Quantum limited sound attenuation in a dilute atomic Fermi gas:

Shaken, not stirred

Thomas Schäfer

Department of Physics, North Carolina State University, Raleigh, NC 27695

Abstract

A new experiment involving resonantly interacting atoms confined by laser beams sheds light on momentum and energy diffusion in quantum fluids.
Ordinary sound is a harmonic oscillation in the density, temperature, and velocity of air. Sound intensity decreases because of the spreading of the sound wave, but ultimately sound attenuation is due to diffusion of momentum and energy from the crest to the trough of the wave. This effect can be characterized in terms of the diffusivity $D$ of sound. In air there is a very large separation of scales between the shortest scale, the distance between molecules, an intermediate scale, the mean free path of air molecules which controls the diffusivity, and the longest scale, the wavelength of the sound mode. A very different, deeply quantum, version of sound attenuation was studied by Patel et al. and is described on page 1222 of Science 370 (2020) 6521 [1]. Their result sheds light on transport properties of strongly correlated quantum fluids [2], and has direct implications for the stability of spinning neutron stars [3].

In the experiment of Patel et al. about two million Lithium atoms are confined in a cylindrical box created by beams of laser light, see Fig. 1. The box is about 100 $\mu m$ long, and 60 $\mu m$ in radius. A typical standing wave studied in the experiment has a wave length which is only about ten times larger than the mean distance between atoms. In order to observe sharp collective modes in this regime the gas must be very strongly correlated. This is achieved by making the gas very cold, and by tuning the interaction between atoms to a resonance. The temperature of the gas is between 50 and 500 nK, which implies that the de Broglie wave length of the atoms is equal to or larger than the mean atomic distance. Here, the de Broglie wave length is the wave length of quantum mechanical wave function of the atoms. The interaction between the atoms is tuned by means of a so-called Feshbach resonance [4]. On resonance we can think of the interaction as having zero range, but infinite scattering length. This means that the wave function of two low-energy atoms is modified by interactions even if the atoms are arbitrarily far apart.

The resonant limit is referred to as the unitary Fermi gas, because the isotropic part of the scattering cross section is as large as conservation of probability (unitarity) in quantum mechanics allows it to be. The unitary Fermi gas is also scale invariant. This means that physical observables are fixed by dimensional analysis and universal functions of dimensionless ratios. We can apply this type of argument to the sound diffusivity. On dimensional grounds, $D$ is proportional to $\bar{h}/m$, where $\bar{h}$ is Planck’s constant, and $m$ is the mass of the atoms.

The constant of proportionality is determined by the detailed mechanism for energy and momentum transfer. If momentum transfer is governed by the diffusion of atoms then
FIG. 1: Sound mode in a cylindrical box filled with a dilute atomic Fermi gas. A cylindrical box made of laser beams, 100 $\mu m$ long and 60 $\mu m$ in radius, contains about $2 \cdot 10^6$ ultracold Lithium atoms. Sound wave are excited by harmonically driving the intensity of the light creating the endcaps. Sound diffusivity is measured by examining the frequency width of the resonant standing waves.

$$D \sim \bar{p}l_{mfp}/m,$$
where $\bar{p}$ is the mean momentum of an atom, and $l_{mfp}$ is the mean free path. In a classical gas $\bar{p}l_{mfp} \gg \hbar$, but we expect that in a strongly correlated gas the product of $\bar{p}$ and $l_{mfp}$ is limited by quantum uncertainty, so that $D$ is of order $\hbar/m$.

This is indeed what the results of Patel et al. demonstrate. In the experiment sound modes are excited by shaking the endcaps of the cylindrical box. The position of resonances determined the speed of sound, and the width of the resonance determines the diffusivity. Patel et al. find that the diffusivity drops as the temperature is lowered, and that it settles at a value around $D \simeq 1.5\hbar/m$ near the transition to a superfluid. This value is consistent with attempts to measure the shear viscosity and thermal conductivity of the unitary Fermi gas individually $[5, 6]$, as well as with theoretical calculations $[7]$. Below the critical temperature the unitary gas forms a superfluid which is roughly analogous to Bardeen-Cooper-Schrieffer (BCS) superconductivity, but with a parametrically large pairing gap and critical temperature. It is interesting to note that there are no sharp features in the diffusivity at the phase
transition temperature.

The results of Patel et al. have direct implications for the structure of spinning neutron stars. The matter in the outer layer of a neutron star (outside the core, but below the crust) is a dilute liquid of neutrons. The neutron-neutron scattering length is much larger than the distance between neutrons, so that the results for the unitary Fermi gas are directly applicable, even though the temperatures and densities are many orders of magnitude larger. What matters is that dimensionless ratios, such as the mean particle distance in units of the thermal de Broglie wave length, are similar.

Neutron stars have many possible modes of oscillations. A special class that arises due the Coriolis force in rotating stars is known as Rossby, or simply r-modes. These modes are unstable, and if they are not damped by momentum or energy diffusion then large amplitude r-modes would lead to strong gravitational wave emission, and a rapid spin-down of the star. Understanding the diffusivity of neutron star matter is crucial to predicting the range of allowed spin frequencies, and possible r-mode signals in gravitational wave detectors.

More generally the results of Patel et al. shed light on the mechanism of transport in other strongly correlated quantum gases, such as the quark-gluon plasma investigated in heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the the Large Hadron Collider (LHC). The quark-gluon plasma is a state of matter that existed microseconds after the Big Bang, at a temperature $T \simeq 2 \cdot 10^{12}$ K. Measurements indicate that the momentum diffusivity of the quark gluon plasma is quite low. In a relativistic setting the mass of the particles is very small, and the natural scale for $D$ is $\hbar c^2/(k_B T)$. Experiments based on the hydrodynamic expansion of the plasma give values as small as $D \simeq 0.1 \hbar c^2/(k_B T)$. This number has been interpreted in terms of holographic models inspired by advances in string theory \cite{8}. However, in relativistic heavy ion collisions the precise mechanism of momentum transport is difficult to determine. This problem can potentially be tackled in future experiments with cold gases, for example by carefully mapping the frequency dependence of the response of the gas to external perturbations.

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