Prediction of the capacitance lineshape in two-channel quantum dots

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We propose a set-up to realize two-channel Kondo physics using quantum dots. We discuss how the charge fluctuations on a small dot can be accessed by using a system of two single electron transistors arranged in parallel. We derive a microscopic Hamiltonian description of the set-up that allows us to make connection with the two-channel Anderson model (of extended use in the context of heavy-Fermion systems) and in turn make detailed predictions for the differential capacitance of the dot. We find that its lineshape, which we determined precisely, shows a robust behavior that should be experimentally verifiable.

Forty years have lapsed since Kondo published his celebrated article [1]. Even today, the Kondo effect continues to be one of the central themes in condensed matter physics. To add to its importance in the context of heavy fermions and localized magnetic moments in metals, one should include now a growing set of experiments that during the last years observed it in quantum dots and other mesoscopic devices [2,3]. The great appeal of such experiments resides in their high degree of control over the different parameters involved that yields unprecedentedly accurate and comprehensive tests for the theories.

An important variant of the Kondo problem is its multi-channel generalization that is known to give rise to non-trivial low-temperature physics characterized by fractional exponents and divergencies [4]. Mainly due to its sensitivity to magnetic fields and channel anisotropy, the experimental examples of multi-channel Kondo physics remain largely controversial. Partly for this reason, as well as for the inherent richness of the physics involved, a large amount of recent theoretical [5, 6, 7, 8, 9, 10, 11, 12, 13, 14] and experimental [15, 16] efforts seek to realize two- or multi-channel Kondo systems in the controlled realm of artificial semiconducting nanostructures and other mesoscopic devices. In this letter we want to contribute to that effort by proposing a highly tunable quantum-dot system that allows access to the two-channel charge-fluctuation physics which lies beyond the scope of an effective Kondo Hamiltonian description.

We propose a set-up involving one quantum dot linked to two mesoscopic systems that we will refer to as grains. The grains are weakly Coulomb blocked as compared to the dot and have a quasi-continuous energy spectrum as against the discrete one of the dot. In Fig. 1 we illustrate the configuration by showing the relative disposition of dot and grains and by indicating the different gates. A set of voltage gates tunes the occupancy of the dot ($V_g$) and the grains ($V_{gL}, V_{gR}$) while another set ($V_{tL}, V_{tR}$) controls the couplings between dot and grains. Although for the most part we envision the grains to be completely decoupled from the two-dimensional electron gas (2DEG) on either side, the indicated gate voltages $V_{pL}$ and $V_{pR}$ allow for controlling the coupling during the tuning stage. Also, for comparison to the conventional set-up of a dot coupled to two leads (that features single-channel physics), the 2DEGs can be allowed to flood the grains by not energizing these two gates.

The purpose of this set-up is to allow controlled experimental measurements of the charging lineshape of the quantum dot for a wide range of $V_g$. Charge fluctuations on the dot are due to the dot-environment coupling which in turn is at the root of complex many-body behaviors such as the single- or multi-channel Kondo effect. In our case the environment is given by the two grains when the plunger voltages ($V_{pL}, V_{pR}$) are completely pinched off in order to decouple the dot-grains system from the 2DEGs. This special environment was carefully engineered to gain a robust access into the rather elusive regime of two-
channel Kondo physics and mixed valence. On one hand the isolated nature of the set-up forbids charge transport experiments, but on the other hand it leaves open the possibility of capacitance lineshape measurements that provide the most accurate and direct information about charge fluctuations and dot-environment interplay.

For making these kind of measurements, the set-up should be prepared as follows. First, let the two 2DEGs flood the grains while energizing $V_a$, so that the charge of the dot is quantized; apply a small bias, and tune $V_g$ while measuring the current until the system is in the middle of a Coulomb peak (a charge degeneracy point of the dot). Second, energize $V_{gx}$ but without isolating the grains and keep adjusting the three voltage gates ($V_g, V_{gL}, V_{gR}$) in order to remain at the center of the Coulomb peak. Slowly increase the values of the plunger while keeping on adjusting as above, until no more current flows and the dot-grains system is isolated from the 2DEGs. This way the dot as well as the grains are all independently tuned near their charge degeneracy points.

As demonstrated in Ref. [17], a high sensitivity charge sensor can be implemented using a metallic Single-Electron Transistor (SET). In our proposed set-up, a “twin SET” (consisting of two SETs arranged in parallel and each associated to one of the two grains) is incorporated directly into the defining gates of the dot-grains system. The central island of each SET is coupled to the source and drain electrodes ($V_s$ and $V_d$) via two small tunnel junctions that make for the leading contribution to the SET capacitance and must be kept as small as possible. One extra gate for each SET ($V_{sL}$ and $V_{sR}$) permits independent fine tuning to achieve the regime of maximum charge sensitivity.

We now turn to the task of writing down a Hamiltonian for this set-up. The largest energy scale involved is the dot charging energy ($E_d$) followed by the charging energies of the grains ($E_{ga}$, $\alpha = L, R$), each responsible for the respective Coulomb blockade. The first step in the modeling should then be to write down what we call the configurational energy of the dot-grains-system:

$$E_{\text{conf}} = E_d [\hat{n}_d - (N_d + y)]^2 + \sum_{\alpha} E_{ga} [\hat{n}_{ga} - (N_{ga} + x_{\alpha})]^2$$

Here we have assumed for the moment that the dot and the grains are all individually isolated and that their respective energies change as the value of the voltage in the corresponding gates is varied (with the linear-function identifications $V_g \mapsto y$ and $V_{ga} \mapsto x_{\alpha}$). The notations $\hat{n}_d$ and $\hat{n}_{ga}$ correspond to the number operators, while $N_d$ and $N_{ga}$ to certain ground state occupations for $y = x_{\alpha} = 0$, of the dot and the grains. Right after preparing the system the way it was described above, we have that $y \approx x_{\alpha} \approx 1/2$ and that $N_d$ and $N_{ga}$ are fixed to certain undefined integer values (we will restrict ourselves to variations of $y$ and $x_{\alpha}$ in the interval $[0, 1]$).

We are going to consider the dot-grains system in isolation; this amounts to imposing the constraint of total charge conservation, that when exactly one extra electron is captured reads:

$$\hat{n} = \hat{n}_d + \sum_{\alpha} \hat{n}_{ga} = (N_d + 1) + \sum_{\alpha} N_{ga} = N + 1 \quad (1)$$

In what follows, we are going to consider $x_{\alpha}$ to be fixed and let only $y$ vary. The charge of the dot fluctuates between $\epsilon(N_d + 1) \leftrightarrow \epsilon N_d$. In the slave-operator language, we associate a fermion operator $\hat{f}^\dagger_{\alpha}$ to the configuration when the extra electron (with spin $\sigma$) is on the dot and a boson operator $\hat{b}_{\alpha}$ to the configuration when the dot is ‘empty’ and the electron is on the grain $\alpha$. The boson does the job of book-keeping by indicating that back tunneling into the dot coming from the opposite grain is blocked ($\hat{a}$ stands for the opposite of $\alpha$). The respective configurational energies read

$$\varepsilon_f = E_d (1 - y)^2 + \sum_{\alpha} E_{ga} (x_{\alpha})^2 \quad (2)$$

$$\varepsilon_{ba} = E_d (y)^2 + E_{ga} (1 - x_{\alpha})^2 + E_{ga} (x_{\alpha})^2 \quad (3)$$

We will make the simplifying assumption that $\varepsilon_{ba} = \varepsilon_{bR} \leftrightarrow \varepsilon_{bL}$; this is true, for instance, in the case of a symmetric set-up (the implications of relaxing this symmetry will be discussed later). The reader can convince himself that all other charge configurations are either forbidden due to the constraint of Eq. (1) or are much higher in energy and can be safely ignored. We will concentrate on the case of $N_d$ even, since, as we shall see, the symmetry of the model is $SU(2) \otimes SU(2)$ and displays two-channel Kondo physics of interest to us. The case of odd $N_d$ is also interesting and will be discussed elsewhere (it gives rise to a single-channel $SU(4)$ model; cf. Ref. [18]).

We shall model the quasi-continuum spectra ($\varepsilon_{ka}^\alpha$) of the grains by a flat density of states ($\rho_{ga}$) and denote the electron creation operator in the grain by $\hat{g}^\dagger_{ka\sigma}$. Writing the configurational energy in Hamiltonian language and adding a term describing the tunneling between dot and grains, we have the total Hamiltonian

$$\hat{H} = \sum_{\alpha} \hat{H}_{\text{grain}} + \hat{H}_{\text{conf}} + \hat{H}_{\text{tun}} \quad (4)$$

$$\hat{H}_{\text{grain}} = \sum_{k\sigma} \varepsilon_{ka}^\alpha \hat{g}^\dagger_{ka\sigma} \hat{g}_{ka\sigma} \quad (5)$$

$$\hat{H}_{\text{conf}} = \sum_{\sigma} \varepsilon_f \hat{f}^\dagger_{\alpha} \hat{f}_\sigma + \sum_{\alpha} \varepsilon_{ba} \hat{b}^\dagger_{\alpha} \hat{b}_{\sigma} \quad (6)$$

$$\hat{H}_{\text{tun}} = \sum_{k\alpha\sigma} t_{ka} \left[ \hat{g}^\dagger_{ka\sigma} \hat{f}_\sigma + \hat{f}^\dagger_{\alpha} \hat{g}_{ka\sigma} \right] \quad (7)$$

The last two terms in the Hamiltonian together with the Hilbert space constraint (cf. Eq. (1))

$$\sum_{\sigma} \hat{f}^\dagger_{\alpha} \hat{f}_\sigma + \sum_{\alpha} \hat{b}^\dagger_{\alpha} \hat{b}_{\sigma} = 1 \quad (8)$$

encode the physics of the Coulomb blockade. As before, we make the simplifying assumption $\varepsilon_{k\sigma}^L = \varepsilon_{k\sigma}^R \mapsto \varepsilon_{k\sigma}$ and $t_{kL} = t_{kR} \mapsto t$. We also define $\Delta_\alpha = \pi \rho_{ga} t^2 \mapsto \Delta$ and $\varepsilon_{\alpha} = (\varepsilon_f - \varepsilon_{ba}) \mapsto \varepsilon$. 
Remarkably, the Hamiltonian $\hat{H}$ turns out to be the same as the one originally studied by Cox in the context of uranium heavy fermions \cite{22}. Substantial progress was made on the analysis of this two-channel Anderson model during recent years \cite{20, 21, 22, 23, 24}. The emerging picture indicates that the low energy physics is governed by a line of boundary conformal-invariant fix points of two-channel non-Fermi-liquid nature. Several thermodynamic properties display low-$T$ logarithmic divergencies and transport properties show $T$-dependencies that deviate from the Fermi-liquid laws of the single-channel models and extend into the mixed-valence regime. In particular, detailed predictions can be made for the charge susceptibility of the dot which, in contrast with the field susceptibilities, displays a regular behavior \cite{22}. Let us define the dot excess charge as $Q \equiv \sum_\sigma < \hat{f}_\sigma^+ \hat{f}_\sigma >$. Its variations caused by the gate voltage determine the differential capacitance of the dot, $\delta \mathcal{C} \propto \partial Q/\partial V_g$, that can be measured using the twin SET set-up. Since $V_g \sim y \sim \varepsilon \equiv \varepsilon_f - \varepsilon_g$, we have $\delta \mathcal{C} \propto \chi_c \equiv - \partial Q/\partial \varepsilon$, which is nothing but the charge susceptibility of the impurity in the two-channel Anderson model. Using Thermodynamic Bethe Ansatz (TBA) one can in fact find an expression for $Q(\varepsilon)$ at zero temperature,

$$Q(\varepsilon) = \int_{-\infty}^{+\infty} \int_{-\infty}^{0} \frac{(2z/\pi) \, dz}{\cosh \left[ \frac{\pi}{4\Delta} (x-z) \right]} \frac{\varepsilon \varphi}{\left[ (x-\varepsilon)^2 + 4\Delta^2 \right]}$$

and from it compute $\chi_c(\varepsilon)$. Alternatively, one can extract these two quantities at an arbitrary finite temperature from the numerical solution of the TBA equations. In Fig. 2 the results for the vanishing-temperature charge susceptibility are given as a function of $\varepsilon$ (open circles). Two commonly used fits are also plotted for illustrative purposes. The dashed line corresponds to a fit to a “purely thermally broadened resonance” that was used successfully to fit the very-weak-coupling regime of a single-grain set-up \cite{16}. The solid line, on the other hand, is a fit to a simple Lorentzian bell-shape. In the figure overlay we show the excess charge as a function of $\varepsilon$, together with the translation of the two fits in the main plot. In Fig. 3 we present the plots of the charge susceptibility (and the fits) as a parametrically defined (via $\varepsilon$) function of $Q$.

Our result can be characterized by two key features. First, one can observe that $\chi_c$ is symmetric about $Q = 0.5$ or $\varepsilon = 0$. This symmetry is a rather special property of the two-channel $SU(2)$ Anderson model, seen to be violated in other cases we shall touch upon later. Second, $\chi_c$ is an universal function of $\Delta$, independent of the quantitative details of the set-up tuning. This is a manifestation of having only one scale in the mixed valence regime; below this scale the physics is governed by the fixed point behavior \cite{22}. The differential conductance lineshape turns out to be insensitive to temperature for a wide range of low temperatures. It only starts to change appreciably for $k_B T \gtrsim \Delta$, when it eventually starts to become ever flatter and quickly approaches the inverted-parabola shape characteristic of thermal broadening; two such curves are shown by the dotted lines in Fig. 3. The story is similar as regards to the effect of an external magnetic field. For low fields the lineshape remains unchanged until $\mu_B H \gtrsim \Delta$. Equivalent to the effect of an
applied field is the one of asymmetries between the tuning and characteristics of the right and left grains. Only when those grain dissimilarities translate into differences between $\varepsilon_{gL}$ and $\varepsilon_{gR}$ of the order of $\Delta$ or larger, then deviations of the capacitance lineshape from the predicted one are expected. If one were to use the Lorentzian form as a first approximation to fit our exact prediction, it would mean that as long as the asymmetries between the two grains and the temperature are well within $\Delta$, one essentially has no free parameter after choosing the scale. This is equivalent to fixing the area $A$ under the curve in the functional form for the Lorentzian,

$$\chi_c(Q) = \frac{2A}{1 + \tan^2[\pi (Q - 1/2)]}$$ (9)

The symmetry of the line-shape which requires a delicate balance between the number of channels and the internal degrees of freedom, combined with the robustness against the effect of temperature and the details of tuning, is what makes our prediction characteristically and clearly distinct for experimental identification. One would like to tune $\Delta$ (via $V_{in}$) to make it as large as possible. This will help also diminish the relative magnitude of channel-hybridization asymmetries. Besides, $\Delta$ should be bigger than the level spacing on the quasi-continuum of states in the grains. The limitation is $\Delta \ll E_d, E_{ga}$ which should be the largest energy scales in the problem. These practical constraints are not too different from the ones for the single-grain set-up of Ref. [16], what takes us to believe that the engineering challenges should be surmountable. For the purpose of comparison, in Fig. 8 we have displayed the fit (dash-dotted line) corresponding to the functional form predicted (and experimentally verified) for the large-conductance regime of the single-grain set-up consisting of a grain connected to a lead (see Refs. [3, 12]). The related theory was based on mapping the charge susceptibility to the spin susceptibility of an effective two-channel Kondo model and was thereby expected to have a logarithmic divergence (required to be cut-off in matching with the experiment). In our proposed set-up the charge-fluctuations are described by a two-channel Anderson model for which we find that the charge susceptibility is a regular quantity. Since the charge degrees of freedom are directly treated, our predictions are not limited by an effective Kondo temperature involved in mapping the two charges at the charge degeneracy point into a spin degree of freedom.

In summary, we have made precise predictions for the capacitance lineshape of two-channel quantum dots, together with a concrete proposal for testing the same. As mentioned earlier, if the dot gate is set during the tuning stage in a way of making $N_d$ odd then an $SU(4)$ single-channel model results. The capacitance lineshape in such a situation, as for all single-channel models, will be very different. In particular, a marked non-zero skewness (making the plot asymmetric) should be seen when the differential capacitance lineshape is plotted as a function of the excess charge. If no assumptions are made, then while tuning the set-up the right hand side of Eq. (11) can also turn out to be $N + 2$. Such a case is equivalent to the one analyzed above; one again has either a two-channel or an $SU(4)$ model but this time for $N_d$ odd or even respectively. Implications of this freedom and the detailed line-shape in the $SU(4)$ case should be the subject of further study and will be presented elsewhere. Exploring the possible use of the proposed set up for transport measurements is an interesting open question, since that might in the future allow access to other, more exotic, aspects of two-channel Kondo physics.

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