Single pump, parametric amplification in randomly-birefringent, unidirectionally spun fibers

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Abstract

A systematic study of the effects of polarization mode dispersion on broad-band and narrow-band, single pump, fiber parametric amplifiers is realized through numerical solutions of the equations governing the interaction. The nonlinear polarization rotation is shown to be a relevant effect that can increase gain randomness when it mixes with polarization mode dispersion. In unidirectionally spun fibers the signal-pump alignment can be highly increased and the gain enhanced. However, in spite of the enhanced alignment, large polarization mode dispersion phase-mismatches the interaction and the gain decreases to zero.

I. INTRODUCTION

Fiber optical parametric amplifiers (FOPAs) have been under investigation for many years. In the single pump configuration both broad-band (BB-FOPA) [1] and narrow-band (NB-FOPA) [2], [3] amplification can be obtained. The former has been mainly addressed for amplification and wavelength conversion [4] while the latter yields a superb technique for slow and fast light generation [5], [6].

FOPAs are highly affected by the fiber random birefringence (polarization mode dispersion - PMD) [7], [8]. In an ideal, perfectly isotropic fiber the waves would maintain their input states of polarization (SOP) over the entire fiber, the gain would be deterministic and it would depend only on the initial SOPs [9]. In low birefringence fibers the evolution of the pump, signal and idler SOPs is random, due to the PMD. Eventually, the gain is reduced and becomes a random quantity, as theoretically analyzed in [7] for BB-FOPA.

Recently, the benefits of unidirectional fiber spinning in Raman [10] and Brillouin fiber amplifiers [11] have been underlined. Here, the same detailed study is extended to single pump, BB- and NB-FOPAs. Note that, differently from the Raman and Brillouin fiber amplifiers, in FOPAs pump polarization scrambling cannot be used to avoid
polarization dependence. Other techniques like dual pumping [12], [13] or polarization diversity [14], [15] were proposed to overcome the problem, but at the cost of an increased set-up complexity. So, the results presented here are particularly relevant because unidirectionally spun fibers can represent a breakthrough for implementing highly performant BB- and NB-FOPA with simple experimental setups.

II. PROPAGATION MODEL

The parametric interaction of the pump, signal and idler monochromatic complex Jones vectors, \( |A_p⟩, |A_s⟩, |A_i⟩ \), in the undepleted pump approximation and neglecting the nonlinear effects of the signal and idler on the pump, is governed by [7]:

\[
\partial_z A_p = j \left( j\alpha_p + k(\omega_p) - \bar{\beta}(\omega_p) \cdot \bar{\sigma}/2 \right) +
+j\gamma/3 \left( 2\langle A_p|A_p⟩ + |A_p^+⟩⟨A_p^+| \right) |A_p⟩,
\]

\[
\partial_z A_{s,i} = j \left( j\alpha_{s,i} + k(\omega_{s,i}) - \bar{\beta}(\omega_{s,i}) \cdot \bar{\sigma}/2 \right) |A_{s,i}⟩ +
+j2\gamma/3 \left( 2\langle A_p|A_p⟩ + |A_p⟩⟨A_p⟩ + |A_p^+⟩⟨A_p^+| \right) |A_{s,i}⟩ +
+j\gamma/3 \left( |A_p^+⟩⟨A_p^+| + 2|A_p⟩⟨A_p^+| \right) |A_{s,i}⟩,
\]

(1)

where \( \gamma = 2 W^{-1} km^{-1} \) is the nonlinear coefficient, \( \alpha_h, h = p, s, i \) are the loss coefficients at the optical pulsations of the pump, signal and idler \( \omega_p, \omega_s, \omega_i \) that satisfy simultaneously the energy \( 2\omega_p = \omega_s + \omega_i \) and the nonlinear phase matching condition \( \Delta k = k(\omega_s) + k(\omega_i) - 2k(\omega_p) \approx k_{2p} (\omega_s - \omega_p)^2 + k_{4p} (\omega_s - \omega_p)^4/12 = -2\gamma P_0 \), \( k_{nh} \) is the \( n \)-th angular frequency derivative of \( k(\omega) \), evaluated at \( \omega_h \) [1] and \( P_0 \) is the pump power). Transition from BB- to NB-FOPA can be obtained by tuning \( \omega_p \) in the vicinity of the fiber zero dispersion wavelength, thus modifying the dispersion coefficient \( k_{2p} \). When the sign of \( k_{2p} \) is negative, gain occurs at \( \omega_s \) within a broad band around the pump frequency. For positive \( k_{2p} \) and if \( k_{4p} \) is negative, the nonlinear matching can be satisfied in two narrow spectral bands, largely detuned from the pump pulsation. The following parameters have been used in the simulations: BB-FOPA, \( k_{2p} = -1.18 \times 10^{-29} \text{s}^2/\text{m}, k_{4p} = 1 \times 10^{-55} \text{s}^4/\text{m} \) [7];NB-FOPA, \( k_{2p} = 7.68 \times 10^{-28} \text{s}^2/\text{m}, k_{4p} = -5 \times 10^{-55} \text{s}^4/\text{m} \) [3].

The Stokes vector \( \bar{\beta}(z, \omega) \) is the local birefringence vector and \( \bar{\sigma} \) is the vector of the Pauli spin matrices [19]. For unspun fibers, \( \bar{\beta}(z, \omega) = \bar{\beta}_{us}(z, \omega) \) can be obtained through the so called random modulus model (RMM) [16], that describes the PMD phenomenon and effects through three characteristic lengths: the beat length \( (L_B) \), the birefringence correlation length \( (L_F) \) and the polarization correlation length \( (L_C) \). The first depends on the pulsation through \( L_B(\omega) = \omega_0 L_B(\omega_0)/\omega; \) the latter is independent of \( \omega \) \( (L_F = 9 \text{m}) \) was fixed in the numerical integrations). The first two contribute to determine the PMD coefficient (PMDC), hereinafter defined as \( D = \sqrt{\langle \Delta\tau^2 \rangle / L} \) where \( \langle \Delta\tau^2 \rangle \) is the fiber PMD mean square differential group delay and \( L (= 2 \text{km}) \) is the fiber length. The polarization correlation length \( L_C \), is the length scale over which the SOP is modified and depends on the previously defined lengths and on the fiber spinning properties. This length scale is a fundamental parameter when nonlinear PMD sets in [16], as in this case.
For unidirectionally spun fibers the birefringence vector becomes \( \vec{\beta}(z, \omega) = R_3[2\phi(z)]\vec{\beta}_{\omega3}(z, \omega) \) where \( R_3 \) is a Mueller matrix representing a rotation around the third component of the Stokes space \( (\vec{u}_3) \). The angle of rotation is given by the spin function \( \phi(z) = 2\pi z/p \) where \( p \) is called the spin pitch. It was found that unidirectional spinning [20]: a) decreases the PMDC and b) modifies the polarization correlation length \( L_C \), according to:

\[
L_C = \frac{L_B^2}{4a\pi^2L_F^2} \left( 1 + \frac{16\pi^2L_F^2}{p^2} \right)
\]

for \( L_B^2 >> 4\pi^2L_F^2/\sqrt{1 + (16\pi^2L_F^2)/p^2} \) and where \( a = 4 \) \((a = 2)\) for linear (circular) input SOPs.

A few thousands statistical realizations of the stochastic processes, using the RMM, and subsequent integrations of Eqs. \( \Pi \) have been performed to determine the mean gain \( \mu_G = \langle \langle A_s(L)|A_s^*(L)\rangle\rangle/\langle A_s(0)|A_s(0)\rangle\rangle \) and the relative gain standard deviation (STD) \( \sigma_G = \sqrt{\langle \langle A_s(L)|A_s^*(L)\rangle\rangle^2 - 1} \). Numerical results are represented in all figures as functions of the PMDC that is calculated from the statistical ensemble.

III. Numerical results

For unspun fibers, the mean gain and STD from numerical solutions are presented (circles) for initially linear and parallel SOPs in Fig. 1 for BB-FOPA \( (\lambda_p \approx 1550.2 \text{ nm}, \lambda_s \approx 1514 \text{ nm}) \) and in Fig. 2 for NB-FOPA \( (\lambda_p \approx 1541 \text{ nm}, \lambda_s \approx 1387 \text{ nm}) \) as a function of \( D \). Mean and STD deviation for other input SOPs are not presented here, for the sake of brevity, but all the results show the same behavior that is described below. When the PMDC is large, the gain decreases down to zero, because PMD tends to phase mismatch the waves [7], [8]. In this regime the predictions of the mean Eqs. of [7] (solid curves in both figures), where nonlinearity was averaged and an effective nonlinear coefficient \( \gamma_c = 8\gamma/9 \) was found [17], [18], are recovered both for BB- and NB-FOPA. The PMD affects more NB-FOPA than BB-FOPA mean gain; this can be explained by the fact that the effects of PMD, at the same value of \( D \), increases with the frequency detuning among the waves.

However, for small PMDC the mean and STD values do not correspond to the theoretical findings. The mean gain is actually larger and the STD presents a peak (see \( \sigma_G \) in Figs. \( \Pi \) and \( 2 \) for \( D \approx 10^{-3} \text{ ps}/\sqrt{\text{km}} \) not predicted by the theory. The discrepancy stems from the nonlinear PMD [16], induced by the nonlinear polarization rotation (NPR) effect, as already briefly described in the case of Raman amplification [21]. In the Stokes space, the pump vector \( \vec{P} = \langle A_p|\vec{\sigma}|A_p\rangle\), is governed by:

\[
\partial_z \vec{P} = [\vec{\beta}(z, \omega_p) + 2\gamma/3(\vec{P} \cdot \vec{u}_3)\vec{u}_3] \times \vec{P}.
\]

When the pump polarization evolution length \( L_C \) is close to the NPR length scale NPR becomes a significant source of randomness [16], which finally leads to enhance signal gain randomness. To prove this affirmation, the theoretical values of \( L_B \) satisfying the condition \( L_C = L_{NPR} \) in Eq. \( 2 \) in function of the spin rate \( \nu = 1/p \), have been calculated for the BB-FOPA and are shown in Fig. 3 (solid curves), in the limit of validity of Eq. \( 2 \) (similar results are obtained for the NB-FOPA). The value of the NPR length was estimated as \( L_{NPR} \approx 9/(2\gamma P_0) = 2.25 \) km in the hypothesis, which is valid if the nonlinear mixing is strong enough, that the mean power in the stochastic vector \( (\vec{P} \cdot \vec{u}_3)\vec{u}_3 \) is one third of the total pump power. In the same Fig. 3 the values of \( L_B \) for which the STD
reaches the relative maximum in the numerical solutions are also shown for comparison (circles). The agreement is very good both for linear (black solid curve and circles) and circular (grey solid curve and circles) input SOPs.

NPR effects become relevant for rapidly spun fibers, for which the averaging of nonlinearity cannot be performed and the effective nonlinear coefficient is \( \gamma \) [22]. In fact, as the spin rate increases, the peak in the STD, due to the NPR, shifts to larger PMDC values because \( D \) does not follow the same scaling law with \( p \) of \( L_C \) [20]. Moreover, the mean gain of both BB- and NB-FOPA increases because of the enhancement in the mean pump-signal SOPs alignment \( \langle \hat{p} \cdot \hat{s} \rangle = \langle \cos(\theta) \rangle \), as already demonstrated in [10], [11]. This effect is shown in Fig. 4 for the NB-FOPA. Finally, observe that highly increasing the spin rate does not lead to a gain enhancement similar to that observed for Raman and Brillouin amplifiers [10], [11]. In fact, PMD affects the pump-signal alignment, restored by spinning, but also the phase matching [7], [8], whose dependence on spinning is actually neglectable.

IV. Conclusions

In conclusion, the effects of polarization mode dispersion and unidirectional fiber spinning in broad-band and narrow-band single pump, fiber optical parametric amplifiers were studied by means of numerical solutions of the governing equations. The effects of nonlinear polarization rotation are relevant in such amplifiers, in particular when unidirectional spinning is applied. The gain can be enhanced by spinning and, in NB-FOPA for \( 10^{-2} \text{ ps}/\sqrt{\text{km}} < D < 4 \cdot 10^{-2} \text{ ps}/\sqrt{\text{km}} \), the gain variance can be decreased. This observation is relevant for slow and fast light generation, where polarization mode dispersion highly affects the interaction [6], [8]. The gain increase stems from the spinning enhancement of the alignment between the pump and the signal along the fiber. However, differently from Raman and Brillouin amplifiers, the phase matching condition is also affected by polarization mode dispersion and the fiber spinning is ineffective to counteract mismatch. So, a limit to gain enhancement is found.

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Fig. 1. Mean gain and STD for initially linear and parallel SOPs for BB-FOPA as a function of $D$. For unspun fibers, the solid curve is the result of the mean equations of [7], while the circles are the result of the numerical integration of Eqs. [1]. The numerical results for spun fibers with spin rate $\nu = 1/p = 0.5, 1, 2$ and 10 turns/m are represented respectively by diamonds, triangles, squares and stars.
Fig. 2. Mean gain and STD for initially linear and parallel SOPs for NB-FOPA as a function of $D$. Symbols are the same of Fig.
Fig. 3. Values of $L_B$ vs. $\nu = 1/p$ satisfying the condition $L_C = L_{NP_R}$ in BB-FOPA: theoretical (solid curves) and numerical (circles) solutions are represented for both linear (black) and circular (gray) input SOPs.

Fig. 4. Evolution of $\langle \cos(\theta) \rangle$ vs. $z$ for in NB-FOPA ($D \simeq 0.02$ ps/$\sqrt{\text{ km}}$). Continuous curves, from top to bottom, refer to spun fibers with decreasing spin rate (10, 2, 0.5 turn/m); the dashed curve refer to unspun fibers. The corresponding mean gain are 26.5, 24.9, 22.9, 16.1 dB, respectively.