An On-line Supply Chain Scheduling Problem with a Constraint of Unavailability on the Machine

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Abstract. In this paper, we focus on a scheduling problem in the on-line environment, in which the number of jobs and all information of every job, such as release time, processing time etc. are not known before each job is released. After a job is released, it can be first manufactured on one machine in a factory. It is noted that there is a given constraint of unavailability on the machine. There may be some job interrupted by the determinate unavailability interval during its process of manufacturing, and the interrupted job must be reprocessed after the machine becomes available again. In order to take full advantage of the unlimited capacity of a vehicle, the completed jobs can be delivered in batches to the customer. Our goal is to minimize the maximum arrival time of the vehicle when the vehicle returns to the factory after it has delivered the last completed job to the customer. The lower bound is proved when we construct a series of intractable instances. And an on-line algorithm for the problem is then developed and the competitive analysis is performed.

Keywords. Supply Chain, On-line, Unavailability constraint.

1. Introduction
In a two-stage supply chain system [1], jobs are first processed on one machine or more machines in a factory in the processing stage and then delivered to one customer or more customers by some transportation facilities in the delivery stage. And in order to utilize effectively the capacity of the vehicle, the jobs can be delivered in batches other than being delivered as soon as they are completed. In the traditional off-line supply chain scheduling problems, job processing times, job release times, the number of jobs and so on can be given in advance, and jobs can be processed on the machines according to all information of jobs. But with the rapid development of the supply chain, more and more on-line problems have been studying by many researchers in recent years. In the on-line supply chain scheduling problems, the number of jobs and all information of every job, such as release time, processing time etc. are not known, or scheduling performs according to the demands at any time. In the problem we discuss in this paper, the jobs are released on line.

It is noted that in the real industrial settings, the machine may become unavailable because of the breakdown or maintenance during the processing period. The job may be interrupted by the unavailability interval and must not be resumable when the machine turns into available again because the unfinished part can’t be continuous, i.e., the preemption is not allowed. Jobs released after that job may be delayed to process on the machine. In this paper, there is an unavailability time interval on the machine, which is determined before the processing begins in the process period.

In the delivery period, there is one vehicle, whose capacity is unlimited, to transport jobs to the customer. A completed job is permitted to compose a delivery batch with some of other completed jobs. The transportation time is a constant between the manufacturer and the customer. Our goal is to minimize the maximum arrival time of the vehicle from the customer back to the factory when the
vehicle delivers all batches of jobs. Therefore, the manager of the factory must consider not only when and in what order to manufacture jobs without any information in advance, but also when and which jobs to deliver in batches to reduce the total cost as little as possible.

2. Literature Review

For the non-resumable off-line supply chain scheduling problem, Hall and Potts [1] considered the problems for the single-customer case and the multi-customer case, and provided the optimal polynomial time dynamic programming algorithms for these two cases respectively. In the latter case, the number of customers is fixed. [2-4] studied the off-line supply chain scheduling problems in different conditions, such as the allowed preemption for jobs, the objective associated with transportation cost, and multiple periodic unavailability intervals etc. Chen [5] surveyed many research results.

With regard to the related on-line models, Hoogeveen and Vestjens [6] and Van den Akker et al. [7] considered almost simultaneously a non-resumable on-line problem in which each job is delivered one by one, and the objective is to make the maximum delivery time of the jobs to minimize. But in [7], the interrupted job must restart to process if it is broken off by some urgent jobs. [8] considered an on-line supply chain scheduling in which the jobs are resumable in the production period on a single machine and the sum of total flow time and total delivery cost of jobs are to be minimized. The researchers of [8] developed an on-line algorithm for the case of a single customer, which is optimal. In the same paper, they also analysed an on-line approximate algorithm for the case of multiple customers. In 2010, an on-line supply chain scheduling problem for a single customer with constraints on vehicle capacity was considered and an on-line optimal algorithm was designed by Averbakh in [9]. Han etc. defined ten on-line supply chain scheduling problems for single-machine and parallel-machine configurations for a single-customer and different vehicle characteristics in [10]. For all those problems, they discussed the same objective function, that is, the sum of makespan and delivery cost. The authors proved the lower bound and proposed on-line algorithms, respectively. Zhang etc. considered the on-line supply chain scheduling problem in [11], which is applied in business-to-customer (B2C) e-commerce. In this special setting, customers generated tasks on-line and the enterprise that received the tasks have to batch goods from the shelves in a warehouse and transport them to the customers in different zones using the vehicles with limited capacities. They developed a 4-competitive algorithm and checked the effectiveness and efficiency of the algorithm by comparison experiments in extensive numerical analyses.

As described in [12], the performance of any on-line algorithm should be evaluated though the competitive analysis. For an instance $I$ of an on-line problem, let $A(I)$ be the objective function value of algorithm $A$ and $OPT(I)$ be the optimal objective function value of the off-line problem, which is obtained by all information of jobs being given in advance. For all $I$ and $r \geq 1$ if $A(I)/OPT(I) \leq r$ is hold, the algorithm $A$ can be called a $r$-competitive algorithm, or the competitive ratio of algorithm $A$ is $r$.

3. Problem Formulation

The manufacturer has a single machine with a determinate unavailability time interval $[T_1, T_2]$ to process jobs. Suppose there is a customer who releases the information of orders at time $r_1, \cdots$, on line to the manufacturer, and the manufacturer processes jobs $J_1, \cdots$, with processing time $p_1, \cdots$, respectively. The job’s processing is non-resumable, that is, the interrupted job by the unavailability interval should restart to process just when the machine returns to available again. Completed jobs should be packed into batches and be delivered to the customer by one vehicle with unlimited capacity. The trip-round delivery time between the manufacturer and the customer is denoted by $T$. Let $D_j$ denote the arrival time of the vehicle back to the factory after job $J_j$ is delivered to the customer for $j = 1, 2, \cdots$. The objective is to minimize the maximum arrival time of all jobs. This on-line supply chain scheduling problem can be represented as
(P): 1. $h_t|nr-a, r_j, online|V(1, \infty), direct|1|D_{\text{max}}.$

The machine configuration is in the first field, in which $h_t$ denotes one unavailability interval. “nr-a” of the second field is used to mark the interrupted job is non-resumable. In the third field, “$V(1, \infty)$” presents the situation of the vehicle. And because there is one customer, the vehicle should transport jobs directly from the manufacturer to the customer. “$D_{\text{max}}$” of the five field is the objective function [2].

4. Lower Bound and the Algorithm

In this section, we firstly analyse the lower bound of the problem (P) according to the transportation time, which deduces the following two theorems.

Theorem 1. No on-line algorithm for the problem (P) can have a competitive ratio less than $\max\{5/3, 1 + T_1/T_2\}$, if the transportation time $T$ is small sufficiently.

Proof. Note that the transportation time $T$ is small sufficiently. Without loss of generality, let $T < \epsilon$. Job $J_1$ with processing time $p_1 = 2\epsilon$ arrives at time $r_1 = 0$. Suppose an on-line algorithm $H$ arranges to manufacture $J_1$ at time $t_1$ and then delivers it as soon as it is completed.

- If $t_1 \geq 2\epsilon$, there is no job coming, then the objective function value of algorithm $H$ is $t_1 + 2\epsilon + T$, denoted by $Z''$. In the optimal schedule, this job is manufactured at time 0 and delivered at time $2\epsilon$. Hence,

$$Z' = 2\epsilon + T,$$

where $Z'$ is used to denote the optimal objective function value. And

$$\frac{Z''}{Z'} = \frac{t_1 + 2\epsilon + T}{2\epsilon + T} \geq 1 + \frac{2\epsilon}{2\epsilon + T} > \frac{5}{3} \quad (1)$$

- If $0 \leq t_1 < 2\epsilon$, a second job $J_2$ with processing time $p_2 = T_1 - t_1 - \epsilon$ arrives at time $r_2 = t_1 + \epsilon$. Algorithm $H$ must arrange job $J_2$ to process after the unavailability internal. The objective function value of algorithm $H$ is no less than $T_2 + T_1 - t_1 - \epsilon + 2T$. But in the optimal schedule, job $J_2$ should be processed before unavailability internal and job $J_1$ should be processed after unavailability internal, that is,

$$Z' = T_1 + 2\epsilon + T.$$

Therefore,

$$\frac{Z''}{Z'} = \frac{T_2 + T_1 - t_1 - \epsilon + 2T}{T_2 + 2\epsilon + T} \to \frac{T_2 + T_1 + 2T}{T_2 + T} \to 1 + \frac{T_1}{T_2} \quad (2)$$

As a result of inequalities (1) and (2), any on-line algorithm has the a competitive ratio no less than $\max\{5/3, 1 + T_1/T_2\}$ for the problem (P).

Theorem 2. For the problem (P), none of on-line algorithms can have a competitive ratio less than $\max\{(1 + \sigma)/2, 1 + T_2/T_1\}$, if all processing times are 0 and the transportation time $T$ is not small sufficiently.

Proof. Job $J_1$ with zero processing time $p_1 = 0$ arrives at time $r_1 = 0$. Suppose an on-line algorithm $H$ arranges to deliver it at time $t_1$.

$$0 \leq t_1 \leq T_1$$

Case 1: If there is no job coming, then the objective function value $Z''$ of algorithm $H$ equals to $t_1 + T$. In the optimal schedule, the job $J_1$ is manufactured at time 0 and delivered at time 0. Hence, $Z' = T$.

Moreover,

$$\frac{Z''}{Z'} = \frac{t_1 + T}{T} = 1 + \frac{t_1}{T} \quad (3)$$
Case 2: If another job $J_2$ with zero processing time $p_2 = 0$ is released at time $r_2 = t_1 + \varepsilon$. The objective function value of algorithm $H$ is no less than $t_1 + 2T$. But in the optimal schedule, job $J_1$ and job $J_2$ should be delivered at time $t_1 + \varepsilon$ in one batch, that is, $Z' = t_1 + \varepsilon + T$. Therefore,

$$
\frac{Z''}{Z'} \geq \frac{t_1 + 2T}{t_1 + \varepsilon + T} \rightarrow 1 + \frac{T}{t_1 + T}.
$$

(4)

It is easy to obtain that the right-hand side of the inequalities (3) and (4) reaches the maximum value $(1 + \sqrt{5})/2$.

$$
t_1 > T_1
$$

There is no job coming in this situation. Job $J_1$ should be processed after unavailability internal. Hence we have

$$
\frac{Z''}{Z'} \geq \frac{T_1 + T}{T} = 1 + \frac{T_1}{T}.
$$

(5)

The following algorithm is developed to solve the case of problem $(P)$ when the transportation time $T$ is small sufficiently.

Algorithm $A$. Jobs are processed as the list schedule ($LS$ rule). Delivery happens as soon as each job is completed.

Theorem 3. For the problem $(P)$, the competitive ratio of Algorithm $A$ is $1 + T_1/T_2$ if $T$ is small sufficiently.

Proof. Because the transportation time $T$ is small sufficiently, the delivery strategy of the optimal solution should be the same with Algorithm $A$.

If the completion time of the last job in the optimal solution is no more than $T_1$, the unavailability interval has no effect on the schedule of jobs in the processing stage. The problem will be equivalent to $1 \mid r_j, online \mid C_{max}$, which is trivial. Hence, Algorithm $A$ can produce the optimal schedule.

If the completion time of the last job in the optimal solution is more than $T_1$, the completion time of the last job is at least $T_2$ in the optimal solution. The completion time of the last job in Algorithm $A$ is no more than $T_1$ plus the completion time of the last job in the optimal solution.

Hence, we have

$$
\frac{Z^A}{Z} \leq 1 + \frac{T_1}{T_2},
$$

(6)

where $Z^A$ is the objective function value of algorithm $A$.

5. Conclusion

An on-line supply chain problem is studied in this paper, in which the information of jobs is unknown. The machine has an unavailability interval which can interrupt the process of some job, and the interrupted job is non-resumable and must re-process just when the machine becomes normal. There is only one vehicle used to deliver completed jobs to the customer. The completed jobs can be delivered in batches to the customer by the vehicle other than being delivered as soon as each one is finished completing. The goal is to minimize the arrival time of the vehicle after it delivers the last delivery and returns to the manufacturer. The lower bound of the problem is analysed due to the transportation time in two cases. And an on-line algorithm is proposed and the competitive analysis is performed.

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References

[1] Hall N and Potts C 2003 Supply chain scheduling: Batching and delivery Operations Research 51 566–584
[2] Fan J 2013 Supply chain scheduling on a single machine with availability constraint Applied Mechanics Materials 380–384
[3] Fan J, Lu X and Liu P 2015 Integrated scheduling of production and delivery on a single machine with availability constraint Theoretical Computer Science 562 581-589
[4] Fan J and Lu X 2015 Supply chain scheduling problem in the hospital with periodic working time on a single machine Journal of Combinatorial Optimization 30 892-905
[5] Chen Z L 2010 Integrated production and outbound distribution scheduling: Review and extensions Operations Research 58 130-148
[6] Hoogeveen J A and Vestjens A P A 2000 A best possible on-line algorithm for minimizing max-imum delivery time on a single machine SIAM Journal of Discrete Mathematics 13 56–63
[7] M Van den Akker, Hoogeveen H and Vakhania N 2000 Restarts can help in the on-line minimization of the maximum delivery time on a single machine Journal of Scheduling 3 333–341
[8] Averbakh I and Xue Z H 2007 On-line supply chain scheduling problems with preemption European Journal of Operational Research 181 500-504
[9] Averbakh I 2010 On-line integrated production-distribution scheduling problems with capacitated deliveries European Journal of Operational Research 200 377-384
[10] Han B, Lu X, Zhang W and Lin Y 2015 On-line supply chain scheduling for single-machine and parallel-machine configurations with a single customer: Minimizing the makespan and delivery cost European Journal of Operational Research 244 704-714
[11] Zhang J, Wang X and Huan K 2018 On-line scheduling of order picking and delivery with multiple zones and limited vehicle capacity Omega 38 104-115
[12] Pruhs K, Sgall J and Torng E 2004 Handbook of Scheduling: Algorithms, Models, and Performance Analysis (US: CRC Press) 235