ηc production in photon - induced interactions at the AFTER@LHC experiment as a probe of the Odderon

V. P. Gonçalves† and W. K. Sauter†
High and Medium Energy Group,
Instituto de Física e Matemática,
Universidade Federal de Pelotas
Caixa Postal 354, CEP 96010-900, Pelotas, RS, Brazil
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One of the open questions of the strong interaction theory is the existence of the Odderon, which is an unambiguous prediction of Quantum Chromodynamics, but still not confirmed experimentally. An alternative to probe the Odderon is the exclusive ηc photoproduction in hadronic collisions. As the Pomeron exchange cannot contribute to this process, its observation would indicate the existence of the Odderon. In this paper we estimate the ηc production in photon - induced interactions in hadronic collisions at the AFTER@LHC experiment. We demonstrate that the experimental analysis of this process is feasible in the AFTER@LHC experiment and that the observation of the ηc production in nuclear collisions is a unambiguous signature of the Odderon.

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I. INTRODUCTION

The AFTER@LHC experiment opens a new kinematical regime where several questions related to the description of the Quantum Chromodynamics (QCD) remain without satisfactory answers [1]. One of these open questions is the Odderon, which is a natural prediction of the QCD, and determines the hadronic cross section difference between the direct and crossed channel processes at very high energies (For a review see Ref. [2]). The current experimental evidence for the Odderon is rather scarce. A recent study of the data on the differential elastic pp scattering shows that one needs the Odderon to describe the cross sections in the dip region [3] (See also Ref. [4, 5]). The difficulties inherent in the description of pp and pp collisions and the lack of further data have made it impossible to establish the existence of the Odderon in these processes beyond reasonable doubt.

An alternative to probe the Odderon is the study of the diffractive photoproduction of pseudoscalar mesons in hadronic collisions [6]. As the real photon emitted by one of the incident hadrons carries negative C parity, its transformation into a diffractive final state system of positive C parity requires the t-channel exchange of an object of negative C parity. In perturbative QCD, the Odderon is a C-odd (C being the charge conjugation) compound state of three reggeized gluons, given by the solution of the Balitsky - Fadin - Kuraev - Lipatov (BFKL) equation [7]. Consequently, the Pomeron exchange cannot contribute to the production of pseudoscalar mesons and this process can only be mediated by the exchange of an Odderon. A particular promising process is the exclusive ηc photoproduction, since the meson mass provides a hard scale that makes a perturbative calculation possible [8, 10].

In what follows we extend the analysis performed in Ref. [6] for the kinematical range probed by the AFTER@LHC experiment. In particular, we estimate the cross sections for the ηc production in photon - induced interactions present in pp, pA and AA collisions. The basic idea is that in hadron-hadron collisions at large impact parameter (b > R_h + R_h) and at ultra relativistic energies the electromagnetic interaction is dominant [11]. In heavy ion collisions, the heavy nuclei give rise to strong electromagnetic fields due to the coherent action of all protons in the nucleus, which can interact with each other. In a similar way, it also occurs when considering ultra relativistic protons in pp collisions. The photon stemming from the electromagnetic field of one of the two colliding hadrons can interact with one photon of the other hadron (two-photon process) or can interact directly with the other hadron (photon-hadron process). Consequently, the ηc can be produced in photon - hadron (γh) and photon - photon (γγ) interactions, with both processes generating two rapidity gaps in the final state. While the ηc production in γh interactions represented in Fig. 1 is a direct probe of the Odderon, its production in γγ interactions (See Fig. 2) is an important background, which should be estimated in order to separate the signal associated to the Odderon.

This paper is organized as follows. In Section II we present a brief review of the main concepts and formulæ used in the description of γγ and γh interactions in

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*Electronic address: barros@ufpel.edu.br
†Electronic address: werner.sauter@ufpel.edu.br
In hadronic collisions at large impact parameter and at ultra relativistic energies the photon-induced cross sections for a given process can be factorized in terms of the equivalent flux of photons of the incident hadrons and the photon-photon or photon-target production cross section \([11]\). The recent experimental results from CDF [12] at Tevatron, STAR [13] and PHENIX [14] at RHIC and ALICE [15, 16] and LHCb [17, 18] at LHC for photon-induced processes in hadronic collisions have demonstrated that a detailed analysis is feasible and that the data can be used to constrain the description of the hadronic structure at high energies [19] as well as to probe possible scenarios for the physics beyond the Standard Model [20]. These results motivate a detailed analysis of other final states in photon-induced interactions.

II. PHOTON-INDUCED INTERACTIONS IN HADRONIC COLLIDERS

In hadronic collisions and in the exclusive \(\eta_c\) photoproduction, which are required to explain our results, which will be presented in Section III. Finally, in Section IV we summarize our main conclusions.

\[ \sigma[h_1 h_2 \rightarrow \eta_c h_1 \otimes h_2] = \int_0^\infty \frac{d\omega_1}{\omega_1} \int_0^\infty \frac{d\omega_2}{\omega_2} F(\omega_1, \omega_2) \tilde{\sigma}_{\gamma\gamma \rightarrow \eta_c}(\omega_1, \omega_2) \]  

where \(\otimes\) represents a rapidity gap in the final state, \(\omega_1\) and \(\omega_2\) the energy of the photons which participate of the hard process and \(\tilde{\sigma}_{\gamma\gamma \rightarrow \eta_c}\) is the cross section for the subprocess \(\gamma\gamma \rightarrow \eta_c\), given by

\[ \sigma_{\gamma\gamma \rightarrow \eta_c} = 8\pi^2(2J + 1)\frac{\Gamma_{\eta_c \rightarrow \gamma\gamma}}{m_{\eta_c}}\delta(4\omega_1\omega_2 - m_{\eta_c}^2) \]  

where \(J, m_{\eta_c}\) and \(\Gamma_{\eta_c \rightarrow \gamma\gamma}\) are the spin, mass and the photon-photon partial decay width of the \(\eta_c\), respectively, and the \(\delta\) function enforces energy conservation. Moreover, the function \(F\) is the folded spectra of the incoming particles (which corresponds to an “effective luminosity” of photons) which we assume to be given by

\[ F(\omega_1, \omega_2) = 2\pi \int_{R_{h_1}}^\infty db_1 b_1 \int_{R_{h_2}}^\infty db_2 b_2 \int_0^{2\pi} d\phi N_1(\omega_1, b_1) N_2(\omega_2, b_2) \Theta(b - R_{h_1} - R_{h_2}) \]

where \(b_i\) are the impact parameters of the hadrons in relation to the photon interaction point, \(\phi\) is the angle
between \( b_1 \) and \( b_2 \), \( R_i \) are the projectile radii and \( b^2 = b_1^2 + b_2^2 - 2b_1b_2 \cos \theta \). The theta function in Eq. (3) ensures that the hadrons do not overlap \( 22 \). The Weizsäcker-Williams photon spectrum for a given impact parameter is given in terms of the nuclear charge form factor \( F(k_\perp^2) \), where \( k_\perp \) is the four-momentum of the quasi-real photon, as follows \( 11 \)

\[
N(\omega, b) = \frac{\alpha_{em} Z^2}{\pi^2 \omega} \left( \frac{\xi}{b} \right)^2 \frac{F \left( \frac{\omega}{2} + \frac{b^2}{2} \right)}{\left( \frac{\omega}{2} + \frac{b^2}{2} \right)} \cdot J_1(bk_\perp) \right] \right)^2 \quad (4)
\]

where \( J_1 \) is the Bessel function of the first kind. For a point-like nucleus one obtains that \( 11 \)

\[
N(\omega, b) = \frac{\alpha_{em} Z^2}{\pi^2} \left( \frac{\xi}{b} \right)^2 \left\{ K_1^2(\xi) + \frac{1}{\gamma} K_0^2(\xi) \right\} \quad (5)
\]

with \( K_{0,1} \) being the modified Bessel function of second kind, \( \xi = \omega \gamma/\gamma v \), \( v \) the velocity of the hadron, \( \gamma \) the Lorentz factor and \( \alpha_{em} \) the electromagnetic coupling constant. This expression has been derived considering a semiclassical description of the electromagnetic interactions in peripheral collisions, which works very well for heavy ions (See e.g. 21). For protons, it is more appropriate to obtain the equivalent photon spectrum from its elastic form factors in the dipole approximation (See e.g. 23). An alternative is to use Eq. (5) assuming \( R_p = 0.7 \) fm for the proton radius, which implies a good agreement with the parametrization of the luminosity obtained in 24 for proton-proton collisions (For a more detailed discussion see Ref. 25).

\[
n_{\gamma/p}(\omega) = \frac{\alpha_{em} Z^2}{2 \pi \omega} \left[ 1 + \left( 1 - \frac{2 \omega}{\sqrt{S_{NN}}} \right)^2 \right] \cdot \frac{\ln(\Omega - \frac{11}{6} + \frac{3}{\Omega} - \frac{3}{2 \Omega^2} + \frac{1}{3 \Omega^3})}{\Omega} \quad (9)
\]

where \( \Omega = 1 + \left[ (0.71 \text{GeV}^2)/Q^2_{\text{min}} \right] \) and \( Q^2_{\text{min}} = 2(1 - \frac{2 \omega}{\sqrt{S_{NN}}} ) \approx (\omega/\gamma_L)^2 \).

The exclusive \( \eta_c \) photoproduction, which is the main input into our calculations [See Eq. (7)] can be obtained using the impact factor representation, proposed by Cheng and Wu (27) many years ago. In this representation, the amplitude for a large-s hard collision process can be factorized in three parts: the two impact factors of the colliding particles and the Green function for the three interacting reggeized gluons, which is determined by the BKP equation and is represented by \( G_{\text{BKP}} \) hereafter. The differential cross section for the process \( \gamma + h \to \eta_c + h \) is given by \( 3 \)

\[
\frac{d\sigma}{dt} = \frac{1}{32\pi^2} \sum_{i=1,2} |A^i|^2 \quad (10)
\]

In the case of the \( \eta_c \) production in photon - hadron interactions the total cross section is given by

\[
\sigma[h_1h_2(\gamma b) \to h_1 \otimes \eta_c \otimes h_2] = \sum_{i=1,2} \int dY \frac{d\sigma_i}{dY} \quad (6)
\]

where \( d\sigma_i/dY \) is the rapidity distribution for the photon-target interaction induced by the hadron \( h_i \), which can be expressed as

\[
\frac{d\sigma_i}{dY} = \omega n_{\gamma/h_i}(\omega) \cdot \sigma_{\gamma h_i \to \eta_c h_i}(W_{\gamma h_i}) \quad (i \neq j) \quad (7)
\]

where \( W_{\gamma h} = 2 \omega \sqrt{s_{NN}} \) and \( s_{NN} \) are the c.m.s energy squared of the photon - hadron and hadron-hadron system, respectively. Moreover, \( n_{\gamma/h_i}(\omega) \) is the \( b \)-integrated photon flux associated to the hadron \( h_i \), which can be obtained considering the requirement that the photon - induced processes are not accompanied by hadronic interaction (ultra-peripheral collision). An analytic approximation for the equivalent photon flux of a nuclei can be calculated, which is given by 11

\[
n_{\gamma/A}(\omega) = \int b_{min} d^2 b N(\omega, b) = \frac{2Z^2 \alpha_{em}}{\pi \omega} \left[ \eta K_0(\eta) K_1(\eta) + \eta^2 U(\eta) \right] \quad (8)
\]

where \( \eta = \omega b_{min}/\gamma_L \) (with \( \gamma_L \) being the Lorentz boost of a single beam), \( b_{min} = R_{h_1} + R_{h_2} \) and \( U(\eta) = K_1^2(\eta) - K_0^2(\eta) \). On the other hand, for proton-proton collisions, we assume that the photon spectrum of a relativistic proton is given by 26.

\[
A^i = \frac{5}{1152} \frac{1}{(2\pi)^8} |<\Phi_{\gamma \eta_c}|G_{\text{BKP}}|\Phi_p>| \quad (11)
\]

Differently from \( \Phi_{\gamma \eta_c} \), that can be calculated perturbatively 10, the impact factor \( \Phi_p \) that describes the coupling of the Odderon to the proton is non-perturbative and should be modelled. In our calculations we consider the model used in Refs. 4 10. Moreover, we assume that the Odderon Green function \( G_{\text{BKP}} \) is described in terms of the solution of the BKP equation 7, with the energy dependence being determined by the Odderon intercept \( \alpha_D \). In particular, we consider the solution obtained by Bartels, Lipatov and Vacca (BLV) 28 that
have found a solution for the BKP equation with intercept $\alpha_0$ exactly equal to one. For comparison we also consider the solution obtained by Kwiecinski and collaborators in Ref. [11] (CKMS model hereafter), which has considered a simplified three gluon exchange model for the Odderon that implies an energy independent cross section. In our calculations we will use a realistic value for $\alpha_s \approx 0.3$.

III. RESULTS

In what follows we present our predictions for the $\eta_c$ production in $\gamma h$ and $\gamma \gamma$ interactions considering the kinematical range which will be probed by the AFTER@LHC experiment. Basically, we assume $\sqrt{s_{NN}} = 115/72/72 \text{ GeV}$ for $pp/Pbp/PbPb$ collisions, which implies that $\sqrt{s_{NN}} \leq 44/12/9 \text{ GeV}$, respectively. Similarly, it is possible to obtain that $\sqrt{s_{NN}} \leq 17/2.0/1.0 \text{ GeV}$. Consequently, the $\eta_c$ production in $\gamma \gamma$ interactions only is present in $pp$ collisions. In other words, the measurement of the exclusive $\eta_c$ production in $Pbp$ and $PbPb$ collisions can be considered a direct probe of the Odderon. Moreover, in our calculations we take into account that the typical rapidity range which is expected to be reachable by the AFTER@LHC experiment is $-3.0 \leq Y_{c.m.} \leq 0.5$.

In Table I we present our predictions for the total cross sections. We predict cross sections for the $\eta_c$ production in $Pbp$ and $PbPb$ collisions that are a factor $\geq 10^4$ larger than the $pp$ predictions. This enhancement is directly associated to the nuclear photon flux and the nuclear dependence of the photon - hadron cross section. As the photon flux is proportional to $Z^2$, because the electromagnetic field surrounding the ion is very larger than the proton one due to the coherent action of all protons in the nucleus, the $Pbp$ and $PbPb$ cross sections are amplified by this factor. Moreover, our predictions for the $\eta_c$ production in $Pbp$ collisions also are amplified by the mass number $A$, since in our calculations for the nuclear case we are assuming in a first approximation that $\sigma(\gamma A \rightarrow \eta_c A) = A \sigma(\gamma p \rightarrow \eta_c p)$. For the exclusive $\eta_c$ production in $pp$ collisions we predict values of the order of a fraction of pb, with the BBVC prediction being a factor of $\approx 6$ larger than the CKMS one. This enhancement is directly associated to the energy dependence present in the BBVC model, which implies that the $\gamma h$ cross section increases at smaller energies, while the CKMS predicts an energy independent cross section. For the $\eta_c$ production in $Pbp$ and $PbPb$ collisions, we predict cross sections of the order of nb for the exclusive $\eta_c$ photoproduction in $Pbp$ collisions at AFTER@LHC experiment. Moreover, we predict that the $Pbp$ cross sections are two orders of magnitude smaller than those predicted for $PbPb$ collisions.

Let now estimate the background associated to the $\eta_c$ production in $\gamma \gamma$ interactions for $pp$ collisions. Assuming that the photon spectrum for the proton is given by Eq. (5), with $R_p = 0.7 \text{ fm}$, $m_{\eta_c} = 2.983 \text{ GeV}$ and $\Gamma(\eta_c \rightarrow \gamma \gamma) = 5.0 \text{ keV}$, we predict that $\sigma[pp(\gamma \gamma \rightarrow p \otimes \eta_c \otimes p)] = 2.2 \text{ pb}$, which is a factor $\gtrsim 8$ larger than the predictions for the $\eta_c$ production in photon - hadron interactions. As both processes generate two rapidity gaps in the final state, the detection of the gaps is not, in a first analysis, an efficient trigger for the separation of the $\gamma h$ production of the $\eta_c$. An alternative is the reconstruction of the entire event with a cut on the summed transverse momentum of the event [22]. As the typical photon virtualities are very small, the hadron scattering angles are very low. Consequently, we expect that a different transverse momentum distribution of the scattered hadron, with $\gamma h$ interactions predicting larger $p_T$ values. In contrast, the background is not present in nuclear collisions, since the maximum $\gamma \gamma$ center-of-mass energies in $Pbp$ and $PbPb$ collisions are smaller than threshold of production.

Considering the design luminosities at AFTER@LHC for $pp$ ($L_{pp} = 2 \times 10^3 \text{ pb}^{-1} \text{yr}^{-1}$), $Pbp$ ($L_{Pbp} = 1.1 \text{ pb}^{-1} \text{yr}^{-1}$) and $PbPb$ collisions ($L_{PbPb} = 7.0 \times 10^3 \text{ pb}^{-1} \text{yr}^{-1}$) we can calculate the production rates (See Table I). Although the cross section for the exclusive $\eta_c$ photoproduction in $Pbp$ collisions is much larger than in $pp$ collisions, the event rates are higher in the $pp$ mode due to its larger luminosity. In particular, we predict that the events rate/year for $pp$ collisions at $\sqrt{s} = 115 \text{ GeV}$ should be larger than 1000. On the other hand, for $Pbp$ and $PbPb$ collisions at $\sqrt{s} = 72 \text{ GeV}$ we predict that the events rate/year should be larger than 30. Although smaller than the $pp$ predictions, the observation of the $\eta_c$ production in nuclear collisions would clearly indicate the existence of the Odderon.

IV. SUMMARY

In the last years, the physics of the Odderon has become an increasingly active subject of research, both from theoretical and experimental points of view. On the theoretical side, the investigation of the Odderon in pQCD has led to discovery of relations of high energy QCD to the theory of integrable models [29] and two leading solutions of the BKP evolution equation were obtained [28, 57], with the intercept being close to or exactly one, depending on the scattering process (See also Refs. [51]). In contrast, on the experimental side, the current evidence for the Odderon is very unsatisfactory.

In Ref. [6] we have proposed the study of the exclu-

| $h_1 h_2$ | CKMS | BBVC |
|-----------|------|------|
| $pp (\sqrt{s} = 115 \text{ GeV})$ | 0.05 pb (1000.0) | 0.30 pb (6000.0) |
| $Pbp (\sqrt{s} = 72 \text{ GeV})$ | 25.1 pb (31.0) | 356.6 pb (393.0) |
| $PbPb (\sqrt{s} = 72 \text{ GeV})$ | 5870.0 pb (41.0) | 74366.0 pb (520.0) |

TABLE I: Cross sections (event rates/year) for the exclusive $\eta_c$ photoproduction in $pp/Pbp/PbPb$ collisions at AFTER@LHC experiment.
sive $\eta_c$ production in hadronic collisions at LHC energies as a probe of the Odderon (For other possibilities see Ref. [32]). In this paper we extend that previous analysis for the kinematical region which would be probed by the AFTER@LHC experiment. As the exclusive $\eta_c$ photoproduction is only possible if the Odderon is exchanged between the vector meson and the hadron, the observation of such processes would clearly indicate the existence of the Odderon. We have estimated the $\eta_c$ cross section considering photon - hadron interactions in $pp/PbPb$ collisions. Moreover, the background associated to the $\eta_c$ production by $\gamma\gamma$ interactions was calculated. We have that the background is only present in $pp$ collisions, which makes the observation of the exclusive $\eta_c$ production in $PbPb$ and $PbPb$ a signature of the Odderon. We predict total cross sections of order of pb (nb) for $pp/PbPb$ collisions and large values for the event rates/year, which makes, in principle, the experimental analysis of this process feasible at AFTER@LHC experiment.

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