Generating Erler–Schnabl-type solution for the
tachyon vacuum in cubic superstring field theory

E Aldo Arroyo

Instituto de Física Teórica, Universidade Estadual Paulista, Caixa Postal 70532-2, 01156-970 São Paulo, SP, Brazil

E-mail: aldohep@ift.unesp.br

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Abstract

We study a new set of identity-based solutions to analyze the problem of tachyon condensation in open bosonic string field theory and cubic superstring field theory. Even though these identity-based solutions seem to be trivial, it turns out that after performing a suitable gauge transformation, we are left with the known Erler–Schnabl-type solutions which correctly reproduce the value of the D-brane tension. This result shows explicitly that a seemingly trivial solution can generate a non-trivial configuration which precisely represents the tachyon vacuum.

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1. Introduction

Recently a simple analytic solution for tachyon condensation was found in open bosonic string field theory [1] as well as in the modified cubic superstring field theory [2]. The main feature of these solutions is that, rather than a discrete sum, the solutions (which we refer as the Erler–Schnabl-type solutions) can be written as a continuous integral over wedge states, where no regularization or phantom term is required. Since these solutions do not require the presence of the phantom term, in contrast to the $B_0$ gauge solutions [3–14], computation of the value of the D-brane tension is very straightforward.

It would be interesting to find a similar Erler–Schnabl-type solution in the case of Berkovits WZW-type superstring field theory [15]. Since the action for this theory is a non-polynomial one, finding the tachyon vacuum solution and computation of the value of D-brane tension seems to be highly cumbersome. Therefore, in order to find some clues for this challenging puzzle, we should analyze the problem in a relatively simple formulation of open superstring field theory.
In the literature, there is an old formulation of open superstring field theory, namely the modified cubic superstring field theory [16, 17]. The action in this formulation is cubic: (instead of the non-polynomial action given by Berkovits [15])

\[
S = -\frac{1}{g^2} \left[ \frac{1}{2} \langle\langle \Phi, Q\Phi \rangle\rangle + \frac{1}{3} \langle\langle \Phi, \Phi \ast \Phi \rangle\rangle \right],
\]

(1.1)

where \( Q \) is the BRST operator, * stands for Witten’s star product [18] and the inner product \( \langle\langle \cdot, \cdot \rangle\rangle \) is the standard BPZ inner product with the difference that we must insert the operator \( Y_{-2} \) at the open string midpoint. The operator \( Y_{-2} \) can be written as the product of two inverse picture-changing operators \( Y_{-2} = Y(i)Y(-i) \), where \( Y(z) = -\partial_{\xi} e^{-2\phi} c(z) \). The string field \( \Phi \) which has ghost number 1 and picture number 0 belongs to the small Hilbert space of the first-quantized matter+ghost open Neveu–Schwarz superstring theory.

As stated in Sen’s first conjecture [19–21], the classical open string field equation of motion should admit a Poincaré invariant solution \( \Psi \equiv \Psi_1 \) which is identified as the tachyon vacuum with no D-branes. This statement means that the energy density of the true vacuum found by solving the equation of motion should be equal to minus the tension of the D-brane. Since the energy density of a static configuration is minus the action, for the case of the cubic action, Sen’s conjecture can be summarized as follows:

\[
\frac{1}{g^2} \left[ \frac{1}{2} \langle\langle \Psi, Q\Psi \rangle\rangle + \frac{1}{3} \langle\langle \Psi, \Psi \ast \Psi \rangle\rangle \right] = -\frac{1}{2\pi^2 g^2}.
\]

(1.2)

The string field equation of motion and Sen’s conjecture allow us to fix the kinetic term

\[
\frac{\pi^2}{3} \langle\langle \Psi, Q\Psi \rangle\rangle = -1.
\]

(1.3)

In this paper, we propose a new prescription for generating a string field \( \Psi_1 \) in cubic string field theories [16–18], which satisfies the equation of motion \( Q\Psi + \Psi \ast \Psi = 0 \) and represents the tachyon vacuum. The procedure of our prescription follows two steps: (i) find a naive identity-based solution [22] of the string field equation of motion, and (ii) perform a gauge transformation [23] over the identity-based solution such that the resulting string field, consistently, represents the tachyon vacuum.²

In order to find identity-based solutions² to the string field equations of motion, we use a basis similar to the one used in [3] with the difference that in our case the operators are inserted on the identity string field \( I \).³ For instance in the case of open bosonic string field theory, to find a solution based on the identity string field, we should use the ansatz

\[
\Psi = \sum_{n,p} f_{n,p} U_1^{\dagger} U_1 \hat{L}^n \hat{c}_p |0\rangle + \sum_{n,p,q} f_{n,p,q} U_1^{\dagger} U_1 \hat{B} \hat{L}^n \hat{c}_p \hat{c}_q |0\rangle,
\]

(1.4)

where \( n = 0, 1, 2, \ldots \), and \( p, q = 1, 0, -1, -2, \ldots \). The operators \( \hat{L}^n, \hat{B} \) and \( \hat{c}_p \) are defined in [3]. Plugging this ansatz (1.4) into the equation of motion will lead to a system of algebraic equations for the coefficients \( f_{n,p} \) and \( f_{n,p,q} \). Analyzing these algebraic equations we discover that many of the coefficients can be set to zero. Therefore, we can use a simpler ansatz than the one given by (1.4); for instance, in this paper we use the following ansatz:

\[
\Psi = \alpha_1 c + \alpha_2 c K + \alpha_3 K c.
\]

(1.5)

¹ By consistently we mean that the solution must reproduce correctly and unambiguously the value of the D-brane tension.

² By identity-based solution we mean a solution which is based on the identity string field \( I \).

³ Let us remember that the identity string field is the wedge state \( I = U_1^{\dagger} U_1 |0\rangle \). A discussion, needed for the purpose of this paper, about the identity string field can be found in [24].
where the basic string fields $K$ and $c$ together with $B$ are defined in terms of the identity string field with the appropriated insertions \cite{9, 10}:

$$K = \frac{1}{\pi} \hat{\mathcal{L}} U_1^U | 0 \rangle, \quad c = U_1^U \hat{c} | 0 \rangle, \quad B = \frac{1}{\pi} \hat{\mathcal{B}} U_1^U | 0 \rangle.$$ (1.6)

As we will prove, the solution obtained from the ansatz (1.5) provides an ambiguous analytic result for the value of the vacuum energy and consequently for the D-brane tension. In general, it can be shown that the energy of an identity-based solution is of the form $0 \times \infty$ \cite{22}, and so it is not well defined. Instead of trying to find a consistent regularization in order to treat our identity-based solution correctly, we show that this particular identity-based solution is equivalent to the recently analytic solution found by Erler and Schnabl \cite{1}.

To show the above statement, we need to find an explicit gauge transformation which relates the identity-based solution with the Erler–Schnabl solution. The explicit form of this gauge transformation is given by

$$\Psi_{E-S} = U QU^{-1} + U \Psi_I U^{-1},$$ (1.7)

where $\Psi_{E-S}$ is the Erler–Schnabl solution \cite{1}, $\Psi_I$ is the identity-based solution and $U$ is an element of the gauge transformation

$$U = 1 + c BK, \quad U^{-1} = 1 - c BK \frac{1}{1 + K}.$$ (1.8)

Since the Erler–Schnabl solution contains a term depending on $1/(1 + K)$, which is crucial for the computation of the vacuum energy \cite{1}, it is not so difficult to guess the form of the gauge transformation which relates our identity-based solution with the Erler–Schnabl solution, and in particular the gauge transformation should contain the $1/(1 + K)$ term.

Nevertheless we can construct different tachyon vacuum solutions by replacing the term $1/(1 + K)$ with other functions of $K$ satisfying the criteria given in \cite{10}. For example the dependence on $K$ in the original Schnabl’s solution \cite{3} is given by $-e^{-K}$ \cite{4, 25}. It is clear that if we want to relate our identity-based solution to these different tachyon vacuum solutions we should use another gauge transformation which will depend on the choice of the function of $K$. The reason why we chose the $1/(1 + K)$ dependence is because the computation of the vacuum energy is simplified. It is well known that the case $-e^{-K}$ which corresponds to the original Schnabl’s solution has subtleties \cite{3, 4, 25}; for instance, in order to compute the right value of the vacuum energy the phantom term must be included.

Finally, we carry out the same analysis for the case of the modified cubic superstring field theory; namely, we show that a solution based on the identity string field can be brought to the tachyon vacuum solution constructed by Gorbachev using a gauge transformation. Our results show explicitly that how a seemingly trivial identity-based solution can generate a non-trivial configuration which precisely represents the tachyon vacuum. Certainly it would be very interesting to extend our results to the case of the non-polynomial Berkovits WZW-type superstring field theory.

This paper is organized as follows. In section 2, we review and further develop some properties of the simple analytic solution for tachyon condensation in open bosonic string field theory. We give an example of an identity-based solution which formally solves the equation of motion. After performing a gauge transformation over this seemingly trivial identity-based solution, we obtain Erler–Schnabl’s solution which correctly reproduces the value of

\footnote{Let us remark that in general we can write expressions for $U$ and $U^{-1}$ depending on a function of $K$:

$$U = 1 + c B f(K), \quad U^{-1} = 1 - c B \frac{f(K)}{1 + f(K)}.$$}
the D-brane tension. We also show that Erler–Schnabl’s solution appears as a particular case of a rather general two-parameter family of solutions. In section 3, we analyze a similar identity-based solution in the modified cubic superstring field theory. As in the bosonic case, by performing a suitable gauge transformation over this identity-based solution, we obtain the known Gorbachev’s solution which correctly reproduces the value of the vacuum energy. A two-parameter family of solutions is discussed as well. In section 4, a summary and further directions of exploration are given. The details related to the explicit construction of identity-based solutions and the gauge transformation are in the appendix.

2. Simple analytic solution in the bosonic case

In this section, after reviewing some aspects of the simple analytic solution for tachyon condensation in open bosonic string field theory [1], we analyze a new identity-based solution which formally solves the equation of motion. Since this solution is an isolated identity-like piece, it may happen that the solution is trivial or inconsistent; in fact, similar identity-based solutions have been proposed in the past, and for such solutions there is no unambiguous analytic calculation for the D-brane tension [22]. However, as we are going to see, after performing a gauge transformation over this seemingly trivial identity-based solution, we obtain the well-known Erler–Schnabl’s solution which correctly reproduces the value of the D-brane tension. We also show that Erler–Schnabl’s solution appears as a particular case of a rather general two-parameter family of solutions.

2.1. Erler–Schnabl’s solution

As derived in Erler–Schnabl’s paper [1] using the methods of [3, 10], the simple analytic solution for tachyon condensation in open bosonic string field theory is

$$\psi_{E-S} = (c + cK B c) \frac{1}{1 + K},$$  \hspace{1cm} (2.1)

where the basic string fields $K$, $B$ and $c$ are given in the split string notation [4, 9, 10], and they can be written, using the operator representation [3], as follows:

$$K \to \frac{1}{\pi} \hat{L} U^i_1 U_1 |0\rangle, \hspace{1cm} (2.2)$$

$$B \to \frac{1}{\pi} \hat{B} U^i_1 U_1 |0\rangle, \hspace{1cm} (2.3)$$

$$c \to U^c_1 U_1 \tilde{c}(0) |0\rangle. \hspace{1cm} (2.4)$$

The operators $\hat{L}$, $\hat{B}$ and $\tilde{c}(0)$ are defined in the sliver frame [8]5, and they are related to the worldsheet energy–momentum tensor, the $b$ and $c$ ghost fields respectively, for instance

$$\hat{L} \equiv L_0 + \hat{L}^1_0 = \oint \frac{dz}{2\pi i} (1 + z^2)(\arctan z + \arccot z) T(z),$$ \hspace{1cm} (2.5)

$$\hat{B} \equiv B_0 + \hat{B}^1_0 = \oint \frac{dz}{2\pi i} (1 + z^2)(\arctan z + \arccot z) b(z).$$ \hspace{1cm} (2.6)

while the operator $U^i_1 U_1$ in general is given by $U^i_1 U_r = e^{\hat{L} \hat{B}} \hat{c}$, so we have chosen $r = 1$; note that the string field $U^i_1 U_1 |0\rangle$ represents the identity string field $1 \to U^i_1 U_1 |0\rangle$ [3, 4, 9, 10].

5 Remember that a point in the upper-half plane $z$ is mapped to a point in the sliver frame $\tilde{z}$ via the conformal mapping $\tilde{z} = \arctan z$. 

4
Using the operator representation (2.2)–(2.4) of the string fields \( K, B \) and \( c \), we can show that these fields satisfy the algebraic relations
\[
\{ B, c \} = 1, \quad [B, K] = 0, \quad B^2 = c^2 = 0, \tag{2.7}
\]
and have the following BRST variations:
\[
QK = 0, \quad QB = K, \quad Qc = cKc. \tag{2.8}
\]
As we can see, solution (2.1) explicitly contains the identity string field and therefore we may think that this solution is not well defined in the sense that for such solution there is no unambiguous analytic calculation for the D-brane tension. However, as shown in [1], Erler–Schnabl’s solution (2.1) correctly and unambiguously reproduces the value of the D-brane tension. To prove this statement it is crucial to write the solution as the following integral:
\[
\Psi_{E-S} = \int_0^\infty dt \, e^{-t}(c + cKBc)\Omega^t; \tag{2.9}
\]
this form of the solution is possible since we can invert \( 1 + K \) using the Schwinger parameterization
\[
\frac{1}{1 + K} = \int_0^\infty dt \, e^{-t(1 + K)} = \int_0^\infty dt \, e^{-t}\Omega^t. \tag{2.10}
\]
As stated in [1], the fact that the solution can be written in terms of a continuous integral over wedge states arbitrarily close to the identity, and not as an isolated identity-like piece, is crucial for the consistency of the solution.

2.2. The identity-based solution and gauge transformation

In this subsection, we analyze a new simple identity-based solution to the equation of motion in open bosonic string field theory; this solution is given by
\[
\Psi = c(1 - K). \tag{2.11}
\]
Using the algebraic relations (2.7) and (2.8), it is easy to show that the string field \( \Psi \) (2.11) satisfies the equation of motion
\[
Q\Psi + \Psi^2 = 0. \tag{2.12}
\]

The next step is to compute the value of the vacuum energy; this computation is crucial if we want to verify Sen’s first conjecture. Using the equation of motion, the computation of the energy can be reduced to the evaluation of the following correlator:
\[
\frac{1}{6g^2} \langle \Psi Q\Psi \rangle = \frac{1}{6g^2} \langle c(1 - K)Q(c(1 - K)) \rangle = \frac{1}{6g^2} \left[ \langle c^2Kc \rangle - \langle cKcKc \rangle + \langle cKcKc \rangle + \langle cKcKc \rangle \right] = 0. \tag{2.13}
\]
The first and third terms vanish because \( c^2 = 0 \), while the second vanishes for the same reason upon using cyclicity of the correlator. The last term
\[
cKcKcKc = cKc\partial cK = c(\partial c)^2K \tag{2.14}
\]
vanishes because \( (\partial c)^2 \). Based on this computation we conclude that the identity-based solution (2.11) seems to be trivial.
Though we have a vanishing result for the value of the vacuum energy, it would be desirable to confirm our calculation by other means, for instance using the $L_0$-level expansion of the identity-based solution (2.11):

$$\Psi = U_1^{\dagger}U_1 \left[ \tilde{c}_1 - \frac{1}{\pi} L_{\tilde{c}_1} + \frac{1}{2} \tilde{c}_0 \right] |0\rangle$$

$$= \sum_{n=0}^{\infty} \left[ \frac{1}{2^n n!} L^n \tilde{c}_1 - \frac{1}{\pi} \frac{1}{2^n n!} L^{n+1} \tilde{c}_1 + \frac{1}{2^{n+1} n!} L^n \tilde{c}_0 \right] |0\rangle$$

$$= \sum_{n=0}^{\infty} \sum_{p=0}^{n} f_{n,p} \tilde{c}_p |0\rangle,$$

where the coefficient $f_{n,p}$ is given by

$$f_{n,p} = \begin{cases} \frac{1}{2^{n+1} n!}, & \text{if } n \geq 0 \text{ and } p = 0 \\ \frac{1}{2^{n+1} n!}, & \text{if } n = 0 \text{ and } p = 1 \\ \frac{1}{2^n n!} - \frac{1}{\pi} \frac{1}{2^{n-1} (n-1)!}, & \text{if } n \geq 1 \text{ and } p = 1. \end{cases}$$

Using the $L_0$-level expansion of our identity-based solution (2.15), we find that the value of the vacuum energy is given by

$$\frac{1}{6 g^2} \langle \Psi Q \Psi \rangle = \frac{1}{6 g^2} \sum_{m=0}^{\infty} \sum_{p=0}^{m} \sum_{q=0}^{p} f_{m,p} f_{m,q} (bpz(\tilde{c}_p) \hat{L}^{m+n} Q \tilde{c}_q)$$

$$= -\frac{1}{6 g^2} \left[ \frac{1}{2} + \frac{2}{\pi^2} \right].$$

Therefore, as we can see, this solution provides an ambiguous analytic result for the value of vacuum energy and consequently for D-brane tension. It is intriguing to note that even though we have obtained a value different from zero (2.17), this value does not coincide with the one given in equation (1.3). To understand this anomaly better, let us consider two general identity-like string fields with $L_0$ eigenvalues equal to $h_1$ and $h_2$ respectively:

$$\tilde{\phi}_1 = U_1^{\dagger}U_1 \tilde{\phi}_1(0)|0\rangle, \quad \tilde{\phi}_2 = U_1^{\dagger}U_1 \tilde{\phi}_2(0)|0\rangle,$$

and compute the correlator $\langle \tilde{\phi}_1, \tilde{\phi}_2 \rangle$:

$$\langle \tilde{\phi}_1, \tilde{\phi}_2 \rangle = \langle 0 | bpz (\tilde{\phi}_1(0)) U_1^{\dagger}U_1 \tilde{\phi}_2(0) |0\rangle$$

$$= \lim_{r \to 1} \langle 0 | bpz(\tilde{\phi}_1(0)) U_r U_r \tilde{\phi}_2(0) |0\rangle$$

$$= \lim_{r \to 1} \left[ \left( \frac{2}{r} \right)^{h_1+h_2} \langle 0 | bpz(\tilde{\phi}_1(0)) U_r \tilde{\phi}_2(0) |0\rangle \right]$$

$$= \lim_{r \to 1} \left[ \left( \frac{1}{r-1} \right)^{h_1+h_2} \langle 0 | bpz(\tilde{\phi}_1(0)) \tilde{\phi}_2(0) |0\rangle, \right.$$ (2.19)

where we have used the definition of $U_r = \left( \frac{2}{r} \right)^{h_0}$. Clearly if the sum of the eigenvalues $h_1 + h_2$ is greater than zero the correlator is divergent, while in the case $h_1 + h_2 = 0$ the correlator is ambiguous. Only when the sum is less than zero we obtain an unambiguous result $\langle \tilde{\phi}_1, \tilde{\phi}_2 \rangle = 0$. Since the case $h_1 + h_2 > 0$ could appear in the computation of the vacuum energy, potential singularities can be present. This analysis also shows the origin of the non-triviality of the identity-based solution.
In this paper, instead of trying to find a consistent regularization in order to treat our identity-based solution correctly, we show that this particular identity-based solution (2.11) is equivalent to the recently analytic solution found by Erler and Schnabl; to show this statement we need to find an explicit gauge transformation which relates the identity-based solution (2.11) with (2.1), and in fact we have found the explicit form of this gauge transformation:

\[
\Psi = U^{-1}(Q + \Psi_{E-S})U
\]

\[
= \left[1 - cBK \frac{1}{1 + K}\right] \left(Q + (c + cKB) \frac{1}{1 + K}(cBK + 1)\right) [cBK + 1]
\]

\[
= \left[1 - cBK \frac{1}{1 + K}\right] \left(Q(cKB + 1) + (c + cKB) \frac{1}{1 + K}(cBK + 1)\right)
\]

\[
= \left[1 - cBK \frac{1}{1 + K}\right] \left(cKcKB - cK^2 + cK \frac{1}{1 + K} + K cKB\right)
\]

\[
= \left[1 - cBK \frac{1}{1 + K}\right] \left(cKcKB - cK^2 + cK \frac{1}{1 + K}(1 - K) + cK cKB\right)
\]

\[
= \left[1 - cBK \frac{1}{1 + K}\right] \left(cKcKB - cK^2 + c(1 - K) + cK cKB\right)
\]

\[
= \left[1 - cBK \frac{1}{1 + K}\right] (cBK + 1)(1 - K)
\]

\[
= c(1 - K). \quad (2.20)
\]

So we just have shown, by explicit computations, that starting with a seemingly trivial identity-based solution (2.11), by a suitable gauge transformation, we arrive to a well-behaved solution which correctly reproduces the desired value of D-brane tension. In the next subsection, we show that the Erler–Schnabl’s solution appears as a particular case of a rather general two-parameter family of solutions.

2.3. Two-parameter family of solutions

As it was mentioned in the introduction section, in order to find identity-based solutions to the open bosonic string field equations of motion, we should use the following ansatz:

\[
\Psi = \sum_{n,p} f_{n,p} U_{1}^{n} U_{1}^{\hat{L}^{n} \hat{c}_{p}} |0\rangle + \sum_{n,p,q} f_{n,p,q} U_{1}^{n} U_{1}^{\hat{B} \hat{L}^{n} \hat{c}_{p} \hat{c}_{q}} |0\rangle, \quad (2.21)
\]

where \( n = 0, 1, 2, \ldots \) and \( p, q = 1, 0, -1, -2, \ldots \). The operators \( \hat{L}^{n}, \hat{B} \) and \( \hat{c}_{p} \) are defined in [3]. Plugging this ansatz (2.21) into the equation of motion will lead to a system of algebraic equations for the coefficients \( f_{n,p} \) and \( f_{n,p,q} \). Analyzing these algebraic equations we discover that many of the coefficients can be set to zero; therefore, we can use a simpler ansatz than the one given by (2.21); for instance, we use the following ansatz:

\[
\Psi = \alpha_{1}c + \alpha_{2}cK + \alpha_{3}Kc. \quad (2.22)
\]

Plugging this ansatz (2.22) into the equation of motion \( Q\Psi + \Psi\Psi = 0 \), we obtain an algebraic equation for the coefficients \( \alpha_{i} \):

\[
1 + \alpha_{2} + \alpha_{3} = 0, \quad (2.23)
\]

and consequently our ansatz (2.22) becomes

\[
\Psi = \alpha_{1}c + \alpha_{2}cK - (1 + \alpha_{2})Kc. \quad (2.24)
\]
A string field $\Psi'$ which identically satisfies the string field equation of motion can be derived by performing a gauge transformation over the identity-based solution (2.24):

$$\Psi' = U(\Psi + Q)U^{-1}. \tag{2.25}$$

Plugging expressions (1.8) for the string field $U$ and its inverse $U^{-1}$ into the definition of the gauge transformation (2.25), we obtain a two-parameter family of solutions:

$$\Psi' = [\alpha_1(c + cBKc) + (\alpha_1 + \alpha_2)(cK + cBKcK) - (1 + \alpha_2)(Kc + KcBKc)]\frac{1}{1 + K}. \tag{2.26}$$

As shown in the appendix, to simplify the calculation of the vacuum energy, we can fix the values of the two parameters $\alpha_1$ and $\alpha_2$. For instance the particular values of the parameters $\alpha_1 = 1$ and $\alpha_2 = -1$ correspond to the Erler–Schnabl’s solution (2.1). Nevertheless at this point it is interesting to ask: do the solutions with different values of the parameters describe the tachyon vacuum? To provide an answer to this question, we should compute the vacuum energy for solution (2.26) with arbitrary values for the parameters $\alpha_1$ and $\alpha_2$.

In order to perform this computation, let us write solution (2.26) as an expression containing an exact BRST term

$$\Psi' = [\alpha_1c + (\alpha_1 + \alpha_2)cKc - (1 + \alpha_2)Kc + Q\{[\alpha_1c + (\alpha_1 + \alpha_2)cKc - (1 + \alpha_2)KcBcK]\frac{1}{1 + K}\}]. \tag{2.27}$$

It turns out that the computation of the vacuum energy can be reduced to the evaluation of the following correlator:

$$\frac{1}{6g^2}\langle \Psi', Q\Psi' \rangle = \frac{1}{6g^2}\left\{ (\alpha_1c + \alpha_2cKc + \alpha_3Kc)\frac{1}{1 + K} (\alpha_1cKc + \alpha_2cKcKc + \alpha_3KcKcK)\frac{1}{1 + K}\right\}, \tag{2.28}$$

where we have defined the coefficients

$$a_1 = \alpha_1, \quad a_2 = \alpha_1 + \alpha_2, \quad a_3 = -1 - \alpha_2. \tag{2.29}$$

Expanding the right-hand side of equation (2.28), we obtain the following expression for the vacuum energy:

$$\frac{1}{6g^2}\left[ a_1^3 \left( c \frac{1}{1 + K} + KcKc \frac{1}{1 + K} \right) + a_1a_2 \left( c \frac{1}{1 + K} + KcKc \frac{1}{1 + K} \right) + a_1a_3 \left( c \frac{1}{1 + K} + KcKc \frac{1}{1 + K} \right) + a_2a_1 \left( cKcKc \frac{1}{1 + K} + KcKc \frac{1}{1 + K} \right) + a_2a_3 \left( cKcKc \frac{1}{1 + K} + KcKc \frac{1}{1 + K} \right) + a_3a_1 \left( cKcKc \frac{1}{1 + K} + KcKc \frac{1}{1 + K} \right) + a_3a_2 \left( cKcKc \frac{1}{1 + K} + KcKc \frac{1}{1 + K} \right) + a_3a_3 \left( cKcKc \frac{1}{1 + K} + KcKc \frac{1}{1 + K} \right) \right]. \tag{2.30}$$

All the correlators involved in the evaluation of the vacuum energy (2.30) can be computed using the basic correlator $\langle \Omega^0c\Omega^0c\Omega^0c\Omega^0c \rangle$:

$$\langle \Omega^0c\Omega^0c\Omega^0c\Omega^0c \rangle = \frac{L^3}{\pi^3} \sin \frac{\pi r_2}{L} \sin \frac{\pi r_1}{L} \sin \frac{\pi (r_2 + r_3)}{L}, \quad L = r_1 + r_2 + r_3 + r_4. \tag{2.31}$$
For instance, the expression for the correlator $\langle Kc \frac{1}{1 + K} Kc \frac{1}{1 + K} \rangle$ is given by
\[
\langle Kc \frac{1}{1 + K} Kc \frac{1}{1 + K} \rangle = -\int_0^\infty \int_0^\infty dt_1 dt_2 e^{-t_1 - t_2} \frac{\partial^3 \langle \Omega^{s_1} c \Omega^{s_2} c \Omega^{s_3} c \Omega^{s_4} c \Omega^{s_5} \rangle}{\partial s_1 \partial s_2 \partial s_3} \bigg|_{s_1 = 0, s_2 = 0, s_3 = 0}
= -\frac{3}{\pi^2}.
\]

Performing similar computations for the rest of the correlators, from (2.30) we get the following expression for the vacuum energy (2.28):
\[
\frac{1}{6g^2} \langle \Psi^\prime, Q \Psi^\prime \rangle = -\frac{1}{2\pi^2 g^2}(-a_1 + a_2 + a_3)^2;
\]
plugging the definition of the coefficients $a_1, a_2, a_3$ (2.29) into equation (2.33), we remarkably obtain the right value for the vacuum energy:
\[
\frac{1}{6g^2} \langle \Psi^\prime, Q \Psi^\prime \rangle = -\frac{1}{2\pi^2 g^2}.
\]
Therefore this last result (2.34) shows that the two-parameter family of solutions (2.26) describes the tachyon vacuum.

3. Simple analytic solution in the superstring case

In this section, we extend our previous results in order to analyze a new identity-based solution in the modified cubic superstring field theory, and as in the case of open bosonic string field theory, by performing a suitable gauge transformation over this identity-based solution, we obtain the known Gorbachev’s solution which correctly reproduces the value of the vacuum energy. A two-parameter family of solutions is discussed as well.

3.1. The identity-based solution and gauge transformation

In the superstring case, in addition to the basic string fields $K, B$ and $c$, we need to include the super-reparametrization ghost field $\gamma$ which, in the operator representation, is given by [11]
\[
\gamma \to U_1^\dagger U_1 \gamma(0)|0\rangle.
\]
Let us remember that in the superstring case the basic string fields $K, B, c$ and $\gamma$ satisfy the algebraic relations [2, 11]
\[
\{ B, c \} = 1, \quad [B, K] = 0, \quad B^2 = c^2 = 0, \quad \partial c = [K, c], \quad \partial \gamma = [K, \gamma], \quad [c, \gamma] = 0, \quad [B, \gamma] = 0,
\]
and have the following BRST variations:
\[
QK = 0, \quad QB = K, \quad Qc = cKc - \gamma^2, \quad Q\gamma = c\partial \gamma - \frac{1}{2} \gamma \partial c.
\]
Employing these basic string fields, we can construct the following identity-based solution:
\[
\Psi = (c + B\gamma^2)(1 - K),
\]
which formally satisfies the equation of motion $Q\Psi + \Psi^2 = 0$, where in this case $Q$ is the BRST operator of the open Neveu–Schwarz superstring theory.

As in the bosonic case, the direct evaluation of the vacuum energy using the identity-based solution (3.4) brings ambiguous result; therefore, we should try to find an explicit gauge transformation in order to construct a well-behaved solution similar to Erler–Schnabl’s solution.
Using the same procedure developed in the previous section, we show that a well-behaved solution \( \Psi_G \) can be generated from our identity-based solution (3.4) by performing a gauge transformation

\[
\Psi = U^{-1}(Q + \Psi_G)U
\]

\[
\Psi = \left[ 1 - cBK \frac{1}{1 + K} \right] \left( Q + (c + cKBc + B\gamma^2) \frac{1}{1 + K} \right) \left( cBK + 1 \right)
\]

\[
\Psi = \left[ 1 - cBK \frac{1}{1 + K} \right] \left( Q(cKB + 1) + (c + cKBc + B\gamma^2) \frac{1}{1 + K} \right) \left( cBK + 1 \right)
\]

\[
\Psi = \left[ 1 - cBK \frac{1}{1 + K} \right] \left( cKcKB - cK^2 + c(1 - K) + cKBc + B\gamma^2(1 - K) \right)
\]

\[
\Psi = \left[ 1 - cBK \frac{1}{1 + K} \right] \left( (cKB + 1)c(1 - K) + B\gamma^2(1 - K) \right)
\]

\[
\Psi = \left[ 1 - cBK \frac{1}{1 + K} \right] \left( cKB + 1 \right) ((c + B\gamma^2)(1 - K))
\]

\[
\Psi = (c + B\gamma^2)(1 - K).
\]

Remarkably, it turns out that the resulting analytic solution \( \Psi_G \) corresponds to the known Gorbachev’s solution [2]

\[
\Psi_G = (c + cKBc + B\gamma^2) \frac{1}{1 + K},
\]

which, as we are going to verify in the next subsection, correctly reproduces the value of the vacuum energy.

Since solution (3.6) looks very similar to the bosonic one (2.1), we should write it as a continuous integral over wedge states by inverting \( 1 + K \) and using the Schwinger parameterization (2.10); this way of writing the solution is important if we are interested in the evaluation of the vacuum energy. Let us mention that the calculation of the vacuum energy for solution (3.6) was already performed in [2] by Gorbachev. Nevertheless, for completeness, we review this important computation in the next subsection.

3.2. Gorbachev’s solution and its vacuum energy

As mentioned in the previous subsection, by inverting \( 1 + K \) and employing the Schwinger parameterization (2.10), we can write solution (3.6) as the following integral:

\[
\Psi_G = \int_0^\infty dt e^{-t} (c + cKBc + B\gamma^2) \Omega';
\]

now acting the BRST operator \( Q \) on the string field \( Bc \): \( Q(Bc) = cKBc + B\gamma^2 \), we can express (3.7) as

\[
\Psi_G = \int_0^\infty dt e^{-t} c\Omega' + Q \left[ \int_0^\infty dt e^{-t} Bc\Omega' \right].
\]

This way of writing solution (3.8) is important to simplify computation of the vacuum energy. Using the equation of motion, the expression for vacuum energy can be reduced to the evaluation of the following correlator:

\[
\frac{1}{6g^2} \langle \langle \Psi_G Q \Psi_G \rangle \rangle = \frac{1}{6g^2} \int_0^\infty dt_1 dt_2 e^{-h_{12}} \langle \langle c\Omega^h (cKc - \gamma^2)\Omega^{\bar{z}z} \rangle \rangle
\]

\[
= \frac{1}{6g^2} \int_0^\infty dt_1 dt_2 e^{-h_{12}} \left[ \langle Y_{-2c}\Omega^h cKc\Omega^{\bar{z}z} \rangle - \langle Y_{-2c}\Omega^{\bar{z}z}\gamma^2\Omega^h \rangle \right]
\]
\[ \langle Y_{\bar{c}}\rangle = -\frac{1}{6} g^2 \int_0^\infty dt_1 dt_2 e^{-t_1-t_2} (Y_{\bar{c}})^2. \] (3.9)

The correlator \( \langle Y_{\bar{c}}^2 \rangle \) can be computed using the methods given in [11]6:

\[ \langle Y_{\bar{c}}^2 \rangle = \frac{(t_1 + t_2)^2}{2\pi^2}. \] (3.10)

Plugging the value of the correlator (3.10) into equation (3.9), we finally obtain

\[ \frac{1}{6} g^2 \langle\langle \Psi G \Psi G \rangle \rangle = -\frac{1}{12\pi^2 g^2} \int_0^\infty dt_1 dt_2 e^{-t_1-t_2} (t_1 + t_2)^2 \]
\[ = -\frac{1}{12\pi^2 g^2} \left( \int_0^\infty du u^2 e^{-u} \right) \left( \int_0^1 dv \right) \]
\[ = -\frac{1}{2\pi^2 g^2}, \] (3.11)

where we have used the change of variables defined as follows [1]:

\[ u = t_1 + t_2, \quad u \in [0, \infty), \]
\[ v = \frac{t_1}{t_1 + t_2}, \quad v \in [0, 1], \] (3.12)
\[ dt_1 dt_2 = u \, du \, dv. \]

Note that the value of the vacuum energy obtained in equation (3.11) is in perfect agreement with the value predicted from Sen’s first conjecture (1.2). We would like to comment that the value of the vacuum energy can be obtained using another means, for instance using the \( L_0 \)-level expansion of the solution; we have confirmed that the value of the vacuum energy is the same as the one computed analytically (3.11). It should be important to confirm this result by using a third option, namely by employing the usual Virasoro \( L_0 \)-level expansion.

### 3.3. Two-parameter family of solutions in the superstring case

Following the same procedures developed in subsection 2.3, in the case of the modified cubic superstring field theory, in the appendix section, we show that Gorbachev’s solution (3.6) appears as a particular case of a rather general two-parameter family of solutions:

\[ \Psi' = [x_1 c + x_2 K + x_3 K c + x_4 B y^2 + x_5 B y^2 K + x_6 K B y^2] \frac{1}{1 + K} \]
\[ + Q \left\{ [\beta_1 B c + (\beta_1 + \beta_2) B c K - (1 + \beta_2) K B c] \frac{1}{1 + K} \right\}. \] (3.13)

where the coefficients \( x_i \) are given by

\[ x_1 = \beta_1, \quad x_2 = \beta_1 + \beta_2, \quad x_3 = -1 - \beta_2, \quad x_4 = 1 - \beta_1, \]
\[ x_5 = -\frac{(\beta_1 - 1)(\beta_1 + \beta_2)}{\beta_1}, \quad x_6 = \frac{(\beta_1 - 1)(\beta_2 + 1)}{\beta_1}. \] (3.14)

Note that Gorbachev’s solution corresponds to the particular values of the parameters \( \beta_1 = 1 \) and \( \beta_2 = -1 \). Nevertheless it is interesting to ask: do solutions with different values of the parameters describe the tachyon vacuum? To provide an answer to this question, we should compute the vacuum energy for the solution (3.13) with arbitrary values for the parameters \( \beta_1 \) and \( \beta_2 \).

---

6 This correlation function has been computed using the normalization \( \langle \xi(x)c\partial_c\bar{c}(\bar{y})e^{-2\phi(z)} \rangle = 2 \).
Since solution (3.13) has an expression containing an exact BRST term, the computation of the vacuum energy can be reduced to the evaluation of the following correlator:

$$\left\langle \frac{1}{6g^2} \langle \langle \Psi', Q\Psi' \rangle \rangle \right\rangle = \frac{1}{6g^2} \langle \langle \Psi_1, Q\Psi_1 \rangle \rangle,$$

(3.15)

where the string field $\Psi_1$ is given by

$$\Psi_1 = [x_1c + x_2\epsilon + x_3Kc + x_4B\gamma^2 + x_5BY^2K + x_6KBY^2] \frac{1}{1 + K}.$$

(3.16)

After a tedious algebra, by plugging the string field defined in (3.16) into (3.15), we obtain the following expression for the vacuum energy:

$$\left\langle \frac{1}{6g^2} \left[ -2 \left\langle \left\langle \frac{c}{K + 1} \gamma^2 \frac{1}{K + 1} \right\rangle \right\rangle x_{1x2} \right. \right.$$

$$\left. - 2 \left\langle \left\langle \frac{c}{K + 1} \gamma^2 \frac{1}{K + 1} \right\rangle \right\rangle x_{1x3} - 2 \left\langle \left\langle \frac{c}{K + 1} \gamma^2 \frac{1}{K + 1} \right\rangle \right\rangle x_{2x3} \right.$$  

$$\left. - \left\langle \left\langle \frac{c}{K + 1} \gamma^2 \frac{1}{K + 1} \right\rangle \right\rangle x_{1x4} + 3 \left\langle \left\langle \frac{c}{K + 1} \gamma^2 \frac{1}{K + 1} \right\rangle \right\rangle x_{1x5} \right.$$  

$$\left. + \left\langle \left\langle \frac{c}{K + 1} \gamma^2 \frac{1}{K + 1} \right\rangle \right\rangle x_{2x4} + 3 \left\langle \left\langle \frac{c}{K + 1} \gamma^2 \frac{1}{K + 1} \right\rangle \right\rangle x_{1x6} \left\rangle \right\rangle \right].$$

(3.17)

All the correlators involved in the evaluation of the vacuum energy (3.17) can be computed using the basic correlators

$$\langle \langle \Omega^\nu c\Omega^{\nu'} c\Omega^{\nu'2} \rangle \rangle = \frac{(r_1 + r_2 + r_3)^2}{2\pi^2},$$

(3.18)

$$\langle \langle \Omega^\nu B\Omega^{\nu'} c\Omega^{\nu'2} c\Omega^{\nu'2} \rangle \rangle = \frac{(r_1 + r_2 + r_3 + r_4 + r_5) r_5}{2\pi^2}. $$

(3.19)

For instance, the expression for the correlator $\langle \langle B\gamma^2 cKc \frac{1}{K+1} \rangle \rangle$ is given by

$$\left\langle \left\langle B\gamma^2 \frac{1}{K + 1} cKc \frac{1}{K + 1} \right\rangle \right\rangle = -\int_0^\infty \int_0^\infty dt_1 dt_2 e^{-t_1 - t_2} \frac{\partial \langle \langle B\gamma^2 \Omega^\nu c\Omega^{\nu2} \rangle \rangle}{\partial s_1} \bigg|_{s_1=0}$$

$$= -\frac{1}{2\pi^2} \int_0^\infty \int_0^\infty dt_1 dt_2 e^{-t_1 - t_2} (t_1 + t_2)$$

$$= -\frac{1}{\pi^2}.$$  

(3.20)

Performing similar computations for the rest of the correlators, from (3.17) we get the following expression for the vacuum energy (3.15):

$$\left\langle \frac{1}{6g^2} \langle \langle \Psi', Q\Psi' \rangle \rangle \right\rangle = -6x_1^2 + (8x_2 + 8x_3 - 4x_4 + 3x_5 + x_6) x_1 - 2 (x_2 + x_3)^2 + (x_2 + 3x_3) x_4,$$

$$\frac{12g^2}{\pi^2};$$

(3.21)
plugging the definition of the coefficients $x_1, x_2, x_3, x_4, x_5, x_6$ (3.14) into equation (3.21), we remarkably obtain the right value for the vacuum energy:

$$\frac{1}{6g^2} \langle \langle \Psi', Q \Psi \rangle \rangle = -\frac{1}{2\pi^2 g^2}.$$  \hspace{1cm} (3.22)

Therefore this last result (3.22) shows that the two-parameter family of solutions (3.13) describes the tachyon vacuum.

4. Summary and discussion

We have studied a new set of identity-based solutions to analyze the problem of tachyon condensation in open bosonic string field theory and cubic superstring field theory; although these solutions seem to be trivial, we have shown that they can be related, by performing a gauge transformation, to well-behaved solutions where in the case of open bosonic string field theory, the resulting solution corresponds to the Erler–Schnabl’s solution [1], while in the case of the modified cubic superstring field theory corresponds to a similar solution [2] which, unlike the known solutions [11, 13, 14], can be written as a continuous integral over wedge states where no regularization or phantom term is required.

Although, after performing the gauge transformation, the resulting Erler–Schnabl-type solutions can be used to correctly compute the value of the vacuum energy, it would be interesting to evaluate directly the vacuum energy using identity-based solutions; this kind of computation should be possible provided that we can find a consistent regularization scheme. Even though our results suggest that these solutions should reproduce the right value for the D-brane tension, the direct calculation persists as one of the most difficult problems in string field theory [22].

The main motivation of this work was to understand how identity-based solutions can be used to generate well-defined solutions which describe the tachyon vacuum in relatively simple cubic string field theories [16–18]. It is important to extend this analysis to the case of Berkovits WZW-type superstring field theory [15], since this theory has a non-polynomial action; finding the tachyon vacuum solution and computation of the value of the D-brane tension seems to be highly cumbersome. Nevertheless, we hope that the ideas developed in this paper can be useful in solving this challenging puzzle.

Another important application of the techniques developed in this paper, as mentioned in [26], could be the extension of the subalgebra generated by the basic string fields $K, B, c$ and $\gamma$ in order to analyze more general string field configurations [27, 28] such as multiple D-branes, marginal deformations, lump solutions as well as time-dependent solutions.

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Appendix. Derivation of the identity-based solution and gauge transformation

In this appendix we provide details related to the derivation of the identity-based solution and gauge transformation. In the case of open bosonic string field theory, we propose the following ansatz:

$$\Psi = \alpha_1 c + \alpha_2 c K + \alpha_3 K c;$$  \hspace{1cm} (A.1)
plugging this ansatz into the equation of motion \( Q\Psi + \Psi' = 0 \) and using the identity \( cK^2c = cKcK + KcKc \), we easily find
\[
\alpha_1(1 + \alpha_2 + \alpha_3)cKc + \alpha_2(1 + \alpha_2 + \alpha_3)cKcK + \alpha_3(1 + \alpha_2 + \alpha_3)KcKc = 0.
\] (A.2)
Therefore, as we can see, the string field equation of motion reduces to the algebraic equation
\[
1 + \alpha_2 + \alpha_3 = 0,
\] (A.3)
and consequently our ansatz (A.1) becomes
\[
\Psi = \alpha_1c + \alpha_2cK - (1 + \alpha_2)Kc.
\] (A.4)

The next step is to perform a gauge transformation over this identity-based solution (A.4). It turns out that the suitable string field \( U \) which will be used to define the gauge transformation is given by
\[
U = 1 + cBK.
\] (A.5)
The inverse of this string field \( U \) can be computed using the power series expansion
\[
U^{-1} = \frac{1}{1 + cBK} = \sum_{n=0}^{\infty} (-1)^n(cBK)^n = 1 + \sum_{n=1}^{\infty} (-1)^n cBK^n = 1 - cBK \frac{1}{1+K}.
\] (A.6)
A string field \( \Psi' \) which identically satisfies the string field equation of motion \( Q\Psi' + \Psi'\Psi' = 0 \) can be derived by performing a gauge transformation over the identity-based solution (A.4)
\[
\Psi' = U(\Psi + Q)U^{-1}.
\] (A.7)
Plugging expressions (A.5) and (A.6) for the string field \( U \) and its inverse \( U^{-1} \) into the definition of the gauge transformation (A.7), we obtain
\[
\Psi' = [\alpha_1(c + cBKc) + (\alpha_1 + \alpha_2)(cK + cBKcK) - (1 + \alpha_2)(Kc + KcBKc)] \frac{1}{1 + K}.
\] (A.8)
To simplify the computation of the vacuum energy, we would like to write Erler–Schnabl-type solution (A.8) in the following way:
\[
\Psi' = [\alpha_1c + \chi] \frac{1}{1 + K}
\] (A.9)
such that \( Q\chi = 0 \), where \( \chi \) is some string field; to satisfy this requirement (A.9) we must impose the following condition on the numerator of equation (A.8):
\[
Q[\alpha_1(c + cBKc) + (\alpha_1 + \alpha_2)(cK + cBKcK) - (1 + \alpha_2)(Kc + KcBKc)] = \alpha_1cKc.
\] (A.10)
From this last equation (A.10), using the BRST variations (2.8), we obtain
\[
(\alpha_1 + \alpha_2)cKcK - (1 + \alpha_2)KcKc = 0,
\] (A.11)
and therefore the values of the coefficients \( \alpha_1 \) and \( \alpha_2 \) are given by
\[
\alpha_1 = 1, \quad \alpha_2 = -1.
\] (A.12)
Finally, plugging the value of these coefficients (A.12) into Erler–Schnabl’s tachyon vacuum solution in open bosonic string field theory [1]:
\[
\Psi_{E-S} = [c + cBKc] \frac{1}{1 + K}.
\] (A.13)
In the case of the modified cubic superstring field theory, we use the following ansatz:

\[
\Psi = \sum_{n,p} f_{n,p} U_1^n \tilde{U}_p \tilde{c}_p \langle 0 \rangle + \sum_{n,p,q} f_{n,p,q} U_1^n \tilde{U}_p \tilde{c}_q \langle 0 \rangle + \sum_{n,t,u} g_{n,t,u} U_1^n \tilde{U}_t \tilde{g}_u \langle 0 \rangle,
\]

(A.14)

where \( n = 0, 1, 2, \ldots, p, q = 1, 0, -1, -2, \ldots \) and \( t, u = \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, \ldots \). Plugging this ansatz (A.14) into the equation of motion will lead to a system of algebraic equations for the coefficients \( f_{n,p}, f_{n,p,q} \) and \( g_{n,t,u} \). Analyzing these algebraic equations we discover that many of the coefficients can be set to zero; therefore, we can use a simpler ansatz than the one given in (A.14), namely

\[
\Psi = \beta_1 c + \beta_2 c K + \beta_3 K c + \beta_4 B y^2 + \beta_5 B y^2 K + \beta_6 K B y^2.
\]

(A.15)

Plugging this ansatz (A.15) into the equation of motion \( Q \Psi + \Psi \Psi = 0 \), we obtain a system of algebraic equations for the coefficients \( \beta_i \):

\[
\begin{align*}
\beta_2 + \beta_3 + 1 &= 0, \\
-\beta_2 \beta_5 + \beta_2 \beta_6 - \beta_3 \beta_6 + \beta_3 \beta_4 + \beta_4 &= 0, \\
\beta_2 - \beta_3 + \beta_4 &= 0, \\
\beta_4 &= 1.
\end{align*}
\]

(A.16) - (A.19)

Solving this system of algebraic equations, we obtain

\[
\beta_3 = -\beta_2 - 1, \quad \beta_4 = 1, \quad \beta_5 = \frac{\beta_2}{\beta_1}, \quad \beta_6 = -\frac{\beta_2 - 1}{\beta_1},
\]

(A.20)

and consequently our ansatz (A.15) becomes

\[
\Psi = \beta_1 c + \beta_2 c K - (\beta_2 + 1) K c + B y^2 + \beta_5 B y^2 K - \frac{\beta_2 + 1}{\beta_1} K B y^2.
\]

(A.21)

A string field \( \Psi' \) which identically satisfies the string field equation of motion can be derived, as in the bosonic case, by performing a gauge transformation over the identity-based solution (A.21):

\[
\Psi' = U (\Psi + Q) U^{-1}.
\]

(A.22)

Plugging expressions (A.5) and (A.6) for the string field \( U \) and its inverse \( U^{-1} \) into the definition of the gauge transformation (A.22), we obtain a two-parameter family of solutions

\[
\begin{align*}
\Psi' &= [x_1 c + x_2 c K + x_3 K c + x_4 B y^2 + x_5 B y^2 K + x_6 K B y^2] \frac{1}{1 + K} \\
&\quad + Q \left[ \left\{ (\beta_1 B c + (\beta_1 + \beta_2) B c K - (1 + \beta_2) K B c) \frac{1}{1 + K} \right\} \right],
\end{align*}
\]

(A.23)

where the coefficients \( x_i \) are given by

\[
\begin{align*}
x_1 &= \beta_1, \\
x_2 &= \beta_1 + \beta_2, \\
x_3 &= -1 - \beta_2, \\
x_4 &= 1 - \beta_1, \\
x_5 &= \frac{(\beta_1 - 1)(\beta_1 + \beta_2)}{\beta_1}, \\
x_6 &= \frac{(\beta_1 - 1)(\beta_2 + 1)}{\beta_1}.
\end{align*}
\]

(A.24)

These coefficients have been defined to simplify the presentation of our equations. Note that Gorbachev’s solution [2]

\[
\Psi_G = (c + c K B c + B y^2) \frac{1}{1 + K}
\]

(A.25)

corresponds to the particular case \( \beta_1 = 1 \) and \( \beta_2 = -1 \).


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