Enhancement of electron-phonon energy transfer and ultrasound attenuation in impure conductors due to Mandelstam-Leontovich relaxational mechanism

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We derive a general relation between the ultrasound attenuation rate $\tau_{ph}^{-1}(\omega)$ due to interaction with conduction electrons and the energy flow $J(T)$ between overheated electrons and phonons at bath temperature (cooling power). We demonstrate an existence of several mechanisms leading to strong enhancement of $\tau_{ph}^{-1}(\omega)$ and $J(T)$ at low frequencies/temperatures in disordered conductors; all of them are related with the Mandelstam-Leontovich relaxation mechanism. The ultrasound attenuation and the electron-phonon energy transfer are found to be sensitive to magnetic field and can be enhanced by the factor $10^2 - 10^3$ at $B \sim 10$ Tesla. They can also be increased strongly in semiconductor heterostructures with an external gate.

The theory of electron-phonon interactions in disordered conductors has a long history. One of its key points, known as ”Pippard ineffectiveness condition” (denoted as PIC below) [1, 2], is that for phonons with small momenta $q$ the rate of inelastic electron-phonon processes is by factor $\sim q l \ll 1$ weaker than in the ballistic limit ($l$ is the elastic scattering length of electrons). In the framework of this theory the ultrasonic attenuation rate $\tau_{ph}^{-1}$ was calculated [1] in the $q l \ll 1$ limit:

$$\frac{1}{\tau_{ph}(q)} = c_\alpha \frac{\nu p_F^2}{\rho_m} D q^2 \sim \frac{n_e}{\rho_m} D q^2, \tag{1}$$

where $\nu$ is electron density of states [4], $p_F = mv_F$ is Fermi momentum, $n_e$ and $\rho_m$ are the density of electrons and the mass density of material, $D = v_F l/d_e$ is the diffusion coefficient, $q$ is the phonon momentum, $d_e$ is the dimensionality of electron motion. The subscript $\alpha$ corresponds to the choice of either transverse (tr) or longitudinal (l) phonons; correspondingly, numerical coefficients $c_\alpha$ are defined as $c_{tr} = 1/(2 + d_e)$ and $c_l = 2(1 - d_e^{-1})/(2 + d_e)$. In the clean limit $q l \gg 1$ coupling to transverse phonons is ineffective, whereas longitudinal phonon attenuation rate scales as $1/\tau_{ph} \propto q$.

Below we will see that phonon attenuation rate $\tau_{ph}^{-1}$ is directly related to the energy flux (cooling power) $J(T_e, T_{ph}) = J(T_s) - J(T_{ph})$ transmitted from electrons with temperature $T_e > T_{ph}$ to phonons. As the temperature decreases below $T_0 = h v_s / l$ wave vectors $q = T / h v_s$ of thermal phonons drop below $l^{-1}$ and crossover takes place from the classical result for the e-ph cooling power (assuming bulk phonons) $J(T) \propto T^5$, to the faster decrease with temperature $J(T) \propto T^6/T_0$. The latter behavior of the cooling power predicted in Ref. [2, 3] corresponds to the result of Ref. [1], Eq.(1) for the phonon decay rate, and was shown to be obeyed very accurately by the data on disordered metal films of Hf and Ti [7].

It is known that PIC does not hold if either a) impurities do not follow the lattice deformations [9], or b) piezoelectric interaction is present [8]; enhancement of the cooling power in semiconductors due to the ”electron-phonon-impurity” interference has been also discussed [10]. Data on the cooling power in some strongly disordered conductors [11–13] where none of the conditions a) or b) are expected to hold, indicate to the presence of a different enhancement mechanism.

In the present Letter we demonstrate an existence of quite general mechanism for the enhancement of both cooling power $J(T)$ and ultrasound attenuation rate $\tau_{ph}^{-1}$ (for longitudinal phonons) at low temperatures. This mechanism is similar to the Mandelstam-Leontovich (ML) relaxation [14] and is effective if lattice motion is able to induce significant oscillations of local densities of certain globally conserved physical quantities. The deviations of these local densities from their equilibrium values are enhanced by slow diffusive character of electron motion (characterized by both small frequency and small momentum) aimed to restore equilibrium. This leads to a significant retardation in the response and thus to the entropy production and dissipation. In contrast to PIC which suppresses the relaxation rate $\tau_{ph}^{-1}$ at strong disorder and small carrier concentration, the ML mechanism is efficient at these conditions. In particular, we found a broad range of frequencies/temperatures where $\tau_{ph}^{-1}$ is $\omega$-independent, whereas $J(T) \propto T^4$.

We will discuss below two specific realizations of such a mechanism: 1) the presence of several types of quasiparticles with different spectra which can be described by the quasi-spin components, and 2) relatively weak Coulomb interaction between electrons, when the local electroneutrality condition is not strictly obeyed. In the first case phonons create non-equilibrium distribution of quasi-spin density, while in the second case it is the charge density. We will show that the physics behind calculations in Ref.[10] is precisely the charge density relaxation at incomplete screening rather than the peculiarity of the momentum-averaged e-ph vertex.

We will use the co-moving frame of reference (CFR) rigidly bound to the lattice. Then electron-phonon inter-
action is encoded mainly in non-uniformity of the coordinate transformation. For a single branch of electrons, the density dependence of the Fermi-energy leads to [1, 15]:

\[ H_{e-ph} = \sum_{p,q} p_n(v_p \nabla \beta u_n)_q \cdot \bar{\psi}_p \psi_{p+q} \]  

(2)

where \( p \) and \( v \) denote electron momentum and velocity, respectively and \( u \) is the lattice displacement. The tensor structure of Eq. (2) is crucial for local processes only, such as those giving rise to the result given in Eq. (1). To study the enhancement of dissipation due to diffusion processes, it is sufficient to average the e-ph vertex over the Fermi surface. For a metal with spherical Fermi-surface one finds \( p_n(v_p \nabla \beta u_n)_q = \Gamma \text{div}u \), where \( \Gamma = \frac{p_F v_F}{d_F} \). In general \( \Gamma \) may contain other contributions. In particular, for semiconductors \( \Gamma \) is known [16] to be much larger than \( E_F \). Under the condition of strict electroneutrality, the scalar vertex \( \Gamma \) is screened out completely and the classical result Eq. (1) is valid. This is not the case, however, when \( N \) different types of quasiparticles are present. Then the interaction can be written as

\[ H_{e-ph} = \sum_{j=1}^{N} \Gamma_j \text{div}u \left( \bar{\psi} \psi \right)_j \]  

(3)

The Coulomb interaction is able to screen out only the single mode corresponding to the total density \( n = \sum_j (\bar{\psi} \psi)_j \), whereas \( N - 1 \) asymmetric modes survive screening [15]. In particular, for the \( N = 2 \) case corresponding to the Zeeman splitting of spin-up and spin-down sub-bands, the mode corresponding to spin density fluctuations is unscreened. As we will see below, coupling to the spin density mode generates diffusion motion of electrons which leads to extra dissipation seen in the increase of both the ultrasound attenuation and of the heat transfer from hot electrons to the phonon bath. The same effect occurs even in the case of a single branch of quasiparticles if Coulomb screening is incomplete. The latter is natural for 2D (or quasi-1D [10]) electron systems where Coulomb interaction mainly propagates through the surrounding dielectric media.

We start by writing down the quantum kinetic equation in order to establish a general relationship between the rates of electron cooling and ultrasonic attenuation. In a typical experiment on electron cooling the electron system possesses its own temperature \( T_{el} \) decoupled from the lattice with the temperature \( T_{ph} \). Assumption of a well defined electron temperature is reasonable if electron-electron inelastic rate \( \tau_{ee}^{-1} \) is much larger than electron-phonon one, \( \tau_{e-ph}^{-1} \), which is the case at low temperatures. Under that assumption of local equilibrium, electron-electron interaction drops out of the theory. Our results for \( \tau_{e-ph}^{-1} \) are presented in [17].

The key point to establish a relationship between electron cooling and phonon relaxation, or ultrasound attenuation, is to consider the kinetic equation for phonons instead of the electron kinetic equation and also to assume conservation of the total energy of the electron and the phonon systems. Thus we consider phonon heating rather than electron cooling, which makes it possible to express heat flow in terms of equilibrium electronic quantities (see [17], Sec. II):

\[ \partial_t B_{ph}(\omega, T_{el}) = \frac{B_{ph}(\omega, T_{el}) - B_{ph}(\omega, T_{ph})}{\tau_{ph}^{-1}(\omega, T_{el})} \]  

(4)

where \( B_{ph}(\omega, T) \equiv B_{ph}(\omega/T) = \frac{1}{\omega} \text{coth}(\omega/2T) - 1 \) is the phonon distribution function. We employed here the relation \( \tau_{ph}^{-1} = \frac{1}{\rho_{ph} \omega} \left[ \text{Im } \Sigma_{R}^{\omega}(\omega, q) \right] \omega = \omega_q \) between the phonon life-time \( \tau_{ph} \) and the retarded phonon self-energy \( \Sigma_{R}^{\omega} \) [17]. Now the energy flow \( J = \frac{\partial B_{ph}}{\partial t} \) can be written as follows:

\[ J = \int_{0}^{\infty} d\omega \omega n_{ph}(\omega) \frac{B_{ph}(\omega/T_{el}) - B_{ph}(\omega/T_{ph})}{\tau_{ph}(\omega, T_{el})} \]  

(5)

where \( n_{ph}(\omega) = \omega^2/(2\pi^2 v_F^2) \) is the phonon density of states for 3D phonons with the sound velocity \( v_s \).

One can see from the relation Eq. (5) that the cooling power is determined by the phonon relaxation rate \( \tau_{ph}(\omega, T_{el}) \) which also determines the ultrasound attenuation. If we do not take into account any correlations induced by electron-electron interactions (charge screening is considered in the RPA approximation only), then \( \tau_{ph} \) does not depend explicitly on electron temperature \( T_{el} \) allowing to separate clearly the incoming and outgoing energy flows: \( J = J(T_{el}) - J(T_{ph}) \). In particular, for the power-law \( \omega \)-dependence of \( \tau_{ph}^{-1}(\omega) \propto \omega^\beta \) the incoming energy flow for the 3D phonons is proportional to \( J(T_{el}) \propto T_{el}^{1+\beta} \), for details see [17], Sec.V.

Below we present a simple derivation of the diffusion-enhanced contribution to the phonon relaxation rate \( \tau_{ph}^{-1} \) in terms of macroscopic equations for the current and density of electrons; alternative diagrammatic derivation is presented in [17], Sec. III. In the CFR the continuity and diffusion equations for each i-th species of quasiparticles read:

\[ \begin{cases} \partial_t n^{(i)} + \text{div}j^{(i)} = 0, \\ j^{(i)} = -D \nabla n^{(i)} - \kappa_i F^{(i)} \end{cases} \]  

(6)

where \( (i) \) stands for the quasiparticle branch number, \( n^{(i)} \) is the electron density, \( j^{(i)} \) is the particle number current, \( \kappa_i = \nu_i D_i \) is the mobility, \( D_i \) is the diffusion coefficient for the i-th branch, \( F^{(i)} = -\nabla U^{(i)} \) and \( U^{(i)} \) is the potential energy. In the simplest derivation we assume no inter-branch scattering and thus the continuity equations in Eq. (6) imply that each of the partial electron densities \( n^{(i)} \) are conserved separately. Generalization to the case where there is mixing between the branches will be done at the end of the paper. The potential energy \( U^{(i)} = U_C + \Phi^{(i)} \) in Eq. (6) consists of the Coulomb part \( U_C \) and
the phonon part \( \Phi^{(i)} = \Gamma^{(i)} \text{div} \mathbf{u} \):

\[
U^{(i)} = \int \mathbf{V}_0(\mathbf{r} - \mathbf{r}') \sum_j \delta n^{(j)}(\mathbf{r}') + \Gamma^{(i)} \text{div} \mathbf{u}
\]  
(7)

where \( \mathbf{V}_0(\mathbf{r}) \) is the bare Coulomb potential acting between conduction electrons; below we use its Fourier-transform \( \mathbf{V}_0(q) \). Note that \( \mathbf{V}_0(q) \) does not include screening by conduction electrons in the sample.

First, we discuss the case of several quasiparticle branches and perfect electroneutrality. The latter formally corresponds to the limit \( \mathbf{V}_0(q) \rightarrow \infty \). For the density modulation \( n^{(i)}(\omega, q) \) induced by the phonon with frequency \( \omega \) and momentum \( q \) one finds from Eqs.(6),(7):

\[
n^{(i)}(\omega, q) = \Pi_i(\omega, q) (\Phi_i(\omega, q) - \Phi_C(\omega, q))
\]  
(8)

where \( \Phi_i(\omega, q) = \Gamma^{(i)} \text{div} \mathbf{u}, \Phi_C = \sum_j \Phi_j \Pi_j / \sum_j \Pi_j \) represents dynamical screening of Coulomb interaction and \( \Pi_i = \nu_i Dq_i^2 / (-i\omega + D_i q^2) \) is the polarization function.

The solution (8) obeys charge-neutrality: \( n_{\text{tot}} = \sum_i n^{(i)} = 0 \). However, there are \( (N - 1) \) independent density modes \( n^{(i)} \) which are excited by phonon modulation. To simplify the expressions we consider below the case \( N = 2 \).

The contribution of ML mechanism to the phonon decay rate may be expressed as \( \tau_{\text{ph,ML}}^{-1} = \left| \frac{Q_l}{E_w} \right| \), where \( Q_l \) and \( E_w \) are the dissipation power and the acoustic wave energy in a unit volume, respectively:

\[
Q_l = \frac{1}{2} \text{Re} (\mathbf{j}^* \cdot \mathbf{F}), \quad E_w = \frac{\rho_m}{2} \omega^2 u_m^2.
\]  
(9)

Here \( u_m \) is an amplitude of ionic displacement and \( \mathbf{u} = (q/q) u_m \exp[-i\omega t + iq \cdot \mathbf{r}] \). Simple calculation leads to the following expression for the "diffusive" contribution to the decay rate of acoustic phonon:

\[
\tau_{\text{ph}}^{-1}(q) = \frac{(\Gamma_1 - \Gamma_2)^2}{2\rho_m} \frac{\nu_s Dq^2}{v_s^2 + D^2 q^2}
\]  
(10)

where \( v_s = \omega / q \) is the sound velocity, while \( \nu_s = (\nu_1^{-1} + \nu_2^{-1})^{-1} \) and \( D_s = (D_1^{-1} + D_2^{-1})^{-1} \) are the effective density of states and diffusion coefficient, respectively.

In particular, application of parallel magnetic field \( H \) to a two-dimensional electron gas in a semiconductor hetero-structure leads to the splitting of spin sub-bands with \( \Gamma_1 - \Gamma_2 = \mu H \) ([17]), where \( \mu = g_i \mu_B \) is the electron magnetic moment. Then, according to Eq.(10), the phonon relaxation rate acquires an \( H \)-dependent contribution that may become dominant at sufficiently strong field and low phonon frequencies. Adding the classical result (1) and the magnetic-field-controlled contribution, Eq.(10), one finds ([17], Sec. III.B, VII) for the full phonon decay rate \( \tau_{\text{ph}}^{-1} = \tau_{\text{ph}}^{(0)} \cdot F_H(q, h) \), where for \( q \) parallel to 2D gas:

\[
F_H(q, h) = 1 + \frac{8}{3} \frac{v_s^2 h^2}{v_s^2 + (Dq)^2}.
\]  
(11)

Here \( \tau_{\text{ph}}^{(0)} \) is given by Eq.(1) for longitudinal phonons and \( h = \mu H/2\varepsilon_F \) (we assume here \( h \) to be relatively small compared to unity). The enhancement factor \( F_H \) can become very large for strong spin polarization, \( h \sim 1 \). In particular, for low phonon momentum, \( q_l \leq v_s/v_F \), the factor \( F_H \) is of the order of inverse adiabatic parameter \( (v_F/v_s)^2 \sim 10^5 \). The same situation may arise in multiple-valley semiconductors (if inter-valley scattering by disorder is weak for some reason), where splitting is present without magnetic field and usually \( \Gamma_1 - \Gamma_2 \sim \Gamma_1 \Gamma_2 \). Another relevant example is provided by ferromagnetic metals with strong intrinsic band-splitting due to the exchange field. In the case of Fe: \( \mu H^* \approx 1.8 \text{eV}, \varepsilon_F = 11.1 \text{eV}, v_F = 1.98 \times 10^8 \text{cm/s}, v_s = 6 \times 10^6 \text{cm/s} \) resulting in the maximum enhancement of the phonon relaxation time as large as \( F_H \sim (\mu H^*/\varepsilon_F)^2(v_F/v_s)^2 \sim 10^4 \).

In the case of a single quasiparticle branch, diffusion-enhanced dissipation may appear due to violation of the charge neutrality condition at a large screening length. To calculate it we employ Eqs. (6,7) and obtain for the density fluctuation \( n = \nu Dq^2 [-i\omega + Dq^2 + 2kq^2 \mathbf{V}_0(q)]^{-1} \Phi(\omega, q) \), which leads to the following contribution to the relaxation rate:

\[
\tau_{\text{ph,C}}^{-1}(q) = \frac{1}{\rho_m \omega^2 + (Dq^2 + 2kq^2\mathbf{V}_0(q))^2} \label{eq:tau_ph_C}
\]  
(12)

Comparison of Eq.(12) with the result of classical theory (1) gives the enhancement factor \( F_C(q) \) in the form:

\[
F_C = 1 + \frac{c_i^{-1}(\Gamma/p_F v_F)^2}{(v_s/v_F)^2 + c_i^{-2}(q^2 l^2)(1 + 2\nu_0\mathbf{V}_0(q))^2}.
\]  
(13)

For the 2D gas with Coulomb interaction and the constant dielectric permittivity \( \varepsilon \) of surrounding media we have \( \mathbf{V}_0(q) = V_2D(q) = 2\pi e^2/\varepsilon q \). In the relevant range of \( q \) parallel to the 2D gas the \( F_C(q) \) factor reduces to a constant (similar result was obtained in [10]):

\[
F_{C_{2D}} = 1 + \frac{c_i^{-1}(\varepsilon l v_F \varepsilon)^2}{\left( \frac{\Gamma}{p_F v_F} \right)^2}.
\]  
(14)

where \( g_{\varepsilon} \) is dimensionless conductance per square in \( e^2/h \) units. Eq.(14) is valid both for 2D \( (c_i^{-1} = 4) \) and for quasi-2D \( (c_i^{-1} = 15/4) \) systems as long as the phonon wavelength \( \lambda \) is much larger than the width of the quasi-2D system. Note the \( e^2 \) dependence of the \( F \)-factor on dielectric susceptibility \( \varepsilon \).

Even larger enhancement arises in the presence of a gate that additionally screens Coulomb interaction and allows the density to fluctuate stronger. For this geometry and \( q \) parallel to 2D gas \( \mathbf{V}_0(q) \rightarrow V_g(q) = V_{2D}(q) (1 - e^{-2\eta b}) \), \( b \) being the distance between the 2d electron gas and the gate. For phonons with the wavelengths \( 1/q \geq b \), the effective potential \( V_g(q) \approx V(q) \cdot 2qb \approx \text{const} \) and the
presence of adiabatic parameter in the denominator of (13) does become important at low enough temperatures:

$$\mathcal{F}_{\text{Gate}} = 1 + \frac{16(\Gamma/pFv_F)^2}{4(v_s/v_F)^2 + (q^2)^2(8\pi ve^2b/\varepsilon)^2},$$  \hspace{1cm} (15)$$

where Coulomb interaction is still assumed to be relatively strong: $2\pi ve^2b/\varepsilon \gg 1$.

Note that Eq.(15) allows to determine the background dielectric constant $\varepsilon$ in a conducting sample (previous example of screening due to an external gate can be considered as a specific example with dielectric function $\varepsilon(q) = \varepsilon/(1 - e^{-2\delta q})$). This part of dielectric function is formed by relatively high-energy processes and is usually added as a phenomenological constant in the theory of electromagnetic response of impure metals and weak Anderson insulators. In band insulators and semiconductors $\varepsilon$ can be measured spectroscopically by studying spectra of F-centers and Wannier-Mott excitons. In impure metals spectroscopic methods fail due to strong absorption by conducting electrons while the direct measurement of static polarizability gives mainly the contribution due to low-energy transitions in the conduction band. One can see from Eq.(15) that the crossover frequency $\omega_{crossover}$ where $\tau_{ph}^{-1} \omega^{-2}$ start to decrease as a function of $\omega$, is proportional to $\varepsilon$: $\omega_{crossover} \tau = (v_s/v_F)^2(4\pi ve^2b)^{-1} \varepsilon$, where $\tau = 1/v_F$ is the elastic scattering time. Thus measuring this crossover frequency one can estimate the background dielectric constant $\varepsilon$ which controls the strength of Coulomb interaction and which is important for the theory of strongly disordered superconductors [18].

Finally, we collect results of both the ML enhancement due to the charge density and the spin density fluctuations, taking also into account mixing of spin projections by the spin-orbit interaction characterized by the parameter $\tau_{SO}^{-1}$. We also consider the dependence of relaxation rate on the direction of phonon propagation relative to 2D gas [17]. Both effects lead to the replacement $Dq^2/(\omega + Dq^2) \Rightarrow Dq^2/(\omega + Dq^2 + \tau_{SO}^{-1})$. It results in the total enhancement factor of the form:

$$\mathcal{F} = \mathcal{F}_{C_{2D}} + \frac{8q^2v_F^2h^2}{(qv_s)^2 + (Dq^2 + 1/\tau_{SO})^2}$$  \hspace{1cm} (16)$$

For 2D electrons and 3D phonons $|\mathbf{q}| = q \sin \theta$ is the phonon momentum component parallel to the 2D system which appears in all the terms originating from electron diffusion. In this case $\mathcal{F}_{C_{2D}}$ is independent of $q$, and $\mathcal{F}$ has a maximum as a function of $\omega$. The spin fluctuation effect given by the second term vanishes at small $\omega$ because of the mixing of branches caused by spin-orbit interaction. It also decreases at large $\omega$ because the dissipation power increases slower with $\omega$ than does the acoustic wave energy. At large enough Zeeman splitting $\hbar$ when the effect of spin fluctuations in its maximum is large, there is a wide frequency region (the falling part of the curve $\mathcal{F} \propto \omega^{-2}$ in Fig.1) where $\tau_{ph}^{-1}$ is almost frequency independent. In this region the cooling/heating rate $J(T) \propto T^4 \ln T$ for the quasi-2D case. This temperature dependence is almost the same as in the case of impurities which are not fully involved in the lattice motion [9]. The extra logarithmic factor arises because of the angular averaging of $1/\tau_{ph}(\theta)$ dominated by the small values of $\theta$. To illustrate this behavior we consider a thin film of semiconductor InSb (g-factor $g \approx 50$). At strong (and parallel to the 2D plane) magnetic fields $g\mu_B H \gg \Delta_{SO}$ classification in terms of the spin sub-bands is still valid approximately, in spite of the Rashba spin-orbit coupling $\Delta_{SO}$. The analysis presented in [17], Sec. IV, VI, leads to Eq.(16) and is summarized in Fig.1.

![FIG. 1: (Color online) The total enhancement factor $\mathcal{F} = \varepsilon_{ph}^{(0)}/\tau_{ph}$ at $\theta = \pi/2$ of ultrasound attenuation in the 2D semiconductor InSb. The parameters taken are $n = 10^{11} cm^{-2}$, $p_{eff} = 50$, $\Delta_{SO} = 0.1 meV$ and magnetic fields are 3T (blue), 5T (red) and 7 T (green). Dashed curves represent the result in the absence of SO relaxation, $\Delta_{SO} = 0$.](image)
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I. HAMILTONIAN

We consider an electron system with several spectral branches (which are labeled by $i \in (1, N)$). Interaction between electrons is considered in the direct (density-density) channel only, and will be treated within the random phase approximation (RPA). Electrons also scatter off local impurities, and we assume this scattering to be identical for all spectral branches, $U_{\text{imp}}(r) = U_{\text{imp}}(r)$. While treating electron-phonon inelastic processes, it is convenient to work in the co-moving frame of reference (CFR), where the Hamiltonian acquires the following form [1–3] in the momentum representation:

$$H = H_{e0} + H_{e-e} + H_{e-ph},$$

$$H_{e0} = \sum_{p,i} \xi_i(p) \psi_{p,i} \psi_{p,i} + \sum_{p,p',i} U_{\text{imp}}(p-p') \psi_{p,i} \psi_{p',i},$$

$$H_{e-e} = \frac{1}{2} \sum_q V_0(q) n_q \overline{n}_q,$$

$$H_{e-ph} = \sum_{q,i} \Gamma_i(\text{div} \mathbf{u})_q n_{q-i},$$

where $U_{\text{imp}}(r)$ is disorder potential, $V_0(q)$ is a bare Coulomb interaction attenuated by the background dielectric constant $\varepsilon$ which could be additionally screened by a metallic gate; $n_{q,i} = \sum_p \overline{\psi}_{p+q,i} \psi_{p,i}$ and $n_q = \sum_i n_{q,i}$ are the $i$-th branch partial- and total densities, respectively. $\Gamma_i$ and $(\text{div} \mathbf{u})_q = i \mathbf{q} \cdot \mathbf{u}$ are the e-ph interaction constant averaged over the Fermi surface and the divergence of ionic displacement field $\mathbf{u}$. The Hamiltonian (S17) contains no inter-branch scattering. We assume these processes to be much weaker than the intra-branch scattering and discuss their role later on in Sec.III C.

II. DERIVATION OF THE PHONON KINETIC EQUATION

Here we derive the phonon quantum kinetic equation via the Keldysh diagrammatic technique (see Ref.4 for details). For simplicity, we consider only the longitudinal phonons; the transverse phonons may be analyzed in a similar way. The Green’s functions are matrices in the Keldysh space:

$$\hat{D} = \begin{pmatrix} D^R & D^R \\ D^A & 0 \end{pmatrix},$$

$$\hat{D}^{-1} = \begin{pmatrix} 0 & (D^R_0)^{-1} \\ (D^A_0)^{-1} & 0 \end{pmatrix},$$

$$\Sigma = \begin{pmatrix} 0 & \Sigma^A \\ \Sigma^R & \Sigma^K \end{pmatrix},$$

where $D_0$ and $D$ are the bare and exact phonon Green’s functions, respectively and $\Sigma$ is the self-energy part. The Keldysh component $(\ldots)^K$ may be parametrized as

$$D^K = D^R \circ F - F \circ D^A$$

where $F$ is related to the phonon distribution function; in equilibrium $F = \coth(\omega/2T)$. The sign $\circ$ means the convolution in the time domain. A bare retarded(advanced) phonon Green’s function is

$$D^{R(A)}_0(\omega, q) = \frac{1}{\hbar^2 \rho_m (\omega \pm i0)^2 - v_q^2 q^2},$$

where $\rho_m$ is the mass density. To derive the kinetic equation one employs the Dyson equation for the matrix Green functions which together with Eq.(S19) results in:

$$(\hat{D}_0^{-1} - \hat{\Sigma}) \hat{D} = 1,$$

$$\Rightarrow \left[D_0^{-1}, F\right] = \Sigma^K - \Sigma^R \circ F - F \circ \Sigma^A.$$  

According to Eq.(S20) $D_0^{-1}$ contains the second derivative with respect to the corresponding time argument of $F = F(t_1, t_2)$. Then the commutator in Eq.(S22) reduces to the first derivative $\partial_t$ with respect to the "slow" combination $t = t_1 + t_2$ multiplied by the Fourier transform $-i\omega$ of the derivative with respect to the "fast" combination $t_1 - t_2$. Averaging over the whole volume of the sample, one finds

$$2i \rho_m \omega \partial_t F = \Sigma^K - \Sigma^R \circ F - F \circ \Sigma^A.$$  

In this equation we assume $F(t_1, t_2) \rightarrow F(\omega; t)$. We also omit the terms describing the external source that pumps energy into electronic system.

A common action of the source term and the e-ph energy relaxation leads to a stationary energy distribution of both electrons and phonons. If the e-ph energy relaxation is much slower than the electron-electron one, a quasi-equilibrium situation with two temperatures $T_{el}, T_{ph}$ is realized [5]. In this case the phonon distribution function is $F(\omega, T_{ph}) = \coth(\omega/2T_{ph})$ while $\Sigma^K$ is a quasi-equilibrium quantity. If in addition a weak e-ph interaction is assumed, electron-phonon interaction enters only trivially as the square of the e-ph matrix element in the Keldysh self-energy part $\Sigma^K$. In this approximation $\Sigma^K$ depends only on the electron temperature, $\Sigma^K = F(\omega, T_{el}) (\Sigma^R - \Sigma^A)$,

$$2i \rho_m \omega \partial_t F(\omega, T_{ph}(t)) = \left[F \cdot (\Sigma^R - \Sigma^A)\right](\omega, q, T_{el}) - \left[F \cdot (\Sigma^K - \Sigma^A)\right](\omega, q, T_{ph}).$$  

Since the phonon decay rate is relatively low, $\tau_{ph}^{-1} \ll \omega$, phonons are well-defined quasiparticles and the quasiparticle distribution function is sharply peaked around the phonon "mass-shell" $\omega = sq$. Thus, the quantities entering in the R.H.S. of Eq.(S24) should be taken at $\omega = sq$, see Ref. [4]. For convenience we introduce a standard
phonon distribution function \( B(\omega) = (1/2)(F(\omega) - 1) \) and define a decay rate \( \tau_{ph}^{-1} \):

\[
\tau_{ph}^{-1}(\omega, T_{el}) = \frac{1}{\rho_{ph}} \text{Im} \Sigma^R(\omega, q, T_{el})|_{\omega = sq}, \quad (S25)
\]

\[
\partial_t B(\omega, T_{ph}(t)) = \frac{B(\omega, T_{el}) - B(\omega, T_{ph})}{\tau_{ph}(\omega, T_{el})}. \quad (S26)
\]

In order to obtain an electron-phonon heat flow, we have to multiply Eq. (S26) by both phonon density of states \( \nu_{ph}(\omega) = \omega^2/2\pi^2v_s^3 \) and by energy, and integrate over \( \omega \):

\[
\mathcal{J} = \int_0^\infty d\omega \nu_{ph}(\omega) \omega \partial_t B(\omega, T_{ph}(t))
\]

\[
= \int_0^\infty d\omega \nu_{ph}(\omega) \frac{\omega}{\tau_{ph}(\omega, T_{el})} [B(\omega, T_{el}) - B(\omega, T_{ph})]
\]

The incoming and outgoing energy flows may be defined as follows:

\[
\mathcal{J}(T_{el}, T_{ph}) = J_+(T_{el}) - J_-(T_{ph}, T_{el}), \quad (S28)
\]

\[
J_+(T_{el}) = \int_0^\infty d\omega \nu_{ph}(\omega) \frac{\omega}{\tau_{ph}(\omega, T_{el})} B(\omega, T_{el}),
\]

\[
J_-(T_{ph}, T_{el}) = \int_0^\infty d\omega \nu_{ph}(\omega) \frac{\omega}{\tau_{ph}(\omega, T_{el})} B(\omega, T_{ph}),
\]

In general, \( J_- \) depends on both \( T_{el} \) and \( T_{ph} \) due to the effect of electron-electron interactions on the decay rate \( \tau_{ph}^{-1} \), see Eq. (S25). However such an effect is absent within the RPA approximation for charge screening which we use here. Thus our result for the phonon lifetime does not depend on temperature explicitly, \( \tau_{ph} = \tau_{ph}(\omega) \). In this case the expressions for heat flows are simplified and \( J_- \) becomes a function of the phonon temperature \( T_{ph} \) only.

The electronic temperature relaxation rate \( \tau_{E}^{-1} \) can be obtained from the expression for the heat flow \( \mathcal{J}_{e-ph} \). Each phonon branch contributes as

\[
\tau_{E,i}^{-1}(\omega, T) = \frac{1}{C_e} \left. \frac{\partial \mathcal{J}_{e-ph}}{\partial T_{el}} \right|_{T_{ph}=T_{el}} \quad (S29)
\]

\[
= \frac{1}{2C_e T} \int_0^\infty \frac{\omega \nu_{ph}(\omega)}{\sinh^2(\omega/2T)} \tau_{ph}^{-1}(\omega, T) d\omega
\]

where \( C_e \propto T \) is the electronic specific heat. The full rate is then \( \tau_{E,f}^{-1} = \tau_{E,T}^{-1} + (d_{ph} - 1)\tau_{E,ir}^{-1} \), where \( d_{ph} - 1 \) is the number of transverse phonon polarizations. Electrons are characterized by the quasi-equilibrium distribution function if their intrinsic inelastic scattering time \( \tau_{ee} \) is much shorter than the temperature relaxation time \( \tau_E \) due to interaction with phonons.

Finally, let us mention that in 2D electron systems the phonon decay rate \( \tau_{ph}(q) \) depends on the angle \( \theta \) between the phonon momentum \( q \) and the direction normal to the plane of the 2DEG. Thus, in the equations like (S27,S28,S29) an angle-averaged decay rate should be used:

\[
\langle \tau_{ph}^{-1} \rangle_\theta(\omega) = \frac{1}{2} \int_0^\pi (\sin \theta d\theta) \tau_{ph}^{-1}(\omega, \theta). \quad (S30)
\]

### III. DIAGRAMMATIC DERIVATION

![Diagram S2: Bare and screened e-ph vertices. Black, curvy red and dashed blue zig-zag lines are electron, phonon and Coulomb interaction propagators respectively. Bold zig-zag lines stand for statically screened Coulomb interaction (with screening by empty electron bubbles only).](image)

We present here the derivation of some of our results in the standard diagrammatic form, as it was done in the most papers on this subject [1–3].

The bare electron-phonon vertex corresponding to the Hamiltonian (S17) is of tensor structure in the space of electron species:

\[
\hat{\Gamma}_{bare} = \begin{pmatrix}
\Gamma_1 \\
\vdots \\
\Gamma_N
\end{pmatrix}. \quad (S31)
\]

The diagonal structure of (S31) corresponds to our assumption about the absence of inter-branch mixing. Now one should take into account static Coulomb screening, which generates scalar counter-term

\[
\hat{\Gamma}_C = -\sum_{l} \frac{\Gamma_l \nu_l}{V_0^{-1}(q) + \sum_i \nu_i} \quad (S32)
\]

The full screened vertex is then a sum \( \hat{\Gamma}_{full} = \hat{\Gamma}_{bare} + \hat{\Gamma}_C \). The structure of these vertices is presented in Fig. S2. The
deformation potentials $\Gamma_i$ averaged over FS are usually approximated as[1]

$$\Gamma_i = [(\Gamma_{bs}(p))]_{FS} + p_F v_F / d_c \right)_{i} \quad (S33)$$

where $\Gamma_{bs}$ represents the lattice-induced deformation potential under the lattice strain and $p_F v_F / d_c$ represents the averaged electron liquid stress tensor. We will consider the simplest model where the lattice contribution is uniform in momentum space $\Gamma_{bs}(p) = \Gamma_{bs}$ and thus is reduced to the shift of electron band (see however Ref. [6]).

\[ \text{FIG. S3: a) Dynamically screened Coulomb interaction, where diffusion is summed up. b) The impurity ladder. Here black dashed line represents impurity correlator } (U(r)U(r')) \text{. c) Diagrams contributing to phonon self energy. This figure encompasses the very general case with arbitrary number of electron types and arbitrary Coulomb interaction strength. Inside the electron bubbles summation goes over all electron branches.} \]

In order to calculate the decay rate and the electron cooling rate we need to calculate the imaginary part of the phonon self energy $\Sigma$. It is represented by the diagrams shown in a Fig.S3. The second diagram is important when the screening is essentially dynamic. Below we demonstrate few particular examples how the diagrammatic description works.

A. Imperfect screening

In this Subsection we consider the case of identical spectral branches, but assuming now that screening is incomplete and electron density variations are allowed. Then the full e-phonon vertex given by the sum of Eqs. (S31) and Eq. (S32) is diagonal:

$$\tilde{\Gamma}_s = \frac{\Gamma}{1 + N_f \nu V_0(q)} \quad (S34)$$

where $N_f$ is a number of identical electron branches. Typically it is equal to $N_f = 2N_v$ where $N_v$ is the number of identical valleys in a semiconductor. For example, $N_v = 6$ for a bulk silicon or $N_f = 4$ for graphene. The first diagram from the Fig.S3c thus gives

$$\Sigma_1 = N_f \left( \frac{\Gamma}{1 + 2\nu N_f V_0(q)} \right) q^2 N_f \nu i\omega \left( -i\omega + Dq^2 \right) \quad (S35)$$

The second diagram turns out to be crucial for a dynamical screening regime, when $\omega \geq Dq^2$:

$$\Sigma_2 = \left( \frac{\Gamma q}{1 + 2\nu N_f V_0(q)} \right)^2 \left( N_f \nu i\omega \left( -i\omega + Dq^2 \right) \right)^2 \times \left( -\frac{1}{V_0^{-1}(q) + N_f \nu Dq^2 / (-i\omega + Dq^2)} \right) \quad (S36)$$

Summing up these contributions we obtain $\Sigma = \Sigma_1 + \Sigma_2$:

$$\Sigma = \frac{\Gamma q^2}{1 + N_f \nu V_0(q)} \times \left( 2N_f \nu \left( i\omega (-i\omega + Dq^2) + (N_f \nu V_0(q) Dq^2) \right) \left( \omega^2 + (Dq^2)^2 (1 + 2N_f \nu V_0(q))^2 \right) \right) \quad (S37)$$

The phonon decay rate is determined by the imaginary part of $\Sigma$:

$$\tau^{-1}_{ph, ML} = \frac{\Gamma^2 q^2}{\rho_m v_s^2 + (Dq^2)^2 (1 + 2N_f \nu V_0(q))^2} \quad (S38)$$

and the corresponding enhancement factor is

$$\mathcal{F}_C(q) = 1 + \frac{1}{\rho_m (v_s / v_F)^2 + d_s^2 (q^2) (1 + 2N_f \nu V_0(q))^2} \quad (S39)$$

These results coincide with the ones in the main text for $N_f = 2$. Thus, we have shown equivalence of the approach using macroscopic kinetic equation and diagrammatic technique.

B. Several spectral branches

Now we switch to the situation of complete screening of Coulomb interaction, so the electro-neutrality condition is obeyed exactly. In such a case the difference in the coupling constants $\Gamma_i$ corresponding to different electron branches is crucial. We consider here the simplest case of two spectral branches. Then the full screened e-phonon vertex given by the sum of Eqs. (S31) and (S32) is traceless:

$$\tilde{\Gamma}_s = \left( \frac{\Gamma_1 - \Gamma_2}{2} \right) \quad (S40)$$

The diagrams shown in Fig.S3c give

$$\Sigma = 2 \times \left( \frac{\Gamma_1 - \Gamma_2}{2} \right)^2 q^2 \nu \left( -i\omega + Dq^2 \right) \quad (S41)$$

Thus, the phonon decay rate due to the Mandelstam-Leontovich mechanism is

$$\tau^{-1}_{ph} = \frac{1}{\rho_m \omega} \left| \text{Im} \Sigma(\omega, q) \right|^{\omega = sq} \quad (S42)$$

$$= \left( \frac{\Gamma_1 - \Gamma_2}{2} \right)^2 \nu Dq^2 \quad \frac{v_s^2 + (Dq)^2}{2\rho_m}$$
and the total decay rate is enhanced (with respect to the classical Pippard result) by the factor

\[ \mathcal{F}(q) = 1 + \frac{1}{2c_1} \left( \frac{\Gamma_1 - \Gamma_2}{p_F v_F} \right)^2 \frac{v_F^2}{v_F^2 + (Dq)^2} \]  \hspace{1cm} (S43)

If the asymmetry in spectral branches arise due to the Zeeman splitting, then \( \Gamma_1 - \Gamma_2 = (2/\hbar) g_\mu H \):

\[ \mathcal{F}_H(q, h) = 1 + \frac{4}{d_c^2 c_1} \frac{v_F^2 h^2}{\omega^2 + (Dq)^2} \]  \hspace{1cm} (S44)

where \( h = g_\mu H/2 \varepsilon_F \ll 1 \) is dimensionless magnetic field (we neglect here \( h \)-dependencies of other parameters, like difference \( D_1 \neq D_2 \), which is negligible at \( h \ll 1 \)).

Note that the effect is absent in zero magnetic field, as the time-reversal symmetry does not allow spin density fluctuations to be excited by phonons.

C. Spin-orbit coupling and interbranch scattering

In this Subsection we revisit the case of the two inequivalent branches of electron spectrum and consider the (previously neglected) role of the interbranch scattering, using the Zeeman-splitting as an example. Qualitatively, the spin-flip scattering with a rate \( \tau_r^{-1} \) leads to non-conservation of the total spin and thus limits the magnitude of any effect which is related to slow spin diffusion. Formally it is described by the modification of the spin diffusion propagator:

\[ \frac{Dq^2}{-i\omega + Dq^2} \rightarrow \frac{Dq^2}{-i\omega + Dq^2 + \tau_r^{-1}} \]  \hspace{1cm} (S45)

which leads to the replacement of Eq.\,(S44) by

\[ \mathcal{F}_H(q, h) = 1 + \frac{4}{d_c^2 c_1} \frac{v_F^2 q^2 h^2}{\omega^2 + (Dq^2 + \tau_r^{-1})^2} \]  \hspace{1cm} (S46).

Below we calculate \( \tau_r \) for the case of 2D electron gas with the Zeeman splitting induced by magnetic field applied in the 2DEG plane in the \( x \)-axis direction. We assume a relatively weak spin-orbit (SO) interaction leading to the spin-orbit band splitting \( \Delta_{SO} \ll \Delta_H = g_\mu H \).

For definiteness we consider the Rashba-type SO coupling with the spin-dependent part of the Hamiltonian being equal to

\[ H_{H+SO} = \frac{\Delta_H}{2} \sigma_x + \frac{\Delta_{SO}}{2} (\sigma_x n_y - \sigma_y n_x) \]  \hspace{1cm} (S47)

where \( n = p/|p| \) is a unit vector in the direction of momentum. The elastic scattering time turns out to be equal for both quasiparticle branches (in the absence of electron-hole asymmetry):

\[ \hat{G}^R(\varepsilon, p) = \sum_{\pm} \frac{\hat{\Pi}_\pm}{\varepsilon - \xi \mp \Delta(n)/2 + i/2\tau}, \]  \hspace{1cm} (S48)

\[ \hat{\Pi}_\pm = \frac{1}{2} \left( 1 \pm \frac{\Delta_H \sigma_x - \Delta_{SO} (\sigma_x n_y - \sigma_y n_x)}{\Delta(n)} \right) \]  \hspace{1cm} (S49)

\[ \Delta(n) = \sqrt{\Delta_H^2 + \Delta_{SO}^2 - 2\Delta_H \Delta_{SO} n_x}. \]  \hspace{1cm} (S50)

where \( \hat{G}^R \) is the retarded electron Green’s function. In order to find the relaxation rate, we evaluate the diffusion self energy for zero frequency and momentum \( (\omega = 0, q = 0) \), Fig.\,S4:

\[ \Xi = \frac{1}{2\pi i \nu_\tau} \int \frac{dp}{(2\pi)^2} \hat{G}^A(0, p) \otimes \hat{G}^R(0, p) \]  \hspace{1cm} (S51)

A simple calculation in a manner similar to that of Ref.\,7 leads to the following result for the diffusion self energy at \( q = \omega = 0 \):

\[ \Xi = \left( 1 - S_x - \frac{1}{1 + \tau^2 \Delta_H^2} - \frac{i\tau \Delta_H}{1 + \tau^2 \Delta_H^2} \right) - \frac{\Delta_{SO}^2}{2\Delta_H^2} \left( \frac{\tau^2 \Delta_H^2}{1 + \tau^2 \Delta_H^2} - \frac{3 + 4\tau^2 \Delta_H^2}{(1 + \tau^2 \Delta_H^2)^3} S_x^2 - \frac{4i\tau^3 \Delta_H^2}{(1 + \tau^2 \Delta_H^2)^3} S_x \right) \]  \hspace{1cm} (S52)

with \( \hat{S} = (\hat{1} \otimes \sigma - \sigma \otimes \hat{1})/2 \) being the total spin of electron-hole pair. We are interested in the \( S_x = 0 \) subspace only as it hosts two eigenvalues of our interest. Naturally, the singlet mode \( (S = 0) \) corresponding to the charge density propagation remains unaffected, \( \Xi_{S=0} = 1 \), while the triplet mode \( (S = 1, S_z = 0) \) representing a spin density diffusion does decay:

\[ \Xi_{S=1, S_z=0} = 1 - \frac{\Delta_{SO}^2}{2 \Delta_H^2 + \tau^2} \]  \hspace{1cm} (S53)
leading to the following result for the spin decay rate
\[
\tau_{r}^{-1} = \tau_{so}^{-1} = \frac{1}{2} \frac{\Delta_{SO}^{2}}{\Delta_{H}^{2} + \tau^{-2}} \tau^{-1} \ll \tau^{-1} \quad (S54)
\]

In the course of derivation of Eq. (S53) we used an identity \(\langle S = 1, S_z = 0 | S_{y}^{2} | S = 1, S_z = 0 \rangle = 1\). We emphasize that Eq. (S54) was derived for weak SO interaction, \(\Delta_{SO} \ll \Delta_{H}\).

**IV. ANGULAR DEPENDENCE OF ULTRASONIC ATTENUATION**

We start here by the quasi-2D case when the thickness of a semiconductor film is much larger than the Fermi wavelength but still smaller than the phonon wavelength, \(\lambda_{F} \ll b \ll \lambda_{ph}\). In this case electron diffusion is two-dimensional and only the component of phonon momentum parallel to the plane enters into the diffusion propagator where a replacement \(Dq_{z}^{2}/(-i\omega + Dq_{z}^{2}) \Rightarrow Dq_{||}^{2}/(-i\omega + Dq_{||}^{2})\) should be made. The result is that Eqs. (S44,S39) should be replaced by
\[
\mathcal{F}_{C}(q) = 1 + \frac{e_{i}^{-1} (\Gamma/p_{F}v_{F})^{2} \sin^{2} \theta}{(u_{s}/v_{F})^{2} + (2/d_{z}) g_{0}^{2} (e^{2}/\varepsilon_{h}v_{F})^{2}} \quad (S55)
\]
\[
\mathcal{F}_{H}(q) = 1 + \frac{2}{\alpha v_{s}^{2} + (Dq_{z})^{2} \sin^{2} \theta} \quad (S56)
\]
where the last equation is given for relatively strong screening \(\nu V_{0}(q) \gg 1\).

The true 2D case, however, should be discussed specially. While the result for the case of imperfect screening at \(\Gamma \gg p_{F}v_{F}\) is identical to Eq. (S55), the magnetic-field induced effect (arising from the momentum-dependent part of the electron-phonon vertex) may behave differently.

For a sufficiently thin film electron motion in the direction perpendicular to the plane is fully quantized, thus the expression for electron-phonon vertex becomes
\[
\Gamma_{1,2} = \Gamma_{bs} \text{div} \mathbf{u} + \frac{1}{m} \langle p_{z}^{2} \rangle (iq_{z}u_{z}) \quad (S57)
\]
\[
+ \frac{p_{||}^{2}}{2m} (iq_{z}u_{x} + iq_{y}u_{y})
\]
\[
= \left( \Gamma_{bs} + \frac{1}{m} \langle p_{z}^{2} \rangle \cos^{2} \theta + \frac{p_{||}^{2}}{2m} \sin^{2} \theta \right) \text{div} \mathbf{u},
\]
where
\[
\frac{p_{||}^{2}}{2m} = \varepsilon_{F} \pm g_{\mu B} H - \frac{\langle p_{z}^{2} \rangle}{2m} \quad (S58)
\]
with \(\pm\) corresponding to the spin-up and spin-down electrons. Here we have taken an average \(\langle \cdot \rangle_{z}\) over the ground state corresponding to the motion in the perpendicular direction and \(p_{||}\) is the momentum of an in-plane motion. We assume that the matrix element \(\langle p_{z}^{2} \rangle_{z}\) does not depend on the spin degree of freedom as well as on the electron density. Thereby we disregard any possible orbital effects of magnetic field and consider the Zeeman interaction only which results in two different momenta \(p_{\parallel,\perp}\) corresponding to the in-plane motion for the up and the down spin projections. The momentum-independent component of the vertex does not contribute to the magnetic-field-controlled relaxation under the condition of perfect screening (according to Eq. (S32) it is screened out completely). Eqs. (S43) explicitly implies that only asymmetric part of the vertex contributes:
\[
\Gamma_{1} - \Gamma_{2} = g_{\mu B} H \sin^{2} \theta \text{div} \mathbf{u} \quad (S59)
\]

However, in real 2D electron systems, such as heterostructures, the lattice strain also affects the quasiparticle mass, generating the additional momentum-dependent contributions:
\[
\delta m^{-1} = m^{-1} (\gamma_{\perp} \cos^{2} \theta + \gamma_{\parallel} \sin^{2} \theta) \text{div} \mathbf{u}, \quad (S60)
\]
\[
\Gamma_{1} - \Gamma_{2} = g_{\mu B} H \times [(1 + \gamma_{\parallel}) \sin^{2} \theta + \gamma_{\perp} \cos^{2} \theta] \text{div} \mathbf{u}.
\]
Thus, the magnetic-field-controlled enhancement becomes equal to
\[
\mathcal{F}_{H}(q, h) = 1 + \frac{8 v_{s}^{2} \sin^{2} \theta}{\alpha v_{s}^{2} + (Dq_{z})^{2} \sin^{2} \theta} [(1 + \gamma_{\parallel}) \sin^{2} \theta + \gamma_{\perp} \cos^{2} \theta]^{2} \quad (S61)
\]

The results (S55,S56,S61) show the angular dependence of ultrasonic attenuation which may exhibit a characteristic cross-like pattern exemplified in Fig. S5.

There are two additional issues which should be addressed to make the above analysis really quantitative: (i) in a general case the incident longitudinal(transverse) acoustic wave reflected off the free surface produces both longitudinal and transverse reflected waves, (ii) in the...
true 2D case diffusion modes could be generated by transverse phonons as well. However, these effects do not seem to lead to any qualitative change of our results and we will postpone the corresponding studies for the future.

V. ELECTRON-PHONON HEAT FLOW

In this Subsection we use previously obtained results for the phonon decay rate, Eqs. (S55-S56), to derive an expression for the electron-phonon heat flow in a true 2D electron gas structure. We start by the spin density diffusion effects. At the lowest temperatures $T \ll T_H^{(1)} = \hbar v_s^2/D$, the enhancement does not depend on temperature and angle, being just a numerical factor:

$$J_H = \frac{2\hbar^2 v_s^2}{6\pi^3 \hbar^2 p_m v_s^2} A_1 T^6,$$

where $A_1 = (48(1 + \gamma_p)^2 + 16(1 + \gamma_p)\gamma_{\perp} + 6\gamma_{\perp}^2)$ and $g_\parallel = k_F l$ is the dimensionless conductance of the 2DEG.

For higher temperatures, $T \gg T_H^{(1)}$, the enhancement factor behaves as $\mathcal{F} \propto T^{-2}$. However, the resulting expression for the e-ph heat flow is a pure power-law in the shape of the angular structure of the vertex (here $A_2 = (2/3)(1 + \gamma_p)(1 + \gamma_p + \gamma_{\perp})$):

$$J_H = \frac{1}{g_\parallel} \frac{p_F^4}{15\pi^3 \hbar^2 p_m v_s^2} \left(A_2 + \gamma_{\perp} \ln T/T_H^{(1)}\right) T^4$$

If the normal strain does not alter electron mass ($\gamma_{\perp} = 0$) the angular dependence $\mathcal{F}(q, \theta)$ does not lead to dominance of small $\theta$ in the corresponding integral. Then the heat flow is a pure power-law $J \propto T^4$. Otherwise, the shape of $\mathcal{F}(q, \theta)$ is rather peculiar (see Fig.S5) and an additional $\ln T$ factor appears due to the contribution of small angles $\theta_H \sim \sqrt{T_H^{(1)}/T}$. Note that the $T^4 \ln T$ behavior is slower than $T^6$ and at temperatures $T > T_H^{(2)} = T_H^{(1)} \sqrt{1 + \frac{\hbar^2 v_F^2}{v_F^2}}$, $T_H^{(1)} = \hbar v_F^2/D$.

the effect of spin fluctuations is smaller than the $H$-independent contribution proportional to $T^6$. In the above Eq.(S64) we denote as $\mathcal{F}_C$ the $H$-independent enhancement factor Eq.(39) averaged over angles $\theta$. The spin-orbit interaction suppresses the effect of spin fluctuations at low temperatures:

$$T < T_{H}^{(SO)} = \frac{\hbar v_s}{\sqrt{D} \pi \sqrt{\kappa}},$$

so that the condition for this effect to be observed is $T_{H}^{(SO)} < T_H^{(2)}$.

The temperature dependence of the e-ph heat flow in the most favorable case $T_{H}^{(SO)} < T_H^{(1)}$ is shown in Fig.S6.

For the effects of charge diffusion on the e-ph heat flow in the 2D electron system with no additional screening of interactions ($\varepsilon = \text{const}$) the situation is similar to that of Eq.(S62):

$$J_{C_{2D}} = \frac{1}{g_\parallel} \frac{\rho_e^{v_F}}{63\pi^3 \hbar^2 p_m v_s^5} \left(\frac{\varepsilon v_F}{2e}\right)^2 \left(\frac{\Gamma_{\varepsilon}}{p_F v_F}\right)^2 T^6,$$

where the effective dielectric constant is an arithmetic mean of the dielectric constants of the media on both sides of the 2D system: $\varepsilon = (\varepsilon_1 + \varepsilon_2)/2$. The enhancement is thus reduced to a temperature-independent factor.

The result is the same for the geometry with additional screening by a metallic gate at temperatures $T \gg \hbar v_s/b$, where $b$ is the distance between the 2D electron plane and the gate. At lower temperature $T \ll \hbar v_s/b$ the effective dielectric permittivity is $q$-dependent: $\varepsilon(q) = \varepsilon(\text{coth} q b + 1)/2 \approx \varepsilon/2 q b$ which transforms Coulomb interaction into a short-range one, $V_0(q) = 4\pi q^2 b$. The behavior becomes similar to Eqs.(S63) however, with the crossover temperature $T_H^{(1)}$ replaced by $T_C^{(1)} = (1/g_\parallel k_F)(v_s/v_F)^2 (2e\varepsilon v_F/2e) E_F$:

$$J_{C_{2D+gate}} = \frac{1}{g_\parallel} \frac{\rho_e^{v_F}}{480\pi^3 \hbar^2 p_m v_s^5} \left(\frac{\varepsilon v_F}{2e}\right)^2 \left(\frac{\Gamma_{\varepsilon}}{p_F v_F}\right)^2 T^4 \ln \frac{T}{T_C^{(1)}}$$

$T \gg T_C^{(1)}$,

$$J_{C_{2D+gate}} = g_\parallel \frac{\rho_e^{v_F}}{63\pi^3 \hbar^2 p_m v_s^5} T^6,$$ $T \ll T_C^{(1)}$.

Similarly to (S63), Eq.(S66) contains $\ln T$ coming from...
the angles \( \theta_C \sim \sqrt{\frac{T_C^{(1)}}{T}} \).

We also note that Eqs. (S66),(S67) represent only the Mandelstam-Leontovich contribution due to charge diffusion. At high enough temperatures

\[
T > T_C^{(2)} \sim T_C^{(1)} \left( \frac{v_F}{v_s} \right)
\]

this contribution is smaller than the \( J_{tr}^{(0)} \) arising from local processes corresponding to the transverse phonon Pippard’s ultrasun attenuation, Eq.(1) in the paper:

\[
J_0 = \frac{q^2}{126\pi^{-6}\rho_m v_{s,t}^2} T^6,
\]

Thus for temperatures \( T > T_C^{(2)} \) the \( T^6 \) law is restored.

We should note that in real heterostructures the deformation potential \( \Gamma \) is in general anisotropic. However, this fact does not lead to any profound changes like suppression of logarithmic behavior \( \propto \ln T \) in Eq.(S66). This would require a highly anisotropic deformation potential \( \Gamma \propto \sin^2 \theta \) which we do not expect.

VI. A PARTICULAR EXAMPLE: THIN FILM OF InSb

Here we consider enhancement on ultrasound attenuation in an InSb thin film with electron density \( n = 10^{11}\text{cm}^{-2} \), and thickness \( d \geq 10 - 20\text{nm} \) on the SiO2 substrate. We will consider the spin effect of parallel magnetic field applied to the film, so its thickness \( d \) is chosen to be relatively small (slightly larger than Fermi wavelength) in order to avoid orbital effects of magnetic field. Phonon wavevector \( \mathbf{q} \) is parallel to the 2DEG plane. Effective mass of InSb is equal to \( m = 0.014m_0 \) [8], spin-orbital band splitting is \( \Delta_{SO} \approx 0.11\text{meV} \) [9, 10] and the electron mean-free-path is supposed to be relatively long, \( p_{FL} = 50 \). At such parameters the Fermi energy \( \varepsilon_F = 0.02\text{eV} \), while the deformation potential \( \Gamma = 13.12\text{eV}[11] \). A SiO2 substrate is characterized by \( v_s = 6 \times 10^5\text{cm/s} \) and the effective dielectric constant \( \varepsilon = (1+3.9)/2 \approx 2.5 \). Finally, we use Eqs.(S46) and (S54) with the specified sample/material parameters. The resulting plots are given in the Fig.1 of the main text.

 VII. A GENERAL EXPRESSION FOR PHONON DECAY RATE: A PHENOMENOLOGICAL DERIVATION

Here we derive a general expression for ultrasonic attenuation using a phenomenological approach of diffusive electron transport. We start by the system of equations

\[
\begin{align*}
\partial_t n^{(i)} + \text{div} j^{(i)} &= 0, \\
j^{(i)} &= -D^{(i)} \nabla n^{(i)} + \kappa^{(i)} \mathbf{F}^{(i)} \\
U^{(i)} &= \int \mathcal{V}_0(\mathbf{r} - \mathbf{r}') \sum_j \delta n^{(j)}(\mathbf{r}') + \Gamma^{(i)} \text{div} \mathbf{u}
\end{align*}
\]

Fourier transforming the set we get

\[
\begin{align*}
\left\{ (-i\omega + D^{(i)} q^2) n^{(i)} = -\kappa^{(i)} q^2 U^{(i)} \\
U^{(i)} &= V_0(q) \sum_j n^{(j)}(q) + (i\mathbf{q} \cdot \mathbf{u}) \Gamma^{(i)}
\end{align*}
\]

Due to Coulomb interaction the solution for \( i \)-th branch depends on the dynamics of total density. Thus, the solution is

\[
n^{(i)} = -\Pi^{(i)}(\omega, q) \left( \Phi^{(i)} - \Phi_C(\omega, q) \right)
\]

where \( \Pi^{(i)}(\omega, q) = \kappa^{(i)} q^2 / (-i\omega + D^{(i)} q^2) \) is a response function, \( \Phi^{(i)} = (i\mathbf{q} \cdot \mathbf{u}) \Gamma^{(i)} \),

\[
\Phi_C = \frac{V_0(q) \sum_i \Pi^{(i)} \Phi^{(i)}}{1 + V_0(q) \sum_i \Pi^{(i)}}.
\]

Here \( \Phi_C \) describes dynamic Coulomb counteraction. To obtain the phonon decay rate we have to evaluate the dissipation power in a unit volume

\[
Q_t = \frac{1}{2} \sum_{\mathbf{i}} \text{Re} \left( j^{(\mathbf{i})} \cdot \mathbf{F}^{*(\mathbf{i})} \right) = \frac{1}{2} \sum_{\mathbf{i}} \text{Re} \left[ -i\omega(\Phi^{(\mathbf{i})} - \Phi_C(\omega, q)) \Pi^{(\mathbf{i})}(\Phi^{(\mathbf{i})} - \Phi_C(\omega, q))^* \right]
\]

\[
= \frac{i}{2} \sum_i \text{Im} \left[ (\Phi^{(i)} - \Phi_C(\omega, q)) \Pi^{(i)}(\Phi^{(i)} - \Phi_C(\omega, q))^* \right].
\]

and the energy of acoustic wave:

\[
E_w = \frac{\rho_m}{2} \omega^2 u_m^2
\]

Finally, for the phonon decay rate we get

\[
\tau_{ph}^{-1} = \frac{Q_t}{E_w} = \frac{q^2 \sum_i \text{Im} \left[ \Gamma_s^{(i)} \Pi^{(i)}(\Gamma_s^{(i)})^* \right]}{\rho_m \omega} = \frac{q^2}{\rho_m \omega} \sum_i \Gamma_s^{(i)} \text{Im} \left[ \Pi^{(i)}(\Gamma_s^{(i)})^* \right]
\]
where $\Gamma_s = (\Phi^{(i)} - \Phi_C(\omega, q))/\text{div} \mathbf{u}$ can be considered as a dynamically screened vertex.

### A. Multiple branches

We analyze here the case of two quasiparticle branches and very strong bare Coulomb potential, $V(q) \to \infty$. The Coulomb counteraction term takes the form

$$\Phi_C = \frac{\sum_i \Pi^{(i)} \Phi^{(i)}}{\sum_i \Pi^{(i)}}\quad (S77)$$

that exactly fixes total density $\delta(n_1 + n_2) = 0$. Thus the phonon decay rate becomes equal to

$$\Gamma(p_F) = \Gamma_0 + p_F \nu_F/2 \to \partial \Gamma/\partial \nu_F = 1\quad (S84)$$

Introducing the dimensionless magnetic field $h = \mu H/2 \nu_F$, we arrive at the result

$$F_H(q, h) = 1 + \frac{8 \nu_F^2 h^2 (1 - h^2)}{v_s^2 + (Dq(1-h^2))^2}\quad (S85)$$

Finally we discuss the effect of inter-branch scattering. In fact, it modifies the response function

$$\frac{D_q^2}{\omega^2 + (D_q q^2)^2} \Rightarrow \frac{D_q^2}{\omega^2 + (D_q q^2 + 1/\tau_{so})^2}\quad (S87)$$

and the final result becomes equal to

$$F_H(q, h) = 1 + \frac{8 \nu_F^2 h^2 (1 - h^2)}{(q v_s)^2 + (Dq(1-h^2) + 1/\tau_{so})^2}\quad (S88)$$

### B. Imperfect screening

Another case of interest is the case of two quasiparticle branches with identical parameters $\Gamma_1(2) = \Gamma$, $\nu_1(2) = \nu$, $D_1(2) = D$ but finite strength of Coulomb interaction. In this case no asymmetry is present and thus asymmetric electron modes cannot be excited. However, finite Coulomb interaction and incomplete screening allows density fluctuations which diffusive relaxation leads to the enhancement of ultrasound attenuation and the e-ph energy flow:

$$\tau_{\text{ph}}^{-1} = \frac{q^2}{\rho_m \omega} \text{Im} \left[ \frac{(\Gamma_1 - \Gamma_2)\Pi_{12}^*}{\Pi_1 + \Pi_2} \left( \frac{(\Gamma_1 - \Gamma_2)\Pi_{12}^*}{\Pi_1 + \Pi_2} \right)^* \right]$$

where $v_s = \omega/q$ is the sound velocity and $\nu_s = (\nu_1^{-1} + \nu_2^{-1})^{-1}$, $D_s = \nu_1^{-1}((\nu_1 D_1^{-1} + \nu_2 D_2^{-1})^{-1}$ are the effective density of states and diffusion coefficient respectively.

To obtain the total ultrasonic attenuation we also have to take into account the PIC result:

$$\tau_{\text{ph}}^{-1}(0) = \alpha \frac{p_F^2}{2 \rho_m} (\nu_1 D_1 + \nu_2 D_2) q^2\quad (S80)$$

This equation coincides with Eq.(1) of the main text if both electron branches are identical $\nu_1 = \nu_2, D_1 = D_2$. Thus, for the total attenuation rate we obtain:

$$\tau_{\text{ph}}^{-1} = \alpha \frac{p_F^2}{2 \rho_m} (\nu_1 D_1 + \nu_2 D_2) q^2 + \frac{(\Gamma_1 - \Gamma_2)^2}{\rho_m} \frac{\nu_s D_s q^2}{v_s^2 + (D_q q^2)^2}\quad (S81)$$

and the enhancement factor $F = \tau_{\text{ph}}^{-1}/(\tau_{\text{ph}}^{-1}(0))$ is

$$F_{MB}(q) = 1 + \frac{2}{c_l} \left( \frac{\Gamma_1 - \Gamma_2}{\nu_F} \right)^2 \frac{\nu_s D_s}{\nu_1 D_1 + \nu_2 D_2} \frac{v_F^2}{v_s^2 + (D_q q^2)^2}\quad (S82)$$

An important particular case is that of the Zeeman splitting by an in-plane magnetic field $H$ for a 2D electron system ($d_e = 2$). Here an asymmetry appears between the spin-up and spin-down electron branches (while $\nu_1 = \nu_2 = \nu$): $\varepsilon_{\uparrow(\downarrow)} = \varepsilon_F \pm \mu H/2$, $D_{\uparrow(\downarrow)} = D(1 \pm \mu H/2 \varepsilon_F)$. However, the most important asymmetry is the one in the vertex $\Gamma$ that arises from the momentum-dependent part of the electron-phonon coupling:

$$\Gamma_1 - \Gamma_2 = (\partial \Gamma/\partial \varepsilon_F) \mu H\quad (S83)$$

In the simplest model, where the only effect of the strain upon electron spectrum is its overall shift, the density-dependent contribution arises from the stress $\langle p_{\alpha} v_{\beta} \rangle_{FS}$
\[
\tau_{ph}^{-1} = \frac{q^2}{\rho_m \omega} \text{Im} \left[ 2 \times \left( \Gamma - \frac{2V_0 \Pi}{1 + 2V_0 \Pi} \right) \Pi \left( \Gamma - \frac{2V_0 \Pi}{1 + 2V_0 \Pi} \right)^* \right] = \frac{q^2}{\rho_m \omega} \text{Im} \left[ \frac{2\Gamma^2 \Pi}{|1 + 2V_0 \Pi|^2} \right] = \frac{\Gamma^2}{\rho_m v_s^2 + (2\nu V_0 + 1)^2(Dq)^2} \tag{S89}
\]

We see that the crossover between \( F \propto \omega^{-2} \) and \( F \propto \omega^0 \) behavior emerges at

\[
\hbar \omega_{cross} \sim \frac{1}{g_\square k_F b} \left( \frac{v_s}{v_F} \right)^2 \left( \frac{\varepsilon h v_F}{e^2} \right) E_F.
\]

This equation can be used for an experimental determination of the background dielectric constant \( \varepsilon \).

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