Rotational terms and quantum degeneracy in black holes

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It is believed that the first law of black-hole mechanics has no independent physical significance and acquires it only after identifying with the first law of thermodynamics. It is argued here that the first law of black-hole mechanics has a direct physical significance: not only the term $\partial W/\partial t$ but all its terms have the same mechanical meaning - the rotational kinetic energy of a black hole in real or in an internal space. Moreover, it is shown that the Kerr-Newman black hole is a system of non-degenerate plane rotors represented by the corresponding terms in the first law of black-hole mechanics. It is found that a degeneracy arises because the energy of a black hole does not depend on where an internal angular momentum of a black hole associated with the black hole area is determined on the horizon.

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I. INTRODUCTION

The statistical interpretation of the black-hole entropy of Bekenstein and Hawking

$$S_{BH} = \frac{A}{4l_p^2}$$

remains a central problem in black hole physics [1]. The majority of approaches to the problem can be reduced to the following main principles suggested by Bekenstein as early as 38 years ago [2]:

- The black hole is a composite object.
- Constituents of a black hole responsible for black hole entropy reside on its event horizon; the area of the horizon is quantized

$$A_n = \Delta A \cdot n, \quad n = 0, 1, 2, \ldots$$

where $\Delta A$ is the quantum of black hole area, $\Delta A \sim l_p^2$, so that the horizon surface consists of $n$ identical patches - constituents of a black hole - each of area $\Delta A$. Then, if every patch can have $k$ states, the total number of states of a black hole is

$$W = k^n.$$  

- From the statistical mechanics point of view, the black hole is a conventional object; in particular, the entropy of a black hole is the logarithm of the number of states associated with its horizon,

$$S = \ln W = n \ln k.$$  

Because the patches are independent of one another, the total entropy of a black hole is just $n$ times the entropy of a single patch $s_1 = \ln k$.

- The area spectrum [2] implies a discrete mass spectrum with the level spacing

$$\Delta M_n \sim \frac{1}{M_n}.$$  

For convenience in the subsequent comparison, we shall refer to the approach (model) based on these principles as the composite black hole approach (model). However these principles raise new puzzles. A puzzle is [3], that locally there is nothing special about the horizon, so it is hard to see why it should behave as an object with the local degrees of freedom [3]. Another puzzle is that we assign an entropy to something which is a classical solution of the gravitational field equations, something which behaves like a soliton, but we do not usually assign an entropy to a soliton [4, 5]. One more puzzle is that the black hole, although additive [1], is nevertheless an indivisible object. The next puzzle is related with the spectrum [6]. It implies a discrete emission spectrum. But if the black hole is a conventional object, then the separation between subsequent energy levels should be exponentially small $\Delta M_n \sim \exp(-S_{BH})$ and the emission spectrum is practically continuous. Finally, the greatest puzzle is the nature of black hole constituents and their quantum states. In connection with these 38-year puzzles a question arises: is it possible that this is not the point that there is not yet quantum theory of gravity, but the point is that our model is not completely adequate to the physical nature of black holes?

In [6] a new approach to the problem of black hole entropy was developed and applied to a Schwarzschild black hole. It is based on the concept of internal angular momentum of a black hole $L_z = A/(8\pi G)$. This approach does not need the concept of constituents for determining the black hole entropy and is free of the difficulties of the composite black hole approach. The basic idea of the approach will be presented below. For convenience, we shall refer to this approach (model) as the fundamental black hole approach (model). The purpose of this paper is to
summarize and extend this approach to a generic Kerr-Newman black hole. The Kerr-Newman case is more difficult than the Schwarzschild one. The point is that a Kerr-Newman black hole has additional degrees of freedom related with rotation and electromagnetism. If the black hole is a fundamental object, then these degrees of freedom can give a contribution to the total degeneracy of the black hole. Note that the statistical interpretation of the black hole is a system of plane rotators represented by the corresponding terms in the first law of black-hole mechanics. That is, the first law of black-hole mechanics has a direct physical significance: not only the term $\Omega dJ$ but all terms in the law have the same mechanical meaning - the rotational kinetic energy of a black hole in real or internal space.

- A black hole has rotational symmetries and no internal structure. Classically it can rotate about any of its axes (the Kerr solution). But from a quantum mechanical point of view such a motion is impossible. It is argued that the rotation of a black hole is nevertheless observable due to the dragging effect. Moreover, it is the dragging effect, that removes the degeneracy with respect to the direction of $J$.

- At first sight it seems that in the fundamental black hole approach all three terms in (6) should give a contribution in the total degeneracy factor of a black hole. But this is not so. The Kerr-Newman black hole is a system of plane rotators represented by the corresponding terms in the first law of black-hole mechanics. That is, the first law of black-hole mechanics should read as

$$dM = \frac{k}{8\pi G} dA + \Omega dJ + \Phi dQ,$$  \hspace{1cm} (6)

rather than (4). Here $\omega_i$ are the angular frequencies and $L_{i\zeta}$ are the $\zeta$-components of the angular momenta of the black hole in the corresponding spaces. The energy levels of rotators are non-degenerate.

- The degeneracy of energy levels of a black hole arises because the energy of a black hole does not depend on where $L_{1z}$ is determined on the horizon. Quantization of $L_{1z} = A/(8\pi G)$ gives the equidistant area spectrum of a black hole with the area quantum $\Delta A = 8\pi l_p^2$. Since the precision with which $L_{1z}$ can be determined equals to the size of the area quantum $\Delta A = 8\pi l_p^2$, the degeneracy of a black hole is

$$W = \frac{A}{8\pi l_p^2}. \hspace{1cm} (8)$$

That is why it is determined by the first term in black hole mechanics.

- The horizon is not a configuration surface with local degrees of freedom but phase space (surface). Once we have accepted this, the black hole entropy is no longer arbitrary and uniquely determined by

$$S = 2\pi W. \hspace{1cm} (9)$$

The organization of the paper is as follows. In Sec. II we consider the degeneracy in Kerr-Newman black holes. We begin with the basic idea suggested early in [6]. Then we consider a trouble with the first law of black-hole mechanics and argue that the law has independent physical significance. We also show that the Kerr-Newman black hole can be viewed as a system of non-degenerate plane rotators. Finally, we find the origin of black hole degeneracy. In Sec. III we consider some applications of our approach.

II. THE BLACK HOLE DEGENERACY

A. The basic idea

In [7] an internal angular momentum of a Schwarzschild black hole $L_z = A/(8\pi G)$ was determined. Quantization of $L_z$ gives the equidistant area spectrum of a black hole $A_m = \Delta A \cdot m, \; m = 0, 1, 2, ..., \; \text{with the area quantum } \Delta A = 8\pi l_p^2$. The number of microstates is intrinsically an integer. But $\exp(2\pi m)$ is not integral [8]. On the other hand, the energy of a black hole does not depend on where $L_z$ is located on the horizon. The precision with which $L_z$ can be determined equals the size of the area quantum. Therefore, the number of states accessible to a black hole can be determined as $m = A/8\pi l_p^2$. However, once we have accepted this, the black hole entropy is uniquely determined by $S = 2\pi (\text{number of states})$ (not by $S = \log (\text{number of states})$). This means that the black hole is a nonadditive object. This agrees with the thermodynamical properties of a black hole. In particular, the Bekenstein-Hawking entropy is not a homogeneous first order function of the black hole energy. Moreover, we cannot divide a black hole into two independent subsystems by a partition as an ideal gas in a box (the area theorem). And the black hole constituents cannot be extracted from a black hole. Therefore the black hole cannot be thought as made up of any constituent subsystems each of them endowed
with its own independent thermodynamics. We have to consider a single black hole as a whole system. On the other hand, the black hole is a vacuum solution of the gravitational equations and can be viewed as a kind of gravitational soliton i.e. as a physical object, localized "within the event horizon" and possessing a mass $M$. Moreover, for $r > R_g$ the Schwarzschild coordinate $t$ is timelike and the coordinate $r$ is spacelike. But for $r < R_g$, $t$ is a spacelike coordinate and $r$ is timelike. So "time" and "radial" coordinates swap character when we cross $r = R_g$. This looks like a twist of spacetime. Thus the black hole should be viewed as a fundamental object like an elementary particle or, rather, a string (because we can assign an entropy to the string). Of course, this idea is not new but rather folklore. 't Hooft had already pointed out that there is no fundamental difference between black holes and elementary particles, and Susskind and other researchers deepened this insight still further. According to this point of view the spectrum of particles does not terminate at the Planck mass but continues on to indefinitely large mass in the form of black holes. It is necessary to stress, however, that in our approach we do not postulate the black hole has additional degrees of freedom associated with rotation and electromagnetism. The fact is that a Kerr-Newman black hole has additional degrees of freedom associated with rotation and electromagnetism. The second and third terms on the right hand side are usually interpreted as changes in the energy due to rotation and electromagnetism. But the first term with $dA$ does not have a direct physical interpretation. As is well known, all basic conservation laws of energy also contain a mix of terms of different nature, but all these terms have clear physical meaning. In contrast to this, the first law of black-hole mechanics has no independent physical significance and becomes meaningful after identifying with the first law of thermodynamics if one assumes the following expressions for the temperature and entropy of a black hole

$$T_H = \frac{k}{2\pi}, \quad S_{BH} = \frac{A}{4l_p^2}$$

The same is true, of course, of the generalized Smarr formula. In what follows it will be convenient to use the formula

$$M = 2B + 2\Omega J + \Phi Q$$

and the additional degrees of freedom do not contribute to the black hole degeneracy. In statistical physics the internal energy of a body is interpreted as the total of the kinetic and potential energy of all the constituents that compose it. If the black hole is a fundamental object, then this concept has no meaning. In this case $M$ is an eigenvalue of a single-particle Hamiltonian and the entropy is a function of the total energy

$$S = S(M).$$

The degeneracy of the energy level $M$ would then be a product of three degeneracy factors, one depending only on $A$, one only on $J$ (usually $2J + 1$), and third only on $Q$. Is this true? To answer the question, we should investigate the nature of terms $k(dA/8\pi G)$, $\Omega dJ$ and $\Phi dQ$ in the first law of black hole mechanics.

### C. The trouble with the first law of black-hole mechanics

It is widely believed that the laws of black hole mechanics have no independent physical significance and acquire it only after identifying with the laws of thermodynamics. For example, the first law of black-hole mechanics reads

$$dM = \frac{k}{8\pi G}dA + 2\Omega J + \Phi Q$$

It is nothing but a statement of energy conservation for a black hole. Here, $dM$, is the change in mass (energy) of a black hole. The second and third terms on the right hand side are usually interpreted as changes in the energy due to rotation and electromagnetism. But the first term with $dA$ does not have a direct physical interpretation. As is well known, all basic conservation laws of energy also contain a mix of terms of different nature, but all these terms have clear physical meaning. In contrast to this, the first law of black-hole mechanics has no independent physical significance and becomes meaningful after identifying with the first law of thermodynamics if one assumes the following expressions for the temperature and entropy of a black hole

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$$S = S(M).$$
terms in (16) have the same structure of the form $2\omega_i L_i$, where $\omega_1 = k$, $\omega_2 = \Omega$, $\omega_3 = \Theta$ and $L_1 = A/(8\pi G)$, $L_2 = J$, $L_3 = Q^2$. The factors $\omega_i$ are in turn very similar and have the same dimensions of frequency. The $L_i$ have dimensions of action. What does this mean?

D. An assumption

It turns out that the first law of black-hole mechanics has a direct physical significance: not only the term $\Omega dJ$ but all terms in the first law of black-hole mechanics have the same mechanical meaning - the rotational energy of a black hole in real or in an internal space. Namely, we assume that the Kerr-Newman black hole can be viewed as a system of plane rotators represented by the corresponding terms in the first law of black-hole mechanics. That is, the first law of black-hole mechanics should read as

$$dM = \omega_1 dL_{1z} + \omega_2 dL_{2z} + \omega_3 dL_{3z} = \sum_i \omega_i dL_{iz}, \quad (19)$$

rather than [14]. Here $\omega_i$ are the angular frequencies and $L_{iz}$ are the $z$-components of angular momenta of a black hole in real or in an internal space. Their concrete expressions will be determined below. Accordingly

$$M = 2\omega_1 L_{1z} + 2\omega_2 L_{2z} + 2\omega_3 L_{3z} = 2\sum_i \omega_i L_{iz}. \quad (20)$$

Moreover, the energy levels of these rotators are non-degenerate, so that the degeneracy factors associated with $L_{1z}$, $L_{2z}$ and $L_{3z}$ do not give a contribution to the total degeneracy of a black hole. There are good reasons for supposing that this is in fact so.

E. Rotational terms: $2k(A/8\pi G)$

The black hole entropy is a concept defined in the rest frame of an external fiducial observer. Moreover, the Kerr-Newman metric extended into the region within the event horizon cannot describe the spacetime inside the black hole [11]. So, since we do not deal with the interior of a black hole, we regard the Euclidean formulation as the more fundamental one. After all calculations with the Euclideanized Kerr-Newman have been completed, we analytically continue the results obtained back to real value of coordinate time $t$. Moreover, since the black hole entropy is associated with the event horizon, we deal only with the Rindler section of the whole Euclidean Kerr-Newman manifold. The important fact is that in the near-horizon approximation the metric of an arbitrary black hole can be reduced to the Rindler form.

For example, the Rindler section of the Euclidean Schwarzschild manifold $R^2 \times S^3$ is an analytic continuation of that part of the Lorentzian geometry that just lies outside or at the event horizon, $r \geq 2GM$. In transforming from Schwarzschild to Euclidean Rindler coordinates the Schwarzschild time $t$ transforms to a variable $\varphi = kt$ in the Euclidean Rindler plane $R^2$, $k = 1/(4GM)$. But the metric has a coordinate singularity corresponding to $r = 2GM$. Regularity is obtained if $\varphi$ is interpreted as an angular coordinate with periodicity $2\pi$. In [2], it was shown that there exists the $z$ component of the angular momentum which is conjugate to this angle, $L_z = i\hbar \partial/\partial \varphi$ with an eigenvalue $L_z = A/(8\pi G)$. We call it Rindler angular momentum. In the Euclidean formulation the Rindler angular momentum and the Hawking temperature have the same origin - the periodicity in the Schwarzschild imaginary time - and, therefore, the universal geometrical nature. The angular momentum $L_z$ is the generator of rotations around $z$ axis (this axis corresponds to $r = 2GM$). Since there exists only one way of rotation, the black hole represents a plane rotator in an internal space. As early as 38 years ago, by proving that the black hole horizon area is an adiabatic invariant, Bekenstein showed [2] that the quantum of black hole area is of the form $\Delta A = 8\pi l_P^2$. In [3], by following the approach used by Susskind [2] to derive the Rindler energy, quantization of the black hole area (and entropy) was obtained from the commutation relation and quantization condition for $L_z$. Namely, since

$$L_z = \frac{A}{8\pi G} = mh, \quad m = 0, 1, 2, \ldots, \quad (21)$$

then

$$A = 8\pi l_P^2 \cdot m, \quad (22)$$

and

$$S = 2\pi \cdot m. \quad (23)$$

The energy of a Schwarzschild black hole in terms of a plane rotator is

$$M = 2kL_z = 2\hbar \omega \cdot m, \quad (24)$$

where $\omega = k$ is the frequency of a black hole in an internal space associated with $L_z$. Note that $L_z$ is an adiabatic invariant as well as $A$.

In [6] it was suggested that quantization of a Schwarzschild black hole is nothing but the Landau quantization and a Schwarzschild black hole represents a two-dimensional isotropic oscillator with an additional interaction $\omega L_z$, where $\omega = k$. But the model of an oscillator presupposes a zero-point energy of a black hole in the absence of the black hole which seems unlikely. In contrast, the spectrum of a plane rotator does not contain a zero-point energy. In what follows we shall use the model of a plane rotator. The model of a harmonic oscillator can be considered only as an approximation; the model reduces to that of a plane rotator in the limit of large $m$.

Setting $t = -i\tau$ and reasoning as in the the Schwarzschild case, we can define the Euclidean Kerr-Newman metric [12]. This metric is complex and it
is asymptotically flat in a coordinate system rotating with the angular velocity of a black hole $\Omega$. The locus $r_+ = M + \sqrt{M^2 - Q^2 - a^2}$ will be a conical singularity unless we identify the point $(r,\theta,\phi)$ with the point $(r + i2\pi k^{-1},\theta,\phi + i2\pi k^{-1})$. As a result, the real coordinate time $t$ transforms to an angular coordinate about the "axis" $r_+ = M + \sqrt{M^2 - Q^2 - a^2}$. Alternatively we can in addition analytically continue in the specific angular momentum $a$ and charge $Q$ to get a real Riemannian metric. In this case the Euclidean Kerr-Newman manifold has a structure identical to that of the Euclidean Schwarzschild manifold $R^4 \times S^2$, and, in particular, it covers only the exterior of a black hole and it also requires the compactification of the Euclidean time in order to eliminate a conical singularity. In an exactly similar manner to the Schwarzschild case, we can determine Rindler angular momentum of a Kerr-Newman black hole with the quantized eigenvalues [7]

$$L_{1z} \equiv \frac{A}{8\pi G} = mh, \quad m = 0, 1, 2, \ldots, \tag{25}$$

where $A$ is the area of a Kerr-Newman black hole. Since the Euclidean Kerr-Newman spacetime has no region corresponding to the region $r < r_+$ in the Lorentzian spacetime, the negative integers $m$ are ruled out (as mentioned in the beginning, continuation of the Lorentzian Kerr-Newman line element inside the surface of the horizon has no physical meaning at all). As in the Schwarzschild case, we can consider the Kerr-Newman black hole as a plane rotator associated with the angular momentum. Analogously, the first term on the right hand side in [10] can be regarded as the rotational energy of a Kerr-Newman black hole

$$2k \frac{A}{8\pi G} = 2\omega_1 L_{1z}, \tag{26}$$

where $\omega_1 = \omega$ is the angular frequency of the black hole in an internal space. As is well known, the energy levels of a plane rotator except the ground state is doubly degenerate (this follows from the fact that for every energy level the rotator can rotate both in positive and in the negative direction). Since there are only positive values of $m$, the plane rotator does not give a contribution to the total degeneracy of a black hole.

F. Rotational terms: $2\Omega J$

At first glance the rotation of a black hole in real space is impossible. A Schwarzschild black hole is spherical in shape apart from quantum fluctuations and has no internal structure. Classically, it can rotate about any of its axes. Analogously, a classical Kerr-Newman black hole can rotate around its axis of symmetry. But from a quantum mechanical point of view the rotation of such objects is unobserved. A black hole cannot rotate, because any rotation leaves its horizon surface invariant and thus by definition does not change the quantum-mechanical state; there is nothing "inside a black hole" to change its position during the rotation and there is nothing marked on its surface to define the orientation. Moreover, no orbiting spots can observed on the horizon, since all radiation from the horizon is infinitely redshifted. But our arguments "against the rotation" are imperfect. Indeed, in Newton’s theory a gravitational field is in no way dependent on the motion of matter. So the gravitational fields of a rotating sphere and the same sphere at rest are completely identical. But in Einstein’s theory this is not so: a rotating sphere drags spacetime around itself. This phenomenon is known as the dragging of inertial frames. It is also known as the Lense-Thirring effect. The effect is differential, stronger near the black hole and weaker at larger distances. Moreover, within the ergosphere the dragging is so strong that no object can remain at rest. It is the dragging effect, that makes the rotation of a black hole observable. Thus, illuminating space by a suitable beam of test particles we may detect rotation of a black hole. And the degree to which the particles rotate is a measure of how rapidly this hole rotates.

The Kerr-Newman black hole is axially symmetric but not spherically symmetric (i.e. rotationally symmetric about one axis only which is the angular-momentum axis). Since the angular velocity $\Omega$ is constant over the horizon, the black hole rotates rigidly. Therefore we can regard the Kerr-Newman black hole as a rigid symmetric rotator or top. According to quantum mechanics [13], the stationary states of a symmetrical top are described by three quantum numbers: the angular momentum $J$ ($J = 0, 1, 2, \ldots, J_z$) and its components along the axis of the top $J_z = K$ ($K = J, J-1, \ldots, -J$) and along the $z$-axis fixed in space $J_z$ ($J_z = J, J-1, \ldots, -J$). The degeneracy of each energy level of the top with $K \neq 0$ is $W = 2(2J+1)$ because $J_z$ can take $2J+1$ different values for a given value of $J$, and $K$ can be either positive or negative, corresponding to the two possible directions of rotation about the axis of angular momentum.

But we have ignored the frame-dragging effect. It turns out that when it is taken into account, the degeneracy factor of a Kerr-Newman black hole is completely removed. To deduce this, it is not in fact necessary to solve the equation $J \psi = J \psi$ in the Kerr-Newman metric; instead, we can argue as follows. First, a black hole drags a fixed coordinate system about its axis of angular momentum. As a result, the stationary states of our symmetrical top are determined only by the quantum number $J_z$. Secondly, the hole’s rotation drags the coordinate system into orbital motion in the same direction as the hole rotates. As a result, the two-fold degeneracy with respect to values of $J_z$ are completely removed. As is well known, in order to compensate the dragging effect and to have a convenient family of observers for which events at the same (Boyier-Lindquist) time are simultaneous, a family of zero angular momentum observers is used. These observers are at fixed $r$ and $\theta$, but have a constant angular velocity $\omega = d\phi/dt$. An observer corotating with the frame-dragging angular velocity $\omega$ is in
a state of zero angular momentum, and experiences no centrifugal forces. From the point of view of such an observer a Kerr-Newman black hole is viewed as a non-degenerate plane rotator rather than a symmetrical top. Thus the second term on the right hand side in (16) is the kinetic energy of rotation of a Kerr-Newman black hole in real space,

\[ 2\Omega dJ = 2\omega_3 L_{3z}, \]  

where

\[ \omega_3 = \Omega, \quad L_{3z} = J_z. \]  

**G. Rotational terms: \( 2\Theta Q^2 \)**

The third term \( 2\Theta Q^2 \) is the change in the electrostatic energy of a black hole. This term is also related with rotations but in an internal space. The electric charge is a conserved quantity. This is a consequence of invariance of the Lagrangian under the one-dimensional group \( U(1) \) of gauge transformations. This group is equivalent to \( O(2) \), the orthogonal group of rotations in a plane. Two-dimensional rotations or gauge transformations belong to the Abelian group. As a result, the electric charge is additive in the same way as the \( z \) component of the angular momentum. On the other hand, the electric charge is quantized. But it is not known for certain why it is quantized. There have been many suggestions, including Kaluza-Klein models [14], magnetic monopoles [15] and grand unified theories [16] to explain the quantization of electric charge. The important fact is that all these suggestions are closely related to the quantization of angular momentum. For example, in the original Kaluza-Klein model charged particles are ones that go round in the fifth curled up dimension (neutral particles do not move in fifth dimension). The charge is proportional to the angular momentum of the motion round the curled up fifth dimension. In quantum theory, angular momentum is quantized, so charge is quantized. Note that when a Kerr-Newman black hole is described in the framework of a five-dimensional Kaluza-Klein model, the quantities \( \Omega \) and \( \Phi \) enter the expressions in a similar manner, and their properties are to a certain extent similar [17].

We shall assume that the electric charge of a black hole is an integer multiple of the fundamental unit of electric charge \( e \). Since the charge operator \( \hat{Q} \) is the generator of the \( U(1) \) gauge transformations, its spectrum of eigenvalues should be of the form

\[ Q = en, \quad n = 0, \pm 1, \pm 2, \ldots \]  

Then

\[ 2\Theta Q^2 = 2\Theta (4\pi an^2 \hbar), \]  

where the definition of the fine structure constant \( \alpha = e^2/(4\pi\hbar) \) has been used. At this point we should note the following. First, as has been stated above, there is an analogy between the electric charge generating gauge transformations and the \( z \) component of the angular momentum generating rotations in a plane. Secondly, as is easily seen from (30), the states of electrostatic energy of a black hole are doubly degenerate due to the factor \( n^2 \). This is just the degeneracy we should expect for a plane rotator. Therefore we can interpret the term \( 2\Theta Q^2 \) as the kinetic rotational energy of a black hole in an internal charge space and express it in terms of a plane rotator

\[ 2\Theta Q^2 = 2\omega_3 L_{3z}. \]

Note that the interpretation of \( 2\Theta Q^2 \) as the rotational energy of a plane rotator has more formal significance than that of \( 2k(A/8\pi G) \) and \( 2\Omega J \). Because of this, we do not determine the frequency \( \omega_3 \) and the angular momentum \( L_{3z} \). Nevertheless, we shall use the form \( 2\omega_3 L_{3z} \) to uniform the term \( 2\Theta Q^2 \) with other rotational terms. Finally we must return to the degeneracy factor. The point is that despite the existence of two kinds of charges with opposite signs in nature, the sign of the black hole charge \( Q \) is given, so the energy levels of the rotator are in fact non-degenerate.

**H. The origin of degeneracy**

As has been shown above, a Kerr-Newman black hole has three rotational degrees of freedom associated with the angular momenta \( L_{1z}, L_{2z} \) and \( L_{3z} \). But these degrees of freedom do not give a contribution to the total degeneracy of a black hole. We did not determine \( L_{3z} \) and \( \omega_3 \) explicitly. If we determined \( L_{3z} \) and \( \omega_3 \) as \( L_{3z} = Q^2 \) and \( \omega_3 = \Theta \), for example, we should have an additional degeneracy associated with \( \omega_3 \). It is clear, however, that in this case all \( \omega_i \) are linearly independent. So, is there a degeneracy in black holes at all? It turns out that the energy of a Kerr-Newman black hole, as in the Schwarzschild case, does not depend on where \( L_{1z} \) is located on the horizon. The precision with which \( L_{1z} \) can be determined equals the size of the area quantum \( \Delta A = 8\pi l_p^2 \). Therefore, the number of states accessible to a black hole can be determined as

\[ W = \frac{A}{8\pi l_p^2}. \]  

We can imagine this as follows. Associate our rotator with a vortex with the area of core \( \Delta A = 8\pi l_p^2 \). Then the number of ways to place the vortex on the horizon is just \( (2\pi)^3 \), the ratio of the area of a black hole to the area of the core. As a result, the (configuration) surface of the event horizon becomes phase space of a black hole, as it should; as is well known, the phase space of a plane rotator is two-dimensional. Therefore, quantization of the black hole area and entropy is nothing but quantization of the phase surface

\[ S = 2\pi m. \]
Note that string theory needs to introduce the notion of a stretched horizon to avoid the problem of the local degrees of freedom. In loop quantum gravity, it is believed that the only possible degrees of freedom on the horizon have to be global or topological, described by a topological quantum field theory. According to the traditional approach we would have to take the logarithm of the entropy, but in this case the generalized second law of black hole thermodynamics (GSL) would be violated [18] (it appears that much earlier, Gour and Mayo showed that the formula for the black hole entropy \( S = f(A) \) with the function \( f(A) = \ln A \) clashes with the GSL and must be excluded). It turns out that once we have accepted the fact that the black hole degeneracy is proportional to the area, the black hole entropy is no longer arbitrary and uniquely determined by

\[ S = 2\pi W \]  

(34)

(not by \( S = \log W \)). This relation is the only way to reconcile the formula \( S = 2\pi m \) with the requirement that \( W \) be integral. The absence of the logarithm means that the black hole is a nonadditive object. Reasoning as in the Schwarzschild case (subsection A), we conclude that the Kerr-Newman black hole is a fundamental object.

### III. SOME APPLICATIONS

#### A. System of black holes (and matter)

To avoid misunderstanding, we should add a clarifying remark concerning a system of black holes. Suppose, for simplicity, that two black holes are far apart and their interaction is negligible, so that they can be viewed as statistically independent. Let \( S_{1(2)} = 2\pi m_{1(2)} \) and \( W_{1(2)} = m_{1(2)} \) be the entropy and degeneracy of the first (second) black hole, respectively. Then the number of states for the combined system is \( W = W_1 W_2 = m_1 m_2 \). What is the entropy of the system? Obviously, we cannot write the total entropy as \( S = 2\pi m_1 m_2 \) because our system is not a single black hole. It seems that we would take the logarithm of \( W \): \( \ln W = \ln m_1 + \ln m_2 \). But in this case, as mentioned above, we cannot interpret \( \ln m_{1(2)} \) as the entropy of the first (second) black hole. Does \( W = m_1 m_2 \) exist? Yes, it does. But in \( m_{1(2)} \) is not the entropy of the first (second) black hole. Despite this failure the laws of thermodynamics are still valid, so we can define the entropy as

\[ S = S_1 + S_2 = 2\pi (m_1 + m_2). \]  

(35)

At this point we refer back to the meaning of the quantum number \( m \). It is the angular (“magnetic”) quantum number. Since interaction is weak, the angular momentum \( L_z \) of the whole system can be regarded as the sum of the angular momenta \( L_{1z} \) and \( L_{2z} \) of its parts,

\[ L_z = L_{1z} + L_{2z}. \]  

(36)

Then it follows that

\[ m = m_1 + m_2. \]  

(37)

According to thermodynamics the entropy of combine system is additive. Thus the law of addition for the Rindler angular momenta is nothing but a law of thermodynamics. We have considered the case of two independent black holes. But we can extend it to an arbitrary number of black holes (and matter).

Note that the relation \( S = 2\pi m \) can be also viewed as the angular momentum quantization condition on the phase of wave-function: if the eigenfunction of \( L_z \) is to be single-valued, it must be periodic in phase, with period \( 2\pi \). So we can consider \( m \) also as a topological number (winding number). As has been shown above, it is an additive number.

#### B. The mean separation between energy levels

From (24) it follows that the separation between energy levels of a black hole is

\[ \Delta M_n = \frac{m_n^2}{2M_n}. \]  

(38)

This value is equal in order of magnitude to the width of energy level \( R_{\gamma}^{-1} \). This value, however, does not agree with estimation obtained from the usual definition of entropy, \( \langle \Delta M_n \rangle \sim \exp(-S_{BH}) \). As mentioned in Introduction, this compounds a problem in the composite black hole approach. The fact is that in this approach the energy levels should split due to unavoidable interactions between the constituents, so the mean separation between energy levels should be really \( \sim \exp(-S_{BH}) \). This implies that the discreteness of the spectrum is very blurred and difficult to see observationally. In contrast, in the fundamental black hole approach there are no constituents, so there is no splitting and the discreteness can be observed.

#### C. The relation \( L_z = \alpha'M^2 \)

Although the black hole is a fundamental object, it is unstable and decays by emitting Hawking radiation (as is well known, stability does not appear to be a criterion of the fundamental nature). Therefore black holes can be viewed as resonant poles in the S-matrix for scattering of stable particles [19]. In this case there should be poles in the complex \( s \)-plane (where the Mandelstam variable \( s \) is the total center-of-mass energy squared) at

\[ s_m = M_m^2 - i\Gamma_m. \]  

(39)

where \( \Gamma_m \) is the width of energy level in the \( s \)-plane and the energy spacing between the subsequent energy levels of a black hole is given by [35]. Srednicki noted
that [68] is very unusual behavior for a set of resonances. Namely, its series never terminates. As is well known, all known laboratory systems (for example, such as nuclei) have dense resonances, but always there is a threshold above which the poles are replaced by a cut. It turns out that the strange behavior of black hole resonances noted by Srednicki can be explained in our model. The Schwarzschild black hole is "the ground state of the Kerr-Newman black hole". So we restrict our consideration to uncharged, non-rotating black holes. The point is that the Rindler angular momentum \( L_z = 2GM^2 \) is proportional to the square of the mass of a black hole and increases without limit

\[
L_z = \alpha' M^2. \tag{40}
\]

Here we have introduced the notation \( \alpha' = 2m_p^{-2} \); the reason for this will be clear in a moment. This resembles the well-known angular momentum - mass relation for hadronic resonances. As is well known, the graph of the angular momentum \( J \) of hadronic resonances against their mass squared falls into lines \( J = \alpha' M^2 \) called Regge trajectories. The constant \( \alpha' \) is known as the Regge slope. Instead of terminating abruptly as in the case of nuclei, the graph continue on indefinitely, implying that quarks don’t fly apart when spun too fast. In contrast to nuclei, there is no a threshold there. This is a manifestation of quark confinement. It has a simple explanation in the string picture where the relation \( J = \alpha' M^2 \) emerges naturally from a rotating open string. In string model, hadrons are modeled by relativistically rotating strings capped with massless quarks at both ends. For example, meson resonances obey the relation \( J = \alpha' M^2 \) with the slope \( \alpha' \sim (1 \text{ GeV})^2 \). Here the slope \( \alpha' \) is inversely proportional to the string tension. But this is an effective string theory. As is well known, to date string theory is considered as the unified theory of particle physics and gravity, so the slope of fundamental strings is determined by the fundamental constants of gravity and quantum theory, \( \alpha' \sim m_p^{-2} \). But it is just the slope of a black hole [40].

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