ON THREE-DIMENSIONAL MULTIVARIATE VERSION OF Q-NORMAL DISTRIBUTION AND PROBABILISTIC INTERPRETATIONS OF ASKEY–WILSON, AL-SALAM–CHIHARA AND Q-ULTRASPHERICAL POLYNOMIALS

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We study properties of compactly supported 4-parameter \((\rho_{12}, \rho_{23}, \rho_{13}, q) \in (-1, 1)^4\) family of continuous 3-dimensional distributions, which have the property that for \(q \to 1\) this family tends to some 3-dimensional Normal distribution. For \(q = 0\) we deal with 3-dimensional generalization of Kesten–McKay distribution. In a very special case when \(\rho_{12} \rho_{13} \rho_{23} = q\), all one-dimensional marginals are identical semicircle distributions. We find all marginal as well as all conditional distributions. Moreover, we find also families of polynomials that are orthogonalized by these marginals and conditional distributions. Consequently, we find both conditional and unconditional moments of such distributions. Among other things, we show that all conditional moments of, say, order \(n\) are polynomials of the same order in the conditioning random variables. In particular, we give yet another probabilistic interpretation of the famous Askey–Wilson polynomials considered at complex but conjugate parameters, Al-Salam–Chicha, \(q\)-Hermite, as well as Rogers polynomials. It seems that this is one of the first papers that give probabilistic interpretation of Rogers (continuous \(q\)-ultraspherical) polynomials.

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