiconductor manufacturing supply chain network by establishing a production and transportation strategy for MPPDS supply network with inherent various volatilities. We defined a PDC-D model that reflected realistic constraints, including capacities of production and air transportation to adjust decisions about production, inventory, and the distribution of products separated by quality. Further, we developed the PDC-MEV stochastic model that reflects both demand and price volatilities and the PDC-RPC deterministic model that considers re-plan costs. This study has evaluated the validity of the models and has shown the significance of the suggested algorithms through various experiments. We expect that more improved decision-making solutions for supply networks can be acquired if the methodology suggested in this study is properly applied to the actual semiconductor manufacturing supply chain.

5. References

1. Ahn H, Kaminsky P. Production and distribution policy in two-stage stochastic push pull system. IIE Transactions. 2005; 7:609–21.
2. Bollapragada R, Rao US. Replenishment planning in discrete time, capacitated, nonstationary, stochastic inventory system. IIE Transactions. 2006; 38:583–95.
3. Gupta A, Maranas CD. Managing demand uncertainty in supply chain planning. Computers and Chemical Engineering. 2003; 27:1219–27.
4. Higle JL, Wallace SW. Sensitivity analysis and uncertainty in linear programming. Interfaces. 2003; 33:53–60.
5. Leung SCH, Wu YA. Robust optimisation model for stochastic aggregate production planning. Prod Plann Contr. 2004; 15:502–14.
6. Leung SCH, Sally OS, Tsang WLN, Wu Y. A robust optimisation model for multi-site production planning problem in an uncertain environment. Eur J Oper Res. 2007; 181:224–38.
7. Sen S, Higle JL. An introductory tutorial on stochastic linear programming models. Interfaces. 1999; 29:33–61.
8. Yildirim I, Tan B, Karaesmen F. A multi period stochastic production planning and sourcing problem with service level constraints. OR Spectrum. 2005; 27:471–89.