Relativistic nature of the EMC-effect

Alexander Molochkov\textsuperscript{a,1}

\textsuperscript{a}Far Eastern National University, 690950 Vladivostok, Russia

Abstract

The deep inelastic scattering of leptons off nuclei is studied within the the Bethe-Salpeter formalism. It is shown that nuclear short-range structure can be expressed in terms of the nucleon structure functions and four-dimensional Fermi motion of the nucleons. The four-dimensional Fermi motion broadens the bound nucleon localization area, what leads to the observation of the nucleon structure change in nuclei – EMC effect. The $^4\text{He}$ to deuteron structure functions ratio is found in good agreement with experimental data. It is shown that the pattern of the ratio is defined by dynamical properties of the nucleon structure and four-dimensional geometry of the bound state.

The European Muon Collaboration (EMC) demonstrated in the experiments on deep inelastic scattering (DIS) of leptons off nuclei that nuclear environment modifies short range nucleon structure [1]. The modifications were observed as deviations from unity of the nuclear and deuteron DIS cross-sections ratio $R^A = 2\sigma^A/(A\sigma^D)$ to the values smaller than one. Since it was widely accepted that soft momentum nucleon-nucleon interaction in nucleii cannot modify a hard momenta distribution of nucleon partons, this phenomena was very unexpected. Later this effect was studied in a wide kinematic range in many experiments (c.f. review [2]). A wide variety of models was proposed to explain these modifications (c.f. reviews [2,3]). In different kinematic ranges the oscillations were considered as the different effects that are shadowing, antishadowing, EMC effect, and Fermi motion. No one of the models provided a quantitative explanation of the effect in the whole kinematic range, what led to the conclusion of the review [2] that origin of the EMC-effect stays unclear.

So, the EMC-effect remains a topical subject up to now. Experimental study of nuclear hard structure provided opportunities to find important regularities that can give additional constrains on the models of the EMC-effect. Study of the EMC-effect dependence on the atomic number of the nucleus (A-dependence) performed in the SLAC experiments [4] showed that amplitude

\footnotesize{1} Partially supported by the Alexander von Humboldt Foundation, Germany
of the effect does not saturate with increase of A. The consequent analysis of the world data for the \(2\sigma^A/(A\sigma^D)\) ratio made in the paper [5] uncovered universality in the EMC-effect A-dependence for the all kinematic ranges. An another important result was obtained in the hard \(pA\) scattering performed in FNAL [6]. Analysis of the nuclear and deuteron Drell-Yan cross-sections ratio showed no excess of the antiquark component in nuclei.

These results provided a basis for the critics of the most obvious explanations of the EMC-effect that were proposed by the binding [7] and mesonic exchange models [8]. In the paper [9] it was stressed that due to the Hugenholtz-van Hove theorem [10] the nuclear binding effect in DIS is defined by the mass defect in the nucleus. Thus, the binding effect has to saturate with increasing of A, while the systematic experimental study of the EMC-effect A-dependence [4] does not show such saturation. Results of the Drell-Yan experiment [6] prove that the mean mesonic field in the nucleus, which presumably consist of \(q\bar{q}\) fields, also cannot explain the EMC-effect [9].

This critics led to the conclusion that nucleon energy-momentum change due to the binding effects cannot be responsible alone for the EMC-effect. The analysis performed within the QMC model [11] pointed that the EMC-effect cannot be explained without introducing the hypothesis that nucleon structure is changed in nuclear media. This conclusion is consistent with the previously obtained resume of the calculation based on the quasi-potential approach [12] and supported recently by the light front analysis [13].

However, despite this critics, the binding model provided an important signal about nucleon structure change in nuclei. This model assumes the bound nucleon mass shift \(m \rightarrow m^* = m - \epsilon\), where \(\epsilon\) is the binding energy of the nucleon. Due to the uncertainty relation the mass-shift changes the observed radius of the nucleon localization area in the four-dimensional (4D) space [14]. Thus, this mass-scale shift leads to the change of the quark confinement 4D-radius and, hence, to distortion of the partonic distribution inside the nucleon. This explanation coincides with the \(Q^2\)-rescaling model [15], which explains the EMC-effect as a change of the quark confinement radius in the bound nucleon. In that way, the explanations of the EMC–effect that are proposed by the binding and \(Q^2\)-rescaling models can be reduced to the distortions of the nucleon 4D-structure in the nuclei. Thus, a fully covariant 4D-treatment is essential for understanding of this phenomenon.

In the present letter I would like to focus on the bound state relativistic properties that are important for understanding of the EMC-effect. To clarify the role played by the relativistic effects it is essential to consider the bound state within an explicitly covariant approach that can be developed in the relativistic field theory framework. Within the covariant field theory the space-time distribution of the nucleons inside the nucleus is defined by the following
amplitude
\[ \phi(x_1, \ldots x_n) = \langle 0 | T \psi(x_1) \ldots \psi(x_n) | A \rangle, \quad (1) \]
where \( \psi \) denotes the nucleon field operators, the four vectors \( x_i \) define positions of the bound nucleons in the space-time. Hence, nucleons inside the nucleus are separated not only by the three dimensional space-intervals \( r_{i,j} = x_j - x_i \), but by the time-intervals \( \tau_{i,j} = x_{0j} - x_{0i} \) as well. The separated in time constituents is the specific relativistic bound state property (it is called \( \tau \)-shift bellow) that was criticized from the very beginning of the relativistic bound state theory development. The critical comments claimed that the shifted in time constituents mean causality violation [16]. In that way the \( \tau \)-shift was considered as a non-physical property, which is not reflected in observables and can be fixed to any value. It stimulated development of the quasi-potential approaches where the \( \tau \)-shift was fixed according to different external conditions, which have to lead to equivalent results in calculations of observable quantities (c.f. reviews [17]). However, this equivalence was proven to be false in the analysis of the relativistic covariance of the quasi-potential approaches performed in the paper [18].

To clarify the role played by the \( \tau \)-shift it is important to note that the quoted above critics has sense only within classical limit, where space-time positions of particles can be detected exactly. A position of a quantum particle in the space-time can be detected within the boundaries that are defined by the uncertainties in detection of the energy and momentum (\( \Delta E \) and \( \Delta p \)) of the particle: \( \Delta x \simeq 1/\Delta p, \quad \Delta t \simeq 1/\Delta E \). Since the uncertainty in energy detection of the nucleon partons defined by the total nucleon energy, \( \Delta E_q = E_N \); the boundaries of the nucleon localization in time are defined as \( \Delta t_N = 1/E_N \). Since the bound nucleon is distributed in the space-time inside the nucleus and energy of the nucleon is shifted from the mass-shell, the boundaries of the bound nucleon are extra blurred: \( \delta \Delta t \simeq 1/(E_N - \Delta E_N) - 1/E_N \), where \( \Delta E_N \) is the energy shift of the nucleon due to the binding and Fermi motion. Thus, the causality violation is unobservable if the \( \tau \)-shift is not larger than the extra blurring area \( \delta \Delta t \) defined by the energy-shift of the bound nucleon:

\[ \tau \leq \frac{\Delta E_N}{E_N(E_N - \Delta E_N)}. \quad (2) \]

If this expression holds true then the effects that come from the \( \tau \)-shift cannot be considered as an evidence of the causality violation. In this case the complete relativistic picture of the nuclear effects has to incorporate the \( \tau \)-shift, which is lost in the conventional nuclear binding models based on the quasi-potential approaches.

For calculation of the \( \tau \)-shift effects an explicitly covariant field theory formal-
ism is essential. In the present paper I use the approach that was developed in the series of the publications [14,19] on the base of the Bethe-Salpeter equation [21].

Let us consider the main line of the calculations. Due to the optical theorem the amplitude of the high-energy inclusive scattering is proportional to the imaginary part of the amplitude for the forward scattering \( \langle A|T(J_\mu J_\nu)|A \rangle \). Within the Bethe-Salpeter formalism this matrix element is defined by the space-time distributions (1) and vacuum average of the T-product of the nucleon fields and nucleon em-current (see Ref. [19]). Following this method we get the expression for the nuclear DIS amplitude \( W_{\mu\nu}^A \). Within the Bjorken limit \((Q^2 \to \infty)\) all terms, except the relativistic impulse approximation term, will come to zero at least as \( 1/(Q^2)^2 \) [19]. The relativistic impulse approximation term, where the lepton scatters off the single nucleon in the nucleus, has the following form:

\[
W_{\mu\nu}^A(P_A, q) = \int \frac{d^4p}{(2\pi)^4} \frac{W_{\mu\nu}^N(p, q) f^{N/A}(P, p)}{(p^2 - m^2)^2 ((P - p)^2 - M_{A-1}^2)}.
\] (3)

This expression gives the nuclear DIS amplitude in terms of the off-mass-shell nucleon DIS amplitude \( W_{\mu\nu}^N \) and the nucleon distribution function \( f^N(P_A, p) \). This distribution function is defined by the amplitudes (1), and together with the denominator it composes the four-dimensional momentum distribution of the struck nucleon inside the nucleus carrying the total momentum \( P_A = (M_A, P) \). In that way Eq. (3) expresses the nucleon blurring that results from the four-dimensional distribution of the nucleon inside the nucleus. By analogy with the non-relativistic 3D momentum distribution I will call it four-dimensional Fermi motion.

Due to the four-dimensional integration in Eq. (3) actual calculations require information about nucleon amplitude \( W_{\mu\nu}^N \) in the kinematic region of the off-mass-shell values of the nucleon energy \( p_0 \) \( (p_0^2 \neq p^2 + m^2) \). The off-mass-shell behavior of \( W_{\mu\nu}^N \) is unobservable, since then explicit microscopic calculations of the amplitude have no experimental reference and strongly model dependent. Thus, Eq. (3) has to be rewritten in terms of measurable quantities such as the DIS structure functions of the physical nucleon, which defined by the total DIS cross-section in the Bjorken limit \( \sigma^N \propto F^N_2 \). The most simple solution of this problem is provided by the integration in Eq. (3) with respect to \( p_0 \).

Analytical properties of the integrand in Eq. (3) give a way to do it explicitly. Within the assumption that \( W_{\mu\nu}^N(p, q) \) and \( f^N(P_A, p) \) are regular with respect to \( p_0 \) the integrand contains a second order pole corresponding to the struck nucleon and a first order pole corresponding to the spectator. These poles lie in

\[2\] Since \( W_{\mu\nu}^N = Im_{\mu\nu}J_{\mu\nu}^{N} \), the first assumption is obvious. The second assumption was proven in the papers [14,20].
the different half-planes of the complex plane \( p_0 \). So, we can choose one of the singularities to perform the integration. To express the nuclear hadron tensor in terms of the nucleon structure functions it is necessary to choose the second order pole \( (p_0 - E_N) \) that corresponds to the struck nucleon. The result of the contour integration in vicinity of the second order pole in Eq.(3) is defined by the derivative of the pole residue with respect to \( p_0 \) at the point \( p_0 = E_N \). Doing the integration and using the relation \( W_{\mu\nu}(P,q) = F_2(x,Q^2)/x \) we get the following expression for the nuclear structure function \( F^A_2 \):

\[
F^A_2(x) = \int \frac{d^3 p}{(2\pi)^3} \left[ \frac{M_A - E_{A-1} - p_3}{E_N} F^N_2(x_N) - \frac{\Delta^N_A}{E_N} x_N \frac{dF^N_2(x_N)}{dx_N} \right] \frac{f^{N/A}(M_A, p)}{8 M_A E_N E_{A-1} \Delta^A_N}, \tag{4}
\]

where \( x = -q^2/(2m_0) \) is the nuclear Bjorken \( x \) normalized on the nucleon mass, \( x_N = x m/(E_N - p_3) \) is the Bjorken \( x \) of the struck nucleon, \( E_N = \sqrt{m^2 + p^2} \) is the struck nucleon on-shell energy, \( E^2_{A-1} = M^2_{A-1} + p^2 \) is the on-shell energy of the nuclear residue, \( \Delta^N_A = M_A - E_N - E_{A-1} \).

The function \( f^{N/A}(M_A, p) \) together with the denominator composes the three dimensional momentum distribution of the struck nucleon inside the nucleus. According to the normalization condition for the Bethe-Salpeter vertex function this distribution satisfies the baryon and momentum sum rules [14] and coincides with the usual nuclear momentum distribution. Thus, the first term in Eq.(4), which results from the derivative of the propagator of the nuclear spectator, expresses contribution of the conventional 3D-Fermi motion.

The second term with \( dF^N_2(x_N)/dx_N \) results from the nucleon DIS amplitude derivative:

\[
\frac{dW^N_{\mu\nu}(p,q)}{dp_0} = \frac{dx_N}{dp_0} \left( \frac{1}{x_N} \frac{dF^N_2(x_N,Q^2)}{dx_N} - \frac{F^N_2(x_N,Q^2)}{x_N^2} \right).
\]

This expression shows the \( \tau \)-shift influence on the observable quantities in nuclear DIS. Its contribution is proportional to the coefficient \( \Delta^A_A \), which, therefore, characterizes contribution of the \( \tau \)-shift. Due to the energy conservation \( \tilde{E}_N + E_{A-1} = M_A \), where \( \tilde{E}_N \) is the off-mass-shell energy of the bound nucleon. Therefore, \( \Delta^N_A \) is equivalent to the to the total energy shift of the struck nucleon due to the binding and Fermi motion \( \Delta^N_A = \tilde{E}_N - E_N \). Thus, the \( \tau \)-shift satisfies the causality condition (2).

It is important to note that the struck nucleon pole gives the factor \( 1/\Delta^2_A \), since then the contribution from all other singularities (for example nucleon self-energy cut or anti-nucleon pole) are suppressed at least as \( (\Delta^N_A/M_A)^2 \) [19]. Since the mean value of the energy shift is small for all nuclei \( (\Delta^N_A/M_A \propto O(10^{-2})) \), Eq.(4) provide nuclear structure function with accuracy up to terms of order \( (\Delta^N_A/M_A)^2 \propto O(10^{-4}) \).
Fig. 1. Ratio of the \(^{4}\text{He}\) and D structure functions. a) Contributions to the ratio of the impulse approximation (dashed curve \(R^{^{4}\text{He}}(IA)\)) and nucleon structure function derivative (dashed-dot curve \(\Delta R^{^{4}\text{He}}\)). The full calculation (full curve) is the sum of these contributions. The experimental values are shown by the dark squares [4] and the light circles [22].

Numerical calculations of the contributions of the different terms in Eq.(4) to the ratio \(R^{^{4}\text{He}} = 2\sigma^{^{4}\text{He}}/(A\sigma^{D}) = F^{^{4}\text{He}}/F^{D}\) are presented at Fig.1 a). The nuclear momentum distribution is taken from [23], the nucleon SF \(F^{N}\) is taken from [24]. The first term \((R^{^{4}\text{He}}(IA))\), which results from the Fermi motion in usual 3D-space, is presented by the monotone increasing dashed curve. The second term \((\Delta R^{^{4}\text{He}})\) resulted from the Fermi motion along the time axis is depicted by the point-dashed curve. Since \(M_{A} < E_{N} - E_{A-1}\) and \(F^{N}_{2}(x_{N})/dx_{N} < 0\) it gives negative contribution to \(F^{A}_{2}\). This term puts the ratio \(R^{^{4}\text{He}}\) bellow unity in the middle \(x\) region and, therefore, produces the EMC-effect. The full curve presents sum of these contributions. It clearly falls to the experimental data within the experimental errors with rather good accuracy (see Fig.1 b)).

Thus, we have obtained that the \(\tau\)-shift affects on the observable quantities in DIS. It broadens the bound nucleon localization area in 4D-space, what leads to the observation of the EMC-effect. This phenomenon resembles the one predicted by the hypothesis of the \(x\)-rescaling and \(Q^{2}\)-rescaling. However, these models assume real change of the bound nucleon structure, while in the present picture the EMC-effect results from the measurement uncertainty due to the the 4D-Fermi motion. This is the key difference between these models and the presented here picture of the EMC-effect. The binding approach that
use separation energy as rescaling parameter clearly underestimates the EMC-effect at the large \( x \) region. The Eq.(4) shows that amplitude of the EMC effect is defined by the geometrical properties of nuclei, which expressed by the parameter \( \Delta N_A \) in the momentum space. It is worth noting that \( A \)-dependence of this quantity explains the \( A \)-dependence of the EMC-effect observed on the experiment [4]. It can be qualitatively shown with the help of an approximated form of \( \Delta N_A \): Assuming weak \( A \)-dependence of the binding energy \( (\epsilon_A \simeq \epsilon_{A-1}) \) and small relative 3-momentum of the bound nucleons \( \langle p^2 \rangle \ll m^2 \), we can rewrite mean value of \( \Delta N_A \) in the form:

\[
\langle \Delta N_A \rangle = \epsilon_A - \langle T \rangle_A \frac{A}{A-1} \tag{5}
\]

where \( \epsilon_A \simeq (M_A - Am)/A \) is the binding energy of the nucleon and \( \langle T \rangle_A \langle p^2 \rangle/(2m) \) is the mean kinetic energy of the bound nucleon. Thus, the amplitude of the deviations from unity of the structure functions ratio is defined by the binding energy \( \epsilon_A \) and mean kinetic energy \( \langle T \rangle_A \) of the nucleon bound in the nucleus \( A \). The binding energy weakly depends on \( A \), since then \( A \)-dependence of the EMC-effect is defined mainly by the nucleon kinetic energy. Due to the uncertainty relation \( \langle T \rangle_A \) is proportional to the mean nuclear density \( \langle T \rangle_A \propto \rho_A^{2/3} \), what explains the observation made in SLAC experiment [4]. Taking into account that the mean density in the central part of the nucleus \( \rho_c \) and on the surface \( \rho_s \) weakly depend on \( A \) and related as \( \rho_s/\rho_c \simeq 0.02 \), we can derive the explicit expression for \( A \)-dependence of the nucleon mean kinetic energy:

\[
\langle T \rangle_A \simeq \langle T_c \rangle \left(1 - 0.98 \frac{N_s(A)}{A} \right). \tag{6}
\]

Here \( N_s(A) \) is the number of nucleons on the surface of the nucleus, \( \langle T_c \rangle \) is the mean kinetic energy of the bound nucleon in the central part of the nucleus \( \langle T_c \rangle \simeq \). This expression corresponds to the world experimental data fit for nuclear DIS made in the paper [5]. Numerical calculation of this estimation is in a good agreement with experimental data (see Fig.2).

In conclusion, the 4D-Fermi motion plays two-fold role in the nuclear DIS. From the one hand the uncertainty in measurement does not allow one to exactly detect nucleon structure in nuclear experiments, so that the information about nucleon structure extracted from the nuclear data strongly depends on the model used for description of the nucleon. It leads to the well known difficulties in extraction of the neutron structure functions from the nuclear experiments [24,25]. From the other hand, Eq. (4) shows that the nucleon structure function derivative defines the pattern of the nuclear to deuteron DIS cross-sections ratio. Thus, the 4D-Fermi motion enables direct access to the information about nucleon structure dynamics. It is worth noting that
Fig. 2. A-dependence of the nuclear to deuteron structure functions ratio calculated at $x = 0.6$. The full line is the calculation according to the Eqs. (6) and (5). The experimental values are shown by the full circles [4].

the proton SF has a large negative slope at small $x$, what can lead to the deviation of $R^A$ from unity producing in that way the effect that coincides with the nuclear shadowing. Thus, the 4D-Fermi motion can give an universal explanation of the EMC-effect, anti-shadowing, and shadowing, what can be confirmed by further studies at small $x$ region.

An another important outcome is the direct link between sub-nucleon structure dynamics and space-time geometry at microscopic distances. Within the covariant 4D approach the space-time is considered as the metric 4-space, the 4D Fermi-motion in which leads to the EMC-effect. The covariant 3D approaches consider 4D space-time as a $3 + 1$ foliated manifold with the time as a factor-space and the metric 3-space as a layer. The 3D Fermi-motion is shift of the particle position in the layers with move along the factor space. To take into account the effects of the Fermi-motion along the time axis, one has to introduce dynamical distortions particle move. Thus, within a such approach the EMC-effect can be explained by dynamical change of bound nucleon structure. For example, the central mechanism in QMC [11] and chiral soliton models [13] to explain the EMC effect is the scalar attractive interaction, which modifies the quark distribution in the nucleon. Influence of the Fermi-motion along time-axis on the observable quantities has scalar character. In that way, the scalar field in the covariant 3D approach can be considered as a three-dimensional dynamical realization of the time component of the 4D-Fermi motion.

In summary, the explicitly covariant 4D approach provides a consistent description of the nuclear short-range structure in terms of the nucleon structure and four-dimensional Fermi motion of the nucleons. The Fermi motion along the time axis broadens the bound nucleon localization area, what lead to the observation of the nucleon structure change in nuclei – EMC effect. The amplitude of the EMC-effect is defined by the binding and kinetic energy of the bound nucleons, thus the $A$-dependence of the EMC-effect can be explained by decrease with $A$ of the number of surface nucleons that have much smaller kinetic energy than nucleons bound in the central part of the nu-
cleus. The pattern of the effect is defined by dynamics of the nucleon structure. Thus, the obtained results are in qualitative consistence with the conclusion of the Drell-Yan experiments [6]. Detailed quantitative comparison need further study with extension of the presented formalism to the Drell-Yan process.

I would like to thank U. Mosel, G.I. Smirnov, and H. Toki for useful discussions.

References

[1] J.J. Aubert et al., Phys. Lett. B123, 275 (1983).
[2] M. Arneodo, Phys. Rep. 240, No. 5–6, 301 (1994).
[3] C.W. Wong, Phys. Rep. 136, 1 (1986); D.F. Geesman, K. Saito, A.W. Thomas, Ann. Rev. Nucl. Part. Sci. 45, 337 (1995); G. Piller, W. Weise, Phys.Rep. 330, 1 (2000).
[4] J. Gomez et al., Phys. Rev. D49, 4348 (1994).
[5] G.I. Smirnov, Phys. Lett. B364, 87 (1995); G.I. Smirnov, Eur.Phys.J. C10, 239 (1999).
[6] D.M. Alde, et al., Phys. Rev. Lett. 64 (1990) 2479.
[7] S.A. Kulagin, Nucl. Phys. A500, 653 (1989); S.V. Akulinichev, Phys. Lett. B357, 451 (1995)
[8] E.L. Berger, F. Coester, R.B. Wiringa, Phys. Rev. D29, 398 (1984)
[9] G.A. Miller, J.R. Smith, Phys. Rev. C65, 015211 (2001); J.R. Smith, G.A. Miller, Phys. Rev. C65, 055206 (2001).
[10] N.M. Hugenholtz, L. van Hove, Physica 24, 363 (1958).
[11] K. Saito, A.W. Thomas, Nucl. Phys. A574, 659 (1994); K. Tsushima et al., nucl-th/0301078
[12] F. Gross, S. Liuti, Phys.Rev. C45, 1374 (1992); C. Ciofi degli Atti, S. Liuti, Phys.Rev. C44, R1269 (1991).
[13] J.R. Smith, G.A. Miller, Phys.Rev.Lett. 91, 212301 (2003).
[14] A.V. Molochkov, Nucl. Phys. A666, 169 (2000).
[15] F.E. Close, R.L. Jaffe, R.G. Roberts, and G.G. Ross, Phys.Rev. D31, 1004 (1985).
[16] A.A. Logunov, A.N. Tavkhelidze, Nuovo Cimento 29, 380 (1963); V.G. Kadyshhevsky, R.M. Mir-Kasimov, N.B. Skachkov, Sov. J. Part. Nucl. 2, No. 3, 40 (1972).
[17] M. Garçon, J.W. van Orden, Adv. Nucl. Phys. 26, 203 (2001); R. Gilman, F. Gross, J. Phys. G28, R37 (2002).

[18] V. Pascalutsa, J.A. Tjon, Phys. Rev. C61 054003 (2000).

[19] V.V. Burov, A.V. Molochkov, G.I. Smirnov, Phys. of Part. and Nucl. 30, No. 6, 579 (1999); V.V. Burov, A.V. Molochkov, Nucl. Phys. A637, 31 (1998).

[20] V.V. Burov, A.V. Molochkov, G.I. Smirnov, Phys. Lett B466, 1 (1999); S. Bondarenko, V. Burov, A. Molochkov, G. Smirnov, H. Toki, Prog. Part. Nucl. Phys. 48, 449 (2002).

[21] E.E. Salpeter, H.A. Bethe, Phys. Rev. 84, 1232 (1951).

[22] NMC, P. Amaudruz, et al., Nucl. Phys. B441, 3 (1995).

[23] C. Ciofi degli Atti, S. Simula, Phys. Rev. C53, 1689 (1996).

[24] V. Burov, A. Molochkov, G. Smirnov, H. Toki, Phys. Lett B587, 175 (2004).

[25] W. Melnitchouk, A.W. Thomas, Phys. Lett., B377, 11 (1996).