Geometric Modeling IN THE Problems OF Lever Mechanism Kinematics Research

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Abstract. General information about the elements of lever mechanisms consisting of kinematic pairs is presented. The classification features of pairs are given: constraining conditions imposed by this pair, their graphic images, as well as brief descriptions of some of them. Two main problems of kinematics are highlighted. There are two main methods for solving the first problem of kinematics related to the solution of mechanism positioning questions: analytical and numerical. The second problem is an inverse one. A variety of methods for solving it is shown in the paper. The choice or development of the method is mainly determined by the complexity of the mechanism. The analysis of the documentary flow on the problem of kinematics of lever mechanisms allowed us to identify the main areas of research in this area. One of the promising areas is geometric computer modeling, which has a number of advantages in comparison with others. Its application makes it possible to study not only coordinate spaces formed by geometric elements of the lever mechanisms themselves, but also objects generated by products involved in these mechanisms functioning. The line of such research allows us to improve directly CAD systems used in modeling and obtain new results in various application areas, which is an urgent task.

Keywords: geometric modeling, lever mechanisms, kinematics, coordinate space, documentary flow analysis

1. Introduction
Plane and spatial lever mechanisms are the most important components of many engineering products. They are widely applied both in industry and agriculture [1]. One of the applications of such mechanisms is industrial manipulators, which are used in various production areas [2]. The process of their construction involves solving a number of problems. One of them is the kinematic analysis of the manipulator [3]. The design of mechanisms kinematics is characterized by high geometric complexity [4], [5], [6]. One of the main components of the kinematics of links is the geometric shape of the elements, their location on the link, and the coordinate sets swept by these elements in movement. Solving problems related to these components involves the use of complex mathematical apparatus and computer algorithms that implement the resulting mathematical models [7].

The analysis of the documentary flow on the topic "Kinematic-geometric modeling of lever mechanisms" showed that there are a lot of studies on it based mainly on mathematical and numerical methods. At the same time, the field of modeling kinematics of spatial mechanisms by means of geometric modeling is, in actual practice, at the stage of formation and is used mainly for solving specific problems, although such modeling has a number of advantages in comparison with other methods, one of which is visualization of the resulting models, which allows avoiding a lot of errors. Based on the analysis, the most promising solution to the problems under consideration is geometric computer modeling, which allows using the capabilities of modern CAD systems to create models of
geometric objects formed by geometric elements of mechanisms, as well as objects generated by
products moved by the mechanism.
In this regard, a research aimed at developing methods of geometric computer modeling of lever
mechanisms in relation to their use in the environment of integrated computer graphics systems is
relevant. Such research makes it possible to improve CAD systems directly, as well as to obtain new
results in various application areas.
The aim of this work is based on the analysis of the documentary flow of existing literature to show
the diversity of kinematic schemes of linkage, to highlight the kinematics of these mechanisms and
basic methods for their solutions, as well as examples to show some possibilities of geometric
computer modeling in the study of coordinate spaces generated as the links of the mechanism and
geometry of these mechanisms.

2. Theory
2.1. General information about the elements of linkage
The basics of the theory of structure and kinematics of mechanisms can be found in the works of
German scientist studied mechanics and mechanical engineering Franz Reuleaux, Russian
mathematician and mechanic, academician P. L. Chebyshev, Russian mechanical scientist, Professor
of the St. Petersburg Institute of technology L. V. Assur [1]. Based on his ideas academician
I. I. Artobolevsky developed a structural classification of mechanisms consisting of kinematic pairs of
the fifth class, and later extended to other mechanisms, including spatial ones [1].
A kinematic pair is a mobile connection of two connecting links. Solid, elastic and flexible bodies are
used as links of the mechanism. The contact of the links can be carried out on the surface; then, such a
pair is called the lowest. A kinematic pair with links touched along a line or at a point, is called a
higher pair. The relative motion of a link in a kinematic pair may be restricted by its interaction with
other links in the mechanism. As a result, the restriction of the mobility of a link in a kinematic pair
depends only on the location of their connection. The restrictions imposed are called binding
conditions in kinematic pairs.
As it is known, the position of an absolutely solid body moving freely in space, which can be set by
some three points, has six degrees of freedom. They include three rotations around the X, Y, and Z
coordinate axes of the fixed OXYZ coordinate system and three translational movements along the
same axes (figure 1).

![Figure 1. Model of a body in a fixed coordinate system](image-url)
The structure of various kinematic mechanisms has general laws that establish the relationship of the number of degrees of freedom (degree of mobility) of the mechanism with the type of its kinematic pairs and the number of links. The mechanism is a kinematic chain with a single fixed link, which is a rack, performing predetermined movements. Such patterns are determined by the structural formula of mechanisms. The degree of mobility of the mechanism \( W \) is the number of independent parameters that determine the position of all the moving parts of the mechanism. These parameters include independent coordinates (generalized coordinates) that determine the positions of input links.

The links of a kinematic pair perform simple movements (rotational and translational) relative to a fixed coordinate system. Since each link has six degrees of freedom, after their contact, i.e. after creating a kinematic pair, the number of independent movements decreases.

The number of constraining conditions \( S \) for the relative motion of each link of the kinematic pair can be found in the range from 1 to 5, i.e. \( 1 \leq S \leq 5 \). Therefore, the number of degrees of freedom \( W \) of a kinematic pair link in relative motion is determined by the dependence

\[
W = 6 - S. \tag{1}
\]

It follows from equality (1) that the number of degrees of freedom of a kinematic pair link in relative motion can vary, as well as the number of constraining conditions from 1 to 5.

Kinematic pairs are classified by the number of binding conditions imposed by this pair [1]. The class number is equal to the number of restrictions. The table shows the classification characteristics of pairs and their computer models.

Here is a brief description of some kinematic pairs.

| Pair class | Number of constraining conditions, \( S \) | Number degrees of freedom, \( W \) | Name of the pair | Computer model | Conventional symbol |
|------------|---------------------------------|-------------------------------|-----------------|-----------------|-------------------|
| I          | 1                               | 5                             | shere-plane     | ![Image](image1.png) | ![Symbol](symbol1.png) |
| II         | 2                               | 4                             | sphere-cylinder | ![Image](image2.png) | ![Symbol](symbol2.png) |
| III        | 3                               | 3                             | Spherical       | ![Image](image3.png) | ![Symbol](symbol3.png) |
| Pair class | Number of constraining conditions, $S$ | Number degrees of freedom, $W$ | Name of the pair | Computer model | Conventional symbol |
|------------|----------------------------------------|-------------------------------|----------------|----------------|---------------------|
| III.       | 3                                      | 3                             | Planar         | ![Planar](image) | ![Planar](image)    |
| IV         | 4                                      | 2                             | Cylindrical    | ![Cylindrical](image) | ![Cylindrical](image) |
| IV         | 4                                      | 2                             | Spherical with a pin | ![Spherical with a pin](image) | ![Spherical with a pin](image) |
| V          | 5                                      | 1                             | Translating    | ![Translating](image) | ![Translating](image) |
| V          | 5                                      | 1                             | Rotational     | ![Rotational](image) | ![Rotational](image) |
| V          | 5                                      | 1                             | Screw          | ![Screw](image)    | ![Screw](image)     |
Pairs of the first class include movable joints that allow helical movement, one translational or one rotational.
Pairs of the second class include movable joints that have two rotational movements or one rotational and one translational movement. In a planar mechanism, there can only be kinematic pairs of the first and second type.
The pairs of the third type include movable connections with: 1) three rotational movements; 2) two rotational and one translational movement; 3) two translational and one rotational movement.
The fourth type pairs include pairs in which two rotational and two translational movements or one translational and three rotational movements are allowed.
Pairs of the fifth type are such movable connections in which three rotational and two translational movements are allowed for its links when they touch each other at a point.
All the movements in the listed pairs must be independent. Despite the fact that, for example, a screw in a nut makes two movements – translational and rotational – simultaneously, since these movements are interconnected, we can set only one movement at random.
To depict the kinematic mechanism in the drawing, a block diagram is used, which incorporates symbols for links and pairs without drawing dimensions. For kinematic or dynamic research, a kinematic scheme with the dimensions required for the corresponding calculations is used.
As an example, figure 2 shows a structural image of a fragment of the bridge opening mechanism [8], and figure 3 – its block diagram.

Figure 2. A fragment of the lifting-bridge mechanism design: a) with dimensioning part position numbers; b) with the indication of kinematic pair nodes
Figure 3. Block diagram of the lifting-bridge mechanism (in two positions: the bridge is opened and the bridge block is rotated 60° relative to the vertical)

A lot of publications have been devoted to solving problems of analysis and synthesis of kinematic mechanisms. Techniques of plane and spatial kinematic mechanisms synthesis have been considered by many researchers. Their results are published in many papers. Since it is not possible to conduct an analytical review of all this material, due to the limited scope of the paper, we will refer only to the review of research given in [9], as well as to the lists and analysis of the literature used in the dissertations [6], [10], [11], [12].

2.2. Problems of kinematics of lever mechanisms and methods of their solution

The problem of mechanisms kinematics is aimed at modeling their location in space as a function of time. The solution of such problems involves establishing a connection between the space of generalized coordinates of the mechanism and the position of its links in the coordinate space.

There are two main tasks of kinematics. The first task is related to solving the problems of positioning the mechanism. It means that the known generalized coordinates of the mechanism and the specified geometric parameters of its links need to determine the position of the output link in the coordinate space. It is called a direct kinematics problem. The second task is an inverse one. There, the geometric parameters of the links are known, so it is necessary to determine the space of generalized parameters that ensure the position of all the links of the mechanism, including the output one, in the coordinate space.

There are two main approaches to solving the first kinematics problem: analytical and numerical ones. In the analytical approach, formulas are derived that explicitly establish the dependence of the output parameters of the mechanism on its input parameters. Establishing such dependencies is possible mainly for fairly simple cases. Consider, for example, a plain mechanism consisting of two links connected by means of rotational kinematic pairs, shown in figure 4. The degree of mobility of such a mechanism is equal to two, since the number of generalized coordinates is two (parameters $\varphi_1$ and $\varphi_2$).

Let the dimensions of the OA and AB links be $L_1$ and $L_2$, respectively. We need to determine the position of points A and B in the fixed OXY coordinate system associated with the rack. Figure 4 shows that the coordinates of these points are determined from the dependencies:

$$x_A = L_1 \times \cos \varphi_1,$$
$$y_A = L_1 \times \sin \varphi_1,$$

and

$$x_B = L_1 \times \cos (\varphi_1 + \varphi_2),$$
$$y_B = L_1 \times \sin (\varphi_1 + \varphi_2).$$

(2)
\[ x_B = x_A + L_2 \times \cos\varphi_2, \]
\[ y_B = y_A + L_2 \times \sin\varphi_2, \]  \hspace{1cm} (3)

or
\[ x_B = L_1 \times \cos\varphi_1 + L_2 \times \cos\varphi_2, \]
\[ y_B = L_1 \times \sin\varphi_1 + L_2 \times \sin\varphi_2 \]  \hspace{1cm} (4)

Note that if the position of point A depends on one parameter (2) – \( \varphi_1 \), the position of point B depends on two parameters (4) – \( \varphi_1 \) and \( \varphi_2 \). These dependencies are simple and are used to calculate the coordinates of the output link of the mechanism using the generalized coordinates \( \varphi_1 \) and \( \varphi_2 \).

When the mechanism has a larger number of links and the resulting dependencies are complex, a numerical approach is used to solve the problem. This approach is based on the derivation of equations that establish the relationship of generalized coordinates with the coordinate space of the mechanism links [7].

Let the kinematic mechanism of the manipulator consist of \( n \) links and be given in the absolute (inertial) coordinate system \( OX_0Y_0Z_0 \) coordinates. Then the center of the grip (a fixed point of the output link) is defined in this system by coordinates
\[ x_{0n} = x(t), \quad y_{0n} = y(t), \quad z_{0n} = z(t), \]
and the orientation of its links relative to each other is set using some generalized coordinate \( q_i \), and \( i = 1, \ldots, n \). If the \( i \)-th link makes a rotational motion relative to the \( (i-1) \)-th, then the generalized coordinate is the angle \( \varphi_i \). In the case of translational motion of the \( i \)-th link relative to the \( (i-1) \)-th, the generalized coordinate is the linear parameter \( S_i \).

In manipulators, the center of the grip is usually the geometric center of the compressed jaws [2]. In general, the position of neighboring links relative to each other is set using some generalized coordinate \( q_i \), and \( i = 1, \ldots, n \). If the \( i \)-th link makes a rotational motion relative to the \( (i-1) \)-th, then the generalized coordinate is the angle \( \varphi_i \). In the case of translational motion of the \( i \)-th link relative to the \( (i-1) \)-th, the generalized coordinate is the linear parameter \( S_i \).

Finally, the mathematical model of the manipulator kinematics can be represented in a general form by a system of equations
\[ x_{0n} = x(q_1, q_2, \ldots, q_n; t); \]
\[ y_{0n} = y(q_1, q_2, \ldots, q_n; t); \]
\[ z_{0n} = z(q_1, q_2, \ldots, q_n; t); \]
\[ \angle x_{0n} z_n = f_1(q_1, q_2, \ldots, q_n; t); \]
\[ \angle y_{0n} z_n = f_2(q_1, q_2, \ldots, q_n; t); \]
\[ \angle x_{0n} y_n = f_3(q_1, q_2, \ldots, q_n; t). \]  \hspace{1cm} (5)

Figure 4. Kinematic scheme of a two-link mechanism
This model is usually formed based on transformations of coordinate systems. The resulting dependencies having the form (5) for a multi-parametric set of generalized coordinates are usually lengthy and are not explicitly solved. In this case, numerical methods are used. For example, a homogeneous 4×4 transformation matrix is used to determine the relative position of adjacent links. In order to uniquely determine the position of one coordinate system relative to another, you need to set six parameters: rotation around three axes and displacement along the same axes. In practical modeling, it is necessary to determine the position of not only the grip, but also the intermediate links. Since the links of the mechanism are connected, it is necessary to solve the problem of positioning for each link of the mechanism in turn. In this case, the Denavit–Hartenberg method is used to determine the position of the manipulator links [13], [14]. This method involves the forming matrices that describe the position of the coordinate system of the i-th link relative to the coordinate system of the (i-1)th link. This approach also allows converting the coordinates of each link specified in the local coordinate system to the base coordinate system.

In accordance with this method, the coordinate system of the i-th link is formed in the following sequence [14]. First, the $z_{i-1}$ axis of the local coordinate system is set so that it is directed along the axis of the i-th junction of the kinematic pair. Then the $x_i$ axis is set to be perpendicular to the $z_{i-1}$ axis, and its direction is set from this axis. The direction of the $y_i$ axis is chosen so that the resulting Cartesian coordinate system is right-handed.

According to this method, only four parameters of each link are used in the process of modeling the position of the manipulator links. These four parameters fully describe any translational or rotational motion.

During the operation of the manipulator, to perform the necessary actions with its object, it is necessary to control the orientation of its grip in space. In this case, it is necessary to solve the problem inverse to the problem discussed above. Here, the initial data are the parameters and joints of the links, as well as the matrix of the position and orientation of the manipulator grip, while the generalized coordinates that ensure the movement of this grip along a given trajectory is to be determined. To solve this problem, a number of methods described foreign literature [15], [16], which have been further developed, are known. So, the method of inverse transformations means creating transition matrices from the coordinate system of one link to the coordinate system of an adjacent link. In this case, the Denavit–Hartenberg method is applied. A number of methods for solving the inverse kinematics problem are based on the use of Jacobi matrices [17], [18], [19]. The method of helical algebra is based on helical calculus [9].

Paper [20] shows the use of several methods, the choice of which is determined depending on the complexity of the manipulator. A hierarchical approach to solving the inverse kinematics problem is shown in [21]. The trigonometric method [13] does not always allow solving the inverse problem. This often happens when the number of degrees of mobility of the mechanism exceeds the number of equations describing its kinematics. This method is preferred for simple manipulators. With a large number of degrees of mobility, for complex manipulators without redundancy, such methods as the method based on the use of Jacobi matrices [17], [18], [19], interval method [22], and Newton methods [23] are used. However, their main disadvantage is the amount time spent on calculations, which makes it difficult to use them in real manipulators.

The problems of designing kinematics in robotics are concentrated today mainly around the Stewart platforms [2] and the tripod [6].

### 3. Geometric computer modeling in the study of kinematics of lever mechanisms

The geometry of the mechanism movement is one of the important fields of kinematics. It examines the movement of points, lines, and surfaces. This section is studied both in classical literature [1] and in scientific literature, including when solving various applied problems. The methods described above are most used in the study of kinematics. Geometric computer modeling is increasingly used in technical problems, both in the design of workpiece and in their shaping. However, the field of modeling kinematics of spatial mechanisms by means of geometric modeling is mainly at the stage of its development. At the same time, one of the features of geometric modeling of kinematics is that its
object of research is not only the mechanism itself, but also the coordinate sets created by the constituent elements of this mechanism. It is developing in several directions. In one of them, we study families of surfaces and their envelopes generated by the movement of bodies moved by various manipulators. Thus, in [6], the study of families of surfaces and their envelopes is carried out in relation to the simulation of wheel movement during the cleaning and release of the aircraft landing gear [5]. Only one-parameter families of surfaces that are defined by a single generalized coordinate are considered here. Mathematical modeling is mainly used to solve these problems.

The second direction of application of geometric modeling is focused on the study of manipulators [3], [4] that move in space with the presence of restricted zones and have redundant connections [24], [25], [26]. These works mainly use analytical methods with the connection of computer graphics tools at certain stages.

Existing modern CAD systems allow designing mechanisms by creating parameterized solid-state computer models, as shown in [27].

In this regard, research aimed at developing methods of geometric computer modeling of lever mechanisms in relation to their use in the environment of integrated computer graphics systems is relevant. Such research makes it possible to improve CAD systems directly, as well as to obtain new results in various application areas.

Based on the conducted experiments, some possibilities of geometric computer modeling are shown below for solving both problems of kinematics of mechanisms and studying the coordinate space generated by an object associated with this mechanism.

4. Experimental results

**Example 1.** Let the mechanism use a three-degree-of-freedom spherical pair (table). Such a pair is a joint that is used only in spatial mechanisms. The class of the pair is 3, because in the relative motion of the links, rotations around three axes of a fixed coordinate system are possible (figure 5). In this case, the number of generalized coordinates of the mechanism is equal to two—these are the angles \( \varphi \) and \( \Theta \). Then the center of the grip \( O_1 \) in its movement will describe a spherical surface, and the moving link, whose model is the segment, will sweep the sphere (figure 6).

![Figure 5. Joint mechanism and its generalized coordinates](image-url)
The parametric equations of a sphere centered at the beginning of the coordinate system will have the following form

\[
\begin{align*}
    x &= R \cdot \sin \Theta \cdot \cos \varphi, \\
    y &= R \cdot \sin \Theta \cdot \sin \varphi, \\
    z &= R \cdot \cos \Theta,
\end{align*}
\] (6)

where \( \Theta \in [0, \pi] \) and \( \varphi \in [0, 2\cdot\pi] \).

Equations (6) of the sphere are used to obtain a computer model of the sphere (figure 7 shows half of the sphere) to study the trajectory of the center of the grip.

So, let some intermediate positions of the grip centers be given by points A, B, and C. You need to plot the shortest path of its movement.

As you know, geodesic lines on a sphere are circles of the largest radius. Then the desired trajectories of the grip center will be arcs of circles passing through these points and lying in the diametric planes of the sphere (figure 7). Since during the operation of the mechanism, not only the center of the grip moves, but also its link, the resulting geometric model can have the form shown in figure 8, in which I, II and III are the compartments of the planes covered by the link of the mechanism.

The resulting models can be used in a visual form not only to select the trajectories of movement of the gripper, but also if necessary to take into account possible restrictions on the space in which the links of the mechanism move.

**Figure 6.** Sphere as a coordinate set swept by a link of a hinge mechanism

**Figure 7.** Sphere, as a coordinate set, swept by the center of the grip of the hinge mechanism
Example 2. Geometric computer modeling of coordinate sets of a plane two-link mechanism components.

The previous section shows an example of using the analytical method for modeling a plane two-link mechanism. Let us give a geometric interpretation to the analytical dependencies (2), (3), and (4). Obviously, dependencies (2) define a circle with radius $R_1 = OA = L_1$ centered at point $O$, and dependencies (3) are also a circle, but of radius $R_2 = AB = L_2$, and its center is located at point $A$ on the first circle (figure 9). The position of point $A$ is determined by the generalized coordinate $\phi_1$. As a result, formulas (4) define a one-parameter family of circles of radius $R_2$ which centers are located on a circle of radius $R_1$.

Consider the family of circles (4) as a graph of mapping some circles in the space $R^3$ to the plane $R^2$ [28]. But then this family in the space $R^3$ sets some auxiliary two-dimensional surface [29]. To model it, we will introduce a third $Z$ coordinate axis perpendicular to the $X$ and $Y$ axes. Let us set this family of circles to move along the $Z$ axis in proportion to the parameter $\phi_1$. Then the equation of the resulting surface has the form

$$x = R_1 \cdot \cos \phi_1 + R_2 \cdot \cos \phi_2,$$
$$y = R \cdot \sin \phi_1 + R_2 \cdot \sin \phi_2,$$
$$z = P \cdot \phi_1,$$
$$
\text{where } P \text{ is a constant.}
\text{The model of the resulting surface is shown in figure 10 and is a tubular screw surface. The family of circles can be moved to space and is proportional to the parameter } \phi_1. \text{ Then we get the equations of the second auxiliary surface}

$$x = R_1 \cdot \cos \phi_1 + R_2 \cdot \cos \phi_2,$$
$$y = R \cdot \sin \phi_1 + R_2 \cdot \sin \phi_2,$$
$$z = P \cdot \phi_2,$$

\text{The resulting second surface (8), like the first (7), is a tubular helical surface and is shown in figure 11. For comparative analysis of the obtained surfaces, they are set in the same coordinate system at different values of radii } R_1 \text{ and } R_2.
The developed computer models of auxiliary surfaces require additional research in order to obtain qualitative characteristics of the features of their display on the plane, determine the space of possible configurations of the mechanism, and select the optimal parameters for moving the grip.

Figure 9. Coordinate sets described by node points of a two-link mechanism

Figure 10. Model of the first auxiliary tubular screw surface

Figure 11. Model of the second auxiliary tubular screw surface
Example 3. Geometric computer modeling can be used not only to study coordinate spaces created by elements of mechanisms, but also spaces generated by geometric objects moved by the mechanism. Let us consider the example of moving a sphere along a helical trajectory [30]. Let a sphere of radius $R$ be defined in a mobile coordinate system and make a helical movement along the $Z_1$ axis of the stationary coordinate system. In this case, a one-parameter family of surfaces in the space $R^3$ will be formed. The computer model of this family is shown in figure 13, its graph in the space $R^4$ is a hypersurface defined as

\begin{align*}
x_1 &= (x + R) \cdot \cos \varphi - y \cdot \sin \varphi, \\
y_1 &= (x + R) \cdot \sin \varphi + y \cdot \cos \varphi, \\
z_1 &= z + p \cdot \varphi, \\
\theta_1 &= p_1 \cdot \varphi.
\end{align*}

The discriminant of the hypersurface (9) is a two-dimensional surface defined by the equations

\begin{align*}
x_1 &= \pm \sqrt{r^2 - z^2 \cdot \left(1 + \frac{P^2}{R^2}\right) + R} \cdot \cos \varphi + \frac{P}{R} \cdot z \cdot \sin^2 \varphi, \\
y_1 &= \pm \sqrt{r^2 - z^2 \cdot \left(1 + \frac{P^2}{R^2}\right) + R} \cdot \sin \varphi - \frac{P}{R} \cdot z \cdot \sin^2 \varphi \cdot \cos \varphi, \\
z_1 &= z + p \cdot \varphi.
\end{align*}

The surface (10) is a tubular screw surface. It is swept by the load as a geometric object during the operation of the mechanism and can be used to analyze the space in order to identify restricted areas of the mechanism movement.
5. Conclusion
The analysis of the documentary flow and the research directions of the modern theory of kinematics of lever mechanisms do not allow us to clearly identify the most significant of them. Such studies are often focused on choosing the optimal method for solving the problem for the corresponding configuration of the kinematic mechanism.
Currently, the development of this theory successfully continues the period in which the capabilities of modern computer graphics tools, including CAD systems, are used. A number of studies also use some results of interlocking theory, as the theory of higher kinematic pairs.
At the same time, research based on geometric computer modeling is particularly relevant. Such modeling is particularly effective when analyzing various coordinate spaces formed not only by geometric objects of the mechanism, but also generated by objects associated with these mechanisms in the course of their functioning. Some of the possibilities of such modeling are shown in the examples of these experiments.
Research based on geometric computer modeling is of scientific interest in terms of modeling new coordinate spaces, as well as of practical one – in order to optimize both the lever mechanisms themselves and their kinematics.

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