Shubnikov-de Haas effect in quasi-two-dimensional compounds

P.D. Grigoriev\(^1,2\), M.V. Kartsovnik\(^3\), W. Biberacher\(^3\) and P. Wyder\(^1\)
\(^1\)Grenoble High Magnetic Field Laboratory, MPI-FKF and CNRS, BP 166, F-38042 Grenoble Cedex 09, France
\(^2\)L.D. Landau Institute for Theoretical Physics, 142432 Chernogolovka, Russia
\(^3\) Walther-Meißner-Institut, Bayerische Akademie der Wissenschaften, Walther-Meißner-Str. 8, D-85748 Garching, Germany

November 13, 2018

Abstract

The Shubnikov-de Haas effect in quasi-two-dimensional compounds is studied. The conductivity is calculated from the Kubo formula. Two effects – the field-dependent phase shift of the beats and the slow oscillations of the conductivity are explained and calculated. The results are applicable to the strongly anisotropic organic metals and other layered compounds.

1 Introduction

The magnetic quantum oscillations (the de Haas - van Alphen (dHvA) and the Shubnikov - de Haas (SdH) effects) have been discovered long ago and were used a lot as a powerful tool of studying the geometry of Fermi surfaces and other electronic properties of different metals \(^1\). Last years the quasi-two-dimensional (Q2D) organic metals \(^2\) attract a great interest and many works have been devoted to the study of magnetic quantum oscillations in these compounds (for a review see e.g. \(^3\)). The quantum oscillations of the magnetization is a thermodynamic effect that is completely determined by the density of states distribution. An exact calculation of the density of states is a very complicated problem but the semi-phenomenological theoretical description of the magnetization oscillations in Q2D compounds was recently provided in a number of papers \(^4, 5, 6, 7, 8, 9, 10\). The chemical potential oscillations and the arbitrary electron reservoir due to the open sheets of the Fermi surface make no principal difficulties \(^11\). Of course, if the e-e interaction drastically changes the ground state (like the FQHE effect) the problem becomes much more complicated. But when the number of the occupied LLs is very large \((n_F > 100\) as in most of Q2D organic metals\) the effect of the e-e interaction is reduced (as in the Fermi liquid) and can be taken into account via the renormalization of the effective mass. Another open question in the theory of the Q2D dHvA effect is the exact shape of the Landau levels. It depends on the particular type of a compound (on the type of impurities, their distribution, the interlayer transfer integral etc.) and makes only some quantitative differences.
The theoretical description of the Q2D quantum conductivity oscillations was not as successful although some works on this subject have appeared last years [6, 11, 12, 13, 14]. There are still some open qualitative questions. One of them is the origin of the phase difference in the beats of the SdH and dHvA effects. It takes place even when the harmonic damping is very strong and only the first harmonic is seen. An explanation and an approximate theoretical description of this phenomenon as well as the more extensive experimental study of this effect has been proposed recently [14].

Another very interesting phenomenon is the slow oscillations of the magnetoresistance that have been observed in a number of Q2D organic metals [3, 15, 16, 17, 18, 19, 20]. The behavior of the slow oscillations resembles that of the SdH effect that has lead to a suggestion of additional, very small Fermi surface pockets in these materials. However, band structure calculations (basically giving a good description of the electron band structure and Fermi surface topology in the organic metals) have shown no evidence of such small pockets in any of these compounds. Moreover, while the slow oscillations are often very pronounced in the magnetoresistance, sometimes even dominating the oscillation spectrum, no analogous observation in oscillating magnetization (dHvA effect) has been reported thus far. Certainly, the slow oscillations must contain a useful information about the compounds but to extract this information one needs some theoretical explanation and, desirably, a quantitative description of the phenomenon.

In this paper we calculate the conductivity starting from the Kubo formula and handling carefully with all oscillating quantities in the expression of the conductivity. The proposed theory explains both the slow oscillations and the phase shift of the beats and gives some quantitative description of these phenomena. It should be noted, however, that our analysis has one important limitation – it is developed only when the magnetic field is perpendicular to the conducting layers. When the magnetic field has substantial tilting angle our results give only a qualitative description.

In section II we calculate the conductivity starting from the Kubo formula. The calculations are described in detail so that they can be easily followed. In Sections III and IV we study the limit of strong harmonic damping in more details. In Sec. III we give the description of the phase shift of beats. In Sec. IV the slow oscillations are considered.

2 The calculation of the conductivity

First, we shall consider the SdH effect in the external magnetic field perpendicular to the conducting planes: \( \vec{H} \parallel \vec{z} \). Then the electron spectrum of Q2D electron gas in magnetic field is given by

\[
\epsilon_{n,k_z} = \hbar \omega_c (n + \frac{1}{2}) - 2t \cos(k_zd)
\]

(1)

where \( \omega_c = eB_z/m^*c \) is the cyclotron frequency, \( k_z \) is the wave number in \( z \)-direction perpendicular to the conducting layers, \( d \) is the interlayer distance and \( t \) is the interlayer transfer integral. This energy spectrum can be easily obtained in the tight binding approximation after we choose the electromagnetic vector potential in the form \( A = (0, B_zx, 0) \). The interlayer bandwidth \( W \) is related to the interlayer transfer integral as \( W = 4t \). We consider the case \( \epsilon_F \gg \hbar \omega_c \). Then the electron-electron interaction does not completely change the energy spectrum (as in the extremely quantum limit of FQHE regime) and we can take the energy spectrum (1) as a zeroth approximation and then treat the impurity scattering using the perturbation theory. If magnetic field is tilted with respect to the conducting planes the formula (1) becomes no more valid. This is the most
important limitation of the analysis that follows. Hence, at the substantial tilting angles our theory gives only qualitative predictions.

Before diving into the calculations let us describe on the physical level the origin of the slow oscillations. They come from the interference of the oscillations of the different quantities, on which the conductivity depends. These quantities are the electron relaxation time, the mean square electron velocity (summed over all states on the FS) and the Dingle temperature. The product of two oscillating factors gives a constant term that is just one half of the product of the amplitudes of these oscillations: \((1 + a \cos x)(1 + b \cos x) = 1 + (a + b) \cos x + (ab/2) \cos 2x + ab/2\) (where in our case \(x \equiv \frac{2\pi\mu}{\hbar\omega_c}\)). The last term \(ab/2\) is responsible for the slow oscillations. In our case the amplitudes \(a, b\) of the fast oscillations (for example, of relaxation time and the mean square velocity) are themselves the slowly oscillating functions of the magnetic field due to the beating. Hence, the conductivity contains the terms \(\sim R^2 D \cos (4\pi t/\hbar\omega_c)^2 = (R^2 D/2)(1+\cos (8\pi t/\hbar\omega_c))\). The last term \((R^2 D/2) \cos (8\pi t/\hbar\omega_c)\) describes the slow oscillations with the frequency equal to the double beating frequency. The amplitude of the slow oscillations contains the square of the Dingle factor. Nevertheless, it can be larger than the amplitude of the fast Shubnikov oscillations. The slow oscillations depend only on the transfer integral \(t\) and are not sensitive to the exact position of the chemical potential. Hence, the amplitude of the slow oscillations does not contain the temperature smearing factor and is larger than the amplitude of the Shubnikov oscillations at high enough temperature (compared to the Dingle temperature).

The magnetization does not have such a slow oscillating term because in the lowest order on damping factors [see Eq. (22) below; the term \((1 + b \cos x)\) comes, for example, from the Dingle factor] it has only the interference of \(a \sin x (1 + b \cos x) = 1 + a \sin x + (ab/2) \sin 2x\) that does not have a constant term \(ab/2\). Hence in the same lowest order on the damping factors the magnetization does not reveal the slow oscillations.

The origin of the phase shift was explained recently in [14] basing on the Boltzmann transport equation. In this paper we shall give more strict calculation of this effect.

To calculate the conductivity we shall use the Kubo formula [21]. The procedure is similar to that in three-dimensional metals without magnetic field ([21], § 7.1.2). In magnetic field only the new set of quantum numbers \(m \equiv \{n, k_z, k_y\}\) instead of the momentum \(\vec{p}\) and the different dispersion relation (1) should be used. The evaluation of the Kubo formula without vertex corrections gives

\[
\sigma_{zz} = \frac{e^2}{V} \sum_m v_z^2(m) \int \frac{d\epsilon}{2\pi} A^2(m, \epsilon) (-n'_F(\epsilon))
\]

where the volume \(V\) is to normalize the sum over quantum numbers \(m\), \(e\) is the electron charge, the limits of the integral over \(\epsilon\) are \((-\infty; \infty)\), \(n'_F(\epsilon)\) is the derivative of the Fermi distribution function:

\[
-n'_F(\epsilon) = 1/\{4T \cosh^2 [(\epsilon - \mu)/2T] \}
\]

and \(A(m, \epsilon)\) is the spectral function that is related to the electron Green’s function \(G^R(m, \epsilon)\) or to the retarded self energy part \(\Sigma^R(m, \epsilon)\):

\[
A(m, \epsilon) \equiv -2\text{Im}G^R(m, \epsilon) = \frac{-2\text{Im}\Sigma^R(m, \epsilon)}{\epsilon - \epsilon(m) - \text{Re}\Sigma^R(m, \epsilon)}\frac{\epsilon - \epsilon(m) - \text{Re}\Sigma^R(m, \epsilon)}{2} + \left[\text{Im}\Sigma^R(m, \epsilon)\right]^2
\]
The formula \(3\) is close to the corresponding formula without magnetic field \([21] \), formula 7.1.10] until the self energy part \(\Sigma^R(m, \epsilon)\) is specified. It arises mainly from the impurity scattering. The constant part of the real part \(\text{Re}\Sigma^R(m, \epsilon)\) of the self energy does not influence the physical effects. It produces only a constant shift of the chemical potential. The small oscillations of \(\text{Re}\Sigma^R(m, \epsilon)\) (that are \(\ll \hbar\omega_c\)) can also be neglected because the final answer is not sensitive to \(\text{Re}\Sigma^R(m, \epsilon)\). Hence, for simplicity, in the later formulas we shall put \(\text{Re}\Sigma^R(m, \epsilon) = 0\).

The imaginary part of the self energy \(\text{Im}\Sigma^R(m, \epsilon)\), on the contrary, is very important since it describes the momentum relaxation of electrons. The main contribution to the resistivity comes from the short range impurity scattering. The matrix element of the point-like impurity scattering is proportional to the number of states to which an electron may scatter, that is to the density of states at the given energy. So, if one considers the point-like impurity scattering in the Born approximation, the imaginary part of the self energy is proportional to the density of states:

\[
-\text{Im}\Sigma^R(m, \epsilon) = C \times \rho(\epsilon)
\]

The constant \(C \approx \pi C_i U^2 I_{\phi z}\) where \(C_i\) is the impurity concentration, \(U = \int V(r)d^3r\) characterizes the strength of the impurity potential and \(I_{\phi z}\) is a number that depends on the electron wave function and the impurity distribution along the z-axis. The physical reason for the proportionality is clear. \(1/\tau(\epsilon) \sim \rho(\epsilon)\). The formula \(3\) violates if one considers the next terms on the small parameter \(UN_{LL}/\hbar\omega_c = f/d\), where \(N_{LL}\) is the LL degeneracy per unit area, \(f - \) is the scattering amplitude (which is constant at small wave vector \(q \ll 1/r_0\), \(r_0\) is the range of the impurity potential) and \(d\) is the interlayer distance. Usually this parameter \(f/d\) is really small.

The constant \(C\) in \(3\) contains many unknown parameters. We shall not calculate them here but note that \(C\) is simply related to the average Dingle temperature \(T_D\):

\[
\langle |\text{Im}\Sigma^R(m, \epsilon)| \rangle = C \cdot \langle r(\epsilon) \rangle = C \cdot (N_{LL}/\hbar\omega_c) (1 + n_R) = \pi k_B T_D
\]

where \(\langle \ldots \rangle\) means the average value of something, \(k_B = 1.38 \times 10^{-16} \text{erg/K}\) is the Boltzmann constant and \(n_R\) is the density of reservoir states, that exist in many organic metals due to the open sheets of the FS.

In the extremely 2D case (\(\hbar\omega_c \gg t\)) the substantial deviations from the formula \(3\) are possible because the strong degeneracy of the LLs makes the simple perturbation theory not applicable. Since we consider the case \(2t > \hbar\omega_c\) (when the beating of the oscillations is visible) we shall take \(3\) for our subsequent calculations that are now straightforward.

Performing the summation over \(k_y\) in \(3\) and changing the integration over \(k_z\) by the integration over energy \(\epsilon(n, k_z)\) we get

\[
\sigma_{zz} = e^2 N_{LL} \sum_n 2 \int_0^{\pi} \frac{d(k_z)}{2\pi} v_z^2(k_z) \int \frac{d\epsilon}{2\pi} A^2(\epsilon(k_z, n), \epsilon) (-n'_F(\epsilon)) =
\]

\[
= e^2 N_{LL} d \int \frac{d\epsilon}{2\pi} \sum n \left| v_z(\epsilon', n) \right| \int \frac{d\epsilon}{2\pi} A^2(\epsilon', \epsilon) (-n'_F(\epsilon))
\]

(7)

where the velocity \(v_z(\epsilon, n)\) is determined from the dispersion relation \(4\)

\[
v_z(\epsilon, n) = \frac{\partial \epsilon(n, k_y, k_z)}{\hbar \partial k_z} = -\frac{2td}{\hbar} \sin(k_zd) =
\]

\[
= \frac{d}{\hbar} \sqrt{4t^2 - (\epsilon - \hbar\omega_c(n + 1/2))^2}
\]

(8)
To go further we have to transform the sum over LLs to the sum over harmonics. This can be done using the Poisson summation formula (Appendix A). Substituting (12) to (3) we get:

\[
\sigma_{zz} = e^2 N_{LL} \int \frac{de'}{2 \pi} \sum_{k=-\infty}^{\infty} \left( -1 \right)^k \frac{2 \pi k t^2}{\hbar k} \exp \left( \frac{2 \pi i k \epsilon'}{\hbar \omega_c} \right) J_1 \left( \frac{4 \pi k t}{\hbar \omega_c} \right) \int \frac{de}{2 \pi} A^2 (\epsilon', \epsilon) (-n'_F(\epsilon)) =
\]

\[
= e^2 N_{LL} \sum_{k=-\infty}^{\infty} \left( -1 \right)^k \frac{2 \pi k t^2}{\hbar k} J_1 \left( \frac{4 \pi k t}{\hbar \omega_c} \right) \int \frac{de}{2 \pi} (-n'_F(\epsilon)) I_z(\epsilon, k)
\]

where for the zeroth harmonic \( k = 0 \) one should use the expansion \( J_1(x) = x/2 \) at \( x \ll 1 \), and the integral \( I_z(\epsilon, k) \) over \( \epsilon' \) can be easily evaluated with the spectral function (14):

\[
I_z(\epsilon, k) = \int \frac{dx'}{2 \pi} A^2 (\epsilon', \epsilon) \exp \left( \frac{2 \pi i k \epsilon'}{\hbar \omega_c} \right) =
\]

\[
\int \frac{dx'}{2 \pi} \left( \frac{-2 \Im \Sigma^R(\epsilon)}{\epsilon - \epsilon'} + \left| \Im \Sigma^R(\epsilon) \right|^2 \right) \exp \left( \frac{2 \pi i k \epsilon'}{\hbar \omega_c} \right) =
\]

\[
= \exp \left( \frac{2 \pi i k \epsilon}{\hbar \omega_c} \right) \left( \frac{1}{\left| \Im \Sigma^R(\epsilon) \right|^2} + \frac{2 \pi k}{\hbar \omega_c} \right) R_D(k, \epsilon)
\]

(9)

where

\[
R_D(k, \epsilon) = \exp \left( -2 \pi |k| \frac{\Im \Sigma^R(\epsilon)}{\hbar \omega_c} \right)
\]

(11)

has the form, similar to that of the usual Dingle factor \( R_D(k) = \exp \left( -2 \pi^2 k k_B T_D/\hbar \omega_c \right) \). Collecting the formulas (1) and (11) we get:

\[
\sigma_{zz} = e^2 N_{LL} \int \frac{de}{2 \pi} (-n'_F(\epsilon)) \sum_{k=-\infty}^{\infty} \left( -1 \right)^k \frac{2 \pi k t^2}{\hbar k} J_1 \left( \frac{4 \pi k t}{\hbar \omega_c} \right) \times
\]

\[
\exp \left( \frac{2 \pi i k \epsilon}{\hbar \omega_c} \right) \left( \frac{1}{\left| \Im \Sigma^R(\epsilon) \right|^2} + \frac{2 \pi k}{\hbar \omega_c} \right) R_D(k, \epsilon)
\]

(12)

We have to consider all the terms in the expression (12) for the conductivity that make an essential contribution to the oscillations. Besides the directly oscillating term \( \exp(2 \pi i k \epsilon/\hbar \omega_c) \) these are the imaginary part of the self energy \( \Im \Sigma^R(\epsilon) \) and the Dingle factor \( R_D(k, \epsilon) \). The imaginary part of the self energy is given by (1) where the density of states is

\[
\rho(E) \equiv \frac{1}{2 \pi} \sum_{m} A(m, E) = \frac{N_{LL}}{(2 \pi)^2} \hbar \sum_{n=0}^{\infty} \frac{2 \pi}{v_z(\epsilon(0), n)} A(\epsilon(m), E)
\]

(13)

The sum over LLs in (13) can be again represented as a harmonic series using the Poisson summation formula. Substituting (14) into (13) we get

\[
\rho(E) = \sum_{k=-\infty}^{\infty} \left( -1 \right)^k \frac{N_{LL}}{\hbar \omega_c} J_0 \left( \frac{4 \pi k t}{\hbar \omega_c} \right) \int \frac{de(m)}{2 \pi \hbar} A(\epsilon(m), E) \exp \left( \frac{2 \pi i k \epsilon(m)}{\hbar \omega_c} \right) =
\]

\[
= \frac{N_{LL}}{\hbar \omega_c} \sum_{k=-\infty}^{\infty} \left( -1 \right)^k J_0 \left( \frac{4 \pi k t}{\hbar \omega_c} \right) \exp \left( \frac{2 \pi i k E}{\hbar \omega_c} \right) R_D(k, E)
\]

(14)
From the formulas (5, 6 and 14), using \( J_0(0) = 1 \) we get

\[
|\text{Im}\Sigma^R(m, \epsilon)| = \pi k_B T D \left( 1 + 2 \sum_{k=1}^{\infty} (-1)^k J_0 \left( \frac{4\pi k t}{\hbar \omega_c} \right) \cos \left( \frac{2\pi k \epsilon}{\hbar \omega_c} \right) R_D(k, \epsilon) \right)
\]

(15)

Together with (11) this is a nonlinear equation on \( \text{Im}\Sigma^R(m, \epsilon) \). We shall obtain explicit results in the next two subsections only in the limit of strong harmonic damping where an expansion over the harmonic damping factors in Eqs. (11) and (15) is possible.

Substituting (15) into (12) we obtain the following expression for the conductivity:

\[
\sigma_{zz} = e^2 N_{LL} \int d\epsilon \left( -n'_F(\epsilon) \right) \frac{2(td)^2}{\hbar^2 \omega_c} \left( 1 + \frac{\hbar \omega_c}{\pi t} \sum_{k=1}^{\infty} (-1)^k \frac{J_1 \left( \frac{4\pi k t}{\hbar \omega_c} \right)}{\frac{2\pi k \epsilon}{\hbar \omega_c}} \right) \cos \left( \frac{2\pi k \epsilon}{\hbar \omega_c} \right) R_D(k, \epsilon) +
\]

\[
+ e^2 N_{LL} \int d\epsilon \left( -n'_F(\epsilon) \right) \frac{4td^2}{\hbar \omega_c} \sum_{k=1}^{\infty} (-1)^k J_1 \left( \frac{4\pi k t}{\hbar \omega_c} \right) \cos \left( \frac{2\pi k \epsilon}{\hbar \omega_c} \right) R_D(k, \epsilon)
\]

(16)

Generally speaking, one should also take into account the oscillations of the chemical potential, that are given by (10), formula 5). The formula (16) is quite huge and without further simplification is applicable only for the numerical calculation of the conductivity. In the next two sections we shall analyze the limit of strong harmonic damping.

3 The first order on damping factors; the phase shift of the beats.

For simplicity we now assume the harmonic damping to be strong and neglect all the terms that have additional powers of the damping factors: the Dingle factor \( R_D \) or the temperature smearing factor

\[
R_T(k) = \left( 2k \pi^2 k_B T / \hbar \omega_c \right) / \sinh \left( 2k \pi^2 k_B T / \hbar \omega_c \right)
\]

that will appear after integration over energy with the Fermi distribution function. So, we rest only the first harmonic in the expression (11) for the conductivity. This approximation is sufficient to obtain the phase shift of the beating. In this approximation the Dingle factor \( R_D(k, \epsilon) \) and the chemical potential can be considered as a constant. The conductivity then becomes

\[
\sigma_{zz} = \frac{e^2 N_{LL}}{2\pi} \int d\epsilon \left( -n'_F(\epsilon) \right) \frac{2(td)^2}{\hbar^2} \frac{1}{\pi k_B T D} \frac{2t}{\hbar \omega_c} \times \frac{1}{1 - 2 \cos \left( \frac{2\pi k \epsilon}{\hbar \omega_c} \right) J_0 \left( \frac{4\pi t}{\hbar \omega_c} \right) R_D} -
\]

\[
- \frac{e^2 N_{LL}}{2\pi} \int d\epsilon \left( -n'_F(\epsilon) \right) \frac{2(td)^2}{\hbar^2} \frac{1}{\pi k_B T D} \frac{2}{\hbar \omega_c} \cos \left( \frac{2\pi k \epsilon}{\hbar \omega_c} \right) J_1 \left( \frac{4\pi t}{\hbar \omega_c} \right) \left( 1 + \frac{2\pi^2 k_B T D}{\hbar \omega_c} \right) R_D
\]

(17)

Since we consider the limit \( R_D \ll 1 \) the constant term is much larger than the oscillating term in the denominator in the first line of Eq. (17). Hence, one can move up the small oscillating term
\[
\sigma_{zz} = \frac{e^2 N_{LL}}{8\pi k_B T_D} \frac{(4td)^2}{\hbar^2 \omega_c} \left\{ 1 + 2 \cos \left( \frac{2\pi \mu}{\hbar \omega_c} \right) R_T R_D \left[ J_0 \left( \frac{4\pi t}{\hbar \omega_c} \right) - \frac{\hbar \omega_c}{2\pi t} \left( 1 + \frac{2\pi^2 k_B T_D}{\hbar^2 \omega_c} \right) J_1 \left( \frac{4\pi t}{\hbar \omega_c} \right) \right] \right\}
\]

(18)

where \( \mu \) is the chemical potential and the temperature smearing factor \( R_T \) is given by the usual L-K expression:

\[
R_T = \frac{2\pi^2 k_B T}{\sinh (2\pi^2 k_B T/\hbar \omega_c)}
\]

If the transfer integral is still large enough \( 4\pi t \gg \hbar \omega_c \) one can use the expansions of the Bessel function at large value of argument:

\[
J_0(x) \approx \sqrt{\frac{2}{\pi x}} \cos (x - \pi/4), \quad x \gg 1
\]

(19)

\[
J_1(x) \approx \sqrt{\frac{2}{\pi x}} \sin (x - \pi/4), \quad x \gg 1;
\]

Then the expression in square brackets of (18) simplifies and making use of the standard trigonometric formulas one can write the oscillating part of the conductivity as

\[
\tilde{\sigma}_{zz} = \frac{e^2 N_{LL}}{8\pi k_B T_D} \frac{(4td)^2}{\hbar^2 \omega_c} \times 2 \cos \left( \frac{2\pi \mu}{\hbar \omega_c} \right) R_T R_D \sqrt{\frac{\hbar \omega_c}{2\pi^2 t}} \sqrt{1 + a^2} \cos \left( \frac{4\pi t}{\hbar \omega_c} - \frac{\pi}{4} + \phi \right)
\]

(20)

where

\[
a = \frac{\hbar \omega_c}{2\pi t} \left( 1 + \frac{2\pi^2 k_B T_D}{\hbar^2 \omega_c} \right)
\]

and \( \phi = \arctan (a) \)

(21)

One important difference of this formula from the usual Lifshitz-Kosevich expression is the phase shift \( \phi \) of the beats of the oscillations. This phase shift can be experimentally observed by comparison with the dHvA oscillations, that possess the phase offset consistent with the standard Lifshitz-Kosevich formula and are proportional to

\[
\tilde{M} \sim \sin \left[ \frac{2\pi \mu}{\hbar \omega_c} \right] R_D(T_D) R_T(T) \cos(4\pi t/\hbar \omega_c - \pi/4)
\]

(22)

The above result concerning the phase of the beats of the SdH oscillations has been recently qualitatively confirmed by a comparative experimental study of the SdH and dHvA oscillations in an organic metal \( \beta_-(\text{BEDT-TTF})_2\text{IBr}_2 \) [14].

The main limitation of the proposed analysis is that the magnetic field is taken to be perpendicular to the conducting layers. A finite tilting angle \( \theta \) of the magnetic field with respect to the normal to the conducting planes may be approximately taken into account by rescaling the Landau level separation: \( \omega_c \rightarrow \omega_c \cos \theta \) and the warping of the Fermi surface [24]: \( t(\theta) = t(0) J_0(k_F d \tan \theta) \), where \( k_F \) is the in-plane Fermi momentum. But this is only a semiclassical approximation based on the assumption that the FS remains the same. Actually, the tilting of the magnetic field changes the dispersion relation and a more profound study of the effect of the tilting magnetic field on the transport properties is required. The quantum mechanical calculation of the dispersion relation in tilted magnetic field in the first order on the transfer integral [25] gives the result close to
that of Yamaji\cite{24}; the correction is that the Bessel’s function should be replaced by the Laguerre polynomial\cite{25}:

\[ J_0(k_F d \tan \theta) \rightarrow e^{- (A/2)^2} \times L_n \left( A^2 / 2 \right), \]

where \( A^2 / 2 = d^2 m \omega_c \tan^2 (\theta) / 2 \hbar \) and \( n = \mu / h \omega_c \).

But the use of the perturbation theory assumes that the transfer integral \( t \) is much less than the Landau level separation \( \hbar \omega_c \). Here we consider the opposite case \( 2t > \hbar \omega_c \) where the beating can be seen and some more profound study of the effect of the tilted magnetic field is needed. For example, the effect of the tilting angle on the conductivity should not be described just by this replacement near the zeros of \( J_0(k_F d \tan \theta) \). So, our analysis is applicable to a nonzero tilting angle only qualitatively.

Other errors may come from the approximate expression for the self energy (5). The Born approximation in Q2D case works quite well, but maybe other scattering mechanisms (especially for the calculation of the DoS) should be taken into account. An accurate study of this problem may depend on a particular type of a compound in hand.

The above analysis does not take into account the vertex corrections. In our case this is valid because, according to the Ward identity, the vertex \( \Gamma(m, E) = \vec{p} + m \vec{\nabla}_p \Sigma_R(m, E) \). Hence, if the retarded self energy depends only on energy, the vertex corrections are zero. The fact that \( \Sigma_R(m, \epsilon) \) is approximately a function of only the energy \( \epsilon \) is a consequence of the short-range (or point-like) impurity potential. More precisely, if one takes a point-like impurity potential and neglects all diagrams with the intersections of the impurity lines in the self energy, then after averaging over randomly and uniformly distributed impurity positions one obtains that \( \Sigma_R(m, \epsilon) = \Sigma_R(\epsilon) \). The neglected graphs with the intersections of the impurity lines describe the coherent scattering on two impurities simultaneously. The contribution of such a scattering is small at large enough interlayer transfer integral. In the three-dimensional case without magnetic field the vertex corrections produce the factor \( 1 - \cos \alpha \) instead of 1 in integrand for the transport scattering relaxation time where \( \alpha \) is the scattering angle. But if the scattering probability is independent of the scattering angle (as for point-like impurities) the additive \( \cos \alpha \) vanishes after the integration over angles and the vertex corrections vanish.

One should note that when the transfer integral is not much greater than the cyclotron energy \( (4 \pi t / \hbar \omega_c \sim 1) \) one cannot use the expansions \( \left[ 19 \right] \) of the Bessel’s functions and should apply the formula \( \left[ 18 \right] \) for the beatings of the conductivity oscillations. This slightly changes the phase shift and makes the beatings not periodic. This should be taken into account in the extraction of the transfer integral from the beating frequency.

### 4 The slow oscillations

The slow oscillations come from the interference of the oscillations of different quantities in \( \left[ 16 \right] \). Hence, one should consider the entanglement of all oscillating quantities and find the non-oscillating term in the lowest order on damping factors. For example, the Dingle factor should no more be considered as constant because the entanglement of the oscillations of the Dingle factor with other oscillating quantities produce the term of the same order as the interference of the oscillations of the mean square velocity and the scattering time. The errors in the self energy now become more important, so, the subsequent analysis may not be quantitatively very accurate. Nevertheless, it describes the main features of the slow oscillations of the conductivity.
Up to the square of the Dingle factor in Eq. (16) we get

\[
\sigma_{zz} = \frac{e^2 N_{LL} (4td)^2}{8\pi k_B T_D \hbar^2 \omega_c} \int d\epsilon (-n'_F(\epsilon)) \times \\
\left\{ 1 - \frac{\hbar \omega_c}{\pi t} J_1 \left( \frac{4\pi t}{\hbar \omega_c} \right) \cos \left( \frac{2\pi \epsilon}{\hbar \omega_c} \right) R_D(\epsilon) \right\} \times \\
\left[ 1 + 2J_0 \left( \frac{4\pi t}{\hbar \omega_c} \right) \cos \left( \frac{2\pi \epsilon}{\hbar \omega_c} \right) R_D(\epsilon) + \left( 2J_0 \left( \frac{4\pi t}{\hbar \omega_c} \right) \cos \left( \frac{2\pi \epsilon}{\hbar \omega_c} \right) R_D(\epsilon) \right)^2 \right] \\
- \frac{2\pi k_B T_D}{t} \frac{\hbar}{\omega_c} \left( \frac{4\pi t}{\hbar \omega_c} \right) \cos \left( \frac{2\pi \epsilon}{\hbar \omega_c} \right) R_D(\epsilon) \right\}
\]

(23)

where \( R_D(\epsilon) \) is now given by (3, 3 and 11)

\[
R_D(\epsilon) \approx \exp \left\{ -\frac{2\pi^2 k_B T_D}{\hbar \omega_c} \left( 1 - 2J_0 \left( \frac{4\pi t}{\hbar \omega_c} \right) \cos \left( \frac{2\pi \epsilon}{\hbar \omega_c} \right) R_D \right) \right\} \approx \\
R_D^0 \left( 1 + \frac{2\pi^2 k_B T_D}{\hbar \omega_c} \right) J_0 \left( \frac{4\pi t}{\hbar \omega_c} \right) \cos \left( \frac{2\pi \epsilon}{\hbar \omega_c} \right) R_D^0
\]

(24)

Combining these two expressions we get:

\[
\sigma_{zz} = \frac{e^2 N_{LL} (4td)^2}{8\pi k_B T_D \hbar^2 \omega_c} \int d\epsilon (-n'_F(\epsilon)) \times (S_1 + S_2)
\]

(25)

where

\[
S_1 \equiv 1 + 2 \cos \left( \frac{2\pi \epsilon}{\hbar \omega_c} \right) \left[ J_0 \left( \frac{4\pi t}{\hbar \omega_c} \right) - \frac{\hbar \omega_c}{2\pi t} \left( 1 + \frac{2\pi^2 k_B T_D}{\hbar \omega_c} \right) J_1 \left( \frac{4\pi t}{\hbar \omega_c} \right) \right] R_D
\]

(26)

describes the constant term and the main term of the usual SdH oscillations (they are the same as in (18)), and

\[
S_2 \equiv R_D^0 \frac{4\pi t}{\hbar \omega_c} \left[ J_0 \left( \frac{4\pi t}{\hbar \omega_c} \right) \left( 1 + \frac{2\pi^2 k_B T_D}{\hbar \omega_c} \right) - \frac{\hbar \omega_c}{2\pi t} J_1 \left( \frac{4\pi t}{\hbar \omega_c} \right) \right] R_D \left( \frac{2\pi^2 k_B T_D}{\hbar \omega_c} \right)^2
\]

(27)

describes the slow oscillations.

We shall consider the case \( 4t \gg \hbar \omega_c \) where our analysis based on the self energy in the form (3) has better accuracy and one can use the large argument expansions (13) of Bessel functions. The formula (27) then simplifies:

\[
S_2 \approx R_D^0 \frac{\hbar \omega_c}{\pi^2 2t} \left( 1 + \frac{2\pi^2 k_B T_D}{\hbar \omega_c} \right) \left[ 1 + \sqrt{1 + a_S^2 \cos \left( \frac{8\pi t}{\hbar \omega_c} - \frac{\pi}{2} + \phi_S \right)} \right]
\]

(28)

where

\[
a_S = \frac{\hbar \omega_c}{2\pi t} \left( 1 + \frac{\left( 2\pi^2 k_B T_D / \hbar \omega_c \right)^2}{1 + 2\pi^2 k_B T_D / \hbar \omega_c} \right) \quad \text{and} \quad \phi_S = \arctan (a_S)
\]

(29)
We now have to consider the temperature and other smearing factors. But the expression (28) does not depend on electron energy (unlike the rapidly oscillating term (26) responsible for the SdH oscillations) and, hence, all types of smearing coming from the averaging over the electron energy do not affect the slow oscillations (they produce a factor 1). Hence, although the slow oscillations have a factor \( R_D^2 \) squared, they do not have the temperature damping factor \( R_T \) entering the SdH oscillations. Therefore, the slow oscillations may be much stronger than the SdH oscillations. This was indeed observed at high enough temperature\[3, 15, 16\].

Now collecting the formulas (20), (25) and (28) and performing the integration over \( \epsilon \) in (25) one obtains up to the second order on damping factors:

\[
\sigma_{zz} = \frac{e^2 N_{LL}}{8\pi k_B T_D} \frac{W^2 d^2}{\hbar^2 \omega_c} \left\{ 1 + 2 \cos \left( \frac{2\pi \mu}{\hbar \omega_c} \right) \sqrt{\frac{2\hbar \omega_c}{\pi^2 W}} \sqrt{1 + a^2 \cos \left( \frac{4\pi t}{\hbar \omega_c} - \frac{\pi}{4} + \phi \right)} R_D R_T + \frac{\hbar \omega_c}{\pi^2 \hbar} \left[ 1 + \sqrt{1 + a_S^2 \cos \left( \frac{8\pi t}{\hbar \omega_c} - \frac{\pi}{2} + \phi_S \right)} \right] R_D^2 \right\}
\]

\[\text{(30)}\]

The formulas (30) and (29) make some predictions about the slow oscillations that can be compared with the experimental data.

1). The frequency \( F_S \) of the slow oscillations (of the conductivity as a function of the inverse magnetic field) is two times larger than the beating frequency. So, the distance between the two nearest nodes of the beatings should be equal to the period of the slow oscillations.

2). The amplitude of the slow oscillations is temperature independent. Of course, if one consider the next terms in the expansion of the damping factor the temperature dependence will appear (for example, from the oscillations of the chemical potential). But these corrections are small. At \( T \gtrsim k_B T_D \) the amplitude of the slow oscillations is larger than the amplitude of the SdH oscillations while at \( T < k_B T_D \) the slow oscillations are hardly distinguishable behind the SdH oscillations.

3). The phase shift of the slow oscillations is given by (28) and (29). It is equal to \( \phi_S / 2 \approx \phi / 2 \) (the factor 1/2 is because the frequency of slow oscillations is two times greater than the beating frequency). Hence the peaks of the slow oscillations must be between the nearest peaks of the beats in the magnetization and conductivity. But this phase is quite sensitive to the exact expression of the self energy. Since we have used the approximate expression for \( \text{Im} \Sigma^R(m, \epsilon) \) (formula 3) the substantial error in the estimate of \( \phi_S \) is possible. For example, the long range crystal imperfections substantially damp the oscillations of the DoS that so that the oscillations of the Dingle factor are much weaker than in formula (24). This long range electron scattering should be taken into account when the \( \text{Im} \Sigma^R(m, \epsilon) \) is calculated.

The temperature independent harmonic damping factor comes not only from the impurity scattering but also from the macroscopic inhomogeneities of a sample. For example, the long-range background potential makes the electron kinetic energy at the Fermi level slightly different over the sample that produces the additional temperature independent harmonic damping. The corresponding part of the Dingle temperature should not be included into (1) and to the phase shifts (22) and (29). This smearing like the temperature smearing does not affect the phase shift and the amplitude of the slow oscillations but causes an additional damping of the SdH or dHvA oscillations. To make a quantitative estimate of the amplitude of slow oscillations one can separate the impurity part of the Dingle temperature from the comparison of the phase-shift and then apply the formulas (24), (25) and (28) without adjustable parameters to compare the amplitudes of the oscillations.
To summarize, we have calculated the main terms in the conductivity $\sigma_{zz}$ in the layered compound in normal magnetic field. The obtained formula describes the slow oscillations and the phase shift of the beatings. But the limitations of the calculation due to zero tilting angle and the approximate form of the self energy may lead to some quantitative deviations in the phase and the amplitude of the slow oscillations.

We are thankful to A.M. Dyugaev and I. Vagner for the encouragement and stimulating discussions. The work was supported by the EU ICN contract HPRI-CT-1999-40013, and grants DFG-RFBR No. 436 RUS 113/592 and RFBR No. 00-02-17729a.

A Appendix

A.1 Transformation of the sums over LLs to the sums over harmonics

To transform the sums over LL number into the harmonic sums we shall apply the Poisson summation formula

$$\sum_{n=n_0}^{\infty} f(n) = \sum_{k=-\infty}^{\infty} \int_{a}^{\infty} e^{2\pi i kn} f(n) \, dn,$$

where $a \in (n_0 - 1; n_0)$ (31)

This formula is valid for arbitrary function $f(n)$. The sum in (7) becomes

$$\sum_{n} |v_z(\epsilon, n)| = \sum_{n=0}^{\infty} \frac{1}{\hbar} \sqrt{4t^2 - \left(\epsilon - \hbar \omega_c \left( n + \frac{1}{2} \right) \right) ^2} =$$

$$= \frac{d}{\hbar} \hbar \omega_c \sum_{k=-\infty}^{\infty} \int_{0}^{\infty} dn \exp \left(2\pi i k \left( n - \frac{1}{2} \right) \right) \sqrt{\left( \frac{2t}{\hbar \omega_c} \right)^2 - \left( \frac{\epsilon}{\hbar \omega_c} - n \right) ^2}$$

$$= \frac{d}{\hbar} \hbar \omega_c \sum_{k=-\infty}^{\infty} (-1)^k \exp \left( \frac{2\pi i k \epsilon}{\hbar \omega_c} \right) \int_{-\infty}^{\infty} dx \exp \left(2\pi i k x\right) \sqrt{\left( \frac{2t}{\hbar \omega_c} \right)^2 - x^2}$$

$$= \sum_{k=-\infty}^{\infty} \frac{dt}{\hbar k} (-1)^k \exp \left( \frac{2\pi i k \epsilon}{\hbar \omega_c} \right) J_1 \left( \frac{4\pi k t}{\hbar \omega_c} \right)$$

(32)

In an analogous way one can also transform the sum

$$\frac{\pi}{N_{LL}} \rho_0(\epsilon) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{4t^2 - \left( \epsilon - \hbar \omega_c \left( n + \frac{1}{2} \right) \right) ^2}} =$$

(33)
\[
\frac{1}{\hbar \omega_c} \sum_{k=-\infty}^{\infty} \int_0^\infty \frac{dn}{\sqrt{2n\hbar}} \exp \left( \frac{2\pi ik (n - \frac{1}{2})}{\hbar \omega_c} \right) = \\
= \frac{1}{\hbar \omega_c} \sum_{k=-\infty}^{\infty} (-1)^k \exp \left( \frac{2\pi ike}{\hbar \omega_c} \right) \int_{-\infty}^{\infty} dx \exp \left( \frac{2\pi ike}{\hbar \omega_c} \right) \left( \frac{4\pi kt}{\hbar \omega_c} \right)^2 \\
= \frac{\pi}{\hbar \omega_c} \sum_{k=-\infty}^{\infty} (-1)^k \exp \left( \frac{2\pi ike}{\hbar \omega_c} \right) J_0 \left( \frac{4\pi kt}{\hbar \omega_c} \right) 
\] 

(34)

References

[1] Shoenberg D. ”Magnetic oscillations in metals”, Cambridge University Press 1984

[2] T. Ishiguro, K. Yamaji and G. Saito, Organic Superconductors, 2nd Edition, Springer-Verlag, Berlin, 1998.

[3] J. Wosnitza, Fermi Surfaces of Low-Dimensional Organic Metals and Superconductors (Springer-Verlag, Berlin, 1996); M.V. Kartsovnik and V.N. Laukhin, J. Phys. I France 6, 1753 (1996); J. Singleton, Rep. Prog. Phys. 63, 1111 (2000).

[4] V.M. Gvozdikov, Fiz. Tverd. Tela 26, p. 2574 (1984) [Sov. Phys. Solid State 26 (9), p. 1560 (1984)]

[5] K. Jauregui, V.I. Marchenko, I.D. Vagner, Phys. Rev. B 41, 12922 (1990).

[6] N. Harrison et al., Phys. Rev. B 54, 9977 (1996).

[7] M.A. Itskovsky, T. Maniv and I.D. Vagner, Phys. Rev. B 61, 14616 (2000).

[8] P. Grigoriev and I. Vagner, Pis’ma Zh. Eksp. Teor. Fiz., 69, 139 (1999) [JETP Letters 69, 156 (1999)].

[9] T. Champel and V.P. Mineev, Phil. Magazine B 81, 55 (2001).

[10] P. Grigoriev, Zh. Eksp. Teor. Fiz, 119(6), 1257 (2001) [JETP 92, 1090 (2001)].

[11] A.E. Datars and J.E. Sipe, Phys. Rev. B 51, 4312 (1995)

[12] P. Moses and R.H. McKenzie, Phys. Rev. B 60, 7998 (1999)

[13] V.M. Gvozdikov, Fiz. Nizk. Temp. 27 (9/10) (2001) [Sov. J. Low Temp. Phys. 27(9/10) (2001)].

[14] P. Grigoriev, M. Kartsovnik, W. Biberacher et al., cond-mat/0108091 (2001).

[15] M. Kartsovnik, V. Laukhin, V. Nizhankovskii and A. Ignat’ev, Pis’ma Zh. Eksp. Teor. Fiz. 47, 302 (1988) [Sov. Phys. JETP Lett. 47, 363 (1988)]; M. Kartsovnik, P. Kononovich, V. Laukhin and I. Schegolev, Pis’ma Zh. Eksp. Teor. Fiz. 48, 498 (1988) [Sov. Phys. JETP Lett. 48, 541 (1988)]; M. Kartsovnik, V. Laukhin and S. Pesotskii, Fiz. Nizk. Temp. 18, 22 (1992) [Sov. J. Low Temp. Phys. 18 13 (1992)].
[16] M. Kartsovnik, P. Kononovich, V. Laukhin, S. Pesotskii and I.F. Schegolev, Pis’ma Zh. Eksp. Teor. Fiz. 49, 453 (1989) [Sov. Phys. JETP Lett. 49, 519 (1989)].

[17] T. Terashima, S. Uji, H. Aoki, M. Tamura, M. Kinoshita and M. Tokumoto, Solid State Commun. 91, 595 (1994).

[18] E. Ohmichi, H. Ito, T. Ishiguro, G. Saito and T. Komatsu, Phys. Rev. B 57, 748 (1998).

[19] B. Narymbetov, N. Kushch, L. Zorina, S. Khasanov, R. Shibaeva, T. Togonidze, A. Kovalev, M. Kartsovnik, L. Buravov, E. Yagubskii, E. Canadell, A. Kobayashi and H. Kobayashi, Eur. Phys. J. B 5, 179 (1998); T. Togonidze, M. Kartsovnik, J. Perenboom, N. Kushch and H. Kobayashi, Physica B 294-295, 435 (2001).

[20] L. Balicas, J. Brooks, K. Storr, D. Graf, S. Uji, H. Shinagawa, E. Ojima, H. Fujiwara, H. Kobayashi, A. Kobayashi and M. Tokumoto, cond-mat/0008287 (2000).

[21] G. Mahan "Many-Particle Physics", 2nd ed., Plenum Press, New York, 1990, (§7.1)

[22] "Standard Mathematical Tables and Formulae", CRC Press, (1996)

[23] P. Grigoriev, JETP, iss. 6 (in press), (2001)

[24] Yamaji K., J. Phys. Soc. Jpn. 58, 1520 (1989)

[25] Yasunari Kurihara, J. Phys. Soc. Jpn. 61, 975 (1992)