DENSITY PROFILES OF GALACTIC DARK HALOS

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Abstract
I propose a modification of the spherical infall model for the evolution of density fluctuations with initially Gaussian probability distribution and scale-free power spectra. I introduce a generalized form of the initial density distribution around an overdense region and cut it off at half the inter-peak separation accounting in this way for the presence of neighbouring fluctuations. Contrary to the original predictions of the model, resulting density profiles within virial radii no longer have power-law dependence on the distance, but are well fitted by the universal formula of changing slope obtained as a result of N-body simulations. The profiles of halos of a given mass are in general flatter than the corresponding ones from the simulations, but the trend of steeper profiles for smaller masses is reproduced. The profiles of galaxy size objects are in general in better agreement with N-body simulations than those of larger ones.

1 Introduction
High-resolution N-body simulations with power-law initial power spectra suggest that density profiles of dark halos in large range of masses are well fitted by a simple universal formula

\[
\frac{\rho(x)}{\rho_{\text{crit},0}} = \frac{\delta_{\text{char}}}{(r/r_s)(1 + r/r_s)^2}
\]

where \(\rho_{\text{crit},0}\) is the present critical density of the Universe and \(\delta_{\text{char}}\) is the characteristic density

\[
\delta_{\text{char}} = \frac{vc^3}{3[\ln(1 + c) - c/(1 + c)]},
\]

with \(v = 200\). The scale radius \(r_s\) is related to the virial radius \(r_v\) (the distance from the center of the halo within which the mean density is \(v\) times the critical density) by \(r_s = r_v/c\) and \(c\) is the concentration, the only fitting parameter in the formula.

The density profile was observed to steepen from \(r^{-1}\) near the center of the halo to \(r^{-3}\) at large distances. This result seemed to contradict the prediction of the analytical spherical infall model (hereafter SIM) which for \(\Omega = 1\) (the only case considered here) finds the profiles to be power-laws of the form \(r^{-(n+3)/(n+4)}\) where \(n\) is the index of the initial power spectrum of density fluctuations.
2 The modified spherical infall model

I will argue here that the discrepancy between the two approaches is mainly due to the oversimplifications applied in the SIM. While such assumptions as spherical symmetry of the initial density distribution and the absence of peculiar velocities will be kept, the shape of the initial density distribution can in fact be made more realistic. This distribution is usually described by the expected overdensity within $r_i$ provided there is a peak (overdense region) of height $a\sigma$ at $r_i = 0$, where $\sigma$ is the rms fluctuation of the linear density field smoothed on scale $R$. If the initial probability distribution of fluctuations is Gaussian and the filter is Gaussian the general form of this quantity as a function of $x_i = r_i / R$ can be found

$$\langle \Delta_i(x_i) \rangle = \frac{6a\sigma}{(n+1)x_i^2} \left[ \frac{1}{2} F_1 \left( \frac{n+1}{2}, \frac{3}{2}, -\frac{x_i^2}{4} \right) - \frac{1}{2} F_1 \left( \frac{n+1}{2}, \frac{1}{2}, -\frac{x_i^2}{4} \right) \right].$$

This function is flat near the center and only at large distances from the peak it approaches the $x_i^{-(n+3)}$ power-law applied in [1].

Although in the flat Universe any overdense region bounds the mass up to infinite distance, in reality there are always neighbouring fluctuations that also gather mass. As a way to emulate this conditions I propose a second modification of the initial density distribution in the form of a cut-off. One can think of two ways of estimating the cut-off scale. First, such scale could be found as a coherence scale of the overdense region defined by the expected overdensity (3) being equal to its rms fluctuation. It turns out however, that a more stringent constraint is induced by the presence of other peaks (see [2]). Therefore here the cut-off will be introduced at the half inter-peak separation $x_{i,pp}/2$ for the most reasonable height of the peak, $a = 3$. In the case of $n = -1$ we have $x_{i,pp}/2 = 6.45$. The generalized initial density distribution with a cut-off will be modelled by

$$\Delta_{i,\text{cut}}(x_i) = \frac{\langle \Delta_i(x_i) \rangle}{1 + e^{(x_i-x_{i,pp}/2)/w}}$$

with the width of the filter $w = 1$.

According to the SIM the subsequent shells numbered by the coordinate $x_i$ will slow down due to the gravitational attraction of the peak, stop at the maximum radius and then collapse by some factor $f$ to end up at the final radius

$$x = x_i f \left[ \Delta_{i,\text{cut}}(x_i) + 1 \right] / \Delta_{i,\text{cut}}(x_i).$$

The simplest versions of the SIM adopt the value $f = 1/2$ motivated by the virial theorem and it will also be assumed here but a more realistic description can be found in [2]. The final profile of the virialized halo is then

$$\frac{\rho}{\rho_{\text{crit},0}} = (1 + a\sigma \varrho)(1 + z_i)^3 \left( \frac{x_i}{x} \right)^2 \frac{dx_i}{dx}$$

where $\varrho = \xi_R(r)/\sigma^2$ is the correlation coefficient.

3 Comparison with the universal profile

Since the measurements of halo properties from N-body simulations [3] were done at the state corresponding to the present epoch, the same condition will be applied for SIM calculations. Once the initial redshift $z_i$ is specified, equating the collapse time to the present age of the
Figure 1: The density profiles of the halo of mass of order $3 \times 10^{12} h^{-1} M_\odot$ for $n = -1$. The solid line shows the prediction of the SIM. The long-dashed one (NFW1) gives the result of the $N$-body simulations with their fitted concentration, while the short-dashed curve (NFW2) presents the formula (1) with concentration fitted to SIM results.

Universe determines the overdensity of the presently virializing shell which ends up at the virial radius of the halo. When we adopt the normalization of the initial power spectrum ($\sigma_8 = 1$) and the conditions $a = 3$ and $aa = 0.1$ (for the linear theory to be valid) choosing the initial redshift $z_i$ for a given spectral index $n$ gives the comoving smoothing scale $R$ with which the overdense regions are identified. The mass of the halo within the virial radius $x_v$ can then also be determined

$$M = \frac{800\pi}{3} \rho_{\text{crit},0} \left( \frac{x_v R}{1 + z_i} \right)^3. \quad (7)$$

Figure 1 shows the density profile of a dark matter halo of galactic mass. The solid line presents the prediction of the SIM obtained from formula (6) for $n = -1$, $z_i = 600$ and $R = 0.188 h^{-1}$ Mpc. The final (virial) proper radius of the halo is $r_v = 0.231 h^{-1}$ Mpc and the mass $M = 2.88 \times 10^{12} h^{-1} M_\odot$ which correspond to a galactic halo. We see that the result of the SIM can be well fitted by formula (1) but the SIM profile is significantly flatter than the corresponding one from the simulations. The concentration parameters which measure the steepness of the profile (the higher $c$ the steeper the profile) are $c = 57.1$ and $c = 19.1$ respectively from the simulations and from the SIM.

One of the main results of $N$-body simulations [3] was the dependence of the shape of the density profiles of halos on their mass. On the other hand, the standard prediction of the SIM gives the same profile independently of mass. However, with the improvements introduced above it is possible to reproduce the dependence of the profiles on mass.

It is sometimes argued that if the density field is smoothed with a given scale $R$ lower peaks end up as galaxies and higher ones as clusters. This, however, would violate the hierarchical way of structure formation since higher peaks collapse earlier. Another argument against such assumption comes from the calculations based on the improved SIM: the reasonable range of peak heights $a$ between 2 and 4, which are most likely to produce halos, leads for a given...
smoothing scale to the range of masses spanning only one order of magnitude, while in $N$-body simulations halos with masses spanning few orders of magnitude are observed. This suggests that the dependence on mass should rather be related to the initial smoothing scale.

The dependence of the shape of the profiles on mass obtained with these assumptions is shown in Figure 2. The solid lines give the values of the concentration parameter $c$ obtained by fitting the formula (1) to the results of SIM for different power spectra and the dashed lines are the corresponding values from the simulations [3]. The overall trend of steeper profiles for smaller masses is reproduced and the agreement between the two approaches is significantly better for smaller masses.

The spherical infall model provides simple understanding of the dependence of the shape of the halo on its mass: smaller halos start forming earlier and by the present epoch their virial radii reach the cut-off scale that accounts for the presence of the neighbouring fluctuations; more massive halos form later and their virial radii are not affected by the cut-off scale, their virialized regions contain only the material that initially was quite close to the peak identified with the smoothing scale corresponding to the mass.

**Acknowledgements.** This work was supported in part by the Polish State Committee for Scientific Research grant No. 2P03D00815.

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