Localized Axion Photon States in a Strong Magnetic Field

E.I. Guendelman

Department of Physics, Ben-Gurion University of the Negev
P.O.Box 653, IL-84105 Beer-Sheva, Israel

Abstract

We consider the axion field and electromagnetic waves with rapid time dependence, coupled to a strong time independent, asymptotically approaching a constant at infinity "mean" magnetic field, which takes into account the back reaction from the axion field and electromagnetic waves with rapid time dependence in a time averaged way. The direction of the self consistent mean field is orthogonal to the common direction of propagation of the axion and electromagnetic waves with rapid time dependence and parallel to the polarization of these electromagnetic waves. Then, there is an effective U(1) symmetry mixing axions and photons. Using the natural complex variables that this U(1) symmetry suggests we find localized planar soliton solutions. These solutions appear to be stable since they produce a different magnetic flux than the state with only a constant magnetic field, which we take as our "ground state". The solitons also have non trivial U(1) charge defined before, different from the uncharged vacuum.

PACS numbers: 11.30.Fs, 14.80.Mz, 14.70.Bh

*Electronic address: guendel@bgumail.bgu.ac.il
I. INTRODUCTION

One of the most interesting ideas for going beyond the standard model has been the introduction of the axion [1], which provided a way to solve the strong CP problem. Since then, the axion has been postulated also as a candidate for the dark matter. A great number of ideas and experiments for the search this particle have been proposed [2].

One particular feature of the axion field \( \phi \) is its coupling to the photon through an interaction term of the form \( g \phi \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \). In fact a coupling of this sort is natural for any pseudoscalar interacting with electromagnetism, as is the case of the neutral pion coupling to photons (which as a consequence of this interaction decays into two photons).

A way to explore for observable consequences of the coupling of a light scalar to the photon in this way is to subject a beam of photons to a very strong magnetic field. This affects the optical properties of light which could lead to testable consequences [3]. Also, the produced axions could be responsible for the "light shining through a wall phenomena ", which is obtained by first producing axions out of photons in a strong magnetic field region, then subjecting the mixed beam of photons and axions to an absorbing wall for photons, but almost totally transparent to axions due to their weak interacting properties which can then go through behind this "wall", applying then another magnetic field one can recover once again some photons from the produced axions [4].

On the other hand, the photon axion system in an external magnetic field shows very interesting field theoretical properties, like the possibility of mass generation [5] and a new possibility for confinement [6]. In particular the results of [5] imply that an external constant magnetic field in the axion photon system is a stable vacuum under small perturbations. This is in contrast to an external electric field, which exhibits tachyonic instabilities [5].

Here we will consider the effects which result from taking into account the back reaction of the axions and electromagnetic waves on this strong "external" magnetic field. It will be shown that when considering the axion photon system in an self consistent, time independent magnetic field, that is one that takes into account the back reaction of the axion photon system on the magnetic field in an averaged mean field approach, we find that there are localized soliton like solutions. The direction of the self consistent mean field we will take orthogonal to the direction of propagation of the axion and propagation of the electromagnetic component with rapid time dependence and parallel to the polarization of the
electromagnetic waves. The resulting planar soliton solutions that are obtained appear to be stable since they produce a different magnetic flux than configuration where the localized axion photon configuration is absent, that is, when we have just the a constant magnetic field all over space (our choice of stable vacuum).

II. ACTION AND EQUATIONS OF MOTION

The action principle describing the relevant light pseudoscalar coupling to the electromagnetic field is,

\[
S = \int d^4x \left[ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{g}{8} \phi \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \right] \quad (1)
\]

We now specialize to the case where we consider a time dependent axion and electromagnetic field with propagation only along the \( z \)-direction and where a time independent magnetic field pointing in the \( x \)-direction is present. This field may have a \( z \) dependence, to be determined later, but it is taken to be time independent. We want however that this field will take into account the back reaction of the time dependent axion and electromagnetic fields in a time averaged way. In the case the magnetic field is constant and external, see for example [5] for general solutions.

Now considering a static strong ”mean” magnetic field pointing in the \( x \) direction having an arbitrary \( z \) dependence, we take \( B_x = -\partial_z A_y \) (since taking also that \( A_z \) depends only on \( z \) leave us only with this contribution) and specializing to \( z \) dependent electromagnetic field perturbations and axion fields. This means that the interaction between the strong mean field, the axion and photon fields (that is the part of the electromagnetic field with fast variations) reduces to

\[
S_I = -\int d^4x \left[ g B_x (z) \phi E_x \right] \quad (2)
\]

Choosing the temporal gauge for the electromagnetic field and considering only the \( x \)-polarization for the electromagnetic time dependent fields (\( A \) will denote the \( x \)-component of the vector potential, so \( E_x = -\partial_t A \)), since only this polarization couples to the axion and static magnetic field. We get the following 1+1 effective dimensional action,

\[
S_2 = \int dz dt \left[ \frac{1}{2} \partial_\mu A \partial^\mu A + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 + g B_x (z) \phi \partial_t A - \frac{1}{2} (\partial_z A_y)^2 \right] \quad (3)
\]
\( (A = A(t, z), \phi = \phi(t, z)) \), which leads to the equations for the \( A \) and \( \phi \) fields, if we assume the mean field \( B_x(z) \) is time independent

\[
\partial_\mu \partial^\mu \phi + m^2 \phi = g B_x(z) \partial_t A
\]  

\[
\partial_\mu \partial^\mu A = -g B_x(z) \partial_t \phi
\]  

The eq. of motion of the mean field \( A_y(z) \) itself will be discussed later. As it is known, in temporal gauge, the action principle cannot reproduce the Gauss constraint and has to be imposed as a complementary condition. However this constraint is automatically satisfied here just because of the type of dynamical reduction employed (which gives both that \( \partial_i E^i = 0 \) and that the charge density that is proportional to \( \partial_i (\phi B^i) = 0 \) also) and does not need to be considered anymore.

III. THE CONTINUOUS AXION PHOTON DUALITY SYMMETRY IN THE MASSLESS AXION CASE AND STATIC STRONG MEAN FIELD

Without assuming any particular \( z \)-dependence for \( B_x(z) \), but still insisting that this "mean field" will be static, we see that in the case \( m = 0 \), we discover a continuous axion photon duality symmetry, as we discussed in Reference [7], where the magnetic field in the \( x \) direction was taken as external, but we only need the time independence of this mean field to observe that,

1. The kinetic terms of the photon and axion allow for a rotational \( O(2) \) symmetry in the axion-photon field space.

2. The interaction term, after dropping a total time derivative (if the mean field is static) can also be expressed in an \( O(2) \) symmetric way as follows

\[
S_I = \frac{1}{2} \int dz dt g B_x(z) \left[ \phi \partial_t A - A \partial_t \phi \right]
\]  

The axion photon symmetry is in the infinitesimal limit

\[
\delta A = \epsilon \phi, \delta \phi = -\epsilon A
\]  

where \( \epsilon \) is a small number. Using Noether’s theorem, this leads to the conserved current \( j_\mu \), with components given by
\[ j_0 = A\partial_t \phi - \phi \partial_t A + \frac{gB_x(z)}{2}(A^2 + \phi^2) \]  

(8)

and

\[ j_i = A\partial_i \phi - \phi \partial_i A \]  

(9)

defining the complex field \( \psi \) as

\[ \psi = \frac{1}{\sqrt{2}}(\phi + iA) \]  

(10)

we see that in terms of this complex field, the axion photon density takes the form

\[ j_0 = i(\psi^* \partial_t \psi - \psi \partial_t \psi^*) + gB_x(z)\psi^* \psi \]  

(11)

We observe that to first order in \( gB_x(z) \), (6) represents the interaction of the magnetic field with the "axion photon density" \( j \), \( j_i \) and also this interaction has the same form as that of scalar QED with an external "electric" field to first order. In fact the magnetic field or more precisely \( gB_x(z)/2 \) appears to play the role of external electric potential.

In terms of the complex field, the axion photon current takes the form

\[ j_k = i(\psi^* \partial_k \psi - \psi \partial_k \psi^*) \]  

(12)

**IV. THE LOCALIZED SOLITON SOLUTIONS**

After the manipulations of the previous section, valid under the approximation that the mean field \( A_y(z) \) is time independent, we can discuss the eq. of motion for this mean field. This is,

\[ \partial_z \left( \frac{ig}{2}(\psi^* \partial_t \psi - \psi \partial_t \psi^*) + B_x(z) \right) = 0 \]  

(13)

The same result can be obtained from the original equations instead of the averaged Lagrangian obtained under the assumption that the mean field \( B_x(z) \) is time independent and there doing a time averaging procedure, using for example that under such time averaging \( \phi \partial_t A \) equals \( \frac{1}{2}(\phi \partial_tA - \partial_t \phi A) \). Equation (13) can be integrated, giving

\[ B_x(z) = -\frac{ig}{2}(\psi^* \partial_t \psi - \psi \partial_t \psi^*) + B_0 \]  

(14)

where \( B_0 \) is an integration constant. The constant \( B_0 \) breaks in fact spontaneously the charge conjugation symmetry of the theory,
or equivalently, changing the sign of $A$, since in such transformation the first term of (14) changes sign, which would be required to leave the interaction term (6) in the action invariant, but the second term does not (since it is a constant). Also, in problems where $B_x$ is taken as an external field [8], the interaction automatically breaks this "charge conjugation" symmetry.

We now consider $\psi$ to have the following time dependence,

$$\psi = \exp(-i\omega t)\rho(z) \quad (16)$$

We want to see now what is the equation of motion for $\rho(z)$, which we take as a real field. We start with the general eq. for $\psi$

$$\partial_{\mu}\partial^{\mu}\psi + igB_x(z)\partial_0\psi = 0 \quad (17)$$

Inserting (16) into (14) and the result into (17), we obtain,

$$\frac{d^2\rho(z)}{dz^2} + \frac{dV_{eff}(\rho)}{d\rho} = 0 \quad (18)$$

where $V_{eff}(\rho)$ is given by

$$V_{eff}(\rho) = \frac{1}{2}(\omega^2 - \omega gB_0)\rho^2 + \frac{1}{4}g^2\omega^2\rho^4 \quad (19)$$

Some comments are required on the nature and signs of the different terms. One should notice first of all that this effective potential is totally dynamically generated and vanishes when taking $\omega = 0$. Concerning signs, all terms proportional to $\omega^2$ are positive, in fact although the $(g\omega)^2$ term is quartic in $\rho$, it has to be regarded as originating not from an ordinary potential of the scalar field in the original action, but rather from a term proportional to a $g^2(-i(\psi^*\partial_t\psi - \psi\partial_t\psi^*))^2$, quadratic in time derivatives, which could have been obtained if we had worked directly with the action rather than with the equations of motion, replacing (14) back into the action (i.e., integrating out the $B_x$ field). Such type of quadratic terms in the time derivatives give a positive contribution both in the lagrangian and in the energy density, unlike a standard (not of kinetic origin) potential, where the contribution to the lagrangian is opposite to that of their contribution in the energy density.
The only term which may not be positive is the $-\omega g B_0$ contribution. This term breaks the charge conjugation symmetry $\text{(15)}$ which for a field of the form $\text{(16)}$ means $\omega \rightarrow -\omega$.

We can in any case choose the sign of $\omega$ such that the $-\omega g B_0$ contribution is negative and choose $B_0$ big enough (or $\omega$ small enough) so that this term makes the first term in the effective potential negative.

Now we are interested in obtaining solutions where $B_x(z) \rightarrow B_0$ as $z \rightarrow \infty$ and also as $z \rightarrow -\infty$, which requires $\rho \rightarrow 0$ as $z \rightarrow \infty$ and also as $z \rightarrow -\infty$. Since the vacuum with only a constant magnetic field is a stable one $\text{(9)}$.

The solution of the equations $\text{(18)}$ and $\text{(19)}$ with such boundary conditions is possible if $\omega^2 - \omega g B_0 < 0$. After solving these analog of the “particle in a potential problem” with zero “energy”, so that the boundary conditions are satisfied, we find that $\rho$ is given by (up to a sign),

$$\rho = \frac{(\sqrt{2(\omega g B_0 - \omega^2)})/g\omega}{cosh(\sqrt{\omega g B_0 - \omega^2}(z - z_0))} \quad (20)$$

where $z_0$ is an integration constant that defines the center of the soliton.

Inserting $\text{(20)}$ and $\text{(16)}$ into the expression for $B_x$ $\text{(14)}$ we find the profile for $B_x$ as a function of $z$. The difference in flux per unit length (that is ignoring the integration with respect to $y$ in the $yz$ plane) through the $yz$ plane of this solution with respect to the background solution $B_x = B_0$ is finite amount. Since magnetic flux is conserved, we take this as an indication of the stability of this solution towards decaying into the $B_x = B_0$ stable “ground state”.

Notice also that the soliton is charged under the $U(1)$ axion photon duality symmetry $\text{(7)}$ and the vacuum is not, another evidence for the stability of these solitons. However for any given soliton, there is no “antisoliton”, since the condition $\omega^2 - \omega g B_0 < 0$ will not be maintained if we reverse the sign of $\omega$. This is due to the fact that the vacuum of the theory, i.e. $B_x = B_0$ spontaneously breaks the charge conjugation symmetry $\text{(15)}$.

There are some similarities, but also some crucial differences with the one dimensional topological solitons considered in Ref. $\text{(9)}$, where also a complex field with a $U(1)$ global symmetry was considered and a time dependence of the form $\text{(16)}$ as well, which means that the soliton is charged under this $U(1)$ global symmetry, as in our case. The difference is in the type of potential used and the origin of the potential. Here, the complete potential
appears when considering non-vanishing $\omega$, while in Ref. [9], there is an original potential and self interaction in the original action and these self interactions appear with a negative sign in the effective potential, contrary to our case where the quartic self interaction enters with a positive sign (because of its kinetic origin as we explained before). Finally, in our case there is also a topological aspect as well, absent in Refs. [9], which is that the magnetic flux of the soliton differs from that of the vacuum.

V. CONCLUSIONS

In the context of a theory of containing a pseudo scalar particle coupled to an electromagnetic field in the form $g\phi\epsilon^\mu\nu\omega_\alpha F_{\mu\nu}F_{\alpha\beta}$ an external constant magnetic field provides a stable vacuum [5], unlike a constant electric field [5]. This one can understand qualitatively since a magnetic field is protected from decay to a lower energy density configuration because the magnetic field cannot be screened, while this is not the case for a constant electric field. We then study a special type of geometry, where all the space dependence is on only one dimension, which we call $z$ and with fast and slow variables. The fast variables being the axion field and the vector potential in the $x$ direction, the slow variable, the magnetic field in the $x$ direction or the $y$ component of the vector potential, this is our "mean field". We then consider time averaging in the equations of motion or in the action so that the mean field is taken to be time independent and then considering the limit of zero axion mass we obtain a continuous axion photon "duality symmetry", and conserved quantities associated.

Introducing complex variables makes the structure of the time averaged equations quite manageable and the planar localized (in the $z$ direction) axion photon solutions are then found. From the profile for $B_x$ as a function of $z$, we see that there is a difference in the flux per unit length (that is ignoring the integration with respect to $y$ in the $yz$ plane) through the $yz$ plane of this solution with respect to the background solution $B_x = B_0$. Since magnetic flux is conserved, we take this as an indication of the stability of this solution towards decaying into the $B_x = B_0$ stable "ground state".

In future research it would be interesting to study generalizations of these solutions and look at the possibility of localizing not only with respect to one dimension (here $z$) but with respect to two. Another area of future research could be the consideration of a vacuum with charge density, which changes the discussion of solitons [10]. Finally one should also
consider the effect of a small axion mass on these solutions.

Acknowledgments

I would like to thank the Department of Physics and Chemistry and in particular the High Energy and Differential Geometry Theory Group of the Southern University of Denmark in Odense, where this work was completed, for great hospitality and support, in particular from Francesco Sannino and Roshan Foadi. I also want to thank Niels Kjaer Nielsen for very interesting discussions.

[1] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. 38, 1440 (1977); S. Weinberg, Phys. Rev. Lett. 40, 223 (1978); F. Wilczek, Phys. Rev. Lett. 40, 279 (1978).

[2] For a review see G.G. Raffelt, hep-ph/0611118.

[3] E. Zavattini, et. al. (PVLAS collaboration), arXiv:hex/0706.3419.

[4] See for example K.Van Bibber et. al., Phys. Rev. Lett. 59, 759 (1987); R. Rabadan, A. Ringwald and K. Sigurson, Phys. Rev. Lett. 96, 110407 (2006) and references here.

[5] S. Ansoldi, E. Guendelman and E. Spallucci, JHEP 0309, 044 (2003).

[6] P. Gaete and E. Guendelman, Mod. Phys. Lett. A20, 319 (2005); P. Gaete and E. Spallucci, J. Physics A39, 6021 (2006); arXiv:0712.2321 [hep-th].

[7] E. Guendelman, arXiv:0711.3685 [hep-th].

[8] E.I. Guendelman, arXiv:0711.3961 [hep-ph].

[9] T.D. Lee, in Proceedings of the Symposium on Frontiers Problems n High energy Physics (in honor of G. Bernardini), edited by L. Fua and L.A. Radicati, Scuola Normale Superiore, Pisa, p. 47 (1976); T.D. Lee, Particle physics and Introduction to Field Theory, (New York, Harwood Academic Publishers, 1981).

[10] J.D. Bekenstein and E.I. Guendelman, Phys. Rev. D35, 716 (1987); E.I. Guendelman, Mod. Phys. Lett. A4,2225 (1989).