Quantum Gravity on Neutrino Mass Square difference

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Abstract

We consider non-renormalizable interaction term as a perturbation of the neutrino mass matrix. We assume that the neutrino masses and mixing arise through physics at a scale intermediate between Planck scale and the electroweak breaking scale. We also assume that, just above the electroweak breaking scale, neutrino masses are nearly degenerate and their mixing is bi-maximal. Quantum gravity (Planck scale effects) lead to an effective $SU(2)_L \times U(1)$ invariant dimension-5 Lagrangian involving neutrino and Higgs fields. On symmetry breaking, this operator gives rise to correction to the above masses and mixing. The gravitational interaction $M_X = M_{pl}$, we find that for degenerate neutrino mass spectrum, the considered perturbation term change the $\Delta^2_{21}$ and $\Delta^2_{31}$ mass square difference is unchanged above GUT scale. The nature of gravitational interaction demands that the element of this perturbation matrix should be independent of flavor indices. In this letter, we study the quantum gravity effects on neutrino mass square difference, namely modified dispersion relation for neutrino mass square differences.

1 Introduction

The existence of neutrino mass and mixings is experimentally well confirmed, therefore the theoretical understanding of these quantities is one of the most important issues for particle physics. One of most sensitive probe of quantum gravity phenomena are neutrinos [1,2]. In recent year the subject of quantum gravity phenomenology has rapid growth complementary theoretical work. Theoretical extension of the standard model of particle physics, which also were expected to explain the origin and the shape of small neutrino mass matrix. There is the possible existence of non-renormalizable gravitational interaction. Those interaction could have an influence on the neutrino sector [3,4,5,6,7]. In
this letter we study the quantum gravity effects on neutrino mass square differences. The outline of the article is as follows. In Section 2, we briefly discuss neutrino mass square difference due to Planck scale effects. In Section 3, we discuss about numerical result. In Section 4, we present our conclusions.

2 Neutrino Mass Square Difference by Perturbation Approach

The neutrino mass matrix is assumed to be generated by the see saw mechanism \[9,10\]. The effective gravitational interaction of neutrino with Higgs field can be expressed as

\[ L\text{grav} = \frac{\lambda_{\alpha\beta}}{M_{pl}}(\psi_{A\alpha}\epsilon\psi_{C})C^{-1}_{ab}(\psi_{B\beta}\epsilon\psi_{D}) + h.c. \] (1)

Here and every where we use Greek indices \(\alpha, \beta\) for the flavor states and Latin indices \(i, j, k\) for the mass states. In the above equation \(\psi_{\alpha} = (\nu_{\alpha}, l_{\alpha})\) is the lepton doublet, \(\phi = (\phi^{+}, \phi^{0})\)is the Higgs doublet and \(M_{pl} = 1.2 \times 10^{19} GeV\) is the Planck mass \(\lambda\) is a \(3 \times 3\) matrix in a flavor space with each elements \(O(1)\). The Lorentz indices \(a, b = 1, 2, 3, 4\) are contracted with the charge conjugation matrix \(C\) and the \(SU(2)_L\) isospin indices \(A, B, C, D = 1, 2\) are contracted with \(\epsilon = i\sigma_{2}, \sigma_{m}(m = 1, 2, 3)\) are the Pauli matrices. After spontaneous electroweak symmetry breaking the Lagrangian in eq(1) generated additional term of neutrino mass matrix

\[ L_{\text{mass}} = \frac{v^{2}}{M_{pl}}\lambda_{\alpha\beta}\nu_{\alpha}C^{-1}\nu_{\beta}, \]

where \(v = 174 GeV\) is the VEV of electroweak symmetric breaking. We assume that the gravitational interaction is”flavor blind” that is \(\lambda_{\alpha\beta}\) is independent of \(\alpha, \beta\)indices. Thus the Planck scale contribution to the neutrino mass matrix is

\[ \mu\lambda = \mu\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \]

where the scale \(\mu\) is

\[ \mu = \frac{v^{2}}{M_{pl}} = 2.5 \times 10^{-6} eV. \]

We take eq(3) as perturbation to the main part of the neutrino mass matrix, that is generated by GUT dynamics. We treat \(M\) as the unperturbed \((0^{th}\) order) mass matrix in the mass eigenbasis. Let \(U\) be the mixing matrix at \(0^{th}\) order. Then the corresponding \((0^{th}\) order mass matrix) \(M\) in flavour space \([4]\) given by

\[ M = U^{*}\text{diag}(M_{i})U^{\dagger}, \]

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where, \( U_{\alpha i} \) is the usual mixing matrix and \( M_i \), the neutrino masses is generated by Grand unified theory. Most of the parameter related to neutrino oscillation are known, the major expectation is given by the mixing elements \( U_{e3} \). We adopt the usual parametrization.

\[
\begin{bmatrix}
|U_{e2}|/|U_{e1}| = \tan \theta_{12}, \\
|U_{\mu 3}|/|U_{\tau 3}| = \tan \theta_{23}, \\
|U_{e3}| = \sin \theta_{13}.
\end{bmatrix}
\]

In term of the above mixing angles, the mixing matrix is

\[
U = \text{diag}(e^{i\theta_1}, e^{i\theta_2}, e^{i\theta_3})R(\theta_{23})\Delta R(\theta_{13})\Delta^* R(\theta_{12})\text{diag}(e^{iu_1}, e^{iu_2}, 1). \tag{9}
\]

The matrix \( \Delta = \text{diag}(e^{\delta 1}, 1, e^{\delta 2}) \) contains the Dirac phase. This leads to CP violation in neutrino oscillation. \( a_1 \) and \( a_2 \) are the so called Majoring phase, which effects the neutrino less double beta decay. \( f_1, f_2 \) and \( f_3 \) are usually absorbed as a part of the definition of the charge lepton field. Due to Planck scale effects on neutrino mixing the new mixing matrix can be written as \[4\]

\[
U' = U(1 + i\delta \theta),
\]

\[
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3}
\end{pmatrix}
\]

\[
+ i \begin{pmatrix}
U_{e2}\delta \theta_{12}^* + U_{e3}\delta \theta_{23}^*, & U_{e1}\delta \theta_{12} + U_{e3}\delta \theta_{23}, & U_{e1}\delta \theta_{13} + U_{e3}\delta \theta_{23}^* \\
U_{\mu 2}\delta \theta_{12}^* + U_{\mu 3}\delta \theta_{23}^*, & U_{\mu 1}\delta \theta_{12} + U_{\mu 3}\delta \theta_{23}, & U_{\mu 1}\delta \theta_{13} + U_{\mu 3}\delta \theta_{23}^* \\
U_{\tau 2}\delta \theta_{12}^* + U_{\tau 3}\delta \theta_{23}^*, & U_{\tau 1}\delta \theta_{12} + U_{\tau 3}\delta \theta_{23}, & U_{\tau 1}\delta \theta_{13} + U_{\tau 3}\delta \theta_{23}^*
\end{pmatrix}. \tag{10}
\]

Where \( \delta \theta \) is a hermition matrix that is first order in \( \mu [11] \). The first order mass square difference \( \Delta M^2_{ij} = M_i^2 - M_j^2 \) get modified \[12\] as

\[
\Delta^\prime_{ij} = \Delta_{ij} + 2(M_i Re(m_{ii}) - M_j Re(m_{jj})), \tag{11}
\]

where

\[
m = \mu U^\dagger \lambda U,
\]

\[
\mu = \frac{v^2}{M_{pl}} = 2.5 \times 10^{-6} eV.
\]

3
The change in the elements of the mixing matrix, which we parametrized by $\delta \theta [16]$, is given by

$$
\delta \theta_{ij} = \frac{i \text{Re}(m_{jj})(M_i + M_j) - \text{Im}(m_{jj})(M_i - M_j)}{\Delta M'_{ij}^2}.
$$

(12)

The above equation determine only the off diagonal elements of matrix $\delta \theta_{ij}$. The diagonal element of $\delta \theta_{ij}$ can be set to zero by phase invariance.

Using Eq(10), we can calculate neutrino mixing angle due to Planck scale effects,

$$
\frac{|U'_{e2}|}{|U'_{e1}|} = \tan \theta'_{12},
$$

(13)

$$
\frac{|U'_{\mu3}|}{|U'_{\tau3}|} = \tan \theta'_{23},
$$

(14)

$$
|U'_{\tau3}| = \sin \theta'_{13}.
$$

(15)

For degenerate neutrinos, $M_3 - M_1 \cong M_3 - M_2 \gg M_2 - M_1$, because $\Delta_{31} \cong \Delta_{32} \gg \Delta_{21}$.

3 Numerical Results

Note from eq(11) that the correction term depends on crucially the type of neutrino mass spectrum. For a hierarchical or inverse hierarchical spectrum the correction is negligible. Hence, we consider a degenerated neutrino Mass spectrum and take the common neutrino mass to be 2 eV, which is upper limit of tritium beta decay spectrum [13]. From definition of the matrix $m$ in eq(11), we find

$$
m_{11} = \mu e^{i2\alpha_1}(U_{e1}e^{if_1} + U_{\mu1}e^{if_2} + U_{\tau1}e^{if_3})^2
$$

$$
m_{22} = \mu e^{i2\alpha_2}(U_{e2}e^{if_1} + U_{\mu2}e^{if_2} + U_{\tau2}e^{if_3})^2
$$

The contribution of the term in the Planck scale correction, $\epsilon = 2(M_i \text{Re}(m_{11}) - M_j \text{Re}(m_{22})$, can be additive or subtractive depending on the values of the phase $\alpha_1$, $\alpha_2$ and phase $f_i$. In our calculation, we used mixing angle as $\theta_{12} = 34^\circ$, $\theta_{23} = 45^\circ$, $\theta_{13} = 10^\circ$, and $\delta = 0^\circ$. We have taken $\Delta_{31} = 0.002eV^2[14]$ and $\Delta_{21} = 0.00008eV^2[15]$. For simplicity, we have set the charge lepton phases $f_1 = f_2 = f_3 = 0$. Since we have set $\theta_{13} = 0$, the Dirac phase $\delta$ drops out of the $0^{th}$order mixing matrix. In table (1.0) we list the modified mass square difference terms for some sample value of $\alpha_1$ and $\alpha_2$. 

4
| $a_1$ | $a_2$ | $\Delta_{21}$ | $\Delta_{31}$ |
|-------|-------|---------------|---------------|
| 0°    | 0°    | $7.6 \times 10^{-5} eV^2$ | $2.0 \times 10^{-3} eV^2$ |
| 0°    | 45°   | $7.3 \times 10^{-5} eV^2$ | $2.0 \times 10^{-3} eV^2$ |
| 0°    | 90°   | $6.9 \times 10^{-5} eV^2$ | $2.0 \times 10^{-3} eV^2$ |
| 0°    | 145°  | $7.3 \times 10^{-5} eV^2$ | $2.0 \times 10^{-3} eV^2$ |
| 0°    | 180°  | $7.6 \times 10^{-5} eV^2$ | $2.0 \times 10^{-3} eV^2$ |
| 45°   | 0°    | $8.3 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 45°   | 45°   | $8.3 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 45°   | 90°   | $7.6 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 45°   | 135°  | $7.9 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 45°   | 180°  | $8.3 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 90°   | 0°    | $9.0 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 90°   | 45°   | $8.6 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 90°   | 90°   | $8.3 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 90°   | 135°  | $8.6 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 90°   | 180°  | $9.0 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 135°  | 0°    | $8.3 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 135°  | 45°   | $7.9 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 135°  | 90°   | $7.6 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 135°  | 135°  | $7.9 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 135°  | 180°  | $8.3 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 180°  | 0°    | $7.6 \times 10^{-5} eV^2$ | $2.1 \times 10^{-3} eV^2$ |
| 180°  | 45°   | $7.3 \times 10^{-5} eV^2$ | $2.0 \times 10^{-3} eV^2$ |
| 180°  | 90°   | $6.9 \times 10^{-5} eV^2$ | $2.0 \times 10^{-3} eV^2$ |
| 180°  | 135°  | $7.3 \times 10^{-5} eV^2$ | $2.0 \times 10^{-3} eV^2$ |
| 180°  | 180°  | $7.6 \times 10^{-5} eV^2$ | $2.0 \times 10^{-3} eV^2$ |

Table 1: The modified mass square difference term for various value of phase. Input value are $\Delta_{31} = 2.0 \times 10^{-5} eV^2$, $\Delta_{21} = 8.0 \times 10^{-8} eV^2$, $\theta_{12} = 34^\circ$, $\theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ$.
4 Conclusions

It is expected that a higher scale generation the neutrino mass matrix, which will eventually produce the presently observed masses and mixings. In an attractive scenario, the mixing pattern generated by higher scale dynamics is predicted to be bimaximal. We consider that the main part of neutrino masses and mixing from GUT scale operator. The gravitational interaction of lepton field with S.M Higgs field give rise to a $SU(2)_L \times U(1)$ invariant dimension-5 effective Lagrangian give originally by Weinberg [8]. On electroweak symmetry breaking this operators leads to additional mass terms. We considered these to be perturbation of GUT scale mass terms. This model predict a value for the modified mass square difference $\Delta_{21} = \Delta_{21} \pm (1.0 + 0.5) \times 10^{-5} eV^2$, which is correspondence to Planck scale $M_{pl} \approx 2.0 \times 10^{19}\text{GeV}$. In addition, the solar mixing angle is predicted [12]. In this letter, we studied how physics from planck scale effects the neutrino mass square difference. We compute the modified neutrino mass square difference due to the additional mass terms for the case of bimaximal mixing. The change in $\Delta_{31}$ due to these planck scale correction are negligible. But the change in $\Delta_{21}$ is enough that final value falls within the experimentally accepted region. This occurs, of course for degenerate neutrino mass with a common mass of about 2eV.

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