Possible S-wave Bound-States of Two Pseudoscalar Mesons

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Abstract

Using the potentials derived from vector-meson-exchanges, the $K\bar{K}$, $DK$, $B\bar{K}$, $D\bar{D}$, $B\bar{B}$, $BD$, $DK$, $BK$ and $BD$ systems are studied. Possible S-wave bound-states of two pseudoscalar mesons are discussed. We find that systems of $DB$, $D\bar{D}$, $BB$ and $DK$ with isospin 0 are likely to form S-wave bound states via strong interactions while the others are unlikely to be bound by the strong interaction sector alone. With the Coulomb potential, the $K^+K^-$ system can form an atomic bound state - the kaonium. The influence of the one-meson-exchange potential on the ground state energy of the kaonium and its decay widths to $\pi\pi$ and $\pi\eta$ are evaluated.

1 Introduction

Since the 1970’s, the possible existence of the nuclear-like bound states of mesons has been one of the interesting subjects in hadron physics. Some hadrons such as the $\psi(4040)$ \textsuperscript{11}, $\eta(1440)$ \textsuperscript{22}, $f_1(1420)$ \textsuperscript{13}, $f_0(1720)$ \textsuperscript{24}, and the $f_0(980)$ and $a_0(980)$ \textsuperscript{35,36,78} were explained as the nuclear-like bound states of mesons. Törnqvist \textsuperscript{9} argued that the one-pion-exchange potential is likely to form a few states composed of two ground-state mesons. In 2003 the Belle Collaboration reported a new narrow charmonium state at $3872 \pm 0.6(stat) \pm 0.5(syst)$ MeV and with a width $< 2.3$MeV(95% C.L.), which has been confirmed by several experiments \textsuperscript{10,11,12,13}. Almost immediately, Törnqvist \textsuperscript{14} claimed that this state is the one he predicted long ago. Indeed the proximity of the X to $D^0\bar{D}^{0*}$ threshold led to a speculation that the X is a $D\bar{D}^*$ resonance \textsuperscript{15,16,17}.

Recently, beginning with the discovery of $D_s(2317)$, more than 10 heavy mesons are reported, such as the $B_c$, $h_c$, $\eta_c$, $D_s(2460)$, $X(3872)$, $X(3940)$, $Y(3940)$, $Z(3930)$ and $Y(4260)$. The new findings have generated much enthusiasm for understanding the nature of the new mesons. A survey of the experimental, phenomenological and theoretical status of the new heavy mesons can be found in a recent review article by Swanson \textsuperscript{18}. Different models are used to study the spectroscopy. For example, Liu, Zeng and Li \textsuperscript{19} suggested that the $Y(4260)$ is a $\chi_{c1}\rho$ molecule state bound by the $\sigma$ exchange. Barnes, Close and Lipkin \textsuperscript{20}}
have suggested that the $D_{s0}$ and $D_{s1}$ may be $DK^{(*)}$ molecules. However, until now, none of these states is well-established as a molecular state.

The existence of so many controversial states strongly suggest possible two-meson structure. However, the dynamics of bound states of two mesons is not well-understood. It is worthy to study systems of two mesons. For example, the understanding for the structure of the $f_0(980)$ has been controversial for many years. It might be a $q\bar{q}$ state, a $q^2\bar{q}^2$ state, or a $KK$ molecule state. Krewald et al. [21] studied the kaon-antikaon system by using the strong interactions generated from vector-meson-exchange in the frame work of the SU(3) invariant effective Lagrangian. They have shown that one-meson exchange potentials derived from this Lagrangian in the non-relativistic limit are sufficient to bind $KK$ into a kaonic molecule with a mass and decay width that closely match the experimental values of the $f_0(980)$ meson. However, in their study the momentum dependent terms of potentials are neglected and a quite large cut-off parameter of the form factor is used.

In present paper, we extend the study to $KK$, $DK$, $B\bar{K}$, $D\bar{D}$, $B\bar{B}$, $BD$, $\bar{D}K$, $BK$, $B\bar{D}$ systems by including the momentum dependent terms of one-vector-meson exchange interactions. The momentum dependent terms result non-local potentials of two mesons in coordinate space. We use these nonlocal potentials to search for possible bound states of two-meson systems.

The model is briefly described in a generalized form in section 2. In section 3 we discuss possible bound states of the two-meson systems. The $KK$ system is discussed in detail. Both strong and Coulomb potential are included in the calculation. The strong interaction and its influence on the ground state energy and decay widths of the kaonium are discussed. We also extend our calculations to some other systems including heavy B and D mesons, where some possible S-wave bound states are found. Finally we give our summary and discussion in section 4.

2 The Model

In order to investigate the possible bound states of the $KK$, $DK$, $B\bar{K}$, $D\bar{D}$, $B\bar{B}$, $BD$, $KK$, $\bar{D}K$, $BK$, $DD$, $BB$ and $B\bar{D}$ systems in the framework of non-relativistic Schrödinger equation, we first derive the interaction potentials between two pseudoscalar mesons from vector-meson exchange diagrams which are found to be the dominant t-channel exchange interactions [21] [22] [23]. The relevant interaction Lagrangian of the pseudoscalar-pseudoscalar-vector coupling can be written in the following form,

$$\mathcal{L}_{P_{P_{P_{V}}} = -\frac{1}{2} i G \nabla Tr(P, \partial_\mu P)V^\mu}.$$  (1)

Usually $P$ and $V$ stand for the fields of pseudoscalar and vector octets, respectively. $P$ and $V$ are $3 \times 3$ matrices,

$$P = \sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & \pi^+ & K^+ \\ \pi^- & \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta_8 \end{pmatrix},$$  (2)

and

$$V = \sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}} \rho^0 + \frac{1}{\sqrt{2}} \omega & \rho^+ & K^{++} \\ \rho^- & \frac{1}{\sqrt{2}} \rho^0 + \frac{1}{\sqrt{2}} \omega & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}.$$  (3)
For a special case of the $K\bar{K}$ system the relevant Lagrangians are

$$L_{KK\rho} = iG_V \{[\bar{K}\tau(\partial_\mu K) - (\partial_\mu \bar{K})\bar{\tau}K] \cdot \bar{p} \}$$

$$L_{KK\omega} = iG_V [\bar{K}(\partial_\mu K) - (\partial_\mu \bar{K})K]\omega^\mu$$

$$L_{KK\phi} = -\sqrt{2}iG_V [\bar{K}(\partial_\mu K) - (\partial_\mu \bar{K})K]\phi^\mu$$

The $K$ and $\bar{K}$ are

$$K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \quad \bar{K} = \begin{pmatrix} K^- \\ \bar{K}^0 \end{pmatrix}$$

Here $G_V$ is the coupling constant, which can be fixed in terms of the experimental value of $\rho\pi\pi$ coupling constant $g_{\rho\pi\pi} = 2G_V$.

In present paper we deal with systems of two pseudoscalar mesons including heavy pseudoscalar mesons $D$ and $B$. We will generalize Eq.(1) to include the $D$ and $B$ pseudoscalar mesons. In this case, the strengths of their couplings to $\rho$, $\omega$ and $\phi$ mesons can be determined from the quark model.

By evaluating the meson-meson scattering amplitude in the Born approximation from the $t$-channel vector meson exchange the interaction potentials of two mesons in momentum space can be obtained. In the case of the $K\bar{K}$ system, one finds

$$V(M_V, \bar{p}, \bar{q}) = -\frac{g_{KKV}^2}{4M_K^2} C_I \frac{1}{\bar{q}^2 + M_V^2} (4M_K^2 + 8\bar{p}^2 + 8\bar{q} \cdot \bar{p} + 3\bar{q}^2).$$

Here subscript $V$ stands for the vector meson $\rho$, $\omega$ or $\phi$. $M_V$ is the mass of the V meson. $M_K$ is the mass of the kaon, $\bar{p}$ is the momentum of the kaon in the center-of-mass frame. $\bar{q}$ is the 3-momentum transfer between $K$ and $\bar{K}$ in the $t$-channel. The relevant coupling constants $g_{KKV}$ are related by SU(3) symmetry relations,

$$g_{KK\rho} = G_V, \quad g_{KK\omega} = G_V, \quad g_{KK\phi} = -\sqrt{2}G_V. \quad (4)$$

$C_I$ is the isospin factor with $I=0, 1$. For the exchange of $\rho$, $\omega$, $\phi$, it is respectively

$$C_0 = \begin{cases} 3 & \text{for } \rho \\
1 & \text{for } \omega \\
1 & \text{for } \phi \end{cases} \quad ; \quad C_1 = \begin{cases} -1 & \text{for } \rho \\
1 & \text{for } \omega \\
1 & \text{for } \phi \end{cases}$$

For each vector-meson exchange, after performing a Fourier transformation of the scattering amplitude the interaction potential in coordinate space can be obtained as

$$V(M_V, \bar{r}) = -\frac{g_{KKV}^2}{4\pi} C_I \left[ U(M_V, r) \frac{2\bar{p}^2}{M_K^2} - \frac{2i}{M_K^2} \nabla U(M_V, r) \cdot \bar{p} - \frac{3}{4M_K^2} \nabla^2 U(M_V, r) + U(M_V, r) \right] \quad (5)$$

with

$$U(M, r) = 4\pi \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{[F^i(\vec{k})]^2}{M^2 + \vec{k}^2} e^{i\vec{k} \cdot \vec{r}}$$

$$= e^{-Mr} - e^{-Ar} \left[ 1 + \frac{1}{16} \left( 11 - \frac{4M^2}{A^2} + \frac{M^4}{A^4} \right) \left( 1 - \frac{M^2}{A^2} \right)^2 \right. \left. (Ar) - \frac{1}{16} \left( 3 - \frac{M^2}{A^2} \right)^2 (Ar)^2 + \frac{1}{48} \left( 1 - \frac{M^2}{A^2} \right)^3 (Ar)^3 \right].$$
Here, a form factor with a cutoff parameter $\Lambda$ for each interaction vertex is added.

$$F^t(\vec{q}) = \left( \frac{\Lambda^2 - M^2}{\Lambda^2 + q^2} \right)^2. \quad (6)$$

The total potential of the $K\bar{K}$ system is a sum of contributions from $\rho$, $\omega$ and $\phi$ exchanges.

$$V(\vec{r}) = V(M_\rho, \vec{r}) + V(M_\omega, \vec{r}) + V(M_\phi, \vec{r}). \quad (7)$$

We assume that the coupling of the vector meson with two pseudoscalar mesons originates from the coupling of the vector meson with quarks inside the pseudoscalar mesons. The $\rho$ and $\omega$ mesons may couple to the the $u$ and $d$ quarks, while the $\phi$ meson only couples to the $s$ quark inside the pseudoscalar meson. From this consideration and by using the quark structure of the heavy mesons, we can generalize the derivation of the vector-meson-exchange potential between $K\bar{K}$ to other two-meson systems including $DK$, $B\bar{K}$, $D\bar{D}$, $BB$, $BD$, $DK$, $BK$ and $BD$. Because the $D$ and $B$ mesons do not contain the $s$ quark, the $\phi$ meson coupling does not present. For $KK$ and $K\bar{K}$ systems $\rho, \omega$ and $\phi$ exchanges all contribute to the interaction. For others systems containing the $D$ or $B$ meson there are only contributions from $\rho$ and $\omega$ exchanges. For these systems, the relevant interaction Lagrangians are written as

$$\mathcal{L}_{BB\rho} = iG_V \{ [\bar{B} i\sigma_\mu B] - (\partial_\mu B) \sigma B \} \cdot \vec{\rho}^\mu$$

$$\mathcal{L}_{BB\omega} = iG_V \{ [\bar{B} B] - (\partial_\mu B) \omega \} \mu$$

$$\mathcal{L}_{DD\rho} = iG_V \{ [D i\sigma_\mu \bar{D}] - (\partial_\mu \bar{D}) \sigma \bar{D} \} \cdot \vec{\rho}^\mu$$

$$\mathcal{L}_{DD\omega} = iG_V \{ [D (\partial_\mu \bar{D})] - (\partial_\mu \bar{D}) \omega \} \mu.$$

We assume that the coupling constant $G_V$ is the same as the usual PPV coupling constant.

$$\bar{D} = \begin{pmatrix} \bar{D}_0 \\ D_- \end{pmatrix}; \quad B = \begin{pmatrix} B^+ \\ B_0 \end{pmatrix}$$

$$D = (D^0, D^+) ; \quad \bar{B} = (B^-, \bar{B}_0)$$

One can get the relevant coupling constants from the Lagrangians

$$g_{DD\rho} = g_{BB\rho} = g_{KK\rho} = G_V \quad g_{DD\omega} = g_{BB\omega} = g_{KK\omega} = G_V \quad (8)$$

From these Lagrangians a non-relativistic Hamiltonian for the two-pseudoscalar-meson system can be obtained as

$$H = \left[ -\frac{1}{2\mu} + a(r) \right] \nabla^2 + b(r) \frac{\partial}{\partial r} + c(r). \quad (9)$$

Here $\mu$ stands for the reduced mass of the two mesons, and

$$a(r) = \sum_V \frac{g_{PPV}^2}{4\pi} C_V \frac{1}{4m_1m_2} \left[ \frac{m_1^2 + m_2^2}{m_1m_2} + 4 \right] U(M_V, r), \quad (10)$$

$$b(r) = \frac{\partial}{\partial r} a(r). \quad (11)$$
\[ c(r) = \sum \frac{g_{PPV}^2}{4\pi} C_1 \left[ \frac{1}{4m_1m_2} \left( \frac{m_1^2 + m_2^2}{m_1m_2} + 1 \right) \left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right) U(MV, r) - U(MV, r) \right], \]  

(12)

with \(m_1, m_2\) the masses of the pseudoscalar mesons.

\(g_{PPV}\) is the pseudoscalar-pseudoscalar-vector coupling constant, \(C_1\) is an isospin factor. By using the quark structure of mesons and comparing with the isospin factors for the \(K\bar{K}\) system we can get the isospin factors.

For \(DK, \bar{B}K, DD, BB\) and \(BD\) systems,

\[
C_0 = \begin{cases} 
3 & \text{for } \rho \\
1 & \text{for } \omega 
\end{cases} \quad C_1 = \begin{cases} 
1 & \text{for } \rho \\
1 & \text{for } \omega 
\end{cases} 
\]  

(13)

For \(\bar{D}K, BK, \bar{D}\bar{D}, BB\) and \(B\bar{D}\) systems,

\[
C_0 = \begin{cases} 
3 & \text{for } \rho \\
-1 & \text{for } \omega 
\end{cases} \quad C_1 = \begin{cases} 
-1 & \text{for } \rho \\
-1 & \text{for } \omega 
\end{cases} 
\]  

(14)

By solving the Schrödinger equation with above Hamiltonian one can search for possible bound states of the two-meson systems.

There are two parameters in the model, the coupling constant \(g_{\rho\pi\pi}\) and the cutoff parameter \(\Lambda\). The coupling constant \(g_{\rho\pi\pi}\) is determined by the decay width \(\Gamma(\rho \to \pi\pi)\) as \(g_{\pi\pi\rho}^2/4\pi = 2.8\). From \(G_V = g_{\pi\pi\rho}/2\) we get \(G_V = 3.0\). So the only free parameter is the cutoff parameter \(\Lambda\) for each system. In general, the cutoff parameters can be different for different pseudoscalar-meson systems. In the analysis of the \(\pi\pi\), and \(\pi K\) phase shifts, the \(\Lambda\) changes ranging from 1.5 GeV to 2.0 GeV \[23\]. We use the same values for all systems in our study.

### 3 Possible bound state

We first discuss the \(K\bar{K}\) system. The strong interaction potential of this system is in the form of Eq.(7). The corresponding Schrödinger equation has a singular point which depends on the reduced mass of two mesons to cause \([-\frac{1}{2p} + a(r)] = 0\) in Eq.(9). Because of the small mass of the \(K\bar{K}\) system and the existence of the singular point, the one meson (\(\rho, \omega, \phi\)) exchange potential alone fails to produce any \(K\bar{K}\) bound state. By solving the Schrödinger equation with Coulomb potential, one finds an atomic bound state of the \(K^+K^-\) system. The binding energy and the root mean square radius of the atomic bound state are \(\epsilon = 6.58\) keV and \(<r^2>^\frac{1}{2} = 190.5\) fm, respectively. Combining the Coulomb potential with the strong interaction potential derived from vector-meson-exchanges, we find that the binding energy and root mean square radius of the \(K^+K^-\) atomic bound state are changed to be \(\epsilon = 7.05\) keV and \(<r^2>^\frac{1}{2} = 175.4\) fm, respectively. In the calculation the cutoff parameter \(\Lambda\) is taken to be a value of 2.0 GeV which is in consistent with that used in Ref. \[23\] and is much smaller than that used in Ref. \[21\] with \(\Lambda = 4\sqrt{2}\) GeV. Even if we take \(\Lambda = 4\sqrt{2}\) GeV or drop momentum-dependent terms but with \(\Lambda = 2\) GeV, we cannot get \(K\bar{K}\) bound state without Coulomb potential. Only when we take \(\Lambda = 4\sqrt{2}\) GeV and drop the momentum-dependent terms to be exactly the same as in Ref. \[21\], we can get the strongly bound \(K\bar{K}\) bound state as in Ref. \[21\].

The atomic state of the \(K^+K^-\) system can decay through the strong interaction of exchanging \(K^+\) to \(\pi\pi\) and \(\pi\eta\). In the following we calculate the decay width from the Feynman
The interaction Lagrangians for the $K^*$ coupling to two pseudoscalar mesons can be written as,

\[
\mathcal{L}_{\pi KK^*} = iG_V \{ (\partial_\mu \bar{K}) \vec{\tau} K^{*\mu} \cdot \vec{\pi} - \bar{K} \vec{\tau} K^{*\mu} \cdot (\partial_\mu \vec{\pi}) + K^{*\mu} \vec{\tau} K \cdot (\partial_\mu \vec{\pi}) - \bar{K}^{*\mu} \vec{\tau} (\partial_\mu K) \cdot \vec{\pi} \} \quad (15)
\]

\[
\mathcal{L}_{\eta KK^*} = \sqrt{3}iG_V \left[ \partial_\mu \eta (\bar{K}^{*\mu} K - \bar{K} K^{*\mu}) + \eta (\partial_\mu \bar{K} K^{*\mu} - \bar{K}^{*\mu} \partial_\mu K) \right]. \quad (16)
\]

With $G_V = g_{\rho\pi\pi}/2$, the coupling constants $g_{\pi KK^*}$ and $g_{\eta KK^*}$ can be determined by the $g_{\rho\pi\pi}$:

\[
g_{\pi KK^*} = G_V, \quad g_{\eta KK^*} = -\sqrt{3}G_V. \quad (17)
\]

The decay width is:

\[
d\Gamma = \frac{1}{32\pi^2} \frac{\lvert \vec{p} \rvert}{M^2} \lvert \mathcal{M}_I (\lvert B \rangle \rightarrow \lvert PP \rangle) \rvert^2 d\Omega, \quad (18)
\]

with

\[
\mathcal{M}_I (\lvert B \rangle \rightarrow \lvert PP \rangle) = \sqrt{\frac{M}{2m_K^2}} \int d^3r \frac{d^3k}{(2\pi)^3} e^{-i\vec{k} \cdot \vec{r}} \psi(\vec{r}) \mathcal{M}_I (\vec{k}, -\vec{k} \rightarrow \lvert PP \rangle).
\]

Here $\psi(\vec{r})$ is the wave function of the bound state and $\vec{p}$ is the momentum of a meson in the final state in the center of mass system. $I$ is the isospin.

For the decay to $\pi\pi$, the isospin of $\pi\pi$ is zero. The decay amplitude is

\[
\mathcal{M}_0 (\lvert B \rangle \rightarrow \lvert \pi\pi \rangle) = -\frac{1}{2} \sqrt{\frac{3}{\pi}} G_V \sqrt{\frac{M}{2m_K^2}} \int dr R_0(r) e^{-\Sigma r} \left[ 4 \left( \frac{1}{r} + \Sigma \right) \cos(pr) + \left( 2(E^2 + E_f^2) + 4p^2 - m_{K^*}^2 + \frac{(m_K^2 - m_\pi^2)^2}{m_{K^*}^2} - \frac{4\Sigma}{r} \right) \frac{\sin(pr)}{p} \right]. \quad (19)
\]

Here $\Sigma$ is defined as

\[
\Sigma = \sqrt{m_{K^*}^2 - (E - E_f)^2},
\]

where $E$ is the energy of the $K$, $E_f$ is the energy of the final pion. $R_0(r)$ is the radial wave function of the $K^+ K^- s$-wave bound state.

We first neglect the influence of the strong interaction to the Coulomb binding energy and wave function. We find that the decay width to $\pi\pi$ is

\[
\Gamma(K^+ K^- \rightarrow \pi\pi) = 0.951 \text{ eV}. \quad (20)
\]
By taking into account the strong interaction, the decay width of the $K^+K^-$ bound state to $\pi\pi$ becomes

$$\Gamma(K^+K^- \to \pi\pi) = 48.4 \text{ eV}.$$  

(21)

In a similar way one can evaluate the amplitude of the $K^+K^- \to \eta\pi^0$ decay as

$$M(|B\rangle \to |PP\rangle) = \frac{1}{2} \sqrt{\frac{3}{\pi}} G_0^2 \sqrt{\frac{M}{2m^2_K}} \int dr R_0(r) \left\{ \begin{array}{l}
(\frac{1}{r^2} + \frac{\Sigma_1}{r}) \cos pr + \left( -\frac{4}{pr^3} - \frac{4\Sigma_1}{pr^2} \right) \sin pr \times e^{-\Sigma_1 r} \\
+ \left[ 4 \left( \frac{1}{r^2} + \frac{\Sigma_2}{r} \right) \cos pr + \left( -\frac{4}{pr^3} - \frac{4\Sigma_2}{pr^2} \right) \sin pr \right] e^{-\Sigma_2 r} \right\},
\right. 

$$

(22)

where

$$\Sigma_1 = \sqrt{m^2_{K^*} - (E - E_\pi)^2} , \quad \Sigma_2 = \sqrt{m^2_{K^*} - (E - E_\eta)^2},$$

with $E_\pi$ and $E_\eta$ as the energies of the the final $\pi$ and $\eta$ mesons, respectively.

Using the $K^+K^-$ Coulomb wave function the calculated strong decay width to $\eta\pi^0$ is:

$$\Gamma(K^+K^- \to \eta\pi^0) = 0.859 \text{ eV}$$

Taking into account the strong correction, $K^+K^-$ bound state decay width to $\eta\pi^0$ becomes

$$\Gamma(K^+K^- \to \eta\pi^0) = 29.8 \text{ eV}$$

One can see that the decay widths are very small, so the $K^+K^-$ atomic bound state cannot have a large mixing with the $f_0(980)$ or $a_0(980)$.

Calculations for other two-pseudoscalar-meson systems are performed with various cutoff parameters $\Lambda$ up to 3.0 GeV. The dependence of the binding energies of the I=0 systems $DB$, $DD$, $BB$, $BK$, $BD$, $DK$ and $DK$ on the choice of the cutoff $\Lambda$ is given in table 1 and Fig. 2. We find that $DB$, $DD$, $BB$ can form possible s-wave bound states with $\Lambda$ smaller than 2.0 GeV. The $BK$ cannot form a bound state with $\Lambda < 3.0$ GeV. For two-pseudoscalar-meson systems with isospin I=1, we fail to find any bound state. For the $KK$, $DD$ and $BB$ pairs, the Bose symmetry demands their orbital s-wave states to have isospin I=1, hence no bound states are found.

The $BK$ and $DK$ have the same isospin factor and coupling constant. One may feel puzzled why the BK cannot form bound state while $DK$ can. The reason is that generally as the reduce mass increases the binding energy of two particles becomes larger. Although the $B$ meson is heavier than $D$ meson, the reduce mass of the $DK$ is bigger than that of $BK$. So we find that the $DK$ bound state is possible with $\Lambda \sim 2.5$ GeV, while the $BK$ system cannot form a bound state.
| Λ(GeV) | E(MeV) | DB   | D̄D  | B̄B  | B̄K  | BD   | D̄K  |
|--------|--------|------|------|------|------|------|------|
| 1.4    | -1.2   | -    | -    | -    | -    | -    | -    |
| 1.5    | -5.7   | -    | -    | -    | -    | -    | -    |
| 1.6    | -13.4  | -    | -    | -    | -    | -    | -    |
| 1.7    | -24.3  | -    | -    | -    | -    | -    | -    |
| 1.8    | -38.1  | -9.3 | -0.8 | -    | -    | -    | -    |
| 1.9    | -54.6  | -17.7| -4.3 | -    | -    | -    | -    |
| 2.0    | -73.6  | -28.7| -10.9| -    | -    | -    | -    |
| 2.1    | -95.0  | -42.4| -21.0| -    | -    | -    | -    |
| 2.2    | -118.6 | -58.8| -35.1| -1.4 | -    | -    | -    |
| 2.3    | -144.2 | -77.9| -53.8| -3.1 | -    | -    | -    |
| 2.4    | -171.8 | -100.0| -78.2| -5.4 | -    | -    | -    |
| 2.5    | -201.3 | -125.1| -110.1| -8.2 | -0.3 | -5.8 |        |
| 2.6    | -232.7 | -153.7| -152.4| -11.6| -2.5 | -55.2|        |
| 2.7    | -265.8 | -186.0| -210.3| -15.5| -6.9 | -302.7|        |
| 2.8    | -300.6 | -222.8| -295.9| -19.9| -13.9|        |        |
| 2.9    | -337.2 | -264.8| -450.9| -24.7| -24.3|        |        |
| 3.0    | -375.6 | -313.3| -30.0 | -39.0|       |        |        |

Table 1: Dependence of binding energies on the cutoff parameter Λ for various isoscalar two-pseudoscalar-meson systems

However, the reduce mass of the system is not the only factor to determine the order of binding energy. In the Hamiltonian given by Eq. (9), \( c(r)/a(r) \) is proportional to something like \( f(m_1, m_2) = (m_1^2 + m_2^2 + m_1 m_2)/(m_1^2 + m_2^2 + 2m_1 m_2) \). One finds \( f(m_D, m_B) > f(m_D, m_K) > f(m_B, m_K) \). This factor also influences the binding energy and may be a reason for the unclear pattern of the binding energy dependence on the mass of two-meson system shown in Table 1.

When extending the value of Λ to be a little larger than 2.0 GeV, we find that the \( DK \) system can also form a bound state. When \( Λ > 2.07 \text{ GeV} \), the \( DK \) binding energy increases very rapidly as shown in Fig. 2. If we assume that the \( D_{s0}(2317) \) is a \( DK \) molecular state, using the masses of \( m_D = 1869 \text{ MeV}, M_K = 493 \text{ MeV} \), the binding energy of \( DK \) is 45 MeV. One finds that \( Λ ≈ 2.103 \text{ GeV} \) is needed in order to generate the \( DK \) bound state. In other word, in this model, the \( D_{s}(2317) \) may be interpreted as a \( DK \) bound state.

4 Summary and discussion

We use the vector-meson-exchange potential between two pseudoscalar mesons to search for possible bound states of the \( KK, DK, BK, D̄D, B̄B, BD, D̄K, BK \) and \( B̄D \) systems. We find that the isoscalar \( DB, D̄D \) and \( BB \) are most likely to form S-wave bound states via strong interactions. The isoscalar \( DK \) and \( BK \) systems are also likely to form S-wave bound state if the cutoff Λ could be larger than 2 GeV. An interesting issue on these possible two-meson bound states is that all of them can mix with heavy-quark-antiquark configuration as well as diquark-antidiquark configuration. For example, the isoscalar \( DK \) system has a
Figure 2: Dependence of the $DK$ binding energy on the cutoff parameter $\Lambda$

quark content of $c\bar{d}d\bar{s}$ which can transit to the $c\bar{s}$ configuration by $d\bar{d}$ annihilation into a gluon and can form diquark-antidiquark configuration $[cd][\bar{d}s]$ by quark re-arrangement. In fact, between the spin zero diquark $[cd]$ and antidiquark $[\bar{d}s]$, the vector-meson-exchange force is the same as between $D$ and $K$ mesons. In addition there is an extra color confinement potential between them. So it is most likely that the $D_{s0}(2317)$ is a mixture of $c\bar{s}$, $[cd][\bar{d}s]$ and $DK$ configurations, with $[cd][\bar{d}s]$ component larger than $DK$ component. For the $c\bar{s}$ configuration, the $c$ and $\bar{s}$ are in relative $P$-wave. For the $[cd][\bar{d}s]$, the $[cd]$ and $[\bar{d}s]$ are in relative S-wave. It is possible that instead of having a quark excited to $P$-wave, the system prefers to drag out a $q\bar{q}$ to make a diquark-antidiquark in relative S-wave. Then the $D_{s0}(2317)$ could be a dominantly 4-quark state as suggested by Cheng and Hou [24] following an earlier idea on the charm-strange 4-quark state by Lipkin [25]. A similar scenario happens also in the baryon sector. For the supposed lowest orbital $l = 1$ excited nucleon state $N^*(1535)$, its properties suggest that it may have very large $[ud][us]\bar{s}$ pentaquark components with diquarks and $\bar{s}$ all in orbital S-wave [26].

If the cutoff parameter $\Lambda$ could be as large as 2.5 GeV, then the $B\bar{D}$ and $D\bar{K}$ may also form bound states. These two systems have quark content of $\bar{b}ud\bar{c}$ and $c\bar{d}us$, respectively. So they cannot mix with any ordinary quark-antiquark configuration. For the corresponding diquark-antidiquark configurations, $[ud][bc]$ and $[cs][ud]$, there is no light-vector-meson-exchange force, but there is color confinement force. Which configuration has lower energy needs further investigation.

Other two-pseudoscalar-meson systems including all isovector ones cannot be bound by the strong interaction sector alone with $\Lambda < 3$ GeV.

For the $K\bar{K}$ system, the calculation by Ref. [21] suggests that the $f_0(980)$ may be a $K\bar{K}$ molecular state bound by the vector-meson-exchange force. In their calculation, the momentum dependent terms in the interactions were dropped and a very large value ($\sim 5.7$ GeV) was assumed for the cutoff parameter $\Lambda$. We find with momentum dependent terms included, the $K\bar{K}$ system cannot be bound by the vector-meson-exchange interaction. The $f_0(980)$ is more likely a dominantly $([us][\bar{d}s] + [ds][\bar{d}s])/\sqrt{2}$ 4-quark state as suggested by Jaffe.
For this kind of diquark-antidiquark configuration, the vector-meson-exchange force is the same as for the $K K$ configuration, meanwhile there is an additional color confinement force to bind them.

With the Coulomb potential, the $K^+ K^-$ system can form an atomic bound state - the kaonium. We find that the binding energy of the kaonium is 6.58 keV. The binding energy of the kaonium is changed to 7.05 keV by including the vector-meson-exchange potential. The decay widths of the kaonium to $\pi\pi$ and $\pi\eta$ are evaluated. It is shown that the decay widths of the $K^+ K^-$ atomic bound state to $\pi\pi$ and $\eta\pi$ are, respectively,

$$\Gamma(K^+ K^- \rightarrow \pi\pi) = 48.4 \text{ eV}, \quad \Gamma(K^+ K^- \rightarrow \eta\pi^0) = 29.8 \text{ eV}.$$ 

One can see that the decay widths are very small. We expect that the mixing of the $K^+ K^-$ atomic bound state with the $f_0(980)$ or $a_0(980)$ is small.

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