Nuclear Structure Studies at the Borders of Stability

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Abstract. We have performed theoretical calculations to describe the structure of nuclei at the extremes of stability, using the nonadiabatic quasiparticle approach. With this approach, for the first time clear evidence has been obtained for partial rotation alignment in a proton emitting nucleus (¹²¹Pr). Recent findings suggest the departure from axial deformation in the region of proton emitting nuclei. Our calculation for ¹⁴⁵Tm, giving the energy spectra of parent and daughter nuclei, half-life and fine structure, confirmed a large triaxiality. Similarly, we have studied decay of ¹⁴¹Ho, the only known nucleus for which fine structure in proton emission from both ground and isomeric states was observed. The interpretation of the data pointed out to the breaking of axial symmetry in this nucleus as well. The present studies provide new and precise theoretical tools to access nuclear structure properties far from the stability domain.

1. Introduction

Proton emission studies focus on utilizing in the best way a fundamental process of quantum mechanical tunneling, in order to understand the structure of exotic nuclei near the proton drip line. In comparison with the well known process of alpha decay, proton emission is a better ground to apply the tunneling theory, due to the unambiguity in the formation probability of the tunneling particle. The coulomb repulsion sets the barrier for the proton to penetrate and the contribution from centrifugal barrier is also quite significant, specially in deformed nuclei. The strong dependance of calculated proton emission widths on details of nuclear wave function, [¹, ², ³] provides us with a versatile probe to study the structure of the extremely proton-rich nuclei. Studying this new phenomenon has become crucial, mainly because of two main reasons namely, (i) proton radioactivity is an unique probe for nuclei around proton drip line and (ii) with the rapid expansion of the nuclear chart, thanks to new experimental facilities [⁴], it is important to test the well established nuclear models in new regimes, and to have broader understanding of nuclear properties.

Proton emission has been interpreted as decay from a resonance very low in the continuum, of the proton in the field of the core nucleus. In the case of deformed nuclei, the simple adiabatic model [¹, ², ⁵], that considers the wave function of the proton as a single-particle Nilsson resonance, has been very successful in determining uniquely deformation and angular momentum of the parent nucleus assumed to be a rotor, with infinite moment of inertia. The nonadiabatic quasiparticle model [³], provides a complete and consistent description of proton
emission by taking into account the finite moment of inertia of the core and the pairing residual interaction.

This approach lead to a good understanding of proton radioactivity from nuclei with axial deformation. Recently, the experimental [6] half-life for proton radioactivity in $^{121}$Pr has been explained by assuming $J^e = 7/2^-$ as decaying state, showing for the first time clear evidence for partial rotation alignment in a proton emitting nucleus [7]. As another facet, recently we have justified that apart from being an unique probe for exotic nuclei beyond the proton drip line, proton emission studies could be an excellent tool to identify the breaking of axial symmetry in deformed nuclei [8, 9, 10].

In the present article, we highlight how the study of decay widths, branching ratios, and specially the fine-structure data could reflect precisely the nuclear properties and hence serve as an accurate probe which could be far better than conventional spectroscopic studies. In the forthcoming section we discuss briefly the formalism and the results are discussed in the case of $^{145}$Tm and $^{141}$Ho. Previous studies of these two nuclei [11, 12, 13, 14], do not take into account correctly the pairing interaction [3].

2. Non adiabatic quasi particle method

For an even $N$ odd $Z$ nucleus, in a particle plus rotor description, the odd proton moves in the triaxially deformed potential of the daughter nucleus which is considered here as a triaxial rotor. Then the total Hamiltonian of the system can be written as the intrinsic Hamiltonian of the valence proton plus a collective part representing the core, $H = H_{\text{in}} + H_{\text{col}}$, where $H_{\text{in}}$ is the triaxial Nilsson Hamiltonian including a deformed spin-orbit term and the residual pairing interaction. To obtain the single-particle energies and wave-functions corresponding to $H_{\text{in}}$ we use potential of the Wood-Saxon type. The residual pairing interaction is treated within the BCS approach employing a constant gap approximation. The Coriolis coupling is thus diagonalized on a basis of deformed quasiparticles states. In this way the yrast levels appear naturally as the lowest in energy, as it should be, and the coherence in the phases of the various components entering the calculation of the decay widths provides the correct branching ratios [3].

The collective Hamiltonian $H_{\text{col}}$ describing the rotations of the triaxial core with respect to the body-fixed axis can be written as

$$H_{\text{col}} = \sum_{\nu=1,2,3} \frac{\hbar^2}{2I_{\nu}} \vec{R}_{\nu}^2$$

with $\vec{R}$ representing the angular momentum of the core which is related to the angular momentum of the nucleus ($\vec{I}$) and of the proton ($\vec{j}$) by $\vec{I} = \vec{R} + \vec{j}$ and $I_{\nu}$ are the moments of inertia given by

$$I_{\nu} = \frac{4}{3} I_0(\nu) \sin^2 \left( \frac{\nu \pi}{3} \right),$$

where $0^\circ \leq \gamma \leq 60^\circ$ is the asymmetry parameter. The moments of inertia may depend on the angular momentum and hence, in the spirit of the variable moment of inertia model (VMI), one has [15] $I_0(\nu) = I_0 \sqrt{1 + b \nu(I + 1)}$, where $b$ is the VMI parameter and the constant $I_0$ is evaluated by fitting the energy of the first excited $2^+$ state [16].

$$E_{2^+} = \frac{3}{4} \frac{\hbar^2}{I_0} \frac{9 - (81 - 72 \sin^2 3\gamma)^{1/2}}{\sin^2(3\gamma)}$$

The above Hamiltonian can be rewritten in terms of operators acting on the degrees of freedom of the rotor and valence particle and a purely kinematic coupling between the degrees of freedom of both, leading to

$$H_{\text{col}} = H_{\text{rot}} + H_{\text{Cr}}$$

(4)

with
\[ H_{\text{rot}} = \frac{1}{2} (A_1 + A_2) \left( I^2 + J^2 - I_+^2 - J_+^2 \right) + A_3 (I_z - J_z)^2, \] (5)

\[ H_C = \frac{1}{4} (A_1 - A_2) \left( I_+^2 + I_-^2 + J_+^2 + J_-^2 \right) - \frac{1}{2} (A_1 + A_2) \left( I_+ J_+ + I_- J_- \right) \]
- \frac{1}{2} (A_1 - A_2) \left( I_+ J_+ + I_- J_- \right), \] (6)

\( I_\pm \) and \( J_\pm \) are the usual angular momentum ladder operators and \( A_\nu = \hbar^2 / 2L_\nu \). All details regarding the wave functions of the parent and daughter nuclei can be found in Ref. [8].

The decay width, corresponding to the outgoing proton with a given spin \( j \) and orbital angular momentum \( l \), is obtained [1] from the overlap of the initial parent state, and the final one which is a coupling between the daughter and emitted proton wave functions. This yields [8] the expression

\[ \Gamma_{ij}^{\mathrm{IR}} = \frac{\hbar^2 k}{\mu} \frac{2(2R + 1)}{2I + 1} \left| \sum_{\sigma, K}^{I} a_{\sigma R K}^{I} \langle j | \frac{G_{R K R}}{G_{I + |F|}}(r) \rangle \right|^2. \] (7)

In the equation above, the symbol \( \sigma \) specifies the single-particle basis states considered, and the prime in the summation stands for the constraint that imposes \( K - \Omega \) to be an even integer. \( F \) and \( G \) are the regular and irregular Coulomb functions, respectively. The coefficients \( a_{\sigma R K}^{I} \) are the components of the eigenvectors of the Coriolis interaction between quasiparticles and \( \phi_{ij}^{\Omega}(r) \) are the radial components of the eigenfunctions of the Nilsson Hamiltonian. The quantity \( \left| u_{\sigma}^{I} \right|^2 \) gives the probability that the proton single-particle level in the daughter nucleus is empty and is obtained from the BCS calculation. In the case of adiabatic calculations the values of \( a_{\sigma R K}^{I} \) are Kronecker symbols since the Coriolis matrix is diagonal. For decay to the ground state \((R = 0)\) of the daughter nucleus, angular momentum conservation imposes that the angular momentum of the escaping proton \((j)\) has to be equal to the angular momentum of the decaying nucleus \((I)\), while if \( R \neq 0 \) then different values of \( j \) are allowed and the total width is calculated as

\[ \Gamma_{I}^{\mathrm{IR}} = \sum_{j=|R-I|}^{R+I} \Gamma_{ij}^{\mathrm{IR}}. \] (8)

The branching ratio for decay to the first excited \( 2^+ \) states is obtained from the relation \( \Gamma_{2}^{\mathrm{IR}} / (\Gamma_{2}^{\mathrm{IR}} + \Gamma_{0}^{\mathrm{IR}}) \).

3. Results and discussion
In the present work we consider proton emission from \(^{145}\text{Tm}\) where the decay populates [17, 11] both the ground state and the first excited \( 2^+ \) state of the daughter nucleus \(^{144}\text{Er}\).

In order to determine the parameters entering in \( H_{\text{coll}} \), one should use the spectrum of \(^{144}\text{Er}\), unfortunately this is not possible since only the energy of one level, the \( 2^+ \), is known experimentally. Nevertheless, the level scheme of the nearest even-even nucleus \(^{142}\text{Dy}\) is available [18] and hence we calculated first its rotational levels assuming a simple rigid triaxial rotor. The small difference in the moment of inertia between \(^{144}\text{Er}\) and \(^{142}\text{Dy}\) will be of no consequence for the calculations of the half-lives, since it can be absorbed by the Coriolis attenuation factor.

With the unique set of parameters \( \beta_2 = 0.25 \) [19], \( \gamma = 30^\circ \) and \( b = 0.01 \) we could achieve a good fit which is evident from the results shown in Fig. 1. The good fit is a direct evidence for the triaxial nature of the rotor and helps us in fixing the VMI parameter at \( b = 0.01 \).
The parameters used in calculations of $^{142}$Dy states are $\beta_2 = 0.25$, $\gamma = 30^\circ$, and $b = 0.01$. For $^{145}$Tm, the additional parameters are $a_\Delta = 1.0$ and $\rho = 0.85$ and $b = 0.01$. However these are not the only parameters yielding good fit with experimental data [see text]. The experimental data are taken from Ref. [18] and [17].

In Fig. 1 we show the spectrum for the case of $^{145}$Tm as well, which is calculated with the quasiparticle plus triaxial rotor model (QPTRM). However now several parameters are involved. The best fit is obtained with $\beta_2 = 0.25$, $\gamma = 30^\circ$, $a_\Delta = 1.0$, $\rho = 0.85$ and $b = 0.01$. Here $a_\Delta$ represents the pairing gap through the relation $\Delta = a_\Delta \times 12/\sqrt{A}$ and $\rho$ is the factor with which the Coriolis interaction is attenuated. It has to be mentioned that this is not a unique parameter set and similar fit could be achieved with few more combination of these parameters, however this choice is justifiable [9]. The set of $\beta_2$, $\gamma$ and $b$ is unique if it has to reproduce the spectrum of $^{142}$Dy and we have used $a_\Delta$ and $\rho$ values which are common in literature.

In the next step we have calculated the decay width for proton emission from $^{145}$Tm using the wave-functions QPTRM which successfully explains the rotations spectrum. The results are shown in the upper panels (a–c) of Fig. 2 where we have considered the effect of different parameters on the proton decay width. In addition to the effect of Coriolis attenuation factor, the pairing gap and triaxial deformation, we have studied the effect of axial deformation also. The corroboration with experimental decay width clearly suggests a strong axial deformation like the one we have assigned. The other parameters ($\beta_2$ and $a_\Delta$) are relatively less effective but their variations are still constrained if we demand a good fit. Though the axially deformed calculation seems to explain the experimental decay width to some extent, the interesting results are those for the branching ratios which we have shown in the lower panels (d–f) of Fig. 2.

The experimental branching ratio can be explained only by a large triaxial deformation with a very narrow window around $\gamma = 30^\circ$. This fact is independent of the choice of various parameters and hence a precise identification of such maximum static triaxial deformation. The identification through the branching ratio is more robust due to the weak dependence on many parameters like that of the mean field, pairing gap and VMI. Even if some parameters entering the calculations to reproduce the experimental rotational energies were not well defined, this fact does not hamper the assignment of triaxial shape through the branching ratio. Our calculations demonstrate that observation of fine structure in proton emission enables us to have

![Figure 1. Rotational spectrum of $^{142}$Dy and $^{145}$Tm. The parameters used in calculations of $^{142}$Dy states are $\beta_2 = 0.25$, $\gamma = 30^\circ$, and $b = 0.01$. For $^{145}$Tm, the additional parameters are $a_\Delta = 1.0$ and $\rho = 0.85$ and $b = 0.01$. However these are not the only parameters yielding good fit with experimental data [see text]. The experimental data are taken from Ref. [18] and [17].](image-url)
Figure 2. Dependence of calculated proton emission width (panels a–c) and branching ratio (panels d–f) on the triaxiality parameter $\gamma$ for the nucleus $^{145}$Tm where the proton decays from $J^\pi = 11/2^-$ state and populates both $0^+$ and $2^+$ states of $^{144}$Er. Different line patterns correspond to the Coriolis attenuation factor $\rho$ (column 1), BCS pairing gap factor $a_\Delta$ (column 2) and the axial deformation parameter $\beta_2$ (column 3), as labelled in the lower panels. The areas shaded in grey correspond to the experimental values including uncertainties in the data. The error bars on the theoretical curves at $\gamma = 20^\circ$ in the first column represents the typical uncertainty in the calculation due to the experimental error in the $Q$-value for proton emission.

unambiguous and parameter free prediction of nuclear structural and decay properties.

We have obtained similar results in the case of $^{141}$Ho [10], the only proton radioactive nucleus where fine structure data is available [20] for both the negative parity yrast state and the positive parity isomeric state. In fact, with our non adiabatic calculations we were able to provide a precise assignment of spin of the ground and isomeric decaying states, and observe the breaking of axial symmetry in this proton radioactive nucleus.

4. Conclusion

In conclusion, we have achieved a solid interpretation of proton radioactive nuclei, and have identified some of the nuclear structure properties of these exotic systems. The observation of fine structure in proton emission leads to have unambiguous and parameter free prediction of structural and decay properties of nuclei near proton drip line. The level of confidence in these predictions is quite high and easily way beyond the conventional probes like the discrete gamma spectroscopy. However, future experimental studies of rotational spectra of proton emitters, along with the study of fine structure in decay to exited states, will yield rich information.
indispensable to a theoretical understanding of nuclei near borders of stability.

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