The assignments of the bottom mesons within the screened potential model and $^3P_0$ model

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In this work, we calculate the mass spectrum of the bottom mesons with a modified nonrelativistic quark model by involving the screening effect, and explore their strong decay properties within the $^3P_0$ model. Our results suggest that the $B_1(5721)$, $B_2(5747)$, $B_1(5840)$, and $B_2(5970)$ could be reasonably assigned as the $B'(1P_2)$, $B(1P_2)$, $B(2S_0)$, and $B(1D_3)$ respectively. The more precise measurements of the excited bottom mesons are crucial to confirm these assignments.

I. INTRODUCTION

Meson spectroscopy is one of the important subjects in hadron physics, and the heavy-light system offers an excellent laboratory for testing the heavy quark symmetry. As we known, most of the charmed and charmed-strange mesons are well established, although there exists some exotic explanations for some states, for example $D_J^*(2317)$ and $D_{s1}(2460)$ [1, 2]. For the family of bottom mesons, the first bottom meson $B$ was observed in 1983 by the CLEO Collaboration [3], and there are only six excited bottom mesons observed experimentally so far, which are $B'$, $B_1(5721)$, $B_J(5732)$, $B_J(5840)$, $B_J(5970)$, and $B_2(5747)$ [4]. Among those bottom states, only $B$ and $B'$ are well assigned as the $S$-wave doublet ($1^1S_0$ and $1^3S_1$).

The $B_J(5732)$, as the first orbitally excited bottom meson, was observed in 1994 by the OPAL detector at LEP [5], and this state has not yet confirmed in the last two decades [4]. It should be stressed that its signal can be interpreted as stemming from several narrow and broad resonances, as discussed in Review of Particle Physics (RPP) [4], thus we will not discuss this state. In 2007, the D0 Collaboration reported two narrow orbitally excited ($L = 1$) bottom mesons, $B_1(5721)$ with $J^P = 1^+$ and $B_J(5747)$ with $J^P = 2^+$ [6], later confirmed by the CDF and LHCb Collaborations [7–9]. The $B_J(5970)$ was first observed in 2013 by the CDF Collaboration [8]. In 2015 the LHCb Collaboration reproted two states $B_J(5840)$ and $B_J(5960)$ [9], where the latter one should be the same state as the $B_J(5970)$ since they have the similar properties. Here we list the masses, widths, and the quantum numbers of $B_1(5721)$, $B_J(5732)$, $B_J(5840)$, $B_J(5970)$, and $B_2(5747)$ in Table I.

Table: Masses, decay widths, and quantum numbers of the bottom mesons [4].

| State          | Mass (MeV) | Width (MeV) | $I(J^P)$ |
|----------------|------------|-------------|----------|
| $B_1(5721)$    | 5698 ± 8   | 128 ± 18    | 1(2^+)  |
| $B_1(5721)^0$ | 5726 ± 1.3 | 27.5 ± 3.4  | 1/2(1^+)|
| $B_J(5747)$    | 5739 ± 0.7 | 24.2 ± 1.7  | 1/2(2^+)|
| $B_J(5840)$    | 5863 ± 9   | 127 ± 40    | 1/2(2^+)|
| $B_J(5970)$    | 5971 ± 5   | 81 ± 12     | 1/2(2^+)|

Based on the experimental measurements of the bottom mesons, there are many theoretical discussions in literatures [10–24]. The $B_J(5747)$ is commonly regarded as the $B(1^3P_2)$ state [15–22], and the $B_1(5721)$ is explained as the $B'(1P_1)$ in Refs. [17, 20–22], $B(1P_1)$ in Refs. [18, 19].

Although the LHCb Collaboration has suggested that the $B_J(5840)$ and $B_J(5970)$ could be the $2^1S_0$ and $2^3S_1$ states, respectively [9], the mass difference between them is about 110 MeV, much larger than the theoretical predictions of the $2S$ mass splitting, which is 39 MeV of Godfrey-Isgur model [11], 30 MeV in alternate relativized (AR) model [16], and 28 MeV in nonrelativistic potential quark model [19]. On the other hand, there are different interpretations for the $B_J(5840)$ and $B_J(5970)$, because their masses and widths can not be simultaneously reasonably reproduced.

As discussed in Ref. [25], the quenched quark models, incorporating a coulomb term at short distances and the linear confining interaction at large distances, will not be reliable for the high excited mesons. This is because the linear potential, which is expected to be dominant in this mass region, will be screened and softened by the vacuum polarization effects of dynamical fermions. It is shown that the screened potential plays an important role in describing the spectra and decay properties of the charmed, charmed-strange mesons and charmonium [26–28]. Thus one expect that the screened potential could improve the description of the bottom mesons.

In this work, we will investigate the possible assignments of the $B_1(5721)$, $B_J(5747)$, $B_J(5840)$, and $B_J(5970)$ by employing the modified nonrelativistic quark model with screening effect, and the $^3P_0$ model to calculate the mass spectrum and decay properties of the bottom mesons.
This article is organized as follows. In Sec. II, we give a brief review about the modified nonrelativistic quark model and the $^3P_0$ model. In Sec. III, we present the numerical results and discuss the possible assignments of the bottom mesons. The summary is given in Sec. IV.

II. THEORETICAL MODELS

A. Modified nonrelativistic quark model

The nonrelativistic quark model [29], as one of the successful quark models, is proposed by Lakhina and Swanson, and has already been used to describe the heavy-light mesons and heavy quarkonium successfully, such as the charmed-strange mesons [30] and the bottom mesons [17]. In the model, the Hamiltonian of a $q\bar{q}$ meson system is defined as [17, 30]

\[
H = H_0 + H_{sd} + C_{q\bar{q}},
\]

where $H_0$ is the zeroth-order Hamiltonian, $H_{sd}$ is the spin-dependent Hamiltonian, and $C_{q\bar{q}}$ is a constant, which will be fixed by experimental data. The $H_0$ can be compressed as

\[
H_0 = \frac{p^2}{M_r} + \frac{4}{3} \frac{r}{M_r} + br + \frac{32\alpha_s \sigma^3 e^{-\sigma^3 r}}{9 \sqrt{3} m_q m_{\bar{q}}} \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}},
\]

where the confinement interaction includes the standard Coulomb potential $-4\alpha_s/3r$ and linear scalar potential $br$, and the last term is the hyperfine interaction that could be treated nonperturbatively. In Eq. (2), $p$ is quark momentum in the system of $q\bar{q}$ meson, $r = |\vec{r}|$ is the $q\bar{q}$ separation, $M_r = 2m_q m_{\bar{q}}/(m_q + m_{\bar{q}})$, $m_q$ ($m_{\bar{q}}$) and $\mathbf{S}_q$ ($\mathbf{S}_{\bar{q}}$) are the mass and spin of the constituent quark $q$ (antiquark $\bar{q}$), respectively.

The spin-dependent term $H_{sd}$ is,

\[
H_{sd} = \left( \frac{S_q}{2m_q^2} + \frac{S_{\bar{q}}}{2m_{\bar{q}}^2} \right) \cdot \mathbf{L} \left( \frac{1}{r} \frac{dV_c}{dr} + \frac{2}{r} \frac{dV_1}{dr} \right) + \frac{S_q \cdot \mathbf{L}}{m_q m_{\bar{q}}} \left( \frac{1}{r} \frac{dV_2}{dr} \right) + \frac{3S_q \cdot \mathbf{r} S_{\bar{q}} \cdot \mathbf{r} - S_q \cdot S_{\bar{q}}}{3m_q m_{\bar{q}}} V_3 + \left[ \frac{S_q}{m_q^2} - \frac{S_{\bar{q}}}{m_{\bar{q}}^2} \right] + \frac{S_{\bar{q}}}{m_q m_{\bar{q}}} \right) \cdot \mathbf{L} \mathbf{V}_4,
\]

with

\[
V_c = -\frac{4}{3} \frac{r}{M_r} + br,
\]

\[
V_1 = -2b r - \frac{2}{9\pi} \frac{r^2}{M_r} \left[ 9 \ln(\sqrt{m_q m_{\bar{q}}}r) + 9\gamma_E - 4 \right],
\]

\[
V_2 = -\frac{4}{3} \frac{r}{M_r} - \frac{1}{9\pi} \frac{r^2}{M_r} \left[ 18 \ln(\sqrt{m_q m_{\bar{q}}}r) + 54 \ln(\mu r) + 36\gamma_E + 29 \right],
\]

\[
V_3 = -\frac{4}{3} \frac{r}{M_r} - \frac{1}{9\pi} \frac{r^2}{M_r} \left[ 36 \ln(\sqrt{m_q m_{\bar{q}}}r) + 54 \ln(\mu r) + 18\gamma_E + 31 \right],
\]

\[
V_4 = \frac{1}{\pi} \frac{r}{M_r} \frac{3m_q}{m_{\bar{q}}},
\]

where $S_+ = S_q + S_{\bar{q}}$, $L$ is the relative orbital angular momentum of the $q\bar{q}$ system. We take Euler constant $\gamma_E = 0.5772$, the scalar $\mu = 1$ GeV, $\alpha_s = 0.5$, $b = 0.14$ GeV$^2$, $\sigma = 1.17$ GeV, $m_q = m_{\bar{q}} = 0.45$ GeV, and $m_b = 4.5$ GeV [17, 30].

Because the coupled-channel effects become more important for higher radial and orbital excitations of the heavy-light mesons, some modified models have been proposed by including the screening effect [25–27], and widely used to calculate mass spectrum of charmed-strange meson [26], charm meson [27], charmonium [25, 31], and bottomonium [28]. The screening effect was introduced by the following replacement [26],

\[
br \rightarrow V_{scr}(r) = \frac{b(1 - e^{-br})}{\beta},
\]

where $V_{scr}(r)$ behaves like $br$ at short distances and constant $b/\beta$ at large distance[26, 27], $\beta$ is parameter which is used to control the power of the screening effect.

The spin-orbit term in the $H_{sd}$ can be decomposed into symmetric part $H_{sym}$ and antisymmetric part $H_{anti}$. These two parts can be written as [17]

\[
H_{sym} = \frac{S_q \cdot L}{2} \left[ \left( \frac{1}{2m_q^2} + \frac{1}{2m_{\bar{q}}^2} \right) \left( \frac{1}{r} \frac{1}{dV_c}{dr} + \frac{2}{r} \frac{dV_1}{dr} \right) \right. + \frac{2}{m_q m_{\bar{q}}} \left( \frac{1}{r} \frac{dV_3}{dr} \right) + \left. \left( \frac{1}{m_q^2} - \frac{1}{m_{\bar{q}}^2} \right) V_4 \right],
\]

\[
H_{anti} = \frac{S_q \cdot L}{2} \left[ \left( \frac{1}{2m_q^2} + \frac{1}{2m_{\bar{q}}^2} \right) \left( \frac{1}{r} \frac{1}{dV_c}{dr} + \frac{2}{r} \frac{dV_1}{dr} \right) \right. + \left. \left( \frac{1}{m_q^2} + \frac{1}{m_{\bar{q}}^2} \right) \frac{2}{m_q m_{\bar{q}}} \right] V_4 \right].
\]
where the $\theta_{\text{det}}$ is the mixing angle.

With above formalisms, one can solve the Schrödinger equation with Hamiltonian of Eq. (1) to get the meson wave functions, which will act as the input for calculating the strong decays of excited bottom mesons in the $^3P_0$ model.

### B. $^3P_0$ model

The $^3P_0$ model was proposed by Micu [32] and further developed by Le Yaouanc [33–36], and it has been widely used to calculate the OZI allowed decay processes [17, 30, 37–50]. In this model, the meson decay occurs through the regroupment between the $q\bar{q}$ of the initial meson and the other $q\bar{q}$ pair created from vacuum with the quantum numbers $J^{PC} = 0^{++}$. The transition operator $T$ of the decay $A \to BC$ in the $^3P_0$ model is given by

$$T = -3\gamma \sum_{m}(1m;1 - m|00) \int d^3p_3d^3p_4d^3(p_3 + p_4)$$

$$\mathcal{Y}_m \left( \frac{p_3 - p_4}{2m} \right) X^{A}_{1-m} \phi^{A}_0(\theta^{0}_A) b_{j}^{M}(p_3)d_{j}^{M}(p_4),$$

where $\mathcal{Y}^m(p) \equiv |p| Y^m(\theta_p, \phi_p)$ is solid harmonic polynomial in the momentum space of the created quark-antiquark pair. $X^{A}_{1-m}$, $\phi^{A}_0$ and $\theta^{0}_A$ are the spin, flavor and color wave functions, respectively. The parameter $\gamma$ is the quark pair creation strength parameter for $u\bar{u}$ and $d\bar{d}$ pairs, and for $s\bar{s}$ we take $\gamma_{s\bar{s}} = \gamma_{u\bar{u}}^{1/2}$ [36]. The parameter $\gamma$ can be determined by fitting to the experimental data. The partial wave amplitude $M^{LS}(P)$ of the decay $A \to BC$ is given by Ref. [51],

$$M^{LS}(P) = \sum_{M_1,\bar{M}_2, M_3, M_4} \langle L M_2 S M_3 | J A M_A \rangle \langle J B J c M_4 M_5 | S M_5 \rangle$$

$$\int d\Omega \mathcal{Y}^*_{L M_1} M^{*A} M_{1a} M_{1c} (P),$$

where $M^{M_A M_{1a} M_{1c}}(P)$ is the helicity amplitude,

$$\langle BC|T|A \rangle = \delta^{j_3}(P_A^o - P_B^o - P_C^o)$$

$$M^{J A M_{1b} M_{1c}}(P).$$

Here, $|A\rangle$, $|B\rangle$, and $|C\rangle$ denote the mock meson states which are defined in Ref. [52]. Then, the decay width $\Gamma(A \to BC)$ can be expressed as

$$\Gamma(A \to BC) = \frac{\pi \Gamma}{4 M_A^{3/2}} \sum_{LS} |M^{LS}(P)|^2,$$

where $P = |P| = \sqrt{M_A^2 - (M_B + M_C)^2} \sqrt{M_A^2 - (M_B - M_C)^2}$, $M_A$, $M_B$, and $M_C$ are the masses of the mesons $A$, $B$, and $C$, respectively. The spatial wave functions of the mesons in the $^3P_0$ model are obtained by solving the Schrödinger equation in Eq. (1).

### III. RESULTS AND DISCUSSIONS

In the modified nonrelativistic quark model, we determine two parameters, $C_{q\bar{q}} = 0.0152$ GeV of Eq. (1) and $\beta = 0.0246$ GeV of Eq. (5), by fitting the masses of the bottom mesons $B$ and $B^*$, which have already been well established as the $B(1^S_0)$ and $B(1^S_1)$, respectively. With these values, we calculated the mass spectrum of the bottom mesons, as shown in Table II, where we also present the predictions with other models without considering the screening effect [10, 12, 13, 15–17]. It should be stressed that the mixing angles are obtained as $\theta_{1P} = -53.5^\circ$, $\theta_{P} = -54.5^\circ$, $\theta_{D} = -50.5^\circ$, $\theta_{D^*} = -50.5^\circ$ and $\theta_{P^*} = -49.0^\circ$ by solving the potential model with the Hamiltonian of Eq. (1), which are close to the heavy quark limit mixing angles $\theta_{1P} = \theta_{P} = -54.7^\circ$, $\theta_{D} = \theta_{D^*} = -50.8^\circ$ and $\theta_{P^*} = -49.1^\circ$ [53].

In Table II, one can find that our results for excited bottom mesons are lower than those of Godfrey-Isgur (GI) [16] and Nonrelativistic quark model (NRQM) [17], which is due to the screening effect. In the modified nonrelativistic quark model, the $B_c^*(5747)$ mass is consistent with the predicted mass of $B(1^3P_2)$, and the $B_c(5721)$ mass is also in well agreement with the one of $B'_c(1P)$. On the other hand, the $B_f(5840)$ can be considered as the candidate of the $B(21^3S_0)$ taking into account the experimental uncertainties, and the $B_f(5970)$ can be assigned as the candidate of the $B_f(31^1D_2)$ or $B(1^3D_2)$. As we known, only the mass information is not enough to make those assignments, and we will calculate their strong decay widths based on these preliminary assignments for the $B_c(5721)$, $B_c(5747)$, $B_f(5840)$, and $B_f(5970)$ to further examine these assignments.

Before presenting the strong decays, we need to determine the quark pair creation strength $\gamma$ firstly. As we discussed above, $B_c(5721)$ may be a mixing state, and the assignments of the $B_f(5840)$ and $B_f(5970)$ are still in debate due to the unknown quantum numbers and poor decay information. In this work we take $\gamma = 0.411$ by fitting to the experimental widths of $B^*_c(5747)$ which is regarded as the $B(1^3P_2)$ state in previous works [17–21]. The decay properties of the state $B^*_c(5747)$ with the assignment of $B(1^3P_2)$ are shown in Table III. The ratio of the decay modes are calculated as,

$$\frac{\Gamma(B^*_c(5747) \to B^{++}\pi^-)}{\Gamma(B^*_c(5747) \to B^{*+}\pi^-)} = 0.95,$$

which is consistent with the experimental data of $1.10 \pm 0.42 \pm 0.31$ [6] and $0.71 \pm 0.14 \pm 0.30$ [9], and also the prediction of the nonrelativistic quark model [17].

The decay widths of the $B_c(5721)$ as the $B_c(1P)$ and $B'_c(1P)$ with the mixing angle $\theta_{1P} = -53.5^\circ$ are listed in Table IV. The total widths of the $B_c(5721)$ as the $B_c(1P)$ and $B'_c(1P)$ are predicted to be 193.5 MeV and 16.5 MeV, respectively. The dependence of the total decay widths of the $B_c(5721)$ as the $B_c(1P)$ and $B'_c(1P)$ on the mixing angle is shown in Fig. 1. One can see that the total width is narrow around $\theta_{1P} = -53.5^\circ$. We can safely rule out the $B_c(1P)$ assignment since the width of this case is much larger than the experimental data. Within the experimental uncertainties, the total width of the $B'_c(1P)$ assignment is in fair agreement with the experimental data, which implies that $B_c(5721)$ could be the $B'_c(1P)$ state.

In the heavy quark limit, the $P$-wave heavy-light mesons could be divided into $j = 1/2(0^+, 1^+)$ doublet and $j =...
TABLE II: The mass spectrum of the bottom mesons by different quark models in the units of MeV. The mixing angles of $B_L - B_L'$ calculated in this work are $\theta_{1\pi} = -53.5^\circ$, $\theta_{2\pi} = -54.5^\circ$, $\theta_{1\rho} = -50.5^\circ$ and $\theta_{2\rho} = 49.0^\circ$.

| State        | PDG  | Gli[16] | ARM[16] | NRQM[17] | EFG[5] | DE[13] | LNR[12] | ZVR[10] |
|--------------|------|---------|---------|----------|--------|--------|---------|--------|
| $B(1^1S_0)$ | 5279.65 ± 0.12 | 5279 | 5312 | 5275 | 5280 | 5280 | 5279 | 5277 | 5280 |
| $B(1^3S_1)$ | 5324.70 ± 0.21 | 5327 | 5371 | 5316 | 5329 | 5326 | 5324 | 5325 | 5330 |
| $B(2^1S_0)$ | 5863 ± 9 | 5870 | 5904 | 5834 | 5910 | 5890 | 5866 | 5822 | 5830 |
| $B(2^3S_1)$ | 5896 | 5933 | 5864 | 5939 | 5906 | 5920 | 5848 | 5870 |
| $B(3^1S_0)$ | 6278 | 6335 | 6216 | 6369 | 6379 | 6320 | 6117 | 6210 |
| $B(3^3S_1)$ | 6297 | 6355 | 6240 | 6391 | 6387 | 6347 | 6136 | 6240 |
| $B(1^3P_0)$ | 5683 | 5756 | 5720 | 5683 | 5749 | 5706 | 5678 | 5650 |
| $B(1^1P_0)$ | 5722 | 5777 | 5738 | 5729 | 5774 | 5700 | 5686 | 5690 |
| $B(1^3P_2)$ | 5726.1 ± 1.3 | 5725 | 5784 | 5753 | 5754 | 5723 | 5742 | 5699 | 5690 |
| $B(2^3P_0)$ | 5739.5 ± 0.7 | 5736 | 5797 | 5754 | 5768 | 5741 | 5714 | 5704 | 5710 |
| $B(2^1P_1)$ | 6104 | 6213 | 6106 | 6145 | 6221 | 6163 | 6010 | 6060 |
| $B(1^3D_1)$ | 6139 | 6197 | 6126 | 6185 | 6281 | 6175 | 6022 | 6100 |
| $B(1^3D_2)$ | 5971 ± 5 | 5959 | 6016 | 6026 | 6014 | 6091 | 5993 | 5871 | 5970 |
| $B(2^3D_1)$ | 6420 | 6475 | 6537 | 6497 | 6534 | 6248 |
| $B(2^3D_2)$ | 6334 | 6450 | 6334 | 6435 | 6554 | 6179 | 6310 |
| $B(3^3D_1)$ | 6433 | 6486 | 6377 | 6513 | 6528 | 6207 | 6320 |
| $B(3^3D_2)$ | 6341 | 6460 | 6347 | 6444 | 6542 | 6140 | 6320 |
| $B(1^1F_2)$ | 6529 | 6387 | 6302 | 6383 | 6412 | 6264 | 6190 |
| $B(1^1F_3)$ | 6157 | 6358 | 6231 | 6236 | 6420 | 6220 | 6180 |
| $B(1^3F_2)$ | 6337 | 6396 | 6316 | 6393 | 6391 | 6271 | 6200 |
| $B(1^3F_3)$ | 6162 | 6364 | 6244 | 6243 | 6380 | 6226 |

3/2$(1^+, 2^+)$ doublet, with $j(=S_q + L)$ is the total angular momentum of the light quark. For the bottom mesons, the decay width is broad for $j = 1/2$ doublet, which couples to $B\pi$ in $S$-wave, and narrow for $j = 3/2$ doublet, which couples to $B\pi$ in $D$-wave. Thus, the $B(1S_0)$ and $B(1S_0')$ should be the $j = 3/2$(1+, 2+) doublet.

FIG. 1: Total decay width of the $B(1S_0)$ as the $B(1P)$ depends on the mixing angle. The vertical red solid line corresponds to the mixing angle $\theta \pi = -53.5^\circ$. and the blue band denotes the experimental width of the $B(1S_0)$ from RPP [4].

The decay widths of the $B(1S_0)$ as the $B(1S_0)$ is shown in Table V, and the predicted total decay width is 109.5 MeV, in well agreement with the experimental measurement 127 ± 40 MeV [4]. In this case, the dominant decay mode is $B^\pi$, and the decay mode $B\pi$ is forbidden for the $B(1S_0)$ assignment. It should be pointed out that the decay mode $B\pi$ has not yet

TABLE III: Decay widths of the $B_2'(5747)$ as the $B(1^3P_2)$ in units of MeV.

| Channel                  | $B(5747)$ |
|--------------------------|-----------|
| $B^+\pi^0$               | 8.2       |
| $B^0\pi^0$               | 4.1       |
| $B^{\ast+}\pi^0$         | 7.8       |
| $B^{\ast0}\pi^0$         | 4.0       |
| Total width              | 24.2      |
| Experiment total width   | 24.2 ± 1.7|

TABLE IV: Decay widths of the $B(1S_0)$ as the $B(1P)$ and $B(1P')$ in units of MeV, with the mixing angle $\theta_{1\pi} = -53.5^\circ$.

| Channel                  | $B(1P)$ |
|--------------------------|---------|
| $B^+\pi$                 | 129.2   |
| $B^0\pi^0$               | 64.3    |
| Total width              | 193.5   |
| Experiment total width   | 165.0   |

TABLE V: Decay widths of the $B(1S_0)$ as the $B(2^1S_0)$ in units of MeV.

| Channel                  | $B(5840)$ |
|--------------------------|-----------|
| $B^+\pi^0$               | 73.0      |
| $B^0\pi^0$               | 36.5      |
| $B^{\ast+}\pi^0$         | 0.004     |
| $B^{\ast0}\pi^0$         | 0.003     |
| Total width              | 109.5     |
| Experiment total width   | 127 ± 40  |
TABLE VI: Decay widths of the $B_J(5970)$ as the $B_2(1D)$ and $B(1D_3)$ in units of MeV with the mixing angle $\theta_{1D} = -50.5^\circ$.

| $B_2(1D)$ | $B(1D_3)$ |
|----------|-----------|
| $B^+\pi$ | 12.6      |
| $B^0\pi^0$ | 6.3      |
| $B^{*0}\pi^0$ | 52.9      |
| $B'^{(0)}\rho_0^0\pi$ | 26.4      |
| $B'^{(1)}\rho_0^0\pi$ | 0.006     |
| $B'^{(1)}\rho_0^0\pi$ | 0.003     |
| $B'^{(2)}\rho_0^0\pi$ | 66.8      |
| $B'^{(2)}\rho_0^0\pi$ | 33.5      |
| $B_1(1P)^0\pi^0$ | 0.005     |
| $B_1(1P)^0\pi^0$ | 0.003     |
| $B_1(1P)^0\pi^0$ | 0.04      |
| $B_1(1P)^0\pi^0$ | 0.03      |
| $B''\eta$ | 0.3       |
| $B''\eta$ | 11.2      |
| $B''K^0_S$ | 0.1       |
| $B''K^0_S$ | 11.1      |
| Total width | 202.5 MeV |
| Experiment | 81 MeV    |

FIG. 2: Total decay width of the $B_J(5970)$ depends on the mixing angle as the $B_2(1D)$ and $B_2(1D)$. The vertical red solid line corresponds to the mixing angle $\theta_{1D} = -50.5^\circ$ and the blue band denotes the experimental width from RPP [4].

Finally, the decay widths of the $B_J(5970)$ as the $B_2(1D)$ and $B(1D_3)$ are listed in Table VI. The predicted total width of the $B_2(1D)$ assignment is 202.5 MeV with mixing angle $\theta_{1D} = -50.5^\circ$, which is about 100 MeV larger than the experimental data $81 \pm 12$ MeV, while the one of the $B(1D_3)$ assignment is 40.1 MeV. The dominant decay modes are $B\pi$ and $B^*\pi$ decay modes, supported by the measurements of the CDF [8] and LHCb Collaborations [9]. The dependence of the decay widths of the $B_J(5970)$ as the $B_2(1D)$ on the mixing angle is shown in Fig. 2, one can find that the total width of $B_2(1D)$ is about 200 MeV around the mixing angle $\theta_{1D} = -50.5^\circ$, and the total width of $B_2'(1D)$ is predicted to be about 50 MeV. Considering the predictive power of the model and the experimental uncertainties, it is reasonable to regard $B_J(5970)$ as the $B(1D_3)$.

IV. SUMMARY

In this paper, we have calculated the bottom meson spectrum with a modified nonrelativistic quark model involving the screening effect. We present a good description of mass spectrum of the bottom mesons, especially for the excited bottom mesons. Furthermore, we also investigate the strong decay properties of the $B_1(5721), B_2'(5747), B_J(5840)$, and $B_J(5970)$ with the $^3P_0$ model.

Based on the mass spectrum and decay properties, the $B_1(5721)$ and $B_2'(5747)$ can be identified as the $B'_1(1P)$ and $B'(1^3P_0)$, respectively. The $B_J(5840)$ could be interpreted as the $B(2^1S_0)$, and the $B_J(5970)$ could be explained as the $B(1D_3)$. Further experimental information, especially the quantum numbers and decay modes of $B_J(5840)$ and $B_J(5970)$, are necessary to confirm these assignments.

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