Comment on “Analytic Structure of One-Dimensional Localization Theory: Re-Examining Mott’s Law”

In a recent Letter A. O. Gogolin has challenged the established point of view that Mott’s prediction for the dynamical conductivity of a localized electron system is correct. The intuitive argument leads in one dimension to a dynamical conductivity of the form \( \omega^2 \ln^2 \omega \). Later, the precise, asymptotical result

\[
\Re \frac{\sigma(\omega)}{\sigma_0} = \nu^2 \left( \ln^2 \nu - \frac{\pi^2}{4} + (2C - 3) \ln \nu - C + \cdots \right)
\]

(eq. (22) in Ref. [1] with \( \sigma_0 = 4 \)). In view of the mentioned variety of works corroborating Mott’s conclusion this is quite unexpected. If Gogolin were right, then one of the thought to be most profound chapters in localization theory would have to be rewritten. In fact, however, as we will demonstrate below, he is not.

Gogolin’s analysis starts from the famous recursion equations derived first by Berezinskii. The equations can be solved in a standard manner by mapping them to a differential equation. Gogolin’s claim is that the previous solution of this equation is incorrect and hence also the conductivity law \( \omega^2 \ln^2 \omega \) derived thereof. He argues that previous authors have not properly taken into account discreteness of the spectrum of the equation.

A simple method to check Berezinskii’s result is to solve the recursion equations for the conductivity numerically. (For details see Ref. [3].) The algorithm is very stable and has been used down to frequencies \( \nu = 5 \times 10^{-6} \) where \( M = 10^8 \) in a calculation with 40 digits (fixed) precision. For even larger \( M = 2 \times 10^8 \) or more digits, e.g. 60, \( \sigma \) does not change implying that rounding errors are irrelevant. Fig. 1 shows our result. The agreement of the numerical data with the Mott/Berezinskii-solution is perfect over more than 3 decades while the data is completely incompatible with Gogolin’s \( \ln^2 \omega \) term.

Fig. 1. Dynamical conductivity from solving the Berezinskii recursion equations (\( \nu = 2 \omega \tau \)). Numerical solution (solid), Berezinskii’s solution, eq. (2) (dashed), Gogolin’s result, eq. (2) (solid). Inset: Determining \( C \) by subtracting first three terms in eq. (2) from numerical data: \( C \approx 0.3 \).

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