THE POMERON IN ELASTIC AND DEEP INELASTIC SCATTERING

M. Bertini\textsuperscript{a}, M. Giffon\textsuperscript{a}, L. Jenkovszky\textsuperscript{b}, F. Paccanoni\textsuperscript{c}, E. Predazzi\textsuperscript{d}

\textsuperscript{a}Institut de Physique Nucléaire de Lyon, IN2P3-CNRS et Université Claude Bernard, 43 Bld du 11 Novembre 1918 F-69622 Villeurbanne Cedex, France

\textsuperscript{b}Bogoliubov Institut for Theoretical Physics, Academy of Sciences of the Ukrain 252143 Kiev, Ukrain

\textsuperscript{c} Dipartimento di Fisica, Università di Padova, INFN Sezione di Padova, via F. Marzolo I-35131 Padova, Italy

\textsuperscript{d}Dipartimento di Fisica Teorica, Università di Torino, Italy and INFN, Sezione di Torino, 10125 Torino, Italy

Abstract

We discuss some properties of the Pomeron in high energy elastic hadron-hadron and deep inelastic lepton-hadron scattering. A number of issues concerning the nature and the origin of the Pomeron are briefly recalled here. The novelty in this paper resides essentially in its presentation; we strive at discussing all these various issues in the following unifying perspective: it is our contention that the Pomeron is one and the same in all reactions. Various examples will be provided illustrating why we do not believe that one should invoke additional tools to describe the data.
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Abstract

We discuss some properties of the Pomeron in high energy elastic hadron-hadron and deep inelastic lepton-hadron scattering. A number of issues concerning the nature and the origin of the Pomeron are briefly recalled here. The novelty in this paper resides essentially in its presentation; we strive at discussing all these various issues in the following unifying perspective: it is our contention that the Pomeron is one and the same in all reactions. Various examples will be provided illustrating why we do not believe that one should invoke additional tools to describe the data. For pedagogical convenience, we list below the topics to be covered in the following.

1. Introduction. How many Pomerons?
2. The Pomeron in the $S$-matrix theory
3. The Pomeron in QCD
4. The Pomeron in deep inelastic scattering
5. The Pomeron structure
6. (Temporary?) Conclusions
1. Introduction. How many Pomerons?

Recent times have witnessed a tremendous revival of interest in diffraction and related issues. A field that had been agonizing for decades, now, has been revitalized by the injection of the great excitement coming from the new HERA experimental data and from the realization that low-$x$ physics in deep inelastic phenomena is strongly and almost directly related to the hadronic high energy soft (i.e. low $p_T$) physics known as diffraction. Hundreds of papers and tens of different approaches have focused especially on the meaning, origin and structure of the Pomeron i.e. of the agent credited to mediate diffraction.

This paper can only partly be considered a real review paper on diffraction or on the Pomeron; more properly, it is the discussion and the review of a selection of presently hot topics related to the problem of diffraction and the Pomeron in which the present authors have been active for a long time. As such, the perspective in which this paper is conceived reflects mostly the view of its authors; even so, we believe that the presentation of this material is pedagogical enough and kept to such a general level as to be especially helpful to the reader who would like to see these fundamental issues reduced to the essential and discussed in as simplified context as possible. The reason why this is not a full size review paper is that, after giving the matter a long and careful consideration, we have decided that the huge number of papers appeared in recent times, would have made it an impossible task that of writing a bona fide, unbiased review paper in which the proper credit would have been given to all authors and approaches. The state of the art is presently such that one should really plan to write a series of review papers each focusing on a different aspect of the problem. The field is in such a rapid swing that it would be impossible to do a good job nor is it possible at this point to predict whether or when the dust will have settled enough as to allow one a definite, unemotional, unbiased and complete vision of the underlying physical problems. All complications arise, obviously, from the fact that conventional diffraction is, inherently and unavoidably, non-perturbative and we have not yet learned to master non-perturbative quantum chromodynamics. In the course of the paper, as we mentioned, we will focus on several issues related to the various ways diffraction manifests itself and we will go deeper in some of them. While we will try to present these issues in as wide as possible perspective, we acknowledge that we will largely ignore the point of view of several very active physicists in the field. We apologize from the start to all those that will feel (rightly so) that their work was not adequately covered; as we have explained, this would not have been possible. The reader who would like to broaden his perspectives beyond what we can do in this paper, is urged to consult, for instance Ref. [1] where a large spectrum of the present days ideas on diffraction was presented and discussed.
There is much confusion about the Pomeron (≡ diffraction). One of the most persistent is that there are two Pomerons - a soft and a hard one. The two-fold interpretation of a single object (phenomenon) however has a clear origin: the conventional Pomeron studied in hadronic physics (as mentioned already, we take this term to be equivalent to diffraction) is a soft phenomenon, outside the range of applicability of perturbative quantum chromodynamics, what has come to be known as the hard Pomeron is something that can be calculated from perturbative quantum chromodynamics. The recent small-$x$ data collected at HERA were interpreted by some authors as a manifestation of the hard Pomeron, and thus as an argument in favor of the existence of two Pomerons.

In our opinion there is only one object and the diversity of its manifestations reflects merely the diversity of the reactions in which it can occur and of the physical and kinematical situations in which it is investigated. More specifically, we visualize the Pomeron as a very complicated entity (which will, in fact, be a function of different sets of variables depending on the reaction one is looking at) which in different dynamical situations may have different manifestations but whose origin is always the same, diffraction,

"In my opinion, the Pomeron - as a leading singularity in the j-plane - is unique, but it contains contributions both at large and small distances. The relative weight of these contributions depends on the given process (at given energies) and hence there is no universality. These contributions mutually renormalize and without a scale it is impossible to determine which one is more important."

The idea of a unique Pomeron is also discussed in the recent talk of T.T. Wu [2a]:

"It should be emphasized that there is only one Pomeron. In particular, the so-called soft Pomeron and the hard Pomeron are merely aspects of the same object."

In the present paper, we will try to clarify these points presenting them first in the framework of the old (but still useful) methods of the analytic S-matrix theory and then combining this description with the quark-parton picture of the hadron structure and with quantum chromodynamics (QCD). We will, thus, start from a brief introduction of the basic properties of the theory of complex angular momenta (by now an almost forgotten language) and then we will rephrase the same ideas of diffraction in the modern QCD language.

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1 Translated from russian and quoted freely from a private communication to one of us, L. L. J., by Lev Lipatov.
2. The Pomeron in the $s$-matrix theory

The Pomeron was introduced in the early sixties in the framework of the complex angular momentum theory, to describe high energy elastic and diffractive hadron scattering\(^2\). Empirically, diffraction implies non-decreasing (with energy) total and elastic cross sections, a very rapidly decreasing (with momentum transfer) forward peak (usually parametrized as an exponential) and an (almost) imaginary forward elastic scattering amplitude.

Let us make a brief pragmatic introduction to the basic ideas leading to the formalism of complex angular momenta. For further reading on this subject we recommend the excellent book by P.D.B.Collins [2b] and several review papers which have appeared recently [2c].

Once the scattering amplitude is expanded in partial waves, if one assumes that the partial wave amplitude $a(j, t)$ is dominated by an isolated, simple, moving pole located in the complex angular momentum plane $j$ at some value $\alpha(t)$

$$a(j, t) = \frac{\beta(j)}{j - \alpha(t)}, \quad (2.1)$$

using a well-defined mathematical prescription known as the Sommerfeld-Watson transform [2b], one finds that the asymptotic behavior of the elastic scattering amplitude $A(s, t)$ in the limit $s \gg m^2, t \approx 0, s \to \infty$ ($s$ and $t$ are the square of the total centre-of-mass energy and four-momentum transfer in the process respectively), is given by

$$A(s, t) = \xi(\alpha)\beta(\alpha)(s/s_0)^\alpha, \quad (2.2)$$

where $\xi(\alpha)$, known as the signature factor, can be written as

$$\xi(\alpha) = 1 + \frac{\xi \exp^{-i\pi \alpha}}{\sin \pi \alpha}, \quad \xi = \pm 1. \quad (2.3)$$

In the above Eq. (2.2), $\beta(t)$ is known as the residue function, $\alpha \equiv \alpha(t)$ is called Regge trajectory and $s_0$ is a scale parameter.

Several comments are in order.

1. The same asymptotic result (2.2) obtains if, instead of an isolated, simple pole (2.1), the partial wave amplitude is endowed with a finite number of isolated moving poles

\[^2\] In the high energy collision of two hadrons, the distinction between diffractive and elastic scattering is that, in the former, one of the initial hadrons has been replaced, in the final state by a hadronic system whose quantum numbers (except, possibly for the spin) are the same as those of the impinging hadron. This gives rise to single diffraction, similarly for double diffraction when the same occurs for both colliding hadrons.
of which (2.1), however, is the one with the *largest real part*. This will be assumed in what follows.

2. Simple Regge pole exchange implies that the scattering amplitude (2.2) factorizes into a "propagator" $s^{\alpha(t)}$ and two vertex functions (Fig. 2.1); hence the residue function $\beta(t)$ may be interpreted as the product of the two vertices $\beta(t) = g_1(t)g_2(t)$.

3. In Regge pole models the $s$-dependence is (asymptotically) rather simple and well-defined, not so, unfortunately, the $t$-dependence. As a matter of fact, the residue functions in Regge pole models are in general quite arbitrary (and so will be the dependence on other variables which may arise in more complicated reactions; such will be the case of the $Q^2$ dependence in *hard diffraction* which will be considered in Section 4). Following dual models, we will assume that the residues depend on $t$ entirely through the trajectories $\alpha(t)$. A simple and reasonable choice is

$$\beta(\alpha) = Ae^{b\alpha(t)}, \tag{2.4}$$

where $A$ and $b$ are free parameters.

4. The numerical value of the scale parameter $s_0$ in Eq. (2.2) is irrelevant since it can be absorbed in the slope parameter $b$ in Eq. (2.4) (this will not be true any more for more complicated Regge singularities); it is reasonable to choose $s_0 = 1 \text{ GeV}^2$ this being the usual scale of strong interactions.

5. Notice that the denominator in the signature factor vanishes at all integer values of $\alpha(t)$. Correspondingly, however, the numerator vanishes for odd and, respectively, even integers according to whether we are considering positive or, respectively, negative signatures. The recurrent blow up of $\xi(\alpha)$ (and, consequently, of the amplitude) every time $\alpha(t)$ crosses the appropriate (even or odd) integer, is at the basis of the statement that a Regge trajectory interpolates among families of resonances. We can thus conclude that a *positive signature trajectory* interpolates between evenly spaced resonances that, (all other quantum numbers being equal), have even spin (and, conversely, a negative signature trajectory interpolates among evenly spaced resonances of odd spin)\(^3\).

The signature factor $\xi(\alpha)$ in Eq. (2.3) may be approximated (for C-parity $C = +1$ of the exchanged particle) by $\xi(\alpha) = e^{-i\pi\alpha/2}$, whenafter the expression for a single Pomeron pole exchange scattering amplitude (2.2) becomes extremely simple

\(^3\) Here we limit ourselves to *mesonic* trajectories for simplicity but the argument could be easily extended to baryonic trajectories.
\[ A(s, t) = Ae^{B(\bar{s})\alpha(t)}, \]  
with \( B(\bar{s}) = b + \ln(\bar{s}), \ \bar{s} = -i\frac{s}{s_0}. \)

6. The trajectories \( \alpha(t) \) extrapolate between the resonance \((t > 0)\) and the scattering \((t < 0)\) regions. They are analytic functions of their argument \( t \) with threshold singularities \([3]\). As it appears from the Chew-Frautschi plot (spins vs (masses)\(^2\)) which follows from fits to the scattering data, the trajectories are nearly linear within a fair range of \( t \) (Fig. 2.2),

\[ \alpha(t) = \alpha(0) + \alpha' t. \]  

\( \alpha(0) \) is known as the intercept and \( \alpha' \) as the slope of the trajectory. The Pomeron is the Regge pole with the largest intercept (arguments have been given that, asymptotically, they must behave logarithmically to meet wide angle scaling behavior in dual models \([3]\)). The fact that the lowest mesonic resonances all lie on essentially the same straight line independently of their quantum numbers (signature included), has come to be known in the literature as exchange degeneracy.

7. The \( S \)-matrix theory, and the Regge pole model in particular, deal with external particles which are on the mass shell, \( p^2 = m^2 \).

8. A single pole exchange (i.e. a simple moving pole in the complex angular momentum plane) is the simplest ingredient normally used as a basic part of the dynamics and, as we saw in Eq. (2.2), it leads to predict a power behavior of the scattering amplitudes. More realistic models may involve more complicated \( j \)-plane structures, either generated by unitarity (Regge cuts) and/or coming from a more involved input (like a double pole\(^4\)). Perturbative QCD calculations \([4]\) indicate that the Pomeron has indeed a complicated \( j \)-plane structure. Phenomenology of hadronic reactions, on the other hand, also points, independently, towards a complicated \( j \)-plane structure whereby integrated hadronic cross sections grow as \( \ln s \) or \( \ln^2 s \) (a kind of growth traditionally attributed to cuts in the complex angular momentum plane).

9. Multiperipheralism is a basic ingredient behind the Regge pole theory. It implies that the dominant production mechanism is the one shown in Fig. 2.3, in which each particle along the chain is produced peripherally, i.e. at small momenta transfer with respect to the adjacent one. To anticipate the forthcoming discussion of hard diffraction, we note that this type of multiple production mechanism is quite different from the one related to QCD

\(^4\) Which we will call a dipole. Similarly, we will briefly call multipoles, poles of multiple order. Increasing the order of poles increases the power of the logarithmic growth with energy of the cross section.
evolution and typical of deep inelastic scattering (but is, on the other hand, quite close to the one used in the so-called BFKL mechanism to be briefly discussed below).

10. Synthetizing: the asymptotic behavior in the (direct) s-channel \( (i.e. \text{ when } s \to +\infty) \) is given by Eq. (2.2) and is determined by the singularity in the complex \( j \)-plane with the rightmost real part \( i.e. \) by the Reggeon with the largest part of \( \text{Re} \alpha \) exchanged in the crossed \( (t \text{ and/or } u) \) channels. This is what makes the theory of complex angular momenta so fruitful for phenomenological analyses.

In a truly asymptotic regime, the leading trajectory (the Pomeron, in fact) is supposed to dominate. In realistic processes, however, \( i.e. \) at subasymptotic energies, all trajectories whose exchange is allowed by quantum number conservation may have to be taken into account if a good description of the data is desired. For example, high-energy \( pp (\bar{p}p) \) scattering can reasonably well be described [5] by the exchange of the following four trajectories

\[
 A_{pp}(s, t) = f \pm \omega + \mathbb{P} \pm O, \tag{2.7}
\]

where the symbols on the r.h.s of (2.7) denote the \( t \) channel exchanges, which are relevant for these reactions, the reggeons \( f \) and \( \omega \), the Pomeron \( \mathbb{P} \) and the Odderon \( O \).

Table 1 shows the \( C \)-parities and intercepts of the 4 trajectories in Eq. (2.7). Trajectories with negative parity contribute differently to particle and antiparticle induced reactions (unfortunately, at high energies only \( \bar{p}p \) data exist, which makes the identification of the Odderon, \( C \)-odd counterpart of the Pomeron, not so straightforward [5]).

The Pomeron has the same (vacuum) quantum numbers as the \( f \)-meson; this makes it sometimes difficult to discriminate between these two exchanges. The intercept of the \( f \)-trajectory is correlated by the spectrum of a family of resonances lying on it (see Fig. 2.2), constraining its intercept to be around the value 0.6. This leaves some flexibility in the discrimination between \( f \) and \( \mathbb{P} \) (\( \mathbb{P} \)-\( f \) mixing).

General unitarity arguments [6], constrain the physical Pomeron intercept, \( \alpha_{\mathbb{P}} \leq 1 \). What has come to be known as the bare Pomeron intercept, however, may be greater than one, (in order to reproduce the rise of the total cross sections with energy). Such a supercritical Pomeron behavior has then to be properly tamed by unitarity before the game is over.

The best way to have a clear signal of the Pomeron is to increase the reaction energy. In fact, for the first time, at the Tevatron the total cross section can be described by the Pomeron only, the relative contribution from the secondary trajectories being smaller than

\footnote{In other reactions (or kinematical domains), other, additional, trajectories may have to be used such as the ones with the quantum numbers of the mesons \( \rho, a_2 \) etc...}
the experimental error bars. The data favor a moderate increase of the total cross section, its rate ranging between \( \ln s \) and \( \ln^2 s \), or - in terms of a supercritical Pomeron - \( s^\epsilon \) with \( \epsilon \approx 0.08 \) [7]. Large values of the parameter \( \epsilon \) (e.g. \( \epsilon \geq 0.3 \), as calculated [4] from perturbative QCD) are ruled out even by fits to the data on total cross sections.

Recently, total cross sections for real photon-proton scattering were measured at HERA. Fig. 2.4 shows the data on proton, antiproton and real photon-proton cross sections with representative fits taken from Ref. [8]. The importance of the HERA data on total cross sections is two-fold. On the one hand, these data reach very high energies (comparable only to the \( \bar{p}p \) Tevatron energies) providing information about the Pomeron and, in addition, they give a direct link between hadron- and lepton-induced reactions; on the other hand, they allow a direct probing of composite structures (the proton) by an elementary probe (the electron).

An important and non-trivial argument in favor of the complex angular momentum theory is the observed shrinkage of the diffraction cone. From its rate, the slope of the Pomeron trajectory is calculated to be \( \alpha' \approx 0.25 \text{ GeV}^2 \). This implies that as the energy increases, more and more particles tend to be scattered in the forward direction (or, to be more precise, inside a cone centered around the forward direction which narrows progressively as the energy increases). This is a prominent feature of hadronic diffraction. At the same time, the near universality of the slope of all Regge trajectories other than the Pomeron makes the latter a rather peculiar trajectory and actually hints at a possibly different (or more complicated) origin (diffraction) or type of complex \( j \)-plane singularity for the Pomeron (may be a cut originated by unitarity rather than a simple moving pole).

Hadron diffraction, mediated by Pomeron exchange(s) may be observed also in single and double diffraction dissociation (Fig. 2.5) (see footnote 2) as well as in a specific configuration of multiperipheral processes called multi-Pomeron exchange (Fig. 2.6). The common feature of all these events is that they are described by the exchange of something with the quantum numbers of the vacuum, whose contribution is peaked near \( t = 0 \) and does not decrease with energy.

While it is rather direct to verify that non-leading Reggeons come from singularities in the complex angular momentum since secondary Regge trajectories are made of valence quarks (and interpolate resonances of a conventional type, see Fig. 2.2), the Pomeron (and Odderon) trajectory is considerably more complicated since it is supposed to be composed mainly of gluons (eventually, with some quark admixture). In any case, one expects that, in analogy

\[6\] The reader should be warned, once again that serious data-fitting cannot avoid fitting together integrated cross sections and angular distributions at the same time. The literature is crammed with wrong statements coming from overlooking such a necessity.
with secondary Reggeons, observable particles (glueballs) should be found on the Pomeron (and Odderon) trajectory for integer values of spin larger than one. Since the parameters of the trajectory are well defined from the scattering region, the predictions for glueball masses (and widths, in the case of nonlinear trajectories [9]) are quite definite. The absence of clear experimental signals of the predicted glueballs is somewhat of an embarrassment for the theory although some evidence has been claimed recently [10] (Fig. 2.7).

Deep inelastic lepton-hadron scattering (DIS) at small values of the Bjorken variable $x$ is another class of reactions where the Pomeron manifests itself; this point will be discussed below.

3. The Pomeron in QCD

Studies of the Pomeron in QCD started in 1975 with the papers of Low [11] and Nussinov [12], who calculated the exchange of two gluons (Fig. 3.1), which has the quantum numbers of the Pomeron and thus may be considered as the Pomeron Born term in QCD.

As a further step, Gunion and Soper [13] introduced a model of Pomeron-hadron coupling by making certain assumptions concerning the hadron structure.

In parallel and nearly simultaneously, intense studies of the perturbative Pomeron were initiated by Lipatov [4] and his collaborators. This has, eventually, developed in what is now called the BFKL Pomeron [14] (see Sec. 4); (recently, very exhaustive and complete series of lectures on this subject have appeared [15]).

Some progress has been achieved in coupling the BFKL Pomeron to hadrons, but in the absence of an infra-red cut-off, the logarithmic slope remained divergent at $t = 0$. Attempts [16] to resolve this problem have demonstrated that the relevant regularization procedure may violate factorization, required to hold (see Sec. 2) in the case of a simple pole exchange.

It may be [17] that a regularization with the correct factorization properties is provided by the vacuum structure of QCD, related to the gluon and fermion condensates. In this way, the Pomeron couples to the quarks like a $C = +1$ photon and, as a consequence, the essential properties of the scattering amplitude are determined by the parameters of the condensate, obtained by lattice calculations.

Several generalizations and phenomenological approaches to the Pomeron have appeared trying also to link together various aspects [18]. Attempts to include non-perturbative contributions in the Pomeron based on models for the relevant gluon propagator have been offered [19-22]. In general, in these models the rigorous properties of Green’s functions are treated approximately in QCD. The lowest order calculations are in qualitative agreement with the data but they fail at higher orders [23]. This is not surprising; it has been shown [24] that
the modification of the gluon propagator by itself is not sufficient for the non-perturbative effects to be accounted for. It is also necessary to modify the corresponding Feynman rules in such a way as to provide self-consistency for the new propagator. Below, we discuss briefly this point by referring for more details to Refs. [24, 25].

Let us consider the transverse gluon propagator in the form

$$D(k^2) = \frac{c}{(k^2 - \mu^2)^2} - \frac{1}{k^2 - M^2}, \quad \mu^2 \ll M^2,$$  \hspace{1cm} (3.1)

which is perhaps the simplest realization of analyticity requirements [26, 27]. The parameters $c, \mu^2$ and $M^2$ depend on the coupling $g$ and vanish when $g \to +0$. In Ref. [24] they were considered as constant and their values determined from the predictions implied by Eq. (3.1). Gluon condensate [19 a)] and heavy quark spectroscopy [19 b)] suggest for these parameters

$$\mu \approx 50\text{MeV}, \quad M \approx 750\text{MeV}, \quad \frac{g^2 c}{6\pi} \approx 0.224 \text{GeV}^2.$$  \hspace{1cm} (3.2)

In Ref. [19 b)] it is shown that the mass spectrum of charmonium and bottonium is successfully reproduced with the non-relativistic potential associated with Eq. (3.1) and with the parameters as in Eq. (3.2).

The first term on the r.h.s. of Eq. (3.1) clearly enhances low frequency modes and will be referred to as the non-perturbative gluon propagator (clearly, other, more complicated forms could be suggested, but the one used here is not only a traditional one but, mostly, serves the purpose of an illustration).

Omitting technical details, the elastic scattering amplitude (for zero mass quarks) is calculated (approximately) at order $g^6$ and, taking into account also the secondary $f$ and $\omega$ reggeons, one can compare the result [24] with the experimental data for total and differential pp scattering. The only parameter left free from spectroscopy can be fixed from the data; this leads to $\alpha_s = \frac{g^2}{4\pi} \approx 0.425$ (or $c \approx 0.79 \text{GeV}^2$). This value is, admittedly, a little too large but the model is sufficiently unsophisticated as to make it not unrealistic.

Fig. 3.2 shows the calculated [24] pp differential cross-section in the energy region, $\sqrt{s} = 19.4 \text{GeV}$, where the Pomeron contribution is nearly constant. The calculation, at order $g^4$, parallels the one by Landshoff and Nachtmann [17] and the result is in good agreement with experiment up to $|t| \leq 0.1 \text{GeV}^2$. The reason for the deviation at larger $|t|$ is mainly due to the momentum space technique used at high energy, many $t$-dependent terms have been dropped.

For the total cross section a more complete calculation yielding a $\ln s$ contribution from the $g^6$ diagrams [24] gives
\[ \sigma_{\text{tot}}(s) = f + w + \alpha + \beta \ln \frac{s}{s_0} + \xi, \]  
(3.3)

where \( f, w \) denotes the standard contribution from non-leading reggeon exchanges defined in Eq. (2.7).

Fig. 3.3 shows the results of a fit [24] to the \( \bar{p}p \) data. The values of the fitted parameters are

\[ \alpha = 0.365 \text{ mb}, \quad \beta = 4.995 \text{ mb}, \quad \xi = 210.03 \text{ mb(GeV)}^2. \]

The agreement with the data could probably be improved (at the price of additional complications) and this could slightly modify the parameters. The point, however, is not the best possible data-fitting; this is not so relevant, more important is the physical meaning and consistency in the numerical values of the parameters. Thus, from Eq. (3.3) one gets \( \alpha_s = 0.386 \) (or \( c = 0.87 \text{ GeV}^2 \) and \( c \) is near its upper bound, see [24]).

Thus, the above model gives a successful description both of quark interaction [24] and heavy quark spectroscopy [19 b)] with all the parameters, except \( \alpha_s \), fixed.

An expansion in higher multipoles (see footnote 4) has also been considered [28]. Reasonable fits to the \( pp \) and \( \bar{p}p \) scattering data are then obtained. The knowledge of the fitted parameters is important for the reconstruction of the assumed perturbative series of multipole Pomeron.

4. The Pomeron in deep inelastic scattering

Consider now the traditional lepton-hadron Deep Inelastic Scattering (DIS) (Fig. 4.1) with the usual definition of the kinematics

\[ k^2 = k'^2 = 0, \quad q = k - k', \quad Q^2 = -q^2, \quad \nu = \frac{pq}{m}, \quad x = \frac{Q^2}{2\nu}, \]

(4.1)

\[ W^2 = (p + q)^2 = Q^2(1/x - 1) + m^2. \]

where \( \nu \) is the energy of the virtual photon and \( x \) the Bjorken variable. The relevant cross section is [29]

\[ \frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4} [2W_1(\nu, Q^2) \sin^2(\theta/2) + W_2(\nu, Q^2) \cos^2(\theta/2)]. \]

(4.2)

\( \theta \) being the scattering angle between the directions of motion of the initial and final leptons. The structure functions \( W_1 \) and \( W_2 \) are related to the forward virtual Compton scattering
and, in analogy to hadronic reactions, one may assume Regge asymptotic behavior to hold for virtual Compton scattering

\[
W_1(\nu, Q^2) \sim \nu \sigma_T \to \sum_i \beta_i^1(Q^2)\nu^{\alpha_i(0)},
\]

\[\nu W_2(\nu, Q^2) \sim (\sigma_T + \sigma_L) \to \sum_i Q^2 \beta_i^2(Q^2)\nu^{\alpha_i(0) - 1}.\]

According to Eq. (4.3), Regge asymptotics holds when \(\nu\) is large. It is compatible with Bjorken scaling if \(Q^2\) is also large and the "residue" behaves like \(\beta(Q^2) \sim Q^{-2\alpha(0)}\) (a dimensional overall constant has been dropped in (4.3)). The applicability and limitations of the Regge pole model for virtual particles scattering is a complicated and yet not completely resolved problem.

In the limit \(Q^2 \to 0\), we have real photon-hadron scattering, which, by vector meson dominance, can be related to hadron-hadron elastic scattering - a classical area for the application of the Regge pole model. The structure functions are related to the virtual Compton scattering total cross section by

\[
\sigma^{\gamma^*p} = \frac{4\pi^2 \alpha (1 + 4m^2x^2/Q^2)}{Q^2(1-x)}F_2(x, Q^2).
\]

As \(Q^2 \to 0\), the structure function should therefore vanish like \(Q^2\) in order to satisfy gauge invariance. This is a very delicate limit and we shall come back to it.

By unitarity, the squared modulus of this process is equal to the imaginary part of the forward virtual Compton scattering amplitude (Fig. 4.2).

While the diagram of Fig. 4.1 corresponds to the usual case of (fully inclusive) DIS, in recent times, HERA has provided lots of very good quality data on the semi-exclusive reaction of Fig. 4.3 i.e. \(e + p \to e'p'X\) (where \(e'\) and \(p'\) are the scattered electron and proton respectively and \(X\) is \textit{a priori} any hadronic system).

This is a much richer reaction which for large values of the total \(\gamma^*p\) energy \(W\) defined in Eq. (4.1) \(i.e.\) for small values of \(x\) is dominated by the exchange of the appropriate reggeons. If, in particular, the quasiparticle \(X\) has the same quantum numbers of the virtual photon \(\gamma^*\), then, for large values of \(W\), the reaction is dominated by Pomeron exchange. Notice that, according to Eq. (4.1), \(W\) will be large when \(1/x\) will be large \(i.e.\) in the domain of small \(x\) which is precisely where HERA is at its best. Aside from \(t\) (as previously defined, \(t = (p - p')^2\) is now the square of the four-momentum transfer of the diffractive reaction \(\gamma^*p \to X (J^{P,C} = 1^{--}) p')\), we must now introduce also \(M_X^2\) (the squared mass of the
hadronic system) and, rewriting \( x = \frac{Q^2}{2p_{pq}} = \frac{Q^2}{W^2 + Q^2 - m^2} \), it has become common practice to introduce also

\[
x_{IP} = \frac{q(p - p')}{q \cdot p} = \frac{M^2_x + Q^2 - t}{Q^2} x,
\]

and

\[
\beta = \frac{x}{x_{IP}}
\]

as the appropriate variables which describe the Pomeron kinematics within the reactions under study.

As we have discussed, in the limit of small-\( x \) the corresponding energy variable of the quasi-hadronic reaction \( \gamma^* p \rightarrow X p' \) becomes large and, in this limit, the complex angular momentum analysis applies and Pomeron exchange dominates; this is the main subject of this discussion.

In view of a previous remark that even the number of variables which define Pomeron exchange may depend on the reaction one studies, it is important to notice the following different choices: in the regime of non-negative fixed \( q^2 \) (\( m^2 \) for hadron-hadron, or 0 for real photon-hadron scattering) and \( t \) variable, we have an (on shell) elastic scattering amplitude in the framework of the analytic \( S \)-matrix theory (see Sec. 2); for \( t = 0 \) but variable (negative) \( q^2 \) we have a case of current-hadron interactions of which the most typical example is DIS, (in this case, as recalled, we use the positive variable, \( Q^2 = -q^2 \)); finally, for variable (negative) \( q^2 \) (\( i.e. \) positive \( Q^2 \)) \textit{and} \( t \) also variable, (or \( x_{IP} \) different from \( x \)), we have the novel experimental situation first encountered at HERA: exclusive (or semi-exclusive) vector meson (virtual) photoproduction over a wide range of \( Q^2 \), \( i.e. \) a reaction in which all three variables \( W^2 \) (or \( s \)), \( t \) and \( Q^2 \) (or a different choice of the previously defined ones) enter as independent variables in the same reaction.

It is the latter regime of Pomeron dominance which has come to be known in the literature as hard diffraction. The terminology arises from the fact that, the experimental investigation is performed in a region in which enough energy is transferred to create a hadronic state \( X \) (or, at FERMILAB, jets) and, yet, the final proton retains almost all of its initial momentum \( i.e. \) is basically not deflected from its original direction so that, to all effects, we are still in a domain dominated by soft physics (the \( \gamma^* p \rightarrow X p \) reaction) but this is the result of a hard interaction whereby the state \( X \) (or a jet) has been emitted at large \( p_T \). All this was made possible within a lepton-hadron DIS by having access to sufficiently low \( x \)’s.

As it is obvious, most of the discussion that follows is confined to the region of hard diffraction \( i.e. \) to the domain of very small \( x \); thus, here, we will totally ignore another interesting limit, \( x \rightarrow 1 \), which corresponds to the transition between DIS structure functions.
(SF) and elastic form factors. Near $x = 1$, the SF are far from the Regge domain and their parametrizations obey the quark counting rules. The interested reader may consult Refs. [29] or [30] for more details.

In Fig. 4.4 we show a typical road map, quoted (with liberal modifications) by various authors, showing different regimes in the behavior of the SF. Here the figure is intended only as a pictorial help to guide the reader. In fact, the definition of these regions and, mostly, the transitions and the connections between them are still far from clear and will be only very briefly discussed below.

A quite general question that imposes itself in the light of the above discussion is: what is the real range of applicability of the Regge pole model in DIS? Intriguing and controversial is the transition in $Q^2$ between the assumed Regge behavior and the QCD evolution. Is this transition smooth, or abrupt (like a phase transition)? Are the two regimes compatible, i.e. does an interface between them exist?

Formally, we know of no upper limit in $Q^2$ for the Regge pole theory to be applied in DIS (but, on the other hand it is not obvious that the usual high energy theorems should hold at $Q^2 \neq 0$). This attitude has been taken in a recent paper [31], where a direct fit to the data including those at the highest $Q^2$ values by a Regge behaved SF was attempted.

To insure Regge behavior to hold in the various kinematical regions we have so far considered, we recall that $W^2 = (q + p)^2$ should be $\gg 0$ (corresponding to the Mandelstam energy variable $s$ in elastic hadron scattering being large, see Fig. 2.1). As remarked already, $W^2$ large requires $x$ small which, together with $Q^2$, is the variable in which the data are normally presented. For this to be possible,

i) the interaction energy ($\sqrt{s}$) must be large enough to ensure that $W$ can be far from the resonance region i.e.

$$W^2 \gg m^2; \quad \text{(4.6)}$$

where $m$ is a typical hadronic mass. Since $W^2 = Q^2 (1/x - 1) + m^2$, if we take $x$ sufficiently small, one has

$$Q^2/m^2 \gg x; \quad \text{(4.7)}$$

ii) The cosine of the scattering angle in the $t$–channel should be large. In conventional DIS, $t = 0$; therefore

$$|\cos \theta_t| = \frac{W^2 + Q^2 - m^2}{2 \sqrt{Q^2m^2}} \gg 1, \quad \text{(4.8)}$$

requiring
\[ Q^2/m^2 \gg x^2. \] (4.9)

The first condition is the strongest if \( x \neq 1 \). Hence, Regge behavior is expected in the region right and upwards from the boundary

\[ Q^2 \geq \frac{x}{1-x}(W_{\text{min}}^2 - m^2), \] (4.10)

shown in Fig. 4.5 where the value \( W_{\text{min}} = 3 \) GeV may be a reasonable choice.

In analogy with hadronic processes, one assumes that the total cross section

\[ \sigma = \sigma_L + \sigma_T = \text{Im}A(W^2, t = 0, Q^2) \] (4.11)

is Regge behaved in the domain specified above.

In \( \gamma p \) scattering, by quantum number conservation, only positive C-parity exchange is allowed, so one can write (see Table 1):

\[ A = \mathbb{P} + f, \] (4.12)

where \( \mathbb{P} \) and \( f \) stand for Pomeron and f-meson exchange.

An immediate consequence of a purely Regge-type fit (as discussed in [31]) is that, in spite of the large flexibility inherent in Regge pole models (especially, in the parametrization of the residue function), it seems very hard to reconcile the mild increase with energy of the cross sections at small \( Q^2 \) with the rapid increase found for \( Q^2 \) around 5-10 GeV\(^2\) (Fig. 4.6). A possible conjecture is that the latter may not have a conventional Regge-like origin, for instance it could be attributed to a \( Q^2 \) dependent slope of the Pomeron (two explicit mechanisms, one assuming it and the other trying to provide an explanation for it, are briefly discussed later in Section 4) or, as we will see in Section 5, this effect could also be blamed to the QCD evolution.

Beyond the (non-perturbative) Regge domain, one expects perturbative QCD to be applicable.

One distinguishes two typical perturbative regimes, namely:

1. If one assumes

\[ \alpha_s(Q_0^2)ln(Q^2/Q_0^2) \sim 1, \quad \alpha_s(Q_0^2) \ll 1, \] (4.13)

in the leading log\((Q^2)\) approximation (LLA) only the terms proportional to \( \alpha_s^n ln^n(Q^2/Q_0^2) \) are retained. The relevant evolution equation \((Q^2\text{-evolution})\) is named after various authors DGLAP [32]. Below we will show how it works in the HERA domain.
2. A different perturbative regime named also after several authors BFKL [14] leads to an \((x\)-evolution) equation by assuming that,

\[
\alpha_s(Q_0^2)\ln(1/x) \sim 1, \quad \text{with} \quad \alpha_s(Q_0^2) \ll 1 \quad (4.14).
\]

In the perturbative decomposition, only leading terms in \(\alpha_s^2(Q_0^2)\ln^n(1/x)\) are retained. The approximate solution of this equation, for \(\alpha_s(Q^2) = \alpha_s\) fixed, leads to

\[
F_2(x) \sim x^{-\Delta}, \quad \Delta = \alpha_{\perp 0}(0) - 1 = (12/\pi)\alpha_s\ln2 > 0.3 \quad (4.15)
\]

which is called the Lipatov Pomeron (remember that for small \(x\), \(x \sim \frac{Q^2}{x^2}\)). It is precisely the relevance of this perturbative (or hard) Pomeron in soft reactions, \(i.e\). when \(Q^2\) is small which is presently the subject of a hot debate still quite unsettled on which we shall focus our attention.

Restating in different words the situation outlined above, while the DGLAP evolution proceeds at constant \(x\) and increasing \(Q^2\) (which corresponds to increasing the resolution with which the hadronic target is probed by the photon), the BFKL equation describes the evolution towards decreasing values of \(x\) and links together hard DIS with soft diffraction. As a consequence, the results of a calculation depend essentially on two things, the first, over which we have little quality control is the choice of the non-perturbative input (its form, its range etc.) and the second, which, on the contrary we can completely control, the procedure of evolution chosen in such a calculation.

To show an example of what we mean, consider the following nonperturbative input SF [33]

\[
F_2(x, Q^2) = A \left(1 + \epsilon \ln \frac{1}{x}\right) \quad (4.16)
\]

based on the expected [34] Regge pole behavior and Pomeron dominance in the small \(x\) and moderate \(Q^2\) region and on the observation [35] that according to the experimental data of NMC [36], the SF \(F_2\) at \(Q^2 = 5\) GeV\(^2\) and in the range of \(10^{-2} \leq x \leq 10^{-1}\) increases like \(\ln(1/x)\). The parameter \(\epsilon\) in (4.16) according to a fit [35] to the NMC data is \(\epsilon \approx 0.1\).

An explicit, approximate solution of the DGLAP equation in the form

\[
F_2(x, Q^2) = B \exp(-4.74\xi) \left(x^{-1.44\xi} + \epsilon \ln(1/x) \exp(3\xi)x^{-0.72\xi}\right), \quad (4.17)
\]

where

\[
\xi = \ln \frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}, \quad (4.18)
\]
was obtained in Ref. [37], where it was shown also that the solution is compatible with
the DGLAP equation only if the power term, implying a *supercritical Pomeron* is present in
(4.17). To reconcile it with the *dipole Pomeron* form (4.16) of Ref. [33] at $Q^2 = 5\ \text{GeV}^2$,
one chooses the scale parameter $Q_0^2$ equal to 5 GeV$^2$, at which point (4.17) reduces to (4.16).
This is the starting point of the evolution, wherefrom the behavior of the SF at higher values
of $Q^2$ is calculated in a parameter-free form. The results are compared with the HERA data
[38, 39] in Fig. 4.7.

The virtue of this exercise is its simple analytic form. Calculations and fits with more
involved inputs and/or more sophisticated or complete forms of the evolution can be found
e.g. in Refs. [40, 41] (see also Refs. [15, 18]).

In 1993 two experimental groups at HERA - H1 and ZEUS - measured the proton
structure function in the hitherto unexplored region $10^{-4} \leq x \leq 10^{-2}$ and $5\ \text{GeV}^2 \leq Q^2 \leq 10^5$
GeV$^2$. The results were first reported at the Moriond meeting in March 1993 [38]. What
became known as the *HERA effect* is the unexpectedly rapid increase of the proton structure
function which occurs with decreasing $x$. The effect becomes more pronounced as $Q^2$ increases
and in a wide range of $Q^2$ it can be parametrized, for instance (but certainly not only) as
$\sim x^{-\delta}$ with $\delta \geq 0.3$. By virtue of the relation $x \sim Q^2/s$ (which follows from (4.1) at small
$x$), this implies that a possible parametrization of the HERA data is obtained with

$$F_2(x) = x^{-\delta}, \ \delta \geq 0.3.$$  \hspace{1cm} (4.19)

For what said above, if one assumes Regge behavior and Pomeron dominance, $\delta$ can be related
to the supercritical Pomeron intercept $\alpha_\text{IP}(0) = 1 + \delta$. The parameter $\delta$ can then be plotted
by fitting the small ($< 10^{-2}$) $x$ data for the various available bins in $Q^2$. We will comment
soon on this procedure but let us for the moment proceed as if no objections could be raised
to it. In this case, the result one obtains [42] is shown in Fig. 4.8. This result is sufficiently
startling that one may comment on it before even criticizing the procedure: two important
and possibly highly non-trivial phenomena are evident from this figure. One is the large (0.3
- 0.4) value of the ”Pomeron intercept” at large $Q^2$, ($Q^2 > 10\ \text{GeV}^2$), (a value tantalizingly
close to the value (4.15) of the *Lipatov Pomeron*) and the other is the very rapid variation of
$\delta$ around $Q^2 = 5\ \text{GeV}^2$. These phenomena have raised several questions that have become
central in the modern studies of diffraction in the context of QCD such as,

1) Is the large $Q^2$ behavior seen at HERA a direct manifestation of the *Lipatov Pomeron*?
The need for an experimental verification of the results of the perturbatively QCD-calculated
Pomeron and the agreement with the predicted value of the perturbative Pomeron intercept
(see Eq. (4.15)) makes this interpretation attractive. On the other hand, other explanations
may be possible (see below) and general theoretical arguments have been given against the
2) Is the rapid change of $\delta$ in $Q^2$ due to a phase transition? [42] and/or is it suggesting that there exist two different Pomerons - a soft one (active at small $Q^2$) and a hard one (active at large $Q^2$)? (this is, really, just another way of restating the previous question).

3) What is the smooth transition between real ($Q^2 = 0$) and virtual ($Q^2 \gg 0$) photon scattering?

In what follows we shall address these questions but the reader should be warned that we are far from a unique answer nor do we have a unified model valid in all kinematical regions; for the time being, only fragmentary understanding of these phenomena is slowly emerging.

Before proceeding further, we have to come back to the cautionary remarks made previously when commenting Fig. 4.8. It is quite clear that treating the data the way it was done to obtain Fig. 4.8 most probably oversimplifies the complexity of the problem; this procedure should be critically revised.

The crucial point is that as $Q^2$ tends to zero, the SF should vanish like $Q^2$ in order to satisfy gauge invariance. This problem has been treated in a number of papers [8c, 42,44]. In particular, let us recall that an early attempt was made to satisfy this condition by using a modified dipole Pomeron [45]

$$F_2(x, Q^2) = A \left[ 1 + \epsilon \ln \left( Q^2 (\frac{1}{x} - 1) + M^2 \right) \right] \ln \left( 1 + \frac{Q^2}{Q^2 + a^2} \right). \quad (4.20)$$

Fig. 4.9 shows a representative fit to the data at low and intermediate values of $Q^2$. This model, however, similar to that of Donnachie and Landshoff [44 b)] based on a supercritical Pomeron, does not reproduce the rapid rise of the SF at large $Q^2$, that may be attributed to the evolution and which was, on the contrary, predicted by other models [41] (see, for instance Fig. 4.10).

A generalization to the above attitude has been taken by Capella et al. [46], who choose a Regge pole model for small $Q^2$ endowing the effective Pomeron intercept with a $Q^2$ dependence of the form

$$F_2(x, Q^2) \sim x^{-\Delta(Q^2)}, \quad \Delta(Q^2) = \Delta_0 \left( 1 + \frac{2Q^2}{Q^2 + d} \right). \quad (4.21)$$

The resulting structure function, assumed to be valid at small and moderate $Q^2$ ($0 \leq Q^2 \leq 5 \text{ GeV}^2$) is

$$F_2(x, Q^2) = A x^{-\Delta(Q^2)} (1 - x)^{n(Q^2) + 4} \left( \frac{Q^2}{Q^2 + a} \right)^{1 + \Delta(Q^2)} + B x^{1-\alpha_R} (1 - x)^{n(Q^2)} \left( \frac{Q^2}{Q^2 + b} \right)^{\alpha_R}. \quad (4.22)$$
where

\[ n(Q^2) = \frac{3}{2} \left( 1 + \frac{Q^2}{Q^2 + c} \right) \]

to obtain at \( Q^2 = 0 \), the same power \((1 - x)^{1.5}\) as in the Dual parton model, \( \alpha_R \) being the secondary reggeon intercept.

From this input the structure function has been calculated for higher values of \( Q^2 \) by means of the QCD evolution. The resulting curves are shown in Fig. 4.11.

A further refinement is found in Ref. [47], where it is shown that, actually, the data at not too large \( x \), \( i.e. \) for \( x \leq 10^{-2} \) can be reconciled with a generalized dipole (or tripole)\(^7\) Pomeron \( i.e. \) which behaves in such a way as not to violate the Froissart-Martin bound [6] while reducing to a power behavior of the Donnachie Landshoff (not Lipatov) type for large \( Q^2 \). Specifically, in [47] one considers (again this should just be taken as an example) either

\[ F_2(x, Q^2) = A_{\text{IP}} \left[ \tilde{x}^\epsilon(Q^2) - (1 + \epsilon(Q^2)\ln(\tilde{x})) \right] \ln \left( 1 + \frac{Q^2}{Q^2 + a_{\text{IP}}^2} \right) \]  \hspace{1cm} (4.23)

or, alternatively

\[ F_2(x, Q^2) = A_{\text{IP}} \left[ \tilde{x}^\epsilon(Q^2) - 1 \right] \ln \left( 1 + \frac{Q^2}{Q^2 + a_{\text{IP}}^2} \right) \]  \hspace{1cm} (4.24)

where \( \tilde{x} = \frac{W^2}{s_0} \), with the hadronic scale taken as \( s_0 = 1 \text{ GeV}^2 \). These forms reduce to the expected behavior if \( \epsilon(Q^2) \) vanishes as \( Q^2 \sim 0 \) because \( \ln(\tilde{x}) \simeq \ln(\frac{1}{x}) \) when \( W^2 \gg Q^2 \).

Choosing for the slope \( \epsilon(Q^2) \)

\[ \epsilon(Q^2) = \frac{\lambda}{\ln^2} \ln \left( 1 + \frac{Q^2}{Q^2 + b^2} \right) \]  \hspace{1cm} (4.25)

Eq. (4.23) leads to a \( \ln^2(\frac{1}{x}) \) behavior in agreement with the Froissart-Martin bound and Eq. (4.24) to a \( \ln(\frac{1}{x}) \). In these equations the parameters \( A_{\text{IP}} \) and \( a_{\text{IP}}^2 \) are fixed by the requirement that the total photoproduction cross section is correctly reproduced. With the specific choice (4.25) made for \( \epsilon(Q^2) \), there are just two adjustable parameters \( \lambda \) and \( b^2 \). Fitting the small-\( x \) data of HERA (specifically up to \( x \leq 5.10^{-3} \)), the result is shown for the case of Eq. (4.23) in Fig. 4.12 with the following values of the parameters \( A_{\text{IP}} = 5.72 \times 10^{-3} \), \( a_{\text{IP}}^2 = 1.12 \) (obtained from a fit to the total photoproduction cross section), \( \lambda = 0.254 \), and \( b^2 = 0.198 \) with a \( \chi^2/(\text{dof})/(58 \text{ HERA data}) \) of about 1.2 (obtained from HERA data). With the use of

\[^7\] Which is the same as saying an increase (at \( Q^2 = 0 \)) compatible with either \( \ln(1/x) \) (dipole form) or with \( \ln^2(1/x) \) (tripole form), see footnote 4.
Eq. (4.24) the results are similar. Two interesting conclusions are in order. First, recall that the HERA data with $x \geq 5.10^{-3}$ are not included in the fit; in spite of this, it is only for very high $Q^2$ that the curve deviates considerably from the data. Second, the asymptotic value of $\epsilon$ as $Q^2$ grows to $\approx 2000$ GeV$^2$ is roughly $= 0.3$ i.e. it reaches the lower limit of what are considered the range of values appropriate for the \textit{Hard Pomeron} (the value of the \textit{soft Pomeron} à la Donnachie and Landshoff, 0.08 being reached for $Q^2$ between 1 and 5 GeV$^2$). If a factor $(1 - x)^{\beta(Q^2)}$ correcting for $x$ not so small is inserted in Eq. (4.23) where one takes

$$\beta(Q^2) = \beta_0 + \beta_1 \tau \quad \text{with} \quad \tau = \ln \left( \frac{\ln \left( \frac{Q^2 + Q^2_0}{A^2} \right)}{\ln \left( \frac{Q^2_0}{A^2} \right)} \right), \quad (4.26)$$

the values of the various parameters are now $A_{IP} = 5.72 \times 10^{-3}$, $a_{IP} = 1.12$, $\lambda = 0.256$, $b^2 = 0.21$, $\beta_0 = 7.0$ and $\beta_1 = 5.6$ with a $\chi^2/(\text{dof})/(67 \text{ HERA data})$ of about 1.55. Notice that, as expected, only the parameters involved in $\beta(Q^2)$ are sensitive to including larger $x$-values in the fit. Fig. 4.13a shows the equivalent of Fig. 4.12 i.e. the variation in $Q^2$ for the various bins in $x$) whereas Fig. 4.13b shows the converse i.e. the variation in $Q^2$ for the various bins in $x$. We conclude that the large disagreement in Fig. 4.12 was due to the lack of an appropriate treatment of the not so small $x$ data.

The approaches of Refs. [42, 45-47], although different in many and also important details, are similar in that a Regge-behaved SF was used in the region of small and intermediate values of $Q^2$. In addition, they all go towards the unifying spirit of saying that a unique form for the Pomeron is responsible for both its \textit{soft} and its \textit{hard} aspects.

Probably, however, the real message of all this discussion is that, in spite of $x$ being so close to zero, the HERA data are not more asymptotic than the corresponding hadronic data and the latter are well known to be capable of being fitted with either a logarithmically increasing or a power-like cross section. As experience with hadronic physics has taught us [48], it is only when angular distributions are also used in the fit that one can really discriminate between a power-like and a logarithmic behavior. Possibly, the same will be the case for the DIS data; we can only hope that angular distributions from ZEUS and H1 will be available soon.

The $Q^2$ dependence of the Pomeron intercept conjectured in Refs. [46, 47] raises an important question, namely that of factorization. We recall (see Sec. 2) that, due to factorization, the scattering amplitude for the exchange of a Regge pole is the product of the vertices and of the propagator $s^{\alpha}$. The trajectory $\alpha$ in this case should not depend on the masses (or virtualities) of external legs. A (presumably) mild and smooth $Q^2$-dependence is expected from t-channel unitarization i.e. from rescattering corrections. In principle this is calculable although technically very complicated; the calculated multi-Pomeron exchanges
depend both on the propagator and on the vertices, involving $Q^2$ dependence. Realistic and quantitative estimates of the expected effect as well as its experimental verification have not yet been carried out. In addition, s-channel unitarization (which is prohibitively complicated to enforce), is probably even harder to estimate. Empirically, we can only interpret the effective $Q^2$-dependent intercept of the Pomeron as an effective way of taking into account unitarity i.e. as the collective effect of rescattering corrections. In this sense, it could be argued that the Pomeron is an atypically complicated angular momentum plane singularity (this, however, has been long recognized, as we have already mentioned earlier).

An original explanation of the apparent $Q^2$-dependence of the Pomeron intercept has been suggested in Ref. [49] where it is shown that the origin of this effect (and the resulting non-factorizability of the Pomeron) may be that (contrary to the common belief) the singlet SF, $F^S_2$, has two different components - one coming from the sea quarks and the other one due to the gluons via the $g \rightarrow q\bar{q}$ process. As a consequence, the singlet SF may be written in the form

$$F^S_2(x, Q^2) \sim \alpha(Q^2)x^{-\delta_{\Sigma}} + \beta(Q^2)x^{-\delta_{G}}$$

(4.27).

From a fit to the data (see Fig. 4.14) the values of the parameters were found [49] to be $\delta_{\Sigma} = 0.2$ and $\delta_{G} = 0.4$. The sum of the two terms may be simulated by $x^{-\delta(Q^2)}$, where $\delta(Q^2)$ varies from 0.2 to 0.4 around $Q^2 = 5 \text{ GeV}^2$. (A similar point of view, although in a different context, has been taken recently by Martin et al. [50])

5. The Pomeron structure

The studies of the Pomeron structure or, alternatively, of hard diffraction have become among the most topical recent subjects. Physically, the motivation for these studies relies on the idea [51] that the Pomeron is a quasi-particle with its own structure, that might be resolved in deep inelastic photon-Pomeron scattering in which it is assumed that X is a $1^{--}$ system (see the discussion preceding Eq. (4.5)).

Formally, the subject is nothing but diffraction dissociation, similar to the case of hadronic reactions (Fig. 2.5), a new degree of freedom being the photon virtuality. For this reason, the appearance of a rapidity gap in virtual photon-proton scattering, observed at HERA [52] (Fig. 5.1) and at the TEVATRON [53], by itself is not surprising but it has been an important signature in order to make sure that we are indeed in the presence of diffraction [54]. What is really intriguing, is the possibility that perturbative QCD calculations may eventually be relevant to the understanding of the soft Pomeron. Before presenting a particular model for the Pomeron structure and its observable consequences, let us briefly discuss
the more general problem of the transition from the small-$Q^2$ Regge behavior to the large-$Q^2$ QCD evolution. Since $W^2 \simeq Q^2 \left(\frac{1}{\sigma} - 1\right)$, one can write the rapidity intervals additively as shown in Fig. 5.2. The lower part of the diagram corresponds to a multiperipheral mechanism of particle production, which by unitarity is related to Regge pole (in particular, Pomeron) exchanges, while its upper part corresponds to the Reggeon’s (e.g. Pomeron’s) inelastic vertex, described by the QCD evolution equation. The transition between the two regimes may be abrupt, similar to a phase transition. The dependence of the \textit{effective Pomeron intercept} on $Q^2$, as extracted from a fit of the structure function $\sim x^{-\alpha(Q^2)}$, is instructive in that it shows the \textit{phase transition} between the low $Q^2$, corresponding to a Pomeron exchange and the large $Q^2$ behavior coming from the QCD evolution in the upper vertex of Fig. 5.2. It is evident from this picture that there is no \textit{hard Pomeron} - the large $Q^2$ behavior comes from the evolution of the upper (inelastic) Pomeron vertex, while the intercept of the Pomeron itself is almost $Q^2$-independent (a mild $Q^2$-dependence is the result of the rescattering or unitarity corrections). The crucial question is \textit{what (if any) is the memory of the Pomeron in the QCD evolution of its vertex?}

The previous discussion raises qualitatively important issues and it is, therefore, necessary to provide evidence that our conjecture can be used as a working hypothesis. Several attempts along these lines have been carried out [1, 55, 56] of which we are just going to illustrate one [55]. Once again, it is not so much the details of the example which are relevant, but the underlying philosophy and the light that the specific example sheds on basic questions which matters.

Let us recall that the differential cross section of a diffractive deep inelastic process is

$$
\frac{d\sigma(ep \rightarrow epX)}{d\zeta dt dx dQ^2} = \frac{4\pi\alpha^2}{xQ^4}(1 - y + \frac{y^2}{2(1 + R)})F_{2}^{\text{diff}}(x, Q^2; \zeta, t),
$$

(5.1)

where $y$ is $\frac{E - E'}{E}$, $\zeta = 1 - x_F$, where $x_F = p_F'/p_z$ and $t = (p - p')^2$. Note that at HERA, $(k + p)^2 = \sqrt{s} = 296$ GeV and the pseudorapidity $\eta$ of the smallest detector angle is $\eta = 4.5$, $\Theta = 1.5$. The cut in $\eta$, $\eta_{max} < 1.5$, distinguishes events with a large rapidity gap and is equivalent to $\zeta_{max} \leq 0.06$. Due to acceptance cuts, the maximum value of $\zeta$ is $\zeta_0 = 2.0 \times 10^{-2}$ for ZEUS [52b]. Factorization

$$
F_{2}^{\text{diff}} \rightarrow F_{IP/p}^{\text{IP}}(\zeta, t)G_{q/IP}(x/\zeta, Q^2)
$$

(5.2)

of the structure function, where $F_{IP/p}$ is the so-called Pomeron flux (see below) and $G_{q/IP}$ is the Pomeron structure function, is an important assumption, adopted in most of the calculations of DIS.
The Pomeron flux

\[ F_{IP/p} = \frac{d\sigma^{diff.}}{d\zeta dt} \frac{1}{\sigma_{tot(IPp)}} \]  

(5.3)

is defined as the diffraction dissociation cross section divided by an abstract quantity called the Pomeron-proton total cross section; the latter is assumed to have a behavior typical of high energy hadron scattering, for instance [55]

\[ F_{P/p}(\zeta, t) = [\exp(B\alpha(t))\zeta^{-\alpha(t)}]^2 \zeta. \]  

(5.4)

Practically, the Pomeron flux can never be completely isolated from non-leading (secondary Reggeon) contributions.

The key point in these and similar calculations is the choice of the Pomeron structure function. A priori, it is not known whether the Pomeron is made of quarks or gluons (most likely - of both). One simply makes a guess, calculates the observable consequences and then compares the results of the calculations with the data.

In Ref. [55] the following simple choice was made for the gluon distribution function, corresponding to a Pomeron made of a few gluons

\[ zG(z, Q^2) = a(Q^2)(1 - z)^{b(Q^2)}. \]  

(5.5)

The parameters

\[ b(Q^2) \simeq 1 + 4\xi/3, \ a(Q^2) \simeq \exp\xi/3, \]  

(5.6)

are calculated [55] from the evolution equation and normalized to \( a(Q_0^2) = 2, \ b(Q_0^2) = 1, \) where \( \xi \) is the usual QCD evolution variable (4.18).

The ratio of the diffractive to all deep inelastic events

\[ r = \frac{L(x, Q^2)}{F_2(x, Q^2)} \]

was then calculated and compared with the HERA data. The result is shown in Fig. 5.3.

A resume of these and similar calculations is that given the large flexibility of the models, coming from various assumptions such as factorization, Pomeron pole dominance etc. as well as the large experimental uncertainties, it is difficult to make a definite discrimination between the existing models of the Pomeron structure [56]. Nevertheless, it is very challenging to try to apply perturbative QCD calculations in studying the structure of an object that is essentially non-perturbative in nature.
A direct signal of the Pomeron structure could be manifest in jets with large transverse energy produced by a hard photon-Pomeron scattering (upper part of the diagram, Fig. 5.2). Experimental searches for such signals are under way both at HERA and the Fermilab Tevatron and are included in various proposals for the LHC.

6. (Temporary) conclusions

As implied in the title of this Section, the conclusions cannot being other than temporary. Much better data, more conclusive analyses and, especially, better theoretical understandings are needed before firm conclusions can be reached.

Presently, there are two different competing approaches to the Pomeron or high energy diffraction.

The first assumes that there are two Pomerons, one called soft the other one hard. In this way one is able to reconcile the formalism and the phenomenology of conventional high energy hadronic physics (see Section 1), with an object calculated in perturbative QCD, the Lipatov Pomeron which might be tempting to relate to the large-$Q^2$ HERA physics. Needless to say, the introduction of a two-component Pomeron increases the degrees of freedom thus making possible excellent fits [8 c)] to the data.

The other alternative is that the Pomeron is a single (but far from simple!) and unique object [2a, 46, 47]. The large-$Q^2$ domain (e.g. the recent HERA data), involving particles with large virtualities could be outside the conventional Regge domain in which case they should be attributed to other mechanism of which we have offered a variety ranging from $Q^2$ effective dependence of the slope or to the parton evolution.

It seems to us that before resorting to introducing more objects to describe the same basic physics (diffraction), one should fully examine all other alternatives. Differently stated:

a) It is neither economic nor aesthetic to violate the law of Occam (*Entia non sunt multiplicanda praeter necessitatem*); before doing so all other possibilities should be fully explored.

b) It is not clear how the two objects could interpolate; the simplest (and most obvious) one could be to assume that the intercept is a function of $Q^2$. This, however, leads to problems with the basic property of factorization of a simple pole (Sec. 2). Factorization is not necessarily valid in the case of more complicated $j$-plane singularities (like the Pomeron is very likely going to be), but the $Q^2$ dependence has to be rather drastic to reconcile this

---

8 Additional entities should not be introduced without necessity.
viewpoint with the data and anyway it is very unlikely that it is ever going to be calculated from \(t\)-channel unitarity (or, even less from \(s\)-channel unitarity).

c) Large-\(Q^2\) may be outside the Regge domain and the power term \(x^{-\lambda}\) may have a different origin from that at low \(Q^2\). The small and large-\(Q^2\) variations of the structure functions could have an entirely different origin so that the relevant formalism may altogether be different: complex angular momentum theory (non-perturbative) in the first case and perturbative QCD (with evolution) in the second.

d) The uniqueness of the object called Pomeron is made stronger i) by the recent analysis by Goulianos [57 d)] of the CDF [54 b)] and DO [54 c)] results suggesting that, in the hard double diffraction dissociation processes, the fraction of rapidity gap dijet events to all dijets events with equal kinematics is the same within errors as the rate expected for soft double diffraction dissociation in which no jets are present and ii) by the analysis from H1 [53] of the deep inelastic diffraction results at HERA where it appears that the same Pomeron is involved in hard and soft collisions.

Hopefully, a transitory region will be found where the (nonperturbative) soft Pomeron is probed by perturbative hard processes. From the practical point of view, the main problem will remain factorization; it will be very difficult to find where it occurs the transition from the controllable perturbative calculations to a (model-dependent) soft Pomeron; among other things, this will make rather ambiguous the signals of a possible Pomeron structure.

Concerning the QCD picture of the Pomeron itself as a moving \(j\)-plane singularity(ies?), this appears essentially non-perturbative and there is no universally established consensus or generally accepted solution for the time being. The main difficulty remains confinement and a unified treatment of the quark interaction (scattering) and hadron structure (spectroscopy) could be very useful. We expect that these topics will be the centre of attention in the near future.
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Figure captions

Fig. 2.1. Diagram for elastic hadron scattering with a single Regge pole exchange, \( s = (p_1 + p_2)^2 \) and \( t = (p_1 - p_2)^2 \) are the Mandelstam variables, \( g_1(t) \), \( g_2(t) \) are the vertex (residue) functions and \( s^{\alpha(t)} \) is the Regge propagator.

Fig. 2.2. Spin of the particles \( J = \text{Re} \, \alpha(t) \) plotted versus their squared masses \( t = m^2 \) (Chew-Frautschi plot). The function \( \alpha(t) \) is called a Regge trajectory; it realizes the analytic continuation from the discrete and positive values of the spin of a particle to a continuum \( t \in ] - \infty, +\infty [ \).

Fig. 2.3. By unitarity, a simple Regge pole exchange may be related to the multiperipheral mechanism of particle production.

Fig. 2.4. Data on the high energy behavior of the hadronic and \( \gamma p \) total cross sections [8 a)]. The curves in \( \gamma p \) correspond to model fits with a supercritical Pomeron (with \( \alpha(0) - 1 = 0.08 \) [8 b]) (dotted line), Froissart saturation (full line), and dipole Pomeron [8 c] (broken line).

Fig. 2.5. Single (a) and double (b) diffraction dissociation are also mediated by a Pomeron exchange.

Fig. 2.6. Multi-Pomeron exchange diagram.

Fig. 2.7. Pomeron trajectory fitted to the elastic scattering data with a candidate glueball on it.

Fig. 3.1. Two-gluon exchange is the "Born term" Pomeron [11, 12] in QCD.

Fig. 3.2. Differential cross section for \( pp \) scattering at \( \sqrt{s} = 19.42 \) GeV calculated in Ref. [24].

Fig. 3.3. \( \bar{p}p \) total cross section calculated in Ref. [24] and fitted to the data.

Fig. 4.1. Diagram for Deep Inelastic lepton-hadron Scattering (DIS), mediated by one-boson exchange.

Fig. 4.2. By unitarity, DIS is related to the imaginary part of the forward virtual Compton elastic scattering amplitude.

Fig. 4.3 Semi-exclusive DIS with a proton in the final state and a vector hadronic system \( X(J^{P,C} = 1^{--}) \) produced.

Fig. 4.4. A pictorial presentation of the "road map" of various regimes (and relevant approaches) to DIS.
Fig. 4.5. Kinematical boundary of the Regge domain. Regge behavior is expected to hold right and upwards from the curve indicated in the figure.

Fig. 4.6. Calculated behavior of the SF in the Regge-type model of Ref. [31].

Fig. 4.7. Behavior of: (a) the input SF (with the assumed [34-36] Regge behavior of the NMC data at $Q^2 = 5 \text{ GeV}^2$), and (b) its QCD evolution according to the GLAP equation, calculated in Ref. [33] and with the HERA data [38,39] superimposed.

Fig. 4.8. $Q^2$-dependence of the ”effective” Pomeron intercept from Ref. [42].

Fig. 4.9. Small-$Q^2$ fit in a model [45] satisfying the $Q^2$ limit. Note that models of that type fail to follow the rapid rise of the SF at large $Q^2$, typical of the HERA data.

Fig. 4.10. Calculated and predicted glue distribution in Ref. [41].

Fig. 4.11. Behavior of the SF calculated in a model [46] combining small-$Q^2$ Regge behavior (with a $Q^2$-dependent Pomeron intercept) and large-$Q^2$ evolution.

Fig. 4.12. Small-$x$ structure function $F_2^p$ from H1 data (triangulated dots) and Zeus data (closed points and stars) plotted as function of $x$ at fixed $Q^2$ compared with the fit of Eq. (4.23). Only data with $x \leq 5 \times 10^{-3}$ have been used in the fit [47].

Fig. 4.13. a) Structure function with the same data of Fig. 4.12 plotted as a function of $x$ at $Q^2$ fixed and b) as a function of $Q^2$ at $x$ fixed. The solid line is obtained with Eq. (4.23) where a factor $(1-x)^\beta$ is included, $\beta$ being given by Eq. (4.26). Only data with $x \leq 5 \times 10^{-3}$ have been used in the fit.

Fig. 4.14. Fits to the data from a model [49] based on a two-component model for the Pomeron.

Fig. 5.1. Large rapidity gap events as first seen at HERA.

Fig. 5.2. The Pomeron in DIS at large $Q^2$. The object (Pomeron) remains the same as in ”soft” scattering, but its vertex becomes deeply inelastic. The rapidity $\ln W^2$ in this case is additive because $W^2 = Q^2(\frac{1}{x} - 1) + m^2 \approx Q^2(\frac{1}{x} - 1)$ at large $W^2$.

Fig. 5.3. The ratio of the diffractive to all deep inelastic events, calculated in Ref. [55] and compared with the HERA data.
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