σ-fields x Chiral Scalars in Nuclear Three Body Potentials

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The scalar-isoscalar field of an effective chiral Lagrangian transforms differently in either linear or non-linear frameworks: in the former case it is the counterpart of the pion whereas in the latter it is chiral invariant on its own. We compare the predictions from these two models for nucleon interactions and find results which are identical for two-body and rather different for three-body potentials. Some qualitative features of three-body interactions are discussed.

I. INTRODUCTION

Since the fundamental work of Yukawa, the meson exchange theory has been successfully applied to the description of the nucleon-nucleon force. Nowadays it is well established that the one pion exchange potential (OPEP) is dominant at large distances, yielding a strong force with spin and isospin dependences.

The next layer of the interaction is associated with the exchange of two pions. This system has the lightest mass beyond the OPEP and involves an intermediate pion-nucleon (πN) amplitude in a kinematical region which is not directly accessible to experiment. Therefore a proper theoretical treatment of this component of the potential requires the use

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of information based on dispersion relations, as pointed out long ago by Cottingham and VinMau [1]. The implementation of this idea led to the Paris potential [2], which is rather successful in describing experimental data.

An important feature of the $\pi N$ amplitude is chiral symmetry, as emphasized more than 25 years ago by Brown and Durso [3]. Recently the application of chiral symmetry to the NN interaction has deserved much attention, initially in the restricted framework of pion and nucleon degrees of freedom [4], and nowadays it is fair to say that the theoretical formulation of this sector of the interaction is free of ambiguities. However, a Lagrangian containing only pions and nucleons cannot reproduce low-energy $\pi N$ data and hence this very reliable part of the model has to be complemented [5,6]. The combination of chiral symmetry and experimental information about the intermediate $\pi N$ amplitude, based on dispersion relations, was brought into this problem in the last two years [6], with a successful description of asymptotic $NN$ scattering data [7,8].

In contrast with the OPEP, the two-pion exchange potential (TPEP) produces an attractive interaction that depends little on spin and isospin, because it is dominated by the exchange of a scalar-isoscalar system. In many phenomenological potentials, the actual two-pion dynamics is simulated by the exchange of an effective scalar-isoscalar meson, with a mass around 550 MeV. In the framework of chiral symmetry, this meson is usually identified with the counterpart of the pion in the linear sigma-model and one obtains a prediction for its coupling constant to the nucleon. The existence of a scalar-isoscalar meson is controversial, the main candidate being the $f_0(400-1200)$, that may be present in $\pi\pi$ scattering [9]. In the case of NN scattering, the TPEP is quite well accounted for by non-linear chiral dynamics, constrained by experimental information, and the scalar-isoscalar meson is unnecessary. According to the unwritten law of quantum mechanics, stating that processes which are not forbidden are compulsory, the exchange of two pions must be considered in any realistic description of the NN interaction and the use of the sigma field to simulate the actual TPEP gives rise to shortcomings. In particular, a scalar field with mass $m_s$, yields a central $NN$ potential proportional to $e^{-m_s r}/r$, whereas the spatial dependence of TPEP is
closer to \((e^{-\mu r}/r)^2\), \(\mu\) being the pion mass [6,8]. Moreover, the TPEP is proportional to \(g^4\), where \(g\) is the \(\pi N\) coupling constant, whereas the exchange of a sigma is proportional to \(g^2\). In spite of all these problems, the use of an effective scalar field may be useful in problems where simplicity is more important than precision. As far as the former is concerned an effective scalar field allows calculations at tree level, whereas the exchange of two pions involves loop integrations.

If one is willing to use an effective scalar field in a calculation, there are two possibilities at hand. The first one consists in employing the usual sigma field of the linear model, which is the chiral partner of the pion. The other possibility is to use a scalar field in the framework of non-linear Lagrangians, which is chiral invariant and appears naturally when the non-linear fields are obtained from the linear ones [10]. These two scalars fields couple differently to pions and nucleons and hence lead to predictions which do not overlap for some specific processes. The main purpose of this work is to explore these predictions in the case of three-body forces. Our presentation is divided as follows: in sect. 2 we compare linear and non-linear Lagrangians and in sect 3 we motivate the physical predictions of the latter. In sect. 4 we derive the kernel of the three-body force, which is fully developed in sect. 5. Finally, in sect 6 we discuss qualitatively some features of the force associated with sigmas and chiral scalars.

II. LAGRANGIANS

In this section we introduce the Lagrangians describing both the sigma and the chiral scalar. In the framework of linear dynamics the field \(\sigma\) is the chiral partner of the pion field \(\pi\) since, for an axial transformation \(\delta^A\), we have \(\delta^A \pi \to \sigma\) and \(\delta^A \sigma \to -\pi\). In the case of non-linear dynamics, the pion field is represented by \(\phi\) and we also consider a new field \(S\), which corresponds to a generalization of the field \(\sigma'\), introduced long ago by Weinberg [10]. The corresponding axial transformation are \(\delta^A \phi = F(\phi^2)\), where \(F(\phi^2)\) is a function of \(\phi^2\) and \(\delta^A S = 0\). The last transformation implies that \(S\) is a chiral scalar.
The linear Lagrangian $\mathcal{L}_\sigma$ has the usual form

$$\mathcal{L}_\sigma = \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \cdot \partial^\mu \pi \right) - \frac{\mu^2}{2} \left( \sigma^2 + \pi^2 \right) + \frac{1}{2} f_\pi \mu^2 \sigma + U \left( \sigma^2 + \pi^2 \right)$$

$$+ \bar{N} i \beta N - g \bar{N} \left( \sigma + i \tau \cdot \pi \gamma_5 \right) N$$  \hspace{1cm} (1)

where $N$ is the nucleon field that transforms linearly, $f_\pi$ is the pion decay constant, $\mu$ is the pion mass, $g$ is the $\pi N$ coupling constant and the term $U \left( \sigma^2 + \pi^2 \right)$ represents self interactions of the mesonic fields. In the framework of the linear sigma model the scalar fluctuations are associated with a field $\epsilon$, related to $\sigma$ by

$$\sigma = f_\pi + \epsilon.$$  \hspace{1cm} (2)

Replacing this into the Lagrangian (1), we obtain

$$\mathcal{L}_\sigma = \frac{1}{2} \left( \partial_\mu \epsilon \partial^\mu \epsilon - m_\epsilon^2 \epsilon^2 \right) + \frac{1}{2} \left( \partial_\mu \pi \cdot \partial^\mu \pi - \mu^2 \pi^2 \right) + U \left( \left( f_\pi + \epsilon \right)^2 + \pi^2 \right)$$

$$+ \bar{N} i \beta N - g \bar{N} \left( f_\pi + \epsilon + i \tau \cdot \pi \gamma_5 \right) N.$$  \hspace{1cm} (3)

In the non-linear approach, the Lagrangian $\mathcal{L}_S$ is written as

$$\mathcal{L}_S = \left[ \frac{1}{2} \left( \partial_\mu S \partial^\mu S - m_s^2 S^2 \right) - U \left( S \right) \right] + \left[ \frac{1}{2} \left( \partial_\mu \phi \cdot \partial^\mu \phi + \partial_\mu f \partial^\mu f \right) + f_\pi \mu^2 f \right] + \mathcal{L}_N.$$  \hspace{1cm} (4)

where $f$ corresponds to the function $f = \sqrt{f_\pi^2 - \phi^2}$. Formally, $U(S)$ describes the self interactions of the scalar field and $\mathcal{L}_N$ represents both the nucleon sector and its interactions with bosonic fields. This part of Lagrangian may be cast in many different forms, two of which are widely employed in the literature. In one of them, the pion-nucleon coupling is pseudo-vector (PV) and, in the other, it is pseudo scalar (PS). For PV coupling, one has

$$\mathcal{L}_{N}^{PV} = \bar{\psi} i \gamma_\mu D^\mu \psi - m \bar{\psi} \psi + \frac{g}{2m} \bar{\psi} \gamma_\mu \gamma_5 \tau \psi \cdot D^\mu \phi - g_s S \bar{\psi} \psi,$$  \hspace{1cm} (5)

where $\psi$ is the nucleon field that transforms non-linearly, $m$ is its mass and $g_s$ represents the coupling of the nucleon to the scalar. In this expression the pion and nucleon covariant derivatives are given by \[\text{[Ref]}\]
\[ D^\mu \phi = \partial_\mu \phi - \frac{1}{f + f_\pi} \partial^\mu f \phi, \quad (6) \]

\[ D^\mu \psi = \left[ \partial^\mu + i \frac{1}{f_\pi (f + f_\pi)} \frac{\tau}{2} \cdot (\phi \times \partial^\mu \phi) \right] \psi. \quad (7) \]

This Lagrangian has been used recently in the study of \( \pi N \) form factors in constituent quarks models \[12\] and it is worth noting that its last term describes a chiral invariant coupling between the scalar and the nucleon.

In the case of \( PS \) coupling, one has

\[ \mathcal{L}^{PS}_N = \bar{N} i \gamma \beta N - g \bar{N} (f + i \tau \cdot \phi \gamma_5) N - \frac{g_s}{f_\pi} \bar{N} (f + i \tau \cdot \phi \gamma_5) N, \quad (8) \]

where \( N \) is the nucleon field that transforms linearly. The last term of this expression has the same meaning as the corresponding one in eq. (5).

On general grounds one knows that, in the framework of chiral symmetry, results should not depend on the choice of \( \mathcal{L}_N \) \[13,14\]. The equivalence between the PS and PV Lagrangians was verified explicitly in the case of TPEP \[6\] and of pion-nucleon interactions in constituent quark models \[12\]. Because of this equivalence and of the similarity with the linear Lagrangian, our discussions are set in PS case. Thus our complete non-linear Lagrangian is written as

\[ \mathcal{L}^{PS}_S = \frac{1}{2} \left( \partial_\mu S \partial^\mu S - m_\pi^2 S^2 \right) + \frac{1}{2} \left( \partial_\mu \phi \cdot \partial^\mu \phi + \partial_\mu f \partial^\mu f \right) + f_\pi \mu^2 f + U (S) \]

\[ + \bar{N} i \gamma \beta N - \left( g + \frac{g_s}{f_\pi} S \right) N (f + i \tau \cdot \phi \gamma_5) N. \quad (9) \]

The interaction terms in eqs (3) and (8) have the same scalar-nucleon and pion-nucleon vertices depicted in fig. 1a. However, the non-linear approach also yields extra vertices, representing seagull scalar-pion-nucleon and pion-pion-nucleon interactions, among others, displayed in fig. 1b. These results indicate that, at tree level, both Lagrangians produce the same \( NN \) potential. On the other hand the extra scalar-pion-nucleon vertex represents a genuine difference between the two approaches and has consequences in processes such as pion absorption by a two-nucleon system or three body forces.
III. EFFECTIVE SEAGULL

The purpose of this section is to discuss the meaning of the effective seagull interaction in terms of more basic processes. In order to do this, we note that, in the framework of non-linear Lagrangians, the TPEP in the pure nucleon sector is given by the five diagrams of fig. 2a, which we may want to associate with an effective scalar exchange. When an external pion is attached to these processes, we have the three possibilities indicated in fig. 2b, the last one corresponding to an effective seagull, whose dynamical meaning is given in fig 2c.

The nine diagrams associated with the effective seagull interaction can be understood as arising from the product of the amplitudes $T^{ba}$ and $P^{bca}$, where the former describes the pion-nucleon scattering $\pi^a(k) N(p) \to \pi^b(k') N(p')$ and latter represents the contribution from pion production, $\pi^a(k) N(p) \to \pi^b(k') \pi^c(q) N(p')$, as indicated in fig 3. The composite amplitude is denoted by $A$ and given by

$$A = -i \int \frac{d^4Q}{(2\pi)^4} T^{ba} P^{bca} \frac{1}{k^2 - \mu^2} \frac{1}{k'^2 - \mu^2},$$

where $Q = \frac{1}{2}(k - k')$.

Using the $\pi N$ vertices obtained from eq. (9), we have

$$P^{bca} = ig^3 \left\{ (\delta_{ab}\tau_c - \delta_{bc}\tau_a - \delta_{ac}\tau_b + i\epsilon_{bac}) \right. \times \bar{u} \left[ \begin{array}{c} \frac{k}{(p' + k')^2 - m^2} \gamma_5 \frac{k}{(p + k)^2 - m^2} \\ \frac{k'}{(p' + k')^2 - m^2} \gamma_5 \frac{1}{g f_\pi} \\ \delta_{ac}\tau_b \frac{k}{(p' + k')^2 - m^2} \gamma_5 \\ \delta_{bc}\tau_a \frac{1}{g f_\pi} \frac{k}{(p + k)^2 - m^2} \gamma_5 \end{array} \right] u.$$\hspace{1cm}(11)

The part of $A$ corresponding to a scalar-isoscalar exchange is associated with the factor $\delta_{ab}$ in the $\pi N$ amplitude and hence it is proportional to

$$\delta_{ab} P^{bac} = ig^3 \tau_c \bar{u} \left[ \begin{array}{c} \frac{k'}{(p' + k')^2 - m^2} \gamma_5 \frac{k}{(p + k)^2 - m^2} \\ \frac{k'}{(p' + k')^2 - m^2} \gamma_5 \frac{1}{g f_\pi} \frac{k}{(p + k)^2 - m^2} \gamma_5 \\ -\frac{k'}{(p' + k')^2 - m^2} \frac{1}{g f_\pi} - \frac{k}{(p + k)^2 - m^2} \gamma_5 \end{array} \right] u.$$\hspace{1cm}(12)
In order to identify the main contribution to this amplitude, we go to the soft pion limit, by using \( k = k' = (k_0, 0) \) and then take the limit \( k_0 \to 0 \). We thus obtain
\[
\delta_{ab} P^{bca}_{\text{soft}} \to -\frac{ig^3}{4m^2} \tau_c \bar{u} \gamma_5 u,
\]
which has the same structure as the effective scalar-nucleon seagull vertex predicted by the non-linear Lagrangian given by (9).

IV. EFFECTIVE \( \pi NN \) VERTEX

As far as NN interactions are concerned both the linear and non-linear Lagrangians, given by eqs. (3) and (9), yield potentials at tree level, which are due to the exchanges of one pion and one scalar meson associated with the diagrams of fig. 1a. In principle the scalar coupling constants predicted by these Lagrangians could be different. However, we are interested only in dynamical distinctions between the sigma and the chiral scalar and, from now on, we set \( g_s = g \) in order to force both models to simulate exactly the same two-body potential.

On the other hand, these models produce different predictions for the effective \( \pi NN \) vertex and in this section we consider the form of this interaction. The basic ingredients are the amplitudes \( T_\sigma \) and \( T_s \), that describe the processes \( \pi N \to \sigma N \) and \( \pi N \to SN \) respectively. The amplitude \( T_\sigma \) is determined by just the first two diagrams of fig. 4a whereas \( T_s \) also includes the seagull term. As the calculations of \( T_\sigma \) and \( T_s \) are quite similar, we denote both amplitudes by \( T \) and identify the seagull contribution by a parameter \( \lambda \), such that \( T_\sigma = T (\lambda = 0) \) and \( T_s = T (\lambda = 1) \). At tree level, \( T \) is given by the diagrams depicted in fig 4a, and we have
\[
T (\lambda) = -ig^2 \tau_c \bar{u} (p') \left[ \frac{\not{p}_d + m}{p_d^2 - m^2} \gamma_5 + \gamma_5 \frac{\not{p}_x + m}{p_x^2 - m^2} + \frac{\lambda}{g f_\pi} \gamma_5 \right] u (p) .
\]
with \( p_d = p + k ; p_x = p' - k \). Using Dirac equation, we rewrite (14) as
\[
T (\lambda) = -ig^2 \tau_c \bar{u} (p') \left[ \frac{k}{p_d^2 - m^2} + \frac{k}{p_x^2 - m^2} + \frac{\lambda}{g f_\pi} \right] \gamma_5 u (p) .
\]
It is worth pointing out that, as shown in [12], this result does not depend on our choice of the $PS$ coupling in non-linear Lagrangian and it would be the same if the $PV$ scheme were adopted.

This amplitude contains positive frequency states, which do not enter in the construction of the proper $\pi NN$ kernel. In order to isolate these contributions, we write the nucleon propagator as

$$\frac{\hat{p} + m}{p^2 - m^2} = \frac{1}{2E} \left[ \frac{1}{p^0 - E} \sum_s u^s(p) \bar{u}^s(p) + \frac{1}{p^0 + E} \sum_s v^s(-p) \bar{v}^s(-p) \right], \tag{16}$$

where $E = \sqrt{p^2 + m^2}$. Thus

$$\left[ \frac{\hat{p} + m}{p^2 - m^2} \right]_{(+)} = \frac{1}{2E} \left[ \frac{\hat{p} + m}{p^0 - E} - \gamma^0 \right] \tag{17}$$

and the positive energy contribution is

$$T_{(+)} = -ig^2 \tau_c \bar{u}(p') \left[ -\frac{k}{2E_d(p_d^0 - E_d)} + \frac{k}{2E_x(p_x^0 - E_x)} \right]$$

$$- \left( \frac{1}{2E_d} - \frac{1}{2E_x} \right) \gamma^0 \gamma_5 u(p), \tag{18}$$

with $\tilde{p}_i = (E_i, \tilde{p}_i)$ for $i = d, x$.

Subtracting (18) from (15) we get $\bar{T}$, the irreducible positive frequency amplitude

$$\bar{T}(\lambda) = -ig^2 \tau_c \bar{u}(p') \left[ -\frac{k}{2E_d(p_d^0 + E_d)} - \frac{k}{2E_x(p_x^0 + E_x)} \right]$$

$$+ \left( \frac{1}{2E_d} - \frac{1}{2E_x} \right) \gamma^0 + \frac{\lambda}{gf_\pi} \gamma_5 u(p). \tag{19}$$

This allows the proper kernel for the process $\pi N_j N_k$ as shown in fig 4b, to be written as

$$K(\lambda) = -ig^3 \left\{ \tau_c \bar{u}(p') \left[ -\frac{k}{2E_d(p_d^0 + E_d)} - \frac{k}{2E_x(p_x^0 + E_x)} \right] \right. \left. \right.$$\right.$$

$$\left. + \left( \frac{1}{2E_d} - \frac{1}{2E_x} \right) \gamma^0 + \frac{\lambda}{gf_\pi} \gamma_5 u(p) \right\}^{(j)}$$

$$\times \frac{1}{q^2 - m_*^2} [\bar{u}(p') u(p)]^{(k)}, \tag{20}$$

where $q = p'_2 - p_2$. 
V. THREE BODY FORCE

In this section we derive the three-nucleon force due to the simultaneous exchanges of one pion and one effective scalar meson and the basic diagram is shown in figure 5. It produces the following three-nucleon amplitude

\[ T_{ijk}(\lambda) = g^4 r^{(i)} \cdot r^{(j)} [\bar{u}(p') \gamma_5 u(p)]^{(i)} \frac{1}{k^2 - \mu^2} \times \left\{ \begin{array}{l}
\bar{u}(p') \left[ -\frac{k}{2E_d (p'_d + E_d)} - \frac{k}{2E_x (p'_d + E_x)} \\
+ \left( \frac{1}{2E_d} - \frac{1}{2E_x} \right) \gamma^0 + \frac{\lambda}{g f_\pi} \right] \gamma_5 u(p) \end{array} \right\} \times \frac{1}{q^2 - m_s^2} [\bar{u}(p') u(p)]^{(k)}. \]  

(21)

In order to obtain the non-relativistic limit of \( T_{ijk}(\lambda) \), denoted by \( t_{ijk}(\lambda) \), we use

\[ \frac{1}{2E_d} - \frac{1}{2E_x} \sim \frac{p^2}{m^3}, \]
\[ \frac{1}{2E_d (p'_d + E_d)} + \frac{1}{2E_x (p'_d + E_x)} \sim \frac{1}{2m^2}, \]

\[ \bar{u}(p') u(p) \to I, \]
\[ \bar{u}(p') \gamma_5 u(p) \to \frac{1}{2m} \sigma \cdot (p - p'), \]
\[ \bar{u}(p') \gamma^0 \gamma_5 u(p) \to \frac{1}{2m} \sigma \cdot (p + p'), \]
\[ \bar{u}(p') \gamma_5 u(p) \to -\sigma \cdot k, \]
\[ \frac{1}{k^2 - \mu^2} \approx -\frac{1}{k^2 + \mu^2}, \]
\[ \frac{1}{q^2 - m_s^2} \approx -\frac{1}{q^2 + m_s^2}, \]

where the arrows indicate that the normalization of the spinors were also changed. Using these results and keeping only terms of the order \( p/m \), we get the non-relativistic amplitude

\[ t_{ijk}(\lambda) = \frac{g^4}{(2m)^2} r^{(i)} \cdot r^{(j)} \sigma^{(i)} \cdot k \frac{1}{k^2 + \mu^2} \times \sigma^{(j)} \cdot \left[ \left( \frac{1}{m} - \frac{\lambda}{g f_\pi} \right) k + \frac{\lambda}{g f_\pi} q \right] \frac{1}{q^2 + m_s^2}. \]  

(22)
where $k = p_i - p'_i$ and $q = p'_k - p_k$. The full non-relativistic amplitude $t_{3N}$ is given by the permutation over all indices $ijk$.

In momentum space the three body potential $W$ is defined by

$$
\langle p'_1p'_2p'_3 | W | p_1p_2p_3 \rangle = -(2\pi)^3 \delta^3 \left( p'_1 + p'_2 + p'_3 - p_1 - p_2 - p_3 \right) t_{3N}.
$$

(23)

We apply Fourier transform to (23) in order to obtain the potential in configuration space and we have

$$
\langle r'_1r'_2r'_3 | W | r_1r_2r_3 \rangle = \delta \left( r'_1 - r_1 \right) \delta \left( r'_2 - r_2 \right) \delta \left( r'_3 - r_3 \right) W.
$$

(24)

and the component $W^{ijk}$ of the local potential $W$ is given by

$$
W^{ijk} = \frac{g^4}{(4\pi)^4 (2m)^2} \frac{\mu m_s}{m} \sigma^{(i)} \cdot \sigma^{(j)}
\times \left[ \left( 1 - \Lambda \right) U \left( \Lambda r_{kj} \right) \sigma^{(i)} \cdot \nabla_i \sigma^{(j)} \cdot \nabla_j U \left( \mu r_{ji} \right)
+ \frac{\Lambda \mu}{g f \sigma} \sigma^{(i)} \cdot \nabla_j U \left( \mu r_{ji} \right) \sigma^{(j)} \cdot \nabla_k U \left( \Lambda r_{kj} \right) \right],
$$

(25)

where $U(\mu r) = \frac{\exp(-\mu r)}{\mu r}$ and $r_{kj} = r_k - r_j$.

We assume the approximate identity $\frac{m}{g f \sigma} \approx 1$ and, similarly to the procedure used in the two pion exchange three-nucleon force [15], we regularize the mesonic exchanges with the following dipole form factor

$$
G \left( k^2 \right) = \left( \frac{\Lambda^2 - \mu^2}{\Lambda^2 - k^2} \right)^2,
$$

(26)

where $\Lambda$ is a cut off parameter.

The regularization changes the Yukawa-type function into

$$
U \left( \mu r, \Lambda \right) = \frac{e^{-\mu r}}{\mu r} - \frac{\Lambda}{\mu} \frac{e^{-\Lambda r}}{\Lambda r} - \frac{1}{2} \frac{\mu}{\Lambda} \left( \frac{\Lambda^2}{\mu^2} - 1 \right) e^{-\Lambda r}
$$

(27)

and its derivatives are given by

$$
\frac{\partial U \left( \mu r, \Lambda \right)}{\partial r_\alpha} = \frac{\mu r_{\alpha}}{r} U_1 \left( \mu r, \Lambda \right),
$$

(28)

$$
\frac{\partial^2 U \left( \mu r, \Lambda \right)}{\partial r_\alpha \partial r_\beta} = \frac{\mu^2}{3} \left[ \delta_{\alpha\beta} \left( U \left( \mu r, \Lambda \right) - G \left( r \right) \right) \right.
+ \left( \frac{3 r_{\alpha} r_{\beta}}{r^2} - \delta_{\alpha\beta} \right) U_2 \left( \mu r, \Lambda \right) \bigg],
$$

(29)
where

\[
U_1 (\mu r, \Lambda) = - \left( \frac{1}{\mu r} + 1 \right) e^{-\mu r} + \frac{\Lambda^2}{\mu^2} \left( 1 + \frac{1}{\Lambda r} \right) e^{-\Lambda r} \\
+ \frac{1}{2} \left( \frac{\Lambda^2}{\mu^2} - 1 \right) e^{-\Lambda r},
\]

(30)

\[
U_2 (\mu r, \Lambda) = \left( 1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2} \right) e^{-\mu r} - \frac{\Lambda^3}{\mu^3} \left( 1 + \frac{3}{\Lambda r} + \frac{3}{\Lambda^2 r^2} \right) e^{-\Lambda r} \\
- \frac{1}{2} \frac{\Lambda}{\mu} \left( \frac{\Lambda^2}{\mu^2} - 1 \right) \left( 1 + \frac{1}{\Lambda r} \right) e^{-\Lambda r},
\]

(31)

\[
G (r) = \frac{1}{2} \left( \frac{\Lambda^2}{\mu^2} - 1 \right) e^{-\Lambda r}.
\]

(32)

Substituting these results into the potential, we have

\[
W^{ijk} = \frac{4}{3} \left( \frac{g \mu}{2m} \right)^4 \frac{m}{(4\pi)^2} \frac{m_s}{\mu} \tau^{(i)} \cdot \tau^{(j)} \\
\times \left\{ (1 - \lambda) \left[ \sigma^{(i)} \cdot \sigma^{(j)} U (m_s r_{kj}, \Lambda) (U (\mu r_{ji}, \Lambda) - G (r_{ji}, \Lambda)) \\
+ S_{ij} (\hat{r}_{ji}, \hat{r}_{kj}) U (m_s r_{kj}, \Lambda) U_2 (\mu r_{ji}, \Lambda) \right] \\
+ \lambda \frac{m_s}{\mu} \left[ S_{ij} (\hat{r}_{ji}, \hat{r}_{kj}) + \hat{r}_{ji} \cdot \hat{r}_{kj} \sigma^{(i)} \cdot \sigma^{(j)} \right] \right\} \\
\times U_1 (\mu r_{ji}, \Lambda) U_1 (m_s r_{kj}, \Lambda),
\]

(33)

where

\[
S_{ij} (\hat{r}_{ji}, \hat{r}_{kj}) = 3 \sigma^{(i)} \cdot \hat{r}_{ji} \sigma^{(j)} \cdot \hat{r}_{kj} - \hat{r}_{ji} \cdot \hat{r}_{kj} \sigma^{(i)} \cdot \sigma^{(j)}.
\]

(34)

VI. DISCUSSION

Initially, we discuss the role of chiral symmetry in the results of the previous section. Inspecting eq.(25), one notes that all the terms of the potential contain two gradients, reflecting the fact that they come from eq.(22), which is a uniform second order polynomial in meson momenta, as expected from a calculation based on chiral symmetry. This feature of the problem is independent of \( \lambda \). On the other hand, the detailed form of the potential is quite sensitive to this parameter. The value \( \lambda = 0 \) corresponds to a scalar field, denoted by
σ, which is the chiral partner of the pion. The value \( \lambda = 1 \), in turn, indicates processes based on a chiral invariant field \( S \). It is worth pointing out that results with \( \lambda = 1 \) are, as they should, independent of the representation adopted for the pion field and, in particular, of the use of either PS or PV pion-nucleon couplings. So the parameter \( \lambda \) describes dynamical processes which are genuinely different.

At tree level, these effects cannot be distinguished in NN interactions, but they manifest themselves in the reactions \( NN \rightarrow \pi NN, \pi d \leftrightarrow NN, \) in axial form factors of nuclei and in three-body forces. In the present work we have considered only the last kind of application.

Our results with \( \lambda = 1 \) coincide with that presented by Coon, Peña and Riska [16]. These authors also obtained a contact interaction, employing a PV pion-nucleon coupling and a scalar-meson coupled to nucleons. In the framework of chiral symmetry, this combination means that they have tacitly used a Lagrangian equivalent of our eq. (5) and hence their \( \sigma \) field corresponds to our \( S \).

In this work we are concerned mainly with the differences between the cases \( \lambda = 0 \) and \( \lambda = 1 \) for three-body forces. The full exploration of this aspect of the problem would require a precise calculation of trinucleon observables, especially the binding energy. In order to produce a preliminary qualitative indication of the role of the force considered in this work, we evaluate the expectation value \( \langle S | W | S \rangle \), where \( |S\rangle \) represents the principal S-state of the trinucleon, which is the basic component of its ground state wave function.

This state is written as [15]

\[
|S\rangle = S(\mathbf{r}_{ji}) \Gamma_{\frac{1}{2}m_{12}}^{m_{12}}(a) \tag{35}
\]

where \( S(\mathbf{r}_{ji}) \) is the spatial component and \( \Gamma_{\frac{1}{2}m_{12}}^{m_{12}}(a) \) is the antisymmetric spin-isospin wave function with z-components \( m \) and \( t \) respectively.

The action of the tensor operators \( S_{ij} \) over \( |S\rangle \) results in states with orbital angular momentum different from zero. Using

\[
\left[ \Gamma_{\frac{1}{2}m_{12}}^{m_{12}}(a) \right]^\dagger \mathbf{\tau}^{(i)} \cdot \mathbf{\tau}^{(j)} \mathbf{\sigma}^{(i)} \cdot \mathbf{\sigma}^{(j)} \Gamma_{\frac{1}{2}m_{12}}^{m_{12}}(a) = -3 \tag{36}
\]
we obtain
\[
\left[ \Gamma_{\frac{1}{2}t} \right] W_{ijk} \Gamma_{\frac{1}{2}t} = -4 \left( \frac{g \mu}{2m} \right)^4 \frac{m}{(4\pi)^2} \frac{m_s}{\mu} 
\times \left\{ (1 - \lambda) U (m_s r_{kj}, \Lambda) [U (\mu r_{ji}, \Lambda) - G (\mu r_{ji}, \Lambda)] 
+ \cos \theta_j \lambda \frac{m_s}{\mu} U_1 (\mu r_{ji}, \Lambda) U_1 (m_s r_{kj}, \Lambda) \right\}
\]
\tag{37}
\]
where \( \cos \theta_j = \hat{r}_{ji} \cdot \hat{r}_{kj} \). Using the results of [13] and neglecting the very short range function \( G (\mu r_{ji}, \Lambda) \) [17], we have
\[
\left[ \Gamma_{\frac{1}{2}t} \right] W_{ijk} \Gamma_{\frac{1}{2}t} = - \left\{ (1 - \lambda) C_\sigma U (m_s r_{kj}, \Lambda) U (\mu r_{ji}, \Lambda) 
+ \lambda C_S \cos \theta_j U_1 (\mu r_{ji}, \Lambda) U_1 (m_s r_{kj}, \Lambda) \right\},
\]
\tag{38}
where \( C_\sigma \) and \( C_S \) are the strength coefficients of sigma and chiral scalar respectively, given by
\[
C_\sigma = 4 \left( \frac{g \mu}{2m} \right)^4 \frac{1}{(4\pi)^2} \frac{m_s}{\mu} m,
\]
\[
C_S = 4 \left( \frac{g \mu}{2m} \right)^4 \frac{1}{(4\pi)^2} \left( \frac{m_s}{\mu} \right)^2 m.
\]
Choosing \( m_s = 550 MeV \) and \( g = 13.5 \), the strength coefficients become \( C_\sigma = 96 MeV \) and \( C_S = 378 MeV \).

In fig. 6 we show equipotential plots for the choices \( \lambda = 0 \) (graph a) and \( \lambda = 1 \) (graph b), constructed by keeping two nucleons 1 fm apart and varying the position of the third one. As the plots are symmetric under rotation around the x-axes the specification of a single quadrant describes the spatial energy distribution. Inspecting this figure one learns that the predictions from both models are rather different, indicating that the effective seagull is very important. In the linear approach, the interaction is attractive over a wide region, whereas the non-linear scalar produces a repulsion around the triangular configuration and these differences should show up in observables.

One is aware that a study based on just S trinucleon waves can provide only rough indications, since it is well known that D waves do play an important role in trinucleons.
Nevertheless, in the absence of a detailed study, we may assume that the trends associated with our equipotential plots would reflect in the binding energy. This assumption is supported by the results of ref. [16] where a three-body force given by our eq.(22) with $\lambda = 1$ was shown to produce a decrease of the binding energy which is rather welcome.

As a final comment, it is important to point out that the discussion presented in section 3 makes us to be biased towards the chiral scalar, but final conclusions must wait until a complete evaluation of the diagrams of fig 2c, which is now in progress.
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[1] W. N. Cottingham and R. Vinh Mau, Phys. Rev. 130 (1963) 735.

[2] W. N. Cottingham, M. Lacombe, B. Loiseau, J-M. Richard and R. Vinh Mau, Phys. Rev. D8 (1973) 800; M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, P. Pires and R. Tourreil, Phys. Rev. D21 (1980) 861.

[3] G. E. Brown and J. W. Durso, Phys. Lett. B435 (1971) 120.

[4] C. Ordóñez and U. van Kolck, Phys. Lett. B291 (1992) 459; L. S. Celenza, A. Pantziris and C. M. Shakin, Phys. Rev. C46 (1992) 2213; J. L. Friar and S. A. Coon, Phys. Rev. C49 (1994) 1272; C. A. da Rocha and M. R. Robilotta, Phys. Rev. C49 (1994); M. C. Birse, Phys. Rev C49 (1994) 2212; C. A. da Rocha and M. R. Robilotta, Phys. Rev C52 (1995) 531.

[5] C. Ordóñez, L. Ray and U. van Kolck, Phys. Lett. 72 (1994) 1982; Phys. Rev. C53 (1996) 2086.

[6] M. R. Robilotta, Nucl. Phys. A595 (1995) 171; M. R. Robilotta and C. A. da Rocha, Nucl. Phys. A615 (1997) 391

[7] J. L. Ballot, M. R. Robilotta and C. A. da Rocha, to appear in Phys.Rev. C.

[8] N. Kaiser, R. Brockmann and W. Weise, preprint nucl-th/9706045

[9] R.M. Barnett et al, Phys.Rev. D54 (1996) 1.

[10] S. Weinberg, Phys. Rev. Lett. 18 (1967) 188.

[11] S. Weinberg, Phys. Rev. 166 (1968) 1568.

[12] C. M. Maekawa and M. R. Robilotta, Phys. Rev. C55 (1997) 2675.

[13] S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2239; C. G. Callan, S. Coleman,
J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2247.

[14] S. A. Coon and J. L. Friar, Phys. Rev. C34 (1986) 1060.

[15] M. R. Robilotta and H. T. Coelho, Nucl. Phys. A460 (1986) 645.

[16] S. A. Coon, M. T. Peña and D. O. Riska, Phys. Rev. C52 (1995) 2925.

[17] M. R. Robilotta and M. P. Isidro Filho, Nucl. Phys. A451 (1986) 581.
Figure Captions

Fig. 1. Interactions of nucleons (full lines), pions (broken lines) and scalar-isoscalar mesons (wavy lines) present in both linear and non-linear models (a) and only in the latter (b).
Fig. 2. Dynamical content of the effective scalar-isoscalar field in (a) nucleon-nucleon interactions, (b) the $\pi NN$ kernel and (c) the seagull term; conventions are the same as in fig. 1 and the crosses in nucleon propagators indicate that they do not contain positive frequency components.
Fig. 3. The effective seagull (a) as composed by the $\pi N \rightarrow \pi N$ (b) and $\pi N \rightarrow \pi\pi N$ (c) amplitudes.
Fig. 4. (a) Amplitudes for the processes $\pi N \to \sigma N (\lambda = 0)$ and $\pi N \to SN (\lambda = 1)$; (b) the corresponding $\pi NN$ kernel.
Fig. 5. Basic diagram for the three-body force.
Fig. 6 Equipotential plots for the expectation value of the three body force in the trinucleon ground state, calculated using eq (36) with $\Lambda = 1.5 GeV$ for the linear (a) and nonlinear (b) models. One of the nucleons is fixed at $x = 0.5 \text{ fm}$, another at $x = -0.5 \text{ fm}$ and the position of the third one is varied.