Towards a mean-field kinetic model of electroweak baryogenesis

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Abstract. We explore the dynamic symmetry breaking between the mass of two annihilating species entrapped within a condensing droplet of a third phase-changing species. The symmetry breaking is induced by different diffusivities and/or different source terms acting at the droplet interface. Potential implications for the problem of electroweak baryogenesis are sketched out.

1. Introduction
The problem of baryogenesis, namely the large asymmetry between matter and antimatter observed in the current Universe, is one of the outstanding issues in modern physics and cosmology [1]. A pioneering investigation by Toussaint and Wilczek [2], pointed out that the time asymptotic behaviour of the mutually annihilating species (A and B for convenience) crucially depends on the initial conditions. The rationale is quite intuitive: if the species mix, they react and both disappear according to the irreversible reaction $A + B \rightarrow P$, where $P$ denotes a set of product species which contains neither $A$ nor $B$. If, on the other hand, by some transport mechanism, they manage to demix or segregate apart, so that the product of their concentrations becomes vanishingly small, then annihilation is quenched, thus spawning a chance for both species to survive much longer than under homogeneous mixing conditions. Besides being of great interest on their own right, the details of such survival may have plenty of applications in chemistry, material science or biology, an example in point being the absorption of drugs within liquid droplets for drug-delivery applications.

In this paper we consider a specific mechanism of segregation associated with the growth of droplets within a phase-changing carrier fluid. By postulating a selective transport of the two species across the droplet interface (membrane), we introduce a symmetry-breaking mechanism which is ultimately responsible for the differential entrapment of the facilitated species (the one with higher transmissivity across the membrane, say A) with respect to the inhibited one (say B). A similar, but more intense, effect is observed by introducing different source terms for the two species. The basic question is: how much mass of species A is entrapped in the growing and moving droplet as a function of time? Once again, this is interesting per-se as a fundamental transport problem in dynamically heterogeneous media, and also for the aforementioned practical purposes. To the best of our knowledge, no detailed account of the hydrodynamic complexity...
associated with a moving and expanding droplet in the presence of transport and chemical reaction, has ever been discussed. This is precisely the aim of the present work, with prospective focus on electroweak baryogenesis.

2. The transport model

We consider three species A, B and C, where C is a fluid carrier undergoing phase-changes, while A and B are passively transported by C and mutually annihilate through chemical reactions. The three species \( k = 1, 2, 3 \) obey a continuity equation of the form

\[
\partial_t \rho_k + \partial_a (\rho_k u_{k,a}) = r_k + s_k
\]

where \( a = x, y, z \) runs over spatial dimensions and obeys Einsteins’s summation rule. Here, \( r_k \) and \( s_k \) denote the density change rate due to chemical reactions and generic sources, respectively.

Species A and B share the same mass, which we set to unity by convention, \( m_A = m_B = 1 \), so that the number and the mass of the species are the same quantity. The two species annihilate at the following rate:

\[
R_A = R_B = -\alpha \rho_A \rho_B
\]

where \( \alpha \) is an adjustable reaction parameter. The C field serves as a carrier for the A and B species and obeys a non-ideal Navier-Stokes equation:

\[
\partial_t (\rho_C u_a) + \partial_b P_{ab}^C = s_C
\]

where

\[ P_{ab}^C = \rho_C u_a u_b + p\delta_{ab} - \sigma_{ab} + \chi \partial_a \rho_C \partial_b \rho_C \]

is the non-ideal momentum flux tensor, including the contributions of inertia, ideal and non-ideal pressure, dissipation and the capillary forces responsible for the first-order phase transition. The right hand side represents an external source of mass.

The species A and B are passively transported by the C-field and diffuse across it with a diffusivity coefficients \( D_A \) and \( D_B \) respectively. The A and B species experience a selective permeability of the droplet interface, so that an excess of A over B accumulates around the interface and further penetrates within the expanding droplet. The actual amount of mass engulfed within the droplet resulting from such complex transport process is highly sensitive to the chemical details, as well as to the hydrodynamic evolution of the system.

Our main aim is to investigate the complex transport phenomena which result from the dynamic competition between advection, diffusion and reaction processes taking place in the framework of a phase-changing carrier fluid. In our stylized model, microscopic symmetry breaking between species A and B is accounted by introducing different diffusivities and source terms, for the two species A and B, respectively.

2.1. Selective diffusivity

We assume a discontinuous jump between the inner and outer value of the diffusion coefficient of the B species, although smoother dependencies could be easily adjusted. In the LB scheme (see appendix), the diffusivity is controlled by the relaxation frequency \( \omega \), hence the jump in diffusivity implies a corresponding change of such frequency across the membrane. The ratio \( D_{in}/D_{out} \) of the B-species is the symmetry-breaking parameter to be varied in the source-free simulations.
2.2. Selective sources
We assume that presence of a fourth species, segregated around the droplet interface and giving rise to further production of both species $A$ and $B$, although at different rates. This effect is modelled by the following source terms:

\[ s_A(x, y) = s_0 W\left(\frac{\vec{r} - \vec{r}_s}{w}\right) \]
\[ s_B(x, y) = \zeta s_0 W\left(\frac{\vec{r} - \vec{r}_s}{w}\right) \]

where $W(\cdot)$ is a piece-wise constant centred around the droplet interface of width $w$, and $0 \leq \zeta \leq 1$ is the symmetry-breaking parameter (SBP).

For standard terrestrial fluids, the hypothetical fourth species could be a solid catalyzer, say a reactive colloid, segregating at the interface, like in bijels. For the case of electroweak baryogenesis (EWBG) we refer to sphalerons, namely hypothetical particles converting baryons to antileptons and antibaryons to leptons, thus changing the baryon number in the process. For a detailed account, see [9].

3. Analytical models
The homogeneous case without space dependence lends itself to some useful analytical considerations which we report in the sequel for both undriven (no sources) and driven case.

3.1. Homogeneous non-driven case
In the homogenous-symmetric scenario $D_A = D_B = 0$, no-phase transitions, no sources and symmetric initial conditions, both species decay according to the nonlinear homogeneous equation:

\[ \frac{d\rho_k}{dt} = -\alpha \rho_k^2, \]

whose analytical solution reads as follows:

\[ \rho_k(t) = \frac{\rho_{k0}}{1 + \alpha \rho_{k0} t}, \quad k = A, B \]

This yields a $\tau_k/t$ decay, where

\[ \tau_k \equiv \frac{1}{\alpha \rho_{k0}} , \quad k = A, B \]

is the density-dependent annihilation time-scale.

In the presence of a symmetry-breaking membrane, the two species are expected to develop different values of $\tau_k$, which we refer as to a dynamic symmetry breaking due to the effect of the selective interface on the species density. Such dynamic symmetry breaking is expected to occur as soon as the droplet starts to grow, i.e. it starts to nucleate out of its initial seed of radius $R_0$. Both species $A$ and $B$ begin to be entrapped within the nucleating droplet and their mass within the droplet grows accordingly, as long as the droplet growth rate exceeds their annihilation rate.

As we shall see, such growth is far from monotonic, but characterized instead by large fluctuations, due to the carrier density waves radiating away from the expanding droplet. Such oscillations do not settle down until the droplet condensation has come to an end, i.e. at $t \sim \tau_g \equiv R/\dot{R}$, where $\tau_g$ defines the condensation time of the droplet, i.e. the time it takes for its mass to reach steady-state.

In the long-term, namely at $t \gg \tau_g$, the densities of the two species are expected to settle to constant values inside and outside the droplet, thus leading to the coexistence of two homogeneous compartments: the droplet and its surrounding environment.
Figure 1. (Color online) Time evolution of the mass of species A, B and C entrapped within the droplet of the carrier species C (Left). The ratio $M_B/M_A$ as a function of time for the three cases $D_{in}/D_{out} = 1, 0.1, 0.01$ (Right). The lower panel at the left reports a time sequence of the density contours of the carrier species C.

Since the droplet is homogeneous, the mass of the entrapped species is expected to follow again a $\tau_k/t$ decay, with two different values of $\tau_A$ and $\tau_B$, due to the aforementioned dynamic symmetry breaking. In the sequel, we shall put these qualitative considerations on quantitative grounds based on the result of extensive numerical simulations.

3.2. Homogeneous driven case

The equations of the mass evolution within the droplet for the homogeneous driven system read as follows:

\[
\begin{align*}
\dot{M}_A &= -\alpha R_{AB} + S_A \\
\dot{M}_B &= -\alpha R_{AB} + S_B
\end{align*}
\]

where we have defined the global density overlap

\[ R_{AB} = \int \rho_A(x,y)\rho_B(x,y)dxdy, \]

and the global mass inputs per unit time

\[ S_k = \int s_k(x,y)dxdy, \quad k = A,B. \]

Subtracting the two equations (9) - (10), delivers:

\[ \dot{M}_A - \dot{M}_B = S_A - S_B \]

This shows that the mass deficit $M_A - M_B$ grows linearly in time.

By multiplying the first by $M_B$, the second by $M_A$ and summing them up, we obtain

\[ \frac{d}{dt}(M_A M_B) = -\alpha R_{AB}(M_A + M_B) + S_B M_A + S_A M_B \]

The right hand side is made zero by imposing

\[ R_{AB} = \frac{1}{\alpha} (S_B \phi_A + S_A \phi_B) \]

where we have defined the mass fractions $\phi_k = M_k/(M_A + M_B)$, $k = A,B$. 


Next, we write \( R_{AB} = \xi \frac{M_AM_B}{V} \), which defines the spatial correlation coefficient, \( \xi \), \( V \) being the droplet volume. As a result, eq. (12) yields:

\[
M_A M_B = \frac{V \xi}{\alpha} S_{AB}
\]

(12)

where we have defined \( S_{AB} = (S_A \phi_B + S_B \phi_A) \).

Solving explicitly for the time dependence of both masses in the asymptotic regime, we obtain:

\[
M_A(t) = s_0 V_{\text{shell}} \frac{(1 - \zeta)t}{2} (+1 + \sqrt{1 + 4\tau^2/t^2})
\]

(13)

\[
M_B(t) = s_0 V_{\text{shell}} \frac{(1 - \zeta)t}{2} (-1 + \sqrt{1 + 4\tau^2/t^2})
\]

(14)

where we have defined the decay time scale

\[
\tau^2 = \frac{M_A M_B}{(S_A - S_B)^2}
\]

which manifestly diverges in the symmetric limit \( S_A = S_B \). Note that since the sources are localized around the interface, the volume \( V_{\text{shell}} \) in the above expression is given by \( V_{\text{shell}} = A w \), where \( A \) is the area of the droplet surface and \( w \) is its thickness.

The upshot of the above analysis is that the majority species grows asymptotically like \( \propto t \) and the minority species decreases like \( \propto 1/t \). As a result the mass ratio \( M_B/M_A \) decays asymptotically in time as:

\[
\frac{M_B}{M_A} \sim \frac{\tau^2}{t^2}
\]

This is potentially far reaching, since it means that even a minuscule asymmetry in the sources is destined to give rise to a power-law (quadratic) extinction of the minority species. As a mere curiosity, to achieve the current value of the matter/antimatter asymmetry ratio \( M_B/M_A \sim 10^{-6} \), with the homogeneous \( 1/t^2 \) decay rate, one should wait a time lapse \( 10^{3/2} \tau \).

Next, let us estimate the decay time as follows. Upon assuming \( \phi_B \ll 1 \) and \( \epsilon \ll 1 \), we can write

\[
S_{AB} \sim S_B
\]

which delivers

\[
\tau^2 = \frac{V \xi}{\alpha S_B \epsilon}
\]

By writing \( S_B = s_0 \zeta V_{\text{shell}} \sim s_0 V_{\text{shell}} \), we finally estimate

\[
\tau^2 = \frac{1}{\alpha s_0 V_{\text{shell}}} \frac{\xi V}{\epsilon^2}
\]

(15)

This expression invites a transparent physical interpretation. The first term is the product of the source time scale and the annihilation time scale, call it \( \tau_{\text{as}}^2 \). The second term on the right hand side is the thickness parameter of the droplet, essentially the width over its effective radius, where "effective" accounts for the inclusion of the correlation coefficient \( \xi \) between the two species. Finally, the third factor is the symmetry breaking source term.

Once \( \tau \) is assessed, any given antimatter/matter ratio \( \mu = M_B/M_A \) is attained on a time scale \( \tau_{\mu} = \tau \mu^{-1/2} \). For the case of EWBG, \( \mu = 10^{-6} \) and \( \tau_{\mu} = 10^6 \tau \). Of course, these numbers are purely qualitative, but serve nonetheless the purpose of showing that, under a quadratic time decay regime, even a tiny source asymmetry could account for the observed matter/antimatter asymmetry if the sources persists long enough in units of the decay time scale \( \tau \).
3.3. Heterogeneous driven case
The previous analysis omits the contribution of diffusion across the droplet membrane, amounting to the mass flux terms \( \int D_k \nabla \rho_k \cdot \hat{n} dA, \ k = A, B \). These can be reabsorbed within an effective source term of the form

\[
 s_k \rightarrow s_k + \frac{D_k}{w} g_k
\]

where \( g_k \) is the surface-averaged density gradient over the droplet interface. With this renormalisation, the previous analysis carries over with no modifications, except that different numerical values are expected, if the diffusive contributions cannot be neglected. On purely dimensional grounds, diffusive effects are expected to be negligible whenever the diffusive time is longer than the source timescale (strongly-driven regime), i.e:

\[
 Da_s \equiv \frac{w^2}{D \tau_s} \gg 1
\]

where \( Da_s \) is the so-called Damköhler number associated with the source term. An analogue Damköhler number applies to annihilation, the condition

\[
 Da_a \equiv \frac{w^2}{D \tau_a} \gg 1
\]

denoting the situation in which the species experience substantial annihilation while traversing the droplet interface (fast-annihilation regime).

4. Numerical set-up
In the sequel we introduce the simulation set-up and present numerical results for the selective source scenario.

4.1. Initial and boundary conditions
Both species are initialised at the same constant density value throughout the computational domain:

\[
 \rho_A = \rho_B = \rho_0 = 1
\]  

(16)

The initial density of the carrier field is defined as follows:

\[
 \rho_C = \rho_{\text{in}} (1 \pm \eta), \quad \text{in a ball of radius } R_0
\]  

(17)

\[
 \rho_C = \rho_{\text{out}} \quad \text{outside the ball}
\]  

(18)

where \( \eta \) is a zero-mean random perturbation with rms \( \delta \rho / \rho = 0.01 \). We take \( \rho_0 = 1, \ \rho_{\text{in}} = 0.2, \ \rho_{\text{out}} = 0.10 \) and \( R_0 = 10 \).

For the phase transition, we choose \( T/T_c = 0.04/0.047 \), corresponding to a coexistence liquid/vapour density ratio of about 0.27/0.027 = 10. The values of \( \rho_{\text{out}} \) and \( \rho_{\text{in}} \) determine the duration of the growth stage, i.e. the time it takes for the droplet to attain the coexistence values of the liquid (l) and vapour (v) phases.

By mass conservation:

\[
 \rho_{\text{in}} V_{\text{in}} + \rho_{\text{out}} V_{\text{out}} = \rho_l V_l + \rho_v V_v = M
\]  

(19)

where \( V_{\text{in}} + V_{\text{out}} = V_l + V_v = V = L^2 \) is the total volume of the system. Clearly, the volume of the droplet grows at increasing the total mass in the system. More specifically, the final value
of the volume fraction, i.e. the ratio of the volume of the liquid droplet to the total volume, is given by:

$$\lambda \equiv \frac{V_l}{V} = \frac{\rho_m - \rho_v}{\rho_l - \rho_v}$$

(20)

where $\rho_m = M/V$ is the average mass density. The diameter of the liquid droplet is thus given by:

$$D = L \sqrt{\frac{4}{\pi} \lambda}$$

(21)

Hence, the maximum droplet diameter, $D = L$, is attained at the volume fraction $\lambda_{\text{max}} = \sqrt{\pi/4} \sim 0.87$.

All species are taken initially at rest, namely:

$$\vec{u}_k = 0, \quad k = A, B, C$$

(22)

and full periodicity is assumed across the four boundaries of the simulation box.

Collisional time-scales are taken as $1/\omega_A = 1/\omega_B = 1/\omega_C = 1$, fixing the fastest timescale in action. This corresponds to an outer diffusivity and carrier viscosity, $D_A = D_B = v_C = 1/6$ in lattice units (see Appendix). The annihilation rate is taken as $\alpha = 0.1$, corresponding to an annihilation timescale $\tau_a = 10$ at unit density $\rho_k = 1$.

5. **Numerical results**

We consider a two-dimensional square with 1024 grid-points per side and run over a timespan of $10^6$ time-steps, thus covering three decades in space and six in time, as it is appropriate for diffusive phenomena. The simulations have been run using a three-component lattice Boltzmann scheme (see Appendix).

To fix ideas for prospective applications to EWBG, let us inspect the physical time and length scales of the simulations. The time span goes from the onset of EWBG, $t_{\text{EWBG}} \sim 10^{-11}$ seconds, to the time of the QCD transition, $t_{\text{QCD}} \sim 10^{-5}$ seconds. With one million timesteps, this fixes the lattice timestep to $\Delta t = t_{\text{EWBH}} = 10$ ps. The corresponding lattice spacing is $\Delta x = c \Delta t = 3 \times 10^{-3}$ meters, which means that we deal with a computational Universe of side $L = 3$ meters, and a Higgs droplet inflating from about 3 mm to 3 meters in diameter.

5.1. **Source-free simulations**

In Figure 1 we present the evolution of the mass of the three species A, B, C for the source free case and different combinations of the diffusion coefficients, namely, outside the droplet $D_A = D_B = 1/6$, while inside $D_B = 1/6, 1/60, 1/600$, so as to break the A-B symmetry [10].

As one can see, after an initial transient, showing large fluctuations (not shown), both species A and B settle close to the expected $\tau_k/t$ decay and with $\tau_A \neq \tau_B$, due to the dynamic symmetry breaking occurring in the transient. By and large, however, the mass ratio $M_B/M_A$ remains $O(1)$, showing that the diffusive symmetry breaking does not appear to be relevant to the EWBG scenario.

5.2. **Source-driven simulations**

Next, we investigate the effects of selective source terms for the species A and B, by changing the asymmetry source coefficient in the range $0.4 \leq \zeta \leq 1$. The main parameters are set as follows: $D_A = 1/6, D_B = 1/6$ and $s_0 = 10^{-3}$. With these parameters, given a radius $R \sim 400$ and a width $\sim 5$, we compute a decay relaxation time $\tau^2 \sim 4 \times 10^5/\epsilon^2$, with $\xi = 1$.

In Figure 2, we present the time evolution of the mass A, B and C within the droplet for the case $\zeta = 0.99$. As clearly shown in the figure, the source term leads to a secular growth of the
Figure 2. Time evolution of the masses of species A, B, and C inside the droplet for the case \(\zeta = 0.99\). Panel (b) reports the corresponding mass ratio \(M_B/M_A\) as a function of time for the cases \(\zeta = 0.999, 0.99, 0.9\) (top to bottom). The whisker in the lower curve is due to a crash of the simulation, due to the large separation between the two species, but the \(t^{-2}\) scaling is nonetheless apparent.

A species, \(M_A(t) \propto t\), versus a homogeneous \(1/t\) decay of the B species. As a result, the mass ratio \(M_B/M_A\) goes indeed asymptotically to zero like \(t^{-2}\).

The time asymptotic behaviour sets in long before the density configuration in space reaches its mechanical equilibrium, the chief condition being that the mass of the droplet be stationary in time, regardless of its shape. Since the majority species follows a linear trend \(M_A(t) \propto t\), while the minority one obeys a reciprocal trend \(M_B(t) \propto 1/t\), their product remains basically asymptotically constant in time, which is indeed confirmed by the numerical simulations (not shown), supporting the assumption made in the analytical model. In particular, with \(\epsilon = 0.001, 0.01, 0.1\), we obtain \(M_B/M_A \sim 10^{-2}, 10^{-4}, 10^{-6}\) after \(10^6\) steps, respectively, which is consistent with a correlation coefficient \(\xi \sim 0.02\).

6. Relevance to electroweak baryogenesis

It is of interest to speculate whether the present transport model can be of any interest in the context of electroweak baryogenesis [6]. To this purpose, let us remind that in our model baryogenesis implies the identification \(A=\text{matter}, B=\text{antimatter}\) and \(C=\text{Higgs fluid}\). It is therefore of interest to assess the plausibility of present model towards the basic requirements for baryogenesis, as as formulated by Sakharov, namely [7]:

i) The existence of an explicit baryon-symmetry breaking mechanism,

ii) Violation of C and CP invariance,

iii) Thermodynamic non-equilibrium

As to i), the baryon symmetry breaking is expressed by the non-unit diffusion jump factor \(J_{AB}\) across the membrane, or an explicit symmetry breaking at the level of source terms, i.e \(\zeta \neq 1\). The microscopic origin of such symmetry breaking is not relevant to the present mean-field model. Item ii) states that the system must be invariant upon a reflection in space, say from \(x\) to \(-x\) across the interface, and charge conjugation. Our model is electrically neutral, hence item i) is basically a requirement that the density of A at location \(x\) be different from the density of \(B\) at the mirror location \(-x\), namely \(\rho_A(x) \neq \rho_B(-x)\). This is certainly true once a non-unit diffusivity jump or source asymmetry factor is in action.

Finally, thermodynamic non-equilibrium implies that species A and B must depart from their local thermodynamic equilibrium, which is certainly true in the presence of density gradients.
Figure 3. The ratio $M_A/M_B$ at $t = 12000$, as a function of $\zeta$ at the time when the droplet attains its equilibrium mass (left). The solid line is the fit $2 \times 10^5 e^{-2.3x} (1 - x)^2$. On the right side, we show a typical density profile of the carrier species across the interface.

across the interface. Thus, at least at a qualitative level, the present model appears compatible with the basic requirements for baryogenesis.

To proceed towards a quantitative analysis, several aspects need to be inspected in detail. First, in the EWBG scenario the Higgs droplet expands much faster than the Universe, until it fills it up entirely, whence its alleged pervasiveness at the current day. In our model, the droplet stops growing once mass equilibrium is attained, typically for density ratios around 10 between the liquid and vapour phases. In addition, our computational Universe is static, as opposed to an expanding Universe. However, both limitations could be significantly mitigated, if needed.

The droplet growth rate in our simulations is $\dot{R} \sim 0.04$ in light speed units, which is about ten times smaller than the credited wall speed of the true Higgs droplets, estimated at $c/2$. Given that no fine-tuning effort has been spent in customising the simulations to the EBWG scenario, the above figures do not appear completely off target. The thickness parameter $w/R \sim 0.01$ also appears reasonable.

Next, let us inspect the values of the matter/antimatter ratio, in our case the ratio $M_A/M_B$ at the time when the droplet reaches its equilibrium mass. In Fig. 3 we report the ratio $M_A/M_B$ at the end of the droplet growth, as a function of the symmetry breaking parameter $\zeta$. We note that with $\zeta \sim 0.5$, ratios around $10^4$ are obtained, which extrapolate to $10^5$ in the limit $\zeta \to 0$. These values are two (one) orders of magnitude above the current value of the matter/antimatter ratio in the Universe, which is estimated at about $10^6$. Although not visible on the scale of the plot, $\zeta = 0.95$ yields a mass ratio around 80, namely two orders of magnitude, in the face of a tiny five percent source asymmetry.

In addition, as shown earlier on, such ratio grows quadratically in time, hence it is very sensitive to the epoch at which sources fade away. As discussed previously, under quadratic decay, the time at which a ratio $10^{-6}$ is obtained is $10^6 \tau$. By choosing $\tau \sim t_{EWBG}$, this is basically the geometric mean between the time of EWBG and QCD time.

Summarizing, it appears reasonable to speculate that, with proper fine-tuning, the present model could prove useful for computational explorations of the strong non-equilibrium spacetime dynamics of EWBG scenarios.

7. Conclusions and outlook

Summarizing, we have analysed the transport of mutually annihilating species within the flow field of a passive carrier experiencing a first-order dynamic phase transition. In particular, we analysed the symmetry-breaking effects on the mass engulfed by the growing droplet as induced by an explicit symmetry breaking of the diffusive and/or source terms. The source-driven
Figure 4. The nineteen discrete velocity lattice in three spatial dimensions (D3Q19).

scenario appears very effective, since any nonzero asymmetry between the two sources turns the $1/t$ decay of the majority species into a secular linear growth. As a result, in the long term, the ratio between minority and majority species decays like $1/t^2$. With suitable adaptations, particularly a more specific modelling of the source terms, the present scheme may permit to gain insights into the strongly non-equilibrium spacetime dynamics of electroweak baryogenesis at a very affordable computational cost.

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Appendix: The Lattice Boltzmann formulation

The transport equations described above are solved by means of a three-species lattice Boltzmann (LB) scheme. The main reason for using LB is its ability of dealing with dynamic phase-transitions in a much handier way than solving the Navier-Stokes equations of non-ideal fluids.

The LB equation takes the following form [4]:

$$ f_i^k(r^i, t + \Delta t) - f_i^k(r^i, t) = -\omega_k \Delta t \left( f_i^k - f_i^{k,eq} \right) + F_i^k \Delta t, \quad k = A, B, C $$

(23)

where $f_i^k(r^i, t)$ represents the probability to find a representative particle of species $k$ at the lattice position $r^i$ and time $t$ with the discrete velocity $c_i^k$. The index $i$ runs over the discrete speeds, $i = 0, 18$ for the present nineteen-velocity three-dimensional lattice.

The local equilibria (a truncated version of Maxwell-Boltzmann distribution) encode the mass-momentum conservation laws. At the moment they are purely classical, but readily extend to quantum statistics. In detail:

$$ f_i^{A,eq} = w_i \rho_A (1 + u_i) $$

(24)

$$ f_i^{B,eq} = w_i \rho_B (1 + u_i) $$

(25)

$$ f_i^{C,eq} = w_i \rho_C (1 + u_i + q_i) $$

(26)

where $u_i = \bar{u} \cdot c_i^k / c_s^2$ and $q_i = u_i^2 - u^2$, $u$ being the magnitude of the net flow of the carrier fluid, namely

$$ \rho \bar{u} = \sum_i f_i^{C,eq} c_i^2 $$

(27)
with
\[ \rho = \sum_i f_i^C \] (29)
the carrier density. Finally, \( w_i \) is the standard set of weights normalised to unity and \( c_s^2 = \sum_i w_i c_i^2 / d \) is the sound speed in \( d \) spatial dimensions. In the present lattice \( c_s^2 = 1/3 \) (Note that the speed of light is \( c = 1 \) in lattice units).

The transport properties are controlled by the relaxation rate, according to the standard LB relations, namely:
\[
D_A = c_s^2 (1/\omega_A - \Delta t/2) \\
D_B = c_s^2 (1/\omega_B - \Delta t/2) \\
\nu_C = c_s^2 (1/\omega_C - \Delta t/2)
\] (30) (31) (32)

Note that A and B equilibria conserve only mass, hence they support mass diffusion, whereas carrier equilibria conserve momentum as well because the local equilibria contain the self-consistent carrier current, see eq. (28). Consequently the carrier relaxation rate controls momentum diffusivity, also known as kinematic viscosity.

For species A and B, the forcing terms are set to zero \( F_i^A = F_i^B = 0 \), so that they obey an ideal equation of state
\[ p_{A,B} = \rho_{A,B} c_s^2. \]
Given that \( c_s^2 = 1/3 \) in lattice units, where \( c = \Delta x / \Delta t = 1 \) is the light speed, this is the equation of state of ultra-relativistic matter, i.e. \( mc^2/kT \ll 1 \), which is reasonable for EWBG, which operates in the regime \( kT \sim 100 \) GeV.

The carrier fluid, however, is subject to self-consistent force resulting from potential energy interactions, according to the standard LB pseudo-potential formulation [5]. Consequently, it obeys a non-ideal equation of state of the form (Carnahan-Starling) [11]
\[
p_C = \rho c_k T (1 + \rho + \rho^2 + \rho^3) - 0.5\rho^2
\] (33)
corresponding to a critical temperature \( T_c = 0.047 \) and density \( \rho_c = 0.066 \) in lattice units.

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