Precise baseline correction using statistical criteria in the measurement of characteristics of laser beams

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Abstract. Laser beam measurement using transmissive diffusion screen is the most appropriate method for wide aperture laser beams. Precise measurement requires minimizing the instrumental and methodical errors. Structure of instrumental errors and its reducing were considered in works [1,2]. In this work the method of precise baseline correction is discussed, which allow to reduce methodical error and to avoid clangers caused by highlights and uneven illumination.

1. Introduction

Laser beaming employed for wireless power transmission application requires precise optical system with high accuracy determined characteristics. The optical system can be characterized by measurement of the laser beam along its propagation. Direct laser beam measurement allows to make a correct qualification of system.

Vast majority of laser beam measurement techniques are well described in Refs. [3–8]. Recently an extension for camera-based methods for wide-aperture laser beams measurement was reported [1,2,9]. Precision of verification methods plays an important role in qualification of precise optical system. Thus an analysis of measurement error and an approach for its compensation is necessary. A number of works in this area is available. Several important results related to instrumental and methodical errors were reported in Refs. [1,4,10]. In particular, baseline offset is a factor with a great impact on the measurement error [11]. In this work we report an algorithm for ranging the data which allows to distinguish a data for high repeatability of measurement.

2. Measurement setup for wide-aperture laser beams

Wide-aperture laser beam measurement technique requires experimental setup based on camera and diffusing screen [1,9]. The scheme of the setup is shown on figure 1.
3. Standard methods of baseline correction

In beam measurement baseline offset is mainly caused by background light and noise of CCD sensor. Detailed description of noises and its calibration are given in Refs. [12,13]. Standard methods of baseline offset calculation are described in Refs. [7,8]. Namely, these methods are: noise map of dark frame (1), mean of dark frame (2), approximation method (mean of unlighted pixels) (3), mean plus dispersion (4), statistical method (5), histogram method (6) or corner area mean (7). Any of these methods can be employed depending on purpose specifics. Each method is discussed in more details below.

Baseline offset as noise map of dark frame ($\overline{E^M_B}$) can be calculates using

$$\overline{E^M_B}(i,j) = \frac{1}{n_B} \sum_{k=1}^{n_B} E_B(i,j)_k,$$

(1)

where $n_B$ is a number of dark frames, $E_B(i,j)_k$ - is a dark frame.

Equation for mean of dark frame $\overline{E^D_B}$ is given by:

$$\overline{E^D_B} = \frac{1}{m^2 \cdot n_B} \sum_{k=1}^{n_B} \sum_{i,j} E_B(i,j)_k,$$

(2)

where $m$ - is resolution of squared frame.

Background calculated in accordance to approximation method $\overline{E^A_B}$ (mean of unlighted pixels) is given by

$$\overline{E^A_B} = \frac{1}{\Omega(i,j)} \sum_{i,j} E(i,j) \cdot \Omega(i,j),$$

(3)

where $E(i,j)$ is a frame with a beam, $\Omega(i,j)$ is a map of unlighted area of frame

$$\Omega(i,j) = \begin{cases} 1 & \text{if } (i,j) \in \Omega \\ 0 & \text{if } (i,j) \notin \Omega \end{cases}.$$

Implementation of this method requires correctness verification using

$$\frac{\overline{E^A_B} - \overline{E^D_B \cdot \Omega}}{\sigma_{E^D_B \cdot \Omega}} < t,$$
where \( t = 2..4 \), \( E_{B}^{Dk} \) is a mean of dark frame in corresponding to unlighted area. The mean of dark frame is given by

\[
E_{B}^{Dk} = \frac{1}{\sum_{i,j} \Omega(i,j)} \sum_{i,j} E_{B}(i,j)\Omega(i,j),
\]

\( N = \sum_{i,j} \Omega(i,j) \).

Method of mean plus dispersion can be implemented using

\[
E_{B}^{MD} = E_{B}^{D} + t \cdot \sigma_{B},
\]

where \( \sigma_{B} \) is a dark frame dispersion given by

\[
\sigma_{\phi} = \left[ \frac{1}{m^{2} \cdot n_{B}} \sum_{k} \sum_{i,j} (E_{B}(i,j)_{k} - E_{B}^{D})^{2} \right]^{1/2}.
\]

Background calculation using statistical method is given by

\[
E_{B}^{H} = \frac{1}{\sum_{i,j} S(i,j)} \sum_{i,j} E(i,j)S(i,j),
\]

where \( S(i,j) \) is defined as

\[
S(i,j) = \theta \left[ E_{B}^{D} - E(i,j) \right],
\]

where \( \theta(x) \) is a Heaviside function, which defines unlighted area.

Method of histogram is given by formula (6).

\[
E_{f} = \frac{1}{E_{T}} \sum_{0}^{E_{T}} E_{i} \cdot I(E_{i}) + t \cdot \sigma_{E_{i}},
\]

where \( I(E_{i}) \) is a histogram of intensity distribution of frame with a beam, \( E_{i} \) is a boundary of a column of histogram, \( E_{T} \) is boundary of clusters of noise and signal, \( \sigma_{E_{i}} \) is a dispersion of noise distribution on the histogram, which is defined as

\[
\sigma_{E_{i}} = \frac{1}{E_{T}} \sum_{0}^{E_{T}} I(E_{i}) \cdot (E_{i}^{T} - E_{i})^{2}.
\]

In many cases it is convenient to define baseline offset at specified area on the frame with the beam. Usually square areas located in corners of frame can be used. Calculation is performs similarly to equation (3):

\[
E_{B}^{Q} = \frac{1}{\sum_{i,j} \Omega(i,j)} \sum_{i,j} E(i,j) \cdot \Omega(i,j)
\]

where \( E(i,j) \) is a frame with a beam, \( \Omega(i,j) \) is a specified area (square areas in corners of the frame),

\[
\Omega(i,j) = \begin{cases} 1 & \text{if } (i,j) \in \Omega \\ 0 & \text{if } (i,j) \not\in \Omega \end{cases}
\]

4. **Baseline correction using ranging experimental data with statistical criteria**

For the precision baseline correction it is necessary to redefine integration area by cutting a region symmetrically with regard to the beam as shown on figure 2. The size of region required depends on
beam profile and varies typically from 2 to 10 times the diameter of a beam. For Gaussian beam profile it is recommended to check that the area is at least 5 times greater than beam.

Algorithm of baseline correction includes 8 steps:
1) evaluation of the background
2) determination of a beam center and diameter
3) cutting the region which is symmetric with regard to the beam
4) verification of relative beam size and optional second cutting
5) baseline correction using mean of dark frame
6) noise measurement in corners of frame
7) statistical comparison of noise values
8) second baseline correction

In order to cut a part of the area symmetric relative to the beam it is necessary to evaluate and correct baseline using dark frame mean or corner area mean of background. Next, beam center and diameter should be evaluated. For calculation of the diameter second moment or percentage of power criteria can be used. Double of minimum distance from center of the beam to the edge of frame is required as a square side, which is cut from the frame as shown on fig 2.

![Figure 2](image.png)

**Figure 2.** A scheme of cutting of the frame area part symmetric relative to beam

For achieving adequate results of beam parameters calculation it is important to check relative beam diameter (diameter to size of frame ratio).

In order to avoid errors caused by size of integration a following ratio should be verified:

$$d_{omn} = \frac{d^0 \sigma}{2L_{min}} < \gamma,$$

where $\gamma = 0.1...0.5$ is the limit value of relative diameter, $d^0 \sigma = 2\sqrt{2} \left(\sigma_x^2 + \sigma_y^2\right)^{0.5}$ is beam diameter according to second moment criteria. If the inequality (8) is not satisfied it is necessary to reject the frame from consideration. In case of high frequency of appearance of reject decision it is better to consider the change in experiment condition.

After cutting the frame it is proposed to correct baseline offset by means of implementation of the method of mean of dark frames (Eq. 2) according to

$$E_{i,j}^O = E_{i,j} - E_D^B,$$

where $E_{i,j}$ is experimental power density distribution, $E_D^B$ is calculated baseline offset, $E_{i,j}^O$ is distribution with corrected baseline.
Result of baseline correction need to be verified by measuring of residual background in corner areas, as shown on figure 3. The first step includes background evaluation $E_{B_{res}}$ using equation (7). Then acquired results should be statistically compared with background of dark frames. Comparing is performs using equations (10) - (13):

$$|E_{B_{res}}| < t \frac{\sigma_B}{\sqrt{N}},$$  \hspace{1cm} (10)

$$\chi_{p_1} < \frac{\sigma_{B_{res}}}{\sigma_B} \sqrt{N - 1} < \chi_{p_2},$$  \hspace{1cm} (11)

where dark frame dispersion $\sigma_B^D$ is defined as

$$\sigma_B^D = \left[ m^{-2} \cdot n_B^{-1} \sum_{k=1}^{n_B} \sum_{i,j} \left( E_B(i,j) - \overline{E_B}^D \right)^2 \right]^{0.5}$$  \hspace{1cm} (12)

and residual dispersion $\sigma_{B_{res}}$ is defined as

$$\sigma_{B_{res}} = \left[ \frac{1}{4h^2} \sum_{\Omega} \left( E_{i,j} - \overline{E_{B_{res}}} \right)^2 \right]^{0.5},$$  \hspace{1cm} (13)

Here $N = 4h^2$ is the area of chosen corner region $\Omega$, $h$ is side of corner square, $t$ is quantile of Student’s distribution, $\chi_{p_1}$ and $\chi_{p_2}$ are quantiles of $\chi^2$-distribution, with $p = p_2 - p_1 = 1 - \alpha = 0.9$, $p_2 = 1 - 0.95 = 0.05$, $p_1 = 0.05$.

To obtain precise results of laser beam measurement it is important to satisfy the inequalities 10 and 11. If inequalities are not satisfied it is necessary to decide whether to reject experimental data or to make second baseline correction using $E_{B_{res}}$ as new level of baseline. After second baseline correction it is necessary to use equation (10)-(13) again.

In order to ensure the background values of all 4 corners, their equality and absence of occasional uneven illumination or highlights the residual background $E_{B_{res}}^q$ of each corner should be compared with each other using equation (14):

$$|E_{B_{res}}^q - E_{B_{res}}^w| < t \frac{\sigma_B^D}{h},$$  \hspace{1cm} (14)

where $q,w = 1...4$ is number of corner, $q \neq w$. 

Figure 3. Areas of residual background checking
If inequality (14) is not satisfied it indicates presence of occasional uneven illumination or highlights or some other factor. It is necessary to provide better experimental conditions if inequality (14) is not satisfied frequently.

5. Example of using criteria for blink detection
Data with high dispersion were obtained during verification of beam forming system. Checking the data processing algorithm doesn’t identify any errors. Then statistical comparison of values of residual background in the corners was performed. Due to performed procedure the highlights in frames with beam was detected as shown on figure 4a and 4b. Amplitude of highlight is 100 times less then maximum value of signal. Thus to show highlight the image transformation using (15) was performed.

\[
f(E) = \begin{cases} 
  E_{i,j} & \text{if } E_{i,j} > 1 \\
  255 & \text{if } E_{i,j} < 1
\end{cases}
\] (15)

![Figure 4](image)

(a) (b)

Figure 4. An example of highlights detected on frames with a beam.

After detection of highlight the lens in measurement setup was changed to lens with a cover appropriate for used wavelength of laser beam.

Statistical comparison can detect such small effects, which may cause error of baseline correction, shift of calculated center of the beam and error of diameter measurement.

6. Conclusion
An algorithm for precise baseline correction with selection of experimental data proposed in this work allows to improve the repeatability and traceability of laser beam measurement. The algorithm is highly useful for verification of beam forming systems. Small dispersion of beam diameter measurement can provide appropriate conditions for adjustment of optical system especially in case of system based on off-axis mirror and extrafocal shifts of laser light source.

The algorithm was tested with a setup for wide aperture laser beam measurement. The setup was used for off-axis beam forming system verification and adjustment in laboratory condition.

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