ON PERFECT LENSES AND NIHILITY

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The canonical problem of a perfect lens with linear bianisotropic materials is formulated. Its solution is shown to be directly connected with the concept of nihility, the electromagnetic nilpotent. Perfect lenses as well as nihility remain unrealizable.

KEYWORDS: Anti–vacuum, Lens, Negative permeability, Negative permittivity, Nihility, Perfect lens

1 Introduction

This communication has been inspired by a report on the theory of a perfect lens by Pendry [1]. This lens is supposedly constructed of a material whose permittivity and permeability, respectively, are exactly negative of the permittivity and the permeability of free space. In a predecessor paper [2], that material was postulated as the anti–vacuum.

2 Canonical Formulation for a Lens

As geometrical optics is applicable for lenses, the canonical formulation for a lens merely involves a linear homogeneous material confined to the region between two parallel planes. Let us, however, generalize the situation in order to understand the issue at greater depth by involving two linear homogeneous materials and four interfaces, as shown in Figure 1. The regions $0 \leq z \leq d_1$ and $d_1 + d_2 \leq z \leq d_1 + d_2 + d_3$ are occupied by a material labelled $a$, and the region $d_1 \leq z \leq d_1 + d_2$ by a material labelled $b$. Both materials are linear, homogeneous, bianisotropic...
and necessarily dispersive, their frequency–domain constitutive relations being as follows:

\[
\begin{align*}
D(x, y, z, \omega) &= \epsilon_0 \left[ \epsilon^{a,b}(\omega) \cdot E(x, y, z, \omega) + \epsilon^{a,b}(\omega) \cdot H(x, y, z, \omega) \right] \\
B(x, y, z, \omega) &= \mu_0 \left[ \beta^{a,b}(\omega) \cdot E(x, y, z, \omega) + \mu^{a,b}(\omega) \cdot H(x, y, z, \omega) \right]
\end{align*}
\] (1)

In these relations, the dielectric properties are delineated by \(\epsilon^{a,b}(\omega)\), the magnetic properties by \(\mu^{a,b}(\omega)\), and the magnetoelectric properties by \(\alpha^{a,b}(\omega)\) as well as \(\beta^{a,b}(\omega)\), all of these dyadics being functions of the angular frequency \(\omega\). The permittivity and permeability of free space (i.e., vacuum) are denoted by \(\epsilon_0\) and \(\mu_0\), respectively.

![Figure 1: Schematic for the canonical lens formulation.](image)

Without loss of essential generality, we can take the spatial Fourier transformations of all electromagnetic phasors with respect to \(x\) and \(y\); thus,

\[
E(x, y, z, \omega) = e(z, \kappa, \psi, \omega) \exp \left[ i \kappa (x \cos \psi + y \sin \psi) \right],
\] (2)
etc., suffice for the present purposes. Wave propagation in the two materials can then be cast in terms of $4 \times 4$ matrix ordinary differential equations as follows [3]:

$$
\frac{d}{dz} [f(z, \kappa, \psi, \omega)] = i [P_{a,b}(\kappa, \psi, \omega)] \cdot [f(z, \kappa, \psi, \omega)].
$$

(3)

Here, $[f] \equiv [e_x; e_y; h_x; h_y]^T$ is a column 4–vector with the superscript $T$ denoting the transpose, while $[P_{a,b}]$ are $4 \times 4$ matrixes. Accordingly, we obtain the basic relation

$$
[f(d_1 + d_2 + d_3, \kappa, \psi, \omega)] = [M(d_1 + d_2 + d_3, \kappa, \psi, \omega)] \cdot [f(0, \kappa, \psi, \omega)],
$$

(4)

where

$$
[M(d_1 + d_2 + d_3, \kappa, \psi, \omega)] = \exp \left\{ id_3 [P_a(\kappa, \psi, \omega)] \right\} \cdot \exp \left\{ id_2 [P_b(\kappa, \psi, \omega)] \right\} \cdot \exp \left\{ id_1 [P_a(\kappa, \psi, \omega)] \right\}.
$$

(5)

Within the confines of continuum electromagnetics, the canonical lens problem involves finding the material $b$ and the thickness $d_2$ for a specified material $a$ and thicknesses $d_1$ and $d_3$, such that

$$
[M(d_1 + d_2 + d_3, \kappa, \psi, \omega)] =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
$$

(6)

for all $\kappa$, $\psi$ and $\omega$.

3 Analysis and Discussion

Obviously, that would be a fruitless endeavor in practice. Hence, lens designers effectively settle for some acceptable ranges of $\kappa$, $\psi$ and $\omega$ in which (3) holds, when material $a$ is air. Deviations from an ideal match introduce aberrations [4].

Mathematically, and at first glance, an excellent candidate for ideal match is the following:

$$
\begin{aligned}
\varepsilon^b(\omega) &= -\varepsilon^a(\omega), \\
\beta^b(\omega) &= -\beta^a(\omega), \\
\alpha^b(\omega) &= -\alpha^a(\omega), \\
\mu^b(\omega) &= -\mu^a(\omega), \\
\end{aligned}
\quad d_2 = d_1 + d_3
$$

(7)
Close inspection of (3), however, shows that (6) is suitable for all \( \kappa \), only if material \( a \) has orthorhombic symmetry, i.e.,

\[
\xi^a = \begin{bmatrix}
\xi_1^a & 0 & 0 \\
0 & \xi_2^a & 0 \\
0 & 0 & \xi_3^a 
\end{bmatrix}, \quad \xi = \epsilon, \alpha, \beta, \mu.
\] (8)

A sandwich of equal thicknesses of materials \( a \) and \( b \) then constitutes a planar realization of a medium named nihility earlier in this journal [2]. Nihility is the postulated electromagnetic nilpotent, with the following constitutive relations:

\[
\begin{align*}
D(x, y, z, \omega) &= 0 \\
B(x, y, z, \omega) &= 0
\end{align*}
\] (9)

Wave propagation cannot occur in nihility, because \( \nabla \times E(x, y, z, \omega) = 0 \) and \( \nabla \times H(x, y, z, \omega) = 0 \) in the absence of sources therein. Whereas the phase velocity and the wavevector of a plane wave in vacuum/anti–vacuum are co–parallel/anti–parallel, the directionality of the phase velocity relative to the wavevector in nihility is a non–issue.

Physically, (7) and (8) are still deficient because the principle of energy conservation has not been considered. With the normal constraint that material \( a \) be passive, it follows from (7) that material \( b \) has to be active. Although their electromagnetic response can be simulated via composites containing active circuit elements [4], I do not think that active materials will provide a realistic option in the near future. Therefore, the passivity constraint on material \( b \) would lead to aberrations due to absorption. To those must be added chromatic aberrations, which would arise from the non–fulfilment of (7) outside some limited range of \( \omega \).

Suppose next that material \( a \) is isotropic and non–magnetoelectric, i.e., \( \xi^a = \epsilon^a \mathbb{I}, \mu^a = \mu^a \mathbb{I}, \) and \( \alpha^a = \beta^a = 0 \), where \( \mathbb{I} \) is the identity dyadic and \( \mathbb{0} \) is the null dyadic. Then, a reasonable match in some frequency range could conceivably be provided by materials that supposedly possess a negative index of refraction [6], so long as absorption is acceptably low.

Pendry [1] actually took material \( a \) to be air, which is optically indistinguishable from vacuum (\( \epsilon^a = \mathbb{I}, \mu^a = \mathbb{I}, \) and \( \alpha^a = \beta^a = 0 \)) for most purposes. Thus, the requirements on material \( b \) became very simple, \( \text{viz.}, \xi^b = -\mathbb{I}, \mu^b = -\mathbb{I}, \) and \( \alpha^b = \beta^b = 0 \). (In other words, material \( b \) has to be anti–vacuum.) But Ziolkowski
R.W. Ziolkowski has recently concluded from two-dimensional computer simulations that even these simple requirements (in some narrow frequency range) cannot be met by realistic meta–materials. Recent correspondence between Pendry and others adds to the debate [8]–[11].

A perfect lens remains unrealizable in my opinion, as does nihility.

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