On an Extended PCAC Relationship

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We explore a consistent way to extend the partially conserved axial vector current (PCAC) relationship and corresponding current algebra results in two strongly correlated directions: 1) towards a search for a set of systematic rules for the establishment of PCAC related relationships in a finite low momentum transfer region and for the extrapolation of the momentum transfer $q^2$ to zero when deriving the low energy PCAC results that can be compared to experimental data and 2) towards taking into account, besides the conventional one, the only other possibility of the spontaneous chiral symmetry breaking, $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$, inside a baryonic system by a condensation (in the sense to be specified in the paper) of diquarks. The paper includes investigations of a chiral Ward–Takahashi identity, the explicit chiral symmetry breaking by a finite current quark mass, the modification of the PCAC relationship and its consequences. It is shown that the signals for a hypothetical diquark condensation inside a nucleon and nucleus is observable in high precision experiments despite the fact that they may evade most of the current observations. We briefly discuss how diquark condensation could provide an answer to the question of where about of pions and quark number in a nucleon and a nucleus, which is raised in explaining puzzles in polarized pion nucleus scattering, violation of Gottfried sum rule and EMC effects.

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I. INTRODUCTION

The partially conserved axial vector current (PCAC) relationship and corresponding current algebra results are in conformity with experimental data within a few percent. This good agreement between theory and experiments is interpreted as due to an underlying approximate chiral $SU(2)_L \times SU(2)_R$ symmetry, which is explicitly broken down by up and down current quark masses of a few MeV much smaller than the hadronic mass scale of 1 GeV. The lightest hadronic particle pions are considered as, in the limit that the current masses of the up and down quarks vanish, the Goldstone bosons of a spontaneous breaking down of the above mentioned symmetry induced by a non-vanishing vacuum expectation value of the quark field bilinear operator $\bar{\psi}\psi$, namely $\langle 0 | \bar{\psi}\psi | 0 \rangle \neq 0$. Equipped with new information on the momentum transfer dependence of one of the axial vector form factors $g_A$, we provide a refined analysis of the old results, which are based on an extrapolation of the physical quantities to $q^2 \rightarrow 0$, to include a wider range of momentum transfer, and, may be more interestingly, to explore possible extensions that are observable and are nevertheless consistent with our present knowledge.

The possibility of the formation of a superconducting phase in a massless fermionic system, which is a realization of one of the possible phases in which the chiral symmetry is spontaneously broken down to an isospin symmetry, is investigated in Ref. [3] base on a 4-fermion interaction model. It is argued that the relativistic superconducting phase might be relevant to the creation of baryons in the early universe due to its quantum mechanical nature. If this scenario reflects the nature at least at a qualitative level, it would be of interest to study the possible existence of such a phase in the hadronic system at the present day condition. Since it is unlikely that the present day strong interaction vacuum at large scale is in the superconducting phase for reasons given in the following, the only regions where the superconducting phase can be found are inside a nucleon, a nucleus, the center region of a heavy ion collision, and an astronomical object like a neutron star, a quasar, etc. Albeit the possible superconducting phase in the baryon creation era of the early universe may have had disappeared entirely at certain previous time during the evolution of the universe, it is still a worthwhile effort to search for such a phase at the present time. One of the reasons is that if such a phase can be found, its properties can be studied in domestic laboratories. Its existence is a reasonable possibility since it is shown [3] that given suitable coupling constants, the Nambu Jona-Lasinio (NJL) phase in which $\langle 0 | \bar{\psi}\psi | 0 \rangle \neq 0$ changes into a superconducting phase as the baryon (or quark) density is raised. The empirical need for such an assumption will be discussed in the conclusion parts of the paper rather than in this section since most of the individual phenomenon considered has its own explanation in terms of conventional picture with various degrees of success. We are interested in a search for a consistent explanation of the observatoins. Motivated by these considerations, we explore some of the phenomenological consequences of a possibility in which the interior
of a nucleon contains diquark condensation, which is enhanced in a nucleus, that spontaneously breaks the chiral symmetry.

The model dependency of the discussion is reduced as much as possible. For that purpose, we classify the spontaneous chiral symmetry belonging to the same chain, namely $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$, into categories characterized by their order parameters introduced in Ref. [3], which are generic in nature. The NJL phase is characterized by a non-vanishing $\sigma$ and vanishing $\phi^\mu$ and $\bar{\phi}^\mu$. The superconducting phase is characterized by non-vanishing $\phi^\mu$ and $\bar{\phi}^\mu$ and possibly a non-vanishing $\sigma$. The generality of the discussion allows the results to be applied to any model with a similar phase structure as the one obtained in Ref. [3].

The paper is organized in the following way. In section 2, a chiral Ward-Takahashi identity is studied by presenting the main results. Section 3 deals with the perturbation of a small current quark mass term in the Lagrangian. Under the assumption that there is a diquark condensation inside a nucleon, we investigate in section 4 the necessity and the consequences of a modification of the PCAC relation and related current algebra results. A comparison with experimental data is made. Section 5 contains a summary and an outlook.

II. THE CHIRAL $SU(2)_L \times SU(2)_R$ WARD-TAKAHASHI IDENTITY

The axial vector current vertex $i A^{5a}_\mu (p', p)$ between single quark states is written as

$$i A^{5a}_\mu (p + \frac{q}{2}, p - \frac{q}{2}) = i \frac{1}{4} \gamma_\mu \gamma^5 \tau^a O_3 + \Gamma^{5a}_\mu (p + \frac{q}{2}, p - \frac{q}{2}),$$

(1)

where the initial and final state (8-component) quark spinors are suppressed and $q_\mu$ stands for the 4-momentum transfer. Here $O_3$ and $O^{(+)}$ in the following are Pauli matrices acting on the upper and lower 4-components of the 8-component spinor $\Psi$ and $\tau^a (a = 1, 2, 3)$ is one of the Pauli matrices acting on the flavor indices of $\Psi$. The radiative part of the axial vector current vertex $\Gamma^{5a}_\mu$ satisfies the chiral Ward-Takahashi identity

$$q^\mu \Gamma^{5a}_\mu (p + \frac{q}{2}, p - \frac{q}{2}) = -\frac{i}{4} (\Sigma \gamma^5 \tau^a O_3 + \gamma \gamma^5 \tau^a O_3 \Sigma),$$

(2)

where $\Sigma = \sigma - \gamma_5 A_\mu O^{(+)} + \gamma_5 \Phi_\mu, \Phi_\mu = \phi_\mu - \bar{\phi}_\mu = 0$ and the superconducting phase where $\phi_\mu \neq 0, \bar{\phi}_\mu \neq 0$ and possibly $\sigma \neq 0$, Eq.2 implies that $\Gamma^{5a}_\mu$ contains a massless Goldstone boson pole due to the fact that its right hand side (r.h.s.) is finite in the $q^\mu \rightarrow 0$ limit. The appearance of this massless pole in the physical excitation spectrum following the spontaneous chiral symmetry breaking is required by the Goldstone theorem.

The chiral Ward-Takahashi identity Eq.2 can determine various properties of the chiral Goldstone boson. We shall consider the case in which $\phi_\mu^c = \bar{\phi}_\mu, \phi_\mu^c \neq 0, \mu^c \neq 0$ and $\sigma = 0$ that has not been discussed in the literature to demonstrate some of the elementary features of the superconducting phase related to the chiral symmetry. Eq.2 becomes

$$q^\mu \Gamma^{5a}_\mu (p + \frac{q}{2}, p - \frac{q}{2}) = -\frac{i}{2} (\gamma \cdot \phi_\nu A^\nu O^{(+)}) \tau^a$$

(3)

in such a case. The propagator of the Goldstone diquark is defined as $G_\delta (q) = -i \delta^{\mu\nu} / \Delta (q)$. The denominator $\Delta (q)$ can be generally parameterized as

$$\Delta (q) = q^2 + a_\delta (\phi \cdot q)^2 / \phi^2$$

(4)

if we choose the phases of $\phi_\mu^c$ and $\bar{\phi}_\mu$ such that $\phi_\mu^c = -\bar{\phi}_\mu$. With the following ansatz, namely

$$t^{\nu}_{\mu \nu} (q \rightarrow 0) = -\frac{\phi_\nu^\mu \phi_\mu^\nu}{\phi^2} \Gamma^{5a}_\nu (q \rightarrow 0) = -\frac{\phi^\mu_\nu \phi^{\nu \mu}}{\phi^2},$$

(5)

and with the definition of the Goldstone diquark-quark vertices given by

$$D^{5a}_{\mu} (p + \frac{q}{2}, p - \frac{q}{2}) = -\frac{i}{2} \delta_{q \mu} \gamma_\mu \tau^a A_\mu O^{(-)}, D^{5a}_{\nu} (p + \frac{q}{2}, p - \frac{q}{2}) = \frac{i}{2} \delta_{q \nu} \gamma_\mu \tau^a A_\mu O^{(+)};$$

(6)
we can obtain the value of $a_5$, the Goldstone diquark-quark coupling constant $g_{\delta q}$ and the Goldstone diquark decay constant $f_\delta$ by separating out the massless pole in $\Gamma_{\mu}^{5a}$, which can be approximately evaluated by using an one loop perturbation calculation. The result is

$$
(\Gamma_{\mu}^{5a})_{\text{pole}} = -2D_\delta^b \left( p + \frac{q}{2}, p - \frac{q}{2} \right) \frac{-i\gamma^\alpha}{\Delta(q)} \frac{i}{2} \left( k - \frac{q}{2} \right) \frac{i}{2} \int \frac{d^4k}{(2\pi)^4} \tilde{D}_\delta^b \left( k - \frac{q}{2}, k + \frac{q}{2} \right) \frac{\gamma \cdot (k - \frac{q}{2}) - \Sigma}{\gamma \cdot (k - \frac{q}{2}) - \Sigma} - H.c.,
$$

where $H.c.$ stands for the hermitian conjugation, $Tr$ stands for the trace operation in the Dirac, flavor, color and upper and lower 4-component spaces of $\Psi$ and the color indices are suppressed. It can be noticed that the symmetry factor for the Feynman diagram is different from the 4-component theory for fermions. After a lengthy process of evaluating the trace and performing the 4-momentum integration, the above equation together with Eq.\[3\] determine the values of $a_5$, $g_{\delta q}$ and $f_\delta$ as functions of $\phi^2$. The numerical values for them are shown in Figs. 1-3. A Goldberger-Treiman relation for single quarks in the superconducting phase exist; it can be expressed as $g_{\delta q}f_\delta = \sqrt{\phi^2}$, which also defines the scale of $f_\delta$.

III. SMALL CURRENT QUARK MASS PERTURBATION

A finite current quark mass that explicitly breaks the chiral $SU(2)_L \times SU(2)_R$ symmetry has non-trivial physical consequences. For simplicity, we shall assume that both the up and down current quarks have an identical mass $m_0$. Some of the consequences of a finite mass for light quarks can be studied base on the Ward-Takahashi identity given by Eq.\[3\] with the term corresponding to the divergence of the axial vector current operator taken into account, namely,

$$
q^\mu \Gamma_{\mu}^{5a} \left( p + \frac{q}{2}, p - \frac{q}{2} \right) = \frac{i}{2} m_0 F_\pi \gamma^5 \tau^a O_3 - \frac{i}{4} \left( \Sigma \gamma^5 \tau^a O_3 + \gamma^5 \tau^a O_3 \Sigma \right),
$$

and

$$
\frac{i}{4} F_\pi \gamma^5 \tau^a O_3 = \int d^4x_1 d^4x_2 e^{i\phi(x_1 - p - q/2)} - e^{i\phi(x_2 - p - q/2)} \langle 0 | T \psi(x_1) \bar{\psi}(x_2) \rangle | 0 \rangle | \text{amp}.\]

Here "T" stands for the time ordering, $j^{5a} = \frac{1}{4} \bar{\psi} \gamma^5 \tau^a O_3 \psi$ and the subscript "amp" denotes the amputation of external fermion lines. $F_\pi$ is a scalar function.

In the NJL phase, if the assumption that $F_\pi(q^2 = 0)$ is dominated by the pion pole, the mass of the pion moves to a finite value, provided that to the first order in $m_0$, the Goldberger-Treiman relation $g_{\delta q}f_\pi = \sigma$ and the Gell-Mann, Oakes and Renner (GOR) relation $f_\delta m_0^2 = \frac{-1}{2} m_0 \langle 0 | \bar{\psi} \psi | 0 \rangle$ hold. This can be checked by evaluating the r.h.s. of Eq.\[3\]. The $\Sigma$ term on the r.h.s. of Eq.\[3\] has a simple diagonal form

$$
\Sigma = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix}
$$

in the NJL phase. It can be shown that Eq.\[3\] is

$$
q^\mu \Gamma_{\mu}^{5a} \left( p + \frac{q}{2}, p - \frac{q}{2} \right) = \frac{i}{2} \left( \frac{m_0 g_{\delta q}}{2} D_\pi(q^2) \langle 0 | \bar{\psi} \psi | 0 \rangle - 1 \right) \gamma^5 \tau^a O_3,
$$

where

$$
D_\pi(q^2) = \frac{1}{q^2 - m_0^2} + \tilde{D}(q^2).
$$

$\tilde{D}(q^2)$ is a smooth function of $q^2$ at small $q^2$ (namely, $q^2 \approx m_0^2$). If the above mentioned Goldberger-Treiman relation and the GOR relation hold and $D(q^2 = 0) \ll 1/m_0^2$, the r.h.s. of Eq.\[11\] vanishes at $q^2 = 0$. Therefore $\Gamma_{\mu}^{5a}$ is regular at $q^2 = 0$. It implies the disappearance of the massless pole in $\Gamma_{\mu}^{5a}$ as well as in the physical spectrum.

In the superconducting phase where $\phi^2 \neq 0$ and possibly $\sigma \neq 0$, the situation is more complicated. In this case, $\Sigma$ term on the r.h.s. of Eq.\[3\] takes the following form
\[
\Sigma = \left( \gamma \cdot \phi_c \gamma^5 A^c - \gamma \cdot \phi^a \gamma^5 A^a \right). \tag{13}
\]

A finite \( m_0 \) for the current quarks in this case does not render the r.h.s. of Eq. \( 8 \) vanish when \( q^\mu \to 0 \). There always remains a finite strength of the massless excitation in \( \Gamma_{\mu}^a \) as long as the mass term is of the form \( \frac{1}{2} m_0 \bar{\psi} \psi \). As a consequence, there are massless excitations in the superconducting phase even if the chiral \( SU(2)_L \times SU(2)_R \) symmetry is explicitly broken by \( m_0 \). There is no GOR type of relation for the Goldstone diquark in the superconducting phase. In addition, certain mixing between two sets of the auxiliary fields \( \pi^a \) and \( (\delta_{\mu}^a, \delta_{\tau}^a) \) is needed to represent the Goldstone boson excitation in the superconducting phase even when \( m_0 \neq 0 \) and \( \sigma = 0 \). The above mentioned mixing provides us with one of the motivations for the following extension of the PCAC relationship.

**IV. THE EXTENSION OF THE PCAC RELATION AND CURRENT ALGEBRA RESULTS**

The present day large scale strong interaction vacuum is expected to be in the NJL phase. There are a few obvious reasons for this statement. First, the overwhelming color confinement at the present day condition prevents the large scale superconducting phase in the strong interaction vacuum scenario from being acceptable. Second, the long range strong interaction force in the superconducting phase due to the massless Goldstone diquark excitation inside the hadronic system is absent in the experimental observations. However, localized superconducting phases inside a baryonic system are not implausible possibilities.

Due to the above consideration and the one given in the introduction, we explore in this section a subset of the phenomenological consequences of the possibility in which the interior of a nucleon contains diquark condensation that spontaneously breaks the chiral \( SU(2)_L \times SU(2)_R \) symmetry down to an isospin \( SU(2)_V \) symmetry. The study of Ref. \([8]\) indicates that the model Lagrangian introduced in Ref. \([3]\) indeed support such a scenario when the coupling constant \( \alpha_3 \) is sufficiently large.

The on shell matrix elements of the axial vector current operator between single nucleon states can be parameterized as

\[
\langle p' | A_{\mu}^a(0) | p \rangle = \bar{U}(p') \left( g_\Lambda \gamma_\mu + g_P q_\mu + g_T \frac{i \sigma_{\mu\nu} q^\nu}{2 m_N} \right) \gamma^5 \pi^a U(p), \tag{14}
\]

with \( q_\mu = (p' - p)_\mu \), \( m_N \) the mass of a nucleon and \( U(p) \) the 4-component nucleon spinor. The longitudinal piece \( g_P q_\mu \) on the r.h.s. of Eq. \( 14 \) is dominated by the contribution of the Goldstone bosons of the spontaneous chiral symmetry breaking. If only the NJL phase is considered, \( g_P \) is given by

\[
g_P(q^2) = -2 \frac{g_{\pi N}(q^2) f_\pi(q^2)}{q^2 - m_\pi^2} + \bar{g}_P(q^2), \tag{15}
\]

with \( \bar{g}_P(q^2) \) the residue term and \( g_{\pi N}(q^2) f_\pi(q^2) \) a slow varying function of both \( m_\pi^2 \) and \( q^2 \). If, however, the assumption that there is a diquark condensation inside a nucleon is made, there would be another longitudinal term in the matrix elements of the axial vector current operator due to the Goldstone diquark excitation inside that nucleon. The expression for \( g_P \) has to be modified to

\[
g_P(q^2) = -2 \left( \frac{g_{\pi N}(q^2) f_\pi(q^2)}{q^2 - m_\pi^2} + z_\delta g_{N \delta}(q^2) f_\delta(q^2) \eta(q^2) \right) + \bar{g}_P(q^2) \tag{16}
\]

after considering this additional excitation. Here \( \eta(q^2) \) is related to the propagator of the Goldstone diquark excitation and \( z_\delta \) is a constant. Similar to the pion, we introduce \( g_{N \delta} \) as the Goldstone diquark-nucleon coupling constant and \( f_\delta \) as the Goldstone diquark decay constant. Albeit there is massless excitation in \( \Gamma_{\mu}^a \), there is no pole behavior in \( \eta(q^2) \) in the small \( q^\mu \) region due to the fact that a Goldstone diquark carries color so that it is confined inside the nucleon.

The PCAC relation is given by

\[
\partial^\mu A^a_\mu = - f_{\pi} m_\pi^2 \phi_\pi^a. \tag{17}
\]

It can be regarded as a definition of the pion field in the low energy regime (when going off the pion mass shell). Taking the matrix elements of Eq. \( 17 \), it can be shown that this definition is inconsistent with Eq. \( 13 \) due to the
additional term added to \( g_P \). We have at least two choices. The first one is to reject Eq.16, which we shall not do in this paper. The second one is to modify Eq.17 when its matrix elements are taken. It can be specified as

\[
\langle p' | \partial^\mu A_\mu^a | p \rangle = -\langle p' | \{(f_\pi m_\pi^2 \phi_\pi^a + f_\mu m_\mu^2 \phi_\mu^a)\} | p \rangle,
\]

with \( s_8 \sim m_\pi^2 \) a parameter proportional to \( m_0 \) and \( \phi_8^a \) a pseudo-scalar \( \phi_8 \) driving field for the Goldstone diquark excitation inside the nucleon.

The r.h.s. of Eq.18 is assumed to be dominated by the chiral Goldstone boson contributions in the sequel. The following relations hold, namely,

\[
g_P(q^2) = -2 \frac{m_N g_A(q^2)}{q^2 - m_\pi^2},
\]

\[
m_N g_A(q^2) = g_{\pi N}(q^2)f_\pi(q^2) + (q^2 - m_\pi^2) \frac{8\delta}{m_\pi^2} g_{\delta N}(q^2)f_\delta(q^2)\eta(q^2),
\]

when we require that the coefficients of \( m_\pi^2 \) and \( q^2 \) vanish separately while assuming that \( (q^2 - m_\pi^2)g_P(q^2) \) and \( g_A(q^2) \) are slow varying functions of both \( m_\pi^2 \) and \( q^2 \). The slow varying assumption of these quantities together with the modified PCAC relation, Eq.18, imply that the residue term \( \tilde{g}_P(q^2) \) in Eq.19 is unimportant in the small \( q^2 \) regime. Eqs. 19, 20 are consistent with Eq.19 provide that \( z_8 = s_8/m_\pi^2 \). The modified Goldberger-Treiman relation is obtained when \( q^2 = m_\pi^2 \) is assumed on the r.h.s. of the second equation of Eqs.19, 20 and \( q^2 \to 0 \) is taken on its left hand side (l.h.s.), namely,

\[
m_N g_A(0) = g_{\pi N}(m_\pi^2)f_\pi(m_\pi^2) + \lim_{q^2 \to m_\pi^2}(q^2 - m_\pi^2)z_8 g_{\delta N}(q^2)f_\delta(q^2)\eta(q^2).
\]

Assuming the results of Ref.1 can be used in the time like region for \( q_{\mu} \), the extrapolation of \( g_A \) from \( q^2 = m_\pi^2 \) to \( q^2 = 0 \) introduces an error of order \( m_\pi^2/M_A^2 \sim 1\% \) with \( M_A \sim 1 GeV \).

The Goldberger-Treiman relation is satisfied to about 5% in experimental observations. This good agreement indicates the validity of the various assumptions combined made above and a rather small value of \( \lim_{q^2 \to m_\pi^2}(q^2 - m_\pi^2)z_8 g_{\delta N}(q^2)f_\delta(q^2)\eta(q^2) \). The \( \eta(q^2) \) term on the pion mass shell actually vanishes since \( \eta(q^2) \) does not have a pole in the low \( q^2 \) region. However, a small value for the \( \eta(q^2) \) term does not follow from the success of the Goldberger-Treiman relation.

The effects of the Goldstone diquark inside a nucleon, if exist, might be comparatively large in the kinematic region off the pion mass shell. Current experimental data considered do not allow any conclusive statement to be made on this point. The value of \( g_P \) can be compared to the PCAC value (Eq.15) to detect the possible effects of the Goldstone diquark. Experimental determinations of \( g_P \) for a nucleon in muon capture experiments involving light nuclei show a systematic increase (of order as large as 100%) of the value of \( g_P \) from its PCAC one \( c_0 \). The world average value of \( g_P \) obtained from the muon capture experiments on a hydrogen is close to the one given by the PCAC one \( c_0 \). However, the interpretation of the results is not unambiguous (Ref.1 and Gnitro and Truol in Ref.16). There are two recent measurements of the muon capture rate on the deuterium. The central value of the first measurement \( c_1 \) implies a value of \( g_P \) smaller than the PCAC one \( c_0 \). The central value of the second measurement \( c_2 \) implies a value of \( g_P \) close to the PCAC one \( c_0 \). The problems related to the value of \( g_P \) are not yet completely settled \( c_0 \). Polarized \( \beta \) decay experiments offer alternative means of measuring the value of \( g_P \) in a different range of \( q^2 \). The experiments are difficult but in principle possible. The \( q^2 \) dependence of \( g_A(q^2) \) and \( g_{\pi N}(q^2)f_\pi(q^2) \) differs from each other (see Eqs.19, 20) if there are Goldstone diquark excitations inside a nucleon. More and better experimental data are needed to investigate such a deviation.

Theoretically, the agreement of the PCAC value for \( g_P \) and the experimentally measured one is expected for a nucleon in free space. This is because Eq.10 implies the deviation of \( g_P \) from the PCAC one is related to the deviation of the value of \( g_A, m_N \) and \( m_\pi \) from the experimentally observed ones, which is not true. Indeed, a recent experimental study \( c_0 \) provides a strong support \( c_0 \) of Eq.10 within momentum transfer of \( 0 < -q^2 < 0.2 GeV^2 \). Therefore, the system in which a deviation of the strength of longitudinal modes in the axial vector current operator from the PCAC one can be observed is inside a nucleus with relatively large number of nucleons. In these systems, the coupling between the pions outside of the nucleons and the Goldstone diquark excitations inside the nucleons could render the properties of pions to change to such a degree that a deviation from PCAC value can be observed.

The modification of some of the current algebra results related to PCAC due to a possible Goldstone diquark excitation inside a nucleon can also be investigated. The Adler-Weisberger sum rule and the nucleon \( \Sigma_N \) term will be studied in this paper base on a Ward identity involving the axial vector current operator \( c_1, c_2 \). In order to obtain
useful information, the low lying longitudinal excitation contributions and the rest part of the axial vector current operator inside a time ordered product are separated in the following way

\[
\langle T \left( \ldots A_u^a \ldots \right) \rangle = \langle T \left( \ldots A_u^a \ldots \right) \rangle + \partial_\mu \langle T \left( \ldots f_\pi \phi_\pi^a \ldots \right) \rangle + \partial_\mu \langle T \left( \ldots z_\phi \phi_\phi^a \ldots \right) \rangle,
\]

(22)

with the second and the third terms on the r.h.s. the longitudinal parts of \( A_u^a \), which is dominated by the low lying chiral Goldstone boson contributions, and \( A_u^a \), which is expected to change slowly with the momentum transfer \( q^\mu \) when its matrix elements are taken between nucleon states, containing the rest part of \( A_u^a \). The matrix elements are evaluated on the pion mass shell [21]. Those of \( A_u^a \) that connect nucleon states by a gradient–coupling [22] are then extrapolated to the kinematic point where \( q^2 = 0 \) in order to compare with experimental data by taking the advantage that they are slow varying function of \( q^2 \). The error of the extrapolation is expected to be of order \( O(\frac{m_\pi^2}{M_\pi^2}) \sim 1\% \). There are contributions from other off (nucleon) shell terms and baryonic excitations in the intermediate states; they are not investigated in detail here. The modified Adler-Weisberger sum rule can be shown to have the following form

\[
\delta_A^2(q^2 = 0) = 1 - 2 \frac{f_\pi^2(m_\pi^2)}{\pi} \int_{m_\pi}^{\infty} dv \frac{g_{\pi N}^{\pi N}(v) + g_{\pi N}^{\pi N}(v)}{(v^2 - m_\pi^2)^{1/2}} - 2 \lim_{q^2 \to m_\pi^2} (q^2 - m_\pi^2)^2 f_\pi^2(q^2) \lim_{v \to 0} \lim_{\frac{z_\pi}{v} \to 0} G_{\pi N}(0, q^2, q^2),
\]

(23)

where \( G_{\pi N}(\cdot, \cdot, \cdot) \) is related to the isospin odd forward Goldstone diquark-nucleon scattering amplitude, which is driven by \( \phi_\pi^a \) without the amputation of the diquark lines.

The value of \( g_A \) from nuclear \( \beta \) decay experiments is \( 1.261 \pm 0.004 \) [22]. The value of the same quantity obtained from the Adler-Weisberger sum rule using pion nucleon scattering cross section (and after estimating other corrections) is about \( 1.24 \pm 0.02 \) [10]. These two numbers, with a central \( (g_A - 1) \) value deviation of order \( 10\% \), barely overlap. Again, the contribution of the second term on the r.h.s. of Eq. [23] is expected to be zero on the pion mass shell. Therefore the good agreement of the Adler-Weisberger sum rule with the observation does not necessarily constitute a fact that is against the additional terms added in Eqs. [13] and [22].

The expression for the nucleon \( \Sigma_N \) term, which is of order \( m_0 \), is modified to

\[
\Sigma_N = f_\pi^2(m_\pi^2)T_{\pi N}^{(+)}(0, 0, m_\pi^2, m_\pi^2) + \lim_{q^2 \to m_\pi^2} (q^2 - m_\pi^2)^2 f_\pi^2(q^2) \frac{z_\pi^2}{2} G_{\pi N}^{(+)}(0, 0, q^2, q^2),
\]

(24)

with \( G_{\pi N}^{(+)} \) related to the isospin even forward Goldstone diquark-nucleon scattering amplitude without the amputation of the diquark lines. The second term on the r.h.s. of Eq. [24] vanishes on the pion mass shell. Experimental verifications of Eq [24] (without the second term on its r.h.s.) have so far been unsuccessful without assuming certain strange quark content of a nucleon, which is discouraged by the OZI rule [23]. A recent review of the nucleon \( \Sigma_N \) problem can be found in Ref. [24]. There is still a sizable discrepancy [20] of order \( 25\% \) between the one obtained from the pion-nucleon scattering data (45MeV) and the one obtained from other available sources (35MeV). The problem is still not well understood.

It should be pointed out here that an additional source of correction need to be considered to relate \( \Sigma_N \), which is proportional to the scalar density of the nucleon on the Cheng-Dashen point [20], to \( \sigma_N \) related to the scalar density of a nucleon. In the Hartree-Fock approximation, the static scalar density measured by \( \sigma \) is free of radiative corrections due to the fact that it satisfies a “gap equation”, which self-consistently adjust the radiative corrections to \( \sigma \) to zero (cancel) by changing its value. This can be proven in the case that \( \sigma \) is space-time independent [2]. We expect it to be true also for \( \sigma_N \). This statement is not true if the matrix elements of the scalar density operator \( \hat{\bar{\Psi}} \Psi \) between states of different 4-momentum are taken. Various corrections due to the interaction have to be considered.

\[ \Sigma_N = \sigma_N(m_\pi^2) \]

term is written as \( \Sigma_N = \sigma_N(0) + \Delta_N \), \( \sigma_N(0) = 25MeV \) can be obtained from the baryon spectrum [20]. \( \Delta_N \) contains various corrections to the extrapolation. If there is diquark condensation in a nucleon, there would be contributions to \( \phi^2 \) coming from the interaction terms. The existence of such a term can be demonstrated at the quark level by considering a second order correction to \( \sigma_N \) due to the interaction, which has the generic form \( \delta \sigma_N(0)^2 \sim \langle \Gamma^p \hat{T} \langle \hat{\bar{\Psi}} \Psi \rangle \langle \hat{\bar{\Psi}} \Psi \rangle \hat{T} \Gamma \hat{\bar{\Psi}} \Psi \rangle \rangle \rho \), with \( \Gamma \) and \( \hat{\bar{\Psi}} \) a pair of the interaction vertices. This term contains \( \phi^2 \) contributions if there is diquark condensation inside a nucleon [25]. Detailed evaluation of these terms can not be proceeded before a specific model is given. Here, we simply parameterize the effects of \( \phi^2 \) as \( \Sigma_N = \sigma_N(0) + \Delta_N(0) + \beta_N \phi_N^2 \), where \( \Delta_N(0) \) is related to the total correction that has already been considered in the literature (see, e.g., Ref. [24]). The new term depending on \( \phi_N^2 \) is due to the possible diquark condensation inside a
nucleon with an average strength measured by $\sqrt{\overline{\sigma_N}}$. It is unclear at present whether the additional term increases ($\beta_N < 0$), decreases ($\beta_N > 0$) or even eliminates the above mentioned discrepancy.

V. SUMMARY AND OUTLOOK

A natural way of extending the PCAC relationship beyond the conventional one is presented in this study. For a relativistic fermionic system, the extension presented here is unique if one assumes that 1) in the low momentum transfer region ($|q^2| < M_A^2 \approx 1$ GeV) the matrix element of the axial vector current operator is dominated by the collective excitations (quasi–Goldstone bosons) related to the spontaneous breaking down of the (approximate) chiral $SU(2)_L \times SU(2)_R$ symmetry and 2) the contribution of those less collective hadronic excitations, which have masses of order $1 \text{ GeV}$ or larger, are relatively small.

From the analysis given above, it can be seen that the effects of a possible Goldstone diquark mode inside a nucleon are suppressed in the kinematic regime where pion is on its mass shell. The effects of such an excitation mode, if exist, could be revealed in the kinematic region off the pion mass shell, where the simple pion pole behavior of certain observables is modified in such a way that can not be explained by conventional pion pole saturation assumption for the divergence of the axial vector current operator. They can be observed in, e.g., high precision observables is modified in such a way that can not be explained by conventional pion pole saturation assumption for the divergence of the axial vector current operator. They can be observed in, e.g., high precision $\beta$ decay [16,27], $\mu$ capture experiments [10,13,15] and in experiments that can compare the momentum transfer dependencies of $g_A$, which is largely known, and $g_{\pi NN}\phi$ (see Eq. 24).

For a large nucleus, in which the nucleons are close together, the effects of the diquark condensation and Goldstone diquarks are expected to be enhanced. The enhancement of the diquark condensation can be attributed to the increase of the volume and baryon density of the system; the enhancement of the effects of the Goldstone diquarks is due to their coupling to the pions outside the nucleons. These effects could shed some light on the question [29] of missing pions, quark number depletion in a nucleus and violation of Gottfried sum rule [30] in a nucleon. Due to the novel nature of the superconducting phenomena in a relativistic massless fermion system, a quantitative study depends first on a model for a nucleon, which is still under investigation, we provide a qualitative picture in the following since if we make the assumption that there is a diquark condensation in a nucleon which is enhanced in a nucleus then the general trend of these phenomena follow quite naturally. Detailed study will be given elsewhere.

The lack of polarization effects observed in Refs. [31] and [32] can be explained, together with other many body effects, by a reduction of pionic effects from its vacuum ones due to an enhancement of contribution of the second term on the r.h.s. of Eq. 24 resulting from the increased Goldstone diquark excitation strength inside a nucleus. The missing polarization phenomenon observed in pion nucleus scattering process is due to cancellation between various components of nucleon–nucleus scattering [33], it is premature to make any definite statement at this point. It is the question of overall consistency rather than fitting of a single experimental observation that prevent us from doing so here.

The depletion of quark number can be related to the missing or spreading of the isoscalar part of the electrical charge of the partons in the superconducting phase discussed in this paper [34]. This phenomenon is known in the study of superconducting condensed matter system [35]. In such a case, the electric charge of some of the partons, which are the up and down quarks, have only isovector charge $\pm 1/2$ in the limit of large strength of diquark condensation. Therefore if there is a diquark condensation that generates a localized superconducting phase in a nucleon, the $F_2(x)$ structure function should be written as

$$F_2(x) = \sum_i \frac{Q_i^2x}{f_i(x)},$$  \hspace{1cm} (25)

with

$$\frac{Q_i}{f_i} = \frac{1}{6} \pm \frac{1}{2},$$  \hspace{1cm} (26)

here $0 \leq \alpha_i \leq 1$ characterizes the strength of the superconducting phase. In case of strong superconductivity, which means some of the $\alpha_i = 0$, $F_{2N}^x(x) \sim F_{2N}^y(x)$. The experimental measurement of $F_{2N}^x(x)$ and $F_{2N}^y(x)$ in the deep inelastic scattering experiments shows such a tendency manifests in the violation of Gottfried sum rule [36]. One of the interpretation is that there is an isospin violation in the sea [35] due to pionic collective excitation. If we use the normal electrical charges for the up and down quark, then there is a reduction of $f_i(x)$, which is interpreted as a violation of the Gottfried sum rule.

In a large nucleus, the effects of diquark condensation are expected to be enhanced; this in general produces the depletion of quark number observed in the EMC effects in a nucleus, which is related to a (enhanced) violation of
Gottfried sum rule in the picture given here, since the antiquark components in a nucleus is known to be small in experimental observations \[36,37\]. It is generally accepted that the EMC effects are still not well understood \[38\].

A detailed study of the effects of these soft longitudinal modes and that of spreading of the isoscalar part of the charge of constituent quarks is beyond the scope of this paper. The above study shows that a coherent picture seems to be emerging if the assumption of a diquark condensation is made. It is an interesting topic to be explored.

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Figure 1: The $\phi^2$ dependence of $a_\delta$. $\Lambda$ is the chiral symmetry breaking scale. In order to obtain a smooth curve, the sharp cut off in the Euclidean momentum space integration is replaced by a smooth one. The functional dependence of the smooth cut off $F(p/\sqrt{\phi^2})$ is plotted on the same graph. Since $a_\delta > 1$, a Goldstone diquark can only propagates outside of the cone $\theta_0 = \cos^{-1}\sqrt{1/a_\delta}$ in the direction of $\vec{\phi}$ in the frame where $\phi^0 = 0$.

Figure 2: The $\phi^2$ dependence of the quark-diquark coupling constant $g_{\delta q}$. The same smooth Euclidean momentum space cut off as the one used in Fig. 1 is used.

Figure 3: The $\phi^2$ dependence of the Goldstone diquark decay constant $f_\delta$. The same smooth Euclidean momentum space cut off as the one used in Fig. 1 is used.
