Multi-resolution analysis of passive cavitation detector signals

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Abstract. Passive cavitation detectors are widely used for measuring acoustic emissions from cavitating bubbles. Acoustic emissions related to the dynamics of oscillating bubbles contain complex time and frequency domain information. Signal processing techniques traditionally used to analyse transient and stationary signals may be of limited value when analysing such acoustic emissions. This paper describes a multi-resolution approach developed for processing acoustic emissions data. The technique consists of the combination of a discrete wavelet transform and of the statistical and spectral analysis to extract cavitation features. These features include broadband emissions and harmonic, sub-harmonic and ultra-harmonic information. The implementation of the technique on experimental datasets demonstrates that this approach provides detailed information about key features of the acoustic signal, especially in complex situations where different types of cavitation occur simultaneously. Furthermore, statistical metrics used in this technique can provide a quantitative means for classifying signatures of cavitation, particularly the broadband segment of the spectrum created by inertial cavitation, which constitutes novel work.

1. Introduction
Acoustic cavitation is the formation and collapse of cavities (bubbles) in a liquid which undergoes a rapid pressure fluctuation generated by an acoustic source [1]. Reference is often made to two types of ultrasound induced cavitation, referred to as inertial and stable cavitation. The former refers to the situation where the created bubbles grow and implode violently within a few cycles of excitation, while in the case of stable cavitation, bubbles sustain and repetitively oscillate for a longer period of time [1]. Several physical effects are associated with each type of cavitation, namely the creation of free radicals, shock waves, shear forces, etc. [1]. These effects are exploited in a range of different applications including: non-invasive therapy [2], sono-chemistry [3], sono-crystallisation [4] and food processing [5]. In order to investigate and study the role of cavitating bubbles in these applications, the cavitation and bubbles oscillations have to be monitored and measured. Taking into account the aforementioned physical effects, several methods have been developed to detect and measure bubble activity [6]. Two main quantitative techniques for cavitation detection and measurement are passive cavitation detection [7] and active cavitation detection [8], which both exploit the acoustic emissions generated by cavitating bubbles. Passive techniques use a single element transducer, which is either focused or planar [9,10], called a Passive Cavitation Detector (PCD), to capture acoustic emissions resulting from cavitation. Passive cavitation detection can be easily implemented and has been widely used over a range
Frequency analysis of the recorded time series acoustic emissions from cavitating bubbles has often been carried out by the aid of the Fast Fourier Transform (FFT) algorithm. However, there exist a number of reasons as to why spectral analysis cannot thoroughly process readings of a PCD and can only extract limited information from such time series:

- Cavitation is a transient phenomenon which only takes place over a short time period relative to that of the fundamental frequency. Transient events are inherently aperiodic, whereas the Fourier transform is best used for analysing signals containing periodic events;
- Multiple phenomena occur simultaneously and appear in the spectrum as sub-harmonics, ultra-harmonics and a broadband segment, but they cannot be readily differentiated from each other and separately analysed through use of the Fourier transform alone;
- When one effect is greater in magnitude than another, e.g. strong ultra-harmonics but weak sub-harmonics, this may mask the signature of other frequency content in the spectrum and consequently affect its detectability; and
- Spectral analysis only provides information about the frequency content of a signal but not the corresponding part of the signal in the time domain.

Another technique which is commonly used to analyse the acoustic emissions from cavitating bubbles relies on the combination of spectral analysis and digital filtering. Spectral analysis is performed to identify different components of the signal in the frequency domain. Digital filters are then designed accordingly to separate each component in the time domain. After temporally windowing the signal, each segment is filtered and represented by the value of its energy. These generated graphs are then used for visually assessing and characterising cavitation activity. This qualitative approach to identifying cavitation signatures also suffers from the shortcomings of spectral analysis. Additionally, it is only applicable to signals with a long exposure time, e.g. a few seconds, to allow for a sufficiently fine frequency resolution within each window.

One way to improve cavitation detection and classification in particular is to utilise Time-Frequency analysis techniques. Time-Frequency techniques are extensively used for analysing non-stationary signals whose frequency content and statistics change over time. Various methods have been developed for use on practical applications such as sonar, ultrasonic applications for non-destructive testing, fault diagnosis, remote sensing and many others. Short Time Fourier Transform (STFT), Bilinear (or quadratic) time-frequency distributions (also called Cohens Class distribution functions) and wavelet transforms, are tools developed for investigating both the temporal and the spectral information of a signal. The STFT is one of the time-frequency techniques mostly used in practice. It has also extensively been applied to acoustic cavitation detection. Perhaps the most novel signal analysis technique in this family is the wavelet transform, which was developed to address the shortcomings of the STFT. Indeed, the main drawback of the STFT, especially in treating transient signals, is its fixed resolution: the broader the time window, the higher the frequency resolution, but the lower the time resolution, and vice versa. In contrast to the STFT, the wavelet transform is a multi-resolution transformation technique which gives good time resolution for high-frequency events and good frequency resolution for low-frequency events. In cavitation detection applications, the Discrete Wavelet Transform (DWT) has already been used for analysing hydraulic cavitation signals. In another application, the DWT was employed to characterise acoustic cavitation generated by an ultrasonic cleaning bath. The wavelet transform has also been used for identification of contrast agent micro-bubbles. In this paper, the DWT was combined with multi-scale spectrum analysis to develop a more powerful technique for acoustic cavitation detection and classification. Furthermore, statistical metrics were utilised for data evaluation and classification at each scale. This approach was implemented on signals obtained
experimentally. The experimental protocol will be described in Section 3. Results demonstrate promising performance of the developed method in comparison with conventional techniques.

2. Materials and Methods

2.1. Signatures of cavitation in a PCD signal

Initially, it is necessary to identify signatures of inertial and stable cavitation in a PCD signal. Stable cavitation is highlighted by tonal components in the time series which are represented by harmonics (that is \(nf\) where \(n = 2, 3, 4, \ldots\) and \(f\) is the excitation frequency) in the frequency domain. These harmonics are explained in terms of the forced nonlinear bubble oscillations though they can be due to nonlinear ultrasound wave propagation in the medium as well as nonlinear vibration of the ultrasound transducer, which can occur at high amplitude driving voltages. Inertial cavitation generates shock waves \([21–23]\) which have impulsive characteristics in the time domain. This manifests itself as a broadband component of the signals spectrum \([1, 12, 24]\). Additionally, the spectrum of acoustic emission may contain sub-harmonics (that is \(fn\) where \(n = 2, 3, 4, \ldots\) and ultra-harmonics (that is \(mf/n\) where \(m > n\) and \(m/n\) is non-integer). Several explanations have been given for these signatures though there is no overall agreement on them. One such explanation associates sub-harmonics with acoustic emissions from pulsating bubbles with equilibrium radii multiple times the size of the resonance radius \([25–27]\). Another study concluded that if bubbles with resonance radii are excited by harmonics of the fundamental frequency, sub-harmonics will appear in the spectrum \([28–30]\). Further work \([31]\) showed that inertial forces, under circumstances where the pressure amplitude is larger than a specific threshold value, can delay the contraction phase of an oscillating bubble and therefore leading to bubble growth for a longer period. Consequently the oscillation frequency of the bubble is different from the excitation frequency. These features of the spectrum of a PCD signal can be summarised as follows:

- harmonics in the spectrum are an indication of oscillating bubbles, considered as stable cavitation;
- broadband noise is an indication of shock waves emitted by inertially cavitating bubbles; and
- ultra-harmonics and sub-harmonics can be assumed to be due to the oscillation of bubbles of either larger or smaller multiples of the resonant bubble size. Ultra- and sub-harmonics may also occurs when the contraction phase of the oscillating bubbles is delayed and consequently bubbles oscillate for more than one pressure cycle prior to collapse. There is no general agreement as to whether ultra- and sub-harmonics should be considered as an indication of inertial or stable cavitation though they are usually interpreted as a threshold for inertial cavitation.

These features can be identified in both time and frequency domains using appropriate statistical indices which will be further elaborated in the following sections.

2.2. Discrete wavelet transform

STFT essentially divides up a signal to different segments through windowing and applies an FFT to each segment. The choice of window function depends on the signal characteristics and its ability to suppress side-lobes and spectral leakage. STFT is mathematically defined as follows \([32]\):

\[
S_x(t,f) = \int_{-\infty}^{\infty} x(t) w(t) e^{-j2\pi ft} \, dt
\]
Figure 1: Mallat Pyramid algorithm, performing a DWT using a bank of filters determined by the wavelet function. \( H(n) \) and \( L(n) \) are transfer functions of highpass and lowpass filters. This figure shows a decomposition up to three scales.

where \( w(t) \) is a window function, \( x(t) \) is the signal in time domain, \( x(t)w(t) \) is the windowed signal and \( f \) is the frequency. The Fourier transform uses the complex \( e^{j2\pi ft} \) function as an analysing function to decompose a signal. In the case of wavelet transform, the analysing function is a wavelet function, \( \psi \). The wavelet transform measures the correlation between signal and various wavelets which are created by shifting and scaling the mother wavelet \( \psi \). This is mathematically written as follows [16]:

\[
W_x(u, s) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{s}} \psi^\ast \left( t - \frac{u}{s} \right) dt 
\]  

(2)

where \( x(t) \) is the time signal, \( s \) and \( u \) are the scaling and translating parameters respectively and \( \ast \) refers to the complex conjugate of the wavelet function. The continuous wavelet transform of a discrete signal can reveal an overwhelming amount of information buried in the signal, which may not be straightforward to interpret. Since in this form of the wavelet transform a continuous wavelet function is used, the scale and translation parameters are \( u \in \mathbb{R} \) and \( s \in \mathbb{R}^+ \) respectively. Theoretically, these can assume infinite amount of values. This usually leads to redundancy as the spectrum changes very little between adjacent scales [16]. This sparked the enthusiasm for discretization of the wavelet transform. The discrete wavelet transform is implemented using a bank of filters where the transfer functions of the filters are determined based on the mother wavelet function. The bandwidths of the highpass and lowpass filters, \( H(n) \) and \( L(n) \), at scale \( i \) are equal to \( \left[ \frac{f_s}{2^{i+1}}, \frac{f_s}{2^i} \right] \) and \( \left[ 0, \frac{f_s}{2^{i+1}} \right] \), respectively, where \( f_s \) is the sampling frequency. This algorithm is illustrated in Figure 1. This method of performing the DWT with the aid of a bank of filters is the cornerstone of developing more complex signal processing techniques which would benefit from the multi-resolution analysis property of the wavelet transform. This algorithm was used to develop a signal processing technique which is suitable for the analysis of cavitation signals and is explained in the following section.

2.3. Multi-resolution cavitation analysis

The DWT algorithm was used to decompose PCD readings into a set of underlying components where each component has a particular frequency characteristic. This is equivalent to splitting up the signal into its basic elements over different frequency ranges. Summing these individual components results in the whole signal being reconstructed. Each component was then analysed separately to extract cavitation features. The simplest processing method is to perform a spectral analysis of each component and to look for aforementioned signatures of cavitation. The energy
of each component can also be calculated as an estimate of the intensity of physical phenomena, i.e. the type of cavitation. In order to pinpoint different regimes of cavitation quantitatively, two statistical indices were employed for feature extraction. Since shock waves created by the inertial cavitation of bubbles appear as short impulses in the time domain, one expects more “peaky” signal when inertial cavitation occurs. On the other hand, the time series would contain more tonal elements which would exist for longer time periods when bubbles oscillate in a stable fashion. Consequently, the signal would be less impulsive. These indices were kurtosis and crest factor which are explained below.

**Kurtosis:** Assuming a signal is a realisation of a real random variable, kurtosis then measures the “peakedness” of the probability distribution of this random variable. This can be interpreted as the proportion of the variance which is the result of infrequent extreme deviations. The mathematical formulation of kurtosis is as follows [33]:

$$K = \frac{\mathbb{E}(x - \mu)^4}{\sigma^4}$$  (3)

where $\mathbb{E}()$ is the expected value operator, $x$ is a random variable (signal) and $\mu$ and $\sigma$ are the mean value and standard deviation of $x$ respectively.

**Crest factor:** Crest factor is the ratio of peak values of a signal to the average value of the signal. The crest factor may be interpreted as a measure of how “extreme” the peaks in the signal are. A higher crest factor tends to indicate more impulsive behaviour of the signal. The mathematical formulation of the crest factor expressed in decibels is as follows [34]:

$$C_f = 10 \log \frac{\max(|x|)^2}{x_{rms}^2}$$  (4)

where $|x|$ is the absolute value of variable $x$. The crest factor is more sensitive to small and short impulses compared to the kurtosis. The kurtosis and crest factor of a pure tone can be found to be about 1.5 and 3 dB, respectively. In the case of a Gaussian noise, the kurtosis is about 3 whereas the crest factor can be any real number as the Gaussian distribution is theoretically unbounded. Given these values, a threshold value for classifying cavitation regimes, however, cannot be readily defined. Nonetheless, these statistical metrics can be utilised for the comparative study of contents of different scales of signal decomposed by the DWT.

The flow chart shown in Figure 2 demonstrates the signal processing technique employed in this study. Taking into account the frequency response of a PCD transducer, those scales of decomposition which fall in the frequency bandwidth of the transducer should be acquired and analysed.

### 3. Experimental Results and Discussion

A schematic of the test set-up is depicted in Figure 3 and briefly explained in Table 1. The transducers were immersed in a tank filled with de-ionised water at room temperature ($22^\circ$C). The gas content of water was measured by an oxygen meter probe and kept about $7.00 \pm 0.2$ mg/L for all trials. The excitation signal was a burst of 100 cycles with a fundamental frequency of 1.1 MHz. Tests were carried out under different excitation amplitudes, the values of which are displayed in Table 2. This table also includes the estimated peak positive and negative pressure amplitude at the focus under these excitation conditions. These estimations were obtained by numerically solving the KZK (Khokhlov-Zabolotskaya-Kuznetsov) equation in MATLAB using the HIFU simulator software [35]. The KZK equation describes the propagation of
Acoustic waves and accounts for effects due to nonlinear wave propagation, diffraction and absorption [36]. The conditions that under which the KZK equation is valid are expressed as follows [37]: i) \( d/a > 1 \) where \( d \) is the active diameter of a focused transducer and \( a \) is the focal depth; and ii) \( ka > (d/a)^{1/3} \) where \( k \) is the wavenumber. The geometry of the HIFU transducer used in this work together with the large wavenumber, i.e. 4600 m\(^{-1}\), satisfy both of these conditions. Acoustic emissions from bubbles at the focus were picked up by two focused transducers which were confocal with the HIFU transducer. These receivers are depicted as R1 and R2 in Figure 3. The acquired voltage signals from the receiving transducers were amplified and digitised at the sampling rate of 200 MHz using an oscilloscope. The effect of the transfer function of each transducer was corrected by performing a deconvolution using digital filters implemented in the time domain. Having measured the pulse-echo transfer function of transducers, deconvolution filters were determined using a least squares algorithm which employs derivative regularization [38]. The Daubechies wavelet function of order 10 [16] was selected and eight scales of decomposition were chosen for the signal processing algorithm. The type of wavelet function was selected in accordance with the physics of cavitation and impulsive nature of desired component, as discussed in Section 2.1. The depth (i.e. the number of scales) of the wavelet transform was chosen by considering the receiving transducers’ bandwidth and the sampling rate used in the measurements. This should be noted that the type of wavelet function and depth of decomposition which are best adapted to process the data are not unique. Indeed, other orthonormal wavelet functions with relatively high order should be able to extract cavitation signatures from the signal.

The spectra of pre-processed signals measured by R1 and R2 transducers are depicted in Figure 4. Figures 5 to 7 show the three underlying components of readings from the second transducer, i.e. within the 0.4 MHz to 3.2 MHz bandwidth, and their corresponding spectra for test condition two. Sub-harmonics are clearly detectable in Figure 5. The crest factor and kurtosis of this component of the signal are 12 dB and 4 respectively, which suggests the existence of an impulsive component in this time series. This is also manifested through a low amplitude erratic segment of the components spectrum, from 0.2 MHz to 0.85 MHz. Figure 6 shows the component of the signal which is merely comprised of the fundamental tone, i.e. the excitation frequency. The very low crest factor and kurtosis also confirm the tonal nature of this component. Figure 7 shows the component of the signal in the frequency range of 1.6 MHz to 3.12 MHz, together with its spectrum. This component consists of the second harmonic of the driving signal, and a set of elements with broadband characteristics. The high crest factor value of 16 dB, together with high kurtosis value of 7.5, suggests an impulsive nature of this component of the signal, which is also reflected in the broadband element of the spectrum.
Figure 3: Schematic test setup. Components are described in Table 1.

Table 1: Specification of the hardware of test setup

| Component    | Type                          | Specification                                                                 |
|--------------|-------------------------------|-------------------------------------------------------------------------------|
| HIFU         | Soniconcept H102              | High Intensity Focused Ultrasound transducer, $f_0 = 1.1$ MHz, -3 dB focal width is 1.33 mm by 13.5 mm |
| R1           | Olympus V327 type             | focused transducer with centre frequency of 10 MHz, -6 dB bandwidth is about 8 MHz |
| R2           | Olympus V392 type             | focused transducer with centre frequency of 1 MHz, -6 dB bandwidth is about 1.2 MHz |
| Power Amplifier | E&I 1040L type             | class AB RF amplifier with nominal gain of 55 dB                              |
| Pre-Amplifier | Stanford Research Systems, SR445A | linear preamplifier with bandwidth of 350 MHz, gain of 7 dB                 |
| Oscilloscope | LeCroy HRO64Zi                 | two channels were used simultaneously for recording data, sampling rate of 200 MHz per channel |
| Oxygen meter | VWR OX4000 Set                | the instrument measures gas content and temperature of water                |
Table 2: Test condition

| Test condition  | 1   | 2   | 3   | 4   |
|-----------------|-----|-----|-----|-----|
| Driving voltage (mVpp) | 80  | 120 | 200 | 280 |
| $P_-$ (MPa)*     | 2.6 | 3.5 | 5.8 | 7.1 |
| $P_+$ (MPa)*     | 3   | 4.5 | 8.7 | 12.5|

* this is an estimated value using the KZK equation solver.

Figure 4: Spectra of signal measured by: left: the 1 MHz transducer, right: the 10 MHz transducer.

Figure 5: Component of the signal measured by the 1 MHz transducer at scale 8 (frequency range of 0.4 MHz to 0.8 MHz) and its spectrum. $C_f = 12$ dB, $K = 4$.

The high value of the statistical metrics along with the broadband element in the spectrum, highlights the probability of occurrence of inertial cavitation which is present in this component of signal. It is difficult to detect the broadband part of the signal in the frequency range of 1.5 MHz to 3 MHz, using conventional Time-Frequency techniques. Indeed the broadband component can easily be masked by the fundamental and/or its harmonics if these are of high enough amplitude. Figure 8 shows the component of the signal picked up by the transducer R1. This component has a very high crest factor, in this case 17 dB and consequently, is likely to contain a broadband spectrum, which is observable in the figure. The high values of the statistical metrics, e.g. kurtosis above 3, indicates the presence of a broadband component in the signal which is produced by the impulsive acoustic emissions of inertial cavitation occurring at the focus. Low values of these metrics, however, do not necessarily refute the possibility of inertial cavitation as the sub-harmonics and ultra-harmonics signatures of inertial cavitation should be analysed too. Another observation which can be made is the existence of multiple broadband segments, with different energy levels, which are difficult to identify by performing a spectral analysis of data on a single scale, i.e. through use of an FFT applied to the time-series.

This analysis was repeated on other data sets acquired under the different test conditions de-
Figure 6: Component of the signal measured by the 1 MHz transducer at scale 7 (frequency range of 0.8 MHz to 1.6 MHz) and its spectrum. $C_f = 8$ dB, $K = 2.3$.

Figure 7: Component of the signal measured by the 1 MHz transducer at scale 6 (frequency range of 1.6 MHz to 3.12 MHz) and its spectrum, $C_f = 16$ dB, $K = 7.5$.

Figure 8: Component of the signal measured by the 10 MHz transducer at scale 4 (frequency range of 6.25 MHz to 12.5 MHz) and its spectrum, $C_f = 17$ dB, $K = 4.7$.

scribed in Table 2. Furthermore, the energy of each sub-harmonic component, ultra-harmonics and broadband features was calculated, as these components have already been differentiated by the aid of the multi-resolution technique developed in this paper. As explained in the Section 2.2, the bandwidth of filters of the DWT varies as a dyadic function of the scale number. At low scale numbers, the bandwidth is broad and it gets narrower as the scale number becomes larger. Consequently, it is possible that in a single scale, particularly in a low scale number, two different spectral components occur simultaneously. In the case of overlap of a broadband component with harmonics or ultra-harmonics, detected using statistical indices, a notch filter was designed to remove tonal components. This enabled the energy of each component to be estimated separately. The bar charts depicted in Figure 9 display the magnitude of the energy, in arbitrary unit, of sub-harmonics and broadband components under all four test conditions. The variance of time domain components were calculated as the measure of the energy of the signal. The results in Figure 9 demonstrate that the raising driving voltage and creating an acoustic...
pressure field of larger magnitude at the focus (see Table 2 for estimated pressure magnitudes at each condition), does not necessarily enhance the broadband characteristic of inertial cavitation resulting from shock waves. Moreover, one can see that at a constant driving frequency, tuning the excitation amplitude not only yields different cavitation regimes, which is expected from theory, but also favours or suppresses different features (or signatures) of the cavitation regime. The latter effects have been numerically demonstrated by Lauterborn [28]. The multi-resolution technique shows promise in being capable of monitoring and quantifying this behaviour using experimental data.

4. Conclusion
A new technique for the application of acoustic cavitation detection and characterisation was described in this paper. The technique utilises multi-resolution analysis together with statistical and spectral analysis of acoustic emissions radiated by cavitating bubbles. The technique uses the discrete wavelet transform to decompose a signal into its underlying components and performs the statistical and spectral analysis on each component to extract cavitation features. These features include broadband emissions and harmonic, sub-harmonic and ultra-harmonic information. It was demonstrated that in complex cases where both types of cavitation occur simultaneously with overlapping spectral features, this technique can identify and quantify each of these successfully. This capability is provided by using statistical metrics and spectral analysis at each scale of the wavelet transform. Moreover, thanks to the multi-resolution property of the DWT, this technique can be applied to signals which are impulsive in nature. This makes the multi-resolution method a useful tool for studying cavitation in short exposure shots. In spite of the improvements in processing acoustic emissions sensed by PCDs aiming for cavitation detection and characterisation, this technique is likely to be further enhanced by using more flexible wavelet transform approaches and feature extraction techniques. Further work is currently in progress which aims to investigate the use of wavelet packet transforms for performing multi-resolution analysis and incorporating more advanced statistical techniques for feature extraction and classification.
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