INTRODUCTION

Shale gas reservoirs are characterized by low porosity and permeability. As a result, conventional development methods cannot be used for commercial production of the gas. Therefore, hydraulic fracturing technology is required to achieve higher yields.1-5 However, this method can seriously threaten the integrity (eg, sealing failure) of the cement sheath, leading to sustained casing pressure (SCP).6 The reason for SCP is that the down-hole fluid migrates to the wellhead through continuous leakage paths, such as a microannulus between the casing and cement sheath or between the cement sheath and formation, microcracks in cement sheath, and casing threads, and then, the gas accumulates at the wellhead.7-9 That is, the formation of SCP has two necessary conditions.10,11 One is the gas/fluid source, and the other is the continuous leakage paths.

Because the cement sheath is an important part of the wellbore barrier, it has been the focus of extensive research. The failure modes of the cement sheath usually include debonding,
shear failure, radial cracking, and disking. These make continuous gas migration paths possible.\textsuperscript{12-14} The stress state of the cement sheath is mainly affected by wellbore pressure fluctuation, temperature variation, tectonic movement, casing shoe leakage or chemical corrosion, poor displacement efficiency, the existence of mud cake on the bonding surface, and poor cementing quality, which may also lead to leakage paths.\textsuperscript{15,16} The tangential and radial stresses in the cement sheath can be obtained based on elastic-plastic theory.\textsuperscript{17} Li et al\textsuperscript{18} provided an analytical solution to simulate the effect of cement/formation stiffness on well integrity evaluation in carbon sequestration projects. Xi et al\textsuperscript{19} combined an integrated method of analytical and numerical simulation to analyze the transient stress variation in the cement sheath when the temperature at the bottom of the well changed. Chu et al\textsuperscript{20} established a new analytical model to study the microannulus mechanism under cyclic casing internal pressure. Xu et al\textsuperscript{21} investigated the thermal stress of the cement sheath for geothermal wells during fracturing by the Gaussian main elimination method. Feng et al\textsuperscript{22} proposed the use of a cohesive zone element (CZM) to simulate the debonding of the bonding surface in carbon dioxide injection wells. Based on the CZM and the Mohr-Coulomb criterion, Yin et al\textsuperscript{6} provided a novel assessment and countermeasure for microannulus initiation during injection/fracturing. In experimental research, Goodwin and Crook\textsuperscript{23} built casing cement sheath equipment to analyze the effects of pressure and temperature on the integrity of the cement sheath. The mechanical properties of cement stone changed significantly under high-temperature and high-pressure conditions. Cyclic stress and temperature cause low cycle fatigue of cement stone.\textsuperscript{24} Zhou et al\textsuperscript{25} built a full-scale physical device to simulate the integrity of the cement sheath under cyclic casing pressure.

The SCP phenomenon after fracturing of shale gas wells in the Sichuan basin of China is significant. The ratio in some fields is more than 50%, but the ratio before fracturing is only 15.84%.\textsuperscript{19} Thus, hydraulic fracturing has a significant impact on the sealing integrity of the cement sheath. This study is different from traditional research because the wellbore cement sheath was taken as a whole, and the effects of single and cyclic fracturing on the integrity of the cement sheath were analyzed. In this study, the failure mechanism and failure mode of the cement sheath at different well depths were obtained, and reasons for the occurrence of continuous leakage paths in shale gas wells were clarified.

The following is the research methodology of this study. The source of wellhead gas was determined by the isotope analysis method first. Then, combined with the in situ stress profile, formation mechanics parameter profile, and casing internal pressure, the casing–cement-sheath–formation assemblies at different well depths were established, the tangential and radial stresses of the cement sheath at different depths were analyzed, and the integrity of the cement sheath was judged by the Mohr-Coulomb failure criterion. Next, a cyclic compression test of cement stone was carried out, and the formation mechanism of a microannulus was analyzed with the results. Finally, through monitoring the volumetric strain of cement stone in a triaxial compression test, the relationship between dilatation and permeability was clarified, and the permeability variations under cyclic compression tests were measured.

2 | NUMERICAL SIMULATION

In this section, the source of the wellhead gas of the shale gas well is analyzed first. Then, an actual shale gas well is taken as an example. Based on the well structure, rock mechanics profile, and in situ stress profile, the two-dimensional numerical calculation models of the casing–cement-sheath–formation assemblies during fracturing were established, and the radial and tangential stress profiles of cement sheath were given. The integrity was analyzed by using the Mohr-Coulomb failure criterion.

2.1 | Gas source analysis

To analyze the source of the gas in the wellhead, CH\textsubscript{4} isotope analysis of gas samples between the surface casing and intermediate casing annulus, intermediate casing, and tubing

| Name      | Gas sample                                  | $\delta^{13}$C (‰) | Gas source                         |
|-----------|---------------------------------------------|--------------------|------------------------------------|
| F7-1HF    | Product gas                                 | −31.52             | Target fracturing formation         |
| F7-1HF    | Between surface casing and intermediate casing | −52.08             | Shallow formation                   |
| F7-1HF    | Between intermediate casing and tubing      | −31.49             | Target fracturing formation         |
| F7-2HF    | Between intermediate casing and tubing      | −31.52             | Target fracturing formation         |
| F20-2HF   | Between intermediate casing and tubing      | −31.11             | Target fracturing formation         |

\textit{TABLE 1} Results of the isotope analysis
annulus was carried out for three SCP wells by measuring the value of $\delta^{13}C_{\text{PDB}}$. The results are shown in Table 1. The value of $\delta^{13}C_{\text{PDB}}$ of the product gas was $-31.52$, which was similar to the value between the intermediate casing and tubing annulus in wells F7-1HF, F7-2HF, and F20-2HF. However, the value of the gas between the surface casing and intermediate casing annulus in well F7-1HF was $-52.08$, which is obviously different from the value of the product gas. Because of the difference of formation conditions and the time of CH$_4$, it can be concluded that the gas between the surface casing and intermediate casing annulus comes from a shallow formation, whereas the gas between the intermediate casing and product casing comes from the targeted fracturing formation.

2.2 Modeling process

A shale gas well in the Sichuan shale gas block of China is taken as example. The measuring depth was 5400 m, and the vertical depth was 3850 m. The depths of the surface casing shoe, intermediate casing shoe, and the kickoff point were 1550, 3150, and 3270 m, respectively, and the designed pump pressure was 90 MPa. In this study, 2D numerical models were used instead of 3D numerical models of the casing–cement-sheath–formation assembly. The numerical models were used to analyze the stress state of cement sheath conditions during hydraulic fracturing, and the analysis was performed with ABAQUS software. This software is designed to perform and postprocess simulations of different well depths.

The diagram of the well structure and the numerical model at different depths are shown in Figure 1.

According to the well structure shown in Figure 1, the finite-element models of the casing–cement-sheath–formation assembly are different at different well depths, and they can be divided into three parts. The first is the finite-element models above the surface casing shoe, which are mainly used to analyze the integrity of the cement sheath from the wellhead to the surface casing shoe. The finite-element model is shown in Figure 1A. The second is the finite-element models between the surface casing shoe and the intermediate casing shoe, which are mainly used to analyze the integrity below the surface casing shoe and above the intermediate casing shoe. The finite-element model is shown in Figure 1B. The third is the casing–cement-sheath–formation assembly, which is mainly used to analyze the integrity from the intermediate casing shoe to the landing point. The finite-element model is shown in Figure 1C. Because the lithology and in situ state in the horizontal section of the shale gas well are basically similar, the target cement sheath in this study was mainly from the wellhead to the landing point. The geometric parameters of the casing, cement sheath, and formation at different depths are presented in Table 2.

2.2.1 Basic assumptions

For the wellbore cement sheath calculation model, some basic assumptions were made, as follows.

![FIGURE 1 Well structure and the corresponding finite-element models](image-url)
The casing–cement-sheath–formation assembly is simplified to be a plane strain model. The reason is that the deformation of the cement sheath in the vertical direction is generally much smaller than the horizontal plane under the effects of in situ stress, casing internal pressure, and thermal stress.

The casing, cement sheath, and formation are all considered as homogeneous isotropic materials.

The mechanical properties of the cement sheath in different annuli and at different well depths are the same.

The casing–cement-sheath–formation assembly is completely cemented. There is no shrinkage or expandable behavior during the hardening process.

The casing and the cement sheath are concentric. The case of casing eccentricity is not considered.

### 2.2.2 Material parameters and load setting

The axial displacement along the wellbore is small and can be neglected. Thus, the numerical models can be simplified into plane strain models. The in situ stress profile and mechanical parameter profiles of formation are shown in Figures 2 and 3.

Figure 2 shows that the in situ stress in three directions of formation increases linearly with the increase in depth, the applied horizontal stresses are 93 and 88 MPa, and the vertical stress is 99 MPa at the landing point (inclination = 90°). Figure 3 shows Young’s modulus and the cohesion increase with the depth caused by the formation compaction. Young’s modulus at 510 m varies, because the formation is in a position of severe lithologic alternation. The law of Poisson’s ratio and internal friction angle of formation vs depth are basically opposite to the law of Young’s modulus.

This study’s focus was on the sealing integrity of the cement sheath. It does not involve the plastic deformation of the casing; therefore, the casing was regarded as an ideal elastic material, and Young’s modulus and the Poisson ratio of the casing were 210 GPa and 0.3, respectively. In this section, the cement sheath is considered as ideal elastic material, and Young’s modulus and Poisson ratio of the cement sheath are 8 GPa and 0.17, respectively. In terms of load setting, Equation (1) is used to calculate the casing internal pressure at different depths.

$$P_i = P_0 + \frac{\rho gh}{10^6}$$  \hspace{1cm} (1)

Here, $P_i$ represents the casing inner pressure, $P_0$ represents pump pressure, $\rho$ represents density, $g$ represents gravity acceleration, and $h$ represents vertical depth. The fluid friction is neglected here.

The in situ stress acting on the formation can be set by a predefined field function in ABAQUS. For the finite-element models above the kickoff point, the maximum horizontal stress and minimum horizontal stresses are set in the X-direction and Y-direction, respectively, and the vertical in situ stress is set in the Z-direction (Figure 1). For the finite-element models in inclined section, the in situ stress needs to be transformed with the coordinate transformation method. The specific transformation method is shown in Equations (2) and (3).

\[
\begin{bmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_{zz}
\end{bmatrix} = [L] \begin{bmatrix}
\sigma_H \\
\sigma_h \\
\sigma_v
\end{bmatrix} [L]^T
\]  \hspace{1cm} (2)

\[
L = \begin{bmatrix}
cos \varphi \cos \Omega & \cos \varphi \sin \Omega & -\sin \varphi \\
-\sin \varphi \cos \Omega & \cos \varphi & 0 \\
\sin \varphi \cos \Omega & \sin \varphi \sin \Omega & \cos \varphi
\end{bmatrix}
\]  \hspace{1cm} (3)

| Vertical depth (m) | Surface casing OD (mm) | Surface casing ID (mm) | Intermediate casing OD (mm) | Intermediate casing ID (mm) | Tubing OD (mm) | Tubing ID (mm) | Wellbore diameter (mm) |
|-------------------|-------------------------|------------------------|----------------------------|---------------------------|----------------|----------------|------------------------|
| 0-1550            | 339.7                   | 315.3                  | 244.5                      | 222.4                     | 139.7          | 118.6          | 444.5                  |
| 1550-3500         | –                       | –                      | 244.5                      | 222.4                     | 139.7          | 118.6          | 311.2                  |
| 3500-3850         | –                       | –                      | –                          | –                         | 139.7          | 118.6          | 215.9                  |

Note: OD refers to the outer diameter, and ID refers to the inner diameter.

**FIGURE 2** In situ stress profile

**TABLE 2** Geometric parameters of casing, cement sheath, and formation
Here, $\varphi$ represents deviation angle, $\Omega$ represents azimuth angle, and the extension direction of the horizontal section is generally perpendicular to the direction of maximum in situ stress, so the azimuth angle here is $90^\circ$, and $\sigma_{ij}$ ($i, j = x, y, z$) are the principle stresses ($i = j$) and shear stresses ($i \neq j$) in different directions after coordinate transformation in megapascals.

2.2.3 Contact model and meshing method

The cohesive zone method (CZM) was used to simulate bonding between casing and cement sheath, and cement sheath and formation. The failure behavior of CZM includes the linear elastic traction-separation model, damage initiation criteria, and damage evolution laws. As shown in Figure 4, the CZM will undergo two stages. The elastic phase: The tensile stress or shear stress increases linearly with the increase in tensile displacement or shear displacement, the slope of the line AB represents the stiffness; at this phase, the damage value of CZM is 0. The damage phase: When the tensile stress or shear stress reaches to the critical strength, there is irreversible damage to the bonding surface; the stiffness is reduced from $K_{nn}$ to $K_n$, and the stiffness decreases with the displacement. The stiffness is reduced to be 0 at the point C. The area enclosed by triangular ABC is the critical fracturing energy.

In the elastic phase, the relationship between the stress and displacement satisfy the linear relationship, and the following is the constitutive relations,

$$ t = \begin{bmatrix} t_n \\ t_s \\ t_t \end{bmatrix} = K \varepsilon = \begin{bmatrix} K_{nn} & K_{ns} & K_{nt} \\ K_{ns} & K_{ss} & K_{st} \\ K_{nt} & K_{st} & K_{tt} \end{bmatrix} \begin{bmatrix} \varepsilon_n \\ \varepsilon_s \\ \varepsilon_t \end{bmatrix} \quad (4) $$

FIGURE 3 Mechanical parameter profiles of formation: (A) Young's modulus, (B) Poisson's ratio, (C) Cohesion stress, and (D) Friction angle

FIGURE 4 Failure process of cohesive zone method; (A) Traction-separation law, (B) Shear-separation law
where \( \mathbf{t} \) is the stress vector such that \( t_r, t_t, \) and \( t_s \) are the stress in the normal, first tangential, and second tangential directions, respectively; \( \mathbf{K} \) is the stiffness matrix; and \( \mathbf{e} \) is the displacement vector such that \( \varepsilon_r, \varepsilon_s, \) and \( \varepsilon_t \) are the displacement in the normal, first tangential, and second tangential directions, respectively.

The failure of CZM is generally affected by both tensile stress and shear stress, the damage occurrence and corresponding softening behavior under composite mode may occur before any stress in one direction reaches their respective simple mode failure criteria.\(^6\) Therefore, the quadratic nominal stress criterion is more applicable than the maximum stress criterion.\(^7,8\) This criterion can be expressed as,

\[
\left( \frac{t_{r0}}{t_{n0}} \right)^2 + \left( \frac{t_{s0}}{t_{t0}} \right)^2 + \left( \frac{t_{t0}}{t_{t0}} \right)^2 = 1 \tag{5}
\]

where \( t_{n0} \) is the critical stress in the normal direction, while \( t_{s0} \) and \( t_{t0} \) are the critical stresses in the first tangential and second tangential directions, respectively; the symbol \( < > \) indicates that the CZM will not damage when subjected to compressive stress. When the stresses in different directions satisfy the Equation (5), the CZM enters the damage evolution stage.

For linear softening, the degree of damage can be described by using the stiffness degradation coefficients, and the damage factor \( D \) can be calculated as,

\[
D = \frac{d_m (d_{m}^{\text{max}} - d_m^0)}{d_{m}^{\text{max}} - d_m^0} \tag{6}
\]

where \( d_{m}^{\text{max}} \) is the maximum displacement in the process of uploading; \( d_m^0 \) is the displacement when the CZM is completely destroyed; and \( d_m^0 \) is the displacement under initial damage.

The failure mode of B-K fracture energy is used to govern damage evolution.\(^9\)

\[
G_c^n + \left( G_s^n - G_c^n \right) \left\{ \frac{G_S}{G_T} \right\}^n = G_c^n + G_s^n + G_t^n \tag{7}
\]

where \( G_c^n, G_s^n, \) and \( G_t^n \) are the energy dissipation due to deformations in the normal, the first shear, and the second shear directions respectively; \( G_c^n, G_s^n, \) and \( G_t^n \) are the critical energies required to cause failure in the normal, the first shear, and the second shear directions, respectively; \( \eta \) is the power factor.

The cohesive properties of casing-cement-formation system are shown in Table 3.\(^6\)

The displacement of the outer edges of the formation is fixed. The outer diameter of the model is a 4.5 m × 4.5 m square, which is ten times greater than that of the borehole, thus allowing the influence of boundary to avoid the effects on the stress. In the meshing method, there is an application of the structured grid and the variable density meshing method, and all the element types are CPE4.

It is necessary to describe how to use Python scripts to generate the finite-element models for the whole wellbore casing–cement–sheath–formation assembly. Firstly, the corresponding INP files are generated with ABAQUS according to the finite-element model in different spuddings. The formation mechanical parameters, in situ stress parameters, and casing internal pressure in INP files are replaced by Python scripts, so that the INP files corresponding to different depth can be generated. The postprocessing code can be generated by Python scripts based on log file from ABAQUS; in terms of output, the maximum tangential and radial stress of cement sheath in different depth should be obtained.

### 2.2.4 Failure criteria

The cement sheath is a typical brittle material, and it is prone to tensile failure under tensile stress and compressive failure or plastic yield under compressive stress. When it is subjected to unidirectional load, the integrity can be judged by the maximum principal stress criterion. However, when both tensile stress and compressive stress exist in different directions, the Mohr-Coulomb failure criterion is usually used to judge the integrity. The Mohr-Coulomb failure criteria in Table 4 can be used to judge the integrity under different stress states in a cylindrical coordinate system, \( \sigma_1 = \sigma_p, \sigma_3 = \sigma_n \), where \( \sigma_\theta \) and \( \sigma_r \) are the tangential and radial stress, respectively.

Here, \( \sigma_1 \) and \( \sigma_3 \) are the maximum and minimum principal stresses, \( \sigma_r \) and \( \sigma_\theta \) are the tensile and compressive strengths, and the cement sheath fails once the judgment value is larger than 1.

| Parameter                      | Casing-cement interface | Cement-formation interface |
|--------------------------------|-------------------------|----------------------------|
| Normal strength (MPa)          | 0.5                     | 0.42                       |
| Shear strength (MPa)           | 2                       | 0.42                       |
| Cohesive stiffness (GPa)       | 30                      | 30                         |
| Critical energy (J/m²)         | 100                     | 100                        |

#### Table 3 Parameters of the cohesive zone method (CZM)
3 | ANALYSIS OF INTEGRITY OF CEMENT SHEATH IN SINGLE-STAGE FRACTURING

In this section, the cement sheath was regarded as an elastic material, and the correctness of the finite-element model was validated by a full-scale cement sheath sealing capacity assessment device. Then, the radial and tangential stresses of the cement sheath at different depths were calculated in single-stage fracturing. This integrity of the cement sheath under the combined effects of tangential and radial stress was judged by the Mohr-Coulomb failure criterion, and the factors affecting the integrity were examined, including Young’s modulus, Poisson’s ratio of the cement sheath, and the pump pressure.

3.1 | Model validation

As shown in Figure 5A, to verify the correctness of the contact mode, loading setting, and meshing method in the numerical model, physical simulation experiments were carried out with a full-scale cement sheath sealing capacity assessment device. The inner-casing–cement-sheath–outer-casing assembly was used to simulate the casing–cement-sheath–formation assembly, as in Figure 5B,C. The pressure system applied periodic loading/unloading pressure to the casing to simulate the stress state of the cement sheath during fracturing. There were stress sensors between the casing and cement sheath to monitor the stress state. The concepts of tangential stress ($\sigma_\theta$) and radial stress ($\sigma_r$) are shown in Figure 5D, where $P_0$ and $P_i$ represent the contact stress in the outer and inner walls of the cement sheath, respectively. The size of the casing was inner diameter, 124.26 mm; outer diameter, 139.7 mm; and height, 1200 mm. The size of the outer casing was inner diameter, 193.7 mm; outer diameter, 244.5 mm; and height, 1200 mm. The grades of the inner and outer casings were P110 and N80, respectively, and Young’s modulus, Poisson’s ratio, tensile strength, and compressive strength of the cement sheath were 8 GPa, 0.17, 4.2 MPa, and 51.9 MPa, respectively.

![Diagram](image)

**TABLE 4** Mohr-Coulomb failure criteria[^19]

| Stress interval | Description of interval | Relationship among principal stresses | Failure standard | Judgment value |
|-----------------|-------------------------|--------------------------------------|-----------------|---------------|
| 1               | Tension-tension-tension | $\sigma_1 \geq \sigma_3 \geq 0$      | $\sigma_1 \geq \sigma_r$ | $\sigma_1/\sigma_r$ |
| 2               | Compression-compression-compression | $0 \geq \sigma_1 \geq \sigma_3$ | $-\sigma_3 \geq \sigma_r$ | $-\sigma_3/\sigma_r$ |
| 3               | Tension-compression-compression; Tension-tension-compression | $\sigma_1 \geq 0 \geq \sigma_3$ | $\frac{\sigma_1}{\sigma_r} - \frac{\sigma_3}{\sigma_r} \geq 1$ | $\frac{\sigma_1}{\sigma_r} - \frac{\sigma_3}{\sigma_r}$ |

**FIGURE 5** Full-scale cement sheath sealing capacity assessment device: (A) testing device, (B) profile of casing–cement-sheath–outer-casing assembly, (C) plain strain model, and (D) concept of tangential stress ($\sigma_\theta$) and radial stress ($\sigma_r$)
Considering the loading capacity of the device, the loading/unloading casing inner pressures were 70 and 0 MPa. The results of the stress sensor between the inner casing and cement sheath are shown in Figure 6.

The finite-element model of the casing–cement-sheath–outer-casing assembly corresponding to the assessment device was established. The numerical model is shown in Figure 5C. The contact model and meshing method were the same as in Sections 2.2.1 and 2.2.2. A liquid column pressure of 70 MPa was applied to the inner wall of the casing. The radial and tangential stresses of the cement sheath are shown in Figure 7.

The stress of the cement sheath shown in Figure 6A varied periodically because of the cyclic pressure. When the pressure was 70 MPa, the radial compressive stress measured by the stress sensor was 22.7 MPa and the tangential tensile stress was 4.6 MPa. As shown in Figure 5B, the cement sheath bears a high tangential tensile stress under high casing inner pressure, which results in tangential tensile cracks.

As shown in Figure 7A,B, the radial and tangential stresses at the inner wall of the cement sheath are 23.26 and 4.74 MPa, respectively. The differences between the assessment device and the numerical calculation results are 2.5% and 3.0%. This proves the correctness of the meshing method, load setting, and contact model of the numerical model.

### 3.2 Stress state of cement sheath in single-stage fracturing

#### 3.2.1 Calculation results

Based on the well structure, in situ stress profile, formation mechanical parameter profiles, and casing inner pressure at

**FIGURE 6** Experimental results: (A) recordings of stress sensor, (B) tensile cracks of cement sheath

**FIGURE 7** Numerical results of cement sheath: (A) radial compressive stress and (B) tangential tensile stress
different depths, the stress state of the cement sheath between the intermediate casing and tubing can be calculated, and the results are shown in Figure 8.

A negative maximum radial stress means that the cement sheath is subjected to radial compressive stress, and a positive maximum tangential stress means that the inner wall of the cement sheath is subjected to tangential tensile stress. Figure 8A shows that the maximum radial stress increases with the depth. The reason is that both in situ stress and casing inner pressure increase with the increase in well depth, which increases the degree of radial compression. Figure 8B shows that the maximum tangential stress decreases with depth. The reason is that the increasing speed of in situ stress with well depth is greater than that of casing internal pressure, so the effect of in situ stress on encapsulation on the casing–cement-sheath–formation assembly is stronger than the expansion effect of casing internal pressure. Figure 8C clearly shows that the judgment value first decreases with the depth and then increases with the depth below the intermediate casing shoe. This is because the judgment value above the intermediate casing shoe is mainly affected by tangential tensile stress, and the value below the intermediate casing shoe is mainly affected by radial compressive stress. The judgment value is larger than 1 in general, so the cement sheath is at risk of failure during the single-fracturing stage. The stress component changes abruptly at the casing shoe, either near the surface casing shoe or near the intermediate casing shoe. For example, at 1550 m, the maximum radial and tangential stresses are 27.9 and 6.799 MPa, respectively.

**FIGURE 8** Integrity analysis of cement sheath in single-stage fracturing: (A) distribution of the radial stress, (B) distribution of the tangential stress, and (C) distribution of the judgment value.
but, when the depth is 1560 m, the corresponding stresses are 31.09 and 5.187 MPa, which are an increase of 11.4% and decrease of 23.7%, respectively. The stress comparisons are shown in Figures 9 and 10.

When the depth of the cement sheath is less than 3150 m and the maximum tangential stress is larger than 3.309 MPa, combined with the radial stress, the cement sheath above the intermediate casing shoe has tangential tensile cracks in the single-stage fracturing, becoming part of the continuous leakage path.

3.2.2 | Sensitivity analysis of factor stress sensitivity

Young's modulus

Because this is one of the most important mechanical parameters of the cement sheath, it is generally believed that the ideal mechanical properties of the cement sheath are high compressive strength and low stiffness, which means that a large elastic deformation range is favorable. The influence of Young's modulus on the stress of the cement sheath or casing has been analyzed by other scholars, but the effect of Young's modulus on the integrity of the whole wellbore cement sheath has not been analyzed. With other parameters remaining unchanged, Young's modulus of the cement sheath was set at 4, 6, 8, and 10 GPa. The calculation results are shown in Figure 11.

Figure 11A,B show that reducing Young's modulus is beneficial for alleviating the maximum radial/tangential stress, and the reduction in Young's modulus under the surface casing shoe is more significant than that over the surface casing shoe; for example, when Young's modulus is reduced from 8 to 4 GPa, the maximum radial stress is reduced from 19.27 to 12.32 MPa at a depth of 0 m, with a decrease of 56.4%. However, the corresponding stresses are 45.14 and 26.65 MPa at the depths of 3155 m, with a decrease of 69.4%. However, the tangential stress above the surface casing shoe is more sensitive to the change in Young's modulus. Therefore, the cement sheath with a low Young's modulus and high tensile strength is better in the vertical section, especially above the surface of the casing shoe. Figure 11C shows that the judgment value can be significantly reduced with a lower Young's modulus cement sheath. The decrease in Young's modulus means the increase in elastic deformation ability, which causes the formation and casing to bear more stress under the same external load. Therefore, the decrease in Young's modulus can relieve the stress state of the cement sheath itself.

Poisson's ratio

To increase the elasticity and toughness of the cement sheath, some elastic materials, such as latex and fibers, are often added to the cement slurry. Poisson's ratio changes as Young's modulus changes. To analyze the influence of Poisson's ratio on the stress state of the cement sheath in the whole wellbore, with other parameters remaining unchanged, Poisson's ratio was set at 0.14, 0.17, 0.20, and 0.23. The calculation results are shown in Figure 12.

As shown in Figure 12A,B, the maximum tangential stress is greatly affected by Poisson's ratio. Increasing Poisson's ratio can significantly reduce the tangential stress and even make the circumferential stress at a certain depth change from tension to compression. This is because the increase in Poisson's ratio means the increase in tangential deformation capacity, and the increase in the tangential deformation ability can decrease the circumferential tensile stress. However, the change in Poisson's ratio has little effect on the maximum radial stress. Figure 12C shows that the judgment value decreases with the increase in Poisson's ratio. When Poisson's ratio is 0.14, the law of judgment value vs depth at a depth of 3150 m is different from that at such Poisson's ratios as...
0.17, 0.20, or 0.23. The main reason is that, when Poisson's ratio is 0.14, the maximum tangential stress under 3150 m is positive, and, when Poisson's ratio is 0.17, 0.20, or 0.23, the maximum tangential stress is negative, which leads to different calculation methods for the judgment value.

Wellhead pump pressure
Because of the strong heterogeneity of shale formation, the wellhead pump pressure varies widely during hydraulic fracturing, so it is necessary to study the influence of different pump pressures on the stress state of the cement sheath. Keeping the other parameters of the numerical model unchanged from those in Section 3.2.1, the wellhead pump pressure was set to 70, 80, 90, and 100 MPa, and the results are shown in Figure 13.

As shown in Figure 13A-C, the maximum radial/tangential stress and the judgment value are basically parallel, and the lower pump pressure is beneficial to the sealing integrity. This is because the reduction in pump pressure eases the outward expansion of the casing, which can obviously reduce the tangential and radial stresses of the cement sheath. However, even if the pump pressure is reduced to 70 MPa, the integrity of the cement sheath above the intermediate casing shoe cannot be guaranteed.

It can be concluded that the cement sheath above the intermediate casing shoe has the risk of tensile failure because of the combined effect of radial stress and tangential stress, and the radial cracks occur during single-stage fracturing. The physical simulation experiments in Section 3.1 also verified this point. However, the cement sheath is regarded as
ideal elastic material, and the results do not involve plastic strain. Therefore, when analyzing the mechanism of the microannulus caused by the plastic strain of the cement sheath in the multistage fracturing process, besides setting Young’s modulus and Poisson’s ratio of the cement sheath, the internal friction angle and cohesion should also be included.

4 | ANALYSIS OF INTEGRITY OF CEMENT SHEATH IN MULTISTAGE FRACTURING

Section 3 showed that the cement sheath above the intermediate casing shoe is susceptible to tangential tensile failure, and the radial compressive stress under the intermediate casing shoe is close to the compressive strength of the cement sheath. In this situation, the microannulus occurs under cyclic casing pressure. In this section, a cyclic compression experiment of the cement stone carried out to clarify the plastic deformation law is described. Then, according to the finite-element model of the casing–cement-sheath–formation assembly, the plastic deformation law of the cement sheath is analyzed.

4.1 | Cumulative plastic strain of cement sheath

4.1.1 | Cyclic compression test

Before performing the cyclic compression test, it is necessary to define the peak deviation stress. The value should be smaller than the compressive strength. Therefore, in this section, a
uniaxial compression test and triaxial compression test were carried out to determine the compressive strength, Young's modulus, Poisson's ratio, cohesion, and internal friction angle. The formula of the well cement slurry was Class G oil well cement (500 g), ganister sand (175 g), liquid dehydrating agent DZJ-Y (20 g), and water (263.5 g). The cement slurry was poured and formed in a cylindrical mold (Φ50 × 120 mm) and cured at 130°C and 20 MPa with a water bath for 3 days. The uniaxial compression test results and the triaxial compression test results are shown in Figure 14.

The mechanical parameters of cement stone can be calculated from the experimental results, and the parameters are shown in Table 5.

The internal friction angle and cohesion can be calculated based on the Mohr-Coulomb criterion. The internal friction angle is 12.64°, the cohesion is 20.78 MPa, and the average values of Young's modulus and Poisson's ratio from the test are the parameters for the numerical calculation: Young’s modulus is 8 GPa and Poisson's ratio is 0.17, which correspond to the parameters in Section 3.

A cyclic compressive test was carried out with a confining pressure of 15 MPa. The maximum and minimum compressive loadings were 40 and 5 MPa, and the loading and unloading rates were 0.5 kN/s. The stress-strain curve and the cumulative plastic strain are shown in Figure 15.

Figure 15A shows that the stress-strain curve is an approximate straight line in the initial loading process, and the unloading and reloading curves form the hysteretic loops. The strain at the lowest point of the unloading curve is defined as the cumulative plastic strain. Figure 15B shows that the
cumulative plastic strain increases approximately linearly with the cycle numbers. The plastic strain after the first unloading is 0.1133%, and the value after the 20th cycle is 0.4315%. The cumulative plastic strain caused by cyclic loading is very obvious.

4.1.2 Plastic strain of cement sheath in multistage fracturing

Because the failure mechanism of the cement sheath above the intermediate casing shoe has been clarified in Section 3.2, the cumulative plastic strain below the intermediate casing shoe is mainly analyzed in this section. Taking the case well in Section 3.2.1 as an example, combined with the mechanical parameters obtained from the experiment described in Section 4.1.1, the cumulative plastic strain of the cement sheath at 90° deviation (the landing point) was analyzed.

Based on the finite-element models established in Figure 1C, the contact relationship between the cement sheath, casing, and formation are the same as in Section 2.2. The constant casing pressure was changed to be cyclic internal pressure, the maximum casing pressure was the wellhead pump pressure (90 MPa) plus the hydrostatic column pressure (38.5 MPa), and the minimum casing pressure was the hydrostatic column pressure (38.5 MPa). The finite-element model and wellhead pump pressure are shown in Figure 16A,B.

To calculate the plastic strain under cyclic loading, the parameters of the internal friction angle and cohesion were added to the cement sheath. The cumulative plastic strain and the microannulus are shown in Figures 17 and 18.

Figure 17 clearly shows that the plastic strain and the size of the microannulus after the first fracturing stage are the largest, and the subsequent plastic strain/width increases linearly with the fracturing stages, which is similar to the experimental results. Figure 18 shows that the size of the microannulus is 17.5 μm after 15 stages of fracturing, which is large enough to enable gas to pass through.32

4.1.3 Sensitivity analysis of factor cumulative plastic strain

Young’s modulus

To analyze the effect of Young’s modulus of the cement sheath on accumulative plastic strain, and keeping the other
parameters constant, Young’s modulus was set to 4, 6, 8, and 10 GPa. The results are shown in Figure 19.

Figure 19 shows that the higher Young’s modulus, the larger the initial plastic deformation, and the cumulative plastic strain can be reduced by decreasing Young’s modulus. This can be explained as follows. First, the tangential and radial stresses to which the cement sheath is subjected can be released when reducing Young’s modulus, and the reduction in external loading can obviously reduce plastic deformation. Second, when Young’s modulus decreases but the strength remains constant, from the perspective of the relationship between stress and strain, it means that the elastic deformation interval becomes longer, and the increase in the elastic deformation capacity is also beneficial to alleviate the plastic deformation.

Poisson’s ratio
Keeping the other parameters constant, as in Section 4.1.2, Poisson’s ratio was set to 0.11, 0.14, 0.17, and 0.23, and the results are shown in Figure 20.

Figure 20 shows that the larger the Poisson ratio, the smaller the initial plastic deformation and plastic deformation increment. This is because increasing Poisson’s ratio is beneficial to decreasing the tangential tensile stress,
even making the stress change from tensile to compressive. Increasing the confining pressure can increase the compressive strength. However, the change in Poisson's ratio has almost no effect on the radial compressive stress (shown in Figure 12); therefore, increasing Poisson’s ratio helps to reduce the plastic deformation.

### Wellhead pump pressure

The wellhead pump pressure has an important influence on the integrity of the cement sheath and keeping the other parameters constant, as in Section 4.1.2, the pump pressures were set to be 70, 80, 90, and 100 MPa. The results are shown in Figure 21.

Figure 21 shows that the plastic strain increases rapidly with the increase in pump pressure. The squeeze of the casing on the cement sheath increases when the pump pressure rises, which obviously increases the plastic deformation of the cement sheath.

### 4.2 The dilatation mechanism of cement sheath

The sealing integrity of cement sheath during single and multistage fracturing was analyzed in Section 3 and Section 4; the cement sheath above the intermediate casing shoe was susceptible to tangential tensile failure and below the intermediate casing shoe was susceptible to accumulative plastic strain under cyclic casing pressure, resulting in microannulus. From the microscopic point of view, the plastic deformation means the dislocation movement of particles and the propagation of microcracks, when the microcracks extend to a certain degree, the permeability can be significantly increased, and the cement stone itself can also become the leakage paths.

#### 4.2.1 Relationship between confining pressure and dilatancy

Under normal conditions, the volume of rock/cement stone decreases continuously due to the compaction of microcracks; however, when the compressive stress reach up to the dilatancy yield stress, the dislocation and slip will occur in the rock and new cracks will expand, the volume then increase with the compressive stress, resulting in dilatancy. The dilatancy is one of the characteristics of rock/cement stone under uniaxial compression test or triaxial compression test. The triaxial compression tests of cement stone under different confining pressures were carried out to monitor the volumetric strain. The results are shown in Figure 22.

Figure 22 shows the volumetric strain generally increases first and then decreases, the stress corresponding to the turning point is the dilatancy yield stress. The dilatancy yield
stress is greatly affected by confining pressure, when the confining pressure is 5, 10, 20, and 25 MPa, the dilatancy yield stress is 47.6, 51.4, 54.3, and 58.7 MPa, respectively, and it can be concluded that the dilatancy yield stress increase with the increase in confining pressure.

4.2.2 Relationship between dilatancy and permeability

During the uniaxial or triaxial compression test, after the elastic deformation stage, the microcracks in cement stone will gradually expand and interconnect with each other with the increasing of compressive stress, and the volumetric strain gradually changes from decreasing to increasing. Therefore, the occurring of dilatancy means that the expanding of microcracks, furthermore, the permeability will change when there are enough microcracks, which will affect the sealing integrity undoubtedly.

The RTR-1500 testing system was used to monitor the change in permeability of cement stone in triaxial compression test, the equipment is shown in Figure 23A, the equipment can carry out uniaxial or triaxial compression test and analyze the dynamic change in permeability. Figure 23B shows the variation in permeability under triaxial compression test with the confining pressure of 15 MPa.

Figure 23B shows the variation in permeability can be divided into 3 stages: (a) The permeability decreases firstly, which means that the original voids or cracks close gradually when loading the confining pressure; (b) The permeability decreases slowly, because the cement stone is in the linear elastic stage, which is gradually compacted under the action of compressive stress; (c) The microcracks in cement stone gradually expand, resulting in the increasing of the permeability, from the line of volumetric strain vs time, it can be concluded that the dilatation phenomenon has occurred. The initial permeability of cement stone is 45 μD, the value is 90 μD with the appearance of dilatation phenomenon, and the value is 230 μD when the cement stone is completely damaged, which significantly reduce the sealing integrity of the cement sheath.

4.2.3 Permeability variation during cyclic compression test

The cyclic compressive experiment on cement stone showed that there was accumulative plastic strain under cyclic compressive loading, the calculation results in Section 4.1.2 show that the accumulative plastic strain can cause microannulus
between casing and cement sheath. However, no analysis has been made on the variation law of cement stone's permeability during cyclic loading. So the cyclic loading tests were carried out with the experimental device (Figure 23A), the loading rate and unloading rate were set at 0.1 MPa/s, the confining pressure was 20 MPa, considering that the radial stress of cement sheath under intermediate casing shoe in case well is greater than 54.8 MPa, the maximum loading pressure were set to be 45 MPa and 55 MPa, the unloading pressure was 0 MPa, and the total cycle numbers were 11, the cement stone was completely crushed at the 11th loading process. The volumetric strain and the permeability of cement stone were monitored during the experiment. The results are shown in Figure 24A,B.

Figure 24 shows the permeability fluctuate with time under cyclic loading conditions, the permeability of cement stone decrease when the pressure is loading, while the permeability increase when the stress is unloading, and the permeability decreases with the loading cycles, this is because the microcracks inside the cement stone are closed during loading, and the microcracks are opened when unloading. When the maximum loading pressure is 45 MPa, which is less than the dilatation yield stress, the permeability tends to decrease on the whole. However, when the maximum loading pressure is 55 MPa, which is greater than the dilatancy yield stress, the microcracks continue to expand with the number of cycles, which leads to the increase in permeability, the permeability is 180 μD after
the 9th unloading time, which is 3.6 times of the initial permeability, thus provide a possibility for the migration of down-hole fluid.

5 | CONCLUSIONS

1. The gas between the product casing and intermediate casing came from the target fracturing formation. The radial stress of cement sheath increased with well depth, while the tangential stress decreased with well depth. The stresses changed abruptly at the surface casing shoe and intermediate casing shoe, and the cement sheath above the intermediate casing shoe had the risk of tangential tensile failure, resulting in tensile cracks, which provided a continuous path for gas migration.

2. The cement stone showed that, in the cumulative plastic strain characteristic in the cyclic compression test, the plastic strain increased linearly with the increase in cycle numbers. The numerical results of the casing–cement–sheath–formation assembly below the intermediate casing shoe indicated that the microannulus would occur under cyclic casing pressure, which would provide a continuous path for gas migration as well.

3. Reducing Young’s modulus of the cement sheath is beneficial for reducing the tangential stress and radial stress. Increasing the Poisson ratio has little effect on the radial stress, but it can reduce the tangential stress. Reducing the pump pressure is beneficial to reducing the radial stress and tangential stress of the whole wellbore cement sheath.

4. The dilatation phenomenon occurred in the triaxial compression test. When the cyclic compressive stress
exceeded the dilatation yield stress, the permeability increased with the increase in cycle numbers, and the increase in permeability provided another leakage path for gas migration.

5. More investigations need to be carried out to verify the failure mechanism of the cement sheath. For example, the effect of temperature on the failure mechanism of the cement sheath should be further investigated, because the temperature at the production layer can be $120^\circ$C. Control methods need to be further discussed as well. Ways to reduce the tangential tensile stress above the intermediate casing shoe and the radial compressive stress below the intermediate casing shoe are key future research topics.

ACKNOWLEDGMENTS

The authors would like to thank the Key Program of National Natural Science Foundation of China (U1762211), The National Natural Science Foundation of China (51674272), for supporting this work and for permission to publish this paper. Thank you for the experimental data provided by Tao Qian and Jinlong Du from Sinopec Engineering and Technology Research Institute, Beijing, China.

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How to cite this article: Lian W, Li J, Tao Q, Du J, Wang L, Xi Y. Formation mechanism of continuous gas leakage paths in cement sheath during hydraulic fracturing. Energy Sci Eng. 2020;8:2527–2547. https://doi.org/10.1002/ese3.684.