From static to evolving geometries –
R-charged hydrodynamics from supergravity

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Abstract

We show that one can obtain asymptotic evolving boost-invariant geometries in a simple manner from the corresponding static solutions. We exhibit the procedure in the case of a supergravity dual of R-charged hydrodynamics by turning on a supergravity gauge field and analyze the relevant thermodynamics. Finally we consider turning on the dilaton and show that electric and magnetic modes in the plasma equilibrate before reaching asymptotic proper times.

1 Introduction

Quark-gluon plasma, the deconfined state of matter produced at RHIC is currently the focus of numerous experimental and theoretical studies. Since it appears that the gauge theory coupling in the plasma is large \cite{1}, it is interesting to develop nonperturbative methods for studying its properties.

A very effective framework for studying nonperturbative phenomena is the AdS/CFT correspondence \cite{2}. Even in its simplest form, for $\mathcal{N} = 4$

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Super-Yang-Mills theory, at finite temperature one may expect many similarities with properties of strongly coupled deconfined QGP.

A lot of work has been done in the study of properties of static plasma at fixed temperature e.g. transport coefficients have been computed \cite{3,4}, the drag force acting on a moving quark was investigated \cite{5}. Much less is known regarding questions related to time-dependent phenomena like the origin of thermalization, hydrodynamic evolution etc. Works investigating these issues include \cite{6,7,8,9,10,11}.

In order to gain more understanding of the above mentioned dynamical processes in strongly interacting gauge theory using AdS/CFT it is worthwhile to investigate the structure of dual time-dependent geometries. In general holographic renormalization \cite{12} gives a prescription for constructing a dual geometry to any given gauge theory energy-momentum spacetime profile. In \cite{8} it was advocated that the requirement of nonsingularity of the dual geometry selects the physical energy-momentum evolution in the gauge theory. At leading order, in a boost invariant setting, this requirement led to late-time perfect fluid hydrodynamical evolution \cite{8}, while carrying out the analysis also for subleading asymptotic times \cite{10} determined the effects of viscosity on the evolution with the exact (shear) viscosity coefficient.

The dual geometry for late asymptotic times constructed in \cite{8} bears a remarkable similarity to the static black hole but with the position of the horizon moving with a specific scaling with proper-time. In this note we would like to perform an analogous construction for the more complicated case with R-charged matter, show that a similar phenomenon also occurs in this case and exhibit the origin of such a behavior.

Another direction of generalization of the evolving geometry is to turn on the dilaton field. This has the physical interpretation on the gauge theory side of allowing for differing expectation values of squares of electric and magnetic fields in the evolving plasma. We find that, for asymptotic proper-times, in order to have a nonsingular geometry one has to have equilibration between electric and magnetic modes.

The plan of this paper is as follows. First in section 2 we will briefly review boost-invariant hydrodynamics with a conserved charge. In section 3, we will construct the dual charged evolving geometry for asymptotic proper-times and explain the origin of its marked similarity to the corresponding static geometry. We will then examine, in section 4, the thermodynamics of

\footnote{And using some results of \cite{11}.}
the resulting evolving system. Finally, in section 5, we will consider turning on the dilaton. We close the paper with concluding remarks.

2 Hydrodynamics with a conserved charge

Perfect fluid hydrodynamics with a conserved charge is described by the energy-momentum tensor and the current

\[ T^{\mu \nu} = (\varepsilon + p)u^\mu u^\nu + p\eta^{\mu \nu}, \quad J^\mu = \rho u^\mu, \]  

which are conserved i.e.

\[ \partial_\mu T^{\mu \nu} = 0, \quad \partial_\mu J^\mu = 0. \]  

Moreover for the conformal theory that we are considering \( T^\mu_\mu = 0 \) which gives \( \varepsilon = 3p \).

Let us now impose the requirement of boost-invariance in the longitudinal plane and no dependence on transverse coordinates. In the natural proper-time/spacetime rapidity coordinates the Minkowski metric has the form

\[ ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2. \]  

Boost-invariance then forces the fluid velocity to be \( u^\mu = (1, 0, 0, 0) \), and energy momentum conservation leads to Bjorken evolution,

\[ \varepsilon(\tau) = \frac{\varepsilon_0}{\tau^4}, \]  

while current conservation leads to

\[ \nabla_\mu J^\mu = \frac{dJ^\tau}{d\tau} + \frac{1}{\tau} J^\tau = 0 \quad \rightarrow \quad J^\mu = \left( \frac{\rho_0}{\tau}, 0, 0, 0 \right). \]  

We will show below that this scaling leads to a nonsingular dual geometry.

3 Supergravity analysis

The 5D Einstein Maxwell action for the the minimal cases is given by

\[ I = \frac{1}{16\pi G_5} \int \left( \sqrt{-g} \left( R + 12 - \frac{1}{4} F_{\alpha \beta} F^{\alpha \beta} \right) - \alpha \epsilon^{\alpha \beta \mu \nu \lambda} F_{\alpha \beta} F_{\mu \nu} A_\lambda \right). \]  

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where \( G_5 = \pi / (2N_c^2) \) and \( \alpha = 1/(48\sqrt{3}) \) in our convention. Any solution of the above action can be consistently embedded into the 10D type IIB supergravity \[13\]. The gravity equations of motion become

\[
R_{\alpha\beta} = -4g_{\alpha\beta} - \frac{1}{12} F_{\mu\nu} F^{\mu\nu} g_{\alpha\beta} + \frac{1}{2} F_\alpha^{\mu} F_{\beta\mu}, \quad (3.2)
\]

\[
\nabla_\alpha F^{\alpha\mu} - \frac{3\alpha}{\sqrt{-g}} \epsilon^{\mu\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta} = 0. \quad (3.3)
\]

The static black hole solution dual to the finite temperature \( \mathcal{N} = 4 \) SYM plasma with R-charge chemical potential is given by

\[
ds^2 = \frac{1}{z^2} (-h(z) dt^2 + d\vec{x}^2 + dz^2 / h(z)), \quad (3.4)
\]

\[
F_{zt} = q z, \quad (3.5)
\]

where \( h(x) = 1 - ax^4 + q^2 x^6 / 12 \) and all other components of the gauge field are vanishing.

In order to analyze the gauge theory energy-momentum tensor in the most convenient manner one usually passes to the Fefferman-Graham coordinates \[12\]. However it turns out that contrary to the uncharged case \( q = 0 \) considered in \[8\], the Fefferman-Graham form of (3.4) cannot be expressed in terms of elementary functions. Therefore it is more convenient to adopt coordinates similar to (3.4) also for the evolving case.

Namely the proper-time dependent metric ansatz will be

\[
ds^2 = \frac{1}{z^2} (-e^{A(z,\tau)} d\tau^2 + \tau^2 e^{B(z,\tau)} dy^2 + dx_\perp^2 + e^{D(z,\tau)} dz^2), \quad (3.6)
\]

\[
F_{z\tau} = K(z, \tau), \quad (3.7)
\]

which is a general ansatz compatible with boost invariance. Note that with respect to the Fefferman-Graham form of coordinates, the \( z \) and \( \tau \) coordinates have to be redefined. With this ansatz, the gauge field equation part can be solved with an integration constant \( q \) by

\[
F_{z\tau} = q z \frac{\tau}{\tau} e^{A+D-B}. \quad (3.8)
\]

Then the scalar \( F^2 \) becomes

\[
F^2 = -2q^2 \frac{z^6}{\tau^2} e^{-B}. \quad (3.9)
\]
To study nontrivial late time scaling behaviors, we take the scaling variable as $v = z/\tau^{s/4}$ and work in the large $\tau$ limit while keeping $v$ fixed. Then in the scaling limit, one has

$$A = a(v) + O(1/\tau^2), \quad B = b(v) + O(1/\tau^2), \quad D = d(v) + O(1/\tau^2), \quad (3.10)$$

$$F^2 = f^2(v) + O(1/\tau^2), \quad (3.11)$$

where $\natural$ denotes some positive power so that the terms can be ignored in the large proper-time limit. One can, of course, consider the case where $f^2 = 0$ but then the analysis will be reduced to the problem of uncharged fluid dynamics.

From (3.9), one concludes then that, in the scaling variable $v = z/\tau^{s/4}$, the power $s$ should be fixed as $4/3$ once we have a nonvanishing charge density in the scaling limit.

Let us rewrite the Einstein equation (3.2) in the following form

$$R^\alpha_\beta = -\delta^\alpha_\beta \left(4 + \frac{1}{12} F^2\right) + \frac{1}{2} F^{\alpha\mu} F_{\beta\mu}. \quad (3.12)$$

The scaling large $\tau$ limit of the diagonal equations ($\tau\tau$, $zz$, $yy$ and $xx$ components) is

$$R^\tau_\tau = -4 + \frac{1}{6} F^2 \quad R^z_z = -4 + \frac{1}{6} F^2 \quad (3.13)$$

$$R^y_y = -4 - \frac{1}{12} F^2 \quad R^x_x = -4 - \frac{1}{12} F^2, \quad (3.14)$$

where the explicit formulas for the $R^\alpha_\beta$ in the scaling limit are given in the appendix. Apart from these equations we also have the leading part of the off-diagonal equation $R^\tau_z = 0$:

$$v(2b''(v) + b'(v)^2) + 2(3a'(v) + 3d'(v) - 2b'(v)) - v(a'(v) + d'(v)b'(v)) = 0. \quad (3.15)$$

Remarkably enough a direct analog of the static solution:

$$a(v) = \log \left(1 - av^4 + \frac{q^2}{12} v^6\right) \quad (3.16)$$

$$b(v) = 0 \quad (3.17)$$

$$d(v) = -\log \left(1 - av^4 + \frac{q^2}{12} v^6\right) \quad (3.18)$$
solves all the above equations! If we recall now the definition of the scaling variable $v = z/\tau^{1/3}$, we see that asymptotically for large $\tau$ the geometry looks like the static charged black hole with

$$h = 1 - a_{\text{eff}}(\tau) z^4 + \frac{q_{\text{eff}}^2(\tau)}{12} z^6$$

(3.19)

but with the parameters $a_{\text{eff}}(\tau)$ and $q_{\text{eff}}(\tau)$ being $\tau$-dependent. In particular the scaling is exactly such that the effective charge behaves like

$$q_{\text{eff}}(\tau) \sim \frac{q}{\tau}$$

(3.20)

as follows from (2.5). We shall return to the more precise determination of charge density later on.

Before we examine in detail the physical properties of the evolving solution let us try to understand why such a simple generalization of the static solution gives an evolving boost invariant solution.

It turns out that the diagonal components of the Ricci tensor $R^\alpha_\beta$ computed for the static metric

$$ds^2 = \frac{1}{z^2} (-e^{a(z)} dt^2 + e^{b(z)} dy^2 + e^{c(z)} dx_\perp^2 + e^{d(z)} dz^2)$$

(3.21)

coincide with the scaling limit of diagonal components of the Ricci tensor $R^\alpha_\beta$ of the boost-invariant evolving metric

$$ds^2 = \frac{1}{z^2} (-e^{a(v)} d\tau^2 + e^{b(v)} \tau^2 dy^2 + e^{c(v)} dx_\perp^2 + e^{d(v)} dz^2)$$

(3.22)

with the substitution of $z$ for $v = z/\tau^{1/3}$. A similar scaling property holds here for the gauge field, hence equations (3.13)-(3.14) for the evolving case coincide with the corresponding equations for the static metric with $v$ interchanged with $z$.

Once equations (3.13)-(3.14) are solved, the off-diagonal equation (3.15) is automatically satisfied without giving any further independent restriction. This can be shown as follows. Note the Bianchi identity,

$$\nabla_{\mu} H^\mu_\nu = 0,$$

(3.23)

where $H^\mu_\nu$ is the tensor appearing in the usual form of the equation of motion

$$H^\mu_\nu \equiv R^\mu_\nu - \frac{1}{2} \delta^\mu_\nu R - 8\pi G_5 T^\mu_\nu = 0$$

(3.24)
with $T_{\mu\nu}$ including the contributions from the gauge fields and the cosmological constant. The equation (3.23) is a mathematical identity once the solution (3.8) of the Maxwell equations is used, which insures the covariant conservation of the tensor $T_{\mu\nu}$. Then $\nu = \tau$ component of the Bianchi identity can be written as
\[ \partial_{z}(\sqrt{-g}H_{\tau}^{z}) + \partial_{\tau}(\sqrt{-g}H_{z}^{\tau}) - \sqrt{-g}\Gamma_{\tau\beta}^{\alpha}H_{\alpha}^{\beta} = 0. \] (3.25)
Note that $H_{\tau}^{z}$ is $O(z/\tau) (= O(v/\tau^{3}))$ in its leading order. Assuming (3.13)-(3.14) are solved in their leading order, $H_{z}^{z} = H_{y}^{y} = H_{x}^{x} = O(v/\tau^{3})$ at most. The terms involving $H_{z}^{z} = H_{y}^{y} = H_{x}^{x}$ in (3.23) have an extra $\tau$ derivative and, therefore, are $v/\tau^{5}$ higher order than the leading terms of $H_{\tau}^{z}$. Hence by setting those higher order terms to zero, one is left with
\[ \partial_{z}(\sqrt{-g}H_{\tau}^{z}) = 0. \] (3.26)
Fixing the integration constant to zero by the asymptotic condition, one does have the relation $H_{\tau}^{z} = R_{\tau}^{z} = 0$. Thus the off-diagonal equation automatically follows from the remaining part of the equations.

Let us emphasize that the above procedure gives a way to obtain the form of asymptotic geometry only for large proper-times from the corresponding static solution. The resulting solution is, however, not an exact solution for smaller $\tau$. The sub-asymptotic form of the metric does not seem to be any longer linked to the static solution. In a way this is not surprising since the nonlinear evolution includes effects of viscosity and even more pronounced deviations from perfect fluid hydrodynamics for small proper-times. What seems to be more surprising is the fact that any such correspondence between static and evolving boost-invariant solutions exists at all.

## 4 Thermodynamics

Let us now analyze the thermodynamic properties of the evolving geometry derived in the previous section.

Let us first evaluate the entropy per unit volume. The boundary volume element here is given by
\[ dV = \tau dydx^{1}dx^{2}. \] (4.1)
We will use just the extrapolated static formulas since in any case the asymptotic geometry is not valid \textit{at} the horizon but rather in the scaling limit
$\tau \to \infty$ with $v$ fixed. The location of the extrapolated horizon is thus obtained by finding a smaller positive root of

$$1 - av_h^4 + \frac{q^2}{12}v_h^6 = 0,$$

so the entropy density is given by

$$s = \frac{S}{V} = \frac{N_c^2}{2\pi} \cdot \frac{1}{z_h^3} = \frac{N_c^2}{2\pi} \cdot \frac{1}{v_h^3 \tau},$$

while the temperature is

$$T = \frac{|h'(z_h)|}{4\pi} = \frac{1 - \frac{q^2}{8a}v_h^2}{\pi v_h (1 - \frac{q^2}{12a}v_h^2)} \cdot \frac{1}{\tau^{\frac{1}{3}}},$$

In order to find the energy density $\varepsilon(\tau)$ one has to pass to Fefferman-Graham coordinates defined by

$$ds^2 = \frac{1}{z_{FG}^2} (-e^{a_{FG}(z_{FG}, \tau)} d\tau^2 + \ldots + dz_{FG}^2)$$

and read off $\varepsilon(\tau)$ from

$$\varepsilon(\tau) = \frac{N_c^2}{2\pi^2} \lim_{z_{FG} \to 0} \frac{-a_{FG}(z_{FG}, \tau)}{z_{FG}^4}.$$

We have to redefine the $z$ variable as

$$z = z_{FG} \left( 1 - \gamma \frac{z_{FG}^4}{\tau^3} + \ldots \right)$$

and requiring

$$\left(1 - a\frac{z^4_{FG}}{\tau^3} + \frac{q^2}{12a}\tau^6\right)^{-1} \cdot \frac{dz_{FG}^2}{z^2} = \frac{dz_{FG}^2}{z_{FG}^2}$$

fixes the coefficient $\gamma = a/8$. At this order and in the scaling limit one does not need to redefine $\tau$. Applying the redefinition (4.7) to the $g_{\tau\tau}$ component of the metric we use (4.6) to get

$$\varepsilon(\tau) = \frac{N_c^2 a - 2\gamma}{2\pi^2} = \frac{N_c^2}{2\pi^2} \cdot \frac{3}{4} \cdot \frac{a}{\tau^{\frac{1}{3}}} = \frac{3N_c^2}{8\pi^2 v_h^4 (1 - \frac{q^2}{12a}v_h^2)} \cdot \frac{1}{\tau^{\frac{1}{3}}}. $$
By the similar procedure for the other component of the metric, one finds that \( p = \epsilon/3 \).

Finally the charge and chemical potential can be found from the behavior of the gauge field. Let us first work out the charge density. The displacement is evaluated as

\[
D^{\tau r} = \frac{1}{16\pi G_5} \sqrt{-g} F^{\tau z} = \frac{N_c^2}{8\pi^2} q,
\]

which is constant. The boundary charge density is then defined by

\[
\rho = \int dy dx^2 D^{\tau r} / V = \frac{N_c^2}{8\pi^2} q / \tau
\]

showing the expected \( \tau \) dependence\(^2\). The chemical potential can be given as the difference between the horizon and the boundary values of the Coulomb potential

\[
\mu = A_\tau(z_h) - A_\tau(z = 0) = \frac{1}{2} \cdot \frac{qv_h^2}{\tau h}
\]

One may check that the Gibbs potential \( \Omega \) satisfies the relation,

\[
\frac{\Omega}{V} = -p = \epsilon - Ts - \mu \rho.
\]

Also the energy density can be written as

\[
\epsilon(s, \rho) = \frac{3s^4}{2(2\pi N_c)^2} \left( 1 + \frac{4\pi^2 \rho^2}{3s^2} \right),
\]

from which one may check that the chemical potential \( \mu \) is indeed conjugated to \( \rho \) by the relation \( \mu = \partial \epsilon / \partial \rho \). Finally there is the requirement of thermodynamic stability leading to the condition \([14, 15, 16]\),

\[
\rho^2 \leq \frac{3s^2}{4\pi^2}.
\]

\(^2\)The boundary charge density and the chemical potential can be determined by studying the boundary behavior of the gauge fields using AdS/CFT dictionary. The results are the same as the ones from the bulk method used here.
5 The dilaton and electric/magnetic equili-bration

It is interesting to consider turning on the dilaton instead of the gauge field. Such a setup would correspond to considering configurations (states) in gauge theory with a nonvanishing expectation value of $\text{tr} \, F_{\mu\nu} F^{\mu\nu}$ (note that this is now the 4D gauge theory field strength and not the 5D supergravity field considered before in this paper). This means that such a configuration of the plasma has

$$\langle \text{tr} \, \vec{E}^2 \rangle \neq \langle \text{tr} \, \vec{B}^2 \rangle. \quad (5.1)$$

The corresponding 5D action in the Einstein frame is given by

$$I = \frac{1}{16\pi G_5} \int \sqrt{-g} \left( R + 12 - \frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi \right). \quad (5.2)$$

The supergravity equations of motion becomes

$$R_{\alpha\beta} = -4g_{\alpha\beta} + \frac{1}{2} \partial_\alpha \phi \partial_\beta \phi, \quad (5.3)$$

$$\nabla^2 \phi = 0. \quad (5.4)$$

As in Ref. [8], we shall use the Fefferman-Graham form of coordinate by setting $D = 0$ this time while allowing a nontrivial $C(z, \tau) = c(v) + O(1/\tau^2)$ with the general scaling variable $v = z/\tau^{s/4}$ with $0 < s < 4$. Also in the boost invariant scaling limit, the scalar field behaves

$$\phi(z, \tau) = \varphi(v) + O(1/\tau^4). \quad (5.5)$$

The scalar field equation in the leading order can be integrated leading to

$$\varphi' = \frac{k v^3}{e^{\frac{a+b}{2}+c}}. \quad (5.6)$$

Using this expression, the remaining Einstein equations become

$$-2v(a' + b' + 2c') + v^2((a')^2 + (b')^2 + 2(c')^2 + 2(a'' + b'' + 2c'')) = 0, \quad (5.7)$$

$$-2v(4a' + b' + 2c') + v^2((a')^2 + a'(b' + 2c') + 2a'') = 0, \quad (5.8)$$
Interestingly these equations can be solved exactly. The steps are as follows. First we add (5.7) and (5.11) to cancel out the scalar contributions. We then linearly combine the resulting expression with (5.8) such that all the second derivatives cancel out. The final result becomes

\begin{equation}
(4 - 3s)a' + (s - 4)b' + 2sc' = 0.
\end{equation}

Using the boundary condition \(a(0) = b(0) = c(0) = 0\), this is solved by

\begin{equation}
a = M - 2m, \quad b = M + (2s - 2)m, \quad c = M + (2 - s)m.
\end{equation}

with \(M(0) = m(0) = 0\). Inserting these expressions into Eqs. (5.8)-(5.10), one finds that the three equations are reduced to

\begin{align}
-7vM' + v^2(2(M')^2 + M'') &= 0, \\
-3vm' + v^2(2m'M' + m'') &= 0.
\end{align}

These are solved by \[8\]

\begin{align}
M &= \frac{1}{2} \ln(1 - \Delta^2 v^8), \\
m &= \frac{1}{4\Delta} \ln \left( \frac{1 - \Delta v^4}{1 + \Delta v^4} \right).
\end{align}

What remains is to satisfy (5.7) with the above expression of \(M\) and \(m\), which leads to

\begin{equation}
\Delta^2 = \frac{3s^2 - 8s + 8}{24} + \frac{k^2}{96}.
\end{equation}

Then finally computing \(R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}\) \[8\], one can check that it is nonsingular only for \(k = 0\) and \(s = 4/3\). Therefore one concludes that the evolving geometry with a nontrivial scalar field is not allowed for the late time.

The above result shows that fluctuations of electric and magnetic modes tend to equilibrate very fast. Perhaps faster than power-like behavior but
note that above result only shows that the scalar field should approach zero faster than $1/\tau^z$ with $z > 0$ for the finite scaling coordinate $v$.

This is analogous to the stability property of the evolving perfect fluid geometry with respect to small scalar perturbations \cite{9} in the linearized theory. These are in effect quasinormal modes which exhibit exponential decay. The above analysis extends this result to states far away from electric/magnetic equilibration.

6 Conclusions

In this note, we obtain the asymptotic evolving boost-invariant geometry involving conserved R-charge. Thereby the boost-invariant late time dynamics of strongly coupled $\mathcal{N} = 4$ Super Yang-Mills with R-charge turned on are studied via the AdS/CFT correspondence. The result shows that the boost invariant late time state has necessarily to be in the perfect-fluid hydrodynamic regime even including the R-charge.

The asymptotic large proper-time boost-invariant evolving geometry can be seen to arise from the corresponding static solution by substituting the scaling variable $v$ for $z$ in the coefficient functions. This property arises due to the specific form of the Ricci tensor in the scaling limit which closely mirrors the Ricci tensor for the analogous static solution. Let us note, however, that the asymptotic geometry is not an exact solution of the Einstein equations and sub-asymptotic corrections exist. These corrections are important as they encode specific time-dependent dynamical effects.

We also discuss the 5D Einstein-scalar theory to show that the late time boost invariant geometry with a nontrivial scalar field is not allowed. In the gauge theory side this implies that a difference between electric and magnetic modes does not survive into the asymptotic proper-time regime.

Acknowledgments. We are grateful to Andreas Karch, Pavel Kovtun, Dam Son and Larry Yaffe for useful discussions and conversations. The work of DB is supported in part by KOSEF ABRL R14-2003-012-01002-0 and KOSEF SRC CQUeST R11-2005-021. This work was initiated during the program ‘From RHIC to LHC: Achievements and Opportunities’ at the Institute of Nuclear Theory, Seattle. RJ would like to thank the INT for hospitality. RJ
was supported in part by Polish Ministry of Science and Information Society Technologies grants 1P03B02427 (2004-2007), 1P03B04029 (2005-2008) and RTN network ENRAGE MRTN-CT-2004-005616.

Appendix. Expressions for the scaling limit of $R^\alpha_\beta$

Here we quote expressions for the diagonal components of the Ricci tensor used in the main text:

\[
R^\tau_\tau = -\frac{1}{4} e^{-d(v)} (16 + \nu^2 (2a''(v) + a'(v))^2 + 2\nu (b'(v) + d'(v) - 4a'(v)) +
+ \nu^2 (b'(v) + d'(v)) a'(v))
\] (a.1)

\[
R^z_z = -\frac{1}{4} e^{-d(v)} (16 + \nu^2 (2a''(v) + a'(v))^2 + \nu^2 (2b''(v) + b'(v))^2) +
- 2\nu (a'(v) + b'(v) - 4d'(v)) - \nu^2 (b'(v) + a'(v)) d'(v))
\] (a.2)

\[
R^y_y = -\frac{1}{4} e^{-d(v)} (16 + \nu^2 (2b''(v) + b'(v))^2 - 2\nu (a'(v) - d'(v)) + 4b'(v)) +
- \nu^2 (d'(v) - a'(v)) b'(v))
\] (a.3)

\[
R^x_x = -\frac{1}{2} (8 + \nu d'(v) - \nu a'(v) - \nu b'(v)).
\] (a.4)

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