Dilepton anisotropy from $p + Be$ and $Ca + Ca$ collisions at BEVALAC energies *

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Abstract

A full calculation of lepton-pair angular characteristics is carried out for $e^+e^-$ pairs created in $p + Be$ and $Ca + Ca$ collisions from 1.0 to 2.1 GeV/A. It is demonstrated that the dilepton decay anisotropy depends sensitively on the different sources and may be used for their disentangling. Due to the dominance of the $\eta$-and $\Delta$-Dalitz decays and only a small anisotropy coefficient for $\pi^+\pi^-$ annihilation, the expected anisotropy coefficients show a decrease with invariant mass of the dilepton pair and change only moderately when comparing $p + Be$ and $Ca + Ca$ reactions at the same bombarding energy.

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Dileptons are quite attractive electromagnetic signals since they provide almost direct information on the hot and dense nuclear phase in heavy-ion collisions at BEVALAC/SIS and SPS energies [1, 2, 3, 4]. The information carried out by leptons may tell us not only about the interaction dynamics of colliding nuclei, but also on properties of hadrons in the nuclear environment or on a possible phase transition of hadrons into a quark-gluon plasma (cf. [5]). However, there are a lot of hadronic sources for dileptons because the electromagnetic field couples to all charges and magnetic moments. In particular, in hadron-hadron collisions, the $e^+e^-$ pairs are created due to the electromagnetic decay of time-like virtual photons which can result from the bremsstrahlung process or from the decay of baryonic and mesonic resonances including the direct conversion of vector mesons into virtual photons in accordance with the vector dominance hypothesis. In the nuclear medium, the properties of these sources may be modified and it is thus very desirable to have experimental observables which allow to disentangle the various channels of dilepton production.

Recently, we have proposed to use lepton pair angular distributions for a distinction between different sources [6, 7]. Indeed, it could be shown that due to the spin alignment of the virtual photon and the spins of the colliding or decaying hadrons, this lepton decay anisotropy turns out to be quite sensitive to the specific production channel (cf. also [8, 9]). Whereas in our previous work [6, 7, 10] we have considered the dilepton anisotropy for elementary nucleon-nucleon or $pd$ collisions, we now evaluate this new observable for the first time for proton-nucleus and nucleus-nucleus collisions from 1 to 2 GeV/A bombarding energy, where the calculated inclusive dilepton spectra can be controlled in comparison to the DLS data [1, 2, 3].

The dynamical evolution of the proton-nucleus or nucleus-nucleus collision is described by a transport equation of the Boltzmann-Uehling-Uhlenbeck type evolving phase-space distribution functions for nucleons, $\Delta$’s, $N^*(1440)$’s, $N^*(1535)$’s, pions and $\eta$’s with their isospin degrees of freedom. The details of the model are discussed in Refs. [11, 12]; here we use the same prescriptions and include additionally the production channels from the direct decay of the vector mesons $\rho$, $\omega$, and $\Phi$ as well as the Dalitz-decay of the $\omega$-meson [13] employing the formfactors from Landsberg [14].

In Fig. 1 we present the calculated dilepton invariant mass spectra $d\sigma/dM$ for $p + ^9Be$ and $^{40}Ca + ^{40}Ca$ collisions at bombarding energies from 1 to 2.1 GeV/A and compare them with the experimental data of the DLS collaboration [1, 2, 3] including the DLS
acceptance filter as well a mass resolution of 50 MeV. In the 'cocktail' plot the $\Delta$ Dalitz decay (labeled by 'Δ'), $N^*$ Dalitz decay ('$N^*$'), proton–neutron bremsstrahlung ('$pn$'), $\pi N$ bremsstrahlung ('$\pi N$') (all without VDM formfactor), $\omega \rightarrow \pi^+\pi^-$ Dalitz decay, $\eta$ Dalitz decay ('$\eta$'), the pion annihilation channel ('$\pi\pi$') as well as the direct decay of the vector mesons ('$\rho, \omega, \Phi$') are separated explicitly while the solid curves (denoted by 'all') represent the sum of all sources. As seen from Fig. 1 the dilepton cross sections agree reasonably well with the experimental data as in [12]. We will use the same cross sections, however, without experimental DLS filter for the calculation of the anisotropy coefficients.

In order to characterize the dilepton decay anisotropy we have introduced in [6, 7] the anisotropy coefficient $B$ which allows to characterize the angular distribution of dileptons created in a hadron-hadron ($h + h$) reaction via

$$S_i(M, \theta) \equiv \frac{d\sigma^{hh}_i}{dM \, d\cos \theta_{hh}} = A_i^{hh}(1 + B_i^{hh} \cos^2 \theta_{hh}). \quad (1)$$

In Eq. (1) $\theta_{hh}$ denotes the angle of the electron momentum $\vec{l}^*$ - measured in the dilepton center-of-mass system ($\vec{q}^* \equiv \vec{l}^- + \vec{l}^+ = 0$) - with respect to the velocity of the $h + h$ c.m.s. (denoted by $\vec{v}^{hh}$) relative to the dilepton c.m.s., i.e. $\vec{v}^{hh} = -\vec{q}^{hh}/q_0^{hh} = -\vec{v}_q^{hh}$; $\cos \theta_{hh} = (\vec{l}^\ast, \vec{v}^{hh}_q)$. Here, the momentum and energy of virtual photon ($q_0^{hh}, \vec{q}^{hh}$) are defined in the c.m.s. of the $h + h$ system while $M$ is the invariant mass of a lepton pair, $(M^2 = q_0^2 - \vec{q}^2)$. $B^{hh}$ describes the anisotropy while $A^{hh}$ determinds magnitude of the cross section.

The total differential cross section for $h + h$ collisions now can be represented as a sum of the differential cross sections for all channels,

$$\frac{d\sigma^{hh}}{dM \, d\cos \theta_{hh}} = \sum_{i=\text{channel}} \frac{d\sigma^{hh}_i}{dM \, d\cos \theta_{hh}} = A^{hh}(M)(1 + B^{hh}(M) \cos^2 \theta_{hh}), \quad (2)$$

which leads to the total anisotropy coefficient:

$$B^{hh}(M) = \sum_{i=\text{channel}} <B_i^{hh}(M)>, \quad <B_i^{hh}(M)> = \frac{d\sigma^{hh}_i}{dM} \cdot \frac{B_i^{hh}}{1 + \frac{1}{3}B_i^{hh}}, \quad (3)$$

where the special weighting factors originate from the necessary angle-integrations. Thus, the anisotropy coefficient $B^{AB}$ for $A + B$ reactions is the sum of the “weighted” anisotropy coefficients ($< B_i^{AB} >$) for each channel $i$ obtained by means of the convolution of $B_i^{AB}$ with the corresponding invariant mass distribution (cf. [10]).
For heavy-ion reactions the situation becomes more complicated due to the nuclear dynamics and the explicit time evolution of the interacting system. Here we start from the point that the form of the angular distribution for all 'elementary' interactions $a + b$, that occur in the nucleus-nucleus reaction $A + B$, are known. For this aim we employ the results of our previous works [6, 7, 10] where the anisotropy coefficients for the $pp$, and $pn$ bremsstrahlung, $NN \rightarrow \Delta \rightarrow NN\gamma^+e^-$ Dalitz decay, $\eta$ Dalitz decay and $\pi\pi$ annihilation channels were calculated explicitly in the hadron-hadron center-of-mass system $a + b$.

The differential angular distribution in elementary $a + b$ collisions – before averaging over the momentum $\vec{q}^{ab}$ – can be represented as:

$$
\frac{d\sigma^{ab}_i}{dM \, d\vec{q}^{ab} \, d\cos\theta^{ab}_{ab}} = A^{ab}_i \left( 1 + B^{ab}_i \cos^2 \theta^{ab}_{ab} \right),
$$

(4)

where the coefficient $B^{ab}_i$ for the elementary process $a + b$ is a function of the dilepton mass $M$, the masses $m_a, m_b$ and the initial invariant energy $\sqrt{s}$ of the hadrons involved in the reaction: $B^{ab}_i = B^{ab}_i(M, \sqrt{s}; m_a, m_b)$. It is important to note that we know the 'elementary' coefficient only in the $a + b$ system. In nucleus-nucleus collisions, however, the direction $\vec{v}^{ab}_q = \vec{q}^{ab}/q^{ab}_0$ is changed in each elementary $a + b$ collision. Thus in order to define an anisotropy coefficient in the latter situation we have to perform an angular transformation from the elementary c.m.s. ($\theta^{ab}_{ab}$) to the c.m.s. of the colliding nuclei ($\theta^{AB}_{AB}$):

$$
\cos \theta^{ab}_{ab} = \cos \theta^{AB}_{AB} \cos \theta_q + \sin \theta^{AB}_{AB} \sin \theta_q \cos (\varphi^{AB}_{AB} - \varphi_q),
$$

(5)

where $\theta_q$ is the angle between the dilepton c.m.s. velocity $\vec{v}^{ab}_q = \vec{q}^{ab}/q^{ab}_0$ in the c.m.s. of $a + b$ and the dilepton c.m.s. velocity $\vec{v}^{AB}_q = \vec{q}^{AB}/q^{AB}_0$ in the c.m.s. of the colliding nuclei $A + B$: $\cos \theta_q = \vec{v}^{ab}_q \cdot \vec{v}^{AB}_q / (|\vec{v}^{ab}_q| \, |\vec{v}^{AB}_q|)$. Here, the vector $\vec{q}^{ab}$ is obtained by a Lorentz transformation of $\vec{q}^{AB}$: $\vec{q}^{ab} = L(\vec{V}^{ab})\vec{q}^{AB}$, where $\vec{V}^{ab} = (\vec{p}_a + \vec{p}_b)/(E_a + E_b)$ is the velocity of the $a + b$ system relative to the $A + B$ system.

Substituting (5) into Eq.(4) and using $d\Omega^{AB} = d\Omega^{ab}$ we get the respective distribution in the observable angle $\theta^{AB}_{AB}$ by integrating over the azimuthal angle $\varphi^{AB}$:

$$
\frac{d\sigma^{ab}_i}{dM \, d\cos \theta^{AB}_{AB}} \sim \int d\vec{q}^{AB} \, \tilde{A}_i(\theta_q) \left( 1 + \tilde{B}_i(\theta_q) \cos^2 \theta^{AB}_{AB} \right),
$$

(6)

$$
\tilde{A}(\theta_q) = 1 + \frac{B^{ab}_i}{2} \sin^2 \theta_q, \quad \tilde{B}(\theta_q) = \frac{B^{ab}_i}{2 \tilde{A}(\theta_q)} (3 \cos^2 \theta_q - 1).
$$

(7)

Obviously, the transformation (5) does not change the quadratic form in $\cos \theta$ of the
angular distribution. From Eqs.(1) and (6) and the normalization condition we finally get

\[ B_{i}^{A^{B}} = \frac{\int d\vec{q}^{A^{B}} \hat{A}(\theta_{q}) \hat{B}(\theta_{q}) \frac{d\sigma_{i}^{ab}}{dM d\vec{q}^{A^{B}}}}{\int d\vec{q}^{A^{B}} \hat{A}(\theta_{q}) \frac{d\sigma_{i}^{0}}{dM d\vec{q}^{A^{B}}}} \quad (8) \]

where the differential cross sections \( d\sigma_{i}^{ab}/(dM d\vec{q}^{A^{B}}) \) for the different channels i are taken from the BUU calculations as described in [11, 12]. The respective 'reduced' differential cross sections \( d\sigma_{i}^{A^{B}}/dM \) are shown in Fig. 1 for the systems to be studied below.

The above definition of the anisotropy coefficient in nucleus-nucleus collisions is valid for all channels except for \( \pi^{+}\pi^{-}-\text{annihilation} \) because there are only two particles in the initial and final states. For this particular reaction we have to use the angle of the pion momentum with respect to the lepton momentum in the c.m.s. of the leptons (or pions, which is the same for this channel), i.e. \( \vec{q}^{ab} = \vec{p}_{a} + \vec{p}_{b} = \vec{l}_{+} + \vec{l}_{-} = 0 \). As was shown in Ref. [6], the elementary anisotropy coefficient in the \( \pi\pi \) c.m.s. then is \( B_{\pi^{+}\pi^{-}} = -1 \). In heavy-ion collisions we can use the same definition for the polar angle as for the other channels because we can reconstruct the direction of \( \vec{q}^{A^{B}} \) in the c.m.s. of the colliding nuclei \( A+B \). Furthermore, we performed the same angular transformation from the c.m.s. of the pions \( a+b \) to the c.m.s. of the nuclei \( A+B \) as in Eq.(5) with the replacement \( \theta_{q} \rightarrow \theta_{\pi} \). Here, \( \theta_{\pi} \) is the angle between the pion momentum \( \vec{p}_{a} \) in the c.m.s. of \( a+b \) and the vector \( \vec{v}_{q}^{A^{B}} = \vec{q}^{A^{B}}/\lambda_{0}^{A^{B}} \). Following the same angular integration as for the previous cases we end up with the expression for the anisotropy coefficient for \( \pi^{+}\pi^{-} \) annihilation:

\[ B_{\pi^{+}\pi^{-}}^{A^{B}} = \frac{\int d\cos \theta_{\pi} \hat{A}(\theta_{\pi}) \hat{B}(\theta_{\pi}) W(\cos \theta_{\pi})}{\int d\cos \theta_{\pi} A(\theta_{\pi}) W(\cos \theta_{\pi})} \quad (9) \]

The quantities \( \hat{A}(\theta_{\pi}), \hat{B}(\theta_{\pi}) \) are defined by Eq.(7) using \( B_{i}^{ab} = -1 \) while \( \cos \theta_{\pi} \) can be computed by the scalar products of the four momenta of the pions \( p_a, p_b \) and colliding nuclei \( P_A, P_B, \)

\[ \cos \theta_{\pi} = \frac{\left(p_{a} - p_{b}\right)\cdot(P_{A} + P_{B}) M}{\sqrt{(M^{2} - 4m_{\pi}^{2}) \left(\left[(p_{a} + p_{b})\cdot(P_{A} + P_{B})\right]^{2} - (P_{A} + P_{B})^{2} M^{2}\right)}} \quad (10) \]

with \( M^{2} = s_{ab} = (p_{a} + p_{b})^{2} \) following O.V. Teryaev [15].

A closer look at Eq.(9) shows that \( B_{\pi^{+}\pi^{-}}^{A^{B}} \) is a constant; its absolute value follows from the pion angular distribution \( W(\cos \theta_{\pi}) \). In case of an isotropic angular distribution, \( W(\cos \theta_{\pi}) = \text{const} \), the anisotropy coefficient \( B_{\pi^{+}\pi^{-}}^{A^{B}} = 0 \) according to Eq.(9). Moreover, in
the BUU calculation the angular distribution \( W(\cos \theta_\pi) \) is also a function of time \( t \) due to the dynamical evolution of the system. Technically we define the distribution \( W(t) \) at time \( t \) (i.e. in the time interval \( [t - \Delta t/2 : t + \Delta t/2] \)) as the ratio of the number of pions with fixed \( \cos \theta_\pi \) produced in the above time interval to the total number of pions produced in the \( A + B \) reaction, i.e. \( W(t, \cos \theta_\pi) = \dot{N}(t, \cos \theta_\pi)/N_{tot} \).

In order to demonstrate the pion angular anisotropy we show in Fig. 2 (l.h.s.) the pion angular distribution \( W(t, \cos \theta_\pi) \) for \( p + ^9Be \) at 2.1 GeV and \(^{40}Ca + ^{40}Ca\) at 1.0 and 2.0 GeV/A at those times \( t \) when the maximal number of pions \( \dot{N}(t) \) was produced during the interval \( \Delta t \); the respective \( \pi \)-production rate \( \dot{N}(t)/N_{tot} \) is illustrated in the r.h.s. of Fig. 2 for the same reactions. Due to the low number of pions produced for \( p + ^9Be \) the angular distribution suffers from low statistics and the error bars – which result from different runs with 8000 testparticles per nucleon – are large. For the further analysis we have used the dotted line in Fig. 2 which represents a fit to the angular distribution. On the other hand, for \(^{40}Ca + ^{40}Ca\) collisions the statistics is good enough and we show only fits to the ‘numerical’ data in terms of the solid and dash-dotted lines, respectively. The anisotropy of \( W(\cos \theta_\pi) \) is most pronounced for \( Ca + Ca \) at 1 GeV/A, because the probability for \( \pi \)’s with transverse momentum is larger then with longitudinal momentum due to a stronger pion absorption in beam (longitudinal) direction. At 2 GeV/A, the pion distribution becomes more isotropic in line with a ‘pionic fireball’ scenario. Since pion absorption effects are only small for \( p + Be \), the resulting angular distribution shows only a very modest anisotropy.

Before going over to the calculation of the anisotropy coefficient for nucleus-nucleus collisions one has to take into account that resonances \((\Delta, N^*)\) can be created in quite different elementary channels than in \(pN\) or \(pd\) reactions. For example, \( \Delta \) production via the \( \pi N \to \Delta \) channel becomes quite important; an elementary channel for which the dilepton anisotropy has not been computed so far. We thus have calculated the \( e^+e^- \) anisotropy using the same vertices, delta-propagator, coupling constants and formfactors as in Refs. [7, 16]. Since this evaluation is straight forward, we do not present the details here.

The results of our microscopic calculation for the anisotropy coefficient for the \( \pi + N \to \Delta \to e^+e^- + X \) channel are displayed in Fig. 3. The anisotropy \( B_{\pi N \to \Delta} \) is a function of the dilepton invariant mass \( M \) and the invariant energy of the interacting particles \( s = (p_\pi + p_N)^2 \equiv M_\Delta^2 \). For small \( \sqrt{s} \) only deltas with \( M_\Delta \approx M_{\Delta 0} = 1.232 \) GeV appear
and the coefficient $B_{\pi^N \to \Delta} \to 1$ as expected for the Dalitz decay of a free delta [6]. With increasing $\sqrt{s}$ more energetic deltas can be created and (for fixed $M$) the phase-space for the final nucleon and virtual photon increases leading to a decrease of $B_{\pi^N \to \Delta}$. We note that we do not take into account the Dalitz decay of the higher nucleon resonances in the further calculations because their statistical weight is too low.

Fig. 4 finally shows the computed weighted anisotropy coefficients $< B_i(M) >$ for $p + ^9Be$ and $^{40}Ca + ^{40}Ca$ collisions at the bombarding energies from 1 to 2.1 GeV/A. The main contributions arise from the $\eta$ and $\Delta$ Dalitz decays due to their large 'elementary' anisotropy coefficients and cross sections (cf. Fig. 1), respectively. The contribution from $pn$ bremsstrahlung is practically zero at all energies due to a smaller 'elementary' anisotropy coefficient and due to a lower cross section as well. The weighted coefficient from $\pi^+\pi^-$ annihilation is rather small ($\approx 0.1$) even for the $Ca+Ca$ reactions and decreases for $M \geq m_{\rho}$ due to the threshold behaviour of the cross section. However, compared to $< B_{\pi^+\pi^-}(M) >$ for $p + ^9Be$, where pion annihilation is very low, a clear (but moderate) enhancement can be extracted.

The contributions of the further channels ($N^*, \pi N$ bremsstrahlung, $\omega \to \pi^0 e^+ e^-$) are also negligible due to their small cross sections (cf. Fig. 1). The cross section from direct decays of the vector mesons becomes compatible with pion annihilation for $M \approx m_{\rho}$ for $p + ^9Be$ at 2.1 GeV, but the anisotropy coefficient for the 'free' vector meson in the vacuum is zero [17]. We do not discuss here a possible modification of the $\rho$-meson properties in the medium that might lead to non-isotropic angular distributions of dileptons because for $p + ^9Be$ at 2.1 GeV one cannot reach sufficiently high baryon or pion densities.

On the other hand, for $^{40}Ca + ^{40}Ca$ collisions at 2.0 GeV and for $p + ^9Be$ at 2.1 GeV we observe a clear anisotropy from the $\eta$ Dalitz decay, while for $p + ^9Be$ at 1 GeV the $\Delta$ Dalitz decay gives the main contribution at small invariant mass due to a dominant $\Delta$ cross section; the $\eta$ coefficient increases at $M$ from 0.4 to 0.5 GeV for the same reason.

For clarity we briefly discuss the steps to 'extract' the anisotropy coefficient from experimental data. In dilepton experiments the four-momenta of leptons in the c.m.s. of the nuclei $A + B$ (or in the laboratory frame, which is connected with the c.m.s. by a simple Lorentz transformation) are measured - $(\varepsilon_{-AB}, \vec{i}_{-AB}), (\varepsilon_{+AB}, \vec{i}_{+AB})$. The four momentum $q = (q_0^{AB}, \vec{q}^{AB})$ of the virtual photon in the $A + B$ c.m.s. then is given by: $q_0^{AB} = \varepsilon_{-AB} + \varepsilon_{+AB}, \quad \vec{q}^{AB} = \vec{i}_{-AB} + \vec{i}_{+AB}$. The angle $\theta_{AB}$ has to be computed via:

$$\cos \theta_{AB} = \frac{|\vec{q}^{AB}| - q_0^{AB} \cos \theta_{qd}}{|\vec{q}^{AB}| \cos \theta_{qd} - q_0^{AB}},$$

(11)
where $\theta_{ql}$ is the angle between the $\vec{q}^{AB}$ and $\vec{l}_AB$, i.e. $\cos \theta_{ql} = \vec{l}_AB \cdot \vec{q}^{AB} / (|\vec{l}_AB| |\vec{q}^{AB}|)$.

In order to extract the anisotropy coefficient one has to count the number of dilepton events $N(M, \cos \theta_{AB})$ with fixed $\cos \theta_{AB}$ and invariant mass $M$. The anisotropy coefficient for the invariant mass $M$ then is simply given by:

$$B^{AB}(M) = \frac{N(M, \cos \theta_{AB} = 1)}{N(M, \cos \theta_{AB} = 0)} - 1.$$  \hspace{1cm} (12)

Thus, summarizing, the calculated anisotropy coefficients for $p + Be$ and $Ca + Ca$ collisions support our suggestion in Refs. [6, 7, 10] that the dilepton decay anisotropy may serve as an additional observable to decompose the dilepton spectra into the various sources. Since the anisotropy vanishes in a hadronic fireball scenario our present results provide valuable information about the nonequilibrium stage of the reactions.

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Figure captions

Figure 1: The dilepton invariant mass spectra $d\sigma/dM$ from $p + ^9Be$ and $^{40}Ca + ^{40}Ca$ collisions at bombarding energies from 1 to 2.1 GeV/A in comparison to the experimental data [1, 2, 3]. The “$\eta$” denotes the contribution of the $\eta$-channel, the “$\Delta$” labels the contribution of the $\Delta$ Dalitz decay, “$N^*$” the $N^*$ Dalitz decay, “$\omega \rightarrow \pi^+\pi^-$” is the $\omega \rightarrow \pi^+\pi^-$ Dalitz decay, “$pn$” the proton–neutron bremsstrahlung, “$\pi N$” the $\pi N$ bremsstrahlung, “$\pi\pi$” the pion annihilation channel, while “$\rho, \omega, \Phi$” is the direct decay of vector mesons. The solid curves (denoted by “all”) show the sum of all sources.

Figure 2: The pion angular distribution $W(t, \cos \theta_\pi)$ for $p + ^9Be$ at 2.1 GeV and $^{40}Ca + ^{40}Ca$ at 1.0, 2.0 GeV and time $t$ (l.h.s.) where the maximal number of pions $N(t)$ was produced. The pion production rate $\dot{N}(t)/N_{tot}$ is shown in the r.h.s. of the figure.

Figure 3: The results of our calculation for the anisotropy coefficient of the $\pi + N \rightarrow \Delta \rightarrow e^+e^- + X$ channel at $M_\Delta = \sqrt{s}$ from 1.232 to 1.8 GeV.

Figure 4: The weighted anisotropy coefficients $< B_i(M) >$ for $p + ^9Be$ and $^{40}Ca + ^{40}Ca$ collisions at bombarding energies from 1 to 2.1 GeV/A. The notation is the same as in Fig. 1.
Fig. 1
Fig. 2
$$\pi N \rightarrow \Delta \rightarrow e^+ e^- + X$$

$M_\Delta = 1.232$ GeV

Fig. 3
Fig. 4