New Strategy for $B_s$ Branching Ratio Measurements
and the Search for New Physics in $B_s^0 \rightarrow \mu^+ \mu^-$

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Abstract

The LHCb experiment at CERN’s Large Hadron Collider will soon allow us to enter a new era in the exploration of $B_s$ decays. A particularly promising channel for the search of “new physics” is $B_s^0 \rightarrow \mu^+ \mu^-$. The systematic key uncertainty affecting the measurement of this – and in fact all $B_s$-decay branching ratios – is the ratio of fragmentation functions $f_d/f_s$. As the presently available methods for determining $f_d/f_s$ are not sufficient to meet the high precision at LHCb, we propose a new strategy using $\bar{B}_s^0 \rightarrow D^+ \pi^-$ and $\bar{B}_d^0 \rightarrow D^+ K^-$. It allows us to obtain a lower experimental bound on $\text{BR}(B_s^0 \rightarrow \mu^+ \mu^-)$ which offers a powerful probe for new physics. In order to go beyond this bound and to determine $f_d/f_s$ with a theoretical precision matching the experimental one it is sufficient to know the $SU(3)$-breaking correction to a form-factor ratio from nonperturbative methods at the level of 20%. Thanks to our strategy, we can detect new physics in $B_s^0 \rightarrow \mu^+ \mu^-$ at LHCb with $5\sigma$ for a branching ratio as small as twice the Standard-Model value, which represents an improvement of the new-physics reach by about a factor of 2 with respect to the current LHCb expectation.
New Strategy for $B_s$ Branching Ratio Measurements and the Search for New Physics in $B_s^0 \to \mu^+\mu^-$

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(Dated: April 22, 2010; published 31 August 2010)

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Keywords: $B_s$ decays, fragmentation function, new physics

I. INTRODUCTION

In this decade, we will enter a new round in the precision testing of the flavor sector of the Standard Model (SM) through $B$-meson decays. Currently the LHCb experiment at CERN’s Large Hadron Collider (LHC) is starting its first physics run. After pioneering results on the $B_s$ system by the CDF and DØ collaborations at the Tevatron, LHCb will allow us to explore this still largely unexplored territory of the flavor-physics landscape [1].

In this respect, one of the most promising channels for detecting signals of “new physics” (NP) is the rare decay $B_s^0 \to \mu^+\mu^-$, which originates in the SM from “penguin” and box topologies, i.e. quantum loop processes. The corresponding branching ratio is predicted as follows [2]:

$$\text{BR}(B_s^0 \to \mu^+\mu^-)|_{\text{SM}} = (3.6 \pm 0.4) \times 10^{-9},$$  \hspace{1cm} (1)

where the error is fully dominated by a nonperturbative “bag parameter” coming from lattice QCD. As is well known, this observable may be significantly enhanced through NP (for a review, see Ref. [2]). The present upper bounds from the CDF and DØ collaborations are still about 1 order of magnitude away from (1) and read as $4.3 \times 10^{-8}$ [3] and $5.3 \times 10^{-8}$ (95% C.L.) [4], respectively.

At LHCb, the extraction of $\text{BR}(B_s^0 \to \mu^+\mu^-)$ will rely on normalization channels such as $B^+_s \to J/\psi K^+$, $B^0_s \to K^+\pi^-$ and/or $B^0_d \to J/\psi K^{*0}$ in the following way:

$$\text{BR}(B_s^0 \to \mu^+\mu^-) = \text{BR}(B_q \to X) \frac{f_q}{f_s} \epsilon_{\mu\mu} N_{\mu\mu} \frac{N_X}{N_X},$$ \hspace{1cm} (2)

where the $\epsilon$ factors are total detector efficiencies and the $N$ factors denote the observed numbers of events. The $f_q$ are fragmentation functions, which describe the probability that a $b$ quark will fragment in a $B_q$ meson ($q \in \{u, d, s\}$). In [2], $f_q/f_s$ is actually the major source of the systematic uncertainty, thereby limiting the ability to detect a $5\sigma$ deviation from the SM at LHCb to $\text{BR}(B_s^0 \to \mu^+\mu^-) > 11 \times 10^{-9}$ [1].

In this estimate, the current experimental knowledge of $f_d/f_s$ was assumed, which we summarize in Section II. In Section III, we propose a new strategy to measure $f_d/f_s$ at LHCb. Its experimental prospects and theoretical limitations are discussed in Sections IV and V respectively. In Section VI, we discuss the implications for the search for NP with $B_s^0 \to \mu^+\mu^-$ branching ratio, while we summarize our conclusions in Section VII.

II. EXPERIMENTAL STATUS OF $f_d/f_s$

The CDF collaboration has estimated the ratio of fragmentation functions through semi-inclusive $B \to D^f \ell^- \nu_\ell X$ decays [5]. The reconstructed $D^f$ signal yields are then related to the number of produced $b$ hadrons by assuming the $SU(3)$ flavor symmetry and neglecting $SU(3)$-breaking corrections (e.g. assuming $\Gamma(B^0_s \to \ell^- \nu_\ell D^+) = \Gamma(B^0_s \to \ell^- \nu_\ell D^+_s)$). Together with an earlier result using double semileptonic decays (containing two muons and either a $K^*$ or a $\phi$-meson) [6] the average value $f_d/(f_d + f_s) = 0.142 \pm 0.019$ is obtained [7].

An alternative approach uses the different mixing probabilities for $B^0_s$ and $B^0_d$ mesons. Despite a 1.8$\sigma$ discrepancy in the time-integrated mixing probability between the LEP and Tevatron data, an average value of $f_s = 0.119 \pm 0.019$ was determined with this method [8].

The CLEO and Belle collaborations have extracted the fraction $f_s$ of $B^+_s/B^+_d$ events among all $b\bar{b}$ events at the $\Upsilon(5S)$ resonance from inclusive $\Upsilon(5S) \to D_s X, \phi X$ de-
BR(Υ(5S) → D_sX,φX) = 2f_sBR(B^0_s → D_sX,φX)
+ (1 − f_s)BR(Υ(4S) → D_sX,φX) \quad (3)

is assumed with BR(B^0_s → D_sX) = (92 ± 11)%, which relies on a variety of assumptions and yields the model-dependent result \( f_s = 0.194 \pm 0.011 \text{(stat)} \pm 0.027 \text{(sys)} \).

It is evident that the fragmentation functions depend on the environment, which becomes apparent when an attempt is made to compare the numerical values for \( f_s \). At the B factories \( f_d + f_s + f_e = 1 \), whereas at hadron colliders the available energy allows the \( b \)-quark to fragment into baryons as well. In addition beam-remnant effects at hadron colliders might affect the \( b \)-hadron fractions depending on \( p_T \) and/or pseudo-rapidity. Consequently, each experiment – and in particular LHCb – should calibrate its own value for this quantity. As a result LHCb cannot directly use the value measured at Tevatron or at LEP. The fragmentation function is not only the major limiting parameter for the determination of \( \text{BR}(B^0_s \rightarrow \mu^+\mu^-) \) at LHCb, but in fact for all \( B_s \)-decay branching ratio measurements at the LHC, the Tevatron, and an \( e^+e^-B \) factory running at \( \text{Υ}(5S) \). As a result, the usage of \( B_s \) modes as normalization channels, obtained from the KEKB runs at \( \text{Υ}(5S) \), also suffer from an imprecise value of \( f_s \), in addition to a large statistical uncertainty [11].

By normalizing the \( B^0_s \rightarrow \mu^+\mu^- \) decay directly to another \( B_s \) decay, the ratio of the fragmentation functions in Eq. (2) would trivially disappear. However, at present the best directly measured \( B_s \) branching ratio is \( \text{BR}(B^0_s \rightarrow D_s\pi) = 3.67^{+0.33}_{-0.33} \text{(stat)}^{+0.43}_{-0.42} \pm 0.49(f_s) \) [11], determined with 23.6 fb\(^{-1} \) of data at \( \text{Υ}(5S) \). Methods are being considered to improve the present knowledge of \( f_s \) at the B factories [12]. However even considering these possible improvements it is unlikely that they will be sufficient to match the required precision of LHCb. A total uncertainty of about 12% could be expected for a sample of 120 fb\(^{-1} \) (corresponding to the total available statistics) and assuming these additional improvements in the determination of \( f_s \) [13]. Moreover the decay \( B^0_s \rightarrow D_s^+\pi^- \) poses experimental difficulties when used as normalization channel for \( B^0_s \rightarrow \mu^+\mu^- \), due to the very different decay topology (hadronic final state, number of tracks, flight distance of the \( D_s \), etc.). A sizable contribution to the uncertainty in the branching ratio estimation due to the ratio of the efficiencies in Eq. (2) must thus be considered. An alternative \( B_s \) decay channel for the direct normalization would be \( B_s \rightarrow J/\psi\phi \), which is however affected by a statistical error twice as large compared to \( B^0_s \rightarrow D_s^+\pi^- \). Assuming the full statistics presently available at the B factories of 120 fb\(^{-1} \), in combination with the possible improvements in the determination of \( f_s \), at best a total relative error of 15% could be expected for this decay.

III. A NEW STRATEGY FOR LHCb

In view of the unsatisfactory situation described in the previous section, we propose a new method for extracting \( f_d/f_s \) at LHCb. The starting point is the following simple expression:

\[
\frac{N_s}{N_d} = \frac{f_s}{f_d} \times \frac{\epsilon(B_d \rightarrow X_2)}{\epsilon(B_d \rightarrow X_1)} \times \frac{\text{BR}(B_s \rightarrow X_1)}{\text{BR}(B_d \rightarrow X_2)} \quad (4)
\]

knowing the ratio of the branching ratios, we could obviously extract \( f_d/f_s \) experimentally. In order to implement this feature in practice, the \( B_s \rightarrow X_1 \) and \( B_s \rightarrow X_2 \) decays have to satisfy the following three requirements:

(1) the ratio of their branching ratios must be easy to measure at LHCb;

(2) the decays must be robust with respect to the impact of NP contributions;

(3) the ratio of their branching ratios must be theoretically well understood within the SM.

At first sight, an obvious choice seems to use semileptonic decays such as \( \bar{B} \rightarrow D^+\mu^-\bar{\nu} \). However, the measurement of such channels at hadron colliders is experimentally challenging since the fully reconstructed \( B \)-mass is not available and various sources of muons in the background have to be controlled. Therefore, we have to focus at non-leptonic decays, where requirement (1) implies to look at decays into charged particles and requirement (2) narrows down the search to channels without penguin contributions, which are flavor-changing neutral-current processes which might well be affected by NP contributions. The third requirement finally guides us to the decays \( B^0_s \rightarrow D_s^+\pi^- \) and \( B^0_s \rightarrow D^+K^- \).

As can be seen in Fig. 1, these channels receive only contributions from color-allowed tree-diagram-like topologies and are related to each other through the interchange of all down and strange quarks, i.e. through the \( U \)-spin subgroup of the \( SU(3) \) flavor symmetry. Moreover, the concept of “factorization” [14] is expected to work well in these transitions. This was expected from “color transparency” already two decades ago [15] [16], while this feature could actually be put on a rigorous

\[ \text{FIG. 1: The } B_s^0 \rightarrow D^+K^- \text{ and } B_s^0 \rightarrow D_s^+\pi^- \text{ decay topologies.} \]
theoretical basis in the heavy-quark limit \cite{17} \cite{18}. Consequently, using these decays, we can calculate the corresponding ratio of their branching ratios entering Eq. (4) up to small, nonfactorizable, $U$-spin-breaking corrections. This feature will be discussed in more detail in Section V.

Let us note that in contrast to the $B^0 \to D^+\pi^-$ mode usually considered in the literature in the context with factorization, the decays in Fig. 1 have the advantage of not receiving additional contributions from “exchange” topologies, which are expected to be small but are not factorizable. Moreover, thanks to the absence of “penguin” topologies, the situation concerning factorization is also much more favourable than in $B \to \pi\pi, \pi K$ decays.

Applying a notation similar to that of Ref. \cite{17}, we write the branching ratios of the decays at hand as

$$\text{BR}(B_q^0 \to D_q^+ P^-) = \frac{G_F^2 (m_{B_q}^2 - m_{D_q}^2)^2 |\bar{q}^s B_q|^2}{16\pi m_{B_q}^2} \times |V_{qs}^* V_{cb}|^2 \left[ f_P f_Q^{(q)}(m_{P}^2) \right]^2 |a_1(D_q P)|^2 \quad (5)$$

with $P = K$ and $\pi$ for $q = d$ and $s$, respectively. Here $G_F$ is Fermi’s constant, the $m$ factors denote meson masses, $\bar{q}$ is the momentum of the final-state $D_q$ and $P$ mesons in the rest frame of the $B_q^0$ meson, $\tau_{B_q}$ is the lifetime of the $B_q^0$ meson, $V_{qs}$ and $V_{ub}$ for $q = d$ and $s$, respectively, contains the relevant elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix, $f_P$ is $P$-meson decay constant, and the form factor $F_Q^{(q)}$ enters the parametrization of the $(D_q^0 | \bar{c} \gamma^\mu b | B_q^0)_s$ matrix element. The quantity $a_1(D_q P)$ describes the deviation from naive factorization. As discussed in detail in Ref. \cite{17}, this parameter is found in “QCD factorization” (QCDF) as a quasi-universal quantity $|a_1| \approx 1.05$ with very small process-dependent nonfactorizable corrections.

We would like to propose to measure the ratio of the $B_d^0 \to D_s^+ \pi^-$ and $B_d^0 \to D^+ K^-$ branching ratios to determine $f_d/f_s$. Neglecting, for simplicity, kinematical mass factors, we have

$$\frac{\text{BR}(B_d^0 \to D_s^+ \pi^-)}{\text{BR}(B_d^0 \to D^+ K^-)} \sim \frac{V_{us}}{V_{ub}} \left[ \frac{\tau_{B_d}}{\tau_{B_s}} \left| V_{ud} \right|^2 \right] \times \left( \frac{f_\pi}{f_K} \right)^2 \frac{\left| F_0^{(s)}(m_{D_s}^2) \right|^2}{\left| F_0^{(d)}(m_{D_d}^2) \right|^2} \frac{|a_1(D_s \pi)|^2}{|a_1(D_d K)|^2} \quad (6)$$

On the other hand, the ratio of the corresponding number of signal events observed in the experiment is given by

$$\frac{N_{D_s \pi}}{N_{D_d K}} = \frac{f_s \epsilon_{D_s \pi}}{f_d \epsilon_{D_d K}} \frac{\text{BR}(B_d^0 \to D_s^+ \pi^-)}{\text{BR}(B_d^0 \to D^+ K^-)} \quad (7)$$

where the $\epsilon$ are again total detector efficiencies. Using \cite{15}, we hence obtain

$$\frac{f_d}{f_s} = 12.88 \times \frac{\tau_{B_d}}{\tau_{B_s}} \left[ \frac{N_0 N_F}{N_{D_s \pi} \epsilon_{D_d K}} \right] \quad (8)$$

with

$$N_a \equiv \left| \frac{a_1(D_s \pi)}{a_1(D_d K)} \right|^2, \quad N_F \equiv \left| \frac{F_0^{(s)}(m_{D_s}^2)}{F_0^{(d)}(m_{D_d}^2)} \right|^2 \quad (9)$$

Let us next first explore the experimental feasibility at LHCb before having a closer look at the theoretical limitations of our new strategy for extracting $f_d/f_s$.

IV. EXPERIMENTAL PROSPECTS AT LHCB

At LHCb, both the $B_s^0 \to D_s^+ \pi^-$ and $B_d^0 \to D^+ K^-$ decay channels can be exclusively reconstructed using the $D^+ \to K^- \pi^+ \pi^+$ and $D_s^+ \to K^+ K^- \pi^+$ final states. An expected $B$-mass resolution of 18 MeV and excellent particle identification capabilities will allow LHCb to select and reconstruct a clean sample of these decays. Since both channels are selected with an identical flavor final state containing the four charged hadrons $K K \pi \pi$, the uncertainty on $\epsilon_{D_s \pi}/\epsilon_{D_d K}$ is expected to be small.

We estimated the corresponding statistical uncertainty on $r \equiv \epsilon_{D_s \pi} N_{D_d K}/(\epsilon_{D_d K} N_{D_s \pi})$ with a toy Monte Carlo, generating a sample equivalent to 0.2 fb$^{-1}$. This is the expected integrated luminosity at the end of 2010, taking a lower $b \bar{b}$ cross section of 250 µb due to the reduced LHC beam energy of 3.5 TeV into account. Following the estimates from full simulation \cite{19}, and assuming a total trigger efficiency of 30% \cite{20}, we expect to select 5500 $B_s^0 \to D_s^+ \pi^-$ and 1100 $B_d^0 \to D^+ K^-$ events, with a background of approximately 6600 $B^0 \to D^+ \pi^-$ events, where one of the three pions is misidentified as a kaon (assuming a 5% probability to mis-identify a pion as a kaon). Combinatorial background from inclusive $b \bar{b}$ events is expected to yield 6000 events inside a mass window $5220 < m < 5420$ MeV around the $B$-mass. We expect a precision of 7.5% on $r$, where the dominant uncertainty originates from $\text{BR}(D_s \to K^+ K^- \pi) = (5.5\% \pm 0.28\%$). With an integrated luminosity of 1 fb$^{-1}$ as expected at the end of 2011, the statistical uncertainty becomes negligible, thereby reducing the total uncertainty to $\sim 5.6\%$.

The ratio $f_d/f_s$ is not only crucial for the precise determination of $\text{BR}(B_d^0 \to \mu^+ \mu^-)$ but actually for the measurement of any $B_s$ branching ratio. Similarly, the general purpose LHC experiments ATLAS and CMS rely on a precise value of $f_d/f_s$ for the determination of $\text{BR}(B_s^0 \to \mu^+ \mu^-)$. Unfortunately our proposed hadronic decays are not ideal for these experiments due to trigger and particle identification requirements. However, we advocate to apply the value of $f_d/f_s$ as determined by LHCb also at ATLAS and CMS, once the dependence of $f_d/f_s$ on $p_T$ and/or rapidity is measured to be small.

V. THEORETICAL LIMITATIONS

In the extraction of $f_d/f_s$ through \cite{8}, we have theoretical uncertainties related to $U$-spin-breaking effects in
\[ N_a \approx 1 + 2\Re(a_1^{\text{NF}}(D_s \pi) - a_1^{\text{NF}}(D_d K)), \]

where the \( a_1^{\text{NF}} \) describe the nonuniversal, i.e. process-dependent, nonfactorizable contributions to the decays at hand. These contributions cannot be calculated reliably. However, they arise as power corrections to the heavy-quark limit, i.e. they are suppressed by at least one power of \( \Lambda_{\text{QCD}}/m_b \), and are \( \cdots \) the decays at hand \( \cdots \) numerically expected at the few percent level [17]. Let us note that within QCD, also final-state interaction effects such as \( B^0_d \rightarrow [D^0 K^0] \rightarrow D^+ K^- \) arise only as nonfactorizable \( \Lambda_{\text{QCD}}/m_b \) corrections.

The nonfactorizable terms can actually be probed through the differential rate of the semileptonic \( B^0 \rightarrow D^\pm \ell^\mp \bar{\nu}_\ell \) which yields the following expression [17]:

\[
\frac{\text{d} \Gamma(B^0_q \rightarrow D^+_q P^-)_{\text{b}}}{\text{d}t(B^0_q \rightarrow D^+_q \ell^- \bar{\nu}_\ell)/dt^{2}_{q^2=m^2_{\ell \bar{\nu}}}} = 6\pi^2 |V_{q1}^2 f_{B^0_q}^2 a_1(D_q P)^2 X_F, \]

where \( X_F \) deviates from 1 below the percent level. Replacing the pseudoscalar mesons \( P \) by their vector-meson counterparts, i.e. \( K^- \rightarrow K^{*-} \) and \( \pi^- \rightarrow \rho^- \), the corresponding \( X_V \) would be exactly given by 1. However, these modes are more challenging for LHCb.

The current experimental value \( \text{BR}(B^0_d \rightarrow D^+ K^-) = (2.0 \pm 0.6) \times 10^{-4} \) [7] agrees well with the number in Ref. [17], although the uncertainty is still too large to probe the nonfactorizable effects. This will be feasible at LHCb by combining the measurement of the \( B^0_d \rightarrow D^+ K^- \) branching ratio described above with measurements of the differential semileptonic \( B^0 \rightarrow D^\ell \ell^- \bar{\nu}_\ell \) rate at \( q^2 = M_{K^0}^2 \) by the \( B \)-factory experiments BaBar and Belle.

It is interesting to note that factorization was already tested in a similar setting at the \( B \) factories, measuring the branching ratios and the \( D(\ell) \) polarization in the \( B^0 \rightarrow D^\ell \ell^- \bar{\nu}_\ell \) decays. Good agreement was found between factorization predictions and the experimental results within the current errors [21, 22].

The deviation of (10) from 1 is actually not only suppressed by \( \Lambda_{\text{QCD}}/m_b \) but also through the feature that this is a \( U \)-spin-breaking difference. In this context, it should again be emphasized that any decay topology contributing to \( B^0_s \rightarrow D^+ K^- \), even the most complicated rescattering topology, has a counterpart in \( B^0_s \rightarrow D^+_s \pi^- \), which is related to the \( B_d \) case through the interchange of all down and strange quarks. Consequently, taking all these considerations into account, we eventually conclude that \( 1 - N_a \) is conservatively expected to be at most a few percent.

The major uncertainty affecting (3) is hence the form-factor ratio \( \mathcal{N}_F \), where \( U \)-spin-breaking corrections arise from \( d \) and \( s \) spectator-quark effects, which were neglected in previous determinations of \( f_s \) [2]. Making the same approximation, we would simply have \( \mathcal{N}_F = 1 \). Unfortunately, the \( B_s \rightarrow D_s \) form factors have so far received only small theoretical attention. In Ref. [23], such effects were explored using heavy-meson chiral perturbation theory, while QCD sum-rule techniques were applied in Ref. [24]. The numerical value given in the latter paper yields \( \mathcal{N}_F = 1.3 \pm 0.1 \).

Interestingly, we can obtain a lower bound on the \( B^0 \rightarrow \mu^+ \mu^- \) branching ratio from our strategy. Using (2) and (3) and assuming \( N_a = 1 \) yields

\[
\text{BR}(B^0_s \rightarrow \mu^+ \mu^-) > \text{BR}(B^0_s \rightarrow \mu^+ \mu^-)_{\text{b}},
\]

which offers an interesting tool for the detection of possible NP contributions to \( B^0_s \rightarrow \mu^+ \mu^- \) at LHCb.

Assuming that we will measure \( \text{BR}(B^0_s \rightarrow \mu^+ \mu^-)_{\text{b}} \), to be 5\% above the SM prediction (1), \( U \)-spin-breaking effects could only enhance the measured branching ratio and could not move it down towards the SM value.

In the long run, we would of course like to measure \( \text{BR}(B^0_s \rightarrow \mu^+ \mu^-) \) as accurately as possible. In order to match the experimental precision for \( r \) of about 5\% discussed above, it is sufficient to know the \( U \)-spin-breaking corrections to the form-factor ratio \( F_0^{(6)}(m_{\ell \bar{\nu}}^2)/F_0^{(d)}(m_{K}^2) \) from nonperturbative calculations, such as lattice QCD, at the level of 20\%. This looks feasible to us, in particular in view of the tremendous amount of work that was invested to study \( B \rightarrow D \) form factors on the lattice for the extraction of \( |V_{cb}| \) from semi-leptonic \( B \rightarrow D \ell \bar{\nu}_\ell \) decays. We are not aware of any lattice calculation of the \( SU(3) \)-breaking corrections to the form-factor ratio entering our strategy; which is due to the fact that such analyses did so far not appear phenomenologically interesting.

Finally, we would like to note that the \( SU(3) \)-breaking effects in Eq. (6) coming from the ratios of decay constants and form factors tend to cancel each other. Assuming \( \mathcal{N}_F = 1.3 \) with \( f_\pi/f_K = 0.8 \), we get an overall \( SU(3) \)-breaking correction to the form-factor ratios of about 10\%, which is surprisingly small and suggests that also the \( SU(3) \) suppression of \( 1 - N_a \) is very efficient.

VI. IMPLICATIONS FOR THE NP REACH IN THE MEASUREMENT OF \( \text{BR}(B_s \rightarrow \mu^+ \mu^-) \)

In Fig. 2 we illustrate the NP discovery potential in \( B^0_s \rightarrow \mu^+ \mu^- \) at LHCb through our method. We show
contours corresponding to a 5σ NP signal with respect to [1] for the bound in [13] and the extracted value of the branching ratio. Here we have assumed that the uncertainty on BR(D_s → K^+K^−π^-) in the determination of f_d/f_s is distributed Gaussian, and likewise for the uncertainty on BR(B_d^0 → J/ψK^+) in the extraction of BR(B_s^0 → μ^+μ^-). We conservatively varied N_F ∈ [1.2, 1.4], resulting in a negligible change to the predicted sensitivity. Similarly, a variation of N_u ∈ [0.97, 1.03] does essentially not affect the contour.

As can be seen in the plot on the right-hand side in Fig. 2 the resulting NP discovery potential is about twice as large as the present LHCb expectation [1] (upper horizontal line) enabling a possible discovery of NP down to BR(B_d^0 → μ^+μ^-) > 6 × 10^-9 (lower horizontal line). In addition to the increased sensitivity in the regime of low branching ratios, even for large values close to the current CDF exclusion limit the possibility of a significant NP discovery would be increased. Thanks to the decrease of the systematical uncertainty, LHCb will be able to fully exploit the statistical improvement, taking full advantage of the accumulated LHCb data up to 10 fb^-1, which corresponds to five years of nominal LHCb data taking.

At a future LHCb upgrade, even the B_d^0 → μ^+μ^- decay will become accessible, and the proposed determination of f_d/f_s will be an important tool for the measurement of BR(B_d^0 → μ^+μ^-)/BR(B_s^0 → μ^+μ^-), which provides an even stronger test of the SM [2].

Let us finally emphasize that a future Super-B factory running at Y(5S) would allow us to check the calculations of the SU(3)-breaking effects in the form factor through the measurement of B_d^0 → D_s^+ℓν_ℓ decays. The possible discovery of NP in B_d^0 → μ^+μ^- at LHCb does not rely on this input, but constraining and even extracting SU(3)-breaking form-factor ratios would lead to a more precise determination of BR(B_s^0 → μ^+μ^-).

VII. CONCLUSIONS

The current experimental knowledge of the ratio f_d/f_s of fragmentation functions is unsatisfactory and affects all absolute B_s-decay branching ratio measurements at hadron colliders. In particular, this quantity is also the major uncertainty for the extraction of the B_s^0 →μ^+μ^- branching ratio from the LHCb data. In view of this situation, we have proposed a new strategy for determining f_d/f_s at LHCb. It uses the pair of the color-allowed, U-spin-related tree decays B_d^0 → D_s^+π^- and B_d^0 → D^+K^-, which are very favorable from an experimental point of view for LHCb, robust with respect to NP contributions and theoretically well understood, thereby offering a precise measurement of f_d/f_s at LHCb. The resulting decrease of the total systematic uncertainty on f_d/f_s allows us to detect a 5σ NP signal in the measured B_s^0 →μ^+μ^- branching ratio for values as small as twice the SM value. This corresponds to an improvement of the corresponding NP reach by a factor of 2 with respect to the present LHCb expectation. Once the dependence of f_d/f_s on p_T and/or rapidity is measured to be small, the value of f_d/f_s, as determined at LHCb by means of our strategy can also be applied at ATLAS and CMS.

Acknowledgements

We would like to thank Wouter Hulsbergen, Marcel Merk and Gerhard Raven for valuable discussions and carefully reading the manuscript, and Barbara Storaci and Remi Louvot for many useful conversations. This work is supported by the Netherlands Organisation for Scientific Research (NWO).

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