No absorption in de Sitter space

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Abstract

We study the wave equation for a minimally coupled massive scalar in D-dimensional de Sitter space. We compute the absorption cross section to investigate its cosmological horizon in the southern diamond. By analogy of the quantum mechanics, it is found that there is no absorption in de Sitter space. This means that de Sitter space is usually in thermal equilibrium, like the black hole in anti de Sitter space. It confirms that the cosmological horizon not only emits radiation but also absorbs that previously emitted by itself at the same rate, keeping the curvature radius of de Sitter space fixed.
I. INTRODUCTION

Recently an accelerating universe has proposed to be a way to interpret the astronomical data of supernova [1–3]. The inflation is employed to solve the cosmological flatness and horizon puzzles arisen in the standard cosmology. Combining this observation with the need of inflation leads to that our universe approaches de Sitter geometries in both the infinite past and the infinite future [4–6]. Hence it is very important to study the nature of de Sitter (dS) space and the dS/CFT correspondence [7,8]. However, there exist difficulties in studying de Sitter space. First there is no spatial infinity and global timelike Killing vector. Thus it is not easy to define conserved quantities including mass, charge and angular momentum appeared in asymptotically de Sitter space. Second the dS solution is absent from string theories and thus we do not have a definite example to test the dS/CFT correspondence. Third it is hard to define the S-matrix because of the presence of the cosmological horizon.

We remind the reader that the cosmological horizon is very similar to the event horizon in the sense that one can define its thermodynamic quantities of a temperature and an entropy using the same way as was done for the black hole. Two important quantities in studying the black hole are the Bekenstein-Hawking entropy and the absorption cross section (=greybody factor). The former relates to the intrinsic property of the black hole itself, while the latter relates to the effect of spacetime curvature. Explicitly the greybody factor for the black hole arises as a consequence of scattering off the gravitational potential surrounding the horizon [9]. For example, the low-energy s-wave greybody factor for a massless scalar has a universality such that it is equal to the area of the horizon for all spherically symmetric black holes [10]. Also the greybody factor measures the Hawking radiation in a semiclassical way. The entropy for the cosmological horizon was discussed in literature [11]. However, there is a few attempts to compute the greybody factor for the cosmological horizon [15].

In this paper we compute the absorption cross section of a massive scalar in the background of D-dimensional de Sitter space. For this purpose we first note that the wave equation is well defined only in the southern diamond. Also we should point out a crucial difference between the cosmological horizon in de Sitter space and the event horizon in the black hole which states that de Sitter space is usually assumed to be in thermal equilibrium [16]. This implies that the cosmological horizon not only emits radiation, but also absorbs radiation previously emitted by itself at the same rate, keeping the curvature radius fixed. On the other hand, one choose either a black hole in thermal equilibrium with a heat bath within a bounded box or a black hole that is truly evaporating. This arises because the black hole (radiation in a bounded box) have a negative (positive) specific heat, whereas the cosmological horizon has a positive specific heat. Two of black hole and heat bath will be in thermal equilibrium if the box is bounded. An example is an eternal black hole in anti de Sitter space (AdS-black hole) [17] because anti de Sitter space is considered as a box. If the box is unbounded the black hole evaporates completely, as the Schwarzschild black hole

\(^{1}\)A similar work for four-dimensional Schwarzschild-de Sitter black hole appeared in [12]. But it considered mainly the black hole temperature. Also the absorption rate for the Kerr-de Sitter black hole was discussed in [13]. Recently, temperature and entropy of the Schwarzschild-de Sitter black hole were discussed in [14]
evaporates. Actually the de Sitter horizon is very similar to the AdS-black hole [18]. In
the previous works [15], we did not consider this stable nature of the cosmological horizon
seriously.

We wish to calculate the outgoing flux near \( r = 0 \). And then we compute the outgoing
flux by using the matching region of overlapping validity near the cosmological horizon of
\( r_c = 1 \). In this work we will not follow the conventional approach for a computation of the
greybody factor of the black hole. As an analog situation we introduce the wave propagation
under the potential step with \( 0 < E < V_0 \) in the quantum mechanics. This gives rise to the
classical picture of what the particle goes on : the total reflection occurs due to the potential
step. The similar situations also occur in the scalar wave propagation in the background of
the cosmological horizon. Even though the cosmological horizon emits radiation, it absorbs
radiation previously emitted by itself at the same rate. Hence there is no net absorption in a
finite time and thus one gets the zero absorption cross section in the semiclassical approach.

The organization of this paper is as follows. In section II we briefly review the wave
equation in de Sitter space. We perform a potential analysis to study the asymptotic region
by introducing a tortoise coordinate \( r^* \) in section III. Also we briefly review the scattering in
the potential step of the quantum mechanics. In section IV we calculate the flux at \( r = 0, 1 \)
to find the greybody factor. Finally we discuss our results in section V.

II. WAVE EQUATION IN DE SITTER SPACE

We start with the wave equation for a massive scalar
\[
(\nabla^2 - m^2)\Phi = 0
\]
(1)
in the background of D-dimensional de Sitter space expressed in the static coordinates
\[
ds^2_{dS} = -\left(1 - \frac{r^2}{l^2}\right)dt^2 + \left(1 - \frac{r^2}{l^2}\right)^{-1}dr^2 + r^2 d\Omega_{D-2}^2.
\]
(2)
Here \( l^2 \) is the curvature radius of de Sitter space and hereafter we set \( l = 1 \) for simplicity
unless otherwise stated. \( d\Omega_{D-2}^2 \) is the metric of the unit sphere \( S^{D-2} \). The above metric
is singular at the cosmological horizon, which divides space into four regions. There are
two regions with \( 0 \leq r \leq 1 \) which correspond to the causal diamonds of observers at the
north and south poles : northern diamond (ND) and southern diamond (SD). An observer
at \( r = 0 \) is surrounded by a cosmological horizon at \( r = 1 \). Two regions with \( 1 < r < \infty \)
containing the future-null infinity \( I^+ \) and past-null infinity \( I^- \) are called future triangle
(FT) and past triangle (PT), respectively. A timelike Killing vector \( \frac{d}{dt} \) is future-directed
only in the southern diamond. To obtain the greybody factor, we have to obtain a definite
wave propagation as time evolves. Hence in this work we confine ourselves to the southern
diamond. This means that our working space is compact, in contrast to the case of the black
hole. This gave rise to an ambiguity to derive the greybody factor in the previous approach
[15].

In connection with the dS/CFT correspondence, one may classify the mass-squared \( m^2 \)
into three cases [8] : \( m^2 \geq 1, \quad 0 < m^2 < 1, \quad m^2 = 0 \). For a massive scalar with \( m^2 \geq 1, \)
one has a non-unitary CFT. A scalar with mass $0 < m^2 < 1$ can be related to a unitary CFT. A massless scalar with $m^2 = 0$ is special and it would be treated separately. For our purpose we consider $m^2$ as a parameter at the beginning. Assuming a mode solution

$$\Phi(r, t, \Omega) = f_\ell(r)e^{-i\omega t}Y_m^\ell(\Omega),$$

Eq.(1) leads to the differential equation for $r$ [19,20]

$$(1 - r^2)f''_\ell(r) + \left(\frac{D - 2}{r} - Dr\right)f'_\ell(r) + \left(\frac{\omega^2}{1 - r^2} - \frac{\ell(\ell + D - 3)}{r^2} - m^2\right)f_\ell(r) = 0,$$

where the prime ($) denotes the differentiation with respect to its argument. Here $Y_m^\ell(\Omega)$ are the hyper-spherical harmonics on $S^{D-2}$ with $\nabla_2^2 Y_m^\ell(\Omega) = -\ell(\ell + D - 3)Y_m^\ell(\Omega)$. In $Y_m^\ell(\Omega)$, $\ell$ is a positive integer including 0 and $m$ is a collective index of $(m_1, m_2, \cdots, m_{D-3})$.

### III. POTENTIAL ANALYSIS

We observe from Eq.(4) that it is not easy to find how scalar waves propagate in the southern diamond. In order to do that, we must transform the wave equation into the Schrödinger-like equation using a tortoise coordinate $r^*$ [21]. Then we can get wave forms in asymptotic regions of $r^* \to \pm\infty$ through a potential analysis. We introduce $r^* = g(r)$ with $g'(r) = 1/r^{D-2}(1 - r^2)$ to transform Eq.(4) into the Schrödinger-like equation with the energy $E = \omega^2$

$$-\frac{d^2}{dr^*^2}f_\ell + V_D(r)f_\ell = Ef_\ell$$

with a D-dimensional potential

$$V_D(r) = \omega^2 + r^{2(D-2)}(1 - r^2)[m^2 + \frac{\ell(\ell + D - 3)}{r^2} - \frac{\omega^2}{1 - r^2}].$$

Considering $r^* = g(r) = \int g'(r)dr$, one finds for three-dimensional (D=3) de Sitter space

$$r^* = \ln r - \frac{1}{2}\ln[(1 + r)(1 - r)], \quad e^{2r^*} = \frac{r^2}{1 - r^2}, \quad r^2 = \frac{e^{2r^*}}{1 + e^{2r^*}}.$$  \(7\)

For D=4 de Sitter space, one has

$$r^* = -\frac{1}{r} + \frac{1}{2}\ln\left[\frac{1 + r}{1 - r}\right].$$  \(8\)

The explicit form of $r^*(r)$ depends on the spacetime dimension D. From the above two expressions we confirm that $r^*$ is a tortoise coordinate such that $r^* \to -\infty (r \to 0)$, whereas

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2 Although this belongs to one of examples of non-unitary theories that are dual to well-behaved stable bulk theory, but the connection between the bulk theory and its non-unitary boundary theory is not understood clearly up to now.
\( r^* \to \infty (r \to 1) \). We can express the potential as a function of \( r^* \) explicitly only for \( D=3 \) case

\[
V_3(r^*) = \omega^2 + \frac{e^{2r^*}}{(1+e^{2r^*})^2}\left[m^2 + \frac{1+e^{2r^*}}{e^{2r^*}}\ell^2 - (1+e^{2r^*})\omega^2\right].
\] (9)

For \( D=3 \), \( m^2 = 1, \ell = 0, \omega = 0.1 \), the shape of this takes a potential barrier (\( \mathcal{V} \)) located at \( r^* = 0 \). On the other hand, for all non-zero \( \ell \), one finds the potential step (\( \mathcal{N} \)) with its height \( \omega^2 + \ell^2 \) on the left-hand side of \( r^* = 0 \). \( V_3(r^*) \) decreases exponentially to zero as \( r^* \) increases on the right-hand side.

From the quantum mechanics we conjecture that traveling waves appear near the cosmological horizon of \( r^* = \infty \). But near the coordinate origin of \( r^* = -\infty (r = 0) \), it is not easy to develop a genuine traveling wave. Near \( r = 0 \) (\( r^* = -\infty \)) one finds the equation

\[
\frac{d^2}{dr^*_2}f_{\ell,-\infty} - \ell(\ell + D - 3)r^{2(D-3)}f_{\ell,-\infty} = 0.
\] (10)

For \( D=3 \) case, this gives us a solution

\[
f_{\ell,-\infty}(r^*) = A_3 e^{\ell r^*} + B_3 e^{-\ell r^*}
\] (11)
which is equivalently rewritten by making use of Eq.(7) as

\[
f_{\ell,r=0}(r) = A_3 r^{\ell} + \frac{B_3}{r^\ell}.
\] (12)

For \( D \geq 4 \) de Sitter space, one obtains

\[
f_{\ell,r=0}(r) = A_D r^{\ell} + \frac{B_D}{r^{\ell + D - 3}}.
\] (13)

In the above two equations, the first terms correspond to normalizable modes at \( r = 0 \) (\( r^* = -\infty \)), while the second terms are non-normalizable, singular modes. As one discards the second terms in Eqs.(11) and (13) for calculating the Bogoliubov transformation [19], the first terms are only needed for our purpose. Hence we set \( B_D = 0 \) for \( D \geq 3 \).

On the other hand, near the cosmological horizon \( r_c = 1(r^* = \infty) \) one obtains a differential equation which is irrespective of \( D, \ell \)

\[
\frac{d^2}{dr^*_2}f_\infty + \omega^2 f_\infty = 0.
\] (14)

This has a solution

\[
f_\infty(r^*) = C_D e^{-i\omega r^*} + E_D e^{i\omega r^*}.
\] (15)

For \( D=3 \) de Sitter space, it is equivalently rewritten as

\[
f_{r=1}(r) = C_3 (1 - r^2)^{\frac{\ell}{2}} + E_3 (1 - r^2)^{-\frac{\ell}{2}}.
\] (16)

The first wave /second wave in Eq.(16) together with \( e^{-i\omega t} \) imply the ingoing (\( \leftarrow \))/outgoing (\( \rightarrow \)) waves across the cosmological horizon. This picture is based on the observer confined
in the southern diamond. In order to obtain a connection between $A_3$ and $C_3$, $E_3$, let us consider the case of $\ell^2 > m^2$ with $V_3(r^*) \approx V_0 = \omega^2 + \ell^2$ for $-\infty < r^* \leq 0$ and $V_3(r^*) \approx 0$ for $0 \leq r^* < \infty$ with the energy $E = \omega^2 < V_0$. It corresponds to the problem for a potential step $V_0$ with $0 < E < V_0$ in the quantum mechanics [22]. Requiring the conservation of flux at $r^* = 0$ leads to the asymptotic relations: $C_3 \approx (\ell - i\omega)A_3/(2i\omega)$, $E_3 \approx (\ell + i\omega)A_3/(2i\omega) = C_3^\ast$. It seems that the flux on the left hand side of $r^* = 0$ is zero due to the real function of Eq.(12), but the flux of the right hand side is not zero because of the traveling wave nature of Eq.(15). For D=4 de Sitter space, from Eqs.(8) and (15) we have

$$f_{r=1}(r) = C_4(1 - r)^{1/2} + E_4(1 - r)^{-1/2}.$$  

(17)

We note that the explicit forms of $f_{r=1}(r)$ for D>4 de Sitter space depend on their spacetime dimensions. However, their asymptotic forms will take the same form as in Eq.(17).

Up to now we obtain asymptotic forms of a scalar wave which propagates in the southern diamond of de Sitter space. In order to calculate the absorption cross section, we need to know an explicit form of wave propagation in $0 \leq r \leq 1$. This can be achieved only when solving the differential equation (4) explicitly.

Before we proceed, we wish to review further the scattering in the potential step for D=3 de Sitter space. To interpret our solution Eq.(15), it is convenient to multiply it by the $1/C_3$ to give a nice form [22]

$$\frac{f_\infty(r^*)}{C_3} = e^{-i\omega r^*} + \frac{E_3}{C_3} e^{i\omega r^*} \approx e^{-i\omega r^*} + \frac{C_3^\ast}{C_3} e^{i\omega r^*}.$$  

(18)

Because of $|C_3^\ast/C_3| = 1$, two waves (the first and second in Eq.(18)) have amplitudes of the same magnitude. As we will see alter, the absolute square of the amplitude of a wave must somewhat be proportional to the flux of particle. Hence we conclude that the wave function of Eq.(18) describes the situation in which an ingoing wave is reflected back to an outgoing wave by the potential step. This interpretation is in accordance with the classical picture of what the particle goes on. The wave function $f_{\ell,-\infty}(r^*)/C_3$ in Eq.(11) with $B_3 = 0$ describes the penetration of the Schrödinger wave into the classically forbidden region of $-\infty < r^* \leq 0$. The amplitude of penetrating wave decreases exponentially as we go further into the forbidden region, and at large distance from the potential barrier the amplitude is for all practical purpose zero in accordance with the classical picture.

### IV. FLUX CALCULATION

In order to solve equation (4), we first transform it into a hypergeometric equation using $z = r^2$. Here the working space still remains unchanged as $0 \leq z \leq 1$ covering the southern diamond. This equation takes a form

$$z(1 - z)f_\ell''(z) - \frac{1}{2}[z(D + 1) - (D - 1)]f_\ell'(z) + \frac{1}{4}\left(\frac{\omega^2}{1 - z} - \frac{\ell(\ell + D - 3)}{z} - m^2\right)f_\ell(z) = 0.$$  

(19)

Here one finds two poles at $z = 0, 1(r = 0, 1)$ and so makes a further transformation to cancel these by choosing a normalizable solution at $z = 0(r = 0)$.
\[ f(z) = z^\alpha (1 - z)^\beta w(z), \quad \alpha = \frac{\ell}{2}, \quad \beta = \frac{i\omega}{2} \] (20)

which is in accordance with Eq.(13) with \( B_D = 0 \) and Eqs.(16) and (17)\(^3\). Then we obtain a hypergeometric equation

\[ z(1 - z)w''(z) + [c - (a + b + 1)z]w'(z) - ab w(z) = 0 \] (21)

where \(a, b\) and \(c\) are given by

\[ a = \frac{1}{2}(\ell + i\omega + h_+), \quad b = \frac{1}{2}(\ell + i\omega + h_-), \quad c = \ell + \frac{D - 1}{2} \] (22)

with

\[ h_\pm = \frac{1}{2}\left[D - 1 \pm \sqrt{(D - 1)^2 - 4m^2}\right]. \] (23)

One regular solution near \( z = 0 \) to Eq.(19) is given by [23]

\[ f_+(z) = A_D z^{\ell/2}(1 - z)^{i\omega/2} F(a, b, c; z) \] (24)

with an unknown constant \( A_D\). For \( D=\)odd dimensions, there is the other solution with a logarithmic singularity at \( z = 0 \) as \( f_-(z) = A_D z^{\ell/2}(1 - z)^{i\omega/2}[F(a, b, c; z) \ln z + \cdots] \). However, both solutions have vanishing flux at \( z = 0 \) because the relevant part \((z^{\ell/2})\) is not complex but real.

Now we are in a position to calculate an outgoing flux at \( z = 0(r = 0, r^* = -\infty) \) which is defined as

\[ \mathcal{F}(z = 0) = 2\frac{2\pi}{i}[f^* \partial_z f - f \partial_z f^*]|_{z=0}. \] (25)

For any kind of real functions near \( z = 0(r = 0) \) including \( f_\pm \), the outgoing \((\rightarrow)/\)ingoing \((\leftarrow)\) fluxes are obviously given by

\[ \mathcal{F}_{out/in}(z = 0) = 0. \] (26)

This means that if a wave form is real near \( z = 0 \), one cannot find any non-zero flux. We choose a regular solution of \( f_+(z) \) for further calculation. To obtain a flux at the horizon of \( z = 1(r = 1) \), we first use a formula:

\[ F(a, b, c; z) = \frac{\Gamma(c)\Gamma(c - a - b)}{\Gamma(c - a)\Gamma(c - b)} F(a, b, a + b - c + 1; 1 - z) \]

\[ + \frac{\Gamma(c)\Gamma(a + b - c)}{\Gamma(a)\Gamma(b)} (1 - z)^{-a - b} F(c - a, c - b, -a - b + c + 1; 1 - z). \] (27)

\(^3\)In [20], to compute the quasi-normal mode spectrum of de Sitter space, the authors considered four cases: i) \( \alpha = 1/2, \beta = i\omega/2 \) ii) \( \alpha = 1/2, \beta = -i\omega/2 \) iii) \( \alpha = -(l + D - 3)/2, \beta = i\omega/2 \) iv) \( \alpha = -(l + D - 3)/2, \beta = -i\omega/2 \). However, i) is relevant to the greybody factor calculation.
Using $1 - z \approx e^{-2r^*}$ near $z = 1$, one finds from Eq.(24) the following form:

$$f_{+, 0 \to 1} \equiv f_{\text{in}} + f_{\text{out}} = H_{\omega, \ell} e^{-i\omega r^*} + H_{-\omega, \ell} e^{i\omega r^*}$$

(28)

where

$$H_{-\omega, \ell} = H_{\omega, \ell}^* = A_D\alpha_{-, \omega, \ell}, \quad \alpha_{-, \omega, \ell} = \frac{\Gamma(1 + \ell)\Gamma(i\omega)2^{i\omega}}{\Gamma[(\ell + i\omega + h_+^*)/2]\Gamma[(\ell + i\omega + h_-^*)/2]}.$$  

(29)

Then we match Eq.(15) with Eq.(28) to yield $C_D = H_{\omega, \ell}$ and $E_D = H_{-\omega, \ell}$ near the cosmological horizon. Finally we calculate its outgoing ($\to$) flux at $z = 1 (r^* = \infty)$ as

$$F_{\text{out}}(z = 1) = \frac{2\pi}{\ell} \left[ f_{\text{out}}^* \partial_{r^*} f_{\text{out}} - f_{\text{out}} \partial_{r^*} f_{\text{out}}^* \right] |r^* = \infty = 4\pi \omega A_D^2 |\alpha_{-, \omega, \ell}|^2.$$  

(30)

On the other hand, the ingoing ($\leftarrow$) flux is given by

$$F_{\text{in}}(z = 1) = \frac{2\pi}{\ell} \left[ f_{\text{in}}^* \partial_{r^*} f_{\text{in}} - f_{\text{in}} \partial_{r^*} f_{\text{in}}^* \right] |r^* = \infty = 4\pi \omega A_D^2 |\alpha_{\omega, \ell}|^2.$$  

(31)

Thus it equals $F_{\text{out}}(z = 1)$ because the modulus of amplitude $|\alpha_{\omega, \ell}|$ is equal to $|\alpha_{-, \omega, \ell}|$.

V. ABSORPTION CROSS SECTION

Up to now we do not insert the curvature radius $l$ of de Sitter space. The correct absorption coefficient can be recovered when replacing $\omega(m)$ with $\omega(lm)$. An absorption coefficient by the cosmological horizon is defined formally by

$$A = \frac{F_{\text{out}}(z = 1)}{F_{\text{out}}(z = 0)}.$$  

(32)

However, one cannot define the absorption cross section for this case because we have zero-flux of $F_{\text{out}}(z = 0) = 0$ for de Sitter wave propagation. Instead, we may follow the black hole approach to obtain the absorption cross section [15]. If we assume an unknown normalization $F_{\text{out}}(z = 0) = H_D$, then $H_D$ may be determined by referring the absorption cross section for the low-energy $s(\ell = 0)$-wave. In this way the absorption cross section in three dimensions may be defined by

$$\sigma_{\text{abs}}^{D=3} = \frac{A_3}{\omega} = \left[ \frac{4\pi l A_3^2}{H_3} \right] |\alpha_{-, \omega, \ell}|^2.$$  

(33)

where

$$|\alpha_{-, \omega, \ell}|^2 = \frac{|\Gamma(1 + \ell)|^2|\Gamma(i\omega l)|^2}{|\Gamma[(\ell + i\omega l + \tilde{h}_+^*)/2]|^2|\Gamma[(\ell + i\omega l + \tilde{h}_-^*)/2]|^2}.$$  

(34)

Here $\tilde{h}_\pm$ is obtained by replacing $m$ by $ml$ at $h_\pm$.

For $D=4$ de Sitter space, one may have

$$\sigma_{\text{abs}}^{D=4} = \frac{\pi A_4}{\omega^2} = \left[ \frac{4\pi^2 l A_4^2}{\omega H_4} \right] |\alpha_{-, \omega, \ell}|^2.$$  

(35)

However, this approach based on the calculation of the black hole-greynbody factor leads to a wrong result to find the absorption cross section for the cosmological horizon. Actually there is no wave propagating truly along “-”axis-direction. As is shown in Eq.(31), the incident (ingoing) wave is totally reflected to give the reflected (outgoing) wave by the approximate potential step at $r^* \approx 0$. This is confirmed by the zero flux of $F(r^* = -\infty) = 0$. 

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VI. DISCUSSION

We study the wave equation for a minimally coupled massive scalar in D-dimensional de Sitter space. We compute the absorption cross section to investigate its cosmological horizon in the southern diamond. By analogy of the quantum mechanics of the wave scattering under the potential step, it is found that there is no absorption of a scalar wave in de Sitter space in the semiclassical approach. This means that de Sitter space is usually stable and in thermal equilibrium, unlike the black hole. The cosmological horizon not only emits radiation but also absorbs that previously emitted by itself at the same rate, keeping the curvature radius of de Sitter space fixed. This can be proved by the relation of $F_{\text{out}}(z = 1) = F_{\text{in}}(z = 1)$ and $F_{\text{out/in}}(z = 0) = 0$. This exactly coincides with the wave propagation of the energy $E$ under the potential step with $0 < E < V_0$ which shows the classical picture of what the particle goes on. Here we find a nature of the eternal de Sitter horizon [18], which means that its cosmological constant $\Lambda_D = (D-2)(D-3)/2l^2$ remains unchanged, as like the eternal black hole in anti de Sitter space [17].

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