Quantum key distribution using superposition of the vacuum and single photon states

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B92-type and BB84-type quantum cryptography schemes using superposed states of the vacuum and single particle states which are robust against PNS attacks are studied. The number of securely transferred classical bits per particle (not per qubit) sent in these schemes is calculated and found to have upper bounds. Possible experimental realizations using the cavity QED or linear optics are suggested.

I. INTRODUCTION

Recent progress in theories and experiments\cite{1, 2, 3, 4, 5} of generation and manipulation of single photons allows one to think the quantum information processing utilizing single particles feasible. The first commercial application of quantum information science at a single qubit level might be the quantum key distribution (QKD)\cite{6}. In the typical QKD scheme a sender (Alice) shares a secret key with a receiver (Bob) by sending superposition of photon polarization states. However, one can encode information not only in particle states but also in the vacuum as shown in some QKD schemes (mainly in double ray schemes)\cite{7, 8, 9}. In other words the vacuum can play a role of an information carrier as particles do. Two of authors had suggested the quantum teleportation and the Bell inequality test using single-particle entanglement which was verified experimentally later\cite{10}. In this direction we proposed\cite{11} the Ekert-type\cite{12} single ray quantum cryptography scheme using the entangled states of the vacuum and the single particle state\cite{10, 13, 14, 15, 16, 17, 18, 19, 20, 21}. The main purpose of this work is to present a single ray B92-type\cite{7} and a BB84-type\cite{22} quantum cryptography schemes using superposed states of the vacuum and single photon states. In our schemes the detection of the superposition state is possible with cavity QED devices or linear optics devices and single photon detectors. Consider a quantum memory with the state $|\phi\rangle = \alpha |0\rangle + \beta |1\rangle$ which is a superposition of the vacuum $|0\rangle$ and a single photon state $|1\rangle$ (optical qubit) which can be prepared by a photon source using parametric down conversion\cite{16} or linear optics\cite{23} with the optical state truncation. By choosing $|\beta|$ small enough we can make the expected number of photons in $|\phi\rangle$ (i.e., $|\beta|^2$) arbitrary small. It implies that we can encode classical bit information in the superposition of the vacuum and single photon state with $|\beta| \ll 1$, which is very faint light. Does this also mean that the legitimate participants can share their secret key through a QKD protocol with an arbitrary faint light source to hide quantum channel itself from eavesdropper? We show that the answer is no at least for straightforward generalizations of B92 and BB84-type QKD with the vacuum-photon superposed state considered in this paper, and there are upper bounds for classical bits shared between parties per particle sent ($K$ defined below, not counting the vacuum) in these schemes.

This paper is organized as follows. In Sec. II we present a B92-type quantum cryptography scheme using the superposition of the vacuum and single particle states. We also calculate the number of classical bits transferred per particle. In Sec. III we extend the arguments to a BB84-type scheme. In Sec. IV possible experimental realizations of our schemes using cavity QED and linear optics are presented, and a security analysis is given. Finally, in Sec. V we present a concluding discussion.
II. B92-TYPE SCHEME

Fig. 1 shows our B92-type quantum cryptography scheme using a cavity QED device. As is well known the B92 protocol exploits the fact that arbitrary two non-orthogonal states cannot be distinguished perfectly. Basically our scheme with a superposition of the vacuum and single photon is just the same as the B92 scheme except for the state and the measuring device used. For clarification we describe the scheme.

(i) Alice sends sequences of states randomly chosen between two non-orthogonal states $|\phi_0\rangle$ and $|\phi_1\rangle$ representing logical 0 and 1, respectively:

$$|\phi_0\rangle = \alpha_0 |0\rangle + \beta_0 |1\rangle,$$
$$|\phi_1\rangle = \alpha_1 |0\rangle + \beta_1 |1\rangle,$$

with normalization $|\alpha_i|^2 + |\beta_i|^2 = 1 \ (i = 0, 1)$.

(ii) At a photon arrival time Bob measures a projection operator randomly chosen between $P_0 \equiv 1 - |\phi_1\rangle\langle \phi_1|$ and $P_1 \equiv 1 - |\phi_0\rangle\langle \phi_0|.$

(iii) After a series of measurements Bob publicly announces to Alice in which instances he obtained a positive result. This happens only when Alice sends $|\phi_0\rangle$ and Bob measures $P_0$ or Alice sends $|\phi_1\rangle$ and Bob measures $P_1$. In other words, with probability $1/2$ the state sent by Alice and the projection operator are correlated. In these cases, applying projection $P_0$ ($P_1$) to $|\phi_0\rangle$ ($|\phi_1\rangle$), Bob obtains a positive result with a probability

$$p = \frac{1}{2}(1 - |\langle \phi_0|\phi_1\rangle|^2) = \frac{1}{2}(1 - |\alpha_0^*\alpha_1 + \beta_0^*\beta_1|^2).$$

Thus, after $N$ trials, the total $n_b = pN < N$ bits of keys are successfully shared, if there have been no eavesdropping or errors.

(iv) To certify the absence of an eavesdropper Alice and Bob sacrifice parts of data to check whether Bob obtained positive results on $P_0$ ($P_1$) measurement or not even in the case that Alice sent $|\phi_1\rangle$ ($|\phi_0\rangle$).

At this point one can pose an interesting question. How many classical bits can Alice transfer per particle sent to Bob in an ideal case without errors or eavesdropping up to the step (iii)? According to the Holevo’s theorem, asymptotically one cannot encode and retrieve reliably more than one bit of classical information per qubit. Note, however, that in this letter we are interested in classical bit information not per qubit but per particle excluding the vacuum, so the bit information per particle is not restricted by the Holevo bound. In our schemes the number of qubits sent is not equal to the number of particles sent, because the states sent are superpositions of the vacuum and single particle states. Let us calculate the ratio $K$. Since Alice should choose randomly between $|\phi_0\rangle$ and $|\phi_1\rangle$, the density matrix for a transmission can be written as $\rho = (|\phi_0\rangle\langle \phi_0| + |\phi_1\rangle\langle \phi_1|)/2$. So the average number of particles sent to Bob is

$$n_p = N \frac{Tr(\rho\hat{n})}{N} = \frac{N}{2}(|\beta_0|^2 + |\beta_1|^2) \leq N,$$
where \( \hat{n} \) is the particle number operator. Then the ratio of bits transferred successfully to the average number of particles sent is therefore

\[
K \equiv \frac{n_b}{n_p} = \frac{1 - |\alpha_0^* \alpha_1 + \beta_0^* \beta_1|^2}{|\beta_0|^2 + |\beta_1|^2}.
\]

(5)

Without loss of generality, using the Bloch representation \((\alpha_i, \beta_i) = (\cos(\theta_i), e^{i\psi_i} \sin(\theta_i))\), \((i = 0, 1)\) we can rewrite \( K \) as

\[
K = \frac{1 - |\cos(\theta_0) \cos(\theta_1) + \sin(\theta_0) \sin(\theta_1) e^{i\psi}|^2}{\sin^2(\theta_0) + \sin^2(\theta_1)}
\]

\[
\leq \frac{1 - (|\cos(\theta_0) \cos(\theta_1)| - |\sin(\theta_0) \sin(\theta_1)|)^2}{\sin^2(\theta_0) + \sin^2(\theta_1)}
\]

\[
\equiv K_{\text{max}}(\theta_0, \theta_1),
\]

where \( \psi \equiv \psi_1 - \psi_0 \) and the equality of the second line is satisfied when \( \psi = 0 \) or \( \pi \) and \( \cos(\theta_0) \cos(\theta_1) \) is opposite in sign to \( \sin(\theta_0) \sin(\theta_1) e^{i\psi} \). The upper bound on \( K \) achieves value 2 asymptotically when \( \sin(\theta_0) = \pm \sin(\theta_1) \to 0 \) (but still nonzero, See Fig. 2). Note that the optimal states are near the vacuum but not the vacuum states. In the case that only one of \( \sin(\theta_0) \) and \( \sin(\theta_1) \) is 0 (i.e., when one of \( |\phi_0\rangle \) and \( |\phi_1\rangle \) is the vacuum state.), \( K \) becomes 1. \( K = 2 \) means that in our QKD scheme Alice and Bob can share secure 2 classical bits of information per single quantum particle transfer on the average in the ideal situation of no error or eavesdropping. The physical reason for this bound is that the more we send the vacuum (i.e. \( |\beta_1| \approx 0 \)), the smaller \( n_p \) is, but then it is harder to distinguish \( |\phi_1\rangle \) from \( |\phi_0\rangle \). One can improve the probability of correct classification into

\[
p = 1 - |\alpha_0^* \alpha_1 + \beta_0^* \beta_1|
\]

(7)

by optimal positive operator valued measures (POVM)\cite{26, 27} using ancilla qubits. In this case a similar argument leads to

\[
K = \frac{n_b}{n_p} = 2 \frac{1 - |\alpha_0^* \alpha_1 + \beta_0^* \beta_1|}{|\beta_0|^2 + |\beta_1|^2},
\]

(8)

which also has a maximum value 2 under the same condition stated below Eq. (6). (However, we will not consider a specific realization of the POVM measurement in this paper.) On the other hand, for ordinary QKD schemes using ordinary particle states \( K \) is usually smaller than 1, because there are always discarded data (i.e., \( n_b < N \)) to prevent Eve from distinguishing Alice’s states perfectly, while to represent a qubit one or more particles are required (i.e., \( n_p \geq N \)). For example the typical B92 scheme using two non-orthogonal photon polarization states has \( n_b = N/2 \) (because the probability to get correct sifted keys is 1/2) and \( n_p = N \), hence \( K = 1/2 \). In this sense, one can say that our B92-type scheme requires a relatively smaller (four times smaller) number of particles to be transferred to send a given classical bit information than the typical B92 quantum cryptography schemes. This ratio \( K \) is similar to the key rate per energy \( \frac{n_b}{n_p} \) but not exactly equal to that, because in QKD schemes usually a sender and a receiver consume additional energy to share their references and to communicate publicly. \( K \) is more likely a “key rate per brightness” which measure how dark the quantum channel is for a fixed information transmission rate. If we have a QKD scheme with very big \( K \), we can use the scheme to hide the quantum channel itself from eavesdroppers, who will encounter a problem to find out where and when the quantum channel opens. This fact can be especially useful for QKD schemes on a moving satellite or free-space systems. The bound for \( K \) for our scheme is non-trivial. If \( K = 2 \) is just the Holevo bound divided by 1/2 (vaguely guessed proportion of particles in the vacuum-single photon superposed states), then we should also have maximal \( K = 2 \) for the quantum memory or ordinary quantum channel but this is not the case (maximal \( K = \infty \) for these cases). Furthermore the optimal \( K \) value for our scheme even does not corresponds to the case where the average particle number in the state is 1/2. Meanwhile the fact that one can store or send multi-bits information per particle is not so surprising. In fact, it is shown that infinite information can be transferred through quantum channel at the cost of infinite entropy\cite{28}. However, what is interesting here is that although one can store or send infinite information per particle in the quantum memory or through a quantum channel using the vacuum-single particle superposition, there is a upper bound for the key rate per particle using QKD schemes with the superposition (at least for the QKD schemes considered in this paper).
FIG. 2: Contour plot of the upper bound on the $K$ values ($K_{\text{max}}$) as a function of $\sin(\theta_0)$ and $\sin(\theta_1)$. Gray level represents the value from black(0) to white(2).

III. BB84-TYPE SCHEME

Compared to the B92 scheme, the BB84 scheme is known to be more robust against the state discrimination attack. It is straightforward to extend our consideration to the BB84-type scheme with superposed states of the vacuum and single particles. As in the typical BB84 scheme, Alice sends one of four states from two classes $\{|\phi_0\rangle, |\phi_1\rangle\}$ and $\{|\phi'_0\rangle, |\phi'_1\rangle\}$ to Bob where $\langle\phi_0|\phi_1\rangle = 0 = \langle\phi'_0|\phi'_1\rangle$ and $|\phi_i\rangle$ ($i = 0, 1$) are not orthogonal to $|\phi'_i\rangle$. Then, Bob measures one of four projection operators $P_i = |\phi_i\rangle\langle\phi_i|$ or $P'_i = |\phi'_i\rangle\langle\phi'_i|$. After basis reconciliation with Alice via a public channel Bob would get the classical bit information with probability $1/2$, hence $n_b = N/2$. The density matrix of Alice’s particle is $\rho = (|\phi_0\rangle\langle\phi_0| + |\phi_1\rangle\langle\phi_1| + |\phi'_0\rangle\langle\phi'_0| + |\phi'_1\rangle\langle\phi'_1|)/4$. Therefore, the ratio of bits shared to the number of particles sent in this scheme is

$$K = \frac{4}{2(|\beta_0|^2 + |\beta_1|^2 + |\beta'_0|^2 + |\beta'_1|^2)},$$

where $|\phi_i\rangle = \alpha_i|0\rangle + \beta_i|1\rangle$ and $|\phi'_i\rangle = \alpha'_i|0\rangle + \beta'_i|1\rangle$. The orthogonality condition $\langle\phi_0|\phi_1\rangle = 0 = \langle\phi'_0|\phi'_1\rangle$ implies $|\beta_0|^2 + |\beta_1|^2 = 1 = |\beta'_0|^2 + |\beta'_1|^2$, so $K = 1$ which is twice the value of $K$ for BB84 schemes with ordinary particles, because for the ordinary BB84 protocol $n_b = N/2$ and $n_p = N$.

IV. APPARATUS AND SECURITY

We may now proceed to the description of the apparatuses for our schemes shown in Fig. 1 for the B92-type scheme and in Fig. 3 for the BB84-type scheme. The setups consist of Alice’s photon source(S) for generation of the superposition of the vacuum and single particle states, and Bob’s projective measurement device using either cavity QED (Fig. 1) or linear optics (Fig. 3) which are considered by many authors. In principle the cavity QED devices and the linear optics devices can be used both for the B92 or the BB84 scheme. The detectors are essentially the same detectors we considered in our previous work, so we will just briefly review here. Let us first consider Fig. 1. By utilizing the parametric down conversion or coherent light, the source(S) generates $\phi_0$ or $\phi_1$ on Alice’s demand. Assuming that at time $t = 0$ a ground state atom $|g\rangle$ is injected into the cavity $C$, the total cavity-atom state is then $|\psi(0)\rangle = |\phi_i\rangle|g\rangle$. The interaction between atoms and photons in the cavity $C$ are described by the Jaynes-Cummings Hamiltonian. With this Hamiltonian and by choosing interaction time appropriately one can transfer the information of photon states $|\phi_i\rangle$ to that of the atoms (See the references for details). Then the projective measurement on the photon state $\alpha|0\rangle + \beta|1\rangle$ can be possible by adjusting appropriately the field in the Ramsey zones ($R$) such that the state undergoes a unitary evolution to the state which registers a click in the state-selective ionization detector $D$. So this setup performs ultimately deterministic (i.e.,
with probability 1 for ideal cases) projection on $|B\rangle \equiv \alpha|0\rangle + \beta|1\rangle$. Let us find $|B\rangle$ such that $P_0$ in Eq. (2) can be written as $|B\rangle\langle B|$. It should satisfies $\langle B|\phi_1\rangle = 0$, because $P_0|\phi_1\rangle = 0$. In other words, to measure $P_0$ Bob should set the fields in the Ramsey zone so that the input photon state with $\alpha = \beta^*_1$ and $\beta = -\alpha^*_1$ (i.e. orthogonal to $|\phi_1\rangle$) corresponds to the click on detector $D$. Similarly Bob can measure $P_1$ by performing projection on $\beta_0^*|0\rangle - \alpha_0^*|1\rangle$.

On the other hand, the projective measurement for the BB84-type scheme using linear optics shown in Fig. 3 is non-deterministic in a sense that the measurement succeeds only probabilistically. This setup is a modification of the projective measurement for the BB84-type scheme shown in Fig. 3. The projective measurement succeeds only probabilistically. This setup is a modification of the BB84-type scheme using linear optics shown in Fig. 3.

Let us now discuss the security of our schemes. Basically our schemes follow the ordinary B92 and the BB84 schemes except for the states and measuring devices used, so one can simply adopt the well known security proof for these ordinary schemes [2] for our schemes also. Another merit of our schemes is that since the superposition of the vacuum and one photon is not a photon number eigenstate, our schemes are robust against the photon number splitting (PNS) attacks [3]. The PNS attacks restrict key rates and distance for many practical QKD schemes such as typical B92 or BB84 QKD schemes with weak coherent states.) Because, even in the case the eavesdropper (Eve) has multiple copies of the state $(|\phi\rangle^\otimes n)$ due to imperfections of the light sources, to do the PNS attack Eve should perform the photon number non-demolition measurement, but our schemes use the superposed states of the vacuum and single photon which is inevitably destroyed by any photon number measurement. This allows Alice and Bob to detect Eve attempting the PNS attack by publicly comparing parts of the qubits sent with the qubits received. So our schemes present yet another way for security against the PNS attacks different from the recently proposed schemes [3].

For our schemes sending the pure vacuum $(|\phi\rangle = |0\rangle)$ as a qubit has an intrinsic problem that Bob can not distinguish the vacuum from channel loss. But fortunately as described above the pure vacuum state is not the optimal state for the maximal $K$ value. So there is no reason to use the pure vacuum state as a qubit for our schemes and we can avoid this problem by simply not using the pure vacuum state. In a practical sense, it is experimentally interesting but

\[
\psi = (\gamma + \delta a^\dagger)(\alpha + \beta b^\dagger)|0\rangle
\]
challenging to implement the detection of a superposition of the vacuum and single-photon states\cite{23, 37}. Recently, there are many related experimental and theoretical works about transferring quantum states using the cavity\cite{38}.

V. DISCUSSION

In summary, we have proposed the B92 and the BB84-type quantum key distribution schemes using superposed states of the vacuum and the single particle state robust against PNS attacks. We showed that in our QKD schemes using the vacuum-photon superposition states the information transferred per particle sent is bounded. So far it is unclear that this restriction has more profound physical reasons. Therefore proving or disproving the existence of an exotic QKD protocol which has a big value of $K$, that is, a QKD scheme with very faint light might be an interesting subject.

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