Configuration entropy description of charmonium dissociation under the influence of magnetic fields

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Abstract

Heavy ion collisions, produced in particle accelerators, lead to the formation of a new state of matter, known as the quark gluon plasma. It is not possible to observe directly the plasma, where quarks and gluons are not confined into hadrons. All the available information comes from the particles that reach the detectors after the strongly interacting matter hadronizes. Among those particles, one that plays an important role is the charmonium $J/\psi$ heavy meson, made of a $c\bar{c}$ quark anti-quark pair. The fraction of such particles produced in a heavy ion collision is related to the dissociation level caused by the plasma. On the other hand, the dissociation of $J/\Psi$ in the plasma is influenced by the temperature and the density of the medium and also by the presence of magnetic fields, that are produced in non central collisions.

A very interesting tool to study stability of physical systems is the configuration entropy (CE). In recent years many examples in various kinds of physical systems appeared in the literature, where an increase in the CE is associated with an increase in the instability of the system. In this article we calculate the CE for charmonium quasistates inside a plasma with a magnetic field background, in order to investigate how the instability, corresponding in this case to the dissociation in the thermal medium, is translated into the dependence of the CE on the field.
I. INTRODUCTION

In the recent years many examples appeared in the literature, where the configuration entropy (CE) \[1-3\] provides information about stability of physical systems. It was found, for diverse systems, as for example \[4-36\], that the more stable, the lower is the value of the CE.

Instability may be a consequence of many different types of transitions, depending on the system one considers. For the particular case that we study here – charmonium quasistates inside a plasma – the transition corresponds to the thermal dissociation in the medium. A natural question to be asked is: why is it important to study the stability of charmonium in a thermal medium? The point is that heavy vector mesons, like charmonium, that is formed by a $c\bar{c}$ valence quark anti-quark pair may survive the dissociation process that affects the light hadrons, when the quark gluon plasma (QGP) is formed in a heavy ion collision. So, the fraction of charmonium detected after a heavy ion collision can be used as a source of information about the QGP \[37, 38\]. Interesting reviews about the QGP are found in, for example: \[39-42\].

The dissociation of charmonium in the plasma is affected by the temperature, by the density and also by the presence of background magnetic fields. It is possible to describe such a behavior using a holographic bottom up model \[43-50\]. In this article we are concerned with the effect of the magnetic field on the charmonium quasistates inside a plasma and the corresponding instability caused by the dissociation in the medium.

The basis for the definition of the configuration entropy is the Shannon information entropy \[51\] that represents the information contained in a variable $x$ that assumes discrete values $x_n$ with probabilities $p_n$:

$$ - \sum_n p_n \log p_n. \quad (1) $$

In order to introduce the configuration entropy \[3\] one takes a normalizable function in coordinate space $\rho(\vec{r})$ (in general the energy density of a physical system) and transforms to momentum space:

$$ \tilde{\rho}(\vec{k}) = \frac{1}{(2\pi)^{d/2}} \int d^d r \rho(\vec{r}) \exp(-i\vec{k} \cdot \vec{r}). \quad (2) $$

Then one defines the modal fraction:

$$ \tilde{\epsilon}(\vec{k}) = \frac{\vert \tilde{\rho}(\vec{k}) \vert^2}{\int d^d k \vert \tilde{\rho}(\vec{k}) \vert^2}, \quad \quad (3) $$
and the CE is introduced as:

\[ S = - \int d^d k \hat{c}(\vec{k}) \log \hat{c}(\vec{k}). \]  

(4)

In contrast to the discrete case, for continuum variables this quantity may be negative. In this case, one can alternatively \[23\] define a different type of modal fraction

\[ \epsilon(\vec{k}) = \frac{|\hat{\rho}(\vec{k})|^2}{|\hat{\rho}(\vec{k})|^2_{\text{max}}}, \]

(5)

where in contrast to eq. (3) we do not normalize the function \( \hat{\rho}(\vec{k}) \) but rather divide by the maximum value of the square of the absolute value: \(|\hat{\rho}(\vec{k})|^2_{\text{max}}\). Then one introduces the so-called differential configuration entropy (DCE):

\[ S = - \int d^d k \epsilon(\vec{k}) \log \epsilon(\vec{k}). \]

(6)

We calculate in this article the DCE for charmonium inside a plasma in the presence of magnetic fields using a holographic model to describe the quasistates in the medium.

This article is organised in the following way: in section II we review the holographic model for charmonium in a plasma in the presence of magnetic fields. Then in section III we develop the calculation of the configuration entropy of the \( J/\psi \) quasistates inside the plasma. In section IV we present the results and section V contains some final conclusions.

II. HOLOGRAPHIC DESCRIPTION OF CHARMONIUM IN A PLASMA WITH MAGNETIC FIELDS

Charmonium \( J/\psi \) vector mesons inside a plasma with magnetic field can be studied using the holographic model of Ref. \[49\]. They are represented by a 5-dimensional dual vector field \( V_m \) with an action integral of the form

\[ I = \int d^4 x dz \sqrt{-g} e^{-\phi(z)} \left\{ -\frac{1}{4g_5^2} F^s_{mn} F^{mn} \right\}, \]

(7)

with \( F_{mn} = \partial_m V_n - \partial_n V_m \).

In the absence of the plasma and of magnetic fields, that means, in the vacuum, the space is just a five dimensional anti-de Sitter one. In this case the masses and decay constants of charmonium states are obtained from the background field \( \phi(z) \):

\[ \phi(z) = k^2 z^2 + M z + \tanh \left( \frac{1}{M} - \frac{k}{\sqrt{1}} \right). \]

(8)
The three energy parameters introduced in the model are interpreted as: $k$ represents the quark mass, $\Gamma$ the string tension of the quark anti-quark interaction and $M$ is a large mass associated with the charmonium non-hadronic decay, when the heavy meson decays into leptons, that involves the matrix element $\langle 0 | J_\mu(0) | J/\psi \rangle = \epsilon_\mu f_n m_n$. The values that provide the best fit to the spectrum are \[47\]:

$$
\begin{align*}
    k_c &= 1.2 \text{ GeV} , \quad \sqrt{\Gamma_c} = 0.55 \text{ GeV} , \quad M_c = 2.2 \text{ GeV} .
\end{align*}
$$

The extension to finite temperature and in the presence of a constant magnetic field $eB$, pointing in the $x_3$ direction, is obtained using in the action of eq. (7) the same scalar field background \[8\] of the vacuum case but with the following black hole geometry\[32\]:

$$
    ds^2 = \frac{R^2}{z^2} \left( -f(z)dt^2 + \frac{dz^2}{f(z)} + (dx_1^2 + dx_2^2 + dx_3^2 d(z)) \right),
$$

where

$$
\begin{align*}
    f(z) &= 1 - \frac{z^4}{z_h^4} + \frac{2 e^2 B^2 z^4}{3} \frac{1}{1.62} \ln \left( \frac{z}{z_h} \right) , \\
    h(z) &= 1 + \frac{8 e^2 B^2}{3} \frac{1}{1.62} \int_{+\infty}^{1/z} dx \frac{\ln(z_h x)}{x^3(x^2 - \frac{1}{z_h^4 x^2})} , \\
    d(z) &= 1 - \frac{4 e^2 B^2}{3} \frac{1}{1.62} \int_{+\infty}^{1/z} dx \frac{\ln(z_h x)}{x^3(x^2 - \frac{1}{z_h^4 x^2})} .
\end{align*}
$$

The plasma temperature is given by:

$$
    T = \frac{|f''(z)|_{z=z_h}}{4\pi} = \frac{1}{4\pi} \left| \frac{4 e^2 B^2 z_h^3}{3} \right| .
$$

One can find interesting alternative holographic studies of heavy flavour hadrons, for example, in \[53-67\]. Also, magnetic field effects in hadronic matter were analysed before in many references, like \[68-76\].

### III. CONFIGURATION ENTROPY OF CHARMONIUM IN A MEDIUM WITH MAGNETIC FIELDS

Now we follow the necessary steps in order to calculate the differential configuration entropy, defined in eq. \[6\], for the case of charmonium in a plasma with magnetic fields. The quantity to be Fourier transformed as in eq. \[2\] and then used to calculate de modal fraction.
using eq. (3) is the energy density of the charmonium \( J/\psi \) quasistate, that corresponds to the \( T_{00} \) component of the energy momentum tensor. In order to represent a meson at rest one considers a solution for the vector field of the form: \( V_{\mu} = \eta_{\mu} v(\omega, z)e^{-i\omega t} \) and choose the radial gauge \( V_{z} = 0 \). So, the energy density depends only on the coordinate \( z \) of the charged AdS Black hole metric of eq. (10). We assume that in our effective model, described by the action integral of eq. (7), the energy momentum tensor \( T_{mn} \) can be obtained as in general relativity:

\[
T_{mn}(z) = \frac{2}{\sqrt{-g}} \left[ \frac{\partial(\sqrt{-g}L)}{\partial g^{mn}} - \frac{\partial}{\partial x^p} \frac{\partial(\sqrt{-g}L)}{\partial (\partial g^{mn}/\partial x^p)} \right].
\]  

(15)

For the vector field action (7) one finds

\[
\rho(z) = T_{00}(z) = e^{-\phi(z)} \left[ g_{00} \left( \frac{1}{4} g^{mp} g^{nq} F_{mn} F_{pq} \right) - g^{mn} F_{0n} F_{0m} \right].
\]  

(16)

The magnetic field is pointing in the \( x_3 \) direction. Its is convenient to separate the analysis in two different cases. The transverse one, when the polarisation is perpendicular to the magnetic field, corresponding to the vector field in the direction of \( \eta_{\mu T} = (0, \cos(\alpha), \sin(\alpha), 0) \) and a longitudinal one, in the direction of \( \eta_{\mu L} = (0, 0, 0, 1) \).

For the transversal case one finds the energy density

\[
\rho_T(z) = \frac{z^2 e^{-\phi(z)}}{2R^2 g_5^2} \left( \frac{1}{d(z)} \right) \left[ \frac{f^2(z)}{|\omega|^2} |E_\alpha'|^2 + |E_\alpha|^2 \right],
\]  

(17)

where we introduced the electric field defined as \( E_\alpha = \omega V_\alpha \) as usual. For the longitudinal case the density reads

\[
\rho_L(z) = \frac{z^2 e^{-\phi(z)}}{2R^2 g_5^2} \left( \frac{1}{h(z)} \right) \left[ \frac{f^2(z)}{|\omega|^2} |E_3'|^2 + |E_3|^2 \right],
\]  

(18)

where, in a similar way \( E_3 = \omega V_3 \).

Our next task is to find the solutions of the vector field equations of motion that represent the quasistates of charmonium. These solutions, called quasinormal modes, must satisfy infalling boundary conditions at the event horizon \( z = z_h \) and must also vanish at the boundary \( z = 0 \) in order to ensure normalizability. This pair of conditions is in general satisfied by complex frequencies \( \omega \) with a real component associated with the thermal mass and an imaginary component associated with the thermal width of the quasistate.

Writing the equations of motion obtained from the action (7) with metric (10) in terms
of the electric fields one finds

$$E''_{\alpha} + \left( \frac{f'(z)}{f(z)} - \frac{1}{z} - \phi'(z) + \frac{h'(z)}{2h(z)} \right) E'_{\alpha} + \frac{\omega^2}{f^2} E_{\alpha} = 0, \quad (\alpha = 1, 2), \quad (19)$$

$$E''_{3} + \left( \frac{f'(z)}{f(z)} - \frac{1}{z} - \phi'(z) + \frac{d'(z)}{d(z)} - \frac{h'(z)}{2h(z)} \right) E'_{3} + \frac{\omega^2}{f^2} E_{3} = 0, \quad (20)$$

where the prime means derivative with respect to $z$.

In order to impose the infalling boundary conditions on the horizon it is helpful to write

the equations of motion in terms of the tortoise coordinate $r_*$ defined by

$$\frac{\partial}{\partial r_*} = f(z) \frac{\partial}{\partial z}$$

with $r_*(0) = 0$, for $z$ in the $0 \leq z \leq z h$. In terms of $r_*$ the solutions split, near the horizon, into a combination of infalling and outgoing waves. Both equations (19) and (20) can be written in the generic form:

$$E'' + a(z) E' + b(z) E = 0, \quad (21)$$

where the functions $a(z)$ and $b(z)$ are different for each polarisation. One searches for a field redefinition of the form $\psi = e^{-\frac{\zeta(z)}{2}} E$ such that equation (21), written in terms of the new function $\psi$ and of the tortoise coordinate, takes the form of a wave equation

$$\partial^2_{r_*} \psi + \omega^2 \psi = U \psi, \quad (22)$$

as long as the derivative of the function $\zeta(z)$ satisfies

$$\zeta'(z) = \frac{f'(z)}{f(z)} - a(z). \quad (23)$$

The potential $U(z)$ has the form

$$U(z) = -f^2(z) \left\{ \left( \frac{\zeta'(z)}{2} \right) + \frac{\zeta''(z)}{2} + a(z) \frac{\zeta'(z)}{2} \right\}. \quad (24)$$

For both transversal and longitudinal polarisations the potential diverges at $z = 0$. So, the normalizability condition for the quasinormal mode solutions require that one must impose the boundary condition $\psi(z = 0) = 0$. At the horizon the potential vanishes: $U(z = z_h) = 0$. So, in the limit $z \to z_h$ the general solution is a combination of infalling $\psi = e^{-i \omega r_*}$ and outgoing $\psi = e^{+i \omega r_*}$ ones. The relevant incoming solutions can be expanded near the horizon in the form

$$\psi = e^{-i \omega r_*}(z) \left[ 1 + c^{(1)}(z - z_h) + \ldots \right]. \quad (25)$$
The potential can also be expanded near the horizon as:

\[ U = (z - z_h)U'(z_h) + \ldots \]  

(26)

The coefficient \( c^{(1)} \) in eq. (25) is given by

\[ c^{(1)} = \frac{U'(z_h)}{f'^2(z_h) + 2i\omega f'(z_h)}. \]  

(27)

Following this approach, the boundary conditions to be satisfied by the electric field solutions corresponding to the quasinormal modes are:

\[ E(0) = 0, \]  

(28)

\[ E(z_h) = e^{-i\omega r_z(z_h) + \frac{c(z_h)}{2}}. \]  

(29)

Equations of motion (19) and (20) do not present analytic solutions, so they are solved numerically. The more convenient way to perform the numerical computations is to impose boundary conditions for the electric field and for the electric field derivative at the horizon:

\[ E(z_h) = e^{-i\omega r_z(z_h) + \frac{c(z_h)}{2}}, \]  

(30)

\[ E'(z_h) = \left(-i\omega r'_z(z_h) + \frac{c'(z_h)}{2} + c^{(1)}\right)E(z_h). \]  

(31)

Then one searches for the lowest complex frequency that lead to a solution vanishing at \( z = 0 \). These type of solutions, that depend on the temperature and on the magnetic field, are the ones that represent the charmonium quasistates. Then, one uses these solutions in the calculation of the energy density in eqs. (17) and (18).

The fields, and consequently the density, depend only on the variable \( z \). So, we need just the Fourier transform of \( \rho(z) \) in coordinate \( z \): \( \tilde{\rho}(k) \). It is convenient, for the computation of the CE, in this one dimensional case, to write \( \tilde{\rho}(k) = (C(k) - iS(k))/\sqrt{2\pi} \), where

\[ C(k) = \int_0^{z_h} \rho(z) \cos(kz)dz, \]  

(32)

\[ S(k) = \int_0^{z_h} \rho(z) \sin(kz)dz. \]  

(33)

In terms of these components, the modal fraction reads:

\[ \epsilon(k) = \frac{S^2(k) + C^2(k)}{[S^2(k) + C^2(k)]_{max}}, \]  

(34)

and the DCE (4) takes the form:

\[ S = -\int_{-\infty}^{\infty} \epsilon(k) \log [\epsilon(k)] \, dk. \]  

(35)
FIG. 1. Differential configuration entropy $S$ for charmonium $J/\psi$ as a function of the magnetic field $eB$ at temperature $T \rightarrow 0$ MeV

FIG. 2. Differential configuration entropy $S$ for charmonium $J/\psi$ as a function of the magnetic field $eB$ at temperature $T = 100$ MeV

IV. RESULTS

The strategy for studying the dependence of the charmonium DCE with the magnetic field is the following. We calculate the fields $E_3$ and $E_\alpha$ ($\alpha = 1, 2$), with complex frequencies, that solve equations (19) and (20) and have the asymptotic form of eq. (31) and vanish at $z = 0$. Then we insert these solutions into the expressions for the energy densities (18), in the longitudinal case, or (17) in the transversal case. Finally, the DCE is obtained using, in this order, equations (32), (33), (34) and (35).

We show in figures 1, 2, 3 and 4 the DCE for charmonium $J/\psi$ as a function of the
FIG. 3. Differential configuration entropy $S$ for charmonium $J/\psi$ as a function of the magnetic field $eB$ at temperature $T = 200$ MeV.

FIG. 4. Differential configuration entropy $S$ for charmonium $J/\psi$ as a function of the magnetic field $eB$ at temperature $T = 300$ MeV, with a zoom of the region $0.3 \leq eB \leq 0.4$ by a factor of 5.

magnetic field $eB$ when the plasma is at temperatures of, respectively, $T = 0, 100, 200, 300$ MeV, for both longitudinal and transversal polarisations. One notes that the DCE increases with the field $eB$ and this effect is more intense for lower temperatures and very similar for the two polarisations. These figures are plotted using the same scale in order to provide a comparison of the DCE variations. In order to make it possible to see that even for the higher temperatures the DCE increases with temperature, we inserted inside figure 4 a small plot with a zoom in the region $0.3$ GeV $\leq eB \leq 0.4$ GeV. The scale was amplified in this part by a factor of 5 so that one can see that indeed the DCE is increasing.
FIG. 5. Differential configuration entropy $S$ for charmonium $J/\psi$ as a function of the magnetic field $eB$, in the longitudinal polarization case, at temperatures $T = 0, 100, 200, 300$ MeV.

Then, in order to make it clear the effect of the temperature, we plot in figure 5 the results for the four temperatures considered in the previous figures for the longitudinal polarisation case. Then, the same thing is shown in figure 6 but for transversal polarisation. One notices that there is a clear increase of the DCE with temperature. This is consistent with the fact that as the temperature increases the charmonium state becomes more unstable against dissociation in the thermal medium.

Now, let us see if the dependence of the DCE on the magnetic field $eB$ can be approximated by some simple form. Searching for polynomial approximations, one finds a very nice
fit using a second order expression of the form:

\[ S = c_0 + c_1(eB) + c_2(eB)^2, \]  

(36)

with \(c_0, c_1, c_2\) varying with the temperature. Tables I and II show the values of the parameters at the temperatures considered previously and the mean absolute error \(R^2_{\text{Adj}}\) for the approximation of the DCE of \(J/\psi\) by relation (36).

| Temperature (GeV) | \(c_0\)       | \(c_1(GeV)^{-1}\) | \(c_2(GeV)^{-2}\) | Mean absolute error |
|-------------------|---------------|--------------------|--------------------|---------------------|
| 0                 | 3.7788 ± 0.0009 | 2.528 ± 0.008      | -1.23 ± 0.02       | 0.0018              |
| 0.1               | 4.476 ± 0.003  | 1.00 ± 0.03        | 0.48 ± 0.02        | 0.007               |
| 0.2               | 5.5399 ± 0.0009| -0.028 ± 0.009     | 0.94 ± 0.02        | 0.0019              |
| 0.3               | 6.5895 ± 0.0007| 0.179 ± 0.006      | 0.07 ± 0.01        | 0.0014              |

**TABLE I.** Coefficients \(c_0, c_1\) and \(c_2\) of eq. (36) for longitudinal polarisation at different temperatures.

| Temperature (GeV) | \(c_0\)       | \(c_1(GeV)^{-1}\) | \(c_2(GeV)^{-2}\) | Mean absolute error |
|-------------------|---------------|--------------------|--------------------|---------------------|
| 0                 | 3.7771 ± 0.0007 | 2.4806 ± 0.007     | -1.43 ± 0.01       | 0.0015              |
| 0.1               | 4.477 ± 0.0007 | 0.983 ± 0.007      | 0.29 ± 0.01        | 0.0011              |
| 0.2               | 5.5393 ± 0.0005| 0.033 ± 0.005      | 0.625 ± 0.009      | 0.0011              |
| 0.3               | 6.5895 ± 0.0007| 0.1863 ± 0.007     | 0.05 ± 0.01        | 0.0014              |

**TABLE II.** Coefficients \(c_0, c_1\) and \(c_2\) of eq. (36) for transversal polarisation at different temperatures.

For the sake of illustrating the quality of the fit of the DCE by the quadratic polynomials of the form given in eq. (36), we show in figure 7 the case of transverse polarisation at \(T = 200\) MeV.

V. CONCLUSIONS

It is known that the presence of \(eB\) magnetic fields enhances the dissociation effect of charmonium in a plasma [49]. The higher is the field, the strongest is the effect. The thermal dissociation corresponds to the disappearance of the charmonium quasistates in the medium.
So, an increase in the dissociation intensity corresponds to an increase in the instability of charmonium. As discussed in the introduction, for many different systems it was observed that the configuration entropy works as an indicator of stability. The more stable is the system, the lower is the value of the configuration entropy. In this article we have calculated the DCE of charmonium quasistates in a plasma in the presence of magnetic fields and obtained the result that it increases with the \( eB \) field. This is consistent with the increase in instability associated with dissociation in the medium. We also found that the DCE increases with the temperature, as it was previously observed in \[32\], and is also consistent with the increase in instability caused by the enhancement of the dissociation effect with the temperature.

A result that also emerged here is that the variation of the DCE with the magnetic field is more intense for lower temperatures. This is consistent with the fact observed in ref.\[49\] that the increase in the imaginary part of the quasinormal mode frequencies caused by the magnetic field is larger for lower temperatures. The imaginary part of the complex frequencies represents the thermal width, that is related to the dissociation level. The larger it is, the more unstable is the quasistate.

It is important to remark that here we considered only the effect of the magnetic field in the thermal medium. It is important to note that it is possible to study the direct effect of the field on the charged constituents of the meson as was considered in \[63\] \[77\] \[78\].

**Acknowledgments:** The authors are supported by CNPq - Conselho Nacional de Desenvolvimento Científico e Tecnológico (N.B by grant 307641/2015-5 and R. M. by a graduate
fellowship). This work received also support from Coordenacao de Aperfeicoamento de
Pessoal de Nivel Superior - Brasil (CAPES) - Finance Code 001.

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