Holographic complexity of ‘black’ non-susy $D3$-brane and the high temperature limit

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Abstract

The holographic complexity of a ‘black’ non-susy $D3$-brane is computed. The difference in the holographic complexity between this geometry and that of the $AdS_5$ geometry is obtained. This is then related to the changes in the energy and the entanglement entropy of the system. We next take the high temperature limit of the change in complexity and observe that it scales with the temperature in the same way as the holographic entanglement entropy. The crossover of the holographic complexity to its corresponding thermal counterpart is similar to the corresponding crossover of the holographic entanglement entropy in the high temperature limit. We further repeat the analysis for $\mathcal{N} = 4$ super Yang-Mills theory and observe a similar behaviour.

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1 Introduction

The computation of entanglement entropy (EE) in conformal field theories [1]-[7] from a bulk gravity theory (which is asymptotically $AdS$) using the $AdS/CFT$ duality [8, 9] has been an area of active research in recent times. The computation of EE holographically is carried out using a formula given in [10, 11]. The formula enunciates that the holographic EE (HEE) of a subsystem $A$ in the gravity dual reads

$$S_E = \frac{\text{Area}(\gamma_A^{min})}{4G_{(N)}}$$

(1)

where $\gamma_A^{min}$ is the $d$-dimensional minimal area surface in AdS$_{d+2}$ spacetime, the boundary of which matches with the boundary of the subsystem $A$ and $G_{(N)}$ is the $(d+2)$-dimensional Newton’s constant.

Interestingly it has been shown that the HEE for a very small subsystem satisfies a relation which looks similar to the first law of thermodynamics when the system is excited [12, 13]. A similar result have also been obtained in [14] where a $(2 + 1)$-dimensional quantum many body system (exhibiting a Lifshitz symmetry [15] near their quantum critical point) is described by a $(3 + 1)$-dimensional dual gravity theory with a negative cosmological constant together with a massive vector field. It was shown that there exists an additional term in the relation analogous to the first law of thermodynamics which owed its origin to the presence of the massive gauge field.

Quantum complexity is yet another important quantity in the field of quantum information which has provided deep insights in the understanding of the properties of horizons of black holes. It has been realised that there may be a possible relation between complexity and fidelity susceptibility between two states appearing in the literature of quantum information [16, 17]. Based on the prescription provided to compute the EE holographically, one may define holographic complexity (HC) following the proposal in [18, 19]

$$C_V = \frac{V(\gamma)}{8\pi RG_{(N)}}$$

(2)

where $R$ is the radius of curvature of the spacetime and $V(\gamma)$ is the volume of the part in the bulk geometry enclosed by the minimal hyper surface involved in the calculation of the HEE. Computation of holographic complexity have also been made recently in [20] in the case of a $(3 + 1)$-dimensional Lifshitz spacetime and the change in complexity has been related to the change in energy, change in EE and change in entanglement chemical potential of the system.

In [21]-[23], a ‘black’ non-susy $D3$-brane has been considered. This solution is known to have a decoupling limit [24] which in turn leads to a gravity dual picture. The gravity dual description is exploited to calculate the EE of the quantum field theory associated with it. The asymptotically $AdS_5$ geometry of the ‘black’ non-susy $D3$-brane in the decoupling limit allows one to write the HEE as a sum of two parts, namely, the HEE associated with the pure $AdS_5$ piece plus an additional piece which can be interpreted
as the HEE of an excited state. The boundary stress tensor was next identified \[25, 26\] using which it was shown that the HEE of the excited state satisfied the relation similar in form to the first law of thermodynamics. It was further demonstrated that at high temperature, the HEE makes a transition to the thermal entropy of the usual ‘black’ D3-brane.

Recently, there has been a study investigating the relation between the HC, HEE and fidelity susceptibility for a spherical shell of D3-branes \[27\]. This provides a motivation to carry out the same investigation for the ‘black’ non-susy D3-brane. In this paper, we obtain the change in complexity of the ‘black’ non-susy D3-brane. We then relate it to the change in HEE and the entanglement temperature. The high temperature limit of this is taken and is found to scale with the temperature in the same way as the HEE does thereby indicating a similar crossover of HC to its corresponding thermal counterpart. The analysis is repeated for \( \mathcal{N} = 4 \) super Yang-Mills theory \[28, 29\] and a similar behaviour is obtained.

The paper is organised as follows. In section 2, we discuss the holographic complexity for ‘black’ non-susy D3-brane. In section 3, we discuss the high temperature limit of the complexity of the ‘black’ non-susy D3-brane and \( \mathcal{N} = 4 \) super Yang-Mills theory. We conclude in section 4.

## 2 Holographic complexity for ‘black’ non-susy D3-brane

In this section, we shall first fix our notations by reviewing the computation of HC for a strip in AdS\(_5\) space. The AdS\(_5\) metric can be written in Poincaré coordinates as follows

\[
ds^2 = \frac{r_1^2}{z^2} \left(-dt^2 + \sum_{i=1}^{3} (dx^i)^2 + dz^2\right)
\]  

(3)

where \(r_1\) is the radius of curvature of the AdS space. We shall now compute the HC by calculating the volume enclosed by the minimal Ryu-Takayanagi (RT) surface \[10, 11\]. We consider the entangling region in the boundary to be a straight belt with width \(\ell\) such that \(-\ell/2 \leq x^1 \leq \ell/2\) and \(0 \leq x^2, x^3 \leq L\), where \(L\) is the extent of the subsystem in the other spatial direction. We parametrize the extremal surface by \(x^1 = x^1(z)\). With this setup, the volume enclosed by the RT minimal surface reads \[18, 19\]

\[
V^{(0)} = 2r_1^4 L^2 \int_{\epsilon}^{z_i^{(0)}} dz \frac{1}{z^4} x_1(z)
\]

(4)

where \(z_i^{(0)}\) is the turning point at which \(dz/dx^1\) vanishes and \(\epsilon\) is the cut-off introduced to prevent the integral from diverging at \(z = 0\). The profile of the minimal surface \(x^1(z)\)
by minimizing the RT area functional considering $z = z(x^1)$. The RT area functional for the metric (3) is given by

$$A(0) = \int_0^L dx_2 \int_0^L dx_3 \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} dx_1 r_1^3 \sqrt{1 + z'(x_1)^2}$$

which in turn gives the minimal surface profile and the width of the strip as follows

$$x_1(z) = \int_z^{z(0)} dz \frac{1}{\sqrt{\left(\frac{z(0)}{z}\right)^6 - 1}}, \quad \frac{\ell}{2} = \frac{\sqrt{\pi} \Gamma\left(\frac{2}{3}\right)}{\Gamma\left(\frac{1}{6}\right)} z(0).$$

Using the above expression for $x_1(z)$ in eq.(4), we obtain

$$V(0) = 2L^2 r_1^4 \int_{\epsilon}^{z(0)} dz \frac{1}{z^4} \int_z^{z(0)} dy \frac{1}{\sqrt{\left(\frac{z(0)}{y}\right)^6 - 1}}$$

$$= \frac{L^2 r_1^4 \ell}{3 \epsilon^3} - \frac{4\pi^2 \Gamma\left(\frac{2}{3}\right)}{9 \Gamma\left(\frac{1}{6}\right) \ell^2}.$$

Therefore, the HC for the $AdS_5$ geometry reads [30, 31]

$$C_V = \frac{V(0)}{8\pi r_1 G(5)}$$

$$= \frac{L^2 r_1^3 \ell}{24\pi G(5) \epsilon^3} - \frac{\sqrt{\pi} \Gamma\left(\frac{2}{3}\right) L^2 r_1^3}{18G(5) \Gamma\left(\frac{1}{6}\right) \ell^2}.$$

With this result in hand, we now look at the ‘black’ non-susy $D3$-brane solution of type IIB string theory in the Einstein frame. The metric of this takes the form [21]-[24]

$$ds^2 = F_1(r)^{-\frac{1}{2}} G(r)^{-\frac{d_4}{2}} \left[-G(r)^{\frac{d_4}{2}} dt^2 + \sum_{i=1}^{3} (dx^i)^2 \right] + F_1(r)^{\frac{1}{2}} G(r)^{\frac{1}{2}} \left[\frac{dr^2}{G(r)} + r^2 d\Omega_5^2 \right]$$

$$e^{2\phi} = G(r)^{-\frac{4d_3}{2} + \frac{1}{4} r_1^2}.$$

$$F_{[5]} = \frac{1}{\sqrt{2}}(1 + *)Q\text{Vol}(\Omega_5)$$

where the functions $G(r)$ and $F(r)$ are defined as

$$G(r) = 1 + \frac{r_0^4}{r^4}, \quad F_1(r) = G(r)^{1/2} \cosh^2 \theta - G(r)^{-1/2} \sinh^2 \theta.$$
δ1, δ2, α1, β1, θ, r0, Q are the parameters characterizing the solution. It is to be noted that the parameters are not all independent but they satisfy the following relations

\[\begin{align*}
\alpha_1 - \beta_1 &= 0 \\
\alpha_1 + \beta_1 &= \sqrt{10 - \frac{21}{2}\delta_2^2 - \frac{49}{2}\delta_1^2 + 21\delta_2\delta_1} \\
Q &= (\alpha_1 + \beta_1)r_0^4\sinh 2\theta.
\end{align*}\] (11)

For δ2 = −2 and δ1 = −\frac{12}{7}, the solution (9) reduces to black D3-brane solution. From now on we will put α1 + β1 = 2 for simplicity. Therefore, from the first relation in eq.(11), we have α1 = 1 and β1 = 1. In this case, the parameters δ1 and δ2 will be related (see the second equation in eq.(11)) by

\[42\delta_2^2 + 49\delta_1^2 - 84\delta_1\delta_2 = 24.\] (12)

The function \(F_1(r)\) given in eq.(10) then reduces to

\[F_1(r) = G(r)^{-\frac{7}{8}}H(r)\] (13)

where

\[H(r) = 1 + \frac{r_1^4\cosh^2\theta}{r^4} \equiv 1 + \frac{r_1^4}{r^4}.\] (14)

Using this form of \(F_1(r)\) in eq.(9) leads to the metric in the Einstein frame to be

\[ds^2 = H(r)^{-\frac{7}{8}}G(r)^{\frac{1}{4}}\left[-G(r)^{\frac{7}{8}}dt^2 + \sum_{i=1}^{3}(dx^i)^2\right] + H(r)^{\frac{1}{2}}\left[\frac{dr^2}{G(r)} + r^2d\Omega_5^2\right]\] (15)

where \(H(r)\) is given in (14). It can be shown that the above reduces to [24]

\[ds^2 = \frac{r_1^2}{r^2}G(r)^{\frac{1}{2}}\left[-G(r)^{\frac{7}{8}}dt^2 + \sum_{i=1}^{3}(dx^i)^2\right] + \frac{r_1^2}{r^2}dr^2 + r_1^2d\Omega_5^2\] (16)

where \(r_1 = r_0\cosh^{\frac{1}{2}}\theta\) is the radius of the transverse 5-sphere which decouples from the five dimensional asymptotically AdS5 geometry. This is possible if one looks into the region

\[r \sim r_0 \ll r_0\cosh^{\frac{1}{2}}\theta\] (17)

where \(\theta \to \infty\) and the function \(H(r) \approx r_1^4/r^4\). In Fefferman-Graham coordinates [32], the asymptotic limit of this metric takes the form [21]

\[ds^2 = \frac{r_1^2}{z^2}\left[-\left(1 + \frac{3\delta_2}{8}z^4/\tilde{z}_0^4\right)dt^2 + \left(1 - \frac{\delta_2}{8}z^4/\tilde{z}_0^4\right)\sum_{i=1}^{3}(dx^i)^2 + dz^2\right]\] (18)

where \(z_0^4 = r_1^8/r_0^4\).
With this set up in place, we are now ready to calculate the HC for ‘black’ non-susy D3-brane. It should be remembered that we are interested in the change in complexity due to small perturbation in the background metric. To see the change in complexity due to change in metric perturbation, we keep the length \( \ell \) fixed. Under this condition the expression for volume enclosed by the RT minimal surface for the above metric (16) for the same entangling region reads

\[
V = 2L^2r^4 \int_\epsilon^{z_t(0)} dz \frac{1}{z^4} \left( 1 - \frac{3\delta_2 z^4}{16 z_0} \right) \int_z^{z_t(0)} dy \frac{1}{\sqrt{\left( \frac{z^{(0)}}{y} \right)^6 - 1}}. \tag{19}
\]

Now as we are interested in the region near the boundary, we keep only the first order terms in the perturbation to write eq.(19) as

\[
V = 2L^2r^4 \int_\epsilon^{z_t(0)} dz \frac{1}{z^4} \left( 1 - \frac{3\delta_2 z^4}{16 z_0} \right) \int_z^{z_t(0)} dy \frac{1}{\sqrt{\left( \frac{z^{(0)}}{y} \right)^6 - 1}} = V(0) - \frac{3\delta_2}{8} L^2 r^4 \int_\epsilon^{z_t(0)} dz \frac{1}{z_0} \int_z^{z_t(0)} dy \frac{1}{\sqrt{\left( \frac{z^{(0)}}{y} \right)^6 - 1}}. \tag{20}
\]

where \( V(0) \) is the volume of the \( AdS_5 \) background. Hence the change in the HC due to a small perturbation in the background metric (16) in terms of change in volume \( \Delta V \) is given by [18, 19]

\[
\Delta C = \frac{\Delta V}{8\pi r_1 G(5)} = -\frac{3\delta_2}{512\pi^2 G(5)} \left( \Gamma \left( \frac{1}{4} \right)^2 \Gamma \left( \frac{5}{4} \right) L^2 r^3 \ell^2 \right). \tag{21}
\]

We now proceed to study the relation between the change in complexity with the stress energy tensor. The stress energy tensor for an asymptotically local AdS metric [32]

\[
ds_{d+1}^2 = \frac{r_1^2}{z^2} \left( dz^2 + g_{\mu\nu} dx^\mu dx^\nu \right), \tag{22}
\]

where \( g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x, z) \) with

\[
h_{\mu\nu}(x, z) = h^{(0)}_{\mu\nu}(x) + h^{(2)}_{\mu\nu}(x) z^2 + \cdots + z^d \left( h^{(d)}_{\mu\nu}(x) + \dot{h}^{(d)}_{\mu\nu}(x) \log z \right) + \cdots \tag{23}
\]

reads

\[
\langle T_{\mu\nu} \rangle = \frac{d}{16\pi G(N)} \frac{r^{d-1}}{16\pi G(N)} h^{(d)}_{\mu\nu}. \tag{24}
\]
One can easily see that for the metric (18) the expectation values of the stress energy tensor reads \[ \langle T_{tt} \rangle = -\frac{3r^3 \delta_2}{32\pi G_{(5)} z_0^4} \] \[ \langle T_{x_ix_j} \rangle = -\frac{\rho_1^3 \delta_2}{32\pi G_{(5)} z_0^4} \delta_{ij} \] where \( i, j = 1, 2, 3 \). Now using the fact that the total energy can be recast as

\[ \Delta E = \int_0^L dx_2 \int_0^L dx_3 \int_{-\frac{l}{2}}^{\frac{l}{2}} \langle T_{tt} \rangle = L^2 \ell \langle T_{tt} \rangle \] (27)

the expression for change in complexity (21) can be restated as follows

\[ \Delta C = \frac{3 \Gamma \left( \frac{5}{6} \right)^2 \Delta E}{2\pi \Gamma \left( \frac{1}{3} \right)^2 T_{ent}} \] (28)

where \( T_{ent} \) is the entanglement temperature [13, 21]

\[ T_E = \frac{24 \Gamma \left( \frac{5}{6} \right) \Gamma^2 \left( \frac{2}{3} \right) \Gamma^2 \left( \frac{1}{6} \right)}{\sqrt{\pi} \Gamma \left( \frac{1}{3} \right) \Gamma^2 \left( \frac{1}{3} \right) \ell} \] (29)

Again we know that the decoupled theory of ‘black’ non-susy D3-brane satisfies the first law of entanglement thermodynamics [21]

\[ \Delta E = T_E \Delta S_E + \frac{3}{5} \Delta P_{x_1x_1} V_3 \] (30)

where \( V_3 = L^2 \ell \) is the volume of the subspace and \( \Delta P_{x_1x_1} = \langle T_{x_1x_1} \rangle \) is the entanglement pressure. We can use this first law to recast eq.(28) as

\[ \Delta C = k_1 \Delta S_E + k_2 \frac{\Delta P_{x_1x_1} V_3}{T_{ent}} \] (31)

where the constants \( k_1 \) and \( k_2 \) are given by

\[ k_1 = \frac{3 \Gamma^2 (5/6)}{2\pi \Gamma^2 (1/3)} \] (32)

\[ k_2 = \frac{3}{5} k_1 \] (33)
3 High temperature limit

In this section, we look at the high temperature limit of the finite part of complexity for ‘black’ non-susy $D3$-brane. Setting $z/z_t = y$, the entangling region $\ell$ can be written as

$$\ell = z_t \int_0^1 dy \frac{y^3 \left( 1 - \frac{3\delta y^4}{8 z_0^4} \right)}{\sqrt{\left( 1 - \frac{3\delta y^4}{8 z_0^4} \right) \left[ 1 - y^6 - \frac{3\delta y^4}{8 z_0^4} y^4 \left( 1 - y^2 \right) \right]}}. \quad (34)$$

The total volume is given by

$$V = 2L^2 \int_t^{z_t} dz \int_z^{z_t} dz_1 \frac{r_1^4}{z^4} \left( 1 - \frac{\delta z^4}{8 z_0^4} \right)^{3/2} \sqrt{1 - \frac{3\delta z_1^4}{8 z_0^4}} \frac{1}{\sqrt{\left( 1 - \frac{3\delta z_1^4}{8 z_0^4} \right) \left[ z_0^6 - 1 - \frac{3\delta z_1^4}{8 z_0^4} \left( \frac{z_1^2}{r_1^2} - 1 \right) \right]}}. \quad (35)$$

This expression for volume is divergent and we are interested only in the behaviour of the finite part of the volume. The finite part of the volume in this case is given by

$$V_{\text{finite}} = \frac{2L^2r_1^4}{3z_t^2} \int_0^1 dy \frac{2F_1(-0.75, -1.5, 0.25, -\frac{\delta z^4 y^4}{8 z_0^4}) \sqrt{1 - \frac{3\delta z_1^4}{8 z_0^4}}}{y^3 \sqrt{\left( 1 - \frac{3\delta z_1^4 y^4}{8 z_0^4} \right) \left[ 1 - y^6 - \frac{3\delta z_1^4}{8 z_0^4} x^4 \left( 1 - x^2 \right) \right]}}. \quad (36)$$

where $2F_1$ is the hypergeometric function. Now in the high temperature limit $z_t \to z_0$, using eq. (34) one can recast $C_{\text{finite}}$ as

$$C_{\text{finite}} = \frac{V_{\text{finite}}}{8\pi r_1 G(5)} \sim \frac{2F_1(-0.75, -1.5, 0.25, -\frac{\delta z^4 y^4}{8 z_0^4}) L^2 r_1^3}{G(5) z_t^3}. \quad (37)$$

Now using the five dimensional Newton’s constant $G(5) = (\pi r_1^3)/(2N^2)$, where $N$ is the number of branes and $1/(\pi z_0) = T$, where $T$ is the temperature of the standard black $D3$-brane, we get from (37)

$$\frac{C_{\text{finite}}}{V_3} = 2F_1(-0.75, -1.5, 0.25, -\frac{\delta z^4}{8}) \frac{\pi}{12} N^2 T^3. \quad (38)$$

To conclude this section, we look at the high temperature behaviour of the AdS black hole geometry in this case dual to the familiar $\mathcal{N} = 4$ super Yang-Mills theory. The AdS black hole geometry is given by [28, 29]

$$ds^2 = r_1^2 \left[ \frac{du^2}{h u^2} + u^2 (-h dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + d\Omega_5^2) \right]. \quad (39)$$
with \( h = 1 - \frac{u_4}{u_3} \) and \( u_0 = \pi T \), where \( T \) is the temperature of the black hole. We now compute the complexity for a straight belt with width \( \ell( -\frac{\ell}{2} \leq x_1 \leq \frac{\ell}{2} ; 0 \leq x_2, x_3 \leq L) \). The length of the strip in this case is

\[
\frac{\ell}{2} = \frac{1}{u_t} \int_1^{\infty} dy \frac{1}{\sqrt{(y^4 - \frac{u_3^4}{u_1^4}) (y^6 - 1)}}. \tag{40}
\]

The total volume enclosed by the Ryu-Takayanagi surface is

\[
V = 2L^2 r_1^4 \int_{u_t}^{\infty} du \frac{u^4}{\sqrt{u^4 - u_0^4}} \int_{u_t}^{u} du_1 \frac{1}{\sqrt{(u^4 - u_0^4) \left( \frac{u_3^4}{u_1^4} - 1 \right)}}. \tag{41}
\]

The finite part of this volume is given by

\[
V_{\text{finite}} = \frac{2}{3} L^2 r_1^4 u_0^2 \int_1^{\infty} dy \frac{2 F_1(-0.75, 0.5, 0.25, \frac{u_3^4}{y^4 u_1^4}) y^3}{\sqrt{(y^4 - \frac{u_3^4}{u_1^4}) (y^6 - 1)}}. \tag{42}
\]

Now in the high temperature limit \( u_t \to u_0 \), using eq.\,(40) we can compute the volume to be

\[
V_{\text{finite}} = \frac{1}{3} 2 F_1(-0.75, 0.5, 0.25, 1) L^2 \ell r_1^4 u_0^3. \tag{43}
\]

Therefore, the finite part of the complexity for this case is given by

\[
\frac{C_{\text{finite}}}{V_3} = 2 F_1(-0.75, 0.5, 0.25, 1) \frac{\pi}{12} N^2 T^3. \tag{44}
\]

It is interesting to note that the high temperature limit of the complexity in both cases scales as \( T^3 \) which is the same scaling behaviour which the HEE exhibits in 5-dimensional bulk theory \[33\].

### 4 Conclusions

In this paper, we compute the holographic complexity of a ‘black’ non-susy D3-brane. From this we obtain the difference in holographic complexity between this geometry and the AdS\(_5\) spacetime. The change in the holographic complexity is then expressed in terms of the changes in the energy and the entanglement entropy of the system. The high temperature limit of this is computed next. It is observed that it scales with the temperature in the same way as the holographic entanglement entropy does \[21\] which in turn displays a similar crossover of holographic complexity to its corresponding thermal counterpart. The analysis is then carried out for \( \mathcal{N} = 4 \) super Yang-Mills theory and a similar behaviour is obtained once again. Further in depth analysis may be required to ascertain if this behaviour of similar crossover for holographic complexity and holographic entanglement entropy is a universal feature in 5-dimensional spacetime geometries.
Acknowledgements

S.G. acknowledges the support by DST SERB under Start Up Research Grant (Young Scientist), File No.YSS/2014/000180. S.G. also acknowledges the support of the Visiting Associateship programme of IUCAA, Pune.

References

[1] L. Bombelli, R. K. Koul, J. Lee and R. D. Sorkin, “A Quantum Source of Entropy for Black Holes”, Phys. Rev. D 34 (1986) 373.

[2] M. Srednicki, “Entropy and area”, Phys. Rev. Lett. 71 (1993) 666.

[3] C. Holzhey, F. Larsen and F. Wilczek, “Geometric and renormalized entropy in conformal field theory”, Nucl. Phys. B 424 (1994) 443.

[4] P. Calabrese and J. L. Cardy, “Entanglement entropy and quantum field theory”, J. Stat. Mech. 0406 (2004) P06002.

[5] P. Calabrese and J. L. Cardy, “Entanglement entropy and quantum field theory: A Non-technical introduction”, Int. J. Quant. Inf. 4 (2006) 429.

[6] P. Calabrese and J. Cardy, “Entanglement entropy and conformal field theory”, J. Phys. A 42 (2009) 504005.

[7] J. Eisert, M. Cramer and M. B. Plenio, “Area laws for the entanglement entropy - a review”, Rev. Mod. Phys. 82 (2010) 277.

[8] J. M. Maldacena, “The Large N limit of superconformal field theories and supergravity”, Int. J. Theor. Phys. 38 (1999) 1113.

[9] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, “Large N field theories, string theory and gravity”, Phys. Rept. 323 (2000) 183.

[10] S. Ryu and T. Takayanagi, “Holographic derivation of entanglement entropy from AdS/CFT”, Phys. Rev. Lett. 96 (2006) 181602.

[11] S. Ryu and T. Takayanagi, “Aspects of Holographic Entanglement Entropy”, JHEP 0608 (2006) 045.

[12] J. Bhattacharyya, M. Nozaki, T. Takayanagi and T. Ugajin, “Thermodynamical Property of Entanglement Entropy for Excited States”, Phys. Rev. Lett. 110 (2013) 091602.

[13] D. Allahbakhshi, M. Alishahiha and A. Naseh, “Entanglement Thermodynamics”, JHEP 1308 (2013) 102.
[14] S. Chakraborty, P. Dey, S. Karar, S. Roy, “Entanglement thermodynamics for an excited state of Lifshitz system”, JHEP 04 (2015) 133.

[15] S.F. Ross and O. Saremi, “Holographic stress tensor for non-relativistic theories”, JHEP 0909 (2009) 009.

[16] M. Alishahiha, “Holographic complexity”, Phys. Rev. D 92 (2015) 126009.

[17] Mohsen Alishahiha, Amin Faraji Astaneh, “Holographic Fidelity Susceptibility”, Phys. Rev. D 96 (2017) 086004.

[18] L. Susskind, “Computational Complexity and Black Hole Horizons”, Fortsch.Phys. 64 (2016) 24, Addendum: Fortsch.Phys. 64 (2016) 44-48.

[19] D. Stanford, L. Susskind, “Complexity and Shock Wave Geometries”, Phys. Rev. D 90 (2014) 126007.

[20] Sourav Karar, Sunandan Gangopadhyay, “Holographic complexity for Lifshitz system”, arXiv: 1711.10887 [hep-th].

[21] Aranya Bhattacharya and Shibaji Roy, “Holographic entanglement entropy and entanglement thermodynamics of ‘black’ non-susy D3 brane”, arXiv: 1712.03740 [hep-th].

[22] J. X. Lu, Shibaji Roy, Zhao-Long Wang, R. J. Wu, “Intersecting non-SUSY branes and closed string tachyon condensation”, Nucl. Phys. B 813 (2009) 259.

[23] Somdeb Chakraborty, Kuntal Nayek and Shibaji Roy, “Wilson loop calculation in QGP using non-supersymmetric AdS/CFT”, arXiv: 1710.08631 [hep-th].

[24] Kuntal Nayek, Shibaji Roy, “Decoupling limit and throat geometry of non-susy D3 brane”, Phys. Lett. B 766 (2017) 192.

[25] Vijay Balasubramanian and Per Kraus, “A Stress tensor for Anti-de Sitter gravity”, Commun. Math. Phys. 208 (1999) 413.

[26] S. de Haro, S.N. Solodukhin and K. Skenderis, “Holographic reconstruction of spacetime and renormalization in the AdS / CFT correspondence” Commun. Math. Phys. 217 (2001) 595.

[27] D. Momeni, M. Faizal, A. Myrzakul, S. Bahamonde, R. Myrzakulov, “A Holographic Bound for D3-Brane”, Eur. Phys. J. C 77 (2017) 391.

[28] Edward Witten, “Anti-de Sitter space and holography”, Adv. Theor. Math. Phys. 2 (1998) 253.

[29] Edward Witten, “Anti-de Sitter space, thermal phase transition, and confinement in gauge theories”, Adv. Theor. Math. Phys. 2 (1998) 505.
[30] Omer Ben-Ami, Dean Carmi, “On Volumes of Subregions in Holography and Complexity”, JHEP 11 (2016) 129.

[31] Pratim Roy, Tapobrata Sarkar, “Note on subregion holographic complexity”, Phys. Rev. D 96 (2017) 026022.

[32] C. Fefferman and C. Robin Graham, “Conformal Invariants”, in “Elie Cartan et les Mathématiques d’aujourd’hui”, Astérisque, 1985, page 95.

[33] X. Dong, S. Harrison, S. Kachru, G. Torroba and H. Wang, “Aspects of holography for theories with hyperscaling violation”, JHEP 1206 (2012) 041.