Applied the Software of MATLAB to Calculate the Critical Clearing Time in Power System

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ABSTRACT

Aims: to avoid improper critical clearing time values due to clerical errors during the artificial calculation process, so that takes advantage of MATLAB application software to compile with those equations such as power angle, swing equation, equal area criterions, etc. to calculate of critical clearing time.

Study Design: This article starts introduction with what the way of the calculation such as power angle, swing equation, equal area criterions, etc in literature, and referred lots of the latest literature to develop.

Place and Duration of Study: The setting of the critical clearing time plays an important role in the power system. The start-up time of any protection relay must be shorter than the critical clearing time; otherwise the fault will expand and cause serious damage when the system fails. Therefore, it is an important question the set time of the protection relay needed to caution in planning design.

Methodology: To use MATLAB application software links with above equations to compile the program the calculation of critical clearing time.

Results: The program has been proved very effective and accurate for calculating the reasonable setting value of proportion relay, the same time it would shortened of assignment time by design planners.

Conclusion: Computerized operating procedures can be used to avoid improper critical clearing time values due to clerical errors during the artificial calculation process.
ABBREVIATIONS

\( X_{eq} \) equivalent reactance
\( X'_{d} \) behind E' ect axis transient reactance
\( X_{TR} \) reactus of transformer
\( X_{12}-X_{13}-X_{14} \) reactus of transmission line
\( E' \) internal voltage of the synchronous machine
\( \delta \) power angle
\( V_{bus} \) infinite bus
\( P_{max} \) power of machine
\( P_{1max} \) normal status of power of machine
\( p_{e} \) power of electricity
\( p_{e1} \) normal status of power of electricity
\( p_{a} \) acceleration power
\( H \) energy at synchronous peed / voltage rating
\( X_{Th} \) Thevenin reactance
\( V_{Th} \) Thevenin voltage source
\( P_{1max} \) abnormal status of power of machine
\( V_{Thp} \) after clearing of Thevenin voltage source
\( X_{Thp} \) after clear of Thevenin reactance
\( P_{2max} \) after clearing status of power of machine
\( p_{e2} \) after clearing status of power of electricity
\( \delta_{0} \) initiate power angle
\( \delta_{1} \) critical clearing angle
\( \delta_{2} \) critical clearing angle
\( \delta_{3} \) maximum power angle
\( \delta_{cr} \) the largest critical clearing angle
\( t_{c} \) critical clearing time
\( t_{cr} \) the largest critical clearing time
\( f \) frequency of operation
\( \rho_f \) power factor

Keywords: Power system; swing equation; critical clearing time (CCT); equal area criterions.

1. INTRODUCTION

Any disturbance occurred on power system which was equipment failure or lightning hitting or unknown case on to leading inevitability. When the power system is disturbed, the protection relay must timely detect and activate the circuit breakers at both ends of the fault point to isolate the fault point from the system. However, the operating time of the circuit breaker is being controlled with the relay protection has a close relationship with the critical clearing time.

The relationship between them is that the time of the circuit breaker opening must be less than the system critical clearing time; otherwise the system disturbance will expand and cause a crash. Speaking of the critical clearance time of the system is obtained through the impedance parameter values of each device take into the swing equation and the equivalent area rule to calculate out. It is learned from the above that the calculation process is extremely tedious and can easily lead to make a slip in writing, so that stimulate to develop a set of programs to perform the calculation steps. This is the main point of this article.

This article divides three paragraphs to introduce as follows:

- Literature description consists of on power system single diagram, power system equivalent circuit diagram, swing equation, equal area criterion, and critical clearing time and to show the calculation steps.
- To validate the results of the proposal program by an example.
- Application and suggestion end the article.

2. LITERATURE REVIEW

This paragraph of literature review will be back with the examples in the textbook of power system analysis to begin and explanation. Fig.1 shows a single diagram of a three-phase 60 Hz synchronous generator connected to an infinite bus through a transformer and two parallel transmission lines. All reactance values are converted to the same value with the same system reference value (p.u). If the infinite bus receives 1.0 p.u of effective power under 0.95 backward power factors, try to solve:

- The internal potential of the generator.
- The power equation provided by the generator at the power angle \( \delta \), the inertia constant (H) of the generator set is 3.

(a) Fig.2 is an equivalent circuit diagram of Fig.1, from which the equivalent reactance value of the internal potential and the infinite bus were been shown [1-7].

\[ X_{eq} = X'_{d} + X_{TR} + \frac{X_{12} \ast (X_{13} + X_{23})}{(X_{12} + X_{13} + X_{23})} = 0.520 \text{ p.u} \]

Therefore, the current flowing into the infinite bus is

\[ I = \frac{P}{V_{bus}(p.f)} = \frac{(1.0)}{(1.0)(0.95)} \ast 0.95 = 1.05263 - 18.95^* \text{ p.u} \]
So the internal potential is

\[ E = E'\angle\delta = V_{\text{bus}} + jX_{\text{eq}} = 1.0\angle0^\circ + (0.520)(1.05263\angle - 18.195^\circ) = 1.2812\angle23.946^\circ \text{ p.u} \]

(b) According to the synchronous generator provides to the effective power supplied to the infinite bus is called the electric power as shown in the formula (1).

\[
p_e = \frac{(E' * V_{\text{bus}})}{X_{\text{eq}}} * \sin \delta = 1.2812(1.0) / 0.520 * \sin \delta = 2.4638 \sin \sin \delta \text{ p.u} \quad (1)
\]

Where,

\[
\omega_{\text{p.u.}}(t) = 1
\]

\[
\frac{2H}{\omega_{\text{sym}}} \frac{d^2\delta(t)}{dt^2} = p_{\text{mp.u.}}(t) - p_{\text{ep.u.}}(t) \quad (2)
\]

Within 3 cycles (Hz) after the fault occurs

\[
p_e = 0
\]

\[
\frac{2H}{\omega_{\text{sym}}} \frac{d^2\delta(t)}{dt^2} = \left( p_{\text{mp.u.}}(t) - 0 \left( p_{\text{ep.u.}}(t) \right) \right) 0 \leq t \leq 0.05s
\]
The above formula is integrated twice and based on initial conditions \( \delta_0 = 0.4179 \) rad and \( \Delta(0)/dt = 0 \) taking
\[
\delta(t) = \frac{\alpha_{\text{sym}} P_m n \cdot t^2 + \delta_0}{4H} \text{ Where } t = 3 \text{ cycles } = 0.05 \text{ s}
\]
\[
\delta(0.05) = ((2\pi 60)/12) \cdot (0.05)^2 + 0.4179 = 0.4964
\]
Therefore, the acceleration zone A1 is shown in Fig. 3:
\[
A_1 = I_0 \int_{\delta_0}^{\delta_1} P_m d\delta = \int_{\delta_0}^{\delta_1} 1.0d\delta = (\delta_1 - \delta_0) = 0.4964 - 0.4179 = 0.0785
\]
When \( t = 0.05 \) seconds, the fault is cleared and the \( \hat{\phi} \) value is immediately incremented from zero to the sinusoid shown in Fig. 3 of the \( \rho-\delta \) relationship, while \( \delta \) continues to increase until the deceleration zone area A2 is equal to A1 so stopping:
\[
A_2 = I_0 \int_{\delta_1}^{\delta_2} (P_m \sin \delta - P_m) d\delta = I_0 \int_{0.0785}^{\delta_2} (2.4638 \sin \delta - 1) d\delta = A_1 = 0.0785
\]
Therefore,
\[
2.4638 \cos (0.4964) - \cos \delta_2 - (\delta_2 - 0.4964) = 0.0785
\]
\[
2.4638 \cos \delta_2 + \delta_2 = 2.5843
\]
The above nonlinear algebraic equation can be solved iteratively to obtain \( \delta_2 = 0.7003 \) rad. Because the critical clearing angle \( \delta_2 \) does not exceed the maximum critical clearing angle \( \delta_{cr} \), so the power system is called in a stable status, as shown in Fig. 4. The other maximum power angle \( \delta_3 = (180^\circ - \delta_0) = 156.05^\circ (\delta_0 = 23.95^\circ) \).

If the fault time increases over the critical clearing time, the power system lies on unstable status. The critical clearing time \( t_{cr} \) is defined as the longest fault duration that can be tolerated in steady status. If the three-phase fault lasts longer than 3 cycles (Hz), try to solve the critical clearing time.

Fig. 4 is shown the relationship of \( \rho-\delta \), know \( \Delta_2 = (180^\circ - \Delta_0) = 2.736 \) RAD
\[
A_1 = A_2
\]
\[
A_1 = \int_{\delta_0}^{\delta_{cr}} P_m d\delta = \int_{\delta_0}^{\delta_{cr}} 1.0d\delta
\]
\[
A_2 = \int_{\delta_{cr}}^{\delta_1} (P_m \sin \delta - P_m) d\delta = \int_{\delta_{cr}}^{2.4638 \sin \delta - 1.0} (2.7236d\delta - 1.0) d\delta
\]
\[
(\delta_{cr} - 0.4179) = 2.4638[\cos \delta_{cr} - \cos (2.7236)] - (2.7236 - \delta_{cr})
\]
\[
2.4638 \cos \delta_{cr} = 0.05402; \delta_{cr} = 1.5489 \text{ rad } = 88.74^\circ
\]
\[
t_{cr} = \frac{4H}{\alpha_{\text{sym}} P_m n} (\delta(0) - \delta_0) = \frac{32}{(2\pi 60)(1.0)} (1.5489 - 0.4179) = 0.1897 \text{ s}
\]

To continue the previous example, a three-phase-to-ground fault occurs on No.3 bus in Fig.1. What time of No.13 and No.32 breakers are opened out to clear the fault point, so that power system remains in steady status, trying to solve the critical clearing angle of power system. Fig. 5 shows three portions-fault equivalent circuit diagram, Thevenin equivalent circuit diagram, and post-fault equivalent circuit diagram.

![Fig. 3. The p-δ diagram for clears 3-cycle](image-url)
A synchronous generator (60 Hz) has a transient reactance of 0.3 p.u and an inertia constant $H$ of 5.5 MJ/MVA. Through a transformer with a reactance of 0.15 p.u crosses...
two transmission lines with each reactance of 0.4 p.u are connected to the infinite bus with a voltage of 1.0 \angle 0^\circ \text{p.u., as shown in Fig. 7. The infinite bus received } P_e = 0.8 \text{ and } Q = 0.05 \text{ p.u of appearance power from the generator.}

Assume that another transmission line (no current) in the Fig. 7 has a temporary three-phase-to-ground bolted fault at F point. Then after No. 13 breaker operated to cut the faulty out, the others transmission lines still remain supplying power [8-13].

- To find what the power angle equation at pre-fault is.
- Please use the equal area criterion to find the critical clearing angle and critical clearing time of the fault point.

Solve (1)

\[ X_{eq} = X_{d}' + X_{TR} + X_{L1}/X_{L2} = 0.650 \text{ p.u.} \]

\[ p.f = \frac{P_e}{\sqrt{(P_e)^2 + Q_e^2}} = \frac{0.8}{\sqrt{(0.8)^2 + (0.074)^2}} = 0.9958 \]

\[ I = \frac{P_e}{(V_{bus})(p.f)} \angle \cos^{-1}(p.f) = 0.8034 \angle -0.0917^\circ \text{ p.u.} \]

The equivalent impedance of Fig. 8 is obtained through previous steps to calculate from Fig. 7.

\[ E' = E' \angle 0^\circ = V_{bus} + jX_{eq} = 1.1281 \angle 0.4813^\circ \text{ p.u} \]

\[ p_{e1} = [(E' \times V_{bus})/X_{eq}] \times \sin \delta = 1.7355 \sin \delta \text{ p.u.} \]

Solve (2)

\[ X_{Th} = X_{d} + X_{TR} = 0.450 \text{ p.u.} \]

\[ V_{Th} = 0 \angle 0^\circ \text{ p.u.} \]

\[ p_e = (E' \times V_{Th})/X_{Th} \times \sin \delta = 0 \]

\[ A_1 = A_2 \]

\[ A_2 = \int_{\delta_0}^{\delta_3} (P_m \sin \delta - p_m) \, d\delta \]

\[ A_1 = \int_{\delta_0}^{\delta_3} (P_m \sin \delta - p_m) \, d\delta \]

\[ A_1 = \int_{\delta_0}^{\delta_3} P_m \, d\delta \text{ when } P_e = 0 \]

\[ A_2 = \int_{\delta_0}^{\delta_3} (P_m \sin \delta - p_m) \, d\delta \]

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δᵦ = 0.8125 rad = 46.576°

δ(t) = \frac{ω₀ \text{synPmpu}}{4H} t² + δ₀

tᵦ = 0.1386 s

Solve (2)

A₁ = A₂

A₁ = \int_{δ₀}^{δ₁} P_m dδ = \int_{0.48}^{31} 1.0 dδ

1.7355 cosδᵦ = 0.6382

δᵦ = 1.1943 rad

δ(t) = \frac{ω₀ \text{synPmpu}}{4H} t² + δ₀

I. Status(Fig. 1)

- The internal potential of the generator at 0.95 pf lagging: 1.281 ∠ 23.9°
- The equation for the electrical power: 2.463sinδ
- During a fault, breakers(disoperation), its maximum power angle: 0.700 rad

II. Table 1. Statue’s results

| Status(Fig. 1) | Results |
|----------------|---------|
| The internal potential of the generator at 0.95 pf lagging | 1.281 ∠ 23.9° |
| The equation for the electrical power | 2.463sinδ |
| During a fault, breakers(disoperation), its maximum power angle | 0.700 rad |

III. Table 2. Results of cases

| Item | Result |
|------|--------|
| Case 1 | [0.3 0.15 0.4 0.2 0.2] |
| the impedance value of device: [Xd|Xtr|X12|X13|X23] | |
| the parameter of relative data: [ P | Vbus | Q | H | f | Pm] | [0.8 1 0.05 5.5 60 1] |
| Case 2 | [0.3 0.15 0.2 0.2 0.2] |
| the impedance value of device: [Xd|Xtr|X26|X13|X23] | |
| the parameter of relative data: [ P | Vbus | Q | H | f | Pm] | [0.8 1 0.05 5.5 60 1] |
| Case 3 | [0.3 0.10 0.4 0.2 0.2] |
| the impedance value of device: [Xd|Xtr|X12|X13|X23] | |
| the parameter of relative data: [ P | Vbus | Q | H | f | Pm] | [0.8 1 0.05 5.5 60 1] |
| δ₀ | 0.4803 | 0.4374 | 0.4483 |
| Pe₁ | 1.7348 | 1.8924 | 1.8494 |
| Pe₂ | 0.4418 | 0.3649 | 0.5201 |
| Pe₃ | 1.3266 | 1.6983 | 1.3870 |
| δ₁ | 0.8069 | 1.2682 | 0.9193 |
| tₙ | 0.138(s) | 0.220(s) | 0.166(s) |
Fig. 7. Single diagram of a design example

Fig. 8. Equivalent circuit diagram of a design example

Fig. 9. Single diagram of a design example
5. AN APPLICATION SOFTWARE PROGRAM

Author had worked as a senior electrical engineer in Taiwan Power Company for many years, managed equipment maintenance, fault analysis and planning design. Therefore, I knew an important question the set time of the protection relay needed to caution in planning design.

In order to avoid errors caused by human factors in the calculation process, this paper developed an application software program for MATLAB, which simply input the impedance values and related parameter data of each device into the program to execute, then the answer will be shown. Other, the procedural steps of computerized program is listed Table 2.

5.1 Case 1

To solve the internal potential voltage of generator, initiate power angle ($\delta_0$), normal status of power of electricity (Pe1), and critical clearing angle ($\delta_t$). To find the critical clearing time ($t_c$) on No.3 bus while a three phase-to-ground-fault occurs.

5.2 Case 2

From Fig 8 (dotted line), if the dotted line paralleled on the transmission line (L16), those parameters are the same of a transmission line (L12), because L12 parallel L16 so that is referred as L26, find the critical clearing time.

5.3 Case 3

In Fig. 8, the transformer is replaced with the same capacity and voltage level except the impedance value is reduced from 0.15 to 1; to find the critical clearing time of the system.

The results of the program execution for case 1, case 2, and case 3 are shown in Table 2.

It is learned from the calculation results of the above cases that any equipment parameter changes or any equipment cut down for maintenance on the system will affect the critical clearance time value of the system.

6. CONCLUSIONS

Any impedance value of power equipment changes (transmission lines or some equipment need to be stopped for maintenance) in power system that will directly affects the critical clearance time of the system, so that the critical clearance time must be carefully considered in the design planning. However, from the above calculation process, it is known that the steps of process are so cumbersome and time-consuming of steps. In order to avoid a slip of writing, a computerized calculation method was designed with MATLAB application software. The proposed computer program has been
repeatedly executed and verified to be accurate and reliable, in fact of which can replace manual calculations and shorten the design operation time. So that it can help those who work as designer engineers in power system.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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APPENDIX

Executive Program

>> clear all
Xeq=input('Please input the impedance value of each device: Xeq= [Xd|Xtr|X12|X13|X23]= '); fprintf('to solve the internal potential value of generator, initiate power angle((y1) = (\delta_o)), normal status of power of electricity (Pe1) , and critical clearing angle ((y2) = (\delta_c))\n\n'); fprintf('to solve the critical clearing time (tc) for a three phase-to-ground-fault occurs on No.3 bus\n
Xeq1=input('Please input the parameter of relative data: Xeq1= [P | Vbus | Q | H | f | Pm]= ');
X6= Xeq(4)+ Xeq(5); X7= (Xeq(3)*X6)/(Xeq(3)+X6); X8= (Xeq(3)*Xeq(4))/(Xeq(3)+Xeq(4)); Xth= Xeq(1)+Xeq(2)+X8; Vth= Xeq(1) (2)*(X8/Xth); Xthp= Xeq(1)+Xeq(2)+Xeq(3);
\[ X_{eq} = X_{eq}(1) + X_{eq}(2) + (X_7) \]
\[ X_{11} = \sqrt{(X_{eq1}(1) \cdot X_{eq1}(1)) + (X_{eq1}(3) \cdot X_{eq1}(3))} \]
\[ X_{12} = \frac{X_{eq1}(1)}{X_{11}} \]
\[ I = \frac{X_{eq1}(1)}{(X_{eq1}(2) \cdot X_{12})} \]
\[ y = \arccos(X_{12}) \cdot \frac{180}{\pi} \]
\[ E = \sqrt{(X_{eq1}(2) \cdot X_{eq1}(2)) + ((I \cdot X_{eq}) \cdot (I \cdot X_{eq}))} \]
\[ y_1(\delta) = \arccos \left( \frac{X_{eq1}(2)}{E} \right) \]
\[ P_{e1} = \frac{(E \cdot X_{eq1}(2))}{X_{eq}} \]
\[ P_{e2} = \frac{(E \cdot V_{th})}{X_{th}} \]
\[ P_{e3} = \frac{(E \cdot X_{eq1}(2))}{X_{thp}} \]

```
syms delta delta_cr
delta_cr_max = (3.14 - \arccos (X_{eq1}(2)/E));
delta_cr_min = \arccos (X_{eq1}(2)/E);
fun_A1_fun = 1.0 - P_{e2} \cdot \sin (delta);
fun_A1_int = int(fun_A1_fun, delta_cr_min, delta_cr);
fun_A2_fun = P_{e3} \cdot \sin (delta) - 1.0;
fun_A2_int = int(fun_A2_fun, delta_cr, delta_cr_max);
fun_A12_ans = solve (fun_A1_int == fun_A2_int);
y_2(\delta_i) = \abs{\text{double} (fun_A12_ans)}
tc = \sqrt{(X_{eq1}(4) \cdot 4 \cdot (y_2 - y_1)/(6.28 \cdot X_{eq1}(5) \cdot X_{eq1}(6)))}
```