Magnetic fields in conformally flat spacetimes

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Abstract. There is a widespread perception in the community that large-scale magnetic fields in Friedmann-Robertson-Walker (FRW) universe always decay adiabatically. This belief follows from the conformal invariance of standard electromagnetism and from the conformal flatness of the FRW spacetimes. Together, they are thought to guarantee that magnetic fields in Friedmann models will always behave as if the space was Minkowski. However, this is only true in spatially flat FRW universes. Although Friedmannian spacetimes with non-Euclidean 3-spaces are also conformally flat, their conformal factors have an additional spatial dependence. This triggers a magneto-curvature term in the magnetic wave equation, which can modify the standard ‘adiabatic’ magnetic decay and lead to a superadiabatic-type amplification of the $B$-field on lengths close to the associated curvature scale. Our aim is to explain the general relativistic nature of this effect and discuss its physical implications.

1. Introduction
Magnetic fields in Friedmann universes are largely believed to decay adiabatically throughout their evolution and regardless of the electrical properties of the cosmic medium. Consequently, any large-scale $B$-fields that might be present are expected to dilute as $a^{-2}$, where $a$ is the cosmological scale factor. This widespread perception derives from the conformal flatness of the FRW spacetimes and the conformal invariance of Maxwell’s equations. When combined, these two are believed to guarantee that the rescaled magnetic vector $\mathcal{B}_a = a^2 B_a$ evolves as in Minkowski space, which then ensures that the magnetic flux remains conserved and finally that $B_a \propto a^{-2}$ irrespective of the electric properties of the cosmos (e.g. see [1, 2] for more details). Thus, a superadiabatic-type amplification of the $B$-field can only be achieved if either the conformally flat FRW spacetimes are abandoned or the conformal invariance of electromagnetism is violated. Given that the current observational data strongly support the Friedmann models, the choice is to follow the latter route. One can break away from standard electromagnetic theory in many ways and therefore the literature is rife with mechanisms that reduce the adiabatic depletion of magnetic fields (see [3]-[6] for a representative, though incomplete list). Some of the proposed scenarios work within the standard model, but the majority of them introduce new physics and almost in every case success is achieved at the expense of simplicity. Breaking away from conventional electromagnetism, however, is not...
always necessary because the above mentioned Minkowski-like, adiabatic decay of the $B$-field is guaranteed only when the Friedmannian background has flat spatial sections. Nevertheless, to the best of our knowledge, this distinction has never been made clear in the literature. As a result, it is now widely perceived that a superadiabatic-type amplification of cosmic magnetic fields is impossible in all FRW universes unless the conformal invariance of Maxwell’s theory is broken.

Although all three Friedmann models are conformally flat, they are far from identical. There are differences in the geometry of the spatial hypersurfaces, which ensure that there is no global one-to-one correspondence between those FRW models with non-Euclidean 3-spaces and the Minkowski space (e.g. see [7]). Another way of putting this is that, when dealing with spatially curved Friedmann universes, the conformal factor no longer coincides with the cosmological scale factor but has an additional spatial dependence. As a result, the wave equation for the previously defined, rescaled $B$-vector does not take a Minkowski-like form when the FRW space is not spatially flat. In particular, the non-Euclidean geometry of the 3-space leads to an extra magneto-curvature term that can modify the adiabatic $a^{-2}$-law, without the need to break away from conventional electromagnetism. Following [8, 9], in models with negative spatial curvature this extra coupling causes the superadiabatic amplification of large-scale magnetic fields during a phase of de Sitter inflation. The amplification occurs on wavelengths close and beyond the curvature scale and leads to fields that are far stronger than any other conventional $B$-field that has survived an epoch of inflation. In particular, assuming a marginally open universe today, with $1 - \Omega_{0} \sim 10^{-2}$, one finds a residual magnetic field of roughly $10^{-35}$ G on a scale of approximately $10^{4}$ Mpc. The latter is almost 20 orders of magnitude stronger than its flat-FRW counterpart and also lies within the current galactic dynamo requirements [10].

2. Magnetic fields on FRW backgrounds

Consider an FRW spacetime, containing a single perfect fluid with energy density $\rho$ and isotropic pressure $p$. Introducing a family of fundamental observers, which move along with the cosmic medium with 4-velocity $u_{a}$, we induce a unique 1+3 threading of the spacetime into time and space. In this decomposition the tensor $h_{ab} = g_{ab} + u_{a}u_{b}$, where $g_{ab}$ is the spacetime metric, projects orthogonal to $u_{a}$ and onto the observers’ 3-D rest space [11, 12]. We perturb this model by allowing for a weak electromagnetic field. Relative to the fundamental observers, the latter splits into an magnetic and an electric component. These are respectively represented by the spacelike vectors $B_{a}$, $E_{a}$ and obey Maxwell’s equations (see [13]-[15] for a covariant description). Combining the latter with Einstein’s formulae leads to the wave equations of the aforementioned electric and magnetic fields [8]. In particular, when linearised around an FRW background, the magnetic wave equation reads [8, 9]

$$
\ddot{B}_{a} - D^{2}B_{a} = -\frac{5}{3}\Theta\dot{B}_{a} - \frac{4}{9}\Theta^{2}B_{a} + \frac{1}{3}(\rho + 3p)B_{a} - R_{ab}B^{b} + \text{curl}J_{a},
$$

where the overdots indicate time differentiation, $D^{2} = h^{ab}\nabla_{a}\nabla_{b}$ is the 3-D Laplacian operator and $\Theta$ is the volume expansion scalar associated with $u_{a}$. Also, $R_{ab}$ is the 3-Ricci scalar that determines the geometry of the background spacial section and $J_{a}$ is the orthogonally projected current density which depends on the electrical properties of the cosmic medium.

For our purposes, the key quantities are the last two in the right-hand side of Eq. (1). The former, the magneto-curvature term, results from the geometrical nature of general relativity and from the vector nature of magnetic fields. This term has purely geometrical origin and has its root in the non-commutativity of the covariant derivatives of vectors in non-Euclidean spaces. Physically speaking, however, the presence of the magneto-curvature stress in (1)
reflects the fact that vector (and tensor) fields ‘feel’ the curvature of the space through the Ricci identities, in addition to the Einstein equations.

The current term in the right-hand side of Eq. (1) depends on the nature of the cosmic medium. In particular, the 3-current is related to the electric field via Ohm’s law, which in the limit of resistive magnetohydrodynamics (MHD) takes the covariant form \[ J^a = \varsigma E^a, \] (2)

where \( \varsigma \) is the electrical conductivity of the fluid. We note the absence of a convective term in the right-hand side of the above. The reason is the covariant form of Maxwell’s formulae, which by construction incorporate the induced electric component (see [15] for further discussion).

When dealing with a perfectly conductive medium, \( \varsigma \to \infty \), the observer sees no electric field and the 3-current is related to the magnetic field vector by \( J^a = \text{curl}B^a \). Substituting this result into expression (1) reduces the latter to \[ \dot{B}^a = -\frac{2}{3} \Theta B^a, \] (3)

which is the familiar magnetic induction equation written on a Friedmannian background (e.g. see [13]-[15]). This means that at the ideal-MHD limit the \( B \)-field decays adiabatically (i.e. \( B^a \propto a^{-2} \)) always and irrespective of the spatial geometry of the FRW host.

The universe is thought to be a very good conductor throughout its standard Big Bang evolution, but not during inflation and the early stages of reheating. In these poorly conductive environments \( \varsigma \to 0 \), which means \( J^a \to 0 \) and that any magnetic fields that might be present at the time evolve free of current effects. In other words, \[ \ddot{B}_a - D^2 B_a = -\frac{5}{3} \Theta \dot{B}_a - \frac{4}{9} \Theta^2 B_a + \frac{1}{3} (\rho + 3p)B_a - R_{ab}B^b, \] (4)

where \( R_{ab} = (6k/a^2)h_{ab} \) and \( k = 0, \pm 1 \) is the curvature index of the background spatial sections. During inflation small-scale quantum fluctuations in the electromagnetic field are stretched well outside the horizon, where they can be treated as classical electromagnetic fields (e.g. see [3]). Also, although inflation drastically increases the curvature radius of the universe, it does not change its spatial geometry. What the de Sitter phase does is stretch the curvature scale well beyond the observer’s horizon. As a result, the universe looks much flatter after inflation than before. Thus, unless the 3-space was perfectly flat at the beginning of the inflationary regime, there will always be a scale where the curvature effects are important. It is on these wavelengths and during the poorly conductive de Sitter phase that the magnetic component of the Maxwell field obeys the wave equation (4).

3. Magnetic fields on spatially curved FRW backgrounds

Let us introduce the decomposition \( B_a = \sum_k B^{(n)}_a Q^{(n)}_a \) for the magnetic field vector, where \( Q^{(n)}_a \) are the standard vector harmonics. This means that \( Q^{(n)}_a = 0 = D_a B^{(n)} \) and \( D^2 Q^{(n)}_a = -(n/a)^2 Q^a \). Then, using Eq. (4), we obtain the following wave-like formula \[ \ddot{B}^{(n)}_a + \frac{n^2}{a^2} B^{(n)}_a = -\frac{5}{3} \Theta \dot{B}^{(n)}_a - \frac{4}{9} \Theta^2 B^{(n)}_a + \frac{1}{3} (\rho + 3p)B^{(n)}_a - \frac{2k}{a^2} B^a, \] (5)

for the n-th magnetic mode. We remind the reader that the Laplacian eigenvalues have a continuous spectrum, with \( n^2 \geq 0 \), when \( k = 0, -1 \) and a discrete one, with \( n^2 \geq 3 \), for
In this notation supercurvature modes in spatially open models have \(0 \leq n^2 < 1\), which guarantees that the physical wavelength of the perturbation is larger than the curvature scale at the time (i.e. \(\lambda_n = a/n > a\)). On the other hand, modes with \(n^2 > 1\) span lengths smaller than the curvature scale and will be therefore termed subcurvature. Note that the supercurvature modes are always larger than the Hubble length and consequently never in causal contact. On the other hand, perturbations on subcurvature scales can be causally connected (see [17] for further discussion).

Recalling that the zero-order Raychaudhuri equation takes the form \(\ddot{a}/a = -\rho(1+w)/6\) and introducing \(\eta\), the conformal time variable with \(\dot{\eta} = 1/a > 0\), Eq. (5) becomes

\[
B''(n) + 4 \left(\frac{a'}{a}\right) B'(n) + 2 \left(\frac{a'}{a}\right)^2 B(n) + 2 \left(\frac{a''}{a}\right) B(n) + n^2 B(n) = -2kB(n),
\]

where primes indicate differentiation with respect to \(\eta\). Finally, employing the ‘magnetic flux’ variable \(B(n) = a^2 B(n)\) the above reduces to

\[
B''(n) + n^2 B(n) = -2kB(n).
\]

When the FRW background is spatially flat the magneto-curvature term vanishes and the rescaled \(B\)-field evolves as in Minkowski space, which guarantees the \(B(a) \propto a^{-2}\) at all times [1, 2]. Despite the general perception, however, the adiabatic decay is not inevitable when the Friedmann universe has non-Euclidean 3-spaces and Eq. (7) has nonzero right-hand side. We will therefore examine whether 3-curvature effects can lead to the superadiabatic amplification of large-scale magnetic fields during the inflationary era, reminding the reader that analogous amplification has so far been obtained only outside the realm of standard electrodynamics.

4. The superadiabatically amplified magnetic field

We begin by noting that Eq. (7) closely resembles the one given in [3] (see Eq. (2.15) there), obtained after introducing an ad hoc coupling between the Maxwell and the Einstein fields in the Lagrangian. The similarity is in the presence of a curvature related source term in both expressions. The difference is that here the magneto-curvature term is a natural and unavoidable consequence of the vector nature of the magnetic field and of the geometrical approach of general relativity. No new physics has been introduced and conventional electromagnetism still holds.

Let us first consider a spatially closed background universe. In this case it is straightforward to verify that the adiabatic depletion rate is preserved despite the presence of a nonzero magneto-curvature term in (7). Indeed, for \(k = +1\) the latter exhibits an oscillatory solution of the form [8]

\[
B(n) = \frac{1}{a^2} \left\{ C_1 \cos \left[ \sqrt{n^2 + 2} \eta \right] + C_2 \sin \left[ \sqrt{n^2 + 2} \eta \right] \right\},
\]

with \(C_1, C_2\) constants. Thus, apart from modifying the oscillation frequency, the magneto-curvature term in Eq. (7) has no significant effect on the evolution of the \(B\)-field when 3-space is positively curved.

When dealing with the hyperbolic geometry of a spatially open FRW model, however, the oscillatory behaviour of \(B(n)\) is not always guaranteed. Indeed, for \(k = -1\) Eq. (7) reads

\[
B''(n) + (n^2 - 2) B(n) = 0,
\]
The adopted normalisation scheme, where \( C = 0 \) and \( \eta < 0 \), has allowed us to streamline the key equations considerably without loss of generality. Within these conventions, \( a \rightarrow 0 \) for \( \eta \rightarrow -\infty \) and \( a \rightarrow +\infty \) as \( \eta \rightarrow 0^- \). The reason is that then the conductivity of the cosmic medium is effectively zero and the magnetic evolution is monitored by Eqs. (4), (7). Also, the most dramatic suppression of the \( B \)-field occurs during inflation and therefore any change in the magnetic depletion rate during that period could prove crucial. Setting \( C = 0 \), which means that \( \eta < 0 \), reduces (12) to \( aH = -\coth \eta \). The latter integrates to give

\[
a = \frac{A_0 e^\eta}{1 - e^{2\eta}},
\]

with \( A_0 = a_0(1 - e^{2\eta_0})/e^{\eta_0} \) a positive constant (see [8] for details).\(^1\) Substituting this result into the right-hand side of Eq. (11) we can express the evolution of the magnetic field in terms of the cosmological scale factor. For simplicity consider the case of \( |k| \rightarrow 1^- \), which corresponds to the largest subcurvature scales with \( n^2 \rightarrow 1^- \). Then, from (11) and (13) we arrive at

\[
B = C_3 \left( 1 - e^{2\eta} \right) a^{-1} + C_4 e^{-\eta} a^{-2},
\]

where \( C_3 \) and \( C_4 \) are constants. Therefore, on the largest subcurvature scales, the dominant magnetic mode never depletes faster than \( a^{-1} \). This decay rate is considerably slower than the typical \( a^{-2} \)-law and holds throughout the inflationary era. Note that the magnetic depletion switches to the adiabatic \( a^{-2} \) rate at the \( \eta \rightarrow 0^- \) limit only.\(^2\) Result (14) immediately implies that, beyond a certain scale, the cosmological magnetic flux increases with time instead of being

\(^1\) The adopted normalisation scheme, where \( C = 0 \) and \( \eta < 0 \), has allowed us to streamline the key equations considerably without loss of generality. Within these conventions, \( a \rightarrow 0 \) for \( \eta \rightarrow -\infty \) and \( a \rightarrow +\infty \) as \( \eta \rightarrow 0^- \).

\(^2\) According to Eq. (9), the curvature effects modify the magnetic evolution on large scales with \( n^2 < 2 \). Expression (14) shows that as \( |k| \rightarrow 1^- \), which corresponds to \( n^2 \rightarrow 1^- \) and the largest subcurvature scales, the magnetic field decays as \( a^{-1} \). When \( n^2 \rightarrow 2^- \), on the other hand, we have \( |k| \rightarrow 0^- \) and \( B \propto a^{-2} \). In particular, expressions (11), (13) combine to provide the general solution

\[
B_{(k)} = C_3 \left( 1 - e^{2\eta} \right) a^{-1} + C_4 \left( 1 - e^{2\eta} \right) a^{-2},
\]

with \( |k| \leq 1 \). Clearly, when \( |k| \) takes its values in the open interval \((0,1)\) the decay rate of the dominant magnetic mode varies between \( a^{-2} \) and \( a^{-1} \), which is always slower than the adiabatic \( a^{-2} \)-law.
5. The residual magnetic field

Following [3], the energy density stored in the n-th magnetic mode as it crosses outside the horizon is \( \rho_B = (M/m_{Pl})^4 \rho \), where \( \rho \approx M^4 \) is the total energy density of the universe and \( m_{Pl} \) is the Planck mass. Then, assuming that \( B^2 \propto a^{-4} \), the energy density in the mode at the end of the inflationary regime is given by [3]

\[
\rho_B = \frac{B^2}{8\pi} \sim 10^{-104} \frac{\gamma - 4}{\lambda_{\text{Mpc}}} \rho_\gamma .
\]  

(16)

Here \( \rho_\gamma \) is the radiation energy density and \( \lambda \) is the comoving scale of the field. The latter is measured in Mpcs and it is normalised so that \( \lambda \) coincides with the physical scale today. Note that the magnetic mode crossed outside the horizon \( N = N(\lambda) \) e-folds before the end of inflation (see [3] for details). The underlying assumption leading to the above result is that any given mode is excited quantum mechanically while inside the horizon and ‘freezes in’ as a classical perturbation once it crosses through the Hubble radius. The dramatic weakness of the residual field demonstrated in Eq. (16), reflects the drastic suppression of the magnetic energy density relative to the vacuum energy, which remains constant throughout the inflationary regime. After inflation \( \rho_\gamma \) also decays as \( a^{-4} \) and the ratio \( r = \rho_B/\rho_\gamma \) does not change.

If the dynamo amplification of large-scale fields is efficient, the strength of the required magnetic seed, as measured at the time of completed galaxy formation, ranges from \( \sim 10^{-19} \) G down to \( \sim 10^{-23} \) G. In addition, the coherence length of the initial field should be at least as large as the size of the largest turbulent eddy, namely no less than \( \sim 100 \) pc. The aforementioned magnetic strengths, which correspond to \( r \sim 10^{-27} \) and \( r \sim 10^{-35} \) respectively, have been obtained in a spatially flat universe with zero cosmological constant. However, if the universe is open or if it is dominated by a dark-energy component, the above quoted requirements are considerably relaxed. In particular, the standard dynamo can produce the currently observed galactic magnetic fields from a seed of the order of \( 10^{-30} \) G, or even less, at the end of galaxy formation [10]. Note that a ‘collapsed’ magnetic field of \( \sim 10^{-30} \) G coherent on approximately 100 pc corresponds to \( r \sim 10^{-27} \) and \( r \sim 10^{-35} \) respectively, instead of \( a^{-4} \)-law. This happens on the largest subcurvature scales (and beyond) when the inflationary patch has negative spatial curvature. Therefore, even a marginally open universe can contain a substantially strong large-scale magnetic field. For a direct comparison with the spatially flat case, it helps to follow the analysis of [3] (see also Eq. (16) above). Consider a typical GUT-scale inflationary scenario with \( M \sim 10^{17} \) GeV and reheating temperature \( T_{\text{RH}} \sim 10^9 \) GeV. Then, for \( B^2 \propto a^{-2} \), the energy density stored in a given magnetic mode by the end of inflation is

\[
\rho_B \sim 10^{-90} M^{8/3} T^{-2/3}_{\text{RH}} \lambda_{\text{Mpc}}^{-2} \rho_\gamma \sim 10^{-51} \frac{\gamma - 2}{\lambda_{\text{Mpc}}} \rho_\gamma ,
\]  

(17)
Thus, on a given scale, the earlier inflation starts and the lower the reheating temperature, the stronger the supearadiabatically amplified residual field. After inflation the high conductivity of the plasma is restored. This ensures that $B^2 \propto a^{-4}$ and consequently that the ratio $r = \rho_B/\rho_\gamma \sim 10^{-51}\tilde{\lambda}_{Mpc}^{-2}$ remains fixed. To proceed we note that $\tilde{\lambda}$ is nearly the curvature scale at the end of inflation. Also, in a universe with nontrivial spatial geometry the effect of curvature in a comoving region remains unchanged, since the curvature scale simply redshifts with the expansion (e.g. see [17]). This means that if $1-\Omega_0$ is of the order of $10^{-2}$, as it appears to be today [18], the current curvature scale is

$$\lambda_k = \frac{\lambda H_0}{\sqrt{1-\Omega_0}} \sim 10^4 \text{ Mpc},$$

where $(\lambda H)_0 = H_0^{-1}$, $H_0 \simeq 2h \times 10^{-42} \text{ GeV}$ and $0.5 \leq h \leq 1$. The above is also the approximate scale of the superadiabatically amplified primordial magnetic field, redshifted to the present. Thus the size of the field is much larger than the minimum coherence scale, of 10 kpc, required for the dynamo to operate. Nevertheless, once the galaxy formation starts, we expect that the fluid motion will force the magnetic force lines to break up and reconnect on lengths similar to the size of a protogalaxy. So, by substituting result (18) into expression (17) we find that

$$r = \rho_B/\rho_\gamma \sim 10^{-59},$$

which corresponds to a $B$-field with current strength around $10^{-35}$ G. We note that although these results depend on the current values of the Hubble and the density parameters, their dependance is relatively weak. Finally, in order to satisfy the conventional causality requirements we have implicitly assumed that the universe was sufficiently open at the onset of inflation. In particular, a relatively mild initial value of $\Omega_i < 0.1$ will suffice for all practical purposes. Such a value ensures that effectively all the largest subcurvature modes are initially inside the horizon and therefore in causal contact when inflation starts.

The first point to underline is that, to the best of our knowledge, magnetic fields with $B_0 \sim 10^{-35}$ G and coherence lengths of $\sim 10^4$ Mpc are greatly stronger than any field obtained within standard electromagnetic theory on such (and much smaller) scales. Moreover, fields with this strength are of astrophysical interest because they can successfully seed the galactic dynamo, as long as the current energy density of the universe is dominated by a dark component; a scenario favoured by recent observations [18]. For a nearly flat universe with the dark energy making up to 70% of the present density parameter, in particular, a seed field of $\sim 10^{-35}$ G is within the lower strength required for the galactic dynamo to operate [10]. We also point out that the above given magnetic strengths do not account for the effects of the physically more realistic scenario of anisotropic protogalactic collapse. The latter is expected to add a few more orders of magnitude to any field obtained through the highly idealised spherical collapse models [19, 20].

For completeness, we will also consider the magnetic evolution on supercurvature scales. During inflation supercurvature modes also obey Eqs. (10) and (14). On these scales the eigenvalue $(n)$ lies in the interval $[0, 1)$, which implies that $1 < k^2 \leq 2$. Then, near the $k^2 = 2$ limit that corresponds to the homogeneous mode, the magnetic decay rate becomes $B \propto a^{-2}$. The latter is considerably slower than the $a^{-1}$-law associated with the largest subcurvature scales. One should keep in mind, however, that supercurvature scales in spatially open FRW cosmologies are always outside the Hubble radius and therefore are not causally connected. Nevertheless, any magnetic field that happens to span over these scales at the onset of inflation will decay much slower than its subcurvature counterparts.
The origin and the evolution of the magnetic fields that we observe almost everywhere in the universe today remains an open issue and a matter of debate. The structure of the large-scale galactic fields points towards a dynamo-type amplification mechanism, but this requires an initial seed field in order to operate. The origin of these seeds is still unknown and may be astrophysical or cosmological. Primordial magnetism is attractive because it can explain all the B-fields seen in the universe today and especially those found in high-redshift protogalaxies. Early magnetogenesis is not problem free however. Typically, the coherence length of post-inflationary magnetic fields is too short, while fields that survived inflation are too weak to sustain the galactic dynamo. The latter requires seeds with strengths between $10^{-12} - 10^{-35} \text{G}$ and a comoving length of approximately 10 kpc on comoving scales. However, even fields as weak as $10^{-35} \text{G}$ are hard to obtain on these scales when the host universe in a Friedmann model. According to the current literature, the root of the problem lies in the conformal invariance of conventional electromagnetism and in the conformal flatness of the FRW spacetimes. The two are thought to guarantee that large-scale magnetic fields are bound to decay adiabatically irrespective of the nature and the electrical properties of the cosmic medium. Although this has only been shown to hold in FRW models with flat spatial sections, it has been de facto extended to all three FRW spacetimes. On these grounds, we have studied the evolution of cosmological magnetic fields in perturbed FRW with non-Euclidean spatial geometry.

The curvature of our universe, whether it is spherical or hyperbolic is an open question of contemporary cosmology [21]. Current observations strongly suggest that the total energy density is close to the critical one, though they stop short from establishing whether the universe is marginally open or marginally closed. Moreover, within the frame of standard general relativity, the probability of a universe with perfectly flat spatial hypersurfaces is effectively zero. Allowing for curved spatial sections, we showed that the adiabatic $B \propto a^{-2}$ law is not always guaranteed because of the linear coupling between the field and the background 3-geometry [8]. When dealing with spatially open FRW models, in particular, the extra curvature-related source term in the magnetic wave equation meant that large-scale fields decay as $a^{-1}$ instead of the standard adiabatic $a^{-2}$-law. This is possible for fields evolving through a period of inflationary expansion, due to the very low electrical conductivity of the latter. As a result, magnetic fields coherent on the largest subcurvature scales at the onset of inflation can survive a de Sitter epoch and still be strong enough to sustain the dynamo process. Assuming that $1 - \Omega \simeq 10^{-2}$ today and that $H_0 = 100h \text{ km/sec-Mpc}$, with $0.5 \leq h \leq 1$, we find a residual field of the order of $10^{-35} \text{G}$ spanning over a region of approximately $10^4 \text{ Mpc}$. Magnetic fields like these are by far stronger than any other large-scale field obtained within standard electromagnetic theory. Moreover, B-fields with the aforementioned properties are of astrophysical interest provided the energy density of our universe is currently dominated by a dark component.

If the universe is marginally open today, our mechanism allows for a simple, viable and rather efficient amplification of large-scale primordial magnetic fields to strengths that can seed the galactic dynamo. Even if the universe is not open, this study still brings about a rather important issue. This is the unique nature and non-trivial properties of magnetic fields and their potential implications in the context of general relativity. Magnetic fields, in particular, are the only vector source that we know that exist in the universe today and in the geometrical framework of Einstein’s theory vectors have different status than scalars. Thus, in addition to the Einstein equations, vector sources are coupled to the curvature of the space through the Ricci identities. This coupling has been largely bypassed in the literature, though its implications are generally non-trivial and in many cases quite counter-intuitive [22, 24]. The
best known example is probably the scattering of electromagnetic waves by the gravitational field, which leads to the violation of Huygens principle [25]. Here, we have considered the implications of this relativistic magneto-geometrical interaction for the evolution of large-scale magnetic fields in FRW universes. We found that, contrary to the widespread perception, a superadiabatic-type amplification of cosmological magnetic fields is possible in conventional cosmological models and within standard electromagnetic theory. In this case, the magneto-geometrical coupling mimics effects that have been traditionally attributed to new physics.

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