Temperature-scaling behavior of the Hall conductivity for Hg-based superconducting thin films

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The Hall conductivities of HgBa$_2$CaCu$_2$O$_{6+\delta}$ and HgBa$_2$Ca$_2$Cu$_3$O$_{8+\delta}$ thin films are investigated for a magnetic field parallel to the c axis. The mixed-state Hall conductivity for these compounds is well described by $\sigma_{xy} = C_1/H + C_2 + C_3H$. The prefactor $C_1$ shows a temperature dependence of the form $C_1 \propto (1 - t)^n$ near $T_c$, where $t = T/T_c$ is the reduced temperature. Contrary to the previous results, $C_2$ also follows a temperature-scaling behavior similar to that of the coefficient $C_1$. The observed value of $n$, 1.8 $\sim$ 2.3, is comparable to the previously observed values for YBa$_2$Cu$_3$O$_{7-\delta}$ and La$_{2-x}$Sr$_x$CuO$_4$.

I. INTRODUCTION

The Hall effect in the mixed state of high-temperature superconductors (HTS) is one of the most interesting and controversial problems related to vortex dynamics. Many experiments have shown that the Hall anomaly occurs not only in HTS, such as YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO), Bi$_2$Sr$_2$CaCu$_2$O$_8$ (Bi-2212), and Ti$_2$Ba$_2$CaCu$_2$O$_8$ (Tl-2223), but also in conventional superconductors, for example, in the thin-film and the single-crystalline forms of Nb, Nb$_3$Sn, and In-Pb alloys.

So far, two kinds of sign reversals have been observed. The first is a simple, single sign reversal as observed in YBCO, Bi-2212, and La$_{2-x}$Sr$_x$CuO$_4$ (LSCO), and the second is a double sign reversal from positive to negative and then to positive again as the temperature decreases. The distinction between the two is that the single sign reversal is observed for the case of relatively low anisotropy while the double sign reversal is observed in the case of relatively high anisotropy, such as Bi-2212, Ti-2223, and Hg-based compounds. The anisotropy ratio is known to be on the order of $10^4$ for Ti-2212, $10^2$ $\sim$ 10$^3$ for LSCO, and 10 $\sim$ 10$^2$ for YBCO, and the ratio for Hg-based superconductors is between the values for Ti-2212 and YBCO.

Recently, even a third sign reversal was observed in the low-temperature region for heavy-ion-irradiated Hg-based compounds. This observation is quite meaningful because this multiple sign reversal was predicted by Kopnin and depended on the behaviors of the density of states and of the gap of the superconductors. This multiple sign reversal is possible if there are localized or almost localized energy states in the superconducting state.

Just after the detection of the Hall effect in Nb crystals, Bardeen and Stephen derived the flux-flow resistivity and the Hall resistivity due to vortex motion. However, in their theory, the sign of the Hall resistivity is always positive. Quite a few other theories, based on the flux-backflow, two-band, or induced-pinning phenomena, have also been developed to explain the Hall effect in mixed states. However, the origin of the Hall anomaly is still not well understood.

In this paper, we report the magnetic-field dependence of the Hall conductivity in the mixed state of HgBa$_2$CaCu$_2$O$_{6+\delta}$ (Hg-1212) and HgBa$_2$Ca$_2$Cu$_3$O$_{8+\delta}$ (Hg-1223) thin films. As expected, a double sign reversal is observed in these highly anisotropic superconductors. The measured Hall conductivities in the mixed states are better fitted by the form $\sigma_{xy} = C_1/H + C_2 + C_3H$, which is different from the case of the less anisotropic YBCO superconductor, where $C_2$ is negligible. In this Hg-based superconductor, $C_1$ and $C_2$ depend strongly on the temperature, but that is not the case for $C_3$. $C_1$ scales as $\sim (1 - t)^n$, which is partially understood from the temperature dependence of the gap and the coupling constant but this understanding is not rigorous. Here, we claim that $C_2$, which appears only for highly anisotropic materials, scales as $C_2 \sim (1 - t)^{n'}$. The critical exponent $n'$ is 2.0 $\pm$ 0.2 for Hg-1223 and 3.2 $\pm$ 0.1 for Hg-1212. We observe, for the first time to the best of our knowledge, this scaling behavior of $C_2$ for Hg-based superconductors. A similar behavior was previously observed in LSCO, but was not analyzed. $C_3$ for these Hg-based thin films weakly depends on the temperature, which is a different behavior than those observed for YBCO and LSCO.

II. THEORETICAL BACKGROUND

Kopnin et al. and Dorsey obtained the Hall conductivity by using the time-dependent Ginzburg-Landau (TDGL) theory in which the relaxation time of the order parameter was taken to be complex. Kopnin et al. claimed that the negative Hall effect was very much related to the energy derivative of the density of states.
According to their theory, the Hall conductivity can be expressed by two contributions. The first contribution due to the vortex motion is proportional to $1/H$ and is dominant in the low-field region. The second contribution, which originates from quasiparticles, is proportional to $H$.

An analysis of the Hall conductivity, based on the TDGL theory, for the YBCO single crystal was reported by Ginsberg and Manson. In their paper, the Hall conductivity $\sigma_{xy}(H)$ was well explained by the sum of $H$- and $1/H$-dependent parts. However, the behavior of $\sigma_{xy}(H)$ varies on a case-by-case basis. For example, the $H$-dependent part for YBCO is replaced by a field-independent part for TI-2212. In the case of LSCO, the Hall conductivity is expressed as the sum of three terms: a $1/H$-dependent term, an $H$-dependent term, and an $H$-independent term. The temperature dependence of the coefficient of each component in the Hall conductivity was also investigated. The coefficient of the $1/H$ term varies as $(1 - t)^n$, where $t = T/T_c$ is the reduced temperature. $n$ is observed to be 2 for YBCO and 2 $\sim$ 3 for LSCO.

Recently, Kopnin et al. calculated the Hall conductivity based on the kinetic equations and the TDGL theory. Their approach included an additional force due to the kinetic effects of changing the quasiparticle densities in the normal core and in the superconducting state. The total Hall conductivity is given by

$$\sigma_{xy}(H) = \sigma_{xy}^{(L)} + \sigma_{xy}^{(D)} + \sigma_{xy}^{(A)},$$  \hspace{1cm} (1)

where $\sigma_{xy}^{(L)}$ comes from localized excitations in the vortex cores, $\sigma_{xy}^{(D)}$ is from delocalized quasiparticles above the gap, and $\sigma_{xy}^{(A)}$ is from the additional force due to the kinetic effects of charge imbalance relaxation. In the vicinity of $T_c$, this term can be expressed as

$$\sigma_{xy}^{(A)} \sim \frac{1}{H \lambda} \left( \frac{d\nu}{dc} \right) \Delta^2,$$  \hspace{1cm} (2)

where $d\nu/dc$ is the energy derivative of the density of states at the Fermi surface, $\lambda$ is the coupling constant, and $\Delta$ is the superconducting energy gap. $\sigma_{xy}^{(L)}$ and $\sigma_{xy}^{(A)}$ depend on $1/H$, while $\sigma_{xy}^{(D)}$ is proportional to $H$. One important thing we have to notice is that first two terms on the right-hand side of Eq. (1) are always positive. For the dirty case, $\sigma_{xy}^{(L)}$ is very small; hence, it can be neglected near $T_c$. A sign reversal can occur when $\sigma_{xy}^{(A)}$ dominates over $\sigma_{xy}^{(D)}$.

III. EXPERIMENTALS

High-quality Hg-1212 and Hg-1223 thin films were grown by using the pulsed laser deposition and postannealing method. The details are reported elsewhere.

The onset-transition temperatures, $T_c$, are 127 K for Hg-1212 and 132 K for Hg-1223. The sizes of the specimens were 3 mm $\times$ 10 mm $\times$ 1 $\mu$m. A 20-T superconducting magnet system (Oxford Inc.) was used for the dc magnetic fields, and a two-channel nanovoltmeter (HP34420A) was used to measure the Hall resistivity ($\rho_{xy}$) and the longitudinal resistivity ($\rho_{xx}$) by using the standard dc five-probe method. The external magnetic field was applied parallel to the $c$ axis of the thin films, and the transport current density was 200 $\sim$ 250 A/cm$^2$. Both the Hall resistivity and the longitudinal resistivity showed Ohmic behavior, i.e., corresponding to the flux-flow region, at the current used in this study.

IV. RESULTS AND DISCUSSION

We measured the longitudinal resistivities and the Hall resistivities of Hg-1212 and Hg-1223 thin films in the magnetic field region $0 \leq H \leq 18$ T, and the results for Hg-1212 are shown in Fig. 1 while those for Hg-1223 are shown in Fig. 2. Compared to most of previous experiments performed at lower fields, we extended the magnetic field up to 18 T. The motivation for doing this was to check whether the previous analysis of the field dependence of the Hall conductivity based on the TDGL theory was valid even at this high field. Figures 1(a) and 2(a) show the field dependences of $\rho_{xx}(H)$ for various temperatures. $\rho_{xx}(H, T)$ increases monotonically with increasing temperature. In Figures 1(b) and 2(b), $\rho_{xy}(H)$ is plotted and has a nearly linear dependence on the field in the high-field region. The sign of the Hall resistivity in the low-field region near the transition temperature becomes negative, which is opposite to the positive sign of the Hall resistivity for the normal state. The range of the field in which sign reversal is observed for Hg-1223 is narrower than that for Hg-1212. The insets of Figs. 1 and 2 show detailed representations of the low-field region.

The Hall conductivity is typically defined as $\sigma_{xy} \sim \rho_{xy}/\rho_{xx}^2$ by assuming $\rho_{xx} \gg \rho_{xy}$. In Fig. 3, the field dependences of the Hall conductivities of Hg-1212 (Fig. 3(a)) and Hg-1223 (Fig. 3(b)) are shown for various temperatures. Based on the theoretical prediction of Kopnin et al., we analyze $\sigma_{xy}(H)$ by using

$$\sigma_{xy}(H) = \frac{C_1}{H} + C_2 + C_3 H,$$  \hspace{1cm} (3)

which is plotted with solid lines in Fig. 3. Compared to YBCO, the component $C_2$ is added for better fitting. The data are well fitted in the region of $115 K \leq T \leq 125 K$ for Hg-1212 and $125 K \leq T \leq 130 K$ for Hg-1223. In this figure, the downward curves, as approaching zero field, show a sign reversal, but the upward curves do not. If the curve is downward, $C_1/H$ is negative, but if the curve is upward, it is positive.
The temperature dependences of \( C_1 \) and \( C_2 \) for Hg-1212 and Hg-1223 are shown in Fig. 4. Experiment shows that \( C_1 \) scales with temperature near \( T_c \) as
\[
C_1 \sim (1-t)^n,
\]
where \( n \) is 2.3\( \pm \)0.2 for Hg-1212 and 1.8\( \pm \)0.3 for Hg-1223, as shown in the Table I. The scaling form of \( C_1 \) can be partially understood from the temperature dependences of the gap and of the coupling constant in Eq. (3) based on the theoretical prediction by Kopnin. However, this has not yet been proven rigorously. Scaling of \( C_1 \) has also been reported for YBCO and \( \text{LSCO} \) with \( n \) values of 2 for YBCO and 2 \( \sim \) 3 for LSCO. These are not significantly different from those for Hg-1212 and Hg-1223. Compared to several other critical exponents, such as the magnetization scaling and the irreversibility lines, \( n \) does not critically depend on the anisotropy.

As shown in Fig. 4(b), \( C_2 \) steeply increases with decreasing temperature. Therefore, we can extract the following scaling form for \( C_2 \) near \( T_c \):
\[
C_2 \sim (1-t)^{n'},
\]
where \( n' \) is 3.2\( \pm \)0.1 for Hg-1212 and 2.0\( \pm \)0.2 for Hg-1223. Differently from the case of YBCO, in which \( C_2 = 0 \), we find that \( C_2 \) is not negligible for Hg-based superconductors. \( C_2 \) seems to be associated with the anisotropy ratio of the material, because the \( C_2 \) part of the resistivity becomes more explicit for the highly anisotropic superconductors, such as LSCO and TI-1212. Specifically, a similar tendency to that shown in Fig. 4(b) for \( C_2 \) was observed in data previously reported for LSCO, but the temperature-scaling behavior was not determined. Not much information is reported for TI-1212; however, the Hall conductivity is well fitted by \( \sigma_{xy} = C_1/H + C_2 \). As explained before, the origins of the \( 1/H \) and \( H \) dependences can be explained by the TDGL theory or the microscopic theory, but neither of them can explain the scaling behavior of \( C_2 \).

The coefficient \( C_3 \) of the term linear in \( H \) shows a weak temperature dependence in both Hg-1212 and Hg-1223. This is different from the cases of underdoped and slightly overdoped LSCO, in which \( C_3 \) becomes more important for highly anisotropic compounds. \( C_2 \) is observed to follow the same scaling form, but with exponent \( n' \sim 3.2 \) and 2.0 for Hg-1212 and Hg-1223, respectively. These scaling behaviors of \( C_1 \) and \( C_2 \) have not yet been explained theoretically.

V. SUMMARY

We investigate the Hall effects for Hg-1212 and Hg-1223 thin films as functions of the magnetic field up to 18 T. The Hall conductivity in the mixed state is expressed well by \( \sigma_{xy}(H) = C_1/H + C_2 + C_3H \). The coefficient \( C_1 \) scales with temperature as \((1-t)^n\) with \( n \approx 2.3 \) and 1.8 for Hg-1212 and Hg-1223, respectively; these values of \( n \) are comparable to the values observed for YBCO and LSCO. We find that \( C_2 \) is more important for highly anisotropic compounds. \( C_2 \) is observed to follow the same scaling form, but with exponent \( n' \sim 3.2 \) and 2.0 for Hg-1212 and Hg-1223, respectively. These scaling behaviors of \( C_1 \) and \( C_2 \) have not yet been explained theoretically.

ACKNOWLEDGMENTS

This work is supported by the Ministry of Science and Technology of Korea through the Creative Research Initiative Program.

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FIG. 1. The field dependences of (a) the longitudinal resistivity and (b) the Hall resistivity for Hg-1212. The inset represents an enlargement of the low-field region.

FIG. 2. The field dependences of (a) the longitudinal resistivity and (b) the Hall resistivity for Hg-1223. The inset represents an enlargement of the low-field region. The temperature and the field regions in which sign reversal occurs for Hg-1223 are narrower than those in which it occurs for Hg-1212.

FIG. 3. The field dependences of the Hall conductivities of (a) Hg-1212 and (b) Hg-1223 for various temperatures. The solid lines are fitting curves obtained by using Eq. (3) for Hg-1212 and Eq. (1) for Hg-1223. The fitting ranges are $t = 0.898 \sim 0.984$ (114 K $\leq T \leq 125$ K) for Hg-1212 and $t = 0.947 \sim 0.985$ (125 K $\leq T \leq 130$ K) for Hg-1223. The fitting parameters are shown in Table I.
TABLE I. The data for $C_1$, $C_2$ and $C_3$ at $t \approx 0.92$ for various samples, where $\Gamma$ is the anisotropy ratio. For LSCO, the values are for the optimal doping case with $x=0.15$ in $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$. The fitting formula $\sigma_{xy} = C_1/H + C_3 H$ is used for YBCO, $\sigma_{xy} = C_1/H + C_2 + C_3 H$ for LSCO, Hg-1212, and Hg-1223, and $\sigma_{xy} = C_1/H + C_2$ for Tl-2212. The dash (−) symbols indicates ‘not applicable’ or ‘not reported.’

|        | $\Gamma$ | $C_1$ (T/Ωcm) | $C_2$ (1/Ωcm) | $C_3$ (1/TΩcm) | $n$ | $n'$ |
|--------|----------|---------------|---------------|---------------|-----|------|
| YBCO   | 50       | $-1.2 \times 10^4$ | 0             | 250           | 2   | −    |
| LSCO   | 1000–1500| −             | −14           | −80           | 2–3 | −    |
| Hg-1212| −        | −540          | 160           | 31            | 2.3±0.2 | 3.2±0.1 |
| Hg-1223| 2500     | −300          | 210           | 55            | 1.8±0.3 | 2.0±0.2 |
| Tl-2212| $10^4$   | −119          | −             | 0             | −   | −    |
Fig. 1 Temperature-scaling behavior of the Hall conductivity...... (PRB) by Wan-seon Kim et al.
Fig. 2 Temperature-scaling behavior of the Hall conductivity...... (PRB) by Wan-seon Kim et al.
Fig. 3 Temperature-scaling behavior of the Hall conductivity...... (PRB) by Wan-seon Kim et al.
Fig. 4  Temperature-scaling behavior of the Hall conductivity...... (PRB) by Wan-seon Kim et al.