Modeling of "Concentric Drum" Tilt Angle Model Based on Mechanical Analysis

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Abstract. "Concentric drums" is a group cooperation project, which requires better tacit understanding between the team members. At the same time, the force exerted on the rope can be accurately controlled, which contains a more complex mechanical relationship. In reality, it is impossible for the players to make force synchronously and the force is difficult to control accurately, which can be attributed to the following two situations: first, there is no situation in which the team has no force in advance but the size is not uniform when the force is applied, so the drum surface is at the level position at the moment when the players start to make force; second, there are individual players who make force in advance, and the strength of each player is different, resulting in the tilted of the drum surface when the players are exerting force. In the first case, the vertical component forces of two opposite members in the same straight line are combined into one force, then the moment of each resultant force is calculated and the vector sum is performed, the angular acceleration is solved according to the rigid body mechanics formula, and finally the tilt angle is calculated by using the time brought in by double integration. In the second case, firstly, consider that some members of the team will start the force ahead of time, and those members who do not start the force will be brought into the model of case I according to the force value of 0 to obtain tilt angle 1. Secondly, the force of the remaining team members is considered. At this time, based on the inclined plane, the force of each member after treatment is brought into the model of case I again to get the inclined angle tilt angle 2 caused by different forces. After two inclinations, three planes with different angles can be obtained. The angle between the final plane and the original plane can be calculated by using the space coordinate normal vector, that is, the bevel angle of horizontal inclination of the drum surface.

Key words: Force analysis; Space vector; Elastic collision.
1. **Introduction**

In this problem, the diameter of the drum surface of the concentric drum $d = 40$ cm, the height of the drum body $n = 22$ cm, the mass of the drum $m_1 = 3.6$ kg, and the mass of the volleyball used $m_2 = 0.27$ kg. The number of members pulling the rope around the drum body $n \geq 8$, and the minimum distance between members $P \geq 60$ cm. The drum surface is horizontal at the initial time, and its initial position is 11 cm lower than that of the rope at the horizontal time. Under actual circumstances, because the players can not precisely control the timing and strength of the force, the drum surface may be tilted, and a mathematical model is established to describe the relationship between the timing and strength of the player’s force and the angle of the drum surface at a particular moment. According to the different timing and strength of the players, the inclination angle of the drum surface is obtained at 0.1 s.

2. **Construction of tilt angle model based on mechanical analysis**

2.1. **Problem analysis**

In the first case, the spatial rectangular coordinate system I is established, and the pulling force $f_n$ of each rope is decomposed to the vertical direction, and the $z$-direction component $f_n \sin \theta_n$ of each rope pulling force is obtained. The symmetrical pulling force in the horizontal direction is synthesized in the $z$-direction component, and four pulling force components along the $z$-direction are obtained, representing the moment $\sum M_n$ and the moment direction of each group of forces; establish the rectangular coordinate system in the horizontal direction II. Four sets of moments are decomposed into $x$-axis and $Y$-axis of coordinate system II, and the moment components on each coordinate axis are synthesized into $\sum M_X$ and $\sum M_Y$ to obtain the total resultant moment $\sum M_{total}$. The total moment causes the drum to rotate. The drum is simplified as a thin-walled drum. The expression of the drum rotation angular acceleration $\alpha$ is listed. The angular acceleration $\alpha$ is integrated to get the drum rotation angle $\delta$. The program 2 is written with MATLAB software to solve the drum tilt angle. In the second case, when $n$ ($n \leq 8$) team members exert force $f_n$ in advance time of $t$ s, the drum inclines at an angle of $\beta_1$. The force is decomposed to the direction perpendicular to the drum surface, and the moment is calculated. According to the moment, the angle $\beta_1$ of $n$ team members exert force in advance of $t$ s, and the angle between each tension and the horizontal direction is $\theta_n$. According to the solution method in the first case, after the force is applied, the angle $\beta$ of the drum caused by all the pulling forces after $8-n$ teams are exerted can be calculated by the improved program 2. Using the above two procedures, we can get the tilt angle of the drum surface at 0.1 s under different starting time and force of Table 1.

### Table 1. symbol description

| symbol | description |
|--------|-------------|
| $f_n$  | Tension of each rope |
| $\theta_0$ | Angle between rope and horizontal plane when drum is suspension still |
| $\theta_n$ | Angle between tension of each rope and horizontal direction, |
| $n$ | Number of players pulling ropes around drum |
| $m_1$ | Drum quality |
| $m_2$ | The quality of Volleyball |

2.2. **Establishment and solution of the model**

It is specified that the number of members who pull the rope around the drum body in an ideal state is $n = 8$, and the force $f$ of each person to control the rope is the same in size and direction, and the drum surface remains horizontal, as shown in Figure 1. The problem can be considered as one-dimensional if only the ball is moving vertically.

The multiple reciprocation of the ball is regarded as multiple motion periods, and I, II, III and IV are recorded as one motion period, which is divided into four stages: static stage I before the movement of drum and ball, motion stage II before the collision, collision stage III and motion stage IV after the collision, as shown in Figure 1.
Static stage I: The conduct force analysis on the state of drum in static stage I before drum and ball movement is as shown in Fig. 2 and Fig. 3. The number of players is $n = 8$, the distance between players is $p = 60$cm, and the horizontal projection of each rope length is $l$ (make $\theta_0 = 45^\circ$):

$$l = \frac{60}{2 \sin(\pi/4)} = 78.4$$

For the first type of situation, the drum is horizontal at the beginning, but after the team members exert force, due to the different magnitude and direction of the force, the drum surface inclines, and a space rectangular coordinate system I is established. The angle of each team member's pulling rope and the applied force are shown in Figure 4, that is, $\phi = (\theta_n, f_n)$. The specific conditions of 8 team members are as follows: $\phi_1 = (\theta_1, f_1)$, $\phi_2 = (\theta_2, f_2), \ldots, \phi_8 = (\theta_8, f_8)$.
Based on the space rectangular coordinate system I, the pull force $f_n$ of each rope is decomposed to the $z$-axis direction, and the component $f_n \sin \theta_n$ of each rope pull force along the $z$-axis direction is obtained. Since the force is vector, there are size and direction, following the principle of vector addition, the pull force components in the $z$-direction at the moment when each member sends the force are synthesized. At the beginning, 8 members are symmetrically standing, i.e. No.1, No.5 and No.2. And No. 6, No. 3, No. 7, No. 4 and No. 8 stand in the same diameter direction respectively, and the angle between the adjacent two members and the line connecting the drum center is $45^\circ$. As shown in Figure 5.

The synthesis result is

$$f_a = f_1 \sin \theta_1 f_5 \sin \theta_5;$$
$$f_b = f_2 \sin \theta_2 f_6 \sin \theta_6;$$
$$f_c = f_3 \sin \theta_3 f_7 \sin \theta_7;$$
$$f_d = f_4 \sin \theta_4 f_8 \sin \theta_8.$$
For the moment expressions of columns $f_a, f_b, f_c, f_d$, the moment composition is as follows:

\begin{align*}
\sum M_1 &= \frac{d}{2} f_1 \sin \theta_1 - f_5 \sin \theta_5 \\
\sum M_2 &= \frac{d}{2} f_2 \sin \theta_2 - f_6 \sin \theta_6 \\
\sum M_3 &= \frac{d}{2} f_3 \sin \theta_3 - f_7 \sin \theta_7 \\
\sum M_4 &= \frac{d}{2} f_4 \sin \theta_4 - f_8 \sin \theta_8
\end{align*}

(1) (2) (3) (4)

The moment $\sum M_n$ and the moment direction of each group of forces are shown in Figure 6. When the moment value is negative, the moment direction is opposite to that in the figure.

Fig. 6 Force and moment diagram of each group of forces

Establish the rectangular coordinate system II in the horizontal direction, and decompose the four groups of moments into the X and Y axes of coordinate system II. As shown in Figure 7, the moment components on each coordinate axis are synthesized into $\sum M_X$ and $\sum M_Y$ to obtain the total resultant moment $\sum M_{total}$.

$$\sum M_{total} = \sqrt{\sum M_X^2 + \sum M_Y^2}$$

(5)

Fig. 7 Diagram of drum moment decomposition
The total moment \( \sum M_{\text{total}} \) always causes the drum to rotate. The drum is simplified as a thin-walled drum. The expressions of the angular acceleration \( \alpha \), the angular velocity \( \omega \) and the rotation angle \( \delta \) of the drum are listed.

\[
\sum M_{\text{total}} = J_z \alpha \tag{6}
\]

\[
J_z = \frac{m_1}{12} [3(r_1^2 + r_2^2) + l^2] \tag{7}
\]

\( \alpha \) is the angular acceleration, \( J_z \) is the moment of inertia around any axis parallel to the drum surface, \( m_1 \) is the mass of the drum, \( r_1 \) is the outer diameter of the drum, \( r_2 \) is the inner diameter of the drum, and \( l \) is the height of the drum.

\[
\frac{dw}{dt} = \alpha \tag{8}
\]

\[
\frac{d\delta}{dt} = \omega \tag{9}
\]

Write a program with MATLAB to find out the tilt angle of the drum caused by the different strength of the players. This mechanical model can be used to describe the relationship between the strength of the players and the inclination angle of the drum surface at a certain time.

For the second kind of situation, some team members exert force ahead of time. Different from the first kind of situation, there are \( n \) (\( n < 8 \)) team members exert force ahead of time \( t_s \), the force of each person pulling rope is \( f_n \), the angle between each force and horizontal direction is \( \theta_n \), the angle of inclination of drum advance \( t_s \) is \( \beta_1 \), establish a straight angle coordinate system, decompose \( n \) forces to the direction perpendicular to drum surface as shown in Figure 8, along the vertical direction of drum. The component is \( f_n \sin \theta_n \cos \beta_1 \).

\[\text{Take } n \text{ forces as the resultant force in the vertical direction } f_a = f_1 \sin \theta_1 - f_3 \sin \theta_3 - f_5 \sin \theta_5; \]
\[f_b = f_2 \sin \theta_2 - f_6 \sin \theta_6; \]
\[f_c = f_3 \sin \theta_3 - f_7 \sin \theta_7; \]
\[f_d = f_4 \sin \theta_4 - f_8 \sin \theta_8. \]

For those who do not exert force in advance, \( f = 0 \) can be taken to calculate the moment \( \sum M_1, \sum M_2, \sum M_3, \sum M_4 \) and the total moment \( \sum M_{\text{total}} \). According to equation (3) (4) (5), the angle \( \beta_1 = \int_{t_0}^{t} w \, dt \) of \( N \) team members' inclining force drum in advance time of \( t_s \) is obtained, and \( \beta_1 \) can be obtained by MATLAB programming.

After \( t_s \), the other 8-\( n \) team members exert force. At this time, the direction of the force of \( n \) individuals who exert force in advance is determined. The angle between each pulling force and the horizontal direction is \( \theta_n \), the 8-\( n \) team members exert force, the component force of each member perpendicular to plane 2 can be calculated by the formula \( f_n \sin \theta_n \cos \beta_1 \), use the first type of model.
again to calculate the inclination angle $\beta_2$ of drum surface relative to plane 2, as shown in Figure 9. Since the two inclination angles are not in the same plane, it is not easy to take the result of adding and subtracting the two angles as the final inclination angle. To solve this problem, the coordinate system is established with plane 2 as the middle plane, and the angle between plane 3 and plane 1 (that is, the horizontal plane) is calculated by using the conclusion that the angle between plane normal vectors is the angle between planes, that is, the total tilt angle. The normal vector of plane 2 can be easily known as $(0, 0, 1)$; the normal vector of plane 1 can be calculated by $(-\cos(\frac{\pi}{2} - \beta_1), 0, \sin(\frac{\pi}{2} - \beta_1))$; for the normal vector of plane 3, the included angle $\theta$ of the inclined direction of plane 2 and plane 3 should be introduced, and the angle can be calculated by the moment coordinate of the drum surface at the second inclination. In conclusion, the normal vector of plane 3 is $(-\cos(\theta), \sin(\theta), \tan(\frac{\pi}{2} - \beta_2))$, and finally use the normal vector to find the dihedral angle formula.

$$\cos(\vec{n}_1, \vec{n}_2)$$

The tilt angle of the last drum can be calculated by MATLAB.

**Fig. 9** Rotation of drum surface when all members apply force after ts advance

3. Conclusion

After the above analysis and modeling, by calling the model, it can be found that if the number of players is 8 people, the 2nd and 5th players will advance 0.1 second in advance, and the 1st and 4th players will have a force of 90N, while the strength of other players is 80N, so the angle of the drum surface is 0.0589281 degrees. Similarly, by giving different initial values, the inclination angle of the drum surface in other cases can also be obtained.

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