NUMERICAL INVESTIGATION ON FREE CONVECTION AROUND ELLIPTICAL CYLINDERS IN A CIRCULAR ENCLOSURE

* 1Parth Patpatiya

1Assistant Professor, School of Automation
Banasthali Vidyapith, Newai, Jaipur, Rajasthan-304022

e-mail- paarth.1098@gmail.com
Mobile No. 9718309037

2Bhavana Yadav 3Soumya Jain, 4Srishti Baura

2, 3, 4 Student, School of Automation
Banasthali Vidyapith, Niwai, Jaipur, Rajasthan-304022

e-mail- yadavbhavana208@gmail.com , soumyajainp@gmail.com , srishtibaura@gmail.com

Abbreviated title: Free Convection around elliptical plate inside a circular enclosure

ORCID NO : 0000-0003-0025-9330

* Corresponding author
Abstract
This paper acquaints with the study of steady state laminar natural convection around heated elliptical plate placed inside a relatively cold circular enclosure subjected to varying number of elliptical plates (2, 3 and 4), Rayleigh number (10^4, 10^5, 10^6) and centre to centre distance between the elliptical plates (xc). The effect of these three parameters are observed and analyzed based on the simulation performed using Finite Volume Method (FVM) based solver. Heat transfer characteristics are analyzed for all the cases in terms of isotherms, streamlines. Plots of average nusselt number along active enclosure walls and along the fringe of the inner Cylinders are illustrated in a graphical manner. Two dimensional study is followed by considering air as the fluid in the enclosure (Prandlt number=0.71). Grid independence test followed by validation of experimental results is carried out. Eventually parametric analysis concludes by analyzing the effects of altering the various parameters on the strength of buoyancy affected flow.

Keywords: Natural Convection, Elliptical plate, Rayleigh number, Circular enclosure

Nomenclature

| Symbol | Description |
|--------|-------------|
| g      | Gravitational acceleration, m/s² |
| Gr     | Grashof number |
| h      | Convective heat transfer coefficient, W/m²-K |
| k      | Thermal conductivity, W/m-K |
| xc     | Distance between the centers of the elliptical plates, m |
| x, y   | Co-ordinates in x and y directions, m |
| a      | Major axis of the inner Cylinders, m |
| b      | Minor axis of the inner Cylinders, m |
| D      | Diameter of the circular enclosure, m |
| Pr     | Prandlt number |
| Nu     | Nusselt number |
| u      | Component of velocity in x directions, m/s |
| v      | Component of velocity in y directions, m/s |
| ρ      | Density, kg/m³ |
| σ      | Temperature in dimensionless form |
| γ      | Thermal diffusivity, m²/s |
| β      | Volumetric coefficient of thermal expansion, K⁻¹ |
| ρ̂     | Dimensionless velocity component in x- and y- directions, m/s |
| ν̂     | Dimensionless velocity component in y directions, m/s |
| β̂     | Dimensionless velocity component in x directions, m/s |
| D̂     | Dimensionless velocity component in z directions, m/s |
| ĝ     | Dimensionless velocity component in x directions, m/s |
| η̂     | Dimensionless velocity component in y directions, m/s |
| θ̂     | Kinematic Viscosity, m²/s |

1. Introduction

Natural convection has attracted a large number of researchers to study the transfer of heat and flow of liquid or air in different type of enclosure with different kind of shapes having different thermal properties and boundary condition. For modeling purpose of various thermal structures, comprehension of flow attributes and heat transfer in natural convention is vital. For various engineering and industrial play, natural convection heat transfer inside an enclosing system is relative, like heat exchangers, solar collectors etc. This caught an eye of a large amount of engineers and thus became a topic of huge interest among them. Large amount of researchers were captivated towards extrapolation of flow of fluid and heat transfer in several fashioned enclosures with different confined liquid and imposing several boundary conditions. Lately, studies on natural convection within a round duct have been accomplished as the Passive Cooling System (PCS). The main focus, from the past, has now been shifted to study of natural convection in enclosures having several differently structured interior objects. Changes in configuration of fluid flow and heat transfer inside the enclosure can be seen, which are due to existence of interior bodies. Few parameters resulting in various problems in this classification are configuration of enclosure, physical appearance, and number of interior objects present, size, and characteristics of enclosed liquid. In this paper analysis of transfer and flow of heat through natural convection in air about heated elliptical Cylinders of different physical magnitudes within a circular enclosure has been carried out. The elliptical Cylinders is kept at intense heat; also the enclosure’s upright walls are kept at frigidity along with cylindrical walls by altering the Rayleigh number. A broad examination of Buoyancy driven flows in closed boundary has been carried out over past years. Jian Zhang et al. [1] numerically investigated laminar Buoyancy driven flows heat devolution from a hemispherical body with invariant heat flux surface and due to Grashof number and adiabatic surface the change in heat transfer and flow motion was studied. P. Mishra et al.[2] altered Reynolds number, Prandlt number, Bingham number and cone angle and concluded that the values of Re, Pr and Bn primarily and cone angle slightly influence the rate of heat transfer. When there is a contact amid the stream and the enclosing boundary [3-7] the buoyancy generated convection procedures get complicated to a greater extent. The change in transfer of heat from adjoining barrel in an arrangement with two or more barrels is higher because of interlinking effects between the flow and enclosure surface. For set of two tube arrays reckon in 1 to 4 disc structured Cylinders mid to mid horizontal of 1.4 to 24 barrel diameters and perpendicular disparity of about 2 to 12 Cylinders diameters simulations were executed. An improved Nusselt number is displayed by M. Corcione analogized to an array of single duct [8], based on position in the arrangement, the
Rayleigh number and the configuration of array. The Rayleigh number is in middle of 10^2 and 10^4. Likewise, the equations of heat transfer and non-dimensional association’s were put forward. The transfer of heat in natural convection around O-void cylinders with an upright semi-major axis was experimentally explored by Yousefi and Ashjaee [9]. Experiments using a Mach-Zehnder, Cylinders separation of approximately two to five major axis lengths and at Rayleigh numbers ranging from 10^3 to 2.5 * 10^3 were performed. They concluded that the dependency of the buoyancy driven flows heat transfer from any particular Cylinders in the matrix on the Rayleigh number, Cylinders spacing mileage and Cylinders location in the array. Association of heat transfer, for solitary ellipsoidal Cylinders, with upright semi-major axis was also put forth by them. Investigation of natural convective heat transfer along upright cylindrical body absorbed in Newtonian fluids like water, mixture of water and ethylene glycol with ratio 75:25 with invariable heat flux condition was carried out. Thermal stratification was perceived [10] in the ambient liquid after the stagnant cases were achieved when the liquid thermal conditions go high with the lengthwise flank on the surface of the boundary layer. It was observed that along the lengthwise direction, Cylinders and fluid temperature goes high and in radial direction the temperature of fluid goes low, also it is unequivocal that thermal stratification is accomplished. The alterations on thermal conductivity enhancements of nanofluids due to nanoparticle volume fractions, heat, carbon nanotube aspect ratio and different kind of surfactant(SDBS, Lignin, Sodium polycarboxylate) were viewed by [11] H. F. Oztop et al. It was seen through their experiments that TC enhancements of nanofluidic particles bring about at very less volume fraction. It is majorly controlled by volume fraction and increase in temperature. Carbon nanotubes aspect ratio and surfactant type invalidly affect the TC enhancement of nanofluidic particles. By dissolving a pint-sized quantity of nanoparticles with large intrinsic conductivity within recently used base liquids, like water, ethylene glycol, oil etc. Nanofuids are acquired. [12] To examine the outcome of this interface on buoyancy driven flow from two horizontal Cylinders kept at high temperature circumscribed in a square boundary with walls being isothermal at the heat sink temperature, a 2-D numerical scrutiny is executed. For Rayleigh numbers lying amid 10^3 and 10^7 and non dimensional horizontal separation of Cylinders between 0.1 and 0.4 the simulations are performed. The conclusions exhibit that if the Rayleigh numbers are less than 10^4 then the Nusselt numbers are mainly dependent on the spacing amid the two Cylinders. Trifling approximations were found on the illations of Cylinders separation on heat transfer when Rayleigh number lies between 10^4 and 10^7. Also, it was perceived that in special spot of Cylinders, the stagnant flow and heat transfer experience periodical oscillation, and eventually disrupted oscillations. In study conducted by Y. G. Park et al. [13], in a cold square enclosure 2-D numerical simulations were carried out for buoyancy driven flow with a plumb arrangement of the aspect ratios of the ellipsoidal cylinders within the limit of 0.25 < aspect ratio < 4.00 and Rayleigh number between 10^4 and 10^7. The immersed boundary method (IBM) was used to pull down the virtual panel border of the Cylinders. The change of state flow regime from stagnant to active state is dependent on the aspect ratio at Rayleigh number =10^6. At minimum value of Rayleigh number 10^4 and 10^5, the surface averaged Nusselt numbers and time extend with the aspect ratio of the sovran elliptical cylinders for the enclosure walls. At the maximum value of Rayleigh number = 10^6, the surface averaged Nusselt number and the time of the enclosure wall increased by 3.9% at aspect ratio=0.50 at Sovran elliptical Cylinders and at pinpoint elliptical Cylinders, at aspect ratio=2.00 compared to 2 circular Cylinders. Buoyancy driven flows [14] is not controlled by the help of extraneous forces but brought about by the non-homogeneous of temperature or concentrated domains in the fluid. It further causes emergence of density gradient and the accompanying buoyancy begin to be the driving force of fluid movement. Because of heat transference process by natural convection’s inherent avails, like economic profit, sufficient protection and less noise/disturbance, etc. The globular configuration continually appears in all types of industrial or engineering operations. From a globular body [15-18] the episodes of natural convection heat transfer is caught in the following areas of heat transference process by natural convection’s inherent avails, like economic profit, sufficient protection and less noise/disturbance, etc. The globular configuration continually appears in all types of industrial or engineering operations.
premises, was examined numerically for a three-dimensional inner free convection. According to the concentric and eccentric positions of sphere there were effects of the premises figure and Rayleigh number on the effluent and heat devolution peculiarity. As per the numerical output, the Rayleigh number exists and on the other hand, the temperature difference increases and the Nusselt number decreases [22]. As the temperature was increasing there was reduction in Rayleigh number and it is attributable that as the temperature increases, both the thermal diffusivity and kinematic viscosity of air increases. In the analysis of nuclear reactor [23], the inboard heated melt pool natural convection behavior is one of the important phenomena. Ethically to the previous tentative, the local heat transfer coefficient varies with the vessel polar angle and also the Rayleigh number and Nusselt number varies with various tests and cases. As it is also necessary to examine in what manner the geometric size could sway the inboard heated melt pool natural convection nature. Rooted on the extant simulation, in the first deed, the water pool with different radius showing the equal natural circulation nature in the heat flux and temperature distribution and also the Nusselt number with inboard Rayleigh number correlations were equal. In the second deed, as the radius of the water pool is increased natural circulation and also the Nusselt number with inboard Rayleigh number correlations both shows another type of nature. By this extant study, it will be helpful for the severe accident management, and in the future also it is helpful to understand in what manner the geometric size could sway the inboard heated melt pool natural convection nature. In this tentative trial of natural convection [24] in an enclosure which is loaded with a muffled bed of relatively huge solid spheres is afflicted on the bottom side and quench at the upper side. Rayleigh number was varied amid $10^7$ and $10^9$ and the Nusselt number was calculated for different sphere conductivities, size and packing. After the calculation of Nusselt number the heat transfer is less for the minimum value of Rayleigh number and for the maximum value of Rayleigh number the asymptotic range occurs, where the maximum heat transfer was for the sphere, on a single curve. For the heat transfer system, the difference between the temperature and velocity region, shows that the velocity volume within the pores in the core zone is maximum in the asymptotic system than for the minimum Rayleigh number the system shows a deeper access of quench and afflicted molten quid and maximum heat transfer.

2. Problem statement and governing equations

The cardinal figures used is of a circular enclosure having elliptical plate placed inside it as shown in figure 1. The number of elliptical plates is varied and the distance between the centers of the plates is also varied which is denoted as $xc$. The diameter of the circular enclosure is denoted as D. The major and minor axis of the enclosure is taken as a and b respectively. Acceleration due to gravity ($g$) is also taken into consideration. The enclosure and plates are kept at different temperatures. To get the best illustrations the temperature of enclosure ($T_{in}$) is kept lower than the temperature of the plate ($T_H$) and the variance between these two temperatures is kept static. In the figure 1 two elliptical plates, in figure 2 three elliptical plates and in figure 3 four elliptical plates are placed inside a circular enclosure. The Rayleigh number is kept constant at the values $10^4$, $10^5$, $10^6$ and it varies as the distance between the 2 Cylinders is changed from 0.3 mm to 0.7 mm and for three Cylinders it alters from 0.4 mm to 0.8 mm and for four Cylinders it varies from 1.5 mm to 1.9 mm and the Prandtl number is kept constant. For the following problem fluid is considered to be steady, incompressible, and laminar and for extracting the best results from the investigation, the viscous dissipation is not considered and the properties of thermal physical are opine to be constant. Bossinque approximation is used to educe the best results. The sets of equations used are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (1)

$$U \frac{\partial u}{\partial x} + V \frac{\partial v}{\partial x} = \frac{1}{\rho} \frac{\partial p}{\partial x} + \nabla \cdot \left( \frac{\partial^2 u}{\partial x^2} \frac{\partial^2 u}{\partial y^2} \right)$$  \hspace{1cm} (2)

$$U \frac{\partial u}{\partial x} + V \frac{\partial v}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nabla \cdot \left( \frac{\partial^2 v}{\partial x^2} \frac{\partial^2 v}{\partial y^2} \right) + g\beta \tau$$  \hspace{1cm} (3)

$$U \frac{\partial u}{\partial x} - V \frac{\partial v}{\partial x} \tau = \gamma \left( \frac{\partial^2 \tau}{\partial x^2} \frac{\partial^2 \tau}{\partial y^2} \right)$$  \hspace{1cm} (4)

Boundary conditions in terms of Governing equations for the above problem are given by $u=0$ and $v=0$ at all the walls. $\tau = T_{in}$ for the enclosure walls and $\tau = T_H$ for the elliptical plates.
3. Results and Discussion

In the current analysis the numerical simulation of the model containing two, three and four elliptical plates kept inside the relatively cold enclosure in a specific order. Analysis is performed and the results are observed in terms of isotherms and flow field. The location and number of plates has a great role in variation of heat transfer rate. The circular enclosure contains air at $Pr=0.71$ (Prandtl number) as the convective medium and its thermo physical properties are kept constant except density. Boussinesq approximation is taken as the assumption for density of air. Cases of elliptical plate with varying quantity distance between them and at different Rayleigh numbers are observed. Results are arranged in a sequential manner to compare the heat transfer rate around a circular enclosure.

3.1 Validation of Result and Grid Independence Test

Validation is performed for the analysis performed by Wei Zhang et al. [14] on the same circular enclosure containing a thin plate placed inside it at different inclination angles and eccentricity ratio $(r/D= 0.00,0.02,0.5)$. The problem is validated on simulation software with temperature of plate kept higher than that of enclosure. Equations are solved on the solver FLUENT and problem is verified using finite volume method using SIMPLE algorithm, Least Square Cell Based, Second Order Pressure, Second Order Upwind Momentum, Second Order Upwind Energy Method [25]. A good concurrence between results is shown in Table1. Grid independence test is conducted for the problem with different pattern of grids as shown in Table 2. The case of thin flat plate kept at 0° at $r/D=0.0$ is observed and plot of surface averaged nusselt number is analyzed. Based on it the best suitable finer pattern of grid is used to attain the results. A similar analysis was performed by Moukalled and Acharya [8] (i.e. natural convection in a high temperature enclosure containing mild temperature symmetrical geometry placed inside it). The problem was solved and validation of the results were done. The results were in a good agreement with the ones obtained using the simulation software keeping the same boundary conditions, fluid properties, temperatures of the enclosure and plate kept inside it. Solution is converged when the sum of the remainder of the residues of each variable (i.e. those used in the equations of boundary conditions: $u ,\nu ,P$) in between two successive iterations is lessthan$10^{-6}$.

3.2 Flow pattern and Temperature distribution

The flow pattern and temperature distribution is arranged in a sequential manner for the different orientation of elliptical plate ($x_c = 0.3-1.9$), varying quantity (2-4) and at three different Rayleigh numbers ($10^4, 10^5$ and $10^6$). Cases of two elliptical plates contained in a circular enclosure at different positions between their centre and at different Rayleigh numbers are conspired and the same analysis is performed for three and four elliptical plates respectively placed inside the same circular enclosure.

3.2.1 Enclosure with pair of two elliptical Cylinders

Figure 2 depicts the creation of temperature distribution and flow field lines around a pair of two elliptical Cylinders kept in a circular enclosure. The distance between the centre of two elliptical plates and Rayleigh number is varied. As the distance between the plates increases the density of the field lines decreases. The temperature flow lines are symmetrical about horizontal and vertical axis. A dense group of isotherms around the plate reflects the intense heat transfer between the plate and air. The buoyancy effect becomes weak as the circulation pace of the fluid carrying heat lessens and the distance between the centers of the elliptical plate increases. Circular enclosure. The incrementing Rayleigh number has an imperative effect on the buoyancy affected flow. The size of the enclosure increases and hence buoyancy effects show a vital change in their behavior. This can be seen from Figure 3, the loop formation increases and flow field lines become intense near the walls of elliptical plate. It divides into two equal halves at Rayleigh number $10^6$ as depicted in Figure 4. The temperature gradient in the flow field shows an increment in its value at high Rayleigh number.
3.2.2 Enclosure with pair of three elliptical Cylinders

Three elliptical plates are arranged in an equilateral triangle with their centre to centre distance changing in all three cases for three different Rayleigh numbers (10^4, 10^5 and 10^6) as shown in figure 5-7. As the centre to centre distance increases massive eddy formation takes place near the walls of the enclosure. Recirculation cells increases and flow becomes uneven. The flow moves upwards near the heated elliptical plates. The flow nearly divides into two equal halves as the Rayleigh number increases. As the distance increases the constant temperature lines starts emerging between the plates which shows enlarged strength of convection. At distance of \( x_c = 0.6 \) and 0.8 between the centre of the plates as shown in figure 6 the eddy formation takes place in two phases i.e. major eddies and minor eddies. In the latest case the major eddy combines with the minor. At Rayleigh number 10^6, the flow becomes more non uniform. Maximum slope in temperature is seen when value of \( x_c \) equals 1.2 as shown in figure 7. The flow lines mirrors itself about the vertical axis with same slope in temperature.

3.2.3 Enclosure with pair of four elliptical Cylinders

Figure 10 depicts a great variation in eddy formation as the quantity of elliptical plates increases to four. Eddy formation takes place in three phases. When the distance \( x_c \) equals 1.9 the flow distorts the most. Temperature declines to a minimum at the space between the plates seen by blue colour of flow lines. Recirculation cells forms at every possible location between the plates thereby mounting the strength of convection. The waviness of the surge lines increases with increasing Rayleigh number. Along the walls of plates strength of convection grows as it can be seen by the density and red colour of flow lines.

3.3 Heat Transfer

The variation of Nusselt number along the active enclosure walls and along the fringe of the inner Cylinders also substantiates this. Local nusselt number is plotted against the wall of enclosure and along the fringe of the inner Cylinders which demonstrates the strength of the buoyancy driven flow. Local Nusselt number’s variation for different Rayleigh number (10^4,10^5,10^6) and varying number elliptical Cylinders is shown in a graphical form.

3.3.1 Enclosure with pair of two elliptical Cylinders

Two elliptical Cylinders for three different Rayleigh numbers (10^4, 10^5,10^6) and three different centre to centre distance are enclosed in an enclosure and heat transfer consequences is monitored. As the space between the Cylinders is incremented as shown in figure 11 the value of the local nusselt number enhances along the wall of enclosure which illustrates the grown strength of convection. The waviness of the graph increases along the wall of enclosure and along the fringe of the inner Cylinders as shown in 12-14. It reaches a maximum value and then dips in case of walls of enclosure. The pattern gets mirrored on the other half of the Cylinders in every case. The Thermal field transits from steady state to unsteady state as the Rayleigh number increases from 10^4 to 10^6. This can be seen from increased rise and falls pattern in the plot of nusselt number against the curve length of Cylinders and wall of enclosure.

3.3.2 Enclosure with pair of three elliptical Cylinders

Effect of incrementing the number of Cylinders can be depicted from figure 11. The asymmetric nature of thermal field prevails over the entire region. Effect of position of the Cylinders can easily be understood from growing size of the tower formed on the closest side as shown in figure11-13 of the enclosure. Maximum value of the nusselt number occurs at the peak of the graph which forms the region between the enclosure and Cylinders. As the horizontal distance between the Cylinders increments space limits between the Cylinders and the walls of enclosure as a result the value of average nusselt number increases as seen from peak formed in figure 15-16. Fluid readily passes through the gap between the inner Cylinders when horizontal distance between the Cylinders is large. This is depicted from the large number of vortices formed between the Cylinders as shown in figure 10. Turbulence in terms of waviness in the graph is maximum when the Rayleigh number reaches a value of 10^6. The peak and valley formation is maximum when \( x_c \) reaches a value of 0.8 as shown in figure 13. In this case the maximum peak forms on the right side of the enclosure (0<\( x_c <0.5 \)) as shown in figure 16.
3.3.3 Enclosure with pair of four Cylinders

As the number of cylinder increases in the circular enclosure the symmetric nature of the graph increases. The figure 11-13 shows the plot of average nusselt number as a function of horizontal distance between the Cylinders and the curve length of enclosure walls and figure 14-16 shows the plot of average nusselt number along the fringe of the inner Cylinders and Rayleigh number. As the number of Cylinders increases to four the size of plume formed increases which illustrates the enhanced fluid flow and hence convection. The rotating motion of the vortices formed also enlarges and plume descends and ascends drastically. The maximum effect is shown when the horizontal distance between the Cylinders \((x_c)\) acquires a value of 1.9. Flow and thermal field with asymmetric structure forms and gradually shows a steady pattern at greater distance between Cylinders. The temperature difference between the flow field increments as the Rayleigh number increases. The flow field acquires symmetry at \(x_c = 1.7\) at Rayleigh number \(10^5\).

4. Summary and Conclusion

This parametric analysis focuses on two dimensional numerical simulation of natural convection inside a circular enclosure containing elliptical Cylinders/Cylinders at different Rayleigh numbers and varying horizontal distance between them. Development of flow field and temperature lines is observed for all the cases inside the enclosure. The results vary for each case. This implies that the buoyancy driven flow is a function of number of elliptical Cylinders, distance between the Cylinders and Rayleigh number. Based on the study following conclusion are drawn

- The incrementing Rayleigh number has an imperative effect on the buoyancy affected flow. The structure of the field lines transits from steady pattern to unsteady pattern.
- Flow and thermal field with asymmetric structure forms and gradually shows a steady pattern at greater distance between Cylinders.
- As the distance increases the constant temperature lines starts emerging between the plates which enlarges the strength of convection.
- Along the walls of plates strength of convection grows as it can be seen by the density and red colour of flow lines.
- For the similar condition the maximum heat transfer rate is achieved for the enclosure containing four elliptical Cylinders inside the circular enclosure at Rayleigh number \(10^5\).
- Further work can be done on optimization techniques for the simulation of the same problem based on second order derivative (PID-DD) scheme based on SOS algorithm used by N. S. Rathore et.al [25].

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**List of Tables:**

| Table 1. Validation of result |
|-------------------------------|
| Surface averaged Nusselt number at flat thin plate |
| α | Current study | Wei Zhang et al.[14] | Difference Percentage |
|---|---------------|---------------------|----------------------|
| 0° | 4.32          | 4.75                | 9.95                 |
| 30° | 4.26          | 4.68                | 7.98                 |
| 60° | 4.11          | 4.15                | 0.24                 |
| 120° | 3.20         | 3.15                | 1.56                 |
Table 2. Grid Independence test for the surface average Nusselt number of elliptical plate at r/D=0.0 and α =0°

| Number of nodes | Nusselt number | Difference Percentage |
|-----------------|----------------|-----------------------|
| 2398            | 8.7036         | -                     |
| 4470            | 8.6504         | 0.6105                |
| 6372            | 8.6187         | 0.3663                |
| 6771            | 8.6188         | 0.0011                |
| 7144            | 8.6153         | 0.0406                |

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Figure 1. Different arrangements of Cylinders and typical meshing of the computational domain

Figure 2. Isotherms and streamlines respectively of free convective flow at Ra= 10^4 for two elliptical plate placed inside circular enclosure.
Figure 3. Isotherms and streamlines respectively of free convective flow at $Ra = 10^5$ for two elliptical plate placed inside circular enclosure.

Figure 4. Isotherms and streamlines respectively of free convective flow at $Ra = 10^6$ for two elliptical plate placed inside circular enclosure.

Figure 5. Isotherms and streamlines respectively of free convective flow at $Ra = 10^4$ for three elliptical plate placed inside circular enclosure.
Figure 6. Isotherms and streamlines respectively of free convective flow at $Ra = 10^5$ for three elliptical plate placed inside circular enclosure.

$x_c = 0.4 \quad x_c = 0.6 \quad x_c = 0.8$

Figure 7. Isotherms and streamlines respectively of free convective flow at $Ra = 10^6$ for three elliptical plate placed inside circular enclosure.

$x_c = 0.8 \quad x_c = 1 \quad x_c = 1.2$

Figure 8. Isotherms and streamlines respectively of free convective flow at $Ra = 10^4$ for four elliptical plate placed inside circular enclosure.

$x_c = 1.5 \quad x_c = 1.7 \quad x_c = 1.9$
Figure 9. Isotherms and streamlines respectively of free convective flow at $Ra = 10^5$ for four elliptical plate placed inside circular enclosure.

Figure 10. Isotherms and streamlines respectively of free convective flow at $Ra = 10^6$ for four elliptical plate placed inside circular enclosure.

List of Graphs:

Figure 11. Local Nusselt number’s variation on the surface of internal Cylinders for $Ra = 10^4$. 

(a) 2Elliptical Cylinders  
(b) 3Elliptical Cylinders  
(c) 4 Elliptical Cylinders
Figure 12 Local Nusselt number’s variation on the surface of internal Cylinders for $Ra = 10^5$

Figure 13 Local Nusselt number’s variation on the surface of internal Cylinders for $Ra = 10^6$

Figure 14 Local Nusselt number’s variation along the wall of enclosure for $Ra = 10^4$

Figure 15 Local Nusselt number’s variation along the wall of enclosure for $Ra = 10^5$
Figure 16 Local Nusselt number’s variation along the wall of enclosure for $Ra = 10^6$