Multistage self-orientating gauge measuring system best fitting parameters for circularity estimation

D Diakov, V Vassilev, H Nikolova
Technical University of Sofia, 1797 Sofia, 8 Blvd. Kl. Ohridski
diakov@tu-sofia.bg, vassilev_v@tu-sofia.bg, hnikolova@tu-sofia.bg

Abstract. The deviation of circularity estimation of large-scaled details in many cases is a serious metrological problem, which can be solved using a measuring system based on a multistage self-orientating gauge. The purpose of this paper is to study the influence of the gauge parameters on the accuracy of measurement of the circularity deviation and optimization thereof in order to ensure the maximum correspondence between the real and the measured profile. The accuracy of the re-creation of individual and group of harmonic components of the profile is examined. Analytical and experimental research results are provided.

1. Introduction
The deviation of circularity measurement of large-scaled details is a serious metrological problem, determined by the specificity of the measured objects - their huge dimensions and mass, and the associated difficulties of a design and exploitative nature.

This problem can be solved successfully by means of a measuring system based on the effect of realization of the median circle of the cross-section profile, where the measuring device is established on a multi-stage self-orientating gauge [2, 4].

The principle scheme of such a system is shown in Figure 1 [4]. It contains the following basic functional elements: a base roller gauge 1 based on the measured detail 2 and a measuring clamp 3. The measuring clamp is mounted on the measured detail 2 on the multi-stages self-orientating gauge. A measuring head 4 is mounted on the clamp. With relative rotation of the clamp to the workpiece due to the aforesaid effect, each point of the housing describes a circle equivalent to the average approximating circle of the profile on which the prism is based. This circle is at the same time the datum of measurement.
The number of contact points depends on the number of the gauge grades \( t \) and is defined by the expression \( k = 2^t + 1 \). Angle \( \alpha \) is the angle between the measuring line and the symmetry of the clamp, \( \mu_0 \) - the angle between the symmetry of the yoke and the symmetry of the first degree, and \( \mu_1, \mu_2, \mu_3 \) - the angles between the symmetries of the respective self-orientating degrees.

The subject of this paper is the optimization of the basic parameters of the system - the angles \( \mu_0, \mu_i \) and \( \alpha \), aiming increasing the accuracy of the realization of the datum in determining the shape of the measured profiles of the details.

2. Mathematical model of recreation of the measured profile when measured by means of a multistage self-orientating gauge

The mathematical model allows clarification of the kinematics of the process of transferring the shape of the measured from the gauge profile \( L \) of the workpiece onto the trajectory \( l \) of the \( P.M \) of the body of the gauge as well as on the obtained \( L_p \) profile as a realization of the distance between the \( P.M \) and \( p.K \) of the measured profile \( L \) (Figure 2).

Measured profile \( L \) in polar coordinates can be described by Fourier order:
Mechanical Science and Technology Update

IOP Conf. Series: Journal of Physics: Conf. Series 1260 (2019) 032013
doi:10.1088/1742-6596/1260/3/032013

\[
R(\varphi) = R_o + \sum_{n=2}^{\infty} \Delta R_n(\varphi)
\]
\[
\Delta R_n(\varphi) = a_n \cos n(\varphi - \theta_n),
\]
(1)

where:

\( R(\varphi) \) - radius-vector of the measured profile \( L \); \( R_o \) - constant; \( a_n \) and \( \theta_n \) - amplitude and initial phase of the \( n \)-th harmonic; \( \varphi \) - polar coordinate.

Generally, when using a \( t \)-stage gauge, the equation of the trajectory \( l \) at point \( M \) in polar coordinates is the following [1]:

\[
r_l(\varphi) = r_{l0} + \sum_{n=2}^{\infty} \Delta r_{ln}(\varphi)
\]
\[
\Delta r_{ln}(\varphi) = \lambda_{nt} a_n \cos \left( \varphi - \theta_n - \Phi_{nt} \right),
\]
(2)

where:

\( r_l(\varphi) \) – radius-vector of trajectory \( l \) of the \( p \). \( M \); \( r_{l0} \) - constant; \( \lambda_{nt} \) - copying coefficient of the amplitude of the \( n \)-th harmonic of the measured profile (\( \lambda_{nt}>0 \)) determined by the expression:

\[
\lambda_{nt} = \prod_{i=1}^{t} \frac{\cos \mu_i}{\cos \alpha_i} \sqrt{\frac{\sin^2 \mu_0 - \sin^2 \alpha}{\cos^2 \alpha}},
\]
(3)

\( \Phi_{nt} \) – angle of phase shifting of the \( n \)-th harmonic from the measured profile when it is reproduced on the trajectory \( l \) at point \( M \), i.e., the angle between the radius vectors \( R_{n \text{ max}} \) and \( r_{ln \text{ max}} \):

\[
\Phi_{nt} = \left\{ \begin{array}{ll}
\frac{\alpha - q_n}{\nu_n} + \pi & \text{if } \prod_{i=1}^{t} \cos n\mu_i \geq 0 \\
\frac{\alpha - q_n}{\nu_n} & \text{if } \prod_{i=1}^{t} \cos n\mu_i < 0
\end{array} \right.,
\]
(4)

where \( q_n \) is determined by: \( \tan nq_n = \tan \alpha \cdot \tan \mu_0 \cdot \tan \alpha \).

by following the signs rules:

\[
\text{sgn}(\sin nq_n) = \text{sgn}(\sin \alpha \sin n\mu_0)
\]
\[
\text{sgn}(\cos nq_n) = \text{sgn}(\cos \alpha \cos n\mu_0)
\]

The radius-vector \( r_p(\varphi) \) of profile \( L_p \) is [2]:

\[
r_p(\varphi) = r_{p0} + \sum_{n=2}^{\infty} \Delta r_{pn}(\varphi)
\]
\[
\Delta r_{pn}(\varphi) = a_{pn} \cos \left( \varphi - \theta_n + \psi_{nt} \right),
\]
(5)

where: \( r_{po} \) - constant; \( a_{pn} \) – amplitude of the profile of the \( n \)-th harmonic; \( a_{pn} = K_{nt} a_n \); \( K_{nt} \) - coefficient of re-creation of the amplitude of the \( n \)-th harmonic (\( K_{nt} \geq 0 \)).
ψ_{nt} – phase shifting difference of the n-th harmonics of L and L_p

\[ tgn\psi_{nt} = \frac{\lambda_{nt} \sin n\Phi_{nt}}{1 - \lambda_{nt} \cos n\Phi_{nt}}, \]

according to the signs rules:

\[ |\text{sgn}(\sin n\psi_{nt})| = |\text{sgn}(\sin n\Phi_{nt})| \]
\[ |\text{sgn}(\cos n\psi_{nt})| = |\text{sgn}(1 - \lambda_{nt} \cos n\Phi_{nt})| \]

The measured profile will be re-created on the profile without distortion and phase shifting at \( K_{nt} = 1 \) and \( \psi_{nt} = 0 \).

3. Research methodology

The study aims to minimize the function \( |K_{nt} - 1| \), reflecting the accuracy of the recreation of the measured profile on the profile diagram for a separately collected harmonics and for a group of harmonics. In the second case, the maximum value of \( |K_{nt} - 1| \) of the harmonics group must be minimal.

For this purpose, the programs have been developed on the basis of the software "Maple V". It varies with the following parameters: \( t = 1 \div 4, \alpha = 0 \div 90^\circ, \mu_0 = 30^\circ \div 80^\circ, \mu_1 = 15^\circ \div 45^\circ, \mu_2 = 7^\circ \div 25^\circ, \mu_3 = 4^\circ \div 16^\circ, \mu_4 = 3^\circ \div 8^\circ \). These limits are in accordance with the possibilities for constructive realization of the gauge.

The studied harmonics (from 2nd to 7th) are dominant in the real technological processes.

4. Study results

Research for a particular harmonic

The analytical examination shows that the necessary and sufficient condition for complete and accurate reflection of the measured profile L, respectively for equal amplitudes and absence of phase shift of the profile L_p, is:

\[ \Delta r_{pn}(\varphi) = \Delta R_n(\varphi) \]

or

\[ a_p \cdot \cos (\varphi - \theta_n + \psi_{nt}) = a_n \cos (\varphi - \theta_n) \]

To be true, the above identity must:

\[ a_p = a_n \]
\[ \cos (\varphi - \theta_n + \psi_{nt}) = \cos (\varphi - \theta_n) \]

\( a_p = a_n \), when the coefficient of recreation \( K_{nt} \) is equal to 1, i.e. when

\[ \lambda_{nt} = 2 \cos n\Phi_{nt} \]
\[ \lambda_{nt} = 0 \]
Hence, in order to reproduce fully and accurately /without distortion/ n-th harmonic, it is necessary and sufficient in the gauge to have an angle $\mu_i$ of any i-th stage that satisfies the condition:

$$n\mu_i = (2k + 1)\pi/2$$
or

$$n\mu_o = (2k + 1)\pi/2 \quad \text{at} \quad \alpha = 0, \pi$$
$$n\mu_o = k\pi \quad \text{at} \quad \alpha = \pm \pi/2$$

$k = 0, 1, 2,\ldots$

Harmonic Group Study

The results of the numerical experiment are represented in Table 1, where $\Delta = \frac{|K_{nt} - 1|}{1}$ is the relative error of recreation of the measured profile. It follows that $\Delta$ decreases as the number of degrees $t$ increases, with $t = 3$ and $t = 4$ not exceeding 2.4%. The larger value of $\Delta$ at $t = 4$ compared to $\Delta$ at $t = 3$ is explained by the additional limitations of constructive character / possibility of constructive realization of the gauge/.

| Table 1. Parameters of t-stage optimized at $K_{nt}$, gauge for $n=2\div7$. |
|---|---|---|---|
| $t$ | $\mu_i$, $\alpha$ [°] | $n$ | $K_{nt}$ | $\Delta$, [%] | $\psi_{nt}$ [°] |
| 1 | $\mu_0 = 56$  
$\mu_1 = 16$  
$\alpha = 78$ | 2 | 0.968 | 3.2 | 30 |
|  |  | 3 | 1.035 | 3.5 | 6 |
|  |  | 4 | 0.873 | 12.7 | 6 |
|  |  | 5 | 1.111 | 11.1 | 2 |
|  |  | 6 | 0.939 | 6.1 | 0 |
|  |  | 7 | 0.883 | 11.7 | -2 |
| 2 | $\mu_0 = 57$  
$\mu_1 = 19$  
$\mu_2 = 13$  
$\alpha = 82$ | 2 | 1.041 | 4.1 | 24 |
|  |  | 3 | 1.040 | 4.0 | 3 |
|  |  | 4 | 0.958 | 4.2 | 2 |
|  |  | 5 | 0.967 | 3.3 | -1 |
|  |  | 6 | 0.960 | 4.0 | 0 |
|  |  | 7 | 1.008 | 0.8 | 0 |
| 3 | $\mu_0 = 77$  
$\mu_1 = 30$  
$\mu_2 = 18$  
$\mu_3 = 10$  
$\alpha = 89$ | 2 | 0.983 | 1.7 | 6 |
|  |  | 3 | 1.000 | 0.0 | 0 |
|  |  | 4 | 1.022 | 2.2 | -2 |
|  |  | 5 | 1.000 | 0.0 | 0 |
|  |  | 6 | 0.995 | 0.5 | 2 |
|  |  | 7 | 1.002 | 0.2 | 0 |
| 4 | $\mu_0 = 77$  
$\mu_1 = 30$  
$\mu_2 = 18$  
$\mu_3 = 8$  
$\mu_4 = 3$  
$\alpha = 89$ | 2 | 0.983 | 1.7 | 6 |
|  |  | 3 | 1.000 | 0.0 | 0 |
|  |  | 4 | 1.024 | 2.4 | -2 |
|  |  | 5 | 1.000 | 0.0 | 0 |
|  |  | 6 | 1.000 | 0.0 | 3 |
|  |  | 7 | 1.003 | 0.3 | 0 |

| Table 2. Parameters of t-stage optimized at $K_{nt}$, gauge for $n=2\div7$ at $\alpha = 0$. |
|---|---|---|---|
| $t$ | $\mu_i$, [°] | $n$ | $K_{nt}$ | $\Delta$, [%] | $\psi_{nt}$ [°] |
| 1 | $\mu_0 = 36$  
$\mu_1 = 18$ | 2 | 0.675 | 32.5 | 0 |
|  |  | 3 | 1.236 | 23.6 | 0 |
|  |  | 4 | 1.325 | 32.5 | 0 |
|  |  | 5 | 1.000 | 0.0 | 0 |
|  |  | 6 | 0.675 | 32.5 | 0 |
|  |  | 7 | 0.764 | 23.6 | 0 |
| 2 | $\mu_0 = 41$  
$\mu_1 = 24$  
$\mu_2 = 15$ | 2 | 0.879 | 12.1 | 0 |
|  |  | 3 | 1.179 | 17.9 | 0 |
|  |  | 4 | 0.925 | 7.5 | 0 |
|  |  | 5 | 0.824 | 17.6 | 0 |
|  |  | 6 | 1.000 | 0.0 | 0 |
|  |  | 7 | 0.889 | 11.1 | 0 |
| 3 | $\mu_0 = 48$  
$\mu_1 = 26$  
$\mu_2 = 14$  
$\mu_3 = 5$  
$\mu_4 = 0$  
$\alpha = 89$ | 2 | 1.096 | 9.6 | 0 |
|  |  | 3 | 1.208 | 20.8 | 0 |
|  |  | 4 | 0.786 | 21.4 | 0 |
|  |  | 5 | 0.829 | 17.1 | 0 |
|  |  | 6 | 1.044 | 4.4 | 0 |
|  |  | 7 | 0.821 | 17.9 | 0 |
| 4 | $\mu_0 = 49$  
$\mu_1 = 26$  
$\mu_2 = 14$  
$\mu_3 = 4$  
$\mu_4 = 3$  
$\alpha = 89$ | 2 | 1.131 | 13.1 | 0 |
|  |  | 3 | 1.219 | 21.9 | 0 |
|  |  | 4 | 0.785 | 21.5 | 0 |
|  |  | 5 | 0.852 | 14.8 | 0 |
|  |  | 6 | 1.059 | 5.9 | 0 |
|  |  | 7 | 0.808 | 19.2 | 0 |
As can be seen from Table 2, the relative error at fixed angle $\alpha=0$ is several times greater. This case, however, is appropriate not only from a constructive point of view, but also because the phase shifting of the individual harmonics is zero. This allows a clearer reflection of the character of the profile.

### Table 3. Experimental results of multistage self-orientating gauge measuring system (MSSOGMS)

|               | CMMS-2 | MSSOGMS | $\alpha = 0^\circ$ | MSSOGMS | $\alpha = 40^\circ$ | MSSOGMS | $\alpha = 60^\circ$ | MSSOGMS | $\alpha = 90^\circ$ |
|---------------|--------|---------|--------------------|---------|---------------------|---------|---------------------|---------|---------------------|
| $K_{\text{atm}}$ | 1.296  | $\psi_{\text{atm}} = 0^\circ$ | $\psi_{\text{atm}} = 0^\circ$ | $\psi_{\text{atm}} = 0^\circ$ | $\psi_{\text{atm}} = 0^\circ$ | 91.0    | $\psi_{\text{atm}} = 0^\circ$ | 1.003   | 11.0    |
| $K_{\text{efk}}$ | 1.273  | $\psi_{\text{efk}} = -1^\circ$ | $\psi_{\text{efk}} = -1^\circ$ | $\psi_{\text{efk}} = -1^\circ$ | $\psi_{\text{efk}} = -1^\circ$ | 70.2    | $\psi_{\text{efk}} = -1^\circ$ | 0.975   | 10.3    |
| $\Delta K$    | 1.8 %  | $\Delta \psi_{\text{efk}} = 0^\circ$ | $\Delta \psi_{\text{efk}} = 0^\circ$ | $\Delta \psi_{\text{efk}} = 0^\circ$ | $\Delta \psi_{\text{efk}} = 0^\circ$ | 1.8 %   | $\Delta \psi_{\text{efk}} = 0^\circ$ | 2.8 %   | 1.0    |
| $K_{\text{ntt}}$ | 1.296  | $\psi_{\text{ntt}} = 0^\circ$ | $\psi_{\text{ntt}} = 0^\circ$ | $\psi_{\text{ntt}} = 0^\circ$ | $\psi_{\text{ntt}} = 0^\circ$ | 1.296   | $\psi_{\text{ntt}} = 0^\circ$ | 1.273   | 1.0     |
| $K_{\text{nte}}$ | 1.296  | $\psi_{\text{nte}} = -1^\circ$ | $\psi_{\text{nte}} = -1^\circ$ | $\psi_{\text{nte}} = -1^\circ$ | $\psi_{\text{nte}} = -1^\circ$ | 1.296   | $\psi_{\text{nte}} = -1^\circ$ | 1.273   | 1.0     |
| $\Delta K$    | 1.8 %  | $\Delta \psi_{\text{nte}} = 0^\circ$ | $\Delta \psi_{\text{nte}} = 0^\circ$ | $\Delta \psi_{\text{nte}} = 0^\circ$ | $\Delta \psi_{\text{nte}} = 0^\circ$ | 1.8 %   | $\Delta \psi_{\text{nte}} = 0^\circ$ | 2.8 %   | 1.0     |
| $K_{\text{ntt}}$ | 1.611  | $\psi_{\text{ntt}} = 0^\circ$ | $\psi_{\text{ntt}} = 0^\circ$ | $\psi_{\text{ntt}} = 0^\circ$ | $\psi_{\text{ntt}} = 0^\circ$ | 1.611   | $\psi_{\text{ntt}} = 0^\circ$ | 1.603   | 1.0     |
| $K_{\text{nte}}$ | 1.611  | $\psi_{\text{nte}} = -1^\circ$ | $\psi_{\text{nte}} = -1^\circ$ | $\psi_{\text{nte}} = -1^\circ$ | $\psi_{\text{nte}} = -1^\circ$ | 1.611   | $\psi_{\text{nte}} = -1^\circ$ | 1.603   | 1.0     |
| $\Delta K$    | 1.0 %  | $\Delta \psi_{\text{nte}} = 0^\circ$ | $\Delta \psi_{\text{nte}} = 0^\circ$ | $\Delta \psi_{\text{nte}} = 0^\circ$ | $\Delta \psi_{\text{nte}} = 0^\circ$ | 1.0 %   | $\Delta \psi_{\text{nte}} = 0^\circ$ | 0.5 %   | 0.5     |
| $K_{\text{ntt}}$ | 1.018  | $\psi_{\text{ntt}} = -6^\circ$ | $\psi_{\text{ntt}} = -6^\circ$ | $\psi_{\text{ntt}} = -6^\circ$ | $\psi_{\text{ntt}} = -6^\circ$ | 1.018   | $\psi_{\text{ntt}} = -6^\circ$ | 1.008   | 1.0     |
| $K_{\text{nte}}$ | 1.018  | $\psi_{\text{nte}} = -7^\circ$ | $\psi_{\text{nte}} = -7^\circ$ | $\psi_{\text{nte}} = -7^\circ$ | $\psi_{\text{nte}} = -7^\circ$ | 1.018   | $\psi_{\text{nte}} = -7^\circ$ | 1.008   | 1.0     |
| $\Delta K$    | 1.0 %  | $\Delta \psi_{\text{nte}} = -1^\circ$ | $\Delta \psi_{\text{nte}} = -1^\circ$ | $\Delta \psi_{\text{nte}} = -1^\circ$ | $\Delta \psi_{\text{nte}} = -1^\circ$ | 1.0 %   | $\Delta \psi_{\text{nte}} = -1^\circ$ | 0.5 %   | 0.5     |

The factors mentioned (permissible measurement error, phase harmonics shifting, possibility of constructive realization) are decisive in choosing one or another schematic solution.
Experimental studies have been carried out with a multi-stage self-orientating measuring gauge system [6] with parameters: \( \mu_0=40^\circ, \mu_1=20^\circ, \mu_2=10^\circ \) and \( \mu_3=5^\circ \).

Table 3 shows the results (profiles and deviations from circularity) from the measurement with MSSOGMS at different values of \( \alpha \) and those from the measurement of the measured profile with CMMS-2 roundness measurement system. The theoretical \( K_{ntt} \) and experimentally predetermined \( K_{nte} \) coefficients of profile recreation and the corresponding shifting angles \( \psi_{ntt} \) and \( \psi_{nte} \) are presented. The correspondence between the theoretical and experimental results is assessed by the difference \( \Delta \psi = \psi_{ntt} - \psi_{nte} \) and the relative \( \Delta K \) difference.

\[
\Delta K = \left( K_{ntt} - K_{nte} \right) / K_{ntt}
\]

\[
K_{nte} = EFK_{\text{meas}} / EFK_{\alpha},
\]

where \( EFK_{\alpha} \) is the deviation from circularity at calibration, but \( EFK_{\text{meas}} \) is the deviation from circularity measured by MSSOGMS /without correction/.

It can be seen from the table that the relative difference between the \( K_{ntt} \) and \( K_{nte} \) values in the worst case \( \alpha = 40^\circ \) is 4.8% and in the other cases is in the range of 0.4 ... 2.8%.

These results confirm the adequacy of the mathematical model. This allows, with a pronounced harmonic of the profile and given parameters of the clamp, the error of the inaccurate recreation of the profile to be corrected by dividing the yield value of the deviation from circularity \( EFK_{\text{meas}} \) with the recreation coefficient \( K_{ntt} \), i.e.

\[
EFK = EFK_{\text{meas}} / K_{ntt},
\]

where \( EFK \) is the value of measured deviation from circularity with correction.

5. Conclusion
The conducted study allows to be made the following conclusions:

- With a pronounced character of the profile (dominant harmonic), a combination of parameters of the measuring system can be selected, ensuring complete and accurate profile recreation.
- For the harmonics group (from 2-nd to 7-th) studied, the maximum error of profile recreation at the optimum combination of the measuring system parameters is 12.7 percent for single-stage, 4.1% for two-stage, 2.2% for three-stage and 2.4% for four-stage self-orientating gauge.
- With a known profile nature, with a given harmonic, it is possible to correlate the measurement result by dividing it with the corresponding \( K_{ntt} \) recreation factor.

References
[1] Gebel. I.D. **Kinematika perenosna nekruglosti bazy na obrabotvayemuyu poverkhnost pri shlifovanii na samoustanavlivayushчисkikh bashmakakh.** Vestnik mashinostroyeniya, 11, (1969), s.52-55.
[2] Gebel. I.D. **Bestsentrovoye izmereniye formy profilya tel vrashcheniya,** Izmeritelnaya tekhnika, 3 (1973), s. 24-27, ISSN 0368-1025.
[3] Dichev, D., Koev, H., Bakalova, T., Louda, P. **An algorithm for improving the accuracy of systems measuring parameters of moving objects.** (2016) Metrology and Measurement Systems, 23 (4), pp. 555-565
[4] Dimitrov D., I. Lazarov, Hr. Radev, Hr. Popov, Ts. Grekov. **Ured za izmervane na otklonenieto ot kraglost na krupno-gabaritni izdeliya.** Standarti i kachestvo, 5 (1980), ISSN 0324-105X.
[5] EN ISO 12181-1:2011 Geometrical product specifications (GPS) - Roundness - Part 1: Vocabulary and parameters of roundness Geometrical product specifications (GPS) - Roundness - Part 1: Vocabulary and parameters of roundness

[6] Radev H., D. Dyakov, P. Bozhinov. Sistema za izmervane na formata i razpolozenieto na vanshni rotatsionni povarhnini na golyamogabaritni detayli. Sb. dokladi ot 9-ti Natsionalen Nauchnen Simpozium s mezhd. uchastie “Metrologiya i metrologichno osiguryavane’99” (Sozopol, 14-16 sept. 99), ISBN 954-438-229-2, s. 120-126

[7] Zhelezarov, I., Analysis of measuring devices and measurement systems. Mechanical Engineering in XXI Century. Nis, Serbia. 2010. p. 199 – 200. ISBN 978-86-6055-008-0