MAJORANA NEUTRINO MASSES CAN SAVE ONE FAMILY TECHNICOLOUR MODELS

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Abstract

We make non perturbative estimates of the electroweak radiative correction parameter $S$ in dynamical symmetry breaking models with Majorana neutrino masses. The Majorana masses are treated as perturbations to a Non Local Chiral Model of the strong interactions. We argue that parameter ranges exist that would allow realistic values of $S$ and $T$ in one family Technicolour models.
1 Introduction

Precision measurements of electroweak radiative corrections have begun to constrain models of electroweak symmetry breaking (EWSB). Any new particles at the TeV scale that transform under $SU(2)_L$ do not decouple from the theory at low energies but contribute to precision electroweak measurements through gauge boson self energies (oblique corrections). When the new particles have masses much larger than the Z boson mass, $M_Z$, their major contribution to the oblique corrections can be described by the parameters $S$ and $T$ introduced by Peskin and Takeuchi.

\[
S = 16\pi \frac{d}{dq^2} \left[ \Pi_{33} (q^2) - \Pi_{3Q} (q^2) \right] \bigg|_{q^2=0} = -8\pi \frac{d}{dq^2} \left[ \Pi_{3Y} (q^2) \right] \bigg|_{q^2=0} \quad (1.1)
\]

\[
\alpha T = \delta \rho_s = \frac{e^2}{s^2 M_W^2} \left[ \Pi_{11}(0) - \Pi_{33}(0) \right] \quad (1.2)
\]

$M_W$ is the W boson mass, $e$ is the electromagnetic charge and $s^2 \equiv \sin^2(\theta_w)$. The $\Pi_{ij}(q^2)$ are the gauge boson self energies with the indices $i, j = 1, 2, 3, Q$ and $Y$ referring to the $SU(2)_L$, electromagnetic and hypercharge currents respectively. The contribution to the self energies from loops of standard model particles are removed to isolate the dependence on the new heavy particles. The parameter $T$ is directly related to the $\rho$ parameter by $\alpha T = \delta \rho_s = \frac{M_Y^2}{M_Z^2} - 1$. Recent measurements suggest that $S \leq 0.6$ and $T \leq 1(\delta \rho_s \leq 0.7\%)$ at the 95% confidence level (c.l.) with slightly negative values ($S$ and $T \approx -0.5$) favoured.

In models such as Technicolour (TC) in which EWS is broken dynamically new fermions are strongly interacting. Estimates have been made of these new fermions' contribution to $S$ and $T$ in the isospin preserving limit by “scaling up” QCD and in a non-local chiral model of the strong interactions. These estimates suggest that the contributions to $S$ and $T$ per fermion doublet are

\[
\Delta S \simeq 0.1 N_{TC}, \quad \Delta T = 0 \quad (1.3)
\]
where $N_{TC}$ is the number of Technicolours. All but the most minimal TC models appear to be ruled out by these estimates of the $S$ parameter.

However, a complete model of EWSB must account for all the fermion masses in the Standard Model. In particular it must explain why these masses break isospin symmetry e.g. $m_t/m_b \geq 20$ and why the neutrinos are massless or nearly so. We might expect that in a realistic TC model isospin would be broken in the techni-fermion sector. In this paper we investigate TC models in which the isospin symmetry is broken by techni-neutrino Majorana masses. Allowing Majorana masses into the theory restricts the TC group to those with real representations.

In one loop perturbation theory it has been shown that Majorana masses can give negative contributions to both $S$ \cite{8} and $T$ \cite{6}. The techni-fermions, however, are strongly interacting and hence a non-perturbative calculation must be performed to estimate their contributions to $S$ and $T$. The contribution to $T$ from Majorana techni-neutrinos has been estimated \cite{9} in Dynamical Perturbation Theory \cite{10} and show the same qualitative behaviour as the perturbative results. The most stringent constraint on TC models comes though from the $S$ parameter. We estimate the contributions to $S$ by perturbing the isospin symmetry in the Non Local Chiral Model \cite{3,7}. We argue that parameter ranges exist that would allow realistic values of $S$ and $T$ in one family TC models.

In Section 2 of this paper we will review the isospin preserving Non Local Chiral Model and the derivation of the estimate of $S$ in Ref \cite{7} (Eq(1.3)). In Section 3 we generalize the model to include Majorana neutrino masses. In Section 4 we present our results for the contribution to $S$ from Majorana techni-neutrinos and present a number of realistic techni-fermion mass spectra in one family TC models. Section 5 concludes the paper.
2 The Non Local Chiral Model

In this section we review the derivation of Eq(1.3) in the Non Local Chiral Model \[5, 7\] approximation to TC dynamics. Consider a techni-fermion doublet, $\Psi = \begin{pmatrix} U \\ D \end{pmatrix}$, which transforms under some isospin preserving TC group. The techni-fermions also transform under the Standard Model electroweak symmetry group $SU(2)_L \otimes U(1)_Y$. The TC group becomes strongly interacting at some scale $\Lambda_{TC}$ and breaks the doublets chiral symmetry

$$SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$$ (2.1)

The techni-fermions acquire a dynamical self energy term, $\Sigma(k^2)$, which acts as the order parameter for chiral symmetry breakdown. There are three Goldstone bosons, $\pi^a$ ($a = 1..3$), associated with the three broken generators which are bound states of the techni-quarks and correspond to the pions of QCD. Below the scale $\Lambda_{TC}$ the effective theory of the Goldstone bosons and $SU(2)_L \otimes U(1)_Y$ gauge bosons can be described by a gauged chiral Lagrangian \[11\]. Following the usual formalism we write the Goldstone fields as

$$U = \exp(\frac{i\pi^a\lambda_a}{f_\pi})$$ (2.2)

where $\lambda_a$ are the broken, axial generators and $f_\pi$ is a dimensionful constant $\approx \Lambda_{TC}$. Under global $SU(2)_L \otimes SU(2)_R$ symmetry transformations $U$ transforms as

$$U \rightarrow R^\dagger UL$$ (2.3)

At low energies we may perform an expansion in powers of momentum and in the notation of Gasser and Leutwyler \[12\] the chiral Lagrangian is given up to fourth order in the (covariant) derivative of $U$ by
\[ \mathcal{L} = \frac{f^2}{4} tr(D^{\mu} U D_{\mu} U^\dagger) + L_1 tr((D^{\mu} U D_{\mu} U^\dagger)(D_{\mu} U D_{\nu} U^\dagger)) \\
+ L_2 tr((D^{\mu} U D_{\mu} U^\dagger)(D_{\nu} U D_{\nu} U^\dagger)) + iL_9 tr(F^R_{\mu\nu} D^{\mu} U D^{\nu} + F^L_{\mu\nu} D^{\mu} U D^{\nu}) \\
+ L_{10} tr(U^\dagger F^R_{\mu\nu} U F^L_{\mu\nu}) + L_{11} tr(D^2 U D^2 U^\dagger) \] (2.4)

The only contribution to the gauge boson self energy \( \Pi_{3Y} \) at order \( q^2 \) is given by \( L_{10} \) and hence the electroweak precision parameter \( S \) is given by

\[ S = -16\pi L_{10} \] (2.5)

The parameter \( L_{10} \) is not determined by the chiral symmetry and must be estimated in some approximation to the full TC dynamics. The Non Local Chiral Model is such an approximation. The model assumes that the following elements of the TC dynamics are responsible for the non perturbative features of TC above the symmetry breaking scale, \( \Lambda_{TC} \):

- The theory is chiral \( ie \) it must possess an \( SU(2)_L \otimes SU(2)_R \) symmetry.
- The techni-fermions, \( \Psi \), acquire a dynamical self energy term, \( \Sigma(k^2) \).
- There are three Goldstone bosons associated with the three generators broken in the chiral symmetry break down.

The Goldstone bosons appear in the theory, just as in the Higgs model, as fluctuations about the order parameter of symmetry break down which here is \( \Sigma(k^2) \). These approximations are born out by the success of the model at reproducing the low energy predictions of QCD \([14]\). The Lagrangian for the techni-fermions is thus given by

\[ \mathcal{L} = \bar{\Psi}(x) \delta(x - y) \delta \Psi(y) + \bar{\Psi}(x) \Sigma(x, y) \Psi(y) \] (2.6)

where the form of \( \Sigma(x, y) \) is determined by the chiral symmetry \([5]\) to be
$$\Sigma_\pi(x, y) = \Sigma(x - y)[1 - \frac{i}{f_\pi} \gamma_5 [\pi(x) + \pi(y)]$$

$$- \frac{1}{2} f_\pi^2 [\pi^2(x) + \pi^2(y) + \pi(x)\pi(y) + \pi(y)\pi(x)] + ...] \quad (2.7)$$

The action for this Lagrangian can be made locally gauge invariant by the insertion of one or more path ordered exponential of a line integral over the gauge field

$$\exp \left[ -ieQ \int_\mathcal{Y} A_\nu dx^\nu \right] \quad (2.8)$$

where $Q$ is the charge matrix for the fermions $\Psi$. This procedure is followed explicitly in Ref[7] and the Feynman rules in Appendix 1 are obtained.

The Non Local Chiral Model has the same chiral symmetry as the Chiral Lagrangian, Eq(2.5). Thus the Chiral Lagrangian may be obtained simply by integrating out the techniquarks from the non-local theory leaving the Goldstone bosons and gauge bosons. We wish to find the coefficient $L_{10}$ in the Chiral Lagrangian; we note that only the term in Eq(2.5) with coefficient $L_{10}$ gives a contribution to the scattering between two vector gauge bosons and two Goldstone bosons of the form $q^2 g^\mu\nu$ where $q$ is the incoming gauge boson momentum. $L_{10}$ is therefore given by the coefficient, $C$, of the $q^2 g_{\mu\nu}$ term in the Taylor expansion of the amplitude given by the sum of diagrams in Fig 1.

$$L_{10}[tr(2\lambda^c\lambda^a\lambda^d\lambda^b + 2\lambda^c\lambda^a\lambda^d\lambda^a) = \frac{f_\pi^2}{8} C[tr(2\lambda^c\lambda^a\lambda^d\lambda^b + 2\lambda^c\lambda^a\lambda^d\lambda^a)$$

$$-tr(\lambda^c\lambda^d\lambda^a\lambda^b - \lambda^c\lambda^a\lambda^b\lambda^d) - tr(\lambda^c\lambda^d\lambda^a\lambda^b - \lambda^c\lambda^a\lambda^b\lambda^d) \quad (2.9)$$

$$-\lambda^c\lambda^d\lambda^a\lambda^a - \lambda^c\lambda^b\lambda^a\lambda^d)] - \lambda^c\lambda^d\lambda^a\lambda^a - \lambda^c\lambda^b\lambda^a\lambda^d)]$$

The calculation is simplified by taking the limit in which the Goldstone bosons’ momenta vanish. The coefficient, $C$, is an integral expression containing over 70 terms involving $\Sigma(k^2)$ and its derivatives which we will not reproduce here (see Refs [13], [16]). We note that the...
two group theory factors provide independent checks on $C$. We will show in Section 4 that for a suitable choice of function for $\Sigma(k^2)$ this estimate for $S$ gives the result in Eq(1.3).

3 Majorana Techni-Neutrinos

We now consider perturbing the Non Local Chiral Model of a techni-lepton doublet, $\psi = \begin{pmatrix} N \\ E \end{pmatrix}$, by explicitly introducing a Majorana mass term, $M$, for the right handed techni-neutrino, $N_R$. We shall assume that $M$ is small relative to the fermions’ Dirac self energy, $\Sigma(k^2)$, so that the chiral symmetry breaking pattern Eq(2.1) is only slightly perturbed. In practice perturbing the effective Chiral Lagrangian of QCD by introducing current masses for the quarks provides a good description of QCD’s low energy interactions even for the strange quark which has a current mass $\approx \Lambda_{QCD}$ (see for example [11]). Whereas the introduction of a Majorana mass for $N_R$ explicitly breaks the custodial $SU(2)$ symmetry and therefore alters the symmetry breaking pattern from Eq(2.1) it is reasonable to assume that Eq(2.1) is a good approximation for Majorana masses $\leq O(\Lambda_{TC})$. We quantify the accuracy of this assumption in Section 4 by comparing our results for $S$ from a lepton doublet with hard mass terms with the perturbative one loop results.

Writing the left and right handed degrees of freedom of $N$ as two Majorana (self conjugate, $\psi^M = C(\bar{\psi}^M)^T$) fields $N_0^0 = \begin{pmatrix} N_L \\ N_C^L \end{pmatrix}$ and $N_0^0 = \begin{pmatrix} N^C_R \\ N_R \end{pmatrix}$ the mass terms are

$$L_M = -\bar{E} \Sigma_E E - \frac{1}{2} (\bar{N}_1^0 \bar{N}_2^0) \begin{pmatrix} 0 & \Sigma_N \\ \Sigma_N & -M \end{pmatrix} \begin{pmatrix} N_1^0 \\ N_2^0 \end{pmatrix}$$

(3.1)

where $\Sigma_E$ and $\Sigma_N$ are the self energies of the $E$ and $N$ fields after chiral symmetry breakdown. The Majorana mass eigenstate fields $N_1$ and $N_2$ with masses $M_1(p)$ and $M_2(p)$ are given, using the notation of Ref[8], as

$$\begin{pmatrix} N_1^0 \\ N_2^0 \end{pmatrix} = \begin{pmatrix} i c_\theta & s_\theta \\ -i s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}$$

(3.2)
where

\[ c_\theta^2 = \frac{M_2}{M_1 + M_2}, \quad s_\theta^2 = \frac{M_1}{M_1 + M_2}, \]  

and

\[ M = M_2 - M_1, \quad \Sigma_N = \sqrt{M_1 M_2} \]  

(3.3)

(3.4)

The currents that give the vertices in Appendix 1 must be written in terms of the mass eigenstate fields, \( N_1 \) and \( N_2 \), according to

\[
\bar{\Psi} \gamma^\mu \Psi = \frac{i}{2} (s_\theta^2 - c_\theta^2) \left( \bar{N}_1 \gamma^\mu \gamma^5 N_1 - \bar{N}_2 \gamma^\mu \gamma^5 N_2 \right) + \frac{i}{2} s_\theta c_\theta \left( \bar{N}_1 \gamma^\mu N_2 - \bar{N}_2 \gamma^\mu N_1 \right)
\]

\[
\bar{\Psi} \gamma_5 \mathcal{O}(\{p\}) \Psi = s_\theta c_\theta \left( \bar{N}_1 \gamma_5 \mathcal{O}(\{p\}) N_1 + \bar{N}_2 \gamma_5 \mathcal{O}(\{p\}) N_2 \right) - \frac{i}{2} \left( s_\theta^2 - c_\theta^2 \right) \left( \bar{N}_1 \mathcal{O}(\{p\}) N_2 + \bar{N}_2 \mathcal{O}(\{p\}) N_1 \right)
\]

\[
\bar{\Psi} \mathcal{O}_\mu(\{p\}) \Psi = \frac{i}{2} \left( \bar{N}_1 \mathcal{O}_\mu(\{p\}) N_2 - \bar{N}_2 \mathcal{O}_\mu(\{p\}) N_1 \right)
\]

\[
\bar{\Psi} \gamma_5 \mathcal{O}_\mu(\{p\}) \Psi = \frac{i}{2} \left( \bar{N}_1 \gamma_5 \mathcal{O}_\mu(\{p\}) N_1 + \bar{N}_2 \gamma_5 \mathcal{O}_\mu(\{p\}) N_2 \right) - \frac{i}{2} \left( s_\theta^2 - c_\theta^2 \right) \left( \bar{N}_1 \mathcal{O}_\mu(\{p\}) N_2 + \bar{N}_2 \mathcal{O}_\mu(\{p\}) N_1 \right)
\]

where \( \mathcal{O}(\{p\}), \mathcal{O}_\mu(\{p\}) \) and \( \mathcal{O}_\mu(\{p\}) \) are general operators which depend on the momenta \( \{p\} \) entering the vertex and where we have used the self conjugacy properties of the Majorana fields \([17]\). Since the Majorana techni-neutrino mass is being treated as a perturbation to the TC symmetry breaking pattern Eq(2.1) the vertices are functions of the dynamical Dirac self energy, \( \Sigma(k^2) \), only.

The two pion-two vector gauge boson scattering amplitude and hence \( L_{10} \) is again given by the diagrams in Fig 1 but with all possible positionings of the neutrino mass eigenstates
in the fermion loops. Remembering that both the Wick contractions \( \langle \bar{\Psi}_M \Psi_M \rangle \) and \( \langle \Psi_M \bar{\Psi}_M \rangle \) are non zero we obtain, for example, the contributions in Fig 2. We extract the generalized coefficient, \( C \), of the \( q^2 g^{\mu \nu} \) term in the Taylor expansion of the scattering amplitude. When Taylor expanding the amplitude we assume that \( c_\theta \) and \( s_\theta \) vary slowly with respect to \( q^2 \) and set all there derivatives to zero. \( L_{10} \) is then given in terms of \( C \) by Eq(2.5). Our full expression for \( L_{10} \) is too long to be reproduced here. However, as a check we have confirmed the results of Ref\[7\] in the limit \( M \to 0 \ (\theta \to \pi/4) \)

4 Numerical Results

In this section we present the numerical results obtained from the expressions derived for \( S \) for a techni-lepton doublet in Sections 2 and 3. We must estimate the form of the Schwinger Dyson equations’ solution for \( \Sigma(p^2) \) in TC. Following Holdom et al. \[5, 7, 14\] we choose a function that behaves as \( 1/k^2 \) for large momenta and is well behaved as \( k^2 \to 0 \)

\[
\Sigma(k^2) = \frac{(A + 1)m^3}{k^2 + Am^2}
\]  

(4.1)

where \( A \) and \( m \) are arbitrary coefficients. The function is normalized so that \( \Sigma(m) = m \). The form of this solution is plotted in Fig 3 for \( A = 1, 2, 3 \) and \( A \to \infty \). \( \Sigma(k^2) \) corresponds to a hard mass \( m \) when \( A \to \infty \) and comparison to experimental data \[14\] suggests that \( 2 \leq A \leq 3 \) gives a good approximation to the solution of the QCD Schwinger Dyson equations. Values of \( A > 3 \) give solutions that fall off more slowly than the QCD solution as is expected in “walking” TC theories.

We first quantify the accuracy of our approximations by comparing the results from
our expression for $S$ when $A \to \infty$ with the perturbative one loop calculation. Fig 4 shows this comparison for a techni-lepton doublet with Dirac mass $m_D$ and techni-neutrino Majorana mass $M$ (the x axis shows the normalized Majorana mass $M/m_D$). The Non Local Chiral Model's prediction for $S$ shows good agreement with the perturbative result until $M/m_D \sim 2–3$. As expected perturbing the isospin preserving chiral symmetry breaking pattern is a good approximation when the symmetry breaking mass terms are $\leq \mathcal{O}(m_D)$. In TC $m_D \sim \mathcal{O}(\Lambda_{TC})$ and hence we expect our results for $S$ to be good estimates when the Majorana mass $\leq \mathcal{O}(\Lambda_{TC})$. Above $M/m_D \sim \mathcal{O}(1)$ the perturbative results diverge as $\log\left(\frac{M_1}{m_D}\right)$ (where $M_1$ is the mass of the lightest Majorana neutrino mass eigenstate) as a result of the divergence of the perturbative diagrams. The diagrams contributing to $S$ in the Non Local Chiral Model (Fig 1) converge as $M/m_D \to \infty$ and hence this behaviour is not reproduced. Note that the $\log\left(\frac{M_1}{m_D}\right)$ term only becomes large when the Chiral Symmetry approximation has broken down.

In Fig 5 we present the results for the contribution to $S$ from a techni-lepton doublet with dynamical Dirac self energy and hard Majorana neutrino mass in the Non Local Chiral Model for various choices of $A$ in the ansatz for $\Sigma(k^2)$. If $1 \leq A \leq 3$ is a good approximation to TC dynamics then in the limit $M = 0$ we find for an $SU(2)_L$ doublet

$$0.07 \leq S \leq 0.1 \quad (4.2)$$

in agreement with the approximation in Eq(1.3). It is commonly assumed in the literature that TC dynamics have the effect of enhancing perturbative estimates for $S$ by a factor of 2. However, our results suggest that the TC dynamics provide a positive shift to the perturbative results, $0.02 \leq \Delta S \leq 0.05$.

If the Majorana mass is dynamically generated then it too will have momentum dependence of the form in Eq(4.1). Fig 6 shows the deviations from the estimates with a hard Majorana mass in the results for $S$ due to this momentum dependence (we show results for
$A = 3$ as an example). The new estimate tends to the result with a hard Majorana mass for low momenta since the momentum dependence is negligible in comparison to the large Dirac self energy. For large Majorana masses the Dirac self energy falls off whilst the Majorana mass is still flat and again the results tend to the hard limit. The deviation in $S$ from the hard limit are at most $\sim 0.01$.

Finally we consider the consequences of these results in one family TC models [1, 18]. Recent work [19] has shown that realistic values for $S$ and $T$ can be obtained from mass splittings in the techni-lepton sector and from Goldstone boson contributions. In models in which the techni-quarks and leptons interact equally with the TC gauge bosons [4] the techni-quark self energy can be enhanced by QCD interactions by a factor of $2 - 5$ relative to the techni-lepton self energy [20]. The $W$ and $Z$ masses are then generated almost exclusively by the techni-quark condensates since, for example

$$M^2_Z = \frac{g^2 + g'^2}{4} \left( \frac{1}{2} f_N^2 + \frac{1}{2} f_E^2 + 3 f_Q^2 \right) \quad (4.3)$$

where $f_N, f_E$ and $f_Q$ are the Goldstone boson decay constants associated with the Goldstone bosons with constituent techni-neutrinos, techni-electrons and techni-quarks respectively. Large mass splittings in the techni-lepton sector can, therefore, give negative contributions to $S$ without contributing a large positive contribution to $\delta \rho_\ast$.

We consider the contributions to $S$ and $\delta \rho_\ast$ from techni-neutrino Majorana masses, $M$, in such a scenario in Table 1. We extrapolate our results for $S$ when $M \leq \mathcal{O}(\Lambda_{TC})$ to larger values of $M$ by shifting the perturbative results by $[S_{NLCM}(M = 0) - S_{pert}(M = 0)]$ as suggested by the results in Fig 3. The contributions to $\delta \rho_\ast$ are estimated using the Dynamical Perturbation Theory [10] results from Ref[3]. The techni-quarks are assumed to have degenerate Dirac self energies as are the techni-leptons. The parameters $m_L, m_Q$ and $M$ in the ansatz for $\Sigma(k^2)$ are scaled in Table 1 by the mass scale $\Lambda$ which must be tuned in a particular TC model to give the experimentally measured $Z$ mass. Three estimates for
$S$ and $T$ are given, the naive results from Eq(1.3) and the Non Local Chiral Model results with $A = 2$ and $A = 3$ in the ansatz for $\Sigma(k^2)$.

The techni-family’s contribution to $S$ can be reduced by a factor of $\sim 4$ without violating the experimental upper bound on $\delta \rho_*$ by including the Majorana mass for the techni-neutrino. The largest negative contributions to $S$ are achieved at the expense of a large Majorana mass $(M/m_D \sim 10)$ in which case the lightest Majorana mass eigenstate may fall below the current experimental lower bound in some models. However, additional negative contributions to $S$ from splitting between the Dirac self energies in the lepton doublet and from Goldstone boson contributions [19] are not included in these estimates. If these additional negative contributions to $S$ were included in a realistic TC model a more reasonable value for $M/m_D$ ($\sim 5$) could be chosen whilst maintaining physical values for $S$ and $T$.

| $m_L/\Lambda$ | $m_Q/\Lambda$ | $M/\Lambda$ | $S_{naive}$ | $\delta \rho_{\ast naive} \%$ | $S_{A=2}$ | $\delta \rho_{\ast A=2} \%$ | $S_{A=3}$ | $\delta \rho_{\ast A=3} \%$ |
|---------------|---------------|-------------|-------------|-----------------|-----------|----------------|-----------|----------------|
| 1             | 1             | 5           | 0.4$N_{TC}$ | 0               | 0.23$N_{TC}$ | 0.69          | 0.19$N_{TC}$ | 0.81          |
| 1             | 1             | 10          | 0.4$N_{TC}$ | 0               | 0.16$N_{TC}$ | 2.43          | 0.12$N_{TC}$ | 2.55          |
| 1             | 2             | 5           | 0.4$N_{TC}$ | 0               | 0.23$N_{TC}$ | 0.26          | 0.19$N_{TC}$ | 0.30          |
| 1             | 2             | 10          | 0.4$N_{TC}$ | 0               | 0.16$N_{TC}$ | 0.76          | 0.12$N_{TC}$ | 0.77          |
| 1             | 3             | 5           | 0.4$N_{TC}$ | 0               | 0.23$N_{TC}$ | 0.16          | 0.19$N_{TC}$ | 0.16          |
| 1             | 3             | 10          | 0.4$N_{TC}$ | 0               | 0.16$N_{TC}$ | 0.34          | 0.12$N_{TC}$ | 0.33          |

Table 1. Values for $S$ and $\delta \rho_*$ for various techni-fermion mass spectra in one family TC models.

5 Conclusions

In this paper we have developed non perturbative estimates for the electroweak radiative correction parameter $S$ in dynamical symmetry breaking models with Majorana neutrino masses. The Majorana mass terms were treated as perturbations to the isospin preserving chiral symmetry breaking pattern in Eq(2.1) of a Non Local Chiral Model of the strong
dynamics that break electroweak symmetry. Comparison with the perturbative one loop results for $S$ with hard mass terms suggest that the approximations made hold when the Majorana mass is below the chiral symmetry breaking scale. Estimates for $S$ in Technicolour theories were then made with a suitable ansatz for the techni-fermion self energies. The results suggest that the TC dynamics have the effect of shifting the perturbative results for $S$ by a factor of $0.02 - 0.05$ per doublet. In walking TC theories the shift would be lower depending on the precise fall off of the techni-fermion self energies. We conclude that Majorana neutrino masses can give negative contributions to the $S$ parameter in non-perturbative theories.

Techni-fermion mass spectra were proposed in one family TC models in which the negative contributions to $S$ from Majorana neutrino masses reduced the estimate for $S$ for the full family by a factor of $\sim 4$ without giving rise to an unphysically large contribution to $\delta \rho_*$. These negative contributions to $S$ are achieved with natural choices of mass scales and without fine tuning. We do require though that the techni-quarks dominate electroweak symmetry breaking as in models such as those in Ref[4]. Extended TC models such as those in Ref[18] in which the large top bottom mass splitting is generated by a large techni-up techni-down mass splitting require that the techni-lepton sector dominates electroweak symmetry breaking. These models will be more tightly constrained by the $\delta \rho_*$ estimates. Estimates of $S$ and $T$ in TC models accounting for mass splittings in the techni-lepton sector, from Goldstone boson contributions [19], and now these results for techni-neutrino Majorana masses suggest that parameter ranges exist in TC models with realistic fermion mass spectrums that give realistic values of $S$ and $T$. 

12
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