Ion sound and dust acoustic waves at finite size of plasma particles

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We consider influence of finite size of ions on properties of classic plasmas. We focus our attention on the ion sound for electron-ion plasmas. We also consider dusty plasmas, where we account finite size of ions and particles of dust and consider the dispersion of dust acoustic waves. Finite size of particles affects classical plasma properties. Finite size of particles gives considerable contribution for small wave lengths, which is area of appearing of quantum effects. Consequently, it is very important to consider finite size of ions in quantum plasmas as well.

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I. INTRODUCTION

Studying plasmas we assume that all particles are point-like objects. It always correct for electrons, since they have not reveal their finite size in any fundamental effects. So they are considered to be point-like even at high energy collisions reaching on accelerators. However, ions have finite radius of order of several of the Bohr radius. Considering classical gas plasma we have dial with waves having wavelength much more interparticle distances, which is more than radius of ions. We have different situations in solids and quantum plasmas, where it is important to consider wavelengths of order of lattice constants, which have same order as size of ions. Consequently it is necessary to consider contribution of radius of ions in plasma of metals and astrophysical quantum plasmas. There are dusty plasmas containing particles of dust with sizes much more of size of a single molecule. Consequently we have to include size of particles of dust to get correct theory of dusty plasmas.

Main goal of this paper is to account size of ions and dust particles in hydrodynamic equations and the quantum hydrodynamics as well. We present a method of finite size consideration for general hydrodynamic equations. We particularly describe consequence of finite sizes for linear waves.

As application of developed approximation we consider linear waves in electron-ion and electron-ion-dusty plasmas in absence of external fields.

Quantum plasmas reveals effects related to the quantum Bohm potential and spin dynamics [1]-[3]. Quantum effects change well-known properties of classic plasmas (for recent examples see Refs. [2]-[11]). Moreover, quantum plasmas open possibility for existence new effects enriching plasmas physics. As examples of new effects we can mention existence of new types of waves due to spin evolution [12], [13], [14], appearance of instabilities caused by neutron beams propagation through magnetized plasmas due to the spin-spin and spin-current interaction of neutron spin with spins and currents in plasmas [14], [15], and existence of quantum vortical structures [16], [17]. Methods of dealing with the short-range interaction between quantum particles was developed in Ref. [15].

Quantum effects and effects of finite size of particles give correction to classical effects particularly important for dense plasmas, where wave length of collective excitations can be comparable with radius of ions. Consequently quantum properties of plasmas can be under stronger influence of ion size than classical properties.

Classical properties of dusty plasmas can be affected by size of particles of dust even more than size of ions due to large average radius of dust particles. Physics of dusty (complex) plasmas reveals to be rather complicated topic, which includes different properties of dust some of them are presented in reviews [19], [20], [21]. Simultaneous consideration of many properties of dust makes study of waves in dusty plasmas almost impossible task. So waves have been studied in terms of simpler approaches.

Hydrodynamic and kinetic models have been used for description of collective excitations in dusty plasmas. Dust has been considered as an extra species of particles having large mass and charge in compare with the mass and charge of ions and neutrals (see for instance [22] for hydrodynamics and [23] for kinetics). Large size of dust leads to a large collision rate between dust and ions and electrons [24], [25], [26]. It can also reveals in sticking or coming off of electrons and ions to dust giving change of charge of particles of dust. The effect of dust size distribution is considered in Refs. [27], [28], [29], [30]. In this model dust is presented as set of several species with different charges $q_{dk}$, masses $m_{dk}$. In this case the Poisson equation has form

$$\Delta \varphi = -4\pi \left( e n_i - e n_e + \sum_k q_{dk} n_{dk} \right). \quad (1)$$

Further development of the model includes that the dust grains size distribution can be considered as continuous. However, the size of dust particles is not included explicitly in this model.
The electron and ion thermal forces acting on a charged grain were discussed in the context of dusty plasmas in Ref. [31], these forces arise due to temperature gradients in dusty plasmas.

Magnetohydrodynamics of dusty plasmas was considered by P. K. Shukla and H. U. Rahman in Ref. [32]. Extended hydrodynamic model including equation for the pressure evolution was also used for study of dusty plasmas [33]. Hydrodynamics of partially ionized dusty magnetoplasmas are presented in Ref. [34].

The charge fluctuation equation for the dust charge is suggested and used in literature since charge of dust particles changes during inelastic collisions and interaction with radiation see Refs. [35], [36]. An example of inelastic collisions consideration in terms of the collision integral in kinetic theory can be found in Ref. [37].

Dusty plasmas reveals existence of sound waves. This topic has been studied for a long time [38]-[41]. Different classical effects on wave properties of dusty plasmas were reviewed by V. E. Fortov et al. in Ref. [20]. See also recent discussion given by A. A. Mamun and P. K. Shukla [42].

Influence of a polarization force on dust acoustic waves has been considered as well [43], [44]. The polarization force \( F_p \) reveals in changing of the electric force \( QnE = -Qn\nabla \varphi \). Considering the electric force and the polarization force together we get dressed electric force \( F_\mathcal{E} = F + Qn\nabla \varphi (1 - \frac{3}{2}Qe/\lambda_D T_i) \cdot (1 - \frac{r^2}{R^2}) \), where \( Q \) is the charge of dust particles, \( T_e(T_i) \) is the temperature of electrons (ions), \( \lambda_D \) (see [46] and discussion in beginning of Ref. [49]).

This paper is organized as follows. In Sec. II we show a method of introduction of finite size of ions formulating new approach. In Sec. III we consider waves in electron-ion plasmas focusing our attention on dispersion of ion-sound. In Sec. IV we calculate dispersion of dust-sound. In Sec. V brief summary of obtained results is presented.

II. MODEL

Using the self-consistent field-Poisson approximation for electric field in plasma we can write hydrodynamic equations for electron-ion plasmas in the following form

\[
\partial_t n_e + \nabla (n_e v_e) = 0, \quad (2)
\]

\[
m_e n_e (\partial_t + v_e \nabla) v_e + \nabla p_e = q_e n_e E, \quad (3)
\]

\[
\partial_t n_i + \nabla (n_i v_i) = 0, \quad (4)
\]

and

\[
m_i n_i (\partial_t + v_i \nabla) v_i + \nabla p_i = q_i n_i E, \quad (5)
\]

with \( m_e \) and \( m_i \) (\( q_e, q_i \)) are the masses (charges) of electrons and ions, \( n_e \) and \( v_e \) are the particle concentrations and velocity fields for electrons \( e = e \) and ions \( a = i \), \( \partial_t \) is the partial derivative on time, \( \nabla \) is the spatial derivative (the gradient operator), \( p_e \) is the thermal pressure, \( E \) is the self-consistent electric field.

We have set of hydrodynamic equations (the continuity and Euler equations) for each species containing self-consistent electric field. This field is created by electrons and ions. Thus hydrodynamic equations are coupled by the quasi-electrostatic Maxwell equations

\[
\nabla E(r, t) = 4\pi \sum_a q_a n_a(r, t), \quad (6)
\]

\[
\nabla \times E(r, t) = 0, \quad (7)
\]

where (6) is the Poisson equation.

At derivation of hydrodynamic equations they appear in an integral form. Non-integral form of hydrodynamic equations is obtained at explicit introducing of the electric field created by electrons and ions

\[
\varphi_a(r, t) = q_a \int G(r, r') n_a(r', t)dr', \quad (8)
\]

with \( E_a(r, t) = -\nabla \varphi_a(r, t) \) as usual. Total electric field \( E = E_e + E_i \) satisfies the Maxwell equations (6) and (7).

For consideration of particles with finite radius we need to go back to integral form of hydrodynamic equations: the continuity equation for electrons

\[
\partial_t n_e + \nabla (n_e v_e) = 0, \quad (9)
\]

the Euler equation (the momentum balance equation) for electrons in the integral form

\[
m_e n_e (\partial_t + v_e \nabla) v_e + \nabla p_e = -q_e n_e (r, t) \left( q_e \int dr' G(r, r') n_e(r', t) \right)
\]

\[
+ q_i \int dr' G(r, r') n_i(r', t), \quad (10)
\]

the continuity equation for ions

\[
\partial_t n_i + \nabla (n_i v_i) = 0, \quad (11)
\]

and the Euler equation for ions in the integral form

\[
m_i n_i (\partial_t + v_i \nabla) v_i + \nabla p_i = -q_i n_i (r, t) \left( q_e \int dr' G(r, r') n_e(r', t) \right)
\]

\[
+ q_i \int dr' G(r, r') n_i(r', t), \quad (12)
\]
where
\[ G(r, r') = \frac{1}{|r - r'|} \]  

is the Green function for the Coulomb interaction. Integrals in equations (10) and (12) are over whole space. Thus a point \( r - r' \) is also included. It corresponds to point like particles. For consideration of finite radius of ions we need to restrict area of integration taking integral over whole space except a sphere of radius \( r_0 = r_i \) for electron-ion interaction, and \( r_0 = 2r_i \) for ion-ion interaction, where \( r_i \) is a radius of ions.

In quantum plasmas there is additional quantum pressure: the quantum Bohm potential. Thus the left-hand side of the Euler equation contains

\[ \nabla p_a \to \nabla p_a - \frac{\hbar^2}{4m} \nabla \delta n + \frac{\hbar^2}{4m} \dot{\vartheta} \left( \frac{\nabla n \cdot \dot{\vartheta} n}{n} \right). \]  

In the linear approximation the quantum Bohm potential gives contribution in the thermal velocity given by the temperature of the system (the Fermi temperature for degenerate plasmas).

### III. LINEAR WAVES IN ELECTRON-ION PLASMAS

In linear approximation on small perturbations of hydrodynamic variables \( \delta n = N \exp(-i \omega t + i k r) \) and \( \delta v = U \exp(-i \omega t + i k r) \) the set of hydrodynamic equations has following form: the linearized form of the continuity equation for electrons

\[ -i \omega \delta n_e + n_0 e \nabla \delta v_e = 0, \]  

the linearized form of the Euler equation for electrons

\[ -i \omega m_e n_0 e \delta v_e + m_e v_e^2 \nabla \delta n_e \]

\[ = -q_e n_0 e \left( q_e \nabla \delta n_e \cdot \int_{0}^{+\infty} dr' G(r, r') e^{ik(r'-r)} + q_i \nabla \delta n_i \cdot \int_{r_0}^{+\infty} dr' G(r, r') e^{ik(r'-r)} \right), \]

the linearized form of the continuity equation for ions

\[ -i \omega \delta n_i + n_0 i \nabla \delta v_i = 0, \]  

and the linearized form of the Euler equation for ions

\[ -i \omega m_i n_0 i \delta v_i + m_i v_i^2 \nabla \delta n_i \]

\[ = -q_i n_0 i \left( q_i \nabla \delta n_i \cdot \int_{r_0}^{+\infty} dr' G(r, r') e^{ik(r'-r)} + \left( -\omega^2 + v_T^2 k^2 + \omega_{Le}^2 \right) \delta n_i = 0, \]

where \( \omega_{Le} = \frac{4 \pi q_e^2 n_0 e}{m_i} \) is the Langmuir frequency.

Dispersion of the ion-sound appears in the following range of frequencies

\[ kv_T i \ll \omega \ll kv_T e. \]

Using this approximation we can neglect by "blue" terms in set of equations \([15] \) and \([16] \) (the first term in the first
group of terms in equation (23) and the second term in the second group of terms in equation (24).

In standard approach of point like ions one finds solution for the dispersion of the ion sound

$$\omega^2 = \frac{\omega_{Li}^2}{1 + \frac{k^2 v_s^2}{v_{Te}^2}} = \begin{cases} k^2 v_s^2 & \text{for } k^2 r_{De}^2 \ll 1 \vspace{1mm} \\
\omega_{Li}^2 & \text{for } k^2 r_{De}^2 \gg 1 \end{cases} \quad (26)$$

where $r_{De} = v_{Te}/\omega_{Le}$ is the electron Debye radius.

General form of the ion-sound dispersion including finite size of ions appears to be

$$\omega^2 = \omega_{Li}^2 \cos(2\pi r_0 k) - \frac{\omega_{Li}^2}{v_{Te}^2 k^2} \sin^2(\pi r_0 k) \quad (27)$$

In “short” wave length limit $r_0 k \gg 1$ (or in other form it looks like $v_{Te} k / \omega_{Le} \gg 1$) we find dispersion of the ion-sound at account of the finite size of ions

$$\omega^2 = \omega_{Li}^2 \cos(2\pi r_0 k) \quad (28)$$

In the large wave length limit $r_0 k \ll 1$ we obtain

$$\omega^2 = v_{Te}^2 k^2 \left( \cos(2\pi r_0 k) - \frac{\omega_{Li}^2}{v_{Te}^2 k^2} \sin^2(\pi r_0 k) \right) \quad (29)$$

where $v_s^2 = m_e v_{Te}^2 / m_i$.

We have found contribution of finite size of ions in dispersion of the ion sound.

### A. Estimation of approximation

Paying attention to quantum plasmas let us make a note on magnitude of possible wavelengths. In classic physics the wave length of matter waves is limited by average interparticle distance. In quantum mechanics we hit the de-Broglie wave nature of particles, hence matter waves can continuously convert into collective quantum excitations with wave lengths smaller than interparticle distance.

In classical plasmas $1/k \gg r_0$. Consequently $\cos(r_0 k)$ is slightly lower than 1. In this limit contribution of the quantum Bohm potential is negligible since it several orders smaller than $v_{Te}^2 k^2$.

In extremely large wave vector limit, which can be reached at high density objects the contribution of the quantum Bohm potential can be comparable with the ”thermal” contribution, which has to be replaced by the contribution of Fermi pressure for degenerate objects.

Getting to quantum limit we see that $1/k$ becomes smaller getting closer to $r_0$, so $r_0 k$ grows up to $\pi$. As consequences we find that contribution of the Langmuir frequencies (all except the electron Langmuir frequency) can be made equal to zero or change sign.

Dispersion equation for small perturbations in quantum electron-ion plasmas allows to consider effects described above as follows

$$\left( -\omega^2 + v_{Te}^2 k^2 + \frac{\hbar^2}{4m_i^2} k^4 + \omega_{Le}^2 \right) \delta n_e = 0, \quad (30)$$

and

$$\frac{4\pi q_e q_i n_0}{m_i} \cos(q_k r_0) \delta n_i = 0. \quad (31)$$

Blue terms in formulas (30) and (31) are to be neglected at consideration of the classic ion-sound without account of finite size of ions. In quantum plasmas we may get to the short wave-length limit and contribution of the quantum Bohm potential becomes considerable. In this case contribution of finite size of particles is also very important and is more considerable than in classic case described above.

At $r_0 k = \pi / 2$ equations (30) and (31) become independent. Conditions for existence of nonzero perturbations $\delta n_e$, $\delta n_i$ are

$$\omega^2 = \omega_{Li}^2 + \frac{\pi^2 v_{Te}^2}{4 r_0^2} + \frac{\pi^4 \hbar^2}{4 m_i^2} \left( \frac{1}{2 r_0} \right)^4 \quad (32)$$

for wave of electrons, and

$$\omega^2 = \frac{\pi^2 v_{Te}^2}{4 r_0^2} + \frac{\pi^4 \hbar^2}{4 m_i^2} \left( \frac{1}{2 r_0} \right)^4 - \omega_{Li}^2 \quad (33)$$

for waves of ions.
At large wave vectors $k \in (\frac{2 \pi}{r_0}, \frac{\pi}{4 r_0})$ formula (27) gives imaginary frequency. Formally this condition arises as $\cos 2 r_0 k - \frac{m}{2 e^2} k^2 \sin^2 r_0 k \approx \cos 2 r_0 k < 0$. The frequency becomes real in area of larger wave vectors $k \in (\frac{2 \pi}{4 r_0}, \frac{\pi}{4 r_0})$.

IV. LINEAR WAVES IN ELECTRON-ION-DUSTY PLASMAS

Let us present a brief description of process have been considered in quantum dusty plasmas. The QHD model was used to study different small amplitude excitations in quantum dusty magnetoplasmas paying attention to the quantum Bohm potential \[54], \[51]. Low frequency non-linear waves in quantum dusty plasmas are considered in Ref. \[52] assuming that electrons and ions obey the Thomas-Fermi distribution. Hydrodynamical equations were applied to dust, but it was done with no account of the quantum Bohm potential. Solitons in quantum plasmas were considered in Ref. \[53] in terms of quantum electrons and ions were considered as inertialess, in quantum dusty plasmas of par-\[54]. Dusty plasmas of particles with internal magnetization were analyzed in Ref. \[54]. However, the quantum Bohm contributions in the Thomas-Fermi distribution. Hydrodynamical equations were applied to dust, but it was done with no account of the quantum Bohm potential. Solitons in quantum plasmas were considered in Ref. \[53] in terms of quantum Bohm potential \[50], \[51]. Low frequency non-linear waves in quantum dusty plasmas are consid-\[52] to dust, but it was done with no account of the quantum Bohm potential. Solitons in quantum plasmas were considered in Ref. \[53] in terms of quantum Bohm potential \[50], \[51]. Low frequency non-linear waves in quantum dusty plasmas are consid-

![FIG. 2: (Color online) The figure shows a system of electrons, ions and dust particles. We have chosen dust particles have charge $Q = 10e$. The electrons, ions, dust particles are presented by small, intermediate, and large circles correspondingly. Size of ions is about $r_0 = 100nm$. Figure does not show rate of radiuses of ions and dust particles. Picture presents the fact that dust particles is larger than ions.](image.png)
We find solution in assumption of high temperature of electrons and ions: \( n_e = n_0 (1 - q_0 \rho^2 / T_0) \). In this case the set of equations (36) and (37) can be represent as

\[
-m_d (\omega^2 - k^2 v_T^2) \delta n_d = -Q n_0 \left( Q k \nabla \int G \delta n_d d\mathbf{r'} \right) + e^2 \left( \frac{n_e}{T_e} + \frac{n_i}{T_i} \right) k \nabla \int G \varphi d\mathbf{r'},
\]

(38)

where we included quasi-neutrality condition \( \int G(\mathbf{r}, \mathbf{r'}) (-e n_0 + e n_0 + Q n_0) d\mathbf{r'} = 0 \).

Taking integrals we get final form of dispersion equation

\[
m_d (\omega^2 - k^2 v_T^2) \delta n_d = -Q^2 n_0 k \cos(k r_d) \left[ \frac{4 \pi}{k^2} k \nabla \delta n_d \right]
- e^2 \left( \frac{4 \pi}{k^2} \right)^2 \cos(2 k r_d) \left( \frac{n_e}{T_e} + \frac{n_i}{T_i} \right) k \nabla \delta n_d \]

(39)

giving following solution for frequency of waves in dusty plasmas

\[
\omega^2 = v_T^2 k^2 + \frac{4 \pi Q^2 n_0}{m_d} \cos(k r_d) \times
\left[ 1 - \frac{4 \pi e^2}{k^2} \cos(2 k r_d) \left( \frac{n_e}{T_e} + \frac{n_i}{T_i} \right) \right].
\]

(40)

Neglecting the first term for cold dust we can rewrite formula (40) in the traditional for the dust acoustic wave form, which was obtained for the first time in Ref. [38] for point-like particles,

\[
\omega^2 = \beta^2 C_s^2 k^2 \left[ 1 + \frac{\lambda^2_D k^2}{(1 + \eta \delta) \cos(2 k r_d)} \right]
\]

(41)

at \( \frac{\lambda^2_D k^2}{1 + \eta \delta} \gg 1 \) and \( k r_d \ll \pi / 2 \). In formula (41) we have use next designations \( \beta = Z (\delta - 1) / (1 + \eta \delta), C_s = \sqrt{T_e / m_d}, \lambda_D = \sqrt{T_e / (4 \pi e^2 n_0)}, \eta = T_e / T_i, \delta = n_0 / n_0 \).

Some interesting consequences of finite size of particle in context of wave dispersion in dusty plasmas can be found from formula (40), when we do not apply conditions \( \frac{\lambda^2_D k^2}{1 + \eta \delta} \gg 1 \) and \( k r_d \ll \pi / 2 \). Size of dust particles is more than size of ions so we can get \( k r_d \in (\pi / 4, \pi) \) in classic regime. So we can consider consequences revealing in damping of the dust acoustic waves.

At \( (1 + \eta \delta) / (\lambda^2_D k^2) < 1 \) we have that the square brackets in formula (40) is positive. And the sign of the second term in the formula is determined by \( \cos(k r_d) \).

So this term becomes negative at \( k r_d \in (\pi / 4, \pi) \). Let us also consider behaviour of terms within the square brackets. Presence of \( \cos(2 k r_d) \) gives decreasing of the second term in square brackets with increasing of \( k r_d \in (0, \pi / 4) \), and it changes sign at \( k r_d = \pi / 4 \) being negative at \( k r_d \in (\pi / 4, 3 \pi / 4) \) and increasing its module at \( k r_d \in (\pi / 4, \pi / 2) \). At \( k r_d \in (\pi / 2, 3 \pi / 4) \) the second term is negative and its module decreased in compare with zero-size particles. Finally we see that at \( k r_d \in (\pi / 2, 3 \pi / 4) \) both the contribution of \( \cos(k r_d) \) and the contribution of square particles becomes negative. Hence damping of dust acoustic waves may not appear.

At \( \xi = (1 + \eta \delta) / (\lambda^2_D k^2) > 1 \) behaviour of \( \cos(2 k r_d) \) gives dramatic contribution of the square brackets. This case allows to reach the following identity \( 1 - \xi \cos(2 k r_d) = 0 \). That makes the second term in formula (40) equals to zero, so frequency appears as \( \omega^2 = v_T^2 k_d^2, \) where \( k_0 \) is given by a transcendental equation \( k_0 = 1 / 2 \pi \arccos(1 / \xi (k_0)). \) Going further we may admit that \( \cos(2 k r_d) > 0 \) at \( 2 k r_d \in [0, \pi / 4) \cup (3 \pi / 4, 5 \pi / 4) \). However it is not enough to reach \( 1 - \xi \cos(2 k r_d) < 0 \). To this end we have to consider \( k r_d \in [0, k_0 r_d) \cup (\pi - k_0 r_d, k_0 r_d). \) We have two consequences of \( 1 - \xi \cos(2 k r_d) < 0 \) for each of two intervals presented above. At small \( k \) when \( \cos(k r_d) > 0 \) we get instability of the spectrum [41]. In area \( k r_d \in (\pi - k_0 r_d, k_0 r_d) \) when \( \cos(k r_d) < 0 \), in opposite, we find stabilization of instability presented above for \( \xi < 1. \)

We can also consider now limit case of spectrum [11] at small \( k \) applying \( \cos(k r_d) \approx 1 - k r_d / 2 \) that gives approximate dispersion dependence of the dust acoustic waves for finite size particles

\[
\omega^2 = \beta^2 C_s^2 k^2 \left[ 1 - \frac{(k r_d)^2}{2} \right] \times
\left[ 1 + \frac{\lambda^2_D k^2}{(1 + \eta \delta)} \left( 1 + 2(k r_d)^2 \right) \right].
\]

(42)

The traditional dust acoustic solution does not exists at \( \cos(k r_d) \approx 0 \) and \( \cos(2 k r_d) \approx 0 \). In the first case the right-hand side of equation (39) equals to zero and we get \( \omega = k v_T d = \frac{\pi v_T}{2 \pi d}. \) In the second of these cases the second term in the second group of terms formula (40) equals to zero and we have \( \omega^2 = \pi v_T^2 / 16 d^2 + 2 \pi Q^2 n_0 / m_d \) at wave vectors \( k \approx \frac{\pi}{4 \sigma}. \)

V. CONCLUSION

We have presented a mechanism changing the electric force in electron-ion plasmas due to finite size of ions. This mechanism is also important for dusty plasmas, where it gives contribution in the force density along with the polarization force. We have considered contribution of the finite size of dust particles for monosized dust particles. We have illustrated contribution of the finite size
of particles in the dispersion of low-frequency waves. We have also shown that the finite size of particles affects properties of quantum plasmas. Presented method consideration of size of ions and dust particles opens possibilities for consideration of various linear and non-linear effects in different classic and quantum plasmas. This method can also be applied for kinetic equation as well.

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