Landau theory of superconducting phases in \( T_h \) crystals

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Abstract

We consider the Landau theory of phase transitions for multiple superconducting phases in \( T_h \) crystals. All possible phase transition sequences involving a single superconducting order parameter are found. These results may be applicable to PrOs\(_4\)Sb\(_{12}\).

Key words: Skutterudites, unconventional superconductivity

Perhaps the greatest descriptive power and beauty of the Landau theory of phase transitions is revealed when successive phase transitions are described by a single, multi-component order parameter (OP). There are many well-known examples of this involving structural OP’s, especially among ferroelectric compounds. But although multi-component OP’s are the hallmark of most unconventional superconductors, so far there are no confirmed examples of multiple superconducting (SC) phases described by a single OP. PrOs\(_4\)Sb\(_{12}\), which displays two SC phases with different symmetries, may be the first.

The application of Landau theory to multiple SC phases described by a single OP was studied previously \cite{1} for various crystal symmetries, and applied to UPt\(_3\). This material displays multiple superconducting phase transitions due to a two-dimensional SC order parameter, but the degeneracy of the order parameter is lifted by antiferromagnetic ordering. In PrOs\(_4\)Sb\(_{12}\), superconductivity occurs at \( T_{c1} = 1.85 \) K; an additional phase transition is observed at \( T_{c2} = 1.75 \) K \cite{2,3}. Thermal conductivity measurements revealed the presence of nodes and a lowering of the symmetry of the gap function from four-fold to two-fold at \( T_{c2} \)\cite{4}, which is highly suggestive of a symmetry lowering phase transition involving the SC order parameter. In this article, we describe the phenomenological Landau theory of multiple SC phases in \( T_h \) crystals, which may be applied to PrOs\(_4\)Sb\(_{12}\).

Especially useful when the microscopic origin of superconductivity is unknown, the phenomenological approach is based entirely on symmetry. Order parameters are classified according to representations of the crystallographic point group, in terms of which the Landau potential is expanded. In principle, all possible phases, their symmetries and phase diagrams, may be obtained in this way, as well as the positions and types of nodes of the corresponding SC gap functions.

The point group \( T_h \) has one-dimensional (1D), two-dimensional (2D) and three-dimensional (3D) representations, called \( A, E \) and \( T \) respectively, which may be even (for singlet pairing) or odd (triplet pairing). The phase diagram for the 1D representation \( A \) involves only one SC phase, therefore we do not consider it any further. For the representations \( E \) and \( T \), we restrict our attention to phases connected by second order phase transitions, as suggested in at least one experiment \cite{2}. Together with the restriction to a single OP, this sets strong constraints on allowed phase transition sequences.

The SC gap function is a \( 2 \times 2 \) matrix, \( \tilde{\Delta}(k) = i\tilde{\sigma}_y \psi(k) \) for singlet pairing and \( \tilde{\Delta}(k) = i(\mathbf{d}(k)\tilde{\sigma})\tilde{\sigma}_y \) for triplet pairing, where \( \tilde{\sigma} = (\tilde{\sigma}_x, \tilde{\sigma}_y, \tilde{\sigma}_z) \) are Pauli matrices, \( \psi(k) \) is an even scalar function and \( \mathbf{d}(k) \) is an odd pseudovector function. The gap in the quasiparticle energy spectrum in the singlet SC state is given by

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\[ \Delta(k) = |\psi(k)|, \text{ while in the triplet state it is } \Delta_2(k) = \sqrt{|\mathbf{d}(k)|^2 + |\mathbf{d}(k) \times \mathbf{d}^*(k)|^2}. \]  

The functions \( \psi(k) \) and \( \mathbf{d}(k) \) are expressed in terms of the components of the OP \( \eta_1 \) as

\[ \psi(k) = \sum_i \eta_i \psi_i(k), \quad \mathbf{d}(k) = \sum_i \eta_i \mathbf{d}_i(k), \quad (1) \]

where \( \psi_i(k) \) and \( \mathbf{d}_i(k) \) are the basis functions for the even and odd representations of the point group, respectively.

The transformation properties of the OP are derived from the basis functions \( \psi_i \) and \( \mathbf{d}_i \), which may be arbitrarily chosen to span the representation. For the 2D representation \( E \), we choose basis functions that are complex conjugate [6]. The Landau potential expanded up to sixth order in the 2D OP is

\[ F = \alpha(\eta_1^2 + \eta_2^2) + \beta_1(\eta_1^4 + \eta_2^4) + 2\beta_2|\eta_1^2|^2|\eta_2| \]

\[ + \gamma_1(\eta_1^3 \eta_2^* + \eta_2^3 \eta_1^*) + \gamma_2|\eta_1|^2|\eta_2|^2|\eta_1|^2|\eta_2|^2, \quad (2) \]

where \( \alpha, \beta_1, \beta_2, \gamma_1, \) and \( \gamma_2 \) are phenomenological parameters. The Landau potential is invariant under \( K \times U \times G \), where \( K \) is time reversal, \( U \) are \( U(1) \) gauge transformations and \( G \) is the space group. Eq. (2) describes second order phase transitions between the normal state and the states \((1,0)\) and \((\phi_1, \phi_2)\), as shown in Fig. 1a. The components of the state \((\phi_1, \phi_2)\) have equal magnitudes and unfixed relative phases. The sixth order expansion is insufficient to describe the additional second order phase transition to the state \((\eta_1, \eta_2)\), which has neither fixed relative magnitudes or phases.

To obtain the third phase \((\eta_1, \eta_2)\) it is necessary to consider additional higher order terms in the Landau potential. However, such an approach is rarely followed in practice. Instead, an effective Landau potential of an effective OP is used to analyse phase transitions between the various SC phases whose symmetries satisfy group-subgroup relations. Second order phase transitions are indicated by the absence of third order terms. In our case, \((\eta_1, \eta_2)\) has symmetry \( D_2 \), which is a subgroup of the symmetry group of \((\phi_1, \phi_2)\), \( D_2 \times K \). Therefore the effective OP is 1D and breaks time reversal symmetry, hence odd order terms in the effective Landau potential are prohibited. It follows that the transition is second order. The symmetry of \((1,0)\) is the group \( T(D_2) \) (see Ref. [6]), which is a supergroup of \( D_2 \). The transition \((1,0) \rightarrow (\eta_1, \eta_2)\) is described by an effective 1D OP which is just \( \eta_2 \). It is clear from Eq. (2) that there will be third order terms in the effective Landau potential of the form \( \eta_2^3 + c.c. \). Therefore the transition \((1,0) \rightarrow (\eta_1, \eta_2)\) is first order.

For the 3D OP, real basis functions which transform under rotation as \((x, y, z)\) are chosen for \( \psi_i \) and \( \mathbf{d}_i \) [6]. The free energy contains second and fourth order terms of the form

\[ \text{References} \]

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