Excitation of zonal flow by intermediate-scale toroidal electron temperature gradient turbulence

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Received 24 November 2020, revised 11 April 2021
Accepted for publication 14 April 2021
Published 4 May 2021

Abstract

On the basis of gyrokinetic theory, we derive nonlinear equations for the zonal flow (ZF) generation in intermediate-scale electron temperature gradient (ETG) turbulence (with wavelength much shorter than the ion Larmor radius but much longer than the electron Larmor radius) in nonuniform tokamak plasmas. Both the spontaneous and forced generation of ZFs are kept on the same footing. The resultant Schrödinger equation for the ETG amplitude is characterized by a Navier–Stokes type nonlinearity, which is typically stronger than the Hasegawa–Mima type nonlinearity resulting from the fluid approximation. The physics underlying the three stages of ZF generation process is clarified, and the role of parallel mode structure decoupling is discussed. It is found that ZFs can be more easily excited in the intermediate-scale ETG turbulence than in the short wavelength regime.

Keywords: gyrokinetics, drift waves, microinstabilities, tokamaks

1. Introduction

Understanding the anomalous electron energy transport is a 'great unsolved problem of tokamak transport physics' [1]. It is of particular importance for future burning plasmas such as ITER, because the energetic fusion alpha particles will mostly heat electrons. Electron temperature gradient (ETG) driven turbulence has been proposed as a prominent candidate for the anomalous electron energy transport in magnetically confined plasmas [2, 3]. Theoretically, it has been shown that, for the ETG turbulence in fluid electron approximation, the zonal flow (ZF) generation is much weaker than its ion temperature gradient (ITG) mode counterpart, and, thereby, a quasi-saturated state characterized by radially extended streamers is gradually reached via the inverse toroidal cascading [4–6]. However, as the ETG toroidal spectrum downshifts, the large-box-size, long-time-scale gyrokinetic simulations report that intermediate-scale ZFs with wavelength much shorter than the ion Larmor radius but much longer than the electron Larmor radius can play a significant role in regulating the ETG turbulence after the quasi-saturated phase, and ultimately become dominant [7–10]. Since the long-time saturated state is more experimentally relevant, a crucial question then naturally arises as to whether the ETG-ZF interaction is more effective in intermediate-scale ETG turbulence. The main motivation of this paper, therefore, is to understand the ZF generation in the intermediate-scale ETG turbulence, and qualitatively illuminate the fundamental difference of the underlying physics with that of short wavelength ETG turbulence. Unlike most treatment available in the literature, here we present a gyrokinetic analysis addressing the spontaneous and forced ZF generation on the same footing. Our gyrokinetic analysis takes the plasma nonuniformity and continuous radial spectrum into account, thereby yielding a nonlinear Schrödinger equation (NLSE) with an integral nonlinearity for the ETG amplitude and emphasizing the important role of parallel mode structure decoupling in ZF generation. The results indicate that the ETG nonlinearity is of a Navier–Stokes form at intermediate-scales,
which is generally much stronger than the Hasegawa–Mima type nonlinearity in fluid limit [5, 6], and, thus, a significant ZF generation is expected for intermediate-scale toroidal ETC turbulence. As a consequence, the ETC turbulence will finally be dominated by intermediate-scale isotropic turbulent eddies. These findings carry the important implication that the ETC turbulence is truly kinetic and multi-scale in nature. Therefore, it becomes essential to properly account for the ZF physics in electron-scale transport models, and perform long-time, multi-scale kinetic simulations for realistic comparisons with experimental observations.

The rest of the paper is organized as follows. In section 2, the gyrokinetic model is presented, while the linear ETC model is briefly reviewed in section 3. The equations describing the ETC-ZF interaction are presented in section 4. Finally, there is a conclusion in section 5.

2. Gyrokinetic model

For simplicity and clarity, we consider an axisymmetric, low-\( \beta \) (plasma to magnetic pressure), large aspect-ratio (\( \epsilon = r/R_0 \ll 1 \)) tokamak with the usual minor radius (r), poloidal (\( \theta \)) and toroidal (\( \zeta \)) coordinates. We examine a single high-\( n \)-ETC mode and associated ZF. Adopting ballooning representation [11], the ETC fluctuation can be written as

\[
\delta \Phi_k = \sum_m e^{i(n\theta - m\zeta)} \int d\theta \delta \eta_k |A_k \delta \Phi_k|,
\]

where the subscript \( k \equiv (n, \theta_k) \) denotes the wavenumber space, and \( n \delta \eta_k \) is a radial envelope wavenumber with \( q(r) \) being the safety factor.

Although the short wavelength modes with \( |k|, \rho_s \approx 1 \) tend to be favored in the linear phase, as the toroidal spectrum gradually downshifts in the early nonlinear evolution via the inverse cascading process [5, 6], long wavelength modes are expected to dominate the quasi-saturated state. Therefore, the nonlinear analysis presented here will focus on intermediate-scale turbulence with \( k^2 \rho_s^2 \ll 1 \ll k^2 \rho_c^2 \), where the Debye shielding is negligible, and the ETC mode is nearly isomorphic to its ion-scale counterpart ITG mode, except for the ion adiabatic response for both ZF and ETC. We can therefore impose the quasineutrality condition

\[
(1 + \tau)\Phi_k + \left( J_k \delta H_k \right)_e = 0,
\]

where \( \tau = T_e/T_i \) is the temperature ratio, \( \mu_k = J_0(k_1 \rho_s v_e) \) is the zeroth-order Bessel function accounting for the finite Larmor radius effect, and \( \delta H_k \) can be derived from the nonlinear gyrokinetic equation [12]:

\[
L_k \delta H_k + (i\delta + \omega_e^2)F_0J_k \Phi_k = \frac{i \mu_k^2 v_e}{2r_n} \sum_{\ell = 1} \left[ \left( J_{k1} \Phi_{k1} \right) \delta H_{k2}^* + \left( J_{k2} \Phi_{k2} \right) \delta H_{k1}^* \right].
\]

Here, we have normalized the electrostatic potential as \( \Phi_k = e\delta \Phi_k/(\rho_c T_e) \) with \( \rho_s = \rho_c/R_n \), and defined the Poisson bracket \([A, B] = (\partial_B A/\partial r)\delta B - (\partial_A \delta B/\partial r) \cdot L_k = [i\delta + \omega_e^2] \) is the inverse phase-space linear propagator, where \( \omega_e = v_e / (qR_n) \) and \( \omega_d = (\gamma^2 + \gamma^2/2)\nu_e^2 (\sin \theta_0 - \cos \theta_0) \) is the electron diamagnetic propagator, and \( \eta_k = \eta_k / \omega_d \), with \( k_0 = m/r, \) and \( r_n \) and \( r_0 \) being, respectively, the equilibrium density and temperature scale lengths. \( F_0 \) is the local Maxwellian. \( k_2 \) satisfies the matching conditions \( k = k_1 - k_2 \).

3. Linear properties

In ballooning space, it is well known that the dominant order linear gyrokinetic equation is an ordinary differential equation parameterized by \( \theta_0 \). To allow a tractable weak turbulence analysis, we assume a local kinetic model [13] to capture essential linear properties of toroidal ETC mode, by replacing the \( \eta \) variable in the transit and magnetic drift frequencies with its parallel-mode-structure-averaged value \( \bar{\eta}_e = \int d\eta \bar{\Phi}^* \bar{\eta}^2 \bar{\Phi}^* / \int d\eta \bar{\Phi}^* \bar{\eta} \bar{\Phi} \). The linear dispersion relation then reduces to an algebraic equation

\[
D_k(\omega, \theta_k, r) \equiv (1 + \tau) - \left( \frac{\omega + \omega_{\text{ZF}}}{\omega + \omega_k + \omega_{\text{ZF}}} \right)^2 F_0 = 0,
\]

where \( \omega_e = -v_e \epsilon_n (2 v_e^2 + v_e^2 \gamma^2) (\sin \theta_0 - \cos \theta_0) \) and \( \omega_d = -\omega_d \epsilon_n (2 v_e^2 + v_e^2 \gamma^2) \). In this case, we can adopt the weak turbulence theory to describe the ETC-ZF interaction and treat electrons kinetically by further assuming the local approximations for \( \omega_d \) and \( \omega_e \). The nonlinear description of ZF can be obtained by taking neoclassical effects into account [4, 14], yielding

\[
[\partial + \gamma e(1 + d_3 k_0^2 \rho_s^2 \gamma^2)] \chi_{\text{ZF}}(\theta_0)
= \sqrt{\frac{n}{2} \left( k_0 \rho_e s \right)^2} \int d\theta_0 \partial_\theta \left[ \alpha_{\text{ZF}}(\theta_0) A_s(\partial - \theta_0) A_s^* \right] - A_s(\partial + \theta_0) A_s^* (\partial \theta_0) A_n,
\]

where the length and time scales are normalized to \( \rho_c \) and \( \omega_{\text{ZF}}^{-1} \), respectively, \( \gamma \approx 3 \gamma_{ee} / (\omega_{\text{ZF}} \sqrt{\gamma}) \) with \( \gamma_{ee} \) being the electron–electron collision frequency, and \( \chi_e = \tau + 1 + 1.6q^2 / \sqrt{\gamma} \rho_c^2 \gamma^2 \). In deriving equation (4), we closely follow reference [15]. The difference in linear terms is that we introduce an ad hoc gyrodiffusive contribution (\( \times d_k \sim O(1) \)) to the ZF collisional damping rate, of which the importance has been emphasized recently [16]. It can enhance the ZF collisional damping,
and, thereby, may suppress the short wavelength ZFs [16]. However, noting that the gyrodiffusive term is algebraic with respect to \( \theta_k \), it turns out that this effect is not effective for the long wavelength ZFs \( (k^2 \rho^2 q^2 \ll 1) \) considered in this work, as shown later. The nonlinear term is related to the Reynolds stress, where we defined a parallel decoupling function \( a_n = a_n(\theta_k, \delta_k) = \int d\eta \hat{\Phi}^*_n(\eta, \delta_k) \hat{\Theta}^n(\eta, \delta_k + \theta_k) \) to measure the parallel correlation of ETG turbulence. It arises from the finite localization of ballooning-type parallel mode structure of the ETG mode, and can reduce the interaction of the ETGs \( \nu_k \) and \( \delta_k + \theta_k \) as their overlap decreases. The calculation of the parallel decoupling function \( a_n \) constitutes the fundamental difference of the current approach with respect to the ITG-ZF model [15]. Specifically, in previous ZF studies in fluid limit [15], \( a_n \) is approximated by its local value \( a_n(0,0) \equiv \int d\eta \hat{\Phi}^*_n(\eta, \delta_k = 0) \langle \partial H_n(\eta, \delta_k + \theta_k = 0) \rangle \) without the parallel decoupling effect, and the nonadiabatic particle response \( \delta H_n \) is further simplified with the fluid approximation. However, since the ETG mode structure \( \hat{\Phi}_n(\eta, \theta_k) \) has a ballooning-type character and is located around \( \eta \approx \theta_k \) in the long wavelength regime with \( |\omega_d| \ll |\omega|, |\omega_e| \), the parallel mode structure, to the lowest order in \( |\omega_d|/\omega \), can then be taken to be of the form [17, 18],

\[
\hat{\Phi}_n(\eta, \theta_k) = \hat{\Phi}_n^0(0, \theta_k) e^{-|\eta - \theta_k|^2 / \sigma^2}.
\]

Here, \( \theta_k \) manifests itself as a tilting angle and \( \eta \) denotes the mode width. Substituting equation (5) into the expression of \( a_n \) and taking the local inverse propagator \( L_k \) to account for the kinetic effects as in equation (3), \( a_n \) can be straightforwardly evaluated as \( a_n = a_n(0,0) \exp(-\theta_k^2 / 2\sigma^2) \). Noting that, for simplicity and clarity of the physics presentation, here we assume no \( \theta_k \) variation of \( \eta^2 \) that, however, can be determined by a systematic numerical investigation of the nonlocal linear eigenmode equation in ballooning space [19]. This is beyond the scope of this study. Therefore, it is evident that the function \( a_n(\theta_k, \delta_k) \) (and the Reynolds stress) decreases exponentially with \( \theta_k^2 \), decoupling the ZF mode from ETGs. Comparing the exponential parallel decoupling effect with the algebraic gyrodiffusive effect, one can anticipate that the parallel decoupling effect plays a major role in suppressing the large \( \theta_k \) ZFs. Furthermore, in the long wavelength regime where \( k^2 \rho^2 q^2 \ll 1 \), the frequency ordering is given by \( |\omega_d| \ll |\omega|, |\omega_e| \), the kinetic theory becomes necessary and we assume the local approximation for \( \delta H_n \) to calculate \( a_n(0,0) \) here.

Solving equation (2) to the next order, the quasineutrality condition straightforwardly produces the following NLSE for the ETG amplitude,

\[
\left[ i(\delta_k - \gamma_n) - b_n k^2 \rho^2 q^2 \theta_k^2 - \frac{c_n}{k^2 \rho^2 q^2} \frac{\partial^2}{\partial \theta_k^2} \right] A_n(\theta_k) = -i k_n \rho \kappa \frac{1}{\sqrt{2\pi}} \int d\nu_k \langle d_\nu d_\nu A_n(\nu_k) A_n(\theta_k - \nu_k) \rangle.
\]

Here, \( \gamma_n \) is the linear growth rate, \( b_n k^2 \rho^2 q^2 \theta_k^2 \) denotes the frequency mismatch, and the \( \propto c_n \) term recovers the correction associated with the plasma nonuniformities in real space. Since the fluid approximation is not applicable for the long wavelength ETG mode, the parameters \( b_n = -\partial H_n^0 / \partial H_n^0 / \partial H_n^0 / \partial H_n^0 \) and \( c_n = \partial H_n^0 / \partial H_n^0 / \partial H_n^0 \) are evaluated numerically from the linear ETG dynamics, i.e., equation (3). Meanwhile, unlike the previous fluid analysis [5, 6], the nonlinear electron response in real space \( \delta H_n^0 = (k_y b_\nu d_\nu / 2) C^{-1} [\partial_\nu A_n \delta H_n - J_n \delta H_n] \) is obtained from equation (2) with the local approximation for \( L \), thus the resultant nonlinear term in equation (6) is essentially of Navier–Stokes type, similar to that for the ITG-ZF interaction [15]. One readily identifies that, the ETG saturation is set by competition between linear growth and ZF-induced scattering to the linearly stable short radial wavelengths.

Equations (4) and (6), along with the complex parameters solely determined by linear ETG properties, fully characterize

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**Figure 1.** Normalized growth rate (dashed line) and real frequency (solid line) vs \( k \rho \theta \) with \( \theta_k = 0, \tau = 1, \epsilon = 0.25, \eta_n = 0.3, \eta = 1.5, \eta_c = 3, q = 1.4 \) and \( s = 0.8 \).
the dynamics of coherent ETG-ZF system, and will hereafter be referred to as the NLSE model. To properly account for the kinetic effects, the conventional fluid limit is not assumed here, and both the forced and spontaneous generation of ZF are kept on the equal footing. We emphasize that the coupling of ZF to ETG is formally of Hasegawa–Mima type in the fluid limit [6, 20], and is \( O(k_z^2\rho_e^2) \) weaker than the Navier–Stokes type nonlinearity in the present gyrokinetic analysis. As a consequence, ZF can more easily regulate the underlying ETG turbulence, and it will be shown later that the threshold condition for spontaneous ZF excitation is reduced by a factor \( O(k_z^2\rho_e^2) \) relative to previous fluid prediction. Noting that the current model accounts only for the intermediate scale ETG-ZF interaction, it does not apply to the early quasi-saturation phase dominated by short wavelength turbulence. Thus, it is not surprising that the current model cannot explain the complete changes of the morphology of fluctuation contours in the fully nonlinear simulations [7–10]. Furthermore, since the early nonlinear saturation phase has been studied extensively by previous theoretical works [5, 6], it is not necessary to go into details here.

It is illuminating to notice that a four-wave model [15] can be straightforwardly extracted from the NLSE model. By ignoring plasma nonuniformities and assuming the narrowband ZF and ETG amplitudes, respectively, as \( A_1(\theta_1 - \theta_2)/W \) and \( A_0(\theta_2/W) + A_1(\theta_1 - \theta_3)/W + A_2(\theta_1 - \theta_4)/W \), one readily obtains

\[
[\partial_t + \gamma_c(1 + d_k^2\rho_e^2)]\chi_k = \sqrt{\pi/2}Wk_z^3\rho_e^4(\chi_{k} + \gamma_{k}\chi_{k} + \gamma_{k}^2\chi_{k}^2),
\]

\[
[\partial_t + i\Delta - \gamma_\eta]A_+ = -Wk_z\rho_eA_0\chi_k/\sqrt{2\pi},
\]

\[
[\partial_t + i\Delta - \gamma_\eta]A_- = Wk_z\rho_eA_0\chi_k/\sqrt{2\pi},
\]

and

\[
[\partial_t - 2\gamma_\eta]A_0 = -Wk_z\rho_e(\chi_{k} + \gamma_{k}\chi_{k} + \gamma_{k}^2\chi_{k}^2)/\sqrt{2\pi}.
\]

Here, \( k_z \) is the usual rectangle function, \( W \) explicitly denotes the bandwidth, \( k_z = k_0\delta_\theta \) is the radial wavenumber, and \( A_0 \) and \( A_\pm \) are, respectively, the pump ETG mode and sidebands produced by the envelope modulation. \( \Delta = Re(b_0)k_z^2\rho_e^2 \) is the frequency mismatch and \( \gamma_e = \gamma_\eta + Im(b_0)k_z^2\rho_e^2 \) is the linear growth/damping rate of sidebands. The four-wave model has the following conservation property

\[
\langle|\partial_t A_0|^2\rangle = (2\gamma_\eta - \delta_\pm)(\langle A_+^2 \rangle + \langle A_-^2 \rangle).
\]

The four-wave model is a dynamical system that displays both weak and strong nonlinear behaviours. We first explore the onset condition of the modulational instability with a constant pump amplitude \( A_0 \). In this case, the system is linear and a dispersion relation can be derived from equations (7)–(9), by letting \( \theta_k \equiv \Gamma_z \), as

\[
[(\Gamma_z - \gamma_\eta)^2 + \Delta^2]^{1/2}\Gamma_z + \gamma_c(1 + d_k^2\rho_e^2)\chi_z = k_z^2\rho_e^2W^2\langle A_0^2 \rangle[\Delta Im(a_0) - (\Gamma_z - \gamma_\eta) Re(a_0)],
\]

which gives the critical threshold condition:

\[
W^2\langle A_0,0^2 \rangle = \left( \frac{\Delta^2 + \gamma_\eta^2}{k_z^2\rho_e^2[\gamma_c Re(a_0) + \Delta Im(a_0)]} \right).
\]

Thus, as discussed earlier, the threshold pump wave intensity is much lower (an order \( O(k_z^2\rho_e^2) \)) than the value from fluid theory [6]. Moreover, after some straightforward algebra, one can show that the threshold \( \langle A_0,0^2 \rangle \) is minimized at

\[
\theta_{z,m} = \gamma_\eta + Im(b_0)k_z^2\rho_e^2/\sqrt{2\pi}.
\]

For typical tokamak parameters, as shown in figure 2, ZFs are more easily excited around \( \theta_{z,m} \sim O(1) \) and the growth rate above the critical amplitude threshold, meanwhile, peaks at \( \theta_{z,m} \sim \theta_{z,m} \). The ETG-ZF interactions, thereby, tend to ultimately isotropize the linear streamers. These effects are mainly attributed to the parallel decoupling between the pump and sidebands in the Reynolds stress term, rather than the gyrodiffusive correction.

Next, consider the temporal evolution of the four-wave model. From equation (11), one can easily show that explosive growth exists for \( \gamma_c, \gamma_\eta > 0 \). On the other hand, for \( \gamma_c, \gamma_\eta < 0 \), we can demonstrate, by using equations (7)–(10), that the four-wave model gives a fixed-point solution with a constant \( A_\pm \), for sufficiently low ZF damping rate, namely,

\[
k_z^2\rho_e^2W^2\langle A_\pm,0^2 \rangle = (\delta - \Delta^2)/(\delta - \gamma_\eta),
\]

and

\[
k_z^2\rho_e^2W^2\langle A_0,0^2 \rangle = \pi(\delta - \Delta) Im(a_0) + \gamma_c Re(a_0).
\]

where \( \delta = \Delta^2/(\gamma_c + \gamma_\eta) \) is the amplitude oscillation frequency of ETGs due to their nonlinear interplay with the ZF. That is, the ETG turbulence is still fluctuating as the ZF converges to a steady-state, consistent with recent numerical results [9]. It also follows from equations (14) and (15) that, as in the case of ITG turbulence [21], \( \langle A_0,0^2 \rangle \) is proportional to the ZF collisional damping, while the ZF level is \( \gamma_\eta \) independent. However, it is important to note that the estimate of the saturated ETG fluctuation level given by equation (15) is only valid for the four-wave model with a single ZF mode. For any realistic system with a spectrum of radial ZF modes, the ETG turbulence will subsequently continue driving ZF with lower threshold condition. Thus, one may use the \( \langle A_0,0^2 \rangle \) value at \( \theta_z = \theta_{z,m} \) to quantitatively estimate the ETG saturation level.

The NLSE model is of integrodifferential nature and generally requires numerical solution. Figure 3 shows the typical time histories for the averaged amplitude \( \langle A_0,0 \rangle \) and dimensionless radial wavenumber \( \langle \theta_{z,m} \rangle \) of the ETG and ZF. In order to demonstrate the ETG saturation due to the ZF shearing, we retain the finite growth rate of the linear ETG mode and, thus, take the passive ZF generation into account [22]. One can identify three stages of the nonlinear evolution of the NLSE model. The first stage is early on before the global ETG mode structure is formed. Although the ZF is being force driven, its spectrum is very sensitive to the specific initial conditions for the ETG and thereby unpredictable.
Figure 2. Normalized critical threshold amplitude $W|A_{0,c}|$ and ZF growth rate $\Gamma_z$ versus $\theta_z$ for $k_0\varphi_e = 0.3$, $\gamma_z = 0.025$ and $d_z = 2$. Equation (3) yields $a_0(0,0) \approx -3 - 2i$ and $b_n \approx -2 - 2.5i$. The rest of the parameters are the same as figure 1.

Figure 3. Time histories of the averaged amplitudes and radial wavenumbers, for $\gamma_z = 0.025$ and $c_n = (-1.25 + 7.5i) \times 10^{-5}$. The other parameters are the same as figure 2.

In the second stage, a global ETG linear mode structure has already been formed, but the ETG nonlinearity is still negligible. In this case, the ZF spectrum can be analytically evaluated as

$$\Gamma_{Z,f} = \frac{i \pi \delta \Omega}{4 \chi \sigma^2} \exp(-\sigma k_1^2 \rho_e^2 - i \delta \Omega t)$$

where $\sigma = -b_n/4c_n$, $\delta \Omega = \gamma_\rho - 2c_n\sigma$ and $\gamma_\rho = \text{Im}(\delta \Omega)$ is the growth rate of the global linear ETG with $A_{Z,f} = A_{e,0} \exp(-\sigma k_1^2 \rho_e^2 - i \delta \Omega t)$. The defining feature of the force-driven process, i.e., an $\exp(2 \gamma_\rho t)$ factor, is readily recognized. It is worthwhile mentioning that, although the spectral shape is deterministic, the ZF intensity will depend on initial conditions. Figure 4 shows that the predicted ETG-ZF spectral shapes at the force-driven stage ($t = 69$) are in qualitative agreement with numerical results. Furthermore, note that although our theoretical model is valid for both passive and spontaneous generation of ZFs, the specific example with unstable ETG is chosen here to vividly show the three different stages of ZF generation and the ETG saturation due to ZF shearing. The corresponding ETG and ZF modes grow exponentially until the nonlinear ETG-ZF interaction dominates. If one considers the long wavelength ETG mode in the quasi-saturated state, which is relevant as the turbulence approaches marginal stability after the initial inverse cascading phase, the linear growth rate of the ETG mode will be negligible. Then equation (4) indicates that the ZF grows slowly and
Figure 4. Snapshots of the ETG-ZF radial spectra at the force-driven \((t = 69)\), early nonlinear \((t = 73)\) and final saturated \((t = 1500)\) phases. The other parameters are the same as figure 3.

When the ETG grows to the threshold intensity, the spontaneous ZF generation starts and the system evolves to the nonlinear saturation stage. As shown in figure 3, ZFs are initially excited around \(|\theta_k| > |\theta_{z,m}| \) in the early nonlinear state \((t = 73)\). Subsequently, a steady state is gradually reached as ZF spectrum shifts toward \(\pm \theta_{z,m}\) and, meanwhile, ETGs are scattered into the linearly stable regime. The effect of long-time-scale ETG-ZF interplay, therefore, is to broaden the ETG radial spectrum, but narrow the ZF spectrum, as vividly illustrated by the snapshots in figure 4. The final steady state is characterized by narrow-band ZFs, and the four-wave model is expected to offer a relevant tool for interpreting numerical results of the complicated NLSE model. Figure 5 shows that the time-averaged ZF-ETG ratio, computed from equations (14) and (15) with \(\langle A_n^2 \rangle = A_{n,p}^2(\theta_s = \theta_{z,m})\) and \(\langle A_z^2 \rangle = (1 + \gamma_n/\gamma_s)A_{z,p}^2(\theta_s = \theta_{z,m})\), indeed agrees quantitatively with numerical results. By taking \(W \simeq 0.8\) by inspection of figure 4, the ZF saturation level \(\langle |A_z| \rangle = 2\) is also in good agreement with the analytical value \(|A_{z,p}| \simeq 2.1\). Furthermore, in order to give a qualitative estimate for the corresponding electron heat transport level, one can evaluate the quasi-linear electron energy flux [2] approximately as \(Q_e/Q_{\text{Boh}} \sim O(|A_z|^2 |k_0 \rho_e|)\), where \(Q_{\text{Boh}} = nT_e v_{te} e^2/2\) is the gyro-Bohm heat flux. Therefore, for typical plasma conditions, this suggests that the electron heat transport caused by final saturated intermediate-scale toroidal ETG turbulence will be \(Q_e/Q_{\text{Boh}} \sim O(0.1) - O(1)\), with a linear dependence on the collisionality \([21]\).

5. Conclusions

To summarize, we have derived a NLSE model for the ZF generation in ETG turbulence, by allowing plasma non-uniformities and properly taking into account the crucial kinetic effects. As the ZF can be generated both passively by the direct beating of linear ETGs and spontaneously via the modulational instability, and it will be a combination of these two mechanisms in reality, the current theoretical model accounts for both the force-driven and spontaneous ZF generation. Furthermore, because of the intrinsic complexity involved in a first-principle theoretical description of the long-time spectral evolution, here we only consider the ZF generation by a single-\(\nu\), long wavelength ETG mode, and so we...
do not examine the full time evolution of whole wavenumber spectra. Nevertheless, by retaining relevant features, the current model still has the capability of capturing qualitative characteristics of the intermediate scale ETG-ZF interaction, and extracting the underlying physics. It is demonstrated that ZF is easily excited by the intermediate-scale toroidal ETG turbulence, and the corresponding threshold condition is lower than previous fluid predictions by at least $O(k^3 \rho_e^2)$, due to the Navier–Stokes type nonlinearity in ETG dynamics. The three-stage evolution of the coherent ETG-ZF system has been addressed, in which the force driven processes, i.e., the first two stages, are shown to crucially depend on the detailed conditions of the ETG turbulence. The parallel decoupling effect is found to be essential for determining the narrow-band ZF in the final saturated state. Conversely, the ETG spectrum is broadband since the saturation is achieved via scatterings to the high-$\theta$ stable regime. Considering typical tokamak parameters, the electron heat transport level expected for the ETG-ZF system is in the range $Q_e \lesssim Q_{eB}$ and proportional to the collisionality. Therefore, ZF generation could be an important nonlinear mechanism for the isotropization and saturation of the intermediate-scale toroidal ETG turbulence.

Finally, the current theory only examines the ZF generation in intermediate-scale ETG turbulence. It is obvious and desirable to extend the present analysis to include other nonlinear saturation channels, such as the nonlinear toroidal cascading. Although such a systematic treatment of the ETG turbulence is beyond the scope intended for this work, we note that the fluid description is applicable to short wavelength ETG turbulence, and that previous studies [5, 6] have shown that the toroidal inverse cascade will dominate over the spontaneous ZF excitation, resulting in streamers. As energy is transferred to intermediate-scale turbulence, however, the ETG-ZF interactions become important and ETG turbulence will be more easily isotropized. Therefore, the role of ZF depends crucially on the characteristic spatial scale of ETG turbulence. Given these considerations, it is worthwhile to point out that this picture also carries significant qualitative implications to the experimental measurements of ETG turbulence. In particular, the theory suggests that the long-time saturated ETG spectrum will peak at the intermediate-scale with $k_{\perp}^3 \rho_e^2 \ll 1$, which could be linearly stable due to the nonlinear toroidal inverse cascade; and the related spectral intensity and electron heat transport will be proportional to collisionality, arising from the ETG-ZF interaction. These features are consistent with the reported experimental observations in tokamak plasmas [23–25], and may offer a useful tool to test the present understanding of ETG driven ZFs in future experiments.

Acknowledgments
This work is supported by the Exascale Computing Project (17-SC-20-SC), a collaborative effort of the US Department of Energy Office of Science and the National Nuclear Security Administration. We thank Prof. Liu Chen, Dr Yang Chen and Dr Junyi Cheng for useful conversations.

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