Revisiting the Meandering Instability During Step-Flow Epitaxy

Yue Chen

Department of Mathematics and Computer Science, Auburn University at Montgomery, Montgomery, AL 36117, USA; ychen5@aum.edu

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Abstract: This paper starts with a generalized Burton, Cabrera and Frank (BCF) model by considering the energetic contribution of the adjacent terraces to the step chemical potential. We use the linear stability analysis of the quasistatic free-boundary problem for a two-dimensional step separated by broad terraces to study the step-meandering instabilities. The results show that the equilibrium adatom coverage has influence on the morphological instabilities.

Keywords: instability; step flow; crystal growth; linear stability analysis

1. Introduction

For molecular-beam epitaxy (MBE), it is well accepted that the thin-film epitaxial growth on a vicinal surface via step flow at a sufficient high temperature constitutes an ideal growth regime [1,2]. The overview and discussion of the crystal growth can be found in References [2–6]. In the step-flow regime, the temperature, while below the surface roughening temperature, is sufficiently high to make adatom diffusion fast and, concomitantly, island nucleation improbable. Thus, the sole mechanism for growth or sublimation is the attachment of adatoms to, or their detachment from, the pre-existing steps. Specifically, during the growth on a surface consisting of terraces separated by steps, the deposited atoms diffuse across the terraces and attach to the step, diffuse along the edge and eventually incorporate into the crystalline bulk (Figure 1).

![Figure 1](https://via.placeholder.com/150)

Figure 1. Schematic view of the steps separated by broad terraces with mean width $l$ on which the deposited adatoms diffuse across the terraces and attach to the step, diffuse along the edge and eventually incorporate into the crystalline bulk.
Therefore, it is important to study the morphological instability of the terrace edge. Reference [7] showed a scanning tunneling microscopy image of a Si(001) surface which consists of broad terraces separated by steps of atomic height. The experimental and theoretical efforts have been worked on the one-dimensional structures, such as quantum wires, as the result of morphological instability during the epitaxy of self-assembling materials [5]. A step-flow model for the heteroepitaxy of a generic, strained, substitutional, binary alloy has been studied [8]. References [9–11] have investigated modeling epitaxial growth by kinetic Monte Carlo simulations. Some applications and experiments concerning epitaxy in thin films have been developed [12–14].

In 1951, Burton, Cabrera and Frank (BCF) [15] developed a model for the growth of crystals and described the crystal growth as a regular flow of edges. In the presence of the Ehrlich–Schwoebel barrier, the meandering of the periodic monatomic steps separated by broad terraces has been discussed in Reference [16]. The classical BCF model, however, does not consider the energetic contribution of the adjacent terraces to the step chemical potential and hence cannot explain some phenomena observed experimentally [5]. Therefore, the classical BCF framework was extended by considering this energetic contribution in Reference [17] which developed a thermodynamically consistent continuum theory for the step-flow epitaxy of single-component crystalline films and studied the step-bunching instabilities of one-dimensional step-flow epitaxy. Since the general theory is two-dimensional, it is appropriate to investigate the step-meandering instabilities of a two-dimensional single step separated by broad terraces. We discuss the details in Section 2.

The generalized BCF model in Reference [17] can be described by the following free-boundary problem that consists of the reaction-diffusion equation on the upper and lower terraces

$$\partial_t \rho = M \Delta \mu + F - \sigma \mu,$$  

with the step-edge boundary conditions

$$\rho^+ V + M (\partial_n \mu)^+ = C_+ (\mu^+ - \mu^s),$$

$$-\rho^- V - M (\partial_n \mu)^- = C_- (\mu^- - \mu^s),$$

and the step evolution equation

$$a \rho^b V = C_+ (\mu^+ - \mu^s) + C_- (\mu^- - \mu^s).$$

Here $\rho$ is the terrace adatom density; $\mu$ is the adatom chemical potential; $M$ is the atomic mobility; $F$ is the deposition flux; $\sigma$ is the desorption coefficient; $n$ is the unit normal to the step pointing to the lower terrace; $\partial_n \mu = \nabla \mu \cdot n$ is the normal derivative of the step chemical potential; $\mu^s$ is the step chemical potential; $\rho^b$ is the bulk atomic density; $a$ is the step height; $V$ is the velocity of the step; $C_+$ ($C_-$) is the kinetic coefficient for the attachment-detachment of adatoms from the lower (upper) terrace onto the step edge; and $(\cdot)^+$ and $(\cdot)^-$ represent the limiting value when approaching the step from the lower and upper terraces respectively.

In this paper, we consider a general step chemical potential derived in Reference [18]

$$\mu^s = \mu^b - \frac{1}{\rho^b} ([\omega] + \Psi^s \kappa),$$

where $\mu^b$ is the (nominal) crystalline chemical potential; $\Psi^s$ and $\kappa$ are the stiffness and curvature of the step respectively; and $[\omega] = \omega^+ - \omega^-$ is the the jump in the terrace grand canonical potential across the step which represents the energy contribution of the adjacent terraces to the step chemical potential. When the time scale for the step migration is large compared to the time scale for the adatom diffusion on the terraces, the reaction-diffusion Equation (1) is approximated by the corresponding steady-state counterpart on the terraces.
\[ M \Delta \mu + F - \sigma \mu = 0, \quad (5) \]

and the step-edge boundary conditions (2) are replaced by

\[ M (\partial_n \mu)^+ = C_+ (\mu^+ - \mu^s), \]
\[ -M (\partial_n \mu)^- = C_- (\mu^- - \mu^s). \quad (6) \]

Hence the step evolution Equation (3) can be formulated as

\[ a \rho^b V = M [ (\partial_n \mu)^+ - (\partial_n \mu)^- ] \quad (7) \]

In the next section, we will discuss the meandering instabilities of the two-dimensional step separated by broad terraces for the free boundary problem consisting of (5)–(7).

2. Stability Analysis

In this paper, we employ a linear stability analysis to study step-meandering. For our convenience, let \( u = \mu - \frac{F}{\sigma} \). The steady-state reaction-diffusion Equation (5) becomes the Helmholtz equation

\[ \Delta u - \frac{u}{x_s^2} = 0, \quad (8) \]

where \( x_s = \sqrt{\frac{M}{\sigma}} \) is the surface diffusion length. We introduce a constant parameter \( \Gamma = \frac{\Psi_s a \rho_b}{\omega_0} \) and a dimensionless parameter \( \Theta = \frac{\rho_{eq}}{a \rho_b} \), the equilibrium adatom coverage, which measures the equilibrium adatom density \( \rho_{eq} \) relative to the bulk atomic density \( a \rho_b \). For the adatom grand canonical potential \( \omega \) in (4), we adopt the linear approximation \( \omega \approx \omega_0 - \mu \rho_{eq} \) with a positive constant \( \omega_0 \) in [17].

Therefore, the step-edge boundary conditions (6) can be written as

\[ M (\partial_n u)^+ = C_+ [u^+ + \Phi - \Theta (u^+ - u^-) + \Gamma \kappa], \]
\[ -M (\partial_n u)^- = C_- [u^- + \Phi - \Theta (u^+ - u^-) + \Gamma \kappa], \quad (9) \]

where \( \Phi = \frac{F}{\sigma} - \mu^b. \) And the evolution equation for \( u \) is

\[ V = MN [ (\partial_n \mu)^+ - (\partial_n \mu)^- ] \quad (10) \]

with \( N = 1/(a \rho^b) \).

The general solution to the Helmholtz Equation (8) is given by the form of

\[ u(x,z) = \sum_k [A_k \sinh(\Lambda_k z) + B_k \cosh(\Lambda_k z)] [\sin(kx) + C_k \cos(kx)], \quad (11) \]

where \( \Lambda_k = \sqrt{\left( \frac{1}{x_s^2} \right) + k^2} \). Assume there is a small amplitude sinusoid perturbation with a wave vector \( k \) for the step. Then the position of the step is given by

\[ z(x) = z + \epsilon \sin(kx). \quad (12) \]

Therefore the curvature \( \kappa = -ek^2 \sin(kx) \) and the normal vector \( n = (-ck \cos(kx), 1)^T \). If we allow the accuracy of the solution \( u \) up to the first order in \( \epsilon \), the solution will be in the form of

\[ u(x,z) = u_0(z) + [A_k \sinh(\Lambda_k z) + B_k \cosh(\Lambda_k z)] \epsilon \sin(kx), \quad (13) \]

where \( u_0(z) = A \sinh(z/x_s) + B \cosh(z/x_s) \) is the solution appropriate for perfectly straight steps, that is, \( k = 0 \) is allowed in (11).
To compute the velocity of the step, we take the time derivative of (12) and get

$$V(x) = V_0 + \frac{de}{dt} \sin(kx).$$  \hfill (14)  

On the other hand, we can also use (10) to compute the velocity. Later we can see it has the form of

$$V(x) = V_0 + \omega(k) \epsilon \sin(kx),$$  \hfill (15)  

where the stability function \( \omega(k) = g(k) - k^2 f(k) \) with certain functions \( f \) and \( g \). Equations (14) and (15) imply \( \frac{de}{dt} = \omega(k) \epsilon \). It shows that the sign of \( \omega(k) \) determines the growth or decay of perturbation.

It is necessary to define the kinetic coefficients \( d_{\pm} = M/C_{\pm} \) for the attachment-detachment of adatoms from the lower and upper terrace onto the step edge and a surface capillary length \( \xi = \frac{\Gamma}{\Phi} \). These notations will be used in the following section.

### 2.1. The \( \Theta \) Dependence of the Stability Function \( \omega(k) \)

Now we consider the general situation that the kinetic coefficients for the attachment-detachment of adatoms from the lower and upper terrace onto the step edge are finite, that is, \( d_{\pm} \) are finite.

To find the solution \( u \) in the form of (13), we first need to obtain \( u_0(z) = A \sinh(z/x_s) + B \cosh(z/x_s) \) for the case of perfectly straight steps where the curvature \( \kappa = 0 \) and the normal vector \( n = (0, 1)^T \). Notice that

\[
  u_0^+ = u_0|_{z=0} = B, \\
  u_0^- = u_0|_{z=-l} = A \sinh(l/x_s) + B \cosh(l/x_s), \\
  (\partial_n u_0)^+ = (\partial_n u_0)|_{z=0} = \frac{A}{x_s}, \\
  (\partial_n u_0)^- = (\partial_n u_0)|_{z=-l} = \frac{A}{x_s} \cosh(l/x_s) + \frac{B}{x_s} \sinh(l/x_s).
\]

Consequently, the boundary condition (9) for the straight step case becomes

\[
  M \frac{A}{x_s} = C_+ (B + \Phi - \Theta [B - A \sinh(l/x_s) - B \cosh(l/x_s)]), \\
  -M \left( \frac{A}{x_s} \cosh(l/x_s) + \frac{B}{x_s} \sinh(l/x_s) \right) = C_- \left( A \sinh(l/x_s) + B \cosh(l/x_s) + \Phi - \Theta (B - A \sinh(l/x_s) - B \cosh(l/x_s)) \right). \hfill (16)
\]

\( A \) and \( B \) in \( u_0 \) can be solved from the linear system (16). Now we allow the accuracy up to the first order of \( \epsilon \).
We have \( u^+ = u|_{z=\epsilon \sin(kx)} \) and \( u^- = u|_{z=-l+\epsilon \sin(kx)} \). Then the first-order Taylor expansion about \( \epsilon \) gives that

\[
\begin{align*}
    u^+ &= B + \epsilon \sin(kx) (\frac{A}{x_s} + B_k), \\
    u^- &= \frac{A}{x_s} \sinh(l/x_s) + \frac{B}{x_s} \cosh(l/x_s) + \epsilon \sin(kx) \left[ \frac{A}{x_s} \cosh(l/x_s) + \frac{B}{x_s} \sinh(l/x_s) \\
    &\quad + A_k \sinh(\Lambda_k l) + B_k \cosh(\Lambda_k l) \right], \\
    \partial_s u^+ &= \frac{A}{x_s} + \epsilon \sin(kx) \left( \frac{B}{x_s} + A_k \Lambda_k \right), \\
    \partial_s u^- &= \frac{A}{x_s} \cosh(l/x_s) + \frac{B}{x_s} \sinh(l/x_s) + \epsilon \sin(kx) \left[ \frac{A}{x_s} \sinh(l/x_s) + \frac{B}{x_s} \cosh(l/x_s) \\
    &\quad + A_k \Lambda_k \cosh(\Lambda_k l) + B_k \Lambda_k \sinh(\Lambda_k l) \right].
\end{align*}
\]

Substituting (17) into the boundary condition (9) and equating the like powers of \( \epsilon \) show that, for the first order in \( \epsilon \), we have

\[
\begin{align*}
    M \left( \frac{B}{x_s} + A_k \Lambda_k \right) &= C_+ \left( \frac{A}{x_s} + B_k - \Theta \left( \frac{A}{x_s} + B_k - \frac{A}{x_s} \cosh(l/x_s) \\
    &\quad - \frac{B}{x_s} \sinh(l/x_s) - A_k \sinh(\Lambda_k l) - B_k \cosh(\Lambda_k l) \right) \right) - \Gamma k^2, \\
    -M \left( \frac{A}{x_s} \sinh(l/x_s) + \frac{B}{x_s} \cosh(l/x_s) + A_k \Lambda_k \cosh(\Lambda_k l) + B_k \Lambda_k \sinh(\Lambda_k l) \right) &= C_- \left( \frac{A}{x_s} \cosh(l/x_s) + \frac{B}{x_s} \sinh(l/x_s) + A_k \sinh(\Lambda_k l) + B_k \cosh(\Lambda_k l) \\
    &\quad - \Theta \left( \frac{A}{x_s} + B_k - \frac{A}{x_s} \cosh(l/x_s) - \frac{B}{x_s} \sinh(l/x_s) - A_k \sinh(\Lambda_k l) - B_k \cosh(\Lambda_k l) \right) \right) - \Gamma k^2.
\end{align*}
\]

The coefficients \( A_k \) and \( B_k \) can be solved from the linear system (18). Therefore, we obtain the solution \( u \) in the form of (13). Substituting \( u \) into (10), we can compute the velocity \( V \) of a single step in the form of

\[ V = V_0 + \omega(k) \epsilon \sin(kx). \]

Here

\[ V_0 = MN \left( \frac{A}{x_s} - \frac{A}{x_s} \cosh(l/x_s) - \frac{B}{x_s} \sinh(l/x_s) \right) \]

and

\[ \omega(k) = MN \left( \frac{B}{x_s} + A_k \Lambda_k - \frac{A \sinh(l/x_s)}{x_s^2} - \frac{B \cosh(l/x_s)}{x_s^2} - A_k \Lambda_k \cosh(\Lambda_k l) - B_k \Lambda_k \sinh(\Lambda_k l) \right) \]

which is in the form of \( g(k) - k^2 f(k) \). From the straight forward calculation, we can get

\[ f(k) = \frac{NM \Lambda_k [\Lambda_k (d_+ + d_-) \tanh(\Lambda_k l) + 2(1 - \text{sech}(\Lambda_k l))] \} \Lambda_k (d_+ - d_-) \text{sech}(\Lambda_k l) - 1 \Theta + (\Lambda_k^2 d_+ d_- + 1) \tanh(\Lambda_k l) + \Lambda_k (d_+ + d_-) \]

and

\[ g(k) = \frac{NM \Phi}{x_s^2} \cdot g, \]
The behavior of the stability function $\omega$ affects the step-meandering instability. It is interesting to see how the equilibrium adatom coverage $\Theta$ affects the step-meandering instability.

The behavior of the stability function $\omega(\mathbf{k})$ for different $\Theta$ is illustrated in Figure 2 with some reasonable parameter values. If $\omega(\mathbf{k})$ is positive, the perturbation grows. If $\omega(\mathbf{k})$ is negative, the perturbation decays. As $\Theta$ increases, the critical value $k_c$ of $\mathbf{k}$ such that $\omega(k_c) = 0$ moves to the left.

![Figure 2](image_url)

**Figure 2.** The stability function $\omega(\mathbf{k})$ for $\Theta = 0, 0.15, 0.25$. Here we take $d_+ / x_s = 5$, $d_- / x_s = 100$, $l / x_s = 0.3$, $\xi / x_s = 0.001$, $\Phi / x_s^2 = 20$.

Next we want to discuss the analytic expressions for this critical value $k_c$. Consider the special case of upper terrace blocking ($d_- \to \infty$). Then

$$f(k) \rightarrow \frac{NM\Lambda_k^2 \sinh(\Lambda_k l)}{\Lambda_k(1 - \cosh(\Lambda_k l))\Theta + \Lambda_k^2 d_+ \sinh(\Lambda_k l) + \Lambda_k \cosh(\Lambda_k l)}$$

(24)
and

\[ g(k) \rightarrow \frac{N\Phi}{x_s^2} \cdot \frac{\Lambda_k \Theta \left[ (1 + \text{sech}(l/x_s)) (\cosh(\Lambda_k l) - 1) - \Lambda_k x_s \sinh(\Lambda_k l) \tanh(l/x_s) \right] + \Lambda_k [\text{sech}(l/x_s) + \Lambda_k] \sinh(\Lambda_k l) \tanh(l/x_s) - \cosh(\Lambda_k l)]}{[(\text{sech}(l/x_s) - 1) \Theta + 1 + d_x/x_s \tanh(l/x_s)] \left[ \Lambda_k (1 - \cosh(\Lambda_k l)) \Theta + \Lambda_k^2 d_x \sinh(\Lambda_k l) + \Lambda_k \cosh(\Lambda_k l) \right]} . \] (25)

Therefore, \( \omega(k) = 0 \), that is, \( k^2 f(k) = g(k) \) gives that

\[ k^2 \Gamma \Lambda_k^2 \tanh(\Lambda_k l) = \frac{\Theta \left[ (1 + \text{sech}(l/x_s)) (1 - \text{sech}(\Lambda_k l)) - \Lambda_k x_s \tanh(\Lambda_k l) \tanh(l/x_s) \right] + \text{sech}(l/x_s) \text{sech}(\Lambda_k l) + \Lambda_k x_s \sinh(\Lambda_k l) \tanh(l/x_s) - \cosh(\Lambda_k l)]}{(\text{sech}(l/x_s) - 1) \Theta + 1 + d_x/x_s \tanh(l/x_s)} . \] (26)

The analytic expressions for \( k_c \) can be obtained from (26) in the limiting cases of very large and very small terrace widths.

When \( l >> x_s \), we have

\[ k_c x_s = \begin{cases} \frac{1}{2} \left( 1 - \frac{2\xi}{x_s} \cdot \frac{1 - \Theta}{1 - \Theta} \right) & \text{if} \ k_c x_s << 1 \\ \frac{\sqrt{2\xi}}{2} \cdot \frac{1 - \Theta}{|1 - \Theta| + d_x/x_s} & \text{if} \ k_c x_s >> 1, \end{cases} \] (27)

where \( \xi = \frac{\Gamma}{\Phi} \) is the surface capillary length. Since \( k_c \) is real, the prerequisite for morphological instability is \( \xi < \frac{x_s (1 - \Theta)}{2(1 - \Theta) + 2d_x/x_s} \). Figure 3 illustrates the dependence of \( \Theta \) on the cutoff \( \xi \) for different values of \( d_x/x_s \).

**Figure 3.** The variation of the cutoff \( \xi \) versus \( \Theta \) for various values of \( d_x/x_s \).

When \( l << x_s \) (and \( l^2 << \lambda_c^2 \), the critical wavelength \( \lambda_c = 2\pi/k_c \)),

\[ k_c x_s = \frac{1}{2\xi} \left( \frac{l}{1 + d_x/x_s} \right) - 1. \] (28)

The cutoff here is \( \frac{x_s^2}{2(x_s^2 + d_x l)} \).
Remark 1. By using a similar method, we can compute for the limiting situation where the step attachment is infinitely fast from the lower terrace and infinitely slow from the upper terrace ($d_\rightarrow \infty, d_+ = 0$). We can show that

$$V_0 = MN \left( \frac{A}{x_s} - \frac{A}{x_s} \cosh(l/x_s) - \frac{B}{x_s} \sinh(l/x_s) \right)$$

(29)

and

$$\omega(k) = MN \left( \frac{B}{x^2} + A_k \Lambda_k - \frac{A \sinh(l/x_s)}{x^2} - \frac{B \cosh(l/x_s)}{x^2} - A_k \Lambda_k \cosh(\Lambda_k l) - B_k \Lambda_k \sinh(\Lambda_k l) \right)$$

(30)

which is in the form of $g(k) - k^2 f(k)$. From the straightforward calculation, we obtain

$$f(k) = -\frac{NM \Gamma \Lambda_k \sinh(\Lambda_k l)}{\Theta(\cosh(\Lambda_k l) - 1) - \cosh(\Lambda_k l)}$$

(31)

and

$$g(k) = -\frac{NM \Phi}{x^2} \times \left\{ \begin{array}{l} \Theta[1 + \sinh(\Lambda_k l) \sinh(l/x_s) \Lambda_k x_s - \cosh(\Lambda_k l) \cosh(l/x_s) - \cosh(\Lambda_k l) + \cosh(l/x_s)] \\ [\Theta(\cosh(\Lambda_k l) - 1) - \cosh(\Lambda_k l)][\Theta(\cosh(l/x_s) - 1) - \cosh(l/x_s)] \\ + \Theta(\cosh(\Lambda_k l) - 1) - \cosh(\Lambda_k l) \sinh(l/x_s) \Lambda_k x_s - \cosh(\Lambda_k l) \cosh(l/x_s) > 0, \end{array} \right.$$  

(32)

Notice that $0 < \Theta < 1$ implies that $\Theta(\cosh(\Lambda_k l) - 1) - \cosh(\Lambda_k l) < 0$ and $\Theta(\cosh(l/x_s) - 1) - \cosh(l/x_s) < 0$. Hence $f(k)$ is positive. Since $1 + \sinh(\Lambda_k l) \sinh(l/x_s) \Lambda_k x_s - \cosh(\Lambda_k l) \cosh(l/x_s) > 0$, it also implies $g(k)$ is positive. As for the critical value $k_c$, when $l \gg x_s$, we find

$$k_c x_s = \begin{cases} \sqrt{\frac{1}{2} \left( 1 - \frac{2l^2}{x_s^2} \right)} & \text{if } k_c x_s << 1 \\ \frac{2l}{\xi} & \text{if } k_c x_s >> 1, \end{cases}$$

(33)

Since $k_c$ is real, we have a prerequisite for morphological instability $\xi < \frac{1}{2}$. when $l << x_s$ and $l^2 << \lambda^2$, we find

$$k_c x_s = \sqrt{\frac{l}{2\xi}} - 1.$$  

(34)

Now the cutoff of $\xi$ is $\frac{1}{2}$.

2.2. The $\Theta$ Dependence of the Critical Capillary Length

We also observe that there is a critical value $\xi_c$ of $\xi = \frac{1}{2}$ so that the morphologically stable step flow occurs if $\xi > \xi_c$ and the morphologically unstable step flow occurs if $\xi < \xi_c$. In order to
investigate $\xi_c$, we expand $\omega(k) = 0$ to get the critical value $\xi_c$ for small and real $k$. The following expansions are needed during the calculation:

$$\Lambda_k \approx \frac{1}{x_s} + \frac{1}{2}k^2x_s,$$

$$(\Lambda_kx_s)^2 \approx 1 + k^2x_s^2,$$

$$\cosh(\Lambda_kl) \approx \cosh(l/x_s) + \frac{1}{2}k^2lx_s\sinh(l/x_s),$$

$$\sinh(\Lambda_kl) \approx \sinh(l/x_s) + \frac{1}{2}k^2lx_s\cosh(l/x_s).$$

(35)

Expand the equation $k^2f(k) = g(k)$ up to $k^2$ by (35) with $f(k)$ and $g(k)$ given by (21) and (22). Then solving for $\xi = \frac{f}{g}$ gives the critical value

$$\xi_c = \frac{-\frac{1}{2}(d_+ - d_-)x_s(x_s \sinh(l/x_s) - l)\Theta}{x_s(d_- - d_+)(1 - \cosh(l/x_s))\Theta + x_s(d_+ + d_-)\cosh(l/x_s) + (x_s^2 + d_+d_-)\sinh(l/x_s)} + \frac{\frac{1}{2}(d_- - d_+)x_s^2\sinh(l/x_s)}{x_s(d_- - d_+)(1 - \cosh(l/x_s))\Theta + x_s(d_- + d_+)\cosh(l/x_s) + (x_s^2 + d_+d_-)\sinh(l/x_s)}.$$

(36)

If $\xi > \xi_c$, then the perturbation decays. If $\xi < \xi_c$, then the perturbation grows. The expression of $\xi_c$ in (36) exhibits the $\Theta$ dependence. The graph of $\xi_c$ for different $\Theta$ is shown in Figure 4. It can be seen that the curve moves downward as $\Theta$ increases.

![Figure 4](image-url)  

Figure 4. The critical value $\xi_c$ for $\Theta = 0, 0.15, 0.25$. Here we take $d_+ / x_s = 5, d_- / x_s = 100$. When $\xi > \xi_c$, the perturbation decays. When $\xi < \xi_c$, morphologically unstable step flow occurs.

3. Conclusions

We started with a generalized BCF model by considering the energetic contribution of the adjacent terraces to the step chemical potential and discussed the step-meandering instabilities of the two-dimensional step separated by broad terraces. Our results show that the single-component crystal growth on a vicinal surface can pass from stable to unstable status. We can also see how the equilibrium adatom coverage $\Theta$ affects the instabilities of the step meandering. It shows that the
increase of $\Theta$ can stabilize the step meandering. In the near future, the step-bunching instabilities of a two-dimensional periodic train of steps will be investigated.

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