MODULATED X-RAY EMISSIVITY NEAR THE STRESS EDGE IN SAGITTARIUS A*
Maurizio Falanga,1,2 Fulvio Melia,1,4 Martin Prescher,3 Guillaume Bélanger,3 and Andrea Goldwurm1,6
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1 CEA Saclay, DSM/IRFU/Service d’Astrophysique, Gif-sur-Yvette, France; mafalanga@cea.fr.
2 AIM-Uniité Mixte de Recherche CEA-CNRS-Université Paris Diderot.
3 Physics Department and Steward Observatory, University of Arizona, Tucson, AZ.
4 Sir Thomas Lyle Fellow and Miegunyah Fellow.
5 ESA/ESAC, Madrid, Spain.
6 UMR Astroparticule et Cosmologie, Paris, France.

ABSTRACT

Sgr A* is thought to be the radiative manifestation of a $\sim3.6 \times 10^6 M_\odot$ supermassive black hole at the Galactic center. Its millimeter/submillimeter spectrum and its flare emission at IR and X-ray wavelengths may be produced within the inner 10 Schwarzschild radii of a hot, magnetized Keplerian flow. The light curve produced in this region may exhibit quasi-periodic variability. We present ray-tracing simulations to determine the general relativistically modulated X-ray luminosity expected from plasma coupled magnetically to the rest of the disk as it spirals inward below the innermost stable circular orbit toward the “stress edge” in the case of a Schwarzschild metric. The resulting light curve exhibits a modulation similar to that observed during a recent X-ray flare from Sgr A*.

Subject headings: accretion, accretion disks — black hole physics — Galaxy: center — instabilities — MHD — plasmas

1. INTRODUCTION

Sgr A*’s time-averaged spectrum is roughly a power law below 100 GHz, with a flux density $S_\nu \propto \nu^{\alpha}$, where $\alpha \sim 0.19–0.34$. In the millimeter/submillimeter region, however, Sgr A*’s spectrum is dominated by a “bump” (Zylka et al. 1992), indicative of two different emission components (Melia et al. 2000; Agol 2000). Higher frequencies correspond to smaller spatial scales (Melia 1992; Narayan et al. 1995), so the millimeter/submillimeter radiation is likely produced near the black hole (BH). X-ray flares detected from Sgr A* (Baganoff et al. 2001; Goldwurm et al. 2003; Porquet et al. 2003; Bélanger et al. 2005) may also have been produced within this compact region, either from a sudden increase in accretion accompanied by a reduction in the anomalous viscosity or from the quick acceleration of electrons near the BH (Liu & Melia 2002; Liu et al. 2004). The energized electrons may also manifest themselves via enhanced emission in a hypothesized jet (Markoff et al. 2001 and references cited therein).

Near-IR flares detected from Sgr A* appear to be modulated with a variable period $\approx 17$ minutes (Genzel et al. 2003; Eckart et al. 2006, 2008; Meyer et al. 2006). The X-ray and near-IR flares may be coupled via the same electron population, so one may expect similarities in their light curves. A long X-ray flare detected with XMM-Newton in 2004 also appears to have a modulated light curve, although not characterized by a fixed period (Bélanger et al. 2008). If real, the modulation in both the near-IR and X-ray events is almost certainly quasi-periodic rather than periodic, with a decreasing cycle from start to end. But are the fluctuations due to a single azimuthal perturbation (i.e., a “hot spot”) or from a global pattern of disturbance with a speed not directly associated with the underlying Keplerian period (Tagger & Melia 2006; Falanga et al. 2007)? In this Letter, we examine the nature of the observed quasi period and focus on its implications for the flow of matter through the innermost stable circular orbit (ISCO). A principal result of this study is a ray-tracing simulation of the general relativistically modulated light curve produced as the disrupted plasma spirals inward toward the disk’s “stress edge” (Krolik & Hawley 2002).

2. BACKGROUND

Magnetohydrodynamic (MHD) simulations of Sgr A*’s disk demonstrate the growth of a Rossby wave instability, enhancing the accretion rate for several hours, possibly accounting for the observed flares (Tagger & Melia 2006). The light curve produced by general relativistic (GR) effects during a Rossby wave–induced spiral pattern in the disk fit the data relatively well, with a quasi period associated with the pattern speed rather than the Keplerian motion (Falanga et al. 2007). However, MHD simulations of black hole accretion suggest that magnetic reconnection might take place within the plunging region, due to the presence of a nonaxisymmetric spiral density structure, initially caused by the magnetorotational instability (MRI) associated with differential rotation of frozen-in plasma (see, e.g., Hawley 2001).

In this case, the accreting flow is no longer Keplerian because of a radial velocity component. If Sgr A*’s quasi period of $\approx 17–25$ minutes is associated with this kind of process rather than a pattern rotation, it would place the corresponding emission region at $(0.73–0.94)r_{\text{isco}}$ radii, below the ISCO (where $r_{\text{isco}} = 3\mathcal{M}/c^2$) for a Schwarzschild BH. Theoretically, we may therefore distinguish the ISCO from the radius at which the inspiraling material actually detaches from the rest of the magnetized disk—the so-called stress edge (Krolik & Hawley 2002). The X-ray modulation would then be associated with the ever-shrinking period of the emitting plasma as it spirals inward from the magnetic flare.

Interest in hot spots began in the early 1980s in connection with quasi-periodic flux modulations observed in BHs accreting from a binary companion. The hot spots are possibly overdense emission regions associated with magnetic instabilities. But even with a hot spot, a Newtonian disk does not produce a modulation since its aspect does not affect the total luminosity observed from it. Other than a dynamical periodicity (such as that due to an azimuthal, radial, or vertical oscillation), only GR effects can produce time-dependent photon trajectories resulting in a modulated light curve (see, e.g., Cunningham & Bardeen 1973; Abramowicz et al. 1991; Karas & Bao 1992;
Fig. 1.—Upper panel: Stress edge radius, \( r_{\text{stress}} \) in units of \( r_{\text{ISCO}} \), as a function of \( \kappa \), the exponent in the power-law formulation of \( \Omega(r) \). The dotted and dashed curves represent a period of 17 and 25 minutes, respectively, using a black hole mass of \( 3.6 \times 10^6 M_\odot \) (Schödel et al. 2003). Lower panel: Corresponding ratio of accreted specific angular momentum, \( j_{\text{ac}} \), to the specific angular momentum at the ISCO.

Hollywood et al. 1995; Falcke et al. 2000; Bromley et al. 2001). Even so, the “standard” disk picture of hot spot modulation has been based on Keplerian motion, for which one then expects a time variability directly related to the Keplerian frequency. Here the modulation is not associated with such a fixed Keplerian frequency but from a shrinking orbit and a monotonically decreasing period (see § 3). The relevance of hot spots has already appeared in other works (Hollywood et al. 1995; Meyer et al. 2006; Eckart et al. 2008; Melia 2007 for review). What is lacking, however, is a non-Keplerian treatment of the motion with the intent of probing the stress edge itself.

So where exactly is the inner edge of the accretion disk in Sgr A*? This is a question asked in a broader context by Krolik & Hawley (2002), whose MHD simulations of the plunging region in a pseudo-Newtonian potential identified several characteristic inner radii. Here we assume a nonspinning BH, so our model pertains solely to the Schwarzschild case.

The monotonic decrease of the period during the flares suggests that we are witnessing the evolution of an event moving inward across the ISCO. The inflow timescale, \( t_{\text{inflow}} \), which determines the rate at which plasma can move from one orbit to another, is given by \( t_{\text{in}} = r_{\text{in}}/v_{\text{inflow}} \approx 9.6(r/r_s)^{1/2} \) minutes (Liu & Melia 2002) and is approximately 23.5 minutes at \( r = 3r_s = 6r_\text{ISCO} \), corresponding to the ISCO for a nonrotating (i.e., \( a/r_s = 0 \)) BH. This timescale does not explicitly depend on a viscosity parameter since the viscosity is directly tied to the MRI physical process via the induced Maxwell stress (Liu & Melia 2002). The inflow timescale defined here characterizes local processes occurring within the innermost portion of the disk during the flares. By comparison, the dynamical timescale, \( t_{\phi} \approx 1.3(r/r_s)^{3/2} \), is roughly 19 minutes at this radius (Liu & Melia 2002). Thus, the azimuthal asymmetry giving rise to the modulated flux during the flare may be due to a transient event associated with either a dynamical or a viscous process close to the ISCO (Melia et al. 2001a, 2001b).

For a BH mass of \( 3.6 \times 10^6 M_\odot \), the inflow timescale at \( r \approx 2.5r_s \) (inferred from the average period) is just slightly larger than the average period, so the event could be due to the sudden reconfiguration of magnetic field lines frozen into plasma rapidly approaching the ISCO and then flowing across it toward the event horizon. Matter flowing past the ISCO may still remain “magnetically” coupled to the outer accretion flow, so a dynamically more meaningful radius is the so-called stress edge, where plunging matter loses dynamical contact with the material farther out (Krolik & Hawley 2002). This may simply be defined as the surface on which the inflow speed first exceeds the magnetosonic speed.

In their simulations, Krolik & Hawley (2002) determined that this surface occurs somewhere between 0.77\( r_{\text{ISCO}} \) and \( r_{\text{ISCO}} \). The specific angular momentum \( j = r^2\Omega(r) \), in terms of the orbital angular frequency \( \Omega(r) \), continues to fall below \( r_{\text{ISCO}} \), although \( \Omega \) may not necessarily trace its Keplerian value, \( \Omega_K(r) \equiv (GM/r^3)^{1/2} \). In the absence of any magnetic coupling across \( r_{\text{ISCO}} \), matter would retain all of its specific angular momentum at the ISCO, so that the accreted value of \( j \), which we call \( j_{\text{ac}} \), would then simply be \( j_{\text{ac}} = j_{\text{ISCO}} \Omega_K(r_{\text{ISCO}}) \). Instead, the MHD simulations show that \( j_{\text{ac}} = 0.95j_{\text{ISCO}} \), for which \( r_{\text{stress}} \) is then \( \sim 0.8r_{\text{ISCO}} \), within the range of values indicated by the location of the transmagnetosonic surface.

If the period in Sgr A* is decreasing monotonically, \( j(r) \) will not follow its Keplerian value below \( r_{\text{ISCO}} \). Therefore, we will adopt the formulation \( \Omega(r) = \Omega_K^\kappa r^{-\kappa} \) to fit the data in § 3. Clearly, \( \kappa = 3/2 \) corresponds to Keplerian rotation; \( \kappa = 2 \) is in the extreme case of angular momentum conservation. A reasonable fit to the data would therefore be associated with \( 3/2 \leq \kappa \leq 2 \). At the boundary \( r_{\text{ISCO}} \), we expect \( \Omega = \Omega_K \), which then forces the constant \( \Omega_K \) to have the value \( c(r/r_s)^{3/2} \). We calculate \( r_{\text{stress}} \) using the quasi periods 17 and 25 minutes emerging from the X-ray light curve (see § 3), and this is plotted as a function of \( \kappa \) in Figure 1. The radius \( r_{\text{stress}} \) falls within the range (0.73–0.96)\( r_{\text{ISCO}} \) for all permitted values of \( \kappa \). The corresponding accreted specific angular momentum, for the same parameters as used before (see Fig. 1), is \( 0.85j_{\text{ISCO}} \leq j_{\text{ac}} \leq j_{\text{ISCO}} \) as a function of \( \kappa \). The ratio \( j_{\text{ac}}/j_{\text{ISCO}} = 0.95 \) from the MHD simulations would require \( \kappa \sim 1.8 \), for which \( r_{\text{stress}} \sim 0.77r_{\text{ISCO}} \). These results are consistent with the MHD simulations, indicating that the infalling plasma below the ISCO remains magnetically coupled to the outer disk, although the dissipation of angular momentum is not quite strong enough in this region to force the gas into Keplerian rotation.

3. THE INSPIRATING PLASMA MODEL

With \( \Omega(r) \) known, we now incorporate strong gravitational effects in a geometrically and optically thin disk, describing the inspiraling disturbance using coordinates \( (r, \theta, \phi) \) in the corotating frame. In Figure 2, we show the inspiraling trajectory and duration of the emitting plasma. The observer is located at infinity with a viewing angle \( i \) relative to the \( z' \)-axis in the nonrotating frame, at (observer) polar coordinates \( (r', \theta', \phi') \). The deflection angle of a photon emitted by plasma in the inspiraling region is \( \psi \), varying periodically with \( \cos \psi = i \cos \phi \), for a disk in the plane \( \theta = \pi/2 \). Also, for \( G = c = 1 \), the BH’s horizon occurs at \( r_s = 2M \), and the last stable orbit is located at \( r_{\text{ISCO}} = 3r_s \).

We calculate the light curve using a full ray-tracing algorithm (see Luminet 1979; Falanga et al. 2007). The disk from \( r_{\text{ISCO}} \) to 90\( r_s \) is an unperturbed, Keplerian flow, with angular velocity \( \Omega_K \), and with specific angular momentum \( j_k = r^2u^\phi/u' \) =
\[ r^2 \Omega_k \] The corresponding four-velocity of the effective flow is then \( u^r, u^\gamma, u^\phi, u^\phi = u(1, 0, 0, \Omega_k) \), where \( u^\phi = (1 - 3M(r)^{-1/2}) \) (Misner et al. 1973). The accretion flow is no longer Keplerian below the ISCO.

Triggering a perturbation induces an azimuthal asymmetry in the region \( r_{ISCO} \approx 0.73 < r < 0.9r_{ISCO} \). Below \( r_{ISCO} \), we use a simple representation of the bulk velocity field, in which \( \Omega(r) = u^\gamma_{se} / u^\phi_{se} \), as described, e.g., in Fukumura & Tsuruta 2004:

\[
v_{sw} = -A e^{-r/r_{ISCO}^1/2} \sin \theta [k(r - r_{\text{stress}}) + m \varphi / 2 - \varphi_{sw} / 2].
\]

In this case, the specific angular momentum is \( j_{sw} = r^2 u^\gamma_{sw} u^\phi_{sw} = \Omega f^{-2} \). The subscript “sw” denotes the spiral wave, and the number \( m \) is the azimuthal wavenumber, fixed to be \( m = 1 \) for a single-armed spiral wave. The constant \( \gamma_0 = 2 \) is the width of the spiral wave, \( A = 0.1 \) and \( A_x = 0.1 \) are the amplitudes chosen to be relatively small, \( k \) characterizes a tightness (i.e., the number of windings) of the spiral, and the effective radial range of the spiral motion is controlled by \( \Delta_{sw} = 30 \), and \( \varphi_{sw} = 0 \) denotes the phase of the spiral. Since \( (u^\gamma_{sw}, u^\phi_{sw}) \) is not axisymmetric, the net velocity field is also nonaxisymmetric. For the effective flow then, \( (u^\gamma_{sw}, u^\phi_{sw}) = (u^\gamma_{sw}, u^\phi_{sw}, u^\phi_{sw}) = u^\gamma_{sw}(1, u^\phi_{sw}, 0, \Omega_k r^{1/4}) \), where \( u^\phi_{sw} = [(1 - 2m) - (1 - 2m(r)/2)]/2) \), corresponding to the four-vector normalization condition \( g_{sw} u^\gamma u^\phi = -1 \).

We consider four GR effects: (1) light-bending; (2) gravitationnal Doppler effect defined as \( (1 + z) \), taking into account the nonaxisymmetric radial and azimuthal components below \( r_{ISCO} \); (3) gravitational lensing, \( \delta \Omega_{obs} = b db dq/dD^2 \) (with \( D \) the distance to the source), expressed through the impact parameter; and (4) the travel time delay. The relative time delay between photons arriving at the observer from different parts of the disk are calculated from the geodesic equation. The first photon arrives from phase \( \varphi = 0 \) and \( r = r_{ISCO} \), and defines the reference time, \( T_0 \), which is set to zero. The observed time is then the orbital time plus the light-bending travel time delays, i.e., \( T_{obs} = T_0 + \delta \).\n
The observed flux at energy \( E' \) is \( F_{obs}(E') = F_{obs}(E) \delta \Omega_{obs} \), where \( F_{obs}(E) \) is the radiation intensity observed at infinity and \( \delta \Omega_{obs} \) is the solid angle on the observer’s sky including relativistic effects. Using the relation \( \delta \Omega_{obs}(E', \alpha') = (1 + z)^{-4}\delta \Omega_{obs}(E, \alpha) \), a Lorentz invariant quantity that is constant along null geodesics in vacuum, the intensity of a light source integrated over its effective energy range is proportional to the fourth power of the redshift factor, \( I_{obs}(\alpha') = (1 + z)^{-4}I_{em}(r, \varphi) \), \( I_{em}(r, \varphi) \) being the intensity measured in the rest frame of the inspiraling disturbance (Misner et al. 1973). The disk radiates an inverse Compton spectrum, \( I_{em} \), calculated using the parameter scalings, rather than their absolute values. The spectrum parameters are (Melia et al. 2001a, 2001b) the disk temperature \( T(r) \), the electron number density \( n_e(r) \), the magnetic field \( B(r) \), and the disk height \( H(r) \). This procedure gives correct amplitudes in the light curve, although not the absolute value of the flux per se.

The synchrotron emissivity is therefore \( j_i \propto B n_e \propto B T_{em} \), where the nonthermal particle energy is roughly in equipartition with the thermal. The X-rays are produced via inverse Compton scattering from the seed photon number flux. Thus, with \( L_{seed} \propto j_i r^2 \), where \( j_i \) is the synchrotron emissivity in units of energy per unit volume per unit time, the soft photon flux scales as the emitted power divided by the characteristic area. That is, \( F_{seed} \propto j_i r^2 \), which is going to be roughly the same scaling as the seed photon density, so \( n_{seed} \propto j_i \propto B T_{em} \). The inverse Compton scattering emissivity is therefore \( j_i \propto n_{seed} \propto (T_{em})^2 r^2 B^2 \). Thus, \( j_i \propto j_i \), and the surface intensity is \( I_{em} \propto j_i d s \propto j_i H \), which gives finally \( I_{em} \propto (T_{em})^2 r^2 BH \).

The flux at a given azimuthal angle \( \varphi \) and radius \( r \) is calculated from a numerical computation of \( \psi(\alpha) \), followed by a calculation of the Doppler shift, lensing effects, and the flux \( F_{obs} \), as a function of the arrival time. For the persistent emission, we use the best-fit spectral parameters to the Chandra data (Melia et al. 2001a, 2001b; Baganoff et al. 2001), described above as a surface emissivity \( I_{em} \). The observed flare normalized flux is modeled with two polynomials, one between 0 and 100 minutes and the second from 100 to 160 minutes (see also Meyer et al. 2006). The value \( k \), is fixed at 11 to have the six observed cycles (see Fig. 3, solid line). The free parameters to fit the data are the inclination angle \( i \) and the \( k \)-value. The integrated flux is calculated for an extended spiral wave 90° long in the azimuthal direction and \( \Delta r = 0.28r_g \) in the radial direction, plus the persistent emission. The MHD simulations show that in the innermost part of the disk a spiral arm often
expands out to \( \sim 90^\circ \) (see, e.g., Hawley 2001). The radial extent of the inspiraling region is set by the observed condition that six cycles should fit within the overall migration of the plasma from the ISCO to the stress edge. In Figure 3 (solid line), we show the best-fit model for \( 72^\circ \pm 3^\circ \) and \( \kappa = 1.7 \pm 0.05 \).

4. CONCLUSION

If we adopt the simple view that the last period corresponds to the ISCO, then Sgr A* with a mass of \( 3.6 \times 10^6 M_\odot \) must be spinning at a rate \( a_{\text{i}} = 0.2 - 0.4 \). With a more realistic analysis of the magnetic coupling between matter in the plunging region and that beyond the ISCO, we conclude that the peak of the instability probably occurs at \( \sim 0.97 r_{\text{ISCO}} \), where the period is \( \sim 25 \) minutes, and the flaring activity continues as the plasma spirals inward, ending several orbits later when the matter crosses the stress edge at \( \sim 0.8 r_{\text{ISCO}} \).

The significance of the fit for an inspiraling disturbance is \( \chi^2/\text{dof} = 92.4/39 \), compared to \( \chi^2/\text{dof} = 285.2/46 \) for a fixed Keplerian period (see dotted curve in Fig. 3). An inspiraling disturbance is preferred over a fixed orbit by a factor of 2.6 in the reduced \( \chi^2 \). The residuals in the lower panels of Figure 3 show that the model using a fixed period produces modulations that are progressively shifted in phase with respect to the data, by as much as \( \sim 16.5 \) minutes by the end of the flare. The inspiraling model, on the other hand, follows the evolution of the flare and therefore fits the data much better. Plasma on such an orbit also produces a constant pulsed fraction \( p \) of \( \sim 9\% \) compared with a linear increase from \( \sim 9\% \) to \( \sim 11\% \) for the inspiraling wave; this effect is due to a radially dependent gravitational lensing effect. Together, these two effects render the inspiraling scenario a better explanation for the data than the fixed orbit disturbance.

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