Derivation of the Einstein Equivalence Principle in a Class of Condensed Matter Theories

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January 4, 2022

Abstract

We consider a class of condensed matter theories in a Newtonian framework with a Lagrange formalism related in a natural way with the classical conservation laws

\[
\begin{align*}
\partial_t \rho + \partial_i (\rho v^i) &= 0 \\
\partial_t (\rho v^i) + \partial_i (\rho v^j v^i + p^{ij}) &= 0
\end{align*}
\]

We show that for an algebraically defined “effective Lorentz metric” \( g_{\mu\nu} \) and “effective matter fields” \( \varphi \) these theories are equivalent to material models of a metric theory of gravity with Lagrangian

\[
L = L_{GR} + L_{\text{matter}} - (8\pi G)^{-1} (\Upsilon g^{00} - \Xi (g^{11} + g^{22} + g^{33})) \sqrt{-g}
\]

which fulfils the Einstein equivalence principle and leads to the Einstein equations in the limit \( \Xi, \Upsilon \rightarrow 0 \).

1 Introduction

General relativity is not only a very successful theory of gravity, but also a very beautiful one. Part of its beauty is the remarkable fact that there are many different ways to general relativity. On one hand, there are remarkable formulations in other variables (ADM formalism \( 3 \), tetrad, triad, and

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Ashtekar variables [1] where the Lorentz metric $g_{\mu \nu}$ appears as derived. Some of them may be considered as different interpretations of general relativity (like “geometrodynamics”). On the other hand we have theories where the Lorentz metric is only an effective metric and the Einstein equations appear as some limit (spin two field in QFT on a standard Minkowski background [11], [29], Sakharov’s approach [22], string theory [19]).

An effective curved Lorentz metric appears also in classical and quantum condensed matter theory, as the “acoustic” metric “seen” by phonons. This has been shown, for example, for an irrotational flow of a barotropic fluid in [25]. Following Unruh [24] such acoustic metrics have been considered as toy models to study Hawking and Unruh radiation (see [21] and references there). Very interesting is also the consideration of superfluid $^3$He – $A$, which shows not only effective gravity, but also effective gauge and chiral fermion fields, where relativistic and gauge symmetry appears in a certain approximation [26] [27]. Following Visser [25] writes that “the aspect of general relativity which does not carry over to the acoustic model is the dynamics – the Einstein equations”. Volovik [27] mentions that the equations are “highly contaminated by many non-covariant terms”.

The results of this paper suggest that a much stronger analogy may be possible. We introduce here a quite general class of condensed-matter-like theories. We require classical conservation laws (continuity equation, Euler equation), Lagrange formalism, and a special relation between them in agreement with Noether’s theorem. This is already sufficient to prove the Einstein equivalence principle for “effective matter fields” on an “effective metric”. We derive a general Lagrangian, which differs from GR in only two additional terms which depend on the Newtonian background frame. As a consequence, in a certain limit we obtain the classical Einstein equations.

The derivation is based on a slightly unorthodox but beautiful variant of covariant description of a theory with preferred frame – we use the preferred coordinates $X^\alpha(x)$ as variables. This gives Euler-Lagrange equations for the preferred coordinates $X^\alpha(x)$. In case of translational symmetry they become conservation laws. This observation motivates our main physical assumption: the identification of the Euler-Lagrange equation for $X^\alpha$ with the classical conservation laws of condensed matter theory. We introduce an effective metric $g_{\mu \nu}$ with an algebraic definition similar to ADM decomposition. This transforms the conservation laws into the harmonic condition. We obtain a remarkable composition of three coordinate-related features: ADM decomposition, harmonic coordinates, and conservation laws. Based on this transformation, the derivation of the EEP and the Lagrangian is surprisingly
Then we discuss the derivation itself and its possible applications. The most surprising thing in this derivation is our set of assumptions. They look quite innocent and natural for a condensed matter theory. Simplicity and beauty of the derivation are remarkable too.

This connection between condensed matter theory and theory of gravity may be used in two directions. First, we may try to apply it in condensed matter theory. While our assumptions seem to be natural for condensed matter theories, they require a Lagrange formalism in unusual form. This suggests that the connection may be used to find new Lagrange formalisms for condensed matter theories starting with GR Lagrangians.

The other direction would be the consideration of condensed matter theory in a Newtonian framework as a candidate for a more fundamental theory behind general relativity. Essentially this would be a revival of old, pre-relativistic ether theory. The ability to derive the Einstein equivalence principle and the beauty of this derivation removes some important arguments against this concept. And there are several independent domains where a Newtonian framework gives advantages: We obtain well-defined local energy and momentum conservation laws. There is no “problem of time” which makes quantization much easier. A preferred frame is also a necessary prerequisite for realistic causal hidden variable theories of quantum theory like Bohmian mechanics \[7\] or Nelson’s stochastics \[18\]. Therefore, a revival of the old ether concept may be not as unreasonable as it sounds.

But, independent of the viability of ether theories of such type, the relation between the equations of classical condensed matter theory and the Einstein equivalence principle is a remarkable and beautiful observation which is worth to be considered in more detail.

\section{Covariant Management of Preferred Coordinates}

Let’s start with the description of our variant of covariant description of theories with a Newtonian framework. We have to be careful here to avoid confusion about the meaning of “covariance”. It is well-known today that “general covariance” is not a special property of general relativity, as initially thought by Einstein. Instead, every physical theory allows a formulation which does not depend on the choice of coordinates \[15\]. The difference between general relativity and other theories is the absence of an a priori, absolute geometry \[4\]. An example of a covariant formulation of Newtonian
mechanics can be found, for example, in [17].

We consider here some class of condensed matter theories in a classical Newtonian framework, thus, theories with an an absolute, a priori geometry. We have an absolute preferred foliation defined by absolute time $T = X^0$, and we have an absolute Euclidean background metric which is $\delta_{ij}$ in our preferred coordinates $X^i$.

To describe this absolute background in a covariant way different variables may be used. One possibility is to handle the constant, flat metric of the background geometry like a variable metric tensor $\gamma_{\mu\nu}(x)$ and to add the covariant equation $R^\mu_{\nu\kappa\lambda}[\gamma] = 0$. Now, our choice of variables is different – we use the preferred coordinates $X^\alpha$ themself as the variables which describe the Newtonian background. Last not least, the preferred coordinates are functions $X^\alpha(x)$ on the manifold and may be used as other scalar functions to describe a physical theory.

An important point is that we require that the $X^\alpha$ should be used in the Lagrangian and the Lagrange formalism as usual scalar fields. A short but sloppy formulation would be that the Lagrangian depends on the preferred coordinates in a covariant way. Unfortunately this may cause confusion. More accurate, the Lagrangian should depend on the functions $X^\alpha(x)$ like a usual covariant scalar Lagrangian depends on scalar fields $U^\alpha(x)$. To minimize confusion, let’s name such a Lagrangian “weak covariant”. Instead, a “strong covariant” Lagrangian in this formalism is also “weak covariant”, but, moreover, does not depend on the preferred coordinates $X^\alpha(x)$ at all – which would be covariant in the usual sense of general relativity.

What it means can be best seen in a simple example. Assume we have a scalar product in the preferred coordinates defined by $(u, v) = \delta_{ij}u^i v^j$. This expression is non-covariant. The same scalar product can be rewritten as $(u, v) = \delta_{ij}X^\mu_i X^\nu_j u^\mu v^\nu$. This expression is already weak covariant. Note that the preferred coordinates $X^\alpha$ themself are scalar functions. Their upper index $\alpha$ is not a spatial index, but simply enumerates the four scalar functions. Thus, $u^0$ is non-covariant because of the open upper spatial index 0, but $X^0_{\cdot\mu} u^\mu$ is already weak covariant.

Despite the special geometric nature of coordinates (see appendix [3]) we can apply the Lagrange formalism as usual and obtain Euler-Lagrange equations for the preferred coordinates $X^\alpha$. Now, these equations have a close connection with the conservation laws for energy and momentum. We know from Noether’s theorem that symmetries of the Lagrangian lead to conservation laws. Especially, conservation of energy and momentum follows from translational symmetry in the preferred coordinates. Now, for translational symmetry $X^\alpha \to X^\alpha + c^\alpha$ we do not need any theorem to find them – we obtain them automatically. The Lagrangian does not depend on the $X^\alpha$.
themself, only on their partial derivatives. Therefore the Euler-Lagrange equations for $X^\alpha$ are already conservation laws:

**Theorem 1** If the Lagrangian has translational symmetry $X^\alpha \rightarrow X^\alpha + c^\alpha$, then the Euler-Lagrange equations for the preferred coordinates $X^\alpha$ are conservation laws:

$$\frac{\delta S}{\delta X^\alpha} = \partial_\mu T^\mu_\alpha = 0$$

We know that in agreement with Noether’s second theorem these conservation laws disappear if we have general covariance, that means, in our formalism, strong covariance. Again, in our formulation we do not need any theorem to see this – if there is no dependence on the $X^\alpha$, the Euler-Lagrange equation for the $X^\alpha$, which are the conservation laws, disappear automatically:

**Theorem 2** If $L$ is strong covariant, then

$$\frac{\delta S}{\delta X^\alpha} \equiv 0$$

In above cases, the proof is so obvious that there seems no need to write it down. Of course, there is not much to wonder about, we have simply used a set of variables $X^\alpha$ appropriate for the symmetry groups we have considered – translational symmetry. Which is, of course, the reason why we have introduced this formalism here. Above symmetry groups as well as the conservation laws play an important role in the following.

In the reverse direction, the last theorem does not hold. There are Lagrangians which are not strong covariant but have covariant Euler-Lagrange equations, like the well-known Rosen Lagrangian \[24\]. This is a consequence of the fact that the Lagrangian is not uniquely defined by its Euler-Lagrange equations. But the Rosen Lagrangian is a well-known GR Lagrangian too. So, what defines the most general GR Lagrangian \[1\] in our formalism is not

\[1\] Here, “most general” means that also higher order derivatives of the metric and non-minimal interactions are allowed. This is in agreement with the modern concept of effective field theory, as, for example, expressed by Weinberg \[30\]: “I don’t see any reason why anyone today would take Einstein’s general theory of relativity seriously as the foundation of a quantum theory of gravitation, if by Einstein’s theory is meant the theory with a Lagrangian density given by just the term $\sqrt{gR}/16\pi G$. It seems to me there’s no reason in the world to suppose that the Lagrangian does not contain all the higher terms with more factors of the curvature and/or more derivatives, all of which are suppressed by inverse powers of the Planck mass, and of course don’t show up at energy far below the Planck mass, much less in astronomy or particle physics. Why would anyone suppose that these higher terms are absent?”
strong covariance, but the weaker property

\[ \frac{\delta S}{\delta X^\alpha} \equiv 0. \]

In the following we assume that the Lagrangian is given in (or can be transformed into) weak covariant form. In appendix A we consider the question if this is a non-trivial restriction or not. Nonetheless, it seems worth to note that until now we have only introduced a general formalism which is in no way restricted to the condensed matter application below.

3 Description of the Condensed Matter Theories

The class of theories we consider here describe something similar to a classical medium in a Newtonian framework – Euclidean space and absolute time. This “medium” is described by steps of freedom typical for condensed matter theory: a positive density \( \rho(x, t) \), a velocity \( v^i(x, t) \), and a symmetrical tensor field \( p^{ij}(x, t) \) similar to pressure, but negative definite. As usual in condensed matter theory, there may be also various material properties \( \varphi^m(x, t) \). These material properties are not specified or restricted in our class of theories.

We require the existence of a weak covariant Lagrange formalism. Now, on one hand from theorem 1 follows that in case of translational symmetry the Euler-Lagrange equations for \( X^\alpha \) are conservation laws. On the other hand, we know the classical conservation laws from condensed matter theory – the continuity equation and Euler equation. It seems quite natural and innocuous to identify them. Thus, we identify the continuity equation with the equation for time \( T = X^0 \):

\[ \frac{\delta S}{\delta X^0} \sim \partial_t \rho + \partial_i (\rho v^i) = 0 \] \hspace{1cm} (1)

and the Euler equation with the equations for spatial coordinates \( X^i \):

\[ \frac{\delta S}{\delta X^i} \sim \partial_t (\rho v^i) + \partial_i (\rho v^i v^j + p^{ij}) = 0 \] \hspace{1cm} (2)

The other equations (the “material equations”) are not specified. That’s already all.
Nonetheless, it should be noted that in this step we have made important physical assumptions: we have only a single, universal medium, and no interactions, especially no momentum exchange with other media or external forces.

4 The Effective Metric and Effective Matter

The new variable we introduce is the “effective metric” $g_{\mu\nu}$. It is defined algebraically by the following formulas:

\[
\hat{g}^{00} = \sqrt{-g} = \rho \\
\hat{g}^{i0} = \sqrt{-g} = \rho v^i \\
\hat{g}^{ij} = \sqrt{-g} = \rho v^i v^j + p^{ij}
\]

Note that this decomposition of $g_{\mu\nu}$ into $\rho$, $v^i$ and $p^{ij}$ is a variant of the ADM decomposition [3]. The signature of $g_{\mu\nu}$ follows from $\rho > 0$ and negative definiteness of $p^{ij}$. It is a global hyperbolic metric — the Newtonian background time is a global hyperbolic function.

Now, a remarkable observation which is essential for our derivation is what happens with our four classical conservation laws in these new variables. They simply (essentially by construction) become the harmonic condition for the effective metric $g_{\mu\nu}$:

\[
\partial_{\mu}(g^{\alpha\mu}\sqrt{-g}) = \Box X^\alpha = 0
\]

That means, the preferred coordinates of the Newtonian background are the harmonic coordinates of the effective metric $g_{\mu\nu}$.

All other (unspecified) material properties $\varphi^m(x)$ of the condensed matter theories are interpreted as “effective matter fields”. That means, different condensed matter theories with different sets of material properties (inner steps of freedom) lead to different effective matter fields. Instead of the original condensed matter Lagrangian $L = L(\rho, v^i, p^{ij}, \varphi^m)$, we have now a Lagrangian $L = L(g_{\mu\nu}, \varphi^m)$ which depends on an effective metric $g_{\mu\nu}$ and effective matter fields $\varphi^m(x)$.

5 Derivation of the General Lagrangian

Now, it’s time to derive the general form of the Lagrangian for our class of condensed matter theories in these effective variables. Let’s at first introduce
some notations for the proportionality factors used in the relations between the Euler-Lagrange equations for the $X^\alpha$ and the continuity and Euler equations. We need two constants $\Xi, \Upsilon$, and we introduce them for convenience with a common factor $(4\pi G)^{-1}$:

$$\frac{\delta S}{\delta T} = -(4\pi G)^{-1} \Upsilon \square T \quad (3)$$
$$\frac{\delta S}{\delta X^i} = (4\pi G)^{-1} \Xi \square X^i \quad (4)$$

Introducing a diagonal matrix $\gamma_{\alpha\beta}$ by $\gamma_{00} = \Upsilon, \gamma_{ii} = -\Xi$ we can write these equations in closed form as

$$\frac{\delta S}{\delta X^\alpha} \equiv -(4\pi G)^{-1} \gamma_{\alpha\beta} \square X^\beta$$

Now, we can easily find a particular Lagrangian which fulfills this condition:

$$L_0 = -(8\pi G)^{-1} \gamma_{\alpha\beta} X^\alpha_{\mu} X^\beta_{\nu} g^{\mu\nu} \sqrt{-g}$$

Then, let’s consider the difference $L - L_0$. We obtain:

$$\frac{\delta \int (L - L_0)}{\delta X^\alpha} \equiv 0$$

But this is simply the condition that the Euler-Lagrange equations of the Lagrangian $L - L_0$ are strong covariant, that means, the equations of general relativity. Thus, $L - L_0$ is simply the most general Lagrangian of general relativity. So we obtain:

$$L = -(8\pi G)^{-1} \gamma_{\alpha\beta} X^\alpha_{\mu} X^\beta_{\nu} g^{\mu\nu} \sqrt{-g} + L_{GR}(g_{\mu\nu}) + L_{\text{matter}}(g_{\mu\nu}, \varphi^m).$$

This Lagrangian fulfills the Einstein equivalence principle (EEP, cf. [31]) in its full beauty – we have a metric theory of gravity. Having derived the

\footnote{Note that the coefficients $\gamma_{\alpha\beta}$ are only constants of the Lagrange density, the indices enumerate the scalar fields $X^\alpha$. They do not define any fundamental, predefined object of the theory. Instead, variables of the Lagrange formalism, by construction, are only $g_{\mu\nu}, \varphi^m$ and $X^\alpha$.}
Lagrangian, we have derived this principle for the whole class of condensed matter theories which fulfill our assumption.

In the preferred coordinates $X^\alpha$ (where we have $X^\alpha_{\mu} = \delta^\alpha_\mu$) we obtain a non-covariant form of the Lagrangian:

$$L = -(8\pi G)^{-1}(\Upsilon g^{00} - \Xi g^{ii})\sqrt{-g} + L_{GR}(g_{\mu\nu}) + L_{\text{matter}}(g_{\mu\nu}, \varphi^m).$$

5.1 Discussion of the Derivation

This extremely simple derivation of exact general-relativistic symmetry in the context of a classical condensed matter theory looks very surprising. It may be suspected that too much is hidden behind the innocently looking relation between Lagrange formalism and the condensed matter equations, or somewhere in our “weak covariant formalism”.

Therefore, let’s try to understand on a more informal level what has happened, and where we have made the physically non-trivial assumption which leads to the very non-trivial result – the EEP.

A non-trivial physical assumption is, indeed, the classical Euler equation. This equation contains non-trivial information – that there is only one medium, which has no interaction, especially no momentum exchange, with other media. This is, indeed, a natural but very strong physical assumption. And from this point of view the derivation looks quite natural: What we assume is a single, universal medium. What we obtain with the EEP is a single, universal type of clocks. An universality assumption leads to an universality result.

From point of view of parameter counting, all seems to be nice too. We have explicitly fixed four equations, and obtain an independence from four coordinates. All the effective matter fields are by construction fields of a very special type – inner steps of freedom, material properties, of the medium. Therefore it is also not strange to see them closely tied to the basic steps of freedom (density, velocity, pressure) which define the effective gravitational field.

The non-trivial character of the existence of a Lagrange formalism seems also worth to be mentioned here. A Lagrange formalism leads to a symmetry property of the equation – they should be self-adjoint. This is the well-known symmetry of the “action equals reaction” principle. It can be seen where this symmetry has been applied if we consider the second functional derivatives. The EEP means that the equations for effective matter fields $\varphi^m$ do not depend on the preferred coordinates $X^\alpha$. This may be proven in this way:
\[
\frac{\delta}{\delta X^\mu} \frac{\delta S}{\delta \varphi^m} = \frac{\delta}{\delta \varphi^m} \frac{\delta S}{\delta X^\mu} = \frac{\delta}{\delta \varphi^m} [\text{cons. laws}] = 0
\]

Thus, we have applied here the “action equals reaction” symmetry of the Lagrange formalism and the property that the fundamental classical conservation laws (related with translational symmetry by Noether’s theorem) do not depend on the material properties \(\varphi^m\).

Thus, our considerations seem to indicate that the relativistic symmetry (EEP) we obtain is explained in a reasonable way by the physical assumptions which have been made, especially the universality of the medium and the special character of the “effective matter” steps of freedom – assumptions which are implicit parts of the Euler equation.

6 Application in Condensed Matter Theory

The connection we have established between condensed matter theory and general relativity may be used in two directions. We can apply condensed matter theory to model gravity – essentially a revival of the old ether idea. And we can use our understanding of general relativity and the powerful differential-geometric machinery which has been developed in this context in condensed matter theory.

Let’s consider first the application to condensed matter theory. If we find a description of a condensed matter theory which fits into our scheme, our derivation gives an important symmetry of the theory – a symmetry which is important for the excitations of the matter itself, but hidden for outside observers.

The continuity equation and the Euler equation we already have in the theory. What we need is therefore an appropriate Lagrange formulation. Unfortunately, it is a hard problem to find Lagrangians for condensed matter theories. The standard Lagrangians (see, for example, the review of Wagner [28]) use Lagrange multipliers to obtain the continuity equation. Now, what we have found is another general way to obtain continuity and Euler equations as Euler-Lagrange equations. What we need now are special Lagrangians for condensed matter theories which follow this general scheme. “Reverse engineering” seems to be a reasonable way: we would have to start with a general-relativistic matter Lagrangian, introduce the condensed matter variables, and try to identify the matter fields with material properties of some type of condensed matter.
In this context, it seems worth to mention that the additional terms of the Lagrangian have a simple and natural form in the original condensed matter variables:

\[
L = L_{GR} + L_{\text{matter}} - (8\pi G)^{-1}(\Upsilon \rho - \Xi(\rho v^2 + p^i))
\]

The Hilbert-Einstein Lagrangian \( R \) may be interpreted as related to inner stress. Indeed, if there is no curvature, there exists a coordinate transformation so that the metric becomes the Minkowski metric, which plays the role of a stress-free reference state. The use of the 3D Euclidean Hilbert-Einstein Lagrangian for the description of stress in the presence of dislocations has been proposed by Malyshev [16]. Our approach suggests that the other, time-dependent parts of the full Hilbert-Einstein Lagrangian have to be taken into account too.

A solvable problem seems to be that the pressure has the wrong sign: it should be negative definite, but usual pressure, as defined in microscopic theories, is positive definite. But in the classical Euler equation pressure is defined only modulo a constant, therefore our pressure should not be identical with microscopic pressure.

An interesting point is the symmetry group of the theories in our class. Looking at the basic conservation laws, which are classical, pre-relativistic equations, it seems natural to assume Galilean symmetry. Instead, the symmetry group is the Poincare group. This Poincare symmetry is defined by the Minkowski metric \( \gamma_{\alpha\beta} \). Thus, if we try reverse engineering, we obtain not classical but special-relativistic condensed matter equations. Classical Galilean invariance we obtain only in a limit \( \Upsilon / \Xi \to \infty \).

7 Application in Fundamental Physics: Revival of the Ether Concept

The other direction would be to try to use a classical condensed matter theory which fits into our scheme as the fundamental “theory of everything”. Our general scheme defines a metric theory of gravity in competition with general relativity, while the special material properties and material equations of the model define the matter fields and matter equations.

We obtain the following equations:

\[
G^\mu_\nu = 8\pi G(T_m)_\mu^\nu + (\Lambda + \gamma_{\kappa\lambda}g^{\kappa\lambda})\delta^\mu_\nu - 2g^{\mu\kappa}\gamma_{\kappa\nu}
\]
as well as the harmonic equations

\[ \partial_\mu (g^{\mu \alpha} \sqrt{-g}) = 0. \]

There is also another form of the energy-momentum tensor – the basic equation may be simply considered as a decomposition of the full energy-momentum tensor \( g^{\mu \kappa} \sqrt{-g} \) into a part \((T_m)_\mu^\nu\) which depends on matter fields and a part \((T_g)_\mu^\nu\) which depends on the gravitational field:

\[ (T_g)_\mu^\nu = (8\pi G)^{-1} \left( \delta_\nu^\rho (\Lambda + \gamma_{\kappa \lambda} g^{\kappa \lambda}) - G_\mu^\rho \right) \sqrt{-g} \]

Thus, the additional non-covariant terms solve the problem of general relativity with local energy and momentum density for the gravitational field. We have even two equivalent forms for the conservation laws, one with a subdivision into gravitational and matter part, and another where the full tensor depends only on the gravitational field.

The consideration of the predictions of the theory which differ from general relativity is outside the aim of this paper and done elsewhere \footnote{They include a “big bounce” instead of a big bang, a homogeneous dark matter term of type \( p = -1/3 \varepsilon \), and stable “frozen stars” instead of black holes}. But from the previous results already follows that the Einstein equivalence principle (which includes local Lorentz invariance, local position invariance, and the weak equivalence principle) holds, and that in the limit \( \Xi, \Upsilon \to 0 \) we obtain the Einstein equations. The overwhelming experimental evidence in favour of the EEP and the Einstein equations (Solar system tests, binary pulsars) as described by Will \footnote{They include a “big bounce” instead of a big bang, a homogeneous dark matter term of type \( p = -1/3 \varepsilon \), and stable “frozen stars” instead of black holes} can give only upper bounds for the parameters \( \Xi, \Upsilon \). Therefore the theory is not in contradiction with observation.

For most physicists it seems more objectionable that this theory is essentially a revival of the old pre-relativistic ether theory. This seems contrary to the overwhelming success of relativity during this century. But most of this progress – especially relativistic quantum field theory up to the standard model – in no way depends on the question if there is a hidden preferred frame or not. And, as our theory shows, the progress in the domain of relativistic gravity can be as well covered by an ether theory too. So, this argument does not seem to be decisive.

The theory we have presented here removes some old arguments against ether theory:

- There was no viable theory of gravity — we have found now a theory of gravity with GR limit which seems viable;
• The assumptions about the Lorentz ether have been ad hoc. There was no explanation of relativistic symmetry, the relativistic terms have had ad hoc character — the assumptions we make for our medium seem quite natural, and we derive relativistic symmetry;

• There was no explanation of the general character of relativistic symmetry, the theory was only electro-magnetic — in the new concept the “ether” is universal, all matter fields describe properties of the ether, which explains the universality of the EEP and the gravitational field;

• There was a violation of the “action equals reaction” principle: there was influence of the ether on matter, but no reverse influence of matter on the stationary and incompressible ether — we have now a Lagrange formalism which guarantees the “action equals reaction” principle and have a compressible, instationary medium;

But not only these old problems of ether theory, which have justified the rejection of the ether concept, have been solved. It is also easy to find advantages of the ether concept – moreover, these advantages can be found in very different domains. We have already mentioned local conservation laws for energy and momentum. There are other domains: compatibility with quantum principles, and compatibility with classical realism and hidden variable theories:

First, some conceptual quantum gravity problems disappear, especially the notorious “problem of time”. This is a well-known fact: “... in quantum gravity, one response to the problem of time is to ‘blame’ it on general relativity’s allowing arbitrary foliations of spacetime; and then to postulate a preferred frame of spacetime with respect to which quantum theory should be written.”

Another argument is related with the violation of Bell’s inequality. It is widely accepted that experiments like Aspect’s show the violation of

\[ 4 \] This way to solve the problem is rejected not for physical reasons, but because of deliberate metaphysical preference for the standard general-relativistic spacetime interpretation: “most general relativists feel this response is too radical to countenance: they regard foliation-independence as an undeniable insight of relativity.” What else can be said about the quantum nature of space and time in a non-relativistic Schrödinger theory. The hopes to find something new, very interesting and fundamental about space and time would be dashed. But nature is not obliged to fulfil such hopes.
Bell’s inequality and, therefore, falsify Einstein-local realistic hidden variable theories. Usually this is interpreted as a decisive argument against hidden variable theories and the EPR criterion of reality. But it can as well turned into an argument against Einstein locality. Indeed, if we take classical realism (the EPR criterion [10]) as an axiom, the violation of Bell’s inequality simply proves the existence of superluminal causal influences. Such influences are compatible with a theory with preferred frame and classical causality, but not with Einstein causality.

We know today that an EPR-realistic, even deterministic hidden variable theory exists – it is Bohmian mechanics [7]. It may be generalized into the special-relativistic domain. In agreement with the general properties of EPR-realistic theories, this requires a preferred frame. This property of Bohmian mechanics explains why it has been widely ignored. Now, an ether theory of gravity which gives a preferred frame allows to extend the concept of Bohmian mechanics into the domain of relativistic gravity.

These considerations suggest that the old ether concept is not as unreasonable as it seems at a first look and is worth to be considered in more detail.

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5 This has been mentioned by Bell [6]: “the cheapest resolution is something like going back to relativity as it was before Einstein, when people like Lorentz and Poincare thought that there was an aether — a preferred frame of reference — but that our measuring instruments were distorted by motion in such a way that we could no detect motion through the aether. Now, in that way you can imagine that there is a preferred frame of reference, and in this preferred frame of reference things go faster than light.”

6 Note also that the existence of Bohmian mechanics proves that QM in itself is compatible with the EPR criterion of reality, so quantum theory gives no additional evidence against the EPR criterion. Thus, the only argument against the EPR criterion is its contradiction with Einstein causality.

7 Note that in all the problems we have considered here (local energy density, problem of time, EPR criterion, Bohmian mechanics) the majority of scientists has made a different choice. In all cases this has been at least partially justified with the contradiction with relativistic philosophy. These have been reasonable decisions in a situation where the ether concept was as dead as possible for a scientific theory and often compared with phlogiston theory. But we discuss here a possible revival of this alternative. Whatever we present as an argument in favour of a preferred frame we present because it is in contradiction with the relativistic paradigm. If the concept has been rejected mainly because it is in contradiction with the relativistic paradigm, this rejection cannot be taken into account in the context of this discussion. It would be circular reasoning.

Thus, to evaluate above arguments in favour of a preferred frame we cannot accept the “majority opinion” as it is. Instead, we have to reevaluate the arguments to find out if there are reasons for the rejection of these concepts which do not depend on their contradiction with the relativistic paradigm. This endeavour is beyond the scope of this article.
8 Conclusion

We have found a remarkable connection between a simple class of classical condensed matter theories and a metric theory of gravity which fulfils the EEP and gives in some limit the Einstein equations of general relativity. It is a simple and beautiful relation which combines in a natural way two very beautiful coordinate-dependent elements of general relativity – harmonic coordinates and the ADM decomposition – with the classical equations of condensed matter theory.

This relation suggests applications in two directions. It may be used to apply the apparatus of general relativity in condensed matter theory. We can also use classical condensed matter theory as a model for fundamental physics below Planck scale. This may be a use as a toy model, similar to the use of acoustic “dumb holes” as models for the study of Hawking radiation [21]. But it seems not unreasonable to think also about a complete revival of pre-relativistic ether concepts. We have found that many old problems of the Lorentz ether are solved by our new approach, and we have also noted several independent arguments in favour of a preferred frame: local energy-momentum densities for gravity, the problem of time in quantum gravity, the EPR criterion of reality and Bohmian mechanics.

Future has to show which of these possibilities will be successful. Anyway, the relation between condensed matter theory and the theory of gravity we have found here seems too beautiful to be without any physical importance.

A Can every Lagrangian be rewritten in weak covariant form?

There has been some confusion about the role of covariance in general relativity. Initially it was thought by Einstein that general covariance is a special property of general relativity. Later, it has been observed that other physical theories allow a covariant description too. The classical way to do this for special relativity (see [12]) is to introduce the background metric $\eta_{\mu\nu}(x)$ as an independent field and to describe it by the covariant equation $R_{\mu\nu\alpha\beta}[\eta] = 0$. For Newton’s theory of gravity a covariant description can be found in [17], §12.4. The general thesis that every physical theory may be reformulated in a general covariant way was proposed by Kretschmann [15].

In our derivation we have to assume that the Lagrangian is given in our “weak covariant” way. Now, the question we want to discuss here is if this is a non-trivial restriction or not. Thus, we have here a similar but more specific problem: We have to consider condensed matter theories with Lagrange
formalism, and have to rewrite them in a covariant way with very special variables – the preferred coordinates $X^\alpha$ – so that every dependence on the Newtonian background is described by these variables $X^\alpha$. Our hypothesis is that this is possible:

*The Lagrangian of every physical reasonable condensed matter theory with Lagrange formalism may be rewritten in “weak covariant” form.*

Unfortunately we don’t know what may appear in the “most general reasonable theory” as a variable. Therefore, in this general form this thesis seems unprovable in principle. All we can do is to present ways to transform some more or less general classes of Lagrangians into weak covariant form.

Now, for the classical geometric objects related with the Newtonian framework it is easy to find a weak covariant formulation:

- The preferred foliation is immediately defined by the coordinate $T(x)$.
- The Euclidean background metric: a scalar product $(u, v) = \delta_{ij} u^i v^j$ may be presented in weak covariant form as $(u, v) = \delta_{ij} X^i_{\mu} X^j_{\nu} u^\mu v^\nu$.
- Absolute time metric: Similarly, the (degenerate) metric of distance in time may be defined by $(u, v) = T_{\mu} T_{\nu} u^\mu v^\nu$.
- The preferred coordinates also define tetrad and cotetrad fields: $dX^\alpha = X^\alpha_\mu dx^\mu$, for the vector fields $\partial/\partial X^\alpha = (J^{-1})^\mu_\alpha (X^\beta_{\nu}) \partial/\partial x^\mu$ we need the inverse Jacobi matrix $J^{-1}$, which is a rational function of the $X^\alpha_{\nu}$.
- Arbitrary upper tensor components $t^{\alpha_1 \ldots \alpha_n}$ may be transformed into $X^\alpha_{\mu_1} \ldots X^\alpha_{\mu_n} t^{\mu_1 \ldots \mu_n}$. For lower indices we have to use again the inverse Jacobi matrix $J^{-1}$.
- Together with the background metrics we can define also the related covariant derivatives.

Now, these examples seem to be sufficient to justify our hypothesis. Of course, if we want to transform a given non-covariant function into a weak covariant form, we have no general algorithm. For example, a scalar in space in a non-relativistic theory may be a spacetime scalar, the time component of a four-vector, and so on for tensors. But this is not the problem we have to solve here. For our hypothesis it is sufficient that there exists at least one way.

Last not least, let’s note that our hypothesis is not essential for our derivation. Simply, if the hypothesis is false, our assumption that the Lagrangian
is given in weak covariant form is really non-trivial. This would be some loss of
generality and therefore of beauty of the derivation, but in no way fatal for
the main results of the paper. The considerations given here already show
that the class of theories which allows a weak covariant formulation is quite
large.

B Remarks about the special geometric na-
ture of coordinates

In our formalism we handle the preferred coordinates like usual fields. Of
course, every valid set of coordinates $X^\alpha(x)$ defines a valid field configura-
tion. But the reverse is not true. To define a valid set of coordinates, the
functions $X^\alpha(x)$ have to fulfil special local and global restrictions: the Jacobi
matrix should be non-degenerated everywhere, and they should fulfil special
boundary conditions.

Thus, we have to be careful and to check how this special geometric nature
of the preferred coordinates influences various questions.

B.1 Justification of the Euler-Lagrange formalism

The description of the Lagrange function in weak covariant form is, in com-
parison with its non-covariant description, only another way to describe the
same minimum problem in other variables. So, to solve this minimum prob-
lem we can try to apply the standard variational calculus.

Now, there is a subtle point which has to be addressed. The point is that
not all variations $\delta X^\alpha(x)$ are allowed – only the subset with the property
that $X^\alpha(x) + \delta X^\alpha(x)$ defines valid global coordinates. Therefore, to justify
the application of the standard Euler-Lagrange formalism we have to check
if this subset of allowed variations is large enough to give the classical Euler-
Lagrange equations.

Fortunately this is the case. To obtain the Euler-Lagrange equations
we need only variations with compact support, so we don’t have a problem
with the different boundary conditions. Moreover, for sufficiently smooth
variations ($\delta X^\alpha \in C^1(\mathbb{R}^4)$ is sufficient) there is an $\varepsilon$ so that $X^\alpha(x) + \varepsilon \delta X^\alpha(x)$
remains to be a system of coordinates: indeed, once we have an upper bound
for the derivatives of $\delta X^\alpha$, we can make the derivatives of $\varepsilon \delta X^\alpha$ arbitrary small, especially small enough to leave the Jacobi matrix of $X^\alpha + \varepsilon \delta X^\alpha$ non-
degenerated.

Now, to obtain the Euler-Lagrange equations we need only small varia-
tions, so, the geometrical restrictions on global coordinates do not influence
the derivation of the Euler-Lagrange equations for the $X^\alpha(x)$. So, for the preferred coordinates we obtain usual Euler-Lagrange equation, as if they were usual fields, despite their special geometric nature.

### B.2 Comparison with “GR with clock fields”

This does not mean that the special geometric nature of the preferred coordinates is unimportant. Instead, this special nature of the fields $X^\alpha(x)$ is a very important part of the definition of our condensed matter theories.

In this context it is useful to compare our condensed matter theories with the theory we obtain by forgetting this special geometric nature. We obtain a general-relativistic theory with four scalar fields $U^\alpha(x)$ and Lagrangian

$$L = -(8\pi G)^{-1} \gamma_{\alpha\beta} U^\alpha_{\mu} U^\beta_{\nu} g^{\mu\nu} \sqrt{-g} + L_{GR}(g_{\mu\nu}) + L_{\text{matter}}(g_{\mu\nu}, \varphi^m).$$

A similar theory with “clock fields” in general relativity has been considered by Kuchar [13]. Now, we have the same Lagrangian and the same Euler-Lagrange equations. Nonetheless, above theories are completely different – they don’t have even a single common solution. Indeed, we have completely different boundary conditions. For a standard field theory with scalar fields, the natural boundary condition is that the solution remains bounded. Instead, the coordinates should be unbounded on a complete solution.

Nonetheless, even if above theories have not even a single common solution, the question how to distinguish above theories by observation remains very complicated.

### References

[1] A. Ashtekar, Phys.Rev.Lett. 57, 2244, 1986 Phys.Rev. D the Ashtekar variables

[2] A. Aspect, J. Dalibard, G. Roger, Experimental test of Bell’s inequalities using time-varying analysers, Phys. Rev.Lett. 49, 1801-1807, 1982

[3] R. Arnowitt, S. Deser, C. Misner, Quantum theory of gravitation: General formalism and linearized theory, Phys. Rev. 113, 745–750, 1959

[4] J.L. Anderson, Principles of relativity physics, Academis Press, New York 1967
[23] I. Schmelzer, General ether theory – a metric theory of gravity with condensed matter interpretation, Proc. of the XXII international workshop on high energy physics and field theory, Protvino, June 23-25, 1999

[24] W.G. Unruh, Phys Rev Lett 46 1351 (1981)

[25] M. Visser, Acoustic black holes: horizons, ergospheres, and Hawking radiation, gr-qc/9712010

[26] G.E. Volovik, Induced gravity in superfluid $^3$He, cond-mat/9806010

[27] G.E. Volovik, Field theory in superfluid $^3$He, cond-mat/9812381

[28] H.-J. Wagner, Das inverse Problem der Lagrangeschen Feldtheorie in Hydrodynamik, Plasmaphysik und hydrodynamischem Bild der Quantenmechanik, Universität Paderborn, 1997

[29] S. Weinberg, Phys.Rev. B138, 988 (1965)

[30] S. Weinberg, What is quantum field theory, and what did we think it is, hep-th/9702027

[31] C.M. Will, The confrontation between general relativity and experiment, gr-qc/9811036