Comparative study of numerical simulation methods for seismic pounding of adjacent girder of curved girder bridges

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Abstract. Under seismic action, non-uniform collision will occur between the main beams of curved beam bridge, which may lead to local damage of front beam or rear beam. At present, the research on collision effect is mainly based on straight beam bridge, and there is still a lack of relevant research on seismic collision of curved beam bridge. Taking two adjacent typical curved bridges as examples, this paper establishes contact elements (linear elastic model, Kelvin model and Hertz model) and solid elements (three-dimensional contact friction model) which can fully reflect the physical characteristics of seismic collision of curved bridges. By comparing the numerical simulation methods of seismic collision, the advantages and disadvantages of the existing numerical simulation methods in the seismic response analysis of curved bridges are evaluated. The results show that the calculation results of Kelvin model and three-dimensional contact friction model have the least error and high calculation efficiency, and are suitable for the seismic analysis of curved beam bridges.

Keywords. Curved girder bridge, earthquake, main girder, pounding, finite element modeling

1. Introduction

The complex physical phenomena such as non-uniform pounding and seismic spatial motion in seismic pounding of adjacent curved bridges, as study by WANG Jun-wen[1] and XU Qiang[2].

In this paper, the contact element method based on spatial member element (point to point contact)[3] [4] and the three-dimensional contact friction element method[5] based on solid element (surface to surface contact) are respectively used to simulate the seismic pounding problem of main girder of curved bridge. In the finite element model, the impact points are connected by contact springs, and the method of simulating pounding by using the relationship between relative displacement and force between two elements is called contact element method. In this paper, OpenSEES software[6] with powerful nonlinear function is used to simulate the pounding of contact elements. Girder element is used to simulate the main girder and the piers. The 3D contact friction model [7] is a refined modeling method based on solid element. The biggest difference between this method and contact element method is that the pounding point is not fixed, and it is automatically calculated according to the situation in earthquake, so it can
simulate the pounding at any points of adjacent main girders. In this paper, the ABAQUS finite element software with implicit dynamic calculation function will be used, in order to establish the solid model of curved girder bridge.

2. Modeling
In this paper, based on the research background of a two segment prestressed reinforced concrete girder bridge of G ramp bridge of an interchange, through the establishment of three-dimensional finite element numerical analysis model, comparative analysis is carried out.

The design parameters are as follows in Table 1. The model of the bridge structure is shown in Fig 1 and Schematic of Mid-span section is shown in Fig 2.

| Table 1. Bridge seismic design parameters |
|------------------------------------------|
| Bridge span                              | Two segment of girder bridge (2x25m + 2x30m) |
| Super-structure                          | Prestressed concrete continuous girder bridge, radius of curvature R = 165m |
| Sub-structure                            | The transition pier is double column pier with cap girder, the diameter of pier is 1.3m, and that of two pile cap foundation is 1.5m; |
|                                          | The middle pier is a single column pier, the pier is consolidated, and the pier diameter is 1.8m; for the foundation of two pile caps, the pile diameter is 1.5m |

![Figure 1. Model of the bridge structure](image)

![Figure 2. Schematic of Mid-span section (unit: cm)](image)

2.1 Simulation by contact element method

The elements are set at the expansion joint position between adjacent girders as shown in Fig 3.
Based on OpenSEES, four typical models are used to establish pounding models. The specific modeling method is as follows.

(1) Linear elastic model

In the linear elastic model, the constitutive relation of linear elastic material obeys the generalized Hooke's law. The linear elastic model is to connect the pounding points with linear spring elements. The schematic diagram of the model is shown in Fig 4. The impact force has a linear relationship with the relative displacement, as shown in Fig. 5.

$$ F_c = k_1 (u_1 - u_2 - g_p), \quad u_1 - u_2 - g_p \geq 0 \quad (1) $$

$$ F_c = 0, \quad u_1 - u_2 - g_p < 0 \quad (2) $$

In the above formula, $F_c$ is the impact force, $k_1$ is the linear spring stiffness of linear elastic model, $u_1$, $u_2$ are displacement between two girders, $g_p$ is the clearance of the expansion joint.
In OpenSEES, PPGap (elastic perfectly plastic gap material) material can be used to simulate the constitutive relationship of line spring model, as shown in Fig. 6. PPGap sets the gap element (GAP) when the parameter is under pressure (negative value).

(2) Kelvin model simulation

Kelvin model can simulate the energy dissipation during seismic pounding. It is essentially a linear damper putted in parallel on the basis of the nonlinear elastic model. The schematic diagram of the model is shown in Fig. 7, and the relationship between the impact force and the relative displacement is shown in Fig. 8. The impact force generated by Kelvin model analysis is as follows:

\[
F_c = k_i (u_i - u_g - g_p) + c_i (v_i - v_g), \quad u_i - u_g - g_p \geq 0 \quad (3)
\]

\[
F_c = 0, \quad u_i - u_g - g_p < 0 \quad (4)
\]

In the above formula: \(k_i\) is the spring stiffness of Kelvin model, \(c_i\) is the damping coefficient of Kelvin model, and \(v_i, v_g\) is the velocity of pounding girder. The meaning of other parameters is the same as that of linear elastic model. In this model, the rate coefficient is \(\alpha = 1\), and the damping coefficient \(C\) is calculated...
from the following formula. Relevant literature studies show that the value range of impact recovery coefficient of concrete bridge is generally 0.7 ~ 0.95.

\[
c_k = 2\xi \sqrt{\frac{k_l m_1 m_2}{m_1 + m_2}}
\]

\[
\xi = \frac{\ln e}{\sqrt{\pi^2 + (\ln e)^2}}
\]

(3) Hertz model simulation

Similar to the linear elastic model, Hertz model cannot consider the energy dissipation, but can consider the nonlinear relationship between the impact force and displacement. It mainly connects the impact points with nonlinear spring elements. The schematic diagram of the model is shown in Figure 9. The relationship between impact force and relative displacement is shown in Fig. 10.

![Figure 9. Model of Linear](Image)

![Figure 10. Hertz model force vs. relative displacement](Image)

The impact force produced by Hertz model analysis is as follows:

\[
F_c = k_h (u_1 - u_2 - g_p)^n
\]

\[
u_1 - u_2 - g_p \geq 0
\]

\[
F_c = 0, \quad u_1 - u_2 - g_p < 0
\]

In the above formula: \(F_c\) is the impact force, \(K_H\) is the Hertz model spring stiffness, literature studies show that the concrete girder bridge, the value range is \(1.2 \times 10^6\) kn / (m) \(3/2\) ~ \(2.6 \times 10^6\) kn / (m) \(3/2\), \(n\) is Hertz coefficient, the value is \(1 ~ 2\), \(u_1, u_2\) is the displacement of girder, \(g_p\) is the gap value.

In OpenSEES software, the Hertz model can be simulated by impact material constitutive model. The equivalent calculation methods of initial stiffness \(K_i\), secondary hardening stiffness \(K_2\).
In the above formula: $K_{\text{eff}}$ is the effective stiffness, $\sigma_m$ is the maximum penetration depth at the time of pounding. The relevant calculation formula is as follows:

$$K_{\text{eff}} = K_h \cdot \sqrt{\sigma_m}$$

$$\sigma_y = a \cdot \sigma_m$$

$a$ is generally taken as 0.1, and the pounding stiffness $K_H$ of Hertz model is generally taken as the axis stiffness $EA / L$ of the short main girder or $1.2 \times 10^6 \text{kn} / (\text{m})^{3/2}$ of the conventional girder bridge.

2.2 Three dimensional contact-Friction model simulation

Different from the contact element method, the Three-dimensional contact friction method model is based on the solid element, and the contact friction model using complex three-dimensional contact is shown in Fig. 11. It is proposed that ABCD is the main section and is a rigid surface, impact point 1 is the point of pounding on the main section, and pounding point 2 is the point of contact between the section and point 1.

**Figure 11.** Model of Three-dimensional contact friction

The position and velocity of impact point 1 are functions of the main cross section:

$$u_i = u_i(a, b, c, d)$$

$$\dot{u}_i = \dot{u}_i(\dot{u}_a, \dot{u}_b, \dot{u}_c, \dot{u}_d)$$

After pounding, there is a $u_{12}$ displacement difference between impact point 1 and impact point 2, and there is a velocity difference between impact point 2 and impact point 1. The formula is expressed as follows:

$$u_{12} = u_2 - u_1$$

$$\dot{u}_{12} = \dot{u}_2 - \dot{u}_1$$
In order to calculate the impact force between the impact point 1 and the impact point 2, a spring with the stiffness of $K_n$ is set between the two points, and a damper is set up in the longitudinal and tangential directions to simulate the energy dissipation. The damping coefficients are $C_n$ and $C_t$ respectively. According to the displacement difference $u_{12}$, the impact force $F_p$ between the two points can be calculated as:

$$F_p = K_n u_{12}$$  \hspace{1cm} (16)

In Fig. 11, $F_{pn}$ represents the impact force component of the impact force $F_p$ in the longitudinal direction of the main section, and $F_{pt}$ represents the component of the impact force $F_p$ in the tangential direction of the main section. When studying the pounding between two points, it is necessary to determine the contact state and calculate the impact force. Considering the requirements of calculation accuracy and efficiency, this paper uses C3D8R hexahedral reduced integral element in ABAQUS. The dynamic implicit algorithm is used to solve the model, as shown in Fig. 12.

![Figure 12. Model of Three-dimensional contact friction](image)

2.3 Seismic wave input

According to the site conditions of the bridge, three artificial seismic waves with a duration of 20s are input in this paper. Through nonlinear time history response analysis, the maximum value of the results is selected as the final data. The design parameters of seismic site are shown in Table 2, and the seismic design response spectrum and acceleration time history of seismic wave are shown in Fig. 13 and Fig. 14.

| Design parameters | Value |
|-------------------|-------|
| Earthquake intensity | E2    |
| Fortification intensity | 8     |
| Site conditions | III   |
| $K_h$ | 0.20 |
| $T_g$ | 0.35s |
| $C_i$ | 0.45 |
| $C_z$ | 0.33 |
3. Comparative analysis

The Three-dimensional contact friction model is a refined modeling method based on solid element, which can simulate the pounding between any point between main girders, and the calculation accuracy is higher than that of contact element method. Therefore, based on the three-dimensional contact friction model, this paper compares the calculation results of the contact element method pounding model and the three-dimensional contact friction model. Then, a new method based on contact element method with high calculation efficiency and accurate data is found. In order to facilitate data identification, the piers and main girders of curved girder bridges are labeled, as shown in Figure 15. There are 8 piers and 2 main girders in the whole bridge.

When the contact element method is used to simulate the pounding, five types of pounding elements are set between adjacent girders (as shown in Fig. 16). When the three-dimensional contact friction model is used to simulate the pounding, the contact surface is set between adjacent girders (as shown in Fig. 17). Figures 18-21 show the time histories of the four different type of models.
Figure 16. Schematic of pounding element

Figure 17. Schematic of interface

Figure 18. Pounding force time history of Linear elastic model

Figure 19. Pounding force time history of Kelvin model
Figure 20. Pounding force time history of Hertz model

Figure 21. Pounding force time history of Three-dimensional contact friction model

It can be seen from Fig. 18-21 that the energy loss is simulated by defining the friction between the contact surfaces in the 3D contact friction model, and the calculation results are very close to the time history law of the pounding force obtained directly by the contact element method.

Figure 22. Total impact force difference ratio of each model

The maximum pounding force of each model is shown in Figure 22. It can be seen from the figure that the maximum pounding force of the model considering energy dissipation is lower than that of the model without considering energy dissipation. The difference between Kelvin model and Three-dimensional contact friction model is the smallest, and the results are the most similar.
In order to reflect the seismic damage caused by pounding, the relative displacements of pier and girder using four models are analyzed. The relative displacement between pier and girder can be divided into: relative displacement between pier and girder along bridge direction (approaching), relative displacement between pier and girder along bridge direction (far away), relative displacement between pier and girder in transverse direction. The relative displacement (approach) between pier and girder along bridge direction refers to that the overlapping length of pier girder is greater than that of original state, as shown in Fig. 23-1; the relative displacement (far away) between pier and girder along bridge direction refers to that the lap length of pier girder is less than that of original state, as shown in figure 23-2.

In Figure 23, $X_1$ is the displacement of main girder and $X_2$ is the displacement of pier top. In order to distinguish the relative displacement between pier and girder along the bridge, the displacement of pier and girder should be positive to the right. When $X_1 - X_2 < 0$ is close, when $X_1 - X_2 > 0$, it is far away. In order to show the difference of results more clearly, based on the data of three-dimensional contact friction model, this paper compares and analyzes the displacement difference ratio of various contact element method models.

It can be seen from Fig.12 that the relative displacement calculated by Kelvin model is the closest to that calculated by 3D contact friction model, by comparing the difference ratio between the calculated values of the three models and the Three-dimensional contact friction model.

**Table 3. Three-dimensional friction model Relative displacement**

| Project Position | Girder 1- pire 1 | Girder 1- pire3 | Girder 2- pire3 | Girder 2- pire5 | Average displacement peak value of each position |
|------------------|------------------|-----------------|-----------------|-----------------|-----------------------------------------------|
| Along the bridge (near) | 182              | 55.7            | 76.1            | 198.5           | 128.1                                        |
| Along the bridge (far)    | 153.9            | 74.3            | 64.4            | 158.4           | 112.8                                        |
The differences between the peak bending moment of pier bottom along bridge direction, the peak moment at pier bottom in transverse direction, the peak shear force at pier bottom along bridge direction and the Three-dimensional contact friction model are shown in Fig. 25. It can be seen from figure 25 that compared with the Three-dimensional contact friction model, the internal force of the pier bottom calculated by Kevin model is closer to that of the Three-dimensional contact friction model. In order to show the difference of results more clearly, based on the data of Three-dimensional contact friction model, this paper compares and analyzes the difference ratio between various contact element method models and their internal force peak sum. (Table 4 bending moment unit is kN m, shear force unit is kN)

### Table 4. Three-dimensional friction model Internal force peak

| position  | Peak moment (anterograde) | Peak moment (transverse) | Peak shear force (anterograde) | Peak shear force (transverse) |
|-----------|---------------------------|--------------------------|-------------------------------|-------------------------------|
| Pier1-1   | 3243.4                    | 2929.2                   | 324.1                         | 299.9                         |
| Pier1-2   | 2333.9                    | 2143.1                   | 385                           | 393.8                         |
| Pier2     | 9306.1                    | 9270.6                   | 1588.3                        | 1520.2                        |
| Pier3-1   | 3885.8                    | 4087.4                   | 632                           | 697.3                         |
| Pier3-2   | 4856.4                    | 3539.8                   | 685.8                         | 710.3                         |
| Pier4     | 9561.5                    | 9672.1                   | 1509.1                        | 1558.9                        |
| Pier5-1   | 2337.4                    | 2006.4                   | 375.4                         | 300.8                         |
| Pier5-2   | 2585                      | 2622.5                   | 391.6                         | 347.9                         |
| average   | 4763.68                   | 4533.89                  | 736.41                        | 728.64                        |
Figure 25. Total displacement difference ratio of each position of the model

4. Conclusions

In this paper, by comparing different pounding simulation methods, the contact element method model and the Three-dimensional contact friction model are compared from three aspects of pounding force, relative displacement of pier girder and internal force of pier bottom, some conclusion can be drawn from below:

(1) When calculating the maximum impact force of bridge, the results of Kelvin model and Three-dimensional friction model are similar, and the simulation of pounding force is the most accurate. Moreover, considering the energy dissipation, the maximum impact force is lower than that of the model without considering the energy dissipation.

(2) The Kelvin model is close to the three-dimensional contact friction model when calculating the relative displacement of bridge between pier and girder and the internal force of pier bottom.

(3) When the contact element method is used to simulate the pounding, Kelvin model can simulate the pounding accurately because of considering the energy loss, and the parameters needed to establish the model are relatively simple.

Based on the above conclusions, Kelvin model is recommended to be used in the simulation of non-uniform impact analysis of curved bridges, which can not only accurately simulate but also facilitate calculation, and is suitable for seismic analysis of curved bridges.

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