Vortex Nucleation Induced Phonon Radiation from a Moving Electron Bubble in Superfluid $^4$He

Dafei Jin$^1$ and Wei Guo$^2$
$^1$Department of Physics, Brown University, Providence, Rhode Island 02912, USA
$^2$Department of Physics, Yale University, New Haven, Connecticut 06520, USA
(Dated: July 12, 2010)

We construct an efficient zero-temperature semi-local density functional to dynamically simulate an electron bubble passing through superfluid $^4$He under various pressures and electric fields up to nanosecond timescale. Our simulated drift velocity can be quantitatively compared to experiments particularly when pressure approaches zero. We find that the high-speed bubble experiences remarkable expansion and deformation before vortex nucleation occurs. Accompanied by vortex-ring shedding, drastic surface vibration is generated leading to intense phonon radiation into the liquid. The amount of energy dissipated by these phonons is found to be greater than the amount carried away solely by the vortex rings. These results may enrich our understanding about the vortex nucleation induced energy dissipation in this fascinating system.

PACS numbers: 67.25.dg, 67.25.dk

I. INTRODUCTION

An electron injected into liquid helium forms a bubble due to the Pauli exclusion between an excess electron and helium atoms.$^{1,2}$ The dissipation mechanisms of a moving electron bubble in superfluid $^4$He has attracted considerable research interests for many years.$^{3-11}$ Above 1 K, the moving bubble experiences a drag force from collisions with thermally excited phonons and rotons.$^3$ Below 1 K, this drag force becomes very small, and even a weak electric field can accelerate the bubble to a high speed until some new dissipation mechanisms set in. In the high pressure regime, roton emission dominates the dissipation owing to the relatively low Landau velocity $v_h$ and the high vortex-ring nucleation critical velocity $v_c.$ In the low pressure regime, vortex-ring nucleation plays the key role. A vortex ring can attach to the bubble to form a bubble-ring complex if the electric field is not overly strong; otherwise, successive vortex rings can be shed away from the bubble surface.$^4,5$

Pioneering simulations using the Gross-Pitaevskii equation (GPE) have demonstrated the above vortex-ring nucleation, trapping, and shedding scenario.$^{12,13}$ But it is known to be difficult for the efficient local GPE to reproduce helium properties.$^{14}$ In contrast, accurate nonlocal density functional (NLDF) theories have been extensively applied to quasiparticles, vortices, and ions-related problems in liquid helium.$^{15,16}$ But they usually require prohibitively high computational cost for dynamic simulations. In this paper, we first introduce our well-constructed semi-local density functional (SLDF), associated with an optimized numerical scheme, that can reconcile both the physical accuracy and the computational efficiency in its applicable regime.$^{17,18}$ We then present our dynamic simulation for a moving electron bubble in pure superfluid $^4$He under low pressures and strong electric fields, where successive vortex-ring shedding is indeed observed. However, we shall point out that although vortex nucleation does trigger the dissipation, the major part of energy loss may come from its induced phonon radiation via surface vibration, rather than purely the shed-away vortex rings.

II. THEORETICAL FORMULATION

We formulate our problem in the framework of a zero-temperature SLDF theory. The system free energy density $G$ consists of the helium part, the electron part, and the helium-electron interaction part,

$$G = G_{\text{He}} + G_e + G_{\text{He-e}}.$$  \hspace{1cm} (1)

The helium part $G_{\text{He}}$ takes the form of

$$G_{\text{He}} = \frac{\hbar^2}{2m_{\text{He}}} |\nabla \psi|^2 - \mu \rho$$
$$+ \frac{1}{2} g_2 |\rho|^2 + \frac{1}{3} g_3 |\rho|^3 + \frac{1}{4} g_4 |\rho|^4$$
$$+ \frac{1}{2} \hbar^2 |\nabla \rho|^2 + \frac{1}{3} h_3 |\nabla \rho|^3 + \frac{1}{4} h_4 |\nabla \rho|^4,$$  \hspace{1cm} (2)

where $\psi$ is the macroscopic helium wavefunction and $\rho \equiv |\psi|^2$ is the local helium number density. The electron part $G_e$ takes the form of

$$G_e = \frac{\hbar^2}{2m_e} |\nabla \phi|^2 - c \varepsilon z \eta,$$  \hspace{1cm} (3)

where $\phi$ is the single electron wavefunction and $\eta \equiv |\phi|^2$ is the local electron number density. The $|\nabla \psi|^2$ and $|\nabla \phi|^2$ terms above are the helium and electron kinetic energy densities, with $m_{\text{He}}$ and $m_e$ being the helium and electron masses, respectively. $\mu$ is the helium chemical potential controlled by a given pressure $p$, which fixes the helium number density $\rho$ in bulk. $\varepsilon$ is the applied electric field, which drives the electron to move along $z$ direction. $g_2$, $g_3$, $g_4$ and $h_2$, $h_3$, $h_4$ are all fitting parameters to be
explained below. The helium-electron interaction part \( G_{\text{He-e}} \) takes the form of a contact collision

\[
G_{\text{He-e}} = f_1 \eta, \tag{4}
\]

in which \( f_1 \) is a fitting parameter chosen so as to produce a 1 Å scattering length and hence a 1.0456 eV potential barrier for an electron to tunnel into homogeneous helium at zero temperature and zero pressure.²

The polynomial function of \( \varrho \) in the second line of Eq. (2) is the helium-helium interaction energy density in the form of two-particle, three-particle and four-particle contact collisions. In the homogeneous case, it gives the helium internal energy density

\[
\varepsilon[\rho] = \frac{1}{2} g_2 \rho^2 + \frac{1}{3} g_3 \rho^3 + \frac{1}{4} g_4 \rho^4, \tag{5}
\]

from which one can derive the equation of state

\[
p[\rho] = \frac{1}{2} g_2 \rho^2 + \frac{2}{3} g_3 \rho^3 + \frac{3}{4} g_4 \rho^4, \tag{6}
\]

the chemical potential

\[
\mu[\rho] = g_2 \rho + g_3 \rho^2 + g_4 \rho^3, \tag{7}
\]

and the sound velocity

\[
c[\rho] = \sqrt{(g_2 + 2 g_3 \rho^2 + 3 g_4 \rho^3) / m_{\text{He}}}. \tag{8}
\]

By choosing \( g_2, g_3 \) and \( g_4 \) according to the well-known Orsay-Trento density functional, the experimentally measured above quantities at zero temperature can be very well produced.

The mixture polynomial function of \( \varrho \) and \( |\nabla \varrho|^2 \) in the third line of Eq. (2) contains our introduced semi-local interactions also up to four-particle collisions. They are the lowest-order gradient-expansion corrections to the local interactions and are essential for incorporating the helium surface tension into our theory

\[
\sigma[\rho] = \int_0^\rho d\varrho \ b[\varrho] \sqrt{\frac{\hbar^2}{8 m_{\text{He}} \varrho}} \frac{2(\varepsilon[\varrho] - \varepsilon[\rho]) - (\mu[\varrho] + \varepsilon[\rho])/\varrho (\varrho - \rho)}{\sqrt{\varepsilon[\varrho] - \varepsilon[\rho]}} - \mu[\rho] (\varrho - \rho), \tag{9}
\]

where the dimensionless function

\[
b[\rho] = \sqrt{1 + \frac{8 m_{\text{He}}}{\hbar^2} \left( \frac{1}{2} h_2 + \frac{1}{3} h_3 \rho + \frac{1}{4} h_4 \rho^2 \right)}. \tag{10}
\]

By choosing \( h_2, h_3 \) and \( h_4 \) appropriately, the experimentally measured surface tension at zero temperature and zero pressure can be produced. In addition, Eq. (9) is a generalized formula to allow not only zero pressure, when the liquid is in equilibrium with vacuum, but also a positive pressure, when the liquid is in contact with an impenetrable wall. In both situations, the liquid density drops from its bulk value \( \rho \) to 0 within a thin interfacial layer. \( \sigma[\rho] \) measures the energy change per unit surface area due to this density bending, in comparison with the internal energy per unit area held by the same number of particles in bulk. Such a generalized definition on the surface tension turns out to be useful for the electron bubble problem, where the bubble boundary pushes away the liquid basically like an impenetrable wall, and so gives rise to a nontrivial surface energy under any finite pressures.

| Parameter | Value          | Unit     |
|-----------|----------------|----------|
| \( f_1 \) | 5.55671 \times 10² | K A⁻¹   |
| \( g_2 \) | -7.18990 \times 10² | K A⁻¹   |
| \( g_3 \) | -3.61779 \times 10⁴ | K A⁻¹   |
| \( g_4 \) | 2.47799 \times 10⁶ | K A⁻¹   |
| \( h_2 \) | 1.16950 \times 10⁴ | K A⁻¹   |
| \( h_3 \) | -1.35048 \times 10⁶ | K A⁻¹   |
| \( h_4 \) | 3.46549 \times 10⁷ | K A⁻¹¹  |

Table I lists all the chosen fitting parameters of our SLDF. Figure 1(a) shows the equation of state. At \( p = 0 \) bar, \( \rho_0 = 0.021836 \) Å⁻³, \( \mu_0 = -7.1500 \) K, and \( c_0 = 237.70 \) m s⁻¹. Figure 1(b) shows the surface tension
versus pressure. At \( p = 0 \) bar, \( \sigma_0 = 0.37554 \text{ erg cm}^{-2} \) with a calculated 5.8 Å surface thickness (defined by 10% to 90% helium density) consistent with experiments and other theories\textsuperscript{15,17,19}.

The excitation spectrum in our theory can be found to be of the Bogoliubov type

\[
h^2 \omega^2 = c^2[\rho] h^2 k^2 + b^2[\rho] \left( \frac{h^2 k^2}{2m_{\text{He}}} \right)^2, \tag{11}
\]

whose pressure dependence is enclosed in the sound velocity \( c[\rho] \) from the local interactions, and the dimensionless function \( b[\rho] \) from the semi-local interactions. Compared with the efficient local GPE, although our SLDF can indeed produce more realistic helium bulk and surface properties, and meanwhile maintain the same computational efficiency, it still cannot incorporate the backflow effect and the roton excitation unless some nonlocal interactions are introduced.\textsuperscript{15} Figure 1(c) shows the excitation spectrum under three typical pressures: the spinodal pressure \(-9.5\) bar, the saturation pressure 0 bar, and the melting pressure 25 bar. The primary reason for employing three semi-local parameters, \( h_2 \), \( h_3 \) and \( h_4 \), in our SLDF instead of a single one, as that done in the well-known Stringari-Treiner density functional,\textsuperscript{15} is to remove some pathological behavior of the excitation spectrum. A single semi-local parameter being used to fit the surface tension at zero pressure will result in too large of a \( b[\rho] \) and hence unrealistically high excitation energies even at just moderate momentums.\textsuperscript{48} This will make the liquid overrigid against density fluctuation and may bring on artifacts in dynamic simulations. With the three semi-local parameters, we can keep \( b[\rho] \) sufficiently small, and also let it slowly decrease with pressure to imitate the tendency of roton gap with pressure in real helium.

We can also find in our theory the rectilinear vortex-line energy per unit length, by calculating the radial number density distribution \( \rho(r) \) under a given pressure,

\[
\frac{E_{\text{line}}}{L} = \int_{0}^{\pi} 2\pi r dr \left( \frac{h^2}{2m_{\text{He}}r^2} \rho + p - \mu \rho \right. \\
\left. + \frac{1}{2} g_2 \rho'^2 + \frac{1}{3} g_3 \rho^3 + \frac{1}{4} g_4 \rho^4 + \frac{h^2}{8m_{\text{He}}r^2} \rho'^2 \right) \tag{12}
\]

Putting them into the hollow-core model\textsuperscript{15}.

\[
\frac{K_{\text{line}}}{L} = \frac{\pi h^2}{m_{\text{He}}} \ln \frac{d}{a},
\]

\[
\frac{U_{\text{line}}}{L} = \frac{\pi h^2}{m_{\text{He}}} \rho_0 \delta,
\tag{13}
\]

we can get the core size \( a = 0.581 \text{ Å} \) and the core parameter \( \delta = 1.893 \), which imply a rather rigid core.

### III. DYNAMIC SIMULATION

We perform the dynamic simulation in the experimental low pressure regime \( p = 0 \sim 10 \) bar and strong electric field regime \( \mathcal{E} \sim 1 \sim 30 \text{ MV m}^{-1} \). Under these conditions, a high-speed electron bubble can nucleate vortex rings. As is well-known, there are two classic models on the formation of a vortex ring and bubble-ring complex.\textsuperscript{2,11} One is the encircling-ring (quantum transition) model, in which a full ring axisymmetrically appears on the bubble equator first, then moves sideways and captures the bubble on its core. The other one is the pinned-ring (peeling) model, in which a small protoring non-axisymmetrically appears on the bubble equator first, then grows up and retains the bubble on its core. Experimental evidence suggests that the encircling-ring model gives a better description at lower temperatures in isotopically purified \( ^4\text{He} \), whereas the pinned-ring model gives a better description at higher temperatures in natural \( ^4\text{He} \), with the existence of thermal rotons or \( ^3\text{He} \) impurities.\textsuperscript{2,12} For an already formed bubble-ring complex, the strength of the electric field determines its subsequent behavior. In a field of the order of \( 10^3 \sim 10^4 \text{ V m}^{-1} \), the ring can keep trapping the bubble and may grow to micron size, which significantly hinders the bubble motion.\textsuperscript{2} But in a field of the order of \( 10^6 \sim 10^7 \text{ V m}^{-1} \), owing to the vanishing escape barrier, the ring should separate from the bubble during several picoseconds with its size almost unchanged. The bubble can then move more freely, as well as successively shed away vortex rings.\textsuperscript{2} Since our simulation is for pure \( ^4\text{He} \) at zero temperature, the axisymmetric vortex-ring nucleation in the encircling-ring model is more suitable. Furthermore, since our electric fields lie in the strong regime, the transient non-axisymmetric capture-and-escape processes should have little influence on the longtime physics. Thus we reduce the original three-dimensional problem into two-dimensional by taking cylindrical symmetry with transverse coordinate \( r \) and longitudinal coordinate \( z \). This simplification makes the numerical integration up to nanosecond timescale computationally practical.

The coupled equations of motion for the macroscopic helium wavefunction \( \psi \) and the single electron wavefunc-
tion $\phi$ can be derived from our SLDF as

$$\dot{\psi} = -\frac{i}{\hbar} \left[ -\frac{\hbar^2}{2m_{\text{He}}} \nabla^2 - \mu + f_1|\psi|^2 
+ g_2|\psi|^2 + g_3|\psi|^4 + g_4|\psi|^6 
- \left( \frac{h_3}{3} + \frac{1}{2}h_4|\psi|^2 \right) (\nabla|\psi|^2 \cdot \nabla|\psi|^2) 
- \left( h_2 + \frac{2}{3}h_3|\psi|^2 + \frac{1}{2}h_4|\psi|^4 \right) \nabla^2|\psi|^2 \right] \psi,$$

$$\dot{\phi} = -\frac{i}{\hbar} \left[ -\frac{\hbar^2}{2m_{\text{e}}} \nabla^2 - e\mathcal{E} z + f_1|\psi|^2 \right] \phi. \tag{14}$$

The fourth-order finite-difference method in space and the fourth-order Runge-Kutta method in time are used to do the numerical integration. The computation grid on the $r$-$z$ plane is $500 \times 1000$ with the space step of 0.2 Å. So the physical space is a pipe of 200 Å in diameter and 200 Å in length. We let it move along to keep the electron in the center. Systematic numerical tests have been done to ensure that 0.2 Å spatial resolution provides enough precision for our problem, even when the vortex core structure is involved.

Due to the big difference between the helium and electron intrinsic timescales of the order of $m_{\text{He}}/m_{\text{e}} \sim 7300$, we adopt the adiabatic approximation. For every instantaneous helium configuration, we first find the electron ground state by evolving the electron with 50 imaginary time steps of 0.001 fs, then develop helium with 1 real time step of 0.001 ps. To avoid the outgoing sound waves from the central region being reflected on the grid boundary, we set a space-dependent damping coefficient by replacing $i$ with $\eta(s) + i$ in the helium equation of motion, where

$$\eta(s) = \frac{\tilde{\eta}}{2} \left[ 1 + \tanh \left( \frac{s - \bar{s}}{\bar{\omega}} \right) \right], \quad (s \equiv \sqrt{r^2 + z^2}). \tag{15}$$

We simply choose $\bar{s} = 80$ Å, $\bar{\omega} = 1.0$ Å, and $\tilde{\eta} = 0.5$. The sound waves can travel freely through the undamped region ($s < \bar{s}$) but are quickly attenuated in the damped region ($s > \bar{s}$). Physically, this looks as if there is a strong zero-temperature heat sink surrounding the system.

Figure 2 shows the calculated radial number density profiles of helium and electron for a spherical static electron bubble under various pressures.

**Figure 2:** (color online). The radial number density profiles of helium and electron for a spherical static electron bubble under various pressures.

---

The acceleration decreases remarkably because of the increase in the effective mass so that $v$ approaches a constant value about 50.6 m s$^{-1}$ and the flow pattern exhibits quite well fore-and-aft symmetry as shown at $t = 450$ ps. After this stage, a narrow edge girdling the bubble waist gradually sticks out signifying the onset of vortex-ring nucleation. This adds a considerable inertia to the bubble motion, making $v$ decrease to 44.9 m s$^{-1}$ at $t = 850$ ps, when the bubble attains its largest equatorial radius about 44 Å and its sharpest edge radius of curvature about 4 Å. We may call the bubble velocity for the first appearance of this characteristic behavior as the vortex-ring nucleation critical velocity $v_c = 44.9$ m s$^{-1}$ under the prescribed conditions. There is no dissipation before, since the electric work mainly transforms to the fluid kinetic energy. But around this time the liquid density starts to fluctuate accompanied by some phonon generation, presumably because the local flow near the bubble waist becomes supersonic and keeps hitting the nucleated vortex core. At $t = 950$ ps, a vortex ring of radius $R_{\text{ring}} \approx 36$ Å is detaching from the bubble surface, while $v$ is 43.8 m s$^{-1}$ and continues to slow down due to the attraction from the vortex ring. The phonon
FIG. 3: (color online). The snapshots of the helium number density profile around the moving electron bubble at a sequence of times under zero pressure and 10 MV m\(^{-1}\) electric field. The bubble travels from left to right along \(z\) direction (the longitudinal direction). Each image corresponds to a physical size of 100 Å \(\times\) 100 Å and is reflected up and down in \(r\) direction (the transverse direction) justified by the imposed cylindrical symmetry. The instantaneous velocity is given for each time. The open and solid arrows indicate the movements of vortex ring and solitary ripplon relative to the bubble as mentioned in the text.

generation appears more intense, particularly peaked at a wavelength of the order of 10 Å. The unexpected large ring radius, almost twice as big as the static bubble radius, is a unique observation of our dynamic SLDF simulation.

In the later stage, the vortex ring falls behind, leaving a drastic surface vibration and velocity oscillation to the bubble in the next hundreds of picoseconds. At \(t = 970\) ps, an annular surface protrusion like a solitary ripplon is formed and sliding backwards, while \(v\) is dragged down to 31.5 m \(s^{-1}\). This solitary ripplon then shrinks on the bubble tail at \(t = 990\) ps, and radiates a large number of phonons backwards into the liquid as shown at \(t = 1030\) ps. This provides a strong forward impulse to speed up the bubble so that \(v\) can even overshoot to 60.5 m \(s^{-1}\) at \(t = 1070\) ps. During this time, the solitary ripplon surviving from the last collision bounces back and propagates forwards. Around \(t = 1080 \sim 1090\) ps, it shrinks on the bubble head and deposits a large number of phonons forwards into the liquid. This provides a strong backward impulse decelerating the bubble velocity to as low as 27.7 m \(s^{-1}\) at \(t = 1170\) ps. Afterwards, some remnant surface vibration and phonon radiation keep happening back and forth, while the bubble velocity oscillatorily increases until a second vortex ring completes its nucleation. Then all the processes repeat.

IV. DISCUSSION

Figure 4 shows the time evolution of the moving electron bubble volume under zero pressure and 10 MV m\(^{-1}\) electric field. The dotted lines mark the moments of the vortex-ring shedding events.

FIG. 4: (color online). The time evolution of the moving electron bubble volume under zero pressure and 10 MV m\(^{-1}\) electric field. The dotted lines mark the moments of the vortex-ring shedding events.

Figure 4 shows the time evolution of the moving electron bubble volume \(V_b(t)/(4\pi R_b^3/3)\) under \(p = 0\) bar and \(E = 10\) MV m\(^{-1}\), obtained from the same evolution shown in Fig. 3 but extended to about 5 ns physical time. Here \(4\pi R_b^3/3\) with \(R_b = 18.52\) Å is the spherical static bubble volume at \(t = 0\) ps. As can be seen, before the dissipation processes arise at \(t \approx 850\) ps, \(V_b(t)\) keeps growing to about 3.6 times the static volume. After that, \(V_b(t)\) oscillates violently between 2.0 and 4.0 (average about 3.0) times the static volume. It approximately implicates the hydrodynamic mass variation, and coincides with the bubble velocity variation described above. The dotted lines mark the moments of the vortex-ring shedding events. During \(t = 800 \sim 5000\) ps, there are six such events in total. Although they do not exhibit a regular periodicity, we may still roughly estimate the average time interval between two successive events \(\tau \approx 700\) ps. Each time when a ring leaves the bubble, there follows a huge volume change immediately and perhaps a series of small volume changes later on. This can be related to
the complex surface motion displayed in Fig. 3.

**Figure 5:** (color online). The drift velocity versus pressure under 2.6, 10, 20, 30 MV m\(^{-1}\) electric fields from our simulation in comparison with the low pressure part of experiment by Nancolas et. al. at 0.3 K under 2.6 MV m\(^{-1}\). The dashed curve gives the Landau velocity for roton emission. The dashed-dotted curve gives the theoretically extrapolated drift velocity from 25 bar under 2.6 MV m\(^{-1}\) if roton emission was the only dissipation mechanism.

Figure 3 shows the drift velocity \(\bar{v}\) versus pressure under \(\mathcal{E} = 2.6, 10, 20, 30\) MV m\(^{-1}\) from our simulation in comparison with the low pressure part of experiment by Nancolas et. al. at 0.3 K under 2.6 MV m\(^{-1}\). Our \(\bar{v}\) is calculated by linearly fitting the displacement of the electron wavefunction-maximum over 2 ns after the bubble has become strongly dissipative. Although the instantaneous velocity \(v\) mentioned above undergoes large oscillation, \(\bar{v}\) is nearly constant depending only on the pressure and electric field for a long time. According to Nancolas et. al., if roton emission was the only dissipation mechanism, then in the limit \(\mathcal{E} \to 0\) V m\(^{-1}\), \(\bar{v}\) should follow the dashed curve of Landau velocity \(v_L\); whereas for \(\mathcal{E} = 2.6\) MV m\(^{-1}\), \(\bar{v}\) should follow the dashed-dotted curve based on the well-known relation \(\bar{v} = v_L + \mathcal{A}\mathcal{E}^{1/3}\), in which \(\mathcal{A}\) takes its value at 25 bar with the pressure dependence ignored. However, the experimentally measured \(\bar{v}\) shows a significant drop from this curve, indicating an additional dissipation mechanism in this low pressure regime believed to be vortex-ring nucleation. Our work supports this general scenario especially for \(p < 3\) bar where the simulation and experimental results quantitatively agree with each other. This suggests that our SLDF can be an efficient and reliable tool to study interesting dynamic problems, particularly under the natural experimental condition \(p \to 0\) bar close to zero temperature. The lack of roton excitation in our SLDF may not be problematic for two reasons that essentially prohibit roton emission in this range. First, the Landau velocity \(v_L \approx 58\) m s\(^{-1}\) is substantially higher than the vortex-ring nucleation critical velocity \(v_c \approx 45\) m s\(^{-1}\). Second, there is experimental evidence that \(\mathcal{A}\) is divergent as \(p \to 3\) bar from above. For \(p > 3\) bar, our simulation still exhibits a qualitatively correct behavior in terms of the increasing \(\bar{v}\) with increasing \(p\) commonly understood as the inverse proportionality of \(v_c\) with respect to the bubble size. The relatively large deviation between simulation and experiment is presumably caused by the lack of roton excitation in our SLDF, whereas in reality vortex-ring nucleation and roton emission coexist in this pressure range.

Although in Fig. 3 we have presented that vortex nucleation is indeed the origin of dissipation, it is nontrivial to investigate how the energy is actually taken away from the system. While this is normally attributed to purely the frequent vortex-ring shedding, our analysis suggests otherwise. The energy associated with a large single vortex ring at zero pressure can be estimated by

\[
E_{\text{ring}} = \frac{\pi \hbar^2}{m_{\text{He}}} \rho_0 2\pi R_{\text{ring}} \left( \ln \frac{8R_{\text{ring}}}{a} - 2 + \delta \right). \tag{16}
\]

Based on our preceding calculation about the vortex properties in our density functional, when \(R_{\text{ring}} \approx 36\) Å as shown in Fig. 3 we have \(E_{\text{ring}} \approx 1147\) K. As a result, the average rate of energy loss to the vortex rings is \(E_{\text{ring}}/\tau \approx 1147\) K / 700 ps = 2.3 \times 10\(^{-11}\) W. On the other hand, the average rate of energy input to the system from the electric work is \(eE\bar{v} \approx 10\) MeV m\(^{-1}\) \times 48 m s\(^{-1}\) = 7.7 \times 10\(^{-11}\) W. Strikingly, the released vortex rings only take away about 30% of the total input energy. According to the processes illustrated in Fig. 3 and discussed above, the remaining energy loss can be primarily attributed to the phonon radiation through surface vibration persistently occurring in the time interval between successive vortex-ring shedding events. Under a fixed electric field with an increasing pressure, we even notice a tendency for such a dissipation channel to become increasingly dominant.

**V. CONCLUSION**

We have carried out dynamic simulation for a moving electron bubble in pure superfluid \(^4\)He under low pressures and strong electric fields, using a zero-temperature semi-local density functional theory that embodies realistic \(^4\)He bulk and surface properties. It visualizes the underlying microscopic processes from picosecond to nanosecond timescale, which cannot yet be directly observed through any experimental apparatus today. Not only have we confirmed that theory and experiment show good agreement on the bubble mobility approaching zero pressure, we have also discovered that the pronounced surface vibration and phonon radiation as a result of vortex nucleation dissipate more energy than the shed-away vortex rings alone. These results may enrich our understanding about the vortex nucleation induced energy dissipation.
sipation in this fascinating system and so warrant further experimental and theoretical studies.

Acknowledgments

The authors are very grateful to Professors H.J. Maris, F. Ancilotto, M. Barranco, M. Pi, and D. Mateo for helpful discussions. This work was supported in part by the National Science Foundation through Grant No. DMR-0605355.

1 W.T. Sommer, Phys. Rev. Lett. 12, 271 (1964); M.A. Woof and G.W. Rayfield, Phys. Rev. Lett. 15, 235 (1965); J. Classen, C.-K. Su, M. Mohazzab, and H.J. Maris, Phys. Rev. B 57, 3000 (1998).
2 B.E. Springett, M.H. Cohen, and J. Jortner, Phys. Rev. 159, 183 (1967); Y. M. Shih and C.-W. Woo, Phys. Rev. A 8, 1437 (1973).
3 L. Meyer and F. Reif, Phys. Rev. 110, 279 (1958); F. Reif and L. Meyer, Phys. Rev. 119, 1164 (1960).
4 A. Phillips and P.V.E. McClintock, Phys. Rev. Lett. 33, 1468 (1974).
5 R.M. Bowley, P.V.E. McClintock, F.E. Moss and P.C.E. Stamp, Phys. Rev. Lett. 44, 161 (1980); G.G. Nancolas and P.V.E. McClintock, Phys. Rev. Lett. 48, 1190 (1982).
6 G.W. Rayfield and F. Reif, Phys. Rev. 136, A1194 (1964).
7 D.M. Strayer and R.J. Donnelly, Phys. Rev. Lett. 26, 1420 (1971).
8 G.G. Nancolas, T. Ellis, P.V.E. McClintock, and R.M. Bowley, Nature 316, 797 (1985).
9 K.W. Schwarz and P.S. Jang, Phys. Rev. A 8, 3199 (1973); C.M. Muirhead, W.F. Vinen and R.J. Donnelly, Phil. Trans. Roy. Soc. A311, 433 (1984).
10 R.J. Donnelly, Quantized Vortices in Helium II (Cambridge University Press, Cambridge, 1991).
11 A.F. Borghesani, Electrons and Ions in Liquid Helium (Oxford University Press, New York, 2007).
12 T. Frisch, Y. Pomeau, and S. Rica, Phys. Rev. Lett. 69, 1644 (1992); T. Winiecki, J.F. McCann, and C.S. Adams, Europhys. Lett. 48, 475 (1999); T. Winiecki and C.S. Adams, Europhys. Lett. 52, 257 (2000).
13 N.G. Berloff and P.H. Roberts, Phys. Rev. B 63, 024510 (2000); N.G. Berloff and P.H. Roberts, J. Phys. A 34, 81 (2001); N.G. Berloff and C.F. Barenghi, Phys. Rev. Lett. 93, 090401 (2004).
14 N.G. Berloff and P.H. Roberts, J. Phys. A 32, 5611 (1999).
15 F. Dalfovo, A. Lastri, L. Pricuapenko, S. Stringari, and J. Treiner, Phys. Rev. B 52, 1193 (1995).
16 L. Giacomazzi, F. Toigo, and F. Ancilotto, Phys. Rev. B 67, 104501 (2003); F. Ancilotto, F. Dalfovo, L.P. Pitaevskii, and F. Toigo, Phys. Rev. B 71, 104530 (2005); D. Mateo, M. Pi, and M. Barranco, Phys. Rev. B 81, 174510 (2010).
17 S. Stringari and J. Treiner, Phys. Rev. B 36, 8369 (1987).
18 D. Jin and H.J. Maris, in the Proceedings of the 2009 International Symposium on Quantum Fluids and Solids, J. Low Temp. Phys., 158, 317 (2010).
19 K. Penanen, M. Fukuto, R.K. Helmann, I.F. Silvera, and P.S. Pershan, Phys. Rev. B 62, 9621 (2000).
20 H.J. Maris, J. Low Temp. Phys. 94, 125 (1994); F. Dalfovo, Phys. Rev. B 46, 5482 (1992).
21 W. Guo and H.J. Maris, in the Proceedings of the 24th International Conference on Low Temperature Physics, AIP Conference Proceedings 850, 161 (2006).
22 See EPAPS Document No. XXX for the continuous movie.