Comparison between Analytical Method and Numerical Model for Footings on Soft Clay Supported by Stone Columns

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Abstract. The increase in population in many locations and the value of land has increased significantly. This has made the use of all areas including soft problematic soils inevitable. Due to the lack of bearing capacity of these soils, different methods of soil improvement techniques such as stone columns are used. In this study, four different analytical methods published before are briefly motioned. Also, three-dimensional (3D) numerical analyses were carried out on a stone column under footing on the soft clay soil. The main point of the current paper is to make a comparison between a numerical model and different analytical methods to represent a footing on the soft clay soil reinforced by a group of stone columns.

1. Introduction
The stone column technique for ground treatment has proven to be successful in improving slope stability of both embankments and natural slopes, increasing bearing capacity, reducing total and differential settlements and decreasing the time of consolidation. These columns were made of granular compacted material. Furthermore, the evidences were presented to support that the stone columns are superior as compared to other ground improvement techniques, with respect to coasts and durability, besides being known as environmentally friendly and easy to install.

Many research works have been carried out to investigate the efficiency of the stone column as a ground-improvement technique and tried to develop an analytical model of the stone column–improved ground \cite{1-6}. Full-scale tests were conducted on soft clay improved with granular piles or stone columns \cite{7-9}. The laboratory tests were conducted on stone column–improved soft clay beneath the footings \cite{10-13}. Finite-element studies have also been conducted on the stone column–improved soil \cite{11, 14, 13, 15, 4, 3 and 6}.

Researches in general concluded that, the stone columns provide the visible improving on the behaviour of the soft ground. The increase of the bearing capacity of the soft soil with stone columns depends mainly on the geometric conditions of columns. The stone columns with smaller spacing distances and smaller diameters have a greater bearing capacity and show smaller settlement as well as low lateral bulging than wider spacing and larger diameters of stone columns. Furthermore, some of researches performed tests for investigating the effect of length stone column at the improving
method. These researches confirmed that the settlement decrease with increasing column lengths and the improving of bearing capacity of improved soil increase with also increasing column lengths. From these results, researcher move to find the optimal normalized column.

The main objective of the present research is to state types of methods calculating the settlement of reinforced ground. Also, develop a numerical model to simulate a group of stone columns below a footing on soft clay and compare the results from this model with analytical methods.

2. Analytical method
In the literature, several methods can be found to predict the amount settlement of reinforced ground. The majority of these methods is based on the unit cell assumption, with a small number based on plane strain or homogenization techniques [4]. Settlement performance tends to be the governing design criterion in these soils, a settlement improvement factor, \( \eta \), which is defined as the ratio of the settlement of an untreated footing (i.e. no stone columns) to that of a treated footing (i.e. with columns) [17]. Figure 1 illustrates the idealization of the unit cell with the foundation pressure on top, the smooth rigid cylinder boundaries (top, bottom and outer boundary) and the direction of uniform vertical displacement.

![Figure 1. Unit cell idealization of stone column [1]](image)

2.1. Priebe's (1995) method
Priebe [2] suggested a method to evaluate settlement improvement factors, \( \eta \). Main assumptions of this method were taken into consider like, soil acts as elastic material, also, column acts as a plastic material. The bulk density of the column and soil is neglected and hence the column yields over full depth as soon as any foundation load is applied and the radial deformation is uniform with depth;

In this method, as the column is much stiffer than the soil, the vertical stress increment in the column is much greater than the vertical stress increment in the soil. At the same time, the horizontal radial stress increment must be the same in both materials (at least at the soil/column interface). As a consequence, the vertical stress is the major principal stress in the column, whereas the horizontal radial stress is the major principal stress in the soil. It also transpires that the ratio of the major
principal stress of minor principal stress is much greater in the column than in the soil. This means that it is always the column rather than the soil that yields first (hence the assumption that the soil is an elastic material whereas the column acts as a plastic material).

Priebe (1995) ends up with an equation for the settlement improvement factor, \( \eta \), given by:

\[
\eta = 1 + Ar \left[ \frac{0.5 + f(v_s, Ar)}{K_{ac} f(v_s, Ar)} - 1 \right]
\]

Where the function \( f(v_s, Ar) \) is given by:

\[
f(v_s, Ar) = \frac{(1 - v_s)(1 - Ar)}{1 - 2v_s + Ar}
\]

\( v_s \), Poisson ratio of soil and (\( K_{ac} \)), the lateral active earth pressure coefficient, given by the Rankine theory as:

\[
K_{ac} = \frac{(1 - \sin \phi')}{(1 + \sin \phi')}
\]

2.2. Castro & Sagaseta, (2009) method:
The main assumptions of this method, soil and column act as elastic material and also, there aren’t any horizontal strains, but the vertical strains are same in both materials [18]. In the results of these assumptions, the equation of vertical equilibrium is known as.

\[
q_a = Ar\Delta \sigma_c
\]

\[
\Delta \varepsilon_{z,a} = \frac{\Delta \sigma_{z,a}}{E_s} = \Delta \varepsilon_{z,c} = \frac{\Delta \sigma_{z,c}}{E_c}
\]

By replacing equation (5) in equation (4) equation (6) will be getting:

\[
\Delta \sigma_{z,c} = \frac{q_a}{1 + Ar (\frac{E_c}{E_s} - 1)}
\]

The vertical strain of the soil with the stone columns:

\[
\Delta \varepsilon_{z,a} = \frac{q_a}{E_s (1 + Ar (\frac{E_c}{E_s} - 1))}
\]

The vertical strain for the soil without columns is given by

\[
\Delta \varepsilon_{z,c} = \frac{q_a}{E_c}
\]

Dividing equations 7 for 8 will be getting the settlement improvement factor, \( \eta \):

\[
\eta = 1 + Ar (\frac{E_c}{E_s} - 1)
\]

2.3. Ballam & Booker (1981) method
This method, like the Castro & Sagaseta, (2008) [18] method, and the difference between them is a horizontal strain which assumed as the horizontal strains in the soil and the column are not assumed to be zero [19]. By this way, they assumed that, the radial stresses in the column and soil must be equal.
at the column/soil interface and the radial displacements in the column and soil must be equal at the column/soil interface and the equilibrium of radial stress:

\[
\begin{align*}
(\Delta \sigma_{r,c})_r &= \frac{d_c}{2} = (\Delta \sigma_{r,s})_r = \frac{d_s}{2} \\
\end{align*}
\] (10)

Compatibility of radial displacement:

\[
(\Delta \varepsilon_{r,c})_r = \frac{d_c}{2} = (\Delta \varepsilon_{r,s})_r = \frac{d_s}{2} \\
\] (11)

The Lamé’s parameters for homogenous isotropic linear elastic materials:

\[
\lambda = \frac{\nu E}{(1-2\nu)(1+\nu)} \\
G = \frac{E}{2(1+\nu)}
\] (12) (13)

And it will be required the transformation of Young’s modulus (E) to the constrained:

\[
E = \frac{(1-\nu)E}{(1-2\nu)(1+\nu)}
\] (14)

They ended up with a parameter \( F \) and the strain of the soil with the stone columns:

\[
F = \frac{\frac{v_c}{(1-v_c)} \frac{E_c}{E_s} - \frac{v_s}{(1-v_s)} (1-Ar)}{\frac{1}{(1-v_c)} \frac{E_c}{E_s} (1-Ar) + \frac{1}{(1-v_s)} (Ar + 1 - 2v_s)}
\] (15)

\[
\Delta \varepsilon_{z,s} = \frac{q_o}{E_s} \left( 1 + Ar \left( \frac{E_c}{E_s} - 1 \right) - \frac{2Ar(1-Ar)}{1} \left[ \frac{v_c}{(1-v_c)} \frac{E_c}{E_s} - \frac{v_s}{(1-v_s)} \right] \right)^2 \\
1 + Ar \left( \frac{E_c}{E_s} - 1 \right) - \frac{1}{(1-v_c)} \frac{E_c}{E_s} (1-Ar) + \frac{1}{(1-v_s)} (Ar + 1 - 2v_s)
\] (16)

Dividing equations 15 by 8 will be getting the settlement improvement factor, \( \eta \):

\[
\eta = 1 + Ar \left( \frac{E_c}{E_s} - 1 \right) - \frac{2Ar(1-Ar)}{1} \left[ \frac{v_c}{(1-v_c)} \frac{E_c}{E_s} - \frac{v_s}{(1-v_s)} \right] \left( 1 + Ar \left( \frac{E_c}{E_s} - 1 \right) - \frac{1}{(1-v_c)} \frac{E_c}{E_s} (1-Ar) + \frac{1}{(1-v_s)} (Ar + 1 - 2v_s) \right)^2 \\
\] (17)

2.4. Castro’s (2014) method

Castro presented an approximate solution to study the settlement of footings resting on a soft soil improved with groups of stone columns [3]. The main assumptions of this method are:

- Soil acted as linear elastic,
- Column is assumed plastic strains as the Mohr–Coulomb yield criterion and a non-associated flow rule.

By this method, the soil profile is divided into slices, and equilibrium of the stresses and compatibility of deformations is imposed in the vertical and horizontal directions. Average constant stress is applied on each slice at depth, \( z \). The applied stress on the footing spreads out at a slope, whose value is 4V:1H from the base of the footing, then comparing with the stress which causes
column yielding. After that, vertical deformation of each slice is calculated either by elastic equation or plastic equation. The summation of the vertical deformation of all of the slices gives the deformation of soil layer.

3. Numerical modelling details

FLAC$^{3D}$, Fast Lagrangian Analysis of Continua in 3 Dimensions was used for the current analysis of stone columns in soft clay. The computation scheme performed by FLAC3D takes an enormous number of estimation steps, each progressively redistributing an unbalanced force caused by changes to stress or displacement boundaries through the mesh [20]. In this section the dimensions of finite difference model; soil properties, dimensions and properties of stone columns and foundation will be presented.

It is required to make an idealization for each part of the model. The footing, stone columns and soil strata were modeled using suitable elements. The dimensions of the model were selected so that the boundaries are far enough to cause any restriction or strain localization to the analysis. This mesh was refined near the footing edges where high stresses and strains are developed as shown in figures (2, 3).

The soil is modeled to behave as a conventional elastic-perfectly plastic model based on Mohr-Coulomb failure criterion in FLAC3D software. Brick elements are used to model the soil. The parameters of soft clay are given in Table (1). The soft clay soil has a constant depth ($H = 10$ m). Groundwater level was assumed to be on top of soil layer.

The stone column is modeled as a massive circular element with an outside interface with soil. The column was divided in a radial direction to four parts. The stone column is modeled to behave as a conventional elastic-perfectly plastic model based on Mohr-Coulomb failure criterion in FLAC3D software. The parameters of stone column are given in Table (1). Length of stone column ($L = 10$ m) fully penetrated soft layers, stone column diameter ($d_c = 0.6$ m), spacing between columns ($S = 2d_c$, $3d_c$ and $4d_c$).

The footing is modeled as square brick elements with 0.7 m thickness, width and length is depended in the spacing between columns. The footing is loaded with distributed stress and the properties of footing are given in Table (1). The area replacement ratio ($Ar$) is defined as the ratio between the sectional areas of stone columns to that of the native loaded soil or to the footing constant area. The area was changed from 25.6% to 8%.

![Figure 2. Typical mesh of the materials for FLAC3D Model](image-url)
Figure 3. Model geometry

Table 1. Physical and mechanical material properties

| Material Properties       | Soft clay | Stone column | Footing |
|---------------------------|-----------|--------------|---------|
| Density (kg/m$^3$)        | 1700      | 1800         | --      |
| Cohesion (kPa)            | 20        | 0            | 0       |
| Friction                  | 0         | 40           | 0       |
| Young’s modulus (MPa)     | 4         | 55           | 25000   |
| Poisson’s ratio           | 0.4       | 0.3          | 0.2     |

4. Comparison of FLAC3D and analytical design method output

The settlement improvement factor of a group of stone columns from FLAC 3D is compared with the four analytical methods which stated in previous section shown in figure 4. The settlement improvement factor, $\eta$, in figure 4 corresponds to the application of a 150 kPa uniform pressure on a footing where the diameter of a stone column, $d_c = 0.6$ m, and with different value of area replacement ratio (Ar).

The comparison in figure 4 showed that the all methods have the same trend and confirmed that as area ratio, Ar, decreases the settlement improvement factor, $\eta$, decreases. As noted also, the Castro's (2014) method was a higher than other methods and the Priebe's (1995) method was a lowest. While the other methods their results are fairly close to same. The settlement improvement factor from FLAC3D is located between the results that calculated from analytical methods. Figure 5 showed the difference between results of FLAC3D and the other analytical methods. As mentioned before the Castro's (2014) method gives highest values than others and Ballam & Booker (1981) method and Castro & Sagaseta, (2009) methods are given closer values than others. This comparison gives quite confidence in a numerical model to represent improving soft clay soil reinforced with stone columns.
5. Conclusion
A numerical model was developed to simulate the case of a system consisted of a footing rests on stone columns embedded in soft clay. In this study a comparison between results from numerical model and four analytical methods. From the results of comparison, conclusions have been drawn and are summarized as follows:

- The results of the analytical methods and numerical model have a same trend.
- The results of the numerical model are closer to the results from Ballam & Booker (1981) method and Castro & Sagaseta, (2009) methods than the other method.
- This comparison may be having significant effect incoming future researches to predict the behaviour of stone column on soft clay soil.

6. Notation
The following symbols are used in this paper:
S = Soil " Subscripts".
C = Column " Subscripts".
Ar = Area replacement ratio.
dec = Equivalent diameter of the cylinder representing the “unit cell”.
dc = Diameter of stone column.
S = Spacing between columns.
L = Length of column.
H = Depth of soft clay layer.
E = Young’s modulus.
G = Shear modulus.
Kac = Lateral active earth pressure coefficient.
qa = Uniform vertical load applied.
ϕ = Friction angle.
λ = Lamé’s constant.
ν = Poisson’s ratio.
σ = Stress.
ε = Displacements.
η = Settlement Improvement Factor.

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