Stratified Global MHD Models of Accretion Disks in Semidetached Binaries

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Abstract

We present results of the first global magnetohydrodynamic simulations of accretion disks fed by Roche-lobe overflow, including vertical stratification, in order to investigate the roles of spiral shocks, magnetorotational instability (MRI), and the accretion stream in disk structure and evolution. Our models include a simple treatment of gas thermodynamics, with orbital Mach numbers at the inner edge of the disk \(M_{\text{in}}\) of 5 and 10. We find mass accretion rates to vary considerably on all timescales, with only the Mach 5 model reaching a clear quasi-stationary state. For Mach 10, the model undergoes an outside-in, magnetically driven accretion event occurring on a timescale of \(\sim 10\) orbital periods of the binary. Both models exhibit spiral shocks inclined with respect to the binary plane, with their position and inclination changing rapidly. However, the time-averaged location of these shocks in the equatorial plane is well fit by simple linear models. MRI turbulence in the disk generates toroidal magnetic field patterns (butterfly diagrams) that are in some cases irregular, perhaps due to interaction with the spiral structure. While many of our results are in good agreement with local studies, we find some features (most notably those related to spiral shocks) can only be captured in global models such as studied here. Thus, while global studies remain computationally expensive—even as idealized models—they are essential (along with more sophisticated treatment of radiation transport and disk thermodynamics) for furthering our understanding of accretion in binary systems.

Unified Astronomy Thesaurus concepts: Semidetached binary stars (1443); Stellar accretion disks (1579); Magnetohydrodynamical simulations (1966); Cataclysmic variable stars (203)

1. Introduction

Semidetached binaries are some of the most interesting sources for studies of disk accretion, due to their well-defined Roche-lobe overflow mass supply, the variety of their observed behavior, and the favorable distances and numbers, allowing for a wealth of observational data to be accessible for a large number of sources. These properties make them natural targets for models of disk accretion, both in a local and global sense.

Of these systems, cataclysmic variables (CVs) are perhaps most accessible to numerical modeling, and a large body of computational studies of their accretion flows has been accrued over the last decades. CVs are close interacting binary systems composed of a Roche-lobe-filling (usually main-sequence) star and a (higher-mass) white dwarf (WD). The binary separation is generally at a few solar radii, and binary periods are of the order of a few hours (Warner 1995). Mass exchange in CVs leads to formation of an accretion disk around the WD, resulting in a range of observational phenomena of interest in the context of accretion physics. The best known of these are the recurring dwarf nova (DN) outbursts, occasionally increasing the brightness of some CVs by up to \(~ 8\) mag for 2–20 days (Lasota 2001). CV accretion disks may also exhibit quasi-periodic oscillations (QPOs; Warner et al. 2003), as well as flickering visible in their rapid photometry (e.g., Bruch 1992; Sokoloski et al. 2001; Woudt et al. 2004). The geometric structure of the accretion disk can be studied via imaging techniques such as eclipse mapping and Doppler tomography (e.g., Steeghs et al. 1997; Baptista et al. 2005; Klingler 2006; Khruzina et al. 2008; Ruiz-Carmona et al. 2020).

1.1. Local Models

Understanding the physics of angular momentum transport and accretion in disks has often made use of the local shearing-box approximation (Goldreich & Lynden-Bell 1965; Hawley et al. 1995). Many of the results from shearing-box simulations are of particular interest in the context of accretion in CVs, especially regarding the DN mechanism and its relation to the magnetorotational instability (MRI; Hawley & Balbus 1992), which is now understood to be the mechanism responsible for angular momentum transport and accretion in fully ionized plasmas. DN outbursts are thought to result from thermal instability within the disk (the disk instability model, or DIM; e.g., Smak 1971; Osaki 1974; Meyer & Meyer-Hofmeister 1981; Faulkner et al. 1983; Lasota 2001), causing hysteresis between the cold and hot stable accretion branches with different hydrogen ionization levels. Assuming the Shakura & Sunyaev (1973) model, researchers have found that DIM requires \(\alpha \sim 0.1\) and \(\alpha \sim 0.01\) in outburst and quiescence, respectively (Mineshige & Osaki 1983; Meyer & Meyer-Hofmeister 1984; Smak 1984, and others). However, connecting these values to the behavior of MRI turbulence has proven challenging. Gammie & Menou (1998) suggested that lower ionization in the cold branch may cause resistivity to reduce angular momentum transport through MRI. Latter & Papaloizou (2012) were able to reproduce the two thermal states corresponding to these changes in their unstratified shearing-box simulations. Adding more sophisticated ionization and opacity prescriptions improved matters further. Hirose et al. (2014), Coleman et al. (2016, 2018), and Scepi et al. (2018a) were able to reproduce the characteristics of DN outbursts in their models, seeing their equilibrium states align with the expected hysteresis S-curve. They found convection to be an important mechanism, increasing MRI-related accretion levels to the \(\alpha \sim 0.1–0.2\) required to match observations (see also Hirose 2015).
1.2. Challenges of Global Modeling in Semidetached Binaries

At first glance, CV systems appear approachable for global numerical modeling. One of the difficulties of global accretion models comes from the dynamical range of a system. In grid models, the integration time step needs to be small enough to resolve the Keplerian motion of the gas at the inner edge ($r_{in}$) of the grid. At the same time, the total simulation time must be large enough to contain a number of Keplerian orbits at the outer edge of the disk ($r_{out}$). The larger the ratio of the two, the more expensive a model is. CVs exhibit fairly accessible dynamic ranges relative to other disk-accretion environments, with $r_{out}/r_{in} \sim 50–100$.

However, a faithful representation would also need to resolve the typical length and timescales of all relevant processes. In the case of CV accretion disks, the main length scales of concern are the disk thermal scale height and the most unstable MRI wavelength. The sonic Mach number at the inner disk edge $M_{in}$ ranges in CVs from $\sim 50$ to $200$ in outburst to $\sim 200–600$ in quiescence (see discussion in Ju 2016). This corresponds to the disk aspect ratio (of thermal scale height to radius) $H/R \sim 1/M_{in}$. Meanwhile, the midplane’s most unstable MRI wavelength is self-consistently set by the MRI dynamo (Brandenburg et al. 1995; Hawley et al. 1996), and it is typically found in stratified shearing-box studies to be $\lambda_{MRI} \sim 0.1–1H$ (e.g., Mamatsashvili et al. 2013; Scepi et al. 2018b; Suzuki et al. 2019). This further increases the requirements for grid resolution—especially given specific needs for resolving MRI turbulence (Hawley et al. 2011, 2013; Sorathia et al. 2012)—to thousands of cells per radian. As a result, realistic values of temperature and related disk aspect ratios remain extremely challenging for numerical studies, especially in the global or stratified context. It is thus common to perform simulations at lower Mach numbers (usually up to $\sim 20$, e.g., Kley et al. 2008; Ju 2016; Arzamasskiy & Rafikov 2018) and apply the understanding built with these models to gain insights about more realistic regimes of temperatures.

To simulate CV disks completely rigorously, one would also need to account for the magnetic diffusion and molecular viscosity timescales. The latter easily reaches $10^7$ yr $\approx 10^{19} P_{orb}$ (orbital periods of the binary), assuming kinetic viscosity $\nu \sim 10^7$ cm$^2$ s$^{-1}$ and disk radius as the typical length scale $\sim 10^3$ cm (Balbus 2003; Johnson 2006). The magnetic diffusion timescale is shorter, due to the small magnetic Prandtl number ($Pm$) expected in CV disks (Balbus & Henri 2008; Potter & Balbus 2014). However, as $Pm$ is likely to remain well above $10^{-5}$ (Gammie & Menou 1998), the relevant timescales remain enormous and only approach the (still large) lower limit of $\sim 10^7 P_{orb}$, for very cold, quiescent states, which are difficult to simulate for other reasons (see previous paragraph). In addition to these enormous times, extremely high resolutions would also be required so that numerical diffusion does not overwhelm the physical extent of these processes. While no existing computational resource would be able to handle such enormous run times, the role of these processes (Balbus & Hawley 1998) has been investigated in high-resolution local studies with artificially high viscosity and magnetic diffusion coefficients at a fixed magnetic Prandtl number (e.g., Fromang et al. 2007; Balbus & Henri 2008; Simon et al. 2011). As a result, a dependency between MRI dynamo efficiency and the Prandtl number has been found for $Pm \sim 1$, which can be of relevance for X-ray binary disks (e.g., Balbus & Lesaffre 2008; Potter & Balbus 2014). In global studies, ensuring a constant nonnumerical $Pm$ can be very challenging, especially for CVs, where a large timescale separation due to $Pm \ll 1$ is expected, and little flexibility in grid resolution is present, due to the already high computational cost. These studies typically rely on numerical viscosity to provide grid-scale dissipation of both turbulence and magnetic fields, which results in $Pm \sim 1$ and $\nu$, $\eta$ that can be difficult to control, especially in the context of fast azimuthal flow and mesh refinement. As a result, large-scale magnetic fields are expected to be overrepresented in these models, as a high $Pm$ promotes efficiency of the MRI dynamo (Brandenburg 2001; Schekochihin et al. 2004; Balbus & Henri 2008). With that in mind, however, the global structure and behavior of these models should be reflected properly, as argued by Balbus et al. (1994) and shown by Sorathia et al. (2012), and they can be very informative of accretion physics, as long as one remembers that such models are not proper tools for studying the small-scale statistics of magnetohydrodynamic (MHD) turbulence (Hawley et al. 2011; Sorathia et al. 2012; Hawley et al. 2013).

In light of the challenges outlined above, all global (and many local) models of CVs performed to date, including this work, are, by necessity, idealizations. However, idealized models have proven to be very effective in building understanding of various physical phenomena, including disk accretion, with perhaps the most prominent example found in the idealized analytical models of Shakura & Sunyaev (1973), which are joined by many numerical studies we review below.

1.3. Global Hydrodynamical Models

There is a large body of research using idealized global hydrodynamical models of CVs, with a number of successful predictions. Perhaps the most notable result of these studies is the importance of spiral-shock angular momentum dissipation as an accretion mechanism in these systems (Lin & Papaloizou 1979; Sawada et al. 1986). Properties of these spirals were thoroughly investigated: from the tidal response’s dependence on disk Mach number (e.g., Savonije et al. 1994), through the efficiency of accretion driving (e.g., Blondin 2000; Ju et al. 2016), factors influencing their opening angles (e.g., Makita et al. 2000), to their occurrence in various environments (e.g., Belvedere & Lanzafame 2002; Lanzafame 2008). Hydrodynamical models also investigated disk-inflow interactions (e.g., Fujiwara et al. 2001; Kunze et al. 2001; Godon 2019) and oscillations related to global dynamical instabilities (Bisikalo 2007; Kley et al. 2008).

1.4. Global MHD Models

However, while the hydrodynamical approach has provided the community with many valuable findings, we now know that the MRI (Hawley & Balbus 1992) is one of the most important drivers of accretion in semidetached binaries. Thus, there is a need for idealized global models of accretion disks with the MRI self-consistently controlled within an MHD framework, as opposed to its subgrid treatment via $\alpha$ prescriptions. Such models, despite their limitations discussed in Section 1.2, are crucial to investigating the interaction between MRI turbulence and global disk structure.

The first global MHD simulations of CVs were performed by Ju et al. (2016, 2017), who used a 3D unstratified setup with realistic treatment of Roche-lobe overflow. They found spiral shocks and MRI to play comparable roles in driving accretion at Mach numbers of $\sim 10$ and disk plasma $\beta \sim 400$ (with $\beta$ defined as the ratio of gas pressure to magnetic pressure), with higher magnetization or Mach numbers causing MRI to dominate. They also reported an interplay between these two
mechanisms, as more vigorous MRI turbulence was seen to enhance accretion through spiral shocks. Observation of these interactions highlights the potential of idealized global MHD models of accretion disks in semidetached binaries for improving our understanding of these systems.

Here, we extend these models with vertical stratification. This allows us to probe the vertical structure of accretion disks and compare it with expectations from stratified shearing-box simulations (e.g., Brandenburg et al. 1995; Stone et al. 1996; Fromang et al. 2013; Salvesen et al. 2016). We also verify the findings of Ju et al. (2016, 2017) regarding spiral shock and MRI accretion in these more physically consistent, albeit still idealized, conditions. Ultimately, our goal is to include realistic radiative cooling and bridge the global MHD models with local studies using more accurate thermodynamics; these extensions will be the topic of future papers.

The paper is structured as follows. In Section 2, we describe basic parameters of our two models. Section 3 describes our numerical setup in Athena++. In Section 4, we report and discuss our results, focusing on global dynamics and comparison with local studies. We briefly discuss possible observational features in Section 4.5. Finally, our findings are summarized in Section 5.

2. Model Parameters

To facilitate comparison with Ju et al. (2016, 2017), we adapt a model that mirrors their unstratified setup. We consider a system with mass ratio \( q = M_2/M_1 = 0.3 \). The equations we solve are defined in dimensionless form, where \( GM_1 + GM_2 = 1 \). The unit of length is the binary separation \( a = 1 \), making the binary orbit \( P_{orb} = 2\pi \). We investigate two models of accretion disks, with the Mach number at the inner edge of the grid \( M_{in} \) equal to 5 and 10 (corresponding to disk aspect ratios of \( H/R \sim 1/M_{in} = 0.2 \) and 0.1). Henceforth, we will refer to these models as “Mach 5” and “Mach 10.”

3. Methods

3.1. Athena++

Our simulations were performed using the finite-volume 3D MHD code Athena++\(^4\) (Stone et al. 2020). It is a higher-order Godunov scheme using a constrained-transport staggered mesh approach to enforce the zero magnetic divergence constraint (Gardiner & Stone 2005, 2008). Athena++ includes a number of features tailored to global MHD simulations, such as a flexible grid structure and adaptive mesh refinement. The code has been extensively tested and benchmarked on parallel systems, showing excellent performance and scaling, with well over 80% parallel efficiency on over half a million threads.

Athena++ has been designed to be highly modular, with a number of wrappers built in to allow for user modification. Most of the physical features of our models were implemented as user-defined source terms. These include source terms due to gravity of the binary and rotating frame of reference (Section 3.2), temperature floor and ceiling (Section 3.2.1), velocity ceiling (Section 3.4.2), and the Alfvén speed ceiling (Section 3.4.3). The hydrodynamics of our boundary conditions (Section 3.5) were set with user-defined boundary functions, while initial conditions (Section 3.6) were given within a “problem generator.” Only the EMF (electromotive force) boundary and initial conditions (Sections 3.5.1 and 3.6) required the base code of Athena++ (the EMF update step) to be edited.

3.2. Equations Solved

To evolve our models, we solve the equations of ideal magnetohydrodynamics (MHD) in a frame of reference corotating with the binary (see Gardiner & Stone 2008; Ju 2016):

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)
\]

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v} - \mathbf{BB} + P^* \mathbf{I}) = -\rho \nabla \Phi_{tot} + \mathbf{F}_l + \mathbf{F}_{Cori}, \quad (2)
\]

\[
\frac{\partial E}{\partial t} + \nabla \cdot ((E + P^*) \mathbf{v} - \mathbf{B} (B \cdot \mathbf{v})) = 0, \quad (3)
\]

\[
\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0, \quad (4)
\]

where \( P^* = P + B^2/2 \), and \( E \) is the total energy density:

\[
E = \frac{P}{\gamma - 1} + \frac{1}{2} \rho (v - \Omega \times (r - r_{bary}))^2 \\
+ \frac{B^2}{2} + \rho \Phi_{tot}. \quad (5)
\]

Here, \( \Phi_{tot} \) is the total gravitational potential of the binary:

\[
\Phi_{tot} = -\frac{GM_1}{r} - \frac{GM_2}{|r - r_e|}, \quad (6)
\]

where \( r_e = 1e_r \) is the location of the binary companion. Frame rotation around the barycenter is included through two apparent forces in Equation (2):

\[
\mathbf{F}_l = \rho \Omega \times (\Omega \times (r - r_{bary})), \quad (7)
\]

\[
\mathbf{F}_{Cori} = -2\rho \Omega \times \mathbf{v}, \quad (8)
\]

where \( \Omega = 1e_z \) is the frame rotation rate, and \( r_{bary} \sim 0.23e_z \) is the location of the barycenter. The remaining symbols have their usual meaning.

3.2.1. Equation of State

The system of Equations (1)–(4) needs to be closed by an equation of state. Ju et al. (2016, 2017), in their unstratified global models, utilized a fixed, locally isothermal temperature profile with local pressure set by

\[
P = \frac{1}{\gamma} c_s^2(R) \rho, \quad (9)
\]

where \( R \) is the cylindrical radius and \( \gamma \) the adiabatic index. The sound speed \( c_s \) (temperature) at each annulus of the disk was given by the Shakura & Sunyaev (1973) model for a gas-pressure-supported disk with opacity dominated by free–free processes:

\[
c_s \propto R^{-3/8}, \quad (10)
\]

\(^4\) The public version of Athena++ is available at https://princetonuniversity.github.io/athena/.
where normalization is set by selecting the Mach number $M_{\text{in}}$ at the inner edge of the grid:

$$c_s(R_{\text{in}}) = \frac{1}{M_{\text{in}}} \sqrt{\frac{GM_\odot}{R_{\text{in}}}}. \quad (11)$$

In our stratified models, we find that fixing the temperature profile makes the inflow very hot and pressurized, causing the injected gas to expand in all directions from the L1 zone instead of forming an inflowing stream. To allow the inflow to remain cool until it is shock-heated on impact at the accretion disk, we turn the temperature profile of Equation (10) into a temperature (pressure) ceiling:

$$P_{\text{ceil}} = \frac{c_s^2(r_{\text{in}})}{\gamma} \left( \frac{r_{\text{in}}}{r} \right)^{3/4} \rho, \quad (12)$$

where $\rho$ is the local density and the gas is otherwise treated as adiabatic. An adiabatic index of $\gamma = 1.1$ was chosen to enable direct comparison with numerical models from the literature (e.g., Ju et al. 2016), which are typically either (locally) isothermal or fully adiabatic. For the latter, low values of $\gamma$ were needed to prevent disks from becoming too hot. Here, low $\gamma$ prevents our disks from adiabatically cooling too far from our Shakura & Sunyaev (1973) ceiling. Even so, some overcooling was seen in our test runs with a temperature ceiling only. As such cooling quenches MRI (by preventing it from being resolved), it has proven necessary to also set a floor on the temperature (pressure) profile:

$$P_{\text{floor}} = \left(1 - f \frac{r - r_{\text{in}}}{r_{\text{out}} - r_{\text{in}}} \right)^k \times P_{\text{ceil}}, \quad (13)$$

where $k = 2, f = 0.95$ (Mach 5) or $k = 1, f = 0.80$ (Mach 10). As a result, the gas is kept hot and close to the Shakura & Sunyaev (1973) model at low radii, while it is allowed to remain cool at large radii.

This simple treatment, with a temperature floor and ceiling limiting an otherwise adiabatic gas, should be sufficient for all processes that depend on the average local temperature (expected to follow Shakura & Sunyaev 1973). However, any effects related to small-scale, nonaxisymmetric, or transient heating—for example, a “hot spot” or thermal structure of spiral shocks—will not be captured. While perhaps of secondary importance to global dynamics, these features are relevant for comparison with observations (see Section 4.5). The region most dynamically affected by our thermodinamics treatment is the inflow’s impact point, where our implementation artificially enforces the fast-cooling scenario of Armitage & Livio (1998; see Section 4.1.2). A future version of our models will address these issues with radiative cooling (see Section 5). This will allow the global dynamical model presented in this work to couple with self-consistent disk thermodynamics.

### 3.3. Simulated Domain and Mesh Layout

We represent our models on a spherical-polar mesh with $r \in [0.05, 0.62]$, limited by $\sim 10$ typical WD radii and the location of the L1 point. It covers a full $4\pi$ of solid angle ($\theta \in [0, \pi], \phi \in [0, 2\pi]$) and uses the “polar” boundary conditions of Athena++ to allow for free movement of gas and magnetic fields over both poles of the grid (see White et al. 2016).

The base grid contains 32 cells in each direction, organized in mesh blocks of $16 \times 8 \times 16$ cells (in the directions of $r$, $\theta$, and $\phi$, respectively). Adaptive mesh refinement (AMR) with up to four (Mach 5) or five (Mach 10) levels of refinement is used to capture the disk and accretion stream. A mesh block is marked for refinement if any of the following conditions are satisfied:

1. It neighbors the L1 inflow zone, that is, intersects the region $|\pi/2 - \theta| < \theta_{\text{disk}}, \phi < \phi_{\text{disk}}$ or $2\pi - \phi < \theta_{\text{disk}}$, $r = r_{\text{max}} = 0.62$;
2. It is close to the midplane ($|\pi/2 - \theta| < \theta_{\text{disk}}$), and any of its cells have high-enough density $\rho > \rho_{\text{AMR}}$;

where we adapt $\theta_{\text{disk}} = 0.3$ and $\rho_{\text{AMR}} = 0.05$. Note that Athena++ ensures that neighboring mesh blocks have refinement levels differing by at most one level (Stone et al. 2020). While our needs could have been satisfied with static mesh refinement, we find AMR to be more flexible and convenient at a negligible additional cost. Our grid is logarithmically spaced in radius, with a cell size ratio of 1:1 in this direction. As a result, the aspect ratio of cells remains close to 1:1:2 throughout the grid, with the elongated part directed azimuthally. In both of our simulations, the accretion disk was always resolved at the highest refinement level and the thermal scale height was resolved by $H/\Delta_z \approx 27$ cells. We discuss our MRI resolving power in Section 4.1.3.

The time step of our simulations was limited by advection of gas with Keplerian velocity at the midplane, at the inner edge of the disk ($r = 0.05, \theta = \pi/2$; however, see Section 3.4.3). Note that, to conserve numerical resources, we increased the inner grid radius from the 0.02 used by Ju et al. (2016, 2017) to 0.05 here.

#### 3.4. Floors and Ceilings

##### 3.4.1. Density Floor

Our simulations contain large volumes of void threaded by magnetic fields, especially in the polar regions. To ensure numerical stability, we find it necessary to use a density floor at $\rho_{\text{floor}} = 10^{-6}$, much smaller than typical disk densities of $\rho \sim 1$. Note that by using a density floor we inject additional mass in a frame of reference corotating with the binary. We find this to be a physically reasonable approach, as some amounts of ambient gas corotating with the disk may be present in astrophysical systems, while a reservoir of mass immobile in the (inertial) observer frame appears unlikely. Thanks to the application of a velocity ceiling (see Section 3.4.2, Equation (19)), the density floor contributes negligibly to the total accretion rates observed in our simulations.

##### 3.4.2. Velocity Ceiling

Near the equatorial plane of our models, gas can readily be supported by Keplerian rotation and pressure gradients. However, near the polar regions, it can at times be in freefall, causing a number of numerical issues common in global models of accretion disks. To mitigate them, we adopt the following velocity ceiling.

If (and only if) the total velocity at a given cell exceeds the critical value, $v > v_{\text{ceil}} = 0.5$, a smooth switch $s_i$ is calculated. It applies the velocity ceiling only to low-density regions, $\rho < \rho_v = 10^{-5}$ (to avoid affecting disk dynamics), with a
smooth transition in velocity:

\[
s_r = \left( \arctan \left( \frac{1 - \rho}{\rho_r} \right) \times 8\pi / \pi + \frac{1}{2} \right) \times \left( \arctan \left( \frac{v}{v_{\text{cell}} - 1.2} \times 8\pi / \pi + \frac{1}{2} \right) \right),
\]

(14)

Factors of \(8\pi\) and 1.2 are chosen to balance the smooth behavior with well-defined application boundaries. Then \(s_r\) is used to compute the adjusted velocity:

\[
v_{\text{new}} = s_r v_{\text{cell}} + (1 - s_r) v,
\]

(15)

which modifies only the radial and poloidal components of the cell velocity:

\[
v_{r,\text{new}} = (v_{\text{new}} / v) \times v_r,
\]

(16)

\[
v_{\phi,\text{new}} = (v_{\text{new}} / v) \times v_{\phi}.
\]

(17)

As all source terms applied to the momenta of the system, the velocity ceiling also includes an adjustment to the total energy density (Equation (5)):

\[
\Delta E = \frac{1}{2\rho} ((M)^2 - (M - \Delta M)^2) = \frac{1}{2\rho} (2M \cdot \Delta M - (\Delta M)^2),
\]

(18)

where \(M\) is the momentum density after velocity ceiling application, and \(\Delta M\) is the vector by which it is changed.

In addition to improving stability, the velocity ceiling also ensures that the accretion rate of material at the density floor from polar regions does not dominate our results. We can approximate the density floor accretion rate as

\[
\dot{M}_{\text{floor}} \approx 4\pi r_{\text{out}}^2 \times \rho_{\text{floor}} v_{\text{cell}} \approx 2.42 \times 10^{-6} \text{ sim.u.},
\]

(19)

which remains well below the observed disk accretion rates, reported in Figure 2 and Section 4.1.1.

While the velocity ceiling does affect the magnetic fields in the polar regions of our grid, we find that the majority of mass and magnetic energy in our models resides in regions of parameter space unaffected by these changes. We confirmed this by inspecting 2D histograms of magnetic energy density and gas density, weighted by these two quantities.

3.4.3. Alfvén Speed Ceiling

In our Mach 10 run, shock structures within the disk sometimes produced very small regions (a few cells each) of density floor permeated by strong magnetic fields. These resulted in small time steps, halting the simulation. To prevent this, we imposed an Alfvén speed ceiling. Its value was set at the Keplerian speed of the inner grid edge (which normally sets the time step), \(v_{A,\text{cell}} = \sqrt{GM/r_{\text{in}}} \approx 3.9\). Whenever it was crossed, the local density was increased for \(v_A\) to match that value. We have measured the rate of mass injection by this modification to the density floor, and it was at most \(10^{-3}\) of the observed physical accretion rate.

3.5. Boundary Conditions

Outside the L1 zone, the inner and outer radial boundary conditions are set as free outflow, no inflow (“diode” boundary conditions). If the local radial velocity is directed outside the simulated grid, both the cell-centered quantities and the edge-centered EMFs (see Gardiner & Stone 2008) are copied to the ghost cells, allowing for free outflow. If the radial velocity is directed into the grid, reflecting boundary conditions are used for cell-centered values and EMFs in the ghost cells are set to zero. The boundary conditions in the \(\phi\) direction are set as periodic. We use special “polar” boundary conditions (White et al. 2016) for the boundaries in the \(\theta\) direction. These allow us to accurately represent the motion of gas and magnetic fields even as they pass through the poles of the grid.

3.5.1. Roche-lobe Overflow Boundary Conditions

In order to simulate the inflow of magnetized gas through the L1 point of the binary system, we designate a special “L1 zone” in the outer radial boundary. It is defined as a circular region at an angle of at most 0.10 rad (~0.5\(^\circ\), Mach 5) or 0.05 rad (~3\(^\circ\), Mach 10) to the binary axis as seen from the grid center.

For L1 zone ghost cells, the density is set to 1 (Mach 5) or 4 (Mach 10), and a small negative radial velocity of \(v_r = -0.01\) is set, while \(v_{\theta}\) and \(v_{\phi}\) are kept at zero. While this boundary \(\rho v_r\) gives us an indirect handle on the mass inflow rate, the actual value of inflow \(M\) depends on the active grid conditions surrounding the L1 zone (mainly gas and ram pressure) and is best measured empirically from the simulation outputs as \(M\) at large radii. A systematically larger inflow \(\rho v_r\) would lead to a denser disk with the same aspect ratio, as long as the disk reaches our temperature ceiling (where \(H/R \sim 1/M_{\text{in}}\)). However, any potential short-timescale fluctuations in inflow \(\rho v_r\) would likely be quickly “forgotten” by the inflow stream, given the level of variability caused by interaction with the gas surrounding our accretion disks (see Section 4.1.1). Such inflow variability can drive some level of turbulence within the disk, but it is likely quickly overridden by MRI effects once magnetic fields are taken into account (as observed for the corresponding hydrodynamical and MHD models of Ju et al. 2016).

Inflowing gas is injected at the local temperature floor (see Section 3.2.1) with the sound speed corresponding to 0.020 and 0.174 of the equatorial sound speed at \(r_{\text{in}}\) for Mach 5 and Mach 10, respectively.

The inflow also contains a magnetic field. We take a zero-net-flux approach with alternating-polarity vertical magnetic loops traveling inside the inflow (the loop axes are in the \(\phi\) direction).\(^5\) We opt to set the magnetic fields of the L1 ghost zone by modifying the EMFs, allowing the code to calculate the corresponding \(B\) fields. This ensures \(\nabla \cdot B = 0\) throughout the grid at all times. The inflow EMF values are set as follows:

1. For a single (circular) row of cells surrounding the L1 zone, the EMF values are always set to zero. This prevents the magnetic fields from “spreading” along the outer radial boundary of the grid and keeps them confined to the inflow.

\(^5\) This orientation prevents numerical reconnection as the loops are sheared down to a few radial rows of cells when the inflow reaches the forming accretion disk.
2. Within the L1 zone, the following condition is used:

\[ E = \pm \left( \frac{2P_{\text{inf}} x_{\text{loop}}}{\beta_{\text{inf}}} \right) v_{\text{inf}} \frac{e_\theta}{\beta_{\text{inf}}}, \]  

with the sign alternating between loops. Here, \( P_{\text{inf}} \) denotes the inflow pressure, \( \beta_{\text{inf}} \) is the minimal plasma \( \beta \) within the loop, and \( x_{\text{loop}} \) describes how far the current loop would have advected into the grid if it were moving at \( q v_{\text{inf}} \):

\[ x_{\text{loop}} = (-qv_{\text{inf}} t \mod 2r_{\text{loop}}) - r_{\text{loop}}, \]

\[ d = \sqrt{(r \cos \theta)^2 + x_{\text{loop}}^2} + 10^{-6}. \]

The “squeezing factor” \( q = 2\pi \) is used to prevent loops from being excessively elongated. The loop radius is set to match the L1 zone radius.

The strength of the injected magnetic field depends on interactions with the active grid, and thus a value that produced a dynamically important (MRI unstable) magnetic field in the disk needed to be set by trial and error. In the data presented here, mass-weighted averages of plasma \( \beta \) over the L1 vicinity are 66.5 (Mach 5) and 343 (Mach 10). These inflows are additionally heated by our temperature floor, which increases the effective plasma \( \beta \) of the gas reaching the disk by another factor of \( (c_{\text{e, disk}}/c_{\text{e, inf}})^2 \) to \( \sim 345 \) and \( \sim 487 \) for Mach 5 and Mach 10, respectively. These final estimates are consistent with typical values used in similar studies (e.g., Ju et al. 2016, 2017). We note that, in the absence of an MRI dynamo, the magnetic fields at the disk midplane would be affected by numerical reconnection due to Keplerian shear and thus decay noticeably within the disk (this is not observed).

3.6. Initial Conditions and Transient Discs

Both simulations are initialized with a thin, magnetized low-density Keplerian disk surrounded by the density floor. For \( r < 0.3 \), the following conditions are used:

\[ \rho = \rho_{\text{init}} \times \xi(r, z) + \rho_{\text{floor}}, \]

\[ v = \frac{\sqrt{GM R}}{r^{3/2}} - \Omega R, \]

\[ \xi(r, z) = \left( \frac{1}{2} - \arctan(16\pi(r - r_{\text{init}}))/\pi \right) \times \left( \arctan \left( \frac{z + H_{\text{init}}}{s} \right) - \arctan \left( \frac{z - H_{\text{init}}}{s} \right) \right)/\pi, \]

where \( r_{\text{init}} = 0.2 \), \( \rho_{\text{init}} = 0.1 \), \( H_{\text{init}} = 0.05 \), \( s = 0.005 \) is the smoothing parameter, \( R \) and \( r \) are the cylindrical and spherical-polar radius, respectively, and \( \Omega = 1 \) is the rotation speed of the frame of reference. The disk is initialized as magnetized with a single magnetic loop defined using

\[ E = vB_{\text{init}} \times \cos \left( \frac{c_{\text{e, inf}}}{2H_{\text{init}}} \right) \sin(\phi) \times \xi(r, z) e_\theta, \]

where \( B_{\text{init}} = 1 \). At \( r > 0.3 \), void conditions are set with \( \rho = \rho_{\text{floor}}, v = 0 \), and \( E = 0 \). The pressure is set according to Equations (10) and (11).

This initial state was first evolved with the density floor and velocity ceiling’s critical density of \( 10^{-4} \) and \( 10^{-3} \), respectively. After four (Mach 5) or 15 (Mach 10) binary periods, well-evolved accretion disks were present. We then set \( \rho_{\text{floor}} \) and \( \rho_{\text{infl}} \) to \( 10^{-6} \) and \( 10^{-3} \), respectively, ensuring that floor accretion is no longer able to affect the accretion rate (see Section 3.4). In order to avoid the influence of transients associated with establishing the accretion flow using this procedure, the initial \( \sim 5P_{\text{orb}} \) of subsequent evolution was ignored, with the remaining \( \sim 9P_{\text{orb}} \) (Mach 5) and \( \sim 11P_{\text{orb}} \) (Mach 10) used to perform the analysis presented in this work.

4. Results and Discussion

As the lengthy discussion of the methods given in the previous section implies, self-consistent modeling of accretion in a global model of a close binary is difficult. Even with advances in computational methods and infrastructure (such as AMR in curvilinear coordinates), we remain very much limited by computational constraints. Nonetheless, even simplified models can provide valuable insights into the behavior of real systems and guide future enhancements to the models. Providing these insights is our goal as we report the findings from our stratified global MHD simulations of an accretion disk with Roche-lobe overflow.

4.1. General Description of the Flow

We plot 3D renderings of our two models in Figure 1. The data are taken from snapshots of Mach 5 (left panel) and Mach 10 (right panel) at \( t = 7.6P_{\text{orb}} \) and \( t = 9.4P_{\text{orb}} \) (after the discarded transients, Section 3.6), respectively. The surface plot corresponds to a density isocontour at 0.30 (Mach 5) and 0.03 (Mach 10) of the central disk density (as measured at \( r = 0.08 \) sim.u., phase A for Mach 10). The disk density isosurfaces readily show signs of spiral structure, which we discuss in detail in Sections 4.1.2 and 4.1.4. The surface color denotes the disk temperature at a given point in sim.u. Note that the factor of two difference in \( M_{\text{infl}} \) between the two models results in a factor of four difference in temperature. The surface temperature confirms that the inflow remains cold until it is shock-heated within the disk into a Shakura & Sunyaev (1973) profile (our temperature ceiling), as intended. The volume rendering of magnetic field strength (seen as light colors surrounding the disk) visualizes the location and shape of magnetic structures. While in Mach 5 strong magnetic fields are seen to fill the entire surroundings of the disk, in Mach 10 they are mostly limited to regions just above the disk surface. We discuss the magnetic field structure in detail in Section 4.1.3.

A number of authors have considered the inflow–accretion disk interactions by means of analytical considerations and with the use of numerical (radiative) hydrodynamics (e.g., Lubow & Shu 1976; Livio et al. 1986; Frank et al. 1987; Lubow 1989; Kunze et al. 2001; Godon 2019). Armitage & Livio (1998) found that if cooling in the system is efficient, the inflow stream can reflect off the rim of the accretion disk, leaving a bulge in the disk downstream of the impact point. In the case of inefficient cooling, the stream is seen to overflow the accretion disk, with smaller streams continuing to slide over the disk surface following near-ballistic trajectories (e.g., Kunze et al. 2001). Our models do not match either of these scenarios exactly, as cooling is applied only at the temperature ceiling (at which point it is nearly infinitely efficient). However, qualitatively, they resemble the efficient cooling scenario of Armitage & Livio (1998) a small, elongated downstream bulge (clearly related to the underlying spiral
4.1.1. Variability

Accretion disks of semidetached binaries exhibit variability at a multitude of timescales and amplitudes: from DN outbursts (e.g., Smak 1971; Osaki 1974; Meyer & Meyer-Hofmeister 1981; Lasota 2001), through various types of QPOs (Warner et al. 2003), to rapid flickering (e.g., Bruch 1992; Sokoloski et al. 2001; Woudt et al. 2004). Nonetheless, it is commonly assumed that, at most times, a sufficiently long time average of accretion disk observables is well described by a steady-state model. Whether or not accretion disks truly reach such quasi-stationary states (and over how long a time) remains a viable question.

In Figure 2, we show the two metrics we use to evaluate whether a steady state is present in our models. First, we consider how radial density profiles of the disk change with time. We use density averaged over $\theta \in [\pi/2 - \theta_H, \pi/2 + \theta_H]$, $\phi \in [0, 2\pi]$ at a number of radii, where $\theta_H = 1/M_{\text{in}}$ roughly corresponds to the local thermal scale height. These time series are then boxcar-averaged over a single binary period to remove high-frequency effects. The resulting density evolution plots are shown in the top panels of Figure 2, color coded by radius. We also calculate instantaneous accretion rates through the selected radii. These are the mass flow rates integrated over the full sphere at given radii, with time series similarly boxcar-averaged over one orbital period. We show these data in the bottom panels of Figure 2.

For comparison, we calculate that the dynamical timescale in our disks ranges from $2 \times 10^{-3}P_{\text{orb}}$ at $r_{\text{in}}$ to $3 \times 10^{-2}P_{\text{orb}}$ at $r = 0.3$, while the effective viscous timescale

$$\tau_{\text{visc}} = \frac{R^2}{\alpha \alpha_s H} \sim \frac{R_{\text{in}}^{1/8} M_{\text{in}}^2}{\alpha \sqrt{\alpha G M_1 R^{11/8}}}$$

(27)

is equal to $\sim 6P_{\text{orb}}$ and $\sim 24P_{\text{orb}}$ for Mach 5 and Mach 10, respectively, assuming $\alpha \sim 0.1$ and $R \sim 0.3$. Thus, given that our simulations were run for $9P_{\text{orb}}$ (Mach 5) and $20P_{\text{orb}}$ (Mach 10), respectively, before the results shown in Figure 2 (see Section 3.6), in each case our simulations cover roughly a viscous time in the disk. However, given that the actual $\alpha$ values we measure in the flow are smaller (Section 4.3), it is clear our models must still be considered exploratory. To fully address long-term dynamical evolution at high Mach numbers, significantly longer (and many times more expensive) simulations are needed. If such ambitious models are attempted in the future, we hope that this work may provide a guideline for what can be expected.

As evident from Figure 2, both of our models show density and accretion rate variability over a wide range of timescales. However, the diagnostics of Mach 5 oscillate around well-defined average levels, describing a valid stationary state with the average $\dot{M}$ roughly independent of radius. In contrast, Mach 10 exhibits variability at all timescales, including that of the analyzed window itself ($\sim 11P_{\text{orb}}$). We find that longest timescale variability to be particularly interesting. Up until $t \sim 5P_{\text{orb}}$ (marked with a vertical dotted line in the top right
panel of Figure 2), the inner parts of the disk \((r \lesssim 0.12)\) operate in a quasi-stationary state, at an accretion level lower than that of the inflow (seen as \(M(r = 0.5)\)). As a result, gas is accumulated throughout the disk, causing density to increase for \(r \lesssim 0.2\). We will refer to this episode of low-level quasi-steady accretion as “phase A.” Around \(t = 5P_{\text{orb}}\), the average densities at larger (bluer) radii start to drop, causing a cumulative increase in density in each consecutive (smaller and redder) radius, until the (so increased) density at that radius drops as well. We observe a runaway, where gas from the outer radii travels inward, gathering mass accumulated in the disk in an outside-in fashion (we refer to this as “phase B”). Once the event passes a given radius, the local density regains its initial value and the accumulation of gas begins anew. This is suggestive of a recurring phenomenon, although longer simulations are needed to verify this hypothesis. Alternatively, phase B may be a nonstationary feature, as a quasi-steady state may exist at a later time beyond the simulation time available to us. Even then, however, it may be present in systems adjusting to a recent change in accretion rate either through the inflow stream or in the disk itself (e.g., during an outburst). We continue our discussion of phases A and B in Section 4.4, where we look closer at the differences between the two regimes.

### 4.1.2. Snapshots of Density and Spiral Structure

In Figure 3, we show density snapshots of our models at \(t = 7.6P_{\text{orb}}\) (Mach 5) and \(t = 9.4P_{\text{orb}}\) (Mach 10), depicted using equatorial and poloidal slices. The latter are taken for \(f = 0^\circ\), \(180^\circ\) for the right and left halves of the plots in the bottom panels, respectively. A zoom-in is used in the \(f = 0^\circ\) plot for Mach 10. The solid black lines in the bottom panels indicate polar angle ranges for the disk’s “main body” at an opening angle of \(\sim 28^\circ\) for Mach 5 and \(\sim 11^\circ\) for Mach 10 \(|\pi/2 - \theta| \in [0, 2.5H/R]\) and \(|\pi/2 - \theta| \in [0, 2H/R]\) for Mach 5 and Mach 10, respectively) and the “corona” further extending to \(\sim 57^\circ\) in both cases \(|\pi/2 - \theta| \in [2.5H/R, 5.0H/R]\) and \(|\pi/2 - \theta| \in [2H/R, 10H/R]\), which we discuss in Section 4.1.3. The inflow enters each accretion disk from the right through an elongated shock structure, seamlessly transitioning into one of the disk’s spiral arms. These can be seen both as overdensities in the
equatorial plots (top panels of Figure 3) and as shock structures in the poloidal slices (bottom panels of Figure 3). Vertically, these spiral shocks are inclined and occasionally broken into multiple parts with different inclinations, as is the case, for instance, for the shock at $r \sim 0.12$, $\phi = 180^\circ$ in the bottom left panel of Figure 3 (Mach 5) or the one at $r \sim 0.125$, $\phi = 0$ in the bottom right panel of Figure 3 (Mach 10). Both the radial position of the spiral arms and the vertical outlines (inclination as a function of height) of the associated shocks change rapidly. For both Mach 5 and Mach 10, the spiral arms regularly deviate by up to $\sim 0.05$ s.i.m.u. from their average positions, and the inclination can rapidly change within the bounds of $\sim [-\pi/4, \pi/4]$, with opposite extrema of this range often seen at consecutive time snapshots, 0.01–0.05 $P_{\text{orb}}$ apart.

The spiral shocks in the Mach 5 model appear to influence the vertical extent of the disk. A difference in height between pre- and postshock regions can be seen in the bottom left plot of Figure 3. Aside from these changes, however, the main body of the Mach 5 disk, extending up to $\sim 3$ vertical scale heights from the midplane, appears to be well mixed, with little vertical structure.

The spiral structure in semidetached binaries has been extensively studied in purely hydrodynamical simulations, using both smoothed-particle hydrodynamics (SPH; e.g., Belvedere & Lanzafame 2002; Lanzafame et al. 2002; Lanzafame 2003, 2010) and grid-based models (e.g., Makita et al. 2000; Fujiwara et al. 2001; Ju et al. 2016; Ryan 2017; Lukin et al. 2017). As magnetic fields are dynamically subdominant in our models, many of the aspects of spiral shocks seen in our models are similar to ones reported in a hydrodynamical framework. The elongated shock structure connecting our inflow to the spiral arms, sometimes dubbed a “hot line,” is a frequent feature in these studies (e.g., Figure 4 of Lukin et al. 2017 and its discussion therein; however, see also Bisikalo et al. 1998). Dependence of the spiral pitch angle on the disk Mach number has been reported and discussed by a number of authors (e.g., Spruit 1987; Hennebelle et al. 2016; Ju et al. 2016), and the spiral pattern we observe in the equatorial plane is similar to ones reported in hydrodynamical studies at similar parameters (e.g., compare our Figure 3 with Figure 3 of Makita et al. 2000 or Figure 14 of Ju et al. 2016). An increase in vertical extent of the disk caused by the underlying spiral structure has also been previously observed in nonmagnetic runs (e.g., Figures 5, 7 of Makita et al. 2000). The pitch angles of spiral arms in semidetached binary disks are usually found to follow linear dispersion relations (Ju et al. 2016; Ryan 2017), which, as we discuss in detail in Section 4.1.4, is also found to be true here for the time-averaged spiral pattern. However, an apparent difference between hydrodynamical studies and our results lies in the level of variability in spiral shock position and vertical inclination. Spiral patterns in hydrodynamical models of accretion disks are generally described as stable, settling into a well-behaved steady state. While there are indications of inclined shocks (see, e.g., Figure 6 of Makita et al. 2000, the discussion in Fujiwara et al. 2001, and Figure 4 in Lanzafame 2003), the vertical patterns appear to be symmetric with respect to the disk midplane. This stands in contrast with our models, where the position and inclination of shocks can change significantly on timescales much smaller than a binary period, and the spiral shocks are often found to be asymmetric with respect to the disk midplane. This is likely caused by interactions with the underlying MRI turbulence. Ju et al. (2016, 2017), in their unstratified MHD simulations, also note some disruption of the spiral pattern by the MRI turbulence, although they do not comment on its variability. The position and inclination of spiral shocks may be more significantly affected in our stratified models, where an additional (vertical) degree of
freedom is introduced (an effect similar to what is seen as distortion of spiral waves in 3D models of Fujiwara et al. 2001). Alternatively, variability of the spiral shock morphology may be related to the high resolution of our models, which resolves a wide range of Kelvin–Helmholtz instabilities in the flow, an effect similar to what is seen in the convergence study of Ju et al. (2016; see their Figure 14).

For Mach 10, the densest regions ($\rho \gtrsim 1$) do not appear to be sensitive to the spiral structure. The disk surface is almost conical, at $\sim 2$ vertical scale heights from the midplane. Above it, a low-density, strongly magnetized “corona” is present (see Section 4.1.3). In density snapshots, shell-like overdensities are produced by parcels of magnetic field buoyantly rising from the disk, as they drive some of the main-body gas over the disk surface.

Interestingly, some of the inflowing gas in our Mach 10 model is initially reflected away at radially supersonic speeds, which results in a series of plumes orbiting around the disk at $r \gtrsim 0.3$, $\phi \lesssim \pi$ (top right panel of Figure 3, most recent plume at $\phi = 3\pi/4$, $r \sim 0.4$), until they settle into the accretion disk at larger $\phi$. These plumes follow a self-regulating cycle. While a plume is present above the inflow stream’s impact point, the reflected gas is shocked not only by the accretion disk’s rim (which causes the initial reflection) but also the plume of previously reflected gas, thus entrapping the inflow and stopping formation of a new plume until the impact point is clear of reflected gas (a process resembling interactions with a circumbinary envelope investigated by Bisikalo et al. 1998). This cycle causes plume formation to occur at regular intervals with each plume $\sim \pi/4$ apart in $\phi$. As discussed in Section 4.1.2, inflow stream reflection has been predicted for efficiently cooling disks by Armitage & Livio (1998), and this scenario appears to be applicable here due to our temperature ceiling being active within the disk region.

In accreting binaries, the disk is influenced by tidal interaction with the binary companion. This results in truncation at a radius $r_{\text{tid}}$ where tidally distorted orbits cross (Paczynski 1977; Hirose & Osaki 1990). For our adopted $q = 0.3$ (e.g., Warner 1995; Harrop-Allin & Warner 1996), this becomes

$$r_{\text{tid}} \approx \frac{0.60}{1 + q} \approx 0.46 \text{ sim.u.} \quad (28)$$

As can be seen in the top row of Figure 3, the outermost parts of both disks do extend only until that radius. However, a distinct densest part of each disk appears to be enclosed within $r \lesssim 0.3$. This may be caused by spiral shocks, which cause gas orbits to be eccentric, increasing chances of orbit crossings and thus lowering the actual outer truncation radius. The radial size of our disk is generally consistent with the corresponding models of Ju et al. (2016, 2017), indicating that stratification does not influence this property of the disk significantly.

### 4.1.3. Magnetic Fields

Figure 4 shows equatorial and poloidal slices of plasma $\beta$ for snapshots at $t = 7.6P_{\text{orb}}$ for Mach 5 (left panels) and $t = 9.4P_{\text{orb}}$ for Mach 10 (right panels). A complex network of high-$\beta$ structures (current sheets) is a sign of vigorous MRI-driven turbulence. The average values of plasma $\beta$ are consistent with zero-net-flux stratified shearing-box simulations, where $\beta \sim 100–1000$ (e.g., Stone et al. 1996; Davis et al. 2010; Salvesen et al. 2016). In an average over the disk body ($r < 0.3$, $|\theta - \pi/2| < 2.5H/R$ or $2H/R$ for Mach 5 and...
Mach 10, respectively) and time, we obtain \( \langle \beta \rangle = 143 \) for Mach 5 and \( \langle \beta \rangle = 157 \) for Mach 10. We note that the grid resolution of our models limits MRI dynamo operation within the disk to \( \beta \lesssim 1000 \), above which the MRI would become unresolved.

A wealth of research has been (and continues to be) conducted on the details of MRI turbulence and the MRI dynamo, often in the context of shearing-box simulations (e.g., Brandenburg et al. 1995; Hawley et al. 1995; Davis et al. 2010; Latter et al. 2010; see also reviews by Balbus & Hawley 1998; Brandenburg & Subramanian 2005). Global models have been studied by Hawley et al. (2011, 2013) and Sorathia et al. (2012), with special attention given to numerical convergence. Hawley et al. (2011) proposed the following quality factors as a measure of whether the MRI is adequately resolved:

\[
Q_l = \frac{\lambda_{MRI}}{\Delta x_l} = 2\pi \sqrt{\frac{16}{15} \frac{v_{A,l}}{\Omega_K(R) \Delta x_l}},
\]

where \( \lambda_{MRI}, v_{A,l} \), and \( \Delta x_l \) are the most unstable MRI wavelength, Alfvén velocity, and cell size in the \( l \)th direction of the grid, respectively, and \( \Omega_K(R) \) is the local Keplerian frequency. In our models, the averages of these quality factors (over \( r \in [0.1, 0.25], |\pi/2 - \theta| \in [-H/2R, H/2R], \) and time) for Mach 5 and Mach 10 were equal to \( Q_z \simeq 3.4, 3.1 \) and \( Q_\phi \simeq 12, 15 \). While small, they are sufficient to describe the global properties of the flow (such as plasma \( \beta \) and the \( \alpha \) parameters), as shown by Sorathia et al. (2012, see their Figures 5 and 8). However, the statistics of MRI turbulence cannot be resolved here. Sorathia et al. (2012) and Hawley et al. (2013) find \( H/\Delta z \gtrsim 32, Q_z \gtrsim 10 \), and \( Q_\phi \gtrsim 25 \) to be necessary to achieve that goal (here, \( H/\Delta z \simeq 27 \); see Section 3.3).

We thus focus on the global features of magnetic field structure. As noted in Section 4.1, the entire body of the Mach 5 disk appears to be fairly well mixed (bottom left panel of Figure 4), with plasma \( \beta \) at a few 100s for most of the volume. Meanwhile, Mach 10 clearly separates into a weakly magnetized main body at \( |\pi/2 - \theta| \lesssim 2H/R (\beta \sim 100-1000) \) and a strongly magnetized (relative to the main body) "corona" at \( |\pi/2 - \theta| \gtrsim 2H/R, 10H/R (\beta \sim \text{a few tens}). \) To avoid potential confusion, we stress that the latter is not the X-ray-emitting corona as understood in observational astrophysics (e.g., Reis & Miller 2013; Wilkins & Gallo 2015; Wilkins et al. 2016). Rather, this term is used as is traditional in the numerical modeling community, denoting a distinct region at the disk surface exhibiting high magnetization relative to the disk body. Whether or not these two definitions are related is an interesting question in itself, one that is, however, beyond the scope of this study. The presence of such a defined "corona" has been nearly ubiquitous in stratified shearing-box simulations since the first models were presented (Brandenburg et al. 1995; Stone et al. 1996). Coronal accretion has also been shown to be an important process in a number of studies, although mainly in the context of nonzero net vertical flux (e.g., Stone & Norman 1994; Beckwith et al. 2009; Guilet & Ogilvie 2012, 2013; Zhu & Stone 2018). We investigate the role of the disk "corona" in driving accretion in Section 4.4.

The shell-like features seen in density snapshots in Figure 3 are also observed in plasma \( \beta \) as strongly magnetized regions surrounded by high-\( \beta \) shells. This supports our interpretation that they correspond to weakly magnetized gas being pushed out of the disk by buoyantly rising magnetic bubbles.

In addition to magnetized coronae, another consequence of magnetic buoyancy observed in stratified shearing-box simulations is the so-called "butterfly diagram" (e.g., Brandenburg et al. 1995; Shi et al. 2009; Salvesen et al. 2016). In the context of magnetic field generation by the MRI dynamo, a (azimuthal) magnetic field is usually produced at the disk midplane (see, however, Begelman et al. 2015). Once the midplane field is strong enough, buoyancy and turbulent motions cause it to rise toward the surface, where Parker instability shapes a corona (Shi et al. 2009). The cycle then restarts with reversed polarity of the field. The typical duration of one such field reversal in stratified shearing-box simulations is \( 6-10 \) local orbital periods. The vertical profile of azimuthally averaged \( B_\phi \) is plotted as a function of time, this results in a characteristic "butterfly" pattern. We plot such (butterfly) diagrams for our models in Figure 5, with each row corresponding to a different radius within the disk. We note that, although Figure 5 shows azimuthally averaged values, the butterfly pattern is also present in our models if poloidal slices of \( B_\phi \) are used.

For Mach 5, the field-reversal patterns are irregular, often asymmetric, and at times rare. However, when they do occur, the recurrence time is consistent with \( 6-10 \) local orbits. In previous studies, convection has been found to be able to quench field reversals in MRI turbulence (Coleman et al. 2017, 2018) by transporting some of the coronal magnetic field to the midplane (other such inhibiting factors include, e.g., the presence of net vertical flux; Salvesen et al. 2016). We speculate that large vertical displacements caused by inclined spiral shocks (see lower left panel of Figure 3 and Section 4.1) may play a similar role. Turbulence itself may also contribute to this process, as the turbulent eddies are quite large in Mach 5.

While the butterfly pattern for Mach 10 is more regular, here too the field reversals are often asymmetric and even missing. Highly inclined spiral shocks are still present and could be responsible. Interestingly, at \( R = 0.15 \) (top right panel of Figure 5), the pattern changes for \( t \in [75, 125]P_{\text{orb}}(R) \). This interval corresponds to a period of enhanced accretion (phase B) we describe in Section 4.1.1 (the transition between phases A and B at \( 5P_{\text{orb}} \) is indicated by vertical black lines in the right panels of Figure 5). Within this time interval, the field reversals are seemingly more frequent, especially below the midplane, although they remain within the usual \( 6-10 \) orbital periods. The frequency of field reversals can be affected by changes in the local shear rate (Gressel & Pessah 2015) and disk aspect ratio (Hogg & Reynolds 2018). An assessment of the significance of the change we observe, however, would strongly benefit from larger samples. We thus leave a closer inspection of this behavior to a future study.

4.1.4. Linear Description of Spiral Structure

Ju et al. (2016) compare the spiral patterns in their unstratified 3D models to those expected from the linear dispersion relation for a compressible wave propagating in a 2D Keplerian flow (e.g., Ogilvie & Lubow 2002). They find a good fit when the propagation speed is equal to the local sound speed \( v_{\text{spl}} = c_s \), and they report further improvement for \( v_{\text{m}} = \sqrt{c_s^2 + v_A^2} \) (where \( v_A \) is the local azimuthally averaged Alfvén speed), which approximates the effect of a (subdominant) magnetic field. It is interesting to see whether these
statements hold when vertical stratification is added. Thus, Figure 6 focuses on the spiral structure of our models.

Mach 10 clearly contains a symmetric $m = 2$ tightly wound spiral. The situation in Mach 5, however, is more complex. There is a strong spiral arm associated with the inflow (top left of the slices; black points in middle left panel of Figure 6; the “black” arm) and a weaker arm to the bottom right of the plots (red points; “red” arm). However, we also see a spiral overdensity just to the right of the “black” arm (most dense at $r \sim 0.1, \phi \sim 0.7\pi$), wound at a different pitch angle. It is a dynamical consequence of the “red” arm. Each particle entering the “red” spiral shock at Keplerian velocity (disks orbit counterclockwise) enters an elliptical orbit with apastron at the shock. The corresponding periastrons are located precisely at the location of the “additional” spiral arm, seen as the change in radial velocity sign in the middle left panel of Figure 6. Recently, Bae & Zhu (2018) formally derived the properties of such “additional” spiral arms in protoplanetary disks as a constructive interference between epicyclic modes excited by the main spiral arms of the system. They found these structures
to merge with the primary spirals for larger (but still planetary) binary mass ratios. In our case, these “additional arms” are only still visible thanks to asymmetry between the “black” and “red” arms of Mach 5, caused by forced alignment of the “black” arm with the gas inflow. If the spiral pattern were symmetric, gas orbits would not be able to pass periastron before reaching the “black” arm and would merge with it as predicted by Bae & Zhu (2018). This highlights the benefits of using a realistic feeding geometry in our models.

In order to compare our spiral patterns with linear theory, we need to measure the location of these shocks. With the exception of the “red” arm for Mach 5, we do so by quasi-automated detection of local maxima in density, proceeding as follows:

Figure 6. Time-averaged spiral structure. Left: Mach 5; right: Mach 10. Top row: time-averaged equatorial density slice. Middle row: time-averaged equatorial slice of radial velocity. Red and black diamond markers in the second and third row show the local minima of the absolute value of radial velocity (“red” arm in Mach 5) or local maxima of density (all other cases), used to track spiral arms. A limited number of points is shown for clarity. The fit of the 2D spiral structure resulting from the linear dispersion relation is plotted as solid lines in the bottom row (see text for details).
1. The equatorial slice of density is split into separate $\rho(\phi)$ tables for each radius.
2. The data is denoised with the Savitzky–Golay filter (Savitzky & Golay 1964). Polynomial order 3 and a window length of $\sim 6\%$ of the $\phi$ range at the given radius are used.
3. The local maxima of the resulting $\rho(\phi)$ function for each radius $r$ are saved as $(r, \phi)$ points.
4. To aid separation of the resulting point collection into spiral arms, we use the clustering algorithm DBSCAN (Ester et al. 1996; Schubert et al. 2017). Clustering was found to be most helpful in (log $r$, $\phi$) space, and $\epsilon = 0.11, 0.15$ were set for Mach 5 and Mach 10, respectively.
5. Finally, the resulting clusters are visually identified with spiral arms and corrected, if needed.

A similar procedure is performed for the “red” arm of Mach 5, except that the local minima of the absolute value of radial velocity have proven to be a better metric for spiral shock detection there. In that case, Savitzky–Golay smoothing is not used, and DBSCAN’s $\epsilon = 0.11$. We plot the resulting spiral arm locations as black and red points in Figure 6 (we only show every fifth point for clarity).

We then proceed to fit the shock locations with predictions from linear models. For a compressible wave with phase velocity $c_\phi$ propagating through a two-dimensional gas disk, the dispersion relation in the linear limit can be written as (e.g., Ogilvie & Lubow 2002)

$$ (m(\Omega - \Omega_p))^2 = \kappa^2 + c_\phi^2 k^2, $$

where $\Omega$ and $\Omega_p$ denote the angular speeds of the local flow and pattern, respectively; $\kappa$ is the epicyclic frequency, $k(R)$ is the local wavenumber, and $R$ denotes the cylindrical radius. After Ju et al. (2016), we use this relationship to calculate the curve of constant phase associated with a spiral wave in a nearly Keplerian disk ($\Omega \approx \kappa$), with the pattern speed set by corotation with the binary companion ($\Omega_p = 1$). For such a pattern, the pitch angle $\xi$ obeys (Ju et al. 2016)

$$ \tan \xi(R) = \frac{c_\phi(R)}{R \sqrt{(\Omega(R) - \Omega_p)^2 - \kappa^2(R)/m^2}} $$

$$ = \frac{1}{M(R) \sqrt{(1 - \Omega_p/\Omega)^2 - 1/m^2}}, $$

where $M(R)$ is the local Mach number. Within our disks, the sound speed is set by the Shakura & Sunyaev (1973) temperature ceiling (Section 3.2.1):

$$ c_s(R) \approx c_{s,0}(R) = \frac{v_k(r_m)}{M_m} \left( \frac{R}{r_m} \right)^{-3/8}, $$

where $v_k(r_m)$ is the Keplerian velocity at the inner edge of the grid. At the same time, the flow is nearly Keplerian:

$$ \kappa \approx \Omega \approx \sqrt{GM_*/R^{3/2}}, $$

Combining Equations (31)–(34), we have

$$ \tan \xi(R) \approx \frac{(R/r_m)^{-1/8}}{M_m \sqrt{(1 - \Omega_p/\Omega)^2 - 1/m^2}}. $$

which fully defines the spiral pattern $R(\phi, \psi)$ in differential form:

$$ \frac{dR}{R d\phi} = \tan \xi(R). $$

The only free parameter is the constant of integration, that is, the azimuthal rotation of the spiral pattern as a whole, $\phi_0$. To fit it to our data, we numerically integrate Equation (36) and minimize the quantity:

$$ \Psi(\phi_0) = \sum_i D^2(P_i|S(\phi_0)), $$

where summation occurs over the spiral arm points in our data $\{P_i\}$, and $D(P_i|S(\phi_0))$ is the Euclidean 2D distance between $P_i$ and its nearest point in the theoretical spiral pattern $S(\phi_0)$.

The resulting fits are shown in the bottom row of Figure 6. Our data are very well described by these analytical considerations, only showing deviations at the outermost points of the respective spiral arms (where our spiral arms become nearly radial). This is somewhat surprising. Our model presents a significant departure from the assumptions of Equation (35). The flow is no longer two-dimensional (even though we do it to its equatorial slice), departures from Keplerian orbits can be significant (as evidenced by the “additional” spiral arm in Mach 5), and the pitch angles are not necessarily small. However, we welcome this finding as a validation of the use of 2D models in the interpretation of physical data. Following Ju et al. (2016), we also performed a fit with $c_\phi$ replaced by $\sqrt{c_s^2 + v_A^2}$, approximating the effect of magnetic fields (note that $v_A \ll c_s$). As in their case, we do see a slight improvement of the fit.

4.2. Vertical Structure of the Disk

One of the main contributions of this work lies in adding vertical structure to the unstratified models of Ju et al. (2016, 2017). Vertical profiles have previously been extensively studied in the local limit of stratified shearing-box simulations. It is thus instructive to compare our results to these local studies.

Figures 7 and 8 show time- and azimuthally averaged vertical profiles at several radii. For density and pressure (dominated by gas pressure), these are clearly Gaussian. This is a common finding in shearing-box simulations (e.g., Stone et al. 1996; Hori et al. 2014; Begelman et al. 2015; Salvesen et al. 2016) and corresponds to the assumption of vertical hydrostatic equilibrium. It is not a trivial result, as our global models could have relied on radial pressure gradients. That this is not the case aligns well with earlier global models of MRI-unstable disks in weakly magnetized environments (e.g., Flock et al. 2017; Hogg & Reynolds 2018; Zhu & Stone 2018; Jiang et al. 2019), as well as the fact that, on average, the midplane regions of our disks remain close to Keplerian rotation. Our findings thus support analytical and local numerical work, where purely vertical equilibrium is often taken as an assumption (Goldreich & Lynden-Bell 1965;}

$^6$ As implemented in the Python SciPy library, Virtanen et al. (2020), version 1.3.2.
$^7$ As implemented in the Python Scikit-Learn library, Pedregosa et al. (2011), version 0.21.3.
Magnetic pressure profiles contain a flattened core caused by magnetic buoyancy (see Section 4.1.3). As a result, the plasma $\beta$ in the midplane is dominated by gas pressure with its Gaussian profile. Both these features are commonly seen in stratified shearing-box simulations (e.g., Stone et al. 1996; Fromang et al. 2013; Salvesen et al. 2016). In the regions farthest from the disk midplane, the vertical profiles of the magnetic field differ between Mach 5 and Mach 10. For Mach 5, both $P_{\text{mag}}$ and plasma $\beta$ show variations due to long-lived turbulent structures high above the disk. We expect such fluctuations to be averaged away in longer simulations. For Mach 10, the vertical profiles of $P_{\text{mag}}$ and $\beta$ corroborate the

Figure 7. Time- and azimuthally averaged vertical profiles for Mach 5. The color of each curve corresponds to the radius at which the vertical profile was measured, as indicated by the legend.
separation into a weakly magnetized main body and a magnetic “corona.” In the plasma \( \beta \) profile (right plot in the third row of Figure 8), the weakly magnetized main body is responsible for the central Gaussian peak, while the “corona” corresponds to a step-like feature at \( \beta \sim 1 \) on either side of the midplane.

Stratified box simulations sometimes report a top-hat profile for the Maxwell stress (Stone et al. 1996; Fromang et al. 2013), which is consistent with our results for Mach 10, as well as the outer radii of Mach 5. In the latter (blue curves in the bottom right panel of Figure 7), we see some evidence of two-peaked profiles, which may indicate coronal accretion. For the inner...
radii of Mach 5, the vertical profile of Maxwell stress becomes more peaked at the midplane. In these inner regions, the disk scale height is not small compared to the local radius of the disk, and thus deviations from the profiles at larger radii, and in shearing-box models, are not unexpected.

Interestingly, we find the Reynolds stress in Figures 7, 8 is highly variable with height and radius and often reaches positive values, acting to prevent accretion in certain regions. After averaging over the disk volume, the cumulative effect of $T_R$ is thus smaller than indicated by its large peak values. Reynolds stress is also seen to act mainly in the midplane, where pressure is large and its influence on accretion rate, measured by $\alpha = T_R/P$, is inhibited. In light of these two observations, although the absolute values of the Reynolds stress in Figures 7 and 8 are about a factor of five larger than the Maxwell stress, their actual influence on disk accretion is comparable (see Figures 9, 10, and the discussion in Section 4.3). We further discuss the relative roles of Reynolds and Maxwell stress in driving accretion in Sections 4.3.1 and 4.4. In both models, Reynolds stress is strongly driven by spiral waves. Since they are a global feature, it is not a surprise that the oscillatory vertical profiles of $T_R$ do not match the flat profiles reported by, for example, Stone et al. (1996) in shearing-box simulations of the MRI.

A final point to make regarding vertical profiles is that at different radii (denoted by curves of different colors in Figures 7 and 8), for some variables the curves are often strikingly self-similar, as especially evident for Mach 10 (Figure 8). This is a strong argument confirming that the vertical structure of the disk is insensitive to radius. While such independence is required by some analytical models (most notably, Shakura & Sunyaev 1973), we note that it is not sufficient for their applicability. Local-only interactions, where each radial annulus of the disk interacts only with neighboring annuli by means of viscosity, may also be necessary. As we discuss in Section 4.3, this assumption is not satisfied here. In addition, the vertical profiles of $T_R$ and $M$ are clearly not self-similar. As they are both at least partially driven by spiral structure, their behavior is specific to the global character of our models.

Ultimately, treatment of radiative cooling is needed to more realistically capture vertical profiles of quantities within the disk. As radiation may transport information about disk (and white dwarf surface) conditions between distant points in the system, including it may change the picture drawn by the results presented here. Realistic disk thermodynamics would also allow convection to occur. If present, it can significantly enhance the Maxwell stress-to-pressure ratio $\alpha_M$ at certain disk temperatures (e.g., Coleman et al. 2016, 2018; Scepi et al. 2018a). We are looking forward to investigating these effects as part of our future work (see Section 5).

### 4.3. Radial Structure of the Disk

As discussed in Section 3.2.1, our temperature ceiling corresponds to a gas-pressure-dominated $\alpha$-disk with free-free opacity. For such a disk, the $\alpha$ prescription predicts that density and pressure will follow $\rho \propto r^{-15/8}$ and $P \propto r^{-21/8}$. These slopes are plotted (with arbitrary normalization) as green dotted lines in Figures 9 and 10. Generally, the Shakura & Sunyaev (1973) model describes the gas (total) pressure profiles of the disk main body (solid lines in Figures 9 and 10) fairly well for $r > 0.12$. However, only the main-body density profile for phase B of Mach 10 (top right panel of Figure 10, solid curve) and the outer radii of the Mach 5 disk (top panel of Figure 9, solid curve) are fit well. The different slope of $\rho$ in phase A of Mach 10 likely results from the difference in accretion rate between the inner and outer radii of the disk (see Section 4.1.1). At $r \lesssim 0.12$, both pressure and density drop inward for all of our models. At that point, the length scales of the problem approach the local radius, which breaks the $\alpha$-model’s locality assumptions. The same applies to
coronal quantities (dashed curves in Figures 9 and 10), where the large vertical extent of the structures (comparable with $R$) is inconsistent with the assumptions of Shakura & Sunyaev (1973). These differences are not unexpected, and we stress that the $\alpha$-disk model remains a good description of the radial structure of our flow in regions where its assumptions are satisfied. This is in line with other studies where an agreement with Shakura & Sunyaev (1973) is reported in appropriate regimes. This is seen,
for instance, in the limit of weak wind-driving in the models of Scepi et al. (2019, see their Figure 2) or the outer regions of the semiglobal simulations of Hogg & Reynolds (2018). We note that the level of alignment with the Shakura & Sunyaev (1973) models may change with a more realistic treatment of the system. On one hand, radiative cooling (planned for our future work; see Section 5) will introduce nonlocal interactions that may break the locality assumption of \( \alpha \)-models. On the other, accretion disks in astrophysical semidetached binaries are colder and geometrically thinner than those presented here (as discussed in Section 1.2), which alleviates some of the tension related to large \( H/R \) ratio, bringing true accretion disks closer to the Shakura & Sunyaev (1973) solution.

### 4.3.1. Spiral Structure Drives Accretion at Very Low Mach Numbers

As we discussed in Section 1.2, numerical resources limit our global models to accretion disks that are hotter (of \( M_{\text{in}} = 5, 10 \)) than real astrophysical systems (e.g., CVs, where \( M_{\text{in}} \geq 50 \)). While the Mach 5 model is unlikely to be realized in nature, it can provide a useful context for the Mach 10 data.

With this in mind, we use Figures 7 and 9 to explain the mechanisms driving accretion in our hot-disk model Mach 5. As seen in Figure 9, the body of the disk (solid brown line, \( \Theta \leq 2.5H/R \)) dominates the accretion rate for most of the grid. Its \( \dot{M} \) has a minimum at \( r \sim 0.15 \), where its impact becomes equal to that of the “corona” (dashed brown line, \( \Theta \in [2.5, 5] H/R \)), the latter’s \( \dot{M} \) being maximal at \( r \sim 0.15 \). This radius also corresponds to the transition from a two-peaked to a triangle-like vertical profile in Maxwell stress (Figure 7; see Section 4.2), a likely related observation. In Figure 9 we see that Reynolds stress dominates over Maxwell stress both within the main body and the “corona.” In the former, the difference is about a factor of 2–3, comparable with the MHD model of Ju et al. (2016). The radial velocity slice in Figure 6 associates this Reynolds-stress-driven accretion with the grand-design spiral structure, pointing to it as the main accretion driver in Mach 5.

### 4.4. Long-term Evolution of the Mach 10 Model

Within our current computational limits on the Mach 10 simulation’s run time, the disk is not seen to reach a statistically steady state. However, as we are only able to study a single viscous timescale of the disk (see Section 4.1.1), it is entirely possible that such a state would be achieved if the model was allowed to continue. Still, the long timescale (\( \sim 10P_{\text{orb}} \)) variability observed in our Mach 10 model may be of interest for understanding disk accretion. Not only is it potentially present during a change in disk accretion rate, but there is some indication that it may be recurring in nature (see Section 4.1.1). In this section, we attempt to understand the differences between phases A and B (Figure 2).

Let us first establish whether the changes are driven by the main body of the disk or by its “corona.” Figure 10 shows the radial profiles of the Mach 10 run, time-averaged separately for phases A and B. The coronal accretion rate (bottom row, dashed brown curve) seems to remain unchanged at a level of \( \sim 2 \times 10^{-5} \). It is driven by Maxwell stress (third row of Figure 10), with \( \alpha_M \sim 2–3 \) times larger than \( \alpha_R \) for \( r \lesssim 0.2 \). As the coronal \( \alpha_M \) and \( \alpha_R \) are similar for phases A and B, it must be the main body of the disk that drives changes in accretion rate. In phase A, \( \alpha_R \) remains nearly constant with radius at \( \sim 1 \times 10^{-3} \), while \( \alpha_M \) is very small close to the inner disk edge, slowly growing with radius up to \( r \sim 0.17 \), where it flattens at \( \sim 2 \times 10^{-4} \). Meanwhile, in phase B, both main-disk \( \alpha \) parameters are nearly equal throughout the disk at \( \sim 2–3 \times 10^{-3} \). We note that this latter situation is very reminiscent of the results of Ju et al. (2017), who found that Reynolds and Maxwell stress played comparable roles in driving accretion in their models at \( M_{\text{in}} = 10 \) (note, however, that even in phase B, the coronal accretion, absent in their models, still drives \( \sim 1/3 \) of our total accretion rate). The enhancement in \( \alpha_M \) is accompanied by a factor of two increase in the main-disk \( M \) in phase B, causing a 50% rise in the total accretion rate. The main-disk magnetic field is also stronger in phase B, as shown in Figure 11.

In Figure 12, the Maxwell-stress vertical profile is seen to change from two-peaked (efficient coronal accretion) for phase A to a top-hat profile (main-body accretion) in phase B (familiar from stratified shearing-box simulations, e.g., Stone et al. 1996). Reynolds stress also increases, pointing to the interplay between MRI turbulence and spiral shocks reported by Ju et al. (2016, 2017).

The transition from phase A to B thus appears to be driven by increased MRI activity in the disk body (Figure 11), which also acts to enhance the role of spiral shocks (Ju et al. 2016, 2017), both of which conspire to raise the accretion rate in the disk midplane. To our knowledge, this is the first time such an event has been observed in a global MRI-unstable model of an accretion disk, and it is likely to be related to our continuous supply of mass through the Roche-lobe overflow (which distinguishes our models from previous MHD work).

The exact underlying cause for this behavior remains elusive to us. The duration of this transition is much longer than MRI saturation times within the disk (which should be limited to at most \( \sim 15 \) local Keplerian orbits), and MRI turbulence is well established in both phases A and B (see Figure 11). It is thus unlikely that the transition is related to any form of MRI turbulence growth. A relation to magnetic field growth through the MRI dynamo also appears unlikely, as many field-reversal cycles are seen in Figure 5 during phase B of Mach 10. At face value, it appears that a “preferred” accretion rate exists within the disk (realized in phase A; see Figure 2), and it is only surpassed once the density and magnetic field accumulate.
beyond a certain level. In our models, this first happens at the outer edge and consequently starts an avalanche as the additional mass accretes through smaller radii, removing accumulated mass (at an accretion rate higher than that of the inflow), restoring initial density levels, and allowing for resumption of accretion at the “preferred” level. Such a disk-specific accretion rate could perhaps be set by magnetization of the inflow or the disk itself, the latter in turn depending on the disk Mach number and interactions with the magnetized “corona.” More realistic cooling and the resulting convection

Figure 12. Time- and azimuthally averaged vertical profiles for the Mach 10 run, where phases A (left) and B (right) are treated separately. The color of each curve corresponds to the radius at which the vertical profile was measured, as indicated by the legend.
could enhance the MRI-driven accretion rate (Coleman et al. 2018; Scepi et al. 2018a), increasing such a “preferred” level of accretion. Thus, if the above explanation is correct, more realistic thermodynamics would either decrease the difference in $M$ between phases A and B or cause their duration to be longer.

If present in real semidetached binaries, such an outside-in mechanism for “waves” of enhanced midplane accretion rate could perhaps work alongside the thermal instability of the disk, assisting in propagation of an ionization front during a state change in dwarf novae. Future studies, with longer run times and higher resolution, are needed to assess what role such episodes may play in true accretion disks and potential steady states of semidetached binaries.

4.5. Observational Appearance

Our models exhibit strong variability at a wide range of timescales (Section 4.1.1). However, it is difficult to assess how the changes in local accretion rates (Figure 2) translate into variability of the observed disk-integrated spectrum. In optically thick disks, it is not obvious how the increase in accretion rate or midplane density would modify the disk appearance (as set at the photosphere). Most likely, the local temperature and position of the disk photosphere are modified, resulting in observational changes. Because our present models feature no radiative transport or realistic thermodynamics, we leave variability assessment to future studies (see Section 5).

Spiral structure has been extensively documented in CVs by means of eclipse mapping and Doppler tomography (e.g., Steeghs et al. 1997; Baptista et al. 2005; Klingler 2006; Khruzina et al. 2008; Ruiz-Carmona et al. 2020). It is then of interest to ask how the spiral structure of our models of semidetached binaries (Sections 4.1.2, 4.1.4) would be seen in such observations. Due to our approximate treatment of temperature (Sections 3.2.1, 4.1), we cannot represent any nonaxisymmetric distribution of surface brightness on our disks. However, we can quantify the influence of the photosphere’s shape and gas velocity on observations. As a proof of concept, we attempt to generate a Doppler diagram from our data. We use isocontours of time-averaged density at $\rho = 0.015$ as an approximate handle on the location of the photosphere. For each isosurface point $P_{i,j}$, we approximate the corresponding area element $\Delta S_{i,j}$:

$$\Delta S_{i,j} \approx 0.5 \left( l(P_{i,j}, P_{i-1,j}) + l(P_{i,j}, P_{i+1,j}) \right) \times 0.5 \left( l(P_{i,j}, P_{i,j-1}) + l(P_{i,j}, P_{i,j+1}) \right),$$

where $l(P_{i,j}, P_{k,l})$ is the 3D distance between points $P_{i,j}$ and $P_{k,l}$, and close indices correspond to neighboring points. We assume the observed surface brightness is proportional to the local $T^4 \propto c_s^2$ (where we use the time-averaged sound speed $c_s$), so the flux associated with each point is proportional to $dS \times c_s^2$. We then calculate the Cartesian velocities of the fluid $v_x$ and $v_y$ for each point on the isocontour (using time-averaged velocities from our simulations), and we bin them into 64 velocity channels for each direction. Finally, we sum the total “flux” $(dS \times c_s^2)$ corresponding to each such bin in velocity space, obtaining a $64 \times 64$ Doppler diagram of the disk seen nearly edge-on.

The resulting Doppler diagrams are shown in Figure 13. The outer edges of the disk appear bright at the center of these velocity-space plots, due to their large emitting surface. The inflow is well visible as a bright line near the diagrams’ centers, starting slightly below $(v_x, v_y) = (0, 0)$ and moving down and to the left. The disk’s Keplerian velocities correspond to the space between the two dashed white circles in the diagrams in Figure 13. Once the inflow reaches these regions, it is seen to connect to the spiral structure, which is also clearly recognizable in the plots. We thus confirm that, at least under the assumptions of this extremely simple observation model, the structure of the disk we describe in Section 4.1.2 could indeed be potentially observable via Doppler tomography. We note that in real semidetached binaries, disk Mach numbers are higher than in our models (Section 1.2), and thus the spiral structure is more tightly wound than can be seen in Figure 13. Moreover, the outer edge of the disk (the innermost regions of Doppler diagrams) is generally too dim to be observed in as much detail as can be seen here, and the diagrams themselves would generally be available in lower resolution than allowed by our models. With these caveats in mind, however, our Figure 13 can be indeed found to be similar to the Doppler diagrams of, for example, V2051 Oph by Rutkowski et al. (2016, see their Figure 7) or EC21178-5417 by Ruiz-Carmona et al. (2020) and Khangale et al. (2020, Figure 6(a)). We also note that our artificial Doppler diagrams are similar to those obtained with hydrodynamical models by, for example, Matsuda et al. (1999), Haraguchi et al. (1999), Steeghs & Stehle (1999), Kunze et al. (2001), Lanzafame (2003), and Foulkes et al. (2004). This supports our discussion on the similarities of the general flow properties with the hydrodynamical models in Section 4.1.

5. Conclusions

While spiral shocks can provide angular momentum transport in accretion disks at very low Mach numbers ($M_{\text{in}} \lesssim 10$), the MRI is likely necessary to drive accretion in colder, fully ionized environments (e.g., cataclysmic variables, Ju et al. 2017), potentially in concert with other mechanisms, such as magnetic interactions with disk winds. Thus, there is a need to augment the large body of work concerning global hydrodynamical models of semidetached binaries with MHD, to include this transport mechanism self-consistently.
To address this need, we have performed the first stratified global MHD simulations of accretion disks fed by an accretion stream due to Roche-lobe overflow. In doing so, we can for the first time observe the MRI turbulence self-consistently interact with a fully three-dimensional global accretion disk structure in idealized models of semidetached binaries. Despite the limitations inherent to large numerical studies (which we discuss in Section 1.2), we find robust global behaviors in our idealized models that may be helpful in understanding accretion in true semidetached binaries, such as cataclysmic variables. Thus, focusing on global dynamics, we report several interesting observations from our results for models with Mach numbers of 5 and 10:

1. The accretion rate through both disks is found to be extremely variable at all timescales. While Mach 5 reaches a quasi-stationary state, Mach 10 exhibits $M$ variability even at the timescale of the entire simulation window we analyze ($\sim 10$ binary orbits).

2. Both disks exhibit a spiral structure, with the position and inclination of spiral shocks changing rapidly. The mid-plane slices of the time-averaged spiral structure, however, are fit extremely well with a two-dimensional linear dispersion relation for a compressible (sound) wave propagating through a Keplerian disk (e.g., Ogilvie & Lubow 2002).

3. Mach 10 is clearly separated into a gas-dominated disk body below $\sim 2$ thermal scale heights and a strongly magnetized (relative to the disk main body) “corona.” Coronal accretion is seen to provide 30%–50% of the total accretion rate.

4. The butterfly diagrams of our models are fairly irregular and asymmetric, with field reversals occasionally absent. We hypothesize that the absence of some field reversals may be related to inclined spiral shocks mixing the disk and coronal magnetic field, similar to convective motions in Coleman et al. (2017).

5. Our results show many similarities to stratified shearing-box models. The vertical profiles of density, gas, and total pressure are Gaussian, while Maxwell stress follows a top-hat vertical profile at most radii. The disk separates into a weakly magnetized main body and a magnetic “corona,” with the butterfly diagram at times showing field reversals every 6–10 local orbital periods. However, some effects (e.g., those attributed to spiral structure) are clearly global. The Reynolds stress is highly oscillatory with radius and height, locally reaching positive values. Close to the disk’s inner edge and in the corona, we see densities and pressures lower than those from the $\alpha$ prescription, and the Maxwell stress vertical profile can become triangular in shape.

6. The longest timescale ($\sim 10^7$-orb) variability in Mach 10 is an outside-in accretion event (akin to an avalanche) seen as a temporary enhancement in midplane accretion through MRI turbulence (which in turn enhances spiral shock accretion; see Ju et al. 2016, 2017). We see some indication of a recurrent nature of these events. Longer, higher-resolution studies are needed to verify their role in true systems and the potential existence of steady states at high Mach numbers.

Finally, we attempt to relate our simulations to observational results, constructing a toy-model observation of a Doppler diagram from our data. Despite the simplicity of this procedure and our use of Mach numbers lower than expected in accretion disks realized in nature, we reach some qualitative agreement with the main features observed in Doppler diagrams of CV disks (e.g., Rutkowski et al. 2016; Khangale et al. 2020; Ruiz-Carmona et al. 2020). While we use CV observations for comparison, we note that our models are not limited to WD primaries and can in principle be used (with their limitations kept in mind; see Section 1.2) to describe accretion disks in other semidetached binaries.

Our current models feature a very simple treatment of disk temperature, where an adiabatic equation of state is limited by a radius-dependent temperature floor and ceiling. In future work, we intend to replace this prescription with realistic radiative cooling using the radiative transfer module of Athena++. In addition to our ability to track disk temperature self-consistently, this extension will enable us to construct mock observations from our data, providing a potent means of comparison with real astrophysical objects.

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The 3D rendering in Figure 1 of this paper has been generated using Mayavi (Ramachandran & Varoquaux 2011).

Software: Athena++ (modified development version, see Section 3; Stone et al. 2020), SciPy (version 1.3.2, Virtanen et al. 2020), SciKit-Learn (version 0.21.3, Pedregosa et al. 2011), Numpy (version 1.17.4, Oliphant 2006; van der Walt et al. 2011), Matplotlib (version 3.1.1, Hunter 2007), Mayavi (version 4.7.1, Ramachandran & Varoquaux 2011), h5py (version 2.10.0, https://www.h5py.org), Pickle (version 0.7.5, van Rossum 2020), Jupyter Notebook (version 6.0.3, Kluyver et al. 2016).

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