The Poincaré-Einstein synchronization: historical aspects and new developments

E. Minguzzi
Dipartimento di Matematica Applicata, Università degli Studi di Firenze, Via S. Marta 3, I-50139 Firenze, Italy
E-mail: ettore.minguzzi@unifi.it

Abstract. An historical introduction to the Poincaré-Einstein synchronization and the problem of its transitivity is given. Inspired by cohomology theory I show that it is possible to improve the Poincaré-Einstein synchronization convention so as to obtain a consistent time foliation even in rotating frames for every assigned measure on space. This fact implies that there is, contrary to a widespread belief, a privileged time foliation of spacetime in several interesting cases.

1. Introduction
In this work I am going to investigate how to consistently synchronize clocks at rest in some frame and distributed in a space $S$. This is an old problem which has several motivations. The presence of a global time is often necessary for legal reasons, for instance if we want to make sense of some contract. It is essential to clarify priorities over some discovery or in order to avoid the mess that would arise from the proliferation of overlapping local times [1]. It is a key ingredient to improve the computational efficiency of computer networks, or to time-order the writing-reading access rights of different processes inside one single computer. Other motivations come from quantum mechanics, quantum cosmology and quantum gravity. Here canonical quantization schemes use time dependent Hamiltonians $H(t)$ which are generically not foliation independent. As a consequence, these methods might run into problem in absence of a privileged time foliation. This is one aspect of the so called problem of time of quantum gravity which would be clarified if we could show that some privileged foliation does indeed exist. This is indeed what I will prove in this work although I will leave some details for a different article. I will also use this opportunity to clarify the historical development of several ideas connected with synchronization and its consistency.

The synchronization problem we are considering will make sense in rather general settings. We wish to synchronize distant clocks by making use of signal exchanges (thus we do not consider slow clock transport synchronization). Nevertheless, we do not need to specify the nature of the signal; it could be sound in air, light in vacuum, or an electric signal propagating along copper wires.

For our mathematical framework we need just a set $S$, whose points are called clocks, and a function $a : S \times S \to \mathbb{R}$ which gives the time $a(s_1, s_2)$ needed by the signal to go from point $s_1$ to point $s_2$. To resynchronize the clocks means to replace the function $a$ with

$$a'(s_1, s_2) = a(s_1, s_2) + c(s_2) - c(s_1)$$

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where $c : S \rightarrow \mathbb{R}$, and $c(s)$ is how much we have moved forward the clock $s$ in the resynchronization. In this mathematical description is already embodied a condition which we denote $z = 0$, namely that the time it takes the signal to move from $s_1$ to $s_2$ does not depend on the instant of departure but only on the initial and final space points. This condition implies that if two signals are emitted at times $t_1$ and $t_1 + \Delta t$ from $s_1$, then they are received at times $t_2$ and $t_2 + \Delta t$ at $s_2$ (thus $a(s_1, s_2) = t_2 - t_1$). The condition $z = 0$ essentially states that clocks $s_1$ and $s_2$ go at the same rate, i.e. they are synchronized. Without this condition the problem of synchronization would not make sense as two clock initially synchronized would run out of sync.

We shall impose some additional conditions on the map $a$. They are, for every $s, s_1, s_2 \in S$,

(i) $a(s_1, s_3) \leq a(s_1, s_2) + a(s_2, s_3)$,

(ii) $a(s, s) = 0$,

(iii) $a(s_1, s_2) + a(s_2, s_1) = 0$ only if $s_1 = s_2$.

Condition (i) states that the time it takes the signal to reach $s_3$ from $s_1$ is equal or smaller than the time it would take by passing through $s_2$. This is not really a condition on the propagation of the signal but on our way of defining the time of arrival. Since the events which happen at the clock $s_3$ are totally ordered by the time of $s_3$, we use the prescription that the time of arrival is that of the first signal (according to the order at $s_3$) that arrives at $s_3$ after emission at $s_1$, independently of the path followed from $s_1$. With this Fermat’s type prescription condition (i) is automatically fulfilled. Condition (ii) states that the signal cannot arrive at $s$ before it is emitted, thus it is a causality condition. By the way this condition was already included in the assumption that $a$ has finite values because, if (ii) was violated then by (i) the only possibility could be $a(s, s) = -\infty$. Finally, condition (iii) states that the speed of the signal is finite in the sense that the two-way time between two distinct point is greater than zero (use (i) and (ii)). This condition could have been omitted by passing to a quotient space, but for simplicity we include it.

Let us now define (radar distance)

$$d_r(s_1, s_2) = \frac{1}{2} [a(s_1, s_2) + a(s_2, s_1)].$$ 

By summing the two equations

$$a(s_1, s_3) \leq a(s_1, s_2) + a(s_2, s_3),$$

$$a(s_3, s_1) \leq a(s_3, s_2) + a(s_2, s_1),$$

we find easily that $d_r$ is a distance on $S$, that is it satisfies the triangle inequality

$$d_r(s_1, s_3) \leq d_r(s_1, s_2) + d_r(s_2, s_3),$$

and $d_r(s_1, s_2) \geq 0$ with equality if and only if $s_1 = s_2$. Furthermore, the equality holds in the triangle inequality if and only if

$$a(s_1, s_3) = a(s_1, s_2) + a(s_2, s_3),$$

$$a(s_3, s_1) = a(s_3, s_2) + a(s_2, s_1). \quad (1)$$

Before we go on with the description of our model, and our proposed synchronization we make a short historical detour in the most known synchronization method: the Poincaré-Einstein synchronization.
1.1. The Poincaré-Einstein synchronization

The Poincaré-Einstein synchronization establishes that clock $s_1$ is synchronized with clock $s_2$ if the time taken by the signal to go from $s_1$ to $s_2$ equals the time taken by the signal to go from $s_2$ to $s_1$, in formulas

$$a(s_1, s_2) = a(s_2, s_1).$$

(3)

Two events are said to be simultaneous with respect to the Poincaré-Einstein convention if the readings of the Poincaré-Einstein synchronized clocks at those events coincide.

In practice in order to synchronize the clocks an observer at $s_2$ sends a signal towards $s_2$ where it is reflected back to $s_1$. Calling $t_1$, $s_1$-time of emission; $t_2$, $s_2$-time of reflection; and $t'_1$, $s_1$-time of reception then the observer at $s_2$ regulates its clock in such a way that

$$t_2 = \frac{1}{2}(t'_1 + t_1).$$

(4)

Of course this regulation can be made only at posteriori when the observer $s_2$ receives a further signal from $s_1$ with the information on the data $t_1$ and $t'_1$ (the follow up signal).

I am going to prove some equivalent ways of stating the same convention. Suppose to define on $S$ the radar metric, and let $s_2$ be a point in the middle of $s_1$ and $s_3$ which means $d_r(s_1, s_3) = 2d_r(s_1, s_2) = 2d_r(s_2, s_3)$. This implies in particular $d_r(s_1, s_3) = d_r(s_1, s_2) + d_r(s_2, s_3)$ thus Eqs. (1)-(2). Their difference can be rewritten

$$a(s_1, s_3) - a(s_3, s_1) = a(s_1, s_2) + a(s_2, s_3) - [a(s_3, s_2) + a(s_2, s_1)]$$

$$= 2[d_r(s_1, s_2) - d_r(s_2, s_3)] - 2[a(s_2, s_1) - a(s_2, s_3)]$$

$$= -2[a(s_2, s_1) - a(s_2, s_3)].$$

This equation can be read as follows:

**Suppose that $s_1$ and $s_3$ admit a middle point $s_2$ according to the radar distance. Clocks $s_1$ and $s_3$ are synchronized according to Poincaré-Einstein, i.e. $a(s_1, s_3) - a(s_3, s_1) = 0$, if and only if a signal sent from $s_2$ reaches $s_1$ and $s_3$ at the 'same' time, i.e. $a(s_2, s_1) - a(s_2, s_3) = 0$.**

The difference between Eqs. (1)-(2) can also be rewritten

$$a(s_1, s_3) - a(s_3, s_1) = a(s_1, s_2) + a(s_2, s_3) - [a(s_3, s_2) + a(s_2, s_1)]$$

$$= -2[d_r(s_1, s_2) - d_r(s_2, s_3)] + 2[a(s_1, s_2) - a(s_3, s_2)]$$

$$= 2[a(s_1, s_2) - a(s_3, s_2)].$$

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As pointed out by Jammer [2] the first historical mention of the Poincaré-Einstein synchronization method is contained in Augustine of Hippo’s Confessions [3, Book 7, Chp. 6]. Augustine relates a story which would prove the unreliability of astrological divinations. According to it two families established that their babies were born at the same time, although they later had very different fortunes in life. The families established the simultaneity of the births by sending messengers to the other family at the time of birth. According to the story the messengers met middle-way and so they had the proof that the babies were born at the ‘same’ time.
Of course, in this story our signal is represented by the messengers. The correspondence with the Poincaré-Einstein synchronization is clear once one considers the last of the equivalent definitions given above.

Even before its formal formulation the Poincaré-Einstein synchronization convention was currently used by telegraphers of the middle of the XIX century to take into account delays in telegraphic signals running on cables laid under the Atlantic ocean [1, 4]. At the time accurate time measurements were needed to get longitudes and draw accurate maps. The first formal statement of the convention was given by Poincaré in the 1900 Leiden lecture [5], where he considers a reference frame moving uniformly with respect to the ether and synchronizing its clocks demanding isotropy. In his words:

Let us suppose that there are some observers placed at various points, and they synchronize their clocks using light signals. They attempt to adjust the measured transmission time of the signals, but they are not aware of their common motion, and consequently believe that the signals travel equally fast in both directions. They perform observations of crossing signals, one traveling from A to B, followed by another traveling from B to A. The local time $t$ is the time indicated by the clocks which are so adjusted.

It is clear that for Poincaré this time, called by him local time, was fictitious. Indeed, he assumed the existence of the ether [6]. In his talk at the St. Louis conference in 1904 he takes up again this idea [7]:

$A$ sends its signal when its clock marks the hour 0, and the station $B$ perceives it when its clock marks the hour $t$. The clocks are adjusted if the slowness equal to $t$ represents the duration of the transmission, and to verify it the station $B$ sends in its turn a signal when its clock marks 0; then the station $A$ should perceive it when its clock marks $t$.

The time-pieces are then adjusted.

Finally, Einstein in his 1905 paper [8] gives the same stipulation. Here the obtained time is no more fictitious because he has dismissed the existence of the ether.

We have not defined a common “time” for $A$ and $B$, for the latter cannot be defined at all unless we establish by definition that the “time” required by light to travel from $A$ to $B$ equals the “time” it requires to travel from $B$ to $A$. Let a ray of light start at the “$A$ time” $t_A$ from $A$ towards $B$, let it at the “$B$ time” $t_B$ be reflected at $B$ in the direction of $A$, and arrive again at $A$ at the “$A$ time” $t_A'$. In accordance with definition the two clocks synchronize if $t_B - t_A = t_A' - t_B$.

Both recognized the theoretical importance of this synchronization when coming to light signals. As Peter Galison points out [4], Poincaré, as a permanent member of the French Bureau of Longitude, was concerned about accurate maps and delay times in telegraphic transoceanic signals. Einstein, while working at the Swiss patent office in Bern, reviewed patents for synchronizing clocks. We might say that they were at the right place at the right time.

Once clocks have been synchronized one can measure the one-way speed of light. It must be noted that its isotropy is imposed by the way we have synchronized clocks. This observation has lead some authors to a common misconception which can be summarized by the following statement:

The one-way speed of light with its isotropy depends on the synchronization convention adopted, so to say that the speed of light is constant on the frame is devoid of physical content.

Poincaré was a conventionalist and would have perhaps shared this view which, however, is incorrect. In order to clarify this point we need first to to learn more on the consistency of
synchronization, because as a matter of fact the Poincaré-Einstein synchronization could not work.

2. The transitivity of synchronization
In this section we wish to answer the following question: Is the Poincaré-Einstein synchronization consistent? Does it really lead to a time foliation? In order to answer this question we need to prove

(a) Clocks once synchronized remain synchronized,
(b1) Reflexivity: clock $s_1$ is synchronized with itself,
(b2) Symmetry: if clock $s_1$ is synchronized with clock $s_2$ then $s_2$ is synchronized with $s_1$,
(b3) Transitivity: if clock $s_1$ is synchronized with clock $s_2$ and $s_2$ is synchronized with $s_3$ then clock $s_1$ is synchronized with clock $s_3$.

Condition (a) is essentially the condition $z = 0$ and thus it is already embodied in our formalism based on the Fermat’s $a$ map. Condition (b1) follows from (ii), namely $a(s, s) = 0$. Condition (b2) is a consequence of our definition of synchronization, namely $a(s_1, s_2) = a(s_2, s_1)$. It remains to establish under which conditions (b3) holds.

Unfortunately, in most recent relativity textbooks the approach to this consistency problem is the same as Einstein’s (1905) who wrote

We assume that this definition of synchronism is free from contradictions, and possible for any number of points; and that the following relations are universally valid [...] [he writes (b2) and (b3)].

This position ignores completely further developments by Von Laue [9], Silberstein [10], Weyl [11], Reichenbach [12, 13, 14], Macdonald [15] and the author [16, 17].

The fundamental idea is that while the one-way time of propagation relies on the synchronization adopted, the time taken by the light signal over a round-trip does not. Some round-trip condition can therefore guarantee that properties (a),(b1)-(b3) hold. The final result can be expressed with the following theorem known to Reichenbach [12, 13] (see also [14, Chap. 4]) (who however did not introduce in a neat way the condition $z = 0$) but clearly stated for the first time by Alan Macdonald [15]

**Theorem 2.1.** The conditions (a),(b1),(b2) and (b3) hold if and only if the following two experimentally verifiable (as synchronization independent) conditions hold

$z = 0$: Clocks are syntonized, that is, if two signals are sent from $s_1$ at times $t_1$ and $t_1'$ and they are respectively received at times $t_2$ and $t_2'$ at $s_2$ then $t_1' - t_1 = t_2' - t_2$.

$\Delta$: (Reichenbach’s round-trip condition) The time it takes light to cover a triangular path (through reflections over suitable mirrors) does not depend on the direction taken around the triangle.

Under the condition $z = 0$ we can apply our initial formalism under which it is possible to restate Reichenbach’s condition by introducing the following Sagnac effect function

$$w(s_1, s_2, s_3) = [a(s_1, s_2) + a(s_2, s_3) + a(s_3, s_1)] - [a(s_1, s_3) + a(s_3, s_2) + a(s_2, s_1)].$$

Then Reichenbach’s condition holds if and only if $w = 0$.

Another even older approach based on a different round-trip condition was started by Max Von Laue [9, p.36], and then taken up again by Silberstein [10, Chap. 4], Weyl [11, Sect. 23] and Macdonald and the author [16, 17]. Basically one imposes that
Theorem 2.2. Let \((S, d)\) be a metric space. The conditions (a),(b1),(b2) and (b3) hold and the one-way speed of light of the so synchronized clocks is a constant \(c\) throughout the frame if and only if the following condition holds:

\[
 \text{L/c law: the average speed of light over a closed (polygonal) path is a constant (c) independent of the path.}
\]

The condition \(\text{L/c}\) is stronger than Reichenbach’s and so are the conclusions that follow from it. Max Von Laue [9], Silberstein [10] and Weyl [11] proved this theorem under the additional tacit assumption that the clocks are syntonized \((z = 0)\), however the condition \(z = 0\) follows already from \(\text{L/c}\) (see [17]).

We can prove that actually the Reichenbach’s round-trip condition and the \(\text{L/c}\) law are essentially the same if as distance is used the radar distance.

Theorem 2.3. Under condition \(z = 0\), let us define on \(S\) the radar distance \(d_r\) and let us measure the distances with it. Then the Reichenbach’s round trip condition holds if and only if the \(\text{L/c}\) law holds.

Proof. It is clear that the \(\text{L/c}\) law implies \(\Delta\). Let us prove the converse. Let \(s_1s_2s_3\cdots s_k\), \(s_k = s_1\) be a polygonal path and let us consider a signal which starts at \(s_1\)-time \(t_0\) from \(s_1\) and covers the path finally returning at \(s_1\) at time \(t_1\). Let us consider a second signal which starts at the same instant but covers the path in the opposite direction to finally reach \(s_1\) at time \(t_2\). Moreover, let us consider the similar propagations but this time over the triangulated path \(s_1s_2s_3s_1s_3s_4s_1s_4s_5s_1\cdots s_k\). Let us denote with \(t'_1\) and \(t'_2\) the corresponding arrival times. We have the identities

\[
\begin{align*}
 t'_1 - t_0 & = t_1 - t_0 + 2[d_r(s_1, s_3) + d_r(s_1, s_4) + d_r(s_1, s_5) + \cdots + d_r(s_1, s_{k-2})] \\
 t'_2 - t_0 & = t_2 - t_0 + 2[d_r(s_1, s_3) + d_r(s_1, s_4) + d_r(s_1, s_5) + \cdots + d_r(s_1, s_{k-2})]
\end{align*}
\]

(6)

(7)

from which we obtain \(t'_1 - t'_2 = t_1 - t_2\). However, since the path \(s_1s_2s_3s_1s_3s_4s_1s_4s_5s_1\cdots s_k\) is the succession of triangles \(s_1s_2s_3s_1, s_1s_3s_4s_1, s_1s_4s_5s_1\cdots\) we can also write

\[
 t'_1 - t'_2 = w(s_1, s_2, s_3) + w(s_1, s_3, s_4) + w(s_1, s_4, s_5) + \cdots
\]

which vanishes because of Reichenbach’s condition \(w = 0\). We have \(t_2 = t_1\) and

\[
 t_1 - t_0 = \frac{1}{2}[(t_1 - t_0) + (t_2 - t_0)] = \frac{1}{2}[a(s_1, s_2) + a(s_2, s_3) + \cdots + a(s_k, s_{k-1} + a(s_{k-1}, s_{k-2}) + \cdots)]
\]

\[
 = d_r(s_1, s_2) + d_r(s_2, s_3) + \cdots + d_r(s_3, s_4) + \cdots = L
\]

because the last expression is the length of the polygonal path. Since \(t_1 - t_0\) is the time it takes the signal to cover the path, the last equality proves the \(\text{L/c}\) law.

\(\square\)

Although these theorems are formulated for light signals, it must be remarked that they do not depend on the nature of the signal.

It seems that in the historical and philosophical literature, as far as these developments are concerned, the contribution of Reichenbach has been overestimated (the very informative book by Jammer [2], which has the merit of pointing out several references, is no exception). While it is certainly true that he put more effort than others in the clarification of the many subtle issues raised by special relativity, his explanations were often lengthy and obscure. This criticism was indeed first moved to him by H. Weyl [18]. Rynasiewicz [19] clarifies the relationship between the two scientists as follows.
their common interest in the foundations of relativity theory and geometry kept them
in communications during the 1920's. By mid-decade, though, they had had it with
one another, exchanging unkind words in print.

The very round-trip condition that goes under Reichenbach’s name is probably not his
own, indeed in the work “Bericht über eine Axiomatik der Einsteinschen Raum-Zeit-Lehre,”
Physikalische Zeitschrift, vol. 22 (1921), pp. 6837, reprinted in [14, Chap. 4], after the
introduction of the round trip condition $\triangle$, in a footnote he states that

Einstein has already pointed out the significance of these axioms for synchronization
in his lectures.

Reichenbach had attended Einstein’s lectures in Berlin and it is known that Einstein gave them
between 1914 and 1917. However, as Jammer points out [2, Chap. 7], Einstein had probably
already read Max Von Laue book [9] back in 1912 when in the manuscript called “Einstein’s 1912
manuscript on the special theory of relativity” he adopts some of his terminologies. Therefore,
as far as we know, the credit for the introduction of the round-trip conditions should be given
to Max Von Laue.

Coming to our earlier issue, we can say that the content of the naive statement “the speed of
light is constant over the frame” is actually “the speed of light is constant over closed paths”, and
this is indeed all we need for synchronization and what is ultimately checked in experiments.
Of course, the condition $L/c$ is not satisfied in all frames, this is the case of the rotating
platform over which Poincaré-Einstein’s synchronization cannot be consistently applied (this
lack of transitivity is related to the Sagnac effect). Indeed, a good definition of inertial reference
frame could be

**Definition 2.4.** An inertial reference frame is a space of clocks $S$ over which the radar distance
is Euclidean and the propagation of light satisfies the $L/c$ law.

The idea that the concept of inertial frame should be related to the geometry of light
propagation has also been proposed by Martin Strauss [2, Chap. 7].

In summary we have learned in this section that the consistency of the Poincaré-Einstein
synchronization method depends on condition $z = 0$ and on some round-trip condition. While
the former is necessary, for otherwise synchronized clocks would run out of synch, the latter does
not seems strictly necessary. Of course it is necessary to get transitivity of Poincaré-Einstein
synchronization but can we generalize the synchronization method so as to remove this additional
assumption? Is there a method that works also when $w \neq 0$, hence for rotating frames? The
answer is affirmative and will be given in the next section.

3. The new synchronization method

In the previous section we have defined the function $w(s_1, s_2, s_3)$ as the difference between the
time it takes the signal to cover the path $s_1s_2s_3s_1$ and the opposite path $s_1s_3s_2s_1$. The function
$w$ is (twice) the Sagnac holonomy and $w = 0$ if and only if Reichenbach’s round-trip condition
holds. From the given expression for $w$, Eq. (5), it follows that $w$ is skew-symmetric and satisfies
the cocycle condition

$$w(s_2, s_3, s_4) - w(s_3, s_4, s_1) + w(s_4, s_1, s_2) - w(s_1, s_2, s_3) = 0.$$ 

Our method works if on the space of clocks $S$ it is defined a natural unit measure $\mu$ (e.g. $S$ is a
Riemannian compact space). In this case, integrating the previous equation we obtain,

$$\delta(s_1, s_2) = \int_S w(s_1, s_2, s) d\mu(s),$$
is a skew-symmetric function such that
\[ w(s_1, s_2, s_3) = \delta(s_1, s_2) + \delta(s_2, s_3) + \delta(s_3, s_1). \] (8)

A skew-symmetric function \( \delta \) which satisfies the latter equality leads to a consistent synchronization (i.e. reflexive, symmetric and transitive) by setting (with the same interpretation of Eq. (4))
\[ t_2 = \frac{1}{2}(t'_1 + t_1) + \frac{1}{2}\delta(s_1, s_2). \]

Indeed, this equation can be rewritten
\[ a(s_1, s_2) - a(s_2, s_1) = \delta(s_1, s_2), \] (9)

which replaces Eq. (3). From this expression reflexivity and symmetry follow from the antisymmetry of \( \delta \). As for transitivity, let \( s_1 \) be synchronized with \( s_2 \) and let \( s_2 \) be synchronized with \( s_3 \) then
\[ a(s_1, s_2) - a(s_2, s_1) = \delta(s_1, s_2), \]
\[ a(s_2, s_3) - a(s_3, s_2) = \delta(s_2, s_3). \]

Summing the two equations and using Eq. (5)
\[ w(s_1, s_2, s_3) = [a(s_3, s_1) - a(s_1, s_3)] = \delta(s_1, s_2) + \delta(s_2, s_3) \]

and using Eq. (8)
\[ a(s_1, s_3) - a(s_3, s_1) = \delta(s_1, s_3), \] (10)

that is, clocks \( s_1 \) and \( s_3 \) are synchronized.

4. Conclusions

I have given an historical account of the origins of the Poincaré-Einstein’s synchronization method, and in particular, of the issue of its transitivity. In the last section I have shown that it can be improved so as to work in more general circumstances. The new synchronization method adds a correction to Poincaré-Einstein’s which amounts to an average of the Sagnac effect (which is observable). Contrary to Poincaré-Einstein’s it is consistent even in absence of round-trip conditions. The new synchronization method reduces to Poincaré-Einstein’s for vanishing Sagnac effect (which is the case in which Poincaré-Einstein’s convention works). Even more it can be proved that with this choice of synchronization the map \( a(s_1, s_2) \) becomes non-negative which means that the so obtained time is non-decreasing over successive causally related events.

As for applications it is not difficult to prove that once applied to the rim of a rotating platform the method gives as simultaneity slices those of the inertial (non-rotating) reference frame. I am presently studying the connection between this time and the one which is usually used in slowly rotating planets (we might call it Kerr time). In general it could be complicated to establish the connection between our proposed time and that displayed in some metrics. This fact does not mean that the method is not useful in practice, on the contrary it is often more practical than a metric approach. For instance it applies readily at the surface of a rotating planet because this surface is equipotential and hence the condition \( z = 0 \) holds there. This surface need not be spherical or axisymmetric, the fact that it is equipotential guarantees that the synchronization method can be applied consistently and lead to a time foliation. The method can be readily applied for instance by modifying the synchronization protocols which are used in computational networks. The existence of such reference total order simplifies the problems mentioned in the introduction. In particular this development proves that in several interesting cases, contrary to a widespread belief, the reference frame does indeed posses a privileged time foliation.
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