Bayer demosaicking using optimised mean curvature over RGB channels

Rui Chen, Huizhu Jia\textsuperscript{a,b}, Xiangfei Wen and Xiaodong Xie

Colour artefacts of demosaicked images are often found at contours due to interpolation across edges and cross-channel aliasing. To tackle this problem, a novel demosaicking method to reliably reconstruct colour channels of a Bayer image based on two different optimised mean-curvature (MC) models is proposed. The missing pixel values in green (G) channel are first estimated by minimising a variational MC model. The curvatures of restored G-image surface are approximated as a linear MC model which guides the initial reconstruction of red (R) and blue (B) channels. Then a refinement process is performed to interpolate accurate full-resolution R and B images. Experiments on benchmark images have testified to the superiority of the proposed method in terms of both the objective and subjective quality.

Introduction: Demosaicking is a technique for reconstructing a full-colour image from the raw image captured by a digital colour camera that utilises a single-image sensor with a colour filter array (CFA). For the popular Bayer CFA pattern, only one colour component is captured at each pixel and the other missing components must be estimated to produce high-quality RGB images. A number of Bayer demosaicking approaches have been developed by mainly exploiting the spatial and spectral characteristics of raw images [1–5]. The high inter-channel and intra-channel correlations are often assumed to restore the unknown colour components. However, for images with sharp colour transition and high colour saturation, the demosaickings performance is subject to the unreliable estimate of local image structures and the extent of spectral correlations. This Letter proposes a novel demosaicking algorithm by transforming into a curvature domain to reconstruct full-resolution RGB channels, which can improve the structure-preserving capability, enhance the weak correlations and reduce the colour artefacts. The whole demosaicking process is shown in Fig. 1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Flowchart of the proposed demosaicking method}
\end{figure}

Reconstruction of green channel: Because original G channel is sampled twice more than the other two channels and preserves much more image structural information, the subsampled image \(G\) is first interpolated to estimate an initial full-resolution image \(\hat{G}\) by utilising an arbitrary colour difference interpolation algorithm. We here adopt a current state-of-the-art algorithm, the gradient-based threshold free [2], to obtain the missing \(G\) pixel values by adding estimated colour differences to \(R\) and \(B\) pixel values, respectively. The inaccurate estimate of colour differences often artefacts.

To prevent possible artefacts and preserve detailed image structures, we use mean curvature of the associated image surface as a good geometric metric to characterise edges and textures. The mean curvature \(\kappa\) can be simplified along level lines passing through smooth surface [6]. Then we define it as

\[
\kappa(I) = \text{div} \left( \frac{\nabla I}{\sqrt{\|\nabla I\|^2 + 1}} \right)
\] (1)

Here \(\nabla\) denotes the gradient operator of the image \(I\) and \(\text{div}\) is the divergence operator. This realisation form of mean curvature can effectively hold geometric features of image surface and differentiate with non-regularity structures. By treating initial interpolation error \(e\) or artefacts as additive noise, we model the expected G-channel image as \(G = \hat{G} + e\). The \(\ell_1\) norm of curvature \(\kappa\) as the regulariser allows jumps of image structures as well as continuous intensity distributions along the contours while minimising absolute value of \(\kappa\) in piecewise smooth regions. These properties suggest that the artefact parts of images can be removed while edges and textures can be preserved. Accordingly, the proposed variational mean-curvature (MC) model is written as follows:

\[
E(\hat{G}) = \min \frac{1}{2} \int_\Omega (G - \hat{G})^2 + \lambda \int_\Omega |\nabla \hat{G}| (2)
\]

To reconstruct final G-channel \(\hat{G} \in BF(\Omega)\), we use augmented Lagrangian method to solve optimisation problem of (2). Let’s first introduce the auxiliary variables: \(q = \text{div}u, n = p/|p| \in R^3, p = [\nabla G, 1]^T \in R^3, m \in R^3\). The associated augmented Lagrangian functional reads as follows:

\[
L(\hat{G}, q, p, n, m; \lambda_1, \lambda_2, \lambda_3, \lambda_4) = \frac{1}{2} \int_\Omega (G - \hat{G})^2 + \lambda_1 \int_\Omega (|p| - p \cdot m) + \lambda_2 \int_\Omega (|q| - q \cdot \text{div}u)^2 + \lambda_3 \int_\Omega (|n| - n \cdot m) + \lambda_4 \int_\Omega (|\nabla \hat{G}|)^2 (3)
\]

where \(\lambda\) is the regularisation parameter. \(\lambda_1, \lambda_2 \in R, \lambda_3, \lambda_4 \in R^3\) are Lagrange multipliers. \(r_1, r_2, r_3, r_4 \in R\) are step parameters. The variable \(m\) is required to lie in the set \(G, \delta_\ell(-)\) is the pulse function on \(G\). The subproblems associated with each variable are derived to find the optimal solution of (3). For \(G\) subproblem, the minimiser is determined by associate Euler–Lagrangian equation. We here employ Gauss–Seidel method to solve this equation and get

\[
\left\{ \begin{array}{l}
G^{k+1}_{i,j} = \tilde{G}_{i,j} + \frac{r_2}{\Lambda_{i,j}} \left[ G^{k+1}_{i-1,j} + G^{k+1}_{i+1,j} + G^{k+1}_{i,j-1} + G^{k+1}_{i,j+1} - 4 \tilde{G}_{i,j} \right]
\end{array} \right.
\] (4)

where \(p = [p_1, p_2, p_3] \) and \(\lambda_2 = [\lambda_{21}, \lambda_{22}, \lambda_{23}]\). \(\tilde{G}^{k+1}_{i,j}\) denotes a updated value of \(k+1\) iteration at the pixel point \((i, j)\). \(\tilde{G}_{i,j}\) denotes the respective value. \(\delta_\ell(-)\) is the discretised spatial size. In a similar way, the variable \(n\) is updated. Other subproblems can be solved using Newton method [5].

Reconstruction of red and blue channels: The obtained \(\hat{G}\) is used to guide the reconstruction of the missing R and B channels. The high accuracy of G values can facilitate the colour interpolation in R and B spaces. Based on the observation that the inter-channel curvature profiles have higher correlation than the usual differences or ratios of colour components, we use the mean curvature measured from full-resolution image \(\tilde{G}\) as a guided mask, which can be further approximated into a linear MC model. For the \((i, j)\)th pixel of the image \(\tilde{G}\), we first compute second-order finite difference approximations of \(\text{div}[\nabla \tilde{G}]\):

\[
\begin{align*}
d_y & = \frac{\Delta_y \tilde{G}_{i,j} + \tilde{G}_{i-1,j} - \tilde{G}_{i+1,j} + \Delta_y \tilde{G}_{i,j-1} + \tilde{G}_{i,j+1} - \Delta_y \tilde{G}_{i,j-1}}{16} \\
d_n & = \frac{\Delta_n \tilde{G}_{i,j} + \tilde{G}_{i,j-1} - \tilde{G}_{i,j+1} + \Delta_n \tilde{G}_{i-1,j} + \tilde{G}_{i+1,j} - \Delta_n \tilde{G}_{i+1,j}}{16} \\
d_E & = d_y \quad d_n = d_E
\end{align*}
\] (5)

So far now we approximate the curvature \(\kappa\) in (1) by a combination of orthogonal difference projections in the \(x\)- and \(y\)-coordinate directions. To smooth local extrema, the gradient magnitude \(\|\nabla \tilde{G}\|\) is weighted the directional curvature terms. Then it follows from (5) and a linear MC model can get

\[
\kappa(\tilde{G}_{i,j}) = \left| \nabla \tilde{G}_{i,j} \right| \cdot \left( \frac{\Delta_x \tilde{G}_{i,j} + 4 \tilde{G}_{i,j} + \Delta_x \tilde{G}_{i,j}}{\|\nabla \tilde{G}_{i,j}\|^2} \right) / \left( \frac{\Delta_x \tilde{G}_{i,j} + 4 \tilde{G}_{i,j} + \Delta_x \tilde{G}_{i,j}}{\|\nabla \tilde{G}_{i,j}\|^2} \right)
\] (6)
where $\Delta_1$ and $\Delta_2$ denote the forward differences. $u_{i,j,W}$, $u_{i,j,E}$, $u_{i,j,S}$ and $u_{i,j,N}$ have
\begin{equation}
\begin{aligned}
u_{i,j,W} &= \frac{2d_{0,E}}{d_{i,j,W} + d_{0,E}}, \quad \nu_{i,j,E} = \frac{2d_{0,W}}{d_{i,j,W} + d_{0,E}} , \\
u_{i,j,S} &= \frac{2d_{0,N}}{d_{i,j,S} + d_{0,N}}, \quad \nu_{i,j,N} = \frac{2d_{0,S}}{d_{i,j,S} + d_{0,N}}.
\end{aligned}
\end{equation}

The weighted coefficients in model (6) can be computed from image $\hat{G}$ based on (7). Since these curvature-related coefficients characterise local image structures in RGB channels, the missing R components are reliably estimated using a set of same coefficients, which measure the importance of neighbouring pixels of whose larger values are assigned to pixels at edges than those in flat areas. We consider that R channel is interpolated in colour difference domain because the interpolation performance is generally affected by the smoothness degree of local structures. The missing component $R_{ij}$ at B sampling position is initially interpolated using four colour differences $\{\tilde{G}_{i,j,W}, \tilde{G}_{i,j,E}, \tilde{G}_{i,j,S}, \tilde{G}_{i,j,N}\}$. Then the estimated R component $\hat{R}_{ij}$ is interpolated as
\begin{equation}
\begin{aligned}
\tilde{G}_{i,j,W} &= \tilde{G}_{i,j,E} = \tilde{G}_{i,j,S} = \tilde{G}_{i,j,N} = 0, \\
\tilde{R}_{i,j} = & \tilde{G}_{i,j,W} + \tilde{G}_{i,j,E} - \tilde{G}_{i,j,S} - \tilde{G}_{i,j,N} \\
\tilde{R}_{i,j} = & \tilde{G}_{i,j,W} + \tilde{G}_{i,j,E} - \tilde{G}_{i,j,S} - \tilde{G}_{i,j,N}.
\end{aligned}
\end{equation}

Since the four nearest neighbouring pixels along the diagonal directions within the R colour plane have stable correlation with the expected value $\tilde{R}_{ij}$, the obtained $\tilde{R}_{ij}$ is further refined by the following equation:
\begin{equation}
\begin{aligned}
\hat{R}_{i,j} = \hat{R}_{i,j} + (1 - \delta) \cdot \sum v_k \mu_k, \\
\mu_k = R_k - \tilde{R}_{i,j}, \quad v_k = 1 / \mu_k
\end{aligned}
\end{equation}

where $k$ represents four neighbouring pixels. The factor $\delta$ is used to adjust the refinement performance. We set it experimentally as a constant value between 0.5 and 0.7 to obtain the optimal reconstruction. The interpolation process for the missing R components at G sampling positions is identical to restoration of the missing R components at B sampling positions. The B channel can be demosaicked by means of the same procedure as R channel.

**Fig. 2 Visual comparison of different demosaicking methods**

- **a** Cropped regions of the original images
- **b** Directional linear minimum square-error estimation (DLMMSE)
- **c** Multi-scale gradient (MSG)
- **d** Multi-directional weighted interpolation (MDWI)
- **e** Minimized Laplacian residual interpolation (MLRI)
- **f** Proposed

**Table 1: Average S-CIELAB and CPSNR (in dB) results**

| Datasets | S-CIELAB | CPSNR | S-CIELAB | CPSNR |
|----------|----------|-------|----------|-------|
| DLMMSE   | 1.436    | 34.46 | 1.309    | 39.58 |
| MSG      | 1.368    | 34.59 | 1.160    | 41.00 |
| MDWI     | 1.081    | 36.42 | 0.717    | 41.96 |
| MLRI + wei | 1.035   | 36.91 | 0.624    | 42.74 |
| Proposed | 1.020    | 37.23 | 0.586    | 43.15 |

**Experimental results:** To evaluate the proposed method, we have conducted testing experiments and compared its performance against four state-of-the-art algorithms: DLMMSE [1], MSG [3], MDWI [4] and MLRI + wei [5].

The representative images are selected from the Kodak and IMAX datasets. The regularisation parameter $\lambda$ is set as $2 \times 10^{-3}$. The spatial step $h$ is set to 5. $r_1$, $r_2$, $r_3$ and $r_4$ are set as 40, 40, 100 and 100, respectively. The stopping condition for iteratively solving (4) is $\|G - \tilde{G}\|^2 < 10^{-4}$. The factor $\delta$ is set as 0.6. From demosaicked images shown in Fig. 2, we can observe that the colour images reconstructed by the proposed method have much higher subjective quality while false colour artefacts, blocking and zipper effect at edges are reduced. To measure the demosaicking quality, the average spatial extension of CIELAB (S-CIELAB) and colour peak signal to noise ratio (CPSNR) values [5] are computed on two image datasets. It can be seen from Table 1 that our method has totally achieved higher reconstruction performance. This is mainly because our MC models are very effective to reduce colour artefacts and much less sensitive to the change of spectral correlations.

**Conclusion:** In this Letter, we propose an efficient demosaicking algorithm by developing two optimised MC models to enhance the interpolation accuracy and suppress colour artefacts. By exploiting the mean curvature of $G$ image surface as a regulariser, the variational MC model is iteratively solved to reconstruct full-resolution $G$ image with high accuracy and powerful structure-preserving capability. Moreover, this model can be incorporated into other demosaicking methods. Based on high correlation in curvature domain, the R and B full-resolution images can be well restored using the linear MC model and a postprocessing refinement. Extensive experiments have fully shown the superior performance of our method.

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**References**
1. Zhang, L., and Wu, X.: ‘Color demosaicking via directional linear minimum square-error estimation’, *Trans. Image Process.*, 2005, **14**, (12), pp. 2167–2178
2. Pekkucuksen, I., and Altunbasuk, Y.: ‘Gradient based threshold free color filter array interpolation’, Proc. IEEE Int. Conf. Image Processing (ICIP), Hong Kong, September, 2010, pp. 137–140
3. Pekkucuksen, I., and Altunbasuk, Y.: ‘Multiscale gradients-based color filter array interpolation’, *Trans. Image Process.*, 2013, **22**, (1), pp. 157–165
4. Chen, X.D., He, L.W., Jeon, G., et al.: ‘Multidirectional weighted interpolation and refinement method for Bayer pattern CFA demosaicking’, *Trans. Circuits Syst. Video Technol.*, 2015, **25**, (8), pp. 1271–1282
5. Kiku, D., Momoy, Y., Tanaka, M., et al.: ‘Beyond color difference: residual interpolation for color image demosaicking’, *Trans. Image Process.*, 2016, **25**, (3), pp. 1288–1300
6. Mylykoski, M., Gioviniosi, R., Karkkainen, T., et al.: ‘A new augmented Lagrangian approach 1-L mean curvature image denoising’, *SIAM J. Imaging Sci.*, 2012, **8**, (1), pp. 95–125