A note on quasinormal modes: A tale of two treatments

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Abstract

There is an apparent discrepancy in the literature with regard to the quasinormal mode frequencies of Schwarzschild–de Sitter black holes in the degenerate-horizon limit. On the one hand, a Poschl–Teller-inspired method predicts that the real part of the frequencies will depend strongly on the orbital angular momentum of the perturbation field whereas, on the other hand, the degenerate limit of a monodromy-based calculation suggests there should be no such dependence (at least, for the highly damped modes). In the current paper, we provide a possible resolution by critically re-assessing the limiting procedure used in the monodromy analysis.

I. INTRODUCTION

For a long time, a fascinating problem in gravitational physics was what happens to small perturbations in an otherwise stationary black hole geometry? Fortunately, at least the basic elements of this problem are now well understood: Such perturbations will essentially be scattered by the gravitational potential and, ultimately, radiated away with a discrete set of complex-valued frequencies. Such behavior, which is reminiscent of the last dying tones of a ringing bell, can be recognized as the quasinormal mode solutions associated with the black hole spacetime. If nothing else, these modes are expected to be significant in the context of gravitational-wave astronomy [1].

Thanks to an intriguing (albeit conjectural) connection between quasinormal modes and quantum gravity, there has been a recent surge of interest into resolving the quasinormal mode spectra for various black hole spacetimes. (See [2,3] for summarized accounts of recent work and the relevant references.) In a sense, such a connection is surprising, given that quasinormal modes represent a purely classical consequence of the black hole gravitational field. Nonetheless, as first advocated by Hod [4], there is reason to believe that quasinormal

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modes can be used to fix the level spacing of the black hole area spectrum. (An evenly spaced area spectrum follows from arguments, as made initially by Bekenstein [5], that the horizon area of a black hole is an adiabatic invariant.) Moreover, this rationale can even be extended into the realm of loop quantum gravity, where it has been suggested by Dreyer [6] that similar considerations can be utilized to fix, unambiguously, the elusive “Immirzi parameter” [7].

In the above circle of ideas, a pivotal role is played by the real part of the quasinormal mode frequency when the modes are highly damped (i.e., when the magnitude of the imaginary part is very large). It is generally believed (or perhaps hoped) that, for a given black hole spacetime, this real part will asymptote to a fixed value, independently of the details of the perturbation. In light of this comment, it is instructive to consider a specific model; let us therefore call upon the Schwarzschild mode spectrum for illustrative purposes. In this case, one obtains (with appropriately chosen boundary conditions) the following set of frequencies for either scalar or axial gravitational perturbations [8–12]:

\[ k_{qnm} = \kappa \left[ i \left( n + \frac{1}{2} \right) + \frac{1}{2\pi} \ln 3 \right] + \mathcal{O}[n^{-1/2}] \quad n = 0, 1, 2, \ldots \tag{1} \]

Note that \( \kappa \) is the (Schwarzschild) surface gravity \(^1\) and this expression becomes increasingly accurate as \( n \to \infty \).

This (asymptotically valid) spectral form has long been known by numerical means (e.g., [8]) and has even, quite recently, been confirmed by analytical treatments. In [11,12], for instance, the authors have invoked a method that is based on calculating the monodromy of the perturbed field when the radial coordinate has been analytically continued to the complex plane. In any event, the above spectrum substantiates that the real part of the frequencies does indeed asymptote to a constant value, independent of any details about the perturbation field itself. \(^2\) As a further point of interest, it is a generic feature of black hole spacetimes that the asymptotic spacing (between the imaginary levels) goes precisely as the surface gravity [13,14].

Let us re-emphasize that the real part of the (highly damped) quasinormal frequencies is a critical ingredient in the proposals made by both Hod [4] and Dreyer [6]. Moreover, the stance of these authors is that this real portion represents a fundamental (transition) frequency associated with the black hole horizon. Hence, if their arguments are to hold up, it is essential that this real part is strictly characterized by the horizon geometry and, hence, not overly sensitive to specifics of the perturbation field. It does, however, remain somewhat controversial as to whether or not this “non-sensitivity” is indeed a universal feature of black hole spacetimes.

\(^1\)Here and throughout, all fundamental constants have been set to unity.

\(^2\)Actually, the type of perturbation field does play a role, as the above form applies, strictly speaking, to only scalar and axial gravitational perturbations. Nonetheless, it could be argued that fundamental considerations should be restricted to a certain class of gravitational perturbations, so this is generally not a concern.
For instance, a viable counterexample is provided by Schwarzschild–de Sitter space when the black hole and cosmological horizon are closely “squeezed” together. To elaborate, a recent study [16] — based on identifying the relevant scattering potential with that of the Poschl–Teller model [17,18] — found a quasinormal spectrum that depends strongly on the orbital angular momentum (\(\ell\)) of the perturbation field. Moreover, in the degenerate limit, the real part of the frequency, for any mode, goes almost linearly with \(\ell\). (In fact, it has since been demonstrated that this behavior is a generic feature of squeezed-horizon spacetimes [15].) However, this is not yet the full story. In a more recent paper [19], the quasinormal mode spectrum was calculated for non-degenerate Schwarzschild–de Sitter space by way of the monodromy method [11,12]. When this form of the spectrum is then subjected to the horizon-degeneracy limit, as was done explicitly in [19], there is absolutely no \(\ell\) dependence in evidence. Hence, what we have is two quite conflicting predictions for precisely the same model.

The purpose of the current paper is to provide a possible resolution for this rather disturbing discrepancy. Our basic point of view is that the Poschl–Teller-inspired calculation is, given its elegant simplicity, most likely correct. Meanwhile, the monodromy-based calculation, although perfectly valid in the non-degenerate regime, can not necessarily be extrapolated up to the point of horizon coincidence. The remainder of the paper contains the analysis in support of our argument (Section II), followed by a pertinent discussion (Section III). Note that we skip over most of the details of these interesting methodologies — “Poschl–Teller” [16,15] and “monodromy” [11,12,19] — and refer the reader to the cited works.

II. ANALYSIS

Before getting to the crux of the matter, let us introduce some necessary formalism. The metric for a Schwarzschild–de Sitter spacetime can be expressed (in a static coordinate gauge) as follows:

\[
ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2d\Omega^2,
\]

where

\[\text{footnote}{3}\]

We often refer to the degenerate (or very nearly degenerate) Schwarzschild–de Sitter model as being in the “squeezed-horizon” limit [15]. Such terminology is meant to distinguish this scenario from one in which the observer is exterior to a pair of (nearly) coincident horizons; that is, exterior to a black hole at (or near) extremality. The two scenarios are, of course, operationally distinct.

\[\text{footnote}{4}\]

Moreover, the Poschl–Teller spectral form for nearly degenerate Schwarzschild–de Sitter space has also been substantiated semi-analytically (particularly for large values of \(\ell\)) by Moss and Norman [20] using continued-fraction techniques [21], as well as by Zhidenko [22] and Konoplya [23] using WKB methodology [24–26]. For other recent studies on Schwarzschild–de Sitter quasinormal modes, see [27–30].
\[ f(r) = 1 - \frac{2M}{r} - \frac{r^2}{a^2}, \]  

with \( M \) representing the black hole mass and \( a \) denoting the de Sitter curvature radius (alternatively, \( \Lambda = \frac{3}{a^2} \) is the positive cosmological constant).

The function \( f(r) \) has three zeroes, two of which locate the black hole horizon, \( r_b \), and the cosmological horizon, \( r_c \), with \( r_b \leq r \leq r_c \) indicating the “observable” portion of the spacetime. [The third zero locates a fictitious negative horizon, \( r_0 = -(r_b + r_c) \).] For future reference, it is useful to take note of the following relations:

\[ a^2 = r_b^2 + r_br_c + r_c^2, \]  

\[ 2Ma^2 = r_br_c (r_b + r_c). \]  

Each horizon is endowed with a surface gravity, which can be evaluated via standard methods (essentially, one half of the derivative of \( f(r) \) evaluated at the appropriate horizon [31]) to yield

\[ \kappa_b = \frac{(r_c - r_b)(2r_b + r_c)}{2a^2r_b}, \]

\[ \kappa_c = \frac{(r_c - r_b)(2r_c + r_r)}{2a^2r_c}, \]

for the black hole and cosmological horizon respectively. Note that these quantities are, by definition, positive definite.

It is often convenient — especially, in the context of quasinormal modes — to introduce a (generalized) “tortoise coordinate”; that is, \( dx = dr/f(r) \) or

\[ x = \int \frac{dr}{f(r)}. \]  

Substituting in equation (3) and integrating, one obtains

\[ x = \frac{1}{2\kappa_b} \ln \left[ \frac{r}{r_b} - 1 \right] - \frac{1}{2\kappa_c} \ln \left[ 1 - \frac{r}{r_c} \right] + \frac{1}{2} \left( \frac{1}{\kappa_c} - \frac{1}{\kappa_b} \right) \ln \left[ \frac{r}{r_b + r_c} + 1 \right]. \]  

Keep in mind that \( r \in (r_b, r_c) \) maps into the region \( x \in (-\infty, +\infty) \).

Let us now focus specifically on the quasinormal mode problem, which entails the study of how small (massless) perturbations of the background spacetime are scattered by the gravitational potential. As has been well documented, one can formally cast this picture into a one-dimensional, Schrodinger-like scattering situation [32,1]. Generically speaking, one obtains an equation that conforms to

\[ \frac{d^2}{dx^2} \psi - V[r(x)] \psi = -k^2 \psi, \]

where \( \psi = \psi[r(x)] \) describes the radial behavior of the perturbation field, \( V[r(x)] \) is a model-dependent “scattering potential”, and \( k \) is the frequency (that is, \( e^{\pm ikt} \) is the time
dependence of the perturbation; with $e^{+ikt}$ then chosen to ensure that the imaginary part of $k$ is positive for an exponentially decaying solution). Note that the potential generally depends on both the spin, $j$, and the orbital angular momentum, $\ell$, of the perturbed field.

For a Schwarzschild–de Sitter spacetime, in particular, it has been shown that the scattering potential takes on the form [33]

$$ V[r] = f(r) \left[ \frac{\ell(\ell+1)}{r^2} + \left(1 - j^2\right) \left(\frac{2M}{r^3} - \frac{2}{a^2}\right) \right], \quad (11) $$

at least for scalar ($j = 0$) and axial gravitational ($j = 2$) perturbations. (Generalizations to other cases are indeed possible [33] but, for simplicity, will not be considered here.)

As a brief but important aside, let us point out that resolving the quasinormal mode problem requires suitably chosen initial conditions. Normally, one imposes “radiation boundary conditions” such that $\psi(x) \propto e^{\pm ikx}$ as $x \to \mp \infty$; that is, an ingoing (outgoing) plane wave at the inner (outer) boundary. It is also necessary, in the case of dual-horizon scenarios, to make a decision as to where the scattering takes place. In the case of black hole scattering, the “incident” wave should be coming in from the outer boundary (i.e., positive infinity in $x$ coordinates) whereas, for cosmological horizon scattering, the incident wave should be coming out from the inner boundary. (For a more quantitative description, see [3].) In the (current) case of nearly coincident horizons, however, this distinction becomes effectively irrelevant.

Since our current interest is specifically with the nearly degenerate (or “squeezed”) horizon scenario, we will now re-express the above formalism as appropriate for this regime. First, let us put the notion of nearly coincident horizons into quantitative terms. This can be accomplished with the introduction of the following “squeezing parameter”:

$$ \Delta \equiv \frac{(r_c - r_b)}{r_b} \ll 1. \quad (12) $$

Given the extent of the relevant manifold ($r_b \leq r \leq r_c$), it immediately follows that, up to corrections of the relative order $\Delta$, $r_b \sim r \sim r_c$ and $\kappa_b \sim \kappa_c$. (For the duration, $\sim$ will always be used to signify corrections of this order.)

In view of the above, the metric function (3), surface gravities (6,7), tortoise coordinate (9) and scattering potential (11) will now simplify as follows:

$$ f(r) \sim 2\kappa_b(r - r_b), \quad (13) $$

$$ \kappa_b \sim \kappa_c \sim \frac{(r_c - r_b)}{2r_b^2}, \quad (14) $$

$$ x \sim \frac{1}{2\kappa_b} \left( \ln \frac{r}{r_b} - 1 \right) - \ln \left[ 1 - \frac{r}{r_c} \right] + \left( \frac{\kappa_b - \kappa_c}{\kappa_b} \right) \ln \left[ \frac{r}{r_b + r_c + 1} \right], \quad (15) $$

$$ V[r] \sim f(r) \frac{\ell(\ell + 1)}{r^2}. \quad (16) $$
Take particular note of the scattering potential; it depends, up to corrections that vanish as $\Delta \to 0$, on the orbital angular momentum but not on the spin of the perturbation.

Given this simplified form of the potential, it turns out that one can directly extract the quasinormal mode frequencies from equation (10). More to the point, as shown in [16] (and later generalized in [15]), the potential $V[x]$ takes on a Poschl–Teller form [17], for which the quasinormal mode spectrum is known exactly [18]. Following this reasoning, one eventually obtains [16]

$$k_{\text{qnm}} = \kappa_b \left[ i \left( n + \frac{1}{2} \right) + \sqrt{\ell(\ell + 1) - \frac{1}{4} + \mathcal{O}[\Delta]} \right] \quad n = 0, 1, 2, \ldots . \quad (17)$$

Next, we would like to know what the monodromy method [11,12] can say about the quasinormal modes for Schwarzschild–de Sitter space when the horizons are closely squeezed. In fact, this monodromy calculation has already been carried out in [19], although under the presumption of non-degenerate horizons. Still, the authors of [19] considered (near) horizon degeneracy as a limiting case of the general analysis. It should, however, be emphasized that the appropriate limit was taken only after the calculation of the mode spectrum was completed. We will now proceed to argue that this particular limiting procedure may not be technically correct.

One of the underlying premises of the monodromy method [11,12] is the viability of an analytic continuation of $r$ (and, by implication, $x$) into the complex-valued plane. This continuation allows the real part of $r$ to enter the “unobserved” region ($r < r_b$) including all the way to the singularity at $r = 0$. However, as pointed out in [3], it is essential to the program that the geometry (specifically, $f(r)$ and related quantities) is first defined in the observable region ($r \geq r_b$) and then analytically continued. That is to say, the “physical” geometry of the black hole interior [i.e., $f(r)$ as defined for $r < r_b$ sans any continuation] must not be allowed to enter into the quasinormal mode problem. This is because, in the original definition of the problem, one sets up strict boundary conditions at the black hole horizon and the cosmological horizon, ¹ thus rendering the black hole interior (and the cosmological horizon exterior) as being operationally irrelevant.

In view of the above discussion, it becomes clear that a “squeezed observer” would find it most appropriate to analytically continue the geometry described by equations (13-16). (Admittedly, this is somewhat naive — see Section III for an elaboration.) Now, considering that the “focal point” of the monodromy calculation is at (or rather near) the singularity, $r = 0$, let us see what happens to these expressions near the spatial origin. Firstly, defining a “shifted tortoise coordinate” or $z = x + \text{constant}$ such that $z(r = 0) = 0$ [11], we are able to deduce [after expanding equation (15) near $r = 0$ and then incorporating equation (14)]

$$z \sim -\frac{3}{2} r + \mathcal{O}[r^2] . \quad (18)$$

Next, let us consider the potential [cf, equations (13,16)] as $r$ or $z$ goes to zero. This is simply

¹For as asymptotically flat spacetime, the boundary conditions are rather fixed at the black hole horizon and spatial infinity.
\[ V[z(r)] \sim -2 \frac{r_b \kappa_b \ell (\ell + 1)}{r^2} + \mathcal{O}[r^{-1}] \]
\[ \sim -\frac{9}{2} \frac{r_b \kappa_b \ell (\ell + 1)}{z^2} + \mathcal{O}[z^{-1}] . \] (19)

Conveniently, this is the same form of near-the-origin potential \((V \propto z^{-2})\) as obtained for the non-degenerate Schwarzschild–de Sitter case [19]. Hence, the rest of the monodromy-based calculation can proceed, in the same manner as [19], but with the necessary substitution

\[ \nu = \frac{1}{2} \sqrt{1 + z^2 V[z \approx 0]} \]
\[ = \frac{1}{2} \sqrt{1 - \frac{9}{2} r_b \kappa_b \ell (\ell + 1)} , \] (20)

where \(\nu\) is the “Bessel-function index” as explicitly defined in equations (12-13) of [19].

Referring the reader again to [19] (also see [11,12]), let us quote the quasinormal mode spectrum as obtained via the prescribed procedure: \(e^{2\pi \kappa_b \ell} = -(1 + 2 \cos 2\pi \nu)\) or

\[ k_{qnm} = \kappa_b \left[ i \left( n + \frac{1}{2} \right) \pm \frac{1}{2\pi} \ln |1 + 2 \cos 2\pi \nu| \right] \quad n = 0, 1, 2, ..., \] (21)

which can be expected to be valid up to corrections of the order \(n^{-1/2}\) (in general) and the relative order \(\Delta\) (in our specific case). Substituting in equation (20) and expanding under the assumption that \(9/2 r_b \kappa_b \ell (\ell + 1)\) is somewhat less than unity, \(^6\) we then have

\[ k_{qnm} = \kappa_b \left[ i \left( n + \frac{1}{2} \right) \pm \frac{91}{32} \pi r_b^2 \kappa_b^2 \ell^2 (\ell + 1)^2 \right] \quad \text{as} \quad n \to \infty \quad \text{and} \quad \Delta \to 0 . \] (22)

**III. DISCUSSION**

Some commentary is, of course, in order. First of all, it is interesting to compare this last (monodromy) calculation with the spectrum obtained from the “Poschl–Teller method” [16] [as depicted in equation (17)]. We immediately see that the asymptotic spacing between the imaginary levels is correctly realized; an outcome which is, in itself, by no means trivial. On the other hand, the real part of the (asymptotic) frequency is somewhat different; in particular, take note of the quartic versus linear dependence of \(\ell\). It is, however, possible to explain this discrepancy as follows (also see the Addendum):

It is not quite clear that equation (16) is truly the correct form of the potential that should extrapolated down to \(r = 0\). More to the point, one can always add additional terms to the potential that do not significantly alter its structure in the observable region but which would, nevertheless, dominate at the origin and, thus, make a significant contribution to the monodromy. That is to say, more care is needed in addressing the contributions from such “neglected” terms. To understand the type of analysis that might be required, \(^7\)

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\(^6\)Since \(r_b \kappa_b \approx \Delta\) and \(\Delta \ll 1\), this assumption must be true as long as \(\ell\) is not too large.
one can turn to a close cousin of the monodromy calculation; namely, the WKB method of quantum mechanics. As discussed in [34], the Stokes and anti-Stokes lines (with three of each emanating from the \( r = 0 \) singularity) need to be addressed. We will defer the fully rigorous treatment of the monodromy calculation until a later time. It does, however, seem feasible that such a treatment could still give back equation (17) [that is, it does not appear to be the naively simple matter of just summing up the various contributions]. Such speculation aside, the point we are really trying to make is that, as \( \Delta \to 0 \), the \( \ell \)-dependent term clearly dominates the potential so that, regardless of any neglected contributions, this \( \ell \)-dependence can not just be blatantly ignored.\(^7\)

Secondly, it is of interest to see how things match up when equation (22) is compared with the spectral form predicted by the ("non-degenerate") monodromy calculation of [19]. To reiterate, these authors obtain equation (21) but with a different expression for the index \( \nu \). To be precise, they have found

\[
\nu = \frac{1}{2} \sqrt{1 + (4M\beta)^2 (j^2 - 1)},
\]

where \( \beta \) is a parameter that depends strictly on \( r_b, r_c, \kappa_b \) and \( \kappa_c \). The philosophy of [19] was that the (nearly) degenerate limit could then be dealt with by directly extrapolating this non-degenerate form for the spectrum. Following this approach, one observes that the quasinormal frequencies of (nearly) degenerate Schwarzschild–de Sitter space can certainly depend on the spin, \( j \), but definitely not on the orbital angular momentum, \( \ell \). How can we make sense of this discrepancy in the \( \ell \) dependence of the frequencies? Well, although a reasonable argument could be made both ways — that is to say, it is not particularly clear what is the rigorously correct limiting procedure — strong support for an \( \ell \)-dependent spectrum follows from the Poschl–Teller calculation. Significantly, this methodology — which is unambiguous and exact in the squeezed-horizon limit — implies that the real part of the frequency can depend only on \( \ell \) and \( \kappa_b \) when the horizons coincide.

Let us now make a couple of general observations. Firstly, we can see from the subtleties of the current case that other quasinormal mode calculations that go on to apply extremal (or near-extremal) limits should probably be viewed with a healthy dose of scepticism. Which is to say, the precise moment (in the calculation) that such a limit can safely be enforced would appear to be a subject worthy of debate. Secondly, let us re-emphasize that a strongly \( \ell \)-dependent quasinormal mode spectrum (particularly, in the real part of the highly damped frequencies) should cast significant doubt on the status of quasinormal modes in quantum gravity. (Let us recall that the real part of the frequency can be used, conjecturally speaking, to fix the spacing of the black hole area spectrum [4] and, by implication, the Immirzi parameter of loop quantum gravity [6].) It is not clear, at least to the current authors, how the fundamental quantum theory can be sensitive to the details of the perturbation field or, alternatively, why would only certain classes of horizons be susceptible to quantization?

Finally, it is worthwhile to point out another discrepancy, in the literature, which centers around the Schwarzschild–de Sitter black hole spacetime. With emphasis on a scenario of

\(^7\)Alternatively, one might consider the quasinormal mode problem strictly in the observable region (as advocated by Born-approximation-inspired methods [13,14]), where the \( \ell \)-dependence is so evidently prominent.
non-degenerate horizons, it has been argued (by the current authors and M. Visser [13] — also see [27]) that both the cosmological and the black hole horizon will “contribute” to the quasinormal mode spectrum. That is, we anticipate one set of modes that goes (asymptotically) as \( i\kappa_b \) and another set that goes as \( i\kappa_c \). Our argument is essentially that, from the perspective of an observer in this spacetime, there would be no reason to give one horizon a preferred status over the other. Hence, for a complete calculation, it would be necessary to deal with two separate scattering problems which can be distinguished by the choice of initial conditions (see the related discussion in Section II). Nonetheless, a contrary opinion has been expressed in [3], where it is claimed that the cosmological horizon scattering conditions would not be physically relevant. The current authors, however, can see no convincing reason why one horizon should be singled out \textit{a priori}. Unfortunately, any study which focuses on the case of horizon degeneracy can offer nothing substantial to this particular argument. Hence, we have nothing to add at this time but hope to readdress the matter, by more rigorous means, in the near future.

ADDENDUM

It has been brought to our attention, in a response to the first archival version of the paper [35,36], that the monodromy calculation used in [19] is not quite correct. The main sticking point seems to be that, upon continuation to the complex plane, the generalized tortoise coordinate does \textit{not} go to infinity as \( r \) goes to \( r_c \); thus invalidating part of the matching procedure used in [19]. Given this breakdown, it is even less surprising that our monodromy result (22) fails to reproduce the Poschl–Teller calculation (17). It should also be emphasized that the invalidation of the original (Schwarzschild–de Sitter) monodromy analysis in no way undermines any of our prior discussion. To elaborate: in [19], the near-the-origin scattering potential depends on \( j \) but not \( \ell \), whereas our analogue depends on \( \ell \) but not \( j \). So, even if equation (21) [that is, the spectrum predicted by the original monodromy calculation] is wrong, our “corrected form” must necessarily differ from that of [19] in a substantial way. Namely, \( \ell \) dependence versus no \( \ell \) dependence.

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