On the theory of scale structural fatigue of metals at the proportional loading

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Abstract. The probabilistic multi-level model of metal fatigue at proportional loadings is presented. A system of hypotheses about the step-by-step grow of metal defects on micro-, meso- and macroscale levels is formulated. Here are proposed constitutive relations for the failure probability and a system of fatigue curves and equations of multilevel damage. The amplitude of the maximum principal stress is chosen as a variable, the material functions are determined according to fatigue tests, taking into account the results of physical research of brittle crack grow and in accordance with experimentally valid criteria of fatigue strength. It is shown the correlation between predicted fatigue strength and experimental data for steel 12XH2A, S135 and some ductile steel at combined tension and torsion.

1. Introduction

The sequential process of metal fatigue is a multi-level and multi-scale kinetic process that is associated with the evolution of the metal structure. This process is characterized by the obligatory stages of initiation and grow of micro-, short and macrocracks. Random volume distributions of structural characteristics make the process statistical, so a probabilistic fatigue theory should be formulated.

An overview of the known physical theories of fatigue failure (structural, energy, statistical, synergetic) shows that physical models establish the laws of the evolution of defects in solids, describe the physical mechanisms of fatigue. Moreover, most theories contain structural parameters and physical failure mechanisms not identified in macroexperiments, and do not allow to determine of the strength fatigue macro-characteristics.

Famous phenomenological theories of fatigue fracture (including gradient, nonlocal, micromorphic continua, etc.) consider the evolution of material damage, but excluding the internal structure of the metal, the physical mechanisms and the sequential nature of the fatigue process [1,2]. According to experimental data, certain states of metal structure correspond to different levels of fatigue failure. Fracture mechanics explores the growing of single macrocracks at the macro level on the basis of strain, stress and energy approaches.

The proposed model is constructed as a phenomenological, covering the whole step-by-step process of fatigue. It is identified the typical state as a defect of certain scale-structural level. The failure probability at the each level is described by a system of recurrent constitutive equations. This approach takes into account the results of experimental and theoretical studies of the metal fatigue in physics of solids, materials science, solid mechanics.
2. Probabilistic modeling approach and fatigue curves of metal structural levels

Based on the analysis of the experimental and theoretical results of metal fatigue at micro-, meso-, and macroscale levels on physics of solids, material science and mechanics of solids (works by L.R. Botvina, V.S. Ivanova, V. F. Terent’ev, A.A. Shaniavsky, N.A. Makhutov, A.P. Soldatenkov, A.A. Lebedev, V.I. Betekhtin, S.A. Golovin, I.I. Novikov, A.M. Glezer, Yu. G. Matvienko, A.N. Romanov, V.V. Panasyuk, V.E. Panin, P.S. Volegov, N.G. Burago, A.D. Nikitin, V.T. Troshchenko, S.Ya. Yarema, C. Bathias, B.L. Averbakh, D. Broek, M.V. Brown, T. Ecobori, E.A. Charkaluk, H. Zenner, C. Miller, Y. Murakami, H. Kitagawa, G. Sih, J. Notta, J. Okamoto, M. Onami, R.P. Skelton, S. Sonsino, A. Spanolli, F. Morel, N. Santier, V.E. Wildeman, C. Hua and others [1–15]) the system of hypotheses for processes of brittle failure of metals and alloys at multi-cycle fatigue \( (N_f \in [5 \times 10^3, 10^4] \text{cycles to fracture}) \) is determined [2, 16–21].

A process of the following proportional loadings:

\[
\sigma_{ik}(\tau) = \alpha_i \sigma_f(\tau), \quad f(\tau) = \sin(\omega \tau + \theta), \quad k = 1, 2, 3, \quad \tau \in [0, t]
\]

\[
|\sigma_{11}| \geq |\sigma_{22}| \geq |\sigma_{33}|, \quad \alpha_i = \sigma_{ik} |\sigma_{11}|^{-1}, \quad \alpha_0 = \sigma_0 |\sigma_{11}|^{-1}, \quad \sigma = (3)^{-1} \sum_{k=1}^{3} \sigma_{ik},
\]

where \( \sigma_0 \) is the amplitude of the maximum principal stress, \( \omega \) and \( \theta \) are the frequency and phase of the stress change, \( \alpha_i \) are ratio of principal stresses, is considered consistently at six the micro-, meso- and macro scale-structural levels corresponding to the different stages of the metal evolution on various physical mechanisms. There is determined a typical state, namely a defect of the scale-structural level, defined by the average dimension \( l_i = l_i(\tau) \) and density \( q_i = q_i(\tau), \quad i = 1...6 \), in the representative volume \( V \) (in which it is possible the initiation of a single macrocrack). The change of defect levels is characterized by the change in the physical mechanisms of fracture.

At the microlevel with increasing of stress cycles the evolution of crystallization defects (vacancy clusters, substitutions and implantations of atoms, Schottky defects, interstitial particles, Frenkel defects, inclusions of foreign atoms, ions, molecules, electron hole defects, submicrocracks, etc.) leads to the initiation of an “self-organizing band substructure” associated with their accumulation. When the critical value of the defect density is reached, it is carried out so-called phase transition to brittle microcracks, their dimensional stability creates the possibility of a significant increase their density. Subsequent fusions of microcracks lead to the initiation of grain size short cracks, which form a “river pattern” on the surface of the sample. At the mesolevel on the French damage lines as a result of short cracks joint there are formed the transcrystallite and grain-boundary macrocracks and further single macrocracks and brittle fracture of the metal. Along with the brittle defects evolution in ductile metals processes of initiation and propagation of dislocation cellular structure on mechanisms of twinning and sliding, which lead to the moving of grain ensembles and initiation of viscous microcracks, take place. Physical mechanisms of the evolution of brittle defect structures (defined as phase-structural inhomogeneities) on different scale levels are investigated in physical and metal science literature.

In the range of \( N_f \in [5 \times 10^3, 10^4] \text{cycles} \) fatigue process occurs at elastic macrodeformation. At the endurance limit the failure process leads to brittle macrofracture on short cracks and mechanisms of intergranular or transcrystalline cleavage. In brittle materials at the range of \( N_f \in [5 \times 10^3, 5 \times 10^6] \text{cycles} \) the fracture process leads to the initiation of brittle single macrocracks at elastic straining and brittle macrofracture. In ductile metals inelastic deformations develop that slows down the propagation of brittle cracks.

It is assumed that the initiation of defects of each level is due to the sequent initiation, growing and merging of defects of all previous levels. Here is considered the continuous increasing representative
function for \( i \)-level \( l_i^* = l_i^* (\tau) \): \( l_i^* (\tau) = l_i (\tau) \left( q_i (\tau) V_i \right)^{\gamma}, \tau \in \left[0, t_i \right] \). \( \gamma \) is the material constant. The defect failure state is determined by the achievement of representative function \( l_i^* = l_i^* (\tau) \) of its limit value \( l_{i,j}^* \), \( i = 1, \ldots, 6 \). Due to the defect random distribution in volume \( V_i \), the process of fatigue is stochastic, and the values \( l_i^* \) at every moment in time \( \tau \), \( \tau \in \left[0, t_i \right] \) are random variables.

Here is introduced a probability distribution function \( F_i = F_i (l_i^*) \), \( 0 \leq F_i \leq 1 \), determining the probability with which a random value \( l_i^* \) takes values less than its limit value \( l_{i,j}^* \) at a time moment \( \tau \): \( F_i = P(l_i^* < l_{i,j}^*) \), \( i = 1, \ldots, 6 \) (\( P(l_i^* = l_{i,j}^*) = 0 \)). It is proposed to consider a truncated normal distribution for function \( F_i = F_i (l_i^*) \) with a distribution density \( f_i = f_i (l_i^*) \): \( F_i = \int_{-\infty}^{l_i^*} f_i (x) dx \), of the following form:

\[
f_i (l_i^*) = \frac{c_i}{\sqrt{2\pi D_i}} \exp \left( \frac{-\left( l_i^* - M_i \right)^2}{2D_i} \right), \quad c_i = \int_{-\infty}^{\infty} f_i (x) dx, \quad i = 1, \ldots, 6
\]  

(2)

where \( M_i = \int_{-\infty}^{\infty} xf_i (x) dx \) and \( D_i = \int_{-\infty}^{\infty} (x - M_i)^2 f_i (x) dx \) are the mathematical expectation and variance of a random variable, respectively.

The probability of brittle fracture on \( i \)-level defects is determined by the function \( Q_i = Q_i (\tau) \), \( i = 1, \ldots, 6, \tau \in \left[0, t_i \right] \), in the form: \( Q_i (\tau) = Q_{i,th} - F_i \left( l_i^* (\tau) \right) \), \( 0 \leq Q_{i,th} \leq 1 \), for which a recurrence system of constitutive relations is formulated. These relations include lifetime moment \( t_{i+1} \), when the defects of the \( i \)-level reach the failure state, the representative function is equal its limit value \( l_{i,j}^* \), the defects of \( i+1 \)-level are born and failure probability reaches the value \( Q_{i,th} \).

So a series of fatigue curves on the certain defect level is determined as the following:

\[
Q_i (t_{i+1}) = Q_{i,th}, \quad i = 1, \ldots, 6
\]  

(3)

The failure process is determined by the loading process on the time \( \tau \in \left[0, t_i \right] \). For loading (1) the failure probability \( Q_i = Q_i (\tau) \), \( i = 1, \ldots, 6 \), is considered as a function of the parameters of the process, and assuming of uniform defect distribution in volume \( V_i \) the following system of constitutive equations for \( Q_i = Q_i (\sigma, n) \), \( i = 1, \ldots, 6 \), is written according the methods of solid mechanics and in accordance with the dimension and similarity theory \( n \) is the number of load cycles) [2]:

at the microlevel, \( i = 1, 2, 3 \), at \( \sigma_0 \geq \sigma_{i,th} \), \( \lg n \geq \lg n_i \left( \sigma_0 \right) \), \( \sigma_0 = 0 \), \( n_i = 1 \)

\[
Q_i = F_i \left( \frac{\sigma_i - \sigma_{i-1}}{\sigma_i - \sigma_{i,th}} \right) R_i \left( \frac{\lg n - \lg n_i \left( \sigma_0 \right)}{\lg N_i - \lg n_i \left( \sigma_0 \right)} \right), Q_i (n_{i+1}) = 1.
\]  

(4)

at the mesolevel, \( \sigma_0 \geq \sigma \), \( \lg n \geq \lg n_i \left( \sigma_0 \right) \)
\[
Q_i = F_i \left( \frac{\sigma_i - \sigma_3}{\sigma_i - \sigma_3} \right) R_i \left( \frac{\lg n - \lg n_i (\sigma_i)}{\lg N_i - \lg n_i (\sigma_i)} \right), \quad Q_i(n_i) = 1, \quad (5)
\]

\[
\sigma_i \leq \sigma_2, \quad \lg n \geq \lg n_i (\sigma_i)
\]

\[
Q_i = F_i \left( \frac{\sigma_i - \sigma_3}{\sigma_i - \sigma_3} \right) R_i \left( \frac{\lg n - \lg n_i (\sigma_i)}{\lg N_i - \lg n_i (\sigma_i)} \right), \quad Q_i = G_i \left( \frac{\sigma_i - \sigma_3}{\sigma_i - \sigma_3} \right) R_i \left( \frac{\lg n - \lg n_i (\sigma_i)}{\lg N_i - \lg n_i (\sigma_i)} \right), \quad Q_0 = 0, \quad Q_0(n_i) = 1, \quad (6)
\]

\[
\sigma_i \leq \sigma_2, \quad \lg n \geq \lg n_i (\sigma_i)
\]

\[
Q_i = F_i \left( \frac{\sigma_i - \sigma_3}{\sigma_i - \sigma_3} \right) R_i \left( \frac{\lg n - \lg n_i (\sigma_i)}{\lg N_i - \lg n_i (\sigma_i)} \right), \quad Q_i = G_i \left( \frac{\sigma_i - \sigma_3}{\sigma_i - \sigma_3} \right) R_i \left( \frac{\lg n - \lg n_i (\sigma_i)}{\lg N_i - \lg n_i (\sigma_i)} \right), \quad Q_0 = 0, \quad Q_0(n_i) = 1 \quad (7)
\]

The proposed model allows to select the form of the functions \( F_i = F_i (\sigma_i) \) and \( R_i = R_i (\sigma_i) \), \( i = 1, \ldots, 6 \), in the expressions (4) - (7) for various materials. Here are selected following functions:

\[
F_i = \left( \frac{\sigma_i - \sigma_{i-1}}{\sigma_i - \sigma_{i-1}} \right)^{\beta_i}, \quad R_i = \left( \frac{\lg n - \lg n_i (\sigma_i)}{\lg N_i - \lg n_i (\sigma_i)} \right)^{\gamma_i}, \quad i = 1, \ldots, 4, \quad F_i = \left( \frac{\sigma_i - \sigma_{i-1}}{\sigma_i - \sigma_{i-1}} \right)^{\beta_i}, \quad R_i = \left( \frac{\lg n - \lg n_i (\sigma_i)}{\lg N_i - \lg n_i (\sigma_i)} \right)^{\gamma_i}, \quad i = 5, 6
\]

when \( \sigma_i = \sigma_i (\alpha_2, \alpha_3, N, \omega), \quad \beta_i = \beta_i (\alpha_2, \alpha_3, N, \omega), \quad \phi_i = \phi_i (\alpha_2, \alpha_3, N, \omega), \quad \chi_i = \chi_i (\alpha_2, \alpha_3, N, \omega) \) are model material functions.

In the recurrence system of relations (4) - (7) the following equation is related to the previous one via number of cycles \( n_{i+1} = n_{i+1} (\sigma_i) \), when the function \( l_i \equiv l_i (n) \) equals the fracture value \( l_i^* \), the \( i \) - level defect reaches the fracture state, and \( (i+1) \) - level defect starts at the given loading.

The model is based on the material constants at three basic types of symmetrical loading under plane stress state, accordingly, uniaxial (\( \alpha_1 = 1, \quad \alpha_2 = \alpha_3 = 0 \)) and biaxial uniform (\( \alpha_2 = \alpha_3 = 1, \quad \alpha_3 = 0 \)) loadings and shear (\( \alpha_1 = 1, \quad \alpha_2 = -1, \quad \alpha_3 = 0 \)), when the \( i \) - level defect reaches the fracture state and the function \( l_i^* \equiv l_i^* (n) \) is equals to its limit value \( l_i^* \) at the number of cycles \( N_i^* \), \( i = 1, \ldots, 6 \). According to these data and the results of brittle cracks studies and taking into account the ratio of the principal stresses on the experimentally valid fatigue criteria for metals, the following system of material functions \( \sigma_i, \quad i = 1, \ldots, 6 \), in relations (4) - (7) is determined as:

\[
\sigma_i (\alpha_2, \alpha_3, N, \omega) = \sigma_i (N, \omega) \tilde{\sigma}_i (\alpha_2, \alpha_3, \tilde{n}_i, \tilde{n}_i), \quad (8)
\]

\[\text{где } \sigma_i (N, \omega) = \sigma_i (\alpha_2 = 0, N, \omega), \quad \tilde{n}_i = \frac{\sigma_i (N, \omega)}{\sigma_i (\alpha_2 = -1, N, \omega)}, \quad \tilde{n}_i = \frac{\sigma_i (N, \omega)}{\sigma_i (\alpha_2 = 1, N, \omega)}, \quad i = 1, \ldots, 6\]

For the microdefects here is the following expressions

\[
\tilde{n}_i = \frac{1}{3} (\tilde{n}_i (i-1) + 4 - i), \quad \tilde{n}_i = \frac{1}{3} (3 \tilde{n}_i (i-1) + 4 - i), \quad i = 1, 2, 3
\]

In expression (8) the functions \( \tilde{\sigma}_i = \tilde{\sigma}_i (\alpha_2, \alpha_3, \tilde{n}_i, \tilde{n}_i) \) are chosen as follows:

for brittle materials

at \( -1 \leq \alpha_2 \leq 0 \)

\[
\tilde{\sigma}_i = \left[ 6 - \tilde{n}_i - \alpha_2 (2 \tilde{n}_i - 6) + \alpha_0 (3 \tilde{n}_i - 15) \right]^{-1},
\]

at \( 0 \leq \alpha_2 \leq 1, \quad \alpha_3 \geq 0 \)

\[
\tilde{\sigma}_i = \left[ 1 + \alpha_2 (\tilde{n}_i - 1) + \alpha_3 (\tilde{n}_i - 1) \right]^{-1},
\]

(9)
at $0 \leq \alpha_2 \leq 1$, $\alpha_3 < 0$ 
\[ \tilde{\sigma}_i = \left[ 6 - \hat{\eta}_i - \alpha_3 (2\hat{\eta}_i - 6) + \alpha_0 \left(3\hat{\eta}_i - 15\right) \right]^{-1}, \]
for ductile materials

at $-1 \leq \alpha_2 \leq 0$ 
\[ \tilde{\sigma}_i = \left[ 3\alpha_0 (1 + \alpha_3) + \left(2^{-1} \hat{\eta}_i^2 \left(1 - \alpha_2 - 3\alpha_0\right) \right) \right]^{-3/2}, \]

at $0 \leq \alpha_2 \leq 1$, $\alpha_3 \geq 0$ 
\[ \tilde{\sigma}_i = \left[ 3\alpha_0 (1 + \alpha_3) + \left(2^{-1} \hat{\eta}_i^2 \left(1 - \alpha_2 - 3\alpha_0\right) \right) \right]^{-3/2}, \] (10)

at $0 \leq \alpha_2 \leq 1$, $\alpha_3 < 0$ 
\[ \tilde{\sigma}_i = \left[ 3\alpha_0 (1 + \alpha_3) + \left(2^{-1} \hat{\eta}_i^2 \left(1 - \alpha_2 - 3\alpha_0\right) \right) \right]^{-3/2}. \]

The expression (8) includes basic material functions $\sigma_j = \sigma_j(N_i, \omega)$, $i = 1,...,6$, to identify these functions series of experiments with microstructure analysis is required. Due to the difficulties and data insufficiency, in most cases this is not possible. Therefore, some propositions are introduced.

Here is considered metals and alloys with a horizontal section of the fatigue curve and unlimited endurance limit. The base amplitude is determined of the known material endurance limit $\sigma_{-4}$ at symmetric axial loading and number of cycles $N_4$. $\sigma_{4}(N_4, \omega) = \sigma_{-4}$, IV-scale level defects reach the fracture state, the function $\hat{l}_4' = \hat{l}_4'(N_4)$ equals the limit value $\hat{l}_4'$ and take place the macrofracture on short cracks with probability $Q_{\text{th}}$.

As a result of analysis of the known data on fatigue microfracture on the microlevel we can assume that the failure probability for defects I, II and III levels are described with a sufficient degree of accuracy the relations (4), $\beta_i = \chi_i = 1$, $\phi_i = 1/2$. A noticeable changes in the metal micro- and macrostructure (the changes of average size and density of micro- and macrocracks) are observed when the number of loading cycles is changed by decimal orders. So for the base numbers of cycles $N_i$ the following expressions are selected on the number of cycles $N_4$:

\[ \lg N_i = \lg N_4 + 4 - i, \quad i = 1,...,4, \quad \lg N_5 = \lg N_4 - 2, \quad \lg N_6 = \lg N_4 - 3. \] (11)

It is assumed the failure state of I–level defects at the number of cycles $N_6$, II–level defects at the number of cycles $N_5$ and III–level defects at the number of cycles $\lg N_4 = 3$ is achieved at the amplitude $\sigma_{-4}$. So at the number of cycles $N_4 = 2\times 10^7$ cycles the basic amplitudes for microlevel defects are determined through the endurance limit $\sigma_{-4}$ in the following form ($Q_{\text{th}} = 1)$:

\[ \sigma_1 = 0.4\sigma_{-4}, \quad \sigma_2 = 0.75\sigma_{-4}, \quad \sigma_3 = 0.86\sigma_{-4}. \] (12)

According to the known results of the fatigue analysis for metals and alloys the base amplitudes for VI–level defects $\sigma_6 = \sigma_6(N_6, \omega)$ can be selected with a good degree of accuracy on the corresponding yield strengths.

Note that it can be considered others values of material constants in the presence of experimental data.

The failure from mesolevel defects are considered as independent events. According the proposed approach the failure probability function $Q = Q(\tau)$ for mesolevel defects is defined in the form:

\[ Q(\tau) = \sum_{i=4}^{6} \frac{Q_i(\tau)}{1 - Q_i(\tau)} \prod_{j=4}^{6} \left(1 - Q_j(\tau)\right), \] (13)

where $Q_i = Q_i(\tau)$ are probability function on (5)–(7) for $i$–level defects, $i = 4,5,6$. Thus the fatigue strength on the mesolevel is determined by the equation:
\[ Q(t_f) = 1 \]  

In equation (14) value \( t_f = t_f(\sigma_f) \) is durability on the mesolevel defects, namely, the time moment of single macrocrack initiation, \( \sigma_f \) is the amplitude of the maximum principal stress at mesolevel fracture.

3. Comparison with known experimental data and known fatigue criteria at axial loading with torsion

In the present section the above approach is applied to some experimental data and known fatigue criteria for steels at combined axial loading and torsion.

For symmetric uniaxial tension - torsion loading \(( -1 \leq \alpha_2 \leq 0, \alpha = \alpha_1 = 0 )\) predicted fatigue curves are well correspond to the criteria of McDiarmid, Matake, Findley, Dang Wang, Zener, Lee, Papadopoulos, Kenmen, Zavoychinskii for brittle materials and Gough, Pollard, Li for ductile materials, and have a satisfactory correlation with the consequences of Sines, Crossland, Kakuno, Kavada, Depero approaches for brittle materials and Carpinteri, Spagnoli criteria for ductile materials, having a wide experimental basis [17,18,20].

Figure 1 and Table 1 present the results of calculations for brittle steel 12XH2A at symmetric tension - torsion loading depending on the ratio \( \alpha_2 \) \(( \sigma_{f,x}, \sigma_{f,y}, \sigma_{f,z}^* \) are the amplitude of the maximum principal stress on the proposed model, the experimental data and the known fatigue criteria, respectively, divided on material endurance \( \sigma_{f} = 390MPa \) ) at different longevity \( N_f \).
steel 12XH2A, 1-6 – material constants [6], $\alpha_3 = 0$; (a) $- \alpha_2 = 0$, (b) $- \alpha_2 = -1$, (c) $- \alpha_2 = 1$ (d) $- \alpha_2 = -0.6$, (f) $- \alpha_2 = 0.6$.

On the basis of the encouraging results herein obtained proposed analytical approach seems to be able to correctly estimate the fatigue life in the range $N_f \in \left[10^5, 5 \times 10^7\right]$ cycles.

**Table 1.** The experimental data, the calculation on the proposed model and on the criteria of McDiarmid, Matake and etc. at various values $\alpha_2$ for steel 12XH2A.

| $N_f$ | $\alpha_2$ = 0 | $\alpha_2$ = -0.2 | $\alpha_2$ = -0.4 | $\alpha_2$ = -0.6 | $\alpha_2$ = -0.8 | $\alpha_2$ = -1 |
|-------|---------------|------------------|------------------|------------------|------------------|------------------|
| 5*10^9 | 1.00          | 1.00             | 0.90             | 0.90             | 0.79             | 0.79             |
| 3*10^9 | 1.00          | 1.00             | 0.90             | 0.90             | 0.79             | 0.79             |
| 10^9   | 1.00          | 1.00             | 0.90             | 0.90             | 0.79             | 0.79             |
| 5*10^9 | 1.00          | 1.00             | 0.90             | 0.90             | 0.79             | 0.79             |
| 5*10^9 | 1.00          | 1.00             | 0.90             | 0.90             | 0.79             | 0.79             |

For ductile structural steel ($\sigma_f = 220\text{MPa}$), the experimental data, the results on the proposed model and on the criteria of Gough, Pollard and Lee as a function of the ratio $\alpha_2$ are presented in Table 2.

**Table 2.** The experimental data, the calculation on the proposed model and on the criteria of Gough, Pollard and Lee as a function of the ratio $\alpha_2$ for ductile steel.

| $N_f$ | $\alpha_2$ = 0 | $\alpha_2$ = -0.2 | $\alpha_2$ = -0.4 | $\alpha_2$ = -0.6 | $\alpha_2$ = -0.8 | $\alpha_2$ = -1 |
|-------|---------------|------------------|------------------|------------------|------------------|------------------|
| 5*10^9 | 1.00          | 1.00             | 0.85             | 0.85             | 0.74             | 0.74             |
| 3*10^9 | 1.00          | 1.00             | 0.85             | 0.85             | 0.74             | 0.74             |
| 10^9   | 1.00          | 1.00             | 0.85             | 0.85             | 0.74             | 0.74             |
| 5*10^9 | 1.00          | 1.00             | 0.85             | 0.85             | 0.74             | 0.74             |
| 5*10^9 | 1.00          | 1.00             | 0.85             | 0.85             | 0.74             | 0.74             |

The results of the fatigue analysis and the microstructure of S135 pipe steel [2] at symmetric axial loading with torsion, $\alpha_3 = -0.72$, are presented on the Figure 2. Photos of the microstructure in Figure 2 (b), corresponding to the red crosses in Figure 2 (a), and in Figure 2 (c) before failure at four amplitudes of the maximum principal stress $\sigma_f$ respectively are presented. Calculated fatigue curve $f(t)$ for brittle defects of the mesoscale lies to the left of the curve of complete failure, developing viscous failure inhibits the grow of brittle cracks and increases the steel durability. It was found the experimental data (red crosses in Figure 2 (a)) are in the field of IV-level defect grow (it is consistent with the structures on the photo of Figure 2 (b) (a) - (c), respectively). For example, according the proposed model at $\sigma_f = 743\text{MPa}$, $n = 10\text{cycles}$ IV-level defects reach the failure state, V-level defects generate (the corresponding structure photo is shown in Figure 2 (b), (d)). At the same amplitude the state of the microstructure before failure is shown on the Figure 2 (c) (a), (c), that corresponds the failure state of V-level defects on the model; in figure 1 (c), (c), (d)) microstructures with the failure state of VI-level defects are represented.
4. Conclusion
The purpose of this work has been to present the A phenomenological model of brittle fatigue fracture of metals and alloys at proportional loading is presented. The model is constructed within the framework of the physical-mechanical approach as a system of hypotheses about the probability of defect growing on different scale structural levels (such as brittle micro- and macro cracks). From the whole scale hierarchy here is considered six scale structural levels in accordance with the stages of the metal structure evolution and different physical mechanisms. Here are formulated constitutive relations for the fracture probability at each level. The amplitude of the maximum principal stress is selected as a variable of these relations, and the material functions are determined on the results of fatigue tests at symmetric uniaxial and biaxial loading and shear and take into account the principal stress relations according to known fatigue strength criteria of metals.

Figure 2. Analysis of fatigue failure for steel S135 (0.32% C) under axial loading with torsion, $\sigma_u = 1197\,MPa$, $\sigma_f = 1112\,MPa$, $d = 0.02\,mm$: (a) – experimental data, area I-VI and the boundaries of defect evolution, $ft$ – mesoscale fatigue curve, $FT$ – the fatigue curve on complete failure, 1-6 – material constants; (b), (c) – steel microstructure

(a) $\sigma_u = 545\,MPa$ $n = 10^3$ cycles (b), $0.9N_f = 1.06 \times 10^6$ cycles (c);
(b) $\sigma_u = 594\,MPa$ $n = 40$ cycles (b), $0.9N_f = 3.13 \times 10^6$ cycles (c);
(c) $\sigma_u = 695\,MPa$ $n = 10$ cycles (b), $0.9N_f = 8.64 \times 10^6$ cycles (c);
(d) $\sigma_u = 743\,MPa$ $n = 10$ cycles (b), $0.9N_f = 5.63 \times 10^7$ cycles (c)
Here were obtained fatigue curves on each of six defect levels and fatigue macro crack formation curves for proportional tension-torsion loading and biaxial loading with different amplitude ratios. It is shown the correlation between calculated and experimental data. To justify the reliability of the proposed approach, the results of an analysis of experimental data on defect evolution at different scale-structural levels of structural steels: carbon, austenitic-martensitic, alloyed; cast irons; metals: molybdenum, nickel, plumbum, titanium, etc.; nickel, magnesium, aluminum, titanium alloys during various processes of proportional loading present in the author’s works [2].

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