Study of Doubly Heavy Baryon Spectrum via QCD Sum Rules

Liang Tang\textsuperscript{a}, Xu-Hao Yuan\textsuperscript{a}, Cong-Feng Qiao\textsuperscript{a,b} and Xue-Qian Li\textsuperscript{a}

\textsuperscript{a} School of Physics, Nankai University, 300071, Tianjin, China
\textsuperscript{b} Department of Physics, Graduate University, the Chinese Academy of Sciences
YuQuan Road 19A, 100049, Beijing, China

In this work, we calculate the mass spectrum of doubly heavy baryons with the diquark model in terms of the QCD sum rules. The interpolating currents are composed of a heavy diquark field and a light quark field. Contributions of the operators up to dimension six are taken into account in the operator product expansion. Within a reasonable error tolerance, our numerical results are compatible with other theoretical predictions. This indicates that the diquark picture reflects the reality and is applicable to the study of doubly heavy baryons.

PACS numbers: 14.20-c, 11.55.Hx, 12.38.Lg
Key words: baryons, QCD sum rules, other nonperturbative calculations

1 Introduction

The considerable success of quark model in interpreting a large amount of hadronic observations has convinced people its undoubted validity for many years. In the quark model, hadrons are constructed according to two configurational schemes: mesons, consisting of a quark and an antiquark ($q\bar{q}$); and baryons, consisting of three quarks ($qqq$). Right after the birth of the quark model, the diquark model was proposed where two quarks constitute a color-anti-triplet which behaves as an independent object in the baryon. In Gell-Mann’s
original paper on the quark model, he discussed the possibility of the existence of free diquarks. The concept of diquarks, has been established in the fundamental theory, and has been invoked to help illuminating a number of phenomena observed in experiments. The systems composed of three quarks should be described by the Faddev equations, but since there are three coupled differential equations, solving them is extremely difficult. As a matter of fact, the three-body problem is still an unsolved subject even in classical physics. It is tempted to consider the diquark-quark structure which turns the three-body system into a two-body one, and the three Faddev equations then reduce to single equation (no matter relativistic or non-relativistic). Thus the problem is greatly simplified and solution concerning baryon physics is obtained. However, for the baryons which are composed of three light quarks, the three Faddev equations have the same weight, so a problem emerges right away, namely which two quarks are combined to compose a diquark while the rest one moves independently. It seems to be an unbeatable difficulty. However, recently the topic on diquarks revives, for it may bring up some direct phenomenological consequences. Especially, when there are two heavy quarks in a baryon, they may constitute a relatively tight structure, a diquark. A diquark has the quantum numbers of a two-quark system. For the ground state, a diquark has positive parity and may be an axial-vector ($S = 1$) or a scalar ($S = 0$). According to the basic principle of QCD, for the two quarks residing in a color anti-triplet, the interaction between them is attractive.

Baryons containing two heavy quarks are important and intriguing systems to study the quark-diquark structure of baryons. The two heavy quarks (b and c) can constitute a stable bound state of $\bar{3}$, namely, a diquark which serves as a source of static color field for light quarks. The SELEX Collaboration reported the first observation of a doubly charmed baryon via the decay process $\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^- \pi^+$, by which its mass of $3519 \pm 1\text{MeV}/c^2$ was determined. Later, this baryon was confirmed by the SELEX Collaboration through the measurement of a different decay mode $\Xi_{cc}^+ \rightarrow pD^+K^-$, whose final state involves a charmed meson. However, both the BABAR and Belle Collaborations did not observe this state in $e^+e^-$ annihilation experiments. This may be due to the distinct beam structures of the two types of experiments, and the reason is worthy of
further and careful studies.

In the theoretical aspect, there have been numerous works in studying the doubly heavy baryons [13, 14, 15, 16, 17]. All these works concern the dynamics which results in the substantial diquark structure.

Although the QCD is proven to be an undisputably valid theory about strong interaction, the non-perturbative QCD which dominates the low energy physics phenomena has not been fully understood yet. Among the the theoretical methods in dealing with the non-perturbative effects, the framework of the QCD Sum Rules which is indeed a bridge between the short-distance and long-distance QCD as initiated by Shifman et al. [18], turns out to be a remarkably successful and powerful technique for computing the hadronic properties. Recently, a number of works have been worked out to interpret the newly observed mesonic resonances within the framework of the QCD sum rules [19, 20, 21]. Meanwhile with the QCD sum rules, a few works were performed in studying the mass spectrum of doubly heavy baryons [22, 23, 24, 25, 26]. In those studies, the authors calculated the correlation function of baryonic currents composed of quark fields by virtue of the operator product expansion (OPE).

Since the correlation of the two heavy quarks is strong, they are tempted to be bound into a diquark which can be regarded to manifest independent degrees of freedom in the baryon. In this work, we no longer treat the two heavy quarks as independent constituents, but a combined sub-system—diquark which behaves as a component of doubly heavy baryons, and the corresponding field is denoted by a new bosonic symbol $\Phi$ with a mass $m_D$. In fact, this picture was recently proposed in Ref.[27]. Then, in this tentative model for calculating the mass spectrum of doubly heavy baryon systems, the light quark $q$ (q=u,d,s) orbits the heavy diquark which is a tightly bound QQ’ (Q=c,b) pair. The application of the diquark can simplify the interpolating currents which are important for obtaining the baryon spectrum in the QCD sum rules. The spin-parity quantum number of a ground-state diquark is either $0^+$ or $1^+$. The former, along with a light $q$, can form the state with $J^P = \frac{1}{2}^+$; the latter can form not only the state with $J^P = \frac{1}{2}^+$, but also $J^P = \frac{3}{2}^+$. That is to say, using the model of the diquark and the QCD sum rules, we can study the doubly heavy baryons with spin-parity $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$. 
The content of the paper is arranged as follows. In Sec. II we derive the formulas of the correlation function of the interpolating currents with proper quantum numbers in terms of the QCD sum rules. In Sec. III, our numerical results and relevant figures are presented. Section IV is devoted to a summary and concluding remarks.

2 Formalism

The method of the QCD Sum Rules is starting with choosing proper correlation function (or Green’s function) both at the quark-gluon level and the hadron level. The correlation function for the doubly heavy baryons reads

\[ \Pi(q^2) = i \int d^4xe^{iq \cdot x}\langle 0|T\{J(x)\bar{J}(0)\}|0\rangle. \] (1)

Considering the spinor structures of baryons, the correlation function has the Lorentz covariant expression as follows \[23,24\]:

\[ \Pi(q^2) = \phi_1(q^2) + \phi_2(q^2). \] (2)

For each invariant function of \( \phi_1(q^2) \) and \( \phi_2(q^2) \) in the doubly heavy baryons one can obtain a sum rule.

Following Refs.\[23,24\] and based on the diquark model, the interpolating currents, which play a crucial role in our analysis, are chosen to be

\[ J(x) = \Phi(x) \Gamma_k q^a(x), \quad \text{for spin } 1/2 \text{ baryon}; \] (3)

\[ J^*(x) = \Phi(x) \Gamma_k q^a(x), \quad \text{for spin } 3/2 \text{ baryon}; \] (4)

\[ J'(x) = \Phi(x) \Gamma_k q^a(x), \quad \text{for spin } 1/2 \text{ baryon}, \] (5)

where \( \Phi(x) \) and \( \Phi^a(x) \) are axial vector and scalar diquarks, respectively. The interpolating current \( J \) corresponds to \( \Xi_{QQ'} \) and \( \Omega_{QQ'} \), \( J^* \) corresponds to \( \Xi_{QQ'}^* \) and \( \Omega_{QQ'}^* \), and \( J' \) corresponds to \( \Xi'_{QQ'} \) and \( \Omega'_{QQ'} \) respectively with \( Q, Q' = c, b \). The concrete definition of \( \Gamma_k \) and \( \Gamma_k \) are presented in Table \[1\]

On the phenomenological side, the correlation function is expressed as a dispersion
Table 1: The choice of $\Gamma_\mu^D$ and $\Gamma_k$. The index D in $J^{P_D}_D$ means the diquark. $\Phi_{QQ'}$ denotes the axial vector diquark and $\Phi_{[QQ]}$ denotes the scalar diquark respectively.

| Baryon | Constituent | $J^{P_D}$ | $J^{P_D}_D$ | $\Gamma_\mu^D$ | $\Gamma_k$ |
|--------|-------------|-----------|-------------|----------------|------------|
| $\Xi_{QQ'}$ | $\Phi_{(QQ')}$ $q$ | $\frac{1}{3}^+$ | $1^+$ | $\gamma^\mu\gamma_5$ | - |
| $\Xi^*_{QQ'}$ | $\Phi_{(QQ')}$ $q$ | $\frac{3}{2}^+$ | $1^+$ | - | 1 |
| $\Omega_{QQ'}$ | $\Phi_{(QQ')}$ $s$ | $\frac{1}{2}^+$ | $1^+$ | $\gamma^\mu\gamma_5$ | - |
| $\Omega^*_{QQ'}$ | $\Phi_{(QQ')}$ $s$ | $\frac{3}{2}^+$ | $1^+$ | - | 1 |
| $\Xi'_{QQ'}$ | $\Phi_{[QQ]}$ $q$ | $\frac{1}{2}^+$ | $0^+$ | - | 1 |
| $\Omega'_{QQ'}$ | $\Phi_{[QQ]}$ $s$ | $\frac{1}{2}^+$ | $0^+$ | - | 1 |

Integral over a physical regime,

$$\Pi(q^2) = \lambda_H^2 \frac{q + M_H}{M_H^2 - q^2} + \int_{s_0}^{\infty} ds \frac{\rho^H(s)}{s - q^2} + \cdots,$$  

where $M_H$ is the mass of the doubly heavy baryon, $\lambda_H$ is the baryon coupling constant and $\rho^H(s)$ is the physical spectral function of the continuum states. When we attain the above expression, the summing relations for the Dirac and Rarita-Schwinger spinors have been used, namely, for spin-3/2 baryons the numerator of the first term in Eq.(6) should be replaced by the proper Lorentz structure [24]

$$\left(q + M_H\right)\left(g_{\mu\nu} - \frac{1}{3}\gamma_{\mu}\gamma_{\nu} + \frac{\eta_{\mu\nu} - q_{\mu}q_{\nu}}{3M_H} - \frac{2q_{\mu}q_{\nu}}{3M_H^2}\right).$$

With the operator product expansion (OPE), the correlation function $\Pi_i(q^2)$ (i=1 or 2) can be written as:

$$\Pi_i(q^2) = \Pi_{i,\text{pert}}(q^2) + \sum_{\text{dim}=3}^6 \Pi_{i,\text{cond, dim}}(q^2).$$  

Here, “pert”, “cond” and “dim” refer to perturbative QCD calculation, the quark or gluon condensates, and the relevant condensate dimensions, respectively. $\Pi_{i,\text{pert}}(q^2)$ is obtained by taking the absorptive part of the Feynman diagram $A$, and $\Pi_{i,\text{cond, dim}}(q^2)$ represents the contributions from various condensates. In this work, we consider the condensates up to dimension six, as people usually do in the literature.
The Feynman diagrams contributing to the correlation function of the doubly heavy baryons are displayed in Fig. 1, and the gluon-diquark vertices are shown in Fig. 2. In Ref. [28], the effective gluon-diquark vertices were given, which will also be used in our later calculations, so we just copy them below:

\[
S_{gS} = i g_s t^a (p_1 + p_2)_\mu F_S(Q^2),
\]

\[
V_{gV} = -g_s t^a \{ g_{\alpha\beta}(p_1 + p_2)_\mu - g_{\mu\alpha}[(1 + \kappa_V)p_1 - \kappa_V p_2]_\beta 
- g_{\mu\beta}[(1 + \kappa_V)p_2 - \kappa_V p_1]_\alpha \} F_V(Q^2).
\]

Here, \(Q^2 = -(p_1 - p_2)^2\), \(g_s = \sqrt{4\pi\alpha_s}\) denotes the QCD coupling constant, \(\kappa_v\) is the anomalous (chromo) magnetic moment of the vector diquark and \(t^a = \lambda^a/2\) is the Gell-Mann color matrix. Furthermore, \(F_S(Q^2)\) and \(F_V(Q^2)\) are the diquark form factors.

The scalar diquark’s propagator is \(\frac{i}{p^2 - m_d^2 + i\epsilon}\), and the axial-vector diquark’s propagator is \(-i\left(\frac{g_{\mu\nu} - \frac{p_\mu p_\nu}{m_d^2}}{p^2 - m_d^2 + i\epsilon}\right)\). Since the diquarks are made up of two heavy quarks, according to the general rule, their condensates are negligible [29].

The Feynman diagrams are computed not only with the regular QCD Feynman rules, but also with the effective vertices displayed above for point-like diquarks. For taking
into account the composite nature of diquarks, phenomenological form factors \( (F_S(Q^2), F_V(Q^2)) \) are introduced. The authors of Ref. [28] suggested the vertex functions as following:

\[
F_S(Q^2) = \delta_S \frac{Q_S^2}{Q_S^2 + Q^2}, \tag{10}
\]

\[
\delta_S = \frac{\alpha_s(Q^2)}{\alpha_s(Q_S^2)} \quad \text{if} \quad (Q^2 \geq Q_S^2),
\]

\[
\delta_S = 1 \quad \text{if} \quad (Q^2 < Q_S^2);
\]

\[
F_V(Q^2) = \delta_V \frac{Q_V^2}{Q_V^2 + Q^2}^2, \tag{11}
\]

\[
\delta_V = \frac{\alpha_s(Q^2)}{\alpha_s(Q_V^2)} \quad \text{if} \quad (Q^2 \geq Q_V^2),
\]

\[
\delta_V = 1 \quad \text{if} \quad (Q^2 < Q_V^2),
\]

where \( Q_S^2 = 3.22\text{GeV}^2 \), \( Q_V^2 = 1.50\text{GeV}^2 \), and the values of these special characteristic quantities are fixed by fitting data.

Supposing the quark-hadron duality, the resultant sum rule for the mass of the doubly heavy baryon reads

\[
m_H = \sqrt{-\frac{R_1}{R_0}}, \tag{12}
\]

with

\[
R_0 = \frac{1}{\pi} \int_{(m_d + m_q)^2}^{m_0} ds \rho_{i}^{\text{pert}}(s)e^{-s/M_B^2} + \tilde{B}[\Pi_i^{\text{cond},3}(q^2)] + \tilde{B}[\Pi_i^{\text{cond},4}(q^2)] + \tilde{B}[\Pi_i^{\text{cond},5}(q^2)] + \tilde{B}[\Pi_i^{\text{cond},6}(q^2)], \tag{13}
\]

\[
R_1 = \frac{\partial}{\partial M_B^2} R_0. \tag{14}
\]

Here, \( m_q(q = u, d, \text{ or } s) \) denotes the masses of the light quarks, \( m_d \) is the mass of the
diquark, $M_B$ is the Borel parameter and $s_0$ is the threshold cutoff introduced to remove the contribution of the higher excited and continuum states [30].

The perturbative contribution $\rho_1(s)$ and non-perturbative contributions

\[ B[\Pi^{\text{cond}, \text{dim}}_3(q^2)] \text{ for } \Xi_{QQ'} \text{ and } \Omega_{QQ'} \text{ in Eq. (13) are shown as follows:} \]

\[ \rho_1(s) = \frac{-3 \left( m_d^2 - m_q^2 - s \right) \sqrt{(m_d^2 - m_q^2 + s)^2 - 4sm_d^2}}{16\pi s^2}, \quad (15) \]

\[ B[\Pi^{\text{cond}, 3}(q^2)] = \langle \Psi \Psi \rangle \frac{m_q(3M_B^2 + m_d^2m_q^2)}{12M_B^6} e^{-\frac{m_q^2}{M_B^2}}, \quad (16) \]

\[ B[\Pi^{\text{cond}, 4, \text{C}}(q^2)] = \langle \alpha_s G^2 \rangle \int_0^1 dx e^{-\frac{m_q^2 + m_d^2}{M_B^2}} \left\{ \frac{3x^2 - 10x + 10}{192\pi M_B^2} + \frac{1}{192\pi (x-1)xM_B^2} \right. \]

\[ \times \left[ x^2 \left( -3x^2 + 10x - 10 \right) + (x - 1)^2 \left( 3x^2 - 7x + 3 \right) m_q^2 \right] \]

\[ + \frac{1}{384\pi (x-1)x^3M_B^6} \left[ 2x^2 \left( -3x^3 + 13x^2 - 21x + 11 \right) m_d^2m_q^2 \right. \]

\[ + x^4 \left( 3x - 10 \right) m_d^4 + (x - 1)^3 \left( 3x^2 - 7x + 8m_q^2 \right) + \frac{1}{384\pi (x-1)^3x^4M_B^8} \]

\[ \left[ (x^2 - 4x + 3) m_q^2 - x^2m_d^2 \right] \left( (x - 1)^2 m_q^2 - x^2m_d^2 \right)^2 \right\}, \quad (17a) \]

\[ B[\Pi^{\text{cond}, 4, \text{D}}(q^2)] = \langle \alpha_s G^2 \rangle \int_0^1 dx e^{-\frac{m_q^2 + m_d^2}{M_B^2}} \left\{ - \frac{1}{512\pi (x-1)^2M_B^6} \right. \]

\[ \left[ 2x^2\kappa_v - x \left( \kappa_v - 2 \right) - 9\kappa_v - 6 \right] - \frac{1}{512\pi (x-1)^2xM_B^6} \left[ x^2m_d^2 \left( -2x\kappa_v + \kappa_v - 2 \right) + (x - 1)^2 m_q^2 \right. \]

\[ \left. + \kappa_v + 2 \right] - \frac{\kappa_v \left( (x - 1)^2 m_q^2 - x^2m_d^2 \right)^2}{512\pi (x-1)^3x^2M_B^8} \right\}, \quad (17b) \]

\[ B[\Pi^{\text{cond}, 4, \text{E}}(q^2)] = \langle \alpha_s G^2 \rangle \int_0^1 dx e^{-\frac{m_q^2 + m_d^2}{M_B^2}} \left\{ \frac{1}{384\pi (x-1)^2M_B^6} \right. \]

\[ \left[ x(x^2(6\kappa_v + 3) + x(4\kappa_v^2 - 4\kappa_v - 2) + 4\kappa_v^2 + 6\kappa_v + 4) \right] + \frac{1}{384\pi (x-1)^3M_B^8} \left[ (x - 1)^2 m_q^2 \left( x(6\kappa_v + 3) + 4\kappa_v^2 + 2\kappa_v + 1 \right) - xm_d^2 \right. \]

\[ \left. \left( x^2(6\kappa_v + 3) + x(4\kappa_v^2 - 4\kappa_v - 2) + 2\kappa_v \right) \right] \]

\[ + \frac{1}{256\pi (x-1)^3xM_B^6} \left[ (2\kappa_v + 1) \left( -x \left( 2x^2 - 3x + 1 \right) m_d^2m_q^2 + x^3m_d^4 \right) \right. \]

\[ \left. + (x - 1)^3 m_q^4 \right] \right\}, \quad (17c) \]
\dot{B}[\Pi^\text{cond.}_1 \{q^2\}] = g_s \langle \bar{\Psi} \sigma \cdot G \Psi \rangle m_q e \frac{m^2_s}{M_B^6} (3M_B^2 (2\kappa_v + 1) - 8m_d^2), \quad (18)

\dot{B}[\Pi^\text{cond.}_1 \{q^2\}] = \frac{g_s^2 \langle \bar{\Psi} \Psi \rangle^2 e \frac{m^2_s}{M_B^6}}{1296M_B^6} (9M_B^2 (2\kappa_v + 1) + 17m_d^2), \quad (19)

where the superscripts (C, D and E) on the left hand of Eq.(17) correspond to the labels in Fig.1.

For concision of the text, the detailed expressions of $R_0$ are given in the Appendix.

3 Numerical Analysis

The numerical parameters used in this work are taken as [31, 32]

\[ \langle \bar{\Psi} \Psi \rangle = -(0.254 \pm 0.015\text{GeV})^3, \quad \alpha_s \langle G^2 \rangle = 0.07 \pm 0.02\text{GeV}^4, \]
\[ g_s \langle \bar{\Psi} \sigma \cdot G \Psi \rangle = m^2_0 \langle \bar{\Psi} \Psi \rangle, \quad \alpha_s \langle \bar{\Psi} \Psi \rangle^2 = (2.1 \pm 0.3) \times 10^{-4}\text{GeV}^6, \]
\[ m_0^2 = 0.8 \pm 0.2\text{GeV}^2, \quad m_s = 0.14\text{GeV}, \]
\[ m_u \simeq m_d = 0.005\text{GeV}, \quad m_c = 1.27\text{GeV}, \]
\[ m_b = 4.19\text{GeV}, \quad m_{\eta_c} = 2.980\text{GeV}, \]
\[ m_{J/\psi} = 3.097\text{GeV}, \quad m_\Upsilon = 9.460\text{GeV}. \] (20)

In our numerical analysis, we find that the effect of the two-gluon condensate is tiny, i.e., if we shift the contributions of two-gluon condensate a little, the final mass of the corresponding baryon hardly changes. We choose it as in Ref[28]: $\kappa_v = 1.39$.

In practice, large uncertainty remains in the evaluation of baryon spectrum due to the input constituent quark masses. The mass of the baryon can be decomposed as $m_D + m_q + \Delta E$, where $m_D$, $m_q$ and $\Delta E$ are the masses of the heavy diquark, light-quark and the binding energy respectively. The binding energy is calculable within certain theoretical framework, whereas the quark, including the diquark, masses are usually not definite input parameters. We have to choose a reasonable strategy to determine the diquark masses, which influence more on the heavy baryon spectrum than light quark masses. It goes as follows.
There are few data on the doubly heavy baryons available, thus we cannot directly extract all necessary information from experimental data so far. Fortunately, the mass of \( \Xi_{cc} \) has been measured. Because of a simple symmetry argument, we know that the \( cc \) must constitute a spin-1 state, thus the \( cc \) diquark in \( \Xi_{cc} \) is an axial vector state. With the measured \( \Xi_{cc} \) mass as input, we determine \( m_{\{cc\}} = 2.77 \) GeV. Then we still need to fix the masses of \( bc \) and \( bb \) diquarks. Generally the effective potential between the two heavy quarks \( Q, Q' \) includes the Coulomb and linear confinement pieces. From the textbooks of the quantum mechanics, we know the solutions of the Schrödinger equations with the Coulomb potential proportional to \( \alpha_s r \) (the solution is the Lauerre polynomial) or the linear potential proportional to \( \kappa r \) (the solution is the Airy function). The binding energy in the case where only the Coulomb piece exists is proportional to the reduced mass \( m \) of the \( QQ' \) system, instead, for the case where only the linear potential exists, the binding energy is proportional to \( (\kappa^2 m)^{1/3} \) if \( \kappa \) is independent of \( m \). Thus for the Schrödinger equation whose potential includes both the Coulomb and linear confinement pieces, the contributions from the two pieces compete and the dependence of the binding energy on the reduced mass is uncertain. Then we would like to invoke the data.

As discussed above, \( m_{\{cc\}} \) and \( \Delta E_{\{cc\}} \) can be directly extracted from the data, then we will use those values for the \( cc \) diquark and some proposed rules to fix \( m_{\{bc\}}, \Delta E_{\{bc\}}, m_{\{bb\}} \) and \( \Delta E_{\{bb\}} \). Now let us make a plausible comparison of the quantities about diquarks with the corresponding mesons of the same flavor and spin structure. We can have \( \Delta E_{\{bc\}} \) and \( \Delta E_{\{bb\}} \) from the relations: \( m_{\{bc\}} = m_b + m_c + \Delta E_{\{bc\}} \) and \( m_{\{bb\}} = 2m_b + \Delta E_{\{bb\}} \). Thus \( \Delta E_{\{cc\}} \) and \( \Delta E_{\{bb\}} \) are obtained as \( M_{J/\psi} - 2m_c \) and \( M_{\Upsilon(1S)} - 2m_b \).

The diquarks \( cc \) or \( bb \) are color-anti-triplet axial vector states, instead the meson \( J/\psi \) or \( \Upsilon(1S) \) are color-singlet vector states of \( c\bar{c} \) and \( b\bar{b} \). The effective potentials between \( QQ \) and \( Q\bar{Q} \) only differ by a color factor, so that we may have

\[
\Delta E_{\{cc\}} : \Delta E_{\{cc\}} = \Delta E_{\{bc\}} : \Delta E_{\{bc\}} = \Delta E_{\{bb\}} : \Delta E_{\{bb\}},
\]

where the dependence of the binding energies on color and constituent masses may cancel. Then we obtain the binding energy for the axial vector \( bb \). Since so far there are no data on \( B_c^\ast \) available yet, we cannot determine \( \Delta E_{\{bc\}} \) in the above scheme, but need to invoke
another way. Since $bc$ diquark is composed of $c$ and $b$ quarks, it is natural to think that an interpolation between $cc$ and $bb$ diquarks would be a good approximation for the $bc$ diquark, thus we write

$$\Delta E_{\{bc\}} = \frac{1}{2}[\Delta E_{\{bb\}} + \Delta E_{\{cc\}}].$$

Unlike the $bb$ and $cc$ diquarks, $bc$ diquark can be either an axial vector or a scalar. Now let us determine the mass of the scalar $bc$ diquark. The mass difference between spin-1 and spin-0 two-quark systems is due to the spin-spin interactions. Such interaction is proportional to $1/(m_Qm_{Q'})$. Since there lack enough data for $b\bar{c}$ mesons, let us first start with the charmmonia which are well measured and then generalize to the $b\bar{c}$ mesons.

The difference of the binding energies of $J/\psi$ and $\eta_c$ is due to the spin-spin interaction between $c$ and $\bar{c}$, and besides a color factor related to the SU(3) Casimir factor which is $\frac{4}{3}$ for a color singlet and $\frac{2}{3}$ for a color-anti-triplet, the case for the $cc$ diquark is the same. Thus we may write

$$\Delta E_{\{cc\}} - \Delta E_{\{cc\}} = (M_{J/\psi} - M_{\eta_c}).$$

Then using

$$\Delta E_{\{cc\}} : \Delta E_{\{cc\}} = \Delta E_{\{bc\}} : \Delta E_{\{bc\}},$$

we fix $\Delta E_{\{bc\}}$.

With above analysis, the diquark masses which will be adopted in the following numerical computations are displayed as following:

$$m_{\{cc\}}^{fit} = 2.77\text{GeV}, \; m_{\{cb\}} = 5.73\text{GeV}, \; m_{\{cb\}} = 5.80\text{GeV}, \; m_{\{bb\}} = 8.83\text{GeV}. \quad (21)$$

For choosing the proper threshold $s_0$ and the Borel parameter $M_B^2$, there are two criteria. First, the perturbative contribution should be larger than the contributions from all kinds of condensates, and another is that the pole contribution should be larger than the continuum contribution\cite{18, 29, 33}. On the other hand, the dependence of the evaluated masses of the doubly heavy baryons is rather unsensitive to variations of the Borel parameter in the Borel windows. For each baryon fortunately we can find an optimal Borel window where the two aforementioned criteria are satisfied and the
Table 2: The mass spectra of doubly heavy baryons. The “pole” stands for the contribution from the pole term to the spectral density. The “cond” stands for the contribution from the condensate terms in the operator product expansion, where the threshold parameter \( s_0 \) takes its central value. \( \Delta m \) is the energy gap between masses of other species of baryons and \( \Xi_{QQ'} \) with the same diquark flavors.

| Baryon | quark | \( J^P \left( J^{P_D}_D \right) \) | Results(GeV) | \( \Delta m \) | \( M_B^2 \) (GeV\(^2\)) | pole | cond |
|--------|-------|-----------------|-------------|----------|-----------------|------|------|
| \( \Xi_{cc} \) | \( \{cc\}q \) | \( \frac{1}{2}^+ \) (1+) | 3.519\(^{fit}\) | 0 | 2.0-3.8 | (53-84)\% | (3-18)\% |
| \( \Omega_{cc} \) | \( \{cc\}s \) | \( \frac{1}{2}^+ \) (1+) | 3.63\(^{+0.06}_{-0.03}\) + \( \delta_{cc} \) | 0.11 | 2.2-4.0 | (58-83)\% | (5-28)\% |
| \( \Xi_{cc}^* \) | \( \{cc\}q \) | \( \frac{3}{2}^+ \) (1+) | 3.62\(^{+0.08}_{-0.09}\) + \( \delta_{cc} \) | 0.10 | 2.4-5.4 | (51-88)\% | (2-5)\% |
| \( \Omega_{cc}^* \) | \( \{cc\}s \) | \( \frac{3}{2}^+ \) (1+) | 3.71\(^{+0.07}_{-0.05}\) + \( \delta_{cc} \) | 0.19 | 3.0-5.5 | (55-82)\% | (3-8)\% |
| \( \Xi_{cb}^* \) | \( \{cb\}q \) | \( \frac{1}{2}^+ \) (0+) | 6.61\(^{+0.08}_{-0.10}\) + \( \delta_{cb}^1 \) | 0 | 5.0-8.0 | (54-79)\% | (2-6)\% |
| \( \Omega_{cb}^* \) | \( \{cb\}s \) | \( \frac{1}{2}^+ \) (0+) | 6.69 \pm 0.06 + \( \delta_{cb}^1 \) | 0.08 | 5.0-8.0 | (58-81)\% | (3-11)\% |
| \( \Xi_{cb} \) | \( \{cb\}q \) | \( \frac{1}{2}^+ \) (1+) | 6.65\(^{+0.07}_{-0.08}\) + \( \delta_{cb}^2 \) | 0 | 3.5-7.0 | (58-90)\% | (3-21)\% |
| \( \Omega_{cb} \) | \( \{cb\}s \) | \( \frac{1}{2}^+ \) (1+) | 6.75\(^{+0.05}_{-0.03}\) + \( \delta_{cb}^2 \) | 0.10 | 4.0-8.0 | (55-87)\% | (4-25)\% |
| \( \Xi_{cb}^* \) | \( \{cb\}q \) | \( \frac{3}{2}^+ \) (1+) | 6.69 \pm 0.08 + \( \delta_{cb}^2 \) | 0.04 | 5.2-9.0 | (50-79)\% | (1-2)\% |
| \( \Omega_{cb}^* \) | \( \{cb\}s \) | \( \frac{3}{2}^+ \) (1+) | 6.77\(^{+0.06}_{-0.04}\) + \( \delta_{cb}^2 \) | 0.12 | 6.0-9.0 | (54-75)\% | (2-4)\% |
| \( \Xi_{bb} \) | \( \{bb\}q \) | \( \frac{1}{2}^+ \) (1+) | 9.80 \pm 0.07 + \( \delta_{bb} \) | 0 | 8.5-11.0 | (63-77)\% | (2-4)\% |
| \( \Omega_{bb} \) | \( \{bb\}s \) | \( \frac{1}{2}^+ \) (1+) | 9.89\(^{+0.04}_{-0.03}\) + \( \delta_{bb} \) | 0.09 | 9.5-12.0 | (73-84)\% | (2-4)\% |
| \( \Xi_{bb}^* \) | \( \{bb\}q \) | \( \frac{3}{2}^+ \) (1+) | 9.84 \pm 0.07 + \( \delta_{bb} \) | 0.04 | 9.5-11.0 | (68-76)\% | 1\% |
| \( \Omega_{bb}^* \) | \( \{bb\}s \) | \( \frac{3}{2}^+ \) (1+) | 9.93\(^{+0.05}_{-0.04}\) + \( \delta_{bb} \) | 0.13 | 10.5-12.0 | (67-74)\% | 2\% |

Results are almost independent of the Borel parameter after all. By the windows we obtain the masses of doubly heavy baryons. The dependence are shown in Figs.(3-9), respectively. The numerical results are collected in Table 2 for various quantum numbers. For a comparison with other theoretical estimates on the baryon masses given in the literature, we also show those results in Table 3. The error bars are estimated by varying the Borel parameters, \( s^0 \) and the uncertainties in the condensates as well. It is noted that the uncertainty caused by introducing the diquark configuration is included in the diquark form factors (Eqs(10) and (11)).

Note that inside the Tables, the mass of the baryon \( \Xi_{cc} \) with superscript “fit” is
Figure 3: Dependence of $\Xi_{cc}$ and $\Omega_{cc}$ masses on the Borel parameter $M_B^2$. The continuum thresholds $s_0$ are taken as $3.9^2, 4.0^2, 4.1^2\text{GeV}^2$ for $\Xi_{cc}$, and $4.0^2, 4.1^2, 4.2^2\text{GeV}^2$ for $\Omega_{cc}$, from down to up, respectively. We deliberately put two vertical lines denoting the chosen Borel window.

Figure 4: Dependence of $\Xi_{cc}^*$ and $\Omega_{cc}^*$ masses on Borel parameter $M_B^2$. The continuum thresholds $s_0$ are taken as $4.0^2, 4.1^2, 4.2^2\text{GeV}^2$ and $4.1^2, 4.2^2, 4.3^2\text{GeV}^2$, from down to up, respectively. We deliberately put two vertical lines denoting the chosen Borel window.

Figure 5: Dependence of $\Xi_{cb}^*$ and $\Omega_{cb}^*$ masses on Borel parameter $M_B^2$. The continuum thresholds $s_0$ are taken as $7.0^2, 7.1^2, 7.2^2\text{GeV}^2$ and $7.1^2, 7.2^2, 7.3^2\text{GeV}^2$, from down to up, respectively. We deliberately put two vertical lines denoting the chosen Borel window.
Figure 6: Dependence of $\Xi_{cb}$ and $\Omega_{cb}$ masses on Borel parameter $M_B^2$. The continuum thresholds $s_0$ are taken as $7.0^2, 7.1^2, 7.2^2\text{GeV}^2$ and $7.1^2, 7.2^2, 7.3^2\text{GeV}^2$, from down to up, respectively. We deliberately put two vertical lines denoting the chosen Borel window.

Figure 7: Dependence of $\Xi_{cb}^\ast$ and $\Omega_{cb}^\ast$ masses on Borel parameter $M_B^2$. The continuum thresholds $s_0$ are taken as $7.1^2, 7.2^2, 7.3^2\text{GeV}^2$ and $7.2^2, 7.3^2, 7.4^2\text{GeV}^2$, from down to up, respectively. We deliberately put two vertical lines denoting the chosen Borel window.

Figure 8: Dependence of $\Xi_{bb}$ and $\Omega_{bb}$ masses on Borel parameter $M_B^2$. The continuum thresholds $s_0$ are taken as $10.7^2, 10.8^2, 10.9^2\text{GeV}^2$ and $10.9^2, 11.0^2, 11.1^2\text{GeV}^2$, from down to up, respectively. We deliberately put two vertical lines denoting the chosen Borel window.
Figure 9: Dependence of $\Xi_{bb}$ and $\Omega_{bb}$ masses on Borel parameter $M_B^2$. The continuum thresholds $s_0$ are taken as 10.4$^2$, 10.5$^2$, 10.6$^2$GeV$^2$ and 10.5$^2$, 10.6$^2$, 10.7$^2$GeV$^2$, from down to up, respectively. We deliberately put two vertical lines denoting the chosen Borel window.

Table 3: Comparison with other theoretical results and the experimental data (if available). The quantities are in GeV.

| Baryon | quark | $J^P(J^{P_D})$ | Our work | [24] | [15] | [16] | Exp. [9] |
|--------|-------|---------------|----------|------|------|------|----------|
| $\Xi_{cc}$ | $\{cc\}q$ | $\frac{1}{2}^+(1^+)$ | 3.519$^{fit}$ | 4.26 | 3.620 | 3.520 | 3.519±0.001 |
| $\Omega_{cc}$ | $\{cc\}s$ | $\frac{1}{2}^+(1^+)$ | 3.63 | 4.25 | 3.778 | 3.619 | - |
| $\Xi^*_{cc}$ | $\{cc\}q$ | $\frac{3}{2}^+(1^+)$ | 3.62 | 3.90 | 3.727 | 3.630 | - |
| $\Omega^*_{cc}$ | $\{cc\}s$ | $\frac{3}{2}^+(1^+)$ | 3.71 | 3.81 | 3.872 | 3.721 | - |
| $\Xi^*_{cb}$ | $[cb]q$ | $\frac{1}{2}^+(0^+)$ | 6.61 | 6.95 | 6.963 | 7.028 | - |
| $\Omega^*_{cb}$ | $[cb]s$ | $\frac{1}{2}^+(0^+)$ | 6.69 | 7.02 | 7.116 | 7.116 | - |
| $\Xi_{cb}$ | $\{cb\}q$ | $\frac{1}{2}^+(1^+)$ | 6.65 | 6.75 | 6.933 | 6.838 | - |
| $\Omega_{cb}$ | $\{cb\}s$ | $\frac{1}{2}^+(1^+)$ | 6.75 | 7.02 | 7.088 | 6.941 | - |
| $\Xi^*_{cb}$ | $\{cb\}q$ | $\frac{3}{2}^+(1^+)$ | 6.69 | 8.00 | 6.980 | 6.986 | - |
| $\Omega^*_{cb}$ | $\{cb\}s$ | $\frac{3}{2}^+(1^+)$ | 6.77 | 7.54 | 7.130 | 7.077 | - |
| $\Xi_{bb}$ | $\{bb\}q$ | $\frac{1}{2}^+(1^+)$ | 9.80 | 9.78 | 10.202 | 10.272 | - |
| $\Omega_{bb}$ | $\{bb\}s$ | $\frac{1}{2}^+(1^+)$ | 9.89 | 9.85 | 10.359 | 10.369 | - |
| $\Xi^*_{bb}$ | $\{bb\}q$ | $\frac{3}{2}^+(1^+)$ | 9.84 | 10.35 | 10.237 | 10.337 | - |
| $\Omega^*_{bb}$ | $\{bb\}s$ | $\frac{3}{2}^+(1^+)$ | 9.93 | 10.28 | 10.389 | 10.429 | - |
taken as inputs to obtain the mass of diquark $m_{cc}$ and then the masses of other baryon states in the table are predicted.

As indicated in above the choice of the diquark masses is based on our postulate about the binding energy, so this strategy would certainly bring up some theoretical uncertainties. To explicitly show how the diquark mass influences the spectrum of doubly heavy baryons, let us shift the corresponding diquark masses by 0.1 GeV, and we find that the uncertainty of the baryon mass lies within $0.047 \sim 0.064$ GeV. One can be convinced that the uncertainty should be no more than 10% as we change the diquark mass within a reasonable range. Moreover, in the forth column of Table 2 we put a term $\delta_{QQ'}$ following the predicted mass to manifest a possible error. In next column of this table, we list the gaps ($\Delta m$) among the concerned baryon masses where the uncertainties cancel out, and hence may make more senses. That means the predictions in this work, especially on the mass gaps, are experimentally testable.

4 Summary and Conclusions

In this work, the masses of various doubly heavy baryons have been studied in terms of the QCD sum rules where the diquark structures are priori assumed. In the calculation we keep the contributions of the condensates up to dimension six in OPE. Our results, in certain tolerance, are in accordance with the theoretical predictions via other models. Especially, it is worth pointing out that our results are reasonably consistent with that calculated in the QCD sum rules without assuming diquark structures.

In the calculation, an effective coupling between diquark and gluon which was phenomenologically introduced is adopted. The form factor at the effective vertex indeed manifests an inner structure of the diquark. But as the diquark is viewed as an independent degree of freedom, this factor performs as an ad-hoc parameter in the given theory and it plays a role just as the quark or gluon condensates in the QCD sum rules which were obtained either from an underlying theory (such as the value of the gluon condensate could be obtained from the dilute gas approximation of instantons) or by fitting data (such as the value of the quark condensate might be gained by fitting the pion decay
Our results imply that the structure of a heavy diquark and a light quark is indeed a reasonable configuration for the doubly heavy baryons. The Large Hadron Collider (LHC) which has already begun running, even at lower energy (7 TeV) and luminosity, will provide a large database of doubly heavy baryons. Once enough data are available, one can further analyze the doubly heavy baryons of various flavors and spins. Comparing our theoretical predictions on their mass spectra with the data, will not only enrich our knowledge on the underlying theory, i.e. the low energy QCD, but also further investigate the diquark structure and applicability for dealing with the processes such as production and decay of the doubly heavy baryons.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China (NSFC) and by the CAS Key Projects KJCX2-yw-N29 and H92A0200S2.

Appendix

The perturbative contribution $\rho_2(s)$ and nonperturbative contributions $\hat{B}[\Pi_2^{\text{cond}, \text{dim}}(q^2)]$ for $\Xi_{QQ'}$ and $\Omega_{QQ'}$ in Eq.(13) are shown as follows:

$$\rho_2(s) = -\frac{3m_q \left(m_d^2 - m_q^2 - s\right) \sqrt{(m_d^2 - m_q^2 + s)^2 - 4sm_q^2}}{8\pi s^2},$$

$$\hat{B}[\Pi_2^{\text{cond}, 3}(q^2)] = -\frac{(2M_B^4 + m_d^2 m_d^2)}{2M_B^2} \langle \bar{\Psi} \Psi \rangle e^{-\frac{m_q^2}{M_B^2}},$$

$$\hat{B}[\Pi_2^{\text{cond}, 4, C}(q^2)] = \langle \alpha_s G^2 \rangle \int_0^1 dx e^{-\frac{m_q^2}{M_B^2}} \left\{ \frac{m_q}{16\pi x^2 M_B^2} - \frac{1}{16\pi(x-1)x^3 M_B^4} \times \left[ m_q \left((x-1)^2 m_q^2 - x^2 m_d^2\right) \right] \right. + \left. \frac{1}{96\pi(x-1)^2 x^4 M_B^6} \times \left[ m_q \left(-4(x-1)^3 m_q^2 m_q^2 + x^4 m_d^4 + (x-1)^3(3x-1)m_q^4\right) \right] \right\},$$
$$\hat{B}[\Pi_2^{\text{cond}, 4, D}(q^2)] = \langle \alpha_s G^2 \rangle \int_0^1 dx e^{-\frac{m_d^2 + m_q^2}{M_B^2}} \left\{ \frac{1}{64\pi(x-1)xM_B^2} \left[ m_q (2\kappa_v + 1) \right] + \frac{1}{256\pi(x-1)^2xM_B^4} \left[ m_q (x^2m_d^2 (\kappa_v + 2) - (x-1)^2m_q^2 (3\kappa_v + 2)) \right] \right. \\
- \frac{1}{256\pi(x-1)^3xM_B^6} \left[ m_q (x-1)^2m_q^2 - x^2m_d^2 \right]^2 \right\},$$

$$\hat{B}[\Pi_2^{\text{cond}, 4, E}(q^2)] = \langle \alpha_s G^2 \rangle \int_0^1 dx e^{-\frac{m_d^2 + m_q^2}{M_B^2}} \left\{ \frac{1}{192\pi(x-1)^2M_B^2} \left[ m_q (8\kappa_v^2 + 8\kappa_v + 5) \right] + \frac{1}{192\pi(x-1)^3xM_B^4} \left[ m_q (2(x-1)^2m_q (2\kappa_v^2 + 2\kappa_v - 1) \\
- xm_d^2 (4\kappa_v^2 + 4\kappa_v - 2) + 3)) \right] \frac{1}{128\pi(x-1)^2xM_B^6} \times [m_q ((x-1)m_q^2 - xm_d^2)^2] \right\},$$

$$\hat{B}[\Pi_2^{\text{cond}, 5}(q^2)] = -g_s \langle \bar{\Psi}T\sigma\cdot G\Psi \rangle e^{-\frac{m_d^2}{M_B^2}} (M_B^2 (2\kappa_v + 1) - 8m_d^2),$$

$$\hat{B}[\Pi_2^{\text{cond}, 6}(q^2)] = 0.$$

The perturbative contributions $\rho_i(s)$ and nonperturbative contributions $\hat{B}[\Pi_i^{\text{cond, dim}}(q^2)]$ for $\Xi_{QQ'}^*$ and $\Omega_{QQ'}^*$ in Eq. (13) are shown as follows:

$$\rho_1(s) = -\frac{3(-m_d^2 + m_q^2 + s) \sqrt{(m_d^2 - m_q^2 + s)^2 - 4sm_d^2}}{8\pi s^2},$$

$$\rho_2(s) = -\frac{3m_q \sqrt{(m_d^2 - m_q^2 + s)^2 - 4sm_d^2}}{4\pi s},$$

$$\hat{B}[\Pi_1^{\text{cond}, 3}(q^2)] = -\frac{m_q (3M_B^4 + m_d^2m_q^2)}{6M_B^6} \langle \bar{\Psi}\Psi \rangle e^{-\frac{m_d^2}{M_B^2}},$$

$$\hat{B}[\Pi_2^{\text{cond}, 3}(q^2)] = \frac{(2M_B^4 + m_d^2m_q^2)}{2M_B^4} \langle \bar{\Psi}\Psi \rangle e^{-\frac{m_d^2}{M_B^2}}.$$
\[
\hat{B}[\Pi_{1}^{\text{cond}, 4, C}(q^2)] = \langle \alpha_s G^2 \rangle \int_0^1 dx \left\{ \frac{m_q^2 + m_d^2}{M_B^2} \left\{ \frac{1}{96\pi M_B^2} \left[ x - 2 \frac{1}{(x - 1)^2} + \frac{1}{192\pi (x - 1)^2 M_B^4} \right] \right. \\
- 2x^3 (x^2 - 3x + 2) m_d^2 \right. + \left. \frac{1}{192\pi (x - 1)^2 M_B^4} \left[ - 2(x - 2)(x - 1) x^2 m_q^2 m_d^2 \right. \\
\left. + x^4 m_d^4 + (x - 3)(x - 1)^3 m_q^4 \right] \right\},
\]

\[
\hat{B}[\Pi_{2}^{\text{cond}, 4, C}(q^2)] = \langle \alpha_s G^2 \rangle \int_0^1 dx \left\{ - \frac{m_q}{48\pi x^2 M_B^2} \\
+ \frac{1}{96\pi (x - 1) x^3 M_B^4} \left[ m_q \left[ 2(x - 1)^2 m_q^2 - x^2 m_d^2 \right] \right. \\
- \frac{1}{96\pi (x - 1) x^4 M_B^6} \left[ m_q^3 \left[ (x - 1)^2 m_q^2 - x^2 m_d^2 \right] \right. \right\},
\]

\[
\hat{B}[\Pi_{1}^{\text{cond}, 4, D}(q^2)] = 0,
\]

\[
\hat{B}[\Pi_{2}^{\text{cond}, 4, D}(q^2)] = 0,
\]

\[
\hat{B}[\Pi_{1}^{\text{cond}, 4, E}(q^2)] = \langle \alpha_s G^2 \rangle \int_0^1 dx \left\{ \frac{m_q^2 + m_d^2}{M_B^2} \left\{ \frac{1}{192\pi (x - 1)^3 M_B^4} \left[ x(-x^2 (4\kappa_v^2 + 4\kappa_v + 7) \right. \\
\left. + 4(\kappa_v^2 + \kappa_v + 1) - 3x) + (x - 1)^2 m_q^2 (3x - (2\kappa_v + 1)^2) \right] + \frac{1}{128\pi (x - 1)^2 x M_B^6} \right. \\
\times \left[ ((x - 1)m_q^2 - xm_d^2)^2 \right) \right\},
\]

\[
\hat{B}[\Pi_{2}^{\text{cond}, 4, E}(q^2)] = \langle \alpha_s G^2 \rangle \int_0^1 dx \left\{ - \frac{1}{192\pi (x - 1)^2 M_B^4} \left[ m_q (8\kappa_v^2 + 8\kappa_v + 5) \right. \\
+ 192\pi (x - 1)^3 x M_B^4 \\
\times \left[ m_q \left( x m_d^2 (x (4\kappa_v^2 + 4\kappa_v - 2) + 3) - 2(x - 1)^2 m_q^2 (2\kappa_v^2 + 2\kappa_v - 1) \right) \right. \\
\left. + \frac{1}{128\pi (x - 1)^2 x^2 M_B^6} \left[ m_q ((x - 1)m_q^2 - xm_d^2)^2 \right) \right\},
\]

19
\[ \hat{B}[\Pi_1^{\text{cond}, 5}(q^2)] = \frac{g_s \langle \bar{T} \sigma \cdot G \Psi \rangle m_d^2 m_q e^{-\frac{m_d^2}{M_B^2}}}{12M_B^6}, \]
\[ \hat{B}[\Pi_2^{\text{cond}, 5}(q^2)] = -\frac{g_s \langle \bar{T} \sigma \cdot G \Psi \rangle m_d^2 e^{-\frac{m_d^2}{M_B^2}}}{4M_B^4}, \]
\[ \hat{B}[\Pi_1^{\text{cond}, 6}(q^2)] = -\frac{g_s^2 \langle \bar{\Psi} \Psi \rangle^2 m_d^2 e^{-\frac{m_d^2}{M_B^2}}}{81M_B^6}, \]
\[ \hat{B}[\Pi_2^{\text{cond}, 6}(q^2)] = 0. \]

The perturbative contributions \( \rho_i(s) \) and nonperturbative contributions \( \hat{B}[\Pi_i^{\text{cond}, \text{dim}}(q^2)] \) for \( \Xi'_{QQ'} \) and \( \Omega'_{QQ'} \) in Eq. (13) are shown as follows:

\[ \rho_1(s) = \frac{3 (-m_d^2 + m_q^2 + s) \sqrt{(m_d^2 - m_q^2 + s)^2 - 4s m_d^2}}{32\pi s^2}, \]
\[ \rho_2(s) = \frac{3m_q \sqrt{(m_d^2 - m_q^2 + s)^2 - 4s m_d^2}}{16\pi s}, \]
\[ \hat{B}[\Pi_1^{\text{cond}, 3}(q^2)] = \frac{m_q (3M_B^2 - m_d^2 m_q^2)}{24M_B^6} \langle \bar{\Psi} \Psi \rangle e^{-\frac{m_d^2}{M_B^2}}, \]
\[ \hat{B}[\Pi_2^{\text{cond}, 3}(q^2)] = -\frac{(2M_B^4 + m_d^2 m_q^2)}{8M_B^6} \langle \bar{\Psi} \Psi \rangle e^{-\frac{m_d^2}{M_B^2}}, \]
\[ \hat{B}[\Pi_1^{\text{cond}, 4, C}(q^2)] = \langle \alpha_s G^2 \rangle \int_0^1 dx e^{-\frac{m_d^2}{M_B^2} - \frac{m_q^2}{M_B^2}} \left\{ \frac{3x^2 - 10x + 10}{384\pi M_B^2} + \frac{1}{768\pi(x - 1)^3 x^4 M_B^4} \right\}, \]
\[ \times \left[ 2(x - 1)^4 x^2 (3x^2 - 7x + 3) m_q^2 - 2(x - 1)^2 x^4 (3x^2 - 10x + 10) m_q^2 \right] + \frac{1}{768\pi(x - 1)^3 x^4 M_B^4} \left[ 2(x - 1) (-3x^3 + 13x^2 - 21x + 11) x^3 m_d^2 m_q^2 \right. \]
\[ + (3x^2 - 13x + 10) x^5 m_d^4 + (x - 1)^3 (3x^3 - 10x^2 + 15x - 8) x m_q^4 \] \[ + \frac{1}{768\pi(x - 1)^3 x^4 M_B^4} \left[ (x - 1)x^4 (3x - 5) m_d^4 m_q^2 \right. \]
\[ + (7 - 3x)(x - 1)^3 x^2 m_d^2 m_q^4 - x^6 m_d^6 + (x - 3)(x - 1)^5 m_q^6 \right\}. \]
\[ \mathcal{B}[\Pi_{\text{cond}, 4, C}(q^2)] = \langle \alpha_s G^2 \rangle \int_0^1 dx e^{-\frac{m_q^2 + m_d^2}{M_B^2}} \left\{ \frac{m_q}{64\pi x^2 M_B^2} - \frac{1}{64\pi (x-1)x^3 M_B^2} \right. \\
\times \left[ m_q \left( (x-1)^2 m_q^2 - x^2 m_d^2 \right) \right] + \frac{1}{384\pi (x-1)^2 x^4 M_B^2} \\
\times \left[ m_q \left( -4(x-1)x^3 m_q^2 m_d^2 + x^4 m_d^4 + (x-1)^3 (3x-1) m_q^4 \right) \right] \\
- \frac{1}{384\pi (x-1)^2 x^5 M_B^8} \left[ m_q^3 \left( (x-1)^2 m_q^2 - x^2 m_d^2 \right)^2 \right] \right\}, \]

\[ \mathcal{B}[\Pi_{\text{cond}, 4, D}(q^2)] = 0, \]

\[ \mathcal{B}[\Pi_{\text{cond}, 4, E}(q^2)] = 0, \]

\[ \mathcal{B}[\Pi_{\text{cond}, 5}(q^2)] = \frac{g_s \langle \bar{\Psi} T \sigma \cdot G \Psi \rangle m_q^2 e^{-\frac{m_q^2}{M_B^2}}}{16M_B^2}, \]

\[ \mathcal{B}[\Pi_{\text{cond}, 5}(q^2)] = -\frac{g_s \langle \bar{\Psi} T \sigma \cdot G \Psi \rangle m_q^2 m_d e^{-\frac{m_d^2}{M_B^2}}}{48M_B^2}, \]

\[ \mathcal{B}[\Pi_{\text{cond}, 5}(q^2)] = \frac{g_s^2 \langle \bar{\Psi} \Psi \rangle^2 m_d^2 e^{-\frac{m_d^2}{M_B^2}}}{324M_B^8}, \]

\[ \mathcal{B}[\Pi_{\text{cond}, 6}(q^2)] = 0. \]

References

[1] M. Gell-Mann, Phys. Lett. 8, 214 (1964).

[2] M. Ida and R. Kobayashi, Prog. Theor. Phys. 36 (1966) 846.

[3] D. B. Lichtenberg, Nuovo Cim. A 28, 563 (1975).

[4] D. B. Lichtenberg, W. Namgung, E. Predazzi and J. G. Wills, Phys. Rev. Lett. 48, 1653 (1982).

[5] R. L. Jaffe, Phys. Rept. 409, 1 (2005) [Nucl. Phys. Proc. Suppl. 142, 343 (2005)].
[6] F. Wilczek, arXiv:hep-ph/0409168.

[7] H. W. Ke, X. Q. Li and Z. T. Wei, Phys. Rev. D 77, 014020 (2008) arXiv:0710.1927 [hep-ph].

[8] A. F. Falk, M. E. Luke, M. J. Savage and M. B. Wise, Phys. Rev. D 49, 555 (1994) arXiv:hep-ph/9305315.

[9] M. Mattson et al. [SELEX Collaboration], Phys. Rev. Lett. 89, 112001 (2002) arXiv:hep-ex/0208014.

[10] A. Ocherashvili et al. [SELEX Collaboration], Phys. Lett. B 628, 18 (2005) arXiv:hep-ex/0406033.

[11] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 74, 011103 (2006).

[12] R. Chistov et al. [BELLE Collaboration], Phys. Rev. Lett. 97, 162001 (2006).

[13] A. Majethiya, B. Patel, A. K. Rai and P. C. Vinodkumar, arXiv:0809.4910 [hep-ph].

[14] S. P. Tong, Y. B. Ding, X. H. Guo, H. Y. Jin, X. Q. Li, P. N. Shen and R. Zhang, Phys. Rev. D 62, 054024 (2000) arXiv:hep-ph/9910259.

[15] D. Ebert, R. N. Faustov, V. O. Galkin and A. P. Martynenko, Phys. Rev. D 66, 014008 (2002) arXiv:hep-ph/0201217.

[16] D. H. He, K. Qian, Y. B. Ding, X. Q. Li and P. N. Shen, Phys. Rev. D 70, 094004 (2004) arXiv:hep-ph/0403301.

[17] V. V. Kiselev, A. K. Likhoded, O. N. Pakhomova, V. A. Saleev, Phys. Rev. D66, 034030 (2002). hep-ph/0206140.

[18] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B147, 385 (1979); ibid, Nucl. Phys. B147, 448 (1979).

[19] Z. G. Wang and X. H. Zhang, Commun. Theor. Phys. 54, 323 (2010) arXiv:0905.3784 [hep-ph]; Z. G. Wang, Y. M. Xu and H. J. Wang, Commun. Theor. Phys. 55, 1049 (2011) arXiv:1004.0484 [hep-ph].
[20] J. R. Zhang and M. Q. Huang, Commun. Theor. Phys. 54, 1075 (2010) [arXiv:0905.4672 [hep-ph]].

[21] C. F. Qiao, L. Tang, G. Hao and X. Q. Li, J. Phys. G 39, 015005 (2012) [arXiv:1012.2614 [hep-ph]].

[22] V. V. Kiselev, A. K. Likhoded, Phys. Usp. 45, 455-506 (2002) [hep-ph/0103169].

[23] E. Bagan, M. Chabab and S. Narison, Phys. Lett. B 306, 350 (1993).

[24] J. R. Zhang and M. Q. Huang, Phys. Rev. D 78, 094007 (2008) [arXiv:0810.5396 [hep-ph]].

[25] R. M. Albuquerque and S. Narison, Nucl. Phys. Proc. Suppl. 207-208, 265 (2010) [arXiv:1009.2428 [hep-ph]].

[26] S. Narison and R. Albuquerque, Phys. Lett. B 694, 217 (2010) [arXiv:1006.2091 [hep-ph]].

[27] K. Kim, D. Jido and S. H. Lee, arXiv:1103.0826 [nucl-th].

[28] R. Jakob, P. Kroll, M. Schurmann and W. Schweiger, Z. Phys. A 347, 109 (1993) [arXiv:hep-ph/9310227].

[29] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. 127, 1 (1985).

[30] P. Colangelo and A. Khodjamirian, in At the frontier of particle physics / Handbook of QCD, edited by M. Shifman (World Scientific, Singapore, 2001), arXiv:hep-ph/0010175.

[31] S. Narison, Nucl. Phys. Proc. Suppl. 207-208, 315 (2010) [arXiv:1010.1959 [hep-ph]].

[32] K. Nakamura et al. [Particle Data Group], J. Phys. G 37, 075021 (2010).

[33] D. S. Du, J. W. Li and M. Z. Yang, Phys. Lett. B619, 105 (2005).