Revisit Multinomial Logistic Regression in Deep Learning: Data Dependent Model Initialization for Image Recognition

Bowen Cheng\textsuperscript{1}*, Rong Xiao\textsuperscript{2}†, Yandong Guo\textsuperscript{3}‡, Yuxiao Hu\textsuperscript{3}‡, Jianfeng Wang\textsuperscript{3}, Lei Zhang\textsuperscript{3}

\textsuperscript{1}University of Illinois at Urbana-Champaign, 
\textsuperscript{2}Ping An Property & Casualty Insurance Company of China, 
\textsuperscript{3}Microsoft

\{bcheng9@illinois.edu, xiaorong283@pingan.com.cn \}
\{yandong.guo@live.com, yuxiaohu@msn.com, \{jianfw, leizhang\}@microsoft.com

Abstract

We study in this paper how to initialize the parameters of multinomial logistic regression (a fully connected layer followed with softmax and cross entropy loss), which is widely used in deep neural network (DNN) models for classification problems. As logistic regression is widely known not having a closed-form solution, it is usually randomly initialized, leading to several deficiencies especially in transfer learning where all the layers except for the last task-specific layer are initialized using a pre-trained model. The deficiencies include slow convergence speed, possibility of stuck in local minimum, and the risk of over-fitting. To address those deficiencies, we first study the properties of logistic regression and propose a closed-form approximate solution named regularized Gaussian classifier (RGC). Then we adopt this approximate solution to initialize the task-specific linear layer and demonstrate superior performance over random initialization in terms of both accuracy and convergence speed on various tasks and datasets. For example, for image classification, our approach can reduce the training time by 10 times and achieve 3.2% gain in accuracy for Flickr-style classification. For object detection, our approach can also be 10 times faster in training for the same accuracy, or 5% better in terms of mAP for VOC 2007 with slightly longer training.

1. Introduction

Training a deep neural network is generally solving a non-convex optimization problem over millions of parameters with no analytical solutions. When there are large scale training data, e.g. ImageNet [28] or millions of face images [11], typically people train a DNN model from scratch with an iterative solver, which takes days, or up to weeks even with the great progresses in computing infrastructure and optimization method. On the other hand, when training a DNN model for a specific task (e.g. object detection, semantic segmentation, fine-grained classification) which tends to have smaller scale of training data, fine-tuning (sometimes called transfer learning) is widely adopted.

In this paper, we focus on improving the fine-tuning method in terms of both accuracy and training speed. Conventionally, in the fine-tuning schema, the parameters of the lower level layers of the model to be trained are transferred directly from a pre-trained model, e.g. AlexNet [19], ResNet [14], which is trained on a much larger scale dataset, e.g. ImageNet-1k [28] classification dataset, while the parameters of the last layer are randomly sampled from certain distributions (usually Gaussian) [31] and are optimized together with previous layers in a multinomial logistic regression manner (i.e. a fully connected layer followed with softmax and cross entropy loss). Examples include Flickr style estimation [17], flower recognition [22], and places recognition [36]. Fine-tuning schema can also be used in other image recognition domain. For example, image classification models trained on ImageNet can also be used to initialize an object detection model, such as YOLO [24] and Faster R-CNN [25], and lead to better performance.

The conventional fine-tuning method successfully leverages the low level visual pattern extractors learned from general tasks (classification), which reduces the over-fitting issue to some extend, and typically converges faster than training from scratch. However, the conventional fine-tuning still suffers from the following two challenges because new layers are randomly initialized. The first is still over-fitting. Even locking the lower level layers and only updating new layers still has the over-fitting issue. The reason might be that the dimension of the input (called features) to the last
layer is generally too high compared with the limited training data, and setting the parameters of new layers randomly at the initial stage might be too far away from the optimal solution. For example, the input of the ‘fc8’ layer in VGG [30] or AlexNet [19] is a tensor with 4096 channels and the input of ’fc’ layer in ResNet-50 [14] is a tensor with 2048 channels. Our experiments with various tasks validate this, as discussed in the experimental results section.

The second challenge of the conventional fine-tuning is the convergence speed. Though for image classification, fine-tuning can converge very fast, for more complex tasks like object detection, it still needs hours or days to fine-tune from pretrained models [25, 24]. This is mainly because with a randomly initialized linear layer, one has to use a smaller learning rate for the parameters in the non-linear feature extraction layers to avoid gradients of randomly initialized parameters in new layers ruining the pre-trained model. This is impractical or inefficient for applications that require frequent model training or prototyping. For example, for a web-based training service like Microsoft Custom Vision1, the model needs to be trained within couple of minutes to guarantee the user experience and the productivity.

The above problems are mainly caused by the fact that the newly added layer is randomly initialized due to the lack of closed-form solution of logistic regression. Recent research [9] has shown that DNN models have near linear decision boundaries. Thus we can assume the pre-trained feature extractor is general enough to obtain near linearly separable features on the new training set. As a result, we can leverage solutions from other linear classifiers, e.g. linear discriminative analysis (LDA), to initialize the linear layer in logistic regression, with the hope that a well-initialized linear layer is closer to the optimum than a randomly initialized linear layer. We have explored several linear classifiers and found all of them give reasonably good results. Among these classifiers, logistic regression gives the best result which makes sense because logistic regression is more consistent with the loss function (cross entropy loss) in DNN training. However, the problem is that logistic regression is time-consuming since it does not have closed-form solution and needs an iterative solver.

In order to tackle the above drawbacks of logistic regression, we first study the properties of logistic regression and find that it has an exponential family distribution. We further show that the exponent is an infinite order polynomial with zero and first order coefficients class-dependent and all other higher order coefficients class-independent. Based on this finding, we can approximate the distribution of the features for each class with a special case of exponential family with polynomial exponent, i.e. Gaussian distributions. To satisfy the coefficient constraints, the Gaussian distribution should have a class-dependent mean vector (first-order coefficients) and a class-independent covariance matrix (second-order coefficients). We derive a closed-form solution of the optimal linear classifier by maximum likelihood (ML) under this approximation named regularized Gaussian classifier (RGC). RGC can be served as an approximate solution to multinomial logistic regression in DNNs while having the advantages that it is fast, it has a closed-form solution, and it is hyper-parameter free. This linear classifier is then used to initialize the last layer of the DNN model. That is, we copy the solution of RGC ($w_k$ and $b_k$ for each of the $K$ categories, $k = 1, \ldots, K$) to initialize the parameters in the last linear layer in the DNN model. Compared with the random initialization algorithm, the proposed algorithm can dramatically reduce the training cost and lead to a better model with negligible initialization cost.

We have applied the proposed model initialization algorithm to both the image classification and object detection problems. Extensive experiment results demonstrate the superiority of the method in both scenarios. For example, for image classification, our approach can reduce the training time by 10 times and achieve 3.2% gain in accuracy for Flickr-style classification. For object detection, our approach can also be 10 times faster in training for the same accuracy, or 5% better in terms of mAP for VOC 2007 with slightly longer training.

2. Related Works

2.1. Model Fine-tuning

A typical deep neural network model can be decoupled into two parts, a non-linear feature extraction part $\phi(\cdot)$ which corresponds to a stack of layers, followed by a linear classification part. The assumption that the feature extraction part can extract some general task-independent features gives rise to the possibility of fine-tuning. Existing fine-tuning schema mainly fall into two categories: 1) randomly initializing the last linear layer and fine-tuning both the linear and non-linear stages (entire model) on the new data set [16, 22, 36], and 2) fixing the model parameters in the feature extraction stage and training a linear classifier, such as linear SVM, or multinomial logistic regression, for the new task [34]. The second category could be very fast for some lightweight image classification tasks, but suffers from suboptimal accuracy on test sets. As shown in [34], the authors retrained the last layer (linear classification stage) with the parameters from the non-linear stage fixed and obtained suboptimal accuracy. In order to overcome the problem of over-fitting, various model regularization methods, for example, weight decay, have been proposed. However, weight decay degrades the efficiency of SGD optimization and may lead to the under-fitting problem. In this paper, we find that our proposed model initialization method prevents models from over-fitting during fine-tuning, without tuning...

---

1http://customvision.ai
any hyper parameters.

2.2. Model Initialization

Previous works initialize models from a predefined distribution (also known as random initialization), e.g. uniform distribution or Gaussian distribution with different statistics (i.e. mean and standard distribution of a Gaussian distribution) to deal with the vanishing gradient problem [31, 7, 13]. More recent works have placed attentions on data-dependent model initialization. [18] proposed to normalize randomly initialized weights according to the training data in order to let parameters to learn at the same rate (activations are equally distributed), finally, they resacle each layer such that the gradient ratio is constant across layers. However, the initialization is still stochastic in [18]. In this paper, we propose a deterministic data-dependent model initialization method and we find our method increase the convergence speed significantly. [29] proposes a PCA-based model initialization method where PCA is first turned into an auto-encoder, then the auto-encoder is trained and the weights are used to initialize the model. However, there is no closed-form solution for the auto-encoder and the training requires tuning many hyper-parameters. Compared with this method, our proposed method has a closed-form solution thus it does not require tuning any hyper-parameter.

3. Logistic Regression in Deep Learning

3.1. Multinomial Logistic Regression Revisit

Softmax with cross-entropy loss is widely used in modern DNN network structures. It is well-known that a single fully connected neural network with Softmax and cross-entropy loss is equivalent to multinomial Logistic regression. Suppose that $x$ is the input of the network, $k \in K$ is the class label, $w_k$ and $b_k$ are network parameters associated with class $k$. Then the probability of $x$ belonging to the class $k$ can be defined by the Softmax function:

$$p(k|x) = \frac{1}{z(x)} \exp(w_k^T x + b_k),$$  

(1)

where $z(x) = \sum_{j=1}^{K} \exp(w_j^T x + b_j)$. Then the likelihood function of class $k$ will be:

$$p(x|k) = \exp(w_k^T x + b_k - \epsilon(x)), $$  

(2)

where terms $w_k$ and $b_k = b_k - \log(p(k))$, which only depend on class $k$, and $\epsilon(x) = \log(z(x)) - \log(p(x))$, which only depends on $x$. Let $F(x, k) = w_k^T x + b_k$, Eq. 2 can be written as

$$p(x|k) = \exp(F(x, k) - \epsilon(x))$$  

(3)

Let the log-likelihood function be $q(x, k) = \log(p(x|k)) = F(x, k) - \epsilon(x)$. We notice that $q(x, k)$ has a class-dependent part $F(x, k)$ which is linearly dependent of class $k$ and a class-independent part $\epsilon(x)$. If we use multivariable Taylor series to represent $q(x, k)$ at point $a$, we will have:

$$q(x, k) = q(a, k) + \nabla q(a, k)(x - a) + \frac{1}{2} (x - a)^T H q(a, k) (x - a) + ...$$  

(4)

Since the class-dependent term is only linearly dependent on class $k$, meaning the second and higher order derivatives in Eq. 4 are independent of class $k$. From Eq. 3 and 4, we can conclude that logistic regression has the following properties:

1. The examples in each class follow a exponential family distribution.
2. The exponent is a polynomial with infinite order and only the coefficients of zero and first order are dependent on class.

3.2. Gaussian Classifier as Logistic Regression

It is well known that there is no analytical solution for logistic regression. However, if we ignore the 3rd and higher order terms, the samples in each class can be assumed to follow a Gaussian distribution with some specific requirements. Since the second order term in a Gaussian distribution only depends on its covariance matrix, a class-independent second order coefficient indicates a class-independent covariance matrix.

If we assume that the features from the same class for the last linear layer come from a Gaussian distribution with a class-specific mean vector and a shared covariance matrix $\Sigma$, then the resulting Gaussian classifier becomes Logistic regression. Based on this assumption, let $\{x_i, y_i\}, i = 1, 2, ..., N, y_i \in K$ denote the features and class labels for the last linear layer. The class centroids $\mu_k$ can be computed as $\mu_k = \frac{1}{C_k} \sum_{i \in C_k} x_i$, where $C_k$ is the set of indices of samples belonging to class $k$. The likelihood function can be evaluated by

$$P(x|k) = |2\pi \Sigma|^{-\frac{1}{2}} \exp \left( \frac{-(x - \mu_k)^T \Sigma^{-1} (x - \mu_k)}{2} \right)$$  

(5)

Although it is only an approximation of logistic regression, Gaussian classifier has the advantage that it has a closed-form solution. Even if the solution is not optimal, it is reasonable to use this solution to initialize the task-specific last layer in fine-tuning because the additional optimization steps can push this sub-optimal solution towards the optimal solution.

Our experimental results show that even without fine-tuning, a Gaussian classifier based model initialization can already achieve a promising accuracy. Moreover, as the DNN model is fully initialized, the model parameters can
be fine-tuned together with the same learning rate, leading to a even faster convergence speed.

4. Model Initialization with RGC

4.1. Regularized Gaussian Classifier (RGC)

Suppose classes are uniformly distributed in real world, then maximum a posterior (MAP) classifier is the same as maximum likelihood (ML) classifier due to the fact that:

\[ p(k|x) = \frac{p(x|k)p(k)}{p(x)} \propto p(x|k) \]

Based on the assumption the second order coefficient in the exponent is class-independent, we can assume features are from a Gaussian distribution with class dependent mean and class independent covariance matrix. Thus, we assign a class label to the sample by maximizing the likelihood function of the underlying Gaussian distribution in Eq. 5:

\[ \hat{y} = \arg \max_{k \in K} p(x|k) \]

(6)

Since the quadratic term \( x^T x \) and the exponential function does not affect the order, we can rewrite Eq. 6 in the linear form:

\[ \hat{y} = \arg \max_k \mu_k^T \Sigma^{-1} x - \frac{1}{2} \mu_k^T \Sigma^{-1} \mu_k \]

(7)

Let

\[ w_k = \Sigma^{-1} \mu_k, \quad b_k = -\frac{1}{2} w_k^T \mu_k \]

(8)

Eq. 7 becomes

\[ \hat{y} = \arg \max_k w_k^T x + b_k \]

(9)

Therefore, Eq. 9 shows an optimal solution to a linear classifier for the problem. However, many real applications do not have sufficient training data. The covariance matrix estimation becomes highly variable, and the weights estimated by Eq. 8 is heavily weighted by the smallest eigenvalues and their associated eigenvectors. In order to avoid this problem, we introduce a regularization term to the covariance matrix. We have \( w_k = (\Sigma + \epsilon I)^{-1} \mu_k \), where \( I \) is an identity matrix and \( \epsilon \) is the regularization term (a pre-chosen small constant, we set it to 0.1 for most of our experiments). In practice, \( w_k \) can be efficiently calculated by solving the following equation,

\[ (\Sigma + \epsilon I)w_k = \mu_k. \]

(10)

Using Eq. 10, we avoid the calculation of matrix inverse, which is usually 2-3 times faster in practice.

4.2. Parameter Calibration

Generally, for any constant \( \alpha > 0, \beta \) and constant vector \( v \), we can define an infinite set of weights and biases \( \{w_k, b_k\} \), where

\[ w_k = \alpha w_k + v, \quad b_k = \alpha b_k + \beta \]

(11)

We can prove that all these parameters are equivalent in terms of classification accuracy. However, their impact on SGD optimization will be different. Note that multi-class logistic regression is normally implemented as fully connected layer followed by Softmax with cross entropy loss layer in most deep learning platforms [16]. If we increase \( \alpha \) by ten times, the cross-entropy loss after the Softmax operation will be changed, and the loss propagated to previous layers will be changed as well. However, there is no analytical solution to finding an optimal set of parameters which can minimize the cross entropy loss. Instead of solving it directly, we use the weights \( \{w_k\} \) of the last linear layer in the pre-trained network as the reference. We want both networks to have similar scales of loss which can be properly propagated to previous layers. Therefore, we align \( \hat{w} \in \{w_k\} \) and \( \hat{b} \in \{b_k\} \) to \( w' \in \{w'_k\} \) and \( b' \in \{b'_k\} \) as follows.

\[ E(\hat{w}) = E(w'), \]

(12)

\[ E(\hat{b}) = E(b'), \]

(13)

\[ E(||\hat{w} - E(\hat{w})||^2) = E(||w' - E(w')||^2), \]

(14)

where \( E(.) \) denotes expectation. From Eq. 11 and Eq. 12-14, we have

\[ v = E(w') - \alpha E(w), \]

(15)

\[ \beta = E(b') - \alpha E(b), \]

(16)

\[ \alpha = \sqrt{\frac{E(||w'||^2) - ||E(w)||^2}{E(||w'||^2) - ||E(w')||^2}}. \]

(17)

Based on Eq. 11, 15-17, we can get the optimal set of weights \( \{\hat{w}_k, \hat{b}_k\} \) to initialize the last FC layer of a DNN model.

4.3. Relationship to Other Data Based Methods

We can find that nearest centroid classifier (NCC) [32] is a special case of RGC when \( \epsilon \to \infty \). When \( \epsilon \to 0 \), RGC is very similar to LDA for binary classification. Compared with the multi-class LDA algorithm, RGC omits the calculation of between-class scatter matrix. Using this trick, RGC avoids two times of SVD calculation, which makes RGC 5-20 times faster in practice than the traditional LDA algorithm. The weakness of RGC is that it is incapable of performing dimension reduction, which is not an issue for the model initialization problem. The regularization used in
RGC is not something entirely new. It have been frequently used in different situations, for example, regularized LDA [6], ridge regression, etc.

There are also many other linear classification algorithms, such as support vector machine and Gaussian Process. Compared with logistic regression with SGD, most of these algorithms are slow, especially in a high dimensional feature space with non-linear separable data.

5. Experimental Results

In this section, we evaluate our method on several tasks including finetuning, e.g. fine-grained recognition and object detection. We compare our method with other data-independent and data-dependent methods thoroughly on these tasks. Experimental results demonstrate the effectiveness of our method. In the following experiments, all speeds are tested on a DGX-1 sever using one NVIDIA Tesla P100 GPU and Intel E5-2698v3 CPU.

5.1. Convergence of class covariance matrices

To demonstrate the covariance matrix of approximated Gaussian classifier is independent on class, we train a ResNet-18 network on the ImageNet dataset for 650k iterations. We use SGD with a mini-batch size of 256. The learning rate starts from 0.1 and divided by 10 for each 150k iterations. For every 20k iterations, we extract DNN features from the “Pool5” layer for every images in the ImageNet test set. Then we use the correlation matrix distance (CMD) [15] metric, which is defined in Eq. 18, to measure the similarity of the covariance matrices between different image categories.

\[ d_{corr}(R_1, R_2) = 1 - \frac{tr\{R_1 R_2\}}{\|R_1\|_f \|R_2\|_f}, \]  

where \(R_1\) and \(R_2\) are two covariance matrices, \(tr\{\cdot\}\) is the trace of a metric, \(\|\cdot\|_f\) is the Frobenius norm of a matrix.

Since there are only 50 images for each category in the ImageNet validation set, dimension reduction is required to get robust estimation of the covariance matrices in the feature space. Thus we (i) pre-process the feature vector by subtracting its corresponding class mean vector to improve the efficiency of the dimension reduction; (ii) use PCA to reduce the dimensionality of feature vector from 512 to 2, (iii) calculate the covariance matrices for each categories and calculate the mean of these covariance matrices; (iv) for each category, calculate the CMD distance between the class covariance matrix and the mean covariance matrix; and (v) calculate the mean and the variance of these CMD scores. The final results are shown in Fig. 1.

From Fig. 1, we find that in the initial training stages, the covariance matrices from different categories are quite similar. The main reason is due to the model random initialization method which is homogeneous. As the training continues, the model starts to fit to the training data, and the mean of the CMD score reaches its maximum at the 320th iteration. After that, the mean and variance of the CMD scores continually drop until converging to the value around 0.04. This result verifies our conclusion that CNN is capable of learning feature to meet the requirement of logistic regression, and as a result the covariance matrices tend to be class-independent under Gaussian approximation. However, it does not converge exactly to zero because the distribution is not exactly Gaussian.

5.2. Fine-grained Classification

We choose the Flickr-style dataset [17] for ablation study to evaluate our method on the fine-grained classification task due to its popularity in transfer learning. Flickr-style dataset contains 80,000 images labeled with 20 different visual styles. We use the same setting as [16]: 80% for training and 20% for testing and fine-tune a AlexNet [19] for fine-grain classification which is pre-trained on ImageNet [28] for 100,000 iterations and achieved the final validation accuracy of 39.16%. It takes 7 hours using Caffe on a K40 GPU.

There are two fine-tuning strategies, one is only fine-tuning the last FC layer while fixing all the previous layers, and the other is fine-tuning both the pretrained layers and the last FC layer together. The first method is usually used when there is not enough data, it treats pretrained layers as feature extractor and train a separate task-specific linear layer, e.g. an SVM, on the new dataset. It is fast but usually suffers from over-fitting. The latter treats pre-trained layers as model initialization and optimize both feature extractor and linear classifier simultaneously but with different rate, the latter is slow but usually generalizes better. If otherwise stated, experiments are done with the second setting.
Table 1: Model initialization accuracies (without fine-tuning) and corresponding costs (not considering feature extraction time). RGC: regularized Gaussian classifier (ours), SVM: support vector machine, LDA: linear discriminative analysis, LR: multinomial logistic regression.

| Algorithm | Accuracy | Time (s) | Hyper-Parameter Selection |
|-----------|----------|----------|---------------------------|
| RGC       | 38.11    | 3.34     | No                        |
| SVM       | 37.27    | 13.78    | Yes                       |
| LDA       | 37.42    | 24.73    | No                        |
| LR        | 38.57    | 97.02    | Yes                       |

Table 2: RGC model initialization for Flickr-style

| Algorithm | Iter  | Acc (%) |
|-----------|-------|---------|
| Baseline  | 16000 | 93.16   |
| RGC       | 0     | 39.16   |
| RGC       | 3000  | 39.20   |
| RGC       | 10000 | 42.39   |

5.2.1 Data-dependent model initialization details

In this section, we discuss the implementation details about data-dependent model initialization methods. First, we drop the last task-specific layer of AlexNet and feed the entire training set to the pre-trained Alexnet to extract features from the output of the “fc7” layer. Then, we use these extracted features to initialize the newly added task-specific layers following Eq. 15-17.

5.2.2 Comparison with other initialization methods

We compare our proposed RGC with different initialization methods, including RAND (the MSRA random initialization method [13]), LDA (linear discriminative analysis), LR (multinomial logistic regression), SVM (linear support vector machine classifier). In this experiment, implementations of LDA, LR and SVM are from the scikit-learn Python package 2. In Tab. 1, we evaluate the accuracy after model initialization without fine-tuning. Comparing RGC with other model initialization methods, RGC is extremely fast, which only takes 3.34 seconds not considering feature extraction time. Another advantage of RGC algorithm is that RGC is hyper-parameter free and insensitive to regularization. Compared with LR and SVM, the RGC algorithm is almost parameter free (we use a fixed $\epsilon = 0.1$ for most scenarios), which is critical for online services when a customer lacks machine learning experience. On the other hand, LR and SVM algorithms require a lot of effort to manually select dataset-dependent hyper-parameters (e.g. learning rate) to get optimal results.

Next, we fine-tune the models after initialization. We conduct experiments with RGC model initialization described in Eq. 15-17 and let all parameters to update with the same learning rate. This setup makes a fair and closer comparison with [16]. More specifically, we follow all settings from [16], except for the following two parameters. First, since our model is properly initialized, we do NOT use 10 times of learning rate to update the parameters in the last linear layer. Second, since the initial model is already close to the optimal solution, we reduce the step size to 3000. The experimental result is shown in Tab. 2. With the help from the RGC model initialization method, the proposed approach achieves 37.96% top-1 accuracy in the initial state, and 39.20% at 3000th-iteration. Finally, the model achieves 42.39% top-1 accuracy, which is significantly higher than the best published results with the same network structure.

We hypothesize that there are two reasons that might contribute to the gain:

1. RGC-initialized weights are more consistent in distribution as pre-trained weights (this is because they have higher accuracy after initialization), which makes all parameters learning at the same rate during fine-tuning. As discovered in [18], it is important to have all parameters in the network to learn at the same rate. Although the learning rate of RAND-initialized layers are scaled by 10, they are not guaranteed to learn at the same rate.

2. We find that RGC-initialized models are less likely to over-fit and we argue that some of the accuracy gap is due to over-fitting. Details about why RGC-initialization reduces over-fitting are discussed in Appendix.

5.2.3 More results

We also evaluate our RGC model initialization method on more fine-grained classification datasets (Flower-102 [22] and Caltech-256 [10]). Compared with state-of-the-art results, using the same pre-trained deep learning model, our model initialization method shows 2-4 times faster in convergence iterations with better results, as shown in Tab. 3 and 4.

Table 3: RGC model initialization for Oxford flower-102

| Algorithm | Iter  | Acc (%) |
|-----------|-------|---------|
| Baseline  | 16000 | 93.04   |
| RGC       | 0     | 81.37   |
| RGC       | 9000  | 93.33   |

2http://scikit-learn.org
Table 4: RGC model initialization for Caltech-256 data set

| Algorithm | Iter | Acc (%) |
|-----------|------|---------|
| Baseline [35] | 7100 | 73.50 |
| RGC | 0 | 63.86 |
| RGC | 1600 | 73.91 |

5.3. Object Detection

Recent object detectors can be categorized into two types: two-stage detectors, e.g. Faster R-CNN [25] and its variants [2, 20, 12, 1, 33], which generates region proposals first and then focuses on the recognition of the proposals; and one-stage detectors, e.g. YOLO [23, 24] and SSD [21], which directly predicts the bounding box coordinates and the classification scores. However, what they have in common is that both types of detectors have separate classifiers to classify regions and bounding box regressors to predict object locations.

In the following experiments, we demonstrate that our method works well for both the two-stage (Faster RCNN) and one-stage (YOLO V2) detectors.

5.3.1 Implementation details

To demonstrate the effect of model initialization for the classification layer, we modify detectors to have class agnostic bounding box regression so that the bounding box regression layer does not depend on object class. Then we use the pre-trained detector (on ImageNet 200 detection set) to initialize bounding box regression layer in all experiments. For our baseline models, we follow their original implementation to initialize the newly added classification layers randomly with Gaussian distribution. We use the RGC initialization to initialize the classification layer as our proposed method. In the following experiments, we demonstrate the effectiveness of our method on YOLO V2 with Darknet-19 backbone [24], and Faster R-CNN with ZF backbone [26].

5.3.2 VOC2007 dataset

The first experiment is conducted on the standard VOC2007 dataset [5] with 20 classes. It contains 5011 images for training and validation and 4952 images for testing. We used VOC2007 trainval set to train the model and evaluate our model on the VOC2007 test set. The accuracy is measured by the standard mean average precision (mAP) at the intersection over union (IoU) of 0.5. Results are shown in Fig 2. RGC model initialization shows significant gains in both object detection algorithms as well as faster convergence speed.

With YOLO V2, the RGC initialized model achieves mAP of 41% after 50 iterations of training, and mAP of 67% after 1,000 iterations, which is 10 times faster than the baseline. After the same number of 10,000 iterations, it outperforms the baseline algorithm by 5% in terms of mAP.

With Faster R-CNN, the RGC initialized model achieves mAP of 44% after 100 iterations of training. Compared with the baseline algorithm, it uses only 2,000 iterations to achieve mAP of 60%, which is 17 times faster. With the same number of 35K iterations, our scheme eventually achieves 65% mAP, which is 4% higher than the baseline in terms of mAP.

5.3.3 Flickr Logo32 dataset

The PASCAL VOC2007 dataset [4] and the ImageNet dataset [3] are very similar in both distribution and context, we are interested to see the performance of our model initialization method when applied to another domain. We use the Flickr-Logo32 [27] for logo detection. It contains 2240 images of 32 brands (we removed 6000 non-logo images). We further split the data set into a training set with 1736 images, and a test set with 504 images. The results are shown in Fig 2(b). RGC model initialization shows significant gains in both object detection algorithms as well as faster convergence speed.
images and a testing set with 434 images.

Results are shown in Fig. 3, we observe that the RGC model initialization method consistently improves YOLO V2 with a 4 times speedup and 2.4% mAP gain in the final model. It also helps the convergence of Faster R-CNN, with a 2.4 times speed up but marginal gain in mAP in the final model. We argue that the decrease of gain in Faster RCNN might be due to its algorithm structure. As pointed out in [23], YOLO is better at transferring to other domains than Faster RCNN. However, the consistent improvements in convergence speed demonstrates the effectiveness of our proposed method.

6. Conclusion

In this paper, we have presented a regularized Gaussian classifier (RGC) with closed-form solution to initializing the last linear layer of a DNN model, which was normally randomly initialized due to the lack of analytical solution of logistic regression. This model initialization algorithm significantly reduces the DNN model fine-tuning cost and also leads to a better model, as validated on both image classification and object detection tasks. We also showed that properly initialized model parameters can reduce the model variance which is the main reason for the performance improvement. There are still some problems not addressed in this paper. For example, 1) how to initialize the model for regression tasks which is crucial for many vision tasks, such as face alignment and bounding box regression; 2) how to regularize the weights which also minimize the cross-entropy loss. These will be our future research directions.

A. Appendix

A.1. Discussion on over-fitting

In this work, we have shown that RGC-based model initialization leads to faster convergence, which is easy to understand as it initializes a CNN model with an approximate rather than random solution. But we also found that this approach also leads to better accuracy, which is counter-intuitive as logistic regression is known to be convex and any initialization should lead to the same global optimal solution.

The understand this, we performed the following two experiments to study why RGC-initialized models are less likely to over-fit.

In the following experiments, we fix all layers other than the newly initialized one. First, we fine-tune the new layer with learning rate 0.01 and weight decay 0.0005 and fine-tune the model for 5000 iterations. Results are shown in Fig. 4(a). We can see that the performance of data-dependent model initialization methods are similar which are better than the RAND initialization by nearly 3.0%.

To demonstrate the degradation of RAND initialization is caused by over-fitting, we perform the second experiment by increasing the weight decay 100 times to impose a stronger regularization and results are shown in Fig. 4(b). Increasing weight decay does not have much effect on data-dependent initialization methods but significantly improves the performance of RAND initialization. It means some of the gap in the first setting is caused by over-fitting.

These experiments show that RGC along with other data-dependent initialization methods prevent model from over-fitting.

Fine-tuning only the last layer is equivalent to solving multinomial logistic regression with an iterative solver (e.g. SGD). Since the cross entropy of an exponential family is always convex, multinomial logistic regression is convex and has a unique global minimum. This is why all data-dependent initialization end up with similar accuracy. However, an interesting problem is why RAND initialization behaves differently?
Figure 4: Fine-tune the last layer for the Flickr-style task with different model initialization methods.

To address this problem, we fix the learning rate to 1e-4 and continue training the model for 20k iterations. Results are shown in Fig. 5 (a), both training losses and testing losses of RGC-initialized model and RAND-initialized model are converged. We also find from Fig. 5 (b) that RGC-initialized model has smaller training loss to testing loss ratio which means it is less likely to over-fit than the RAND-initialized model.

Compared with Fig. 4 (a) and Fig. 4 (b), we find that the data-dependent model initialization methods reduce the risk of over-fitting. We observed such phenomena consistently in different experiments, which can be explained in Fig. 6. SGD is a local greedy optimization algorithm. It gradually seeks for path to reduce the loss on the training set. However, due to the variance between training and testing set, SGD may be stuck early in a local minimum of the loss function defined on the training set, which is possibly not even close to the global optimal solution of the test set. A good model initialization method has the potential to move the SGD initial condition closer to the test set optima and lead to a better result. Especially when using a approximate solution, it may even place the initial condition within the same local minimum region. In contrary, RAND initialization places initial condition far from global optimal, since fine-tuning uses small learning rate, SGD is likely to stuck in a local minimum far from global optimum.

References

[1] B. Cheng, Y. Wei, H. Shi, R. Feris, J. Xiong, and T. Huang. Revisiting rcnn: On awakening the classification power of faster rcnn. In ECCV, 2018.
[2] J. Dai, Y. Li, K. He, and J. Sun. R-fcn: Object detection via region-based fully convolutional networks. In NIPS, pages 379–387, 2016.
[3] J. Deng, W. Dong, R. Socher, L.-J. Li, K. Li, and L. Fei-Fei. Imagenet: A large-scale hierarchical image database. In
Figure 6: Better initialization method helps SGD optimization. $O$ is the minimum on the training set, $G$ is the minimum on the test set, $W$ is the randomly initialized weight, $W^*$ is the RGC-initialized weight.

\[ 1 \]

M. Everingham, L. Van Gool, C. K. I. Williams, J. Winn, and A. Zisserman. The PASCAL Visual Objects Classes Challenge 2007 (VOC2007) Results. http://www.pascal-network.org/challenges/VOC/voc2007/workshop/index.html.

\[ 2 \]

M. Everingham, L. Van Gool, C. K. I. Williams, J. Winn, and A. Zisserman. The pascal visual object classes (voc) challenge. International Journal of Computer Vision, 88(2):303–338, June 2010.

\[ 3 \]

J. H. Friedman. Regularized discriminant analysis. Journal of the American Statistical Association, 84(405):165–175, 1989.

\[ 4 \]

X. Glorot and Y. Bengio. Understanding the difficulty of training deep feedforward neural networks. In In Proceedings of the International Conference on Artificial Intelligence and Statistics (AISTATS10). Society for Artificial Intelligence and Statistics, 2010.

\[ 5 \]

J. Goode. Caffe cnns for the oxford 102 flower dataset. https://github.com/jimgoo/caffe-oxford102, 2015.

\[ 6 \]

I. J. Goodfellow, J. Shlens, and C. Szegedy. Explaining and harnessing adversarial examples. arXiv preprint arXiv:1412.6572.

\[ 7 \]

G. Griffin, A. Holub, and P. Perona. Caltech-256 object category dataset. Technical Report 7694, California Institute of Technology, 2007.

\[ 8 \]

Y. Guo, L. Zhang, Y. Hu, X. He, and J. Gao. MS-Celeb-1M: A dataset and benchmark for large scale face recognition. In ECCV. Springer, 2016.

\[ 9 \]

K. He, G. Gkioxari, P. Dollár, and R. Girshick. Mask r-cnn. In ICCV, pages 2980–2988, 2017.

\[ 10 \]

K. He, X. Zhang, S. Ren, and J. Sun. Delving deep into rectifiers: Surpassing human-level performance on imagenet classification. In ICCV, pages 1026–1034, 2015.

\[ 11 \]

K. He, X. Zhang, S. Ren, and J. Sun. Deep residual learning for image recognition. In Proceedings of the IEEE conference on computer vision and pattern recognition, pages 770–778, 2016.

\[ 12 \]

S. Karayev, M. Trentacoste, H. Han, A. Agarwala, T. Darrell, A. Hertzmann, and H. Winnemoeller. Recognizing image style. In BMVC, 2014.

\[ 13 \]

P. Krähenbühl, C. Doersch, J. Donahue, and T. Darrell. Data-dependent initializations of convolutional neural networks. In ICLR, 2016.

\[ 14 \]

A. Krizhevsky, I. Sutskever, and G. E. Hinton. Imagenet classification with deep convolutional neural networks. In NIPS, pages 1097–1105. MIT Press, 2012.

\[ 15 \]

T.-Y. Lin, P. Dollár, R. Girshick, K. He, B. Hariharan, and S. Belongie. Feature pyramid networks for object detection. In CVPR, volume 1, page 4, 2017.

\[ 16 \]

W. Liu, D. Anguelov, D. Erhan, C. Szegedy, S. Reed, C.-Y. Fu, and A. C. Berg. Ssd: Single shot multibox detector. In ECCV, pages 21–37, 2016.

\[ 17 \]

M.-E. Nilsback and A. Zisserman. Automated flower classification over a large number of classes. In Proc. of the Indian Conf. on Computer Vision, Graphics and Image Processing, 2008.

\[ 18 \]

J. Redmon, S. Divvala, R. Girshick, and A. Farhadi. You only look once: Unified, real-time object detection. In CVPR, pages 779–788, 2016.

\[ 19 \]

J. Redmon and A. Farhadi. Yolo9000: better, faster, stronger. In CVPR, 2017.

\[ 20 \]

S. Ren, K. He, R. Girshick, and J. Sun. Faster r-cnn: Towards real-time object detection with region proposal networks. In NIPS, pages 91–99, 2015.

\[ 21 \]

S. Ren, K. He, R. Girshick, and J. Sun. Faster r-cnn: Towards real-time object detection with region proposal networks. In NIPS, pages 91–99, 2015.

\[ 22 \]

S. Ren, K. He, R. Girshick, and J. Sun. Faster r-cnn: Towards real-time object detection with region proposal networks. In NIPS, pages 91–99, 2015.

\[ 23 \]

S. Romberg, L. G. Pueyo, R. Lienhart, and R. van Zwol. Scalable logo recognition in real-world images. In Proc. of the 1st ACM Int'l Conf. on Multimedia Retrieval, pages 25:1–25:8, New York, NY, USA, 2011.

\[ 24 \]

O. Russakovsky, J. Deng, H. Su, J. Krause, S. Satheesh, S. Ma, Z. Huang, A. Karpathy, A. Khosla, M. Bernstein, A. C. Berg, and L. Fei-Fei. ImageNet Large Scale Visual Recognition Challenge. International Journal of Computer Vision, 115(3):211–252, 2015.

\[ 25 \]

M. Seuret, M. Alberti, M. Liwicki, and R. Ingold. Pca-initialized deep neural networks applied to document image analysis. In Document Analysis and Recognition (ICDAR), 2017 14th IAPR International Conference on, volume 1, pages 877–882. IEEE, 2017.

\[ 26 \]

K. Simonyan and A. Zisserman. Very deep convolutional networks for large-scale image recognition. arXiv preprint arXiv:1409.1556, 2014.
[31] I. Sutskever, J. Martens, G. Dahl, and G. Hinton. On the importance of initialization and momentum in deep learning. In *ICML*, pages 1139–1147, 2013.

[32] R. Tibshirani, T. Hastie, B. Narasimhan, and G. Chu. Diagnosis of multiple cancer types by shrunken centroids of gene expression. *Proceedings of the National Academy of Sciences*, 99(10):6567–6572, 2002.

[33] Y. Wei, Z. Shen, B. Cheng, H. Shi, J. Xiong, J. Feng, and T. Huang. Ts2c: Tight box mining with surrounding segmentation context for weakly supervised object detection. In *European Conference on Computer Vision*, 2018.

[34] J. Yosinski, J. Clune, Y. Bengio, and H. Lipson. How transferable are features in deep neural networks? In *NIPS*, pages 3320–3328, 2014.

[35] M. D. Zeiler and R. Fergus. Visualizing and understanding convolutional networks. In *European conference on computer vision*, pages 818–833. Springer, 2014.

[36] B. Zhou, A. Lapedriza, J. Xiao, A. Torralba, and A. Oliva. Learning deep features for scene recognition using places database. In *NIPS*, pages 487–495, Cambridge, MA, USA, 2014.