Vortex Lines or Vortex-Line Chains at the Lower Critical Field in Anisotropic Superconductors?

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Abstract

The vortex state at the lower critical field, $H_{c1}$, in clean anisotropic superconductors placed in an external field tilted with respect to the axis of anisotropy (c-axis) is considered assuming two possible arrangements: dilute vortex-lines or dilute vortex-line chains. By minimizing the Gibbs free energies in the London limit for each possibility we obtain the corresponding lower critical fields as a function of the tilt angle. The equilibrium configuration at $H_{c1}$ for a given tilt angle is identified with that for which $H_{c1}$ is the smallest. We report results for parameter values typical of strong and moderate anisotropy. We find that for strong anisotropy vortex-line chains are favored for small tilt angles ($<7.9^\circ$) and that at $7.9^\circ$ there is coexistence between this configuration and a vortex-line one. For moderate anisotropy we find that there is little difference between the vortex-line and the vortex-chain lower critical fields.

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Motivated by the discovery of high-$T_c$ superconductivity in the cuprates, calculations of vortex properties in uniaxially anisotropic superconductors were carried out by several workers [1,2]. One surprising result is that the interaction between a pair of straight vortex lines parallel to each other, tilted and coplanar with respect to the c-axis is attractive at distances larger than a value of the order of the penetration depth, as was first found by Grishin, Martynovich and Yampol’skii [3–6]. As a consequence, a chain of such lines has lower energy than the same vortex lines placed far apart. On the basis of this result it was suggested that at $H_{c1}$ the mixed state consists of a vanishing small density of vortex-line chains instead of vortex lines [4,6].

However, this is not correct in general for tilted vortex lines. The reason is that in order to determine the $H_{c1}$ configuration it is necessary to minimize the Gibbs free energy in the limit of vanishing vortex density for each candidate vortex arrangement and, from it, obtain the corresponding $H_{c1}$. The equilibrium configuration is the one with the smallest $H_{c1}$. In this paper we carry out this calculation in detail assuming two possible arrangements: dilute vortex lines (DVL) or dilute vortex-line chains (DVLC).

The Gibbs free-energy minimization for the DVL was carried out by Sudbø, Brandt and Huse [7] in the London limit with the goal of studying coexistence of vortex states at $H_{c1}$. They find that, over a range of parameter values, there is one external field tilt angle for which two DVL states, differing by the vortex-line orientation with respect to the c-axis, coexist at $H_{c1}$. In this paper we also investigate how these results are modified by vortex-line chain states.

To carry out the Gibbs free-energy minimization we proceed along lines similar to those developed by Sudbø, Brandt and Huse [7]. We obtain $H_{c1}$ as a function of the external field tilt angle for parameter values typical of strong and moderate anisotropy, both for DVL and DVLC, and determine which one of these is the equilibrium one at $H_{c1}$. For strong anisotropy we predict coexistence between DVL and DVLC at a particular external field tilt angle.

We consider a bulk uniaxially anisotropic superconductor placed in a magnetic field $H$, with magnitude $H$ and tilted with respect to the c-axis by an angle $\alpha$. We assume that the vortex lines are straight and point in a direction making an angle $\theta$ with the c-axis.

In the London limit the superconductor is characterized by the penetration depths $\lambda_{ab}$ and $\lambda_c$, for currents parallel to the ab-plane and to the c-direction, respectively. The free energy per unit length of a generic arrangement of these vortex lines, $F$, can be written as the sum of pairwise interactions [4]

$$F = \frac{\Phi_0^2}{8\pi} \sum_{i,j} f(r_i - r_j) ,$$

where $r_i$ is the i-th line position vector in the plane perpendicular to the vortex line direction and $f(r)$ is the Fourier transform of

$$f(k) = e^{-2g(k)} \frac{1 + \lambda^2_\theta k^2}{(1 + \lambda^2_{ab} k^2_x)(1 + \lambda^2_\theta k^2_x + \lambda^2_y k^2_y) } ,$$

where $\lambda^2_\theta = \lambda^2_{ab} \sin^2 \theta + \lambda^2_c \cos^2 \theta$, $x$ and $y$ refer to two directions perpendicular to one another and to the vortex lines, with $x$ coplanar with the c-axis, and $g(k)$ is the vortex core cutoff function [2,8,9].

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For both DVL and DVLC, $F$ can be cast in the form

$$F_0(n_v, \theta) = n_v A \varepsilon(\theta),$$

where $n_v$ is the vortex-line density and $A$ is the sample area perpendicular to the vortex lines direction. For the DVL we can neglect interactions between lines, so that $\varepsilon(\theta) = \varepsilon_{sf}(\theta)$, the vortex-line self-energy ($\varepsilon_{sf}(\theta) = (\Phi_0^2 / 8 \pi f(0))$). For DVLC we can neglect interchain interactions, so that $\varepsilon(\theta) = \varepsilon_{sf}(\theta) + \varepsilon_{ch}(\theta)$, where $\varepsilon_{ch}(\theta)$ is the energy of interaction per vortex-line of a chain running along the $x$-direction ($\varepsilon_{ch}(\theta) = (\Phi_0^2 / 8 \pi \sum_{n \neq 0} f(x = n L_1, y = 0)$, $L_1$ being the vortex-chain period).

In order to calculate $H_{c1}(\alpha)$ we have to minimize the Gibbs free energy with $H$ and the sample volume fixed. The Gibbs free energy per volume for the above described vortex states is given by

$$G_0(B, \theta; H, \alpha) = \frac{B}{\Phi_0} [\varepsilon(\theta) - \frac{\Phi_0}{4 \pi} H \cos(\theta - \alpha)],$$

where $B = \Phi_0 n_v$ is the modulus of the magnetic induction.

We adopt for $\varepsilon_{sf}(\theta)$ the expression derived by Sudbø and Brandt [10], using an elliptic core cutoff function,

$$\varepsilon_{sf}(\theta) = \varepsilon_0 \frac{\lambda_0}{\lambda_c} [\ln(\kappa \gamma) + \frac{\lambda_c^2 \cos^2 \theta}{\lambda_c^2 \cos^2 \theta + \lambda_0^2} \ln \frac{\gamma^2 (\lambda_c^2 + \lambda_0^2)}{2 \lambda_0^2}],$$

where $\varepsilon_0 = (\Phi_0 / 4 \pi \lambda_{ab})^2$, $\gamma = \lambda_{ab} / \lambda_c$, $\kappa = \lambda_{ab} / \xi_{ab}$, $\xi_{ab}$ being the ab-plane coherence length. This line energy was derived from the anisotropic Ginzburg-Landau theory using the Klemm-Clem transformations [11]. The elliptical core cut off has semimajor axis $\xi_{ab}^{-1}$, and semiminor axis $(\xi_{ab}^2 \cos^2 \theta + \xi_{ab}^2 \sin^2 \theta)^{-1/2}$.

Taking the derivatives of $G_0$ with respect to $B$ and $\theta$ and equating to zero we obtain, respectively

$$\varepsilon(\theta) - \frac{\Phi_0}{4 \pi} H \cos(\theta - \alpha) = 0$$

$$\frac{d\varepsilon(\theta)}{d\theta} + \frac{\Phi_0}{4 \pi} H \sin(\theta - \alpha) = 0.$$

The first equation is the familiar condition $G_0 = 0$ at $H_{c1}$, whereas the second results from torque balance.

To solve this system of equations we first obtain $H$ from Eq. (6) and substitute it in Eq. (7). The following relationship between $\alpha$ and $\theta$ results

$$\alpha = \theta + \tan^{-1} \left[ \frac{1}{\varepsilon(\theta)} \frac{d\varepsilon(\theta)}{d\theta} \right].$$

Finally, we obtain $H_{c1}(\alpha)$ by substituting the function $\theta(\alpha)$ obtained by inverting Eq. (8) into Eq. (6).
\[ H_{c1}(\alpha) = \frac{4\pi\varepsilon[\theta(\alpha)]}{\Phi_0 \cos[\theta(\alpha) - \alpha]} \quad (9) \]

As we shall see shortly, \( \theta(\alpha) \) may be a multivalued function. Multivalued solutions for \( H_{c1} \) in anisotropic materials have been first noticed by Klemm and Clem [11]. If this is so, according to Eq. (9), for each \( \alpha \) there are several, generally distinct, \( H_{c1}(\alpha) \) and corresponding vortex configurations satisfying the minimization conditions. The physical solution is the one with the smallest \( H_{c1}(\alpha) \).

Now we obtain numerically the solutions of these equations for the following choices of parameters, typical of strong and moderate anisotropy:

**Moderate anisotropy:** \( \kappa = 50, \gamma = 1/5 \)

**Strong anisotropy:** \( \kappa = 10, \gamma = 1/\sqrt{200} \)

The results are as follows.

i) *Dilute arrangement of vortex lines (DVL).* As expected, our results are identical to those obtained by Sudbø, Brandt and Huse [7]. For strong anisotropy there is a region where \( \theta(\alpha) \) is multivalued: for each \( \alpha \) there are three values of \( \theta \) and of \( H_{c1}(\alpha) \) as shown in Fig. [1]. The physical solution for each \( \alpha \) is the one with the smallest \( H_{c1}(\alpha) \) (Fig. [2]). At \( \alpha = 7.5^\circ \) the solutions \( \theta_1 = 35^\circ \) and \( \theta_2 = 86^\circ \) give the same \( H_{c1} \) ((Fig. [2]), indicating coexistence of DVL with vortex lines oriented in these directions. The region \( \theta_1 < \theta < \theta_2 \) is forbidden for vortex-line tilt angles. For moderate anisotropy there is a single \( \theta \) and \( H_{c1}(\alpha) \) for each \( \alpha \) as show in Fig. [3].

ii) *Dilute arrangement of vortex-line chains (DVLC).* First we calculate \( \varepsilon_{ch}(\theta) \) by numerically minimizing the energy per vortex line of a single chain with respect to the chain period, \( L_1 \), using a simulated annealing algorithm based on a fast convergent series for the energy [13,14]. The results for \( \varepsilon_{ch}(\theta) \) and for \( L_1 \) are shown in Fig. [3]. Our results are similar to those obtained in Ref. [4].

It turns out that \( \varepsilon_{ch}(\theta) \) is small compared to \( \varepsilon_{sf}(\theta) \) for all \( \theta \): less than 6% for strong anisotropy and less than 0.5% for moderate anisotropy. However, as we shall discuss in detail shortly, this small correction has non-trivial consequences for strong anisotropy.

It is interesting to analyze in some detail the chain structure. In Fig. [3] we show the interaction energy of a vortex-line pair and the equilibrium positions of the six nearest neighbors of a given vortex line for strong anisotropy and \( \theta = 70^\circ \). We note that the chain period is considerably smaller than the distance where the vortex-line pair interaction is minimum. This results from the long range of the attractive interaction between vortex lines. Indeed, in order to reproduce our calculated value for \( \varepsilon_{ch} \) in this case it is necessary to add the contributions of all six neighbors shown in Fig. [3].

We accurately fit our numerical data for \( \varepsilon_{ch}(\theta) \) to an analytical function and obtain the solutions of Eqs. (3) and (4) with \( \varepsilon(\theta) = \varepsilon_{sf}(\theta) + \varepsilon_{ch}(\theta) \) following the same steps as in i).

For moderate anisotropy we find that the \( \theta(\alpha) \) and \( H_{c1}(\alpha) \) for the DVLC are practically identical with the same quantities for the DVL. This results from the very small \( \varepsilon_{ch}(\theta) \) in this case.

For strong anisotropy, we find that \( \theta(\alpha) \) for the DVLC is multivalued in a range of \( \alpha \) values, with three values of \( \theta \) and \( H_{c1}(\alpha) \) for each \( \alpha \), similarly to the DVL case. The smallest \( H_{c1}(\alpha) \), and the corresponding \( \theta(\alpha) \), are the full curves shown in Fig. [4]. Comparing the results for the two configurations we conclude that for \( \alpha < 7.9^\circ \) \( H_{c1}(\alpha) \) for DVLC is smaller than that for lines, whereas for \( \alpha > 7.9^\circ \) there is practically no difference between the
$H_{c1}(\alpha)$ for these configurations (the same is true for $\alpha \sim 0^\circ$). At $\alpha = 7.9^\circ$ there is coexistence between a DVLC arrangement with $\theta_3 = 61^\circ$ and a DVL one with $\theta_4 = \theta_1 = 86^\circ$. The region $\theta_3 < \theta < \theta_4$ is forbidden for vortex-line tilt angles.

In conclusion then, we determine if a dilute arrangement of vortex-lines or of vortex-line chains is the equilibrium vortex configuration at lower critical field by calculating $H_{c1}$ for each case and by identifying the equilibrium configuration with that for which $H_{c1}$ is the smallest. We find that the vortex-line chain configuration is favored for strong anisotropy and small external field tilt angles ($< 7.9^\circ$). For larger tilt angles the vortex lines are nearly parallel to the a-b plane, where there is little difference between the two configurations. For moderate anisotropy we find no significant difference between the two $H_{c1}$. The values chosen in our calculation of the anisotropy parameter $\gamma$ for what we call moderate and strong anisotropy are typical of YBCO and BSCCO, respectively. The parameter $\kappa$ for moderate anisotropy is typical of YBCO, but for strong anisotropy is about five times smaller than those typical of BSCCO\cite{1}. This difference does not alter significantly the above stated conclusions because the chain interaction energy does not depend on $\kappa$. Only the self-energy does.

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FIG. 1. Dilute vortex-line arrangement for strong anisotropy.  a) $\theta(\alpha)$ curve. Dashed line: solution of Eq. (8). Arrow indicates coexistence point ($\alpha = 7.5^\circ$). Full line: physical solution with smallest $H_{c1}$. b) Smallest $H_{c1}(\alpha)$ curve. Insets: detail of region where $\theta(\alpha)$ and $H_{c1}(\alpha)$ are multivalued. Arrows in b) indicate point where $H_{c1}(\alpha)$ curve changes branches.
FIG. 2. Dilute vortex-line arrangement for moderate anisotropy.
FIG. 3. Numerical results for chains. Triangles: moderate anisotropy. Squares: strong anisotropy. a) Chain period. Inset: interaction energy of a vortex-line pair along the x-direction (full line) and chain nearest neighbors positions (full circles). b) Chain interaction energy per vortex line.
FIG. 4. Dilute vortex-line chain and vortex-line arrangements for strong anisotropy compared. Dashed lines: (smallest value) dilute vortex-line arrangement (same as in Fig. 1). Full line: $\theta(\alpha)$ and $H_{c1}(\alpha)$ (smallest value) for dilute vortex-line chain arrangement. Arrow indicates coexistence point ($\alpha = 7.9^\circ$).