Optimal quantum resource distribution in quantum dense metrology

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Abstract

Quantum entanglement can engineer the statical distribution of photons and then lead to the enhancement of measurement sensitivity. Quantum emitter, i.e., optical parametric amplifier (OPA), is the device to produce entanglement. However, the generated entanglement couldn’t be infinite, and therefore it is indispensable to distribute quantum resources to the measured observable reasonably. Quantum dense coding and metrology are proposed to equally divide quantum resource to two conjugate physical quantities for achieving the joint measurement of multiple observable. Here we present a variation of quantum dense metrology (QDM), in which we place two degenerate optical parametric amplifiers (DOPAs) at the unused port of input and dark of output port of a linear interferometer respectively. We achieve the quantum enhancement at arbitrary quadrature through reasonably controlling the angle in phase-sensitive amplifiers to create what is called SU(1,1) interferometry (SUI). Profit from the advantage of loss tolerant from SUI, we can separate the output of the second DOPA into two beams and achieve QDM with quantum resource distribution at any desired ratios.
I. INTRODUCTION

Phase is an extremely significant physical quantity in sensing the information from the external environment. Optical interferometer was invented for phase measurement hundreds of years ago and has become the most precise instrument in the world[1]. Technical noise is the dominant source, which ultimately limits such type devices to high absolute sensitivity for a long time. With the development of technology, this kind of noise is massively reduced. Thus interferometers are not limited by technical noise but instead defined by the classically statistical distribution of photons, named photon shot noise[2].

The phase uncertainty of an interferometer with classical resources is bounded by standard quantum limit (SQL) that scales as $1/\sqrt{N}$. The phase sensitivity in principle can be infinite through increasing the photon number $N$ inside the interferometer. However, too strong power of light inside interferometer will cause various problems, e.g., The Laser Interferometer Gravitational-Wave Observatory (LIGO) suffers the radiation pressure noise from ultra-high power light hitting the mirrors[2]. In the field of biology physics, high intensity of probe light is not allowed since the sample is usually intolerable to excessively powerful energy[3].

A possible way to keep the high absolute sensitivity at a relatively low light intensity is to engineer the distribution of photons, i.e., the vacuum fluctuation of a classical interferometer can be reduced effectively through replacing the unused input port with a single-mode squeezed light and leading to a higher sensitivity[3]. In the past, squeezed light was used in the quantum enhancement of a single parameter[5–7]. In recent years, two-mode squeezed light is demonstrated to be capable of embedding two or more non-commuting observables in information with precision beyond the standard quantum limit (SQL), which are named quantum dense coding and metrology in the area of quantum information science and quantum metrology respectively[8–11].

An alternate way to achieve quantum-enhanced sensitivity is noiselessly amplifying the signal[12–14]. A well-known device is SU(1,1) interferometer (SUIR), which is firstly proposed by Yurke et al. more than thirty years ago[15]. Recently, SUIRs are experimentally demonstrated to perform a quantum-enhanced phase measurement and simultaneously possess detection loss tolerance advantage compare to the conventional squeezed light interferometer[16–19]. Nowadays, SUIR is demonstrated to be capable of measuring multiple
noncommuting parameters and beating the standard quantum limit simultaneously\cite{9,11}. Although SUI has achieved such a lot of merit, compared to the conventional interferometer, however, all of the improvement are based on quantum light source as the phase sensing field. The low conversion efficiency of nonlinear media result in low phase sensing light intensity ($I_{ps}$). Thus the absolute sensitivity of such type interferometer is still not comparable to classical interferometer.

A prospective scheme to boost the $I_{ps}$ in SUIR is not treating parametric amplifier (PA) as splitting and recombination elements but regarding it as a kind of entangled source to engineer the photon distribution in a linear system. Caves firstly propose such configuration aim to deal with the detection loss problem in laser gravitational-wave detectors\cite{2}. In this paper, we consider a more practical scheme, embedding a linear interferometer between PA1 and PA2. We will analyze the performance of QDM through utilizing SU(1,1) interferometry (both degenerate and non-degenerate condition). Moreover, we will compare the performance of SUIR to conventional QDM.

II. LINEAR INTERFEROMETER WITH QUADRATURE MEASUREMENT

A. Direct quadrature measurement

An optical field can be described in a rotation frame and composed of two non-commuting observables. The quantum state can be written as $\hat{a} = \hat{X}_1 + i\hat{X}_2$, where $\hat{X}_1$ and $\hat{X}_2$ are quadrature amplitude and phase operators respectively\cite{20}. To acquire all of the information coded in the optical field, the most common way is utilizing optical interference, so-called homodyne detection (HD), to read the light in any direction of quadrature. Fig. 1a shows the configuration of balanced homodyne detection, the strength of differential photocurrents is proportional to the quadrature of the probe beam: $\hat{X}(\theta) = \hat{a}_{in} e^{-i\theta} + \hat{a}_{in}^{\dagger} e^{i\theta}$, where $\theta$ is the relative phase between the probe light and LO. If we apply a weak phase modulation of $\delta \ll 1$ and weak amplitude modulation of $\epsilon \ll 1$ for coherent light, the output field can be expressed as $\hat{a} = \hat{a}_{in} e^{i\delta - \epsilon} \approx \hat{a}_{in} (1 + i\delta - \epsilon)$. The noise of a coherent state is $\langle \Delta \hat{X}^2(\theta) \rangle = 1$, which is independent to the phase of LO. The signal is defined by $\langle \hat{X}(\theta) \rangle^2 = 4e^{-2\epsilon} \cos^2(\theta - \delta) I_{ps}$, where $I_{ps} \equiv |\alpha^2|$ is the intensity of light used to probe the signal. Obviously we find the maximal SNR for the weak phase modulation $\delta$ is $4I_{ps}\delta^2$ when $\theta = (n + 1)\pi/2$ and $n \in \mathbb{Z}$,
corresponding to phase quadrature $X_2$. And the maximal SNR of the weak amplitude modulation $\epsilon$ is $4I_{ps}\epsilon^2$ when $\theta = n\pi$ corresponding to amplitude quadrature $X_1$.

**B. Quadrature measurement in MZI**

Although HD is, in principle, sensitive to the variance of phase and amplitude, however, it is not a good candidate to work as a sensor. To achieve quadrature measurement should satisfy the power of LO much higher than the probe, which makes the system very difficult to reach a high absolute sensitivity (high $I_{ps}$). Interferometer is the most well-known device to sense the variance of phase. In this section, we will analyze the SNR of an ordinary MZI with a variable beam splitter (VBS). The input states are shown in Fig. 1 where $\hat{a}_{in}$ is a coherent state and $\hat{b}_{in}$ is a vacuum state. Here we give the input-output relation of the BS in Fig. 1:

$$\hat{A} = \sqrt{T_1}\hat{a}_{in} + \sqrt{R_1}\hat{b}_{in}, \quad \hat{B} = \sqrt{T_1}\hat{b}_{in} - \sqrt{R_1}\hat{a}_{in}$$

$$\hat{b}_{out} = \sqrt{T_2}\hat{B}e^{-\epsilon^2}e^{i\phi} + \sqrt{R_2}\hat{A}, \quad \hat{a}_{out} = \sqrt{T_2}\hat{A} - \sqrt{R_2}\hat{B}e^{-\epsilon^2}e^{i\phi} \tag{1}$$

So for Fig. 1b, we have the input-output relation
\[ \hat{a}_{\text{out}} = (\sqrt{T_1 T_2} + \sqrt{R_1 R_2} e^{-\epsilon e^{i\varphi}}) \hat{a}_{\text{in}} + (\sqrt{R_1 R_2} - \sqrt{T_1 T_2} e^{-\epsilon e^{i\varphi}}) \hat{b}_{\text{in}} \]
\[ \hat{b}_{\text{out}} = (\sqrt{T_1 R_2} - \sqrt{R_1 T_2} e^{-\epsilon e^{i\varphi}}) \hat{a}_{\text{in}} + (\sqrt{R_1 R_2} + \sqrt{T_1 T_2} e^{-\epsilon e^{i\varphi}}) \hat{b}_{\text{in}} \] (2)

These are the operator relations for a linear interferometer. We will engineer the input state differently and analyze the SNR performance for phase and amplitude modulated signal. Now we give the expression of observable in an interferometer with homodyne measurement (HD):

\[ X \equiv X(\theta) = e^{-i\theta} \hat{\gamma} + e^{i\theta} \hat{\gamma}^\dagger \] (3)

Where \( \theta \) is the relative phase between the local oscillator and the \( \hat{\gamma} \) represent the field to be measured. For a interferometer, we choose the quadrature of phase \( (X_2 = X(\pi/2) = i(\hat{\gamma}^\dagger - \hat{\gamma})) \) as the physical quantity for our sensitivity analysis. Then we give a general definition of SNR:

\[ \text{SNR} \equiv \frac{\langle \Delta X \rangle^2}{\langle \Delta^2 X \rangle} = \frac{\langle \Delta \varphi \rangle^2}{\langle \Delta^2 X \rangle} \frac{\partial X}{\partial \varphi}^2 = \frac{\langle \Delta X \rangle^2}{\langle \Delta^2 X \rangle} \] (4)

Note \( \varphi = \varphi_0 + \delta \varphi \) where \( \varphi_0 \) is the set point of interferometer, usually at dark fringe \( (\varphi_0 \rightarrow \pi) \) in our case. And \( \delta \varphi \) is the weak phase shift we want to measure. For the observable from HD, the signal referred to the output is \( \langle \Delta X \rangle = \langle X(\varphi_0 + \Delta \varphi) \rangle - \langle X(\varphi_0) \rangle \).

Noise is \( \langle \Delta^2 X \rangle \) which is evaluated at \( \varphi = \varphi_0 \). Then we can give the optimal SNR of MZI at phase and amplitude quadrature when the interferometer is locked at dark fringe:

\[ \text{SNR}_{\hat{a}_{\text{out}}} (\hat{X}_1) = 4R_2 I_{ps} \epsilon^2, \text{SNR}_{\hat{a}_{\text{out}}} (\hat{X}_2) = 4R_2 I_{ps} \delta^2 \]
\[ \text{SNR}_{\hat{b}_{\text{out}}} (\hat{X}_1) = 4T_2 I_{ps} \epsilon^2, \text{SNR}_{\hat{b}_{\text{out}}} (\hat{X}_2) = 4T_2 I_{ps} \delta^2 \] (5)

Where \( I_{ps} = R_1 \alpha^2 \). From Fig. [1] easily find, the interferometer need to satisfy the symmetry condition: \( R_1 = R_2, T_1 = T_2 \) to allow it’s capable of arriving complete dark fringe. And when \( T_2 \rightarrow 1 \), we get the optimal SNR at the dark port. Different from using HD measure a beam directly, the SNR varies with the transmission of the second VBS since part of the
information is separated into another output of the interferometer. Note the optimal SNR is not from the quantum enhancement, but profit from the unbalanced splitting ratio \((T_1 = T_2 \rightarrow 1)\) helps us distribute the information between the two output port of interferometer and devote all the resource into one detected port. Therefore the sum of two outputs of the interferometer for phase and amplitude quadrature are \(4I_{ps}\delta^2\) and \(4I_{ps}\epsilon^2\) respectively, which are equal to the SNR of single beams with HD as shown in Eq. [5]. However, the balanced condition is still the most practical scheme due to the very small phasing sensing light intensity \((I_{ps} = R_1\alpha^2, R_1 \rightarrow 0)\) in an unbalanced condition. So we will set the linear interferometer at a balanced condition \((R = T = 1/2)\) in the next part.

III. QUANTUM ENHANCED LINEAR INTERFEROMETER WITH QUADRATURE MEASUREMENT

It is well-known, squeezed light can be used to replace the unused injected port of a linear interferometer to suppress the vacuum noise and increase the SNR in quadrature measurement. Recently, quantum dense metrology (QDM) was demonstrated to be capable of measuring two non-orthogonal quadrature and maintain the quantum enhancement simultaneously. Later, SUIR was theoretically proposed and experimentally investigated the ability to detect multiple quadrature phase amplitude at an arbitrary angle with quantum enhanced precision. In this section, we will analyze the performance of joint measurements in different schemes. Firstly, we will review the original scheme of QDM from Steinlechner et.al. Then we will apply vacuum injected SUI (consider both degenerate and non-degenerate configuration) to linear interferometers and compare the advantages of SNR enhancement and joint measurement.

A. Quantum dense metrology

The most straightforward method to split one observable into two is using a linear BS, as we have talked above. However, the vacuum noise injected into the unused port that will decrease the SNR. The basic idea of QDM is encoding the information at one beam of two-mode squeezed light (TMS) and utilizing the twin beams to cancel the correlated noise at the BS used to split the signal, thus avoiding the vacuum noise involved.
The configuration of QDM is shown in Fig. 2a. The input state of MZI’s unused port is replaced by one of the TMS. We use a non-degenerate optical amplifier (NOPA) to generate TMS, here we give the input-output relation of NOPA:

\[ \hat{b} = G\hat{b}_0 + g\hat{a}_0^\dagger, \hat{C} = G\hat{a}_0 + g\hat{b}_0^\dagger \] (6)

Where the input state \( \hat{a}_0 \) and \( \hat{b}_0 \) are vacuum state. Considering the input-output relation Eq. 2 then we give the output of QDM:

\[ \hat{d}_1 = \frac{1}{\sqrt{2}} e^{i\phi} (G\hat{a}_0 + g\hat{b}_0^\dagger) - \frac{1}{2\sqrt{2}} \left[ (1 + e^{-e^{-\epsilon}} e^{i\phi})(G\hat{b}_0 + g\hat{a}_0^\dagger) + (1 - e^{-e^{-\epsilon}} e^{i\phi})\hat{a}_m \right] \]
\[ \hat{d}_2 = \frac{1}{\sqrt{2}} e^{i\phi} (G\hat{a}_0 + g\hat{b}_0^\dagger) + \frac{1}{2\sqrt{2}} \left[ (1 + e^{-e^{-\epsilon}} e^{i\phi})(G\hat{b}_0 + g\hat{a}_0^\dagger) + (1 - e^{-e^{-\epsilon}} e^{i\phi})\hat{a}_m \right] \] (7)
The first two terms are correlated noise, which can be canceled at a balanced BS ($T = R \to 1/2$) when phase $\phi$ is set on an optimal position (Here we set $\phi \to 0$). When the MZI is locked at dark fringe ($\varphi \to 2n\pi, n \in \mathbb{Z}$), obviously the minimal noise for HD1 and HD2 are when the phase of LO locked at $\theta_1 \to 0$ (amplitude quadrature) and $\theta_2 \to \pi/2$ (phase quadrature) simultaneously. Then we give the optimal noise performance of the two HD:

$$\langle \Delta^2 \hat{X}_{1,d_1} \rangle = \frac{1}{(G + g)^2}$$

$$\langle \Delta^2 \hat{X}_{2,d_2} \rangle = \frac{1}{(G + g)^2}$$

(8)

The vacuum noise is suppressed by the correlation from EPR entanglement. The term with $\hat{a}_{in}$ are signal, we give the optimal power of signal,

$$\langle \hat{X}_{1,d_1} \rangle^2 = I_{ps}\epsilon^2, \langle \hat{X}_{2,d_2} \rangle^2 = I_{ps}\delta^2$$

(9)

Note the power splitting from BS make us losing half of the information compare to conventional MZI. Finally the QDM give the quantum enhanced SNR for phase and amplitude quadrature:

$$SNR_{QDM}(\hat{X}_{1,d_1}) = (G + g)^2 I_{ps}\epsilon^2,$$

$$SNR_{QDM}(\hat{X}_{2,d_2}) = (G + g)^2 I_{ps}\delta^2$$

(10)

B. QDM with non-degenerate SU(1,1) interferometry

In this section, we replace the BS used to cancel the correlated noise with a non-degenerate optical parametric amplifier (NOPA) in Fig. 2b. The correlated noise can be canceled out at NOPA2 due to destructive quantum interference, also called SU(1,1) interferometry, which will help us noiseless amplify the signal from MZI at each output of SUI. The input-output relation can be described as

$$\hat{d}_1 = G_2 e^{i\phi}(G_1\hat{b}_0 + g_1\hat{a}_0^\dagger) + \frac{g_2}{2}[(1 + e^{-\epsilon}e^{i\varphi})(G_1\hat{b}_0 + g_1\hat{a}_0^\dagger) + (1 - e^{-\epsilon}e^{i\varphi})\hat{a}_{in}]^\dagger$$

$$\hat{d}_2 = g_2 e^{i\phi}(G_1\hat{b}_0 + g_1\hat{a}_0^\dagger)^\dagger + \frac{G_2}{2}[(1 + e^{-\epsilon}e^{i\varphi})(G_1\hat{b}_0 + g_1\hat{a}_0^\dagger) + (1 - e^{-\epsilon}e^{i\varphi})\hat{a}_{in}]^\dagger$$

(11)
The optimal strength of the two observables at HD1 and HD2 are given by

\[ \left\langle \hat{X}_{1,d_1} \right\rangle^2 = 2G_2^2 I_{ps} \epsilon^2; \left\langle \hat{X}_{2,d_2} \right\rangle^2 = 2g_2^2 I_{ps} \delta^2 \]  \tag{12}

Where \( I_{ps} = |\alpha|^2 / 2 \). The noise can be optimized through adjusting relative phase between the pump and the two input state of NOPA2, we find the point of quantum destruction interference is when phase \( \phi \) is set at \( \pi \). Then we give the minimum noise of output:

\[ \left\langle \Delta \hat{X}_{1,d_1}(\theta_1) \right\rangle^2 = \left\langle \Delta \hat{X}_{2,d_2}(\theta_2) \right\rangle^2 = (G_2 G_1 - g_1 g_2)^2 + (G_1 g_2 - G_2 g_1)^2 \]  \tag{13}

Compare to conventional QDM scheme, the phase of LO must be locked at a specific angle to maximal the SNR. In the configuration of SUI, the noise is independent to the quadrature phase angle \( \theta_1 \) and \( \theta_2 \). According to Eqs. 12 and 13, we have the maximal SNRs of phase and amplitude quadrature simultaneously measured at HD1 and HD2:

\begin{align*}
SNR_{NSUI}(\hat{X}_{1,d_1}) &= \frac{2G_2^2 I_{ps} \epsilon^2}{(G_2 G_1 - g_1 g_2)^2 + (G_1 g_2 - G_2 g_1)^2} \\
SNR_{NSUI}(\hat{X}_{2,d_2}) &= \frac{2g_2^2 I_{ps} \delta^2}{(G_2 G_1 - g_1 g_2)^2 + (G_1 g_2 - G_2 g_1)^2}
\end{align*}

\tag{14}

When \( G_2 \to \infty \), we rewrite the SNR as

\begin{align*}
SNR_{DSUI}(\hat{X}_{1,d_1}) &= (G_1 + g_1)^2 I_{ps} \epsilon^2 \\
SNR_{DSUI}(\hat{X}_{2,d_2}) &= (G_1 + g_1)^2 I_{ps} \delta^2
\end{align*}

\tag{15}

C. QDM with degenerate SU(1,1) interferometry

In the last section, we found the joint measurement through the method in A and B will lead enhanced factors \( G \) and \( g \) to corresponding quadrature modulation simultaneously. However, both the scheme in Fig. 2a and b split the quantum resource to roughly half, and devote half to phase quadrature and another half to amplitude quadrature. In the practical operation of interferometer, the phase quadrature is usually more critical than amplitude quadrature since the phase is the physical quantity to be measured, and amplitude signal
is the spurious signal in a system need to be removed. Therefore it’s better to devote more quantum source to the quadrature of phase than amplitude. Here we propose to use a degenerate optical parametric amplifier (DOPA) to achieve joint measurement. Profit from the DOPA, which can be regarded as a phase-sensitive amplifier, we can selectively distribute quantum source to phase and amplitude modulation measurement. Benefit by the advantages of loss tolerant from SU(1,1) interferometry, and we place a BS at the output of the second DOPA to achieve QDM, as shown in Fig. 2 c. Firstly, we give the input-output relation of DOPA

$$\hat{\zeta}_{\text{out}} = G\hat{\zeta}_{\text{in}}e^{i\psi} + e^{-i\phi}\hat{\zeta}_{\text{in}}^\dagger e^{-i\psi}$$

(16)

Where $\hat{\zeta}_{\text{in}}$ and $\hat{\zeta}_{\text{out}}$ are the input and output state of DOPA, $G$ and $g$ are the gain factor, which satisfy $G^2 - g^2 = 1$. The output noise of DOPA, different from NOPA, which is
dependent on the initial phase of input state $\psi$ and the phase $\phi$ inside DOPA.

Then the input-output relation of QDM with degenerate SU(1,1) interferometry can be described as

$$\hat{d}_1 = \frac{1}{\sqrt{2}} \hat{v} - \frac{1}{\sqrt{2}} (G_2 \hat{b}_{out} + e^{i\phi_2} g_2 \hat{b}_{out}^\dagger)$$
$$\hat{d}_2 = \frac{1}{\sqrt{2}} (G_2 \hat{b}_{out} + e^{i\phi_2} g_2 \hat{b}_{out}^\dagger) + \frac{1}{\sqrt{2}} \hat{v}$$

(17)

Where $\hat{b}_{out} = [(1 + e^{-\epsilon} e^{i\phi})(G_1 \hat{b}_0 e^{i\psi_1} + e^{i\phi_1} g_1 \hat{b}_0^\dagger e^{-i\psi_1}) + (1 - e^{-\epsilon} e^{i\phi}) \hat{a}_{in}] e^{i\psi_2}/2$. The optimal strength of the two observables at HD1 and HD2 are given by

$$\langle \hat{X}_{1,d_1} \rangle^2 = \frac{1}{2} (G_2 \cos(\psi_2) + g_2 \cos(\psi_2 - \phi_2))^2 I_{ps} \epsilon^2,$$
$$\langle \hat{X}_{2,d_2} \rangle^2 = \frac{1}{2} (G_2 \sin(\psi_2) + g_2 \sin(\psi_2 - \phi_2))^2 I_{ps} \delta^2$$

(18)

Where $I_{ps} = |\alpha|^2/2$. (a)-(d) in Fig. 3 we give the evolution of state under phase-space representation. (a) show the initial vacuum state; (b) DOPA1 unsqueeze the vacuum state at the selective direction of quadrature; (c) the SU(2) interferometer encoding a weak phase signal $|\alpha|\delta$ in the input beam; (d) DOPA2 squeeze the selective direction of quadrature and noiseless amplify the signal. For a DOPA with vacuum injection, the noise level of output is $G^2 + g^2 + 2Gg \cos(2\theta - \phi)$. We find the noise is determined by the relative phase $\phi$ between the pumps and measured phase of quadrature $\theta$. Here we give a differential phase $\Theta$, and rewrite the noise with $G^2 + g^2 + 2Gg \cos(2\theta - \phi + \Theta)$. As shown in Fig. 3 f and g, we set $\Theta = 0$ and $\pi$, then result in DOPA1 and DOPA2 unsqueezing and squeezing at the chosen quadrature respectively. Finally we give the minimum noise of output:

$$\langle \Delta \hat{X}_{1,d_1}(\theta_1) \rangle^2 = \langle \Delta \hat{X}_{2,d_2}(\theta_2) \rangle^2 = \frac{1}{2} [(G_2 G_1 - g_1 g_2)^2 + (G_1 g_2 - G_2 g_1)^2] + \frac{1}{2}$$

(19)

Compare to conventional QDM scheme, the phase of LO must be locked at a specific angle to maximal the SNR. In the configuration of SUI, the noise is independent to the quadrature phase angle $\theta_1$ and $\theta_2$. According to Eqs. 18 and 19, we have the maximal SNRs of phase and amplitude quadrature simultaneously measured at HD1 and HD2.
FIG. 4. Quantum resource distribution of QDM. Green line show the two output port of original and non-degenerate SU(1,1) QDM. Violet and blue lines represent the two output port of degenerate SU(1,1) QDM. In the calculation, \( G_1 = 2 \), \( g_1 = \sqrt{3} \), \( G_2 = 10 \), \( g_2 = \sqrt{99} \) and \( I_{ps} \delta^2 = I_{ps} \epsilon^2 = 1/2 \). Black line is the standard quantum limit (SQL).

\[
\begin{align*}
SNR_{DSU1}(\hat{X}_{1,d_1}) &= \frac{(G_2 \cos(\psi_2) + g_2 \cos(\psi_2 - \phi_2))^2 I_{ps} \epsilon^2 / 2}{1/2[(G_2 G_1 - g_1 g_2)^2 + (G_1 g_2 - G_2 g_1)^2] + 1/2} \\
SNR_{DSU1}(\hat{X}_{2,d_2}) &= \frac{(G_2 \sin(\psi_2) + g_2 \sin(\psi_2 - \phi_2))^2 I_{ps} \delta^2 / 2}{1/2[(G_2 G_1 - g_1 g_2)^2 + (G_1 g_2 - G_2 g_1)^2] + 1/2}
\end{align*}
\]  

(20)

When \( G_2 \to \infty \) and \( \phi_2 \to n\pi \), \( n \in \mathbb{Z} \), we rewrite the SNR as

\[
\begin{align*}
SNR_{DSU1}(\hat{X}_{1,d_1}) &= 2 \cos^2(\psi_2) (G_1 + g_1)^2 I_{ps} \epsilon^2 \\
SNR_{DSU1}(\hat{X}_{2,d_2}) &= 2 \sin^2(\psi_2) (G_1 + g_1)^2 I_{ps} \delta^2
\end{align*}
\]

(21)

In Fig. 4, we plot the comparison of different QDM methods. Green lines show original QDM and non-degenerate SU(1,1) QDM distribute quantum resource equally to the two output port, therefore result in equal SNR. Violet and blue lines represent the two output port of degenerate SU(1,1) QDM. Obviously, we can optimize the ratio of quantum resource distribution through detuning the phase \( \psi_2 \). We note the total of output quantum enhancement, in all the methods above, always equal to the input quantum resource from the initial parametric amplifier, which satisfy the principle of quantum resource distribution.
IV. CONCLUSION

In summary, we have theoretically investigated the quantum resource distribution at any desired quadrature and joint measurement of multiple noncommuting observables. We find degenerate SU(1,1) interferometry is capable of devoting all of the quantum resources to a selected quadrature, also called noiseless amplification. The projection of the noiseless amplified signal results in quantum enhancement sensitivity of multiple noncommuting observables. We note a recent work from LIGO Collaboration[22], they experimentally demonstrated a joint quantum enhancement in phases of laser beams and positions of mirrors through choosing proper squeezed angle of the initial squeezer. The basic idea is also devote quantum resource partly to each quadrature. The difference is we focus on simultaneously measurement of multiple observables in this paper. We take advantage of loss tolerance from SU(1,1) interferometry, and split the output field while almost maintaining the same SNR, and thus measure multiple noncommuting observables simultaneously.

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[1] The LIGO Scientific Collaboration, LIGO: The Laser Interferometer Gravitational-Wave Observatory, Science 256, 325 (1992).

[2] C. M. Caves, Quantum-mechanical noise in an interferometer, Phys. Rev. D. 23, 1693 (1981).

[3] M. A. Taylor and W. P. Bowen, Quantum metrology and its application in biology, Physics Reports. 615, 1-59 (2016).

[4] R. Schnabel, Squeezed states of light and their applications in laser interferometers, Physics Reports. 684, 1-57 (2017).

[5] Min Xiao, L.A. Wu, and H.J. Kimble, Precision measurement beyond the shot-noise limit, Phys. Rev. Lett. 59, 278 (1987).

[6] The LIGO Scientific Collaboration, A gravitational wave observatory operating beyond the quantum shot-noise limit, Nature Physics. 7, 962 (2011).

[7] Hidehiro. Yonezawa, Daisuke Nakane, Trevor A. Wheatley, Kohjiro Iwasawa, Shuntaro Takeda, Hajime Arao, Kentaro Ohki, Koji Tsumura, Dominic W. Berry, Timothy C. Ralph, Howard M. Wiseman, Elanor H. Huntingdon, Akira Furusawa, Quantum-enhanced optical phase tracking, Science 337, 1514 (2012).

[8] X. Li, Q. Pan, J. Jing, J. Zhang, C. Xie, and K. Peng, Quantum dense coding exploiting a bright Einstein-Podolsky-Rosen beam, Phys. Rev. Lett. 88, 047904 (2002).

[9] S. Steinlechner, J. Bauchrowitz, M. Meinders, H. Müller-Ebhardt, K. Danzmann, and R. Schnabel, Quantum dense metrology, Nature Photonics. 7, 626 (2013).

[10] J. Li, Y. Liu, L. Cui, N. Huo, S. M. Assad, X. Li, and Z.Y. Ou, Quantum dense coding exploiting a bright Einstein-Podolsky-Rosen beam, Phys. Rev. A. 97, 052127 (2018).

[11] Y. Liu, J. Li, L. Cui, N. Huo, S. M. Assad, X. Li, and Z.Y. Ou, Loss-tolerant quantum dense metrology with SU(1,1) interferometer, Optics Express. 21, 27705 (2018).

[12] Z.Y. Ou, Quantum amplification with correlated quantum fields, Phys. Rev. A. 48, R1761(R) (1993).

[13] N. V. Corzo, A. M. Marino, K. M. Jones, and P. D. Lett, Noiseless Optical Amplifier Operating on Hundreds of Spatial Modes, Phys. Rev. Lett, 109, 043602 (2012).

[14] J. Kong, F. Hudelist, Z.Y. Ou and W. Zhang, Cancellation of Internal Quantum Noise of an Amplifier by Quantum Correlation, Phys. Rev. Lett. 111, 033608 (2013).
[15] B. Yurke, S. L. McCall, and J. R. Klauder, SU(2) and SU(1,1) interferometer, Phys. Rev. A. 33, 4033-4054 (1986).

[16] Jietai Jing, Cunjin Liu, Zhifan Zhou, Z. Y. Ou, and Weiping Zhang, Realization of a nonlinear interferometer with parametric amplifiers, Appl. Phys. Lett. 99, 011110 (2011).

[17] F. Hudelist, Jia Kong, Cunjin Liu, Jietai Jing, Z. Y. OU and Weiping Zhang, Quantum metrology with parametric amplifier-based photon correlation interferometers, Nat. Commun. 5, 3049 (2014).

[18] Mathieu Manceau, Gerd Leuchs, Farid Khalili and Maria Chekhova, Detection loss tolerant supersensitive phase measurement with an SU(1,1) interferometer, Phys. Rev. Lett. 119, 223604 (2017).

[19] Brian E. Anderson, Prasoon Gupta, Bonnie L. Schmittberger, Travis Horrom, Carla Hermann-Aviglano, Kevin M. Jones, and Paul D. Lett, Phase sensing beyond the standard quantum limit with a variation on the SU(1,1) interferometer, Optica. 4, 7 (2017).

[20] C. W. Helstrom, Quantum Detection and Estimation Theory (Academic Press, New York, 1976).

[21] C. F. McCormick, A. M. Marino, V. Boyer, and P. D. Lett, Strong low-frequency quantum correlations from a four-wave-mixing amplifier, Phys. Rev. A. 78, 043816 (2008).

[22] LIGO Scientific Collaboration, Quantum correlations between the light and kilogram-mass mirrors of LIGO, arXiv:2002.01519v1 (2020).