Osp(1|8)-Gravity

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We analyze a new MacDowell-Mansouri $R^2$-type supergravity action based on the superalgebra $Osp(1|8)$. This contribution summarizes the work of ref. [1].

Recently, there has been renewed speculation that further supergravity theories might exist in $d = 11$ and $d = 12$ dimensions [2]-[6], which might provide a unifying origin for the supergravities appearing as low-energy limits of string- or M-theory. The idea of supergravities beyond $d=11$ was already explored fifteen years ago, but no conventional supergravity theory was found [7], even though in $d = (10,2)$ dimensions Majorana-Weyl spinors exist, and dimensional reduction to $d = (10,1)$ would therefore lead to a $N = 1$ supergravity theory. The search for principles underlying string theory leads us to investigate the superalgebraic foundations of supergravities. While these are well understood in simple and usually low-dimensional cases, for higher $N$ or higher dimension one only suspects that they exist. The indications we have of relations between all superstrings prompt us to reopen the investigation.

1. Algebras

The recent spate of such ideas is based on the fact that p-branes couple naturally to $(p+1)$-form gauge potentials via the currents [2,3]:

\[ J^{\mu_1\ldots\mu_{p+1}}(x) = \frac{1}{\sqrt{g}} \int d\tau \int d^p\sigma \delta^d(x - X(\tau, \sigma)) \epsilon^{\tau_1\ldots\tau_{p+1}} \]

These currents are conserved:

\[ \partial_{\nu}(\sqrt{g} J^{\mu_1\ldots\mu_{p}}(x)) = 0 \] (2)

and give rise to tensor charges

\[ Z^{\mu_1\ldots\mu_{p}} = \int d^{d-1}\Sigma J^{\mu_1\ldots\mu_{p}}(x) . \] (3)

They appear then in (maximally) extended supersymmetry algebras as follows [3]: for the case of IIA supersymmetry in $d=(1,9)$ one has

\[ \{ Q_{a}, Q_{b} \} = \Gamma_{a\beta}^{\mu} P_{\mu} + \Gamma_{a\beta}^{\mu} Z_{\mu} \]

\[ + \Gamma_{a\beta}^{\mu_{1}\ldots\mu_{5}} Z_{\mu_{1}\ldots\mu_{5}} \] (4)

\[ \{ Q_{a}, Q^{\beta} \} = \Gamma_{a\beta}^{\mu} P_{\mu} - \Gamma_{a\beta}^{\mu} Z_{\mu} \]

\[ + \Gamma_{a\mu_{1}\ldots\mu_{5}} Z_{\mu_{1}\ldots\mu_{5}} \]

\[ \{ Q_{a}, Q^{\beta} \} = \delta_{a}^{\beta} Z + \Gamma_{a\beta}^{\mu_{1}\ldots\mu_{4}} Z_{\mu_{1}\ldots\mu_{4}} \]

\[ + \Gamma_{a\mu_{1}\ldots\mu_{4}} Z_{\mu_{1}\ldots\mu_{4}} \]

where the 16-dimensional Majorana-Weyl spinors $Q_{a}$ and $Q^{a}$ have opposite chirality. This algebra may be given a $(1,10)$-d interpretation in terms of real 32-component spinors:

\[ \{ Q_{a}, Q_{b} \} = \Gamma_{ab}^{M} P_{M} + \Gamma_{ab}^{MN} Z_{MN} \]

\[ + \Gamma_{ab}^{M_{1}\ldots M_{5}} Z_{M_{1}\ldots M_{5}} \] (5)

or even a $(2,10)$-d one in terms of 32-component Majorana-Weyl spinors:

\[ \{ Q_{a}, Q_{b} \} = \Gamma_{ab}^{MN} M_{MN} \]

\[ + \Gamma_{ab}^{M_{1}\ldots M_{6}} Z_{M_{1}\ldots M_{6}} \] (6)

The type IIB algebra in $d=(1,9)$ reads...
\{Q_{\alpha i}, Q_{\beta j}\} = \Gamma_{\alpha \beta}^{\mu} \Sigma_{ij}^{\mu} Z_{\lambda \mu} + \Gamma_{\alpha \beta}^{\mu \nu} \epsilon_{i j} Z_{\mu \nu \rho \sigma} + \Gamma_{\alpha \beta}^{\mu \nu} \Sigma_{ij}^{\mu \nu} Z_{\lambda \mu \nu | \rho \sigma},

where we use the conventions \(\Sigma_{ij}^{\mu} = \epsilon_{\mu} \Sigma_{ij}^{R}, \Sigma_0 = -i \sigma^2 \Sigma_1 = -\sigma^1 \) and \(\Sigma_2 = \sigma^3\). These matrices satsify \(\Sigma_{ij} \Sigma_{jk} = \eta_{ij} + \epsilon_{ij \ell} \Sigma_{k \ell} = \eta_{ij} + \epsilon_{ij \ell} \eta^{\ell k} \Sigma_{k},\) with \(\eta_{ij} = (- + +)\), \(\epsilon_{012} = 1\), and hence generate \(SL(2,\mathbb{R})\). In all cases the Z-charges fit the respective brane-scan, and all cases form some decomposition of the Q-Q part of \(Osp(1|32)\).

| D=11 | \(\mathbf{2}\) | \(\mathbf{5}\) |  |
|-----|-----|-----|-----|
| IIA | \(0_D\) | \(1_F\) | \(2_D\) | \(4_D\) | \(5_S\) | \(6_D\) |
| IIB | \(1_F, 1_D\) | \(3^+_D\) | \(5_S\) | \(5_D\) |
| Type I | \(1_D\) | \(5_D\) |
| Het | \(1_F\) | \(5_S\) |

**Branescan:** the subscripts \(F, D\), or \(S\), denote ‘Fundamental’, ‘Dirichlet’ or ‘Solitonic’ branes.

We will conjecture, for the purposes of this paper, that the rest of the algebra completes (possibly some contraction of) \(Osp(1|32)\):

\[\{Q_a, Q_b\} = J_{ab}\] (7)

\[\{J_{ab}, Q_c\} = -C_{c(a} Q_{b)}\]

\[\{J_{ab}, J_{cd}\} = 2C_{a(c} J_{b)d)}\]

which may be decomposed in terms of \(SO(2,10)\) covariant tensors to yield the extended (1,9)-d superconformal algebra of van Holten and van Proeyen [3], namely

\[\{Q_a, Q_b\} = -\frac{1}{128} \Gamma^{MN}_{ab} J_{MN}\]

(8)

\[\{J_{MN}, Q_a\} = -(\Gamma_{MN})^b_a Q_b\]

\[\{J_{M1...M6}, Q_a\} = -(\Gamma_{M1...M6})^b_a Q_b\]

\[\{J_{MN}, J_{KL}\} = 8 \delta^K_N J_{[M} | L]\]

\[\{J_{MN}, J_{M1...M6}\} = 24 \delta^K_N J_{[M} | J_M]_{M1...M6}\]

\[\{J_{N1...N6}, J_{M1...M6}\} =
-12 \cdot 6! \delta^{|M_1...M_5|}_{[N_1...N_5} J_{M_6]} + 12 \epsilon_{N_1...N_6} [M_1...M_5 | R J_R | M_6].\]

In the (1,10)-d context this algebra was studied by D’Auria and Fré [4]. We will try to take some first steps towards constructing a conformal supergravity theory based on that type of algebra.

The signature of the vector space that appears in the above algebra is \((2,10)\). This provides another hint of a connection to string-theoretic ideas, as Vafa’s [5] argument shows: \(SL(2,Z)\)-duality of type IIB strings may be explained via D-strings. The zero-modes of the open strings stretched between such D-strings determine the worldsheet fields of the latter. We have

\[\Psi_{1/2} | k > \mu = 0,1 \quad \text{2-d vect. field} (9)\]

\[\Psi_{-1/2} | k > m = 2, \cdots, 9 \quad \text{transv. scalars} (10)\]

and hence we find on the D-string an extra U(1) gauge field. In \(d=2\) this is nondynamical, of course, but it leaves, after gauge fixing, a pair of ghosts \(B, C\) with central charge \(c = -2\). The critical dimension is hence raised by two, and the no-ghost theorem [10], which states that the BRST cohomology effectively eliminates those extra dimensions, forces us to assume the existence of a nullvector in the extra dimensions, and that means they must have signature \((1,1)\).

Taking the idea of strings moving in a 12-dimensional target space more seriously, we are immediately led to the puzzle of why strings oscillate in only 10 of these dimensions, but never in the extra 2. If one has conformal symmetry in mind, there is a natural answer: the 12 dimensions are those in which the conformal group is linearly realized, but only a 10-dimensional null hypersurface in real projective classes of these coordinates is physical. The extra two dimensions “don’t really exist”. The idea that strings might have some sort of target space conformal symmetry is not new [12], but as of now no model exists that can be convincingly linked to the string theories known today.

At least part of the problem is the fact that a conventional superconformal algebra in \(d=(1,9)\) does not exist, and while one can write a confor-
mal supergravity action, the fields one uses are subject to differential constraints \[13\]. In contrast, for \( d=1,3 \), conformal \( N=1 \) supergravity and its superalgebraic \( SU(2,2|1) \)-underpinnings are understood, and therefore we will restrict ourselves to an analysis of \( Osp(1|8) \), which may be interpreted as a variant superconformal algebra. We note that \( SU(2,2|1) \) is not a subalgebra of \( Osp(1|8) \). This is most clearly seen by analyzing their embedding in \( Osp(2|8) \) \[14\]: let the oscillators \( a_A = (a^K, \pi_K, a, \bar{a}) \) have the (anti)commutation relations \[ a^K, \pi_L] = \delta^K_L, \{ a, \bar{a} \} = 1 \]. Here \( \pi_K = \eta_{KL}a^L \) is up to the \( SU(2,2) \)-metric \( \eta_{KL} \) the complex conjugate of \( a_K \). A real \( Sp(8) \)-spinor is represented by the complex pair \( (a^K, \pi_K) = a_a \). \( Osp(2|8) \) has a total of 16 real supersymmetry charges, namely the \( SU(2,2|1) \)-underpinnings \( \{ Q_K, \bar{Q}_L \} = J_{KL} \). One might think that we simply have to set \( J_{KL} = 0 = J^{KL} \) in order to obtain the ordinary superconformal algebra. This true up to a factor -3 in the \( Q^K \)-\( Q_L \) anticommutator, which is the trace of the \( SU(2,2|1) \)-metric. In the oscillator representation this factor appears as follows: the bosonic generators of \( SU(2,2|1) \) are given by \( J^K_L = \frac{1}{2}\{ a^K, \pi_L \} - \frac{i}{2}\delta^K_L\{ a^N, \pi_N \} \) and \( J = \frac{1}{2}\{ a^N, \pi_K \} = \frac{1}{2}\{ a, \bar{a} \} \) (which implies a non-trivial trace condition on the total Hilbert space) and hence

\[
\{ Q^K, \bar{Q}_L \} = \frac{1}{2}\{ a^K, \pi_L \} - \frac{i}{2}\delta^K_L\{ a, \bar{a} \} \tag{11}
\]

\[ = J^K_L - \frac{i}{4}\delta^K_L J, \]

while for \( Osp(1|8) \) we obtain

\[
\{ Q^K, \bar{Q}_L \} = \frac{1}{2}\{ a^K, \pi_L \} \tag{12}
\]

\[ = J^K_L + \frac{i}{4}\delta^K_L J, \]

where we have defined \( J = \frac{1}{2}\{ a, \bar{a} \} \) in the same fashion. Apart from this factor, and of course the generators \( J^{KL} = a^K a^L \) and \( J_{KL} = \pi_K \pi_L \), the two algebras are identical.

Let us present \( Osp(1|8) \) in a \( SO(2,4) \)-covariant form. First, we define \( \{ a_a, a_b \} = -C_{ab} \) with \( a, b = 1, \ldots, 8 \). We obtain \( \{ Q_a, Q_b \} = a_a a_b \) and use the Fierz identity

\[
\delta^{cc}_{\{ a \} a^{ed}} = -\frac{1}{8} \left\{ \Gamma^{\gamma} ab \Gamma_{cd}^{\gamma} + \frac{1}{2} \Gamma^{MN}_{ab} \Gamma^{cd}_{MN} \right. \]

\[ + \frac{1}{6} \Gamma^{LMN}_{ab} \Gamma_{LMN}^{cd} \right\} \tag{13} \]

to rewrite this as

\[
\{ Q_a, Q_b \} = \frac{1}{8} \left\{ \Gamma^{\gamma} ab a^{\gamma} cd \right. \]

\[ + \frac{1}{2} \Gamma^{MN}_{ab} a^{\gamma} MN_{cd} \]

\[ + \frac{1}{6} \Gamma^{LMN}_{ab} a^{\gamma} L MN_{cd} \}

\[ = \frac{1}{4} \left\{ \Gamma^{\gamma} ab J_7 + \frac{1}{2} \Gamma^{MN}_{ab} J_{MN} \tag{15} \]

\[ + \frac{1}{6} \Gamma^{LMN}_{ab} J_{LMN} \right\}. \]

The \( SO(1,3) \)-decomposition of the Gamma - matrices we use reads

\[
\Gamma^m = -\gamma^m \otimes \sigma^3, \Gamma^7 = -\gamma^5 \otimes \sigma^3 \]

\[ \Gamma^0 = \frac{1}{\sqrt{2}} \left\{ \begin{array}{c} 1 \otimes \sigma^+ \end{array} \right\}, \Gamma^\circ = \frac{1}{\sqrt{2}} \left\{ \begin{array}{c} 1 \otimes \sigma^- \end{array} \right\} \tag{16} \]

\[ \{ \Gamma^M, \Gamma^N \} = 2\eta^{MN} = (-+++), \]

where \( \Gamma^M = \Gamma^M_{\;ab} \) and we have chosen \( \eta_{\Theta\Theta} = 1 \). We raise and lower indices as follows: \( a^a = C_{ab} a_b \), \( \Gamma^a_{ab} = \Gamma^{ac}_{\;\;b} C_{cb} = C_{ac} \Gamma^{c\;b} \), \( \Gamma^a_{ab} = \Gamma^{a\;c}_{bc} = C_{ac} \Gamma^{c\;b} \), \( \Gamma^{ab} = (\gamma^b \otimes \sigma^1)^{ab} \) is the \( 8 \times 8 \) charge conjugation matrix introduced above. With these conventions, among the matrices \( \Gamma^a_{ab} \) we find \( \Gamma^7, \Gamma^MN \) and \( \Gamma^{MN} \) symmetric under interchange of \( a \) and \( b \), while \( C, \Gamma^M, \Gamma^{MN}PQ \) and \( \Gamma^{MN}PQR \) are antisymmetric. Similarly, the real \( 4 \times 4 \) matrices \( \gamma^a_{\alpha\beta} \) are split into the symmetric \( \gamma^m, \gamma^{mn} \) and the antisymmetric \( C^4, \gamma^{mp} \) and \( \gamma^5 \).

The remaining sectors of \( Osp(1|8) \)

\[
\begin{align*}
[ J^*, Q_a ] &= -\Gamma^*_{\;ab} b Q_b \\
[ J^7, J^{MN} ] &= \frac{1}{2} \delta^{MNPQRST}_{[J^7]} J_{RST} \\
[ J^{MN}, J_{RS} ] &= 8 \delta^{[M}_{[J^7]} J^{MRN]}_{RST} \\
[ J^{MN}, J_{RST} ] &= 12 \delta^{[M}_{[J^7]} J^{MRN]}_{RST} \\
[ J^{MPN}, J_{RST} ] &= 2 \delta^{[M}_{[J^7]} J^{MPN}}_{RST} \\
&- 36 \delta^{[M}_{[J^7]} J^{PQRST}}_{RST} 
\end{align*} \tag{17-21} \]

are now straightforward to decompose under \( SO(1,3) \), and we use then the notation
\[ P^m = J^\oplus m \quad K^m = J^\ominus m \quad E^{mn} = J^\ominus mn \]
\[ M^{nm} = \frac{1}{2} J^{mn} \quad D = J^\ominus \quad Z^m = -\frac{1}{3} e^{mnpq} J_{npq} . \]

2. Curvatures

In SO(2, 4)-covariant language the connection 1-forms are written as \( h = h_t J_t + \frac{1}{2} h_{MN} J^{MN} + \frac{1}{3} h_{MNP} J^{MNP} + \psi^a Q_a \), with \( \psi^a = (\delta^a, \psi^a) \), and the curvatures \( R = dh + hh \) are given by

\[ R = \left\{ \begin{array}{l}
\frac{1}{2} \epsilon^{MNP} R_{MNP} \lambda_{RST} \\
\frac{1}{2} \epsilon_{MNPST} R_{MNP} \lambda_{RST}
\end{array} \right\} J^7
\]

\[ + \frac{1}{3} \left\{ dh_{MN} + 2 h_M K^N + h_{RST} \right\} J^{MN}
\]

\[ + \frac{1}{5} \left\{ dh_{MNP} + 6 h_M K^N + h_{RST} \right\} J^{MNP}
\]

The gauge transformations \( \delta h = d\lambda + [h, \lambda] \) imply \( \delta R = [R, \lambda] \), i.e.

\[ \delta R = \left\{ \begin{array}{l}
\frac{1}{16} \epsilon^{MNP} R_{MNP} \lambda_{RST} \\
\frac{1}{2} \epsilon_{MNPST} R_{MNP} \lambda_{RST}
\end{array} \right\} J^{7}
\]

\[ + \frac{1}{2} \left\{ 4 R_{MK} \lambda_K N - 2 R_{MS} \lambda_{RST} \right\} J^{MN}
\]

\[ + \frac{1}{5} \left\{ - \frac{1}{3} \epsilon^{MNPST} R_{T} \lambda^{RST} \\
\frac{1}{5} \epsilon^{MNPST} R_{T} \lambda^{RST}
\end{array} \right\} J^{MN}
\]

\[ + 6 R_{MK} \lambda_K N - 6 R_{MN} \lambda_P K
\]

\[ - \frac{1}{2} \epsilon^{MNPab} \lambda^{P} \\
\frac{1}{2} \epsilon_{MNPab} \lambda^{P}
\end{array} \right\} J^{MN}
\]

\[ + \left\{ \Gamma^a_{ab} + \frac{1}{6} R_{MN} \Gamma^{MN}_{ab} \lambda^{b} \\
+ \lambda \Gamma^a_{ab} R^b + \frac{1}{2} \lambda_{MN} \Gamma^{MN}_{ab} R^b
\end{array} \right\} Q_a . \]

The \( SO(1, 3) \)-decomposition results in

\[ R(P)^m = dE^{mn} \psi^m - 2 E^{mn} \psi^n + 2 \psi^m - 2 E^{mn} n^m \]

\[ - 2 E^{mn} n^m - \frac{1}{4 \sqrt{2}} \gamma^m \psi \]

\[ R(E)^m = dE^{mn} - 2 \omega^{[m} n^{k]} + 2 h E^{mn} + 2 \epsilon^{mnpq} \phi_{pq} \\
+ \frac{1}{2} \psi^m \phi \]

\[ R(Q) = d\psi + \left( - a \gamma^5 + b + \frac{1}{2} \omega^{m} \gamma_m \right) \psi
\]

\[ - \epsilon^{m} \gamma_m + z^m \gamma_m \psi
\]

\[ + \left( \sqrt{2} \epsilon^{m} \gamma_m + \frac{1}{2} \psi^{m} \phi \right) \phi \]

\[ R(M)^m = dE^{mn} - \omega^{[m} \omega^{n]} + 4 \epsilon^{m} \psi^{n} + 4 \frac{2}{2} \left( \psi^n \right)
\]

\[ - 8 \epsilon^{[m} \psi^{n]} - 8 \epsilon^{m} \psi^{n]}
\]

\[ + \frac{1}{4} \psi^{m} \phi \]

\[ R(D) = db - 2 E^{m} f_m - E^{m} F_m
\]

\[ + \frac{1}{4} \psi^{m} \phi \]

\[ R(A) = da - 2 E^{m} z_m - E^{m} F_m
\]

\[ + \frac{1}{4} \psi^{m} \phi \]

\[ R(V)^m = dE^{m} + \omega^{m} \psi + 2 z^m \psi
\]

\[ + 2 E^{m} f_m + E^{m} F_m
\]

\[ + \frac{1}{4} \psi^{m} \phi \]

\[ R(Z)^m = dE^{m} + \omega^{m} z_m - 2 v^m \psi
\]

\[ + 2 E^{m} f_m + E^{m} F_m
\]

\[ + \frac{1}{4} \psi^{m} \phi \]

\[ R(S) = d\phi + \left( a \gamma^5 + b + \frac{1}{2} \omega^{m} \gamma_m \right) \phi
\]

\[ - \epsilon^{m} \gamma_m + z^m \gamma_m \phi
\]

\[ + \left( - \sqrt{2} \epsilon^{m} \gamma_m + \frac{1}{2} \psi^{m} \phi \right) \phi \]

\[ R(K)^m = dE^{m} + \omega^{m} \psi + 2 z^m \phi
\]

\[ + 2 E^{m} f_m - E^{m} z_m
\]

\[ + \frac{1}{4} \psi^{m} \phi \]

\[ R(F)^m = dE^{m} - \omega^{m} \psi + 2 \omega^{mn} [F_n]^k - 2 b F^{mn}
\]

\[ + 2 \epsilon^{mnpq} f_{pq} + 2 \psi^{m} \phi \]

The duals \( E^{m} \) are defined by \( \psi^{m} \equiv (1/2) \epsilon^{mnpq} X_{pq} \), \( \bar{X}^{mn} = - X^{mn} \) and the bars on the Majorana fermions are defined by \( \bar{\psi} = \psi^\gamma C_4 = \psi^\gamma \gamma^0 \).

In order to obtain the curvatures of a \( SU(2, 2|1) \)-gauge theory, we simply drop the fields \( E_{mn}, \nu_m, z_m \) and \( F_{mn} \) and change the fermionic curvatures to

\[ ^{2} \text{We remind the reader that in Minkowski space.} \]
$R(Q) = d\psi + (3\alpha\gamma^5 + b + \frac{1}{4}\omega^{mn}\gamma_{mn})\psi$
$+ \sqrt{2}e^m\gamma_m\phi$  \hspace{1cm} (36)
$R(S) = d\phi - (3\alpha\gamma^5 - b + \frac{1}{4}\omega^{mn}\gamma_{mn})\phi$
$- \sqrt{2}f^m\gamma_m\psi$ .  \hspace{1cm} (37)

The only difference to (27) and (28) is the additional factor $-3$ in the terms $a\gamma^5\psi$ and $a\gamma^5\phi$.

3. Actions

We now construct an affine action quadratic in curvatures and invariant under the symmetries $S, K_m$ and $F_{mn}$. By affine we mean that no vierbeins are used to contract indices, but only constant Lorentz tensors such as $\epsilon^{\mu\nu\rho\sigma}$, $\eta_{\mu\nu}$ and Dirac matrices. The most general parity-even, Lorentz-invariant, dilaton-weight zero, mass dimension zero affine action ($S = \int_M \mathcal{L}$ for some four-manifold $M$) reads

\[-\mathcal{L} = \alpha_0\epsilon_{mnpq}R(M)^{mn}R(M)^{pq} + \alpha_1R(A)R(D) + \alpha_2R(V)^mR(Z)_m + \alpha_3\epsilon_{mnpq}R(E)^{mn}R(F)^{pq} + \beta R(Q)\gamma^5 R(S) .\]  \hspace{1cm} (38)

This action is, of course, manifestly general coordinate invariant since the integration measure $\epsilon^{\mu\nu\rho\sigma}$ is a tensor density under general coordinate transformations. The term $\alpha_2 R(V)^mR(Z)_m$ is not $A$-invariant like all the other terms and one could therefore consider setting the coefficient $\alpha_2 = 0$ already at this point. Since we are interested in a theory of gravity we set $\alpha_0 = 1$ (in fact, no nontrivial solution exists for $\alpha_0 = 0$).

The requirement that the action in (38) be invariant under the symmetries $S, K$ and $F$ yields $\alpha_0 = 1, \alpha_1 = -32, \alpha_2 = 0, \alpha_3 = 8, \beta = -8,$ \hspace{1cm} (39)
as well as the following constraints on the field strengths:

$R(P)^m = 0$  \hspace{1cm} (40)
$R(E)^{mn} = -R(E)^{mn}$  \hspace{1cm} (41)
$R(Z)^m = R(V)^m$  \hspace{1cm} (42)
$R(Q) = -\gamma^5 R(Q) , \hspace{1cm} (43)$

which are in turn invariant under $S, K$ and $F$. The sign on the right hand side is in principle at our disposal. The above choice guarantees that the constraints can be solved algebraically if the vierbein is assumed to be invertible.

In order to compare with the $SU(2,2|1)$-case we again write down the most general parity even, dilaton weight zero, affine action and fix the coefficients and constraints by requiring invariance with respect to the symmetries $K$ and $S$. The results are

\[-\mathcal{L} = \epsilon_{mnpq}R(M)^{mn}R(M)^{pq} + 32R(A)R(D) - 8R(Q)\gamma^5 R(S) \]  \hspace{1cm} (44)

with the constraints

$R(P)^m = 0$  \hspace{1cm} (45)
$R(Q) = -\gamma^5 R(Q) \hspace{1cm} (46)$
$R(A) = R(D) . \hspace{1cm} (47)$

These constraints are again invariant under the symmetries $S$ and $K$ and algebraically solvable. We note that the actions (42) and (43) are the most complicated ones in a series of gauge theories covering Anti-de-Sitter gravity based on $Sp(4)$, its supersymmetrized $Osp(1|4)$-version and of course ordinary conformal gravity based on $SU(2,2)$. The complexity of the constraints increases with the size of the algebra, however in each case, a kinematical study of the gauge algebra shows that the constraints are exactly such that gauge transformations $\delta h = d\lambda + [h, \lambda]$ are modified precisely so that the gauge algebra closes onto general coordinate transformations, rather than $P^m$ gauge transformations (i.e. gauge transformations generated by the translation generator $P_m$) (16) In fact one can adopt a purely kinematical approach in which one derives the constraints through the requirement that the algebra closes onto general coordinate transformations. If one makes a similar kinematical study of the $Osp(1|8)$ algebra, one is quickly led to the conclusion that no set of algebraically solvable con-

\footnote{The action is Hermitean and the curvatures are real if one takes the reality condition for Majorana spinors $\psi = \psi^\dagger C_4 \psi = \psi^\dagger i\gamma^5 \psi$. We note the left hand side of the Minkowski action in (45) by $-\mathcal{L}$ to stress that we are using the metric ($\gamma^5 \gamma^5$) rather than the Euclidean notation of (42). The sign $-\mathcal{L}$ ensures that the kinetic terms for the vierbein have the correct sign, see, for example, reference [34].}
constraints exists such that the algebra closes onto general coordinate transformations. However, in
the hope that the model could again be made consistent through further generalizations of the \( E_{mn} \)
gauge symmetries and super gauge symmetries \( (Q) \) along with the usual trade between \( P_m \) gauge
transformations and general coordinate transformations, we followed the dynamical affine action
approach which has enjoyed considerable success as evidenced by the string of models given above.

4. Constraints

The constraints \([13] - [15]\) are necessary but not yet sufficient for obtaining conformal super-
gravity. For an irreducible representation of the conformal superalgebra we should try and express
as many fields as possible algebraically in terms of a minimal set. In the conformal case, the maximal
set of solvable constraints is

\[
0 = R(P)_{\mu\nu}^m \\
0 = \bar{R}(M)^{mn}\epsilon_{mn} + 2R(A)\epsilon^m \\
- \frac{1}{2\sqrt{2}} \gamma^5 \gamma^m R(Q),
\]

as well as

\[
\gamma^\mu R(Q)_{\mu\nu} = 0 ,
\]

which can be shown \([17]\) to be necessary for \( Q \)-supersymmetry of the action \([14]\).

For the \( Osp(1|8) \)-case, we summarize in figure 1 the curvature components that, when constrained to zero, lead to algebraic equations for connection pieces. We have found the following maximal set of solvable constraints:

\[
0 = R(P)_{\mu\nu}^m \\
0 = R(E)_{\rho[\mu\nu]}^\rho \\
0 = R(E)_{\mu\nu}^\rho \\
0 = \epsilon^{\mu\nu\rho\sigma} R(E)_{\mu\nu\rho\sigma} \\
0 = R(E)_{\mu\nu}^{mn} + \ast R(E)_{\mu\nu}^{mn} \\
0 = R(Z)_{\mu\nu}^m - \ast R(V)_{\mu\nu}^m \\
0 = \gamma^\mu R(Q)_{\mu\nu} \\
0 = R(M)_{\rho\mu}^\rho - \frac{1}{2} \theta_{\rho\mu} R(M)_{\rho\sigma}^\sigma \\
+ 2 \ast R(A)_{\mu\nu} + \frac{1}{2\sqrt{2}} R(Q)_{\rho\nu} \gamma^\rho \psi^\mu \\
\]

\[
R(P)_{\mu\nu}^m = 24 = 16 + 4 + 4 \\
R(E)_{\mu\nu}^{mn} = 36 = 1 + 10 + 9 + 9 + 6 + 1 \\
R(Q)_{\mu\nu} = 24 = 8 + 12 + 4 \\
R(M)_{\mu\nu}^{mn} = 36 = 1 + 10 + 9 + 9 + 6 + 1 \\
R(D)_{\mu\nu} = 6 \\
R(V)_{\mu\nu}^m + \ast R(Z)_{\mu\nu}^m = 24 = 16 + 4 + 4 \\
R(V)_{\mu\nu}^m - \ast R(Z)_{\mu\nu}^m = 24 = 16 + 4 + 4
\]

Figure 1. Lorentz irreducible pieces of the “solvable” curvatures. The ticks “\( \sqrt{\cdot} \)” and crosses “\( \times \)” indicate those Lorentz irreducible pieces of curvatures that may or may not, respectively, be solv-
ably constrained.

\[-2 \ast R(E)_{\rho\sigma\mu} z^{\rho\sigma} - 2R(E)_{\rho\sigma\mu} v^{\rho\sigma} \\
+ 2R(V)_{\rho\sigma\mu} E^{\rho\sigma} \mu + 2R(Z)_{\rho\sigma\mu} \bar{E}^{\rho\sigma} \mu . \tag{58}\]

All further constraints follow from this set, either algebraically or, for example, by Bianchi identities.
Unlike the superconformal case, this set does not guarantee all the symmetries necessary for
consistency of the action \([38]\).

5. Problems

The set \([51] - [58]\) of constraints does not allow us to express explicitly all fields in terms of
a minimal set. Rather, we obtain a coupled set of equations, which determine, say, \( \omega_{\mu}^{mn}, F_{\mu}^{mn}, \phi_{\mu}^a, z_{(\mu\nu)}, v_{(\mu\nu)} \) and \( z_{[\mu\nu]}, v_{[\mu\nu]} \) in terms of \( \epsilon^m, E_{\mu}^{mn}, \psi_{\mu}, b_{\mu}, a_{\mu} \) and \( z_{[\mu\nu]}, -v_{[\mu\nu]} \). We may try to solve these equations iteratively, but in order to investigate the symmetries of the action, an explicit solution is not necessary. However, then the same problem occurs when we examine the invariance of the action under the remaining symmetries \( V, Z, E \) and \( Q \). These symmetries need to be modified when acting on dependent fields such that they leave \([51] - [58]\) invariant, and
again the constraints provide only a coupled set of equations for the extra transformations of the dependent fields. In order to calculate further, one can make a consistent expansion in the number of fields and study the model in the lowest order in this expansion.

Let us consider the constraint $R(P)^m = 0$ and some $\delta \in \{V, Z, E, Q\}$. On independent fields $\delta$ acts simply as a gauge transformation

$$\delta h^A_{\text{indept.}} = \delta A^A + \epsilon_C h^B f_{BC} = \delta A^A_{\text{indept.}}.$$

However acting on dependent fields we have

$$\delta h^A_{\text{dept.}} = \delta A^A + \epsilon_C h^B f_{BC} + \delta h^A_{\text{dept.}}.$$

where the extra transformations $\delta h^A_{\text{dept.}}$ are determined by requiring that the constraints are invariant under $\delta$, for example

$$0 = \delta R(P)^m = \delta A^A_{\text{dept.}} + \delta V^mn c_n - 2 E^{mn} \delta v_n - 2 E^{mn} \delta z_n.$$

Note that in [61], the extra transformations of three dependent fields appear, so that neither is uniquely determined. In contrast, for conformal supergravity only $\delta v^mn$ is present and can then be determined. One may write down similar expressions for all other constraints which in principle uniquely determine all extra transformations of the dependent fields.

In practice, to write down a solution for the extra transformations of the dependent fields, we solve the set of coupled equations for the extra transformations iteratively. Namely, we make an expansion order by order in the number of independent fields, where one counts the vierbein as a Kronecker delta (i.e. field number zero). This means that to first order, we ignore the terms $-2 E^{mn} \delta v_n - 2 E^{mn} \delta z_n$ in [61]. To linear order one then obtains

$$e^n \hat{\delta}_V \hat{\omega}_{mn} = -2 R(E)_{mn} e^n$$

$$e^n \hat{\delta}_Z \hat{\omega}_{mn} = 2 R(E)_{mn} e^n$$

$$e^m \hat{\delta}_\gamma \hat{\omega}_{mn} = -\frac{1}{\sqrt{2}} \gamma_m R(Q) e^n$$

$$e^m \hat{\delta}_Z \hat{\gamma} \phi = \frac{1}{\sqrt{2}} \gamma_m R(Q) e^n$$

and our action is then indeed invariant under $V$ and $Z$. However, at linear order, the action is not invariant under $E_{mn}$ and $Q$ symmetries although many terms do cancel. The $E$-transformations leading to non-vanishing variations are

$$\hat{\delta}_E \phi_\mu = -\frac{1}{4} \left[ \gamma^\rho \gamma^\mu \gamma^\sigma R(S)_{\rho\sigma} + \frac{1}{6} \gamma_\nu \gamma^\mu \gamma^\sigma R(S)_{\rho\sigma} \right] \epsilon_{mn}$$

$$\hat{\delta}_E \omega^0_{\mu
u} = R(V)_{mn} \epsilon_{\mu\nu} - R(V)_{\mu\nu} \epsilon_{mn} + R(V)_{\mu\nu} \epsilon_{mn} + R(Z)_{mn} \epsilon_{\mu\nu} - R(Z)_{mn} \epsilon_{\mu\nu}$$

$$\hat{\delta}_E \omega_{\mu} = 2(1 - \beta) \epsilon_{mn} R(Z)^n + \epsilon_{mn} R(V)^n$$

while the relevant extra $Q$-transformations read

$$\hat{\delta}_Q \phi_\mu = \frac{1}{3} \sqrt{2} \left[ R(D)_{\mu\nu} + R(A)_{\mu\nu} \gamma^5 \right]$$

$$\hat{\delta}_Q \phi_\mu = \frac{1}{2} (R(V)_{\mu\nu} + R(Z)_{\mu\nu} \gamma^5 \gamma_{\mu}) \gamma^\rho \epsilon$$

$$\hat{\delta}_Q \phi_\mu = \frac{1}{2} \sqrt{2} \left[ R(D)_{\mu\nu} \right]$$

This means the action [68] is not consistent. In a flat gravitational and otherwise trivial background the fields $\psi^\alpha$ and $E_{mn}$ enter the quadratic part of the action only in terms of their linearized field strengths $db_\mu$ and $E_{mn}$, hence the associated gauge invariances are necessary for obtaining invertible kinetic terms. Since they do not survive at the interacting level, we conclude that the theory does not exist in the way we have for-
mulated it, unless one can find generalizations of $Q$ and $E_{mn}$ symmetries under which the action is invariant.

6. Conclusions

Even though $Osp(1|8)$ seems to fit naturally into a pattern of (super)gravity theories in $d=(1,3)$, the affine action has serious deficiencies. Since the affine action does not suffice either for theories with a higher number of supersymmetries, we may speculate that one should add an appropriate number of non-gauge fields. This is also borne out by the spectrum of (conformal) supergravities in high dimensions. At this point it is not clear precisely what we should add and how one systematically derives then appropriate constraints. Work on these issues is in progress.

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