Output property in loss-noise model of single mode laser

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Abstract. In order to optimize communication and output stability of laser system, with linear approximation method and loss-noise model of single mode laser driven by colored pump noise and quantum noise whose real and imaginary part are cross-correlated, the steady-state mean intensity correlation function and signal-to-noise ratio (SNR) are calculated with an input additive periodic signal. The new physical quantity $K$ is defined to describing output property of laser system. The maximums are existed between the relationship of $K_1$ and net-gain coefficient, self-saturation coefficient, cross-correlated coefficient of real and imaginary part of quantum noise. That is the best match of the laser communication system. Also the decrease of noise intensity and increase of input signal frequency all can optimize the output property of laser system.

1. Introduction

The research of stochastic resonance has attracted a great deal of attention both experimentally and theoretically in the recent years. Generally we use SNR to describe stochastic resonance. It is to say the maximums which exist in the SNR imply the phenomenon of stochastic resonance. Besides as we know, the fluctuation of laser field is usually caused by two kinds of noise. One is the quantum noise caused by spontaneous radiation, the other is the pump noise caused by environmental disturbance. Moreover, reducing the fluctuation of output can enhance the stability of laser system. In order to study the output properties of laser system better, we define a specific value, that is $K = \frac{R}{C(0)}$. The maximum is appeared when there is some match between the stochastic resonance and fluctuation. That is the optimal configuration in laser system.

By adopting the linear approximation method, we study the time evolution of intensity correlation function, mean intensity relative fluctuation and signal-to-noise ratio for a single-mode laser driven by both pump noise and the quantum noise with the cross-correlation between its real and imaginary parts. We get some new conclusions by analyzing the specific value $K$ versus the parameters to optimize the output properties.

2. Calculation

The equation of intensity for a loss-noise model of a single mode laser with a periodic signal is given by

$$\frac{dI}{dt'} = 2a_0 I - 2A I^2 + Q(1 - |\lambda_\epsilon|) + 2Ip_\phi(t') + 2\sqrt{I} \epsilon_\phi(t') + B \cos \Omega t'$$

Where the pump noise and quantum noise are correlated in the following forms
\[ \langle p_R(t') \rangle = \langle \varepsilon_r(t') \rangle = 0 \]
\[ \langle p_R(t') p_R(s) \rangle = \frac{P}{2\tau} e^{-\frac{t'-s}{\tau}} \]
\[ \langle \varepsilon_r(t') \varepsilon_r(s) \rangle = Q(1+|\lambda_q|)\delta(t'-s) \]
\[ \langle p_R(t') \varepsilon_r(s) \rangle = 0 \]  \hspace{1cm} (2)

In Eqs. (1) and (2), \( a_0 \) and \( A \) represent the net gain and the self-saturation, \( I \) is the laser intensity, \( B \) and \( \Omega \) are the amplitude and frequency of input signal, \( p_R(t') \) is the real part of the pump noise, \( \varepsilon_r(t') \) is the quantum noise of phase lock, \( P \) and \( Q \) are the intensities of the pump noise and the quantum noise respectively, \( \tau \) is self-correlation time of pump noise, and \( \lambda_q \) is the cross-correlation coefficient between the real and imaginary part of the quantum noise, and the range is \(-1 \leq \lambda_q \leq 1\).

Let \( I = I_0 + \delta(t') \), where \( I_0 = \frac{a_0}{A} \) is the deterministic steady-state intensity, we linearize Eq. (1) around the deterministic steady-state intensity \( I_0 \), we get
\[
\frac{d\delta(t')}{dt} = -\gamma \delta(t') + 2I_0 p_R(t') + 2\sqrt{I_0} \varepsilon_r(t') + Q(1-|\lambda_q|) + B \cos \Omega t'
\]  \hspace{1cm} (3)

Where \( \gamma = 2a_0 \), according to the steady-state mean intensity correlation function (SSMICF) defined by
\[
C(t) = \lim_{t' \to \infty} \frac{\langle I(t') I(t'+t) \rangle - \langle I(t') \rangle^2}{\langle I(t') \rangle^2}
\]
the result by solving Eq.(3) is
\[
C(t) = \frac{A^2 Q^2 (1-|\lambda_q|)^2}{4a_0^4} + \left( \frac{AQ(1+|\lambda_q|)}{a_0^2} - \frac{P}{a_0(4a_0^2 \tau^2 - 1)} \right) e^{-2a_0|t'|} + \frac{2P \tau}{4a_0^2 \tau^2 - 1} e^{-\frac{|t'|}{\tau}}
\]  \hspace{1cm} (4)

Let \( t = 0 \), the mean intensity relative fluctuation is
\[
C(0) = \frac{A^2 Q^2 (1-|\lambda_q|)^2}{4a_0^4} + \frac{AQ(1+|\lambda_q|)}{a_0^2} - \frac{P}{a_0(4a_0^2 \tau^2 - 1)} + \frac{2P \tau}{4a_0^2 \tau^2 - 1} + \frac{A^2 B^2}{2a_0^2 (4a_0^2 + \Omega^2)}
\]  \hspace{1cm} (5)

Then, translate SSMICF into power spectrum by Fourier transform
\[
S(\omega) = S_1(\omega) + S_2(\omega)
\]  \hspace{1cm} (6)

Where \( S_1(\omega) \) and \( S_2(\omega) \) are output power spectra of the signal and noise, respectively.
\[
S_1(\omega) = \frac{\pi B^2}{(\gamma^2 + \omega^2)^2} \left[ \delta(\omega-\Omega) + \delta(\omega+\Omega) \right]
\]  \hspace{1cm} (7)
\[
S_2(\omega) = \frac{4I_0 Q(1+|\lambda_q|)}{\gamma^2 + \omega^2} - \frac{4I_0^2 P}{(\gamma^2 + \omega^2)(\gamma^2 \tau^2 - 1)} + \frac{4I_0^2 P \tau^2}{(\gamma^2 \tau^2 - 1)(1 + \omega^2 \tau^2)}
\]  \hspace{1cm} (8)

The total output power is
\[
P_s = \int_0^\infty S_1(\omega) \, d\omega
\]  \hspace{1cm} (9)
The SNR is defined as the ratio of the output signal power to the average power of unit noise spectrum at \( \omega = \Omega \) (only the spectrum of \( \omega > 0 \) is kept). We have signal-to-noise ratio

\[
R = \frac{P_S}{S_2(\omega = \Omega)}
\]  

(10)

Let \( S_2(\omega = \Omega) \) and \( P_S = \int_0^\infty S_1(\omega)d\omega \) substitute into Eq. (10)

\[
R = \frac{\pi B^2}{8I_0Q(1+|\hat{\lambda}_q|)+\frac{8I_0^2P}{\Omega^2\tau^2+1}}
\]  

(11)

For the reason of applying the unified colored noise approximation, we will discuss the condition only on the short-time correlation (\( \tau \ll 1 \)). Defining the specific ratio \( K \) of SNR to the fluctuation

\[
K = \frac{R}{C(0)}
\]  

and then putting Eq.(5) and Eq.(11) into, we get

\[
K = \frac{\pi A^2B^2\left(\Omega^2\tau^2+1\right)}{8a_0^4AQ\left(1+|\hat{\lambda}_q|\right)+\frac{P}{a_0^2}\left(4a_0^2\tau^2-1\right)+\frac{2P\tau}{4a_0^2\tau^2-1}+\frac{A^2B^2}{2a_0^2\left(4a_0^2+\Omega^2\right)}}
\]  

(12)

3. Descriptions

By virtue of the Eq. (12), the curves of \( K \) versus different system coefficients are plotted from Fig.1 to Fig.3. In the followings, we will discuss the version of \( K \) in the single mode laser.

Choosing the intensity of pump noise \( P \) and quantum noise \( Q \) as parameters, the curves of \( K \) versus net gain \( a_0 \) are plotted in Fig.1(a) and Fig.1(b). The curve of \( K - a_0 \) exhibits one maximum, single peak, that is optimization once. Then the curve decreases monotonously, after that, tends to be stable with net gain \( a_0 \) increasing. As we know in Eq. (12), the change of parameters have influences on the position and variation of the peak.

In Fig.1(a), the value of peak increases as the intensity of pump noise \( P \) decreasing. When intensity of pump noise \( P \) decreases by the same value, the peak value of the curve doesn’t increase by the same value, but accelerately increase. Meanwhile, the half-height width of the peak also increase and the peak position shifts slightly to the right. Choosing intensity of quantum noise \( Q \) as a parameter, we plot the curves of \( K - a_0 \) in Fig.1(b). With the increase of \( Q \), the peak value of the curve decrease. In the case of \( 0.001 \leq Q \leq 0.002 \), the peak value lowers down quickly. But when \( 0.002 \leq Q \leq 0.005 \),
the peak value decreases slowly. The peak value is almost disappeared at $Q = 0.005$, and the peak position of the curve remains basically unchanged.

Choosing the intensity of pump noise $P$ and frequency of input signal $\Omega$ as parameters, the curves of $K$ versus self-saturation $A$ are plotted in Fig.2. We find that the curve of $K - A$ also exhibits one maximum, single peak. Then the curve decreases monotonously with $A$ increasing. As $A$ increases, the curves tend to the same value with the parameter $P$, but with the parameter $\Omega$ the curves have different values. These can be seen clearly in the space diagram.

In Fig.2(a), the value of peak increases with the intensity of pump noise $P$ decreasing. When intensity of pump noise $P$ decreases by the same value, the peak value of the curve doesn’t increase by the same value. At the same time, the half-height width of the peak also decreases and the peak position shifts slightly to the left. We plot the curve of $K - A$ as the parameter of input frequency $\Omega$ in Fig.2(b). The value of the peak increases with $\Omega$ increasing. When the frequency $\Omega$ of input signal increases by the same value, the peak value of the curve also increases by the same value.
Choosing the intensity of pump noise $P$ and quantum noise $Q$ as parameters, the curves of $K$ versus correlation coefficient $q$ with the real and imaginary part of quantum noise are plotted in Fig.3. Under the given parameters, the curves exhibit maximums all at $q = 0$. Every curve has single peak, and the peak is sharp and distributed symmetrically. With the parameter of $P$ in the Fig.3(a), the value of curve increase equivalently with intensity of pump noise $P$ decreasing by the same value. It is also in the Fig.3(b) as the parameter of intensity of quantum noise $Q$. From these, we can compare that the change in the value of $Q$ which affects the peak value of curve more. These can be seen clearly in the space diagram.
Fig. 3(c) $a_0 = 0.1, \tau = 0.02, A = 1, B = 1, \Omega = 20$

The peak value of curve decreases as the parameter $Q$ increasing in Fig. 3(b). The shape of curves are shown in Fig. 3(c) when $0.045 \leq Q \leq 0.055$. The peak of the curve is not so sharp as that in Fig. 3(b), and the value of peak doesn’t change by the same value.

4. Conclusions
In summary, we discuss the influences on the curves of $K - a_0, K - A, K - \lambda_{ij}$ with different parameters, such as the intensity of pump noise $P$, quantum noise $Q$, input signal frequency $\Omega$. They have different effects especially the maximums existed on the curves. Meanwhile decreasing the intensity of pump noise $P$ and quantum noise $Q$ or increasing the input signal frequency $\Omega$, can all increase the peak value. It is to say the decrease of noise intensity and the increase of input signal frequency can optimize the output properties of the laser communication system.

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