Forward Kinematics Solution of the Bogie Test Bench Based on the IPSO

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Abstract. In order to control the motion pose of the bogie test-bench more accurately, an improved particle swarm optimization (IPSO) is proposed to solve the forward kinematics of the six degree of freedom (6-DOF) motion platform of the bogie test bench. According to the geometry and parameters of the parallel 6-DOF motion platforms, the mathematical model for the forward kinematics of the platform is established by the homogeneous transformation matrix. Then, the built objective function is optimized based on the IPSO and the pose parameters of the 6-DOF platform are obtained by MATLAB simulation. Finally, in order to verify the correctness and feasibility of this method, the inverse kinematics simulation model of the 6-DOF platform is established by MATLAB/Simulink, comparing the actual measured value and the calculated value of the rod length, the error is within the allowable range, which meet the requirements of precision control.

1. Introduction

As an important component of rail vehicles, it is of great significance to detect various performance parameters of bogies for rail vehicle safety [1]. Jilin university automobile transportation engineering institute has designed and developed the bogie indoor test bench [2-3], which can realize bogie dynamic simulation under various complex lines. The bench is a parallel mechanism composed of two 6-DOF motion platforms, and each platform is driven by seven hydraulic actuators. The accuracy of the forward kinematics solution plays an important role in realizing the attitude control of the parallel manipulator. At present, in view of the forward kinematics solution problem of a parallel mechanism, many scholars set up the nonlinear equations with the pose as variable based on the constraint condition of rod length, and solve the forward kinematics solution by using the homotopy continuous method [4], newton iterative method [5], neural network method[6] and other methods. However, the numerical iteration method has a strong dependence for the initial value and a slow convergence rate that cannot meet the requirement of real time. And the standard PSO or genetic algorithm (GA) has low convergence accuracy and is easy to fall into local optimum.

In this paper, an IPSO, that is the multi-group parallel particle swarm optimization, is proposed to solve the forward kinematics of the 6-DOF motion platform of the bogie test bench. Based on the kinematics analysis of the test bench, the objective function of the forward kinematics solution of the 6-DOF motion platform is established, and the specific solution steps of the forward kinematics solution by using the IPSO are given. Finally, the simulation calculation is carried out by MATLAB, and the
pose parameters of the 6-DOF motion platform are obtained, which lays the foundation for the dynamic simulation of the bogie test bench.

2. Structure of the Bogie Test Bench

In this article, the bogie test bench is a parallel mechanism with two 6-DOF motion platforms, driven by electro-hydraulic servo system with 14 actuators, as shown in Fig.1.

![Figure 1. Geometric model of the bogie test bench](image1)

It is convenient for kinematic analysis to define the forward direction of the train as the X axis, the transverse direction as the Y axis, and the vertical axis as the Z axis. The global coordinate system and the local coordinate system are introduced to describe the pose of 6-DOF motion platform at any time [7-8]. Combined with the geometric model of the bogie test bench, the intersection of the transverse symmetrical section and the longitudinal symmetrical section of the two upper platform of the bogie test bench on the track horizontal plane is selected as the origin, then the global coordinate system $O-XYZ$, that is, the absolute coordinate system is established. The intersection point of the two longitudinal horizontal actuators' center line and the two lateral actuators' centerline on the track horizontal plane is selected as the origin of the local coordinate system, then the local coordinate system $O_1-X_1Y_1Z_1$ of the front platform and the local coordinate system $O_2-X_2Y_2Z_2$ of the behind platform are set up respectively. The X, Y and Z directions of the local coordinate system is the same as that of the global coordinate system. Fig.2 and Fig.3 are the left and front view simplified drawings of the bogie test bench.

![Figure 2. Left view simplification of the bogie test bench](image2)
Figure 3. Front view simplification of the bogie test bench

3. Kinematic analysis

Reference Fig.1 and Fig.2 and taking the front 6-DOF motion platform as an example, the coordinate of the upper hinge point attached to every actuator $A_i$ ($i = 1, 2, \ldots, 7$) with respect to the local coordinate can be expressed in vector form.

$$
A = \begin{bmatrix}
-\frac{l_2}{h_1} & \frac{l_2}{h_1} & l_1 & l_1 & l_6 & l_6 + l_4 & l_6 + l_4 & 0 \\
-\frac{v_2}{h_1} & \frac{v_2}{h_1} & v_1 & v_1 & v_1 & v_1 & v_1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}
$$

(1)

The coordinate of the lower hinge point attached to every actuator $B_i$ ($i = 1, 2, \ldots, 7$) with respect to the global coordinate can be expressed in vector $B$.

$$
B = \begin{bmatrix}
-\frac{l_2}{h_1} & \frac{l_2}{h_1} & l_1 & l_1 & l_3 & l_3 & l_4 & l_4 \\
-\frac{v_2}{h_1} & \frac{v_2}{h_1} & v_1 & v_1 & v_1 & v_1 & v_1 & v_1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}
$$

(2)

The rotational and translational variable quantity around and along X, Y, Z of the local coordinate with respect to the static coordinate could be described by $q = (q_1, q_2, q_3, q_4, q_5, q_6)$. The homogeneous transformation matrix $T$ of ZYX rotation space can be expressed as follows.

$$
T = \begin{bmatrix}
cq_2cq_3 & -cq_2s_4 + sq_2cq_4 & sq_2cq_3 & cq_2s_4 + cq_4c_4 & cq_2c_4 & 0 & cq_2c_3 & q_3 \\
cq_2s_4 & cq_2c_4 & sq_2s_4 + sq_2c_4 & cq_2s_4 + cq_4c_4 & -sq_4c_4 & qq_3 & cq_2s_3 & q_3 \\
-sq_2c_4 & sq_2s_4 & cq_2s_4 + cq_4c_4 & -sq_4c_4 & cq_2s_3 & 0 & sq_2s_3 & q_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

(3)

Where, $c$ means cos and $s$ means sin.

The coordinate matrix $A$ of the upper hinge points and the coordinate matrix $B$ of the lower hinge points have different reference systems, it is necessary to transform the matrix $A$ into the global coordinate system through transformation matrix. Therefore, the length of the 7 actuators of the front 6-DOF motion platform could be determined by the distance between the upper and lower hinge points.
4. Forward kinematics solution based on the IPSO

4.1. Objective function

The forward kinematics problem of the 6-DOF motion platform is that to determine the output pose of the platform according to the flex quantity of the rod length of the 7 actuators which is known. Therefore, it is converted to a mathematical problem to solve the unknown parameters \( q \) with the known parameters \( L_i \).

The maximum rotation amount of the 6-DOF motion platform around X, Y and Z axis is ±8 degrees, the maximum translational amount along the X axis is ±100mm, and the maximum translational amount along the Y axis is ±100mm, and the maximum translational amount along the Z axis is ±250mm. The pose parameters \( q \) are regarded as particles, and when the platform at the initial position, the range of the variable value of the particles are as follows, \( q_1 \in [-8, 8] \), \( q_2 \in [-8, 8] \), \( q_3 \in [-8, 8] \), \( q_4 \in [-100, 100] \), \( q_5 \in [-100, 100] \), \( q_6 \in [-250, 250] \).

Formula (4) can be expressed as follows,

\[
F(q, L) = L_j - \left\| T \cdot A - B \right\| = 0
\]

According to formula (5), it can be transformed into a mathematical model waiting for optimization, that is, the fitness function of the IPSO.

\[
\text{fitness} = \min f(q) = \sum_{i=1}^{7} f_i^2(q) \\
st. \quad -8 \leq q_1, q_2, q_3 \leq 8 \\
\quad \quad \quad -100 \leq q_4, q_5 \leq 100 \\
\quad \quad \quad -250 \leq q_6 \leq 250
\]

Formula (6) is further expressed as,

\[
\text{fitness} = \min f(q) = \sum_{i=1}^{7} \left[ ((T_{i1} A_x + T_{i2} A_y + T_{i3} A_z + T_{i4} - B_x))^2 \\
+ (T_{i5} A_x + T_{i6} A_y + T_{i7} A_z + T_{i8} - B_y))^2 \\
+ (T_{i9} A_x + T_{i10} A_y + T_{i11} A_z + T_{i12} - B_z))^2 \right] - L_j^2 \\
st. \quad -8 \leq q_1, q_2, q_3 \leq 8 \\
\quad \quad \quad -100 \leq q_4, q_5 \leq 100 \\
\quad \quad \quad -250 \leq q_6 \leq 250
\]

4.2. Standard PSO

All particles of the standard PSO have a fitness value determined by an optimized function, each particle has a velocity determining its direction of flight and distance, and then the other particles follow the current best particle to search in the solution space. Each particle in the POS updates its speed and position according to the following formulas [9].

\[
v_{id}^{t+1} = \omega v_{id}^{t} + c_1 r_1 (p_{id}^{t} - x_{id}^{t}) + c_2 r_2 (p_{gd}^{t} - x_{gd}^{t})
\]
Where, \( i = (1, 2, \cdots N), d = (1, 2, \cdots D) \); \( c_1 \) and \( c_2 \) are the acceleration constants; \( \omega \) is the inertia weight; \( r_1 \) and \( r_2 \) are mutually independent random numbers obeying the uniform distribution on \((0,1)\); \( v_{id}^t \) is the velocity of the particle \( i \) iteration to the \( t \) generation; \( v_{id}^{t+1} \) is the velocity of the particle \( i \) iteration to the \( t+1 \) generation; \( x_{id}^t \) the position of the particle \( i \) iteration to the \( t \) generation; \( x_{id}^{t+1} \) is the position of the particle \( i \) iteration to the \( t+1 \) generation.

Each particle in standard PSO algorithm updates its speed and position according to the formula (8) and formula (9) respectively. In the process of the iterative evolution, the movement of particles is toward the current optimal particle, and all particles are easily attracted to the current optimal position of the population, and the diversity of the population is rapidly reduced. Therefore, if there are many local optima in the solution space, the standard PSO will easily fall into the local optimum in the search process.

4.3. Multi-group PSO

The particle swarm in the IPSO is decomposed into three species. The particles in the first population have a good exploring ability to extensive search in the solution space; the particles in the second population can balance the exploration and development ability, which facilitate the iterative optimization of particles in their respective regions; the particles in the third population focus on developing ability, which accelerate converge after finding the best advantage of the particle. In the whole search process, the extreme points of the search neighborhood are found one by one, the characteristics of the particle flight are different in different populations, and the diversity of the population is guaranteed.

According to the above analysis and reference [10], the three species can be evolved iteratively by the “Cognition only” population, the standard model particle population and the “Social only” population, respectively. The particle swarm with different characteristics has its own iterative evolution formula, and its evolution equation is as follows.

The formula (10) and (11) are the evolution equations of “Cognition only” population.

\[
v_{id}^{t+1} = \omega v_{id}^t + c_1 r_1 (\hat{p}_d^t - x_{id}^t) \\
x_{id}^{t+1} = x_{id}^t + v_{id}^{t+1}
\]  

The formula (12) and (13) are the evolution equations of “Social only” population.

\[
v_{id}^{t+1} = \omega v_{id}^t + c_2 r_2 (\hat{p}_d^t - x_{id}^t) \\
x_{id}^{t+1} = x_{id}^t + v_{id}^{t+1}
\]  

The specific implementation steps based on the multi-group parallel particle swarm optimization algorithm are as follows.

Step 1: The particles are divided into three groups with the same size and initialized, after the iteration of the \( N \) generation, the worst particle of the second population is replaced by the best particle of the first population, the worst particle of the second population is replaced by the best particle of the third population, and the counter \( i \) is cleared.

Step 2: The fitness values of all particles in each population are calculated by the fitness function (7).
Step 3: Comparing the current fitness values of each particle with the individual historical optimum value $p_b(k)$, if the current individual's fitness value is better, then $p_b(k)$ is updated.

Step 4: Comparing the individual historical optimum value $p_b(k)$ of each particle with its population optimum value $p_p(k)$, If $p_b(k)$ is superior to $p_p(k)$, then $p_p(k)$ is updated.

Step 5: Case I, if $i < N$, the particles of the second population are updated according to the formula (8) and (9); the particles of the third population are updated according to the formula (12) and (13); the particles of the first population are updated according to the formula (10) and formula (11). Case II, if $i \geq N$, Two particles with the worst fitness value in the second population are found to be replaced by the best particles of the first population and the third population respectively, and the counter $i$ is cleared.

Step 6: To determine whether the iteration termination condition is satisfied. If the termination condition is satisfied, the optimal solution is output. Otherwise, turn to step 2.

5. Simulation and verification

In this paper, the simulation calculation is carried out by MATLAB software, which realized the forward kinematics solution of the 6-DOF motion platform based on the IPSO algorithm. The following is the setting of the parameters in the program, there are six independent variables in the pre-optimized function, so the space dimension is $D=6$; the inertia weight is reduced from 0.9 linear to 0.4; accelerating constants are $c_1=c_2=1.496$; the maximum number of iterations is 1000; the threshold value of the algorithm termination condition is $\varepsilon=1e-9$ (convergence precision); the population size is 30 particles, and each sub population size is 10 particles.

![Figure 4. Optimization process of the forward kinematics of the front 6-DOF platform](image)

The test platform can control the bogie to move at any pose, so it is necessary to calculate the pose of the 6-DOF platform when it is in the change of the multi-DOF pose. The actuators’ length $L_i$ of the platform in the 6-DOF pose are measured through experiments, and $L_i = (2978.79, 3018.05, 3045.17, 3005.92, 2633.79, 2810.89, 2897.04)$. The length of each actuator is used as the input parameter of the IPSO algorithm, and the optimization process, as shown in Fig.4, is completed by running the MATLAB master program, and the result of the calculation is $q=(0.01746, 0.03426, 0.05233, 9.75185, 40.011, 80.09179)$. 

In order to verify the accuracy of the forward kinematics optimization model of the 6-DOF platform and the IPSO algorithm proposed in this paper, the inverse kinematics Simulink model of the front 6-DOF platform based on the above kinematic analysis, as shown in Fig.5. The result $q=(0.01746, 0.03426, 0.05233, 9.75185, 40.011, 80.09179)$ of the forward kinematics solution is used as the input of the inverse kinematics Simulink model, then the seven rod length values of the platform are obtained, and they are $L_1=2979.113$, $L_2=3018.374$, $L_3=3044.977$, $L_4=3005.727$, $L_5=2634.064$, $L_6=2811.105$, $L_7=2897.04$, respectively. Finally, compared with the measured data, the maximum error is $0.009\%$, which is within the allowable range and meets the control accuracy requirement.

6. Conclusion
This paper proposed a method for solving the forward kinematics of 6-DOF motion platform based on the IPSO. The forward kinematics of the platform mathematical model for optimizing is established by kinematic analysis. The length of each actuator measured is used as the input parameter of the IPSO, and the pose parameters of the platform are calculated. In this paper, the inverse kinematics model is used to verify and analyze, the pose parameters obtained by the IPSO are used as the input parameters of the inverse kinematics model, then the length of the rod is calculated and compared with the measured value, and the correctness of the method this paper proposed is verified.

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