Optimal design of advanced 3D printed composite parts of rocket and space structures

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Abstract. In this article the method of structural and topology optimization for 3D printed composite parts reinforced with continuous fibers is presented. Within this method part shape is changed simultaneously with material orientation in order to minimize the mass of a part under its strength constraints. The presented method is demonstrated on example of cantilever plate. Quadratic stress criteria is used as strength constraint. Algorithm is implemented as python code interacted with Ansys software. The obtained optimized design is shown. Finite element analysis of the optimized plate is performed.

Key words: topology optimization, fiber steering, composite 3D printing, optimal design of composites

1. Introduction
Mass minimization of advanced parts of rocket and space structures is an important problem solved by designers. The reduction of mass enables to use less fuel which is a great economic and ecologic benefit. Continuous fiber reinforced composite materials have high mechanical properties along fibers and are lightweight. To realize these benefits of composites to the fullest extent composite parts should be designed so that the load is taken along the fibers. The proper use of composites provides obtaining lightweight parts with high mechanical properties. Lattice structures are example of such use of composites [1]. Composite Payload Lattice Adapter is shown in Figure 1.
Continuous fiber reinforced composite 3D printing technology has developed rapidly during last years [2-7]. It enables high flexible fiber steering so that parts with almost any topology and internal structure can be produced. Anisoprint company has developed 3D printers which embed reinforcing fiber pre impregnated with thermoset polymer into molten thermoplastic polymer and produce the so-called bi-matrix composites [8-10]. Continuous fiber 3D printed Side Panel Element produced by Anisoprint is shown in Figure 2.
In order to design composite part in the best way structural and topology optimization is needed. Topology optimization is the process of finding the best material distribution within the given design space, for given loads and boundary conditions. Structural optimization is the process of finding the best orientation of fibers within the given topology. To get the optimal design for anisotropic part these optimizations should be performed simultaneously.

Mass minimization problem with strength constraints has been successfully solved for isotropic materials [11-21]. One of the algorithms for solving this problem is Proportional Topology Optimization method (hereinafter referred to as PTO) [20]. This is a non-gradient algorithm which implies easy implementation and small computational cost.

As we deal with anisotropic material we need to optimize material orientation as well. For this purpose, a fiber steering technique was proposed [22]. Within this approach reinforcing fibers are oriented along the principal stress direction. This orientation implies minimum local compliance of the structure and assures that the load is directed along the fiber. Another approach was suggested by Kentaro Sugiyama et al. [23].

In this article a new method of mass minimization with strength constraints for 3D printed composite parts is presented. It combines PTO method with fiber steering technique. The method is demonstrated on a rectangular cantilever plate with applied concentrated force. Optimal shape and layout are obtained for the plate. Strength constraint is satisfied. Finite element analysis is performed on the optimized model. Stresses appear to be uniformly distributed.

2. Design variables

Design variables for PTO method are chosen using Solid Isotropic Material with Penalization (SIMP) technique. Design space is supposed to be divided into finite elements (FE). The so-called density $\rho$, varying from 0 to 1, is assigned to each element. It states for the presence of material in particular element. So, $\rho = 0$ means that the element is empty i.e. does not contain material, while $\rho = 1$ means that the element is full, i.e. it is completely filled with material. Only 0 and 1 values are physically essential. But we allow intermediate densities as well in order to have continuous problem. Within SIMP method elastic moduli $E$ and $G$ of material in element with density $\rho$ are calculated through the real moduli $E_{\text{real}}$ and $G_{\text{real}}$ of material in the following way:

$$
E = \rho^p E_{\text{real}} \\
G = \rho^p G_{\text{real}}
$$

$p > 1$ is a penalty factor. The increase of $p$ makes it ineffective to have intermediate densities.

Optimization variables for fiber steering technique are angles of material orientation $\theta$. So, besides the density, the angle is assigned to each finite element. Thus, the number of optimization variables is twice as big as the number of elements.

3. Problem formulation.

Using chosen optimization variables, the problem of mass minimization with strength constraints can be written as follows:
min $\sum_{i=1}^{N_{el}} d_i v_i \rho_i$

$\sigma^* \leq 1$

$0 \leq \rho_i \leq 1, \quad i = 1, ..., N_{el}$

$0 \leq \theta_i \leq 180^\circ, \quad i = 1, ..., N_{el}$

Here $N_{el}$ is a total number of elements in the model, $d_i$ is the density of material in the $i$th element, $v_i$ is the volume of the $i$th element, $\sigma^*$ is some function of stresses representing strength constraint.

4. The PTO algorithm.

Algorithm of PTO is described in [20]. Slightly modified version used in this work is shown in Figure 3. To start some initial densities need to be chosen, say, all densities are equal to the same value. Sum of all densities in the current iteration is called target material amount. Before the first iteration it is calculated using initial densities.

Then a cycle begins and continues until chosen convergence criteria is satisfied. Each loop of the cycle begins with FE analysis of the current model. Stresses are obtained and stress measures of elements are calculated. In case of isotropic material von Mises stresses were taken [20]. Then a strength constraint is checked. If it is satisfied the target material amount is increased. Otherwise it is decreased. Target material amount is then distributed using an inner cycle. On each iteration of this cycle some part of target material amount remains. This part is called remaining material amount. At first it is made equal to target material amount. The inner cycle continues until remaining material amount is small enough so that almost all target material is distributed on elements. Current material amount is used to keep information about material amount left to be distributed. At first it is equal to target material amount. On each inner iteration current material amount is distributed proportional to element stress measures. After this distribution some densities can appear to exceed 1. Such densities are made equal to 1 and material deleted by this procedure is acquired to remaining material. After the inner cycle densities are updated using historical coefficient $\alpha$ and densities from previous iteration $\rho^{prev}$ in the following way:

$$\rho := \alpha \rho^{prev} + (1 - \alpha) \rho$$

Historical coefficient $\alpha$ represents inertia of densities changing. In case of $\alpha = 0$ densities do not depend on the previous iteration at all. In case of $\alpha = 1$ densities are not changed at all. Finally filtering
technique is applied to densities. Instead of element density $\rho_i$ weighted average of all densities inside the circle with the center in this element and radius $r$ are taken:

$$\rho_i^{\text{filter}} = \frac{\sum_{j=1}^{N} w_{ij} \rho_j}{\sum_{j=1}^{N} w_{ij}}$$

$w_{ij}$ – are weights inversely proportional to the distance between the $i$th and the $j$th elements. After densities are filtered one outer iteration ends and another one begins.

5. Fiber steering technique.
Within fiber steering technique angles are changed iteratively in the following way. On each iteration principal stress directions $\mathbf{n}_\alpha$ are found for all elements. In case of 2D problem there are two in-plane principal stress directions $\mathbf{n}_1$ and $\mathbf{n}_2$ for an element. Local compliance corresponding to principal stress direction $\mathbf{n}_\alpha$ is calculated by the following formula:

$$c = \frac{1}{2} \{\sigma_{xy}\}^T C^{-1}(\mathbf{n}_\alpha) \{\sigma_{xy}\}$$

Here $\{\sigma_{xy}\}$ is the vector of stress components for the element. $C(\mathbf{n}_\alpha)$ is a stiffness matrix for material which reinforcement coincides with principal stress direction $\mathbf{n}_\alpha$. Principal stress direction corresponding to minimum local compliance is chosen. Current material orientation is moved towards the chosen direction.

6. Presented algorithm
The algorithm presented in this work combines PTO method with fiber steering technique and is described in Figure 4.

![Figure 4](image_url)

**Figure 4.** Combination of PTO method and fiber steering technique

The usual procedure of PTO takes place and fiber steering technique is added at the end of each outer loop. Finite element analysis is conducted after filtering and stresses are calculated in all elements. Using
this information principal stress directions with minimum compliance are chosen and angles are moved towards them.

Quadratic failure criterion containing the difference between tensile and compressive strengths is applied to strength evaluation in this work:

\[
\sigma_1 \left( \frac{1}{\sigma_1^2} - \frac{1}{\sigma_1^2} \right) + \sigma_2 \left( \frac{1}{\sigma_2^2} - \frac{1}{\sigma_2^2} \right) + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} + \frac{\sigma_2^2}{\sigma_2^2 + \sigma_1^2} + \left( \frac{\tau_{12}}{\tau_{12}} \right)^2 \leq 1
\]

Reserve factors are calculated using the equation:

\[
n = \frac{2}{s + \sqrt{s^2 + 4\tau^2}}
\]

where

\[
s = \sigma_1 \left( \frac{1}{\sigma_1^2} - \frac{1}{\sigma_1^2} \right) + \sigma_2 \left( \frac{1}{\sigma_2^2} - \frac{1}{\sigma_2^2} \right), \quad \tau^2 = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} + \frac{\sigma_2^2}{\sigma_2^2 + \sigma_1^2} + \left( \frac{\tau_{12}}{\tau_{12}} \right)^2
\]

Inverse reserve factors are taken as element stress measures. In the work of Emre Biyikli and Albert C. To strength constraint bounds maximal stress measure to be less or equal to 1 [20]. In the current work p-norm of stress measures is used instead of maximum in order to ignore local stress peaks. P-norm is used by Erik Holmberg, Bo Torstenfelt and Anders Klarbring in the work [19]. P-norm of some values \(a_1, a_2, \ldots, a_n\) looks like the following:

\[
\|a_1, a_2, \ldots, a_n\|_p = \left( \frac{\sum_i (a_i)^p}{n} \right)^{\frac{1}{p}}
\]

P-norm of some set of values tends to the maximal value when \(p\) tends to infinity:

\[
\lim_{p \to \infty} \|a_1, a_2, \ldots, a_n\|_p = \max_i a_i
\]

In this work \(P = 20\) in which case only view element stress measures exceeded the value of P-norm.

7. Test problem formulation.

In order to demonstrate described algorithm of optimization a rectangular plate (see Figure 5) is considered.

\[
\text{Figure 5. Cantilever plate with applied concentrated force } F
\]

The left edge of the plate is clamped. A concentrated force is applied at the center of the right edge as shown in Figure 5, and directed down. Plate width is 0.2 m. Plate height is 0.1 m. Plate thickness is 0.005 m. The magnitude of the force is 100 N. The following material properties are used:

\[
E_1 = 64100000000 \quad E_2 = 4100000000 \quad G_{12} = 11700000000 \quad v_{12} = 0.023
\]

Such properties correspond to material obtained using Composite Filament Co-Extrusion (CFC) Anisoprint technology. The plate was meshed into 2048 square elements.

8. Results.

To solve the problem Python code interacting with Ansys software package is written. Initial mass of the part is 141 g. The result of optimization is a frame shown in Figure 6.
The lower ribs of the frame are thicker than the upper one. It follows from the fact that compressive strength is lower than tension strength. In the central part of the frame the strut elements can be seen. These elements prevent distortion of the frame shape under shear load. Material is oriented along the ribs of the frame. Weight of the obtained part is 35.9 g which is 25.5% of the fully filled space. FE analysis is performed. Stresses $\sigma_1$, $\sigma_2$ and $\tau_{12}$ in material coordinate system are shown in Figures 7(a)–7(c). Stresses are close to uniform in the most part of the plate. On the left side of the plate near rigid support stresses achieve their maximum. Reserve factors are shown in Figure 7(d). In the most part of the plate they exceed 1 quite a lot and are close to 1 near the rigid support. Strength criterion is satisfied but the method still needs to be improved so that all reserve factors are close to 1.

9. Conclusion.
New algorithm for mass minimization with strength constraints for Anisoprint CFC technology is proposed. This algorithm combines PTO method and fiber steering technique. Optimization problem for cantilever plate with concentrated force is solved using described algorithm. Quadratic failure criterion containing the difference between tensile and compressive strengths is used as strength constraint. The optimal design of the plate is obtained. Strength constraint is satisfied. Finite element analysis is conducted for the plate. Stresses are shown to be close to uniform which proves optimality of the
solution. The proposed method enables to get lightweight strong parts and can be applied to design of rocket and space structure where mass is of prime importance [24-26].

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