Corrugated capillary as THz Cherenkov Smith-Purcell radiator

K V Lekomtsev\textsuperscript{1,2}, A S Aryshev\textsuperscript{1}, A A Tishchenko\textsuperscript{2}, A A Ponomarenko\textsuperscript{2}, V M Sukharev\textsuperscript{2}, N Terunuma\textsuperscript{1}, J Urakawa\textsuperscript{1} and M N Strikhanov\textsuperscript{2}

\textsuperscript{1}High Energy Accelerator Research Organisation (KEK), Tsukuba, Japan
\textsuperscript{2}National Research Nuclear University (MEPhI), Moscow, Russia

E-mail: konstlek@post.kek.jp

Abstract. In this article we discussed Particle In Cell electromagnetic simulations and mechanical design of dielectric capillaries that produce THz Cherenkov Smith-Purcell radiation (ChSPR), arising when a femtosecond electron multi-bunch beam propagates through corrugated and non-corrugated dielectric capillaries with metallic radiation reflectors. We investigated the influence of the four-bunch beam on the SPR field spectrum and on the ChSPR power spectrum, and the influence of the non-central beam propagation on the ChSPR power spectrum. We also discussed the design and assembly of the capillaries, constructed as sets of cylindrical rings.

1. Introduction

In the past decade accelerator based sources of THz radiation have seen progress in generation of short femtosecond duration electron bunches as well as in THz radiator development. LUCX accelerator at High Energy Accelerator Research Organisation (KEK) produces short, hundreds of femtosecond duration electron multi-bunch beam with variable distance between bunches \cite{1,2}. This type of multi-bunch beam may be used to generate tunable SPR. Capillary structures capable of producing powerful Cherenkov radiation (ChR) wakefields have been used for THz radiation generation \cite{3} as well as for energy modulation of electron beams \cite{4}. Characterization of coherent ChR sources and high gain antennas for such sources are currently being discussed as well \cite{5}. Introduction of the corrugation into a dielectric capillary increases spectral tunability of THz radiation emitted due to the mechanism of SPR \cite{6}.

In this paper we discuss a corrugated capillary as a source of THz radiation based on coherent ChSPR mechanism, and a non-corrugated capillary as a source of THz radiation based on coherent ChR. We consider the geometries constructed as sets of cylindrical dielectric rings enclosed in metallic radiation reflectors. Designs of corrugated and non-corrugated capillaries, holders, and radiation reflectors are explained. We also present laser microscope scans of the outer surfaces of the corrugated and non-corrugated capillaries, which were used to check assembly accuracy. Particle In Cell (PIC) simulations are performed for the different distances between bunches and for off-central propagation of the beam through the capillaries. The spectral response and the radiation power produced by both types of the geometries are compared for the different values of the distance between bunches and for off-central propagation of the four-bunch beam.
2. PIC Simulations

The electromagnetic simulations were performed using Computer Simulation Technologies (CST) PIC solver that calculates propagation of charges and fields in a calculation domain [7]. We reported in [6] the spectral-angular distribution of THz ChSPR from a corrugated channel in infinite dielectric. In [8] we compared the theoretical model [6] with the PIC simulation of the electron bunch propagation in a corrugated channel in quasi-infinite medium. Because the spectral angular distribution could not be directly applied to the geometry of a capillary with limited dimensions and a reflector, PIC simulations were required to investigate the characteristics of ChSPR. In [9] we presented an extension of the study and showed a comparison between the simulation model and the theory for the SPR peak, as well as the optimisation of corrugation parameters, to achieve maximum radiation intensity for the capillary with a radiation reflector.

Figure 1 schematically shows the corrugated capillary and the radiation reflector placed in the calculation domain. We considered propagation of four bunches separated by the distance $d$ through the capillary. In figure 1 the solid lines show reflective borders and the dashed lines show outer non-reflective borders of the calculation domain. Values of the electric field were calculated at each discretization point in the calculation domain. The electric field values at the outer non-reflective borders were extrapolated in order to obtain electric field values in the wave zone at the location of the electric field probe. The method used for the extrapolation is described in [10]. The electric field probe measured all three component of electric field, including each component’s real and imaginary part. Simulation parameters are shown in table 1. The values of $r_2$ and $a$ were chosen based on the optimization discussed in [9]. The value of $r_1$ was chosen to accommodate electron beam with $\sigma_{\text{transv}}$ up to 500 $\mu$m. The capillary material was assumed to be loss free Fused Quartz with the real part of dielectric permittivity $\varepsilon_{\text{real}} = 3.75$. As shown in [11] $\varepsilon_{\text{real}}$ stays relatively constant throughout the frequency range 100 - 700 GHz.

Table 1. Simulation parameters.

| Parameter name                  | Value   |
|---------------------------------|---------|
| Beam Lorentz - factor, $\gamma$| 16      |
| Bunch charge                    | 0.1 nC  |
| Bunch long. dimension, $\sigma_{\text{long}}$| 0.03 mm |
| Bunch transv. dimension, $\sigma_{\text{transv}}$| 0.3 mm  |
| Number of bunches               | 4       |
Distance between bunches, $d$ & Variable
Simulated frequency range & up to 700 GHz
Capillary material & Fused Quartz (loss free)
Fused Quartz diel. perm. $\varepsilon$ (real part) & 3.75
Material of the Radiation reflector & Copper (pure)
Internal radius, $r_1$ & 2 mm
External radius, $r_2$ & 2.7 mm
Corrugation depth, $a$ & 0.2 mm
Number of corrugation periods & 30
Corrugation width, $l$ & 0.5 mm
Corrugation period, $p = 2l$ & 1 mm
Bunch offset, $h$ & Variable
Capillary length, $L$ & 30 mm

2.1. Smith-Purcell radiation characteristics for different distances between bunches

In this paper we discuss characteristics of the SPR emitted at 90 deg, which as a primary source of THz radiation at large angles $\theta$. On the contrary, ChR is reflected by the outside surface of the corrugated capillary and directed in the forward direction at small angles $\theta \approx 10$ deg. Efficient extraction of the radiation produced by both mechanisms is a separate topic requiring thorough investigation, and it is left for future publications.

Figure 2 shows the spectra of the SPR field calculated at the electric field probe, which is located at $\theta = 90$ deg, at the distance $\approx \gamma^2 \lambda$. The spectra were calculated for $\gamma = 16$ and $\lambda = 1$ mm. The contribution of the non-corrugated capillary with $r_1 = 2.2$ mm and $r_2 = 2.7$ mm was subtracted. The absolute value of the field at the probe location is first calculated as a function of time, and then by taking Fourier transform we obtain a frequency representation of the electric field. The spectra are shown for four different distances between bunches $d = 0.25, 0.5, 0.75, 1$ mm. In the case of the $d = 1$ mm (equal to the period of the corrugation) one may observe the SPR of (-1) and (-2) diffraction orders, however when the distance is $d = 0.5$ mm (equal to the half period of the corrugation) only the SPR of (-2) diffraction order may be observed. For the distances $d = 0.25$ mm and $d = 0.75$ mm no peaks of SPR are present in the considered frequency range.

**Figure 2.** SPR spectra at probe location at $\theta = 90$ deg, $\phi = 90$ deg, and distance to probe $\gamma^2 \lambda$, calculated for $\gamma = 16$ and $\lambda = 1$ mm.

**Figure 3.** Form factors of four-bunch beam with different bunch spacing, and form factor of one bunch.
The fact that diffraction orders may be suppressed for different values of the bunch spacing can be explained by the power spectrum (form-factor) of four-bunch beam, shown in figure 3. The power spectra were calculated for four bunches with the bunch spacing \(d = 0.25, 0.5, 0.75, 1 \text{ mm}\) and for one bunch in the calculation domain without a capillary. When the corrugated capillary is added in the calculation domain the radiation distribution and the spectrum are defined by the modulated power spectrum of the four-bunch beam and by the dispersion relation (1). For example, in the case of \(d = 0.5 \text{ mm}\) above 200 GHz the power spectrum of the beam has only one peak around 600 GHz, therefore giving only one peak in the spectrum of the SPR in figure 2. In the case of \(d = 1 \text{ mm}\) in both figures 2 and 3 the peaks are present at the frequencies 300GHz and 600GHz.

The diffraction orders of the Cherenkov peak and the SPR peaks satisfy the following dispersion relation:

\[
\cos(\theta) = \frac{2\pi m}{k p} + \frac{1}{\beta / \sqrt{\varepsilon(\omega)}},
\]

where \(\theta\) is the polar angle in figure 1; \(\beta\) is the charge speed in terms of the speed of light; \(k\) is the wave number in the dielectric; \(p\) is the corrugation period; and \(m\) is a diffraction order. The value of \(m = 0\) corresponds to the Cherenkov peak, and the values of \(m = \pm n; n = 1,2,3 \ldots\) correspond to the diffraction orders of SPR. This dispersion relation is only valid for the radiation propagation inside the dielectric material, since the theory was developed for the case of a corrugated channel in infinite dielectric. At a capillary border the radiation follows Snell’s law of refraction.

Figure 4 shows the power radiated through the surface of the outside boundary \(A\) of the calculation domain during the simulation time \(\Delta t\). The calculated power for each frequency is given by the following expression:

\[
P(\omega) = \left| \int_0^{\Delta t} \oint S(\omega) \cdot n \, dA \, dt \right|;
\]

where \(S(\omega)\) is the Poynting vector, \(n\) is a unity vector in the outward normal direction from the boundary \(A\) of the calculation domain. The radiated power was calculated for the non-corrugated capillary (black dashed curve) and for the corrugated one (red solid curve).

**Figure 4.** Power spectrum of ChSPR radiated in all directions outside of calculation domain, for corrugated and non-corrugated capillaries (four bunches and one bunch).

**Figure 5.** Power spectrum of ChSPR radiated in all directions outside of calculation domain, for corrugated capillary and off-centre beam propagations: \(h = -1,0,1 \text{ mm}\).
The distance between bunches was equal to the corrugation period $p = d = 1\, \text{mm}$, and the beam was propagating at the offset $h = 1\, \text{mm}$ (figure 1). When these equalities are satisfied, the power radiated from the corrugated capillary is approximately three times more intense than from the non-corrugated one. The power increase occurs at 300GHz, corresponding to the SPR of (-1) order, and at 600 GHz, corresponding to the SPR of (-2) order (figure 4). In the case of the non-corrugated capillary two less intense radiation peaks also present, however it is important to clarify that they originate not due to SPR, but due to the ChR from the four-bunch beam. To prove this point figure 4 also shows the power spectrum of one bunch (blue dotted curve) that demonstrates relatively flat response at all frequencies.

2.2. Smith-Purcell radiation characteristics for different beam positions

In addition to the investigation of the influence of the bunch spacing on the radiation characteristics, it is also interesting to understand how the radiation characteristics change if the beam propagates not through the centre of a capillary, but at an offset from it. As a measure of the offset we can introduce parameter $h$ (figure 1). When the beam propagates through the centre of a capillary, $h$ equals zero, when the beam is shifted towards the positive values of the axis $y$, $h$ is positive, and when the beam is shifted towards the negative values of the axis $y$, $h$ is negative. Figure 5 shows the comparison of the SPR power spectra for $h = -1, 0, 1\, \text{mm}$. When the beam travels at $h = 1, -1\, \text{mm}$ SPR is more intense than for the central propagation.

2.3. ChSPR power patterns

Using the capability of CST Particle Studio to calculate radiation power patterns based on the fields in the calculation domain, we obtained 3D power patterns of the ChSPR for fixed frequencies.

Figure 6 demonstrates the power pattern of the ChSPR from the corrugated capillary at 300GHz for the beam propagation at the offset $h = 1\, \text{mm}$. There are two effects that may be separated: Cherenkov radiation is reflected by the outside boundaries of the corrugated capillary and directed at the small angles $\theta \approx 10\, \text{deg}$, for the chosen capillary parameters SPR of (-1) order is emitted at $\theta = 90\, \text{deg}$. The power pattern changes when the beam propagates off-centre. Figure 7

Figure 6. 3D power pattern of ChSPR at 300GHz for off-central propagation of four-bunch beam at $h = 1\, \text{mm}$.

Figure 7. Polar cross-sections of SPR peak power patterns at $\theta = 90\, \text{deg}$, for three different non-central beam propagations.
demonstrates this behaviour, it shows the cross-sections of the SPR power pattern for the two different beam offsets and for the central beam position \((x, y) = (0, 0)\). While \(\theta\) is fixed at 90 deg, \(\varphi\) is varied in the range \((0; 360)\) deg. The values of the peak power were obtained from the expression (2) as \(P_{\text{peak}} = P(\omega)/\Delta t\), where \(\Delta t = 0.13\) ns is the simulation time. When the beam is shifted along the axis \(y\) by 1 mm, the radiation has the pattern depicted by the red curve. When the beam is shifted along the axis \(x\) by 1 mm, the maximum of the radiation pattern is shifted by approximately 40 deg. towards the smaller values of the angle \(\varphi\) (green dash dot-line). For the beam shift along the axis \(x\), changes in the patterns are not necessarily the same for all frequencies. Depending on the considered frequency the radiation can experience reflections inside the capillary in different ways, subsequently changing the power pattern considerably.

3. Capillary design

The capillaries, both corrugated and non-corrugated, were constructed as sets of cylindrical rings manufactured from Fused Quartz. A composite design was used to allow for greater flexibility and for potential modifications. For example, the cylindrical rings may be produced from different materials, the internal radius of the rings can be designed to be gradually decreasing or increasing along the capillary length. These changes may be used to achieve higher radiation intensities, or to investigate their influence on electron beam dynamics.

Figure 8. Image of Fused Quartz cylindrical ring obtained using electronic microscope.

Figure 9. Corrugated capillary assembly mounted on frame.

Figure 8 shows the image of one of the dielectric rings, taken using an electronic microscope. The image demonstrates some material chipping in the order of 50 to 100 micron at the ring edges (white areas on the light blue background). For both corrugated and non-corrugated capillaries the holders are designed to be identical. Figure 9 shows the assembly of the holders, the radiation reflector, and the corrugated capillary. Using the radiation reflector as an alignment tool, the cylindrical rings were assembled and then secured by two holders on both sides of the reflector.

3.1. Measurements of assembly accuracy

We used a laser scanning setup to check the assemblies’ accuracies. A laser beam was scanned along the outer surfaces of each capillary. The light reflected from the surfaces was detected by an array detector to obtain the vertical off-set of the laser beam. We obtained a dependence of the vertical offset as a function of the horizontal travel range. The manufacturing accuracies of the inside and outside surfaces of the rings were assumed to be roughly the same.
Figure 10 shows the laser scan for the non-corrugated capillary, the top plot shows the scan for 25 mm long section, the bottom plot shows a close-up of the scan for the range of the horizontal scanning position from 15 to 20 mm. The bottom plot shows that the width of each cylindrical ring corresponds to $500 \pm 50 \mu m$, the resolution of 50 μm is defined by the minimum step of the scan. The data points in the top plot were fitted with a line, the tilt of the line is -0.045 deg., which shows that the beginning and the end of the capillary were horizontally levelled within a fraction of degree. Similar scan over the outside surface was performed for the corrugated capillary. It was horizontally levelled with the tilt of 0.21 deg. over the length of 25 mm.

4. Conclusions
In this paper we considered the capillary structures as THz radiators optimised for multi-bunch femtosecond electron beams. Using CST PIC solver we investigated the change in the spectrum of the SPR radiation at $\theta = 90$ deg for the four different values of bunch spacing. For (-1) and (-2) diffraction orders of SPR it was shown, that the corrugated capillary emits more intense radiation than the non-corrugated one. For the given corrugated capillary parameters we identified that more efficient generation of SPR was achieved for the non-central propagation of the four-bunch beam.

We discussed a composite design of the capillaries constructed as sets of cylindrical rings, and showed the assembly accuracy checks, which were performed using the laser-scanning setup. Both capillaries were horizontally levelled with the accuracy of less than 0.3 deg. over the length of 25 mm.

As was pointed out earlier, current design allows generating only SPR at large angles $\theta$. After initial experimental studies we will consider other designs of the radiation reflector and the cylindrical rings in order to achieve extraction of hybrid ChSPR at large angles $\theta$. The hybrid mechanism should provide more intense radiation, compared to only SPR mechanism.

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