Constraints on large-scale inhomogeneities from WMAP5 and SDSS: confrontation with recent observations

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Accepted 2009 September 6. Received 2009 September 2; in original form 2009 February 4

ABSTRACT
Measurements of the Type Ia supernovae Hubble diagram which suggest that the Universe is accelerating due to the effect of dark energy may be biased because we are located in a 200–300 Mpc underdense ‘void’ which is expanding 20–30 per cent faster than the average rate. With the smaller global Hubble parameter, the Wilkinson Microwave Anisotropy Probe 5 data on cosmic microwave background (CMB) anisotropies can be fitted without requiring dark energy if there is some excess power in the spectrum of primordial perturbations on 100 Mpc scales. The Sloan Digital Sky Survey (SDSS) data on galaxy clustering can also be fitted if there is a small component of hot dark matter in the form of 0.5 eV mass neutrinos. We show however that if the primordial fluctuations are Gaussian, the expected variance of the Hubble parameter and the matter density are far too small to allow such a large local void. Nevertheless, many such large voids have been identified in the SDSS Luminous Red Galaxy survey in a search for the late integrated Sachs–Wolfe effect due to dark energy. The observed CMB temperature decrements imply that they are nearly empty, thus these real voids too are in gross conflict with the concordance Λ cold dark matter model. The recently observed high peculiar velocity flow presents another challenge for the model. Therefore, whether a large local void actually exists must be tested through observations and cannot be dismissed a priori.

Key words: cosmic microwave background – cosmological parameters – cosmology: theory – dark matter – large-scale structure of Universe.

1 INTRODUCTION

The Einstein–de Sitter (E–deS) universe with Ωm = 1 is the simplest model consistent with the spatial flatness expectation of inflationary cosmology. However, Type Ia supernovae (SNe Ia) at redshift z ≃ 0.5 appear ~25 per cent fainter than expected in an E–deS universe (Riess et al. 1998; Perlmutter et al. 1999). Together with measurements of galaxy clustering in the 2df survey (Efstathiou et al. 2002) and of cosmic microwave background (CMB) anisotropies by the Wilkinson Microwave Anisotropy Probe (WMAP) (Spergel et al. 2003), this has established an accelerating universe with a dominant cosmological constant term (or other form of ‘dark energy’) which presumably reflects the present microphysical vacuum state. This ‘concordance’ Λ cold dark matter (ΛCDM) cosmology (with ΩΛ ≃ 0.7, Ωm ≃ 0.3, h ≃ 0.7) has passed a number of cosmological tests, including baryonic acoustic oscillations (BAOs) (Eisenstein et al. 2005) and measurements of mass fluctuations from clusters and weak lensing (e.g. Contalba, Hoekstra & Lewis 2003). Further observations of both SNe Ia (Riess et al. 2004; Astier et al. 2006; Wood-Vasey et al. 2007) and the WMAP 3-year results (Spergel et al. 2007) have continued to firm up the model. However, there is no physical basis for this model, in particular there are two fundamental problems with the notion that the universe is dominated by vacuum energy. The first is the notorious fine-tuning problem of vacuum fluctuations in quantum field theory – the energy scale of the cosmological energy density is ~10^-12 GeV, many orders of magnitude below the energy scale of ~10^2 GeV of the standard model of particle physics, not to mention the Planck scale of ~10^19 GeV (see Weinberg 1999). The second is the equally acute coincidence problem: since ρΛ/ρm evolves as the cube of the cosmic scalefactor a, there is no reason to expect it to be of O(1) today, yet this is apparently the case. In fact what is actually inferred from observations is not an energy density, just a value of O(H_0^2) for the otherwise unconstrained Λ term in the Friedmann equation. It has been suggested that this may simply be an artefact of interpreting cosmological data in the (oversimplified) framework of a perfectly homogeneous universe in which H_0 ~ 10^{-42} GeV ~ (10^{28} cm)^{-1} is the only scale in the problem (Sarkar 2008).

In fact the WMAP results alone do not require dark energy if the assumption of a scale-invariant primordial power spectrum is relaxed. This assumption is worth examining given our present
ignorance of the physics behind inflation. We have demonstrated (Hunt & Sarkar 2007) that the temperature angular power spectrum of an E–deS universe with \( h \approx 0.44 \) matches the WMAP data well if the primordial power is enhanced by \( \sim 30 \) per cent in the region of the second and third acoustic peaks (corresponding to spatial scales of \( k \approx 0.01–0.1 \) Mpc\(^{-1}\)). This alternative model with no dark energy actually has a slightly better \( \chi^2 \) for the fit to WMAP3 data than the ‘concordance power-law ΛCDM model’ and, in spite of having more parameters, has an equal value of the Akaike information criterion (AIC) used in model selection. Other E–deS models with a broken power-law spectrum (Blanchard et al. 2003) have also been shown to fit the WMAP data. Moreover, an E–deS universe can fit measurements of the galaxy power spectrum if it includes an \( \sim 10 \) per cent component of hot dark matter (HDM) in the form of massive neutrinos of mass \( \sim 0.5 \) eV (Blanchard et al. 2003; Hunt & Sarkar 2007). Clearly the main evidence for dark energy comes from the SNe Ia Hubble diagram.

A mechanism that sets \( \Lambda = 0 \) is arguably more plausible than one which leads to the tiny energy density \( \rho_\Lambda \approx 10^{-47} \) GeV\(^4\) associated with the concordance cosmology.\(^1\) If \( \Lambda \) is indeed zero, then perhaps some effect fools us into wrongly deducing the existence of dark energy by mimicking a non-zero cosmological constant. It is natural to connect this effect with inhomogeneities since cosmic acceleration and large-scale non-linear structure formation appear to have commenced simultaneously. This approach offers the possibility of solving the cosmological constant problems within the framework of general relativity and keeps the introduction of new physics to a minimum.\(^2\) Several different ways in which inhomogeneities could potentially mimic dark energy have been considered in the literature – for reviews see Celerier (2007), Buchert (2008) and Enqvist (2008). In an inhomogeneous universe, averaged quantities satisfy modified Friedmann equations which contain extra terms corresponding to ‘backreaction’ since the operations of spatial averaging and time evolution do not commute (Buchert 2000). The backreaction terms depend upon the variance of the local expansion rate, and hence increase as inhomogeneities develop. Whether backreaction can indeed account for the apparent cosmological acceleration is hotly debated and remains an open question at present (Wetterich 2003; Ishibashi & Wald 2006; Khosravi, Kourkchi & Mansouri 2007; VanderVeld, Flanagan & Wasserman 2007; Will 2007; Behrend, Brown & Robbers 2008; Leith, Ng & Wiltshire 2008; Paranjpe & Singh 2008; Rasana 2008).

Another possibility is that inhomogeneities affect light propagation on large scales and cause the luminosity distance-redshift relation to resemble that expected for an accelerating universe. This has been investigated as a Swiss-cheese” universe in which voids modelled by patches of Lemaître–Tolman–Bondi (LTB) space–time are distributed throughout a homogeneous background. However, the results seem to be model-dependent: some authors find the change in light propagation to be negligible because of the cancellation effects (Biswas & Notari 2008; Brouzas & S. Sarkar 2007) that the temperature angular power spectrum of an E–deS universe with \( h \approx 0.44 \) matches the WMAP data well if the primordial power is enhanced by \( \sim 30 \) per cent in the region of the second and third acoustic peaks (corresponding to spatial scales of \( k \approx 0.01–0.1 \) Mpc\(^{-1}\)). This alternative model with no dark energy actually has a slightly better \( \chi^2 \) for the fit to WMAP3 data than the ‘concordance power-law ΛCDM model’ and, in spite of having more parameters, has an equal value of the Akaike information criterion (AIC) used in model selection. Other E–deS models with a broken power-law spectrum (Blanchard et al. 2003) have also been shown to fit the WMAP data. Moreover, an E–deS universe can fit measurements of the galaxy power spectrum if it includes an \( \sim 10 \) per cent component of hot dark matter (HDM) in the form of massive neutrinos of mass \( \sim 0.5 \) eV (Blanchard et al. 2003; Hunt & Sarkar 2007). Clearly the main evidence for dark energy comes from the SNe Ia Hubble diagram.

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1. “Quintessence” models, which attempt to address the coincidence problem, also assume that every other contribution to the vacuum energy cancels apart from that of the quintessence field.
2. In models that seek to explain the observations through modifications of gravity, the relevant scale of \( H_0 \) has to be introduced by hand, just as in quintessence models the quintessence field has to be given a mass of the order of \( H_0 \) these are technically unnatural choices since this is an infrared scale for any microphysical theory.
(Rudnick, Brown & Williams 2007). Recently, a large number of voids of varying sizes have been identified in the SDSS Luminous Red Galaxy (LRG) catalogue in a search for the late integrated Sachs–Wolfe (ISW) effect due to dark energy (Granett, Neyrinck & Szapudi 2008b).

How likely is the existence of such huge voids according to standard theories of structure formation? Statistical measures of the void distribution such as the void probability function and underdense probability function have been estimated from the Two-degree Field Galaxy Redshift Survey (2dFGRS), SDSS and Deep Extragalactic Evolutionary Probe 2 (DEEP2) galaxy redshift surveys (Croton et al. 2004; Hoyle & Vogeley 2004; Conroy et al. 2005; Patiri et al. 2005; Tikhonov 2006; Tinker et al. 2008; Tikhonov 2007; von Benda-Beckmann & Mueller 2007). Void probability statistics have also been examined theoretically using analytical methods (Sheth & van de Weygaert 2004; Furlanetto & Piran 2006; Shandarin et al. 2006) and N-body simulations (Little & Weinberg 1994, Schmidt et al. 2000; Arbabí-Bidgoli & Mueller 2002; Benson et al. 2003; Padilla, Ceccarelli & Lambas 2005). However, such studies have been restricted to voids with radii of 10–30 Mpc. The scales of the large voids we are considering lie in the linear regime where the variance of the Hubble contrast is directly related to the matter power spectrum $P_m(k)$. It has been noted (using results from Turner, Cen & Ostriker 1992) that above 100 Mpc linear theory predictions agree well with N-body simulation results, although on smaller scales the Hubble contrast is underestimated due to non-linear effects (Shi, Widrow & Dursi 1996). Applying the linear theory and using the measured CMB dipole velocity, Wang, Spergel & Turner (1998) obtained the model-independent result $\delta_{\text{lin}}^2(r) < 10.5 h^{-1} \text{Mpc}/r$ in a sphere of radius $r$. This ought to be an acceptable procedure up to scales of the order of 800 $h^{-1}$ Mpc – on larger scales, relativistic corrections become increasingly important.) In this paper, we update these results by determining the probability distribution of $\delta_{\text{lin}}$ and the density contrast on various scales using constraints on $P_m(k)$ from WMAP 5-year data (Komatsu et al. 2008) and the SDSS galaxy power spectrum (Tegmark et al. 2004). We find that even the ‘minimal local void’ is extremely unlikely if the primordial density perturbation is indeed Gaussian as is usually assumed and the other LTB model voids even less so. However by the same token, the ISW effect due to the voids seen in the SDSS LRG survey (Granett et al. 2008b) appears to be too strong. Moreover, observed large-scale peculiar velocities appear to be much higher than expected (Kashlinsky et al. 2008; Watkins, Feldman & Hudson 2008). It would appear that the standard model of structure formation itself needs re-examination, hence the existence of a large local void cannot be dismissed on these grounds.

2 MODELS

We study variations of the Hubble parameter in the context of two different cosmological models, both of which fit the WMAP and SDSS data but have different amounts of power on spatial scales of $\mathcal{O}(100)$ Mpc. The intention is to examine whether previous conclusions concerning the magnitude of such variations (Wang et al. 1998) can be circumvented in an unorthodox model.

Our first model is the standard $\Lambda$CDM concordance model with a power-law primordial power spectrum. The spectral index and amplitude $P_R$ of the comoving curvature perturbation spectrum are evaluated at a pivot point of $k = 0.05$ Mpc$^{-1}$. The second model is dubbed the ‘CHDM bump model’ since it has both cold and hot dark matter and a ‘bump’ in the primordial spectrum. It was developed by us (Hunt & Sarkar 2007) based upon the supergravity multiple inflation scenario in which ‘flat direction’ fields undergo gauge symmetry-breaking phase transitions during inflation triggered by the fall in temperature (Adams, Ross & Sarkar 1997; Hunt & Sarkar 2004). Each flat direction $\psi$ has a gravitational strength coupling to the inflaton $\phi$, giving a contribution to the potential of the form $V \sim \frac{1}{2} \lambda \phi^2 \psi^2$. The flat directions are lifted by supergravity corrections and non-renormalizable superpotential terms. Thus, when a phase transition occurs the flat direction evolves rapidly from the origin where it was trapped by thermal effects to the global minimum of the potential. Each phase transition changes the effective inflaton mass from $m_0^2$ to $m_1^2 - \lambda \langle \psi \rangle^2$. Since the primordial power spectrum is very sensitive to the inflaton mass, this can introduce features into the spectrum. We showed that two flat directions $\psi_1$ and $\psi_2$ which cause successive phase transitions about two e-folds apart and create a small bump in the power spectrum centred on $k \approx 0.03$ h Mpc$^{-1}$, allow an E–deS model with $h = 0.44$ to fit the WMAP data (Hunt & Sarkar 2007). The effective scalar potential is

$$V(\phi, \psi_1, \psi_2) = \begin{cases} V_0 - \frac{1}{2} m_0^2 \phi^2, & t < t_1, \\ V_0 - \frac{1}{2} m_0^2 \phi^2 - \frac{1}{2} \mu_1^2 \psi_1^2 + \frac{\lambda_1}{m_0^2} \phi \psi_1, & t_1 \leq t \leq t_2, \\ V_0 - \frac{1}{2} m_0^2 \phi^2 - \frac{1}{2} \mu_2^2 \psi_2^2 + \frac{\lambda_2}{m_0^2} \phi \psi_2, & t_2 \leq t, \end{cases}$$

where $t_1$ and $t_2$ are the times at which the first and second phase transitions begin, $\lambda_1$ and $\lambda_2$ are the couplings between $\phi$ and the flat directions, $\gamma_1$ and $\gamma_2$ are the coefficients of the non-renormalizable terms of the order of $n_1$ and $n_2$, and $V_0$ is a constant which dominates the potential. In the slow-roll approximation, the height of the bump is $P_R^{(1)}$ and the amplitude of the primordial perturbation spectrum to the left and right of the bump is $P_R^{(0)}$ and $P_R^{(2)}$, respectively, where

$$P_R^{(0)} = \frac{9 H^6}{4 \pi^2 m^2 \phi_0^2},$$

$$P_R^{(1)} = \frac{P_R^{(0)}}{(1 - \Delta m_1^2)^2},$$

$$P_R^{(2)} = \frac{P_R^{(0)}}{(1 - \Delta m_1^2 + \Delta m_2^2)^2},$$

where $\phi_0$ is the initial value of $\phi$ and

$$\Delta m_1^2 = \frac{\lambda_1}{m_0^2} \left( \frac{\mu_1^2 M_0^{n_1-4}}{n_1 \gamma_1} \right)^{2/(n_1-2)}$$

and

$$\Delta m_2^2 = \frac{\lambda_2}{m_0^2} \left( \frac{\mu_2^2 M_0^{n_2-4}}{n_2 \gamma_2} \right)^{2/(n_2-2)}$$

are the fractional changes in the inflaton mass-squared due to the phase transitions. The bump lies approximately between the wavenumbers $k_1$ and $k_2$, where $k_2 = k_1 e^{i(\nu-\nu_1)}$. In this paper, we set $\gamma_1$ and $\gamma_2$ equal to unity, $m_0 = 0.005 H^2$, $\phi_0 = 0.01 M_0$, $\mu_1^2 = 3 H^2$ and $\lambda_1 = \lambda_2 = H^2/M_0^2$ throughout as in our earlier work (Hunt & Sarkar 2007). In fitting to the WMAP5 data, we also consider continuous (non-integral) values of $n_1$ and $n_2$ to determine
whether a different shape of the ‘bump’ gives a better fit, keeping in mind that its physical origin may be different from multiple inflation (Chung et al. 2000; Lesgourgues 1999; Easther et al. 2001; Kaloper & Kaplinghat 2003; Gong 2005; Wang et al. 2005; Ashoorian & Krause 2006; Bean et al. 2008).

A pure cold dark matter (CDM) model exhibits excessive galaxy clustering on small scales. Therefore, it is necessary to include an HDM component which suppresses structure formation below the free-streaming scale. We obtain a good match to the shape of the SDSS galaxy power spectrum with three neutrino species of mass \( \sim 0.5 \) eV. Hence, the CHDM bump model has \( \Omega_\text{b} \simeq 0.1, \quad \Omega_\text{v} \simeq 0.1 \) and \( \Omega_\Lambda \simeq 0.8 \) (Hunt & Sarkar 2007).

3 THE DATA SETS

We fit to the WMAP 5-year (Nolta et al. 2008) temperature–temperature (TT), temperature–electric polarization (TE), and electric–electric polarization spectra. Compared to the WMAP3 results, the WMAP5 measurement of the TT spectrum is \( \sim 2.5 \) per cent higher in the region of the acoustic peaks due to the revised beam transfer functions, and the third acoustic peak is determined more accurately. Polarization measurements are improved by the use of data from an additional bandpass.

We also fit the linear matter power spectrum \( P_m(k) \) to the measurement of the real-space galaxy power spectrum \( P_g(k) \) in the SDSS (Tegmark et al. 2004).

4 METHOD

The Hubble contrast \( \delta_H \) smoothed over a sphere of radius \( R \) is (Shi et al. 1996)

\[
\delta_H(x) = \int d^3 y \frac{v(y)}{H_0} \frac{y - x}{|y - x|^2} W_R(y - x),
\]

where \( v \) is the peculiar velocity field and \( W_R \) is the ‘top hat’ window function,

\[
W_R(x) = \begin{cases} 
3/(4\pi R^3), & |x| \leq R, \\
0, & |x| > R. 
\end{cases}
\]

Using the linear perturbation theory (Peebles 1993), it can be shown that the variance of \( \delta_H \) is related to the matter power spectrum as (Wang et al. 1998)

\[
\langle \delta_H^2 \rangle R = \frac{f^2}{2\pi^2} \int_0^\infty dk \, k^2 P_m(k) W_H^2(k R).
\]

Here, the window function \( W_H \) is

\[
W_H(k R) = \frac{3}{k R^3} \left( \sin k R - \int_0^R dy \frac{\sin y}{y} \right),
\]

and the dimensionless linear growth rate \( f \) for an \( \Lambda \)CDM universe can be approximated by (Lahav et al. 1991; Hamilton 2001)

\[
f(\Omega_m, \Omega_\Lambda) \simeq \Omega_m^{1/7} + \Omega_\Lambda \frac{1 + \Omega_m}{2}.
\]

Similarly, the variance of the density contrast \( \delta \equiv (\rho_m - \rho_{\text{out}}) / \rho_{\text{out}} \) in a sphere of radius \( R \) is

\[
\langle \delta^2 \rangle R = \frac{1}{2\pi^2} \int_0^\infty dk \, k^2 P_m(k) W^2(k R),
\]

where the window function

\[
W(k R) = \frac{3}{k R^3} \left( \sin k R - k R \cos k R \right)
\]

is the Fourier transform of \( W_R \).

The variance of the peculiar velocity is given by

\[
\langle v^2 \rangle R = \frac{f^2 H_0^2}{2\pi^2} \int_0^\infty dk \, k^2 P_m(k) W^2(k R).
\]

Finally, we also consider \( \Omega_m = 8\pi G \rho_m / H_0^2 \), the ratio of the matter density to the critical density as measured locally by an observer inside the void (Wang et al. 1998). The variance of the perturbation \( \delta_{\text{2D}}(\Omega_m - \Omega_m) / \Omega_m \) is then

\[
\langle \delta_{\text{2D}}^2 \rangle R = \frac{1}{2\pi^2} \int_0^\infty dk \, k^2 P_m(k) W^2_2(k R),
\]

where

\[
W_2(k R) = \frac{3}{k R^3} \left[ (2f - 1) \sin k R \\
+ k R \cos k R + 2f \int_0^{k R} dy \frac{\sin y}{y} \right].
\]

We use the Monte Carlo Markov Chain (MCMC) approach to cosmological parameter estimation, which is a method for drawing samples from the posterior distribution \( P(\omega \mid \text{data}) \) of the cosmological parameters \( \omega \), given the data. For a discussion of the MCMC likelihood analysis, see appendix B of Hunt & Sarkar (2007). Given \( n \) samples \( \omega^{(i)} \) the best estimate for the distribution is

\[
P(\omega \mid \text{data}) \simeq \frac{1}{n} \sum_{i=1}^{n} \delta^2(\omega - \omega^{(i)}),
\]

where \( \delta^2 \) is the Dirac delta function.

4 http://camb.info

5 http://cosmologist.info/cosmomc/
The priors adopted on the base Monte Carlo parameters of the various models, as well as on the derived parameters: the Hubble constant and the age of the Universe.

| Parameter | Model | Lower limit | Upper limit | Lower limit | Upper limit | Lower limit | Upper limit |
|-----------|-------|-------------|-------------|-------------|-------------|-------------|-------------|
| $\Omega_b h^2$ | $\Lambda$CDM power law | 0.005 | 0.1 | 0.005 | 0.1 | 0.005 | 0.1 |
| $\Omega_c h^2$ | | 0.01 | 0.99 | | | | |
| $\theta$ | | 0.5 | 10.0 | 0.5 | 10.0 | 0.5 | 10.0 |
| $\tau$ | | 0.01 | 0.8 | 0.01 | 0.8 | 0.01 | 0.8 |
| $f_v$ | | 0.01 | 0.3 | 0.01 | 0.3 | 0.01 | 0.3 |
| $n_s$ | | 0.5 | 1.5 | | | | |
| $10^4 k_1 (\text{Mpc}^{-1})$ | | 0.01 | 600 | 0.01 | 600 | | |
| $10^4 k_2 (\text{Mpc}^{-1})$ | | 0.01 | 800 | 0.01 | 1100 | | |
| $\ln\left(10^{10} P_R\right)$ | | 2.7 | 4.0 | | | | |
| $\ln\left(10^{10} P_R^{(0)}\right)$ | | 2.0 | 6.0 | 2.0 | 6.0 | 2.0 | 6.0 |
| $\ln\left(10^{10} P_R^{(1)}\right)$ | | 2.0 | 6.0 | | | | |
| $\ln\left(10^{10} P_R^{(2)}\right)$ | | 2.0 | 6.0 | | | | |
| $h$ | | 0.4 | 1.0 | 0.1 | 1.0 | 0.1 | 1.0 |
| Age (Gyr) | | 10.0 | 20.0 | 10.0 | 20.0 | 10.0 | 20.0 |

Using equation (17), this is approximated by

$$P(\delta|\text{data})_R = \frac{1}{n} \sum_{i=1}^{n} P\left[\delta_\text{H}^{(i)} | \mathbf{w}\right]_R,$$

(19)

where

$$P(\delta_\text{H} | \mathbf{w})_R = \frac{1}{\sqrt{2\pi \langle \delta^2_\text{H} \rangle_R}} \exp\left(-\frac{\delta_\text{H}^2}{2 \langle \delta^2_\text{H} \rangle_R}\right).$$

(20)

We calculate the probability distribution $P(\delta|\text{data})_R$ in the same way.

Flat priors are used on the parameters listed in Table 1. Here, $\theta$ is the ratio of the sound horizon to the angular diameter distance to last scattering (multiplied by 100), $\tau$ is the optical depth (due to reionization) to the last scattering surface and $f_v = \Omega_v / \Omega_d$ is the fraction of dark matter in the form of neutrinos, where the total dark matter density is $\Omega_d \equiv \Omega_c + \Omega_\nu$. We assume the chains have converged when the Gelman–Rubin ‘R’ statistic falls below 1.02. We evaluate the sum in equation (19) when post-processing the chains.

5 RESULTS

The mean values of the marginalized cosmological parameters together with their 68 per cent confidence limits are listed in Table 2. As in our previous work (Hunt & Sarkar 2007), we also list the value of the AIC relative to the $\Lambda$CDM power-law model. Recall that the AIC is defined as $\text{AIC} \equiv -2 \ln \mathcal{L}_{\text{max}} + 2N$ (Akaike 1974), where $\mathcal{L}_{\text{max}}$ is the maximum likelihood and $N$ is the number of parameters. It is a commonly used guide for judging whether additional parameters are warranted given the increased model complexity, and quantifies the compromise between improving the fit and adding extra parameters.

The CHDM ‘bump’ model with $n_1 = 12$ and $n_2 = 13$ has a $\chi^2$ equal to the $\Lambda$CDM power-law model. Allowing $n_1$ and $n_2$ to vary freely further improves the fit to the data with the consequence that the CHDM model with $n_1$ and $n_2$ continuous is lavoured over the $\Lambda$CDM model according to the AIC. The primordial power spectrum of the models is shown in Fig. 1 together with the fit to the WMAP TT and TE spectra and the SDSS galaxy power spectrum.

The uncertainties of the derived parameters are smaller compared to those derived from the WMAP 3-year results, as would be expected for higher quality data. For example, the optical depth due to reionization for the CHDM model with continuous $n_1$ and $n_2$ has gone from $\tau = 0.075^{+0.012}_{-0.010}$ to $\tau = 0.0771^{+0.0003}_{-0.0003}$ due to the more accurate polarization measurements. The shape of the ‘bump’ in the primordial power spectrum for the CHDM model with continuous $n_1$ and $n_2$ is slightly changed by the new data. Although the quantity $\ln\left[10^{10} P_R^{(0)}\right]$ is almost unaltered, $\ln\left[10^{10} P_R^{(1)}\right]$ has increased slightly from a 3-year value of $3.429^{+0.043}_{-0.044}$ to a 5-year value of $3.462^{+0.036}_{-0.036}$ because of the increased amplitude of the TT spectrum for multipoles $\ell > 200$. Due to the increased height of the third acoustic peak, $\ln\left[10^{10} P_R^{(2)}\right]$ has increased slightly from $3.091^{+0.067}_{-0.067}$ to $3.183^{+0.043}_{-0.043}$ and $10^4 k_1$ fallen from $585^{+51}_{-42} \pm 82$ to $500^{+35}_{-31} \pm 82$ Mpc$^{-1}$. The increased amplitude of the primordial power spectrum on small scales has raised $\sigma_8$ from a value of $0.662^{+0.003}_{-0.004}$ to $0.700^{+0.008}_{-0.008}$.

The mean values of the variances $\langle \delta^2_{R_H} \rangle_R$, $\langle \delta^2_{R_1} \rangle_R$, $\langle \delta^2_{R_2} \rangle_R$ and $\langle \delta^2_{R_3} \rangle_R$, together with their 1σ limits, are plotted in Fig. 2. The different variances in the two models can be understood with reference to the matter power spectrum. From the relativistic Poisson equation, a given density perturbation leads to a larger curvature perturbation in a higher density universe. Since the amplitude of the primordial curvature perturbation is similar in both models (as can be seen from Fig. 1), the density contrast during the early matter dominated era is greater in the $\Lambda$CDM universe than in the higher density CHDM universe. Although the growth of density perturbations at late times is suppressed in a low-density universe, this means that the matter...
Table 2. The marginalized cosmological parameters for the various models (with 1σ limits). The 12 parameters in the upper section of the table are varied by COSMOMC, while those in the lower section are derived quantities. The χ² of the fit is given, as is the AIC relative to the power-law ΛCDM model.

| Parameter | ΛCDM power law | CHDM bump with n₁, n₂ = 12, 13 | CHDM bump with n₁, n₂ continuous |
|-----------|----------------|---------------------------------|---------------------------------|
| Ω₀h²      | 0.022 ±0.00060 | 0.016 ±0.00041                 | 0.017 ±0.00095                  |
| Ω₀h²      | 0.114 ±0.00046 | 0.084 ±0.0005                  | 0.085 ±0.00015                  |
| θ         | 1.039 ±0.0029  | 1.031 ±0.0039                  | 1.033 ±0.0048                  |
| τ         | 0.084 ±0.0077  | 0.072 ±0.0069                  | 0.077 ±0.0071                  |
| fν        | 0.114 ±0.012   | 0.085 ±0.015                   |                                  |
| nₙ        | 0.961 ±0.014   |                                 |                                  |
| 10⁴k₁(Mpc⁻¹) | 81.7 ±8.5     | 87 ±11                         |                                  |
| 10⁴k₂(Mpc⁻¹) | 442 ±37       | 500 ±23                        |                                  |
| ln [10²P_R(0)] | 3.078 ±0.037  | 3.294 ±0.031                   | 3.274 ±0.048                   |
| ln [10²P_R(1)] | 3.94 ±0.31    | 3.46 ±0.036                    |                                  |
| ln [10²P_R(2)] | 3.813 ±0.041 |                                  |                                  |
| Ω₀h²      | 0.1450 ±0.0079 | 0.156 ±0.012                   |                                  |
| Ω₀h²      | 0.163 ±0.0042 | 0.170 ±0.0074                  |                                  |
| h         | 0.695 ±0.021  | 0.424 ±0.0052                  | 0.433 ±0.0093                  |
| Age (Gyr) | 13.78 ±0.14   | 15.36 ±0.20                    | 15.05 ±0.32                    |
| Ωₘ        | 0.284 ±0.025  | 0.284 ±0.025                   |                                  |
| Ωₐ        | 0.716 ±0.025  | 0.716 ±0.025                   |                                  |
| σₗ        | 0.817 ±0.027  | 0.617 ±0.059                   | 0.700 ±0.098                   |
| τ_reion   | 11.0 ±1.4     | 13.0 ±2.0                      | 13.4 ±2.1                      |
| Δm₁      | 0.074 ±0.0046 | 0.089 ±0.020                   |                                  |
| Δm₂      | 0.151 ±0.015  | 0.136 ±0.016                   |                                  |
| H (t₂ - t₁) | 1.68 ±0.12   | 1.73 ±0.14                     |                                  |
| χ²        | 1339.9        | 1339.9                         | 1330.2                         |
| ΔARC     | 0             | 6.0                            | -3.7                           |

power spectrum of the ΛCDM universe is larger on all scales than that of the CHDM universe, when measured in units of h⁻¹ Mpc³. (This is not evident in Fig. 1 where the galaxy power spectrum is shown – the galaxies are more biased in the CHDM universe than in ΛCDM so the matter power spectrum occurs at a larger scale.) Thus, the quantity f²P_m(k) which appears in equation (9) is greater for the ΛCDM universe for wavenumbers below k_cross ≃ 0.01 h Mpc⁻¹ but is greater for the CHDM universe for wavenumbers above k_cross. The window function W(k) (10) makes (δᵏ²)R sensitive to the value of k²P_m(k) for the wavenumber k ≃ π/R. Consequently, the (δᵏ²)M and R(k) curves for the two models cross at the scale π/k_cross ≃ 300 h⁻¹ Mpc as seen in Fig. 2. The two (δᵏ²)R curves cross at a smaller scale of about 100 h⁻¹ Mpc. This is because the integral (14) for the variance in the peculiar velocity (v²)R is more strongly weighted towards small wavenumbers than the corresponding expression equation (9) for the variance in the Hubble contrast (δ²)R, which has an additional factor of k². Finally for (δ²)R the situation is intermediate between that for the variance in the density contrast and the variance in the Hubble contrast, since only some of the terms in W contain factors of k.

The scale dependence of (δ²)R is the reason that the P(δ²)[data] is broad and the CHDM ‘bump’ models on scales above and below 300 h⁻¹ Mpc, respectively, as shown in Fig. 3. Similarly, the P(δ²)[data] distribution is broader for the ΛCDM model on all scales, as seen in Fig. 4. To illustrate our findings, we calculate the probability of a fluctuation in the Hubble contrast greater than or equal to a given value.
Figure 1. The top-left panel shows the primordial perturbation spectrum for the CHDM bump model (with $n_1 = 12$ and $n_2 = 13$) and for the ΛCDM power-law model with $n_s \simeq 0.96$. The top-right and bottom-left panels show the best fits for both the models to the WMAP5 TT and TE spectra, while the bottom-right panel shows the best fits to the SDSS galaxy power spectrum.

Figure 2. The variation with increasing void radius of the variance of the Hubble parameter, the density contrast, the density parameter and the peculiar velocity for the ΛCDM power-law and CHDM bump models, given the WMAP5 and SDSS data (with 1σ limits).
\[ \delta^{0}_H \text{ in a sphere of radius } R, \text{ given by} \]

\[
\text{Probability } (\delta_H \geq \delta^{0}_H)_R = \int_{\delta^{0}_H}^{\infty} P(\delta_H|\text{data})_R \, d\delta_H. \quad (21)
\]

Since \( P(\delta_H|\text{data})_R \) is symmetric, this is also equivalent to the probability of a fluctuation being less than or equal to \(-\delta^{0}_H\). As seen in Fig. 5, the probability of a large excursion in \(\delta_H\) is largest on small scales, in accordance with physical intuition. Note that the probability on all scales tends to a value of half for small \(\delta^{0}_H\), because the fluctuation has an equal probability of being positive or negative. The probability is greater for the CHDM model than for the \(\Lambda\)CDM model on small scales because the \( P(\delta_H|\text{data})_R \) distribution is broader for the CHDM model on these scales. Conversely since the distribution is broader on large scales for the \(\Lambda\)CDM model, the probability is greater there for this model.

Similarly we calculate the probability of a fluctuation in the density contrast less than or equal to a given value \(-\delta^{0}\) in a sphere of
Figure 4. The probability distribution of the density contrast (with 1σ limits), given the WMAP5 and SDSS data, for the ΛCDM power-law and CHDM bump models, for spherical voids of radius \( R = (40, 70, 100, 150, 200, 300, 500, 800) \times h^{-1} \text{ Mpc} \).

radius \( R \), which is given by

\[
\text{Probability} \ (\delta \leq -\delta^0) \_R = \int_{-\infty}^{-\delta^0} P(\delta|\text{data}) \_R d\delta.
\] (22)

This probability is greater for the ΛCDM model on all scales as seen in Fig. 6, due to the broader \( P(\delta|\text{data}) \_R \) distribution.

Moreover, we can determine the probability of one or more voids with comoving volume \( V_1 \) occurring within some larger comoving volume \( V_2 \). If the ratio \( V_2/V_1 \) is \( N \) to the nearest integer and \( p \) is the probability of a void with volume \( V_1 \), then the probability of \( n \) voids within \( V_2 \) is \( \binom{N}{n} p^n (1-p)^{N-n} \), where \( \binom{N}{n} \) is the binomial coefficient. The expected number of voids within \( V_2 \) is \( Np \).

6 DISCUSSION

A void with \( \delta_H \simeq 0.2–0.3 \) and a radius exceeding \( 100 h^{-1} \text{ Mpc} \) is required to fit the supernova data without dark energy (Tomita 2001c; Alexander et al. 2009; Biswas et al. 2007). The probability that we are situated in such a void is less than \( 10^{-12} \) as can be seen
from Fig. 5. The probability is exponentially smaller for the larger voids of Gpc size that have also been considered (Alnes et al. 2006; Clifton et al. 2008; Garcia-Bellido & Haugboelle 2008a).\footnote{There is a further constraint on Gpc scale voids from the observed absence of a ‘γ-distortion’ in the spectrum of the CMB (Caldwell & Stebbins 2008) and from the ‘kinetic Sunyaev–Zeldovich’ effect observed for X-ray-emitting galaxy clusters (Garcia-Bellido & Haugboelle 2008b). However, this has no impact on smaller voids.}

However, before we dismiss the possibility of a local void on these grounds, we should also evaluate the probability of voids which have actually been claimed to exist elsewhere in the universe. For example, it has been argued that a void with radius 200–300 h\(^{-1}\) Mpc and a density contrast of \(\delta = -0.3\) at \(z \approx 1\) can account for the WMAP ‘cold spot’ in a \(\Lambda CDM\) universe (Inoue & Silk 2006). Even if we conservatively take the radius to be 150 h\(^{-1}\) Mpc (and the same underdensity), the probability that one or more such voids lie within the volume out to \(z = 1\) is only 1.05\(^\pm\)5.24 \times 10\(^{-10}\).

It has been argued that the WMAP cold spot may not be a localized feature (Naselsky et al. 2007) and there may be no matching void in the NRAO VLA Sky Survey (NVSS) radio source catalogue (Smith & Huterer 2008); however, an equally striking anomaly arises if we consider the large number of voids which have been identified in the SDSS LRG survey in a search for the late ISW effect (Granett, Neyrinck & Szapudi 2008a; Granett et al. 2008b). These are of angular radius \(\sim 4^\circ\) corresponding to a (comoving) radius of \(\sim 50 h^{-1} \text{ Mpc}\) and are tabulated as having 1\(\sigma\), 2\(\sigma\) or 3\(\sigma\) underdensities. These numbers relate to the detection significance (the likelihood of detecting the void by chance out of a Poisson distribution) rather than the likelihood of finding such underdensities in a Gaussian field which we have computed in this paper (Granett, private communication). Moreover, the observed LRGs are biased with regard to the dark matter, hence the underdensities in dark matter are likely to be smaller than the quoted values.

However, if Granett et al. (2008a,b) have indeed detected the late ISW effect as they assert, we can simply circumvent these uncertainties by requiring that the voids be large enough and/or underdense enough to yield the observed CMB temperature decrements. To calculate the late ISW effect, we consider the propagation of CMB photons to us from the last scattering surface through an intervening void. The photon temperature change caused by the void is

\[
\frac{\Delta T}{T} = \frac{2}{c^2} \int_{a_{nw}}^{a_{sa}} \frac{d\Phi}{da} da, \tag{23}
\]

where \(a_{nw}\) is the scalefactor when the photon crossed the far side of the void and \(a_{sa}\) is the scalefactor when the photon crossed the near side of the void. The gravitational potential of a void with proper radius \(r\) is

\[
\Phi = \frac{4\pi G}{3} r^2 \rho_b \delta (a), \tag{24}
\]

where the background density is given by \(\rho_b = 3H_0^2 \Omega_m / 8\pi G a^3\) and the density perturbation is given by \(\delta (a) = D(a) \delta (a_0)\) where \(D\)
is the linear growth factor. Hence,
\[
\frac{\Delta T}{T} = \Omega_\text{m} \left( \frac{R}{c/H_0} \right)^2 \frac{D(a_{\text{far}}) - D(a_{\text{near}})}{a_{\text{far}} - a_{\text{near}}} \delta. \tag{25}
\]

Using this we calculate the expected ISW signal for the 50 highest significance voids in table 4 of Granett et al. (2008a), employing the concordance ΛCDM cosmology to determine \(a_{\text{far}}\) and \(a_{\text{near}}\) for each void from the void redshift measurements. The ISW signal is found to be only \(-0.42\,\mu\text{K}\) on average if the dark matter underdensities are smaller than the observed underdensities in the LRG counts by the bias factor of 2.2 (taking \(\sigma_8 = 0.8\)). This is in contrast to the detected mean signal of \(-11.3\,\mu\text{K}\) which is over 20 times bigger! We must therefore conclude that the void radii and/or underdensities have been significantly underestimated. The void radii can at most be increased by a factor of 1.75 within the quoted uncertainties so the observed signal of \(-11.3\,\mu\text{K}\) can be matched only if the underdensities are increased by a factor of 5 (implying a bias factor of 0.2). The CMB temperature decrements of such model voids calculated using equation (25) are shown in Fig. 7 and are (by construction) similar to the actual measurements shown in Fig. 2 of Granett et al. (2008b). While such an underbias for the observed LRGs may seem plausible, we emphasize that this is the only way in which the temperature decrements observed by Granett et al. (2008a,b) can be accounted for as being due to the late ISW effect.

Fig. 7 displays a histogram of the probabilities for finding such voids in the SDSS LRG survey volume \((5\,h^{-3}\,\text{Gpc}^3)\) in the redshift range \(0.4 < z < 0.75\). The most improbable void is at \(z = 0.672\) – in order to yield the observed average CMB temperature decrement it must have a density contrast of \(-0.72\) (quoted galaxy underdensity of \(-0.316\) multiplied by \(5/2.2\)) and a radius of \(230\,h^{-1}\,\text{Mpc}\) (radius derived from the quoted volume of \(10^7\,h^{-3}\,\text{Mpc}^3\) and multiplied by \(1.75\)). The probability of such a void is \(1.9 \times 10^{-24}\) according to our calculations. Although the linear theory may not be applicable for such a deep void, it is clear that its existence is in gross conflict with the standard theory of structure formation from Gaussian primordial density perturbations.

This conclusion is strengthened by the recent detection of very large peculiar velocities on large scales. As seen in Fig. 2, the expected variance of the peculiar velocity as calculated by equation (14) is about 200 km s\(^{-1}\) on a scale of \(100\,h^{-1}\,\text{Mpc}\), whereas the measured value is at least five times higher, and the discrepancy is even bigger on larger scales up to 300 Mpc (Kashlinsky et al. 2008).

It is also seen from Fig. 5 that if a determination of the Hubble constant is required with say 1 per cent accuracy, then measurements extending out to at least \(150\,h^{-1}\,\text{Mpc}\) must be made to overcome local fluctuations. A similar estimate was made by Li, Seikel & Schwarz (2008) who noted that the observed variance in measurements of \(h\) is in accord, thus consistent with the assumption of a Gaussian density field. However, the voids observed in the SDSS LRG survey (Granett et al. 2008b) call this assumption into question. In particular whether there is a large local void is then an issue that must be addressed observationally and not dismissed on the grounds that it is inconsistent with Gaussian perturbations. The Hubble flow is presently poorly measured in the redshift range \(0.1 \lesssim z \lesssim 0.3\) – just where the effects of such a local void would be most apparent (Alexander et al. 2009). Given that dark energy may well be an artefact of such a void, this issue needs urgent attention.

The question of how such voids can have been generated without conflicting with the CMB observations is beyond the scope of the present work. Some suggestions have been made in the context of multifield inflationary models (Occhionero et al. 1997; DiMarco & Notari 2006; Itzhaki 2008).

ACKNOWLEDGMENTS

This work was supported by an STFC Senior Fellowship award to SS (PPA/C506205/1) and by the EU Marie Curie Network ‘UniversaNet’ (HPRN-CT-2006-035863).

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