Electron-capture Rates in $^{20}\text{Ne}$ for a Forbidden Transition to the Ground State of $^{20}\text{F}$ Relevant to the Final Evolution of High-density O–Ne–Mg Cores

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Abstract

Electron capture on $^{20}\text{Ne}$ is critically important for the final stage of evolution of stars with initial masses of 8–10 $M_\odot$. In the present paper, we evaluate electron-capture rates for a forbidden transition $^{20}\text{Ne} (0_{gs}^+) \rightarrow ^{20}\text{F} (2_{gs}^+)$ in stellar environments by the multipole expansion method with the use of shell-model Hamiltonians. These rates have not been accurately determined in theory as well as in experiments. Our newly evaluated rates are compared with those obtained by a prescription that treats the transition as an allowed Gamow–Teller transition with the strength determined from a recent $\beta$-decay experiment for $^{20}\text{F} (2_{gs}^+) \rightarrow ^{20}\text{Ne} (0_{gs}^+)$. We find that different electron energy dependence of the transition strengths between the two methods leads to sizable differences in the weak rates of the two methods. We also find that the Coulomb effects, that is, the effects of screening on ions and electrons are nonnegligible. We apply our electron-capture rates on $^{20}\text{Ne}$ to the calculation of the evolution of high-density O–Ne–Mg cores of 8–10 $M_\odot$ stars. We find that our new rates affect the abundance distribution and the central density at the final stage of evolution.

Key words: nuclear reactions, nucleosynthesis, abundances – stars: AGB and post-AGB

1. Introduction

The evolution and final fates of stars depend on their initial masses $M_1$ (e.g., Nomoto et al. 2013) and are also subject to some uncertainties involved in stellar mass-loss, mixing processes, and nuclear transition rates. A strongly electron-degenerate O–Ne–Mg core is formed after carbon burning in stars with $M_1 = 8–10 M_\odot$, which can end up in various ways, that is, as O–Ne–Mg white dwarfs, as electron-capture (e-capture) supernovae, or as Fe core-collapse supernovae (Miyaji et al. 1980; Nomoto 1984, 1987; Nomoto & Hashimoto 1988). The evolutionary changes in the central density and temperature of the degenerate O–Ne–Mg core are determined by the competition among the contraction, cooling, and heating processes.

Nuclear URCA processes, especially in nuclear pairs with $A = 23$ and 25, are found to be important for the cooling of the O–Ne–Mg cores after carbon burning (Jones et al. 2013; Toki et al. 2013; Schwab et al. 2017; Schwab & Rocha 2019). Electron-capture reactions and successive gamma emissions in nuclei with $A = 24$ and 20 are important for the contraction and heating of the core in later stages leading to an electron-capture supernova. The fate of the stars depends sensitively on the nuclear electron-capture and $\beta$-decay rates. Accurate evaluations of the weak rates for high densities and temperatures in fine steps are important for a proper treatment of cooling and heating processes (Suzuki et al. 2016).

The weak rates for nuclei with $A = 24$ and 20 are examined in detail in Martinez-Pinedo et al. (2014) by taking into account the forbidden transitions between $^{20}\text{Ne} (0_{gs}^+)$ and $^{20}\text{F} (2_{gs}^+)$. The forbidden transition was usually not taken into account to obtain the weak rates (Takahara et al. 1989). The forbidden transitions are found to give nonnegligible contributions at $\log_{10} T < 9.0$ in a density region; $9.3 < \log_{10} (\rho Y_e) < 9.6$. Here $Y_e$ is the proton fraction, namely, the lepton-to-baryon ratio. The forbidden transitions, however, were treated as if they were allowed Gamow–Teller (GT) transitions, and the $B(GT)$ value was taken to be the largest one corresponding to the lower limit of the mean value of the $\beta$-decay: $ft = 6147/B(GT)$. The experimental transition rate for the $\beta$-decay was not well determined: a lower limit of $\log(ft) > 10.5$ is given in NNDC. Recently, a new measurement on the $\beta$-decay has been carried out, and the transition rate is determined to be $\log ft = 10.47 \pm 0.11$ (Kirsebom et al. 2018). The mean value is very close to the lower limit value of $\log ft = 10.5$ (see footnote 5), and the difference is only by 7%. However, in general for forbidden $\beta$-decays, shape factors are energy-dependent and the prescription to use constant shape factors as in allowed transitions is an approximation.

Here we treat the forbidden transitions between $^{20}\text{Ne} (0_{gs}^+)$ and $^{20}\text{F} (2_{gs}^+)$ properly and evaluate the weak rates by using the multipole expansion method (Walecka 1975). We compare the rates with those obtained by the prescription of using a constant $B(GT)$ value assuming the transitions as allowed ones. We also investigate the effects of the screening effects on the rates. The aim of the present paper is to point out the difference in the transition strengths and weak rates between the multipole expansion method and the prescription assuming allowed GT transitions. We discuss origins and reasons that cause the differences.

In Section 2, we discuss e-capture rates of the forbidden transition on $^{20}\text{Ne}$. We also discuss $\beta$-decay rates of the forbidden transition from $^{20}\text{F}(2_{gs}^+)$. In Section 3, the dependence of the evolution of the O–Ne–Mg core on the e-capture rates is investigated in the later heating stages. Summary is given in Section 4.

$^5$ National Nuclear Data Center online retrieval system, http://www.nndc.bnl.gov.
2. Electron-capture Rates on $^{20}$Ne

We discuss e-capture rates for the forbidden transition, $^{20}$Ne ($0_{1/2}^+ \rightarrow ^{20}$F ($2^+_2$)). Formulae for the e-capture rate for finite density and temperature are given as (O’Connell et al. 1972; Walecka 1975; Parr et al. 2009; Fanitina et al. 2012)

$$\lambda^\text{cap}(T) = \frac{V_{\text{sd}}^2 q^5 c}{\pi^2 (hc)^3} \int_{E_{\text{th}}}^{\infty} \sigma(E, T) E dE$$

$$\sigma(E, T) = \sum_i \frac{(2J_i + 1) e^{-E_i/kT}}{G(Z, A, T)} f(E_i)$$

$$G(Z, A, T) = \sum_i (2J_i + 1) e^{-E_i/kT},$$ (1)

where $V_{\text{sd}} = \cos \theta_C$ is the up-down element in the Cabibbo–Kobayashi–Maskawa quark mixing matrix with $\theta_C$ as the Cabibbo angle, $g_V = 1$ is the weak vector coupling constant, $E_e$ and $p_e$ are electron energy and momentum, respectively, $E_{\text{th}}$ is the threshold energy for the electron capture, and $f(E_i)$ is the Fermi–Dirac distribution for electron. The electron chemical potential is determined from $\rho Y_e$ with $\rho$ the baryon density and $Y_e$ the proton fraction. Here, $i$ denotes the initial state with excitation energy $E_i$ and angular momentum $J_i$, and $f$ specifies the final state. The cross section $\sigma_{f,i}(E_e)$ from an initial state with $E_i$ and $J_i$ to a final state with excitation energy $E_f$ and angular momentum $J_f$ is evaluated with the multipole expansion method (O’Connell et al. 1972; Walecka 1975):

$$\sigma_{f,i}(E_e) = \int \left( \frac{d\sigma}{d\Omega} \right)_{f,i} d\Omega$$

$$\left( \frac{d\sigma}{d\Omega} \right)_{f,i} = \frac{G_F^2 F(Z, E_e)}{2\pi (2J_f + 1)} \sum_{J_{\text{f}}} W(E_{\text{f}}) \{ (1 - \nu \cdot \hat{q})(\beta \cdot \hat{q}) \}$$

$$\times [ ] |J_f||T_j^\text{mag}||L_j|^2 + |J_f||T_j^\text{elec}||L_j|^2$$

$$- 2\hat{q} \cdot (\nu - \beta)\text{Re} \{ J_f||T_j^\text{mag}||L_j||T_j^\text{elec}||L_j^* \}$$

$$+ \sum_{J_{\text{f}} \neq 0} W(E_{\text{f}}) \{ (1 - \nu \cdot \beta + 2(\nu \cdot \hat{q})(\beta \cdot \hat{q})\} |J_f||L_f||L_f|^2$$

$$+ (1 + \nu \cdot \beta)|J_f||M_f||L_f|^2$$

$$- 2\hat{q} \cdot (\nu + \beta)\text{Re} \{ J_f||L_f||L_f||M_f||L_f^* \},$$ (2)

where $q = \nu - k$ is the momentum transfer with $\nu$ and $k$ the neutrino and electron momentum, respectively, $\hat{q}$ and $\nu$ are the corresponding unit vectors, and $\beta = k/E_e$. $G_F$ is the Fermi coupling constant, $F(Z, E_e)$ is the Fermi function, and $W(E_{\text{f}})$ is the neutrino phase space given by

$$W(E_{\text{f}}) = \frac{E_{\text{f}}^2}{1 + E_{\text{f}}/M_F},$$ (3)

where $E_{\text{f}} = E_e - Q + E_e - E_f$ is the neutrino energy and $M_F$ is the target mass. The $Q$ value is determined from $Q = M_i - M_f$, where $M_i$ and $M_f$ are the masses of parent and daughter nuclei, respectively. The Coulomb, longitudinal, transverse magnetic, and electric multipole operators with multipolarity $J$ are denoted as $M_J$, $L_J$, $T_J^\text{mag}$, and $T_J^\text{elec}$, respectively.

For a $0^+ \rightarrow 2^+$ transition, the transition matrices for Coulomb, longitudinal, and electric transverse operators from a weak vector current as well as an axial magnetic operator from weak axial-vector current with multipolarity $J = 2$ contribute to the rates:

$$M_2(q) + L_2(q) = F_1^V (q^2) \frac{q^2}{q^2} j_2(qr) Y^2$$

$$T_2^\text{elec}(q) = \frac{q}{M} F_1^V (q^2) \left\{ \frac{3}{5} j_1(qr) \left[ Y^3 \times \nabla \right]^2 \right\}$$

$$- \frac{2}{5} j_1(qr) \left[ Y^3 \times \left( \frac{\nabla}{q} \right)^2 \right]$$

$$+ \frac{1}{2} \mu \nu (q^2) j_2(qr) Y^2 \times \sigma^2,$$ (4)

where $F_1^V$, $\mu$, and $F_A$ are the nucleon vector (Dirac), magnetic, and axial-vector form factors, respectively (Kuramoto et al. 1990).

Here, we evaluate the electron-capture rates for the forbidden transition, $^{20}$Ne ($0_{1/2}^+ \rightarrow ^{20}$F ($2^+_2$)), with the USDB shell-model Hamiltonian (Brown & Richter 2006) within the sd shell as well as the YSOX Hamiltonian (Yuan et al. 2012). The YSOX Hamiltonian, designed to be used in $p - sd$ shell configuration space, can reproduce well the ground state energies and energy levels, electric quadrupole properties, and spin properties of boron, carbon, nitrogen, and oxygen isotopes. Calculated e-capture rates for the forbidden transition obtained with the USDB and YSOX Hamiltonians are shown in Figure 1 for $\log_{10}(T(K)) = 8.6$. Here, the quenching factors for the axial-vector coupling constant $g_A$ are taken to be $q = 0.764$ (Richter et al. 2008) and $q = 0.85$ (Yuan et al. 2012) for the USDB and

![Figure 1. Calculated e-capture rates for $^{20}$Ne ($e^-, \nu_e$) $^{20}$F ($2^+_2$) at $T = 10^{8.6}$ (K) obtained with the shell-model USDB (Brown & Richter 2006) and YSOX (Yuan et al. 2012) Hamiltonians as well as the GT the prescription that treats the transition as a GT one with $b(\text{GT}) = 1.04 \times 10^{–6}$ determined from the inverse $\beta$-decay rate of $log_{10}(T) = 10.47$ (Kisboson et al. 2018).](image-url)
YSOX, respectively. Harmonic oscillator wave functions with a size parameter $b = 1.85$ fm are used. Calculated rates obtained as an allowed transition with a $B(GT)$ value corresponding to $\log_{10} \beta = 10.47$ (Kirsebom et al. 2018), that is, $B(GT) = 1.04 \times 10^{-6}$, are also shown in Figure 1. We refer to this method as “GT prescription” hereafter. A sizable difference is found between the two methods. The rates obtained by the GT prescription are found to be enhanced (reduced) compared with those with the USDB and YSOX at $\log_{10}(\rho Y_e) < (>) 9.9$.

These tendencies are due to the difference in the electron energy dependence of the reaction cross section $\sigma(E_e)$ between the two methods. $\sigma(E_e)$ for the shell-model calculations with USDB and YSOX as well as for the prescription of using the constant $B(GT)$ value are shown in Figure 2(a). The cross section for the constant $B(GT)$ is proportional to the neutrino phase-space factor $W(E_e)$, that is, it increases nearly proportional to $E_e^2$ as the electron energy $E_e$ increases. Different electron energy dependence of the cross sections is found for the shell-model results: the cross sections are reduced (enhanced) at $E_e < (>) 9.9$ MeV compared with the $B(GT)$ prescription. In the case of the shell-model calculations, the contributions from the axial magnetic and transverse electric terms are dominant at low $E_e$ regions, while those from the Coulomb and longitudinal terms increase as $E_e$ increases: the latter contributions become 26.9% (14.5%), 48.0% (30.1%), 55.7% (37.1%), and 59.1% (41.3%) of the total ones at $E_e = 9, 11, 13, \text{and} \ 15 \text{MeV, respectively, for USDB (YSOX), and they finally reach almost constant fractions of 62% (44%) at } E_e \geq 20 \text{MeV.}$

The Fermi distribution of electron multiplied by the electron phase-space factor $p_\nu E_f(E_e)$ is shown in Figure 2(b) for the densities $\log_{10}(\rho Y_e)$ (g cm$^{-3}$) = 9.0–10.0 in steps of 0.2. As the density increases, the chemical potential of electron increases and the region of $E_e$ that can contribute to the e-capture rates increases. At $\log_{10}(\rho Y_e) < 9.8$, where the electron chemical potential is below 10 MeV, the shell-model cross sections are smaller than the $B(GT)$ one, and the shell-model rates also remain smaller than the $B(GT)$ one. At $\log_{10}(\rho Y_e) \geq 9.9$, the electron energy larger than 10 MeV can contribute to the rates, and the shell-model rates begin to exceed the $B(GT)$ rate.

Next, we study the effects of the Coulomb effects. Screening effects on both electrons and ions are taken into account for the Coulomb effects (Juodagalvis et al. 2010; Toki et al. 2013; Suzuki et al. 2016). The screening effects of electrons are evaluated by using the dielectric function obtained by relativistic random phase approximation (Itoh et al. 2002). The effect is included by reducing the chemical potential of electrons by an amount equal to the modification of the Coulomb potential at the origin, $V_i(0)$ (Juodagalvis et al. 2010), where

$$V_i(r) = Ze^2(2k_F)J(r)$$

$$J(r) = \frac{1}{2k_F r} \left[ 1 - \frac{2}{\pi} \int \frac{\sin(2k_F qr)}{q^2 e(q, 0)} dq \right].$$

The screening coefficient $J$ is tabulated in Itoh et al. (2002).

The other Coulomb effect is the change of the threshold energy

$$\Delta Q_C = \mu_c(Z - 1) - \mu_c(Z),$$

where $\mu_c(Z)$ is the Coulomb chemical potential of the nucleus with charge number $Z$ due to the interactions of the ion with other ions in the electron background (Slattery et al. 1982; Ichimaru 1993). The Coulomb chemical potential in a plasma of electron number $n_e$ and temperature $T$ is given by

$$\mu_c(Z) = kT f(\Gamma),$$

where $f(\Gamma)$ is the Fermi-Dirac distribution function.
with \( \Gamma = Z^{5/3} \Gamma_e, \Gamma_e = \frac{e^2}{kT_e}, \) and \( a_e = \left( \frac{3}{4kT_e} \right)^{1/3}. \) The function \( f_1 \) for the strong-coupling regime, \( \Gamma > 1, \) is given by Equation (A.48) in Ichimaru (1993), while for the weak-coupling regime an analytic function given by Equation (A.6) in Juodagalvis et al. (2010) is used for \( \Gamma < 1 \) (Yakovlev & Shalybkov 1989). The threshold energy gets larger for e-capture processes. The e-capture (\( \beta \)-decay) rates are thus reduced (enhanced) by both the Coulomb effects.

Calculated results for the e-capture rates for the USDB Hamiltonian with and without the Coulomb effects are shown in Figure 3(a). The Coulomb effects shift the e-capture rates toward a higher-density region due to an increase of the \( Q \) value. The e-capture rates with the Coulomb effects for the USDB, YSOX Hamiltonians as well as the GT prescription are shown in Figure 3(b). The shell-model rates are reduced compared with the GT one at \( 9.6 < \log_{10}(\rho Y_e) < 9.9. \) The difference of the rates of the two methods is at most about by 5 times at \( \log_{10}(\rho Y_e) = 9.6. \)

Total e-capture rates on \( ^{20}\text{Ne} \) for the USDB and the GT prescription for the forbidden transition are shown in Figure 4 for the case with the Coulomb effects. Contributions from Gamow–Teller transitions from \( 0^+_{\text{scr}} \) and \( 2^+ \) states in \( ^{20}\text{Ne} \) to \( 1^+, 2^+ \) and \( 3^+ \) states in \( ^{20}\text{F} \) evaluated with the USDB are included as well as the forbidden transition, \( 0^+_{\text{scr}} \longrightarrow 2^+_{\text{ex}}. \) The difference of the two methods is at most about by 3 (3–4) times at \( \log_{10}(\rho Y_e) = 9.5–9.7 \) for \( T = 10^{8.6} \) (\( 10^{8.5} \)) (K). In case of \( T = 10^{9.0} \) (K), the difference disappears. As the calculated rates for USDB and YSOX are similar, we show results for the USDB only hereafter.

Now we discuss \( \beta \)-decay rates for the forbidden transition, \( ^{20}\text{F} (2^+_{\text{ex}}) \longrightarrow ^{20}\text{Ne} (0^+_{\text{scr}}). \) The \( \beta \)-decay rate for finite density and temperature is given as (O’Connell et al. 1972; Walecka 1975)

\[
\lambda^\beta(T) = \frac{2V_{\text{eff}}^2 c}{\pi^2 (\hbar c)^3} \int_{m_e^2}^{Q} S(E_e, T)E_e \rho c \times (Q - E_e)^2 (1 - f(E_e))dE_e
\]

\[
S(E_e, T) = \sum_i \frac{(2J_i + 1)e^{-E_i/\kappa T}}{G(Z, A, T)} \sum_f S_{f,i}(E_e)
\]

\[
S_{f,i}(E_e) = \int \frac{1}{4\pi d\Omega_e} \int d\Omega_e \frac{G_f^2}{2\pi} F(Z + 1, E_e) \times \left( \sum_{J_i \geq 1} \left( (1 - (\hat{\nu} \cdot \hat{q})(\beta \cdot \hat{q})) ||(J_f||T^{\text{mag}}||J_i)||^2 + ||(J_f||T^{\text{elec}}||J_i)||^2 \right)^2 \right.
\]

\[
+ \left. \left| \langle J_f||T^{\text{elec}}||J_i \rangle \right|^2 \right) + \left( \hat{\nu} \cdot \hat{q} \right) \left( \hat{q} \cdot \beta \right) \left( \hat{q} \cdot \beta \right) \right)
\]

\[
- 2 \hat{q} \cdot (\hat{\nu} + \beta) Re \left( \langle J_f||L_i||J_i \rangle \langle J_f||M_i||J_i \rangle \right)
\]

where \( q = k + \nu, \) and the factor \( 1 - f(E_e) \) denotes the blocking of the decay by electrons in high-density matter.

The log \( ft \) value for a \( \beta \)-decay transition is given as (Oda et al. 1994; Langanke & Martinez-Pinedo 2001)

\[
ft = \ln 2 \frac{I}{\lambda^\beta}
\]

\[
I = \int_{m_e^2}^{Q} E_e \rho c (Q - E_e)^2 F(Z + 1, E_e)(1 - f(E_e))dE_e
\]
Woods with USDB and GT prescription. The strength for USDB obtained with (Equation 8) is also shown. The Astrophysical Journal, 2018 December 27 Suzuki et al.

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Here, \( \lambda^\beta \) is the \( \beta \)-decay rate for the transition, and \( I \) is the phase-space integral. In the case of \( \beta \)-decay in the vacuum at \( T = 0 \) or in low-density matter at low temperature, the term \((1 - f(E_e))\) can be replaced by 1.

The transition strengths multiplied by phase-space factors are compared in Figure 5 for USDB and the GT prescription. A small dent seen around \( E_e = \frac{1}{2}Q \) for the GT prescription arises from the lepton kinematical factor for the transverse multipoles, \[ f_T = 1 - (\hat{u} \cdot \hat{q})(\beta \cdot \hat{q}). \] It can be expressed as \[ f_T = (1 - \beta^2 \cos^2(\theta/2)) + (1 + \beta^2 \sin^2(\theta/2) \cos^2(\beta \cdot \hat{q})) \]. Here, \( \omega = k + \nu (k = |k|) \) and \( \nu = |\nu| \) is the energy transfer, and \( \beta = |\beta| = k/E_e \). An integral \[ f = \int_0^\infty f_T 2\pi \sin \vartheta d\vartheta \] is a function of \( E_e \) with a minimum at \( E_e = Q/2 \) and maxima at \( E_e = m_e \) and \( Q \), where \( m_e \) is the electron mass, leading to a divot at \( E_e = \frac{1}{2}Q \).

However, this behavior of the strength little affects the \( \beta \)-decay transition rate. The difference in the decay rate (log \( f_T \) value) from that obtained by a standard formula for allowed \( \beta \)-decay, where \( f \) is taken to be a constant, is as small as 15% (0.06).

The strength of USDB is reduced compared with the GT prescription in the whole energy region. The summed strength is also small for the shell-model case, and leads to a log \( f_T \) value for USDB larger by 0.65 compared with the GT prescription, that is, log \( f_T \) = 11.18. The branching ratio is obtained to be 2.15 \( \times \) 10\(^{-6} \), which is 19.5% of the observed value (Kirsebom et al. 2018). Sensitivity to radial behavior of wave functions is examined by using Woods–Saxon wave functions obtained with standard parameters of Bohr & Mottelson (1969). The difference from the harmonic oscillator case is rather small except at \( E_e < 2 \) MeV as shown in Figure 5. The summed strength is increased only by 5%. We also give calculated results for YSOX; log \( f_T \) = 11.24 and the branching ratio is 1.87 \( \times \) 10\(^{-6} \). The difference of the strengths between the two methods can be ascribed to that in the energy dependence of the strengths, which prove to be important in explaining the difference in the e-capture rates.

3. Evolution of High-density O–Ne–Mg Cores

Now, we study the effects of the forbidden transition in the electron-capture processes on \( ^{20}\text{Ne} \) on the evolution of the high-density electron-degenerate O–Ne–Mg cores. Heating of
the core due to $\gamma$ emissions succeeding the double e-capture reactions, $^{20}\text{Ne} (e^-, \nu_e) ^{20}\text{F} (e^-, \nu_e) ^{20}\text{O}$, is important in the final stage of the evolution of the core. Four transition rates obtained with (1) USDB with the Coulomb effects, (2) the GT prescription with the Coulomb effects, (3) USDB without the Coulomb effects, and (4) the GT prescription without the Coulomb effects are used for the forbidden transition, $^{20}\text{Ne} (0^+_{\text{g.s.}}) \rightarrow ^{20}\text{F} (2^+_{\text{g.s.}})$, and the sensitivities of the evolution of the core on the rates are investigated.

We calculate the growth of an O–Ne–Mg core toward the Chandrasekhar mass limit using Modules for Experiments in Stellar Astrophysics (MESA; Paxton et al. 2011, 2013, 2015), revision 8118. The O–Ne–Mg core is prepared by evolving an 8.4 $M_\odot$ nonrotating solar-metallicity star as in Jones et al. (2013), removing its envelope right before thermal pulses of He shell burning start. We increase the O–Ne–Mg core mass at a constant rate of $10^{-6} M_\odot$ yr$^{-1}$, similar to Schwab & Rocha (2019). URCA and electron-capture processes are included using the rates in Suzuki et al. (2015). For convective stability, the Ledoux criterion is adopted. We switch off the mixing-length treatment for convection by the control \texttt{mlt\_option = ‘none’} when electron capture on $^{24}\text{Na}$ starts, as done in Schwab et al. (2017), to avoid numerical nonconvergence. The evolution with the Schwarzschild criterion needs more computational efforts to follow, and the results will be shown elsewhere (Zha et al. 2019).

In Figure 6(a), we show the temperature distribution as a function of the enclosed mass $M_*$ for the four different rates (1)–(4) defined above. The evolution of the central density and temperature of the O–Ne–Mg core is shown in Figure 6(b) for the four rates. In Figure 6(a), the temperature inversion appears in the central region because of the following reason. The electron-capture rate on $^{20}\text{Ne}$ with the second forbidden transition is not high enough to cause a rapid heating of the center, and then the central region is cooled down by the $^{25}\text{Na}$$^{25}\text{Ne}$ Urca shell cooling around $\log \rho_c = 9.85$–9.9 as seen in Figure 6(b). At $\log \rho_c > 9.9$, the temperatures in these layers increase due to e-capture on $^{20}\text{Ne}$ as well as core contraction. The heating effect of e-capture on $^{20}\text{Ne}$ is slightly higher in the outer layers because the outer layer contains the larger mass fraction of $^{20}\text{Ne}$ than the inner layer when their densities reach around $\log \rho = 9.9$ during the contraction. This leads to the formation of the temperature inversion. Eventually oxygen is ignited in the outer shell as seen in Figure 6(a). The off-center oxygen ignition occurs at $6.9 \times 10^{-3} M_\odot$ (33 km), $2.3 \times 10^{-2} M_\odot$ (50 km), $2.3 \times 10^{-1} M_\odot$ (23 km), and $1.2 \times 10^{-2} M_\odot$ (88 km) for the four different rates, respectively. Here the ignition occurs when the nuclear energy generation rate exceeds the thermal neutrino loss. The heating due to oxygen ignition forms a convectively unstable region even for the Ledoux criterion; the resulting convective energy transport will slow down the increase in the temperature due to oxygen burning.

Further contraction of the core to the higher central density will continue before the thermonuclear runaway. The final central density and position of oxygen ignition are important for the subsequent hydrodynamical behavior and flame propagation, and thus the fate of the O–Ne–Mg core (Jones et al. 2016; Leung et al. 2019; Takahashi et al. 2019). The ignition closer to the center for the USDB rates with Coulomb effects may favor neutron star formation as the final outcome (Nomoto & Leung 2017; Leung & Nomoto 2019). The detailed study of the complete evolution with these new rates, as well as how the theoretical uncertainties affect the evolutionary path, will be reported elsewhere (Zha et al. 2019).

4. Summary

We evaluated e-capture rates for the forbidden transition, $^{20}\text{Ne} (0^+_{\text{g.s.}}) \rightarrow ^{20}\text{F} (2^+_{\text{g.s.}})$, by the multipole expansion method of 

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**Figure 6.** (a) The temperature profiles as a function of the enclosed mass ($M_*$) at the moment of the oxygen ignition for the four different rates described in the text. Rate (1) USDB with the Coulomb effects, (2) the GT prescription with the Coulomb effects, (3) USDB without the Coulomb effects, and (4) the GT prescription without the Coulomb effects. (b) The evolution of central temperature ($T_*$) and density ($\rho_c$) of an O–Ne–Mg core, starting from the end of $^{25}\text{Mg}(e^-, \nu_e)^{25}\text{Na}(e^-, \nu_e)^{25}\text{Ne}$ up to the ignition of oxygen.
O'Connell et al. (1972) and Walecka (1975). The Coulomb, longitudinal, transverse electric, and axial magnetic multipoles with $J^\pi = 2^+$ contribute to the transitions. The e-capture rates at stellar environments obtained by the multipole method with the USDB and YSOX Hamiltonians are compared with the GT prescription that treats the transitions as allowed Gamow–Teller transitions with the $B(GT)$ value determined from the $\beta$-decay $ft$ value. Sizable differences are found between the rates obtained by the two methods for both cases with and without the Coulomb effects. Origin and density dependence of these differences in the rates are shown to be explained by the difference in the electron energy dependence of the transition strengths.

The four e-capture rates on $^{20}$Ne obtained by the multipole method with the USDB and the GT prescription, with and without the Coulomb effects, are used to study the evolution of the high-density O–Ne–Mg cores. Sizable sensitivity of the heating process in the O–Ne–Mg core on the e-capture rates is found in the final stage of the evolution of the core. It is thus important to evaluate the weak rates of forbidden transitions properly with the multipole expansion method, which gives rise to the energy dependence of the transition strengths.

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**References**

Bohr, A., & Mottelson, B. R. 1969, Nuclear Structure (New York: Benjamin)
Brown, B. A., & Richter, W. A. 2006, PhRvC, 74, 034315
Fanitina, A. F., Khan, E., Colo, G., Paar, N., & Vietenar, D. 2012, PhRvC, 86, 036805
Ichimaru, S. 1993, RvMP, 65, 255
Itoh, N., Tomizawa, N., Tamamura, M., & Wanajo, S. 2002, ApJ, 579, 380
Jones, S., Hirschi, R., Nomoto, K., et al. 2013, ApJ, 772, 150
Jones, S., Röpke, F. K., Pakmor, R., et al. 2016, A&A, 593, A72
Juodagalvis, A., Langanke, K., Hix, W. R., Martinez-Pinedo, G., & Sampaio, J. M. 2010, NuPhA, 848, 454
Kirsebom, O. S., Hukkanen, M., Kankainen, A., et al. 2018, arXiv:1805.08149
Kuramoto, T., Fukugita, M., Kohyama, Y., & Kubodrra, K. 1990, NuPhA, 512, 711
Langanke, K., & Martinez-Pinedo, G. 2001, ADNDT, 79, 1
Leung, S.-C., & Nomoto, K. 2019, PASA, 36, e006
Leung, S.-C., Nomoto, K., & Suzuki, T. 2019, arXiv:1901.11438
Martinez-Pinedo, G., Lam, Y. H., Langanke, K., Zegers, R. G., & Sullivan, C. 2014, PhRvC, 89, 045806
Miyaji, S., Nomoto, K., Yokoi, K., & Sugimoto, D. 1980, PASJ, 32, 303
Nomoto, K. 1984, ApJ, 277, 791
Nomoto, K. 1987, ApJ, 322, 206
Nomoto, K., & Hashimoto, M. 1988, PhR, 163, 13
Nomoto, K., Kobayashi, C., & Tomonaga, N. 2013, ARA&A, 51, 457
Nomoto, K., & Leung, S.-C. 2017, in Handbook of Supernovae, Vol. 1, ed. A. W. Alsabti & P. Murdin (Berlin: Springer), 483
O’Connell, J. S., Donnelly, T. W., & Walecka, J. D. 1972, PhRvC, 6, 719
Oda, T., Hino, M., Muto, K., Takahara, M., & Sato, K. 1994, ADNDT, 56, 231
Parr, N., Colo, G., Khan, E., & Vietenar, D. 2009, PhRvC, 80, 055801
Paxton, B., Bildsten, L., Dotter, A., et al. 2011, ApJS, 192, 3
Paxton, B., Cantiello, M., Arras, P., et al. 2013, ApJS, 208, 4
Paxton, B., Marchant, P., Schwab, J., et al. 2015, ApJS, 220, 15
Richter, W. A., Mkhdze, S., & Brown, B. A. 2008, PhRvC, 78, 064302
Schwab, J., Bildsten, L., & Quataert, E. 2017, MNRAS, 472, 3390
Schwab, J., & Rocha, K. A. 2019, ApJ, 872, 131
Slattery, W. L., Doolen, G. D., & DeWitt, H. E. 1981, PhRvD, 26, 2255
Suzuki, T., Toki, H., & Nomoto, K. 2015, ArXiv:1509.04724
Suzuki, T., Toki, H., & Nomoto, K. 2016, ApJ, 817, 163
Takahara, M., Hino, M., Oda, T., et al. 1989, NuPhA, 504, 167
Takahashi, K., Sumiyoshi, K., Yamada, S., et al. 2019, ApJ, 871, 153
Toki, H., Suzuki, T., Nomoto, K., Jones, S., & Hirschi, R. 2013, PhRvC, 88, 015806
Walecka, J. D. 1975, in Muon Physics, ed. V. M. Hughes & C. S. Wu (New York: Academic), 113
Yakovlev, D. G., & Shalybkov, D. A. 1989, SSRvE, 7, 311
Yuan, C., Suzuki, T., Otsuka, T., Xu, F., & Tsunoda, N. 2012, PhRvC, 85, 064324
Zha, S., Leung, S.-C., Suzuki, T., & Nomoto, K. 2019, arXiv:1907.04184