Suppression of Flavor Symmetry Breaking in $B$ Decay Sum Rules

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ABSTRACT

While flavor symmetries are useful for studying hadronic $B$ decays, symmetry relations for amplitudes and decay rates are usually violated by first order symmetry breaking corrections. We point out two cases in which first order symmetry breaking is suppressed by a small ratio of amplitudes: (1) An isospin sum rule for four $B \to K\pi$ decays, where isospin breaking is shown to be negligible. (2) An SU(3) sum rule for pairs of $B \to K\pi$ and $B \to K\eta_8$, generalized to pairs of $B \to K\pi$, $B \to K\eta$ and $B \to K\eta'$. 

Charmless hadronic $B$ decays provide valuable tests for the pattern of CP violation in the Cabibbo-Kobayashi-Maskawa (CKM) framework. Several methods, using isospin symmetry and flavor SU(3) symmetry relations for decay amplitudes, have been developed for extracting CKM phases [1, 2]. Flavor symmetry relations for amplitudes are expected to involve corrections which are in most cases linear in symmetry breaking parameters. The parameters describing isospin and SU(3) breaking are respectively of order

\[ \epsilon_I \equiv \frac{m_d - m_u}{\Lambda_{\text{QCD}}} \sim 0.03 \quad \text{and} \quad \delta_{\text{SU}(3)} \equiv \frac{m_s - m_d}{\Lambda_{\text{QCD}}} \sim 0.3 . \]  

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Thus, corrections at corresponding levels have been shown to affect the determination of $\alpha$ in $B \to \pi\pi, \rho\pi, \rho\rho$ when applying methods based on isospin [3, 4] and flavor SU(3) [5, 6].

Isospin symmetry has been employed also to derive an approximate sum rule, relating penguin-dominated $B \to K\pi$ decay rates involving a charged pion with those containing a neutral pion [7, 8],

$$\Gamma(B^+ \to K^0\pi^+) + \Gamma(B^0 \to K^+\pi^-) \approx 2[\Gamma(B^+ \to K^+\pi^0) + \Gamma(B^0 \to K^0\pi^0)] \quad (2)$$

Terms quadratic in small ratios of amplitudes, which may violate this sum rule, have been calculated, implicitly in the isospin symmetry limit, and were found to be between one and five percent [9, 10, 11]. Corrections of similar order are expected to originate from isospin symmetry breaking if it is first order, as anticipated on general grounds. These corrections must be considered in order to provide evidence for physics beyond the Standard Model if a deviation from the sum rule is observed.

The purpose of this Letter is to show that the leading terms in the $B \to K\pi$ sum rule do not involve first order isospin breaking. That is, first order isospin breaking corrections are suppressed by a small ratio of tree and penguin amplitudes and may be safely neglected. We will also consider a sum rule combining decay rates for $B^+ \to \pi^+\pi^0, B^+ \to \pi^+\eta$ and $B^0 \to \pi^0\eta$, where both SU(3) breaking and isospin breaking are suppressed by small ratios of amplitudes [12].

We start our discussion with the four $B \to K\pi$ decay amplitudes. There are three isospin-invariant amplitudes,

$$B_{1/2} \equiv (1/2|\mathcal{H}_W^{1/2}|1/2), \quad A_{1/2} \equiv (1/2|\mathcal{H}_W^{1/2}|1/2), \quad A_{3/2} \equiv (3/2|\mathcal{H}_W^{1/2}|1/2). \quad (3)$$

(A fourth one, $C_{3/2} \equiv (3/2|\mathcal{H}_W^{3/2}|1/2)$ vanishes in the isospin limit). The amplitudes are

$$-A(B^0 \to K^+\pi^-) = B_{1/2} - A_{1/2} - A_{3/2},$$

$$A(B^+ \to K^0\pi^+) = B_{1/2} + A_{1/2} + A_{3/2},$$

$$-\sqrt{2}A(B^+ \to K^+\pi^0) = B_{1/2} + A_{1/2} - 2A_{3/2},$$

$$\sqrt{2}A(B^0 \to K^0\pi^0) = B_{1/2} - A_{1/2} + 2A_{3/2}. \quad (4)$$

This implies a well-known quadrangle amplitude relation which is exact in the isospin symmetry limit (for compact notation we denote amplitudes from now on by final states alone) [13]:

$$A(K^0\pi^+) - A(K^+\pi^-) + \sqrt{2}A(K^+\pi^0) - \sqrt{2}A(K^0\pi^0) = 0. \quad (5)$$

We will consider first order isospin breaking in this equation and in Eq. (2).
It is useful to express the isospin amplitudes in terms of contributions corresponding to flavor SU(3) topologies for final states involving two members of the light pseudoscalar octet \([14]\),

\[
B_{1/2} = p + \frac{1}{2}(t + A), \quad A_{1/2} = \frac{1}{6}(-t + 2c + 3A), \quad A_{3/2} = -\frac{1}{3}(t + c). \tag{6}
\]

(In the appendix we perform an equivalent calculation using a tensor language.) Each contribution may involve more than one CKM factor. The terms \(p, t\) and \(c\), contain respectively a penguin amplitude \(P\), a “color-favored” tree amplitude \(T\), and a “color-suppressed” tree amplitude \(C\), each appearing in a given linear combination with either a “color-favored” \((P_{EW})\) or a “color-suppressed” \((P_{EW}^C)\) electroweak penguin amplitude \([15]\),

\[
p \equiv P - \frac{1}{3}P_{EW}^C, \quad t \equiv T + P_{EW}^C, \quad c \equiv C + P_{EW} \tag{7}.
\]

The above contributions obey a simple hierarchy. The dominant term is the penguin amplitude \(P\). Flavor SU(3) and the measured decay rates for \(B \rightarrow K\pi\) and \(B \rightarrow \pi\pi\) \([16]\) show that the two tree amplitudes \(T\) and \(C\) are much smaller than \(P\). For instance \([17, 18]\)

\[
\frac{|T + C|}{|P|} \approx \sqrt{2}V_{us}f_K \frac{B(B^+ \rightarrow \pi^+\pi^0)}{B(B^+ \rightarrow K^0\pi^+)} = 0.19 \pm 0.02. \tag{8}
\]

The two electroweak penguin contributions, \(P_{EW}\) and \(P_{EW}^C\), may be related by SU(3) to \(T\) and \(C\) \([19]\), and are found somewhat smaller than the latter. The smallest contribution is the annihilation amplitude \(A\), which is expected to be suppressed by \(1/m_b\) relative to the two tree amplitudes \(T\) and \(C\) in an approach based on Soft Collinear Effective Theory \([20]\),

\[
\frac{|A|}{|T|} \sim \frac{|A|}{|C|} \sim \frac{\Lambda_{QCD}}{m_b/2} \sim 0.2. \tag{9}
\]

Thus, we will use

\[
|A| \ll |t|, |c| \ll |p|, \tag{10}
\]

where the successive hierarchy suppression factor is about 0.2.

The isosinglet amplitude \(B_{1/2}\) is the only one containing the dominant amplitude \(p\), while the isovector amplitudes \(A_{1/2}\) and \(A_{3/2}\) are subdominant. Eq. (3) is obeyed separately for the dominant and subdominant isospin amplitudes. The vanishing of the three contributions \(B_{1/2}, A_{1/2}\) and \(A_{3/2}\) in this linear combination of hadronic matrix elements holds for any \(\Delta I = 0\) and \(\Delta I = 1\) operators. This is the crucial point in understanding why first order isospin breaking vanishes in the leading terms in Eq. (3).

Consider isospin breaking due to the \(d\) and \(u\) quark mass difference or due to their charge difference. The “spurion” operator representing all these differences transforms like a sum of \(\Delta I = 0\) and \(\Delta I = 1\) operators. The \(\Delta I = 0\) operator in the effective Hamiltonian, with the dominant \(B \rightarrow K\pi\) matrix element \(B_{1/2}\), becomes sum of \(\Delta I = 0\) and \(\Delta I = 1\) operators. The linear combination of matrix elements appearing in Eq. (3) vanishes for these operators as it would for any combination of \(\Delta I = 0\) and \(\Delta I = 1\) operators.
To illustrate this general behavior, let us consider first order isospin breaking effects, which can be visualized diagrammatically in terms of quark mass insertions on quark lines [21]. We will distinguish between four types of effects: (i) A spectator effect in \( p, t, c \) which we denote by parameters \( \epsilon_S^p, \epsilon_S^t, \epsilon_S^c \), each representing a sum of \( \Delta I = 0 \) and \( \Delta I = 1 \) corrections. Thus, for a spectator \( d \)-quark one has \( p_d \equiv p, t_d \equiv t, c_d \equiv c \) while for a spectator \( u \)-quark, \( p_u \equiv (1 + \epsilon_S^p)p, t_u \equiv (1 + \epsilon_S^t)t, c_u \equiv (1 + \epsilon_S^c)c \). (ii) A \( q\bar{q} \) pair-production (“popping”) effect in \( p \), distinguished similarly by a parameter \( \epsilon_p^q \). (iii) Mixing of \( \pi_3 \equiv (d\bar{d} - u\bar{u})/\sqrt{2} \) with an isospin singlet pseudoscalar, \( \pi_1 \equiv (d\bar{d} + u\bar{u})/\sqrt{2} \), given by a \( \Delta I = 1 \) mixing parameter \( \epsilon_1 \), where \( \pi_0 = \pi_3 + \epsilon_1 \pi_1 \). (iv) The isosinglet state \( \pi_1 \) couples to a new penguin amplitude \( s \) (possibly comparable to \( p \)) involving a pair-production (“popping”) of a light \( q\bar{q} \) pair which is a singlet under both isospin and color. (See discussion below of amplitudes related to the SU(3) singlet \( \eta_1 \)). We expect all five isospin breaking parameters to be roughly equal in magnitude to \( \epsilon_I \) in Eq. (11).

Using these notations, the four physical amplitudes including first order isospin breaking corrections are:

\[
A(K^0\pi^+) = (1 + \epsilon_S^p)p + A,
\]

\[
-A(K^+\pi^-) = (1 + \epsilon_p^q)p + t,
\]

\[
-\sqrt{2}A(K^+\pi^0) = (1 + \epsilon_S^t + \epsilon_p^q - \epsilon_1)p - \epsilon_1 s + (1 + \epsilon_S^t - \epsilon_1)t + (1 + \epsilon_S^c - \epsilon_1)c + A,
\]

\[
\sqrt{2}A(K^0\pi^0) = (1 + \epsilon_1)p + \epsilon_1 s - (1 - \epsilon_1)c.
\]

This implies

\[
A(K^0\pi^+) - A(K^+\pi^-) + \sqrt{2}A(K^+\pi^0) - \sqrt{2}A(K^0\pi^0) = -(\epsilon_S^t - \epsilon_1)t - \epsilon_S^c c .
\]

We see that the leading first order isospin breaking contributions originating in \( p \) (and \( s \)) cancel, as has been argued above. The remaining isospin breaking terms are suppressed by \(|t|/|p|\) and \(|c|/|p|\) relative to these contributions.

Consider now the sum rule (2) for \( B \to K\pi \) decay rates. Using Eqs. (11) we note that first order isospin breaking in the dominant \(|p|^2\) terms (and potential \( \text{Re}(p^*s) \) terms) cancel in this sum rule. Omitting a few subleading terms common to both sides and neglecting terms involving \( A \), the sum rule reads (for simplicity of expressions we will assume everywhere that symmetry breaking parameters are real):

\[
2|p|^2 = 2|p|^2 + 2|c|^2 + \text{Re}(c^*t) + 2(\epsilon_S^p + \epsilon_S^t - 2\epsilon_1)\text{Re}(p^*t) - 2\epsilon_1 \text{Re}(s^*t)
\]

\[
+ 2(\epsilon_S^p + \epsilon_p^q + \epsilon_S^c - 2\epsilon_1)\text{Re}(p^*c) - 4\epsilon_1 \text{Re}(s^*c) .
\]

The first terms, \( 2|c|^2 + \text{Re}(c^*t) \), which violate the sum rule (2) in the isospin symmetry limit, have been estimated to correspond to a ratio \(|c|^2 + \text{Re}(c^*t)|/|p|^2\) in the range \( 1 - 5\% \) [9][10][14]. The remaining subleading isospin breaking terms, of forms \( 2\epsilon_1 \text{Re}(p^*t), 2\epsilon_1 \text{Re}(p^*c), 2\epsilon_1 \text{Re}(s^*t) \) and \( 2\epsilon_1 \text{Re}(s^*c) \), are expected to be smaller by about a factor \( \epsilon_I|p|/|c| \approx 0.15 \) relative to these terms. The neglected terms involving \( A \) are even smaller. Therefore, isospin breaking corrections in the sum rule are negligible in the Standard Model.
We now turn to an SU(3) quadrangle relation for amplitudes involving pairs of $B \to K\pi$ and $B \to K\eta_s$ decays:

$$A(K^+\pi^-) + A(K^0\pi^+) + \sqrt{6}A(K^0\eta_s) - \sqrt{6}A(K^+\eta_s) = 0 \ .$$  \ 

(14)

This relation reads, in terms of SU(3) invariant amplitudes and including leading SU(3) breaking corrections

$$[-p-t] + [p+A] + [p-c] - [p-c-t+A] = 0 \ .$$  \ 

(15)

First order SU(3) breaking caused by the $s$ and $d$ (or $u$) quark mass-difference occurs in the dominant amplitude $p$ and in the doubly suppressed amplitude $A$, while there is no SU(3) breaking in $t$ and $c$. Symmetry breaking in $p$ and $A$ is due to $s\bar{s}$ popping in $B \to \bar{K}\eta_s$ versus $u\bar{u}$ or $d\bar{d}$ popping in $B \to K\pi$. We denote penguin and annihilation amplitudes by the popping $q\bar{q}$ pair, using $\delta^p$ and $\delta^A$ to describe SU(3) breaking in $p$ and $A$. Thus, one has $p_u = p_d \equiv p$, $p_s = (1 + \delta^p)p$, $A_u = A_d \equiv A$, $A_s = (1 + \delta^A)A$, where one expects $|\delta^p| \sim |\delta^A| \sim \delta_{SU(3)} \sim 0.3$. We find that terms multiplying $p$ involving the parameter $\delta^p$ cancel in Eq. (14),

$$[-p-t] + [p+A] + [(1 + 2\delta^p)p - c] - [(1 + 2\delta^p)p - c - t + (1 + 2\delta^A)A] = 2\delta^A A \ .$$  \ 

(16)

Higher order SU(3) breaking corrections in $p$ caused by the $s$ and $d$ (or $u$) quark mass-difference are given by an arbitrary number of $s\bar{s}$ insertions. Since $s\bar{s}$ and the operator contributing to $p$ are both $I = 0$ operators, SU(3) breaking in $p$ from this source alone without isospin breaking cancels in (14) to all orders.

The remaining first order SU(3) breaking correction proportional to $\delta^A$ is suppressed by $|A|/|p| \sim (0.2)^2$ relative to each of the four amplitudes which are dominated by $p$. At this low level the corrections from first order isospin breaking effects could be of the same order. Including isospin breaking spectator corrections in $p, t$ and $c$ as discussed above, Eq. (14) now reads:

$$A(K^+\pi^-) + A(K^0\pi^+) + \sqrt{6}A(K^0\eta_s) - \sqrt{6}A(K^+\eta_s) = 2\delta^A A - 2\delta^p\epsilon^p_s|p\epsilon^c_s t + \epsilon^c_s c \ .$$  \ 

(17)

The right-hand side consists of four terms representing first order SU(3) breaking in $A$, first order SU(3) breaking and first order isospin breaking in $p$, and first order isospin breaking in $t$ and $c$. All four terms are expected to be of comparable magnitudes and are comparable to the isospin breaking corrections appearing in the $K\pi$ quadrangle relation Eq. (12).

The decay rates of the four processes in Eq. (14) obey an approximate sum rule, relating processes involving a neutral kaon with those containing a charged kaon:

$$\Gamma(K^0\pi^+) + 6\Gamma(K^0\eta_s) \approx \Gamma(K^+\pi^-) + 6\Gamma(K^+\eta_s) \ .$$  \ 

(18)

In terms of SU(3) invariant amplitudes and including leading SU(3) breaking corrections this reads, after omitting subleading terms which are common to both sides:

$$2|p|^2 = 2||p|^2 + |t|^2 + \text{Re}(c^*t) - 2\delta^p \text{Re}(p^*t)\ .$$  \ 

(19)
Indeed, first order SU(3) breaking corrections in the dominant \( |p|^2 \) term cancel in the \([18]\). The remaining SU(3) breaking in the subleading term Re(\(pt\)) is comparable to the two quadratic subleading terms, \(|t|^2\) and Re(\(ct\)), which violate the sum rule in the SU(3) symmetry limit.

So far we have considered decays to states involving the unphysical SU(3) octet state \(\eta_8\). SU(3) breaking mixes the \(\eta_8\) with the SU(3) singlet \(\eta_1\) in the physical \(\eta\) and \(\eta'\). In order to discuss relations for the physical states analogous to Eqs. \([17]\) and \([19]\) for the unphysical case, we use the single mixing angle parametrization, \(\eta = \cos \theta \eta_8 + \sin \theta \eta_1\), \(\eta' = \cos \theta \eta_1 - \sin \theta \eta_8\). \(\theta\) is connected by two gluons attached to quark lines in all possible ways. The annihilation and exchange amplitudes, in which the two gluons are attached to the spectator quark are topologically equivalent to tree amplitudes, \(t_s\) and \(c_s\). For instance, in \(t_s\) a spectator \(u\) and a \(\bar{u}\) from \(b \to \bar{u}\) are connected to \(\eta_1\) by two gluons. The singlet annihilation and exchange amplitudes, which have been often neglected \([23, 24]\), have recently been shown to be of leading order in \(1/m_b\) \([25]\) and will be included here. Of these two amplitudes only \(t_s\) contributes to strangeness changing \(B^0\) and \(B^+\) decays. We expect \(s\) to be dominant and \(t_s\) to be subdominant in \(B \to K\eta_1\), similar to \(p\) and \(t\) in \(B \to K\pi\).

Neglecting the \(1/m_b\)-suppressed contribution \(A\), we write expressions for amplitudes in the SU(3) symmetry limit while keeping \(\theta\) as the only SU(3) breaking parameter (we denote \(S_\theta \equiv \sin \theta, C_\theta \equiv \cos \theta\)) \([26]\):

\[
\begin{align*}
\sqrt{6}A(K^+\eta) &= (C_\theta + 2\sqrt{2}S_\theta)p + 3\sqrt{2}S_\theta s - (C_\theta - \sqrt{2}S_\theta)c - C_\theta t + \sqrt{2}S_\theta(t + t_s) , \\
\sqrt{6}A(K^0\eta) &= (C_\theta + 2\sqrt{2}S_\theta)p + 3\sqrt{2}S_\theta s - (C_\theta - \sqrt{2}S_\theta)c , \\
\sqrt{6}A(K^+\eta') &= (2\sqrt{2}C_\theta - S_\theta)p + 3\sqrt{2}C_\theta s + (\sqrt{2}C_\theta + S_\theta)c + \sqrt{2}C_\theta(t + t_s) + S_\theta t , \\
\sqrt{6}A(K^0\eta') &= (2\sqrt{2}C_\theta - S_\theta)p + 3\sqrt{2}C_\theta s + (\sqrt{2}C_\theta + S_\theta)c .
\end{align*}
\]  

This implies an amplitude relation,

\[
\sqrt{6} \left( C_\theta [A(K^+\eta) - A(K^0\eta)] - S_\theta [A(K^+\eta') - A(K^0\eta')] \right) = A(K^0\pi^+) + A(K^+\pi^-) ,
\]  

which becomes Eq. \([14]\) in the SU(3) symmetry limit, \(\theta = 0\).

Taking the physical \(\eta\) to replace \(\eta_8\) in \([18]\) the approximation

\[
\Gamma(K^0\pi^+) + 6\Gamma(K^0\eta) \approx \Gamma(K^+\pi^-) + 6\Gamma(K^+\eta) .
\]  

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Table I: Branching ratios [16] entering into tests of decay rate sum rules.

| Decay process | $B$ (units of $10^{-6}$) |
|---------------|--------------------------|
| $B^+ \to K^0\pi^+$ | $24.1 \pm 1.3$ |
| $B^+ \to K^+\pi^0$ | $12.1 \pm 0.8$ |
| $B^0 \to K^+\pi^-$ | $18.9 \pm 0.7$ |
| $B^0 \to K^0\pi^0$ | $11.5 \pm 1.0$ |
| $B^+ \to K^+\eta$ | $2.5 \pm 0.3$ |
| $B^0 \to K^0\eta$ | $0.9 \pm 0.6 \pm 0.1$ \(^a\) |
| $B^+ \to K^+\eta'$ | $69.4 \pm 2.7$ |
| $B^0 \to K^0\eta'$ | $63.2 \pm 3.3$ |

\(^a\) Reference [26]

now reads, with the same omissions on both sides as in [19] and to first order in SU(3) breaking,

$$2|p|^2 = 2|p|^2 + |t|^2 + \text{Re}(c^t t - (2\delta p + \sqrt{2} S_\theta)\text{Re}(p^* t) + \sqrt{2} S_\theta \text{Re}(p^* t_s) - 3 \sqrt{2} S_\theta \text{Re}(s^* t)) \right) \right). \tag{24}$$

Flavor SU(3) studies of $B$ decays into two pseudoscalars have shown that $|s| < |p|$ [24], and we have $|S_\theta| \sim |\delta p|$. Therefore this approximation is qualitatively similar to (19).

Finally, we turn to a sum rule obtained from Eqs. [21] for decay rates involving $K\pi$, $K\eta$ and $K\eta'$,

$$\frac{\Gamma(K^0\eta)}{1 - \sqrt{2} \tan \theta} + \frac{\Gamma(K^0\eta')}{1 + \sqrt{2} \cot \theta} + \frac{\Gamma(K^0\pi^+)}{6} \approx \frac{\Gamma(K^+\eta)}{1 - \sqrt{2} \tan \theta} + \frac{\Gamma(K^+\eta')}{1 + \sqrt{2} \cot \theta} + \frac{\Gamma(K^+\pi^-)}{6}. \tag{25}$$

While the leading terms on both sides include dominant terms $|p|^2, |s|^2$ and $\text{Re}(p^* s)$, we neglect on the right-hand side much smaller terms, $|t|^2$ and $\text{Re}(c^* t)$, as we did in Eqs. [18] and [23]. In fact, in the SU(3) symmetry limit (including $\theta = 0$) Eq. [25] becomes identical to Eq. [18]. SU(3) breaking corrections in the above three dominant terms cancel. First order SU(3) breaking corrections occurring in subleading terms in the sum rule [25] are $(2/3)\delta p \text{Re}(p^* t) + \sqrt{2} S_\theta \text{Re}(p^* t_s) + \sqrt{2} S_\theta \text{Re}(s^* t)$. These terms are formally similar in magnitude to the SU(3) breaking corrections in Eq. [24]. Since $|s| < |p|$ [24], they would be smaller if also $|t_s| < |t|$, as has been often assumed.

We conclude with a brief discussion of the present status of the sum rules [21], [23] and [25]. Similar sum rules are obeyed by $B^+$ and $B^0$ branching ratios if one corrects for the $B^+/B^0$ lifetime ratio [16] $\tau_+/\tau_0 = 1.076 \pm 0.008$, for instance by multiplying all $B^0$ branching ratios by this ratio. We shall adopt this procedure.

Using the latest averages for branching ratios [16] quoted in Table II one finds that, in units of $10^{-6}$, the sum rule [2] may be written

$$44.4 \pm 1.5 = 48.9 \pm 2.7, \tag{26}$$

which is violated at the level of $1.5\sigma$. More precise data are needed in order to test the Standard Model expectation [3, 10, 11] that the sum rule should hold to an accuracy of
between one and five percent. The isospin breaking effects discussed here were shown to be considerably smaller.

The sum rule (23) reads

$$29.9 \pm 4.1 = 35.3 \pm 2.0 \quad ,$$  \hspace{1cm} (27)

where the error on the left-hand side is dominated by the uncertainty in $B(B^0 \to K^0\eta)$. The sum rule is satisfied and there is no evidence for large SU(3) breaking effects. Indeed, we did anticipate in Eq. (24) a suppression of these effects by $O(|t/p|)$.

For $\theta = \sin^{-1}(-1/3) \approx -19.47^\circ$, consistent with measurements [22], the sum rule (25) reads

$$\frac{2}{3}\Gamma(K^0\eta) - \frac{1}{3}\Gamma(K^0\eta') + \frac{1}{6}\Gamma(K^0\pi^+) = \frac{2}{3}\Gamma(K^+\eta) - \frac{1}{3}\Gamma(K^+\eta') + \frac{1}{6}\Gamma(K^+\pi^-) \quad ,$$  \hspace{1cm} (28)

or

$$-18.0 \pm 1.3 = -18.1 \pm 0.9 \quad .$$  \hspace{1cm} (29)

Here the experimental errors on the $K\eta'$ branching ratios are the dominant source of uncertainties. As we have argued, SU(3) breaking corrections in this sum rule are suppressed, occurring only in subleading terms. Assuming $|t_s| < |t|$, one would expect these corrections to be smaller than those affecting the sum rule (23).

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## A Symmetry breaking with isospin invariants

In this work we have used the language of amplitude topologies to discuss isospin and SU(3) breaking effects. The same discussion can be conducted also in a language of isospin and SU(3) invariant amplitudes. This approach isolates isospin and SU(3) breaking terms transforming as an isospin triplet and SU(3) octet, respectively. This isolation may also be achieved in the graphical approach. In the case of isospin breaking the spectator effect is then given by $a_u \equiv (1 + \epsilon_S^u)a$, $a_d \equiv (1 - \epsilon_S^d)a$ for $a = p$, $t$, $c$ and similar expressions apply to the “popping” effect in $p$.

In this Appendix we derive the results related to the isospin sum rule again, using symmetry invariants language. Using tensor language we consider the isospin invariant tensors

$$K_i = (K^+ K^0), \quad \Pi_j^i = \begin{pmatrix} -\pi^0/\sqrt{2} & \pi^+ \\ -\pi^- & \pi^0/\sqrt{2} \end{pmatrix}, \quad B^i = \begin{pmatrix} B^+ \\ B^0 \end{pmatrix},$$  \hspace{1cm} (30)

together with the Hamiltonian singlet operator $H_0$ and the Hamiltonian triplet operator $H_{ij}^i$. To these ingredients we now add the isospin breaking triplet which can most generally be parametrized by

$$M_j^i = \begin{pmatrix} -\epsilon/\sqrt{2} & 0 \\ 0 & \epsilon/\sqrt{2} \end{pmatrix},$$  \hspace{1cm} (31)
with $\epsilon \sim \epsilon_I$ of (1). By considering every possible contraction of the invariant tensors indices above, one can build the effective Hamiltonian for all $B \to K\pi$ decays [27].

Considering first the contraction of the singlet Hamiltonian operator $H_0$ with $M_j^i$. The combination is obviously an isospin triplet $H_0 M_{j}^{i}$. Upon contracting with the meson tensors, which are combined to create an $I = 1/2$ and an $I = 3/2$ representation, two reduced matrix elements result. (More technical details can be found in [28].) We denote these reduced matrix elements by $A_{1/2}^{\epsilon,0}$ and $A_{3/2}^{\epsilon,0}$, where the subscripts indicate the $K\pi$ representation and the superscript reminds us that these terms are a result of combining the isospin breaking triplet $M_j^i$ with the singlet Hamiltonian operator $H_0$. Those reduced matrix elements transform as $A_{1/2}$ and $A_{3/2}$ respectively. In the same way we have $B_{1/2}^{\epsilon,1}$, which comes from the singlet combination $H_{1/2}^{1} M_{1}^{i}$. In principle, there could also be $A_{0}^{\epsilon,1}$ and $A_{3/2}^{\epsilon,1}$ which come from the triplet combination $H_{1}^{i} M_{1}^{k}$. However, since both $H_1$ and $M$ have only an $I_z = 0$ component the $I = 0$ combination of them is zero. There is one additional reduced matrix element which transforms as a $\Delta I = 2$ term under isospin. We denote it by $C_{3/2}^{\epsilon,1}$, coming from the combination $H_{1}^{1} M_{1}^{k}$.

Using these matrix elements, we write the four $B \to K\pi$ amplitudes as

$$A(B^+ \to K^0\pi^+) = B_{1/2} + A_{1/2} + A_{3/2} + B_{1/2}^{\epsilon,1} + A_{1/2}^{\epsilon,0} + A_{3/2}^{\epsilon,0} + C_{3/2}^{\epsilon,1}$$

$$-A(B^0 \to K^+\pi^-) = B_{1/2} - A_{1/2} - A_{3/2} + B_{1/2}^{\epsilon,1} - A_{1/2}^{\epsilon,0} - A_{3/2}^{\epsilon,0} + C_{3/2}^{\epsilon,1}$$

$$-\sqrt{2} A(B^+ \to K^+\pi^0) = B_{1/2} + A_{1/2} - 2A_{3/2} + B_{1/2}^{\epsilon,1} + A_{1/2}^{\epsilon,0} - 2A_{3/2}^{\epsilon,0} - 2C_{3/2}^{\epsilon,1}$$

$$\sqrt{2} A(B^0 \to K^0\pi^0) = B_{1/2} - A_{1/2} + 2A_{3/2} + B_{1/2}^{\epsilon,1} - A_{1/2}^{\epsilon,0} + 2A_{3/2}^{\epsilon,0} - 2C_{3/2}^{\epsilon,1} \quad (32)$$

The amplitude sum rule now reads

$$A(K^0\pi^+) - A(K^+\pi^-) + \sqrt{2} A(K^+\pi^0) - \sqrt{2} A(K^0\pi^0) = 6 C_{3/2}^{\epsilon,1} \quad (33)$$

While from the group theoretical point of view, the (now seven) reduced matrix elements are all independent, the transformation properties of the symmetry breaking terms lead us to expect the approximate relations

$$B_{1/2}^{\epsilon,1} \sim \epsilon A_{1/2}, \quad A_{1/2}^{\epsilon,0}, A_{3/2}^{\epsilon,0} \sim \epsilon B_{1/2}, \quad C_{3/2}^{\epsilon,1} \sim \epsilon A_{3/2} \quad (34)$$

Since the dominant penguin term is in $B_{1/2}$ we see that the isospin breaking in the amplitude sum rule is suppressed.

We next write the relation for the rates, dropping terms of $\mathcal{O}(\epsilon^2)$. We get

$$\Gamma(K^0\pi^+) + \Gamma(K^+\pi^-) - 2\Gamma(K^+\pi^0) - 2\Gamma(K^0\pi^0) = 12 \text{Re}(A_{3/2}^{*} A_{1/2} - 6 |A_{3/2}|^2$$

$$+ 12 \text{Re}(A_{1/2}^{*} A_{3/2}^{0}) + 12 \text{Re}(A_{3/2}^{*} A_{1/2}^{0}) - 12 \text{Re}(A_{3/2}^{*} A_{3/2}^{0}) + 12 \text{Re}(B_{1/2}^{*} C_{3/2}^{\epsilon,1}) \quad (35)$$

A linear breaking term in the dominant amplitudes would have been of order $\sim \epsilon |B_{1/2}|^2$. Such a term can only come from either Re($B_{1/2}^{*} A_{1/2}^{0}$) or Re($B_{1/2}^{*} A_{3/2}^{0}$). We see that no such terms exist.

We have also performed a similar analysis for the SU(3) sum rule of Eq. (14). Technical details for such a calculation can be found in [28]. The results serve as a check to those derived using the graphical method as described above.
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