A Completed Chiral Fermionic Sector Model with Little Higgs

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Abstract

The Standard Model has some intrinsic beauty in the sector of fermions and gauge bosons. Its scalar sector, though minimal, is however haunted by the hierarchy problem. The fermionic spectrum also have two major problems, the flavor problem with its fundamental notion about why there are three families, and the phenomenological limitation of massless neutrinos. We present here a completed chiral fermionic sector model, based on a little Higgs model, that has the plausible potential of addressing all these problems of the SM at an accessible energy scale, and comment briefly on its phenomenology. The focus here is not on the little Higgs part, but rather on the electroweak quarks and leptons from the model, which of course from an important part of the full model.

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Particle theorists have been working on extending the Standard Model (SM) for some thirty years. A major theme in such model-building works is to extend the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry, as exemplified by the classic $SU(5)$ grand unification model. The attempts to go beyond the SM are motivated by many limitations of the SM itself, as much as by our desire to see some other layer of structure in nature. For that matter, the grand desert spanning the next fourteen orders of magnitude in energy/length scale as suggested by the (supersymmetric) gauge unification idea certainly sounds a bit boring to some of us. Does nature has more excitement to offer at the next energy/length scale? Supersymmetry (SUSY) is a beautiful idea, but its low energy implementation does leave much to be desire. However, the supersymmetric SM stays popular over the decades for some good reasons. In our opinion, the major part of it is the fact that it offers a solution to the hierarchy problem (of stabilizing the Higgs masses) without quite compromising the perfect beauty of the (one-family) chiral fermionic spectrum of the SM, while maintaining its phenomenological viability in front of all the precision electroweak data. All those are done by incorporating one basic symmetry — supersymmetry into the SM. To contemplate a really competitive alternative, one needs to take this to the heart.

The first central ingredient of the SM, or its supersymmetric extension, is its gauge symmetry. One perspective of extending that gauge symmetry leads to the “exceptional” embedding sequence of

$$SU(3)_C \times SU(2)_L \times U(1)_Y \subset SU(5) \subset SO(10) \subset E(6) \subset E(7) \subset E(8).$$

From the perspective of the fermionic spectrum, the embedding is vertical, \textit{i.e.} a one-family unification. The highly nontrivial chiral gauge anomaly cancellation among the different fermionic multiplets of the quarks and leptons in the SM is sort of trivialized in the embedding at the $SO(10)$ level, while new fermionic states also started to be required. The most intriguing aspect of the spectrum, the fact that there are three families, remains unexplained. Adding SUSY only make the flavor problem more complicated from the phenomenological point of view. There are also in the literature many horizontal (or family) symmetry models, in which extra symmetries are added to describe flavor physics. However, it is fair to say that only such kind of models with a nonabelian gauge horizontal symmetry address, to some extent, why there are three families. This is again through the issue of nontrivial chiral gauge anomaly cancellation.
Nontrivial three-family \([SU(5)\text{ based}]\) unification models also has an early history\[4\]. Attempts in model-building with an extended gauge symmetry that incorporates a nontrivial family structure are however less popular. Such models, if not unification based, should be more interesting. They could provide much more accessible phenomenology while explaining the flavor structure. And such (flavor) models offer the potential of tackling the hierarchy problem and other limitations of the SM at the same time — the new model we present below may be the first example with some success along the line. We would like to name two particular examples of the kind of models, an \(SU(3)_C \times SU(3)_L \times U(1)_X\) (331) model\[5\] without family changing gauge boson, and an \(SU(4)_A \times SU(3)_C \times SU(2)_L \times U(1)_X\) model and generalizations\[6\] with family changing gauge bosons.

One other limitation of the SM is its massless neutrinos. To explain the neutrino oscillation data, we need beyond SM properties of neutrinos — either couplings with or without extra (singlet) neutrino states. While it can be argue that the generic supersymmetric SM, without the \textit{ad hoc} R parity imposed, naturally solve the problem\[7\], most alternatives requires having extra singlet (or called right-handed) neutrino states at some scale.

We now start the discussion of the construction of a new model that has the potential to address the hierarchy problem, flavor problem, and neutrino mass problem, altogether at an accessible energy scale. The desirability of such a model is well illustrated by the above discussions. The starting point of our construction is a simple, though less than perfect, little Higgs model from Ref.\[8\].

The little Higgs idea is an interesting alternative solution to the hierarchy problem\[9\]. The SM Higgs boson is here to be identified as pseudo-Nambu Goldstone bosons (PNGB) of some global symmetry. Two separate global symmetries, each to be broken by a Higgs vacuum expectation value (VEV), are to be arranged such that a 1-loop (SM) Higgs mass diagram is protected by the (residue) symmetries to be free from quadratic divergence. The idea was motivated by dimensional deconstruction\[10\], though the mechanism may not necessarily follows from the strong interaction dynamics behind as suggested\[11\]. Recently, simple group theoretical constructions of little Higgs models are attempted\[8, 9\]. little Higgs construction looks a bit like a heavy machinery to fix only the hierarchy problem, and would remain a toy model without a completed description of the SM fermionic embedding. The necessarily extended gauge symmetry, to provide quadratic divergent Higgs mass contribution cancellation to the electroweak gauge bosons, means that the embedding
into a consistent model is highly nontrivial, due to gauge anomalies. Building such a model embedding looks similar to the model-building works for the flavor problem discussed above. This is the major focus of the present letter. The consistent fermionic spectrum is also the basis for further studies of what implications the little Higgs structure really have for flavor physics.

We take the simplest model with a $SU(3)_L \times U(1)_X$ gauge symmetry in Ref. [8]. The $t$-$b$ quark doublet of $SU(2)_L \times U(1)_Y$ is to be embedded into a $SU(3)_L$ triplet as follows

$$3_L = \begin{bmatrix} t^a \\ b^a \\ T^a \end{bmatrix}$$

with $X$-charge being $\frac{1}{3}$. The third state is another top-like quark $T$, with the usual electric charge of $Q = \frac{2}{3}$; and $a$ represents the $SU(3)_C$ index. In fact, we have here

$$Q = \frac{1}{2} \lambda_8^a - \frac{1}{2\sqrt{3}} \lambda_8^8 + X ;$$

or $Y = X - \frac{1}{2\sqrt{3}} \lambda_8^8$ ($Q = T_3 + Y$). The vector like QCD spectrum is to be recovered by having the Dirac partners in $SU(3)_L \times U(1)_X$ singlets as

$$1_L = \bar{b}^a, \bar{T'}^a.$$

with $Q = X = \frac{1}{3}$ and $-\frac{2}{3}$ respectively. So far, this is what had been suggested in Ref. [8]. The extra top quark $T$ is exactly what is needed to cancel the 1-loop quadratic divergence in the Higgs mass(es) as required by the little Higgs idea. The electroweak Higgs doublets are embedded into $SU(3)_L \times U(1)_X$ anti-triplets ($\Phi_i$’s) of $X = \frac{1}{3}$. The $\Phi_i$’s bear VEVs that break the gauge symmetry to that of the SM at scale $f$ of about 1 TeV. One expect the Yukawa couplings

$$L_{\text{top}} = y_h \bar{\nu}^a \Phi_1 Q^a + y_2 \bar{T'}^a \Phi_2 Q^a$$

$$= f (y_h \bar{\nu} + y_2 \bar{T'}) T + \frac{i}{\sqrt{2}} (y_h \bar{\nu} - y_2 \bar{T'}) h \begin{pmatrix} t \\ b \end{pmatrix} + \cdots$$

$$= m_T \bar{T} T - i y_h \bar{\nu} h \begin{pmatrix} t \\ b \end{pmatrix} + \cdots$$

Here $Q^a$ denotes the triplet of Eq.(1); and we suppress the color indices after the first line. Both $y_h$ and $y_2$ are expected to be of order one to produced the phenomenological top mass
from electroweak symmetry breaking. The part of the model discussed in this paragraph is all taken from Ref. [8]. We have no much to add, in this short letter, to the little Higgs story but rather prefer to focus on the fermionic part — the full set of SM quarks and leptons. Without further dwelling on the little Higgs part of the model, we refer the readers to Ref. [8] for the details of how the quadratic divergence cancellation works here. We would like to bring the readers attention to a potential limitation of the current model as a little Higgs model though. This is the difficulty with getting a good Higgs quartic coupling. In fact, the latter motivated the authors of Ref. [8] go to the construction of a similar but more preferable $SU(4)_L \times U(1)_X$ model. We will report of constructions of consistent $SU(4)_L \times U(1)_X$ fermion spectra in a coming publication [12].

A question of paramount importance not handled in Ref. [8] is the complete fermionic spectrum under $SU(3)_L \times U(1)_X$.\footnote{In fact, Ref. [8] has a bit of discussion on the other SM fermions, which we find unsatisfactory from the current model-building point of view.} We emphasize again that the issue is nontrivial, due to the chiral gauge anomaly cancellation required. For example, it is quite obvious that a similar quark embedding for the lighter two families does not work. While all representation of $SU(2)$ are real, the complex nature of the $SU(3)_L$ fundamental representation process nontrivial anomaly. The nine $3_L$ representations has anomaly added up that required maybe the same number of $\bar{3}_L$ fermions to cancel it. Adding the large number of fermionic states to the spectrum is highly undesirable. Moreover, one would still has to add more states to take care of the SM leptonic sector. It is not simpler about adding enough representations under the full gauge group $SU(3)_C \times SU(3)_L \times U(1)_X$ to incorporate all the SM chiral fermions and taking care of the $SU(3)_L$ chiral anomaly either. The full fermionic spectrum better be vectorlike at the QCD and QED level or risk predicting the existence of extra massless fermions. And finally, all chiral anomalies, including also the $SU(3)_C$ and $U(1)_X$ parts and all the mixed anomalies, have to be canceled. If a judicially chosen fermionic spectrum satisfying the above cannot be found, the little Higgs model remains a toy model for the scalar sector, incapable of being a consistent particle physics model extending the SM.

However, the family universal structure mentioned above is not necessary, nor desirable. The idea of treating the third family different from the lighter two has been a favorable tool in the kit of family symmetry model-builders, as mentioned in the introduction above. With
the complex $3_L$, one may also used a $\bar{3}_L$ to house a SM doublet. The bonus is that we can have nontrivial anomaly cancellation among the three families, hence bring another major goal of model-builders into the little Higgs game.

Here, we present in this letter a completed chiral fermionic sector model having exactly the above discussed features. We have the quark doublets of the other two families embedded as follow:

$$\bar{3}_L = \begin{bmatrix} d^a \\ u^a \\ D^a \end{bmatrix}, \quad \begin{bmatrix} s^a \\ c^a \\ S^a \end{bmatrix},$$

(5)

$X = 0$ gives the right $U(1)$ charges. The new quarks $D$ and $S$ are also down-type quark (with $Q = -\frac{1}{3}$). Each of the quark states has a matching Dirac partner to keep QCD vectorlike, i.e. we have the singlets

$$1_L = \bar{u}_a, \bar{d}'_a, \bar{D}'_a, \bar{c}_a, \bar{s}'_a, \bar{S}'_a;$$

(6)
each with their $X$-charge given exactly by the electric charge $Q$ [cf. Eq.(2)]. We have then, within the quark sector, three $\bar{3}_L$ in excess. The latter is to be canceled by three (colorless) $3_L$ representations housing the leptonic doublets in a family universal pattern. Explicitly, we have

$$3_L = \begin{bmatrix} \nu_e \\ e^- \\ N_e \end{bmatrix}, \quad \begin{bmatrix} \nu_\mu \\ \mu^- \\ N_\mu \end{bmatrix}, \quad \begin{bmatrix} \nu_\tau \\ \tau^- \\ N_\tau \end{bmatrix},$$

(7)

with $X = -\frac{1}{3}$. The full spectrum is completed with the three leptonic singlets as

$$1_L = e^+, \mu^+, \tau^+,$$

(8)

with $X = 1$.

One can easily check that all the potentially dangerous triangle anomalies, $(3_e)^3$, $(3_\mu)^2X$, $(3_\tau)^3$, $(3_\mu)^2X$, $X$-trace, and $X^3$ — notation self explanatory, do cancel. We have illustrated the cancellation of the $(3_e)^3$ anomaly in the construction above. The $(3_e)^3$ anomaly is absent for we do require the QCD spectrum to be vectorlike. The nontrivial $(3_\mu)^2X$ anomaly contribution from the $Q^a$ triplets is canceled by that from the three families of leptonic triplets [cf. Eqs.(1) and (7)]. The cancellation of the remaining anomalies are quite nontrivial and take some algebra. The basic feature of cancellation among the three families persists. In particular the $(3_e)^3$, $(3_\mu)^2X$, and $X^3$ anomalies for each family is nonvanishing. The spectrum
and anomaly cancellation structure share quite a bit of similarity with the 331 model\cite{5}, which is a major inspiration for our model construction. Apart from the little Higgs perspective incorporated in our new 331 model, there are other major differences between the two fermionic model spectra. The present fermionic spectrum is, arguably, phenomenologically more interesting — an aspect that we will then turn our discussion to.

Before going into the different phenomenological predictions of our new 331 model, however, we will discuss briefly the similarity in the group theoretical structure it shares with the original 331 model\cite{5}. This may shed some light on model-building. To put it in an oversimplifying statement, one can say that our model is nothing more than the original 331 model-building idea with a twist — a different enlargement of the quark representations dictated by our interest in accommodating the little Higgs. Taking the $Q^a$ triplet with the 331 construction scheme, the rest more or less follows. The nontrivial anomaly cancellation among the three families is intrinsic to the basic strategy of the $SU(2)_L \times U(1)_Y$ into $SU(3)_L \times U(1)_X$ embedding with new quarks. It works, largely because the 3 in $SU(3)_C$ is the same as the number of SM families. The fact that such construction works, in either case, is quite amazing.\footnote{See, however, detailed analysis of the gauge anomaly structure of the kind of SM embeddings in our forthcoming publication\cite{12}.}

A major phenomenological difference between our new model and the original 331 model is the fermionic content. The old 331 model has extra quarks of electric charges $\frac{5}{3}$ and $\frac{-4}{3}$ as the only fermions beyond the three-family SM spectrum. In our model, there is the same number of extra quarks. They are, however, just some duplications of the existing $t$, and $d$ and $s$ quarks — or rather two more down-sector quarks, without exotic electric charges. Recall that the extra top quark $T$ is demanded by the little Higgs mechanism\cite{8}. There has been many discussions in the literature about extra down-sector quarks from both theoretical\cite{13} point of view, as well as experimental in which one tries to explain the issues related to the $b$ quark $Z$-width\cite{14}. Hence, the different quark content of the new model actually looks very desirable. In the new model, we have also extra leptonic states — three new neutral fermions $N_e$, $N_\mu$, and $N_\tau$, which are essentially singlet neutrinos. This, together with extra interactions of the SM neutrinos, may provide the base for interesting beyond SM properties of neutrinos. The gauge boson sector of course plays an important role in the little Higgs

\footnote{See, however, detailed analysis of the gauge anomaly structure of the kind of SM embeddings in our forthcoming publication\cite{12}.}
mechanism, and is also well discussed in that aspect in Ref. [8]. While the original 331 model has doubly charged gauge bosons that may provide interesting experimental signature [5], the present model has no extra gauge bosons of exotic charges. The five extra gauge bosons are rather a pair of $W'$ and three extra $Z'$s.

Next we look into the possible couplings of the scalar $\Phi_i$ multiplets to the fermions. Such couplings are responsible for the SM Yukawa couplings, and the basic properties of the extra singlet quarks and neutrinos. Besides the top, the bottom quark has to get its Yukawa coupling from the $\bar{b} \Phi_i^\dagger \Phi_j^\dagger Q^a$ term, hence the desired suppression in its mass (after electroweak symmetry breaking). For the first two families the $1_L \Phi_i^\dagger \bar{3}_L$ terms are naively allowed for all the down type quarks. Compare against Eq. (4), the couplings, if allowed, might need some fine-tuning to keep the down and strange quarks light. In fact, a splitting in mass between $d$ and $D$ as well as $s$ and $S$ is necessary for fitting SM phenomenology. This is an important issue to be investigated in detail. Similar to the case of the bottom quark, up and charm quark have their masses from couplings of the form $1_L \Phi_i^\dagger \bar{3}_L$ at lowest order. These structures are not enough to produce the hierarchical quark mass pattern. However, the little Higgs structure has considerations of a different global $SU(3)$ for each $\Phi_i$ (which is not respected by the gauge symmetry itself). One may consider a full description of these global symmetries for the full Lagrangian and see if they can be used to help getting a more viable phenomenology without employing un-natural small couplings for some of the terms here discussed admitted by the gauge symmetry along. As for the leptonic sector, the representation structure is family universal. The lowest order admissible coupling is of the form $\ell^+ \Phi_i^\dagger \Phi_j^\dagger L$ for $\ell^+$ and $L$ representing the singlet and triplet leptons. Neutrino mass constructions looks nontrivial, and may have to be considered at loop level. The above is limiting the scalar sector to the two $\Phi_i$’s. One may consider adding extra $SU(3)_L \times U(1)_X$ to $SU(2)_L \times U(1)_Y$ symmetry breaking Higgs multiplets, which would likely change the picture. Care would have to be taken to ensure the little Higgs mechanism is preserved intact though. Our discussion on the phenomenology for the fermions has to stop here in this short letter.

In summary, we have presented a consistent fermionic sector model of $SU(3)_C \times SU(3)_L \times U(1)_X$ with chiral fermions giving rise to quarks and leptons of the three-family SM plus some extra quarks and neutrinos that are singlets under electroweak symmetry. The extra quarks are a top-like quark and two down-sector quarks. Group theoretically speaking, the model is a simple twist of the old 331 model with the same gauge symmetry. The new model
is however motivated by solving the hierarchy problem using the little Higgs mechanism, and seems also to be having more desirable phenomenological properties apart from that. It may provide, from the theoretical point of view, a TeV scale solution that address both the hierarchy problem and the flavor problem successfully; and may also gives feasible solution to the experimentally required beyond SM properties of neutrinos such as mass oscillation. In our opinion, more model-building of the type, and careful phenomenological studies of the successful models, deserve up most attention from particle physics.

NOTE: After posting the first draft of our manuscript, we came to realize that the 331 model fermion spectrum obtained here has actually been available in the literature\textsuperscript{15}. These earlier works having no connection with the little Higgs idea which motivates our rediscovery though. Accordingly, the scalar spectrum and Yukawa couplings discussed are not the same as that discussed here, as required by the little Higgs mechanism.

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\[1\] S. Glashow and H. Georgi, Phys. Rev. Lett. \textbf{32}, 438 (1974).

\[2\] See, for example, A. Aranda, C.D. Carone, and, P. Meade, Phys. Rev. \textbf{D65}, 013011 (2002) and references therein.

\[3\] For examples of (vertical) unification and non-unification type models, see P.H. Frampton and O.C.W. Kong, Phys. Rev. Lett. \textbf{77}, 1699 (1996) and P.H. Frampton and O.C.W. Kong, Phys. Rev. Lett. \textbf{75}, 781 (1995), respectively.

\[4\] See, for examples, H. Georgi, Nucl. Phys. \textbf{B156}, 126 (1979); P.H. Frampton and S. Nandi, Phys. Rev. Lett. \textbf{43}, 1460 (1979).

\[5\] P.H. Frampton, Phys. Rev. Lett. \textbf{69}, 2889 (1992); F. Pisano and V. Pleitez, Phys. Rev. \textbf{D46}, 410 (1992).

\[6\] O.C.W. Kong, Mod. Phys. Lett. \textbf{A11}, 2547 (1996); Phys. Rev. \textbf{D55}, 383 (1997).

\[7\] See, for example, O.C.W. Kong, Int. J. Mod. Phys. A (2003) \textit{to be published}\textsuperscript{a}, for a discussion of the perspective, and S.K. Kang and O.C.W. Kong, \texttt{hep-ph/0206009} for a detailed analysis of possible neutrino mass contributions.

\[8\] D.E. Kaplan and M. Schmaltz, \texttt{hep-ph/0302049}
[9] See, for a recent review, M. Schmaltz, hep-ph/0210415, and references therein.

[10] N. Arkani-Hamed, A.G. Cohen, and H. Georgi, Phys. Rev. Lett. 86, 4757 (2001).

[11] K. Lane, Phys. Rev. D65, 115001 (2002).

[12] O.C.W. Kong, NCU-HEP-k010, manuscript in preparation.

[13] For examples, from $E_6$ unification [see J.L. Hewett and T.G. Rizzo, Phys. Rept. 183, 194 (1989)] and strong CP considerations [see P.H. Frampton and T.W. Kephart, Phys. Rev. Lett. 66, 1666 (1991) O.C.W. Kong and B.D. Wright, Phys. Rev. D58, 015002 (1998)].

[14] C.-H V. Chang, D. Chang, and W.-Y. Keung, Phys. Rev. D54, 7051 (1996); D. Chang, W.-F. Chang, and E. Ma, Phys. Rev. D61, 037301 (2000).

[15] M. Singer, J.W.F. Valle, and J. Schechter, Phys. Rev. D22, 738 (1980); R. Foot, H.N. Long, and T.A. Tran, Phys. Rev. D50, R34 (1994).