Large-Flavor QCD on the Lattice*

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Abstract

We study the nature of the QCD vacuum at general number of flavors $N_F$ by numerical simulations on the lattice. Combining the results with those of the perturbation theory, we propose the following picture: 1) For $N_F \geq 17$, there exists only one IR fixed point at vanishing gauge coupling, i.e., the theory in the continuum limit is trivial. 2) For $16 \geq N_F \geq 7$, there is a non-trivial fixed point. Therefore, the theory is non-trivial with anomalous dimensions, however, without quark confinement. 3) For $N_F \leq 6$, theories satisfy both quark confinement and spontaneous chiral symmetry breaking in the continuum limit.

1. Introduction

The beta function of QCD is universal up to two-loop in the perturbation theory: $\tilde{\beta}(g) = -b_0 g^3 - b_1 g^5$, with $b_0 = \frac{1}{16 \pi^2} \left(11 - \frac{2}{3} N_F\right)$ and $b_1 = \frac{1}{(16 \pi^2)^2} \left(102 - \frac{38}{9} N_F\right)$. When the number of flavors ($N_F$) exceeds $16\frac{1}{2}$, $b_0$ becomes negative, so that the asymptotic freedom of QCD is lost. In this case, the origin $g = 0$ becomes an IR fixed point, i.e. the low energy limit of QCD is a free theory. Quarks cannot be confined and the chiral symmetry cannot be broken spontaneously. $N_F > 16\frac{1}{2}$ is a sufficient, but not a necessary condition for these. A natural question is, whether confinement and spontaneous breaking of the chiral symmetry are satisfied at all $N_F$ below $16\frac{1}{2}$.

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It is well-known that the second coefficient $b_1$ changes its sign already at $N_F \approx 8.05$. Therefore, a non-trivial IR fixed point may appear at finite $g$. From the two-loop beta function, this happens for $N_F = 9-16$. At least for $N_F \sim 16$, the IR fixed point shows up in a perturbative region. Therefore, it is plausible that the full beta function has a non-trivial IR fixed point for $N' \leq N_F \leq 16$ with some $N' \leq 16$. When such an IR fixed point exists, the coupling constant cannot become arbitrarily large in the IR region — this will imply that quarks are not confined. In particular, for $N_F \sim 16$, QCD is perturbative also in the IR limit, i.e., we cannot expect confinement.

In this paper, we study the $N_F$-dependence of the QCD vacuum. A non-perturbative investigation is required. Lattice QCD is the only systematic method that enables us to study non-perturbative properties of QCD. We performed a series of lattice simulations for a wide range of $N_F$, from 2 to 300, to study the phase structure of QCD. When the phase diagram becomes clear, we are able to see the nature of the QCD vacuum in the continuum limit, and eventually answer the question about the condition on $N_F$ for confinement and spontaneous breakdown of the chiral symmetry.

In 1982, Banks and Zaks published a pioneering paper on the $N_F$-dependence of the QCD vacuum \[1\]. Based on an early result obtained on the lattice \[2\], they assumed that, in the strong coupling limit, the theory is confining and the beta function is negative, for any value of $N_F$. Using the perturbative results for the beta function discussed above, they conjectured Fig. 1(a) as the simplest $N_F$-dependence of the beta function, and studied the phase structure of QCD based on this beta function. The assumption of negative beta function in the strong coupling region leads to an additional non-trivial UV fixed point for $N_F \geq N'$. Due to this UV fixed point, their conjecture for the phase structure is complicated. [For different approaches see Refs. \[3, 4, 5, 6\].]

In the arguments of Banks and Zaks, the assumption of confinement and negative beta function in the strong coupling limit plays an essential role. Here, it should be noted that the lattice study on which this assumption was based, is a study of a pure gauge theory \[2\]. With dynamical quarks, the vacuum structure can be much more complicated. Actually, in the realistic cases, there exist no proofs of confinement for general $N_F$ even in the strong coupling limit.\[^{1}\] A non-

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\(^{1}\) We can formally rewrite the theory in terms of mesons and baryons. We may argue that, when the resulting effective action for hadrons is a well-defined action of weakly interacting particles, it is plausible that quarks are confined — the hadrons are good variables to describe the dynamical contents of the theory. In the strong coupling limit, the effective action can be computed either in the large $N_c$ limit \[7\], using a meanfield approximation ($1/d$ expansion) \[8, 9\], or using a heavy quark mass expansion \[9, 10\]. In these cases, the effective action for mesons and baryons seems to lead to a well-defined world with spontaneously broken chiral symmetry.
perturbative investigation on the lattice is required.

In a previous study, we performed simulations of QCD in the strong coupling limit at various $N_F$ \[1\]. We found that, when $N_F \geq 7$, light quarks are not confined and chiral symmetry is restored, even in the strong coupling limit. Here, we extend this study to weaker couplings and to a wider range of $N_F$ up to 300. Combining the lattice results and perturbative arguments, we conjecture Fig. 1(b) for the non-perturbative beta function of QCD. This leads to the following $N_F$-dependence of the QCD vacuum.

- When $N_F$ is smaller than a critical value, the beta function is negative for all values of $g$. Quarks are confined and the chiral symmetry is spontaneously broken at zero temperature. We find that the critical number of flavors is 6. (Corresponding critical value of $N_F$ is 2 for $N_c = 2$, to be compared with the two-loop value 5 \[12\].)

Fig. 1. Renormalization group beta function of QCD. (a) Conjecture by Banks and Zaks \[1\] assuming confinement in the strong coupling limit for all $N_F$. (b) Our conjecture deduced from the results of lattice simulations.
On the other hand, when \( N_F \) is equal or larger than 17, we conjecture that the beta function is positive for all \( g \), in contrast to the conjecture by Banks and Zaks shown in Fig. 1(a). The theory is trivial in this case.

When \( N_F \) is between 7 and 16, the beta function changes sign from negative to positive with increasing \( g \). Therefore, the theory has a non-trivial IR fixed point, i.e., the theory is non-trivial with anomalous dimensions. We conjecture that quarks are not confined in this case.

Our results are in part reported in Refs. [12] and [13].

This report is organized as follows: In Sec. 2, we describe our lattice model and simulation parameters. Results for the phase structure in the strong coupling limit are given in Sec. 3. The phase structure at finite coupling is discussed in Sec. 4. Finally, in Sec. 5, we summarize our conclusions and conjectures.

### 2. Model and simulation parameters

We consider a 4-dimensional Euclidian lattice with the lattice spacing \( a \) and finite lattice volume \( (N_s a)^3 \times (N_t a) \), where \( N_s \) and \( N_t \) are the number of lattice sites in spatial and temporal directions. The action consists of the gauge part and the quark part, \( S = S_{gauge} + S_{quark} \). For the gauge part, we use the standard one plaquette action:

\[
S_{gauge} = -\frac{\beta}{6} \sum_{x,\mu \neq \nu} \text{Tr} \left( U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\mu}+\hat{\nu},\mu} U^\dagger_{x+\hat{\nu},\nu} \right),
\]

where \( U_{x,\mu} \) is the gauge connection between the site \( x \) and the neighboring site in the positive \( \mu \) direction, and \( \beta = 6/g^2 \).

For the quark part, there are several alternatives. Two conventional choices are the staggered (Kogut-Susskind) fermion action [14] and the Wilson fermion action [10]. Staggered quark has been preferably used in previous studies because a part of the chiral symmetry is preserved in this formulation. On the other hand, the flavor structure is quite complicated: The action is local only when \( N_F \) is a multiple of 4. Off the continuum limit, there exists a lattice artifact mixing these 4 degenerate flavors. For the more realistic cases \( N_F = 2 \) and 3, we modify by hand the power of the fermionic determinant in the numerical path-integration. This necessarily makes the action non-local, that sometimes poses conceptually and technically difficult problems.

Another conventional choice for the quark action is the Wilson fermion action, which we adopt in the followings:

\[
S_{quark} = \sum_{f,x} \left[ \bar{\psi}_x^f \psi_x^f - K \left\{ \bar{\psi}_x^f (1-\gamma_\mu) U_{x,\mu} \psi_{x+\hat{\mu}}^f + \bar{\psi}_{x+\hat{\mu}}^f (1+\gamma_\mu) U^\dagger_{x,\mu} \psi_x^f \right\} \right],
\]
The index \( f \) \((= 1, \cdots, N_F)\) is for the flavors. We assume that all \( N_F \) quarks are degenerate with the bare mass \( m_0 \), which is related to the hopping parameter \( K \) by \( m_0 = (1/2a)(1/K - 1/K_c) \), where \( K_c = 1/8 \) is the point where the bare mass vanishes. In this formulation, the chiral symmetry is explicitly broken at finite \( a \). On the other hand, the flavor symmetry as well as the C, P and T symmetries are manifestly satisfied without mixing etc. also on the lattice.

In this study, we use the Wilson fermion formalism because this is the only formalism that preserves manifest flavor symmetry. This feature will be important in a study of \( N_F \)-dependence in QCD. Concerning the chiral symmetry, a perturbative study of Ward identities shows that the effects of the \( O(a) \) chiral breaking terms can be absorbed by appropriate renormalizations, including an additive renormalization of quark mass [16]. As a result, the location of the chiral limit \( K_c \) shifts from the tree value \( 1/8 \) as a function of \( \beta \).

In order to take the additive mass renormalization into account, we define the current quark mass \( m_q \) in terms of an axial vector Ward identity [17, 16].

\[
2m_q \langle 0 \mid P \mid \pi(\vec{p} = 0) \rangle = -m_\pi \langle 0 \mid A_4 \mid \pi(\vec{p} = 0) \rangle
\]  

(3)

where \( P \) is the pseudoscalar density, \( A_4 \) the fourth component of the local axial vector current, and \( m_\pi \) the pion (screening) mass. A multiplicative renormalization factor for the axial current, which is not important in this study, is absorbed into the definition of the quark mass. We then define \( K_c \) by the condition \( m_q = 0 \). Numerical simulations for small \( N_F \) shows that \( K_c(\beta) \) is a smooth curve connecting \( 1/8 \) at \( \beta = \infty \) and \( 1/4 \) at \( \beta = 0 \).

In QCD, even when the confinement is realized at low temperatures, we expect deconfinement at high temperatures [18]. Therefore, we have to study the temperature dependence of the results to distinguish between the finite temperature deconfinement transition/crossover due to high temperatures and a bulk transition at zero temperature due to the effects of many flavors. On a lattice with \( N_t \) sites in the Euclidian time direction, the temperature is given by \( T = 1/N_t a \). When the beta function is negative, as in the case of small \( N_F \), the lattice spacing \( a \) is a decreasing function of \( \beta = 6/g^2 \). In this case, \( T \) increases with increasing \( \beta \) for a fixed finite \( N_t \). On the other hand, when the beta function is positive, \( T \) decreases with increasing \( \beta \). Therefore, in order to study the temperature dependence for general \( N_F \), we have to compare the results at various \( N_t \).

Numerical study shows that the value of \( m_q \) defined through the axial vector Ward identity does not depend on whether the system is in the high or the low temperature phase when \( \beta \) is large; \( \beta \geq 5.5 \) for \( N_F = 2 \) [19]. When we define the pion decay constant \( f_\pi \) by \( \langle 0 \mid A_4 \mid \pi(\vec{p} = 0) \rangle = m_\pi f_\pi \), either \( m_\pi = 0 \) or \( f_\pi = 0 \) has to be satisfied in the chiral limit. The chiral symmetry is restored when
Fig. 2. Plaquette at $\beta = 0$ as a function of $1/K$. (a) For $N_F = 7$–300 at $N_t = 4$. (b) For $N_F = 240$ at $N_t = 4$–16.

$f_\pi = 0$ in the chiral limit, while $m_\pi = 0$ if the chiral symmetry is spontaneously broken [20].

We perform simulations on lattices $8^2 \times 10 \times N_t$ ($N_t = 4$, 6, and 8), $16^2 \times 24 \times N_t$ ($N_t = 16$) and $18^2 \times 24 \times N_t$ ($N_t = 18$). We vary $N_F$ from 2 to 300. For each $N_F$, we study the phase structure in the coupling parameter space ($\beta, K$). It should be noted that, in QCD with dynamical quarks, there are no order parameters for quark confinement. We discuss about confinement by comparing the screening pion mass and the lowest Matsubara frequency, and, simultaneously, consulting the values of plaquette and the Polyakov loop. In the followings, we call the pion screening mass simply the pion mass, and similarly for the quark mass. Further details about our simulation are described in Refs. [12, 13]

3. Strong coupling limit $\beta = 0$

In a previous paper [11], we have shown that, when $N_F \geq 7$, light quarks are deconfined and chiral symmetry is restored at zero temperature even in the strong coupling limit. For $N_F \leq 6$, we have only one confining phase from the heavy quark limit $K = 0$ up to the chiral limit $K_c = 0.25$. On the other hand, for $N_F \geq 7$, we find a strong first order transition at $K = K_d$:

- When quarks are heavy ($K < K_d$), both plaquette and the Polyakov loop are small, and $m_\pi$ satisfies the PCAC relation $m_\pi^2 \propto m_q$. Therefore, quarks are confined and the chiral symmetry is spontaneously broken in this phase. We find that $m_q$ in the confining phase is non-zero at the transition point $K_d$. While an extrapolation in $1/K$ shows that $m_\pi^2$ decreases towards zero at $K = 0.25$, the chiral limit does not belong to this phase ($K_d < K_c$).
Fig. 3. The transition point $1/K_d$ at $\beta = 0$ versus $N_F$ for $N_t = 4$ and $N_t \geq 8$. For clarity, data at $N_t = 8$ for $N_F = 300$ is slightly shifted to a larger $N_F$ in the figure.

- On the other hand, when quarks are light ($K > K_d$), plaquette and the Polyakov loop are large. In this phase, $m_\pi$ remains large in the chiral limit and is almost equal to twice the lowest Matsubara frequency $\pi/N_t$. This implies that the pion state is a free two-quark state and, therefore, quarks are not confined in this phase. The pion mass is nearly equal to the scalar meson mass, and the rho meson mass to the axial vector meson mass. The chiral symmetry is also manifest within corrections due to finite lattice spacing.

See [11] for details.

We extend the study to larger $N_F$. We show the results of plaquette at $N_t = 4$ for $N_F = 7$–300 in Fig. 2(a). Clear first order transition can be seen at $K$ below 0.25 ($1/K > 4$). We then study the $N_t$ dependence of the results, as shown in Fig. 2(b) for the case of $N_F = 240$. We find that, although the transition point shows a slight shift to smaller $1/K$ when we decrease $T = 1/N_ta$ from $N_t = 4$ to 8, it stays at the same point for $N_t \geq 8$. The chiral limit, $1/K = 4$, cannot be achieved in the confining phase even at $T = 0$. Similar result for $N_F = 7$ is given in Ref. [11]. Therefore, we conclude that the transition is a bulk transition.

From these results, we obtain the phase diagram at $\beta = 0$ shown in Fig. 3. The first-order transition line separating the confining phase and the deconfining phase reaches $1/K = 4$ (chiral limit for small $N_F$) between $N_F = 6$ and 7. Note that the critical value of the inverse critical hopping parameter is greatly increased from $1/K = 4.08$ to 7.75 when $N_F$ increases from 7 to 240. Recall that $1/K = 8$ is the point for massless free quarks at $\beta = \infty$. 
4. Phase structure at finite $\beta$

Let us now study the case $\beta > 0$. We perform simulations at $\beta = 0.0–6.0$ varying $N_F$ from 2 to 300. On a lattice with a fixed finite $N_t$, we have the finite temperature deconfining transition when the quark mass is sufficiently heavy. In particular, at $K = 0$, we have the first order deconfinement transition of the pure gauge theory at finite $\beta$ irrespective to the value of $N_F$.

For $N_F \leq 6$, the finite temperature transition/crossover line crosses the $K_c$ line at finite $\beta$ [20]. A schematic diagram of the phase structure for this case is shown in Fig. 4. (In the followings, we call the transition/crossover line simply as transition line.) We have a monotonous RG flow towards the strong coupling limit also on the $K_c$ line. Therefore, the whole finite temperature transition line moves towards a larger $\beta$ as $N_t$ is increased. In the limit $N_t = \infty$ (i.e., at $T = 0$), only the confining phase is left.

For $N_F \geq 7$, we have seen that there exist a bulk deconfining transition in the strong coupling limit. We have to clarify the relation between the finite temperature transition at small $K$ and the bulk transition at $\beta = 0$. From our
simulations, we obtain the phase diagrams shown in Fig. 5 for $N_F = 12$ and 18, and in Fig. 6 for $N_F = 240$. When $N_t = 4$ or 8, the phase boundary line between confining and deconfining phase bends down at finite $\beta$ due to the finite temperature phase transition of the confining phase. The darker shaded lines are our conjecture for the phase boundary between the confining phase and the deconfining phase at zero temperature ($N_t = \infty$).

In Figs. 5 and 6, the dashed lines in the deconfining phase represent the chiral limit, $m_q = 0$. This line also corresponds to the minimum point of $m_q^2$. We have checked that, for the case $N_F = 240$, the quark propagator in the Landau gauge actually shows the chiral symmetry, $\gamma_5 G(z) \gamma_5 = -G(z)$, on the $m_q = 0$ line [12]. For $N_F \gtrsim 240$, we find that the $m_q = 0$ line in the deconfining phase, which starts at $1/K = 8$ in the weak coupling limit $\beta = \infty$, reaches the strong coupling limit, as shown in Fig. 6 for $N_F = 240$. For a smaller $N_F \lesssim 100$, because the bulk transition line $K_d$ in the strong coupling region shifts toward larger $K$ with decreasing $N_F$, the $m_q = 0$ line in the deconfining phase hits the $K_d$ line at finite $\beta$. For example, in the case $N_F = 18$, it hits at $\beta = 4.0 - 4.5$, as shown in Fig. 5(b).

4.1. **RG-flow for $N_F \geq 17$**

The $m_q = 0$ point at $\beta = \infty$ is a trivial IR fixed point for $N_F \geq 17$. The phase diagram shown in Fig. 5 suggests that there are no other fixed points on the $m_q = 0$ line at finite $\beta$.

Figure 7(a) shows the results of $m_\pi^2$ and $2m_q$ in the deconfining phase for $N_F = 240$ at $N_t = 4$. We find that the shape of $m_\pi^2$ and $2m_q$ as a function of $1/K$ only slightly changes for $1/K < 8$ when the value of $\beta$ decreases from $\infty$ down 0. We obtain similar results also for $N_t = 8$. This suggests that quarks are almost free down to $\beta = 0$ in the deconfining phase. The results for $N_F = 300$
are essentially the same, except for very small shifts of the transition point.

We make a Monte Carlo renormalization group (MCRG) study along the $m_q = 0$ at $N_F = 240$. Performing a block transformation with scale factor 2, we estimate the quantity $\Delta \beta = \beta(2a) - \beta(a)$: We generate configurations on an $8^4$ lattice on the $m_q = 0$ points at $\beta = 0$ and 6.0 and make twice blockings. We also generate configurations on a $4^4$ lattice and make once a blocking. Then we calculate $\Delta \beta$ by matching the value of the plaquette at each step. We obtain $\Delta \beta \simeq 6.5$ at $\beta = 0$ and 10.5 at $\beta = 6.0^\ddagger$. The value obtained from the two-loop perturbation theory is $\Delta \beta \simeq 8.8$ at $\beta = 6.0$. The signs are the same and the magnitudes are comparable. This suggests that the direction of the RG flow on the $m_q = 0$ line at $\beta = 0$ and 6.0 is the same as that at $\beta = \infty$. This further suggests that there are no fixed points at finite $\beta$. These imply that the theory is trivial for $N_F = 240$.

The area of the deconfining phase decreases with decreasing $N_F$ in the strong coupling region. However, the shape and the values of $m_\pi^2$ and $m_q$ are

$\ddagger$ In order to get a more precise value of $\Delta \beta$, one has to make many steps of block transformations, together with a fine tuning of parameters of the block transformation, and do matching using several types of Wilson loops. We reserve elaboration of this point for future works. For $N_F = 240$, because the velocity of the RG flow is large, we will be able to obtain the correct sign and approximate value of $\Delta \beta$ by a simple matching.
quite similar when we vary $N_F$ from 300 down to 17. Fig. 7(b) shows $m_\pi^2$ and $2m_q$ for $N_F = 18 - 300$ at $\beta = 6.0$ and 0. At $\beta = 6.0$, the shapes of $m_\pi^2$ are almost identical to each other, except for a small shift toward smaller $1/K$ as $N_F$ is decreased.

These facts suggest that, for $N_F \geq 17$, the nature of physical quantities in the deconfining phase is almost identical to that observed for $N_F = 240$ and 300, i.e., quarks are almost free. Therefore, we conjecture that the direction of the RG flow at $\beta \lesssim 6$ is identical to the case $N_F = 240$. Combining this with the perturbative result that $\beta = \infty$ is an IR fixed point for $N_F \geq 17$, we conjecture that the RG flow along the massless quark line in the deconfining phase is uniformly directing to smaller $\beta$, as in the case of $N_F = 240$. When this is the case, the theory has only one IR fixed point at $\beta = \infty$, i.e. the theory is trivial for $N_F \geq 17$.

4.2. $16 \geq N_F \geq 7$

As discussed in Sec. 3, quark confinement is lost for $N_F \geq 7$ at $\beta = 0$. We have performed simulations for $N_F = 12$ and 7. The phase diagram for $N_F = 12$ is shown in Fig. 5(a). At $\beta \lesssim 6.0$ we studied, the gross feature of the phase diagrams for $N_F = 7$ and 12 is quite similar to the case $N_F = 18$ shown in Fig. 5(b). Physical quantities at $\beta \lesssim 6.0$ also show behavior similar to that shown in Fig. 4 for $N_F \geq 17$. Therefore, we consider it probable that the direction of the RG flow in the deconfining phase at small $\beta$ is towards a larger $\beta$ as in the case of $N_F \geq 17$. However, the direction of the RG flow at $\beta = \infty$ is opposite to that for $N_F \geq 17$ because the theory is asymptotically free for $N_F \leq 16$. This means that we have an IR fixed point somewhere at a finite value of $\beta$. The continuum limit is governed by this IR fixed point.

On the $m_q = 0$ line around $\beta = 4.5$, we find that $m_\pi$ is roughly twice the lowest Matsubara frequency. This implies that the quarks are not confined and almost free. Therefore, the anomalous dimensions are small at the IR fixed point, suggesting that the IR fixed point locates at finite $\beta$. Unfortunately, as shown in Fig. 2, the massless line hits the boundary in the coupling parameter space. Without much space in the strong coupling region where simulation is easy, it is hard to study numerically the location of the IR fixed point. A detailed study of RG flow in a wider coupling parameter space is required. We reserve these studies for future works.
Fig. 8. Phase structure and the RG flow at $T = 0$ for general number of flavors.

Thin dashed lines in the phase diagrams for $N_F \geq 7$ represent the location of the chiral limit for $N_F \leq 6$ as a guide for eyes.

5. Conclusions

Summarizing our numerical results and conjectures, we propose Fig. 8 for the phase structure of QCD at $T = 0$: When $N_F \leq 6$, quarks are confined for any values of the current quark mass and $\beta$. The chiral limit $K_c(\beta)$, where the current quark mass $m_q$ vanishes, belongs to the confining phase. When $N_F \geq 7$, there is no chiral limit in the confining phase. There is a line of a first order phase transition from the confining phase to a deconfining phase at a finite current quark mass for all values of $\beta$. The chiral limit exists only in the deconfining phase, and, therefore, can not be taken from the confining phase.

In order to see the orientation of the RG flow on the chiral limit line in the deconfining phase, we performed a MCRG study at $N_F = 240$, because, in this case, we have sufficiently long massless quark line in the strong coupling region where numerical simulation is easy. We found that the RG flow along the
massless quark line at small $\beta$ is consistent with the result of the perturbation theory. This suggests that the direction of the RG flow is uniform for all values of $\beta$, as shown in Fig. 8. Therefore, there is only one IR fixed point at $\beta = \infty$, so that the theory in the continuum limit is trivial. Accordingly, we find that, in the deconfining phase, the $K$-dependences of pion screening mass and current quark mass at small $\beta$ are almost identical to those of free Wilson quarks.

Also for smaller values of $N_F \geq 7$, the general features of the phase diagram and the $K$-dependence of physical quantities are quite similar to the case of large $N_F \sim 240$. Therefore, we conjecture that the direction of the RG flow is the same as in the case of $N_F = 240$, at least for $\beta \lesssim 6.0$ we have studied.

Together with the results of the perturbation theory, we conjecture the following picture.

- For $N_F \geq 17$, because the RG flow in the weak coupling limit has the same direction due to the lack of asymptotic freedom, we conjecture a uniform RG flow towards the trivial IR fixed point, as in the case of $N_F = 240$. Thus the theory is trivial for $N_F \geq 17$.

- When $7 \leq N_F \leq 16$, because the theory is asymptotically free, the direction of the RG flow is opposite at large $\beta$. Therefore, we expect a non-trivial IR fixed point at finite $\beta$. In this case, the theory in the continuum limit is a non-trivial theory with anomalous dimensions. Because the coupling constant in the IR limit is finite, quarks are not confined in this case.

- When $N_F \leq 6$, quarks are confined and the chiral symmetry is spontaneously broken for any values of the current quark mass and $\beta$. The chiral limit belongs to the confining phase.

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