Sensitivity to sterile neutrino mixings and the discovery channel at a neutrino factory

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Sensitivity of a neutrino factory to various mixing angles in a scheme with one sterile neutrino is studied using \( \nu_e \rightarrow \nu_\mu \), \( \nu_\mu \rightarrow \nu_\mu \), \( \nu_e \rightarrow \nu_\tau \) and \( \nu_\mu \rightarrow \nu_\tau \). While the “discovery-channel” \( \nu_\mu \rightarrow \nu_\tau \) is neither useful in the standard three flavor scheme nor very powerful in the sensitivity study of sterile neutrino mixings, this channel is important to check unitarity and to probe the new CP phase in the scheme beyond the standard neutrino mixing framework.

Keywords: neutrino oscillation; sterile neutrino; neutrino factory

1. Introduction

It is known that the deficit of the solar and atmospheric neutrinos are due to neutrino oscillations among three flavors of neutrinos, and these observations offer evidence of neutrino masses and mixings. The standard three flavor framework of neutrino oscillations are described by six oscillation parameters: three mixing angles \( \theta_{12}, \theta_{13}, \theta_{23} \), the two independent mass squared differences \( \Delta m^2_{21}, \Delta m^2_{31} \), and one CP violating phase \( \delta \), where \( \Delta m^2_{jk} \equiv m_j^2 - m_k^2 \) and \( m_j \) stands for the mass of the neutrino mass eigenstate. From the solar neutrino data, we have \( \sin^2 2\theta_{12}, \Delta m^2_{21} \simeq (0.86, 8.0 \times 10^{-3} \text{eV}^2) \), and from the atmospheric neutrino data we have \( \sin^2 2\theta_{23}, \Delta m^2_{31} \simeq (1.0, 2.4 \times 10^{-3} \text{eV}^2) \). On the other hand, only the upper bound on \( \theta_{13} \) is known \( \sin^2 2\theta_{13} \leq 0.19 \)[3] and no information on \( \delta \) is known at present.

To determine \( \theta_{13} \) and \( \delta \), various neutrino long baseline experiments have been proposed[1] and the ongoing and proposed future neutrino long baseline experiments with an intense beam include conventional super ( neutrino ) beam experiments such as T2K[8], NO\( \nu \)A[11], LBNE[12], T2KK[9,10], the \( \beta \) beam proposal[13] which uses a \( \nu_e \) ( \( \bar{\nu}_e \) ) beam from \( \beta \)-decays of radioactive isotopes, and the neutrino factory proposal[14] in which \( \bar{\nu}_e \) and \( \nu_\mu \) ( \( \nu_e \) and \( \bar{\nu}_\mu \) ) are produced from decays of \( \mu^- \) ( \( \mu^+ \) ). As in the

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[1] In Refs. 2, 3, 4, 5, 6, a global analysis of the neutrino oscillation data has been performed, in which a non-vanishing best-fit value for \( \theta_{13} \) is found. This result, however, is compatible with \( \theta_{13} = 0 \) at less than 2\( \sigma \), and it is not yet statistically significant enough to be taken seriously.
case of the B factories, precise measurements in these experiments allow us not only to determine the oscillation parameters precisely but also to probe new physics by looking for deviation from the standard three flavor scheme. In particular, test of unitarity is one of the important subjects in neutrino oscillations, and tau detection is crucial for that purpose. Among the proposals for future long baseline experiments, the neutrino factory facility produces a neutrino beam of the highest neutrino energy, and it is advantageous to detect $\nu_\tau$, because of the large cross section at high energy.

New physics which has been discussed in the context of neutrino oscillation includes sterile neutrinos, the non-standard interactions during neutrino propagation, the non-standard interactions at production and detection, violation of unitarity due to heavy particles etc. These scenarios, except the non-standard interactions during neutrino propagation, offer interesting possibilities for violation of three flavor unitarity. Among these possibilities, phenomenological bound of unitarity violation is typically of $O(1\%)$ in the case of non-standard interactions at production and detection, and it is of $O(0.1\%)$ in the case of unitarity violation due to heavy particles. On the other hand, the bound in the case of sterile neutrinos is of $O(10\%)$ which comes mainly from the constraints of the atmospheric neutrino data, so scenarios with sterile neutrinos seem to be phenomenologically more promising to look for than other possibilities of unitarity violation.

In this talk I will discuss phenomenology of schemes with sterile neutrinos at a neutrino factory. Schemes with sterile neutrinos have attracted a lot of attention since the LSND group announced the anomaly which suggest neutrino oscillations with mass squared difference of $O(1eV^2)$. The reason that we need one extra neutrino to account for LSND is because the standard three flavor scheme has only two independent mass squared differences, i.e., $\Delta m_{21}^2 = \Delta m_{32}^2 \approx 8 \times 10^{-5}eV^2$ for the solar neutrino oscillation, and $|\Delta m_{31}^2| = \Delta m_{atm}^2 \approx 2.4 \times 10^{-3}eV^2$ for the atmospheric neutrino oscillation, and it does not have room for the mass squared difference of $O(1eV^2)$. And the reason that the extra state has to be sterile neutrino, which is singlet with respect to the gauge group of the Standard Model, is because the number of weakly interacting light neutrinos has to be three from the LEP data. The LSND anomaly has been tested by the MiniBooNE experiment, and it gave a negative result for neutrino oscillations with mass squared difference of $O(1eV^2)$. While the MiniBooNE data disfavor the region suggested by LSND, Ref. gave the allowed region from the combined analysis of the LSND and MiniBooNE data, and it is not so clear whether the MiniBooNE data alone are significant enough to exclude the LSND region. On the other hand, even if the MiniBooNE data are taken as negative evidence against the LSND region, there still remains a possibility for sterile neutrino scenarios whose mixing angles are small enough to satisfy the constraints of MiniBooNE and the other negative results. The effect of these scenarios could reveal as violation of three flavor unitarity in the future neutrino experiments. So in this talk I will discuss sterile neutrino schemes as one of phenomenologically viable possibilities for unitarity violation, regardless of whether the LSND anomaly...
is excluded by the MiniBooNE data or not.

It has been known that sterile neutrino schemes may have cosmological problems (see, e.g., Ref. 29). However, these cosmological discussions depend on models and assumptions, and I will not discuss cosmological constraints in this talk. Also it has been pointed out in Ref. 30 that some sterile neutrino models have absorption effects even for neutrino energy below 1 TeV, but I will not take such effects into consideration for simplicity.

2. Schemes with sterile neutrinos

For simplicity I will discuss schemes with four neutrinos, although phenomenology of the schemes with two or three sterile neutrinos have also been discussed. Denoting sterile neutrinos as $\nu_s$, we have the following mixing between the flavor eigen states $\nu_\alpha$ ($\alpha = e, \mu, \tau$) and the mass eigenstates $\nu_j$ ($j = 1, \cdots, 4$):

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau \\
\nu_s
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} & U_{e4} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\
U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\
U_{s1} & U_{s2} & U_{s3} & U_{s4}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
\nu_4
\end{pmatrix}.
\]

There are two kind of schemes with four neutrinos, depending on how the mass eigenstates are separated by the largest mass squared difference. One is the (2+2)-scheme in which two mass eigenstates are separated by other two, and the other one is the (3+1)-scheme in which one mass eigenstate is separated by other three (cf. Fig. 1).

![Fig. 1. The two classes of four-neutrino mass spectra, (a): (2+2) and (b): (3+1).](image-url)

2.1. (2+2)-schemes

In this scheme the fraction of sterile neutrino contributions to solar and atmospheric oscillations is given by $|U_{s1}|^2 + |U_{s2}|^2$ and $|U_{s3}|^2 + |U_{s4}|^2$, respectively, where the

\[b\] I would like to thank J. E. Kim for calling my attention to the possibility of the absorption effect due to the transition magnetic moments of neutrinos.
mass squared differences $\Delta m^2_{21}$ and $|\Delta m^2_{43}|$ are assumed to be those of the solar and atmospheric oscillations. The experimental results show that mixing among active neutrinos give dominant contributions to both the solar and atmospheric oscillations. In particular, in Fig. 19 of Ref. [38] we can see that at the 99% level $|U_{s1}|^2 + |U_{s2}|^2 \leq 0.25$ and $|U_{s3}|^2 + |U_{s4}|^2 \leq 0.25$, from the solar and atmospheric oscillations, respectively, and this contradicts the unitarity condition $\sum_{j=1}^{4} |U_{sj}|^2 = 1$. In fact the (2+2)-schemes are excluded at 5.1$\sigma$ CL. This conclusion is independent of whether we take the LSND data into consideration or not and I will not consider (2+2)-schemes in this talk.

2.2. (3+1)-schemes

Phenomenology of the (3+1)-scheme is almost the same as that of the standard three flavor scenario, as far as the solar and atmospheric oscillations are concerned. On the other hand, this scheme has tension between the LSND data and other negative results of the short baseline experiments. Among others, the CDHSW [33] and Bugey [34] experiments give the bound on $1 - P(\nu_\mu \rightarrow \nu_\mu)$ and $1 - P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, respectively, and in order for the LSND data to be affirmative, the following relation has to be satisfied [35, 36]:

$$\sin^2 2\theta_{\text{LSND}}(\Delta m^2) < \frac{1}{4} \sin^2 2\theta_{\text{Bugey}}(\Delta m^2) \cdot \sin^2 2\theta_{\text{CDHSW}}(\Delta m^2),$$  

(1)

where $\theta_{\text{LSND}}(\Delta m^2)$, $\theta_{\text{CDHSW}}(\Delta m^2)$, $\theta_{\text{Bugey}}(\Delta m^2)$ are the value of the effective two-flavor mixing angle as a function of the mass squared difference $\Delta m^2$ in the allowed region for LSND ($\bar{\nu}_\mu \rightarrow \bar{\nu}_e$), the CDHSW experiment ($\nu_\mu \rightarrow \nu_\mu$), and the Bugey experiment ($\bar{\nu}_e \rightarrow \bar{\nu}_e$), respectively. The (3+1)-scheme to account for LSND in terms of neutrino oscillations is disfavored because eq. (1) is not satisfied for any value of $\Delta m^2$. This argument has been shown quantitatively by Ref. [38] including the atmospheric neutrino data and other negative results. In Fig.2 the right hand side of the lines denoted as “null SBL 90% (99%)” is the excluded region at 90% (99%) CL by the atmospheric neutrino data and all the negative results of short baseline experiments, whereas the allowed region by the combined analysis of the LSND and MiniBooNE data at 90% (99%) CL is also shown.

In the following discussions I will assume the mass pattern depicted in Fig.1(b) because the inverted (3+1)-scheme is disfavored by cosmology, and I will also assume for simplicity that the largest mass squared difference $\Delta m^2_{41}$ is larger than $O(0.1\text{eV}^2)$, so that I can average over rapid oscillations due to $\Delta m^2_{41}$ in the long baseline experiments as well as in the atmospheric neutrino observations.

3. Sensitivity of a neutrino factory to the sterile neutrino mixings

3.1. Neutrino factories

Unlike conventional long baseline neutrino experiments, neutrino factories use muon decays $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ and $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ to produce neutrinos. In the setup suggested in the Physics Report [4] of International Scoping Study for a future Neutrino
Factory and Super-Beam facility, muons of both polarities are accelerated up to $E_\mu = 20$ GeV and injected into one storage ring with a geometry that allows to aim at two far detectors, the first located at 4000 km and the second at 7500 km from the source. The reason to put far detectors at two locations is to resolve so-called parameter degeneracy. The useful channels at neutrino factories are the following:

- $\nu_e \rightarrow \nu_\mu$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$: the golden channel
- $\nu_\mu \rightarrow \nu_\mu$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$: the disappearance channel
- $\nu_e \rightarrow \nu_\tau$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$: the silver channel
- $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$: the discovery channel

At neutrino factories, electrons and positrons produced out of $\nu_e$ and $\bar{\nu}_e$ create electromagnetic showers, which make it difficult to identify their charges. On the other hand, charge identification is much easier for $\mu$ detection, so the golden channel $\nu_e \rightarrow \nu_\mu$ is used unlike the conventional long baseline neutrino experiments which use $\nu_\mu \rightarrow \nu_e$. The golden channel turns out to be powerful because of very low backgrounds. The disappearance channel is also useful because of a lot of statistics. The golden and disappearance channels are observed by looking for muons with magnetized iron calorimeters. On the other hand, the silver and discovery channels are observed by looking for $\tau$’s with emulsion cloud chambers (nonmagnetized or

\footnote{It has been known that this channel is not useful in the standard three flavor framework. On the other hand, once one starts studying physics beyond the standard three flavor scenario, this channel becomes very important. This is the reason why it is called the discovery channel.}
magnetized, and the statistics of the silver channel is limited. The silver channel is useful to resolve parameter degeneracy. Combination of the golden, disappearance and discovery channels is expected to enable us to check unitarity.

3.2. Sensitivity of a neutrino factory with far detectors

Ref. [39] studied sensitivity of a neutrino factory with far detectors to sterile neutrino mixings. The setup is the following: the muon energy is 20GeV, the number of useful muons is $5 \times 10^{20}$ $\mu^-$'s and $\mu^+$'s per year, the measurements are supposed to continue for 4 years, the baseline lengths are $L=4000$km and $L=7500$km, the volume of each magnetized iron calorimeter at the two distances is 50kton, that of each magnetized emulsion cloud chamber at the two distances is 4kton, and the statistical as well as systematic errors and the backgrounds are taken into account.

At long baseline lengths such as $L=7500$km, matter effects become important. The oscillation probability in constant-density matter can be obtained by the formalism of Kimura, Takamura and Yokomakura. [55,56] The oscillation probability in matter can be written as

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{j<k} \text{Re}(\tilde{X}^\beta_j \tilde{X}^{\beta*}_k) \sin^2(\Delta E_{jk} L/2)$$

$$- 2 \sum_{j<k} \text{Im}(\tilde{X}^\beta_j \tilde{X}^{\beta*}_k) \sin(\Delta E_{jk} L),$$

where $\tilde{X}^\alpha_j \equiv \tilde{U}_{\alpha j} \tilde{U}^*_{\beta j}$, $\Delta E_{jk} \equiv E_j - E_k$, $E_j$ and $\tilde{U}_{\alpha j}$ are the energy eigenvalue and the neutrino mixing matrix element in matter defined by $U \text{diag}(0, \Delta E_{21}, \Delta E_{31}, \Delta E_{41}) U^{-1} + \text{diag}(A_e, 0, 0, A_n) = U \text{diag}(\tilde{E}_1, \tilde{E}_2, \tilde{E}_3, \tilde{E}_4) \tilde{U}^{-1}$ ($\Delta E_{jk} \equiv E_j - E_k \simeq \Delta m^2_{jk} / 2E \equiv (m_j^2 - m_k^2) / 2E$). The matter potentials $A_e, A_n$ are given by $A_e = \sqrt{2} G_F N_e$, $A_n = G_F N_n / \sqrt{2}$, where $N_e$ and $N_n$ are the density of electrons and neutrinos, respectively. The neutrino energy $E$ and the baseline length $L$ which are typical at a neutrino factory satisfy $|\Delta m^2_{31} L / 4E| \sim O(1)$, $|\Delta m^2_{31} L / 4E| \ll 1$ and $|\Delta m^2_{31} L / 4E| \gg 1$, and the energy eigenvalues in this case to the lowest order in the small mixing angles and to first order in $(|\Delta m^2_{31}| / |\Delta m^2_{41}|)$ are $\tilde{E}_1 \sim \Delta E_{31}, \tilde{E}_2 \sim 0, \tilde{E}_3 \sim A_e, \tilde{E}_4 \sim \Delta E_{41}$. It can be shown that the 4-th component $X^{\alpha\beta}_4$ in matter is the same as that in vacuum: $X^{\alpha\beta}_4 \equiv U_{\alpha j} U^{*\beta j}$ has been also introduced for the quantity in vacuum. On

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\[d\] In Ref. [39] an analysis was performed also for the case of muon energy 50GeV and the baseline lengths $L=3000$km and $L=7500$km, and it was shown that sensitivity with $e$ detectors increases for 50GeV because of higher statistics. In this talk, however, I will only mention the results for the neutrino factory with muon energy 20GeV for simplicity.

\[e\] Another proof of the KTY formalism was given in Refs. [55,56] and it was extended to four neutrino schemes in Refs. [57,56] Analytic forms of the oscillation probability in the (3+1)-scheme were also given in Ref. [58].
the other hand, other three components are given by
\[
\begin{align*}
\tilde{X}_1^{\beta\alpha} &= -\Delta E_{21}^{-1} \tilde{E}_{31}^{-1} \{ X_4^{\beta\alpha} \tilde{E}_3 \tilde{E}_3 + (\tilde{E}_2 + \tilde{E}_3) P^{\beta\alpha} + Q^{\beta\alpha} \} \\
\tilde{X}_2^{\beta\alpha} &= +\Delta E_{21}^{-1} \tilde{E}_{32}^{-1} \{ X_4^{\beta\alpha} \tilde{E}_3 \tilde{E}_1 + (\tilde{E}_4 + \tilde{E}_1) P^{\beta\alpha} + Q^{\beta\alpha} \} \\
\tilde{X}_3^{\beta\alpha} &= -\Delta E_{31}^{-1} \tilde{E}_{32}^{-1} \{ X_4^{\beta\alpha} \tilde{E}_1 \tilde{E}_2 + (\tilde{E}_1 + \tilde{E}_2) P^{\beta\alpha} + Q^{\beta\alpha} \},
\end{align*}
\]
where
\[
\begin{align*}
P^{\beta\alpha} &= \{ A(X_4^{ee} + X_4^{ss}/2) - A_{\alpha\alpha} - A_{\beta\beta} \} X_4^{\beta\alpha} + \Delta E_{31} X_3^{\beta\alpha} + \Delta E_{21} X_2^{\beta\alpha} \\
Q^{\beta\alpha} &= X_4^{\beta\alpha} \{ A_{\alpha\alpha} + A_{\alpha\beta} + A_{\beta\beta} + A_{\alpha\alpha} + A_{\beta\beta} - A(A_{\alpha\alpha} + A_{\beta\beta})(X_4^{ee} + X_4^{ss}/2) \} \\
&- \Delta E_{31}(\Delta E_{31} + A_{\alpha\alpha} + A_{\beta\beta})X_3^{\beta\alpha} \\
&- \Delta E_{21}((\Delta E_{21} + A_{\alpha\alpha} + A_{\beta\beta})X_2^{\beta\alpha} \\
&+ A\Delta E_{31}(X_4^{\beta\alpha} X_4^{\alpha\alpha} + X_3^{\beta\alpha} X_4^{\alpha\alpha} + X_4^{\beta\alpha} X_3^{\alpha\alpha} + X_3^{\beta\alpha} X_4^{\alpha\alpha}) \\
&+ A\Delta E_{21}(X_4^{\beta\alpha} X_2^{\alpha\alpha} + X_2^{\beta\alpha} X_4^{\alpha\alpha} + X_4^{\beta\alpha} X_2^{\alpha\alpha} + X_2^{\beta\alpha} X_4^{\alpha\alpha}).
\end{align*}
\]
In Eq. (3) $A_{\alpha\alpha} = A_{e\alpha\alpha} + A_{e\alpha\alpha}$ is the matrix element of the matter potential, and no sum is understood over the indices $\alpha, \beta$. If the sterile neutrino mixings $X_i^{\alpha\beta}$ ($\alpha = e, \mu, \tau$) are small, then $\tilde{X}_j^{\beta\alpha}$ ($j = 1, 2, 3$) reproduce those for the standard three flavor case. These sterile neutrino mixings $X_i^{\alpha\beta}$ appear in the coefficients $\tilde{X}_j^{\beta\alpha}$ ($j = 1, 2, 3$) in front of the sine factors $\sin^{2}(\Delta E_{jk}L/2)$, so we can get information on the sterile neutrino mixings from precise measurements of the coefficients of the oscillation mode $\sin^{2}(\Delta E_{jk}^2L/4E)$ ($j, k = 1, 2, 3$), which are the dominant contribution to the probability. We have evaluated sensitivity numerically by taking matter effects into account, and the results are given in Figs. 2-5. Since we have assumed $\Delta m_{41}^{2} > O(0.1 eV^{2})$, the results for $\Delta m_{41}^{2} < 0.1 eV^{2}$ are not given in the figures. The advantage of measurements with the far detectors is that sensitivity is independent of $\Delta m_{41}^{2}$ and it is good even for lower values of $\Delta m_{41}^{2}$ in most cases. In particular, in the case of the golden channel $\nu_{e} \rightarrow \nu_{\mu}$, the far detectors improve the present bound on $4|U_{e4}|^2|U_{\mu4}|^2$ by two orders of magnitude for all the values of $\Delta m_{41}^{2} > O(0.1 eV^{2})$. The neutrino factory with far detectors, therefore, can provide a very powerful test of the LSND anomaly. Their disadvantage of measurements with the far detectors is that sensitivity is not as good as that of the near detectors, which will be described in the next subsection, at the peak.

### 3.3. Sensitivity of a neutrino factory with near detectors

In my talk I skipped the discussions on sensitivity of a neutrino factory with near detectors, but because of recent interest on the near detector issues, I will describe sensitivity of measurements with near detectors for the sake of completeness. In Ref. 40 sensitivity of a neutrino factory with near detectors to sterile neutrino mixings was studied. The setup used in this analysis is the following: the muon flux is 20 GeV, the number of useful muons is $2 \times 10^{20} \mu^{-} s^{-1}$ per year, the measurements are supposed to continue for 5 years, the volume of a magnetized iron calorimeter at the distance $L=40$ km is 40 kton, that of an emulsion cloud chamber.
Fig. 3. Sensitivity to $4|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2)$ in the (3+1)-scheme of a 20 GeV neutrino factory with Far Detectors or with Near Detectors. The excluded regions by CDHSW, by CCFR and by the MiniBooNE $\nu_\mu$ data are also shown.

Fig. 4. Sensitivity to $4|U_{e4}|^2|U_{\tau 4}|^2$ in the (3+1)-scheme of a 20 GeV neutrino factory with Far Detectors or with Near Detectors. The excluded regions by NOMAD and by CHORUS are also shown.

At $L=1\text{km}$ is 1kton, and the statistical errors and the backgrounds are taken into account.

At such short baselines, $|\Delta m_{31}^2 L/2E| \sim \mathcal{O}(1) \gg |\Delta m_{31}^2 L/2E| \gg |\Delta m_{21}^2 L/2E|$

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$^5$ In this analysis the effects of the systematic errors were not taken into account. Their results can be refined in the future.
is satisfied, so the only relevant mass squared difference is $\Delta m_{41}^2$. So we have the following oscillation probabilities:

$$P(\nu_e \to \nu_\mu) \simeq 4 |U_{e4}|^2 |U_{\mu 4}|^2 \sin^2(\Delta m_{41}^2 L/4E)$$

$$P(\nu_\mu \to \nu_\mu) \simeq 1 - 4|U_{\mu 4}|^2 (1 - |U_{\mu 4}|^2) \sin^2(\Delta m_{41}^2 L/4E)$$

$$P(\nu_e \to \nu_\tau) \simeq 4 |U_{e4}|^2 |U_{\tau 4}|^2 \sin^2(\Delta m_{41}^2 L/4E)$$

$$P(\nu_\mu \to \nu_\tau) \simeq 4 |U_{\mu 4}|^2 |U_{\tau 4}|^2 \sin^2(\Delta m_{41}^2 L/4E)$$

Thus we can determine $4|U_{\alpha 4}|^2|U_{\beta 4}|^2$ or $4|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2)$ from the coefficient of the dominant oscillation mode $\sin^2(\Delta m_{41}^2 L/4E)$. The results are shown in Figs.2 and 5. The mass squared difference for which this neutrino factory setup has the best performance depends on the baseline length $L$, and in the present case it is approximately $10eV^2$. The advantage of measurements with the near detectors is that sensitivity to the sterile neutrino mixings is very good at the peak while their disadvantage is that sensitivity becomes poorer for lower values of $\Delta m_{41}^2$. From these results, we conclude that the near and far detectors are complementary in their performance.

4. The CP phases due to new physics

The results in the previous section suggest that the discovery channel $\nu_\mu \to \nu_\tau$ may not be so powerful in giving the upper bound on the mixing angles. To see the role of the discovery channel, let us now consider the effects of the CP phases in neutrino oscillations.
4.1. T violation in four neutrino schemes

In matter T violation $P_{\alpha\beta} - P_{\beta\alpha} \equiv P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha)$ is more useful than CP violation $P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$, so I will discuss T violation in four neutrino schemes\(^6\).

In the three flavor scheme it is known that T violation is given by

$$P_{\alpha\beta} - P_{\beta\alpha} = -16 \text{Im}(\tilde{X}_1^{\beta\alpha} \tilde{X}_2^{\beta\alpha*}) \sin \frac{\Delta E_{21} L}{2} \sin \frac{\Delta E_{31} L}{2} \sin \frac{\Delta E_{32} L}{2}. \quad (4)$$

The Jarlskog factor $\text{Im}(\tilde{X}_1^{\beta\alpha} \tilde{X}_2^{\beta\alpha*})$ can be written as\(^7\)

$$\text{Im}(\tilde{X}_1^{\beta\alpha} \tilde{X}_2^{\beta\alpha*}) = \text{Im}(X_1^{\beta\alpha} X_2^{\beta\alpha*}) \Delta E_{21} \Delta E_{31} \Delta E_{32}/\Delta E_{21} \Delta E_{31} \Delta E_{32}. \quad (5)$$

If $|\Delta E_{31} L| \sim O(1)$, then the differences of the eigenvalues in this case are all of $O(\Delta E_{31})$ in the zeroth order in $\sin^2 \theta_{13}$. In that case the product of the sine factors $\prod_{j<k} \sin(\Delta E_{j,k} L/2)$ is of $O(1)$, but in Eq.\((5)\) we have a suppression factor $|\Delta E_{21}/\Delta E_{21}| \sim |\Delta m_{21}^2/\Delta m_{31}^2| \sim 1/30$.

On the other hand, in the case of the four neutrino schemes, Eq.\((4)\) is replaced by

$$P_{\alpha\beta} - P_{\beta\alpha} = -16 \text{Im}(\tilde{X}_1^{\beta\alpha} \tilde{X}_2^{\beta\alpha*}) \sin \frac{\Delta E_{21} L}{2} \sin \frac{\Delta E_{41} L}{2} \sin \frac{\Delta E_{42} L}{2}$$

$$-16 \text{Im}(\tilde{X}_1^{\beta\alpha} \tilde{X}_3^{\beta\alpha*}) \sin \frac{\Delta E_{31} L}{2} \sin \frac{\Delta E_{41} L}{2} \sin \frac{\Delta E_{43} L}{2}$$

$$-16 \text{Im}(\tilde{X}_2^{\beta\alpha} \tilde{X}_3^{\beta\alpha*}) \sin \frac{\Delta E_{32} L}{2} \sin \frac{\Delta E_{42} L}{2} \sin \frac{\Delta E_{43} L}{2}. \quad (6)$$

In the case of neutrino energy with $|\Delta E_{31} L| \sim O(1)$, the dominant contribution in Eq.\((6)\) to the leading order in the small mixing angles is given by

$$P_{\alpha\beta} - P_{\beta\alpha} \simeq \sum_{(j,k)=(1,2),(1,3),(2,3)} 4 \text{Im}(\tilde{X}_j^{\beta\alpha} \tilde{X}_k^{\beta\alpha*}) \sin \Delta E_{j,k} L, \quad (7)$$

where we have averaged over rapid oscillations due to $\Delta m_{41}^2$, i.e., $\lim_{x \rightarrow \infty} x \sin x \sin(x + \theta) = \cos \theta/2$. To compare T violation in the different schemes or in different channels, we have only to compare the Jarlskog factors $\text{Im}(X_j^{\beta\alpha} X_k^{\beta\alpha*})$ in Eqs.\((5)\) and \((6)\), since the sine factors $\prod_{j<k} \sin(\Delta E_{j,k} L/2)$ in Eq.\((4)\) and $\sin \Delta E_{j,k} L$ in Eq.\((7)\) are both of $O(1)$. If the sterile neutrino mixing angles are roughly as large as $\theta_{13}$,

\(^6\) Note that we are not claiming that T violation can be measured experimentally for all the channels. The oscillation probability can be always decomposed into T conserving and T violating terms, $P_{\alpha\beta} = (P_{\alpha\beta} + P_{\beta\alpha})/2 + (P_{\alpha\beta} - P_{\beta\alpha})/2$, and the second term is proportional to $\sin \delta$ in the standard three flavor framework\(^8\) in constant-density matter, as in the case in vacuum. So T violation is phenomenologically suitable to examine $\delta$. In the case of CP violation, on the other hand, CP is violated in matter even if the CP phase vanishes. In practice, people perform a numerical analysis by fitting the hypothetical oscillation probability to the full data including neutrinos and anti-neutrinos, instead of measuring $P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$ or $P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha)$. So discussions on CP violation or T violation should be regarded as tools to help us understand the results intuitively.

\(^7\) Since $\text{Im}(\tilde{X}_1^{\beta\alpha} \tilde{X}_2^{\beta\alpha*}) = 0$, we must consider $\text{Im}(\tilde{X}_1^{\beta\alpha} \tilde{X}_3^{\beta\alpha*})$ or $\text{Im}(\tilde{X}_2^{\beta\alpha} \tilde{X}_3^{\beta\alpha*})$.\(^8\)
then it turns out the dominant contribution to the Jarlskog factor comes from
\( \text{Im}(X_3^\beta \alpha X_4^{\beta \alpha^*}) \), which appears in \( \text{Im}(X_j^{\beta \alpha} X_k^{\beta \alpha^*}) \) with coefficients of \( \mathcal{O}(1) \) if we plug
Eq. (2) in Eq. (7). Furthermore, let us introduce a parametrization for the \( 4 \times 4 \) mixing matrix with three CP phases \( \delta_i \):

\[
U = R_{34}(\theta_{34}, 0) R_{24}(\theta_{24}, 0) R_{23}(\theta_{23}, \delta_3) R_{14}(\theta_{14}, 0) R_{13}(\theta_{13}, \delta_2) R_{12}(\theta_{12}, \delta_1),
\]

where \( R_{ij}(\theta_{ij}, \delta_i) \) are the \( 4 \times 4 \) complex rotation matrices defined by

\[
[R_{ij}(\theta_{ij}, \delta_i)]_{pq} = \begin{cases} 
\cos \theta_{ij} & p = q = i, j \\
1 & p = q \neq i, j \\
\sin \theta_{ij} e^{-i \delta_i} & p = i; q = j \\
-\sin \theta_{ij} e^{i \delta_i} & p = j; q = i \\
0 & \text{otherwise}.
\end{cases}
\]

\( \theta_{14} \) stands for the mixing angle in short baseline reactor neutrino oscillations, and
\( \theta_{24} (\theta_{34}) \) represents the ratio of the oscillation modes due to \( \Delta m_{21}^2 \) and \( \Delta m_{24}^2 \) (the ratio of the active and sterile neutrino oscillations) in the atmospheric neutrinos, respectively. In the limit when these extra mixing angles \( \theta_{j4} (j = 1, 2, 3) \) become zero, \( \delta_2 \) becomes the standard CP phase in the three flavor scheme. The explicit forms of the mixing matrix elements \( U_{\alpha j} \) can be found in the Appendix A in Ref. [39].

From the constraints of the short baseline reactor experiments and the atmospheric neutrino data, these angles are constrained as [33] \( \theta_{14} \lesssim 10^\circ \), \( \theta_{24} \lesssim 12^\circ \), \( \theta_{34} \lesssim 30^\circ \). If we assume the upper bounds for \( \theta_{j4} (j = 1, 2, 3) \) and \( \theta_{13} \), for which we have \( \theta_{13} \lesssim 13^\circ \), then together with the best fit values for the solar and atmospheric oscillation angles \( \theta_{12} \simeq 30^\circ \), \( \theta_{23} \simeq 45^\circ \), we obtain the following Jarlskog factor:

\[
4 |\text{Im}(X_3^{\mu} X_4^{\mu*})|_{\text{4flavor}} \simeq 4 |s_{23}s_{13}s_{14}s_{24} \sin(\delta_3 - \delta_2)| \lesssim 0.02 |\sin(\delta_3 - \delta_2)|
\]

\[
4 |\text{Im}(X_3^{\tau} X_4^{\tau*})|_{\text{4flavor}} \simeq 4 |s_{23}s_{13}s_{24} \sin \delta_3| \lesssim 0.2 |\sin \delta_3|
\]

for the \( (3+1)\)-scheme, where \( c_{jk} \equiv \cos \theta_{jk} \) and \( s_{jk} \equiv \sin \theta_{jk} \). These results should be compared with the standard Jarlskog factor:

\[
4 |\text{Im}(X_1^{\mu} X_2^{\mu*})|_{\text{3flavor}} = (1/2) |c_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \sin \delta_1| \lesssim 0.2 |\sin \delta_1| \sim \mathcal{O}(1),
\]

Notice that the Jarlskog factor is independent of the flavor \( (\alpha, \beta) \) in the three flavor case. Assuming that all the CP phases are maximal, i.e., \( |\sin \delta_3| \sim |\sin(\delta_3 - \delta_2)| \sim |\sin \delta| \sim \mathcal{O}(1) \), the ratio of T violation in the \( (3+1)\)-scheme for the two channel and that in the standard three flavor scheme is given by

\[
|P_{\mu \tau} - P_{\mu e}|_{\text{4flavor}} : |P_{\mu \tau} - P_{\mu e}|_{\text{3flavor}} : |P_{\mu \mu} - P_{\mu e}|_{\text{3flavor}} \sim 0.02 : 0.2 : 0.006.
\]

Note that the term which would reduce to the three flavor T violation in the limit \( \theta_{j4} \rightarrow 0 \) \((j = 1, 2, 3)\) is contained in Eq. (7) as a subdominant contribution which is suppressed by \( |\Delta m_{21}^2/\Delta m_{24}^2| \sim 1/30 \). From this we see that dominant contribution to T violation in the \( (3+1)\)-scheme could be potentially much larger when measured with the discovery channel than that in the \( (3+1)\)-scheme with the golden channel or than that in the standard three flavor scheme. In fact it was shown in Ref. [39] by a detailed analysis that the CP phase may be measured using the discovery and disappearance channels.
4.2. CP violation in unitarity violation due to heavy fields

In generic see-saw models the kinetic term gets modified after integrating out the right handed neutrino and unitarity is expected to be violated. In the case of the so-called minimal unitarity violation, in which only three light neutrinos are involved and sources of unitarity violation are assumed to appear only in the neutrino sector, deviation from unitarity is strongly constrained from the rare decays of charged leptons. Expressing the nonunitary mixing matrix $N$ as $N = (1 + \eta)U$, where $U$ is a unitary matrix while $\eta$ is a hermitian matrix which stands for deviation from unitarity, the bounds are typically $|\eta_{\alpha\beta}| < \mathcal{O}(0.1\%)$. The CP asymmetry in this scenario in the two flavor framework can be expressed as:

$$\frac{P_{\alpha\beta} - P_{\bar{\alpha}\bar{\beta}}}{P_{\alpha\beta} + P_{\bar{\alpha}\bar{\beta}}} \approx \frac{-4|\eta_{\alpha\beta}| \sin(\arg(\eta_{\alpha\beta}))}{\sin(2\theta) \sin(\Delta m^2/2)}.$$ 

The constraint on $\eta_{\mu\tau}$ is weaker than on $\eta_{e\mu}$, and it was shown that the CP violating phase $\arg(\eta_{\alpha\beta})$ may be measured at a neutrino factory with the discovery channel.

5. Summary

In this talk I described sensitivity of a neutrino factory to the sterile neutrino mixings. The golden channel $\nu_e \to \nu_\mu$ improves the present upper bound on $4|U_{e4}|^2|U_{\mu4}|^2$ by two orders of magnitude, and provides a powerful test for the LSND anomaly. It is emphasized that $\tau$ detection at a neutrino factory is important to check unitarity, and the discovery channel $\nu_\mu \to \nu_\tau$ is one of the promising channels to look for physics beyond the standard three flavor scenario. We may be able measure the new CP violating phase using this channel in the sterile neutrino schemes and in the scenario with unitarity violation due to heavy particles.

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References

1. C. Amsler et al. [Particle Data Group], Phys. Lett. B 667 (2008) 1.
2. G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A. M. Rotunno, Phys. Rev. Lett. 101 (2008) 141801 [arXiv:0806.2649 [hep-ph]].
3. G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A. M. Rotunno, arXiv:0809.2936 [hep-ph].
4. H. L. Ge, C. Giunti and Q. Y. Liu, arXiv:0810.5443 [hep-ph].
5. G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A. M. Rotunno, arXiv:0905.3549 [hep-ph].
6. M. C. Gonzalez-Garcia, M. Maltoni and J. Salvado, arXiv:1001.4524 [hep-ph].
7. A. Bandyopadhyay et al. [ISS Physics Working Group], Rept. Prog. Phys. 72, 106201 (2009) [arXiv:0910.4947 [hep-ph]].
8. Y. Itow et al. [The T2K Collaboration], arXiv:hep-ex/0106019.
9. M. Ishitsuka, T. Kajita, H. Minakata and H. Nunokawa, Phys. Rev. D 72, 033003 (2005) [arXiv:hep-ph/0504026].
10. K. Hagiwara, N. Okamura and K. i. Senda, Phys. Lett. B 637, 266 (2006) [Erratum-ibid. B 641, 486 (2006)] [arXiv:hep-ph/0504061].
11. D. S. Ayres et al. [NOvA Collaboration], arXiv:hep-ex/0503053.
12. J. Maricic [LBNE DUSEL Collaboration], J. Phys. Conf. Ser. 203, 012109 (2010).
13. P. Zucchelli, Phys. Lett. B 532 (2002) 166.
14. S. Geer, Phys. Rev. D 57 (1998) 6989 [Erratum-ibid. D 59 (1999) 039903] [arXiv:hep-ph/9712290].
15. Some of the references on sterile neutrinos are found at the Neutrino Unbound web page, by C. Giunti and M. Laveder, http://www.nu.to.infn.it/Sterile_Neutrinos/.
16. Belle experiment, http://belle.kek.jp/.
17. Babar experiment, http://www-public.slac.stanford.edu/babar/.
18. L. Wolfenstein, Phys. Rev. D 17, 2369 (1978).
19. M. M. Guzzo, A. Masiero and S. T. Petcov, Phys. Lett. B 260 (1991) 154.
20. E. Roulet, Phys. Rev. D 44 (1991) 935.
21. Y. Grossman, Phys. Lett. B 359 (1995) 141 [arXiv:hep-ph/9507344].
22. S. Antusch, C. Biggio, E. Fernandez-Martinez, M. B. Gavela and J. Lopez-Pavon, JHEP 0610 (2006) 084 [arXiv:hep-ph/0607020].
23. S. Antusch, J. P. Baumann and E. Fernandez-Martinez, Nucl. Phys. B 810, 369 (2009) [arXiv:0807.1003 [hep-ph]].
24. A. Donini, M. Maltoni, D. Meloni, P. Migliozzi and F. Terranova, JHEP 0712, 013 (2007) [arXiv:0704.0388 [hep-ph]].
25. C. Athanassopoulos et al. [LSND Collaboration], Phys. Rev. Lett. 77 (1996) 3082 [arXiv:nucl-ex/9605003].
26. C. Athanassopoulos et al. [LSND Collaboration], Phys. Rev. Lett. 81 (1998) 1774 [arXiv:nucl-ex/9709006].
27. A. Aguilar et al. [LSND Collaboration], Phys. Rev. D 64 (2001) 112007 [arXiv:hep-ex/0104049].
28. A. A. Aguilar-Arevalo et al. [The MiniBooNE Collaboration], Phys. Rev. Lett. 98, 231801 (2007) [arXiv:0704.1590 [hep-ex]].
29. A. Y. Smirnov and R. Zukavovich Funchal, Phys. Rev. D 74, 013001 (2006) [arXiv:hep-ph/0603009].
30. J. E. Kim, Phys. Rev. Lett. 41, 360 (1978).
31. M. Sorel, J. M. Conrad and M. Shaevitz, Phys. Rev. D 70, 073004 (2004) [arXiv:hep-ph/0305255].
32. M. Maltoni and T. Schwetz, Phys. Rev. D 76, 093005 (2007) [arXiv:0705.0107 [hep-ph]].
33. F. Dydk et al., Phys. Lett. B 134, 281 (1984).
34. Y. Decais et al., Nucl. Phys. B 434, 503 (1995).
35. N. Okada and O. Yasuda, Int. J. Mod. Phys. A 12, 3669 (1997) [arXiv:hep-ph/9606411].
36. S. M. Bilenky, C. Giunti and W. Grimus, Eur. Phys. J. C 1, 247 (1998) [arXiv:hep-ph/9607372].
37. G. Karagiorgi, Z. Djurcic, J. M. Conrad, M. H. Shaevitz and M. Sorel, Phys. Rev. D 80, 073001 (2009) [Erratum-ibid. D 81, 039902 (2010)] [arXiv:0906.1997 [hep-ph]].
38. M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. 6, 122 (2004) [arXiv:hep-ph/0405172v6].
39. A. Donini, K. i. Fuki, J. Lopez-Pavon, D. Meloni and O. Yasuda, JHEP 0908, 041 (2009) [arXiv:0812.3703 [hep-ph]].
40. A. Donini and D. Meloni, Eur. Phys. J. C 22, 179 (2001) [arXiv:hep-ph/0105089].
41. J. Tang and W. Winter, Phys. Rev. D 80, 053001 (2009) [arXiv:0903.3039 [hep-ph]].
42. Main Injector Non Standard Interactions Search, 
http://www-off-axis.fnal.gov/MINSIS/
43. Madrid Neutrino NSI Workshop, UAM, Madrid, 10-11 December 2009,
http://www.ft.uam.es/workshops/neutrino/default.html
44. J. Burguet-Castell, M. B. Gavela, J. J. Gomez-Cadenas, P. Hernandez and O. Mena, 
Nucl. Phys. B 608, 301 (2001) [arXiv:hep-ph/0103258].
45. H. Minakata and H. Numakawa, JHEP 0110, 001 (2001) [arXiv:hep-ph/0108085].
46. G. L. Fogli and E. Lisi, Phys. Rev. D 54 (1996) 3667 [arXiv:hep-ph/9604415].
47. V. Barger, D. Marfatia and K. Whisnant, Phys. Rev. D 65, 073023 (2002) [arXiv:hep-ph/0112119].
48. A. Donini, talk at the 0th IDS plenary Meeting, CERN 29-31 March 2007, 
http://www.hep.ph.ic.ac.uk/ids/communication/cern-2007-03-29/slides/ 
IDstalk-Donini.pdf.
49. A. Cervera-Villanueva, AIP Conf. Proc. 981, 178 (2008).
50. A. Donini, D. Meloni and P. Migliozi, Nucl. Phys. B 646 (2002) 321 [arXiv:hep-ph/0206034].
51. D. Autiero et al., Eur. Phys. J. C 33 (2004) 243 [arXiv:hep-ph/0305185].
52. T. Abe et al. [ISS Detector Working Group], [arXiv:0712.4129 [physics.ins-det]].
53. K. Kimura, A. Takamura and H. Yokomakura, Phys. Lett. B 537, 86 (2002) [arXiv:hep-ph/0203099].
54. K. Kimura, A. Takamura and H. Yokomakura, Phys. Rev. D 66, 073005 (2002) [arXiv:hep-ph/0205295].
55. Z. z. Xing and H. Zhang, Phys. Lett. B 618 (2005) 131 [arXiv:hep-ph/0503118].
56. O. Yasuda, [arXiv:0704.1531 [hep-ph]].
57. H. Zhang, Mod. Phys. Lett. A 22, 1341 (2007) [arXiv:hep-ph/0606040].
58. A. Dighe and S. Ray, Phys. Rev. D 76, 113001 (2007) [arXiv:0709.0383 [hep-ph]].
59. P. Astier et al. [NOMAD Collaboration], Nucl. Phys. B 611, 3 (2001) [arXiv:hep-ex/0106102].
60. E. Eskut et al. [CHORUS Collaboration], Nucl. Phys. B 793, 326 (2008) [arXiv:0710.3361 [hep-ex]].
61. I. E. Stockdale et al., Phys. Rev. Lett. 52, 1384 (1984).
62. V. A. Naumov, Int. J. Mod. Phys. D 1, 379 (1992).
63. E. Fernandez-Martinez, M. B. Gavela, J. Lopez-Pavon and O. Yasuda, Phys. Lett. B 649, 427 (2007) [arXiv:hep-ph/0703098].
64. S. Antusch, M. Blennow, E. Fernandez-Martinez and J. Lopez-Pavon, Phys. Rev. D 80, 033002 (2009) [arXiv:0903.3986 [hep-ph]].