A Possible Test of the $J_2c^{-2}$ General Relativistic Orbital Effects with Juno

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Abstract. For the first time, the 1PN $J_2c^{-2}$ effects could be measured by the Juno mission in the gravitational field of Jupiter during its nearly yearlong science phase thanks to the high eccentricity ($e = 0.947$) of the spacecraft’s orbit and to the huge oblateness of Jupiter ($J_2 = 1.47 \times 10^{-2}$). A numerical analysis shows that the expected $J_2c^{-2}$ range-rate signal for Juno should be as large as $\approx 280$ microns per second ($\mu$m s$^{-1}$) during a typical 6 h pass at its closest approach to Jupiter. The radio science apparatus of Juno should reach an accuracy in Doppler range-rate measurements of $\approx 1 - 5 \mu$m s$^{-1}$ over such passes. The range-rate signature of the classical even zonal perturbations is different from the $J_2c^{-2}$ one. Thus, further investigations, based on covariance analyses of simulated Doppler data and dedicated parameters estimation, are worth of further consideration.

1. Introduction
The quadrupole mass moment $J_2$ of the gravitational field of an oblate body of mass $M$ and equatorial radius $R$ has an impact on the motion of a test particle orbiting it also at the first Post-Newtonian (1PN) level, causing non-vanishing long-term orbital perturbations of order $J_2c^{-2}$ on all its Keplerian orbital elements [1–5]. So far, such effects did not receive much attention since it was always believed that they are too small to be detectable in any foreseeable future.

As recently shown in [6], recent developments in space sciences may soon favorably overturn such opinions.

2. Perspectives for a detection in the gravitational field of Jupiter with Juno
Juno$^1$ [7] is a spacecraft en route to Jupiter where its arrival is scheduled for July 2016. Its science phase has a nominal duration of almost 1 yr (10 November 2016-5 October 2017) [8]. During it, Juno will move along a highly elliptical, wide orbit characterized by an eccentricity $e = 0.947$, a semimajor axis $a = 20.03R$, an orbital period of $P_b = 11.07$ d, and an inclination to the Jupiter’s equator $I = 90$ deg. Such a polar trajectory is designed to accurately map, among the other things, the gravitational field of Jupiter [9] through the perturbations suffered due the departures from spherical symmetry of the Jovian gravitational potential.

$^1$ See http://www.nasa.gov/mission_pages/juno/main/index.html on the WEB.
The science measurements will be performed with the Juno’s radio science system\(^2\) (X-Band, Ka-Band) providing an accurate determination of the Doppler shift \(\Delta \dot{\rho}\) \([8, 10]\) during many\(^3\) of the scheduled 31 orbits in the science phase, mainly at the perijove passages lasting about 6 hr. Indeed, as pointed out in \([10]\), large longitude-keeping maneuvers would compromise the dynamical coherence of the orbit, making, thus, practically infeasible an analysis based on steady time series of the Keplerian orbital elements. The expected overall range-rate accuracy should be of the order of \(1 - 5 \mu m \ s^{-1}\) over time scales of \(10^3\) s \([8, 10]\) for the Ka-Band apparatus, while the spacecraft’s position and velocity should be known with an uncertainty of about 10 m and 1 mm s\(^{-1}\), respectively, in the three spatial directions \([8]\).

The huge Jupiter’s oblateness \((J_2 = 1.47 \times 10^{-2})\), combined with the notable eccentricity of the Juno’s orbit and with its accurate radio science Ka-Band system, may offer a unique opportunity to detect, for the first time, the \(J_2c^{-2}\) orbital effects.

In order to make a close connection with the actual direct observable, in the right panel of Figure 1 we plot a numerically integrated Earth-Juno range-rate signal \(\Delta \dot{\rho}\) due to the \(J_2c^{-2}\) component of the acceleration of Juno over a 6 hr pass centered about the first perijove passage after the beginning of the science phase. The mathematical model of the \(J_2c^{-2}\) acceleration used in our numerical integration of the equations of motion of Juno has no restrictions on the spatial orientation of the Jupiter’s spin axis \(\hat{k}\). That is fine since it allows to use different coordinate systems. Indeed, it is likely that the real data will be actually processed in the ICRF frame; in it, the Jupiter’s spin axis is \(k_x = -0.0146021, k_y = -0.430337, k_z = 0.90255\), as it can be retrieved from the celestial coordinates of its north pole of rotation at epoch \(J2000.0\) \([11]\). The resulting \(J_2c^{-2}\) range-rate signal amounts to about 280 \(\mu m s^{-1}\), thus supporting the feasibility of our proposed measurement. From the left panel of Figure 1, we also note that the \(J_2c^{-2}\) range shift \(\Delta \rho\) of Juno is as large as \(\approx 20\) m over the same interval.

We stress that this is just a preliminary sensitivity analysis aimed to explore the possibility of a detection of the effect considered. It should be accompanied by a dedicated full covariance study with the simulated data of the real Doppler range-rate measurements at the perijove passages, and by the estimation of dedicated solve-for parameters explicitly accounting for the \(J_2c^{-2}\) acceleration in the dynamical models. Nonetheless, the encouraging outcome of the investigations in \([8, 10]\) concerning the possibility of an accurate measurement of the Lense-Thirring effect, proposed in \([12]\), makes us confident that also for the \(J_2c^{-2}\) effect a percent determination with Juno may be possible.

Here we give just a concise list of potentially competing orbital effects whose impact on the proposed measurement may be the object of future dedicated investigations. The odd zonals \(J_\ell, \ell = 3,5,\ldots\) should, in principle, be considered in more detail with respect to \([12]\). Indeed, if, on the one hand, they do not produce long-term perturbations on the semimajor axis \(a\), on the other hand they change all the other Keplerian orbital elements with long-term harmonic shifts depending on the period of the perijove \(\omega\) \((P_\omega \approx 500\) yr\). In principle, the tidal effects of Saturn on the wide orbit of Juno might be of some relevance. An order-of-magnitude evaluation of their importance can be performed by looking at the magnitude of the Kronian tidal acceleration on Juno. At the distance of Saturn from Jupiter expected at the beginning of the Juno science phase \((d_{JS} = 9.73\) au\), the nominal Kronian tidal acceleration on Juno should be of the order of \(A_{\text{tide}} \approx Gm_S\tau d_{JS}^3 = 2 \times 10^{-11}\) m s\(^{-2}\). It would yield a nominal displacement of about \(\approx 10^4\) m over \(T\). Thus, since the Saturn’s gravitational parameter \(Gm_S\) is nowadays known with a

\(^2\) While the Ka-Band system is for gravity science, the X-Band apparatus is used for the spacecraft orbit determination and navigation \([8]\). As far as the Doppler range-rate measurements are concerned, the X-Band system is less accurate than the Ka-Band one by a factor of about \(10 - 100\) \([8]\).

\(^3\) It depends on the pointing of the Juno’s high-gain antenna \([8]\); if it does not point to the Earth, as in the microwave radiometry passes when it will point to Jupiter, Doppler tracking is impossible. According to Helled et al. \([8]\), about 22 useful perijove passes should be available during the scheduled science phase.
Figure 1. From the left to the right: Earth-Juno range $\Delta \rho$, in m, and range-rate $\dot{\Delta \rho}$, in $\mu$m s$^{-1}$, due to the $J_2 c^{-2}$ component of the Juno’s acceleration. Both were calculated by numerically integrating the equations of motion of the Earth, Jupiter and Juno with and without the $J_2 c^{-2}$ term over 1 yr in a coordinate system with the ICRF equator as reference $\{x,y\}$ plane. In it, the Jupiter’ spin axis is $\hat{k}_x = -0.0146021, \hat{k}_y = -0.430337, \hat{k}_z = 0.90255$, as it can be inferred from the ICRF equatorial coordinates at epoch J2000.0 of its north pole of rotation [11]. Both the integrations shared the same initial conditions corresponding to the beginning of the science phase (10 November 2016) [8]; for Jupiter and the Earth they were retrieved from the WEB interface HORIZONS by NASA/JPL. For Juno we adopted $x_0 = -0.0444405 R, y_0 = -1.3097 R, z_0 = 2.74684 R, \dot{x}_0 = 19.4951$ km s$^{-1}, \dot{y}_0 = -11.9024$ km s$^{-1}, \dot{z}_0 = 23.5485$ km s$^{-1}$ corresponding to an initial polar orbital configuration with $\omega_0 = 5.7$ deg with respect to the Jupiter’s equator [8]. The time interval of the plots, covering 6 hr, is centered about the first perijove passage after the beginning of the science phase.

fractional accuracy of the order of $\approx 3 \times 10^{-8}$ [13], it can be reasonably concluded that the impact of the Kronian tides on the Juno’s path is quite negligible for our purposes. Among the non-gravitational perturbations, whose impact on the proposed measurement is beyond the scope of this paper, particular attention should be paid to the drag from the Jovian atmosphere and to possible spurious thrusts from thermal out-gassing from the spacecraft potentially capable of affecting $a$.

3. Summary and conclusions
We looked at the 1PN effects on the motion of a test particle orbiting an oblate central body in order to check if recent advancements in space science and technology make them potentially measurable.

The answer is cautiously positive thanks to the ongoing Juno mission to Jupiter. Indeed, its peculiar orbital configuration, the accuracy of its radio-tracking apparatus and the huge Jovian oblateness make the perspective of measuring the $J_2 c^{-2}$ effects on the Juno’s orbit feasible during
its scheduled yearlong science phase in 2016-2017. Indeed, the numerically integrated $J_2 c^{-2}$ range-rate shifts at the perijove passes are expected to be as large as \( \approx 280 \, \mu m \, s^{-1} \); the onboard Ka-Band Doppler system should be accurate at a \( 1 - 5 \, \mu m \, s^{-1} \) level during such passes. The signatures of other competing effects such as the classical shifts due to the even zonal harmonics of the Newtonian component of the aspherical gravitational potential of Jupiter are different, thus likely allowing for an adequate separation of the signal we are interested in. Most of the positive results of independent investigations previously made by other researchers in view of a possible measurement of the smaller Lense-Thirring effect with Juno may be valid for the $J_2 c^{-2}$ effects as well, thus making us reasonably confident about a successful determination of them. Nonetheless, a dedicated covariance analysis implying simulations of Doppler measurements and parameter estimation is required to further support the promising results of our preliminary sensitivity study.

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