Calculation of the optimal number of redundant elements of power systems using the Lagrange multipliers method and information theory

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Abstract. The article analyzes the applicability of information theory tools in solving problems of analysis of functioning engineering networks. This applicability is justified by the laws of distribution of random variables. There is a need to develop methods for determining the number of redundant elements. A method for calculating the optimal number of redundant elements of an engineering network based on the method of Lagrange multipliers taking into account the measure of information uncertainty is proposed. At the end of the article, an example of calculating the number of redundant elements that should be added to the system to increase the system reliability level to the required value is shown.

1. Introduction

To date, among the current tasks to improve the quality of analysis of functioning systems of various kinds, including electrical networks, we can note the development of innovative methods for analyzing these networks. In practice, there is often a situation when a built and functioning system already has failures of certain elements, due to stochastic and uncertain perturbations and external influences on them. Speaking about the reliability of distribution networks, it is worth noting that in Russia, the accident rate of these networks is 6 times higher than in foreign countries, since this is certainly caused by the climate component. At the same time, in case of failures that affect the loss of power supply to certain consumers, these failures must be eliminated in the shortest possible time, depending on the reliability category. Adding redundant elements to the system is one of the methods that minimize the risk of long-term lack of power supply to electricity consumers. While the number of these redundant elements directly depends on the reliability category of the consumer supplied with electricity (category I, II or III). All this supports the development of innovative methods for calculating the number of redundant elements using new tools and methods in calculations. One of the promising areas is the use of information theory.
2. The applicability of the tools of information theory
A suitable approach to solving this problem can be called the use of tools from such a field of knowledge as information theory [1-3]. The main tools of information theory include the amount of information and the measure of information uncertainty. These concepts were first compared in 1948 by Claude Elwood Shannon in [4]. It should be emphasized that C. Shannon understood information as useful signals for the recipient. Shannon considered the noise and interference useless. If the output signals are similar to the input signals, then this indicates that there is no information entropy. The absence of noise indicates the presence of maximum information.

The relationship between entropy and information was quantitatively justified by Leon Brillouin in [5], while the amount of information in his works is called negentropy (the presence of a negative prefix neg (from negative) indicates the opposite of entropy). The paper [6] provides a justification for the relationship between entropy and negentropy in the case of a change in the state of the engineering network. The essence of this relationship is that a change in one component inversely affects the other.

The formula for calculating the amount of information in the case of different probabilities of events was derived by Claude Shannon in the middle of the XX century by generalizing the formula of Ralph Winton Lyon Hartley [7], derived in 1928 and applicable only for equally probable events. The Claude Shannon's formula [8] has the following form:

$$ I = -\sum_{i=1}^{n} p_i \log p_i \text{, bit,} $$  

(1)

where \( I \) is the amount of information; \( p_i \) is the probability of occurrence of the event \( i = 1, 2, \ldots, n \).

However, Claude Shannon himself attributed the applicability of this formula only to the areas of signal transmission and encoding. Later, the vast applicability of this expression was repeatedly confirmed, and the Soviet scientist Yevgeny Sedov published the book "One formula and the whole world. A book about entropy" in 1982 [9], which affirms the universality of the concept of the formula for calculating information entropy.

The validity of the application of the measure of information uncertainty is due to the fact that almost all the data used by various technical systems are more or less uncertain, that is, for their analysis, you can use the formula developed by Claude Shannon, which allows you to remove this uncertainty and express the amount of information in quantitative form and thereby distinguish between indicators on a qualitative basis. The second reason for the validity of the applicability of the information uncertainty measure is that these measurements are subject to the laws of distribution of random variables. In particular, even classical calculations of structural reliability are based on the exponential law, where the base of the logarithm is the Napier's constant \( e \).

3. Calculation of the optimal number of redundant elements of the electrical network using the method of Lagrange multipliers
As mentioned earlier, one of the important points in analyzing the structure of a functioning engineering network is the addition of redundant elements to the system. This addition is justified by the fact that it is likely that the degree of system reliability will not be at a high enough level. In articles [10-11], a restriction was formed taking into account the number of reserving elements. The system of these restrictions is presented below:
\[
\begin{align*}
\sum_{i=1}^{n} x_i q_i^{l_i} \cdot (-\log_2 q_i) &\leq H^0(Q_{01}); \\
\sum_{i=1}^{n} x_i q_i^{l_i} \cdot (-\log_2 q_i) &\leq H^0(Q_{02}); \\
&\vdots \\
\sum_{i=1}^{n} x_i q_i^{l_i} \cdot (-\log_2 q_i) &\leq H^0(Q_{0L}),
\end{align*}
\]

where \( x_j \geq 0, \ j=1,2,\ldots,n, \ x_l \) — number \( i \)-th of redundant elements on the network section (connection 0-1); \( v \) — the power source; \( n \) — number of elements belonging to the connection 0-1; \( H^0(Q_{0l}) = q_i^l \log_2 q_i^l \) — the boundary value of the entropy of a non-operable state of the connection 0-1 (if this value isn’t met, the condition for achieving the required number of redundant elements is violated), \( q_i^l \) — the probability of a possible break in power supply of consumer \( l \); \( q_i \) — the probability of the non-operable state of element \( i \) belonging to the connection 0-1.

The objective function has the following form:

\[
V(x) = \sum_{j=1}^{n} v_j x_j \to \min,
\]

where \( V \) is the economic cost of building an engineering network; \( v \) is the cost of building elements of the engineering network.

Next, apply the method for calculating the number of redundant elements. It is important to understand that each redundant element leads to unnecessary economic costs, so it is very important to correctly calculate the minimum required number of these elements, but at the same time that they allow you to keep the system at the required level of reliability.

Let’s demonstrate an example of determining the number of redundant elements of an engineering network using the method of Lagrange multipliers, taking into account the measure of information uncertainty. In order to apply this method, the following assumption must be taken into account: constraints are an equation, not inequalities.

Suppose we have a simple network consisting of two series-connected elements, in this case, the objective function and the constraint will take the following form:

\[
V(x) = v_1 x_1 + v_2 x_2, \quad q_1^{\prime} \log_2 q_1 = H^0(Q_1), \text{ take the following value: } v_1 = v_2 = 20000\$, \ q_1 = 0.081, \text{ and } q_2 = 0.082.
\]

Let’s take the boundary values of the annual average duration of a non-operative state for electroreceiver 1: \( M_{q1} = 80 \) hours.

Determine the probabilities of the no-break power supply and allowable failure in electrical power supply:

\[
p_1^0 = \frac{M_{q1}}{T} = \frac{8680}{8760} = 0.9909; \quad q_1^0 = \frac{M_{q1}}{T} = 0.009;
\]

\[
H^0(Q_1) = -q_1^0 \log_2 q_1^0 = -0.009 \log_2 0.009 = 0.0612;
\]

\[
-0.081^{q_1} \log_2 0.081 - 0.082^{q_2} \log_2 0.082 = 0.0612;
\]

\[
0.081^{q_1} \cdot 3.62 + 0.082^{q_2} \cdot 3.6 = 0.0612;
\]

\[
3.62 \cdot (0.081^{q_1} + 0.082^{q_2}) = 0.0612;
\]

\[
0.081^{q_1} + 0.082^{q_2} = 0.017.
\]
As a result of a series of transformations, the Lagrange’s formula takes the following form:

\[ L(x_1, x_2, \lambda) = 200 \cdot x_1 + 200 \cdot x_2 + \lambda \cdot (0.081^{x_1} + 0.082^{x_2} - 0.017). \]

Next, we differentiate by all unknowns and find \( \lambda \), calculate the number of reserves, if there are 2 possible reserves, then find the one that will be at the minimum cost:

\[
\frac{\partial L(x_1, x_2, \lambda)}{\partial x_1} = 200 + \lambda \cdot (0.081^{x_1})' = 0
\]
\[
\frac{\partial L(x_1, x_2, \lambda)}{\partial x_2} = 200 + \lambda \cdot (0.081^{x_2})' = 0
\]
\[
\frac{\partial L(x_1, x_2, \lambda)}{\partial \lambda} = 0.081^{x_1} + 0.082^{x_2} - 0.017 = 0
\]
\[
L'(x_1) = 200 + (\lambda \cdot 0.081^{x_1})' = 0
\]
\[
L'(x_2) = 200 + (\lambda \cdot 0.082^{x_2})' = 0
\]
\[
L'(\lambda) = -0.082^{x_2} + 0.017 \cdot 0.081^{x_2} + 0.018 = 0
\]

\[
L'(x_1) = 200 + (\lambda \cdot 0.081^{x_1})' = 0
\]
\[
\lambda \cdot (0.081^{x_1})' = -200
\]
\[
(0.081^{x_1})' = \frac{-200}{\lambda}
\]
\[
0.081^{x_1} \cdot \ln(0.081) = \frac{-200}{\lambda}
\]
\[
0.081^{x_1} = \frac{-79.5}{\lambda}
\]
\[
x_1 \approx \log_{0.08} 80 \frac{80}{\lambda}
\]

\[
L'(x_2) = 200 + (\lambda \cdot 0.082^{x_2})' = 0
\]
\[
\lambda \cdot (0.082^{x_2})' = -200
\]
\[
(0.082^{x_2})' = \frac{-200}{\lambda}
\]
\[
0.082^{x_2} \cdot \ln(0.082) = \frac{-200}{\lambda}
\]
\[
0.082^{x_2} = \frac{-80}{\lambda}
\]
\[
x_2 = \log_{0.082} 80 / \lambda
\]

\[-0.082^{x_1} + 0.017 \cdot 0.081^{x_2} + 0.018 = 0\]

Substitute \( x_1 \) and \( x_2 \) and find \( \lambda \):
\[-0.08 \log_{0.08} \frac{80}{\lambda} + 0.017 - 0.082 \log_{0.08} \frac{80}{\lambda} + 0.018 = 0;\]
\[-\frac{80}{\lambda} + 0.017 - \frac{80}{\lambda} + 0.018 = 0\]
\[-\frac{160}{\lambda} = -0.034\]
\[\lambda = \frac{160}{0.034} = 4705.\]

Determined \(x_1\) and \(x_2\): \[x_1 = \log_{0.08} \frac{80}{\lambda} = \log_{0.08} \frac{80}{4705} = \log_{0.08} 0.017 = 1.613 \approx 2\]
\[x_2 = \log_{0.082} \frac{80}{\lambda} = \log_{0.082} 0.017 = 1.629 \approx 2\]

Thus, the required number of redundant elements is two.

It is worth noting that this solution is applicable only if the constraints are equalities, but if the constraints are inequalities, then it is necessary to use the Kuhn-Tucker theorem and find the saddle point of the Lagrange function.

4. Conclusion

As a result, it can be stated that the scope of application of the measure of information uncertainty isn’t limited to the field of information theory, since an increasing number of areas are beginning to apply it in solving their problems. Ongoing research in the field of developing methods for analyzing the structural reliability of the electric network allows us to conclude that it is possible to express the structural reliability in a quantitative form through a measure of information uncertainty. This expression allows you to analyze the network structure and identify the least reliable parts of the network. The developed method based on the method of Lagrange multipliers allows calculating the optimal number of redundant elements needed to maintain the system at the required level of reliability.

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