Exponentially spread dynamical Yukawa couplings from non-perturbative chiral symmetry breaking in the dark sector

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We propose a new paradigm for generating exponentially spread standard model Yukawa couplings from a new $U(1)_F$ gauge symmetry in the dark sector. Chiral symmetry is spontaneously broken among dark fermions that obtain non-vanishing masses from a non-perturbative solution to the mass gap equation. The necessary ingredient for this mechanism to work is the existence of higher derivative terms in the dark $U(1)_F$ theory, or equivalently the existence of Lee-Wick ghosts, that (i) allow for a non-perturbative solution to the mass gap equation in the weak coupling regime of the Abelian theory; (ii) induce exponential dependence of the generated masses on dark fermion $U(1)_F$ quantum numbers. The generated flavor and chiral symmetry breaking in the dark sector is transferred to the standard model Yukawa couplings at one loop level via Higgs portal type scalar messenger fields. The latter carry quantum numbers of squarks and sleptons. A new intriguing phenomenology is predicted that could be potentially tested at the LHC, provided the characteristic mass scale of the messenger sector is accessible at the LHC as is suggested by naturalness arguments.

I. INTRODUCTION

After the discovery [1] of the Higgs boson [2] at the LHC, the only unexplained sector in the standard model (SM) is the flavour sector. While gauge couplings, such as the electric charge, are fundamental constants of nature following from the gauge symmetry principle, the SM Yukawa couplings seem not to be connected to any known local or global symmetry. Instead, they resemble arbitrary dimensionless numbers spanning over 6 orders of magnitude for charged fermions and at least 12 orders of magnitude if the SM neutrinos are Dirac particles. All quark flavour and CP-violation experiments over the last 40 years have confirmed the correctness of the SM description of flavour observables via the Yukawa interactions [3]. Lepton flavour observables may indicate new physics [4], such as the seesaw mechanism [5], but neutrinos can also be Dirac particles, exactly as the quarks and charged leptons. Despite the huge amount of experimental information, constructing the theory of flavour is one of the biggest challenges in modern physics since the physics principles behind it are not known.

There are only two generic classes of attempts to address the huge spread of the SM Yukawa couplings, each consisting of hundreds of concrete models. The first class is based on the Froggat-Nielsen mechanism [6], that introduces $U(1)_F$ flavour symmetric non-renormalizable operators involving a large number of scalar flavon fields that are suppressed by a large cut-off scale $\Lambda$. When the flavon $\phi$ obtains a vacuum expectation value (VEV) $\langle \phi \rangle$, effective Yukawa couplings $Y$ are induced as powers of the expansion parameter $\lambda \sim \langle \phi \rangle / \Lambda$ as $Y \sim \lambda^n$, depending on the particle quantum numbers under $U(1)_F$. Since $\lambda \sim 0.2$ to explain the Cabibbo angle, explaining Yukawa couplings within 6 or 12 orders of magnitude requires constructing operators with very high dimensionality. It is not clear what underlying physics is responsible for those operators and whether this paradigm is testable.

The second attempt is based on confining different fermions in different branes that are located in different places in extra dimensions [7]. The Yukawa couplings are induced due to overlaps of the fermion wave-functions with the Higgs wave-function in extra dimensions. This scenario allows for exponential parameterisation of the measured Yukawa couplings, but does not explain why their values are what they are. Neither of the attempts is completely satisfactory theoretically, and none have any experimental support at present.

In this work we propose a new, predictive paradigm for generating exponentially spread SM Yukawa couplings from gauge quantum numbers in the dark sector. We assume that in addition to the globally $U(1)_N$ flavour symmetric SM there exists a dark sector with complicated internal dynamics manifested today by the existence of dark matter (DM) [8]. The origin of flavour symmetry breaking is the chiral symmetry breaking (ChSB) due to non-perturbative dynamics in the dark sector. We present a concrete model with new $U(1)_F$ gauge symmetry in the dark sector that generates masses for dark fermions (singlets under the SM gauge group) non-perturbatively a la the Nambu-Jona-Lasinio (NJL) mechanism [9] [10]. While the original NJL mechanism operates in the strong coupling regime of the theory, our mechanism operates in the weak coupling regime. This is achieved by assuming the existence of Lee-Wick [11] [12] type higher derivative terms in the dark $U(1)_F$ theory that are equivalent to the existence of negative norm Lee-Wick ghosts [11][15]. Due to the existence of a massless (or light) dark photon, the generated masses depend ex-
ponentially on the $U(1)_F$ quantum numbers [10]. As an additional bonus, when the Lee-Wick theory is generalized to the scalar fields, the Lee-Wick ghosts cancel the quadratic divergences, providing a natural solution to the SM hierarchy problem [17,21]. Thus the dynamics and spectacular new features of our mass generating mechanism rely on the Lee-Wick proposal.

The generated dark fermion mass spectrum is the source of chiral and flavour symmetry breaking. In our proposal this spectrum is transferred to the SM Yukawa couplings at one loop level via Higgs portal type messenger fields, by requiring the spontaneous symmetry breaking (SSB) of the discrete Higgs parity symmetry. Then, the SM Yukawa couplings will be dynamically generated in perturbation theory as finite quantities at one loop. In addition to dark quantum numbers, the messenger fields must also carry SM quantum numbers that are similar to the ones of supersymmetric squarks and sleptons. As a result, we obtain effective and finite SM Yukawa couplings of the schematic form

$$Y^i \sim \exp \left\{ -\frac{\gamma}{\alpha q_f^2} \right\}, \quad (1)$$

where $\alpha$ is the strength of the dark $U(1)_F$ interaction, $q_f$ are the $U(1)_F$ quantum numbers of the fermions $f_i$, $\gamma$ is some universal constant related to the anomalous dimension of the fermion mass operator and $i$ denotes flavour. Then, the non-universality of the quark and lepton Yukawa couplings results from the non-universality of the $U(1)_F$ fermion charges $q_f$ in the dark sector.

By means of Eq. (1), we are able to explain the exponential spread of SM Yukawa couplings with order one generation dependent $U(1)_F$ charges. Interesting sum rules are predicted for the mass spectrum as a consequence of Eq. (1), which are directly related to the $U(1)_F$ charges. Incidentally, as we will show numerically, this framework can actually explain the observed charged fermions mass hierarchies within a few % level accuracy, by using a simple integer sequence for the $U(1)_F$ charges. It can also accommodate the observed patterns of particle mixing.

The proposed framework predicts rich collider phenomenology that can be potentially tested at the LHC and in future colliders. As already stated, the messengers themselves must carry SM quantum numbers similarly to those of squarks and sleptons of supersymmetric theories. If kinematically accessible, those new particles can be produced and discovered at the LHC. Since they couple to the Higgs boson and contribute to the Higgs mass at one loop, naturalness arguments require the messenger mass scale to be below 10 TeV. An exact replica of a rescaled SM fermion spectrum is also expected in the dark sector, as a consequence of the flavor universality of the messenger fields and their couplings to SM fields. The important message to stress is that our scenario is, in principle, directly testable.

We are aware that there is a long way to go towards a more complete understanding of the theory of flavour in this approach. In general, there might be different realizations of both the chiral symmetry breaking mechanism in the dark sector as well as the messenger mechanism presented in this paper. However, we believe that the general features of our proposal are new and could motivate further studies of dynamical flavour breaking mechanism in the presented framework.

The paper is organised as follows. In next section we present details of non-perturbative chiral symmetry breaking in Lee-Wick type models with $U(1)_F$ gauge symmetry. The model for generating the SM Yukawa couplings dynamically is presented in section III. In section IV we present the analysis of the naturalness and vacuum stability bounds. Phenomenology and direct tests of our proposal are discussed in section IV. We conclude in section V.

II. NON-PERTURBATIVE CHSB MECHANISM FROM $U(1)_F$ GAUGE INTERACTION

In the seminal papers [11,12] Lee and Wick proposed a new approach to quantum field theories that prompted the construction of more general theories in which the S-matrix is fully unitary, although the Lagrangian is not Hermitian. This required the introduction of negative norm states which are associated with massive unstable particles. However, despite the presence of an indefinite metric, unitarity can be recovered, provided the negative norm states are massive and have a finite decay width [11,12,15]. As an advantage, ultraviolet divergences may indeed cancel out in the loops due to the indefinite metric of the Hilbert space. In principle, problems related to the microscopic violation of Lorentz invariance, that could also arise due to the presence of an indefinite metric in the Hilbert space [13], can be circumvented too [14]. Indeed, as shown by Cutkosky et al. [15], a relativistic and unitary S-matrix can be defined provided a new prescription for the deformed energy contour in the Feynman integrals is implemented. Although, this prescription is not derived from the first principles of the field theory approach, it is well defined in perturbation theory [12,15]. There is no rigorous proof yet that the Lee-Wick extensions could also work at the non-perturbative level. Nevertheless, there are studies in this direction leading to a consistent non-perturbative approach on the lattice [22,24].

The original model, satisfying all these requirements, was the one proposed by Lee and Wick in the framework of quantum electrodynamics (QED) [12]. In particular, if one replaces the standard photon field $A_\mu$ by a complex gauge field $\phi_\mu = A_\mu + iB_\mu$, where $B_\mu$ is a massive boson field with negative norm, it is possible to remove all infinities in QED from the electromagnetic mass differences between charged particles. This procedure is equivalent to the introduction of a higher (gauge-invariant) derivative term in the Lagrangian of a primary $U(1)_F$ gauge field. Then, the mass of the ghost field
turns out to be proportional to the new physics scale $\Lambda$ connected to the higher derivative term. In order to render charge renormalization finite, new higher derivative terms should be introduced for the fermion fields as well. Although, this procedure shares some similarities with the Pauli-Villars regularization scheme, in the Lee-Wick approach the massive ghosts are not fictitious artefact of the regularization scheme, but are physical objects associated with observable particle resonances.

Recently, the Lee-Wick approach to a finite theory of QED has been reconsidered in view of its generalization to the SM. This approach leads to a new SM theory which is naturally free of quadratic divergencies, thus providing an alternative way to the solution of the hierarchy problem \cite{17–20}.

A new interesting feature of the Lee-Wick theories has been recently noticed in \cite{16}. In particular, if we add to a massless Dirac field $\psi$, minimally coupled to a $U(1)_{F}$ gauge theory, a higher derivative term in the pure gauge sector of the $U(1)_{F}$ Lagrangian $\mathcal{L}$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \gamma^{\mu} D_{\mu} \psi + \frac{1}{\Lambda^2} \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta}$$

it can be shown that this term can trigger spontaneous chiral symmetry breaking at low energy in the weak coupling regime \cite{16}. In the above equation $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$ and $D_{\mu} = \partial_{\mu} + ig A_{\mu}$ are the $U(1)_{F}$ field strength and corresponding covariant derivative respectively. This result has been derived by following the approach of the NJL mechanism \cite{9, 10}. In the NJL approach, the fermion mass term arises as a non-trivial solution of the self-consistent mass gap equation, namely

$$m = \Sigma(p, m)|_{p=m} = \frac{\alpha m}{2\pi} \int_{0}^{1} dx (2 - x) \log \left( \frac{m^2 (1 - x)^2}{\Lambda^2} \right) + \mathcal{O}(m^2/\Lambda^2) \tag{4}$$

In the above equation we neglected terms of order $\mathcal{O}(m^2/\Lambda^2)$ since we are interested to see if there is a non-trivial mass-gap solution corresponding to the case in which $m \ll \Lambda$. As we can see, this equation admits two solutions. One trivial, corresponding to $m = 0$ and related to the perturbative vacuum, and a non-trivial one $m \neq 0$ corresponding to the non-perturbative vacuum. Following the arguments exposed in \cite{9, 10}, it can be shown that the vacuum state associated with the minimum energy, is the one corresponding to the massive solution. Hence, the true vacuum corresponds to the phase of ChSB and is orthogonal to the perturbative vacuum.

Finally, by solving the mass-gap equation in \cite{16} at the leading order, we get the following the results \cite{16},

$$m = \Lambda \exp \left\{ -\frac{2\pi}{3\alpha} + \frac{1}{4} \right\}, \tag{5}$$

where $\Lambda$ is the scale associated with the higher derivative term and $\alpha = g^2/4\pi$ is the effective fine-structure constant. In order to include the resummation of the leading log terms $\alpha^n \log^n(\Lambda/m)$, expected to come from higher order contributions in perturbation theory, $\alpha$ appearing in Eq.\cite{5} should be substituted with the running coupling constant $\alpha(\mu)$ evaluated at the high energy scale $\Lambda$, namely \cite{1}

$$m = \Lambda \exp \left\{ -\frac{2\pi}{3\alpha(\mu)} + \frac{1}{4} \right\} \left( \frac{\Lambda}{\mu} \right)^\frac{\pi}{4}, \tag{6}$$

This relation can be also be expressed as a function of $\alpha(\mu)$ evaluated at an arbitrary renormalization scale $\mu < \Lambda$, as follows

$$m = \Lambda \exp \left\{ -\frac{2\pi}{3\alpha(\mu)} + \frac{1}{4} \right\} \left( \frac{\Lambda}{\mu} \right)^\frac{\pi}{4}, \tag{7}$$

where the $U(1)$ one-loop beta-function has been used. It is easy to check that the r.h.s. of Eq.\cite{7} is independent on $\mu$, consistently with the beta-function evaluated at the leading order in $\alpha$. As we can see from the exponential dependence of the coupling constant $\alpha$, the solution in Eq.\cite{6} is a truly non-perturbative one. However, notice that this solution always exists in the weak coupling regime, $\alpha \ll 1$, since its consistency requires that $\alpha \ll 8\pi/3$. Remarkably, in the original NJL solution, derived by introducing ad hoc chiral symmetric four-fermion contact interaction, a strongly coupled regime was required to break the chiral symmetry. A generalization of this result for the corresponding non-abelian $SU(N)$ interaction can be found in \cite{16}.

\footnote{Notice that, in \cite{16} $\alpha$ has been set at the scale $m$ in the corresponding solution for the mass-gap equation, missing the proper resummation of the leading log terms. This led to an inconsistent condition, namely that this solution was allowed only for $N_f < 2$, with $N_f$ the number of fermions charged under $U(1)_{F}$, which was just a consequence of the incorrect scale at which $\alpha$ inside Eq.\cite{5} was evaluated.}
The main difference between the solution in \([5]-[6]\) and the corresponding one in the NJL model is due to the fact that in our case the fundamental interaction has a \(U(1)_F\) local gauge symmetry. The fact that there exists a non-trivial mass solution is actually peculiar to the \(U(1)_F\) and \(SU(N)\) gauge interactions and does not hold in general. Indeed, in the case in which the chiral invariant interaction is replaced by a massless scalar and pseudoscalar fields coupled to the fermion field in a chiral invariant way, supplied by a higher derivative term in the kinetic part of the scalar Lagrangian as in Eq.\((2)\), but with the fermionic term replaced by a massless scalar and pseudoscalar term of the charge operator \(\hat{Q}\) is still \(U(1)_F\) gauge invariant after the spontaneous ChSB, since the fermion mass matrix is a function of the charge operator \(\hat{Q}\).

Notice that the solution in Eq.\((6)\) implies a relation between \(\alpha(\Lambda)\) and \(\alpha(m)\), which in the case of \(N_f\) fermions with unity charge minimally coupled to \(U(1)_F\), is given by

\[
\alpha(\Lambda) = \alpha(m) \left(1 + \frac{4}{9} N_f \right),
\]

where in deriving the above expression the \(U(1)\) \(\beta\)-function at one-loop has been used and the \(1/4\) factor inside the exponent of Eq.\((6)\) has been neglected in the weak coupling regime \(\alpha(\Lambda) \ll 1\).

Now, it is tempting to speculate whether this ChSB pattern for the fermion masses could be consistent with the observed mass spectrum of quarks and leptons. Let us consider first the charged lepton mass spectrum. Due to the mass hierarchy in Eq.\([10]\), we should expect \(q_\tau > q_\mu > q_e\). For example, we can extract the values of \(\alpha\) and \(\Lambda\) from the measured masses and assumed \(U(1)_F\) charges of the electron and muon, namely

\[
\alpha^{-1} = \frac{3}{2\pi} q_e^2 q_\mu^2 \log \left(\frac{m_e}{m_\mu}\right),
\]

\[
\Lambda = m_e \left(\frac{m_\mu}{m_e}\right) \frac{q_e^2}{q_\mu^2}. 
\]

If we assign, for example, the charges in the lepton sector as a sequence of integer numbers as \(q_e = 4\), \(q_\mu = 5\), \(q_\tau = 6\), we get

\[
\alpha^{-1}(\Lambda) \simeq 113, \quad \Lambda \simeq 1.4 \text{ TeV}. 
\]

This will give the following prediction for the tau lepton mass

\[
m_\tau \simeq 1.9 \text{ GeV}. 
\]

Thus, this charge pattern gives the mass of the \(\tau\) lepton within 7\% accuracy, without invoking any order one coefficient that, in principle, could be present.

Similarly, assuming Dirac neutrino masses with \(q_{\nu_3} = 3\) as the charge of the heaviest light neutrino, and using exactly the same values for the interaction strength and the new physics scale as for the charged leptons given by Eq.\((13)\), we obtain a prediction for the neutrino mass scale

\[
m_{\nu_3} \simeq 5 \text{ eV}. 
\]

Although this value is just a bit too large to be consistent with the direct neutrino mass measurements and with cosmological constraints on the neutrino mass scale, it might not to be totally unrealistic. However, as we will show in the following, the tree-level coupling of \(U(1)_F\) to SM fermions cannot be a realistic model, and this
problem would require a different implementation of the main idea.

Incidentally, for the quark spectrum, we found that a good fit is obtained also following the sequence of 4, 5, 6, integer charges separately for up- and down-quark sectors. The corresponding mass predictions are within 20-40\% accuracy. This indicates that additional corrections of order $O(1)$ are needed in the quark sector, as is the case also for the Frogatt-Nielsen mechanism.

Clearly, this example should not be taken as a realistic model of flavor, since there are several flaws showing that it cannot be phenomenologically acceptable. First of all, an exact $U(1)_F$ gauge interaction cannot be simply coupled at tree-level with SM fermions, unless it is extremely weak, which is not the case here. Notice that in the above example the effective strength at the $\Lambda$ scale of this new interaction coupled to electrons is $q_e^2\alpha(\Lambda) \sim 0.14$, with $q_e = 4$, which is almost twenty times stronger than EM interactions. Second, if we require that quarks and lepton masses arise from the SM Higgs mechanism, as is confirmed by the global fits to the LHC and Tevatron data [25], this extra contribution to their masses would spoil the tree-level relation of SM Yukawa couplings with masses, and eventually the unitarity of the SM.

However, we will see that there is actually a phenomenologically viable way to implement this mechanism to generate the hierarchy of the SM fermion masses. The main idea is to assume that this mechanism is acting on fundamental fermions which belong to a dark sector. These fermions must be singlet under the SM gauge group. Then, the flavor and ChSB of the dark sector is transferred to the SM Yukawa couplings by Higgs portal type messenger fields. We will see that the Yukawa couplings can be actually generated by finite radiative corrections and be proportional to the masses of dark fermions. The latter ones play now the role of a primary source of flavor and ChSB in the SM. In the next section we shall present a model for the messenger sector, and provide predictions for the finite one loop SM Yukawa couplings.

### III. Generation of the SM Yukawa Couplings from Dark Dynamics

In this section we present the Lagrangians for the dark and messenger sectors, following the model building guidelines of the previous section, and compute the induced SM Yukawa couplings. Let us start with the dark sector.

The dark sector is assumed to be composed by Dirac fermions $Q^{U_i, D_i}$ which are similar to replica of the SM fermions, although they are singlet under the SM gauge interactions, where $i, j$ indicate the flavor. We will focus here only on the quark sector, the generalization to the leptonic sector will be straightforward. These fermions are assumed to be massless at tree-level and satisfy an exact dark $U(1)_F$ gauge symmetry. The pure gauge sector will be supplemented by a Lee-Wick term as in Eq.\[2\], in order to dynamically trigger spontaneous ChSB at low energy. Notice that chiral symmetry is assumed to be an exact symmetry of the Lagrangian, that is spontaneously broken by the $U(1)_F$ gauge interaction. This assumption avoids to introduce generic tree-level mass terms for the fermions, that would explicitly break chiral symmetry and eventually spoil the predictions of exponentially spread mass gaps.

Then, the Lagrangian of the dark sector is given by

\[
\mathcal{L}_{DS} = i \sum_i (\overline{Q}^{U_i} D^U_{\mu} \gamma^\mu Q^{U_i} + \overline{Q}^{D_i} D^D_{\mu} \gamma^\mu Q^{D_i}) + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{\Lambda^2} \partial^\mu F_{\mu\alpha} \partial_e F^{e\alpha}, \tag{16}
\]

where $D_\mu = \partial_\mu + i g A_\mu$ is the covariant derivative associated with the $U(1)_F$ gauge field, with $\vec{Q}$ the charge operator acting on the fermion fields $\overline{Q}^{U_i}, \overline{Q}^{D_i}$, and $F_{\mu\alpha}$ is the corresponding $U(1)_F$ field strength tensor. In order to explain the large mass splitting, we assume that the $U(1)_F$ quantum charges are not degenerate, and indicate them with $q_{U_i}, q_{D_i}$ corresponding to the fields $\overline{Q}^{U_i}, \overline{Q}^{D_i}$ respectively. The Lagrangian Eq.\[16\] is invariant under the corresponding $U(1)_F$ gauge transformations given in Eq.\[4\]. Therefore, the flavor symmetry is explicitly broken by the non-universality of $U(1)_F$ quantum charges.

Given the particle content and the $U(1)_F$ gauge interaction in the dark sector, the dark fermions obtain masses as described in the previous section. Those generation dependent masses are exponentially spread according to their gauge quantum numbers. We assume that this is the origin of chiral and flavour symmetry breaking in nature that is communicated to the SM. As we will see, this will necessarily require us to introduce Higgs portal type interactions, mediated by scalar messenger fields.

Basically, the main idea is the following. We assume that the SM structure remains the same at low energies, while the Yukawa couplings should emerge (as finite contributions) at one loop order due to the interaction of the SM fields with the messenger sector. The SM fermions acquire mass by means of the SM Higgs mechanism, but now the splitting between the Yukawa couplings is naturally explained by the large mass differences of fermions in the dark sector. Although there might be other ways to implement the messenger sector for generating finite Yukawa couplings, the requirements of both having finite Yukawa couplings at one loop and renormalizable $SU(2)_L \times U(1)_Y \times U(1)_F$ invariant interactions in the messenger sector, will strongly reduce many other potential choices.

Before entering into the details of the structure of the messenger sector, we would like to discuss some relevant issues. A crucial constraint that must be imposed in order to radiatively generate the Yukawa couplings is to avoid the presence of SM Yukawa couplings at the tree-level. This can by simply achieved by imposing a discrete Higgs parity symmetry, namely $H \rightarrow -H$, where
$H$ stands for the SM Higgs boson doublet under $SU(2)_L$. Indeed, the SM Yukawa couplings are the only interaction terms in the SM in which $H$ appears linearly. Therefore forbidding the SM Yukawa interaction terms is technically natural.

In order to generate (finite) Yukawa couplings at one loop level, this parity symmetry must be broken. Thus we need to introduce a singlet scalar field $S_0$ (under SM gauge interactions) which is coupled to the Higgs field, and that transforms as $S_0 \rightarrow -S_0$ under the Higgs parity transformation $H \rightarrow -H$. This implies that the Yukawa couplings are always proportional to the VEV $\mu = < S_0 >$ of the scalar field associated with the SSB of this discrete symmetry $^2$.

In the case in which the scalar messenger masses are much larger than the dark fermion masses, by using dimensional analysis, the generated Yukawa couplings are expected to be of the form

$$Y_i \sim \frac{M_{Q_i} \mu L}{m^2}, \quad (17)$$

where $M_{Q_i}$ is the mass of the dark fermion, which plays the role of the primary ChSB source, and $m$ is an average mass of the messenger fields. Here $L$ is a dimensionless constant, expected to be $L \ll 1$, which absorbs all loop factors and products of perturbative coupling constants in the messenger sector. While $M_{Q_i}$ and $m$ are masses of dynamical particles, the singlet VEV $\mu$ is an external mass scale. The latter property allows us to have an extra free parameter necessary for adjusting the normalization of $Y_i$ at the right phenomenological scale. Then, we can see that the hierarchy of fermion masses $M_{Q_i}$ in the dark sector is directly translated to the hierarchy of SM Yukawa couplings, provided the scalar messenger sector is heavier than the dark fermion one. A similar conclusion is achieved in the opposite case when $M_{Q_i} \gg m$. In this case the scaling properties of Eq. (17) should be replaced by

$$Y_i \sim \frac{\mu L}{M_{Q_i}}, \quad (18)$$

reversing the hierarchy of the Yukawa couplings as a function of the dark fermion masses. As we will show in the following, the latter realization would be phenomenologically disfavoured since, due to the conservation of the $U(1)_F$ charge, some messenger fields that are charged under the SM gauge group might become stable.

The total tree-level Lagrangian can be expressed as follows

$$\mathcal{L} = \mathcal{L}^Y_{SM} + \mathcal{L}_{MS} + \mathcal{L}_{DS}, \quad (19)$$

where $\mathcal{L}^Y_{SM}$ is the SM Lagrangian with vanishing tree-level Higgs Yukawa couplings, $\mathcal{L}_{MS}$ is the Lagrangian containing the messenger sector with its couplings to the SM and dark fields, and $\mathcal{L}_{DS}$ is the Lagrangian in Eq. (16). The $\mathcal{L}_{MS}$ Lagrangian communicates the ChSB of the dark sector to the SM observable one through the generation of Higgs Yukawa couplings at one loop.

In order to have a Higgs portal type messenger sector, which is invariant under the SM gauge group and under the $U(1)_F$ gauge theory, the minimum set of messenger fields required is the following

- $2N_f$ complex scalar $SU(2)_L$ doublets: $\hat{S}^L_{ui}$ and $\hat{S}^L_{di}$,
- $2N_f$ complex scalar $SU(2)_L$ singlets: $\hat{S}^R_{ui}$ and $\hat{S}^R_{di}$,
- one real $SU(2)_L \times U(1)_Y$ singlet scalar: $S_0$,

where $\hat{S}^L_{ui}, \hat{S}^R_{ui} \in SU(2)_L$ doublets; $\hat{S}^R_{ui}, \hat{S}^R_{di} \in SU(2)_L$ singlets; $S_0 \in U(1)_Y$ singlet scalar. $N_f = 3$ and $i = 1, 2, 3$ stand for the flavor index. It is understood that the messenger and corresponding dark fermion fields associated with the leptonic sector will follow the same pattern as for the quark sector, assuming that neutrinos are of Dirac type. In the following we will discuss only the quark sector, the extension to the leptonic sector will be straightforward.

Notice that the messenger fields $\hat{S}^L_{ui}, \hat{S}^R_{ui}$, $\hat{S}^R_{ui}, \hat{S}^R_{di}$ carry the SM quantum numbers of quarks, where the labels $L, R$ stand for the corresponding chirality structure of the SM fermions. Therefore, they couple both to the electroweak gauge bosons and to the gluons in the standard way. In this respect they resemble the squarks of the supersymmetric extensions of the SM. Analogous conclusions hold in the case of extensions of the messenger field content to the lepton sector.

The quantum numbers of the messenger fields are reported in Table I. Corresponding entries in the columns of $SU(2)_L$ and $SU(3)_C$ refer to the group representations, namely $1/2$ and $3$ for doublets and triplets respectively, while the entries in $U(1)_Y$ and $U(1)_F$ columns stand for the corresponding quantum numbers for hypercharge $Y$ and $U(1)_F$ dark sector $q_f$, respectively. The electromagnetic (EM) quantum charges, in units of the electric charge $e$, are given by the SM relation $Q_{EM} = t_3 + Y/2$, with $t_3$ the corresponding eigenvalue of the $SU(2)_L$ diagonal generator, namely $t_3(S^L_{i1}) = 1/2$, $t_3(S^L_{i2}) = -1/2$, and $t_3(S^R_{i1}) = 0$.

Finally, for the interaction Lagrangian $\mathcal{L}^I_{MS}$ of the mes-

\footnotesize

\begin{tabular}{|c|c|c|c|c|c|}
\hline
Label & $SU(2)_L$ & $SU(3)_C$ & $U(1)_Y$ & $U(1)_F$ & \hline
$\hat{S}^L_{ui}$ & $1/2$ & $3$ & $-1$ & $0$ & \hline
$\hat{S}^R_{ui}$ & $1/2$ & $3$ & $-1$ & $0$ & \hline
$\hat{S}^R_{di}$ & $1/2$ & $3$ & $-1$ & $0$ & \hline
\hline
\end{tabular}

\footnotesize

\textsuperscript{2} The spontaneous breaking of discrete $Z_2$ symmetry may generate cosmological problems because of domain walls. A solution is to break the discrete symmetry explicitly with a small parameter so that the model features are not changed numerically \cite{29}. Here we assume that the $Z_2$ symmetry is explicitly broken by small parameters in the scalar sector of the model, like $\rho S H^2$, with $\rho \ll M_H$, that does not change our results. Alternatively, if the scale of inflation is below $< S_0 >$, the domain walls are diluted by inflation. In the following we assume that the domain wall problem is solved in our model.
In the messenger sector with quarks and SM Higgs boson we have

\[
\mathcal{L}^I_{MS} = g_L \left( \sum_{i=1}^{N_f} \left[ \bar{q}_L^i Q_R^i \right] S_{L}^{u_i} + \sum_{i=1}^{N_f} \left[ \bar{q}_L^i Q_R^i \right] \tilde{S}_{L}^{D_i} \right) + \\
+ g_R \left( \sum_{i=1}^{N_f} \left[ \bar{D}_R^i Q_L^i \right] S_{R}^{u_i} + \sum_{i=1}^{N_f} \left[ \bar{D}_R^i Q_L^i \right] \tilde{S}_{R}^{D_i} \right) + \\
+ \lambda_S S_0 \left( \bar{H}^I S_{L}^{u_i} S_R^{D_i} + H^I S_{L}^{u_i} \tilde{S}_{R}^{D_i} \right) + h.c.,
\]

(20)

where contractions with color indices are understood and \(S_0\) is a real singlet scalar field. Here \(q_L^i\) and \(Q_R^i\), \(D_R^i\), \(D_R^i\), indicate the SM fermion fields, and \(S\) is the SM Higgs doublet, with \(H = i\sigma_2 H^*\). We do not report here the subdominant scalar terms needed to avoid the domain wall problem, see the discussion above. We also do not report the expression for the interaction Lagrangian of the messenger scalar fields with the SM gauge bosons since the corresponding Lagrangian follows from the universal structure of gauge interactions. Furthermore, the messenger fields are also charged under \(U(1)_F\) and carry the same \(U(1)_F\) charges as the corresponding dark fermions.

In principle, there is no reason why the masses of the up and down-scalar messenger fields should be flavor independent. However, if one assumes that the only source of flavor breaking comes from the quantum charge sector, then imposing the flavor universality for the free Lagrangians in the up- and down-scalar sector separately turns out to be a minimal and natural choice. Unavoidably, the flavor breaking contained in the gauge sector is then communicated to the scalar sector at one loop level. However, since this effect will be suppressed by \(U(1)_F\) gauge coupling and loop effects, the flavor dependence in the messenger mass-sector should be considered as a small deviation from flavor universality. We will neglect this small effect in our analysis and assume, as a minimal choice, four flavor-universal free mass parameters \(m_{U_L}, m_{U_R}, m_{D_L},\) and \(m_{D_R}\), corresponding to the mass terms of the \(S_{L}^{u_i}, S_{R}^{u_i}, S_{L}^{D_i},\) and \(S_{R}^{D_i}\) fields, respectively.

As explained before, the following discrete symmetry \(H \rightarrow -H\) and \(S_0 \rightarrow -S_0\) must be imposed to the whole Lagrangian in order to avoid tree level Yukawa couplings. However, in order to radiatively generate the SM Yukawa couplings we have to require that the singlet scalar field \(S_0\) acquires a VEV, namely \(\langle S_0 \rangle = \mu\). There is no problem with the unwanted massless Goldstone boson in this case, since this is a discrete symmetry.

In Fig. 2 we show the relevant Feynman diagrams which contribute to the SM Yukawa couplings at one loop order. These diagrams are finite at one loop order, and in general at any order in perturbation theory, due to the structure of the renormalizable interaction in Eq. (20) and the SSB of the discrete parity symmetry \(H \rightarrow -H\) and \(S_0 \rightarrow -S_0\).

By computing the Feynman diagrams in Fig. 2 the SM Yukawa couplings at zero transferred momenta can be extracted by using the standard procedure as follows. We match the results of the Feynman diagrams in Fig. 2 where the external momenta are set to zero, with the corresponding effective Yukawa operators evaluated at \(q^2 = 0\). In the calculation of the one loop diagrams we assume for simplicity that the masses of the scalar fields running in the loop are flavor independent and their masses \(m\) are degenerate between the left and right scalars. Finally, by following the above procedure, we get

\[
Y^{u_i} = \frac{\lambda_S g_L g_R \mu M_{Q^{u_i}}}{16\pi^2 \bar{m}^2} C_0(x_i),
\]

(21)

and analogously for the \(Y^{d_i}\) sector, where \(x_i = M_{Q^{u_i}}^2 / \bar{m}^2\) and \(M_{Q^{u_i}} = \Lambda \exp \left( -\frac{2\pi}{\alpha_M^2} \right)\), where \(\alpha\) stands for the fine structure constant of \(U(1)_F\) gauge interaction. Here the
function \( C_0(x) \) is defined as
\[
C_0(x) = \frac{1 - x (1 - \log x)}{(1 - x)^2},
\]
where \( C_0(1) = 1/2 \), while for small \( x \ll 1 \) it can be approximated as \( C_0(x) \approx 1 + (1 + \log(x))x + O(x) \). In the opposite limit of large \( x \gg 1 \), one has \( C_0(x) \sim 1/x \).

Therefore, from these results we can see that, as expected from the decoupling theorem, in the limit of \( \Lambda \to \infty \) all the Yukawa couplings tend to zero.

Finally, after EWSB, the SM fermions get the same mass pattern as in Eq.\((10)\), as in the example discussed in section II, namely
\[
m_i = \Lambda_{\text{eff}} \exp\left(-\frac{2\pi}{3\alpha q_i}\right)
\]
where now \( q_i \) is the corresponding \( U(1)_F \) charge of the corresponding dark fermion partner, and the \( \Lambda_{\text{eff}} \) is related to the Lee-Wick scale \( \Lambda \) of \( U(1)_F \) by
\[
\Lambda_{\text{eff}} \sim \left(\frac{\mu \Lambda}{\bar{m}}\right) \frac{\lambda_{\text{SUGRA}} C(x_i)}{16\pi^2},
\]
with \( \mu \) the Higgs vev, \( \bar{m} \) an average mass of the associated messengers fields and \( g_{L,R} \) the corresponding messenger couplings to SM left-handed and right-handed fermions. Since there is no reason why the messenger scalar fields in the lepton and quark sectors should have the same mass and couplings, it is possible to choose different masses and couplings \( g_{L,R} \) for the messengers fields in the lepton and quark sector, in order to set the appropriate scales \( \Lambda_{\text{eff}} \) for the lepton and quark sectors.

In the case in which the messenger sector is flavor independent one can obtain interesting sum rules that connect the various Yukawa couplings in the up- or down-quark sectors. By means of Eqs.\((10), (21)\) we get
\[
Y_{ij} = Y_{vi} \exp\left\{\frac{2\pi}{3\alpha q_i} \frac{q^2_{ij} - q^2_{vi}}{q^2_{ij}} \right\} \frac{C_0(x_j)}{C_0(x_i)},
\]
Analogous results hold for the down-sector Yukawa couplings \( Y^{\nu\nu}_i \), with \( q_{\nu_i} \), charges replaced by the corresponding \( q_{\nu_i} \), ones. Clearly, if the messenger sector is flavor universal in both up- and down sector, the above relations in Eq.\((25)\) can be generalized to mix the up- and down-sector Yukawa couplings. As explained above, in order to avoid stable heavy charged particles in the spectrum, messenger masses should be always heavier than the corresponding dark fermion ones. In the case of a large mass gap between the messenger and dark fermion sector, the last term multiplying the exponential in Eq.\((25)\) can be well approximated by \( C_0(x_j)/C_0(x_i) \sim 1 \). In the following we will restrict our phenomenological analysis to this particular scenario.

The next issue to address is the origin of flavour mixing. In our framework the generated Yukawa couplings are proportional to the fermion masses in the dark sector. There are two logical possibilities, either the observed flavour mixings are present already among the dark fermions or, alternatively, they are generated by the radiative transfer mechanism. The first possibility requires dynamical breaking of the dark \( U(1)_F \) symmetry since the charge conservation requires the mixing to be either zero (different \( U(1)_F \) charges for different generations) or maximal (same \( U(1)_F \) charges for different generations). As long as the dark photon acquires a small mass, much smaller than the generated fermion masses, the exponential dependence of masses on the \( U(1)_F \) quantum numbers is not spoiled \([10]\). However, such a dynamical breaking requires an additional mechanism and we do not consider it here. Instead, we assume that the small Cabibbo-Kobayashi-Maskawa (CKM) type mixings are due to a mismatch between the dark sector masses and the SM masses. Thus, they are generated by the scalars mediating the dark fermion masses to the SM sector. To achieve that, we have to relax the assumption of flavour universality of the messenger sector. However, due to the smallness of CKM mixing angles this is just a small mismatch effect originating from the flavour non-diagonal messenger couplings and from the messenger mass non-universality. Thus the CKM matrix can always been accommodated in our mechanism.

Finally, we would like to comment about the phenomenological implications of the spontaneous ChSB in the dark sector. In the case of degenerate \( U(1)_F \) charges, there is a global symmetry of the Lagrangian in the dark sector which corresponds to \( U(N)_L \times U(N)_R \). After the spontaneous ChSB, induced by the higher derivative term in the \( U(1)_F \) gauge sector, this symmetry breaks down to an exact \( U(N)_V \) global symmetry. According to the Nambu-Goldstone theorem, there should then appear in the spectrum \( N^2 \) massless Nambu-Goldstone pseudoscalar bosons, in this case would correspond to the condensates of elementary dark fermions. Some of these composite states would be also charged under the \( U(1)_F \) gauge group. On the other hand, if the \( U(1)_F \) charges are all non-degenerate, the \( U(1)_F \) gauge interaction term will play the role of an explicit \( SU(N)_L \times SU(N)_R \) breaking term. Then, according to general arguments, we expect that of the \( N^2 \) Nambu-Goldstone particles of the degenerate case, only one will remain massless, while the other \( N^2 - 1 \) ones will acquire a mass term proportional to the splitting of the \( U(1)_F \) charges. Clearly, a rigorous analysis is mandatory in order confirm these naive expectations, and this might be the subject for future investigations.

IV. NATURALNESS AND VACUUM STABILITY BOUNDS

The radiative generation of the Yukawa couplings of light quarks has already been extensively considered in the literature in the context of supersymmetry \([27, 31]\).
In this framework the radiative generation of the top quark mass was considered to be impossible because the supersymmetry breaking scale was believed to be below 1 TeV, and generating a particle mass of 173 GeV at one loop seems impossible. As already discussed above, in our case we can choose the singlet VEV \( \mu \) and the mass scales large enough to overcome the smallness of the loop factor. Thus all SM Yukawa couplings can be generated with our mechanism.

However, large values of \( \mu \), required to generate the top-quark Yukawa coupling, can in principle spoil naturalness in the Higgs sector. This is due to the fact that the trilinear coupling of the Higgs and messenger sector can induce one loop contributions to the Higgs mass square \( \delta m^2_H \), which is of order

\[ \delta m^2_H \sim \frac{\lambda^2 \mu^2}{16\pi^2}. \]

(26)

In this expression we have neglected the loop function since we are just interested in a rough estimate of the contribution to the Higgs boson mass. By using Eqs. (26) and (21), and approximating the top Yukawa coupling by \( Y^t \sim 1 \), the one loop radiative contribution to the Higgs mass square \( \delta m^2_H \) is given by

\[ \delta m^2_H \sim \frac{16\pi^2 m^2}{(g_L g_R)^2 x_L C_0(x_L)}, \]

(27)

where \( x_L = M_{Q_L}^2/\bar{m}^2 \). From these results we can see that in order to avoid large fine-tuning in the Higgs sector, large couplings of \( g_L \) and \( g_R \) are needed. Contrary to the radiative generation of Yukawa couplings in SUSY models, in our framework the messenger couplings to the Higgs boson are not constrained by any symmetry, and we can allow the \( g_L, g_R \) couplings to be large. If we assume that the mass of the dark fermion partner of the top-quark is of the same order as the messenger mass scale \( \bar{m} \), namely \( x_L \sim 1 \), and assume \( g_L, g_R \sim 1 \), we get

\[ \delta m^2_H \sim 4 \times 10^4 \left( \frac{\bar{m}}{\text{TeV}} \right)^2 m_H^2, \]

(28)

for the Higgs mass \( m_H = 126 \) GeV. This implies that for the messenger mass scale of order \( \bar{m} \sim 1 \) TeV, a \( 10^{-4} \) fine-tuning is required in the Higgs sector.

A potential solution to the fine tuning problem might be provided by extending the Lee-Wick ghosts to the SM fields, including the Higgs field, which is actually one of the main motivations for this proposal [17,20]. Another possibility is to consider the supersymmetric extension of our scenario, that would necessarily require also the supersymmetric extension of the dark sector.

Now we derive the lower bounds on the dark fermion masses by using vacuum stability bounds in the messenger scalar sector. In order to simplify the analysis, we assume the messenger masses to be degenerate, that is \( m_{S_L} \sim m_{S_R} = \bar{m} \). After electroweak symmetry breaking the interaction term \( \lambda S_H S_L S_R \) generates a mixing term in the mass-square matrix of the \( S_L \) and \( S_R \) scalar fields, which is equal to \( \lambda S_H S_L S_R \). If this mixing term is too large, one of the eigenvalues of the scalar mass-square matrix becomes negative and tachyons are generated, inducing vacuum instability. Then, in order to avoid tachyons in the messenger sector, we must require that

\[ \lambda S_H S_L S_R < \bar{m}^2, \]

(29)

where \( v \) is the VEV of the Higgs field.

The SM fermion masses are generated as in the SM after the electroweak symmetry breaking. We get from Eq. (21)

\[ \frac{m_i}{v} = \frac{L \lambda i M_{Q_i} C_0(x_i)}{\bar{m}^2}. \]

(30)

Substituting Eq. (31) into Eq. (29) we get

\[ M_{Q_i} > \frac{m_i \bar{m}^2}{v L M_{Q_i} C_0(x_i)}, \]

(32)

that provides a lower bound on the dark fermion mass in terms of the corresponding SM fermion partner. Notice that, in the case of heavy messengers \( x_i \ll 1 \), the lower bound depends only on the fermion masses and coupling constants.

Some comments about relation Eq. (32) are in order. In the case in which the dark fermion associated with the top-quark has a mass of the same order as the messenger ones, namely \( x_i \sim 1 \), we get

\[ M_{Q_i} \gtrsim \left( \frac{55}{g_L g_R} \right) \text{TeV}. \]

(33)

In the case of large couplings \( g_L, g_R \sim 1 \), but still perturbative, the heaviest dark fermion should have a mass not smaller than \( 55 \) TeV to avoid problems with the vacuum stability. On the other hand, for the lightest quarks, assuming their mass to be of order \( 10 \) MeV, we get

\[ M_{Q_i} \gtrsim \left( \frac{1.6}{g_L g_R} \right) \text{GeV}. \]

(34)

Clearly, if the masses of the dark fermions are just a rescaling of the the SM fermion masses, as suggested by our scenario, the bound Eq. (34) automatically holds once the bound Eq. (33) is satisfied.

These results show that the lightest dark fermions could be relatively light for strongly coupled messenger fields and can be produced at the LHC in the decays of (heavy) messenger fields. However, in order for the
messengers to be kinematically accessible at collider experiments, the bound Eq. (33) should be relaxed. It is possible that the messenger masses for the quarks and leptons are different. For the lepton partners the equivalent bound is rescaled by the ratio of Yukawa couplings squared, allowing them to be kinematically reachable at colliders. We will discuss the phenomenological implications of this scenario at the LHC in the next section.

Finally, we would like to comment on the fact that this scenario can easily pass all the tests from electroweak precision observables and flavor physics. For instance, due to the fact that the messenger fields are charged under the $SU(1)_L \times U(1)_Y$ gauge group, they can contribute at one loop level to the $\rho$ parameter. However, since the messenger masses may be as large as 50 TeV and also degenerate, we expect them not to contribute significantly to the $\rho$ parameter and to the other electroweak precision observables. The same conclusions hold for the contribution to rare processes in flavor physics induced at one loop. Since the messenger fields enter in flavor-changing neutral current (FCNC) loops, this will induce a tiny contribution to the relevant FCNC operators, being suppressed by a typical scale which should be associated with the messenger masses. However, due to the fact that $g_{L,R}$ might be large, an accurate analysis of these new contributions to the FCNC sector is needed in order to assess this issue more precisely.

V. PHENOMENOLOGY AND DIRECT TESTS

The dark sector of our theory contains an unbroken $U(1)_F$ gauge group. Thus there must exist massless dark photons that may have cosmological implications if the dark matter of the Universe is charged under this gauge group \[32\]. Recently there has been a revival of interest to this possibility \[33\]. The dark matter self interactions may solve problems of small scale structure formation that seem to deviate from the simple N-body simulation results. Spectacular signatures of this scenario include formation of dark discs of galaxies \[34\] that can be observable. If the dark photon is exactly massless, there is no kinetic mixing with the electromagnetic photon – there are two orthogonal states that must be identified accordingly. In our scenario the natural candidate for dark matter is the lightest dark fermion that is charged under the $U(1)_F$ gauge group. If, however, the dark matter is neutral under $U(1)_F$, the dark photons are very difficult to observe in laboratory experiments.

While probing the dark sector particles at colliders is a very challenging task, neutrino physics may offer unexpected possibilities. Namely, some of the dark fermions, for example the ones corresponding to the lightest SM neutrinos, may be light enough to play the role of an additional sterile neutrino. The existence of $O(10)$ eV sterile neutrinos may be hinted by the LSND \[35\] and Mini-BooNE \[36\] experiments. To mix the dark and the SM fields, the dark gauge symmetry must be broken.

Thus the phenomenology of our scenario may also affect neutrino physics. However, it is not yet clear if this simple scheme, which would assume Dirac neutrinos, could explain the correct scales for the neutrino masses and mixing. A more close inspection of this model in the neutrino sector is necessary and this is beyond the purpose of the present paper. Perhaps extended versions could be necessary in the neutrino sector to make this model more realistic.

However, by far the most promising way to test our model is to search for direct or indirect effect of this scenario at the LHC and in future colliders. As already stated, the messengers themselves must carry SM quantum numbers similarly to the squarks and sleptons of supersymmetric theories. If kinematically accessible, those new particles can be produced and discovered at the LHC. For example, the coloured messengers can be pair produced at the LHC by the gluon fusion mechanism

$$gg \rightarrow SS^\dagger,$$  \hspace{1cm} (35)

or by the quark fusion mechanism

$$q\bar{q} \rightarrow SS^\dagger,$$  \hspace{1cm} (36)

where $S$ stands for a generic scalar messenger. The latter process can be enhanced by the potentially large $g_{L,R}$ couplings. For colourless messengers only the process \[36\] can take place, mediated by the SM gauge bosons. This phenomenology would somewhat resemble the one of supersymmetric squarks and sleptons. However, there are many differences between those scenarios. While in supersymmetric theories searches for squarks assume that they are produced in gluino cascade decays, our model does not contain coloured fermions and the scalars must be produced directly. This implies smaller cross sections and lower mass reach than in supersymmetry, especially for colourless particles like the messengers of leptons. Moreover, the masses of messenger fields are expected to be large, as follows from the bound Eq. (33) coming from the very large top Yukawa coupling. Therefore, it is likely that the quark messenger cannot be produced on-shell at the LHC (unless the flavour universality assumption is relaxed that we do not consider in this work). However, the lepton messengers can be much lighter and accessible at the LHC. Once produced, the colourless scalars decay to SM leptons and to dark fermions. The lightest dark fermion is stable, manifested at collider experiments by the signature of missing energy. Thus the experimental signature of our scenario is a pair of SM leptons and large missing energy. The latter can be used to trigger on the events at the LHC. Thus the LHC searches for supersymmetry could also be used to test our flavour model.

Due to the direct coupling of the messenger sector with the Higgs boson, effects on the $H \rightarrow \gamma\gamma$ and gluon-decay amplitude $H \rightarrow gg$ can affect the present measurements of the 126 GeV Higgs-like resonance observed at the LHC. We study how the radiative Higgs decay rates can be used...
to set direct bounds on the masses of particles in the messenger and dark fermion sector. Since the present measurements are in good agreement with SM predictions, one can use these results to set indirect lower bounds on the new particle spectra. In particular, the messenger fields could contribute to the $H \rightarrow \gamma\gamma$ amplitude at one loop, where inside the loop the $S_{Q}^{(1),i}$ and $S_{R}^{(1),i}$ fields are circulating, together with their potential counterparts in the leptonic sector. By dimensional analysis, we estimate that this contribution is proportional to

$$A(H \rightarrow \gamma\gamma) = \frac{\lambda_{3H}\mu\alpha}{m^{2}4\pi}L_{F}\hat{F}_{\mu\nu},$$

where $m$ is the average messenger mass, $L_{F}$ is the loop function, which is expected to be of order $O(1)$, and $\hat{F}_{\mu\nu}$ is the Fourier transform of the EM field strength. Now, if we extract the $\lambda$ term from the requirement of generating the top Yukawa coupling at the right scale by using Eq. (21), we get that the amplitude for $H\gamma\gamma$ will be of order

$$A(H \rightarrow \gamma\gamma) \sim \frac{4\pi\alpha}{g_{L}\mu_{R}M_{Q}, C_{0}(x_{l})}L_{F}\hat{F}_{\mu\nu}\hat{F}_{\mu\nu},$$

while the corresponding SM contribution is proportional to

$$A(H \rightarrow \gamma\gamma)_{SM} \sim \frac{\alpha g}{4\pi m_{W}}L_{F}^{SM}\hat{F}_{\mu\nu}\hat{F}_{\mu\nu},$$

where $m_{W}$ is the W-boson mass, $g$ the weak coupling, and $L_{F}^{SM}$ the corresponding SM loop function, which is a term of order $O(1)$. Since the vacuum stability bounds are restrictive, see Eqs. (33), the messenger contribution to $H \rightarrow \gamma\gamma$ is expected to be suppressed with respect to the SM one. The same conclusions hold for the new contribution to the Higgs production mechanism by gluon-gluon fusion. Notice, that these estimates are based on pure dimensional analysis, and the precise calculation of the bounds from the Higgs boson analysis at the LHC would require a dedicated study of these effects that goes beyond the purpose of the present paper.

VI. CONCLUSIONS

We have proposed a new paradigm for the dynamical generation of exponentially spread SM Yukawa couplings. The new idea we advertise is that exponentially spread fermion masses are generated non-perturbatively in the dark sector. The resulting chiral and flavour symmetry breaking is transferred to the SM via the messenger fields presented in Table 1. The important ingredient for our mechanism to work is the existence of Lee-Wick negative norm ghosts in the dark sector, allowing the NJL type mechanism to be operative in the weak coupling regime of the theory and producing an exponential mass spectrum. The interaction that generates the non-perturbative effect is the unbroken dark $U(1)_{F}$ gauge interaction. Since the Abelian group can have different integer or fractional charges for different generations, flavour symmetries are broken exponentially by the $U(1)_{F}$ charges. As a result, our mechanism offers a natural explanation to the observed SM fermion mass spectrum. If the light neutrinos will turn out to be Dirac particles, explaining the extreme smallness of their Yukawa couplings becomes natural in our framework.

We have presented an explicit model of flavour achieving those tasks. It contains a scalar messenger sector consisting of particles with the SM quark and lepton quantum numbers, thus resembling the supersymmetric squark and slepton sector. We have shown that, due to the large top Yukawa coupling, quark messengers must likely be very heavy. However, the lepton messengers can be orders of magnitude lighter. If kinematically accessible, those particles can be discovered at the LHC, offering direct tests of our model.

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