(α′)⁴ CORRECTIONS IN HOLOGRAPHIC LARGE $N_c$ QCD AND $\pi - \pi$ SCATTERING

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Abstract

We calculate the $\alpha'^4$ corrections to the non-Abelian DBI action on the $D8$-brane in the holographic dual of large $N_c$ QCD proposed by Sakai and Sugimoto. These give rise to higher derivative terms, in particular, four derivative contact terms for the pion field with the coupling uniquely determined. We calculate the pion-pion scattering amplitude near threshold. The results respecting unitarity are in qualitative agreement with the experimental values.
A holographic dual of QCD with $N_f$ massless quarks using a $D4/D8$-brane configuration in Type-II-A string theory, within the framework of AdS/CFT, has been proposed by Sakai and Sugimoto [1,2]. This describes the low energy phenomena of large $N_c$ QCD such as the spontaneous breaking of chiral $U(N_f)_L \times U(N_f)_R$ symmetry to the diagonal subgroup $U(N_f)L + R$. In this model, the ingredients are an $U(N_f)$ five-dimensional Yang-Mills and Chern-Simons theory on a curved background, originating from the low energy effective action on the probe $D8$-branes embedded into the $D4$ background [3]. The entire tower of vector mesons and the pions are in a single $U(N_f)$ gauge field in five dimensions, simplifying possible interaction structures among mesons.

In the AdS/CFT correspondence [4,5,6] scenario, the baryons are constructed by $D4$-branes wrapped on non-trivial cycles [7,8]. In the $D4/D8$ model [1,2], baryons are identified as $D4$-branes wrapped on a non-trivial 4-cycle in the $D4$ background, realising $D4$-brane as a small instanton configuration in the world-volume gauge theory on the probe $D8$-brane. The pion effective action obtained from the 5-dimensional YM theory is the Skyrme action [9] in which baryons appear as solitons, with the identification of the baryon number as a winding number and equivalent to the instanton number in the 5-d YM theory.

The chiral lagrangian derived from the $D4/D8$-brane model [1,2] is found to describe the axial coupling $g_A$ and the electromagnetic form factors of the nucleon [10,11]. Using the hedgehog ansatz [9,12] for the chiral field in the Skyrme model, the mass and the root-mean-square radius of the brane-induced Skyrmion compare well with the standard Skyrmion [13]. In the studies of the $D4/D8$ model [1,2] thus far, only the leading terms in the non-Abelian Dirac-Born-Infeld (DBI) action of the $D8$-brane, i.e., order of $(2\pi\alpha')^2$, have been considered to obtain a 4-dimensional action for pions, although higher order corrections have been considered in the fluctuation analysis in background intersecting branes [14] and in the computation of soliton mass [11].

In this work, we include higher-order terms of the DBI-action up to $(2\pi\alpha')^4$ to obtain a 5-dimensional $D8$-brane action. When this is applied to pions, we obtain, besides the Skyrme action, two terms involving four pion fields, precisely of the form suggested by Weinberg [15] in his phenomenolog-
ical approach, thus bringing the holographic dual QCD in closer connection with the realistic QCD. The coupling strength of these terms are dimensionless and their values uniquely determined. It is to be noted that while in the phenomenological lagrangian approach, it was necessary to arrange all the pion couplings as derivative interactions, to suppress the incalculable graphs in which soft pions would be emitted from internal lines of a hard particle processes, in our case, all these pion couplings naturally come as derivative interactions. As an application, we have evaluated the \( \pi - \pi \) scattering amplitude \( R_0 \) defined in Sannino and Schechter [16] and find that the inclusion of the higher order derivative terms ameliorate the \( R_0 \) curve, avoiding violation of unitarity, bringing it closer to Roy’s curve [17] as given in [16].

Within the \( \alpha'^2 \) terms of the \( D_8 \)-brane DBI action, it was shown in [2] that when an infinite tower of massive vector mesons are included, the low energy \( \pi - \pi \) scattering is governed only by the chiral lagrangian for pions. We are here including the \( \alpha'^4 \) corrections from the \( D_8 \)-brane DBI action to the chiral lagrangian for pions.

Briefly, in the SS model [1], the \( D_4 \) background consisting of \( N_c \) flat \( D_4 \)-branes with one of the spatial world-volume directions \( \tau \) compactified on \( S^1 \), is given by the supergravity solution [18]

\[
(ds)^2 = \left( \frac{U}{R} \right)^{\frac{3}{4}} (\eta_{\mu\nu} dx^\mu dx^\nu + f(U) d\tau^2) + \left( \frac{R}{U} \right)^{\frac{3}{4}} \left( \frac{dU^2}{f(U)} + U^2 d\Omega_2^2 \right),
\]

\[
e^\phi = g_s \left( \frac{U}{R} \right)^{\frac{3}{4}}; F_4 = \frac{2\pi N_c}{V_4} \epsilon_4; f(U) = 1 - \frac{U_{KK}^3}{U^3},
\]

where \( x^\mu (\mu = 0, 1, 2, 3) \) and \( \tau \) are the directions along which the \( D_4 \)-brane is extended, \( d\Omega_4, \epsilon_4, V_4 = \frac{8\pi^2}{3} \) are the line element, volume form and the volume of unit \( S^4 \), \( R \) and \( U_{KK} \) are parameters, the coordinate \( U \) is bounded from below \( (U \geq U_{KK}) \), \( U = U_{KK} \) corresponds to a horizon in the supergravity solution, \( g_s(= e^{\phi_0}) \) is the string coupling constant and \( R^3 = \pi g_s N_c \ell_s^3 \), \( \ell_s \) being the string length. The 4-form is \( F_4 = dC_3 = \frac{2\pi N_c}{V_4} \epsilon_4 \) with \( \epsilon_4 \) as the volume form of \( S^4 \). With \( \tau \) periodic, the conical singularity at \( U = U_{KK} \) is avoided by having the period \( \delta \tau \) of \( \tau \) as \( \frac{4\pi}{3} \frac{R^3}{U_{KK}^2} \). The Kaluza-Klein mass scale is \( M_{KK} = \frac{2\pi}{\delta \tau} = \frac{3}{2} \frac{U_{KK}^\frac{1}{2}}{R^\frac{3}{2}} \), below which the dual gauge theory is effectively the same as 4-dimensional YM theory, with \( g^2_{YM} = 4\pi^2 g_s \ell_s / \delta \tau \). Then the
The induced metric on the $D8$-brane, embedded in the $D4$-background (1) with $U = U(\tau)$ is

$$(ds)^2_{D8} = \left(\frac{U}{R}\right)^{\frac{3}{2}} \eta_{\mu\nu} dx^\mu dx^\nu + \left\{ \left(\frac{U}{R}\right)^{\frac{3}{2}} f(U) + \left(\frac{R}{U}\right)^{\frac{3}{2}} \frac{U''}{f(U)} \right\} d\tau^2 + \left(\frac{R}{U}\right)^{\frac{3}{2}} U^2 d\Omega_4^2, \quad (3)$$

where $U' \equiv \frac{dU}{d\tau}$. The $N_f$ $D8 - \bar{D8}$ pairs are separately placed along the anti-podal points of $\tau$ (see figures 1, 2 of [13]) and are smoothly interpolated with each other. Then, the $U(N_f)_{D8} \times U(N_f)_{\bar{D8}}$ gauge symmetry breaks in to $U(N_f)$ gauge symmetry which is interpreted as a holographic manifestation of chiral symmetry breaking in QCD, realizing the chiral symmetry breaking by the geometrical connection of $D8$ and $\bar{D8}$ branes. As the probe $D8$-brane world volume is on a plane of constant $\tau$, the induced metric (3) can be written as

$$(ds)^2_{D8} = \left(\frac{U(z)}{R}\right)^{\frac{3}{2}} \eta_{\mu\nu} dx^\mu dx^\nu + 4 \left(\frac{R}{U(z)}\right)^{\frac{3}{2}} \frac{U_{KK}}{U(z)} dz^2 + \left(\frac{R}{U(z)}\right)^{\frac{3}{2}} U^2(z) d\Omega_4^2, \quad (4)$$

with $U^3 = U^3(z) = U_{ KK}^3 + U_{ KK} z^2$. The $D8$-brane extends along $x^\mu(\mu = 0, 1, 2, 3)$ and $z$ directions, wrapping around $S^4$.

The gauge field on the probe $D8$-brane has nine components, $A_\mu$, $A_z$ and $A_i$ ($i = 5, 6, 7, 8$, the coordinates on $S^4$). The non-Abelian DBI-action on $D8$-brane is

$$S_{D8}^{DBI} = T_8 \int d^9x e^{-\phi} \sqrt{-\det(g_{MN} + (2\pi\alpha') F_{MN})}, \quad (5)$$

where $\alpha' = l_s^2$ is the Regge slope parameter and $T_8 = \frac{1}{(2\pi)^8 l_s^8}$ is the surface tension of the $D8$-brane. $F_{MN} = \partial_M A_N - \partial_N A_M + i[A_M, A_N]$, $g_{MN}$ is the induced metric on $D8$-brane given in (4) and $M, N$ take values $(0, 1, 2, 3, \ldots, 8)$. In (5) $STr$ is the symmetric trace. From (4), we have $e^{-\phi} \sqrt{-\det g_{MN}} =$
\[ \frac{2}{3} \hat{R}^2 U_{KK}^{\frac{1}{2}} U_s^{(z)} g_s^{-1} \] . The gravitational energy of the D8-brane in general coordinates is \( S_{D8}^{DBI} \mid_{A_M=0} \) and subtracting the gravitational energy as a vacuum energy relative to the gauge sectors,

\[ S_{D8}^{DBI} - S_{D8}^{DBI} \mid_{A_M=0} = T_8 \int d^3 x e^{-\phi} STr\{ \sqrt{-det(g_{MN} + (2\pi\alpha') F_{MN})} - \sqrt{-detg_{MN}} \}. \tag{6} \]

This is expanded as in [19] to give (we denote the left side of (6) by \( \tilde{S}_{D8}^{DBI} \))

\[
\tilde{S}_{D8}^{DBI} = \frac{T_8}{4} (2\pi\alpha')^2 \int d^3 x e^{-\phi} \sqrt{-detg_{MN}} T r[F_{MN} F^{MN} - \frac{1}{3} (2\pi\alpha')^2 (F_{MN} F^{RN} F_{ML} F_{RL} + \frac{1}{2} F_{MN} F^{RN} F_{RL} F^{ML} - \frac{1}{4} F_{MN} F^{MN} F_{RL} F^{RL} - \frac{1}{8} F_{MN} F^{RL} F^{MN} F_{RL}) + O(\alpha^4)] \tag{7} \]

Now restricting to \( SO(5) \) singlets, we set \( A_t = 0 \) and take \( A_\mu \) and \( A_z \) to be independent of the coordinates of \( S^4 \). The integration over the \( S^4 \) coordinates is performed. Then, the full D8-brane DBI action up to \( (\alpha')^4 \) terms becomes,

\[
\tilde{S}_{D8}^{DBI} = \left( \frac{2}{3} T_8 \hat{R}^2 U_{KK}^{\frac{1}{2}} V_4 g_s^{-1} \right) (2\pi\alpha')^2 \int d^4 x dz \frac{R^3}{4 U(z)} \eta^{\mu\nu} \eta^{\lambda\sigma} F_{\mu\lambda} F_{\nu\sigma} + \frac{9 U_3}{8} U_{KK} \eta^{\mu\nu} F_{\mu z} F_{\nu z} - \frac{1}{12} (2\pi\alpha')^2 \left\{ \frac{R^6}{U^4(z)} \eta^{\mu\nu} \eta^{\lambda\sigma} \eta^{\rho\delta} \eta^{\alpha\beta} \left[ F_{\mu\lambda} F_{\delta\sigma} F_{\nu\beta} F_{\rho\alpha} - \frac{1}{8} F_{\mu\lambda} F_{\delta\beta} F_{\nu\sigma} F_{\rho\alpha} \right] + \frac{1}{2} F_{\mu\lambda} F_{\delta\sigma} F_{\alpha\beta} F_{\nu\rho} - \frac{1}{4} F_{\mu\lambda} F_{\nu\sigma} F_{\alpha\rho} \right\} + \frac{9}{4} \frac{R^3}{U_{KK}} \eta^{\mu\nu} \eta^{\lambda\sigma} \eta^{\alpha\beta} \left[ 2 F_{\mu\lambda} F_{\beta\sigma} F_{\nu\beta} F_{\alpha z} + F_{\mu\lambda} F_{\sigma\gamma} F_{\nu\beta} F_{\alpha z} + F_{\mu\lambda} F_{\sigma\gamma} F_{\beta z} F_{\alpha z} - \frac{1}{2} F_{\mu\lambda} F_{\beta z} F_{\nu\sigma} F_{\alpha z} + \frac{1}{2} F_{\mu\lambda} F_{\beta z} F_{\nu\sigma} F_{\alpha z} \right] + \frac{81}{16} \frac{U^4(z)}{U_{KK}^2} \eta^{\mu\nu} \eta^{\lambda\sigma} \left[ \frac{1}{2} F_{\mu z} F_{\sigma z} F_{\nu z} F_{\chi z} + F_{\mu z} F_{\sigma z} F_{\chi z} F_{\nu z} \right] \right], \tag{8} \]

where we follow the normalization \( T r(T^a T^b) = \frac{1}{2} \delta^{ab} \).

The action in (8) is general up to \( \alpha'^4 \) terms. The 5-dimensional gauge fields \( A_\mu(x, z) \) and \( A_z(x, z) \) can be expanded using complete sets of functions
of $z$ and a 4-dimensional action can be obtained by integrating over $z$. We use the gauge $A_z = 0$ (see [1] and [13] for details of realization of this gauge choice). We are interested in the pions only and so we expand as in [1],

$$A_\mu(x, z) = U^{-1}(x) \partial_\mu U(x) \psi_+(z),$$

where $\psi_+(z) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1}\left(\frac{z}{U_{KK}}\right)$ and $U(x) = e^{f_\pi(x^\mu)}$, with $f_\pi$ as a parameter (at this stage) and $\pi(x^\mu)$ is the pion field. The function $\psi_+(z)$ is closely related to the implementation of the gauge $A_z = 0$ [1]. From (9) it follows that

$$F_{\mu\nu} = [U^{-1} \partial_\mu U, U^{-1} \partial_\nu U] \psi_+(z)(\psi_+(z) - 1),$$

$$F_{z\mu} = U^{-1} \partial_\mu U (\partial_z \psi_+(z)) \equiv U^{-1} \partial_\mu U \hat{\phi}_0(z),$$

where $\hat{\phi}_0(z) = \frac{U_{KK}^2}{\pi} \frac{1}{U(z)}$ where $U(z)$ is defined below (4). Substituting (10) in (8), we encounter the following $z$-integrals which are numerically evaluated.

$$\frac{R^3}{4} \int_{-\infty}^{\infty} \frac{1}{U(z)} \psi_+^2(\psi_+ - 1)^2 dz = 15.2463,$$  \quad (11)

$$\frac{9}{8U_{KK}} \int_{-\infty}^{\infty} U^3(z) \hat{\phi}_0^2(z) dz = \frac{9U_{KK}}{8\pi},$$  \quad (12)

$$\int_{-\infty}^{\infty} U^4(z) \hat{\phi}_0^4(z) dz = \frac{U_{KK}}{\pi^4} \times 1.275,$$  \quad (13)

$$\int_{-\infty}^{\infty} \psi_+^2(\psi_+ - 1)^2 \hat{\phi}_0^2(z) dz = \frac{1}{U_{KK} \pi^6} \times 7.4545,$$  \quad (14)

$$\int_{-\infty}^{\infty} \frac{1}{U^4(z)} \psi_+^4(\psi_+ - 1)^4 dz = \frac{1}{U_{KK}^3 \pi^8} \times 43.7376.$$  \quad (15)

Then, the D8-brane DBI action in four dimensions becomes

$$S = \int d^4x Tr\left\{\frac{f_\pi^2}{4} L_\mu L^\mu + \frac{1}{32\pi^2} [L_\mu, L_\nu]^2\right\}$$
\[ T_8(2\pi \alpha')^2 \left( -\frac{1}{12}(2\pi \alpha')^2 \right) \int d^4x Tr \left[ \left( \frac{2R^6}{\pi^8 U_{KK}^3} \times 43.7376 \right) \eta^{\mu \nu} \eta^{\lambda \sigma} \eta^{\rho \delta} \eta^{\alpha \beta} \{ \{ L_\mu, L_\lambda \}[L_\delta, L_\sigma][L_\nu, L_\rho][L_\alpha, L_\beta] \right. \\
- \frac{1}{8}[L_\mu, L_\lambda][L_\beta, L_\delta][L_\nu, L_\sigma][L_\alpha, L_\rho] \\
+ \frac{1}{2}[L_\mu, L_\lambda][L_\beta, L_\sigma][L_\alpha, L_\rho][L_\nu, L_\delta] \\
- \frac{1}{4}[L_\mu, L_\lambda][L_\nu, L_\sigma][L_\alpha, L_\rho][L_\beta, L_\delta] \right. + \left. \left( \frac{9R^3}{2U_{KK}^3 \pi^6} \times 7.4545 \right) \eta^{\mu \nu} \eta^{\lambda \sigma} \eta^{\rho \delta} \eta^{\alpha \beta} \{ 2[L_\mu, L_\lambda][L_\beta, L_\sigma]L_\nu L_\alpha \\
+ [L_\mu, L_\lambda]L_\sigma[L_\nu, L_\beta]L_\alpha + L_\mu[L_\sigma, L_\nu]L_\beta[L_\lambda, L_\alpha] \\
- \frac{1}{2}[L_\mu, L_\lambda][L_\beta, L_\nu][L_\sigma, L_\alpha] + \frac{1}{2}[L_\mu, L_\lambda][L_\beta, L_\sigma][L_\alpha, L_\nu] \\
+ \frac{1}{2}[L_\mu, L_\lambda]L_\sigma L_\alpha[L_\nu, L_\beta] + \frac{1}{2}L_\mu L_\beta[L_\alpha, L_\lambda][L_\nu, L_\sigma] \\
+ \frac{1}{2}L_\beta L_\mu[L_\nu, L_\sigma][L_\alpha, L_\lambda] - [L_\mu, L_\lambda][L_\nu, L_\sigma]L_\alpha L_\beta \right) \\
+ \left( \frac{81 \times 1.275}{8U_{KK}^3 \pi^4} \right) \eta^{\mu \nu} \eta^{\lambda \sigma} \left\{ \frac{1}{2}L_\mu L_\sigma L_\nu L_\lambda + L_\mu L_\nu L_\sigma L_\lambda \right\}, \tag{16} \right. \\
\] 

where \( T_8 = \frac{2}{3} R^2 U_{KK}^{\frac{1}{2}} T_8 V_4 g_s^{-1} \), \( L_\mu = U^{-1} \partial_\mu U \) and

\[ T_8(2\pi \alpha')^2 = \frac{\pi f_\pi^2}{9U_{KK}}, \]

\[ = \frac{1}{32e^2} \frac{2\pi^4}{15.2463 \times R^3}. \tag{17} \]

For \( N_f = 2 \), a lagrangian describing massless pions up to \( \alpha' = 4 \) and four derivatives of \( U(x) \) is given by the first two and the last two terms in (16) which should describe the properties of pion and \( \pi - \pi \) scattering. The first two terms reproduce the Skyrme model and the static properties of the pion are well described by the hedgehog ansatz [12]. The second term in (16) is the familiar Skyrme term introduced by Skyrme to stabilize the soliton. The holographic dual model [1] has this term naturally. When massive vector mesons (infinite tower) are introduced, as said in the beginning, the contribution from the Skyrme term for \( \pi - \pi \) scattering gets cancelled and the
resulting lagrangian for the pions is just the chiral lagrangian [2]. Thus the pion-pion scattering here will be described by the first and the last two terms in (16) which are precisely the terms in the phenomenological lagrangian of Weinberg [15], with the coefficients (coupling constants) fixed by the parameters of the holographic model. The sixth and eighth derivative terms in (16) will contribute to $\pi\pi$ scattering leading to four and six pions.

Now we consider the $\pi\pi \rightarrow \pi\pi$ scattering. Weinberg [20] obtained the $\pi - \pi$ scattering amplitude for $\mathcal{L} = -\frac{f^2}{4} Tr(L\mu L^\mu)$ (the first term in (16)) as

$$A(s,t,u) = \frac{s}{f_\pi^2},$$

where $s, t, u$ are the Mandelstam variables, $s + t + u = 0$. In holographic QCD, pion mass can be realized by introducing instantons on the $S^4$ [21] which will not affect (16) except for a mass term for the pions. Then, (18) reads as

$$A(s,t,u) = \frac{s-m^2_\pi}{f_\pi^2}$$

with $s + t + u = 4m^2_\pi$. Sannino and Schechter [16] found that the dependence of $R_0^0(s)$ on $\sqrt{s}$ did not follow the Roy curves [17] and violated unitarity. Following the phenomenological lagrangian of Weinberg [15], they [16] introduced four-derivative contact terms (which are the last two terms in (16)) with arbitrary coefficients, and adjusted them so as to have vanishing contribution from these for threshold scattering. Notice that in the holographic model, the couplings are fixed uniquely and they involve only the Yang-Mills coupling $\frac{g}{\sqrt{2}Y_M}$. Further details of pion-pion scattering can be found in [21,22]. The chiral lagrangian for pions in the holographic model, up to $\alpha'^4$ corrections and up to four derivatives are given from (16) as

$$\mathcal{L}^{pion}_{eff} = \frac{f^2_\pi}{4} Tr(\partial_\mu U \partial^\mu U^\dagger)$$

$$- C_4 Tr\{\frac{1}{2} \partial_\mu U \partial_\rho U^\dagger \partial^\mu U \partial^\rho U^\dagger + \partial_\mu U \partial^\mu U^\dagger \partial_\rho U \partial^\rho U^\dagger\}. \quad (19)$$

The $(\alpha')^4$ corrections give the four derivative contact interaction in (19) with the dimensionless coupling constant $C_4$ as

$$C_4 = \frac{\tilde{T}_8 (2\pi\alpha')^4}{96 U_{KK} \pi^4} \frac{81 \times 1.275}{g^2_Y} = \frac{1.173 \times 10^{-3}}{g^2_Y}, \quad (20)$$

using (2) and (17).
The pion-pion scattering amplitude from (19) is

\[
A(s,t,u) = \frac{s-m^2_{\pi}}{f^2_{\pi}} - \frac{2C_4}{f^4_{\pi}} [(t-2m^2_{\pi})^2 + (u-2m^2_{\pi})^2]
+ (s-2m^2_{\pi})^2, \tag{21}
\]

for which the partial wave amplitude \(T_0^0(s)\) is

\[
T_0^0(s) = \frac{1}{64\pi} \sqrt{1 - \frac{4m^2_{\pi}}{s}} \times \int_{-1}^{1} d\cos\theta T_0^0(s,t,u), \tag{22}
\]

where \(T_0^0(s,t,u) = 3A(s,t,u)+A(t,s,u)+A(u,t,s)\) with \(s = 4(\vec{p}^2+m^2_{\pi})\); \(t = -2\vec{p}^2(1-\cos\theta)\); \(u = -2\vec{p}^2(1+\cos\theta)\); \(\vec{p}\) the 3-momentum and \(\theta\) the scattering angle of the pion. Then it is straightforward to obtain

\[
R_0^0(s) = T_0^0(s) = \frac{1}{64\pi} \sqrt{1 - \frac{4m^2_{\pi}}{s}} \left[ \frac{2}{f^2_{\pi}} (2s-m^2_{\pi}) \right]
- \frac{10C_4}{f^4_{\pi}} \left[ 2(s-2m^2_{\pi})^2 + s^2 + \frac{1}{3}(s-4m^2_{\pi})^2 \right]. \tag{23}
\]

In Figure 1, we have plotted \(R_0^0(s)\) as a function of \(\sqrt{s}\) with and without \(\alpha'^4\) corrections using \(f_{\pi} = 95\, \text{MeV}\) and for two representative values of \(g_{YM}^2 = 4\pi\alpha_s\). Curve I is without the \(\alpha'^4\) corrections. Curve II is with the \(\alpha'^4\) corrections using \(g_{YM}^2 = 4\pi\alpha_s\) with the value of \(\alpha_s = 0.12\) at the Z-boson mass [23]. This value for \(g_{YM}^2\) is for real QCD at short distances. In view of the holographic model used here, it will be consistent to use large \(N_c\) value for \(g_{YM}^2\). By writing \(g_{YM}^2\) as \(\left(\frac{\lambda N_c}{N_c}\right)\), where \(\lambda = g_{YM}^2 N_c\), the 'tHooft coupling parameter, we adopt the fit for \(g_A\) in [10] with \(\lambda N_c \simeq 26\) and shift the denominator \(N_c\) by \(N_c + 2\) following Dashen and Manohar [24]. The amplitude \(R_0^0\) with these is displayed in Figure 1 as curve III. From the figure, it is seen that the \(\alpha'^4\) corrections are important to be consistent with the unitarity. Curve III further respects the unitarity bound \(|R_0^0| \leq \frac{1}{2}\). The numerical values for \(R(s)\) are in qualitative agreement with the real part of the \(I = 0; \ell = 0\) partial wave amplitude using the phase shifts given in [22]. We find the \(I = 1\) amplitude \(T^1(s,t,u) = A(t,s,u)-A(u,t,s)\), using (21) is independent of the \(\alpha'^4\) corrections. The \(I = 2\) amplitude \(A^2(s,t,u) = A(t,s,u)+A(u,t,s)\) is used to calculate the partial wave amplitudes \(R_0^2(s)\) and \(R_2^2(s)\) for \(\ell = 0, 2\).
respectively. It is noted that the $I = 2; \ell = 2$ partial wave scattering amplitude $R^2_2(s)$ involves only the $\alpha'^4$ terms after the angular integration. The experimental phase shifts from [25] are used to find these amplitudes using $R^2_2(s) = \frac{1}{2} \sin(2\delta^2_2)$ with $\eta^{(2)}_I$ the $I = 2$ inelasticity parameter set equal to unity and compared with our theoretical values in Tables 1 and 2. As the experimental phase shifts are available over a bin for $\sqrt{s}$, we have taken the median values.

### Table. 1

The results for $R^2_0(s)$. The second column is our theoretical values and the third column is using the phase shifts from [22,25]

| $\sqrt{s}$ (GeV) | Theory | $\frac{1}{2}\sin(2\delta^2_0)$ |
|------------------|--------|-------------------------------|
| 0.35             | -0.07  | -0.069±0.035                  |
| 0.45             | -0.155 | -0.137±0.015                  |
| 0.60             | -0.268 | -0.18±0.027                   |
| 0.64             | -0.323 | -0.26±0.023                   |

### Table. 2

The results for $R^2_2(s)$. The second column is our theoretical values and the third column is using the phase shifts from [22,25]

| $\sqrt{s}$ (GeV) | Theory | $\frac{1}{2}\sin(2\delta^2_2)$ |
|------------------|--------|-------------------------------|
| 0.75             | -0.0082| -0.015±0.005                  |
| 0.80             | -0.011 | -0.04±0.005                   |
| 1.0              | -0.03  | -0.035±0.01                   |

The numerical values are in reasonable agreement with the results using the experimental phase shifts.
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Figure 1: $R_0(s) = R$ and $x = \sqrt{s}$ in GeV. Curve I is without the $\alpha'^4$ corrections. Curves II and III are with these corrections for two representative values of $g_{YM}^2$ (see text).