Pulse shape effects in high-field Bethe–Heitler pair production

K Krajewska 1,* , J Z Kamiński 1 and C Müller 2

1 Institute of Theoretical Physics, Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland
2 Institut für Theoretische Physik I, Heinrich-Heine-Universität Düsseldorf, Universitätsstr. 1, 40225 Düsseldorf, Germany
* Author to whom any correspondence should be addressed.
E-mail: Katarzyna.Krajewska@fuw.edu.pl

Keywords: Bethe–Heitler process, electron–positron pair creation, strong laser pulses, pulse shape effects, monochromatic limit

1. Introduction

Shortly after the construction of the first laser in 1960, theoreticians started to investigate quantum electrodynamic processes in strong laser fields. In particular, nonlinear Compton scattering involving multiphoton absorption from an intense laser field as well as the crossing channel of (Breit–Wheeler) electron–positron (e−e+) pair production by a high-energy γ quantum propagating through an ultrastrong laser field were examined [1, 2]. Also the strong-field Bethe–Heitler process of e−e+ pair production by a nuclear Coulomb field and a superintense laser wave was considered [3]. The strong-field Breit–Wheeler and Bethe–Heitler processes are related in the sense that pairs are produced in the presence of an intense laser wave by a real non-laser photon in the first case, and by a virtual photon (from the spectrum of virtual photons representing a Coulomb field) in the second case. In these pioneering calculations, the laser fields were always modeled as monochromatic plane waves of infinite extent. At that time, when lasers generated field intensities on the order of 10⁶ W cm⁻² [4], these problems were of purely theoretical interest because a sizable probability for multiphoton Compton scattering requires laser field amplitudes E₀ large enough to promote the Lorentz invariant parameter μ = |eE₀|/(mₑcωₑ) close to unity, which corresponds to field intensities on the order of 10¹⁸ W cm⁻² in the optical regime. Here, e denotes the electron charge, mₑ the electron mass, c the speed of light, and ωₑ the carrier laser frequency. For laser-induced pair production, the requirements are even more severe, as the parameter χ = μℏωₑ/(mₑc²), with the reduced Planck constant ℏ, needs to approach unity.

It took more than three decades of further development of laser technology to reach the required field intensity level. In the mid 1990s, both multiphoton Compton scattering [5] and Breit–Wheeler pair production [6] were observed at the Stanford Linear Accelerator Center (SLAC) utilizing an optical laser pulse of picosecond duration and 10¹⁸ W cm⁻² intensity leading to μ ≈ 0.4. The parameter χ was boosted into the required range by using the highly relativistic electron beam at SLAC to trigger the pair production. In the rest frame of the projectile electrons the laser field strength is amplified by a factor ≈ 2γ where γ denotes the relativistic Lorentz factor of the beam.

Nowadays, new experimental prospects for studying strong-field quantum electrodynamics (QED) in laser fields are offered by upcoming high-field laboratories such as the Extreme Light Infrastructure (ELI) [7] and the Exawatt Center for Extreme Light Studies (XCELS) [8]. Field intensities of 10²⁴ W cm⁻² are
envisaged which are generated in very short laser pulses of femtosecond duration. These novel laser sources will allow to investigate hitherto unexplored domains of high-intensity laser–matter interactions.

These developments have stimulated, accordingly, substantial theoretical activities on $e^−e^+$ pair production and other QED processes in high-intensity laser fields during recent years [9–11]. In particular, since the ultrahigh laser intensities will be produced in short pulses, theoreticians have recently started to calculate strong-field QED processes in laser fields of finite extent. The focus has been laid on nonlinear Compton scattering [12–23] and Breit–Wheeler pair production [24–33]. Effects stemming from the multi-frequency composition of the pulse, its carrier-envelope phase and shape effects have been revealed. We note that analogous phenomena are also investigated in laser-atom interactions at much lower intensities, in particular with regard to photoionization in short laser pulses [34]. It is worth mentioning in this context that strong-field photoionization and pair production in intense laser fields are known to share a number of similarities which can be traced back to the formal similarity of the corresponding quantum mechanical transition amplitudes.

Similarly to the experiment at SLAC, $e^−e^+$ pair production via the strong-field Bethe–Heitler process could be realized by bringing a relativistic nuclear beam into collision with an intense laser field. To this end, the large Lorentz factors ($\gamma \sim 10^3–10^4$) available at the Large Hadron Collider at CERN could be exploited in conjunction with a superstrong laser beam of intensity $\gtrsim 10^{21}$ W cm$^{-2}$. Alternatively, at the ELI facility, corresponding studies could already be conducted at significantly lower nuclear beam energies ($\gamma \sim 10–100$).

These experimental perspectives have motivated theoreticians to thoroughly investigate the strong-field Bethe–Heitler pair production in relativistic nucleus-laser collisions. Total pair production rates as well as energy and angular distributions of the created particles were obtained in a broad range of field parameters [35–42]. Effects related to bound atomic states [43], nuclear recoil [44, 45], electron spin [46], multiple Coulomb centers [47], and bichromatic laser fields [48–51] have also been explored. Furthermore, we have studied modifications of the strong-field Bethe–Heitler process when the laser field is a finite pulse or a periodic train of pulses [52]. The envelope of a single pulse was assumed to be sin$^2$ shaped.

In this paper, we continue our investigation of $e^−e^+$ pair production by the high-field Bethe–Heitler process in laser pulses of finite length. Our main goal is to establish a connection with the earlier studies by examining the transition from a monochromatic laser wave of infinite extent to a short laser pulse. To this end, we introduce a suitable envelope function with a flat top which mimics a monochromatic plane wave for relatively long durations. By varying the extent of the flat portion of the pulse at a fixed duration of its ramp-on and ramp-off portions, or vice versa, we examine characteristic pulse signatures and the transition to the monochromatic plane wave limit. Our systematic analysis is relevant for future experimental studies on strong-field QED phenomena in short laser pulses as it indicates under which conditions well-established monochromatic theories may still provide good approximations and keep their predictive power.

Throughout the paper, we use the notation and mathematical convention introduced in our previous paper on the electron–positron pair creation [52]. In particular, in formulas below we keep $\hbar = 1$ but our numerical results are presented in relativistic units (r.u.) such that $\hbar = c = m_e = 1$.

2. Theory

The process of electron–positron pair creation via collisions of relativistic protons with an ultra-intense laser pulse is investigated. We focus on a single proton response, treating it as an infinitely massive particle. Such a treatment allows us to study purely laser-field-related effects on the pair creation process. Note that the same approach was used in our previous paper [52]. While in reference [52] we focused on aspects such as sensitivity of pair creation to a driving pulse duration or to its carrier-envelope phase, in the current paper we investigate pulse shape effects.

As discussed in reference [52], we describe a driving laser field as a plane-wave pulse, the way it was originally introduced in the context of Compton scattering [53]. If the driving field is linearly polarized, it is described by the following four-vector potential

$$A(\mathbf{k} \cdot \mathbf{x}) = A_0 e f(\mathbf{k} \cdot \mathbf{x}),$$

(1)

where the polarization four-vector $\varepsilon = (0, \varepsilon)$ is such that $\varepsilon^2 = -1$ and $\mathbf{k} \cdot \varepsilon = 0$. Here, $\mathbf{k}$ is the wave four-vector $\mathbf{k} = (\omega/c)n \equiv (\omega/c)(1, \mathbf{n})$, $\omega$ denotes the fundamental field frequency, whereas $\mathbf{n}$ determines the direction of pulse propagation. In equation (1), we have also introduced $A_0$ which describes the peak potential and which can be conveniently expressed in terms of the intensity parameter $\mu$, since $\mu = |eA_0|/(mc^2)$. In the pulsed plane wave approximation, one assumes a plane wave with a finite extent in
the direction of propagation. This can be satisfied by choosing a shape function \( f(k \cdot x) \) in equation (1) such that
\[
f(k \cdot x) = 0 \quad \text{for} \quad k \cdot x < 0 \quad \text{and} \quad k \cdot x > 2\pi.
\] (2)
Hence, one can understand that the fundamental frequency \( \omega \) is related to the pulse duration \( T_p = 2\pi/\omega \).
This is in contrast to the carrier laser frequency \( \omega_c \), as introduced in section 2.1. Note that \( f(k \cdot x) \) and its first two derivatives should be smooth functions of the argument, \( k \cdot x \). This will guarantee that the electric and magnetic components of the field,
\[
E(k \cdot x) = -\frac{\omega A_0}{c} f'(k \cdot x),
\]
\[
B(k \cdot x) = -\frac{\omega A_0}{c} (n \times e) f'(k \cdot x),
\] (3) (4)
are well defined, along with the Maxwell’s equations that they satisfy. (Here, ‘prime’ means a derivative with respect to \( k \cdot x \).)
In reference [52], we performed calculations for a pulsed field with a sine-squared envelope which is typically used in strong-field QED calculations (see, also references [16, 24, 26]). As we demonstrated in [52], the results obtained for this pulse envelope cannot be compared in a meaningful way with the corresponding results obtained for a plane wave field. Therefore, in our investigations of pulse shape effects on pair creation we will use a more sophisticated envelope, as introduced below.

2.1. Pulse model
We consider a pulse propagating in the z-direction (\( n = e_z \)), with its linear polarization vector along the x-axis (\( e = e_x \)). Moreover, we choose the shape function in equation (1) such that
\[
f(\phi) = F(\phi) \sin(N_{osc} \phi + \vartheta),
\] (5)
where \( \phi = k \cdot x \) is the field phase, \( N_{osc} \) denotes a number of cycles within a pulse, and \( \vartheta \) is a carrier-envelope phase. Here, we introduce the carrier laser frequency \( \omega_c = N_{osc} \omega \), which relates to duration of one laser cycle. For the pulse shape function, we take
\[
F(\phi) = \begin{cases} 
0 & \phi < 0 \\
\sin^2 \left( \frac{\phi}{4\xi} \right) & 0 \leq \phi \leq 2\pi \xi \\
1 & 2\pi \xi < \phi < 2\pi(1 - \zeta) \\
\sin^2 \left( \frac{2\pi - \phi}{4\xi} \right) & 2\pi(1 - \zeta) \leq \phi \leq 2\pi \\
0 & \phi > 2\pi.
\end{cases}
\] (6)

Note that, for \( \xi \neq 0 \) and \( \zeta \neq 0 \), this function and its derivatives are smooth functions of the phase \( \phi \) which guarantees that the vector potential (1) and the fields, equations (3) and (4), are physically relevant. Equation (6) defines a flat-top pulse envelope with, in general, asymmetric wings characterized by parameters \( \xi \) and \( \zeta \). These parameters are related to the number of field oscillations within each wing. If we denote by \( N_u \) and \( N_d \) the number of cycles in the raising and falling wing, respectively, we can write down that \( \xi = N_u/N_{osc} \) and \( \zeta = N_d/N_{osc} \). With this interpretation, one can understand that \( \xi + \zeta \leq 1 \). For completeness, let us also introduce a variable \( N_{flat} = N_{osc} - N_u - N_d \), which denotes a number of cycles in the pulse plateau, i.e. for \( 2\pi \xi < \phi < 2\pi(1 - \zeta) \). Note also that in the case when \( \xi + \zeta = 1 \) the pulse has no flat top, whereas for \( \xi = \zeta = 0 \) equation (6) represents a rectangular pulse with no wings (the latter will be analyzed separately in section 3). In the following, we shall also consider an infinite train comprising of—potentially asymmetric—flat-top pulses. For this, we shall repeat the aforementioned model of a pulse periodically.

2.2. Probability distribution of pair creation
We consider the electron–positron pair creation in collision of a laser pulse with a proton; thus, resulting from the superposition of the laser and the nuclear fields. Following the standard approach to the problem, we treat both fields classically, implying that they remain undistorted by the pair production process. For the laser field this is justified because depletion effects can be ignored due to the extremely large photon number in a high-intensity laser pulse [54–56]. Hence, the overall electromagnetic four-vector potential in the proton rest frame equals
\[
A''(x) = A'(k \cdot x) + \frac{1}{ec} V(x)\delta',
\] (7)
where \( A(k \cdot x) \) is given by equation (1), whereas \( \frac{1}{2} V(x) = -\frac{\alpha}{\beta x} \) with the fine structure constant \( \alpha \). Note that the same approach relates to ions of charge \(-Z e > 0\), in which case \( \alpha \) has to be replaced by \( Z\alpha \) in \( V(x) \) and all proceeding formulas describing the probability distributions have to be multiplied by \( Z^2 \). According to the Feynman rules, the process can be represented by the transition matrix element between the negative energy Volkov state \( \psi_{p_- \lambda_-}^{(-)}(x) \) and the positive energy Volkov state \( \psi_{p_+ \lambda_+}^{(+)}(x) \) due to the interaction with the nuclear field,

\[
S_0 = -i \int d^4x \psi_{p_- \lambda_-}^{(+)}(x) \frac{1}{c} V(x) \gamma^0 \psi_{p_+ \lambda_+}^{(-)}(x).
\]

Here, the Volkov states represent the exact solutions of the Dirac equation coupled to the laser field \[57\], and are labeled by the positron and the electron asymptotic momenta \( p_{\pm e} \) and their spin projections \( \lambda_{\pm e} \). Thus, in essence, the matrix element (8) accounts for the laser field to all orders in the Volkov states, while treating the nuclear Coulomb field to the leading order of the perturbation theory.

As introduced in reference \[52\], two different formulations are required when studying the nonlinear Bethe–Heitler process by a single pulse and by a train of pulses. Departing from equation (8), we have shown in \[52\] that the differential probability distribution of pair creation by a single pulse can be defined as a Fourier series in terms of the fundamental frequency \( \omega \). Thus, taking the form

\[
\frac{d^6P^{(p)}}{dE_{p_-}dE_{p_+}d\Omega_{p_-}d\Omega_{p_+}} = \frac{\alpha^2 m_e^2}{4\pi^4} \sum_{\{\lambda\}} \frac{|P_{\pm e}||P_{\pm e}|^*}{[(P_{\pm e} + P_{\pm e})^2 + (n \cdot P_{\pm e} + n \cdot P_{\pm e})^2]^2} \times \sum_{N} C_{N}^0 \left(\frac{1 - e^{-2\pi i(N - N_{\text{eff}})}}{k^0(N - N_{\text{eff}})}\right)^2.
\]

Since the fundamental frequency \( \omega \) is related to the pulse duration (not to the duration of one cycle), \( N \) cannot be interpreted as a number of laser photons leading to pair creation. This makes the formulation presented in \[52\] and, hence, equation (9) general in the sense that they are valid for arbitrary pulse durations. Here, \( p_{\pm e} = \langle E_{p_{\pm e}} / \epsilon, p_{\pm e} \rangle \) stands for a positron or an electron final four-momentum, where \( p_{\pm e} \) may have in general the longitudinal \( p_{\parallel e} = p_{\pm e} \cdot n \) and the transverse \( p_{\perp e} = p_{\pm e} - p_{\parallel e} n \) components. In addition, \( N_{\text{eff}} \) is expressed in terms of laser-dressed momenta, \( p_{\pm e} = \langle E_{p_{\pm e}} / \epsilon, p_{\pm e} \rangle \), such that

\[
N_{\text{eff}} = \frac{E_{p_{+ e}} + E_{p_{- e}}}{\omega}.
\]

These four-momenta are defined according to

\[
p_{\pm e} = p_{\pm e} \pm \frac{\mu m_e c}{k \cdot p_{\pm e}} (f) k + \frac{1}{2} \left( \frac{\mu m_e c}{k \cdot p_{\pm e}} \right)^2 (f^2) k,
\]

where \( \langle \ldots \rangle \) means the average over the entire pulse duration (see, equation (23) of reference \[52\]). \( \sum_{\{\lambda\}} \) in equation (9) denotes summation over the final spin degrees of freedom of product particles. The coefficients \( C_{N}^{0} \) are expressed as combinations of generalized Bessel functions (for more details, see equation (28) of reference \[52\]).

We stress that although the laser-dressed momenta (11) depend on a gauge, \( N_{\text{eff}} \) and the expression (9) are gauge-invariant \[52\]. It is also worth mentioning that under condition that \( N_{\text{eff}} \approx N \), meaning that

\[
E_{p_{+ e}} + E_{p_{- e}} \approx N\omega,
\]

the probability distribution of electron–positron pair creation (9) becomes maximum. This will be further analyzed numerically in section 3, where a comparison of pair creation spectra induced by a single pulse and an infinite sequence of pulses will be made. In doing so, we need to transform equation (9) such that

\[
\frac{d^6P^{(p)}}{dN_{\text{eff}}dE_{p_-}d\Omega_{p_-}d\Omega_{p_+}} = \left| \frac{\partial E_{p_{+ e}}}{\partial N_{\text{eff}}} \right| \frac{d^6P^{(p)}}{dE_{p_-}dE_{p_+}d\Omega_{p_-}d\Omega_{p_+}}.
\]

This quantity is an analog of the probability distribution of pair creation stimulated by one pulse from a train \[52\]

\[
\frac{d^6P^{(i)}}{dN dE_{p_-}d\Omega_{p_-}d\Omega_{p_+}} = \frac{(\alpha m_e c)^2}{\pi^2 \epsilon c^3} \sum_{\{\lambda\}} \frac{|P_{\pm e}||P_{\pm e}|}{D(p_{\pm e})} \left| \frac{G_N}{(p_{\pm e} + p_{\pm e} - N k)^2} \right|_{p_{\pm e} = \epsilon p_{\pm e}}.
\]
3. Numerical illustrations

Our purpose is to examine the validity of a monochromatic plane wave approximation in laser-induced Bethe–Heitler process. As introduced in the previous section, we consider two cases—when the process is induced by a single pulse and by an infinite sequence of identical pulses. For numerical illustrations, we take a flat-top pulse with the envelope given by equation (6) such that $\mu = 2$, $\omega_1 = 0.1 m_e c^2$ (i.e. $\chi = 0.2$), and $\vartheta = 0$. For a train, we repeat this pulse infinitely many times. Regarding the chosen value of $\omega_1$, we note that, when a proton with Lorentz factor $\gamma = 10^4$ and a laser pulse with 2.5 eV photon energy collide head-on, such a laser frequency is obtained in the rest frame of the proton. As already mentioned in section 2, the results presented in this paper refer to the proton rest frame.

In figure 1, we compare the energy spectra of positrons calculated for the case when the Bethe–Heitler process occurs under the influence of an isolated pulse and a train of pulses assuming that an individual pulse has one cycle in each wing, $N_u = N_d = 1$, while 14 cycles in the plateau, $N_{\text{flat}} = 14$. The results based on equation (13) for a single pulse are plotted as the blue solid line, whereas the results based on equation (14) for a train of pulses are plotted as black dots (they are connected for visual purposes only). In this figure, we also present the results for a monochromatic plane wave. Note that such a wave can be considered as an infinite train of rectangular pulses, and, therefore, it can be described by equation (6) where we assume that $N_u = N_d = 0$. The corresponding results are presented as a stem plot in figure 1. The spectra are for the final positron direction specified by the angles $\theta_{e+} = 0.1\pi$ and $\varphi_{e+} = 0$. The accompanying electron with the final momentum $|p_{e-}| = m_e c$ is detected at $\theta_{e-} = 0.1\pi$ and $\varphi_{e-} = \pi$.

As it follows from figure 1, for a long flat-top pulse with rather sharp turn-on and turn-off wings as well as for a train of such pulses, which are driving the pair creation, there is a very good agreement with the results calculated for a monochromatic plane wave. The latter consists of a discrete spectrum at subsequent multiphoton orders between 56 and 84 in terms of $\omega_1$. The agreement between the presented spectra

This time, $N$ corresponds to a number of photons exchanged with the pulse train whereas the laser-dressed momenta of created particles, $\tilde{p}_{e\pm}$, are gauge-independent,

$$\tilde{p}_{e\pm} = p_{e\pm} + \frac{1}{2} (\mu m_e c^2) \frac{(f^2)}{k \cdot \tilde{p}_{e\pm}} k.$$  \hspace{1cm} (15)

Here, $\langle \ldots \rangle$ means the average over an individual pulse from the train. In addition, the quantities $G_N$ and $D(p_{+})$ are defined by equations (36) and (43) in reference [52], with $G_N$ expressed in terms of generalized Bessel functions. Moreover, as it follows from equation (14), the spectrum is calculated at certain positron energies $E_{p_{+}}^{\nu}$. The latter solve the energy conservation condition for the Bethe–Heitler process induced by an infinite train of pulses, in which case equation (12) has to be strictly satisfied. This indicates that, under certain conditions which will be analyzed in the next section, the distributions (13) and (14) may possibly coincide. Note also that equation (14) is valid for pair creation by a monochromatic plane wave.
concerns the location of multiphoton peaks as well as their modulations. Still, the peaks in the positron spectrum obtained for a single laser pulse are higher than the corresponding peaks for a monochromatic plane wave and a train of pulses. Moreover, this difference increases with increasing the positrons energy. As one can see in figure 1, the multiphoton peaks are not equidistant which follows from the laser-field dressing of produced particles (see equation (12)). Note that the dressing is, in principle, defined differently for a finite pulse (equation (11)) and for an infinite train of pulses (equation (15)) (the latter concerns also the case of a monochromatic plane wave). This difference is negligible in the current case as all distributions acquire maxima at the exactly same positron energies. The reason being that for a long enough pulse the term with $\langle f \rangle$ in equation (11) is negligible, resulting in the same laser-field dressing as in the case of a monochromatic plane wave (15). The same seems to be still true for the case presented in figure 2, where the flat portion of the driving pulse consists of six oscillations ($N_{\text{flat}} = 6$). As it follows from our calculations, starting with $N_{\text{flat}} = 3$ one can observe, however, more pronounced differences. Going back to equation (12), we adapt this condition to figures 1 and 2 (taking $\langle f \rangle \approx 0$) and write it explicitly as

$$E_{p_{\text{e}}} - E_{p_{\text{e}}} + \frac{1}{2}(\mu m_{e}c^{2})^{2} \left( f^{2} \right) \left( \frac{1}{E_{p_{\text{e}}} - cp_{\text{e}}} + \frac{1}{E_{p_{\text{e}}} + cp_{\text{e}}} \right) \approx N\omega.$$  \hfill (16)\end{eqnarray*}

Note that this is a nonlinear condition with respect to either positron or electron energy which explains why the multiphoton peaks in the energy spectra of produced particles (like the ones presented in figures 1 and 2) are not equidistant.

In order to compare the distributions for an isolated pulse and a pulse train (that includes also a monochromatic plane wave), we have used equation (13). The respective comparison resulted in figures 1 and 2, which show a roughly one order of magnitude difference between the results. Let us note, however, that the formula (13) contains $|dE_{p_{\text{e}}} / dN_{\text{eff}}|$ which could, in principle, contribute to such difference. Another way to compare the results is by plotting the cumulative probability distributions. Such cumulative distributions for the same parameters as in figure 1 are presented in figure 3. Here, however, we do not account for $|dE_{p_{\text{e}}} / dN_{\text{eff}}|$. In other words, we present the cumulative distribution functions for the probability densities defined by equation (9) for the pulse (top panel) and by equation (14) for both the pulse train (middle panel) and monochromatic plane wave (bottom panel). One can observe that each curve exhibits a typical step-like behavior. Namely, for energies for which we observe multiphoton peaks, the cumulative probability distributions increase significantly. Rather than that, the distribution functions exhibit multiple plateaus where their values are nearly constant.

In figure 4, we present the energy distributions of created positrons for the case when the flat-top laser pulse collides with a proton for the same parameters as in figure 1, except that now we decrease the number of cycles in the flat portion of the pulse. This is presented for (a) $N_{\text{flat}} = 14$, (b) $N_{\text{flat}} = 6$, (c) $N_{\text{flat}} = 3$, and (d) $N_{\text{flat}} = 1$. The data in the top panel have been already plotted in figure 1. We know, therefore, that the narrow major maxima correspond to the monochromatic peaks. Note also that, while the spectra in figure 1 were presented in logarithmic scale, now we keep the linear scale in the figure. Hence, the previously observed rapid side oscillations accompanying the monochromatic peaks are not very pronounced, even though still present. If the process occurs in a long laser pulse with a flat-top, we observe a continuous spectrum with regular and narrow peaks. With decreasing the pulse duration, the peaks become broader, and for still shorter driving pulses the regular peak structure is lost. These are general features observed for

![Figure 2. The same as in figure 1 but for $N_{\text{flat}} = 6$.](image)
other processes as well, when induced by a laser field in both relativistic and nonrelativistic domains. The link between these processes is the energy-time uncertainty relation. It follows from this relation that, when the process is driven by an infinitely long plane wave, the energy interval over which the system undergoes a significant change has to be zero. This leads to discrete energy distributions, as the one presented by the stem plots in figures 1 and 2. For a finite but long laser pulse driving the process, the corresponding energy interval has to be narrow, which results in very narrow peaks in the energy spectrum (figure 4(a)). With decreasing the pulse duration, the system changes substantially over a broader energy interval, which is observed in figure 4 as wider peaks. To make our analysis more quantitative, let us check how the areas under a certain peak in the energy spectra of positrons compare when the plateau length of the driving laser pulse is varied. The point being that in the monochromatic case the production probability grows linearly with the interaction time. It is interesting to check, whether this holds for our data as well. For this reason, we have integrated the probability distributions across the width of one of the peaks in figure 4(a)—taking as the example the peak below $1.5 \, m_e c^2$—and then we have taken integrals of our data presented in other panels over the same energy interval. In doing so, we have obtained $4.89 \times 10^{-15} \text{ r.u.}$ for $N_{\text{flat}} = 14,$
Figure 4. Positron energy distributions calculated for the Bethe–Heitler process induced by a flat-top pulse. The pulse shape is defined by equation (6) where we assume that there is one cycle in the turn-on and off parts of the pulse, \( N_u = N_d = 1 \). Different panels correspond to different number of cycles in the pulse plateau: (a) \( N_{\text{flat}} = 14 \), (b) \( N_{\text{flat}} = 6 \), (c) \( N_{\text{flat}} = 3 \), and (d) \( N_{\text{flat}} = 1 \). The remaining laser field parameters are \( \omega_L = 0.1 m_e c^2 \) and \( \mu = 2 \). The spectra are for the final positron direction specified by the angles \( \theta_e^+ = 0.1 \pi \) and \( \phi_e^+ = 0 \). The accompanying electron with the final momentum \( |p_e^-| = m_e c \) is detected at \( \theta_e^- = 0.1 \pi \) and \( \phi_e^- = \pi \).

\[
2.16 \times 10^{-15} \, \text{r.u. for } N_{\text{flat}} = 6, \ 1.13 \times 10^{-15} \, \text{r.u. for } N_{\text{flat}} = 3, \ \text{and } 4.56 \times 10^{-16} \, \text{r.u. for } N_{\text{flat}} = 1. \text{ Note that the obtained quantity resembles the integral resonance strength which is often considered in atomic physics. Most importantly, it does scale linearly with the duration of the flat-top portion of the pulse for as long as } N_{\text{flat}} \gtrsim 3. \text{ However, already for } N_{\text{flat}} = 2 \text{ (which is not presented in figure 4) the integral resonance strength for the chosen peak equals } 7.96 \times 10^{-16} \, \text{r.u.}, \text{ showing more pronounced deviation from the linear dependence on } N_{\text{flat}} \text{ than in the previous cases. This tendency manifests itself even more strongly for } N_{\text{flat}} = 1. \text{ The exactly same conclusion can be drawn when calculating the integral probability across other peaks as well.}

We note that the distributions presented in figure 4 exhibit a strong modulation of the strength of multiphoton peaks. Specifically, we observe a strong suppression of peaks at positron energies of roughly \( 1.1 m_e c^2 \) and \( 1.3 m_e c^2 \). As we have checked numerically, those modulations vary with parameters of the laser field, i.e. with \( \mu \) and \( \omega_L \). The same happens if we change the emission geometry of created particles, but keep the parameters of the laser field fixed. To explain such behavior, we go back to equation (9) defining the probability distribution of positrons created in a proton–laser-pulse collision. It contains the square of a
Figure 5. The same as in figure 4 but we keep the number of cycles in the flat portion of the driving pulse fixed $N_{\text{flat}} = 6$, whereas we change the number of cycles in the ramp-on and off portions of it such that in panel (a) $N_d = N_u = 3$ and in (b) $N_d = N_u = 12$.

Figure 6. The same as in figure 4 for an asymmetric flat-top pulse with a fixed duration of plateau equal to $N_{\text{flat}} = 3$. The green line is for $N_u = 2$ and $N_d = 7$, the blue dots are for $N_u = 3$ and $N_d = 6$, whereas the magenta curve is for $N_u = 4$ and $N_d = 5$.

sum over $N$ wherein each term involves a sinc function, having finite width. Thus, it may happen that contributions from different values of $N$ around $N_{\text{eff}}$ add up either constructively or destructively, depending on their phases and the sign of the coefficients $C_n^{(0)}$. This is the origin of interference, which results in modulations (and frequently suppression) of peaks in the positron energy spectra.

In regard to figure 4, it is interesting to estimate the total probability of pair creation $P_{P}$. For the cases such that $N_{\text{flat}} \gtrsim 3$, we have seen that the integral resonance strength is linearly proportional to the pulse length. Therefore, it seems reasonable to estimate the total probability by multiplying the known pair
production rate in an infinitely extended plane laser wave by the duration $T_p$ of our pulses. Since $\mu = 2$ and $\chi = 0.2$, we can use a tunneling-like formula for the total rate, e.g., equation (42) in reference [38], which leads to a scaling law: $P^{(0)} = 4.4 \times 10^{-15} \cdot N_{\text{osc}}$. Hence, for the cases presented in figures 4(a)–(c), the total probability of pair creation (per one proton) is on the order of $10^{-14} - 10^{-13}$. For the case plotted in figure 4(d), on the other hand, a better estimate would be provided by Monte Carlo methods introduced in references [37, 39].

In figure 5, we present the energy spectra of positrons for the same parameters as in figure 4 except that, this time, both panels correspond to the same number of cycles in the flat-top portion of the driving pulse ($N_{\text{flat}} = 6$) while we change the number of cycles in its wings. More specifically, panel (a) is for $N_d = N_u = 3$ whereas panel (b) is for $N_d = N_u = 12$. Besides, the case $N_{\text{flat}} = 6, N_u = N_d = 1$ can be seen in figure 4(b). As we have checked for other parameters as well, the overall behavior is such that with increasing $N_u$ and $N_d$ the monochromatic peaks are still present but the accompanying side oscillations become more abrupt. This is similar to side-peak structures observed in nonlinear Compton and Thomson scattering, as described for instance in references [12, 13]. In order to interpret these results let us note that, for a monochromatic plane wave field driving the pair production, the regular peaks in the energy spectrum of created particles result from constructive intercycle interferences. If the process is driven by a finite flat-top pulse with a flat portion of $N_{\text{flat}} \gtrsim 3$ and sharp raising and falling wings, we observe that the spectrum is dominated by monochromatic peaks on the quantitative level. This has been proven when comparing the integral resonance strengths for figure 4. Moreover, as we have checked that, with increasing the duration of the pulse wings the corresponding integral probability distribution calculated across the given monochromatic peak is also increasing. Specifically, it equals $2.23 \times 10^{-13}$ r.u. for panel (a) and $2.96 \times 10^{-15}$ r.u. for panel (b), and has to be compared with $2.16 \times 10^{-15}$ r.u. for the case presented in figure 4(b). This shows that the intercycle interferences originating from the ‘monochromatic’ portion of the driving field are dominant. However, if the driving pulse does not raise and fall sharply, interferences originating from the wings of the pulse become increasingly more important.

Finally, in figure 6 we present the energy spectra of positrons generated by asymmetric pulses with the fixed flat-top portion equal to $N_{\text{flat}} = 3$. In contrast, the ramp-on and off wings of the pulse are varied in duration. Specifically, the green line is for $N_u = 2$ and $N_d = 9$, the blue dots are for $N_u = 3$ and $N_d = 8$, whereas the magenta curve is for $N_u = 4$ and $N_d = 7$. In the upper panel, the distribution over a large positron energy interval is shown. It forms a highly oscillating pattern with the envelope maximum at positron energy of roughly $3m_e c^2$. The enlarged portion of the distribution is presented in the lower panel over the energy interval that spans across five multiphoton

Figure 7. The same as in figure 6 for an asymmetric flat-top pulse such that $N_{\text{flat}} = 1$. Moreover, the green line is for $N_u = 2$ and $N_d = 9$, the blue dots are for $N_u = 3$ and $N_d = 8$, whereas the magenta curve is for $N_u = 4$ and $N_d = 7$. 

as we illustrate in figure 8. Here, we present the positron energy distribution for an eight-cycle rectangular pulse ($N_{\text{flat}} = 8$ and $N_u = N_d = 0$). The remaining parameters are the same as in figure 4. In the upper panel, the distribution over a large positron energy interval is shown. It forms a highly oscillating pattern with the envelope maximum at positron energy of roughly $3m_e c^2$. The enlarged portion of the distribution is presented in the lower panel over the energy interval that spans across five multiphoton
peaks. Note that the oscillations covering the energy interval of a single multiphoton peak are grouped in series of eight. Thus, they are separated by a fundamental frequency of the pulse $\omega = \omega_L/N_{\text{osc}}$, which in our case equals $\omega = \omega_L/N_{\text{flat}} = 0.0125 m_e c^2$. We would like to stress that there is no physical reason for such pattern to appear. As we have indicated below equation (2), in order for the vector potential to be physically meaningful it has to be at least class $C^2$ function. Note that the rectangular vector potential does not satisfy this condition, which results in a spurious behavior of probability distributions of created particles.

4. Conclusions

We have theoretically investigated the laser-induced electron–positron pair creation via the Bethe–Heitler mechanism. The idea was to compare the probability distributions for the case when the laser field is modeled either as a finite pulse, an infinite pulse train or a monochromatic plane wave. The point of such comparison was to determine under which conditions the well-known approximation, i.e. describing the laser field as a monochromatic plane wave, is physically valid. For such comparison we have assumed a flat-top pulse envelope.

It follows from our studies that if the laser pulse has sharp turn-on and off portions, the distributions of created particles are, in general, very similar to those obtained in the monochromatic plane wave field approximation. This is particularly true if the flat-top portion of the pulse is relatively long, namely, for $N_{\text{flat}} \gtrsim 3$. If, however, the pulse plateau contains fewer cycles the differences become not only quantitative but also qualitative.

On the other hand, one could expect that turning on and off the laser pulse more adiabatically, i.e. by increasing the number of cycles in each arm, would make the results even more similar to those obtained for a monochromatic plane wave. As we have demonstrated numerically, this is not necessarily the case, as the spectra exhibit additional side lobes with increasing the duration of the ramp-up and the ramp-down portions of the pulse.

Finally, in the limiting case of a rectangular pulse, we have observed that the resulting energy distributions of created particles exhibit nonphysical behavior. This has been attributed to the fact that the

Figure 8. The positron energy distribution generated in collision of a rectangular laser pulse with a proton. Here, $N_{\text{flat}} = 8$ whereas $N_d = N_u = 0$. The remaining parameters are the same as in figure 4. In the lower panel we demonstrate an enlarged portion of the distribution presented in the upper panel.
rectangular vector potential is physically irrelevant, leading to discontinuous electric and magnetic components of the laser field.

As an outlook, we note that our investigation has been carried out within a plane-wave model for the laser pulses. In experiment, very high field intensities are usually achieved by strong laser focusing. The more tightly focused a laser pulse is, the more it deviates from a plane wave. Focusing effects have so far been examined for nonlinear Breit–Wheeler pair production. A numerical example for $\mu \approx 10$ showed that the shape of the angle-resolved positron energy spectra is affected only moderately, whereas the total production probability is reduced substantially by the field focusing [28]. While qualitatively similar focusing effects may be expected for the strong-field Bethe–Heitler process as well, a corresponding calculation has yet to be performed.

Acknowledgments

This work has been supported by the National Science Center (Poland) under Grant Nos. 2014/15/B/ST2/02203 and 2018/31/B/ST2/01251, and by the Deutsche Forschungsgemeinschaft (DFG) under Grant No. 416699545 within the Research Unit FOR 2783/1.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

ORCID iDs

K Krajewska https://orcid.org/0000-0003-2636-6222
J Z Kamiński https://orcid.org/0000-0003-0970-3477

References

[1] Reiss H R 1962 J. Math. Phys. 3 59
[2] Nikishov A I and Ritus V I 1964 Zh. Eksp. Teor. Fiz. 46 776
Nikishov A I and Ritus V I 1964 Sov. Phys. JETP 19 529
[3] Yakovlev V P 1965 Zh. Eksp. Teor. Fiz. 49 318
Yakovlev V P 1966 Sov. Phys. JETP 22 223
[4] Franken P A, Hill A E, Peters C W and Weinreich G 1961 Phys. Rev. Lett. 7 118
[5] Bula C et al 1996 Phys. Rev. Lett. 76 3118
[6] Burke D et al 1997 Phys. Rev. Lett. 79 1626
[7] Mourou G A, Korn G, Sandner W and Collier J I 2013 ELI Whitebook https://eli-laser.eu/media/1019/eli-whitebook.pdf
[8] Institute of Applied Physics RAS 2009 Exawatt Center for Extreme Light Studies (XCELS) http://xcels.iapras.ru/img/XCELS-Project-english-version.pdf
[9] Ehlotzky F, Krajewska K and Kamiński J Z 2009 Rep. Prog. Phys. 72 046401
[10] Di Piazza A, Müller C, Hatsagortsyan K Z and Keitel C H 2012 Rev. Mod. Phys. 84 1177
[11] Narozhny N B and Fedotov A M 2015 Contemp. Phys. 56 249
[12] Boca M and Florescu V 2009 Phys. Rev. A 80 053403
Boca M and Florescu V 2011 Eur. Phys. J. D 61 449–62
[13] Heinzl T, Seipt D and Kämpfer B 2010 Phys. Rev. A 81 022125
Seipt D and Kämpfer B 2011 Phys. Rev. A 83 022101
[14] Mackenroth F and Di Piazza A 2011 Phys. Rev. A 83 032106
[15] Voroshilo A I, Roshchupkin S P and Nedoroshina V N 2011 Laser Phys. 21 1675
[16] Krajewska K and Kamiński J Z 2012 Phys. Rev. A 85 062102
[17] Seipt D, Khairin V, Rykovyan S, Surzyhkov A and Fritzschke S 2016 J. Plasma Phys. 82 655820203
[18] Blackburn T G, Seipt D, Bulanov S V and Marklund M 2018 Phys. Plasmas 25 083108
[19] Chen Y, Li J, Hatsagortsyan K Z and Keitel C H 2018 Phys. Rev. Lett. 121 074801
[20] Heinzl T, King B and MacLeod A J 2020 Phys. Rev. A 102 063110
[21] Li Y, Shaisultanov R, Chen Y, Wan F, Hatsagortsyan K Z, Keitel C H and Li J 2020 Phys. Rev. Lett. 124 014801
[22] Ilderton A, King B and Tang S 2020 Phys. Lett. B 804 135410
[23] King B 2021 Phys. Rev. D 103 036018
[24] Heinzl T, Ilderton A and Marklund M 2010 Phys. Lett. B 692 250
[25] Titov A I, Takabe H, Kämpfer B and Hosaka A 2012 Phys. Rev. Lett. 108 240406
[26] Krajewska K and Kamiński J Z 2012 Phys. Rev. A 86 052104
[27] Ipp A, Evers J, Keitel C H and Hatsagortsyan K Z 2011 Phys. Lett. B 702 383
[28] Di Piazza A 2016 Phys. Rev. Lett. 117 213201
[29] Meuren S, Keitel C H and Di Piazza A 2016 Phys. Rev. D 93 085028
[30] Jansen M J A, Kamiński J Z, Krajewska K and Müller C 2016 Phys. Rev. D 94 013010
[31] Titov A I, Takabe H and Kämpfer B 2018 Phys. Rev. D 98 036022
[32] Lv Q Z, Dong S, Li Y T, Sheng Z M, Su Q and Grobe R 2018 Phys. Rev. A 97 022515
[33] Wan F, Wang Y, Guo R T, Chen Y Y, Shaisultanov R, Xu Z F, Hatsagortsyan K Z, Keitel C H and Li J X 2020 Phys. Rev. Res. 2 032049
[34] Milošević D B, Paulus G G, Bauer D and Becker W 2006 J. Phys. B: At. Mol. Opt. Phys. 39 R203
[35] Müller C, Voitkiv A B and Grün N 2003 Phys. Rev. A 67 063407
[36] Averissian H K, Averissian A K, Mkrtchian G F and Sedrakian K V 2003 Nucl. Instrum. Methods A 507 582
[37] Sieczka P, Krajewska K, Kamiński J Z, Panek P and Ehlotzky F 2006 Phys. Rev. A 73 053409
[38] Milstein A I, Müller C, Hatsagortsyan K Z, Jentschura U D and Keitel C H 2006 Phys. Rev. A 73 062106
[39] Kamiński J Z, Krajewska K and Ehlotzky F 2006 Phys. Rev. A 74 033402
[40] Kuchiev M Y and Robinson D J 2007 Phys. Rev. A 76 012107
[41] Krajewska K and Kamiński J Z 2008 Laser Phys. 18 185
[42] Di Piazza A and Milstein A I 2012 Phys. Lett. B 717 224
[43] Müller C, Voitkiv A B and Grün N 2003 Phys. Rev. Lett. 91 223601
[44] Müller S J and Müller C 2009 Phys. Rev. D 80 053014
[45] Krajewska K and Kamiński J Z 2011 Phys. Rev. A 84 033416
[46] Müller T O and Müller C 2012 Phys. Rev. A 86 022109
[47] Fillion-Gourdeau F, Lorin E and Bandrauk A D 2013 Phys. Rev. Lett. 110 013002
[48] Roshchupkin S P 2001 Phys. At. Nucl. 64 243–52
[49] Lebed A A and Roshchupkin S P 2011 Laser Phys. 21 1613–20
[50] Di Piazza A, Lötstedt E, Milestein A I and Keitel C H 2010 Phys. Rev. A 81 062122
[51] Krajewska K and Kamiński J Z 2012 Phys. Rev. A 85 043404
[52] Krajewska K and Kamiński J Z 2012 Phys. Rev. A 86 021402
[53] Augustin S and Müller C 2013 Phys. Rev. A 88 022109
[54] Krajewska K, Müller C and Kamiński J Z 2013 Phys. Rev. A 87 062107
[55] Neville R A and Rohrlich F 1971 Phys. Rev. D 3 1692
[56] Bergou J and Varró S 1981 J. Phys. A: Math. Gen. 14 2281
[57] Seipt D, Heintel T, Marklund M and Bulanov S S 2017 Phys. Rev. Lett. 118 154803
[58] Ilderton A and Seipt D 2018 Phys. Rev. D 97 016007
[59] Volkov D M 1935 Z. Phys. 94 250–60