Gauge coupling unification and light Exotica in String Theory

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In this letter we consider the consequences for the LHC of light vector-like exotica with fractional electric charge. It is shown that such states are found in orbifold constructions of the heterotic string. Moreover, these exotica are consistent with gauge coupling unification at one loop, even though they do not come in complete multiplets of SU(5).

The LHC era is about to begin. The minimal supersymmetric standard model [MSSM] is the premier candidate for new physics beyond the Standard Model [SM], providing a plethora of new signatures for LHC physics. Many attempts have been made to derive the MSSM from string theory. Such attempts typically lead to the MSSM spectrum plus additional states, termed exotics.

Vector-like exotics can by definition obtain mass at an arbitrary scale without breaking any Standard Model gauge symmetry. Simple examples of vector-like exotics include additional pairs of Higgs doublets (±D̄), transforming under the SM as (3, 1/2) + (1, 2), transforming under the SM as (3, 1/2) + (1, 2), transforming under the SM as (3, 1/2) + (1, 2) -1 [D̄c + L] + c.c. In the first example, these states contribute to the renormalization group running of the SM gauge coupling in such a way as to change the GUT scale. If these states are light, with mass of order the weak scale, only after electroweak symmetry breaking. In the worst of circumstances, the chiral exotics remain massless. Hence these theories with chiral exotics are severely constrained by precision electroweak data.

Recently, heterotic string constructions using orbifold compactifications have provided models which have only the MSSM spectrum or the MSSM spectrum with the addition of vector-like exotics. Vector-like exotics can by definition obtain mass at an arbitrary scale without breaking any Standard Model gauge symmetry. Simple examples of vector-like exotics include additional pairs of Higgs doublets (H_u + H_d), a pair of states with the quantum numbers of a down quark, D̄c, and its charge conjugate or complete multiplets under SU(5) such as 5̄ + 5, transforming under the SM as (3, 1/2) + (1, 2) -1 [D̄c + L] + c.c. In the first example, these states contribute to the renormalization group running of the SM gauge coupling in such a way as to change the GUT scale. If these states are light, with mass of order the weak scale, then they will destroy the nice agreement of the LEP data and gauge coupling unification in SUSY GUTs. Hence, in order to preserve the nice features of SUSY GUTs, one typically argues that these states have mass of order the GUT scale. On the other hand, if the vector-like exotics come in complete SU(5) multiplets, then at least at one loop they do not affect gauge coupling unification, even if they are light. In fact, light vector-like exotics in complete SU(5) multiplets have been used as messengers for low energy gauge-mediated SUSY breaking.

In this paper we point out that it is also possible to have light vector-like exotics, which are not in complete SU(5) multiplets, however still preserving the nice features of SUSY GUTs. Moreover, in the simple example, these states have fractional electric charge and thus it would be very interesting to search for such states at the LHC. Such states are present in many heterotic orbifold constructions.

I. STRINGY EXOTICA

In Ref. [3, 5] the E_8 × E_8 heterotic string was compactified on a particular six torus modded out by the discrete symmetry Z_3-III = Z_3 × Z_2. By taking one compactified dimension to be much larger than the other 5, it was shown in one of the examples that the Z_3 orbifold by itself produced a low energy effective field theory (below the string scale) equivalent to an E_6 orbifold GUT in 5 space-time dimensions. The final Z_2 orbifolding of the one extra dimension, along with one Wilson line, reduced the E_6 gauge symmetry to Pati-Salam [PS] SU(4) × SU(2)_L × SU(2)_R. The resulting orbifold described a line segment with two end-of-the-world branes. At one end the gauge symmetry was SO(10), while at the other end it was SU(6) × SU(2)_R. Whereas Standard Model families resided on the SO(10) brane, it was found that vector-like exotics in the representations, [(6, 1) + (1, 2)] + charge conj., were localized on the SU(6) × SU(2)_R brane. Under PS, i.e.

\[ SU(6) \times SU(2)_R \rightarrow SU(4) \times SU(2)_L \times SU(2)_R, \]

these states transform as

\[ [(6, 1) + (1, 2)] + c.c. \]

\[ \rightarrow [(4, 1, 1) + (1, 2, 1) + (1, 1, 2)] + c.c. \]

Assuming that PS spontaneously breaks to the Standard Model, i.e.

\[ SU(4) \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y, \]

we finally obtain

\[ [(4, 1, 1) + (1, 2, 1) + (1, 1, 2)] + c.c. \]

\[ \rightarrow [(3, 1)_1 + (1, 1) -1 + (1, 2)_0 + (1, 1)_{\pm 1}] + c.c. \]

\[ = [Q + E_+ + L + E_\pm] + c.c. \]
with \( Y = (B - L) + 2T_{3R} \) and electric charge \( Q = T_{3L} + Y/2 \). Note, these states are explicitly vector-like. They are all located in the same twisted sector of the string. Thus it is not unreasonable to expect that they all obtain mass (when SM singlet fields get supersymmetric vacuum expectation values) of the same order. We shall therefore explore the possibility that they have a gauge invariant, supersymmetric mass \( M \) of order the electroweak scale.\(^3\) In addition, they are very exotic; henceforth we refer to them as exotica. They have fractional electric charge given by

\[
\begin{align*}
[Q_{1/6} + E_{-1/2} + L_{\pm 1/2} + E_{\pm 1/2}] + \text{c.c.}
\end{align*}
\]  

The exotic leptons have charges \( \pm 1/2 \), while the bound states of exotic quarks with SM quarks form an iso-vector and iso-scalar multiplet of baryons

\[
\Sigma^{+3/2}, \Sigma^{+1/2}, \Sigma^{-1/2}, \Lambda^{1/2}_Q,
\]
defined by

\[
\begin{align*}
[Quu]_{3/2}, [Q(ud)]_{1/2}, [Qdd]_{-1/2}, [Q(ud)]_{1/2},
\end{align*}
\]

and an iso-doublet of mesons

\[
\begin{align*}
Q^+_{u1/2} = [Qu]_{+1/2}, Q^-_{d1/2} = [Qd]_{-1/2}.
\end{align*}
\]

Searches for fractionally charged heavy particles exclude them with mass less than 200 GeV \(^2\)\(^\dagger\)\(^\dagger\). Nevertheless they can be produced at the Tevatron or the LHC via Drell-Yan processes. The fermionic exotica are expected to be lighter than their scalar partners, due to soft SUSY breaking contributions to the scalar masses. Therefore the scalar exotica will decay to their fermionic partners and a gaugino (either a gluino, chargino or neutralino depending on the quantum numbers of the (s)exotica). Moreover, unless the flavor symmetries of the leptonic exotica are broken via Yukawa couplings to the MSSM Higgs bosons, they will all be stable.\(^3\)

The lightest fractionally charged color singlet state will also be stable. However, due to exothermic processes\(^3\) such as

\[
\bar{Q}^{+1/2}_d + p \rightarrow \Sigma^{+3/2}_Q + \pi^0
\]

we would expect the baryonic exotica to be more abundant than the mesons. In addition we expect the beta decay processes

\[
\Sigma^{+1/2}_Q \rightarrow \Lambda^{+1/2}_Q + e^- + \bar{\nu}_e
\]
to occur with the lifetime \( \tau_{\Sigma^{+1/2}} \sim 10^{-5} \) sec., of order \( \tau_{\Sigma^{+}} / Br(\Sigma^{+} \rightarrow \Lambda^{0} + e^+ + \nu_e) \). Thus the \( Q \)s will be produced at the Tevatron or the LHC predominantly via gluons, they will hadronize and the color singlet bound state exotica will likely be stopped in the hadronic calorimeter. Note, the electric charge on one fractionally charged state cannot be screened. However two fractionally charged states can have their charges screened by the surrounding normal matter.

The novel feature of these exotica is that even though they do not come in complete representations of SU(5), they nevertheless preserve gauge coupling unification at one loop.

### A. Gauge coupling unification

Consider first the evaluation of \( \sin^2(\theta_W) \) at \( M_{\text{GUT}} \). In general we have

\[
\sin^2(\theta_W)|_{M_{\text{GUT}}} = 1/(1 + C^2) \quad \text{with} \\
C^2 = \frac{\text{Tr}(Y^2/4)}{\text{Tr}(\Sigma^2)}.
\]

For example, if we take the trace over the 5-plet of SU(5) we find \( C^2 = 5/3 \) and \( \sin^2(\theta_W)|_{M_{\text{GUT}}} = 3/8 \). However, we find the same value of \( C^2 = 5/3 \) for the exotica in Eqn. \([2]\). Hence the GUT boundary conditions are unchanged with the addition of these states.

Moreover the RG running below the GUT scale is also unchanged. At one loop we have

\[
\frac{d\alpha_i}{dt} = -\frac{b_i}{2\pi} \alpha_i^2, \quad t = \ln(\mu/\mu_0)
\]

with

\[
b_i = 3C_2(G_i) - nT(r).
\]

In this equation, \( n \) is the number of chiral multiplets in the representation \( r \) of the gauge group \( G_i \). \( \text{Tr}(T_AT_B) = T'(r)\delta_{AB} \), where \( T_A \) is the gauge generator for the chiral multiplet. \( T'(r) = 1/2 \) for the \( N \) dimensional representation of SU(\( N \)) and \( T'(3/5) = 4/4 \) for U(1)$_Y$. Finally for SU(\( N \)), the quadratic Casimir for the adjoint representation, \( C_2(\text{SU}(\( N \))) = N \).

Let \( \Delta b_i \) be the contribution of the exotica to the RG running parameters, Eqn. \([5]\). We find \( \Delta b_i = -1 \) for all \( i \).

Hence, gauge coupling unification is unaffected by these states at one loop, even if they have weak scale masses.

One may inquire, whether these novel exotica are generic in string model constructions. We have looked

\(^1\) We assume here that all these exotics obtain the same mass \( M \), however this is not a priori necessary.

\(^2\) Note, bounds on stable heavy hydrogen are very severe. For example, Ref. \([20]\) (see also Ref. \([21]\)) finds the relative abundance to baryons less than of order \( 10^{-28} \). Although these fractionally charged exotics are stable, they can annihilate. We do not consider the cosmological evolution of these states in this paper.

\(^3\) Similar exothermic processes for R-hadrons was considered in Refs. \([22, 23, 24]\).
at the exotica found in the “mini-landscape” search of Ref. 10. In many cases (of order 5%) we find exotica with properties similar to those in Eqn. 2. However, unlike the previous exotica, these states transform non-trivially under a hidden sector gauge group. Consider the following two examples:

- \[ 2 \times [\mathbf{3}, \mathbf{1}, \mathbf{1}]_{-1/3} + (\mathbf{1}, \mathbf{1}, \mathbf{1})_1 \]  
  +2 \times [\mathbf{1}, \mathbf{2}, \mathbf{1}]_0 + (\mathbf{1}, \mathbf{1}, \mathbf{1})_{\pm 1} \]  
  +2 \times (\mathbf{1}, \mathbf{1}, \mathbf{1})_{-1} + (\mathbf{1}, \mathbf{1}, \mathbf{1})_{0} + (\mathbf{1}, \mathbf{1}, \mathbf{2}) \] 
  
transforming under

\[ \text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_A \times U(1)_Y, \] 

and

- \[ [\mathbf{3}, \mathbf{2}, \mathbf{1}, \mathbf{1}]_{1/3} + 5 \times [\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1}]_{2/3} \]  
  +2 \times (\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1})_0 + 4 \times (\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1})_{\pm 1} \]  
  +\[ [\mathbf{3}, \mathbf{2}, \mathbf{1}, \mathbf{1}]_{-1/3} + 5 \times (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{-2/3} \]  
  +2 \times (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{0}) + 4 \times (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2})_{\pm 1} \] 
  
transforming under

\[ \text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_A \times \text{SU}(2)_B \times U(1)_Y. \] 

These states can all be given a supersymmetric mass \( M \) without breaking any gauge symmetries. Moreover the low energy theory is anomaly free. Although the exotica do not affect the value of the GUT scale or the low energy prediction from gauge coupling unification at one loop, they do increase the value of \( \alpha_{GUT} \). In the first example (Eqn. 2) we find \( \alpha_{GUT}^{-1} = 19 \). In the second example (Eqn. 10) we find \( \alpha_{GUT}^{-1} = 14 \). These two cases are consistent with perturbative unification. However, in the third example (Eqn. 11), there is a problem since we reach the Landau pole before the GUT scale. Hence, if we demand perturbative unification, this last case is excluded.

In these examples, the hidden SU(2) gauge symmetry can become strong forming SU(2) singlet bound states. We consider a more general example below.

\section*{B. Exotica with Hidden sector charge}

As discussed in the above examples it is possible in string theory for the exotica to transform non-trivially under a hidden sector gauge group. Here we consider a generalized example, not obtained from a particular string construction, which has interesting phenomenology. Consider a hidden sector gauge group \( SU(N) \) with the exotica transforming as

\[ [(\mathbf{6}, \mathbf{1}, \mathbf{N}) + (\mathbf{1}, \mathbf{2}, \mathbf{N})] + \text{c.c.} \]  

under

\[ \text{SU}(6) \times \text{SU}(2)_R \times \text{SU}(N). \]  

Or under

\[ \text{SU}(4) \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(N), \]  

the exotica transform as

\[ [(\mathbf{4}, \mathbf{1}, \mathbf{1}, \mathbf{N}) + (\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{N}) + (\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{N})] + \text{c.c.} \]  

Note, pursuant to the previous section, values of \( N > 3 \) are excluded by demanding perturbative unification.

Assuming the hidden sector gauge coupling gets strong at a scale \( \Lambda_N \gg M_Z \), the exotica will form \( SU(N) \) singlet bound states with mass of order \( \Lambda_N \). Once again these states will not affect gauge coupling unification at one loop. The phenomenology of such \( SU(N) \) singlet “baryons” and “mesons” will depend on the values of \( N \) and \( \Lambda_N \).

At the scale \( \Lambda_N \) we can expect the \( N \)-exotica (Eqn. 2) and their charge conjugates to form chiral condensates.\(^4\) Neglecting the Standard Model gauge interactions, this theory has a \( U(8)_L \times U(8)_R \) chiral symmetry which is broken to \( U(8)_\text{vector} \) via the condensates. Hence we obtain 64 pseudo-Nambu-Goldstone [PNG] bosons, expected to be the lightest particles of the strong \( SU(N) \) gauge interactions. For an analysis of the PNG spectrum, see [23, 24, 27]. Most of these states transform non-trivially under QCD and are expected to have mass \( m \approx \left( \frac{\alpha_{GUT}}{2} \right) \Lambda_N \) \( \sim \Lambda_N \) where \( C_3(r) \) is the quadratic Casimir for the pseudo-NG boson in representation \( r \) of QCD. On the other hand, some states with only electroweak quantum numbers will be much lighter. These states typically have mass squared of order

\[ m^2 \approx \alpha_2(\Lambda_N) \Lambda_N^2. \]  

However, a few of them (i.e. bound states \( [(\mathbf{1}, \mathbf{2}, \mathbf{1}, \overline{\mathbf{N}}) \otimes (\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{N})] \) under \( [\text{Pati-Salam} \times SU(N)] \) have mass squared

\[ m^2 \approx 3 \alpha \frac{\Lambda_2^2}{4 \pi} M_2^2 \ln(\Lambda_N^2/M_2^2). \]  

These states have identical electroweak quantum numbers to the pair of Higgs doublets in the MSSM, i.e. \( H_u, H_d \) and, hopefully without confusion, we use this notation to apply to these exotica here. Finally the lightest states are axion-like with no Standard Model gauge quantum numbers.

These light states significantly constrain the lower bound on \( \Lambda_N \). In fact, bounds on an invisible axion would apply here and we would therefore expect \( \Lambda_N > 10^9 \) GeV, from astrophysical constraints on an invisible axion [28]. However, the charged states \( H_u^+ \) and \( H_d^- \) would have mass

\(^4\) Unlike technicolor theories, these chiral condensates do not break the Standard Model gauge symmetries.
less than $M_Z$ even for $\Lambda_N \sim M_{\text{GUT}}$ (see Eqn. [17]). They would most likely have been observed at LEP or the Tevatron via the Drell-Yan production of the exotica through an off-shell photon or $Z$. The only way out is to also give these exotica (Eqn. [12]) a gauge invariant mass $M \geq 200$ GeV. Then all the $\text{SU}(N)$ gauge singlets get mass of order $M$.

We then can consider two possibilities, either $\Lambda_N \geq M$ or $\Lambda_N \ll M$. The first case is comparable to QCD with all quark masses less than or equal to $\Lambda_{\text{QCD}}$. The PNG bosons will then have two contributions to their mass squared, i.e. the radiative contributions considered previously and the explicit mass contribution given by $\delta m^2 = MA_N^2/F_N^2$ where $F_N$ is the axion decay constant. A more novel scenario occurs if we take $\Lambda_N \ll M$. These exotica have properties similar to the “quirks” introduced in Ref. [29]. They can be produced at the LHC. When the exotics are produced they can separate by large distances in the detector before forming the bound state, since their effective string tension is so much smaller than their mass. In addition, we expect $\bar{H}_u^+$ and $\bar{H}_d^-$ to be heavier than their neutral weak doublet partners $H_u^0$ and $H_d^0$. Thus the charged states will decay via the beta process $\bar{H}_d^{-} \rightarrow H_u^0 + e^- + \nu_e$. The neutral ones are stable and may be dark matter candidates. Finally, the lightest $\text{SU}(N)$ singlet exotics can be defined as follows. Let $\Psi = (8, N)$ define a Dirac spinor for the exotica of Eqn. [12] in an $\text{SU}(8) \times \text{SU}(N)$ notation. Then the axial currents $J_{\mu}^A = \bar{\Psi} \gamma_{\mu} \gamma^5 T^A \Psi$, with $T^A$ given by the 4 generators in the Cartan sub-algebra of $\text{SU}(8)$ but not in $\text{SU}(3)_c \times \text{SU}(2)_L$, create these light PNG bosons from the vacuum. Two of these can either decay to two photons or two gluons through the triangle anomaly. Clearly, all these exotica would have very interesting signatures at the LHC.

II. CONCLUSION

It is well known that the prediction of gauge coupling unification in traditional SUSY GUTs is unaffected at one loop by the presence of light vector-like states in complete multiplets of $\text{SU}(5)$. In this letter we have identified a class of vector-like exotica which

- do not come in complete $\text{SU}(5)$ multiplets,
- do not affect gauge coupling unification at one loop,
- can have fractional electric charge, and
- are found in orbifold constructions of the heterotic string.

Such states provide an interesting challenge for the LHC.

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