ON GLUON MASSES, COLOUR SYMMETRY, AND A POSSIBLE MASSIVE NINTH GLUON

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Abstract Gluons acquire a determinate mass from quark loop radiative corrections and self-interaction, when these are recomputed with the symmetrical theory of generalised functions. The colour symmetry may be $U(3)$ instead of $SU(3)$. The symmetry breaks spontaneously and the ninth colour $SU(3)$ singlet gluon acquires a very large mass. There may be some experimental support for its existence.

1. Introduction

A new theory of generalised functions has been constructed [1,2]. The available simple model allows the multiplication of all generalised functions needed in quantum field theory. Standard concepts of analysis, such as limit, derivative, and integral, have to be extended to make multiplication of generalised functions possible.

Integration of symmetrical generalised functions between arbitrary limits is always possible and yields well-defined finite results. Infinity of integrals is replaced by the less restricted concept of determinacy, which is related to the scale transformation properties of the integrand. In contrast to all regularization schemes the results are not arbitrary by finite renormalizations.

Conversely, all results in quantum field theory that depend on the use of any particular method of regularization are invalid by the standard of the symmetrical theory of generalised functions. In particular, every result that is dependent on the use of dimensional regularization to obtain it, disagrees with the corresponding generalised function result [3]. This is not merely a mathematical subtlety, it has physical consequences for observable quantities [4,5].

Renormalizability is no longer a relevant criterion for quantum field theories, and gauge invariance may be broken dynamically.

The starting point is the strong sector of the usual standard model Lagrangian. The $SU(2)$ symmetry is broken by hand, by giving the top quark a non-zero mass, leaving the bottom quark (effectively) massless. Note: It is inevitable that one fermion mass is introduced to set the mass scale. Gauge boson masses can result from radiative corrections, conversely fermion masses cannot be generated by interaction with massive gauge bosons.

Since only the mass terms have to be computed it is not necessary to consider ghosts. We can compute conveniently in the unitary gauge, which is free of unphysical fields. The calculations are very similar to the corresponding calculations [5] in the electro-weak sector, so only an outline is given.
2. Quark contributions

The fermionic contribution to the gluon self-energy is found from the fundamental two gluon-quark vertex and the corresponding loop diagram, which is needed for the present purpose only at gluon momentum $k = 0$. The gluons have pure vector coupling, so we obtain the usual \[^{2}\text{photon mass integral},\]

\[
g \cdot \bar{t} \cdot g = \Pi_{\mu\nu}(0) = -3\frac{g_s^2}{4} \left\{ \frac{\lambda^\alpha \lambda^\beta}{(2\pi)^4} \int\! \frac{d^4 p}{(2\pi)^4} \frac{\gamma_\mu (\not{p} + m_t) \gamma_\nu (\not{p} + m_t)}{(p^2 - m_t^2)^2} \right\},
\]

multiplied by an additional factor three for summing over the quark colours. After contracting with $g_{\mu\nu}$ and evaluating the trace, both over the $SU(3)$ generators and $\gamma$-matrices, one obtains

\[
\Pi^{\mu}_{\mu}(0) = \frac{3\alpha_s}{\pi^2} \int\! d^4 p \left\{ \frac{p^2 - 2m_t^2}{(p^2 - m_t^2)^2} \right\},
\]

with $\alpha_s = g_s^2 / 4\pi$ as usual. This corresponds to the photon mass integral found before \[^{5}\]. The imaginary part of the integral equals $i\pi^2$, so

\[
\Delta m_g^2 = \frac{1}{4i} \Pi^{\mu}_{\mu}(0) = \frac{3\alpha_s}{4\pi} m_t^2.
\]

The result can not be interpreted as a quantitative prediction, since it is unclear what value of the strong coupling constant one should use in an integral over all energies. This point is discussed below.

The gluons acquire a large mass by interacting with the top quark. Consequently they now also have a self-interaction contributing to their mass.

3. Gluon self-interaction

The cubic and quartic terms in the free field part of the Lagrangian give rise to gluon self-interaction. As long as the gluons are massless these terms cannot contribute a mass term by self-interaction. As soon as the gluons have acquired a mass (by any mechanism) there will be self-mass corrections from the mass term in the gluon propagator.

There are two Feynman diagrams contributing to the self-energy, the loop diagram obtained from the three-gluon vertex, and the bubble diagram obtained from the four-gluon vertex.

In the loop diagram the $6^{th}$ degree terms involving $p^4 p_\mu p_\nu$ cancel, leaving upon contraction with $g^{\mu\nu}$ the terms

\[
g \cdot g \cdot g = \Pi_{\mu}^{\mu}(0)_{\text{loop}} = -3\frac{g_s^2}{m_g^2} f^{\alpha\gamma\delta} f_{\beta\gamma\delta} \int\! \frac{d^4 p}{(2\pi)^4} \frac{p^4 - 3p^2 m^2}{(p^2 - m^2)^2},
\]

taking a symmetry factor $1/2!$ into account. The integral taken by itself is indeterminate.
The 4-gluon vertex gives the bubble diagram, which yields upon substitution of a gluon propagator

\[ g \cdot g = \Pi_\mu(0)_{\text{bubble}} = + \frac{3 g_s^2}{m_g} f^{\alpha\gamma\delta} f^{\beta\gamma\delta} \int \frac{d^4 p}{(2\pi)^4} \frac{p^2 - 4m_g^2}{p^2 - m_g^2}, \]

again with a symmetry factor \(1/2!\) included.

For the \(SU(3)\) structure constants we have \([6]\) \(f^{\alpha\gamma\delta} f^{\beta\gamma\delta} = 3\). The quartic terms cancel in the sum of the two diagrams, so the total gluon self-mass correction is the sum of (4) and (5)

\[ \Delta m_{g_{\text{self}}}^2 = \frac{1}{4t} \Pi_\mu(0)_{\text{total}} = - \frac{3 g_s^2}{32\pi^2} \int d^4 p \frac{p^2 - 2m_g^2}{(p^2 - m_g^2)^2}, \]

which is again proportional to the same determinate quadratic integral (2) found previously. The total self-mass correction for the gluons is

\[ \Delta m_{g_{\text{self}}}^2 = - \frac{9\alpha_s}{8\pi} m_g^2. \]

It is not clear what value one should take for the strong coupling constant \(\alpha_s\) in (7), but the pre-factor can be of order one. The gluon self-mass correction is of the same order of magnitude as the gluon mass itself. Whatever mass the gluons may acquire by any mechanism is largely annihilated again by self-interaction. This large self-interaction may perhaps justify the usual treatment of gluons as massless particles. Perturbation theory is clearly not adequate in this strongly non-linear case. Quark and gluon contributions should not be considered separately. The problem cannot be resolved without an adequate understanding of confinement and the origin of the masses, which is at present lacking.

Nevertheless the result may be taken as a qualitative indication of what to expect from a more adequate treatment.

4. A ninth singlet gluon?

Traditionally the colour symmetry group has been taken to be \(SU(3)\) instead of \(U(3)\). There are only eight \(SU(3)\)-octet gluons \(g_1 \cdots g_8\) carrying colour charge. The ninth gluon, \(g_0\), being the colour singlet generated by the identity matrix with colour factor

\[ g_0 = \frac{1}{\sqrt{3}} (r\bar{r} + g\bar{g} + b\bar{b}), \]

would not be confined by the colour force, and it could be exchanged directly between nucleons, which also are colour singlets.

As long as the gluons are (forced to be) massless there are good reasons [7] for the choice of \(SU(3)\) rather than \(U(3)\). The singlet gluon \(g_0\) would carry a strong long range force between nucleons, in obvious disagreement with the existence of the world as we know
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The self-mass correction (7) applies for the traditional $SU(3)$ octet gluons only. When the symmetry group is $U(3)$ the additional generator is the identity matrix. It always commutes, and the additional structure constants $f^{0\alpha\beta} = 0$ are all zero. The ninth gluon has neither self-interaction nor interaction with the octet gluons. It retains the full quark generated gluon mass, renamed to $m_0$. The $U(3)$ symmetry is spontaneously broken, with the singlet much heavier than the octet.

Once broken $SU(3)$ is accepted we can also regard the singlet gluon mass as a free parameter, to be measured in suitable experiments. Perturbation theory is probably inadequate to give a quantitative prediction in this strongly interacting case. Moreover, our understanding of the strong interaction is not adequate at present to exclude other possible sources of singlet gluon mass.

The singlet gluon has some properties in common with the proposed [8,9] very heavy $Z'$-boson, which would also couple to quarks and be relevant to the same experimental data. Having one eliminates the need for the other. The singlet gluon involves less change to the structure of the standard model.

6. Indications from experiment

The current understanding of the standard model of the strong interaction is believed to be in agreement with experiment. Yet we cannot be completely sure that an additional singlet gluon with a mass of tens of GeV would spoil the agreement, since the experiments have been fitted self-consistently to complicated higher order calculations.

A massive ninth gluon will cause a very short range Yukawa interaction between quarks. It will not cause additional quark confinement. The Coulomb-like part of the Yukawa potential will cause additional hard scattering. Experimentally a massive ninth gluon will manifest itself by an increase in the strength of the hard, Coulomb-like part of the strong force by a factor $9/8$ at quark energies well above the singlet gluon mass.

It might be thought that the massive singlet gluon should be easily observable as a resonance in quark/antiquark scattering. This is true, but only marginally so.

In the first place the singlet effects are masked by the octet gluons. Then the singlet gluon resonance must be very broad. The singlet gluon will decay strongly to quark/antiquark pairs, broadening and lowering the resonance. The decay width to quark pairs will to be given by ( $N_f$ flavours, quark mass ignored)

$$\Gamma_{g_0} = \frac{1}{12} N_f \alpha_s m_0 \approx m_0/20,$$

with $\alpha_s = \alpha_s(m_0)$ the effective strong coupling constant at the singlet mass. This will make the $g_0$ the shortest lived particle ever, with a lifetime of order $10^{-25} \ldots 10^{-26}$ s, and a range before decay much shorter than the proton radius.

The already broad resonance will be further broadened by being folded with the parton distribution functions inside the proton. It remains to be seen whether the proton scattering data can be fitted, with recomputed parton distribution functions, to strong interaction including the ninth gluon. The recent claim [10] of the possible discovery of
quark constituents, based on an observed excess of hard events at very high energy, may actually be an indication of the existence of a massive gluon. If this is indeed the case the mass of the singlet gluon must be at least 300 GeV, assuming half the momentum of the proton is carried by the quarks. This is much higher than the second order perturbation theory result.

The experimental errors at the highest available energy are very large, and the interpretation of the data is based self-consistent fitting of parameters, so a firm conclusion cannot be drawn at present. Moreover, the fitting has been done on basis of formulae ignoring singlet gluons, which may introduce a bias. More accurate data at higher energy beyond the singlet resonance, and reinterpretation of the data should settle this point. At present the interpretation of the data is too controversial to allow conclusions to be drawn.

7. Conclusions

1 In second order gluons acquire a determinate mass by interaction with the top quark. This mass will be greatly reduced by self-interaction for the usual $SU(3)$ octet gluons.

2 The colour symmetry of the strong interaction may be (spontaneously broken) $U(3)$ instead of $SU(3)$.

3 The ninth gluon will be a very massive colour singlet without gluon self-interaction. Second order perturbation theory cannot be trusted to predict gluon masses quantitatively.

4 Once the ban on gluon masses is lifted, the existence of a singlet gluon and its mass becomes a matter to be settled by experiment.

5 The existence of a sufficiently massive singlet gluon does not immediately conflict with experiment, and may even be confirmed by the highest energy data at FNAL.
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