Web-based Supporting Materials for
”Measurement errors in control risk regression: a comparison of correction techniques” by
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Web Appendix A: Additional simulation results for the log odds case

This web appendix reports a portion of the results of the simulation study carried out to compare the performance of the competing approaches in control risk regression, as described in Section 4 of the main manuscript. The reference is to simulation scenario i) in the main manuscript.
Table S1: Bias and standard deviation (SD) of the estimates of $\beta_0$, and average of the estimated standard errors (SE) obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk normally distributed.

| $\tau^2$ | Method       | Bias  | SD   | SE   | Bias  | SD   | SE   | Bias  | SD   | SE   |
|----------|--------------|-------|------|------|-------|------|------|-------|------|------|
| 0.1      | NAIVE        | -0.002| 0.157| 0.083| 0.003 | 0.137| 0.088| 0.001 | 0.076| 0.048|
|          | LIKELIHOOD   | -0.001| 0.162| 0.128| 0.002 | 0.120| 0.108| 0.000 | 0.067| 0.064|
|          | SKEW-NORMAL  | -0.001| 0.162| 0.129| 0.002 | 0.121| 0.109| 0.001 | 0.067| 0.064|
|          | CORRECTED SCORE | -0.002| 0.152| 0.135| -0.003| 0.123| 0.110| 0.000 | 0.069| 0.064|
|          | CONDITIONAL SCORE | -0.001| 0.148| 0.134| -0.002| 0.121| 0.110| 0.000 | 0.069| 0.064|
|          | SIMEX        | -0.001| 0.155| 0.135| -0.001| 0.126| 0.116| 0.001 | 0.075| 0.069|
| 0.5      | NAIVE        | 0.001 | 0.283| 0.255| 0.000 | 0.235| 0.226| 0.002 | 0.163| 0.137|
|          | LIKELIHOOD   | 0.004 | 0.275| 0.227| -0.001| 0.187| 0.176| 0.002 | 0.112| 0.110|
|          | SKEW-NORMAL  | 0.001 | 0.276| 0.228| -0.001| 0.189| 0.177| 0.003 | 0.112| 0.110|
|          | CORRECTED SCORE | 0.006| 0.271| 0.228| -0.003| 0.190| 0.175| 0.003 | 0.112| 0.110|
|          | CONDITIONAL SCORE | 0.005| 0.270| 0.227| -0.003| 0.189| 0.174| 0.003 | 0.112| 0.110|
|          | SIMEX        | 0.004 | 0.274| 0.241| -0.003| 0.198| 0.188| 0.003 | 0.118| 0.116|
| 0.9      | NAIVE        | 0.003 | 0.385| 0.436| -0.024| 0.376| 0.375| -0.007| 0.202| 0.221|
|          | LIKELIHOOD   | 0.004 | 0.353| 0.290| -0.011| 0.230| 0.215| 0.003 | 0.143| 0.139|
|          | SKEW-NORMAL  | 0.008 | 0.349| 0.289| -0.011| 0.232| 0.216| 0.003 | 0.144| 0.139|
|          | CORRECTED SCORE | 0.005| 0.346| 0.292| -0.012| 0.231| 0.215| 0.003 | 0.144| 0.139|
|          | CONDITIONAL SCORE | 0.005| 0.345| 0.291| -0.011| 0.231| 0.215| 0.003 | 0.144| 0.139|
|          | SIMEX        | 0.005 | 0.354| 0.309| -0.012| 0.241| 0.229| 0.003 | 0.151| 0.147|
| 1.5      | NAIVE        | -0.014| 0.460| 0.711| 0.015 | 0.380| 0.570| 0.015 | 0.220| 0.350|
|          | LIKELIHOOD   | -0.015| 0.420| 0.374| 0.008 | 0.294| 0.276| 0.008 | 0.177| 0.177|
|          | SKEW-NORMAL  | -0.018| 0.425| 0.373| 0.007 | 0.296| 0.278| 0.009 | 0.175| 0.176|
|          | CORRECTED SCORE | -0.013| 0.420| 0.370| 0.009 | 0.296| 0.277| 0.008 | 0.177| 0.177|
|          | CONDITIONAL SCORE | -0.014| 0.419| 0.370| 0.009 | 0.295| 0.277| 0.008 | 0.177| 0.177|
|          | SIMEX        | -0.013| 0.432| 0.396| 0.011 | 0.310| 0.296| 0.008 | 0.186| 0.186|
Table S2: Bias and standard deviation (SD) of the estimates of $\beta_0$, and average of the estimated standard errors (SE) obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk as a mixture of Normals.

| $\tau^2$ | Method       | Bias | SD  | SE   | Bias | SD  | SE   | Bias | SD  | SE   |
|---------|--------------|------|-----|------|------|-----|------|------|-----|------|
|         | $n = 10$     |      |     |      |      |     |      |      |     |      |
| 0.1     | NAIVE        | 0.006| 0.141|0.078 |-0.001|0.119|0.065 | 0.001|0.094|0.044 |
|         | LIKELIHOOD   | -0.010| 0.143|0.121 |-0.005|0.100|0.094 |-0.005|0.066|0.063 |
|         | SKEW-NORMAL  | -0.015| 0.147|0.121 |-0.007|0.101|0.095 |-0.008|0.067|0.064 |
|         | CORRECTED SCORE | -0.009| 0.140|0.130 |-0.001|0.101|0.096 |-0.002|0.067|0.063 |
|         | CONDITIONAL SCORE | -0.008| 0.141|0.129 |-0.001|0.103|0.100 |-0.002|0.067|0.063 |
|         | SIMEX        | -0.001| 0.139|0.128 |-0.001|0.126|0.116 |-0.001|0.070|0.066 |
| 0.5     | $n = 10$     |      |     |      |      |     |      |      |     |      |
|         | NAIVE        | -0.012| 0.265|0.249 | 0.003|0.212|0.196 | 0.010|0.165|0.132 |
|         | LIKELIHOOD   | -0.034| 0.248|0.218 |-0.007|0.167|0.163 |-0.002|0.107|0.108 |
|         | SKEW-NORMAL  | -0.034| 0.247|0.219 |-0.005|0.169|0.165 |-0.004|0.112|0.109 |
|         | CORRECTED SCORE | -0.028| 0.248|0.218 | 0.001|0.169|0.163 | 0.004|0.108|0.107 |
|         | CONDITIONAL SCORE | -0.027| 0.247|0.218 | 0.001|0.169|0.163 | 0.004|0.108|0.107 |
|         | SIMEX        | -0.019| 0.250|0.226 | 0.002|0.176|0.172 | 0.004|0.114|0.113 |
| 0.9     | $n = 10$     |      |     |      |      |     |      |      |     |      |
|         | NAIVE        | 0.013| 0.501|0.510 |-0.012|0.279|0.333 | 0.001|0.187|0.216 |
|         | LIKELIHOOD   | -0.031| 0.329|0.278 |-0.016|0.227|0.210 |-0.012|0.142|0.138 |
|         | SKEW-NORMAL  | -0.030| 0.328|0.278 |-0.009|0.230|0.212 |-0.013|0.150|0.138 |
|         | CORRECTED SCORE | -0.017| 0.323|0.274 |-0.008|0.227|0.210 |-0.004|0.142|0.138 |
|         | CONDITIONAL SCORE | -0.016| 0.323|0.274 |-0.008|0.227|0.210 |-0.004|0.142|0.138 |
|         | SIMEX        | -0.010| 0.333|0.288 |-0.008|0.236|0.223 |-0.004|0.150|0.145 |
| 1.5     | $n = 10$     |      |     |      |      |     |      |      |     |      |
|         | NAIVE        | 0.022| 0.450|0.685 | 0.010|0.543|0.630 | 0.009|0.315|0.361 |
|         | LIKELIHOOD   | -0.009| 0.415|0.354 |-0.008|0.276|0.267 |-0.003|0.179|0.175 |
|         | SKEW-NORMAL  | -0.010| 0.425|0.358 |-0.011|0.283|0.266 |-0.004|0.180|0.174 |
|         | CORRECTED SCORE | 0.009| 0.411|0.352 | 0.000|0.278|0.265 | 0.005|0.179|0.174 |
|         | CONDITIONAL SCORE | 0.008| 0.411|0.352 | 0.001|0.278|0.265 | 0.005|0.179|0.174 |
|         | SIMEX        | 0.018| 0.420|0.374 | 0.002|0.292|0.282 | 0.005|0.189|0.184 |
Table S3: Bias and standard deviation (SD) of the estimates of $\beta_0$, and average of the estimated standard errors (SE) obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk as a Skew-Normal.

| $\tau^2$ | Method       | Bias  | SD   | SE   | Bias  | SD   | SE   | Bias  | SD   | SE   |
|----------|--------------|-------|------|------|-------|------|------|-------|------|------|
| 0.1      | NAIVE        | 0.057 | 0.182| 0.087| 0.074 | 0.280| 0.113| 0.069 | 0.160| 0.055|
|          | LIKELIHOOD   | -0.008| 0.188| 0.152| -0.011| 0.146| 0.128| 0.001 | 0.081| 0.080|
|          | SKEW-NORMAL  | -0.007| 0.191| 0.156| -0.011| 0.150| 0.131| 0.001 | 0.082| 0.080|
|          | CORRECTED    | -0.013| 0.192| 0.169| -0.016| 0.158| 0.139| 0.000 | 0.086| 0.078|
|          | SCORE        | -0.004| 0.184| 0.164| -0.008| 0.153| 0.135| 0.000 | 0.085| 0.077|
|          | SIMEX        | 0.009 | 0.187| 0.168| 0.005 | 0.156| 0.147| 0.010 | 0.090| 0.086|
| 0.5      | NAIVE        | 0.057 | 0.354| 0.264| 0.100 | 0.328| 0.236| 0.058 | 0.246| 0.147|
|          | LIKELIHOOD   | -0.021| 0.347| 0.274| -0.006| 0.246| 0.215| -0.002| 0.141| 0.134|
|          | SKEW-NORMAL  | -0.017| 0.354| 0.269| -0.008| 0.249| 0.214| -0.002| 0.142| 0.135|
|          | CORRECTED    | -0.024| 0.340| 0.282| -0.005| 0.245| 0.211| -0.004| 0.142| 0.132|
|          | SCORE        | -0.023| 0.341| 0.274| -0.001| 0.241| 0.206| -0.004| 0.142| 0.131|
|          | SIMEX        | 0.004 | 0.343| 0.294| 0.022 | 0.252| 0.231| 0.006 | 0.148| 0.143|
| 0.9      | NAIVE        | 0.068 | 0.481| 0.448| 0.082 | 0.452| 0.390| 0.091 | 0.284| 0.237|
|          | LIKELIHOOD   | -0.024| 0.446| 0.364| -0.003| 0.303| 0.270| 0.005 | 0.179| 0.175|
|          | SKEW-NORMAL  | -0.023| 0.456| 0.356| -0.003| 0.304| 0.274| 0.006 | 0.179| 0.172|
|          | CORRECTED    | -0.016| 0.430| 0.360| -0.004| 0.306| 0.268| 0.006 | 0.182| 0.169|
|          | SCORE        | -0.011| 0.427| 0.353| -0.002| 0.307| 0.263| 0.007 | 0.181| 0.168|
|          | SIMEX        | 0.010 | 0.439| 0.382| 0.022 | 0.313| 0.288| 0.017 | 0.187| 0.182|
| 1.5      | NAIVE        | 0.045 | 0.570| 0.685| 0.075 | 0.573| 0.636| 0.080 | 0.376| 0.367|
|          | LIKELIHOOD   | -0.045| 0.584| 0.449| -0.003| 0.386| 0.345| 0.004 | 0.227| 0.218|
|          | SKEW-NORMAL  | -0.058| 0.588| 0.455| 0.000 | 0.385| 0.348| 0.005 | 0.227| 0.216|
|          | CORRECTED    | -0.037| 0.557| 0.455| 0.000 | 0.387| 0.342| 0.002 | 0.229| 0.213|
|          | SCORE        | -0.034| 0.554| 0.450| 0.004 | 0.384| 0.336| 0.003 | 0.227| 0.211|
|          | SIMEX        | -0.014| 0.576| 0.481| 0.032 | 0.394| 0.367| 0.012 | 0.237| 0.228|
Figure S1: Mean squared error of the estimators of $\beta_0$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk normally distributed.
Figure S2: Mean squared error of the estimators of $\beta_0$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a mixture of Normals.
Figure S3: Mean squared error of the estimators of $\beta_0$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a Skew-Normal.
Figure S4: Empirical coverage probabilities of confidence intervals for $\beta_0$ from uncorrected approach (NAIVE), likelihood approach under a Normal specification (LIKELIHOOD) or a Skew-Normal specification (SKEW-NORMAL) for the underlying risk distribution, SIMEX, corrected score and conditional score, on the basis of 1,000 replicates of simulation scenario $i$. Underlying risk normally distributed.
Figure S5: Empirical coverage probabilities of confidence intervals for $\beta_0$ from uncorrected approach (NAIVE), likelihood approach under a Normal specification (LIKELIHOOD) or a Skew-Normal specification (SKEW-NORMAL) for the underlying risk distribution, SIMEX, corrected score and conditional score, on the basis of 1,000 replicates of simulation scenario $i$. Underlying risk distributed as a mixture of Normals.
Figure S6: Empirical coverage probabilities of confidence intervals for $\beta_0$ from uncorrected approach (NAIVE), likelihood approach under a Normal specification (LIKELIHOOD) or a Skew-Normal specification (SKEW-NORMAL) for the underlying risk distribution, SIMEX, corrected score and conditional score, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a Skew-Normal.
Table S4: Empirical coverage probabilities (multiplied by 1,000) of confidence intervals for $\beta_0$ and associated Monte Carlo standard error (in parentheses, multiplied by 1,000) obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a Normal, Mixture of Normals, or Skew-Normal.

| $\tau^2$ | Method   | 10   | 20   | 50   | 10   | 20   | 50   | 10   | 20   | 50   |
|---------|----------|------|------|------|------|------|------|------|------|------|
| 0.1     | NAIVE    | 898  | 883  | 902  | 909  | 902  | 852  | 906  | 695  | 727  |
|         | LIKELIHOOD | 802  | 879  | 931  | 834  | 911  | 935  | 807  | 866  | 937  |
|         | SKEW-NORMAL | 797  | 873  | 931  | 827  | 898  | 929  | 787  | 865  | 932  |
|         | CORRECTED SCORE | 902  | 911  | 920  | 888  | 885  | 919  | 914  | 935  | 937  |
|         | CONDITIONAL SCORE | 901  | 907  | 921  | 884  | 884  | 919  | 911  | 934  | 937  |
|         | SIMEX    | 879  | 913  | 921  | 875  | 923  | 926  | 888  | 930  | 931  |
| 0.1     | NAIVE    | 904  | 891  | 845  | 913  | 897  | 839  | 890  | 810  | 739  |
|         | LIKELIHOOD | 851  | 917  | 946  | 875  | 930  | 944  | 831  | 905  | 936  |
|         | SKEW-NORMAL | 859  | 910  | 949  | 872  | 929  | 933  | 828  | 900  | 931  |
|         | CORRECTED SCORE | 870  | 905  | 944  | 888  | 925  | 946  | 851  | 890  | 930  |
|         | CONDITIONAL SCORE | 872  | 901  | 943  | 888  | 925  | 946  | 850  | 889  | 930  |
|         | SIMEX    | 887  | 912  | 941  | 897  | 930  | 945  | 868  | 918  | 937  |
| 0.1     | NAIVE    | 882  | 761  | 835  | 718  | 883  | 871  | 867  | 778  | 775  |
|         | LIKELIHOOD | 861  | 920  | 940  | 864  | 917  | 936  | 859  | 891  | 941  |
|         | SKEW-NORMAL | 872  | 922  | 936  | 869  | 914  | 920  | 850  | 894  | 939  |
|         | CORRECTED SCORE | 870  | 919  | 939  | 882  | 914  | 935  | 877  | 897  | 929  |
|         | CONDITIONAL SCORE | 869  | 919  | 939  | 884  | 917  | 935  | 868  | 895  | 930  |
|         | SIMEX    | 873  | 923  | 944  | 883  | 922  | 934  | 885  | 903  | 940  |
| 0.1     | NAIVE    | 904  | 864  | 891  | 881  | 652  | 741  | 893  | 770  | 727  |
|         | LIKELIHOOD | 876  | 937  | 940  | 876  | 930  | 938  | 844  | 905  | 929  |
|         | SKEW-NORMAL | 876  | 931  | 942  | 876  | 920  | 937  | 844  | 910  | 930  |
|         | CORRECTED SCORE | 882  | 927  | 943  | 886  | 929  | 939  | 848  | 907  | 925  |
|         | CONDITIONAL SCORE | 880  | 925  | 943  | 886  | 929  | 940  | 845  | 904  | 926  |
|         | SIMEX    | 894  | 931  | 941  | 897  | 934  | 937  | 864  | 913  | 933  |
Table S5: Bias and standard deviation (SD) of the estimates of $\beta_1$, and average of the estimated standard errors (SE) obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$. Underlying risk normally distributed.

| $\tau^2$ | Method       | Bias  | SD   | SE  | Bias  | SD   | SE  | Bias  | SD   | SE  |
|---------|--------------|-------|------|-----|-------|------|-----|-------|------|-----|
|         |              | $n = 10$ |      |     | $n = 20$ |      |     | $n = 50$ |      |     |
| 0.1     | NAIVE        | -0.070 | 0.189 | 0.083 | -0.092 | 0.146 | 0.088 | -0.055 | 0.083 | 0.048 |
|         | LIKELIHOOD   | 0.028  | 0.209 | 0.144 | 0.023  | 0.152 | 0.129 | -0.003 | 0.076 | 0.073 |
|         | SKEW-NORMAL  | 0.032  | 0.216 | 0.142 | 0.024  | 0.156 | 0.131 | -0.003 | 0.077 | 0.073 |
|         | CORRECTED SCORE | -0.001  | 0.221 | 0.153 | -0.026  | 0.157 | 0.130 | -0.029 | 0.082 | 0.074 |
|         | CONDITIONAL SCORE | -0.016  | 0.204 | 0.149 | -0.033  | 0.149 | 0.126 | -0.029 | 0.080 | 0.073 |
|         | SIMEX        | -0.013  | 0.205 | 0.139 | -0.005  | 0.154 | 0.126 | 0.000  | 0.088 | 0.079 |
| 0.5     | NAIVE        | -0.051  | 0.302 | 0.255 | -0.073  | 0.240 | 0.226 | -0.035 | 0.161 | 0.137 |
|         | LIKELIHOOD   | 0.042  | 0.339 | 0.256 | 0.024  | 0.230 | 0.201 | 0.004  | 0.122 | 0.120 |
|         | SKEW-NORMAL  | 0.044  | 0.363 | 0.259 | 0.025  | 0.231 | 0.200 | 0.003  | 0.123 | 0.121 |
|         | CORRECTED SCORE | 0.008  | 0.322 | 0.234 | -0.028  | 0.228 | 0.185 | -0.028 | 0.122 | 0.116 |
|         | CONDITIONAL SCORE | 0.002  | 0.319 | 0.231 | -0.033  | 0.226 | 0.183 | -0.029 | 0.121 | 0.117 |
|         | SIMEX        | -0.003  | 0.316 | 0.245 | -0.007  | 0.233 | 0.193 | 0.004  | 0.126 | 0.122 |
| 0.9     | NAIVE        | -0.054  | 0.399 | 0.436 | -0.004  | 0.378 | 0.375 | -0.063 | 0.197 | 0.221 |
|         | LIKELIHOOD   | 0.049  | 0.446 | 0.328 | 0.017  | 0.277 | 0.232 | -0.002 | 0.156 | 0.150 |
|         | SKEW-NORMAL  | 0.042  | 0.428 | 0.326 | 0.017  | 0.277 | 0.231 | -0.004 | 0.156 | 0.151 |
|         | CORRECTED SCORE | 0.003  | 0.406 | 0.299 | -0.014  | 0.271 | 0.219 | -0.040 | 0.152 | 0.144 |
|         | CONDITIONAL SCORE | -0.001  | 0.402 | 0.298 | -0.017  | 0.269 | 0.220 | -0.040 | 0.153 | 0.145 |
|         | SIMEX        | -0.005  | 0.404 | 0.317 | 0.011  | 0.276 | 0.231 | -0.005 | 0.159 | 0.152 |
| 1.5     | NAIVE        | -0.080  | 0.492 | 0.711 | -0.072  | 0.367 | 0.570 | -0.061 | 0.217 | 0.350 |
|         | LIKELIHOOD   | 0.025  | 0.519 | 0.417 | 0.047  | 0.350 | 0.313 | -0.006 | 0.205 | 0.190 |
|         | SKEW-NORMAL  | 0.024  | 0.522 | 0.417 | 0.042  | 0.342 | 0.315 | -0.011 | 0.203 | 0.190 |
|         | CORRECTED SCORE | -0.018  | 0.504 | 0.378 | -0.012  | 0.337 | 0.286 | -0.042 | 0.197 | 0.182 |
|         | CONDITIONAL SCORE | -0.025  | 0.496 | 0.376 | -0.017  | 0.329 | 0.286 | -0.043 | 0.196 | 0.182 |
|         | SIMEX        | -0.023  | 0.499 | 0.398 | 0.007  | 0.335 | 0.299 | -0.008 | 0.203 | 0.190 |
Table S6: Bias and standard deviation (SD) of the estimates of $\beta_1$, and average of the estimated standard errors (SE) obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a mixture of Normals.

| $\tau^2$ | Method               | Bias  | SD    | SE    | Bias  | SD    | SE    | Bias  | SD    | SE    |
|----------|----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.1      | NAIVE                | -0.100| 0.220 | 0.078 | -0.131| 0.175 | 0.065 | -0.095| 0.130 | 0.044 |
|          | LIKELIHOOD           | 0.058 | 0.289 | 0.184 | 0.005 | 0.166 | 0.143 | -0.004| 0.105 | 0.100 |
|          | SKEW-NORMAL          | 0.056 | 0.302 | 0.181 | 0.001 | 0.168 | 0.144 | -0.015| 0.104 | 0.100 |
|          | CORRECTED SCORE      | 0.033 | 0.280 | 0.207 | -0.022| 0.174 | 0.150 | -0.022| 0.108 | 0.101 |
|          | CONDITIONAL SCORE    | 0.011 | 0.257 | 0.194 | -0.028| 0.166 | 0.145 | -0.024| 0.106 | 0.097 |
|          | SIMEX                | 0.008 | 0.255 | 0.187 | 0.024 | 0.180 | 0.152 | -0.007| 0.112 | 0.105 |
|          |                      |       |       |       |       |       |       |       |       |       |
| 0.5      | NAIVE                | -0.105| 0.401 | 0.249 | -0.104| 0.292 | 0.196 | -0.068| 0.233 | 0.132 |
|          | LIKELIHOOD           | 0.051 | 0.474 | 0.354 | 0.021 | 0.282 | 0.254 | 0.013 | 0.175 | 0.172 |
|          | SKEW-NORMAL          | 0.060 | 0.524 | 0.347 | 0.021 | 0.290 | 0.253 | 0.007 | 0.177 | 0.170 |
|          | CORRECTED SCORE      | 0.023 | 0.463 | 0.340 | 0.021 | 0.286 | 0.248 | -0.016| 0.174 | 0.169 |
|          | CONDITIONAL SCORE    | 0.012 | 0.451 | 0.326 | -0.007| 0.280 | 0.242 | -0.015| 0.173 | 0.167 |
|          | SIMEX                | -0.023| 0.429 | 0.336 | -0.003| 0.289 | 0.256 | 0.011 | 0.181 | 0.177 |
|          |                      |       |       |       |       |       |       |       |       |       |
| 0.9      | NAIVE                | -0.158| 0.658 | 0.510 | -0.140| 0.365 | 0.333 | -0.097| 0.253 | 0.216 |
|          | LIKELIHOOD           | 0.067 | 0.589 | 0.436 | 0.008 | 0.358 | 0.323 | -0.007| 0.228 | 0.217 |
|          | SKEW-NORMAL          | 0.066 | 0.614 | 0.431 | 0.008 | 0.373 | 0.329 | -0.006| 0.227 | 0.214 |
|          | CORRECTED SCORE      | 0.015 | 0.538 | 0.407 | -0.023| 0.348 | 0.313 | -0.035| 0.226 | 0.212 |
|          | CONDITIONAL SCORE    | 0.009 | 0.529 | 0.395 | -0.028| 0.343 | 0.310 | -0.036| 0.224 | 0.210 |
|          | SIMEX                | -0.012| 0.537 | 0.419 | -0.019| 0.352 | 0.327 | -0.007| 0.233 | 0.222 |
|          |                      |       |       |       |       |       |       |       |       |       |
| 1.5      | NAIVE                | -0.082| 0.685 | 0.685 | -0.073| 0.675 | 0.630 | -0.086| 0.394 | 0.361 |
|          | LIKELIHOOD           | 0.088 | 0.800 | 0.554 | 0.025 | 0.457 | 0.404 | -0.001| 0.287 | 0.277 |
|          | SKEW-NORMAL          | 0.095 | 0.859 | 0.556 | 0.028 | 0.463 | 0.402 | -0.008| 0.282 | 0.273 |
|          | CORRECTED SCORE      | 0.000 | 0.689 | 0.522 | -0.003| 0.454 | 0.391 | -0.029| 0.281 | 0.268 |
|          | CONDITIONAL SCORE    | -0.004| 0.683 | 0.512 | -0.011| 0.446 | 0.386 | -0.029| 0.280 | 0.267 |
|          | SIMEX                | 0.000 | 0.719 | 0.543 | 0.003 | 0.460 | 0.410 | 0.007 | 0.290 | 0.281 |
Table S7: Bias and standard deviation (SD) of the estimates of $\beta_1$, and average of the estimated standard errors (SE) obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a Skew-Normal.

| $\tau^2$ | Method       | Bias  | SD   | SE   | Bias  | SD   | SE   | Bias  | SD   | SE   |
|----------|--------------|-------|------|------|-------|------|------|-------|------|------|
|          |              | $n = 10$ |      |      | $n = 20$ |      |      | $n = 50$ |      |      |
| 0.1      | NAIVE        | -0.097 | 0.212 | 0.087 | -0.111 | 0.263 | 0.113 | -0.099 | 0.167 | 0.055 |
|          | LIKELIHOOD   | 0.028  | 0.242 | 0.180 | 0.025  | 0.188 | 0.151 | 0.001  | 0.103 | 0.094 |
|          | SKEW-NORMAL  | 0.029  | 0.244 | 0.179 | 0.023  | 0.195 | 0.155 | -0.001 | 0.104 | 0.095 |
|          | CORRECTED SCORE | -0.003 | 0.251 | 0.190 | -0.011 | 0.213 | 0.172 | -0.031 | 0.109 | 0.099 |
|          | CONDITIONAL SCORE | -0.017 | 0.242 | 0.184 | -0.029 | 0.197 | 0.155 | -0.031 | 0.107 | 0.095 |
|          | SIMEX        | -0.026 | 0.235 | 0.177 | -0.016 | 0.199 | 0.160 | -0.016 | 0.113 | 0.101 |
| 0.5      |              | $n = 10$ |      |      | $n = 20$ |      |      | $n = 50$ |      |      |
|          | NAIVE        | -0.085 | 0.370 | 0.264 | -0.144 | 0.329 | 0.236 | -0.084 | 0.231 | 0.147 |
|          | LIKELIHOOD   | 0.051  | 0.420 | 0.311 | 0.036  | 0.295 | 0.250 | 0.003  | 0.166 | 0.150 |
|          | SKEW-NORMAL  | 0.046  | 0.418 | 0.300 | 0.034  | 0.299 | 0.250 | 0.001  | 0.168 | 0.151 |
|          | CORRECTED SCORE | 0.021  | 0.408 | 0.296 | -0.015 | 0.291 | 0.242 | -0.037 | 0.164 | 0.148 |
|          | CONDITIONAL SCORE | 0.012  | 0.393 | 0.285 | -0.022 | 0.281 | 0.230 | -0.037 | 0.162 | 0.145 |
|          | SIMEX        | -0.010 | 0.388 | 0.304 | -0.021 | 0.285 | 0.247 | -0.015 | 0.170 | 0.153 |
| 0.9      |              | $n = 10$ |      |      | $n = 20$ |      |      | $n = 50$ |      |      |
|          | NAIVE        | -0.079 | 0.496 | 0.448 | -0.123 | 0.406 | 0.390 | -0.108 | 0.286 | 0.237 |
|          | LIKELIHOOD   | 0.078  | 0.567 | 0.412 | 0.016  | 0.363 | 0.308 | -0.002 | 0.206 | 0.192 |
|          | SKEW-NORMAL  | 0.074  | 0.595 | 0.408 | 0.012  | 0.368 | 0.307 | -0.005 | 0.206 | 0.190 |
|          | CORRECTED SCORE | 0.015  | 0.504 | 0.370 | -0.034 | 0.353 | 0.287 | -0.044 | 0.201 | 0.184 |
|          | CONDITIONAL SCORE | 0.006  | 0.497 | 0.358 | -0.043 | 0.342 | 0.278 | -0.045 | 0.199 | 0.182 |
|          | SIMEX        | 0.004  | 0.530 | 0.395 | -0.038 | 0.342 | 0.295 | -0.019 | 0.206 | 0.190 |
| 1.5      |              | $n = 10$ |      |      | $n = 20$ |      |      | $n = 50$ |      |      |
|          | NAIVE        | -0.105 | 0.601 | 0.685 | -0.110 | 0.518 | 0.636 | -0.114 | 0.337 | 0.367 |
|          | LIKELIHOOD   | 0.046  | 0.708 | 0.506 | 0.048  | 0.443 | 0.393 | -0.001 | 0.250 | 0.234 |
|          | SKEW-NORMAL  | 0.051  | 0.722 | 0.505 | 0.035  | 0.442 | 0.394 | -0.002 | 0.248 | 0.234 |
|          | CORRECTED SCORE | -0.007 | 0.637 | 0.460 | -0.017 | 0.421 | 0.366 | -0.043 | 0.242 | 0.224 |
|          | CONDITIONAL SCORE | -0.012 | 0.630 | 0.451 | -0.024 | 0.411 | 0.355 | -0.044 | 0.240 | 0.221 |
|          | SIMEX        | -0.014 | 0.654 | 0.496 | -0.027 | 0.409 | 0.376 | -0.018 | 0.245 | 0.232 |
Figure S7: Mean squared error of the estimators of $\beta_1$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk normally distributed.
Figure S8: Mean squared error of the estimators of $\beta_1$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a mixture of Normals.
Figure S9: Mean squared error of the estimators of $\beta_1$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a Skew-Normal.
Table S8: Empirical coverage probabilities (multiplied by 1,000) of confidence intervals for $\beta_1$ and associated Monte Carlo standard error (in parentheses, multiplied by 1,000) obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a Normal, Skew-Normal, or Mixture of Normals.

| $\tau^2$ | Method          | 10  | 20  | 50  | 10  | 20  | 50  | 10  | 20  | 50  |
|----------|-----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|          |                 | Normal | Mixture | Skew-Normal | Normal | Mixture | Skew-Normal | Normal | Mixture | Skew-Normal |
| 0.1      | NAIVE           | 823 (12) | 748 (14) | 773 (13) | 860 (11) | 741 (14) | 728 (14) | 830 (12) | 669 (15) | 620 (15) |
|          | LIKELIHOOD      | 770 (13) | 873 (11) | 926 (8)  | 742 (14) | 861 (11) | 923 (8)  | 774 (13) | 855 (11) | 928 (8)  |
|          | SKEW-NORMAL     | 759 (15) | 863 (11) | 922 (9)  | 719 (15) | 864 (12) | 897 (12) | 779 (14) | 862 (12) | 925 (8)  |
|          | CORRECTED SCORE | 818 (13) | 848 (12) | 893 (10) | 831 (12) | 878 (11) | 914 (9)  | 818 (13) | 866 (12) | 906 (9)  |
|          | CONDITIONAL SCORE | 824 (13) | 843 (12) | 896 (10) | 817 (13) | 868 (11) | 908 (9)  | 808 (14) | 848 (12) | 899 (10) |
|          | SIMEX           | 770 (13) | 863 (11) | 912 (9)  | 809 (12) | 866 (11) | 926 (8)  | 804 (13) | 869 (11) | 905 (9)  |
| 0.5      | NAIVE           | 887 (10) | 834 (12) | 821 (12) | 885 (10) | 845 (11) | 789 (13) | 875 (10) | 762 (13) | 729 (14) |
|          | LIKELIHOOD      | 830 (12) | 889 (10) | 935 (8)  | 797 (13) | 910 (9)  | 927 (8)  | 814 (12) | 891 (10) | 902 (9)  |
|          | SKEW-NORMAL     | 822 (13) | 883 (11) | 930 (8)  | 789 (14) | 902 (11) | 924 (10) | 807 (13) | 900 (10) | 901 (10) |
|          | CORRECTED SCORE | 823 (12) | 855 (11) | 918 (9)  | 814 (12) | 886 (10) | 923 (8)  | 812 (12) | 884 (10) | 880 (10) |
|          | CONDITIONAL SCORE | 817 (12) | 840 (12) | 923 (8)  | 805 (13) | 880 (10) | 915 (9)  | 802 (13) | 862 (11) | 885 (10) |
|          | SIMEX           | 828 (12) | 862 (11) | 928 (8)  | 822 (12) | 888 (10) | 927 (8)  | 848 (11) | 878 (10) | 899 (10) |
| 0.9      | NAIVE           | 876 (10) | 773 (13) | 822 (12) | 775 (13) | 850 (11) | 849 (11) | 876 (10) | 797 (13) | 738 (14) |
|          | LIKELIHOOD      | 840 (12) | 880 (10) | 935 (8)  | 818 (12) | 902 (9)  | 927 (8)  | 829 (12) | 878 (10) | 924 (8)  |
|          | SKEW-NORMAL     | 839 (12) | 886 (10) | 934 (8)  | 815 (13) | 895 (11) | 929 (10) | 826 (13) | 876 (11) | 920 (9)  |
|          | CORRECTED SCORE | 824 (12) | 866 (11) | 912 (9)  | 812 (12) | 896 (10) | 916 (9)  | 815 (12) | 851 (11) | 908 (9)  |
|          | CONDITIONAL SCORE | 817 (12) | 865 (11) | 915 (9)  | 801 (13) | 893 (10) | 919 (9)  | 800 (13) | 844 (11) | 903 (9)  |
|          | SIMEX           | 836 (12) | 881 (10) | 927 (8)  | 823 (12) | 902 (9)  | 924 (8)  | 848 (11) | 870 (11) | 917 (9)  |
| 1.5      | NAIVE           | 885 (10) | 852 (11) | 867 (11) | 886 (10) | 746 (14) | 769 (13) | 890 (10) | 800 (13) | 760 (14) |
|          | LIKELIHOOD      | 850 (11) | 892 (10) | 918 (9)  | 793 (13) | 897 (10) | 929 (8)  | 835 (12) | 896 (10) | 923 (8)  |
|          | SKEW-NORMAL     | 848 (12) | 897 (10) | 922 (9)  | 801 (13) | 894 (11) | 939 (9)  | 831 (12) | 889 (10) | 924 (8)  |
|          | CORRECTED SCORE | 824 (12) | 876 (10) | 903 (9)  | 803 (13) | 879 (10) | 926 (8)  | 806 (13) | 872 (11) | 911 (9)  |
|          | CONDITIONAL SCORE | 814 (12) | 866 (11) | 904 (9)  | 789 (13) | 877 (10) | 922 (8)  | 800 (13) | 861 (11) | 906 (9)  |
|          | SIMEX           | 834 (12) | 887 (10) | 917 (9)  | 812 (12) | 895 (10) | 933 (8)  | 847 (11) | 898 (10) | 920 (9)  |
Figure S10: Mean squared error of the estimators of $\tau^2$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk normally distributed.
Figure S11: Mean squared error of the estimators of $\tau^2$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a mixture of Normals.
Figure S12: Mean squared error of the estimators of $\tau^2$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a Skew-Normal.
**Table S9:** Bias and standard deviation (SD) of the estimates of $\beta_0$, $\beta_1$, $\tau^2$, and average of the estimated standard errors (SE) obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$), with $n = 100$. Underlying risk normally distributed.

| $\tau^2$ | Method          | $\beta_0$ Bias | $\beta_0$ SD | $\beta_0$ SE | $\beta_1$ Bias | $\beta_1$ SD | $\beta_1$ SE | $\tau^2$ Bias | $\tau^2$ SD | $\tau^2$ SE |
|---------|----------------|----------------|-------------|-------------|----------------|-------------|-------------|---------------|-------------|-------------|
| 0.1     | NAIVE          | -0.003         | 0.053       | 0.051       | -0.072         | 0.061       | 0.049       | 0.159         | 0.045       | 0.037       |
|         | LIKELIHOOD     | -0.002         | 0.049       | 0.046       | -0.002         | 0.057       | 0.054       | -0.005        | 0.029       | 0.018       |
|         | SKEW-NORMAL    | -0.002         | 0.049       | 0.047       | -0.003         | 0.056       | 0.054       | -0.005        | 0.029       | 0.018       |
|         | CORRECTED SCORE| -0.003         | 0.054       | 0.048       | -0.005         | 0.066       | 0.057       | 0.012         | 0.043       | 0.081       |
|         | CONDITIONAL SCORE| -0.003      | 0.054       | 0.048       | -0.005         | 0.066       | 0.057       | 0.012         | 0.043       | 0.081       |
|         | SIMEX          | -0.003         | 0.054       | 0.051       | -0.005         | 0.066       | 0.061       | 0.012         | 0.044       | 0.044       |
| 0.5     | NAIVE          | 0.000          | 0.085       | 0.084       | -0.070         | 0.085       | 0.079       | 0.183         | 0.106       | 0.098       |
|         | LIKELIHOOD     | 0.001          | 0.081       | 0.079       | -0.001         | 0.088       | 0.086       | -0.034        | 0.087       | 0.119       |
|         | SKEW-NORMAL    | 0.001          | 0.081       | 0.078       | -0.002         | 0.088       | 0.086       | -0.034        | 0.087       | 0.119       |
|         | CORRECTED SCORE| 0.000          | 0.086       | 0.079       | -0.002         | 0.092       | 0.085       | 0.034         | 0.105       | 0.079       |
|         | CONDITIONAL SCORE| 0.000       | 0.086       | 0.079       | -0.002         | 0.092       | 0.085       | 0.034         | 0.105       | 0.079       |
|         | SIMEX          | 0.000          | 0.086       | 0.083       | -0.002         | 0.092       | 0.090       | 0.034         | 0.104       | 0.107       |
| 0.9     | NAIVE          | -0.004         | 0.105       | 0.106       | -0.075         | 0.106       | 0.100       | 0.195         | 0.173       | 0.156       |
|         | LIKELIHOOD     | 0.000          | 0.099       | 0.100       | -0.011         | 0.111       | 0.108       | -0.069        | 0.149       | 0.259       |
|         | SKEW-NORMAL    | 0.000          | 0.100       | 0.099       | -0.012         | 0.111       | 0.108       | -0.069        | 0.149       | 0.259       |
|         | CORRECTED SCORE| -0.004         | 0.105       | 0.100       | -0.009         | 0.114       | 0.105       | 0.045         | 0.172       | 0.094       |
|         | CONDITIONAL SCORE| -0.004      | 0.105       | 0.100       | -0.009         | 0.114       | 0.106       | 0.045         | 0.172       | 0.095       |
|         | SIMEX          | -0.004         | 0.105       | 0.106       | -0.009         | 0.114       | 0.111       | 0.043         | 0.171       | 0.166       |
| 1.5     | NAIVE          | 0.006          | 0.134       | 0.133       | -0.073         | 0.131       | 0.126       | 0.220         | 0.265       | 0.246       |
|         | LIKELIHOOD     | 0.014          | 0.127       | 0.126       | -0.008         | 0.141       | 0.134       | -0.136        | 0.229       | 0.520       |
|         | SKEW-NORMAL    | 0.014          | 0.127       | 0.125       | -0.009         | 0.140       | 0.134       | -0.137        | 0.229       | 0.521       |
|         | CORRECTED SCORE| 0.007          | 0.134       | 0.125       | -0.008         | 0.140       | 0.128       | 0.066         | 0.269       | 0.177       |
|         | CONDITIONAL SCORE| 0.007       | 0.134       | 0.125       | -0.008         | 0.140       | 0.128       | 0.066         | 0.269       | 0.17       |
|         | SIMEX          | 0.004          | 0.134       | 0.132       | -0.008         | 0.140       | 0.136       | 0.053         | 0.260       | 0.257       |
Figure S13: Mean squared error of the estimators of $\beta_0$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$), with $n = 100$. Underlying risk normally distributed.
Figure S14: Mean squared error of the estimators of $\beta_1$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$), with $n = 100$. Underlying risk normally distributed.
Figure S15: Mean squared error of the estimators of $\tau_2$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$), with $n = 100$. Underlying risk normally distributed.
Figure S16: Empirical coverage probabilities of confidence intervals for $\beta_0$ from uncorrected approach (NAIVE), likelihood approach under a Normal specification (LIKELIHOOD) or a Skew-Normal specification (SKEW-NORMAL) for the underlying risk distribution, SIMEX, corrected score and conditional score, on the basis of 1,000 replicates of simulation scenario $i$), with $n = 100$. Underlying risk normally distributed.
Figure S17: Empirical coverage probabilities of confidence intervals for $\beta_1$ from uncorrected approach (NAIVE), likelihood approach under a Normal specification (LIKELIHOOD) or a Skew-Normal specification (SKEW-NORMAL) for the underlying risk distribution, SIMEX, corrected score and conditional score, on the basis of 1,000 replicates of simulation scenario $i$), with $n = 100$. Underlying risk normally distributed.
Web Appendix B: Simulation results for the log event rate case

This web appendix reports a portion of the results of the simulation study carried out to compare the performance of the competing approaches in control rate regression, as described in Section 4 of the main manuscript. The reference is to simulation scenario ii) in the main manuscript.
Table S10: Bias and standard deviation (SD) of the estimates of $\beta_0$, and average of the estimated standard errors (SE) obtained from uncorrected approach (NAIVE), likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $ii)$. Underlying risk normally distributed.

| $\tau^2$ | Method          | Bias  | SD   | SE   | Bias  | SD   | SE   | Bias  | SD   | SE   |
|----------|-----------------|-------|------|------|-------|------|------|-------|------|------|
|          | $n = 10$        |       |      |      |       |      |      |       |      |      |
| 0.1      | NAIVE           | -0.007| 0.198| 0.115| -0.029| 0.194| 0.110| -0.049| 0.142| 0.072|
|          | LIKELIHOOD      | -0.012| 0.182| 0.131| -0.013| 0.128| 0.106| -0.004| 0.064| 0.058|
|          | SKEW-NORMAL     | -0.009| 0.179| 0.130| -0.013| 0.130| 0.106| -0.002| 0.062| 0.057|
|          | CORRECTED SCORE | 0.007 | 0.168| 0.126| 0.003 | 0.129| 0.103| 0.003 | 0.082| 0.071|
|          | CONDITIONAL SCORE | 0.007| 0.168| 0.126| 0.004 | 0.129| 0.103| 0.003 | 0.082| 0.071|
|          | SIMEX           | -0.002| 0.166| 0.132| -0.017| 0.128| 0.108| -0.016| 0.079| 0.073|
|          | $n = 20$        |       |      |      |       |      |      |       |      |      |
| 0.5      | NAIVE           | 0.000 | 0.537| 0.411| -0.033| 0.284| 0.242| -0.047| 0.406| 0.196|
|          | LIKELIHOOD      | -0.002| 0.273| 0.229| -0.019| 0.199| 0.175| -0.005| 0.114| 0.113|
|          | SKEW-NORMAL     | -0.008| 0.273| 0.228| -0.017| 0.199| 0.173| -0.002| 0.113| 0.112|
|          | CORRECTED SCORE | 0.016 | 0.269| 0.227| 0.000 | 0.197| 0.172| 0.016 | 0.115| 0.115|
|          | CONDITIONAL SCORE | 0.016| 0.268| 0.227| 0.000 | 0.197| 0.172| 0.016 | 0.116| 0.115|
|          | SIMEX           | 0.002 | 0.271| 0.239| -0.027| 0.198| 0.180| -0.011| 0.117| 0.118|
|          | $n = 50$        |       |      |      |       |      |      |       |      |      |
| 0.9      | NAIVE           | -0.059| 0.576| 0.599| -0.032| 0.448| 0.432| -0.027| 0.297| 0.252|
|          | LIKELIHOOD      | -0.027| 0.352| 0.296| 0.007 | 0.237| 0.221| -0.008 | 0.151| 0.144|
|          | SKEW-NORMAL     | -0.027| 0.350| 0.296| 0.007 | 0.235| 0.221| -0.006 | 0.151| 0.143|
|          | CORRECTED SCORE | -0.004| 0.341| 0.296| 0.029 | 0.235| 0.217| 0.017 | 0.150| 0.143|
|          | CONDITIONAL SCORE | -0.005| 0.341| 0.296| 0.029 | 0.235| 0.217| 0.017 | 0.150| 0.143|
|          | SIMEX           | -0.021| 0.345| 0.313| -0.006| 0.242| 0.228| -0.014| 0.152| 0.148|
|          | $n = 10$        |       |      |      |       |      |      |       |      |      |
| 1.5      | NAIVE           | -0.022| 0.717| 0.943| -0.040| 0.582| 0.673| -0.013| 0.371| 0.379|
|          | LIKELIHOOD      | -0.008| 0.408| 0.340| 0.002 | 0.296| 0.277| 0.005 | 0.187| 0.179|
|          | SKEW-NORMAL     | -0.012| 0.421| 0.340| 0.003 | 0.297| 0.276| 0.006 | 0.187| 0.179|
|          | CORRECTED SCORE | 0.003 | 0.409| 0.344| 0.024 | 0.293| 0.272| 0.030 | 0.185| 0.176|
|          | CONDITIONAL SCORE | 0.002| 0.409| 0.344| 0.024 | 0.293| 0.272| 0.030 | 0.185| 0.176|
|          | SIMEX           | -0.025| 0.435| 0.369| -0.020| 0.304| 0.288| -0.012| 0.190| 0.183|
Table S11: Bias and standard deviation (SD) of the estimates of $\beta_0$, and average of the estimated standard errors (SE) obtained from uncorrected approach (NAIVE), likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $ii$). Underlying risk distributed as a mixture of Normals.

| $\tau^2$ | Method    | Bias SD SE | Bias SD SE | Bias SD SE |
|----------|-----------|------------|------------|------------|
|          |           | $n = 10$   | $n = 20$   | $n = 50$   |
| 0.1      | NAIVE     | -0.007 0.180 0.123 | -0.069 0.361 0.172 | -0.052 0.148 0.084 |
|          | LIKELIHOOD| -0.023 0.164 0.123 | -0.031 0.139 0.114 | -0.006 0.070 0.060 |
|          | SKEW-NORMAL| -0.024 0.164 0.122 | -0.025 0.197 0.138 | 0.002 0.129 0.089 |
|          | CORRECTED SCORE | -0.010 0.159 0.119 | -0.018 0.136 0.111 | -0.009 0.087 0.077 |
|          | CONDITIONAL SCORE | -0.009 0.159 0.120 | -0.018 0.137 0.111 | -0.009 0.087 0.077 |
|          | SIMEX     | -0.014 0.158 0.123 | -0.036 0.133 0.114 | -0.027 0.082 0.078 |
| 0.5      | NAIVE     | -0.022 0.289 0.281 | -0.055 0.263 0.250 | -0.034 0.215 0.161 |
|          | LIKELIHOOD| -0.023 0.255 0.218 | -0.031 0.194 0.176 | -0.016 0.123 0.117 |
|          | SKEW-NORMAL| -0.023 0.255 0.214 | -0.029 0.192 0.171 | -0.012 0.128 0.110 |
|          | CORRECTED SCORE | -0.006 0.251 0.215 | -0.012 0.194 0.174 | 0.005 0.123 0.115 |
|          | CONDITIONAL SCORE | -0.006 0.251 0.215 | -0.012 0.194 0.174 | 0.005 0.123 0.115 |
|          | SIMEX     | -0.013 0.252 0.224 | -0.038 0.195 0.182 | -0.019 0.123 0.118 |
| 0.9      | NAIVE     | -0.039 0.763 0.799 | -0.048 0.342 0.378 | -0.046 0.229 0.244 |
|          | LIKELIHOOD| -0.013 0.328 0.280 | -0.020 0.238 0.219 | -0.010 0.151 0.143 |
|          | SKEW-NORMAL| -0.014 0.325 0.279 | -0.018 0.242 0.216 | -0.007 0.151 0.141 |
|          | CORRECTED SCORE | 0.006 0.324 0.279 | -0.001 0.235 0.217 | 0.014 0.150 0.144 |
|          | CONDITIONAL SCORE | 0.006 0.324 0.280 | -0.001 0.235 0.217 | 0.014 0.150 0.144 |
|          | SIMEX     | -0.005 0.328 0.292 | -0.033 0.240 0.228 | -0.017 0.153 0.149 |
| 1.5      | NAIVE     | -0.013 0.625 0.894 | -0.023 0.777 0.837 | -0.051 0.493 0.421 |
|          | LIKELIHOOD| -0.014 0.417 0.351 | 0.010 0.294 0.274 | -0.001 0.189 0.177 |
|          | SKEW-NORMAL| -0.015 0.413 0.352 | 0.011 0.291 0.269 | 0.004 0.185 0.175 |
|          | CORRECTED SCORE | 0.008 0.410 0.352 | 0.028 0.294 0.272 | 0.023 0.187 0.177 |
|          | CONDITIONAL SCORE | 0.009 0.410 0.353 | 0.027 0.293 0.272 | 0.023 0.187 0.177 |
|          | SIMEX     | -0.004 0.416 0.369 | -0.015 0.302 0.287 | -0.018 0.192 0.184 |
Table S12: Bias and standard deviation (SD) of the estimates of $\beta_0$, and average of the estimated standard errors (SE) obtained from uncorrected approach (NAIVE), likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $ii$). Underlying risk distributed as a Skew-Normal.

| $\tau^2$ | Method          | Bias  | SD   | SE  | Bias  | SD   | SE  | Bias  | SD   | SE  |
|---------|-----------------|-------|------|-----|-------|------|-----|-------|------|-----|
|         | $n = 10$        |       |      |     |       |      |     |       |      |     |
| 0.1     | NAIVE           | 0.022 | 0.283| 0.121| 0.029 | 0.244| 0.091| -0.006| 0.237| 0.076|
|         | LIKELIHOOD      | 0.014 | 0.239| 0.149| 0.032 | 0.195| 0.129| 0.019 | 0.118| 0.075|
|         | SKEW-NORMAL     | 0.005 | 0.234| 0.149| 0.028 | 0.219| 0.143| 0.020 | 0.118| 0.075|
|         | CORRECTED SCORE | 0.016 | 0.238| 0.146| 0.046 | 0.201| 0.123| 0.059 | 0.178| 0.099|
|         | CONDITIONAL SCORE | 0.017 | 0.238| 0.147| 0.045 | 0.202| 0.124| 0.058 | 0.178| 0.100|
|         | SIMEX           | 0.012 | 0.237| 0.146| 0.035 | 0.203| 0.129| 0.043 | 0.177| 0.114|
| 0.5     | NAIVE           | -0.012| 0.509| 0.322| 0.005 | 0.489| 0.286| 0.004 | 0.282| 0.159|
|         | LIKELIHOOD      | -0.001| 0.355| 0.268| 0.015 | 0.266| 0.208| 0.029 | 0.182| 0.148|
|         | SKEW-NORMAL     | -0.015| 0.343| 0.261| 0.014 | 0.267| 0.207| 0.035 | 0.180| 0.145|
|         | CORRECTED SCORE | 0.006 | 0.341| 0.270| 0.030 | 0.267| 0.208| 0.069 | 0.196| 0.143|
|         | CONDITIONAL SCORE | 0.007 | 0.341| 0.271| 0.031 | 0.267| 0.208| 0.069 | 0.196| 0.144|
|         | SIMEX           | -0.003| 0.353| 0.276| 0.016 | 0.269| 0.213| 0.047 | 0.197| 0.156|
| 0.9     | NAIVE           | -0.007| 0.629| 0.524| 0.017 | 0.468| 0.382| 0.019 | 0.398| 0.255|
|         | LIKELIHOOD      | 0.018 | 0.458| 0.350| 0.028 | 0.312| 0.260| 0.042 | 0.206| 0.180|
|         | SKEW-NORMAL     | 0.008 | 0.452| 0.343| 0.027 | 0.310| 0.257| 0.054 | 0.212| 0.179|
|         | CORRECTED SCORE | 0.020 | 0.452| 0.354| 0.049 | 0.315| 0.260| 0.081 | 0.221| 0.175|
|         | CONDITIONAL SCORE | 0.020 | 0.452| 0.355| 0.049 | 0.315| 0.260| 0.081 | 0.221| 0.176|
|         | SIMEX           | 0.012 | 0.458| 0.360| 0.030 | 0.320| 0.266| 0.054 | 0.226| 0.187|
| 1.5     | NAIVE           | 0.061 | 0.824| 0.872| -0.003| 0.655| 0.659| -0.004| 0.695| 0.466|
|         | LIKELIHOOD      | 0.026 | 0.552| 0.431| 0.039 | 0.375| 0.332| 0.057 | 0.251| 0.219|
|         | SKEW-NORMAL     | 0.022 | 0.545| 0.430| 0.045 | 0.372| 0.326| 0.072 | 0.249| 0.219|
|         | CORRECTED SCORE | 0.031 | 0.541| 0.438| 0.056 | 0.373| 0.329| 0.095 | 0.256| 0.213|
|         | CONDITIONAL SCORE | 0.031 | 0.541| 0.439| 0.056 | 0.373| 0.330| 0.096 | 0.256| 0.214|
|         | SIMEX           | 0.019 | 0.559| 0.449| 0.028 | 0.383| 0.339| 0.058 | 0.263| 0.227|
Figure S18: Mean squared error of the estimators $\beta_0$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk normally distributed.
Figure S19: Mean squared error of the estimators $\beta_0$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i)$. Underlying risk distributed as a mixture of Normals.
Figure S20: Mean squared error of the estimators $\beta_0$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a Skew-Normal.
Table S13: Bias and standard deviation (SD) of the estimates of $\beta_1$, and average of the estimated standard errors (SE) obtained from uncorrected approach (NAIVE), likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $ii)$. Underlying risk normally distributed.

| $\tau^2$ | Method        | Bias  | SD   | SE   | Bias  | SD   | SE   | Bias  | SD   | SE   |
|----------|---------------|-------|------|------|-------|------|------|-------|------|------|
|          |               | $n = 10$ |      |      | $n = 20$ |      |      | $n = 50$ |      |      |
| 0.1      | NAIVE         | -0.055 | 0.228 | 0.115 | -0.053 | 0.219 | 0.110 | -0.062 | 0.191 | 0.072 |
|          | LIKELIHOOD    | -0.038 | 0.230 | 0.129 | -0.051 | 0.186 | 0.105 | -0.024 | 0.085 | 0.061 |
|          | SKEW-NORMAL   | -0.033 | 0.225 | 0.129 | -0.050 | 0.183 | 0.105 | -0.025 | 0.087 | 0.061 |
|          | CORRECTED SCORE | -0.046 | 0.223 | 0.115 | -0.061 | 0.193 | 0.100 | -0.080 | 0.157 | 0.087 |
|          | CONDITIONAL SCORE | -0.048 | 0.222 | 0.116 | -0.062 | 0.193 | 0.101 | -0.081 | 0.157 | 0.087 |
|          | SIMEX         | -0.043 | 0.225 | 0.122 | -0.047 | 0.199 | 0.109 | -0.067 | 0.159 | 0.093 |
|          |               | $n = 20$ |      |      | $n = 50$ |      |      | $n = 100$ |      |      |
| 0.1      | NAIVE         | -0.056 | 0.466 | 0.411 | -0.069 | 0.285 | 0.242 | -0.056 | 0.342 | 0.196 |
|          | LIKELIHOOD    | -0.029 | 0.324 | 0.231 | -0.067 | 0.249 | 0.179 | -0.039 | 0.153 | 0.122 |
|          | SKEW-NORMAL   | -0.018 | 0.316 | 0.232 | -0.068 | 0.248 | 0.181 | -0.044 | 0.160 | 0.124 |
|          | CORRECTED SCORE | -0.045 | 0.313 | 0.212 | -0.082 | 0.247 | 0.168 | -0.088 | 0.182 | 0.129 |
|          | CONDITIONAL SCORE | -0.047 | 0.312 | 0.213 | -0.084 | 0.247 | 0.169 | -0.088 | 0.181 | 0.130 |
|          | SIMEX         | -0.044 | 0.315 | 0.226 | -0.064 | 0.253 | 0.178 | -0.067 | 0.186 | 0.135 |
|          |               | $n = 50$ |      |      | $n = 100$ |      |      | $n = 100$ |      |      |
| 0.1      | NAIVE         | -0.050 | 0.581 | 0.599 | -0.034 | 0.483 | 0.432 | -0.097 | 0.326 | 0.252 |
|          | LIKELIHOOD    | -0.058 | 0.426 | 0.299 | -0.066 | 0.304 | 0.226 | -0.057 | 0.190 | 0.154 |
|          | SKEW-NORMAL   | -0.052 | 0.418 | 0.300 | -0.064 | 0.299 | 0.227 | -0.066 | 0.197 | 0.157 |
|          | CORRECTED SCORE | -0.060 | 0.405 | 0.279 | -0.074 | 0.297 | 0.210 | -0.096 | 0.207 | 0.156 |
|          | CONDITIONAL SCORE | -0.063 | 0.404 | 0.280 | -0.076 | 0.296 | 0.211 | -0.097 | 0.207 | 0.156 |
|          | SIMEX         | -0.056 | 0.410 | 0.298 | -0.050 | 0.305 | 0.222 | -0.074 | 0.211 | 0.162 |
|          |               | $n = 20$ |      |      | $n = 50$ |      |      | $n = 100$ |      |      |
| 0.1      | NAIVE         | -0.058 | 0.714 | 0.943 | -0.039 | 0.601 | 0.673 | -0.081 | 0.359 | 0.379 |
|          | LIKELIHOOD    | 0.025  | 0.485 | 0.380 | -0.037 | 0.356 | 0.282 | -0.071 | 0.224 | 0.188 |
|          | SKEW-NORMAL   | 0.011  | 0.489 | 0.385 | -0.036 | 0.354 | 0.283 | -0.077 | 0.225 | 0.190 |
|          | CORRECTED SCORE | 0.011  | 0.488 | 0.362 | -0.046 | 0.342 | 0.263 | -0.099 | 0.223 | 0.184 |
|          | CONDITIONAL SCORE | 0.009  | 0.487 | 0.365 | -0.048 | 0.341 | 0.264 | -0.100 | 0.223 | 0.184 |
|          | SIMEX         | 0.031  | 0.498 | 0.390 | -0.019 | 0.353 | 0.279 | -0.069 | 0.230 | 0.193 |
Table S14: Bias and standard deviation (SD) of the estimates of $\beta_1$, and average of the estimated standard errors (SE) obtained from uncorrected approach (NAIVE), likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $ii)$. Underlying risk distributed as a mixture of Normals.

| $\tau^2$ | Method         | $n = 10$ |     |     |     | $n = 20$ |     |     |     | $n = 50$ |     |     |     |
|---------|----------------|----------|-----|-----|-----|----------|-----|-----|-----|----------|-----|-----|-----|
| 0.1     | NAIVE          | -0.079   | 0.309 | 0.123 | -0.079 | 0.414 | 0.172 | -0.133 | 0.259 | 0.084 |
|         | LIKELIHOOD     | -0.047   | 0.299 | 0.166 | -0.092 | 0.254 | 0.148 | -0.043 | 0.142 | 0.090 |
|         | SKEW-NORMAL    | -0.031   | 0.278 | 0.168 | -0.088 | 0.331 | 0.185 | -0.057 | 0.223 | 0.132 |
|         | CORRECTED SCORE| -0.047   | 0.290 | 0.154 | -0.099 | 0.255 | 0.143 | -0.158 | 0.240 | 0.129 |
|         | CONDITIONAL SCORE| -0.054  | 0.287 | 0.154 | -0.101 | 0.255 | 0.143 | -0.159 | 0.237 | 0.129 |
|         | SIMEX          | -0.063   | 0.284 | 0.154 | -0.088 | 0.260 | 0.151 | -0.143 | 0.240 | 0.136 |
| 0.5     | NAIVE          | -0.062   | 0.480 | 0.281 | -0.106 | 0.380 | 0.250 | -0.126 | 0.336 | 0.161 |
|         | LIKELIHOOD     | -0.016   | 0.471 | 0.307 | -0.080 | 0.323 | 0.244 | -0.065 | 0.225 | 0.173 |
|         | SKEW-NORMAL    | -0.013   | 0.457 | 0.304 | -0.066 | 0.314 | 0.237 | -0.078 | 0.224 | 0.167 |
|         | CORRECTED SCORE| -0.037   | 0.445 | 0.285 | -0.105 | 0.317 | 0.233 | -0.148 | 0.271 | 0.180 |
|         | CONDITIONAL SCORE| -0.042  | 0.442 | 0.286 | -0.107 | 0.316 | 0.234 | -0.149 | 0.270 | 0.180 |
|         | SIMEX          | -0.045   | 0.451 | 0.295 | -0.087 | 0.325 | 0.248 | -0.127 | 0.277 | 0.188 |
| 0.9     | NAIVE          | -0.044   | 0.959 | 0.799 | -0.114 | 0.490 | 0.378 | -0.134 | 0.369 | 0.244 |
|         | LIKELIHOOD     | -0.009   | 0.551 | 0.390 | -0.089 | 0.398 | 0.301 | -0.082 | 0.274 | 0.216 |
|         | SKEW-NORMAL    | 0.003    | 0.555 | 0.390 | -0.077 | 0.387 | 0.300 | -0.095 | 0.271 | 0.211 |
|         | CORRECTED SCORE| -0.030   | 0.533 | 0.363 | -0.109 | 0.398 | 0.287 | -0.154 | 0.296 | 0.215 |
|         | CONDITIONAL SCORE| -0.036  | 0.530 | 0.363 | -0.112 | 0.396 | 0.287 | -0.156 | 0.295 | 0.216 |
|         | SIMEX          | -0.035   | 0.542 | 0.378 | -0.087 | 0.410 | 0.305 | -0.128 | 0.303 | 0.227 |
| 1.5     | NAIVE          | -0.020   | 1.019 | 0.894 | 0.012  | 0.956 | 0.837 | -0.083 | 0.605 | 0.421 |
|         | LIKELIHOOD     | 0.004    | 0.733 | 0.500 | -0.066 | 0.469 | 0.381 | -0.067 | 0.306 | 0.264 |
|         | SKEW-NORMAL    | 0.003    | 0.723 | 0.498 | -0.058 | 0.464 | 0.374 | -0.081 | 0.309 | 0.256 |
|         | CORRECTED SCORE| -0.034   | 0.674 | 0.474 | -0.081 | 0.459 | 0.363 | -0.136 | 0.320 | 0.256 |
|         | CONDITIONAL SCORE| -0.039  | 0.669 | 0.473 | -0.084 | 0.458 | 0.364 | -0.138 | 0.319 | 0.257 |
|         | SIMEX          | -0.026   | 0.695 | 0.494 | -0.052 | 0.472 | 0.387 | -0.103 | 0.332 | 0.271 |
Table S15: Bias and standard deviation (SD) of the estimates of $\beta_1$, and average of the estimated standard errors (SE) obtained from uncorrected approach (NAIVE, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $ii$). Underlying risk distributed as a Skew-Normal.

| $\tau^2$ | Method          | Bias   | SD    | SE    | Bias   | SD    | SE    | Bias   | SD    | SE    |
|----------|-----------------|--------|-------|-------|--------|-------|-------|--------|-------|-------|
| 0.1      | NAIVE           | -0.058 | 0.271 | 0.121 | -0.079 | 0.258 | 0.091 | -0.066 | 0.230 | 0.076 |
|          | LIKELIHOOD      | -0.050 | 0.255 | 0.140 | -0.073 | 0.235 | 0.126 | -0.042 | 0.134 | 0.074 |
|          | SKEW-NORMAL     | -0.038 | 0.242 | 0.141 | -0.061 | 0.261 | 0.139 | -0.040 | 0.141 | 0.076 |
|          | CORRECTED SCORE | -0.038 | 0.246 | 0.119 | -0.076 | 0.240 | 0.107 | -0.112 | 0.216 | 0.105 |
|          | CONDITIONAL SCORE | -0.040 | 0.246 | 0.120 | -0.075 | 0.242 | 0.109 | -0.109 | 0.217 | 0.106 |
|          | SIMEX           | -0.038 | 0.245 | 0.132 | -0.071 | 0.244 | 0.120 | -0.103 | 0.219 | 0.11  |
| 0.5      | NAIVE           | -0.053 | 0.487 | 0.322 | -0.048 | 0.456 | 0.286 | -0.069 | 0.281 | 0.159 |
|          | LIKELIHOOD      | -0.043 | 0.415 | 0.262 | -0.063 | 0.312 | 0.212 | -0.074 | 0.213 | 0.152 |
|          | SKEW-NORMAL     | -0.030 | 0.398 | 0.258 | -0.062 | 0.317 | 0.211 | -0.080 | 0.217 | 0.151 |
|          | CORRECTED SCORE | -0.034 | 0.384 | 0.242 | -0.063 | 0.295 | 0.191 | -0.115 | 0.230 | 0.150 |
|          | CONDITIONAL SCORE | -0.035 | 0.384 | 0.242 | -0.063 | 0.295 | 0.192 | -0.115 | 0.229 | 0.151 |
|          | SIMEX           | -0.039 | 0.391 | 0.261 | -0.056 | 0.297 | 0.204 | -0.105 | 0.233 | 0.160 |
| 0.9      | NAIVE           | -0.008 | 0.623 | 0.524 | -0.090 | 0.477 | 0.382 | -0.084 | 0.362 | 0.255 |
|          | LIKELIHOOD      | -0.029 | 0.496 | 0.346 | -0.074 | 0.346 | 0.257 | -0.087 | 0.241 | 0.185 |
|          | SKEW-NORMAL     | -0.016 | 0.490 | 0.340 | -0.076 | 0.341 | 0.255 | -0.104 | 0.250 | 0.185 |
|          | CORRECTED SCORE | -0.026 | 0.475 | 0.315 | -0.086 | 0.336 | 0.237 | -0.128 | 0.255 | 0.180 |
|          | CONDITIONAL SCORE | -0.027 | 0.474 | 0.315 | -0.087 | 0.336 | 0.237 | -0.127 | 0.255 | 0.181 |
|          | SIMEX           | -0.023 | 0.478 | 0.340 | -0.078 | 0.342 | 0.249 | -0.114 | 0.260 | 0.190 |
| 1.5      | NAIVE           | -0.057 | 0.879 | 0.872 | -0.050 | 0.655 | 0.659 | -0.050 | 0.628 | 0.466 |
|          | LIKELIHOOD      | -0.032 | 0.614 | 0.436 | -0.083 | 0.421 | 0.329 | -0.098 | 0.282 | 0.225 |
|          | SKEW-NORMAL     | -0.029 | 0.606 | 0.441 | -0.087 | 0.411 | 0.329 | -0.120 | 0.287 | 0.226 |
|          | CORRECTED SCORE | -0.039 | 0.570 | 0.395 | -0.088 | 0.404 | 0.304 | -0.131 | 0.289 | 0.217 |
|          | CONDITIONAL SCORE | -0.040 | 0.570 | 0.395 | -0.089 | 0.403 | 0.304 | -0.131 | 0.288 | 0.217 |
|          | SIMEX           | -0.026 | 0.598 | 0.430 | -0.074 | 0.412 | 0.320 | -0.112 | 0.294 | 0.226 |
Figure S21: Mean squared error of the estimators $\beta_1$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk normally distributed.
Figure S22: Mean squared error of the estimators $\beta_1$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a mixture of Normals.
Figure S23: Mean squared error of the estimators $\beta_1$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a Skew-Normal.
Table S16: Bias and standard deviation (SD) of the estimates of $\tau^2$, and average of the estimated standard errors (SE) obtained from uncorrected approach (NAIVE, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $ii$). Underlying risk normally distributed.

| $\tau^2$ | Method          | Bias   | SD    | SE     | Bias   | SD    | SE     | Bias   | SD    | SE     |
|----------|----------------|--------|-------|--------|--------|-------|--------|--------|-------|--------|
|          |                | $n = 10$ |       |        | $n = 20$ |       |        | $n = 50$ |       |        |
| 0.1      | NAIVE          | 0.131  | 0.497 | 0.115  | 0.230  | 0.452 | 0.110  | 0.252  | 0.298 | 0.072  |
|          | LIKELIHOOD     | 0.017  | 0.148 | 0.048  | 0.055  | 0.189 | 0.092  | 0.037  | 0.156 | 0.053  |
|          | SKEW-NORMAL    | 0.012  | 0.137 | 0.044  | 0.047  | 0.176 | 0.084  | 0.029  | 0.132 | 0.042  |
|          | CORRECTED SCORE| 0.081  | 0.453 | 0.089  | 0.139  | 0.388 | 0.102  | 0.164  | 0.263 | 0.112  |
|          | CONDITIONAL SCORE| 0.081 | 0.453 | 0.089  | 0.139  | 0.388 | 0.102  | 0.164  | 0.263 | 0.112  |
|          | SIMEX          | 0.129  | 0.562 | 0.203  | 0.176  | 0.430 | 0.202  | 0.186  | 0.276 | 0.170  |
| 0.5      | NAIVE          | 0.323  | 0.832 | 0.411  | 0.227  | 0.493 | 0.242  | 0.461  | 0.644 | 0.196  |
|          | LIKELIHOOD     | -0.075 | 0.265 | 0.249  | -0.022 | 0.228 | 0.238  | 0.079  | 0.240 | 0.281  |
|          | SKEW-NORMAL    | -0.081 | 0.259 | 0.244  | -0.025 | 0.225 | 0.236  | 0.075  | 0.240 | 0.269  |
|          | CORRECTED SCORE| -0.021 | 0.472 | 0.136  | 0.071  | 0.421 | 0.131  | 0.137  | 0.289 | 0.113  |
|          | CONDITIONAL SCORE| -0.020 | 0.473 | 0.136  | 0.071  | 0.421 | 0.131  | 0.137  | 0.289 | 0.113  |
|          | SIMEX          | 0.123  | 0.596 | 0.399  | 0.168  | 0.475 | 0.311  | 0.192  | 0.302 | 0.225  |
| 0.9      | NAIVE          | 0.299  | 0.962 | 0.599  | 0.397  | 0.674 | 0.432  | 0.334  | 0.434 | 0.252  |
|          | LIKELIHOOD     | -0.173 | 0.364 | 0.524  | -0.076 | 0.325 | 0.498  | 0.041  | 0.277 | 0.461  |
|          | SKEW-NORMAL    | -0.176 | 0.369 | 0.526  | -0.074 | 0.326 | 0.502  | 0.045  | 0.282 | 0.462  |
|          | CORRECTED SCORE| -0.088 | 0.570 | 0.172  | -0.012 | 0.442 | 0.151  | 0.086  | 0.316 | 0.118  |
|          | CONDITIONAL SCORE| -0.087 | 0.570 | 0.172  | -0.012 | 0.442 | 0.151  | 0.086  | 0.316 | 0.118  |
|          | SIMEX          | 0.160  | 0.734 | 0.619  | 0.145  | 0.507 | 0.413  | 0.177  | 0.340 | 0.281  |
| 1.5      | NAIVE          | 0.387  | 1.350 | 0.943  | 0.520  | 1.003 | 0.673  | 0.355  | 0.502 | 0.379  |
|          | LIKELIHOOD     | -0.522 | 0.417 | 0.825  | -0.226 | 0.392 | 0.903  | -0.059 | 0.338 | 0.754  |
|          | SKEW-NORMAL    | -0.529 | 0.425 | 0.820  | -0.226 | 0.392 | 0.907  | -0.057 | 0.338 | 0.756  |
|          | CORRECTED SCORE| -0.369 | 0.523 | 0.209  | -0.118 | 0.525 | 0.182  | -0.002 | 0.389 | 0.133  |
|          | CONDITIONAL SCORE| -0.369 | 0.523 | 0.209  | -0.118 | 0.525 | 0.182  | -0.002 | 0.389 | 0.133  |
|          | SIMEX          | 0.031  | 0.705 | 0.879  | 0.142  | 0.612 | 0.604  | 0.156  | 0.417 | 0.384  |
Table S17: Bias and standard deviation (SD) of the estimates of $\tau^2$, and average of the estimated standard errors (SE) obtained from uncorrected approach (NAIVE, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $ii$). Underlying risk distributed as a mixture of Normals.

| $\tau^2$ | Method           | Bias  | SD    | SE    | Bias  | SD    | SE    | Bias  | SD    | SE    |
|----------|------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|          |                  | $n = 10$ |       |       | $n = 20$ |       |       | $n = 50$ |       |       |
| 0.1      | NAIVE            | 0.146 | 0.622 | 0.123 | 0.416 | 0.846 | 0.172 | 0.310 | 0.337 | 0.084 |
|          | LIKELIHOOD       | 0.007 | 0.093 | 0.038 | 0.091 | 0.248 | 0.154 | 0.063 | 0.220 | 0.085 |
|          | SKEW-NORMAL      | 0.000 | 0.082 | 0.035 | 0.164 | 0.362 | 0.209 | 0.190 | 0.434 | 0.171 |
|          | CORRECTED SCORE  | 0.096 | 0.555 | 0.090 | 0.199 | 0.467 | 0.123 | 0.228 | 0.314 | 0.132 |
|          | CONDITIONAL SCORE| 0.096 | 0.555 | 0.090 | 0.201 | 0.469 | 0.123 | 0.228 | 0.314 | 0.132 |
|          | SIMEX            | 0.151 | 0.693 | 0.233 | 0.244 | 0.518 | 0.266 | 0.254 | 0.325 | 0.216 |
|          |                  | $n = 0.1$ |       |       | $n = 10$ |       |       | $n = 20$ |       |       |
| 0.5      | NAIVE            | 0.062 | 0.557 | 0.281 | 0.251 | 0.495 | 0.250 | 0.290 | 0.330 | 0.161 |
|          | LIKELIHOOD       | -0.064| 0.247 | 0.251 | 0.004 | 0.228 | 0.268 | 0.010 | 0.264 | 0.307 |
|          | SKEW-NORMAL      | -0.082| 0.252 | 0.244 | -0.023| 0.254 | 0.264 | 0.056 | 0.302 | 0.286 |
|          | CORRECTED SCORE  | -0.011| 0.523 | 0.136 | 0.117 | 0.454 | 0.143 | 0.154 | 0.305 | 0.116 |
|          | CONDITIONAL SCORE| -0.011| 0.523 | 0.136 | 0.117 | 0.454 | 0.142 | 0.154 | 0.305 | 0.115 |
|          | SIMEX            | 0.128 | 0.657 | 0.401 | 0.220 | 0.504 | 0.349 | 0.211 | 0.317 | 0.232 |
|          |                  | $n = 0.5$ |       |       | $n = 10$ |       |       | $n = 20$ |       |       |
| 0.9      | NAIVE            | 0.698 | 2.221 | 0.799 | 0.235 | 0.565 | 0.378 | 0.295 | 0.368 | 0.244 |
|          | LIKELIHOOD       | -0.171| 0.356 | 0.527 | -0.065| 0.318 | 0.515 | 0.084 | 0.251 | 0.508 |
|          | SKEW-NORMAL      | -0.177| 0.366 | 0.528 | -0.096| 0.322 | 0.496 | 0.041 | 0.323 | 0.490 |
|          | CORRECTED SCORE  | -0.095| 0.597 | 0.175 | 0.033 | 0.483 | 0.160 | 0.120 | 0.326 | 0.122 |
|          | CONDITIONAL SCORE| -0.095| 0.597 | 0.175 | 0.033 | 0.483 | 0.160 | 0.120 | 0.326 | 0.122 |
|          | SIMEX            | 0.140 | 0.763 | 0.624 | 0.193 | 0.543 | 0.451 | 0.220 | 0.344 | 0.295 |
|          |                  | $n = 1.5$ |       |       | $n = 10$ |       |       | $n = 20$ |       |       |
| 1.5      | NAIVE            | 0.288 | 1.351 | 0.894 | 1.011 | 1.421 | 0.837 | 0.562 | 0.703 | 0.421 |
|          | LIKELIHOOD       | -0.459| 0.449 | 0.822 | -0.180| 0.396 | 0.939 | -0.009| 0.336 | 0.793 |
|          | SKEW-NORMAL      | -0.479| 0.450 | 0.809 | -0.232| 0.422 | 0.893 | -0.080| 0.385 | 0.753 |
|          | CORRECTED SCORE  | -0.222| 0.777 | 0.211 | -0.041| 0.576 | 0.186 | 0.034 | 0.372 | 0.133 |
|          | CONDITIONAL SCORE| -0.222| 0.777 | 0.211 | -0.041| 0.576 | 0.186 | 0.034 | 0.372 | 0.133 |
|          | SIMEX            | 0.144 | 0.994 | 0.926 | 0.231 | 0.677 | 0.633 | 0.203 | 0.413 | 0.389 |
Table S18: Bias and standard deviation (SD) of the estimates of $\tau^2$, and average of the estimated standard errors (SE) obtained from uncorrected approach (NAIVE, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario ii). Underlying risk distributed as a Skew-normal.

| $\tau^2$  | Method       | Bias  | SD    | SE    | Bias  | SD    | SE    | Bias  | SD    | SE    |
|-----------|--------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.1       | NAIVE        | 0.142 | 0.566 | 0.121 | 0.174 | 0.407 | 0.091 | 0.272 | 0.336 | 0.076 |
|           | LIKELIHOOD   | 0.010 | 0.114 | 0.038 | 0.055 | 0.157 | 0.079 | 0.048 | 0.176 | 0.060 |
|           | SKEW-NORMAL  | 0.006 | 0.104 | 0.036 | 0.089 | 0.258 | 0.110 | 0.047 | 0.188 | 0.053 |
|           | CORRECTED SCORE | 0.077 | 0.473 | 0.081 | 0.118 | 0.356 | 0.089 | 0.189 | 0.285 | 0.114 |
|           | CONDITIONAL SCORE | 0.077 | 0.473 | 0.081 | 0.118 | 0.356 | 0.089 | 0.189 | 0.285 | 0.114 |
|           | SIMEX        | 0.126 | 0.592 | 0.189 | 0.151 | 0.400 | 0.162 | 0.211 | 0.298 | 0.174 |
| 0.5       | NAIVE        | 0.145 | 0.809 | 0.322 | 0.358 | 0.785 | 0.286 | 0.281 | 0.342 | 0.159 |
|           | LIKELIHOOD   | -0.094 | 0.231 | 0.227 | -0.007 | 0.223 | 0.246 | 0.106 | 0.249 | 0.301 |
|           | SKEW-NORMAL  | -0.127 | 0.231 | 0.204 | -0.016 | 0.239 | 0.247 | 0.090 | 0.249 | 0.278 |
|           | CORRECTED SCORE | -0.004 | 0.560 | 0.137 | 0.083 | 0.425 | 0.130 | 0.158 | 0.291 | 0.114 |
|           | CONDITIONAL SCORE | -0.004 | 0.560 | 0.137 | 0.083 | 0.425 | 0.130 | 0.158 | 0.291 | 0.114 |
|           | SIMEX        | 0.133 | 0.706 | 0.413 | 0.164 | 0.474 | 0.301 | 0.209 | 0.308 | 0.227 |
| 0.9       | NAIVE        | 0.148 | 0.818 | 0.524 | 0.247 | 0.623 | 0.382 | 0.351 | 0.469 | 0.255 |
|           | LIKELIHOOD   | -0.189 | 0.359 | 0.496 | -0.077 | 0.313 | 0.481 | 0.077 | 0.288 | 0.491 |
|           | SKEW-NORMAL  | -0.219 | 0.357 | 0.471 | -0.097 | 0.313 | 0.462 | 0.076 | 0.288 | 0.483 |
|           | CORRECTED SCORE | -0.114 | 0.577 | 0.167 | 0.018 | 0.478 | 0.152 | 0.127 | 0.331 | 0.122 |
|           | CONDITIONAL SCORE | -0.114 | 0.577 | 0.167 | 0.018 | 0.478 | 0.152 | 0.127 | 0.331 | 0.122 |
|           | SIMEX        | 0.101 | 0.726 | 0.577 | 0.149 | 0.537 | 0.415 | 0.209 | 0.345 | 0.292 |
| 1.5       | NAIVE        | 0.244 | 1.142 | 0.872 | 0.476 | 0.907 | 0.659 | 0.783 | 1.034 | 0.466 |
|           | LIKELIHOOD   | -0.467 | 0.458 | 0.817 | -0.175 | 0.415 | 0.926 | -0.008 | 0.339 | 0.790 |
|           | SKEW-NORMAL  | -0.485 | 0.462 | 0.802 | -0.204 | 0.430 | 0.899 | -0.007 | 0.345 | 0.792 |
|           | CORRECTED SCORE | -0.258 | 0.709 | 0.206 | -0.036 | 0.590 | 0.184 | 0.062 | 0.392 | 0.135 |
|           | CONDITIONAL SCORE | -0.258 | 0.709 | 0.206 | -0.036 | 0.590 | 0.184 | 0.062 | 0.392 | 0.135 |
|           | SIMEX        | 0.093 | 0.913 | 0.870 | 0.184 | 0.668 | 0.612 | 0.204 | 0.424 | 0.395 |
Figure S24: Mean squared error of the estimators $\tau^2$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk normally distributed.
Figure S25: Mean squared error of the estimators $\tau^2$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a mixture of Normals.
Figure S26: Mean squared error of the estimators $\tau^2$ obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $i$). Underlying risk distributed as a Skew-Normal.
Figure S27: Empirical coverage probabilities of confidence intervals for $\beta_0$ from uncorrected approach (NAIVE), likelihood approach under a normal specification (LIKELIHOOD) or a Skew-Normal specification (SKEW-NORMAL) for the underlying risk distribution, SIMEX, corrected score and conditional score, on the basis of 1,000 replicates of simulation scenario $ii$). Underlying risk normally distributed.
Figure S28: Empirical coverage probabilities of confidence intervals for $\beta_0$ from uncorrected approach (NAIVE), likelihood approach under a Normal specification (LIKELIHOOD) or a Skew-Normal specification (SKEW-NORMAL) for the underlying risk distribution, SIMEX, corrected score and conditional score, on the basis of 1,000 replicates of simulation scenario $ii$). Underlying risk distributed as a mixture of Normals.
Figure S29: Empirical coverage probabilities of confidence intervals for $\beta_0$ from uncorrected approach (NAIVE), likelihood approach under a Normal specification (LIKELIHOOD) or a Skew-Normal specification (SKEW-NORMAL) for the underlying risk distribution, SIMEX, corrected score and conditional score, on the basis of 1,000 replicates of simulation scenario ii). Underlying risk distributed as a Skew-Normal.
Table S19: Empirical coverage probabilities (multiplied by 1,000) of confidence intervals for $\beta_0$ and associated Monte Carlo standard error (in parentheses, multiplied by 1,000) obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $\pi$. Underlying risk distributed as a Normal, Skew-Normal, or Mixture of Normals.

| $\tau^2$ | Method | 10 | 20 | 50 | n | 10 | 20 | 50 | n | 10 | 20 | 50 | n |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | NAIVE | 851 (9) | 806 (13) | 764 (13) | Normal | 810 (12) | 838 (13) | 867 (14) | Skew-Normal | 575 (16) | 617 (16) | 659 (17) | Mixture |
| | LIKELIHOOD | 881 (14) | 909 (9) | 939 (9) | Normal | 933 (9) | 924 (9) | 918 (9) | Skew-Normal | 838 (12) | 859 (12) | 879 (13) | Mixture |
| | SKewed-NORMAL | 887 (10) | 909 (9) | 926 (9) | Normal | 939 (9) | 924 (9) | 918 (9) | Skew-Normal | 838 (12) | 859 (12) | 879 (13) | Mixture |
| | CORRECTED SCORE | 883 (10) | 908 (9) | 928 (9) | Normal | 932 (9) | 924 (9) | 918 (9) | Skew-Normal | 838 (12) | 859 (12) | 879 (13) | Mixture |
| | SIMEX | 897 (10) | 925 (8) | 948 (7) | Normal | 942 (7) | 924 (7) | 918 (7) | Skew-Normal | 847 (11) | 874 (10) | 891 (11) | Mixture |
| 0.5 | NAIVE | 554 (16) | 792 (13) | 437 (16) | Normal | 831 (11) | 729 (14) | 688 (15) | Skew-Normal | 575 (16) | 729 (14) | 688 (15) | Mixture |
| | LIKELIHOOD | 871 (11) | 909 (9) | 926 (9) | Normal | 939 (9) | 924 (9) | 918 (9) | Skew-Normal | 838 (12) | 859 (12) | 879 (13) | Mixture |
| | SKewed-NORMAL | 876 (10) | 909 (9) | 926 (9) | Normal | 939 (9) | 924 (9) | 918 (9) | Skew-Normal | 838 (12) | 859 (12) | 879 (13) | Mixture |
| | CORRECTED SCORE | 878 (10) | 908 (9) | 928 (9) | Normal | 932 (9) | 924 (9) | 918 (9) | Skew-Normal | 838 (12) | 859 (12) | 879 (13) | Mixture |
| | SIMEX | 888 (10) | 925 (8) | 948 (7) | Normal | 942 (7) | 924 (7) | 918 (7) | Skew-Normal | 847 (11) | 874 (10) | 891 (11) | Mixture |
| 0.9 | NAIVE | 692 (15) | 631 (15) | 680 (15) | Normal | 680 (15) | 631 (15) | 680 (15) | Skew-Normal | 617 (16) | 659 (17) | 691 (18) | Mixture |
| | LIKELIHOOD | 902 (9) | 913 (9) | 936 (9) | Normal | 902 (9) | 913 (9) | 936 (9) | Skew-Normal | 788 (11) | 821 (12) | 854 (13) | Mixture |
| | SKewed-NORMAL | 904 (9) | 906 (9) | 932 (9) | Normal | 904 (9) | 906 (9) | 932 (9) | Skew-Normal | 788 (11) | 821 (12) | 854 (13) | Mixture |
| | CORRECTED SCORE | 903 (9) | 907 (9) | 933 (9) | Normal | 907 (9) | 907 (9) | 933 (9) | Skew-Normal | 788 (11) | 821 (12) | 854 (13) | Mixture |
| | SIMEX | 911 (9) | 925 (8) | 948 (7) | Normal | 923 (9) | 940 (7) | 969 (7) | Skew-Normal | 847 (11) | 874 (10) | 891 (11) | Mixture |
| 1.5 | NAIVE | 641 (15) | 631 (15) | 680 (15) | Normal | 617 (16) | 659 (17) | 691 (18) | Skew-Normal | 788 (11) | 821 (12) | 854 (13) | Mixture |
| | LIKELIHOOD | 902 (9) | 913 (9) | 936 (9) | Normal | 902 (9) | 913 (9) | 936 (9) | Skew-Normal | 788 (11) | 821 (12) | 854 (13) | Mixture |
| | SKewed-NORMAL | 904 (9) | 906 (9) | 932 (9) | Normal | 904 (9) | 906 (9) | 932 (9) | Skew-Normal | 788 (11) | 821 (12) | 854 (13) | Mixture |
| | CORRECTED SCORE | 903 (9) | 907 (9) | 933 (9) | Normal | 907 (9) | 907 (9) | 933 (9) | Skew-Normal | 788 (11) | 821 (12) | 854 (13) | Mixture |
| | SIMEX | 911 (9) | 925 (8) | 948 (7) | Normal | 923 (9) | 940 (7) | 969 (7) | Skew-Normal | 847 (11) | 874 (10) | 891 (11) | Mixture |
Figure S30: Empirical coverage probabilities of confidence intervals for $\beta_1$ from uncorrected approach (NAIVE), likelihood approach under a Normal specification (LIKELIHOOD) or a Skew-Normal specification (SKEW-NORMAL) for the underlying risk distribution, SIMEX, corrected score and conditional score, on the basis of 1,000 replicates of simulation scenario $ii)$. Underlying risk normally distributed.
Figure S31: Empirical coverage probabilities of confidence intervals for $\beta_1$ from uncorrected approach (NAIVE), likelihood approach under a Normal specification (LIKELIHOOD) or a Skew-Normal specification (SKEW-NORMAL) for the underlying risk distribution, SIMEX, corrected score and conditional score, on the basis of 1,000 replicates of simulation scenario $ii$). Underlying risk distributed as a mixture of Normals.
Figure S32: Empirical coverage probabilities of confidence intervals for $\beta_1$ from uncorrected approach (NAIVE), likelihood approach under a Normal specification (LIKELIHOOD) or a Skew-Normal specification (SKEW-NORMAL) for the underlying risk distribution, SIMEX, corrected score and conditional score, on the basis of 1,000 replicates of simulation scenario $i_i$). Underlying risk distributed as a Skew-Normal.
Table S20: Empirical coverage probabilities (multiplied by 1,000) of confidence intervals for $\beta_1$ and associated Monte Carlo standard error (in parentheses, multiplied by 1,000) obtained from naive analysis, likelihood analysis under a Normal or a Skew-Normal specification of the distribution of $\xi$, corrected score, conditional score, SIMEX, on the basis of 1,000 replicates of simulation scenario $ii$). Underlying risk distributed as a Normal, Skew-Normal, or Mixture of Normals.

| $\tau^2$ | Method   | $n$       |       |       |       |       |       |       |       |       |
|----------|----------|-----------|-------|-------|-------|-------|-------|-------|-------|-------|
|          |          | 10        | 20    | 50    | 10    | 20    | 50    | 10    | 20    | 50    |
| 0.1      | NAIVE    | 808 (12)  | 723 (14) | 693 (15) | 795 (13) | 590 (16) | 630 (15) | 766 (13) | 724 (14) | 631 (15) |
|          | LIKELIHOOD | 806 (13)  | 842 (12) | 927 (8)  | 756 (14) | 823 (12) | 882 (10) | 777 (13) | 802 (13) | 869 (11) |
|          | SKEW-NORMAL | 812 (13)  | 851 (12) | 924 (8)  | 769 (14) | 827 (12) | 865 (11) | 785 (14) | 810 (12) | 864 (11) |
|          | CORRECTED SCORE | 775 (13)  | 820 (12) | 899 (10) | 752 (14) | 809 (12) | 772 (13) | 737 (14) | 768 (13) | 761 (14) |
|          | CONDITIONAL SCORE | 778 (13)  | 827 (12) | 897 (10) | 752 (14) | 809 (12) | 772 (13) | 737 (14) | 768 (13) | 761 (14) |
|          | SIMEX    | 791 (13)  | 836 (12) | 909 (9)  | 747 (14) | 816 (12) | 801 (13) | 786 (13) | 791 (13) | 819 (12) |
| 0.5      | NAIVE    | 699 (15)  | 773 (13) | 513 (16) | 822 (12) | 791 (13) | 678 (15) | 760 (14) | 637 (15) | 649 (15) |
|          | LIKELIHOOD | 808 (12)  | 846 (11) | 910 (9)  | 760 (14) | 843 (12) | 882 (10) | 764 (13) | 802 (13) | 842 (12) |
|          | SKEW-NORMAL | 816 (13)  | 838 (12) | 906 (9)  | 773 (13) | 855 (11) | 867 (11) | 776 (13) | 797 (13) | 822 (12) |
|          | CORRECTED SCORE | 785 (13)  | 823 (12) | 884 (10) | 743 (14) | 817 (12) | 806 (13) | 762 (14) | 786 (13) | 793 (13) |
|          | CONDITIONAL SCORE | 792 (13)  | 820 (12) | 881 (10) | 744 (14) | 817 (12) | 804 (13) | 766 (14) | 787 (13) | 799 (13) |
|          | SIMEX    | 796 (13)  | 832 (12) | 892 (10) | 755 (14) | 833 (12) | 821 (12) | 784 (13) | 809 (12) | 823 (12) |
| 0.9      | NAIVE    | 778 (13)  | 635 (15) | 625 (15) | 634 (15) | 755 (14) | 700 (14) | 735 (14) | 696 (15) | 649 (15) |
|          | LIKELIHOOD | 790 (13)  | 858 (11) | 894 (10) | 779 (13) | 817 (12) | 866 (11) | 769 (13) | 823 (12) | 851 (11) |
|          | SKEW-NORMAL | 794 (13)  | 864 (11) | 884 (10) | 787 (13) | 838 (12) | 860 (11) | 775 (13) | 821 (12) | 836 (12) |
|          | CORRECTED SCORE | 782 (13)  | 829 (12) | 863 (11) | 766 (13) | 801 (13) | 811 (12) | 741 (14) | 793 (13) | 816 (12) |
|          | CONDITIONAL SCORE | 781 (13)  | 834 (12) | 860 (11) | 770 (13) | 803 (13) | 814 (12) | 741 (14) | 790 (13) | 816 (12) |
|          | SIMEX    | 799 (13)  | 841 (12) | 881 (10) | 776 (13) | 815 (12) | 828 (12) | 794 (13) | 799 (13) | 830 (12) |
| 1.5      | NAIVE    | 697 (15)  | 639 (15) | 661 (15) | 745 (14) | 585 (16) | 581 (16) | 711 (14) | 683 (15) | 510 (16) |
|          | LIKELIHOOD | 818 (67)  | 865 (11) | 883 (10) | 759 (14) | 853 (11) | 875 (10) | 782 (13) | 836 (12) | 853 (11) |
|          | SKEW-NORMAL | 812 (69)  | 865 (11) | 878 (10) | 764 (13) | 859 (11) | 867 (11) | 801 (13) | 845 (11) | 844 (11) |
|          | CORRECTED SCORE | 818 (67)  | 863 (15) | 871 (11) | 760 (14) | 832 (12) | 846 (11) | 770 (13) | 822 (12) | 822 (12) |
|          | CONDITIONAL SCORE | 818 (67)  | 863 (15) | 869 (11) | 758 (14) | 835 (12) | 841 (12) | 770 (13) | 824 (12) | 819 (12) |