K-BANHATTI AND K-HYPER BANHATTI INDICES OF DOMINATING DAVID DERIVED NETWORK

WEI GAO, BATSHA MUZAFFAR¹, WAQAS NAZEER

ABSTRACT. Let \( G \) be connected graph with vertex \( V(G) \) and edge set \( E(G) \). The first and second K-Banhatti indices of \( G \) are defined as \( B_1(G) = \sum_{ue} \left[ d_G(u) + d_G(e) \right] \) and \( B_2(G) = \sum_{ue} \left[ d_G(u) d_G(e) \right] \), where \( ue \) means that the vertex \( u \) and edge \( e \) are incident in \( G \). The first and second K-hyper Banhatti indices of \( G \) are defined as \( HB_1(G) = \sum_{ue} \left[ d_G(u) + d_G(e) \right]^2 \) and \( HB_2(G) = \sum_{ue} \left[ d_G(u) d_G(e) \right]^2 \). In this paper, we compute the first and second K-Banhatti and K-hyper Banhatti indices of Dominating David Derived networks.

Mathematics Subject Classification : 05C05, 05C07, 05C35.
Key words and phrases : K-Banhatti index; K-hyper Banhatti index; Dominating David Derived networks.

1. Introduction

Chemical graph theory is a branch of graph theory in which a chemical compound is represented by simple graph called molecular graph in which vertices are atoms of compound and edges are the atomic bounds. A graph is connected if there is at least one connection between its vertices. Throughout this paper we take \( G \) a connected graph. If a graph does not contain any loop or multiple edges then it is called a network. Between two vertices \( u \) and \( v \), the distance is the shortest path between them and is denoted by in graph \( G \). For a vertex \( v \) of \( G \) the degree is number of vertices attached with it. The edge connecting the vertices \( u \) and \( v \) will be denoted by \( uv \). Let \( d_G(e) \) denote the degree of an edge \( e \) in \( G \), which is defined by \( d_G(e) = d_G(u) + d_G(v) - 2 \) with \( e = uv \). The degree and valence in chemistry are closely related with each other. We refer the book [1] for more details. Now a day another emerging field is Cheminformatics, which helps

¹ Corresponding Author

© 2017 Wei Gao, Batsha Muzaffar, Waqas Nazeer. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.
to predict biological activities with the relationship of Structure-property and quantitative structure-activity. Topological indices and Physico-chemical properties are used in prediction of bioactivity if underlined compounds are used in these studies [2,3]. A number that describe the topology of a graph is called topological index. In 1947, the first and most studied topological index was introduced by Weiner [4]. For more details about this index can be found in [5,6]. In 1975, Milan Randić introduced the Randić index [7]. Bollobas et al. [8] and Amic et al. [9] in 1998, working independently defined the generalized Randić index. This index was studied by both mathematicians and chemists [10]. For details about topological indices, we refer [11,12]. The first and second K-Banhatti indices of $G$ are defined as:

$$B_1(G) = \sum_{ue} [d_G(u) + d_G(e)],$$

and

$$B_2(G) = \sum_{ue} [d_G(u) \times d_G(e)],$$

where $ue$ means that the vertex $u$ and edge $e$ are incident in $G$. The first and second K-hyper Banhatti indices of $G$ are defined as

$$HB_1(G) = \sum_{ue} [d_G(u) + d_G(e)]^2,$$

and

$$HB_2(G) = \sum_{ue} [d_G(u) \times d_G(e)]^2.$$ 

We refer [13] for details about these indices. The David derived and dominating David derived network of dimension $n$ can be constructed as follows [14]: consider a $n$ dimensional star of David network. Insert a new vertex on each edge and split it into two parts, we will get David derived network $DD(n)$ of dimension $n$. 

Figure 1.

Dominating David derived network of the first type $D_1(2)$
The dominating David derived network of the first type of dimension $n$ which can be obtained by connecting vertices of degree 2 of $DDD(n)$ by an edge that are not in the boundary and is denoted by $D_1(n)$ [14].

The dominating David derived network of the second type of dimension $n$ can be obtained by subdividing the new edges in $D_1(n)$ [14] and is denoted by $D_1(2)$.

![Figure 2. Dominating David derived network of the second type $D_2(2)$](image)

The dominating David derived network of the third type of dimension $n$ denoted by $D_3(n)$ can be obtained from $D_1(n)$ by introducing a parallel path of length 2 between the vertices of degree two that are not in the boundary [14, 15].

![Figure 3. Dominating David derived network of the third type $D_3(2)$](image)

In this article, we compute first and second K-Banhatti index and first and second hyper K-Banhatti index of Dominating David derived networks of first, second and third type. Throughout this paper $E_{m,n} = \{e = uv \in E(G): d_u = m, d_v = n\}$ and $|E_{m,n}(G)|$ is the number of elements in $E_{m,n}(G)$.

**Theorem 1.1.** Let $G = D_1(n)$ be the dominating David derived network of 1st type. Then the first and the second K-Banhatti indices of $D_1(n)$ are

$$B_1[D_1(n)] = 1485n^2 + 1624n - 1002,$$

$$B_2[D_1(n)] = 3204n^2 + 764n - 3292.$$
Proof. Let \( G = D_1(n) \) be the dominating David derived network of 1\(^{st} \) type. From figure 1, the edge partition of dominating David derived network of 1\(^{st} \) type \( D_1(n) \) based on degrees of end vertices of each edge is give in table 1. First

Table 1. Edge partition of Dominating David derived network of first type

| \((d_u, d_v); e = uv \in E(G)\) | Number of edges | Degree of Edges | \(d_G(e) = d_G(u) + d_G(v) - 2\) |
|-----------------------------|----------------|----------------|---------------------------------|
| \((2, 2)\)                 | \(4n\)          | \(2\)          |                                 |
| \((2, 3)\)                 | \(4n - 4\)      | \(3\)          |                                 |
| \((2, 4)\)                 | \(28n - 16\)    | \(4\)          |                                 |
| \((3, 3)\)                 | \(9n^2 - 13n + 24\) | \(4\)          |                                 |
| \((3, 4)\)                 | \(36n^2 - 56n + 24\) | \(5\)          |                                 |
| \((4, 4)\)                 | \(36n^2 - 56n + 20\) | \(6\)          |                                 |

K-Banhatti index of \( D_1(n) \) is calculated as

\[
B_1[D_1(n)] = \sum_{ue} [d_G(u) + d_G(e)] \\
= \sum_{ue \in E_{2,2}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
+ \sum_{ue \in E_{2,3}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
+ \sum_{ue \in E_{2,4}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
+ \sum_{ue \in E_{3,3}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
+ \sum_{ue \in E_{3,4}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
+ \sum_{ue \in E_{4,4}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
= 4n[(2 + 2) + (2 + 2)] + (4n - 4)[(2 + 3) + (3 + 3)] \\
+ (28n - 16)[(2 + 4) + (4 + 4)] \\
+ (9n^2 - 13n + 24)[(3 + 4) + (3 + 4)] \\
+ (36n^2 - 56n + 24)[(3 + 5) + (4 + 5)] \\
+ (36n^2 - 56n + 20)[(4 + 6) + (4 + 6)] \\
= 1458n^2 + 1624n - 1002.
\]
Second K-Banhatti index of $D_1(n)$ is calculated as

$$B_2[D_1(n)] = \sum_{ue} [d_G(u)d_G(v)]$$

$$+ \sum_{ue \in E_{2,2}} [(d_G(u)d_G(e)) + (d_G(v)d_G(e))]$$

$$+ \sum_{ue \in E_{2,3}} [(d_G(u)d_G(e)) + (d_G(v)d_G(e))]$$

$$+ \sum_{ue \in E_{2,4}} [(d_G(u)d_G(e)) + (d_G(v)d_G(e))]$$

$$+ \sum_{ue \in E_{3,3}} [(d_G(u)d_G(e)) + (d_G(v)d_G(e))]$$

$$+ \sum_{ue \in E_{3,4}} [(d_G(u)d_G(e)) + (d_G(v)d_G(e))]$$

$$+ \sum_{ue \in E_{4,4}} [(d_G(u)d_G(e)) + (d_G(v)d_G(e))]$$

$$= 4n[(2 + 2) + (2 \times 2)] + (4n - 4)[(2 + 3) + (3 \times 3)]$$

$$+ (28n - 16)[(2 \times 4) + (4 \times 4)]$$

$$+ (9n^2 - 13n + 24)[(3 \times 4) + (4 \times 4)]$$

$$+ (36n^2 - 56n + 24)[(3 \times 5) + (4 \times 5)]$$

$$+ (36n^2 - 56n + 20)[(4 \times 6) + (4 \times 6)]$$

$$= 3204n^2 + 764n - 3292.$$
Second K-hyper Banhatti index of $D_1(n)$ is calculated as

$$ HB_2[D_1(n)] = \sum_{ue \in E_{2,2}} [(d_G(u) + d_G(e))^2 + (d_G(v) + d_G(e))^2] $$

$$ + \sum_{ue \in E_{2,3}} [(d_G(u) + d_G(e))^2 + (d_G(v) + d_G(e))^2] $$

$$ + \sum_{ue \in E_{2,4}} [(d_G(u) + d_G(e))^2 + (d_G(v) + d_G(e))^2] $$

$$ + \sum_{ue \in E_{3,3}} [(d_G(u) + d_G(e))^2 + (d_G(v) + d_G(e))^2] $$

$$ + \sum_{ue \in E_{3,4}} [(d_G(u) + d_G(e))^2 + (d_G(v) + d_G(e))^2] $$

$$ + \sum_{ue \in E_{4,4}} [(d_G(u) + d_G(e))^2 + (d_G(v) + d_G(e))^2] $$

$$ = 4n[(2 + 2)^2 + (2 + 2)^2] + (4n - 4)[(2 + 3)^2 + (3 + 3)^2] $$

$$ + (28n - 16)[(2 + 4)^2 + (4 + 4)^2] $$

$$ + (9n^2 - 13n + 24)[(3 + 4)^2 + (3 + 4)^2] $$

$$ + (36n^2 - 56n + 24)[(3 + 5)^2 + (4 + 5)^2] $$

$$ + (36n^2 - 56n + 20)[(4 + 6)^2 + (4 + 6)^2] $$

$$ = 66564n^2 - 89092n + 33892. $$
Theorem 1.3. Let \( G = D_2(n) \) be the dominating David derived network of \( 2^{nd} \) type. Then the first and the second K-Banhatti indices of \( D_2(n) \) are

\[
B_1[D_2(n)] = 1530n^2 - 1810n + 650, \\
B_2[D_2(n)] = 32584n^2 - 4127n + 1506.
\]

Proof. Let \( G = D_2(n) \) be the dominating David derived network of \( 2^{nd} \) type. Table 2 shows the edge partition of dominating David derived network of \( 2^{nd} \) type \( D_2(n) \) based on degrees of end vertices of each edge.

Table 2. Edge partition of Dominating David Derived Network of second type

| \((d_u, d_v); e = uv \in E(G)\) | Number of edges | Degree of Edges \(d_G(e) = d_G(u) + d_G(v) - 2\) |
|---------------------------------|-----------------|----------------------------------|
| \((2, 2)\)                     | \(4n\)          | 2                                |
| \((2, 3)\)                     | \(18n^2 - 22n + 6\) | 3                                |
| \((2, 4)\)                     | \(28n - 16\)    | 4                                |
| \((3, 4)\)                     | \(36n^6 - 56n + 24\) | 5                                |
| \((4, 4)\)                     | \(36n^6 - 56n + 20\) | 6                                |

First K-Banhatti index of \( D_2(n) \) is calculated as

\[
B_1[D_2(n)] = \sum_{ue} [d_G(u) + d_G(v)] \\
= \sum_{ue \in E_{2,2}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
+ \sum_{ue \in E_{2,3}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
+ \sum_{ue \in E_{2,4}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
+ \sum_{ue \in E_{3,4}} [(d_G(u) + d_G(e)) + (d_G(v) + d_G(e))] \\
= 4n[(2 + 2) + (2 + 2)] + (18n^2 - 22n + 6)[(2 + 3) + (3 + 3)] \\
+ (28n - 16)[(2 + 4) + (4 + 4)] \\
+ (36n^6 - 56n + 24)[(3 + 5) + (4 + 5)] \\
+ (36n^6 - 56n + 20)[(4 + 6) + (4 + 6)] \\
= 1530n^2 - 1810n + 650.
\]
Second K-Banhatti index of $D_2(n)$ is calculated as

$$B_1[D_2(n)] = \sum_{ue} [d_G(u)d_G(v)]$$

$$= \sum_{ue \in E_{2,2}} [(d_G(u)d_G(e)) + (d_G(v)d_G(e))]$$

$$+ \sum_{ue \in E_{2,3}} [(d_G(u)d_G(e)) + (d_G(v)d_G(e))]$$

$$+ \sum_{ue \in E_{2,4}} [(d_G(u)d_G(e)) + (d_G(v)d_G(e))]$$

$$+ \sum_{ue \in E_{3,4}} [(d_G(u)d_G(e)) + (d_G(v)d_G(e))]$$

$$= 4n[(2 \times 2) + (2 \times 2)] + (18n^2 - 22n + 6)(2 \times 3) + (3 \times 3)$$

$$+ (28n - 16)(2 \times 4) + (4 \times 4)$$

$$+ (36n^6 - 56n + 24)(3 \times 5) + (4 \times 5)$$

$$+ (36n^6 - 56n + 20)(4 \times 6) + (4 \times 6)$$

$$= 32584n^2 - 4127n + 1506.$$
K-Banhatti and K-hyper Banhatti indices of Dominating David Derived network

\[\sum_{ue \in E_{4,4}} [(d_G(u) + d_G(e))^2 + (d_G(v) + d_G(e))^2] + \sum_{ue \in E_{4,3}} [(d_G(u) + d_G(e))^2 + (d_G(v) + d_G(e))^2] + \sum_{ue \in E_{4,2}} [(d_G(u) + d_G(e))^2 + (d_G(v) + d_G(e))^2] + \sum_{ue \in E_{4,1}} [(d_G(u) + d_G(e))^2 + (d_G(v) + d_G(e))^2]
\]

\[= 4n[(2 + 2)^2 + (2 + 2)^2 + (18n^2 - 22n + 6)(2 + 3)^2 + (3 + 3)^2] + (28n - 16)[(2 + 4)^2 + (4 + 4)^2]
+ (36n^6 - 56n + 24)[(3 + 5)^2 + (4 + 5)^2] + (36n^6 - 56n + 20)(4 + 6)^2 + (4 + 6)^2]
\]

\[= 1351n^2 - 1693n + 6246.\]

Second K-hyper Banhatti index of \(D_2(n)\) is calculated as

\[HB_2[D_2(n)] = \sum_{ue} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2]
\]

\[= \sum_{ue \in E_{2,2}} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2] + \sum_{ue \in E_{2,3}} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2] + \sum_{ue \in E_{2,3}} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2] + \sum_{ue \in E_{2,4}} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2] + \sum_{ue \in E_{2,4}} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2]
\]

\[= 4n[(2 \times 2)^2 + (2 \times 2)^2 + (18n^2 - 22n + 6)(2 \times 3)^2 + (3 \times 3)^2] + (28n - 16)[(2 \times 4)^2 + (4 \times 4)^2]
+ (36n^6 - 56n + 24)[(3 \times 5)^2 + (4 \times 5)^2] + (36n^6 - 56n + 20)(4 \times 6)^2 + (4 \times 6)^2]
\]

\[= 22606n^2 - 28486n + 10582.\]

\[\square\]

**Theorem 1.5.** Let \(G = D_3(n)\) be the dominating David derived network of 3\(^rd\) type. Then the first and the second K-Banhatti indices of \(D_3(n)\) are

\[B_1[D_3(n)] = 1944n^2 - 2128n + 600,\]

\[B_2[D_3(n)] = 4320n^2 - 8224n + 2112.\]

**Proof.** Let \(G = D_3(n)\) be the dominating David derived network of 3\(^rd\) type. Table 3 shows the edge partition of dominating David derived network of 3\(^rd\) type \(D_3(n)\) based on degrees of end vertices of each edge. First K-Banhatti index of \(D_3(n)\) is calculated as

\[B_1[D_3(n)] = \sum_{ue} [d_G(u) + d_G(v)]\]
Table 3. Edge partition of Dominating David derived network of third type

| $(d_u, d_v); e = uv \in E(G)$ | Number of edges | Degree of Edges $d_G(e) = d_G(u) + d_G(v) - 2$ |
|-------------------------------|----------------|----------------------------------|
| $(2, 2)$                      | $4n$           | $2$                              |
| $(2, 4)$                      | $36n^2 - 20n$  | $4$                              |
| $(4, 4)$                      | $72n^2 - 108n + 44$ | $6$                              |

\[
= \sum_{ue \in E_{2,2}} [(d_G(u) + d_G(v)) + (d_G(v) + d_G(e))] \\
+ \sum_{ue \in E_{2,4}} [(d_G(u) + d_G(v)) + (d_G(v) + d_G(e))] \\
+ \sum_{ue \in E_{4,4}} [(d_G(u) + d_G(v)) + (d_G(v) + d_G(e))] \\
= 4n[(2 + 2) + (2 + 2)] + (36n^2 - 20n)[(2 + 4) + (4 + 4)] \\
+ (72n^2 - 108n + 44)[(2 + 6) + (4 + 6)] \\
= 1944n^2 - 2128n + 600.
\]

Second K-Banhatti index is calculated as

\[
B_2[D_3(n)] = \sum_{ue} |d_G(u) + d_G(v)| \\
= \sum_{ue \in E_{2,2}} [(d_G(u)d_G(v)) + (d_G(v)d_G(e))] \\
+ \sum_{ue \in E_{2,4}} [(d_G(u)d_G(v)) + (d_G(v)d_G(e))] \\
+ \sum_{ue \in E_{4,4}} [(d_G(u)d_G(v)) + (d_G(v)d_G(e))] \\
= 4n[(2 \times 2) + (2 \times 2)] + (36n^2 - 20n)[(2 \times 4) + (4 \times 4)] \\
+ (72n^2 - 108n + 44)[(2 \times 6) + (4 \times 6)] \\
= 4320n^2 - 8224n + 2112.
\]

\[\square\]

**Theorem 1.6.** Let $G = D_3(n)$ be the dominating David derived network of 3rd type. Then the first and the second K-hyper Banhatti indices of $D_3(n)$ are

\[
HB_1[D_3(n)] = 18000n^2 - 23472n + 8800, \\
HB_2[D_3(n)] = 94464n^2 - 130688n + 50688.
\]
Proof. Let $G = D_3(n)$ be the dominating David derived network of $3^{rd}$ type. Then the first K-hyper Banhatti index is calculated as

$$HB_1[D_3(n)] = \sum_{ue} (d_G(u) + d_G(v))^2$$

$$= \sum_{ue \in E_{2,2}} [(d_G(u) + d_G(e))^2 + (d_G(v) + d_G(e))^2]$$

$$+ \sum_{ue \in E_{2,4}} [(d_G(u) + d_G(e))^2 + (d_G(v) + d_G(e))^2]$$

$$+ \sum_{ue \in E_{4,4}} [(d_G(u) + d_G(e))^2 + (d_G(v) + d_G(e))^2]$$

$$= 4n[(2 + 2)^2 + (2 + 2)^2] + (36n^2 - 20n)(2 + 4)^2 + (4 + 4)^2]$$

$$+ (72n^2 - 108n + 44)(2 + 6)^2 + (4 + 6)^2]$$

$$= 18000n^2 - 23472n + 8800.$$

Second K-hyper Banhatti index of $D_3(n)$ is calculated as

$$HB_1[D_3(n)] = \sum_{ue} (d_G(u) + d_G(v))^2$$

$$= \sum_{ue \in E_{2,2}} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2]$$

$$+ \sum_{ue \in E_{2,4}} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2]$$

$$+ \sum_{ue \in E_{4,4}} [(d_G(u)d_G(e))^2 + (d_G(v)d_G(e))^2]$$

$$= 4n[(2 \times 2)^2 + (2 \times 2)^2] + (36n^2 - 20n)(2 \times 4)^2 + (4 \times 4)^2]$$

$$+ (72n^2 - 108n + 44)(2 \times 6)^2 + (4 \times 6)^2]$$

$$= 94464n^2 - 130688n + 50688.$$

\[\square\]

2. Conclusion

In the present report, we have computed First and second K-Banhatti and K-hyper Banhatti indices of Dominating David derived networks of first, second and third type.

Competing Interests

The author do not have any competing interests in the manuscript.
1. West, D. B. (2001). Introduction to graph theory (Vol. 2). Upper Saddle River: Prentice Hall.
2. Rcker, G., & Rcker, C. (1999). On topological indices, boiling points, and cycloalkanes. Journal of chemical information and computer sciences, 39(5), 788-802.
3. Klavar, S., & Gutman, I. (1996). A comparison of the Schultz molecular topological index with the Wiener index. Journal of chemical information and computer sciences, 36(5), 1001-1003.
4. Wiener, H. (1947). Structural determination of paraffin boiling points. Journal of the American Chemical Society, 69(1), 17-20.
5. Dobrynin, A. A., Entringer, R., & Gutman, I. (2001). Wiener index of trees: theory and applications. Acta Applicandae Mathematicae, 66(3), 211-249.
6. Gutman, I., & Polansky, O. E. (2012). Mathematical concepts in organic chemistry. Springer Science & Business Media.
7. Randić, M. (1975). Characterization of molecular branching. Journal of the American Chemical Society, 97(23), 6609-6615.
8. Bollobás, B., & Erds, P. (1998). Graphs of extremal weights. Ars Combinatoria, 50, 225-233.
9. Amic, D., Belo, D., Luc?ic, B., Nikolic, S., & Trinajstic, N. (1998). The vertex-connectivity index revisited. Journal of chemical information and computer sciences, 38(5), 819-822.
10. Hu, Y., Li, X., Shi, Y., Xu, T., & Gutman, I. (2005). On molecular graphs with smallest and greatest zeroth-order general Randić index. MATCH Commun. Math. Comput. Chem, 54(2), 425-434.
11. Sardar, M. S, Zafar, S., & Farahani, M. R. (2017). the generalized zagreb index of capra-designed planar benzenoid series $ca_b (c_0)$, Open J. Math. Sci., 1(1), 44-51.
12. Rehman, H. M, Sardar, R., Raza, A. (2017). computing topological indices of hex board and its line graph. Open J. Math. Sci., 1(1), 62-71.
13. Kulli, V. R., Chalvaraju, B., & Boregowda, H. S. (2017). Connectivity Banhatti indices for certain families of benzenoid systems. Journal of Ultra Chemistry, 13(4), 81-87.
14. Imran, M., Baig, A. Q., & Ali, H. (2015). On topological properties of dominating David derived networks. Canadian Journal of Chemistry, 94(2), 137-148.
15. Simonraj, F., & George, A. (2012). Embedding of poly honeycomb networks and the metric dimension of star of david network. International Journal on Applications of Graph Theory in Wireless Ad Hoc Networks and Sensor Networks, 4(4), 11.

Wei Gao
School of Information Science and Technology,
Yunnan Normal University, Kunming 650500, China.
e-mail: gaweijingy@ynnu.edu.cn

Batshá Muzaffar
Department of Mathematics and Statistics,
University of Lahore Lahore 54590, Pakistan.
e-mail: batshamuzaffar41@gmail.com

Waqas Nazeer
Division of Science and Technology,
University of Education, Lahore-54590, Pakistan.
e-mail: nazeer.waqas@ue.edu.pk