Generalized Mirror Symmetry 
and 
Quantum Black Hole Entropy 

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ABSTRACT

We find general relations between the on-shell gravitational trace anomaly $A_N$, and the logarithmic correction $\Delta S_N$ to the entropy of “large” BPS extremal black holes in $\mathcal{N} \geq 2$ supergravity theories in $D = 4$ space-time dimensions (recently computed by Sen [1]). For (generalized) self-mirror theories (all having $A_N = 0$), we obtain the result $\Delta S_N = -\Delta S_{8-N} = 2 - N/2$, whereas for generic theories the trace anomaly $\hat{A}_N$ of the fully dualized theory turns out to coincide with $2\Delta S_N$, up to a model-independent shift: $\hat{A}_N = 2\Delta S_N - 1$. We also speculate on $\mathcal{N} = 1$ theories displaying “large” extremal black hole solutions.
1 Introduction

Recently, a generalized notion of mirror symmetry was suggested \cite{2}, under which
\begin{equation}
A_N = - \frac{1}{24} \rho,
\end{equation}
 occurring in the on-shell\footnote{As given by Eq. (1.2), we call “on-shell” anomaly the one concerning the square of $R_{\mu\nu\rho\sigma}$, following \cite{2} (see also e.g. \cite{4} for an extensive list of Refs.).} gravitational trace anomaly \cite{3,4,5}
\begin{equation}
g_{\mu\nu} \langle T^{\mu\nu} \rangle = A_N \frac{1}{32\pi^2} R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma},
\end{equation}
changes sign.

In $M$-theory compactified on a seven-manifold $X^7$ with Betti numbers $(b_0, b_1, b_2, b_3)$, $\rho$ is defined as \cite{2}
\begin{equation}
\rho \equiv 7b_0 - 5b_1 + 3b_2 - b_3,
\end{equation}
and $\rho \to -\rho$ under the generalized mirror symmetry \cite{2}
\begin{equation}
(b_0, b_1, b_2, b_3) \to (b_0, b_1, b_2 - \rho/2, b_3 + \rho/2).
\end{equation}

In Ref. \cite{2} it was shown that $D = 4$, $N \geq 3$ extended supergravity theories are generalized self-mirror\footnote{It should be pointed out that this is not the same as the anomaly computed on the supergravity equations of motion. Indeed, while the coefficient of the Gauss-Bonnet term is always proportional to $n_s + 62n_V + \frac{1}{2}n_{MF}$ ($n_s$, $n_V$ and $n_{MF}$ respectively standing for the number of scalar, vector and Majorana spinor massless fields), in a conformally flat background (as is the Bertotti-Robinson $AdS_2 \times S^2$ near-horizon geometry of the extremal black hole), the term proportional to the square of the Weyl tensor does vanish (see e.g. \cite{4}, and also \cite{1} for a recent discussion).}. On the other hand, for $N = 1, 2$ theories the generalized self mirror condition imposes some constraints on the matter content.

As we will see below, results crucially depend on the dualization of 3- and 2- form fields, which naturally arise from $M$-theory compactifications; it is remarkable that the trace anomaly coefficient $A_N$ of the undualized theory does vanish for $N = 8, 6$ and $5$ “pure” supergravities, if the corresponding graviton multiplet is properly defined as containing also form fields of degree higher than one \cite{6} (see also \cite{2}). This is still true in matter coupled $N = 3$ and $4$ theories, if at least $n_V = 2$ resp. $3$ massless vector multiplets in the dualized theory (corresponding to $2$ resp. $3$ massless 2-form multiplets in the undualized avatar) are taken into account. Self-mirror $N = 2$ theories and generalized self-mirror $N = 1$ theories all have $\rho = 0$, which in the fully dualized framework respectively contrains the matter content as follows \cite{2}:
\begin{align}
N = 2 & : n_H = n_V + 1; \quad (1.5) \\
N = 1 & : n_c = 3n_V + 7, \quad (1.6)
\end{align}
where $n_V$, $n_c$ and $n_H$ respectively denote the number of vector, chiral and hyper massless multiplets.

On the other hand, Sen et al. \cite{7,8,1} recently computed the coefficient $\Delta S$ of the logarithmic correction to the Bekenstein-Hawking \cite{9} entropy of “large” BPS extremal black holes (BHs), in particular achieving the following result for a generic $N = 2$ supergravity:
\begin{equation}
\Delta S_{N=2} = \frac{1}{24} (23 + n_H - n_V). \quad (1.7)
\end{equation}
For self-mirror $N = 2$ theories ($n_H = n_V + 1$), Eq. (1.7) yields
\begin{equation}
\text{self-mirror : } \Delta S_{N=2} = 1, \quad (1.8)
\end{equation}
as it holds for the self-mirror *stu* model \[^5\], characterized by \( n_V = 3 \) and \( n_H = 4 \).

Up to some irrelevant \( \mathcal{O}(1) \) terms, the following structure is known to hold in general (see *e.g.* \[^7\] \[^8\] \[^1\], and Refs. therein):

\[
S = S_0 + \Delta S \ln \left( \frac{A_H^4}{\pi V} \right),
\]

where \( S_0 = A_H/4 \) is the Bekenstein-Hawking entropy of the “large” BPS extremal BH under consideration, whose non-vanishing event horizon area is denoted by \( A_H \). Due to the Attractor Mechanism \[^10\] \[^11\], \( A_H \) is given by the BH effective potential \( V_{BH} \) computed at its attractor points \[^11\]: \( A_H = 4\pi V_{BH}\bigg|_{V_{BH}=0} \). Actually, in any theory with \( N \geq 3 \) the scalar manifold is a symmetric coset \( \frac{G_{1, N}}{H_{1, N}} \) and it holds that \( A_H = \sqrt{I_4} \), where \( I_4 \) is the unique independent polynomial invariant (quartic in electric and magnetic charges) constructed with the BH charge irrepr. of \( G_{1, N} \). The symmetric coset structure of the scalar manifold also characterizes \( N = 2 \) minimally coupled and \( N = 3 \) matter coupled theories, but in such theories \( \sqrt{I_4} = |I_2| \). In general, the scalar manifold of \( N = 2 \) and \( N = 1 \) theories, despite being characterized by Kähler geometry (of special type in \( N = 2 \)), is not necessarily symmetric nor homogeneous, and \( I_4 \) may thus not exist at all.

Aim of the present note is to clarify the relation between \( A_N \) and \( \Delta S_N \) for \( N \geq 2 \), and consider, within some consistency conditions, also \( N = 1 \) theories of gravity exhibiting “large” extremal BH solutions.

Two main general results are achieved in this investigation:

**I**] (Generalized) self-mirror theories exhibit \( N \)-dependent values of \( \Delta S_N \) related by a fermionic symmetry:

\[
\text{(gen.) self-mirror : } \Delta S_N = -\Delta S_{N-2} = 2 - \frac{N}{2}.
\]

(1.10)

this result can be made explicit by the following symmetric pattern, centered at \( N = 4 \):

\[
\begin{array}{cccccccc}
N & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \\
\Delta S_N & -2 & -\frac{5}{2} & -1 & -\frac{1}{2} & 0 & \frac{1}{2} & 1 & \frac{3}{2} & 2?
\end{array}
\]

(1.11)

suggesting a possible “generalized self-mirror” \( N = 0 \), \( D = 4 \) gravity theory with \( \Delta S_{N=0} = 2 \).

**II**] In generic theories, the gravitational trace anomaly \( \tilde{A}_N \) of the fully dualized theory is nothing but \( \Delta S_N \) itself, up to a model-independent shift:

\[
\tilde{A}_N = 2\Delta S_N - 1.
\]

(1.12)

Note that \( \tilde{A}_N \) is the value of the on-shell gravitational trace anomaly coefficient as computed in “standard” \( D = 4 \) supergravity theories, with only physical spin degrees of freedom (see App. A of \[^2\] for a detailed treatment). In \( M \)-theory on \( X^7 \), the degrees of freedom \( f \), the number \( \# \) and the contribution to \( \tilde{A}_N \) of the various types of massless fields in the fully dualized \( N \geq 1 \), \( D = 4 \) supergravity theory are given in Table XX of \[^2\], which we partially report in Table 1.

By virtue of (1.10), for (generalized) self-mirror theories (1.12) can be recast as

\[
\text{(gen.) self-mirror : } \tilde{A}_N = 3 - N,
\]

(1.13)

thus curiously yielding \( \tilde{A}_{N=3} = 0 \) (as noted long time ago in \[^5\]). It is worth observing that \( \tilde{A}_{N=8} = -5 \) matches the result of \[^12\]; in particular, as given by the general formula (1.12), \( \tilde{A}_{N=8} \) is not proportional to \( \Delta S_{N=8} \).

The plan of this note is as follows.

In Sec. \[^2\] starting from some results recently obtained in \[^7\] \[^8\] \[^1\], the massless multiplet content of fully dualized \( N \geq 3 \), \( D = 4 \) supergravity theories is decomposed in terms of the various types of \( N = 2 \) multiplets, whose contributions to \( \Delta S_{N=2} \) are then explicitly computed.

General relations, involving \( \Delta S_N \), the undualized trace anomaly \( A_N \) and the fully dualized trace anomaly \( \tilde{A}_N \), are obtained in Sec. \[^2\].
Table 1: Degrees of freedom $f$, contribution to $\tilde{A}_N$ and number $\#$ of the various massless fields in a fully dualized $\mathcal{N} \geq 1$, $D = 4$ supergravity theory obtained as compactification of $M$-theory on $X^7$ with Betti numbers $(b_0, b_1, b_2, b_3)$ [2].

| Field | $f$ | $360\tilde{A}_N$ | $\#$ |
|-------|-----|----------------|------|
| $g_{\mu\nu}$ | 2 | 848 | $b_0$ |
| $A_\mu$ | 2 | $-52$ | $b_1 + b_2$ |
| $\phi$ | 1 | 4 | $2b_3$ |
| $\psi_{\mu}$ | 2 | $-233$ | $b_0 + b_1$ |
| $\chi$ | 2 | 7 | $b_2 + b_3$ |

Two classes of $\mathcal{N} = 1$ theories are treated in Sec. 4. Subsec. 4.1 deals with consistent $\mathcal{N} = 1$ truncations of $\mathcal{N} = 2$ theories, and a general formula for $\Delta S_{\mathcal{N}=1}$ is obtained; this is the class of $\mathcal{N} = 1$ theories for which the general results derived in Sec. 3, specified for $\mathcal{N} = 1$, hold. Another class of $\mathcal{N} = 1$ theories, which we dub “minimally coupled”, is then considered in Subsec. 4.2, and the corresponding $\Delta S_{\mathcal{N}=1}^{\text{mc}}$ is computed; by performing a proper $\mathcal{N} = 0$ limit, the result $\Delta S_{\mathcal{N}=0}$, recently computed in [1], is recovered.

2 $\mathcal{N} = 2$ Multiplet Decomposition of $\mathcal{N} \geq 3$ Theories

A crucial step in the treatment given in [1] is the fact that $\Delta S_{\mathcal{N}=4} = 0$ (2.1) for any number $n$ of coupled matter (vector) massless multiplets.

The various types of $\mathcal{N} = 2$ massless multiplets will be referred to as $G_{\lambda_{\text{max}}}$, where $\lambda_{\text{max}}$ denotes the maximal helicity of the multiplet ($G \equiv G_2$, $G_{3/2}$, $G_V \equiv G_1$ and $G_H \equiv G_{1/2}$ respectively stand for the graviton, gravitino, vector and hyper multiplets), in Table 2 the multiplet content of any fully dualized $\mathcal{N} \geq 3$, $D = 4$ supergravity theory in terms of these building blocks is given (see e.g. [13, 14], and Refs. therein).

By denoting the contribution of $G$, $G_{3/2}$, $G_V$ and $G_H$ to the coefficient $\Delta S_{\mathcal{N}=2}$ (recall (1.9)) by $\Delta S_2$, $\Delta S_{3/2}$, $\Delta S_V$ and $\Delta S_H$ respectively, the general $\mathcal{N} = 4$ result (2.1) implies the following two relations:

$$\Delta S_V = -\Delta S_H;$$

$$2\Delta S_{3/2} = -\Delta S_2 - \Delta S_V.\quad (2.3)$$

Therefore, by using the decompositions reported in Table 2 as well as the results (1.8) (for self-mirror $\mathcal{N} = 2$ stu model), (2.1) and (8)

$$\Delta S_{\mathcal{N}=8} = -2,\quad (2.4)$$

one can compute the contribution of each type of massless $\mathcal{N} = 2$ multiplet to the total $\Delta S_{\mathcal{N}=2}$:

$$\Delta S_2 = \frac{23}{24}; \quad \Delta S_{3/2} = -\frac{11}{24}; \quad \Delta S_V = -\frac{1}{24}; \quad \Delta S_H = \frac{1}{24},\quad (2.5)$$

consistent with (1.7) and (2.2)-(2.3). Thus, by exploiting results (2.5), Table 2 allows one to compute the total $\Delta S_{\mathcal{N}}$ for all $\mathcal{N} \geq 3$ theories; in particular, the curiously simple result (1.10) for $\mathcal{N} \geq 3$ is obtained.
Table 2: Decomposition of the massless multiplet content of $N \geq 3$, $D = 4$ supergravities in terms of $N = 2$ multiplets. $n$ denotes the number of matter (vector) multiplets in $N = 3, 4$ matter coupled theories. (Massless) gravitino multiplets are not considered.

| $N$ | Decomposition |
|-----|----------------|
| 8   | $G + 6G_{3/2} + 15G_V + 10G_H$ |
| 6   | $G + 4G_{3/2} + 7G_V + 4G_H$ |
| 5   | $G + 3G_{3/2} + 3G_V + G_H$ |
| 4   | $G + 2G_{3/2} + G_V + n(G_V + G_H)$ |
| 3   | $G + G_{3/2} + n(G_V + G_H)$ |

3 General Relations between $A_N$, $\tilde{A}_N$ and $\Delta S_N$

In order to derive Eq. (1.12), one just needs to combine the results (1.10) (for $N \geq 3$), (2.5) (for $N = 2$) with the explicit computation of the coefficient $A_N$ of the on-shell gravitational trace anomaly of the fully undualized theories, whose field content is defined in the $M$-theoretical setting of [2] (see e.g. Table I therein). Nicely, the following simple and completely general formula is achieved:

$$A_N = 2\Delta S_N + N - 4.$$  \hspace{1cm} (3.1)

Note that, for (generalized) self-mirror theories, Eq. (3.1) consistently reduces to the result (1.10) (made explicit in (1.11)). Therefore, (3.1) is nothing but a generalization of (1.10) for completely generic theories. Note that (3.1) can actually be extended to include $N = 1$ theories obtained as truncation of $N = 2$ theories, which are treated in Subsec. 4.1 where the result (4.5) is derived.

Let us here recall that the coefficients $\tilde{A}_N$ of the on-shell gravitational trace anomaly of the fully dualized theories, in which only physical spin degrees of freedom are present, are computed in detail in App. A of [2]. By comparing $\tilde{A}_N$ with its undualized counterpart $A_N$, one obtains the simple and general relation

$$\tilde{A}_N - A_N = 3 - N,$$  \hspace{1cm} (3.2)

which, by using (3.1), finally yields the general result (1.12). Note that (3.2) is independent on the matter content of $N \leq 4$ theories.

For all (generalized) $N \geq 1$, $D = 4$ self-mirror theories, which all have $A_N = 0$ [2], (1.12) reduces to (1.13). Furthermore, for such theories it also holds

$$\tilde{A}_N = -\tilde{A}_{8-N} - 2,$$  \hspace{1cm} (3.3)

which is a consequence of the fermion symmetry displayed by Eq. (1.12). Eq. (3.3) can also be

\footnote{It should be pointed out that the quantity $K_0$ given by Eq. (7.3) of [1] is nothing but $\tilde{A}_N$ itself.}
summarized by the following symmetric pattern, centered at $N = 3$:

$$
\begin{array}{cccccccccc}
N : & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \\
\tilde{A}_N : & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
\end{array}
$$

providing an hint for a possible “generalized self-mirror” $N = 0$, $D = 4$ gravity theory with $\tilde{A}_{N=0} = 3$.

4  \( \mathcal{N} = 1 \) Theories with Extremal Black Holes

4.1  \( \mathcal{N} = 1 \) as Truncation of $\mathcal{N} = 2$

One can further decompose $\mathcal{N} = 2$ massless multiplets $\{G_{\lambda_{\text{max}}}\}$ in terms of the $\mathcal{N} = 1$ multiplets $\{g_{\lambda_{\text{max}}}\}$ ($\lambda_{\text{max}} = 2, 3/2, 1, 1/2$), obtaining (see e.g. $[15, 14]$, and Refs. therein)

$$
\begin{align*}
G &= g + g_{3/2}; \\
G_{3/2} &= g_{3/2} + g_V; \\
G_V &= g_V + g_c; \\
G_H &= 2g_c,
\end{align*}
$$

where $g \equiv g_2$, $g_{3/2}$, $g_V \equiv g_1$ and $g_H \equiv g_{1/2}$ stand for the graviton, gravitino, vector and chiral massless $\mathcal{N} = 1$ multiplets, respectively.

It should be stressed that our treatment of $\mathcal{N} = 1$ theories relies on at least four assumptions:

1. in order to display “large” extremal BH solutions, $\mathcal{N} = 1$ theories should at least contain one vector field : $n_V \geq 1$ [17];

2. for $n_c \geq 1$, an attractor dynamics takes place in the near-horizon geometry [17];

3. the results for $\mathcal{N} = 1$ theories can be derived from the $\mathcal{N} = 2$ ones in a purely kinematical way (i.e. by multiplet decomposition). In particular, fermionic bilinear terms coupled to 2-form field strengths (see e.g. [16]) should generally appear for our analysis to make sense;

4. the results on $\Delta S_{\mathcal{N} \geq 2}$ for “large” extremal BPS BHs can be used to derive results on $\Delta S_{\mathcal{N} = 1}$ of “large” extremal BHs in $\mathcal{N} = 1$ theories, in which there is no central extension of the local supersymmetry algebra, and thus no BPS notion, at all$^4$.

Also as a consequence of assumptions 1-4, we are therefore assuming that the kinematical consistent truncation procedure $\mathcal{N} = 2 \to \mathcal{N} = 1$ properly takes into account the corresponding change in the species of bilinear fermionic interaction terms with 2-form field strengths (as understood in Secs. 2 and 3; this issue does not arise for $\mathcal{N} \geq 2$-extended supergravities, which all have the same Lagrangian structure).

As discussed in [17], at least those $\mathcal{N} = 1$ theories obtained as consistent truncations of $\mathcal{N} = 2$ ones do satisfy the conditions of points 1 and 2.

By using Eqs. (2.5) and (4.1)-(4.4), the contribution of each $\mathcal{N} = 1$ multiplet to the coefficient $\Delta S_{\mathcal{N} = 1}$ (recall (1.9)) can be computed; denoting the contribution of $g$, $g_{3/2}$, $g_V$ and $g_c$ to $\Delta S_{\mathcal{N} = 1}$ by $\Delta s_2$, $\Delta s_{3/2}$, $\Delta s_V$ and $\Delta s_c$ respectively, one obtains

$$
\begin{align*}
\Delta s_2 &= \frac{65}{48}, \\
\Delta s_{3/2} &= -\frac{19}{48}, \\
\Delta s_V &= -\frac{3}{48}, \\
\Delta s_c &= \frac{1}{48},
\end{align*}
$$

$^4$Short $\mathcal{N} = 2$ BPS massive multiplets are the same as $\mathcal{N} = 1$ massive multiplets; for example, a massive hypermultiplet is the same as a massive charged chiral multiplet [13]. Thus, the multiplet structure of $\mathcal{N} = 2$ BPS BHs is the same as $\mathcal{N} = 1$ (necessarily non-BPS) BHs [17].
thus yielding the general formula:

\[ \Delta S_{N=1} = \frac{1}{48} (65 + n_c - 3n_V). \]  

(4.6)

For generalized self-mirror \( N = 1 \) theories \( (n_c = 3n_V + 7) \) \[2\], Eq. (4.6) yields

\[ \text{gen. self-mirror : } \Delta S_{N=1} = \frac{3}{2}. \]  

(4.7)

consistent with the \( N = 1 \) case of Eq. (1.10).

4.2 “Minimally Coupled” \( N = 1 \)

On the other hand, (at least) another class of \( N = 1 \), \( D = 4 \) theories, complementary to the one discussed above, can be considered. Such a class, which we will dub “minimally coupled” (mc), cannot be obtained as consistent truncation of \( N = 2 \) theories, and its kinetic vector matrix is constant: \( f_{IJ} \sim \delta_{IJ} \) \((I,J = 1,\ldots,n_V \geq 1)\). This implies that the complex scalar fields from the \( n_c \) chiral multiplets are not involved in an attractor dynamics in the near horizon geometry of the “large” extremal BH under consideration\[4\], which at bosonic level can thus be regarded as a Reissner-Nördstrom (RN) extremal BH coupled to a set of spectator scalar fields and uncharged vectors.

For “minimally coupled” \( N = 1 \) theories, the contributions to \( \Delta S'_{N=0} \) split into two parts:

1. The \( N = 1 \) supersymmetrization \( \Delta S_{N=1}^{RN} \) of the RN contribution \( \Delta S_{RN}^{extr} \), which is composed by an \( N = 1 \) graviton multiplet and an \( N = 1 \) vector multiplet. By making use of Eq. (4.5), the resulting contribution to the logarithmic correction coefficient can be computed to be:

\[ \Delta S_{N=1}^{RN} = \Delta s_2 + \Delta s_V = \frac{31}{24}. \]  

(4.8)

This \( N = 1 \) supersymmetrization of the RN contribution can also be justified by observing that \( N = 3 \) “pure” supergravity \[18, 19\] displays a \((\frac{1}{3})\)BPS extremal dyonic RN BH solution, with entropy \[20\]

\[ S_0 = \frac{\pi}{2} \sum_{i=1}^{3} \left[ (p_i)^2 + q_i^2 \right]. \]  

(4.9)

Since there are no scalars, from this system one can derive two \( N < 3 \) Maxwell-Einstein systems, namely \( N = 2 \) “pure” supergravity \[21\] and \( N = 1 \) supergravity coupled to 1 vector multiplet \[22\], the two theories being related by exchanging one gravitino with one gaugino (with related interactions; see also e.g. the discussion in \[17\]).

2. The \( N = 1 \) supersymmetrization \( \Delta S_{N=1}^{GB} \) of the Gauss-Bonnet (GB) contribution \( \Delta S_{GB}^{extr} \), which is a well-defined, independent invariant in \( N = 1 \), \( D = 4 \) superspace \[23\]. We start from the non-supersymmetric expression (see e.g. \[1\], and Refs. therein)

\[ \Delta S_{GB} = -\frac{1}{360} \left( n_s + 62n_V + \frac{11}{2}n_{MF} \right), \]  

(4.10)

where \( n_s, n_V \) and \( n_{MF} \) respectively denote the number of real scalar, vector and \( \lambda = 1/2 \) Majorana spinor massless fields. By recalling the helicity content of \( N = 1 \) massless multiplets, it is immediate to re-express the right-hand side of (4.10) in terms of only \( n_c \) and \( n_V \), where the latter now stands for the number of \( N = 1 \) massless vector multiplets other than the one

\[ ^{5}\text{In fact, they behave as hypermultiplets' scalars in } N = 2 \text{ theories.} \]
contained in the $\mathcal{N} = 1$ supersymmetrization of RN term. Thus, one obtains the consistent $\mathcal{N} = 1$ supersymmetrization of $\Delta S_{\text{GB}}$:

$$\Delta S_{\mathcal{N}=1}^{\text{GB}} = -\frac{1}{360} \left(6n_c + \frac{135}{2}n_V\right).$$  \hspace{1cm} (4.11)

Summing all up, in the class of $\mathcal{N} = 1$ theories under consideration, the total contribution to the logarithmic correction coefficient reads as follows ($n_V \geq 0$):

$$\Delta S_{\mathcal{N}=1}^{\text{mc}} = \Delta S_{\mathcal{N}=1}^{\text{RN \ extr}} + \Delta S_{\mathcal{N}=1}^{\text{GB}} = -\frac{1}{360} \left(-465 + 6n_c + \frac{135}{2}n_V\right),$$ \hspace{1cm} (4.12)

which is completely different from Eq. (4.6).

Note that in “minimally coupled” $\mathcal{N} = 1$ supergravities the vector multiplets participating in the $\mathcal{N} = 1$ supersymmetrization of the RN term stands on a different footing with respect to the other $n_V$ vector multiplets.\footnote{The Noether supercurrent coupling to the gravitino \cite{22} $J_\mu^\alpha \bar{\psi}_\alpha$ introduces a fermionic bilinear term proportional to the flux of the RN vector field (see also \cite{13}). Note that only the gaugino of the $\mathcal{N} = 1$ vector multiplet of the $\mathcal{N} = 1$ RN term interacts with the gravitino; indeed, all other vector fields are minimally coupled, and they have vanishing fluxes of the corresponding 2-form field strengths. This explains why the $J \bar{\psi} \sim \text{Re}(\bar{\chi}\psi F)$ interactions do not contribute to the $\mathcal{N} = 1$ supersymmetrization of the GB term (discussed at point 2 above).}

Furthermore, by construction, the $\mathcal{N} = 0$ limit of the expression (4.12) corresponds to the $\Delta S_{\mathcal{N}=0}$ given by Eq. (1.3) of \cite{1}; indeed, when $f_{IJ} \sim \delta_{IJ}$, by setting $\psi = 0$ all bilinear fermionic terms coupled to the 2-form vector field strengths vanish. In particular, the RN limit of (4.12), which amounts to setting $n_c = n_V = 0$ and to removing the gravitino and gaugino contained in the $\mathcal{N} = 1$ supersymmetric RN term, correctly yields the non-supersymmetric RN contribution \cite{1}

$$\Delta S_{\text{RN \ extr}}^{\text{RN}} = -\frac{241}{90}.$$  \hspace{1cm} (4.13)

Finally, let us shortly comment on the physical significance of our results.

For generic theories, the result (1.12) expresses the fact that the entropy correction is the same as the (on shell) gravitational anomaly, up to an universal shift. On the other hand, for (generalized) self-mirror theories, Eq. (1.10) implies that $\Delta S_{\mathcal{N}}$ is odd under the fermionic symmetry $\mathcal{N} \rightarrow 8 - \mathcal{N}$, and that it is given by the lowest helicity component of the gravity multiplet, namely $\lambda_{\text{min}} = 2 - \mathcal{N}/2$. Thus, Eqs. (1.9) and (1.10) yield the following correction to the Bekenstein-Hawking entropy-area formula of (generalized) self-mirror theories:

$$S = \frac{A_H}{4} + (2 - \mathcal{N}/2) \ln (A_H^2).$$ \hspace{1cm} (4.14)

This is universal, because it only depends on $\mathcal{N}$, and it increases or decreases the classical Bekenstein-Hawking entropy depending on $\mathcal{N} < 4$ or $\mathcal{N} > 4$ (it is vanishing for $\mathcal{N} = 4$).

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