More about F-term uplifting

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Abstract

We study moduli stabilization and a realization of de Sitter vacua in generalized F-term uplifting scenarios of the KKL\ T-type anti-de Sitter vacuum, where the uplifting sector $X$ directly couples to the light Kähler modulus $T$ in the superpotential through, e.g., stringy instanton effects. F-term uplifting can be achieved by a spontaneous supersymmetry breaking sector, e.g., the Polonyi model, the O’Raifeartaigh model and the Intriligator-Seiberg-Shih model. Several models with the $X$-$T$ mixing are examined and qualitative features in most models even with such mixing are almost the same as those in the KKL\ T scenario. One of the quantitative changes, which are relevant to the phenomenology, is a larger hierarchy between the modulus mass $m_T$ and the gravitino mass $m_{3/2}$, i.e., $m_T/m_{3/2} = \mathcal{O}(a^2)$, where $a \sim 4\pi^2$. In spite of such a large mass, the modulus F-term is suppressed not like $F^T = \mathcal{O}(m_{3/2}/a^2)$, but like $F^T = \mathcal{O}(m_{3/2}/a)$ for $\ln(M_{Pl}/m_{3/2}) \sim a$, because of an enhancement factor coming from the $X$-$T$ mixing. Then we typically find a mirage-mediation pattern of gaugino masses of $\mathcal{O}(m_{3/2}/a)$, while the scalar masses would be generically of $\mathcal{O}(m_{3/2})$.

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1 Introduction

It is quite important to realize the real world based on string theory [1], though we have lots of difficulties to be overcome. We do not have a definite answer in string theory why the dimension of our spacetime is four or a definite scenario to derive the standard model (SM), the $SU(3) \times SU(2) \times U(1)$ gauge symmetry, three families of quarks and leptons, experimental values of gauge couplings and fermion masses and mixing angles, etc. Many attempts have been tried to solve these problems in several aspects step by step.

One of the most severe difficulties to be solved, is the moduli stabilization problem. This problem always occurs when one assumes that the observed spacetime dimension is realized in string theory. Non-perturbative effects, such as gaugino condensations [2] were used to be considered for the moduli stabilization [3]. Recently a new mechanism of moduli stabilization has been brought to the attention. That is a compactification with closed string flux [4] with orientifold planes and D-branes, which can be a source of the fluxes and cause non-perturbative effects.

Advantages of such flux compactification are that the internal compact space can be warped by flux generating large hierarchies, and also that we can fix a lot of moduli, e.g. a number of $\mathcal{O}(100)$, simultaneously. For example, in type IIB orientifold model on Calabi-Yau, the dilaton, complex structure and D7-brane moduli are stabilized by an imaginary self dual flux. If our world can be described by string theory with D-branes, it is natural to consider both fluxes and non-perturbative effects as sources of moduli fixing.

Furthermore, moduli fields can play important roles in the low energy phenomenology. Moduli fields determine compactification scales of the internal space. Then, the moduli generically couple to four-dimensional kinetic terms of gauge fields, and their vacuum expectation values (vevs) determine gauge couplings and similarly other couplings like Yukawa couplings. Moreover, F-components of moduli superfields, which are also given by nonvanishing vevs of moduli themselves, can be a source of supersymmetry (SUSY) breaking and induce soft SUSY breaking terms in the visible sector. SUSY can be broken nonperturbatively through the moduli fixing procedure. Therefore, if we find any signatures of SUSY breaking at near future experiments, it is very interesting not only from the phenomenological viewpoint but also from the viewpoint of string theory.

Recently the authors of [5] proposed a semi realistic scenario based on the flux compactification with D-branes and an anti D-brane, that is called KKL T scenario. In this scenario, all moduli are fixed by flux and non-perturbative effects on D-branes and the de Sitter/Minkowski vacuum is realized. SUSY can be broken moderately by a red shifted anti D-brane which is sitting at the tip of warped throat and is well separated from the light modulus as well as the visible sector. Because of this sequestering structure, the scalar potential of the modulus, which has a SUSY minimum with a negative vacuum energy before adding the uplifting effect, can be easily uplifted allowing a tuning of the cosmological constant. The minimum becomes a SUSY breaking metastable vacuum for closed string sector.

In this scenario, F-components of Kähler moduli are suppressed compared with the gravitino mass [6]. This results in the fact that moduli mediation and anomaly mediation [7] are comparable [8], that is, the so-called mirage mediation. Thus, the gravitino mass is of $\mathcal{O}(100) \text{ TeV}$ to realize the low-energy SUSY breaking in this scenario. This
kind of the mediation mechanism causes a distinctive pattern of sparticle spectra at the TeV scale [8, 9] and naturally solves the SUSY CP problem [6, 8], though it may have a gravitino overproduction problem [10]. It is also known that, with the mirage mediation, the so-called little hierarchy problem can be avoided within the minimal SUSY SM (MSSM) [11], where the mirage unification of the wino and the gluino masses at the TeV scale is important [12].

The source of the uplifting is the anti-brane in the original KKLT scenario. However, it can be replaced by a dynamical SUSY breaking. Recently, metastable vacua with dynamical SUSY breaking have been studied in field-theoretical model building [13, 14]. Also, including realization of these field-theoretical models, metastable models of a dynamical SUSY breaking have been studied not only in the closed string sector but also in the open string sector [15]. By introducing such SUSY breaking sector into the KKLT model, we can construct F-term uplifting scenarios [16, 17, 18, 19, 20], where the Polonyi model [21], the O’Raifeartaigh model [22] and the Intriligator-Seiberg-Shih (ISS) model [13] have been considered as the F-term uplifting sector. F-term uplifting models are more interesting than other uplifting schemes, because the size of SUSY breaking is controllable and a small gravitino mass, which is comparable to the electroweak scale, can be realized. In the D-term uplifting [23] and the Kähler uplifting [25] schemes, to control the size of SUSY breaking is not simple, and they would naturally lead to a large SUSY breaking scale, which is comparable to the Planck scale.

In the F-term uplifting scenario, we find slight differences from the original KKLT predictions with the anti-brane, although the qualitative features are not changed. For example, the ratio of the anomaly mediation to the modulus mediation takes a different value depending on the model and as a consequence the prediction of sparticle mass spectra at a low energy scale is different from the original KKLT scenario.

In this paper, we generalize the F-term uplifting scenarios such that the uplifting sector \( X \) directly couples to the light Kähler modulus \( T \) in the nonperturbative superpotential induced by, e.g., stringy instanton effects. We will show that in most cases the qualitative features of the KKLT scenario can still be almost the same even with the modulus mixing to the uplifting sector. One of the quantitative changes, which would be phenomenologically relevant, is a larger hierarchy between the modulus mass and the gravitino mass than one in the original KKLT model. We will typically find mirage-mediation type gaugino masses, while the scalar masses are of the order of the gravitino mass in general.

We arrange the sections of this paper as follows. In Sec. 2 we study the Polonyi-KKLT model and the ISS-KKLT model as concrete examples of the F-term uplifting. We introduce a mixing between the light modulus \( T \) and the uplifting sector \( X \) in the superpotential, and find the minimum of the scalar potential based on the perturbation from a reference point where both the Polonyi/ISS-type structure and the KKLT-type structure would be realized for \( X \) and \( T \), respectively. Here we assume that constants of the Polonyi/ISS models are (dynamically) generated by flux or stringy instantons [26] depending on the light modulus. Then, in Sec. 3 we analyze the SUSY breaking order

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1 The ISS model can correspond to the O’Raifeartaigh model after integrating out heavy modes in both models.
2 See also [24].
3 For Affleck-Dine-Seiberg-KKLT model [27], see [28].
parameters and the masses of light modes. After deriving some general expressions in Sec. 3.1, we show some results in several concrete examples in Sec. 3.2 and 3.3. Sec. 4 is devoted to conclusions and discussions.

2 Moduli involved F-term uplifting

For concreteness, we consider type IIB supergravity with the dilaton $S$, complex moduli $U$ and a single Kähler modulus $T$. Our analysis can be extended to the case with several Kähler moduli. Here and hereafter we use the mass unit with $M_{Pl} = 1$, where $M_{Pl}$ is the four-dimensional Planck mass. As in the original KKLT model [5], we assume that the dilaton $S$ and complex structure moduli $U$ are stabilized by the flux induced superpotential

$$W_{\text{flux}}(S, U) = \int G_3 \wedge \Omega,$$

because of large supersymmetric masses $\langle \partial_S \partial_U W_{\text{flux}} \rangle \sim 1$ [29]. Then, the dilaton and the complex structure moduli are much heavier than the Kähler modulus $T$, and these do not affect the low energy dynamics of $T$ and the visible sector.

In the KKLT model, the Kähler modulus $T$ is stabilized by a nonperturbative effect on a $D7$-brane, that is, the following superpotential is considered,

$$W_{\text{KKLT}} = w_0 - Ae^{-aT},$$

with the Kähler potential $-3 \ln(T + \bar{T})$, where the constant $w_0$ in the superpotential originates from the flux induced superpotential $W_{\text{flux}}$ or a gaugino condensation which depend on heavy moduli such as $S$ [30], and the second term is due to the nonperturbative effect. The potential minimum corresponds to a SUSY AdS vacuum. In order to uplift the AdS minimum to a Minkowski one, anti $D3$-branes are introduced at the tip of the warped throat, which is well sequestered from the Kähler modulus $T$ as well as the visible sector, and the effect of anti $D3$-branes just appears as an ($T$-dependent) uplifting potential, which is an explicit SUSY breaking term in terms of the $N = 1$ supergravity, added to the standard F-term scalar potential of $T$.

Instead of adding such anti D-branes, we can uplift the SUSY AdS minimum by adding a superpotential term $W_{\text{lift}}(T, X)$, which leads to a nonvanishing F-term of the hidden sector field $X$. That is the F-term uplifting. In the original F-term uplifting scenarios [16, 17, 18, 19, 20], the uplifting sector is basically assumed to be well separated from the light modulus as anti $D3$-branes in the KKLT scenario, i.e. $\partial W_{\text{lift}}(T, X)/\partial T = 0$. As the spontaneous SUSY breaking sector $X$, the Polonyi model, O’Raifeartaigh model and ISS model have been considered. Here, we study generalized F-term uplifting scenarios where the hidden sector field $X$, which is responsible for the nonvanishing F-term, is directly couples to the light Kähler modulus due to, e.g., stringy instanton effects [26]. Such instanton effects for instance induce a mass term or a tadpole term of the hidden sector field $X$ which depends on the light Kähler modulus. Similar situation has been studied based on heterotic and M-theoretical models with multiple light moduli [28, 31].

\[4\] See also [32] for a model with similar properties.
We will show that the F-term uplifting is still valid in most cases without changing the qualitative features of both the light modulus (KKLT) sector and the uplifting (dynamical SUSY breaking) sector.

2.1 Polonyi-KKL T model

One of the simplest models for the F-term uplifting is the Polonyi-KKL T model \cite{18, 20, 33}. The Kähler potential $K$ and the superpotential $W$ are given by

\begin{align}
K &= \Omega(T, \bar{T}) + Z(T, \bar{T})|X|^2, \\
W &= w_0 - A e^{-a T} + B e^{-b T} X,
\end{align}

where $\Omega(T, \bar{T})$ is the Kähler potential of overall volume (Kähler) modulus, which is typically given by $-3 \ln(T + \bar{T})$, and $Z(T, \bar{T}) = K_{XX}$ is the Kähler metric of $X$. In the superpotential, we assume a typical magnitude of parameters $|a|, |b| \sim 4\pi^2$.

When $A = 0$ and $b = 0$, the above superpotential corresponds to the Polonyi model, i.e. $W_{\text{Polonyi}} = w_0 + BX$, and leads to spontaneous SUSY breaking with nonvanishing $F^X$. The third term in the right hand side is the mixing between $X$ and $T$, and such a mixing can be induced by sting instanton effects. The Polonyi-KKL T model without $X$-$T$ mixing, i.e. $b = 0$, has been studied in \cite{18, 20, 33}.

From these Kähler potential and superpotential, we can derive the $F$-term scalar potential $V$ using the standard $N = 1$ supergravity formula:

\begin{align}
V &= e^G(G^{IJ}G_I G_J - 3) = K_{IJ} F^I \bar{F}^J - 3e^K |W|^2, \\
G &= K + \ln |W|^2, \quad F^I = -e^{K/2} K^{IJ} D_J \bar{W}, \quad D_I W = W_I + K_I W.
\end{align}

We try to find a minimum of the potential by a perturbation from the reference point. We choose the reference point $(X, T) = (X_0, T_0)$ which satisfies the following conditions

\begin{align}
V_X|_0 = V|_0 = 0, \quad D_T W|_0 = 0, \quad X_0 = \bar{X}_0, \quad T_0 = \bar{T}_0.
\end{align}

At this reference point, the KKLT-like modulus property and the Polonyi-like SUSY breaking property would be realized. We tune our parameters to obtain almost vanishing vacuum energy $V = 0$. In this Polonyi-KKL T model, the reference point (4) is characterized by

\begin{align}
X_0 &\sim T_0 \sim \partial_T \partial_T^m \Omega|_0 \sim \partial_T^n \partial_T^m Z|_0 \sim O(1), \\
W|_0 &\sim W_T|_0 = -K_T W|_0, \quad |\partial_T^{n+2} W|_0 \sim a^{n+2} |W|,
\end{align}

where $n, m = 0, 1, 2, \ldots$.

\footnote{Here, because the number of complex parameters in superpotential is less than four, we can always make the vevs of fields real by field redefinitions (shifts or rotations).}
The true minimum of the potential is denoted by
\[ \langle \Phi' \rangle = \Phi'|_0 + \delta \Phi', \]
where \( \Phi' = (X, T) \). Assuming \( \delta \Phi' / \Phi'|_0 \ll 1 \), we expand \( V_I \) as
\[
V_I = V_I|_0 + V_{IJ}|_0 \delta \Phi^J + V_{IJK}|_0 \delta \Phi^J + \mathcal{O}(\delta \Phi^2)
\]
where \( \hat{V}_{IJ} = V_{IJ} + V_{IJ}|_0 \) and \( \delta \Phi^I = \delta \Phi^J \). The stationary condition \( V_I = 0 \) results in
\[
\delta \Phi^I = -\hat{V}^{IJ} V_J|_0 + \mathcal{O}(\delta \Phi^2),
\]
where \( \hat{V}_{IJ} \hat{V}^{JK}|_0 = \delta^I_0 \). For \( V_{IJ} > V_{IJ}|_0 \), we find \( X_0 = \sqrt{3} - 1 \), \( T_0 = \mathcal{O}(1) \),
\[
V_T|_0 \sim b e^G|_0,
\]
and
\[
\hat{V}_{IJ}|_0 \sim \left( \begin{array}{cc} V_{X\bar{X}} & V_{X\bar{T}} \\ V_{T\bar{X}} & V_{T\bar{T}} \end{array} \right) \sim \left( \begin{array}{ccc} b^2 + 1 & b(m_T/m_{3/2} + 1) & b(m_T/m_{3/2} + 1) \\ b(m_T/m_{3/2} + 1) & m_T^2/m_{3/2} + b^2 & m_T^2/m_{3/2} + b^2 \\ b^2 + 1 & ab^2 + b & ab^2 + b \\ ab^2 + b & a^2b^2 + b^2 & a^2b^2 + b^2 \end{array} \right) m_{3/2}^2.
\]
where \( m_{3/2} = e^{G/2}|_0 \) and
\[
m_T = -e^{K/2}K^{TT}W_{TT}|_0 \sim ab(T_0 + \bar{T}_0)^2 m_{3/2}.
\]
Then, for \( a \sim b \neq 0 \), we find that the Hessian can be positive, and obtain
\[
\frac{\delta T}{T_0} \sim \frac{1}{b^2} \ll 1, \quad \frac{\delta X}{X_0} \sim \frac{1}{b^2} \ll 1.
\]
This result should be compared with \( \delta T/T_0 \approx a^{-2} \) and \( \delta X/X_0 \approx a^{-1} \) derived in the case that \( B e^{-bT} \) is replaced by a \( T \)-independent constant in the superpotential. (When one sets \( b = 1 \) in the above expressions, the results of such case are obtained.) This difference originates from the fact that the superpotential of modulus is not effectively a KKL T type but rather a racetrack type because of \( \langle X \rangle = \mathcal{O}(1) \). From this result, we find that the true minimum resides in a perturbative region from the reference point (4). Then, we expect that the SUSY breaking structure of the Polonyi-KKL T model is not affected qualitatively by the mixing between the Polonyi sector \( X \) and the KKL T sector \( T \), although quantitatively the modulus mass becomes the racetrack-type (6) for \( b \sim a \) [6].
Finally we comment that a considerable Kähler mixing $|Z^{-1}\partial_T Z| = \mathcal{O}(1)$ at the reference point would affect the above order estimations. For example, we obtain $V_T|_0 \sim b(1 + aX_0\partial_T Z)e^G|_0 \sim b^2 e^G|_0$ and $\delta T/T_0 \sim 1/b^2$, $\delta X/X_0 \sim 1/b$, assuming that $T_0$ and $X_0$ are of $\mathcal{O}(1)$. Then without a tuning between $a$ and $b$, we may find

$$F^X \sim D_X W = D_X W|_0 + W_{XT}|_0 \delta T + K_X W_X|_0 \delta X + \cdots$$

$$\sim m_{3/2} \left(1 + \frac{1}{b} + \cdots\right),$$

$$F^T \sim D_T W = W_{TT}|_0 \delta T + W_{TX}|_0 \delta X + W_{XTT}|_0 \delta X \delta T + \frac{1}{2} W_{TTT}|_0 (\delta T)^2 + \cdots$$

$$\sim m_{3/2} \left(1 + \frac{1}{b} + \cdots\right),$$

where we wrote both $1/a$ and $1/(a - b)$ as $1/b$. If $a - b$ is of $\mathcal{O}(1)$, the above expansion in some cases may not converge, and the perturbation may be invalid as in the ISS-KKLT model shown later. For the case in which it converges ($a - b = \mathcal{O}(4\pi^2)$ etc.), we leave a concrete study as a future work.

### 2.2 ISS-KKLT model

Another interesting source of uplifting is the ISS model \cite{13} and in this subsection we consider the ISS-KKLT model \cite{17, 18}. After heavy modes are integrated out around the SUSY breaking minimum, the ISS model leads to the same superpotential as the Polonyi model, where the field $X$ corresponds to the meson field and the tadpole term of $X$ corresponds to a mass term of quarks $q$ and $\bar{q}$ in the dual side, i.e. $X \sim q\bar{q}$. A $T$-dependent mass term of $q$ and $\bar{q}$ can be generated by string instanton effect like $q\bar{q} e^{-bT}$. That can be an origin of the third term of the right hand side in Eq. (2). At any rate, we use the same superpotential as Eq. (2). However, the Kähler potential of $X$ receives a one-loop correction from the heavy modes. The relevant part can be written as \cite{19, 34}

$$K = K - \frac{1}{\Lambda^2} Z^{(1)}(T, \bar{T})|X|^4,$$

where $K$ is the tree level Kähler potential given by Eq. (1). We assume the same superpotential (2) as before. The O’Raifeartaigh model leads to the same Kähler potential and superpotential after heavy modes are integrated out.

The scalar potential is then written as

$$V = e^G (G^{IJ} G_I G_J - 3),$$

$$= e^G (G^{13} G_I G_J - 3) + m_X^2 (T, \bar{T})|X|^2 + \cdots,$$

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\*\*\* If we really adopt the ISS model itself, $\Lambda$ would be dependent of $T$ like $\Lambda^2 \sim 16\pi^2 Be^{-bRe(T)}$ because the mass of heavy modes is also given by $Be^{-bT}$. However we consider $\Lambda$ as a constant which comes from the vevs of heavy moduli, that is, the “ISS model” represents some proper O’Raifeartaigh model in this paper. Note that, even if $\Lambda$ is dependent of $T$, the following results would not be changed as far as the ISS-like vacuum is stable.
where $G = \mathcal{K} + \ln |W|^2$, $G$ is given by Eq. (1) and

$$m_X^2(T, \bar{T}) = \frac{4B^2}{A^2} e^{-b(T+\bar{T})} \frac{Z^{(1)}(T, \bar{T})}{Z(T, \bar{T})}.$$ 

We execute a similar analysis to the Polonyi-KKLT model based on the reference point (4) with the order estimation (5) but now $X_0 \simeq (\sqrt{3}Z/6Z^{(1)})X^2 \sim 10^2 m_{3/2} \ll 1$. The important difference between the Polonyi uplifting scenario and the ISS uplifting scenario is the size of $X_0$, that is, we have $X_0 = \mathcal{O}(1)$ in the Polonyi uplifting scenario, while $X_0$ is much smaller in the ISS uplifting scenario. Because of the smallness of $X_0$, the following results are not affected by the Kähler mixing $\partial_T Z \neq 0$. In this case, we can estimate the order of $V_T|_0$, $V_{IJ}|_0$ and $V_{IJ}|_0$, and find

$$V_T|_0 \sim b e^G|_0,$$

and

$$\hat{V}_{IJ}|_0 \sim \begin{pmatrix} V_X \bar{X} & V_{X T} \\ V_{X \bar{T}} & V_{\bar{T}} \end{pmatrix} \sim \begin{pmatrix} 1/X_0 + b^2 & b(m_T/m_{3/2} + 1) \\ b(m_T/m_{3/2} + 1) & m_T^2/m_{3/2}^2 + b^2 \end{pmatrix} \frac{m_{3/2}^2}{m_T}.$$

where $m_{3/2} = e^{G/2}|_0$ and $m_T = -e^{K/2} K_{TT} W_T|_0 \sim a(T_0 + \bar{T}_0)m_{3/2}$. Because $X_0$ is much smaller than the Planck scale, the effective superpotential for the modulus is not the racetrack type but the KKLT type. Then, for $a \sim b \neq 0$, we obtain

$$\frac{\delta T}{T_0} \simeq a^{-1} \ll 1, \quad \frac{\delta X}{X_0} \simeq a \gg 1.$$ 

This results should be compared with the results $\delta T/T_0 \simeq a^{-2}$ and $\delta X/X_0 \simeq a^{-1}$ for the case that $B e^{-bT}$ is replaced by a $T$-independent constant. (When one sets $b = 1$, one can obtain the result for such case.) We conclude that, with the superpotential mixing between the ISS sector and the KKLT sector, the reference point (4) is far from the true minimum. Then, the SUSY breaking structure would be quite different from the ISS-KKLT model without the $X$-$T$ mixing.

### 2.3 ISS-racetrack model

We can evade the problem in the previous subsection by adding another nonperturbative effect, $Ce^{-cT}(c \neq a)$, to the ISS-KKLT superpotential.\(^7\)

$$W = w_0 - Ae^{-aT} + Ce^{-cT} + Be^{-bT} X. \quad (7)$$

We call this ISS-racetrack model. In this case, we find $V_T|_0 \sim b e^G|_0$ and $\hat{V}_{IJ}|_0 \sim \begin{pmatrix} 1/X_0 & abc \\ abc & a^2 + b^2 \end{pmatrix} e^G|_0$.\(^8\)

\(^7\) For $w_0 \neq 0$, we cannot make the vevs of fields real without a fine-tuning of complex parameters, because in such case the number of complex parameters is four.

\(^8\) This argument depends on parameters in the superpotential. For example, when the vacuum of modulus is almost supersymmetric Minkowski vacuum, that is, $w_0 \sim A e^{-a(T) - c(T)} \gg \langle W \rangle$, the ISS vacuum is more stable than the usual racetrack vacuum, because $m_T$ is much heavier than $a(T + \bar{T})^2 m_{3/2}$ for such case.
Note that the modulus mass is estimated as

$$m_T \sim ac(T_0 + \bar{T}_0)^2 m_{3/2},$$  \hspace{1cm} (8)

without a fine-tuning among $a$, $b$ and $c$. Therefore, we typically find

$$\frac{\delta T}{T_0} \sim \frac{b}{a^2c^2}, \quad \frac{\delta X}{X_0} \sim \frac{b^2}{ac};$$

and the reference point is stable so far as $b^2 < ac$. Here, we consider the case that $b \lesssim c \lesssim a$. The Kähler potential mixing can be safely introduced into the ISS-racetrack model without affecting the structure of the minimum due to the smallness of $X_0$. Finally, in this ISS-racetrack model, the vacuum would be metastable. However its life time would be sufficiently long [13, 17, 35], because $Be^{-b(T)} \ll 1$.

### 3 SUSY breaking order parameters

In the previous section, we have studied generalized F-term uplifting scenarios where the hidden sector field $X$, which is responsible for the nonvanishing F-term, is directly couples to the light Kähler modulus $T$, e.g., because of stringy instanton effects [26]. As we can see from Eqs. (6) and (8), the $X$-$T$ mixing generically produce a larger hierarchy $m_T/m_{3/2} \sim a^2$ between the gravitino mass and the modulus mass than one in the original KKLT model $m_T/m_{3/2} \sim a$, because the modulus superpotential is effectively given by the racetrack-type [6]. Otherwise, the reference point is unstable and the KKLT-type structure that the modulus is somehow heavier than the gravitino, might be spoiled as shown in the ISS-KKLT model.

In this section, we analyze the SUSY breaking order parameters in detail. We will find that, despite the large modulus mass $m_T \sim a^2 m_{3/2}$, we can still obtain the same size of the modulus F-component $F^T \sim m_{3/2}/a$ as one in the original KKLT scenario, because of the enhancement by the $X$-$T$ mixing in the superpotential. In the following, we first derive general expressions for the order parameters $F^T$ and $F^X$ in terms of modulus and gravitino masses, and then apply them to several typical models.

#### 3.1 General result

For the evaluation of $F^T$, we expand the relevant part of the scalar potential around the reference point defined by (4) with $W|_0 \neq 0$.

$$V = e^K(K^{IJ}D_I W \overline{D_J W} - 3|W|^2)$$

$$\approx V|_0 + [e^K K^{XX} \overline{D_X W}(K_{XT} W + K_X W_T + W_{XT})]|_0 \delta T$$

$$+ [e^K K^{TT} (K_{TT} W + K_T W_T + W_{TT})]|_0 \overline{D_TW} \delta T - 3[e^K \overline{W W_T}]|_0 \delta T + \cdots$$

$$\approx [e^K K^{XX} \overline{D_X W}(-K_X K_T W + W_{XT})]|_0 \delta T$$

$$+ [e^K K^{TT} W_{TT}]|_0 \overline{D_TW} \delta T + 3[K_T e^K |W|^2]|_0 \delta T + \cdots,$$

As shown in [28], in such a case we may have a nice property that the smallness of the gravitino mass $m_{3/2} \ll M_{Pl}$ could be a natural consequence of a tiny cosmological constant.
where ellipses denote complex conjugates and terms that are of order \((\delta X)^2, (\delta \bar{X})^2, (\delta T)^2, (\delta \bar{T})^2\) and \(|\delta X|^2\). Here we have assumed
\[
|K_{XT}|, |K_{X\bar{T}}| < \left| \frac{W_{XT}}{W_{TT}} \right| \sim \mathcal{O}(a^{-1}) < K_{TT}, K_{X\bar{X}} \sim 1,
\]
\[
|K_{TT}W|, |K_TW| = | − K_T^2 W| \ll |W_{TT}|,
\]
\[
|V_{TT}| \ll |V_{TT}|,
\]

at the reference point. An important point is that the expansions of \(m\) at the reference point. From Eq. (9), we easily find that
\[
\delta T \bar{F} \text{ of the same order as the one in the original KKL T model}
\]
where the effect of the
\[
X \text{ have used } \sqrt{K_{XX} D_X W} = \sqrt{3} W.
\]

Using the expressions \(m_T \simeq −e^{K/2}K_{TT}W_{TT}\) and \(F_T \simeq −e^{K/2}K_{TT}D_T W\), one can rewrite the above scalar potential as
\[
V \approx \delta T \left[ m_T^2 \left\{ \sqrt{3K_{XX}} \left( \frac{W_{TX}}{W} - K_{X}K_T \right) + 3K_T \right\} + K_{TT}m_T F_T \right] + \cdots,
\]
where we have omitted the symbols \(0\) and we have used \(\sqrt{K_{XX}} D_X W = \sqrt{3} W\). With an equation of motion for \(\delta T\), we obtain
\[
F_T \simeq \frac{m_T^2}{m_T} \left( \frac{-K_T}{K_{TT}} \right) \left[ \sqrt{3} (\sqrt{3} - \sqrt{K_{XX}} K_X) + \sqrt{3K_{XX}} \left( \frac{W_{TX}}{K_TW} \right) \right]
\]
\[
= \frac{m_T^2}{m_T} \left( \frac{-K_T}{K_{TT}} \right) \sqrt{3} (\sqrt{3} - \sqrt{K_{XX}} K_X) \left( 1 + \frac{\partial_T \ln(W_X)}{K_T} \right), \quad (9)
\]

where the effect of the \(X-T\) mixing is encoded in \(\partial_T \ln W_X = b\) as well as \(m_T\). Here we have used \(\sqrt{K_{XX}} D_X W = \sqrt{3} W\) and
\[
\frac{W_{TX}}{W} = \frac{W_{TX}}{W_X} \cdot \frac{W_X}{W} = \frac{W_{TX}}{W_X} \cdot \frac{(\sqrt{3} - \sqrt{K_{XX}} K_X)}{\sqrt{K_{XX}}},
\]
at the reference point. From Eq. (9), we easily find that \(F_T \simeq m_{3/2}/a\) for the typical modulus mass \(m_T/m_{3/2} \sim ab\) shown in Eqs. (6) and (8). This should be compared with the original KKLT model and the F-term uplifting scenarios without the mixing \(\partial_T \ln W_X = 0\), which result in \(F_T \sim m_{3/2}/a\) with \(m_T/m_{3/2} \sim a\). The ratio \(F_T/m_{3/2}\) can be of the same order as the one in the original KKLT model \(F_T/m_{3/2} \sim 1/a\), although our models have a larger hierarchy between the modulus and the gravitino masses. This is due to the enhancement factor \(\partial_T \ln W_X = b\) in Eq. (9). On the other hand, around the reference point, we simply find
\[
F_X \simeq −\sqrt{3/K_{XX}} m_{3/2}.
\]

Here, we comment on the shift of field vevs from the reference point. From the above expressions, we can roughly estimate as \(F_T \sim \delta T \partial_T F_T \sim m_T \delta T \sim bm_{3/2}/m_T\) where \(b = −\partial_T \ln(W_X)\). Then we find
\[
\frac{\delta T}{T_0} \sim b \frac{m_{3/2}}{m_T},
\]

9
where we have assumed $T_0 = O(1)$. Furthermore, $\delta X$ in later examples can be typically given by $\delta X \sim -\delta T(V_{XT}/V_{XX}) \sim -\delta T \frac{b m_T m_{3/2}}{m_{3/2}(1+b^2X_0)/X_0}$. Then we find

$$\frac{\delta X}{X_0} \sim - \frac{b^2 m_{3/2}}{m_T (1+b^2X_0)}.$$

These expressions agree with results in the previous sections and are useful for checking the stability of the reference point in each model.

In the following subsections, we show several concrete examples. In some cases, we will assume that the gauge kinetic function of the hidden sector $f_{hid}$, which is responsible for the nonperturbative superpotential terms, is given by the mixture of the heavy and the light moduli [30], e.g., $f_{hid} = w T + m \langle S \rangle$, for generality. All the parameters in the models are taken to be positive and real, and we will take $b < a$ in the Polonyi-KKLT model and $b \leq c < a$ in the ISS-racetrack model for concreteness.

### 3.2 Polonyi-KKLT model

First examples are some simplified versions of the Polonyi-KKLT model [2]. As we emphasized, the perturbation around the Polonyi vacuum $X_0 \sim O(1)$ generates effectively a racetrack-type superpotential for the light modulus $T$. Then, we can simply drop the constant piece $w_0$ in the superpotential [2] just for stabilizing $T$ unlike the case without the $X$-$T$ mixing. Instead, we introduce a moduli mixing in the hidden sector gauge kinetic function [30].

**3.2.1 Model 1**

We start from the Kähler potential and the superpotential given by

$$K = -3 \ln(T + \bar{T}) + |X|^2, \quad W = A e^{-c\langle S \rangle + a T} + B e^{-b T} X,$$

where the dilaton $S$ is assumed to be stabilized at a much higher scale by, e.g., the flux induced superpotential. In this case, we find

$$X_0 = \sqrt{3} - 1, \quad T_0 \approx \frac{1}{a+b} \ln \left[ \frac{b B X_0}{a A} e^{c \langle S \rangle} \right] \sim \frac{c}{a+b} \langle S \rangle,$$

$$\langle W_{TT} \rangle \approx ab \langle W \rangle, \quad \langle W \rangle \approx A e^{-c\langle S \rangle} \left( \frac{b B X_0 e^{c \langle S \rangle}}{a A} \right)^{\frac{1}{a+b}} \left( 1 + \frac{a}{b} \right),$$

$$F^T \approx -\sqrt{3} \frac{m_{3/2}}{a}, \quad m_T = -\frac{ab(T + \bar{T})^2}{3} m_{3/2}.$$

Therefore the condition for the Polonyi vacuum, $W_X|_0 = W|_0$, i.e. $V|_0 = 0$ at $X_0 = \sqrt{3} - 1$, requires $b/a \approx 1/X_0 - 1 \approx 0.37$, and the positive sign $+a T$ in the first exponent is

\[\text{10}\text{Except for model 5, we can always assume the positive and real parameters up to the overall phase of the superpotential. In model 5, we have to tune some of these complex parameters for such assumption.}\]
necessary in the superpotential in order for the Polonyi-like vacuum to be compatible with the racetrack vacuum.

To evaluate the ratio of the anomaly mediation to the modulus mediation, we define $\alpha$ as

$$
\alpha \equiv \frac{1}{\ln(M_{Pl}/m_{3/2})} \cdot \frac{F^C/C_0}{F^T/(T + \bar{T})},
$$

where $C_0 = e^{K/6}$ is the lowest component of the conformal compensator superfield $C$, and $F^C$ is F-component of $C$, i.e.

$$
\frac{F^C}{C_0} = m_{3/2} + \frac{1}{3} \sum I K_I F^I.
$$

Note that $F^C/C_0$, in general, has contributions due to the gravitino mass and F-components $F^I$ of the SUSY breaking sector, although we have $F^C/C_0 = m_{3/2}$ in the original KKL T scenario. We obtain in this example

$$
\alpha = -\frac{2}{3} \frac{a}{b} \approx -\frac{2}{3}(\sqrt{3} + 1) \approx -1.82.
$$

where $\ln(M_{pl}/m_{3/2}) \approx \ln(1/W|_0) \approx b T_0$ has been adopted.

### 3.2.2 Model 2

We can also consider the case with a moduli mixing in the term which is responsible for the $X$-$T$ mixing through, e.g., stringy instanton effects. Then we analyze the following model:

$$
K = -3 \ln(T + \bar{T}) + |X|^2, \quad W = Ae^{-aT} + Be^{-c(S)+bT}X.
$$

Note that the positive sign $+bT$ in the second exponent is necessary in order for the Polonyi-like vacuum to be compatible with the racetrack vacuum as in the previous model. In this case, we obtain

$$
X_0 = \sqrt{3} - 1, \quad T_0 \approx \frac{1}{a + b} \ln \left[ \frac{aA}{bB X_0} e^{c(S)} \right] \approx \frac{c}{a + b} \langle S \rangle,
$$

$$
\langle W_{TT} \rangle \approx ab \langle W \rangle, \quad \langle W \rangle \approx A \left( \frac{aA e^{c(S)}}{bB X_0} \right)^{-\frac{a}{a+b}} \left( 1 + \frac{a}{b} \right),
$$

$$
F^T \approx \sqrt{3} \frac{m_{3/2}}{a}, \quad m_T = -\frac{ab(T + \bar{T})^2}{3} m_{3/2}.
$$

\[11\] In the ISS-KKLT model with $W = Ae^{-aT} + Be^{+bT}X$, we cannot satisfy this kind of condition $W_X|_0 \approx \sqrt{3}|W|_0$, that is, $b/a \approx 1/(\sqrt{3}X_0) \approx 1 \sim 10^{12}$.

\[12\] Contributions to $F^C/C_0 = m_{3/2}/\sqrt{3}$ come from the gravitino mass and $K_X F^X/3 = -(1-1/\sqrt{3})m_{3/2}$ in the Polonyi-KKLT model. The latter was not taken into account in [18], and the value of $\alpha$ was evaluated for $F^C/C_0 = m_{3/2}$, but that should be replaced by $F^C/C_0 = m_{3/2}/\sqrt{3}$.
The condition for realizing the Polonyi vacuum is the same as the previous example, \( b/a \simeq 1/X - 1 \). The anomaly/modulus mediation ratio \( \alpha \) in this model is found as

\[
\alpha = \frac{2}{3} \frac{1}{b/a + c \langle S \rangle T_0} = \frac{2}{3}
\]

where we applied \( \ln(M_p/m_{3/2}) = c \langle S \rangle - bT_0 \simeq aT_0 \).

### 3.2.3 Model 3

Finally, we show the results in the Polonyi-KKL T model (2) with the minimal Kähler potential:

\[
K = -3 \ln(T + \bar{T}) + |X|^2, \quad W = w_0 - A e^{-aT} + B e^{-bT} X,
\]

where we find

\[
X_0 = \sqrt{3} - 1, \quad T_0 \simeq \frac{1}{a-b} \ln \left[ \frac{aA}{bB X_0} \right],
\]

\[
\langle W_{TT} \rangle \simeq -abX_0 \left( 1 - \frac{b}{a} \right) \langle W \rangle, \quad \langle W \rangle \simeq B \left( \frac{aA}{bB X_0} \right)^{\frac{a}{b+}}.
\]

\[
F^T \simeq \frac{\sqrt{3} m_{3/2}}{(a-b)X_0}, \quad m_T = \frac{ab(T + \bar{T})^2}{3} \left( 1 - \frac{b}{a} \right) X_0 m_{3/2}.
\]

Here we adopted \( w_0 \simeq A e^{-aT_0} + B (1 - X_0) e^{-bT_0} \) coming from the condition for the Polonyi-like vacuum, \( W|_0 = W_X|_0 = B e^{-bT_0} \).

In this case, one finds that the shift of fields are given by \( \delta T/T_0 \sim 1/b(a-b)^2 \) and \( \delta X/X_0 \sim 1/b(a-b) \). Then the expansion of the potential around the reference point is done in terms of \( 1/(a-b)^2 \), where

\[
D_X W = D_X W|_0 + \sum_{n=1} \frac{1}{n!} (\partial^n_T W_X)|_0 (\delta T)^n + \cdots
\]

\[
\sim m_{3/2} \left( 1 + \frac{1}{(a-b)^2} + \frac{1}{(a-b)^4} + \cdots \right),
\]

\[
D_T W = \sum_{n=1} \frac{1}{n!} (\partial^n_T W_T)|_0 (\delta T)^n + \delta X \sum_{n=1} \frac{1}{(n-1)!} (\partial^n_T W_X)|_0 (\delta T)^{n-1} + \cdots
\]

\[
\sim \frac{m_{3/2}}{(a-b)} \left( 1 + \frac{1}{(a-b)^2} + \frac{1}{(a-b)^4} + \cdots \right).
\]

Therefore it is a good approximation when \( a-b = \mathcal{O}(10) \). Note that only if \( B = e^{-c \langle S \rangle} \) with \( c \sim a \sim b \), the difference \( a-b \) can be of \( \mathcal{O}(10) \) for \( T_0 \sim \langle S \rangle = \mathcal{O}(1) \).

In such a case, the mirage mediation is important for the gaugino masses. For \( c \neq 0 \) we find the anomaly/modulus mediation ratio \( \alpha \) as

\[
\alpha = \frac{2}{3} \frac{(a-b)X_0}{b + c \langle S \rangle T_0} = \frac{2}{3} \left( \sqrt{3} - 1 \right) \left( 1 - \frac{b}{a} \right) < 0.49,
\]

\[ \text{12} \]
where we substituted $\ln(M_p/m_{3/2}) \simeq c(S) + bT_0 \simeq aT_0$.

For $c = 0$, $F^T$ is of $O(m_{3/2})$ through a tuning $(a-b)^2 = O(1)$ in order to make $T_0$ sufficiently large. However, in this case the perturbation around the reference point becomes unstable. Thus, generically it is required that $\delta T/T_0 \leq 1/b^2$ and $\delta X/X_0 \leq 1/b$ for the convergence.

### 3.3 ISS-racetrack model

#### 3.3.1 Model 4

Next we analyze the ISS-racetrack model (1). Similarly we first omit the constant piece $w_0$ in the superpotential and start with

$$
K = -3 \ln(T + \bar{T}) + |X|^2 - \frac{|X|^4}{\Lambda^2}, \quad W = -Ae^{-aT} + Ce^{-cT} + Be^{-bT}X.
$$

Then we find the following results.

$$
X_0 = \frac{\sqrt{3}}{6} \Lambda^2 \sim 16\pi^2 Be^{-bT_0} \simeq 10^2 m_{3/2} \ll 1, \quad T_0 \simeq \frac{1}{a-c} \ln \left[ \frac{aA}{cC} \right],
$$

$$
\langle W_{TT} \rangle \simeq -ac \langle W \rangle, \quad \langle W \rangle \simeq \frac{B}{\sqrt{3}} \left( \frac{aA}{cC} \right)^{-\frac{b}{a+c}} \simeq A \left( \frac{a}{c} - 1 \right) \left( \frac{aA}{cC} \right)^{-\frac{a}{a+c}},
$$

$$
F^T \simeq \frac{3b}{ac} m_{3/2}, \quad m_T = \frac{ac(T + \bar{T})^2}{3} m_{3/2},
$$

where the condition for obtaining the ISS-like vacuum $W|_0 \simeq D_X W|_0/\sqrt{3} \simeq W_X|_0/\sqrt{3}$, i.e. $V|_0 = 0$, has been adopted. The anomaly-to-modulus mediation ratio (10) is now given by

$$
\alpha = \frac{2acT_0}{3b \ln(M_p/m_{3/2})} = \frac{2ac}{3b^2},
$$

where we applied $\ln(M_p/m_{3/2}) \simeq bT_0$. Note that $ac/b^2 > 1$ is required in order to make the reference point of the ISS model stable. For the case with $ac/b^2 = 3$, we obtain $\alpha = 2$. For the case with $b = c$, $B = C = O(1)$, we need a large value of $A$, e.g., $A = e^{-d(S)} \gg 1$ and the condition that $b/a \simeq 1 - 1/\sqrt{3} \simeq 0.42$ to make the reference point stable, and then find $\alpha \simeq 1.58$.

#### 3.3.2 Model 5

Finally we turn on the constant $w_0$ in the previous example:

$$
K = -3 \ln(T + \bar{T}) + |X|^2 - \frac{|X|^4}{\Lambda^2}, \quad W = w_0 - A e^{-aT} + C e^{-cT} + B e^{-bT}X.
$$

Then we obtain the following results,

$$
X_0 = \frac{\sqrt{3}}{6} \Lambda^2 \simeq 10^2 m_{3/2} \ll 1, \quad T_0 \simeq \frac{1}{a-c} \ln \left[ \frac{aA}{cC} \right],
$$

13
\[ \langle W \rangle \simeq -ac \left( \frac{a}{c} - 1 \right) Ae^{-aT_0}, \quad \langle W \rangle \simeq \frac{B}{\sqrt{3}} e^{-bT_0} \simeq \frac{B}{\sqrt{3}} \left( \frac{aA}{cC} \right)^{-\frac{b}{a-c}}, \]

\[ F^T \simeq \frac{\sqrt{3}bB}{(a-c)aA} e^{-(a-b)T_0} m_{3/2} \simeq \frac{\sqrt{3}bB}{(a-c)aA} \left( \frac{aA}{cC} \right)^{\frac{1}{a-c}} m_{3/2} \]

\[ m_T = \frac{ac(T+\bar{T})^2}{3} m_{3/2} \left[ \frac{\sqrt{3}A}{B} \left( \frac{a}{c} - 1 \right) e^{-(a-b)T_0} \right], \]

where we applied the condition for realizing the ISS-like vacuum, \( w_0 \simeq (B/\sqrt{3})e^{-bT_0} + Ae^{-aT_0} - Ce^{-cT_0}. \)

The anomaly/modulus mediation ratio (10) is found as

\[ \alpha = \frac{2(a-c)aA}{\sqrt{3}b^2} e^{-(a-b)T_0} \frac{T_0}{\ln \left( \frac{M_p}{m_{3/2}} \right)}. \]

Note that \( \ln(M_p/m_{3/2}) \simeq \ln(B^{-1}e^{bT_0}) \). We find that \( \alpha \) is sensitive to the adjustment of \( B \). For \( B = \mathcal{O}(1) \), we obtain \( \ln \left( \frac{M_p}{m_{3/2}} \right) \simeq bT_0 \). Therefore,

\[ \alpha = \frac{2(a-c)aA}{\sqrt{3}b^2} e^{-(a-b)T_0}. \]

In the case that \( A \) and \( C \) are of \( \mathcal{O}(1) \), a value of \( a - c \) must be also of \( \mathcal{O}(1) \) for large \( T_0 \). Thus, the value of \( \alpha \) becomes typically very small such as \( \alpha \sim b^{-1}e^{-(a-b)T_0} \). However in this case it is not valid to use the reference point for the analysis. For example, we take the parameters as \( c = b \) and \( C = B = \mathcal{O}(1) \) and one finds that \( m_T \sim a(a-b)m_{3/2} \) and then \( \delta T/T_0 \sim b/a^2(a-b)^2 \sim 1/b, \delta X/X_0 \sim b^2/a(a-b) \sim b \). Therefore the perturbation of the potential around the reference point does not converge. On the other hand, if \( A = e^{d(S)} \) with \( d = \mathcal{O}(10) \), a value of \( a - c \) can be of \( \mathcal{O}(10) \). For example, in the case with \( b = c \) and \( B = C \), we find \( \alpha \simeq (2/\sqrt{3}) (a/b - 1) = \mathcal{O}(1) \) and the reference point is stable.

For \( B = Ce^{-d(S)} \ll 1 \) with \( d = \mathcal{O}(1-10) \), we obtain \( \ln(M_p/m_{3/2}) \simeq d(S) + bT_0 \) and find

\[ \alpha = \frac{2(a-c)aA}{\sqrt{3}bC} \frac{1}{b + d^{(S)}} e^{d(S) - (a-b)T_0}. \]

In this case, the magnitude of \( \alpha \) can be strongly dependent on \( e^{d(S)} \). For example, when \( A, C, a - b = \mathcal{O}(1) \) and \( b = c \), one obtains \( F^T = \sqrt{3}e^{d(S)}m_{3/2}/(a - b) \) and

\[ \alpha = \frac{2(a-b)}{\sqrt{3}} \frac{1}{b + d^{(S)}} e^{d(S)}. \]

For the case with \( b \lesssim e^{d(S)} \), i.e. \( d(S) = \mathcal{O}(1) \), the value of \( \alpha \) becomes of \( \mathcal{O}(1) \) and shifts of fields are given by \( \delta T/T_0 \sim e^{-2d(S)}/b(a-b)^2 \lesssim 1/b^3 \) and \( \delta X/X_0 \sim b^{-1}e^{d(S)}/(a-b) \lesssim 1/(a-b) \lesssim 1 \).

On the other hand, for the case with \( b \ll e^{d(S)} \), i.e. \( d(S) = \mathcal{O}(10) \), one can obtain \( \alpha \sim e^{d(S)} \gg 1 \) and find that the anomaly mediation is dominant compared with the
modulus mediation. In the latter case, note that mass of the modulus can be much heavier than the gravitino mass such as

\[ m_T \simeq \frac{(a-b)b(T+\bar{T})^2}{\sqrt{3}} e^{d(S)} m_{3/2} \sim e^{d(S)} m_{3/2} \gg ab(T+\bar{T})^2 m_{3/2}, \]

and the magnitude of \( w_0 \) becomes almost \( Ae^{-aT_0} - Ce^{-cT_0} \) which can be much larger than \( Be^{-bT_0} \) like the model in [36, 19]. In this case, the vacuum of the modulus can be stable during the inflation and then the modulus-induced gravitino problem can be avoided. The Polonyi problem may not occur if we have a low scale inflation like a new inflation. In that case, if one changes the gravitino mass of \( \mathcal{O}(100) \) TeV to \( \mathcal{O}(1) \) GeV, one could realize the gauge mediation model, which is studied in [34, 37]. However that is beyond the scope of this paper.

### 3.4 Phenomenological aspects

We have studied several concrete models. Here we comment on their phenomenological aspects like SUSY spectra in the visible sector. In most of models except model 5, the modulus mass \( m_T \) is quite large compared with the gravitino mass \( m_{3/2} \), i.e.

\[ \frac{m_T}{m_{3/2}} = \mathcal{O}(a^2), \]

with \( a \sim 4\pi^2 \). Model 5 can derive much heavier modulus mass.

There are three important sources of SUSY breaking, \( F_X, F_T \) and \( F_C \), where SUSY breaking through \( F_C \) appears as the anomaly mediation. Most of models except model 5 predict similar ratios among \( F_X, F_T \) and \( F_C \). Thus, first we concentrate ourselves to models 1-4. These models have \( F_X = \mathcal{O}(m_{3/2}) \) and

\[ \frac{F_T}{m_{3/2}} = \mathcal{O}(a^{-1}). \]

The ratio of the anomaly mediation to the modulus mediation is denoted by a value of \( \alpha \) defined in Eq. (10). Models 1-4 lead to \( \alpha = \mathcal{O}(1) \), that is, the modulus mediation and anomaly mediation are comparable.

Now, let us evaluate soft SUSY breaking terms in the visible sector. For such purpose, we have to fix couplings between SUSY breaking sources and the visible sector. We assume that gauge kinetic functions of the visible sector \( f_v \) are obtained as

\[ f_v = w_v T + m_v S, \]

where \( w_v \) and \( m_v \) are constants. In the simplest case with \( w_v = 1 \) and \( m_v = 0 \), gaugino masses in the visible sector appear as the mirage mediation with the values of \( \alpha \), which are shown in the previous subsections. In generic case, gaugino masses are obtained as the mirage mediation by replacing \( \alpha \) by \( \alpha \cdot \frac{f_v}{w_v(T+\bar{T})} \). For example, we could derive the value of \( \alpha \approx 2 \) in the simplest case with \( w_v = 1 \) and \( m_v = 0 \), e.g. in model 4 with \( ac/b^2 = 3 \). Also, other cases with nonvanishing values of \( w_v \) and/or \( m_v \) would lead to \( \alpha \cdot \frac{f_v}{w_v(T+\bar{T})} \approx 2 \). In
these models, gaugino masses at the TeV scale are given by the (tree level) pure modulus mediation, that is, the TeV scale mirage unification of gaugino masses \[8, 11\]. If the $X$ field couples to heavy modes, which are charged under the SM gauge group, effects due to $F^X$ could appear through loop-effects, that is, the gauge mediation. Its contribution is the same size as the anomaly mediation when mass of the messenger fields is Planck scale. Also, if the $X$ field does not sequestered from visible matter fields in the Kähler metric and the VEV of the $X$ field is comparable with the Planck scale, effects due to $F^X$ could appear in the gaugino mass through the Konishi/Kähler anomaly \[39\]. At any rate, the gaugino masses are of $O(m_{3/2}/a) \sim 1$ TeV unless a direct coupling appears in the gauge kinetic function.

Now, we also consider the couplings between the SM matter fields and $X$. In our framework, the hidden sector field $X$ directly couples to the modulus $T$. Thus, it would be natural that $X$ might live in the Calabi-Yau space rather than in the warped throat. Also the SM lives on D-branes which are wrapping on bulk Calabi-Yau. Thus, the contact terms between $X$ and the SM matter fields $Q$,

$$\int d\theta^4 c|X|^2|Q|^2,$$

would not be suppressed. Then, we would obtain SUSY braking scalar mass and A-terms of $O(m_{3/2}) \sim 100$ TeV. In this case, we have a large hierarchy between gaugino masses $M_a$ and scalar masses $m_i$ as

$$M_a = O(m_{3/2}/a), \quad m_i = O(m_{3/2}),$$

with $a = O(4\pi^2)$. That would have several phenomenological interesting aspects \[38\]. If the SM matter fields are sequestered from $X$ by any reason and the above contact terms are suppressed sufficiently, the mirage mediation would also be dominant in scalar masses and A-terms.

The situation in model 5 is different from others. Model 5 has a rich structure in the modulus mass and ratios among $F^X$, $F^T$ and $F^C$. A heavier modulus mass could be realized in model 5, and a value of $\alpha$ can vary from values like $\alpha = O(1)$ to large values like $\alpha = O(10)$. In the latter case with $\alpha = O(10)$, the anomaly mediation would be dominant in gaugino masses in the visible sector. Scalar masses would be of $O(m_{3/2})$ unless the SM matter fields are sequestered from $X$ by any reason.

Finally, we comment on the phases of F-components. The phases are aligned as $\text{Arg}[F^C] = \text{Arg}[F^X] = \text{Arg}[F^T] = \text{Arg}[\bar{W}]$ as long as $\text{Arg}[W_T] = \text{Arg}[\bar{W}]$. Thus, we can always solve the SUSY CP problem except for model 5. In model 5, we need a find-tuning for the alignment unless $\alpha = O(10)$.

4 Conclusion and discussion

We have studied modulus stabilization and realization of de Sitter vacua through F-term uplifting by a dynamically generated F-term. Here the uplifting sector $X$ directly couples
to the light Kähler modulus $T$ in the superpotential through, e.g., a stringy instanton effects. In the Polonyi-KKLT model, the perturbation from the reference point, where the KKLT-type modulus properties and the SUSY breaking structure are realized, is stable under the existence of such superpotential mixing, though the Kähler mixing can spoil the stability in general. Contrary, in the ISS-KKLT model, the superpotential mixing makes the perturbation unstable, that is, the true minimum is far from the reference point. This instability can be avoided by introducing another nonperturbative effect into the ISS-KKLT superpotential, i.e., by considering the ISS-racetrack model.

In the case that the perturbation from the reference point is stable, the qualitative features of the KKLT scenario are preserved even with such modulus mixing to the uplifting sector. One of the quantitative changes, which are phenomenologically relevant, is a larger hierarchy between the modulus mass $m_T$ and the gravitino mass $m_{3/2}$, i.e. $m_T/m_{3/2} \sim \mathcal{O}(a^2)$. Even with such a large mass, the modulus F-term keeps a moderate value, $F_T \sim \mathcal{O}(m_{3/2}/a)$ for $\ln(M_{Pl}/m_{3/2}) \sim a$, thanks to the enhancement factor originating from the $X$-$T$ mixing in the superpotential. Then we typically find a mirage-mediation pattern of gaugino masses of $\mathcal{O}(m_{3/2}/a)$. The scalar masses and the A-terms are generically of $\mathcal{O}(m_{3/2})$ because of possible direct couplings with $X$.

In the ISS-racetrack model we can realize $m_T \sim 10^8$ GeV and $m_X \sim 10^{10}$ GeV. In such case we could avoid the gravitino overproduction problem \[1\] by these scalar fields when the inflation has a low Hubble parameter $H_{inf} \sim 10^7$ GeV. This may be also important for the explanation of the dark matter abundance, and in this scenario the dark matter candidate is likely to be gaugino. Stability of the vacuum during the inflation epoch may suppress the non-thermal production of the gaugino (neutralino\[14\]) and the gravitino through the modulus and $X$ decays. With a tuning of $B$, we can partially obtain a sparticle spectrum of the anomaly-mediation type, and both the modulus $T$, whose mass can be much heavier than $10^8$ GeV, and the SUSY breaking field $X$ may be stable during the low scale inflation such as a new inflation model.

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**References**

[1] M. B. Green, J. H. Schwarz and E. Witten, *Cambridge, Uk: Univ. Pr. (1987)* 469 P. (Cambridge Monographs On Mathematical Physics); Cambridge, Uk: Univ. Pr. (1987) 596 P. (Cambridge Monographs On Mathematical Physics); J. Polchinski, *Cambridge, UK: Univ. Pr. (1998)* 402 p; Cambridge, UK: Univ. Pr. (1998) 531 p

\[14\] As for the thermal production in the mirage-type models, see Refs. [11].
[2] M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. B 156, 55 (1985).

[3] H. P. Nilles, arXiv:hep-th/0402022.

[4] S. B. Giddings, S. Kachru and J. Polchinski, Phys. Rev. D 66, 106006 (2002) [arXiv:hep-th/0105097]; S. Kachru, M. B. Schulz and S. Trivedi, JHEP 0310, 007 (2003) [arXiv:hep-th/0201028]; A. Giryavets, S. Kachru, P. K. Tripathy and S. P. Trivedi, JHEP 0404, 003 (2004) [arXiv:hep-th/0312104]; P. K. Tripathy and S. P. Trivedi, JHEP 0303, 028 (2003) [arXiv:hep-th/0301139]; O. DeWolfe and S. B. Giddings, Phys. Rev. D 67, 066008 (2003) [arXiv:hep-th/0208123]; F. Denef, M. R. Douglas, B. Florea, A. Grassi and S. Kachru, Adv. Theor. Math. Phys. 9, 861 (2005) [arXiv:hep-th/0503124]; S. Kachru and A. K. Kashani-Poor, JHEP 0503, 066 (2005) [arXiv:hep-th/0411279]; O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, JHEP 0507, 066 (2005) [arXiv:hep-th/0505160]; M. R. Douglas and S. Kachru, arXiv:hep-th/0610102; F. Denef, M. R. Douglas and S. Kachru, arXiv:hep-th/0701050.

[5] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D 68, 046005 (2003) [arXiv:hep-th/0301240].

[6] K. Choi, A. Falkowski, H. P. Nilles and M. Olechowski, Nucl. Phys. B 718, 113 (2005) [arXiv:hep-th/0503216]; K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski and S. Pokorski, JHEP 0411, 076 (2004) [arXiv:hep-th/0411066].

[7] L. Randall and R. Sundrum, Nucl. Phys. B 557, 79 (1999) [arXiv:hep-th/9810155]; G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP 9812, 027 (1998) [arXiv:hep-ph/9810442].

[8] K. Choi, K. S. Jeong and K. i. Okumura, JHEP 0509, 039 (2005) [arXiv:hep-ph/0504037].

[9] M. Endo, M. Yamaguchi and K. Yoshioka, Phys. Rev. D 72, 015004 (2005) [arXiv:hep-ph/0504036].

[10] S. Nakamura and M. Yamaguchi, Phys. Lett. B 638, 389 (2006) [arXiv:hep-ph/0602081]; M. Endo, K. Hamaguchi and F. Takahashi, Phys. Rev. Lett. 96, 211301 (2006) [arXiv:hep-ph/0602061]; Phys. Rev. D 74, 023531 (2006) [arXiv:hep-ph/0605091]; T. Asaka, S. Nakamura and M. Yamaguchi, Phys. Rev. D 74, 023520 (2006) [arXiv:hep-ph/0604132].

[11] K. Choi, K. S. Jeong, T. Kobayashi and K. i. Okumura, Phys. Lett. B 633, 355 (2006) [arXiv:hep-ph/0508029]; R. Kitano and Y. Nomura, Phys. Lett. B 631, 58 (2005) [arXiv:hep-ph/0509039]; R. Kitano and Y. Nomura, Phys. Rev. D 73, 095004 (2006) [arXiv:hep-ph/0602096]; K. Choi, K. S. Jeong, T. Kobayashi and K. i. Okumura, Phys. Rev. D 75, 095012 (2007) [arXiv:hep-ph/0612258].

[12] H. Abe, T. Kobayashi and Y. Omura, Phys. Rev. D 76, 015002 (2007) [arXiv:hep-ph/0703044].
[13] K. Intriligator, N. Seiberg and D. Shih, JHEP 0604, 021 (2006) [arXiv:hep-th/0602239].

[14] S. Forste, Phys. Lett. B 642, 142 (2006) [arXiv:hep-th/0608036]; M. Dine, J. L. Feng and E. Silverstein, Phys. Rev. D 74, 095012 (2006) [arXiv:hep-th/0608159].

[15] S. Franco and A. M. Uranga, JHEP 0606, 031 (2006) [arXiv:hep-th/0604136]; H. Ooguri and Y. Ookouchi, Nucl. Phys. B 755, 239 (2006) [arXiv:hep-th/0606061]; Phys. Lett. B 641, 323 (2006) [arXiv:hep-th/0607183]; R. Kitano, H. Ooguri and Y. Ookouchi, Phys. Rev. D 75, 045022 (2007) [arXiv:hep-ph/0612139]; V. Braun, E. I. Buchbinder and B. A. Ovrut, Phys. Lett. B 639, 566 (2006) [arXiv:hep-th/0606166]; JHEP 0610, 041 (2006) [arXiv:hep-th/0606241]; S. Franco, I. Garcia-Etxebarria and A. M. Uranga, JHEP 0701, 085 (2007) [arXiv:hep-th/0607218]; A. Amariti, L. Girardello and A. Mariotti, JHEP 0612, 058 (2006) [arXiv:hep-th/0608063]; I. Bena, E. Gorbatov, S. Hellerman, N. Seiberg and D. Shih, JHEP 0611, 088 (2006) [arXiv:hep-th/0608157]; C. Ahn, Class. Quant. Grav. 24, 1359 (2007) [arXiv:hep-th/0608160]; Phys. Lett. B 647, 493 (2007) [arXiv:hep-th/0610025]; JHEP 0705, 053 (2007) [arXiv:hep-th/0701114]; R. Argurio, M. Bertolini, S. Franco and S. Kachru, JHEP 0701, 083 (2007) [arXiv:hep-th/0610212]; JHEP 0706, 017 (2007) [arXiv:hep-th/0703236]; M. Aganagic, C. Beem, J. Seo and C. Vafa, [arXiv:hep-th/0610249]; J. J. Heckman, J. Seo and C. Vafa, [arXiv:hep-th/0702077]; R. Tatar and B. Wetenhall, JHEP 0702, 020 (2007) [arXiv:hep-th/0611303]; S. Hirano, JHEP 0705, 064 (2007) [arXiv:hep-th/0703272]; A. Giveon and D. Kutasov, [arXiv:hep-th/0703135]; S. Franco, A. Hanany, D. Krefl, J. Park, A. M. Uranga and D. Vehg, [arXiv:hep-th/0707.0298]; E. Dudas, J. Mourad and F. Nitti, [arXiv:hep-th/0706.1269]; C. Angelantonj and E. Dudas, [arXiv:hep-th/0704.2553]; J. Marsano, K. Papadodimas and M. Shigemori, [arXiv:hep-th/0705.0983].

[16] A. Saltman and E. Silverstein, JHEP 0411, 066 (2004) [arXiv:hep-th/0402135]; M. Gomez-Reino and C. A. Scrucca, JHEP 0605, 015 (2006) [arXiv:hep-th/0602246]; O. Lebedev, H. P. Nilles and M. Ratz, Phys. Lett. B 636, 126 (2006) [arXiv:hep-th/0603047]; Z. Lalak, O. J. Eytin-Williams and R. Matyszskiewicz, JHEP 0705, 085 (2007) [arXiv:hep-th/0702026].

[17] E. Dudas, C. Papineau and S. Pokorski, JHEP 0702, 028 (2007) [arXiv:hep-th/0610297].

[18] H. Abe, T. Higaki, T. Kobayashi and Y. Omura, Phys. Rev. D 75, 025019 (2007) [arXiv:hep-th/0611024].

[19] R. Kallosh and A. Linde, JHEP 0702, 002 (2007) [arXiv:hep-th/0611183].

[20] O. Lebedev, V. Lowen, Y. Mambrini, H. P. Nilles and M. Ratz, JHEP 0702, 063 (2007) [arXiv:hep-ph/0612035].

[21] J. Polonyi, Hungary Central Inst Res - KFKI-77-93 (77,REC.JUL 78) 5p.
[22] L. O’Raifeartaigh, Nucl. Phys. B 96, 331 (1975).

[23] C. P. Burgess, R. Kallosh and F. Quevedo, JHEP 0310, 056 (2003) [arXiv:hep-th/0309187]; A. Achucarro, B. de Carlos, J. A. Casas and L. Doplicher, JHEP 0606, 014 (2006) [arXiv:hep-th/0601190]; S. L. Parameswaran and A. Westphal, JHEP 0610, 079 (2006) [arXiv:hep-th/0602253]; arXiv:hep-th/0701215.

[24] K. Choi and K. S. Jeong, JHEP 0608, 007 (2006) [arXiv:hep-th/0605108].

[25] A. Westphal, JHEP 0703, 102 (2007) [arXiv:hep-th/0611332].

[26] B. Florea, S. Kachru, J. McGreevy and N. Saulina, JHEP 0705, 024 (2007) [arXiv:hep-th/0610003]; R. Blumenhagen, M. Cvetic and T. Weigand, Nucl. Phys. B 771 (2007) 113 [arXiv:hep-th/0609191]; L. E. Ibanez and A. M. Uranga, JHEP 0703, 052 (2007) [arXiv:hep-th/0609213]; M. Buican, D. Malyshev, D. R. Morrison, H. Verlinde and M. Wijnholt, JHEP 0701, 107 (2007) [arXiv:hep-th/0610007].

[27] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B 241, 493 (1984).

[28] B. S. Acharya, K. Bobkov, G. L. Kane, P. Kumar and J. Shao, [arXiv:hep-th/0701034].

[29] H. Abe, T. Higaki and T. Kobayashi, Phys. Rev. D 74, 045012 (2006) [arXiv:hep-th/0606095].

[30] H. Abe, T. Higaki and T. Kobayashi, Phys. Rev. D 73, 046005 (2006) [arXiv:hep-th/0511160].

[31] M. Serone and A. Westphal, [arXiv:0707.0497] [hep-th].

[32] H. Abe, T. Higaki and T. Kobayashi, Nucl. Phys. B 742, 187 (2006) [arXiv:hep-th/0512232].

[33] M. Dine, R. Kitano, A. Morisse and Y. Shirman, Phys. Rev. D 73, 123518 (2006) [arXiv:hep-ph/0604140].

[34] R. Kitano, Phys. Lett. B 641, 203 (2006) [arXiv:hep-ph/0607090].

[35] A. Westphal, [arXiv:0705.1557] [hep-th].

[36] R. Kallosh and A. Linde, JHEP 0412, 004 (2004) [arXiv:hep-th/0411011].

[37] S. P. de Alwis, [arXiv:hep-th/0703247].

[38] J. D. Wells, Phys. Rev. D 71, 015013 (2005) [arXiv:hep-ph/0411041]. M. Ibe, T. Moroi and T. T. Yanagida, Phys. Lett. B 644, 355 (2007) [arXiv:hep-ph/0610277].

[39] J. A. Bagger, T. Moroi and E. Poppitz, JHEP 0004, 009 (2000) [arXiv:hep-th/9911029]; P. Binetruy, M. K. Gaillard and B. D. Nelson, Nucl. Phys. B 604, 32 (2001) [arXiv:hep-ph/0011081].

[40] K. Choi and H. P. Nilles, JHEP 0704, 006 (2007) [arXiv:hep-ph/0702146].
[41] H. Baer, E. K. Park, X. Tata and T. T. Wang, JHEP 0608, 041 (2006) [arXiv:hep-ph/0604253]; K. Choi, K. Y. Lee, Y. Shimizu, Y. G. Kim and K. i. Okumura, JCAP 0612, 017 (2006) [arXiv:hep-ph/0609132]; H. Baer, E. K. Park, X. Tata and T. T. Wang, arXiv:hep-ph/0703024; H. Abe, Y. G. Kim, T. Kobayashi and Y. Shimizu, arXiv:0706.4349 [hep-ph].