Ferromagnet-superconductor proximity effect: The clean limit

Miloš Božović and Zoran Radović
Department of Physics, University of Belgrade, P.O. Box 368, 11001 Belgrade, Serbia and Montenegro

We study theoretically the influence of ferromagnetic metals on a superconducting film in the clean limit. Using a self-consistent solution of the Bogoliubov–de Gennes equation for a ferromagnet-superconductor-ferromagnet double junction we calculate the pair potential and conductance spectra as a function of the superconducting layer thickness \( d \) for different strengths of ferromagnets and interface transparencies. We find that the pair potential and the critical temperature are weakly perturbed by the exchange interaction and do not drop to zero for any finite \( d \). On the other hand, for thin superconducting films charge transport is spin polarized and exhibits a significant dependence on the ferromagnetic strength and magnetization alignment.

I. INTRODUCTION

Rapid advancement of nanofabrication technology in the past decade has reinvigorated interest in understanding the effects inherent to clean superconducting heterostructures. Apart from potential device applications, particularly in quantum information storage and processing, a variety of phenomena makes systems consisting of superconductor sandwiched between two ferromagnets interesting for both experimental and theoretical investigation (see, for example, refs. 2-8 and 9-13). The interplay of ferromagnetism and superconductivity results in characteristic proximity effects near the contacts between two metals: the Andreev reflection is suppressed in the presence of a ferromagnet; conversely, the superconducting correlations, described via the pair amplitude, extend in the ferromagnetic material in an oscillatory way.

The proximity effect in heterostructures consisting of a superconductor (S) in contact with a normal, nonmagnetic (N) or ferromagnetic (F), metal has been widely studied and well understood in the dirty limit, where the electron mean free path \( l \) is much smaller than the superconducting coherence length \( \xi_0 \). In particular, a strong depairing effect is found in FSF trilayers, which results in a superconducting-to-normal phase transition at a finite thickness of the S layer. Furthermore, using a spin-valve setup for these junctions, spontaneous transition from parallel (P) to antiparallel (AP) ferromagnetic orientation could be triggered.

However, in recent experiments with FSF trilayers it was found that the difference between the superconducting critical temperatures for AP and P alignment is less by two orders of magnitude than theoretical predictions for the dirty limit. A possible reason for this disagreement could be that these samples were actually much cleaner than the ones used in previous experiments. Namely, as a consequence of phase coherence of electron wave functions quasiparticle spectrum of clean trilayers differs substantially from the BCS result for bulk superconductor: density of states is practically gapless and Andreev reflection is reduced in thin S layers. As a result, depairing induced by ferromagnets appears to be much weaker in the clean limit, when \( l \gg \xi_0 \).

The purpose of this paper is to clarify the ferromagnet-superconductor inverse proximity effect in the clean limit, within the framework of the BCS theory. The model we study is an FISIF heterostructure with an S film in contact with massive F metals, where \( I \) denotes the interface potential barrier of arbitrary transparency. To describe the equilibrium and transport properties of such a system, we use a numerical procedure to solve the Bogoliubov–de Gennes equation self-consistently for spatial-averaged pair potential. We find that neither the magnitude of exchange interaction in the ferromagnets nor the relative orientation of the magnetizations has any significant influence on the pair potential and the critical temperature of the superconductor. On the other hand, transport properties of thin S films exhibit a significant dependence on the ferromagnetic strength and alignment from parallel (P) to antiparallel (AP) ferromagnetic moment orientation could be triggered. Unlike the case of diffusive mesoscopic superconducting bilayers and trilayers, we found a BCS type of the pair potential temperature dependence. A peculiar temperature and alignment dependence of the proximity effect is recently obtained for a model of FSF trilayers with atomic thickness.

II. EQUILIBRIUM PROPERTIES

In this section we study the equilibrium properties of an \( s \)-wave superconductor sandwiched in between two ferromagnets. We consider a simple model of FISIF double junction consisting of a superconducting layer of thickness \( d \) connected to ferromagnetic metals by interfaces of arbitrary transparency, fig. 1. Assuming that the metals are clean, quasiparticle propagation is described by the Bogoliubov–de Gennes equation

\[
\begin{pmatrix}
H_0(r) - \rho_{\sigma} h(r) \\
\Delta^*(r) + \rho_{\sigma} h(r)
\end{pmatrix}
\begin{pmatrix}
u_{\sigma}(r) \\
v_{\sigma}(r)
\end{pmatrix} =
\begin{pmatrix}
\Delta(r) \\
-H_0^*(r) + \rho_{\sigma} h(r)
\end{pmatrix}
\begin{pmatrix}
u_{\sigma}(r) \\
v_{\sigma}(r)
\end{pmatrix} =
E
\begin{pmatrix}
u_{\sigma}(r) \\
v_{\sigma}(r)
\end{pmatrix},
\]

where \( H_0(r) = -h^2 \nabla^2 / 2m + W(r) - E_F \), \( E_F \) is the Fermi energy, \( \sigma \) is the quasiparticle spin orientation (\( \sigma = \uparrow, \downarrow \)), and \( E \) is the quasiparticle energy with respect to the Fermi level. The interface potential is
modeled by \( W(r) = \hat{W}\{\delta(z) + \delta(z - d)\} \), where the \( z \) axis is perpendicular to the layers and \( \delta(z) \) is the Dirac delta-function. For simplicity, the electron effective mass \( m \) and the Fermi velocity, \( v_F = \sqrt{2mE_F/\hbar^2} \), are assumed to be constant through the junction. The superconducting pair potential is given by \( \Delta(z) = \Delta(z)\Theta(z)\Theta(d - z) \), where \( \Theta(z) \) is the Heaviside step function. The exchange potential \( h(r) \) is given by \( h_0[\Theta(-z) + \Theta(z - d)] \) for the P [AP] alignment, and \( \rho_s \) is \( 1 \) (\(-1\)) for \( \sigma = \uparrow \) (\( \downarrow \)); a uniform magnetization is assumed to be parallel to the layers. The parallel component of the wave vector, \( q|| \equiv (q_x, q_y, 0) \), is conserved, and the spinor \( \{u_\sigma(r), v_\sigma(r)\}^T \) satisfies the appropriate boundary conditions for the wave function and its first derivative at \( z = 0 \) and \( z = d \).

**FIG. 1:** The geometry of FISIF heterostructure in P (AP) alignment of magnetizations.

For a constant pair potential, solution of eq. \( \text{I} \) has the following form in the superconductor, \( 0 < z < d \),

\[
\begin{bmatrix}
    u_\sigma(r) \\
    v_\sigma(r)
\end{bmatrix} = \exp(iq|| \cdot r) \times \left[ \begin{array}{c}
    [c_1(E, q||) \exp(-iq^+_z z) + c_2(E, q||) \exp(-iq^-_z z)] \begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix} \\
    [c_3(E, q||) \exp(iq^+_z z) + c_4(E, q||) \exp(iq^-_z z)] \begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix}
\end{array} \right].
\]

Here, \( \bar{u} = \sqrt{(1 + \Omega/E)/2} \) and \( \bar{v} = \sqrt{(1 - \Omega/E)/2} \) are the BCS coherence factors, \( \Omega = \sqrt{E^2 - \Delta^2} \), and the modulus of the wave vector \( |q^\pm| = \sqrt{(2m/\hbar^2)(E_F \pm \Omega)} \). Coefficients \( c_1 \) through \( c_4 \) are obtained from boundary conditions in the scattering problem for FISIF heterostructure. Wave-vector components have to be normalized by using the condition for canonical transformation

\[
\int_V d^3r \left[ |u_\sigma(r)|^2 + |v_\sigma(r)|^2 \right] = 1,
\]

where the integration is performed over the volume \( V \) of the superconductor. The self-consistency condition for the pair potential is given by\(^28\)

\[
\Delta(z) = \lambda \sum_{q} \left\{ u_\uparrow(r)v^*_\uparrow(r)[1 - f_0(E)] - u_\downarrow(r)v^*_\downarrow(r)f_0(E) \right\}.
\]

Here, \( \lambda \) is the coupling constant and \( f_0 \) is the Fermi distribution function at temperature \( T \). By performing the summation over \( q \), we get

\[
\Delta(z) = \lambda N(0)V \int_0^{\pi/2} d\theta \sin \theta \cos \theta \times
\]

\[
\int_0^{\hbar\omega_D} d\Omega \left\{ u_\uparrow(r)v^*_\uparrow(r)[1 - f_0(E)] - u_\downarrow(r)v^*_\downarrow(r)f_0(E) \right\},
\]

where \( N(0) = mK_F/2\pi^2\hbar^2 \) is the normal-metal density of states (per spin orientation) at the Fermi level, \( \theta \) is the angle between \( q^\perp \) and the \( z \) axis, and \( \hbar\omega_D \) is the upper cutoff in integration over quasiparticle kinetic energy \( \Omega \).

**FIG. 2:** Zero-temperature pair potential \( \Delta \), normalized to the bulk value \( \Delta_0 \), as a function of the S film thickness \( d \) for NSN \( (Z = 0) \) and NISIN \( (Z = 10) \) junctions. The same curves describe dependence of \( T_c/T_{c0} \) on \( d \).

We solve eq. \( \text{I} \) using the stepwise approximation for \( \Delta(z) \). We calculate the spatial average of \( \Delta(z) \) following a standard iteration procedure

\[
\bar{\Delta}_{i+1} = \frac{1}{d} \int_0^d \bar{\Delta}_i(z)dz
\]

by setting \( E = \sqrt{\Omega^2 + \Delta^2} \) in eq. \( \text{I} \) in order to obtain \( \bar{\Delta}_i(z) \) in the \( i \)-th iteration\(^29\) Starting from the bulk value \( \Delta_0 \) we repeat this procedure until the difference between \( \bar{\Delta}_i+1 \) and \( \bar{\Delta}_i \) becomes sufficiently small. This procedure is justified both for thin S films, \( d/\xi_0 \lesssim 1 \), where the spatial variation of the pair potential is small, and thick S films, \( d/\xi_0 \gg 1 \), where \( \Delta(z) \) is practically flat at \( \Delta_0 \), except in a narrow region of the order of \( \xi_0 \) near the interfaces.

Numerical results obtained by the self-consistency algorithm described above are shown in figs. 2 and 3. The average pair potential \( \bar{\Delta} \), normalized to the bulk value \( \Delta_0 \), is shown in fig. 2 as a function of \( d/\xi_0 \), where \( \xi_0 = \hbar v_F/\pi\Delta_0 \) for an NISIN junction at zero temperature and for two strengths of the interface barriers, \( Z = 2W/\hbar v_F \). As the transmissivity of the interfaces decreases (i.e., as \( Z \) increases), the normal reflection of incoming electrons becomes more probable, which makes
FIG. 3: Zero-temperature pair potential $\bar{\Delta}$, normalized to the bulk value $\Delta_0$, for (a) FSF ($Z = 0$) and (b) FISIF ($Z = 10$) junctions as a function of the S film thickness $d$ for $X = 0.5$ and for parallel and antiparallel magnetization alignments. The corresponding NISIN curves ($X = 0$) are shown for comparison (solid lines). The same curves describe dependence of $T_c/T_{c0}$ on $d$.

Recent experiments also imply that the difference between the superconducting critical temperatures in AP and P alignment is off by about two orders of magnitude with respect to the theoretical predictions for the dirty limit. A possible reason for this disagreement could be that these samples were much cleaner than what was the case in earlier experiments (refs. 2 and 3) where data fit very well to the dirty-limit theory.

Qualitatively, the fact that $\Delta$ weakly depends on $X$ in the clean limit can easily be shown using the quasiclassical approximation. By neglecting the wave vectors outside the small interval around the Fermi surface of radius $k_F$, so that $\hbar^2q^2/2m \simeq -E_F \pm \hbar^2k_F|q|/m$, the one-electron Hamiltonian could be linearized, $H_0 \simeq \pm i(h^2k_F/m)(\partial/\partial z) - 2E_F$. After normalization given by eq. (4), coefficients in eq. (2) become

$$
c_1(E, q_\parallel) = \frac{\bar{u}}{\sqrt{V(1 - \bar{u}^2\bar{v}^2\zeta^{-1}\sin\zeta)}}; \quad c_2(E, q_\parallel) = 0,
$$

$$
c_3(E, q_\parallel) = -\frac{\bar{v}_e\zeta^{-1}}{\sqrt{V(1 - \bar{u}^2\bar{v}^2\zeta^{-1}\sin\zeta)}}; \quad c_4(E, q_\parallel) = 0
$$

where $\zeta \equiv d(q_\parallel^2 - q_F^2)$. Therefore, solutions $u_r(r)$ and $v_e(r)$ of linearized eq. (1) are independent of $X$ and exactly match the solutions of the Bogoliubov–de Gennes equation for transparent NSN junctions. We emphasize that a weak influence of the exchange potential is inherent for clean S and F metals. The dependence of the critical temperatures on the mutual orientations of ferromagnetic moments is also hardly observable in dirty FISIF hybrids with finite interface transparency. However, for such structures with higher transparency the inverse proximity effect is significant.

We have also verified that the temperature dependence of $\Delta(T)$ remains to be of the BCS type well described by

$$
\Delta(T) = \Delta(0) \tanh \left( 1.74\sqrt{T_c/T - 1} \right)
$$

for all $d$, where $T_c$ is the critical temperature of the S film. In addition, the $d$-dependence of the critical temperature normalized to the bulk value, $T_c/T_{c0}$, coincides with $\Delta(0)/\Delta_0(0)$ vs. $d$ curves (figs. 2 and 3), being greater for AP than for P alignment of magnetizations, which is in agreement with theoretical predictions and experimental results. In diffusive mesoscopic superconducting bilayers and trilayers, however, a more complex dependence is predicted. Moreover, for finite F layers the critical temperature, as well as the pair potential, oscillate with the F-layer thickness.

III. TRANSPORT PROPERTIES

The properties of quasiparticle transport are commonly described by differential conductance spectrum. For an FISIF double-barrier junction at zero temperature conductance per orbital transverse channel can be...
FIG. 4: Zero bias conductances of FSF ($Z = 0$, top panel) and FISIF ($Z = 1$, bottom panel) junctions at zero temperature as a function of the S film thickness $d$ for $X = 0.5$. The corresponding NSN and NISIN spectra ($X = 0$, dashed lines) are shown for comparison.

The conductance $G(E)$ is given by

$$G(E) = G_0 \sum_{\sigma=\uparrow,\downarrow} P_\sigma \times \int_0^{\pi/2} d\theta \sin \theta \cos \theta \ [A_\sigma(E,\theta) + C_\sigma(E,\theta)] \tag{6}$$

where $G_0 = 2e^2/h$ is the conductance quantum and $P_\sigma = (1 + \rho_\sigma X) / 2$. Analytical results for the probabilities of Andreev reflection and direct quasiparticle transmission, $A_\sigma(E,\theta)$ and $C_\sigma(E,\theta)$, respectively, are presented and discussed in ref.\textsuperscript{13}. Contribution of evanescent propagation to $G(E)$ is included as well. Here, we only point out the strong influence of ferromagnetism on charge transport through a thin S film, in contrast to the weak proximity effect on equilibrium properties.

As an illustration, we calculate the zero bias conductance, $G(0)$, using the self-consistent pair potential obtained in the previous section. The results are shown in fig.\textsuperscript{4}. The subgap transport through the superconductor changes by virtue of two principal mechanisms: firstly, due to the presence of the ferromagnets, and secondly, due to the decrease of thickness $d$. The zero-bias conductance for an FSF junction with $X = 0.5$ is significantly below the NSN value of unit conductance per channel, and splits for the P and the AP magnetization alignment as the S film becomes thinner. The subgap conductance is greater in P than in the AP alignment as a result of a strong magnetoresistive effect in thin S layers: when $d \lesssim \xi_0$ the direct transmission of spin polarized quasiparticles across the superconductor becomes a dominant transport mechanism.\textsuperscript{13,15} Hence, in contrast to the NSN trilayers, the conductance of an FSF system shows a strong size effect. Conductances become significantly suppressed for finite interface transparency ($Z = 1$), both for NISN and FISIF junctions, due to the increase of normal and decrease of Andreev reflection probability.

IV. CONCLUSION

We have studied the ferromagnet-superconductor proximity effect in clean FISIF heterostructures with thin S layers and massive F metals. We have found that the superconducting order parameter is weakly affected by the exchange interaction and has practically the same dependence on the S film thickness as in the corresponding NISIN structures. On the other hand, quasiparticle dynamics within an FISIF heterojunction is substantially changed with respect to the corresponding NISIN system, the more so as the S film becomes thinner.

While in clean ferromagnet-superconductor hybrids with massive F layers the ferromagnetism has negligible inverse proximity effect on equilibrium properties, the opposite behavior is previously found for the dirty ones,\textsuperscript{3} where the pairing potential may be very sensitive to the vicinity of the F layer(s). Moreover, in dirty hybrids the critical value of superconducting layer thickness at which destruction of superconductivity occurs is strongly dependent on the ferromagnetic exchange potential.\textsuperscript{16} On the other hand, charge transport is diffusive in this case, and consequently the conductance spectrum is practically independent on the S film thickness and alignment of magnetizations.

In summary, we have found that in the clean limit the BCS self-consistent solution for a thin superconducting film in contact with massive ferromagnets shows very weak depairing effect of the exchange interaction. While the equilibrium properties are practically unaffected, charge transport is strongly influenced by proximity of ferromagnets.

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The case of arbitrary orientation of ferromagnetic moments requires the introduction of triplet components of the anomalous Green’s function.

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