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To cite this article: Hyung Won Lee et al JHEP10(1999)014

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Tachyon and fixed scalars of $D^5_{\pm} - D^1_{\pm}$ black hole in type 0B string theory

Hyung Won Lee and Yun Soo Myung

Department of Physics, Inje University
Kimhae 621-749, Korea
E-mail: hwlee@physics.inje.ac.kr, ysmyung@physics.inje.ac.kr

Jin Young Kim

Department of Physics, Kunsan National University
Kunsan 573-701, Korea
E-mail: jykim@kunsan.ac.kr

ABSTRACT: In the type 0B string theory, we discuss the role of tachyon ($T$) and fixed scalars ($\nu, \lambda$). The issue is to explain the difference between tachyon and fixed scalars in the $D^5_{\pm} - D^1_{\pm}$ black hole background. For this purpose, we perform the semiclassical calculation. Here one finds a mixing between ($\nu, \lambda, T$) and the other fields. Using the decoupling procedure, one finds the linearized equation for the tachyon. From the potential analysis, it turns out that $\nu$ plays a role of test field well, while the tachyon induces an instability of Minkowski space vacuum. But the roles of $\nu$ and $T$ are the same in the near-horizon geometry. Finally we discuss the stability problem.

KEYWORDS: D-branes, Black Holes in String Theory, Black Holes.
1. Introduction

Recently type 0 string theories attract much interest in the study of non-supersymmetric gauge theories [1, 2, 3]. Type 0 string theories can be obtained from the worldsheet of type II string theories by performing a non-chiral GSO projection [4]. The resulting theories have world sheet supersymmetry but no space-time supersymmetry. The crucial differences of type 0 theories with type II theories is to have the doubling of Ramond-Ramond (RR) fields and the tachyon. Thus they all have twice as many D-branes. The application of this model to the study of non-SUSY gauge theory was realized by Klebanov and Tseytlin [2]. They considered the theory of N coincident electrically charged D3-branes. Using the near-horizon D3-brane, they constructed an SU($N$) gauge theory with six adjoint scalars and studied its behavior. They observed that the doubled RR flux in type 0 dual background stabilizes the tachyon. Since then a number of papers on this mode appeared [5]. In a subsequent paper, Klebanov and Tseytlin [6] also considered the theory of N electric D3$_+$-branes coincident with N magnetic D3$_-$-branes. The non-SUSY theory is the SU($N$) × SU($N$) gauge theory coupled to six adjoint scalars of the first SU($N$), six adjoint scalars of the second SU($N$), and fermions in the bifundamental representation (four Weyl spinors in the $(N,\bar{N})$ and another four in the $(\bar{N},N)$). The D$p_{\pm}$-brane bound states of the type 0 theories are somewhat similar to those of the type II theories. However, the essential difference with type II theories comes from the existence of tachyon. In the previous work of one of the authors [7], the low energy scattering of fields in the type 0B theory for electric D3-branes attempted. In this analysis the dilaton field can be used as a test field.
One of the simplest way to see the role of tachyon in type 0 theories is to consider the intersecting $D_p$-branes. The $D_{p\pm}$-brane bound states can be intersected according to the same rules of the type II theories. The $D_{5\pm}$-$D_{1\pm}$ brane black hole thus is constructed to show that the corresponding near-horizon geometry is $AdS_3 \times S^3 \times T^4$ and it has asymptotically flat space at infinity. It is shown that the tachyon field can be stabilized in $AdS_3 \times S^3 \times T^4$ \cite{6,8}. In this paper, we will study the role of the tachyon in $D_{5\pm}$-$D_{1\pm}$ brane black hole, by comparing it with the fixed scalars. Further, we will present a complete solution to the stability by analyzing the potentials surrounding the $D_{5\pm}$-$D_{1\pm}$ brane black hole.

The organization of the paper is as follows. In section 2, we briefly sketch the $D_{5\pm}$-$D_{1\pm}$ brane black hole in the type 0B string theory. We set up the perturbation for all fields around this black hole background in section 3. Here we choose the harmonic gauge and use all linearized equations to decouple $(\nu, t)$ from the remaining fields. section 4 is devoted to analyzing their potentials. Finally we discuss our results in section 5.

2. $D_{5\pm}$-$D_{1\pm}$ brane black hole

Here we consider a class of 5D black holes representing the bound state of the $D_{5\pm}$-$D_{1\pm}$ brane system compactified on $T^5 = T^4 \times S^1$. This black hole can also be obtained as a solution to the semiclassical action of type 0B string compactified on $T^5$. The effective action for a 5D black hole is given by \cite{8}

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left\{ R - \frac{4}{3} (\nabla \lambda)^2 - 4 (\nabla \nu)^2 - \frac{1}{4} (\nabla T)^2 - \frac{m^2}{4} e^{-2\lambda/3} T^2 - \frac{1}{4} e^{\frac{3}{2}\lambda} F^{(K)2} - \frac{1}{4} e^{-\frac{3}{2}\lambda + \nu} \left( f_+ (T) F_+^2 + f_- (T) F_-^2 \right) - \frac{1}{4} e^{-\frac{3}{2}\lambda - 4\nu} \left( f_+ (T) H_+^2 + f_- (T) H_-^2 \right) \right\}, \quad (2.1)$$

where $F^{(K)}_{\mu\nu}$ is the Kaluza-Klein (KK) field strength along the string direction ($S^1$), $F_{\pm\mu\nu}$ are the electric components of the Ramond-Ramond (RR) three-form $F_{3\pm}$ and $H_{\pm\mu\nu}$ are dual to the magnetic components of the RR three-form $F_{3\pm}$. Here we omit the analysis of the 6D dilaton $\phi_6$, since it is just a minimally decoupled scalar. On the other hand, the scalars $\nu$ and $\lambda$ interact with the gauge fields and are examples of the fixed scalar. $\nu$ is related to the scale of the internal torus ($T^4$), while $\lambda$ is related to the scale of the KK circle ($S^1$). $f_{\pm} (T) = 1 \pm T + \frac{1}{2} (T^2)$ and the tachyon mass is given by $m^2 = -2/\alpha'$. Comparing with the results of type IIB theory \cite{13,14,15}, the new ingredients are the tachyon and the doubling of the RR fields. $\kappa_5^2$ is the 5D gravitational coupling constant ($\kappa_5^2 = 8\pi G_5^N, G_5^N = 5D$ Newtonian constant). This can be determined by $G_5^N = G_N^1 / V_5 = 8\pi^6 g^2 / (2\pi)^5 VR = \pi g^2 / 4VR$.
with \( V = R_5 R_6 R_7 R_8 \) (volume of \( T^4 \)), \( R = R_9 \) (radius of \( S^1 \)), \( \alpha' = 1 \), and \( g = 10D \) string coupling constant. We wish to follow the MTW conventions [11].

The equations of motion for action (2.1) are given by

\[
R_{\mu\nu} - \frac{4}{3} \partial_\mu \lambda \partial_\nu \lambda - 4 \partial_\mu \partial_\nu \lambda - \frac{1}{4} \partial_\mu T \partial_\nu T - \frac{1}{12} m^2 e^{-2\lambda/3} T^2 g_{\mu\nu} - e^{\frac{2}{3} \lambda} \left( \frac{1}{2} F^{(K)}_{\mu\rho} F^{(K)}_{\rho\nu} - \frac{1}{12} F^{(K)2} g_{\mu\nu} \right) - e^{-\frac{4}{3} \lambda + 4\nu} \left\{ f_+(T) \left( \frac{1}{2} F^{\mu\rho}_{+\nu} F^{\rho}_{+\nu} - \frac{1}{12} F^2 g_{\mu\nu} \right) + f_-(T) \left( \frac{1}{2} F^{\mu\rho}_{-\nu} F^{\rho}_{-\nu} - \frac{1}{12} F^2 g_{\mu\nu} \right) \right\} -
\]

\[
8 \nabla^2 \nu - e^{-\frac{4}{3} \lambda + 4\nu} \left( f_+(T) F^2_+ + f_-(T) F^2_- \right) + e^{\frac{4}{3} \lambda - 4\nu} \left( f_+(T) H^2_+ + f_-(T) H^2_- \right) = 0,
\]

\[
8 \nabla^2 \lambda - 2 e^{\frac{2}{3} \lambda} F^{(K)2} + e^{-\frac{4}{3} \lambda + 4\nu} \left( f_+(T) F^2_+ + f_-(T) F^2_- \right) + e^{-\frac{4}{3} \lambda - 4\nu} \left( f_+(T) H^2_+ + f_-(T) H^2_- \right) = 0,
\]

\[
\nabla_\mu \left( e^{\frac{2}{3} \lambda} F^{(K)\mu\nu} \right) = 0,
\]

\[
\nabla_\mu \left( f_+(T) e^{-\frac{4}{3} \lambda + 4\nu} F^{\mu\nu}_+ \right) = 0,
\]

\[
\nabla_\mu \left( f_-(T) e^{-\frac{4}{3} \lambda + 4\nu} F^{\mu\nu}_- \right) = 0,
\]

\[
\nabla_\mu \left( f_+(T) e^{-\frac{4}{3} \lambda - 4\nu} H^{\mu\nu}_+ \right) = 0,
\]

\[
\nabla_\mu \left( f_-(T) e^{-\frac{4}{3} \lambda - 4\nu} H^{\mu\nu}_- \right) = 0,
\]

\[
\nabla^2 T - m^2 e^{-2\lambda/3} T - \frac{1}{2} e^{-4\lambda/3 + 4\nu} \left( f'_+(T) F^2_+ + f'_-(T) F^2_- \right) - \frac{1}{2} e^{-4\lambda/3 - 4\nu} \left( f'_+(T) H^2_+ + f'_-(T) H^2_- \right) = 0,
\]

where the prime (') denotes the differentiation with respect to its argument. In addition, we need the remaining Maxwell equations as five Bianchi identities [10].

\[
\partial_\mu F^{(K)}_{\rho\sigma} = \partial_\mu F_{\pm\rho\sigma} = \partial_\mu H_{\pm\rho\sigma} = 0.
\]

The black hole solution is given by the background metric

\[
ds^2 = -df^{-\frac{2}{3}} dt^2 + d^{-1} f^\frac{2}{3} dr^2 + r^2 f^\frac{2}{3} d\Omega^2
\]
and

\[
e^{2\lambda} = \frac{f_K}{\sqrt{f_1 f_5}}, \quad e^{4\varphi} = \frac{f_1}{f_5}, \quad f = f_1 f_5 f_K.
\] (2.13)

\[
F_{tr}^{(K)} = \frac{2\tilde{Q}_K}{r^3 f_K}, \quad F_{\pm tr} = \frac{2Q_{1\pm}}{r^3 f_1^2}, \quad \tilde{H}_{\pm tr} = \frac{2Q_{5\pm}}{r^3 f_5^2}, \quad T = 0.
\] (2.14)

Here four harmonic functions are defined by

\[
f_1 = 1 + \frac{r_1^2}{r^2}, \quad f_5 = 1 + \frac{r_5^2}{r^2}, \quad f_K = 1 + \frac{r_K^2}{r^2}, \quad d = 1 - \frac{r_0^2}{r^2},
\] (2.15)

with \( r_i^2 = r_0^2 \sinh^2 \sigma_i, \quad i = 1, 5, K \). \( Q_{1\pm}, Q_{5\pm} \) and \( \tilde{Q}_K \) are related to the characteristic radii \( r_1, r_5, r_K \) and the radius of horizon \( r_0 \) as

\[
Q_{j+} = Q_{j-} = Q_j, \quad j = 1, 2,
\]

\[
\tilde{Q}_i = \frac{1}{2} r_0^2 \sinh 2\sigma_i, \quad i = 1, 2, K,
\]

\[
\tilde{Q}_j = 2Q_j^2 = Q_{j+}^2 + Q_{j-}^2 = r_j^2 (r_j^2 + r_0^2), \quad j = 1, 2,
\]

\[
\tilde{Q}_K = r_K^2 (r_K^2 + r_0^2),
\] (2.16)

where

\[
r_i^2 = \sqrt{\tilde{Q}_i^2 + \frac{r_0^4}{4} - \frac{r_0^2}{2}}, \quad i = 1, 2, K.
\] (2.17)

The background metric (2.12) is just a 5D Schwarzschild one with time and space components rescaled by different powers of \( f \). The event horizon (outer horizon) is clearly at \( r = r_0 \). When all five charges are nonzero, the surface of \( r = 0 \) becomes a smooth inner horizon (Cauchy horizon). If one of the charges is zero, the surface of \( r = 0 \) becomes singular. The extremal case corresponds to the limit of \( r_0 \to 0 \) with the boost parameters \( \sigma_i \to \pm \infty \) keeping \( \tilde{Q}_i \) fixed. Here one has \( \tilde{Q}_1 = r_1^2, \quad \tilde{Q}_5 = r_5^2, \) and \( \tilde{Q}_K = r_K^2 \). In this work we are very interested in the limit of \( r_0, r_K \ll r_1, r_5 \), which is called the dilute gas approximation. This corresponds to the near-extremal black hole and its thermodynamic quantities are given by

\[
M_{\text{next}} = \frac{2\pi^2}{\kappa_5^2} \left( r_1^2 + r_5^2 + \frac{1}{2} r_0^2 \cosh 2\sigma_K \right),
\] (2.18)

\[
S_{\text{next}} = \frac{4\pi^3 r_0}{\kappa_5^2} r_1 r_5 \cosh \sigma_K,
\] (2.19)

\[
\frac{1}{T_{H,\text{next}}} = \frac{2\pi}{r_0} r_1 r_5 \cosh \sigma_K.
\] (2.20)

The above energy and entropy are actually those of a gas of massless 1D particles. In this case the effective temperatures of the left and right moving string modes are given by

\[
T_L = \frac{1}{2\pi} \left( \frac{r_0}{r_1 r_5} \right) e^{\sigma_K}, \quad T_R = \frac{1}{2\pi} \left( \frac{r_0}{r_1 r_5} \right) e^{-\sigma_K}.
\] (2.21)
The Hawking temperature is given by their harmonic average

$$\frac{2}{T_H} = \frac{1}{T_L} + \frac{1}{T_R}. \quad (2.22)$$

3. Perturbation analysis

Here we start with the perturbation around the black hole background as in [10]

$$F_{tr}^{(K)} = \tilde{F}_{tr}^{(K)} + \mathcal{F}_{tr}^{(K)} = \tilde{F}_{tr}^{(K)}[1 + \mathcal{F}(t,r,\chi,\theta,\phi)], \quad (3.1)$$

$$F_{\pm tr} = \tilde{F}_{\pm tr} + \mathcal{F}_{\pm tr} = \tilde{F}_{\pm tr}[1 + \mathcal{F}(t,r,\chi,\theta,\phi)], \quad (3.2)$$

$$H_{\pm tr} = \tilde{H}_{\pm tr} + \mathcal{H}_{\pm tr} = \tilde{H}_{\pm tr}[1 + \mathcal{H}(t,r,\chi,\theta,\phi)], \quad (3.3)$$

$$\lambda = \tilde{\lambda} + \delta\lambda(t,r,\chi,\theta,\phi), \quad (3.4)$$

$$\nu = \tilde{\nu} + \delta\nu(t,r,\chi,\theta,\phi), \quad (3.5)$$

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}, \quad (3.6)$$

$$T = 0 + t. \quad (3.7)$$

Here \( h_{\mu\nu} \) is given by

$$h_{\nu} = \left(\begin{array}{cccc}
h_1 & h_3 & 0 & 0 \\
-d^2h_3/\rho & h_2 & 0 & 0 \\
0 & 0 & h_3^\chi & h_3^\phi \\
0 & 0 & h_\theta^\chi & h_\theta^\phi & h_\phi^\phi
\end{array}\right). \quad (3.8)$$

This seems to be general for the s-wave calculation.

One has to linearize (2.23)–(2.10) in order to obtain the equations governing the perturbations as
\[-\frac{1}{6}e^{-\frac{4}{3}\lambda + 4\nu} \bar{F}_{\mu\nu} \bar{F}_{\nu} - \frac{1}{2}e^{-\frac{4}{3}\lambda + 4\nu} \bar{F}_{\mu\nu} \bar{F}_{\rho} g_{\mu\rho} + e^{-\frac{4}{3}\lambda + 4\nu} \bar{F}_{\mu} g_{\mu\nu} \left( -\frac{1}{9}\delta \lambda + \frac{1}{3}\delta \nu \right) + \frac{1}{12}e^{-\frac{4}{3}\lambda + 4\nu} \bar{F}_{\mu} h_{\mu\nu} + \frac{1}{2}e^{-\frac{\lambda}{3} - 4\nu} H_{+\mu\nu} h_{\rho\alpha} - e^{-\frac{\lambda}{3} - 4\nu} H_{\mu\nu} \mathcal{H}_{+\rho} \right) + \frac{1}{6}e^{-\frac{\lambda}{3} - 4\nu} \bar{H}_{\mu\nu} \eta_{\mu\rho} g_{\nu\rho} - e^{-\frac{\lambda}{3} - 4\nu} H_{\mu\nu} \bar{g}_{\mu\nu} \right) + \frac{1}{12}e^{-\frac{\lambda}{3} - 4\nu} \bar{H}_{\mu\nu} h_{\mu\nu} + \frac{1}{2}e^{-\frac{\lambda}{3} - 4\nu} \bar{H}_{\mu\nu} \bar{H}_{\rho\alpha} - e^{-\frac{\lambda}{3} - 4\nu} \bar{H}_{\mu\nu} \mathcal{H}_{-\rho} \right) + \frac{1}{6}e^{-\frac{\lambda}{3} - 4\nu} \bar{H}_{\mu\nu} \bar{H}_{\nu} \mathcal{H}_{-\nu} \bar{g}_{\mu\nu} \right) - \frac{1}{6}e^{-\frac{4}{3}\lambda - 4\nu} \bar{H}_{\rho\nu} \bar{H}_{\nu} \bar{H}_{\mu\rho} g_{\mu\nu} - e^{-\frac{4}{3}\lambda - 4\nu} \bar{H}_{\mu} \bar{H}_{\nu} \bar{H}_{\nu} \left( \frac{1}{9}\delta \lambda + \frac{1}{3}\delta \nu \right) + \frac{1}{12}e^{-\frac{4}{3}\lambda - 4\nu} \bar{H}_{\mu} h_{\mu\nu} = 0, \]

\[(\nabla^2 \delta \nu - h^{\mu\nu} \nabla_{\mu} \nabla_{\nu} \bar{\eta} - \bar{g}_{\mu\nu} \delta_{\mu\nu} \bar{h}_{\rho} \bar{h}_{\rho} - \frac{1}{4}e^{-\frac{4}{3}\lambda + 4\nu} \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} + \frac{1}{4}e^{-\frac{4}{3}\lambda + 4\nu} \bar{F}_{\mu\nu} \bar{F}^{\mu} h_{\rho} \delta_{\mu\rho} - \frac{1}{8}e^{-\frac{4}{3}\lambda + 4\nu} \bar{F}^{2} \left( -\frac{1}{3}\delta \lambda + \frac{1}{2}\delta \nu \right) + \frac{1}{4}e^{-\frac{4}{3}\lambda - 4\nu} \bar{H}_{\mu\nu} \mathcal{H}_{\mu\nu} - \frac{1}{4}e^{-\frac{4}{3}\lambda - 4\nu} \bar{H}_{\mu\nu} \bar{H}_{\nu} \bar{H}_{\nu} \left( \frac{1}{9}\delta \lambda + \frac{1}{3}\delta \nu \right) + \frac{1}{8}e^{-\frac{4}{3}\lambda - 4\nu} \bar{H}_{\mu\nu} h_{\mu\nu} = 0, \]

\[(\nabla^2 \delta \lambda - h^{\mu\nu} \nabla_{\mu} \nabla_{\nu} \bar{\lambda} - \bar{g}_{\mu\nu} \delta_{\mu\nu} \bar{\lambda}_{\rho} \bar{\lambda}_{\rho} - \frac{1}{2}e^{-\frac{4}{3}\lambda} \bar{F}^{(K)} \bar{F}^{(K)\mu\nu} + \frac{1}{2}e^{-\frac{4}{3}\lambda} \bar{F}^{(K)} \bar{F}^{(K)\nu} h_{\rho} \delta_{\mu\rho} - \frac{2}{3}e^{-\frac{4}{3}\lambda} \bar{F}^{2} \delta \lambda + \frac{1}{4}e^{-\frac{4}{3}\lambda} \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} - \frac{1}{4}e^{-\frac{4}{3}\lambda} \bar{F}_{\mu\nu} \bar{F}^{\mu} h_{\rho} \delta_{\nu\rho} + \frac{1}{8}e^{-\frac{4}{3}\lambda} \bar{F}^{2} \left( -\frac{1}{6}\delta \lambda + \frac{1}{2}\delta \nu \right) + \frac{1}{4}e^{-\frac{4}{3}\lambda} \bar{H}_{\mu\nu} \mathcal{H}_{\mu\nu} - \frac{1}{4}e^{-\frac{4}{3}\lambda} \bar{H}_{\mu\nu} \bar{H}_{\nu} \bar{H}_{\nu} \left( \frac{1}{9}\delta \lambda + \frac{1}{3}\delta \nu \right) + \frac{1}{8}e^{-\frac{4}{3}\lambda} \bar{H}_{\nu} h_{\mu\nu} = 0, \]

\[(\nabla_{\mu} + \frac{8}{3} \partial_{\mu} \bar{\lambda}) \left( \bar{F}^{(K)\mu\nu} - \bar{F}^{(K)\nu} h_{\alpha}^{\mu} - \bar{F}^{(K)} \bar{h}_{\beta}^{\mu} \right) + \bar{F}^{(K)\mu\nu} \left( \delta \Gamma_{\sigma}^{\mu\nu} \left( h_{\mu\nu} \right) + \frac{8}{3} \partial_{\nu} \bar{\lambda} \right) = 0, \]

\[(\nabla_{\mu} - \frac{4}{3} \partial_{\mu} \bar{\lambda} + 4 \partial_{\mu} \bar{\nu}) \left( \bar{F}^{\mu\nu} - \bar{F}^{\nu} h_{\mu}^{\nu} - \bar{F}^{\mu} h_{\nu}^{\nu} \right) + \bar{F}^{\mu\nu} \left( \delta \Gamma_{\sigma}^{\mu\nu} \left( h_{\mu\nu} \right) - \frac{4}{3} \partial_{\nu} \bar{\lambda} + 4 \partial_{\nu} \bar{\delta} \nu + \partial_{\nu} t \right) = 0, \]
We have to examine whether there exists any choice of gauge which can simplify propagation of fields. For this purpose, since we start with full degrees of freedom (3.8), we choose a gauge to study the equations (3.10), (3.14), and (3.17) lead to

$$
\nabla^2 \delta \nu + \left( - \frac{Q^2}{r^3 f^{1/3}} \right) (2 \mathcal{F}_+ + 2 \mathcal{F}_- - 2 \mathcal{H}_+ - 2 \mathcal{H}_- + 16 \delta \nu) = 0 \tag{3.21}
$$

where

$$
\delta R_{\mu\nu}(h) = -\frac{1}{2} \nabla^2 h_{\mu\nu} - \frac{1}{2} \nabla_\mu \nabla_\nu h_{\rho\rho} + \frac{1}{2} \nabla^\rho \nabla_\nu h_{\rho\mu} + \frac{1}{2} \nabla^\rho \nabla_\mu h_{\nu\rho} \tag{3.18}
$$

$$
\delta \Gamma^\rho_{\mu\nu}(h) = \frac{1}{2} \delta^{\rho\sigma} (\nabla_\nu h_{\mu\sigma} + \nabla_\mu h_{\nu\sigma} - \nabla_\sigma h_{\mu\nu}) \tag{3.19}
$$

Since we start with full degrees of freedom (3.8), we choose a gauge to study the propagation of fields. For this purpose, \( \delta R_{\mu\nu} \) can be transformed into the Lichnerowicz operator [42]

$$
\delta R_{\mu\nu} = -\frac{1}{2} \nabla^2 h_{\mu\nu} + R_{\sigma(\nu} h^{\rho)}_{\mu) - R_{\nu\rho\sigma\mu} h^{\rho\sigma} + \nabla_\nu \nabla_\rho h_{\mu\nu} \tag{3.20}
$$

We have to examine whether there exists any choice of gauge which can simplify eqs. (3.10) and (3.11). Conventionally, we choose the harmonic (transverse) gauge \( \nabla_\mu h^{\mu\nu} = \tilde{g}^{\mu\nu} \delta \Gamma^\nu_{\mu\nu} = 0 \).

Considering the harmonic gauge and \( Q_1 = Q_5 \) case for simplicity, eqs. (3.10), (3.11) and (3.17) lead to
\[ + \frac{2Q_1^2}{r^6 f^2 f^{1/3}} \left( 2h_1 + 2h_2 - F_+ - F_+ + H_+ - H_+ + \frac{8}{3} \delta \lambda \right) = 0, \]  
\[ \nabla^2 t + \frac{2}{\alpha'} \left( \frac{f_1}{f_K} \right)^{1/3} t + \frac{8Q_1^2}{r^6 f^2 f^{1/3}} (2t + F_+ - F_+ + H_+ - H_-) = 0, \]  

Now we attempt to disentangle the mixing between \((\delta \nu, \delta \lambda, t)\) and other fields by using both the harmonic gauge and U(1) field equations in eqs. (3.12)–(3.16). After some calculations, one finds the relations

\[ 2F^{(K)} = h_1 + h_2 - h^0_{\theta_1} - \frac{16}{3} \delta \lambda, \]  
\[ 2(F_{\pm} t) = h_1 + h_2 - h^0_{\theta_1} + \frac{8}{3} \delta \lambda - 8 \delta \nu, \]  
\[ 2(H_{\pm} t) = h_1 + h_2 - h^0_{\theta_1} + \frac{8}{3} \delta \lambda + 8 \delta \nu, \]  

where \(h^0_{\theta_1} = h_\chi + h_\phi + h_\phi.\) Using (3.24)–(3.26), one obtains the equations for \(\delta \nu, \delta \lambda, t\) as

\[ \nabla^2 \delta \nu - \frac{8\tilde{Q}_1^2}{r^6 f^2 f^{1/3}} \delta \nu = 0, \]  
\[ \nabla^2 \delta \lambda - \frac{d}{f^{1/3}} h^{rr} \partial^2 \tilde{\lambda} + \frac{d}{f^{1/3}} h^{\mu \nu} \Gamma^r_{\mu \nu} \partial_r \tilde{\lambda} + \]  
\[ + \frac{2}{r^6 f^{1/3}} \left( \frac{\tilde{Q}_1^2}{f_1^2} - \frac{\tilde{Q}_K^2}{f_K} \right) h^0_{\theta_1} - \frac{8}{3r^6 f^{1/3}} \left( \frac{\tilde{Q}_1^2}{f_1} + 2 \frac{\tilde{Q}_K^2}{f_K} \right) \delta \lambda = 0, \]  
\[ \nabla^2 t + \left\{ \frac{2}{\left( \frac{f_1}{f_K} \right)^{1/3}} - \frac{8\tilde{Q}_1^2}{r^6 f^2 f^{1/3}} \right\} t = 0. \]  

We wish to point out that \((\delta \nu, t)\)-equations are decoupled completely but \(\delta \lambda\)-equation still remains a coupled form. In this sense the role of \(\delta \lambda\) remains obscure. Hence we no longer consider this field. Here we observe from (3.27) and (3.29) that if the mass term in (3.29) is absent, two equations are exactly the same form.

4. Potential analysis

From the Bianchi identities (2.11) one has

\[ \partial_\chi F^{(K)} = \partial_\theta F^{(K)} = \partial_\phi F^{(K)} = 0, \]  
\[ \partial_\chi F_{\pm} = \partial_\theta F_{\pm} = \partial_\phi F_{\pm} = 0, \]  
\[ \partial_\chi H_{\pm} = \partial_\theta H_{\pm} = \partial_\phi H_{\pm} = 0. \]  

This implies either \(F^{(K)} = F^{(K)}(t, r), F_{\pm} = F_{\pm}(t, r), H_{\pm} = H_{\pm}(t, r)\) or \(F^{(K)} = F_{\pm} = H_{\pm} = 0.\) The former together with (3.24)–(3.26) means that all higher modes of
\(l \geq 1\) are forbidden in this scheme. Hence we consider only the s-wave \((l = 0)\) propagations. Then the relevant fields become \(\delta \nu(r, t) = \delta \nu(r)e^{i\omega t}\) and \(t(r, t) = t(r)e^{i\omega t}\), but the graviton modes \(h_{\mu\nu}\) are irrelevant to our interest. For \(r_1 = r_5 = R\), the equations \((3.27)\) and \((3.29)\) lead to

\[
\begin{align*}
\left\{ \begin{array}{l}
-3 \partial_r (dr^3 \partial_r) - d^{-1} f \partial_t^2 - \frac{8R^4}{r^2(r^2 + R^2)^2} \left( 1 + \frac{r_0^2}{R^2} \right) \delta \nu = 0, \\
-3 \partial_r (dr^3 \partial_r) - d^{-1} f \partial_t^2 + \frac{2}{(1 + \frac{r_0^2}{r^2})^{3/2} (1 + \frac{R^2}{r^2})^{1/3}} - \frac{8R^4}{r^2(r^2 + R^2)^2} \left( 1 + \frac{r_0^2}{R^2} \right) t = 0,
\end{array} \right. \\
\end{align*}
\]

(4.2)

Considering \(N = r^{-3/2} \tilde{N}\), for \(N = \delta \nu, t\) and introducing a tortoise coordinate \(r^* = \int (dr/d(r)) = r + (r_0/2) \ln ((r - r_0)/(r + r_0))\), then the equation takes the form

\[
\frac{d^2 \tilde{N}}{dr^{*2}} + (\omega^2 - \tilde{V}_N) \tilde{N} = 0.
\]

(4.3)

Here we take \(r_0 = r_K\) for simplicity. In the dilute gas limit \((R \gg r_0)\), \(\tilde{V}_N(r)\) is given by

\[
\tilde{V}_\nu(r) = -\omega^2(f - 1) + h \left\{ \frac{3}{4r^2} \left( 1 + \frac{3r_0^2}{r^2} \right) + \frac{8R^4}{r^2(r^2 + R^2)^2} \right\},
\]

(4.4)

\[
\tilde{V}_t(r) = -\omega^2(f - 1) + h \left\{ \frac{3}{4r^2} \left( 1 + \frac{3r_0^2}{r^2} \right) + \frac{8R^4}{r^2(r^2 + R^2)^2} - \frac{2}{(1 + \frac{r_0^2}{r^2})^{3/2}(1 + \frac{R^2}{r^2})^{1/3}} \right\},
\]

where

\[
f - 1 = \frac{r_0^2 + 2R^2}{r^2} + \frac{2r_0^2 + R^2}{r^4} + \frac{r_0^4R^4}{r^6}.
\]

(4.5)

We note that \(\tilde{V}_N\) depends on two parameters \((r_0, R)\) as well as the energy \((\omega)\). As \((4.3)\) stands, it is far from the Schrödinger-type equation. The \(\omega\)-dependence is a matter of peculiar interest to us compared with the Schwarzschild black hole potentials \((V_{RW}, V_Z, V_p)\). This makes the interpretation of \(V_N\) as a potential difficult. This arises because \((f - 1)\) is very large as \(10^6\) for \(r_0 = 0.01, R = 0.3\) in the near-horizon. In order for \(\tilde{V}_N\) to be a potential, it is necessary to take the low energy limit of \(\omega \to 0\). It is suitable to be \(10^{-3}\). And \(\omega^2(f - 1)\) is of order \(O(1)\) and thus it can be ignored in comparison with the remaining ones. Now we can define a potential \(V_N = \tilde{V}_N + \omega^2(f - 1)\). Hence, in the low energy limit \((\omega \to 0)\), eq. (4.3) becomes as the Schrödinger-type equation.

**Figure 1:** The graph of \(V_\nu(r), V_t(r)\) in the near-horizon region with \(r_0 = 0.01, R = 0.3\).
First we consider the near-horizon geometry, which corresponds to the limit of $r \to r_0$ such that the dilute gas limit ($R \gg r_0$) holds. In this limit one finds $\text{AdS}_3 \times S^3 \times T^4$. In the near-horizon, as is shown in fig. 1, the potential of the fixed scalar ($V_\nu(r)$) takes exactly the same form of the tachyon ($V_t(r)$). This means that in the near-horizon the roles of the fixed scalar ($\nu$) and the tachyon ($T$) are the same. Now let us observe the far-region behavior of their potentials. In the asymptotic region ($r \gg R$) $V_\nu$ approaches to zero, whereas $V_t$ goes to $-2$ (see fig. 2). With $V_t$ one finds an exponentially growing mode ($e^{\omega t}, \omega = -i\alpha$) from eq. (4.3). This implies that $\nu$ plays a role of the good test field, while $t$ induces an instability of the flat space.

5. Discussions

Let us first discuss the role of a fixed scalar $\nu$ in the greybody factor calculation (Hawking radiation). Although $\nu$ is related to the scale of $T^4$, it turns out to be the 10D dilaton ($\phi_{10}$) when $\phi_6 = \phi_{10} - 2\nu = 0$. For $Q_1 = Q_5$ case, one finds the same linearized equation for the harmonic, dilaton gauge, and K-K setting. This means that the fixed scalar ($\nu$) gives us a gauge-invariant result. In the low energy limit ($\omega \to 0$), the s-wave semiclassical greybody factor takes the form

$$
\sigma_\nu^{\nu} = \frac{A_H}{4} \left( \frac{r_0}{R} \right)^4.
$$

On the other hand, $\lambda (= \nu_5 - \phi_6/2)$ is entirely determined by the scale ($\nu_5$) of the KK circle ($S^1$) when $\phi_6$ is turned off. The semiclassical result of its greybody factor takes the form

$$
\sigma_\lambda^{\lambda} = \frac{9}{4} A_H \left( \frac{r_0}{R} \right)^4.
$$

However, in the previous work we found out that $\lambda$ depends on the gauge choice. The fixed scalar $\nu$ is clearly understood as a good test field for studying the D5±-D1± brane black hole. On the other hand, the role of $\lambda$ as a test field is obscure because it is a gauge-dependent field and gives rise to some disagreement for the cross section. The tachyon plays the same role of $\nu$ in the near-horizon geometry. But this induces an instability of Minkowski space (see fig. 2). Hence the tachyon seems not to be a good test field to investigate the quantum aspect of the D5±-D1± brane black hole.
Finally let us comment on the stability problem of the near-horizon geometry. It is known that while the Minkowski vacuum is unstable in type 0 string theory, the $\text{AdS}_5 \times S^5$ with self-dual 5-form flux should be a stable background for sufficiently small radius $\text{AdS}_3 \times S^3 \times T^4$. The RR fields work to stabilize the tachyon in the near-horizon. It is clear from Fig. 1 that the $\text{AdS}_3 \times S^3 \times S^1$ is stable because $V_\nu(r)$ and $V_t(r)$ take the shapes of the potential barrier. From eq. (3.25), if there do not exist the RR fields ($F_\pm, H_\pm$), one finds a potential well for the tachyon, which induces an instability in the near-horizon $V_\nu(r)$. However, thanks to the RR fields, one obtains a potential barrier. This shows obviously that the RR fields can work to stabilize the tachyon in the near-horizon.

Acknowledgments

This work was supported by the Brain Korea 21 Program, Ministry of Education, Project No. D-0025.

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