Constraining and constructing mass distributions of Primordial Black Holes from 21cm signal

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The evaporation of Primordial Black Hole (PBH) via Hawking radiation influences the evolution of Inter Galactic Medium by heating up the latter and consequently affects the 21cm signal originated from the neutral Hydrogen atoms. In this work, we have considered EDGES observational data of 21cm line corresponding to cosmic dawn era to constrain the mass and the abundance of PBHs. In this context, three different PBH mass distributions namely, monochromatic, power law and lognormal mass distributions are considered to estimate the effects of PBH evaporation on the 21cm brightness temperature $T_{21}$. The impacts of Dark Matter - baryon interactions on $T_{21}$ are also considered in this work along with the influences of PBH evaporation. Further an attempt has been made in this work to formulate a mass distribution expression for the PBHs in the context of the 21cm results considered here. The mass distribution best suited for the present purpose is found to be a combination of an error function and Owen function. Bounds on Dark Matter mass are also calculated in this work by considering the PBH mass distributions.

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I. INTRODUCTION

The 21cm hydrogen spectrum is obtained due to the transition between two hyperfine spin states \((s = 0\) and \(1\)) of neutral hydrogen atoms. Since hydrogen occupies about 75% of the baryonic mass of the Universe, the 21cm hydrogen spectrum could be an important probe to Cosmos in general and cosmic processes of the dark age and reionization epoch in particular. The observational results of 21cm line is generally expressed in terms of the brightness temperature \(T_{21}\) of 21cm line which is dependent on the background radio temperature (in this work the background temperature is CMB temperature \(T_\gamma\)), spin temperature of the hydrogen gas \(T_s\) (\(T_s\) is the excitation temperature of the hydrogen gas) and optical depth \(\tau\) of the gas and it is defined as,

\[
T_{21} = \frac{T_s - T_\gamma}{1 + z}(1 - e^{-\tau}) .
\]

Therefore, 21cm spectrum show absorption or emission signal if \(T_s < T_\gamma\) and \(T_s > T_\gamma\) respectively.

Observational outcomes of EDGES’s (Experiment to Detect the Global EoR Signature) has become a remarkable probe in exploration of several unknown cosmic phenomena of the cosmic dark age. EDGES experiment reported excess absorption trough in \(T_{21}\) signal corresponding to cosmic dawn epoch around \(z \approx 17\). According to the standard cosmological scenario, the brightness temperature at \(z \approx 17\) is obtained about \(-200\) mK but EDGES experiment observed that 21cm brightness temperature during cosmic dawn is \(T_{21} = -500^{+200}_{-500}\) mK at \(z \approx 17\). Consequently, in order to explain the observational outcomes of EDGES, a certain amount of additional cooling is required, which essentially appears in the form of baryon-DM scattering [1]. However, other heating/cooling effects such as PBH evaporation [2–4], annihilation [5–8], and decay of dark matter candidates [4, 9, 10] etc. may perturb the global 21-signal remarkably. In spite of its popularity, the EDGES results are suffered from controversies [11–13]. Recently, SARAS 3 has challenged the EDGES results with 95.3% confidence level [14]. Despite of this recent controversy, in this work we use EDGES result to represent any global 21cm excess absorption signal [15].

On the other hand, Primordial Black Holes (PBHs) has been the centre of interest for decades in several aspects of astrophysics and cosmology. PBHs are believed to be generated as an outcome of the collapse of the over density regions in the early epochs of the Universe [16–19]. However, besides the standard scenarios, there are few alternative conjectures for addressing the formation of PBHs, namely, collapse of domain walls and cosmic strings [20–22], fragmentation of scalar condensation [23–25] etc. PBHs may be substantially smaller in mass in comparison to the stellar-mass black holes [3, 4, 15, 26]. As a result, the Hawking radiation [27] from such black holes (BHs)
are significantly prominent. Therefore, the phenomenon of PBH evaporation via Hawking radiation may be a promising tool in the exploration of several aspects of this hypothetical candidate of the black hole. The emitted particles in the form of Hawking radiation heat up the baryonic medium and thus could influence the global 21cm signature.

In the present analysis, we attempt to study the mass distribution of PBHs in the context of the global 21cm signal. As it is suggested from recent studies that extended mass distribution of PBH could be favourable, we essentially focus on two special types of distribution function namely, power-law mass distribution \cite{28, 29} and lognormal mass distribution \cite{28, 29} and find constraints on the distribution parameters using the observed limit of EDGES experiment \((-500^{+200}_{-500}\ mK)\).

It should be mentioned here that in this work the effects of Dark Matter - baryon (DM - baryon) scattering on the 21cm spectrum are also taken into account. The cooling effect due to baryon - DM interaction essentially governed by the baryon - DM scattering cross-section, given by \(\bar{\sigma} = \sigma_0(v/c)^n\), where \(v\) is the velocity and \(c\) denotes the velocity of light in space. The index \(n\) in the above expression depends on different physical processes. In the case of DM candidates having magnetic dipole moment, the index \(n\) is considered as \(n = +2, -2\) and \(n = 2, 1, 0, -1\) while the scattering in presence of Yukawa potential \cite{30} is considered. On the other hand, for millicharged DM candidate \cite{31, 32}, \(n = -4\) is chosen. The variation of baryon-DM cross-section is addressed in Ref. \cite{33} for a wide mass range of DM. Similar studies have also been carried out in Ref. \cite{34–36}. In the present analysis, cross-section \(\bar{\sigma}\) is parameterized as \(\bar{\sigma} = \sigma_0(v/c)^{-4}\) \cite{1, 37, 38} where the term \(\sigma_0\) is the absolute scattering cross-section of DM - baryon scattering. Several recent phenomenological studies on the global 21cm signal also suggest the similar velocity dependence \((n = -4)\) of the baryon-DM cross-section \cite{1, 35, 36, 38, 39}. However in this work we consider \(\sigma_0 \sim 10^{-41}\ cm^2\).

Along with constraining the parameters of two possible mass distributions of PBHs (power law and lognormal) with EDGES like results, we also propose a new distribution of PBH mass based on the EDGES observation. In this particular case, we evaluate the weight factor at every PBH mass by computing the brightness temperature of 21cm hydrogen line and comparing that with the EDGES result.

We discuss the imprints of PBH on 21cm line in section II while in section III the impacts of DM - baryon scattering on the spectrum are discussed and temperature evolutions are briefly described in section IV. The calculations and results are furnished in section V and finally in section VI summary and discussions are given.
II. IMPRINTS OF PRIMORDIAL BLACK HOLES ON THE 21CM LINE

In this section, we give a brief account of the energy injection of PBHs due to Hawking radiation on IGM. This can affect the evolution of IGM and thus the 21cm brightness signal.

A. Energy Injection Effects of Primordial Black Holes on IGM

Evaporation of PBHs through Hawking radiation can be a possible steady source of electrons/positrons and photons. These particles, emitted from relatively low mass PBHs \( M_{BH} < 10^{15} \) g, can interact with IGM and thus consequently modify the 21cm brightness temperature [3, 4, 40].

The rate of mass loss of the PBHs due to Hawking radiation is given by [41, 42],

\[
\frac{dM_{BH}}{dt} \approx -5.34 \times 10^{25} \sum_i \phi_i \left( \frac{M_{BH}}{g} \right)^{-2} \text{g sec}^{-1},
\]

where the coefficient \( \phi_i \) denotes the evaporation fraction of the \( i \)-th particle. The total evaporation fraction is calculated as [43],

\[
\sum_i \phi_i = 1.569 + 0.569\exp \left( -\frac{0.0234}{T_{PBH}} \right) + 3.414\exp \left( -\frac{0.066}{T_{PBH}} \right) + 1.707\exp \left( -\frac{0.11}{T_{PBH}} \right) + 0.569\exp \left( -\frac{0.394}{T_{PBH}} \right) + 1.707\exp \left( -\frac{0.413}{T_{PBH}} \right) + 1.707\exp \left( -\frac{22}{T_{PBH}} \right) + 0.963\exp \left( -\frac{0.1}{T_{PBH}} \right).
\]

Therefore, the total evaporation rate is dependent on the temperature of PBH \( T_{PBH} \) which is defined as \( T_{PBH} \approx 1.06 \times \left( \frac{10^{13} \text{g}}{M_{BH}} \right) \) GeV [27].

The energy injection rate per unit volume due to the evaporation of PBH is computed as [43],

\[
\frac{dE}{dV dt} \bigg|_{PBH} = -\frac{dM_{BH}}{dt} n_{PBH}(z),
\]

where \( n_{PBH}(z) \) is the number density of PBHs at redshift \( z \) and defined by [3],

\[
n_{PBH}(z) \approx 1.46 \times 10^{-4} \beta_{BH}(1 + z)^3 \left( \frac{M_{PBH,i}}{g} \right)^{-3/2} \text{cm}^{-3}.
\]

In the above equation, \( M_{PBH,i} \) represents the initial mass of PBH and \( \beta_{BH} \) is the initial mass fraction of PBH.
B. Mass Distribution Functions of Primordial Black Holes

In order to compute the energy injection of PBHs by using the above Eq. 4, it is considered that PBHs would have monochromatic mass distribution i.e., all the PBHs would be of identical masses, but some recent studies suggest that extended mass distributions would rather be favourable [28]. For such extended mass functions of PBHs the energy injection rate per unit volume is calculated as,

\[ \frac{dE}{dVdt}\bigg|_{\text{total}} = \int_{\gamma M_{PBH}} dM_{PBH} \, g(M_{PBH}) \, \frac{dE}{dVdt}\bigg|_{PBH}, \]  

with \( g(M_{PBH}) \) being the mass distribution function of PBHs. Hence, in this work we have considered two theoretically motivated mass distribution functions of PBHs namely, power law mass distribution [28] and lognormal mass distribution [28].

Power law mass distribution of PBHs arises from the scale invariant density fluctuations or from the cosmic string collapse [28] and this mass distribution function is expressed as [28, 29, 44],

\[ g(M_{BH}) = \frac{\gamma}{M_{\max}^{\gamma} - M_{\min}^{\gamma}} M_{BH}^{\gamma - 1}, \]  

where \( \gamma \) represents the power law index and \( M_{\max} \) and \( M_{\min} \) denote maximum mass limit and minimum mass limit of PBHs respectively. The power law index (\( \gamma \)) is related to the equation of state (\( \omega \)) at the time of PBH formation with the relation \( \gamma = -\frac{2\omega}{1 + 2\omega} \) [45]. Since PBH formations are assumed to take place at post inflationary time, the values of power index \( \gamma \) would be \( \gamma \in \{-1, 1\} \).

On the other hand, lognormal mass distribution of PBHs is considered when PBHs are formed from a smooth symmetric peak in the inflationary power spectrum [46]. The lognormal mass function is defined by [28, 29, 44],

\[ g(M_{BH}) = \frac{1}{\sqrt{2\pi\sigma M_{BH}}} \exp \left( -\frac{\ln^2(M_{BH}/\mu)}{2\sigma^2} \right), \]  

while \( \mu \) and \( \sigma \) are respectively the mean and standard deviation of the lognormal distribution. Such mass function of PBHs is first observed in Ref. [47] to address a mechanism of PBH formation for a model of baryogenesis. Later some authors have discussed this type of PBH mass distribution both theoretically and numerically [48].

III. IMPACTS OF DARK MATTER - BARYON INTERACTION ON THE 21CM LINE

In this work we have also considered the impacts of the interaction between Dark Matter (DM) and baryonic matter on the 21cm signal. In literature it is discussed that due to DM - baryon
interaction, baryon can transfer heat to the colder DM fluid and hence influence the evolution of 21cm brightness temperature [38, 49]. Moreover, the relative velocity \( V_{χb} \) between DM and baryon fluid would also affect the 21cm line [1] as the tendency to damp their relative velocity will heat up both of the fluids. Hence, in this work we have considered both of the above mentioned effects of DM - baryon interaction on the 21cm line. The heating rate of baryon can be evaluated from Ref. [1] as,

\[
\frac{dQ_b}{dt} = \frac{2m_bρ_χσ_0e^{-r^2/2}(T_χ - T_b)}{(m_χ + m_b)^2\sqrt{2πu_{th}^3}} + \frac{ρ_χ}{ρ_m} \frac{m_χm_b}{m_χ + m_b}V_{χb}D(V_{χb}),
\]

where \( ρ_χ, \rho_m \) are energy densities of DM and total matter respectively while \( T_χ \) and \( T_b \) denote DM temperature and baryon temperature respectively. The masses of DM and baryon are represented by \( m_χ \) and \( m_b \). In the above equation, the first term on the r.h.s arises from the temperature difference of DM and baryon \( (T_χ - T_b) \) and the second term originates due to the velocity difference \( (V_{χb}) \) between them. The drag term \( D(V_{χb}) \) is calculated as [1],

\[
D(V_{χb}) \equiv -\frac{dV_{χb}}{dt} = \frac{ρ_mσ_0}{m_b + m_χ} \frac{1}{V_{χb}^2}F(r),
\]

where \( r \equiv V_{χb}/u_{th}, \frac{V_{χb}^2}{u_{th}^2} = \frac{T_b}{m_b} + \frac{T_χ}{m_χ} \) and \( F(r) \equiv \text{erf} \left( \frac{r}{\sqrt{2}} \right) - \sqrt{\frac{2}{π}}e^{-r^2/2}r \). The parametrization \( \bar{σ} = σ_0v^{-4} \) for interaction cross section of DM and baryon fluid is considered for this calculation. The heating rate of DM \( (\frac{dQ_χ}{dt}) \) can be obtained by interchanging \( χ \leftrightarrow b \) in Eq. 9.

**IV. TEMPERATURE EVOLUTIONS AND 21CM SIGNAL**

In this section, we calculate the evolutions of temperatures \( (T_b, T_χ) \) and 21cm signal by including the effects of the energy injection of PBHs and DM - baryon interaction. The temperature evolutions of DM and baryon can be calculated by solving the following coupled differential equations [50],

\[
\frac{dT_χ}{dz} = \frac{2T_χ}{1 + z} - \frac{2\dot{Q}_χ}{3(1 + z)H(z)}, \tag{11}
\]

\[
\frac{dT_b}{dz} = \frac{2T_b}{1 + z} + \frac{Γ_c}{(1 + z)H(z)}(T_b - T_γ) - \frac{2\dot{Q}_b}{3(1 + z)H(z)} - \frac{2}{3k_bH(z)(1 + z)} \frac{K_{PBH}}{1 + f_{He} + x_e}. \tag{12}
\]

Here, \( T_γ = 2.725(1 + z) \) K is the photon temperature and \( Γ_c = \frac{8σ_T a_s T_γ^4 x_e}{π(1 + f_{He} + x_e)m_e c} \) denotes the Compton interaction rate where \( σ_T \) and \( a_s \) are the Thomson scattering cross section and the radiation constant respectively. The fractional abundance of He is denoted by \( f_{He} \) while the free electron
abundance is \( x_e = n_e / n_H \) (\( m_e \) and \( c \) is the electron mass and the speed of light). The third term on the r. h. s of Eq. 12 includes the effect of energy injection from PBHs due to Hawking radiation where \( K_{PBH} \) is expressed as [4, 40, 51–53],

\[
K_{PBH} = \chi_h f(z) \left( \frac{1}{n_b} \frac{dE}{dV dt} \right)_{total},
\]  

with \( \chi_h = (1 + 2x_e)/3 \) being the fraction of the emitted energy contributes to the heating of IGM and the parameter \( f(z) \), stands for the ratio of the total amount of deposited energy to the energy injected to the medium due to PBH evaporation [54–58].

To compute the evolution of baryon temperature, the evolution of free electron fraction \( x_e \) is needed to be calculated simultaneously. The evolution equation of \( x_e \) is expressed as [59]

\[
\frac{dx_e}{dz} = \frac{C_P}{(1+z)H(z)} \left( n_H A_B x_e^2 - 4(1-x_e)B_B e^{-4\gamma} \right) - \frac{1}{(1+z)H(z)} I_{PBH}(z),
\]  

where the Peebles C-factor [60] is represented by \( C_P \), \( E_0 \) denotes the ground state energy of Hydrogen (\( E_0 = 13.6 \) eV) while the effective recombination coefficient and the effective photoionization rate to and from the excited states are \( A_B \) and \( B_B \) respectively [61]. In the above Eq. 14, the \( I_{PBH} \) denotes the ionization rate caused by the energy injection of PBHs and it is defined by [4, 40, 51–53],

\[
I_{PBH} = \chi_i f(z) \left( \frac{1}{n_b} \frac{1}{E_0} \frac{dE}{dV dt} \right)_{total},
\]  

with \( \chi_i = (1 - x_e)/3 \) is the fraction of injected energy influences ionization of the IGM.

To obtain the evolutions of \( T_b \) and \( T_\chi \) the variations of the relative velocity between DM and baryon should also be simultaneously calculated with the differential equation [1],

\[
\frac{dV_{\chi b}}{dz} = \frac{V_{\chi b}}{1+z} + \frac{D(V_{\chi b})}{(1+z)H(z)}.  
\]  

Since Eqs. 2, 11, 12, 14 and 16 are all coupled, we need to solve these five equations simultaneously with proper initial conditions to compute the evolutions of baryon temperature \( T_b \) and thus to obtain the 21cm brightness temperature. Now, the spin temperature \( T_s \), defined by the ratio of the number densities of Hydrogen atoms in spin triplet and spin singlet states \( \left( n_1/n_0 = g_1 / g_0 \exp(-h\nu/kT_s) \right) \), is calculated from the expression [62],

\[
T_s^{-1} = \frac{T_\gamma^{-1} + y_c T_b^{-1} + y_\alpha T_\alpha^{-1}}{1 + y_c + y_\alpha}.  
\]
Here, \( y_\alpha \) and \( y_c \) denote the Lyman-\( \alpha \) coupling parameter and collisional coupling parameter respectively [63–65] while \( T_\alpha \) is temperature of the Lyman-\( \alpha \) background which is identical to baryon temperature \( T_b \) for \( z \approx 20 \) [66].

As mentioned in Sect. I, the 21cm brightness temperature \( T_{21} \) can now be calculated from the definition,

\[
T_{21} = \frac{T_s - T_{\gamma}}{1 + z} (1 - e^{-\tau}) \approx \frac{T_s - T_{\gamma}}{1 + z} \tau ,
\]

where \( \tau \) is the optical depth expressed as \( \tau = \frac{3}{32\pi} \frac{T_s}{T_\alpha} n_{\text{HI}} \lambda_{21}^3 \frac{A_{10}}{H(z)} \) [62] (here \( A_{10} \) signifies the Einstein coefficient for spontaneous emission due to the transition from triplet to singlet state [59, 61], wavelength of 21cm line is denoted by \( \lambda_{21} \) while \( n_{\text{HI}} \) is the number density of neutral Hydrogen and \( T_s \) represents the 21cm photon transition temperature).

### V. CALCULATIONS AND RESULTS

In this section, we describe our calculations and results using the formalism described in Sect. II - IV. Evolutions of \( T_s \), \( T_b \) and \( T_{21} \) are calculated for two mass distributions of PBH (lognormal mass distribution and power law mass distribution) and bounds on model parameters are computed in this context with EDGES like observational limit. Constraints on DM mass \( m_\chi \), initial mass fraction of PBH \( \beta_{\text{BH}} \) and parameters of PBH mass distributions are estimated for the above mentioned cases with EDGES’s results. Moreover, a mass distribution function of PBH is derived in such a way that it can predict the EDGES limit \( T_{21} = -500^{+200}_{-500} \) mK at reionization epoch (discussed is Subsect. V C).

#### A. Lognormal Mass Distribution of Primordial Black Hole

Lognormal distribution of PBH masses is considered in this section to study the effects of PBH energy injections and DM - baryon interactions on 21cm brightness temperature. In Fig. 1(a) evolutions of \( T_b \) (solid lines in the plot) and corresponding spin temperature \( T_s \) (dashed lines in the plot) with redshift \( z \) are plotted for different values of DM mass \( (m_\chi = 0.5 \text{ GeV}, 1 \text{ GeV}) \) and different mean values of the distribution \( (\mu = 5 \times 10^{14} \text{ g and } 1.5 \times 10^{14} \text{ g}) \). It can be noted from Fig. 1(a) that a smaller gas temperature \( T_b \) (and \( T_s \)) is obtained at the reionization epoch when a larger value of \( \mu \) \((\mu=5.0 \times 10^{14} \text{ g})\) is considered for a fixed value of \( m_\chi \) \((m_\chi = 0.5 \text{ GeV})\). It indicates the fact that the energy injection rates of PBHs with smaller masses are higher than the same with larger masses. Hence, PBH mass distribution with a lower mean value can inject larger amount
FIG. 1. (a) Evolutions of $T_s$ (dashed lines) and $T_b$ (solid lines) with $z$ for different mean values $\mu$ of lognormal distribution of PBH and different DM mass $m_\chi$. (b) variations of $T_{21}$ with $z$ for different mean $\mu$ and variance $\sigma$ values of the mass distribution of PBH.

of energy in the IGM compared to the distribution with a higher $\mu$. From Fig. 1(a) it can also be observed that IGM temperature decreases with the decrease of $m_\chi$. This is however expected from Eq. 9 since the cooling rate of the baryon is inversely proportional to DM mass [37].

Similar comments can be made from Fig. 1(b) where the variations of 21cm brightness temperature $T_{21}$ with $z$ are shown for different mean $\mu$ and variance $\sigma$ values. Here also it can be observed that more negative values of $T_{21}$ can be obtained for larger values of $\mu$ as energy injection rates are smaller for heavier PBHs. It can also be observed from Fig. 1(b) that larger $T_{21}$ is obtained when $\sigma = 0.3$ is considered (indigo line in the Fig. 1(b)) when compared with the same with $\sigma = 0.6$ (green line in Fig. 1(b)) where the mean value is fixed at $\mu = 2 \times 10^{14}$ g. It can be mentioned here that we have repeated the calculation by fixing $\mu = 10^{15}$ g and have found that smaller $T_{21}$ is obtained for $\sigma = 0.3$ than when $\sigma = 0.6$ is considered. This indicates that PBHs with smaller masses $\lesssim 10^{14}$ g evaporate before $z \sim 17.2$ and thus contributions for lighter PBHs are not significant. But for $\mu = 10^{15}$ g, the distribution with larger variance $\sigma = 0.6$ includes larger range of PBH masses with significant contributions from the smaller masses.

One of our main focuses of the current work is to provide bounds on the parameters of PBH mass distributions and on the initial mass fraction of PBH $\beta_{BH}$. In Fig. 2 and Fig. 3 the allowed regions of $\beta_{BH} - \mu$ plane and $\beta_{BH} - \sigma$ plane are shown for different values of $m_\chi$ ((a) $m_\chi = 0.1$ GeV, (b) $m_\chi = 0.3$ GeV, (c) $m_\chi = 0.5$ GeV and (d) $m_\chi = 1.0$ GeV) by considering EDGES observational results. Upper bounds and lower bounds of $\beta_{BH}$, $\mu$ and $\sigma$ are estimated by using the EDGES limit on brightness temperature of 21cm line of reionization epoch i.e., at $z \simeq 17.2$ value of
is $T_{21} = \frac{-500^{+200}_{-500}}{mK}$. We compare our calculated values of $T_{21}$ at $z \simeq 17.2$ with EDGES limit ($T_{21} = \frac{-500^{+200}_{-500}}{mK}$) to compute the constraints on the parameters and hence show the calculated $T_{21}$ at $z \simeq 17.2$ with a different notation $\Delta T_{21}$ with colour bars in Figs. 2, 3. It is clear from Fig. 2 that larger initial mass fractions of PBH can be probed for mass distributions with higher mean values. It is expected as energy injection rate of PBH is inversely proportional to their mass value and hence larger abundance of heavier PBHs are still compatible with the EDGES results. It can also be noted from the figure that $\beta_{BH}$ decreases with the decrement of $\mu$ up to a certain value $\mu \sim 2 \times 10^{14}$ g and then it starts to slightly increase. This is showing that PBHs with mass less than $\sim 2 \times 10^{14}$ g evaporate before $z \lesssim 20$ and thus their contributions in IGM heating at $z \lesssim 20$ are comparatively lower. From Fig. 2 it is also noted that for lower values of $m_\chi$ the allowed region (the coloured region showing the allowed range between the upper limits and lower limits of the parameters) in $\beta_{BH} - \mu$ is very narrow but the lower limit increases significantly with the increment of DM mass while the upper limit varies very slightly with $m_\chi$. Similar constraints on $\beta_{BH} - M_{PBH}$ plane are observed for monochromatic mass distribution of PBH in Ref. [67]. For smaller $m_\chi$ ($m_\chi = 0.1$ GeV, 0.3 GeV) the effects of DM - baryon interaction on $T_{21}$ are very high and hence $T_{21}$ fall beyond the EDGES's lower limit (less than -1000 mK) and consequently the lower limits of $\beta_{BH} - \mu$ plane become stringent. As larger DM - baryon interaction rate can be acquired for smaller $m_\chi$, larger PBH abundance can be probed in these cases and hence the upper limit slightly decreases when $m_\chi$ increases. Similar conclusion can be drawn from Fig. 3 that allowed regions in $\beta_{BH} - \sigma$ plane increase with mass of DM $m_\chi$. In Fig. 3 it can be noted that larger values of $\beta_{BH}$ are allowed at smaller values of $\sigma$. This is expected because larger variance ($\sigma$) indicates the inclusion of larger mass range of PBHs with lower mass values and higher evaporation rate. It can be mentioned that DM - baryon interaction cross section $\sigma_{41}$ (in the unit of $10^{-41}$ cm$^2$) is kept at $\sigma_{41} = 1$ for these plots (Fig. 2 and Fig. 3), while in Fig. 2 and Fig. 3 we choose $\sigma = 0.5$ and $\mu = 5 \times 10^{14}$ g respectively. The same computations are repeated for other values of $\sigma_{41}$ (say for $\sigma_{41} = 5$) and extended allowed ranges of the parameters are obtained for larger $\sigma_{41}$ values.

B. Power Law Mass Distribution of Primordial Black Hole

In this section, upper and lower limits of initial mass fraction of PBH $\beta_{BH}$, power index $\gamma$ and DM mass $m_\chi$ are obtained from EDGES experimental results by considering the power law mass distribution of PBH.

In Fig. 4, allowed regions (coloured zones) in the $\beta_{BH} - \gamma$ parameter space that satisfy the
FIG. 2. Allowed zones in the $\beta_{\text{BH}} - \mu$ plane for lognormal distribution, where different values of DM masses are considered ((a) $m_\chi = 0.1$ GeV, (b) $m_\chi = 0.3$ GeV, (c) $m_\chi = 0.5$ GeV and (d) $m_\chi = 1.0$ GeV).

EDGES results are shown. The calculated value of $T_{21}$ at redshift $z \simeq 17.2$ is denoted by $\Delta T_{21}$ and is represented by the colour bars in the plots. The allowed zones of the $\beta_{\text{BH}} - \gamma$ parameter space are estimated for different chosen values of DM masses (Fig. 4(a) $m_\chi = 0.1$ GeV, Fig. 4(b) $m_\chi = 0.3$ GeV, Fig. 4(c) $m_\chi = 0.5$ GeV and Fig. 4(d) $m_\chi = 1.0$ GeV). It can be observed that for $m_\chi = 0.1$ GeV, the narrowest allowed region of $\beta_{\text{BH}} - \gamma$ is obtained among the four cases and the lower limits of the allowed region drop significantly with the increment of DM mass values. This can have similar explanation as in Fig. 2 and Fig. 3. Moreover, it can be observed from Fig. 4 that $\beta_{\text{BH}}$ slightly decreases as the power index $\gamma$ increases and hence $\beta_{\text{BH}}$ depends on the formation time of the PBH. The value of $\gamma$ is varied from $-\frac{1}{2}$ to $\frac{1}{2}$ in Fig. 4 which corresponds to the variation of the equation of state $\omega$, or the formation epoch of PBHs, from $\frac{1}{3}$ to $-\frac{1}{2}$. It can be noted from Fig. 4 that maximum value of $\beta_{\text{BH}}$ is obtained for $\gamma = -\frac{1}{2}$ or $\omega = \frac{1}{3}$ which corresponds to the equation of
FIG. 3. Allowed zones in the $\beta_{BH} - \mu$ plane for lognormal distribution, where different values of DM masses are considered ((a) $m_\chi = 0.1$ GeV, (b) $m_\chi = 0.3$ GeV, (c) $m_\chi = 0.5$ GeV and (d) $m_\chi = 1.0$ GeV).

state of the radiation dominated epoch. Therefore, the Fig. 4 estimates that a larger initial mass fraction of PBH (thus larger initial abundance of PBH) is obtained if the formation time of PBH is radiation dominated epoch and the abundance decreases slightly if the PBHs formation take place at later epochs ($\gamma > -\frac{1}{2}$).

C. Mass Distribution of Primordial Black Hole Calculated from Excess 21cm Absorption Results

Two analytical distribution models for PBH mass distribution(e.g. Subsect. V A and V B) have been studied in the previous sections. We proceed by proposing an analytical formula for the variation of $T_{21}$ with PBH masses during the epoch of reionization. This is essentially motivated by the EDGES observational results and its interpolation taking into consideration the PBH evaporation effects along with DM - baryon intercation. As has been discussed earlier, the EDGES observation
FIG. 4. Allowed zones in the $\beta_{BH} - \gamma$ plane for power law mass distribution of PBH, where different values of DM masses are considered ((a) $m_\chi = 0.1$ GeV, (b) $m_\chi = 0.3$ GeV, (c) $m_\chi = 0.5$ GeV and (d) $m_\chi = 1.0$ GeV).

has reported the brightness temperature at the redshift $z \sim 17.2$ is $T_{21} = -500^{+200}_{-500}$ mK with 99% confidence level. Consequently, the probabilities of different brightness temperature within the range $-1000$ mK $\leq T_{21} \leq -300$ mK can be computed at that epoch. In this approach, a basic skew normal distribution for $T_{21}$ (see Eq. 19) is fitted in such a way that, the peak of the distribution lie at $-500$ mK and 99% of this distribution lies within the range $-1000$ mK $\leq T_{21} \leq -300$ mK while the individual probabilities at $T_{21} = -300$ mK and at $-1000$ mK are equal.

The fitted skew normal distribution (SND) function given by,

$$G_{21} = \left(1 + \text{erf} \left( \frac{\alpha_1 (\Delta T_{21} - \mu_1)}{\sqrt{2} \sigma_1} \right) \right) \exp \left( -\frac{(\Delta T_{21} - \mu_1)^2}{2\sigma_1^2} \right), \tag{19}$$

where the fitted parameters are $\mu_1 = -405.708$, $\sigma_1 = 223.35$ and $\alpha_1 = -3.90017$ (in the above equation $\Delta T_{21}$ is $T_{21}$ at redshift $\sim 17.2$). It is to be mentioned that, the above expression (Eq. 19)
FIG. 5. The fitted function (Eq. 19) of $G_{21}$ using EDGES observational result. The green solid line denotes $G_{21}$ as function function of $\Delta T_{21}$. The pink region represents the uncertainty of the EDGES result (i.e. $-1000 \sim -300$ mK), while the the solid green region occupies the 99% confidence level.

describes only the probabilities of brightness temperature $\Delta T_{21}$ (i.e., $T_{21}$ at $z \simeq 17.2$). The relation between PBH mass and $\Delta T_{21}$ is to be calculated numerically for different chosen DM mass and PBH initial mass fractions in order to estimate corresponding population of PBHs. Eq. 19 is graphically described in Fig. 5. In this plot the 99% C.L. region of the probability distribution function $G_{21}$ for $\Delta T_{21}$ in the range $-1000 \, \text{mK} \leq \Delta T_{21} \leq -300 \, \text{mK}$ are shown by the green region. It is to be noted that $G_{21}$ at both the boundaries (i.e. $-300$ mK and $-1000$ mK) are kept equal.

In order to obtain the analytical form of the mass distribution of PBHs, the values of $\Delta T_{21}$ are numerically calculated for different PBH masses ($M_{\text{BH}}$) by solving five mutually coupled equations (Eqs. 2, 11, 12, 14 and 16) for each possible combinations of PBH parameters. In this case we adopt the procedure and corresponding equations as introduced in Ref. [67]. In the entire calculation we choose the DM mass $m_{\chi} = 0.5$ GeV and $\sigma_{41} = 1$ as benchmark values.

In Fig. 6 the variations of $\Delta T_{21}$ and corresponding $G_{21}$ for different masses $M_{\text{BH}}$ of PBHs are graphically described. Fig. 6(a) addresses the variation of $\Delta T_{21}$ vs PBH mass $M_{\text{BH}}$ for different initial mass fractions of primordial black holes $\beta_{\text{BH}}$. Different values of $\beta_{\text{BH}}$ are represented by the lines of different colours, where the corresponding values of $\beta_{\text{BH}}$ are shown by the colour bar, given at the right of top panel of Fig. 6. The corresponding $G_{21}$ vs $M_{\text{BH}}$ plots are furnished in Fig. 6(b). Fig. 6(a) and 6(b) are similar except that in Fig. 6(b) the variations of $G_{21}$ with $M_{\text{BH}}$ are shown. The plots if Fig. 6(b) are generated by identifying $\Delta T_{21}$ value corresponding to $G_{21}$ value (from Fig. 5 or Eq. 19) and then following the same procedure to obtain Fig. 6(a). Similar representations of $\Delta T_{21}$ vs $\beta_{\text{BH}}$ and corresponding $G_{21}$ vs $\beta_{\text{BH}}$ for different chosen values of $M_{\text{BH}}$ are shown in 6(c) and 6(d) respectively. From Fig. 6(a) (and also from Fig. 6(b)), it can be noticed that, for lower
FIG. 6. Variations of $\Delta T_{21}$ (left column) and corresponding $G_{21}$ (right column) with $M_{\text{BH}}$ for different values of $\beta_{\text{BH}}$ are shown in the plots of upper row (Fig. 6(a) and 6(b)), while the similar variations with $\beta_{\text{BH}}$ for different are plotted in the graphs of lower row (Fig. 6(c) and 6(d)). Different chosen values of $M_{\text{BH}}$ ($\beta_{\text{BH}}$) are represented by different colours, as mentioned in the colour bar at the end of upper (lower) row.

values of $\beta_{\text{BH}}$s all the $\Delta T_{21}$ vs $M_{\text{BH}}$ graphs suffer certain discontinuities near $M_{\text{BH}} \approx 1.7 \times 10^{14}$g. Such nature arises as the PBHs of mass $M_{\text{BH}} \approx 1.7 \times 10^{14}$g had completely evaporated at $z \sim 17$. Similar features are also obtained from Fig. 6(c) and 6(d) for lower values of $\beta_{\text{BH}}$.

In the present analysis our attempt is to find an approximately fitted analytical form of $G_{21}$ (for $M_{\text{BH}} \gtrsim 1.7 \times 10^{14}$g) as a function of PBH mass $M_{\text{BH}}$ and initial PBH mass fraction $\beta_{\text{BH}}$. In this regard we first propose a form for $\Delta T_{21}$ as,

$$\Delta T_{21}(M_{\text{BH}}, \beta_{\text{BH}}) = (a_1 + a_2) \times \text{CDF}_{\text{SQN}}(\mu_f, \sigma_f, \alpha_f) - a_2, \quad (20)$$

where $a_1$, $a_2$, $\mu_f$, $\sigma_f$, $\alpha_f$ are parameters obtained by fitting this equation with the results obtained in Fig. 6. The cumulative distribution $\text{CDF}_{\text{SQN}}$ (in Eq. 20) is given by

$$\text{CDF}_{\text{SQN}}(\mu_f, \sigma_f, \alpha_f) = \frac{1}{2} \text{erfc} \left( \frac{\mu_f - \beta_{110}}{\sqrt{2} \alpha_f} \right) - 2 \text{OwenT} \left( \frac{\beta_{110} - \mu_f}{\sigma_f}, \alpha_f \right). \quad (21)$$
FIG. 7. Dotted lines are showing values obtained with Eq. 19 and 20 ans solid lines are representing values obtained by solving suitable equations of sections II - IV.

In the above expression, $\beta_{10} = \log_{10} \beta_{BH}$ and erfc($x$) and OwenT($i,j$) are the complementary error function and the Owen function [68] respectively, given by

$$\text{erfc}(x) = 1 - \text{erf}(x)$$

$$\text{OwenT}(x,a) = \frac{1}{2\pi} \int_{0}^{a} \exp \left(-\frac{x^2(1+t^2)/2}{1+t^2}\right) dt.$$  \hfill (23)

The parameters are obtained as $a_1 = 32.83$, $a_2 = 903.54$, $\sigma_f = 0.8$ and $\alpha_f = -1.7$ We also note that value of $\mu_f$ varies with the PBH mass $M_{BH}$ while other two parameters $\sigma_f$ and $\alpha_f$ remain almost unchanged. The variations of $\mu_f$ with $M_{BH}$ is found to be approximated as,

$$\mu_f(M_{BH}) \approx 13.2206 - 1103.94 (M_{BH})^{-0.1}.$$  \hfill (24)

It should be mentioned here that, the form of the fitted function in Eq. 20 is obtained by trial and $\chi^2$-fitting. It can now be seen that, replacing $\Delta T_{21}$ in Eq. 19 with the expression of $\Delta T_{21}$ obtained in Eq. 20, a form for a PBH mass distribution follows (i.e. $G_{21}(M_{BH}) = f(M_{BH})$). But it is to be noted here that numerical values of the parameters of this distribution function would change for different chosen DM mass $m_\chi$.

Since the evolution of the spin temperature and consequently the brightness temperature of 21 cm line also depend on DM mass $m_\chi$, we repeat the entire calculation with different chosen values of dark matter mass $m_\chi$ (in the previous case we use fixed value of dark matter particle $m_\chi = 0.5$ GeV). However, the values of fitted parameters $a_1$, $a_2$, $\mu_f$, $\sigma_f$ and $\alpha_f$ may be modified with different choices of $m_\chi$ but the functional form of $G_{21}$ remains same. Instead of showing the variation of those parameters with $m_\chi$, in Fig. 8 the allowed region in the $\beta_{BH} - m_\chi$ parameter space is plotted.
FIG. 8. The allowed region in the $\beta_{BH} - m_\chi$ space that satisfies the 21cm brightness temperature limit $(-500^{+200}_{-500})$ (EDGES result). The black dashed lines represent the upper and lower limits for the same, while the different colours of the contour plot denote different values of $\Delta T_{21}$ in mK.

In this particular case, the mass distribution of PBHs is considered as mentioned in Eq. 19-24. It is also clear from Fig. 2 and Fig. 3 that $m_\chi$ influences the upper and lower bounds of $\beta_{BH} - \mu$ and $\beta_{BH} - \sigma$ planes. Therefore, in Fig. 9(a) the allowed region in $\beta_{BH} - m_\chi$ parameter space obtained in Fig. 8 is compared with the lower and upper bounds of the same ($\beta_{BH} - m_\chi$) calculated by considering lognormal distribution of PBHs. The coloured dashed lines of same colour denote the upper and lower bounds of $\beta_{BH}$ and $m_\chi$ for different chosen values of $\mu$. Similarly the evolution of $T_{21}$ and the allowed ranges computed in Fig. 4 depend on the value of $m_\chi$. Hence, in Fig. 9(b) the upper and lower limits on $\beta_{BH} - m_\chi$ from EDGES data are plotted by considering power law mass distribution of PBHs (the coloured dashed lines of same colour denote the upper and lower bounds of $\beta_{BH}, m_\chi$ for different chosen values of $\gamma$). Further such limits from power law mass distribution of PBHs are compared with the allowed region of $\beta_{BH}, m_\chi$ computed in Fig. 8.

VI. SUMMARY AND DISCUSSIONS

In this work, we use the EDGES 21cm results as a representative of any global 21cm excess trough line to constrain different possible mass distributions of PBHs in the Universe. To this end, three types of possible PBH mass distribution (discussed in the literature) are considered and these are monochromatic mass distribution (only unique mass of PBH), lognormal mass distribution and power law mass distribution. Then doing this, relevant coupled differential equations are solved simultaneously for the evolution of spin temperature $T_s$, the evolution of baryon temperature $T_b$, and
DM temperature $T_\chi$ and other quantities required to calculate brightness temperature of 21cm line $T_{21}$. It is to be noted that the contributions of Dark Matter - baryon interaction, PBH evaporation are included for the computation of temperature evolutions of $T_{21}$. From these analyses the PBH mass distribution parameters are constrained using the 21cm results considered in this work. The effects of DM of different masses also play a major role for the constrained parameter space of PBHs. To this end, the allowed regions of the PBH parameter spaces such as $\beta_{BH} - \mu$, $\beta_{BH} - \sigma$ and $\beta_{BH} - \gamma$ are computed for different fixed values of DM mass. Here, $\beta_{BH}$ represents initial mass fraction of PBHs, $\mu$ and $\sigma$ are mean and variance of lognormal mass distribution respectively and $\gamma$ denotes the power law index of power law mass distribution. It is found the initial mass fraction of PBH $\beta_{BH}$ slightly varies with $\gamma$ and hence estimates that $\beta_{BH}$ depends on the formation epoch of PBHs.

From the 21cm results considered here an attempt has been made in this work to formulate an expression for the mass distribution expression for the PBHs. For this purpose, the range of the $T_{21}$ results around its measured central value (at reionization epoch) is first considered and the weights of each $\Delta T_{21}$ value (i.e., value of $T_{21}$ at $z \sim 17.2$) within these experimentally obtained range are estimated and a distribution function to this effect is constructed. It appears that a skew normal distribution ($G_{21}$) is best fitted for these probabilities. Using this $G_{21}$ distribution the variation of $\Delta T_{21}$ with PBH mass $M_{BH}$ for a fixed chosen value of DM mass $m_\chi$ are obtained for different fixed values of $\beta_{BH}$. An analytical form of $\Delta T_{21}$ in the reionization epoch (as a function of $M_{BH}$) is then proposed from this analysis. This form contains certain parameters, the numerical values of which
are found out by suitable $\chi^2$-fitting. It is to be mentioned that in obtaining the analytical form, DM - baryon interaction and PBH evaporations are taken into account. From the expression of $\Delta T_{21}$ and using the expression of $G_{21}$, a functional form of PBH mass distribution at reionization epoch can be obtained. The variation of $\Delta T_{21}$ is found to be a combination of an error function and Owen function (Eq. 23). Using this distribution the allowed region of variation of $\beta_{\text{BH}}$ and DM mass $m_\chi$ are obtained. This $\beta_{\text{BH}} - m_\chi$ allowed region is then compared with the same when lognormal mass distribution and power law mass distribution for PBH are considered.

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