An Empirical Study on Patient Queuing after Medical Staff Supporting Disaster Areas in Northwest China

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ABSTRACT

Recently, the new coronavirus has brought great disaster to human beings, so we have to take strong measures to suppress the large-scale outbreak of the disease. In this paper, by looking up the data of medical staff supporting Wuhan area in Northwest China, we build a queuing model of $M|M|c\infty$ to analyze the waiting time and staying time of patients. Secondly, due to the increase of patients, the burden of outpatient service is gradually increasing, which leads to the speed of epidemic spread greatly accelerated. Therefore, SIR model is constructed to analyze the relationship between patients and healers. The experimental results show that: (1) at the beginning of the data of more than 1000 medical staff, the patients were served for too long, which led to low efficiency. When they were supported, the efficiency was increasing with the increase of support, and the time was shortened, which was very helpful to relieve the medical pressure of outpatient. (2) With the increase of patients, at the same time, the number of healers is increasing, of course, there are also healthy people in it. At this time, we should focus on finding a suitable node, reducing the number of patients and increasing the number of healers, so as to effectively control the epidemic.

Keywords-- New Crown Epidemic Situation, $M|M|c\infty$ Queuing Model, Waiting Time, SIR Model

I. INTRODUCTION

Under the new epidemic situation, great changes have taken place in the economic operation of outpatients, doctors' medical behaviors and diagnosis and treatment paths. Waiting in line is something we all have to experience, such as waiting for payment [1], waiting for dinner, waiting for elevator [2], etc. If the medical service capacity needed by the patient has been occupied by the time he arrives at the clinic, the patient will have to wait in line. Patients are different from healthy people, patients are very reluctant to accept waiting in line. Waiting in line is not good for the physical and mental health of patients, and it will also produce crowd aggregation, which is not conducive to the prevention of outpatient work. With the increase of people's awareness of disease prevention, the acceleration of life rhythm and the improvement of medical service level, the organizational management of queuing is particularly important. If the outpatient makes the patient wait, it will lead to dissatisfaction or possible loss of the patient, or even cross infection [3]. Therefore, based on the problem of patients waiting in line, this paper establishes a model to provide appropriate help for epidemic control.

The only condition for waiting to never happen is to stipulate that the patient arrives at a fixed time interval and the service time is certain. Because the medical service ability [4] cannot match the demand completely, for example, most outpatient clinics are limited by objective conditions such as cost, facilities, personnel, etc., it is not easy to add equipment and personnel to adapt to and match the changes of patients’ demand, or the medical demand is difficult to predict and the medical service ability lacks corresponding flexibility. In this paper, the queuing theory model is established to study the change of waiting time of patients after supporting medical staff in Wuhan area in Northwest China. Because of the strong infectivity and wide spread of the new coronavirus, a large number of patients are likely to have human to human transmission in the process of outpatient queuing, which needs to be contained. In this paper, based on queuing theory, SIR model is proposed to simulate the transmission process of new coronavirus. Through data statistics, a mathematical model is established to predict infectious diseases, which is convenient to prevent at a certain time node and prevent large-scale outbreak. Of course, the predicted model is ideal, so the outpatient service is considered as a whole area, which is more practical.

II. ESTABLISHMENT OF $M|M|c\infty$ QUEUING MODEL AND SIR MODEL

It is assumed that all departments of patients are fever clinics; patients will not leave in the middle of the process due to the excessive number of people in the queue until the completion of treatment at the service desk [5], otherwise they will always wait in the queue; because there are tens of thousands of fever patients, in order to reduce the amount of computer calculation, it is assumed that each
service desk has 1000 outpatient doctors serving the patients, that is, in thousands of people.

2.1 Establishment of $M|M|c|\infty$ Queuing Model

Firstly, the arrival process of patients obeys the Poisson distribution [6]: $p(x = k) = \frac{\lambda^k}{k!} e^{-\lambda}$ ($k = 0, 1, 2, ...$). Where $\lambda$ is the effective average arrival rate. The service time of patients obeys the negative exponential distribution: $B(t) = 1 - e^{-\mu t}$ ($t \geq 0$), where $\mu > 0$ and $\mu$ are constant, representing the average service rate per unit time. The service window contains multiple service desks, and the number of service desks is $c$ ($c \geq 2$), and each service desk is parallel. Assuming that each service desk works independently, and its average service rate $\mu$ is the same, then there is $\mu_1 = \mu_2 = \cdots = \mu_c$. The queuing rule adopts the waiting system: that is to say, when the patients arrive, if all the service desks are not free, they will wait in line, and receive services first, then. And suppose that the patient source is unlimited, that is, the patient's length $L \in (0, \infty)$ and $L \in Z$.

The steady state [7] probability of the system is:

$$p_n = \left\{ \begin{array}{ll} \frac{1}{n!} \rho^n p_0 & (n \leq c) \\ \frac{1}{c^n} c^n e^{-\lambda} & (n > c) \end{array} \right.$$  

$\lambda = \frac{\lambda}{c\mu}$, when $\rho \geq 1$, the queuing system is considered to be unsteady, which indicates that the number of patients arriving is greater than the average service level of outpatient service, that is, the service window opened is not enough to meet the patients coming to see a doctor, and the patients need to wait in a queue. With the increase of time, the queue becomes longer and longer, and finally forms an infinite queue in a certain time; when $\rho < 1$, the corresponding indexes can be calculated by $M|M|c|\infty$ model.

The service time of patients is $W_s = \frac{\lambda}{\mu}$, the average waiting time of the patient, which is $W_q = \frac{\lambda}{\mu}$, the total visit time of the patient, which is $W_t = W_s + W_q = t_q + t_c$, where $t_q$ is the departure time of the patient, $t_c$ is the arrival time of the patient.

Next, the $M|M|c|\infty$ queuing model is established. The basic structure of the $M|M|c|\infty$ queuing system is composed of four parts: arrival input, service time, service window and queuing rules. The working diagram is as follows.

![Figure 1: $M|M|c|\infty$ queue structure of patients](image-url)
people and patients at the initial time is $s_0$ and $y_0$, then the differential equation model is established as follows:

$$\begin{align*}
\frac{ds}{dt} &= \lambda s y - \mu y \\
\frac{dy}{dt} &= -\lambda s y \\
s(0) &= s_0, y(0) = y_0
\end{align*}$$

Since the differential equation group cannot be solved, then the nature of the solution is discussed on the phase plane so $y$. The domain $(s, y) \epsilon D$ of phase trajectory should be: $D = \{(s, y)|s \geq 0, y \geq 0, s + y \leq 1\}$. By eliminating $dt$ from the system of equations and $\frac{\lambda}{\mu} = \delta$, we can get:

$$\begin{align*}
\frac{dy}{ds} &= \frac{1}{s_0} - \frac{1}{y} \\
y(s_0) &= y_0
\end{align*}$$

that is, $y(s) = (s_0 + y_0) - s + \frac{1}{\delta} \ln \frac{s}{s_0}$.

### III. APPLICATION OF PATIENT QUEUING PROBLEM IN M|M|c|∞ QUEUING MODEL AND SIR MODEL

#### 3.1 The Solution of M|M|c|∞ Queuing Model

#### 3.1.1 Situation after the Support of Qinghai Province and Ningxia in Northwest China

Next, the above models are analyzed. First of all, the number of medical support personnel in Qinghai Province and Ningxia Region in Northwest China is 235 and 782 respectively, a total of 1017. The average arrival rate of patients is $\lambda = 10$, the average service rate of outpatient department is $\mu = 4$, and the total simulation time is 20. MATLAB software is used to simulate the queuing system of $M|M|c|\infty$, and the results are shown in the following figure 2.

![Figure 2: Waiting and staying time of patients after support in Qinghai and Ningxia](image)

It is not difficult to find that when the number of medical staff is 1017, the arrival time curve of patients is far from the departure time curve. When the queue length Lq is 50~70, the waiting time and stay time range of patients is 6.65~12.22, which shows that the current number of outpatient doctors is insufficient, leading to the longer stay time of patients in the outpatient clinic. The problems are as follows: (1) in the process of queuing for a long time, patients are prone to cross infection; (2) for patients with fever who have a serious illness, long queuing may lead to aggravation of the illness.

In order to avoid the above two situations, for a certain place, only 1017 medical and nursing personnel are obviously not enough to allow tens of thousands of patients to see a doctor, so more medical personnel are in urgent need of support. From the above right figure 2, we can see that the longer the queue length is Lq, the longer the patient's waiting time and stay time are, and the longer the patient's total visit time $W_T = t_l - t_c$, which is in line with our usual cognition of queuing, so the rationality of the simulation results is verified.

#### 3.1.2 Situation of New Xinjiang and Gansu Provinces in Northwest China after Support

Next, consider increasing the number of medical staff. Under the support of 1017 medical staff in the original two provinces, 386 medical staff and 786 medical staff were respectively supported in Xinjiang and Gansu, that is to say, the total number of medical staff increased to 2189.
The above figure3 shows the simulation results when \( c = 2 \). It can be seen from the above figure 3 that with the increase of time, the queue length \( L_q \) increases, and the number of patients increases gradually. The longer the queue length is, the longer the total time for patients to see a doctor is. This trend is the same as that of \( c = 1 \), which is reasonable. However, different from \( c = 1 \), when the queue length \( L_q \) is 50~70, the waiting time and stay time range of patients is 2.21~5.42, which is significantly shorter than 6.65~12.22 when \( c = 1 \).

It can be seen that when the number of well-known aid is 2189, the treatment time of patients has been significantly shortened, so that the efficiency of patients' treatment has been greatly improved. Furthermore, it greatly reduces the probability of cross infection between patients and the mortality of critical patients, and at the same time, it can also relieve the pressure of local outpatient to a certain extent.

**3.1.3 Situation after the Support of the Whole Northwest China**

In addition to the four provinces mentioned above, there is another Shaanxi Province in Northwest China, and the number of people supporting Shaanxi Province is 1400. Therefore, the analysis and calculation of the whole northwest China's support to Wuhan will be made. The results showed that the total time of treatment was greatly shortened when the number of people who were rescued increased from 2189 to 3589, while other initial values remained unchanged. Even if the queue length is long, the relative shortening of the total visit time of patients can be satisfied.
Compared with the previous, if the number of people supporting Wuhan in Northwest China reaches 5000, the waiting time of patients will be greatly shortened and the efficiency of patients’ treatment will be greatly improved. In this way, for many patients with serious disease, through timely treatment can greatly improve their survival rate, and greatly alleviate the pressure of outpatient treatment in a certain place. It can be seen that in this battle against the new coronavirus, as long as the people of the world unite as one, they can defeat the epidemic.

### 3.2 Solution of SIR Model

For $y(s) = (s_0 + y_0) - s + \frac{1}{\delta} \ln \frac{s}{s_0}$, in the domain of definition, $(s_0, y_0)$, making an image in the same rectangular coordinate system: when $\delta = 1$, Fetch point($0.65, 0.3$), ($0.35, 0.4$), ($0.45, 0.5$), ($0.25, 0.7$).

Regardless of the initial conditions of $s_0$ and $y_0$, the proportion of patients is getting smaller and smaller, and finally disappeared. In $y(s) = (s_0 + y_0) - s + \frac{1}{\delta} \ln \frac{s}{s_0}$, when $y(s_\infty) = 0$, the single root of $(s_0 + y_0) - s_\infty + \frac{1}{\delta} \ln \frac{s_\infty}{s_0} = 0$ is $s_\infty$, which indicates the proportion of healthy people who are not infected at last. In the image, $s_\infty$ is the abscissa of the intersection of the phase trajectory and the $s$ axis in ($0, \frac{1}{\delta}$). When $s_0 \leq \frac{1}{\delta}$, infectious diseases will not spread, such as the leftmost curve. With the increase of $s$, the number of patients is decreasing. Therefore, to improve the level of medical and health care, make the daily contact rate $\lambda$ decrease or the daily cure rate $\mu$ increase, so that $\frac{1}{\delta} = \frac{s}{\lambda \mu}$ increase, and also reduce $s_0$. If $y_0 = 0$ and $s_0 + r_0 = 1$, then $s_0 = 1 - r_0 \leq \frac{1}{\delta}$, then the proportion of the immunized will increase. This is actually more difficult, for example, $\delta = 5, r_0 \geq 80\%$.
IV. CONCLUSION

Patients' queuing is inevitable at all levels of outpatient service, but outpatient service should not passively accept it, but should actively create conditions, take measures to shorten the waiting time, reduce the negative impact of waiting on patients. This paper is based on $M|M|c|=\infty$ queuing model defines the number of service units for medical staff in thousands of people. The number of service units in the initial state is 1, that is, the number of medical staff to support at the beginning, and then compared with the number of reinforcements. Through fitting analysis, the waiting time and stay time of patients become better and better with the increase of the number of service units, and the efficiency is the highest after the number of service units is increased at the beginning.

Infectious medical records are the enemy of human health. It is an important problem to predict the transmission process of new coronavirus in epidemic control[9]. Traditional time series analysis and prediction methods are generally difficult to simulate the actual transmission process. Then this paper proposes a SIR model to simulate the transmission process of the new pneumonia virus, and establishes a mathematical model to describe the infectious diseases in a macroscopic way, which is helpful to analyze the change trend of the number of infected people from a quantitative perspective, master the epidemic law of infectious diseases, so as to control the epidemic situation in time, provide scientific data, recognize the basic elements of infection, and provide the necessary basis for disease prevention It is also simple and easy to operate, so it is of great significance.

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