Heralded amplification of path entangled quantum states

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Keywords: quantum communication, heralded photon amplifier, path entanglement, device-independent QKD

Abstract

Device-independent quantum key distribution (DI-QKD) represents one of the most fascinating challenges in quantum communication, exploiting concepts of fundamental physics, namely Bell tests of nonlocality, to ensure the security of a communication link. This requires the loophole-free violation of a Bell inequality, which is intrinsically difficult due to losses in fibre optic transmission channels. Heralded photon amplification (HPA) is a teleportation-based protocol that has been proposed as a means to overcome transmission loss for DI-QKD. Here we demonstrate HPA for path entangled states and characterise the entanglement before and after loss by exploiting a recently developed displacement-based detection scheme. We demonstrate that by exploiting HPA we are able to reliably maintain high fidelity entangled states over loss-equivalent distances of more than 50 km.

1. Introduction

The distribution of entanglement is a key resource for quantum communication [1]. Single-photon entanglement, also called path entanglement, represents possibly the simplest form of entanglement to generate [2, 3], and one that lies at the heart of some of the most efficient quantum repeater architectures [4]. Nonetheless, in the context of quantum communication, a means of measuring these states in distributed scenarios has proven difficult. The measurement, and subsequently the characterisation, of these states has been recently addressed by adopting the relatively old idea of what we call displacement-based detection [5–8]. This scheme combines aspects of discrete-(photon counting) and continuous-(local oscillator) variable detection and has recently been used to perform entanglement witness [9] and detection loophole-free EPR steering [10] experiments.

Transmission loss is critical for all quantum communication scenarios, however, heralded photon amplification (HPA) [11], or noiseless linear amplification, provides a potential solution to mitigate its impact. This is a teleportation-based protocol that has been experimentally studied in the case of both polarisation [12] and time-bin [13] qubits as well as for single photons [14, 15] and in the continuous variables regime [16, 17]. In the context of entangled systems, loss represents a fundamental limit for the detection loophole-free Bell test [18–20] needed for device-independent quantum key distribution (DI-QKD) [21]. Heralded amplification has been proposed as a means to overcome loss in the critical case of DI-QKD [22–25]. More generally, it can also be seen as an entanglement distillation protocol, whereby the herald for the amplifier announces, or selects, a subset of states that have a higher degree of entanglement than before. First experiments in this direction for path entanglement relied on joint measurements [14, 15] due to the difficulty in measuring these states in a distributed scenario, i.e. with local measurements. Continuous variable entangled systems, using homodyne measurements, have also used HPAs to distil entanglement, demonstrating the flexibility of this concept [26, 27].

Here, we bring together all three of these concepts: heralded path entanglement, displacement-based detection and HPA, all operating at telecom wavelengths. We demonstrate how HPA can be exploited to overcome transmission loss, up to a loss-equivalent distance of around 50 km. The initial and final states are measured using a displacement-based detection scheme and characterised by their Fidelity with respect to a maximally entangled state.
2. Concept and theory

Figure 1 illustrates the scheme for generating and distributing path entanglement as well as how we incorporate the heralded photon amplifier. One should note the conceptual similarity to an entanglement swapping scheme. In the following, we elaborate on the theoretical description of this system and how we can characterise and compare the entanglement before and after the HPA. Alice starts by sending a photon through a beamsplitter (BS) with splitting ratio \( \tau : 1 - \tau \), and creates the entangled state \( |\psi(\tau)\rangle = \sqrt{\tau} |10\rangle_{ab} + \sqrt{1 - \tau} |01\rangle_{ab} \) in the photon number basis. Part of the entangled state is sent to Bob via a lossy link with transmission \( L_h \). The initial state, after passing through this link, can be written as

\[
|\psi_i\rangle = \frac{\sqrt{\tau} |10\rangle_{ab} \pm \sqrt{1 - \tau} |01\rangle_{ab}}{\sqrt{\tau + (1 - \tau)}}.
\]

describing the state shared between modes \( a \) and \( b \) as indicated in figure 1.

Bob then uses a HPA [11] to compensate the lossy link on his part of the entangled state. The heralded photon amplifier works in the following way: Bob uses an auxiliary photon that impinges on a highly-transmissive (\( t \)) beamsplitter (BS) before performing a Bell state measurement (BSM) between his half of the entangled state (\( b \)) and part of the auxiliary photon (\( c \)). A click in one of the BSM detectors heralds the loss-reduced state into modes \( a \) and \( d \).

A successful BSM heralds a final state of the form:

\[
|\psi_f\rangle = \sqrt{\tau(1 - t)} |10\rangle_{ad} + \sqrt{1 - \tau} |01\rangle_{ad}.
\]

where \( \lambda \) is given by

\[
\lambda = (1 - \tau)(1 - t) \text{ defines the loss, or vacuum, component of the heralded state and}
\]

\[
N = (1 - \tau) t + (1 - t) \text{ gives the state normalisation. From this we can determine the probability of a successful BSM, } P_{BSM} = N/2, \text{ for either of the BSM detectors. We note that } \rho_f \text{ holds for binary detectors; the use of photon-number-resolving detectors can further improve the performance of the HPAs [13, 15]. The HPA acts on the initial lossy state in two ways. We see from equation (3) where, by varying the HPA’s transmission

\[
\rho_f = \frac{1}{N} \left\{ \lambda |00\rangle \langle 00| + (N - \lambda) |\psi_f\rangle \langle \psi_f| \right\},
\]

Note that using the other outcome (detector) of the BSM produces a phase-flipped version of this state.
parameter $t$, we can make $\lambda$ small, thus overcoming the loss. If we look at equation (4), we see that loss can also affect the degree of entanglement, which again, can be overcome by varying $t$ and $\tau$.

We characterise the initial (i) and final (f) states $\rho_{if}$ by employing a measurement scheme [9, 10, 28] that uses small displacements $D(\alpha) = e^{\alpha n^{d} - n^{a}}$ operating on each of Alice’s and Bob’s modes [29], followed by binary detectors. The resulting detection click (C) and no-click (0) probabilities are given by

$$P_0 = \text{Tr}[\rho |\alpha\rangle \langle \alpha|], \quad P_C = 1 - P_0,$$

where $\rho$ represents the reduced state for either Alice or Bob. We can use these measurements to obtain the conditional probabilities $P_{00}$, $P_{01}$, $P_{10}$, $P_{11}$, which are dependent on the measurement (phase and amplitude) settings of both Alice’s and Bob’s displacements, $\sigma_{ai}$, which gives us access to approximate $\sigma_{x}$, $\sigma_{y}$ and $\sigma_{z}$ Pauli matrix operators [9, 10, 28]. We also note that all measurement outcomes are considered, i.e. there is no post-selection, nor do we truncate the Hilbert space as the matrix operators [9, 10, 28].

The Fidelity is given by $F = |d| + 0.5[P_{01} + P_{10}]$, which is obtained by considering the joint probabilities $P_{mn}$ to have outcomes $m$ for Alice and $n$ for Bob. The off-diagonal coherence term $d = \langle \{01\} | \rho | \{10\} \rangle$ can be extracted from the correlator obtained with a displacement $D(\alpha_{A}) \otimes D(\alpha_{B})$, which is given by

$$\langle \sigma_{x} \otimes \sigma_{y} \rangle = e^{2\theta^2}[8d^2 + P_{00}(e^{i\theta} - 2) + (e^{i\theta} - 2)[e^{i\theta}(P_{10} + P_{10} + P_{11}) - 2(P_{10} + P_{10} + P_{11} + P_{11} + P_{11} + P_{11} + P_{11})]].$$

To simplify the notation we define $\alpha = re^{i\theta}$, with $\theta = 0$ and the $P_{mn}$ correspond to the probabilities without displacement.

The probabilistic nature of the BSM for the amplification imposes a trade-off, as high values of $F_f$ implies low heralding rates. It is possible to obtain $F_f \rightarrow 1$ when $\tau \rightarrow 1 + \eta_0(1 - t)$ and $t \rightarrow 1$, however, the probability of success also then tends to zero. Note that in the case of $\tau = 1$ or $t = 1$ the state is separable. Hence we need to verify both that the protocol increases the Fidelity and that it preserves the entanglement. This is done by comparing the measured Fidelities $F_f$ to the maximum Fidelity that can be obtained with a separable state, $F_{sep}$, i.e. where $\rho$ stays positive under partial transposition in the $\{0, 1\}$ subspace [30, 31]. The separable Fidelity is then given by $F_{sep} = \sqrt{P_{00}P_{11}} + 0.5[P_{10} + P_{01}]$. In practice, for both $F$ and $F_{sep}$, we measure the probabilities of detection / non-detection, that take into account the contribution of multiphoton events.

3. Experiment

A simplified experimental schematic is shown in figure 2. Two heralded single photon sources (HSPSs) are used to create both the path entangled states on Alice’s side, between modes $a$ and $b$, and the auxiliary photons on Bob’s side, which are also non-maximally entangled between modes $c$ and $d$. The HSPSs are realised by detecting one photon from each pair generated in a double pass spontaneous parametric down-conversion (SPDC) scheme, although to simplify figure 2, these are shown as independent sources. The sources are based on a PPKTP nonlinear crystal pumped with a 76 MHz pulsed (ps) laser at 772 nm that generates pure photons.
(>90%) in telecom-band without spectral filtering [32]. The degree of entanglement for Alice’s initial heralded entangled state is determined by rotating a half-wave plate (HWP) before a polarisation beamsplitter (PBS), which is equivalent to a BS with a ratio $r \equiv 1 - t$. A gated InGaAs single photon detector, with a detection efficiency of $\sim 25\%$, heralds entangled states at a rate of 50 kHz, with one half directed locally to Alice’s measurement setup and the other coupled into fibre and sent to Bob. The transmission through this link is attenuated (Att.) to simulate distance denoted by $\eta_L$.

On Bob’s side we use a modified set-up for the HPA compared to figure 1. The second HSPS generates the auxiliary photon which passes through another HWP and PBS, denoted with $\star$, corresponding to a BS splitting ratio $t : 1 - t$ for the HPA. As explained previously, $\tau$ and $\gamma$ can be used to optimise the Fidelity of the final amplified state as a function of channel transmission. Bob’s half of the initial entangled state is sent straight through PBS($\star$) and combined with the reflected, and orthogonally polarised, component $(1 - t)$ of the auxiliary photon. The BSM then consists in rotating these, via another HWP, and interfering them at another PBS. A single detection—a successful BSM—then heralds a loss-reduced version of the state that was shared between modes $a$ and $b$, into Alice and Bob’s output modes $a$ and $d$.

Path entangled state analysis is performed via a displacement-based measurement scheme [8–10]. Experimentally, this is realised by interfering the coherent state $|$; $|$ with each mode of the entangled state and then using single photon counting detectors. This approach can be seen as somewhere in between continuous and discrete variable measurement schemes, where the weak coherent state gives us access to phase dependent measurements in the photon counting regime, i.e. to make measurements in the $|0\rangle \pm |1\rangle$ basis. The value of $\alpha$ is chosen around 0.7 which corresponds to the point where the influence of $d$ is maximised. An important requirement for this scheme is that there are no phase fluctuations or drifts between the coherent state and the path entangled state to be analysed, e.g. from path-length dilation of transmission fibres due to changes in temperature. To ensure this, the coherent state is input to the system at the first PBS on Alice’s side so that it can co-propagate with the entangled state; the two states are orthogonally polarised. The use of polarisation allows us to reflect the coherent state towards Bob’s output mode $d$ at the PBS($\star$), thus co-propagating with the heralded, and amplified, state in mode $d$.

As illustrated in figure 2 the coherent state used for the displacement-based measurements is generated using difference-frequency generation (DFG) between the pulsed pump laser at 772 nm and a cw laser at the same wavelength (1546 nm) as the heralding photon, using a PPLN nonlinear crystal [33]. Another HWP and PBS couples this into modes $a$ and $b$, with orthogonal polarisation to the entangled state and with a fixed temporal delay. This delay is introduced to avoid saturating the BSM detector due to finite polarisation extinction at PBS ($\star$). Gated detectors are then used to only open when the entangled state is expected and not the coherent state, avoiding false detections that would introduce errors in the final state.

Finally, the two states are sent to unbalanced Mach–Zehnder interferometers, where a PBS directs them into the two different paths to erase the timing information. In the interferometers the polarisation of the coherent state is rotated, by a HWP, before interfering with the entangled state at the 99:1 fibre BSs, which correspond to the displacement operations. The interferometers are actively, and independently, stabilised and piezo-actuated mirrors allow us to perform measurements with well-defined phase differences between Alice and Bob.

A motorised delay line between the two HSPs is used to ensure that the photons arrive at the BSM at the same time. The exact position of the delay line is set by performing a HOM interference measurement. Similarly, additional delay lines between each HSP and the DFG ensure their synchronous arrival at the 99:1 BSs for the displacement operations. The indistinguishability, or overlap, between the photons and the coherent state is essential for the displacement operation and is measured directly from several $P_C$ measurements: without the coherent state; without the reduced (entangled) state of Alice and Bob, and then with both. This provides a more direct and faster measurement than a HOM interference measurement; we determine an overlap of $0.90 \pm 0.02$.

We have an initial entanglement heralding rate of around 50 kHz, however, the final, amplified, heralding rates are significantly lower due to the probabilistic nature of the two SPDC sources, which simultaneously generate $\approx 30$ path entangled states and auxiliary photons per second. In the case with amplification, the heralding rate is also dependent on the loss, achieving rates of 0.25 Hz, 0.19 Hz and 0.15 Hz, respectively, for decreasing $\eta_L$ of approximately 0.28, 0.17 and 0.09, corresponding to equivalent transmission distances in standard telecom fibre (0.2 dB km$^{-1}$) of around 30, 40 and 50 km. The transmission $\tau = 0.6$ was used to create the initial path entangled state to be amplified, while the transmission of the auxiliary photon through the PBS ($\star$) was set to $\tau = 0.93$. These two values were maintained for all measurements, as they provide a reasonable trade-off between measurement time and Fidelity $F_f$ for all tested cases.
4. Results

We characterise and compare the Fidelities for the entangled state, with and without the HPA as the total transmissions $\eta_L$ is varied. The main results are shown in figure 3(a). In the current configuration we use InGaAs detectors, with detection efficiencies around 25%, and the corresponding raw Fidelity is shown on the left vertical axis. On the right vertical axis we re-scale these results, based on unit detection efficiency, to help differentiate the effect of limited detection efficiency from the performance of the state generation, displacement-based measurements and HPA.

The Fidelities, $F$, $F_{\text{exp}}$, are obtained from the measured joint probabilities $P_m$ without and with the HPA at $\eta_L = 0.09$. Using the measurement results for $P_m$ in the case where we use the HPA, we can calculate the off-diagonal coherence term $d = (4.8 \pm 1.6) \times 10^{-2}$. The error bars for the experimental results in figure 3(a) for the amplified case are quite large and are dominated by the measurement of the off-diagonal coherence term $d$, which is a function of all the probabilities $P_{mn}$ and terms exponential in $|\alpha|^2; |\alpha|$ can vary over time resulting in errors larger than just the statistical case. We limit these measurements to around $2.6 \times 10^3$ events, compared to $38 \times 10^3$ events without the coherent state, for each attenuation. Measurements with the coherent state, while having a higher $P_{\text{coh}}$ are less stable as the polarisation of the coherent state is not actively stabilised before injecting into the set-up and can rotate, as such this is re-aligned and we take many measurements of around one hour and accumulate these over a few days. The measurements without coherent state do not suffer from any stability problem and can run continuously for several days, giving rise to better statistics. Figure 3(a) also shows the theoretically expected Fidelities with and without amplification as a blue (upper) and pink (lower) curves, respectively. They take into account the coupling loss of the photons, the detection and transmission efficiencies for Alice and Bob as well as any imperfections in the spectral overlap between the heralded photons and the coherent state. The error bars of the model are determined by considering
the uncertainties on the experimental parameters, such as the measured BS ratios, and values for the transmission and detection efficiencies.

One can observe that the measured Fidelities not only match the theory in both cases, but a clear increase in Fidelity is achieved after amplification. While a significant increase in Fidelity is demonstrated, one must also ensure that the final state preserves entanglement. This is realised by comparing the measured Fidelities $F_f$ to the separable state $F_{sep}$. We note that $F_{sep}$ corresponds to the case where the coherence is lost, i.e. we have a mixed state of vacuum, $|10\rangle\langle10|$ and $|01\rangle\langle01|$ terms, is larger than the minimal Fidelity shown in figure 3(a). The minimal Fidelity corresponds to the case where Bob’s photon is completely lost and we approach a mixed state of vacuum and $|10\rangle\langle10|$. The right hand side of figure 3(b) shows the reconstructed density matrices for the initial (top) and final (bottom) states, assuming unit detection efficiencies, where we more clearly see the effect of the HPA. These results are for a transmission of $\eta_L = 0.09$, which is equivalent to more than 50 km of fibre transmission. In the case of the initial state, before the HPA, the $|01\rangle_{lb}$ component is greatly attenuated and there is a significant vacuum contribution. After the HPA we see this vacuum contribution is noticeably reduced and we recover a state close to the maximally entangled state. This final state also reveals that the $\tau$ and $t$ parameters could be further optimised to improve the Fidelity.

Table 2 compares $F_{sep}$ with the amplified Fidelities for each measurement with different $\eta_L$. We find $F > F_{sep}$ for each measurement, the respective confidence levels are about 1.2, 2.4 and 2.2 standard deviations, respectively. By combining all results it is possible to calculate a probability of around $5 \times 10^{-6}$ that $F < F_{sep}$, showing with a high confidence level that the HPA is preserving entanglement.

In this demonstration we use standard gated InGaAs SPADs for photon detection, which are limited to 25% detection efficiency. While this allows us to clearly demonstrate the potential of this approach, it is not sufficient to overcome the detection efficiency threshold necessary for device-independent protocols. The current system efficiency is easily determined from the $P_{QA}$ value $(8.8 \times 10^{-2})$ divided by the detector efficiency $(0.25)$ and BS ratio $(0.6)$, which gives around 60%. One significant improvement would be to use superconducting nanowire single photon detectors (SNSPD), however, in this configuration we cannot exploit their full potential, in terms of detection efficiency, due to detecting photons from the weak coherent states when no entanglement has been heralded. These detections saturate the detectors and reduce their effective efficiency. Previously, some of us performed a detection loophole-free EPR steering experiment—a one-sided semi-device-independent protocol—using SNSPDs [10], however the clock rate of the system had to be significantly reduced to match the relatively slow recovery time of the SNSPDs. To exploit the high detection efficiencies of SNSPDs their recovery time has to be reduced [34] or a means of operating them in a gated mode needs to be developed [35], which are both ongoing areas of research for this technology. Nonetheless, one could use independent fibres for the coherent and entangled states, which would require that their paths were actively phase stabilised. As discussed in [10], improved coupling, fibre splices rather than connectors, and higher transmission polarising BS, efficiencies sufficient for violations with non-maximally entangled states [19, 20] are feasible. Alternatively, multipartite scenarios offer another avenue for reducing the required detection thresholds. Nonetheless, there are interesting approaches using event-ready schemes [36] for one-sided DI-QKD [37] that would be feasible based on [10] and these results.

5. Conclusion

We have brought together three concepts: heralded path entanglement generation, displacement-based detection and HPA, demonstrating that high Fidelity entanglement can be distributed over distances equivalent to up to 50 km. The original proposal for DI-QKD, based on heralded photon amplifiers [22], was for polarisation entangled photons, conceptually, however, path entanglement provides a much simpler implementation. This is the first experimental demonstration of a system suitable for implementing this DI-QKD protocol. The current distance limitation is governed primarily by the probabilistic nature of the SPDC sources and was already alluded to in the initial proposal [22]. Nonetheless, 50 km would already be a significant distance for DI-QKD. In combination with recent experiments demonstrating heralded path entanglement and
displacement-based detection \cite{9, 10}, the combination with HPA represents a fascinating and flexible toolbox for testing concepts of device-independent protocols and entanglement distribution in quantum networks.

Acknowledgments

The authors thank N Bruno and N Sangouard for useful discussions. This work was supported by the Swiss National Science Foundation (Grant No. 200021_159592) and by Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO) (VIDI).

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