Experimental update on the exclusive determination of $|V_{cb}|$

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In this talk a review on the exclusive determination of $|V_{cb}|$ is presented. Updated values of this quantity obtained from $\overline{B} \rightarrow D^{(*)} \ell^+ \nu_\ell$ and $B \rightarrow D^{(*)} \ell^+ \nu_\ell$ decays are given. New measurements from B-factories are expected to come soon and the main challenges to improve the accuracy on $|V_{cb}|$ extracted from these decays are discussed.

1 Introduction

One way to determine the CKM matrix element $|V_{cb}|$ is by exclusively measuring the differential decay width of the $\overline{B} \rightarrow D^{(*)} \ell^+ \nu_\ell$ or $B \rightarrow D^{(*)} \ell^+ \nu_\ell$ processes as a function of the kinematic variable $w$, the product of the four-velocities of the initial and final mesons. The latter decay channel has the experimental advantage of a larger branching ratio and less background. Even more profitable is the fact that this decay does not suffer from helicity suppression near $w = 1$, the point where the charmed meson is produced at rest. At this point the theoretical description is well controlled by the Heavy Quark Effective Theory (HQET) [1] and $|V_{cb}|$ can be determined with higher accuracy. However, measurements of $|V_{cb}|$ through $\overline{B} \rightarrow D^{(*)} \ell^+ \nu_\ell$ decays, although more difficult to perform, are also important to check the consistency of the theory.

2 $|V_{cb}|$ from $\overline{B} \rightarrow D^{(*)} \ell^+ \nu_\ell$

The differential decay width of the $\overline{B} \rightarrow D^{(*)} \ell^+ \nu_\ell$ process can be expressed as [2]:

$$\frac{d\Gamma(\overline{B} \rightarrow D^{(*)} \ell^+ \nu_\ell)}{dw} = \frac{G_F^2|V_{cb}|^2}{48\pi^3} \mathcal{K}_{D^{(*)}}(w) \mathcal{F}_{D^{(*)}}^2(w)$$

where $\mathcal{K}_{D^{(*)}}(w)$ is a phase space function and $\mathcal{F}_{D^{(*)}}(w)$ is the form factor describing the $\overline{B} \rightarrow D^{(*)}$ transition. The shape of the form factor cannot be predicted by the theory but it can be constrained by using dispersion relations [2]. HQET gives the normalization of $\mathcal{F}_{D^{(*)}}(w)$ at zero recoil ($w = 1$), where the B and D* wave functions are completely overlapped, to be unity. Taking into account $1/m_Q$ and QCD corrections to the heavy quark limit, the normalization yields [3]:

$$\mathcal{F}_{D^{(*)}}(1) = 0.91 \pm 0.04.$$  \hspace{1cm} (2)

If $\mathcal{K}_{D^{(*)}}(w)$ was not zero at $w = 1$, $|V_{cb}|$ could be directly extracted from the measured differential decay width at this point. As the phase space vanishes in this region, the differential decay width has to be extrapolated, the quality of the extrapolation depending on the quality of the reconstruction efficiency near to the zero recoil point.

2.1 Signal reconstruction

Measurements of $|V_{cb}|$ by reconstructing $\overline{B} \rightarrow D^{(*)} \ell^+ \nu_\ell$ decays have been performed by ALEPH [4], DELPHI [5,6] and OPAL [7] collaborations from $Z \rightarrow bb$ decays, and by CLEO [8] and BELLE [9] collaborations, from $B\overline{B}$ pairs coming from $\Upsilon(4S)$ decays. When the B meson is coming from a $Z$ decay, a large boost is given to the B and to its decay products. Secondary vertices can be better determined than in case of a $\Upsilon(4S)$ decay where the two B mesons are produced practically at rest. On the contrary, the energy of a B coming from a $Z$ cannot be so well determined as if it was coming from a $\Upsilon(4S)$ decay, and the resolution on the $w$ variable deteriorates. To measure the $\overline{B} \rightarrow D^{(*)} \ell^+ \nu_\ell$ differential decay width, leptons and D* candidates are selected. The D* meson is reconstructed by its decay into a D meson and a soft pion, the latter is produced almost at rest in the D* rest frame. The D meson can be reconstructed using several decay channels. In case of experiments working at the Z, the good vertex separation allows the D* to be inclusively reconstructed by mainly detecting the soft pion and few particles from the D0, thus increasing the available statistics. When the B is produced at rest, the soft pion cannot be so well detected and the efficiency for charged pions decreases as $w$ goes to 1. This does not happen for $D^0 \rightarrow D^{(*)}\pi^0$ decays where the soft $\pi^0$ is identified by its decay into two photons. The CLEO collaboration analyzes this channel in addition to the $D^{(*)} \rightarrow D^0\pi^{(*)}$ one.

2.2 Background

The most difficult source of background in $\overline{B} \rightarrow D^{(*)} \ell^+ \nu_\ell$ decays, is due to D*’s coming from excited D** states. Decay properties of resonant and non-resonant D** decays are not well established yet and these decays introduce an important uncertainty in the determination of $|V_{cb}|$. At the $\Upsilon(4S)$ energy, kinematic variables such as the cosine between the B and the D*–lepton system, which are obtained making use of the missing energy measurement and the beam energy as a constraint, can be used to eliminate D**
decays. The contamination from $D^{**}$ is larger at the $Z$ energy and it is more difficult to separate. Experiments use topological variables based on vertex separation and charge correlation to identify this source of background.

2.3 $|V_{cb}|$ measurements

The shape of the form factor $F_D(w)$ entering in $d\Gamma/dw$ is usually expressed in terms of the form factor slope $\rho_D^2$ and of the form factor ratios $R_1$ and $R_2$ [1]. A parameterization of $F_D(w)$, constrained by dispersion relations has been proposed in [2]. The $R_1$ and $R_2$ form factor ratios have been measured by the CLEO collaboration [10]. Experiments use these values and fit $d\Gamma/dw$ to extract $\cal F_D(1)^2|V_{cb}|^2$ and the form factor slope $\rho_D^2$. Figure 1 shows the fit of the unfolded distribution of $\cal F_D(w)|V_{cb}|$ measured by the BELLE collaboration [9] using the form factor of expression given in [2] or a linear parameterization.

Averaging the results of the different experiments, the values of \cal F_D(1)|V_{cb}| and $\rho_D^2$ have been found to be [11]:

$\cal F_D(1)|V_{cb}| = (38.8 \pm 0.5 (stat) \pm 0.9(sys)) \times 10^{-3}$

$\rho_D^2 = 1.49 \pm 0.05(stat) \pm 0.14(sys)$.

Figure 2 shows the results and the world average of the different analyses, scaled to common inputs. Using the value of $F_D(1)$ given in Eq. (2), it yields:

$|V_{cb}| = (42.6 \pm 0.6(stat) \pm 1.0(sys) \pm 2.1(\text{theo})) \times 10^{-3}$.

2.4 Uncertainties

The dominant uncertainty on $|V_{cb}|$ is coming from theory. It is expected that lattice computations will improve the accuracy of the $F_D(1)$ value during the next years. The statistical error will also decrease below 1% as soon as the systematic errors will also decrease below 0.5% as soon as the dominant uncertainty on $|V_{cb}|$ is coming from theory.

![Figure 1](image1.png)

**Figure 1.** Unfolded distribution of $\cal F_D(w)|V_{cb}|$ as function of $w$ measured by the BELLE collaboration.

![Figure 2](image2.png)

**Figure 2.** The $\cal F_D(1)|V_{cb}|$ world average performed by the Heavy Flavor Averaging Group [11], after rescaling the analyses to common input parameters. The labels 'partial reco.' and 'excl.' have been used to distinguish analyses where the $D^0$ is partially or exclusively reconstructed, respectively. The label 'Prel.' refers to the preliminary result of [6]. Only published results are taken into account in the average.

![Figure 3](image3.png)

**Figure 3.** Correlation between $\cal F_D(1)|V_{cb}|$ and $\rho_D^2$ measurements. The ellipses correspond to one sigma deviation. The world average has been performed by the Heavy Flavor Averaging Group [11], after rescaling the analyses to common input parameters. The label 'Prel.' refers to the preliminary result of [6]. Only published results are taken into account in the average.
BABAR and BELLE analyze their total available statistics. The other systematic uncertainties are coming from different sources. The most important contribution originates from errors correlated between the different sources. For $|V_{cb}|$ these are the measurements of $\mathcal{B}(b \to \overline{B}^0_d)$ and $\mathcal{B}(\Upsilon(4S) \to \overline{B}^0_d)$ rates, the $D^{**}$ contribution and the branching fractions of the $D$ decay channels, whereas for $\rho_D^2$ the measurements of $R_1$ and $R_2$ form factor ratios are the dominant error source.

3. $|V_{cb}|$ from $\overline{B} \to D\ell^+\nu_\ell$

$|V_{cb}|$ can also be extracted from the differential width:

$$\frac{d\Gamma(\overline{B} \to D\ell^+\nu_\ell)}{dw} = \frac{G^2_F|V_{cb}|^2}{48\pi^3}\mathcal{K}_D(w)\mathcal{G}_D^2(w)$$

where, analogously to the $\overline{B} \to D^{*+}\ell^-\nu_\ell$ case, $\mathcal{K}_D(w)$ is a phase space function and $\mathcal{G}_D(w)$ is the form factor for the $\overline{B} \to D$ transition. The experimental difficulty in measuring $|V_{cb}|$ from this decay is coming from the large D contribution, from $D^{*}$ decays, to the background, especially at $w \approx 1$, where the $\overline{B} \to D\ell^+\nu_\ell$ decay rate is helicity suppressed. In addition, up to now, the theoretical control on $\mathcal{G}_D(1)$ is weaker than on $F_{D^{*}}(1)$ since the uncertainties coming from lattice computations have not been completely determined. Calculations using sum rules or the quark model find that $\mathcal{G}_D(1)$, unlike $F_{D^{*}}(1)$ which benefits from the Luke’s theorem, is affected by the first order $1/m_Q$ corrections. Averaging the ALEPH [4], CLEO [13] and BELLE [14] measurements, gives [11]:

$$\mathcal{G}_D(1)|V_{cb}| = (42.4 \pm 3.7) \times 10^{-3}, \rho_D^2 = 1.14 \pm 0.16.$$

The contribution of each experiment, scaled to common inputs, can be seen in Figures 4 and 5.

Using $\mathcal{G}_D(1) = 1.04 \pm 0.06$ [11], it yields:

$$|V_{cb}| = (40.8 \pm 3.6(exp) \pm 2.3(theo)) \times 10^{-3}.$$

which is compatible with the value obtained from $\overline{B} \to D^{*+}\ell^-\nu_\ell$ decays.

4. Conclusions

$|V_{cb}|$ has been exclusively measured by different experiments using $\overline{B} \to D\ell^+\nu_\ell$ and $\overline{B} \to D^{*+}\ell^-\nu_\ell$ decays.

The averaged value obtained from $\overline{B} \to D^{*+}\ell^-\nu_\ell$ decays is:

$$|V_{cb}| = (42.6 \pm 1.1(exp) \pm 2.1(theo)) \times 10^{-3}$$

and from $\overline{B} \to D\ell^+\nu_\ell$:

$$|V_{cb}| = (40.8 \pm 3.6(exp) \pm 2.3(theo)) \times 10^{-3}.$$
The dominant uncertainty is coming from the theoretical determination of the form factors at zero recoil, $F_D(1)$ and $G_D(1)$, which are expected to be improved by lattice calculations during the next few years. The experimental uncertainty is limited by systematics due to input parameters such as $\mathcal{B}(b \to B_d)$ and $\mathcal{B}(\Upsilon(4S) \to B_d)$ rates, the $D^{**}$ contribution and the branching fractions of the $D$ decay channels. These quantities have to be better determined to improve the $|V_{cb}|$ accuracy measured from exclusive decays.

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