The ground states of double-charm hyperons form a spin 1/2 isospin 1/2 multiplet analogous to that of nucleons. Their main strong interaction may be inferred directly from the corresponding nucleon-nucleon interaction by multiplication of the interaction components by the appropriate fractional difference between interaction strengths for pairs of light flavor quarks and pairs of triplets, e.g., nucleons, of light flavor quarks. By construction of the interaction between the recently discovered double-charm hyperons by this method from several realistic nucleon-nucleon interaction models it is shown that double-charm hyperons are likely to form bound (or possibly meta-stable) states akin to the deuteron in the spin triplet state. Double beauty baryons would form corresponding deeply bound states. Nucleons and double charm (beauty) hyperons will also form bound states. The existence of hypernuclei with double-charm and double-beauty hyperons, which are stable against the strong decay, is very likely.

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The SELEX collaboration has recently discovered several double-charm hyperons, the lowest one of which is the ground state multiplet with the $\Xi_{cc}^+$ at 3.46 GeV/$c^2$ [1, 2]. The ground state multiplet $\Xi_{cc}^+$, $\Xi_{cc}^{++}$ forms a spin 1/2 isospin 1/2 multiplet, with the valence quark configuration $d\bar{c}c$ and $ucc$ [3].

The main color-neutral strong interaction between double-charm hyperons is due to their light flavor quark component. This may be inferred from the nucleon-nucleon interaction, by multiplication of the components of the nucleon-nucleon interaction by appropriate fractional coefficients, which relate the interaction strength between pairs of light flavor quarks to that between such triplets. The interaction between the charm quark pairs in different hadrons is expected to be weaker than that between light flavor quarks in different hadrons, as the latter either arises from the short range interaction that is mediated by the exchange of charmonia or from the color van der Waals interaction. Because of the isospin 1/2, the $\Xi_{cc}$ interactions include a long range pion exchange component, in contrast to the $\Lambda_{cc}^+$, which has isospin 0. A priori hypernuclei with double charm hyperon should therefore be more likely than hypernuclei with single charm hyperons [4].

Although the strong interaction between double-charm hyperons is weaker than that between nucleons, this is partially compensated by their larger mass which weakens the repulsive effect of their kinetic energy. The latter may be schematically illustrated by a calculation of the binding energy of the deuteron-like state, which is obtained by replacement of the nucleon mass by the mass of the $\Xi_{cc}$ (3.46 GeV/$c^2$), with the realistic nucleon-nucleon interaction models in Refs. [3, 4, 7], all of which reproduce the experimental binding energy of the deuteron and provide a quantitative description of nucleon-nucleon scattering data. The corresponding binding energies are $-71$ MeV, $-75$ MeV and $-58$ MeV respectively (the scatter between these values reflects the differences in the detailed behavior of these interaction models at short range, which is not constrained by nucleon-nucleon scattering data).

In operator form the nucleon-nucleon interaction is given in terms of rotational invariants of spin and isospin as well as momenta and angular momenta. The quark model scaling factors between the matrix elements of the spin-isospin invariants for $\Xi_{cc}$ and nucleon states may be derived from the quark model matrix elements of light flavor quark operators [5]:

$$\langle \Xi_{cc}^+ | 1 | \Xi_{cc}^+ \rangle = \frac{1}{3} \langle N | 1 | N \rangle ,$$

$$\langle \Xi_{cc}^+ | \sum_q \sigma^a_q | \Xi_{cc}^+ \rangle = -\frac{1}{3} \langle N | \sum_q \sigma^a_q | N \rangle ,$$

$$\langle \Xi_{cc}^+ | \sum_q \tau^a_q \tau^b_q | \Xi_{cc}^+ \rangle = -\frac{1}{3} \langle N | \sum_q \tau^a_q \tau^b_q | N \rangle .$$

(1)

Here $N$ represents the nucleon.

The interaction between two double-charm hyperons (and that between double-charm hyperons and nucleons) that arises from the interaction between the light flavor quarks may be determined from any realistic nucleon-nucleon interaction model, that is given in operator form. We here consider the models of Refs. [3, 4, 7] mentioned above. From these, after dropping the (small) flavor symmetry breaking terms, which are inapplicable to hyperons (from those which include such), the corresponding interactions between double-charm hyperons may be derived by application of the appropriate downscaling of the strengths of the corresponding interaction components.

In Table I the scaling factors for all the $NN$ operators that appear in the $NN$ potential models considered are listed. Where necessary, the larger scaling factor corresponding the operator has been taken. The main qualitative difference between the nucleon-nucleon interaction and that between double-charm hyperons is the weakening by the factor 9 of the central spin and isospin inde-
be written as, the bound state equation can be stated as, and

\[ \left\langle q l s j | V | q l^\prime s j \right\rangle. \]

For the purpose of the present investigation a mesh of 80 Gauss-Legendre points was sufficient for stable results. As a test of the numerical method the deuteron binding, \(-2.2\) MeV, was calculated with all the models, AV18, Nijmegen II and Paris potential.

In Table \(\text{II}\) the calculated binding energies obtained for the deuteron-like states of double-charm hyperons are given. The scatter between the calculated values provides an estimate of the theoretical uncertainty that derives from the different short range behavior of the nucleon-nucleon interaction models.

It is natural to assume that the experimentally discovered double-charm hyperon state \(\Xi^+_{cc}(3520)\) represents the double-charm spin \(3/2\) state, which is analogous to the \(\Delta(1232)\). The 60 MeV splitting between this state and the \(\Xi^+_{cc}(3460)\) is in line with quark model estimates, although slightly smaller than numerical lattice method based estimates. As the coupling to this state can only increase the calculated binding energy, the binding energies in Table \(\text{III}\) may be viewed as lower bounds.

Two-baryon states formed of double-charm hyperons can in principle couple to states with a single charm and a triple-charm \(\Omega_{ccc}\) by the quark rearrangement interaction. If the latter states have lower energy the former are metastable rather than bound. This depends on the size of the binding energy as compared to the mass difference:

\[ \Delta^c \equiv M_{ccc} + M_{cll} - 2M_{cel}. \]  

Here \(l\) represents a light quark. For many non-relativistic quark models the inequality \(\Delta^c < 0\) holds. Corresponding numerical estimates suggest that \(\Delta \approx [130 - 158]\) MeV. Adoption of those values imply that the double charm hyperons form bound states with the AV18 potential, but only metastable states with the Nijm II potential.

To further explore the theoretical uncertainty of the calculated binding energies these have also been calculated using the class “AVn” of systematically simplified versions of the V18 \(\text{F}\) interaction models. These results are shown in Table \(\text{III}\). The number of each potential designates the number of operator structures present, ordered as in the first column of Table \(\text{II}\). These potential models are reprojections of the full AV18 potential, of which all but AV1’ and AVX’ reproduce the deuteron binding energy.

The results show a considerable spread in the calculated binding energies. No bound state is found with the AVX’ potential, which already includes tensor and spin orbit forces. With the simpler AV2’, AVX’, AV4’ and AV6’ models bound states appear. Consequently nucleon-nucleon interaction models, which give the correct deuteron binding energy, with (AV6’) and without tensor or spin-orbit forces, predict a bound state in the

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Operator & Scaling factor \\
\hline
1 & 1/9 \\
\tau\cdot\tau_j & 1 \\
\sigma\cdot\sigma_j & 1/9 \\
\langle\sigma\cdot\sigma_j\rangle(\tau\cdot\tau_j) & 1/25 \\
S_{ij} & 1/9 \\
S_{ij}(\tau\cdot\tau_j) & 1/25 \\
L\cdot S & -1/9 \\
L\cdot S(\tau\cdot\tau_j) & -1/5 \\
L^2(\tau\cdot\tau_j) & 1/9 \\
L^2(\sigma\cdot\sigma_j) & 1/9 \\
L^2(\sigma\cdot\sigma_j)(\tau\cdot\tau_j) & 1/25 \\
(L\cdot S)^2 & 1/9 \\
(L\cdot S)^2(\tau\cdot\tau_j) & 1 \\
\hline
\end{tabular}
\caption{Quark scaling factors for the components of the interaction models of the form \(\bar{c}c\).}
\end{table}
components differ notably at ranges less than 0.6 fm. These differences are of little significance for nucleon-nucleon scattering observables at low energy, because of the very strong short range repulsion. The reduction of the strength of the repulsive central interaction by a factor 9 enhances the role of the short range behavior of the state dependent components of the interaction in the case of double-charm hyperons.

Double beauty-hyperons analogous to, but heavier than, the double-charm hyperons are expected to exist in the mass range above twice the B-meson mass. The interaction between the ground state multiplets of double-beauty hyperons with the quark configurations \((bbu)\) and \((bbd)\) is expected to be very similar to that of the double charm hyperons, as their main interaction is that between their light flavor quarks. Their binding energy will however be much larger than that of double-charm hyperons in view of their larger mass. This is illustrated in Table III where the binding energy of two double-beauty hyperons is estimated by the same method as used above for double charm hyperons with the assumption that their mass is: \(M_{bbu} = M_{bdl} \approx 9000\) MeV were used. The value in brackets corresponds to a second bound state.

Finally we have also explored the possibility of deuteron-like bound states of nucleons and double-heavy hyperons: \(N - \Xi_{cc}\) and \(N - \Xi_{bb}\). Such bound states were found with the AV18 and the Nijm II potential models. The AV18 potential gives bound states at \(-388\) MeV and \(-494\) MeV for the \(N - \Xi_{cc}\) and \(N - \Xi_{bb}\) systems respectively. The Nijm II potential gives bound states at \(-35\) MeV and \(-76\) MeV for the \(N - \Xi_{cc}\) and \(N - \Xi_{bb}\) systems respectively.

In summary it has been shown that bound states of the recently discovered double-charmed hyperons are likely. Their binding energies were estimated by construction of their main strong interaction by the quark-model scaling relations from several realistic phase-equivalent nucleon-nucleon interaction models. The \(\Xi_{cc}\) hyperons are likely to be bound in the spin triplet state by more than 30 MeV. The \(\Xi_{bb}\) hyperons are expected to be bound by more than 50 MeV in the triplet state, while pairs of \(\Xi_{cc}\) and \(\Xi_{bb}\) baryons are expected to be bound with binding energies of that order. The qualitative nature of these estimates reflects the uncertain short range behavior of the extant realistic nucleon-nucleon interaction models.

### TABLE II: Binding energies for the \(\Xi^{+} - \Xi^{-}\) and \(\Xi_{c}^{0} - \Xi_{b}^{0}\) systems obtained with Argonne V18, AV18, Nijmegen II and Paris potentials. For the double-beauty baryons the mass values: \(M_{bbu} = M_{bdl} \approx 9000\) MeV were used. The value in brackets corresponds to a second bound state.

| Potential | \(\Xi_{c}\) - \(\Xi_{c}\) | \(\Xi_{c}\) - \(\Xi_{b}\) | \(\Xi_{b}\) - \(\Xi_{b}\) | deuteron |
|-----------|------------------|------------------|------------------|-----------|
| AV8’      | -                | -                | -                | -2.2      |
| AV6’      | -1               | -5               | -15              | -2.2      |
| AV4’      | -30              | -41              | -58 (-4)         | -2.2      |
| AVX’      | -15              | -22              | -33              | -0.4      |
| AV2’      | -26              | -48              | -88 (-4)         | -2.2      |
| AV1’      | -                | -                | -                | -0.4      |

### TABLE III: Estimated binding energies for the pairs \(\Xi_{cc} - \Xi_{cc}\), \(\Xi_{cc} - \Xi_{bb}\) and \(\Xi_{bb} - \Xi_{bb}\) of double-heavy together with the deuteron as obtained with the class of Argonne potentials in Ref. 13.

\(\Xi^{+} - \Xi^{-}\) system in the spin triplet state. The case of the AV2’ potential, for which the quark model scaling is simply a factor 1/9 of the 1 part, and which predicts a bound state of 26 MeV, is particularly telling. The binding energies obtained with the AVn’ class of potentials are considerably smaller than the one obtained with the full AV18 potential. In particular, with the numerical estimates in Ref. 13 based on the mass inequalities, see Eq. 8, these bound states would be metastable against decay to the single-charm – triple-charm hyperon state.

The origin of the large binding energy given by the AV18 interaction model may be traced to its large isospin dependent squared spin-orbit interaction, which acts in the \(D\)–state. The relative significance of this interaction component derives from the large quark model scaling factor (Table I). The general reason for the large spread in the calculated binding energies obtained with the different interaction models is the fact that their
The authors want to thank Frank Frömel for pointing out an algebraic error in the original version of this manuscript. B.J.-D thanks J. M. Richard for stimulating discussions and the European Euridice network for support (HPRN-CT-2002-00311). Research supported in part by the Academy of Finland through grant 54038.

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