QCD prerequisites for extra dimension searches

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Abstract

For the dilepton production at hadron collider in TeV scale gravity models, inclusion of QCD corrections to NLO stabilises the cross section with respect to scale variations. The K-factors for the various distributions for the ADD and RS model at both LHC and Tevatron are presented.

1 Introduction

Extra dimension scenarios are now essential part of the studies of physics beyond the Standard Model (SM). They provide an alternate view of the hierarchy between the electroweak and the Planck scale. Some of these extra dimension models invoke the brane world scenarios to hide the extra spacial dimensions from current observation. Two such models that are phenomenologically widely studied are the Arkani-Hamed, Dimopoulos and Dvali (ADD) \cite{1} and the Randall-Sundrum (RS) \cite{2} models.

In the ADD case the compactified extra dimensions could be large and the large volume of the compactified extra spacial dimension would account for the dilution of gravity in 4-dimensions and hence the hierarchy. In this model, new physics can appear at a mass scale of the order of a TeV. A viable mechanism to hide the extra spacial dimension, is to introduce a 3-brane with negligible tension and localise the SM particles on it. Only the graviton is allowed to propagate the full 4 + d dimensional space time. As a consequence of these assumptions, it follows from Gauss Law that the effective Planck scale $M_P$ in 4-dimension is related to the 4 + d dimensional fundamental scale $M_S$ through the volume of the compactified extra dimensions \cite{1}. The extra dimensions are compactified on a torus of common circumference $R$. The space time is factorisable and the 4-dimensional spectrum consists of the SM confined to 4-dimensions and a tower of Kaluza-Klien (KK) modes of the graviton propagating

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the full 4 + d dimensional space time. The interaction of the KK modes $h_{\mu \nu}^{(\vec{n})}$ with the SM fields localised on the 3-brane is given by

$$\mathcal{L}_{\text{int}} \sim -\frac{1}{M_P} \sum_{\vec{n}=0}^{\infty} T^{\mu \nu}(x) h_{\mu \nu}^{(\vec{n})}(x) ,$$

where $T^{\mu \nu}$ is the energy-momentum tensor of the SM fields on the 3-brane. The zero mode corresponds to the usual 4-dimensional massless graviton. The KK modes are all $M_P$ suppressed but the high multiplicity could lead to observable effects at present and future colliders.

In the RS model there is only one extra spacial dimension and the extra dimension is compactified to a circle of circumference $2L$ and further orbifolded by identifying points related by $y \rightarrow -y$. Two branes are placed at orbifold fixed points, $y = 0$ with positive tension called the Planck brane and a second brane at $y = L$ with negative tension called the TeV brane. For a special choice of parameters, it turns out that the 5-dimensional Einstein equations have a warped solution for $0 < y < L$ with metric $g_{\mu \nu}(x^0, y) = \exp(-2ky) \eta_{\mu \nu}$, $g_{\mu y} = 0$ and $g_{yy} = 1$. This space is not factorisable and has a constant negative curvature—AdS$_5$ space-time. $k$ is the curvature of the AdS$_5$ space-time and $\eta_{\mu \nu}$ is the usual 4-dimensional flat Minkowski metric. In this model the mass scales vary with $y$ according to the exponential warp factor. If gravity originates on the brane at $y = 0$, TeV scales can be generated on the brane at $y = L$ for $kL \sim 10$. The apparent hierarchy is generated by the exponential warp factor and no additional large hierarchies appear. The size of the extra dimension is of the order of $M_P^{-1}$.

Further it has been showed that [3] the value of $kL$ can be stabilised without fine tuning by minimising the potential for the modulus field which describes the relative motion of the two branes. In the RS model graviton and the modulus field can propagate the full 5-dimensional space time while the SM is confined to the TeV brane. The 4-dimensional spectrum contains the KK modes, the zero mode is $M_P$ suppressed while the excited modes are massive and are only TeV suppressed. The mass gap of the KK modes is determined by the difference of the successive zeros of the Bessel function $J_1(x)$ and the scale $m_0 = k e^{-\pi kL}$. As in the ADD case the phenomenology of the RS model concerns the effect of massive KK modes of the graviton, though the spectrum of the KK mode is quite different. In the RS model the massive KK modes $h_{\mu \nu}^{(n)}(x)$ interacts with the SM fields

$$\mathcal{L}_{\text{int}} \sim -\frac{1}{M_P} T^{\mu \nu}(x) h_{\mu \nu}^{(0)}(x) - \frac{e^{\pi kL}}{M_P} \sum_{n=1}^{\infty} T^{\mu \nu}(x) h_{\mu \nu}^{(n)}(x) .$$

The masses of $h_{\mu \nu}^{(n)}(x)$ are given by $M_n = x_n k e^{-\pi kL}$, where $x_n$ are the zeros of the Bessel function $J_1(x)$. In the RS model there are two parameters which are $c_0 = k/M_P$, the effective coupling and $M_1$ the mass of the first KK mode. Expect for an overall warp factor the Feynman rules of RS are the same as those of the ADD model.
These models are being tested at the Tevatron [7, 8] and will be tested at the LHC in the near future. One of the most important channels that could probe the signatures of these extra-dimensional models is large invariant mass di-lepton pair production, (also called Drell-Yan(DY) production) in hadronic colliders. In this process, the gravitons which are KK modes of the theory can appear in the intermediate state to produce di-lepton pairs because gravity couples to anything and everything. In ADD model, even though the coupling of gravity to standard model particles is small, the collective effect coming from all the KK modes can result in observable effect in some kinematic domains that can be probed. In RS, the warp factor compensates the small coupling making the effect feasible. Since the process under study is initiated by quarks and gluons, there is a large theoretical uncertainty coming from factorisation and renormalisation scales and also from missing higher order corrections resulting from strong interaction dynamics namely Quantum Chromodynamics(QCD). It is important for such searches to have some quantitative estimate of the effects of higher order QCD corrections and here we review the recent results on the NLO QCD corrections for various distributions of the di-lepton pair production both at the Tevatron and LHC.

2 Di-lepton pair production at hadron collider

Consider the collision of hadrons $P_1, P_2$ to leptonic final states $P_1(p_1) + P_2(p_2) \rightarrow \ell^+(l_1) + \ell^-(l_2) + X(P_X)$, where $X$ denotes the final inclusive hadronic state and carries the momentum $P_X$. In the QCD improved parton model, the hadronic cross section can be expressed in terms of partonic cross sections convoluted with appropriate parton distribution functions

$$2\hat{s} \frac{d\sigma_{P_1P_2}}{dQ^2} (\tau, Q^2) = \sum_{ab=q,\bar{q},g} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 f_a(x_1) f_b(x_2) \times \delta(\tau - z x_1 x_2) \cdot \delta(\tau - z x_1 x_2).$$

The partonic level process is mediated by the SM $\gamma$ or $Z$ boson and the graviton. Interestingly, the interference between the SM and graviton diagrams identically vanish when the phase space integration is performed [4]. To NLO the following subprocesses involving graviton contribute

$$q + \bar{q} \rightarrow G + g, \quad q + \bar{q} \rightarrow G + \text{one loop},$$

$$q + g \rightarrow G + q, \quad \bar{q} + g \rightarrow G + \bar{q},$$

$$g + g \rightarrow G + g, \quad g + g \rightarrow G + \text{one loop}.$$
The cross sections beyond LO involve the computation of one loop virtual gluon corrections and real gluon bremsstrahlung contributions to LO processes. We also include processes with a gluon in the initial state. Since we are dealing with energy momentum tensor coupled to gravity expressed in terms of renormalised fields and masses, there is no overall ultraviolet renormalisation required. In other words, the operator renormalisation constant for the energy momentum operator is identical to unity to all orders in perturbation theory. But we encounter soft and collinear divergences in our computation. These divergences are regulated using the dimensional regularisation. We define $n = 4 + \varepsilon$ where $n$ is the space-time dimension. With this procedure all divergences appear as $1/\varepsilon^\alpha$ where $\alpha = 1, 2$. The soft divergences coming from virtual gluons and bremsstrahlung contributions cancel exactly according to the Bloch-Nordsieck theorem. The remaining collinear divergences are removed by mass factorisation which is performed in the $\overline{MS}$ scheme. The Drell-Yan coefficient function after mass factorisation, denoted by $\Delta^{i}_{ab}(z, Q^2, \mu^2_F)$, is computed by

$$\bar{\Delta}^{i}_{ab}(z, Q^2, 1/\varepsilon) = \sum_{c,d} \Gamma_{ca}(z, \mu^2_F, 1/\varepsilon) \otimes \Gamma_{db}(z, \mu^2_F, 1/\varepsilon) \otimes \Delta^{i}_{cd}(z, Q^2, \mu^2_F),$$

where $\Delta^{i}_{ab}(z, Q^2, 1/\varepsilon)$ is the bare partonic coefficient function before mass factorisation is carried out. The factorisation scale is given by $\mu_F$ and the kernel $\Gamma_{cd}(z)$ in the $\overline{MS}$ scheme is given by

$$\Gamma_{cd}(z, \mu^2_F) = \delta_{cd}\delta(1-z) + a_s \frac{1}{\varepsilon} \left( \frac{\mu^2_F}{\mu^2} \right)^{\varepsilon/2} P_{cd}^{(0)}(z),$$

where $a_s = \alpha_s(\mu^2_F)/4\pi$ and $P_{cd}^{(0)}(z)$ are the leading order Altarelli-Parisi splitting functions. Expanding Eq. (5) up to order $a_s$, one can compute the coefficient function $\Delta^{i}_{ab}(z, Q^2, \mu_F)$ from the bare $\Delta^{i}_{ab}(z, Q^2, 1/\varepsilon)$ and the known Altarelli-Parisi kernels $P_{ab}^{(0)}$. Finally we have to fold these finite $\Delta^{i}_{ab}(z, Q^2, \mu_F)$ with the appropriate partonic distribution functions to arrive at the $Q^2$ distribution for the DY pair. For completeness we present the results for the beyond SM effects coming due to the graviton exchange

$$2S \frac{d\sigma^{P_1 P_2}}{dQ^2}(\tau, Q^2) = \sum_q \mathcal{F}_G \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dz \delta(\tau - z x_1 x_2) \times \left[ H_{q\bar{q}}(x_1, x_2, \mu^2_F) \left( \Delta_{q\bar{q}}^{(0),G}(z, Q^2, \mu^2_F) + a_s \Delta_{q\bar{q}}^{(1),G}(z, Q^2, \mu^2_F) \right) \right. + \left. \left( H_{gg}(x_1, x_2, \mu^2_F) + H_{gq}(x_1, x_2, \mu^2_F) \right) a_s \Delta_{gg}^{(1),G}(z, Q^2, \mu^2_F) \right. + \left. H_{gg}(x_1, x_2, \mu^2_F) \left( \Delta_{gg}^{(0),G}(z, Q^2, \mu^2_F) + a_s \Delta_{gg}^{(1),G}(z, Q^2, \mu^2_F) \right) \right],$$
the factors $F_G$ correspond to pure gravity part, $H_{ab}$ are the renormalised parton distributions and $\Delta_{ab}^{(0,1)}$ are given in [4].

3 Discussion

The NLO-QCD corrections have been calculated in the ADD case for various distributions and for forward-backward asymmetry of the invariant lepton pair at both LHC and Tevatron [4]. It was further extended to the RS model in [5]. The un-integrated distribution with respect to $\cos \theta^*$ (the angle of a lepton with respect to the hadron in the $c.m$ frame of the lepton pair) is particularly important as the interference between the SM and gravity is not zero any more [6]. The inclusion of the higher order QCD corrections reduces the dependence on the factorisation scale significantly making the theoretical predictions for the relevant observable more reliable for physics studies at high energies in hadron colliders. The dominant production mechanism for di-lepton production is through gluons in the initial state in the case of LHC and quarks in the case of Tevatron. The parton distribution functions corresponding to these initial states at LHC energies usually have large factorisation scale dependence. We find that our results on NLO QCD corrections to the observable mentioned in this paper do reduce the factorisation scale dependence coming from the parton distribution functions as expected. We also find that the NLO QCD corrections quantified by the $K$ factor

![Figure 1: The various contributing sub-process to the invariant mass distribution of the dilepton pair for ADD model at LHC and Tevatron.](image-url)
are not small. The K-factor at large invariant mass is found to be as large as 1.6 at the LHC while at the Tevatron, it is similar to the SM value [4]. This large value at LHC is again due to the initial state gluons which have large flux at LHC energies. The behavior of the K-factor for RS model can be understood from the fact that only in the resonance region the gravity part contributes, so at off resonance it is the SM K-factor while on resonance it is dominated by the gravity part.

Figure 2: Invariant mass distribution of the dilepton pair for RS model at LHC and the corresponding K-factor.

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