Gravitational wave diagnosis of a circumbinary disk

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When binary black holes are embedded in a gaseous environment, a rotating disk surrounding them, the so-called circumbinary disk, will be formed. The binary exerts a gravitational torque on the circumbinary disk and thereby the orbital angular momentum is transferred to it, while the angular momentum of the circumbinary disk is transferred to the binary through the mass accretion. The binary undergoes an orbital decay due to both the gravitational wave emission and the binary-disk interaction. This causes the phase evolution of the gravitational wave signal. The precise measurement of the gravitational wave phase thus may provide information regarding the circumbinary disk. In this paper, we assess the detectability of the signature of the binary-disk interaction using the future space-borne gravitational wave detectors such as \textit{DECIGO} and \textit{BBO} by the standard matched filtering analysis. We find that the effect of the circumbinary disk around binary black holes in the mass range $6M_\odot \leq M \lesssim 3 \times 10^3 M_\odot$ is detectable at a statistically significant level in five year observation, provided that gas accretes onto the binary at a rate greater than $\dot{M} \sim 1.4 \times 10^{17} \text{[g s}^{-1}] \, j^{-1} (M/10M_\odot)^{33/23}$ with 10% mass-to-energy conversion efficiency, where \( j \) represents the efficiency of the angular momentum transfer from the binary to the circumbinary disk. We show that $O(0.1)$ coalescence events are expected to occur in sufficiently dense molecular clouds in five year observation. We also point out that the circumbinary disk is detectable, even if its mass at around the inner edge is by over 10 orders of magnitude less than the binary mass.

\textit{Subject headings:} black hole physics - accretion, accretion disks - gravitational waves - stars: evolution
1. Introduction

It is widely accepted that coalescing binary compact objects, \textit{i.e.}, those consisting of black holes, neutron stars, or white dwarfs, are important sources of gravitational waves (GWs). Ground-based detectors with higher sensitivity are currently under construction, \textit{e.g.}, Advanced-LIGO, Advanced-VIRGO, and LCGT, while there are plans for the space-borne interferometers such as \textit{LISA} \cite{Danzmann1997}, \textit{DECIGO} \cite{Seto2001} and \textit{BBO} \cite{Phinney2003}. The significant improvement of sensitivity will make it possible to directly detect GWs in near future. \textit{LISA} has its best sensitivity at around $10^{-3} - 10^{-2}$ Hz and one of its main targets is a merger event of binary massive black holes. On the other hand, \textit{DECIGO/BBO} has its best sensitivity at around $0.1 - 1$ Hz. Its ultimate goal is to detect primordial GW backgrounds, but it has promising astrophysical sources such as binary stellar-mass black holes. It is also sensitive to merging signals from binary intermediate-mass black holes.

Recently, much attention has been paid to the surrounding environment of the GW sources. Several types of possible electromagnetic (EM) signatures associated with GW emissions have been proposed, such as afterglows \cite{Milosavljevic2005}, precursors \cite{Chang2009, Bode2010} and periodic emission \cite{Hayasaki2007, Bogdanovic2008, MacFadyen2008, Cuadra2009} in the context of a massive black hole coalescence. Those EM signatures will be also very useful in localizing the position of a GW source in the sky and in identifying its redshift. By combining the redshift information with a luminosity distance determined by the GW signal, one can obtain a distance-redshift relation. This will be one of the best observational probes of the dark energy \cite{Schutz1986, Holz2003}. However, there are not so many works discussing EM signals from coalescing binary black holes in the stellar/intermediate mass range $6 - 10^4 M_\odot$. Obviously, no black holes can emit EM
radiation, unless they interact with their environmental gas. Thus, our interest is naturally led to binary stellar/interstellar-mass black holes in a dense gaseous medium.

Narayan (2000) investigated the effect of hydrodynamic interactions between an advection-dominated accretion flow onto a supermassive black hole and inspiraling stars on GW signals. This effect was shown to be negligible as long as the advection-dominated accretion flow is concerned. The effects of the interaction with a gaseous disk on the GW phase evolution have been also discussed in the context of an extreme mass ratio inspiral (EMRI), where a compact object inspirals into a supermassive black hole (Barausse & Rezzolla 2008; Kocsis et al. 2011; Yunes et al. 2011b). In this case the situation is more similar to a planet migration. It has been claimed that the existence of an accretion disk around a supermassive black hole is detectable for a certain parameter range with LISA.

In contrast, a binary composed of black holes with nearly equal masses residing in a gaseous environment will form a triple disk system: two accretion disks around the respective black holes and one circumbinary disk surrounding both of them (Hayasaki et al. 2007, 2008). Then, the binary black holes and the circumbinary disk should mutually interact, which inevitably affects the orbital evolution of the binary. The gas surrounding the binary acts on the orbital motion through a drag force (tidal/resonant interaction), leading to the angular momentum transfer from the binary to the gas. Conversely, the angular momentum is carried from the circumbinary disk to the binary by gas accretion. Since DECIGO/BBO will detect many cycles of GWs from the binary inspiral, a rather tiny correction in the GW phase is detectable. Therefore signature of the circumbinary disk can arise in the GW signal even if the circumbinary disk is extremely less massive by many orders of magnitude than the binary. In this paper, we give an estimate how large the mass accretion rates of the circumbinary disk should be, so that we can detect the
effect of the binary-disk interaction on the GW phase evolution at a statistically significant level with DECIGO/BBO. Note that we evaluate the effect of the binary-disk interaction at the first order in a mass accretion rate of the circumbinary disk throughout this paper. Although our current understanding of the binary-disk interaction is poor, we assume here that we knew how the GW phase evolution is modified and hence the standard matched filtering analysis is applicable. Based on our estimate of the mass accretion rate required for detection, we also give a prediction for the number of merger events under detectable influence of the circumbinary disk.

2. Measuring mass accretion rates with GW detectors

In this section, we first briefly explain the basics about how the phase of GWs from a binary evolves as a result of the radiation reaction due to the GW emission. Second, we present the picture that we have in mind about the interaction between the binary and the circumbinary disk, providing a quantitative estimate for the correction due to the binary-disk interaction in the GW phase evolution. We relate the magnitude of the correction to the mass accretion rate of the circumbinary disk. Then, we evaluate the determination accuracy of the mass accretion rate of the circumbinary disk.

2.1. Standard gravitational wave phase evolution

Let us first briefly review the evolution of the GW phase of a coalescing binary, neglecting the interaction between the binary and the circumbinary disk. Close enough binaries are expected to evolve their orbits by losing the energy and angular momentum due to the GW emission. Here, we assume that binary black holes are in a circular orbit, because the orbital eccentricity decreases rapidly in proportion to $a^{19/12}$ (Peters 1964),
where $a$ is the semi-major axis of the binary. At the leading order of the Post-Newtonian (PN) expansion, the binding energy and orbital angular momentum of a circular binary are, respectively, given by

$$
E = -\frac{\mu c^2}{2} x, \quad J = \frac{\mu c^2}{\Omega_{\text{orb}}} x,
$$

(1)

where $c$ is the speed of light, $\mu \equiv M_1 M_2 / M$ is the reduced mass with $M \equiv M_1 + M_2$ representing the total mass of binary black holes, $x = (GM\Omega_{\text{orb}}/c^3)^{2/3}$ is a non-dimensional quantity of $O((v/c)^2)$, and $\Omega_{\text{orb}} = \sqrt{GM/a^3}$ is an orbital frequency of the binary. The leading terms in the GW energy and angular momentum fluxes emitted from a binary are given by the well-known quadrupole formulas as

$$
\dot{E}_{\text{gw}} = \frac{32c^5}{5G} \eta^2 x^5, \quad \dot{J}_{\text{gw}} = \frac{32c^5}{5G\Omega_{\text{orb}}} \eta^2 x^5
$$

(2)

with $\eta \equiv \mu / M$ and a dot represents differentiation with respect to time. For an equal-mass binary, $\eta$ is set to be 0.25. The orbit of the binary gradually decays as a result of the loss of its energy and angular momentum due to the GW emission. Equating the time variation of the orbital energy or angular momentum with the loss due to the GW emission, one can derive, to the lowest PN order, $\dot{x} = 64c^3 \eta x^5 / 5GM$, which is translated into the evolution equation for the GW frequency $f = \Omega_{\text{orb}} / \pi$ as

$$
\dot{f} = \frac{96\pi}{5} f^2 \left( \frac{\pi GM f}{c^3} \right)^{5/3},
$$

(3)

where we introduced the chirp mass, $\mathcal{M} \equiv \eta^{3/5} M$.

Integrating this equation with the coalescence time set to $t_0$, the time evolution of the GW frequency is computed as

$$
f(t) = \frac{c^3}{\pi GM} \left( \frac{256 c^3 (t_0 - t)}{5 \frac{GM}{\eta}} \right)^{-3/8}
\sim 9.3 \times 10^{-2}[\text{Hz}] \eta_{0.25}^{-3/8} m_{10}^{-5/8} \left( \frac{t_0 - t}{1\text{yr}} \right)^{-3/8},
$$

(4)
where we use $\eta_{0.25} \equiv \eta/0.25$ and $m_{10} \equiv M/10M_\odot$ for brevity. One can translate this frequency evolution into the evolution of $a$, using the relation $(\pi f)^2 = GM/a^3$, that is,

$$a(t) = \frac{GM}{c^2} \left( \frac{256 c^3 (t_0 - t) \eta}{5 GM} \right)^{1/4} \approx 2.5 \times 10^9[\text{cm}] \eta_{0.25}^{1/4} m_{10}^{3/4} \left( \frac{t_0 - t}{1\text{yr}} \right)^{1/4}. \quad (5)$$

As the orbit shrinks with time, the GW frequency gradually increases and finally reaches the frequency at the inner-most stable circular orbit (ISCO)

$$f_{\text{ISCO}} = \frac{c^3}{6^{3/2} \pi GM} \sim 440[\text{Hz}] m_{10}^{-1}, \quad (7)$$

which is here approximated by the expression in the EMRI limit irrespective of the mass ratio.

The number of the orbital cycles before the coalescence is then calculated as

$$N_{\text{cyc}} \equiv \int_t^{t_0} \frac{f(t)}{2} dt \sim 2.3 \times 10^6 \eta_{0.25}^{-3/8} m_{10}^{-5/8} \left( \frac{t_0 - t}{1\text{yr}} \right)^{5/8}. \quad (8)$$

Our main interest is in binary black holes within the mass range $10^2 - 10^4 M_\odot$. When the mass of each black hole is $10 M_\odot$ and the observation starts one year before the coalescence, the initial GW frequency and the initial semi-major axis are $\sim 0.1 \text{Hz}$ and $\sim 0.04 R_\odot$, respectively. In the typical case one can observe $N_{\text{cyc}} \sim 10^6$ cycles of GWs. Such a large number of orbital cycles allow us to precisely measure the GW phase correction possibly to the level sensitive to the tiny effect of the circumbinary disk.

The observed waveform of GWs from the binary at the leading PN order is given by (Culter & Flanagan 1994)

$$h(t) = \left( \frac{384}{5} \right)^{1/2} \pi^{2/3} Q \frac{(GM)^2}{a(t) D_L c^4} \eta \cos \phi(t), \quad (9)$$

(Culter & Flanagan 1994)
where $Q$ is a factor of at most order unity that depends on the direction and orientation of the binary and $D_L$ is the distance from the earth to the GW source. The GW phase $\phi(t)$ is defined by

$$\phi(t) = \phi_0 + 2\pi \int_{t_0}^{t} f dt,$$

where $\phi_0$ is the GW phase at the coalescence.

### 2.2. Correction to the GW phase due to binary-disk interaction

In this subsection, we consider the correction to the phase evolution of GWs emitted from an inspiralling binary caused by the interaction with the circumbinary disk. Such binary-disk interaction induces additional extraction of the orbital angular momentum from the binary. This effect will be tiny compared to the phase evolution driven by the GW emission when the binary separation is close enough to be observable by future GW detectors. Even in that case, however, the correction might be observable because of a large number of cycles of GWs in the frequency range of observations. For simplicity, we assume that the orbital evolution is dominated by the radiation reaction due to the GW emission. In this case it would be natural to assume that the orbit is well approximated by a quasi-circular one. Although the circumbinary disk is not necessarily aligned to the binary orbital plane, here we also assume a simple aligned configuration.

The exchange between the orbital angular momentum of the binary, $J$, and the angular momentum of the circumbinary disk is in two ways. One is the angular momentum transfer through the tidal/resonant interaction (Armitage & Natarajan 2002). The other is through the overflow of the gas from the inner edge of circumbinary disk onto the central binary (Artymowicz & Lubow 1996, Hayasaki et al. 2007). The transferred gas will form accretion disks around respective black holes (Hayasaki et al. 2008). During
this accretion process, some fraction of the angular momentum of the transferred gas will temporarily go to the accretion disks associated with respective black holes. However, since the specific angular momentum of gas at the inner edges of the accretion disks is so tiny, only little fraction of the angular momentum of the accretion disks is transferred to the black hole spins. Thus, most of this angular momentum is transferred outward via the viscous process and is finally added to the orbital angular momentum of the binary $J$ via the tidal interaction (Hayasaki 2009). Therefore we can neglect the angular momentum that ends up with the spins of the black holes. Thus, the net angular momentum extracted from the binary orbital one due to interaction with the circumbinary disk, $\dot{J}_{\text{disk}}$, should be almost identical to the net angular momentum flow inside the circumbinary disk, $\dot{J}_{\text{disk}}$, which simply follows from the conservation law. (We take the flow of the angular momentum to be positive when it flows outward.)

Now, we roughly estimate the order of magnitude of $\dot{J}_{\text{disk}}$, without relying on a specific model of the circumbinary disk. For a quasi-stationary disk, $\dot{J}_{\text{disk}}$ should be independent of the radius $r$, which should be equal to the sum of the flows of angular momentum by the viscous torque $\dot{J}_{\text{vis}}(>0)$ and by the mass accretion $\dot{J}_{\text{acc}}(<0)$,

$$
\dot{J}_{\text{disk}} = \dot{J}_{\text{vis}}(r) + \dot{J}_{\text{acc}}(r) = \text{const.},
$$

(11)

except near the inner edge of the disk located at $r = r_{\text{in}}$, where the tidal/resonant torque is non-negligible. Note that $\dot{J}_{\text{acc}}(r)$ is negative when the mass is accreting inward. Since $|\dot{J}_{\text{acc}}(r)|$ increases with $r$ like $\propto \sqrt{r}$ for a Keplerian disk, $\dot{J}_{\text{vis}}(r)$ and $|\dot{J}_{\text{acc}}(r)|$ approximately balance with each other except near the inner edge of the circumbinary disk. If we were discussing an accretion disk around a single central object, the radius of the inner edge of the disk would be much smaller than that of the circumbinary disk. In this case, $\dot{J}_{\text{vis}}(r)$ should be almost balanced with $|\dot{J}_{\text{acc}}(r)|$ near $r = r_{\text{in}}$. In the present situation, however, the binary exerts the tidal/resonant torque, which truncates the disk at $r = r_{\text{in}}$. Then, the
surface density of the disk is enhanced near the inner edge by a factor of two or so compared with the case without the tidal/resonant torque (e.g., see Figure 3 of Hayasaki et al. 2007). Accordingly, the viscous torque is also locally enhanced by about the same ratio. This implies that $\dot{J}_{\text{disk}} \approx |\dot{J}_{\text{acc}}(r_{\text{in}})|$. Thus, the rate of the loss of the binary orbital angular momentum is estimated as

$$\frac{\dot{J}_{\text{disk}}}{\mathcal{J}} \approx \frac{|\dot{J}_{\text{acc}}(r_{\text{in}})|}{\mathcal{J}} = \frac{\dot{M} r_{\text{in}}^2 \Omega_{\text{in}}}{\mu a^2 \Omega_{\text{orb}}} \approx \sqrt{2} \frac{\dot{M}}{\mu},$$

(12)

where $\Omega_{\text{in}}$ is the Keplerian frequency of the circumbinary disk measured at $r_{\text{in}}$ and $\dot{M}$ is set to be positive when the mass flux is inward. In the last approximate equality $r_{\text{in}}$ was approximated by $2a$ (Artymowicz & Lubow 1994). Since currently we are not able to make the estimate of $\dot{J}_{\text{disk}}$ more precise, we introduce a parameter

$$j \equiv \frac{\dot{J}_{\text{disk}}}{\mathcal{J}} \left(\frac{\dot{M}}{\mu}\right)^{-1},$$

(13)

that represents the efficiency of angular momentum transfer from the binary to the circumbinary disk (recall that $\mu = \eta M$).

The angular momentum transfer from a circular equal-mass binary to the circumbinary disk has been recently studied by performing hydrodynamic simulations with $\alpha$-prescription (MacFadyen & Milosavljević 2008) and magneto-hydrodynamics (MHD) simulations (Shi and Krolik et al. 2011). In both simulations the sound velocity was set by hand instead of solving the energy equation. In such a numerical setup, one can freely scale the binary mass $M$, the semi-major axis $a$, and the surface density of the circumbinary disk $\Sigma$. MacFadyen & Milosavljević (2008) derived the averaged mass accretion rate $\langle \dot{M} \rangle = 2.5 \times 10^{-4} (GMa)^{1/2} \Sigma_0$ and the angular momentum flux carried by the circumbinary disk $\langle \dot{J}_{\text{disk}} \rangle \approx 1.4 \times 10^{-3} G Ma \Sigma_0$ from the simulated result after 4000 binary orbital cycles, where $\Sigma_0$ is a typical value of $\Sigma$ introduced for the normalization. On the other hand, Shi and Krolik et al. (2011) derived $\langle \dot{M} \rangle = 1.8 \times 10^{-2} (GMa)^{1/2} \Sigma_0$ and
$\langle \dot{J}_{\text{disk}} \rangle \simeq 1.2 \times 10^{-2} GM a \Sigma_0$ after $\simeq 77$ binary orbital cycles. Substituting these values of $\langle \dot{M} \rangle$ and $\langle \dot{J}_{\text{disk}} \rangle$ into equation (13), we can estimate the efficiency parameter $j$ to be $\sim 3.6$ for the former simulation and $\sim 0.69$ for the latter simulation, independent of $a$, $M$, and $\Sigma_0$. The scatter of $j$ between these two simulations will be mainly attributed to the difference in the strength of shear viscosity. MacFadyen & Milosavljević (2008) assumed that the Shakura-Sunyaev viscosity parameter $\alpha_{\text{SS}}$ to be 0.01, whereas the effective value of $\alpha_{\text{SS}}$ derived from the MHD simulation by Shi and Krolik et al. (2011) was $\sim 0.2$. Namely, $j$ becomes larger for smaller viscosity. One can explain this tendency qualitatively as follows: when the mass accretion rate is fixed, the circumbinary disk becomes more massive for smaller viscosity, while a more massive disk gains larger resonant torque from the binary.

The black hole growth due to the mass accretion from the circumbinary disk to each of the black holes via accretion disks can also modify the binary orbital evolution. We note from equation (13) that the growth rate of binary mass $\dot{M}/M$ is the same order as the rate of the angular momentum transfer $\dot{J}_{\text{disk}}/J$, if $j$ is the order of unity. Hence, for simplicity, we absorb this mass growth effect into the coefficient $j$ in equation (13) and neglect the time variation of the binary mass in the following discussion.

From the angular momentum conservation

$$\frac{dJ}{dt} = -(\dot{J}_{\text{gw}} + \dot{J}_{\text{disk}}), \quad (14)$$

we can derive the increasing rate of the GW frequency from equations (1), (2), (13), and (14) as

$$\dot{f} = \frac{96\pi}{5} f^2 \left( \frac{\pi G M f}{c^3} \right)^{5/3} \times \left[ 1 + \frac{5}{32} \eta^2 \frac{GM}{c^3} x^{-4} + \cdots \right]. \quad (15)$$

The first term in the square brackets of equation (15) is the leading contribution from the quadrupole GW radiation given in equation (3). The second term represents the effect of
the circumbinary disk, and the ellipsis stands for the higher order PN corrections. Recalling that $x \equiv (\pi G M f / c^3)^{2/3} = O((v/c)^2)$, we note that the correction due to the binary-disk interaction has “-4PN” frequency dependence relative to the leading term for a constant $j$.

By integrating equation (15) with respect to time, we can express $f$ as a function of $t$, which in turn leads to the expression of $t$ as a function of $f$:

$$t(f) = t_0 - \frac{5}{256\pi} \frac{1}{f} \left( \frac{\pi G M f}{c^3} \right)^{-5/3} \times \left[ 1 - \frac{5}{64} \frac{j GM}{c^3} x^{-4} + \cdots \right].$$

(16)

The GW phase defined by equation (10) then becomes

$$\phi(f) = \phi_0 - \frac{1}{16} \left( \frac{\pi G M f}{c^3} \right)^{-5/3} \times \left[ 1 - \frac{25}{416} \frac{j GM}{c^3} x^{-4} + \cdots \right].$$

(17)

From the Fourier transformation of equation (9), we finally find that the sky-averaged GW waveform in the Fourier domain under the stationary phase approximation (Culter & Flanagan 1994; Berti et al. 2005) is given by

$$\tilde{h}(f) = \frac{1}{2\sqrt{10\pi^{2/3}}} \frac{c}{D_L} \left( \frac{G M}{c^3} \right)^{5/6} f^{-7/6} e^{i \Psi(f)},$$

(18)

where the sky-averaged GW phase:

$$\Psi(f) = 2\pi f t_0 - \phi_0 - \frac{\pi}{4} + \frac{3}{128} \left( \frac{\pi G M f}{c^3} \right)^{-5/3} \times \left[ 1 - \frac{25}{832} \frac{j GM}{c^3} x^{-4} + \cdots \right].$$

(19)

2.3. Determination accuracy of mass accretion rates: rough estimate

In this section, we analytically estimate the determination accuracy of the mass accretion rate of the circumbinary disk with DECIGO/BBO. The determination accuracy
of the mass accretion rate is mainly governed by the low frequency side in the observation band, where the effect of the circumbinary disk is the largest. The lower cutoff frequency is determined by the duration of the observation or the detector sensitivity,

\[ f_{\text{min}} = \max(f(t = t_{\text{ini}}), f_{\text{low}}), \quad (20) \]

where \( f(t = t_{\text{ini}}) \) is given by equation (4) into which we substitute an appropriate observation period \( t_0 - t_{\text{ini}} \). We also adopt \( f_{\text{low}} = 10^{-3}\text{Hz} \) for the cutoff due to the detector sensitivity of \( \text{DECIGO/BBO} \). Then, \( f_{\text{min}} \) turns out to be determined by \( f(t = t_{\text{ini}}) \) for \( t_0 - t_{\text{ini}} = 5 \text{yr} \) in the mass range \( 6M_{\odot} < M \leq 5 \times 10^3M_{\odot} \).

The square of the signal-to-noise ratio is defined by

\[ SN^2 \equiv \int_{f_{\text{min}}}^{f_{\text{max}}} \vartheta^2(f) d\ln f, \quad (21) \]

where

\[ \vartheta^2(f) \equiv 4N_{\text{dev}} \frac{|\tilde{h}(f)|^2 f}{S_n(f)} \quad (22) \]

and \( N_{\text{dev}} = 8 \) denotes the effective number of interferometers for \( \text{DECIGO/BBO} \) (Cutler & Holz 2009) and \( S_n(f) \) represents a total noise spectral density. As given in equation (36) of Yagi et al. (2011), we adopt the non sky-averaged instrumental noise spectral density for \( \text{DECIGO/BBO} \) as

\[ S_{n}^{\text{inst}}(f) = 1.8 \times 10^{-49} \text{[Hz}^{-1}] \left( \frac{f}{1\text{Hz}} \right)^2 + 2.9 \times 10^{-49} \text{[Hz}^{-1}] \\
+ 9.2 \times 10^{-52} \text{[Hz}^{-1}] \left( \frac{f}{1\text{Hz}} \right)^{-4}. \quad (23) \]

This is obtained by multiplying the sky-averaged noise spectrum, which is shown in equation (15) of Cutler & Holz (2009), by \( 3/20 \) (Berti et al. 2005). The third term of the above equation dominates over the other terms at low frequencies so that \( S_{n}^{\text{inst}}(f) \approx 9.2 \times 10^{-52} \text{[Hz}^{-1}] (f/1\text{Hz})^{-4} \). For simplicity, we assume in this subsection that

\[ S_n(f) = 9.2 \times 10^{-52} \text{[Hz}^{-1}] \left( \frac{f}{1\text{Hz}} \right)^{-4} \quad (24) \]
by neglecting the white dwarf confusion noise from the total noise spectral density. Note that the value of $f_{\text{max}}$ is irrelevant in the following discussion of this subsection.

To assess how accurately we can constrain the mass accretion rate, we extract the phase correction coming from the interaction between the binary and the circumbinary disk in equation (19), i.e.,

$$\Psi_{\text{disk}}(f) = -\frac{25}{128 \times 72^2/5} \frac{j \dot{M}}{c^3} \left(\frac{\pi G \dot{M}}{c^3}\right)^{-13/3}. \quad (25)$$

If there are no degeneracies between the mass accretion rate and the other binary parameters, we can roughly claim that the effect of the circumbinary disk is detectable at the first order in the mass accretion rate as long as the inequality

$$|\varrho(f)\Psi_{\text{disk}}(f)| \gtrsim 1 \quad (26)$$

is satisfied. This inequality is most likely satisfied at $f = f_{\text{min}}$ because $\Psi_{\text{disk}}(f)$ is proportional to a large negative power of $f$. Therefore, we apply $f = f_{\text{min}}$ to equation (26). As a typical source of DECIGO/BBO, we obtain $\varrho(f_{\text{min}}) \approx 108 \left(D_L/3\text{ Gpc}\right)^{-1}((t_0 - t_{\text{ini}})/5\text{ yr})^{-1/2}$ for five year observation. Since $\varrho(f)$ is found to be proportional to $M^{5/6} f^{4/3}$ from equations (18), (22), and (24), it exhibits no dependence on $M$ as long as we adopt $f_{\text{min}} = f(t_{\text{ini}}) \propto M^{-5/8}$.

Then, the condition (26) gives an analytic estimate for the determination accuracy of the mass accretion rate, $\Delta \dot{M}$, as

$$\frac{\Delta \dot{M}}{\dot{M}_{\text{Edd}}} \approx 4.5 \times 10^{-4} j^{-1} \epsilon_{0.1} \eta_{0.25}^{-13/8} m_{10}^{5/8} \left(\frac{t_0 - t_{\text{ini}}}{5\text{ yr}}\right)^{-13/8} \quad (27)$$

in the mass range $6M_\odot \leq M \lesssim 3 \times 10^3M_\odot$, where we normalized the determination accuracy by the Eddington accretion rate:

$$\dot{M}_{\text{Edd}} = \frac{1}{\epsilon} \frac{L_{\text{Edd}}}{c^2} \simeq 1.4 \times 10^{19}[\text{g s}^{-1}] \epsilon_{0.1}^{-1} m_{10}, \quad (28)$$

and $L_{\text{Edd}} = 4\pi GMm_p c/\sigma_T$ is the Eddington luminosity with $m_p$ and $\sigma_T$ denoting the proton mass and Thomson scattering cross section, respectively. The mass-to-energy conversion efficiency $\epsilon$ is set to 0.1 in the following discussion.
The relative accuracy in comparison with the Eddington rate decreases with the total mass of the binary but increases in proportion to \( j \). This can be understood by the facts that the Eddington rate is proportional to \( M \) and that \( \Psi_{\text{disk}}(f_{\text{min}}) \propto j M^{-13/8} \) in equation (25), which gives \( \Delta \dot{M} \propto j^{-1} M^{13/8} \). We will compare this analytic estimate with a more precise numerical one in the succeeding subsection.

2.4. Determination accuracy of mass accretion rates: more precise estimate

We derive the determination accuracy of the mass accretion rate by applying the matched filtering analysis (Culter & Flanagan 1994). We adopt the phase correction given in equation (19) as templates, although we do not know the exact form of the correction to the GW phase. More detailed study for more reliable prediction of the phase correction is left for future work. By neglecting the black hole spins, the determination accuracy of the binary parameters \( \theta^a = \{ \ln M, \ln \eta, t_0, \phi_0, D_L, \dot{M} \} \) can be estimated by

\[
\Delta \theta^a = \sqrt{(\Gamma^{-1})_{aa}/N_{\text{dev}}},
\]

where

\[
\Gamma_{ab} \equiv 4\text{Re} \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{\partial \tilde{h}}{\partial \theta^a} \frac{\partial \tilde{h}^*}{\partial \theta^b} S_n(f) df
\]

is the Fisher matrix. Here,

\[
f_{\text{max}} = \min(f_{\text{ISCO}}, f_{\text{high}})
\]

is the cutoff frequency on the higher frequency side, where the cutoff frequency determined by the detector sensitivity of DECIGO/BBO is taken as \( f_{\text{high}} = 100 \text{Hz} \) and \( f_{\text{ISCO}} \) is the GW frequency at the ISCO given in equation (7). For the noise spectrum \( S_n(f) \), we adopt the total noise spectral density (see equation (36) of Yagi et al. (2011)), which includes the confusion noise of WD/WD binaries that masks the instrumental noise below \( f < 0.2 \text{ Hz} \) (Farmer & Phinney 2003). We also assumed that the NS/NS foreground noise can be cleaned down to the level below the instrumental noise.
Figure 1 shows to what extent one can constrain the mass accretion rate of coalescing binary black holes from the GW observation with DECIGO/BBO. The vertical axis shows the standard error of the mass accretion rate normalized by the Eddington rate, whereas the horizontal axis shows the total mass of binary black holes. The mass accretion rate is measurable at a statistically significant level, if it is greater than the determination accuracy $\Delta \dot{M}$.

The solid, dashed, and dotted curves represent $\Delta \dot{M}/\dot{M}_{\text{Edd}}$ for the cases of the observation time of $t_0 - t_{\text{ini}} = 1$, 3, and 5 yr, respectively. The condition $\Delta \dot{M} \leq \dot{M}_{\text{Edd}}$ is realized in the mass range $6M_\odot \leq M \lesssim 3 \times 10^3 M_\odot$ for one year observation and in the mass range $6M_\odot \leq M \lesssim 8 \times 10^3 M_\odot$ for both three year and five year observations. The determination accuracy is higher for binaries at a closer distance from the earth and is also higher with a longer observation time. We also note that the dependence of the determination accuracy on the observation time is in good agreement with $(t_0 - t_{\text{ini}})^{-13/8}$ shown in equation (27) in the mass range $50M_\odot \lesssim M \lesssim 3 \times 10^3 M_\odot$. From Figure 1, we can estimate the mass dependence of the determination accuracy in five year observation as

$$\frac{\Delta \dot{M}}{\dot{M}_{\text{Edd}}} \approx 1.0 \times 10^{-2} j^{-1} m_{10}^{10/23}$$

in the mass range $6M_\odot \lesssim M \lesssim 3 \times 10^3 M_\odot$. By comparing this estimate with equation (27), we find that the analytic estimate tends to give about one order of magnitude higher determination accuracy and that its dependence on the binary mass is slightly different. These are because we have neglected the contribution from the white dwarf confusion noise in the analytic estimate.
3. Expected number of merger events

In this section, we estimate the expected number of merger events that have detectable signatures of the binary-disk interaction in GWs.

3.1. Circumbinary disk formation

The presence of a circumbinary disk around binary black holes with sufficiently large accretion rate will not be ubiquitous. To achieve such a large accretion rate as shown in equation (31), it will be required that binary black holes are embedded in a dense gaseous environment like a molecular cloud which exists in a star forming region of a galaxy. Its typical length scale of the molecular cloud is $10^{2-3}$ pc and its molecular hydrogen number density is distributed over the range of $10^{3-6}$ cm$^{-3}$.

If we consider a binary traveling through a dense molecular cloud, the circumbinary disk will be formed around the binary through the Bondi-Hoyle-Lyttleton (BHL) accretion. The gas within the BHL radius expressed by

$$r_{\text{BHL}} = \frac{2GM}{v_\infty^2} \sim 2.2 \times 10^{-4} [\text{pc}] \, v_{20}^2 m_{10}$$

will accrete onto the binary, where $v_{20} \equiv v_\infty/(20 \text{ km s}^{-1})$ and $v_\infty \equiv \sqrt{v_{\text{bulk}}^2 + c_s^2}$, with the sound velocity $c_{s,\infty} = 0.2 \text{ km s}^{-1}$ for typical molecular clouds. Here, we also adopted $v_{\text{bulk}} = 20 \text{ km s}^{-1}$ as a fiducial bulk velocity of the binary relative to the ambient medium. This is indicated from the observations that the radial velocities of some X-ray binaries including GRO J0422+32 in our galaxy are $\sim 10 \text{ km s}^{-1}$ (see Table 1 of Nelemans & Van den Heuvel (1999)). Since the BHL radius is much less than a typical size of a molecular cloud core $\sim 1 \text{ pc}$, the circumbinary disk can be formed even in the core of a molecular cloud.
While the outer edge radius of the circumbinary disk should be less than the BHL radius, its inner edge radius is approximately given by \( r_{in} \approx 2a(t) \) (Artymowicz & Lubow 1994). The consistency condition \( r_{BHL} > r_{in} \) requires \( a(t) < a_{BHL} \sim 1.1 \times 10^{-5} \text{[pc]} \, v_{20}^{-2} m_{10} \), where \( a(t) \) can be read from equation (6). At five years before the coalescence, this condition is easily satisfied.

The rate of the BHL accretion is given by (Bondi & Hoyle 1944)

\[
\dot{M}_{BHL} = 4\pi (GM)^2 \rho_{\infty} v_{\infty}^{-3} \\
\sim 2.8 \times 10^{17} \text{[g s}^{-1}] \rho_{-19} v_{20}^{-3} m_{10}^2 ,
\]

(33)

where \( \rho_{-19} \equiv \rho_{\infty}/(10^{-19} \text{g cm}^{-3}) \) with the density of the molecular cloud \( \rho_{\infty} \). Its fiducial value \( 10^{-19} \text{g cm}^{-3} \) corresponds to a typical density of dense molecular cloud cores. In Figure 1, the dash-dotted line represents this BHL rate normalized by the Eddington rate. The determination accuracy is higher than \( \dot{M}_{BHL} \) in the mass range \( M \gtrsim 100M_\odot \) for one year observation and in the almost whole mass range for both three year and five year observations. It can happen that the mass accretion rate is in the range between \( \Delta \dot{M} \) and \( \dot{M}_{BHL} \), and then the effect of the circumbinary disk is detectable at a statistically significant level with \text{DECIGO/BBO}.

From equations (28) and (33), the lowest number density of the molecular cloud for a BHL rate to be measured with the determination accuracy \( \Delta \dot{M} \) is given by

\[
n_{MC} \sim 1.5 \times 10^6 \text{[cm}^{-3}] \epsilon_{0.1}^{-1} v_{20}^3 m_{10}^{-1} \left( \frac{\Delta \dot{M}}{\dot{M}_{\text{Edd}}} \right),
\]

(34)

where we substitute \( \Delta \dot{M} \) into \( \dot{M}_{BHL} \).
3.2. Estimate of merger events

Following Mii & Totani (2005), the volume fraction of molecular clouds with \( n \gtrsim n_{MC} \) in our galaxy can be modeled as

\[
\begin{align*}
    f_{MC} = & \frac{(\beta - 2)\langle \Sigma_{MC} \rangle}{2\mu m_p n_{\text{min}}^2 H_g} \int_{n_{\text{min}}}^{n_{\text{max}}} \left( \frac{n}{n_{\text{min}}} \right)^{-\beta} dn \\
    \sim & \quad 5.2 \times 10^{-10} \epsilon_{0.1}^{-9/5} v_{20}^{-27/5} m_{10}^{9/5} \left( \frac{\Delta \dot{M}}{\dot{M}_{\text{Edd}}} \right)^{-9/5},
\end{align*}
\]

(35)

where \( n \) is the molecular hydrogen number density, \( n_{\text{min}} = 10^2 \text{cm}^{-3} \) and \( n_{\text{max}} = 10^5 \text{cm}^{-3} \) are the minimum and maximum number densities of typical molecular clouds, \( \beta \) is the index of power-law distribution, \( \langle \Sigma_{MC} \rangle \) is the mean surface mass density of molecular clouds, and \( H_g \) is the scale-height of the Galactic disk. Here, we adopted the values of \( \beta = 2.8 \) (Agol & Kamionkowski 2002), \( \langle \Sigma_{MC} \rangle = 29 M_\odot/\text{pc}^2 \), and \( H_g = 75 \text{pc} \) (Sanders et al. 1984). From the condition that \( n_{\text{max}} \) should not exceed \( n_{MC} \), we obtain the upper limit of \( \Delta \dot{M}/\dot{M}_{\text{Edd}} \) as

\[
\frac{\Delta \dot{M}}{\dot{M}_{\text{Edd}}} \lesssim 6.7 \times 10^{-2} \epsilon_{0.1} m_{10} v_{20}^{-3}.
\]

(36)

The merger rate of binary stellar-mass black holes due to the GW emission in our galaxy is given by \( f_{BH} = 2.56 \times 10^{-5} \text{yr}^{-1} \) by “Model A” of Table 4 of Belczynski et al. (2002). The expected number of merger events for the binary black holes within 3 Gpc for five year observation is then estimated as

\[
\begin{align*}
    N \sim & \quad \frac{4\pi}{3} n_{\text{gal}} f_{BH} f_{MC} \left( \frac{D_L}{1 + z} \right)^3 \frac{t_0 - t_{\text{ini}}}{1 + z} \\
    \sim & \quad 6.0 \times 10^{-2} j^{9/5} \epsilon_{0.1}^{-9/5} v_{20}^{-27/5} n_{gal,0.01} m_{10}^{117/115},
\end{align*}
\]

(37)

where \( n_{gal,0.01} \equiv n_{gal}/(0.01 \text{Mpc}^{-3}) \) with the number density of galaxies \( n_{gal} \) and the redshift \( z \) is set to 0.5 corresponding to \( D_L = 3 \text{Gpc} \). We also estimate \( f_{MC} \) in the above equation by substituting equation (31) into equation (35). Note that \( \Delta \dot{M}/\dot{M}_{\text{Edd}} \) satisfies the condition given by equation (36). We have not taken into account the redshift evolution of the
merging rate of the binaries per galaxy in the above analysis. When this effect is added, the number of events is expected to be a few times larger. We conclude that the feasibility of detection of circumbinary disks around binary stellar-mass black holes is marginal in five year observation with DECIGO/BBO. We emphasize that our analysis so far does not rely on specific models of the circumbinary disk structure.

4. Astrophysical implications of binary black holes with circumbinary disks

In this section, we further discuss some astrophysical implications of binary black holes with a circumbinary disk.

4.1. Evolution of binary black holes in the molecular cloud

The binary transfers its orbital energy and angular momentum to the circumbinary disk through the tidal/resonant interaction, which leads to the rapid orbital decay of the binary. If the binary evolution towards the coalescence is accelerated significantly during its stay in the molecular cloud, the merger rate of binary black holes would be increased in comparison with the previous estimates of Belczynski et al. (2002). In this subsection, we examine this possibility.

Let us consider a circular binary whose coalescence timescale due to the GW emission given by (Peters 1964)

\[
t_{gw} = \frac{5}{64} \left( \frac{a}{r_g} \right)^4 \frac{r_g}{c} \frac{1}{\eta}
\]

\[
\sim 1.0 \times 10^{-12} \text{ [yr]} \, m_{10} n_{0.25}^{-1} \left( \frac{a}{r_g} \right)^4
\]

(38)

exceeds the Hubble time. The binary-disk interaction will be one of the key mechanisms to force such binary to merge within the Hubble time. The orbital decay timescale due to
the tidal/resonant interaction corresponds to the inverse transfer rate of the orbital angular momentum, which is given by equation (13) as

\[ t_{\text{tide/res}} = \frac{1}{j \dot{M}} \sim 1.1 \times 10^7 \text{[yr]} j^{-1} \epsilon_0.1 \eta_{0.25} \dot{m}^{-1}. \]  

(39)

This is consistent with equation (17) of Hayasaki et al. (2010) when \( j = 1.0 \) is adopted.

Owing to this tidal/resonant interaction, the orbital evolution is accelerated even when \( t_{gw} > t_{\text{tide/res}} \) holds, i.e., in the range \( a_{\text{tide/res}} < a \leq a_{\text{BHL}} \) with

\[ \frac{a_{\text{tide/res}}}{r_g} = \left( \frac{64 c t_{\text{tide/res}}}{5 r_g \eta} \right)^{1/4} \sim 6.9 \times 10^4 j^{-1/4} \epsilon_0.1 \eta_{0.25} m_{10}^{-1/4} \dot{m}^{-1/4}, \]  

(40)

if the binary stays in the molecular cloud for a sufficiently long term.

Since the typical size of a giant molecular cloud is \( r_{MC} \sim 100 \text{pc} \), the crossing time of binary black holes in the molecular cloud is \( t_{\text{cross}} \sim 5 \times 10^6 \text{[yr]} r_{MC,100} (v_{\text{bulk}}/20 \text{km s}^{-1})^{-1} \), where \( r_{MC,100} \equiv r_{MC}/(100 \text{pc}) \). During this crossing time, the binary can keep interacting with the molecular cloud gas. The condition \( t_{\text{cross}} \geq t_{\text{tide/res}} \) leads to \( \dot{M} \geq \dot{M}_c \) with the critical accretion rate

\[ \dot{M}_c \sim 2.3 j^{-1} \epsilon_{20} \eta_{0.1} \dot{m}^{r_{MC,100}^{-1}} \dot{M}_{\text{Edd}}. \]  

(41)

If \( \dot{M} \geq \dot{M}_c \), binary black holes can merge within the Hubble time, even if the initial separation is too large to coalesce within the Hubble time by means of the radiation reaction due to the GW emission alone.

4.2. Inner-most disk mass

In this subsection, we evaluate the mass of the circumbinary disk with the aid of the assumption that the structure of the circumbinary disk is described by the standard
disk model (Shakura & Sunyaev 1973). The circumbinary disk is assumed to be a steady, axisymmetric, geometrically thin, gas-pressure and electron-scattering opacity dominated, Keplerian disk. Here, we use the following solution of the standard disk (Kato et al. 2008):

\[
\Sigma(r) \approx 4.1 \times 10^5 \text{[g cm}^2\text{]} \times \alpha_{\text{SS}}^{-4/5} \epsilon_{0.1}^{-3/5} m_1^{1/5} \dot{m}^{3/5} \left( \frac{r}{r_g} \right)^{-3/5},
\]

\[
H(r) \approx 1.6 \times 10^4 \text{[cm]} \times \alpha_{\text{SS}}^{-1/10} \epsilon_{0.1}^{-1/5} m_1^{9/10} \dot{m}^{1/5} \left( \frac{r}{r_g} \right)^{21/20},
\]

\[
c_s(r) \approx 1.7 \times 10^8 \text{[cm s}^{-1}\text{]} \times \alpha_{\text{SS}}^{-1/10} \epsilon_{0.1}^{-1/5} m_1^{-1/10} \dot{m}^{1/5} \left( \frac{r}{r_g} \right)^{-9/20},
\]

where \(\Sigma\), \(H\), \(c_s\), \(r_g \equiv GM/c^2\) are the surface density, the scale-height of the disk, the sound velocity measured at the mid-plane temperature of the disk, the gravitational radius, and the normalized mass accretion rate, respectively (recall that \(\alpha_{\text{SS}}\) is the Shakura-Sunyaev viscosity parameter).

As it is not the whole mass of the circumbinary disk but the mass at around the inner edge of the circumbinary disk that affects the GW phase evolution, we estimate

\[
M_{d,\text{in}} = \pi r_{\text{in}}^2 \Sigma(r_{\text{in}})
\]

and refer to it as an inner-most disk mass. It can be evaluated by substituting \(r_{\text{in}} = 2a\) into the expression of \(\Sigma\) given in equation (42).

Figure 2 shows the inner-most disk mass of the circumbinary disk that is detectable by GWs. They are evaluated at the beginning of the observation period of \(t_0 - t_{\text{ini}} = 1, 3,\) and \(5\) yr, respectively. We also adopt \(\alpha_{\text{SS}} = 0.1\). It shows that the existence of the circumbinary disk can be confirmed, even if the disk mass is less than the binary mass by many orders of magnitude. This is because we can observe many \((N_{\text{cyc}} \sim 10^6)\) cycles of GWs.
We can understand the binary mass dependence of $M_{d,\text{in}}/M$ in Figure 2 in the following way. The semi-major axis for a fixed value of $t_0 - t_{\text{ini}}$ is proportional to $M^{3/4}$ from equation (6) and the surface density at the inner edge of the circumbinary disk is proportional to $M^{1/5} M^{3/5} a^{-3/5}$ from equation (42). When $M$ is given in equation (27), we find

$$\frac{M_{d,\text{in}}}{M} \approx 2.2 \times 10^{-12} m_{10}^{49/40}$$  \hspace{1cm} (46)

in the mass range $6M_\odot \leq M \lesssim 3 \times 10^3 M_\odot$ for five year observation. This is roughly in agreement with the dotted curve in Figure 2.

### 4.3. Decoupling from the circumbinary disk

When $t_{\text{gw}}$ of the binary is shorter than the viscous timescale evaluated at the inner edge of the circumbinary disk, the binary is decoupled from the circumbinary disk. After the decoupling, no binary-disk interaction occurs so that it is impossible to detect the effect of the circumbinary disk by the GW observation.

The viscous timescale measured at the inner edge of the circumbinary disk is given by

$$t_{\text{vis}}(a) = \frac{r_{\text{in}}^2}{\nu} \approx t_{\text{vis},0} \left( \frac{a}{r_g} \right)^{7/5},$$  \hspace{1cm} (47)

where we used the relation for the disk viscosity $\nu = \alpha_{\text{SS}} c_s H$ and $t_{\text{vis},0} \equiv t_{\text{vis}}(r_g) \approx (1/\alpha_{\text{SS}})(c_r/c_s^2(r_g))$ with equations (13) and (14). The decoupling radius at which $t_{\text{gw}}(a) = t_{\text{vis}}(a)$ is given by

$$\frac{a_d}{r_g} = \left( \frac{64 c r_{\text{vis},0}}{5 r_g \eta} \right)^{5/13} \approx 1.1 \times 10^2 \alpha_{\text{SS}}^{-4/13} \eta_{0.25}^{5/13} m_{10}^{2/13} m^{-2/13}. \hspace{1cm} (48)$$

Even when $t_{\text{gw}}$ is shorter than $t_{\text{tide/res}}$, the binary continues to gravitationally interact with the circumbinary disk until the binary orbit decays down to $a_d$. In the mass range
of $6M_\odot \leq M \lesssim 2 \times 10^4 M_\odot$, the condition $a \gtrsim a_d$ is satisfied even at one year before the coalescence. Namely, the binary continues to gravitationally interact with the circumbinary disk, as we described, at least at the early stage of the GW phase evolution during the observation period. Thus, although we take no account of the decoupling effect in Figure 1, this neglect has been justified in the mass range $M \lesssim 10^3 M_\odot$. For the larger mass range, the determination accuracy is deteriorated by taking into account the decoupling effect. However, for $M > 1.4 \times 10^4 M_\odot$, DECIGO/BBO is completely insensitive to the circumbinary disk when the mass accretion rate is smaller than the Eddington rate.

The BHL accretion rate can exceed the Eddington rate depending on parameters: $\rho_\infty$, $v_\infty$, and $M$. X-ray observations have detected the bright X-ray sources, such as GRS 1915+105 in our galaxy, with a luminosity over the Eddington one (cf. Done et al. 2007). Such large luminosities can be explained by a supercritical accretion flow onto stellar-mass black holes. If the circumbinary disk is in a super-critical state, equation (48) does not apply. Since the accretion timescale is much shorter than the viscous timescale in the super-critical state, the binary will not be decoupled from the circumbinary disk until the coalescence, although it is still unclear how such a circumbinary disk evolves (Tanaka & Menou 2010).

5. Summary and discussion

We have derived the minimum mass accretion rate of a circumbinary disk around binary stellar/intermediate-mass black holes necessary for the detection by future space-borne GW detectors such as DECIGO/BBO. Our main conclusions are summarized as follows:

1. The circumbinary disk with $\dot{M} \sim 1.4 \times 10^{17} [\text{g s}^{-1}] j^{-1} (M/10M_\odot)^{33/23}$ with 10% mass-to-energy conversion efficiency, where $j$ represents the efficiency of the angular
momentum transfer defined by equation (13), within 3 Gpc from the earth is detectable at a statistically significant level with DECIGO/BBO by observing for five years before the coalescence. The determination accuracy of the mass accretion rate is higher at a closer distance from the earth, and is also higher for a longer observation time.

2. If binary stellar-mass black holes are residing in sufficiently dense molecular clouds, the influence of the circumbinary disk will be detectable. The number of merger events of such binaries within 3 Gpc for five year observation, is approximately estimated to be $\mathcal{O}(0.1)$ by assuming typical values for the event rate of the black hole coalescence and that molecular clouds have the number density from $10^{-5}$ cm$^{-3}$ to $n_{MC}$ given in equation (34). This suggests that there might be a possibility to detect the circumbinary disk around binary stellar-mass black holes with DECIGO/BBO.

In the estimate presented in this paper, we still have a lot of uncertainties, e.g., the formation rate of binary black holes, the molecular hydrogen number density, the relative velocity to the molecular cloud, the efficiency of angular momentum transfer ($j$: see equation (13)), and so on. Therefore, we should understand that there remains a possibility that the event rate is one or two orders of magnitude larger than our estimate.

For the idealized situation that we discussed in this paper, the correction to the GW phase evolution shows $-4\mathrm{PN}$ frequency dependence. This dependence is the same as the one for a braneworld model discussed in Yagi et al. (2011). Similarly, the cosmic acceleration (Seto et al., 2001) and the alternative theories of gravity in which the gravitational constant $G$ is not a constant (Yunes et al., 2010) also give the corrections with $-4\mathrm{PN}$ frequency dependence. Furthermore, the acceleration acting on a binary in time-independent external gravitational field gives the correction with the same frequency dependence (Yunes et al., 2011a). Therefore, once the deviation from the standard template
is discovered, discriminating the effect of the circumbinary disk from the others will be a difficult task. However, for example, the binary-disk interaction rapidly decays after the decoupling of the circumbinary disk as we discussed in section 4.3. This is a signature unique to the interaction with the circumbinary disk. In addition, there can be a method to distinguish the effect of the circumbinary disk from that of modified gravity theory by detecting the GW signals from multiple sources. This is because the effect of modified gravity theory is universal, whereas that of circumbinary disk is not. Therefore, further detailed studies may provide some ways of distinctions.

In this paper we have assumed that the binary is in a circular orbit. The orbital eccentricity also gives the frequency dependence of negative PN order (-19/6PN) (Cutler & Harms 2006). Inclusion of the orbital eccentricity as a parameter of the waveform reduces the determination accuracy of -4PN order term by some factors. However, we expect that we can determine both the orbital eccentricity and the mass accretion rate for the large mass parameter range with errors of $O(0.1)$ for typical binary parameters. Key & Cornish (2011) calculated the determination accuracies of full seventeen binary parameters including an initial orbital eccentricity for binary massive black holes with LISA, using a Bayesian analysis under Markov Chain Monte Carlo simulations. They conclude that the LISA could distinguish between circular orbit and eccentric orbit with a very small orbital eccentricity $e \sim 10^{-3}$.

Furthermore, we would like to point out that the effect of the circumbinary disk can be important, even if it is suppressed to the undetectable level for each event. The primary mission of the space-borne GW antennas such as DECIGO/BBO is to measure the primordial GW background radiation. In order to achieve this goal, abundant binary GW sources become the foreground noises. To remove them, the expected GW signals for the events identified by the matched filtering will be subtracted from the data. However, if the
theoretical templates contain some unknown elements, the subtraction leaves residuals that remain to contribute to confusion noises (Cutler & Harms 2006; Harms et al. 2008).

Finally, we briefly mention the relevance of our discussion to the ground-based GW detectors. With these detectors, the distance to which we can detect GWs from binary black holes is reduced, and hence the expected event rate is extremely small. Using the third-generation-ground-based GW detector: Einstein Telescope (ET), and assuming high signal-to-noise ratio corresponding to an accidentally nearby event at $D_L = 100$Mpc, the effect of the circumbinary disk will be detectable only for the mass accretion rate $10^4$ times higher than the Eddington rate. Even if stellar-mass binary black holes are at rest in a dense molecular cloud core, the expected BHL accretion rate is $10^{3-4}$ times as much as the Eddington rate. Hence, we would be able to safely conclude that the effect of the circumbinary disk becomes relevant only for space-borne GW detectors such as DECIGO/BBO.

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Fig. 1.— Determination accuracy $\Delta \dot{M}$ of mass accretion rates of the circumbinary disk around coalescing binary black holes within $D_L = 3\,\text{Gpc}$. Here, $j = 1$ is adopted (see equation (13) for the definition of $j$). The determination accuracy is normalized by the Eddington accretion rate with 10% mass-to-energy conversion efficiency. $M$ is the total mass of binary black holes in units of the solar mass. The solid, dashed and dotted curves correspond to the determination accuracies with the observation time, $t_0 - t_{\text{ini}} = 1, 3, \text{and } 5\,\text{yr}$, respectively. The dash-dotted line shows the Bondi-Hoyle-Lyttelton accretion rate normalized by the Eddington rate, where the BHL rate is given by equation (33). The mass accretion rate can be measured from GW observations at a statistically significant level if $\Delta \dot{M}$ is less than the fiducial accretion rate $\dot{M}$. The horizontal solid thin line represents $\Delta \dot{M}$.
Fig. 2.— The inner-most disk mass that gives detectable effect on the GW phase evolution is plotted as a function of the total mass of binary black holes. The vertical axis is normalized by the total mass of binary black holes, and the inner-most disk mass is defined by $M_{d,\text{in}} = \pi r_{\text{in}}^2 \Sigma(r_{\text{in}})$, where $r_{\text{in}}$ and $\Sigma$ are the inner edge radius of circumbinary disk and the surface density of the circumbinary disk evaluated at around $r_{\text{in}}$, respectively (see equation [45] in the text). The solid, dashed and dotted curves correspond to $M_{d,\text{in}}/M$ for $t_0 - t_{\text{ini}} = 1, 3,$ and $5$ yr, respectively.
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