Lorentz Violation and the Higgs Mechanism

Brett Altschul

Department of Physics and Astronomy
University of South Carolina
Columbia, SC 29208

Abstract

We consider scalar quantum electrodynamics in the Higgs phase and in the presence of Lorentz violation. Several equivalent formulations of this theory exist, related by coordinate redefinitions. Spontaneous breaking of the gauge symmetry may give rise to Lorentz-violating gauge field mass terms. Such mass terms may cause the longitudinal mode of the gauge field to propagate superluminally. However, a number of properties of this theory are quite analogous to those seen in a conventional Abelian gauge theory with spontaneous symmetry breaking. The theory may be quantized by the Faddeev-Popov procedure, although the Lagrangian for the ghost fields also needs to be Lorentz violating. We compare these results to some other quantum field theories with Lorentz violation.
1 Introduction

Lorentz violation is currently a topic of significant interest in particle physics and other areas. No particularly strong evidence for a deviation from Lorentz invariance has been found, but experimental Lorentz tests are constantly being refined. The study of Lorentz symmetry remains an active area of research—both experimentally and theoretically—because if any violation of Lorentz invariance were to be found, that would be a discovery of premier importance.

Lorentz violating can be readily studied using quantum field theory. Violations of the symmetry may be described in an effective field theory called the standard model extension (SME). The SME contains Lorentz-violating corrections to the standard model, parameterized by small tensor-valued background fields \[1, 2\]. The most frequently considered subset of the SME is the minimal SME, which contains only gauge-invariant, superficially renormalizable forms of Lorentz violation. The minimal SME has become the standard framework used for parameterizing the results of experimental Lorentz tests.

Recent searches for Lorentz violation have included studies of matter-antimatter asymmetries for trapped charged particles \[3, 4, 5\] and bound state systems \[6, 7\], measurements of muon properties \[8, 9\], analyses of the behavior of spin-polarized matter \[10\], frequency standard comparisons \[11, 12, 13, 14\], Michelson-Morley experiments with cryogenic resonators \[15, 16, 17, 18, 19\], Doppler effect measurements \[20, 21\], measurements of neutral meson oscillations \[22, 23, 24, 25, 26, 27\], polarization measurements on the light from cosmological sources \[28, 29, 30, 31\], high-energy astrophysical tests \[32, 33, 34, 35, 36\], precision tests of gravity \[37, 38\], and others. The results of these experiments set constraints on the various SME coefficients, and up-to-date information about most of these constraints may be found in \[39\].

The one-loop renormalization of various sectors of the minimal SME has been studied. This has included analyses of Abelian \[40\], non-Abelian \[41\], and chiral \[42\] gauge theories with spinor matter, as well as scalar field theories with Yukawa interactions \[43\]. Notably absent from this list is a full treatment of gauge theories with charged scalar fields. Such theories play a crucially important role in the standard model, but they are complicated by the possibility of spontaneous gauge symmetry breaking.

This paper shall look at several aspects of Lorentz-violating scalar quantum electrodynamics (SQED), with particular emphasis on spontaneous symmetry breaking and the Higgs mechanism. Normally, the Higgs mechanism is responsible for a mass term for the gauge field. There are other ways to endow an Abelian gauge boson with mass, but the Higgs mechanism is the most important, because only it has a straightforward generalization to non-Abelian gauge theories. Normally, the mass term produced by the Higgs mechanism resembles a Proca term in the action. However, if the dynamics of the scalar field responsible for the symmetry breaking are not Lorentz invariant, it is possible to have mass terms with different structures. Any Lorentz violation in the scalar sector will be transferred to the gauge sector when the Goldstone boson of the spontaneously broken
symmetry is “eaten” by the gauge field—becoming the longitudinal component of the massive vector excitation. There has been some previous discussions of spontaneous symmetry breaking in the context of the full electroweak sector of the SME [2, 44]. However, prior work has not focused on how the Lorentz violation affects the gauge boson mass terms that arise through the Higgs mechanism. Analysis of the Lorentz-violating mass terms that arise in SQED will be one of the primary topics of this study. Since Lorentz violation is a small effect, we shall only work to first order in the SME coefficients.

This paper is organized as follows. In section 2, we shall introduce the SQED Lagrange density with dimensionless Lorentz-violating coefficients. After including the effects of gauge symmetry breaking, we examine several sectors of the theory, paying particular attention to the structure of the gauge boson mass terms. Section 3 discusses the quantization of the spontaneously broken gauge theory, including the introduction of interacting Faddeev-Popov ghosts, which are associated with Lorentz-violation coefficients related to those in the matter sector. Section 4 recasts these results using coordinate redefinitions, which can be used to move certain types of Lorentz violation from one sector of the theory to another. Finally, section 5 summarizes the paper’s conclusions.

2 Lorentz-Violating Lagrangians

2.1 Lagrangian Structure

The Lagrange density for our study of Lorentz-violating SQED is

\[ \mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} k^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + (g^{\mu\nu} + k^{\mu\nu}_{\Phi}) (D_{\mu} \Phi)^* (D_{\nu} \Phi) + \mu^2 \Phi^* \Phi - \frac{\lambda}{2} (\Phi^* \Phi)^2. \]  

(1)

\[ D_{\mu} = \partial_{\mu} + ieA_{\mu} \]

is the usual covariant derivative, and we will use \( V(\Phi) = -\mu^2 \Phi^* \Phi + \frac{\lambda}{2} (\Phi^* \Phi)^2 \) to denote the scalar field potential. The Lorentz violation enters through the coefficients \( k_{F} \) in the gauge sector and \( k_{S} \) in the scalar sector. Both of these background tensors are dimensionless. \( k_{F} \) has the symmetries of the Riemann tensor and a vanishing double trace.

The structure of \( k_{S} \) is a bit more subtle. Reality of the action requires that

\[ k_{\Phi}^{\mu\nu} = k_{S}^{\mu\nu} + ik_{A}^{\mu\nu}, \]

where \( k_{S}^{\mu\nu} = k_{S}^{\nu\mu} \) is symmetric and traceless in its Lorentz indices, while \( k_{A}^{\mu\nu} = -k_{A}^{\nu\mu} \) is antisymmetric. The discrete symmetries of \( k_{S} \) are quite similar to those of \( k_{F} \). Different components of the Lorentz-violating tensors \( k_{F} \) and \( k_{S} \) have different behaviors under parity (P) and time reversal (T). In a Lorentz-violating theory, the three spatial reflections that together constitute P are generally inequivalent. Components of the tensors \( k_{F} \) and \( k_{S} \) are odd under a reversal of a specific spacetime coordinate if that coordinate appear as an Lorentz index an odd number of times. Overall, a particular coefficient \( k_{F}^{\mu\nu\rho\sigma} \) acquires a sign \((-1)^{\mu}(-1)^{\nu}(-1)^{\rho}(-1)^{\sigma}\) under either a P or T transformation, where \((-1)^{\mu} = 1 \) if \( \mu = 0 \) and \((-1)^{\mu} = -1 \) if \( \mu = 1, 2, \) or 3. The transformation of \( k_{S}^{\mu\nu} \) is similarly associated with the sign \((-1)^{\mu}(-1)^{\nu}\). For example, \( k_{S}^{00} \) is even under T.
and all spatial reflections. Moreover, both $k_F$ and $k_S$ are even under charge conjugation (C) and the combined operation CPT. However, $k_A$ has a different symmetry structure. The appearance of the additional factor of $i$ changes the transformation under $C$ and $T$. Under $P$, $k_A^{\mu\nu}$ transforms as $(-1)^{\mu}(-1)^{\nu}$, just as does $k_S^{\mu\nu}$. However, under $T$ it transforms as $-(1)^{\mu}(1)^{\nu}$, with an additional negative sign because $T$ is anti-linear. The full action is invariant under CPT, which means that $k_A$ must additionally be odd under C. Moreover, since $k_A$ is antisymmetric, it can only couple to expressions with exactly one derivative and one factor of the gauge field. Using integration by parts, the $k_A$ term in $L$ is equivalent to $\frac{1}{2}ek^{\mu\nu}_A\Phi^*F_{\mu\nu}$. (We notice that, since $\Phi^*\Phi$ is even under $C$, $P$, and $T$ separately, writing the $k_A$ term in this form is the easiest way to find its discrete symmetries; they must be the same as those of the gauge field strength $F_{\mu\nu}$.) The fact that the $k_A$ term in a minimally coupled but Lorentz-violating $L$ can be written in this form means that any additional, non-minimal, dimension-4 couplings between the scalar and gauge fields are redundant; their effects are already completely contained in $k_A$. It is not clear whether this equivalence has been entirely appreciated in all earlier work.

There are potentially also CPT-odd operators in both the scalar and vector sectors. However, the CPT-odd scalar coefficients $a_\mu^\phi$ are unobservable in a theory with only a single species of charged matter; they can be eliminated by a redefinition of the matter field. The gauge coefficients $k_A^{\mu\nu}$ are not so trivial; they generate birefringence in the gauge field propagation and may actually destabilize the theory. However, this birefringence does not interact with the Higgs mechanism in any particularly interesting fashion, and so $k_A^{\mu\nu}$ will be neglected.

Because of the “wrong-sign” mass term in $L$, there are static solutions to the field equations with nonzero values of $\Phi$. The Lorentz violation, which appears only in the kinetic terms, does not affect these solutions, which are derived from

$$\left. \frac{\delta L}{\delta \Phi^*} \right|_{static} = \mu^2 \Phi - \lambda (\Phi^* \Phi) \Phi = 0. \quad (2)$$

The static solutions $\Phi_0$ must satisfy $|\Phi_0| = \frac{\mu}{\sqrt{\lambda}} \equiv v$. Such solutions obviously break the $U(1)$ gauge invariance associated with the gauge transformation

$$\Phi \rightarrow \Phi' = e^{i\alpha} \Phi \quad (3)$$
$$\Phi^* \rightarrow \Phi'^* = e^{-i\alpha} \Phi^* \quad (4)$$
$$A_\mu \rightarrow A'_\mu = A_\mu - \frac{1}{\epsilon} \partial_\mu \alpha. \quad (5)$$

However, it is possible to make $\Phi_0$ real by a gauge rotation and then decompose the field into its vacuum expectation value and excitations,

$$\Phi = v + \frac{1}{\sqrt{2}}(h + i\varphi); \quad (6)$$
$h$ is the Higgs field and $\varphi$ represents the Goldstone boson.

The original Lagrange density $\mathcal{L}$ may be expanded in terms of these new variables, giving

$$
\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}k_{F}^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{1}{2}(g^{\mu\nu} + k_{\Phi}^{\mu\nu}) \left\{ (\partial_{\mu}h)(\partial_{\nu}h) + (\partial_{\mu}\varphi)(\partial_{\nu}\varphi) + e^{2}h^{2}A_{\mu}A_{\nu} + e^{2}\varphi^{2}A_{\mu}A_{\nu} + 2\sqrt{2}e^{2}vhA_{\mu}A_{\nu} + 2e^{2}v^{2}A_{\mu}A_{\nu} + i([\partial_{\mu}h](\partial_{\nu}\varphi) - (\partial_{\mu}\varphi)(\partial_{\nu}h)) + i\sqrt{2}ev[(\partial_{\mu}h)A_{\nu} - A_{\mu}(\partial_{\nu}h)] + ie[(\partial_{\mu}h)(A_{\nu}h) - (A_{\mu}h)(\partial_{\nu}h)] + ie[(\partial_{\mu}\varphi)(A_{\nu}\varphi) - (A_{\mu}\varphi)(\partial_{\nu}\varphi)] - e[(\partial_{\mu}h)(A_{\nu}\varphi) + (A_{\mu}\varphi)(\partial_{\nu}h)] + \sqrt{2}ev[A_{\mu}(\partial_{\nu}\varphi) + (\partial_{\mu}\varphi)A_{\nu}] + e[(A_{\mu}h)(\partial_{\nu}\varphi) + (\partial_{\mu}\varphi)(A_{\nu}h)] \right\} - V(h, \varphi).
$$

The expansion of the potential around $v$ takes the standard form,

$$
V(h, \varphi) = -\frac{\mu^{4}}{\lambda} + \mu^{2}h^{2} + \mu \sqrt{\frac{\lambda}{2}}h(h^{2} + \varphi^{2}) + \frac{\lambda}{8}(h^{2} + \varphi^{2})^{2}.
$$

Symmetry considerations limit the effects that $k_{S}$ and $k_{A}$ can have on the propagation of free particles. Ultimately, the physical excitations of the theory will be the (massive) gauge field $A$ and the Higgs $h$. There is actually no way for a dimensionless, two-index antisymmetric $k_{A}^{\mu\nu}$ to modify the separate propagation of these real fields. For instance, there is no nonzero operator that is linear in $k_{A}$ and includes two derivatives of $h$, as $k_{A}^{\mu\nu}(\partial_{\mu}h)(\partial_{\nu}h)$ clearly vanishes. However, it is possible for $k_{A}$ to mix $h$ and $A$ states. Although the Higgs and gauge fields are, respectively, even and odd under $C$, they can be mixed through $k_{A}$. In fact because $k_{S}$ and $k_{F}$ (and all the terms in the Lorentz-invariant Lagrangian) are invariant under $C$, such mixing can only be generated by $k_{A}$.

Rather similar restrictions on the behavior of different forms of Lorentz violation also exist in the fermion sector of the SME, where the analogue of $k_{\Phi}$ is a tensor $c$, appearing in

$$
\mathcal{L}_{\psi} = \bar{\psi}[i(g^{\mu\nu} + c^{\mu\nu})\gamma_{\nu}\partial_{\mu} - m]\psi.
$$

While the contribution of $c_{A}^{\mu\nu} = \frac{1}{2}(c^{\mu\nu} - c^{\nu\mu})$ to the Lagrangian is not identically zero, there is again no room in the energy-momentum relation for such an antisymmetric object (at leading order). It turns out that the only physically observable combinations of the $c$ coefficients are $c^{\mu\nu} + c^{\nu\mu} + c^{\alpha\nu}c_{\alpha}^{\mu}$. Orthogonal combinations of the $c$ coefficients merely correspond to redefinitions of the Dirac matrices.

The Goldstone boson field $\varphi$ does not have physical excitations. By working in the unitarity gauge, we may choose the gauge parameter $\alpha$ so as to make $\Phi$ everywhere real (at the classical level). This eliminates $\varphi$ from external states. However, quantum fluctuations in the $\varphi$ field cannot be entirely eliminated, and the Goldstone boson field will appear as a virtual intermediary in loop calculations. In fact, the presence of $\varphi$ is a crucial ingredient in some cancellations that are necessary for the renormalizability and unitarity of the Lorentz-invariant version of SQED.
2.2 Propagation and Interactions

Propagation of physical fields is governed by the portion $\mathcal{L}_2' = \mathcal{L}_{2,h}$ of $\mathcal{L}$ that is bilinear in just $A$ and $h$. This is

$$\mathcal{L}_2' = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \frac{1}{4} k_F^{\mu\rho\sigma} F_{\mu\rho} F_{\sigma\nu} + \frac{1}{2} (g^{\mu\nu} + k_S^{\mu\nu}) \left( (\sqrt{2}\epsilon) \right)^2 A_\mu A_\nu + \frac{1}{2} (g^{\mu\nu} + k_S^{\mu\nu}) (\partial_\mu h)(\partial_\nu h) - \frac{1}{2} (\sqrt{2} \epsilon)^2 h^2 + \frac{1}{\sqrt{2}} \epsilon v k_A^{\mu\nu} h F_{\mu\nu},$$  \hspace{1cm} (10)

where an integration by parts has again been performed on the last term. Written in this form, the mass term for the gauge field is obviously Lorentz-violating. Although we shall show in section 4 that it is possible to recast the Lagrangian in a form that moves all the gauge-sector Lorentz violation into the $k_F$ term, the form (10) for $\mathcal{L}_2'$ may be somewhat unexpected. To the extent that the longitudinal component of the massive gauge field is really the Goldstone boson of the broken symmetry, we would expect the kinetic term for this field to be modified, so that the longitudinal $A$ should propagate like $\Phi$. However, this phenomenon is not seen in (10).

Nevertheless, an examination of the full bilinear Lagrange density $\mathcal{L}_2$, which includes the Goldstone bosons, shows how the kinetic part of the action for $\varphi$ combines with the Lorentz-violating mass term for $A$ to preserve the transversality of the gauge propagator. The gauge part of $\mathcal{L}_2$

$$\mathcal{L}_{2,A} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \frac{1}{4} k_F^{\mu\rho\sigma} F_{\mu\rho} F_{\sigma\nu} + \frac{1}{2} (g^{\mu\nu} + k_S^{\mu\nu}) \left( (\sqrt{2}\epsilon) \right)^2 A_\mu A_\nu$$  \hspace{1cm} (11)

is manifestly transverse, except for the term with the gauge boson mass $m_A = \sqrt{2}\epsilon v$, which is certainly not. However, there is also a vertex that mixes the gauge and Goldstone boson propagators; it comes from

$$\mathcal{L}_{2,\varphi} = \mathcal{L}_{2,A} + \frac{1}{2} (g^{\mu\nu} + k_S^{\mu\nu}) \left\{ (\partial_\mu \varphi)(\partial_\nu \varphi) + m_A [A_\mu (\partial_\nu \varphi) + (\partial_\mu \varphi) A_\nu] \right\}.$$  \hspace{1cm} (12)

To second order in $m_A$ and first order in $k_S$, there are two possible insertions from $\mathcal{L}_{2,\varphi}$ that contribute to the polarization tensor $i\Pi^{\mu\nu}(q)$. The first is the photon mass insertion, which contributes $im_A^2 (g^{\mu\nu} + k_S^{\mu\nu})$. The second insertion involves two of the $A-\varphi$ vertices, with a $\varphi$ propagator between them. This propagator is

$$D_{\varphi}(q) = \frac{i}{q^2} \left( 1 - k_S^{\gamma\delta} \frac{q_\gamma q_\delta}{q^2} \right),$$  \hspace{1cm} (13)

making the polarization tensor

$$i\Pi^{\mu\nu}(q) = im_A^2 (g^{\mu\nu} + k_S^{\mu\nu}) + [m_A (g^{\mu\alpha} + k_S^{\mu\alpha})]q_\alpha \left( \frac{i}{q^2} \left( 1 - k_S^{\gamma\delta} \frac{q_\gamma q_\delta}{q^2} \right) \right) [m_A (g^{\beta\nu} + k_S^{\beta\nu}) (-q_\beta)]$$  \hspace{1cm} (14)

$$= im_A^2 \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} + k_S^{\mu\nu} - k_S^{\alpha\nu} q_\alpha q^\nu - k_S^{\mu\beta} q_\beta q^\mu + k_S^{\gamma\delta} q_\mu q^\nu q_\gamma q_\delta / (q^2)^2 \right).$$  \hspace{1cm} (15)
Although its structure is rather complicated, this tensor is transverse, \( q_\mu \Pi^{\mu\nu} = 0 \).

There are also terms in \( \mathcal{L}_2 \) that mix \( h \) with the other fields. However, they are less important, for two separate reasons. An insertion with an intermediate Higgs involves a massive propagator; without a pole at \( q^2 = 0 \), this cannot affect the pole structure of the gauge propagator. Moreover, any mixing of \( h \) with \( A \) or \( \varphi \) violates \( C \). Since \( k_A \) is the only source of \( C \) violation in the theory, any modification of the \( A \) or \( \varphi \) propagator by a virtual \( h \) insertion will necessarily be second order in the Lorentz violation.

Special examples of Lorentz-violating mass terms of the general form

\[
M^{\mu\nu} A_\mu A_\nu = \frac{1}{2} (g^{\mu\nu} + k_S^{\mu\nu}) m^2 A_\mu A_\nu
\]  

have previously been studied. Forms considered have been an isotropic but boost-invariance-violating \(-\frac{1}{2} m^2 A_j A_j\), as an alternative to the Proca mass term \[45, 46\]; or \[-\frac{e^2}{24\pi^2} (b^2 g^{\mu\nu} + 2 b^\mu b^\nu)\], which could be generated by unusual radiative corrections \[47\]. (Note however, that while the gauge boson mass in these situations is assumed to be small, the Lorentz-violating and Lorentz-invariant parts of the mass term are of comparable size.) Most recently, Lorentz-violating Stueckelberg mass terms have also been considered \[48\]. The previous analyses of these models have demonstrated another aspect of the unusual interplay between Lorentz-violating mass terms for Abelian gauge bosons and the the kinetic part of the gauge-sector Lagrangian. In all concrete examples that were considered, there were only two distinct eigenvalues in the mass squared matrix \( M^{\mu\nu} \). It was found that if the eigenvalue \( \frac{1}{2} m^2_0 \) corresponding to the timelike direction is smaller in magnitude than a spacelike eigenvalue \( \frac{1}{2} m^2_1 \), there could be propagation with signal and group velocities greater than 1 and as large as \( m_1 \). However, this superluminal propagation is limited to modes that are approximately longitudinal. These properties may well be general features of theories with \( M^{\mu\nu} A_\mu A_\nu \) mass terms.

The existence of a Lorentz-violating mass term can therefore have profound effects on the propagation of gauge bosons with momenta far above the apparent mass scale. The mass term (which might be expected to be important only in the infrared) affects the ultraviolet behavior of the theory through its influence on the gauge. Requiring charge conservation forces \( A \) to obey a gauge condition \( M^{\mu\nu} \partial_\mu A_\nu = 0 \). The relative sizes of the elements of the mass matrix \( M^{\mu\nu} \) determine the required gauge. However, the absolute magnitude of the matrix components are irrelevant; the gauge condition produced by a mass matrix \( \zeta M^{\mu\nu} \) is independent of \( \zeta \). Since the gauge condition contains derivatives, it affects the propagation of the longitudinal mode of the gauge field even at the highest energies.

When a Lorentz-violating gauge field mass term arises through the Higgs mechanism, there is a clear physical mechanism underlying superluminal propagation. If the timelike eigenvalue of \( k_S^{\mu\nu} \) is \( \lambda_0 \) and the largest spacelike eigenvalue is \( \lambda_1 > \lambda_0 \), the free \( \Phi \) field has a kinetic term that supports propagation up to speeds of \( \sqrt{\frac{\lambda_1}{\lambda_0}} \). When the Goldstone boson is eaten by the gauge field, this possibility for superluminal propagation is transferred to
the gauge field, although the Lorentz-violating term that makes this possible is part of the mass term in $\mathcal{L}_A$, rather than the kinetic term.

In addition to the propagation governed by $\mathcal{L}_2$ (or $\mathcal{L}'_2$), there are also interaction vertices in the theory. For tree-level calculations, only those vertices involving $A$ and $h$ are needed. These vertices are given by the interaction Lagrange density

$$\mathcal{L}'_I = \frac{1}{2} (g^{\mu\nu} + k_S^{\mu\nu}) e^2 (h^2 + 2vh) A_\mu A_\nu - \frac{1}{2} e k_A^{\mu\nu} [h(\partial_\mu h) A_\nu - h(\partial_\nu h) A_\mu] - \mu \sqrt{\frac{\lambda}{2}} h^3 - \frac{\lambda}{8} h^4. \quad (17)$$

This includes the usual Higgs self-interaction terms from $\Phi^4$ theory, as well as a seagull vertex (involving two Higgs and two gauge fields) and a related three-particle vertex with one of the Higgs fields replaced by the vacuum expectation value $v$. The seagull and three-field vertices have their Lorentz structures modified by $k_S$ in precisely the same way as the gauge boson mass term.

The remaining terms in $\mathcal{L}'_2$ and $\mathcal{L}'_I$ are the C-violating terms involving $k_A$. These can be expressed in terms of the gauge field strength. The three-field interaction is equivalent to $\frac{1}{2} e k_A^{\mu\nu} h^2 F_{\mu\nu}$. There is no Lorentz-invariant analogue for such a term. Terms involving $k_S$ are similar to Lorentz-invariant terms, in that they involve replacing the Minkowski metric tensor $g^{\mu\nu}$ with an arbitrary symmetric $k_S^{\mu\nu}$. In contrast, there is no Lorentz-invariant, antisymmetric, two-index tensor to be contracted with $F_{\mu\nu}$, so the $k_A^{\mu\nu} F_{\mu\nu}$ interactions have a uniquely Lorentz-violating structure. Moreover, these terms are gauge invariant, in spite of the spontaneous symmetry breaking. They depend on the gauge field $A$ only through the field strength tensor and descend directly from the term $\frac{1}{2} e k_A^{\mu\nu} \Phi^* \Phi F_{\mu\nu}$ in the original $\mathcal{L}$.

3 Quantization and Ghost Fields

Calculation of quantum corrections for a theory with spontaneously broken gauge symmetry requires the introduction of a gauge fixing term in the action, which leads naturally to the inclusion of Faddeev-Popov ghosts. The gauge fixing term serves two purposes. It can eliminate the zero modes in the gauge field action, something which is necessary for the derivation of a well-defined propagator; the gauge fixing term fulfills this function in all gauge theories, whether or not they involve spontaneous symmetry breaking. However, when the gauge symmetry is broken, the gauge fixing terms is also used to eliminate another obstacle to the construction of a self-contained gauge propagator; the gauge fixing function may be chosen to remove any terms that mix the gauge and Goldstone boson fields.

To quantize the gauge field according to the Faddeev-Popov procedure [49], we begin with the gauge-invariant functional integral for the theory and insert the identity, in the form

$$1 = \int \mathcal{D}\alpha(x) \delta[G(A', h', \varphi') - \omega] \det \left[ \frac{\delta G(A', h', \varphi')}{\delta \alpha} \right]. \quad (18)$$
where $A'_\mu = A_\mu - \frac{1}{e} \partial_\mu \alpha$, $h' = h - \alpha \varphi$, and $\varphi' = \varphi + \alpha(\sqrt{2}v + h)$ are the infinitesimally gauge transformed fields from (3)–(5). The gauge-fixing function $[G(A, h, \varphi) - \omega]$ is then integrated over a Gaussian distribution of $\omega$ values. It is convenient to absorb the width $\xi$ of the Gaussian into $G$ itself and take

$$G = \frac{1}{\sqrt{\xi}} \left( (g^{\mu
u} + k_G^{\mu
u}) \partial_\mu A_\nu - \sqrt{2} \xi e v \varphi \right).$$

(19)

The Lorentz-invariant terms in (19) are identical to those in the gauge fixing function for the $R_\xi$ gauge, for which we shall derive a Lorentz-violating generalization. $k_G$ is an (as yet undetermined) Lorentz-violating tensor coefficient. It is not possible to include Lorentz violation in the $\varphi$ part of $G$ without introducing higher derivatives into the final ghost action.

The Faddeev-Popov procedure introduces two new sets of terms into the Lagrange density. The first set is the result of the integration over $\omega$,

$$- \frac{1}{2} G^2 = - \frac{1}{2\xi} \left( (g^{\mu
u} + k_G^{\mu
u}) \partial_\mu A_\nu \right)^2 - \sqrt{2} \xi e v (g^{\mu
u} + k_G^{\mu
u}) (\partial_\mu \varphi) A_\nu - \xi e^2 v^2 \varphi^2.$$  

(20)

The Lorentz violation $k_G$ in the gauge fixing should be chosen to eliminate the $A\cdot \varphi$ mixing term in $L_2 - \frac{1}{2} G^2$. We see that this requires $k_G^{\mu\nu} = k_S^{\mu\nu}$. Then the gauge part of $L_k$ becomes

$$L_{2,A} - \frac{1}{2\xi} \left( (g^{\mu
u} + k_G^{\mu
u}) \partial_\mu A_\nu \right)^2 = - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2\xi} (\partial_\mu A_\mu)^2 + \frac{1}{2} m_A^2 A_\mu A^\mu$$

(21)

$$- \frac{1}{4} k_F^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} - \frac{1}{\xi} k_S^{\mu\nu} (\partial_\mu A_\nu)(\partial_\rho A_\rho) + \frac{3}{4} k_S^{\mu\nu} m_A^2 A_\mu A^\nu,$$

to leading order in the Lorentz violation. The Lorentz-violating kinetic terms can be recast as $-k_\xi^{\mu\nu\rho\sigma} (\partial_\mu A_\nu)(\partial_\rho A_\sigma)$, where $k_\xi^{\mu\nu\rho\sigma} = k_F^{\mu\nu\rho\sigma} + \frac{1}{\xi} g^{\mu\nu} k_S^{\rho\sigma}$; this utilizes the explicit symmetries of $k_F$. The $(\partial_\mu A_\mu)^2$ term combines with the Maxwell and Proca terms to produce the usual propagator

$$D_A^{\mu\nu}(q) = \frac{-i}{q^2 - m_A^2} \left[ g^{\mu\nu} - (1 - \xi) \frac{q^\mu q^\nu}{q^2 - \xi m_A^2} \right].$$

(22)

and the Lorentz-violating terms may be treated as vertices. The $\xi$-dependent part of the $k_\xi$ vertex is superficially similar in structure to the $k_F$ part. However, while the $k_F$ term in $L_2$ only involves the physical fields contained in $F^{\mu\nu}$, the gauge fixing part involves only purely gauge degrees of freedom, since $k_G$ couples solely to the symmetric part of $\partial_\mu A_\nu$.

The other terms that the Faddeev-Popov procedure adds to $L$ come from the determinant in (18). Since

$$\delta G \over \delta \alpha = \frac{\delta G}{\delta A_\mu} \left( \frac{1}{e} \partial_\mu \right) + \frac{\delta G}{\delta \varphi} (v + h)$$

(23)

$$= \frac{1}{\sqrt{\xi}} \left[ (g^{\mu\nu} + k_S^{\mu\nu}) \left( -\frac{1}{e} \partial_\mu \partial_\nu \right) - \xi m_A (\sqrt{2}v + h) \right].$$

(24)
\[ \det[\delta G/\delta \alpha] \] may be exponentiated as a part of the action by introducing ghost fields \( c \) and \( \bar{c} \) with Lagrange density

\[
\mathcal{L}_c = (g^{\mu\nu} + k_S^{\mu\nu}) (\partial_\mu \bar{c})(\partial_\nu c) - \xi m_A^2 \left( 1 + \frac{h}{\sqrt{2}v} \right) \bar{c}c.
\]  

(25)

The (gauge-dependent) mass term for the Faddeev-Popov ghosts is, unlike the photon mass term, unaffected by the Lorentz violation; the interaction vertex with the Higgs field is also unchanged. However, the ghosts do acquire a modification to their kinetic term, equivalent to the \( k_S \) for the original scalar field \( \Phi \). For each of the spinless fields \((h, \varphi, \text{and } c)\), the Lorentz violation may again be treated as a vertex to be inserted along propagation lines. Several loop diagrams involving the \( k_S \) in the ghost sector have already been evaluated [50].

The quantum implementation of the unitarity gauge comes from taking the limit \( \xi \to \infty \), which assigns the ghost and Goldstone boson propagators infinite masses. This is nearly sufficient to eliminate the unphysical degrees of freedom from the theory. However, this limit needs to be taken after the unphysical fields are used to cancel some of the theory’s leading perturbative divergences [51].

4 Coordinate Redefinitions

The appearance of the same Lorentz-violating coefficients \( k_S \) in the Higgs, Goldstone boson, and ghost sectors may be unsurprising, because of the structure of the \( k_S \) term. If \( k_F \) vanishes in the original Lagrange density \( \mathcal{L} \), then \( k_S \) describes a mismatch between the natural coordinates for describing the gauge and matter fields. Having a vanishing \( k_F \) means that the chosen coordinates are natural for the gauge field. However, redefining coordinates according to

\[
x^\mu \to x'^\mu = x^\mu - \frac{1}{2} k_{S \nu} x^\nu
\]  

(26)

will transform the Lorentz violation coefficients in \( \mathcal{L} \) to

\[
k_F^{\mu \nu} \to k_F^{\prime \mu \nu} = i k_A^{\mu \nu}
\]

\[
k_F^{\mu \nu \rho \sigma} \to k_F^{\prime \mu \nu \rho \sigma} = k_F^{\mu \nu \rho \sigma} - \frac{1}{2} \left( g^{\mu \rho} k_S^{\nu \sigma} - g^{\mu \sigma} k_S^{\nu \rho} - g^{\nu \rho} k_S^{\mu \sigma} + g^{\nu \sigma} k_S^{\mu \rho} \right).
\]  

(28)

If this transformation is made prior to the calculations, many of the Lorentz-violating terms that could appear after spontaneous symmetry breaking are actually absent. By eliminating \( k_S \) prior to quantization and spontaneous symmetry breaking, we can ensure that there is no Lorentz-violating modification of the gauge field mass term, nor is any Lorentz violation required in the ghost sector.

In fact, it is straightforward to see how a transformation that eliminates \( k_S \) from the kinetic term for \( \Phi \) likewise eliminates \( k_S \) from the ghost kinetic term. Both terms have the
same basic scalar kinetic structure, and a transformation that carries \((g^{\mu\nu} + k^{\mu\nu}_S)\partial_\mu \partial_\nu \rightarrow \partial^\mu \partial_\mu\) will have the same effect in either sector. The transformation of the gauge boson mass term is almost as simple. Accompanying the redefinition of the coordinates \((26)\) must be a similar linear reshuffling of the gauge fields; the transformed \(A'_\mu\) must be exactly what enters in conjunction with \(\partial'_\mu \equiv \frac{\partial}{\partial x'_\mu}\) in the covariant derivative. So \((26)\) simply takes \((g^{\mu\nu} + k^{\mu\nu}_S)A_\mu A_\nu \rightarrow A^\mu A_\mu\).

Each of \(k_\Phi\) and \(k_F\) contains a portion that can be eliminated via this kind of field redefinition. Respectively, these are \(k_S\) and the portion of \(k_F\) with the structure appearing on the right-hand-side of \((28)\): 

\[
\frac{1}{2} (g^{\mu\rho} k_{F\alpha}^{\nu\sigma} - g^{\mu\sigma} k_{F\alpha}^{\nu\rho} - g^{\nu\rho} k_{F\alpha}^{\mu\sigma} + g^{\nu\sigma} k_{F\alpha}^{\mu\rho}).
\]

There are natural physical reasons why the remainder of each background tensor cannot be so eliminated. The part of \(k_F\) that cannot be defined away has the structure of the Weyl tensor, and it gives rise to birefringent propagation by the gauge field. No redefinition of the coordinate system can change the fact that polarizations are traveling at different rates. Meanwhile, \(k_A\) is associated with C-violating non-minimal interactions between gauge and scalar field components, and a change in coordinates cannot mask the fact that particle identities might change because of the effects of these interactions.

Since it is possible to define away the \(k_S\) Lorentz violation, we might be tempted to dismiss analyses that include \(k_S\) entirely, as adding needless complexity to an already subtle physical situation. However, since the transformation that eliminates \(k_S\) is a global redefinition of the coordinates, it can only be used to eliminate this type of Lorentz violation from a single sector. This is already evident from the fact that removing \(k_S\) from the matter sector introduces it into the \(k_F\) of the gauge sector. Ultimately, physical observables that depend on \(k_S\) need to involve differences. In pure SQED, the only observable difference is \(k^{\mu\nu}_S - k_{F\alpha}^{\mu\nu}\); if SQED is embedded in a larger theory that includes fermions, combinations such as \(k^{\mu\nu}_S - c^{\mu\nu} - c^{\nu\mu}\) also become relevant. If tree-level Lorentz violation only existed in the scalar sector the standard model, it would certainly be most natural to use conventional coordinates, limiting the SME coefficients to a single \(k_\Phi\) for the Higgs. However, this would also mean using Lorentz-violating mass terms for the intermediate vector bosons, so understanding the effects of such mass terms is indeed important.

If both the Lorentz violation and the mass are small enough to be treated as perturbations, it is straightforward to determine the dispersion relations for the gauge field modes in the coordinate system with all Lorentz violation moved into the gauge sector. For the transverse polarization states with wave vector \(\vec{q}\), the frequencies are [52]

\[
q_0^\pm = |\vec{q}| \left[1 + \rho (\hat{q}) \pm \sigma (\hat{q})\right] + \frac{m_A^2}{2 |\vec{q}|},
\]

where \(\rho (\hat{q}) = -\frac{1}{2} \tilde{k}^\alpha_{,\alpha}\), and \(\sigma^2 (\hat{q}) = \frac{1}{2} \tilde{k}^{\alpha\beta} \tilde{k}_{\alpha\beta} - \rho^2 (\hat{q})\), with \(\tilde{k}^{\alpha\beta} = k_{F}^{\alpha\mu\beta\nu} \hat{q}_\mu \hat{q}_\nu\) and \(\hat{q}^\mu = (1, \vec{q}/ |\vec{q}|)\). The result \((29)\) simply represents the conventional dispersion relation, plus the usual perturbations due to the \(k_F\) Lorentz violation and the mass \(m_A \ll |\vec{q}|\).
However, there is also a longitudinal polarization state, whose energy is not affected by $k'_F$ at leading order,

$$q_0 = |\vec{q}| + \frac{m_A^2}{2|\vec{q}|}. \quad (30)$$

The reason that $k'_F$ does not affect this dispersion relation is that the presence of the mass term forces $A$ to obey the Lorentz gauge condition $q^\mu A_\mu = 0$. The longitudinal propagation mode can have only $A_0$ and $\vec{A} \cdot \hat{q}$ nonzero, so the gauge condition implies that $A^\mu$ must be proportional to $q^\mu$ (which may still be considered lightlike, since the perturbatively small mass $m_A$ may be neglected in the analysis of the Lorentz-violating terms). This causes the $k'_F$ term in the equation of motion to vanish. The lack of any dependence on $k'_F$ might initially seem puzzling, but it is actually quite natural. Since $m_A^2$ was treated as a perturbation, (30) applies only in the high-energy regime, when the momentum $|\vec{q}|$ is large compared with the Higgs mass scale. In that regime, the longitudinal component of the gauge field essentially becomes indistinguishable from the uneaten Goldstone boson. The propagation of the longitudinal mode should therefore be governed only by the Lorentz-violating tensor $k'_S$ in the Higgs sector, and in the transformed coordinates used to derive (30), $k'_S$ vanishes.

Of course, these dispersion relations may be transformed back into the original coordinates with nonzero $k_S$ simply by inverting the coordinate redefinition (26), so that $q^\mu \rightarrow q^\mu - \frac{1}{2} k^\mu_S \nu q^\nu$. The result for the longitudinal mode is

$$q_0 = |\vec{q}| \left[ 1 - \frac{1}{2} k^0_S + 2 k^0_S \hat{q}_j \hat{q}_j + \frac{1}{2} k^j_S i \hat{q}_j \hat{q}_j \right] + \frac{m_A^2}{2|\vec{q}|}. \quad (31)$$

This exhibits exactly the same kind of potentially superluminal behavior for the longitudinal mode as was discussed in section 2.2, with the limiting speed controlled by the relative sizes of the spacelike and timelike eigenvalues of $k^\mu_S \nu$.

This analysis also provides insight into another feature of the non-Higgs mass models discussed in [47]. The normal modes of propagation in the presence of the Lorentz-violating mass term do not involve orthogonal polarization vectors. This is related to the non-orthogonal nature of the transformation (26); a coordinate redefinition that moves Lorentz violation from the gauge field kinetic term to the mass term changes an orthogonal basis of polarization states into a non-orthogonal one. In fact, the transformation required to turn a Lorentz-invariant Proca mass term into the term $-\frac{e^2}{24\pi^2} (b^2 g^{\mu\nu} + 2b^\mu b^\nu)$ from [47] would produce extremely skewed coordinates. This is a reminder that, while the gauge boson mass parameters in [45, 46, 47] may be small, the Lorentz violation for the theories involved is, in a meaningful sense, quite large—with the equivalent of $k_S$ being $\mathcal{O}(1)$.
5 Conclusion

The focus of this paper has been on SQED, with new Lorentz-violating but CPT-preserving SME terms included in the Lagrangian. At least with a single scalar field $\Phi$ and one Abelian gauge field $A$, all possible forms of renormalizable, CPT-even Lorentz violation are captured in the coefficients $k_\Phi$ and $k_F$, with minimal coupling of the gauge and matter fields through the covariant derivative $D_\mu$. Any additional non-minimal couplings would be redundant—equivalent to the imaginary, antisymmetric part $k_A$ of $k_\Phi$.

In standard SQED, spontaneous breaking of the $U(1)$ gauge symmetry makes the gauge boson massive. The Lorentz-violating theory includes an analogous mass term, with a generalization of the Proca form. Such generalized mass terms have previously been studied in rather different Lorentz-violating theories. The structure of the Higgs mechanism mass term, and its relationship to the structure of Lorentz-violating terms in other sectors, was found to display several interesting features.

We have derived several important Lagrange densities relevant in this kind of theory. $L'$ governs the classical behavior of the physical excitations: the gauge and Higgs fields. However, we have also demonstrated how to quantize the theory using the Faddeev-Popov procedure, by introducing ghost fields and a Lorentz-violating ghost Lagrange density.

In conventional gauge theories with spontaneously broken symmetry, the virtual transformation of massive gauge particles into Goldstone bosons is crucial to the preservation of the Ward identity $q_\mu \Pi^{\mu\nu}(q) = 0$. Although the structure (15) of $\Pi^{\mu\nu}$ is more complex in the presence of Lorentz violation, the polarization tensor is still transverse. Maintaining this transverse structure requires a precise cancellation between the Lorentz violation in the gauge field mass and in the $\phi$ propagator.

Moreover, if the fundamental scalar field $\Phi$ can propagate with speeds greater than 1, then both the Higgs field $h$ and the massive gauge field $A$ also support superluminal propagation. This is not, in itself, surprising, since both $h$ and the physical part of $A$ (in the unitarity gauge) are constructed, at least in part, from components of $\Phi$. What is perhaps unexpected is that the superluminal behavior of the gauge field does not arise from a change to the kinetic part of $L_A$ but from the Lorentz-violating photon mass

$$\frac{1}{2}(g_{\mu\nu} + k^S_{\mu\nu}) m_A^2 A_\mu A_\nu.$$  

Quantum field theories involving gauge interactions with charged scalar matter are important; in the standard model, the Higgs sector is responsible for the existence of many particle masses. The full treatment of quantum corrections in Lorentz-violating extensions of such theories is an important problem in the theoretical study of Lorentz violation. This work presents a step toward the full understanding of such Lorentz-violating models, giving the quantized Lagrangian for Lorentz-violating SQED in the Higgs phase and illustrating several unexpected features of this theory.
References

[1] D. Colladay, V. A. Kostelecký, Phys. Rev. D 55, 6760 (1997).

[2] D. Colladay, V. A. Kostelecký, Phys. Rev. D 58, 116002 (1998).

[3] R. Bluhm, V. A. Kostelecký, N. Russell, Phys. Rev. Lett. 79, 1432 (1997).

[4] G. Gabrielse, A. Khabbaz, D. S. Hall, C. Heimann, H. Kalinowsky, W. Jhe, Phys. Rev. Lett. 82, 3198 (1999).

[5] H. Dehmelt, R. Mittleman, R. S. Van Dyck, Jr., P. Schwinberg, Phys. Rev. Lett. 83, 4694 (1999).

[6] R. Bluhm, V. A. Kostelecký, N. Russell, Phys. Rev. Lett. 82, 2254 (1999).

[7] D. F. Phillips, M. A. Humphrey, E. M. Mattison, R. E. Stoner, R. F. C. Vessot, R. L. Walsworth, Phys. Rev. D 63, 111101(R) (2001).

[8] R. Bluhm, V. A. Kostelecký, C. D. Lane, Phys. Rev. Lett. 84, 1098 (2000).

[9] V. W. Hughes, et al., Phys. Rev. Lett. 87, 111804 (2001).

[10] B. R. Heckel, E. G. Adelberger, C. E. Cramer, T. S. Cook, S. Schlamminger, U. Schmidt, Phys. Rev. D 78, 092006 (2008).

[11] C. J. Berglund, L. R. Hunter, D. Krause, Jr., E. O. Prigge, M. S. Ronfeldt, S. K. Lamoreaux, Phys. Rev. Lett. 75, 1879 (1995).

[12] V. A. Kostelecký, C. D. Lane, Phys. Rev. D 60, 116010 (1999).

[13] D. Bear, R. E. Stoner, R. L. Walsworth, V. A. Kostelecký, C. D. Lane, Phys. Rev. Lett. 85, 5038 (2000).

[14] P. Wolf, F. Chapelet, S. Bize, A. Clairon, Phys. Rev. Lett. 96, 060801 (2006).

[15] H. Müller, et al., Phys. Rev. Lett. 99, 050401 (2007).

[16] S. Herrmann, A. Senger, K. Möhle, E. V. Kovalchuk, A. Peters, in CPT and Lorentz Symmetry IV, edited by V. A. Kostelecký (World Scientific, Singapore, 2008), p. 9.

[17] S. Herrmann, et al., Phys. Rev. D 80, 105011 (2009).

[18] Ch. Eisele, A. Yu. Nevsky, S. Schiller, Phys. Rev. Lett. 103, 090401 (2009).

[19] H. Müller, Phys. Rev. D 71, 045004 (2005).
[20] G. Saathoff, S. Karpuk, U. Eisenbarth, G. Huber, S. Krohn, R. Muñoz Horta, S. Reinhardt, D. Schwalm, A. Wolf, G. Gwinner, Phys. Rev. Lett. 91, 190403 (2003).

[21] C. D. Lane, Phys. Rev. D 72, 016005 (2005).

[22] V. A. Kostelecký, Phys. Rev. Lett. 80, 1818 (1998).

[23] V. A. Kostelecký, Phys. Rev. D 61, 016002 (1999).

[24] Y. B. Hsiung, Nucl. Phys. Proc. Suppl. 86, 312 (2000).

[25] K. Abe et al., Phys. Rev. Lett. 86, 3228 (2001).

[26] J. M. Link et al., Phys. Lett. B 556, 7 (2003).

[27] B. Aubert et al., Phys. Rev. Lett. 96, 251802 (2006).

[28] S. M. Carroll, G. B. Field, Phys. Rev. Lett. 79, 2394 (1997).

[29] V. A. Kostelecký, M. Mewes, Phys. Rev. Lett. 87, 251304 (2001).

[30] V. A. Kostelecký, M. Mewes, Phys. Rev. Lett. 97, 140401 (2006).

[31] V. A. Kostelecký, M. Mewes, Phys. Rev. Lett. 99, 011601 (2007).

[32] F. W. Stecker, S. L. Glashow, Astropart. Phys. 16, 97 (2001).

[33] T. Jacobson, S. Liberati, D. Mattingly, Nature 424, 1019 (2003).

[34] B. Altschul, Phys. Rev. Lett. 96, 201101 (2006).

[35] B. Altschul, Phys. Rev. D 74, 083003 (2006).

[36] F. R. Klinkhamer, M. Risse, Phys. Rev. D 77, 016002 (2008); addendum Phys. Rev. D 77, 117901 (2008).

[37] J. B. R. Battat, J. F. Chandler, C. W. Stubbs, Phys. Rev. Lett. 99, 241103 (2007).

[38] H. Müller, S. Chiow, S. Herrmann, S. Chu, K.-Y. Chung, Phys. Rev. Lett. 100, 031101 (2008).

[39] V. A. Kostelecký, N. Russell, \texttt{arXiv:0801.0287}

[40] V. A. Kostelecký, C. D. Lane, A. G. Pickering, Phys. Rev. D 65, 056006 (2002).

[41] D. Colladay, P. McDonald, Phys. Rev. D, 75, 105002 (2007).

[42] D. Colladay, P. McDonald, Phys. Rev. D, 79, 125019 (2009).
[43] A. Ferrero, B. Altschul, Phys. Rev. D 84, 065030 (2011).
[44] D. L. Anderson, M. Sher, I. Turan, Phys. Rev. D 70, 016001 (2004).
[45] G. Gabadadze, L. Grisa, Phys. Lett. B 617, 124 (2005).
[46] G. Dvali, M. Papucci, M. D. Schwartz, Phys. Rev. Lett. 94, 191602 (2005).
[47] B. Altschul, Phys. Rev. D 73, 036005 (2006).
[48] M. Cambiaso, R. Lehnert, R. Potting, arXiv:1201.3045
[49] L. D. Faddeev, V. N. Popov, Phys. Lett. B 25, 29 (1967).
[50] B. Altschul, Phys. Rev. D 73, 045004 (2006).
[51] L. Dolan, R. Jackiw, Phys. Rev. D 9, 2904 (1974).
[52] V. A. Kostelecký, M. Mewes, Phys. Rev. D 66, 056005 (2002).