Involute Teeth with Variable Base Diameter

A Seregin and A Kravtsov*
Orenburg’s State University, 13 Victory avenue, Orenburg, Russia

*aasdom@yandex.ru

Abstract. The article presents the results of theoretical and experimental research. The reserve of increasing the durability of gears of machines and mechanisms and reducing the costs of their maintenance and repair is shown to reduce wear of the surfaces of the teeth. The dependences for the calculation and construction of the profile of the teeth of the wheels, providing a decrease in the coefficient of relative slip. Using formulas in polar coordinates for the angle of deployment of the tooth profile involute, a new shape of the lateral profile of the gear teeth is obtained.

1. Introduction
In the field of mechanical engineering, along with the continuous improvement of involute gears, research continues to find new types of gears that provide increased wear resistance of gear elements or obtain other technological and operational advantages.

2. Theoretical research
The main ways to reduce wear are to improve the quality of the lubricant and change the nature of the contact of the teeth (see technical solutions set forth in patent 67425 of the USSR and patent 1471023 of the USSR, as well as US patent 3438279).

Technical solution for patent 1471023 of the USSR compares favorably with patent 67425 and US patent 3438279 in that it provides a reduction in the coefficient of relative sliding of the teeth by 21% while maintaining a constant gear ratio and providing a coefficient of profile overlap of $1 < \varepsilon < 2$.

However, profile shaping according to the indicated technical solution is possible only by copying. Therefore, to reduce the complexity of calculating the profile of modular end mills, its automation is necessary.

The radius of the base circle of the tooth profile should be equal to:

$$\rho_C = r_0 + (\Omega - r_0) \cdot \Theta^{-\frac{1}{2}} \cdot \psi_C,$$

(1)

$$\rho_C, \psi_C$$ - coordinates of the C-th point of the polar system (radius and angle) with a pole on the axis of the smallest of the wheels; $r$ - the radius of the main circle, which is the evolute of the tooth profile of a similar wheel, but with a constant evolution. C is the point of contact of the teeth.

Multiplier

$$\Omega = mn^{-\frac{1}{2}} (A + \delta A)^{\frac{1}{2}} \cdot (R + \delta R)$$
Multiplier

\[ \Omega = \pi m^{-1/2} \left( A + \delta A \right)^{1/2} \cdot \left( R + \delta R \right) \]

\( A; \delta A \) - center distance and its tolerance; \( R; \delta R \) - the outer radius of the gear and its tolerance; \( m \) - gearing module.

With increasing radius vector of the contact points of the gear teeth, the relative sliding coefficient of the teeth decreases, since the latter is inversely proportional to the radius vector according to the formula:

\[ \lambda = \left( 1 + i_{12} \right) p_c C \cdot \rho_c^{-1} \]

\( i_{12} \) - gear ratio; \( P_o C \) - the distance from the pole of the engagement \( P_o \) to the \( C \)-th point of contact of the teeth.

It should be noted that the increase in the radius vector is limited by the value of the required value of the coefficient of profile overlap of the teeth \( \varepsilon > 1 \). Approximately the value of the tooth overlap coefficient can be determined by replacing the length of the practical line of engagement with the length of the theoretical line:

\[ \varepsilon = A \cdot R^2 \left( \pi m r_o^2 \right)^{-1} \]

To keep the gear ratio constant and insensitive to mounting errors, it is necessary that the ratio of the radii of the conditional main circles is always equal to the gear ratio of gearing at all contact points of the teeth of the practical gearing line. Based on the condition \( \varepsilon > 1 \), the increase in the radius of the conditional base circles lies in the range:

\[ r_o \leq r \leq \Omega. \]  \hspace{1cm} (2)

Using dependence (2) as applied to polar coordinates, and taking the angle \( \psi_c \) of deployment of the tooth profile involute as a parameter, we obtain dependence (1).

Thus, when the gear pair is rotated, the instantaneous contact of the teeth moves along the profile until the next tooth of the wheel engages with the gear tooth, creating permanent engagement. The maximum contact of the face and flank of the tooth is provided at any moment of rotation of the gear and gearwheel. At the same time, it is possible to achieve the maximum possible increase in the tooth contact spot with the positive feature that this contact spot is distributed evenly over the entire height of the teeth in contact, since the radius of the conditional circles of the tooth face and the radius of the tooth flank have an equivalent increment over the entire surface of the mating gearwheel and gear. This advantage allows you to significantly increase the contact endurance of the working surfaces of the teeth.

To eliminate possible jamming from heating during the operation of the gearing of the proposed design, it is necessary to provide for the expansion of the depressions, as well as a chamfer along the entire length of the top land of the tooth.

To increase the wear resistance and durability of gears, it is necessary that the completeness of contact of the mating lateral surfaces of the gear teeth is greatest. With incomplete and uneven fit of the teeth, the bearing surface area of their contact decreases, contact stresses and lubricant are unevenly distributed, which leads to intensive wear of the teeth.
To ensure the necessary completeness of the contact of the teeth and the distribution of lubricant in the transmission of the experimental design, the smallest dimensions of the permissible total contact spot are established. When operating in conditions of limited lubrication, the experimental gear train has increased wear resistance of the teeth and reduced friction losses in gearing.

![Figure 1](image)

**Figure 1.** Standard (top) and experimental (bottom) gears.

Changing the radii of the base circles of the teeth of the wheels involved in the transmission, reduces the coefficient of sliding of the teeth. The slip coefficient decreases for small-modular gearwheels from 2% at the tooth face to 7% at the flank, for large-modular wheels from 6.5% at the face to 21% at the flank. The slip coefficient is changed by replacing the relative slippage of the teeth of the gearwheel and gear by rolling. This is achieved by changing the radius of curvature of the tooth surface (see Figure 1) within the limits allowed by the overlap coefficient. The amount of wear on the lateral surfaces of the gear teeth decreases in proportion to the slip coefficient. As can be seen from Figure 1, the head of the tooth of the wheel by patent 1471023 of the USSR (the gearwheel is located below) has a smaller narrowing to the outer diameter. This simple fact helps to reduce the wear of the gear teeth, ceteris paribus. Since the process of mutual rolling of the gearwheels is involved during wear, then with the increase in wear the evolutionoid profile tends to involute.

The proposed evolutionoid gearing is supposedly stronger than the involute gearing when the teeth work on the face crushing, since the ratio of the thickness of the tooth face to its flank for involute gearing is from 0.47 to 0.65, and for the evolutionoid gearing from 0.73 to 0.85.
3. Experimental Works and Discussion

One of the important parameters that ensure reliable operation of gears is the contact surface area. The contact spot was evaluated as part of the active lateral surface of the tooth of the wheel, on which there are traces of the fit of the teeth of the twin wheels (traces of nadir or paint) in the assembled gear after rotation under load.

One of the main operational indicators of gears of kinematic chains of metal-cutting machine tools is smooth operation [1], [2], that is, the absence of cyclic errors repeated many times per wheel revolution. With increasing speed, the requirements for smooth operation increase.

The smoothness of the transmission was determined by parameters, the errors of which cyclically occur during the rotation of the gearwheel and also form part of the kinematic error. Analytically, the kinematic error can be represented in the form of a spectrum of harmonic components, the amplitude and frequency of which depend on the nature of the component errors. For example, deviation of the meshing pitch (main pitch) causes fluctuations in the kinematic error with a frequency equal to the frequency of the gear teeth entering the meshing.

Cyclic error is the main reason for the violation of the smoothness of gears, consisting of spur gears. Spin gears dominate the kinematic chains of metal cutting machines. Cyclic errors usually cause an increase in noise characteristics, and the level of noise power increases with increasing transmission speed.

The transmission should work silently and without vibration [3], which can be achieved with minimal errors in the shape and relative position of the teeth [4]. The wheels of such gears usually have medium modules. For them, the noise characteristics of a working transmission, vibration, static and dynamic imbalance of rotating masses, etc. are also often limited. The noise was measured in a muffled noise measuring chamber using a sound meter-vibrometer model "Octave-110A" which is a class 1 device. Instruments of this model can analyze the spectrum of noise and vibration with an accuracy of 3.0 dB. As a result of a number of tests, it was determined that transmissions with an experimental profile have slightly inflated noise characteristics relative to standard designs.

Figure 2. Diagrams of wear of the teeth of the gears of the standard (right) experimental (left) gear.

Figure 2 presents a plot of the wear of a gear tooth, obtained on the basis of the analysis of experimental data taking into account the impact of abrasive particles. The main line indicates the
nominal contour of the tooth profile. The thin line is the newly formed profile due to wear. In the area of the pole of engagement, the amount of wear is minimal. Parallel thin lines show the ablation of the tooth metal mass due to wear.

Depreciation was assessed by the change in tooth thicknesses measured at 10 profile points from the flank to the tooth face. The profile of the tooth was tested to an accuracy of 0.001 mm on the coordinate measuring machine model XO 55 TM 1 manufactured by Wenzel. Dependences of wear in the zone of the pole are revealed. The results of bench tests showed that a notch of a cycloidal profile is formed on the gear teeth in the area of the tooth face in front of the pole, and the changes on the mating tooth of the wheel do not obey any specific laws. The small wear at the tooth flank is explained by the difficulty of access of abrasive particles in this zone. Increased wear was observed on the face of the tooth. According to theoretical calculations on the tooth face, the slip coefficient is maximum.

According to the degree of wear or damage to the working surface of the tooth that occurred during a certain period of time, the transmission resource is usually estimated. Using the methodology for predicting the reliability of products [5], we can approximately estimate the term for increasing the resource. The experimental design of the gear engagement allows you to maintain the kinematic accuracy of transmission for a period of time longer by two or three inter-repair periods than the standard one.

4. Conclusion
The experimental profile of the gearshift is a new generation mechanism with high technical and economic indicators that can improve its reliability and durability.

References
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