In this paper we investigate weak decays of heavy mesons in the framework of a covariant quark model, which is based on the Bethe-Salpeter equation in instantaneous approximation. Apart from a phenomenological confinement potential, a residual interaction induced by instantons is adopted. Masses and many decay observables of light mesons have already been described successfully in this model [4,5]. An appropriate extension allows a unified description of light and heavy systems. Using a set of parameters which are fixed by the mass spectra, we evaluate the form factors of semileptonic decays of charmed and bottom mesons. In the heavy quark limit these can be reduced to the Isgur-Wise function, which is calculated. Finally the form factors are used to determine the non-leptonic decay rates of B mesons in the factorization approximation.

1. INTRODUCTION

In the last few years new and improved data on the spectra and the decays of charmed and bottom mesons have become available. The observations of the $D_s^*$ and the radial excited $D^{*'}$ and $B'$ have been published recently (see [23] and references therein). The dominant semileptonic decays of $B$, $D$ and $D_s$ mesons have been measured with good precision in the meantime, and data for double Cabibbo-suppressed channels are also available. Yet many new results will be provided by the B-factories BABAR, BELLE, HERA-B and LHC-B within the next years.

For the theoretical description of these masses and decays various ansätze are pursued. Lattice gauge theory gives good results for the transition form factors for high momentum transfer $q^2$, while QCD sum rules are suitable for the low $q^2$ regime. Heavy quark effective theory (HQET), which is based on a systematic expansion in the inverse mass of the heavy quark, provides some model independent predictions, i.e. approximate degenerated mass doublets according to different orientation of the heavy quark spin, and a connection between the semileptonic partial decay rate of the pseudoscalar and pseudovector decay channels, which has been experimentally confirmed. Unfortunately it can not predict the masses and decay rates itself, and corrections due to the finite quark mass, at least for the charm quark, are expected to be substantial. Hence for a consistent description of meson masses and decays over the full kinematic region, constituent quark models, even if the connection to the underlying theory is not quite clear, are still the most successful tool.

In previous papers we have developed a relativistic constituent quark model for light mesons, which is based on the Bethe-Salpeter equation in instantaneous approximation. Apart from a phenomenological confinement potential we adopt a residual interaction induced by instantons. In this model, a very good description of the light meson masses, from the ground state nonet up to highest angular momenta, has been achieved. Also many decay observables have been calculated in reasonable agreement with the experimental data (see [4] for a recent update). Motivated by this success, the model has been extended for heavy flavours [6]. Thereto we do not apply the One Gluon Exchange, which is known to give a good description of heavy quarkonia and even of the whole meson spectrum with a suitable inclusion of relativistic effects [17]. Instead we generalize the instanton induced interaction in a naive way. This is done in order to keep the model simple and unified: the parametrization of confinement should be valid for all flavours, and only one residual interaction is introduced. Also the question should be raised, when and how the model for light mesons fails, if higher quark and meson masses are involved. It turns out, that heavy-light mesons and, to a certain extent, heavy quarkonia can be described in that way. The resulting spectra together with a brief review of our model are discussed in section II. Knowing the meson amplitudes we then calculate the semileptonic decays without further parameters in section III. This is done in order to test these amplitudes rather than to determine decay rates.
or CKM matrix elements with high precision. Therefore we concentrate on the dominant processes $B \to D^{(*)} \ell \bar{\nu}$ and on the CKM-favoured decay channels of $D$ and $D_s$ mesons. Finally in section IV we investigate non-leptonic decays of $B$ mesons, and section V contains our summary.

II. THE BETHE-SALPETER MODEL

Since the model has been described earlier in greater detail \[11\], we will only briefly review the main features. The model is based on the Bethe-Salpeter equation for $q\bar{q}$ bound states

$$\chi^P(p) = -i S^F_1 \left( \frac{P}{2} + p \right) \int \frac{d^4p'}{(2\pi)^4} K(P;p,p') \chi^{P'}(p') S^F_2 \left( -\frac{P}{2} + p' \right)$$

(1)

for the Bethe-Salpeter amplitude $\chi^P(p) := \langle 0 | T \psi(p) \bar{\psi}(\bar{p}) | P \rangle$. Here $|P\rangle$ denotes a bound state with mass $M$ and total momentum $P$, $P^2 = M^2$. In our ansatz the full quark propagators $S^F_i$ are approximated by free fermion propagators $S^F_i(p) \approx i/(\gamma - m_i + i\epsilon)$, where the constituent quark masses $m_i$ are introduced, which are treated as free parameters of the model. Furthermore, the irreducible interaction kernel $K$ is assumed to be instantaneous in the rest frame of the meson, $K(P;p,p')|_{P=(M,0)} = V(\bar{p},p')$. These assumptions lead to the (full) Salpeter equation

$$\Phi(\bar{p}) = \Lambda^-_1(\bar{p}) \gamma^0 \left[ \int \frac{d^3p'}{(2\pi)^3} \frac{V(\bar{p},p') \Phi(p')}{M + \omega_1 + \omega_2} \right] \gamma^0 \Lambda^+_1(\bar{p}) \gamma^0 - \Lambda^-_1(\bar{p}) \gamma^0 \left[ \int \frac{d^3p'}{(2\pi)^3} \frac{V(\bar{p},p') \Phi(p')}{M - \omega_1 - \omega_2} \right] \gamma^0 \Lambda^+_1(\bar{p}) \gamma^0$$

(2)

for the Salpeter amplitude $\Phi(\bar{p}) := \int \frac{d^3p'}{(2\pi)^3} \chi^P(p) |_{P=(M,0)}$. Here $\omega_i := \sqrt{\bar{p}^2 + m_i^2}$ and $\Lambda^\pm$ denotes the projector on positive and negative energy solutions of the Dirac equation.

As ansatz for the interaction kernel we apply the sum of a phenomenologically motivated confinement potential and a residual interaction, which is induced by instantons. Confinement is parametrized by a linearly rising potential in configuration space with an adequate Dirac structure, symbolically written as

$$V_C(r) = (a_c + b_c \cdot r) \Gamma \otimes \Gamma.$$

To estimate the influence of the Dirac structure, two possibilities are taken into account, in the following referred to as model $A$ and model $B$: a combination of scalar and time-like vector, which has already been discussed in [11] and which is known to minimize spin-orbit splittings, and a chirally invariant combination of scalar, pseudoscalar and vector type, previously investigated by [8] and [9].

The additional instanton induced residual interaction is based on the work of 't Hooft [10] and Shifman et al. [11]. The effect of QCD instantons on the quarks leads to an effective two body interaction, which can be expressed by the following kernel in momentum space

$$\int \frac{d^3p'}{(2\pi)^3} V_{int}(\bar{p}, p') \Phi(\bar{p}) = 4G \int \frac{d^3p'}{(2\pi)^3} R_\Lambda(\bar{p}, p') \cdot (\text{tr} [1 \Phi(p')]) \cdot (\text{tr} \left[ \gamma^5 \Phi(p') \right] \gamma^5).$$

(3)

The strengths of the coupling are collected in the flavour coupling matrix

$$G_{f_2 f_3 f_1 f_4} = \begin{cases} -g_{f_2 f_3} : f_1 = f_3 \neq f_2 = f_4 \\ g_{f_2 f_3} : f_1 = f_2 \neq f_3 = f_4 \\ 0 : \text{ otherwise,} \end{cases}$$

(4)

with flavour indices $f_i$. The originally point like interaction is regularized by the function $R_\Lambda$, for which a gaussian form is used.

Heavy mesons have been included in this model to achieve a unified description of all mesons. Especially the parametrization of confinement should be universal. As shown in [11], this is done most successfully by extending the instanton induced residual interaction in a naive way for heavy-light systems. This is done at the level of the interaction kernel eq. (3) by simply allowing $f_i \in \{ u, d, s, c, b \}$ in (4). It has to be stressed that this extension is purely phenomenologically motivated, since the derivation leading to (3) is strictly valid for light quarks only. Also it should be mentioned that this use of the instanton induced interaction allows the mixing of heavy and light flavours.
in the scalar and pseudoscalar flavour-neutral sectors. This has been investigated in [13] but will be neglected in this paper, since the effect turned out to be very small.

Due to the $J = 0$ selection rule of the residual interaction, a gradual fit of the model parameters to the light meson masses has been performed as explained in [13], leading to the parameter sets in table II. The resulting spectra of the isovector mesons are shown in fig. (1). The Regge trajectory is reproduced excellently in both models. The spectra of the light isoscalars and kaons are of similar quality. In particular the position of $\eta$ and $\eta'$ can be accounted for. The most significant difference between model $A$ and $B$ are the masses of the scalar mesons, esp. the $a_0(980)$ in fig. (1).

For a detailed discussion of these spectra we refer to [14] and [14].

Based on this parameter set the masses of the heavy quarks have been adjusted to the $J \neq 0$ heavy mesons. Finally the additional couplings $g_{f_i f_j}$ have been adjusted to reproduce the heavy-light pseudoscalar masses. We want to stress that in this way all heavy mesons except for the pseudoscalar $D_1$, $D_s$, $B$ and $B_s$ are described by fitting the heavy quark masses only. The resulting spectra are shown in figs. (12) and (13). We find good agreement for the heavy-light mesons in both models with small advantages in model $B$ due to a larger spin-orbit splitting. Also the gross structure of the heavy quarkonia spectra can be reproduced, but fine and hyperfine splittings come out to small. This is due to the missing of a substantial spin-orbit and spin-spin interaction, as provided e.g. by the One Gluon Exchange potential. The radial excitations of the $\Upsilon$, on the other hand, are excellently reproduced in model $B$ up to the $5s$ state. Thus we think that we have reached a good overall agreement with the experimental heavy masses and therefore have gained a good estimate for the heavy meson amplitudes. To test these amplitudes further we investigate the semileptonic decays of heavy to light mesons. The relevant current matrix elements are calculated in lowest order after the prescription of Mandelstam [12]. In our formalism this leads in general to

$$\langle P'|J^\mu(q)|P \rangle = \int \frac{d^4p}{(2\pi)^4} \text{Tr} \left[ \frac{\Gamma^{\nu}(p - \frac{q}{2})S\mu^{F}_{\nu}(\frac{P + p - q}{2} + P)\Gamma^{\nu}(p)S\mu^{F}_{\nu}(-\frac{P + p}{2})}{(2\pi)^4} \right]$$

(5)

with the vertex function $\Gamma^{\nu}(p) := [S\mu^{F}_{\nu}(\frac{P}{2} + p)]^{-1} \chi^{\nu}(p) [S\mu^{F}_{\nu}(-\frac{P}{2} + p)]^{-1}$. In the instantaneous approximation the vertex function in the rest frame of the meson can be easily reconstructed from the Salpeter amplitude according to

$$\Gamma^{\nu}(p) |_{p=(M,\vec{0})} = -i \int \frac{d^4p'}{(2\pi)^4} V(p',p') \Phi(p').$$

(6)

Thus the full 4-dimensional vertex function is known in the rest frame. Formal covariance of our model then allows to calculate the vertex function of any meson on its mass shell, which is essential for the calculation of meson decays.

### III. SEMILEPTONIC DECAYS

The effective Lagrangian for the semileptonic decays, e.g. $b \rightarrow c$ transitions, after integrating out the W boson, has the usual V-A current-current form

$$\mathcal{L}_{cb}^{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{cb} \bar{c} \gamma^\mu(1 - \gamma^5)b \bar{\ell} \gamma_\mu(1 - \gamma^5)\nu$$

(7)

with the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{cb}$. Whereas the matrix elements of the leptonic current can be calculated exactly, those of the vector ($V^\mu$) and axial vector ($A^\mu$) hadronic currents are parametrized by form factors, reflecting the hadronic structure. A common parametrization is

for a $0^- \rightarrow 0^-$ transition, e.g. $B \rightarrow D \ell \nu_\ell$

$$\langle D|V^\mu|B \rangle = f_+(q^2)(P_B^\mu + P_D^\mu) + f_-(q^2)(P_B^\mu - P_D^\mu)$$

(8)

and a $0^- \rightarrow 1^-$ transition, e.g. $B \rightarrow D^* \ell \nu_\ell$

$$\langle D^*|V^\mu|B \rangle = \frac{2i}{m_B + m_{D^*}} \epsilon^{\nu}_{\rho\sigma\alpha} \epsilon^{*\nu} P_B^\rho P_D^\sigma V(q^2)$$

(9)
alignment of the heavy quark spin. Recent experimental results show evidence for this degeneracy in the spectrum freedom. For the spectra of heavy hadrons this will lead to degenerated doublets, corresponding to the two possible transitions. In particular, in the infinite quark mass limit, the transition matrix elements can be expressed by a single function only, the Isgur-Wise function \( \xi \), which is normalized to unity at maximum recoil. Therefore, a new set of form factors is used, defined by

\[
0^- \rightarrow 0^-
\]

\[
\langle D^* | A^\mu | B \rangle = 2m_D \frac{\varepsilon^\ast \cdot q}{q^2} q^\mu A_0(q^2) + (m_B + m_{D^*}) \left( \varepsilon^\ast \mu - \frac{\varepsilon^\ast \cdot q}{q^2} q^\mu \right) A_1(q^2) - \frac{\varepsilon^\ast \cdot q}{m_B + m_{D^*}} \left( P^\mu_B + P^\mu_{D^*} - \frac{m^2_B - m^2_{D^*}}{q^2} q^\mu \right) A_2(q^2),
\]

(10)

where \( q = P_B - P_{D^*} \) is the momentum transfer and \( \varepsilon \) the polarization vector of the vector meson. In the limit of vanishing lepton mass only 4 of these 6 form factors contribute to the decay rates, namely \( f_+, V, A_1 \) and \( A_2 \), since terms proportional to \( q^\mu \) can be neglected. Then the differential decay rates, expressed in terms of these form factors, are

\[
\frac{d\Gamma^{0^- \rightarrow 0^-}}{dq^2} = |V_{cb}|^2 \frac{G_F^2}{24\pi^3} P^3_{D^*} |f_+|^2,
\]

(11)

\[
\frac{d\Gamma^{0^- \rightarrow 1^-}}{dq^2} = |V_{cb}|^2 \frac{G_F^2}{(2\pi)^3 12m^2_B} (|H_+|^2 + |H_-|^2 + |H_0|^2),
\]

(12)

where we have introduced the helicity amplitudes

\[
H_\pm(q^2) := (m_B + m_{D^*}) A_1(q^2) \mp \frac{2m_B P_{D^*}}{m_B + m_{D^*}} V(q^2)
\]

(13)

\[
H_0(q^2) := \frac{m_B^2 - m_{D^*}^2 - q^2}{2m_{D^*} \sqrt{q^2}} (m_B + m_{D^*}) A_1(q^2) - \frac{2m_B^2 P_{D^*}}{m_B + m_{D^*} \sqrt{q^2}} A_2(q^2).
\]

(14)

With regard to the processes \( B \rightarrow D^{(*)}\ell \bar{\nu} \), that is the transition between heavy flavours, we want to summarize the predictions of heavy-flavour symmetry. For these transitions, the Heavy Quark Effective Theor y (HQET) provides the appropriate framework. It is based on a systematic expansion in the inverse quark mass and has been worked out by Isgur and Wise. In the limit \( m_Q \rightarrow \infty \) the theory will become independent of the heavy degrees of freedom. For the spectra of heavy hadrons this will lead to degenerated doublets, corresponding to the two possible alignments of the heavy quark spin. Recent experimental results show evidence for this degeneracy in the spectrum of the \( D \) mesons. For semileptonic decays HQET predicts a connection between the form factors of \( 0^- \rightarrow 0^- \) and \( 0^- \rightarrow 1^- \) transitions. In particular, in the infinite quark mass limit, the transition matrix elements can be expressed by a single function only, the Isgur-Wise function \( \xi \), which is normalized to unity at maximum recoil.

In this heavy-quark limit it is more natural to express the decay amplitudes in terms of velocities rather than momenta and to introduce the dimensionless variable \( \omega := v_B v_{D^*} = (m_B^2 + m_{D^*}^2 - q^2) / (2m_B m_{D^*}) \) instead of the momentum transfer \( q^2 \). Therefore, a new set of form factors is used, defined by

\[
0^- \rightarrow 0^-:
\]

\[
\frac{\langle D | V^\mu | B \rangle}{\sqrt{m_B m_{D^*}}} = h_+(\omega)(v^\mu_B + v^\mu_{D^*}) + h_-(\omega)(v^\mu_B - v^\mu_{D^*});
\]

(15)

\[
0^- \rightarrow 1^-:
\]

\[
\frac{\langle D^* | V^\mu | B \rangle}{\sqrt{m_B m_{D^*}}} = i\varepsilon^\nu_{\mu\rho\sigma} \varepsilon^{\ast\nu} v^\rho_B v^\sigma_{D^*} h_V(\omega),
\]

\[
\frac{\langle D^* | A^\mu | B \rangle}{\sqrt{m_B m_{D^*}}} = \varepsilon^{\ast\mu}(\omega + 1) h_{A_1}(\omega) - v^\mu_B \varepsilon^{\ast\nu} v_{D^*} h_{A_2}(\omega) - v^\mu_{D^*} \varepsilon^{\ast} \cdot v_B h_{A_3}(\omega).
\]

(16)

with

\[
m_Q \rightarrow \infty : \quad h_V = h_{A_1} = h_{A_3} = h_+ = \xi \\
h_{A_2} = h_- = 0
\]

(17)

in the infinite quark mass limit. The differential decay rates are given in this framework most conveniently by
In table V our results for polarization ratios, defined by

\[
\frac{d\Gamma^{0\rightarrow1-}}{d\omega} = \frac{G_F^2}{48\pi^3}m_B^3(m_B + m_D)^2(\omega^2 - 1)^{3/2}|V_{cb}|^2G^2(\omega)
\]

\[
\frac{d\Gamma^{0\rightarrow0-}}{d\omega} = \frac{G_F^2}{48\pi^3}m_B^3(m_B - m_D)^2\sqrt{\omega^2 - 1}(\omega + 1)^2 \left[ 1 + \frac{4\omega}{\omega + 1} \left( 1 - r^2 \right)^2 \right]|V_{cb}|^2F^2(\omega)
\]

with \( r = m_D/m_B \), where the two form factors \( G \) and \( F \) are functions of \( h_+ \) and \( h_V, h_A \) respectively. This parametrization has the advantage that for \( m_Q \rightarrow \infty \) \( F \) and \( G \) become equal and coincide with the Isgur-Wise function \( \xi \).

In the following sections our results for semileptonic B and charmed meson decays are compared to the available experimental data and to the results of other models: the relativised constituent quark model of Isgur and Scora (ISGW2 [13]), the relativistic calculation of Wirbel, Stech and Bauer in the infinite-momentum frame (WSB [13]) and the relativistic dispersion relation approach of Melikhov and Stech (MS [24]). Since the experiments are not able to extract form factors from their measurements due to missing statistics, these are usually parametrized according to theoretical predictions. Therefore we find it convenient to parameterize our results in the same way, allowing a comparison with experimental data. The accuracy of these parameterizations is always indicated.

A. Semileptonic decays of B mesons

The decays \( B \rightarrow D\ell\bar{\nu}_\ell \) and \( B \rightarrow D^*\ell\bar{\nu}_\ell \) have been measured by CLEO [25,24] and, more recently, by OPAL [32] and CLEO [28]. Figs. [3] and [4] show our results for the differential decay rate compared to the CLEO data 23,24. We find good overall agreement with the experimental data for both decays, using a CKM matrix element of \(|V_{cb}| = 0.034 \pm 0.001 \) and \(|V_{cb}| = 0.035 \pm 0.001 \) for model \( A \) and \( B \) respectively, which has been determined by a \( \chi^2 \) fit. These values are somewhat smaller than the PDG average of \(|V_{cb}| = 0.037 \) to 0.043. The resulting decay rates are

- **model A**: \( \Gamma(B \rightarrow D\ell\bar{\nu}_\ell) = 1.22 \cdot 10^{10} \text{s}^{-1} \), \( \Gamma(B \rightarrow D^*\ell\bar{\nu}_\ell) = 3.21 \cdot 10^{10} \text{s}^{-1} \)
- **model B**: \( \Gamma(B \rightarrow D\ell\bar{\nu}_\ell) = 1.14 \cdot 10^{10} \text{s}^{-1} \), \( \Gamma(B \rightarrow D^*\ell\bar{\nu}_\ell) = 3.24 \cdot 10^{10} \text{s}^{-1} \)

The comparison with the current world average of the Particle Data Group [1]

- PDG: \( \Gamma(B^+ \rightarrow \bar{D}^0\ell^+\nu_\ell) = 1.30 \pm 0.13 \cdot 10^{10} \text{s}^{-1} \), \( \Gamma(B^+ \rightarrow \bar{D}^0\ell^+\nu_\ell) = 3.21 \pm 0.48 \cdot 10^{10} \text{s}^{-1} \)
- \( \Gamma(B^0 \rightarrow D^-\ell^+\nu_\ell) = 1.36 \pm 0.12 \cdot 10^{10} \text{s}^{-1} \), \( \Gamma(B^0 \rightarrow D^-\ell^+\nu_\ell) = 2.97 \pm 0.17 \cdot 10^{10} \text{s}^{-1} \)

as well as the recent data of CLEO [25] and OPAL [32]

- CLEO: \( \Gamma(\bar{B}^0 \rightarrow D^{*+}\ell^-\bar{\nu}_\ell) = 3.66 \pm 0.18 \pm 0.23 \cdot 10^{10} \text{s}^{-1} \)
- OPAL: \( \Gamma(\bar{B}^0 \rightarrow D^{*+}\ell^-\bar{\nu}_\ell) = 3.40 \pm 0.13 \pm 0.30 \cdot 10^{10} \text{s}^{-1} \)

shows satisfying agreement. But the new data tends to higher values of \( \Gamma \), which would lead to higher values of \(|V_{cb}| \) from our calculation.

In table V our results for polarization ratios, defined by

\[
\frac{\Gamma_L}{\Gamma_T} = \frac{\int dq^2q^2P_D^2|H_0(q^2)|^2}{\int dq^2q^2P_D^2([H_+^2(q^2)]^2 + [H_-^2(q^2)])} \quad \frac{\Gamma_+}{\Gamma_-} = \frac{\int dq^2q^2P_D^2|H_+(q^2)|^2}{\int dq^2q^2P_D^2|H_-(q^2)|^2},
\]

are summarized. In these (\(|V_{cb}| \) independent) quantities we find good agreement with the experimental values as well.

We also show the results for the form factor ratios

\[
R_1 := \frac{h_V(\omega)}{h_{A_1}(\omega)} = \left( 1 - \frac{q^2}{(m_B + m_D)^2} \right) \frac{V(q^2)}{A_1(q^2)}
\]

\[
R_2 := \frac{h_{A_4}(\omega) + \frac{m_D}{m_B}h_{A_2}(\omega)}{h_{A_1}(\omega)} = \left( 1 - \frac{q^2}{(m_B + m_D)^2} \right) \frac{A_2(q^2)}{A_1(q^2)}.
\]

In the heavy quark limit these ratios are predicted to be constant and equal to unity. In reality, due to corrections to this limit, \( R_1 \) and \( R_2 \) do depend on \( \omega \), but at the scale of the \( b \)-quark mass this dependency is expected to be
very weak. Therefore these ratios are preferred in the analysis of \( B \to D^* \ell \nu \), where they are assumed to be constant, whereas \( h_{A_1} \) is approximated by a linear or quadratic function

\[
h_{A_1}(\omega) \approx h_{A_1}(1) (1 - \rho_{A_1}^2 (\omega - 1) + \lambda_{A_1} (\omega - 1)^2).
\]

(23)

For our form factors the \( \omega \)-dependency of \( R_{1/2} \) is less than 2%, and the mean values are shown in table I. The form factor \( h_{A_1} \) can be described by the parametrization (24) with an accuracy of 0.1% (0.3%) for model \( A(B) \), yielding

model \( A \) : \( h_{A_1}(1) = 0.97 \quad \rho_{A_1}^2 = 0.73 \quad \lambda_{A_1} = 0.27 \)

model \( B \) : \( h_{A_1}(1) = 1.01 \quad \rho_{A_1}^2 = 0.98 \quad \lambda_{A_1} = 0.48 \),

which is in good agreement with the experimental data shown in table I.

Two recent measurements of \( B \to D^* \ell \nu \) by the Cleo [28] and the Opal [32] collaborations have been published, where a different parametrization of the form factor based on dispersion relations has been used. These analyses expand \( h_{A_1} \) in the variable \( z(\omega) = (\sqrt{\omega + 1} - \sqrt{2}) / (\sqrt{\omega + 1} + \sqrt{2}) \) and use the parametrization [21]

\[
h_{A_1}(\omega)/h_{A_1}(1) = 1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3
\]

with the (inconsistent) results

\[
\text{Opal} [32] : \rho^2 = 1.21 \pm 0.12 \pm 0.20, \quad \text{Cleo} [28] : \rho^2 = 1.67 \pm 0.11 \pm 0.22.
\]

A similar fit to our form factor gives

model \( A \) : \( \rho^2 = 0.83 \), model \( B \) : \( \rho^2 = 1.06 \)

with an accuracy of 1% and 0.4% for model \( A \) and \( B \), respectively. Thus our calculation is rather compatible with the Opal result.

To connect our results with the description in the framework of HQET, we have calculated the form factors \( F \) and \( G \) (see eq. [13]) and their slopes at \( \omega = 1 \) by fitting a quadratic function analogous to (24). This is possible with an accuracy of 0.2% (0.3%) for model \( A(B) \) and gives the result

model \( A \) : \( G(1) = 1.01, \quad \rho_G^2 = 0.70, \quad \lambda_G = 0.22; \quad F(1) = 0.97, \quad \rho_F^2 = 0.65, \quad \lambda_F = 0.20 \)

model \( B \) : \( G(1) = 1.01, \quad \rho_G^2 = 0.85, \quad \lambda_G = 0.31; \quad F(1) = 1.01, \quad \rho_F^2 = 0.91, \quad \lambda_F = 0.56 \)

These quantities \( G(1) \) and \( F(1) \) have been calculated in the HQET. Current values are [12]

\[
G(1) = 1.00 \pm 0.07, \quad F(1) = 0.92 \pm 0.05.
\]

Finally we have performed the heavy quark limit numerically in our model by scaling the quark masses with a large factor. We find that the form factors then indeed coincide or vanish (see eq. [17]). The resulting universal function \( \tilde{\xi} \), which we identify as the Isgur-Wise function of our model, can be parametrized up to deviations of less than 0.2% and 0.5% for model \( A \) and \( B \) as

\[
\tilde{\xi}(\omega) = \tilde{\xi}(1) (1 - \rho_{\tilde{\xi}}^2 (\omega - 1) + \lambda_{\tilde{\xi}} (\omega - 1)^2),
\]

where

model \( A \) : \( \tilde{\xi}(1) = 1.00 \quad \rho_{\tilde{\xi}}^2 = 0.78 \quad \lambda_{\tilde{\xi}} = 0.31 \)

model \( B \) : \( \tilde{\xi}(1) = 1.00 \quad \rho_{\tilde{\xi}}^2 = 1.06 \quad \lambda_{\tilde{\xi}} = 0.50 \).

In particular, we find that \( \tilde{\xi} \) is indeed normalised to \( \tilde{\xi}(1) = 1 \). The \( \omega \)-dependence is of course model dependent.
B. Semileptonic decays of charmed mesons

The semileptonic decays of charmed mesons, induced by the quark level process $c \rightarrow s$, have been measured for the $D \rightarrow K^{(*)}$ as well as for the $D_s \rightarrow \eta/\eta'/\phi$ transitions. The results, averaged over isospin, are shown in table VII. For the $D$ meson decay $D \rightarrow K \ell \nu$ the comparison with our calculation shows reasonable agreement for both models. The decay to the $K^*$ final state however is overestimated by about a factor of 2. The polarization observables on the other hand are comparable with the experimental result, whereby model $\mathcal{A}$ gives better agreement than model $\mathcal{B}$. This is a well known problem of constituent quark models. Our results are rather comparable with those of Wirbel et al. (WSB [15]). With respect to the ISGW2 predictions it is interesting to note that, whereas the inclusion of relativistic corrections was one of the main ingredients in their model to decrease the $D \rightarrow K^*$ decay rate, this problem still exists in our full relativistic calculation.

Apart from the decay rates and the polarization observables, the form factor ratios at zero momentum transfer, defined by

$$ r_V := \frac{V(0)}{A_1(0)}, \quad r_2 := \frac{A_2(0)}{A_1(0)}, $$

are considered. To extract these ratios from experiment the form factors are usually parametrized by a simple pole ansatz with a pole mass of 2.1 GeV for the vector form factor and 2.5 GeV for the axial form factors. We have performed such a fit to our calculations, which works up to deviations of about 6% for the form factors and of 4% for their ratios, and have extracted the form factor ratios for $D$ meson decays from these parametrizations. The results in table VII show that the axial form factors are generally overrated, while our vector form factor parameters agree with the values extracted from experiment. Thus the failure in the $D \rightarrow K^{(*)}$ decay rate can be traced back to this overestimation of the axial form factors.

In this connection it is interesting to estimate the effect of the full relativistic treatment. To do this we neglect the negative energy or 'lower' components of the Salpeter amplitude when reconstructing the vertex function. This procedure changes the norm of the Salpeter amplitude by a factor of

$$ \frac{\langle \Phi^{++} | \Phi^{++} \rangle}{\langle \Phi | \Phi \rangle} = 0.55 $$

for the $D$ meson in model $\mathcal{A}$. Here $\Phi^{++} := \frac{1+\gamma^0}{2} \Phi \frac{1-\gamma^0}{2}$ and $\langle . | . \rangle$ is the scalar product induced by the Salpeter equation. This shows the importance of the negative energy components for heavy-light mesons. With this reduced vertex function the form factor ratios $r_V$ and $r_2$ are calculated. Hereby we concentrate on model $\mathcal{A}$, since, as has been discussed in [13], the positive and negative energy components of the Salpeter amplitude decouple in the non-relativistic reduction of the Salpeter equation. We find that these ratios slightly rise: $r_V$ changes from 1.48 to 1.55, $r_2$ goes from 0.78 up to 0.84 while the $q^2$ dependence is hardly changed. Thus the modification of a form factors by the full relativistic treatment seems to be more complex than the intuitive correction anticipated in [10].

The results on the semileptonic decays $D_s \rightarrow \eta$ and $D_s \rightarrow \eta'$ are shown in table VII. Here it has to be stressed that the flavour mixing of $\eta$ and $\eta'$ had already been fixed by the mass fit. No additional mixing parameter is necessary. Although the differences between the results of our two models are rather large, the experimental data do not allow to prefer one of our parameter sets.

Our results on $D_s \rightarrow \phi$ show the same behaviour as those on $D \rightarrow K^*$: While the decay rate is overestimated by a factor of 2, the polarization observable is in good agreement with the experimental data and the form factor ratio $r_V$ tends to be too small. Concerning the ratio of axial form factors it is interesting to note that the experimental value of $r_2$ is 4 standard deviations higher than the corresponding value for $D \rightarrow K^*$, which is in contradiction to an (approximate) flavour $SU(3)$ symmetry. This result, if it should be confirmed, is clearly a challenge for any theoretical description.

Finally, to conclude the discussion of charmed meson decays, we report our results on the Cabibbo suppressed process $c \rightarrow d$ in tables VIII and IX. The form factors again have been parametrized by a pole ansatz with pole masses of 2.0 GeV and 2.4 GeV for the vector and axial form factors respectively, which works up to deviations of 8% for $A_1(0)$ and 5% for the ratios. The current experimental situation however allows no evaluation of our description of these decays.

\footnote{These are the calculated values, not to results of the pole ansatz parametrization in table VII.}
Thus in summary we find excellent agreement in the description of heavy to heavy transitions $B \to D^{(*)}$ over the whole kinematic regime. We find also consistency with the description of these processes in the framework of the HQET. The results on heavy to light transitions are mostly in agreement with the experimental data, but the common problem of quark models to overestimate the axial form factors is also present here.

IV. NON-LEPTONIC WEAK DECAYS

To extract further information from our meson amplitudes we finally investigate non-leptonic decays. On tree level, non-leptonic decays are mediated by a single W-boson emission. Hard and soft gluonic effects however might play a significant role in these processes. These corrections have been calculated with great effort in the last years in order to extract model independent Cabibbo-Kobayashi-Maskawa matrix elements or signatures for CP violation in B decays from experimental data. Thereby the hard and soft gluon contributions are separated by a Wilson operator product expansion, which results in the effective Lagrangian, e.g. for $B \to D\pi$ transitions,

$$\mathcal{L}_{B \to D\pi}^{\text{eff}} = -\frac{G_F}{\sqrt{2}}V_{cb}V_{du}^* \left( \frac{1}{2} \left( a_1(\mu)(\bar{c}b)_{V-A}^\mu(\bar{d}u)_{\mu V-A} + a_2(\mu)(\bar{d}b)_{V-A}^\mu(\bar{c}u)_{\mu V-A} \right) \right. \right.$$  
$$\left. + \frac{1}{2}C_1(\mu)(\bar{d}\lambda^A b)_{V-A}^\mu(\bar{c}\lambda^A u)_{\mu V-A} + \frac{1}{2}C_2(\mu)(\bar{c}\lambda^A b)_{V-A}^\mu(\bar{d}\lambda^A u)_{\mu V-A} \right)$$

(25)

with $(\bar{c}\lambda^A b)_{V-A}^\mu := \bar{c}\gamma^\mu(1-\gamma^5)\lambda^A b$ etc., $a_1 := C_1 + \frac{2}{3}C_2$ and $a_2 := C_2 + \frac{1}{3}C_1$, where $C_{1/2}$ are the (scale dependent) Wilson coefficients, and $\lambda^A$ the SU(3) color Gell-Mann matrices. Here we found it convenient to use an expression symmetric under Fierz transformation. The second line obviously does not contribute for color singlet states.

This lagrangian gives rise to $W$ emission diagrams. Contributions by weak annihilation and internal $W$ exchange, which are suppressed by powers of $\Lambda_{QCD}/m_b$, are neglected.

Thus the matrix element of a product of currents has to be evaluated. This is usually done using the ”factorization approximation”, where one assumes that the amplitude is dominated by its factorizable part. Then it is given by the product of two current matrix elements, e.g. for the transition $B^+ \to \bar{D}^0 \pi^+$

$$A(B^+ \to \bar{D}^0 \pi^+) = \frac{G_F}{\sqrt{2}}V_{cb}V_{du}^* \left( a_1(\pi^+|h_{\pi d}|0)\langle \bar{D}^0 |h_{\pi b}^\mu B^+ \rangle + a_2(\bar{D}^0 |h_{\pi d c}|0)\langle \pi^+ |h_{\pi a b}^\mu |B^+ \rangle \right).$$

(26)

In this way the decay amplitude can be expressed by the decay constant and a form factor of the semileptonic decay at the relevant $q^2$, e.g.

$$\langle \pi^+ |J^\mu(0)\langle \bar{D}^0 |J_{\mu'}^\nu|B^+ \rangle = (m_B^2 - m_{\pi}^2)f_{\pi}F_{0}(m_{\pi}^2)$$

(27)

$$\langle \pi^+ |J^\mu(0)\langle \bar{D}^0 |J_{\mu'}^\nu|B^+ \rangle = 2\epsilon^* \cdot p_B m_{\bar{D}}f_{\pi}A_0(m_{\pi}^2)$$

(28)

with the decay constants $\langle 0|J^\mu(0)|0^- \rangle = i f_{\rho} \rho$, $\langle 0|J^\mu(0^-)|b^- \rangle = m_{\rho} F_{\rho}$. It should be noted that the relevant form factors are $F_0$ and $A_0$, which are unimportant for semileptonic decays due the smallness of the lepton mass. Hence non-leptonic decays offer a possibility to access the remaining semileptonic form factors, at least within the framework of the factorization approximation. The factorization assumption has been proven recently at two loop order $[8]$ for a special class of decays.

In this approach, however, the resulting amplitudes are scale dependent due to the $\mu$-dependence of the Wilson coefficients, which is not canceled by the scale independent matrix elements. To deal with this difficulty the coefficients $a_1$ and $a_2$ are often treated as effective and free parameters, corresponding to some factorization scale, to be extracted from the data. But since we are interested in an estimate of the quality of our form factors, we find it sufficient to neglect all strong gluonic effects, which results in $C_1 = 1$, $C_2 = 0 \equiv a_1 = 1$, $a_2 = \frac{1}{3}$ and restrict our calculation to the decays of $B$ mesons via the emission of a $W$ meson, usually called Type I decays.

The weak decay constants are shown in table XIII. Since these are generally overestimated in our models we have used the experimental values, which are extracted from leptonic decay or $\tau$–decay and are summarized in table XIII. For the vector mesons $D^*$ and $D_{s*}$, where no data are available yet, we use $F_{D_{s*}} \approx F_{D_{s}}$, which is valid in the heavy quark limit. Deviations from this limit are expected to be about 10-20% $[22]$. The relevant CKM-matrix elements are taken from $[3]$ except for $V_{cb}$ where we use the fit results from section $[11]$.
Our results for non-leptonic $B$-decays are shown in table XII compared with the experimental data from [1] as well as the calculation of Neubert et al. [22]. We find good agreement with the data available so far for both our models; and for those decays, which are not measured yet, our results are comparable with [22].

In order to stress the influence of our form factors we show the ratios of decay rates (in which the decay constants as well as the coefficient $a_1$ cancel) in table XIII. We also find good agreement with the experimental data, however the errors are quite large.

V. SUMMARY

In this paper we have calculated the masses and the exclusive semileptonic and non-leptonic decays of heavy mesons in a constituent quark model based on the Bethe-Salpeter equation in instantaneous approximation. Apart from a linear confinement potential with two different phenomenological Dirac structures, a flavour dependent residual interaction motivated by instanton effects is adopted, which has been naively generalized to heavy flavours. Thus extending a very good description of light mesons, which has recently been updated in [3], we also find good overall agreement with the data on heavy meson masses.

The resulting meson amplitudes are used to calculate semileptonic decay rates. We find excellent agreement in the description of heavy to heavy transitions $B \rightarrow D^{(*)}$ over the whole kinematic regime. Our results are also consistent with the description of these processes in the framework of the HQET. The results on heavy to light transitions are mostly in agreement with the experimental data, but the common problem of quark models to overestimate the axial form factors is also found here.

Finally non-leptonic decays have then been calculated in the approximation of factorizing matrix elements. In spite of this simple picture we find good agreement with the experimental results on $B$ meson decays.

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FIG. 1. The spectrum of the isovector mesons. The centric column shows the experimental data from [1]. Errors are indicated by shadowed boxes. The left and right columns show our results with model $A$ and $B$, respectively.

FIG. 2. The spectra of the charmed $D$ and $D_s$ mesons. The centric column shows the experimental data from [1]. Errors are indicated by shadowed boxes. The levels marked with $\dagger$ are taken from [26]. The left and right columns show our results with model $A$ and $B$, respectively. Note that the $1^+$ states are twofold degenerated in our calculation corresponding to the total spin $S = 0$ and $S = 1$. 
FIG. 3. The spectra of the bottom $B$ and $B_s$ mesons. The centric column shows the experimental data from [1]. Errors are indicated by shadowed boxes. The levels marked with $^\dagger$ are taken from [23]. The left and right columns show our results with model $A$ and $B$, respectively. Note that the $1^+$ states are twofold degenerated in our calculation corresponding to the total spin $S = 0$ and $S = 1$.

FIG. 4. The spectra of heavy quarkonia. The centric column shows the experimental data from [1]. Errors are indicated by shadowed boxes. The left and right columns show our results with model $A$ and $B$, respectively.
FIG. 5. The differential decay rate for $B \to D\ell\bar{\nu}$. The data are taken from [25]. The values of $|V_{cb}|$ corresponding to our calculations are $|V_{cb}| = 0.034$ and $|V_{cb}| = 0.035$ for model A and B, respectively.

FIG. 6. The differential decay rate for $B \to D^*\ell\bar{\nu}$. The data are taken from [24]. The values of $|V_{cb}|$ corresponding to our calculations are $|V_{cb}| = 0.034$ and $|V_{cb}| = 0.035$ for model A and B, respectively.
### TABLE I.
The parameters of the confinement force, the 't Hooft interaction and the constituent quark masses in the models $A$ and $B$.

| Parameter                  | Model $A$ | Model $B$ |
|----------------------------|-----------|-----------|
| $m_n$ [MeV]                | 306       | 380       |
| $m_s$ [MeV]                | 503       | 550       |
| $m_c$ [MeV]                | 1835      | 1780      |
| $m_b$ [MeV]                | 5240      | 5092      |
| $g_{nn}$ [GeV$^{-2}$]      | 1.73      | 1.62      |
| $g_{ns}$ [GeV$^{-2}$]      | 1.54      | 1.35      |
| $g_{nc}$ [GeV$^{-2}$]      | 1.11      | 1.58      |
| $g_{ob}$ [GeV$^{-2}$]      | 0.53      | 1.07      |
| $g_{sc}$ [GeV$^{-2}$]      | 0.65      | 1.27      |
| $g_{sb}$ [GeV$^{-2}$]      | 0.00      | 0.76      |
| $\Lambda$ [fm]            | 0.30      | 0.42      |
| $a_c$ [GeV]                | -1.751    | -1.135    |
| $b_c$ [GeV fm$^{-1}$]      | 2.076     | 1.300     |

Spin structure

$$\frac{i}{4} \left( \mathbb{1} \otimes \mathbb{1} - \gamma^0 \otimes \gamma^0 \right)$$

$$\frac{i}{4} \left( \mathbb{1} \otimes \mathbb{1} - \gamma^5 \otimes \gamma^5 - \gamma^\mu \otimes \gamma^\mu \right)$$
### TABLE II. Masses of the charmed $D$ and $D_s$ mesons in MeV, calculated in model $A$ and $B$; $n$ denotes the radial excitation. Note that the $1^+$ states are twofold degenerated corresponding to the total spin $S = 0$ and $S = 1$.

| Meson ($J^\pi$) | n  | Model $A$ | Model $B$ |
|------------------|----|-----------|-----------|
| $D(0^-)$         | 0  | 1869      | 1869      |
|                  | 1  | 2677      | 2578      |
|                  | 2  | 3258      | 3041      |
| $D^*(1^-)$       | 0  | 1993      | 2034      |
|                  | 1  | 2769      | 2648      |
|                  | 2  | 2822      | 2679      |
|                  | 3  | 3327      | 3079      |
|                  | 4  | 3344      | 3101      |
| $D_0^*(0^+)$     | 0  | 2519      | 2375      |
|                  | 1  | 3115      | 2884      |
| $D_1(1^+)$       | 0  | 2464      | 2420      |
|                  | 1  | 2464      | 2420      |
|                  | 2  | 3075      | 2908      |
|                  | 3  | 3075      | 2908      |
| $D_2^*(2^+)$     | 0  | 2475      | 2469      |
|                  | 1  | 3086      | 2930      |
|                  | 2  | 3128      | 2957      |
| $B(0^-)$         | 0  | 5279      | 5279      |
|                  | 1  | 6002      | 5869      |
|                  | 2  | 6517      | 6279      |
| $B^*(1^-)$       | 0  | 5325      | 5346      |
|                  | 1  | 6035      | 5900      |
|                  | 2  | 6108      | 5947      |
|                  | 3  | 6543      | 6296      |
|                  | 4  | 6575      | 6322      |
| $B_0^*(0^+)$     | 0  | 5796      | 5675      |
|                  | 1  | 6341      | 6120      |
| $B_1(1^+)$       | 0  | 5770      | 5696      |
|                  | 1  | 5770      | 5696      |
|                  | 2  | 6322      | 6136      |
|                  | 3  | 6322      | 6136      |
| $B_2^*(2^+)$     | 0  | 5771      | 5711      |
|                  | 1  | 6324      | 6140      |
|                  | 2  | 6391      | 6186      |

### TABLE III. Masses of the bottom $B$ and $B_s$ mesons in MeV, calculated in model $A$ and $B$; $n$ denotes the radial excitation. Note that the $1^+$ states are twofold degenerated corresponding to the total spin $S = 0$ and $S = 1$.

| Meson ($J^\pi$) | n  | Model $A$ | Model $B$ |
|------------------|----|-----------|-----------|
| $B(0^-)$         | 0  | 5279      | 5279      |
|                  | 1  | 6002      | 5869      |
|                  | 2  | 6517      | 6279      |
| $B^*(1^-)$       | 0  | 5325      | 5346      |
|                  | 1  | 6035      | 5900      |
|                  | 2  | 6108      | 5947      |
|                  | 3  | 6543      | 6296      |
|                  | 4  | 6575      | 6322      |
| $B_0^*(0^+)$     | 0  | 5796      | 5675      |
|                  | 1  | 6341      | 6120      |
| $B_1(1^+)$       | 0  | 5770      | 5696      |
|                  | 1  | 5770      | 5696      |
|                  | 2  | 6322      | 6136      |
|                  | 3  | 6322      | 6136      |
| $B_2^*(2^+)$     | 0  | 5771      | 5711      |
|                  | 1  | 6324      | 6140      |
|                  | 2  | 6391      | 6186      |

TABLE III. Masses of the bottom $B$ and $B_s$ mesons in MeV, calculated in model $A$ and $B$; $n$ denotes the radial excitation. Note that the $1^+$ states are twofold degenerated corresponding to the total spin $S = 0$ and $S = 1$. 
| Meson \((J^{pc})\) | n | Model A | Model B | Meson \((J^{pc})\) | n | Model A | Model B |
|-----------------|---|---------|---------|-----------------|---|---------|---------|
| \(\eta_c(0^{-+})\) | 0 | 3037 | 3114 | \(\eta_b(0^{-+})\) | 0 | 9497 | 9562 |
|                 | 1 | 3789 | 3708 |                  | 1 | 10055 | 9883 |
|                 | 2 | 4363 | 4147 |                  | 2 | 10503 | 10318 |
| \(J/\Psi(1^{--})\) | 0 | 3039 | 3133 | \(\Upsilon(1^{--})\) | 0 | 9497 | 9565 |
|                 | 1 | 3792 | 3719 |                  | 1 | 10055 | 9885 |
|                 | 2 | 3846 | 3754 |                  | 2 | 10103 | 10020 |
|                 | 3 | 4366 | 4155 |                  | 3 | 10503 | 10319 |
|                 | 4 | 4398 | 4176 |                  | 4 | 10534 | 10342 |
|                 | 5 |       |       |                  | 5 | 10892 | 10609 |
|                 | 6 |       |       |                  | 6 | 10914 | 10625 |
|                 | 7 | 11250 | 10871 |                | 7 | 11250 | 10871 |
| \(h_c(1^{+})\) | 0 | 3482 | 3483 | \(h_b(1^{+})\) | 0 | 9824 | 9811 |
|                 | 1 | 4110 | 3965 |                  | 1 | 10304 | 10171 |
| \(\chi_{c0}(0^{++})\) | 0 | 3482 | 3449 | \(\chi_{b0}(0^{++})\) | 0 | 9824 | 9805 |
|                 | 1 | 4110 | 3943 |                  | 1 | 10304 | 10166 |
| \(\chi_{c1}(1^{++})\) | 0 | 3482 | 3483 | \(\chi_{b1}(1^{++})\) | 0 | 9824 | 9811 |
|                 | 1 | 4110 | 3965 |                  | 1 | 10304 | 10171 |
| \(\chi_{c2}(2^{++})\) | 0 | 3485 | 3508 | \(\chi_{b2}(2^{++})\) | 0 | 9824 | 9816 |
|                 | 1 | 4114 | 3983 |                  | 1 | 10304 | 10175 |
|                 | 2 | 4164 | 4015 |                  | 2 | 10352 | 10210 |

**TABLE IV.** Masses of the heavy quarkonia in MeV, calculated in model \(A\) and \(B\); \(n\) denotes the radial excitation.

| Parameter | Exp. [1] | Model \(A\) | Model \(B\) | ISGW2 [16] | WSB [19] | MS [20] |
|-----------|---------|-------------|-------------|------------|-------|--------|
| \(\Gamma(B \to D) \left[10^{13}|V_{cb}|^2s^{-1}\right]\) | — | 1.05 | 0.93 | 1.19 | 0.808 | 0.86 |
| \(\Gamma(B \to D^*) \left[10^{13}|V_{cb}|^2s^{-1}\right]\) | — | 2.78 | 2.64 | 2.48 | 2.19 | 2.28 |
| \(\Gamma_L/\Gamma_T\) | 1.24 ± 0.16 [2] | 1.14 | 1.20 | 1.04 | — | 1.11 |
| \(\Gamma_+/\Gamma_-\) | — | 0.23 | 0.27 | — | — | — |
| \(R_1\) | 1.18 ± 0.30 ± 0.12 | 1.18 | 1.10 | 1.27 | 1.09\(^\ast\) | — |
| \(R_2\) | 0.71 ± 0.22 ± 0.07 | 0.94 | 0.87 | 1.02 | 1.06\(^\ast\) | — |
| \(\rho_3^A\) | 0.91 ± 0.15 ± 0.06 | 0.75 | 1.02 | — | — | — |

**TABLE V.** \(B \to D^{(*)}\bar{\nu}\) decay observables and form factor parameters. We use \(|V_{cb}| = 0.034(A)\) resp. \(|V_{cb}| = 0.035(B)\), as described in section IIIA.

\(^\ast\) taken from [4]
| Parameter | Exp. [1] | Model A | Model B | ISGW2 [16] | WSB [19] | MS [20] |
|-----------|----------|---------|---------|-------------|----------|---------|
| \( \Gamma(D \rightarrow K) \left[ 10^{10} \text{s}^{-1} \right] \) | 7.97 ± 0.36 | 7.51 | 7.26 | 10.0 | 8.26 | 9.7 |
| \( \Gamma(D \rightarrow K^*) \left[ 10^{10} \text{s}^{-1} \right] \) | 4.55 ± 0.34 | 7.64 | 10.08 | 5.4 | 9.53 | 6.0 |
| \( \Gamma_L/\Gamma_T \) | 1.14 ± 0.08 | 1.29 | 1.48 | 0.94 | 0.91 | 1.28 |
| \( \Gamma_+/\Gamma_- \) | 0.21 ± 0.04 | 0.23 | 0.34 | — | — | — |
| \( A_1(0) \) | 0.56 ± 0.04 | 0.69 | 0.81 | — | 0.88 | 0.66 |
| \( r_V \) | 1.82 ± 0.09 | 1.54 | 1.18 | 2.0* | 1.44 | 1.56 |
| \( r_2 \) | 0.78 ± 0.07 | 0.81 | 0.62 | 1.3* | 1.31 | 0.74 |

* taken from [29]

**TABLE VI.** \( D \rightarrow K^{(*)}\ell\bar{\nu} \) decay observables and form factor parameters. The experimental decay rates are averaged over isospin. We use \(|V_{cs}| = 0.975\) [1].

| Parameter | Exp. [1] | Model A | Model B | ISGW2 [16] | WSB [19] | MS [20] |
|-----------|----------|---------|---------|-------------|----------|---------|
| \( \Gamma(D_s \rightarrow \eta) \left[ 10^{10} \text{s}^{-1} \right] \) | 5.24 ± 1.41 | 4.05 | 3.11 | 3.5 \text{ a)} | — | 5.0 |
| \( \Gamma(D_s \rightarrow \eta^*) \left[ 10^{10} \text{s}^{-1} \right] \) | 1.80 ± 0.69 | 1.27 | 1.75 | 3.0 \text{ a)} | — | 1.85 |
| \( \Gamma(D_s \rightarrow \phi) \left[ 10^{10} \text{s}^{-1} \right] \) | 4.03 ± 1.01 | 7.89 | 9.67 | 4.6 | — | 5.1 |
| \( \Gamma_L/\Gamma_T \) | 0.72 ± 0.18 | 1.20 | 1.42 | 0.96 | — | — |
| \( \Gamma_+/\Gamma_- \) | — | 0.20 | 0.33 | — | — | — |
| \( A_1(0) \) | — | 0.66 | 0.79 | — | — | 0.65 |
| \( r_V \) | 1.92 ± 0.32 | 1.77 | 1.30 | 2.1* | — | 1.71 |
| \( r_2 \) | 1.60 ± 0.24 | 0.85 | 0.63 | 1.3* | — | 0.72 |

* taken from [19]

**TABLE VII.** \( D_s \rightarrow \eta/\eta^*/\phi\ell\bar{\nu} \) decay observables and form factor parameters. The experimental decay rates are averaged over isospin. We use \(|V_{cs}| = 0.975\) [4].

\text{a) } A \eta - \eta^* \text{ mixing angle of } -20^\circ \text{ is assumed. An angle of } -10^\circ \text{ would lead to 5.3 and 2.3.}
| Parameter  | Exp. 1 | Model A | Model B | ISGW2 16 | WSB 19 | MS 20 |
|------------|--------|---------|---------|-----------|--------|-------|
| $\Gamma(D^+ \rightarrow \pi^0)$ | 2.9 ± 1.4 | 1.03 | 0.99 | 2.4 | 3.6 | 4.8 |
| $\Gamma(D^0 \rightarrow \pi^0)$ | 9.0 ± 1.5 | 2.06 | 1.99 | 4.8 | 7.1 | 9.6 |
| $\Gamma(D^+ \rightarrow \rho^0)$ | 2.1 ± 0.8 | 2.25 | 3.36 | 1.2 | 3.4 | 2.1 |
| $\Gamma(\rho^0 \rightarrow \pi^0)$ | — | 1.30 | 1.55 | 0.67 | 0.91 | 1.16 |
| $\Gamma(\rho^0 \rightarrow \pi^-)$ | — | 0.15 | 0.26 | — | — | — |
| $A_1(0)$ | — | 0.58 | 0.72 | — | 0.78 | 0.6 |
| $r_V$ | — | 1.68 | 1.26 | — | 1.58 | 1.48 |
| $r_2$ | — | 0.77 | 0.59 | — | 1.18 | 0.82 |

**TABLE VIII.** $D \rightarrow \pi/\rho\ell\bar{\nu}$ decay observables and form factor parameters. We use $|V_{cd}| = 0.222$.  

| Parameter  | Exp. 1 | Model A | Model B | ISGW2 16 | WSB 19 | MS 20 |
|------------|--------|---------|---------|-----------|--------|-------|
| $\Gamma(D \rightarrow \eta)$ | — | 0.79 | 0.95 | 1.5 $^a$ | — | — |
| $\Gamma(D \rightarrow \eta')$ | — | 0.14 | 0.16 | 0.3 $^a$ | — | — |
| $\Gamma(D \rightarrow \omega)$ | — | 2.23 | 3.35 | 1.2 | — | — |
| $\Gamma_L/\Gamma_T$ | — | 1.31 | 1.55 | 0.68 | — | — |
| $\Gamma_+/\Gamma_-$ | — | 0.15 | 0.26 | — | — | — |
| $A_1(0)$ | — | 0.41 | 0.51 | — | — | — |
| $r_V$ | — | 1.68 | 1.27 | — | — | — |
| $r_2$ | — | 0.77 | 0.59 | — | — | — |

**TABLE IX.** $D \rightarrow \eta/\eta'/\omega\ell\bar{\nu}$ decay observables and form factor parameters. We use $|V_{cd}| = 0.222$.  

$^a$ A $\eta - \eta'$ mixing angle of $-20^\circ$ is assumed. An angle of $-10^\circ$ would lead to 1.1 and 0.4.

| Parameter  | Exp. 1 | Model A | Model B | ISGW2 16 | WSB 19 | MS 20 |
|------------|--------|---------|---------|-----------|--------|-------|
| $\Gamma(D_s \rightarrow K)$ | — | 3.42 | 3.15 | 4.4 | — | 6.4 |
| $\Gamma(D_s \rightarrow K^*)$ | — | 2.71 | 4.54 | 2.2 | — | 3.9 |
| $\Gamma_L/\Gamma_T$ | — | 1.24 | 1.51 | 0.76 | — | 1.21 |
| $\Gamma_+/\Gamma_-$ | — | 0.14 | 0.29 | — | — | — |
| $A_1(0)$ | — | 0.43 | 0.58 | — | — | 0.57 |
| $r_V$ | — | 1.86 | 1.26 | — | — | 1.83 |
| $r_2$ | — | 0.76 | 0.56 | — | — | 0.74 |

**TABLE X.** $D_s \rightarrow K/K^*\ell\bar{\nu}$ decay observables and form factor parameters. We use $|V_{cd}| = 0.222$.  

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| Decay ratio | Exp. $|\pm|$ | Model $A$ | Model $B$ | NRSX $|\pm|$ |
|-------------|-------------|-----------|-----------|-------------|
| $\Gamma(B^0 \rightarrow D^- \pi^+)$/$\Gamma(B^0 \rightarrow D^+ \pi^+)$ | $1.09 \pm 0.17$ | $1.00$ | $1.05$ | $1.04$ |
| $\Gamma(B^0 \rightarrow D^- \rho^+)$/$\Gamma(B^0 \rightarrow D^+ \rho^+)$ | $1.16 \pm 0.62$ | $0.87$ | $0.92$ | $0.88$ |
| $\Gamma(B^0 \rightarrow D^0 \rightarrow D^+ D^-$ | $0.83 \pm 0.43$ | $1.42$ | $1.28$ | $1.47$ |
| $\Gamma(B^+ \rightarrow D^{0*} D^-)$/$\Gamma(B^+ \rightarrow D^{0*} D^-)$ | $1.08 \pm 0.56$ | $0.50 \pm 0.31$ | $0.41$ | $0.36$ | $0.39$ |
| $\Gamma(B^0 \rightarrow D^- D^0)$/$\Gamma(B^0 \rightarrow D^- D^0)$ | $0.50 \pm 0.31$ | $0.41$ | $0.36$ | $0.39$ |
| $\Gamma(B^0 \rightarrow D^- D^0)$/$\Gamma(B^+ \rightarrow D^{0*} D^-)$ | $0.33 \pm 0.19$ | $0.41$ | $0.36$ | $0.39$ |

**TABLE XI.** Ratios of non-leptonic $B$ meson decay rates

| Decay mode | Exp. $|\pm|$ | Model $A$ | Model $B$ | NRSX $|\pm|$ |
|------------|-------------|-----------|-----------|-------------|
| $B^0 \rightarrow D^- \pi^+$ | $1.94 \pm 0.26$ | $2.21$ | $1.97$ | $1.94$ |
| $B^0 \rightarrow D^- \rho^+$ | $5.10 \pm 0.90$ | $5.68$ | $6.13$ | $4.84$ |
| $B^0 \rightarrow D^- \pi^+$ | $1.78 \pm 0.14$ | $2.22$ | $1.88$ | $1.87$ |
| $B^0 \rightarrow D^- \rho^+$ | $4.4 \pm 2.2$ | $6.57$ | $6.67$ | $5.48$ |
| $B^0 \rightarrow D^- K^+$ | — | $0.17$ | $0.15$ | $0.15$ |
| $B^0 \rightarrow D^- K^{*+}$ | — | $0.31$ | $0.30$ | $0.26$ |
| $B^0 \rightarrow D^- K^*$ | — | $0.16$ | $0.14$ | $0.14$ |
| $B^0 \rightarrow D^- K^{*+}$ | — | $0.36$ | $0.34$ | $0.32$ |
| $B^0 \rightarrow D^- D^+$ | — | $0.53$ | $0.49$ | $0.23$ |
| $B^0 \rightarrow D^- D^{*+}$ | — | $0.37$ | $0.34$ | $0.23$ |
| $B^0 \rightarrow D^- D^+$ | — | $0.39$ | $0.41$ | $0.17$ |
| $B^0 \rightarrow D^- D^{*+}$ | $0.40^{+0.36}_{-0.23}$ | $0.86$ | $0.89$ | $0.54$ |
| $B^+ \rightarrow D^{0*} D^+$ | $7.9 \pm 2.4$ | $8.73$ | $8.27$ | $6.61$ |
| $B^+ \rightarrow D^* D^+$ | $5.2 \pm 1.9$ | $8.73$ | $8.27$ | $6.61$ |
| $B^+ \rightarrow D^* D^{*+}$ | $5.4 \pm 2.4$ | $6.05$ | $5.49$ | $6.15$ |
| $B^0 \rightarrow D^- D^{*+}$ | $6.5 \pm 3.2$ | $6.05$ | $5.49$ | $6.15$ |
| $B^+ \rightarrow D^{0*} D^+$ | $7.3 \pm 3.0$ | $6.14$ | $6.47$ | $4.49$ |
| $B^+ \rightarrow D^* D^+$ | $6.2 \pm 2.2$ | $6.14$ | $6.47$ | $4.49$ |
| $B^+ \rightarrow D^* D^{*+}$ | $16.3 \pm 6.0$ | $14.68$ | $15.24$ | $15.8$ |
| $B^0 \rightarrow D^- D^{*+}$ | $12.9 \pm 4.5$ | $14.68$ | $15.24$ | $15.8$ |

**TABLE XII.** Non-leptonic decay rates $\Gamma [\text{ns}^{-1}]$. We use $|V_{cb}| = 0.975$, $|V_{td}| = 0.975$, $|V_{us}| = 0.223$ and $|V_{ub}| = 0.034$ resp. $|V_{cb}| = 0.035$ (B), as described in section III A. The results of [22] have been adapted to the decay constants used in our calculation.
| Meson | Exp. | Model $A$ | Model $B$ |
|-------|------|-----------|-----------|
| $\pi$ | $130.7 \pm 0.1 \pm 0.36$ | 212 | 219 |
| $K$   | $159.8 \pm 1.4 \pm 0.44$ | 248 | 238 |
| $D$   | $300^{+180+80}_{-150-40}$ | 293 | 263 |
| $D_s$ | $280 \pm 19 \pm 28 \pm 34$ | 315 | 284 |
| $\rho$ | $216 \pm 5$ | 470 | 717 |
| $\omega$ | $195 \pm 3$ | 472 | 726 |
| $\phi$ | $237 \pm 4$ | 475 | 685 |
| $K^*$ | $214 \pm 7$ | 467 | 695 |
| $D^*$ | — | 339 | 409 |
| $D_s^*$ | — | 378 | 445 |

TABLE XIII. Decay constants of pseudoscalar and pseudovector mesons in MeV