LARGE ENERGY DEPENDENCE OF CURRENT NOISE IN SUPERCONDUCTING/NORMAL METAL JUNCTIONS

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Interference of electronic waves undergoing Andreev reflection in diffusive conductors determines the energy profile of the conductance on the scale of the Thouless energy. A similar dependence exists in the current noise, but its behavior is known only in few limiting cases. We consider a metallic diffusive wire connected to a superconducting reservoir through an interface characterized by an arbitrary distribution of channel transparencies. Within the quasiclassical theory for current fluctuations we provide a general expression for the energy dependence of the current noise. We derive closed analytical expressions for large energy.

Introduction

Interference of electronic waves in metallic disordered conductors is responsible for weak localization corrections to the conductance. If these are neglected, the probability of transferring an electron through the diffusive medium is given by the sum of the modulus squared of the quantum probability amplitudes for crossing the sample along all possible paths. This probability is denoted as semiclassical, since quantum mechanics is necessary only for establishing the probability for following each path independently of the phases of the quantum amplitudes. In superconducting/normal metal hybrid structures, interference contributions are not corrections, they may actually dominate the above defined semiclassical result for temperatures and voltages smaller than the superconducting gap. This is seen experimentally as an energy dependence of the conductance on the scale of the Thouless energy. Indeed, the energy dependence comes from the small wavevector mismatch, linear in the energy of the excitations, between the electron and the Andreev reflected hole. This is responsible for the phase difference in the amplitudes for two different paths leading to interference. The effect is well known and explicit predictions and measurements exist for a number of systems.

Interference strongly affects the current noise too. The largest effects are ex-
pected in the tunneling limit, when the transparency of the barrier is small and its resistance is much larger than the resistance of the diffusive normal region. Then, the conductance has a strong non linear dependence at low bias (reflection-less tunneling). This is actually the case, but the zero-temperature noise (or shot noise) does not give any additional information on the system since it is simply proportional to the current, as shown quite generally in Ref. 6. In the more interesting case of a diffusive metal wire in contact with a superconductor through an interface of conductance $G_B$ much larger than the wire conductance $G_D$, Belzig and Nazarov found that the differential shot noise, $dS/dV$, shows a reentrant behavior, as a function of the voltage bias, similar, but not identical, to the conductance one. (The extension of the Boltzman-Langevin approach to the coherent regime in Ref. 5 neglects this difference.) In order to compare quantitatively with actual experiments and to gain more insight in the interference phenomenon, it is necessary to obtain the energy dependence of noise in more general situations. The numerical method used in Ref. 7 is, in principle, suitable to treat more general cases, notably the case when $G_D \sim G_B$, but only if all channel transparencies, $\{\Gamma_n\}$, that characterize the interface are small. In Ref. 12 we presented an analytical solution for the diffusion-type differential equation for the noise within the theory of current fluctuations in the quasiclassical dirty limit. It allows to treat the general case of arbitrary values for $\{\Gamma_n\}$ and $G_B/G_D$. In the present paper we present the minimal set of equations necessary to obtain the noise. We then exploit them to obtain closed analytical expressions for the noise at large energy.

1. Equations to obtain the noise

In Ref. 12 we have developed an analytical theory to calculate the current noise in a diffusive wire of length $L$, diffusive constant $D$, conductance $G_D$, connected to a normal reservoir on one side (with a transparent junction) and to superconductor on the other side through an arbitrary interface characterized by a set of channel transparencies $\{\Gamma_n\}$. These equations are obtained by exploiting the semiclassical theory proposed by Nazarov to calculate the full counting statistics of charge transfer. Details are given in Ref. 12. Let us summarize in a compact form the equations necessary to obtain the noise.

The first step is to obtain the conductance. This depends on $f_T(x)$ and $\theta(x)$, parameterizing the fermion distribution and the superconducting correlations, respectively. The variable $x$ varies between 0 and $L$, $\theta = \theta_1 + i \theta_2$ is a complex number and satisfies the following equation:

$$\hbar D \theta''(x) + 2i \varepsilon \sinh \theta(x) = 0,$$

with boundary conditions $\theta(L) = 0$ and $\theta'(0) = H[\theta(0)]$ where

$$H[\theta] = \frac{i}{L r} \left( \frac{\cosh \theta}{1 + \frac{1}{2} (i \sinh \theta - 1)} \right).$$
We defined $\langle \psi (\Gamma) \rangle = \sum_n \Gamma_n \psi (\Gamma_n) / \sum_n \Gamma_n$ and $r = G_D/G_B$, with $G_B = (2e^2/h) \sum_n \Gamma_n$. For $f_T$ we have the simpler equation $(\cosh^2 \theta_1(x) f_T'(x))' = 0$ with boundary conditions $f_T(L) = f_{T0}$ and

$$f_T'(0) = \frac{f_T(0) \theta_1'(0)}{\cosh \theta_1(0) \sinh \theta_1(0)}.$$  \hspace{1cm} (3)

Here $f_{T0} = f_+ - f_-$, $f_{k}(\varepsilon) = f(\varepsilon \pm eV)$, $f$ is the Fermi function at temperature $T$, and $V$ is the voltage bias. Then the current is given by:

$$I = \frac{1}{2} \int d\varepsilon G(\varepsilon) f_{T0}(\varepsilon)$$

and $C^{-1}(\varepsilon) = 1/L \int_0^L ds / \cosh^2 \theta_1(s)$. At low temperatures, $k_B T \ll eV$, $G_{\text{diff}}(V) = G(eV)$ is the differential conductance. These equations have been recently exploited in Ref. [13] to discuss the conductance.

To obtain the noise we need an additional parameter $a(x)$. It parameterizes the first correction in the counting field to the Usadel Green’s function. Other parameters intervene, but we do not need them to calculate the noise.[12]

The complex parameter $a = a_1 + ia_2$ satisfies the following linear differential equation:

$$hD a''(x) + 2i \varepsilon a(x) \cosh \theta(x) = -2E_T \frac{\sinh \theta_1(x)}{\cosh \theta_1(x)} \frac{G(\varepsilon)^2}{G_D^2},$$  \hspace{1cm} (5)

with $E_T = hD/L^2$. The boundary conditions are $a(L) = 0$ and $L a'(0) = \alpha a(0)/r + \beta/r$ with

$$\alpha = \left\langle \frac{i \sinh \theta - \Gamma (i \sinh \theta - 1)/2}{[1 + \Gamma (i \sinh \theta - 1)/2]^2} \right\rangle$$  \hspace{1cm} (6)

and

$$\beta = \frac{ic^2}{8} \frac{2 \Gamma^2 \cosh \theta^* + 8(\Gamma - 1) \cosh \theta - 2i \Gamma (\Gamma - 2) \sinh \theta \cosh \theta^*}{[1 + \Gamma (i \sinh \theta - 1)/2]^2 (1 + \Gamma (i \sinh \theta - 1)/2)},$$  \hspace{1cm} (7)

both evaluated at $x = 0$ and we defined $r = G_D/G_B$. The low frequency noise is finally given by:

$$S = \int d\varepsilon G(\varepsilon) \{ 1 - f_{L0}^2(\varepsilon) - [1 - F(\varepsilon)] f_{T0}^2(\varepsilon) \},$$  \hspace{1cm} (8)

where

$$F(\varepsilon) = \frac{2}{3} (1 + c(0)^3) + \frac{2G(\varepsilon)}{G_D} \int_0^1 \frac{\sinh \theta_1 a_1}{\cosh \theta_1} ds - c(0) \left( \frac{G_D a_1'(0)c(0)}{G(\varepsilon) \tanh \theta_1(0)} + \frac{2a_1(0)}{\sinh 2 \theta_1(0)} \right).$$  \hspace{1cm} (9)

Here the parameter $c$ is simply proportional to $f_T(x)$: $c(x) = -f_T(x)/f_{T0}$ with $c(0) = 1 - G(\varepsilon)/|G_D C(\varepsilon)|$, and $f_{L0} = 1 - f_+ - f_-$. We have all ingredients to calculate explicitly the noise for arbitrary values of the ratio $r$, of the energy $\varepsilon$, or of the transparency set $\Gamma_n$. 

2. Large energy limit for \( r \neq 0 \)

In the following we discuss the \( \varepsilon \gg E_T \) analytical limit. If \( r \neq 0 \) for large \( \varepsilon \) the parameter \( \theta \) is very small, it is thus possible to set up the following expansion:

\[
\theta(x) = \frac{\theta^{(0)}(x)}{k} + \frac{\theta^{(1)}(x)}{k^2} + \frac{\theta^{(2)}(x)}{k^3} + \ldots
\]  

(10)

where we introduced the large parameter of the expansion \( k = \sqrt{\varepsilon} \). We can now substitute (10) into (1) and into the boundary conditions. In collecting terms of the same order in \( 1/k \) one has to remember that each derivative with respect to \( x \) introduce a \( k \) factor. At lowest order we obtain

\[
D \theta^{(m)''}(x) + 2ik^2 \theta^{(m)} = 0
\]

(11)

\[
D \theta^{(2)''}(x) + 2ik^2 \left[ \theta^{(2)} + \theta^{(0)3} \right] = 0
\]

(12)

where \( m = 0, 1 \). For the boundary conditions we obtain \( \theta^{(m)}(L) = 0 \) for all \( m \), \( \theta^{(0)} = k H(0) \), \( \theta^{(1)} = k H'(0) \theta^{(0)} \), and \( \theta^{(2)} = k H''(0) \theta^{(1)} + k H'''(0) \theta^{(0)}^2 / 2 \). To obtain the conductance to order \( 1/k^2 \) we need also the boundary condition for \( \theta^{(3)'} \), but we do not need to solve the associated differential equation. Solving the differential equations (11) and (12) and substituting the result into Eq. (11) we obtain for the conductance up to second order in \( 1/k \) the following expression:

\[
G = \frac{H'}{1 + H'} + \frac{H'H''}{2k(1 + H')^2} + \frac{H'H''(2H' + 2H'^2 - HH'')}{4k^2(1 + H')^3} + \ldots.
\]

(13)

Equation (13) holds for any distribution of channel transparency, it suffices to calculate the appropriate average in the definition of \( H \), \( H' \), and \( H'' \).

The procedure to obtain the noise is similar. This time we need an expansion of the parameter \( a \) which has the same form of (10). Actually the differential equation for \( a^{(0)} \) and \( a^{(1)} \) coincide with those for \( \theta^{(0)} \). The equation for \( a^{(2)} \) reads:

\[
hD a^{(2)''} + 2ik^2 a^{(2)} = -2k^2 [ia^{(0)} \theta^{(0)}]^2 + f_T a \theta^{(0)} + B \theta^{(0)}^2 \theta^{(0)}].
\]

(14)

The boundary conditions for \( a \) read: \( a^{(0)} + k \beta^{(0)} / r = 0 \), \( a^{(1)} + k[\alpha(0)a^{(0)} + \beta^{(1)}] / r = 0 \), \( a^{(2)} + k[\alpha(0)a^{(1)} + \alpha(1)a^{(0)} + \beta^{(2)}] / r = 0 \), where the same kind of development has been performed on \( \alpha \) and \( \beta \). Substituting these expressions into (15) we can obtain the differential Fano factor \( F(\varepsilon) = (dS/dV)(\varepsilon)/2eG(\varepsilon) \). The lowest order, i.e. the incoherent contribution has a simple expression:

\[
F_{inc} = \frac{2}{3} \left[ 1 + (2 - 3 \left< \frac{\Gamma^3}{(2-\Gamma)^3} \right> \left( \frac{G_D^3}{G_D + 2G_B \left< \frac{\Gamma^3}{(2-\Gamma)^3} \right>^3} \right) \right]
\]

(15)

This can be understood by a comparison with the classical calculation of Ref. 1. for a wire connected to normal reservoirs. Eq. (16) coincide with the Fano factor given there when the substitution \( e \rightarrow 2e \), \( G_D \rightarrow G_D / 2 \) and \( \Gamma_n \rightarrow \Gamma_n^A \). This is consistent with the expectation that phase coherence becomes irrelevant at high energy (see also the discussion in Ref. 15).
The expression for the quantum correction to Eq. (15) is cumbersome in the general case and we will not present it. Simpler expression is obtained when all transparencies are the same (\(\Gamma_n = \Gamma\)):

\[
F^{(1)} = \frac{4r^2(\Gamma - 2)^2}{[r(\Gamma - 2) + 2\Gamma]^4} \left[ -64 + \Gamma(\Gamma - 2)(-64 + r(\Gamma - 2)(\Gamma^2 + 12\Gamma - 12)) \right]
\]

and \(F(\varepsilon) = F_{inc} + F^{(1)}/\sqrt{\varepsilon}\).

3. Conclusions

We presented a theory to calculate the energy dependence of the noise in a wire connecting a normal with a superconducting reservoir. The theory allows to obtain closed analytical expressions in different relevant limits. We considered here in some details the large energy case. The classical incoherent result appears for energy much larger than the Thouless energy. Quantum corrections are explicitly evaluated when all transparencies have the same value.

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References

1. B. L. Altshuler and A. G. Aronov, in Electron-electron interactions in disordered systems, Eds. A. L. Efros and M. Pollak, (North-Holland, Amsterdam) (1985).
2. A. F. Volkov, A. V. Zaitsev, and T. M. Klapwijk, Physica C 210, 21 (1993).
3. F. W. J. Hekking and Yu. V. Nazarov, Phys. Rev. Lett. 71, 1625 (1993).
4. C. W. J. Beenakker, Rev. Mod. Phys. 69, 731 (1997).
5. Ya. M. Blanter and M. B"uttiker, Phys. Rep. 336 1 (2001).
6. F. Pistolesi, G. Bignon, and F. W. J. Hekking, Phys. Rev. B, (in press)
7. W. Belzig and Yu. V. Nazarov, Phys. Rev. Lett. 87, 067006 (2001).
8. M. Houzet and V. P. Mineev, Phys. Rev. B 67, 184524 (2003).
9. X. Jehl, M. Sanquer, R. Calemczuk, and D. Mailly, Nature 405, 50 (2000).
10. A. A. Kozhevnikov, R. J. Schoelkopf, and D. E. Prober, Phys. Rev. Lett. 84, 3398 (2000).
11. F. Lefloch, C. Hoffmann, M. Sanquer, and D. Quirion, Phys. Rev. Lett. 90, 067002 (2003).
12. M. Houzet and F. Pistolesi, Phys. Rev. Lett. 92, 107004(2004).
13. L. S. Levitov, H. W. Lee, and G. B. Lesovik, J. Math. Phys. 37, 4845 (1996).
14. Yu. V. Nazarov, Superlatt. Microstruct. 25, 1221 (1999).
15. A. V. Zaitsev, Sov. Phys. JETP 59, 1163 (1984).
16. Y. Tanaka, A. A. Golubov, and S. Kashiwaya, Phys. Rev. B 68 054513 (2003).
17. M. J. M. de Jong and C. W. J. Beenakker, Physica A 230, 219 (1996).
18. P. Samuelsson and M. B"uttiker, Phys. Rev. B 66, 201306 (2002).