Change points estimation for equipment degradation process based on fuzzy c-means clustering

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Abstract. The working equipment would experience a degrading process until it runs to failure. Many works have been done to illustrate and describe this process as accurate as possible. In the real world applications, the monitored data are unlabelled data. In this paper, we aim to automatically assign degrading states to the monitored. To achieve this target, the change points between adjoining states should be evaluated. Instead of using the conventional hard clustering methods, we adopted the fuzzy c-means clustering, where the monitored data can be assigned to two adjoining states, to estimate the change points. Additionally, the degrading process of an intermittent working equipment is utilized to verify the effectiveness the evaluation method. The experiment results show that fuzzy c-means clustering is suitable for assigning state labels to the unlabeled data and evaluating the change point of involved equipment.

1. Introduction
The degrading process of systems has been a researching hotspot [1][2]. Estimating the states of working equipment is one of the key issues to predict the remaining useful life (RUL). Linear degrading as well as nonlinear degrading model is developed to attain this target [3][4]. Recently, many reaches found that some equipment show strong two-phase or multi-phase degrading character [5][6]. Moreover, even for the linear degrading equipment, to divide the degrading process into different state is helpful to accurately monitor the working equipment. To achieve this target, conventional clustering methods are adopted to deal with monitored unlabelled data.

However, using the conventional clustering process, such as c-means clustering, each monitored data is assigned to one clustering center. However, for the data that scattered at the boundary of clusters, it is unreasonable to assign them to one cluster center. Moreover, in conventional clustering methods, the data is supposed to belong to only one cluster with 100% certainty. But this assumption is not reasonable in real world application. Thus, in this paper, we adopted the fuzzy c-means clustering (FCM) method, where fuzzy membership is introduced to better represent the uncertainty.

The remaining of this paper is organized as follows: Section 2 illustrates the degradation process and framework of FCM, Section 3 gives the description of data and experiment results, Section 4 concludes the paper.
2. Change point estimation using FCM

2.1. Equipment degradation process
As discussed in the Introduction section. The working equipment are in a gradually wearing out process. If the state evaluation is not in time or not accurate, the possible catastrophic may happen at any working time. In [7] and [8], the degradation process is divided into normal state, mild degradation state, moderate degradation state, and near-to-failure degradation state, which is reasonable for that too less division would result in imbalance. In this paper, we also assume that the degrading process is composed of these 4 state.

2.2. Fuzzy c-means clustering

2.2.1. Basic assumption. In practical application, it is impossible for the users to directly obtain the degrading state. We suppose that the center of degrading state are \( V = \{v_1, v_2, \ldots, v_c\} \), \( v_i \subset \mathbb{R}^p \). The monitored \( X = \{x_1, x_2, \ldots, x_n\} \), \( x_j \subset \mathbb{R}^p \) and \( x_j \) represents the monitored unlabeled data. Thus to evaluate the degrading process and select the change points between different states, we have to solve an unsupervised learning process, where the unlabeled data would be labeled by different degrading states. In our paper, let \( v_i \) represent the \( i \)th clustering center. For the predetermined number of clustering centers, let \( U \) be the membership matrix of \( n \) samples to \( c \) clusters, which is defined as

\[
U = \begin{pmatrix}
 u_{1,1} & \cdots & u_{1,c} \\
 \vdots & \ddots & \vdots \\
 u_{n,1} & \cdots & u_{n,c}
\end{pmatrix}
\]

Using FCM, Euclidean distance is adopted to measure the similarity between the \( j \)th measured data to the \( i \)th clustering center. The distance is defined as

\[
d^2_{ij} = \|x_j - v_i\|_\lambda^2
\]

wherein \( \|x\|_\lambda = \sqrt{\langle x, x \rangle_\lambda} = \sqrt{x^T A x} \).

2.2.2. Objective function. Through clustering process, we try to obtain the result that can assign each data to some certain clusters with corresponding memberships. In this paper, to evaluate the degrading state to which the monitored data belongs, the object is to obtain minimum fuzzy membership weighted distance from data to clusters. Thus, the objective function can be expressed as:

\[
\min \left( J_m(U, V) \right) = \min \left( \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^m \|x_j - v_i\|_\lambda^2 \right)
\]

Additionally, the total fuzzy membership of each data to different cluster centers equals to. Therefore, the aforementioned objective function is subject to \( \sum_{i=1}^{c} u_{ij} = 1 \) and \( \forall u_{ij} \geq 0 \). The objective function is an extremum problem with constraints. The Lagrangian function obtained from the function and constrains is as follows:

\[
\Gamma = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^m \|x_j - v_i\|_\lambda^2 + \sum_{j=1}^{n} \lambda_j \left( \sum_{i=1}^{c} u_{ij}^m - 1 \right)
\]

Zeroing the gradient of Lagrangian function with respect to \( V \)
\[
\begin{align*}
\frac{\partial}{\partial U} L &= \sum_{ij} \left( d_{ij} / m_{ij} \right)^{\frac{2}{m-1}} = 0 \\
\sum_{ij} \left( d_{ij} / m_{ij} \right)^{\frac{2}{m-1}} &= 0
\end{align*}
\]

Zeroing the gradient of Lagrangian function with respect to \( U \) and \( V \):

\[
\begin{align*}
\mathbf{v}_i &= \left( \sum_{j=1}^{n} u_{ij}^m \mathbf{x}_j / \sum_{j=1}^{n} u_{ij}^m \right) \\
\mathbf{u}_j &= \left[ \sum_{k=1}^{c} \left( d_{kj} / d_{kj} \right)^{\frac{2}{m-1}} \right]^{-1}
\end{align*}
\]

Repeat the aforementioned procedures, we can update the membership matrix and clustering centers. The predetermined termination threshold can be \( t = T \) or \( \| \mathbf{V}_t - \mathbf{V}_{t-1} \| \leq \varepsilon \), which means that the iterating process can be terminated after a predetermined number of cycles or the change of clustering centers are less than certain threshold. After the iterating process, we can obtain the final value of \( U \) and \( V \).

2.2.3. Performance evaluation. The performance of clustering methods can be evaluated by calculating the inner cluster distances and inter cluster distances. To evaluate the performance of FCM, equation (6) is adopted, where the former one is sum of the within fuzzy cluster fluctuations and the latter one is the sum of the between fuzzy cluster fluctuations.

\[
\begin{align*}
\min P(c) &= \min \left( \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^m \left( \| \mathbf{x}_j - \mathbf{v}_i \|^2 \right) \right) - \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^m \left( \| \mathbf{v}_i - \bar{x} \|^2 \right) \\
\bar{x} &= \frac{1}{n} \sum_{k=1}^{n} \mathbf{x}_k 
\end{align*}
\]

where \( \bar{x} \) is the average of all the measured data.

2.3. Change point estimation

For some two-phase degrading process, the change point is quite obvious. In [9], the degrading process of liquid coupling devices obviously performs inverse ‘L’ shape, where the change point estimation can be easily and reasonably set as the turning point of this shape. However, for most of equipment or components, the change point estimation is not intuitive. Thus, for this kind of time-series, the change point can be obtained by the membership matrix. In our paper, considering that we are dealing with continuous degrading data rather than randomly scattered data sets, we assume that each monitored data can only be assigned to belongs to two adjoining states. Thus, we set the data whose absolute differential membership values (denoted as \( \varepsilon_m \)) to two adjoining degrading states less than 0.1 as change point of two adjoining states. The setting can be expressed as:

\[
\begin{align*}
u_{ij} - u_{ij+1} \leq \varepsilon_m \quad (\forall i = 1, \cdots c, j = 1, \cdots n)
\end{align*}
\]

3. Case study

3.1. Data description

In this paper, we aims to select the change point and evaluate the degrading state of an intermittent working equipment. The equipment operates in two states, the storage state, where the degrading process is quite slow due to the stable environment, and the operating state, where the degrading process is sharp due to the open environment and random strong shock. The degrading process of this equipment are as depicted in Figure 1, where the longitudinal axis represent the accumulative degradation and transverse axis represent the corresponding monitoring point (396 times in total). It can be seen that the degrading process is composed of 4 working cycle and the failure threshold is set as 4.4.
3.2. Clustering output
To label the monitored unlabeled data, fuzzy c-means clustering method, as described in subsection 2.2 is utilized. Note that the experiment is conducted using the Y_FCMC MATLAB Toolbox V1.0 on an Intel(R) Core(TM) i7-6500 U CPU 2.50 GHz using Windows 10. And the output is as showed in Figure 2. As we have illustrated in our paper, the clustering output show that the normal state is represent by fuchsia points, mild degradation state is blue stars, moderate degradation state is green circles, and near-to-failure degradation state is red crosses. The corresponding clustering are mark with bold dark symbols. Also, in Figure 3, we can see that the termination measure gradually decrease to a very low level and the iterating process ended at the 67th iteration, where the termination threshold $\varepsilon$ is satisfied.

3.3. Membership and change point selection
To select appropriate change points, the final membership of monitored data to different clusters in the given time series are as showed in Figure 4. Also to better illustrate the membership, a stem depiction (Figure 5) is provided. It can be seen that the membership of points to a certain clustering center gradually increase to nearly 1 and then gradually decrease to nearly zero. With the decreasing process, the membership of these points to adjoin clustering centers increases, which implies that the state of equipment gradually turns to another one.
According to the definition of change point, we can obtain that change points are 96-99, 198-202 and 301-305.

4. Conclusion
In this paper, we manage to use the FCM algorithm to automatically assign labels to the monitored unlabelled data, which denotes the different degrading process of an intermittent working equipment. The results show that FCM is of good performance in estimating change points.

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