On local vertex irregular reflexive coloring of graphs

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Abstract. Let $\chi(G)$ be a chromatic number of proper coloring on $G$. For an injection $f : V(G) \rightarrow \{0, 2, \ldots, 2k_v\}$ and $f : E(G) \rightarrow \{1, 2, \ldots, k_e\}$, where $k = \max\{k_e, 2k_v\}$ for $k_v, k_e$ are natural number. The associated weight of a vertex $u, v \in V(G)$ under $f$ is $w(u) = f(u) + \sum_{uv \in E(G)} f(uv)$. The function $f$ is called a local vertex irregular reflexive $k$-labeling if every two adjacent vertices has distinct weight. When we assign each vertex of $G$ with a color of the vertex weight $w(uv)$, thus we say the graph $G$ admits a local vertex irregular reflexive coloring. The smallest number of vertex weights needed to color the vertices of $G$ such that no two adjacent vertices share the same color is called a local vertex irregular reflexive chromatic number, denoted by $\chi_{lrvs}(G)$. Furthermore, the minimum $k$ required such that $\chi_{lrvs}(G) = \chi(G)$ is called a local reflexive vertex color strength, denoted by $lrvcs(G)$. In this paper, we will obtain the $lrvcs(G)$ and characterize the existence of a graph with given its local reflexive vertex color strength.

1. Introduction

By a simple graph $G$, we mean a pair of set $(V(G), E(G))$, where $V(G)$ is non empty vertex set and $E(G)$ is non-ordered pair set of two distinct vertices $u, v$ in $V(G)$. The graph $G$ is connected if there exists a path for every two distinct vertices of $G$. We consider a graph in this paper are connected and un-directed graph $G(V, E)$, i.e. a graph without loops and parallel edges [8]. One of the problems that appear while studying graph theory is a labeling of graphs. A labeling of a graph is a map that carries graph elements to the numbers (usually to the positive or non-negative integers). The concept and also some results of graph labeling study can be found in [9]. When we assign labels to only all vertices (called vertex labeling), assign labels to only all edges (called edge labeling), but when we assign labels to both vertices and edges (called total labeling). In many cases it is interesting to consider the sum of all labels associated with graph elements. This will be called a weight of the graph elements denoted by $w$.

In this paper we focus to study a new type of labeling namely an irregular labeling. When we assign positive integer labels to the edges or vertices of a connected graph, it will become irregular if the weight (labels sum) on each vertex is distinct. The minimum label of the largest label over all such irregular assignments is known as the irregularity strength of graph. The results of irregularity strength of graph can be seen in [1]-[5],[6, 11, 12]. Baca et al obtained the lower bound and the upper bound of total vertex irregularity strength on graph $G$, see [7].
By a vertex irregular reflexive labeling, we define, for a graph $G$, a $k$-labeling assigns numbers $\{1,2,\ldots,k\}$ to the elements of graph. Let $k$ be a natural number, a function $f : V(G) \cup E(G) \rightarrow \{1,2,3,\ldots,k\}$ is called total $k$ irregular labeling. Irregular $k$-labeling extends to many types one of them is a vertex irregular reflexive total $k$-labeling. Tanna et. al. [13] give the detail definition of vertex irregular reflexive labeling. Total $k$-labeling is a function $f_v$ from the edge set to the first natural number $k_v$ and a function $f_e$ from the vertex set to the non negative even number up to $2k_v$, where $k_v = \max\{k_e,2k_v\}$. A vertex irregular reflexive $k$-labeling of the graph $G$ is the total $k$-labeling, if for every two different vertices $x$ and $y$ of $G$, $wt(x) \neq wt(y)$, where $wt(x) = f_v(x) + \Sigma_{xy \in E(G)}f_e(xy)$. The minimum $k$ for graph $G$ which has a vertex irregular reflexive $k$-labeling is called the reflexive vertex strength of the graph $G$, denoted by $\text{lrves}(G)$.

Furthermore, a coloring of graph is called a vertex coloring if every two adjacent vertices have different colors. In other term, we call it as a proper $k$-coloring. The smallest number of $k$ colors needed to get a proper vertex coloring is called the chromatic number of the graph, denoted by $\chi(G)$ [10].

The last, by a local vertex irregular reflexive $k$-labeling, we mean a function $f$ that maps $f : V(G) \rightarrow \{0,2,\ldots,2k_v\}$ and $f : E(G) \rightarrow \{1,2,\ldots,k\}$, where $k = \max\{k_e,2k_v\}$ for $k_v,k_e$ are natural number, such that for every two adjacent vertices have a distinct weight. The associated weight of a vertex $u,v \in V(G)$ under $f$ is $w(u) = f(u) + \Sigma_{uv \in E(G)}f(uv)$. When we assign each vertex of $G$ with a color of the vertex weight $w(u)$, thus we say the graph $G$ admits a local vertex irregular reflexive coloring. The smallest number of vertex weights needed to color the vertices of $G$ such that no two adjacent vertices share the same color is called a local vertex irregular reflexive chromatic number, denoted by $\chi_{lrves}(G)$. Furthermore, the minimum $k$ required such that $\chi_{lrves}(G) = \chi(G)$ is called a local reflexive vertex color strength, denoted by $\text{lrves}(G)$. In this paper, we will obtain the $\text{lrves}(G)$ and characterize the existence of a graph with given its local reflexive vertex color strength.

2. The lower bound of $\text{lrves}(G)$

**Lemma 1** Let $G$ be a connected graph of minimum degree $\delta$, maximum degree $\Delta$, and chromatic number $\chi(G)$. The local reflexive vertex color strength of graph $G$ is

$$\left\lceil \frac{\chi(G) + \delta - 1}{1 + \Delta} \right\rceil.$$  

**Proof.** Let $G$ be a connected graph of minimum degree $\delta$, maximum degree $\Delta$, and chromatic number $\chi(G)$. Since we require $k$-minimum for the local vertex irregular reflexive $k$-labeling of the graph $G$, we should choose $\delta$ of the vertex with possibly having a minimum vertex weight. Secondly, the set of a vertex-weight should be consecutive and the number of different weights is equal to chromatic number $\chi(G)$. Thus, the set of a vertex-weight is $wt(x) = \{\delta,\delta + 1,\delta + 2,\ldots,\delta + (\chi(G) - 1)\}$. Whilst, the maximum possible vertex weight of graph $G$ under the local vertex irregular reflexive $k$-labeling is obtained at the vertex with having the maximum degree $\Delta$. The vertex weight will be $2k_v + \Delta k_e$. Since $k' = \max\{k_e,2k_v\}$ is the local reflexive vertex color strength, it implies $2k_v + \Delta k_e \leq k'(1 + \Delta)$. Furthermore, we also have

$$k'(1 + \Delta) \geq \delta + (\chi(G) - 1)$$

$$\frac{k'}{1 + \Delta} \geq \frac{\delta + \chi(G) - 1}{1 + \Delta}$$

$$\text{lrves}(G) \geq \frac{\delta + \chi(G) - 1}{1 + \Delta}.$$
Since \( \text{lrvecs}(G) \) should be integer, and we need a sharpest lower bound, it implies

\[
\text{lrvecs}(G) \geq \left\lceil \frac{\delta + \chi(G) - 1}{1 + \Delta} \right\rceil
\]

It completes the proof. \( \square \)

By having the above lower bound of the local reflexive vertex color strength, we are ready to show our new results in the format of lemmas and theorems.

3. The reflexive vertex strength of some graphs

The following obtained theorems will show the local reflexive vertex color strength of cycle graph, star graph, wheel graph, friendship graph, and ladder graph. They are all initial research findings in this paper.

**Theorem 1** Let \( C_n \) be a cycle graph. For every positive integers \( n \geq 3 \), \( \text{lrvecs}(C_n) = 2 \).

**Proof.** The graph \( C_n \) is a connected graph with vertex set \( V(C_n) = \{v_i; 1 \leq i \leq n\} \), \( |V(C_n)| = n \) and edge set \( E(C_n) = \{v_i v_{i+1}; 1 \leq i \leq n-1\} \cup \{v_1 v_n\} \), \( |E(C_n)| = n \). The graph \( C_n \) is regular graph, such that the minimum and the maximum degree of \( C_n \) are 2. To determine the lower bound of the local reflexive vertex color strength of graph \( C_n \), we have two cases:

**Case 1.** For even \( n \), \( \chi(C_n) = 2 \).

Based on Lemma 1, we have the following lower bound.

\[
\text{lrvecs}(C_n) \geq \left\lceil \frac{\chi(C_n) + \delta - 1}{1 + \Delta} \right\rceil = \left\lceil \frac{2 + 2 - 1}{1 + 2} \right\rceil = 1
\]

Suppose 1 is \( \text{lrvecs}(C_n) \), 1 will be the largest label under the local vertex irregular reflexive labeling on graph \( C_n \). Since 1 is the largest label, then the vertex label and the edge label must be 0 and 1, respectively. Thus with that condition will give the same weight for all vertices. It contradicts with the condition for any two adjacent vertices must have different weights. Hence \( 1 \neq \text{lrvecs}(C_n) \), such that \( \text{lrvecs}(C_n) > 1 \). Thus, it concludes that \( \text{lrvecs}(C_n) \geq 1 + 1 = 2 \).

**Case 2.** For odd \( n \), \( \chi(C_n) = 3 \).

Based on Lemma 1, we have the following lower bound.

\[
\text{lrvecs}(C_n) \geq \left\lceil \frac{\chi(C_n) + \delta - 1}{1 + \Delta} \right\rceil = \left\lceil \frac{3 + 2 - 1}{1 + 2} \right\rceil = \left\lceil \frac{4}{3} \right\rceil = 2
\]

Furthermore, we will show the upper bound of vertex irregular reflexive 2-labeling by defining the injection \( f \) and \( g \) in the following:

\[
f(v_i) = \begin{cases} 0, & \text{if } i \text{ odd}, \\ 2, & \text{if } i \text{ even}, \end{cases} \quad g(v_i v_{i+1}) = \begin{cases} 1, & \text{if } i \text{ odd}, \\ 2, & \text{if } i \text{ even}, \end{cases} \quad g(v_1 v_n) = 2
\]

Based on the functions \( f \) and \( g \), it gives \( \text{lrvecs}(C_n) \leq 2 \). The vertex weights of \( C_n \) are

\[
wt(v_i) = \begin{cases} 3, & \text{for } n \text{ even}, \text{if } i \text{ odd}, \\ 5, & \text{for } n \text{ even}, \text{if } i \text{ even}, \end{cases} \quad wt(v_i) = \begin{cases} 3, & \text{for } n \text{ odd}, \text{if } i \text{ odd}, \\ 4, & \text{for } n \text{ odd}, \text{if } i = n, \\ 5, & \text{for } n \text{ odd}, \text{if } i \text{ even}. \end{cases}
\]

It is easy to see that the number of different vertex weights is equal the chromatic number of \( C_n \). Since \( \text{lrvecs}(C_n) \geq 2 \) and \( \text{lrvecs}(C_n) \leq 2 \), it concludes that \( \text{lrvecs}(C_n) = 2 \). \( \square \)
Based on Lemma 1, we have the following lower bound.

For Case 1.

\[ lrvcs(S_n) \geq \left\lceil \frac{\chi(S_n) + \delta - 1}{1 + \Delta} \right\rceil = \left\lceil \frac{2 + 1 - 1}{1 + n} \right\rceil = \frac{2}{1 + n} = 1. \]

Furthermore, we will show the upper bound of vertex irregular reflexive 1-labeling by defining the injection \( f \) and \( g \) in the following:

\[ f(A) = 0 \quad f(v_i) = 0 \quad g(Av_i) = 1 \]

Based on the functions \( f \) and \( g \), it gives \( lrvcs(S_n) \leq 1 \). The vertex weights of \( S_n \) are

\[ wt(A) = n \quad wt(v_i) = 1 \]

It is easy to see that the number of different vertex weights is equal the chromatic number of \( S_n \). Since \( lrvcs(S_n) \geq 2 \) and \( lrvcs(S_n) \leq 2 \), it concludes that \( lrvcs(S_n) = 1 \).

\[ \square \]

**Theorem 2** Let \( S_n \) be a star graph. For every positive integers \( n \geq 3 \), \( lrvcs(S_n) = 1 \).

**Proof.** The graph \( S_n \) is a connected graph with vertex set \( V(S_n) = \{ A, v_i; 1 \leq i \leq n \} \)

\[ |V(C_n)| = n + 1 \quad \text{and edge set } \quad E(S_n) = \{ Av_i; 1 \leq i \leq n \}, \quad |E(C_n)| = n. \]

The minimum degree of \( S_n \) is 1, the maximum degree of \( S_n \) is \( n \), and \( \chi(S_n) = 2 \). Based on Lemma 1, we determine the lower bound of the local reflexive vertex color strength of graph \( S_n \) as follows.

\[ lrvcs(S_n) \geq \left\lceil \frac{\chi(S_n) + \delta - 1}{1 + \Delta} \right\rceil = \left\lceil \frac{2 + 1 - 1}{1 + n} \right\rceil = \frac{2}{1 + n} = 1. \]

Case 1. For \( n = 3 \), \( \chi(W_3) = 4 \).

Based on Lemma 1, we have the following lower bound.

\[ lrvcs(W_n) \geq \left\lceil \frac{\chi(W_n) + \delta - 1}{1 + \Delta} \right\rceil = \left\lceil \frac{3 + 3 - 1}{1 + 3} \right\rceil = \frac{3}{1 + 3} = 2 \]

Case 2. For odd \( n \geq 4 \), \( \chi(W_n) = 4 \).

Based on Lemma 1, we have the following lower bound.

\[ lrvcs(W_n) \geq \left\lceil \frac{\chi(W_n) + \delta - 1}{1 + \Delta} \right\rceil = \left\lceil \frac{4 + 3 - 1}{1 + n} \right\rceil = \frac{6}{1 + n} = 1 \]

Case 3. For even \( n \), \( \chi(W_n) = 3 \).

Based on Lemma 1, we have the following lower bound.

\[ lrvcs(W_n) \geq \left\lceil \frac{\chi(W_n) + \delta - 1}{1 + \Delta} \right\rceil = \left\lceil \frac{3 + 3 - 1}{1 + n} \right\rceil = \frac{3}{1 + n} = 1 \]

Suppose 1 is \( lrvcs(W_n) \), 1 will be the largest label under the local vertex irregular reflexive labeling on graph \( W_n \). Since 1 is the largest label, then the vertex label and the edge label must be 0 and 1, respectively. Thus, with that condition will give the same weight for all vertices \( v_i \). It contradicts with the condition for any two adjacent vertices must have different weights. Hence \( 1 \neq lrvcs(W_n) \), such that \( lrvcs(W_n) > 1 \). Thus, it concludes that \( lrvcs(W_n) \geq 1 + 1 = 2 \).
Figure 1. (a) a local vertex irregular reflexive coloring of $W_3$ (b) a local vertex irregular reflexive coloring of $W_4$.

Furthermore, we will show the upper bound of vertex irregular reflexive 2-labeling by defining the injection $f$ and $g$ in the following. We construct the labeling on the vertices and edges of $W_3$ and $W_4$ in Figure 1.

$$f(A) = \begin{cases} 2, & \text{if } n = 5, 6, \\ 0, & \text{if } n \text{ otherwise.} \end{cases}$$

$$f(v_i) = \begin{cases} 0, & \text{if } i \text{ odd,} \\ 2, & \text{if } i \text{ even.} \end{cases}$$

$$g(v_iv_{i+1}) = \begin{cases} 1, & \text{if } i \text{ odd,} \\ 2, & \text{if } i \text{ even.} \end{cases}$$

$$g(v_1v_n) = 2, g(Av_i) = 1$$

Based on the functions $f$ and $g$, it gives $lrvc(W_n) \leq 2$. The vertex weights of $W_n$ are

$$wt(A) = \begin{cases} n + 2, & \text{if } n = 5, 6, \\ n, & \text{if } n \text{ otherwise.} \end{cases}$$

$$wt(v_i) = \begin{cases} 4, & \text{for } n \text{ even, if } i \text{ odd,} \\ 6, & \text{for } n \text{ even, if } i \text{ even.} \end{cases}$$

$$wt(v_i) = \begin{cases} 4, & \text{for } n \text{ odd, if } i \text{ odd,} \\ 5, & \text{for } n \text{ odd, if } i = n, \\ 6, & \text{for } n \text{ odd, if } i \text{ even.} \end{cases}$$

It is easy to see that the number of different vertex weights is equal the chromatic number of $W_n$. Since $lrvc(W_n) \geq 2$ and $lrvc(W_n) \leq 2$, it concludes that $lrvc(W_n) = 2$. □

**Theorem 4** Let $F_n$ be a friendship graph. For every positive integers $n \geq 2, lrvc(F_n) = 2$.

**Proof.** The graph $F_n$ is a connected graph with vertex set $V(F_n) = \{A, v_i; 1 \leq i \leq 2n\}$, $|V(F_n)| = 2n + 1$ and edge set $E(F_n) = \{v_iv_{i+1} : 1 \leq i \leq n - 1, i \text{ is odd} \} \cup \{Av_i : 1 \leq i \leq n\}, |E(F_n)| = 3n$. The minimum degree of $F_n$ is 2, the maximum degree of $F_n$ is $2n$, and $\chi(F_n) = 3$. Based on Lemma 1, we determine the lower bound of the local reflexive vertex color strength of graph $F_n$ as follows.
Suppose 1 is \( lrvcs(F_n) \), 1 will be the largest label under the local vertex irregular reflexive labeling on graph \( F_n \). Since 1 is the largest label, then the vertex label and the edge label must be 0 and 1, respectively. Thus, with that condition will give the same weight for all vertices \( v_i \). It contradicts with the condition for any two adjacent vertices must have different weights. Hence \( 1 \neq lrvcs(F_n) \), such that \( lrvcs(F_n) > 1 \). Thus, it concludes that \( lrvcs(F_n) \geq 1 + 1 = 2 \).

Furthermore, we will show the upper bound of vertex irregular reflexive 2-labeling by defining the injection \( f \) and \( g \) in the following:

\[
\begin{align*}
    f(A) &= 2 \\
    f(x_i) &= 0 \\
    f(y_i) &= 2 \\
    g(Ax_i) &= g(Ay_i) = g(x_iy_i) = 1
\end{align*}
\]

Based on the functions \( f \) and \( g \), it gives \( lrvcs(F_n) \leq 2 \). The vertex weights of \( F_n \) are

\[
\begin{align*}
    wt(x_i) &= 2 \\
    wt(y_i) &= 4 \\
    wt(A) &= 2n + 2
\end{align*}
\]

It is easy to see that the number of different vertex weights is equal the chromatic number of \( F_n \). Since \( lrvcs(F_n) \geq 2 \) and \( lrvcs(F_n) \leq 2 \), it concludes that \( lrvcs(F_n) = 2 \). \( \Box \)

**Theorem 5** Let \( L_n \) be a ladder graph. For every positive integers \( n \geq 3 \), \( lrvcs(L_n) = 2 \).

**Proof.** The graph \( L_n \) is a connected graph with vertex set \( V(L_n) = \{u_i, v_i; 1 \leq i \leq n\} \), \( |V(L_n)| = 2n \) and edge set \( E(L_n) = \{u_iu_{i+1}, v_iv_{i+1}; 1 \leq i \leq n - 1\} \cup \{u_iv_i; 1 \leq i \leq n\} \), \( |E(L_n)| = 3n - 2 \). The minimum degree of \( L_n \) is 2, the maximum degree of \( L_n \) is 3, and \( \chi(L_n) = 2 \). Based on Lemma 1, we determine the lower bound of the local reflexive vertex color strength of graph \( L_n \) as follows.

\[
lrvcs(L_n) \geq \left\lfloor \frac{\chi(L_n) + \delta - 1}{1 + \Delta} \right\rfloor = \left\lfloor \frac{2 + 2 - 1}{1 + 2n} \right\rfloor = 1.
\]

Suppose 1 is \( lrvcs(L_n) \), 1 will be the largest label under the local vertex irregular reflexive labeling on graph \( L_n \). Since 1 is the largest label, then the vertex label and the edge label must be 0 and 1, respectively. Thus, with that condition will give the same weight for all vertices apart from \( \{u_1, u_n, v_1, v_n\} \). It contradicts with the condition for any two adjacent vertices must have different weights. Hence \( 1 \neq lrvcs(L_n) \), such that \( lrvcs(L_n) > 1 \). Thus, it concludes that \( lrvcs(L_n) \geq 1 + 1 = 2 \).

Furthermore, we will show the upper bound of vertex irregular reflexive 2-labeling by defining the injection \( f \) and \( g \) in the following:

\[
\begin{align*}
    f(x_i) &= \begin{cases} 
        0, & \text{if } i \text{ even}, \\
        2, & \text{if } i \text{ odd}.
    \end{cases} \\
    f(v_i) &= \begin{cases} 
        0, & \text{if } i \text{ odd}, \\
        2, & \text{if } i \text{ even}.
    \end{cases} \\
    g(u_iv_i) &= \begin{cases} 
        1, & \text{if } 2 \leq i \leq n - 1, \\
        2, & \text{if } i = 1, n.
    \end{cases} \\
    g(u_iu_{i+1}) &= g(v_iv_{i+1}) = 1
\end{align*}
\]

Based on the functions \( f \) and \( g \), it gives \( lrvcs(L_n) \leq 2 \). The vertex weights of \( L_n \) are
\[
wt(u_i) = \begin{cases} 
3, & \text{if } i \text{ even}, \\
5, & \text{if } i \text{ odd},
\end{cases} 
\]

\[
f(v_i) = \begin{cases} 
3, & \text{if } i \text{ odd}, \\
5, & \text{if } i \text{ even},
\end{cases} 
\]

It is easy to see that the number of different vertex weights is equal the chromatic number of \( L_n \). Since \( lrvcs(L_n) \geq 2 \) and \( lrvcs(L_n) \leq 2 \), it concludes that \( lrvcs(L_n) = 2 \). □

4. Concluding Remark
In this paper, we have determined the exact values of the local reflexive vertex color strength of some graphs, namely cycle graph, star graph, wheel graph, friendship graph, and ladder graph. They are all initial research findings in this paper. Since obtaining the local reflexive vertex color strength of some graphs are not easy, even this problem raise to be very complex problem. Thus finding the local reflexive vertex color strength of any other graphs are still widely open, especially doing characterization of the existence of the local vertex irregular reflexive coloring of any graphs. Thus, we propose the following open problems.

**Open Problem 1** Determine the exact value of the local reflexive vertex color strength of some other type of graphs.

**Open Problem 2** Characterize the existence of the local vertex irregular reflexive coloring of any graphs.

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References
[1] Agustin I H, Dafik, Utoyo M I, Slamin, and Venkatachalam M 2021 The Reflexive Edge Strength on Some Almost Regular Graphs *Heliyon* 7
[2] Agustin I H, Utoyo M I, Dafik, and Venkatachalam M 2020 Edge irregular reflexive labeling of some tree graphs *Journal of Physics: Conference Series* 1543
[3] Agustin I H, Utoyo M I, Dafik, Venkatachalam M and Surahmat 2020 On the construction of the reflexive vertex \( k \)-labeling of any graph with pendant vertex *International Journal of Mathematics and Mathematical Sciences* 2020 7812812
[4] Alfarisi R, Ryan J, Siddiqui M K, Dafik and Agustin I H 2021 Vertex irregular reflexive labeling of disjoint union of gear and book graphs *Asian-European Journal of Mathematics* 14(5) 2150078
[5] Ali A and Bača M 2013 On Vertex Irregular Total Labelings *Ars Combinatoria* 112:129-139
[6] A’yun Q, Dafik, Adawiyah R, Agustin I H and Alibirri E R 2021 On the irregular coloring of bipartite graph and tree graph families *J. Phys.: Conf. Ser.* 1836 012024
[7] Bača M, Irfan M, Ryan J, Semaničová-Fešáková A, Tanna D 2017 On Edge Irregular Reflexive Labellings for the Generalized Friendship Graphs *Mathematics* 5(67)
[8] Chartrand G, Zhang P 2016 *Graphs & digraphs, sixth ed.* (Taylor & Francis Group)
[9] Gallian J A 2017 A Dynamic Survey of Graph Labeling *The Electronic Journal of Combinatorics*, 20:1-432.
[10] Levin O 2019 Discrete Mathematics an Open Introduction, 3rd Edition (Greeley: University of Northern Colorado)
[11] Mursyidah I L, Dafik, Adawiyah R, Kristiana A I and Agustin I H 2021 On local irregularity vertex coloring of comb product on star graphs *J. Phys.: Conf. Ser.* 1836 012023
[12] Nisviasari R, Dafik and Agustin I H 2020 The total H-irregularity strength of triangular ladder graphs *Journal of Physics: Conference Series* 1465 012026
[13] Tanna D, Ryan J, Semaničová-Fešáková A and Bača M 2018 Vertex Irregular Reflexive Labeling of Prisms and Wheels *AKCE International Journal of Graphs and Combinatorics*