Reheating constraints on Tachyon Inflation

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Abstract

Tachyon inflation is one of the most attractive models of noncanonical inflation motivated by string theory. In this work we revisit the constraints on tachyon inflation with inverse cosh potential and exponential potential considering reheating. Although the phase of reheating is not well understood, it can be parameterized in terms of reheating temperature $T_{re}$, number of e-folds during reheating $N_{re}$ and effective equation of state during reheating $w_{re}$, which can be related to the parameters of the tachyon potential, spectral index $n_s$ and tensor-to-scalar ratio $r$. For various reheating scenarios there is a finite range of $w_{re}$ and the reheating temperature should be above electroweak scale. By imposing these conditions, we find that both the inverse cosh potential and exponential potential are disfavored by Planck observations. We also find that $w_{re}$ for both these potentials should be close to 1 to satisfy Planck-2015 joint constraints on $n_s$ and $r$.

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1. INTRODUCTION

Inflation not only provides solution to the problems of big bang cosmology, but also generates seeds for CMB anisotropy and structures in the universe. It predicts adiabatic, nearly scale invariant and Gaussian perturbations which are confirmed by various CMB observations like COBE, WMAP and Planck. In standard scenario the potential energy of a scalar field, named as inflaton, dominates the energy density of universe for a short period of time that causes the rapid expansion of the universe. The inflaton field \( \phi \), whose dynamics can be determined by the Klein-Gordon action, rolls slowly through its potential. During this period the quantum fluctuations in the scalar field, which are coupled to the metric fluctuations, generate the primordial density perturbations. There are also vacuum fluctuations of the metric during inflation that generate primordial gravitational waves (tensor perturbations). The power spectra of the primordial density perturbations and tensor perturbations generated during inflation depend on the potential of the inflaton, which can be obtained from particle physics models or string theory.

There are some alternative to the standard inflationary models, named as K-inflation, where the inflation is achieved by non-standard kinetic term in the Lagrangian of the inflaton. One attractive and popular model of K-inflation is tachyon inflation, which can be realized in Type-II string theory where the tachyon signals the instability of unstable and uncharged D-branes of tension \( \lambda \). In this case the tachyon action is of the Dirac-Born-Infeld form (see for different approaches). It was also pointed out by Sen that the rolling tachyon can have the dust like equation of state, which raised the possibility of tachyon providing a unified description of inflation and dark matter. However, tachyon as inflaton has been criticized as the string theory motivated values of the parameters are incompatible with the slow-roll conditions and observed amplitude of the scalar perturbations. Despite the criticism and regardless the string theory motivation, tachyon inflation has been studied phenomenologically as an example of noncanonical inflation model. In it was shown that the consistency relation in case of tachyon inflation is different than the standard single field inflation and an observational signature of this deviation can lead to distinguish between the two models.

Inflation leaves the universe in a cold and highly nonthermal state without any matter content. Universe needs to be in a thermalized state at a very high temperature for hot big bang picture. This is achieved by reheating which is a transition phase between the end of inflation and the start of radiation dominated era. During reheating the energy density of inflaton is converted to the thermal bath, at a reheating temperature \( T_{re} \), that fills the universe at the beginning of radiation dominated era (see for detailed review). In simplest case reheating can occur via perturbative decay of inflaton into standard model matter particles, while inflaton is oscillating around the minimum of its potential, but this scenario was criticized as it does not take into account the coherent nature of the inflaton field. In other scenarios reheating is preceded by phase of preheating during which the particle production occurs via non-perturbative processes such as parametric resonance decay, tachyonic instability and instant preheating. Preheating leaves the universe in highly nonthermal state which is thermalized by scattering and the universe is left with a blackbody spectrum at a temperature \( T_{re} \), named as reheating temperature, which corresponds to the temperature at the beginning of the radiation dominated epoch.
It is difficult to constrain the reheating temperature $T_{re}$ from CMB and LSS observations, but it is considered that $T_{re}$ should be above the electroweak scale: so that the weak scale dark matter can be produced. If we adopt a conservative approach, $T_{re}$ must be greater than 10MeV for the big bang nucleosynthesis. The upper bound on reheating temperature is obtained by assuming reheating to be an instantaneous process which reheats the universe to the scale of inflation i.e. $10^{16}$GeV by considering the Planck upper bound on tensor-to-scalar ratio. The evolution of the energy density of the cosmic fluid during reheating depends on effective equation of state $w_{re}$, which, in general, depends on time. Its value at the end of inflation is $-\frac{1}{3}$ and reaches $\frac{1}{3}$ at the beginning of radiation dominated epoch. In case where the reheating occurs due to perturbative decay of massive inflaton, the effective equation of state during reheating $w_{re}$ is $w_{re} = 0$ and for instant reheating $w_{re} = \frac{1}{3}$. A numerical study performed for various reheating scenario [34] shows that $w_{re}$ can vary between 0 to 0.25.

Another important parameter to describe reheating is its duration, which can be defined in terms of number of e-foldings $N_{re}$ from the end of inflation to the onset of radiation dominated epoch. In general, this is incorporated by providing a range for $N_{k}$, the number of e-folds from the time when a Fourier mode $k$ corresponding the the horizon size of our observable universe leaves the inflationary horizon to the end of inflation. $N_{k}$ depends on the potential of inflaton and it should be between 46 to 70 to solve the horizon problem. The upper bound on $N_{k}$ comes from assuming instantaneous reheating and the lower bound arises from considering reheating temperature at electroweak scale.

These three parameters of reheating can be used to obtain constraints on various inflationary models [35–37]. Demanding that the equation of state during reheating lies between 0 and 0.25, one can get bounds on the spectral index $n_s$ and $N_{k}$, which translates to bounds on tensor-to-scalar ratio $r$. In this work we use this approach to constrain tachyon inflation with inverse cosh and exponential potentials. We obtain $T_{re}$ and $N_{re}$ as a function of spectral index as in [37] by assuming $w_{re}$ to be constant during reheating. We obtain the allowed regions for these potentials in $n_s - r$ plane for various values of $w_{re}$. We also use Planck-2015 1σ bounds on $n_s$ and $r$ to determine the equation of state during reheating for these potentials.

The work is organized as follows: in Sec. 2 we give a brief review of tachyon inflation providing the expressions for $N_{k}$, scalar power spectrum, spectral index $n_s$ and tensor to scalar ratio $r$. In Sec. 3 we provide the derivation for reheating temperature $T_{re}$ and number of e-folds during reheating $N_{re}$ for constant effective equation of state $w_{re}$. In Sec. 4 we obtain $T_{re}$, and $N_{re}$ for tachyon inflation with inverse cosh and exponential potential for various choices of equation of state $w_{re}$ and use these three parameters to constrain tachyon inflation. In Sec. 5 we conclude our work.

2. TACHYON INFLATION

Tachyon inflation is a class of $K$-inflation models where inflation is achieved by noncanonical kinetic term. the action for tachyon inflation is given by

$$S_T = -\int d^4x\sqrt{-g}(V(T)(1 + g^\mu\nu\partial_\mu T\partial_\nu T)^{\frac{1}{2}}$$

(1)
and the metric has signature $-\,+,\,+,\,+$.
$T$ represents the tachyon field with dimension of length and $V(T)$ represents its potential. Various choices for the potential have been derived using string theory \cite{11-17}. Here we consider the inverse hyperbolic potential \cite{11, 12, 38} and exponential potential \cite{39, 40} given by

$$V(T) = \frac{\lambda}{\cosh \left( \frac{T}{T_0} \right)}$$  \hspace{1cm} (2)

and

$$V(T) = \lambda \exp \left( -\frac{T}{T_0} \right)$$  \hspace{1cm} (3)

The action for the tachyon-gravity system is given by

$$S = \int d^4x \sqrt{-g} \, \frac{R}{16\pi G} + S_T$$  \hspace{1cm} (4)

The energy-momentum tensor can be obtained by varying this action as

$$T_{\mu\nu} = -V(T) g_{\mu\nu} \sqrt{1 + \partial^\mu T \partial_\mu T} + \frac{V(T)}{\sqrt{1 + \partial^\mu T \partial_\mu T}} \partial_\mu T \partial_\nu T$$  \hspace{1cm} (5)

The energy density and pressure for the background part of the tachyon field in a homogeneous and isotropic universe is given as

$$\rho = \frac{V(T)}{\left( 1 - \dot{T}^2 \right)^{\frac{1}{2}}}$$  \hspace{1cm} (6)

$$p = -V(T) \left( 1 - \dot{T}^2 \right)^{\frac{1}{2}}$$  \hspace{1cm} (7)

Hence the Friedmann equation becomes

$$H^2 = \frac{1}{3M_p^2} \frac{V(T)}{\left( 1 - \dot{T}^2 \right)^{\frac{1}{2}}},$$  \hspace{1cm} (8)

and the equation of motion for the background part of the tachyon field can be obtained using the conservation of energy-momentum tensor as

$$\frac{\ddot{T}}{\left( 1 - \dot{T}^2 \right)} + 3H \dot{T} + (\ln V)' = 0$$  \hspace{1cm} (9)

The conditions to achieve inflation can be obtained by using Friedmann equation

$$\frac{\dot{a}}{a} = -\frac{1}{6M_p^2} \left( \rho + 3p \right) = \frac{1}{3M_p^2} \frac{V}{\left( 1 - \dot{T}^2 \right)^{\frac{1}{2}}} \left( 1 - \frac{3}{2} \dot{T}^2 \right) > 0,$$  \hspace{1cm} (10)
which gives $\dot{T}^2 < \frac{3}{2}$. And also for inflation to last sufficiently longer $\ddot{T}$ should be smaller than the friction term in the equation of motion for tachyon field Eq. (9)

$$\ddot{T} < 3H\dot{T}$$

Hence during inflation

$$\dot{T} \sim -\frac{(\ln V)'}{3H}, \quad H^2 \sim \frac{V}{3M_p^2}$$

To analyze the dynamics of inflation the slow-roll parameters can be defined in terms of the Hubble flow parameters \[41\] as

$$\epsilon_0 \equiv \frac{H_k}{H}$$

$$\epsilon_{i+1} \equiv \frac{d\ln|\epsilon_i|}{dN}, \quad i \geq 0,$$

where $H_k$ is the Hubble constant during inflation when a particular mode $k$ leaves the horizon and $N$ is the number of e-foldings

$$N \equiv \int_t^{t_e} H(t)dt,$$

where $t_e$ is the end of inflation. We also have

$$\dot{\epsilon}_i = \epsilon_i \epsilon_{i+1}.$$  \[16\]

In terms of $T$ the slow-roll parameters defined by Eqs. (13) and (14) can be written as

$$\epsilon_1 = \frac{3}{2} \dot{T}^2$$

$$\epsilon_2 = \sqrt{\frac{2}{3\epsilon_1}} \frac{\epsilon'}{H} = 2 \frac{\ddot{T}}{HT}$$

For conditions \[12\] to be satisfied, $\epsilon_1, \epsilon_2 \ll 1$ and inflation ends when $\epsilon_1 = 1$.

The power spectra for scalar and tensor perturbations, spectral index and tensor to scalar ratio is these models are given as \[9, 21\]

$$P_\zeta = \left. \frac{H^2}{8\pi^2 M_p^2 c_S \epsilon} \right|_{c_S k = aH}$$

$$P_h = \left. \frac{2}{\pi^2 M_p^2} \right|_{k = aH}$$

$$n_s = 1 - 2\epsilon_1 - \epsilon_2$$

$$r = 16c_S^2 \epsilon$$

Here $c_S$ is the effective sound speed given as

$$c_S^2 = \frac{\partial P/\partial \dot{T}^2}{\partial \rho/\partial \dot{T}^2} = 1 - \dot{T}^2$$
The effective sound speed for these models is very close to 1. The power spectrum $P_\zeta$, spectral index $n_s$ and tensor-to-scalar ratio $r$ are all evaluated at the pivot scale $k = k_0$ which is 0.05Mpc$^{-1}$ for Planck observations. All these parameters depend on the choice of the tachyon potential $V(T)$ and are constrained by CMB and LSS observations. In Sec. 1 we will discuss how reheating can be used to limit our choice of the potentials. Before that we discuss the relation between reheating parameters and inflationary parameters in the next section.

3. REHEATING

Models of reheating can be parameterized in terms of thermalization temperature $T_{re}$ at the end of reheating, duration of reheating $N_{re}$ and equation of state during reheating $w_{re}$ \[36, 37\]. We consider $w_{re}$ to be constant during reheating and it should be larger than $-\frac{1}{3}$ for inflation to end and should be smaller than 1 to the causality to be preserved.

If the equation of state remains the same during reheating, the change in the scale factor can be related to the change in energy density by using $\rho = a^{-3(1+w)}$ as

$$\frac{\rho_{end}}{\rho_{re}} = \left(\frac{a_{end}}{a_{re}}\right)^{-3(1+w_{re})}. \tag{24}$$

Here the subscripts $end$ and $re$ denote the values of the quantity at the end of inflation and at the end of reheating respectively. Eq. (24) can be expressed in terms of $N_{re} = \ln \left(\frac{a_{re}}{a_{end}}\right)$ as

$$N_{re} = \frac{1}{3(1+w_{re})} \ln \frac{\rho_{end}}{\rho_{re}} = \frac{1}{3(1+w_{re})} \ln \left(\frac{3}{2} \frac{V_{end}}{\rho_{re}}\right). \tag{25}$$

Here we have substituted $\rho_{end} = \frac{3}{2} V_{end}$ as the equation of state at the inflation is $-\frac{1}{3}$. The relation between the reheating temperature $T_{re}$ and $N_{re}$ can be obtained by expressing the energy density at the end of reheating $\rho_{re}$ in terms of $T_{re}$ as

$$\rho_{re} = \frac{\pi^2}{30} g_{re} T_{re}^4. \tag{26}$$

So from Eq. (25) we get

$$N_{re} = \frac{1}{3(1+w_{re})} \ln \left(\frac{30 \frac{3}{2} V_{end}}{\pi^2 g_{re} T_{re}^4}\right). \tag{27}$$

Using entropy conservation the temperature at the end of reheating can be related to the CMB temperature today as

$$T_{re} = T_0 \frac{a_0}{a_{re}} \left(\frac{43}{11 g_{re}}\right)^{\frac{4}{3}} = T_0 \frac{a_0}{a_{eq}} e^{N_{RD}} \left(\frac{43}{11 g_{re}}\right)^{\frac{4}{3}}, \tag{28}$$
where $T_0$ is the CMB temperature today, $a_0$ is the scale factor today, $N_{RD}$ is the number of e-foldings during radiation dominated epoch and $a_{eq}$ is the scale factor at matter-radiation equality. The ratio $\frac{a_0}{a_{eq}}$ can be expressed as

$$\frac{a_0}{a_{eq}} = \frac{a_0}{a_{eq}} \frac{a_k}{a_{eq}} a_{re} = \frac{a_0 H_k}{k} e^{-N_k} e^{-N_{RD}},$$  

(29)

where $a_k$ and $H_k$ are the values of scale factor and the Hubble constant during inflation when the Fourier mode $k$ leaves the horizon, $N_k$ is the number of e-foldings from this time to the end of inflation and $k = a_k H_k$ for horizon exit. Now using Eqs. (29) and (28) the relation between $T_{re}$ and $N_{re}$ can be expressed as

$$T_{re} = T_0 \frac{a_0 H_k}{k} \left( \frac{43}{11 g_{re}} \right)^{\frac{1}{3}} e^{-N_k} e^{-N_{re}},$$  

(30)

Substituting this into Eq. (27) we obtain the expression for $N_{re}$ as

$$N_{re} = \frac{4}{3 (1 + w_{re})} \left( \frac{1}{4} \ln \left( \frac{3^2 \cdot 5}{\pi^2 g_{re}} \right) + \ln \left( \frac{V_{end}^\frac{1}{3}}{H_k} \right) + \ln \left( \frac{k}{T_0 a_0} \right) + \frac{1}{3} \ln \left( \frac{11 g_{re}}{43} \right) + N_k + N_{re} \right).$$  

(31)

If we consider $w_{re} \neq \frac{1}{3}$, Eq. (31) can be solved to obtain $N_{re}$ as

$$N_{re} = \frac{4}{1 - 3w_{re}} \left( -\frac{1}{4} \ln \left( \frac{3^2 \cdot 5}{\pi^2 g_{re}} \right) - \frac{1}{3} \ln \left( \frac{11 g_{re}}{43} \right) - \ln \left( \frac{V_{end}^\frac{1}{3}}{H_k} \right) - \ln \left( \frac{k}{T_0 a_0} \right) - N_k \right).$$  

(32)

For $w_{re} = \frac{1}{3}$ reheating occurs instantaneously leaving the universe at grand unification scale and hence parameters of reheating cannot be used to constrain models of inflation. Using Eqs. (32) and (30) the temperature at the end of reheating $T_{re}$ can be expressed as

$$T_{re} = \left( \left( \frac{43}{11 g_{re}} \right)^{\frac{1}{3}} a_0 T_0 H_k e^{-N_k} \left( \frac{3^2 \cdot 5 V_{end}}{\pi^2 g_{re}} \right)^{\frac{1}{3 (1 + w_{re})}} \right)^{\frac{3 (1 + w_{re})}{3 w_{re} - 1}}.$$  

(33)

The main results of this section are expressions for reheating temperature $T_{re}$ Eq. (33) and number of e-folds during reheating Eq. (32) that depend on the inflationary parameters $H_k$, $N_k$ and $V_{end}$. In next section we obtain these parameters for tachyon inflation with inverse cosh and exponential potential in terms of amplitude of scalar perturbations $A_s$ and spectral index $n_s$. With this $T_{re}$ and $N_{re}$ can be expressed as function of $n_s$ and are used to constrain tachyon inflation by demanding $w_{re}$ between $-\frac{1}{3}$ and 1.

4. CONSTRAINTS ON TACHYON INFLATION

In this section we constrain tachyon inflation with inverse cosh (2) and exponential (3) potential from reheating. To simplify our calculations we define $x \equiv \frac{T_{re}}{T_0}$ and a constant dimensionless ratio $X_0^2 \equiv \frac{\lambda T_0^2}{M_{pl}^2}$. 
4.1. Inverse cosh potential

The inverse cosh potential (2) can be obtained from string theory [11, 12, 38] and is the most popular choice for tachyon potential. In terms of $x$ it can be written as

$$V = \frac{\lambda}{\cosh x}$$

The two slow-roll parameters $\epsilon_1$ and $\epsilon_2$ for this model can be obtained using Eq. (12), Eq. (17) and Eq. (18) as

$$\epsilon_1 = \frac{1}{2} \frac{\sinh^2 x}{X_0^2 \cosh x}$$

$$\epsilon_2 = \frac{1}{X_0^2} \frac{\cosh^2 x + 1}{\cosh x}$$

We can obtain the value of the tachyon field at the end of inflation by putting $\epsilon_1 = 1$ and it gives

$$\cosh x_{\text{end}} = X_0^2 + \sqrt{X_0^4 + 1},$$

which gives $x_{\text{end}} = \ln 4X_0^2$ for $X_0 > 1$. The number of e-foldings $N_k$ during inflation from the time when mode $k$ leaves the horizon to the end of inflation can be obtained using Eqs. (15) and (12) as

$$N_k = \int_{T_k}^{T_{\text{end}}} H(t) dt = \int_{T_k}^{T_{\text{end}}} \frac{H}{T} dT$$

$$= -\frac{1}{M_p^2} \int_{T_k}^{T_{\text{end}}} \frac{V^2}{V'} dT,$$

which for the inverse cosh potential becomes

$$N_k = X_0^2 \int_{x_k}^{x_{\text{end}}} \frac{1}{\sinh x} dx = X_0^2 \ln \left( \frac{\tanh \frac{x_{\text{end}}}{2}}{\tanh \frac{x_k}{2}} \right)$$

Using Eq. (37) and $X_0 > 1$ it can be shown that $\tanh x_{\text{end}} \sim 1$ so the value of tachyon field at the time when mode $k$ leaves the inflationary horizon can be given as

$$\tanh \frac{x_k}{2} = e^{-\frac{N_k}{X_0}}.$$ 

From Eq. (40) we obtain

$$\sinh x_k = \frac{1}{\sinh \left( \frac{N_k}{X_0} \right)}, \quad \cosh x_k = \frac{1}{\tanh \left( \frac{N_k}{X_0} \right)}.$$ 

The scalar power spectrum $P_\zeta$ (19), spectral index $n_s$ (21) and tensor-to-scalar ratio (22) are evaluated at the horizon crossing $c_s k = aH$ for pivot scale $k = k_o$, which we choose
0.05\text{Mpc}\text{^{-1}}\text{ as in Planck.} P_k \text{ at } k_0 \text{ is equal to the amplitude of scalar perturbations } A_s \text{ so we can express the Hubble constant } H_k \text{ using Eq. (19) as}

\[ H_k = \pi M_p \sqrt{8 A_s e \epsilon c_s}. \] (42)

The spectral index (21) for the potential Eq. (34) can be obtained using Eqs. (35) and (36) as

\[ n_s = 1 - \frac{2}{X_0^2} \cosh x_k. \] (43)

The tachyon potential at the end of inflation can be obtained as

\[ V_{\text{end}} = \frac{\lambda}{\cosh x_{\text{end}}} = 3 M_p^2 H_k^2 \cosh x_k \cosh x_{\text{end}} \] (44)

Using Eqs. (12,37,43) we get

\[ V_{\text{end}} = 3 \frac{1}{4} M_p^2 H_k^2 \left(1 - n_s \right). \] (45)

We can also write \( N_k \) in terms of \( n_s \) using Eqs. (41) and (43) as

\[ N_k = X_0^2 \tanh^{-1} \left( \frac{2}{X_0^2 \left(1 - n_s \right)} \right). \] (46)

Both the slow-roll parameters and \( c_S \) can be expressed in terms of spectral index \( n_s \) and so the Hubble constant \( H_k \) can also be expressed in terms of \( n_s \) as

\[ H_k = 2 \pi M_p \frac{1}{X_0} \sqrt{A_s \left( \frac{\frac{1}{2} \left(1 - n_s \right)}{X_0^2} - \frac{2}{(1 - n_s) X_0^2} \right) \left(1 - \frac{1}{2} \left(1 - n_s \right) \frac{X_0^2}{6 X_0^2} - \frac{2}{(1 - n_s) X_0^2} \right)} \] (47)

It can be seen from Eqs. (45,47,46) that \( V_{\text{end}}, H_k \) and \( N_k \) are all expressed in terms of amplitude of scalar perturbations \( A_s \) and spectral index \( n_s \). Hence \( T_{\text{re}} \) and \( N_{\text{re}} \) can be obtained as a function of \( A_s \) and \( n_s \) by putting these expressions in Eqs. (32) and (33). We use Planck-2015 values \( A_s = 2.20 \times 10^{-9} \) (central value) and \( n_s = 0.9645 \pm 0.0049 \) for our analysis. The small error bars on \( A_s \) does not affect the results.

Fig. 1 shows the variation of temperature at the end of reheating \( T_{\text{re}} \) and the number of e-folds during reheating \( N_{\text{re}} \) as a function of spectral index \( n_s \). We chose four values of \( w_{\text{re}} \) between \(-\frac{1}{3}\) to 1. The curves for all \( w_{\text{re}} \) meet at a point that corresponds to \( w_{\text{re}} = \frac{1}{3} \), which is defined as instant reheating (\( N_{\text{re}} \to 0 \)). The curve of \( w_{\text{re}} = \frac{1}{3} \) would pass through this point and be vertical. As depicted in the figure for \( X_0 = 7 \) the values of \( T_{\text{re}} \) and \( N_{\text{re}} \), for all choices of \( w_{\text{re}} \), completely lie outside the Planck bounds on \( n_s \). So to satisfy the observations \( X_0 > 1 \), which justifies our assumption used in our calculations. We chose physically plausible values for \( w_{\text{re}} \) i.e. \( 0 \leq w_{\text{re}} \leq 0.25 \) obtained in [34] and demand that the reheating temperature \( T_{\text{re}} \) should be larger than 100GeV (shown by light purple region in Fig. 1) for production of weak scale dark matter. This gives bounds on \( n_s \) which are stronger than the Planck 1\( \sigma \) bounds (shown by light pink region in Fig. 1) for large value of \( X_0 \). These bounds on \( n_s \) correspond to the bounds on \( N_k \), which can be obtained using Eq. (46) and are listed in Table I.
FIG. 1: Figure shows $N_{re}$ and $T_{re}$, the length of reheating and temperature at the end of reheating respectively, as a function of $n_s$ for three different values of $X_0$ for inverse cosh potential. Here vertical light pink region represents Planck bounds on $n_s$ and dark pink region represents a precision of $10^{-3}$ from future experiments \cite{42}. Horizontal dark purple region represents $T_{re}$ of 10MeV from BBN and light purple region represents 100GeV of electroweak scale. Red dotted line corresponds to $w_{re} = -\frac{1}{3}$, blue dashed line corresponds to $w_{re} = 0$, green solid line corresponds to $w_{re} = 0.25$ and black dotted line corresponds to $w_{re} = 1$. For $X_0 = 7$ both $N_{re}$ and $T_{re}$ lie outside the Planck bound.

It can be seen from Table. \ref{table1} that for physically plausible values of $w_{re}$ i.e between 0 and 0.25 the number of e-folds should have values between $N_k = 46$ to $N_k = 55$. If we consider $w_{re} \leq 1$, we can allow $N_k$ to be around 67. These upper bounds on $N_k$ and $n_s$ can be transferred into lower bounds on tensor-to-scalar ratio $r$.

The tensor-to-scalar ratio (22) for this model can be obtained from Eqs. (35) and (40) in terms of $N_k$ as
Equation of state during reheating

\( 0 \leq w_{re} \leq 0.25 \)
\( 0.25 \leq w_{re} \leq 1 \)
\( 0 \leq w_{re} \leq 0.25 \)
\( 0.25 \leq w_{re} \leq 1 \)
\( 0 \leq w_{re} \leq 0.25 \)
\( 0.25 \leq w_{re} \leq 1 \)
\( 0 \leq w_{re} \leq 0.25 \)
\( 0.25 \leq w_{re} \leq 1 \)

\( 7 \)
\( 10 \)
\( 15 \)
\( 20 \)

TABLE I: The allowed values of spectral index \( n_s \) and number of efolds \( N_k \) for various values of \( X_0 \) for inverse cosh potential considering \( T_{re} \geq 100 \text{GeV} \).

\[
\frac{16}{X_0^2} \left( \frac{1}{\sinh \left( \frac{2N_k}{X_0^2} \right)} \right),
\]

and the spectral index can also be expressed in terms of \( N_k \) by inverting Eq. (46) as

\[
 n_s = 1 - \frac{2}{X_0^2} \frac{1}{\tanh \frac{N_k}{X_0^2}}.
\]

The predictions for \( r \) and \( n_s \) can be obtained for various values of \( X_0 \) and \( N_k \) using Eqs. (48) and (46) for the potential Eq. (34). Fig. 2 shows \( N_k \) and \( r \) as a function of \( n_s \) corresponding to different values of equation of state during reheating \( w_{re} \) along with joint 68\%CL and 95\%CL Planck-2015 constraints. It can be seen from Fig. 2 that the physically plausible value of the equation of state \( 0 \leq w_{re} \leq 0.25 \), which corresponds to \( 46 \leq N_k \leq 55 \) is disfavored by Planck observations and \( w_{re} \) for these models should be close to 1 to satisfy Planck constraints on \( r \) and \( n_s \) for any value of \( X_0 \). For \( n_s-r \) predictions of tachyon inflation with inverse cosh potential to fall within Planck-2015 1\( \sigma \) bounds, one requires the equation of state \( w_{re} \) to be larger than 1, which violates causality.

4.2. Exponential potential

Another string theory motivated potential for tachyon inflation is the exponential potential (3), which was studied by \([39, 40]\). In terms of variable \( x \equiv \frac{T}{T_0} \) it can be expressed as

\[
V(x) = \lambda e^{-x}
\]

The slow-roll parameters for this model can be expressed using Eqs. (17) and (18) as

\[
\epsilon_1 = \frac{\epsilon_2}{2} = \frac{1}{2X_0^2} e^x.
\]
FIG. 2: $N_k$ vs $n_s$ and $r$ vs $n_s$ predictions for inverse cosh potential along with joint 68%CL and 95%CL Planck-2015 constraints. In both figures the orange region corresponds to $w_{re} < 0$, the green region corresponds to $0 < w_{re} < 0.25$, the yellow region corresponds to $0.25 < w_{re} < 1$ and the purple region corresponds to $w_{re} > 1$. In second figure dashed blue lines corresponds to $N_k = 46$, solid green lines corresponds to $N_k = 55$ and dashdotted black lines corresponds to $N_k = 67$.

To find the value of tachyon field at the end of inflation we put $\epsilon_1 = 1$ and we get

$$x_{\text{end}} = \ln \left(2X_0^2\right)$$

Using Eq. (38) the number of e-foldings $N_k$ for this potential can be obtained as

$$N_k = X_0^2 = \left(e^{-x_k} - e^{-x_{\text{end}}}\right) = X_0^2 = \left(e^{-x_k} - \frac{1}{2X_0^2}\right)$$

(53)

One can see from this equation that $X_0^2 \geq (N_k + \frac{1}{2}) \geq N_k$. This is in contrast to the inverse cosh potential, where sufficient number of e-foldings can be obtained with any value of $X_0$. The value of the tachyon field at the time when the mode $k$ leaves the inflationary horizon can be obtained using Eq. (53) as

$$x_k = \ln \frac{X_0^2}{N_k + \frac{1}{2}}.$$  

(54)

The spectral index $n_s$ for this model is expressed using Eqs. (21) and (51) as

$$n_s = 1 - \frac{2}{X_0^2} e^{x_k},$$

(55)

The relation between $n_s$ and $N_k$ can be obtained using Eqs. (54) and (55) as

$$N_k = \frac{2}{1 - n_s} - \frac{1}{2}.$$ 

(56)
The value of the potential at the end of inflation for this case can be expressed as

\[ V_{\text{end}} = \lambda e^{-x_{\text{end}}} = 3M_p^2 H_k^2 e^{-x_{\text{end}}}, \quad (57) \]

which using Eqs. (52) and (55) becomes

\[ V_{\text{end}} = \frac{3}{4} M_p^2 H_k^2 (1 - n_s). \quad (58) \]

Hubble constant at time of horizon exit of mode \( k \) can be expressed in terms of scalar amplitude \( A_s \) and spectral index \( n_s \) using Eqs. (19, 51, 55) as

\[ H_k = \pi M_p \sqrt{4A_s (1 - n_s) \left(1 - \frac{1}{12} (1 - n_s)^2\right)}. \quad (59) \]

FIG. 3: Figure shows \( N_{\text{re}} \) and \( T_{\text{re}} \), the length of reheating and temperature at the end of reheating respectively, as a function of \( n_s \) for exponential potential. Here all curves and shaded regions are same as Fig. 1.

Again for the potential (50), \( N_k, V_{\text{end}} \) and \( H_k \) are expressed in terms of \( A_s \) and \( n_s \) and one can obtain the temperature at the end of reheating \( T_{\text{re}} \) and the number of e-folds during reheating \( N_{\text{re}} \) as a function of \( n_s \) using Eqs. (52) and (53), which are shown in Fig. 3. As in the case of inverse cosh potential we have again chosen four values of \( w_{\text{re}} \) between \( -\frac{1}{3} \) to 1. For the physically plausible value of \( w_{\text{re}} \) i.e. \( 0 \leq w_{\text{re}} \leq 0.25 \) and \( T_{\text{re}} \geq 100\text{GeV} \), the value of \( n_s \) is restricted between \( 0.958 \leq n_s \leq 0.964 \). This again corresponds to \( 47 \leq N_k \leq 55 \). If we chose \( 0 \leq w_{\text{re}} \leq 1 \), it can be seen from the Fig. 3 that \( 0.958 \leq n_s \leq 0.971 \) for \( T_{\text{re}} \geq 100\text{GeV} \), which gives \( 47 \leq N_k \leq 68 \).

The tensor-to-scalar ratio (22) for this potential is given as

\[ r = \frac{8e^{x_k}}{X_0^2} = 4 (1 - n_s), \quad (60) \]
FIG. 4: $N_k$ vs $n_s$ and $r$ vs $n_s$ predictions for exponential inflation along with joint 68%CL and 95%CL Planck-2015 constraints. In both the figures orange portion of the curve represents allowed $r$ and $n_s$ for $w_{re} < 0$, green portion corresponds to $w_{re}$ between 0 and 0.25, yellow portion corresponds to $w_{re}$ between 0.25 and 1 and purple portion corresponds to $w_{re} > 1$. It can be seen from the figure that this model is ruled out by Planck observations at 2σ for physically allowed equation of state during reheating $0 \leq w_{re} \leq 0.25$.

where we have used Eq. (55) in the last step. The plots for $N_k$ and $r$ as a function of $n_s$ are shown along with joint 68%CL and 95%CL Planck-2015 constraints in Fig. 4. The bounds on $n_s$ obtained by imposing the condition $0 \leq w_{re} \leq 0.25$ and $T_{re} \geq 100$GeV provide bounds on $r$ as $0.144 \leq r \leq 0.168$. If we consider the broader range for $w_{re}$ i.e. $0 \leq w_{re} \leq 1$, the bounds on $r$ become $0.116 \leq r \leq 0.168$, which is slightly above than the Planck 2015 bound $r \leq 0.1 \ [7]$. As depicted in Fig. 4 the effective equation of state during reheating $w_{re}$ for this choice of potential should lie between 0.25 and 1 to satisfy Planck constraints on $r$-$n_s$. Hence tachyon inflation with exponential potential (50) is disfavored if physically plausible value of reheating equation of state $0 \leq w_{re} \leq 0.25$ is considered.

5. CONCLUSIONS

Tachyon inflation [10–17] is one of the most attractive models of $K$-inflation [8, 9] motivated by string theory. In this work we have analyzed tachyon inflation by imposing constraints from reheating. This technique was earlier used to constrain various models of canonical inflation [35–37]. Here we chose inverse cosh potential [11, 12, 38] and exponential potential [39, 40] for our analysis. We compute reheating temperature $T_{re}$ and number of e-folds during reheating $N_{re}$ as a function of spectral index $n_s$ for these potentials by assuming the effective equation of state during reheating $w_{re}$ to be constant. $w_{re}$ was obtained for various reheating scenarios [44] and it was found that $0 \leq w_{re} \leq 0.25$. By demanding $0 \leq w_{re} \leq 0.25$ and $T_{re} \geq 100$GeV we find bounds on $n_s$ and number of e-folds $N_k$ from the time when mode $k$ corresponding to pivot scale $k_0 = 0.05\text{Mpc}^{-1}$ leaves inflationary horizon.
to the end of inflation. These bounds restrict the allowed regions in $n_s$-$r$ plane for these potential.

For inverse cosh potential (2), as shown in Fig. 2, we find that $N_k$ should lie between 46 and 55 for $0 \leq w_{re} \leq 0.25$. If we choose a broader range $0 \leq w_{re} \leq 1$, $N_k$ can lie between 46 and 67. The $n_s$-$r$ predictions for inverse cosh potential lie outside the Planck-2015 bounds for physically plausible values $0 \leq w_{re} \leq 0.25$.

For exponential potential (3), the condition $0 \leq w_{re} \leq 0.25$ and $T_{re} \geq 100\text{GeV}$ gives bounds on $n_s$ as $0.958 \leq n_s \leq 0.964$, which corresponds to $47 \leq N_k \leq 55$. For $0 \leq w_{re} \leq 1$ we obtain $47 \leq N_k \leq 68$. As shown in Fig. 4 for this model also, the $n_s$-$r$ predictions lie outside the Planck-2015 bounds for $0 \leq w_{re} \leq 0.25$. We also find that $r \geq 0.116$ for this model for $w_{re} \leq 1$, which is slightly higher than the Planck bound $r \leq 0.116$. Both exponential potential and inverse cosh potential are disfavored by Planck-2015 bounds on $n_s$-$r$ for the physically plausible values of effective equation of state during reheating $0 \leq w_{re} \leq 0.25$. For both these models $w_{re}$ close to 1 is required to satisfy Planck bounds on $n_s$ and $r$. With tachyon potentials derived from string theory reheating is not well understood [43]. So this work can be helpful in determining correct mechanism for reheating in tachyon inflation.

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[1] A. H. Guth, Phys. Rev. D 23, 347 (1981). doi:10.1103/PhysRevD.23.347
[2] V. F. Mukhanov and G. V. Chibisov, JETP Lett. 33, 532 (1981).
[3] A. A. Starobinsky, Phys. Lett. B117, 175 (1982).
[4] A. H. Guth and S.-Y. Pi, Phys. Rev. D32, 1899 (1985).
[5] G. F. Smoot, C. L. Bennett, A. Kogut, E. L. Wright, J. Aymon, N. W. Boggess, E. S. Cheng, G. De Amici et al., Astrophys. J. 396, L1-L5 (1992).
[6] E. Komatsu et al. [ WMAP Collaboration ], Astrophys. J. Suppl. 192, 18 (2011). arXiv:1001.4538 [astro-ph.CO].
[7] P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 594, A20 (2016) doi:10.1051/0004-6361/201525898 [arXiv:1502.02114 [astro-ph.CO]].
[8] C. Armendariz-Picon, T. Damour, V. F. Mukhanov, Phys. Lett. B458, 209-218 (1999). hep-th/9904176.
[9] J. Garriga, V. F. Mukhanov, Phys. Lett. B458, 219-225 (1999). hep-th/9904176.
[10] G. W. Gibbons, Phys. Lett. B 537, 1 (2002) doi:10.1016/S0370-2693(02)01881-6 hep-th/0204008.
[11] A. Sen, JHEP 9910, 008 (1999) doi:10.1088/1126-6708/1999/10/008 hep-th/9909062.
[12] M. R. Garousi, Nucl. Phys. B 584, 284 (2000) doi:10.1016/S0550-3213(00)00361-8 hep-th/0003122.
[13] E. A. Bergshoeff, M. de Roo, T. C. de Wit, E. Eyras and S. Panda, JHEP 0005, 009 (2000) doi:10.1088/1126-6708/2000/05/009 hep-th/0003221.
[14] J. Kluson, Phys. Rev. D 62, 126003 (2000) doi:10.1103/PhysRevD.62.126003 hep-th/0004106.
[15] A. Sen, Int. J. Mod. Phys. A 18, 4869 (2003) doi:10.1142/S0217751X03015313 hep-th/0209122.
[16] D. Kutasov and V. Niarchos, Nucl. Phys. B 666, 56 (2003) doi:10.1016/S0550-3213(03)00498-
X [hep-th/0304045].
[17] K. Okuyama, JHEP 0305, 005 (2003) doi:10.1088/1126-6708/2003/05/005 [hep-th/0304108].
[18] A. Sen, JHEP 0204, 048 (2002) doi:10.1088/1126-6708/2002/04/048 [hep-th/0203211].
[19] A. V. Frolov, L. Kofman and A. A. Starobinsky, Phys. Lett. B 545, 8 (2002) doi:10.1016/S0370-2693(02)/02582-0 [hep-th/0204187].
[20] L. Kofman and A. D. Linde, JHEP 0207, 004 (2002) doi:10.1088/1126-6708/2002/07/004 [hep-th/0205121].
[21] D. A. Steer and F. Vernizzi, Phys. Rev. D 70, 043527 (2004) doi:10.1103/PhysRevD.70.043527 [hep-th/0310139].
[22] N. Barbosa-Cendejas, J. De-Santiago, G. German, J. C. Hidalgo and R. R. Mora-Luna, JCAP 1803, no. 03, 015 (2018) doi:10.1088/1475-7516/2018/03/015 [arXiv:1711.06693 [astro-ph.CO]].
[23] R. Allahverdi, R. Brandenberger, F. Y. Cyr-Racine and A. Mazumdar, Ann. Rev. Nucl. Part. Sci. 60, 27 (2010) doi:10.1146/annurev.nucl.012809.104511 [arXiv:1001.2600 [hep-th]].
[24] L. F. Abbott, E. Farhi and M. B. Wise, Phys. Lett. 117B, 29 (1982). doi:10.1016/0370-2693(82)90292-1
[25] A. D. Dolgov and A. D. Linde, Phys. Lett. 116B, 329 (1982). doi:10.1016/0370-2693(82)90292-1
[26] E. V. F. Abrecht, P. J. Steinhardt, M. S. Turner and F. Wilczek, Phys. Rev. Lett. 48, 1437 (1982). doi:10.1103/PhysRevLett.48.1437
[27] J. H. Traschen and R. H. Brandenberger, Phys. Rev. D 42, 2491 (1990). doi:10.1103/PhysRevD.42.2491
[28] A. D. Dolgov and D. P. Kirilova, Sov. J. Nucl. Phys. 51, 172 (1990) [Yad. Fiz. 51, 273 (1990)].
[29] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. Lett. 73, 3195 (1994) doi:10.1103/PhysRevLett.73.3195 [hep-th/9405187].
[30] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D 56, 3258 (1997) doi:10.1103/PhysRevD.56.3258 [hep-ph/9704452].
[31] B. R. Greene, T. Prokopec and T. G. Roos, Phys. Rev. D 56, 6484 (1997) doi:10.1103/PhysRevD.56.6484 [hep-ph/9705357].
[32] J. F. Dufaux, G. N. Felder, L. Kofman, M. Peloso and D. Podolsky, JCAP 0607, 006 (2006) doi:10.1088/1475-7516/2006/07/006 [hep-ph/0602144].
[33] G. N. Felder, L. Kofman and A. D. Linde, Phys. Rev. D 59, 123523 (1999) doi:10.1103/PhysRevD.59.123523 [hep-ph/9812289].
[34] D. I. Podolsky, G. N. Felder, L. Kofman and M. Peloso, Phys. Rev. D 73, 023501 (2006) doi:10.1103/PhysRevD.73.023501 [hep-ph/0507096].
[35] L. Dai, M. Kamionkowski and J. Wang, Phys. Rev. Lett. 113, 041302 (2014) doi:10.1103/PhysRevLett.113.041302 [arXiv:1404.6704 [astro-ph.CO]].
[36] J. B. Munoz and M. Kamionkowski, Phys. Rev. D 91, no. 4, 043521 (2015) doi:10.1103/PhysRevD.91.043521 [arXiv:1412.0566 [astro-ph.CO]].
[37] J. L. Cook, E. Dimastrogiovanni, D. A. Easson and L. M. Krauss, JCAP 1504, 047 (2015) doi:10.1088/1475-7516/2015/04/047 [arXiv:1502.04673 [astro-ph.CO]].
[38] N. D. Lambert, H. Li and J. M. Maldacena, JHEP 0703, 014 (2007) doi:10.1088/1126-6708/2007/03/014 [hep-th/0303139].
[39] A. Sen, Mod. Phys. Lett. A 17, 1797 (2002) doi:10.1142/S0217732302008071 [hep-th/0204143].
[40] M. Sami, P. Chingangbam and T. Qureshi, Phys. Rev. D 66, 043530 (2002) doi:10.1103/PhysRevD.66.043530 [hep-th/0205179].

[41] D. J. Schwarz, C. A. Terrero-Escalante and A. A. Garcia, Phys. Lett. B 517, 243 (2001) doi:10.1016/S0370-2693(01)01036-X [astro-ph/0106020].

[42] L. Amendola et al., Living Rev. Rel. 21, no. 1, 2 (2018) doi:10.1007/s41114-017-0010-3 [arXiv:1606.00180 [astro-ph.CO]].

[43] J. M. Cline, H. Firouzjahi and P. Martineau, JHEP 0211, 041 (2002) doi:10.1088/1126-6708/2002/11/041 [hep-th/0207156].