Late-time Entropy Production from Scalar Decay and Neutrino Decoupling

Paramita Adhya $^{a,1}$ and D. Rai Chaudhuri $^{a,2}$

$^a$Department of Physics, Presidency College, 86/1, College Street, Calcutta 700073, India

Abstract

Late-time entropy production from scalar decay arises in scenarios like thermal inflation, proposed to dilute long-lived, massive fields like the gravitino and the moduli. The scalar decay may continue into Mev-scale temperatures and affect BB nucleosynthesis. The effect of such entropy production on electron neutrino decoupling is studied. A lower bound of about $10^{-22}$ Gev is estimated for the scalar decay constant, such that, for higher values of the decay constant, standard electron neutrino decoupling is unaffected.

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1 Introduction

Supersymmetric and string theory models throw up long-lived, massive fields like the gravitino, the Polonyi, the moduli and the dilaton [1]. Stability of the corresponding particle may lead to over-abundance, while decay during baryogenesis or BBN nucleosynthesis may disturb $\eta$ or nuclear abundances too much [2]. Thermal inflation [3] has been proposed as a method of diluting away troublesome fields. A scalar field, the flaton, is used to start inflation at a temperature of about $10^7$ Gev. The inflation stops at a temperature of the order of the flaton mass, typically $\leq 10^3$ Gev. Such a scalar field goes on decaying into Mev-scale temperatures with potential trouble for BBN nucleosynthesis.

The effect of flaton decay on the effective number of neutrino species, $N_{\text{eff}}$, and the neutrino distribution function during nucleosynthesis has been recently studied in detail [4, 5]. Here, we study the effect of scalar decay on decoupling of the usual left-handed, massless neutrinos. As the phenomenological details of such a decaying particle are still quite open, we consider the general effect of late time entropy production on electron neutrino decoupling, assuming that the decaying scalar once dominated the energy density of the universe. The electron neutrino decoupling temperature is a sensitive input of BBN, and we estimate lower bounds on the scalar decay constant, demanding that this decoupling temperature should not be disturbed.

2 Entropy Generation and Decoupling

The entropy generation rate is calculated from

$$dS_\Phi = -\frac{d(a^3 \rho_\phi)}{T},$$

where $\rho_\phi$ is the scalar energy density at radiation temperature $T$, all massless particles being collectively called radiation. Let $\Gamma$ be the scalar decay constant. Writing $\Phi = a^3 \rho_\phi$, the equation for the evolution of the scalar energy density is $\dot{\Phi} = -\Gamma \Phi$, with the solution $\Phi = \Phi_E e^{-\Gamma(t-t_E)}$, where $\Phi_E$ is the value of $\Phi$ at a fiducial time $t = t_E$. This gives

$$\dot{S}_\Phi = \frac{\Gamma}{T} \Phi_E e^{-\Gamma(t-t_E)}.$$  \hspace{1cm} (1)

What is the standard situation in the absence of scalar decay? In the era of neutrino decoupling, the electron neutrinos interact via charged and neutral currents, while muon and taon neutrinos interact via neutral currents alone. So, the muon and taon neutrinos decouple at a slightly higher temperature than the electron neutrinos. Then, the change in electron neutrino ($\nu$ ) number occurs due to the process $\nu + \bar{\nu} \rightarrow e^- + e^+$. The decoupling of electron neutrinos is governed by the integrated Boltzmann equation [7, 8]

$$\dot{n} + 3Hn = -<\sigma|v|> (n^2 - n_{\text{EQ}}^2),$$  \hspace{1cm} (2)

where $n$ is the number density of the electron neutrinos, $n_{\text{EQ}}$ their equilibrium number density, and $<\sigma|v|$ the thermally averaged cross-section times relative velocity. Assuming
an absence of neutrino degeneracy, we use the Boltzmann distribution function to give
\[ n_{EQ} = \frac{T^3}{\pi^2}. \] (3)

Following the usual method of calculation [8, 9, 10], neglecting the masses of the neutrino and the electron,
\[ < \sigma|v| > = \frac{8}{\pi} G_F^2 [(C_{Ve} + 1)^2 + (C_{Ae} + 1)^2]T^2 \]
\[ = \frac{4.112 \times 10^{-10}}{Gev^4} T^2. \] (4)

The decoupling may be taken to start when the following relation just holds.
\[ - < \sigma|v| > (n^2 - n_{EQ}^2) < 3Hn. \] (5)

Denoting \((a^3)\) times entropy density as \(S\), the net contribution, into the \(\nu, \bar{\nu}\) sector, of the covariant divergence \(1/(a^3) \dot{S}\) of the usual entropy density current, due to the process \(e^- + e^+ \rightarrow \nu + \bar{\nu}\), is \(-2\alpha < \sigma|v| > (n^2 - n_{EQ}^2)\) [11], where
\[ n = n_{EQ}e^{-\alpha}. \] (6)

If there is additional entropy generation due to scalar decay, there will be a corresponding extra term on the RHS of (2). To find a lower bound to \(\Gamma\), such that scalar decay does not affect neutrino decoupling, we require this additional term to be small compared to the term \(- < \sigma|v| > (n^2 - n_{EQ}^2)\) on the RHS, at decoupling. To find the lower bound on \(\Gamma\), we consider the worst case scenario when the entire additional entropy due to scalar decay is transferred\(^4\) to the neutrinos. Then, if decoupling is to be kept undisturbed, we can require that the contribution, to the \(\nu, \bar{\nu}\) sector, of the covariant divergence of the entropy current due to scalar decay should be less than that due to the usual annihilation process, or
\[ \frac{1}{a^3} \dot{S}_\Phi < -2\alpha < \sigma|v| > (n^2 - n_{EQ}^2), \]
when (5) holds. This means that the required lower bound to \(\Gamma\) is to be obtained from the criterion
\[ \frac{1}{a^3} \dot{S}_\Phi < 2\alpha (3Hn), \] (7)

\(^3\)In [11], the distribution function of the decoupling particle is taken as \(f(p) = e^{-\alpha(t)\beta(t)\xi(p)}(1 + \xi(p))\). For sufficiently strong elastic collisions, it is shown that \(\xi \ll 1\), and the process is well represented by the time-dependent parameter \(\alpha(t)\). Despite the resemblance, \(\alpha\) is not a chemical potential. It parametrises the distribution and is the same for particle and anti-particle, instead of being different in sign.

The 2 factor in \(2\alpha\) arises because our equation (2) considers the change in particle density alone, while \(\dot{S}\) considers net entropy transferred to both particle and antiparticle. [11] does not have the 2 factor as particle and anti-particle are there considered together as a single species.

\(^4\)In current models, it is usually assumed that the scalar cannot decay directly to neutrinos [4]. Any massive decay products of the scalar quickly decouple and the entropy due to scalar decay passes to the \(e^-, e^+, \gamma\) sector, and thence to the neutrinos via the annihilation process \(e^- + e^+ \rightarrow \nu + \bar{\nu}\). As electromagnetic interactions keep the \(e^-, e^+, \gamma\) sector very near equilibrium, almost all the entropy may be supposed to pass to the neutrinos.
where $\alpha$ corresponds\textsuperscript{5} to its decoupling value, in the absence of scalar decay, worked out from the equality corresponding to (5).

3 Estimating the Entropy Generation

(1) indicates that $\dot{S}_\Phi$ depends on $\Phi_E$. In the absence of definite phenomenological values, $\Phi_E$ is to be estimated indirectly. We are interested in the era when radiation domination has begun, and (1) is dominated by the exponential. Then, the pre-exponential may be estimated approximately, to an order of magnitude. So, in the pre-exponential, we define the initial epoch $t_E$ by putting $\rho_{\phi E} = \rho_{RE}$, $\rho_R$ being the radiation energy density. Radiation domination is taken to start after this epoch. Further, we neglect entropy generation while estimating the pre-exponential in (1), and put

$$\Phi_E = a^3_E \rho_{\phi E} = a^3_E \rho_{RE} = (\pi^2/30) g^* a^3 E T_E^4 = (\pi^2/30) g^* a^3 T^3 T_E,$$

where $g^*, a, T$ refer to the epoch of decoupling.

3.1 Estimate of $T_E$

We assume that when the neutrinos decouple, $\rho_\phi \ll \rho_R$, such that $\rho_\phi/\rho_R$ cannot be neglected, but its higher powers can. It is possible to show [12], in fact, that decoupling is not possible when $\rho_\phi \gg \rho_R$. In the era $\rho_\phi \ll \rho_R$ of incomplete radiation domination, the Friedmann equation is put in the form

$$H^2 = \frac{8\pi R}{3M^2_P a^4} (1 + \frac{\Phi a}{R}),$$

where $R = a^4 \rho_R$. If, well into this epoch, the correction term $\Phi a/R$ on the RHS of (9) is neglected, the full radiation domination relations are found:

$$H = \frac{1}{2t}, \text{ and } a = A t^{\frac{1}{2}},$$

$A$ being a constant.

The $\Phi$ evolution equation has, as solution, a falling exponential in $t$, viz. $\Phi \sim e^{-\Gamma t}$. Instead of taking the falling exponential in $t$ directly, a suitable approximation to the

\textsuperscript{5}We assume that the electron neutrino distribution function continues to be parametrised as $f(p) = e^{-\alpha(t) - \beta(t) E(p)}$ even when there is scalar decay. It is shown in meticulous detail in ref.[4] that, for low values of the scalar decay constant, the distribution is not thermalised. There is a deficit from the thermal distribution. So, our assumption is tantamount to an approximation of the deficit by a uniform factor $e^{-\alpha}$, with $\alpha > 0$. 

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correction term $\Phi a/R$ on the RHS of (9) is first worked out. Let $t_0$ be a sufficiently late epoch, when $\Phi = \Phi_0 \approx 0$. Then, for use only in the correction term $\Phi a/R$, one takes
\[
\Phi - \Phi_0 = \Phi\left(\frac{1}{t}\right) - \Phi\left(\frac{1}{t_0}\right) = \frac{d\Phi}{dt}\bigg|_{t_0}\left(\frac{1}{t} - \frac{1}{t_0}\right).
\]
Neglecting $\Phi_0, 1/t_0$ compared to $\Phi, 1/t$, respectively, an approximation
\[
\Phi \approx \frac{B}{t},
\]
will be used only in the correction term $\Phi a/R$, i.e., in the correction term, the falling exponential will be approximated by a rectangular hyperbola. $B$ is a constant.

A similar approximation is considered for $R$. From
\[
\frac{\partial}{\partial t}\left[a^3(\rho_{\phi} + \rho_R)\right] + p_R \frac{\partial}{\partial t}a^3 = 0,
\]
one obtains
\[
\dot{R} = a\Gamma \Phi.
\]
It ought to be mentioned that $R$ refers to the total radiation present, and not only to that produced by decay. However, the change in $R$ is due to $\phi$ decay and consequent entropy production. In the absence of this decay, $\dot{R} = 0$. Using (10) and (11) in (12), and, integrating, one obtains approximately, for use only in the correction term $\Phi a/R$,
\[
R - R_E \approx 2AB\Gamma(\frac{1}{2} - \frac{1}{2}t_E^2).
\]
If $t_E$ is sufficiently early compared to $t$, and there is sufficiently copious radiation production since $t_E$, it is sufficient to take
\[
R \approx 2AB\Gamma t^\frac{1}{2}
\]
in the correction term $\Phi a/R$. (10) and (11) are now used to give, in the correction term,
\[
x = \frac{\Gamma}{H} \\
\approx \frac{R}{\Phi a}
\]
Introducing the variable $x$ in (9), we get (9) in the form
\[
H = \frac{\Gamma}{x} = \frac{4.461 \times 10^{-19}}{Gev} T^2(1 + \frac{1}{x^2})^{\frac{1}{2}},
\]
putting $g^* = 10.75$. (14) is a good equation for $x \gg 1$. But, we approximate $T_E$ in the pre-exponential term $\Phi_E$ of (1), by putting $x = x_E = 1$ in (14) (corresponding to $\rho_{\phi} = \rho_R$, or $\Phi a = R$, at $t = t_E$) to get
\[
T_E = \frac{\sqrt{\Gamma}}{\sqrt{2} \times 4.461 \times 10^{-19}/Gev}
\]
putting $g^* = 10.75$. (14) is a good equation for $x \gg 1$. But, we approximate $T_E$ in the pre-exponential term $\Phi_E$ of (1), by putting $x = x_E = 1$ in (14) (corresponding to $\rho_{\phi} = \rho_R$, or $\Phi a = R$, at $t = t_E$) to get
\[
T_E = \frac{\sqrt{\Gamma}}{\sqrt{2} \times 4.461 \times 10^{-19}/Gev}
\]
4 Numerical Results and Conclusions

Now, BB nucleosynthesis is consistent with a neutrino decoupling temperature $T_D \sim$ a few Mev. Here, we take $T_D = 1, 2, 3$ Mev, and find corresponding values of $\alpha$, in the absence of scalar decay, from the equality corresponding to (5), using (3), (6) and \( H = 4.461 \times 10^{-19}T^2 \), the last being the usual radiation domination value of $H$. The results are shown in the first two columns of Table I.

Next, we consider scalar decay. The value of $T_E$ from (15) is used in (8) and the latter is put into (1). Utilising (14) to relate the temperature and $x$ where necessary, and estimating $t_E$ from $H = 1/(2t)$, (13), and $x_E = 1$, the equality corresponding to the relation (7) is now solved for $x = x_D$, keeping the same values of $\alpha$ already found to correspond to decoupling temperatures $T_D = 1, 2, 3$ Mev in the absence of scalar decay. The results are in the third column of Table I. Finally, $\Gamma$ is found corresponding to these values of $x_D$ from (14) for $T_D = 1, 2, 3$ Mev. The form of $\dot{S}_\Phi$ and the inequality (7) show that these values correspond to a lower bound on $\Gamma$.

We conclude that if the entropy produced by scalar decay is not to disturb the neutrino decoupling temperature, the lower bound on the scalar decay constant must be of the order of \( 8 \times 10^{-24}Gev \), \( 26.5 \times 10^{-24}Gev \), \( 60 \times 10^{-24}Gev \), corresponding to neutrino decoupling temperatures of 1, 2, 3 Mev. So, neutrino decoupling is unaffected, and BB nucleosynthesis is unaffected on this count, if the scalar decay constant is greater than about \( 10^{-22} \) Gev, corresponding to a reheating temperature of 8.6 Mev.

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