 Supernova Neutrino Nucleosynthesis of Light Elements with Neutrino Oscillations

Takashi Yoshida 1, Toshihata Kajino 2,3, Hidekazu Yokomakuara 4, Keiichi Kimura 4, Akira Takamura 5, and Dieter H. Hartmann 6
1 Astronomical Institute, Graduate School of Science, Tohoku University, Miyagi 980-8578, Japan
2 National Astronomical Observatory of Japan, and The Graduate University for Advanced Studies, Tokyo 181-8588, Japan
3 Department of Astronomy, Graduate School of Science, University of Tokyo, Tokyo 113-0033, Japan
4 Department of Physics, Graduate School Science, Nagoya University, Aichi 464-8602, Japan
5 Department of Mathematics, Toyota National College of Technology, Aichi 471-8525, Japan
6 Department of Physics and Astronomy, Clemson University, Clemson, South Carolina 29634, USA
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Light element synthesis in supernovae through neutrino-nucleus interactions, i.e., the $\nu$-process, is affected by neutrino oscillations in the supernova environment. There is a resonance of 13-mixing in the O/C layer, which increases the rates of charged-current $\nu$-process reactions in the outer He-rich layer. The yields of $^7$Li and $^{11}$B increase by about a factor of 1.9 and 1.3, respectively, for a normal mass hierarchy and an adiabatic 13-mixing resonance, compared to those without neutrino oscillations. In the case of an inverted mass hierarchy and a non-adiabatic 13-mixing resonance, the increase in the $^7$Li and $^{11}$B yields is much smaller. Observations of the $^{7}$Li/$^{11}$B ratio in stars showing signs of supernova enrichment could thus provide a unique test of neutrino oscillations and constrain their parameters and the mass hierarchy.

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A tremendous number of neutrinos are released from a core-collapse supernova (SN). These neutrinos interact with nuclei in the surrounding stellar envelope and thereby affect the synthesis of new elements. This so-called $\nu$-process may be a major contributor to the production of several light isotopes, such as $^7$Li, $^{11}$B, $^{19}$F, as well as a few heavy isotopes, such as $^{138}$La and $^{180}$Ta. However, the yields of these isotopes may depend on the effects of neutrino oscillations, which was not taken into consideration in the above cited studies.

Recent neutrino experiments on atmospheric, solar, and reactor neutrinos significantly constrain most of the neutrino oscillation parameters. However, only an upper limit on $\theta_{13}$ is obtained and the mass hierarchy remains unknown. Theoretical studies of neutrino oscillations in SNe have been used to suggest potential constraints on $\theta_{13}$ and the mass hierarchy based on observed SN neutrino spectra. These studies indicate that the neutrino spectra from SNe strongly depend on $\theta_{13}$ and the assumed mass hierarchy. When the resonance of the 13-mixing is adiabatic, substantial conversion $\nu_e \leftrightarrow \nu_{\mu,\tau}$ occurs in the O/C layer for a normal mass hierarchy and conversion $\nu_e \leftrightarrow \nu_{\mu,\tau}$ occurs for an inverted hierarchy. These direct methods are of course limited by the fact that nearby core-collapse SNe occur rarely in the small detection volume given by current detector sizes and methods.

Here we suggest an alternative method to study the effects of neutrino oscillations, by considering light element synthesis in SNe. Neutrino energy spectra change as they are transported through SN ejecta. This change will affect the production of light elements via the $\nu$-process. The thermal neutrinos emitted from a cooling protoneutron star have a well-known, but not yet rigorously established, energy hierarchy: $\langle \varepsilon_{\nu_e} \rangle < \langle \varepsilon_{\nu_\mu} \rangle < \langle \varepsilon_{\nu_\tau} \rangle$ (e.g., [14]). Neutrino oscillations could thus increase the average energies of $\nu_e$ and $\bar{\nu}_e$, and consequently the rates of charged-current $\nu$-process reactions could be much larger than expected from models without oscillations. Therefore, the yields of the light elements may increase significantly.

We investigate nucleosynthesis of light elements $^7$Li and $^{11}$B through the $\nu$-process in SNe taking neutrino oscillations into account. Since the other $\nu$-process elements are mainly produced in the O-rich layers, they are not expected to be affected by neutrino oscillations. The $^7$Li and $^{11}$B yields in SNe can thus be used as probes of neutrino oscillations. We show that the $^7$Li yield increases significantly through neutrino oscillations. The dependence of the $^{7}$Li/$^{11}$B ratio on the mixing parameter, $\theta_{13}$, provides an observable signature that could be used to constrain its absolute value and the neutrino mass hierarchy.

Neutrino luminosities are assumed to decrease exponentially in time, with a decay time scale of $\sim 3$ s. The total neutrino energy is assumed to be fixed at $3 \times 10^{53}$ erg. The neutrino energy spectra at the neutrino sphere are approximated with Fermi-Dirac (FD) distributions with zero chemical potential. The neutrino temperatures of $\nu_e$, $\bar{\nu}_e$, and ($\nu_{\mu,\tau}$ and $\bar{\nu}_{\mu,\tau}$) are set to be 3.2, 5.0, and 6.0 MeV as adopted in [14]. These energy spectra change during the subsequent passage through the outer stellar layers by neutrino oscillations.

*Electronic address: takashi.yoshida@nao.ac.jp
In order to evaluate the reaction rates of the $\nu$-process, we need the cross sections as functions of neutrino energy because the spectra changed by neutrino oscillations no longer follow the FD shape. We assume that the cross sections of the charged-current reactions of $^4$He and $^{12}$C, i.e., $^4$He($\nu_e, e^- p$)$^3$He, $^4$He($\bar{\nu}_e, e^+ n$)$^3$H, $^{12}$C($\nu_e, e^- p$)$^{11}$C, $^{12}$C($\bar{\nu}_e, e^+ n$)$^{11}$B, $^{12}$C($\nu_e, e^- \gamma$)$^{12}$N, and $^{12}$C($\bar{\nu}_e, e^+ \gamma$)$^{12}$B, are expressed as a power law $\sigma(\varepsilon_\nu) = \sigma_0(\varepsilon_\nu - \varepsilon_{\text{th}})^\alpha$, where $\varepsilon_{\text{th}}$ is the threshold energy. Coefficients of the functions are determined such that the reaction rates deduced using these cross sections (and assuming FD energy distributions) fit the rates tabulated in [15]. Details are provided in [16]. For the other $\nu$-process reactions, we use the reaction rates with FD distribution of the neutrino spectra.

Recent neutrino experiments [8, 13, 11] have determined most of the values of the mass squared differences $\Delta m^2_{ij} = m^2_i - m^2_j$ and the mixing angles $\theta_{ij}$. Based on these results, we use $\Delta m^2_{23} = 7.9 \times 10^{-5}$ eV$^2$, $\Delta m^2_{31} = \pm 2.4 \times 10^{-3}$ eV$^2$, and $\sin^2 2\theta_{12} = 0.816$, $\sin^2 2\theta_{13} = 1.0$, $0 \leq \sin^2 2\theta_{13} \leq 1 \times 10^{-1}$. The positive value of $\Delta m^2_{31}$ corresponds to “normal hierarchy”, i.e., $m_1 < m_2 < m_3$ and the negative value corresponds to “inverted hierarchy”, i.e., $m_3 < m_1 < m_2$. We numerically solve the mixing probabilities of neutrinos for each neutrino energy by Runge-Kutta methods and using the exact solutions of the oscillations described in [17]. By convolving the mixing probabilities and the neutrino spectra at the neutrino sphere, we evaluate the neutrino energy spectra taking neutrino oscillations into account. We do not include CP phase $\delta$. Based on [18] we assume that the effect of CP violation will not be seen because the spectra of $\nu_\mu$ ($\bar{\nu}_\mu$) and $\nu_\tau$ ($\bar{\nu}_\tau$) emitted from the neutrino sphere are the same. The change of the spectra due to oscillations is calculated using the density profile of a presupernova model.

We use the same SN explosion model as in [3, 7]. The presupernova model is the 14E1 model constructed for SN 1987A in [19]. The SN explosion is calculated using piecewise parabolic method code [20, 21]. The explosion energy and the location of the mass cut are set to be $1 \times 10^{51}$ erg and 1.61 $M_\odot$. The detailed nucleosynthesis in the SN is calculated using a nuclear reaction network including 291 species [2].

Figure 1 shows the mass fraction distributions of $^7$Li and $^{11}$B in the SN ejecta with neutrino oscillations of $\sin^2 2\theta_{13} = 2 \times 10^{-2}$ and for those without oscillations. In the case of a normal hierarchy, the mass fraction of $^7$Be with the neutrino oscillations is much larger than that without oscillations in the He layer. There is a 13-mixing resonance for neutrinos in the O/C layer and the resonance is adiabatic in this case. Thus, the energy spectrum of $\nu_\tau$ in the He/C layer becomes almost the same as that of $\nu_\mu, \tau$ in the O-rich layer. Beryllium 7 is produced through $^4$He($\nu_e, \nu' n$)$^3$He($\alpha, \gamma$)$^7$Be. Owing to the neutrino oscillations, the reaction rate of $^4$He($\nu_e, e^- p$)$^3$He becomes larger than that of $^4$He($\nu_e, \nu' n$)$^3$He. The mass fraction of $^7$Li including neutrino oscillations is also larger, but the increment is much smaller than that for $^7$Be. The main production process of $^7$Li is $^4$He($\nu, \nu' p$)$^3$H($\alpha, \gamma$)$^7$Li and the corresponding charged-current reaction is $^4$He($\bar{\nu}_e, e^+ n$)$^3$H. However, there are no resonances for antineutrinos.

The effect of neutrino oscillations on the mass fraction distributions of $^{11}$B and $^{11}$C is similar to that for $^7$Li and $^7$Be. The mass fraction of $^{11}$C with the neutrino oscillations is larger than that without oscillations in the He layer. During the $\nu$-process, $^{11}$C is produced through $^{12}$C($\nu_e, e^- p$)$^{11}$C and partly through $^{12}$C($\nu_e, e^- \gamma$)$^{11}$C. The reaction rate of $^{12}$C($\nu_e, e^- p$)$^{11}$C with the oscillations becomes larger than that without oscillations by about one order of magnitude. The mass fraction of $^{11}$B with oscillations is only slightly larger than that without oscillations. The main production process of $^{11}$B is $^3$He($\nu_e, e^+ n$)$^3$H($\alpha, \gamma$)$^{11}$B. The corresponding charged-current reaction is $^3$He($\bar{\nu}_e, e^+ n$)$^3$H.
About 12% – 16% of $^{11}\text{B}$ in the He layer is produced from $^{12}\text{C}$ through $^{12}\text{C}(\nu_{e},\nu^{p})^{11}\text{B}$ and $^{12}\text{C}(\bar{\nu}_{e},e^{-}n)^{11}\text{B}$. The $^{11}\text{B}$ abundant region in the He layer is inside the $^{7}\text{Li}$ abundant region because of the decrease in peak shock temperature as one moves outward in the star. The increase in the $^{11}\text{B}$ production through $^{12}\text{C}(\bar{\nu}_{e},e^{-}n)^{11}\text{B}$ is not as large because of the absence of resonances for antineutrinos, as mentioned above. In the O-rich layers, light element production is not influenced by the neutrino oscillations. The oscillation amplitude in these layers is too small because of high densities (e.g., $^{12}\text{C}$).

In the case of an inverted hierarchy, mass fractions of $^{7}\text{Li}$ and $^{11}\text{B}$ are larger than those for a normal hierarchy. The reaction rates of $^{4}\text{He}(\bar{\nu}_{e},e^{-}n)^{3}\text{H}$ and $^{12}\text{C}(\bar{\nu}_{e},e^{-}n)^{11}\text{B}$ become larger owing to an adiabatic resonance of $\bar{\nu}_{e} \leftrightarrow \bar{\nu}_{\mu,\tau}$. However, the increase of the mass fractions of $^{7}\text{Li}$ and $^{11}\text{B}$ is less pronounced than that of $^{7}\text{Be}$ and $^{11}\text{C}$ for a normal hierarchy. This is because the average energy of $\bar{\nu}_{e}$ is larger than $\nu_{e}$ at the neutrino sphere and, therefore, the difference from the average $\nu_{\mu,\tau}$ energy is smaller. On the other hand, the mass fractions of $^{7}\text{Be}$ and $^{11}\text{C}$ are slightly larger in the mass range $M_{c} \geq 4.5M_{\odot}$ and slightly smaller inside the range of the He layer. There is no 13-mixing resonance for $\nu_{e}$, so that substantial conversion of $\nu_{e} \leftrightarrow \nu_{\mu,\tau}$ does not occur. At the same time, some $^{7}\text{Be}$ and $^{11}\text{C}$ capture neutrons produced through $^{4}\text{He}(\nu_{e},e^{-}n)^{3}\text{H}$.

Figure 2 shows the ratios of the $^{7}\text{Li}$ and $^{11}\text{B}$ yields with neutrino oscillations in comparison to those without oscillations, hereafter called yield ratios, as a function of $\sin^{2}2\theta_{13}$. The yields of $^{7}\text{Li}$ and $^{11}\text{B}$ without the oscillations are $2.36 \times 10^{-7}M_{\odot}$ and $6.26 \times 10^{-7}M_{\odot}$. The yield ratio of $^{7}\text{Li}$ is at most 1.88 in the case of $\sin^{2}2\theta_{13} \geq 2 \times 10^{-3}$ and normal hierarchy. This increase in the yield is due to the adiabatic 13-mixing resonance.

The dependence of the $^{11}\text{B}$ yield ratio on $\sin^{2}2\theta_{13}$ is similar to that of $^{7}\text{Li}$. The $^{11}\text{B}$ yield ratio is about 1.27 even in the case of adiabatic range of $\theta_{13}$ and normal hierarchy. This value is much smaller than that of $^{7}\text{Li}$. Neutrino oscillations raise the rate of $^{12}\text{C}(\nu_{e},e^{-}p)^{11}\text{C}$ and the $^{11}\text{C}$ yield. However, the increased $^{11}\text{C}$ yield is still small for the total yield of $^{11}\text{B}$. In the inverted hierarchy case, the maximum yield ratio of $^{11}\text{B}$ is slightly smaller than that in the normal hierarchy. In this case the contribution of $^{12}\text{C}(\nu_{e},e^{-}n)^{11}\text{B}$ and $^{4}\text{He}(\nu_{e},e^{-}n)^{3}\text{H}$ increases. As shown in $^{7}\text{Li}$ case, however, the increase is small due to a small difference of the average energy between $\nu_{e}$ and $\nu_{\mu,\tau}$. In the limit of $\sin^{2}2\theta_{13} = 0$, the $^{7}\text{Li}$ and $^{11}\text{B}$ yields still slightly increase due to the residual mixing other than 13-mixing as shown in Fig. 2.

We solve for neutrino energy spectra changed by neutrino oscillations in the density profile of a presupernova star. We expect that the influence on the spectral changes due to neutrino oscillations caused by the passing shock is small. When the shock front is in the O-rich layers, the density behind the shock front is still so high that the shock wave does not affect the oscillations. After the shock front passes through the O/C layer, the change of the density profile affects the mixing probability of neutrino oscillations. However, most neutrinos have passed before the shock arrival at the O/C layer. Details are discussed in [16].

In our previous studies [3, 6], we constrained the neutrino temperature with Galactic chemical evolution (GCE) arguments (e.g., [22]) for $^{11}\text{B}$. However, the remaining model uncertainties still render the effects on the observed $^{7}\text{Li}$ and $^{11}\text{B}$ abundance trends from neutrino oscillations somewhat ambiguous. Still, the possibility to obtain, or at least constrain, fundamental neutrino properties from these observations encourage us to pursue these arguments further. We consider the dependence of the $^{7}\text{Li}/^{11}\text{B}$ ratio on $\sin^{2}2\theta_{13}$ taking account uncertainties of neutrino energy spectra. We consider two additional spectral parameter sets: $(T_{\nu_{e}}, T_{\nu_{\mu,\tau}}, E_{\nu})$ = (3.2 MeV, 5.0 MeV, 6.6 MeV, 2.4 $\times 10^{53}$ erg) and (3.2 MeV, 4.3 MeV, 5.2 MeV, 3.5 $\times 10^{53}$ erg). The obtained $^{11}\text{B}$ yields for these two cases without neutrino oscillations are $7.3 \times 10^{-7}M_{\odot}$ and $3.3 \times 10^{-7}M_{\odot}$, corresponding to the maximum and minimum values satisfying the GCE models for $^{11}\text{B}$ [6]. The corresponding $^{7}\text{Li}$ yields are $2.9 \times 10^{-7}M_{\odot}$ and $1.3 \times 10^{-7}M_{\odot}$.

Figure 3 shows the number ratio of $^{7}\text{Li}/^{11}\text{B}$ as a func-

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**FIG. 2:** The yield ratios of $^{7}\text{Li}$ and $^{11}\text{B}$ with the relation of $\sin^{2}2\theta_{13}$. Thick solid line and thick dotted line are the yield ratio of $^{7}\text{Li}$ in the cases of a normal hierarchy and inverted hierarchy, respectively. The thin solid line and thin dotted line are that of $^{11}\text{B}$ in the cases of normal and inverted hierarchy. The case of $\sin^{2}2\theta_{13} = 0$ is also calculated (see text).
the corresponding charged-current reactions. Data analysis, the dependence of neutral-current reaction rates on yields on the $\nu$ in the inverted hierarchy. We should note that uncertainties however, that they are largely canceled out when we take the neutrino energy spectra deduced from the calculations using three sets of neutrino temperatures and total neutrino energies (see text).

The number ratio of $^7\text{Li}/^{11}\text{B}$ with the relation of $\sin^2 2\theta_{13}$. The uncertainty due to neutrino spectra is included as shaded regions. The $^7\text{Li}/^{11}\text{B}$ ratio in the case of adiabatic 13-mixing resonance and normal hierarchy is larger than that without neutrino oscillations, even with the spectral uncertainties included. Thus, the enhancement of observed $^7\text{Li}/^{11}\text{B}$ ratio may constrain the lowest value of $\theta_{13}$ and eliminate the possibility of inverted hierarchy. We should note that uncertainties in the $\nu$-process cross sections still remain. We expect, however, that they are largely canceled out when we take the $^7\text{Li}/^{11}\text{B}$ ratio. Since $^7\text{Li}$ and $^{11}\text{B}$ are mainly produced through the $\nu$-process from $^4\text{He}$, the dependence of their yields on the $\nu$-process reaction rates is similar. In addition, the dependence of neutral-current reaction rates on the neutrino temperature is not so different from that of the corresponding charged-current reactions. Data analysis of SN 1987A and future observations of SN relic neutrinos may provide additional information on the $\bar{\nu}_e$ spectrum. The effect of neutrino oscillations on the analyzed $\bar{\nu}_e$ signal should be taken into account, and the evaluation of the $\bar{\nu}_e$ spectrum will lead to a more precise evaluation of $^7\text{Li}/^{11}\text{B}$ ratio in SNe.

Recent observational efforts to obtain Li and B abundances in stars which may have formed in regions directly affected by prior generations of massive stars and their subsequent SNe (e.g., [23], may have detected the signature of the $\nu$-process in $^{11}\text{B}$-enriched stars. [20]. The combination of SN nucleosynthesis theory and observations of light elements may ultimately provide powerful constraints on mass hierarchy and the mixing angle $\theta_{13}$.

In summary, we investigated light element synthesis in SNe through the $\nu$-process including the change of neutrino spectra due to neutrino oscillations. In the case of adiabatic 13-mixing resonance and normal hierarchy, the $^7\text{Li}$ yield increases by about a factor 1.9 compared to the case without oscillations. This increase may be accessible to high resolution spectroscopic studies of stars in young, star-forming regions. The $^7\text{Li}$ yield in other cases and the $^{11}\text{B}$ yield are scarcely affected by neutrino oscillations. The adiabaticity of the 13-mixing resonance and the mass hierarchy affect robust determinations of $^7\text{Li}/^{11}\text{B}$ ratios in SNe.

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