Performance and limitations of dual-comb based ranging systems: supplement

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1. TRADE-OFF BETWEEN PRECISION AND NON-AMBIGUITY RANGE

Since the information of the distance is estimated from the slope of the phases differences of each frequency modes of the beating radio-frequency (RF) comb, the non-ambiguity range is defined by the distance corresponding to a $\pi$ phase difference between two modes for unwrapping concerns. Considering that the phase of each frequency mode $m$ is shifted by $\Delta \Phi_m = 2\pi f_r z$, and as only half of the traveled round-trip distance is measured we define:

$$NAR = \frac{c}{4f_r} = \frac{\pi}{2\alpha} \tag{S1}$$

For an optical comb of repetition rate $10 \text{GHz} - \Delta f \simeq 10 \text{GHz}$ sent to the target, we obtained a NAR of $7.5 \text{mm}$ in the first repetition rate limited setup.

Regarding the precision of our measurement, knowing the relationship between the phases and the distance we infer that

$$\sigma_d = \frac{\sigma_s}{\alpha} \tag{S2}$$

with $\sigma_s$ the standard deviation of the slopes of the phases differences $\frac{\Delta\Phi_m}{\Delta m}$. The slope being determined by linear regression, its standard deviation is calculated through a weighted sum of the standard deviations of each point of the slope [1]. Knowing that the phase variance of each frequency mode is directly defined by its signal-to-noise ratio (SNR) [2] we obtain:

$$\sigma_s = \sqrt{\sum_m c_m^2 \left( \frac{1}{\text{SNR}_{m,\text{meas}}} + \frac{1}{\text{SNR}_{m,\text{ref}}} \right)} \tag{S3}$$

With $c_m = \frac{m - <m>}{\sigma_m}$, the linear regression coefficients, $\text{SNR}_{m,\text{meas}}$, the SNR of frequency mode $m$ at the measurement photodiode and $\text{SNR}_{m,\text{ref}}$, the SNR at the reference photodiode. The unavoidable trade-off between ambiguity range and precision is thus defined by combining Eq. S1 and Eq. S2:

$$\sigma_d = \frac{2NAR}{\pi} \sigma_s \tag{S4}$$

According to Eq. S3, the SNR of high orders frequency modes have a greater influence on $\sigma_s$. Insofar as the optical combs cannot be flattened, their shape have a direct influence on $\sigma_s$, as verified experimentally. Therefore the precision of the system is entirely described by $\sigma_s$. The $\alpha$ factor, modified by adjusting the repetition rate of the signal comb, tunes the trade-off between precision and non-ambiguity range. It allows us to characterize the performances of the dual comb ranging system and to predict the achievable results with different repetitions rates and different combs’ amplitude envelopes.

2. ANALYSIS ON THE $\epsilon_{\psi,M}$ FUNCTION

The optical shift induced by a reflective target at a distance $d_i$ from the collimator on the frequency order $m$ of the signal optical comb is:

$$\psi_{i,m,s} = \frac{4\pi d_i f_{s,m}}{c} \tag{S5}$$

Moreover, we know that the phase $\psi$ of the interference between two waves at the same frequency, with amplitudes $A_1, A_2$ and phases $\psi_1, \psi_2$ is given by:

$$\tan(\psi) = \frac{A_1 \sin(\psi_1) + A_2 \sin(\psi_2)}{A_1 \cos(\psi_1) + A_2 \cos(\psi_2)} \tag{S6}$$
Therefore, if two reflections coming from distances \(d_1\) and \(d_2\), with a relative amplitude \(p = \frac{A_2}{A_1+A_2}\), interfere, the resulting phase shifts \(\tilde{\psi}_{m,s}\) are given by:

\[
\tilde{\psi}_{m,s} = \arctan\left(\frac{(1-p)\sin(\psi_{1,m,s}) + p\sin(\psi_{2,m,s})}{(1-p)\cos(\psi_{1,m,s}) + p\cos(\psi_{2,m,s})}\right)
\]  

(S7)

We define \(\epsilon_{\psi,m}\) as the differences between \(\tilde{\psi}_{m,s}\) described above and the optical phases shifts \(\psi_{1,m}\) induced by the first reflection at a distance \(d_1\):

\[
\epsilon_{\psi,m} = \tilde{\psi}_{m,s} - \psi_{1,m,s}
\]  

(S8)

This way, by considering the reflection at distance \(d_2\) as a spurious reflection, \(\epsilon_{\psi,m}\) describes the optical phase shift error created by a spurious reflection at a distance \(d_2\), with a relative amplitude described by \(p\), on a main reflection at a distance \(d_1\).

Since \(\tilde{\psi}_m = \psi_{1,m,s} + \epsilon_{\psi,m}\), the phases of the beating comb at the measurement photodiode \(\Phi_{m,\text{meas}}\), resulting from the differences of the optical phases between the signal and the local oscillator (not impacted by the spurious reflection), are described by:

\[
\Phi_{m,\text{meas}} = \Phi_{m,\text{ref}} + \epsilon_{\psi,m}
\]  

(S9)

with \(\Phi_{m,\text{meas}}\) the beating comb phases at the measurement photodiodes expected for a single reflection at a distance \(d_1\). Finally, the difference between the phases of the beating combs at the reference and measurement photodiodes \(\Delta \Phi_m = \Phi_{m,\text{meas}} - \Phi_{m,\text{ref}}\) are in the same way shifted by \(\epsilon_{\psi,m}\), since \(\Phi_{m,\text{ref}}\) is not impacted by the spurious reflection:

\[
\Delta \Phi_m = \Delta \Phi_m + \epsilon_{\psi,m}
\]  

(S10)

with \(\Delta \Phi_m\) the difference between the beating combs phases expected for a single reflection at a distance \(d_1\). The distance is extracted from the slope of the values of \(\Delta \Phi_m\) adjusted by linear regression. Therefore, the effect of \(\epsilon_{\psi,m}\) on the measured distance is described by [1]:

\[
d = \frac{c \sum_m (\epsilon_{\psi,m}\epsilon_m)}{4\pi f_{s,s}}
\]  

(S11)

To better understand the non-linear effect of the interference between two reflections, it is useful to analyze the function \(\frac{\partial \epsilon_{\psi,m}}{\partial m}\). We remind that:

\[
\epsilon_{\psi,m} = \arctan\left(\frac{(1-p)\sin\left(\frac{4\pi d_1 f_{s,m}}{c}\right) + p\sin\left(\frac{4\pi d_2 f_{s,m}}{c}\right)}{(1-p)\cos\left(\frac{4\pi d_1 f_{s,m}}{c}\right) + p\cos\left(\frac{4\pi d_2 f_{s,m}}{c}\right)}\right)
\]  

(S12)

By introducing \(f_1(m) = (1-p)\sin\left(\frac{4\pi d_1 f_{s,m}}{c}\right) + p\sin\left(\frac{4\pi d_2 f_{s,m}}{c}\right)\) and \(f_2(m) = (1-p)\cos\left(\frac{4\pi d_1 f_{s,m}}{c}\right) + p\cos\left(\frac{4\pi d_2 f_{s,m}}{c}\right)\) we find, from Eq. S12:

\[
\frac{\partial \epsilon_{\psi,m}}{\partial m} = \frac{f_1(m)f_2(m) - f_1(m)f_2(m)}{f_1^2(m) + f_2^2(m)} - \frac{4\pi d_1 f_{s,s}}{c}
\]  

(S13)

By inserting the values of \(f_1(m)\), \(f_2(m)\) and their derivatives in Eq.S13 it comes that:

\[
\frac{\partial \epsilon_{\psi,m}}{\partial m} = \frac{4\pi f_{s,s}}{c} [(d_1 + d_2)(p(1-p)\cos\left(\frac{4\pi d_1 f_{s,m}}{c}\right) + 2d_1(1-p)^2 + 2p^2 d_2)] - \frac{4\pi d_1 f_{s,s}}{c}
\]  

(S14)

Finally, by differentiating Eq. S14 we find:

\[
\frac{\partial^2 \epsilon_{\psi,m}}{\partial m^2} = \frac{\left(4\pi f_{s,s}\right)^2 p(1-p)\Delta d^2 \sin\left(\frac{4\pi d_1 f_{s,m}}{c}\right)^2 |p|^2 - (1-p)^2}{|(1-p)^2 + p^2 + 2p(1-p)\cos\left(\frac{4\pi d_1 f_{s,m}}{c}\right)^2 |}
\]  

(S15)

From Eq. S15 it comes that the linearity is rigorously preserved only if \(\frac{\partial^2 \epsilon_{\psi,m}}{\partial m^2} = 0\) for each frequency mode \(i.e\) for all \(m\), therefore only if \(\Delta d = 0\) (if the two target are at the same distance), or if
\[ p = 0.5 \text{ (in this case the measured distance is exactly equal to the mean of the two distances).} \]

Nevertheless, since in the short range configuration \( \hat{d}_{s,m} \) is approximately the same for all \( m \) \((f_0 >> f_{rs})\), the measurement stays accurate for values of \( \Delta d \) such as:

\[
\sin\left(\frac{4\pi\Delta d}{c} \hat{d}_{s,m}\right) = 0
\]

i.e values of \( \Delta d \) in the vicinity of \( \Delta d = k\frac{\lambda_0}{2} \) with \( k \) being an integer. We note that, in the disambiguated setup, by using the RF comb of frequency \( f_{IF,m} = f_{AOM} + m\Delta f \), this approximation is no longer relevant since \( f_{AOM} \) and \( \Delta f \) have the same order of magnitude.

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