Fundamental Bounds on Radio Localization Precision in the Far Field

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Abstract—This paper experimentally and theoretically investigates the fundamental bounds on radio localization precision of far-field Received Signal Strength (RSS) measurements. RSS measurements are proportional to power-flow measurements time-averaged over periods long compared to the coherence time of the radiation. Our experiments are performed in a novel localization setup using 2.4GHz quasi-monochromatic radiation, which corresponds to a mean wavelength of 12.5cm. These experiments show for the first time that time-averaged far-field RSS measurements are not independent but cross-correlated over a spatial region. We experimentally and theoretically show that the minimum radius of the cross-correlated region approaches the diffraction limit, which equals half the mean wavelength of the radiation. Measuring RSS beyond a sampling density of one sample per half the mean wavelength is shown not to increase localization precision, as the Root-Mean-Squared-Error (RMSE) converges asymptotically to roughly half the mean wavelength. This adds to the evidence that the diffraction limit determines the fundamental lower bound on the RMSE rather than the spread in independent noise as is usually assumed in Cramer-Rao Lower Bound (CRLB) analyses on RSS and Time-Of-Flight (TOF) signals. For the first time, we experimentally validate the theoretical relations between Fisher information, CRLB and uncertainty, where uncertainty is lower bounded by diffraction as derived from coherence and speckle theory.

I. INTRODUCTION

Radio localization involves the process of obtaining physical locations using radio signals. Radio signals are exchanged between radios with known and unknown positions. Radios at known positions are called reference radios. Radios at unknown positions are called blind radios. Localization of blind radios reduces to fitting these measured radio signals to appropriate propagation models. Propagation models express the distance between two radios as a function of the measured radio signals. In the field of radio localization, these measured radio signals are often modeled as the true radio signals with independent noise using a large variety of empirical statistical models.¹ We adopt the widely used empirical Log-Normal Shadowing Model (LNSM) for modeling the Received Signal Strength (RSS) over distance decay.¹ The LNSM assumes independent Gaussian noise on the RSS. Radio localization involves non-linear numerical optimization techniques that fit the parameters in the propagation model given the statistical model of the independent noise. The independence of the noise in these models has not been experimentally verified on space and time scales that are of the order of half an oscillation period. One uses these independent noise models to determine the fundamental bounds on radio localization precision.¹,¹⁴,¹⁹,²¹,²⁴

This paper investigates the fundamental bounds on localization precision of radio localization systems operating in the far field. Fundamental bounds are expressed as the minimum resolvable position of electromagnetic signals between radios. This minimum resolvable position depends on whether the measured radio signals contain phase information. Phase information can only be retrieved from measurements that are instantaneous on a time scale that is short relative to the oscillation period of the signals. The smallest resolvable position depends on how well phase can be resolved. This is usually limited by the speed and noise of the electronics of the system. Less complex and less expensive localization systems are based on measurements of time-averaged power flows. Time-averaging is usually performed over timespans that are large compared to the coherence time of the radios, so that all phase information is lost. RSS localization is an example of such less-complex systems. When determining the fundamental bounds on localization precision, it makes sense to distinguish between the time scales of the signal measurements, i.e. between instantaneous and time-averaged signal and noise processing. The performance bounds of all radio localization systems ultimately depend on how well radiation and its noise over space and time can be resolved.²

The propagation of noise, i.e. fluctuations on signals, is governed by the same macroscopic equations of motions over [space, time] as signals. For electromagnetic signals, these motions are governed by the Maxwell equations with the boundary conditions on the electromagnetic field components when contrasting surfaces are present and with Sommerfeld’s radiation condition. Noise that propagates with the signal and manifests itself in the radiation is called radiation noise. The origin of far-field radiation noise is expressed at the contrasting surfaces from which the radiation is emitting and diffracting. Existing coherence and speckle theory show that radiation noise is always cross-correlated over a so-called coherence region for [space, time] measurements. Within this region, noise cannot be considered as statistically independent. In all practical cases of interest,¹⁷ and²⁷ show that the lower bound on the size of the radius of this far-field region is of the order of half the mean wavelength of the radiation. This lower bound on the coherence region even holds for the far fields of sources that are regarded as spatially completely incoherent, e.g. blackbody sources.²⁷ §13. This fundamental lower bound is called the diffraction limit or Rayleigh criterion.¹⁶,¹⁷,¹⁸,²⁷,²⁹. Correlated noise

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over a bounded region determines the fundamental bound on localization precision. Further resolution of this region of correlated signals can be established by non-linear signal processing like homodyning and heterodyning. Maxwell’s equations scale with wavelength, so that optical coherence and speckle theory also apply to the Radio Frequency (RF) regime as a fundamental model for signal analysis.

The well-established rationale of cross-correlated electromagnetic far-field radiation noise appears to contradict the well-established rationale of existing fundamental bound analyses in the field of radio localization, usually based on Cramer-Rao Lower Bound (CRLB) analysis \[14, 19, 24\]. CRLB analysis is assumed to provide a fundamental bound for the covariance of unbiased estimators of unknown deterministic wave variables like time or position. In the field of radio localization, the noise on these deterministic wave variables is modeled as independent over [space, time], so that the correlation region is assumed to be infinitely small. The independence of the noise implies that there exists an infinite sample size of independent measurements over finite measuring range in [space, time]. Hence, the minimum resolvable position approaches zero with increasing ensemble size over a finite measuring range. The existence of an infinite ensemble size of independent measurements over finite time and space is in contradiction with the fundamental physics of electromagnetic far-field radiation noise, except when the wavelength of the radiation is assumed to approach zero relative to the linear dimensions of contrasting planar interfaces present in measurement space. The zero-wavelength approximation is commonly called the geometrical-optics approximation in electromagnetic theory \[16\].

A. First-Order and Second-Order Coherence, Speckles, Uncertainty and Fisher Information

In the context of this paper, radios either measure instantaneous cross-correlations of electrical field strengths or time-averaged power flow of radiation. Power flow is theoretically obtained from the time-averaged far-field Poynting vector. Instantaneous cross-correlations of far-field electrical field strengths are used when phases are measured or when temporal coherence properties of the fields need to be accounted for. The coherence properties of the radiation are usually defined by the first-order temporal coherence function and second-order spatial coherence function. First-order temporal coherence is calculated from cross-correlations of instantaneous field strengths as described in the signal analysis given by \[27\]. Second-order spatial coherence is calculated from cross-correlations of the instantaneous Poynting vector rather than from instantaneous field strengths \[18, 27, 29\]. Second-order spatial coherence was first observed and theoretically explained by Hanbury Brown and Twiss \[2\]. Their extended source was a distant star emitting quasi-mono-chromatic radiation with random phases and amplitudes from its small surface elements. The instantaneous far-field superposition of the radiation from all these surface elements determines the coherence properties of the radiation of the star. Our experimental work determines the second-order spatial coherence of the time-averaged rather than of the instantaneous Poynting vector of the quasi-mono-chromatic far field transmitted by an antenna and reflected by contrasting surfaces in localization space as described by speckle theory \[17\]. Speckle theory originates from the same mathematical representations as second-order spatial coherence theory. A finite temporal coherence length in combination with a spatially random or spatially uniform source introduces uncertainty and generates a region of spatial far-field correlations \[27 \S 4.2.2\]. As stated above, \[17 \S 4\] and \[27 \S 4.4.4 - \S 13.1\] show that the radius of this correlation region has a lower bound of the order of half the mean wavelength of the radiation, irrespective of the degree of temporal coherence of the transmitter and timescale of the measurements.

Coherence and speckle properties of radiation are usually governed by the spreads of Fourier pairs of wave variables that show up as independent variables in the classical propagation models of electromagnetic waves. The products of these spreads of Fourier pairs express uncertainty relations. Coherence and speckle theory set the diffraction limit as the fundamental lower bound on one of the spreads of the wave variables of such pairs. \[1\] mathematically establishes a relationship between Fisher information, the CRLB and uncertainty. As uncertainty is lower bounded by diffraction, CRLB is lower bounded and Fisher information is upper bounded. Our experimental work validates this theoretical work for the first time.

B. Phase Interpolation and Time-Averaged Power Flow

As stated before, the minimum resolvable position depends on whether phase information is included. In radio and sound-wave ranging (RADAR and SONAR), Doppler shift is usually used to establish a certain degree of phase interpolation \[20\]. In the optical regime of the electromagnetic spectrum, phase is often measured by Zeeman splitting in combination with Doppler shift \[5\]. In Fourier Optics, chromatic resolving power \[29 \S 4.7\] is usually expressed as the ratio between wavelength (\(\lambda\)) and resolvable fraction of the wavelength or minimum resolvable position (\(\delta \lambda\)). In Radar positioning, upper bounds on chromatic resolving power have been measured up to \(\frac{\Delta \lambda}{\delta \lambda} = 20\) \[25\]. In the optical regime, phase interpolation of Zeeman-split laser beams has been pushed to an upper bound of about \(\delta \lambda = 100\) \[5\].

RSS-based localization is an example of a less-complex localization system that measures power in Decibel-milliwatts (dBm), usually time-averaged over timespans that are large compared to the coherence time of the radios, so that all phase information is lost. In most cases of practical interest, speckle noise is present, so that the diffraction limit determines the fundamental bounds on localization precision. Thus, the smallest resolvable position with RSS localization is expected to be half the mean wavelength.

An example of a system where this fundamental bound is evident, is when using a high numerical-aperture parabolic mirror. Such imaging systems are used both at optical and radio frequencies. Reflection of an infinitely extended plane electromagnetic monochromatic wave by a high numerical-aperture parabolic mirror of finite diameter reveals that diffraction limits longitudinal and transverse resolution of the focal
spot to roughly half the mean wavelength $12\mu$s. This lower bound on spatial resolution achieved by a high-numerical-aperture parabolic mirror also equals the diffraction limit. Even for partially or incoherent incident radiation, the lower bound on the radius of the coherence region remains of the order of half the mean wavelength. Sub-diffraction limits can be achieved when the energy is exchanged through non-linear electromagnetic interaction processes as described in \cite{22, 23, 27}.

C. Experimental Setup

In our experiments, the source is an antenna transmitting quasi-monochromatic radio waves in the Radio-Frequency (RF) regime at 2.4GHz or 12.5cm wavelength with a typical coherence time of $25\mu$s or with a temporal frequency bandwidth of roughly 40KHz. The antenna is a typical dipole antenna with an uniformly radiating surface with a length of half a wavelength and width of one twentieth of a wavelength. Our receivers perform time-averaging of these quasi-monochromatic signals over time spans of $125\mu$s.

We determine the spatial correlations in the far field between power-flow measurements time-averaged over timespans that are large compared to the coherence time of the radiation. Such spatial correlations have not been measured or determined before in radio localization systems. An illustrative example of such a spatial correlation experiment is shown in Figure 1.

The purpose of this paper is to define an experimental RSS localization setup that could support either rationale to hold: is radio localization precision of RSS systems determined by the spread of independent noise as assumed by existing CRLB analyses or by spatial correlations between RSS measurements? To this aim, we performed 60 million multiplexed time-averaged power-flow measurements, 2,400 of which were performed manually and the remaining set was performed automatically. The manual measurements comprised of 2,400 equidistant time-averaged power-flow measurements by the reference radio along the circumference of a $3 \times 3m^2$ square. This means that the reference radio sampled RSS with a density of 25 samples per wavelength around the circumference, sufficient to compute the spatial correlations within the region of a wavelength. In addition, at each of these 2,400 positions the reference radio automatically repeated the time-averaged power-flow measurements 500 times to enable statistical CRLB analysis on this ensemble of 25,000 repeated, non-overlapping, multiplexed measurements. The fixed blind transmitter was positioned somewhere inside the square, with the reference radio always in its far field. The relatively small range between blind and reference radio ensures that the Signal-to-Noise Ratio (SNR) is relatively high. Our experimental setup investigates the fundamental bounds on localization precision in two dimensions. Our theoretical analysis generalizes the obtained results to three dimensions.

D. Organization Paper

This paper is organized as follows. Section III describes the experimental setup. Section IV gives the fundamental theoretical rationale. Section V presents the rationale of CRLB analysis and MLE along with the empirical LNSM. Section VI presents the experimental results in terms of spatial correlations and upper bounds on localization precision. Finally, Section VII provides a discussion and Section VIII summarizes the conclusions.

II. EXPERIMENTAL SETUP

Figures 2 and 3 show the two-dimensional experimental setup. Figure 2 shows a square of $3 \times 3m^2$, which represents the localization surface. We distinguish between two types of radios in our measurement setup: one reference radio and one blind radio. The reference radio is successively placed at known positions and is used for estimating the position of the blind radio. The crosses represent the 2,400 manual positions of the reference radio $(x_1, y_1 \ldots; x_{2400}, y_{2400})$ and are uniformly distributed along the circumference of the square (one position every half centimeter).

Due to the general wave character of our radio signals, this setup is expected to provide good localization performance. This expectation is based on Green’s theorem for wave amplitudes where a three-dimensional volume integral reduces to a surface integral enclosing the localization volume. In the two dimensions of our experimental setup, the volume integral becomes a surface integral, the surface integral becomes a contour integral, and the Green function becomes the Hankel function. Hence, rather than placing a two-dimensional array of reference radios inside the $3 \times 3m^2$ square, it may suffice to place a much lesser number on the circumference of the rectangle and get similar localization performance. Whether sampling on circumferences instead of sampling across two-dimensional surfaces suffices has yet to be verified by experiment. This paper aims to show the practical feasibility of this novel technique, which was first proposed by \cite{28} §5.6.
The red circle represents the position of the blind radio. We place the blind radio at an unsymmetrical position, namely \((0, 0)\), as symmetry could influence the localization performance. We only use one blind radio and one reference radio to minimize influence of hardware differences between radios. The blind and reference radios are both main powered to minimize voltage fluctuations. The reference radio has a power amplifier and broadcasts messages with maximum power allowed by ETSI to maximize SNR. Both blind and reference radios have an external dipole antenna. The antennas have the same vertical orientation and are in Line-Of-Sight (LOS) for best reception. The length of the antenna is half a wavelength. Its diameter is roughly one twentieth of a wavelength. We keep the relative direction of the printed circuit boards on which the antennas are mounted constant by realigning them every 25 cm.

In order to minimize interference from ground reflections, we place the radios directly on the ground, so that their antennas are within one wavelength height. The ground floor consists of a reinforced concrete floor covered by industrial vinyl. We minimize interference from ceiling reflections by placing a 50 x 50 cm\(^2\) aluminum plate one centimeter above the blind radio antenna. The ground and the aluminum plate minimize the influence from signals in the \(z\)-direction, so that we only have to consider signals in two dimensions. All reference radio positions are in the far field of the blind radio. A photo of our localization setup is shown in Figure 3.

At each of the 2, 400 reference radio positions, the reference and blind radio perform a measurement round. A measurement round consists of 500 x 50 repetitive multiplexed RSS measurements to investigate and quantify the measurement noise and apply CRLB analysis to this measurement noise. Each measurement round consists of 50 measurement sets that consist of 500 RSS measurements on an unmodulated carrier transmitted by the blind radio. Between each measurement set of 500 RSS measurements, the reference and blind radios automatically turn on and off (recalibrate radios). Although we did not expect to find any difference from these two different forms of multiplexing RSS signals, our experiments should verify this, which they did.

Hence, a measurement round consists of 50 measurement sets, each consisting of 500 repetitive multiplexed RSS measurements. Measurement rounds and measurement sets are represented by \(P_{l,m,n} = P_{1,1,1} \ldots P_{2400,50,500}\). Index \(l\) identifies the position of the reference radio and thus the measurement round, index \(m\) identifies the 50 measurement sets, and index \(n\) identifies the 500 individual RSS measurements in each measurement set. \(P_{l,m,n}\) represents the measured power in dBm and is time-averaged over 125 \(\mu s\) according to the radio chip specification [15]. The averaging time of 125 \(\mu s\) is a factor of five larger than the specified coherence time of the carrier wave of a typical 802.15.4 radio [31]. Practically, this means that these power measurements lose all phase information [29, §1.5]. In theory, this means that we measure the time-averaged power flow or Poynting vector as the cross-sections of all antennas are the same.

The blind radio transmits in IEEE 802.15.4 channel 15 in order to minimize interference with Wi-Fi channels 1 and 6. The reference radio performs power-flow measurements in the same channel and sends the raw data to a laptop over USB, which logs the data. We use Matlab to analyze the logged data. Between each measurement round, we change the position of the reference radio by 0.5 cm and push a button to start a new measurement round. Note that 0.5 cm is well within the \(\lambda/2\) diffraction limit of half the mean wavelength.

In summary, each measurement set takes one second; each measurement round takes roughly one minute (50 x 1 seconds). The experiment consists of 2, 400 measurement rounds and takes in total 40 hours (2, 400 minutes \(\approx 40\) hours). In practice, we spend almost three weeks in throughput time to perform this complete data collection.

Care has to be taken, because we have to make sure that we are exactly measuring what we intend to measure and that the experiment runs stable for 40+ hours. At the beginning of each measurement set, the reference and blind radios synchronize via wireless communication. When communication is successful, synchronization precision is of the order of microseconds (several clock ticks). Guard times (unreliable clock) and backup mechanisms (unreliable wireless commu-
III. PROPAGATION AND NOISE MODEL FOR FUNDAMENTAL BOUNDS

This section analyzes the fundamental bound on localization precision of radio localization systems operating in the far field. We use the fact that radio localization is based on electromagnetic signals with a periodic wave character, having a non-zero frequency bandwidth. We also use the fact that radio localization is based on source and noise processes that are statistically stationary. We distinguish between three types of sources:

- Primary extended sources, i.e. the known excited electromagnetic surface currents on the transmitters in localization space.
- Secondary extended but induced sources consisting of induced electromagnetic surface currents on other contrasting surfaces in localization space.
- Scattering from thermal noise in the atmosphere and electronics of the system.

This implies that all signal noise, considered in this paper, is propagated in the radiation of these three sources and finally received by the receiving radios that suffer from thermal and quantum noise. It is relatively straightforward to derive the fundamental characteristics of the radiated signals and noise. The governing dynamics are fundamentally determined by the Maxwell equations, the boundary conditions, Sommerfeld’s radiation condition, the constitutive relations between radiation and matter, and the stochastics of the noise processes. As we shall see, in all practical situations of interest, radiation noise in its far field is always correlated over [space, time], never independent, even not in multiplexed, time-averaged signal measurements like in the experiments described in this paper. The radius of the space over which the signals are correlated has a lower bound that approaches the diffraction limit and is directly related to the wavelength of the radiation. This even holds for black-body radiation in an unechoed room. The size of this region is a direct measure for the inverse of localization precision as we will show.

A. Propagation Model

For a concise analysis on wave propagation, we refer to [30]. The electric and magnetic field strengths, \( \vec{E}(k, \vec{r}_p, \omega, t, \tau_c) \) and \( \vec{H}(k, \vec{r}_p, \omega, t, \tau_c) \), propagate towards the point of observation, \( \vec{r} \), from an extended primary source of closed surface area, \( A = A(\vec{r}) \), with \( \vec{r}_p \) denoting the surface positions on the source surface [30, 18.10.13 and 18.10.14]:

\[
G(k, R) = \frac{\exp[i(kR)]}{4\pi R}, \tag{1}
\]

so that

\[
\vec{E}(k, \vec{r}_p, \omega, t, \tau_c) = \text{Re}\left\{ f(\omega, t, \tau_c) \int_A \exp[i\phi(\vec{r}, t)] \right\}, \tag{3}
\]

and

\[
\vec{H}(k, \vec{r}_p, \omega, t, \tau_c) = \text{Re}\left\{ f(\omega, t, \tau_c) \int_{A'} \exp[i\phi(\vec{r}', t)] \right\}, \tag{4}
\]

with a slowly varying envelope approximation [10], [11], [18] $f(\omega, t, \tau_c) = \exp \left[ -\frac{\pi}{2} \left( \frac{t}{\tau_c} \right)^2 \right] \exp[-i\omega t].$ (5)

In (3) and (4), \( \hat{n}(\vec{r}) \) denotes the outward unit normal to the source surface, and \( \vec{E}(\vec{r}) \) and \( \vec{H}(\vec{r}') \) represent the electric and magnetic field strengths on the source area \( A = A'(\vec{r}) \). The electric field strength is expressed in V/m and magnetic field strength in A/m. In addition, \( k \) represents the wavenumber or spatial frequency; \( \omega \) represents the carrier angular frequency; \( t \) the time, and \( \phi(\vec{r}, t) \) represents the fluctuating phase on the surface, which we shall later identify with the noise process. At this point it suffices to note that the noise process originates from phase fluctuations on the surface \( A(\vec{r}) \), which we refer to as phase noise. Phase noise is either instantaneous in nature, \( \phi(\vec{r}, t) = \phi(t) + \phi(\vec{r}) \), due to random fluctuations over [space, time], or static in nature, \( \phi(\vec{r}, t) = \phi(\vec{r}) \) due to the random surface roughness of the surface area \( A(\vec{r}) \). The field representations of (3) and (4) also hold for the general case where the integrations over space include all contrasting surfaces of secondary sources in localization space. Each surface may then generate its own characteristic phase noise. Finally, a Gaussian damping term is included to account for the angular frequency spread, \( \Delta \omega \), of the resonance oscillator. This spread is related to the coherence time, \( \tau_c \), as described by [11, 18, §2.3] and [29, §10.7]:

\[
\tau_c = \frac{(8\pi \ln 2)^{1/2}}{\Delta \omega}, \tag{6}
\]

where we assume a narrow bandwidth relative to the carrier frequency as is the case for quasi-monochromatic radiation

\[
\frac{\Delta \omega}{\omega} \ll 1. \tag{7}
\]

Time and position are connected by the constant speed of light, \( c \), as are angular frequency and wavelength

\[
\lambda = 2\pi c / \omega = \frac{2\pi}{k}. \tag{8}
\]
B. Far-Field Approximation

Equations (3) and (4) represent a signal model of extended primary and secondary sources. In our experimental setup, the radio transmitter is an extended primary source of linear dimensions \( h \times \delta h \) and closed surface area \( A \). Our antenna transmits radio waves using the UHF band at 2.4GHz (\( \lambda = 12.5\text{cm} \)) in IEEE 802.15.4 channel 15 in order to minimize interference with Wi-Fi channels 1 and 6. It has a coherence time of \( 25\mu \text{s} \) and a cross-section of roughly half a wavelength \( (\delta h) \). The extended source area acts as a diffracting finite surface area, from which each infinitesimal surface element, \( dA \), acts as a partially coherent point source located at \( \vec{r} \) as expressed by (3) and (4). To approximate the far field of this extended source, the far-field condition is assumed to hold

\[
r_p \gg r, \quad \text{with} \quad r_p = |\vec{r}_p| \quad \text{and} \quad r = |\vec{r}|
\]

so that the Green function in the integrals in (3) and (4) assumes its far-field approximation

\[
G(k, \vec{r}_p, \vec{r}) \simeq \frac{\exp[ik(r_p - \vec{r}_p \cdot \vec{r}/r_p)]}{4\pi r_p} \left( 1 + O\left( \frac{r}{r_p} \right) \right).
\]

(10)

(4) describes the general 3D-case of rewriting Maxwell’s differential equations to integral representations using Green’s theorem and solve those with an iterative global Root-Mean-Squared-Error (RMSE) technique. Their formalism reduces to (3) and (4) where the surface integrations need to be taken as the sum of integrations over all primary emitting source surfaces and the secondary induced source surfaces of all contrasting obstacles in localization space. One of the advantages of integral representations is that they visualize Huygens geometrical principle. Huygens principle determines how a wave front propagates towards a point of observation by just adding the geometrical contributions of each elementary spherical wavelet on that wave front towards the point of observation. The Green function represents such an elementary Huygens spherical wavelet. In the far-field, when one substitutes (10) in (3) and (4), these integral representations simplify to spatial Fourier transforms of the surfaces of the source and obstacles, the foundation of Fourier Optics with its spatial filtering and signal processing [16].

In the far-field approximation of (3) and (4), one usually makes the assumption that all diffracting surfaces are good conductors, so that the tangential electric field components in (3) are assumed to be zero

\[
\vec{n}(\vec{r}) \times \vec{E}(\vec{r}) = 0 \quad \text{when} \quad \vec{r} \in A.
\]

(11)

The tangential electric field components are also zero upon grazing incident radiation with polarization that is perpendicular to contrasting surfaces. Hence, mainly magnetic surface currents remain in (3) to account for the diffraction effects, which are proportional to the normal derivative \( \vec{n}(\vec{r}) \cdot \nabla \vec{E}(\vec{r}) \).

This effect was analyzed by [13] and finds it further justification in the wire-grid polarizer. This polarizer blocks radiation polarized parallel to the wires due to maximum diffraction effects as the tangential component of the electric field strength must be zero on the wire surface and its normal derivative increases rapidly towards edges parallel to the polarization.

For radiation polarized perpendicular to the wires, the radiation flows through the wires as barely any diffraction occurs.

In coherence theory as formalized by [27], the far-field radiation fluctuations or radiation noise originates from instantaneous surface-phase noise \( \phi(\vec{r}, t) = \phi(t) + \phi(\vec{r}) \) in (3) and (4), which is assumed to be uniformly distributed over \([0, 2\pi]\) in the [space, time] domains. Instantaneous cross-correlations of the electric far-field strengths over [space, time] are derived from (3), and [27, §4.4.4 - §13.1] shows that the far-field radiation noise has a lower bound on its correlation region of about the mean wavelength of the radiation. The lower bound on the radius of such a region is usually called the Rayleigh criterion of maximum spatial resolution and equals the diffraction limit of half of the mean wavelength. Hence, radiation noise is not independent or uncorrelated as is usually assumed in RSS and TOF localization. Whether an ensemble of repetitive, multiplexed measurements can be regarded as ergodic needs to be experimentally verified. For such an ensemble, the ensemble average may be replaced by a time average, and we arrive at the integral representations of (3) and (4). For a single, near-instantaneous measurement, the noise becomes correlated as its propagation model comprises of Maxwell’s equations with its boundary conditions and Sommerfeld’s radiation condition, or its equivalent integral equations as expressed by Green’s theorem. [27, §4.2.3] shows that within this [space, time] correlation region, the photons become indistinguishable as was first observed by Hanbury Brown and Twiss [2] for an extended star as a source with a uniformly distributed phase over the surface of the star. [27, §4.3.3] links this observation of the spatial coherence properties of the far-field radiation to the uncertainty principle. This link is also apparent from Fourier pairs \([\omega, t]\) in (5) and \([k, r]\) in (10) as we show later at the end of this section.

C. Linear Interaction between Radiation and Matter

In localization setups, the interaction between electromagnetic radiation and matter is assumed to be linear, time-invariant and spatially varying, and the statistical properties of the sources are assumed to be stationary and spatially uniform. This time invariance and spatial variance is usually expressed by the linear constitutive relations that are assumed to hold in matter that is interacting with the radiation

\[
D(t, \vec{r}) = \int_{-\infty}^{t} \epsilon_0 \varepsilon_r(t - t', \vec{r}) E(t', \vec{r}) dt'
\]

(12)

or

\[
D(\omega, \vec{r}) = \varepsilon_0 \varepsilon_r(\omega, \vec{r}) E(\omega, \vec{r}).
\]

(13)

In (12), \( D(t, \vec{r}) \) is called the electric flux density expressed in \( \text{C/m}^2 \), \( \varepsilon_0 \) is the free-space permittivity expressed in \( \text{F/m} \), and \( \varepsilon_r(t - t', \vec{r}) \) is the relative permittivity of matter that is interacting with the radiation. A similar constitutive relation holds between the magnetic flux density and the magnetic field strength, but we assume the properties of localization space to be non-magnetic and purely dielectric or conducting. The interesting part of (12) and (13) is that although the interaction is assumed to be time-invariant, its spatial variance may
induce non-local time-invariant field variations through the electromagnetic boundary conditions imposed at contrasting boundaries of obstacles, radio transmitters and receivers in localization space as described by [6].

Radiation noise caused by atmospheric fluctuations in the density are usually modelled as stochastic fluctuations in the macroscopic permittivity as expressed by

$$\varepsilon_r(t - t', \vec{r}) = \varepsilon_r(t - t', \vec{r}) + \frac{\partial \varepsilon_r}{\partial \rho} \Delta \rho(t, \vec{r}).$$  \hfill (14)

Equation (14) represents a non-linear response function where $\rho(t, \vec{r})$ denotes the locally fluctuating density of the atmosphere. The governing dynamics of these density fluctuations are determined by the Navier-Stokes equations [4], so that the resulting noise is spatially correlated over the thermal diffusion length in the atmosphere. Maxwell equations allow for solutions of evanescent waves parallel to interfaces. Such waves are present in absorbing and conducting media, or in general, when the angle of incidence is beyond total internal reflection like in the claddings of optical fibers. It has been shown in theory and experiment that even those evanescent waves contain noise of fluctuations that are governed by the Navier-Stokes equations [4] and that are scattered into the far field. In most localization setups of practical interest, the scattering from atmospheric noise or more general from any noise of thermal-acoustic origin can be neglected.

D. **Power Flows and Spatial Correlations**

RSS and TOF localization is based on power-flow measurements $P$ transferred into the “receiving” antenna surface. Power is defined as the energy per unit time (J/s=W), so that the power flow transferred into the antenna surface is proportional to power at a given distance $r_p$ from the source. Power flow is defined as the power transferred per unit area (W/m$^2$). The time-averaged power flow at any far-field point of observation, $r_p$, is obtained from the Poynting vector averaged over time as described by [29, §1.5], so that the time-averaged transmitted power flow is given by

$$\bar{P}(r_p, T_m) = \frac{1}{T_m} \int_{-T_m/2}^{T_m/2} \vec{E}(k, r_p, \omega, t, \tau_c) \times \vec{H}(k, r_p, \omega, t, \tau_c) \, dt.$$  \hfill (15)

The far-field electromagnetic field strengths in (15) are given by [9, 10] and [10], where the integrals are summed over all primary and secondary source surfaces, each of which may have its own characteristic phase noise. In (15), $T_m$ represents averaging time. For time-averaged power flows, (15) represents the time-averaged power flow diffraction from static surface roughness, $\phi(\vec{r})$, which is assumed to be uniformly distributed over $[0, 2\pi]$ for all contrasting surfaces in localization space.

For averaging times, $T_m \gg \tau_c$, the far-field cross-correlations, $\gamma(\delta r_p)$, of the time-averaged Poynting vector are readily determined from

$$\gamma(\delta r_p) = \langle (\bar{P}(r_p) - \langle P(r_p) \rangle) (\bar{P}(r_p + \delta r_p) - \langle P(r_p + \delta r_p) \rangle) \rangle,$$  \hfill (16)

where

$$P(r_p) = |\bar{P}(r_p, T_m)| \quad \text{for} \ T_m \gg \tau_c.$$  \hfill (17)

E. **The Fundamental Lower Bound**

The formalism of calculating the cross-correlations of far-field power flows from static randomly distributed surface roughness is equivalent to the speckle theory given by [17, 4.51 and 4.52]. [17] derives a lower bound of the transverse far-field spatially correlated power flows of roughly the mean wavelength of the quasi-monochromatic source. Hence, the radius of this spatially correlated region is again the diffraction limit. This time-averaged radiation noise is not independent and its distribution is additive and not multiplicative as is usually assumed for the measurement noise in TOF and RSS localization.

In our measurement setup, measuring time is large compared to coherence time ($T_m \gg \tau_c$), but the extended primary source is not larger than half a wavelength. The spatial frequencies of the surface roughness of the antenna should be smaller than the inverse of the diffraction limit to enable the observation of far-field spatial correlations. The far-field plane waves of (3) and (4) can be considered as being incident on any contrasting surface with random surface roughness $\phi(\vec{r})$ over a large area. As long as this area is large compared to the diffraction limit, and the surface roughness has spatial frequencies below the inverse of the diffraction limit, this static phase noise generates far-field spatially correlated radiation fluctuations. The grazing incident far-field of the transmitting blind antenna is reflected by the ground floor of our measuring chamber and received by the reference radio. Hence, the receiver observes the spatially correlated speckle noise of this grazing reflection from the ground floor. The size of this correlated far-field region should have a lower bound that equals the diffraction limit of the radiation. This lower bound causes a lower bound on the global RMSE, hence, an upper bound on the localization precision.

Equations (3) and (4) with (5) and (10) contain two products of the Fourier pairs [angular frequency $\omega$, time $t$] and [spatial frequency $k$, position $\vec{r}$]. These two sets of conjugate wave variables naturally lead to an uncertainty in physics called Heisenberg Uncertainty principle. Hence, uncertainty is a basic property of Fourier transform pairs as described by [13, §2.4.3]

$$\sigma_\omega \sigma_t \geq 0.5,$$  \hfill (18)

and

$$\sigma_k \sigma_r \geq 0.5.$$  \hfill (19)

Equations (18) and (19) are the well-known uncertainty relations for classical wave variables of the Fourier pairs [energy, time] and [momentum, position]. In these Fourier pairs, energy and momentum are proportional to angular frequency and wavenumber, the constant of proportionality being the reduced Planck constant when we consider the uncertainty principles in quantum mechanics. In information theory, they are usually only given in the time domain and are called bandwidth measurement relations as described by [9]. In Fourier optics, they are usually given in the space domain as described by [16]. Heuristically, one would expect the uncertainty in time, $\sigma_t$, and position, $\sigma_r$, to correspond to half an oscillation cycle, as half a cycle is the lower bound on the period of energy exchange between free-space radiation and a receiving or
transmitting antenna giving a stable time-averaged Poynting
vector.
When one defines localization performance or precision as
\[ \frac{1}{\sigma_r}, \]  
(20)
one would expect this performance not to become infinitely
large by just keep adding repetitive multiplexed measurements.
When the density of measurements crosses the spatial region of
spatial coherence as computed from (16) and (17), the power
measurements become mutually dependent. This region of
spatial coherence can be made visible in the optical regime
of the spectrum with the aid of an optical instrument with high
resolving power as described by [12]
\[ \sigma_r \gtrsim \lambda/2NA. \]  
(21)
In (21), NA stands for the Numerical Aperture of the imaging
system. For a high-numerical aperture parabolic mirror, this
minimum resolvable position is calculated from the surface
integrals in (3) and (4) over the entrance pupil of the mirror
with the far-field condition of (10). [12] derives that the
longitudinal as well as the transverse radius of the focus
approach the diffraction limit \( \lambda/2 \) as the Numerical Aperture
is close to one of such a mirror. Such mirrors are also applied
in the RF regime of the frequency spectrum. Equation (21)
can be considered as a lower bound on spatial resolution
on localization precision when only energy without phase is
measured of the radio signals.

F. Fundamental Bound on Uncertainty and CRLB
In conclusion, we showed in this section that in classical
propagation theory of electromagnetic radiation, spatial
coherence, speckle size, bandwidth, and uncertainty are all
interrelated concepts in physics and are bounded by the funda-
mental bound of the diffraction limit or the Nyquist-Shannon
sampling rate of half the oscillation period. This fundamental
bound can only be further resolved with additional a priori
information of the localization setup like non-linear mixing
of radio signals. Non-linear interactions such as homodyning
and heterodyning are commonly used in signal processing
to resolve the phase of the signals. As we shall see in the
next section, in the field of radio localization CRLB analysis
is based on empirical propagation models with independent
noise. A link between uncertainty and Fisher Information was
first established by [11]. [11] shows on mathematical grounds
that the Uncertainty Principle, Fisher Information and thus
the CRLB are based on the same fundamental inequalities.
[11] shows in the scalar and single measurement case that the
square of uncertainty, \( \sigma_r^2 \), and the CRLB on variance, \( \sigma_{\text{CRLB}}^2 \),
are related by:
\[ \frac{1}{\sigma_{\text{CRLB}}^2} \sigma_r^2 \geq 1, \]  
(22)
where the equality holds for Gaussian distributions. Uncer-
tainty, \( \sigma_r \), has a lower bound that equals the diffraction limit.
Hence, in case of Gaussian distributions the CRLB on variance
has a fundamental lower bound that equals the square of the
diffraction limit. Equation (22) holds for variances of single
measurements, because the number of measurements does not
alter the respective uncertainties in (13) and (19) with their
fundamental bounds. Our experiments quantify the difference
between the CRLB using the rationale in [11] and the measured
fundamental bound on RMSE, which appears to be 2-3%
Section V-C). A quantitative experimental verification of [11]
has not been given up to today.

IV. EMPIRICAL PROPAGATION AND NOISE MODEL FOR
MAXIMUM LIKELIHOOD ESTIMATOR AND CRAMER-RAO
LOWER BOUND
This section describes the empirical propagation model,
MLE and CRLB usually applied in the field of RSS-based
radio localization [14], [19], [21], [26]. The localization space
of our experimental setup is considered as two-dimensional
in the \((x, y)\) plane. The extension to three dimensions is
straightforward. We use the notations introduced in Sections
[11] and [11]. The experimental setup consists of a reference
radio that measures power at 2,400 positions \((x_1, y_1) =
(x_1, y_1 \ldots x_{2400}, y_{2400})\) of an unmodulated carrier transmitted
by the blind radio. At each position, the reference radio
performs \(50 \times 500 = 25,000\) repetitive multiplexed power
measurements \(P_{l,m,n} = P_{1,1,1} \ldots P_{2400,50,500}\). These mea-
surements are used to estimate the blind radio position, which
is located at the origin \((x, y) = (0, 0)\).

A. Empirical Propagation Model
We adopt the empirical LNSM for modeling the power over
distance decay of our RSS measurements added with indepen-
dent noise. As the cross-sections of the blind transmitter and
reference receiver are given and equal, the power as well as the
power-flow measurements are assumed to satisfy the empirical
LNSM [7]
\[ P(r_p) = P_{r_0} - 10\eta \log_{10} \left( \frac{r_p}{r_0} \right) + X_{\sigma_{dB}}. \]  
(23)
In (23), \(P(r_p)\) represents measured power at distance \(r_p\)
in dBm. \(P_{r_0}\) represents power at reference distance \(r_0\)
in dBm. \(\eta\) represents the path-loss exponent. \(X_{\sigma_{dB}}\) represents
the Additional White Gaussian Noise (AWGN) of the model
in dB due to shadowing effects and is invariant with distance.
\(X_{\sigma_{dB}}\) follows a zero-mean normal distribution with standard
deviation \(\sigma_{dB}\).
Usually, the three parameters \([P_{r_0}, \eta, \sigma_{dB}]\) of the LNSM
are calibrated for a given localization setup [14], [19], [26].
The blind radio position is assumed to be known when the
LNSM is calibrated. The LNSM assumes that \(\sigma_{dB}\) is equal for
each RSS measurement, so that \(P_{r_0}\) and \(\eta\) can be estimated
independent of \(\sigma_{dB}\):
\[ [P_{r_0}^{cal}, \eta^{cal}] = \arg \min_{[P_{r_0}, \eta]} \sum_{l \in L} \sum_{m \in M} \sum_{n \in N} \left( P_{l,m,n} - \left( P_{r_0} - 10\eta \log_{10} \left( \frac{r_l}{r_0} \right) \right) \right)^2, \]  
(24)
where
\[ r_l = \sqrt{(x_l - x)^2 + (y_l - y)^2}. \]  
(25)
In (23), \( \sigma_{dB}^{cal} \) is defined as the standard deviation between the measurements and the fitted LNSM using \( P_{r_0}^{cal} \) and \( \eta^{cal} \). In (24), \( r_l \) represents the geometrical distance between reference radio position \( l \) and the true blind radio position \((0,0)\). Equation (24) gives for all our measurements \( P_{l,m,n} \) the best fit when the LNSM parameters are calibrated at
\[
P_{r_0}^{cal} = -16.7 \text{dBm}, \eta^{cal} = 3.36, \sigma_{dB}^{cal} = 1.68 \text{dB}.
\] (26)

The far-field power flow \( P(r_p) \) radiated from a source at known position (in the origin) can be calculated in any far-field position using (24) setting the last term to zero. We use these calibrated LNSM parameter values to calculate the bias and efficiency of our estimator from simulations. In addition, we use these calibrated LNSM parameter values to calculate the cross-correlations in the far-field and measurement variance of the wave variables.

B. Maximum Likelihood Estimator

We use the MLE as proposed by (20) to estimate the blind radio position:
\[
[x_{mle}, y_{mle}, P_{r_0_{mle}}, \eta_{mle}] = \arg \min_{[x,y,P_{r_0},\eta]} \sum \sum \sum (P_{l,m,n} - (P_{r_0} - 10 \cdot \eta \cdot \log_{10} (\frac{r_l}{r_0})))^2
\]
subject to
\[
\eta > 0
-1 \leq x \leq 2
-2 \leq y \leq 1,
\] (27)
where
\[
r_l = \sqrt{(x_l - x)^2 + (y_l - y)^2}.
\] (28)

In (28), \( r_l \) represents the calculated geometrical distance between reference radio position \( l \) and the estimated blind radio position.

C. Cramer-Rao Lower Bound

CRLB analysis provides a fundamental performance bound on the spreads of unbiased estimators of unknown deterministic parameters. This lower bound implies that the covariance of any unbiased estimator, \( \text{COV} (\hat{\theta}) \), is bounded from below by the inverse of the Fisher Information Matrix (FIM) \( F \) as given by (8)
\[
\text{COV} (\hat{\theta}) \geq \text{F}^{-1} (\theta).
\] (29)

In (29), the elements of \( F(\theta) \) are computed as
\[
F_{a,b} = -E \left( \frac{\partial^2 \ln (PDF(S; \theta))}{\partial \theta_a \partial \theta_b} \right).
\] (30)

In (30), \( PDF \) represents the Probability Density Function (PDF) of the set of independent power measurements \( S \), where the PDF is parameterized by the set of unknown parameters \( \theta \). Note that in evaluating (30), the true value of \( \theta \) is used. The elements of the \( 4 \times 4 \) FIM associated with the MLE defined by (27), \( \hat{\theta} = [x_{mle}, y_{mle}, P_{r_0_{mle}}, \eta_{mle}] \), are given by (19). The elements of \( F \) consists of all permutations in set \( \theta \). After some algebra, this leads to
\[
F_{x,x} = b \sum \frac{(x - x_l)^2}{r_l^4}
\]
\[
F_{x,y} = b \sum \frac{(y - y_l)(x - x_l)}{r_l^4}
\]
\[
F_{x,\eta} = b \sum \frac{(x - x_l)}{r_l^2} \ln(r_l)
\]
\[
F_{x,P_{r_0}} = \frac{10 \eta}{\sigma_{dB}^2 \ln(10)} \sum \left( \frac{x - x_l}{r_l^2} \right)
\]
\[
F_{\eta,\eta} = \frac{\sigma_{dB}^2 \ln(10)}{\eta^2} \sum \left( \ln(r_l) \right)^2
\]
\[
F_{\eta,P_{r_0}} = \frac{10}{\sigma_{dB}^2 \ln(10)} \sum \ln(r_l)
\]
\[
F_{P_{r_0},P_{r_0}} = \frac{1}{\sigma_{dB}^2},
\] (31)
where
\[
b = \frac{10 \eta}{\sigma_{dB} \ln(10)}.
\] (32)

The elements \( F_{y,y} \), \( F_{x,\eta} \) and \( F_{x,P_{r_0}} \) can be computed in a similar way as \( F_{x,x} \), \( F_{x,\eta} \) and \( F_{x,P_{r_0}} \). We use (31) to calculate the CRLB of the MLE expressed by (27) using the calibrated LNSM parameter values given by (26): \( \sigma_{dB}^{cal}, \eta = \eta^{cal} \).

In principle, CRLB analysis offers a computational scheme for any ensemble of measurements as long as the estimator is unbiased and efficient, and the measurements are independent. For unbiased estimators, the covariance or standard deviation of the ensemble average of blind radio position estimates approaches zero with increasing ensemble size as is computed from
\[ RMSE = \sqrt{E((x_{mle} - x)^2 + (y_{mle} - y)^2)} \geq \sqrt{tr(F^{-1} \theta)}. \]

Here \(tr(\cdot)\) represents the trace of the matrix.

Figure 4 shows the lower bound on the RMSE calculated by (31) and (33) as a function of the number of independent RSS measurements. In this figure, we assume that RSS measurements at each reference radio position are independent. Reference radio positions are equidistantly positioned along the circumference of the measurement setup. Note that a similar result can be obtained when we assume that RSS measurements over time are independent. Figure 4 shows that the RMSE decreases to zero with an ever increasing number of independent RSS measurements, which is in accordance with the theoretical analysis presented by [14, 19, 21, 24].

The theoretical results in Figure 4 reveal the untenability of adding independent measurements up to 60 million in a finite measuring range to increase localization precision to over \(\lambda/100\). The first caveat is that the central limit theorem only holds for a given single domain, so combining measurements from different domains should not further increase localization precision. Secondly, truly independent measurements are uncorrelated and equally spread in the measurement domain. However, part of that measurement domain is reserved for correlated signals and noise as all electromagnetic radiation noise is spatially correlated over at least the mean wavelength of the radiation [27, §4.4.4 - §13.1]. The applicability of existing CRLB analyses in the field of radio localization to estimate performance bounds is limited to idealized situations where the mean wavelength of the radiation is assumed to be zero.

D. Simulations

To validate CRLB analysis on its internal consistency, we need to quantify the difference between the covariance of our estimator and CRLB. We performed 10,000 simulation runs using the measurement setup described in Figure 2. We used these simulated measurements for evaluating (1) the bias and (2) the efficiency of the MLE used to estimate the position of the blind radio. Note that these simulations do not provide an experimental validation of the LNSM and the independence of the noise. An estimator is unbiased if [8]

\[ E(\hat{\theta}) - \theta = 0. \]

We are interested in the bias of the estimated blind radio position \((x_{mle}, y_{mle})\), so we define the bias as

\[ \text{BIAS}(x_{mle}, y_{mle}) = \sqrt{(E(x_{mle}) - x)^2 + (E(y_{mle}) - y)^2}. \]

Simulations show that the bias is of the order of \(\text{BIAS}(\hat{x}, \hat{y}) \approx 0.1\)mm. Estimator efficiency is defined as the difference between the covariance of the estimator and CRLB. We quantify this difference by

\[ \text{Estimator efficiency} = |RMSE_{simulated} - \sqrt{tr(F^{-1} \theta)}|. \]

Simulations show that the estimator and CRLB differs at most by Estimator efficiency \(\leq 1\)mm. Hence, we could validate the internal consistency of our CRLB analysis only on two out of three counts: negligible bias and high efficiency, but the independence of the noise model has not been addressed.

V. EXPERIMENTAL RESULTS

This section presents the experimental results of the two-dimensional measurement setup described in Section II. The first subsection presents the RSS signal decay over distance. Then, we present the measured RSS signals along the circumference of the localization area. In the second subsection we determine the far-field spatial cross-correlation function of the RSS signals and estimate the spread of these spatial correlations. We then determine the spatial Fourier transform of these cross-correlations to look for an upper bound at a spatial frequency of \(k = 2\pi / (\lambda / 2)\), in line with Fourier Optics that spatial resolution is bounded by the diffraction limit. Finally, we determine the global RMSE of (33) and show its asymptotic behavior to the diffraction limit with the increase of the density of sampling points. We compare this experimentally determined RMSE with the RMSE determined by the CRLB using (31) and show that the CRLB underestimates the experimentally determined RMSE in all cases.

A. RSS Signal over Distance and Circumference

Figure 5 shows the RSS over distance decay. The spread in red dots represents the deviations from the LNSM.

Figure 6 shows the RSS signals along the circumference of localization space. The apparently small difference between the red and black curves indicates that noise on repetitive RSS signals over time is relatively small.

B. Spatial Correlations and Spatial Frequency Distribution

Figure 7 shows spatial cross-correlations between power flows as a function of correlation distance in wavelengths.
Fig. 6. Measured RSS signals along the circumference of localization space. The blue curve represents the average of 25,000 RSS signals at each of the 2,400 reference radio positions; the red curve represents 1 RSS signal at each of the 2,400 reference radio positions.

Fig. 7. Second-order spatial cross-correlations of RSS power flows as given by (16) as a function of correlation distance in wavelengths. The black curve shows the measured cross-correlations using all 60 million RSS measurements. The red curve shows the measured cross-correlations using 1 RSS measurement per reference radio position.

as given by (16). The average power flows \(\langle P(\vec{r}_p)\rangle\) in (16) are calculated using the LNSM expressed by (23) and calibrated propagation parameters expressed by (26). Hence, spatial cross-correlations are distance independent by cross correlating the deviations from the calibrated LNSM. Figure 7 shows that the spatial cross-correlations go to a minimum over a distance of roughly half the mean wavelength, which corresponds to the diffraction limit. The small difference between the black and red curve indicates that noise resulting from repetitive multiplexed measurements over time is negligible compared to the radiation noise.

In the case of Fisher Information and thus CRLB analysis, information is additive when measurements are independent

[8]. In Section IV LNSM, MLE and CRLB assume that all RSS signal measurements are independent. Hence, localization precision increases with an ever increasing amount of independent measurements over a finite measuring range. However, when measurements are correlated, information and localization precision gain decrease with increasing correlations. Figure 7 shows that when space-measurement intervals become as small as the diffraction limit, measurements become spatially correlated and mutually dependent. Our measurements show that RSS signal measurements are spatially correlated over a single-sided region of roughly half the mean wavelength. Therefore, increasing reference radio density beyond one per half a wavelength has a negligible influence on Fisher information gain and thus on localization precision. Our experimental results in the next subsection appear to support this. In case of independent measurements, Figure 7 would show an infinitely sharp pulse (Dirac delta function).

Figure 8 shows the power spectrum, \(|\gamma(k)|\), i.e. the spatial Fourier transform of (16). This figure shows that the energy is mainly located in lower spatial frequencies and it diminishes over a single-sided interval of \(k \leq 2\pi/(\lambda/2)\). This upper bound corresponds to the diffraction limit. The Nyquist-Shannon sampling rate provides an estimate of the minimum sampling rate to reconstruct the power-flow signal over space without loss of information, which equals the single-sided bandwidth of our power spectrum. The vertical black line represents the spatial frequency associated with the Nyquist-Shannon sampling rate, which equals \(k = 2\pi/(\lambda/2)\). Our experimental results in the next subsection appear to support this by showing that the localization precision does not increase by sampling beyond this sampling rate. In case of independent measurements, Figure 8 would show a uniform distribution.
C. Localization Precision

Figure 9 shows the experimental results of our measurement setup described in Section II. Localization precision is given as the inverse of RMSE, which is calculated from (33).

The red curve in Figure 9 shows the measured RMSE as a function of the number of measurements per wavelength. The measured RMSE decreases with increasing number of RSS measurements over space until sufficient measurements are available. Then, the fundamental bound on localization becomes of interest. The measured RMSE (red and black curves) converges asymptotically to roughly half the mean wavelength as one would expect from the diffraction limit.

The measured RMSE represented by the red curve is based on processing all 25,000 RSS signal measurements per reference radio position. The measured RMSE represented by the black curve is based on processing one RSS signal measurement per reference radio position. The negligible difference between the red and black curves shows that the number of RSS signal measurements per reference radio has a negligible influence on the measured RMSE.

The CRLB starts deviating from the measured RMSE (red and black curves) when sampling density is increased beyond one RSS signal per half the mean wavelength (see black dotted curve), as one would expect from the diffraction limit and the Nyquist sampling rate over space. Spatial correlations between RSS signals increase rapidly with increasing RSS measurement density beyond one sample per half the mean wavelength (Figure 7). Correlated RSS signals cannot be considered as independent. [1] has shown that Fisher information is upper bounded by uncertainty as expressed by (22). Coherence and Speckle theory have shown that uncertainty is lower bounded by the diffraction limit. Hence, the CRLB at a sampling density of one sample per half the mean wavelength (vertical dotted black curve) should equal the measured fundamental bound on RMSE. We define the measured fundamental bound on RMSE as the measured RMSE processing all RSS measurements. Our measurements show that the difference between this CRLB and the measured fundamental bound on RMSE is 2-3% (1mm). Hence, our experiments validate the theoretical concepts introduced by [1]. Our experiments reveal evidence that the CRLB indeed has a fundamental lower bound, which cannot be further decreased by increasing the number of measurements. On the other hand, at 25 RSS signal measurements per wavelength, the measured RMSE is a factor of four higher than the one calculated by the CRLB. This difference cannot be explained by the difference between the covariance of the estimator and the CRLB (see Section IV).

The green curve shows localization precision calculated by the CRLB that regards all 2, 400 × 50 × 500 RSS signals over time and space as independent. The measured RMSE has no resemblance with this CRLB.

All 60 million power measurements and Matlab files are arranged in a database at Linköping University [32].

VI. DISCUSSION

Our novel localization setup of using reference radios on the circumference of a localization area instead of setting them up in a two-dimensional array worked well. This implies that one does not have to know the phases and amplitudes on the closed surface around an extended source to reconstruct the position of the extended source. Our experiments show that it suffices to measure time-averaged power flows when a localization precision of about half a wavelength is required (in our case 6 cm). Time averaging on a time scale that is long compared to the coherence time is usually employed in RSS localization, so that phase information is lost. We expect such a setup to work well when all radio positions are in LOS. For a practical implementation in NLOS environments, we refer to [28] [5].

Our experiments consist of an extensive amount of RSS measurements over space and time to investigate the assumption of independent noise over [space, time] of existing fundamental bound analyses in the field of radio localization [14], [19], [24]. In order to investigate whether RSS measurements are independent over time, the reference radio performed 25,000 repetitive multiplexed RSS measurements over time at each of the 2, 400 reference radio positions in space. Our experiments show that there is a negligible difference in our experiments with 1 to up to 25,000 independent RSS measurements at each of the 2, 400 reference radio positions in space. In addition, the measured localization precision shows no resemblance with the CRLB analysis treating the 25,000 RSS measurements over time as independent measurements.

In section III on fundamental propagation of signals and noise, we distinguished between primary and secondary radiation noise, and thermal noise. In our experimental setup where signal levels are large compared to noise levels, it is reasonable to neglect thermal and quantum noise. As the size of our transmitter is half a wavelength, the surface roughness has spatial frequencies in excess of 2π/(λ/2). Hence, no primary radiation noise from antenna surface roughness is to be expected, because the radius of the spatial cross-correlation
region is $\lambda/2$. However, the primary grazing incident radiation is scattered by the surface roughness of the large area between transmitter and receiver. The stochastic properties of this surface roughness must be the same for all 2,400 transmitter-receiver positions. Hence, our 2, 400 time-averaged RSS measurements are correlated over space as was verified by our experiments.

As discussed above, the noise in our measurements originates from a variety of sources and can be significant in their respective frequency regimes. In our setup where only energy is exchanged and signals below the diffraction level are not time-resolved, the noise level of the antenna plus electronics on a typical 802.15.4 radio is about 35dBm below signal level as specified in [31]. The independent noise in the LNSM is practically not present in our measurements. The reproducible ripples on our RSS signals originated in part from small interference effects from undue reflections from mostly hidden metallic obstacles in the measuring chamber. Such obstacles like armored concrete pillars could not be removed. In a real indoor office environment, these interference effects usually dominate the RSS signals. As long as time scales are long compared to the coherence time of the transmitters, phase information is lost and the diffraction limit determines the fundamental bound on localization precision. Multi-path effects do not change the stochastic properties of the contrasting surfaces where the phase noise is generated.

CRLB analysis has been applied to RSS and TOF localization systems as a purely statistical tool, encouraging to increase the number of independent measurements across different domains in the expectation to decrease the lower bound on measurement variance. In that sense, it is a handy tool to give a performance measure for localization systems as long as the noise is independent. In existing CRLB analyses applied to RSS and TOF localization systems [14], [19], [24], the measured power flow is modeled as the true power flow with independent noise. Independent noise does not propagate with the signal over time and space. The challenge to experimentally verify this model has always been difficult as the majority of the available measurements were noise limited. Achieving a localization performance of the order of half the mean wavelength was not as trivial as it sounds. With the arrival of relatively inexpensive Gigahertz radios, far-field conditions are now generally met, so that spatial correlations can easily be determined from far-field RSS signals.

In coherence and speckle theory, the far-field signals and noise of primary and secondary sources are governed by the same equations of motion. For electromagnetic signals, the dynamics are determined by Maxwell’s equations plus boundary conditions on the tangential field components and Sommerfeld’s radiation condition. Maxwell equations are differential equations that transform to integral representations by applying Green’s theorem. One of the advantages of using integral instead of differential equations is that they visualize Huygens geometrical principle. Huygens principle determines how a wave front propagates towards a point of observation by adding geometrical contributions of each elementary spherical wavelet on that wave front towards the point of observation. In the far field, these integral representations simplify to spatial Fourier transforms of the surface of the source, the foundation of Fourier Optics with its spatial filtering and signal processing.

The second order coherence function links temporal coherence to spatial coherence, first observed and theoretically explained by Hanbury Brown and Twiss in 1958 [2], later formalized by [27]. Our experimental work determines the spatial coherence of the time-averaged rather than of the instantaneous Poynting vector of the far-field quasi-monochromatic radiation as is described by speckle theory [17]. In all cases, the theoretically expected lower bound on the size of spatial coherence region equals the diffraction limit. Our experiments revealed this lower bound in a two dimensional setup. Within this coherence region, noise is correlated and not independent. Our global measured RMSE calculated from RSS signal measurements showed that with increasing sampling density, localization precision did not further increase beyond the inverse of the diffraction limit, in line with what one would expect from the lower bound on the size of the coherence region. Our theoretical analysis generalizes the obtained results to three dimensions.

With the mathematical work of [1], a rigorous link between Fisher Information, CRLB and Uncertainty could be established. As uncertainty has a fundamental lower bound according to Coherence and Speckle theory, so must Fisher Information have a fundamental upper bound, so that the CRLB has a fundamental lower bound, all when the noise processes are assumed to be Gaussian distributed. Our experiments for the first time were able to validate those theoretical concepts with a 97-98% accuracy.

Finally, it took almost three weeks in throughput time to perform the 60 million measurements, generating $2400 \times 50 \times 500$ repetitive multiplexed measurements to test the asymptotic behavior of the localization performance to the diffraction limit with increasing sampling density. We tried to minimize multi-path effects by avoiding the interfering influence of ground and ceiling reflections. Making sure to minimize reflections of other metallic obstacles in the measuring chamber were challenging but could be overcome. In the end, the repeatability of our measurement sets was much better as estimated by CRLB analysis. This allowed us to reveal a performance limit in theory and experiment well-known in physical optics, but not yet achieved in RSS- and TOF-based localization without phase interpolations.

VII. CONCLUSION

Our novel two-dimensional localization setup where we positioned the reference radios on the circumference of the localization area rather than spreading them out in a two-dimensional array over this area worked well in our LOS setup. Our measurements show that localization performance does not increase indefinitely with an ever increasing number of multiplexed RSS signals (space and time), but is limited by the spatial correlations of the far-field RSS signals. When sufficient measurements are available to minimize the influence of measurement noise on localization performance, the fundamental bound on localization precision is revealed as the
region of spatially correlated far-field radiation noise. The determination of this region of spatial correlations is straightforward and can be directly calculated from RSS signals. Within this region of correlated RSS signals, existing assumptions of independent measurements are invalidated, so that the existing CRLB underestimates the fundamental bounds on radio localization precision. The CRLB is linked to the uncertainty principle as measurement variance is directly related to this principle as we showed. Existing fundamental bound analyses on radio localization are only valid in the idealized model of the geometrical-optics approximation, where wavelength is assumed to be zero and unlimited information density is possible. The fundamental bounds on the precision of RSS- and TOF-based localization are expected to be equal and of the order of half a wavelength of the radiation as can be concluded from our experiments and underlying theoretical modeling. Sampling beyond the diffraction limit or the Nyquist-Shannon sampling rate does not further resolve the oscillation period unless near-instantaneous measurements are performed with the a priory knowledge that signal processing is assumed to be on non-linear mixing. Future research is aimed at the inclusion of strong interference effects such as show up in practically any indoor environment.

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