Real teaching situations to encourage the learning of fractions from physics

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Abstract. The research aims to implement real didactic situations for learning fractions and physics in students from educational institutions in “San José de Cúcuta, Norte de Santander, Colombia”. From the theoretical point of view, it starts from one of the ideas that have been emerging in recent times regarding the teaching of physics, such as the development of physical-mathematical thinking by inserting the paradigm of didactic situations into the educational environment. The research is part of a field study, quasi-experimental with self-control. The treatment plan was organized in five didactic situations on theoretical and practical aspects of physics. The results of the pretest reflect deficiencies in the use of mathematical contents associated to the description of physical phenomena in the population under study, which allowed the design and implementation of a pedagogical intervention plan. The post-test showed an increase in the execution of mathematical models in the application to physical phenomena, which inferred the effectiveness of the intervention plan, whose activities allowed innovation in the classroom to achieve the students' attention. It is concluded that the didactic proposal was an enriching experience with the active participation of the students.

1. Introduction

Human progress has been linked to the need to solve problems. Therefore, problems associated with mathematics and physics arise when people are presented with the situation of measuring lengths, areas, volumes, weights, displacements, speeds and other kinds of measurements in everyday life. Furthermore, when the need to find another way of representing dynamic situations is observed, mathematical models present some degree of difficulty to students. However, in basic education students, the concept of fraction, the basis for describing physical phenomena, has only one meaning: it is used to express a part of a whole and its meaning depends on the context in which it is applied. Therefore, understanding the concept of fraction requires that teachers have full mastery of the various contexts, as well as that their classroom activities be coherent and cover a diversity of situations, where the student can differentiate the context and therefore the meaning of the fraction.

Likewise, students in the primary education grades should acquire knowledge about fractions that will allow them to learn mathematics well and, therefore, its application to physics, which is necessary for obtaining knowledge in different areas and at different levels, including higher education. In this regard, teachers should seek viable teaching strategies so that the learning of fractions helps students to develop skills to understand and pose various problems and exercises in both mathematics and physics.

In this order of ideas, this research that aims to implement real didactic situations for the learning of fractions in students of the fifth grade of basic primary education. From the theoretical point of view, it is based on one of the ideas that has been emerging in recent times regarding the teaching of
mathematics, the approach of Brosseau [1], who has inserted his paradigm of didactic situations into the educational environment. He bases his proposals on the cognitive, constructivist and humanist approaches, since his ideas consider the triad of teacher, knowledge and student as an interactive process in the development of classes, which includes respect for the ideas of the students. It is also based on the proposals for teaching physics at basic levels [2] through experimentation. From the methodological point of view, the research is framed under a field, quasi-experimental study with a single group to which an intervention or treatment plan was applied [3,4] and evaluated before and after its application, the previous results being the basis for the design of the intervention and the final evaluation of results.

The theory of didactic situations proposed by Brousseau establishes that the student learns by adapting to an environment that is a factor of contradictions, of difficulties, of imbalances, a little like human society. This knowledge, the fruit of the student's adaptation, manifests itself as new responses that are the test of learning [5]. The learning of mathematics and physics constitutes one of the objects of knowledge that can be mediated by the teacher on the basis of Brousseau's [6] approaches, related to teaching situations. Pedagogical mediation can accompany learning by building bridges between what the student knows and the new knowledge, since the socio-cultural approach of teaching processes represents a new epistemological vision, in the sense that it proposes that teaching and learning are not processes of transmission and reception of contents, but of mediated construction of meanings [7].

However, in Brousseau's proposals the term interaction appears, which makes us think that his conceptions come from the Piagetian and Vigotskian constructivist theories. In this sense, some of Brousseau's assumptions are brought [6] where he states that the term situation is a model of interaction of a subject with a certain environment that determines a given knowledge as the resource available to the subject to reach or preserve a favorable state in this environment. Some of the situations require the previous acquisition of all the necessary knowledge and schemes, but there are others that offer a possibility to the subject to construct by himself a new knowledge in a genetic process [8,9].

Therefore, teaching practices developed from real situations represent one of the most important aspects of teachers' mediated didactic interventions, beyond the teaching materials provided to students, since it is the doing of them that will lead them to a meaningful learning of mathematical and physical concepts. Designing real teaching situations does not mean merely stating certain activities. It implies recognizing the students' cognitive abilities and processes in order to select, from a range of possibilities, those that promote the integral development of the student and to identify the didactic methods and resources appropriate to the subject area to be taught [10]. In this order of ideas, we can say that the paradigmatic aspects of the different ways of teaching determine the nature of knowledge [11,12].

2. Method
The research is part of a field and quasi-experimental study. It is field based because the strategy followed by the research was based on methods that allowed data to be collected directly from reality. Field research constitutes a systematic, rigorous and rational process of data collection, treatment, analysis and presentation, based on a strategy of direct collection of the reality of the information needed for the research [13].

The investigation starts from the diagnosis and recognition of the initial situation. The process began with the identification of the problem area based on preliminary ideas to arrive at the subject matter. Subsequently, the instruments were developed, the pilot test was applied, the pretest was applied, and the data were ordered, grouped, arranged and related, in accordance with the objectives of the research; in other words, the information was prepared in order to proceed with its analysis and interpretation in both the problem statement component and the pretest. Based on these results, an intervention plan is drawn up. When we know what is happening, i.e. a situation has been diagnosed, we have to decide what to do. During this moment an action plan is elaborated, to improve what is already happening. In the action plan, the pedagogical priorities are studied and established, and options for possible alternatives are proposed. Implementation involves acting to put the plan into practice and observing its effects in the context in which it takes place. Finally, a post-test is applied in order to evaluate the result of the intervention.
3. Results
The results of the research are organized in three sections: analysis of the pre-test, description of the implementation of the intervention plan and analysis of the results of the post-test.

3.1. Analysis of the results of the pre-test
In the pre-test, the items of equivalent fraction were answered correctly by 48% of the students. These items try to reason the problem, understanding reasoning as the action of ordering ideas in the mind to reach a conclusion; the process of reasoning is directed to the network of relationships between mathematical concepts and structures that is practically inexhaustible. Mathematical reasoning allows the continuous generation of new procedures and algorithms; it is not possible, therefore, to terminate the mastery of any concept in a short period of time, nor to pretend that a significant connection between a new knowledge and those previously established knowledges is automatically achieved [14].

The pictorial representation of the fraction applied to physics was correctly identified by 61%. In this sense, several authors agree that representation in mathematics has to do with some aspects where the student gives an account of the how and why of the processes followed to reach conclusions [15].

The fraction as part of a whole is correctly identified by 77%. By means of this type of exercise, the student is expected to reflect, analyze and exchange interpretations; a process that will also allow him/her, through the confrontation of conjectures, to express them with the language of mathematics.

The addition of fractions is done correctly by 58% of the students. Fraction addition exercises play an important role in the mathematics class, but on condition that it is not understood simply as the transcription of a symbolic language through which the teacher, who possesses the codes of mathematical language, tries to communicate them to his or her students, whose role is limited to being mere receivers, or that the oral and written interactions that take place in the mathematics class are not reduced to simply giving small answers [16]. The subtraction of fractions was only done correctly by 19%. When the teaching of fractions in mathematics is deficient, the class can generally be reduced to the transcription of a symbolic language that lacks meaning. This does not allow students to develop their mathematical thinking, through the processes of particularization, generalization, conjecture and conviction; thus, with the practice of fractions, mathematical thinking processes are developed where students are stimulated to learn [17].

Fraction comparison is done correctly by 38% of the students. Therefore, for the teaching of fractions, he states that teaching is of capital importance, since it is the negotiation of meanings that should take precedence. Teaching refers to the interaction between the different subjects in a class, using their own language, which is a mixture of everyday language and mathematical language; thus, for the teaching of fractions, the negotiation of meanings must appear naturally, which refers to the way in which students and teachers expose to each other their way of understanding mathematical concepts and processes, refine them and adjust them to mathematical knowledge [19].

The fraction as a ratio is correctly calculated by 77% of the students. This implies that the students, in their great majority, solve the exercises in a positive way. Therefore, it may be that the element that is having a positive impact is teaching; as Lerner and Sadovsky [20] state, the mission of the teacher in information-rich environments is that of a facilitator, a guide and advisor on appropriate sources of information, a creator of habits and skills in the search, selection and treatment of information. In these environments, experience, in some cases, is more important than the information itself, which is accessible by other, more efficient means. Students, for their part, must adopt a much more important role in their training, not only as mere passive recipients of what is generated by the teacher, but as active agents in the search, selection, processing, and assimilation of information.

The operator fraction is correctly identified by 52% of the students. This result refers to calculation using fractions; therefore, it is related to Castro's study [21] on the meaning of fractions in school mathematics and concludes that knowledge of mathematical content and didactic knowledge of content about the school notion of fraction based on the part-to-all relationship depends on teaching techniques.
3.2. Intervention plan

The intervention proposal was developed in a time frame of five weeks. Five real didactic situations were created, where the diagnosed contents were integrated into the pretest; each one of the didactic situations was organized according to Brousseau's [1] exposition as described below.

Action situations: It is also related to the initial moment of the structure of the school guide, the real situation is presented to the student and he or she makes decisions by himself/herself and acts on the means that the teacher proposes, according to what he/she knows.

Communication situations: This corresponds to the praxicological part, the student communicates with his or her colleagues, exposes and defends his or her ideas, develops and carries out the problems posed, and through practice discusses the solutions, as well as giving shape to the situation.

Validation situations: It is supported by concepts already given to validate the way in which he arrived at the solution of the situations, being work in groups, compares his answers and the decisions he made with the other groups, where the points of view of each group are put in debate, between each of the groups justifications or demonstrations of what was done during the process are requested.

Institutionalization situations: Although the main actor of the intervention plan is the student, the teacher, besides propitiating the real didactic situations with the objective that they learn a knowledge, must also verify the final process, therefore, enters the teacher initiates a dialogue with the student, about what they did, clarifies the doubts, and relates the proposals of each group with the theory or scientific knowledge.

The evaluation is constant during the whole intervention, since the student is always faced with questions in relation to the situation that force him/her to verify or correct his/her decisions. These situations are operationalized from previous activities, interaction activities, application activities and extension activities.

3.3. Analysis of the results of the post-test

A test of difficulty and characteristics similar to the pre-test was applied in order to compare results once the intervention was applied. It was found that in general the answers reflect a significant improvement in the different aspects evaluated. Sixty-two percent of the students correctly answered the items related to equivalent fraction, 74% correctly answered the fraction representation items, 94% correctly answered the fraction items as part of the whole, 71% correctly answered the fraction addition items and 29% correctly answered the fraction subtraction items, 52% correctly answered the fraction items as operator and 91% correctly answered the fraction items as reason. Therefore, it can be said in this sense that the treatment plan was significant.

The understanding of the concept of fraction requires that the teacher has full mastery of the various contexts, as well as that his or her classroom activities are coherent and cover a diversity of situations, where the student can differentiate the context and therefore the meaning of the fraction. In this sense, working with two- and three-dimensional objects and their movements and transformations allows the integration of fraction notions with other mathematical contents [22].

On the other hand, Esquinas [23], mentions that students may be found who, in spite of having a good performance in some contents such as the relation part all in fractional, do not manage to exhibit a good performance in other contents. In the same vein, Socas, Camacho, Palarea and Hernández [24] state that, in order to develop content associated with fractional numbers, one must work with real and meaningful environments, since students have notions of concepts, with the aim of using them to reason critically in situations of uncertainty or risk. The fraction has only one meaning, since it is used to express a part of a whole and its meaning depends on the context in which it is applied. Brousseau's ideas [1] in his theory of didactic situations, state that teaching should include situations of interaction between students and their life contexts, constituting the axis of action of pedagogical practices. For this reason, turning the classroom into a situational space is one of Brousseau's [25] premises, who has called a situation a model of interaction of a subject with a certain environment that determines a given knowledge, such as real teaching situations.
4. Discussion

When analyzing the results, a significant overall difference between the pre-test and post-test can be observed. That is to say, in the questions related to the evaluated contents, the students improved their performance, after the treatment plan. This is interesting, since it can be established that the intervention proposal was significant. According to the research, the development of the activities generated in the student’s enthusiasm and participation, which leads to think that the teacher's strategies based on didactic situations are essential for the learning of the fractions.

In this respect, Brousseau's ideas [1] refer to the fact that the theory of didactic situations makes it possible to design and explore a set of class sequences conceived by the teacher in order to have a means of learning. According to this expert, the teacher must be a reflective professional, who decides, designs, implements and experiments with action strategies to achieve the learning of his or her students. Thus, learning mathematics is not reduced to remembering formulas or definitions to solve problems with attachment to the methods illustrated in school texts. The theory of didactic situations proposes the study of the conditions in which mathematical knowledge is constituted; and it is considered that control of these conditions will make it possible to reproduce and optimize the processes of knowledge acquisition.

With respect to the theoretical assumptions, Godino and Font [26] refer that the subject of fractions implies representing, generalizing and formalizing patterns and regularities in any aspect of mathematics. As this is developed, progress is made in the performance in mathematics necessary to raise the grade point average in that subject. Romero and Aguilar [2] refer to experimentation, in the sense of the application of field work to the problems proposed, in order to reach their understanding and therefore, from the observation of physical phenomena, mathematics can be linked to achieve a good development of physical-mathematical skills. For Esquinas [23], teachers should help students to build a different view of fractions in mathematical activity, and on how to develop this theme throughout the different levels. Godino and Font [26] propose some characteristics of fractions, which can be generalized to mathematics that are simple to acquire by students and therefore should be known by teachers in training. They can be recognized, extended or generalized. The same pattern can be found in many different ways.

In this sense, most teachers at the secondary level teach mathematics and physics in a routine, expository, and tedious manner; they do not apply methods, techniques, and learning strategies that are striking to the student and still follow the traditionalist model. Therefore, in the pedagogical practice developed from real situations, collaborative work and interaction between students, students and teachers was sought. As Cardoso and Cerecedo [27] point out, the implementation of mathematical and physical activities in the classroom that make sense to students improves performance in these areas of knowledge. So, in the dimension interpretation of fractions as reason, relation part all, quotient and operator in different contexts, reason of change, use in speed calculation and others; the activities of the proposal, were effective for most of the thematic contents. In this sense, in the teaching processes the cognitive development of each student is put at stake, so that the teacher, in some cases, cannot solve all the learning problems. Therefore, cognitive development and learning are basically the result of a process of construction, which the human fact cannot be understood as the deployment of a unique program for each student, nor as the result of an accumulation and absorption of experiences.

According to Brousseau [1], interaction is conceived as an activity of exchange between the student's thought and knowledge, between teachers and students through various exchanges, between each actor in education and a resource internal or external to the subject. The theory of didactic situations is currently presented as a scientific instrument that seeks the unification and integration of the contributions of other disciplines and provides a greater understanding of the possibilities of mathematics teaching; the experiences with the different forms of use of the usual operations such as addition, subtraction, multiplication and division, for the calculation of fractions, generate combinations, separations, groupings or distributions in this type of exercise. From this perspective, Resnick and Ford [22] rescue the notion of a didactic situation that corresponds to the interaction of a subject with a certain medium that determines a given knowledge. Some of these situations require the acquisition of the necessary prior knowledge. By didactic situation we mean a situation intentionally constructed by the
teacher in order to make the students acquire a certain knowledge or knowledge in the process of being constituted. Therefore, each didactic situation in this study was planned based on real activities, whose need to be solved implied the previous mathematical knowledge to give sense to the class.

To this end, according to Brosseau [1], the teacher must make sure that the student takes responsibility for working on the solution of mathematical problems with applications to physics, and if he or she does not reach a solution, at least indicate certain approaches according to the proposed objectives. For this reason, school mathematical activity is modelled on the notion of situation, which is a set of specific knowledge situations that allow a field of problems to be solved. The teacher must imagine and propose to the students applied situations that they can experience, which provoke the emergence of genuine problems related to physics and mathematics and in which the knowledge in question appears as an optimal solution to these problems, with the additional condition that this knowledge is constructible by the students.

The teacher has to offer a possibility to the subject to build by himself a new knowledge in a constructive process. The same word situation serves, in its ordinary sense, to describe both the set of conditions that frame an action, and the theoretical and eventually formal model that serves to study it. Thus, the situations, activities and experiences from which the teacher draws the pedagogical path that his or her students must necessarily follow with him or her in order to construct and reconstruct their own knowledge must be adjusted to the context. Thus, through joint and interactive activities, mathematics and physics teachers help students to develop construction areas so that they can appropriate the knowledge, thanks to their contributions and structured aids in school activities following a certain intentional direction. In their practice, teachers should try to create flexible and strategic scaffolding systems. For this, collaborative work is fundamental, since with this type of strategy the participants share interaction, exchange ideas and knowledge by actively participating.

5. Conclusions
The design of a treatment plan with real didactic situations for learning fractions and their application in physics in fifth grade students was a task that took dedication and effort. The development of each of the activities took time and dedication. To do so, the researchers were confronted with the reading of various materials and consulted experts. This is important because the design was basically oriented to understanding the educational reality where the research put the plan into practice. This allowed for five real educational situations where activities, time, resources and evaluation are made explicit.

Regarding the implementation of a treatment plan with real didactic situations for learning fractions in fifth grade students. In general terms, the activities allowed for innovation in the classroom to achieve student attention; in addition, having used what the students handle on a daily basis and maintain as a permanent reference, such as the activities of their own context, allowed for greater participation in the activities.

The didactic proposal became an enriching experience. The activities developed allowed an active participation of the students. Thus, with an appropriate pedagogical approach, it allowed the students to find new ways of learning, since they felt motivated, interested and attracted by the activities. The integration in the learning of physics explained from the mathematical model was quite attractive for students, since they not only develop physical-mathematical thinking but also find in physics an experimental model where mathematics becomes very relevant.

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