A possible explanation of the stair-step brittle deformation evolutionary pattern of a rockslide

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ABSTRACT

The focus of the present study is to explain the stair-step brittle deformation evolutionary pattern of the landslide and reveal the corresponding internal physical mechanism. Based on the concept of high strength geological body (HSGB), similar to rock bridges in the intermittence jointed rock masses, a conceptual geological model and the corresponding mechanical model are successively outlined. Through reanalysing typical stair-step displacement records of a wedge rockslide, it was found that the HSGB model can reasonably explain the stair-step deformation feature and reveal its internal essence. Finally, a displacement criterion is derived through combining the two-dimensional renormalization group model with the shear creep constitutive model. By means of this displacement criterion, one can quantitatively describe the displacement ratio between the onset of the accelerating creep and brittle rupture of HSGB. In order to make the reader understand this displacement criterion better, a detailed application process is demonstrated through a case study. To some extent, the present study has shed light on the failure prediction of the HSGB-type landslide, whose stability is mainly controlled by HSGB.

1. Introduction

In the fields of engineering geology and rock engineering, the failure mechanism and the failure time prediction of landslides are always attractive topics. Much effort has been devoted to creating various prediction models, such as Saito’s model (Saito 1965), Fukuzono’s model (Fukuzono 1985) and Voight’s model (Voight 1988, 1989). It is widely recognized that when involving the failure prediction issue, the first step is to choose a certain physical quantity as a predictor, such as displacement, velocity and acceleration. Usually displacement is preferred and its possible evolutionary patterns have been described in detail and classified into several types in the literature (Allison and Brunsden 1990; Crosta and Agliardi 2002, 2003; Petley et al. 2005; Rose and Hungr 2007), which mainly includes the chaotic, periodical and brittle patterns.

Usually, the global deformation trend of a slope could be identified through using monitoring data from multiple monitoring stations. However, there are often some individual monitoring data that show a random, chaotic, even contradictory movement characteristic compared with the global deformation trend. This evolutionary pattern is termed the chaotic pattern, which only reflects the local movement and makes no sense for the failure prediction of a slope. The so-called periodical pattern means that displacement records usually show some regular fluctuations. For example, the precipitation...
changes with the seasons, which may cause alternating acceleration and deceleration deformation behaviours of the landslide. This is a kind of common deformation evolutionary pattern which is characterized by a long span of time and the progressive displacement change. However, these two kinds of evolutionary patterns are not research emphases and we will focus on the brittle deformation evolutionary pattern in the present study. As illustrated in Figure 1, it can be easily observed that displacement records of a rockslide, whose background information will be introduced in the next section, show a stair-step shape with alternating nearly horizontal segments (i.e. b-c and d-e) and almost vertical upward abrupt segments (i.e. a-b, c-d and e-f), which is the most prominent feature of the brittle deformation evolutionary pattern. Thus, some doubts usually arise as follows:

1. the catastrophic failure of a landslide usually occurs when deformation increases very sharply. However, it is not the case for a-b, c-d and e-f segments in Figure 1. How do we explain qualitatively this phenomenon?
2. what is the internal physical mechanism of the brittle deformation evolutionary pattern? At least it could account for Figure 1.

Reasonable explanations for the above doubts have not been obtained up to now. Therefore, the present study attempts to do some effort. The structure of this paper is as follows: In Section 2, we present a brief background information about Figure 1 and carry out an analysis about the displacement records. In Section 3, a conceptual geological model is proposed to qualitatively explain the brittle deformation evolutionary pattern. In Section 4, a mechanical model is established to reveal the mechanical behaviour of the brittle deformation evolutionary pattern. In Section 5, a displacement criterion is derived, through which the brittle deformation evolutionary pattern can be described quantitatively. Furthermore, the application process of the displacement criterion above is demonstrated in Section 6 through reanalysing the displacement records in Figure 1. Finally, conclusions and future work are outlined in Section 7.
2. Background and analysis about the wedge rockslide 1971

2.1 Background information of the wedge rockslide 1971

The monitoring data in Figure 1 were recorded by extensometer L-7, which is a kind of wire-type multiple position borehole extensometer, and clearly reflected movement history of the wedge rockslide 1971, which is located in the left abutment of Libby Dam in northwestern Montana, USA (see Figure 2). Following Voight (1979), there exist several sets of dominant joints and a set of well-defined bedding plane in the left bank of Libby Dam. The intersection between them resulted in the formation of many potential wedge-shaped unstable blocks as shown in Figure 2, such as notch 1# (~4800 m³), notch 2# (~800 m³), wedge rockslide 1967 (~6000 m³), wedge rockslide 1971 (~33,000 m³), prehistoric wedge rockslides 925# (~270,000 m³) and 930# (~2,400,000 m³), which may have been triggered by the excavation at the toe of slope, the change of water pressure, thermal expansion of the rock mass, blasting, felt earthquake or other unknown external factors (Voight 1979 and references therein). Moreover, it can be observed from Figure 3 that the topographic change in the left bank of Libby dam is obvious with many undulating ridges and valleys.

The wedge rockslide 1971 is formed by the intersection between joint A and the bedding plane DS+122 as illustrated in Figure 2(b). It begun to slide at approximately 6:04 am (Mountain Standard Time) on 31 January 1971 and the sliding direction is nearly parallel to the dam axis. With the exception of some property loss, it is very fortunate that no casualties were reported. As mentioned by Voight (1979), the construction period of Libby Dam is from 1966 to 1973, while extensometer L-7 was installed in 1967. Therefore, the actual monitoring period of the extensometer L-7 is from June 1967 to final failure as shown in Figure 1.

2.2 Analysis about the displacement records of the wedge rockslide 1971

As shown in Figure 1, there exist some characteristic points in four displacement records, i.e. a, b, c, d, e, f and g. When we treat the points c or e as a reference point (see Figure 4), it can be easily observed that all the four displacement records are almost overlapping except a-b and b-c segments, which is especially true for c-d-e-f-g segment. Therefore, it can be intuitively inferred that the wedge rockslide 1971 may move in the form of global deformation.

In order to further confirm the inference above, an in-depth quantitative analysis about the displacement records in Figure 1 was carried out. The deformation data of different characteristic points are extracted from Figure 1, with an error range of less than two pixels, and listed in Table 1. It is assumed that the difference between a set of data can be ignored when the range, \( x_{\text{max}} - x_{\text{min}} \), is not greater than 0.5. Thus, some quantitative results can be obtained as follows:

(I) for the displacement increment between points a and b (\( \Delta ab \)), it can be obviously observed that \( \Delta ab_1 \gg \Delta ab_2 > \Delta ab_3 \), where the subscripts represent the corresponding monitoring curve labelled in Figure 1;

(II) \( \Delta bc_1 \gg \Delta bc_2 > \Delta bc_3 \), among which the range is larger than 0.5;

(III) \( \Delta cd_1 \approx \Delta cd_2 \approx \Delta cd_3 \), among which the range is less than 0.5;

(IV) \( \Delta de_1 \approx \Delta de_2 \approx \Delta de_3 \), among which the range is equal to 0.5;

(V) \( \Delta ef_1 \approx \Delta ef_2 \approx \Delta ef_3 \), among which the range is less than 0.5.

Obviously, it was found that the global deformation hypothesis above seems to be reasonable according to cases (III), (IV) and (V), but is contradictory with cases (I) and (II). How do we explain this phenomenon? Following Voight (1979), it was known that the wedge rockslide 1967 resulted from the pre-split excavation of Montana State Highway (MSH) 37 in 1967 and the termination date of blasting is indicated by TB in Figures 1 and 4, which is almost consistent with point b. Therefore, we speculated that the sharp increments in a-b segment of four displacement records may be
Figure 2. Overview of the study area.
Note 1: Here it is only an approximate installation diagram of extensometer L-7, compared with the accurate location of extensometer L-7 as illustrated by Voight (1979). Note 2: Because the pictures above are captured from Google Earth from different view angles, a unified scale bar is missing. If necessary, one can directly enter the latitude and longitude (48.410N,115.313W) to observe the study area by means of Google Earth.
attributed to the excavation. Considering that an excavation disturbed zone may subsequently develop around the excavated surface and the excavation effect on the stress and strain fields of the wedge rockslide 1971 would gradually weaken from near field to far field, we can explain the phenomenon that the increments $\Delta ab$ and $\Delta bc$ in cases (I) and (II) are gradually decreased along the direction from sensor H to sensor 3 of extensometer L-7. Meanwhile, considering that the relative displacement records of different sensors gradually become consistent as time goes on, just like cases (III), (IV) and (V), we thought that the excavation effect can be ignored after point c and displacement records in c-d-e-f-g segment may reflect a global deformation trend of the wedge rockslide 1971. For segment a-b mentioned above, its movement may be a mixed mode, which includes local and global deformations. In this case, it is easy to understand that the global deformation of the

Figure 3. Topographic map of the vicinity of the Libby Dam.
wedge rockslide 1971 during segment a-b should be less than or equal to the increment $\Delta a b_3$. Similarly, the increment $\Delta b c_3$ is the upper bound of the global deformation during segment b-c.

It should be noted that a felt earthquake, which is indicated by EQ in Figures 1 and 4, occurred on 1 April 1969 according to Voight (1979), nearly two month before the point c. Considering the possible rock damage induced by this earthquake, it was inferred that this earthquake event may have a little effect on the sharp increment in segment c-d, or at least it may be an external trigger factor.

Table 1. Displacement records of different characteristic points (unit: mm).

|       | Monitoring curve in Figure 1 |
|-------|-----------------------------|
|       | H 1 | 2 | 3 |
| a     | 0   | / | / |
| b     | 21.0 (21.0) | 8.4 (8.4) | 4.9 (4.9) | 3.8 (3.8) |
| c     | 22.8 (1.8) | 9.5 (1.1) | 5.5 (0.6) | 3.8 (0) |
| d     | 31.4 (8.6) | 17.9 (8.4) | 13.8 (8.3) | 12.1 (8.3) |
| e     | 33.2 (1.8) | 19.4 (1.5) | 15.1 (1.3) | 13.6 (1.4) |
| f     | 42.6 (9.4) | 28.5 (9.1) | 24.5 (9.4) | 22.9 (9.3) |
| g     | 43.6 | 29.2 | 24.6 | 22.9 |

Note: The values in the parentheses mean the increments of a-b, b-c, c-d, d-e and e-f segments of different monitoring curve, respectively.
3. Conceptual geological model of the brittle deformation evolutionary pattern

In order to explain the stair-step deformation characteristics of the wedge rockslide 1971 as shown in Figure 1, we attempt to establish a geological conceptual model.

According to Voight (1979) and references therein, there exist a number of asperities in both the bedding plane DS+122 and joint A. The amplitude of asperities distributed in the DS+122 is less than 1.9 cm, while that of joint A ranges from centimetre scale to metre scale, with an average of about 0.6 m. Thus, we inferred that joint A should be discontinuous before the failure and there exist three unbroken high strength geological bodies (HSGBs), similar to rock bridges in the intermittence jointed rock masses (see Figure 5). In this case, the whole damage evolution process of the wedge rockslide 1971 can be inferred as follows: at first, the pre-split excavation results in the abrupt increment of segment a-b. However, due to the existence of three unbroken HSGB, the potential unstable wedge is still stable after segment a-b. Afterwards, due to the stress redistribution and the rock damage of HSGB, induced by the excavation in 1967 and the earthquake in 1969, one of three unbroken HSGB firstly breaks, which causes the abrupt increment of segment c-d. Similarly, the potential unstable wedge still keeps stable due to the bearing capacity of the other two unbroken HSGB. In this case, through the stress redistribution again, one of two unbroken HSGB will lose the bearing capacity and cause the sharp increment of segment e-f. However, the failure of the potential unstable wedge still cannot happen and the deformation trend evolves into the horizontal segment f-g again because of the existence of the last unbroken HSGB. After point g, the last unbroken HSGB begins to break quickly and the wedge rockslide 1971 ultimately occurs. For convenience, HSGB that breaks firstly is hereinafter referred to as HSGB 1#, the second broken HSGB is referred to as HSGB 2#, and so on.

It is easy to understand that HSGB may have a dual role during the overall failure process of the wedge rockslide 1971. On one hand, HSGB acts like welded contacts, through which the potential unstable wedge was connected to the bottom bedrock. In this case, HSGB indeed hinders the downward trend of the potential unstable wedge. Only when the last HSGB was sheared off could the final

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**Figure 5.** Conceptual model of the brittle deformation evolutionary pattern of the wedge rockslide 1971. (a) Schematic diagram of the wedge rockslide 1971. (b) View along the line of intersection. (c) and (d) Schematic diagrams of high strength geological bodies from different view angles.
failure along the bedrock happen. On the other hand, HSGB is indeed the medium of the nucleation, initiation and propagation of new cracks. Considering that HSGB counteracts a large part of the sliding force, it will store more and more strain energy, which inevitably results in the stress concentration phenomenon. As the local stress continues to increase, some weak rock elements in HSGB will firstly break. With more and more rock elements broken, HSGB will gradually lose the bearing capacity, which will further drive fracture propagation along joint A until HSGB was sheared off. Meanwhile, the deformation of HSGB will accelerate. Sometimes, the duration of the acceleration stage is very short, which makes the displacement trend increase sharply just like c-d and e-f segments in Figure 1. It seems that the answer to the first question, which was mentioned in the introduction, has been gained and the stair-step brittle deformation evolutionary pattern can be reasonably explained by the HSGB conceptual model.

4. Mechanical model of the brittle deformation evolutionary pattern

As mentioned in Section 3, the brittle deformation evolutionary pattern can be reasonably explained by the HSGB conceptual model. Only when HSGB is sheared off, the potential unstable wedge will begin to slip. Therefore, it is very necessary to investigate the process which caused HSGB to be sheared off in order to reveal the internal physical mechanism. Considering that displacement record is usually considered as a medium of failure prediction of a landslide, it will be necessary to reveal the internal connection between the shear mechanical behaviour of HSGB and the corresponding deformation monitoring data.

4.1 Description of the ideal shear stress–shear strain curve

Usually the ideal shear stress–shear strain curve of rock samples is very much like the compressive mechanical behaviour, which can be divided into five stages: (I) crack closure, (II) elastic deformation, (III) crack initiation and stable crack growth, (IV) crack damage and unstable crack growth, (V) failure and post-peak behaviour (Brace et al. 1966; Bieniawski 1967a, 1967b; Lajtai and Lajtai 1974; Martin and Chandler 1994; Xue, Qin, Sun, et al. 2014; Xue et al. 2015a). On the basis of previous literature (Barton 1973; Grasselli 2001; Budi et al. 2014), the ideal shear stress–shear strain curve of strain-softening rocks is also divided into five stages in the present study (see Figure 6). The distinguishing characteristics of each stage can be described as follows:

![Ideal shear stress–shear strain behaviour of strain-softening rocks.](image)

Note: A, B, C, D, E and F are case sensitive in the present paper.
Stage I (OA): the shear stress–shear strain response is usually non-linear (convex upwards), which can be attributed to the closure of pre-existing flaws.

Stage II (AB): the mechanical behaviour is linear elastic with constant stiffness. During this stage, there is almost no initiation of new cracks and no propagation of old cracks, which indicates that cumulative damage is small.

Stage III (BC) and Stage IV (CD): the shear stress–shear strain response along segment B-D is non-linear (concave downwards). Therefore, segment B-D is different from the elastic segment A-B. Meanwhile, segment B-D obviously should not belong to the post-peak segment D-E-F. In this case, it seems that there is no need to further subdivide segment B-D, as described by Grasselli (2001). However, according to previous studies (Crosta and Agliardi 2003; Amitrano and Helmstetter 2006; Damjanac and Fairhurst 2010; Xu et al. 2013), it seems that in the shear creep test there exists a shear stress threshold $\tau_{\text{creep}}$. And when the shear stress $\tau < < \tau_{\text{creep}}$, only instantaneous creep (see vertical line 0 in Figure 7) occurs. As the shear stress increases but still remains lower than $\tau_{\text{creep}}$ i.e. $\tau < \tau_{\text{creep}}$, the creep curve is almost flat and no tertiary creep stage appears (see curve 1 in Figure 7). When $\tau_{\text{creep}} \leq \tau < < \tau_{\text{short}}$ (i.e. the short-term shear strength that corresponds to the shear stress at point D in Figure 6), the standard creep curve with three stages may occur (see curve 2 in Figure 7). Thereafter, as the shear stress further increases, approaching or even exceeding $\tau_{\text{short}}$, the primary and secondary creep stages gradually fade away and the creep curve becomes steeper and steeper, with the appearance of a dominant tertiary creep stage (see curve 3 in Figure 7). Obviously, only when the shear stress reaches the threshold $\tau_{\text{creep}}$, the tertiary creep stage appears, which inevitably results in the final creep failure. Considering that there is almost no cumulative damage in the elastic segment A-B, which means that the tertiary creep stage will not appear, we postulate that $\tau_{\text{creep}}$ should not belong to the elastic segment A-B. Meanwhile, as mentioned above, $\tau_{\text{creep}}$ is less than $\tau_{\text{short}}$. In this case, we think that during the damage evolution process of segment B-D, there must exist a special point in which the corresponding shear stress is $\tau_{\text{creep}}$. As shown in Figure 6, this special point is indicated by a capital letter C. According to the analysis above, the crack propagation should be stable for segment B-C and unstable for segment C-D, which makes the point C look like a phase transition point. In view of this, we think that it is very necessary to subdivide segment B-D into two separate stages, i.e. Stage III (BC) and Stage IV (CD).

Figure 7. Different evolution processes of shear creep curve under different shear stress levels. See text for details.
Stage V (D-E-F): The prominent feature of this stage is that the curve firstly drops rapidly with a negative slope like segment D-E in Figure 6 and then gradually becomes horizontal like segment E-F. In the present study, D-E and E-F segments are regarded as a same stage, i.e. the post-peak stage. If necessary, they can also be considered as two separate stages, as described by Grasselli (2001).

4.2 Description of the ideal shear creep curve

It is well-known that a typical creep test curve usually includes primary creep stage, secondary creep stage and tertiary creep stage. According to Qin, Wang, et al. (2010), Qin, Xu, et al. (2010), Xu, et al. (2011) and Xue, Qin, Li, et al. (2014), an ideal creep curve can be further subdivided through several feature points as illustrated in Figure 8, where the secondary creep includes two sub-stages, i.e. A1-B1 and B1-C1 segments, while the tertiary creep can be divided into three sub-stages, i.e. initial accelerated segment (C1D1), medium accelerated segment (D1E1) and critical failure segment (E1F1).

4.3 Mechanical model of the brittle deformation evolutionary pattern

Figure 9(a) is a schematic diagram of the single HSGB conceptual model, in which the bedrock is assumed to be rigid, while the potential unstable wedge and HSGB are deformable. As stated in Section 4.1, the mechanical behaviour of the process that HSGB is sheared off can be described by Figure 9(b). Moreover, although there are some differences between the actual deformation process of HSGB and the ideal shear creep curve, the displacement record of HSGB could still be approximately regarded as a shear creep behaviour as shown in Figure 9(c). Thus, relying on previous studies (Qin, Wang et al. 2010; Xue, Qin, Li, et al. 2014), a single HSGB mechanical model can be proposed through establishing a connection between the shear creep curve and the shear stress–shear strain curve of strain-softening materials as shown in Figures 9(b,c).

As mentioned in Section 3, displacement records of the wedge rockslide 1971 could be qualitatively explained by a multiple HSGB conceptual model. In this case, its physical mechanism may be further revealed by means of a multiple HSGB mechanical model as shown in Figure 10. If comparing Figure 10 with Figure 9, it could be easy to understand that the abrupt increment segment c-d is
Figure 9. Single HSGB mechanical model of the brittle deformation evolutionary pattern. (a) Schematic diagram of the single HSGB conceptual model. (b) Ideal shear mechanical behaviour of the single HSGB conceptual model. (c) Ideal displacement record of the single HSGB conceptual model. See text for details.

Note: A, B, C, D, E, F, A1, B1, C1, D1, E1 and F1 are case sensitive in the present paper; Dashed line D-E-F means an ideal post-peak mechanical behaviour, which may be restricted or terminated due to the existence of other unbroken HSGB; Dashed line O1-A1 means that it is not always observed because it may have occurred prior to monitoring; Dashed line D1-E1-F1 means it is not always observed for some reasons as follows: (1) in a short time, the deformation may be very large in D1-E1-F1 segment, which causes the malfunction of the monitoring equipment. (2) The damage evolutionary process in D1-E1-F1 segment may be restricted or terminated due to the existence of other unbroken HSGB.
approximately equivalent to the initial accelerated segment \((C_1D_1)\) of HSGB 1#. However, due to the existences of unbroken HSGB 2# and 3#, not only the accelerating deformation trend of segment \(c-d\) does not further evolve into the medium accelerated segment \((D_1E_1)\) and critical failure segment \((E_1F_1)\) as shown in Figure 9(c), but also the displacement record eventually evolves into an almost horizontal segment \(d-e\). Similarly, the abrupt increment segment \(e-f\) is approximately equivalent to the initial accelerated segment \((C_1D_1)\) of HSGB 2#. In this case, the expected failure of the potential unstable wedge still does not happen because of the existence of unbroken HSGB 3#. Afterwards, the deformation trend once again evolves into an almost horizontal segment \(f-g\). When the point \(g\) is reached, the unstable crack propagation of HSGB 3# begins. Because HSGB 3# is the last one, the supporting force of HSGB 3# will disappear rapidly and the expected failure of the wedge rockslide 1971 ultimately occurs. Obviously, the answer to the second question as described in the introduction has been gained and it is just the fracture process of multiple HSGB that results in the stair-step brittle deformation evolutionary pattern of the wedge rockslide 1971.

5. Displacement criterion of the brittle deformation evolutionary pattern

Through HSGB conceptual and mechanical models as presented above, the brittle deformation evolutionary pattern can be reasonably explained and easily understood. In this case, if a quantitative relationship between points C and D in the inset of Figure 10 can be established, the quantitative relationship between points \(c\) and \(d\) or between points \(e\) and \(f\), as shown in Figure 10, can also be obtained. Thus, the brittle deformation evolutionary pattern can be described quantitatively. From this point of view, we firstly attempt to carry out an investigation about the points C and D, respectively.
5.1 Mathematical description of the point D

As suggested by Chen et al. (2006) and Qin et al. (2006), the strain-softening mechanical behaviour of the shear stress–shear strain curve like Figure 9(b) can be described by the constitutive model as follows:

\[ \tau = G_s \times \varepsilon \times e^{\left(-\varepsilon / \varepsilon_0\right)^m}, \]  

(1)

where \( G_s \) is initial shear modulus, \( \varepsilon_0 \) is a measurement of the average strain and \( m \) is a Weibull modulus.

Considering that the slope of the constitutive curve is equal to zero at the peak point D in Figure 9(b), which can be described as follows:

\[ \frac{\partial \tau}{\partial \varepsilon} \bigg|_{\varepsilon = \varepsilon_D} = G_s \times e^{\left(-\varepsilon / \varepsilon_0\right)^m} \times \left[ 1 - m \times \left(\frac{\varepsilon_D}{\varepsilon_0}\right)^m\right] = 0, \]

(2)

Rearranging Equation (2) gives

\[ \frac{\varepsilon_D}{\varepsilon_0} = \left(\frac{1}{m}\right)^{\frac{1}{m}}, \]

(3)

where \( \varepsilon_D \) is the shear strain at the peak point D.

5.2 Mathematical description of the point C

As shown in Figure 11(a), it was assumed that the potential shear fracture surface of HSGB is composed of numerous elementary cells that cannot be divided further. When subjected to an external load, most cells are unbroken at first. Due to the damage evolution, more and more cells become broken. As mentioned in Section 4.1, point C looks like a phase transition point in the damage evolution process and can distinguish the stable crack propagation from the unstable crack propagation. In other words, the critical crack density, \( p^* \), has been reached at the point C, where the deformation

![Figure 11. Illustration of the two-dimensional renormalization group model of HSGB. (a) Schematic diagram of the potential shear fracture surface of HSGB. (b) Two-dimensional renormalization group model. (c) Evolutionary process of cells. See text for details.](image-url)
of HSGB begins to accelerate markedly as illustrated in Figure 9(c). Assuming the percentage of broken cells at point C is equivalent to \( p^* \). If we can solve \( p^* \), the point C could be described mathematically. According to numerous previous studies (Qin, Wang, et al. 2010; Qin, Xu, et al. 2010; Xue 2011; Zhang et al. 2013; Xue, Qin, Li, et al. 2014; Xue 2015), it was argued that the phase transition point C is equivalent to the unstable fixed point in renormalization group theory (Wilson 1983; Turcotte 1997). Therefore, it is possible to mathematically describe the essence of the point C based on the RG theory.

Assuming that four elementary cells form a first-order block and four first-order blocks form a second-order block, and so on, thus a two-dimensional renormalization process can be constructed (see Figure 11). For an elementary cell as illustrated in Figure 11(a), there exist only two kinds of states, i.e. broken (coloured box) or unbroken (white box). In this case, there are five possible combinations for a first-order block, i.e. B4U0, B3U1, B2U2, B1U3 and B0U4 as shown in Figure 11(c), in which the capital letters B and U mean broken and unbroken states respectively, while the subsequent figure indicates the number of cells with the corresponding state. Meanwhile, it was assumed that only when all four cells embedded in a first-order block are broken, the corresponding first-order block can be thought to be broken. Following this rule, the first-order block with a combination B4U0 is broken, the first-order block with a combination B0U4 is unbroken, while the state of the first-order blocks with B3U1, B2U2 or B1U3 are also unbroken if the interaction among cells is ignored. However, it is known to all that when a rock element breaks, adjacent unbroken rock elements will inevitably be affected in the actual fracture process of rock due to the stress transfer. Therefore, the interaction between cells should be considered. In this case, the unbroken first-order blocks with B3U1, B2U2 or B1U3 may become broken ultimately. In other words, when the interaction among cells is considered, the state of the first-order blocks with B3U1, B2U2 or B1U3 may have two options, i.e. broken or unbroken (see Figure 11(c)).

Following Smally et al. (1985), a stress transfer mechanism that can approximately describe the interaction between cells is considered in the present study. In this case, according to previous studies (Xue 2015; Xue et al. 2015b), the total broken probability of the first-order block, \( p_1^{(2)} \), can be obtained as follows:

\[
P_1^{(2)} = p_1^4 + C_2^1 p_1^3 (1-p_1)p_{4,1} + C_2^2 p_1^2 (1-p_1)^2[p_{2,1}^2 + C_1^1 p_{2,1} (1-p_{2,1})p_{4,2}]
+ C_4^1 p_1^2 (1-p_1)^3 \left[ \frac{3}{4} p_{3,1}^3 + C_3^3 C_4 p_{4,3,1}^2 (1-p_{4,3,1})p_{4,4,3} \right]
\]

where \( p_1 \) means the broken probability of a cell and is assumed to follow the Weibull distribution, while \( p_{a,b} \) means the conditional probability that an unbroken cell with local stress \( b \sigma \) will fail when subjected to an additional stress \( (a-b) \sigma \) from a broken cell.

For higher order blocks, an iteration equation can be obtained in the following form:

\[
P_1^{(n+1)} = (p_1^{(n)})^4 + C_2^1 (p_1^{(n)})^3 (1-p_1^{(n)})p_{4,1}^{(n)}
+ C_2^2 (p_1^{(n)})^2 (1-p_1^{(n)})^2[p_{2,1}^{(n)}]^2 + C_1^1 p_{2,1}^{(n)} (1-p_{2,1}^{(n)})p_{4,2}^{(n)}]
+ C_4^1 p_1^{(n)} (1-p_1^{(n)})^3 \left[ \frac{3}{4} p_{3,1}^{(n)} + C_3^3 (p_{4,3,1}^{(n)})^2 (1-p_{4,3,1}^{(n)})p_{4,4,3}^{(n)} \right]
\]

where \( n \) is the order of the block.

Equation (5) is actually the formula of the two-dimensional renormalization group model. By means of the graphical method proposed by Xue, et al. (2015b), the approximate numerical solution
of Equation (5), i.e. $p^*$, can be obtained. Meanwhile, it was found that $p^*$ is a function of Weibull modulus $m$ (Smalley, et al. 1985; Qin, Xu, et al. 2010; Xue, et al. 2015b).

Furthermore, as reported by Xue, et al. (2015b), the broken probability of a cell, i.e. $p_1$ as mentioned above, can be described as follows:

$$ p_1 = 1 - e^{-\left(\frac{m}{\bar{m}}\right)^{m}} $$

(6)

thus when substituting $p^*$, which has been solved through Equation (5), into Equation (6), one can get

$$ \frac{\varepsilon_C}{\varepsilon_0} = [-\ln(1 - p^*)]^{\frac{1}{m}} $$

(7)

where $\varepsilon_C$ is the shear strain at the peak point C.

5.3 Displacement criterion of the brittle deformation evolutionary pattern

According to the Equations (3) and (7), it is easy to obtain the quantitative relationship between $\varepsilon_D$ and $\varepsilon_C$ as follows:

$$ \frac{\varepsilon_D}{\varepsilon_C} = [-m \ln(1 - p^*)]^{-\frac{1}{m}} $$

(8)

As mentioned earlier, $p^*$ is a function of Weibull modulus $m$. Therefore, the ratio between $\varepsilon_D$ and $\varepsilon_C$ is also a function of Weibull modulus $m$. As illustrated in Figure 12, although both $p^*$ and $\varepsilon_D/\varepsilon_C$ decrease gradually with increasing Weibull modulus $m$, it should be noted that they seem to be insensitive to $m$ especially when $m$ is larger.

Considering the inherent mechanical relationship as shown in Figures 9(b) and 9(c), one can get

$$ \frac{S_{D1}}{S_{C1}} \approx [-m \ln(1 - p^*)]^{-\frac{1}{m}} $$

(9)

where $S_{D1}$ and $S_{C1}$ are the real global displacement at points D1 and C1.

![Figure 12. The ratio of $\varepsilon_D/\varepsilon_C$ versus the shape parameter m. Note: Subscript capital letters, such as C and D, are case sensitive in the present paper.](image)
Obviously, if the point $C_1$ (i.e. onset of tertiary creep) can be identified in advance, the displacement at point $D_1$, $SD_1$, can be predicted according to Equation (9). Therefore, the quantitative relationship in Equation (9) can be regarded as the displacement criterion of the brittle deformation evolutionary pattern.

6. Application demonstration of displacement criterion and discussion

In this section, we will introduce the application process of the displacement criterion in Equation (9) through quantitatively reanalysing the displacement records of the wedge rockslide 1971.

First, taking the displacement record of sensor H in Figure 1 for example, the monitoring data at points c, d, e and f have been listed in the $S_{moni}$ column of the Table 2. As inferred in Section 2.2, local deformation dominates the movement trend of segment a-b, while the c-d-e-f-g segment mainly reflects a global deformation, especially the abrupt increments in c-d and e-f segments. In other words, $S_{moni}$ is actually a superposition of local and global displacement records. However, it should be noted that the global displacement record should be adopted when the Equation (9) is employed. In this case, when the point c is a reference point, the approximate global displacement increment, $S_{incr}$, can be obtained through deducting the local displacement $S_{moni-c}$ from the monitoring data $S_{moni}$. The values of $S_{incr}$ at c, d, e and f have also been listed in Table 2.

Second, for the wedge rockslide 1971, the global movement trend may have occurred prior to the installation of the monitoring equipment in 1967. These unrecorded global displacements, denoted by $S_{miss}$, should also be considered when the Equation (9) is employed. As illustrated in Figure 10, the brittle evolutionary process of HSGB 1# should obey Equation (9), that is to say,

$$\frac{S_{incr-d} + S_{miss}}{S_{incr-c} + S_{miss}} \approx \left( \frac{\varepsilon_D}{\varepsilon_C} \right)_{HSGB1#},$$

where $(\varepsilon_D/\varepsilon_C)_{HSGB1#}$ means the theoretical value of $\varepsilon_D/\varepsilon_C$ of HSGB 1#. $S_{incr-c}$ and $S_{incr-d}$ are the values of $S_{incr}$ at points c and d, respectively. When the point c is regarded as a reference point, $S_{incr-c}$ is actually equal to zero.

Rearranging Equation (10), one can get

$$S_{miss} \approx S_{incr-d} - (\varepsilon_D/\varepsilon_C)_{HSGB1#} \times S_{incr-c},$$

which can be used to estimate the unrecorded global displacement of HSGB 1 prior to monitoring.

Considering that the actual $m$ value of HSGB 1# cannot be determined at present, the actual theoretical value of $\varepsilon_D/\varepsilon_C$ of the HSGB 1# cannot also be obtained. In view of this, we assume that the $m$ value of HSGB 1# ranges from 1 to 15, then the results of $\varepsilon_D/\varepsilon_C$ and $S_{miss}$ can be calculated based on the Equations (8) and (11) and have been listed in the Table 2.

Third, in order to reflect the whole evolutionary process of HSGB 1#, including the global displacement after monitoring and the estimated unrecorded global displacement before monitoring, the revised global displacement record $S_{revi}$ is introduced, which can be calculated through the sum of $S_{incr}$ and $S_{miss}$. When $m$ is in the range of 1 to 15, the corresponding $S_{revi}$ at c, d, e and f have been listed in Table 2.

Finally, $(S_d/S_c)_{HSGB1#}$, which means the displacement criterion of the brittle deformation evolutionary pattern of HSGB 1#, can be approximately calculated through the following formula:

$$\left( \frac{S_d}{S_c} \right)_{HSGB1#} \approx \frac{S_{revi-d}}{S_{revi-c}}.$$
Table 2. Data processing demonstration of the displacement criterion.

| Point | Weibull modulus $m$ | Theoretical value of $e_D/e_C$ | $S_{\text{miss}}$ (mm) | $S_{\text{incr}}$ (mm) | $S_{\text{revi}} = S_{\text{incr}} + S_{\text{miss}}$ (mm) |
|-------|---------------------|------------------------------|------------------------|------------------------|--------------------------------------------------|
| c     | 1.44                | 1.47                         | 1.39                   | 1.33                   | 1.31                                             |
| d     | 1.41                | 1.39                         | 1.35                   | 1.33                   | 1.31                                             |
| e     | 1.41                | 1.39                         | 1.36                   | 1.33                   | 1.31                                             |
| f     | 1.41                | 1.39                         | 1.37                   | 1.33                   | 1.31                                             |

Notes: $S_{\text{mono}}$: displacement monitoring data of sensor H as shown in Figure 1. The value of $S_{\text{mono}}$ at a certain point, such as point c, can be denoted by $S_{\text{mono}}-c$. $S_{\text{incr}}$: displacement increment. Because the point c is regarded as a reference point in the present table, $S_{\text{incr}}$ can be calculated by means of the formula: $S_{\text{incr}} = S_{\text{mono}} - S_{\text{mono}}-c$. Similarly, the value of $S_{\text{incr}}$ at a certain point, such as point c, can be denoted by $S_{\text{incr}}-c$. $S_{\text{miss}}$: unrecorded global displacement before monitoring, which can be estimated based on Equation (11); $S_{\text{revi}}$: revised global movement record, which can be calculated through the sum of $S_{\text{incr}}$ and $S_{\text{miss}}$. It indeed reflects a global displacement during the whole evolutionary process. Similarly, the value of $S_{\text{revi}}$ at a certain point, such as point c, can be denoted by $S_{\text{revi}}-c$. $\left(S_d/S_c\right)_{\text{HSGB1#}}$: it reflects the displacement criterion of the brittle deformation evolutionary pattern of HSGB 1#, which can be estimated based on Equation (12); $\left(S_f/S_e\right)_{\text{HSGB2#}}$: it reflects the displacement criterion of the brittle deformation evolutionary pattern of HSGB 2#, which can be estimated based on Equation (13); $S_{\text{pred}}-f$: Predicted displacement at point f; $S_{\text{error}}-f$: Error between $S_{\text{pred}}-f$ and $S_{\text{revi}}-f$. 

|                  | $S_{\text{mono}}$ (mm) | $S_{\text{incr}}$ (mm) | $S_{\text{revi}}$ (mm) | $(S_d/S_c)_{\text{HSGB1#}} = S_{\text{revi-}d}/S_{\text{revi-c}}$ | $(S_f/S_e)_{\text{HSGB2#}} = S_{\text{revi-}f}/S_{\text{revi-c}}$ | $S_{\text{pred}-f} = S_{\text{revi-}f} \times (S_d/S_c)_{\text{HSGB1#}}$ (mm) | $S_{\text{error}-f} = S_{\text{pred}-f} - S_{\text{revi-}f}$ (mm) |
|------------------|------------------------|------------------------|-----------------------|---------------------------------------------------------------|---------------------------------------------------------------|--------------------------------------------------------------------|---------------------------------------------------------------|
| $c$              | 22.8                   | 0.0                    | 22.1                  | 23.2                                                          | 24.6                                                          | 26.1                                                               | 26.9                                                          |
| $d$              | 31.4                   | 8.6                    | 21.0                  | 22.1                                                          | 24.6                                                          | 26.1                                                               | 27.7                                                          |
| $e$              | 33.2                   | 10.4                   | 19.5                  | 21.0                                                          | 22.1                                                          | 22.1                                                               | 27.7                                                          |
| $f$              | 42.6                   | 19.8                   | 39.3                  | 41.9                                                          | 43.0                                                          | 44.4                                                               | 45.9                                                          |
Similarly, \((S_f / S_e)_{\text{HSGB2#}}\) can be approximately estimated as follows:

\[
\left( \frac{S_f}{S_e} \right)_{\text{HSGB2#}} \approx \frac{S_{\text{revi-f}}}{S_{\text{revi-e}}}. \tag{13}
\]

Considering that the precondition that \((S_d / S_e)_{\text{HSGB1#}}\) can be estimated is that the brittle evolutionary process of HSGB 1# obeys the Equation (9), it is easy to understand that \((S_d / S_e)_{\text{HSGB1#}}\) is consistent with the theoretical value of \(\varepsilon_{D} / \varepsilon_{C}\) of the HSGB 1# as shown in Table 2. Meanwhile, it can also be observed that \((S_f / S_e)_{\text{HSGB2#}}\) is different from \((S_d / S_e)_{\text{HSGB1#}}\) for the same \(m\) value, but they are gradually becoming consistent as the increasing \(m\). In a sense, this changing trend may make no sense considering that the actual \(m\) values of HSGB 1# and HSGB 2# are unknown. Even so, we can split this issue up into two cases so that it can be discussed.

1. It may be a fact that the \(m\) values of HSGB 1# and HSGB 2# are indeed different, which is the reason why \((S_f / S_e)_{\text{HSGB2#}}\) is different from \((S_d / S_e)_{\text{HSGB1#}}\) as shown in Table 2. In this case, even if \((S_d / S_e)_{\text{HSGB1#}}\) can be perfectly described by Equation (9), it is actually meaningless, at least technically for the prediction of the brittle evolutionary segment e-f of HSGB 2#, because \((S_f / S_e)_{\text{HSGB2#}}\) is still an unknown unless the actual \(m\) value of HSGB 2# can be obtained in advance.

2. It may also be true that the \(m\) values of HSGB 1# and HSGB 2# should be the same or have smaller differences. In other words, \((S_d / S_e)_{\text{HSGB1#}}\) is approximately equal to \((S_f / S_e)_{\text{HSGB2#}}\). Under this circumstance, the revised global displacement at point f in Figure 10 can be predicted in advance by means of the following formula:

\[
S_{\text{pred-f}} \approx S_{\text{revi-e}} \times \left( \frac{S_d}{S_e} \right)_{\text{HSGB1#}}. \tag{14}
\]

The predicted results have been listed in Table 2. It can be found that the predicted values \(S_{\text{pred-f}}\) are always higher than \(S_{\text{revi-f}}\) which looks like a systematic error, but the errors between them,
become smaller and smaller as the increasing \( m \). Furthermore, when the displacement records of sensors 1, 2, and 3 are reanalysed, the similar changing trend can also be observed (see Figure 13). It is inferred that the reasons that \( S_{\text{pred}}\) is always higher than \( S_{\text{revi}}\) can be attributed to two aspects. On one hand, it is indeed a simplified method that the shear fracture surface of HSGB is described by a two-dimensional renormalization group model as illustrated in Figure 11. On the other hand, the parameter \( p^* \) in Equation (9) was obtained through the graphical method as mentioned in Section 5.2. Therefore, it indeed is a numerical solution rather than an analytical solution, which will inevitably cause a systematic error when the Equation (9) is employed.

It should be noted again that the displacement criterion, i.e. Equation (9), is a function of Weibull modulus \( m \). When it is employed, the \( m \) value should be obtained firstly. In the laboratory scale, the determination method of \( m \) has been discussed in the literature (Wang et al. 2007; Li et al. 2012; Singh et al. 2015; Xue, et al. 2015b). However, it is difficult to obtain the actual \( m \) value of the in-situ large-scale HSGB based on the method above. In view of this, we must remind the reader that the purpose of this section is only a demonstration about the application process of the displacement criterion and the predicted results above are not the focus of this section.

7. Conclusions

Aiming at making a reasonable qualitative explanation of the stair-step brittle deformation evolutorial pattern of the landslide, a conceptual geological model is firstly outlined based on HSGB. Subsequently, in order to reveal the internal physical mechanism of the stair-step displacement records, a mechanical model is proposed through investigating the internal connection between the shear mechanical behaviour of HSGB and the corresponding temporal sequence of displacement monitoring. Finally, a displacement criterion, which establishes a quantitative formula to describe the displacement ratio between the onset of the accelerating creep and brittle rupture, is derived through combining the two-dimensional renormalization group model with the shear creep constitutive model. To some extent, the present study has shed light on the failure prediction of the HSGB-type landslides, whose stability is mainly controlled by HSGB.

However, it should be pointed out that at the present stage there still exist some difficulties when the displacement criterion above is put into the practical application. The first problem which we have to solve is how to probe HSGB in the field. A preliminary idea has been outlined in the previous literature (Pan et al. 2014) and further study is ongoing. The second urgent problem is how to obtain the actual \( m \) value of the in-situ large-scale HSGB. This is indeed a challenging issue because many published studies mainly focus on the laboratory-scale determination method of \( m \) (Wang, et al. 2007; Li, et al. 2012; Singh, et al. 2015; Xue, et al. 2015b). Moreover, as mentioned earlier, the global displacement record is recommended, when the displacement criterion above is employed. Therefore, the third problem is how to distinguish local displacement and global displacement. It may be an effective method through the combination of the deep and shallow deformation monitoring networks.

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