Cognitive Psychology

Strategic Thinking: A Random Walk Into the Rabbit Hole

David J. Grüning¹, Joachim I. Krueger²

¹ Psychology Department, Heidelberg University, Heidelberg, Germany, ² Cognitive, Linguistic & Psychological Sciences, Brown University, Providence, Rhode Island, US

Keywords: strategy, social projection, game theory

https://doi.org/10.1525/collabra.24921

Collabra: Psychology

Vol. 7, Issue 1, 2021

At its best, strategic thinking yields an advantage needed to beat an opponent. At the least, it protects the person from exploitation. In four studies, conducted in two countries, we used a simple number-guessing game, in which one respondent wins by guessing the number chosen by another. We show that people generate numbers nonrandomly, and, on the basis of this finding, we predict and find that nonrandom strategic choice is advantageous to the guesser if the chooser does not randomize either. As expected, respondents in the role of the guesser preferred to play a game in which they were to actively think of a number instead of randomizing if the chooser had to think of a number, too. Guessers did not prefer thinking if the chooser selected a number randomly. Having shown these limitations to strategic reasoning, we close with the observation that successful strategic reasoning may—at times—require the breaking of rules and being the first to do so.

No battle plan ever survives contact with the enemy.
- Helmuth von Moltke the Elder, in translation and paraphrasis

It is […] impossible to win the victory unless you dare to battle.
- Richard M. DeVos the Elder, American business billionaire

Eminent military figures, such as Helmuth von Moltke (the Elder) or Themistocles of Athens might be intrigued by the prevalence of military jargon and imagery in current discussions of strategy, especially in contexts of business or economics (cf. R. M. DeVos). The word ‘stratos’ is Greek for ‘army,’ and its derivative ‘strategia’ refers to a battle plan. A battle plan is different from, say, a building plan. The latter includes a rational sequence of steps, which, if followed faithfully, produces a structure that won’t collapse. Such a plan represents an architect’s game against nature, its materials and elements. A battle plan, by contrast, must allow for a thoughtful opponent who also has a battle plan. The better battle plan wins. A plan is good to the extent that it anticipates the opponent’s moves. However, as both parties seek to outwit the other, no battle can guarantee victory.¹ Still, a military leader is condemned to plan, for otherwise the battle is lost before it begins. And so it is in other strategic contexts.

Penalty shots in soccer highlight the dilemma in a stylized environment (Bar‐Eli et al., 2007; Bar‐Eli & Azar, 2009; Chiappori et al., 2002). The shooter tries to predict what the goalkeeper will do—jump right, jump left, or stay in the center—and aims elsewhere. The goalkeeper tries to predict where the shooter will aim and chooses accordingly. Whereas the goalkeeper’s strategy is to match his move to the shooter’s aim, the shooter seeks to frustrate this very strategy, and both know that this is so. Assuming no difference in competence and depth of foresight, neither player possesses a sure-fire strategy. Their attempts to outpredict the other are infinitely regressive; they descend into a mental rabbit hole.

In cognitive psychology, theories of k-level reasoning address this kind of strategic reasoning (Nagel, 1995; Stahl & Wilson, 1995). The simplest kind of decision-making, at k = 0, is hardly strategic as it contains no assumptions about the opponent’s reasoning. It does not involve mentalization, simulation, or theory of mind. Each additional level introduces another layer of strategic sophistication. Most individuals manage to think only a few layers deep; yet many think they are one step ahead of their opponents (as speculated by Burchardi & Penczynski, 2014; Camerer et al., 2004). Few understand the concept of infinite regress. Limited strategizing need not be irrational, but the belief that the depth of one’s own reasoning is greater than the opponent’s is incoherent in the absence of evidence. Players who merely assume they can strategically outthink the other suffer an illusion of superiority. When all of them do, the outcome of the contest will prove half of them wrong.

¹ This mutual nullification of strategies holds unless one party has extraneous advantages, such as time, luck, or, as we will argue below, a cunning willingness to break the rules.
A famous scene from *The Princess Bride* (see Goldman, 1973, for the book, and Scheinman & Reiner, 1987, for the film) puts the issue in comic relief. Seeking to free Princess Buttercup from the clutches of Vizzini, the evil Sicilian, Westley, Buttercup’s humble suitor, appears disguised as a knight, and challenges Vizzini to a *Battle of Wits*. Placing two cups of wine between Vizzini and himself, he announces that the contents of one cup are poisoned. He dares Vizzini to drink one cup, assuring him that he, Wesley, will drink from the other. Trying to make a rational choice, Vizzini narrates his unfolding thought processes, thereby supplying an inside view of *k*-level reasoning. When reaching the sixth level, Vizzini appears to be satisfied with his efforts, drinks from one cup, and promptly dies.

With Vizzini’s self-assurance and the comparative sophistication of his reasoning, his death is more comical than tragic. Research participants rarely match his determination, with few reasoning beyond the second level (Arad & Rubinstein, 2012; Benndorf et al., 2017; Camerer et al., 2004; Nagel, 1995). At $k = 2$, reasoning requires assumptions about the reflected appraisals of the opponent’s inferences about one’s own prediction. Here, the predictor might say “She thinks that I think she thinks that I think she will do X.” Thinking being hard and likely to cross the line into confusion, many individuals settle for $k = 0$ thinking, which requires only a simple prediction of the other’s choice. First-level reasoning ($k = 1$) involves one step of mentalization of the competitor’s psychological world. Here, the predictor might say “She thinks that I think she will do X.” At $k = 2$, reasoning splits into three layers. Figure 1 shows the first three layers of reasoning in linguistic and graphical form.

Any expectation of being able to outsmart an opponent requires the assumption that the opponent fails to match one’s own depth of reasoning. Barring any direct evidence for this, a player confident of victory is necessarily overconfident (Moore, 2020). There is no normative defense for the assumption that the average opponent is less sophisticated than the average strategic agent. Without justification, the belief in the superiority of one’s own strategic reasoning efforts amounts to unwarranted self-enhancement (Alicke & Govorun, 2005; Heck & Krueger, 2015).

**DN’s thought experiment**

Dixit and Nalebuff (hereafter DN), in their influential *The art of strategy* (2008), sought to illustrate the difference between nonstrategic and strategic reasoning with a thought experiment. They first introduce a simple guessing game, in which a number between 1 and 100 is drawn at random and the respondent guesses what it is. If the first guess is right, the respondent receives $100. If it is wrong, the respondent is told whether the true number is larger or smaller than the guess, and they then guess again. If the second guess is right, the reward is $80; if it is not, the game continues with gradual $20 reductions in the reward until there is either a match or no money left. It is easy to see that the best strategy is to split, at each stage of the game, the range of possible numbers. The responder would begin by guessing 50 and then proceed to guess 25 or 75 respectively if 50 was too high or too low, and so forth.

DN then modify this game such that it is now a person, say Professor Dixit, who chooses a number that is to be guessed. The intuition is that this two-person guessing task is more difficult than the original, random, one, and that a strategic instead of a random approach is needed. The guesser may now assume that Professor Dixit would rather not part with the money, and thus avoid the mid-range numbers precisely because these numbers are the optimal picks in the non-strategic context ($k = 1$). Thinking further, the guesser might anticipate that Professor Dixit knows that the guesser assumes the professor to avoid the mid-range numbers and will therefore pick them after all ($k = 2$). A rabbit hole of recursive inferences now opens up. If any strategy can be anticipated, so can any anticipation be anticipated. There is no logical stopping point, only fatigue or befuddlement (as presumably experienced by Vizzini). Indeed, DN consider it paradoxical that both sides of a game might think that “they can outsmart the other” (p. 24). Of course, both sides can think they can outsmart the other, but only one side can be proven right.

When the strategic game collapses into the random game, and if a random choice of a number re-emerges as the best equilibrium strategy, one wonders if strategic thinking can retain a unique, non-random element. The answer seems to be no. For the number chooser, randomness is the safest strategy, and therefore, the number guesser in a multi-stage game can do no better than using the same mid-range strategy regardless of whether the number generator is a machine or Professor Dixit. Indeed, DN do not reveal if or how the strategically minded guesser might depart from the mid-range guessing protocol. They merely state that the chooser (DN) who thinks one level ahead of the guesser (reader) wins. This conclusion begs the question of why it should be the chooser and not the guesser who is one step ahead.

In this article, we argue that the guesser can find an advantage if the chooser fails to recognize the conservative

Figure 1. Verbalized reasoning model with the highest level being $k = 2$.

2 See Appendix A for Vizzini’s complete monologue.
rationality of randomness. Any departure from randomness on the part of the chooser amounts to a bias, and being a systematic source of variance, a bias is in principle predictable. To bring this potential advantage for the guesser to light, we introduce a modified version of the number-guessing game.

Taking the ‘thought’ out of the thought experiment

DN’s thought experiment can be simplified, without loss or acuity, to a set of numbers from 1 to 6. Chooser and guesser each have the option of either randomizing or ‘thinking’ when generating a number. If the chooser randomizes by, say, casting a die, the guesser might do the same. There is no point trying to intuit a random number by ‘thinking.’ A guesser who elects to generate a number thoughtfully betrays an illusion of control (Langer, 1975; Rothbart & Snyder, 1970). In contrast, if the chooser has thought of a number, that number might be predictable inasmuch as biases in number generation are shared among individuals. There is indeed evidence for “population stereotypes” favoring certain numbers. Asked to name a number from 1 to 12, the modal response is 7 (Teigen, 1985). There is a general bias against the highest and the lowest numbers. A guesser who knows that the chooser generated a number ‘thoughtfully’ may thus find it wise to pick 3 or 4, simply because these are the numbers that would first come to the guesser’s mind too. ‘What comes to the chooser’s mind,’ the guesser might reason, ‘is probably what comes to my mind.’ Projecting the contents of one’s own mind to the mind of the other takes advantage of shared biases and may thus give a competitive edge to the guesser (Krueger, 1998; Krueger & Acevedo, 2007). In turn, a clever chooser realizes that projection can only serve the guesser and disables it by reverting to randomization. In short, the guesser can reliably win in this strategic contest only if the chooser makes the mistake to think.

To date, there has been little research on how people reason their way through the kinds of psychological challenges inherent in the number-guessing game. Some authors have proposed that respondents proceed naively, as did Rubin and colleagues (1997) when exploring games of hide and seek. More recently, Crawford and Iriberri (2007) hypothesized that players use advanced k-level reasoning (for an experimental extension, see Heinrich & Wolff, 2012). We use the modified number guessing game introduced above and ask whether guessers understand that they might profit from an opponent’s suboptimal preference for thoughtful play. We show that guessing games cannot be won by brute-force strategic mentalizing. Such efforts descend into a psychological rabbit hole. One cannot hope to outsmart a competitor who is as rational as oneself.

The present research

In two studies (experiments 1 and 3), we cast the respondents in the role of the guesser to consider four versions of the modified number game. In two versions, they were told that the chooser selected a number randomly, and in the other two they were told that the chooser thought of a number. This variation was crossed with the guesser’s own strategy. That is, in two versions, the respondents were instructed to guess a number randomly (by throwing a die), and in the other two they were told to think of a number. For each of the four games, respondents stated their willingness to pay (WTP) for the opportunity to play. We predicted that when respondents believed the choosers to be rational, by randomly choosing a number, there would be no variation in the average WTP. However, if guessers were informed that choosers thought about their numbers, they could potentially improve their prospects by also thinking of a number. If so, guessers should be willing to pay more for the chance to play when being able to think themselves. This is the focal hypothesis for these experiments.

In another two studies (experiments 2 and 4), we explored respondents’ number selection. In one condition, respondents simply thought of a number from 1 to 6. We expected a non-equal frequency distribution with a hump around the mid-range (see this effect for numbers, Teigen, 1983; multiple choice tests, Attali & Bar-Hillel, 2003; or bathroom stalls, Christenfeld, 1995). The other condition was designed to shine some light into the cognitive rabbit hole. In a modified beauty-contest design (Keynes, 1956/2018), we asked respondents to pick the number they thought the fewest respondents would pick (hereafter referred to as meta thinking).

It seemed improbable that the quasi-Keynesian prediction task would reproduce the frequency distribution obtained in the baseline condition. Suppose a respondent generated the number 4 in the baseline condition. With projection, this respondent would conclude that many others chose the same number. To then choose the number 4 again in the contest in hopes of a strategic gain, the respondent would have to disable projection and believe that the others did what the respondent did not do, namely change the number. Although a few individuals might reason this way, it is hard to imagine this line of thought as being widespread (Krueger, 2013).

Most likely, respondents change their own number selection while assuming that others do not. This mindset would be consistent with the idea that respondents think they can think a step ahead of others. Yet, there is the same logical problem, such that more respondents will have chosen popular than unpopular numbers. Middling numbers were more popular than end-point numbers in the baseline condition. The reverse is true in the meta-prediction condition. It is easy to imagine that respondents make a switch towards initially unpopular numbers (k = 1), while failing to see that if the majority switches, the initially unpopular numbers turn into the popular ones. This is our focal hypothesis for these experiments.

The third possibility is that respondents understand the logical constraint built into the contest. They might realize that in the aggregate it cannot be true that most people select the number that the fewest respondents select, and that there is no strategy that would allow them, and only them, to place themselves into the lucky minority that finds the minority number. At this point, respondents might realize that randomization is the only remaining strategy. This understanding of the game is probably rarely achieved.

To review, we conducted four experiments to see if
guessed prefer a thoughtful number selection when the chooser is known to do so (experiments 1 and 3) and if respondents generate numbers in a biased and nonrandom way (experiments 2 and 4). We sampled respondents in the United States (experiments 1 and 2) and in Germany (experiments 3 and 4) to test these hypotheses.

**Experiment 1**

We predicted that guessers who are allowed to think of a number are willing to pay more for a game if the chooser also thinks of a number instead of generating it randomly. If a chooser fails to deploy a rational equilibrium strategy, it is normatively defensible for a self-regarding guesser to pay more for a game with a higher expected value.

**Method**

**Participants and Design.** A total of 188 participants were recruited by students at Brown University (Rhode Island, USA) via direct and social media targeting. The sample comprised 102 (54.8%) female and 85 (45.2%) male participants, ranging from 18 to 62 years of age (M = 21.23, SD = 4.06). The experimental design crossed the chooser’s and the guesser’s mandated strategy (randomize vs. think) within participants.

**Procedure.** After giving their consent, participants were presented with four different scenarios. In each they had to imagine playing a game. There was a ‘chooser’ who generated a number from one to six, either by rolling a die or by thinking of a number. Participants were asked to assume the role of the ‘guesser’ who would also generate a number by either thinking of one or rolling a die. These four scenarios resulted from the crossing of the two players’ strategies. For each game, participants were asked to imagine they would win $60 if the number they guessed matched the chooser’s number. They were then, for each game separately, asked to indicate how much money they would be willing to pay (WTP) for the opportunity to play. After providing some standard demographical data, respondents were thanked and debriefed about the study’s purpose.

**Results and Discussion**

We refer to the four conditions of the study with the following designations: r_r refers to the game in which both players randomly select a number; r_t refers to the game in which only the guesser thinks of a number; t_r refers to the game in which only the chooser thinks of a number; t_t, finally, refers to the game in which both players actively think of a number.

We computed an analysis of variance (ANOVA) with two withins-factors (chooser vs. guesser) and two strategy levels each (randomly choosing a number vs. thinking of a number). The chooser’s strategy produced no main effect (p = .258), that is, respondents were on average indifferent as to whether the chooser was randomizing or thinking. In contrast, the guesser's strategy produced a significant difference, F(1, 187) = 30.59, p < .001, η² = .061, such that respondents were willing to pay more for the game if they could think of a number (M = 9.47) than if they were obliged to guess randomly (M = 7.85). The critical prediction, however, called for an interaction effect. This effect was indeed significant, F(1, 187) = 13.53, p < .001, η² = .061, and it took the theoretically expected shape, as shown in Figure 2. Participants preferred a game that allowed them to think of a number, if the chooser was also thinking (t_t vs. t_r), F(1, 187) = 36.26, p < .001, η² = .162. The preference for thinking was also statistically significant if the chooser was randomizing, but this effect was much smaller (r_t vs. r_r), F(1, 187) = 6.56, p = .011, η² = .034.

A Bayesian paired-samples t-test analysis corroborated this interpretation. For the comparison of the two games with a thinking chooser, the evidence against the null hypothesis of no difference was very strong (BF₁₀ > 1,000,000), whereas the evidence was very weak for the comparison of the games with a randomizing chooser, BF₁₀ = 1.96 (Lee & Wagenmakers, 2014, for conventional benchmarks of Bayes factor interpretation).

**Experiment 2**

Guessers’ attraction to games in which choosers think (vs. randomize) is rational if there are shared preferences for certain numbers. We proceeded to look for such shared preferences. A non-uniform number distribution is likely a priori, but its shape needs to be found empirically. Judging from the literature, we expected central numbers to be more popular than marginal numbers (Teigen, 1985). We then explored meta-thinking, that is, the first rung into the predictive rabbit hole (i.e., k = 1 level reasoning). We tested – and expected to reject – the hypothesis that respondents would generate an equal frequency distribution when trying to guess the least popular number. Granting that respondents could succeed in this task if they all randomized their responses, we considered this outcome unlikely. If respondents thought they could identify the least popular number – and if many thought likewise – the resulting distribution would again be unequal. That is, what we sought to show was that strategic reasoning is self-nullifying if it is shared among players. The resulting distribution would be the in-
verse of the distribution obtained with simple number sele-
ction, with minimum (1) and maximum (6) numbers now
being favored. We culled this prediction from the finding
that most respondents believe they are smarter than aver-
age (Heck et al., 2018), a belief that seduces people into
thinking that they are able to peer more deeply into the rab-
bit hole than the average person.

Methods

Participants and Design. A total of 183 participants
were recruited by students at Brown University, again, via
direct and social media targeting. There were 94 (51.4%) fe-
male and 87 (47.5%) male participants. Two (1.1%) selected
the "nonbinary" option. Age ranged from 18 to 82 years
(M = 24.70, SD = 11.40). The experimental design featured
one within-subjects factor, namely number thinking (sim-
ple vs. meta thinking), and one between-subjects factor in-
troduced to counterbalance the order of the two tasks of
simple and meta thinking.

Procedure. After giving consent, participants were pre-
SENT with two tasks, namely to "simply think of a number
from 1 to 6" (simple thinking) and to "think of a number be-
tween 1 and 6, knowing that many other study participants
are asked the same. You are asked – as are the others –
to select the number that you think will be selected by the
fewest participants in this study" (meta thinking). The pre-
sentation of the two items was counterbalanced. Questions
about the participants’ demographics followed. Lastly, par-
ticipants were thanked and debriefed.

Results and Discussion

Analyses focused on the independent variable with two
levels, simple thinking and meta thinking. We aggregated
over the factor order, computing simple thinking first and
second as one level and did the same with meta thinking. The
dependent variable was the number, from 1 to 6, generated
by a respondent.

Choice distribution. The left half of Figure 3 shows that
respondents who were simply thinking, were more likely to
select mid-range numbers than floor or ceiling numbers. A
Kolmogorov-Smirnov test indicated that the observed dis-
btribution is unlikely under the null hypothesis of equality,
K-S = .104, p < .05. The right half of Figure 3 shows that re-
spondents in the meta thinking condition favored the num-
bers 1 and 5 and neglected the numbers 3 and 4. This dis-
tribution, too, significantly departed from an equal
distribution, too, significantly departed from an equal
K-S = .12, p < .02. The two empirical distributions
were also significantly different from each other, K-S
= .17, p < .02.

Improved hit rates and monetary gains. We can now
ask how much a thinking guesser stands to gain when facing
a thinking chooser. How much does a shared bias raise the
probability of matching numbers above the random base-

![Figure 3](http://online.ucpress.edu/collabra/article-pdf/7/1/24921/469135/collabra_2021_7_1_24921.pdf)

**Figure 3.** Visualization of numbers generated in the simple thinking (blue) and meta thinking (orange) condition (N = 183), with the horizontal dotted line showing the values expected for an equal distribution.

![Figure 4](http://online.ucpress.edu/collabra/article-pdf/7/1/24921/469135/collabra_2021_7_1_24921.pdf)

**Figure 4.** Cumulative probability of chooser and guesser selecting the same number, from 1 to 6, in the simple thinking condition (blue trend), meta thinking condition (orange), and assuming random number selection by both players (grey); f² = (frequency of a selected number)².

To obtain the guesser’s hit rate, we multiplied each number-choice frequency, that is, the frequencies of choice from one, two, three, four, five and six, with itself and summed these six squared frequencies. The result is about 16.7% assuming no bias (see Figure 4, grey trend). However, when assuming that both chooser and guesser are thinking of a number, the guesser’s hit rate increases by 2.5% to 19.2% (Figure 4, red dotted line). If chooser and guesser simply thought of a number from 1 to 6, the guesser’s hit rate would be 19.3% (Figure 4, blue trend), and 19.1% (orange trend) if both tried to think of a number that others would least choose (meta thinking).

A shared bias favors the guesser, but how much is it
worth? Bias-enabled increases in the guesser’s hit rate yield
larger expected values for the game than the expected value
assuming no bias, E(x) = $10.3 Because the guessing game
was designed along the lines of players simply thinking of a
number, bias matching on average increases the expected

\[ E = \frac{x}{y} \]

where x is the amount of money the guesser wins when guessing correctly and y corresponds to the number of options the
guesser can choose from. Knowing that the guesser can win 60$ and has six options to choose from it follows that, E = 60$ / 6.
value by 16%, to E(x) = $11.58. If stakes were higher, and many choosers were willing to play, a sophisticated guesser could exploit naive or overconfident opponents.

Experiments 3 and 4

We sought to replicate the findings of experiments 1 (with experiment 3) and 2 (experiment 4) in one preregistered study within a German sample (aspredicted.org/blind.php?x=hn9gb3).

Participants and Design. A total of 163 participants were recruited via a student-recruitment platform from the department of Social Psychology at the University of Mannheim (website: forschung-erleben.de). The sample consisted of 119 (73.0%) female and 41 (25.2%) male participants, while three respondents (1.8%) did not indicate an answer or preferred not to say. Age ranged from 18 to 62 years (M = 25.09, SD = 8.53). As in experiment 1, participants first completed a full within-participants design comprising two factors (chooser vs. guesser) with two levels each (randomize vs. think). Second, as in experiment 2, participants were randomly assigned to one of two conditions in a subsequent between-subjects design (simple thinking first vs. second).

Procedure. The procedures of experiments 3 and 4 were the same as the procedures of experiments 1 and 2 respectively. Instructions were in German.4

Results and Discussion

To repeat, the four conditions of the WTP study (experiment 3) had the following designations: r_r as the game in which both players randomly select a number; r_t as the game in which only the guesser thinks of a number; t_r as the game in which only the chooser thinks about a number; t_t, finally, refers to the game in which both players think instead of randomizing. The dependent variable was, again, the guesser’s willingness to pay (WTP)5 to participate in the game.

An ANOVA with two within-factors (chooser vs. guesser and random vs. thinking selection strategy) yielded a significant effect for the guesser’s choice of strategy, F(1, 162) = 10.33, p = .002, η² = .016, indicating that guessers were sensitive to the chooser’s number-generating strategy (think, M = 5.34 vs. random, M = 5.71). The significant effect for the guesser’s strategy choice, F(1, 162) = 4.33, p = .039, η² = .015, indicated that guessers were willing to pay more if they themselves could think of a number (M = 5.70) compared to random selection (M = 5.55). Again, however, the interaction effect was critical for our hypothesis, F(1, 162) = 8.22, p = .005, η² = .008. The pattern of means shown in Figure 5 corroborates the prediction.

Respondents preferred a game that allowed them to think of a number if the chooser was also thinking (t_t vs. t_r), F(1, 162) = 13.40, p < .001, η² = .076. The corresponding difference between the guesser’s strategies was not significant if the chooser selected a number randomly (r_t vs. r_r), p = .636. A Bayesian paired-samples t-test showed very strong evidence against the null hypothesis for games with thinking choosers (BF_{10} > 49), and strong evidence for the null hypothesis for games with randomizing choosers, BF_{10} = .098.

Choice distribution. In experiment 4, we focused on participants’ number selection in the simple thinking and the meta thinking condition. As shown in Figure 6 (left panel), for the simple thinking condition, respondents preferred mid-range numbers over floor or ceiling numbers, K-S = .160, p < .001.

In contrast, respondents in the meta thinking condition favored the numbers 1 and 2 and neglected the numbers 3 and 6 (see Figure 6, right panel). Here, the test against dis-

---

4 Oriented on the TRAPD-procedure of translation (Harkness et al., 2003), the English original was first translated into German by one of the authors (DJG). Second, the translation was reviewed by a first adjudicator and proposed changes were discussed with the translator. Finally, the resulting translation was backtranslated by a second adjudicator to check for translation errors.

5 German participants indicated their WTP in Euros (€). The exchange rate approximates to 1.00€ = $1.20.
tributional equality was marginally significant, K-S = .102, p < .10. The two empirical distributions, as in experiment 2, were significantly different from each other, K-S = .262, p < .001.

**Improved hit rates and monetary gains.** Similar to the English-language sample, the overall hit rate for the guesser increased by 5.0% to 19.7% (Figure 7, red dotted line), compared to the no-bias baseline with 16.7% (grey trend) if both the chooser and the guesser think. More specifically, if chooser and guesser simply thought of a number from 1 to 6, the guesser’s hit rate would be 21.6% (Figure 7, blue trend), and 17.9% (orange trend) if both tried to think of a number that others would be least likely to choose (meta thinking). With matching biases, the average expected value increased to E(x) = €12.96. This amounts to a monetary gain of nearly one third, 29.6% to be exact, of the prior expected value, E(x) = €10.

**General discussion**

As respondents generate numbers in a biased and non-random way (experiments 2 and 4), it is in the guesser’s interest to also think of a number if the chooser thinks. Consistent with this notion, respondents were willing to pay more to play the game as guessers if the chooser thought of a number and if they were also free to think of a number (experiments 1 and 3). Respondents appeared to realize that they stood to gain from matched number-generation biases. When in the shoes of the guesser against a human chooser, actively guessing a number is a strategy that weakly dominates randomization. That is, if there is any chance that the chooser thinks of a number, the guesser is better off also thinking, but the guesser cannot be worse off than when randomizing. Note, however, that even in this condition participants were not overpaying relative to the expected value of $10 or €10. In contrast, when gambling (Garrett & Sobel, 2004; Griffiths & Wood, 2001) or buying insurance (Pauly, 1990), people routinely pay in excess of the expected value.

**Cultural differences and future research.** The findings obtained with the U.S. American sample and the German sample were similar, but there is one noteworthy difference. For American respondents, the difference in WTP between t_t (both players think of a number) and r_r (only the guesser thinks) arose from the former being the most preferred option. For German respondents, the difference mainly emerged from the latter, t_r, being the least preferred option. We suspect that American respondents might have been more promotion focused, whereas German respondents might have been more prevention focused. Different regulatory foci (Higgins, 1997, 2012) might have affected which one of the four options served as the comparison default. Being more focused on promotion, American participants might have set the gain-option t_t as their default against which they evaluated the other options. In contrast, German participants might have used the loss-option t_r as the default. As a result, American participants embraced the option t_t, whereas German participants undervalued t_r. Consistent with this conjecture, American participants reported a higher WTP overall (M = $8.63) than German participants (M = 5.52€). This difference is consistent with Hofstede and colleagues’ (2010) finding of comparatively low uncertainty avoidance in the U.S. In an early study, Hofstede (1984) reported a comparatively high preference for gambling and other forms of risk-taking among U.S. American respondents. In contrast, uncertainty-avoidance in Germany is relatively high.

As noted by a reviewer, the descriptively lower WTP among participants playing as a randomizing guesser against a thinking chooser (t_r) instead of a randomizing chooser (r_r) might have been caused by anticipated frustration in the former situation. When the chooser thinks of a number, participants might be frustrated to not be able to also think because they are confident to win such a battle of wits. When the chooser randomizes, outplay is not possible and it is, hence, not frustrating for the guesser to also randomize. If this is true, participants in Germany, according to the present results (Figure 6), are more frustration averse than American participants (Figure 2). This conjecture is consistent with our assumption that German participants have a stronger prevention focus.

Our guessing game stylizes the dynamics of a sequential battle of wits. We introduced respondents to the game as guessers and told them to imagine a chooser who first has to generate a number. The temporal order provided by this design might have influenced perceptions and responses in unknown ways. In theory, choosers could make their selection after guessers make theirs. Such an inverted order would not affect the logic of what it is to be a guesser – only the guesser benefits from matched numbers – but it might affect the psychological construal of the game.

---

6 The amount of €5.52 equals an amount of $6.62.
Likewise, the chooser’s perspective is also of interest. Smart choosers will realize that they are exploitable by thinking guessers, but only if they themselves actively think of a number. If so, choosers should offer the smallest amount of money for a game in which both players are barred from randomizing and forced to think. But there is an intriguing alternative hypothesis. If choosers, like guessers, are overconfident and self-enhance, they might in fact prefer the think-think variant of the game the most and be willing to pay more in order to play it. In other words, the guesser might be able to outthink a thinker because the forces of rational inference, social projection, and overconfidence are aligned. The chooser might prefer the t-t version of the game only if they are overconfident. Rational inference and social projection would dissuade them from this preference. Future research should settle this issue.

Lastly, we note the need for future research to replicate the present findings in the context of real interaction between a chooser and a guesser playing for money.

Implications for reasoning. Research on k-level reasoning presents ex post (‘after the event’) distributions of players’ choices, and thus has little to say about the nature of the decision-making processes (Arad & Rubinstein, 2012). To our knowledge, theorists have assumed rather than shown that players believe themselves to be able to think at least one k-level past their competitors (Burchardi & Penczynski, 2014; Camerer et al., 2004; Stahl & Wilson, 1995). Burchardi and Penczynski (2014, p. 55) assert that “players do not expect other players to be of the same or a higher level of reasoning.” The axiomatic assumption of self-enhancement in strategic reasoning has been justified with appeals to, for example, research on overconfidence and limited cognitive resources. “The brain,” Camerer, Ho, and Chong (2004, p. 864) declare, “does not always understand its own limits,” and Burchardi and Penczynski (p. 48, footnote) report that “out of 12 players with a lower bound on K greater or equal to 2, the messages of 9 players [were] classified to have a degenerate population belief.” That is, 25% showed no sign of the expected belief. Gaining more insight into how people relate their own level of reasoning to the level of others is important to understand the role of recursive mentalization in multiple player games (e.g., Thomas et al., 2016).

So far, theories of k-level reasoning conflate sophisticated strategic reasoning with the self-enhancing idea that competitors think with less sophistication. If both players reason at the same level, no matter how deep, neither has the advantage. In fact, the higher the k-level, the greater the psychological cost of reaching and maintaining it. Given these difficulties, the modest engagement of $k = 0$ reasoning is efficient if both players limit themselves to it. Spiliopoulos and Hertwig (2020) show that a $k = 0$ reasoning performs well as a satisfying heuristic in many task ecologies. The number-guessing game explored in the present research is one such environment. Further, both the chooser and the guesser can avoid exploitation and burning cognitive resources if randomizing ab initio. Still, the guesser is free to self-enhance by simulating the chooser’s thoughts and hope that the chooser fails to randomize. A self-enhancing guesser chooses rationally by not randomizing.

When strategic choice requires a random event, the question is: Can people create random events? The answer is a qualified ‘no.’ A respondent who offers the number 2 as a random choice cannot demonstrate to others (or to the self) that the number was generated randomly. The psychological random number generator in the brain – if it exists – is not open to conscious inspection. A convincing argument for randomness requires a statistical evaluation of sequences of multiple events. With proper reinforcements, pigeons can learn to randomize their pecking (Page & Neuringer, 1985), and with the help of algorithmic feedback, humans can learn to produce unpredictable number sequences (Neuringer, 1986). Many prey animals produce unpredictable behavior in the wild. Fleeing from a predator, gazelles zigzag in ways hungry hunters find impossible to anticipate (Furuichi, 2002; Schaller, 1976). Truly random single-event human behavior, however, remains the province of fiction. In a cult novel, George Cockcroft (a.k.a. Luke Rinehart) described a “Diceman,” who liberated (and endangered) himself and others by relying on dice to determine his choices after he had laid down the array of options and the probability of each (Rinehart, 1971).

Conclusion

The battle of wits in the numbers game is, as Dixit and Nalebuff (2008) suggested, a simple (thought) experimental treatment of a larger issue. If the point of strategic thinking is competitive advantage, how can there be any advantage if both players are equally informed about the strategic options? We suggest that there cannot be any. We have argued and shown that the guesser’s competitive advantage emerges only when choosers fail to understand that their best hope is to keep themselves unpredictable by acting randomly. It is not without irony that DN’s attempt to introduce readers to the difference between non-strategic and strategic reasoning failed. DN did not see that choosers peering into the predictive rabbit hole will return to that strategy (i.e., randomization) DN present as being characteristic of nonstrategic games.

What then is the hallmark of strategic reasoning? Can it be more than the ability to take advantage of less rational opponents? There is a special role for mentalization and creativity, which lies outside the traditional purview of game theory. As a family of formal mathematical models, game theory takes it for granted that the rules of the game are understood, respected, and stable (von Neumann & Morgenstern, 1944). Psychological theories have no such constraints. These theories are more flexible in that they allow players to generate shifting construals of the games.

---

7 The belief that other people are on the k-1 level at most.

8 Students are impressed and perplexed when asked to raise their left hand with a probability of .3.
This flexibility opens the door to higher-order kinds of strategizing such as cunning (Krueger et al., 2020). The player who is the first to realize this meta-level reframing of the game can take the advantage. Such an initiative may not be fair; but it is creative and strategic. It wins by taking the element of surprise.

We return to the Princess Bride for an illustration of this critical point. Knight Westley has an initial advantage because he is both the architect of the game and a player. Vizzini’s rational response would be to refuse to play. But he played and died, never knowing what occurred. Westley later reveals to Princess Buttercup that he had poisoned both cups and swallowed an antidote. He solved the strategic challenge, which was unsolvable by conventional terms, by transcending the rules of the game. Creatively short-cutting the descent into the rabbit hole is one way not to get sucked in. The possibility of unfair playing has received little attention in the literature, but it has not gone completely unnoticed (Schelling, 1960). This strikes us as surprising because cheating and other kinds of creative innovation are not uncommon outside the lab. Used car dealers, business negotiators in their war rooms, and high-performing athletes are no strangers to it. Military men of historic rank made ample use of cheating, too. Arminius the German and Themistocles of Athens beat their foes by pretending to spy for them. They played dirty, but theory and research on strategic reasoning should not look away because such tactics are morally repugnant, for they reveal what can happen in real life. In the end, successful strategic reasoning may require the breaking of rules and being the first to do it. The latter is a strategic game in itself.

Contributions

Contributed to conception and design: DJG, JIK
Contributed to acquisition of data: DJG, JIK
Contributed to analysis of data: DJG
Contributed to interpretation of data: DJG, JIK
Drafted and/or revised the article: DJG, JIK
Approved the submitted version for publication: DJG, JIK

Funding information

The authors received no specific funding for this work.

Competing interests

The authors declare no competing interests.

Data accessibility statement

All the stimuli, presentation materials, participant data, and analyses can be found on this paper’s project page on OSF, Open Science Framework (https://osf.io/4r958/).

Submitted: January 30, 2021 PDT, Accepted: June 16, 2021 PDT
REFERENCES

Alicke, M. D., & Govorun, O. (2005). The better-than-average effect. In M. D. Alicke, D. A. Dunning, & J. I. Krueger (Eds.), The Self in Social Judgment (pp. 85–106). Psychology Press.

Arad, A., & Rubinstein, A. (2012). The 11-20 money request game: A level-k reasoning study. American Economic Review, 102(7), 3561–3573. https://doi.org/10.1257/aer.102.7.3561

Attali, Y., & Bar-Hillel, M. (2005). Guess where: The position of correct answers in multiple choice test items as a psychometric variable. Journal of Educational Measurement, 40(2), 109–128. https://doi.org/10.1111/j.1745-3984.2005.tb01099.x

Bar-Eli, M., & Azar, O. H. (2009). Penalty kicks in soccer: An empirical analysis of shooting strategies and goalkeepers’ preferences. Soccer & Society, 10(2), 183–191. https://doi.org/10.1080/1466070802601654

Bar-Eli, M., Azar, O. H., Ritov, I., Keidar-Levin, Y., & Schein, G. (2007). Action bias among elite soccer goalkeepers: The case of penalty kicks. Journal of Economic Psychology, 28(5), 606–621. https://doi.org/10.1016/j.joep.2006.12.001

Benndorf, V., Kübler, D., & Normann, H.-T. (2017). Depth of reasoning and information revelation: An experiment on the distribution of k-levels. International Game Theory Review, 19(4), 1750021. https://doi.org/10.1142/s0219198917500219

Burchard, K. B., & Penczynski, S. P. (2014). Out of your mind: Eliciting individual reasoning in one shot games. Games and Economic Behavior, 84, 39–57. https://doi.org/10.1016/j.geb.2013.12.005

Camerer, C. F., Ho, T.-H., & Chong, J.-K. (2004). A cognitive hierarchy model of games. The Quarterly Journal of Economics, 119(3), 861–898. https://doi.org/10.1162/0033553041502225

Chiappori, P.-A., Levitt, S., & Groseclo, T. (2002). Testing mixed-strategy equilibria when players are heterogeneous: The case of penalty kicks in soccer. American Economic Review, 92(4), 1138–1151. https://doi.org/10.1257/00028280260344678

Christenfeld, N. (1995). Choices from identical options. Psychological Science, 6(1), 50–55. https://doi.org/10.1111/j.1467-9280.1995.tb00304.x

Crawford, V. P., & Iriberri, N. (2007). Level-k auctions: Can a nonequilibrium model of strategic thinking explain the winner’s curse and overbidding in private-value auctions? Econometrica, 75(6), 1721–1770. https://doi.org/10.1111/j.1468-0262.2007.00810.x

Dixit, A. K., & Nalebuff, B. (2008). The art of strategy: A game theorist’s guide to success in business & life. Norton & Company.

Furuichi, N. (2002). Dynamics between a predator and a prey switching two kinds of escape motions. Journal of Theoretical Biology, 217(2), 159–166. https://doi.org/10.1006/jtbi.2002.3027

Garrett, T. A., & Sobel, R. S. (2004). State lottery revenue: The importance of game characteristics. Public Finance Review, 32(3), 313–330. https://doi.org/10.1177/1091142104264423

Goldman, W. (1975). The Princess Bride: S. Morgenstern’s classical tale of true love and high adventure. Harcourt.

Griffiths, M., & Wood, R. (2001). The psychology of lottery gambling. International Gambling Studies, 1(1), 27–45. https://doi.org/10.1080/14459800108732286

Harkness, J. A., Van de Vijver, F. J. R., & Mohler, P. (2005). Cross-cultural survey methods. Wiley.

Heck, P. R., & Krueger, J. I. (2015). Self-enhancement diminished. Journal of Experimental Psychology: General, 144(5), 1003–1020. https://doi.org/10.1037/xe0000105

Heck, P. R., Simons, D. J., & Chabris, C. F. (2018). 65% of Americans believe they are above average in intelligence: Results of two nationally representative surveys. PLOS ONE, 13(7), e0200103. https://doi.org/10.1371/journal.pone.0200103

Heinrich, T., & Wolff, I. (2012). Strategic reasoning in hide-and-seek games: A note [WORKINGPAPER]. http://kops.uni-konstanz.de/handle/123456789/18877

Higgins, E. T. (1997). Beyond pleasure and pain. American Psychologist, 52(12), 1280–1500. https://doi.org/10.1037/0003-066x.52.12.1280

Higgins, E. T. (2012). Regulatory focus theory. In P. A. Van Lange, A. W. Kruglanski, & E. T. Higgins (Eds.), Handbook of theories of social psychology (pp. 483–504). Sage Publications. https://doi.org/10.4135/9781446249215.n24
Hofstede, G. (1984). Cultural consequences: International differences in work related values (Rev. 5th ed). Sage Publications.

Hofstede, G., Hofstede, G. J., & Minkov, M. (2010). Cultures and organizations: Software of the mind (Rev. 3rd ed.). McGraw-Hill.

Keynes, J. M. (2018). The general theory of employment, interest, and money. Springer. https://doi.org/10.1007/978-3-519-70344-2 (Original work published 1936)

Krueger, J. I. (1998). On the perception of social consensus. Advances in Experimental Social Psychology, 30, 165–240. https://doi.org/10.1016/s0065-2601(08)60384-6

Krueger, J. I. (2013). Social projection as a source of cooperation. Current Directions in Psychological Science, 22(4), 289–294. https://doi.org/10.1177/0963721413481352

Krueger, J. I., & Acevedo, M. (2007). Perceptions of self and other in the prisoner’s dilemma: Outcome bias and evidential reasoning. The American Journal of Psychology, 120(4), 593–618. https://doi.org/10.2307/20445427

Krueger, J. I., Heck, P. R., Evans, A. M., & DiDonato, T. E. (2020). Social game theory: Preferences, perceptions, and choices. European Review of Social Psychology, 31(1), 322–353. https://doi.org/10.1080/10463283.2020.1778249

Langer, E. J. (1975). The illusion of control. Journal of Personality and Social Psychology, 32(2), 511–528. http://doi.org/10.1037/0022-3514.32.2.511

Lee, M. D., & Wagenmakers, E.-J. (2014). Bayesian cognitive modeling: A practical course. Cambridge University Press. https://doi.org/10.1017/cbo9781139087759

Moore, D. A. (2020). Perfectly confident: How to calibrate your decisions wisely. Harper Business.

Nagel, R. (1995). Unraveling in guessing games: An experimental study. The American Economic Review, 85(5), 1315–1326. http://www.jstor.org/stable/2950991

Neuringer, A. (1986). Can people behave “randomly?”: The role of feedback. Journal of Experimental Psychology: General, 115(1), 62–75. https://doi.org/10.1037/0096-3445.115.1.62

Page, S., & Neuringer, A. (1985). Variability is an operant. Journal of Experimental Psychology: Animal Behavior Processes, 11(3), 429–452. https://doi.org/10.1037/0097-7403.11.3.429

Pauly, M. V. (1990). The rational nonpurchase of long-term care insurance. Journal of Political Economy, 98(1), 153–168. https://doi.org/10.1086/261673

Rinehart, L. (1971). The diceman. W. Morrow.

Rothbart, M., & Snyder, M. (1970). Confidence in the prediction and postdiction of an uncertain outcome. Canadian Journal of Behavioural Science/Revue canadienne des sciences du comportement, 2(1), 38–43. https://doi.org/10.1037/h0082709

Rubinstein, A., Tversky, A., & Heller, D. (1997). Naive strategies in competitive games. In W. Albers, W. Güth, P. Hammerstein, B. Moldovanu, & E. van Damme (Eds.), Understanding strategic interaction: Essays in Honor of Reinhard Selten (pp. 394–402). Springer. https://doi.org/10.1007/978-3-642-60495-9_30

Schaller, G. B. (1976). The serengeti lion: A study of predatory-prey relations. University of Chicago Press. https://doi.org/10.7208/chicago/9780226736600.001.0001

Scheinman, A. (Producer), & Reiner, R. (Director). (1987). The Princess Bride [Motion Picture]. Act III Communications.

Schelling, T. C. (1960). The strategy of conflict. Harvard University Press.

Spiliopoulos, L., & Hertwig, R. (2020). A map of ecologically rational heuristics for uncertain strategic worlds. Psychological Review, 127(2), 245–280. http://doi.org/10.1037/rev0000171

Stahl, D. O., & Wilson, P. W. (1995). On players’ models of other players: Theory and experimental evidence. Games and Economic Behavior, 10(1), 218–254. https://doi.org/10.1006/game.1995.1051

Teigen, K. H. (1983). Studies in subjective probability I: Prediction of random events. Scandinavian Journal of Psychology, 24(1), 13–25. https://doi.org/10.1111/j.1467-9450.1983.tb00471.x

Thomas, K. A., De Freitas, J., DeScioli, P., & Pinker, S. (2016). Recursive mentalizing and common knowledge in the bystander effect. Journal of Experimental Psychology: General, 145(5), 621–629. https://doi.org/10.1037/xge0000153

von Neumann, J., & Morgenstern, O. (1944). Theory of games and economic behavior. Princeton University Press.
Appendix

Appendix A. Vizzini’s full monologue as an illustration of k-level reasoning.

"[I]t's so simple," Vizzini begins confidently, "all I have to do is divine from what I know of you. Are you the sort of man who would put the poison into his own goblet, or his enemy's? Now, a clever man would put the poison into his own goblet, because he would know that only a great fool would reach for what he was given. I'm not a great fool, so I can clearly not choose the wine in front of you [here Vizzini advances from $k = 0$ to $k = 1$]. But you must have known I was not a great fool; you would have counted on it, so I can clearly not choose the wine in front of me [$k = 2$]." He continues: "Iocaine [poison] comes from Australia, as everyone knows. And Australia is entirely peopled with criminals. And criminals are used to having people not trust them, as you are not trusted by me. So I can clearly not choose the wine in front of you [$k = 3$]. [...] And you must have suspected I would have known the powder's origin, so I can clearly not choose the wine in front of me [$k = 4$]. [...] You're exceptionally strong. So, you could have put the poison in your own goblet, trusting on your strength to save you. So I can clearly not choose the wine in front of you [$k = 5$]. But, you've also bested my Spaniard which means you must have studied. And in studying, you must have learned that man is mortal so you would have put the poison as far from yourself as possible, so I can clearly not choose the wine in front of me [$k = 6$]." At this point, Vizzini triumphantly declares: "It has worked – you've given everything away – I know where the poison is."
SUPPLEMENTARY MATERIALS

Peer review history
Download: https://collabra.scholasticahq.com/article/24921-strategic-thinking-a-random-walk-into-the-rabbit-hole/attachment/63371.docx?auth_token=RVo1zYjwDCQtwlqCMsM