Electric Field and Light Field Modulated Josephson Effect in Silicene-Based SNS Josephson Junction: Andreev Reflection and Free Energy

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Abstract

We investigate the Josephson effect in superconductor-normal-superconductor junction (SNS) based on an unbiased silicene under the perpendicular electric field and off-resonance circularly polarized light both analytically and numerically. The Andreev reflection (during the subgap transport), free energy, Josephson current, and the reversal of the Josephson effect are explored. The Andreev reflection is complete (for tunnel limit) in the NS interface even for the clean sample and without the Fermi wave vector mismatch, which is opposite to the case of ferromagnet-superconductor interface. The dynamical polarization of the related degrees of freedom is also related to the $0 - \pi$ transition and the generation of $\phi_0$-junction. In the short junction limit, the approximated results about the Andreev level and free energy are also discussed. Beside the low-energy limit of the tight-binding model, the finite-size effect is needed to be taken into account. The effect of the time-dependent drive and the Green's function method also represented in the end of this paper.

Keywords
Josephson effect · Silicene · Andreev bound state · Josephson current

1 Introduction

Silicene is a topological material which attract much attention both theoretical [1–5] and experimental [6, 7] due to its favorable properties including the strong spin-orbit coupling and the gate- or light-controllable band gap [8–10]. The silicene also provides us a good platform for the investigation of local or nonlocal electronic transport [11, 12]. In the low-temperature limit, the dc Josephson effect in the silicene-based SNS Josephson junction is worth to study for the exploration of the superconducting physics as well as the applications on the spintronics, valleytronics and optoelectronics. Here the middle normal part is the semimetal for intrinsic silicene. In this paper, we consider a silicene-based SNS Josephson junction with the silicene nanoribbon lies along the $k_x$ (zigzag) direction in a two-terminal geometry as shown in Fig. 1, where an incoming quasiparticle from conduction band and a hole due to the lost electron in valence band are indicated. The two superconducting leads weakly linked by the middle normal region or by a microbridge. The weak link effect can also be realized by using a (half) metal, (topological) insulator, or the ferromagnet [13, 14] as described by the sinusoidal current-phase relation. The silicene is divided into three parts, the middle part is deposited on the SiO$_2$ substrate, and thus with the dielectric constant $\epsilon = 2.45$ ($\epsilon_{SiO_2} = 3.9$), while the left- and right-side parts are deposited on the conventional superconductor electrodes with the $s$-wave superconductors realized by the superconducting proximity effect. The perpendicular electric field and off-resonance circularly polarized light are applied to the middle normal region, and the resulting equilibrium phase difference may making a $\psi$-state differs from the standard 0- or $\pi$-state, where the transport critical current is harder to estimated [15]. In the presence of impurity, the diffusive transport can be described by the Usadel equation [16], besides, due to the existence non-resonance light, the Stokes (or anti-Stokes) scattering in the middle normal region also emerges.
The low-energy effective Dirac Hamiltonian of the silicene (for the normal part) reads [21–23]

\[
H = \hbar v_F (\eta \tau_y k_y + \tau_z k_z) + \eta \lambda_{SOC} \tau_z \sigma_z + \eta \tau_z \sigma_z (k_x \sigma_x - k_y \sigma_y) \\
\approx \frac{A}{2} E_\perp \tau_z + \frac{\lambda_{R}}{2} (\eta \sigma_y \tau_x - \sigma_z \tau_y) + M_s \sigma_z + M_c - \eta \tau_z \hbar v_F^2 \frac{A^2}{\Omega},
\]

where \(E_\perp\) is the perpendicularly applied electric field, \(a = 3.86\) is the lattice constant, \(A = eA_{0}/\hbar\) is describes the illumination effect. \(A\) is the buckled distance in \(z\)-direction between the upper sublattice plane and lower sublattice plane. \(M_s\) is the spin-dependent exchange field and \(M_c\) is the charge-dependent exchange field. \(\lambda_{SOC} = 3.9\) meV is the strength of the intrinsic spin-orbit coupling (SOC) and \(\lambda_{R} = 0.7\) meV is the intrinsic Rashba coupling which has been found that linear with the applied electric field in our previous works [21, 22, 24–26], which as \(\lambda_{R} = 0.012E_\perp\). For circularly polarized light, the electromagnetic vector potential reads \(A(t) = A(\pm \sin \Omega t, \cos \Omega t)\), where \(\pm \) denotes the right and left polarization, respectively. Due to the perpendicular electric field \(E_\perp\) and the off-resonance circularly polarized light which with frequency \(\Omega > 1000\) THz, the Dirac-mass and the corresponding approximated energy spectrum of the normal region are

\[
m_{D}^{\eta \sigma_z \tau_z}\approx|\eta \lambda_{SOC} \tau_z - \frac{A}{2} E_\perp \tau_z + M_s \sigma_z + M_c - \eta \hbar v_F^2 \frac{A}{\Omega}|, \\
e \approx s \sqrt{(\eta^2 v_F^2 k_y^2 + (m_{D}^{\eta \sigma_z \tau_z})^2 + s^2 \mu_n^2)},
\]

respectively, where the dimensionless intensity \(A = eA_{0}/\hbar\) is in a form similar to the Bloch frequency, and \(s = \pm 1\) is the electron/hole index, and the subscript \(e\) and \(h\) denotes the electron and hole, respectively. The off-resonance circularly polarized light results in the asymmetry band gap in two valleys (see Ref. [26]) and breaks the time-reversal symmetry in the mean time, and thus provides two pairs of the different incident electrons that may leads to the josephson current reversal due to the valley-polarization. The superconducting gap (complex pair potential) \(\Delta_s\) which obeys the BCS relation can be estimated as \(\Delta_s = \Delta_0 \tan(1.74 \sqrt{\Delta_T/\Delta_0 - 1}) e^{\phi/2}\) (here we only consider the right superconducting lead) [15, 27] with \(\phi\) the macroscopic phase-difference between the left and right superconducting leads, and \(\Delta_0\) is the zero-temperature superconducting bulk gap which estimated as 0.001 eV [18] here and \(T_c\) is the superconducting critical temperature which estimated as \(5.66 \times 10^{-4}\) eV. Obviously, the quasienergy spectrum here is distinct from the one obtained by the Floquet technique in

by creating a electron-hole part with the electron inelastically scattered between K and K’ valleys [17]. We set a finite chemical potential \(\mu_n\) in the middle region by slightly doping, while at the superconducting regions, the chemical potential \(\mu_s\) required by the high carriers density is much larger than the Fermi wave-vector \(k_F\) and the Dirac-mass \(m_D^{\eta \sigma_z \tau_z}\) (here \(\sigma_z\) and \(\tau_z\) are the spin and sublattice (pseudospin) degrees of freedom, respectively, \(n\) is the valley index), which reads \(k_F = \sqrt{\mu_n^2 - (m_D^{\eta \sigma_z \tau_z} - U)^2}/\hbar v_F\) with \(U\) the electrostatic potential induced by the doping or gate voltage in the superconducting region, which breaks the electron-hole symmetry, and lifts the zero-model (between the lowest conduction band and the highest valence band) above the Fermi level (imaging the zero Dirac-mass here) [18]. The \(U\) has been experimentally proved to be valid in controlling the phase shift of the \(\phi_0\)-junction as well as the 0 – \(\pi\) transition [19] like the bias voltage. Note that the Dirac-mass here is related to the band gap by \(\Delta = 2m_D^{\eta \sigma_z \tau_z}\).

Thus we can know that \(\mu_s \gg \sqrt{(m_D^{\eta \sigma_z \tau_z} - U)^2}\) and the incident angle is larger than the transmission angle due to the relation \(k_F \sin \theta_h = \mu_s \sin \theta_h\) [20] where \(\theta_h\) is the incident angle from the normal region and \(\theta_i\) is the transmission angle in the superconducting region. Furthermore, we can estimate the transmission angle as \(\theta_i \approx 0\) to obtain the zero scattering angle and the smooth propagation.
where $T$ obtained, by solving the above DBdG equation with for the travelling electron and hole in normal region can be seen from the illumination term (see Appendix.A):

$$\psi^+ = (1, e^{i\theta}, 0, 0)^T e^{ik_y x},$$

$$\psi^- = (1, e^{-i\theta}, 0, 0)^T e^{-ik_y x},$$

$$\psi^h_+ = (0, 0, 1, e^{i\theta})^T e^{ik_y x},$$

$$\psi^h_- = (0, 0, 1, e^{-i\theta})^T e^{-ik_y x},$$

where $\theta$ is the incident angle of electrons and $\theta'$ is the angle of Andreev reflected holes:

$$\theta = \arccos \frac{\hbar v_F k}{\sqrt{\varepsilon + m_D^{++} + \sqrt{\varepsilon - m_D^{++} \tau_c}},}$$

$$\theta' = \arccos \frac{\hbar v_F^h k}{\sqrt{\varepsilon + m_D^{+-} - \sqrt{\varepsilon - m_D^{+-} \tau_c}}}.$$  

(6)

The factor $e^{ik_y x}$ is factored out where the momentum $k_y$ is quantized and dependent on the sample width. Note that for the Andreev reflection in normal region, due to the illumination effect, the group velocity of the electron and hole are different, i.e., $v_F^e \neq v_F^h$, which implies the broken of electron-hole symmetry. Such anisotropic effect is particularly obvious in the low frequency case, as can be seen from the illumination term (see Appendix.A):

$$v_F^e/v_F^h = \frac{\lambda_{SOC} + s \frac{\Delta^2}{\varepsilon}}{\lambda_{SOC} - s \frac{\Delta^2}{\varepsilon}}.$$  

That is in contrast to the case of intrinsic silicene, or the one (gated) epitaxially grow on the Ag(111) which with ambipolar symmetry [29]. At the N/S interface ($x = 0$), the wave function of the normal part and superconducting part are identical.

As revealed by the Floquet physics, the electron-hole symmetry is usually broken in the $p-d$ Dirac model [29, 30] by the high frequency drive. The difference between the silicene-based SNS Josephon junction and the ones base on other topological insulators (like graphene) is the broken of the valley-degeneracy in silicene under the external electric field as well as the off-resonance circularly polarized light (time periodic field). That not only allows the application of the valley-polarized Josephson currence, but also helpful to the many-terminal measurements using the Floquet technique in the presence of spin/pseudospin-momentum locking. For irradiated silicene, its time-dependence is embedded in the eigenvectors correspond to the Hamiltonian given above (see Appendix.A for details). As shown in Appendix.A, when the small intrinsic Rashba coupling $\lambda_{R_e}$ is neglected, the illumination term (and the resulting Dirac-mass) related only to the valley and pseudospin (sublattice) indices, and thus breaks the time-reversal symmetry ($THT^{-1} \neq H$) and lifts the valley degeneracy, results in the reversible supercurrents. This inspire us that a similar effect may induced also by a strong dynamical (static) magnetic field, which can breaks the time-reversal symmetry and creates the charge pumping by nonstatically (statically) breaks the charge-balance as happened in the junction with ring geometry [31, 32].

2 Andreev Bound State

The Andreev reflection (AR) exists in the middle normal region (or in an insulating barrier) for the case that the applied bias voltage [18] is smaller than $\Delta_s$ and $U$ (the gate voltage). We consider the quasi-normal incidence of the electrons from normal region, i.e., with constant $k_y$ and non-constant $k_x$. The Andreev bound state, which is common in the $d$-wave superconductor [33], exists in the middle normal region unless there is a band insulator, and can be easily realized by control the electric field and the light field which takes effect in the middle normal region. For SNS Josephson junction, in contrast to the normal reflection (NR) whose reflection and transmission coefficients obey the relation $|r_e|^2 + |t_e|^2 = 1$, the AR process obeys $|r_h|^2 + |t_h|^2 = 1$ (for a ideal interface with very high transparency) where $r_h$ is the reflection coefficient of the reflected hole from conduction band (retro-reflection) or valence band (specular reflection), and $t_e$ is the transmission coefficient of the electron-like quasiparticle. Note that for the nonlocal transport like the elastic cotunneling (ECT) process in the ferromagnet (normal)/superconductor/ferromagnet (normal) junction, it relies on the coherent superposition of states through the, e.g., quantum dot orbit in the parallel configuration between...
for a propagating scattering waves and a evanescent scattering waves (in the superconducting region for SNS junction), respectively, and we have (for $\varepsilon \approx \Delta_s$)

$$e^{i\beta} = \frac{\varepsilon \mp \sqrt{\varepsilon^2 - \Delta_s^2}}{\Delta_s},$$

(11)

where the sign "" takes "" + "" for $|\varepsilon| < \Delta_s$, and takes "" - "" for $|\varepsilon| > \Delta_s$. Then the normal reflection coefficient can be obtained as

$$r_e = \sqrt{1 - \left|\frac{k_F^n \cos \theta_n}{\sqrt{\varepsilon \Delta_s^2}}\right|^2},$$

(12)

where Im denotes the imaginary part. Both the reflection coefficient and transmission coefficient implies that the AR is complete in the NS interface (as depicted in Fig. 1 since we consider the tunnel limit) even when the interface is clean (without impurity) and without the Fermi wave vector mismatch, that’s opposite to the case of ferromagnet-superconductor interface [35]. In N-S junction, the Andreev reflection requires the excitation gap in normal part be smaller than the superconducting gap [20], i.e., only the subgap excitation energy is allowed, which would leads to the coherent superposition of the electron and hole excitations at small excitation energy (but not zero). And that is easily to realized in silicene. This is in contrast to the topological superconductor junction with two Majorana bounded states [36], where the Majorana zero modes are the zero-energy superpositions of the electron and hole, and form the so-called fractional Josephson effect [37]. The Majorana zero modes have also been explored in black phosphorus based-junction [38]. While at s-wave superconductor, the Majorana fermions can’t be found due to the absence of SOC. Such effect can also be found by a topological insulator-mediated junction [32]. When the band structures or the fermi levels between the normal region and the superconducting region are with great difference, the tunnel limit can be reached. In the case of $\varepsilon < \Delta_s$, electrons can enter into the superconductor lead only by forming a Cooper pair which consisted of a electron with up-spin in K valley and a electron with down-spin in L valley, otherwise it can not penetrate into the superconductor lead due to the small excitation energy [35]. We note that, for extreme spin-imbalance case (e.g., induced by nonzero exchange field) where the two spin components have mismatched fermi surfaces, like the half metal phase, the Fulde-Ferrell-Larkin-Ovchinikov pairing is a typical pairing mechanism where only the odd-frequency (triplet) Cooper pair can be penetrated into the middle part [14, 39]. Then it is possible to acts like a spin filter and produces a dissipationless spin-polarized current as seen in the single-Dirac-cone systems. Further, when $\varepsilon > \mu_n + m_D$, the AR is specular (interband), and it’s retro-like (intra-band) for $\varepsilon < \mu_n + m_D$. For the former one, the large excitation
energy also results in the thermal transport between two superconducting leads by the propagating model [40] and it’s related to the Fermi distribution term tanh(\(\varepsilon/2k_BT\)), while for the latter one, it contributes only to the localized model.

Then we focus on the dispersion of the Andreev bound level, which reads

\[
e_A = \frac{\Delta_f}{\sqrt{2}} \sqrt{1 - \frac{A(C - \cos \phi) + \sin \phi}{A^2 + B^2}},
\]

where we define (for \(\mu_s - \varepsilon + |D| > 0\))

\[
A = C_1 C_2 + \frac{(S_1 S_2 (\frac{f_1}{f_2} + 1) (\frac{f_1}{f_2} - 1)) \sqrt{f_1 f_2}}{4 \sqrt{1 - \frac{h^2 v_F^2 k^2}{f_1 f_2}} \sqrt{1 - \frac{h^2 v_F^2 k^2}{f_1 f_2}} \sqrt{f_3 f_4}}
\]

\[
B = \frac{S_1 C_2 (\frac{f_1}{f_2} + 1) \sqrt{f_1 f_2}}{\sqrt{1 - \frac{h^2 v_F^2 k^2}{f_1 f_2} f_3 f_4}} - \frac{C_1 S_2 (\frac{f_1}{f_2} - 1) \sqrt{f_1 f_2}}{\sqrt{1 - \frac{h^2 v_F^2 k^2}{f_1 f_2} f_3 f_4}}
\]

\[
C = \begin{cases} 
S_1 S_2 \frac{h v_F k}{\sqrt{1 - \frac{h^2 v_F^2 k^2}{f_1 f_2} f_3 f_4}} \left(1 - \frac{H_{\alpha}^2 k^2}{f_1 f_2}\right) \\
\sqrt{1 - \frac{h^2 v_F^2 k^2}{f_1 f_2} f_3 f_4} \end{cases}
\]

which distinguish the two kinds of AR: retroreflection and specular AR, and thus makes this expression valid for both of these two case. The wave vectors \(f_1 \sim f_4\) and parameters \(C_1, C_2, S_1, S_2\) are defined as

\[
f_1 = m^{\mu_s, \varepsilon} + \varepsilon + \mu_s, \quad f_2 = -m^{\mu_s, \varepsilon}, \quad f_3 = -\varepsilon - m^{\mu_s, \varepsilon} + \mu_s, \quad f_4 = m^{\mu_s, \varepsilon} - \varepsilon + \mu_s.
\]

\[
C_1 = \cos \left( \frac{L}{\hbar v_F} \sqrt{1 - \frac{h^2 v_F^2 k^2}{f_1 f_2} f_3 f_4} \right),
\]

\[
C_2 = \cos \left( \frac{L}{\hbar v_F} \sqrt{1 - \frac{h^2 v_F^2 k^2}{f_1 f_2} f_3 f_4} \right),
\]

\[
S_1 = \sin \left( \frac{L}{\hbar v_F} \sqrt{1 - \frac{h^2 v_F^2 k^2}{f_1 f_2} f_3 f_4} \right),
\]

\[
S_2 = \sin \left( \frac{L}{\hbar v_F} \sqrt{1 - \frac{h^2 v_F^2 k^2}{f_1 f_2} f_3 f_4} \right).
\]

Note that although the detail sign of the superscripts of Dirac mass (the degrees of freedom) is not shown, it’s indeed important since the photoinduced term related to the valley and sublattice indices while the intrinsic SOC term related to the spin index additionally. The \(x\)-component of the wave vector for the electron channel and hole channel, \(k_{xe}\) and \(k_{xh}\), are incorporated in the above wave vectors, specially, the electron and hole wave vectors here are both complex, which implies the inclusion of the subgap solutions with the evanescent scattering waves [18], and it has \(\frac{\Delta_f}{\sqrt{2}} 2 \cos \beta = 1\) for \(|\varepsilon| < \Delta_s\). In this paper we consider only the bulk state of the silicene, however, for electric field \(E_\perp < \lambda_{SOC}/\Delta = 0.017\) eV (don’t consider the effect of the off-resonance light), the silicene will exhibits the helical edge states, and the evanescent wave carries the supercurrent when the chemical potential of the middle region is zero [41, 42]. Note that we only consider the dc Josephson effect in the thermodynamic equilibrium state here rather than the ac Josephson effect which with time-dependent phase difference (e.g., by a time-dependent bias voltage). Base on above results, the Josephson current at zero-temperature limit can be obtained as \(J = \frac{\pi e}{4} \sum_n \frac{\partial J^{\mu_s}}{\partial \phi} \), by summing over the negative Andreev energies (the occupied Andreev states) in equilibrium regime, where \(n\) denotes the number of Andreev bound states (the resonant electron-hole state).

There exist a critical angles for AR process. When the incident angle of the scattering wave exceed this critical angle, it becomes exponentially-decayed. The critical angle for AR process is

\[
\theta_{AR} = \arcsin \left( \frac{f_3 f_4}{f_1 f_2} \right),
\]

when the quasienergy and chemical potential \(\mu_n\) are much smaller than the Dirac-mass \(m^{\mu_s, \varepsilon}\), the above critical angle can be simplified as \(\theta_{AR} = \arcsin \left( \frac{f_3 f_4}{f_1 f_2} \right)\), which is consistent with the result of Ref. [43]. For the case of heavily doping in the middle normal region, that’s \(\mu_n \gg \varepsilon\), then the AR is dominated by the retro one. In this case, when the chemical potential is comparable with the Dirac-mass, the AR process is suppressed by the destructive interband interference [44] among the Cooper pairs due to the large Dirac-mass (and thus the large band gap) which brings a large dissipative effect and reduce the AR. The minimum free energy in this SNS planar junction is \(\phi\)- and temperature-dependent, and local st the 0 or \(\pi\)-junction. With the defined incident angle \(\theta_i\) for the quasiparticle injected from the one of the superconducting electrodes to the middle normal region, the minimum free energy with AR process can be written as

\[
E(\phi, T) = -k_B T \sum_{\phi_s, \tau_s} \ln \left[ 2 \cosh \left( \frac{e_A}{2k_BT} \right) \right].
\]

The reversal of the Josephson effect will be caused by the valley, spin, and pseudospin polarization, which with dramatic dipole oscillation between the two components induced by the off-resonance circularly polarized light as we discussed [26], and results in a nonzero center-of-mass (COM) wave vector, just like the Josephson current realized by a ferromagnetic middle silicene (in superconductor/ferromagnet/superconductor ballistic Josephson
It’s found that the oscillation of valley polarization is related to the carrier-phonon scattering due to the photoexcitation which has a relaxation time in picosecond range \([45]\) (since the frequency of light is setted in the terahertz range) and larger than that of the electron-electron scattering. On the other hand, since the silicene in normal region is been deposited on a SiO\(_2\) substrate, the relaxation of the valley-polarization is also associated with the screened scattering by the charged impurities within the substrate. It’s found that for a certain concentration of the impurity and the vertical distance to the middle silicene layer \(d\), the relaxation time can be obtained in a dimensionless form as

\[
\tau = \left[ \frac{n_{\text{imp}} \varepsilon_0}{\hbar^2 v_F^2} \left( \frac{2\pi \varepsilon^2}{\varepsilon_0 \varepsilon} + \frac{1 + N_f}{\pi^2 \varepsilon_0 \varepsilon} \Pi (q, \omega) \right)^2 \right]^{1/2},
\]

where \(\varepsilon_0\) and \(\varepsilon\) are the vacuum dielectric constant and background dielectric constant, respectively. \(N_f\) is the number of the degenerate which can be treated as background dielectric constant, respectively. \(N_f\) is the number of the degenerate which can be treated as background dielectric constant, respectively. \(g_s\) and \(g_v\) are the partial Klein tunneling of Cooper pairs (AR does not requires the zero Dirac-mass) [51].

which can be further simplified as \(\tilde{\varepsilon}_0^* = \Delta_x \cos \phi\) in the case that the \(\varepsilon\) is small and that’s consistent with the result of Refs. [49, 50], where the transmission coefficient equals 1 here due to the partial Klein tunneling of Cooper pairs which happen at zero Dirac-mass [51] (while the AR does not requires the zero Dirac-mass).

\[\varepsilon_A^* = \Delta_x \cos \left[ \phi - L \left( \frac{-f_3}{\hbar v_F} + \frac{f_2}{\hbar v_F} \right) \right],\]

where \(\phi\) is the phase-correlation in nonequilibrium Josephson setup leads to an anomalous conductance [35]. In short junction limit \((\Delta_0 L / \hbar v_F \ll 1 \ll W)\), the Andreev level can be approximatively obtained as

\[\tilde{\varepsilon}_A = \Delta_x \cos \left[ \phi - L \left( \frac{-f_3}{\hbar v_F} + \frac{f_2}{\hbar v_F} \right) \right],\]

\[\varepsilon_A^* = \Delta_x \cos \phi\]

which can be further simplified as \(\tilde{\varepsilon}_0^* = \Delta_x \cos \phi\) in the case that the \(\varepsilon\) is small and that’s consistent with the result of Refs. [49, 50], where the transmission coefficient equals 1 here due to the partial Klein tunneling of Cooper pairs which happen at zero Dirac-mass [51] (while the AR does not requires the zero Dirac-mass).

3 Results and Discussion

Due to the presence of the degrees of freedom \(\eta, s_x\) and the electron/hole index \(s\), the each Andreev state has \(2^3 = 8\) levels under a certain electric field and light field, although there exists the level-degeneracy. The Josephson current is a nonlocal supercurrent carried by the Cooper pairs in the superconducting leads (while it’s carried by the quasiparticles in the middle normal region), and its slope is affected by the dynamical polarizations of the degrees of freedom [26] when, e.g., \(\sqrt{\varepsilon - m_{Dn}^{++}} \neq \sqrt{\varepsilon - m_{Dn}^{--}}\). For a Cooper pair penetrated into the bulk state, the macroscopic wave function is affected by the scattering process by the charged impurity, and thus has \(\Psi(x) \propto e^{-x/\xi} e^{iL/\xi} e^{i\phi/2}\), where \(\xi = \sqrt{\hbar D/\Delta_0}\) is the superconducting coherent length with \(D\) the diffusion constant for elastic scattering, and \(x \ll L\) is the mean free path in the bulk region. The Josephson effect here only consider the low-temperature condition which is dominated by the elastic scattering, while at higher temperature, the rised inelastic scattering may leads to the switching of the Fermi parity [32], and the frequency-dependent noise which is induced by the current-current correlation in nonequilibrium Josephson setup leads to the \(4\pi\) period of the Josephson current due to the existence of Majorana bound states. That also implies
that the dissipation plays an important role during the quasiparticle transition.

For simulation, we set the parameters as follows: The frequency of the off-resonance light is set larger than 3000 THz which is much higher than the critical value. The critical value here is \( \nu_c = 4.8 \text{ eV} = 1200 \text{ THz} \) for silicene. In the case that frequency is 3000 THz, the dimensionless intensity is \( A = 0.3 \) which is much larger than the SOC parameter \( \lambda_{SOC} \). The length of the normal region \( L \) is much shorter than the superconducting coherence length \( L \ll \hbar \nu_F / \Delta_0 \), i.e., \( L \ll \Delta_0 \). Here we estimate \( \nu_F = \sqrt{3} \Delta_0 \text{ at } \Delta_0 = 5.35 \text{ eV} \) here. The \( y \)-component of the momentum as well as the off-resonance circularly polarized light (which we only use the right-polarization from now on) is conserved due to the

![Fig. 3](Color online) Andreev bound level versus phase difference \( \phi \) for different electric field strength. The temperature set here is 0.3\( T_c \) to ensure the existence of superconducting effect

![Fig. 4](Color online) Dirac mass as a function of electric field strength. Other parameter setting is similar to Fig. 3. Here we present only two possible configurations: \( \eta = 1, \sigma_z = -1, \tau_z = 1 \) and \( \eta = -1, \sigma_z = 1, \tau_z = 1 \). As can be seen, (a) and (c) have only one curve due to the missing \( A \) term, but (b) and (d) have two curves due to the presence of \( A \) term (the brown line and blue line correspond to these two configurations respectively)
translational invariance, thus it’s a good quantum number in the computation, and we set $k_y = 2$ meV. $k$ is in an order of 0.01 eV here. While the detail setting of the parameters are labeled in the each following plot.

Figure 3 depicts the dispersion of the Andreev bound level for the AR process in 0 state (we don’t consider the breaking of inversion symmetry by the Rashba-coupling here since it’s very small). We can easily see than the period of Andreev level is $2\pi$.

We detect the effects of the electric field, off-resonance circularly polarized light, chemical potential, and length of the normal region to the Andreev bound level numerically. We focus on the short junction limit with $L = 20$ Å, where only the subgap Andreev states contribute to the supercurrent [52]. We found that, in Fig. 3a-b, the slope of the Andreev level is always negative in the range $[0, \pi)$ and positive in the range $[\pi, 2\pi]$, while in Fig. 3c-d, the negative and positive slope appear in both regions, where the sign reversal happen. For the case $\mu_s = 2$ eV, the absolute value of the slope of Andreev level increase monotonously with the increase of electric field, but such phenomenon disappear when the light turns on as shown in Fig. 3b. While for other levels, the sign change can happen
only at $\phi = \pi$. Although the spin- and valley-splitting of the Andreev level dispersion are not shown in the Figures, they are similar to the dispersion splitting of the Dirac fermions dependent on the change of Dirac mass. Figures 4a-d, 5a-d, and 6a-d show the Dirac masses, Andreev bound level with different junction length, and the free energies, respectively. From Fig. 6, we see that for $\mu_s = 2$ eV, the period of free energy is $\pi$, while it becomes $2\pi$ for $\mu_s = 2.5$. And most of the free energy curves locate in the negative region. The 0-\pi transition caused by change of electric field can be seen in the presence (Fig. 6c) or absence (Fig. 6d) of the $A$ term, by looking at the positions of the minimum free energy. The different effects between chemical potential $\mu_s = 2$ eV and $\mu_s = 2.5$ eV can be seen more clearly from the plot of $e_A/\Delta_s$, as shown in Fig. 7, where for $\mu_s = 2$ eV the slope of the Andreev level is very small and even becomes nearly flat line when $A = 2$, while for $\mu_s = 2.5$ eV the slope is much larger and the 0-\pi transition (sign reversal)
can be easily seen. We can obtain that the change of electric field and the intensity of light is valid for the generation of $0 - \pi$ transition because of the sublattice degree of freedom and the valley degree of freedom consider here, respectively. Thus if the sublattice degree of freedom of electric field is not considered, the variety of electric field won’t support the $0 - \pi$ transition or the transition to the $\phi_0$-junction, which is consistent with the result in Ref. [15, 42]. The Josephson current are presented in Fig. 8, which have period $2\pi$. Interestingly, we found that for $E_\perp = 1.517$ eV and $E_\perp = 1.217$ eV the currents have very large peaks compared to other cases, as shown in Figs. 8e and 8f, respectively.

In the absence of the electric field, off-resonance light and the antiferromagnetic exchange field, it is always be the 0-junction due to the presence of time-reversal invariance (and chiral symmetry), and the Josephson supercurrent vanishes at $\phi = n\pi$ ($n$ is integer) in this case. However, the broken symmetry or chiral leads to the nonzero supercurrent at $\phi = n\pi$ and the $\phi_0$-junction which can also be implemented by a external magnetic field in nanowire quantum dots-based junction (SQUID) [19].

The approximated Andreev level for short length $L$ is shown in Fig. 9 according to Eq. 19, where the $0 - \pi$ transition (see the gray line and blue line) and the $\phi_0$-junction can be implemented by changing the electric field, $A$, and the length $L$. Phenomenologically, through Fig. 9, the Eq. 19 can be rewritten as $\varepsilon_A^* = \Delta_s \cos(\phi + \phi_0)$ with $\phi_0 \in [0, 2\pi)$, the $\phi_0$ here is controlled by the variables presented in the plot, and this expression is similar to that of anomalous switching current [19]. The usual current-phase relation can be deduced from the derivative of $\varepsilon_A^*$ as $J = -\frac{2e}{\pi} \sin(\phi + \phi_0)$ in low-temperature limit. Note that these results (including the anomalous Josephson effects) are all base on the first-order perturbation theory with

![Fig. 9](image_url) (Color online) Andreev bound level versus phase difference according to the approximation result of Eq. 19 with short junction length. The temperature setted here is $0.5T_c$. The corresponding setting of the parameters are labeled in the plot.

![Fig. 10](image_url) (Color online) Free energy obtained by the approximated Andreev level in Eq. 19, where we show the free energy in one period. The temperature is setted as $1.5T_c$. The corresponding setting of the parameters are labeled in the plot. a-c show the free energy with different temperatures, while the other parameters as setted as: $E_\perp = 0.034$ eV, $A = 0.3$, $\mu_n = 0.2$ eV, $L = 50$ Å. The dash-line in a-c indicate the position of minimum free energy.
show the free energy within one period, $2\pi$. It’s obvious that the free energy exhibits the characteristic of the $\phi_0$-junction with the minimum free energy (i.e., the maximum value of the $-E(\phi, T)$ in the plot) changing their positions under different parameters. By comparing the black, purple, yellow lines in Fig. 7a, we can obviously see the phase shift induced by the variety of length of normal region. While for the temperature near the critical one $T_c$, as shown in Fig. 7b, it doesn’t affect the existence of 0($\pi$)-junction and the time reversal invariance, and thus the minimum free energy is always located at $\phi = 3.5$. For temperature lower than the critical one $T_c$ as shown in Fig. 7c, the minimum free energy located at $\phi = 5.24$ for temperature lower than $0.6T_c$, while it located at $\phi = 5.48$ for temperature larger or equals $0.6T_c$ as shown in Fig. 6b. In contrast to the Rashba-coupling-induced helical $p$-wave spin-triplet superconductor, the Josephson current of conventional $s$-wave superconductor saturates at low-temperature (e.g., at $T < 0.6T_c$) [14], and the free energy doesn’t change gradually with the variational temperature.

4 Conclusion

In the interface between the normal nanoribbon (strip) region and the superconducting region, the AR is related to the width $W$ (the number of edge states along the armchair direction) of the ribbon: For $W$ is even, the AR suppressed by the opposite pseudospin degree of freedom between first sublattice and last sublattice in the $k_y$ direction, while for $W$ is odd, the AR is allowed. In this strip scenario, the model spacing between bands above the Dirac level depends on $W$ and the NN hopping when consider the finite-size effect, the model spacing is $\delta \Delta = 3\pi t/(3W - 2)$ for $W$ is even and $\delta \Delta = \pi t/(W - 1)$ for $W$ is odd. The band structure of the zigzag silicene nanoribbon in strip geometry is shown in Fig. 11, where the model spacing $\delta \Delta$ is indicated in the inset. The finite-size effect here is important in large energy range, but it can be ignored in low-energy limit as depicted in the Sec.1. For the strong SOC case (including the intrinsic SOC and the electric field-induced Rashba-coupling), the anomalous Andreev reflection [53, 54] can happen where a hole is retro reflected in conduction band with the preserved spin direction. The anomalous Andreev reflection requires small quasienergy $\varepsilon \leq \mu + m^{\sigma}D$ and thus it can be found only during the retro AR in the strong SOC system.

In this paper we investigate the Josephson effect as a spin or valley polarized subgap transport in the superconductor-normal-superconductor junction base on the doped silicene with a dc Josephson current in zero voltage. The $0-\pi$ transition and the $\phi_0$-junction realized by changing the length of the normal region or the electric field are also investigated. We found that the dynamical polarizations of the degrees of freedom mentioned above can induce the $0-\pi$ transition and the emergence of the $\phi_0$-junction. For example, the change of the electric field or off-resonance circularly polarized light may induce the $0-\pi$ transition by the pseudospin degree of freedom or valley degree of freedom, respectively. Further, the interaction between antiferromagnetic exchange field and the SOC may induce the $0-\pi$ transition by the valley polarization [42], and the interaction between the internal exchange field and Josephson superconducting current may induce the $\phi_0$-junction when the normal region is noncentrosymmetric [55]. The scattering by the charged impurity in the substrate affects the transport properties in the bulk as well as the valley relaxation, which can be taken into consideration by using the (phenomenological) macroscopic wave function. The cold atom technique also used in the setup of Josephson junction, like the two-dimensional electron gas-mediated one, or the weak tunneling coupling between two superfluid fermi gas [56]. In such case, the formation of the conducting polarons [57, 58] can also be taken into account, that may also be a useful experimental tool to detecting the quantum effect within the Josephson junction.

Appendix A: Time-Dependent Potential Induced by the Off-Resonance Circularly Polarized Light

The circularly polarized light-induced electromagnetic vector potential is $A(t) = A(\pm \sin \Omega t, \cos \Omega t)$ where $\pm$ denotes the right and left polarization, respectively. In this paper, we use the radiation field which has frequency $\Omega = 3000$ THz $\approx 12$ eV, and it is certainly in the off-resonance region, thus the periodic gauge vector potential is $A =$
where $\varphi_0/\Omega = 0.078$ where $\varphi_0$ is the radiation field amplitude, and the radiation field here satisfies $E(t) = -\frac{i}{\epsilon} \partial_t A(t)$.

According to the second-order effect of the effective photocoupling process, where the quasienergy is shifted by absorbing (emitting) a certain quantity photons, as $\varphi \pm \hbar \Omega/2$, i.e., it’s shifted by a half-integer multiples of $\hbar \Omega/2$. The transport properties during such process can be explored by the Green’s function with complex variable (quasienergy), and the decimation method [59]. For the perturbation from the time-dependent periodic off-resonance light $V(T + t) = V(t)$ ($T = 2\pi/\Omega$), the monochromatic-perturbated Hamiltonian can be obtained by the simple Schrödinger equation $H_V \varphi = E \varphi$ using the Floquet technique, with Floquet Hamiltonian in a tridiagonal form as

$$H_V = \begin{pmatrix}
\ddots & & & & \\
& V_{-N} & H_{-N} & V_{+N} & 0 & 0 & \cdots \\
& 0 & V_{-N} & H_0 & V_{+N} & 0 & \cdots \\
& 0 & 0 & V_{-N} & H_{+N} & \ddots & \cdots \\
& \cdots & & & & & \ddots \\
\end{pmatrix},$$

and

$$\varphi = (\ldots, \varphi_{-N}, \varphi_0, \varphi_{+N}, \ldots)^T$$

thus the $H_V$ can be obtained as

$$H_V = H_0 + V_{-N} \frac{1}{\epsilon - H_{-N}} V_{+N} + V_{+N} \frac{1}{\epsilon - H_{+N}} V_{-N} + O(V^2_N)$$

$$\approx H_0 + \frac{[H_{-1}, H_{+1}]}{\hbar \Omega} + O(A^4/\Omega^5),$$

(22)

with the interaction term $V_N = \frac{1}{T} \int_0^T H(t) e^{-iN\hbar \omega t} (N$ is the Floquet mode), and the eigenvector of $H(t)$ can be written as $\psi = e^{-i\epsilon t} \sum_n \varphi_n e^{i n \Delta t}$ where $\varphi_n$ is the Floquet states. The above perturbated effective Hamiltonian can be related to the evolution operator with the effect of Berry curvature by [60]

$$H_V = \frac{i\hbar}{T} \log U,$$

(23)

with

$$U = \mathcal{T} e^{-\frac{i}{\hbar} \int_0^T H(t) dt} \mathcal{P} e^{\frac{i}{\hbar} \int_0^T \mathcal{A}(k) dk},$$

(24)

where $T$ is the time-order operator $P$ is the path operator of the electron contour $\mathcal{C}$ in the phase-space. While for the five-diagonal one, we can using the following equation with real space renormalization group

$$\begin{pmatrix}
H_{-N} & V_{+N} & V_{+N+1} \\
V_{-N} & H_0 & V_{+N} \\
V_{-N-1} & V_{+N-1} & H_{+N} \\
\end{pmatrix} \begin{pmatrix}
\varphi_{-N} \\
\varphi_{-N+1} \\
\varphi_{-N+2} \\
\end{pmatrix} = \epsilon \begin{pmatrix}
\varphi_{-N} \\
\varphi_{-N+1} \\
\varphi_{-N+2} \\
\end{pmatrix},$$

(25)

with the Fourier index $N$ and the Bloch wave function in zeroth Fourier model is

$$\varphi_0 = \frac{V_{-N}}{\epsilon - H_0} \varphi_{-N} + \frac{V_{+N}}{\epsilon - H_0} \varphi_{+N}. $$

(26)

The fast oscillation term can be described as [60–62]

$$H_V - H_0 \equiv H'$$

$$= -\frac{e^2 A^2}{\hbar \Omega} \eta \tau_z \hbar^2 v_F^2 - a^2 \lambda^2 \sigma_z$$

$$- a \lambda R^2 \hbar \nu F(\eta \tau_x \sigma_y - \tau_y \sigma_x)$$

(27)

which can also be written in the form of matrix as

$$H'_K = \begin{pmatrix}[1.5] i a \lambda R^2 (A_x + i A_y) & 2 \hbar \nu F (A_x - i A_y) \\
2 \hbar \nu F (A_x + i A_y) & i a \lambda R^2 (A_x - i A_y) \end{pmatrix},$$

$$H'_K' = \begin{pmatrix}[1.5] -i a \lambda R^2 (A_x + i A_y) & 2 \hbar \nu F (A_x + i A_y) \\
-2 \hbar \nu F (A_x - i A_y) & -i a \lambda R^2 (A_x - i A_y) \end{pmatrix},$$

(28)

Through this, we can also find that, in the case of time-reversal-invariance (TRI; i.e., in the absence of Rashba coupling), $H'_K = H'_K'$. However, that can be possible only for the linear-polarization case which with much smaller frequency, since the circularly polarized light will breaks the TRI. A further discussion about the in Floquet system is presented in Refs. [63].

**Appendix B: The Possible Triplet Supercurrent and the Green’s Function Method**

It’s well known than the long-range proximity-induced triplet pairs can diffuse into the middle ferromagnetic region in SFS junction much longer than the singlets [64, 65], where the triplet correlation vector (locked with the spin vectors) needed to be noncollinear with the magnetization axis. While for the S/N interface, the singlet-
triplet conversion is possible through a spin-active S/N interface [66, 67], and the leakage of the proximity-induced spin-triplet pairings into the normal region is related to the potential barrier near the interface [68]. The realization of the long-range triplet pairs needs the local SU(2) gauge potential with the local SU(2) gauge invariant SOC. Note that such SU(2) gauge potential (induced by, e.g., the SU(2) electric field) is unlike the electrostatic potential which applied only in the superconducting sides, but needed to be applied to the middle (quasiclassical diffusive) region.

In the presence of SU(2) gauge invariance, the gauge-covariant Wigner transform can be carried to obtained the quasiclassical Green’s function. Firstly, we introduce the hole and electron propagators in space-time

\[ G_h(i, j, t, t') = -\frac{i}{\hbar}(c(i, t)c^\dagger(j, t')), \]
\[ G_e(i, j, t, t') = \frac{i}{\hbar}(c^\dagger(j, t)c(i, t)), \]  
\[ G_{\text{Keldysh}}(i, j, t, t') = \frac{i}{\hbar}(c^\dagger(j, t)c(i, t)), \]

respectively. Then the full Green’s function in Nambu space reads (for the retarded block)

\[ G_r(i, j, t, t') = \begin{pmatrix} G_r^{\uparrow\uparrow}(i, j, t, t') & F(i, j, t, t') \\ F^\dagger(i, j, t, t') & -G_r^{\downarrow\downarrow}(i, j, t, t') \end{pmatrix}, \]  

where \( F(i, j, t, t') \) is the anomalous Green’s function. Although the elements in above matrix contain all spin degrees of freedom, the diagonal elements should have identical spin index for triplet channel, then according to the properties of the anomalous Green’s function, we know that the non-diagonal elements should also have the spin index identical to the diagonal elements. Similarly, for the singlet channel, we can obtain there are only two kinds of spin configurations (for Green’s function in Nambu@spin space)

\[ \tilde{G}_r(i, j, t, t') = \begin{pmatrix} G_r^{\uparrow\uparrow}(i, j, t, t') & F^{\uparrow\downarrow}(i, j, t, t') \\ F^{\downarrow\uparrow}(i, j, t, t') & -G_r^{\downarrow\downarrow}(i, j, t, t') \end{pmatrix}. \]  

Before utilizing this Nambu space Green’s function, we introduce another Green’s function formed simply by the electron/hole propagators

\[ \tilde{G}_r(i, j, t, t') = \begin{pmatrix} G_e(i, j, t, t') & G_h(i, j, t, t') \\ G_h(i, j, t, t') & G_e(i, j, t, t') \end{pmatrix}. \]

Then by using the Keldysh rotation [69–71] to extend to the Keldysh space, we can obtain for the case of \( t > t' \)

\[ \tilde{G}_r(i, j, t, t') \mapsto \frac{1}{\sqrt{2}} \left( \begin{array}{cc} 1 & -1 \\ 1 & 1 \end{array} \right) \sigma_z \tilde{G}_r(i, j, t, t') \frac{1}{\sqrt{2}} \left( \begin{array}{cc} 1 & 1 \\ -1 & 1 \end{array} \right) \]

\[ \tilde{G}_r(i, j, t, t') = \frac{1}{2} \begin{pmatrix} G_e(i, j, t', t) + G_h(i, j, t, t') - G_e(i, j, t, t') - G_h(i, j, t', t) & G_e(i, j, t', t) + G_h(i, j, t, t') + G_e(i, j, t, t') + G_h(i, j, t', t) \\ G_e(i, j, t', t) - G_h(i, j, t, t') - G_e(i, j, t, t') + G_h(i, j, t', t) & G_e(i, j, t', t) - G_h(i, j, t, t') + G_e(i, j, t, t') - G_h(i, j, t', t) \end{pmatrix} \]

\[ = \frac{1}{2} \begin{pmatrix} G_h(i, j, t, t') - G_e(i, j, t, t') & G_h(i, j, t, t') + G_e(i, j, t, t') \\ 0 & G_h(i, j, t, t') + G_e(i, j, t, t') \end{pmatrix} \]

\[ = \frac{1}{2} \begin{pmatrix} G_r(i, j, t, t') & G_K(i, j, t, t') \\ 0 & G_A(i, j, t, t') \end{pmatrix}. \]  

Then by substituting the above Nambu space form of the Green’s functions into above matrix, we can obtain the final full Gor’kov Green function [72, 73]. Since in this paper we consider the ballistic limit of the transport in SNS junction with the Andreev bound states, a averaging of the Green’s function is valid in the presence of the diffusive mean free path much smaller than the superconducting coherence length and the Cooper pair decay length [2, 3, 74, 75], then in mean-field approximation, the Gor’kov equation reads

\[ \left( i\hbar \partial_t - H(i) \right) \tilde{G}(i, j, t, t') = \delta(i - j)\delta(t - t') + \int_{\mathcal{C}_i} dt'' \int_{\mathcal{C}_j} dx'\Sigma(i, x, t, t')\tilde{G}(x, j, t'', t'), \]  

where \( \Sigma(i, x, t, t') = \text{diag}(\Sigma) \).
where $H(x)$ is the Hamiltonian of the normal part’s silicene in real space, which can be obtained by the matrix

$$H(k) = \begin{pmatrix} H^k & 0 \\ 0 & H^{k'} \end{pmatrix},$$

$$H^k = \begin{pmatrix} m_{++}^{++} & h_{F}(k_x - i k_y) & i a \lambda R_2(k_x - i k_y) & 0 \\ -i a \lambda R_2(k_x + i k_y) & m_{-+}^{--} & -i \lambda R_1 & 0 \\ 0 & i a \lambda R_2(k_x + i k_y) & h_{F}(k_x + i k_y) \\ -i \lambda R_1 & 0 & m_{-+}^{--} & h_{F}(k_x + i k_y) \end{pmatrix},$$

$$H^{k'} = \begin{pmatrix} m_{+}^{++} & h_{F}(k_x - i k_y) & i a \lambda R_2(k_x - i k_y) & 0 \\ -i a \lambda R_2(k_x + i k_y) & m_{-}^{--} & -i \lambda R_1 & 0 \\ 0 & i a \lambda R_2(k_x + i k_y) & h_{F}(k_x + i k_y) \\ -i \lambda R_1 & 0 & m_{-}^{--} & h_{F}(k_x + i k_y) \end{pmatrix}. \tag{36}$$

After using the minimal (Peierls) substitution $k \to (-i \hbar \frac{d}{dx} + \frac{e}{\hbar} A(x))$, the self-energy $\Sigma(i, x, t, t')$ here (Hartree-Fock type) is in fact the BdG term in self-consistent Born approximation in the presence of zero pair potential [76], which represents the coupling between the superconducting leads and the normal part and is dependent on the relaxation time and the quasiclassical Green’s function, i.e., the $\Delta_s$ which couples the time-reversed partners.

For the impurity scattering potential in the Born approximation (within the diagrammatic perturbation theory [57, 58]), the dressed Green’s function reads

$$\hat{G}^{-1}(i, j, t, t') = \hat{G}_0^{-1}(i, j, t, t') - \Sigma(i, j, t, t'), \tag{37}$$

where the self-energy is defined by the power series expansion of the Feynman diagrams. Thus we also have

$$\hat{G}_0^{-1}(i, j, t, t') = \delta(t - j) \delta(t - t') + \begin{pmatrix} i \hbar \delta t - H(i) & 0 \\ 0 & -i \hbar \delta t - H^*(j) \end{pmatrix}. \tag{38}$$

Note that for the superconducting regions [4], the non-diagonal matrix elements of the matrix within above Gor’kov equation should be replaced by the s-wave pairing potential $\Delta_s \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. 

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