Abstract

We point out that the rare decay mode $W^\pm \rightarrow \gamma + D_s^\pm$ could be spectacularly enhanced, with a branching ratio around $10^{-6}$ which is three order of magnitude larger than previous predictions. Its observation will determine the $W$ boson mass with great accuracy, providing additional high precision tests of the standard model, as well as reveal eventual deviation from the trilinear non-abelian gauge coupling.
Possible huge enhancement in the radiative decay of the weak W boson into the charmed $D_s$ meson.

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ABSTRACT

We point out that the rare decay mode $W^\pm \rightarrow \gamma + D_s^\pm$ could be spectacularly enhanced, with a branching ratio around $10^{-6}$ which is three order of magnitude larger than previous predictions. Its observation will determine the W boson mass with great accuracy, providing additional high precision tests of the standard model, as well as reveal eventual deviation from the trilinear non-abelian gauge coupling.

In this paper, we present a study of the exclusive radiative decay mode of the weak charged boson $W^\pm$ into a photon and a pseudoscalar meson $D_s^\pm(1969)$, which is a $c\bar{s}$ bound state of charm and strange quarks. The branching ratio $B(W^\pm \rightarrow \gamma + D_s^\pm)$ we obtain is about $10^{-6}$ which is thousand times larger than previous estimates by Arnellos, Marciano and Parsa [1], referred as AMP in the following. Matched to the inclusive radiative decay $W^+ \rightarrow \gamma + c + \bar{s}$ for which the branching ratio is calculated [2] to be around $5 \cdot 10^{-4}$, our result suggests that the probability for a pair of $c + \bar{s}$ quarks to form a pseudoscalar $D_s^+$ is few per mille, which is quite plausible. For such radiative ( and rare ) decay mode, this branching ratio is large and could be observable with the increase in statistics in the future. If detected, this exclusive process provides useful informations regarding electroweak and strong dynamics, its interests lie in the following reasons:

1. The trilinear non-abelian gauge coupling, manifestly involved in this decay (Fig.1), and its eventual deviation from the Yang-Mills structure can be tested.

2. Detection of the two-body decay mode $W^\pm \rightarrow \gamma + D_s^\pm$, combined with the measurement of the photon energy, provides a very precise determination of the W boson mass which is still poorly known within $\pm 260$ MeV errors at the present time ( to be compared with $\pm 7$ MeV for the $Z^0$ ). The accuracy value of $M_W$ obtained from this mode will represent a big jump in the high precision test program of the Standard Model.

3. In the triangular diagrams ( Fig 2, 3 ) that we employ to compute the rate ( what could be else ? ), an effective Yukawa coupling $\lambda$ between the pseudoscalar meson $P$ and the quark pair $\bar{q}q$ is introduced via the interaction $\lambda \bar{q}\gamma_5 q P$, and $\lambda$ must be determined. Using, as a guide, the pion-up quark effective coupling $\lambda_{\pi\pi^0} = \sqrt{2} m_u/F_\pi$ which is the Goldberger-Treiman ( G-T ) relation extended to the quark level[61], and
We note that the coupling \( \lambda_{\eta \pi \pi} \) is in perfect agreement with the Adler-Bell-Jackiw anomaly\(^{[3,4]} \) and can be derived within the Nambu, Jona-Lasinio model\(^{[5,6]} \).

will be discussed later in details, then the mode \( W^\pm \rightarrow \gamma + D_s^\pm \) could be used to test our model (i.e. the triangular diagrams together with the extended G-T relation ) and consequently the dynamics of hadronization from quarks.

The total decay amplitude is described by the three Feynman diagrams of Figs 1, 2 and 3 :

\[ Q_1 = -i(eg/2\sqrt{2})V_{cs}\varepsilon^\mu(P)\varepsilon^{\nu}(k)\{g_{\mu\nu}(P + k)_\alpha - 2g_{\nu\alpha}k_\mu - 2g_{\mu\alpha}P_\nu\} \]
\[ \{(g^\alpha_\beta + (P - k)\alpha(P - k)\beta) / ((P - k)^2 - M_W^2)\}iF_D(P - k)_\beta \}. \]

In Eq.(1), the conditions \( P_\mu\varepsilon^{\nu}(P) = k_\nu\varepsilon^{\nu}(k) = 0 \) being taken account, \( V_{cs} \) is the Cabibbo-Kobayashi-Maskawa charm-strange transition, \( F_D \) is the \( D_s \) decay constant ( analogous to the pion one : \( F_\pi \simeq 132 \text{ MeV} \) ) involved in \( D_s \rightarrow \tau \nu \) for example and \( g = e/sin\theta_W \) is the SU(2) gauge coupling.

Contracting the indices \( \alpha \) and \( \beta \), the amplitude \( Q_1 \) reduces to a very simple expression\(^{[1]} \):

\[ Q_1 = (eg/2\sqrt{2})V_{cs}F_D\varepsilon^\mu(P)\varepsilon^{\nu}(k)g_{\mu\nu} \]. \( \text{(2)} \)

We remark that this reduction holds only if the tri-linear coupling obeys the standard non-abelian structure as explicited in Eq.(1).

For the diagrams of Figs(2) and (3), let us denote by \( \lambda_D \) the effective coupling of the \( cs \) pair to the \( D_s \) meson through the Yukawa interaction \( \lambda_D \bar{c} \gamma_5 s D_s \), then the amplitudes given by the two triangular loops can be written as :

\[ Q_2 + Q_3 = \lambda_D(eg/2\sqrt{2})V_{cs}\varepsilon^\mu(P)\varepsilon^{\nu}(k)T_{\mu\nu}(P, k) \]. \( \text{(3)} \)
where $T_{\mu\nu}$ has the following form:

$$T_{\mu\nu}(P, k) = K_1 g_{\mu\nu} + K_2 P_\nu k_\mu + K_3 P_\mu P_\nu + K_4 k_\mu k_\nu + K_5 P_\mu k_\nu + iK_6 \varepsilon_{\mu\nu\alpha\beta} P^\alpha k^\beta.$$  \hspace{1cm} (4)

In Eq.(4), the six terms $K_i(M^2_W, m^2_D, m_c, m_s), i = 1,6$ are functions of the external masses $(P^2 = M^2_W, (P-k)^2 = m^2_D)$ as well as of the internal fermions masses $m_c, m_s$ in the loops. The terms $K_3, K_4,$ and $K_5$ do not contribute to the amplitude when they are contracted with $\varepsilon^{\mu}(P)$ and $\varepsilon^\nu(k)$, the terms $K_2$ and $K_6$ are finite, while $K_1$ turns out to be logarithmically divergent. However when the gauge-invariant condition is imposed to the full amplitude $Q = Q_1 + Q_2 + Q_3$, (i.e. the amplitude $Q$ must vanish under the replacement $\varepsilon^\nu(k)$ by $k^\nu$), then the $K_1$ term is completely fixed by $F_D$ and $K_2$, its computation is therefore unnecessary. The constraint required by gauge-invariance is given by:

$$F_D + \lambda_D[K_1 + (P \cdot k)K_2] = 0.$$ \hspace{1cm} (5)

This condition is reminiscent of the similar problem arises in the three indices $R_{\mu\nu\tau}$ amplitude met long time ago by Rosenberg [7] in the triangular loop integration: among the eight terms in the development of $R_{\mu\nu\tau}$, there are two divergent ones that can be related by gauge-invariance to other finite terms, their computations can therefore be avoided. This condition has been employed in the Adler paper [8].

Putting together Eqs.(2), (3), (4) and (5), the full amplitude $Q$ has a simple form with only two finite terms $K_2$ and $K_6$:

$$Q = \lambda_D(eg/2\sqrt{2})V_c\varepsilon^\mu(P)\varepsilon^\nu(k)\left\{[P_\nu k_\mu - g_{\mu\nu}(P \cdot k)]K_2(M^2_W, m^2_D, m_c, m_s) + i\varepsilon_{\mu\nu\alpha\beta} P^\alpha k^\beta K_6(M^2_W, m^2_D, m_c, m_s)\right\}.$$ \hspace{1cm} (6)

The $K_2$ and $K_6$ are calculated to be:

$$K_2(M^2_W, m^2_D, m_c, m_s) = N[Q_c A^e(P, k) + Q_s A^s(P, k)] = 2A^c - A^s,$$

$$K_6(M^2_W, m^2_D, m_c, m_s) = N[Q_c V^e(P, k) + Q_s V^s(P, k)] = 2V^c - V^s.$$ \hspace{1cm} (7)

where $N$ is the number of colors, $Q_{c,s}$ are respectively the charges (in units of $|e|$) of the charm and strange quarks coupled to the photon, finally the $A^{c,s}(P, k)$ and $V^{c,s}(P, k)$ terms are related to the contributions of the axial-vector and vector-current of the quarks coupled to the W boson.

They are given by:

$$A^c(M^2_W, m^2_D, m_c, m_s) = \frac{1}{4\pi^2} \int^1_0 dx \int^1_{-x} dy \frac{m_c - x(m_c + m_s) - 2xy(m_c - m_s)}{M^2_W(x(1 - y) + m^2_Dxy - m^2_s(1 - x) - m^2_sx)}.$$ \hspace{1cm} (8)

$$A^s(M^2_W, m^2_D, m_c, m_s) = \frac{1}{4\pi^2} \int^1_0 dx \int^1_{-x} dy \frac{m_c + x(m_c + m_s) - 2xy(m_c - m_s)}{M^2_W(x(1 - y) + m^2_Dxy - m^2_s(1 - x) - m^2_sx)}.$$ \hspace{1cm} (9)
\[ V^c(M_W^2, m_D^2, m_c, m_s) = \frac{1}{4\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{m_c - x(m_c - m_s)}{M_W^2 x(1-x-y) + m_D^2 xy - m_c^2(1-x) - m_s^2 x} . \]  

(10)

\[ V^s(M_W^2, m_D^2, m_c, m_s) = \frac{1}{4\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{m_s + x(m_c - m_s)}{M_W^2 x(1-x-y) + m_D^2 xy - m_s^2(1-x) - m_c^2 x} \]

\[ = V^c(M_W^2, m_D^2, m_s, m_c) . \]  

(11)

which can be reexpressed in terms of the integral representation :

\[ A^{c,s} = \frac{m_c - m_s}{M_W^2 - m_D^2} + \frac{F^{c,s}}{(M_W^2 - m_D^2)^2} . \]  

(12)

\[ V^{c,s} = \pm \frac{m_c - m_s}{M_W^2 - m_D^2} G_0^{c,s} - \frac{m_{c,s}}{M_W^2 - m_D^2} G_1^{c,s} . \]  

(13)

The + and − signs in Eq.(13) are associated to \( V^c \) and \( V^s \) respectively.

\[ F^{c,s}(M_W^2, m_D^2, m_c, m_s) = \frac{1}{4\pi^2} \int_0^1 dx \frac{\alpha_{c,s}(x)}{x} \log[1 - x(1-x)\beta_{c,s}(x)] . \]  

(14)

\[ G_j^{c,s}(M_W^2, m_D^2, m_c, m_s) = \frac{1}{4\pi^2} \int_0^1 dx \frac{1}{x^j} \log[1 - x(1-x)\beta_{c,s}(x)] \quad (j = 0, 1) . \]  

(15)

where :

\[ \alpha_{c,s}(x) = 2(m_c - m_s)[M_W^2 x(1-x) - m_c^2(1-x) - m_s^2 x] \]

\[ - (M_W^2 - m_D^2)[m_{c,s}(1\mp x)\pm m_{s,c} x] . \]  

(16)

\[ \beta_{c,s}(x) = \frac{M_W^2 - m_D^2}{M_W^2 x(1-x) - m_{c,s}(1-x) - m_{s,c} x} . \]  

(17)

In Eq.(16), the − and + signs correspond respectively to the first and the second lower index i, j of \( m_{i,j} \). In Eq.(6), the three couplings \( \lambda_D, e, gV_{cs} \) associated respectively to \( D_s, \gamma \) and \( W \) are explicitly factorized out, all the dynamics are contained in the \( K_2 \) and \( K_6 \) quantities through Eqs.(7)...(11) that reflect the one loop integration in which one recognizes the \( \frac{1}{4\pi^2} \) factor together with the internal quarks masses \( m_c, m_s \) as well as the external ones \( M_W^2, m_D^2 \). Squaring the amplitude Eq.(6), summing and averaging over the polarizations of the photon and the \( W \), carrying out the phase-space integrations, one gets :

\[ \Gamma(W^\pm \rightarrow \gamma + D_s^\pm) = \lambda_D^2 e^2 g^2 |V_{cs}|^2 M_W^3 (1 - \frac{m_D^2}{M_W^2})^2 (|K_2|^2 + |K_6|^2) . \]  

(18)
We remark that $F_{c,s}$ and $G_{j}^{c,s}$ and consequently $K_2$ and $K_6$ are complex functions because of the logarithm in the integrands: the complexity of these functions can be easily understood since the energy $M_W$ is far beyond the threshold $m_c + m_s$, then the W boson can first decay into a real $c + s$ pair which are converted afterwards to the final hadronic states, therefore the triangle graph develops an imaginary part. Numerically we get, using $m_c = 1.5$ GeV, $m_s = 0.2$ GeV:

$$K_2(P, k) = (1.56 - 1.92i) \times 10^{-4} \text{ GeV}^{-1}$$

$$K_6(P, k) = (2.20 - 2.22i) \times 10^{-4} \text{ GeV}^{-1}. \quad (19)$$

These numbers are rather insensitive to the variation of $m_c$ taken between 1.3 - 1.7 GeV, $m_s$ between 0.15 - 0.5 GeV. The only unknown parameter for the width in Eq.(18) is the dimensionless effective coupling $\lambda_D$ to which is devoted the following section.

The cases of $\pi^0 \rightarrow \gamma \gamma$, $Z^0 \rightarrow \pi^0 + \gamma$, $W^+ \rightarrow \gamma + \pi^+$ and the effective hadron-quarks coupling.

We first reconsider the classical $\pi^0 \rightarrow \gamma \gamma$ case as an illustration for the determination of the effective pseudoscalar meson quark-antiquark coupling. The triangle diagrams we employ to calculate the decay amplitude $W^\pm \rightarrow \gamma + D^\pm_s$ are nothing else but a copy of the famous $\pi^0 \rightarrow \gamma \gamma$ ones computed long time ago by Steinberger [8] in which the fermion circulating in the loop is the proton and $\lambda$ is the strong pion-nucleon coupling $g_{NN\pi^0}$ which is related to the proton mass $M_N$ and the pion decay constant $F_\pi$ via the Goldberger-Treiman relation $g_{NN\pi^0} = \frac{\sqrt{2} M_N}{F_\pi}$. Today, in the modern QCD language, the proton is naturally replaced by the up-down quarks and the pion-nucleon coupling $g_{NN\pi^0}$ becomes an effective coupling between the $\pi^0$ and the quarks involved in the triangle diagrams:

$$\lambda_u \equiv g_{uu\pi^0} = \frac{\sqrt{2} m_u}{F_\pi}, \quad \lambda_d \equiv g_{dd\pi^0} = -\frac{\sqrt{2} m_d}{F_\pi}. \quad (20)$$

where $m_{u,d} = m$ is the up, down quark mass taken to be equal for simplicity. Since only the vector currents (and not the axial-vector ones) are relevant to photons in $\pi^0 \rightarrow \gamma \gamma$, its amplitude analogous to our Eq.(6) is given by the vector part $V^q$ of the quarks.

$$Q(\pi^0 \rightarrow \gamma \gamma) = ie^{2\gamma - (P)\varepsilon(k)\varepsilon_{\mu\nu\alpha\beta}P^\alpha k^\beta} \{ \sum_{q=u,d} (\lambda_q Q^2 V^q) N \}. \quad (21)$$

where $V^q$ is twice the corresponding $V^{c,s}$ of our Eqs.(10),(11) with appropriate modifications of the arguments: $(M_W \rightarrow 0, m_D \rightarrow m_\pi, m_{c,s} \rightarrow m_{u,d} = m)$. The origin
of the factor two is that for each triangle graph, there must be added a second one in which the two photons are crossed due to the Bose statistic. If the pion, as a Goldstone boson, is taken to be massless, then Eq.(10) or (11), after appropriate replacements of the arguments as given above, becomes:

\[ V(0,0,m,m) = -\frac{1}{8\pi^2} \frac{1}{m} \]

such that

\[ V^u = V^d = -\frac{1}{4\pi^2} \frac{1}{m}. \]  \hfill (22)

Putting Eqs.(20) and (22) into Eq.(21), we get:

\[ Q(\pi^0 \rightarrow \gamma\gamma) = -ie^2 \varepsilon^\mu(P)\varepsilon^\nu(k)\varepsilon_{\mu\nu\alpha\beta} P^\alpha k^\beta \frac{\sqrt{2}}{4\pi^2 F_\pi}. \]  \hfill (23)

in perfect agreement with data, justifying a posteriori, the correctness of the effective coupling \( \lambda_q \) as given by the extended G-T relation i.e. Eq.(20).

Two remarks are in order:

(i). The amplitude Eq.(23) is independent on \( m \), the mass of the internal fermion circulating in the loop. This property is only a particular case due to the fact that the masses of the external fields are neglected. Indeed, the \( m \) dependence of \( V(0,0,m,m) \) as given by the denominator in Eq.(22) is exactly cancelled by the same \( m \) dependence in the numerator of the coupling \( \lambda_q \) via Eq.(20). This property, which is also true in the case of \( W^+ \rightarrow \gamma^+ \) PseudoGoldstone of technicolor model (because all the external masses are negligible compared to the techniquark masses in the loop) \cite{F2}, no longer holds when the external masses are dominant as in the cases \( W^\pm \rightarrow \gamma^+ \pi^\pm, W^\pm \rightarrow \gamma^+ D_s^\pm \) that will be discussed later. This property is also summarized in Table 1.

(ii). The effective coupling \( \lambda_q \) as given by the G-T relation Eq.(20) can also be derived\cite{6} in the Nambu, Jona-Lasinio theory\cite{5}, a prototype which dynamically generates masses for both fermions and bosons.

We now go further and consider the cases \( Z^0 \rightarrow \pi^0 + \gamma, W^\pm \rightarrow \gamma + \pi^\pm \) that have been thoroughly analysed by AMP. In some sense, these modes are the inverse of the \( \pi^0 \rightarrow \gamma\gamma \) case, in which one photon \( \gamma \) is off-mass shell. Consider first the \( W^\pm \rightarrow \gamma + \pi^\pm \), the case \( Z^0 \rightarrow \pi^0 + \gamma \) is similar\cite{6}.

\[ [F2]. In the last part of the AMP paper, the authors implicitly use the scheme we advocate here (triangular diagrams + effective hadron quark coupling) to calculate \( W^\pm \rightarrow \gamma + \text{Technipion}. \]
The $W^\pm \rightarrow \gamma + \pi^\pm$ amplitude is again given by Eq.(6) with appropriate substitution in the coupling $\lambda$ and in the arguments of $K_2, K_6$:

$$\lambda_D \rightarrow \lambda_\pi \equiv g_{\pi^+ u d} = \sqrt{2} g_{\pi^0 u d} = \frac{2m}{F_\pi}. \quad (24)$$

Also the internal fermion masses $m_{u,s}$ in Eq(8)...(11) are replaced by the common up-down quark mass $m$, such that the corresponding $K_2, K_6$ for the $W^\pm \rightarrow \gamma + \pi^\pm$ case, after appropriate substitution mentioned above, can be explicitly calculated and put in an analytic form:

$$K_2(P^2 = M_W^2, m_\pi^2 = 0, m, m) = \frac{1}{4\pi^2} \frac{m}{M_W^2} \left[ L^2 \frac{m^2}{2} - 6\beta L + 12 \right]. \quad (25)$$

$$K_6(P^2 = M_W^2, m_\pi^2 = 0, m, m) = \frac{1}{4\pi^2} \frac{m}{M_W^2} \frac{L^2}{2}$$

where

$$L(M_W^2, m^2) = \log \frac{1 + \beta}{1 - \beta} - i\pi \quad \text{and} \quad \beta = \sqrt{1 - \frac{4m^2}{M_W^2}} \quad (\text{for } M_W^2 \gg m^2)$$

The presence of the imaginary part is already discussed. Putting together Eqs(24) and (25) into Eq.(6) we get for the $W^\pm \rightarrow \gamma + \pi^\pm$ amplitude:

$$Q(W^\pm \rightarrow \gamma + \pi^\pm) = \frac{eg/2\sqrt{2}}{F_\pi} V_{ud} \varepsilon^\mu(P) \overline{\varepsilon}(k) \cdot \left\{ [P_\mu k_\mu - g_{\mu\nu}(P \cdot k)] K_2 + i\varepsilon_{\mu\nu\alpha\beta} P^\alpha k^\beta K_6 \right\}. \quad (26)$$

where

$$K_2 \equiv \lambda_\pi K_2 = \frac{1}{4\pi^2} \frac{1}{F_\pi} \frac{m^2}{M_W^2} \left[ 24 - 12\eta + \eta^2 \right]. \quad (27)$$

$$K_6 \equiv \lambda_\pi K_6 = \frac{1}{4\pi^2} \frac{1}{F_\pi} \frac{m^2}{M_W^2} \eta^2 \quad \text{(with } \eta = \log \frac{M_W^2}{m^2} - i\pi \text{)} \quad (28)$$

Absorbing the effective coupling $\lambda_\pi$ into the dynamical parts $K_2, K_6, (\overline{K}_i = \lambda_\pi K_i)$ we are now in a position to compare our Eq.(26) with the AMP paper, namely their Eq.(2.11). These authors do not use, neither the triangle graphs nor the effective hadron-quarks coupling as we do in this paper, but instead they assume the Brodsky-Lepage (BL) asymptotic off-shell photon $\pi^0 \gamma \gamma$ form factor\(^{[10]}\) calculated in QCD. In our notation, they simply assume:

$$\overline{K}_6 = \frac{F_\pi}{M_W^2}. \quad (29)$$
where the right-hand side of Eq.(29) $\frac{F_\pi}{M_W^2}$ is the asymptotic ($P^2 \to \infty$) BL form factor. Also AMP assume that asymptotically $K_2 = K_6$. For the mass scale in question, i.e. when $P^2 = M_W^2$, $F_\pi$ is in agreement asymptotically with the BL form factor $\frac{F_\pi}{M_W^2}$. The work of BL suggests that as $P^2 \to \infty$, the form factor $K_6$ decreases with a power law behaviour $(P^2)^{-n}$ with $n = 1$, and the residue is fixed by the scale $F_\pi$ representing the non-perturbative QCD dynamics of the pion. In our model, Eq.(28) indicates that at large $P^2$ the form factor $K_6$ also behaves like $(P^2)^{-1}$, however the residue, obtained from the triangle loop integral, is equal to $\frac{1}{4\pi^2} \frac{m^2}{F_\pi} [(\log \frac{P^2}{m^2})^2 + \pi^2]$. Here, we take the absolute value of $|K_6|$ since it is complex, instead of a real number $F_\pi$ in the BL work. The residue $\frac{1}{4\pi^2} \frac{m^2}{F_\pi} [(\log \frac{P^2}{m^2})^2 + \pi^2]$ tells us that the dynamic in $W^\pm \to \gamma + \pi^\pm$ still remembers (through the $m$ dependence) the quarks bound inside the pion. Moreover if we try to identify our residue with the Brodsky-Lepage one (i.e. $F_\pi$), then we get an equation that determines the mass $m$:

$$\frac{1}{4\pi^2} \frac{m^2}{F_\pi} [(\log \frac{P^2}{m^2})^2 + \pi^2] = F_\pi.$$  \hspace{1cm} (30)

or equivalently:

$$4\pi^2 x = (\log x)^2 + 2(\log \frac{P^2}{F_\pi^2}) \log x + \pi^2 + (\log \frac{P^2}{F_\pi^2})^2.$$  \hspace{1cm} (31)

with $x = \frac{P^2}{m^2}$

This non-linear equation has an unique solution $m \simeq 55$ MeV which could be considered as the running mass of the up-down quark at the scale $M_W$: $m(\mu = M_W) \simeq 55$ MeV.

It is a good surprise that our model, when matched with the BL one, gives a plausible value $m = 55$ MeV to the up-down quark mass. In other words, our amplitude $W^\pm \to \gamma + \pi^\pm$ as given by Eqs.(26)..(28) is the same as the AMP one if we take $m = 55$ MeV. For such value of $m$, the effective coupling $\lambda_\pi$ as given by Eq.(24) is equal to 0.83. Also the numerical values of the dynamical quantities $K_{2,6}$ given by Eq.(25) become:

$$K_2 = (0.56 - 0.58i) \cdot 10^{-5} GeV^{-1}$$
$$K_6 = (2.16 - 0.99i) \cdot 10^{-5} GeV^{-1}.$$  \hspace{1cm} (32)

These values for $W^\pm \to \gamma + \pi^\pm$ are to be compared with the numerical ones of the $W^\pm \to \gamma + D^+_s$ case given in Eq.(19).

Encouraged by the above results where the use of the effective pion quark-antiquark $\lambda_\pi$ coupling (together with the triangle diagram) turns out to be satisfactory in $\pi^0 \to \gamma\gamma$, as well as in $W^\pm \to \gamma + \pi^\pm$, $Z^0 \to \gamma + \pi^0$ for which experimental
limits are known\cite{11}, we come back to our decay $W^\pm \to \gamma + D^\pm_s$ in Eq.(6). The dynamical parts symbolized by $K_{2,6}$ have been computed and their numerical values are given in Eq.(19), it remains the coupling $\lambda_D$.

We conjecture that $\lambda_D$ is of the same order as $\lambda_\pi = 0.83$ calculated above. Indeed, in the old days of the strong interaction with SU(3) flavour symmetry, all the couplings: pion-nucleon, Kaon-hyperons etc... as compiled in Ref\cite{12} turn out to be similar, the inclusion of charm with SU(4) symmetry seems plausible.

It is an ascertainment as a fact that the strong couplings are similar in the flavour world of particles, it is their masses and their weak interaction properties that differentiate them.

Then let us assume $\lambda_D \simeq \lambda_\pi$, in such case we obtain $B(W^\pm \to \gamma + D^\pm_s) \simeq 10^{-6}$. On the other hand, if we try the extreme case in which the $D_s$ is considered as a Goldstone boson on the same footing as the pion ( at the W mass scale the $D_s$ is still very light ), then the G-T relation extended to charm would be $\lambda_D = \frac{m_c + m_s}{F_D} \simeq 8$. This extreme case which is not firmly justified, leads to an unplausible large value $5 \cdot 10^{-5}$ for the branching ratio.

Another guess

$$\frac{\lambda_D}{\lambda_\pi} = \frac{(m_c + m_s)/F_D}{(\overline{m}_u + \overline{m}_d)/F_\pi}$$

where $\overline{m}_{u,d}$ are the constituent\cite{F3} up-down quark mass $\simeq 300$ MeV - would give a branching ratio $\simeq 2 \cdot 10^{-6}$, which is also our preferred value.

Compared to the AMP work, the main difference with ours rests in the dynamical quantities $K_2, K_6$. By assuming the BL asymptotic form-factors, AMP argue that all radiative decay modes of $W^\pm$ into a pseudoscalar meson $P^\pm : \pi^\pm, K^\pm, D^\pm, D^\pm_s, B^\pm$ are approximately equal (beside the Cabibbo-Kobayashi-Maskawa CKM mixing ). Since their form-factors $\overline{K}_{2,6}$, as fixed by $F_\pi, F_K, F_D, F_B$ are similar, the branching ratios turn out to be $3 \cdot 10^{-9}$ for $\pi^\pm$ and $D^\pm_s$ and much smaller for $K^\pm, D^\pm, B^\pm$ by CKM suppression.

On the contrary, in our model, the dynamical terms $K_2, K_6$ (given by triangular loop integral) still remember the quarks - via their masses - inside the $P^\pm$. Therefore in our scheme, the rate $W^\pm \to \gamma + D^\pm_s$ is much larger than the $W^\pm \to \gamma + \pi^\pm$ one by at least a factor $(m_c/m_u)^2$. (See Table 1 and our Eqs (19), (31)).

\[ F_3 \] It is quite possible that the BL form factors - employed by AMP in the calculation of the $W^\pm \to \gamma + \pi^\pm$ mode - underestimate its rate, since it corresponds to the use of a value 55 MeV (quasi current mass) for the up down quark. If instead we take the constituent mass $\simeq 300$ MeV, then the $W^\pm \to \gamma + \pi^\pm$ branching ratio would be $10^{-7}$ (instead of $3 \cdot 10^{-9}$ in the AMP calculation).
In the AMP scheme, the effective coupling $\lambda_P$ must decrease with the quark-mass like $\frac{1}{m_q}$ in order to compensate the dependence like $m_q$ of $K_2, K_6$, such that the amplitudes $\bar{K}_{2,6} = \lambda_q K_{2,6}$ are independent on $m_q$. We don’t understand why the effective coupling of K meson with strange quark, D meson with charm quark and B meson with b quark is smaller and smaller like $\frac{1}{m_q}$, while the ones of pion with up-down quark or technipion with techniquark, as given by the G-T relation, are proportional to $m_q$.

In conclusion, we present in details the reasons - summarized in Table 1 - why the mode $W^\pm \rightarrow \gamma + D_s^\pm$ is strongly enhanced compared to $W^\pm \rightarrow \gamma + \pi^\pm$, and could be observable with a branching ratio around $10^{-6}$. But the last words, as always, are on the experimental side. Only experiments can tell us whether or not the mode discussed here has a viable chance of being detected, and we are eager to learn from experiments the possible observation of photon and $D_s$ in the W decay. The interests for this mode are mentioned from the beginning.
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Table 1

| \( \pi^0 \to \gamma + \gamma \) | \( W^\pm \to \gamma + P_{Tech}^\pm \) | \( W^\pm \to \gamma + \pi^\pm \) | \( W^\pm \to \gamma + D_s^\pm \) |
|-----------------|-----------------|-----------------|-----------------|
| \( \lambda_{\pi} \sim \frac{m_u}{F_\pi} \) | \( \lambda_{P_{Tech}} \sim \frac{M_{Techniquark}}{F_{Technipion}} \) | \( \lambda_{\pi} \sim \frac{m_u}{F_\pi} \) | \( \lambda_{D} \geq \lambda_{\pi} \) |
| \( K_{2,6} \sim \frac{1}{m_u} \) | \( K_{2,6} \sim \frac{1}{M_{Techniquark}} \) | \( K_{2,6} \sim \frac{m_u}{M_W^2} \) | \( K_{2,6} \sim \frac{m_c}{M_W^2} \) |
| \( \overline{K}_{2,6} \equiv \lambda K_{2,6} \sim \frac{1}{F_\pi} \) | \( \overline{K}_{2,6} \sim \frac{1}{F_{Technipion}} \) | \( \overline{K}_{2,6} \sim \frac{m_u^2}{M_W^2} \frac{1}{F_\pi} \) | \( \overline{K}_{2,6} \geq \left( \frac{m_c m_u}{M_W^2} \right) \frac{1}{F_\pi} \) |

Figures captions: Feynman diagrams for the process \( W^\pm \to \gamma + D_s^\pm \).

Table 1 caption: Treatment of the four decays \( \pi^0 \to \gamma \gamma \), \( W^\pm \to \gamma P^\pm_{Technipion} \), \( W^\pm \to \gamma \pi^\pm \), \( W^\pm \to \gamma D_s^\pm \) by a common approach.