Cosmological neutrino entropy changes due to flavor statistical mixing

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Abstract – Entropy changes due to delocalization and decoherence effects should modify the predictions for the cosmological neutrino background (CνB) temperature when one treats neutrino flavors in the framework of composite quantum systems. Assuming that the final stage of neutrino interactions with the γe−e+ radiation plasma before decoupling works as a measurement scheme that projects neutrinos into flavor quantum states, the resulting free-streaming neutrinos can be described as a statistical ensemble of flavor-mixed states. Even not corresponding to an electronic-flavor pure state, after decoupling the statistical ensemble is described by a density matrix that evolves in time with the full Hamiltonian accounting for flavor mixing, momentum delocalization and, in case of an open-quantum-system approach, decoherence effects. Depending on the quantum measurement scheme used for quantifying the entropy, mixing associated to dissipative effects can lead to an increase of the flavor-associated von Neumann entropy for free-streaming neutrinos. The production of the von Neumann entropy mitigates the constraints on the predictions for energy densities and temperatures of a cosmologically evolving isentropic fluid, in this case, the cosmological neutrino background. Our results state that the quantum mixing associated to decoherence effects are fundamental for producing an additive quantum entropy contribution to the cosmological neutrino thermal history. According to our framework, it does not modify the predictions for the number of neutrino species, \( N_\nu \sim 3 \). It can only relieve the constraints between \( N_\nu \) and the temperature ratio, \( T_{\nu}/T_\gamma \), by introducing a novel ingredient to re-direct the interpretation of some recent tantalizing evidence that \( N_\nu \) is significantly larger than by more than 3.

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The question of late-time entropy production that leads to changes in the cosmological neutrino background (CνB) temperature [1–4] has been recently posed as theoretical puzzle on the foundations of the cosmological standard model. The textbook literature sets that, due to the electron-positron (e+e−) annihilations which heated the background radiation after neutrino-radiation decoupling, the CνB temperature should be lower by a factor

\[
\frac{T_{\nu 0}}{T_{\gamma 0}} = \left( \frac{4}{11} \right)^{1/3}
\]

when compared to the cosmological microwave background (CMB) temperature, where \( T_{\nu 0} \) and \( T_{\gamma 0} \) are the CνB and CMB temperatures at present, respectively.

Speculative factors which may cause some slight departure from the standard value of \( T_{\nu 0}/T_{\gamma 0} = (4/11)^{1/3} \) produce an overall increase of the neutrino energy density of an order of 1%, i.e. a tiny effect when included in practical calculations. It is originated either from corrections due to finite-temperature quantum field theories, which lead to an additional slight heating of the neutrinos [5], or from a secondary heating due to e+e− annihilation prior to decoupling [2].

The questions posed in this letter concern therefore how flavor mixing effects could introduce additional corrections to \( T_{\nu 0}/T_{\gamma 0} \), which become effective just some time after decoupling, deep inside the free-streaming regime, and how it affects the constraints over the number of neutrino species. Flavor-related von Neumann entropies may therefore be relevant in such a context. In particular, on describing neutrino flavor oscillations in the framework of composite quantum system [6]. Any relative contribution to the entropy changes of a cosmologically evolving isentropic fluid,
namely the CνB, could modify the predictions for its corresponding temperature which would affect the rapport to the energy densities. Since the von Neumann entropy for a composite quantum system of well-defined flavor quantum numbers can be obtained in the framework of the generalized theory of quantum measurement, the arena of cosmological neutrinos is indeed suitable for obtaining a functional characterization of the corresponding flavor-associated entropies.

Let us recapitulate that the main difference between $T_\nu$ and $T_\gamma$ must arise from $e^+e^-$ annihilations after neutrino decoupling from the $\gamma e^+e^-$ radiation plasma. Prior to annihilations, the total entropy density that includes all the ultra-relativistic fermionic and bosonic species is given by [7]

$$s_1(a) = \frac{2\pi^2}{45} T_\nu^3 \left[ 2g_\gamma + 3 \left( g_\nu + g_\rho \right) \right],$$  \hspace{1cm} (1)

where $g_\gamma = 2$, $g_\nu = 4$, and $g_\rho = 6$ are the respective numbers of degrees of freedom for photon, electron/positron and neutrino/antineutrino according to the Standard Model, and $T_\nu$ is the temperature of the radiation plasma. After annihilations, the electrons and positrons have gone away and the photon and neutrino temperatures are no longer identical. The total entropy density is thus given in terms of the above-defined temperatures, $T_\nu$ and $T_\gamma$, by

$$s_f(a) = \frac{2\pi^2}{45} T_\nu^3 \left[ g_\gamma \frac{T_\gamma^3}{T_\nu^3} + 3 \left( \frac{7}{8} g_\nu \right) \right].$$  \hspace{1cm} (2)

Assuming the isentropic evolution over eqs. (1) and (2), one sets $a^3 s_1(a) = a^3 s_f(a)$ and easily obtains the temperature ratio $T_{\nu,0}/T_{\nu,0} = (4/11)^{1/3}$, which can be substituted into the energy and particle number density definitions, $\rho$ and $n$, in order to give $\rho_\nu/\rho_\gamma = N_\nu/(7/8)/(4/11)^{1/3}$ and $n_\nu/n_\gamma = 3/11$, where $N_\nu$ is the number of neutrino species [7]. One should notice that the value of $g_\rho = 6$ is introduced in the absence of right-handed neutrinos. As the right-handed neutrinos are not produced in the early universe (the masses are so small to be irrelevant), only one spin state is produced and $g = 6$ (see footnote 1). Meanwhile, it is straightforward to verify that the number of $\nu$ degrees of freedom do not affect the rapport between $T_\nu$ and $T_\gamma$.

Moreover, as usually noticed in the literature, the temperature ratio of radiation and neutrinos is assumed to be constant. Once neutrinos are massive they may enter the non-relativistic regime below a scale factor $a$ of about $10^{-4}$. The reference [8] provides us with a suitable explanation for the consequence of finite neutrino thermal speed: the initial phase-space density for massive neutrinos is a relativistic Fermi-Dirac distribution, preserved from the time when the neutrinos decoupled in the early universe. Decreasing the temperature with time is compensated by relating proper momentum to co-moving momentum. Therefore, ignoring perturbations, the present-day distribution for massive neutrinos is the relativistic Fermi-Dirac distribution—not the equilibrium non-relativistic distribution—because the phase-space distribution was preserved after neutrino decoupling. The neutrinos' distribution function is similar to that for a massless particle and neutrinos temperature scales as $a^{-1}$.

Assuming that free-streaming flavor oscillating neutrinos evolve isentropically straightforward to the era of low temperatures, the maximal free-streaming neutrino entropy may change due to decoherence effects. It is parameterized by $\delta S_{\nu}$ into the following relation for neutrino temperatures:

$$g_\nu \left( \frac{7}{8} \right) \frac{2\pi^2}{45} T_\nu^3 = \left( \frac{7}{8} \right) \frac{2\pi^2}{45} T_\nu^3 \pm n_\nu \delta S_{\nu}.$$  \hspace{1cm} (3)

By observing that $n_\nu = (9/5)\pi^{-2}T_{\nu,0}^2$, and assuming that $g_\nu = g_\nu^{\prime}$, i.e. that the number of degrees of freedom is conserved, eq. (3) results in

$$T_\nu = T_\nu \left( 1 \pm \frac{324}{\pi^4} \delta S_{\nu} \right)^{1/3},$$  \hspace{1cm} (4)

where, as we shall notice, $\delta S_{\nu}$ stands for the associated von Neumann entropy per flavor quantum ensemble in a volume $d^3p_\nu d^3q_\nu$ of the phase space, so that $\rho_\nu \delta S_{\nu}$ corresponds to the total von Neumann entropy density.

The modifications introduced by eq. (4) are guaranteed by the subadditivity of entropies associated to density matrices that describe independent quantum systems $A$ and $B$, i.e. systems with completely or partially uncorrelated quantum properties. For cosmological neutrino ensembles, the quantum property related to the system $A$ is approximated by the momentum, $p$, which is read as a quantum phase-space element through its relation to the classical momentum established by the distribution function, $f(p)$, utilized for computing averaged quantities like $\rho_\nu$ and $n_\nu$ [7]. The quantum property related to the system $B$ is obviously the flavor quantum number. The triangle inequality given by

$$|S_A - S_B| \leq S_{AB} \leq |S_A + S_B|,$$  \hspace{1cm} (5)

sets upper and lower limits for the total entropy, $S_{AB}$, that describes a composite quantum system with the above-mentioned characteristics. While according to Shannon’s theory the entropy of a composite system can never be lower than the entropy of any of its parts [9], in quantum mechanics the triangle inequality sets that the entropy of the joint system can be less than the sum of the entropy of its components due to the possibility of entanglement. It happens when the localization character introduced by the momentum distribution function, $f(p)$, changes the pattern of flavor oscillations, as an intrinsic decoherence mechanism promoted by some dynamics of the delocalization effect similar to those caused by the space-time
evolution of mass-eigenstate wave packets. In parallel, the right-hand inequality can be interpreted as saying that the entropy of a composite system is maximized when its components are completely uncorrelated, i.e., when the entanglement disappears.

Turning back to the main point of our analysis, let us report about some foundations on quantum statistics in order to quantify $\delta S_{\text{VN}}$ into eq. (4). The density matrix representation of a composite quantum system of three flavor species, namely $e$, $\mu$, and $\tau$, is given by

$$
\rho(t) \equiv \rho = \sum_{\alpha=e,\mu,\tau} w_{\alpha} M_{\alpha}^0, \quad \sum_{\alpha=e,\mu,\tau} w_{\alpha} = 1, \quad (6)
$$

where $w_{\alpha}$ are the statistical weights, and $M_{\alpha}^0(t)$ are the $\alpha$-flavor projection operators which depend on the flavor-associated mixing angles and are constrained by the unitarity condition $\sum_{\alpha} M_{\alpha}^0(t) = 1$.

The von Neumann entropy provides an important functional defined in terms of the density matrix as

$$
S(\rho) = -\text{Tr}\{\rho \ln(\rho)\}, \quad (7)
$$

where the Boltzmann constant, $k_B$, was set equal to unity. The entropy $S(\rho)$ quantifies the departure of a composite quantum system from a pure state, i.e. it implicitly measures the entanglement of an ensemble of flavor states describing a given finite system. As one can expect, quantum measurements induce modifications on the von Neumann entropy of the system. The entropy change due to a non-selective measurement scheme described by operations parameterized by the projection operators $M_{\alpha}^0$ [9] is given by

$$
\Delta S = S(\rho') - S(\rho) \geq 0, \quad (8)
$$

where

$$
S(\rho') = S\left(\sum_{\alpha} P_{\alpha}^0(\rho_{\alpha})\right), \quad (9)
$$

with $\rho_{\alpha} = (P_{\alpha}^0(\rho_{\alpha})^{-1} M_{\alpha}^0(\rho_{\alpha}) M_{\alpha}^0(\rho_{\alpha})$, and where $P_{\alpha}^0$ are the probabilities of measuring $\alpha$-flavor eigenstates at time $t$.

In the density matrix, one has

$$
P_{\alpha}(t)^0 = \text{Tr}\{M_{\alpha}^0(\rho_{\alpha})\} = \sum_{\beta=e,\mu,\tau} w_{\beta} \text{Tr}\{M_{\alpha}^0(\rho_{\beta}) M_{\beta}^0\} = \sum_{\beta=e,\mu,\tau} w_{\beta} P_{\alpha \rightarrow \beta}(t), \quad (10)
$$

with $P_{\alpha \rightarrow \beta}(t) = |\langle \nu_{\alpha}^0 | \nu_{\beta}^{\alpha}(t) \rangle|^2$ describing the $\alpha$- to $\beta$-flavor conversion probabilities in the single-particle quantum mechanics framework.

We assume that $\delta S_{\text{VN}}$ quantifies the level of flavor-mixing during the evolution of cosmological neutrino ensembles either from pure states to statistical mixtures, or from statistical mixtures to maximal statistical mixtures. It results into a kind of late-time entropy production.

The above conceptions related to the von Neumann entropy bring up important insights into the scope of distinguishing measurement procedures [9] and quantifying the degree of mixture of statistical ensembles. In order to distinguish possible decoherence effects, i.e., those caused by dissipative mechanisms (extrinsic decoherence) from those due to delocalization characteristics (intrinsic decoherence), we shall parameterize $\delta S_{\text{VN}}$ by two different ways. Both effects lead to increasing entropy density values after neutrino decoupling.

The cosmological standard model prescription for neutrino decoupling in the early universe sets that the three neutrino species ($e$, $\mu$, $\tau$) are kept in thermal contact with the radiation plasma through the elastic scattering process with background electrons (positrons). Different flavor neutrinos ($e$, $\mu$, $\tau$) coexist with the same averaged temperature: neutrinos corresponding to the same volume of the phase space (that is constrained by some momentum distribution), reach the thermal equilibrium through the elastic scattering which, in the language of quantum mechanics, corresponds to a measurement scheme. The proportion between the corresponding cross-sections, $\sigma_\nu$, is given by

$$
\sigma_e : \sigma_\mu : \sigma_\tau \Leftrightarrow 1 : 0.16 : 0.16. \quad (11)
$$

After scattering terminates, one should have an averaged statistical ensemble described by

$$
1 : 0.16 : 0.16 \Leftrightarrow w_e : w_\mu : w_\tau, \quad (12)
$$

where we have introduced the statistical weights $w_{\alpha}$, with $\alpha = e, \mu, \tau$. Since it does not correspond to the maximal statistical mixture, $w_e = w_\mu = w_\tau$, decoherence effects may lead the free-streaming flavor ensemble to the maximal entropy configuration.

The composite quantum system of neutrino flavors certainly reaches the configuration of a maximal statistical mixture corresponding to $S = \ln(3)$ before entering the non-relativistic regime. Due to some extrinsic decoherence mechanism, one identifies the variation of the von Neumann entropy as a deviation from the entropy of such a maximal statistical mixing through $\delta S_{\text{VN}} = S(N_3) - S(\rho_{t=0})$, for which $t = 0$ is defined as the time of neutrino decoupling and $S(\rho_{t=0})$ is computed in terms of the statistical weights $w_{\alpha}$.

The alternative way of quantifying such an entropy increase after decoupling is through the parametrization of $\delta S_{\text{VN}}$ by $\delta S_{\text{DL}}^{\text{VN}} = S(\rho_{\text{time}}) - S(\rho_{t=0})$. Assuming that some intrinsic (dynamical) decoherence mechanism suppresses the time dependence of the non-diagonal elements of the density matrix, the delocalization (DL) effects can be reproduced by time-averaging the density matrix: $(\rho_{\text{time}})$.

Both parameterizations can be conceptually modified by adding to $\delta S_{\text{DL}}^{\text{VN}}$ the entropy change due to a non-selective measurement scheme given by $\Delta S(t = 0)$ from eq. (8). As one can see from fig. 1, the non-selective measurements do not change $\delta S_{\text{VN}}$ at $t = 0$. By assuming the next-to-standard phenomenological values for the neutrino flavor mixing angles, namely the tri-bimaximal approximation.
That quantum mixing is fundamental for introducing the additive quantum entropy. It is convenient to notice that the entropy change due to a non-selective measurement performed over a maximal statistical mixture is null, i.e. $S(\rho) = S(\rho')$. For the values corresponding to the rapport from eq. (12), one should have $w_\nu \sim 0.68$. This leads to corrections of the order of 3–4% on the maximal bounds.

Finally, one should observe that the mixing entropy assumes its maximum value which, in case of an $n$-level system, corresponds to $\delta S_m = \ln(n)$. For a three-flavor system of neutrinos, it corresponds to $\delta S_{\nu N} = 0$. For maximal statistical mixing, the non-selective measurement does not change either the energy or the entropy of the system, while the selective measurement changes the entropy [9]. This means that the von Neumann entropy and the above-related quantities gain relevance in the study of the measurement procedures which take into account the flavor eigenstate correspondence to measurable energies.

Observing that the entropy increase follows the level of mixing of the systems, the maximal variation for the ratio $T_{\nu o}/T_{\nu o} = (4/11)^{1/3}$ can be set through the reading of eq. (4) by means of eq. (5) that results in

$$\left(1 - \frac{324}{7\pi^4} \ln 3\right) T_{\nu 0}^\nu \lesssim \left(1 + \frac{324}{7\pi^4} \ln 3\right)^\nu \Rightarrow$$

$$0.86 \lesssim \frac{T_{\nu 0}^\nu}{T_{\nu 0}} \lesssim 1.28$$

(13)

in case of realistic values given by the tri-bimaximal mixing. Even assuming an upper limit for $\theta_{13}$ such that $\sin^2(\theta_{13}) \sim 0.1$, which has been tested from several (reactor) experiments [10] in the last two years, the results here obtained would only slightly deviate from the results for the tri-bimaximal approach. In this case, it does not add any incremental interpretation of results obtained from the tri-bimaximal approach. The above values can be mitigated if one considers the $\delta S_{\nu N}^D$ in place of $\delta S_{\nu N}$.

In the same way of some eventual indirect exotic coupling of neutrinos to electrons or photons that could have kept neutrinos longer in equilibrium with photons, entropy changes due to flavor mixing introduce a novel ingredient that suggests that the $\nu - \gamma$ number density ratio could not be diluted by 4/11. The possibility of attenuating the constraints on the late-time entropy production from the large-scale structure and CMB anisotropies has already been considered [11–14]. Herein the referred entropy modifications can change the rapport between the effective number of neutrino families, $N_\nu$, and any parameter phenomenologically depicted from the pattern of large-scale structures and CMB anisotropies.

To clear up this point, let us assume that the standard value for the $\nu - \gamma$ energy density ratio, 7/8, leads to the following relation between the red-shift of matter-radiation equality and $N_\nu$:

$$1 + z_{eq} \propto \left[1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_\nu\right]^{-1}.$$  

(14)
To keep the ratio \( \rho_\nu/\rho_\gamma \approx N_\nu(7/8)(4/11)^{4/3} \) consistent with the phenomenology, the modifications introduced by the growing von Neumann entropy discussed above introduce the upper and lower bounds to \( N_\nu \), through the modified parameter

\[
N'_\nu \sim N_\nu \left(1 \pm 324 \frac{\delta S_{VN}(w_e = 1)}{7\pi^4} \right)^{-\frac{4}{3}} \tag{15}
\]

that, from this point on, has to be interpreted as novel phenomenological parameter, and not as the number of neutrino species. For \( N_\nu \sim 3 \), one should have the bounds

\[
1.7 \lesssim N'_\nu \lesssim 8.1, \tag{16}
\]

where the bounds are for neutrino ensembles being produced as pure states after decoupling and evolving to a maximal statistical mixture in the free-streaming regime.

Essentially, we are not modifying the predictions for the number of neutrino species, \( N_\nu \), which was already accurately computed, for instance, at [15], where the distortions in the \( \nu_e \) and \( \nu_\mu/\nu_\tau \) phase-space distribution that arise in the standard cosmology due to electron-positron annihilations have been considered. Our results just relieve the constraints on the value of the parameter \( N'_\nu \) that re-enters into the expression derived from matter-radiation equality (cf. eq. (14)) and that could lead to some phenomenological tension [2,16].

Our result enlarges the range of phenomenological agreement for tantalizing cosmological and terrestrial evidences that suggest that the number of light neutrinos may be greater than three [17,18]. A recent re-examination of cosmological bounds on extra light species have been performed [18] in order to consider the cosmological scenario with two sterile neutrinos and explore whether partial thermalization of the sterile states can mitigate the conflict between apparently ambiguous cosmological constraints on the number of neutrino species.

Accurately computed values of helium abundance depicted from the Big Bang nucleosynthesis (BBN) formalism constrain the number of relativistic neutrino species present during nucleosynthesis, while measurements of the CMB angular power spectrum constrain the values of the effective energy density of relativistic neutrinos and photons. Therefore, scenarios where new sterile neutrino species may have different contributions to \( N_\nu^{(\text{eff})} \), respectively from BBN and CMB data, can be reconciled through the entropy corrections to \( C_eB \) computed through the approach that we have introduced. The same argument can be reported on the analysis of increasing the effective number of neutrino species, \( N_\nu \), in the early universe, focussed on introducing extra relativistic species (hot dark matter or dark radiation [19]).

The above results does not change the significance and the magnitude of the finite-temperature electromagnetic corrections to the energy density of the \( \gamma e^+ e^- \) radiation plasma [15,20,21] or of the finite-temperature QCD corrections [22]. They are of the same order of magnitude of flavor mixing corrections upon the averaged temperature of decoupling for different neutrino species, which also depend on the mixing parameters [22]. Although the above limits relieve the constraints on the possible values for \( N_\nu \), neutrino heating/freezing corrections can be a
little larger than those predicted by the finite-temperature quantum field theories, for instance, late-time entropy production due to some weakly interacting scalar field decay [1,2].

To conclude, one should observe that it is commonly assumed that ultra-relativistic thermal relics are in perfect equilibrium state even after decoupling. For photons in CMB this has been established with a very high degree of accuracy. Thus, the same assumption has been made about free-streaming neutrinos. Our line of reasoning calls into attention the premise of coherent flavor eigenstates at the epoch of neutrino decoupling from the radiation background. It has been assumed that the free evolution of flavor ensembles leads to some spontaneous lowering of the coherence interference effects, associated with the destruction of the oscillation pattern and with the vanishing of the 3-partite quantum entanglement. Some previous studies have addressed the issue of the decoherence history of the cosmological neutrinos and its implications in probing the best values for neutrino masses and in modifying the power spectrum of large-scale structures [22–24]. In the single-particle quantum mechanics framework, it has been supposed that flavor wave packets could have spatial extents that would be comparable to the space-time curvature scale of the universe itself. The delocalization effects lead to decoherence in the same way as the one that we have presented in this letter (cf. fig. 2).

Likewise, it could also result from a some suitable prescription for some extrinsic dissipative mechanism that results into the wave function collapse, for instance, as a consequence of describing the flavor ensemble as an open quantum system. Assuming that some kind of decoherence mechanism results in increasing the level of mixing for neutrino flavor ensembles in the cosmological background, the analysis developed here would not have only addressed to the entropy issues related to neutrinos, but would also suggest some new insights into the role of quantum coherence and decoherence in the history of these relic particles.

Finally, our results states that the flavor quantum mixing of neutrino mass eigenstates associated to decoherence effects are fundamental for producing an additive contribution of quantum entropy to the cosmological neutrino thermal history. According to our framework, it does not modify the predictions for the number of neutrino species, $N_{\nu} \approx 3$. It can only relieve the constraints between $N_{\nu}$ and the neutrino to the radiation temperature ratio, $T_{\nu}/T_r$, by introducing a novel ingredient to re-direct the interpretation of some recent tantalizing evidence that $N_{\nu}$ is significantly larger than by more than 3. Obtaining the neutrino entropy changes well inside the free-streaming propagation regime is therefore a relevant aspect that has to be considered while computing cosmological neutrino properties, namely the cosmic energy density, the constraints related to the precise number of neutrino species, $N_{\nu}$, the specific entropy itself, and eventually, the neutrino mass values [6].

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