MAGNETIC FIELDS AND COSMIC RAYS IN GRBs: A SELF-SIMILAR COLLISIONLESS FORESHOCK

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ABSTRACT

Cosmic rays accelerated by a shock form a streaming distribution of outgoing particles in the foreshock region. If the ambient fields are negligible compared to the shock and cosmic ray energetics, a stronger magnetic field can be generated in the shock upstream via the streaming (Weibel-type) instability. Here we develop a self-similar model of the foreshock region and calculate its structure, e.g., the magnetic field strength, its coherence scale, etc., as a function of the distance from the shock. Our model indicates that the entire foreshock region of thickness $\sim R/(2\Gamma_{\text{sh}}^3)$, being comparable to the shock radius in the late afterglow phase when $\Gamma_{\text{sh}} \sim 1$, can be populated with large-scale and rather strong magnetic fields (of subgauss strengths with the coherence length of order $10^{16}$ cm) compared with the typical interstellar medium magnetic fields. The presence of such fields in the foreshock region is important for high efficiency of Fermi acceleration at the shock. Radiation from accelerated electrons in the foreshock fields can constitute a separate emission region radiating in the UV/optical through radio band, depending on time and shock parameters. We also speculate that these fields being eventually transported into the shock downstream can greatly increase radiative efficiency of a gamma-ray burst afterglow shock.

Key words: cosmic rays – gamma rays: bursts – magnetic fields – shock waves

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1. INTRODUCTION

Do gamma-ray bursts (GRBs) accelerate cosmic rays (CRs)? There are arguments in favor of such an idea (Dermer & Atoyan 2004). It is supposed that CRs are accelerated via the Fermi mechanism in which a particle crosses the shock many times and gradually gains its energy. For an ultrarelativistic shock, a steady-state universal power-law energy distribution of particles shall form (Kirk et al. 2000; Achterberg et al. 2001), which seems to be consistent with GRB observations. The shock shall accelerate all particle species, but the electrons being much lighter than protons and ions will also lose their energy via synchrotron cooling. This radiation is thought to be observed as the delayed afterglow emission of GRB sources. A problem immediately arises here from a simple estimate: if the X-ray afterglow observed on a day timescale after the prompt burst is indeed the synchrotron radiation from the shock-accelerated electrons, then the preshock medium has to be highly magnetized with fields of milligauss strengths (Li & Waxman 2006). Thus, either the magnetic field is somehow generated in the shock upstream, or the conventional paradigm of the GRB afterglow needs revision.

In this paper, we present a self-similar model of the large region in front of a relativistic shock—the foreshock. This region is populated with shock-accelerated particles, which stream away from the shock into the collisionless ambient medium and generate a magnetic field via a streaming (Weibel-type) instability. The model predicts the generation of strong, subgauss, magnetic fields in the entire foreshock whose thickness is $\sim R/(2\Gamma_{\text{sh}}^3)$ and is comparable to the shock radius, $\sim 10^{17}$–$10^{18}$ cm, in the afterglow phase a day or more after the explosion, when the shock is weakly relativistic or nonrelativistic. The fields are sustained against dissipation by the anisotropy of newly accelerated particles. Moreover, these fields are relatively large scale, with the coherence length being as large as a fraction of the foreshock size, $\sim 10^{16}$ cm, which makes them effectively decoupled from dissipation. We speculate that these mesoscale magnetic fields being ultimately advected into the shock downstream can significantly increase the radiative efficiency of GRB afterglows and, perhaps, explain the origin of the magnetic field in an external shock of a GRB. We remark here, however, that our study is analytical and cannot account for a number of nonlinear feedback effects of the generated fields and pre-conditioned external medium onto the shock structure and particle acceleration. Kinetic and hybrid computer modeling is essential for better and more accurate understanding of the foreshock structure.

2. THE MODEL

Overall, our model is as follows. A shock is a source of CRs which move away from it, thus forming a stream of particles through the ambient medium, say, the interstellar medium (ISM). If the ISM magnetic fields are negligible, i.e., their energy density is small compared to that of CRs, the streaming instability (either the pure magnetostatic Weibel or the mixed-mode electromagnetic oblique Weibel-type instability, depending on conditions) is excited (Bret et al. 2005a, 2005b) and stronger magnetic fields are quickly generated. These fields further isotropize (thermalize) the CR stream. Since less energetic particles, having a greater number density and carrying more energy overall, are thermalized closer to the shock, the generated B-field will be stronger closer to the shock and fall off away from it, whereas its correlation length will increase with the increasing distance from the shock. More energetic particles keep streaming because of their larger Larmor radii and produce the magnetic field further away from the shock. This process stops at distances where either the CR flux starts
decreasing (because of the finite distance the CR particles can get away from a relativistic shock or because of the shock curvature causing CR density to decrease as $r^{-3}$ if the shock is sub- or nonrelativistic) or where the generated magnetic fields become comparable to the ISM field and the instability ceases. Thus, a large upstream region—the foreshock—is populated with magnetic fields. We now derive its self-similar structure. We work in the shock comoving frame unless stated otherwise.

Let us consider a relativistic shock moving along $x$-direction with the bulk Lorentz factor $\Gamma_0$; the shock is plane-parallel and lies in the $yz$-plane, and $x = 0$ denotes the shock position. The shock continuously accelerates cosmic rays, which then propagate away from it into the upstream region. We conventionally assume that the CR distribution over the particle Lorentz factor is described by a power law:

$$n_{CR} = n_0(\gamma/\gamma_0)^{-s}$$

for $\gamma > \gamma_0$ and zero otherwise. Here the index $s = p - 1$ is approximately equal to 1.2 for ultrarelativistic shocks and $n_0$ is the normalization.\textsuperscript{4} We assume that the above energy distribution is the same everywhere in the upstream, that is, we neglect the nonlinear feedback of magnetic fields onto the particle distribution. Moreover, we assume that the formation of the CR power-law distribution is cotemporaneous with the shock formation itself, so we neglect the finite acceleration time of CRs, which may be quite important for higher energy CRs. Accurate inclusion of this effect would require solution of the convection–diffusion equation with diffusion being calculated self-consistently from the self-generated fields, which are not steady at the beginning of the shock formation; all these issues are beyond the scope of the present paper. The CR momentum distribution exhibits strong anisotropy: the parallel ($\parallel$) components of CR momenta are much greater than their thermal spread in the perpendicular ($\perp$) plane. Indeed, for a particle to move away from the shock, it should have the $x$-component of the velocity exceeding the shock velocity. Since both the shock and the particle move nearly at the speed of light, this puts a constraint on their relative angle of propagation to be less than $1/\Gamma_0$ in the lab (observer) frame. Hence, the transverse spread of the CR particle’s momenta is $p_\perp \lesssim p_\parallel/\Gamma_0 \ll p_\parallel$. This is also seen in numerical simulations (Spitkovsky 2005).

The CR particles propagate through the self-generated fore-shock fields and scatter off them. Lower energy particles are deflected in the fields more strongly and, therefore, isotropize faster than the higher energy ones, as having larger Larmor radii. At a position $x > 0$ the CR distribution can roughly be divided into isotropic (thermalized) component with $\gamma < \gamma_r(x)$ and streaming component with $\gamma > \gamma_r(x)$, where $\gamma_r(x)$ is the minimum Lorentz factor of the streaming particles at a location $x$; it is also the maximum Lorentz factor of the randomized component at this location. The streaming component is Weibel-unstable with a very short $e$-folding time $\tau \sim \omega_{rel}^{-1}$, where $\omega_{rel} = (4\pi e^2 n(\gamma)/m_p c^2)^{1/2}$ is the relativistic plasma frequency, $n(\gamma)$ is the density of streaming particles of the Lorentz factor $\gamma$ (tilde denotes streaming particles). Note that the Weibel instability growth rate depends on $n$ of the lower density component—cosmic rays, in our case—measured in the center of mass frame of the streaming plasmas. For the lower

\textsuperscript{4} Conventionally the distribution is given as $dn/dy \propto \gamma^{-p}$ with $p$ being $\sim 2.2–2.3$ for relativistic shocks; hence the density of particles of energy $\sim \gamma$ is $n(\gamma) \propto \gamma^{-p} \delta y \propto \gamma^{-p+1}$.
further away from the shock. This field will be of weaker and of larger scale because of the lower density of the streaming particles $\dot{n}(\gamma) \ll \dot{n}(\gamma_r)$, according to Equations (1)–(3).

Finally, the number density of streaming CR particles at $\gamma_r$ is $\dot{n}(\gamma_r) = n_0(\gamma_r/\gamma_0)^{-s}$. Therefore,

$$\lambda(\gamma_r) \sim (m_p c^2 \gamma_0 / 4 \pi e^2 n_0)^{1/2} (\gamma_r / \gamma_0)^{(1+s)/2} \equiv \lambda_0(\gamma_r / \gamma_0)^{(1+s)/2},$$

where $\lambda_0$ is the inertial length of the lowest energy CR “plasma.” Inverting this expression yields

$$\gamma_r \sim \gamma_0 [\lambda(\gamma_r) / \lambda_0]^{2/(1+s)} = \gamma_0 (2 \xi_B x_r / \lambda_0)^{2/(1+s)}.$$ (8)

Hereafter, the subscript “r” can be omitted without loss of clarity.

In a steady state, this field is continuously advected toward the shock (in the shock comoving frame since the center of mass frame of the foreshock plasma differs from the shock frame) and may affect the onset and the saturation level of the Weibel instability. In addition, the current filaments producing the fields merge with time, so that $B$ and $\lambda$ change while being advected. These nonlinear feedback effects are difficult to properly account for in a theoretical model; hence they are omitted in the current study. PIC simulations can help us to quantify the effects as well as to confirm or disprove our assumption that the shock and the foreshock do form a self-sustained, steady state structure.

3. THE SELF-SIMILAR FORESHOCK

The self-similar structure of the foreshock immediately follows from Equations (1), (2), (6), and (8). The magnetic field correlation length is proportional to the upstream distance from the shock

$$\lambda(x) \sim x (2 \xi_B),$$

and its strength decreases with the distance as

$$B(x) \sim B_0 (x/x_0)^{-s},$$

where $B_0 = (8 \pi \xi_B m_p c^2 n_0 \gamma_0)^{1/2}$ and $x_0 = \lambda_0 / (2 \xi_B) = (m_p c^2 \gamma_0 / 4 \pi e^2 n_0)^{1/2} / (2 \xi_B)$. In this estimate we neglected the advected fields $B(\gamma')$ as subdominant compared with $B(\gamma_r)$ for $\gamma > \gamma_r$. We note here that the idea of self-similarity of the Weibel turbulence has been first proposed by Medvedev et al. (2005) and then further elaborated by Katz et al. (2007), whose results are in agreement with the above scalings. The $\xi_B$ parameter expresses the field energy normalized to the shock kinetic energy. The energy of cosmic rays is $U_{\text{CR}} = \int n(\gamma / \gamma_0)(m_p c^2 \gamma) d(\gamma / \gamma_0) \sim m_p c^2 n_0 \xi_B$ and constitutes a fraction $\xi_{\text{CR}}$ of the total shock energy, $U_{\text{sh}}$. The efficiency of cosmic ray acceleration, $\xi_{\text{CR}}$, can be as high as several tens percent, perhaps, up to $\xi_{\text{CR}} \sim 0.5$, as follows from the nonlinear shock modeling (Vladimirov et al. 2006; Ellison et al. 2007). The scaling of $\xi_B$ is

$$\xi_B \sim \xi_{\text{CR}} \xi_B \left( x / x_0 \right)^{-2 s / (1+s)}.$$ (11)

These scalings hold while the shock can be treated as planar and while the ISM magnetic fields are negligible compared to the Weibel-generated fields. If the shock is relativistic, CR particles can occupy a narrow region in front of it. Assuming CR to propagate nearly at the speed of light, their front is ahead of the shock at the distance $\Delta r = c t_{\text{rel}} = c (R / c - R / v_{\text{sh}}) \simeq R - R / [1 - 1/(2 \Gamma_{\text{sh}}^2)]$ measured in the lab (observer) frame, that is at the distance $\sim \Delta r \Gamma_{\text{sh}} \sim R / (2 \Gamma_{\text{sh}}^2)$ in the shock frame. Also, when the radial distance in the lab frame $\Delta r = x / \Gamma_{\text{sh}}$ becomes comparable to the shock radius $\Delta r \sim R$ the curvature of the shock can no longer be neglected: the density of CR particles, which was assumed to be constant in our model, starts to fall as $\sim r^{-2}$. This leads to a steeper decline of $B$ with distance. Obviously, the first constraint is more stringent for a relativistic shock, whereas both are very similar (within a factor of two) for a nonrelativistic shock. Hence we use the first constraint hereafter. Meanwhile, at some distance $X$, the Weibel-generated fields can become comparable to the ambient magnetic field, $B(X) \sim B_{\text{amb}}$ and the Weibel instability ceases; here we used that the ambient field in the shock frame is $B_{\text{amb}} \sim B_{\text{ism}1,\Gamma_{\text{sh}}} \sim B_{\text{ISM} \Gamma_{\text{sh}}}$. PIC simulations (Spitkovsky 2005) indicate that for low magnetizations $\sigma < 0.01$, i.e., $B(X) / B_{\text{amb}} > 0.1$, the shock behaves as unmagnetized and the Weibel instability dominates. Although there is no sharp threshold, one sees the Weibel instability to be suppressed for lower values of $B(X) / B_{\text{amb}}$. Within the order of magnitude, we set $B_{\text{ISM} \Gamma_{\text{sh}}} \sim B(X) \sim B_0 (X / x_0)^{-2(1-s)/(1+s)}$, therefore $X \sim x_0 [B_0 / (B_{\text{ISM} \Gamma_{\text{sh}}})]^{(1+s)/(4-1)}$. To conclude, the scalings (9)–(11), hold at $x \lesssim x_{\text{max}}$, where

$$x_{\text{max}} = \min \{ R / (2 \Gamma_{\text{sh}}), X \} = \min \{ R / (2 \Gamma_{\text{sh}}), x_0 (B_0 / B_{\text{ISM} \Gamma_{\text{sh}}})^{4(1-s)/(4-1)} \}.$$ (12)

The region filled with the magnetic field in front of the shock is large, so is the region where radiation is emitted by the CR electrons. The power emitted by a relativistic electron in a magnetic field is $P_B = (4/3) \sigma_T c \gamma_e^2 (B^2 / 8\pi)$, where $\sigma_T$ is the Thompson cross section and $\gamma_e$ is the Lorentz factor of the emitting electron. This expression is accurate for both synchrotron and jitter radiation (Medvedev 2000). For the distribution of electrons (Equation (1)) homogeneously populating the foreshock, the power is dominated by the lowest energy particles with $\gamma_e \sim \epsilon_0 (m_p/m_e) \gamma_p \sim \epsilon_0 (m_p/m_e) \gamma_0 \sim \epsilon_0 (m_p/m_e) \Gamma_{\text{sh}}$ (here $\epsilon_0$ is the efficiency of the electron heating) and the density $n_e \sim n_0 \sim n_{\text{ISM} \Gamma_{\text{sh}}}$. Then $P_{\text{tot}} = \int P_B(x) n_0 dx$, where the volume element is $dV = 4\pi R^2 dx$, so

$$P_{\text{tot}} \propto \int_0^{x_{\text{max}}} (x / x_0)^{-2 s / (1+s)} dx \sim x_0 (x_{\text{max}} / x_0)^{1/(1+s)}.$$ (13)

Note that the comoving radiating power increases with the foreshock thickness, $P_{\text{tot}} \propto x_{\text{max}}^{(3-s)/(1+s)}$, thus emission is not localized in a thin layer near the shock and is, in fact, dominated by large distances:

$$P_{\text{tot}} \sim L_{\text{CR}} \xi_{\text{CR}} \xi_B \left( x / x_0 \right)^{2 (3-s)/(1+s)}.$$ (14)

where $L_{\text{CR}} = 4\pi R^2 (m_p c^2 n_0 / \gamma_0) \epsilon_0$ is the kinetic luminosity of cosmic rays, which is a fraction $\xi_{\text{CR}} < 1$ of the total kinetic luminosity of a GRB, $L_{\text{GRB}} = \xi_{\text{CR}} L_{\text{GRB}}$. $R$ is the shock radius, and the comoving CR density is of order the density of the incoming ISM plasma, $n_0 \sim n_{\text{ISM} \Gamma_{\text{sh}}}$. The foreshock electrons are radiating in the synchrotron regime: the jitter parameter $\delta$ (Medvedev 2000; Medvedev et al. 2007), which is the average deflection angle of an electron in the foreshock fields, $\theta_e \sim (e B(x) / m_p c) \lambda / (\gamma e c) \| \lambda / (\gamma e c) \| / c$ over the radiation beaming angle, $\sim 1 / \gamma e$, is always much larger than unity:

$$\delta(x) \sim (e B(x) \lambda / (m_p c^2)) \sim (m_p / m_e) \gamma_0 \sqrt{2 \xi_B c} \left( x / x_0 \right)^{(2s+1)/(1+s)} \gg 1.$$ (15)
Although consideration of the postshock fields is beyond the scope of the present paper, we can estimate the magnetic field spectrum at and after the shock jump as long as dissipation is not playing a role. The magnetic field of different correlation scales created in the foreshock is advected toward the shock, so a broad spectrum is accumulated:

\[ B_b \propto \lambda^{-\frac{1}{5}} \sim \lambda^{-0.091}, \]  

(16)

where Equations (9) and (10) were used and \( s = p - 1 \sim 1.2 \) was assumed.

4. THE AFTERGLOW FORESHOCK

The relation between the shock radius \( R \) and its Lorentz factor \( \Gamma_{sh} \) follows from a simple energy argument: the energy of an explosion is \( E \sim (4\pi/3)R^3m_pc^2n_{ISM}\Gamma_{sh}^2 \), therefore

\[ R \sim (10^{18} \text{ cm}) E_{52}^{1/3} n_{ISM}^{-1/3} \Gamma_{sh}^{-2/3} \]  

(17)

or \( \Gamma_{sh} \sim E_{52}^{2/3} n_{ISM}^{-1/2} R_{18}^{-3/2} \), where \( E_{52} = E/10^{52} \text{ erg} \) and similarly for other quantities. The observed time of photons emitted at radius \( R \) is the time since the very first photons (i.e., emitted at \( R \approx 0 \)) arrive, \( t_{ob} = R/c - R_{sh}/v_{sh} \approx R/c - R/\[c(1 - 1/2\Gamma_{sh}^2)\] \), that is \( t_{ob} \sim R/(2\Gamma_{sh}^2c) \). Using the equation for \( R \), one obtains

\[ \Gamma_{sh} \sim 3.7 E_{52}^{1/8} n_{ISM}^{-1/8} t_{day}^{-3/8} \]  

(18)

\[ R \sim (4.2 \times 10^{17} \text{ cm}) E_{52}^{1/4} n_{ISM}^{-1/4} t_{day}^{-1/4} \]  

(19)

Here we assumed a local GRB with \( z = 0 \); to include the redshift time dilation is trivial.

The comoving density is \( n_0 \sim n_{ISM}\Gamma_{sh} \) (assuming the CR efficiency \( \xi_{CR} \approx 0.5 \sim 1 \)) and the minimum Lorentz factor of CR protons is \( \gamma_0 \sim \Gamma_{sh} \). Hence, the length scale \( x_0 \sim \lambda_0/(2\xi_B) \) (\( \lambda_0 \) is the skin length) and the field \( B_0 \) in the shock comoving frame become

\[ x_0 \sim (2 \times 10^7 \text{ cm}) n_{ISM}^{-1/2} / (2\xi_B) \sim (10^9 \text{ cm}) n_{ISM}^{-1/2}, \]  

(20)

\[ B_0 \sim (0.2 \text{ gauss}) \xi_B^{1/2} n_{ISM}^{1/2} \Gamma_{sh} \sim (1 \text{ gauss}) E_{52}^{1/2} R_{18}^{-3/2}, \]  

(21)

where we assumed \( \xi_B \approx 0.01 \). Assuming \( p = 2.2 \) and the interstellar fields, \( B_{ISM} \), to be of order microgauss, we estimate \( X \) as

\[ X \sim x_0 \left[ 0.2(\xi_B n_{ISM})^{1/2} B_{ISM}^{1/3} \right]^{1/2} \sim 2 \times 10^{17} x_0 n_{ISM}^{5.5} B_{ISM}^{-11/6}, \]  

(22)

independent of \( \Gamma_{sh} \). On the other hand,

\[ R/(2\Gamma_{sh} \lambda) \sim (5 \times 10^{17} \text{ cm}) E_{52}^{1/3} n_{ISM}^{-1/3} \Gamma_{sh}^{-5/3} \ll X, \]  

(23)

indicating that the ambient field is relatively unimportant, even for very steep energy spectra \( p \sim 3.5 \) rarely observed in prompt GRBs. The foreshock thickness is

\[ x_{max} \sim R/(2\Gamma_{sh} \lambda) \sim 5 \times 10^9 x_0 E_{52}^{1/3} n_{ISM}^{-1/3} \Gamma_{sh}^{-5/3}. \]  

(24)

Therefore, the typical field in the foreshock is of subgauss strength:

\[ B(x_{max}) \sim (0.2 \text{ gauss}) E_{52}^{0.45} n_{ISM}^{0.09} R_{18}^{-1.3}. \]  

(25)

This field is relatively large scale, as its comoving correlation scale is

\[ \lambda(x_{max}) \sim x_{max}/(2\xi_B) \sim (10^{16} \text{ cm}) E_{52}^{1/2} n_{ISM}^{1/2} R_{18}^{5/2}. \]  

(26)

The power emitted by CR electrons from the foreshock amounts to

\[ P_{tot}^{obs} \sim (2 \times 10^{39} \text{ erg s}^{-1}) E_{52}^{2.1} L_{CR,45} n_{ISM}^{1/2} R_{18}^{-5.5}, \]  

in the observer’s frame and is emitted at a peak (synchrotron) frequency

\[ \nu_{m}^{obs} \sim (10^{11} \text{ Hz}) E_{52}^{2.0} n_{ISM}^{-1.4} R_{18}^{-5.8}, \]  

(27)

(28)

which corresponds to the IR band at about one day after the explosion, where \( R(t) \) is given in Equation (19), so that

\[ P_{tot}^{obs} \propto t_{day}^{-6} \propto t_{day}^{-1.4} \text{ and } \nu_{m}^{obs} \propto t_{day}^{3/2} \propto t_{day}^{-1.4}. \]

5. DISCUSSION

Here we presented a model of a self-similar foreshock produced by protons scattered by a relativistic shock into the unmagnetized or weakly magnetized external medium. It is immediately applicable to the external shock producing a GRB afterglow. The model predicts that a large region in front of the shock, of thickness of order the shock radius, shall be filled with relatively strong and large-scale magnetic fields. The upstream magnetic field strength and correlation length depend on the distance from the shock and the power-law index of accelerated protons (cosmic rays) and are given by Equations (9), (10), and (21) in the shock comoving frame. The overall energetics of the field is dominated by large distances; hence the average foreshock B-field is of subgauss strength and is increasing toward the shock while its typical coherence length is of order of few percent of the shock radius and is decreasing toward the shock.

The results presented here may seem counterintuitive because the Weibel instability is always acknowledged as the microscale length of the lower density plasma component—CRs in our case—which is rapidly increasing with particle energy; it is \( \propto (n_{CR}/\gamma)^{-1/2} \propto (\gamma^{-2}/\gamma^{-1/2}) \propto \gamma^{-3/2}. \) The instability is driven by streaming (anisotropic) particles only. Lower energy particles dominate the free energy budget near the shock, but got isotropized at larger distances. Hence, higher energy particles start to provide free energy to the instability at larger distances and the corresponding skin length rapidly increases with distance from the shock as well. The strength of the fields is decreasing with distance accordingly, but for the spectrum with \( p \sim 2.2 \) this decrease is rather slow; note that the spectrum with \( p = 2 \) contains equal energy per logarithmic interval and high-energy particles carry as much energy as low-energy ones.

The result is interesting, especially in the light of observational constraints on the particle acceleration in GRB afterglows. It has been shown that a few-milligauss magnetic fields are needed in front of the shock in order to efficiently Fermi-accelerate the electrons to the energies required to produce the observed X-ray emission (Li & Waxman 2006). Our model provides a possible and rather natural mechanism for generation of such fields in the far upstream medium. We also make a
prediction that the shock-accelerated electrons will be radiating in the foreshock fields at a characteristic synchrotron frequency
given by Equations (28). For reference, $\nu_{m} \sim 10$ THz at about a day after the burst. It is possible that nonlinear effects omitted in this analysis (see discussion below) may limit the field strength to lower values compared to the present analysis, so the synchrotron peak can move to cm/mm-wave band. We can speculate that the foreshock can be an emission region separate from the afterglow shock and show up in the X-ray/optical band in the early afterglow phase and in the radio at the very late times.

The presented results can have interesting implications for the radiative efficiency of external shocks. Unlike internal shocks, where electron cooling is extremely fast and a thin shock layer of thickness of a hundred ion inertial lengths may be enough to produce the observed prompt emission (Medvedev & Spitkovsky 2008), the Weibel shock model (Medvedev & Loeb 1999) seems to face the efficiency problem when it is applied to an external shock. In such a shock, magnetic fields shall occupy a much larger region, perhaps the entire downstream region, in order for the shock to produce enough photons that will be observed as a delayed afterglow emission. This is not very likely (though not proved to be impossible, yet) provided that the small-scale fields generated at the shock can be subject to rapid dissipation. However, dissipation shall be of much lesser importance for the foreshock fields that have a (much) larger coherence length, Equation (9). In the steady state, the fields generated in the upstream are advected to the shock and their strength is maintained against dissipation by the anisotropy of the continuously “refreshed” CR distribution. Hence, one can expect that the magnetic field near the shock will have the spectrum given in Equation (16). Once these fields pass though the shock into the downstream, they are enhanced by the shock compression and begin to decay. Commonly, dissipation is proportional to the inverse gradient scale squared $\propto \partial_{x}^{-2} \propto \lambda^{-2}$, so that the skin-length-scale fields may eventually disappear. However, the fields above a certain coherence length, $\lambda_{diss}$, shall survive and fill up the postshock medium. The mechanism of dissipation is not yet understood in detail, so it is premature to make any quantitative conclusions about $\lambda_{diss}$, but it will certainly be much smaller than $\lambda(x_{\text{max}}) \lesssim R$, see Equation (26).

A simple estimate can be made here. The diffusion/dissipation time follows from $\tau_{d} \sim D \partial_{x}^{-2}$ as $\tau_{diss} \sim \lambda^{2}/D$. From the dimensional analysis, the diffusion time at the skin scale, $\lambda_{0}$, is of order $\lambda_{0}/c \sim \omega_{m}^{-1}$, so that $D \sim c\lambda_{0}$. If such a diffusion coefficient holds at other scales, one gets the field dissipation time at scale $\lambda$ to be $\tau_{diss} \sim (\lambda_{0}/c)(\lambda/\lambda_{0})^{2}$. The minimum coherence scale of the field is, therefore, a function of time and the distance from the shock into the downstream: $\lambda_{\text{min}} \sim \lambda_{0}(ct/\lambda_{0})^{1/2} \propto |x/x_{0}|^{1/2}$. For a typical GRB shock $\tau_{diss} \sim (10^{-3} \text{ s})(\lambda/10^{7} \text{ cm})^{2}$, so that at $t \sim 10^{7}$ s, the shocked gas shall contain magnetic fields with coherence scales above $\lambda \sim 10^{12}$ cm. Since $B_{\gamma}$ is a weak function of $\lambda$, one can speculate that relatively strong fields, perhaps of order tens or hundreds milligauss, can occupy the postshock medium.

We want to note that a number of simplifying assumptions have been made in our analysis. In particular, nonlinear feedback effects of the upstream magnetic field on the particle distribution, on the shock structure, and on Fermi acceleration were omitted. The inclusion of these effects is hardly possible in any analytical model. We neglected in this model that the accelerated electrons can loose their energy and cool down in the self-generated fields while the protons keep their energy, thus causing electric fields and, possibly, currents to build up in the region and modify the foreshock structure. We also assumed that a steady state exists for the shock–foreshock system at hand. Apparently, it is not at all clear whether the steady state is at all possible or the system exhibits an intermittent behavior. One can envision a scenario in which the CRs overproduce upstream magnetic fields leading to enhanced particle scattering and the overall preheating of the ambient medium, which, in turn, can cause the shock to weaken, disappear and then reappear in a different place further upstream. Presently available two-dimensional PIC simulations of an electron–position shock do show the upstream field amplification and no steady state has been achieved: both upstream and downstream fields continue to grow for the duration of the simulations (Keshet et al. 2008) though no shock reformation has so far been observed. We argue that extensive PIC or/and hybrid simulations of a shock are imperative for further study.

Finally, we mention that our model complements other models of the magnetic field generation. It is reasonable to...
expect that the field can be amplified by vortical motions produced by the Richtmeyer–Meshkov instability, if the ambient medium is clumpy or if the shock velocity is not perfectly uniform (Goodman & MacFadyen 2007; Sironi & Goodman 2007; Milosavljevic et al. 2007). On the other hand, if the ambient magnetic fields are strong enough, the fields can be generated via nonresonant (Bell & Lucek 2001; Bell 2004; Pelletier et al. 2008a) or resonant (Diamond & Malkov 2006, 2007; Zweibel 2003; Vladimirov et al. 2006; Ellison et al. 2007) mechanisms, or both (Reville et al. 2008).

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