Unification of Couplings and Proton Decay

in SUSY GUTs *

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Abstract

We show that gauge couplings unify more precisely in the region of SUSY parameter space already preferred by Yukawa unification. While proton decay due to dimension 5 operators is maximally suppressed in this region, the contribution from dimension 6 operators is enhanced as a consequence of lower unification scale.

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Introduction

It has been pointed out that assuming high degree of universality in soft SUSY breaking terms and positive $\mu$ (preferred by $b \to s\gamma$ and the muon anomalous magnetic moment) top-bottom-tau Yukawa coupling unification (motivated by SO(10) symmetry) can be satisfied only in a narrow region of SUSY parameter space \cite{1}. In that analysis the minimal SO(10) boundary conditions for soft SUSY breaking parameters at the GUT scale were considered: universal squark and slepton masses – $m_{16}$; universal gaugino masses – $M_{1/2}$; universal trilinear couplings – $A$; and non-universal Higgs $H_u, H_d$ masses (non-universality in Higgs masses is well motivated by GUT scale threshold corrections \cite{1}). The last two parameters can be exchanged for $\mu$ (SUSY Higgs mass) and $m_A$ (CP odd Higgs boson mass) by requiring radiative electroweak symmetry breaking with resulting values of $\mu$ and $m_A$. The region preferred by Yukawa coupling unification is specified by $\mu, M_{1/2} \ll m_{16}, A \sim -2m_{16}$ and $\tan \beta \sim 50 \pm 2$. The fit is improving and the region of allowed $\mu$ and $M_{1/2}$ grows with increasing $m_{16}$. These results can be understood by studying SUSY threshold corrections \cite{1} and have been verified in independent analysis \cite{2}. The squark and slepton masses have to be quite heavy, $m_{16} \gtrsim 1.4$ TeV, preferably few TeV. Nevertheless, it was shown that these boundary conditions lead to maximal hierarchy between first two generations and the third generation due to RG running \cite{3}. Therefore, taking $m_{16} \simeq 10$ TeV may still be considered natural, since the mass of the stop will be in a TeV region, roughly $m_{16}/10$. Masses of the first two generations of squarks and sleptons are close to $m_{16}$.

It is well known that heavier superpartners lead to more precise unification of gauge couplings \cite{4, 5}. Since heavy superpartners are already required by Yukawa unification, it is interesting to see how gauge coupling unification improves in the same region of SUSY parameter space.

Gauge coupling unification

Precise analysis of gauge coupling unification, including two-loop renormalization group equations, leading-log \cite{4} and finite \cite{5} weak scale threshold corrections, together with improved measurements of electroweak data reveals that gauge couplings miss each other with a significant discrepancy. If we define the GUT scale as a scale at which the first two
gauge couplings meet, $\alpha_1(M_G) = \alpha_2(M_G) \equiv \alpha_G$, the value of the third gauge coupling at the weak scale can be predicted. With mSUGRA boundary conditions and requiring $m_0 < 2$ TeV the predicted $\alpha_s$ is $0.125 < \alpha_s(M_Z) < 0.143$ [5, 6, 7], which should be compared with the current experimental value of the strong coupling constant, $\alpha_s(M_Z) = 0.1172 \pm 0.002$ [8]. This discrepancy between predicted and measured value of $\alpha_s$ can be parametrized by $\epsilon_3 \equiv (\alpha_3(M_G) - \alpha_G)/\alpha_G$. The above mentioned results for $\alpha_s(M_Z)$ translate into $-5\% < \epsilon_3 < -2.5\%$. Let us see what we get in the region suggested by Yukawa unification.

The solid line in Fig. 1 (left) represents $\epsilon_3$ as a function of $m_{16}$ with $\mu = M_{1/2} = m_A = 0.1 m_{16}$, $\tan \beta = 50$ and $A$ being linearly increased from $-1.7 m_{16}$ for $m_{16} = 1$ TeV to $-2.0 m_{16}$ for $m_{16} = 15$ TeV (to avoid negative stop mass squared which would occur if we took $A = -2 m_{16}$ for $m_{16} \lesssim 2$ TeV with other parameters fixed as explained). In the same figure (right) we see further variation of $\epsilon_3$ for one point from the plot on the left corresponding to $m_{16} = 5$ TeV. For different values of $\mu$ and $M_{1/2}$ (up to 0.2 $m_{16}$), $\epsilon_3$ varies by $\sim 0.8\%$ and this variation corresponds to dashed lines in the plot on the left. The shaded band correspond to $1\sigma$ region for gauge coupling unification. With our choice of parameters, the region $m_{16} \lesssim 2$ TeV is excluded due to chargino and/or stop masses being below experimental bounds. For $m_{16} = 15$ TeV the mass of the stop is below 2 TeV and so it still might be acceptable. We call the region of $m_{16} = [2, 15]$ TeV with other parameters varied as above the region with “acceptable spectrum”. Thus we see that in this region $\epsilon_3$ varies from $-5\%$ to $-1\%$ and gets close to the band where gauge couplings unify within $1\sigma$. 

FIG. 1: Value of $\epsilon_3$ in % as a function of $m_{16}$ with all other parameters fixed as explained in the text (left); and as a function of $(\mu, M_{1/2})$ for $m_{16} = 5$ TeV and $A_0 = -1.8 m_{16}$ (right).
FIG. 2: Value of $M_G$ ($10^{16}$ GeV) as a function of $m_{16}$ with all other parameters fixed as explained in the text (left); and as a function of ($\mu$, $M_{1/2}$) for $m_{16} = 5$ TeV and $A_0 = -1.8 m_{16}$ (right).

**Scale of unification and proton decay**

It is well known, see for example Ref. 9, that the amplitude squared for proton decay due to dimension 5 operators (mediated by color triplet higgsinos) scales approximately as $M_{1/2}^2/m_{16}^4$. Therefore, as far as SUSY spectrum is concerned, proton decay is maximally suppressed in the region required by Yukawa coupling unification.

Perhaps more interesting thing related to proton decay is the value of the GUT scale itself. With increasing $m_{16}$ gauge couplings not only unify more precisely, they unify at lower scale. Fig. 2 (left) shows $M_G$ as a function of $m_{16}$ with all other parameters fixed as in Fig. 1. In the region with “acceptable spectrum” $M_G$ varies between $3.5 \times 10^{16}$ and $1.5 \times 10^{16}$ GeV. In Fig. 2 (right) we can see further variation of $M_G$ for one point corresponding to $m_{16} = 5$ TeV. For different values of $\mu$ and $M_{1/2}$ it varies by $\sim 0.5 \times 10^{16}$ GeV so the curve in Fig. 2 (left) can be shifted up and down by this amount. Comparing with Fig. 1 we see that in the region where gauge couplings start to unify within 1σ, the corresponding value of the GUT scale is $M_G \simeq 1.0 \times 10^{16}$ GeV. This however highly enhances proton decay due to dimension 6 operators (mediated by heavy gauge bosons). The amplitude squared is proportional to $1/(M_G)^4$. For $M_G = 1.0 \times 10^{16}$ GeV the proton lifetime is $\sim 5 \times 10^{34}$ yr! This is just an order of magnitude from the current experimental bound and about 100 times larger than the lifetime obtained with usual assumption of $M_G \simeq 3.0 \times 10^{16}$ GeV corresponding to TeV scale SUSY breaking parameters.
Conclusions

I think it is highly non-trivial that there exists a region of SUSY parameter space which is simultaneously favored by gauge coupling unification, Yukawa coupling unification and proton decay. There is no reason why improving the situation with one feature of GUTs should not go in a wrong direction for another. Moreover, the same region is also favored when considering natural suppression of flavor and CP violation \[3\], and can provide the right amount of neutralino relic density \[10\]. Perhaps the only thing which does not favor this region is the naturalness of EWSB.

Finally, it is interesting to note, that gauge coupling unification, Yukawa coupling unification and proton decay received some attention within higher dimensional GUTs. Sometimes problems associated with these features in four dimensional GUTs are used as a motivation for higher dimensional models \[11\]. Comparing to these, having somewhat heavy superpartners might not look so crazy anymore. :)

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