Jiang, Wenshuai; Naber, Aaron  
$L^2$ curvature bounds on manifolds with bounded Ricci curvature. (English)  
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Authors’ abstract: Consider a Riemannian manifold with bounded Ricci curvature $|\text{Ric}| \leq n - 1$ and the noncollapsing lower volume bound $\text{Vol}(B_1(p)) > v > 0$. The first main result of this paper is to prove that we have the $L^2$ curvature bound $f_{B_r(p)}(\text{Rm})^2(x) < C(n,v)$, which proves the $L^2$ conjecture. In order to prove this, we will need to first show the following structural result for limits. Namely, if $(M^n, d_j, p_j) \to (X, d, p)$ is a GH-limit of noncollapsed manifolds with bounded Ricci curvature, then the singular set $\mathcal{S}(X)$ is $n-4$ rectifiable with the uniform Hausdorff measure estimates $H^{n-4}(\mathcal{S}(X) \cap B_r) < C(n,v)$ which, in particular, proves the $(n-4)$-finiteness conjecture of Cheeger-Colding. We will see as a consequence of the proof that for $n-4$ a.e. $x \in \mathcal{S}(X)$, the tangent cone of $X$ at $x$ is unique and isometric to $\mathbb{R}^{n-4} \times C(S^3/\Gamma_x)$ for some $\Gamma_x \subseteq O(4)$ that acts freely away from the origin.

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MSC: 
53B20 Local Riemannian geometry  
35A21 Singularity in context of PDEs

Keywords: 
Ricci; curvature; stratification; singularity

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