Implication of the Hubble tension for the primordial Universe in light of recent cosmological data

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Abstract

In prerecombination resolutions of the Hubble tension, such as early dark energy, new physics before recombination shifts the values of relevant cosmological parameters so that the models can fit with cosmic microwave background and baryon acoustic oscillations observations as well as ΛCDM does. In this paper, we clarify how the parameter shifts are related with $\delta H_0$, particularly we find the shift of primordial scalar spectral index scales as $\delta n_s \simeq 0.4 \frac{\delta H_0}{H_0}$ by performing the Monte Carlo Markov chain analysis with the Planck2018+BAO+Pantheon+R19+Keck Array/BICEP dataset. A novel point of our result is that if the current $H_0$ measured locally is correct, complete resolution of the Hubble tension seems to be pointing to a scale invariant Harrison-Zeldovich spectrum, i.e. $n_s = 1$ for $H_0 \sim 73 \text{km/s/Mpc}$.

PACS numbers:
I. INTRODUCTION

The Hubble constant $H_0$ quantifies the current expansion rate of our Universe. It can be predicted based on observations of anisotropies in the cosmic microwave background (CMB) and other early universe physics such as baryon acoustic oscillations (BAO). Assuming the standard cosmological model ($\Lambda$CDM), the Planck collaboration has reported $H_0 = 67.4 \pm 0.5\text{km/s/Mpc}$ [1]. Recently, $H_0$ has been also measured by lots of local observations (up to percent level accuracy). However, almost all yield $H_0 \sim 73\text{km/s/Mpc}$, which is in stark ($> 4\sigma$) tension with that reported by Planck collaboration based on the $\Lambda$CDM model [2, 3], usually dubbed the Hubble tension. The origin of this tension is still under investigation, but it is unlikely to be explained by unknown systematic errors [4–6].

Recently, it has been widely thought that the Hubble tension is suggesting new physics beyond $\Lambda$CDM [2, 3, 7–10], see Ref.[11] for a thorough review on various ideas, see also, e.g, Ref.[12–14] for some more recent discussions. Reducing the sound horizon $r_s^* = \int_{z_*}^{\infty} c_s/H(z)dz$ ($z_*$ is the redshift at recombination), as in early dark energy [15–33] (non-negligible only for a short epoch decades before recombination) or early modified gravity models [34–38], is a promising road towards the complete resolution of the Hubble tension. Since probes of the early universe, such as CMB and BAO, set the angular scales $\theta_s^* \equiv r_s^*/D_A^* \left(D_A^* \sim 1/H_0 \right.$ is the angular diameter to last scattering surface), a smaller $r_s^*$ naturally brings a larger $H_0$. It should be mentioned that the beyond-$\Lambda$CDM modifications after recombination are difficult to reconcile with low redshift data (light curves and BAO) [5, 6, 39, 40]; see also e.g. [18, 41–45].

In corresponding early dark energy (EDE) models, $H_0 \gtrsim 70\text{km/s/Mpc}$, moreover, the existence of anti de-Sitter (AdS) vacua around recombination can further lift $H_0$ to $\sim 73\text{km/s/Mpc}$ [18, 24]. Although many of the early resolutions of the Hubble tension have been found to fit with CMB, BAO and light curve observations as well as $\Lambda$CDM does, the cost of compensating for the impact of new physics before recombination is that the values of relevant parameters $\omega_{\text{cdm}}, \omega_b$ and $n_s$ must be shifted [15–17, 24]. The parameter shifts not only make the corresponding early resolution models tested by upcoming CMB experiments, but also have potential implications to the inflation and primordial Universe, see also [46–50] for relevant studies. Thus it is significant to have a full insight into the shift patterns of parameters, specially the shift of the spectral index $n_s$ (the primordial perturbation spectrum...
$P_s \sim (k/k_{\text{pivot}})^{n_s}$.

To identify the common pattern of parameter shifts in different models, we first have to marginalize over the model-specific information. We focus on the prerecombination resolutions (referred to as early resolutions) of the Hubble tension satisfying the following:

- Reduce $r^*_s$ to lift $H_0 \cdot (r^*_s H_0) \sim \text{const.}$.
- The evolution after recombination is described by $\Lambda$CDM.
- The recombination process is not modified \(^1\).

We show how the parameter shifts in early resolution models are related scalingly with $H_0$. Specially, for $n_s$, we get

$$\delta n_s \simeq 0.4 \frac{\delta H_0}{H_0}, \quad (1)$$

see also Fig.1 for the Monte Carlo Markov chain (MCMC) analysis with joint Planck2018+BAO+Pantheon+R19 dataset, as well as recent Keck Array/BICEP data \[^55\]. (1) explains how the early resolutions of the Hubble tension bring about a larger $n_s$ than $\Lambda$CDM, as observed in e.g. Refs. \[^15\text{–}17, 24\], in which $n_s \gtrsim 0.98$ for $H_0 \gtrsim 71 \text{km/s/Mpc}$.

The exact scale invariant primordial spectrum ($n_s = 1$), i.e. the Harrison-Zeldovich spectrum proposed first in \[^56\text{–}58\], has been strongly ruled out in $\Lambda$CDM (suffering Hubble tension) at 8.4σ \[^59\]. However, Refs. \[^49, 50, 60\] point out the possibility of fully ruling out $n_s = 1$ is actually connected with the solution to the Hubble tension by noticing qualitatively some possible correlation between a larger $n_s$ and a larger $H_0$ in $N_{\text{eff}}$ (and/or $Y_{\text{He}}$)+$\Lambda$CDM models. According to (1), the novel point of our result is that if the current $H_0$ measured locally is correct, complete resolution of the Hubble tension seems to be pointing to a scale invariant Harrison-Zeldovich spectrum, i.e.$n_s \simeq 1$ for $H_0 \sim 73 \text{km/s/Mpc}$, see also Fig.2.

In section-II, we identify the physical sources behind the parameter shifts and show the corresponding scaling relations, which are then confronted with the MCMC results of early resolution models in section-III. We conclude our results in section-IV.

\(^1\) There are also proposals modifying the recombination process, e.g. \[^51\text{–}53\], see also \[^54\] for the primordial magnetic fields.
FIG. 1: The Hubble constant vs. $n_s$ plot. The EDE models: the $n = 3$ Axion model [15], the $n = 2$ Rock ‘n’ model [16] (called $\phi^4$ for simplicity) and the AdS-EDE model [24]. Dataset: Planck2018+Keck Array/BICEP2015+BAO+Pantheon+R19, also for Figs. 2, 3 and Tables I, II. We see that the ΛCDM and EDE models are consistent with the $n_s$-$H_0$ scaling relation (1).

II. PARAMETER SHIFT

In this section we identify the physics relevant to the shifts of $\omega_{cdm}$, $\omega_b$, and $n_s$ in the early resolutions (refer to section-I for our definition of “early resolution”). Parameters in early resolutions are expected to shift by $\lesssim 15\%$ to fully resolve the Hubble tension; thus it is sufficient to work around the fiducial ΛCDM bestfit point. Regarding CMB power spectra, we look for cosmological parameter shifts in early resolutions that restore the shapes of spectra to those in the fiducial model.
FIG. 2: The tensor-to-scalar ratio $r$ vs. $n_s$ plot. The pivot scale $k_{\text{pivot}} = 0.05\text{Mpc}^{-1}$. Scattered points correspond to the AdS-EDE model with a color coding for $H_0$. As indicated by the contours and scattered points, $n_s \simeq 1$ for $H_0 \sim 73\text{km/s/Mpc}$.

A. Shift in $\omega_{\text{cdm}}$

The reason behind the $\omega_{\text{cdm}}$ shift has been presented in Ref.[24], see also [61] and the result is $\omega_{\text{cdm}}H_0^{-2} \sim \text{const.}$ (applicable to any early resolutions compatible with CMB and BAO data e.g.[24]), or equivalently $\Omega_{\text{cdm}} \sim \text{const.}$ Thus considering $\theta_s^* = r_s^*/D_A^* \sim \text{const.}$ and $r_s^*H_0 \sim \text{const.}$, we have

$$\frac{\delta H_0}{H_0} \simeq -\frac{\delta D_A}{D_A} \sim 0.5 \frac{\delta \omega_{\text{cdm}}}{\omega_{\text{cdm}}}. \quad (2)$$

B. Shift in $n_s$

In this subsection we show how $n_s$ is shifted in response to the change in $\omega_b$.

Suppose $\omega_b$ has some fractional deviation from its $\Lambda$CDM value, it changes the effectiveness of baryon drag, particularly the relative heights between even and odd TT acoustic peaks. Baryon drag is affected by $\Psi_s^*$, the Newtonian potential near last scattering, as well, but $\Psi_s^*$ is also constrained independently by the (integrated) SW effect(s). Thus $\Psi_s$ at the last-scattering surface is preserved in the early resolutions. The peak height (PH) of the
first two (1 and 2) TT peaks numerically respond to $\omega_b$ according to

$$\frac{\delta P_{H_1}}{P_{H_1}} \simeq 0.3 \frac{\delta \omega_b}{\omega_b}, \quad \frac{\delta P_{H_2}}{P_{H_2}} \simeq -0.4 \frac{\delta \omega_b}{\omega_b}. \quad (3)$$

Data fitting will pin the pivot $C_i^{TT}(k = k_{pivot})$, which is close to the second peak, to the observed value by adjusting $A_s$ and $\tau_{reion}^2$, resulting in a cumulative excess of power in the first peak $\frac{\delta P_{H_1}}{P_{H_1}} \simeq 0.7 \frac{\delta \omega_b}{\omega_b}$. This is compensated by $n_s$ according to

$$\left(1 + 0.7 \frac{\delta \omega_b}{\omega_b}\right) \left(\frac{k_1}{k_{pivot}}\right)^{\delta n_s} \sim 1 \quad (4)$$

where $k_1 \sim 0.021 \text{Mpc}^{-1}$ corresponds to the first TT peak while $k_{pivot} = 0.05 \text{Mpc}^{-1}$ is the pivot scale. This suggests

$$\delta n_s \sim 0.8 \frac{\delta \omega_b}{\omega_b}. \quad (5)$$

C. Shift in $\omega_b$

The damping angular scale $l_D \sim k_D D_A$ is fixed by CMB observation [62], thus $k_D$ must respond to the fractional change in $D_A$ according to

$$\frac{\delta k_D}{k_D} \simeq -\frac{\delta D_A}{D_A}. \quad (6)$$

To have an insight into the sensitivity of $k_D$ to the background evolution brought by the new physics shortly before recombination, we look at a simple example. Consider new physics (e.g. dark radiation, EDE) excited at $z_c > z_*$, which can be approximated as a fluid with $p = w \rho$ ($w > 1/3$ so it redshifts faster than radiation) at the background level, we have

$$k_D^{-2} = \int_0^{n_\ast} \frac{d\tilde{n}}{6(1 + R)n_\ast \sigma_T a(\tilde{n})} \left[\frac{R^2}{1 + R} + \frac{8}{9}\right]$$

$$\simeq k_D^{-2}(z_c) + \frac{27 \lambda \omega_b \sigma_T}{4} \left(\frac{3}{4}\right)^{-1} \int_{a_c}^{a_\ast} \left[\omega_r(a/a_0)^{-4} + \omega_m(a/a_0)^{-3} + f_c(a/a_c)^{-3(w+1)}\right]^{-1/2} da. \quad (7)$$

where $R \equiv 3 \rho_b/4 \rho_\gamma$ is the baryon-to-photon energy ratio, and $f_c$ is the energy fraction of new physics at $z_c$. $n_c \simeq \lambda \omega_b (a/a_0)^{-3}$, $\lambda$ being a dimensionful proportional coefficient, and

\footnote{The overall amplitude at $l \gtrsim 1500$ is suppressed by the streaming of extra non-tightly coupled degree of freedom (e.g. $\Delta N_{eff}$ or scalar field) in the early resolutions, so that the increment in $A_s e^{-2 \tau_{reion}}$ does not spoil the fit at high $l$.}
\(\sigma_T\) are the free electron density and Thomson cross-section respectively. We approximately have \(R = 0\) since \(z_c > z_s\). We set \(a_c = a_0\omega_r/\omega_m \simeq a_{eq}\) (the matter-radiation equality point), and expand around the ΛCDM model

\[
\left| \frac{\delta k_D}{k_D} \right| \lesssim f_c \int_{1}^{y_s} y^{6-(w+1)}(1+y)^{-3/2} \, dy \ll f_c
\]

(8)

where \(y_s = a_s/a_c\), which shows that \(k_D\) is insensitive to the background evolution brought by the new physics before recombination. Physically, eq. (8) represents the fact that, whatever the new physics, its energy density redshifts fast enough that it is negligible on the last scattering surface. The major contribution to the integration determining \(k_D\), eq. (7), comes from the last scattering surface thus the background modification induced by the new physics has negligible effect on \(k_D\). This suggests that the shift in \(k_D\) required by Eq. (6) is essentially encoded in shifts of cosmological parameters. According to \(k_D \propto \omega_b^{1/2} \omega_{cdm}^{1/4}\) (within ΛCDM) and Eq. (2), we get

\[
\frac{\delta \omega_b}{\omega_b} \simeq - \frac{1}{2} \frac{\delta \omega_{cdm}}{\omega_{cdm}} - 2 \frac{\delta D_A}{D_A} \sim - \frac{\delta D_A}{D_A}.
\]

(9)

In ΛCDM, both the sound horizon \(r_s\) (corresponding to angular scale \(l \sim 200\)) and the damping scale \(k_D\) (important for the damping tail \(l > 1500\)) are tuned by one single parameter \(\omega_b\). Early resolutions break this correlation by introducing new physics before recombination which only prominently affects the larger scale, i.e. \(r_s\). Actually, the increment in \(\omega_b\) will be less than Eq. (9), since compared with the ΛCDM model some extra damping is needed to compensate for the excess power at high \(l\) brought by a larger \(n_s\). However, since the high \(l\) CMB data is not as precise as the first few acoustic peaks, it is difficult to speculate the corresponding effects in an analytical way. To this end, we marginalize over this effect with the parameter \(0 < \alpha < 1\) and rewrite (9) as

\[
\frac{\delta \omega_b}{\omega_b} \sim -(1 - \alpha) \frac{\delta D_A}{D_A}.
\]

(10)

III. \(n_s-H_0\) SCALING RELATION AND MCMC RESULTS

We confront the scaling relations shown in section-II with the MCMC results. As concrete examples of early resolution models, we limit ourself to the EDE. The EDE models we consider are the \(n = 3\) Axion model \(V(\phi) = V_0(1 - \cos(\phi/f))^3\) [15, 20], the \(n = 2\) Rock ‘n’ model \(V = V_0(\phi/M_p)^4\) [16] (called \(\phi^4\) for simplicity) and the AdS-EDE model with
fixed AdS depth, see [18] for details. In addition to the six ΛCDM cosmological parameters 
\{ω_b, ω_{cdm}, H_0, \ln 10^{10} A_s, n_s, τ_{reion}\}, all EDE models have two additional MCMC parameters 
\{\ln(1 + z_c), f_{ede}\}, with z_c being the redshift at which the field φ starts rolling and f_{ede} the 
energy fraction of EDE at z_c. The Axion model varies yet one more MCMC parameter Θ_i, 
the initial position of the scalar field, see [15] for details.

According to Eqs.(2), (5) and (10), the shift of parameters \{ω_{cdm}, ω_b, n_s\} can be straightly 
related to \( f_{ede} \). Generally, all components (baryon, dark matter, radiation\(^3\) and early dark 
ergy) contribute to \( r_s^* \). Assuming the energy injection near matter-radiation equality 
z_c ≈ z_{eq}, which is valid for almost all EDE models, we numerically evaluate the response of 
r_s to \( f_{ede}, \omega_{cdm} \) and \( ω_b \) around the ΛCDM bestfit \( f_{ede} = 0 \)
\[
- \frac{δr_s}{r_s} ≃ 0.3 f_{ede} + 0.2 \frac{δω_{cdm}}{ω_{cdm}} + 0.1 \frac{δω_b}{ω_b}.
\]

Compatibility with (2) implies \( \frac{δω_{cdm}}{ω_{cdm}} ≃ f_{ede} + 0.33 \frac{δω_b}{ω_b} \). This is equivalent to adjusting \( f_{ede} \) 
and \( ω_{cdm} \) such that near recombination \( Φ^{EDE}(l) ≃ Φ^{LCDM}(l) \) up to data uncertainty for the 
first few peaks. Thus we have
\[
\frac{δω_b}{ω_b} ∼ 0.6 f_{ede}(1 - 1.2\alpha), \tag{12}
\]
\[
δn_s ∼ 0.5 f_{ede}(1 - 1.2\alpha), \quad \frac{δH_0}{H_0} ≃ 0.5 \frac{δω_{cdm}}{ω_{cdm}} ∼ 0.6 f_{ede}(1 - 0.2\alpha). \tag{13}
\]

Thus \( H_0 \) is lifted proportionally to \( f_{ede} > 0 \). However, the cost of making EDE still fit CMB, 
BAO and light curve observations as well as ΛCDM does (as is confirmed with the MCMC 
analysis) is that the relevant parameters must be shifted \( (∼ f_{ede}) \).

To clearly see the effect of the Hubble tension on the parameters \( n_s \) and \( r \) of the primordial 
Universe, where the tensor-to-scalar ratio \( r = A_T/A_s (k_{pivot} = 0.05\text{Mpc}^{-1}) \), we use Planck 
low-l EBBB and Keck Array/BICEP 2015 data [55], and set recent SH0ES result \( H_0 = 74.03 \pm 1.42\text{km/s/Mpc} \) [65] (R19) as a Gaussian prior. In addition, our datasets consist of 
the Planck18 high-l TTTEEE and low-l TT likelihoods as well as Planck lensing [1], the 
BOSS DR12 [66] with its full covariant matrix for BAO as well as the 6dFGS [67] and MGS 
of SDSS [68] for low-z BAO, and the Pantheon data [69].

It should be underlined that the fiducial model we consider is ΛCDM with its six cos-
mological parameters and the MCMC results for ΛCDM depend on dataset. Our results in

\(^3\) The radiation energy density is fixed by the \( T_{0,FIRAS} \) [63, 64], which is compatible with EDE [24].
| Model       | $f_{ede}$ | $100\omega_b$ | $\alpha$ | $H_0$   | $\omega_{cdm}$ | $n_s$   | $r$       |
|-------------|-----------|----------------|----------|---------|----------------|--------|----------|
| $\Lambda$CDM | -         | 2.246          | -        | 68.1    | 0.1184         | 0.969  | $< 0.0636$ |
| $\phi^4$    | 0.070     | 2.274          | 0.59     | 70.3(70.6) | 0.1271(0.127) | 0.980(0.979) | $< 0.0603$ |
| Axion       | 0.094     | 2.295          | 0.51     | 70.9(71.5) | 0.1295(0.13)  | 0.987(0.987) | $< 0.066$   |
| AdS-EDE     | 0.115     | 2.336          | 0.35     | 72.6(72.5) | 0.1346(0.134) | 0.997(1)    | $< 0.0574$ |

TABLE I: The mean values of parameters in corresponding models and the 95% upper bounds on the tensor-to-scalar ratio $r$. In the parenthesis are the analytic estimations made by Eq.(13), which are consistent with the MCMC results.

section-II are based on Eq.(2), which suggests that the corresponding MCMC dataset must include CMB and BAO data at least [24].

We modified the Montepython-3.3 [70, 71] and CLASS [72, 73] codes to perform the MCMC analysis. Table-I presents the MCMC results for $\Lambda$CDM and EDE models (Axion, $\phi^4$ and AdS-EDE), see also the corresponding $H_0$-$n_s$ and $r$-$n_s$ contours in Figs.1 and 2, respectively. As expected, all early resolution models fit to CMB and BAO as well as $\Lambda$CDM does, see Table.II for the bestfit $\chi^2$ per experiment. The existence of the AdS region in AdS-EDE actually sets a physical lower bound on the EDE energy fraction $f_{ede}$, because the field would fail to climb out of the AdS region if $f_{ede}$ is too small. However, as is clear in Fig.1, 2, 3 and Table.II, the MCMC chain is not hard capped by this bound and converges around the bestfit point well. The upper bound on $r$ in the AdS-EDE model is slightly smaller than that in other models, see Fig.2 and Table-I.

Actual numerical results of parameter shifts are plotted in Fig.3, which are consistent with Eqs.(2) and (5). Though the detailed shape of CMB spectra is affected by all cosmological parameters in complicated ways, the analytic approximations Eqs.(2) and (5) have clearly captured the common pattern of parameter shifts.

As representative early resolutions models, all EDE models in Figs.1 and 2 show $n_s \gtrsim 0.98$, which is actually a “universal” prediction of early resolutions of the Hubble tension, in particular the AdS-EDE model allows $n_s = 1$ at 1$\sigma$ region. To see this, Eqs.(2), (5) and (10) in combination relates $n_s$ with $H_0$

$$\delta n_s \sim 0.8(1 - \alpha) \frac{\delta H_0}{H_0}.$$  \hfill (14)

Here, $\alpha$ can be set by Eq.(12) with MCMC results of $f_{ede}$ and $\omega_b$, see Table-I. Generally, we
FIG. 3: Analytic approximations (2) and (5) confronted with MCMC 68% and 95% contours for EDE models. Left panel: $\omega_{cdm}$ vs. $H_0$. Yellow line plots Eq.(2). Right panel: $\omega_b$ vs. $n_s$. Yellow line plots Eq.(5).

have $0.4 \lesssim \alpha \lesssim 0.6$. According to (14), we approximately get (1).

The argument in section-II is valid for early resolutions, as defined in section-I, with rather weak dependence on the specific physical model. According to (14), we have $n_s \approx 1$ for $H_0 \approx 73$ km/s/Mpc. Thus it is intriguing to speculate that, contrary to $\Lambda$CDM, early resolutions of the Hubble tension might play well with $n_s \approx 1$.

IV. CONCLUSION

The early resolutions of the Hubble tension, such as EDE, have been found to fit with CMB, BAO and light curve observations as well as $\Lambda$CDM does, however, the cost of compensating for the impact of new physics before recombination is that the values of relevant parameters must be shifted. We have identified the major physical source behind the parameter shifts. The patterns of parameter shifts are represented by a set of linear response equations (2), (5), (10) and (14), which are confirmed by performing the MCMC analysis with Planck2018+BAO+Pantheon+R19 dataset as well as Keck Array/BICEP data. Our results are common to early resolutions defined in section-I, which not only bring new in-
sight into the physics behind the early resolutions, but also highlight the significance of other model-independent probes of \(n_s, \omega_{\text{cdm}},\) etc. for falsifying early resolutions, e.g.\([74–77].\)

Specially, the shift of \(n_s\) with respect to \(\delta H\) implies that data (Planck2018+Keck Array/BICEP+BAO+Pantheon+R19) allow for much larger \(n_s\) in the early resolutions than in \(\Lambda\text{CDM}.\) Interestingly, if the current local measurement \(H_0 \sim 73\text{km/s/Mpc}\) is correct, it seems to favor an \(n_s = 1\) cosmology in the early resolutions, which has been strongly ruled out in \(\Lambda\text{CDM}\) \([59].\) This might have profound implication to inflation and the early Universe physics. Compared with Planck 2018 result \([59],\) a scale-invariant primordial spectrum together with the current upper bound on tensor-to-scalar ratio \(r \lesssim 0.06\) (Fig.2) does not favor e.g. Starobinski inflation, hilltop inflation. Theoretical implications of our observation is yet to be explored, however, it would be expected that the primordial universe with \(n_s = 1\) might be radically different from the popular paradigm of slow-roll inflation.

Another potentially interesting point is the degeneracy between \(n_s\) and diffusion damping, as mentioned in section-II C. Precise CMB power spectra observations at \(l \gtrsim 2000\) might help break this degeneracy. See, for example, Refs.\([23, 26, 28]\) for current proceedings with SPT \([78]\) or ACT \([79].\)

**Acknowledgments**  YSP is supported by the National Natural Science Foundation of China Grants Nos.12075246, 11690021. BH is supported by the National Natural Science Foundation of China Grants No. 11973016. To produce results of the \(\phi^4\) and Axion model

| Experiment          | \(\Lambda\text{CDM}\) | \(\phi^4\) | Axion | AdS-EDE |
|---------------------|-----------------------|------------|-------|---------|
| Planck high \(l\) TTTEE | 2347.1               | 2347.1     | 2346.4 | 2349.9  |
| Planck low \(l\) TT   | 22.7                 | 21.9       | 20.8   | 21.1    |
| Planck low \(l\) EEBB | 785.9                | 785.7      | 785.9  | 785.5   |
| Planck lensing       | 10.2                 | 9.7        | 10.9   | 10.4    |
| BK15                 | 736.5                | 737.8      | 738.8  | 734.1   |
| BOSS DR12            | 3.4                  | 4.3        | 3.9    | 3.5     |
| BAO low-\(z\)        | 1.9                  | 1.3        | 1.4    | 1.8     |
| Pantheon             | 1026.9               | 1027.3     | 1027.2 | 1026.9  |
| \(H_0\) prior        | 15.6                 | 8.3        | 6.2    | 1.3     |

TABLE II: Bestfit \(\chi^2\) per experiment.
we make use of the publicly available codes class_rnr (https://github.com/franyancr/class_rnr) and AxiCLASS (https://github.com/PoulinV/AxiCLASS).

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