MEASUREMENT AS ABSORPTION OF FEYNMAN TRAJECTORIES: COLLAPSE OF THE WAVE FUNCTION CAN BE AVOIDED

by

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ABSTRACT

We define a measuring device ( detector) of the coordinate of quantum particle as an absorbing wall that cuts off the particle’s wave function. The wave function in the presence of such detector vanishes on the detector. The trace the absorbed particles leave on the detector is identified as the absorption current density on the detector. This density is calculated from the solution of Schrödinger’s equation with a reflecting boundary at the detector. This current density is not the usual Schrödinger current density. We define the probability distribution of the time of arrival to a detector in terms of the absorption current density. We define coordinate measurement by an absorbing wall in terms of 4 postulates. We postulate, among others, that a quantum particle has a trajectory. In the resulting theory the quantum mechanical collapse of the wave function is replaced with the usual collapse of the probability distribution after observation. Two examples are presented, that of the slit experiment and the slit experiment with absorbing boundaries to measure time of arrival. A calculation is given of the two dimensional probability density function of a free particle from the measurement of the absorption current on two planes.
1. Introduction

The concept of measurement is essential for the consistent mathematical formulation of quantum mechanics. Several different theoretical approaches to measurement exist in the quantum mechanics literature [1]-[5]. The postulates of measurement theory in QM [6] describe a coordinate measuring device as a machine that “measures” the coordinate by collapsing instantaneously the wave function to a point at a given moment of time. Repeated measurements of identical particles result in a histogram of the coordinate at the given time. According to QM this histogram is $|\Psi(x,t)|^2$, where $\Psi(x,t)$ is the solution of Schrödinger’s equation. It is, however, universally agreed that there is no known Hamiltonian that brings about such a sharp and instantaneous transformation so that the description of this experiment may not be contained in quantum mechanics. This quantum mechanical collapse of the wave function is not the classical collapse of the probability distribution of a random variable after it is observed. The difference lies in the assumption that the coordinate of the particle was undefined before the collapse of the wave function occurred while the random variable had a definite value regardless of whether it was observed or not.

This is the result of confusing the concept of the coordinate of a particle at a given time with that of the probability amplitude to find it there and then. Actually, there is no agreed mathematical definition of what the instantaneous spatial coordinate of a quantum particle is, although the expectation value is well defined. Obviously, it is impossible to assign probability to an undefined event.

This confusion is illustrated by the following one-dimensional example. Consider a particle on a linear segment bounded by two detectors. At the initial time the wave function of the particle is prepared to be symmetrically distributed about the center of the interval in the shape of two packets traveling in opposite directions with equal momenta. According to standard quantum mechanics (QM), at all times prior to detection the wave function evolves according to Schrödinger’s equation, ignoring the detectors, and assigns equal probabilities to detection at either end. The two nearly separate packets are often interpreted, in the absence of a better interpretation, as the particle being spread in various parts of the interval up to the instance of detection. This interpretation is forced by the QM view that a quantum particle does not have a trajectory [2]. However, it is detected at only one end of the interval, resulting in the collapse of its wave function from a double packet to a delta function at one point. Could it be said that the other half of the packet reached that end in zero time? This is one of the paradoxes that ensue from the concept of collapse: the description of a particle as a wave packet implies that parts of the packet may move to the detector in zero time.

The interpretation of the square of the modulus of the wave function, $|\Psi(x,t)|^2$, as the histogram obtained from repeatedly detecting the coordinate of identical particles, emitted under identical conditions, at a given moment of time, $t$, ignores the detector and its dynamics as part of the evolution of the particle. Ignoring the detector makes the collapse of the wave function at the moment of detection incompatible with the quantum descrip-
tion of the particle. Thus, the concept of making a measurement without a measuring device is an oxymoron. Actually, detecting particles by an impermeable screen, such as a photographic plate or a fluorescent screen, cannot produce $|\Psi(x,t)|^2$ for a particle in the entire space, because $|\Psi(x,t)|^2$ represents, according to quantum theory, the probability density of all particles at $(x,t)$, including those that went across the screen and returned to it at time $t$. Thus, in QM there is no probability density of the points where particles appear on the detector, e.g., on a photographic plate or a fluorescent screen. Thus, in one dimension, the collapse machine is “transparent to particles” in the sense that even if the source of particles is on one side of the machine, it detects particles whose wave function does not vanish on the other side of the machine. In terms of Feynman trajectories, this means that Feynman trajectories can traverse the collapse machine in either direction.

The concept of a quantum measuring device, as proposed by von Neumann [3], for example, does not resolve the above mentioned problem. It is necessary to use a measuring device to observe the measuring device thus putting the first quantum measuring device in contact with a classical system that observes it. Therefore, observing one quantum system with another does not resolve the problem of measurement. The quantum measuring device, when observed, is in one and only one state while its wave function spreads it among different states, as is the case with detecting a particle [4, Ch.1].

Furthermore, there is no definition of the notion of the time of arrival of a particle at a detector (or any other point) [3], [7]-[10]. These are all manifestations of the collapse problem.

The same ambiguity arises with the concept of a probability current density

$$J(x,t) = 2\Im m \bar{\Psi}(x,t) \nabla \Psi(x,t). \quad (1.1)$$

It represents the net flow of particles at the point $x$ at time $t$ from all directions. This is inconsistent with the concept of an impermeable or absorbing screen (or detector) such as a photographic plate. The equation of continuity implies, as in the case of diffusion [1, Sect.5.4.2], [2], that the current on an absorbing screen must be unidirectional and therefore cannot be given by eq.(1.1). Thus a unidirectional current has to be defined for the purpose of describing such a measurement. Furthermore, on the one hand the observed pattern on an absorbing screen is expected to show the unidirectional current density on the screen, but on the other hand the postulates of measurement [4, 5, 6] assert that what is detected is $|\Psi(x,t)|^2$. These two views seem to be incompatible with each other.

We consider now an ideal photographic plate or a fluorescent screen as a measuring device (detector) of the coordinate of a quantum particle. The ideal plate is not transparent to particles as can be seen from the experiment of putting two parallel plates one behind the other. The plate on the side opposite to the source will detect a negligible fraction of the emitted particles. This is in contrast to the QM collapse machine that is transparent in the sense that two collapse machines in a row, when moved along the $x$-axis will produce the same histogram of a packet of free particles, that is, their histograms are independent of one another. A photographic plate differs from the QM collapse machine.
in other aspects as well. First, it cuts the wave function by making it vanish on the
opposite side to that of the source, that is, it does not permit particles through. Second,
within the resolution of the observer, the plate records continuously the time of arrival of
particles. Although, upon the arrival of a particle to the plate it instantaneously collapses
the wave function to a point mass, it is not a collapse machine in the sense of QM. The
collapse brought about by the detection of a particle in a plate is the same as that in
probability theory. That is, the probability distribution, that is spread in space prior to
detection, collapses into a point mass upon measurement.

It can be assumed that an ideal photographic plate (or a grounded screen) instantane-
ously absorbs the detected particles. Thus the measurement process in such a device
consists in irreversibly absorbing particles at a surface. In addition, the $(t, x)$ histogram
of arrivals produced by moving the plate along the $x$-axis does not represent $|\Psi(x, t)|^2$ of
the freely propagating packet, but rather the unidirectional absorption flux density on
the plate.

It is the purpose of this paper to replace the postulates of QM by a set of postulates
that contain an absorbing wall (detector). To this end, we adopt Feynman’s formulation
of QM in terms of trajectories and the Feynman integral. The postulates assert that

1. A quantum particle has a trajectory

2. A detector of spatial coordinate is a surface that absorbs Feynman trajectories when
they reach it for the first time.

3. At any time the wave function of a quantum particle in the presence of an absorbing
wall is the Feynman integral over all trajectories that have not reached the absorbing wall
by that time.

4. At the time of arrival the wave function of the particle is described by the absorbing
surface (not by Feynman’s integral).

Postulate 1 implies that the trajectories of a quantum particle are Feynman trajec-
tories. The view expressed in [2], that an electron cannot have a trajectory, is based on the
argument that tighter measurements of the coordinate of an electron at fixed time inter-
vals lead to wilder vacillations in the position of the electron. This view requires revision.
According to [4], the measurement process interacts with the electron, thus modifying its
trajectory. The tighter is the measurement the stronger is the interaction. Thus tighter
measurements measure different trajectories than do looser measurements. Consider for
example a measurement by a circular hole in a wall followed by a photographic plate.
The smaller is the hole the more spread is the distribution of the spots that electrons
filtered through the hole leave on the plate. This, contrary to the view expressed in [4,
Ch.1,§7], does not imply that an electron has no trajectory, because the measurements
by various size holes measure different trajectories, due to the different interactions of the
different holes with the electron. This is an expression of the uncertainty principle. It
cannot be argued that the same trajectory passes, unmodified, through the various size
holes, because, as stated in [2, Ch.1, §1], the influence of the smaller hole on the electron
(and therefore on its trajectory, if it has one) is stronger than that of the bigger hole.

In [2, Ch.1, §1] it is stated that trajectory (coordinates) measurements at shorter time
intervals apart gives closer results for the coordinate measurement. This is compatible
with the notion of continuous trajectories of the electron.

Postulate 2 does not imply that the absorbed particle disappears. It should be under-
stood in the following sense. Once a particle has reached the absorbing wall its (Feynman)
trajectory is terminated, but a new trajectory begins, possibly at the same place and the
future evolution of the wave function is subject to another Hamiltonian. The initial value
of the wave function after absorption may be concentrated at the terminal position of
the particle on the absorbing wall. As described in [13]-[15], Feynman trajectories that
have been absorbed in the absorbing boundary not longer interfere with those that have
not been absorbed so far. This means that the Feynman integrals over the surviving
trajectories and over the absorbed trajectories are orthogonal. That is, setting

$$
\psi_{\text{surviving}}(x,t) = \int_{\text{surviving trajectories}} \exp \left\{ \frac{i}{\hbar} S[x(\cdot), t] \right\} \mathcal{D}x(\cdot)
$$

$$
\psi_{\text{absorbed}}(x,t) = \int_{\text{absorbed trajectories}} \exp \left\{ \frac{i}{\hbar} S[x(\cdot), t] \right\} \mathcal{D}x(\cdot),
$$

we assume that for all \( t \)

$$
\int \mathbb{R}e \{ \psi_{\text{surviving}}(x,t) \bar{\psi}_{\text{absorbed}}(x,t) \} \, dx = 0. \tag{1.2}
$$

We call eq. (1.2) a separation principle.

According to Postulate 3, \( |\psi(x,t)|^2 \) represents the conditional probability density that
the particle is at position \( x \) at time \( t \), given that it has not reached the absorbing wall
by time \( t \). It was shown in [13] and [14] that postulates 2 and 3 imply that \( |\psi(x,t)|^2 \)
is the solution of Schrödinger’s equation with zero boundary conditions on the absorbing
wall. The joint probability density that the particle is at position \( x \) at time \( t \) and the
particle has not reached the absorbing boundary by time \( t \) is \( |\Psi(x,t)|^2 = S(t)|\psi(x,t)|^2 \),
where \( S(t) \) is the survival probability of the particle. That is, \( S(t) \) is the probability that
the particle has not reached the absorbing boundary by time \( t \). The survival probability
\( S(t) \) has been calculated in [13] and [14] and is discussed below in the context of the
concept of time of arrival. The histogram constructed on the detector (the absorbing
wall) is neither \( |\psi(x,t)|^2 \) nor \( |\Psi(x,t)|^2 \), but rather the unidirectional current density of
the absorbed Feynman trajectories (particles), that is, the absorption current density.

While \( \psi(x,t) \) is obtained from a Hamiltonian theory with infinite potential behind the
wall, the discounted wave function \( |\Psi(x,t)|^2 = S(t)|\psi(x,t)|^2 \) is not. It does not preserve
probability in the domain bounded by the absorbing boundary because

$$
\int |\Psi(x,t)|^2 \, dx = S(t) \int |\psi(x,t)|^2 \, dx = S(t),
$$
which decays in time. Obviously, if the absorbed Feynman trajectories are tracked after absorption (i.e., are included in the Feynman integral, but without interference with the trajectories that have not been absorbed so far), total probability is preserved. This is in contrast to complex valued Hamiltonians that lead to loss of total probability.

According to postulate 4, the joint statistics of the location of the point where the particle arrives at the detector and the time of arrival are determined, as shown below, by the unidirectional probability current density on the wall, not by $|\psi(x, t)|^2$ at the wall. Thus, in one dimension, the probability density that the particle arrives at the detector placed at $x = 0$ at time $t$ is given by eqs.(1.1) and (1.3), whereas QM predicts that the density is $|\Psi(0, t)|^2$. In higher dimensions the statistics of the point and time of arrival of the particle on the detector (wall) are given by eqs.(5.2) and (1.3), whereas QM predicts $|\Psi(x, t)|^2$ on the detector. If the surface of a two dimensional detector is the plane $x = 0$, the quotient of the two statistics is only a function of $t$. That is, both theories predict the same pattern on the detector screen at any time $t$. According to QM, there is no definition of the pattern obtained by repeatedly firing electrons at the screen because there is no definition of the time of arrival. Instead, this pattern is approximated by $|\Psi(x, t)|^2$ on the detector at some mean time obtained from semi-classical considerations [16]. According to the theory obtained from the above postulates, the pattern is obtained by integrating the flux density on the screen over all times.

We refer to the quantum mechanics defined by the above postulates measurable quantum mechanics (MQM). It is obvious, that QM is recovered from MQM by moving the absorbing wall to infinity. This renders MQM and extension of QM. In MQM the Feynman trajectories are interpreted as the possible trajectories of a particle. The description of a particle in MQM adopts the language of stochastic processes: a particle is the set of all continuous functions (trajectories), a $\sigma$-algebra of measurable sets of trajectories, and a Feynman integral defined on measurable sets of trajectories. This is analogous to the theory of diffusion processes and their sample paths [17] which defines a diffusing particle (in the sense of a stochastic process) as the set of all continuous functions (trajectories), a $\sigma$-algebra of measurable sets of trajectories, and a Wiener integral. It so happens that the Wiener integral defines a measure in function space and can be applied in particular to diffusion theory with absorbing boundaries [11, 12, 18]. The Feynman integral is countably additive set function and is therefore a vector valued measure. The probability defined by the Feynman integral, however, is not a countably additive set function. This is the feature of the Feynman integral that gives rise to interference. Thus, interference pattern is observed in the double slit experiment when the particles are sent one at a time.

The problem of absorption in quantum systems was considered by us in [13] and [14]. Absorbing boundaries in a finite interval $[a, b]$ are described by the following assumptions:

1. A trajectory that reaches $x = a$ or $x = b$ is instantaneously absorbed.
2. The population inside the interval $(a, b)$ is reduced by the probability absorbed at the boundary.
3. The absorption process at time $t$ is the limiting process as $\Delta t \rightarrow 0$ of absorbing
trajectories that survived in the interval \([a, b]\) till time \(t\), and propagated into the boundary in the time interval \([t, t + \Delta t]\).

4. The probability of the absorbed trajectories in the time interval \([t, t + \Delta t]\) is proportional to the p.d.f. to propagate into the boundary in this time interval. The proportionality constant is a characteristic length.

It was shown in [13] that for an absorbing wall at the origin, the survival probability of a quantum particle with an absorbing wall (detector) at the origin is given by

\[
S(t) = \exp \left\{ -\frac{\lambda h}{m \pi} \int_0^t \left| \frac{\partial \psi_B(0, t')}{\partial x} \right|^2 dt' \right\},
\]

(1.3)

where \(\psi_B(x, t)\) is the wave function obtained from Schrödinger’s equation with a reflecting wall at the origin (zero boundary conditions at \(x = 0\) or an infinite potential behind the wall). Actually, \(\psi_B(x, t) = 0\) for \(x \geq 0\) and \(\psi_B(x, t) \neq 0\) for \(x < 0\).

2. A mathematical model of quantum mechanics with measurements

In Bayesian probability theory [19] it is impossible to assign probability to events that cannot be observed. The assignment of probabilities to events is based on prior observations. Otherwise, all events are equally likely. Thus measurements (observations) are the basis for constructing a probability theory that describes any experiment. These facts apply to quantum mechanics (QM) as well. The most basic concepts of QM are probability amplitude and probability density. These have never been measured under the condition of absence of a measuring device (e.g., photographic plate, fluorescent screen, etc.) whereas the measuring device always influences the electron in a drastic way. Furthermore, according to standard quantum mechanics (QM), the measurement process influences the wave function in a drastic way as well, namely, the wave function “collapses”.

Thus, to be mathematically consistent, the accumulated data from measuring the location of a single electron as a point in time and space cannot be used for the construction of QM in which a measuring device is completely neglected, as is common practise in QM. Therefore, the introduction of a mathematical model of a measuring device in the mathematical formulation of QM is inescapable.

The fundamental requirement of a mathematical model of a quantum measuring device (QMD) is that it reflects what is actually measured. Thus, we define a QMD (for the purpose of this discussion) as a device that measures the electron as a point in time and space. Our postulate is that prior to measurement the electron is described by the wave function and at the moment of measurement it is described by the QMD. This reflects our uncertainty about the location of the electron as long as it has not been observed, however, when it is observed, our uncertainty is instantaneously replaced with certainty. Note that this statement implies that the electron actually has a location at all times prior to measurement, which in turn implies that it actually has a trajectory, in contravention to
the commonly held view in QM [2]. This difference, between the description of a random variable or random process prior to observation and at observation time, is fundamental in probability theory and therefore should be no surprise in QM. Obviously, this formulation of the mathematical model of quantum mechanics eliminates the notion of collapse of the wave function.

In view of the above discussion, we adopt Feynman’s formulation of QM, which is based on the concept of trajectories, although the Feynman trajectories have not been interpreted so far as actual trajectories of electrons. Feynman’s formulation of QM is equivalent to Schrödinger’s formulation and therefore suffers from the same difficulties concerning the concept of measurement. Our interpretation of Feynman’s QM is as follows. The mathematical model of an electron is a stochastic (random) process with continuous trajectories with a vector valued density, called the wave function. According to the standard methodology of probability theory [20], such a random process consists of the space of trajectories (all continuous functions of time with values in $\mathbb{R}^3$).

### 3. Trajectories of a diffusion process

We begin with a brief review of relevant notions from diffusion theory. In diffusion theory [17] the motion of a Brownian particle in a domain $\mathcal{D}$ is modeled as a stochastic process. That is, by definition, a Brownian particle is, mathematically, the set of all trajectories, $\Omega$, (all continuous paths) and a Wiener integral defined on them (or a Wiener measure defined on the sample space of continuous functions). It can be shown that the Wiener integral assigns probability zero to all trajectories that have a finite velocity at any time. The Wiener integral over all trajectories, $\omega = x(t)$, that start at time zero at the point $x_0$ and reach the point $x$ at time $t$, denoted $p(x,t|x_0)$, satisfies the Fokker-Planck equation

$$\frac{\partial}{\partial t} p(x,t|x_0) = -\nabla \cdot J(x,t|x_0),$$

where $J$ is the probability current density of trajectories at the point $x$ at time $t$, that started out at $x_0$. At an absorbing boundary, $\Gamma$, say, $p(x,t|x_0)|_\Gamma = 0$, and $J(x,t|x_0)|_\Gamma$ is the uni-directional current of trajectories into the absorbing boundary $\Gamma$ [12]. The function

$$p(x|x_0) = \int_0^\infty p(x,t|x_0) \, dt$$

is the mean time trajectories that start out at $x_0$ spend at $x$ prior to absorption. It satisfies the stationary Fokker-Planck equation with a source at $x_0$ and vanishes on $\Gamma$.

Denoting by $\tau(\omega)$ the first passage time of a trajectory $\omega \in \Omega$ to $\Gamma$ (thus $\tau(\omega)$ is a random variable defined on trajectories), we have

$$\text{Pr}\{\tau(\omega) > t | x_0\} = \int_{\mathcal{D}} p(x,t|x_0) \, dx.$$  \hfill (3.2)

It follows from the Fokker-Planck equation that

$$\text{Pr}\{\tau(\omega) = t | x_0\} = \oint_{\Gamma} J(x,t|x_0) \cdot n(x) \, dS_x,$$  \hfill (3.3)
where $\mathbf{n}(\mathbf{x})$ is the unit outer normal to $\Gamma$ at the point $\mathbf{x}$ and $dS_x$ is a surface area element on $\Gamma$. Thus,

$$\oint_{\Gamma} \mathbf{J}(\mathbf{x}, t | \mathbf{x}_0) \cdot \mathbf{n}(\mathbf{x}) \, dS_x$$

is the probability density function of the first passage time of a trajectory to $\Gamma$. That is, the normal component of $\mathbf{J}(\mathbf{x}, t | \mathbf{x}_0)|_\Gamma$ is the probability density of points where Brownian trajectories hit the boundary at time $t$ (for the first and last time!).

If a Brownian particle is released at the point $\mathbf{x}_0$ at time $t = 0$ and is detected (observed) for the first time when it reaches that absorbing boundary $\Gamma$, it appears there as a single point. The probability density function of the points on $\Gamma$ where the particle is observed (measured, absorbed) at time $t$ (per unit area and unit time) is

$$\Pr \{ \mathbf{x}(t) = \mathbf{x}, \tau(\omega) = t | \mathbf{x}_0 \} = \mathbf{J}(\mathbf{x}, t | \mathbf{x}_0) \cdot \mathbf{n}(\mathbf{x}).$$

Thus the normal component of the probability flux density represents the joint probability density of two events: the time of arrival of the measured particle is $t$ and the point where it appears on the screen is $\mathbf{x}$. The probability density function of the points where it leaves a trace on the detector (a photographic plate, say) is

$$\Pr \{ \mathbf{x}(\tau(\omega)) = \mathbf{x} | \mathbf{x}_0 \} = \mathbf{J}(\mathbf{x} | \mathbf{x}_0) \cdot \mathbf{n}(\mathbf{x}) = \int_0^\infty \mathbf{J}(\mathbf{x}, t | \mathbf{x}_0) \cdot \mathbf{n}(\mathbf{x}) \, dt,$$

where $\mathbf{J}(\mathbf{x} | \mathbf{x}_0)$ is the current density calculated from the function $p(\mathbf{x} | \mathbf{x}_0)$ (see eq. (3.1)). All these densities can be constructed from histograms of the outcomes of repeated identical experiments.

We adopt the same approach for a quantum particle, with the only difference that the Wiener integral is replaced with the Feynman integral. All these notions are generalized to Feynman integrals (see [13], [14], [21]), though their calculation is different from that of their Wiener integral counterparts.

4. Uni-directional current and time of arrival

In order to complete the construction of the description of measurement, we introduce the concept of a unidirectional probability current density. Since the wave function vanishes at and beyond the absorbing wall [13] the Schrödinger current vanishes there. We define, therefore, the instantaneous unidirectional current into an absorbing wall as the square of the modulus of the Feynman integral over trajectories that propagate into the wall per unit time. This notion can be generalized to the situation where the wave function or its gradient suffer a discontinuity across a wall [15]. It was shown in [21] that the uni-directional current at an absorbing wall at the origin (in one dimension) is given by

$$\mathcal{J}(0, t) = \frac{\hbar \lambda}{m \pi} \left| \frac{\partial \psi(0, t)}{\partial x} \right|^2 = \frac{\hbar \lambda}{m \pi} \left| \frac{\partial \psi_B(0, t)}{\partial x} \right|^2 \exp \left\{ - \frac{\hbar \lambda}{m \pi} \int_0^t \left| \frac{\partial \psi_B(0, t')}{\partial x} \right|^2 \, dt' \right\}$$

(4.1)
(see eq. (1.3)). It represents the probability per unit time \( \Delta t \) of Feynman trajectories that propagate in the time interval \([t, t + \Delta t]\) into a point \((x = 0)\) in the wall. In diffusion theory this definition gives the usual expression for the probability current at an absorbing boundary \([11, 12]\).

It was shown in \([13, 14]\) that the probability distribution of the time of arrival of a particle at a detector is determined by the relation

\[
Pr\{\tau > t\} = \int_0^{\infty} |\psi(x, t)|^2 \, dx = \exp\left\{-\frac{\lambda \hbar}{m \pi} \int_0^t \left| \frac{\partial \psi_B(0, t')}{\partial x} \right|^2 \, dt' \right\}.
\]

In the experiment of measuring the time of arrival of a particle at a detector, \(\tau\), the information of non-arrival of the particle at the detector is available continuously all the time. Thus, the discounting in the time interval \([t, t + \Delta t]\) has to be conditioned on the information that the particle has not arrived at the detector prior to time \(t\), that is, on the event \(\{\tau > t\}\). The conditional probability is given by

\[
Pr\{t < \tau < t + \Delta t \mid \tau > t\} = \frac{Pr\{t < \tau < t + \Delta t\}}{Pr\{\tau \geq t\}} = \frac{\lambda \hbar}{m \pi} \Delta t \left| \frac{\partial \psi_B(0, t)}{\partial x} \right|^2 + o(\Delta t).
\]

At each time \(s\) prior to the arrival of the particle at the detector the probability distribution of the arrival time, \(\tau\), is conditioned on \(\tau \geq s\). That is, for \(t \geq s\),

\[
Pr\{\tau > t \mid \tau > s\} = \frac{Pr\{\tau \geq t \cap \tau \geq s\}}{Pr\{\tau \geq s\}} = \frac{Pr\{\tau \geq t\}}{Pr\{\tau \geq s\}} = \exp\left\{-\frac{\lambda \hbar}{m \pi} \int_s^t \left| \frac{\partial \psi_B(0, t')}{\partial x} \right|^2 \, dt' \right\}.
\]

It follows that the conditional probability density of the time of arrival of the watched detector for any time \(t \geq s\) is

\[
Pr\{\tau = t \mid \tau \geq s\} = \frac{\lambda \hbar}{m \pi} \left| \frac{\partial \psi_B(0, t)}{\partial x} \right|^2 \exp\left\{-\frac{\lambda \hbar}{m \pi} \int_s^t \left| \frac{\partial \psi_B(0, t')}{\partial x} \right|^2 \, dt' \right\}.
\]

Hence, the rate of arrival of the particle at the detector at time \(s\) is

\[
f_\tau(s) = \lim_{t \to s} Pr\{\tau = t \mid \tau \geq s\} = \frac{\lambda \hbar}{m \pi} \left| \frac{\partial \psi_B(0, s)}{\partial x} \right|^2.
\]

As an application of this theory, we consider the following experiment. A particle is started with initial wave function \(\psi(x, 0)\) and a detector placed at the origin. The detector records the time between the release of the particle and its arrival at the detector. Repeated recordings of identical experiments form a histogram of the times of arrival. Adopting the approach that the recordings and the histogram are independent of whether the detector is watched or not, we calculate below the probability distribution obtained from the histogram in the limit of large number of experiments. According to the adopted
approach, we assume that the detector is watched continuously. Thus information of non-arrival of the particle at the detector is available continuously.

The measurement of the point of arrival of a quantum particle at a detector requires a higher dimensional formulation. In higher dimensions the discounted wave function in a domain $D$ in the presence of an absorbing boundary $\Gamma$ is given by

$$
\psi(x, t) = \psi_B(x, t) \exp \left\{ -\frac{\lambda h}{2m\pi} \int_0^t \oint_\Gamma \left| \frac{\partial \psi_B(x', t')}{\partial n} \right|^2 dS_{x'} dt' \right\}, \quad (4.3)
$$

where $\psi_B(x, t)$ is the solution of Schrödinger’s equation in $D$ with zero boundary condition on $\Gamma$. Adopting the interpretation of the squared modulus of the wave function as the probability density of finding a particle at the point $x$ at time $t$ (whatever that means), the discounted wave function can be used to calculate the joint probability density of surviving by time $t$ and finding the particle at $x$ at the same time. The squared modulus of the wave function, conditioned on surviving by time $t$ is found by dividing the joint probability density $|\psi(x, t)|^2$ by the probability of the condition, $S(t)$. In the multi-dimensional case at hand

$$
S(t) = \exp \left\{ -\frac{\lambda h}{m\pi} \int_0^t \oint_\Gamma \left| \frac{\partial \psi_B(x', t')}{\partial n} \right|^2 dS_{x'} dt' \right\}. \quad (4.4)
$$

It follows from eqs. (4.3) and (4.4) that the conditioned wave function is $\psi_B(x, t)$.

Applying the method of propagation into the wall and the discounting procedure, as in [13], we find that the normal component of the multi-dimensional probability current density at any point $x$ on $\Gamma$ is given by

$$
J(x, t) \cdot n(x)|_\Gamma = \frac{\lambda h}{m\pi} \left| \frac{\partial \psi_B(x, t)}{\partial n} \right|^2 \exp \left\{ -\frac{\lambda h}{m\pi} \int_0^t \oint_\Gamma \left| \frac{\partial \psi_B(x', t')}{\partial n} \right|^2 dS_{x'} dt' \right\}. \quad (4.5)
$$

As in the case of diffusion, the pdf of the arrival time is given by

$$
Pr\{\tau = t\} = \oint_\Gamma J(x, t) \cdot n(x) dS_x. \quad (4.6)
$$

The probability density function of the particle hitting the point $x$ on the detector is given by

$$
Pr\{x(\tau) = x\} = J(x) \cdot n(x) = \int_0^\infty J(x, t) \cdot n(x) dt. \quad (4.7)
$$

Obviously, the integral converges and

$$
\oint_\Gamma Pr\{x(\tau) = x\} dS_{x'} = 1 - \exp \left\{ -\frac{\lambda h}{m\pi} \int_0^\infty \oint_\Gamma \left| \frac{\partial \psi_B(x', t')}{\partial n} \right|^2 dS_{x'} dt' \right\}.
$$

Thus, if

$$
\int_0^\infty \oint_\Gamma \left| \frac{\partial \psi_B(x', t')}{\partial n} \right|^2 dS_{x'} dt' = \infty,
$$

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then
\[ \Pr \{ \tau < \infty \} = \int_{\Gamma} \Pr \{ \mathbf{x}(\tau) = \mathbf{x} \} \, dS_{\mathbf{x}'} = 1, \quad (4.8) \]
which means that the particle is absorbed in finite time with probability 1. However, if
\[ \int_{0}^{\infty} \int_{\Gamma} \left| \frac{\partial \psi_B(\mathbf{x}', t')}{\partial n} \right|^2 \, dS_{\mathbf{x}'} \, dt' < \infty, \]
then
\[ \Pr \{ \tau < \infty \} = \int_{\Gamma} \Pr \{ \mathbf{x}(\tau) = \mathbf{x} \} \, dS_{\mathbf{x}'} < 1, \quad (4.9) \]
so that
\[ \Pr \{ \tau = \infty \} > 0. \]
That is, there is finite probability that the particle is never absorbed at the detector.

5. Collapse of the wave function and measurement

We begin the discussion of the collapse of the wave function with the analysis of an analogous situation in diffusion theory, where this concept has been well understood for a long time now. A Brownian particle released at a point appears (is observed) as a point at a random time and at a random coordinate on an absorbing wall that is used as a detector. Its probability density function evolves according to the Fokker-Planck equation which takes into account the absorbing wall as a boundary condition: it vanishes there at all times \[11, 12, 18\]. When the Brownian particle (its trajectory) reaches the wall it is observed as a point on the wall so that its probability density function collapses instantaneously to a delta function there and then. There is no mechanism in the Fokker-Planck equation to effect this sudden catastrophe and there cannot be one. This apparent paradox is resolved through the connection between the Brownian trajectories and the Wiener integral (the solution of the Fokker-Planck equation). This probability density function represents our uncertainty about the location of the Brownian trajectory at all times prior to absorption in the wall and assigns densities to all possible Brownian trajectories. It assigns a probability density to the point and time of arrival of the Brownian particle (that is, its trajectory) at the wall by means of the probability current density on the wall, as described in Section 2 above. This density on the wall can be constructed from a histogram of the points of arrival of identical Brownian particles, as mentioned there.

According to MQM, the situation with a quantum particle is quite similar. For a one-dimensional quantum particle in the presence of detectors at the ends of a given interval (absorbing walls, say), as long as its trajectory has not reached either detector its wave function evolves according to the Schrödinger equation with boundary conditions given at the endpoints of the interval, where it vanishes at all times \[13, 21, 15\] and is discounted as described in Section 1 and given by eq.\,(1.3). When the particle (its trajectory) reaches either end it is observed there. Prior to this observation the particle’s
wave function assigns a probability amplitude to all possible Feynman trajectories that have not reached the boundaries. Actually, it vanishes outside the interval at all times. The wave function assigns a probability density to the point and time of arrival of the quantum particle (that is, of its trajectory) at the endpoints by means of the probability current density there, as described in Sections 3,4 above.

In summary, a quantum particle (that is, its trajectory) arrives at a detector with the probability density defined by the uni-directional current at the detector, computed from the discounted wave function that is constructed from the solution of the Schrödinger equation with zero boundary conditions at the detector by eq.(1.3). This description eliminates the need for the notion of the QM collapse of the wave function. The QM collapse is replaced in MQM by the usual collapse of probability theory.

In higher dimensions the wave function in a domain $D$ in the presence of an absorbing boundary $\Gamma$ is given by

$$
\psi(x,t) = \psi_B(x,t) \exp \left\{-\frac{\lambda \hbar}{2m \pi} \int_0^t \left| \frac{\partial \psi_B(x,t')}{\partial n} \right|_\Gamma^2 dt' \right\},
$$

(5.1)

where $\psi_B(x,t)$ is the solution of Schrödinger’s equation in $D$ with zero boundary condition on $\Gamma$. The probability current density on $\Gamma$ is given by

$$
\mathcal{J}(x,t)|_\Gamma = \frac{\lambda \hbar}{m \pi} \left| \frac{\partial \psi(x,t)}{\partial n} \right|_\Gamma^2
= \frac{\lambda \hbar}{m \pi} \left| \frac{\partial \psi_B(x,t')}{\partial n} \right|_\Gamma^2 \exp \left\{-\frac{\lambda \hbar}{m \pi} \int_0^t \left| \frac{\partial \psi_B(x,t')}{\partial n} \right|_\Gamma^2 dt' \right\},
$$

(5.2)

The notions of current density at $\Gamma$ and time of arrival are defined in an analogous manner, as described in Sections 2-4. This is illustrated in the slit experiment application below.

6. Application to the slit experiment

We consider the following experimental setup. A planar screen is placed in the plane $x = 0$ and another screen it placed in the plane $x = x_0$ and it is slit along a line parallel to the $z-$axis. Due to the invariance of the geometry of the problem in $z$ the mathematical description of the slit is, following [10], an initial truncated Gaussian wave packet in the $(x,y)-$plane, concentrated around the initial point, $(x_0,0)$. To describe the interference pattern on the screen, we assume it is an absorbing line on the $y-$axis and apply the formalism developed above. The wave function, as given by our formalism, evolves from the initial packet according to eq.(5.1), as

$$
\psi(x,y,t) = \psi_B(x,y,t) \exp \left\{-\frac{\lambda \hbar}{2m \pi} \int_0^t \int_{-\infty}^{\infty} \left| \frac{\partial \psi_B(0,y',t')}{\partial x} \right| dy' dt' \right\},
$$
where $\psi_B(x, y, t)$ is the solution of the Schrödinger equation in the half plane $x > 0$ with $\psi_B(0, y, t) = 0$ and $\psi_B(x, y, 0)$ is the given initial packet. The probability distribution of the time to arrival of the Feynman trajectories to the absorbing line is determined from the equation

$$\Pr\{\tau > t\} = \int_0^\infty \int_{-\infty}^\infty |\psi(x, y, t)|^2 \, dx \, dy = \exp\left\{-\frac{\lambda \hbar}{m \pi} \int_0^t \int_{-\infty}^\infty \left|\frac{\partial \psi_B(0, y', t')}{\partial x}\right|^2 \, dy' \, dt'\right\}. $$

Measurement in time gives

$$\mathcal{J}(0, y, t) = \frac{\lambda \hbar}{m \pi} \left|\frac{\partial \psi(0, y, t)}{\partial x}\right|^2 \exp\left\{-\frac{\lambda \hbar}{m \pi} \int_0^t \int_{-\infty}^\infty \left|\frac{\partial \psi_B(0, y', t')}{\partial x}\right|^2 \, dy' \, dt'\right\} \quad (6.1)$$

This is the probability density of a collapse of the wave function occurring at the point $y$ on the screen at time $t$. The total current

$$\mathcal{J}(y) = \frac{\lambda \hbar}{m \pi} \int_0^\infty \left|\frac{\partial \psi(0, y, t)}{\partial x}\right|^2 \, dt \quad (6.3)$$

is the probability density that the collapse of the wave function occurs at the point $y$ on the screen (ever).

In a real experiment the measurement is neither instantaneous nor infinite in time. That is, an integral over a finite time interval is observed rather than (6.2) or (6.3). If a packet of particles is sent out eq. (6.2) is the probability density of Feynman trajectories that propagate instantaneously at time $t$ into the point $(0, y)$ in the screen and is seen as the density of light intensity on the (ideal) fluorescent screen at time $t$. The function (6.3) represents the cumulative (in time) probability current density of Feynman trajectories absorbed in the wall and is seen as the density of the trace the initial packet eventually leaves on the screen (e.g., photographic plate).

To determine the patterns (6.2) and (6.3), we have to calculate first the two-dimensional wave function with zero boundary condition on the $y$-axis. It can be written as

$$\psi_B(x, y, t) = \psi_1^h(x, t)\psi_2(y, t),$$

where, using the method of images, we find that

$$\psi_1^h(x, t) = \int_{-\infty}^0 \frac{e^{-(z-x_0)^2/2\sigma_x^2} e^{-im(z-x)^2/2\hbar t}}{\sqrt{2\pi i\sigma_x}} \frac{e^{-im(z-x)^2/2\hbar t}}{\sqrt{2\pi i\hbar t/m}} \, dz - \int_0^\infty \frac{e^{-(z+x_0)^2/2\sigma_x^2} e^{-im(z-x)^2/2\hbar t}}{\sqrt{2\pi i\sigma_x}} \frac{e^{-im(z+x)^2/2\hbar t}}{\sqrt{2\pi i\hbar t/m}} \, dz \quad (6.4)$$
and
\[
\psi^2(y, t) = \int_{-\infty}^{\infty} \frac{e^{-z^2/2\sigma_y^2} e^{-im(y-z)^2/2\hbar t}}{\sqrt{2\pi i \sigma_y}} d\zeta.
\]
Evaluation of the integral gives
\[
|\psi^2(y, t)|^2 = \frac{1}{2\pi \sigma_y} \left( \frac{\hbar^2 t^2}{\sigma_y^2 m^2} + \sigma_y^2 \right)^{-1/2} \exp \left\{ -\frac{y^2}{\hbar^2 t^2} \frac{\sigma_y^2 m^2}{\sigma_x^2 m^2 + \sigma_y^2} \right\}
\]
If \( \sigma_x << |x_0| \), the upper limit of integration in eq. (5.6) can be replaced by \( \infty \) with a transcendentally small error. Thus, we write
\[
\psi(x, y, t) \approx \psi^2(y, t) \int_{-\infty}^{\infty} \frac{e^{-(z-x_0)^2/2\sigma_x^2}}{\sqrt{2\pi i \sigma_x}} \left[ \frac{e^{-im(x-z)^2/2\hbar t}}{\sqrt{2\pi i \hbar t / m}} - \frac{e^{-im(x+z)^2/2\hbar t}}{\sqrt{2\pi i \hbar t / m}} \right] d\zeta.
\]
According to eq. (5.2), the instantaneous absorption rate at time \( t \) at a point \( (0, y) \) on the screen is given by
\[
J(0, y, t) = \frac{4\lambda m}{2\pi \sigma_x^2} \left( \frac{\hbar^2 t^2}{\sigma_x^2 m^2} + \sigma_x^2 \right)^{-3/2} \exp \left\{ -\frac{x_0^2}{\hbar^2 t^2} \frac{\sigma_x^2 m^2}{\sigma_x^2 m^2 + \sigma_y^2} \right\} |\psi^2(y, t)|^2.
\]
To compare eq. (5.3) with that given in [10], we reproduce the derivation of [10] with an initial two-dimensional Gaussian wave packet. The result gives the wave function as
\[
\psi_F(x, y, t) = \psi^1_F(x, t) \psi^2(y, t)
\]
and probability density at the screen at time \( t \) as
\[
|\psi_F(0, y, t)|^2 = \frac{1}{2\pi \sigma_x} \left( \frac{\hbar^2 t^2}{\sigma_x^2 m^2} + \sigma_x^2 \right)^{-1/2} \exp \left\{ -\frac{x_0^2}{\hbar^2 t^2} \frac{\sigma_x^2 m^2}{\sigma_x^2 m^2 + \sigma_x^2} \right\} |\psi^2(y, t)|^2.
\]
The instantaneous intensity of the diffraction pattern in the absence of an absorbing screen, given in [10], is defined as \( |\psi_F(0, y, t)|^2 \). Thus, the introduction of an absorbing screen, according to these interpretations, gives the relative brightness as
\[
\frac{\lambda \hbar}{m\pi} \left\{ \frac{\partial |\psi_F(0, y, t)|^2}{\partial x} \right\}^2 = \frac{4\lambda m}{2\sigma_x^6} \left( \frac{\hbar^2 t^2}{\sigma_x^2 m^2} + \sigma_x^2 \right).
\]
The decay in time of the quotient reflects the fact that the absorbing screen depresses the entire wave function in time. Thus, the part of the packet that arrives later is already attenuated by the preceding absorption, relative to the unattenuated wave function in the absence of absorption.

Next, we propose a simple device for performing a measurement of times of arrival of particles at an absorbing boundary in one dimension, as well as that of the unidirectional current at the absorbing boundaries. In addition, the proposed device demonstrates the effect of additional absorbing boundaries on the slit experiment. The proposed measurement can discriminate between the various modes of absorption described above.

Consider the setup of the slit experiment enclosed between two parallel absorbing walls, symmetric with respect to the slit and perpendicular to the planes of the screen and the slit. For example, the walls can be made of photographic plates. Particles are given a constant initial velocity, \( v_x \), in the \( x \) direction (perpendicular to the planes of the screen and slit), within the constraints of uncertainty. The time a particle leaves the slit is also measurable within the constraints of uncertainty.

The initial packet is Gaussian in the \( x \) direction and is uniform inside the slit (in the \( y \) direction). This means that the initial velocities in the \( y \) direction have the density

\[
|\Psi (k)|^2 = \left| \sin \frac{\pi k}{2} \right|^2. \tag{6.7}
\]

The plane of the slit is \( x = x_0 \), the plane of the screen is \( x = 0 \), the slit is the interval \( -\pi/2 < y < \pi/2 \). The absorbing planes are \( y = \pm y_0 \) with \( y_0 > \pi/2 \). In this setup the motion of the particles in the \( x \) direction is independent of that in the \( y \) direction. The latter is the object of the proposed experiment.

Particles that hit the planes \( y = \pm y_0 \) leave traces at points \( x_1, x_2, ..., x_N \). These distances are proportional (within the constraints of uncertainty) to the times of arrival at the absorbing walls of particles that start out in the interval \( -\pi/2 < y < \pi/2 \) with initial velocities distributed as in eq.(6.7). The histogram obtained from these points, on an axis normalized with the velocity \( v_x \), is that of the times of arrival of one dimensional particles moving on the \( y \) axis.

The wave function for this configuration is given by

\[
\psi (x, y, t) = \psi_1(x, t)\psi^2(y, t),
\]

where \( |\partial\psi_1(0, t)/\partial x|^2 \) was calculated in [14] and

\[
\psi^2(y, t) = \sum_{n=1}^{\infty} \frac{2}{n\pi^{3/2}} \cos \frac{n\pi}{4y_0} \sin \frac{n\pi}{y_0} y \exp \left\{ -\frac{i n^2 \pi^2}{\hbar y_0^2} t \right\}.
\]

According to eq.(4.6), the histogram of the times of arrival is also that of the unidirectional current on each wall. According to MQM, this pattern is given by

\[
J(x, \pm y_0) = \int_0^\infty J(x, \pm y_0, t) \, dt \tag{6.8}
\]
\[
\int_0^\infty \left| \frac{\partial \psi_1(x, t)}{\partial x} \right|^2 \left| \psi^2(\pm y_0, t) \right|^2 dt.
\]

Next, we consider the pattern observed on the wall \(x = 0, -\pi/2 < y < \pi/2\). The histogram of the traces of the particles on the screen at \(x = 0\) is given by

\[
J(y) = \int_0^\infty \left| \frac{\partial \psi_1^1(0, t)}{\partial x} \right|^2 \psi^2(0, t) y_0 \exp \left\{ -\frac{i n \pi^2}{\hbar y_0^2} t \right\} dt.
\]  

If the velocities in the \(x\) direction are concentrated around \(v_x\), the histogram will be approximately

\[
J(y) \approx J(y, \bar{t}) = \left| \frac{\partial \psi_1^1(0, \bar{t})}{\partial x} \right|^2 \psi^2(y, \bar{t})
\]

\[
= \left| \frac{\partial \psi_1^1(0, \bar{t})}{\partial x} \right|^2 \sum_{n=1}^\infty \frac{2}{n \pi^{3/2}} \cos \frac{n \pi^2}{4 y_0} \sin \frac{n \pi}{y_0} y \exp \left\{ -\frac{i n^2 \pi^2}{4 \hbar y_0^2} \right\}^2.
\]

This is not the same as the expression obtained in eq.(6.3). The difference is due to the effect of the lateral absorbing boundaries. Thus, according to MQM, absorbing boundaries cannot be ignored, as done in QM.

### 7. Recovering the free particle probability density function from measurements of unidirectional currents

The wave function of a freely propagating initial Gaussian packet (Gaussian slit) can be evaluated from measurements by a detector in the form of an absorbing screen. It suffices to rotate the screen to obtain two measurements that reproduce the entire wave function of the free packet as follows.

It was shown above that \(|\psi_F(0, y, t)|^2\) can be recovered, up to a multiplicative function of time, from measurements of the unidirectional current in the Gaussian slit experiment,

\[
|\psi_F(0, y, t)|^2 = \frac{J(0, y, t)}{\phi(t)},
\]

where eq.(6.6) gives

\[
\phi(t) = \frac{4 x_0^2 \lambda m}{2 \pi \sigma_x^2} \left( \frac{\hbar^2 t^2}{\sigma_x^2 m^2} + \sigma_x^2 \right)^{-3/2} \exp \left\{ -\frac{x_0^2}{\hbar^2 t^2 \left( \frac{\sigma_x^2}{\sigma_x^2 m^2} + \sigma_x^2 \right)} \right\}.
\]

Now, we rotate the screen plane about the point \((0, 0)\) on the screen by an angle \(\theta\) and introduce a new coordinate system about the same origin,

\[
x' = \alpha x + \beta y, \quad y' = \gamma x + \delta y,
\]

\[
(7.1)
\]
where \( \alpha = \cos \theta, \beta = \sin \theta, \gamma = -\sin \theta, \delta = \cos \theta \). We write
\[
|\psi_F(x, y, t)|^2 = |\tilde{\psi}_F(x', y', t)|^2
\]
so that
\[
\psi_F^1(x, t)\psi^2(y, t) = \tilde{\psi}_F^1(x', t)\tilde{\psi}^2(y', t). \tag{7.2}
\]
According to our theory,
\[
|\tilde{\psi}^2(y', t)|^2 = \frac{\tilde{J}(a', y', t)}{\tilde{\phi}(t)},
\]
where \( a' \) is the distance from the center of the initial packet to the rotated screen and
\[
\tilde{\phi}(t) = \frac{4a'^2\lambda m}{2\pi\sigma^2} \left( \frac{h^2t^2}{\sigma^2m^2} + \sigma_x^2 \right)^{-3/2} \exp \left\{ -\frac{a'^2}{h^2t^2} \left( \frac{m}{\sigma_x^2} + \sigma_x^2 \right) \right\}.
\]
The current \( \tilde{J}(a', y', t) \) is given by
\[
\tilde{J}(a', y', t) = \frac{4a'^2\lambda m}{2\pi\sigma^2} \left( \frac{h^2t^2}{\sigma^2m^2} + \sigma_x^2 \right)^{-3/2} \exp \left\{ -\frac{a'^2}{h^2t^2} \left( \frac{m}{\sigma_x^2} + \sigma_x^2 \right) \right\} |\tilde{\psi}^2(y', t)|^2.
\]
It should be borne in mind that \( J(0, y, t) \) and \( \tilde{J}(a', y', t) \) are measurable on the screen. Now, eqs.(7.2) and (7.1) give
\[
|\tilde{\psi}_F^1(a', t)\tilde{\psi}^2(y', t)|^2 = \left| \psi_F^1 \left( \frac{a' - \beta y}{\alpha}, t \right) \psi^2(y, t) \right|^2.
\]
It follows that
\[
|\psi_F^1 \left( \frac{a' - \beta y}{\alpha}, t \right)|^2 = \frac{\tilde{J} \left( a', \gamma \frac{a' - \beta y}{\alpha} + \delta y, t \right) \phi(t) |\tilde{\psi}_F^1(a', t)|^2}{\tilde{J}(0, y, t)\tilde{\phi}(t)},
\]
hence
\[
|\psi_F(x, y, t)|^2 = \left| \psi_F^1(x, t)\psi^2(y, t) \right|^2 \tag{7.3}
\]
\[
= \frac{\tilde{J} \left( a', \gamma x + \delta \frac{a' - \alpha x}{\beta}, t \right) J(0, y, t) |\tilde{\psi}_F^1(a', t)|^2}{\tilde{J} \left( 0, \frac{a' - \alpha x}{\beta}, t \right) \phi(t)}.
\]
Thus, up to a time dependent normalization factor, the wave function in the absence of the screen can be found from measurable currents on the two screens. Since
\[
\int \int |\psi_F(x, y, t)|^2 \, dx \, dy = 1,
\]
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we can rewrite eq. (7.3) as
\[
|\psi_F(x, y, t)|^2 = \mathcal{N}^{-1}(t) \frac{\tilde{J}(a', \gamma x + \frac{\delta'(a - \alpha x)}{\beta}, t) \mathcal{J}(0, y, t)}{\mathcal{J}(0, \frac{a' - \alpha x}{\beta}, t)},
\]
where
\[
\mathcal{N}^{-1}(t) = \int \int \frac{\tilde{J}(a', \gamma x + \frac{\delta'(a - \alpha x)}{\beta}, t) \mathcal{J}(0, y, t)}{\mathcal{J}(0, \frac{a' - \alpha x}{\beta}, t)} \, dx \, dy.
\]

8. Summary

A measuring device of the coordinate of a particle is modeled as an absorbing wall that cuts off the wave function. This is a more physically realistic model than von Neumann’s collapse machine or other quantum collapse devices. The proposed detector modifies the wave function of the particle in an irreversible manner. It replaces the QM collapse of the wave function with the usual collapse of the probability distribution after observation. The proposed MQM is based on postulates that assume well defined trajectories of quantum particles. The calculation of the wave function of a particle in the presence of such detector is based on the Feynman integral and a discounting procedure of the probability of absorbed trajectories. The resulting MQM is Hamiltonian prior to absorption but the absorption process is not. The fundamental notion in MQM is the absorption probability current on the absorbing wall. This is used to define the probability distribution of the arrival time and of the arrival point.

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