How dense does parton matter get in Pb + Pb Collisions at the CERN SPS?

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Received 1 January 1996; revised version 1 January 1997

Abstract. We examine the qualitative features of parton production through materialization in heavy-ion collisions within perturbative QCD, and estimate the magnitude of the resulting parton density created during the early stage of the collisions. The implications for “anomalous” $J/\psi$ suppression observed in Pb+Pb collisions at the CERN SPS are discussed. We argue that the $A$-dependence of absorption of $J/\psi$ by (partonic) comovers is steeper than assumed in most phenomenological models, because the absorption process is dominated by quasi-perturbative QCD interactions. Our argument is supported by results recently obtained in the framework of the parton cascade model. We predict significant “anomalous” suppression for Pb+Pb collisions at the CERN-SPS, but not for S+U collisions.

Pacs No.: 25.75.-q, 12.38.Mh

1. Introduction

The creation of high-density QCD matter consisting of quarks and gluons that are being liberated in ultra-relativistic heavy-ion collisions, is one of the most fundamental issues for investigating the new physics regime of deconfined parton matter and the quark-gluon plasma. In this context, the question whether the NA50 data \cite{1} on $J/\psi$ production in Pb + Pb collisions at the CERN-SPS signal the presence of a novel nuclear suppression mechanism is a matter of heated controversy. Some theoretical papers \cite{2,3} have argued that the additional suppression observed in Pb + Pb, compared with S + U, is “anomalous”, i.e. provides evidence of a new mechanism, most likely dissociation of the $c\bar{c}$ quark pair in a dense deconfined medium, or quark-gluon plasma \cite{4}. Others \cite{5,6} have argued that “conventional” mechanisms, such as absorption of $J/\psi$ on light hadrons emitted at the same rapidity, can
explain the increased suppression in Pb + Pb collisions.

It is not our goal here to add another phenomenological analysis of the NA50 data to the existing ones. Rather, we will make an attempt to analyze on the basis of the established concepts of perturbative QCD, whether Pb + Pb collisions \((P_{\text{beam}} = 158 \text{ A GeV})\) are indeed so different from S + U collisions at almost the same energy \((P_{\text{beam}} = 200 \text{ A GeV})\) that the former system might result in the formation of a thermalized, color-deconfined state whereas the latter does not. Our reasoning is based partly on qualitative arguments first announced a decade ago \cite{10} and partly on quantitative results \cite{11} recently obtained in the framework of the parton cascade model of high-energy nuclear collisions \cite{12,13}.

2. \(A\)-dependence of parton production

We begin with the qualitative arguments. Energy deposition at large transverse momentum \(p_{\perp}\) in hadronic collisions can be described in the framework of perturbative QCD through jet or mini-jet production. The crucial question is how low in \(p_{\perp}\) this description remains reliable. There is an increasing amount of evidence \cite{14} that the perturbative domain of QCD extends down to (spacelike) momenta of the order of 1 GeV/\(c\). Contrary to hadron collisions at presently accessible energies, in heavy-ion collisions the applicability of perturbative methods involves an additional scale namely the parton density in the colliding nuclei. For large nuclei even at moderately small values of the Bjorken variable \(x\), the Lorentz contraction of the nucleons may lead to overlapping partons with the net effect that parton densities appear to be suppressed as compared to isolated hadrons. Hence, not only the momentum transfer \(p_{\perp}\) of mini-jet production, but also the nuclear mass number \(A\) determine the perturbative regime of parton dynamics.

In order to estimate the \(A\)-dependence of the density of partons produced in heavy nucleus collisions and to pin down the point of critical density where the parton densities begin to deviate, we follow ref. \cite{10} by considering the number of mini-jets produced in a nucleus-nucleus collision \(A + B\) at impact parameter \(b\) \cite{15}:

\[
x \frac{dN(b)}{dp_{\perp}^2 dx} = 2T_{AB}(b) \sum_{ab} \int_{x'}^{1} \frac{dx'}{x'} x' f_{a/A}(x', p_{\perp}^2) x f_{b/B}(x, p_{\perp}^2) \frac{d\hat{\sigma}_{ab}}{dp_{\perp}^2} \tag{1}
\]

where \(T_{AB}(b) = \int d^2 s T_A(s) T_B(b - s)\) with the nuclear profile functions \(T_A\) and \(T_B\) normalized to unity. The functions \(f_{a/A}\) and \(f_{b/B}\) are the distribution of partons \(a, b = g, q, \bar{q}\) in the colliding nuclei \(A\) and \(B\), which depend on the energy fractions \(x, x'\) and the transverse momentum \(p_{\perp}\) of the partonic scattering with cross-section \(\hat{\sigma}_{ab}(x, x', p_{\perp}^2)\). For purpose of simplicity, we consider now symmetric collision systems, i.e. \(A = B\), but the arguments below generalize straightforwardly to asymmetric systems \(A \neq B\).

To proceed, we recall that for sufficiently large values of \(p_{\perp}\) and not too small values of \(x\), the nuclear parton distribution functions \(f_{a/A}\) are approximately given
by

$$f_{a/N}(x, p_{\perp}^2) \approx A f_{a/N}(x, p_{\perp}^2),$$

(2)

where $f_{a/N}$ is the number density of parton species $a$ in the nucleon $N$. However, as $x$ and $p_{\perp}$ are decreased, the parton density in a heavy nucleus is increasingly reduced ("shadowed") because partons from different nucleons overlap spatially and begin to fuse. Since the small-$x$ regime is strongly dominated by the rise of the gluon density with quarks being negligible, the behavior of the gluon distribution $f_{g/N}(x, p_{\perp}^2)$ with decreasing $x$ and $p_{\perp}$ may be viewed as representative indicator for shadowing effects to become significant. For gluons, the characteristic momentum $p_{\text{crit}}$ where this happens is determined by the number of gluons per unit area in the transverse plane of the colliding nuclei at this momentum scale \[10\],

$$p_{\text{crit}}^2 = \frac{9}{16} \pi \alpha_s C_A A x f_{g/N}(x, p_{\text{crit}}^2),$$

(3)

where $C_A = 3$ is the value of the quadratic Casimir operator of SU(3) in the adjoint representation, and $R_A = 1.2 A^{1/3}$ fm is the nuclear radius. Notice that $p_{\text{crit}}^2$ enters as argument of $f_{g/N}$ on the right side; therefore (3) is a non-trivial implicit equation. Nevertheless, we may get a rough estimate of the condition (3) by taking for instance $p_{\text{crit}} = 1 \text{ GeV}$ and $x f_{g/N}(x, 1 \text{ GeV}^2) = 3$ at $x \approx 0.1$ with $\alpha_s \approx 0.3$ at $p_{\text{crit}}^2$, in which case one finds that shadowing corrections would become significant only for nuclei with $A > 240$.

Since $R_A \sim A^{1/3}$, the critical transverse momentum grows approximately as $p_{\text{crit}} \sim A^{1/6}$. The effect of shadowing is to reduce the number of mini-jets with transverse momenta $p_{\perp} < p_{\text{crit}}$. For larger values of $p_{\perp}$, the mini-jet yield grows as $A^{4/3}$, because $f_{g/A} \sim A f_{g/N}$ and $T_{AA}(b) \sim R_A^{-2} \sim A^{-2/3}$. However, approximating the glue-glue cross section by its small angle limit

$$\frac{d\sigma_{\text{gg}}}{dp_{\perp}^2} \approx \frac{\alpha_s C_A^2 \pi}{2 p_{\perp}^4},$$

(4)

one finds that the integrated mini-jet yield for all momenta $p_{\perp} > p_{\text{crit}}$ only grows linearly with $A$:

$$\int_{p_{\text{crit}}^2}^{\infty} dp_{\perp}^2 x \frac{dN(b)}{dp_{\perp}^2 dx} \sim A^{4/3} \frac{1}{p_{\text{crit}}^2} \sim A.$$

(5)

This agrees with the result of Blaizot and Mueller [14], who found that for two heavy nuclei the total yield of gluon mini-jets per unit of rapidity is

$$\frac{dN(b)}{dy} = 2 A x f_{g/N} \frac{T_{AA}(b)}{T_{AA}(0)}.$$

(6)

The same considerations apply when the mini-jet production is mostly due to quark-quark or quark-gluon scattering, because the differential parton cross sections all have the same functional form (4) at small angles.
To summarize, we have the following picture describing the mini-jet production at central rapidity in nucleus-nucleus collisions: Above a critical transverse momentum $p_{\text{crit}}$, which itself grows as $A^{1/6}$, the mini-jet multiplicity grows like $A^{4/3}$. The total mini-jet yield, as well as the multiplicity in the saturation regime below $p_{\text{crit}}$, only grows linearly with $A$. This result implies that the area density of mini-jets grows as $A^{2/3}$ above $p_{\text{crit}}$, but only as $A^{1/3}$ below $p_{\text{crit}}$.

3. Implications for $J/\psi$ suppression

We now return to the problem of the observed strong suppression of $J/\psi$ production in Pb + Pb collisions. In particular, we want to address the question of the $A$-dependence of the absorption of the $J/\psi$ by comoving produced hadronic matter. In the framework developed by Bhanot and Peskin [16], interactions between light hadrons and deeply bound, heavy quarkonium status, such as the $J/\psi$, are mediated by short-range color dipole interactions. The relevant momentum transfers $p_{\perp}^2$ for these interactions lie in the perturbative domain of QCD, $p_{\perp}^2 \geq 1 \text{ GeV}^2$.

The key element in the following arguments is the fact that, for most nuclear collision systems at the relatively low center-of-mass energy of the CERN-SPS experiments, perturbative mini-jet production is largely due to quark-quark scattering, because the typical value of Bjorken-$x$ probed is $\langle x \rangle > 0.1$, where the gluon density is small. This implies that shadowing effects are negligible, i.e. the relation (2) holds. Furthermore, according to (3) the critical momentum $p_{\text{crit}}$ lies below the perturbatively accessible range of momenta $p_{\perp} \geq 1 \text{ GeV}$ at the SPS energy, and may just begin to reach into it for the Pb + Pb system. If this is correct, then all mini-jet production involving momenta $p_{\perp} \geq 1 \text{ GeV}$ lies safely above $p_{\text{crit}}$, where eq. (2) holds. As immediate consequence, the density of comovers which can effectively interact and absorb the $J/\psi$ at the SPS grows like $A^{2/3}$, or $(A_1A_2)^{1/3}$ in asymmetric collisions. This is a much stronger $A$-dependence than naively expected and embodied in most comover suppression models [9]. It also implies a stronger impact parameter or $E_T$-dependence of comover suppression than predicted by existing models.

Quantitative support for our speculations comes from recent calculations [11] of secondary particle production in the parton cascade model [17]. This model is based on perturbative parton scattering with dynamic, medium dependent cut-off mechanisms of the infrared divergences of QCD. The model also incorporates a density dependent clustering mechanism for the parton-to-hadron transition. The calculations predict that the energy density $\epsilon$ produced by scattering partons (mini-jets) at central rapidity grows by a factor of more than 2 between S + U and Pb + Pb, compatible with the scaling law $\epsilon \sim (A_1A_2)^{1/3}$, but much faster than expected in the usual Glauber model approach which predicts $\epsilon \sim (A_1^{1/3} + A_2^{1/3})$, in which case $\epsilon$ increases only by 30%.

In Pb + Pb collisions the initial partonic energy density $\epsilon_p$ is predicted to reach $5 \text{ GeV/fm}^3$ at times $\tau < 1 \text{ fm/c}$ in the comoving reference frame (see Fig. 1). For
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A thermalized gas of free gluons and three flavors of light quarks this corresponds to an initial temperature $T_i \approx 230$ MeV, clearly above the critical temperature $T_c \approx 150$ MeV predicted by lattice-QCD calculations [13] shown in Fig. 2. On the other hand, the parton density $\epsilon_p \approx 2$ GeV/fm$^3$ predicted for S + U collisions (Fig. 1), just lies at the upper end of the transition region in the equation of state from lattice-QCD.

The temperature $T_D$ required for dissociation of the $J/\psi$ bound state due to color screening is known [19] to be higher than $T_c$, namely $T_D \approx 1.2T_c = 180 – 200$ MeV. The parton cascade model results are therefore compatible with the experimental finding that there appears to exist no significant comover-induced suppression of $J/\psi$ in S + U collisions, but a large and strongly impact parameter dependent effect is observed in Pb + Pb collisions.

In conclusion, we have argued that the density of comovers relevant for $J/\psi$ suppression in nucleus-nucleus collisions grows more rapidly as function of nuclear mass than assumed in previous studies based on the Glauber model. Recent results obtained in the framework of the parton cascade model are therefore consistent with the interpretation that the “anomalous” $J/\psi$ suppression observed in Pb + Pb collisions is caused by color screening in a dense partonic medium. The same calculations predict that there is no such effect in S + U collisions. It must be stressed that these model results do not contradict the experimental data collected at the SPS, as one might suspect, since it is usually claimed that those are fully consistent with the Glauber picture. In fact, the detailed study of particle spectra in Ref. [11], comparing model and experiment, shows that the data are compatible with a parton cascade picture. In particular, the magnitude of transverse energy production visible in the final-state hadron spectra, is in fair agreement, although the maximum achieved energy density during the very early stage in Fig. 1 is substantially larger than previously estimated.

Acknowledgement

This work was supported in part by grants DE-FG02-96ER40495 and DE-AC02-76H00016 from the U.S. Department of Energy.

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Fig. 1. Time evolution of the energy density $\varepsilon$ of the partonic matter in the central slice of the collision systems $S + U$ and $Pb + Pb$ at CERN-SPS beam energies of 200 $A\cdot GeV$ and 158 $A\cdot GeV$, respectively. The model calculations were obtained with the MC implementation \[17\] of the parton cascade model \[12, 13\].
Fig. 2. Equation of state for two-flavor QCD from lattice calculations by the MILC collaboration [18], showing energy density $\epsilon$ and pressure $p$ as a function of temperature $T$. Indicated in the figure are also the energy densities corresponding to several values of the temperature $T$, as well as the location of the "melting" points of the $J/\psi$ and $\Upsilon$ states, where these bound states disappear due to dissociation [19].