Control of spontaneous emission dynamics in microcavities with chiral exceptional surfaces

Q. Zhong\(^1\), A. Hashemi\(^1\), Ş. K. Özdemir\(^2,3\) and R. El-Ganainy\(^1,4\)

\(^1\)Department of Physics, Michigan Technological University, Houghton, Michigan, 49931, USA
\(^2\) Department of Engineering Science and Mechanics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA
\(^3\) Materials Research Institute, The Pennsylvania State University, University Park, Pennsylvania 16802, USA
\(^4\) Henes Center for Quantum Phenomena, Michigan Technological University, Houghton, Michigan, 49931, USA

We investigate the process of spontaneous emission from a quantum emitter located within the mode volume of a microring resonator that features chiral exceptional points. We show that this configuration offers enough degrees of freedom to exhibit a full control to either enhance or suppress the emission process. Particularly, we demonstrate that the Purcell factor can be enhanced by a factor of two beyond its value in an identical microring operating at a diabolic point. Our conclusions, which are derived using a non-Hermitian Hamiltonian formalism, are confirmed by employing full-wave simulations of realistic photonic structures and materials. These results offer a straightforward route to improve the performance of single photon sources using current photonics technology without the need for building optical resonators with ultra-high quality factors or nanoscale volumes.

Introduction:— Quantum engineering seeks to utilize quantum mechanics to build a new generation of computing machines, encryption schemes, and sensing devices with unprecedented performance in terms of computational power, security strength and sensitivity, among other applications. In the pursuit to achieve these goals, several material platforms provide complementary solutions to overcome various practical hurdles. These include trapped atoms [1], superconducting circuits [2], and photonics [3]. The latter is particularly interesting due to its mature technology and natural interface with current optical communication systems. At the heart of modern quantum optics technology is the ability to control light-matter interaction at the quantum level for various applications such as building non-classical light sources [4], optical transistors [5], and quantum memory [6]. In this regard, efforts have been recently dedicated for building efficient single photon sources that can produce individual photons on demand at high repetition rates [7]. This progress was enabled by engineering various optical resonator geometries that can support small modal volumes and large quality factors in order to tailor the photonic local density of states surrounding quantum emitters, and hence controlling their spontaneous emission (SE) rates as quantified by the Purcell factor (PF) [8] (see [9] for detailed discussions). Examples include planar photonic crystals, vertical Bragg reflectors microdisks and plasmonic structures. For comprehensive reviews and performance comparison, see [10]. Despite these promising results, the aforementioned arrangements are not easy to mass-produce or integrate with other photonics components. An attractive alternative in terms of mass-fabrication and large scale integration is microring resonators [11]. On the downside, however, microring resonators suffer from relatively large mode volumes and limited quality factors [10]. It will be thus of interest to device new route for controlling spontaneous emission in microring resonators and possibly enhance their PF beyond their current performance.

Motivated by this goal, here we study the interaction between light and a quantum emitter in a new family of microring resonators whose design is tailored to operate in the vicinity of or at a special type of non-Hermitian singularities known as chiral exceptional points (for recent reviews on the physics of non-Hermitian optical systems and exceptional points see [13–16]). In...
contrast to recent investigations of SE in optical configurations with EPs arising from inhomogeneous loss distribution [12, 19], which is basically a lossy version of PT symmetric arrangements [20, 24]. Here instead the EP arises via unidirectional coupling between two optical modes in an optical resonator [25, 29]. A robust realization of chiral EPs that relies on a microring resonator evanescently to a waveguide with an end mirror was recently proposed [12, 30, 31]. Here, we will focus on this particular structure. In addition to its robustness against fabrication imperfections [12], this geometry offers several advantages in terms of controlling Purcell factor (by suppressing it completely or enhancing its value by a factor of two compared to microring resonators operating at diabolic points or DPs) and integration with a waveguide channel to collect the emitted photon from a predetermined port.

Formalism:— The optical platform we consider in this study is shown in Fig. 1. It consists of a microring resonator coupled to a waveguide terminated by a mirror at one of its ports. This structure was previously shown to exhibit an exceptional surface and was proposed for optical sensing [12], optical amplifiers [31] and directional absorbers [12]. In these previous studies, the system was externally excited through the waveguide, and the response was studied by monitoring the transmission or reflection spectra under different conditions. Here, we study the emission properties of a quantum emitter (QE) located within the mode volume of the resonator (Fig. 1) by monitoring the emitted optical power collected by the waveguide ports. We assume that QE is driven to its excited state optically or electrically.

In order to simplify the analysis and gain insight into the problem, we will employ a technique based on a non-Hermitian Hamiltonian \( \hat{H} = \hat{H}_R + \hat{H}_E + \hat{H}_I \) (with the sub-indices \( R, E \) and \( I \) referring to radiation, emitter and interaction respectively) and neglect the effect of quantum jumps. The system shown in Fig. 1 can be represented by the Hamiltonian:

\[
\begin{align*}
\hat{H}_R &= \hbar (\omega_o - i\gamma_W)(\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}) + \hbar K a^\dagger b^\dagger, \\
\hat{H}_E &= \hbar \omega_e |e\rangle \langle e|, \\
\hat{H}_I &= \hbar (\omega_{CW} \hat{a}^\dagger + \omega_{CCW} \hat{b}^\dagger) |g\rangle \langle e| + \text{h.c.}
\end{align*}
\]

where \( \hbar \) is Planck’s constant divided by 2\( \pi \), \( \omega_o \) is the resonant frequency, \( \gamma_W \) and \( \gamma_{GW} \) are the loss rate of the waveguide mode and the coupling to the waveguide, respectively, \( \hat{a}^\dagger \) and \( \hat{b}^\dagger \) are the annihilation operators of the clockwise (CW) and counterclockwise (CCW) optical modes, and \( \omega_e \) and \( \omega_{CW,CCW} \) are the frequency of the excited state and the coupling frequencies, respectively. The creation and annihilation operators of the clockwise (CW) and counterclockwise (CCW) modes are denoted by \( \hat{a}^\dagger \) and \( \hat{b}^\dagger \), respectively.

In the radiation Hamiltonian \( \hat{H}_R \), \( \hat{a}^\dagger \) and \( \hat{b}^\dagger \) are the creation and annihilation operators of the clockwise (CW) and counterclockwise (CCW) optical modes, both assumed to have a resonant frequency \( \omega_o \) and loss rate \( \gamma_W \), due to the coupling to the waveguide. The unidirectional coupling from the CW to the CCW mode is denoted by \( \kappa = -2i\gamma_W |r| e^{i\phi_W} \). Here, \( |r| \) is the field reflection amplitude from the mirror, and \( \phi_W = 2\beta L + \phi_r \) is the propagation phase in the waveguide, \( L \) is the distance between the waveguide-ring junction and the mirror, and \( \phi_r \) is the field reflection phase from the mirror (i.e. \( r = |r| e^{i\phi_r} \)). Note that \( \hat{H}_R \) (which can be inferred from Eq. (1) in [12] by elevating the classical field variables into operators) is not Hermitian. In general, the Hamiltonian \( \hat{H}_R \) will exhibit an exceptional point of order two in the single-photon subspace. In the emitter Hamiltonian \( \hat{H}_E \), \( \hbar \omega_e \) denotes the transition frequency between the ground state \( |g\rangle \) and excited state \( |e\rangle \). We assume that the emitter does not radiate efficiently into free space directly, which we will justify later when discussing implementations. In the interaction Hamiltonian \( \hat{H}_I \), \( \omega_{CW,CCW} = \tilde{\mu} \cdot \hat{E}_{CW,CCW}(\vec{r}) \) are the coupling constants between the CW/CCW optical modes and an emitter having an electric dipole moment \( \tilde{\mu} \) and located at \( \vec{r} \), h.c. stands for Hermitian conjugate, and \( \hat{E}_{CW,CCW}(\vec{r}) \) are the normalized electric fields of CW and CCW modes at the location of the emitter \( \vec{r} \). For a microring resonator with traveling modes, the magnitude of the electric field at a particular transverse position does not vary along the angular direction. Instead, the field just acquires a phase. If we chose the ring-waveguide junction as a reference point, we can then write: \( \hat{J}_{CW} = J e^{-i\phi_W} \), and \( \hat{J}_{CCW} = J e^{i\phi_W} \) (see Fig. 2).

Within the single excitation subspace, which is relevant to the spontaneous emission process, the general wavefunction can be written as \( |\psi(t)\rangle = a(t) |1, 0, g\rangle + b(t) |0, 1, g\rangle + c(t) |0, 0, e\rangle \), where the coefficients \( a(t), b(t) \) and \( c(t) \) are the probability amplitudes of finding the excitation either in the CW mode, CCW mode or in the quantum emitter. Importantly, it is straightforward to show that \( [\hat{N}, \hat{H}] = 0 \), where \( \hat{N} = \hat{a} \hat{a}^\dagger + \hat{b} \hat{b}^\dagger + |e\rangle \langle e| \) is a generalized number operator that accounts for the total excitations in the bosonic modes and the quantum emitter. In other words, the Hamiltonian \( \hat{H} \) conserves the number of excitations. However, as a result of its non-Hermitian character, it does not conserve the probability of finding the excitation trapped inside the system, i.e. \( p(t) = |a(t)|^2 + |c(t)|^2 + |b(t)|^2 \leq 1 \). By substituting \( \psi(t) \) in the Schrödinger’s equations \( i\hbar \frac{d\psi(t)}{dt} = \hat{H} \psi(t) \) and projecting on the states \( |1, 0, g\rangle, |0, 1, g\rangle \) and \( |0, 0, e\rangle \), we obtain:

\[
i \frac{d\vec{v}}{dt} = \hat{H} \vec{v}, \quad \hat{H} = \begin{pmatrix}
\omega_o - i\gamma_W & 0 & J e^{-i\phi_W} \\
\kappa & \omega_o - i\gamma_W & J e^{i\phi_W} \\
J e^{i\phi_W} & J e^{i\phi_W} & \omega_e
\end{pmatrix}, \tag{2}
\]

where \( \vec{v} = (a(t), b(t), c(t))^T \), with the superscript \( T \) indicates matrix transpose. Note that, consistent with the expression for \( \hat{H} \), the effective discrete Hamiltonian \( \hat{H} \) is non-Hermitian and exhibits an exceptional surface when \( J = 0 \). In order to study spontaneous emission using Eq. (2), we consider the initial condition \( a(0) = b(0) = 0 \) and \( c(0) = 1 \), i.e. the emitter is initially in the excited state and the optical modes are in the vacuum state.

Results:— In order to ensure that the system operates in the vicinity of the EP, we consider the weak coupling regime \( J \ll \gamma_W \) with negligible Markovian effects. In other words, a photon emitted from the QE into the photonic mode will escape quickly to the waveguide environment before it is able to couple back to the QE.
think of $A$ and $B$ as the steady state field amplitudes of the CW and CCW modes under excitation by a driven classical dipole antenna. The output signal from ports 1 and 2 will be thus $s_{EP}^{(1)} = -\sqrt{2}\gamma_W |A| e^{i\phi_W} + B$ and $s_{EP}^{(2)} = -\sqrt{2}\gamma_W (|t| e^{i(\phi + \phi_W)}) A$, where $\phi_i$ is the phase associated with the field transmission coefficient $t$ from the mirror, i.e. $t = |t| e^{i\phi_W}$. In the following, we define $\eta = P_E/P_{DP}$ as the Purcell factor enhancement, which quantifies the change in the Purcell factor when the waveguide-coupled ring resonator shown in Fig. 1 is operated at an EP compared to its operation at a DP:

$$\eta \equiv \frac{P_E}{P_{DP}} = \frac{|\chi + |r| e^{i\Delta \phi}|^2 + |t|^2}{2}, \quad (5)$$

where $P_E$ and $P_{DP}$ are the total optical powers emitted in the waveguide ports for the EP and DP cases, i.e. $P_{EP,DP} = |s_{EP,DP}^{(1)}|^2 + |s_{EP,DP}^{(2)}|^2$ with $P_{DP}|_{r=0} = \frac{4\gamma_W J^2}{\Delta + i\gamma_W}$. In Eq. (4), $\Delta \phi = 2\theta_E - \phi_W$ and $\chi \equiv \frac{e^{-i\phi_W}}{\Delta + i\gamma_W} = \frac{e^{-|r| \gamma_W}}{\Delta + i\gamma_W}$. When $r = 0$ and $t = 1$, the system operates at a DP and hence, we have $\eta = 1$, as expected. In the limit of $r = 1$ (i.e. perfect mirror) the system is at the EP and for $\Delta = 0$ we have resonant emission with $\chi = -2$. Under these conditions, the enhancement $\eta$ is given by $\eta = 1 - \cos \Delta \phi$, which attains its maximum value $\eta_{\text{max}} = 2$ at $\Delta \phi = (2m + 1)\pi$ for integer $m$. On the other hand, for $\Delta \phi = (2m \pm 1/2)\pi$, we obtain $\eta = 1$, i.e. equivalent to the DP case at the same resonant frequency condition. Interestingly for $\Delta \phi = 2m\pi$, we find that $\eta$ attains its minimum value of $\eta_{\text{min}} = 0$. This analysis shows that the platform shown in Fig. 1 provides enough degrees of freedom to tune the enhancement factor $\eta$ across its two limits $\eta_{\text{min}} = 0$ and $\eta_{\text{max}} = 2$. In other words, we can tune the system between suppressed SE regime to enhanced SE regime where SE enhancement at the EP can be as high as twice the enhancement at a DP, where conventional systems operate. This is also evidenced in Fig. 2 which depicts $\eta$ as a function of both $|r|$ and $\Delta \phi$. An interesting feature of Eq. (1) becomes evident by writing $B = \frac{J e^{i\phi_W}}{(\Delta + i\gamma_W)^2} \times (\Delta + i\gamma_W - 2|t| e^{-i\Delta \phi})$. When $\cos \Delta \phi = 1/(2|t|)$ and $\Delta = \pm \gamma_W \sqrt{|t|^2 - 1}$ (the sign depends on the value of $\Delta \phi$) with $|r| \geq 0.5$, we have $B = 0$, and $\eta = 0.5$. In this case, the spontaneous emission rate is suppressed compared to that associated with the DP, but the emission becomes chiral with the photon emitted only in the CW mode. This feature, which arises due to destructive interference between the CCW mode and the back-reflected wave in the waveguide inside the ring, has been recently observed experimentally in PT-symmetric microwave and acoustic setups. 

Next, we confirm the above predictions by performing 2D full-wave finite-difference time-domain (FDTD) simulations using realistic structure dimensions and material systems. In our simulations, the microring

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**FIG. 2.** Plot of the enhancement of PF, $\eta$, as a function of the mirror field reflectivity amplitude $|r|$ and $\Delta \phi$ (characterizing the change in position of the mirror or the QE) under the resonant condition $\Delta = 0$. The absorption of the mirror in the above plot is assumed to be zero. The maximum value for the enhancement is $\eta = 2$ which occurs at $\Delta \phi = \pi$. For more realistic mirrors with a finite absorption coefficient, the peak enhancement will be less than its maximum possible value attainable in the ideal case.
FIG. 3. Plots of (a) The value of PF enhancement, $\eta$, as a function of the normalized frequency detuning $\Delta \gamma W$ for $\Delta \phi = \pi$, and $|r| = 0.976$; and (b) $\eta$ as a function of the mirror (or QE) position as parametrized by $\Delta \phi$, under resonant condition $\Delta = 0$. In producing this plot, we used $|r| = 0.976$ as obtained for our realistic implementation of the mirror. Red solid line represents the results obtained by using Eq. 5 together with the optical parameters of the device, obtained as outlined in the Appendix. Black dots represent the results obtained by full-wave simulations.

The resonator has a refractive index of $n_r = 3.47$ embedded in a background with $n_b = 1.44$. The outer radius of the microring is taken to be $R = 5 \mu$m and its width is $w = 0.25 \mu$m. The edge-to-edge separation between the ring and the waveguide is $d = 0.2 \mu$m and its width is identical to that of the ring waveguide. The mirror at end of the waveguide is made of a 100-nm-thick silver layer. These design parameters lead to the following optical properties for the TE optical modes: effective refractive index $n_{eff} = 2.93$, $\omega_0 = 1216$ THz or equivalently $\lambda_0 = 1549$ nm, $\gamma_W = 124$ GHz, corresponding to $Q = 4900$, and $|r| = 0.976$. Due to the optical absorption of silver, we find $|t| = 0.008$. The numerical evaluation of the above parameters is presented in detail in the Appendix. The QE in our simulations is located as shown in Fig. 1, i.e. at one quarter of the perimeter as measured from the ring-waveguide junction in the clockwise direction. Its dipole moment tensor is assumed to have a component perpendicular to the ring plane and thus couples only to the TE optical mode. In the FDTD simulations, the QE is modeled by using a classical dipole (for more on the analogy between quantum and classical emitters, see [34, 35]).

Figure 3 shows $\eta$ as a function of $\Delta/\gamma_W$ for the optimal mirror position that maximizes the Purcell factor with $|r| = 0.976$ (see Appendix for more details about finding the mirror position) as obtained using Eq. 5 in conjunction with the above parameters (denoted as Theory in Fig. 3), as well as by directly calculating the output optical power in the waveguide ports from the FDTD simulations (with and without the mirror) and using the definition of $\eta$ (denoted as Simulation in Fig. 3). Evidently good agreement is observed especially at the resonant frequency. Due to the finite reflectivity of the mirror, the maximum enhancement here is $\eta = 1.95$.

Next, we plot the values of $\eta$ as a function of $\Delta \phi$ by scanning the mirror position around its optimal value. Particularly, we varied the mirror position in the range $440 \text{ nm}$ with a step $20 \text{ nm}$. Again, we observe good agreement between theory and numerical simulations.

In order to gain insight into these results, we investigate the electric field distribution associated with various mirror positions as shown in Fig. 4, focusing on the cases when $\Delta = \pi$ (optimal mirror position) and $\Delta = 0$ (trapping condition). In our simulations, these values correspond to $L = 5050 \text{ nm}$ and $L = 5180 \text{ nm}$, respectively. From Fig. 4(a), we observe that the field in the left port of the waveguide is uniform, indicating...
an escaping traveling wave as expected. The field inside the ring forms an imperfect standing wave pattern (the amplitude of the CCW component three times larger than that of the CW component) having a peak at the QE location, which explains the enhancement of the Purcell factor. On the other hand, the field distribution in Fig. 4(b) demonstrates a perfect standing wave inside the ring with a node located at the QE position. As a result, the photonic mode effectively decouples from the QE, leading to near zero PF. In this latter scenario, the microring and the right section of the waveguide with its end mirror form a resonator that traps the excitation, with the lifetime determined by the mirror reflectivity and the radiation rate from the ring to free space.

Conclusion:— In summary, we have investigated the interaction between light and a quantum emitter in microring optical resonator exhibiting a chiral EP. Our analysis based on the non-Hermitian Hamiltonian approach shows that in the weak coupling regime, the presence of an EP can enhance the Purcell factor by a factor of two. Furthermore, implementing the chiral EP by side coupling the resonator to a waveguide terminated by a mirror can offer enough degrees of freedom to even suppress the spontaneous emission process significantly. These conclusions are confirmed by using full wave FDTD simulations. Our results open the door for designing more efficient single photon sources and lead to even more interesting questions about the prospect of using these setups to enhance photon-photon interactions.

Appendix: Numerical evaluation of the optical parameters of the structure

In the main text, we have compared the results obtained by using FDTD to those estimated using the coupled mode theory (CMT) as represented by the Hamiltonian $\hat{H}$. To use this later approach, however, one must evaluate the essential optical parameters characterizing the system, such as the coupling between the microring resonator and the waveguide (together with the related optical loss from the ring to the waveguide port), the mirror complex reflectivity as well as its transmission (and hence its optical absorption). In our work, these parameters were computed numerically using FDTD. Here we outline the details of these calculations.

In order to extract the decay rate from the microring to the waveguide ($\gamma_W$), we consider an add-drop ring resonator filter geometry as shown in Fig. 5(a). The amplitude of the power transmission coefficient as a function of the input wave frequency is then obtained using FDTD (Fig. 5(b)). By using the relation $\text{FWHM} = 4\gamma_W$, we estimate that, in our case, $\gamma_W = 124$ GHz, which corresponds to a quality factor of $Q=4900$ for the all-pass structure.

Next, we calculate the optical parameters of the mirror by removing the ring resonator altogether and using FDTD simulations to evaluate the field reflection and transmission coefficient from the mirror. In these simulations, an incident optical mode having a free space wavelength of $\lambda_o = 1549$ nm is launched into the waveguide from the left. The absolute values of the reflection and transmission coefficient are easily obtained by numerically measuring the reflected and transmitted optical powers, respectively. Doing so gives $|r| = 0.976$ and $|t| = 0.008$. To evaluate the phase of the reflection coefficient, we plot the absolute value of the steady state electric field $|E|$ distributed at the incident wave side as shown in Fig. 6, and use curve fitting based on the analytical expression:

![Image](image_url)
where here \(|E_0|\) is the incident wave amplitude, \(|r|\) is the field amplitude reflection coefficient, \(L = 3.95 \, \text{µm}\) is the distance between the mirror and the waveguide-ring junction, \(\phi_r\) is the reflection phase, and \(\beta = \frac{2\pi m}{\lambda} = 11.891 \, \text{µm}^{-1}\). Based on these values, we find that \(\phi_r = 3.56\) gives the best fit between Eq. (A.1) and the data in Fig. 6. However, in order to obtain the best match between CMT and FDTD for the full structure, we used \(\phi_r = 3.61\), which is an accepted discrepancy given the approximate nature of CMT.

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