Ramsey’s Method of Separated Oscillating Fields and its Application to Gravitationally Induced Quantum Phaseshifts

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We propose to apply Ramsey’s method of separated oscillating fields to the spectroscopy of the quantum states in the gravity potential above a vertical mirror. This method allows a precise measurement of quantum mechanical phaseshifts of a Schrödinger wave packet bouncing off a hard surface in the gravitational field of the earth. Measurements with ultra-cold neutrons will offer a sensitivity to Newton’s law or hypothetical short-ranged interactions, which is about 21 orders of magnitude below the energy scale of electromagnetism.

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I. INTRODUCTION

The system of a Schrödinger quantum particle with mass \( m \) bouncing in a linear gravitational field is known as the quantum bouncer \[ \{1, 2, 3\} \] and "Quantum wave packet revivals" can be found in \[ 4 \]. Gravity tests with neutrons as quantum objects or within the classical limit are reviewed in \[ 5 \]. Above a vertical mirror, the linear gravity potential leads to discrete energy eigenstates of a bouncing quantum particle. Such quantum states have been demonstrated at the Institut Laue-Langevin with ultra-cold neutrons in a previous collaboration \[ 6, 7, 8, 9 \]. Above a vertical mirror, linear gravity potential leads to discrete energy eigenstates of a bouncing particle. The lowest energy eigenvalues \( E_n \), \( n = 1, 2, 3, 4, 5 \), are 1.41 peV, 2.46 peV, 3.32 peV, 4.09 peV, and 4.78 peV. The energy levels together with the neutron density distribution are shown in Fig. 1. The idea of observing quantum effects in such a gravitational cavity was discussed with neutrons \[ 10 \] or atoms \[ 11 \].

An important feature of the quantum bouncing ball – in contrast to the harmonic oscillator problem – is the fact that levels are not equidistant in energy. A combination of any two states can therefore be treated as a two-level system. The energy eigenstates in the gravity potential can be coupled to a mechanical or magnetic oscillator field. Transitions between quantum states in the gravitational field of the earth i.e. a change of the state occupation can therefore be induced similar to magnetic transitions, which occur when the oscillator frequency equals one of the Bohr frequencies of the system. This magnetic resonance method was in the original conception for measurements of nuclear magnetic moments \[ 12, 13 \], but soon it became a very general technique for radiofrequency spectroscopy \[ 14 \]. Ramsey developed his method of separated oscillating fields in which the oscillatory field is confined to a region at the beginning and a region at the end with no oscillating field in between \[ 15 \]. Variations of Ramsey’s method is inherently connected with precision measurements ranging from atomic clocks \[ 16 \] to atom interferometry \[ 17 \], from NMR \[ 18 \] to quantum-metrology \[ 19 \], or the related spin-echo technique \[ 20 \]. That method has also been used to measure the precession frequency of atoms, molecules or neutrons in a weak magnetic field, for example in a search for permanent atomic or neutron electric-dipole moments and in constructions of sensitive magnetometers. The sensitivity is extremely high, because a quantum mechanical phase shift is converted into a frequency measurement. The sensitivity reached so far \[ 21 \] in a search for the electric dipole moment of the neutron is \( 6.8 \times 10^{-22} \) eV, or one Bohr rotation within 6 days.

In analogy to these examples from electrodynamics, we discuss here an application of Ramsey’s method to probe the eigenstates in the gravity potential. Such a technique should open a new way to precision gravity experi-
ments and we propose to apply it to quantum states of neutrons or atoms in the gravitational field of the earth. This method will allow a precise measurements of energy differences with a precision similar to the magnetic resonance technique. Here, we are sensitive to energy shifts of a Schrödinger wave packet bouncing off a hard surface. Such energy shifts are expected from hypothetical gravity-like forces in the light of recent theoretical developments in higher-dimensional field theory and will allow searches for pseudo-scalar coupling of axions in the previously experimentally unaccessible astrophysical axion window [22,23], see Sec. III.

II. RAMSEY’S METHOD AND ITS APPLICATION TO GRAVITY POTENTIALS

A quantum mechanical system that is described by two states can be understood in analogy to a spin 1/2 system (assuming two states of a fictitious spin in the multiplet, similarly to spin up and spin down states). The time development of such systems is described by the Bloch equations. In magnetic resonance of a standard spin 1/2 system, the energy splitting results in the precession of quantum states in the gravity field. Transitions between the two states are driven by a transverse magnetic field, and the energy splitting results in the precession of the related magnetic moment in the magnetic field. Transitions between the two states are driven by a transverse magnetic radio frequency field. Similar concepts can be applied to any driven two level system, e.g. in optical transitions with light fields. Here we apply this picture to quantum states in the gravity field.

We start with a short description of Rabi’s method [12] to measure the energy difference between a two-level system with a coupled oscillating field. With ωpq, the frequency difference between the two states, ω, the frequency of the driving field, ΩR, the Rabi frequency and the time t, the Hamiltonian H is given by

$$H = \left(\frac{\hbar \omega_{pq}}{2} \sin^2 \left(\frac{\Omega_R t}{2}\right) - \frac{\hbar \omega_{pq}}{2} \sin^2 \left(\frac{\Omega_R t}{2}\right)\right).$$ (1)

The probability of being found in the excited state as a function of time is

$$P(t) = \left(\frac{\Omega_R}{\Omega_R}\right)^2 \sin^2 \left(\frac{\Omega_R t}{2}\right),$$ (2)

where the effective Rabi frequency is

$$\Omega_R = \sqrt{\Omega_R^2 + (\omega_{pq} - \omega)^2} = \sqrt{\Omega_R^2 + \delta^2},$$ (3)

with detuning δ from resonance. The sinusoidal population transfer is referred to as Rabi flopping. It has been proposed to measure the energy levels of a neutron in the gravitational field of the Earth with this method (GRANIT experiment [24,25]). The periodic drive is given by neutrons moving through a spatially oscillating magnetic field created by horizontal conducting wires.

As we will show below, one can drive transitions between quantum states in gravity above the mirror by vibrating the mirror surface.

Let’s consider the motion of ultracold neutrons in the gravitational field above a mirror. We assume the gravitational force to act in -z-direction, while the mirror is aligned with the xy-plane, vibrating with amplitude a in z-direction. The motion in x- and y-direction is free and completely decouples from that in z-direction. It suffices therefore to consider the time-dependent Schrödinger equation restricted to the z-direction

$$\left\{ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + mgz + V_0 \Theta(-z + a \sin \omega t) \right\} \psi = i\hbar \frac{\partial \psi}{\partial t}. \quad (4)$$

Here, g is the acceleration of gravity, m is the mass of the neutron and Θ is the Heaviside step function. The potential $V_0 \approx 100 \text{ neV}$ associated with the substance of the mirror is repulsive and much larger than eigenenergies of the lowest quantum states in the gravitational field. Therefore Eq. (4) must be solved with the boundary condition $\psi(z=a \sin \omega t, t) = 0$. For further considerations it is preferable to introduce $\tilde{z} = z - a \sin \omega t$ and to transform Eq. (4) into the rest frame of the mirror,

$$\{H_0 + W(\tilde{z}, t)\} \Psi = i\hbar \frac{\partial \Psi}{\partial t} \quad (5)$$

where

$$H_0 = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial \tilde{z}^2} + mg \tilde{z} + V_0 \Theta(-\tilde{z}), \quad (6)$$

$$W(\tilde{z}, t) = a \left[ mg \sin \omega t + i\hbar \omega \cos \omega t \frac{\partial}{\partial \tilde{z}} \right] \quad (7)$$

and $\tilde{\Psi}(\tilde{z}, t) = \Psi(z, t)$. The hamiltonian $H_0$ describes the neutron in the gravitational field above a mirror at rest. The second term $W(\tilde{z}, t)$ accounts for the vibration of the mirror.

The solution can be expressed in terms of the eigenfunctions $\psi_n(z)$ of $H_0$

$$\Psi(z, t) = \sum_n C_n(t) e^{-iE_n t/\hbar} \psi_n(z) \quad (8)$$

with time-dependent coefficients $C_n(t)$. Projection of Eq. (5) on the eigenstates of $H_0$ yields a system of differential equations for the coefficients $C_n(t)$

$$\frac{d}{dt} C_n(t) = i \hbar \sum_k \langle \psi_n | W | \psi_k \rangle \cdot C_k(t) \cdot e^{i \omega_{nk} t}. \quad (9)$$

The transitions between different quantum states is governed by the matrix elements of $W(\tilde{z}, t)$ defined in (7)

$$\langle \psi_n | W | \psi_k \rangle = a \left[ mg \delta_{n,k} \sin \omega t + i\hbar \omega Q_{n,k} \cos \omega t \right] \quad (10)$$

with

$$Q_{n,k} = \int_0^\infty dz \psi_n(z) \frac{d}{dz} \psi_k(z). \quad (11)$$
The relevant overlap integrals $Q_{n,k}$ for the transitions between the lowest eigenstates in the gravitational field are given in Table I.

| $k$ = 1 | $k$ = 2 | $k$ = 3 | $k$ = 4 | $k$ = 5 |
|--------|--------|--------|--------|--------|
| $n$ = 1 | 0.00000 | 0.09742 | -0.05355 | 0.03831 | -0.03040 |
| $n$ = 2 | -0.09742 | 0.00000 | 0.11894 | -0.06314 | 0.04419 |
| $n$ = 3 | 0.05355 | -0.11894 | 0.00000 | 0.13458 | -0.07031 |
| $n$ = 4 | -0.03831 | 0.06314 | -0.13458 | 0.00000 | 0.14724 |
| $n$ = 5 | 0.03040 | -0.04419 | 0.07031 | -0.14724 | 0.00000 |

**TABLE I:** Relevant overlap integrals $Q_{n,k}$ defined in Eq. (1) for the five lowest eigenstates in the gravitational field in $\mu m^{-1}$.

The physics behind the transitions between the energy eigenstates of the quantum bouncer caused by a vibrating mirror or an oscillating potential is related to earlier studies of energy transfer when matter waves bounce of a vibrating mirror [26, 27, 28, 29] or on a time dependent crystal [30, 31, 32]. In the later cases the transitions are between continuum states, in the quantum bouncer between discrete eigenstates. Most interesting for our proposal to drive transitions between eigenstates of the quantum bouncer with a vibrating mirror is the physics of reflection of a neutron by an oscillating potential step as has been investigated at the research reactors Munich and Geesthacht (FRM and FRG) [28], however in a different energy regime.

Applying Ramsey’s resonance method with separated oscillating fields will allow a careful measurement of the energy eigenstates states of the quantum bouncer [33]. We propose to implement it with neutrons by traversing five regions as shown in Fig. 2. The horizontal direction in space is considered as free motion, while the vertical one is described by a one-dimensional time dependent Schrödinger equation (see e.g. Eq. (I)).

To implement Ramsey’s method, one has to realize (1) a state selector, (2) a region, where one applies a $\pi/2$ pulse creating the superposition of the two states, whose energy difference should be measured, (3) a region, where the phase evolves, (4) a second region to read the relative phase by applying a second $\pi/2$ pulse, and finally (5) a state detector.

In the following we will describe all these components as they are shown in Fig. 2.

In region one, neutrons are prepared in a specific quantum state $|p\rangle$ in the gravity potential following the procedure demonstrated in [6]. A polished mirror on bottom and a rough absorbing scatterer on top at a height of about 20 $\mu$m is a realization of a state selector. It prepares neutrons into the ground state. Neutrons in higher, unwanted states are scattered out of the system and absorbed i.e. $C_1 = 1$ and $C_n = 0$ for $n > 1$. A quantum mechanical description of such a system can be found in [3]. The neutron passage through a mirror-scatterer system has also been studied in a frame, where the rough scatterer surface has been treated as a time-dependent variation of the scatterer position [34].

In region two of length $l$, the first of two identical oscillators is installed. Here, transitions between quantum states $|p\rangle$ and $|q\rangle$ are induced within a time $\tau$ according to Equ. (2). The oscillator frequency at resonance for a transition between states with energies $E_p$ and $E_q$ is

$$\omega_{pq} = \frac{(E_q - E_p)}{\hbar}. \quad (12)$$

The squared ratio of $\Omega_R$ and $\Omega_R$ as a function of the driving field $\omega$ for transitions $|1\rangle \rightarrow |3\rangle$, $|1\rangle \rightarrow |2\rangle$ and $|2\rangle \rightarrow |3\rangle$ is shown in Fig. 3. For the $|1\rangle \rightarrow |2\rangle$ transition, which we chose as an example for transitions in a two level system, $\omega_{12} = \omega_2 - \omega_1 = 2\pi \times 254 \text{ s}^{-1}$.

![Figure 2: Sketch of the proposal. Region 1: Preparation in a specific quantum state, e.g. state one with polarizer. Region 2: Application of first $\pi/2$-flip. Region 3: Flight path with length $L$. Region 4: Application of second $\pi/2$-flip. Region 5: State analyzer.](image)

![Figure 3: Ratio $\{\Omega_R/\Omega_F\}$ as a function of $\omega [s^{-1}]$ for transition $|1\rangle \rightarrow |3\rangle$, $|1\rangle \rightarrow |2\rangle$ and $|2\rangle \rightarrow |3\rangle$. The strength of the vibration was set to $1.0 \text{ m/s}^2$.](image)
Driven on resonance ($\omega = \omega_{pq}$), this oscillator drives the system into a coherent superposition of state $|p\rangle$ and $|q\rangle$. A $\pi/2$-pulse, that is one with pulse area $\Omega R \tau = \pi/2$, creates an equal superposition between state $|p\rangle$ and $|q\rangle$. This can be done by using oscillating magnetic gradient fields or by vibrating mirrors i.e. a modulation of the mirror potential in height.

In the intermediate region three, a non-oscillating mirror with a neutron flight path of $L$ and flight time $T$ follows. It might be convenient to place a second mirror on top of the bottom mirror at a certain height $h$. It allows to tune the resonance frequency between $|a\rangle$ and $|b\rangle$ due to the additional potential, and it provides an effective doubling of sensitivity in a search for hypothetical axion induced phase shifts or other fifth forces, see section III.

Subsequently, in region four a second oscillator in phase with the oscillator in region two is placed. If the oscillating $\omega$ is equal $\omega_{pq}$ than the system is at resonance and we have a complete reversal of the state occupation between $|p\rangle$ and $|q\rangle$. There is no change in the relative phase of the oscillator and the quantum state of the neutron independent of the neutron velocity. In the other cases for $\omega \neq \omega_{pq}$, a velocity dependent relative phase shift builds up, since a slower neutron is in the region three longer and experiences a greater shift than a faster neutron.

Afterwards in section five, such a phase shift can be measured by transmission through a second state selector.

This method can be realized with some modifications to the previous setup in the following way: Neutrons are taken from the ultracold neutron installation PF2 at ILL with a measured horizontal velocity $v = 3.2$ m/s $< v < 20$ m/s. At the entrance of the experiment, a collimator absorber system limits the transversal velocity to an energy in the pico-eV range. The experiment itself is mounted on a polished plane granite stone with an active and passive antivibration table underneath. This stone is leveled using piezo translators \cite{35}. Inclinometers together with the piezo translators in a closed loop circuit guarantee leveling with a precision better than 1 $\mu$rad \cite{36}. A solid block with dimensions $10$ cm $\times 15$ cm $\times 3$ cm composed of optical glass serves as a mirror for neutron reflection. The neutrons see a surface that is essentially flat. In region one, an absorber/scatterer that is a rough mirror with a surface roughness of about 0.4 $\mu$m is placed above the first mirror at a height of 27 $\mu$m in order to select the first quantum state. The other states are efficiently removed, except for the second state, which is still present with a contribution of a few percent. In region two, a second mirror is placed after the first one. Piezo elements attached underneath induce a fast modulation of the surface height with amplitude $a$ according to Eq. \ref{eq:4}.

As an example, we consider transitions between state $|1\rangle \rightarrow |2\rangle$ for the most probable velocity at the PF2/UCN beam position, $6$ m/s. The length $l = 15$ cm of this mirror is chosen in such a way to provide a neutron in a superposition of these two quantum states after $\tau = 25.0$ ms. Region three has a flight path of $L = 80$ cm on a single mirror between the two oscillators in region two and, identical to region two, in region four. In region five, a state selector as an analyzer is placed, identical to the selector in region one but with a neutron detector behind for counting the transmitted neutrons. Calculated transition probabilities \cite{12} for $|p\rangle$ and $|q\rangle$ as a function of $\omega$ is shown in Fig. 4 for different parts of the measured velocity spectrum.

This method can also be applied to stored ultra-cold neutrons. Fig. 5 shows the theoretical Ramsey signal for a neutron storage time of 100 s. The appeal of a neutron storage lies in a very narrow resonance line. A search for phase shifts are suggested in the next section.

![FIG. 4: Transition probability for a neutron velocity 5.5 m/s $< v < 6.5$ m/s (red) and 3.0 m/s $< v < 9.0$ m/s (blue) as a function of $\omega [s^{-1}]$. The strength of the vibration was set to 1.6$m/s^2$.](image)

![FIG. 5: Theoretical Ramsey signal for a neutron storage time of 100 s. Again, the strength of the vibration was set to 1.6$m/s^2$.](image)
III. PHASE SHIFTS FROM HYPOTHETICAL GRAVITY-LIKE FIFTH FORCES

Theoretical considerations arising from higher-dimensional gravity, gauge forces or massive scalar fields suggest that the Newtonian gravitational potential for masses \( m_i \) and \( m_j \) and distance \( r \) should be replaced by a more general expression including a Yukawa term,

\[
V(r) = -G \frac{m_i m_j}{r} (1 - \alpha \cdot e^{-r/\lambda}),
\]

where \( \lambda \) is the Yukawa distance over which the corresponding force acts and \( \alpha \) is a strength factor in units of Newtonian gravity. \( G \) is the gravitational constant. Most interesting, from the experimental point of view, are scenarios, where the strength of the new force is expected to be many orders of magnitude stronger than Newtonian gravitation. Such forces are possible via abelian gauge fields in the bulk \[37, 38, 39, 40, \] (see also \[41, 42\] for explicit realizations in string theory). The strength of the new force would be \( 10^6 < \alpha < 10^{12} \) stronger than gravity, independent of the number of extra dimensions \( n \) \[39\].

The observation of quantum states already tests speculations of this kind on large extra dimensions of submillimeter size of space-time \[22, 43, 44\]. Most recent theoretical developments support the original proposal of large extra dimensions with bulk gauge fields and more specific predictions for a high interaction strength can be made. One proposal of Callin et al. \[45\] predicts deviations from predictions for a high interaction strength can be made. Our approach of probing Newtonian Gravity at the micron scale with the help of Ramsey’s Method of Separated Oscillating Fields is advantageous because of its small systematic effects. In contrast to atoms the electrical polarizability of neutrons \[53\] inducing such Casimir effects or van der Waals forces is extremely low. This together with its electric neutrality the neutron provides the key to a sensitivity of more than 10 orders of magnitude below the background strength of atoms.

The dynamics of such a quantum mechanical wave packet combines quantum theory with aspects of Newtonian mechanics at short distances. When a neutron with mass \( m \) approaches the mirror, the mass of this extended source might modify the earth acceleration \( g \), when strong non-Newtonian forces with range \( \lambda \) and strength \( \alpha \) are present. For small neutron distances \( z \) from the mirror, say several micrometres, we consider the mirror as an infinite half-space with mass density \( \rho \). By replacing the source mass \( m_i \) by \( dm_i \) and integrating over \( dm_i \), the modified Newtonian potential \( \Delta V(z) \) is having the form

\[
\Delta V(z) = 2\pi m \rho \lambda^2 G e^{-|z|/\lambda} = 8.47 \times 10^{-14} \alpha \lambda^2 e^{-|z|/\lambda} \text{peV}
\]

with \( \rho = 19 \text{ g/cm}^3 \) (gold or tungsten coating) and \( \lambda \) given in \( \mu \text{m} \). Taking these gravity-like forces into account, first order perturbation theory predicts a shift of the \( n \)-th energy eigenvalue \[22\],

\[
\Delta E_n = \langle \psi_n | \Delta V(z) | \psi_n \rangle.
\]

They differ from state to state in the range of interest. The sensitivity can be seen in Fig. 6, where the energy shift as a function of range \( \lambda \) is plotted for a fixed \( \alpha = 10^{12} \).

Newtonian gravity and hypothetical fifth forces evolve with different phase information in the non-oscillating region. We expect the following sensitivity for 50 days of beam time at the PF2-UCN beam position at the ILL: With a count rate of 0.1 \( \text{s}^{-1} \) for neutrons in the ground state, we will have \( N = 430000 \) registered neutrons. Due to the uncertainty principle \( \Delta \Phi \Delta N \geq 2\pi \), we estimate a minimal detectable phase shift of \( 9.6 \times 10^{-13} \) radians. For an estimate of \( T = 130 \text{ ms} \) interrogation time (flight path between the oscillators), the minimal resolvable energy shift is

\[
\Delta E = \Delta \Phi \hbar / T = 0.096\hbar / s = 4.8 \times 10^{-5} \text{peV}.
\]

Together with Eq. (14), this corresponds to a sensitivity of \( \alpha = 7.4 \times 10^7 \), which is about three orders of magnitude better than existing limits right now. In principle,
The spin-dependent part in presence of matter given by \[\text{(57)}\] is cosmological data (see e.g. \[\text{(55, 56)}\]). The CP-violating allowed by the otherwise stringent constraints posed by the otherwise stringent constraints posed by \(\text{orders of magnitude}. The statistical sensitivity of the new method is therefore around

\[
\Delta E = 4.8 \times 10^{-21} \text{eV} \tag{17}
\]
or

\[
\alpha < 7.6 \times 10^3. \tag{18}
\]

This is orders of magnitude better than existing limits right now.

Limits for hypothetical fifth forces can be easily interpreted as bounds of the strength of the matter couplings of axions. Axion interactions with a range within \(20 \mu m < \lambda < 200 \text{nm}\) (corresponding to axion masses \(10^{-6} \text{eV} \leq m_a < 10^{-2} \text{eV}\)), the “axion window”, are still allowed by the otherwise stringent constraints posed by cosmological data (see e.g. \[\text{(55, 56)}\]). The CP-violating spin-dependent part in presence of matter given by \[\text{(57)}\] is related to the geometry of the macroscopic matter configuration.

Integrating this potential over the geometry of region three \[\text{(22)}\], we arrive at \(\lambda = 5 \mu m\) at a limit of

\[
\frac{g_a g_p}{\hbar c} \leq 5.3 \times 10^{-23}. \tag{20}
\]

for the dimensionless axion coupling strength. This is again eight orders of magnitude better than the only existing limit \[\text{(22, 23)}\] in the axion window from the previous experiment with neutrons.

### IV. SUMMARY

In conclusion, we discussed an application of Ramsey’s method of oscillating fields to the quantum bouncer. It will allow high precision spectroscopy of the energy eigenstates of a neutron bouncing on a flat vertical surface. Such Ramsey type interference measurements will improve the sensitivity for neutron’s coupling to gravity, to hypothetical short ranged forces or the influence of the cosmological constant. A sensitivity of 21 orders of magnitude below the strength of electromagnetism is found, when the energy \(\Delta E = 4.8 \times 10^{-21} \text{eV}\) of Eq. (17) is compared with the Rydberg energy of 13.6 eV, which is the energy scale of electromagnetically bound quantum systems. Such an energy change corresponds to a strength \(\alpha \sim 7.6 \times 10^3\) compared to gravity or to \(\frac{g_a g_p}{\hbar c} \sim 5.3 \times 10^{-23}\), the axion coupling strength, at a range \(\lambda = 5 \mu m\).

The new method profits from small systematic effects in such systems, mainly due to the fact that in contrast to atoms, the electrical polarizability of neutrons is extremely low. Neutrons are not disturbed by short range electric forces such as van der Waals or Casimir forces. Together with its neutrality, this provides the key to a sensitivity of several orders of magnitude below the strength of electromagnetism.

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[1] R. L. Gibbs, Am. J. Phys. 43 25 (1975).
[2] H. C. Rosu, [arXiv:quant-ph/0104003](http://arxiv.org/abs/quant-ph/0104003).
[3] Julio Gea-Banacloche, Am. J. Phys. 67 776 (1999).
[4] R.W. Robinett, Phys. Reports 392 1 (2004).
[5] H. Abele, Progress in Particle and Nuclear Physics 60 1 (2008).
[6] V. Nesvizhevsky et al., Nature, 415 297 (2002).
[7] V. Nesvizhevsky et al., Eur. J C40 479 (2005).
[8] V. Nesvizhevsky et al., Phys. Rev. D, 67 1022002 (2003).

![Fig. 6: The relative frequency shift vs. range \(\lambda\). The strength of the modified Newtonian potential \(\Delta V(z)\) of Eq. (15) is set to \(\alpha = 10^{12}\).](chart.png)
[9] A. Westphal et al., Eur. Phys. J. C 51 367 (2007). [arXiv:hep-ph/0612093].
[10] V.I. Luchshkov and A.I. Frank, JETP Lett. 28 559 (1978).
[11] H. Wallis et. al., Appl. Phys. B 54 407 (1992).
[12] I. L. Rabi et al., Phys. Rev. 55 526 (1939).
[13] J. M. B. Kellogg et al., Phys. Rev. 56 728 (1939).
[14] J. M. B. Kellogg et al., Phys. Rev. 57 677 (1940).
[15] N. F. Ramsey, MOLECULAR BEAMS, Oxford at the Clarendon Press 1956.
[16] N. F. Ramsey, Rev. Mod. Phys. 62 3, p. 541-552 (1990)
[17] A. Cronin, J. Schmiedmayer, D. Pritchard, Rev. Mod. Phys. in print (2009). [arXiv:0712.37031]
[18] L. M. K. Vandersypen and I. L. Chuang, Rev. Mod. Phys. 76 4, 1037 (2005).
[19] C. F. Roos et al., Nature 443, 316 (2006).
[20] M. DeKieviet et al., Phys. Rev. Lett. 75, 1919 (1995).
[21] C.A. Baker et al., Phys. Rev. 97 131801 (2006), [arXiv:hep-ex/0602020].
[22] A. Westphal et al., [arXiv:hep-ph/0703108].
[23] S. Baeßler et al., Phys. Rev. D 75, 075006 (2007).
[24] M. Kreuz et al., [arXiv:0902.0156].
[25] Pignol et al., [arXiv:0708.2541].
[26] W. A. Hamilton et al., Phys. Rev. Lett. 58, 2770 (1987).
[27] J. Felber et al., Physica B 162, 191 (1990).
[28] J. Felber et al., Phys. Rev. A 53 319 (1996).
[29] T. Hils et al., Phys. Rev. D 58, 4784 (1998).
[30] A. Steane et al., Phys. Rev. Lett. 74, 4972 (1995).
[31] P. Szriftgiser et al., Phys. Rev. Lett. 77, 4 (1996).
[32] S. Bernet et al., Phys. Rev. Lett. 77, 5160 (1996).
[33] H. Abele, Symposium on precision measurements at low energies, ETH Zurich, 2 June 2008.
[34] A. Yu. Voronin et al., Phys. Rev. D 73 044029 (2006).
[35] T. Jenke, Diploma Thesis, University of Heidelberg, 2006, unpublished.
[36] D. Stadler, Diploma Thesis, University of Heidelberg, 2009, unpublished.
[37] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429, 263 (1998) [arXiv:hep-ph/9803315].
[38] I. Antoniadis et al., Phys. Lett. B 436, 257 (1998) [arXiv:hep-ph/9804398].
[39] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Rev. D 59, 086004 (1999) [arXiv:hep-ph/9807344].
[40] I. Antoniadis, Lect. Notes Phys. 631, 337 (2003).
[41] D. Cremades, L. E. Ibanez and F. Marchesano, Nucl. Phys. B 643, 93 (2002) [arXiv:hep-th/0205074].