Non-linear convection flow of micro polar nanofluid past an isothermal sphere

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Abstract

In this study, steady laminar two dimensional boundary layer flow of non-linear convection micropolar nanofluid over an isothermal sphere is examined. The mathematical developing for the flow problem has been made. By means of appropriate similarity transformation and dimensionless variable, the governing non-linear boundary value problems were reduced into combined high order non-linear ordinary differential equations. Then, solution for velocity, microrotation, temperature and concentration has been obtained numerically. The equations were calculated using method bvp4c from matlab software for various quantities of main parameters. The influences of different parameters on skin friction coefficient, wall duo stress coefficient the Nusselt number, Sherwood number as well as the velocities, temperature, and concentration are analyzed and discussed through the tables and plotted graphs. A comparison with previous paper obtainable in the literature has been performed and an excellent agreement is obtained. The finding results indicate the raise in either the values of thermal Grashof number \(Gr\) or Brownian motion parameter \(Nb\) or Schmidt number \(Sc\) allows to bringing down the kinematic viscosity of the fluid causes improve the temperature profile distribution within the boundary layer. On the other hand, it can be diminished by the growth in either the value of solutal Grashof number \(Gm\) or solutal non linear convection parameter \(s\) which agrees to enlarge fluid density.

Nomenclature

\begin{align*}
a & \quad \text{Constant b Radius of sphere [m]} \\
C_f & \quad \text{Skin friction coefficient} \\
C & \quad \text{Concentration} \\
C_w & \quad \text{Ambient concentration} \\
D_T & \quad \text{Thermophoretic diffusion coefficient} \\
D_B & \quad \text{Brownian diffusion [m}^2\text{/s]} \\
g & \quad \text{Dimensionless microrotation velocity} \\
Gr & \quad \text{Grashof number} \\
k_0 & \quad \text{Thermal jump factor [1/m] K Conductivity [W/mK]} \\
m_w & \quad \text{Wall couple stress} \\
N & \quad \text{Microrotation at surface} \\
N_t & \quad \text{Thermophoresis parameter} \\
n & \quad \text{Local Nusselt number} \\
q_w, q_m & \quad \text{Heat and mass fluxes [W/m}\^2, \text{ Kg/m}\^2\] \\
S_h & \quad \text{Microrotation slip parameter} \\

\end{align*}
1. Introduction

Studies dealing with non-Newtonian fluids are a general field of exploration with numerous industry and engineering applications. In general term the holding up of small body in fluid and colloidal fluid elements together are called as micropolar fluids. The idea of the new non-Newtonian kind of fluid for which the classical Navier–Stokes theory is insufficient for its full description is called micropolar fluids was given for the first time by Eringen [1]. The mathematical equations describing the micropolar fluid was given by the same author.

Following that, many researchers had examined the micropolar fluid under different physical conditions by considering various parameters. Accordingly, Hamzeh [2] has evaluated the steady natural convective flow with MHD hanging micropolar Casson fluid past a solid sphere. Moreover, the influences of nonlinear convection on flow of micropolar fluid, an exponentially touching stream about an exponentially stretching sheet have been illustrated by Mandal and Mukhopadhyay [3].

The effects of MHD with thermal conductivity on micropolar nanofluid flow over radiative widening surface and the influences of heat generation/absorption on an axisymmetric MHD flow of nanofluid between two concentric cylinders have been presented by Srinivas and Kishan [4] and Naseem et al [5]. The study revealed that nano fluid particles concentration reduced by rising Lewis number, thermophoresis and Brownian motion parameters. The effects of viscous dissipation reduced by rising Lewis number, thermophoresis and Brownian motion parameters. The effects of viscous dissipation together with thermophoresis on time varied magnetic field on free convection flow over a slopping permeable plate were discussed by Deepa [6]. Further, Raju et al [7] have presented the changeable viscosity on unsteady dissipation Carreau fluid resting on a reduced cone filled with Titanium alloy nano-particles. These works indicated that inconsistent viscosity significantly surges viscous drag and rate of heat transfer. But the viscous dissipation parameters have tendency to diminish the rate of heat transfer and expands the temperature filed as seen by Salina et al [8].

Noor et al [9] have computed numerically the slip effects in the presence of thermal buoyancy force on micropolar nanofluid flow at stagnation point along a vertically elongating surface. Similarly, by using Keller-box method, the slip effects on MHD flow of a Williamson and effects of radiation and thermal slip on non-Newtonian fluid starting an isothermal sphere were reported by Nagendra et al [10] and Madhavi et al [11] respectively. The result shows that with very large velocity and thermal slip, temperature profiles near sphere surface has reduced significantly.

Further, Abdul et al [12] and Amanulla [13] have computed the influences of magnetic field and convective heating condition on non-Newtonian fluid flow and heat transfer past an isothermal sphere.
Moreover, Sandeep and Kumar [14] examined the characteristics of heat and mass transfer in MHD nanofluid flow about an inclined stretching sheet including volume fraction of dust and nanoparticles. The results illustrated that an increasing in the inclined angle, decreasing in wall friction useful in cooling management, in oil recovery management. Also, decline temperature profiles and boost in heat transfer rate is most valuable in production industries. Furthermore, the effects of changeable Brownian and thermophoretic diffusion coefficients including temperature and concentration at wall on MHD mixed convection movement of a nanofluid above nonlinear enlarging sheet have been evaluated by Sumalatha and Shanker [15] and Kala et al [16] have analyzed Dufour and Soret effects on MHD micro polar fluid above a linearly stretching sheet about a non-Darcy porous medium. The results illustrated that the influence of Grashof number drops the thermal diffusion of the fluid.

Saleem et al [17, 18] have presented the effects of heat source, chemical reaction, conductive heat and mass transfer rate on magneto Jeffrey fluid and magneto Walter’s B nanofluid flow with time variation past rotating cone. Furthermore, Sadiq et al [19] have addressed the behavior of MHD oscillatory sloping stagnation point surge micropolar nano fluid. The finding shows that the thickness of velocity boundary layer is thin for weak concentration as compared to strong concentration.

The influence of slip conditions and joule heating including Brownian motion and Thermophoresis diffusions over exponentially stretched surface and MHD Walter-B non-liquid on mixed convection flow of nonlinear power law, nonlinear radiation and nonlinear thermally radiating Carreau nano fluid flow have been analyzed by Saleem et al [20], Ijaz Khan et al [21] and Waqas et al [22] correspondingly. The results show that enhancing the values of Brownian motion parameter, thermophoresis parameter, magnetic parameter, increase the temperature distribution within the boundary layer. Temperature based thermal conductivity together with changeable thickness, homogeneous- heterogeneous reaction in stagnation points along a nonlinear stretching surface have been addressed by Hayat et al [23, 24] and Ijaz Khan et al [25]. The finding indicates that the boost in the values of Prandtl number decreases the temperature distribution within thermal boundary layer.

All of the above research papers have been discussing the flow over a plane, over a parabolic boundary. But in this paper, we examine numerically the two-dimensional non-linear convection flow of micro polar nano fluid over an isothermal sphere in the presence of velocity and micro rotation slips, thermal and solutal jumps, non-linear convection effects, using bvp4c from matlab. The effects of physical parameters on fluid velocity, micro rotation, temperature and concentration were discussed and presented in graphs and tables as well.

2. Mathematical formulation

Consider a steady, laminar, two dimensional boundary layer flow of a viscous micropolar nano fluid over an isothermal spherical body of fixed radius b. The velocity at the surface of sphere is given by \( u = \frac{b}{a} \) where, \( a > 0 \) is a constant and the ambient fluid velocity \( u_{\infty} \) past an isothermal spherical body. The surface temperature of the sphere \( T_w \) is taken greater than the ambient temperature of the fluid (heated sphere). The coordinate x and y are selected such that x measures the distance along the surface of the sphere from the lower point and y measures the distance normal to the surface of the sphere as shown in the figure 1.

Following Chandra et al [3], Noor et al [9] and Amanulla et al [13] the differential equations governing the present problem are given as follows:

\[
\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial y} = 0 \tag{1}
\]
\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left( \nu + \frac{\kappa}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho} \frac{\partial N}{\partial y} + g \left( \beta_0 (T - T_\infty) + \beta_1 (T - T_\infty)^2 \right) + g \left( \beta_0 (C - C_\infty) + \beta_1 (C - C_\infty)^2 \right) \sin \left( \frac{x}{b} \right)
\]

(2)

\[
\frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \varphi \frac{\partial^2 N}{\partial y^2} - \frac{\kappa}{\rho} \left( 2N + \frac{\partial u}{\partial y} \right)
\]

(3)

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho \varphi} \frac{\partial^2 T}{\partial y^2} + \mathcal{T} \frac{\partial C}{\partial y} + \mathcal{D} \frac{\partial^2 T}{\partial y^2}
\]

(4)

\[
\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_T \frac{\partial^2 C}{\partial y^2} + D_T \frac{\partial^2 T}{\partial y^2}
\]

(5)

Subject to the proper boundary condition:

\[
u = N_0 \frac{\partial u}{\partial y}, \quad v = 0, \quad N = -n \frac{\partial u}{\partial x}, \quad N = -n \frac{\partial u}{\partial x}, \quad T = T_w + k_0 \frac{\partial T}{\partial y}, \quad \text{aty} = 0
\]

(6)

\[
u \rightarrow u_\infty, \quad N \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad \text{asy} \rightarrow \infty.
\]

(7)

where \(u\) and \(v\) are the components of velocity in the \(x\) and \(y\) axes correspondingly, \(N_0\) stands for momentum slip factor, \(k_0\) represents thermal jump factor, \(s_0\) is solutal jump factor, \(N\) is the component of micro rotation perpendicular to the \(xy\) plane. Here \(n\) is a constant which lies between 0 and 1 stands for different category of variations for concentration. When \(n = 0\) that is the movement particles concentration is concentrated that the micro elements near to the wall surface are incapable to rotate, when \(n = \frac{1}{2}\), the anti-symmetric part of the stress tensor is disappearing that stands for puny concentration. The case \(n = 1\) is meant for modeling the turbulent boundary layer flows as reported by Noor et al.\[9\]. \(\mu\) stands for the coefficient of fluid viscosity, \(\rho\) is the density \(\nu = \frac{2}{3}\) is kinematic viscosity, \(c_\rho\) stands for the specific heat, \(j = \frac{\nu}{c_\rho}\) is the microinertia per unit mass, \(\varphi = (\mu + \frac{\kappa}{\rho})\) is spin gradient viscosity, \(\kappa\) is the vortex viscosity, \(T\) stands for temperature, \(K\) is the thermal conductivity of the fluid, \(T_w (= T_\infty + \Delta T)\) and \(C_w (= C_\infty + \Delta C)\) represent the variables temperature and concentration at surface, where, \(\Delta T\) and \(\Delta C\) being constants which give the rate of growth of temperature and concentration alongside the surface and \(T_\infty, \ C_\infty\) stand for the uniform temperature, concentration of the free stream, \(g_0\) represents for the gravity, \(\frac{\nabla \mu}{\rho} (= \beta_0 (T - T_\infty) + \beta_1 (T - T_\infty)^2 + \beta_0 (C - C_\infty) + \beta_1 (C - C_\infty)^2)\), \(\beta_0\) and \(\beta_1\) are constants, \(\beta_0\) and \(\beta_1\) are the constant coefficients of thermal and volumetric expansion respectively. This relation will be non-linear density temperature and concentration (NDTC) variation. \(\gamma = \frac{\rho_0}{\rho}\) is the ratio between the effective heat capacity of the nanoparticle material and the heat capacity of the fluid, \(D_T\) stands for the thermophoretic diffusion coefficient. Let \(r(x) = b \sin \left( \frac{x}{b} \right)\) be the radial distance from the symmetrical axis to the surface of the sphere.

Following Amanulla et al.\[13\] the non-dimensional variables such as:

\[
\xi = \frac{x}{b}, \quad \eta = \frac{Gr^\frac{1}{3}}{b}, \quad \tau = \frac{r(x)}{b}, \quad f = \frac{\psi}{\nu \xi (Gr)^\frac{1}{3}}, \quad g = \frac{b^2}{\nu \xi (Gr)^\frac{1}{3}} N, \quad \theta = \frac{T}{T_\infty} - \frac{T_\infty}{T_w - T_\infty}
\]

(8)

\[
\Omega = \frac{C - C_\infty}{C_a - C_\infty}
\]

Introducing a stream function \(\psi(x, y)\) and \(u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}\)

That satisfies the continuity equation equation (1). Hence, equations (2)–(5) can be written in the following non linear system of ODEs.

\[
(1 + \beta) f'' + \left( 1 + \xi \cot(\xi) \right) f + \xi \frac{\partial f}{\partial \xi} f'' - \xi \frac{\partial f}{\partial \xi} f' - (f')^2 + \beta g' + \theta \omega^2 \sin(\xi) + \frac{Gm}{\Omega + s\Omega^2} \sin(\xi) \frac{\partial f}{\partial \xi} = 0.
\]

(9)

\[
\left( 1 + \frac{\beta}{2} \right) g'' + (1 + \xi \cot(\xi)) f g' - \beta (2g + f'') + \xi \left( \frac{\partial f}{\partial \xi} g' - \frac{\partial g}{\partial \xi} f' \right) = 0.
\]

(10)
\[
\frac{1}{Pr} \frac{\partial \theta''}{\partial \eta} + (1 + \xi \cot(\xi)) \frac{\partial \theta'}{\partial \eta} + \xi \left( \frac{\partial^2 \theta'}{\partial \xi^2} - \frac{\partial \theta'}{\partial \xi} \frac{\partial \xi}{\partial \eta} \right) + Nt(\theta')^2 + Nb\Omega\theta' = 0.
\]
\[\tag{11}\]

\[
\frac{1}{Sc} \frac{\partial \Omega''}{\partial \eta} + \frac{Nt}{ScNb} \frac{\partial \theta''}{\partial \eta} + \left(1 + \xi \cot(\xi)\right) \frac{\partial \Omega'}{\partial \eta} - \xi \frac{\partial \Omega}{\partial \xi} \frac{\partial \xi}{\partial \eta} = 0.
\]
\[\tag{12}\]

Subject to the boundary conditions

\[
\alpha \eta = 0: f'(0) = s_r f''(0), \quad \frac{\partial f(0)}{\partial \xi} = 0, \quad g(0) = -nf''(0), \quad \frac{\partial g(0)}{\partial \xi} = S_k f''(0),
\]
\[
\theta(0) = 1 + s_b \theta'(0), \quad \frac{\partial \theta(0)}{\partial \xi} = Gr^\frac{1}{4} \theta'(0), \quad \Omega(0) = 1 + s_m \Omega'(0), \quad \frac{\partial \Omega(0)}{\partial \xi} = Gr^\frac{1}{4} \Omega'(0),
\]
\[
as \eta \to \infty: f' = 0, \quad g = 0, \quad \theta = 0, \quad \Omega = 0.
\]
\[\tag{13}\]

where, prime represent differential with respect to \(\eta\), \(\xi\) stands for Dimensionless stream-wise coordinate, \(s_r = \frac{NcGr^\frac{1}{4}}{b}\) is velocity slip parameter, \(S_k = nb + \frac{n^2}{\xi}\) denotes microrotation slip parameter, \(s_t = \frac{k_5 Gr^\frac{1}{4}}{b}\) stands for thermal parameter, \(s_m = \frac{k_6 Gr^\frac{1}{4}}{b}\) withstands for solutal jump parameter, \(\beta = \frac{\kappa_2}{\mu}\) is material parameter, \(Gr = \frac{\nu^2}{\nu^2 (T_w - T_r)}\) represents the ratio of the buoyancy force arise from temperature difference to the viscous force times inertia force to viscous force termed as thermal Grashof number, \(Gm = \frac{\nu^2}{\nu^2 (C_w - C_r)}\) represents the ratio of the buoyancy force arise from concentration difference to the viscous force times inertia force to viscous force termed as solutal Grashof number, \(\lambda = \frac{\kappa_3(T_w - T_r)}{s_3}\) and \(s = \frac{\kappa_3(C_w - C_r)}{s_3}\) are thermal and solutal nonlinear convection parameters correspondingly. We observe that for \((\lambda = s = 0)\), the flow of equation (9) becomes convective micropolar nanoliquid. Also, the relation of buoyancy force (Grashof:Gr) is seen in the this equation is the dimensionless parameter indicating the ratio between the buoyancy force caused by concentration change and the buoyancy force owing to temperature change. Further, \(Gm = 0\) is for the case of non-buoyancy influence because of mass diffusion, \(Gr \to \infty\) i.e. \(\frac{Gm}{Gr} \approx 0\) is for non-buoyancy effect owing to thermal diffusion and \(\frac{Gm}{Gr} = 1\), is for thermal and mass buoyancy forces of the same strength. \(Pr = \frac{\nu^2}{\nu^2}\) is Prandtl number, \(Sc = \frac{\nu}{\nu^2}\) is Schmidt number \(Nb = \gamma \frac{\nu^2}{\nu^2} (C_w - C_r)\) stands for Brownian motion parameter, \(Nt = \gamma \frac{\nu^2}{\nu^2} (T_w - T_r)\) represents thermophoresis parameter.

It can be appeared that the lower point of the sphere \(\xi \approx 0\), equations (9)–(12) reduced to the next nonlinear system of ordinary differential equations.

\[
(1 + \beta)f'' + 2ff' - (f')^2 + \beta \theta' + \theta + \lambda \theta^2 + \frac{Gm}{Gr} (\Omega + s\Omega') = 0.
\]
\[\tag{14}\]

\[
\left(1 + \frac{\beta}{2}\right) g'' + 2fg' - gf' - \beta (2g + f') = 0.
\]
\[\tag{15}\]

\[
\frac{1}{Pr} \theta'' + 2\beta \theta' + Nt(\theta')^2 + Nb \Omega \theta' = 0.
\]
\[\tag{16}\]

\[
\frac{1}{Sc} \Omega'' + \frac{Nt}{ScNb} \theta'' + 2f \Omega' = 0.
\]
\[\tag{17}\]

The boundary equation (13) converts

\[
\eta = 0: f'(0) = s_r f''(0), \quad g(0) = -nf''(0), \quad \theta(0) = 1 + s_b \theta'(0), \quad \Omega(0) = 1 + s_m \Omega'(0),
\]
\[
as \eta \to \infty: f' = 0, \quad g = 0, \quad \theta = 0, \quad \Omega = 0.
\]
\[\tag{18}\]

The physical quantities of awareness in this problem are the limited skin friction coefficient \(C_f\) Wall couple stress \(m_w\), the Nusselt number Nu and the Sherwood Sh number and they can be written as:

\[
C_f = \frac{\tau_w}{\rho u_w}, \quad Nu = \frac{q_w}{K(T_w - T_r)} , \quad Sh = \frac{q_m}{Dh(C_w - C_r)}, \quad m_w = \left(\frac{\nu + \kappa}{2\rho}\right) \left(\frac{\partial N}{\partial y}\right)_{y=0}.
\]
\[\tag{19}\]

Where, \(\tau_w = (\mu + \kappa)\left(\frac{\partial \nu}{\partial y}\right)_{y=0} + \kappa(N)_{y=0}, \quad q_w = -K\left(\frac{\partial T}{\partial y}\right)_{y=0}, \quad q_m = -Dh\left(\frac{\partial C}{\partial y}\right)_{y=0} \). Using the non-dimensional variables (8) and the boundary conditions (13) the limited skin friction coefficient \(C_f\), wall couple
stress $m_w$, the Nusselt number $Nu$ and the Sherwood number $Sh$ are

\[
\begin{align*}
f''(0) &= \frac{C_f N_0 \xi Gr^\frac{1}{2}}{\nu b(1 + \beta(1 - n))}, \quad g'(0) = \frac{m_w b}{\nu^2 \xi N_0 \sqrt{\frac{Gr}{Gr^2}}}, \quad Nu = \frac{b}{Gr^\frac{1}{2}} \\
\lambda &= -\theta'(0), \quad Sh = -\frac{b}{Gr^\frac{1}{2}}.
\end{align*}
\]

(20)

The pair four harmonized high order ordinary differential equations (9)–(12), subjected to the boundary conditions, equation (13), are answered numerically using the function bvp4c from matlab software for various values of physical parameters, that is, slip parameters $Sv$, Prandtl number $Pr$, thermal Grashof number $Gr$, Solutal Grashof number $Gm$, Brownian motion parameter $Nb$, thermophoresis parameter $Nt$, Schmidt number $Sc$, thermal nonlinear convection parameter $s$, material parameter $b$ and microrotation parameter $n$, dimensionless stream-wise coordinate $x$.

Statistical results are found using Matlab BVP solver bvp4c from matlab which is a finite difference code that realize the three-stage Lobatto IIIa formulation. To apply bvp4c from matlab, first, equations (9)–(12) are converted into a system of first-order equations. Second, up and doing a boundary value problem (bvp) and use the bvp solver in matlab to numerically solve this system, including the above boundary condition and income on a suitable finite value for the far field boundary condition, that is, $\eta \to \infty$, say $\eta_{\infty} = 10$ and the step-size is taken as $\Delta \eta = 0.01$, $\Delta \xi = 0.00001$ are obtaining the numerical results; and accuracy to the fifth decimal place as the measure of convergence. In solving the BVP by means of matlab, bvp4c has only three points of views: a function ODEs for calculation of the residual in the boundary conditions, and a building solint that provides a guess for a mesh. The ODEs are handled exactly as in the Matlab IVP solvers. Further clarification on the procedure of bvp4c is found in the book by Shampine et al. [26].

Let $y(1) = f$, $y(2) = f'$, $y(3) = f''$, $y(4) = g$, $y(5) = g'$, $y(6) = \theta$, $y(7) = \theta'$, $y(8) = \Omega$, $y(9) = \Omega'$, $y(10) = \frac{\partial \Omega}{\partial \xi}$, $y(11) = \frac{\partial \theta}{\partial \xi}$, $y(12) = \frac{\partial \Omega}{\partial \xi}$, $y(13) = \frac{\partial \Omega}{\partial \xi}$, $y(14) = \frac{\partial \Omega}{\partial \xi}$ and $y = [f, f', f'', g, g', \theta, \theta', \Omega, \Omega', \frac{\partial \Omega}{\partial \xi}, \frac{\partial \Omega}{\partial \xi}, \frac{\partial \Omega}{\partial \xi}, \frac{\partial \Omega}{\partial \xi}, \frac{\partial \Omega}{\partial \xi}, \frac{\partial \Omega}{\partial \xi}, \frac{\partial \Omega}{\partial \xi}, \frac{\partial \Omega}{\partial \xi}, \frac{\partial \Omega}{\partial \xi}]$ gives

Figure 2. Graph (a) $f'$ profile for different values of $Gr$ when $Gm = 90$, $Sv = 0.25$, $s = 0.1$, $\lambda = 0.2$, $\beta = 2$, $\xi = 12$, and (b) $f'(0)$ $\lambda = 0.5$, $Gm = 0.15$, $\beta = 0.1 S = 0.2$, $\xi = 3$. 

3. Numerical solution

Solutal nonlinear convection parameter $s$, material parameter $\beta$ and microrotation parameter $n$, dimensionless stream-wise coordinate $\xi$, Statistical results are found using Matlab BVP solver bvp4c from matlab which is a finite difference code that realize the three-stage Lobatto IIIa formulation. To apply bvp4c from matlab, first, equations (9)–(12) are converted into a system of first-order equations.
4. Results and discussion

In this section, the results of various governing physical parameters on non-dimensional velocity, temperature, concentration, microrotation, skin friction and wall couple stress coefficients, limited Nusselt and Sherwood numbers have been examined.
4.1. Velocity and microrotation profiles

The dimensionless velocity profile graphs of $f(\eta)$, Skin friction graphs $f'(0)$, wall couple stresses-$\Omega'(0)$ and angular velocity profile graphs of $g(\eta)$ for different values of the microrotation slip parameter $Sg$, solutal nonlinear convection $s$, microrotation $n$ and material parameters $\beta$, dimensionless stream-wise coordinate $\xi$, thermal and solutal Grashof numbers ($Gr$, $Gm$) are represented in figures 2–6. Figures 2 and 5, illustrate that upsurge in values of $Gr$ and $\beta$, rise the acceleration due to gravity and vortex viscosity of the fluid flow which raise resistance to flow the fluid that result decrease in the fluid velocity profiles $f(\eta)$, angular velocity profiles $g(\eta)$ and their boundary layer thicknesses, as well as skin friction coefficients, $f'(0)$ at the wall. However, figures 4 and 6 show that enhance either in the magnitude of $Sg$, $Gm$, or $n$, reduce the viscosity of the fluid flow which add opposition to flow the fluid that result in increasing the angular velocity profiles and their boundary layer thicknesses within boundary, Skin friction coefficient $f'(0)$ and Wall Couple Stresses coefficients-$\Omega'(0)$ at the surface. Next, figure 3 indicates that an increase in the denomination of $s$, result in rising of skin friction.
coeficient, $f'(0)$ at the wall but the fluid velocity profile $f'(\eta)$ and its boundary layer thickness declined as $\xi$ growth. This influence is happen due to the velocity boundary layer increase which raises opposition to flow the fluid.

4.2. Temperature and concentration profiles

The consequences of thermal jump parameter $St$, thermal and solutal Grashof numbers ($Gr$ and $Gm$), Schmidt and Prandtl numbers (Sc and Pr), Brownian motion, thermophoresis, thermal nonconventional, solutal non-linear convection parameters ($Nb$, $Nt$, $\lambda$ and $s$), on temperature and concentration sketches are given in figures 7–12. Figures 7 and 8 indicate that increase in the values of thermal jump parameter $St$, drop the Nusselt number as well as raise in the magnitude of solutal Grashof number $Gm$, decline the temperature distribution and its boundary layer thickness. These effects are happen due to either increase thermal or solutal volumetric expansion coefficient of the fluid which fall thermal conductivity to diffuse the temperature that results drop in the thermal diffusion, associate with lessen Nusselt number and temperature dissemination and its boundary layer thickness. However, enlarge the values of $Nt$, drops kinematic viscosity of the fluid which rise solutal
diffusion to diffuse the concentration that results increase in Sherwood number and concentration profile. Moreover, figures 9 and 11 show that as the values of Gr and Nb grow result in bringing down the kinematic viscosity of the fluid which fall resistance to diffuse the temperature that causes the thermal diffusion rise, associate with increase temperature distribution and its boundary layer thickness. Furthermore, growth in the amount of Prandtl number Pr and Brownian motion constraint Nb, drop the thermal diffusion, the viscosity of the fluid respectively and refusal to go along to diffuse the temperature which lower the kinematic viscosity of the fluid that associate through increase in temperature distributions and their boundary layer thicknesses, that results decrease the Nusselt number. On the other hand, figure 10 indicates that increases in the values of Solutal and thermal non linear convection parameters, s and \( \lambda \), decrease the temperature and its boundary layer thickness and growth the Nusselt number. These effects are happen due to escalation in the volumetric

Figure 8. Graph (a) - \( \Omega'(0) \) profile for different values of Nt when St = 0.6, Sm = 0.1, Nb = 1, Sc = 0.2, Pr = 5 and (b) \( \Omega \) when St = 0.7, Sm = 0.6, Nb = 0.3, Sc = 0.23, Pr = 0.51.

Figure 9. Graph (a) - \( \rho'(0) \) profile for different values of Pr when St = 0.6, Sc = 0.2, Nb = 1, and (b) \( \theta \) for different values of Gr when Gm = 23, St = 0.7, Sm = 0.6, \( \lambda = 0.1 \), s = 0.3, Sc = 0.2, Pr = 9, Nb = 0.19, Nt = 0.8.

Figure 8.
enlargement coefficient of the fluid which reduces thermal conductivity to diffuse the temperature that results drop in the thermal diffusion, associate with decrease temperature dissemination and its boundary layer thicknesses which result the enlargement of Nusselt number.

Figure 12, indicates that increase value of Sc, lower the diffusion coefficient that result bring down in concentration distribution and their boundary layer thickness. On the contrary, rise quantities of Pr; enhance thermal diffusion which causes rise in temperature distribution and their boundary layer thicknesses.

Table 1 drawn to compare the exactness of the method used association with previously presented data feasible in the literature has been done. From table 1 it can be seen that the numerical values of the Nussalt number $-\theta' (0)$ in this paper for different values of $\beta$ and $\xi$ when $n = 0.5$, $Pr = 0.7$ is in an excellent agreement with the pervious outcomes of published paper by Nazar et al [18]. The results indicate that the numerical used in the report is truthful and highly accurate.
4.3. Skin frictions

Tables 2 and 3 point out that upsurge in values of \((G_m, \lambda, S_v, S_m, s)\), causes enhancement in the skin friction coefficient. However, as the magnitudes of \((S_t \text{ and } \xi)\) increase, reduces the viscosity of the fluid flow which lowers opposition to flow the fluid that results reduction of skin friction coefficients.
4.4. Wall couple stresses

Tables 2, 3 illustrate that an increase in denominations of \(G_m, \lambda, S_v, S_m\) and \(s\), causes growth in the wall couples coefficient. On the other hand, as the values of \(S_t\) and \(x\) grow, reduces the viscosity of the fluid flow which moderates resistance to rotate the fluid that results bargain of wall couples coefficients.

4.5. Nusselt number

Tables 2 and 3 show that an increase in quantities of \(G_m, \lambda, S_v, S_m\) and \(s\), causes rise in the Nusselt number on contra sty, as the magnitudes of \(S_t\) and \(x\) increase, bring down the viscosity of the fluid flow which reduces opposition to rotate the fluid that results fall of Nusselt number.

4.6. Sherwood number

Table 2 explain that an increase in values of \(G_m, \lambda, S_v\), causes enhancement in the Sherwood number. But, as the amounts of \(S_m\) rise, decreases the viscosity of the fluid flow which lowers opposition to diffuse the fluid that results lessening of Sherwood number.

5. Conclusion

This study considers the effects of thermal and solutal nonlinear convection as well as manifold slip conditions on two dimensional boundary layer flow of a micropolar nano fluid past an isothermal sphere were discussed. The boundary layer equations of the flow problem are reduced into a duo of high order non-linear ordinary differential equations by means of the similarity transformation. The obtained differential equations are calculated numerically using bvp4c from matlab software. Numerical outcomes are acquired for different main parameters of the flow problem. Comparison with previously published work was made and excellent agreement was obtained. The effects of governing parameters are presented using figures and tables. The main findings are:

1. An increasing in either the values of thermal Grashof number \(Gr\) or dimensionless stream-wise coordinate \(\xi\) agrees to reduce fluid density results decrease the velocity boundary layer thickness near the sphere surface.

2. The increment performance of microrotation profile for raising values of the microrotation parameter \(n\) is due to reduction of fluid density in the boundary while an inverse result is observed when the material parameter \(\beta\) is improved.

3. The existence of the microrotation slip parameter \(s_g\) allows diminishing viscosity of the fluid causes enhancing the wall couple stress near the sphere surface.

4. Skin friction coefficient \(f''(0)\) exhibits decreasing behavior for increasing either thermal Grashof number \(Gr\) or material parameter \(\beta\) while an inverse effect is pragmatic when the values of either solutal non linear

**Table 3.** The computed values of skin friction coefficient \(f''(0)\), wall couple stress \(g'(0)\), Nusselt number \(-\theta'(0)\) when \(S_g = S_v = S_m = \lambda = 0.1, Gr = G_m = 50\), for various values of \(x, S_t\) and \(s\).

| \(S_t\) | \(S\) | \(\xi\) | \(f''(0)\) | \(g'(0)\) | \(-\theta'(0)\) |
|---|---|---|---|---|---|
| 0.0 | 0.1 | \(10^{-12}\) | 0.0380 | 0.0154 | 0.6836 |
| 0.1 | 0.1 | \(10^{-12}\) | 0.0368 | 0.0148 | 0.6558 |
| 0.3 | 0.1 | \(10^{-12}\) | 0.353 | 0.0141 | 0.6038 |
| 0.6 | 0.1 | \(10^{-12}\) | 0.0338 | 0.0133 | 0.5364 |
| 0.8 | 0.1 | \(10^{-12}\) | 0.0330 | 0.0129 | 0.4973 |
| 0.1 | 0.0 | \(10^{-12}\) | 0.0382 | 0.0155 | 0.6564 |
| 0.1 | 0.3 | \(10^{-12}\) | 0.0434 | 0.0188 | 0.6840 |
| 0.1 | 0.6 | \(10^{-12}\) | 0.0493 | 0.0202 | 0.7201 |
| 0.1 | 0.8 | \(10^{-12}\) | 0.0534 | 0.0244 | 0.7404 |
| 0.1 | 1.0 | \(10^{-12}\) | 0.0572 | 0.0267 | 0.7604 |
| 0.1 | 0.1 | 10 | 0.0137 | 0.0042 | 0.0252 |
| 0.1 | 0.1 | 15 | 0.0120 | 0.0035 | 0.0203 |
| 0.1 | 0.1 | 25 | 0.0106 | 0.0028 | 0.0168 |
| 0.1 | 0.1 | 35 | 0.0084 | 0.0025 | 0.0115 |
convection parameter $s$ or solutal Grashof number $G_m$ is enhanced. These effects are happened due to an increment of thermal or volumetric expansion in the boundary layer.

5. Raise in either the values of thermal Grashof number $G_r$, Brownian motion parameter $N_b$, or Schmidt number $S_c$ allows to bringing down the kinematic viscosity of the fluid causes improve the temperature profile distribution within the boundary layer. On the other hand, it can be diminished by the growth in either the value of solutal Grashof number $G_m$ or solutal non linear convection parameter $s$ which agrees to enlarge fluid density.

6. The local heat transfer rate is enlarged with boost in the thermal non linear convection parameter $\lambda$ while an inverse outcome is observed when the thermal jump $S_t$, Brownian motion parameters and prandtl number $Pr$ are raised.

7. An increase in the values of thermophoresis $N_t$ tolerates allows to drop viscosity of the fluid causes raise both the concentration boundary layer thickness and the local mass transfer rate.

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