Implementing a Unit Commitment Power Market Model in FICO Xpress-Mosel

René Brandenberg
Technische Universität München

Matthias Silbernagl
Technische Universität München

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Abstract

Starting from a basic Unit Commitment problem as published in [3], we develop a fully functional, running implementation for Xpress Mosel, useable in power market modeling. The constraints of the model are discussed in details and solutions to left open implementation problems are presented, guidelines on how to handle data input (from both, databases and Excel), as well as data output (to Excel) are given.
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1 Introduction

The deregulation of the energy market in the last two decades attracted the researchers’ attention, as electricity producers constantly needed to optimize their production to stay competitive. The raising volatility in energy production, mostly due to the renewable resources, even increases the potential for optimization and therefore the interest in power market models.

We consider a Unit Commitment (UC) model based on the mixed integer linear program (MILP) presented in [3], which assumes perfect competition, and as such approximates the typical oligopolistic power market in deregulated countries. Its main advantage over possibly more sophisticated game theoretical or stochastic approaches is its capability to handle real-world problem sizes with an accurate physical model of the power units, combined with its relatively good comprehensibility.

To make the model in [3] suitable for real-life application, we need to modify it in a few key aspects. In particular, we focus on

- the reduction of the prohibitively large number of constraints needed to model the start-up costs,
- the incorporation of storage units,
- the analysis of causes of infeasibility and how to avoid them, and
- the incorporation of daily fuel prices, as fuel costs make up the better part of the production costs.

After a short theoretical presentation of the model, we put our main focus on a discussion of the constraints and a thorough guide on their implementation.

Finally, we show how to read the needed input data (which may be considerably huge, if long time period real world data is used) from either an SQL database or an Excel spreadsheet. The output of the results is written to an Excel spreadsheet, for further processing and analysis.

We use the Xpress-Mosel modeling language [1], which is part of the Xpress Optimization Suite [2].
2 The Unit Commitment Problem

In the Unit Commitment problem one has to satisfy a certain energy demand over a number of periods, using a set of given power units, while minimizing the total incurred costs. A typical Unit Commitment problem can roughly be split into three parts:

1. **The modeling of the physical constraints of each unit (subsection 2.2)**
   In this part the technical limitations of each individual unit are considered. We include
   - the feasible range for the production level,
   - the feasible range for the change rate of the production level,
   - the minimal up- and downtime and
   - the storage constraints.

2. **The modeling of the power grid (subsection 2.3)**
   A model of the underlying power grid may be considered, too, which we exclude in this paper. Hence, the overall production is just connected to the demand, a typical assumption in many models, usually described by the keyword “big copper plate”.

3. **The modeling of the costs (subsection 2.4)**
   Besides the obvious fuel costs, we include start up and shutdown costs and some penalties.

In the remaining of this section we discuss the components of our model in the above listed order (subsections 2.2, 2.3), give a short note on how to handle infeasibilities 2.5 and include summaries of the parameters 2.6 and the model 2.7. The latter may be used as a convenient reference in the following sections, since it is cross-referenced with the discussion and implementation over there.

2.1 Basic notation

We denote the set of time periods as $K = \{1, \ldots, T\}$, and the set of units as $J$. The remaining parameters will be introduced “as we go”, and summarized in subsection 2.6. We use the shortcut notations $[a..b]$ to denote the discrete set $[a, b] \cap \mathbb{Z}$, and $[b]$ to denote $[1..b]$.

2.2 Physical Constraints of the Units

2.2.1 Unit Variables (constraints 2.2 and 2.3 in subsection 2.7)

In any period $k \in K$, the units $j \in J$ are modeled by their operational state (on/off) $v_j^k$, their current production $p_j^k$, and their maximal possible production $p_j^k$:

$$\forall j \in J, k \in K : \quad v_j^k \in \{0, 1\}, \quad p_j^k, p_j^k \in \mathbb{R}^+,$$

where $v_j^k = 1$ iff unit $j$ is operational in period $k$ and may produce power.
The maximal possible production $p^k_j$ is used to measure the spinning reserve: In case of a power outage, the power grid must be stabilized by ramping up the currently operational units. The available additional production capacity $p^k_j - p^k_j$ is called spinning reserve of unit $j$ and a general grid constraint regulates the minimal need of spinning reserve over all units (see subsection 2.3 below).

### 2.2.2 Minimal Up- and Downtime (constraints 2.7 to 2.10 in subsection 2.7)

Units cannot be started up and shut down arbitrarily. For example, for a coal unit, after being shut down, an appropriate cool-off time has to be kept.

To model this, minimal up- and downtime parameters $UT_j$ and $DT_j$ for each unit are given, denoting the minimal time a unit has to stay operational after a start up, and the minimal cool-off time after a shutdown, respectively. Since we do not know when a unit has last been started up or shut down prior to the modeled time range, we further expect to be given the two parameters $IUT_j$ and $IDT_j$, which tell us for how long a unit needs to be operational or shut down initially. Given these parameters, we model the minimal uptime as

$$\forall j \in J, k \in [IUT_j]: \quad v^k_j = 1$$

for the first periods, and as

$$\forall j \in J, k \in [(IUT_j + 2)\ldots T], i \in [UT_j - 1] \cap [T - k]: \quad v^{k+i}_j \geq v^k_j - v^{k-1}_j$$

for the remaining periods. Consider the right-hand side $v^k_j - v^{k-1}_j$ of the latter constraint: it always lies in $\{-1, 0, 1\}$, and is equal to 1 if and only if unit $j$ starts up in period $k$. Thus, in the start-up case, the variables $v^{k+i}_j$ on the left-hand side are forced to 1, while the constraint is always fulfilled otherwise. Similarly, the minimal downtime is modeled as

$$\forall j \in J, k \in [IDT_j]: \quad v^k_j = 0$$

$$\forall j \in J, k \in [(IDT_j + 2)\ldots T], i \in [DT_j - 1] \cap [T - k]: \quad v^{k+i}_j \leq 1 - (v^k_j - v^{k-1}_j)$$

Note that we expect $IDT_j, IUT_j \in [0..T]$ and $DT_j, UT_j \in [T]$.

### 2.2.3 Minimal and Maximal Production (constraint 2.11 in subsection 2.7)

If a unit $j$ is not operational, it may not produce; otherwise both the actual and maximal possible production level have to lie in a specific interval, defined by the two parameters $P^j$ (minimal production) and $\overline{P}^j$ (maximal production). All this can be expressed by

$$\forall j \in J, k \in K: \quad P^j v^k_j \leq p^k_j \leq \overline{P}^j v^k_j$$

Note that storage units may have a negative minimal production $P^j$. However, $p^k_j \geq 0$ only captures power production, while power consumption is modeled by $c^k_j$ (see section 2.2.5 below).
2 THE UNIT COMMITMENT PROBLEM

2.2.4 Ramping (constraints \([2.12\text{ to } 2.14\text{ in subsection }2.7]\))

Of course the production level can not change arbitrarily either. The parameters \(RU_j\) / \(RD_j\) denote the maximal speed when increasing / decreasing the production level of an operating unit. For example, a unit with \(RU_j = 50\text{MW/h}\) would be able to increase its production level from 120MW to 170MW in one hour, but not to 171MW. This may simply be modeled as

\[
\forall j \in J, k \in [2..T] : \\
p_j^k \leq p_j^{k-1} + L \cdot RU_j \\
p_j^k \geq p_j^{k-1} - L \cdot RD_j v_j^k
\]

where \(L\) is a parameter denoting the period length. However, at start up and shutdown units are typically able to change their production levels faster. This higher ramping speed is denoted by the two parameters \(SU_j\) (maximal production level at start up) and \(SD_j\) (maximal production level before shutdown).

Thus for up ramping we get the constraint

\[
\forall j \in J, k \in [2..T] : \\
p_j^k \leq p_j^{k-1} + L \cdot RU_j v_j^k - SU_j (1 - v_j^k) \\
+ \min\{SU_j, \max\{\bar{P}_j, 0\} + L \cdot RU_j \cdot (1 - v_j^k)\}
\]

The first three terms on the right-hand side assert that the production may only increase by \(L \cdot RU_j\) if the unit is already running, or at start up. The last term does not change the right hand side in the case \(v_j^k = 0\). Consider the constraint without the last term:

- Case \(v_j^k = 0, v_j^{k-1} = 0\) : \(\bar{P}_j \leq p_j^{k-1} + L \cdot RU_j v_j^{k-1} + SU_j (1 - v_j^{k-1}) = SU_j\)
- Case \(v_j^k = 0, v_j^{k-1} = 1\) : \(\underline{P}_j \leq p_j^{k-1} + L \cdot RU_j v_j^{k-1} + SU_j (1 - v_j^{k-1}) = p_j^{k-1} + L \cdot RU_j\)

Thus, in case of \(v_j^k = 0\) we could tighten the constraint by subtracting the term \(\min\{SU_j, p_j^{k-1} + L \cdot RU_j\}\) on the right hand side. However, since we need to multiply the term by \((1 - v_j^k)\) to leave the case \(v_j^k = 1\) unchanged, to avoid non-linearities, it should not contain a variable. Hence we have to replace \(p_j^{k-1}\) by its best lower bound \(\max\{\underline{P}_j, 0\}\). This leads exactly to the last term in the rampup constraint. This term is not necessary for a correct model, but tightens the linear relaxation, possibly improving the solution time.

The rampdown is modeled analogously:

\[
\forall j \in J, k \in [2..T] : \\
p_j^k \geq p_j^{k-1} - L \cdot RD_j v_j^k - SD_j (1 - v_j^k) \\
+ \min\{SD_j, \max\{\underline{P}_j, 0\} + L \cdot RD_j \cdot (1 - v_j^{k-1})\}
\]
Finally, we need to put a limit on the possible maximal production in case of an imminent shutdown. A shutdown scheduled for period $k + 1$ can not be postponed, even in case of a power outage. Thus, in case of a shutdown in $k + 1$, the unit cannot produce more than $SD_j$ in period $k$:

$$\forall j \in J, k \in [T - 1]: \quad p^k_j \leq SD_j(v^k_j - v^{k+1}_j) + P_j v^{k+1}_j$$

The term $SD_j(v^k_j - v^{k+1}_j)$ expresses the desired limit, while the term $P_j v^{k+1}_j$ keeps the constraint valid in each of the other three settings of $v^k_j$ and $v^{k+1}_j$.

### 2.2.5 Storage (constraints 2.4, 2.15 to 2.19 in subsection 2.7)

Storage units are used to even out the demand. The classical examples for storage units are water pumping stations and, to a lesser extent, batteries. We indicate a storage unit by a negative minimal production $P_j$, meaning that the maximal storage inflow is $-P_j$.

Not being thermal units, the nowadays relevant storage units have very high ramping speeds, which allow us to neglect them, i.e. we assume $RU_j = RD_j = SD_j = SU_j = -P_j + (-P_j)$ for each storage unit $j$.

In our model storage units are characterized by five parameters:

- their storage capacity $SC_j$,
- their storage efficiency $SE_j$,
- their constant inflow $SIF_j$,
- and their initial and final storage fill, $SI_j$ and $SF_j$,

and two new variables:

- the current storage fill $\forall j \in J, k \in K: s^k_j \in \mathbb{R}^+$ and
- the power consumption $\forall j \in J, k \in K: c^k_j \in \mathbb{R}^+$.

The maximal storage inflow and the storage capacity are now easily formulated as

$$\forall j \in J, k \in K: \quad s^k_j \leq SC_j$$

$$\forall j \in J, k \in K: \quad c^k_j \leq \max\{0, -P_j\}$$

The storage fill change from period $k - 1$ to $k$ has to account for the prior storage, the stored and consumed energy, and a possible constant inflow (for example a stream feeding a reservoir):

$$\forall j \in J, k \in [2..T]: \quad s^k_j = s^{k-1}_j + L \cdot (SE_j c^{k-1}_j - p^{k-1}_j + SIF_j)$$
The initial and final storage fill parameters $SI_j$ and $SF_j$ are set outside the model (possibly for raising the total storage when demands and prizes are expected to increase after the considered total time period). They apply to the storage fill at $k = 1$ and $k = T + 1$:

$\forall j \in J: s^1_j = SI_j$

$\forall j \in J: SF_j = s^T_j + L \cdot (SE_j c^T_j - p^T_j + SIF_j)$

### 2.3 Power Grid Constraints (constraints 2.20 and 2.21 in subsection 2.7)

All units together have to satisfy the energy demand in each period, given as $D^k$:

$\forall k \in K: \sum_{j \in J} (p^k_j - c^k_j) = D^k$

Moreover, to be failure-tolerant, the units should be able to compensate a power outage, caused for example by a failing unit. This is expressed by the need to keep a given reserve $R^k$ capacity available:

$\forall k \in K: \sum_{j \in J} (p^k_j - p^{k+1}_j + c^{k+1}_j) \geq R^k$

Typically $R^k$ is constant over time. However, a dependency e.g. on $D^k$ maybe reasonable, too.

### 2.4 Cost Constraints

#### 2.4.1 Objective Function (constraints 2.1 and 2.5 in subsection 2.7)

The objective is to minimize the overall costs consisting of production costs $cp^k_j \geq 0$ (see subsection 2.4.2 below) as well as start-up and shutdown costs, $cu^k_j, cd^k_j \geq 0$ (see 2.4.3):

$$\min \sum_{j \in J} \sum_{k \in K} cp^k_j + cu^k_j + cd^k_j.$$

#### 2.4.2 Production Costs (constraint 2.22 in subsection 2.7)

In $[3]$ a convex production cost function is assumed and approximated by piecewise affine linear functions. However, since the efficiency of a power unit usually increases with its production level, we assume a concave production cost function. Fortunately, the increase in efficiency is typically quite small, allowing a nearly affine linear approximation

$\forall j \in J, k \in K : cp^k_j = A \cdot L \cdot p^k_j + B \cdot L \cdot v^k_j$,

with parameters $A$ and $B$. $A$ can be interpreted as the variable cost, incurred for every additional MWh of production, and $B$ as the fixed cost, incurred for running the unit for one hour.
In our model we further split the parameters \( A \) and \( B \) in two parts, one part which is attributed to the needed fuel, and is thus dependent on the current fuel price, while the other part is attributed to operation and maintenance, and thus fixed. We expect the fuel price to be given as the parameter \( FC_{F_j}^{k} \), where \( F_j \) denotes the fuel type used by unit \( j \).

Therefore, we replace \( A \) and \( B \) by their parts, the time and fuel-type dependendent \( FA_j \) and independent \( FB_j \) fuel needs and the time and fuel-type dependendent \( PA_j \) and independent \( PB_j \) costs. This leads to

\[
\forall \ j \in J, \ k \in K : \ cp_k^j = (FA_j \cdot FC_{F_j}^{k} + PA_j) L \cdot p_k^j + (FB_j \cdot FC_{F_j}^{k} + PB_j) L \cdot v_k^j.
\]

Note that due to the affine, non-linear cost function, we do not have a constant production efficiency as modeled in many other papers; instead, the modeled production efficiency increases with the production and is concave, which is typical for thermal units.

### 2.4.3 Startup and Shutdown Costs (constraints 2.23 and 2.24 in subsection 2.7)

Every unit incurs costs when starting up or shutting down, the former increasing with the offline time (e.g. for a thermal unit, the start-up costs are partially attributed to the need for reheating it), the latter constant.

We expect the start-up cost of unit \( j \) after an offline time of \( t \) periods to be given as \( CU_t^j \) and the shutdown costs to be given as a single constant \( CD_j \). The start-up and shutdown costs can then be modeled as

\[
\forall \ j \in J, \ k \in [2..T] : \ cd_k^j \geq CD_j(v_k^{k-1} - v_k^j)
\]

\[
\forall \ j \in J, \ k \in K, \ t \in [k-1] : \ cu_k^j \geq CU_t^j \left(v_k^j - \sum_{n=1}^{t} v_k^{k-n}\right).
\]

Here, the term \( v_k^j - \sum_{n=1}^{t} v_k^{k-n} \) is 1 exactly if unit \( j \) starts up in period \( k \) and was offline in periods \( [(k-1)..(k-t)] \), otherwise it is less or equal to 0. Thus, the latter constraint is equivalent to

\[
\forall \ j \in J, \ k \in K, \ t \in \{t' \in [k-1] : \forall \ i \in [t'] : v_j^{k-i} = 0\} : \ cu_k^j \geq CU_t^j.
\]

Furthermore, since \( CU_t^j \) is increasing with \( t \), this again is equivalent to

\[
\forall \ j \in J, \ k \in K, \ t = \max\{t' \in [k-1] : \forall \ i \in [t'] : v_j^{k-i} = 0\} : \ cu_k^j \geq CU_t^j.
\]

Since we minimize the costs, \( cu_k^j = CU_t^j \) for start-ups in an optimal solution, as intended.

\footnote{\( F_j \) may not contain special characters and spaces, see subsection 4.3}
2.4.4 Thinning Out the Startup Cost Function

The number of constraints needed in [3] to describe the start-up cost function is quite substantial. While all the other constraints amount to about $12 |J||K|$, the model needs about $|J||K|^2$ constraints to model the start-up cost function as discussed in the last sections. Of course it is not necessary to model the start-up cost function for $|K|$ cooldown periods, but only for the first periods, when the cost changes significantly. Still, for typical thermal units the relevant timespan is about 2 to 3 days, leading to between $48 |J||K|$ and $72 |J||K|$ constraints if we assume hourly periods, for example.

If we want to reduce the number of constraints further, we have to use a single step for a whole group of cooldown periods, which amounts to assigning the same start-up cost to each of these periods. As an example, let us have a look at a typical start-up cost function with a fixed cost of about 70% and a variable cost of about 30%. It is possible to reduce the number of steps from 71 to 9, while maintaining a relative error of less than 5%:

![Figure 1: Approximation of the start-up cost function with a tolerance of 5%](image)

Thus, two questions are to be answered: How should we group the periods together to obtain a relative error as small as possible, and which start-up cost should we assign to each of these group?

The second question is answered easily: If we are given a group of periods with cooldown times $t_a$ to $t_b$, it is straight-forward to show that the minimal relative error

$$\text{bestError}(CU_j^{t_a}, CU_j^{t_b}) = \frac{CU_j^{t_b} - CU_j^{t_a}}{CU_j^{t_b} + CU_j^{t_a}}$$

is obtained with the step value

$$\text{bestStep}(CU_j^{t_a}, CU_j^{t_b}) = \frac{2 \cdot CU_j^{t_a} \cdot CU_j^{t_b}}{CU_j^{t_b} + CU_j^{t_a}},$$

and 0, if $CU_j^{t_a} = CU_j^{t_b} = 0$. 
To answer the first question: let STARTUP_TOL be a parameter giving the maximal error tolerance. Then the periods may be grouped iteratively by starting the first group at \( t = 1 \), expanding the group as long as its relative error is less than the tolerance, and continuing with the next group on the remaining \( t \)'s. In pseudo-code, for given \( j \):

| Algorithm 1: startupCostThinning |
|----------------------------------|
| 1 \( t_a, t_b \leftarrow 1 \)  // Start and end index of current group |
| 2 \[ \text{while } t_a \leq |K| \] do |
| \[ // Expand the group as long as the next relative error is less than STARTUP_TOL \] |
| 3 \[ \text{while } t_b + 1 \leq |K| \land \text{bestError}(CU^t_j, CU^{t+1}_j) < \text{STARTUP_TOL} \] do |
| 4 \[ t_b \leftarrow t_b + 1 \] |
| 5 \[ CU^t_j \leftarrow \text{bestStep}(CU^t_j, CU^{t+1}_j) \]  // Calculate optimal step value |
| 6 \[ \forall t \in [(t_a+1), t_b]: CU^t_j \leftarrow 0 \]  // Delete other step values |
| 7 \[ t_a, t_b \leftarrow t_b + 1 \]  // Continue with next group |

It can be proven that the number of groups produced by this algorithm is the minimal number of groups needed to fulfill the tolerance requirements.

### 2.5 Dealing with Infeasibilities

The main cause of infeasibilities is an excessive demand or reserve capacity,

\[ \exists k : \sum_{j \in J} p_{j,k} < D^j + R^j. \]

In most cases the demand not satisfiable by the units within the model is covered by energy sources which are not part of the model (e.g. by power imports from external markets or backup units). Therefore it is advisable to soften the demand and reserve constraints by measuring the underproduction and underreserve and applying a penalty to it. The softened demand and reserve constraints thus are:

\[ \forall k \in K : \sum_{j \in J} (p^k_j - c^k_j) + P^k = D^k \]

\[ \forall k \in K : \sum_{j \in J} (p^k_j - p^k_j + c^k_j) + R^k \geq R^k \]

Given the two parameters UPP (penalty factor for underproduction) and URP (penalty factor for underreserve), we replace the objective function by

\[
\min \sum_{j \in J} \sum_{k \in K} cp^k_j + cu^k_j + cd^k_j + \sum_{k \in K} \text{UPP} \cdot P^k + \text{URP} \cdot R^k.
\]
The second most common cause is a demand profile too volatile to be satisfied by the units, or in other words, units too slow to follow the demand profile. This infeasibility also originates from energy sources not present in the model. Although the softening against underproduction is enough to prevent infeasibilities, one may want to penalize overproduction less (in real world, overproduction may for example be dumped on external markets). So we obtain

$$\forall k \in K : \sum_{j \in J} (p^k_j - c^k_j) + P^k_- - P^k_+ = D^k$$

$$\forall k \in K : \sum_{j \in J} (p^k_j - p^k_j + c^k_j) + R^k_- \geq R^k$$

and, given the parameter OPP (penalty factor for overproduction),

$$\min \sum \sum_{j \in J, k \in K} cp^k_j + cu^k_j + cd^k_j + \sum_{k \in K} UPP \cdot P^k_- + URP \cdot R^k_- + OPP \cdot P^k_+$$

There are a few other inconsistencies in the input data which could lead to infeasibilities or unwanted behavior, namely

- simultaneous positive initial down- and uptime:
  $$\exists j \in J : IUT_j > 0 \text{ and } IDT_j > 0,$$

- impossible production and ramping limits:
  $$\exists j \in J : P_j > \overline{P}_j \text{ or } P_j > SU_j \text{ or } P_j > SD_j,$$

- decreasing start-up costs:
  $$\exists j \in J, k \in K : CU^k_j > CU^{k+1}_j,$$

- non fulfillable storage constraints:
  $$\exists j \in J : SIF_j > P_j$$
  $$\exists j \in J : SE_j \notin [0, 1]$$
  $$\exists j \in J : SI_j > SC_j \text{ or } SF_j > SC_j$$
  $$\exists j \in J : SI_j + L \cdot T \cdot (SE_j P_j + SIF_j) < SF_j \text{ or } SI_j + L \cdot T \cdot (\overline{P}_j + SIF_j) > SF_j$$

All those are errors in the input data, which have to be avoided for meaningful optimization.
# 2.6 Parameters summary

## 2.6.1 General parameters

| Name        | Domain | Scale unit | Description                             |
|-------------|--------|------------|-----------------------------------------|
| L           | $\mathbb{R}^+$ | h          | Period length                           |
| UPP         | $\mathbb{R}^+$ | cost/MWh   | Penalty for underproduction             |
| URP         | $\mathbb{R}^+$ | cost/MWh   | Penalty for underreserve                |
| OPP         | $\mathbb{R}^+$ | cost/MWh   | Penalty for overproduction              |
| STARTUP_TOL | [0,1]  |            | Startup cost error tolerance             |

## 2.6.2 Parameters for each period

| Name | Domain | Scale unit | Description                             |
|------|--------|------------|-----------------------------------------|
| $D^k$ | $\mathbb{R}^+$ | MW | Power demand                           |
| $R^k$ | $\mathbb{R}^+$ | MW | Needed reserve capacity                 |
| $FC_f$ | $\mathbb{R}^+$ | cost/MWh | Cost of a MWhs worth of a fuel type   |

## 2.6.3 Parameters for each power unit

| Name | Domain | Scale unit | Description                             |
|------|--------|------------|-----------------------------------------|
| $UT_j$ | $[T]$ | periods | Minimal uptime after starting up          |
| $DT_j$ | $[T]$ | periods | Minimal downtime after shutting down      |
| $IUT_j$ | $[0..T]$ | periods | Initial uptime (unit is online in periods 1, ..., IUT_j) |
| $IDT_j$ | $[0..T]$ | periods | Initial downtime (unit is offline in periods 1, ..., IDT_j) |
| $P_j$ | $\mathbb{R}$ | MW | Minimal production in online state       |
| $\bar{P}_j$ | $\mathbb{R}^+$ | MW | Maximal production in online state       |
| $RU_j$ | $\mathbb{R}^+$ | MW/h | Maximal production increase per hour, in online state |
| $RD_j$ | $\mathbb{R}^+$ | MW/h | Maximal production decrease per hour, in online state |
| $SU_j$ | $\mathbb{R}^+$ | MW | Maximal production increase at start-up   |
| $SD_j$ | $\mathbb{R}^+$ | MW | Maximal production decrease on shutdown   |
| $SC_j$ | $\mathbb{R}^+$ | MWh | Storage capacity (Zero for most units)    |
| $SE_j$ | $[0, 1]$ | MWh/MWh | Storage efficiency (Zero for most units)  |
| $SIF_j$ | $\mathbb{R}^+$ | MW | Storage inflow (Zero for most units)      |
| $SI_j$ | $\mathbb{R}^+$ | MWh | Initial storage fill                     |
| $SF_j$ | $\mathbb{R}^+$ | MWh | Final storage fill                       |
| $F_j$ | fuel   |            | Used fuel type $^1$                      |
| $FA_j$ | $\mathbb{R}^+$ | MWh/MWh | Fuel need increase per MW of production  |
| $FB_j$ | $\mathbb{R}^+$ | MWh/h | Fixed fuel need in online state          |
| $PA_j$ | $\mathbb{R}^+$ | cost/MWh | Cost increase per MWh of production     |
| $PB_j$ | $\mathbb{R}^+$ | cost/h | Fixed cost in online state               |
| $CU_j$ | $\mathbb{R}^+$ | cost | Startup cost after t offline periods      |
| $CD_j$ | $\mathbb{R}^+$ | cost | Shutdown cost                            |

$^1$F_j may not contain special characters and spaces, see subsection 4.3
2.7 Model summary

Objective function (discussion in 2.4.1 and 2.5, implementation in 3.7):

$$\min \sum_{j \in J} \sum_{k \in K} cp_j^k + cu_j^k + cd_j^k + \sum_{k \in K} UPP \cdot P^k + URP \cdot R^- + OPP \cdot P^k$$  \hspace{1cm} (2.1)

Physical unit variables (discussion in 2.2.1 and 2.2.5, implementation in 3.2):

$$\forall j \in J, k \in K : \quad v^k_j \in \{0, 1\} \hspace{1cm} (2.2)$$

$$\forall j \in J, k \in K : \quad p^k_j, P_j^k \geq 0 \hspace{1cm} (2.3)$$

$$\forall j \in J, k \in K : \quad s^k_j, v^k_j \geq 0 \hspace{1cm} (2.4)$$

Cost variables (discussion in 2.4, implementation in 3.2)

$$\forall j \in J, k \in K : \quad cp_j^k, cd_j^k, cu_j^k \geq 0 \hspace{1cm} (2.5)$$

Period variables (discussion in 2.3 and 2.5, implementation in 3.2):

$$\forall k \in K : \quad P_k^-, P_k^+, R^k_- \geq 0 \hspace{1cm} (2.6)$$

Minimal up- and downtime (discussion in 2.2.2, implementation in 3.3):

$$\forall j \in J, k \in [IUT_j] : \quad v^k_j = 1 \hspace{1cm} (2.7)$$

$$\forall j \in J, k \in [IDT_j] : \quad v^k_j = 0 \hspace{1cm} (2.8)$$

$$\forall j \in J, k \in ([IUT_j + 2), T], i \in [UT_j - 1] \cap [T - k] : \quad v^{k+i}_j \geq v^k_j - v^{k-1}_j \hspace{1cm} (2.9)$$

$$\forall j \in J, k \in ([IDT_j + 2), T], i \in [DT_j - 1] \cap [T - k] : \quad v^{k+i}_j \leq 1 - (v^k_j - v^{k-1}_j) \hspace{1cm} (2.10)$$

Minimal and maximal production (discussion in 2.2.3, implementation in 3.3):

$$\forall j \in J, k \in K : \quad P_j v^k_j \leq p^k_j \leq P_j v^k_j \hspace{1cm} (2.11)$$

Ramping constraints (discussion in 2.2.4, implementation in 3.3):

$$\forall j \in J, k \in [2, T] : \quad p^k_j \leq p^{k-1}_j + L \cdot RU_j v^{k-1}_j + SU_j (1 - v^{k-1}_j) \hspace{1cm} (2.12)$$

$$- \min\{SU_j, P_j + L \cdot RU_j\} \cdot (1 - v^k_j)$$

$$\forall j \in J, k \in [2, T] : \quad p^k_j \geq p^{k-1}_j - L \cdot RD_j v^k_j - SD_j (1 - v^k_j) \hspace{1cm} (2.13)$$

$$+ \min\{SD_j, P_j + L \cdot RD_j\} \cdot (1 - v^{k-1}_j)$$

$$\forall j \in J, k \in [T - 1] : \quad p^k_j \leq P_j v^{k+1}_j + SD_j (v^k_j - v^{k+1}_j) \hspace{1cm} (2.14)$$
Storage constraints (discussion in 2.2.5, implementation in 3.3):

\( \forall j \in J, k \in K : \quad s^k_j \leq SC_j \quad (2.15) \)

\( \forall j \in J, k \in K : \quad c^k_j \leq \max\{0, -P_j\} \quad (2.16) \)

\( \forall j \in J, k \in [2..T] : \quad s^k_j = s^{k-1}_j + L \cdot (SE_j c^{k-1}_j - P^{k-1}_j + SIF_j) \quad (2.17) \)

\( \forall j \in J : \quad s^1_j = SI_j \quad (2.18) \)

\( \forall j \in J : \quad SF_j = s^T_j + L \cdot (SE_j c^T_j - P^T_j + SIF_j) \quad (2.19) \)

Demand and reserve (discussion in 2.3 and 2.5, implementation in 3.4):

\( \forall k \in K : \quad \sum_{j \in J} (p^k_j - c^k_j) + P^k_k - P^k_{k-1} = D^k \quad (2.20) \)

\( \forall k \in K : \quad \sum_{j \in J} (p^k_j - p^k_j + c^k_j) + R^k_k \geq R^k \quad (2.21) \)

Production costs (discussion in 2.4.2, implementation in 3.5):

\( \forall j \in J, k \in K : \quad cp^k_j = \left( FA_j \cdot FC^{k}_j + PA_j \right) L \cdot p^k_j \quad (2.22) \)

\( + \left( FB_j \cdot FC^{k}_j + PB_j \right) L \cdot v^k_j \)

Startup and shutdown costs (discussion in 2.4.3 and 2.4.4, implementation in 3.6):

\( \forall j \in J, k \in [2..T] : \quad cd^k_j \geq CD_j (v^{k-1}_j - v^k_j) \quad (2.23) \)

\( \forall j \in J, k \in K, t \in [k - 1] : \quad cu^k_j \geq CU_j \left( v^k_j - \sum_{n=1}^{t} v^{k-n}_j \right) \quad (2.24) \)
3 Implementing the Model

In this section, we show how to implement the model discussed in the last section in Mosel [1], i.e. how the model composed by parameters, variables and constraints is stated and solved. The how-to on the surrounding tasks, data input and output, is given in the following sections.

For the whole implementation, we assume the following options to be used:

```
options noimplicit, explterm, keepassert;
```

The option noimplicit disallows implicit variable creation, and thus forces the programmer to declare each variable. Mosel’s automatic line termination requires us to wrap multi-line formulas after an operator, therefore we deactivate it with explterm. We check the input data with assert(), which by default is only active in debug mode; keepassert enables it in all modes.

3.1 Parameters (subsection 2.6)

The fitting datatype is clear from the parameters table: real parameters are represented as real, integer parameters as integer and fuel types as string. The index sets of the parameters are either J, K, the fuel types or a combination of two of these sets. For performance reasons, the order of the index sets should be the same as in the forall loops defined over these sets. Here, this means that the set K always comes last.

Based on these design decisions, we declare the parameters as

```
declarations
  J: set of integer;                               ! Set of units
  K = 1..T;                                       ! Set of periods
  Fuels: set of string;                          ! Available fuel types (index set for CF)
  UPP, URP, OPP: real;                           ! Penalties [cost/MWh]
  D, R: array(K) of real;                        ! Demand and reserve [MW]
  FC: dynamic array(Fuels) of array(K) of real;  ! Cost of a fuel type [cost/MWh]
  UT, DT: array(J) of integer;                   ! Minimal up-/downtime [periods]
  IUT, IDT: array(J) of integer;                 ! Initial up-/downtime [periods]
  P_min, P_max: array(J) of integer;             ! Minimal and maximal production [MW]
  RU, RD: array(J) of integer;                   ! Maximal upwards/downwards ramping [MW/h]
  SU, SD: array(J) of integer;                   ! Startup/shutdown ramping [MW]
  SC: array(J) of real;                          ! Storage capacity [MWh]
  SE: array(J) of real;                          ! Storage efficiency [MWh/MWh]
  SIF: array(J) of real;                         ! Storage inflow [MW]
  SI, SF: array(J) of real;                      ! Initial and final storage fill [MWh]
  F: array(J) of string;                        ! Fuel type of a unit [Fuels]
  FA, PA: array(J) of real;                      ! Variable prod. costs [MW/MW],[cost/MW]
  FB, PB: array(J) of real;                      ! Fixed production costs [MW],[cost]
  CU: dynamic array(J, range) of real;           ! Startup costs [cost]
  CD: array(J) of real;                          ! Shutdown costs [cost]
end-declarations
```
3 IMPLEMENTING THE MODEL

The parameters $T$ (number of periods) and $L$ (period length) are declared in the data input section since they need to be known for selecting the data to be read.

The size of $\text{CU}'s$ second index set range is unknown, since we do not know in advance how many different start-up costs are available for each unit. We therefore declare this array as dynamic.

$\text{FC}$ has to be a dynamic array too, since the set of fuels is not known in advance. Usually, we would declare such an array as

$\text{FC: dynamic array(Fuels, K) of real;}$

However, using the more specific declaration

$\text{FC: dynamic array(Fuels) of array(K) of real;}$

has an important advantage: This way we may pass individual columns of the matrix, the subarrays $\text{FC}(f)$ to the SQL functions, see subsection 4.3. Conveniently, the elements of $\text{FC}$ can still be accessed as $\text{FC}(f, k)$.

3.2 Variables

Again, the index sets of the variables are determined from the used indices. In Mosel, the usual datatype for variables is $\text{mpvar}$ (mathematical programming decision variable). The only exception is the objective function, which as a linear function is declared as $\text{linctr}$ (linear constraint, also used for linear functions).

As the number of storage units is usually quite small, we declare the storage and consumption variable arrays as dynamic and create the individual variables only for them, thus saving variables.

\begin{verbatim}
declarations
    p: array(J, K) of mpvar;       ! Production [MW]
    p_max: array(J, K) of mpvar;   ! Maximal possible production [MW]
    v: array(J, K) of mpvar;       ! On-Off state [0/1]
    s: dynamic array(J, K) of mpvar; ! Storage [MWh]
    c: dynamic array(J, K) of mpvar; ! Consumption [MW]
    cp: array(J, K) of mpvar;      ! Production costs [cost]
    cu, cd: array(J, K) of mpvar;  ! Startup/shutdown costs [cost]
    overallCosts: linctr;          ! Overall costs [cost]
    p_under, p_over: array(K) of mpvar;  ! Under-/overproduction [MW]
    r_under: array(K) of mpvar;    ! Underreserve [MW]
end-declarations

! Create storage and consumption variables for storage units
forall(j in J, k in K | P_min(j) < 0) do
    create(s(j, k));
    create(c(j, k));
end-do
\end{verbatim}

By default, these variables are constrained to take positive real values. This fits all variables except the on-off-state $v$, which should be binary:

\begin{verbatim}
forall(j in J, k in K) v(j, k) is_binary;
\end{verbatim}
3 IMPLEMENTING THE MODEL

3.3 Physical Constraints of the Units (constraints 2.7 to 2.14)

The Mosel equivalent of the minimal up- and downtime constraints (2.7 to 2.10) is canonical:

forall(j in J, k in 1..IUT(j)) v(j, k) = 1;
forall(j in J, k in 1..IDT(j)) v(j, k) = 0;

for the initial up- and downtime, and

forall(j in J, k in IUT(j)+2 .. T, i in 1..minlist(UT(j)-1, T-k))
  v(j, k + i) >= v(j, k) - v(j, k-1);
forall(j in J, k in IDT(j)+2 .. T, i in 1..minlist(DT(j)-1, T-k))
  v(j, k + i) <= 1 - (v(j, k-1) - v(j, k));

for the interperiod minimal up- and downtime.

The jointed constraints for minimal and maximal production (2.11) have to be separated:

forall(j in J, k in K) do
  P_min(j) * v(j, k) <= p(j, k);
p(j, k) <= p_max(j, k);
p_max(j, k) <= P_max(j) * v(j, k);
end-do

The ramping limits (2.12 to 2.14) are straightforward to implement:

forall(j in J, k in 2..T) do
  p_max(j, k) <= p(j, k-1) + L * RU(j) * v(j, k-1) + SU(j) * (1 - v(j, k-1)) - minlist(SU(j), P_min(j) + L * RU(j)) * (1 - v(j, k));
p(j, k) >= p(j, k-1) - L * RD(j) * v(j, k) - SD(j) * (1 - v(j, k)) + minlist(SD(j), P_min(j) + L * RD(j)) * (1 - v(j, k-1));
end-do
forall(j in J, k in 1..T-1) do
  p_max(j, k) <= P_max(j) * v(j, k+1) + SD(j) * (v(j, k) - v(j, k+1));
end-do

Same as with the storage variables, the storage constraints (2.15 to 2.19) are implemented only for storage units.

forall(j in J | P_min(j) < 0) do
 forall(k in K) do
    s(j, k) <= SC(j);
c(j, k) <= maxlist(0, -P_min(j));
  end-do
 forall(k in 2..T) do
    s(j, k) = s(j, k-1) + L * (SE(j)*c(j, k-1) - p(j, k-1) + SIF(j));
  end-do
  SI(j) = s(j, 1);
  SF(j) = s(j, T) + L * (SE(j)*c(j, T) - p(j, T) + SIF(j));
end-do
3.4 Power Grid Constraints (constraints 2.20 and 2.21)

Since the consumption variables are only created for storage units, we use $\exists$ to sum only the existing variables.

\[
\text{forall}(k \in K) \text{ do}
\sum_{j \in J} p[j, k] - \sum_{j \in J | \exists(c[j, k])} c[j, k] + p_{\text{under}}(k) - p_{\text{over}}(k) = D(k);
\sum_{j \in J} (p_{\text{max}}[j, k] - p[j, k]) - \sum_{j \in J | \exists(c[j, k])} c[j, k] + r_{\text{under}}(k) \geq R(k);
\text{end-do}
\]

3.5 Production Cost Constraints (constraint 2.22)

The model could be reduced by replacing every use of $cp[k,j]$ by the right-hand side of constraints 2.22, and thus removing the variables $cp[k,j]$ completely. Fortunately, the Xpress Optimizer automatically applies this reduction at the presolve stage and enables us to use the canonical implementation for better readability:

\[
\text{forall}(j \in J, k \in K) \quad cp[j, k] = (FA[j] \times FC(F[j], k) + PA[j]) \times p[j, k] + (FB[j] \times FC(F[j], k) + PB[j]) \times v[j, k];
\]

3.6 Startup and Shutdown Costs (constraints 2.23 and 2.24)

The start-up and shutdown costs can be implemented in Mosel as

\[
\text{forall}(j \in J, k \in K) \text{ do}
\text{cd}[j, k] \geq CD[j] \times (v[j, k-1] - v[j, k]);
\text{forall}(j \in J, k \in K, t \in 1..k-1 | \exists(CU[j, t]))
\text{cu}[j, k] \geq CU[j, t] \times (v[j, k] - \sum_{n \in 1..t} v[j, k-n]);
\text{end-do}
\]

The $\exists$ operator is again used to enumerate only relevant indices. Especially the number of constraints depending on $t$ may be further reduced from considering only a subset (compare subsection 2.4.4), accepting a loss in accuracy of the start-up costs.

3.6.1 Thinning Out the Startup Cost Function

The pseudo-code of the start-up cost thinning can be translated one-to-one into Mosel code:

\[
\text{function bestError(CU}_{j, ta}: \text{real}, \text{CU}_{j, tb}: \text{real}): \text{real}
\text{if(CU}_{j, ta} = 0 \text{ and } \text{CU}_{j, tb} = 0) \text{ then}
\text{returned} := 0;
\text{else}
\text{returned} := \text{abs(CU}_{j, ta} - \text{CU}_{j, tb})/(\text{CU}_{j, ta} + \text{CU}_{j, tb});
\text{end-if}
\text{end-function}
\]
function bestStep(CU\_j\_ta: real, CU\_j\_tb: real): real
    if(CU\_j\_ta = 0 and CU\_j\_tb = 0) then
        returned := 0;
    else
        returned := 2 * CU\_j\_ta * CU\_j\_tb / (CU\_j\_ta + CU\_j\_tb);
    end-if
end-function

procedure startupCostThinning(j: integer)
    declarations
        t\_a, t\_b: integer;
    end-declarations
    ! Start the first group
    t\_a := 1;
    t\_b := 1;
    repeat
        ! Expand the group as long as the next
        ! relative error is less than STARTUP\_TOL
        while(exists(CU\{j, t\_b + 1\}) and
            bestError(CU\{j, t\_a\}, CU\{j, t\_b+1\}) < STARTUP\_TOL) do
            t\_b := t\_b+1;
        end-do
        ! Calculate optimal step value
        CU\{j, t\_a\} := bestStep(CU\{j, t\_a\}, CU\{j, t\_b\});
        ! Delete other step values
        forall(t in t\_a+1..t\_b) do
            delcell(CU\{j, t\});
        end-do
        ! Continue with next group
        t\_a := t\_b + 1;
        t\_b := t\_b + 1;
    until(not exists(CU\{j, t\_a\}));
end-procedure

Of course, the thinning has to be applied to every unit \( j \in J \):

forall(j in J) do
    startupCostThinning(j);
end-do

### 3.7 Objective Function (constraint 2.1)

First, we have to define the cost function

\[
\text{overallCosts} := \sum_{j \in J, k \in K} (cp(j, k) + cd(j, k) + cu(j, k)) + \sum_{k \in K} (UP\_p\_under(k) + UR\_p\_r\_under(k) + OPP*p\_over(k));
\]

and then call minimize:

minimize(overallCosts);
3 IMPLEMENTING THE MODEL

3.8 Dealing with Infeasibilities (subsection 2.5)

The penalties have already been incorporated in the power grid constraints and the objective function. To detect errors in the input data, we use Mosel’s `assert` procedure, which stops the program if a condition is not met.

```mosel
assert(and(j in J) IUT(j) in 0..T, "Initial uptime out of range!");
assert(and(j in J) IDT(j) in 0..T, "Initial uptime out of range!");
assert(and(j in J) IUT(j)*IDT(j) = 0, "Simultaneous initial down- and uptime!");
assert(and(j in J) UT(j) in 1..T, "Initial uptime out of range!");
assert(and(j in J) DT(j) in 1..T, "Initial downtime out of range!");
assert(and(j in J) P_min(j) <= P_max(j), "Impossible production limits!");
assert(and(j in J) P_min(j) <= SU(j), "Some unit is not able to start up!");
assert(and(j in J) P_min(j) <= SD(j), "Some unit is not able to shutdown!");
declarations
  lastCU: real;
end-declarations
forall(j in J) do
  lastCU := 0;
  forall(k in K | exists(CU(j, k))) do
    assert(lastCU <= CU(j, k),
          "The start-up costs are not monotonically increasing!");
    lastCU := CU(j, k);
  end-do
end-do
assert(and(j in J) SIF(j) <= P_max(j), "Storage inflow leads to overcapacity!");
assert(and(j in J) (0 <= SE(j) and SE(j) <= 1), "Invalid storage efficiency!");
assert(and(j in J) SI(j) <= SC(j), "Invalid initial storage fill!");
assert(and(j in J) SF(j) <= SC(j), "Invalid final storage fill!");
assert(and(j in J) SI(j) + L*T*(SE(j)*maxlist(0, -P_min(j)) + SIF(j)) >= SF(j),
          "Some storage constraints are not fulfillable!");
assert(and(j in J ) SI(j) + L*T*(-P_max(j) + SIF(j)) <= SF(j),
          "Some storage constraints are not fulfillable!");
```

As noted at the beginning of this section (see 3), we need to use the option `keepassert` to activate `assert` outside of debug mode.
4 Data Input

4.1 General Parameters

We use the two parameters \texttt{UNITS\_SOURCE} and \texttt{PERIODS\_SOURCE} to store the sources for the unit parameters and the period data. Depending on whether one wants to access a database or a spreadsheet, these parameters contain the name of a database table or a spreadsheet.

Once we know where to get the period data, we have to know a start time \texttt{START} and the number of periods \texttt{T}. Since the datatype \texttt{datetime} can not be used for parameters, \texttt{START} will be a \texttt{string}, formatted with the standard SQL format \texttt{YYYY-MM-DD HH:MM:SS} (example: June 30th, 2007 at 7:12 PM is written as \texttt{2007-06-30 19:12:00}). The parameter \texttt{PERIOD\_LENGTH} should be given in seconds.

Summarising, the used parameters are:

\begin{verbatim}
parameters
UNITS\_SOURCE = "Units.xls";; ! Source of the unit parameters
PERIODS\_SOURCE = "Periods.xls";; ! Source of the period data
DSN = "Server=?;Database=?;UID=?;PWD=?;";; ! Example DSN
START = "2009-05-11 00:00:00";; ! Start of the modeled timespan
T = 168;; ! Number of modeled periods
L = 1;; ! Period length [h]
STARTUP\_TOL = 0.05;; ! Tolerance for modeling of start-up cost
end-parameters
\end{verbatim}

Important: parameters have to be declared with a default value to define their data type.

4.2 Unit Parameters

The parameters describing the power units are too many to be read from the command line, so we have to read them from a different data source. Mosel supports database access, including Excel spreadsheets, over ODBC.

Now, databases are superior to spreadsheets in performance, reliability and scalability, making them a good choice for a Mosel model used in day-to-day operations. Excel spreadsheets on the other hand are easy to create and modify, making them a good choice for the development and experimental stage.

Fortunately, the Mosel \texttt{initializations} block hides most of the differences between real databases and Excel spreadsheets, allowing us to switch between them with little effort. For details on how to use the \texttt{initializations} block, please refer to the FICO Whitepaper “Using ODBC and other database interfaces with Mosel”, which should reside in the Xpress installation directory at \ldots/\ldots XpressMP/docs/mosel/mosel\textunderscore odbc/moselodbc.pdf [4].

We expect two tables in our database,

\begin{itemize}
  \item one table with the unit parameters with index set \texttt{J} (all except start-up costs),
  \item one table with the start-up costs with index sets \texttt{J} and \texttt{K}.
\end{itemize}
The name of the first table should be given as the parameter `UNITS_SOURCE`, whereas the second table should have the same name with an appended “/_CU” (in reference to the start-up costs name `CU`). The names of the index and of the columns should be the same as in our model.

The Excel spreadsheet is set up in the same way, except that we store each set of units in their own spreadsheet file. Thus the `UNITS_SOURCE` parameter now denotes the spreadsheet file, while the two sheets are always called “Units” and “Units_CU”.

Finally, we need the ODBC data source name (DSN), also called “Connection String”, which differs with the brand of database server used. We expect the DSN to be given as `DSN`. Connection strings can be found in the database’s manual. A collection of connection strings for popular databases is also available at [www.connectionstrings.com](http://www.connectionstrings.com).

Now, we can setup the parameters needed by `initializations`:

```mosel
declarations
unitsDSN: string;                ! ODBC data source name
unitsTable: string;              ! Table containing the unit parameter
unitsCUTable: string;            ! Table containing the start-up costs
end-declarations
if(isExcelDocument(UNITS_SOURCE)) then
  unitsDSN := "mmodbc.excel:skiph;" + UNITS_SOURCE;
  unitsTable := "Units";
else
  unitsDSN := "mmodbc.odbc:" + DSN;
  unitsTable := UNITS_SOURCE;
end-if
unitsCUTable := unitsTable + "_CU";
```

The function `isExcelDocument` used here is listed in Appendix A and works by checking the file extension.

In the `initializations` block, we read all unit parameters with index set `J` from the first table:

```mosel
initializations from unitsDSN
  [UT, DT, IUT, IDT, P_min, P_max, RU, RD, SU, SD, SC, SE, SIF, SI, SF, F, FA, FB, PA, PB, CD]
  as unitsTable + "(j,UT,DT,IUT,IDT,P_min,P_max,RU,RD,SU,SD,SC,SE,SIF,SI,SF,F,FA,FB,PA,PB,CD)";
```

and the start-up costs from the second table:

```mosel
  CU as unitsCUTable + "(j,k,CU)";
end-initializations
```

Mosel automatically uses the columns `j` and `k` as the unit and period indices. Once the units have been read, we are able to define the `Fuels` set:

```mosel
Fuels := union(j in J) {F(j)};
finalize(Fuels);
```
4.3 Period Data

Since we only want to get data for the periods in our timespan, not the full dataset, and we (possibly) do not know all used fuel types, the number of arrays to be read must be variable, which is not possible within one initializations. An elegant solution to this problem is the direct access to the database using Mosel’s SQL functions.

The period data is stored in a single table which holds the demand $D$, the reserve $R$ and the fuel costs. For every used fuel, we expect the apposite fuel cost column to be called $FC_f$, where $f$ stands for the actual name of the fuel type. Therefore, the fuel name may not contain special characters and spaces.

To access the database, we have to setup a connection first, so the ODBC data source name is needed again:

```mosel
if isExcelDocument(PERIODS_SOURCE) then
    SQLconnect("DSN=Excel Files;HDR=Yes;DBQ="+expandpath(PERIODS_SOURCE));
    assert(getparam("SQLsuccess"), 'SQLconnect failed!');
else
    SQLconnect(DSN);
    assert(getparam("SQLsuccess"), 'SQLconnect failed!');
end-if
```

Note that `expandpath` expands the path of PERIODS_SOURCE, since the ODBC driver’s working directory might be different from ours.

To send an SQL query to the database now, we have to build it up first. The following variables are used for this:

```mosel
declarations
    periodsIndex: text; ! Index column
    reindexedPeriods: text; ! Period source with converted index
    periodsColumns: list of text; ! Columns to read
    periodsArrays: list of array(K) of real; ! Arrays to fill
    periodsQuery: text; ! Assembled SQL query
end-declarations
```

The first column to be read is the period index $k$. How to derive this index depends on the indexing of the periods in the database. For our model, we expect the database periods to be indexed by the column $t$ with data type TIMESTAMP. We also assume the database periods to have the same length as the model periods, $T$; data with different period lengths needs to be interpolated beforehand.

Given all these informations, we can derive our index $k$ as

$\text{periodsIndex} := \{\text{fn FLOOR}( ((t-\{\text{ts '"START"'}\})*24+0.1)/"+L" ) \} + 1$;

- $\{\text{ts '"START"'}\}$ is the start time START as a timestamp
- $(t-\{\text{ts '"START"'}\})$ is the elapsed time since START in days
- $((t-\{\text{ts '"START"'}\})*24+0.1)$ is the elapsed time in hours
- $((t-\{\text{ts '"START"'}\})*24+0.1)/"+L")$ is the elapsed time in periods $\frac{T}{L}$
- $\{\text{fn FLOOR}(((t-\{\text{ts '"START"'}\})*24+0.1)/"+L")\}$ is the period index $\in [0..T-1]$
- $\{\text{fn FLOOR}(((t-\{\text{ts '"START"'}\})*24+0.1)/"+L")\} + 1$ is our index $k \in [T]$
In a fully-fledged database, it would be possible to assign the name \( k \) to this converted index, and to use it by this name. In Excel, this assignment has to be done in a subquery. At the same time, we have to specify the source of the periods:

\[
\text{reindexedPeriods := "SELECT \"periodsIndex\" as k, \"\";} \\
\text{if(isExcelDocument(PERIODS_SOURCE)) then} \\
\text{reindexedPeriods += \" FROM Periods\";} \\
\text{else} \\
\text{reindexedPeriods += \" FROM \" + PERIODS_SOURCE;} \\
\text{end-if}
\]

The subquery `reindexedPeriods` now represents the periods table, with a converted and renamed index column `k`.

With the index settled, we can select our first data arrays for reading: the demand `D` and the reserve `R`.

\[
\text{periodsColumns += \[\"D\", \"R\\\];} \\
\text{periodsArrays += \[D, R\];}
\]

Here, the last statement actually does not copy the `D` and `R` arrays. Since they are complex datatypes, Mosel just copies their references to the list. So, when initialising the arrays of this list, we actually initialize our original arrays.

The procedure for the fuel costs is similar, but since the `FC` array is dynamic, we first have to create the cost subarray for each used fuel type:

\[
\text{forall(f in Fuels) do} \\
\text{create(FC(f));} \\
\text{periodsColumns += \"FC\_{f}\\];} \\
\text{periodsArrays += \[FC(f)\];} \\
\text{end-do}
\]

Finally, we can assemble the SQL query and specify the needed periods:

\[
\text{periodsQuery := \"SELECT k, \" + join(periodsColumns, \",\", \")\);} \\
\text{periodsQuery += \" FROM \" + reindexedPeriods\"\);} \\
\text{periodsQuery += \" WHERE 1 <= k AND k <= \" + T;} \\
\]

The `join` function joins the elements of `periodsColumns`, its implementation can be found in Appendix A.

In conclusion, an assembled query for example may equate to

\[
\text{SELECT k, D, R, FC\_gas, FC\_oil, FC\_coal} \\
\text{FROM \{} \\
\text{SELECT \{fn FLOOR((t-{ts '2009-05-11 00:00:00'})*24+0.1)/1\} + 1 as k, \"} \\
\text{FROM Periods} \\
\text{\}} \\
\text{WHERE 1 <= k AND k <= 84}
\]

We now initialize our parameters using `SQLexecute`:

\[
\text{SQLexecute(periodsQuery, periodsArrays);} \\
\text{assert(getparam("SQLsuccess"), \"SQLexecute failed\")};
\]
After the initialization, we have to close the connection to the database server:

```sql
SQLdisconnect;
```

When working with big Excel spreadsheets, this approach is a good alternative to multiple `initializatons` blocks. Excel reopens the spreadsheet for every `initializations` block, which can be quite time consuming. In contrast, when using SQL functions, Excel opens the spreadsheet just once at `SQLconnect`. 
5 Data Output

Same as with the data input sources, the destination file is given as a parameter:

```plaintext
parameters
    RESULTS_DEST = "Results.xls";  ! Destination of the results
end-parameters
```

5.1 Electricity price

Once the cost function has been minimized within the model, we can derive a basic estimate of the electricity price by observing:

- The price is set the highest accepted bid of any unit.
- In a market with perfect competition, a unit will bid in at marginal costs.
- The marginal cost of a unit is $FA_j \cdot FC^k_{F_j} + PA_j$.

This can be calculated in Mosel as:

```plaintext
declarations
    price: array(K) of real;
end-declarations
forall(k in K) do
    price(k) := max(j in J | v(j, k).sol >= 0.5) (FA(j) \cdot FC(F(j), k) + PA(j));
end-do
```

5.2 Postprocessing

The maximal production and ramping constraints just impose an upper limit on the maximal possible production. Thus, $p_{\text{max}}$ may actually be lower than the real maximal possible production. We can fix this by deriving the maximal possible production from the production variables and the operational state:

```plaintext
declarations
    exact_p_max: array(J, K) of real;
end-declarations
forall(j in J, k in K) do
    exact_p_max(j, k) := P_max(j).sol \cdot v(j, k).sol;
    if(k > 1) then
        exact_p_max(j, k) := minlist(exact_p_max(j, k),
            p(j, k-1).sol + RU(j) \cdot v(j, k-1).sol
            + SU(j) \cdot (1 - v(j, k-1).sol) + P_max(j) \cdot (1 - v(j, k).sol));
    end-if
    if(k < T) then
        exact_p_max(j, k) := minlist(exact_p_max(j, k),
            P_max(j) \cdot v(j, k+1).sol + SD(j) \cdot (v(j, k).sol - v(j, k+1).sol));
    end-if
end-do
5 DATA OUTPUT

5.3 Data Output to Excel

We want to output the optimizer variables for

1. the operational state $v^k_j$,
2. the production variables $p^k_j, \overline{p}^k_j$,
3. the price $p^j$ and
4. the costs $c^k_j, C^k_j, C^d_{j,k}$.

We will output each variable to its own sheet (inside the same file). This allows us to

- change the sizes of $J$ and $K$ and
- add new variables to the model

without having to change the position of the existing variables. Since there are usually more periods than units, and Excel supports more rows than columns\(^2\) we associate the periods with rows and the units with columns. Together with the actual data, we have to output the index sets $J$ and $K$. Since the period indices are relative to START and $L$, we will also output the timestamp of each period.

Now, it would be convenient to implement the output in a separate function, which is to be called for each variable. Unfortunately, one initializations block per function call would be needed, reopening the Excel spreadsheet each time. Depending on the size of the spreadsheet, this may be quite time-consuming. Therefore we use a single initializations block, but still perform the preparation of each variable in a separate function.

We need two functions: one for variables with index set $K$, and one for variables index sets $J$ and $K$. They both should return a two-dimensional array of text, which subsequently will be written to an Excel sheet:

```plaintext
declarations
    PeriodsSheet = dynamic array(rows: range, singleCol: range) of text;
    UnitsPeriodsSheet = dynamic array(rows, cols: range) of text;
end-declarations

function createSheet(data: array(K) of real, name: string): PeriodsSheet ...
end-function

function createSheet(data: array(J, K) of real): UnitsPeriodsSheet ...
end-function
```

\(^2\)Excel 2007 or higher: \(2^{20}\) rows vs. \(2^{14}\) columns; prior to Excel 2007: \(2^{16}\) rows vs. \(2^8\) columns or less
In the body of the `createSheet` function for variables with index set $K$, we need to:

1. Fill the first two columns with the index and the starting time of each period:

   ```
   forall(k in K) do
       returned(k + 1, 1) := text(k);
       returned(k + 1, 2) := text(datetime(START) + (k-1) * L * 3600);
   end-do
   ```

2. Fill the first row with the given variable name:

   ```
   returned(1, 3) := name;
   ```

3. Fill the space between period indices and titles with the actual data:

   ```
   forall(k in K) do
       returned(k + 1, 3) := text(data(k));
   end-do
   ```

In the function for variables with index sets $J$ and $K$, the time series for all units $J$ should be written to adjacent columns. Since the index set $J$ is not necessarily of the form $\{1, \ldots, |J|\}$, we first construct a mapping from $J$ to $\{1, \ldots, |J|\}$:

```plaintext
declarations
    mapping: dynamic array(J) of integer;
end-declarations
forall(j in J) do
    mapping(j) := getsize(mapping) + 1;
end-do
```

The remainder of the function is similar to the previous function for index set $K$. We need to:

1. Fill the first two columns with the index and the starting time of each period:

   ```
   forall(k in K) do
       returned(k + 1, 1) := text(k);
       returned(k + 1, 2) := text(datetime(START) + (k-1) * L * 3600);
   end-do
   ```

2. Fill the first row with the indices of the units,

   ```
   forall(j in J) do
       returned(1, mapping(j)) := text(j);
   end-do
   ```

3. Fill the space between period indices and titles with the actual data:

   ```
   forall(j in J, k in K) do
       returned(k + 1, mapping(j)) := text(data(j, k));
   end-do
   ```
These functions only take real-valued time series; For convenience, we implement them for mpvar-valued time series as well:

```plaintext
function createSheet(data: array(K) of mpvar, name: string): PeriodsSheet
    returned := createSheet(array(k in K) data(k).sol, name);
end-function

function createSheet(data: array(J, K) of mpvar): UnitsPeriodsSheet
    returned := createSheet(array(j in J, k in K) data(j, k).sol);
end-function
```

To perform the output to Excel, we still need to specify the region to which each variable needs to be written. For this, we will use the cellRange function, implemented in our ExcelFunctions package (Appendix A), which generates Excel ranges like [Sheet$A1:Q10].

```plaintext
initializations to "mmodbc.excel:noindex;" + RESULTS_DEST
    evaluation of createSheet(v)
        as cellRange("v", 1, 1, getsize(J)+2, getsize(K)+1);
    evaluation of createSheet(p)
        as cellRange("p", 1, 1, getsize(J)+2, getsize(K)+1);
    evaluation of createSheet(exact_p_max)
        as cellRange("p_max", 1, 1, getsize(J)+2, getsize(K)+1);
    evaluation of createSheet(price, "Price")
        as cellRange("price", 1, 1, 3, getsize(K)+1);
    evaluation of createSheet(s)
        as cellRange("s", 1, 1, getsize(J)+2, getsize(K)+1);
    evaluation of createSheet(c)
        as cellRange("c", 1, 1, getsize(J)+2, getsize(K)+1);
    evaluation of createSheet(cp)
        as cellRange("cp", 1, 1, getsize(J)+2, getsize(K)+1);
    evaluation of createSheet(cu)
        as cellRange("cu", 1, 1, getsize(J)+2, getsize(K)+1);
    evaluation of createSheet(cd)
        as cellRange("cd", 1, 1, getsize(J)+2, getsize(K)+1);
end-initializations
```
6 Remarks

This whitepaper presents a detailed model for the Unit Commitment problem. We have given its practical implementation and discussed how to reduce the number of start-up cost constraints and how to analyze the cause of infeasibilities. Finally, we have shown how to manage data input and output, and have given a basic estimate of the electricity price.

One major drawback of the implemented model is the lacking consideration of market power. To remedy this, usually a so-called “uplift” function is added to the model’s electricity price, which is derived by statistically analyzing the disagreement between the outcome of this model and the real price on historical data.

Of course, it would be better to consider the market power directly as part of the model. Different approaches in achieving this are discussed in [5]. The two most popular ones are game theoretical approaches:

- the Cournot equilibrium and
- the Supply Function equilibrium.

They differ in the modelling of the strategies of the power generating companies. While in a Cournot model a company may only decide on its power output, a company’s strategy in a Supply Function equilibrium is described by a function mapping the market price to its power supply. The greater freedom in the choice of a strategy, however, comes at the expense of a higher computational effort, which in turn forces the use of less detailed models.

For a comparison of the practical performance of Cournot and Supply Function models on the German power market, we refer to [6].

The second drawback is the neglect of the underlying power grid. The production schedule resulting from our model may not be feasible on a real power grid, i.e. the power grid may not be able to transport the electricity from the producing units to the consumers. While this was typically not a fundamental issue so far, it becomes more and more important due to the increasing energy production from renewable, more volatile resources causing bottlenecks in the power exchange between different regions and thus necessitate an explicit modelling of the power grid.

Also, most of today’s power markets are connected to one or more neighboring markets through interconnectors. The interconnectors are used to transport energy from a market with higher price to a market with lower price, thus diminishing the price difference. The effective price change is typically small due to the small capacity of the interconnectors, but may not be negligible.
References

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[5] M. Ventosa, A. Baillo, A. Ramos, and M. Rivier. Electricity market modeling trends. *Energy Policy*, 33(7):897–913, 2005.

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A Excel Functions

The Excel functions are implemented as part of the ExcelFunctions package. Therefore, the functions to be used in other packages or the main model have to marked as public.

A.1 Function isExcelDocument

The function isExcelDocument checks if the given data source is an Excel file. This is done by checking the file extension:

```plaintext
public function isExcelDocument(source: string): boolean
    declarations
        S = getsize(source);
    end-declarations
    returned := strtolower(copytext(text(source), S-4+1, S)) = '.xls' or 
               strtolower(copytext(text(source), S-5+1, S)) = '.xlsx';
end-function
```

Unfortunately, we also have to implement the strtolower function for ourselfs, which converts a string to lower case:

```plaintext
public function strtolower(str: text): text
    returned := str;
    forall(i in 1..getsize(returned)) do
        if(65 <= getchar(returned, i) and
            getchar(returned, i) <= 90) then
            setchar(returned, i, getchar(returned, i) + (97-65));
        end-if
    end-do
end-function
```

A.2 Referencing cells in Excel

Excel supports two notations for referencing cells,

- the A1 notation (ex: C4, R7, Z80) and
- the R1C1 notation (ex: R4C3, R7C18, R80C26).

While the R1C1 notation is easier to use programmatically, the A1 notation may be more familiar to end users. Therefore, we offer the boolean useA1notation which switches from R1C1 to A1 notation:

```plaintext
public declarations
    useA1notation: boolean; ! If true, the A1 notation is used
end-declarations

Depending on this setting, the coordinates of a cell can be determined with a call to cellCoords:
```
A.2.1 Column index conversions in the A1 notation

The column numbering scheme in the A1 notation is a bit unusual and needs special conversion functions.

Conversion from integer to A1 column index (1 → A, 2 → B, ...)

```plaintext
public function columnToA1(index: integer): string
declarations
c: text;  ! Column name as text (needs mmsystem)
end-declarations
while(index > 0) do
    c := " + c;
    setchar(c, 1, 65  (! = A !) + (index-1) mod 26));
    index := (index-1) div 26;
end-do
returned := string(c);
end-function
```

Conversion from A1 column index to integer (A → 1, B → 2, ...)

```plaintext
public function cellCoords(column: integer, row: integer): string
    if(useA1notation) then
        returned := columnToA1(column) + row;
    else
        returned := "R" + row + "C" + column;
    end-if
end-function
```

Cell ranges are described by the coordinates of their top-left and bottom-right cells, divided by a colon (ex. C4:R7, R4C3:R7C18). They can be determined with a call to `cellRangeCoords`:

```plaintext
public function cellRangeCoords(column: integer, row: integer,
                                 width: integer, height: integer): string
    returned := cellCoords(column, row) + ":" + cellCoords(column + width - 1, row + height - 1);
end-function
```

When referencing Excel cells through ODBC, it is usual to include the cell’s sheet name too. This is done by prepending the sheet name to the coordinates, divided by a dollar sign, and by enclosing the reference in square brackets. The two following functions generate such references to a cell or respectively to a cell range:

```plaintext
public function cell(sheet: string, column: integer, row: integer): string
    returned := ";" + sheet + "$" + cellCoords(column, row) + "];
end-function
public function cellRange(sheet: string, column: integer, row: integer,
                          width: integer, height: integer): string
    returned := ";"+sheet+$cellRangeCoords(column, row, width, height) + ";";
end-function
```
public function a1ToColumn(column: string): integer
    forall(i in 1..getsize(column)) do
        returned := returned * 26 + getchar(column, i) - 65 (! = A !) + 1;
    end-do
end-function

A.3 Function Join

Joins the specified separator string between each element of the specified set of strings, yielding a single joined string. Example:

join(["A", "D", "B", "C"], ", ") -> "A, D, B, C"
join(["A"], ", ") -> "A"
join(["A", "D", "B", "C"], ", ") -> "A, B, C, D"

Implementations for lists and sets of string:

public function join(strings: list of text, separator: string): text
    forall(s in strings) do
        returned := returned + s + separator;
    end-do
    returned -= separator;
end-function

public function join(strings: set of text, separator: string): text
    forall(s in strings) do
        returned := returned + s + separator;
    end-do
    returned -= separator;
end-function