Weak and Strong Forms of $\omega$-Perfect Mappings

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Abstract:
In this paper, we introduce weak and strong forms of $\omega$-perfect mappings, namely the $\theta$-$\omega$-perfect, weakly $\theta$-$\omega$-perfect and strongly $\theta$-$\omega$-perfect mappings. Also, we investigate the fundamental properties of these mappings. Finally, we focused on studying the relationship between weakly $\theta$-$\omega$-perfect and strongly $\theta$-$\omega$-perfect mappings.

Keywords: weakly $\theta$-$\omega$-perfect mappings and strongly $\theta$-$\omega$-perfect mappings.

1. Introduction
In 1943, Formin [1] introduced the concepts of $\theta$-continuous mappings. In 1966, Bourbaki [2] defined perfect mappings. In 1968, Velicko[3] introduced the concepts of $\theta$-open and $\theta$-closed subsets, while in 1968 Singal [4] introduced the notion of almost continuous mappings. In 1981, Long and Herrington [5] introduced the notion of strongly continuous mappings, in 1989, Hdeib [6] introduced the concepts of $\omega$-continuous mappings. In 1991, Chew and Tong [7] introduced the notion of weakly continuous mappings. In this work, $(G, \tau)$ and $(H, \sigma)$ stand for topological spaces. For a subset $K$ of $G$, the closure of $K$ and the interior of $K$ will be denoted by $\text{cl}(K)$ and $\text{int}(K)$, respectively. Let $(G, \tau)$ be a space and $K$ be a subset of $G$, then a point $g \in G$ is called a condensation point of $K$ if, for each $S \in \tau$ and $g \in S$, the set $S \cap K$ is uncountable. $K$ is called to be $\omega$-closed [6] if it contains all its condensation points. The complement of $\omega$-closed set is called to be $\omega$-open. It is well known that a subset $W$ of a space $(G, \tau)$ is $\omega$-open if and only if, for each $g \in W$, there exists $S \in \tau$, such that $g \in S$ and $S-W$ is countable. The family of all $\omega$-open sets of a space $(G, \tau)$, denoted by $\omega(G)$, forms a topology on $G$ finer than $\tau$. The $\omega$-closure and $\omega$-interior, that can be known in the same way as $\text{cl}(K)$ and $\text{int}(K)$, respectively, will be denoted by $\omega\text{cl}(K)$ and $\omega\text{int}(K)$, respectively. Several characterizations of $\omega$-closed sets were provided in previous articles [8-16]. A point $g$ of $G$ is called $\theta$-cluster point of $K$ if $\text{cl}(S) \cap K \neq \emptyset$, for all open sets $S$ of $G$ containing $g$. The set of all $\theta$-cluster points

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of $K$ is called $\theta$-closure of $K$ and is denoted by $\text{cl}_\theta(K)$. A subset $K$ is called $\theta$-closed if $K = \text{cl}_\theta(K)$ [3]. The complement of $\theta$-closed set is called $\theta$-open. A point $g$ of $G$ is called an $\omega$-$\theta$-cluster point of $K$ if $\omega\text{cl}(S) \cap K \neq \emptyset$ for every $\omega$-open set $S$ of $G$ containing $g$. The set of all $\omega$-$\theta$-cluster points of $K$ is called $\omega$-$\theta$-closure of $K$ and is denoted by $\omega\text{cl}_\theta(K)$. A subset $K$ is called $\omega$-$\theta$-closed if $K = \omega\text{cl}_\theta(K)$. The complement of $\omega$-$\theta$-closed set is called $\omega$-$\theta$-open. The $\omega$-$\theta$-interior of $K$ is defined by the union of each $\omega$-$\theta$-open sets contained in $K$ and is denoted by $\omega\text{int}_\theta(K)$. A mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is called $\omega$-continuous (see[16]) (resp., almost weakly $\omega$-continuous[see[11]]) if for each $g \in G$ and each open set $T$ of $H$ containing $\lambda(g)$, there exists an open subset $S$ in $G$, such that $\lambda(S) \subseteq T$ (resp., $\lambda(S) \subseteq \text{cl}(T)$). A mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is called almost $\omega$-continuous[12] (resp., $\theta$-$\omega$-continuous (see[13])), strongly $\omega$-$\theta$-continuous (see[7])) if, for each $g \in G$ and for each regular open set $T$ (resp., open) of $H$ containing $\lambda(g)$, there exists an open subset $S$ in $G$, such that $\lambda(S) \subseteq T$ (resp., $\lambda(\omega\text{cl}(S)) \subseteq \text{cl}(T)$, $\lambda(\omega\text{cl}(S)) \subseteq T$). A mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is called $\theta$-continuous (resp., continuous[16]), if for all an open set $T$ in $H$, $\lambda^{-1}(T)$ is an $\theta$-open (resp., open) set in $G$. A mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is called weakly (resp., strongly) continuous[3] if, for each $g \in G$ and all open set $T$ of $H$ containing $\lambda(g)$, there is an open set $S$ of $G$, such that $\lambda(S) \subseteq \text{cl}(T)$ (resp., $\lambda(\text{cl}(S)) \subseteq T$). A mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is called almost continuous[7] if $\lambda^{-1}(T)$ is open in $G$ for all regular open set $T$ of $H$. A mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is called weakly (resp., strongly) $\theta$-continuous if, for each $g \in G$ and all open set $T$ of $H$ containing $\lambda(g)$, there is an open set $S$ in $G$, such that $\lambda(S) \subseteq \text{cl}(T)$ (resp., $\lambda(\text{cl}(S)) \subseteq T$). A topological space $G$ is called a regular[14] if, for all closed set $F$ and for each point $g \in G$, there exist disjoint open sets $S$ and $T$ such that $g \in S$ and $F \subseteq T$. A topological space $G$ is called a semi-regular[15] if, for all point $g \in G$ and all open set $S$ containing $g$, there is an open set $T$ such that $g \in S \subseteq \text{int}(\text{cl}(T)) \subseteq S$. A topological space $G$ is called $\omega$-regular (resp., $\omega$-regular[12]) if, for all closed (resp., closed) set $F$ and for each point $g \in G$, there are disjoint $\omega$-open sets $S$ and $T$ such that $g \in S$ and $F \subseteq T$. Also we introduce several results and examples concerning deferent forms of $\omega$-perfect mappings.

2. Weakly $\theta$-$\omega$-Perfect Mappings

In this section, we study the weakly $\theta$-$\omega$-perfect mappings and several related theorems.

**Definition 2.1.** A mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is said to be weakly $\theta$-$\omega$-continuous at $g \in G$ if, for every open subset $T$ of $H$ containing $\lambda(g)$, there exists an $\omega$-$\theta$-open subset $S$ in $G$ containing $g$, such that $\lambda(S) \subseteq \text{cl}(T)$. If $\lambda$ is weakly $\omega$-$\theta$-continuous at every $g \in G$, it is said to be weakly $\theta$-$\omega$-continuous.

**Definition 2.2.** A mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is said to be perfect mapping (resp., $\omega$-perfect mapping, $\theta$-$\omega$-perfect mapping, almost $\omega$-perfect mapping, weakly $\theta$-$\omega$-perfect mapping, almost weakly $\omega$-perfect mapping, $\theta$-perfect mapping) if it is continuous (resp., $\omega$-continuous, $\theta$-$\omega$-continuous, almost $\omega$-continuous, weakly $\theta$-$\omega$-continuous, almost weakly $\omega$-continuous, $\theta$-continuous), closed, and, for every $h \in H$, $\lambda^{-1}(h)$, compact. The relationships among the weakly $\omega$-perfect mappings are given by the following figure:

- $\omega$-perfect mapping
- almost weakly $\omega$-perfect mapping
- weakly $\theta$-$\omega$-perfect mapping
- almost $\omega$-perfect mapping
- $\theta$-$\omega$-perfect mapping

In the figure above, the converses are not true, as demonstrated by the following examples.

**Example 2.3.** Let $\lambda : (G, \tau) \rightarrow (G, \tau)$ be a mapping such that $G = \{K, L, M\}$, and $\tau = \{\emptyset, G, \{K\}, \{L\}, \{K, L\}\}$ such that $\lambda(K) = \lambda(L) = \lambda(M) = M$. Then $\lambda$ is $\theta$-$\omega$-perfect mapping but it is not almost $\omega$-perfect mapping.
Example 2.4. Let $\lambda : (\mathbb{R}, \tau) \to (H, \sigma)$ be a mapping such that $\mathbb{R}$ be a real line with topology $\tau = \{ \varnothing, \mathbb{R}, (0, 1) \}$. Let $H = \{ u, v, w \}$ and $\sigma = \{ H, \varnothing, \{ v \}, \{ w \}, \{ v, w \} \}$. 
\[
\lambda (g) = \begin{cases} 
  u, & \text{if } g \in [0, 2] \\
  v, & \text{if } g \notin [0, 2]
\end{cases}
\]
Then, $\lambda$ is weakly $\theta$-perfect but it is not $\theta$-perfect.

Example 2.5. As in example 2.4, $\lambda$ is weakly $\theta$-perfect mapping, but it is not $\omega$-perfect mapping. Also, $\lambda$ is weakly $\theta$-perfect mapping, but not almost $\omega$-perfect mapping, and $\lambda$ is almost weakly $\omega$-perfect mapping, but it is not $\theta$-perfect mapping.

Example 2.6. Let $\lambda : (G, \tau) \to (G, \sigma)$ be a mapping such that $G = \{ u, v, w \}$, and $\tau = \{ G, \varnothing, \{ u, v \} \}$ and $\sigma = \{ G, \varnothing, \{ v, w \} \}$, such that $\lambda (u) = \lambda (v) = \lambda (w) = u$. Then $\lambda$ is almost $\omega$-perfect mapping but it is not $\omega$-perfect mapping.

Example 2.7. Let A be the upper half of a plane and B be the X-axis. Let $X = A \cup B$. If $\tau_{\text{disc}}$ be the half disc topology on X and $\tau$, be the relative topology that X inherits by virtue of being a subspace of $\mathbb{R}^2$. Then, the identity of the mapping $\lambda : (X, \tau) \to (X, \tau_{\text{disc}})$ is that it is an almost weakly $\omega$-perfect mapping but it is not $\omega$-perfect mapping.

Example 2.8. Let $\lambda : (G, \tau) \to (G, \sigma)$ be a mapping such that $G = \{ K, L, M \}$ and $\tau = \{ G, \varnothing, \{ K, \} \}$. Let $\sigma = \{ G, \varnothing, \{ K, L \} \}$, such that $\lambda (K) = \lambda (L) = \lambda (M) = M$. Then $\lambda$ is almost weakly $\omega$-perfect mapping but it is not $\omega$-perfect mapping.

Lemma 2.9. [13] A topological space $G$ is $\omega$-regular (resp., $\omega^\ast$-regular) if and only if, for all $S \in o\omega(G)$ (resp., $S \in O(G)$) and all point $g \in S$, there is $T \in o\omega(G, g) ; g \in T \subseteq ocl(T) \subseteq S$.

Theorem 2.10. Let $\lambda : (G, \tau) \to (H, \sigma)$ be a mapping such that $G$ be an $\omega$-regular space. If $\lambda$ is almost weakly $\omega$-perfect mapping then it is $\theta$-perfect mapping.

Proof: Assume that $\lambda$ is almost weakly $\omega$-perfect mapping. It suffices to be demonstrated that $\lambda$ is $\theta$-continuous, let $g \in G$ and $T$ be an open set containment $\lambda (g)$ in $H$. Because $\lambda$ is almost weakly $\omega$-continuous, there is an $\omega$-open set $S$ containment $g$, such that $\lambda (S) \subseteq cl(T)$. Since $G$ is an $\omega$-regular space, by Lemma 2.9, there is $W \in o\omega(G, g)$ such that $g \in W \subseteq ocl(W) \subseteq S$. Therefore, $\lambda(o\omega(W)) \subseteq cl(T)$. Then $\lambda$ is $\theta$-continuous, so $\lambda$ is $\theta$-perfect mapping.

Corollary 2.11. Let $(G, \tau)$ be $\omega$-regular spaces. The mapping $\lambda : (G, \tau) \to (H, \sigma)$ is almost weakly $\omega$-perfect if and only if it is $\theta$-perfect.

Theorem 2.12. Let $\lambda : (G, \tau) \to (H, \sigma)$ be an $\omega$-perfect mapping, and let $\mu : (H, \sigma) \to (I, \psi)$ be almost weakly $\omega$-perfect. Then $\mu \circ \lambda : (G, \tau) \to (I, \psi)$ is almost weakly $\omega$-perfect.

Proof: Assume that $g \in G$ and $W$ is an open set containment $(\mu \circ \lambda) (g)$ in $I$. Since $\mu$ is almost weakly $\omega$-continuous, there is an open set $T$ containment $\lambda (g)$ in $H$ such that $\mu (T) \subseteq cl(W)$. Since $\lambda$ is $\omega$-continuous, then for each $g \in G$ and each open set $T$ of $\lambda (g) = h$, there is an open $S$ of $g$ in $G$ such that $\lambda (S) \subseteq T$, so $\mu (\lambda (S)) \subseteq \mu (T)$ also $(\mu \circ \lambda) (g) \subseteq \mu (T)$, then $(\mu \circ \lambda) (g) \subseteq cl(W)$. Also $\mu \circ \lambda$ is almost weakly $\omega$-continuous. Hence, $\mu \circ \lambda$ is almost weakly $\omega$-perfect mapping.

Theorem 2.13. Let $\lambda : (G, \tau) \to (H, \sigma)$ be a mapping, such that $G$ be an $\omega$-regular space. If $\lambda$ is weakly $\theta$-perfect mapping then it is $\omega$-perfect mapping.

Proof: Let $\lambda$ be a weakly $\theta$-perfect mapping. It suffices to be demonstrated that $\lambda$ is $\omega$-continuous, let $g \in G$ and $T$ be an open set containment $\lambda (g)$ in $H$. Since $H$ is an $\omega$-regular space, $\lambda (g) \in TJ$ and $cl(T) \subseteq T$. Since $\lambda$ is weakly $\theta$-continuous, there is an open $S$ of $g$ with $\lambda (S) \subseteq cl(T)$. It follows that $\lambda (S) \subseteq T$, therefore $\lambda$ is $\omega$-continuous. Hence $\lambda$ is $\omega$-perfect mapping.

Corollary 2.14. Let $(G, \tau)$ be $\omega$-regular spaces. The mapping $\lambda : (G, \tau) \to (H, \sigma)$ is weakly $\theta$-perfect if and only if it is $\omega$-perfect.

Theorem 2.15. Let $\lambda : (G, \tau) \to (H, \sigma)$ be a mapping, such that $G$ be an $\omega$-regular space. If $\lambda$ is $\theta$-perfect mapping then it is almost $\omega$-perfect mapping.

Proof: Let $\lambda$ be a $\theta$-perfect mapping. It suffices to be demonstrated that $\lambda$ is almost $\omega$-continuous, let $g \in G$ and $T$ be an open set containing $\lambda (g)$ in $H$. Because $\lambda$ is $\theta$-continuous, $y$nd $\lambda$ is an $\omega$-open set $S$ containing $g$, such that $\lambda (ocl(S)) \subseteq cl(T)$. Because $\lambda (ocl(S)) \subseteq int(cl(T)) \subseteq cl(T)$, then $\lambda (ocl(S)) \subseteq int(cl(T)) \subseteq cl(T)$, then $\lambda (ocl(S)) \subseteq cl(T)$. Also $G$ is $\omega$-regular space, and there is an $\omega$-open set $SI$ in
Almost such that \( g \in S I \) and \( \text{cl}(SI) \subseteq S \), so \( \lambda(\text{ocl}(SI)) \subseteq \lambda(S) \) and \( \text{int}(\text{cl}(T)) \subseteq \text{cl}(T) \). It follows that \( \lambda(S) \subseteq \text{int}(\text{cl}(T)) \). So \( \lambda \) is almost \( \omega \)-continuous. Hence, consider that \( \lambda \) is almost \( \omega \)-perfect mapping.

**Corollary 2.16.** Let \((G , \tau)\) be a \( \omega \)-regular spaces. The mapping \( \lambda : (G , \tau) \to (H , \sigma) \) is \( \theta\omega \)-perfect if and only if it is almost \( \omega \)-perfect.

**Theorem 2.17.** Let \( \lambda : (G , \tau) \to (H , \sigma) \) be a mapping such that \( H \) be an \( \omega \)-regular space. If \( \lambda \) is almost weakly \( \omega \)-perfect mapping on \( G \), then it is \( \omega \)-perfect mapping on \( G \).

**Proof:** Let \( \lambda \) be almost weakly \( \omega \)-perfect mapping. It suffices to be demonstrated that \( \lambda \) is \( \omega \)-continuous, let \( g \in G \) and \( T \) be an open set containing \( \lambda(g) \) in \( H \). Since \( H \) is an \( \omega \)-regular space, there is an open set \( TI \) in \( H \) such that \( \lambda(g) \in TI \) and \( \text{cl}(TI) \subseteq T \). Since \( \lambda \) is almost weakly \( \omega \)-continuous, there is an open-open set \( S \) containment \( g \), such that \( \lambda(S) \subseteq \text{cl}(T) \). It follows that \( \lambda(\text{ocl}(S)) \subseteq \text{cl}(T) \), therefore \( \lambda \) is \( \omega \)-continuous. Hence, \( \lambda \) is \( \omega \)-perfect mapping.

**Corollary 2.18.** Let \((H , \tau)\) be \( \omega \)-regular spaces. The mapping \( \lambda : (G , \tau) \to (H , \sigma) \) is almost weakly \( \omega \)-perfect if and only if it is \( \omega \)-perfect.

**Theorem 2.19.** Let \( \lambda : (G , \tau) \to (H , \sigma) \) be a mapping, such that \( G \) be an \( \omega \)-regular space. If \( \lambda \) is weakly \( \theta\omega \)-perfect mapping then it is \( \theta\omega \)-perfect mapping.

**Proof:** Let \( \lambda \) be weakly \( \theta\omega \)-perfect mapping. It suffices to be demonstrated that \( \lambda \) is \( \theta\omega \)-continuous, let \( g \in G \) and \( T \) be an open set containment \( \lambda(g) \) in \( H \). Since \( G \) is an \( \omega \)-regular space, there is an open set \( TI \) in \( H \) such that \( \lambda(g) \in TI \) and \( \text{cl}(SI) \subseteq S \). Since \( \lambda \) is weakly \( \theta\omega \)-continuous, there is an open-open set \( S \) containment \( g \), such that \( \lambda(S) \subseteq \text{cl}(T) \). Then \( \lambda(S) \subseteq \text{cl}(T) \), \( H \) is \( \omega \)-regular space, and there is an open-open set \( SI \) in \( G \), such that \( g \in SI \) and \( \text{cl}(TI) \subseteq T \), so \( \lambda(S) \subseteq \text{cl}(TI) \subseteq T \). It follows that \( \lambda(S) \subseteq T \). So \( \lambda \) is \( \omega \)-continuous. Hence, consider that \( \lambda \) is \( \omega \)-perfect mapping.

**Corollary 2.20.** Let \((G , \tau)\) be \( \omega \)-regular spaces. The mapping \( \lambda : (G , \tau) \to (H , \sigma) \) is \( \omega \)-perfect if and only if it is \( \theta\omega \)-perfect.

**Theorem 2.21.** Let \( \lambda : (G , \tau) \to (H , \sigma) \) be a mapping, such that \( H \) be an \( \omega \)-regular space. If \( \lambda \) is almost \( \omega \)-perfect mapping then it is \( \omega \)-perfect mapping.

**Proof:** Let \( \lambda \) be an almost \( \omega \)-perfect mapping. It suffices to be demonstrated that \( \lambda \) is \( \omega \)-continuous, let \( g \in G \) and \( T \) be an open set containment \( \lambda(g) \) in \( H \). Because \( \lambda \) is almost \( \omega \)-continuous, there is an open-open set \( S \) containment \( g \), such that \( \lambda(S) \subseteq \text{int}(\text{cl}(T)) \). Because \( \text{int}(\text{cl}(T)) \subseteq \text{cl}(T) \), then \( \lambda(S) \subseteq \text{int}(\text{cl}(T)) \subseteq \text{cl}(T) \). Then \( \lambda(S) \subseteq \text{cl}(T) \), \( H \) is \( \omega \)-regular space, and there is an open-open set \( SI \) in \( G \), such that \( g \in SI \) and \( \text{cl}(TI) \subseteq T \), so \( \lambda(S) \subseteq \text{cl}(TI) \subseteq T \). It follows that \( \lambda(S) \subseteq T \). So \( \lambda \) is \( \omega \)-continuous.

**Corollary 2.22.** Let \((G , \tau)\) be \( \omega \)-regular spaces. The mapping \( \lambda : (G , \tau) \to (H , \sigma) \) is \( \omega \)-perfect if and only if it is \( \theta\omega \)-perfect.

**Theorem 2.23.** Let \( \lambda : (G , \tau) \to (H , \sigma) \) be a mapping such that \( H \) be an \( \omega \)-regular space. If \( \lambda \) is weakly \( \theta\omega \)-perfect mapping then it is \( \omega \)-perfect mapping.

**Proof:** Let \( \lambda \) be weakly \( \theta\omega \)-perfect mapping. It suffices to be demonstrated that \( \lambda \) is almost \( \omega \)-continuous, let \( g \in G \) and \( T \) be an open set containment \( \lambda(g) \) in \( H \). Since \( H \) is an \( \omega \)-regular space then it is an open-open set \( TI \) in \( H \) such that \( \lambda(g) \in TI \) and \( \text{cl}(TI) \subseteq T \). Since \( \lambda \) is weakly \( \theta\omega \)-continuous, there is an open-open set \( S \) containment \( g \), such that \( \lambda(S) \subseteq \text{cl}(T) \). Also, \( \text{int}(\text{cl}(T)) \subseteq \text{cl}(T) \). It follows that \( \lambda(S) \subseteq \text{int}(\text{cl}(T)) \subseteq \text{cl}(T) \), therefore \( \lambda(S) \subseteq \text{int}(\text{cl}(T)) \). So \( \lambda \) is almost \( \omega \)-continuous on \( G \). Hence \( \lambda \) is almost \( \omega \)-perfect mapping.

**Corollary 2.24.** Let \((H , \tau)\) be \( \omega \)-regular spaces. The mapping \( \lambda : (G , \tau) \to (H , \sigma) \) is weakly \( \theta\omega \)-perfect if and only if it is \( \theta\omega \)-perfect.

**Theorem 2.25.** Let \( \lambda : (G , \tau) \to (H , \sigma) \) be a mapping and \( \mu : G \to G \times H \) be the graph mapping of \( \lambda \) defined by \( \mu(g) = (g , \lambda(g)) \) for every \( g \in G \). Then \( \mu \) is \( \theta\omega \)-perfect if and only if \( \lambda \) is \( \theta\omega \)-perfect.

**Proof:** Necessity. Assume that \( \mu \) is \( \theta\omega \)-perfect mapping. It suffices to be demonstrated that \( \lambda \) is \( \theta\omega \)-continuous, let \( g \in G \) and \( T \) be an open set containment \( \lambda(g) \). Then \( G \times T \) is an open set of \( G \times H \) containment \( \mu(g) \). Because \( \mu \) is \( \theta\omega \)-continuous, there is \( S \in \omega O(G , g) \) such that \( \mu(\text{ocl}(S)) \subseteq \text{cl}(G \times T) = G \times \text{cl}(T) \). Therefore, \( \lambda(\text{ocl}(S)) \subseteq \text{cl}(T) \), therefore \( \lambda \) is \( \theta\omega \)-continuous. So \( \lambda \) is \( \theta\omega \)-perfect mapping.
Sufficiency. Assume that $\lambda$ is $\theta$-$\omega$-perfect mapping. It suffices to be demonstrated that $\lambda$ is $\theta$-$\omega$-continuous, let $g \in G$ and $W$ be an open set of $G \times H$ containment $\mu(g)$. There are the open sets $SI \subseteq G$ and $T \subseteq H$ such that $\mu(g) = (g, \lambda(g)) \in SI \times T \subseteq W$. Because $\lambda$ is $\theta$-$\omega$-continuous, there is $S2 \in \omega O(G, g)$ such that $\lambda (\text{occl}(S2)) \subseteq \text{cl}(T)$. Assume that $S = SI \cap S2$, then $S \in \omega O(G, g)$. Therefore, $\mu (\text{occl}(S)) \subseteq \text{cl}(SI) \times \lambda (\text{occl}(S2)) \subseteq \text{cl}(SI) \times \text{cl}(T) \subseteq \text{cl}(W)$. Then $\mu$ is $\theta$-$\omega$-continuous. So $\mu$ is $\theta$-$\omega$-perfect mapping.

**Theorem 2.26.** For a mapping $\lambda : G \rightarrow H$ and $H$ is regular, the following properties are equivalent.

(a) $\lambda$ is weakly $\theta$-$\omega$-perfect.

(b) $\lambda$ is $\omega$-perfect.

(c) $\lambda$ is almost $\omega$-perfect.

(d) $\lambda$ is $\theta$-$\omega$-perfect.

(e) $\lambda$ is almost $\omega$-perfect.

3. **Strongly $\theta$-$\omega$-Perfect Mappings**

In this section we study the strongly $\theta$-$\omega$-perfect mappings and some of their theorems.

**Definition 3.1.** A mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is said to be almost strongly $\omega$-continuous if, for each $g \in G$ and each regular open set $T$ of $H$ containing $\lambda(g)$, there exists an $\omega$-open subset $S$ in $G$, such that $\lambda(S) \subseteq T$.

**Definition 3.2.** A mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is said to be strongly $\theta$-$\omega$-perfect mapping (resp., almost strongly $\omega$-perfect mapping) if it is strongly $\theta$-$\omega$-continuous (resp., almost strongly $\omega$-continuous), closed, and, for every $h \in H$, $\lambda^{-1}(h)$,compact.

The relationships among the strongly $\omega$-perfect mappings are given by the following figure:

\[\text{\textbf{$\omega$-perfect mapping}} \rightarrow \text{\textbf{almost strongly $\omega$-perfect mapping}} \rightarrow \text{\textbf{strongly $\theta$-$\omega$- perfect mapping}} \rightarrow \text{\textbf{$\theta$-$\omega$-perfect mapping}} \rightarrow \text{\textbf{$\omega$-perfect mapping}}\]

In the figure above, the converses are not to be right, as demonstrated by the following examples:

**Example 3.3.** Let $\lambda : (G, \tau) \rightarrow (G, \tau)$ be a mapping such that $G = \{K, L, M\}$, and $\tau = \{\emptyset, G, \{K\}, \{L\}, \{K, L\}\}$ such that $\lambda(K) = \lambda(L) = K$, $\lambda(M) = M$. Then $\lambda$ is $\omega$-perfect mapping but is not strongly $\theta$-$\omega$-perfect mapping.

**Theorem 3.4.** Let $\lambda : (G, \tau) \rightarrow (H, \sigma)$ be a mapping such that $H$ be an regular space. If $\lambda$ is $\omega$-perfect mapping then it is strongly $\theta$-$\omega$-perfect mapping.

**Proof:** Let $\lambda$ be an $\omega$-perfect mapping. It suffices to demonstrate that $\lambda$ is strongly $\theta$-$\omega$-continuous. Let $g \in G$ and $T$ be an open set containment $\lambda(g)$ in $H$. Because of $H$ is an regular space, there is an open set $W$ such that $\lambda(g) \in W \subseteq \text{cl}(W) \subseteq T$. Since $\lambda$ is $\omega$-continuous, then $\lambda^{-1}(W)$ is an $\omega$-open set and $\lambda^{-1}(\text{cl}(W))$ is an $\omega$-closed. Assume that $S = \lambda^{-1}(W)$, then $g \in \lambda^{-1}(W) \subseteq \lambda^{-1}(\text{cl}(W))$, $S \in \omega O(G, g)$ and $\text{occl}(S) \subseteq \lambda^{-1}(\text{cl}(W))$. We have $\lambda(\text{occl}(S)) \subseteq \text{cl}(W) \subseteq T$, therefore $\lambda$ is strongly $\theta$-$\omega$-continuous. Hence $\lambda$ is strongly $\theta$-$\omega$-perfect mapping.

**Corollary 3.5.** Let $(H, \tau)$ be regular spaces. The mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is $\omega$-perfect if and only if it is strongly $\theta$-$\omega$-perfect.

**Example 3.6.** Let $\lambda : (\mathcal{R}, \tau) \rightarrow (\mathcal{R}, \tau)$ be a mapping where $\lambda(g) = g$, and let $(\mathcal{R}, \tau)$ where $\tau$ is the topology with a basis whose members are of the form $(a, b)$ and $(a, b) \cdot N$ such that $N = \{1/n ; n \in \mathbb{Z}^+\}$. Then $(\mathcal{R}, \tau)$ is a Hausdorff but not $\omega$-regular. Then $\lambda$ is $\omega$-perfect but not almost strongly $\omega$-perfect mapping.

**Theorem 3.7.** Let $\lambda : (G, \tau) \rightarrow (H, \sigma)$ be a mapping such that $G$ be an $\omega$-regular space. If $\lambda$ is $\omega$-perfect mapping then it is almost strongly $\omega$-perfect mapping.
Proof: Let $\lambda$ be an $\omega$-perfect mapping. It suffices to demonstrate that $\lambda$ is almost strongly $\omega$-continuous. Let $g \in G$ and $T$ be an open set containment $\lambda (g)$ in $H$. Since $\lambda$ is $\omega$-continuous, there is an $\omega$-open set $S$ containment $g$ in $G$ such that $\lambda (S) \subseteq T \subseteq \text{cl}(T)$. Since $G$ is $\omega$-regular, there is an $\omega$-open set $S_1$ in $G$ such that $g \in S_1$ and $\text{cl}(S_1) \subseteq S_1$, so $\lambda (\text{cl}(S_1)) \subseteq \lambda (S)$, $\lambda (S) \subseteq \text{cl}(T)$ and $\text{int}(\text{cl}(T)) \subseteq \text{cl}(T)$. It follows that $\lambda (\text{cl}(S_1)) \subseteq \text{int}(\text{cl}(T))$, therefore $\lambda$ is almost strongly $\omega$-continuous. Hence $\lambda$ is almost strongly $\omega$-perfect mapping.

Corollary 3.8. Let $(G , \tau)$ be $\omega$-regular spaces. The mapping $\lambda : (G , \tau) \rightarrow (H , \sigma)$ is $\omega$-perfect if and only if it is almost strongly $\omega$-perfect.

Example 3.9. Let $G = \{ u , v , w \}$ and $\lambda : (G , \tau) \rightarrow (G , \sigma)$, such that $\tau = \{ G , \varphi , \{ u , v \} , \sigma =\{ G , \varphi , \{ v , w \} \}$, and $\lambda (u) = \lambda (w) = w$, $\lambda (v) = v$. Then $\lambda$ is $\theta$-$\omega$-perfect mapping but not strongly $\theta$-$\omega$-perfect mapping.

Theorem 3.10. Let $\lambda : (G , \tau) \rightarrow (H , \sigma)$ be a mapping such that $H$ be an regular space. If $\lambda$ is $\theta$-$\omega$-perfect mapping then it is strongly $\theta$-$\omega$-perfect mapping.

Proof: Let $\lambda$ be an $\theta$-$\omega$-perfect mapping. It suffices to demonstrate that $\lambda$ is a strongly $\theta$-$\omega$-continuous, let $g \in G$ also $T$ be an open set containment $\lambda (g)$ in $H$. Since $\lambda$ is $\theta$-$\omega$-continuous, there is an $\omega$-open set $S$ containment $g$ in $G$ such that $\lambda (\text{cl}(S)) \subseteq \text{cl}(T)$. Since $H$ is regular, there is an open set $W$ such that $\lambda (g) \in W \subseteq \text{cl}(W) \subseteq T$, then $\lambda (\text{oclc}(S)) \subseteq \text{cl}(W) \subseteq T$, therefore $\lambda (\text{cl}(S)) \subseteq T$. So $\lambda$ is strongly $\omega$-continuous. Hence $\lambda$ is strongly $\theta$-$\omega$-perfect mapping.

Corollary 3.11. Let $(H , \tau)$ be regular spaces. The mapping $\lambda : (G , \tau) \rightarrow (H , \sigma)$ is $\theta$-$\omega$-perfect if and only if it is strongly $\theta$-$\omega$-perfect.

Theorem 3.12. A space $G$ is $\omega^*$-regular if and only if, for any space $H$, any perfect mapping $\lambda : (G , \tau) \rightarrow (H , \sigma)$ is strongly $\theta$-$\omega$-perfect mapping.

Proof: Sufficiency. Let $\lambda : G \rightarrow G$ be the identity mapping. Then $\lambda$ is continuous and strongly $\theta$-$\omega$-continuous by our hypothesis. For any open set $S$ of $G$ and for any point $g$ of $S$, we have $\lambda (g) = g \in S$. Also, there is $T \subseteq \omega O(G , g)$ such that $\lambda (\text{oclc}(T)) \subseteq S$, therefore $g \in T \subseteq \omega O(T) \subseteq S$. It follows from Lemma 2.9 that $G$ is $\omega^*$-regular.

Necessity. Assume that $\lambda : G \rightarrow H$ is continuous and $G$ is $\omega^*$-regular. For any $g \in G$ and any open neighborhood $T$ of $\lambda (g)$, $\lambda^{-1}(T)$ is an open set of $G$ containing $g$. Since $G$ is $\omega^*$-regular, there is $S \subseteq \omega O(G)$ such that $g \in S \subseteq \text{oclc}(S) \subseteq \lambda^{-1}(T)$ by Lemma 2.9. Therefore, $\lambda (\text{oclc}(S)) \subseteq T$. Hence $\lambda$ is strongly $\theta$-$\omega$-perfect.

Example 3.13. Let $\lambda : (G , \tau) \rightarrow (G , \tau)$ be a mapping such that $G = \{ K , L , M \}$ and $\tau = \{ \varphi , G , \{ K \}, \{ L \}, \{ K , L \} \}$, such that $\lambda (K) = \lambda (L) = \lambda (M) = M$. Then $\lambda$ is $\theta$-$\omega$-perfect mapping, but not almost strongly $\omega$-perfect mapping.

Theorem 3.14. Let $\lambda : (G , \tau) \rightarrow (H , \sigma)$ be a mapping such that $H$ be an $\omega$-regular space. If $\lambda$ is $\theta$-$\omega$-perfect mapping then it is almost strongly $\omega$-perfect mapping.

Proof: Let $\lambda$ be an $\theta$-$\omega$-perfect mapping. It suffices to demonstrate that $\lambda$ is almost strongly $\omega$-continuous, let $g \in G$ and $T$ be an open set containment $\lambda (g)$ in $H$. Since $\lambda$ is $\theta$-$\omega$-continuous, there is an $\omega$-open set $S$ containment $g$ in $G$ such that $\lambda (\text{cl}(S)) \subseteq \text{cl}(T)$. Since $H$ is an $\omega$-regular, there is an $\omega$-open set $T_1$ in $H$ such that $\lambda (g) \in T_1$, also $\text{cl}(T_1) \subseteq T$ and $\text{int}(\text{cl}(T_1)) \subseteq \text{cl}(T_1)$. It follows that $\lambda (\text{cl}(S)) \subseteq \text{int}(\text{cl}(T)), therefore \lambda$ is almost strongly $\omega$-continuous. So $\lambda$ is almost strongly $\omega$-perfect mapping.

Corollary 3.15. Let $(G , \tau)$ be $\omega$-regular spaces. The mapping $\lambda : (G , \tau) \rightarrow (H , \sigma)$ is $\omega$-perfect if and only if it is almost strongly $\omega$-perfect mapping.

Example 3.16. Let $\lambda : (G , \tau) \rightarrow (H , \sigma)$ such that $G = \{ u , v , w \}$, $H = \{ a , b , c \}$, $\tau = \{ G , \varphi , \{ u \}, \{ v \}, \{ u , v \} \}$ and $\sigma =\{ H , \varphi , \{ a \}, \{ b \}, \{ c \}, \{ a , b \}, \{ a , c \}, \{ b , c \} \}$, such that $\lambda (u) = b$, $\lambda (v) = w$ and $\lambda (w) = a$. Then $\lambda$ is almost $\omega$-perfect mapping, but not almost strongly $\omega$-perfect mapping.

Theorem 3.17. Let $\lambda : (G , \tau) \rightarrow (H , \sigma)$ be a mapping such that $G$ be an $\omega$-regular space. If $\lambda$ is almost $\omega$-perfect mapping then it is almost strongly $\omega$-perfect mapping.

Proof: Let $\lambda$ be almost $\omega$-perfect mapping. It suffices to demonstrate that $\lambda$ is almost strongly $\omega$-continuous, let $g \in G$ and $T$ be an open set containment $\lambda (g)$ in $H$. Since $\lambda$ is almost $\omega$-continuous, there is an $\omega$-open set $S$ containment $g$ in $G$ such that $\lambda (S) \subseteq \text{int}(\text{cl}(T))$. Since $G$ is $\omega$-regular, there is an $\omega$-open set $S_1$ in $G$ such that $g \in S_1$, also $\text{cl}(S_1) \subseteq S$, so $\lambda (\text{cl}(S_1)) \subseteq \lambda (S)$, then $\lambda (\text{cl}(S_1)) \subseteq \lambda (S) \subseteq \text{cl}(T)$.
int(cl(T)). It follows that \( \lambda(cl(SI)) \subseteq int(cl(T)) \), therefore \( \lambda \) is almost strongly \( \omega \)-continuous. So \( \lambda \) is almost strongly \( \omega \)-perfect mapping.

**Corollary 3.18.** Let \( (G, \tau) \) be \( \omega \)-regular spaces. The mapping \( \lambda : (G, \tau) \to (H, \sigma) \) is almost \( \omega \)-perfect if and only if it is almost strongly \( \omega \)-perfect.

**Lemma 3.19.** Let a mapping \( \lambda : G \to H \) be strongly \( \theta \)-\( \omega \)-perfect and \( \mu : H \to L \) be perfect. Then \( \mu \circ \lambda \) is strongly \( \theta \)-\( \omega \)-perfect.

**Theorem 3.20.** Let \( \lambda : (G, \tau) \to (H, \sigma) \) be a mapping and \( \mu : G \to G \times H \) the graph mapping of \( \lambda \) defined by \( \mu(g) = (g, \lambda(g)) \) for each \( g \in G \). Then \( \mu : G \to G \times H \) is strongly \( \theta \)-\( \omega \)-perfect if and only if \( \lambda : (G, \tau) \to (H, \sigma) \) is strongly \( \theta \)-\( \omega \)-perfect and \( G \) is an \( \omega \)-regular.

**Proof:** By Lemma 3.19, \( \lambda \) is strongly \( \theta \)-\( \omega \)-perfect if the graph mapping \( \mu \) is strongly \( \theta \)-\( \omega \)-perfect. Also it follows that \( G \) is regular. To prove the converse, assume that \( \lambda \) is strongly \( \theta \)-\( \omega \)-perfect. Let \( g \in G \) and \( W \) be an open set of \( G \times H \) containment \( \mu(g) \). There are the open sets \( S1 \subseteq G \) and \( T \subseteq H \) such that \( \mu(g) = (g, \lambda(g)) \in S1 \times T \subseteq W \). Since \( \lambda \) is strongly \( \theta \)-\( \omega \)-continuous, there is \( S2 \in \omega O(G, g) \) such that \( \lambda(ocl(S2)) \subseteq T \). Because \( G \) is an \( \omega \)-regular and \( S1 \cap S2 \in \omega O(G, g) \), there is \( S \in \omega O(G, g) \) such that \( g \in S \subseteq ocl(S) \subseteq S1 \cap S2 \) (by Lemma 2.9). Therefore, \( \mu(ocl(S)) \subseteq S1 \times \lambda(ocl(S2)) \subseteq S1 \times T \subseteq W \). Then \( \mu \) is strongly \( \theta \)-\( \omega \)-continuous. So \( \mu \) is strongly \( \theta \)-\( \omega \)-perfect mapping.

**Example 3.21.** Let \( \lambda : (G, \tau) \to (H, \sigma) \), such that \( G = H = \{ u, v, w \} \) and \( \tau = \{ \emptyset, G, \{ u \}, \{ v \}, \{ u, v \} \} \), \( \sigma = \{ \emptyset, H, \{ w \} \} \), defined by \( \lambda(u) = \lambda(v) = \lambda(w) = w \). Then \( \lambda \) is strongly \( \theta \)-\( \omega \)-perfect doesn't the mappings \( \mu \) of the \( \lambda \). Then \( \mu(g) = (g, \lambda(g)) \), then it is not strongly \( \theta \)-\( \omega \)-perfect mapping at \( u \) and \( v \).

**Example 3.22.** From in Example 3.9, \( \lambda \) is almost \( \omega \)-perfect mapping, but not strongly \( \theta \)-\( \omega \)-perfect mapping.

**Theorem 3.23.** Let \( \lambda : (G, \tau) \to (H, \sigma) \) be a mapping, such that \( H \) be an \( \omega \)-regular space. If \( \lambda \) is almost \( \omega \)-perfect mapping then it is strongly \( \theta \)-\( \omega \)-perfect mapping.

**Proof:** Let \( \lambda \) be almost \( \omega \)-perfect mapping. It suffices to demonstrate that \( \lambda \) is strongly \( \theta \)-\( \omega \)-continuous, let \( g \in G \) and \( T \) be an open set containment \( \lambda(g) \) in \( H \). Since \( \lambda \) is almost \( \omega \)-continuous, there is an \( \omega \)-open set \( S \) containment \( g \) in \( G \) such that \( \lambda(S) \subseteq int(cl(T)) \). Since \( G \) is \( \omega \)-regular, there is an \( \omega \)-open set \( S1 \) in \( G \) such that \( g \in S1 \) and \( cl(S1) \subseteq S \). So \( \lambda(cl(S1)) \subseteq \lambda(S) \), also int(cl(T)) \( \subseteq cl(T) \). It follows that \( \lambda(cl(S1)) \subseteq T \), therefore \( \lambda \) is strongly \( \theta \)-\( \omega \)-continuous. So \( \lambda \) is strongly \( \theta \)-\( \omega \)-perfect mapping.

**Corollary 3.24.** Let \( (G, \tau) \) be a \( \omega \)-regular spaces. The mapping \( \lambda : (G, \tau) \to (H, \sigma) \) is almost \( \omega \)-perfect if and only if it is strongly \( \theta \)-\( \omega \)-perfect.

**Theorem 3.25.** For a mapping \( \lambda : (G, \tau) \to (H, \sigma) \) and \( H \) is regular space, the following properties are equivalent :

(a) \( \lambda \) is almost strongly \( \theta \)-\( \omega \)-perfect.

(b) \( \lambda \) is \( \omega \)-perfect.

(c) \( \lambda \) is almost \( \omega \)-perfect.

(d) \( \lambda \) is \( \theta \)-\( \omega \)-perfect.

4. Relationship between Weak and Strong Forms of \( \omega \)-Perfect Mappings

In this section, we study the relationship between weakly \( \theta \)-\( \omega \)-perfect mappings and strongly \( \theta \)-\( \omega \)-perfect mappings and some theorems concerning them.

**Definition 4.1.** A mapping \( \lambda : (G, \tau) \to (H, \sigma) \) is said to be super (resp., weakly, strongly) \( \omega \)-continuous if for each \( g \in G \) and each open neighborhood (resp., open set) \( T \) of \( H \) containing \( \lambda(g) \), there exists an \( \omega \)-open neighborhood (resp., \( \omega \)-open set) \( S \) of \( G \), such that \( \lambda(int(cl(S))) \subseteq T \) (resp., \( \lambda(S) \subseteq T \)).

**Definition 4.2.** A mapping \( \lambda : (G, \tau) \to (H, \sigma) \) is said to be almost weakly (resp., almost strongly) continuous if for each \( g \in G \) and each open (resp., regular open) set \( T \) of \( H \) containing \( \lambda(g) \), there exists an open set \( S \) in \( G \), such that \( \lambda(S) \subseteq cl(T) \).

**Definition 4.3.** A mapping \( \lambda : (G, \tau) \to (H, \sigma) \) is said to be weakly \( \theta \)-continuous if for each \( g \in G \) and each open set \( T \) of \( H \) containing \( \lambda(g) \), there exists an open set \( S \) in \( G \), such that \( \lambda(S) \subseteq cl(T) \).

**Definition 4.4.** A mapping \( \lambda : (G, \tau) \to (H, \sigma) \) is called to be super \( \omega \)-perfect mapping (resp., weakly \( \omega \)-perfect mapping, strongly \( \omega \)-perfect mapping, almost weakly perfect mapping, almost strongly perfect mapping, weakly \( \theta \)-perfect mapping) if it is super \( \omega \)-continuous (resp., weakly \( \omega \)-continuous mapping, strongly \( \omega \)-perfect mapping, almost weakly perfect mapping, almost strongly perfect mapping, weakly \( \theta \)-perfect mapping).
strongly \(\omega\)-continuous, almost weakly continuous, almost strongly continuous, weakly \(\theta\)-continuous), closed, and, for every \(h \in H, \lambda^{-1}(h)\), compact.

The relationships weakly and strongly \(\omega\)-perfect mappings are given by the following figure:

\[
\begin{array}{c|c|c|c|}
\text{strongly } & \text{weakly } & \text{weakly } & \text{weakly } \\
\theta\omega\text{-perfect } & \omega\text{-perfect } & \theta\omega\text{-perfect } & \theta\omega\text{-perfect } \\
\downarrow & \downarrow & \downarrow & \downarrow \\
\text{super }\omega\text{-perfect } & \text{almost }\omega\text{-perfect } & \text{almost }\omega\text{-perfect } & \text{almost }\omega\text{-perfect } \\
\downarrow & \uparrow & \downarrow & \uparrow \\
\omega\text{-perfect } & \text{perfect } & \text{perfect } & \text{perfect } \\
\downarrow & \downarrow & \downarrow & \downarrow \\
\theta\omega\text{-perfect } & \text{perfect } & \text{perfect } & \text{perfect } \\
\downarrow & \downarrow & \downarrow & \downarrow \\
\text{weakly }\theta\text{-perfect } & \text{weakly }\omega\text{-perfect } & \text{weakly }\omega\text{-perfect } & \text{weakly }\omega\text{-perfect } \\
\downarrow & \uparrow & \downarrow & \uparrow \\
\omega\text{-perfect } & \text{weakly }\omega\text{-perfect } & \text{weakly }\omega\text{-perfect } & \text{weakly }\omega\text{-perfect } \\
\downarrow & \downarrow & \downarrow & \downarrow \\
\end{array}
\]

In the figure above, the converses are not to be right as demonstrated by the following examples:

**Example 4.5.** Let \(\lambda : (G, \tau) \rightarrow (G, \tau)\), such that \(G = \{u, v, w\}\) and \(\tau = \{\emptyset, G, \{u\}, \{v\}, \{u, v\}\}\) defined by \(\lambda(u) = u, \lambda(v) = v, \lambda(w) = w\). Then \(\lambda\) is super \(\omega\)-perfect mapping but it is not strongly \(\theta\omega\)-perfect mapping.

**Theorem 4.6.** Let \(\lambda : (G, \tau) \rightarrow (H, \sigma)\) be a mapping, such that \(G\) be a regular space. If \(\lambda\) is super \(\omega\)-perfect mapping then it is strongly \(\theta\omega\)-perfect mapping.

**Proof:** Let \(\lambda\) be a super \(\omega\)-perfect mapping. It suffices to demonstrate that \(\lambda\) is strongly \(\theta\omega\)-continuous, let \(g \in G\) and \(T\) be an open set containment \(\lambda (g)\) in \(H\). Because of \(\lambda\) is a super \(\omega\)-continuous, there is a regular open set \(S\) containment \(g\), such that \(\lambda (S) \subseteq T\). Because \(\text{int(cl}(T)) \subseteq \text{cl}(T)\), then \(\lambda (S) \subseteq \text{int(cl}(T)) \subseteq \text{cl}(T)\), then \(\lambda (S) \subseteq \text{cl}(T)\). Also \(G\) is a regular space, there is an open set \(W\) such that \(g \in W \subseteq \text{cl}(W) \subseteq S\), so \(\lambda(\text{cl}(W)) \subseteq T\). Therefore \(\lambda\) is strongly \(\theta\omega\)-continuous. Hence consider that \(\lambda\) is strongly \(\theta\omega\)-perfect mapping.

**Corollary 4.7.** Let \((G, \tau)\) be regular spaces. The mapping \(\lambda : (G, \tau) \rightarrow (H, \sigma)\) is super \(\omega\)-perfect if and only if it is strongly \(\theta\omega\)-perfect.

**Example 4.8.** Let \(\lambda : (G, \tau) \rightarrow (H, \sigma)\) be a mapping, such that \(G = \{u, v, w\}\), \(H = \{a, b\}\), and \(\tau = \{\emptyset, G, \{u\}, \{v\}, \{u, v\}\}, \sigma = \{H, \emptyset, \{a\}\}\) defined by \(\lambda(u) = \lambda(w) = b, \lambda(v) = a\). Then, \(\lambda\) is \(\omega\)-perfect but it is not super \(\omega\)-perfect.
Theorem 4.9. Let $\lambda : (G, \tau) \rightarrow (H, \sigma)$ be a mapping, such that $G$ be a regular space. If $\lambda$ is $\omega$-perfect mapping then it is super $\omega$-perfect mapping.

Proof: Let $\lambda$ be $\omega$-perfect mapping. It suffices to demonstrate that $\lambda$ is super $\omega$-continuous, let $g \in G$ and $T$ be an open set containment $\lambda(g)$ in $H$. Because of $\lambda$ is $\omega$-continuous, there is $S \in oO(G, g)$, such that $\lambda(S) \subseteq T$. Also, $\text{int}(\text{cl}(S)) \subseteq \text{cl}(S)$, then $\lambda(\text{int}(\text{cl}(S))) \subseteq \lambda(\text{cl}(S))$. Also $G$ is a regular space, there is an open set $S_1$ such that $g \in S_1 \subseteq \text{cl}(S_1) \subseteq S$, so $\lambda(\text{int}(\text{cl}(S_1))) \subseteq \lambda(S_1)$ also $\lambda(S) \subseteq T$. So $\lambda(\text{int}(\text{cl}(S))) \subseteq T$, then $\lambda$ is super $\omega$-continuous. Hence consider that $\lambda$ is super $\omega$-perfect mapping.

Corollary 4.10. Let $(G, \tau)$ be regular spaces. The mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is $\omega$-perfect if and only if it is super $\omega$-perfect.

Example 4.11. Let $\lambda : (\mathcal{Y}, \tau) \rightarrow (\mathcal{Y}, \tau)$ be a mapping, such that $\lambda(g) = g$, and let $(\mathcal{Y}, \tau)$ where $\tau$ is the topology with a basis whose members are of the form $(a, b)$ and $(a, b) - N$, such that $N = \{1\in n ; n \in Z^+ \}$. Then $(\mathcal{Y}, \tau)$ is a Hausdorff but not $\omega$-regular. Then $\lambda$ is perfect but it is not strongly perfect mapping.

Theorem 4.12. Let $\lambda : (G, \tau) \rightarrow (H, \sigma)$ be a mapping such that $G$ be an regular space. If $\lambda$ is perfect mapping then it is strongly perfect mapping.

Proof: Let $\lambda$ be perfect mapping. It suffices to demonstrate that $\lambda$ is strongly continuous, let $g \in G$ and $T$ be an open set containment $\lambda(g)$ in $H$. Since $\lambda$ is continuous, there is an open set $S$ containing $g$ in $G$ such that $\lambda(S) \subseteq T$. Since $G$ is regular space, there is an open set $S_1$ in $G$ such that $g \in S_1$ and $\text{cl}(S_1) \subseteq S$, so $\lambda(\text{int}(\text{cl}(S_1))) \subseteq \lambda(S)$. Then $\lambda(\text{cl}(S)) \subseteq T$, therefore $\lambda$ is strongly continuous. So $\lambda$ is strongly perfect mapping.

Corollary 4.13. Let $(G, \tau)$ be regular spaces. The mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is perfect if and only if it is strongly perfect.

Theorem 4.14. Let $\lambda : (G, \tau) \rightarrow (H, \sigma)$ be a mapping such that $H$ be a regular space. If $\lambda$ is weakly perfect mapping then it is perfect mapping.

Proof: Let $\lambda$ be weakly perfect mapping. It suffices to demonstrate that $\lambda$ is continuous, let $g \in G$ and $T$ be an open set containment $\lambda(g)$ in $H$. Since $\lambda$ is weakly continuous, there is an open set $S$ containing $g$ in $G$ such that $\lambda(S) \subseteq T$. Since $G$ is regular space, there is an open set $S_1$ in $G$ such that $g \in S_1$ and $\text{cl}(S_1) \subseteq S$, so $\lambda(\text{int}(\text{cl}(S_1))) \subseteq \lambda(S)$. Then $\lambda(\text{int}(\text{cl}(S))) \subseteq T$, therefore $\lambda$ is continuous. So $\lambda$ is perfect mapping.

Corollary 4.15. Let $(G, \tau)$ be regular spaces. The mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is weakly perfect if and only if it is perfect.

Example 4.16. A mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ such that $G = \{a, b, v, w\}$, $H = \{a, b\}$, $\tau = \{G, \varphi, \{a, u\}, \{v\}, \{u, v\}, \{v, w\}\}$, $\sigma = \{H, \varphi, \{a\}\}$, defined by $\lambda(u) = \lambda(v) = \lambda(w) = b$. The mapping $\lambda$ is almost $\omega$-perfect mapping but it is not super $\omega$-perfect mapping.

Theorem 4.17. Let $\lambda : (G, \tau) \rightarrow (H, \sigma)$ be a mapping, such that $G$ and $H$ are semi-regular spaces. If $\lambda$ is almost $\omega$-perfect mapping then it is super $\omega$-perfect mapping.

Proof: Let $\lambda$ be an almost $\omega$-perfect mapping. It suffices to demonstrate that $\lambda$ is super $\omega$-continuous, let $g \in G$ and let $T$ be an open set containment $\lambda(g)$ in $H$. Because of $\lambda$ is almost $\omega$-continuous, there is an $\omega$-open set $S$ containing $g$, for each regular open set $T$ of $H$ containment $\lambda(g)$ such that $\lambda(S) \subseteq T$. So $\lambda(S) \subseteq \text{int}(T)$. Because the space $G$ is semi-regular space, there is an open set $S_1$ in $G$ such that $g \in S_1$ and $T \subseteq \text{int}(T)$, so $\lambda(S) \subseteq \lambda(\text{int}(T)) \subseteq \lambda(S)$. Also $\lambda(S) \subseteq \text{int}(T)$. Then $\lambda(\text{int}(T)) \subseteq \lambda(S) \subseteq \text{int}(T)$. Also the space $H$ is semi-regular space, there is an open set $T_1$ in $H$ such that $\lambda(g) \in T_1$, and $S \subseteq \text{int}(S)$, so $\lambda(S) \subseteq \lambda(\text{int}(S))$. It follows that $\lambda(\text{int}(S)) \subseteq T$. Then $\lambda$ is super $\omega$-continuous. Hence $\lambda$ is super $\omega$-perfect mapping.

Corollary 4.18. Let $(G, \tau)$ and $(H, \sigma)$ be semi-regular spaces. The mapping $\lambda : (G, \tau) \rightarrow (H, \sigma)$ is almost $\omega$-perfect if and only if it is super $\omega$-perfect mapping.

Example 4.19. A mapping $\lambda : (G, \tau) \rightarrow (G, \tau)$ such that $G = \{u, v, w\}$, $\tau = \{G, \varphi, \{u\}, \{v\}, \{u, v\}\}$, $\lambda(u) = \lambda(v) = u$, and $\lambda(w) = w$, then $\lambda$ is almost weakly perfect mapping but it is not almost strongly perfect mapping.

Theorem 4.20. Let $\lambda : (G, \tau) \rightarrow (H, \sigma)$ be a mapping, such that $G$ is a regular space. If $\lambda$ is almost weakly perfect mapping then it is almost strongly perfect mapping.

Proof: Let $\lambda$ be almost weakly perfect mapping. It suffices to demonstrate that $\lambda$ is almost strongly continuous, let $g \in G$ and let $T$ be an open set containment $\lambda(g)$ in $H$. Because of $\lambda$ is almost weakly
continuous and \( g \in G \) for each open set \( T \) of \( H \) containment \( \lambda(g) \), there is an open set \( S \) containment \( g \), such that \( \lambda(S) \subseteq \text{cl}(T) \). Because the space \( G \) is a regular space, there is an open set \( S_1 \) in \( G \) such that \( g \in S_1 \) also \( \text{cl}(S_1) \subseteq S \), so \( \lambda(\text{cl}(S_1)) \subseteq \lambda(S) \). Also \( \lambda(S) \subseteq \text{cl}(T) \). Then \( \lambda(\text{cl}(S_1)) \subseteq \text{cl}(T) \) and \( \text{int}(\text{cl}(T_1)) \subseteq \text{cl}(T_1) \). Then \( \lambda(\text{cl}(S_1)) \subseteq \text{int}(\text{cl}(T_1)) \). It follows that \( \lambda \) is almost strongly continuous. Hence \( \lambda \) is almost strongly perfect mapping.

**Corollary 4.21.** Let \( (G , \tau) \) and \( (H , \sigma) \) are regular spaces. The mapping \( \lambda : (G , \tau) \rightarrow (H , \sigma) \) is almost weakly perfect if and only if it is almost strongly perfect.

**Theorem 4.22.** Let \( \lambda : (G , \tau) \rightarrow (H , \sigma) \) and \( (H , \sigma) \) be regular spaces, then the following properties are equivalent:

(a) \( \lambda \) is strongly perfect.

(b) \( \lambda \) is perfect.

(c) \( \lambda \) is weakly perfect.

**Theorem 4.23.** Let \( \lambda : (G , \tau) \rightarrow (H , \sigma) \) be a mapping with a regular space and \( \mu : G \rightarrow G \times H \). where the \( \lambda \) defined by \( \mu(g) = (g , \lambda(g)) \) for each \( g \in G \). If \( \lambda : (G , \tau) \rightarrow (H , \sigma) \) is strongly perfect , then \( \mu : G \rightarrow G \times H \) is strongly perfect.

**Proof:** Assume that \( \lambda \) is strongly perfect, let \( g \in G \) and \( W \) be an open set of \( G \times H \) containment \( \mu(g) \).

Yood represents open sets \( S_1 \subseteq G \) and \( T \subseteq H \) such that \( \mu(g) = (g , \lambda(g)) \in S_1 \times T \subseteq W \). Since \( \lambda \) is strongly continuous and \( G \) is a regular space, an open set \( S \) containing \( g \) in \( G \) such that \( \text{cl}(S) \subseteq S_1 \) and \( \lambda(\text{cl}(S)) \subseteq T \). Therefore \( \mu(\text{cl}(S)) \subseteq S_1 \times T \subseteq W \), then \( \mu \) is strongly continuous. So the mapping \( \mu = \text{id}_{\Delta} \lambda : G \rightarrow G \times H \) maps \( G \) homeomorphically onto the graph \( \mu(g) \) which is a closed subset of \( G \times H \). So \( \mu \) is perfect, and because \( G \) is regular, then \( G \times H \) is regular by theorem 4.22. Hence \( \mu : G \rightarrow G \times H \) is strongly perfect.

**Theorem 4.24.** For a mapping \( \lambda : (G , \tau) \rightarrow (H , \sigma) \) and since \( H \) is a regular space, the following properties are equivalent:

(a) \( \lambda \) is almost strongly \( \theta\omega \)-perfect.

(b) \( \lambda \) is \( \omega \)-perfect.

(c) \( \lambda \) is almost \( \omega \)-perfect.

(d) \( \lambda \) is \( \theta\omega \)-perfect.

(e) \( \lambda \) is almost weakly \( \omega \)-perfect.

**References**

1. Formin, S. 1943. "Extension of topological spaces," *Annals of Mathematics*. Second Series, 44: 471-480, 1943.

2. Bourbaki, N. 1966. *General Topology*, Part I, Addison-Wesley, Reding, Mass.

3. Velicko, N.V. 1968. H-closed topological spaces, *Amer. Math. Soc. Transl.,* 78: 103-118. Current address: Selcuk University Faculty of Science and Arts Department of Mathematics.

4. Singal, M.K. and Singal, A.R. 1968. Almost-continuous mappings, *Yokohama Math. J.*, 16: 63-73.

5. Long, P.E. and Herrington, L.L. 1981. Strongly \( \theta \)-continuous functions, *J.of the Korean Math. Soc.*, 18(1): 21-28.

6. Hdeib, H.Z. 1989. "\( \omega \)-continuous functions" *Dirasat*, 16(2): 136-142.

7. Chew, J. and Tong, J. 1991. Some Remarks on Weak continuity, *American Mathematical Monthly*, 98: 931-934.

8. Noiri, T., Al-Omari, A. and Noorani, M.S.M. 2009. "Weak forms of \( \omega \)-open sets and decomposition of continuity ", *E.J.P.A.M.* 2(1): 73-84.

9. Noiri, T. 1980. On \( \delta \)-continuous functions. *J. Korean Math. Soc.*, 16: 161-166.

10. Noiri, T. 1989. " On almost continuous function," *Indian Journal of pure and Applied Mathematics*, 20(6): 571-576.

11. Al-Omari, A. and Noorani, M.S.M. 2007. " Contra-\( \omega \)-continuous and almost contra-\( \omega \)-continuous," *International Journal of Mathematics and Mathematical Sciences*, Vol. 2007, Article ID 40469 , 13 pages.
12. Al-Omari, A. and Noorani, M.S.M. 2007. "Regular generalized $\omega$-closed sets," *International Journal of Mathematics and Mathematical Sciences*, Vol. Article ID 16292, 2007, 11 pages.

13. Al-Omari, A. and Noorani, M.S.M. 2009. "Weak and Strong form of $\omega$-continuous" *International Journal of Mathematics and Mathematical Sciences*, Vol. Article ID 174042, 12 pages.

14. Bourbaki, N. 1989. "Regular Space. " in *Elements of Mathematics: General Topology*. Berlin: Springer-Verlag, pp. 80-81.

15. Stone, M.H. 1937. Applications of the theory boolean rings to General Topology. *Trans. Am. Math. Soc.*, 41: 375-481.

16. Devi, R., Balachan dran, K. and Maki, H. 1995. on Generalized $\alpha$-continuous maps, *Far.East J. Math.*, 16: 35-48.