DIAGNOSTIC SIGNATURES OF RADIO AND HARD X-RAY EMISSION ON PARTICLE ACCELERATION PROCESSES IN THE COMA CLUSTER

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1. INTRODUCTION

Radio halos in galaxy clusters are diffuse radio sources possessing large sizes and steep spectra. The radio emission must be produced by the synchrotron radiation of relativistic electrons. Nonetheless, the sources of these relativistic electrons are still unclear. In the intracluster medium (ICM), relativistic electrons lose energy on the timescale of order \( \sim 10^{10} \) yr because of inverse Compton and synchrotron losses (e.g., Ip & Axford 1999). Because of the short lifetimes of the relativistic electrons, it is difficult to interpret the large size of radio halos as the result of the diffusion of the relativistic electrons injected from radio galaxies (Jaffe 1977). Consequently, for the formation of radio halos, a significant level of reacceleration might be involved. The secondary electron model first proposed by Dennison (1980) provides a different scenario for the origin of the radio halo and can avoid the problem of reacceleration for the relativistic electrons in the primary electron models. However, this model encounters serious problems when comparing with observations (for a recent review, see Brunetti 2003).

Coma C in the Coma Cluster is the prototype of radio halos in galaxy clusters. Being the best-studied example, Coma C has been observed at many different radio wavebands (e.g., Schlickeiser, Sievers, & Thiemann 1987; Kim et al. 1990; Venturi, Giovannini, & Feretti 1990). Giovannini et al. 1990) and 326 MHz (Venturi et al. 1990) to derive the radial distribution of the spectral index in Coma C, and they found a central "plateau" with a size of \( \sim 15' \) for the spectral-index distribution. In the central region, the value of the spectral index is \( \sim 0.8 \); in the outside region, the spectral index strongly steepens as the radius increases.

Observations with BeppoSAX and the Rossi X-Ray Timing Explorer (RXTE) have detected a hard X-ray (HXR) excess with respect to thermal emission from the Coma Cluster (Fusco-Femiano et al. 1999; Rephaeli, Gruber, & Blanco 1999; Rephaeli & Gruber 2002). The HXR excesses from several other clusters have also been reported (Kaastra et al. 1999; Fusco-Femiano et al. 2000, 2001; Gruber & Rephaeli 2002). The most favored mechanism for the HXR excess is inverse Compton scattering (ICS) of the cosmic microwave background (CMB) photons by relativistic electrons. Since these electrons with energy \( \gamma \sim 10^4 \) will also produce radio synchrotron radiation, the HXR excesses and radio halos may originate from the same electron population. In the Coma Cluster, the volume-averaged values of magnetic fields deduced from the comparison of the radio with the HXR excess emission is \( \sim 0.1-0.3 \) \( \mu G \) (Fusco-Femiano et al. 1999; Rephaeli et al. 1999; Rephaeli & Gruber 2002). These low values of field strength are not consistent with those deduced from the measurements of Faraday rotation. Clarke, Kronberg, & Böhinger (2001) have shown that many clusters have relatively large (\( \sim 4-8 \) \( \mu G \)) fields. For the Coma Cluster, Kim et al. (1990) found a central field strength of \( 1.7 \pm 0.9 \) \( \mu G \), and Feretti et al. (1995) estimated the strength to be \( 6 \pm 1 \) \( \mu G \). It is thus very important to see whether it is possible to produce the observed HXR excess via the inverse Compton mechanism with such a high central magnetic field. Nonetheless, some uncertainties in both methods may lead to the discrepancy (Newman, Newman, & Rephaeli 2002). An alternative interpretation of the HXR excess is nonthermal bremsstrahlung from superthermal electrons (Ensslin, Lieu, & Biermann 1999; Blasi 2000; Dogiel 2000; Sarazin & Kempner 2000). However, a huge amount of energy is necessary in this model to produce the observed HXR excess (Petrosian 2001; Blasi 2000).

Brunetti et al. (2001) proposed a two-phase model to interpret the radial steepening of the spectral-index
distribution in Coma C. In this two-phase model, the relativistic electrons were injected during a first phase in the past and reaccelerated during a second phase up to present time. In such a reacceleration model, there must be a cutoff in the electron spectrum because of the balance between the loss and the gain of the electron energy. A cutoff in the electron spectrum should lead to a cutoff in the emissivity of synchrotron radiation and then in the integrated radio spectrum. If the magnetic fields have a radial-decrease profile, the cutoff frequencies will decrease with the increasing radius. Consequently, a radial steepening of the spectral index between two fixed frequencies will appear. Brunetti et al. (2001) successfully reproduced the radial steepening of the spectral index, the radio spectrum steepening at high frequencies, and the HXR excess in the Coma Cluster; however, the central “plateau” in the spectral-index distribution was not well explained. The authors adopted a central field strength of \( \lesssim 3 \) \( \mu \)G in their models. As pointed in their work, the spectral index would be very steep in low magnetic fields and flat in moderate ones. It would be important to know whether a stronger central magnetic field can reproduce the central “plateau” in the spectral-index distribution without violating the observational constraints from the radio and HXR emission.

Cluster mergers are very violent events and release a large amount of energy (\( \sim 10^{64} \) ergs). Merger shocks and violent turbulence must play an important role in the generation and reacceleration of relativistic electrons (Sarazin 2001). Nonetheless, Gabici & Blasi (2003) claimed that the shocks generated by major mergers, mergers between clusters with comparable mass, is too weak to account for the spectral slopes of the nonthermal emission. The Mach numbers of the shocks in major mergers are of order unity in their simulations and roughly consistent with the Mach number \( \sim 2 \) observed in Cygnus A (Markevitch, Sarazin, & Vikhlinin 1999). However, if a significant level of reacceleration is involved, the evolved spectra of relativistic electrons may be able to account for the observed spectra of nonthermal emission.

In this paper, we investigate the radio and HXR excess emission in the Coma Cluster assuming magnetic fields possessing a central field strength of \( 6 \) \( \mu \)G and a radial-decrease profile. The effects of different Mach numbers of merger shocks on the formation of radio halos and HXR excesses are also investigated. We assume that relativistic electrons are injected by merger shocks and reaccelerated by ensuing violent turbulence. The turbulence is expected to be more violent at the moment right after the merger shocks and then gradually decay with time (Ricker & Sarazin 2001); therefore, particles in the ICM might obtain stronger reacceleration in the early period of mergers and then, following the decay of turbulence, the strength of reacceleration would decrease. In this work, both time-independent and time-dependent reacceleration models are considered. The radial variation of the spectral index of synchrotron radiation in the Coma Cluster is computed to test the scenarios of the cosmic-ray electron reacceleration. In particular, we try to obtain a central “plateau” in the spectral-index distribution that is in agreement with observational results reported by Giovannini et al. (1993). We also investigate the production of an HXR excess with the assumption that the HXR excess is due to ICS of the CMB photons by the cosmic-ray electron distribution that we obtained from the spectral index fitting. Comparing the modeling results with the observations, we find that the Mach numbers of the merger shocks have to be in a very small range to form the radio halo and the HXR excess observed in the Coma Cluster.

In this paper, \( H_0 = 50 \) km s\(^{-1}\) Mpc\(^{-1}\) is assumed. The redshift \( z \) of the Coma Cluster is \( \sim 0.0233 \), so that the distance is \( \sim 140 \) Mpc and \( 1' \) corresponds to \( \sim 40 \) kpc. The virial radius of 3.28 Mpc (Girardi et al. 1998) is adopted as the radius of the Coma Cluster.

### 2. PARTICLE ACCELERATION MODELS

The energy variation of particles can be expressed as

\[
\frac{d\gamma}{dt} = b_0 + b_1\gamma + b_2\gamma^2 ,
\]

where \( b_0 = b_{\text{Coul}} + b_{\text{sync}}, \) \( b_1 = b_{\text{brem}} - b_{\text{acc}}, \) and \( b_2 = b_{\text{sync}} + b_{\text{TC}}. \) The coefficients of the loss rates due to the Coulomb and bremsstrahlung losses can be approximated as (Sarazin 1999)

\[
b_{\text{Coul}} \approx 1.2 \times 10^{-12} n_{\text{gas}} \left[ 1.0 + \frac{\ln(\gamma/n_{\text{gas}})}{75} \right] \text{s}^{-1}
\]

and

\[
b_{\text{brem}} \approx 1.51 \times 10^{-16} n_{\text{gas}} [\ln(\gamma) + 0.36] \text{s}^{-1},
\]

where \( n_{\text{gas}} \) is the gas density. For the Coma Cluster, it can be defined as

\[
n_{\text{gas}}(r) = n_0 f_{\text{gas}}(r) = n_0 \left[ 1 + \left( \frac{r}{r_{\text{gas}}} \right)^2 \right]^{-3/2} \text{s}^{-1},
\]

where \( n_0 = 2.89 \times 10^{-3} \text{ cm}^{-3}, r_{\text{gas}} = 10.5 \approx 0.42 \text{ Mpc}, \) and \( \beta_{\text{gas}} = 0.75 \) (Briel, Henry, & Boehringer 1992). We set \( \gamma = 10^3 \) and \( n_{\text{gas}} = 10^{-3} \text{ cm}^{-3} \) into the logarithms for simplicity.

The coefficients in the loss function of inverse Compton scattering and synchrotron radiation can be expressed as (Sarazin 1999)

\[
b_{\text{TC}} = 1.37 \times 10^{-20} (1 + z)^4 \text{s}^{-1}
\]

and

\[
b_{\text{sync}} = 1.3 \times 10^{-21} \left( \frac{B}{\mu \text{G}} \right) \text{s}^{-1},
\]

where \( z \ll 1 \) is assumed in these calculations.

To account for the temporal dependence of the electron reacceleration due to violent turbulence, the general form of the acceleration function \( b_{\text{acc}} \) is given by

\[
b_{\text{acc}}(r, t) = b_\alpha(r) g(t) ,
\]

where

\[
b_\alpha(r) = a_0 + a_1 f_\alpha(r) ,\]

\[
g(t) = 1 + Ae^{-Dt} .
\]

The parameters \( A \) and \( D \) are the amplification factor and the dissipation factor, respectively. The cases with \( A = D = 0 \) are called time-independent acceleration, and the cases with \( A \neq 0 \) and \( D \neq 0 \) are called time-dependent acceleration. According to the numerical work of Ricker & Sarazin (2001), the maximum temperature of mergers is \( \lesssim 3 \) times the initial temperature before merging and returns to that when the
times passed after merging are greater than 1 Gyr. Because the sources for heating gas are turbulence and shocks generated by merging, it is reasonable to relate the temperature variation with the turbulence strength, i.e., the strength of reacceleration. Thus, we choose the values of the amplification factor $A$ and the dissipation factor $D$ under the constraint that the maximum value of $g(t)$, i.e., $1 + A$, is $\leq 3$ and the value of $g(t)$ is $\sim 1$ when $t = 1$ Gyr in our models.

The reacceleration model in the two-phase model proposed by Brunetti et al. (2001) is a time-independent acceleration model in our classification. Brunetti et al. (2001) suggested that the reacceleration function $\chi(r)$ in their eq. (16), corresponding to $b_\chi(r)$ here, can be parameterized as the sum of a uniform large-scale component and a small-scale component; they assumed that the uniform large-scale component is caused by shocks and/or turbulence, possibly generated during a recent merger, and the small-scale component, assumed to be proportional to the inverse of the typical distance between galaxies, is due to the amplification of these shocks/turbulence by the motion of the massive galaxies in the cluster core. Following Brunetti et al. (2001), we assume that the acceleration component $a_0$ in the reacceleration function $b_\chi(r)$ is due to turbulence and spatially uniform in the cluster. The lower hybrid wave turbulence proposed by Eilek & Weatherall (1999) is a possible scenario for this kind of reacceleration. They showed that large-amplitude Alfvén waves do generate lower hybrid waves that collapse to produce localized, intense wave packets, and even a modest level of lower hybrid turbulence can be very effective at accelerating relativistic particles. For the $a_1 f_\chi(r)$ component (corresponding to the small-scale component in the two-phase model), we assume that this is caused by the orbiting motion of galaxies. According to the work of Deiss & Just (1996), the excited turbulent motions of ICM are related to the galaxy density in the cluster and vary as $\nu_{\text{turb}}^2 \propto n_{\text{gal}}$, so that we assume $f_{\chi}(r) \propto n_{\text{gal}}$. The $\beta$-model is adopted for the galaxy distribution in the Coma Cluster:

$$n_{\text{gal}}(r) \propto f_{\text{gal}}(r) = \left[ 1 + \left( \frac{r}{r_{\text{gal}}} \right)^2 \right]^{-3 \beta_{\text{gal}}/2},$$

where $r_{\text{gal}} \approx 0.18$ Mpc and $\beta_{\text{gal}} = 0.86$ (Girardi et al. 1998). We define $f_\chi(r) = f_{\text{gal}}(r)$.

For time-independent acceleration, the analytic solution of equation (1) with the condition that $\gamma$ and $n_{\text{gas}}$ are constant in the logarithms of the Coulomb and bremsstrahlung losses are (see, e.g., Brunetti et al. 2001)

$$\tau = \frac{2}{\sqrt{|\eta|}} \left( \tanh^{-1} y - \tanh^{-1} y_0 \right),$$

where

$$\tau = t - t_0,$$

$$\eta = 4 b_\chi b_2 - b_1^2,$$

$$y = \frac{2 b_\chi \gamma + b_1}{\sqrt{|\eta|}},$$

$$y_0 = \frac{2 b_\chi \gamma_0 + b_1}{\sqrt{|\eta|}}.$$  

The notation $\gamma_0$ is the energy of electrons at time $t_0$.

This solution can be rewritten as

$$\gamma(t) = \frac{\sqrt{|\eta|}}{2 b_2} \left( y_0 + \tanh x \right) - \frac{b_1}{2 b_2},$$

where

$$x = \frac{\sqrt{|\eta|}}{2} \tau.$$  

The cutoff energy is defined by setting $\gamma_0 = \infty$ at time $t_0$ in equation (8) and can be expressed as

$$\gamma_c(t) = \frac{\sqrt{|\eta|}}{2 b_2} \tanh x - \frac{b_1}{2 b_2}.$$  

3. INJECTION AND EVOLUTION OF THE ELECTRON SPECTRUM

In our models, the relativistic electrons are assumed to be injected by merger shocks. A power-law spectrum of relativistic electrons is expected from diffuse shock acceleration. We assume that the injected spectrum has the form

$$n_e(\gamma, r) = f_e(r) K_e \gamma^{-\alpha},$$

where $f_e(r)$ is assumed to be proportional to the gas distribution and is normalized to be equal to $f_{\text{gas}}(r)$ as described in equation (2), i.e., $f_e(r) = f_{\text{gas}}(r)$. The parameter $K_e$ can be determined by normalizing the theoretical radio spectrum to the observed data. The power-law index $\alpha$ is related to the Mach number $M$ of the merger shocks by the expression $s = 2 (\alpha^2 + 1)/(\alpha^2 - 1)$ (e.g., Gabici & Blasi 2003). For studying the effects of the Mach numbers of merger shocks on the nonthermal emission in galaxy clusters, we adopt different values for the power-law index $s$: 2.5, 3.3, 4.0, and 4.7, corresponding to Mach numbers 3, 2, 1.73, and 1.58, respectively. Without loss of generality, we ignore the initial variation of the power-law index in the cluster for simplicity.

Cosmic-ray protons can also generate secondary electrons. Nonetheless, the contribution of the secondary electrons to the relativistic electrons in galaxy clusters is still unclear. In our calculation, we consider only the electrons injected by merger shocks, and assume that the secondary electrons are negligible (Kuo, Hwang, & Ip 2003).

The evolution of the electron population is described by the kinetic equation

$$\frac{\partial n_e(\gamma)}{\partial t} = \frac{\partial}{\partial \gamma} [b(\gamma) n_e(\gamma)] + q(\gamma),$$

where $b(\gamma) = b_0 + b_1 \gamma + b_2 \gamma^2$. Here we assume no continuous injections, i.e., $q(\gamma) = 0$. The analytic solution of equation (11) for time-independent acceleration is then

$$n_e(\gamma, t) = n_e(\gamma_0, t_0) \left[ 1 - \tanh^2 x \right] \left[ (1 - \tanh x)^2 \right],$$

where $x$ and $y$ are defined in equations (8) and (7), respectively.

4. DISTRIBUTION OF MAGNETIC FIELDS IN COMA

Using the techniques of dimensional analysis for studying turbulent structures, Jaffe (1980) derived a magnetic field
model determined by the gas and galaxy distributions and described by

$$B(r) = B_0 f_B(r) = B_0 [f_{\text{gas}}(r)]^m [f_{\text{gal}}(r)]^n,$$

where $f_{\text{gas}}(r)$ and $f_{\text{gal}}(r)$ are defined in equations (2) and (6), and $(m, n) = (0.5, 0.4)$. For the Coma Cluster, there are other constraints that can be used to determine the profile of magnetic fields; the observed spectral-index distribution possesses a central “plateau” and strongly steepens outside this region. It is expected that the profile of the magnetic field strongly affected the size of the central “plateau” and the steepness of the spectral-index distribution outside this region. We assume that the central field strength $B_0$ is 6 $\mu$G. The profile with $(m, n) = (0.7, 0.3)$ shown in Figure 1 is chosen in our paper in order to make the size of the central “plateau” and the steepness of the spectral-index distribution outside this region calculated in the acceleration models agree with the observations.

The orbiting motion of galaxies might amplify the seed fields to grow into the present magnetic field in galaxy clusters (Jaffe 1980; Roland 1981; Ruzmaikin, Sokoloff, & Shukurov 1989). However, De Young (1992) has shown that the turbulent dynamo driven by the galaxy motion is difficult to produce the present microgauss fields. Cluster merging, a very energetic process, may offer a possibility to produce the observed fields. Because the decay time of the magnetic field is very long, the magnetic field will gradually build up under successive merging (Tribble 1993). It is reasonable to assume that the observed fields at the present time are the amplified fields after the last merging. This is the reason for the assumption that the magnetic field is time invariant in our models.

We note that the profile of $(m, n) = (0.7, 0.3)$ is adopted for the purpose of reproducing the observed spectral-index distribution and might be different from a real distribution of cluster magnetic fields. For examples, the profiles of simulated magnetic fields are flat in the core region, and the fields are $\sim 1$ $\mu$G (Dolag, Bartelmann, & Lesch 2002). From the Faraday rotation measurements, the magnetic fields are $\sim 4$–8 $\mu$G out to $\sim 0.75$ Mpc from cluster centers (Clarke et al. 2001). The adopted central magnetic field is higher than the simulated results but lower than the observational ones.

5. Modeling Procedure

In this section, we briefly describe the methods for calculations. We assume that the electron spectra have a power law at time $t = 0$, which corresponds to the moment right after the main merger shock. The electron spectra have an initial power-law index $s$ according to the Mach number of the shock and would then evolve according to the formulation described in $\S$ 2 and $\S$ 3. Since a flat central region in the spectral-index distributions of the radio-halo emission may exist only in a short period during the evolution of radio halos, we calculate the brightness distributions at two fixed frequencies, 326 and 1380 MHz, for different evolution timescales and derive the spectral-index distribution from them. To fit and compare our models with observations, we first vary the acceleration function $b_{\text{acc}}(r, t)$ and judge whether the corresponding spectral-index distributions are consistent with the observations according to two criteria: (1) there is a flat central region with $\alpha_{326} = 0.8$ as reported by Giovannini et al. (1993) and (2) $\alpha_{326} = 1.8$ is located in the range of 16′–18′ to agree with the results obtained by Deiss et al. (1997). The spectral-index distributions at different time $t$ with a step of 0.2 Gyr are computed until consistency with the criteria has been obtained. Second, we also calculate the spatial brightness distributions at 326 MHz to compare with the observed one at 90 cm (Govoni et al. 2001). Third, we normalize the theoretical radio spectrum to the observed data at 430 MHz and then determine a parameter $K_x$ in equation (10). Finally, the HXR spectrum is calculated in a region with a projected radius of 50′ and compared with the observational data from BeppoSAX (Fusco-Femiano et al. 1999) and OSSE (Rephaeli, Ulmer, & Gruber 1994). The thermal temperature of the cluster is assumed to be 8.21 keV (Hughes et al. 1993) in our calculations.

6. Model Results for Coma C

6.1. Time-Independent Reacceleration

We study the time-independent reacceleration models by assuming that the amplification and dissipation factors in the acceleration function are zero. Different initial power-law indices corresponding to different Mach numbers are considered. The values of the parameters in the time-independent models are listed in Table 1.

Figure 2 shows the results from the models with power-law index $s = 2.5$ (models A1 and A2). The radii of the flat central regions in the spectral-index distributions are $\sim 6''$ in both models, which is roughly consistent with the observations. Nonetheless, the spatial brightness distributions are much more concentrated toward the central regions than in observations; the integrated radio spectra also deviate from the measured data at both the low- and high-frequency parts. The HXR spectra are too low to compare with the observations. These results show that a
merger shock with a Mach number $M \approx 3$ cannot produce the radio and HXR emission observed in the Coma Cluster.

Figures 3 and 4 show the results with power-law index $s = 3.3$ (models A3 and A4) and the power-law index $s = 4$ (models A5 and A6), respectively. All these models can produce a flat spectral-index distribution with a radius of $\approx 5\prime-6\prime$ in the central regions. The brightness distributions and the HXR spectra are all roughly consistent with the observations in these models, although distributions seem to be better described by the $s = 4$ models than the $s = 3.3$ ones. These models can also match the integrated radio spectra very well; the spectra also seem to be better fitted by the models with $s = 4$ than by those with $s = 3.3$. These results indicate that merger shocks with Mach numbers $M \approx 2$ can produce the main features of the radio and HXR emission observed in the Coma Cluster.

Figure 5 shows the results with the power-law index $s = 4.7$ (models A7 and A8). We cannot obtain spectral-index distributions with spectral index $\alpha_{1380} = 1.8$ in the region $16\prime-18\prime$ that satisfy the second criterion described in § 5 for models with power-law index $s = 4.7$ or higher when we fit the central flat spectral index to $\alpha_{1380} \approx 0.8$. Nonetheless, we try to find a flat central distribution in the spectral index distribution for these models by ignoring the second criterion. The results are shown in Figure 5. The radii of the flat central regions are $\approx 4\prime$ in model A7 and $\approx 6\prime$ in model A8. The spectral index $\alpha_{1380} = 1.8$ locates at radius larger than $20\prime$ in both models. The spatial brightness distributions show a flat distribution in the central regions; this is particularly obvious for model A8. Neither model fits the integrated radio spectral data at high frequencies; model A8

![Fig. 2.](image2)

![Fig. 3.](image3)

![Fig. 4.](image4)

![Fig. 5.](image5)

### TABLE 1

| Model  | $t$ (Gyr) | $a_0$ ($10^{-16}$ s$^{-1}$) | $a_1$ ($10^{-16}$ s$^{-1}$) |
|--------|-----------|----------------------------|----------------------------|
| A1     | 1.6       | 1.87                       | 1.52                       |
| A2     | 1.8       | 1.84                       | 1.53                       |
| A3     | 1.4       | 2.07                       | 1.35                       |
| A4     | 1.6       | 2.18                       | 1.23                       |
| A5     | 0.8       | 3.40                       | 0.62                       |
| A6     | 0.6       | 2.88                       | 0.85                       |
| A7     | 0.4       | 7.05                       | 0.01                       |
| A8     | 0.6       | 5.05                       | 0.30                       |

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also overproduces the HXR excess. These results indicate that it is difficult to produce all observed features of the radio and HXR emission in the Coma Cluster by mergers shocks with Mach numbers \( M < 1.6 \).

According to the results shown in Figures 2–5, a central “plateau” in the spectral-index distribution can be produced for all these models. Nonetheless, only the models with the initial electron power-law index \( s > 3.3 \) can produce results that match all observed features of the radio and HXR emission in the Coma Cluster; this corresponds to a very narrow range for the Mach numbers of the merger shocks, i.e., \( 1.6 < \mathcal{M} < 2 \).

### 6.2. Time-Dependent Reacceleration

In this section, we show the effects of time-dependent reacceleration by assuming that the amplification factor \( A = 1 \) and the dissipation factor \( D = 2 \text{ Gyr}^{-1} \). We note that the values of \( A \) and \( D \) are constrained by the conditions \( 1 + A \lesssim 3 \) and that \( A \mathcal{e}^{-D t} \) becomes sufficiently small as \( t \) approaches 1 Gyr. Since we just want to investigate the effects of the time-dependent reacceleration on our current models, we fix the values of \( A \) and \( D \) for simplicity. A suitable but different choice of \( A \) and \( D \) could affect the results, particularly on the evolving timescales that fit the observations. The values of the parameters in the time-dependent models are listed in Table 2. Since the results from the time-independent models with \( s = 2.5 \) and 4.7 strongly deviate from the observations, we consider only the cases of \( s = 3.3 \) and 4 in the time-dependent models.

Figure 6 shows the results with the power-law index \( s = 3.3 \) (models B1 and B2). The flat central regions in the spectral-index distributions have radii of \( \lesssim 0.1 \text{ Gyr} \) in both models B1 and B2. However, we note that in model B2 the spectral index \( \alpha_{\text{obs}} = 1.8 \) is located at \( \approx 20' \), which does not satisfy the second criterion stated in § 5. The brightness distributions are concentrated to the central regions, resembling those from models A1 and A2. These results indicate that models with time-dependent reacceleration have effects similar to those with flatter initial electron spectra. This also shows that merger shocks with Mach numbers \( \mathcal{M} \geq 2 \), i.e., \( s \leq 3.3 \), cannot produce the radio emission observed in the Coma Cluster.

Figure 7 shows the results with the power-law index \( s = 4 \) (models B3 and B4). The flat central regions in the

### Table 2

| Time-dependent Reacceleration Models with \( A = 1 \) and \( D = 2 \text{ Gyr}^{-1} \) |
|--------------------------|-----------|-----------|-----------|
| Model       | \( t \) (Gyr) | \( a_0 \) \( \times 10^{-16} \text{ s}^{-1} \) | \( a_1 \) \( \times 10^{-16} \text{ s}^{-1} \) |
| \( s = 3.3 \)      |           |           |           |
| B1           | 1.4       | 1.90      | 1.26      |
| B2           | 1.6       | 2.02      | 1.16      |
| \( s = 4.0 \)      |           |           |           |
| B3           | 0.8       | 2.46      | 0.57      |
| B4           | 1.0       | 2.73      | 0.86      |
spectral-index distributions have a radius of \( \approx 4' \) in model B3 and \( \approx 6' \) in model B4. The integrated radio spectra and the HXR spectra obtained from both models are all in very good agreement with the observations. The brightness distributions also agree very well with the observations in the inner part of the cluster, but are slightly lower than those observed at radii greater than 18'.

We find that the radio spatial brightness distribution is very sensitive in these time-dependent models. Our results indicate that the models with time-dependent reacceleration have effects on the brightness distribution similar to those with flatter initial electron spectra. These effects can be understood because the time-dependent reacceleration function provides a larger reacceleration to the electrons at the early stage of the evolution; this is similar to having a flatter initial electron spectrum. Nonetheless, time-dependent models can still fit the integrated radio spectra and the HXR excess emission better. Combining the results from the time-independent and time-dependent models, we find that only merger shocks with Mach numbers, \( 1.6 < \mathcal{M} < 2 \), i.e., power-law index \( 4.7 > s > 3.3 \), can reproduce results agreeing with the observations of the Coma Cluster.

7. DISCUSSION

As shown in the modeling results, a central “plateau” in the spectral-index distribution can be produced in clusters with a central field strength of 6 \( \mu \text{G} \). To understand the origin of this characteristic feature of the radio emission of Coma C, we use the results of model A6 as an example and show the electron spectra and the radio emissivity of this model at different locations in Figures 8 and 9, respectively. It is obvious that the electrons in the central regions are strongly accelerated to the cutoff energies, and flat spectra can be formed within the central 10' regions. The ratios between the radio emissivity at 326 and at 1380 MHz are almost equal within the central regions, and at these two frequencies the radio emissivity within the central regions dominates over that outside these regions; thus, a central “plateau” may form in the spectral-index distribution. Outside the central region, the electron spectra are not very different from one another because the effects are mainly from the inverse Compton losses and the spatially uniform reacceleration, and then the differences between the radio emissivity at different locations are mainly due to the variations of the magnetic fields. As the radius increases, the high-frequency emissivity gradually decreases because of weaker magnetic fields; therefore, the progressive steepening of the spectral index is presented. Naturally, a decline at high frequencies in the integrated radio spectrum should also exist, as proposed by Schlickeiser et al. (1987).

It is interesting to note that the size of the plateau is almost equal to the region where the magnetic field is \( \gtrsim 3 \mu \text{G} \) (Fig. 1). This fact might indicate that the formation of this plateau is related to the situation that the synchrotron losses dominate over the inverse Compton losses (e.g., Brunetti et al. 2001). The plateau should be produced by the combination of the radial setting of reacceleration and that of magnetic fields. As discussed in previous paragraph, when the inverse Compton losses dominate, it is impossible to obtain a flat plateau because the radius-dependent magnetic fields will cause the high-frequency emissivity to decrease as the radius increase. In other words, it is possible to obtain a flat spectral index plateau in our models only when the synchrotron losses dominate over the inverse Compton losses. If a central plateau is present in the spectral-index distribution, the central strength of the cluster magnetic field should be greater than 3 \( \mu \text{G} \). Note that since a central plateau can exist only in a period in the evolution of radio halos, a central plateau in the spectral-index distribution may be absent even in a cluster possessing a high central field.

The emissivity of the HXR excess of model A6 is shown in Figure 10. Obviously, the HXR excess is mainly contributed from the outer regions of the cluster volume (>30'), as
from model A6. The emissivity at $r=30$ from the outside and low magnetic field regions of the cluster volume, as shown in Figure 9. It is thus inad-

It is expected that the profile and strength of magnetic fields will affect our results. From the Faraday rotation measurements, the magnetic fields are $\sim4$--$8 \mu G$ out to $\sim0.75$ Mpc from cluster centers (Clarke et al. 2001). We note that the magnetic fields in galaxy clusters might have a higher value and a flatter profile in the core region than the adopted model. A higher magnetic field requires a stronger reacceleration to sustain the radio emission. To produce a plateau distribution of the spectral index for a higher magnetic field with a flatter profile, the reacceleration function needs to be almost constant over the region. This indicates that the reacceleration caused by the galaxy motions might be negligible.

The ICS of the CMB photons by relativistic electrons has also been suggested as the mechanism of the extreme-ultraviolet (EUV) excess in the Coma Cluster (Hwang 1997; Ensslin & Biermann 1998). However, the EUV excess possesses a narrow radial profile and concentrates in the inner regions of the cluster (Bowyer & Berghofer 1998), while the HXR excess is mainly contributed from the outer regions of the cluster volume, as shown in Figure 9. It is thus inad-}

We have reproduced all the main features in the radio and HXR excess emission of the Coma Cluster in our models. We have obtained a central “plateau” in the spectral-index distribution as observed in the Coma Cluster with a high central field strength of $6 \mu G$ in our models. The size of the plateau is almost equal to the region where the synchrotron loss dominates over the inverse Compton loss, i.e., the magnetic field is $\gtrsim3 \mu G$. Our models can naturally produce the observed radial steepling of the radio spectral index. We can fit the integrated radio spectra very well; in particular, we can reproduce the high-frequency decline feature in the spectra as measured by Thierbach et al. (2003). These same models can also produce the observed HXR excess, which is mainly from the outer regions of the cluster volume, as pointed out by Brunetti et al. (2001).

We have also shown that only merger shocks with Mach numbers in a very small range can produce results agreeing with the observations of the radio and HXR emission in the Coma Cluster. The Mach numbers are $1.6 < \mathcal{M} < 2$, corresponding to the initial electron power-law index $4.7 > \beta > 3.3$, which are consistent with simulations and observations of merger shocks.

We have adopted an artificial magnetic field distribution in our calculation. The real magnetic field might be higher and have a flatter distribution than the adopted model. It is possible to obtain similar results for a higher magnetic field with a stronger reacceleration. However, it might have problems producing the observed HXR excess within the ICS scheme for such a high magnetic field.

8. SUMMARY

Time-independent and time-dependent reacceleration models are studied to investigate the formation of the radio halo and HXR excess in the Coma Cluster with a high central field strength of $6 \mu G$. In these models, the relativistic electrons are injected by merger shocks and reaccelerated by ensuing violent turbulence. The effects of the strength of merger shocks on the formation of the radio halo and HXR excess are also considered.

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Fig. 10.—HXR excess emissivity at different radii. The parameters are from model A6. The emissivity at $r = 0$ (thick solid curve), 10' (solid curve), 30' (dash-dotted curve), 50' (dashed curve), and 70' (dotted curve) is shown.

indicated by Brunetti et al. (2001). This might reconcile the problem that the volume-averaged magnetic fields derived from the HXR excesses are usually much lower than the magnetic fields from the Faraday rotation measurements. However, we note that the average magnetic field in the core regions adopted here is $\lesssim2 \mu G$, while the magnetic fields from the Faraday rotation measurements are $\approx6 \mu G$. It might be possible that a further fine-tuned version of our model could solve the problem of the high field--ICS discrepancy.

We have also shown that only merger shocks with Mach numbers in a very small range can produce results agreeing with the observations of the radio and HXR emission in the Coma Cluster. The Mach numbers are $1.6 < \mathcal{M} < 2$, corresponding to the initial electron power-law index $4.7 > \beta > 3.3$, which are consistent with simulations and observations of merger shocks.

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Blasi, P. 2000, ApJ, 532, L9
Bowyer, S., & Berghöfer, T. W. 1998, ApJ, 506, 502
Briel, U. G., Henry, J. P., & Böhringer, H. 1992, A&A, 259, L31
Brunetti, G. 2003, in ASP Conf. Ser. 301, Matter and Energy in Clusters of
Galaxies, ed. S. Bowyer, & C.-Y. Hwang (San Francisco: ASP), in press
(astro-ph/0208074)
Brunetti, G., Setti, G., Feretti, L., & Giovannini, G. 2001, MNRAS, 320, 365
Clarte, T. E., Kronberg, P. P., & Böhringer, H. 2001, ApJ, 547, L111
Deiss, B. M., & Just, A. 1996, A&A, 305, 407
Deiss, B. M., Reich, W., Lesch, H., & Wielebinski, R. 1997, A&A, 321, 55
Dennison, B. 1980, ApJ, 239, L93
DeYoung, D. S. 1992, ApJ, 387, 464
Dolag, K., Bartelmann, M., & Lesch, H. 2002, A&A, 387, 383
Eilek, J. A., & Weatherall, J. C. 1999, in Proc. of Ringberg Workshop
on Diffuse Thermal and Relativistic Plasma in Galaxy Clusters, ed.
H. Böhringer, L. Feretti, & P. Schuecker (MPE Rep. 271; Garching:
MPE), 249
Ensslin, T. A., & Biermann, P. L. 1998, A&A, 330, 90
Ensslin, T. A., Lieu, R., & Biermann, P. L. 1999, A&A, 344, 409
Feretti, L., Dallacasa, D., Giovannini, G., & Tagliani, A. 1995, A&A, 302, 680
Fusco-Femiano, R., Dal Fiume, D., Feretti, L., Giovannini, G., Grandi, P.,
Molendi, S., & Santangelo, A. 1999, ApJ, 513, L21
Fusco-Femiano, R., Dal Fiume, D., Orlandini, M., Brunetti, G.,
Feretti, L., & Giovannini, G. 2001, ApJ, 552, L97
Gabici, S., & Blasi, P. 2003, ApJ, 583, 695
Giovannini, G., Feretti, L., Venturi, T., Kim, K.-T., & Kronberg, P. P.
1993, ApJ, 406, 399
Girardi, M., Giuricin, G., Mardorossian, F., Mezzetti, M., & Boschin, W.
1998, ApJ, 505, 74
Govoni, F., Enßlin, T. A., Feretti, L., & Giovannini, G. 2001, A&A, 369, 441
Gruber, D., & Rephaeli, Y. 2002, ApJ, 565, 877
Hughes, J. P., Butcher, J. A., Stewart, G. C., & Tanaka, Y. 1993, ApJ, 404, 611
Hwang, C.-Y. 1997, Science, 278, 1917
Ip, W.-H., & Axford, W. I. 1999, Astrophys. Space Sci., 264, 437
Jaffe, W. J. 1977, ApJ, 212, 1
———. 1980, ApJ, 241, 925
Kaasstra, J. S., Lieu, R., Mittaz, J. P. D., Bleeker, J. A. M., Mewe, R.,
Colafrancesco, S., & Lockman, F. J. 1999, ApJ, 519, L119
Kim, K.-T., Kronberg, P. P., Dewdney, P. E., & Landecker, T. L. 1990,
ApJ, 355, 29
Kuo, P.-H., Hwang, C.-Y., & Ip, W.-H. 2003, ApJ, submitted
Markevitch, M., Sarazin, C. L., & Vikhlinin, A. 1999, ApJ, 521, 526
Newman, W., Newman, A., & Rephaeli, Y. 2002, ApJ, 575, 755
Petrosian, V. 2001, ApJ, 557, 560
Rephaeli, Y., & Gruber, D. 2002, ApJ, 579, 587
Rephaeli, Y.; Gruber, D., & Blanco, P. 1999, ApJ, 511, L21
Rephaeli, Y., Ulmer, M., & Gruber, D. 1994, ApJ, 429, 554
Ricker, P. M., & Sarazin, C. L. 2001, ApJ, 561, 621
Roland, J. 1981, A&A, 93, 407
Ruzmaikin, A. A., Sokoloff, D., & Shukurov, A. 1989, MNRAS, 241, 1
Sarazin, C. L. 1999, ApJ, 520, 529
———. 2001, in Merging Processes in Clusters of Galaxies, ed. L. Feretti,
& T. M. Gioia, & G. Giovannini (Dordrecht: Kluwer), 1
Sarazin, C. L., & Kempner, J. C. 2000, ApJ, 533, 73
Schlickeiser, R., Severs, A., & Thiemann, H. 1987, A&A, 182, 21
Thierbach, M., Klein, U., & Wielebinski, R. 2003, A&A, 397, 53
Tribble, P. C. 1993, MNRAS, 263, 31
Tsay, M. Y., Hwang, C.-Y., & Bowyer, S. 2002, ApJ, 566, 794
Venturi, T., Giovannini, G., & Feretti, L. 1990, AJ, 99, 1381