THE GALILEO SATELLITE CONSTELLATION AND MODIFICATIONS TO THE INVERSE-SQUARE LAW FOR GRAVITATION

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ABSTRACT

We consider the impact of a power-law correction to the Newtonian potential, inspired by ungravity or \( f(R) \) extensions of the Standard Model, and draw conclusions on the possibility of measuring the relevant parameters through observables made available by the Galileo satellite positioning system.

Key words: Power-law additions, Ungravity, \( f(R) \) theories.

1. INTRODUCTION

The Galileo positioning system poses a great opportunity, not only for the improvement and development of new applications in navigation monitoring and related topics, but also possibly for fundamental research in physics. Indeed, together with the already deployed Global Positioning System (GPS) and Glonass, satellite navigation may be considered the first practical application where relativistic effects are taken into account, not from the usual experimental scientific point of view, but as a regular engineering constraint on the overall design requirements. Indeed, effects arising from special and general relativity (GR) – gravitational blueshift, time dilation and Sagnac effect – may account to as much as \( \sim 40 \mu s/day \), which is many orders of magnitude above the accuracy of the onboard clock deployed in these systems. Moreover, the gravitational Doppler effect, of the order of \( V_N/c^2 \sim 10^{-10} \) (where \( V_N = GM_E/R_E \) is the Newtonian potential, \( G \) is Newton’s constant, \( M_E \approx 6.0 \times 10^{24} \) kg is the Earth’s mass, \( R_E \approx 6.4 \times 10^6 \) m is its radius and \( c \) is the speed of light) falls within the \( 10^{-12} \) frequency accuracy of current space-certified clocks, and must therefore be taken into account. In the GPS, this is done by imposing an offset in the onboard clock frequency, while in the Galileo Navigation Satellite System (GNSS) this correction should be accounted by the receiver. For further details, the reader is directed to Refs. [1, 2, 3, 4] and references therein.

This said, it is not clear as to what extent the accuracy of the Galileo positioning system may be improved – which is designed to offer pinpoint localization within an error margin of 1 m, against the 10 m margin of previous the GPS – so to provide clues to the nature of models beyond the current gravitational physics knowledge (see Refs. [5, 6] for updated surveys). In this study, we aim to establish bounds on the detectability of extensions to the recently discussed unparticle extension of the Standard Model of particle interactions [7] that manifest themselves through power-law corrections to the Newtonian potential [8]. These corrections are associated to the exchange of spin-2 states that do not correspond to particle states, usually referred to as unparticles. The exchange of unparticles leads to a correction to Newton’s potential usually dubbed as ungravity.

This paper is organized as follows: firstly, we assess the main relativistic effects that are present in the GNSS. We proceed and consider the possibility of measuring the discussed corrections using the GNSS.

2. MAIN RELATIVISTIC EFFECTS

2.1. Frame of reference

Assuming that all time-dependent effects are of cosmological origin, and hence of order \( H_0^{-1} \), where \( H_0 \) is Hubble’s constant, one may discard these as too small within the timeframe of interest; hence, one assumes a static, spherically symmetric scenario, posited by the standard Schwarzschild metric. In isotropic form, this is given by the line element
where \( dV = ds^2 + d\Omega^2 \) is the volume element, and \( V \) is the gravitational potential. In the standard GR scenario, the latter coincides with the Newtonian potential \( V = V_N = -(1 + \sum_{n=1}^{\infty} J_n)GM_E/r \), where the \( J_n \) multipoles account for the effect of geodesic perturbations and density profiles.

To this picture, one must introduce the rotation of the Earth with respect to this fixed-axis reference frame, with angular velocity \( \omega = 7.29 \times 10^{-5} \text{ rad/s} \); by doing a coordinate shift \( t' = t, r' = r, \theta' = \theta \) and \( \phi' = \phi - \omega t' \), one gets the Langevin metric, given by the line element:

\[
\begin{align*}
\text{ds}^2 &= -\left[ 1 + \frac{2V}{c^2} \right] (c \, dt)^2 + \frac{1}{1 + 2\frac{V}{c^2}} dV \quad (1) \\
&\simeq -\left[ 1 + \frac{2V}{c^2} \right] (c \, dt)^2 + \left( 1 - \frac{2V}{c^2} \right) dV ,
\end{align*}
\]

Finally, if one gets back to a non-rotating frame, the metric is given by the line element

\[
\begin{align*}
ds^2 &= -\left[ 1 + \frac{2(V - \Phi_0)}{c^2} \right] (c \, dt)^2 + \left( 1 - \frac{2V}{c^2} \right) d\Omega . \quad (4)
\end{align*}
\]

### 2.2. Constant and periodic clock deviation

One may now consider the difference between the time elapsed on the ground and the satellite clock; keeping only terms of order \( c^{-2} \), one finds that the proper time increment on the moving clock is given approximately by

\[
d\tau = ds/c = \left( 1 + \frac{V - \Phi_0}{c^2} - \frac{v^2}{2c^2} \right) dt . \quad (5)
\]

Considering an elliptic orbit with semi-major axis \( a \), and taking \( V = V_N \simeq GM_E/r \), this may be recast into the form \([1]\)

\[
\begin{align*}
d\tau &= ds/c = \left[ 1 + \frac{3GM_E}{2ac^2} + \frac{\Phi_0}{c^2} - \frac{2GM_E}{c^2} \left( \frac{1}{a} - \frac{1}{r} \right) \right] dt . \quad (6)
\end{align*}
\]

The constant correction terms in the above amount to

\[
\frac{3GM_E}{2ac^2} + \frac{\Phi_0}{c^2} = -4.7454 \times 10^{-10} , \quad (7)
\]

for the GNSS, and \(-4.4647 \times 10^{-10}\), for the GPS; this indicates that the orbiting clock is beating faster, by about \(41 \mu s/day\), for the GNSS, and \(39 \mu s/day\), for the GPS. For this reason, the GPS has a built in frequency offset of this magnitude, while the increased computational capabilities made available to current and future receivers of the GNSS leave this correction to the user. The residual periodic corrections, proportional to \(1/r - 1/a\), have an amplitude of order \(49 ns/day\), for the GNSS, and \(46 ns/day\), for the GPS.

### 2.3. Shapiro time delay and the Sagnac effect

The so-called Shapiro time delay, a second order relativistic effect due to the signal propagation is given by \([1]\)

\[
\begin{align*}
\Delta t_{delay} &= \frac{\Phi_0}{c^3} + \frac{2GM_E}{c^3} \ln \left( 1 + \frac{l}{R_E} \right) , \quad (8)
\end{align*}
\]

\[
\begin{align*}
\Delta t_{delay} &= \frac{\Phi_0}{c^3} + \frac{2GM_E}{c^3} \ln \left( 1 + \frac{l}{R_E} \right) , \quad (8)
\end{align*}
\]
where we have integrated over a straight line path of (proper) length \( l \). Evaluating this delay, one concludes that this effect amounts to \( 6.67 \times 10^{-11} \) s.

Also, one must consider the so-called Sagnac effect, which arises from the difference between the gravitational potential \( V \) and the effective potential \( \Phi \), when proceeding from a non-rotational to a rotational frame. Hence, one gets the additional time delay

\[
\Delta t_{\text{Sagnac}} = \frac{\omega}{c^2} \int_{\text{path}} r^2 \, d\phi = \frac{2\omega}{c^2} \int_{\text{path}} dA_z, \tag{9}
\]

where \( dA_z \) is the ortho-equatorial projection of the area element swept by a vector from the rotation axis to the satellite. For the GNSS, this yields a maximum value of 153 ns, while for the GPS, one gets 133 ns.

One concludes this section by recalling the main effects affecting the considered global positioning systems: a frequency shift of order \( 10^{-10} \) and a propagation time delay (Shapiro plus Sagnac effect) of the order \( 10^{-7} \) s. In what follows, one will compute the additional frequency shift and propagation time delay induced by ungravity inspired corrections with the above quantities, plus the frequency accuracy of \( 10^{-12} \) and the time accuracy of Galileo, of order \( 10^{-9} \) s, which corresponds to an optimistic spatial accuracy of 30 cm.

3. DETECTION OF A POWER-LAW ADDITION TO THE newTONIAN POTENTIAL

Having studied the measurability of an array of hypothetical perturbations to the Newtonian potential in a previous work [9] (including the effect of the cosmological constant, of a Yukawa term [10] and a constant acceleration such as the one reported in the so-called Pioneer anomaly — see Ref. [11] and references therein for a review, and Ref. [12] for a conventional explanation), one now considers a power-law perturbation of the form

\[
V_P = \frac{GM_E}{r} \left( \frac{R}{r} \right)^n, \tag{10}
\]

where \( n \) is a (possibly non-integer) exponent and \( R \) is a characteristic length scale arising from the underlying physical theory.

A power-law perturbation to the Newtonian poses an interesting scenario for several reasons: phenomenologically, it provides a convenient alternative to the more usual Yukawa parameterization of modifications to gravity, thus enabling one to probe potential candidates for extensions and modifications of GR. Furthermore, some current proposals give rise to the existence of induced power-law effects at astrophysical scales. As mentioned, corrections of this type arise from the exchange of spin-2 unparticles from which one can set bounds, for instance, from stellar stability considerations [13] and cosmological nucleosynthesis [14]; in this context, the exponent is related to the scaling dimension of the unparticle operators \( d_U \) through \( n = 2d_U - 2 \), and the lengthscale \( R \) reflects the energy scale of the unparticle interactions, the mass of exchange particles and the type of propagator involved.

Another relevant candidate for detection through a power-law parameterization is the so-called \( f(R) \) class of theories [15], which relies upon a generalization of the Einstein-Hilbert action (including a non-minimal coupling of geometry with matter [16]); in an astrophysical setting, these give rise to power-law additions to the Newtonian potential, which may be relevant in the context of the galaxy rotation puzzle [17, 18].

In the above Eq. (10), the Newtonian potential \( \Phi_N \) is recovered by setting \( R = 0 \) (for positive \( n \)) or \( R \rightarrow \infty \) (for negative \( n \)). The limit \( n \rightarrow 0 \) is ill-defined, since then the additional term \( V_P \) does not vanish, but is identified with the Newtonian potential, \( V_P = \Phi_N \): this indicates that one should rewrite the gravitational constant in terms of an effective coupling, leading to the full potential

\[
\Phi = \frac{G_P M_E}{r} \left[ 1 + \left( \frac{R}{r} \right)^n \right], \tag{11}
\]

with

\[
G_P = \frac{G}{1 + \left( \frac{R}{R_0} \right)^n}, \tag{12}
\]

where \( R_0 \) signals the distance at which the full gravitational potential matches the Newtonian one, \( \Phi(R_0) = \Phi_N(R_0) \).

This additional length scale \( R_0 \) should arise as an integration constant when solving the full field equations derived from the fundamental theory of gravity beyond the considered power-law correction. For simplicity, this study assumes that \( (R/R_0)^n \ll 1 \), so that this may be safely discarded — at the cost of neglecting the regime \( n \rightarrow 0 \), for which this condition fails (notice that this is complementary to the approach followed in a previous study [13]). Hence, the subsequent section uses \( G_P = G \) freely, albeit with the above caveat in mind.
3.1. Relative frequency shift

The relative frequency shift of a signal emitted at a distance from the origin \( r = R_E + h \) (for the GNSS, \( h = 23.222 \times 10^3 \) km) and received at a distance \( r = R_E \) is given by

\[
\epsilon = 1 - \frac{f_{\text{Earth}}}{f_{\text{Sat}}} = 1 - \sqrt{\frac{900 \text{ Earth}}{900 \text{ Sat}}} = (13)
\]

\[
1 - \sqrt{\frac{1 - 2V(R_E)/c^2}{1 - 2V(R_E + h)/c^2}} \approx \frac{V(R_E) - V(R_E + h)}{c^2}.
\]

Hence, one may compute the additional frequency shift induced by the extra contribution to the potential, through

\[
\left( \frac{\Delta f}{f} \right)_p = \frac{V_P(R_E) - V_P(R_E + h)}{c^2} \approx \frac{GM_E}{c^2 R_E} \left( \frac{R}{R_E} \right)^n \left[ 1 - \left( \frac{R_E}{R_E + h} \right)^{n+1} \right] = 6.96 \times 10^{-7} \xi^n [1 - 0.22^{n+1}],
\]

where a dimensionless ratio \( \xi \equiv R/R_E \) has been defined. One clearly has two asymptotic regimes: for \( n \gg -1 \), one may approximate the above by

\[
\left( \frac{\Delta f}{f} \right)_p = 6.96 \times 10^{-7} \xi^n, \quad (15)
\]

so that, considering the accuracy \( \epsilon_{\xi} \approx 10^{-12} \) of the Galileo constellation, one has \( \xi^n \lesssim 1.44 \times 10^{-6} \). Conversely, if \( n \ll -1 \), one obtains

\[
\left( \frac{\Delta f}{f} \right)_p = -1.50 \times 10^{-7} (0.22 \xi)^n, \quad (16)
\]

which, upon comparison with the accuracy \( \epsilon_{\xi} = 10^{-12} \), yields \( (0.22 \xi)^{-n} \gtrsim 1.50 \times 10^5 \). Since the r.h.s. is larger than unity, this imposes the rather strong bound \( \xi > (0.22)^{-1} \approx 4.64 \) when \( n \to -\infty \).

In the vicinity of \( n = -1 \), one expands Eq. (14) as

\[
\left( \frac{\Delta f}{f} \right)_p = 1.07 \times 10^{-6} \left( \frac{n + 1}{\xi} \right), \quad (17)
\]

and the considered accuracy yields \( \xi \gtrsim 1.07 \times 10^6 |n + 1| \). The different regimes are depicted in Fig. 1.

3.2. Propagation time delay

Besides the relative frequency shift discussed above, a power-law correction to the gravitational Newtonian potential also induces a modification to the propagational time delay, given by

\[
\Delta t_p = \frac{1}{c} \int_{R_E}^{R_E + h} \frac{GM_E}{r} \left( \frac{R}{r} \right)^n dr \approx \frac{GM}{nc} \left( \frac{R}{R_E} \right)^n \left[ 1 - \left( \frac{R_E}{R_E + h} \right)^n \right]
\]

\[
\approx 1.33 \times 10^{6} \xi^n \left( 1 - 0.22^n \right).
\]

Equating this to the \( 10^{-9} \) s time resolution of the GNSS, yields the constraint

\[
\xi^n \left| 1 - 0.22^n \right| \leq 7.52 \times 10^{-16}. \quad (19)
\]

Analogously to the previous discussion, for the extreme negative power-law regime \( n \ll 0 \) this expression reads \( |0.22\xi|^n \lesssim 7.52 \times 10^{-16} |n| \). Since, for sufficiently large (negative) \( n \), the r.h.s. is larger than unity, one recovers the stronger bound, \( \xi > (0.22)^{-1} \approx 4.64 \), obtained above.

The allowed region for the \( \xi, n \) parameters is depicted in Fig. 2.
Asides from the obtained range for these quantities, the main result of this study lies in the exclusion of a definite range of values for $R$, namely those lying between $0 \leq R \leq 4.64 R_E$. Recall, however, that condition $(R/R_0)^n \ll 1$ leads to the limit $R_0 < R$ when $n \to -\infty$, yielding an additional requirement for the normalization constant $R_0$ in the case of a large negative exponent $n$. Also, the above results are not valid in the vicinity of $n = 0$, due to the lack of knowledge concerning the effective gravitational coupling $G_P$.

4. CONCLUSIONS

In this contribution, one has assessed the possibility of detecting signals of new physics through the use of the GNSS. This application could be valuable, as any unexpected new phenomenology could provide further insight into what lies beyond the Standard Model of particle interactions and GR. One has specifically looked at the propagation time delay and frequency shift induced by a putative correction to the Newtonian potential, namely one of the a power-law type. This study complements the one where the GNSS was used to examine a constant contribution, cosmological constant induced and Yukawa-type corrections to the Newtonian potential [9].

As can be seen from Figs. 1 and 2, one cannot lift the degeneracy in the $(R_E, n)$ parameter space through an intersection of the relative frequency shift and propagation time delay observables, since these produce rather similar allowed regions for the mentioned parameters.

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