1 INTRODUCTION

Composite construction systems consider the rational structural composition of the right materials at the right places in order to optimally exploit their properties. Composite structures have the widest application in large-span engineering constructions [12], but they can be applied successfully in residential and commercial buildings. By adequate coupling of the constructive elements of the same or different physical-mechanical characteristics into an integral cross-section, the basic goal of the procedure is achieved, i.e. the capacity of the system is increased in relation to the individual elements. Depending on the applied materials, composite structures that are often in use in the construction industry generally are timber-timber, concrete-concrete, steel-concrete and timber-concrete.

Since the composite structures, made by different materials and methods of joining, have reached very high level of application in the construction industry in last several decades, there is a demand for their more precise analysis and design. It is known that the type of used fasteners mostly influence the overall behaviour of the coupled structures. Therefore, it is of crucial importance how to introduce the problem of the connection behaviour between coupled materials into the analysis and design.
Spajanje konstitutivnih elemenata može se postići na različite načine, pri čemu jedan od najčešćih postupaka jeste upotreba diskretno postavljenih sredstava za sprezanje (npr. mehanička spojna sredstva, sidra). Spojna sredstva treba da obezbjeđe vezu dva različita materijala, prenoseći smičuće sile između dva elementa, s ciljem obezbeđenja kompozitnog sadevstva konstrukcije. S obzirom na to što je primena štapastih spojnih sredstava najčešća u spregnutim konstrukcijama drvo–beton (SDB), a pošto ponašanje celokupne konstrukcije zavisi od njihovog ponašanja, interes istraživača i projektanata, kao i brojnih studija i istraživačkih radova, odnosi se na ove tipove sredstava za sprezanje. Upotreba mehaničkih spojnih sredstava za sprezanje dva različita materijala, kao što su drvo i beton, ukazuje na to da je ponašanje SDB konstrukcija veoma složeno, budući da spojna sredstva dozvoljavaju izvesno klizanje u spoju koje dovodi do dimenzionalne interakcije (elastično sprezanje). Prema tome, analiza i proračun SDB konstrukcija zahteva da se imaju u vidu klizanja u spoju između elemenata.

Razmatrajući jednodimenzionalni problem, prve teorije elastičnog sprezanja kod greda izloženih statičkom dejstvu razvili su Newmark (1943,1951), Granholm (1949), Pleshov (1952) i Goodman (1967). Teoriju elastičnog sprezanja primenili su Girhammar i Göp (1991) u analizi stubova s klizanjem u spoju, izloženih odgovarajućim slučajevima klizanja, koja je kasnije proširena i generalizovana u njihovom daljem radu. Na osnovu prethodnih istraživanja i analiza, oni su prikazali tačni analitički postupak statičke analize elastično spregnutih nosača s klizanjem u spoju [7], a u narednim radovima [8], [9], [10] predložili su tačne analitičke i pojednostavnjene metode za analizu elastično spregnutih sistema s primenom na grede i stubove. U Srbiji, u oblasti sprezanja drvo–beton, teorijske osnove za analizu dimenzionalno spregnutih sistema uz eksperimentalne rezultate dao je B. Stevanović (1994) [17], kao i Lj. Kozarić [11] i R. Cvetcikov [3].

Teorija parcijalnog (elastičnog) sprezanja zasniva se na odgovarajućim pretpostavkama teorije elastičnosti i uzima u obzir klizanje spoja/veze pri njihovom proračunu. Analitički proračun elastičnog sprezanja podrazumeva rešavanje diferencijalnih jednačina, gde se rešenja u zatvorenom obliku mogu formulisati samo za pojedina (jednostavnije) slučajeve konturnih uslova i opterećenja.

U EN1995 [5] usvojen je pojednostavljeni manuelni postupak proračuna ("γ-metod"), koji se u praksi široko primenjuje. Ovaj metod prvobitno je primenio Mohler (1956), raspazmatrući problem klizanja u spoju između spregnutih elemenata (drvo–drvo), s mehaničkim spojnim sredstvima, ali uz odgovarajuće modificacije, ovaj postupak se može primeniti i na druge tipove spregnutih konstrukcija, kao što su konstrukcije tipa drvo–beton. "Gama" metod razvijen je za statički sistem proste grede izložene sinusoidnom opterećenju $q(x) = q_0 \cdot \sin(\pi \cdot x/L)$. U ovom slučaju postoji jednostavno rešenje u zatvorenom obliku, koje se može primeniti i na druge vrste opterećenja, a zbog malog odstupanja od tačnog analitičkog rešenja diferencijalne jednačine. Ova metoda zasniva se na efektnoj krutosti spregnutog sistema i teoriji elastičnog sprezanja, imajući u vidu konzervativni efekat raspodele sila unutar nosača, i skoro u potpunosti pokriva sve parametre koji utiču na Coupling of the constitutive elements can be achieved in different ways, where one of the most common procedure is the use of number of individual shear connectors (mechanical fasteners, anchors,...). Shear connectors should ensure bond of two different materials, transferring the shear forces between two elements, enabling the composite action of the structure. The interest of researchers and constructors as well as numerous studies and research works refer to these types of fasteners since the application of dowel type connectors is the most common in timber-concrete composite structures (TCC), and additionally the behaviour of the overall construction depends on their behaviour. The use of mechanical fasteners for coupling two different materials such as timber and concrete shows that the behaviour of the TCC system is very complex, since the fasteners allow certain interlayer slip that leads to partially interaction (elastic composite action). Therefore, the analysis and design of TCC structures requires consideration of the interlayer slip between the sub-elements.

Considering one-dimensional problem, the first theories for partial composite action for beams subjected to static loads were developed by Newmark (1943,1951), Granholm (1949), Pleshov (1952) and Goodman (1967). The application of partial composite action theory was performed by Girhammar and Göp (1991) in analysis of columns with interlayer slip subjected to one particular axial loading case which was extended and generalized in their further work. Based on previous research and analysis, they presented an exact static analysis of partial composite structures with interlayer slip and afterwards in papers [8], [9], [10] they proposed an exact and simplified methods for analysis of the partial composite structures applied to the beams and columns. In Serbia, in the field of timber-concrete composites, the theoretical basis for analysis of partially composite system was given by B.Stevanović (1994) [17] and later on by Lj.Kozarić [11] and R.Cveticikovic [3], which was followed by experimental data.

The theory of partial (elastic) composite action is based on the corresponding assumptions of the theory of elasticity and takes into account the interlayer slip in the connection at their calculation. The exact calculation of the partial composite action implies solving differential equations where closed form solutions can be formulated only for some particular (simple) cases of boundary and loads conditions.

In EN1995 [5] the simplified manual design procedure ("γ-method") widespread in practice is adopted. This method was originally applied by Mohler (1956), considering the problem of interlayer slip between composite members (timber-timber) coupled with mechanical fasteners, but, with appropriate modifications, this procedure can be applied to the other types of composite constructions such as timber-concrete system. "Gamma" method was developed in the case of simply supported beam subjected to sinusoidal load $q(x) = q_0 \cdot \sin(\pi \cdot x/L)$. In this case, there is a simple closed-form solution, that could be applied to the other types of loads as well, due to a slight deviation from the exact analytical solution of the differential equation. This method is based on the effective stiffness of the composite system and on the theory of elastic
Takोđе, за прораčun spregnutih sistema, moguće je primeniti aproksimativne metode zasnovane na diferencijalnoj [14] ili varijacionoj formulaciji [13].

Diferencijalna formulacija zasnova se na izvođenju diferencijalnoj jednačine, koje opisuju probleme u određenom domenu, gde rešenje zavisi od graničnih uslova. U rešavanju problema, potrebno je najprije nepoznatu funkciju koja će zadovoljiti diferencijalnu jednačinu, kao i granične uslove. Rešavanjem izvedenih diferencijalnih jednačina, dobija se analitičko rešenje problema, gde se rešenja u zatvorenom obliku mogu dobiti samo za ograničen broj jednostavnih proračunskih modela. Ako je proračunski model kompleksan, tada se najčešće primenjuju aproksimativne metode, pogodne za dobijanje prihvatljivog rešenja. Metode reziduuma u takvim slučajevima jesu pogodan način za formulisanje numeričkog rešenja.

U varijacionoj formulaciji problema, potrebno je najprije nepoznatu funkciju ili više funkcija koje zadovoljavaju uslove stacionarnosti funkcionala, gde u tom slučaju nepoznata funkcija mora da zadovoljava odgovarajuće dodatne uslove koji nisu implicitno sadržani u funkcionalu. Da bi se primenila varijaciona formulacija, neophodno je da za razmatrani problem postoji funkcional.

Na osnovu diferencijalne i varijacione formulacije problema, razvijene su brojne metode i postupci za određivanje približnih rešenja, pri čemu je od metoda reziduuma najzastupljenija Galerkinova metoda, dok je od varijacionih to Ritz-ova metoda.

Metod konačnih elemenata (MKE) jeste jedan od najčešće korišćenih numeričkih metoda u strukturnoj analizi, pri čemu se formulacija konačnog elementa zasniva na rešenju diferencijalnih jednačina metodama reziduuma ili korišćenjem varijacione formulacije. MKE zasnovana na Galerkinovoj metodi (ili drugim metodama reziduuma) može se primeniti na mnogo širim skup diferencijalnih jednačina, jer nije potrebno imati odgovarajuće varijacionu formulu, kao što je slučaj kada se koristi MKE bazirana na Rayleigh-Ritz-ovoj metodi [1]. Na osnovu prethodnog izlaganja, može se zaključiti da je primena pojednostavljenih i/ili aproksimativnih numeričkih metoda za analizu i proračun SDB konstrukcija dobrodošla i preporučena. Upravo iz tog razloga, približne metode zasnovane na diferencijalnoj ili varijacionoj formulaciji [16] imaju široki primenom, jer mogu biti implementirane u programe za strukturnu analizu, kako bi se obezbedilo poseban alat inženjerima za proračun elastično spregnutih konstrukcija.

U radu je prikazana Galerkinova metoda u analizi SDB konstrukcija [14]. Analiziran je izbor probnih funkcija koje opisuju problem elastičnog sprezanja, kao i njihov uticaj na konačne rezultate. Za poredenje dobijenih rezultata, sprovedene su i analize prema analitičkim rešenjima [17] i „gama“ postupku [5]. Na osnovu predloženih numeričkih modela, model koji najbolje opisuje problem elastičnog sprezanja izabran je za dodatnu komparativnu analizu sa eksperimentalnim podacima [18]. Pored toga, predstavljena je i upotreba Ritz-ove metode u analizi SDB konstrukcija. Dobijeni rezultati prema Ritz-ovoj metodi, s različitim probnim funkcijama, analizirani su i upoređeni sa analitičkim rešenjem i „gama“ postupkom. Sve analize su sprovedene upotrebom programa MATLAB [15].

coupling, taking into account the conservative effect of the distribution of forces within the girders, and so far most fully covers all the parameters that affect the behaviour of RCC.

Also, for the calculation of composite systems, it is possible to apply approximate methods based on the differential [14] or the variation formulation [13].

The differential formulation is based on the derivation of differential equations that describe the problem in a particular domain, where the solution depends on the boundary conditions. In solving the problem, it is necessary to find unknown function that satisfies differential equation as well as the boundary conditions. By solving the derived differential equations, an analytical solution of the problem arises, where the closed-form solution can be obtained only for a limited number of simpler design models. If the design model is complex, then the approximate methods are most commonly used and suitable for obtaining an acceptable solution. Residue methods are in such cases a convenient way to formulate a numerical solution.

In the variational formulation of the problem, it is necessary to find unknown function or several functions that satisfy the requirement of functional stationarity, where the unknown function must also satisfy the corresponding additional conditions that are not implicitly contained in the functional. In order to apply the variational formulation, it is necessary that functional exists for considered problem.

Numerous methods and procedures for determination of approximate solutions have been developed based on the differential and variational formulation of the problem. The Galerkin method is the most frequently used one from the residue methods, while the Ritz method is most often used for variational formulation.

The finite element method (FEM) is one of the most used numerical methods in structural analysis where the final element formulations is based on the solution of differential equations by residual methods or using the variation formulation.FEM based on the Galerkin method (or other weighted residual methods) can be applied to a much broader set of differential equations because it is not necessary to have a proper variational form as it is the case when using Rayleigh-Ritz based FEM [1]. Based on the previous exposition, it can be concluded that the application of simplified and/or approximate numerical methods for the analysis and design of RCC structures is welcome and recommended. Therefore, the approximate methods based on differential or variational formulation [16] are widely used, because they can be implemented in structural analysis software in order to provide a specific tool for engineers for designing partial composite structures.

This paper presents the Galerkin method in the analysis of the RCC system [14]. The selection of trial functions that describe the problem of elastic composite action as well as their influence on the final results was analyzed. For comparison of the obtained results, analysis were performed according to analytical solution [17] and the „gamma“ method [5]. On the basis of the proposed numerical models, a model that best describes the problem of elastic coupling was chosen for further comparative analysis with the experimental data [18]. In addition, the use of the Ritz method was also presented in the analysis of the RCC system. The obtained results...
2 FORMULACIJA JEDNAČINA SDB SISTEMA

Za proračun spregnutih nosača od drveta i betona, gde se koriste mehanička spojna sredstva, primenjuje se teorija elastičnog sprezanja [17], [11].

Osnovne pretpostavke teorije elastičnosti koje se uvode jesu sledeće:

- drvo i beton su izotropni, elastični materijali – važi Hukov zakon;
- važi Bernoullijeva hipoteza, odnosno ravni preseci i posle deformacije ostaju ravni i upravni na deformisanu osu preseka;
- spojna sredstva postavljena su na određenom razmaku i mogu se smatrati ekvivalentnom kontinualnom vezom s konstantnom elastičnosti spoja duž celog nosača;
- poprečni preseci betona i drveta su konstantni duž raspona;
- drvo i beton imaju jednaku ugibnu dužnosću;
- aksijalna sila deluje u težištu betonskog preseka.

Prij savijanju SDB nosača, nastaje pomeranje (klizanje) u spojnoj ravni dva materijala. Klizanje elemenata sprečeno je spojnim sredstvima, što uzrokuje pojavu silu klizanja (silmicu sila u kontaktnoj ravni) $T_s$, koja izaziva silu pritiska $N_1$ i momenat savijanja $M_1$ – u gornjem, a silu zatezanja $N_2$ i momenat savijanja $M_2$ – u donjem elementu nosača, slika 1 (gde su $A$ i $I$ obeležene geometrijske karakteristike poprečnih preseka gornjeg i donjeg elementa, a $E$ moduli elastičnosti primenjenih materijala). Intenziteti sila zavise od krutosti i deformabilnosti spojnog sredstva, odnosno njegovog modula pomerljivosti $K$ [2].

2 GOVERNING EQUATION OF TCC SYSTEM

The theory of elastic coupling [17], [11] is used for the calculation of TCC structures, where mechanical fasteners are used.

The basic assumptions of the theory of elasticity that are introduced are as follows:
- timber and concrete are isotropic, elastic materials and Hook’s law applies,
- Bernoulli’s hypothesis is valid, i.e. plane sections initially perpendicular to the midsurface will remain plane and perpendicular on deformed axis,
- coupling means are set at certain distances and can be considered as equivalent continuous connection with the constant elasticity along the beam,
- cross sections of concrete and timber are constant along the span,
- concrete and timber have equal deflections at each point of the connection,
- axial force acts at the centre of gravity (centroid) of the concrete section.

In TCC structure, one element slips ($v$) over the other along TC interface in the case of bending. Sliding of elements is prevented by the coupling means with appearance of interlayer slip (shear in contact interface) force $T_s$, with compression force $N_1$ and the bending moment $M_1$ in the upper and the tensile force $N_2$ and the bending moment $M_2$ in the lower element of the structure, Figure 1 (where notation $A$ and $I$ represent geometrical properties of cross-sections of upper and lower element, while $E$ represent the modulus of elasticity of applied materials). The intensities of forces depend on the stiffness and deformability of the coupling means and its slip modulus $K$ [2].
Kada se razmatra spregnuta greda drvo–beton, statičkog sistema proste grede, opterećena ravnomerno raspodeljenim opterećenjem \( q(x) \), bez spoja aksijalne sile, problem elastičnog sprezanja može se predstaviti diferencijalnom jednačinom drugog reda u aksijalne sile u betonu:

\[
N'_1''(x) - \alpha^2 N_1(x) = \beta M(x)
\]  

(1)

where are:

\[
\alpha^2 = k \cdot \left( \frac{1}{A_1 E_1} + \frac{1}{A_2 E_2} + \frac{r^2}{(EI)_0} \right)
\]

(2)

\[
\beta = \frac{k \cdot r}{(EI)_0}
\]

(3)

\( M(x) \) - moment kruto spregnutog preseka \((k \to \infty)\);  
\( k \) - krutost spoja ("raspodeljen" modul pomerljivosti) \([N/m^2]\), \( k = K/s \);  
\( K \) - modul pomerljivosti spojnog sredstva \([N/m]\), određen ispitivanjima;  
\( s \) - rastojanje spojnih sredstava za sprezanje;  
\( r \) - rastojanje između težišta betonskog i drvenog dela preseka.

Takođe, problem SDB nosača može se izraziti putem diferencijalne jednačine četvrtog reda u funkciji vertikalnog pomeranja:

\[
w''''(x) - \alpha^2 \cdot w''(x) = \frac{\alpha^2 \cdot M(x)}{(EI)_0} - \frac{M''(x)}{(EI)_\infty}
\]

(4)

where is:

\[
(EI)_0 = \frac{\alpha^2 \cdot (EI)_0}{\alpha^2 - \beta \cdot r} = \frac{E_1 I_1 + E_2 I_2 + \frac{r^2 \cdot E_1 A_1 \cdot E_2 A_2}{E_1 A_1 + E_2 A_2}}
\]

(5)

\((EI)_0\) i \((EI)_\infty\) predstavljaju savojnu krutost za nespregnutu \((k \to 0)\) i kruto spregnutu \((k \to \infty)\) gredu, respektivno.

Rešavanje diferencijalnih jednačina (1 ili 4) predstavlja složen zadatak, i to naročito za različite slučajeve opterećenja i/ili granične uslove. U literaturi, za različite slučajeve opterećenja i uslove oslanjanja, mogu se pronaći analitička rešenja. U radu [17], analitička rešenja za aksijalnu silu \( N \), silu klijanja \( T_s \) i vertikalno pomeranje \( w \), za statički sistem proste grede i kontinualno opterećenje, data su jednačinama (6–8).

Aksijalna sila

\[
N(x) = \frac{\beta}{\alpha^2} M(x) \left[ 1 - 2 \frac{\cosh \alpha \frac{l}{2} - \cosh \alpha \frac{l - x}{2}}{\sinh \alpha \frac{l}{2}} \right]
\]

(6)

Silu klijanja

\[
T_s(x) = \frac{\beta}{\alpha^2} T(x) \left[ 1 - \frac{\sinh \alpha \frac{l}{2} - \sinh \alpha \frac{l - x}{2}}{\sinh \alpha \frac{l}{2}} \right]
\]

(7)

The problem of partial composite action could be represented with differential equation of the second order in the function of the axial force in concrete while observing the composite timber-concrete simply supported beam system with uniformly distributed load \( q(x) \) without an external axial force:

\[
N'_1''(x) - \alpha^2 N_1(x) = \beta M(x)
\]  

(1)

where are:

\[
\alpha^2 = k \cdot \left( \frac{1}{A_1 E_1} + \frac{1}{A_2 E_2} + \frac{r^2}{(EI)_0} \right)
\]

(2)

\[
\beta = \frac{k \cdot r}{(EI)_0}
\]

(3)

\( M(x) \) - the moment of the fully composite section \((k \to \infty)\);  
\( k \) - the slip modulus per-unit length ("smeared" slip modulus) \([N/m^2]\), \( k = K/s \);  
\( K \) - the slip modulus \([N/m]\), determined by testing,  
\( s \) - the spacing between connections,  
\( r \) - the distance between centroid of flange and web elements.

In addition, the problem of the TCC beam could be expressed through the differential equation of the fourth order in function of vertical displacement:

\[
w''''(x) - \alpha^2 \cdot w''(x) = \frac{\alpha^2 \cdot M(x)}{(EI)_0} - \frac{M''(x)}{(EI)_\infty}
\]

(4)

where is:

\[
(EI)_0 = \frac{\alpha^2 \cdot (EI)_0}{\alpha^2 - \beta \cdot r} = \frac{E_1 I_1 + E_2 I_2 + \frac{r^2 \cdot E_1 A_1 \cdot E_2 A_2}{E_1 A_1 + E_2 A_2}}
\]

(5)

\((EI)_0\) and \((EI)_\infty\) are the bending stiffness of non-composite \((k \to 0)\) and fully composite \((k \to \infty)\) beam, respectively.

Solving the differential equations (1 or 4) is a complex task, especially for different load cases and/or boundary conditions. In the literature, analytical solutions for different load cases and support conditions could be found. According to [17], analytical solutions for axial force \( N \), interlayer slip force \( T_s \) and vertical displacement \( w \), for a simply supported beam system and continuous load, are given by the equations (6-8).
Vertikalno pomeranje

$$w(x) = w_\infty(x) + \frac{q \cdot \beta \cdot r}{\alpha^5 \cdot (EI)_{\infty}} \cdot f(x)$$

$$w_\infty(x) = \frac{q \cdot L^4}{24 \cdot (EI)_{\infty}} \cdot \left( \frac{x}{L} \right) \cdot \left( 1 - 2 \cdot \left( \frac{x}{L} \right)^2 + \left( \frac{x}{L} \right)^3 \right)$$

$$f(x) = \frac{\cosh \left( \alpha \cdot \left( x - \frac{L}{2} \right) \right)}{\cosh \left( \alpha \cdot \frac{L}{2} \right)} + \frac{x \cdot L \cdot \alpha^2}{2} - \frac{x^2 \cdot \alpha^2}{2} - 1$$

(8)

Prikaz kvalitativne promene aksijalne sile N i to jest sile klizanja u spoju T_s SDB nosača, dat je na slici 2.

A demonstration of the qualitative change of the axial force N i.e. slip force T_s in TCC system is shown in Figure 2.

Slika 2. Prikaz kvalitativne promene normalne tj. sile klizanja u SDB sistemu [17]

Figure 2. Demonstration of qualitative change of the axial i.e. slip forces in TCC [17]

3 APROKSIMATIVNE METODE

Problemi teorije elastičnosti opisani su pomoću diferencijalne formulacije (diferencijalne jednačine i odgovarajućih graničnih uslova) ili u varijacionoj formulaciji u obliku funkcionala. Iako rešenja ovih problema u matematičkom smislu egzistiraju kao jednoznačna, nalaženje analitičkih rešenja predstavlja zahtevan i često nerešiv zadatak. Stoga, približne metode često se koriste prilikom određivanja rešenja za ove probleme. Posebno značajne jesu one metode gde se kao polazna osnova koristi pretpostavka o rešenju u obliku aproksimativnih ili probnih funkcija, pri čemu je jedna od najčešće primenjivih metoda reziduuma Galerkinova metoda, dok je od varijacionih metoda to najčešće Ritz-ova metoda.

Nepoznata funkcija $u(x)$ diferencijalne jednačine problema aproksimira se približnim rešenjem $\tilde{u}(x)$, izraz (9), koje se može predstaviti kao suppozicija proizvoda poznatih baznih funkcija $\Phi_m$ i nepoznatih koeficijenata $c_m$.

$$\tilde{u}(x) = \sum_{m=1}^{n} c_m \cdot \Phi_m(x_m)$$

(9)

gde je:

- $\Phi_m$ - skup izabranih linearno nezavisnih funkcija $\Phi_m(x_m)$;
- $c_m$ - nepoznati parametri, konstante ili funkcije, koje treba odrediti.

Najčešći oblici probnih funkcija jesu polinomi ili trigonometrijske funkcije. Funkcije $\Phi_m$ unapred se usvajaju, imajući u vidu granične uslove po...

3 APROXIMATE METHODS

The problems of the theory of elasticity are described by means of the differential formulation (differential equations and corresponding boundary conditions) or in the variational formulation in the form of the functional. Although the solutions to these problems in the mathematical sense exist as unambiguous, finding analytical solutions is a delicate and often unsolvable task. Therefore, the approximate methods are often used to find solutions to these problems. Of particular interest are those methods in which the assumption of a solution in the form of approximate or trial function is used as the baseline, wherein one of the most commonly applied weight residual methods is Galerkin method, and commonly applied variational method is Ritz method.

The unknown function $u(x)$ of problem's differential equation has to be approximated by the approximate solution $\tilde{u}(x)$, expression (9), that could be represented as a superposition of products of known basis functions $\Phi_m$ and unknown coefficients $c_m$.

$$\tilde{u}(x) = \sum_{m=1}^{n} c_m \cdot \Phi_m(x_m)$$

where are:

- $\Phi_m$ - set of chosen linearly independent functions $\Phi_m(x_m)$;
- $c_m$ - unknown parameters, constants or functions to be determined.

The most common trial functions are polynomials or trigonometric functions. Functions $\Phi_m$ are adopted in advance by taking into account of essential boundary
pomeranjima. Kada je reč o drugim uslovima, izbor funkcija \( \Phi_m \) uglavnom je proizvoljan, ali kvalitet rešenja umnogome zavisi baš od izbora funkcija \( \Phi_m \). Poželjno je da funkcije \( \Phi_m \) zadovoljavaju i granične uslove po silama, te da njihov oblik kvalitativno odgovara tačnom analitičkom rešenju. Dakle, kvalitativno poznavanje prirode rešenja veoma je korisno da bi se izbegao pogeršan izbor funkcija koje po svom obliku predstavljaju grubo odstupanje od analitičkog rešenja.

Galerkinova metoda ima širu primenu od Ritz-ove metode, jer se može primeniti pri rešavanju onih problema za koje funkcional ne postoji. U mehanici deformabilnih tela, ove dve metode su ekvivalentne, jer daju rezultate iste tačnosti. Izboram istih probnih funkcija u Ritz-ovoj i Galerkin-ovoj metodi, dobijaju se isti koeficijenti \( c_m \) (ista rešenja).

3.1 Metoda reziduuma

Neka je posmatrani fizički problem, u domenu \( \Omega \), koji može da bude 1D do 3D, definisan diferencijalnom jednačinom:

\[
L(\Phi) - f_\Omega = 0
\]

gde je:

\[L - \text{odgovarajući linearni diferencijalni operator};\]
\[\Phi(x) - \text{uznan ili odgovarajući sa silam};\]
\[f_\Omega - \text{vrednost funkcije na granici domena} \Omega;\]
\[\Phi(x) - \text{vektor koji zadovoljava integralnu jednačinu}.\]

Nepoznata funkcija problema \( \Phi(x) \) aproksimiraju probleme, koja zavisi od koordinate \( x \) unutar prostora \( \Omega \), pri čemu funkcija \( \Phi(x) \) zadovoljava date granične uslove na granicama domena \( \Omega \).

Ideja metode je da se vektor ostatka svede na nulti vektor "u srednjem smislu". Stoga, uvede se linearno nezavisne težinske funkcije \( W(\Phi) \), uz uslov da integral skalarnog proizvoda vektora težinskih funkcija i vektor s ostatka unutar domena \( \Omega \) bude jednak nuli:

\[
L(\Phi) - f_\Omega = R(\Phi) \equiv 0
\]

Kako je jednačina (10) sistem jednočina, odnosno matrična jednačina, otkriva se \( \Phi(x) \) vektor. Naravno, kada bi \( \Phi(x) \) bilo tačno analitičko rešenje, onda bi vektor ostatka \( \Phi(x) \) bio jednak nultom vektoru. Ideja metode je da se vektor ostatka svede na nulti vektor "u prosečnom smislu". Stoga, uvede se linearno nezavisne težinske funkcije \( W(\Phi) \), uz uslov da integral skalarnog proizvoda

\[
\int_{\Omega} W^T(\Phi) \cdot R(\Phi) \cdot d\Omega = \int_{\Omega} W^T(\Phi) \cdot (L(\Phi) - f_\Omega) \cdot d\Omega = 0
\]

Skalarni proizvod dva vektora jednak je nuli ukoliko su ti vektori međusobno ortogonalni. Prema tome, integralna jednačina (12) predstavlja uslov ortogonalnosti vektor s ostatka na izabranu vektor težinskih funkcija. Međutim, intenzivnost funkcija \( \Phi(x) \) za koje \( \Phi(x) \) je integralna jednačina (12) biti zadovo-

3.1 Weighted residual method

A physical problem is observed in the domain \( \Omega \), which can be 1D to 3D, defined with a differential equation:

\[
L(\Phi) - f_\Omega = 0
\]

where are:

\[L - \text{corresponding linear differential operator};\]
\[\Phi(x) - \text{unknown function of the problem, that depends on the coordinate x within the domain} \Omega;\]
\[f_\Omega - \text{given force term in domain} \Omega.\]

The unknown function of the problem \( \Phi(x) \) is approximated with the approximate function \( \Phi(x) \), equation (9), which satisfies the boundary conditions upon the displacements (essential conditions), but does not have to satisfy the conditions by forces (natural conditions). As \( \Phi(x) \) is approximate solution of the equation (10), the residue or residuum is obtained:

\[
L(\Phi) - f_\Omega = R(\Phi) \equiv 0
\]

Since equation (10) is a system of equations i.e. a matrix equation, than the residue \( R(\Phi) \) is a vector. Of course, if \( \Phi(x) \) would be the exact solution, then the residue vector \( R(\Phi) \) would be equal to the zero vector. The idea behind the method is to reduce the residue vector to the zero vector "in the average sense". Because of that, linearly independent weight functions \( W(\Phi) \) are introduced with the condition that the integral of the scalar product of the weight function vector and the residual vector within the domain \( \Omega \) is equal to zero:

\[
\int_{\Omega} W^T(\Phi) \cdot R(\Phi) \cdot d\Omega = \int_{\Omega} W^T(\Phi) \cdot (L(\Phi) - f_\Omega) \cdot d\Omega = 0
\]

The scalar product of the two vectors is equal to zero if these vectors are mutually orthogonal. Accordingly, the integral equation (12) is a condition of the orthogonality of the residual vector to the selected vector of weight functions. Residue methods consist of finding functions \( \Phi(x) \) for which the integral equation (12) will be satisfied. If the
ljena. Ako je jednačina (12) zadovoljena za bilo koji vektor težinskih funkcija, onda će se vektor ostatka približavati nulom vektoru.

Na taj način, približno rešenje \( \hat{u}(x) \) apsorpira tačno rešenje \( u(x) \). Sva rešenja \( \hat{u}(x) \) koja zadovoljavaju (10) moraju da zadovoljavaju i (12), bez obzira na izbor težinskih funkcija. Dimenzija vektora težinskih funkcija odgovara broju nepoznatih koeficijenata \( c_m \) razmatranog problema.

Kao jedna od osnovnih varijanti metode reziduuma, koja usvaja težinske funkcije kao bazne funkcije \( \Phi_m \) kojima je apsorpirano traženo rešenje, jeste Galerkinova metoda [16].

Na osnovu diferencijalnih jednačina elastičnog sprezanja (1 ili 4) i uslova (12), moguće je definisati sledeće relacije za određivanje problema SDB nosača u funkciji aksijalne sile u betonu ili u funkciji pomeranja za slučaj SDB grede opterećene kontinualnim opterećenjem \( q \), prema sledećim izrazima:

\[
\int_{0}^{L} \Phi_m(x) \cdot \left[ N''(x) - \alpha^2 N(x) - \beta M(x) \right] \cdot dx = 0 \quad , \quad m = 1, 2, ..., n
\]

Integralna formulacija koja u sebi implicitno sadrži diferencijalnu jednačinu problema, naziva se slaba formulacija (13 ili 14) koja izražava uslove i rešenje koje moraju biti zadovoljene u prosečnom ili integralnom smislu.

Kako je diferencijalna jednačina problema parnog reda \((2r = 4)\), parcijalnom integracijom izraza (12) red izvoda \( r \) u probnim funkcijama moguće je smanjiti sa \( r = 4 \) na \( r = 2 \). Parcijalnom integracijom izraza (13 ili 14) postiže se da odabrane probne funkcije moraju zadovoljavati samo granične uslove po pomeranjima, koji moraju biti zadovoljeni izborom samih probnih funkcija, dok su uslovi po silama već uključeni u formulaciju problema parcijalnom integracijom.

Rešavanjem integrala, dobija se sistem od \( n \) jednačina po nepoznatim koeficijentima \( c_m \) i približno rešenje za traženu funkciju \( u(x) \) može se dobiti određivanjem koeficijenata \( c_m \).

### 3.2 Varijaciona metoda

Kako se Ritz-ova metoda zasniva na variacionoj formulaciji, potrebno je zadovoljiti uslov stacionarnosti funkcionala koji opisuje razmatrani problem. Za rešavanje problema u mehanici deformabilnih tela, funkcional je jednak ukupnoj potencijalnoj energiji, a stacionarna vrednost odgovara njenoj minimalnoj vrednosti. U Teoriji konstrukcija, ovaj metod je najpoznatiji varijacioni postupak. Razlog jeste to što postoji funkcional u obliku potencijalne energije [13].

Kada se posmatra jednodimenzionalni linijski problem s domenom definisanosti \( x \in [x_1, x_2] \), funkcional (potencijalna energija) izražava se putem integrala \( I(u) \) u celom domenu:

\[
\text{equation (12) is satisfied for any weight functions vector, then the residue vector will approach the zero vector.}
\]

In this way, the approximate solution \( \hat{u}(x) \) approximates the exact solution \( u(x) \). All solutions \( \hat{u}(x) \) that satisfy (10) must satisfy (12) regardless of weight functions' choice. The dimension of the weight functions vector corresponds to the number of unknown coefficients \( c_m \) of the considered problem.

As one of the basic variants of the residual method, which adopts weight functions as basis functions \( \Phi_m \) for which the required solution is approximated, is Galerkin's method [16].

Based on the differential equations of the elastic coupling (1 or 4) and the condition (12), it is possible to define the following relations for determining the problem of the TCC girder through the axial force in the concrete or trough displacements for the case of a TCC beam loaded with continuous load \( q \), according to following expressions:

\[
\int_{0}^{L} \Phi_m(x) \cdot \left[ N''''(x) - \alpha^2 N''(x) - \beta^2 M(x) \right] \cdot dx = 0 \quad , \quad m = 1, 2, ..., n
\]

An integral formulation that implicitly contains a differential equation of the problem is called a weak formulation (13 or 14) that expresses the conditions and relations that must be satisfied in the average, or in an integral sense.

Since the differential equation of the problem is of even order \((2r = 4)\), it is possible to reduce the required order of derivation in the trial functions by partial integration of the expression (12) from \( r = 4 \) to \( r = 2 \). By partial integration of expressions (13 or 14) is achieving that selected trial functions must satisfy only the essential conditions, that have to be satisfied by the selection of trial functions themselves, while the force conditions are already included into the formulation of the partial integration problem.

By solving the integrals, a system of \( n \) equations by unknown coefficients \( c_m \) is obtained and an approximate solution for the required function \( u(x) \) could be derived by determination of coefficients \( c_m \).
gde je: \( \Pi(\ldots) \) – funkcional funkcija \( u(x) \), \( du(x)/dx \), \( d^2u(x)/dx^2 \), ...

Uслов stacionarnosti funkcionala prikazuje se uslovom da je prva varijacija funkcionala jednak nuli:

\[ \delta \Pi = 0 \]  \tag{16} 

ili zapisano u razvijenom obliku:

\[ \delta \Pi = \frac{\partial \Pi}{\partial c_1} \delta c_1 + \frac{\partial \Pi}{\partial c_2} \delta c_2 + \ldots + \frac{\partial \Pi}{\partial c_n} \delta c_n = 0 \]  \tag{17} 

Kako su koeficijenti \( c_1, c_2, \ldots, c_n \) međusobno nezavisni parametri, onda se \( \delta \Pi = 0 \) svodi na sledeći uslov:

\[ \frac{\partial \Pi}{\partial c_m} = 0 \quad (m = 1, 2, \ldots, n) \]  \tag{18} 

što predstavlja sistem algebarskih jednačina po nepoznatim koeficijentima \( c_m \).

Na osnovu diferencijsalne formulacije problema elastičnog sprezanja, moguće je definisati funkcional na osnovu opših varijacionih principa [8]. Pošto se najčešće koriste mehanička spoja sredstva za SDB nosača, usled spoja opterećenja javljaju se izvesna pomeranja (i.e. klizanje u spoju) između drveta i betona. Pored rada unutrašnjih sila \( (M_1, N_1, M_2, N_2) \), potrebno je uzeti u obzir i deformacioni rad usled klizanja u spoju. Funkcional, ili ukupna potencijalna energija spregnutog sistema, u slučaju proste grede na koju deluje raspodijeljeno opterećenje \( q(x) \), može se prikazati u sledećem obliku [19]:

\[ I = W_t - W_e \]  \tag{19} 

\[ W_t = \frac{1}{2} \int_0^l \frac{M_1^2(x)}{E_1 I_1} \, dx + \frac{1}{2} \int_0^l \frac{M_2^2(x)}{E_2 I_2} \, dx + \frac{1}{2} \int_0^l \frac{N_1^2(x)}{EA^*} \, dx + \frac{1}{2} \int_0^l \frac{(N'(x))^2}{k} \, dx \]  \tag{20} 

\[ W_e = \int_0^l w(x) \cdot q(x) \cdot dx \]  \tag{21} 

gde je:

\[ W_t \] – potencijalna energija deformacije;

\[ W_e \] – potencijal sila.

Kako moment savijanja \( M(x) \) možemo izraziti preko pomeranja \( w(x) \), koristeći uslov jednakih rotacija spregnutih elemenata (drveta i betona), uvodeći odnos \( g(x) \), izraz (20) prikazujemo u sledećem obliku:

\[ I(u) = \int_1^x \Pi \left( x, u(x), \frac{du(x)}{dx}, \frac{d^2u(x)}{dx^2}, \ldots \right) \, dx \]  \tag{15} 

where is:

\[ \Pi(\ldots) \] – represents the functional of functions \( u(x) \), \( du(x)/dx \), \( d^2u(x)/dx^2 \), ...

Extremum of a functional is represented by requirements that the first variation of the functional be zero:

\[ \delta \Pi = 0 \]  \tag{16} 

or shown in the developed form:

\[ \delta \Pi = \frac{\partial \Pi}{\partial c_1} \delta c_1 + \frac{\partial \Pi}{\partial c_2} \delta c_2 + \ldots + \frac{\partial \Pi}{\partial c_n} \delta c_n = 0 \]  \tag{17} 

Since \( c_1, c_2, \ldots, c_n \) are mutually independent parameters, then \( \delta \Pi = 0 \) is represented by following condition:

\[ \frac{\partial \Pi}{\partial c_m} = 0 \quad (m = 1, 2, \ldots, n) \]  \tag{18} 

which represents a system of algebraic equations with unknown coefficients \( c_m \).

Based on the differential formulation of the partially composite problem, it is possible to define a functional according to variation principles [8]. As the mechanical fasteners are commonly used for coupling in TCC, a certain displacements (i.e. an interlayer slip) occur on the TC interface due to the external load. Besides the strain energy due to internal forces \( (M_1, N_1, M_2, N_2) \), it is also necessary to take into account the strain energy due to interlayer slip. Functional, or total potential energy of the composite system, in the case of simply supported beam with uniform distributed load \( q(x) \), can be shown in the following form [19]:

\[ I = W_t - W_e \]  \tag{19} 

\[ W_t = \frac{1}{2} \int_0^l \frac{M_1^2(x)}{E_1 I_1} \, dx + \frac{1}{2} \int_0^l \frac{M_2^2(x)}{E_2 I_2} \, dx + \frac{1}{2} \int_0^l \frac{N_1^2(x)}{EA^*} \, dx + \frac{1}{2} \int_0^l \frac{(N'(x))^2}{k} \, dx \]  \tag{20} 

\[ W_e = \int_0^l w(x) \cdot q(x) \cdot dx \]  \tag{21} 

where is:

\[ EA^* = \frac{E_1 A_1 \cdot E_2 A_2}{E_1 A_1 + E_2 A_2} \]  \tag{22} 

\( W_t \) – strain energy due to internal forces,

\( W_e \) – potential energy due to external forces.

As the bending moment \( M(x) \) can be expressed by deflection \( w(x) \), using the condition of equal rotations of the composite members (timber and concrete), introducing the relation \( g(x) \), the expression (20) is represented in the following form:
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\[ W_i = \frac{(EI)_0}{2} \int_0^l [w''(x)]^2 \, dx + \frac{(E_2 I_2)^2}{EA^*} \int_0^l \left[ \frac{w''(x)}{g(x)} \right]^2 \, dx + \frac{1}{2} \cdot k \int_0^l \left( -\frac{E_2 I_2 \cdot w''(x)}{g(x)} \right)^2 \, dx \]  

(23)

gde je:

\[ g(x) = \frac{M_2(x)}{N(x)} = \frac{E_2 I_2}{r} \left( \frac{1}{EA^*} - \frac{N''(x)}{N(x)} \right) \]  

(24)

\[ N(x) = \frac{M_2(x)}{g(x)} = -\frac{E_2 I_2 \cdot w''(x)}{g(x)} \]  

(25)

Uvedeni odnos \( g(x) \) izveden je iz uslova kompatibilnosti pomeranja na spoju dva elementa, koji može da se zapiše u sledećem obliku:

\[ \frac{N(x)}{EA} - \frac{M_2(x) \cdot r}{E_2 I_2} = \frac{N''(x)}{k} \]  

(26)

Poznajući rad unutrašnjih sila \( W_i \), određen je funkcional za elastično spregnut SDB nosač. Kako se u izrazu \( g(x) \) javlja normalna sila \( N(x) \), za rešavanje problema, pored pretpostavljanja probne funkcije za pomeranje \( w(x) \), potrebno je pretpostaviti i probnu funkciju za normalnu silu \( N(x) \). Primenom varijacionih principa na funkcional, Ritz-ovom metodom, možemo rešiti problem elastičnog sprežanja, odnosno odrediti pomeranje nosača i unutrašnje sile u spregnutom nosaču.

4 APROKSIMACIJA REŠENJA – PROBNE FUNKCIJE

Pogodne, a samim tim i najčešće, probne funkcije su polinomi ili trigonometrijske funkcije. Probne funkcije treba da zadovolje sledeće uslove:

- da su neprekidne i do potrebnog reda diferencijabilne;
- pored esencijalnih graničnih uslova, treba da zadovolje prirodne granične uslove;
- treba da oblikom kvalitativno odgovaraju tačnom rešenju;
- da budu potpune, npr. u slučaju polinoma određenog stepena, takođe treba da budu uključeni i svi članovi nižeg stepena.

Na osnovu dobro poznatih rešenja iz literature, jednačine (6) i (8), kao i kvalitativnog poznavanja oblika rešenja (slika 2), u ovom radu za probne funkcije izabrane su tri funkcije (hiperbolična, sinusna i funkcija polinoma).

Usovjeđene probne funkcije opisuju zakon promene aksijalne sile \( N \), sile klizanja u spoju \( T_s (N') \) i pomeranje \( w \) duž spregnutog nosača i kvalitativno odgovaraju analitičkim rešenjima (slika 2). Pomoću izraza (27, 28 i 29), date su odabrane probne funkcije za \( N/ii \) w; dok je na slici 3 prikazan oblik funkcije i njen prvi izvod duž nosača.

Knowing the strain energy due to internal forces \( W_i \), functional for a partial TCC system is determined. As in the expression \( g(x) \) the normal force \( N(x) \) appears, for solving the problem, beside assumed trial function of displacement \( w(x) \), it is also necessary to assume the trial function for \( N(x) \). By applying variation principles to a functional, with Ritz method, the problem of partially composite system can be solved, which means to determine displacement and internal forces in the composite members.

4 APPROXIMATION OF THE SOLUTION – TRIAL FUNCTIONS

Suitable, and therefore, most often, trial functions are polynomials or trigonometric functions. Trial functions should satisfy the following conditions:

- to be continuous and differentiable till the necessary order,
- in addition to essential, they also have to satisfy natural boundary conditions,
- to correspond qualitatively by the form to the analytical solution,
- to be complete, e.g. in the case of polynomials of a certain degree, all members of the lower degrees should also be included.

Based on the well-known solutions from the literature, equations (6) and (8), as well as on the qualitative flow of the solution (Fig. 2), three functions (hyperbolic, sinusoidal and polynomial functions) were selected for trial functions in this paper.

The adopted trial functions describe the law of the change of the axial force \( N \), slip forces \( T_s (N') \) and displacement \( w \) along the composite girder and qualitatively correspond to the solutions (Fig. 2). By means of expressions (27, 28 and 29), selected trials for \( N/ii \) w are given, while on Fig. 3 the shape of the function and its first derivative along the beam are shown.
1. Hiperbolična funkcija / Hyperbolic function:
\[ \bar{u}(x) = c_1 \cdot \Phi_1(x) = c_1 \cdot \left( \frac{\cosh \left( \frac{x - L}{2} \right)}{\cosh \left( \frac{L}{2} \right)} + \frac{x \cdot L}{2} - \frac{x^2}{2} - 1 \right) \] (27)

2. Sinusna funkcija / Sinusoidal function:
\[ \bar{u}(x) = c_1 \cdot \Phi_1(x) = c_1 \cdot \sin \left( \frac{\pi \cdot x}{L} \right) \] (28)

3. Polinom / Polynomial function:
\[ \bar{u}(x) = c_1 \cdot \Phi_1(x) = c_1 \cdot \left( \frac{x}{L} \right) \cdot \left( 1 - 2 \cdot \left( \frac{x}{L} \right)^2 + \left( \frac{x}{L} \right)^3 \right) \] (29)

U Galerkinovoj metodi, jednu od tri predložene funkcije treba usvojiti za probnu funkciju (za \( N \) ili \( w \)) prema odabranoj integralnoj formulaciji (jednačine 13 ili 14). U Ritz-ovoj metodi, potrebno je usvojiti dve probne funkcije (za \( N \) i \( w \)).

5 NUMERIČKE ANALIZE I POTVRDA REZULTATA

5.1 Opis analiziranog modela

Razmatrana je SDB konstrukcija tavanice za numeričku analizu metodama Galerkina i Ritz-a. Raspored elemenata i spojnih sredstava, kao i njihove dimenzije i svojstva primenjenih materijala prema evropskim standardima, prikazani su na slici 4.

In Galerkin method, one of three suggested trial functions has to be adopted (for \( N \) or \( w \)) according to chosen integral formulation (Eq 13 or 14). In Ritz method it is necessary to adopt two trial functions (for \( N \) and \( w \)).

5 NUMERICAL ANALYSIS AND VERIFICATION OF THE RESULTS

5.1 Description of analyzed structural model

The TCC floor structure is considered for the numerical analysis by Galerkin and Ritz method. The disposition of the elements and fasteners, as well as their dimensions and properties of the applied materials according to European standards, are shown in Fig. 4.
The floor structure consists of a glulam beams that are coupled with concrete slab by vertically arranged dowel type fasteners. In this paper, the slip modulus $K$ is determined by the Gelfi model [6]. The floor structure is loaded by the self-weight of the structural elements $g$, by additional permanent load $\sigma_0$, as well as by the imposed load $p$. It is considered that the timber glulam beams will be supported in the stage of pouring and hardening of the concrete slab, and the composite section will receive imposed and total permanent load. It is possible to analyze the part of the composite floor structure separately (glulam beam with the effective width of the concrete slab), because in analyzed TCC floor system all concrete slabs are one-way and glulam beams are simply supported with uniformly distributed load. Numerical analysis according to Galerkin and Ritz method of TCC structure was performed and several subprograms/codes are written in MATLAB 2014 [15]. The simplified “γ-procedure” was also performed in order to obtain the referent values suggested by Eurocode.

5.2 Numerical analysis by Galerkin method

For the purpose of numerical analysis, two groups of models are defined on the basis of selection of trial functions and integral formulation (eqs. 13 or 14) for axial force $N(x)$ or deflection $w(x)$: Models of Group A (N-HIP, N-SIN, N-POL) and Models of Group B (w-HIP, w-SIN, w-POL).

Vertical displacements $(w)$, moments $(M_1, M_2)$, axial forces $(N_1, N_2)$ and stresses $(\sigma_1, \sigma_2)$ for the cross-section of concrete / timber element (top and bottom) in the middle of the beam span were calculated, as well as shear forces ($F_2$) in connectors and slip force $(T_2)$ values at the concrete-timber contact in support zones. Results of performed numerical analysis for Models of group A and B were compared with analytical solution and their percentage deviations are shown at Figures 5, 6. Results of simplified “γ-procedure” were also compared with analytical solution.

From Figs. 5 and 6, it can be noticed that numerical results of group A models have smaller differences in relation to the analytical solution than the models of group B.

Analysis of results obtained by different trial functions for the approximate solution of the normal force $N$ shows that a minimal deviation occurs when a hyperbolic function is adopted, and the maximum one if it is the sinusoidal function. The models N-POL and N-SIN have significant deviations in stresses $(\sigma_1, \sigma_2)$ even up to 21%, and minor deviations in the slip and shear forces $(T_2, F_2)$ up to 6.5%. All values are smaller than those obtained by analytical solutions. Comparing the Group A models with results of the “γ” method, none of the models has greater deviations in absolute sense, but it can be noticed that the N-POL and N-SIN models give smaller values.

Analysis of results obtained by different trial functions for the approximate displacement $w$ solution shows that a minimal deviation occurs if the polynomial function is adopted, and the maximum one if it is a hyperbolic function. The w-HIP and w-SIN models have significant deviations in stresses $(\sigma_1, \sigma_2)$ even up to 41%, and for slip and shear forces $(T_2, F_2)$ even up to 28%. Model w-HIP shows significantly smaller values.
vrednosti u poređenju sa analitičkim rešenjem. Poređenje modele grupe B s rezultatima „γ-metode”, može se uočiti je da najbolje podudaranje sa „γ-metodom” ima model w-POL.

Comparing to analytical. By comparing the models of group B with results of the “γ-method”, it can be seen that the best match with the “γ-method” has the w-POL model.

**Slika 5. Procentualna odstupanja modela grupe A u odnosu na analitičko rešenje**

**Figure 5. Percentage deviations of models of group A in relation to the analytical solution**

**Slika 6. Procentualna odstupanja modela grupe B u odnosu na analitičko rešenje**

**Figure 6. Percentage deviations of models of group B in relation to the analytical solution**
5.3 Numerical analysis by Ritz method

For the purpose of numerical analysis, the group of models is defined on the basis of simultaneous selection of two trial functions for axial force \( N(x) \) and shear force \( w(x) \): Models of Group C (w-POL,N-POL; w-POL,N-SIN; w-SIN,N-SIN). All effects analyzed by Galerkin method are calculated according Ritz as well and their percentage deviations compared to analytical solution are presented on Fig 7. Results of simplified “\( \gamma \)-procedure” were also compared with analytical solution.

Na slici 7, može se primetiti da minimalno odstupanje od analitičkog rešenja pokazuje varijantni model (w-POL,N-SIN), dok se maksimalna odstupanja javljaju kod varijantnog modela (w-SIN,N-SIN). Rezultati određeni varijantnim modelom (w-POL,N-SIN) jesu na strani sigurnosti, jer daju neznatno veće vrednosti (do 3%) za unutrašnje sile i pomeranja, dok odstupanja za normalne napone i sile klizanja / smišćenja sile, dostižu vrednosti do 19% i 15% respektivno, u odnosu na analitičko rešenje. Razlog za takvo povećanje leži u činjenici da su normalni naponi i sile klizanja / smišćenja sile izvedene veličine osnovnih nepoznatih \( w(x) \) i \( N(x) \), pa su kumulativne greške veće. Očigledno je da izbor probnih funkcija za \( w(x) \) i \( N(x) \) ima značajan uticaj na konačni rezultat, kao i na izvedene statičke veličine. Takođe, može se primetiti da model (w-POL,N-SIN) ima najbolje poklapanje s pojednostavljenim “\( \gamma \)-metodom”, predloženim u EN 1995. Iako varijantni model (w-POL,N-POL) daje manja odstupanja od varijantnog modela (w-SIN,N-SIN), poredeci ih sa analitičkim rešenjem, može se uočiti da dobijene vrednosti precentuju ili potcenjuju analitičke, te ovi modeli nisu na strani sigurnosti.

From Fig. 7, it can be noticed that the minimum deviation from the analytical solution shows the variant model (w-POL,N-SIN), while the max deviation occurs in variant model (w-SIN,N-SIN). The results obtained by variant model (w-POL,N-SIN) are on the safe side because they give a slightly higher values (up to 3%) for internal forces and displacements, while deviations for normal stresses and slip/shear forces, arise up to 19% and 15% respectively, comparing to analytical solution. The reason for such increase lays in the fact that normal stresses and slip/shear forces are derived values from baseline unknowns \( w(x) \) and \( N(x) \), so the cumulative errors are higher. It is obvious that the selection of trial functions for \( w(x) \) and \( N(x) \) has the significant impact on final result, as well as on derived statical values. It can be also noted that model (w-POL,N-SIN) has the best match with the approximate “\( \gamma \)-method” proposed in EN 1995. Although variant model (w-POL,N-POL) shows smaller deviations from variant model (w-SIN,N-SIN), comparing these two models with analytical solutions it can be seen that obtained values overestimate or underestimate analytical ones, but both models are not on the safe side.
5.4 Verification of Galerkin method

In order to verify the application of Galerkin's method in TCC system, the comparison of numerical obtained values and experimental data was done. As it was shown that the N-HIP model describes the problem of TCC partial composite action on the best way, this model was applied for comparative analysis with experimental test results of TCC (EP1 and EP2) beams [18], where mechanical fasteners were used. The diagrams in Figure 8 show the stresses in cross-section of constitutive elements of TCC beams in the middle of the span, in relation to force intensity throughout experimental loading phases (F = 6, 12, 18, 24, 30 and 36 kN). The shaded surfaces represent the envelopes of the results obtained from experiments for beams EP1 and EP2, while the full and dashed lines represent the results of the numerical analysis by N-HIP model and by "γ-method" respectively. Considering the stresses (σ₁,b and σ₂,t) on the contact of two materials, deviations from the experimental results can be noticed, significantly for stress σ₂,t at loads F = 18 and 24 kN. A good match with experimental results on the top and bottom sides of the cross-section as well as in bottom of the concrete slab is obvious.

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Slika 8. Uporedna analiza rezultata dobijenih N-HIP modelom i eksperimentalnih podataka

Figure 8. Comparative analysis of numerical model N-HIP and experimental data
6 ZAKLJUČAK

Na osnovu predstavljenih analiza primenom Galerkinove i Ritz-ove metode, može se zaključiti da izbor probnih funkcija u formulaciji problema ima najveći uticaj na konačne vrednosti dobijenih i prikazanih rezultata. U Galerkinovoj metodi, značajan uticaj ima i sam izbor osnovnih nepoznatih (w i N). Takođe, kvalitativno poznavanje prirode rešenja može značajno doprineti smanjenju odstupanja rezultata (greške) od analitičkog rešenja. U prikazanoj analizi primenom Galerkinove metode, predočeni N-HIP model smatra se najpogodnijim za rešavanje problema elastičnog sprezanja. Kada se primenjuje varijaciona formulacija, funkcional može biti definisan putem jedne promenljive ili više njih (silapomernjanje), dok će nepoznate koje su izabrane za osnovne biti određene s većom tačnošću od ostalih izvedenih veličina.

Kako su metode reziduuma ili varijancije metode prilično uobičajeni oblik formulacije u MKE, predstavljene metode Galerkinova i Ritz-a mogu se uspešno primeniti prilikom definisanja spregnutog KE za elastično spregnute konstrukcije drvo–beton, a time se dobija efikasan inženjerski alat u praksi. Uvođenjem Evrokoda za spregnute sisteme drvo–beton [4], očekuje se preciznije definisanje osnovnog ulaznog parametra za različita sredstva spajanja – modula pomerljivosti spojnog sredstva K a time i realniji odgovor spregnutih nosača u numeričkim analizama.

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6 CONCLUSION

Based on the presented analysis using Galerkin's method and Ritz method, it can be concluded that the selection of trial functions in problem formulation has the major influence on the final effect values of obtained and presented results. An important influence is also the choice of baseline unknowns (w and N) for Galerkin's method. Also, the qualitative knowledge of solution nature can significantly contribute to the reduction of errors in obtained results related to analytical solution. In presented analysis by Galerkin's method the proposed N-HIP model qualifies as the most appropriate in order to solve the problem of partial coupling. When using a variational formulation, functional could be defined trough one or more unknowns (forces/displacements), while the unknowns that are chosen as basic will be determined with more accuracy than the derived ones.

As the weighted residual method or the variation formulation in the FEM is quite usual form, presented the Galerkin and Ritz methods could be successfully applied when defining a composite FE for partially timber-concrete composite systems, thereby enabling an efficient engineering tool in practice. With introduction of Eurocode for timber-concrete composite structures [4], it is expected that more precise definition of basic input parameter - slip modulus for different types of fasteners K will contribute to the more realistic response of composite beams in numerical analysis.

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**REZMINE**

**PRIMENA NUMERIČKIH METODA U ANALIZI SPREGNUTIH KONSTRUKCIJA DRVO–BETON**

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Analiza i proračun spregnutih drvo–beton (SDB) konstrukcija, gde je veza konstitutivnih elemenata ostvarena mehaničkim spojnim sredstvima, predstavlja kompleksan zadatak zbog uzimanja u obzir pomerljivosti sredstava za sprezanje tj. klizanja na kontaktu dva materijala. Primena pojednostavljenih postupaka i metoda u analizi SDB nosača predstavlja pogodan i poželjan način proračuna, koji inženjerima u praksi omogućava efikasan alat. Siroko rasprostranjena, pojednostavljen proračun tzv. γ-metod dat je u EN 1995. Metode zasnovane na diferencijalnoj ili varijacionoj formulaciji često su u upotrebi kada su u pitanju programi za strukturalnu analizu konstrukcija. U radu je prikazana Galerkin-ova i Ritz-ova metoda za analizu i proračun SDB nosača za slučaj proste grede izložene raspodjeljenom opterećenju. Analiziran je izbor probnih funkcija koje opisuju problem elastičnog sprezanja, kao i njihov uticaj na konačne rezultate. Za potrebe numeričke analize, na osnovu predloženih numeričkih modela, napisani su kodovi u MATLAB-u. Model primenjen u analizi Galerkinovom metodom, koji najbolje opisuje problem elastičnog sprezanja, izabrano je za komparativnu analizu sa eksperimentalnim podacima.

**Ključne reči:** Ritz-ova metoda, Galerkinova metoda, γ-metod, sprezanje drvo–beton, elastično sprezanje, klizanje u spoju.

**SUMMARY**

**APPLICATION OF NUMERICAL METHODS IN ANALYSIS OF TIMBER CONCRETE COMPOSITE SYSTEM**

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The analysis and design of composite timber-concrete (TCC) structures, where the connection of the constituent elements is achieved by dowel type fasteners, is a complex task due to taking into account the slip of the coupling means, i.e. interlayer slip on the contact surface of two materials. The application of simplified procedures and methods in the analysis of the TCC system is a convenient and desirable way of design that enables efficient tool for engineering practice. Widespread simplified calculation procedure, so called "γ-method", is adopted in EN 1995. Methods based on differential or variational formulation are commonly applied when software for structural analysis are used. Galerkin's and Ritz’s methods for analysis and design of TCC systems in the case of simply supported beam loaded with uniformly distributed load are shown in this paper. The selection of trial functions that describe the problem of elastic composite action as well as their influence on the final results were analyzed. For the purposes of numerical analysis, based on the proposed numerical models, several codes are written in MATLAB. The model applied in analysis by Galerkin’s method, that best describes the problem of elastic coupling, was chosen for further comparative analysis with experimental data.

**Key words:** Ritz's method, Galerkin's method, γ-method, Weak form, Numerical analysis, FEM, Timber-concrete composite, Partial interaction, Interlayer slip.