Students’ specializing in mathematics problem solving

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Abstract. The ability to think mathematically in solving problems is an important goal of schooling. It is an ultimate goal of teaching that students will be able to conduct a mathematical investigation by themselves and that they will be able to identify where the mathematics they have learned is applicable in real-world situations. The first strategy of mathematical thinking is specializing. Specializing means turning to some examples to learn more about the main goal of a concept or problem and these examples are specific and particular instances of more a general situation in a concept or problem. This study based on students’ difficulties to think mathematically in mathematics problem solving. There are still many students who feel confused when solving problem. The purpose of this study is to investigate the specializing of two sixth grade students in solving mathematics problems. They were selected from ten students taking mathematical skills and problem solving tests. They were classified in high and low mathematics abilities. Then they were interviewed based on their work of the given task. When solving the problem, the subject with high mathematics ability was able to describe the known, the way to determine the solution and try to look at the other examples. While the subject with low mathematics ability knows about a concept that can be used to help the problems but she didn’t know the way to determine the solution. Even though taking more time, by applying both a structured-scaffolding and using specializing systematically that fit the problem they succeeded in solving tem. These results may contribute mathematics teachers in a way how to help students use specializing in solving problem.

Keywords: Mathematical thinking, specializing, problem solving, mathematics education.

1. Introduction
The ability to think mathematically and to use mathematical thinking to solve problems is an important goal of schooling. Besides that, mathematical thinking is also important as a way of learning mathematics, and for teaching mathematics [12]. Focusing on mathematical thinking in schools provides a better learning of mathematical content, as well as, a confidence in systematic and multiple ways of thinking and reasoning abilities [13]. Most researchers, mathematicians, and mathematics educators define mathematical thinking as a process, which contains at least one of the mental and math-related activities such as reasoning, abstracting, conjecturing, representing and switching between different representations, visualizing, deducing, inducing, analyzing, synthesizing, connecting, proving, specializing, convincing, generalizing [1,2,4,7,9,11,14]. The first strategy of mathematical thinking is Specializing [7]. Specializing means turning to some example to learn more about the main goal of concept or problem and these examples are specific and particular instances of the more general situation in a concept. Specializing determines teacher, student and a person who involved teaching and learning process; what she/he knows about a concept that past experiences can be used to help in this case,
what he/she wants to do and what he/she can introduce [3]. It should be considered that the specialization must be done systematically. Specializing alone is unlikely to solve the problem but it does provide a starting point and can present cases of broader situations in the question. The purpose of specializing is to get an idea of what answers are possible from a question and at the same time, can develop the mind why the answer is correct. In other words, specializing means simplifying the question, making it more specific or more special until some progress is possible [7].

Furthermore, thinking and problem-solving are two things that cannot be separated. By thinking, someone will be able to solve the problem. Problem-solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program [8]. Student will be able to solve mathematical problems if they have an appropriate scheme for problem solving. The scheme is constructed from meaningful knowledge of relevant concepts, previous experiences in solving problem, understanding of the problems being solved, and knowledge of problem solving approaches or strategies [5,6]. But, there are still many students who feel confused when solving problem. Mathematical thinking process which involves specializing has been applied supporting difficulties in mathematical problem solving. Yudariah (1997) has been used mathematical thinking process to change students’ attitude toward calculus among Malaysian undergraduate students [16]. Moreover, Roselainy (2008) has tried to use mathematical thinking overcome students’ difficulties in multivariable derivation and multiple integral [10]. The characteristics of the problem such as it must begin where the student is, the problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn, it must require justification and explanation for answers and methods [15]. Specializing in problem solving can be appeared by drawing diagrams, constructing tables, making symbols and organization [3].

Based on this definition, students’ specializing in mathematics Problem solving can be known from the way students solve problems and understand the question. So, the question raised by researchers in this study is “How do students understand the problem? How is the students’ specializing in mathematics problem-solving?”

2. Method

This study was a qualitative research with descriptive approach. At the data analysis stage, the activities conducted by researchers were (1) transcribing the data obtained from interview, (2) focusing on important thing, removing the unnecessary ones, (4) describing the specializing in mathematics problem solving, and (5) conclusion.

| Mathematical thinking strategy | Indicators                                                                                                                                 |
|-------------------------------|------------------------------------------------------------------------------------------------------------------------------------------|
| Specializing                  | 1. Students can devise details of known and ask in the problem.                                                                            |
|                               | 2. Students can understand the concept that past experience can be used to help problem-solving tasks.                                    |
|                               | 3. Students can find some examples and related to some information to solve the problem.                                                   |

The sample of this research was 6th-grade students of NRC Course. This class consists of 6 female and 4 male students. To get a research subject, all students were given 10 questions on math skills test (TKM) and 1 questions on problem-solving tasks, researchers will choose the subject who has high and low mathematics abilities. Two volunteer students were selected as a research subject, one subject
had a high mathematics ability, and one subject had a low mathematics ability. Then they were interviewed based on their work of the given tasks.

3. Results and Discussion
Based on the results of the analysis of the Problem solving Task answer sheets and interviews, we got data on the specializing undertaken by students in solving problem.

3.1. Specializing by Student of High Mathematics ability (S1)
Subject S1 was able to describe the known, the way to determine the solution and try to look at the other examples. Subject S1 observed cases by observing and counting the number of border tiles that required for a garden. S1 tried some specific case by drawing diagram until the length of the garden 12 meter. In the interview, it will be known student’s specializing.

Q: Can you understand the problem easily?
S1: Not easy, but after reading this problem twice, I can understand the problem.
Q: What is the first to do when solving this problem?
S1: Finding out what information is asked and known first.
Q1: Have you given more examples when solving this problem.
S1: Yes, I tried to gave examples by drawing diagram like this until I found the answer that was 30 border tiles.
Q1: Can you explain how many border tiles are required for a garden of length “n”?
S1: The top and bottom has n+2 tiles, the sides have +1. Its mean that the number of border tiles that required for a garden with length “n” was $(n + 2) \times 2 + 2$

Figure 1. The answer of student with high mathematics ability (S1) in problem solving task

From the answer, subject S1 identified the number of border tiles in the top and bottom was n+2 which “n” is equal to the length of the garden, and the sides always have 2 border tiles

Figure 2. General rule of the number of border tiles which the length of the garden 3 meter.
Specializing helped Subject S1 understand the question by forcing S1 to clarify the idea of a problem, later on systematic specializing exposed a pattern and so gave subject S1 an idea of why the result was true.

3.2. Specializing by Student Low Mathematics ability (S2)
Unlike S1, this student S2 knows about the concept that can be used to help the problems but she didn’t know the way to determine the solution. The student didn’t show the completion process at all. She being stuck about the problem because she didn’t understand the way to determine the solution. In the interview below, it will be known by the students just answer limitedly.

Q: Can you understand the problem easily?
S2: Quite difficult, I have never encountered a problem like that.
Q: How do you determine a problem solving strategy?
S2: Reading only, I know this is about pattern, but I have no prescribed or memorized rules or methods.
Q: Have you tried to give more examples to solve this problem?
S2: Yes, but I couldn’t doing this task until finished.

Subject S2 tried some examples and she gets the number of the border tiles bigger by two every time the garden gets bigger by one. But she feel confused when determine the number of border tiles are required for a garden of length “n”. She tried by counting the perimeter of rectangular until she being stuck about the problem.
Figure 4. The answer of student with low mathematics ability (S2) in problem solving task before applying a structured-scaffolding and used specializing systematically

The specializing shown by student S2 of the interview result was known if the student had low ability to understand the problem. How to answer students who provide an answer with misconception can also indicate the low ability to specialize. Even though taking more time, by applying both a structured-scaffolding and used specializing systematically that fit the problems they succeeded in solving them.

Figure 5. The answer of student with low mathematics ability (S2) in problem solving task after applying a structured-scaffolding and used specializing systematically

The result of this research represents that student with high mathematics ability has a specializing better than the student with low mathematics ability. Specializing helped students to understand the question by forcing them to clarify the idea of a pattern. It also led students to discover the form of a pattern. Later on, systematic specializing exposed a pattern and so gave an idea of why the result was true. Testing whether that pattern was correct (it was not) involved further specializing. It is because it can be used so effectively, so easily, and in so many ways that specializing is basic to mathematical thinking [7]. The students with good specializing and done systematically can easily solve problems [3].

4. Conclusion
The lack of specializing ability can affect student learning outcomes. Put another way, by doing examples students make the question meaningful to themselves and students may also begin to see an
underlying pattern in all the special cases which will be the clue to resolving the question completely. Students with low abilities require special handling or methods and learning models tailored to the material that will be accepted students. So it can be said that the creativity of teachers such as our strategy to help student use specializing in solving problems can help students to get good learning outcomes.

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