2D-CWT IP core design and implementation for fringe pattern analysis

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Abstract. In order to meet the demand of high-speed or real-time analysis for digital interferograms, an advanced hardware accelerated method based on the field programmable gate array (FPGA) is proposed by using the two-dimensional continuous wavelet transform (2D-CWT) technique. In this paper, a 2D-CWT IP core is designed and implemented. It consists of a digital interferograms acquisition module, an interferograms data buffer module, a configuration module, a 2D-CWT operation module, and an output module. When the mother wavelet is selected, e.g., the Morlet wavelet, and the scale factors and rotation angles are setting, the spectrums of fringe patterns are obtained in the 2D-CWT operation module and transferred to the computer through the output module for further processing. Then, the phase distribution of the interferogram is extracted by using the wavelet ridge detection algorithm. Real experiments results show that the 2D-CWT IP core analysis method can reduce the time of phase extraction, and verify the validity and effectiveness of the proposed method.

1. Introduction

Since abundant physical information is contained in digital interferograms, physical quantity information can be obtained by studying the phase distribution of the interferograms. Hence, the phase extraction technology becomes an important part of digital interferograms analysis. At present, the phase extraction methods mainly include the phase shifting (PS) technique, Fourier transform (FT) technique, windowed Fourier transform (WFT) technique, wavelet transform (WT) technique, and so on. The PS technique requires at least three interferograms to extract the phase, the other methods require only one interferogram [1]. The PS technique belongs to the time domain processing technique. When extracting the phase of the noisy interferograms, it is sensitive to noise, and it is not suitable for the real-time measurement and vibration analysis [2]. The FT technique belongs to the frequency domain processing technology, the carrier frequency of the fringe patterns needs to be accurately obtained. Furthermore, the FT is a global operation, and the technique cannot deal with the local information of the fringe patterns. The WFT method can analyze the interferograms in the time-frequency domain, and solve the global problem of the FT technique in some aspects [3]. It can perform local analysis for fringe patterns, but it has the disadvantage of single resolution [4]. The WT method is not sensitive to noise, because it has a flexible time-frequency analysis window that can be adjusted, and analyzes localized information in the time-frequency domain [5].
Among all phase extraction techniques mentioned above, the WT technique has obvious advantages. As an excellent analytical technique, the WT technique has undoubtedly attracted widespread attention in recent years. Utilizing the concept of wavelet ridge, the one-dimensional continuous wavelet transform (1D-CWT) method is successfully introduced to analyze different interferograms or fringe patterns. Compared with the 1D-CWT technique, the 2D-CWT technique is more suitable for fringe pattern analysis due to anti-noise capability [6]. Therefore, the technique has been further developed in analyzing fringe patterns. However, the convolution operation is involved in the 2D-CWT technique, which is a challenge for the phase extraction processing time, and the 2D-CWT analysis technique and its application are also limited.

Traditionally, the digital interferograms phase extraction technique is implemented by a computer. It cannot meet the demand of the speed and real-time of digital interferograms in practical applications[7]. In order to reduce the processing time of the 2D-CWT technique, the 2D-CWT IP core is designed and implemented using the Verilog programming language for phase extraction of interferograms. The two-dimensional fast Fourier transform (2D-FFT) algorithm is used to implement the design of 2D-CWT IP core. In the implementation, the 2D-CWT is calculated by the multiplication of the two-dimensional pattern spectrum and the two-dimensional wavelet kernel function spectrum. This method can greatly reduce the time of the convolution operation.

In this paper, an advanced method of designing and implementing the 2D-CWT IP core on FPGA platform is proposed for high-speed phase extraction of the fringe patterns. Through real moiré interference experiments, the phase extraction time is reduced effectively, and the validity and effectiveness of the proposed method is verified.

2. 2D-CWT technique

Generally, the intensity of a digital interferogram can be simplified as:

\[ I(x) = I_b(x) + I_a(x) \cos[\phi(x)] \]

where \( x \) is the spatial coordinates, \( I \) is the intensity of a interferogram, \( I_b \) is the background intensity, \( I_a \) is the modulation amplitude; \( \phi \) is the phase distribution of the fringe patterns. Specifically, the 2D-CWT needs to be normalized in terms of amplitude or energy, which is usually referred to in mathematics as \( L^1 \)-normalization or \( L^2 \)-normalization, respectively. under \( L^1 \)-normalization, the 2D-CWT is defined as:

\[ W(b, s, \theta) = s^{-2} \int_{-\infty}^{\infty} I(x) \psi^*[s^{-1}r_\theta(x-b)]d^2x \]

where \( W(b, s, \theta) \) is the wavelet coefficient, \( s \) is the scale factor, \( b \) is the translation parameter, \( \theta \) is the rotation angle, * denotes the complex conjugate operator, \( \psi \) is the mother wavelet [8], \( \omega \) represents the frequency coordinates, and the symbol \( \hat{\ } \) denotes the Fourier transform operator. According to equation (2), the 2D-FFT method is used instead of the convolution operation. In addition, \( r_\theta \) is the \( 2 \times 2 \) rotation matrix corresponding to \( \theta \) in equation (3).

\[ r_\theta = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \]

The Morlet wavelet is used as the mother wavelet for fringe pattern analysis in this paper. It should be pointed out that the Morlet wavelet is actually a kind of bandpass filter, and the center frequency can be easily modulated to \( \omega_0 \) [9]. The wavelet is defined as follows:
\[ \psi_M^s(x) = C_0 \exp \left( -\frac{|x|^2}{2\sigma^2} + i\omega_0 \cdot x \right) \quad \omega_0 \geq 5 \quad (4) \]

A series of wavelet coefficients at each pixel location can be obtained by different scale factors and rotation angles. In order to find the maximum wavelet coefficients for each point \( b \) by calculating and comparing them. The maximum number is called wavelet ridge [10], can be expressed as follows:

\[ W(b)_{ridge} = W(b, s_{ridge}, \theta_{ridge}) \quad (5) \]

where

\[ (s_{ridge}, \theta_{ridge}) = \arg \max \left[ |W(b, s, \theta)| \right] \quad (6) \]

Consequently, the phase of the fringe patterns at \( b \) can be calculated from:

\[ \phi(b) = \tan^{-1} \left( \frac{\text{Im}[W(b)_{ridge}]}{\text{Re}[W(b)_{ridge}]} \right) \quad (7) \]

where \( \text{Im} \) and \( \text{Re} \) denote imaginary and real parts of \( W \), respectively.

3. Design of 2D-CWT phase extraction IP core

The block diagram of 2D-CWT IP core is shown in Figure 1, which mainly consists of a digital interferograms acquisition module, an interferograms data buffer module, a configuration module, a 2D-CWT operation module, and an output module. The core part of the design is the 2D-CWT operation module.
Figure 2. RTL schematic of the 2D-CWT IP core.

In this design, the digital interferograms obtained by the experiment is converted into a .mif file, and then buffered in the RAM for subsequent spectrum calculation of the interferogram. It should be reminded that the calculation of the interferogram spectrum requires Fourier transforms twice for the rows and columns, respectively. The FFT IP core provided by Altera can only implement the one-dimensional fast Fourier transform (1D-FFT) algorithm. Therefore, the 2D-FFT algorithm is decomposed into 1D-FFT algorithm in the row and column directions, respectively [11]. In addition, the spectrum is required to perform the Fourier transform twice, and the exponential module is the sum of the exponents of the Fourier transforms. Finally, the 2D-CWT can be completed by the 2D-FFT and 2D-IFFT operations in the FPGA. The RTL schematic of the 2D-CWT IP core is shown in Figure 2, and the summary report of resource utilization is shown in Table 1.

Table 1. The summary report of resource utilization.

| Content                        | Utilization percent |
|--------------------------------|---------------------|
| Total logic elements           | 4452 / 114480 (4%)  |
| Total pins                     | 273 / 529 (52%)     |
| Total memory bits              | 3424000 / 3981312 (86%) |
| Embedded multiplier elements   | 28 / 532 (5%)       |

4. Experiments

4.1. Experimental results

The phase extraction experiment is carried out by using Matlab on the computer and using the 2D-CWT IP core on the FPGA platform to verify the validity and reliability of the design. As shown in Figure 3, the MI3D-3 moiré interferometer is used to apply the longitudinal tension on a perforated aluminum sheet with a size of 60mm×10mm, and then the original moiré interferogram is obtained with the size of 256×256 pixels.

The phase distribution of the original moiré interferogram in Figure 3 is obtained by using the 2D-CWT ridge detection algorithm, and the parameters are set to: \( s = 50, \theta = \pi/2 \), and the wrapped phase map obtained by Matlab is shown in Figure 4(a). The result obtained with the 2D-CWT IP core on the FPGA is shown in Figure 4(b).
4.2. Data analysis

The root mean square error (RMSE) and phase error map of the extracted phase are used for error analysis. The RMSE is calculated as follows:

$$RMSE = \left[ \frac{\sum (\phi_i - \phi_{\text{ideal}})^2}{N} \right]^{1/2}$$

(8)

where $\phi_i$ is the phase extracted by the 2D-CWT IP core on the FPGA platform, and $\phi_{\text{ideal}}$ is the phase extracted by the 2D-CWT algorithm using Matlab. $i \in [0, N - 1]$ , $N$ is the total number of pixels. With different parameters are selected in the 2D-CWT technique, the RMSE of the phase of the moiré interferogram is calculated and shown in Table 2. It can be concluded that the results obtained by the 2D-CWT IP core meet the error demand, and the reliability and accuracy of the 2D-CWT IP core are verified.

| Scale factor | Rotation angle | 0      | π/8    | π/4    | π/2    |
|--------------|----------------|--------|--------|--------|--------|
| 10           | 1.8159         | 1.8115 | 1.8114 | 1.8128 |
| 50           | 1.8158         | 1.8146 | 1.8137 | 1.8132 |
| 90           | 1.8138         | 1.8138 | 1.8138 | 1.8138 |
| 130          | 1.8137         | 1.8138 | 1.8138 | 1.8139 |
| 170          | 1.8137         | 1.8138 | 1.8138 | 1.8137 |

In order to analyze the accuracy of the phase extraction as a whole, the phase error of the 2D-CWT IP core processing moiré interferogram is calculated by the following equation:
where $\Delta \phi$ is the phase error, and the phase error map is obtained using equation (9), and shown in Figure 5. It is observed that the result corresponds to the error range in Table 2, which meets the requirements of phase error in practical applications. The validity of the 2D-CWT IP core was further verified.

![Figure 5. Phase error of Moiré pattern.](image)

The 2D-CWT technique is used to process the Moiré pattern of 256×256 pixels on the computer and on the FPGA, respectively. The comparison of execution time is shown in Table 3. It can be seen that the FPGA platform has obvious high performance. When the clock frequency is 200MHz, the time of processing the moiré pattern is less than 10ms, and the goal of high-speed processing is achieved.

| Platform               | Calculation time |
|------------------------|------------------|
| Matlab                 | 1.004s           |
| 2D-CWT IP core         | 7.99ms           |

5. Conclusion
An advanced 2D-CWT IP core analysis method is proposed for phase extraction of the fringe patterns. The core part of this method is the design and implementation of the 2D-CWT operation module and configuration module. Through the moiré interferometry test and phase error analysis, the results show that the 2D-CWT IP core analysis method can reduce the time of phase extraction for the fringe patterns. The effectiveness of implementing the 2D-CWT IP core analysis method on the FPGA platform has been verified. It provides a viable solution for dynamic measurements.

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References
[1] Takeda, M., Ina, H., Kobayashi, S. (1982) Fourier-transform method of fringe-pattern analysis for computer-based topography and interferometry. J. Opt. Soc. Am., 72: 156–160.
[2] Xu, Y.Y., Wang, Y.W., Han, H., et al.(2018) Fast phase retrieval with four-quadrant analysis in phase-shifting interferometry with blind phase shifts. Opt. Commun., 407: 169-174.
[3] Zhao, M., Qian, K.M. (2015) Multicore implementation of the windowed Fourier transform algorithms for fringe pattern analysis. Applied Optics, 54: 587-594.
[4] Qian, K.M. (2007) Two-dimensional windowed Fourier transform for fringe pattern analysis: Principles, applications and implementations. Opt. Lasers Eng., 45: 304-317.
[5] Li, S.K., Su, X.Y., Chen, W.J. (2011) A new wavelet transform for reliability-guided phase unwrapping of optical fringe patterns. Opt. Commun., 284: 4879–4883.

[6] Abid, A. (2013) Fringe pattern demodulation using the one-dimensional continuous wavelet transform: field-programmable gate array implementation. Applied Optics, 52: 1468-1471.

[7] Jiao, F.M., Ma, J., Xu, C.F., et al. (2018) High-speed moiré patterns analysis technique and its implementation with FPGA. Journal of Guilin University of Electronic Technology, 38: 35-40.

[8] Ma, J., Wang, Z.Y., Pan, B., et al. (2011) Two-dimensional continuous wavelet transform for phase determination of complex interferograms. Applied Optics, 50: 2425–2430.

[9] Zhang, M.Z., Mou, J.H., Liu, Y., et al. (2012) Phase extraction for fringe patterns based on complex Morlet wavelet transform. Optics and Precision Engineering, 20: 643-650.

[10] Ma, J., Wang, Z.Y., Vo, M., et al. (2011) Parameter discretization in two-dimensional continuous wavelet transform for fast fringe pattern analysis. Applied Optics, 50: 6399-6408.

[11] Ma, J., Ma, Y.Y., Jiao, F.M., et al. (2018) High-speed phase extraction technique for interferograms by WFTM-IP core implementation. Journal of Electronic Measurement and Instrumentation, 32: 124-130.