Quantum Spacetime, from a Practitioner’s Point of View

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Abstract. We argue that theories of quantum gravity constructed with the help of (Causal) Dynamical Triangulations have given us the most informative, quantitative models to date of quantum spacetime. Most importantly, these are derived dynamically from nonperturbative and background-independent quantum theories of geometry. In the physically relevant case of four spacetime dimensions, the ansatz of Causal Dynamical Triangulations produces – from a fairly minimal set of quantum field-theoretic inputs – an emergent spacetime which macroscopically looks like a de Sitter universe, and on Planckian scales possesses unexpected quantum properties. Important in deriving these results are a regularized version of the theory, in which the quantum dynamics is well defined, can be studied with the help of numerical Monte Carlo methods and extrapolated to infinite lattice volumes.

Keywords: quantum gravity, nonperturbative quantization, causal dynamical triangulations, dynamical triangulations, lattice gravity

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INTRODUCTION

What unites cosmologists and researchers working on quantum gravity is their wish to understand the nature and physical properties of spacetime, and how they are influenced by and in turn influence the matter located in space and time. Although the work of cosmologists is usually associated with the behaviour of the universe on the very largest scales (10^{25} m and beyond), and that of quantum gravitators with ultra-short distances of the order of the Planck length, \( \ell_{Pl} \approx 10^{-35} m \), there are nevertheless phenomena which conceivably require input from both. These fall under the umbrella of what one may call "quantum spacetime phenomenology" [1], the (so far putative) observable consequences for cosmology and astrophysics of a quantum microstructure of spacetime, described by a (so far unknown) quantum theory of gravitation. The challenges one faces in developing such a phenomenology have to do with the incredible weakness of the gravitational forces and the associated smallness of the characteristic Planck scale at which quantum effects are expected to become important. Conversely, in the absence of any obvious phenomenology we have little reliable guidance in our search for a viable theory of quantum gravity.

Well-known obstacles must be overcome when trying to quantize classical general relativity, because of its highly nonlinear character and the dimensionality of the gravitational coupling or "Newton's" constant, which implies that a perturbative quantization of gravity becomes useless as one approaches Planckian distances. At the same time, nonperturbative candidate theories for quantum gravity, which provide more or less concrete quantum models of spacetime and the gravitational excitations at the Planck scale, face great difficulties in deriving any predictions – observable or not – because of their current incompleteness and a scarcity of quantitative computational tools to bridge the gap to more accessible macroscopic scales. This usually makes it difficult to verify that one’s favourite quantum-gravitational theory reproduces any aspects of the classical theory (as it must in a suitable large-scale limit), let alone predict specific quantum corrections to Einstein’s theory.

Despite these daunting prospects, there is a lot of activity and exciting progress to report about in this area. The development and further sophistication of nonperturbative computational tools has in recent times enabled us to derive quantitative results (yes, numbers!) in full\(^1\), four-dimensional quantum gravity. In so far as they relate to the classical limit, these numbers can be used to rule out (or rule in) specific models as promising candidates for quantum gravity. In so far as they concern \( \hbar \)-effects, they cannot as yet be related to any true quantum spacetime phenomenology, but can already be used to distinguish between different candidate theories. In the ansatz of Causal Dynamical Triangulations, a prominent example of such a theory, one meets concrete instances of both situations. The use of effective computational methods has also exhibited a number of generic pitfalls in

\(^1\) that is, without making any additional symmetry assumptions to reduce the degrees of freedom drastically, as is done, for example, in quantum cosmology
constructing a nonperturbative quantum gravity theory, which could not have been anticipated from either the classical or the perturbative theory, thus giving us valuable insights into what works and what does not.

QUANTUM SPACETIME

Our working definition of a quantum spacetime is a "spacetime" (in the most general sense) with quantum properties on Planckian scales, which in a suitable macroscopic limit can be approximated by a classical curved spacetime \( (M^{(4)}, g_{\mu\nu}(x)) \), consisting of a differentiable manifold \( M^{(4)} \) and a metric tensor \( g_{\mu\nu}(x) \) of Lorentzian signature. We do not know a priori which of the geometric or other properties of a classical spacetime will continue to be meaningful at the Planck scale; this is precisely one of the interesting questions the quantum theory should be able to answer.

There are essentially two ways to arrive at a quantum spacetime, a kinematical and a dynamical one. In the first case, one chooses a particular classical ("background") geometry and "embellishes" it with quantum fluctuations or, alternatively, arranges some microscopic quantum degrees of freedom such that they "approximate" a given classical background geometry. This can be thought of as a generalization of standard perturbative quantum gravity, with the difference that the assumed Planckian quantum fluctuations will typically look very different from linearized quantum fields. Since many researchers believe in a fundamental discreteness of spacetime at short distances, these degrees of freedom will often be of a discrete nature, involving sets of points or one-dimensional lattice-like structures, possibly with additional relations or labels. Other examples of kinematical, "fuzzy" spacetimes are given by certain noncommutative spaces.

The other, much harder way of obtaining a quantum spacetime is as the solution to the dynamics of a full theory of quantum gravity, say, as the extremum of some path integral or a solution to some Wheeler-DeWitt equation. The crucial difference with the kinematical approach is that no distinguished background spacetime is put in by hand. Instead, similar to what happens in the classical theory, a physical (quantum) spacetime is obtained only by solving (quantum) equations of motion.

Contrary to what the label "background-independent quantization" may suggest, there are specific choices involved in trying to set up such a nonperturbative quantum theory of gravity, most importantly of (i) the microscopic degrees of freedom, representing whatever becomes of "gravity" at the Planck scale, (ii) the kinematical principles (causality, locality, etc.), symmetries and other algebraic structures, and (iii) a dynamical principle – whatever constitutes the "quantum equations of motion". There is little way of telling which are the correct choices and to what extent they will influence the final outcome, without actually constructing the theories and computing some concrete numbers from them.

The trouble with a "kinematical" spacetime is that one cannot estimate to what extent it approximates a true, dynamical quantum spacetime of the full theory before having solved the latter. In other words, one does not know whether there is a nonperturbative theory of quantum gravity which has it as a stable solution or ground state. In the absence of dynamical solutions, kinematical quantum spacetimes may serve as useful models of the type of "quantum effects" that may be present, and one can try to study their phenomenological implications. Almost all quantum spacetimes that have been studied are kinematical and/or rely on an a priori symmetry reduction in the dynamical degrees of freedom, where the symmetry is suggested by that of a particular classical solution (for example, homogeneity, isotropy or spherical symmetry). This implies that they carry a considerable degree of classical background structure by construction, which makes it rather unsurprising if one can "rederive" a classical spacetime from them in a suitable large-scale limit.

This is very different from a truly background-independent derivation, where the "emergence" of classical spacetime – if it can be shown to occur – is highly nontrivial. As far as we are aware, the only fully nonperturbative candidate theory of quantum gravity

\[2\] A good measure of how little we know about "quantum spacetime" is how few pictures there are on Google, where one finds just a handful of artistic impressions of "spacetime foam", like that shown in Fig. 1.
which has produced a quantum spacetime with genuine classical large-scale properties is that of Causal Dynamical Triangulations (CDT), mentioned in the Introduction, and to be discussed further below. Like any other formulation of quantum gravity, this approach involves working hypotheses on the relevant ingredients (i)-(iii) above, which ultimately can only be justified a posteriori.

Isn’t this just as much guesswork as goes into the construction of a kinematical quantum spacetime? The answer is no, because the presence of dynamics provides crucial additional information about the system. Already the need for a quantum configuration space – a domain of the path integral or a Hilbert space – on which a dynamics is well defined (as opposed to being just "formal") imposes strong restrictions. Moreover, the presence of dynamics exhibits which region(s) of phase space the quantum-geometrical system is driven towards, and whether there are instabilities or other unexpected dynamical features. It is not surprising that it was in the context of Dynamical Triangulations (DT), which provides a concrete calculational framework of this kind, that generic, nonperturbative dynamical instabilities (associated with highly degenerate geometries) of systems of strongly fluctuating higher-dimensional quantum geometry were first discovered and quantified [2, 3].

A dynamical approach – even before all of its implications are understood and analyzed – can also give valuable "pre-geometric" information about the quantum spacetime, for example, its dimensionality. On Planckian scales, this may in principle deviate from the classical value of four. Again, one should be careful to distinguish between genuine dynamical properties of the spacetime and those which are simply consequences of some kinematical choices put in by hand. For example, if one postulates that spacetime is fundamentally discrete and chooses a particular type of fundamental "building block" or quantum excitation whose size is the Planck length, all properties of the quantum spacetime at that scale will strongly depend on this choice. This is not really good enough if one is interested in a fundamental description of quantum gravity at the Planck scale, because in that case one still needs to argue why a particular choice of discrete structure is better than the infinitely many other choices one could have made instead.

One way to obviate the need for seemingly arbitrary guesses about "physics at the Planck scale" is to work with a regularization – which is necessary at an intermediate stage to make calculations well defined – whose associated cut-off scale is sent to zero in the end or, from a practical point of view, taken to be at least significantly smaller than the physical scale one is interested in describing. A well-known mechanism for removing the dependence on arbitrary regularization details is that of "universality", realized under favourable conditions when approaching a phase transition point (of order 2 or higher) in the space of bare coupling constants of a regularized statistical model underlying the candidate theory of quantum gravity under consideration. If an ultraviolet fixed point and an associated scaling limit exist, the theory is said to possess a "continuum limit". Despite the name, such a construction is as a matter of principle not necessarily incompatible with fundamental discreteness. For example, it may happen that a distinguished length scale is generated dynamically, and appears as the minimal eigenvalue of a quantum operator measuring length, area or some other geometric quantity, similar to what happens in the kinematical sector of loop quantum gravity, say. However, there are to our knowledge no compelling arguments for the existence of a minimal length scale in quantum gravity, as opposed to wishfully thinking that it should exist to provide a "natural cut-off" to momentum integrals.

There is strong evidence that the scenario sketched

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3 By depicting particular aspects of quantum geometry, both DT and CDT quantum gravity theories have produced pictures that are not only pretty, but convey information about their dynamical content (Fig. 2).

4 usually defined in terms of a divergent correlation length of the theory

5 How to give an operationally well-defined meaning to the notion of a minimal length in a nonperturbative theory of quantum gravity is of course yet another nontrivial issue.
above – a regularized quantum field-theoretic framework for gravity with a set of minimalist ingredients (i), (ii) and (iii), from which a full theory of quantum gravity emerges, along with a specific "quantum spacetime" as its ground state – is indeed realized in the ansatz of four-dimensional CDT. – In the remainder of this article, we will only give the briefest of summaries of the ingredients and some recent results of this approach to quantum gravity. It should be read in conjunction with our companion article [4] in the same volume, which contains some further technical details. In what follows, we will specifically highlight some new results concerning the phase structure of CDT quantum gravity. For a much more comprehensive exposition of this material and a more complete bibliography, we refer the interested reader to several recent reviews of the subject [5, 6, 7].

QUANTUM SPACETIME FROM CAUSAL DYNAMICAL TRIANGULATIONS

Tools and ingredients

"Quantum gravity from CDT" relies on few ingredients and a minimal set of prior assumptions. In terms of the ingredients listed in the previous section, we have

(i) microscopic degrees of freedom: curved Lorentzian geometries of fixed topology, encoded in four-dimensional, piecewise flat triangulations ("simplicial manifolds") with curvature concentrated at two-dimensional sub-simplices (the triangles of the triangulation).

(ii) kinematical principles and other structures: quantum superposition principle, locality, causality; notions of proper time and Wick rotation.

(iii) dynamical principle: path integral over the domain (i) with weights given by the Einstein-Hilbert action with a cosmological constant term, implemented nonperturbatively using standard tools from quantum field theory, including appropriate renormalization of couplings in the limit as the ultraviolet lattice cut-off (edge lengths of the triangulations) is taken to zero.

This minimalist and rather conservative starting point, following closely the dynamical ingredients of the classical theory, without postulating any new symmetries or degrees of freedom, is all that is necessary to produce an array of truly interesting results. Key to obtaining an emergent classical geometry are causality conditions imposed on the individual triangulations, with the help of a global proper-time slicing [8, 9], such that the simplicial building blocks are arranged in layers, see Fig. 3. This particular definition of the domain of the path integral is not "pulled out of a hat", but motivated by previous investigations, most importantly, in the much-studied Euclidean DT version of the formulation. The calculational set-up can be seen as the gravitational analogue of a lattice quantum field theory (like lattice QCD), although not on a given fixed (hypercubic, say) lattice, but incorporating a sum over all inequivalent triangular lattice geometries, reflecting the dynamical nature of spacetime. In addition, one can of course add other matter degrees of freedom by assigning field variables to the elements of the simplicial lattices and including them in the path integral. Of crucial importance in all cases is the availability of numerical methods to evaluate the regularized, nonperturbative path integrals (after Wick rotation), using Monte Carlo techniques and finite-size scaling. A wealth of results from studying DT and CDT models in dimensions two, three and four, with and without matter, have revealed a large degree of universality, the highly desirable property of the continuum limits of these theories to be largely independent of the details of the implementation and regularization. Universality covers various regularity conditions on the path integral configurations, and, to a large degree, the form of the action and the measure, but not the signature of the geometries. Choosing Euclidean or Lorentzian signature leads to inequivalent continuum theories, as was first demonstrated in two-dimensional gravity [8], an early key insight obtained by using dynamical triangulations.

Some Results

As a consequence of the regularization chosen and a subsequent Wick rotation, the original Lorentzian grav-
that the minimal assumption leads to an interesting continuum theory, tunable bare coupling constant in the full, effective action of the theory. The bare action only means that these terms will not be associated with a tunable bare coupling constant in the full, effective action of the theory. That this minimal assumption leads to an interesting continuum theory, by fine-tuning at most \( \kappa_0 \) and/or \( \Delta \), obviously needs to be verified.

It turns out that in all DT models, fine-tuning the bare cosmological constant to its critical value is tantamount to taking the infinite-volume limit (in lattice units), a necessary step for obtaining a theory in the continuum. This leaves us with two free parameters for the case of CDT quantum gravity in four dimensions, namely, \( \kappa_0 \) and \( \Delta \).\(^6\) The qualitative structure of the relevant \( \kappa_0-\Delta \) phase diagram (Fig. 4) has been known for some time [12]. There are three phases, A, B, and C, which can be distinguished with the help of an "order parameter", given by the overall shape of the quantum spacetime that emerges as ground state in the given phase. This so-called volume profile \( V_3(t) \) is simply the three-volume of space (by definition compact, with the topology of a three-sphere) at proper time \( t \). The quantity \( V_3(t) \) for a given path integral geometry is of course not an observable, only its expectation value \( \langle V_3(t) \rangle \) in the ensemble is. Nevertheless, "typical" path integral histories in the different phases exhibit very distinct characteristics, as illustrated by the three small figures included in the phase diagram of Fig. 4. Only in phase C can the average volume profile \( \langle V_3(t) \rangle \) be matched with that of a cosmological solution to the classical Einstein equations, namely, the de Sitter universe [10, 11]. This can be done with quite spectacular precision (cf. Figs. 3 and 4 in [4]), including the quantum fluctuations, which can be matched to those computed from a related minisuperspace model.

In order to establish whether we can understand quantum gravity within the standard framework of critical statistical-mechanical systems, where continuum limits are associated with critical points at second- or higher-order transitions, we have recently performed a detailed analysis of the phase transition lines bordering the physical phase C. The numerical determination of the order of the phase transitions is a subtle affair, because of the characteristic slow-down of the simulations near the transition, which make the simulations computationally very expensive. One also has to determine which standard indicators of the order of a phase transition can be adapted to the case of fluctuating geometry, and how. Only by combining several such criteria have we been able to determine the order of the phase transitions with confidence [14].

More specifically, we have (i) used a histogram analysis of selected quantities conjugate to the couplings controlling the approach to the respective transition lines, (ii) extracted so-called shift exponents from measuring the location of the maxima of the susceptibilities associated with these quantities, and (iii) measured a variety of Binder cumulants. The histograms exhibit a clear two-peak structure near either transition line of the CDT system (see Fig. 5), the pertinent question being whether in the limit of large volumes they fuse into a single one (indicative of a second-order transition) or become more pronounced (in line with a first-order transition).

The combined outcomes of these measurements constitute strong evidence that the A-C transition, between

\(^6\) To address a frequent misconception, let us point out that our choice of the Einstein-Hilbert does not mean that high-curvature terms are ignored or suppressed. These will all be present in the path integral and are contained in the "entropy" of path-integral configurations at a given value of the bare action. Not including higher-order curvature terms in the bare action only means that these terms will not be associated with a tunable bare coupling constant in the full, effective action of the theory.
the oscillatory and de Sitter phase, is first order, and the B-C transition between the "time-collapsed" and the physical de Sitter phase is second order. This finding is potentially very significant, because all points along a second-order line are potential candidates for the existence of continuum theories. We also note two distinguished points along the B-C line, namely, an apparent end point for small \( \kappa_0 \), as well as a triple point where all three phases meet. The finding of a second-order phase transition, so far unique in four-dimensional quantum gravity, suggests that in order to probe quantum spacetime at even smaller scales than what has been achieved until now – where the de Sitter universes have had a linear extension of about 12-20 Planck lengths across – one needs to perform simulations closer to the B-C transition line. Making the algorithms more efficient in the neighbourhood of this transition line presents a considerable technical challenge and is currently under investigation. Physical statements extracted for the infinite-volume limit based on measurements taken inside phase C, especially those referring to large-scale structure, are not invalidated by this result. However, we have noted before that our current "resolution" is not sufficient to analyze certain true Planck-scale details of geometry like, for example, deviations of the volume profile \( \langle V_3(t) \rangle \) near its end points, where \( V_3(t) \) is small and quantum corrections to the potential energy of the relevant minisuperspace model should come into play.

**SUMMARY AND OUTLOOK**

After discussing the status of "quantum spacetime", and possible ways of obtaining it in a theory of quantum gravity, we have focussed on the approach of Causal Dynamical Triangulations, which arguably has made the most progress in demonstrating that and how a solution to the classical Einstein equations can emerge from a nonperturbative and background-independent formulation in a large-scale limit. The investigation of this candidate theory of quantum gravity is still ongoing. The results obtained so far on the short-scale, Planckian structure of the theory are quite remarkable, because they have revealed totally unexpected (from a perturbative point of view) properties, most prominently that of dimensional reduction, as described in the companion paper [4] (see especially Fig. 5 there, illustrating the scale-dependent spectral dimension of quantum spacetime). CDT, together with its nonperturbative computational toolbox, gives us unprecedented glimpses of this highly nonclassical regime, which so far cannot be accessed reliably by other methods. Other insights include the pivotal role played by "entropy", the number of microscopic realizations for a given value of the action, which – amongst other things – is responsible for curing the problem of unboundedness of the path integral and the associated instability [7].

To demonstrate that the quantum theory obtained from CDT is the correct (and hopefully unique) theory of quantum gravity, further aspects of the classical theory need to be "rederived", and quantum properties at Planckian scales to be probed in more detail. All of these require the identification and spectral analysis of suitable "observables", on top of the already known ones. Finding observables which are well defined in absence of any background geometry, as is the case in a nonperturbative formulation, is of course one of the central challenges of any approach to quantum gravity. The advantage of the CDT formulation discussed here is that the issue takes on a concrete form, thanks to the presence of a well-defined computational set-up where observables can be operationally implemented, measured and interpreted. In quantum gravity, this is just about as good as it comes.

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