Mathematical modeling of grain mixtures in optimatization tasks of the dump bunker`s kinematic parameters

I L Rogovskii¹, L S Shymko¹, S A Voinash², V A Sokolova³, A A Rzhavtsev³ and A V Andronov³

¹National University of Life and Environmental Sciences of Ukraine, 15, Heroiv Oborony str., Kyiv, 03041, Ukraine
²Federal State Budgetary Educational Institution of Higher Education "Novosibirsk State Agrarian University", 160 Dobrolyubova str., Novosibirsk, 630039, Russian Federation
³Federal State Budgetary Educational Institution of Higher Education “St. Petersburg State Forestry University named after S.M. Kirov”, Institutskiy Per. 5, Saint-Petersburg, 194021, Russian Federation

E-mail: sergey_voi@mail.ru

Abstract. Typically, in the technological processes of agricultural production, machines and equipment are used, where self-loading containers, bunkers, operating on the principle of natural gravitational leakage of friable, grain materials are used for transportation or overload of bulk materials. But the use of such components is justified only for use in the process of harvesting maize ears in the swaths and provided that the start is fed to the collecting hopper of the combine harvester, and after filling it is overloaded in the technological transport by the principle of "dump truck". When trying to go beyond these limitations to increase transportation or overload of bulk materials, there are objective difficulties that arise from the freezing and sticking of the bulk material in the absence of the gravitational leakage phenomenon. As part of the study described in this article, an analytical description of overcoming these difficulties is proposed. It is based on the fact that the analytical equations of motion of a grain mixture, such as quasi-liquids, allow taking into account the bunker's turn around the fastening axis during unloading of the grain mixture. This analytical approach allows us to further substantiate the kinematic parameters of self-propelled bunkers of harvesting machines. The mode of the technological process of unloading the dump of the hopper with grain material with a minimum unloading time is analyzed. This means that the obtained scientific result in the form of the equation of motion of a grain mixture, as quasi-liquids, on a sloping plane is interesting from the theoretical point of view. From the practical point of view, the analytical equations of motion of the grain mixture, such as quasi-liquids, allow us to determine the conditions of use of combine harvesters in the technology of plant production. Thus, the applied aspect of using the obtained scientific result is the possibility of improving the typical technological process of harvesting agricultural crops. This is a prerequisite for the transfer of technological solutions to agro-industrial production.

1. Introduction

Unfortunately, modern combine harvesters have a number of certain shortcomings [1]. These disadvantages are related to ensuring the technological cycles of grains accumulation and its unloading
into technological vehicles [2]. First of all, the complex design of unloading devices [3]. Secondly, insufficient stability of the process of unloading material from bunkers-drives [4]. Thirdly, the low productivity of grain discharging with different physical and mechanical properties [5]. Fourthly, significant (up to 15%) time spending on unloading from bunkers [6]. Fifthly, noticeable damages of grain and plant material [7]. The development of more top-of-the-line structures of harvester’s mechanism is impossible without theoretical justification of rational parameters of working processes and engineering calculations of unloading devices [8]. The laws of mechanics of bulk materials had been studied and formed of scientists. Their contribution to the formation of the scientific principles of the calculation theory has led to the appearance of various designs of unloading mechanisms [9]. However, in view of the still existing imperfection of the process and the widespread distribution and use of an unloading systems for harvester’s variety and a large number of variants of their designs, the scientific problem of justifying the parameters and modes of unloading devices requires further improve.

Free flowing granular materials is a set of disjointed individual particles of more or less the same size. According to Robert Brown and Richards, the term "bulk materials" can mean powders consisting of particles in the size of 0.1 mm (100 μm), granules in size from 0.1 to 3.0 mm, and pieces of fragmented solids larger than 3.0 mm in size [10]. In addition, powders can be divided into very small (0.1...1.0 microns) and very small (1...10 microns) and classified as capable of free-flowing or clinging.

The analysis of studies [11] shows that in the shifting flow of loose grain material, three main causes of the emergence of tension are distinguished: dry friction; impulse movement due to the displacement of particles from one layer to another; impulse motion due to collisions between particles.

In our study, the physical model of bulk materials is based on the principle of continuum [12]. In connection with this, the following assumptions are made:

- the collected material, further we will use definition the bulk material that consists of grains, those grain particles are so small in size compared to the grain hopper can be considered as a continuum [9];
- in accordance with the above, by the law we can use the concept of tension in the bulk materials is similar to the use of this term in the mechanics of continuum [13];
- there is compressive stress σ and the tensile shear stress τ in the body of the bulk material [14];
- the stresses σ and τ can occur simultaneously in one region, which is looked out in the focus of the bulk material [15];
- the particles forming the bulk material, have the properties of elasticity [16];
- particles of bulk material are characterized by strength, which results in the absence of plastic deformations of particles in the presence of specified stresses σ and τ [17].

An analysis of scientific and literary sources covering the theory of bulk materials showed that for dry and lightly loose cargoes (grain), the coefficient of internal friction is determined by the graph of boundary tangential stresses for an ideal bulk material. For those environment, it is equal to the shearing resistance coefficient. The angle of the natural repose of a perfectly bulk material is equal to the angle of internal friction. The determination of the angle of the natural deviation of the grain materials of the main crops allows us to substantiate the design parameters of the dump truck unloading devices of the combine harvesters necessary for the construction of the physical model of the dump bunker. The installation of dump bunker on the modern harvesters creates opportunities for increasing the efficiency of the technological process of harvesting grain crops by increasing the variable productivity of combines and vehicles, reducing the time for unloading grain, avoiding grain damage when unloading it, simplifying the design of the harvesting combine, reducing material consumption, improving the reliability of unloading unit in grain harvesters.

2. The aim and objective of research
The conducted researches were aimed at analyzing the behavior of the quasiridine model for a grain mixture in the technological process of unloading from a dump bunker to a capacity of a technological
vehicle. For further substantiation of the working parameters and design features of the unloading devices combine harvesters. According to the aim, we must follow tasks that should be solved: to analyze the working processes of the combine harvesting devices; to research review of the models of quasi-liquids for the grain mix main description; to design mathematical models of grain mix in problems of kinematic parameters optimization of a combine dump bunker.

3. Materials and methods

In accordance to the our aims and objectives of the study, consider (figure 1) a layer of grain mixture (thickness h), which is limited to the free surface from the top and from the bottom – a fixed plane, inclined at an angle \( \alpha \) to the horizon.

![Figure 1. The calculation scheme of the problem (within the framework of the quasilicate model for grain material).](image)

Should determine, the movement of the grain mixtures from the influence of the gravitational field. Where is \( \ddot{g} \) an acceleration of free fall. We select the stationary lower plane as a plane (x, y), the x-axis is directed along the direction of the grain mix, and the z axis is perpendicular to the plane (x, y). We seek a solution that depends only on the z coordinate. The Navier-Stokes equation with \( \vartheta(z) = \vartheta(\zeta) \) the presence of a gravitation field has the form:

\[
\begin{aligned}
\eta \cdot \frac{d^2 \vartheta}{dz^2} + \rho g \sin \alpha &= 0; \\
\frac{dp}{dz} + \rho g \cos \alpha + 2 \rho f \cdot \frac{d\vartheta}{dt} \cdot \vartheta &= 0
\end{aligned}
\]  

(1)

where: \( \eta \) – is dynamic viscosity \((kg(m \cdot s)^{-1})\); \( \rho \) – is density of grain \(kg/m^3\); \( \vartheta \) – is speed \((m/s)\), \( f \) – is coefficient of external friction, \(2\rho f \cdot \frac{d\vartheta}{dt} \cdot \vartheta \) is the Coriolis force, due to, \( g \gg 2f \cdot \vartheta \frac{d\vartheta}{dt} \). The conditions subsequent \((z = h)\) should be fulfilled for the free surface of the grain mixture:

\[
\sigma_{zz} = -p = -p_0; \quad \sigma_{xz} = \eta \frac{d\vartheta}{dz} = 0; 
\]  

(2)

where: \(\sigma_{zz}, \sigma_{xz}\) – are components of the stress tensor, \(p_0\) – is atmospheric pressure. If \(z = 0\) then should be \(\vartheta = 0\).

The task solution (1), which satisfies these conditions, has the form:

\[
p = p_0 + \rho g \cos \alpha \cdot (h - z); \quad \vartheta = \rho g \sin \alpha \cdot (2\eta)^{-1} \cdot (2h - z). 
\]  

(3)

If we use the expression \( \nu = \eta \cdot \rho^{-1} \), where \(\nu\) is the kinematic grain viscosity \((m^2/s)\), then the amount of the grain mixture (quasi-liquids) unloaded per unit time due to the cross-section of the grain layer (per unit length along the y axis) is determined from the following equation:

\[
\bar{Q} = \rho \cdot \int_0^h \vartheta dz = \rho g \sin \alpha \cdot \frac{1}{2\nu} \int_0^h (2h - z)^2 dz = \frac{\rho g \sin \alpha}{2\nu} \cdot (2h \cdot z^2 \frac{z^2}{2} - \frac{z^3}{3})_0^h = \frac{\rho g \sin \alpha}{2\nu} \cdot (3h^3 - \frac{h^3}{3}) = \rho gh^3 \sin \alpha \cdot \frac{1}{3\nu}, 
\]  

(4)

where: \(\bar{Q}\) – is unloading productivity \((kg/s)\).

We will find dependence \(h(t)\) for the following reasons. Namely, in the technological process of unloading the collected material, at any moment of time – \(t\), the amount of grain mixture contained in the hopper can be determined from the ratio:
\[ V_r = \left[ L_x \cdot L_y \cdot h_0 - g h^3(t) \sin \alpha(t) \cdot L_y \cdot (3v)^{-1} \cdot t \right] = L_x \cdot L_y \cdot h(t); \]  

where: \( h_0 \) – the initial height of the grain mixture in the dump bunker (\( t = 0 \)).

We believe that in the technological process of unloading the grain mix from the dump bunker, \( t \in (0, T) \) inequality is in force:

\[ g \cdot (3v \cdot L_x)^{-1} \cdot h^2(t) \sin \alpha(t) \cdot t \gg 1. \]  

The necessity of the existence of (6) allows us to get rid of certain inconveniences associated with the solution of the cubic equation (5). Then, under condition (6), the solution (5) has the form:

\[ h(t) \equiv \left[ h_0 \cdot 3v \cdot L_x \cdot [g \cdot \sin \alpha(t) \cdot t]^{-1/3} \right] \]  

The law of rotation of the dump bunker \( \alpha(t) \) (7) is optimal. The functional (5) had been minimized, in accordance with the reduction of the total time spent on the process of discharging the hopper, within the "quasi-liquid" model of leakage of grain material under the action of gravity and the movable hopper tray, due to rotation around the axle fixation.

From expression (7) it is easy to find the angular velocity of a dumper bunker rotation:

\[ \omega(t) = \frac{d\alpha}{dt} = 2 \cdot \omega_0 \cdot e^{\omega_0 \cdot t} \cdot \tan \left( \frac{\alpha_0}{2} \right) \left( 1 + e^{\omega_0 \cdot t} \cdot \tan \left( \frac{\alpha_0}{2} \right)^2 \right)^{-1}. \]  

Accordingly, the angular acceleration of the dump bunker is determined by differentiation of the expression (8) with respect to time. For an angular acceleration of a bunker \( \varepsilon(t) \) we have the expression:

\[ \varepsilon(t) = \frac{d\omega}{dt} = 2 \omega_0^2 \cdot \tan \left( \frac{\alpha_0}{2} \right) \cdot e^{\omega_0 \cdot t} \cdot \left[ 1 - e^{2\omega_0 \cdot t} \cdot \tan \left( \frac{\alpha_0}{2} \right)^2 \right] \left[ 1 + e^{2\omega_0 \cdot t} \cdot \tan \left( \frac{\alpha_0}{2} \right)^2 \right]^{-1}. \]  

### 4. Results and discussion

For different values \( \alpha_0 \) and \( \omega_0 \), using the appropriate software tools, dependency charts were constructed: \( \alpha(t) \), \( \omega(t) \), \( \varepsilon(t) \) (figure 2, figure 3), that is characterized the optimal mode of technological process of unloading grain materials from dump bunker (for a minimum discharge time), where is \( \omega_0 \) – is the velocity of a dumper bunker rotation around the axis, \( (rad/s) \); \( \alpha_0 \) – is the initial angle of the bunker (its bottom) relative to the horizon, rad.
The dependencies of the design parameters is shown on figure 4 and figure 5, respectively, there are the height of the lid-tray side – $a$ and the width of the tray – $b$ of the combine harvester dump hopper from $\varphi$ – the angle of natural slope of the particular grain crop’s material at different values of the grain volume weight $\rho$, that is, the theoretical study of the height layer of grain mixture in the tray, in which $v_{av}>0$, when the grain material could freely unloaded.

As can be seen from the constructed graphs (figure 5), the height of the hopper-tray’s sides in the dump-bunker in the range from 200 mm to 500 mm has a straight line, at a given range of values $\varphi$ from $22^\circ$ to $38^\circ$, almost linear dependence on the angle of the natural slope of the bulk agricultural crops and inverse to the bulk weight and width of the transport tray.

The obtained dependencies are important for further substantiation of the design parameters of the dump bins of harvest machines. In particular, they can be used to increase the productivity of the technological process of harvesting crops and to study the unloading of grain materials from the harvester’s dump hopper into the vehicle and to rationally select the structures of bunker designs.
Figure 4. The graph of the lid-tray body side height dependence – \( \alpha \) on \( \varphi \) is the angle of the natural slope at different values of grain volume weight – \( \rho \) and the width of the tray – \( b \).

Figure 5. Schedule of the lid-tray’s height body sides dependence – \( \alpha \) on the width of the lid-tray – \( b \) taking into account the mechanical and technological properties of the main grain crops.
5. Conclusions
In each of the variants of the working parameters of the grain material’s unloading process, the law of bunker’s rotation around the axis is calculated: \( \alpha(t) \), the law of the change in the velocity of this motion is \( \omega(t) \), and also the law of changes in the time of the angular acceleration \( \epsilon(t) \), \( \text{rad/s}^2 \). For \( \alpha(t) \), in each calculation variant it is established that there is an inflection point, where \( \frac{d^2 \alpha}{dt^2} = 0 \). For \( \omega(t) \) in each calculation variant it is established that there is an extremum of the type of maximum, which corresponds to \( \omega_0 = 0.05 \text{ rad/s} \). For \( \epsilon(t) \), in each of the calculation options, there are two extremes of the maximum type a 5.1 \( \text{rad/s}^2 \) and a minimum of \(-5.1 \text{ rad/s}^2 \).

The mathematical model of a grain mixture in the form of a quasiridine using the Navier-Stokes equations is developed. In this case, the optimal structural parameters of the sloping lid-tray are defined: width 2.1 m, length 2.7 m and height of sides 0.4 m for the grain mix (peas, oats, wheat, corn, barley, millet, rape).

References
[1] Kavka M, Mimra M and Kumhála F 2016 Sensitivity analysis of key operating parameters of combine harvesters Research in Agricultural Engineering 62(3) 113-21
[2] Redreev G V, Okunev G A and Voinash S A 2020 Efficiency of usage of transport and technological machines. Lecture Notes in Mechanical Engineering Proceedings of the 5th International Conference on Mechanical Engineering ICIE 625-31
[3] Mirzazadeh A, Abdollahpour S, Mahmoudi A and Ramazani B 2012 Intelligent modeling of material separation in combine harvester’s thrasher by ANN International Journal of Agriculture and Crop Sciences 4(23) 1767-77
[4] Špokas L, Adamčuk V, Bulgakov V and Nodzrovický L 2016 The experimental research of combine harvesters Research in Agricultural Engineering 62(3) 106-12
[5] Calcente A, Fontanini L and Mazzotto F 2013 Coefficients of repair and maintenance costs of self-propelled combine harvesters in Italy Agricultural Engineering International: CIGR Journal 15 141-7
[6] Mimra M, Kavka M and Kumhála F 2017 Risk analysis of the business profitability in agricultural companies using combine harvesters Research in Agricultural Engineering 63 99-105
[7] Artyunin A I and Eliseyev S V 2013 Effect of “crawling” and peculiarities of motion of a rotor with pendular self-balancers Applied Mechanics and Materials 71(4) 38-42
[8] Parkhomenko G G, Voinash S A, Sokolova V A, Krivonogova A S and Rzhavtsev A A 2019 Reducing the negative impact of undercarriage systems and agricultural machinery parts on soils IOP Conference Series: Earth and Environmental Science 316 012049
[9] Rogovskii I, Titova L, Trokhaniak V, Trokhaniak O and Stepanenko S 2019 Experimental study on the process of grain cleaning in a pneumatic microbiocature separator with apparatus camera. Bulletin of the Transilvania University of Brasov Series II: Forestry, Wood Industry, Agricultural Food Engineering 12(61) 117-28
[10] Brown R and Richards A 2018 Engineering principles of agricultural machinery ASABE 84(2) 1120-32
[11] Yousif A L, Dahab H M and El-Ramlawi R H 2013 Crop-machinery management system for field operations and farm machinery selection Journal of Agricultural Biotechnology and Sustainable Development 8 84-90
[12] Markov V A, Sokolova V A, Rzhavtsev A A and Voinash S A 2019 Research of wear resistance of the composite coverings applied by a method of electric contact sintering IOP Conference Series: Materials Science and Engineering International 416 32060
[13] Miu P I, Kutzbach H D 2008 Modelling and simulation of grain threshing and separation in threshing units Computers and Electronics in Agriculture 60(1) 96-104
[14] Amirkhani M, Mayton H S, Netravali A N and Taylor A G 2019 A seed coating delivery system for bio-based biostimulants to enhance plant growth *Sustainability* 11(5304) 1-16
[15] Vlăduț D, Biriş S, Vlăduț V, Cujbescu D, Ungureanu N and Găgeanu I 2018 Experimental researches on the working process of a seedbed preparation equipment for heavy soils *INMATEH Agricultural Engineering* 55(2) 27-34
[16] Hemisa M and Choudhary R 2012 A coupled mathematical model for simultaneous microwave and convective drying of wheat seeds *Biosystems Engineering* 112 202-9
[17] Sirotenko A N, Partko S A and Voinash S A 2020 Research of pneumodrive with energy recovery into additional volume *Lecture Notes in Mechanical Engineering Proceedings of the 5th International Conference on Industrial Engineering ICIE* 1325-33