A Three-Loop Model of Neutrino Mass with Dark Matter

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We propose a model in which the origin of neutrino mass is dependent on the existence of dark matter. Neutrinos acquire mass at the three-loop level and the dark matter is the neutral component of a fermion triplet. We show that experimental constraints are satisfied and that the dark matter can be tested in future direct-detection experiments. Furthermore, the model predicts a charged scalar that can be within reach of collider experiments like the LHC.

I. INTRODUCTION

The observed neutrino mixing provides concrete evidence that the Standard Model (SM) is incomplete. Similarly, the need to explain dark matter (DM) motivates the addition of a long-lived particle species to the SM. It is natural to ask if these two short-comings of the SM could have a unified explanation; are the DM and neutrino mass problems related?

A simple model connecting the origin of neutrino mass to the existence of DM was proposed by Krauss, Nasri and Trodden (KNT) [1]. The basic idea was to extend the SM to include new fields, one of which was the DM, such that neutrino mass was radiatively generated at the three-loop level (for detailed studies see Refs. [2–5]). In this model DM played a key role in enabling neutrino mass; if the DM is removed the loop diagram simply does not manifest and neutrinos remain massless.

In recent years, a number of alternative models were proposed that similarly predicted relationships between radiative neutrino mass and DM (for a review see Ref. [6]). A particularly simple one-loop model was proposed by Ma [7] and further studied in Ref. [8]. Other related models also appeared [9–13], including a colored version of the KNT model which employed leptoquarks [14]. In both the KNT and Ma models the SM is extended to include gauge-singlet fermions. Normally these fermions couple to SM leptons and generate neutrino mass at tree-level. However, in the models of KNT and Ma the coupling to the SM does not occur, due to a discrete symmetry. This avoids a tree-level (Type-I) seesaw mechanism and ensures DM longevity.

It is well known that the seesaw mechanism can be generalized to a triplet (Type-III) variant by using $SU(2)_L$-triplet fermions, instead of singlet fermions [15]. Similarly, the Ma model can be generalized to a triplet variant [16]. In this paper we present a new model motivating a connection between the origin of neutrino mass and DM. The model is essentially a triplet variant of the KNT model, and employs triplet fermions to generate radiative masses at the three-loop level. We show that viable neutrino masses are obtained, and that constraints from flavor-changing decays, the anomalous magnetic moment of the muon, and neutrino-less double-beta decay can be satisfied. Viable DM is also obtained, in the form of the neutral component of a triplet fermion. This candidate should produce signals in next-generation direct-detection experiments. The model contains a new charged scalar that can also be within reach of collider experiments.

The plan of this paper is as follows. In Section II we describe the model and detail the origin of neutrino mass. Various constraints are analyzed in Section III and we consider DM in Section IV. Conclusions appear in Section V.
II. THREE-LOOP RADIATIVE NEUTRINO MASSES

A. The Model

The SM contains the gauge symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$. In this work we extend the particle spectrum of the SM to include a charged scalar, $S^+ \sim (1, 1, 2)$, a triplet scalar, $T \sim (1, 3, 2)$, and real fermion triplets, $E_i \sim (1, 3, 0)$, where $i = 1, 2, 3$ labels generations. A $\mathbb{Z}_2$ symmetry with action $\{ T, E_i \} \rightarrow \{ -T, -E_i \}$ is also imposed, while all other fields are $\mathbb{Z}_2$ even. The $SU(2)_L$ triplet fields are written as symmetric matrices, $T_{ab}$ and $E_{ab}$, with components

$$T_{11} = T^{++}, \quad T_{12} = T_{21} = \frac{1}{\sqrt{2}} T^+, \quad T_{22} = T^0,$$

$$E_{11} = E_L^+, \quad E_{12} = E_{21} = \frac{1}{\sqrt{2}} E_L^0, \quad E_{22} = E_L^- \equiv (E_R^+)^c.$$

With these elements, the Lagrangian includes the terms

$$\mathcal{L} \supset \mathcal{L}_{\text{SM}} + \left\{ f_{\alpha \beta} \tilde{T}_\alpha T^\beta + g_{i \alpha} \tilde{E}_i T e_{\alpha R} + \text{H.c} \right\} - \frac{1}{2} E_1^\dagger M_{ij} E_j - V(H,S,T),$$

where $L_\alpha \sim (1, 2, -1)$ and $e_{\alpha R} \sim (1, 1, -2)$ are the SM leptons and $f_{\alpha \beta} = -f_{\beta \alpha}$ are Yukawa couplings. Lowercase greek letters label lepton flavors, $\alpha, \beta \in \{ e, \mu, \tau \}$. The singlet leptons couple to the exotics through a Yukawa matrix $g_{i \alpha}$, and the superscript “c” denotes charge conjugation.

Due to the discrete symmetry the triplet fermions do not mix with the SM at any order in perturbation theory. The triplet mass term gives

$$-(E_i^\dagger)_{ab} M_{ij} (E_j)_{cd} \epsilon^{ac} \epsilon^{bd} = -(E_{i R}^\dagger M_{ij} E_{jL} - \frac{1}{2} (E_{i L}^\dagger)^c M_{ij} E_{j L}^0).$$

Without loss of generality we work in a diagonal basis with $M_{ij} = \text{diag}(M_1, M_2, M_3)$, where $M_1$ is the lightest triplet-fermion mass. The charged and neutral components of $E_i$ are degenerate at tree-level, though radiative corrections involving SM gauge bosons lift this degeneracy, making the charged component heavier. For $M_E \sim \text{TeV}$ the mass splitting is $\Delta M_E \approx 167 \text{ MeV}$ [17]. For the most part we can neglect this small splitting. To bring the neutral fermion mass-term to the correct sign, one defines the Majorana fermions as $E_1^0 = E_{1L}^0 - (E_{1 R}^0)^c$ (see Appendix A). The DM candidate is the lightest neutral fermion $E_1^0$ with mass $M_{DM} = M_1$. When the mass splitting is neglected, one can denote the mass for all members of the lightest triplet simply as $M_{DM}$.

B. Neutrino Mass

The scalar potential contains the terms

$$V(H,S,T) \supset \frac{\lambda_S}{4} (S^-)^2 T_{ab} T_{cd} \epsilon^{ab} \epsilon^{cd} + \frac{\lambda_S}{4} (S^+)^2 (T^+)^{ab} (T^+)^{cd} \epsilon^{ab} \epsilon^{cd},$$

which, in combination with the new Yukawa Lagrangian shown in Eq. (2), explicitly break lepton number symmetry. Therefore, the three vertices should appear simultaneously in the Majorana mass diagram. As a result, neutrino masses are generated at the three-loop level, as shown in Figure 11 where there are three distinct diagrams, corresponding to the sets $\{ T^+, E_0^0, T^0 \}$, $\{ T^+, (E^+)^c, T^0 \}$ and $\{ T^+T^-, E^0, T^0 \}$ propagating in the inner loop. Note that the intermediate fermion is a triplet instead of the singlet field in the KNT model. The triplet scalars are nondegenerate at tree-level due to the term $\lambda_{HT} H^+ T^+ T^0 \subset V(H,S,T)$. However, this splitting is small for the parameter space of interest here, provided $|\lambda_{HT}| < 1$. Thus, one can neglect the mass-splitting among members of both the scalar and fermion triplets when calculating the mass diagram. The calculation gives

$$(M_\nu)_{\alpha \beta} = \frac{3 \lambda_S}{(4 \pi^2)^3} \frac{m_{\alpha \delta} m_{\delta \beta}}{M_T} f_{\alpha \gamma} f_{\beta \delta} g_{i \alpha}^2 g_{i \beta}^* \times F \left( \frac{M_1^2}{M_T^2}, \frac{M_2^2}{M_T^2} \right).$$

Here $m_{\alpha \beta}$ denote SM charged-lepton masses, and $F(x, y)$ is a function encoding the loop integrals (see Appendix B). $M_T$ is the charged-singlet mass and $M_T$ is the triplet scalar mass.

The elements of the neutrino mass matrix can be related to the mass eigenvalues and the Pontecorvo-Maki-Nakawaga-Sakata (PMNS) mixing matrix [18] elements in the standard way

$$(M_\nu)_{\alpha \beta} = [U_\nu \cdot \text{diag}(m_1, m_2, m_3) \cdot U^\dagger_\nu]_{\alpha \beta}.$$
The PMNS matrix is parametrized as

$$U_{\nu} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta_D} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta_D} & c_{12}c_{23} - s_{13}s_{23}e^{i\delta_D} & s_{13}s_{23}e^{i\delta_D} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_D} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta_D} & c_{13}c_{23} \end{pmatrix} \times U_p, \quad (7)$$

where $U_p = \text{diag}(1, e^{i\theta_\alpha/2}, e^{i\theta_\beta/2})$ contains the Majorana phases $\theta_\alpha$ and $\theta_\beta$, $\delta_D$ is the Dirac phase, and $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$. The best-fit experimental values for the mixing angles and mass-squared differences are $s_{12}^2 = 0.320^{+0.016}_{-0.017}$, $s_{23}^2 = 0.53^{+0.02}_{-0.03}$, $s_{13}^2 = 0.025^{+0.003}_{-0.003}$. $\Delta m_{21}^2 = 7.62^{+0.19}_{-0.16} \times 10^{-5}\text{eV}^2$, and $|\Delta m_{12}^2| = 2.55^{+0.06}_{-0.05} \times 10^{-3}\text{eV}^2$ [19]. Within these ranges, one can determine the regions of parameters space where viable neutrino masses are compatible with experimental constraints, and in agreement with the measured DM relic density.

### III. EXPERIMENTAL CONSTRAINTS

We shall discuss DM in detail in Section 15. For the moment we note that the lightest $Z_2$-odd field is a stable DM candidate. There are two possibilities for the DM, namely $T^0$ and $E_0^0$. However, $T$ has nonzero hypercharge so that $T^0$ couples to the Z boson. As the CP-even and CP-odd components of $T^0$ are mass-degenerate, stringent direct-detection constraints apply and, in fact, exclude $T^0$ as a DM candidate [20]. Consequently the neutral fermion $E_0^0$ is the only viable DM candidate in the model and $M_T > M_{DM}$ is required to ensure DM stability. In earlier models with triplet-fermion DM [16, 17], the observed relic density was obtained via $SU(2)_L$ (co-)annihilation channels and required a DM mass around $M_{DM} \sim 2.3-2.4\text{ TeV}$. However, we shall see that in our model the additional annihilation channels required the DM mass to be in the range $M_{DM} \sim 2.35-2.75\text{ TeV}$. Thus, both $T$ and $E_0$ are too heavy to be produced at the LHC, though the $Z_2$-even field $S$ could be within the reach of the LHC, as we discuss below.

The Yukawa couplings $g_{ai}$ induce flavor changing processes like $\mu \to e + \gamma$. At the one-loop level the exotic triplets give four distinct diagrams, as shown in Figure 2. For two of these diagrams the photon is attached to the internal fermion line. In the limit that the mass-splitting between $T^0$ and $T^--$ vanishes, the amplitudes for these diagrams differ by an overall sign. Thus, in this limit, the coherent sum of these amplitudes vanishes, and to good approximation the diagrams can be neglected. Therefore we need only calculate the diagrams in Figure 2a. There is also a diagram mediated by the singlet scalar $S$ that must be included. Putting all this together, the branching ratio for $\mu \to e + \gamma$ is

$$B(\mu \to e\gamma) = \frac{\Gamma(\mu \to e + \gamma)}{\Gamma(\mu \to e + \nu + \bar{\nu})} \approx \frac{\alpha v^4}{384\pi} \times \left\{ \frac{|f_{\mu\tau}^* f_{\tau\gamma}|^2}{M_S^4} + \sum_i g_{i e}^* g_{i \mu} F_2(M_i^2/M_T^2) \right\}^2, \quad (8)$$

where $F_2(R) = [1 - 6R + 3R^2 + 2R^3 - 6R^2 \log R]/[6(1 - R)^4]$. Eq. (8) can also be used to determine $B(\tau \to \mu + \gamma)$ by simply changing flavor labels.

Replacing the final-state electron with a muon, the diagrams in Figure 2 contribute to the muon magnetic moment. Similar arguments hold for the calculation of the muon magnetic moment; the diagrams with the photon attached to the internal fermion cancel when the mass splitting for the scalar triplet is neglected. The result for the muon anomalous magnetic moment is

$$\delta a_{\mu} = \frac{m_{\mu}^2}{16\pi^2} \left\{ \sum_{|\alpha|>\mu} |f_{\mu\alpha}|^2 6M_S^2 + \sum_i 3|g_{i \mu}|^2 F_2(M_i^2/M_T^2) \right\}. \quad (9)$$
FIG. 2: Diagrams for $μ \to e + γ$ due to the $Z_2$-odd fields $E \sim (1, 3, 0)$ and $T \sim (1, 3, 2)$. Two additional diagrams, with the photon attached to the lower line, and a diagram due the singlet scalar $S \sim (1, 1, 2)$ are also shown.

FIG. 3: Left: Viable regions of parameter space for the Yukawa couplings $f_{αβ}$ and $g_{iα}$. Here, neutrino mass/mixing and lepton flavor-changing constraints are satisfied, and the correct DM relic abundance is obtained. Right: Branching fractions for lepton flavor violating decays $μ \to e + γ$ and $τ \to μ + γ$ versus the anomalous magnetic moment of the muon. Branching fraction limits appear as horizontal lines and all points easily satisfy the magnetic moment constraint (which is too large to appear in the figure).

After matching the calculated neutrino mass-matrix elements with the neutrino mixing data, one finds a significant region of parameter space that is consistent with low-energy constraints, as shown in Figure 3. We find that the Yukawa couplings $f_{αβ}$ lie in the range $|f_{αβ}| \sim 0.02 − 0.5$, and the couplings $g_{iα}$ are expected to be $O(1)$. We restrict the latter to the perturbative region with $|g_{iα}| \lesssim 3$. From the left panel, one notices that the constraint from $μ \to e + γ$ is most severe. The null-results from searches for neutrino-less double-beta decay provide an additional constraint of $(M_ν)_{ee} \lesssim 0.35$ eV [21], though this is easily satisfied. Next generation experiments will improve this bound to the level of $(M_ν)_{ee} \lesssim 0.01$ eV [22, 23].

We note that when considering only one or two generations of the triplet fermions, no solution that simultaneously accommodates the neutrino mass and mixing data, low-energy flavor physics constraints, and the DM relic density, could be found. Thus a minimum of three generations of exotic triplet-fermions are required.

IV. DARK MATTER

A. Relic Density

The lightest neutral fermion, $E_0^0$, is a stable DM candidate. There are two classes of interactions that can maintain thermal contact between the DM and the SM in the early Universe. The first class of interactions are mediated by the triplet scalar, while the second involve $SU(2)_L$ gauge interactions. In addition to annihilation processes like $E^0E^0 \to SM$, there are coannihilations like $E^0E^± \to SM$. Due to the small mass-splitting between neutral
and charged triplet-fermions, and the fact that coannihilation cross sections can be larger than annihilation ones, coannihilations cannot be neglected.

The annihilation channels mediated by triplet scalars give

$$
\sigma(2E^0 \to \ell_\beta^+ \ell_\alpha^-) \times v_r = \frac{|g_1^a g_1^\beta|^2}{48\pi} \frac{M_{DM}^2 (M_{DM}^4 + M_T^4)}{(M_{DM}^2 + M_T^2)^4} \times v_r^2 \equiv \sigma_{00}^{\alpha\beta} \times v_r, \tag{10}
$$

where $v_r$ is the relative velocity of DM particles in the center-of-mass frame. As expected for Majorana DM, there are no s-wave annihilations in the limit where final-state fermion masses are neglected. There are no coannihilations mediated by $T$ but one must include the charged fermion annihilations:

$$
\sigma(E^- E^+ \to \ell_\beta^+ \ell_\alpha^-) \times v_r = \frac{|g_1^a g_1^\beta|^2}{48\pi} \frac{M_{DM}^2 (M_{DM}^4 + M_T^4)}{(M_{DM}^2 + M_T^2)^4} \times v_r^2 \equiv \sigma_{-+}^{\alpha\beta} \times v_r. \tag{11}
$$

For the $SU(2)_L$ channels we work in the $SU(2)_L$-symmetric limit, neglecting gauge-boson masses. This is justified a posteriori as we find $M_{DM} \sim \text{TeV}$ is required. We can therefore use standard results in the literature [24].

Ignoring the tiny mass-splitting, we combine annihilation and coannihilation channels in the standard way [25], giving

$$
\sigma_{\text{eff}}(2E \to SM) \times v_r = \frac{1}{g_{\text{eff}}^2} \left[ \sigma_W \times v_r + \sum_{\alpha,\beta} \left\{ g_0^2 \sigma_{00}^{\alpha\beta} + 2g_{\pm} \sigma_{-+}^{\alpha\beta} \right\} \times v_r \right]. \tag{12}
$$

Here $g_0 = g_\pm = 2$ and $g_{\text{eff}} = g_0 + 2g_\pm$. The $SU(2)$ channels are denoted by

$$
\sigma_W \equiv \sigma(2E \to W^- W^+ SM) \sim \frac{\pi a_0^2}{2M_{DM}^2 v_r} \left\{ 222 + \frac{51}{2} v_r^2 \right\}, \tag{13}
$$

while the $T$-exchange cross sections are defined above.\(^1\)

We find that the DM mass is of the same order of magnitude as previous results in the literature [16]. The $p$-wave annihilation channels into charged leptons shift the DM mass from the value $M_{DM} \approx 2.55 \text{ TeV}$ (blue line in the right panel of Figure 4) into the range $M_{DM} = 2.35 \sim 2.75 \text{ TeV}$, though the difference is not particularly large. If the triplet fermion only contributes a fraction of the total DM abundance, the requisite mass range increases accordingly.

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\(^1\) If $p$-wave processes are neglected and $T$-exchange channels ignored, our result matches the s-wave expression of Ref. [16].
For example, for a triplet fermion that only comprises 50% of the observed relic abundance, the mass should lie in the range \( M_{DM} = 3.3 \sim 3.8 \) TeV.

In principle the cross sections are subject to a Sommerfeld enhancement due to \( SU(2)_L \) gauge boson exchange. However, at the freezeout temperature \( T_f \sim M_{DM}/25 \), the electroweak symmetry is broken. When calculating the Sommerfeld enhancement one should therefore consider massive mediators. It is well known that the enhancement from a massive mediator turns off for \( M'/(\alpha' M_{DM}) \gtrsim O(1) \), where primes denote the coupling and mass of the mediator (see e.g. Ref. \[26\]). With \( \alpha_2 \approx 1/30 \), we have \( M_W/(\alpha_2 M_{DM}) \approx 1 \), and the enhancement gives a modest correction (less than \( O(1) \)) that can be neglected, at least in an initial treatment. Previous works found an increase in \( M_{DM} \) of roughly 500 GeV, due to the enhancement \[27, 28\]. We checked numerically that with \( M_{DM} \sim 3 \) TeV all other constraints can be satisfied, so a small increase in \( M_{DM} \) will not spoil our conclusions. A consistent treatment requires both s-wave and p-wave enhancements \[29\] for a massive mediator; such an analysis is beyond the scope of this paper.

### B. Direct Detection

The triplet-fermion DM has vanishing isospin and consequently does not couple to SM quarks at tree-level. Also, because the DM is a Majorana fermion, there are no radiatively-induced magnetic dipole interactions with SM gauge bosons. However, \( W \) boson exchange generates three one-loop diagrams relevant for direct-detection experiments; see Figure 5. The resultant scattering has both spin-dependent and spin-independent contributions, though the former are suppressed by the DM mass. The dominant interaction type is therefore spin-independent scattering with cross section

\[
\sigma_{SI}(E^0 N \to E^0 N) \approx \frac{\pi \alpha_2^4 M_A^4 f^2}{M_W^4} \left[ \frac{1}{M_W^2} + \frac{1}{M_h^2} \right]^2. \tag{14}
\]

Here the DM scatters off a target nucleus \( A \) with mass \( M_A \), \( \alpha_2 \) is the \( SU(2)_L \) fine-structure constant and we use the standard matrix-element parametrization for the nucleon:

\[
\langle N \sum q m_q \bar{q} q | N \rangle = f m_N, \tag{15}
\]

with \( m_N \) being the nucleon mass. We take \( f \approx 1/3 \), though this is subject to standard QCD uncertainties. This gives a cross section of roughly \( \sigma_{SI} \approx 10^{-47} \text{cm}^2 \) per nucleon, which is beyond the current sensitivity of experiments like LUX \[30\], but could be within reach of future experiments \[31\]. Future prospects for probing this DM candidate are therefore promising.

### C. Indirect Detection

Dark Matter candidates with non trivial electroweak quantum numbers can generate observable signals at indirect-detection experiments. Though subject to greater uncertainties than direct-detection experiments, indirect searches can give useful constraints. For triplet-fermion DM, perhaps the strongest indirect constraints come from gamma-ray searches from the Galactic center. Gamma rays are produced via annihilations like \( 2E^0 \to 2\gamma \) and \( 2E^0 \to \gamma Z \), which occur at the one-loop level. The cross sections are dominated by box diagrams with \( W \) bosons and are largely insensitive to the DM mass. The strongest constraints come from photon-line searches of the Galactic center by Fermi-Lat \[32\] and HESS \[33\]. The total rate for gamma-ray production depends on the DM profile at the galactic
center so the severity of the constraints depends on the assumptions about the halo structure. Recent analyses of
wino DM in supersymmetric models \[34–36\] find that thermal wino DM is consistent with the data for DM halos with
a significant core, while for the more cuspy Einasto and Navarro-Frenk-White (NFW) distributions the parameter
space with $10^2 \text{ GeV} \lesssim M_{DM} \lesssim 3 \text{ TeV}$ is excluded. For a cored profile like the Burkert 10 kpc profile, a smaller region
of parameter space with $2.25 \text{ TeV} \lesssim M_{DM} \lesssim 2.45 \text{ TeV}$ is excluded but wino DM remains otherwise viable \[34, 36\].
Higher-order effects are expected to weaken the bounds by a $O(1)$ factor but this should not modify the conclusion
that thermal wino DM is excluded for the NFW profile \[34\]. These constraints hold to good approximation for the
triplet-fermion DM in the present model. Together, the analysis suggests that thermal DM requires a cored profile
and a DM mass in the range $M_{DM} \gtrsim 2.45 \text{ TeV}$. In the case where the triplet fermion does not supply the fu-
ll DM abundance these constraints are relaxed.

V. CONCLUSION

We presented a new model in which the origin of neutrino mass is connected to the existence of DM. The model
is related to the KNT proposal but has a number of distinguishing features. The DM should be somewhat heavy,
$M_{DM} \sim \text{ TeV}$, and can be probed in next-generation direct-detection experiments. We showed that viable neutrino
masses appear at the three-loop level and that experimental constraints can be satisfied. Flavor changing effects
like $\mu \to e + \gamma$ could manifest at next-generation experiments, and the model predicts a singly-charged scalar
that can be within reach of collider experiments. Discovery of the singlet scalar, in conjunction with the observation of
direct-detection signals from the triplet-fermion DM, would provide strong evidence for this model. In a future work
we shall study the impact of the exotics on: the Higgs decay channels $h \to \gamma \gamma, \gamma Z$, the possible enhancement of the
triple-Higgs coupling, the electroweak phase transition strength, and the collider phenomenology \[37\], to determine if
further signals are possible.

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Appendix A: Triplet Fermion Couplings

In terms of this Majorana fermion $E^0 = E^0_L - (E^0_R)^c$, the charged-current interactions take the form

$$\mathcal{L}_{W,E} = \frac{g}{\sqrt{2}} \left( E^+ \gamma^\mu \sigma^\mu E^0_{\mu} + \frac{1}{2} \sigma^0 E^0 W^+ E^- \right),$$

(A1)

where $E^+ = E^+_L + E^+_R$, while the coupling to the triplet scalar is

$$g_{\alpha \beta} (\overline{E}_i)_{ab} e_{\alpha R} = g_{\alpha \beta} \left\{ E^+_{iL} T^{++} + \overline{E}^0_{iL} T^+ + (E^+_{iR})^c T^0 \right\} e_{\alpha R}$$

$$= g_{\alpha \beta} \left\{ \overline{E}^0_i P_R e_{\alpha} T^{++} + \overline{E}^0_i P_R e_{\alpha} T^+ + c_{\alpha R} P_R E^+_i T^0 \right\}$$

$$= g_{\alpha \beta} \left\{ \overline{E}^0_i P_R e_{\alpha} T^{++} - e_{\alpha R} P_R E^0 T^+ + c_{\alpha R} P_R E^+_i T^0 \right\}.$$  

(A2)

Note the extra negative sign in last form, which plays a role in the loop-mass calculation. This sign difference results
from the negative sign in the Majorana mass terms in Eq. (3).

Appendix B: Radiative Neutrino Mass

The Majorana neutrino masses are calculated to be

$$\langle M_{\nu} \rangle_{\alpha \beta} = \frac{3 \lambda_S}{4 \pi^2} \frac{m_e m_\delta}{M_T} \int f_{\alpha \gamma} f_{\beta \delta} g_{\gamma i} g_{\delta i} \times F \left( \frac{M^2}{M_T^2}, \frac{M^2}{M_T^2}, \frac{M^2}{M_T^2} \right).$$

(B1)
where

\[
F(\alpha, \beta) = \frac{\sqrt{\alpha}}{8\beta^{3/2}} \int_{0}^{\infty} \frac{r}{r + \alpha} \left( \int_{0}^{1} dx \ln \frac{x(1-x)r + (1-x)\beta + x}{x(1-x)r + x} \right)^2. \tag{B2}
\]

In obtaining this form, the lepton masses that would otherwise appear in the function \(F\) have been neglected. The expression for \(F(\alpha, \beta)\) is the same as that found in the KNT model \[.]

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