Clustering aspects of sd shell nuclei studied by AMD

Masaaki Kimura‡, Yasutaka Taniguchi‡, Yoshiko En’yo§ and Hisashi Horiuchi‡

a) Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
b) Department of Physics, Kyoto University, Kitashirakawa, Kyoto 606-8502, Japan

Abstract. The new clustering aspects of sd-shell nuclei found by the recent application of the antisymmetrized molecular dynamics are reported. In this paper we present two topics, “superdeformed state of $^{32}$S and $^{16}$O+$^{16}$O clustering” and “molecular-orbital and di-nuclei states in $^{22}$Ne”. In the first topic, it will be shown that the superdeformed band of $^{32}$S has a considerable amount of $^{16}$O+$^{16}$O cluster component, and can be regarded as to belong to a family of the $^{16}$O+$^{16}$O molecular bands. In the second topic, the presence of the molecular-orbital band which has an $\alpha+^{18}$O cluster core surrounded by two covalently neutrons is suggested together with an $\alpha+^{16}$O di-nuclei band.

1. Introduction
The nuclear clustering is one of the most essential degrees of freedom of nuclear excitation especially in light N=Z nuclei such as p-shell nuclei. In these decades, the domain of the cluster studies has been expanding toward heavier nuclei and N≠Z nuclei. In this mass region, the nucleus shows many interesting aspects since the coexistence and coupling between cluster and shell (mean-field) structure enriches a variety of the nuclear structure. The theoretical framework of the antisymmetrized molecular dynamics is one of the powerful method to investigate the interplay between cluster and shell structure and has revealed many new clustering aspects in sd-pf shell and lighter nuclei.

In this paper, we discuss two topics, “superdeformed state of $^{32}$S and $^{16}$O+ $^{16}$O clustering” and “molecular-orbital and di-nuclei states in $^{22}$Ne”.

2. Superdeformed band of $^{32}$S and $^{16}$O+$^{16}$O clustering
Recently, a rather conclusive result of the properties of $^{16}$O+$^{16}$O molecular resonance was obtained by macroscopic models [1] using the unique potential. These studies gave the following results for the lowest three rotational bands whose principal quantum numbers are N=24, 26 and 28. The lowest Pauli-allowed rotational band (N=24) starts from the 0+ state located at approximately 9 MeV in the excitation energy, and the energy gap between the N=24 and N=26 bands and that between N=26 and N=28 bands are both approximately 10 MeV. In Ref. 1), it is proposed that the observed $^{16}$O+$^{16}$O molecular states correspond to the third band (N=28).

Aside from the molecular bands, the superdeformed state of $^{32}$S has also been studied by many authors using the mean-field theories [2, 3], and one of their interests is the relationship between the superdeformed state of $^{32}$S and the $^{16}$O+$^{16}$O molecular structure. It is notable that these calculations predict that the superdeformed band starts from the 0+ located at approximately 8 to 10 MeV which agrees with the energy of the N=24 band head obtained from the unique
Figure 1. Energy curves obtained after the angular momentum projection. Solid lines show the curves obtained by the deformed-basis AMD. Dashed line shows that obtained by the $^{16}\text{O}+^{16}\text{O}$ cluster wave function.

Figure 2. Superdeformed band (N=24 $^{16}\text{O}+^{16}\text{O}$ molecular band), N=26 and 28 $^{16}\text{O}+^{16}\text{O}$ molecular bands obtained by the deformed-basis AMD+GCM (solid lines) and $^{16}\text{O}+^{16}\text{O}$ cluster wave function.

optical potential. Therefore, it is sufficiently conceivable and has been pointed out by some
authors[1] that the superdeformed band and the lowest Pauli-allowed $^{16}\text{O}+^{16}\text{O}$ molecular band ($N=24$) are identical.

In the present study, we aim at clarifying the relationship between the superdeformed state and the $^{16}\text{O}+^{16}\text{O}$ molecular structure based on the deformed-basis AMD calculation. The objectives of the present study are summarised as follows. (1) To what extent are the superdeformed state and the $^{16}\text{O}+^{16}\text{O}$ molecular structure related? In the unique optical potential analysis, the factors which distort the $^{16}\text{O}+^{16}\text{O}$ cluster structure, such as the effect of the spin-orbit force and the formation of the deformed mean-field, are renormalised into the optical potential. Therefore, when we treat these factors accurately using the full microscopic model, the pure $^{16}\text{O}+^{16}\text{O}$ structure will be distorted and can be understood as the superdeformed state of $^{32}\text{S}$. (2) Do the excited states exist in which the excitation energy is spent to excite the relative motion between the clusters? Do they correspond to the $N=26$ and 28 bands obtained from the unique optical potential? When we consider that the superdeformed states of $^{32}\text{S}$ have a considerable amount of the $^{16}\text{O}+^{16}\text{O}$ component, we can expect the excitation mode in which the excitation energy is used to excite the relative motion between clusters.

In the following, we mention the prominent feature of our results, and readers are directed to Ref. 5) for more detailed discussions. In Fig.1, the obtained energy curves after the angular momentum projection are plotted. We can see that each curve has a well developed superdeformed minimum around $\beta=0.7$. And the wave function of the superdeformed minimum shows a $^{16}\text{O}+^{16}\text{O}$ nature (see density plot in Fig. 1). However, the obtained superdeformed wave function shows the large effect of the distortion of the $^{16}\text{O}+^{16}\text{O}$ cluster structure. Namely, the superdeformed state obtained by the deformed-base AMD (solid lines in Fig. 1) is more deeply bound than the pure $^{16}\text{O}+^{16}\text{O}$ state (dashed line in Fig. 1) by about 10 MeV because of the energy gain in the density-dependent force and spin-orbit force. The non vanishing expectation value of the spin-orbit force at the superdeformed minimum directly indicates the distortion of the $^{16}\text{O}+^{16}\text{O}$ structure and the formation of the deformed one-body field. However, the obtained superdeformed wave function still has a considerable $^{16}\text{O}+^{16}\text{O}$ component which amounts to approximately 50%. By superposing the obtained wave functions, we have obtained three rotational bands which correspond to the $N=24$, 26 and 28 bands (Fig. 2). The lowest band ($N=24$) has a mixed character of the $^{16}\text{O}+^{16}\text{O}$ structure and deformed mean-field structure and its excitation energy agrees well with the superdeformed state obtained by the mean-field calculations. In the $N=26$ and $N=28$ bands, we have found that the amount of the $^{16}\text{O}+^{16}\text{O}$ component increases markedly and these two bands have an almost pure $^{16}\text{O}+^{16}\text{O}$ structure. At the same time, these band members are fragmented into several states and the assignment of the $N=28$ band members to the observed $^{16}\text{O}+^{16}\text{O}$ molecular resonances appears plausible.

3. Molecular-orbital and di-nuclei states in $^{22}\text{Ne}$

The successful description of Be isotopes by assuming a molecular-orbital structure (2$\alpha$ cluster core plus covalently valence neutrons) suggests a new type of clustering which is peculiar to neutron-rich isotopes. It will be of importance to investigate whether the molecular-orbital structure exists not only in Be isotopes but also in heavier systems.

Ne isotopes are one of the promising candidates of such heavier systems, since $^{20}\text{Ne}$ has a prominent $\alpha+^{16}\text{O}$ clustering in its ground state and many excited states. We have investigated the structure of $^{22}\text{Ne}$ focusing on the $\alpha$ clustering by combining the theoretical framework of the deformed-basis AMD and $\alpha+^{18}\text{O}(0^+_1)$ cluster model. By calculation, we have obtained eight bands of $^{22}\text{Ne}$. Among the obtained bands, the spectra of the five excited bands which show considerable $\alpha$ clustering are plotted in Fig. 3, together with that of the ground band. In the following, we focus on these bands with prominent $\alpha$-clustering and do not discuss other bands. But we note that almost all of the known experimental data such as the excitation energy and $E2$ transition probability of the ground band and low-lying band that are not shown in Fig. 3
are reproduced well.

The excited bands with the prominent $\alpha$ clustering are categorized into two types. The first type is the molecular-orbital band in which the system has an $\alpha+^{16}$O cluster core and two valence neutrons covalently bound around the core. The $K^\pi=0^+_2$, $0^-_1$ and $1^-_1$ bands belong to this type.

The $K^\pi=0^+_2$ and $0^-_1$ bands have similar intrinsic wave functions in which two valance neutrons occupy the orbital that is elongated along the $\alpha+^{16}$O clustering of the core. In Fig. 4, the density

**Figure 3.** Observed and calculated level schemes of excited bands with $\alpha$ clustering and ground band.

**Figure 4.** Density plot for $\sigma$-orbital (gray scaled plot) and for $\alpha+^{16}$O core (contour lines) of $1^-$ state ($K^\pi=0^-_1$).
plot of the neutron single particle orbitals of $1\frac{1}{2}$ ($K^\pi=0^-$) state is presented. Here the single particle orbitals are derived by constructing the Hartree-Fock single particle Hamiltonian from the deformed-basis AMD wave function. In the $1^-\sigma^s$ state, there are six independent neutron single particle orbitals and two neutrons occupy each orbital. They are labeled from No.1 to 6 in order of the single particle energy. The most deeply bound orbital (No. 1) shows, of course, a $0s_{1/2}$ nature. But the center of the single particle orbital is not located at the center of the entire system, but at the center of $^{16}$O cluster. Therefore, the neutron orbital No.1 roughly corresponds to the $0s_{1/2}$ orbital of $^{16}$O cluster. In the same way, we can roughly see that the orbitals No. 2, 3 and 5 correspond to $0p_{3/2,1/2}$ orbital of $^{16}$O and No. 4 corresponds to $0s_{1/2}$ orbital of $\alpha$ cluster. The weak mixing between the single particle orbital of $^{16}$O and $\alpha$ means that the $\alpha$ and $^{16}$O are not interacting strongly and may not bound to each other. On the contrary, most weakly bound neutron orbital (No. 6) is spatially expanded to the whole system and interact with both of $\alpha$ and $^{16}$O clusters. Thus, the structure of the single particle orbitals suggest that the two valence neutrons bond the unbound $\alpha$ and $^{16}$O clusters in this band. The orbital No. 6 can be decomposed into the $1s0d$ orbital around $^{16}$O and the $0p$ orbital around $\alpha$, and is considered as an analogue of the $\sigma_{1/2}$-molecular-orbital of Be isotopes. The most important difference between the present $\sigma$-orbital and that of the Be isotopes is the parity asymmetry of the present one which is due to the parity asymmetry of the cluster core. The prominent parity-asymmetry of the total system of the $\sigma_2$ molecular-orbital states leads to the existence of the parity-doublet bands of $K^\pi=0^+$ and $0^-$ in $^{22}$Ne with almost the same intrinsic wave function. In the $K^\pi=1^-$ band, one of the valence neutrons occupies the $\sigma$-orbital and the other occupies the parity-symmetric orbital which has the dominant component of the $0d_{5/2}$ orbital. These three bands are built on the states around the $\alpha+^{18}$O threshold energy and correspond to the states with the prominent $\alpha$ reduced widths experimentally observed in the $\alpha$ transfer reaction [4]. Indeed the calculated and observed excitation energies and $\alpha$ reduced widths show good agreement.

The second type is the di-molecule band in which the system is divided into $\alpha$ and $^{18}$O clusters. The $K^\pi=0^+$ and $0^-$ bands belong to this type. Their excitation energies are higher than that of the molecular-orbital band and the $\alpha+^{18}$O threshold energy. In these bands, two valence neutrons orbit only around the $^{16}$O. We consider that these bands correspond to the observed $\alpha+^{18}$O molecular resonances[5], since the calculated and observed $\alpha$ reduced widths show qualitative agreement.

From the present results, we suggest the two different kinds of $\alpha$ cluster structure in $^{22}$Ne. The first one is the molecular-orbital structure and the second one is the di-molecule structure.

4. Summary
We have applied the deformed-basis AMD+GCM method to $^{32}$S and $^{22}$Ne.

By the application to $^{32}$S, we have shown that the superdeformed band obtained from the HF(B) calculations and the Pauli-allowed lowest N=24 band of the $^{16}$O+$^{16}$O molecular bands are essentially identical. In this band, $^{16}$O+$^{16}$O molecular structure is distorted, but superdeformed states still have a considerable amount of the $^{16}$O+$^{16}$O molecular structure. We have obtained two excited bands which are generated by the excitation of the relative motion between two $^{16}$O clusters. In the excited N=26 and 28 bands, the distortion is less important and the band members have the prominent molecular structure.

By the application to $^{22}$Ne, we have found two different cluster structure. First one is the molecular-orbital structure in which two valence neutrons are covalently bound around the $\alpha$ and $^{16}$O clusters. Second one is the di-nuclei structure in which system is divided into two clusters, $\alpha$ and $^{18}$O.

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