Anomalous reflection and excitation of surface waves in metamaterials

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We consider reflection of electromagnetic waves from layered structures with various dielectric and magnetic properties, including metamaterials. Assuming periodic variations in the permittivity, we find that the reflection is in general anomalous. In particular, we note that the specular reflection vanishes and that the incident energy is totally reflected in the backward direction, when the conditions for resonant excitation of leaking surface waves are fulfilled.

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Introduction Materials with new kinds of dielectric and magnetic properties, e.g. left-handed metamaterials, have stimulated much recent work, see e.g. [1]. In the light of the anomalous electromagnetic characteristics [1, 2, 3, 4, 5, 6, 7, 8, 9, 10], it is necessary to reconsider many well known problems, accounting also for metamaterials. Due to their extraordinary properties, left-handed materials are believed to become highly important in various technological applications [11, 12]. The first examples of metamaterials had their left-handed properties in the microwave range, and then later for infrared waves [11]. Recently materials showing anomalous refractive behaviour have been constructed also in the optical regime [2].

It has previously been shown in Ref. [12] that certain layered dielectric structures give rise to peculiar reflection phenomena, provided surface waves [7, 8, 13, 14, 15] can be excited. Such waves have previously been considered in connection with layered structures [3, 10, 12, 16], open plasma wave guides [17, 18], and fusion plasmas [19].

In the present paper, we consider the reflective properties of a dielectric and magnetic medium with arbitrary permittivity and permeability, which is covered by a dielectric layer. We show that for a homogeneous medium, we have the familiar specular reflection independently of the material parameters. However, with periodic variations in the dielectric permittivity, as can be found in nature [20, 21] and constructed in laboratories [1, 2, 11], we find that the incident energy can be completely converted into a backscattered energy flux, a phenomenon which is associated with the resonant excitation of leaking surface waves. The necessary conditions for obtaining backward reflection will be deduced, and possible applications of this peculiar mirror effect are consequently pointed out.

Homogeneous media We first consider a semi-infinite \((z > z_0, \text{region } a)\) material with arbitrary relative permittivity \(\varepsilon_a\) and permeability \(\mu_a\). Specifically we will allow these constants to have arbitrary signs, such that for example metamaterials with negative values of both \(\varepsilon\) and \(\mu\) are included as special cases. This material is assumed to be covered by a dielectric layer with arbitrary permittivity \(\varepsilon_b\) and \(\mu_b = 1\) in the region \(0 < z < z_0\) (region \(b\)), whereas we have vacuum at \(z < 0\). Furthermore, we let a \(p\)-polarized wave with a magnetic field \((B_0 y / 2) \exp(ik_z x + ik_z z - i\omega t) + c.c.\), be incident on this structure. Here c.c. denotes complex conjugate, \(B_0\) is the wave amplitude, and \(k_{z,z} > 0\). We denote the angle of incidence by \(\theta\) (given by \(k_x = (\omega / c) \sin \theta\) or \(k_x = (\omega / c) \cos \theta\)). As a prerequisite for the more complicated calculations in the next section, we here study the fields induced in the materials for the special case where the waves are evanescent in both regions \(a\) and \(b\).

The magnetic field in each of the three regions \(\varepsilon_0 z > z > z_0\), \(z < z_0\) and \(z > z_0\) is thus written in the form

\[
B = (\hat{y} / 2) (B_0 e^{-ik_z z} + R_0 e^{-ik_z z} + R_b e^{-ik_z z} + B_0 e^{-ik_z z}) + c.c.
\]

where \(R_b\) and \(R_a\) are constants which represent the specularly and backward reflected waves, respectively. Normally, one would expect \(R_b\) to be zero. However, we note that metamaterials may behave differently compared to normal materials. Furthermore, inclusion of the \(R_b\)-term will be useful for the calculations made in the next section. From energy conservation, evanescent waves in regions \(a\) and \(b\) imply \(R_b + R_a = 1\). Next, in region \(b\) we solve the equation

\[
\partial_x^2 B + \partial_z^2 B + \frac{\omega^2}{c^2} B = 0, \tag{1}
\]

which has solutions

\[
B = B_0 e^{ik_z x + ik_z z} + B_0 (R_e e^{ik_z x} + R_b e^{-ik_z x}) e^{-ik_z z}, \tag{2}
\]

and write the solution in the form

\[
B = \frac{B_0}{2} \left( 1 - \frac{ik_z \varepsilon_b}{\kappa_b} \right) e^{ik_z x} (e^{-\kappa_b z} + r e^{\kappa_b z}) + (R_e e^{ik_z x} + R_b e^{-ik_z x}) (e^{\kappa_b z} + r e^{-\kappa_b z}), \tag{4}
\]

where \(\kappa_b = (\omega / c) (\sin^2 \theta - \varepsilon_b) / 2\), and where \(r = (1 + ik_z \varepsilon_b / \kappa_b) / (1 - ik_z \varepsilon_b / \kappa_b)\) represents the reflectivity at \(z = 0\). We note that the standard boundary conditions expressed in terms of \(B\), namely the continuity of \(B\) and
\( (1/\varepsilon) \partial_z B \) are satisfied at \( z = 0 \) for arbitrary values of \( R_s \) and \( R_b \). In region \( a \) we use the equation
\[
\partial_z^2 B + \partial_z^2 B + \frac{\omega^2 \varepsilon_a \mu_a}{c^2} B = 0, 
\]
and write the solution as
\[
B = (B_{+} e^{ik_x x} + B_{-} e^{-ik_x x}) e^{-\kappa_a (z-z_0)}, \tag{6}
\]
with \( \kappa_a = (\omega/\varepsilon) (\sin^2 \theta - \varepsilon_a \mu_a)^{1/2} \). Since both \( \mu \) and \( \varepsilon \) vary across the boundary, the boundary conditions are now the continuity of \( B/\mu \) and \( (1/\varepsilon) \partial_z (B/\mu) \). Matching the solutions in regions \( a \) and \( b \), we obtain
\[
\frac{B_0}{2} \left( 1 - \frac{ik \varepsilon_b}{\kappa_b} \right) \left[ (e^{-\kappa_a z_0} + re^{\kappa_a z_0}) \right.
+ (e^{\kappa_a z_0} + re^{-\kappa_a z_0}) R_s] = \frac{B_+}{\mu_a}, \tag{7}
\]
\[
\frac{B_0}{2} \left( 1 - \frac{ik \varepsilon_b}{\kappa_b} \right) (e^{\kappa_a z_0} + re^{-\kappa_a z_0}) R_b = \frac{B_{-}}{\mu_a}, \tag{8}
\]
and as well as
\[
\frac{\kappa_a B_0}{2} \left( 1 - \frac{ik \varepsilon_b}{\kappa_b} \right) \left[ (e^{-\kappa_a z_0} + re^{\kappa_a z_0}) \right.
+ (e^{\kappa_a z_0} - re^{-\kappa_a z_0}) R_s] = -\frac{\kappa_a \varepsilon_b B_+}{\varepsilon_a \mu_a}, \tag{9}
\]
\[
\frac{\kappa_b B_0}{2} \left( 1 - \frac{ik \varepsilon_b}{\kappa_b} \right) (e^{\kappa_a z_0} - re^{-\kappa_a z_0}) R_b = -\frac{\kappa_b \varepsilon_b B_{-}}{\varepsilon_a \mu_a}. \tag{10}
\]

Here we note that the equations involving \( B_{-} \) couple only to the backward reflection coefficient \( R_b \), and that the incident wave does not act as a source for this part. Thus, we can interpret this contribution as a leaking surface wave \([23, 24]\) that is localized around the boundary \( z = z_0 \) and decays exponentially away from it, but couples to a propagating ("leaking") part in the boundary region. However, no matter what kind of material properties we have in region \( a \), this mode cannot be excited by the incident wave, as the boundary conditions do not allow it to couple to the incident wave. The field profile of the surface mode is shown in Fig. 1. Furthermore, as will be demonstrated in the next section, the properties of the surface wave are crucial for the reflection properties when the media in the different regions are not homogeneous.

Due to the decoupling of the surface waves in a homogeneous medium, however, we here obtain \( R_s = 1 \) from the boundary conditions, and hence we always have a purely specular reflection with this geometry. Let us stress that for a metamaterial, the surface wave localized around \( z = z_0 \) carries energy in the opposite direction along the \( x \)-axis, as compared to the incident wave. However, as shown above, this does not affect the direction of the reflected energy in the vacuum region.

**Periodic media**  We next extend our previous analysis to consider a semi-infinite \((z > z_0)\) medium with a periodically varying permittivity \( \varepsilon_a = \varepsilon_{a0} + \varepsilon_{a1} \cos(2k_x x) \), where \( \varepsilon_{a0} \) and \( \varepsilon_{a1} \) are constants. We note that such a medium can be constructed in laboratories \([1, 2, 11]\), but also that such structures are common in nature \([20]\). The medium in region \( b \) is the same as in the previous section. For simplicity, the permeability \( \mu_a \) in region \( a \) is kept constant. Generally, the interaction between the incident wave and the periodic dielectric structure in medium \( a \) will result in the generation of new waves corresponding to the sum and difference of the wave vectors. Here, we choose the \( x \)-component of the incident wave vector such that it is equal to half the wave number of the periodic modulation in region \( a \). This choice of incident angle \( \theta \) means that the backscatter generation mechanism is resonant and thus much more important than the corresponding interaction mechanism for other values of \( \theta \). Since the medium and thereby the wave equations are the same as in the previous analysis in the vacuum region and region \( b \), we can adopt the corresponding solutions \([31, 32]\) and \([33, 34]\). Furthermore, for region \( a \) we can closely follow the analysis in Ref. \([12]\). Including the spatial dependence of the permittivity, the wave equation \((5)\) is then generalized to
\[
\frac{1}{\varepsilon_a \mu_a} \partial_z^2 B + \partial_z \left( \frac{1}{\varepsilon_a \mu_a} \partial_z B \right) + \frac{\omega^2}{c^2} B = 0. \tag{11}
\]
Similarly, the ansatz \((3)\) is generalized to
\[
B = B_{+}(z) e^{ik_x x} + B_{-}(z) e^{-ik_x x}. \tag{12}
\]
Assuming that \( \varepsilon_{a1} \) is much smaller than \( \varepsilon_{a0} \), we then obtain the two coupled equations
\[
\partial_z^2 B_{\pm} + \left( \frac{k_a^2 - \varepsilon_{a1} \omega^2}{c^2 \varepsilon_{a0} \mu_a} \right) B_{\pm}
= -\frac{\varepsilon_{a1}}{2 \varepsilon_{a0} \mu_a} \left[ \partial_z^2 B_{\mp} - (k_x^2 + 2 \kappa_a^2) B_{\mp} \right], \tag{13}
\]
with the boundary conditions
\[
(B_{+})_{z = z_0} = B_0 \left[ r + R_s + (1 + r R_s) e^{-2\kappa_a z_0} \right], \tag{14}
\]
(B_\perp)_{z=z_0} = \tilde{B}_0 R_b \left(1 + r e^{-2\kappa_b z_0}\right), \quad (15)

(\partial_z B_+)_{z=z_0} = \tilde{B}_0 \frac{\kappa_b}{\varepsilon_b} \left\{ \varepsilon_{a0} \mu_a \left[ r + R_a - (1 + r R_a)e^{-2\kappa_a z_0} \right] + \frac{\varepsilon_{a1}}{2} R_b \left(1 - r e^{-2\kappa_a z_0}\right) \right\}, \quad (16)

and

(\partial_z B_-)_{z=z_0} = \tilde{B}_0 \frac{\kappa_b}{\varepsilon_b} \left\{ \varepsilon_{a0} \mu_a R_b \left(1 - r e^{-2\kappa_a z_0}\right) + \frac{\varepsilon_{a1}}{2} \left[ r + R_a - (1 + r R_a)e^{-2\kappa_a z_0} \right] \right\}, \quad (17)

where \(\kappa_a = (\omega/c)(\sin^2 \theta - \varepsilon_{a0}\mu_a)^{1/2}\) and \(\tilde{B}_0 = (B_0/2)(1-ikz\varepsilon_b/\kappa_b)\exp(\kappa_b z_0)\). The solution of \((13)\) can be found by expanding \(B_\pm\) in powers of the small parameter \(\varepsilon_{a1}\). After lengthy but straightforward calculations, we obtain the solution

\[ B_\pm \approx C_\pm e^{-\kappa_a (z-z_0)} - \frac{\varepsilon_{a1}(k_x^2 + \kappa_a^2)}{4\kappa_a^2 \varepsilon_{a0} \mu_a} C_\mp e^{-\kappa_a (z-z_0)}, \quad (18)\]

where \(C_\pm\), as well as \(R_b\) and \(R_a\), are complex constants that can be found from \((14)-(17)\). As \(\varepsilon_{a1}/\varepsilon_{a0} \ll 1\), only weakly damped leaking surface waves are of interest here.

Thus, from now on we focus our attention on the regime \(\exp(-\kappa_a z_0) \ll 1\). We then obtain the simple approximate solution

\[ R_a \approx -r \frac{|D|^2}{D^2} \left[ 1 + \frac{\varepsilon_{a1} \omega^2 (1 - r^2)e^{-2\kappa_a z_0}}{8 \kappa_a^2 \varepsilon_{a0} \mu_a r |D|^2 D} \right], \quad (19)\]

and

\[ R_b \approx \varepsilon_{a1} \frac{\omega^2}{2 \kappa_a^2 c^2} \frac{(1 - r^2)}{D^2} e^{-2\kappa_a z_0}, \quad (20)\]

where

\[ D \approx 1 + \frac{\kappa_b \varepsilon_{a0} \mu_a}{\kappa_a \varepsilon_b} + 2r e^{-2\kappa_a z_0}. \quad (21)\]

In the absence of dielectric modulations, i.e. if \(\varepsilon_{a1} = 0\), we recover from \((19)\) the well known formula \(R_a^{(0)} = -r |D|^2/D^2\), i.e. \(|R_a^{(0)}| = 1\). Obviously, \(R_b^{(0)} = 0\) for this case. However, from Eqs. \((19)\) and \((20)\), we also find the interesting result that \(R_a \approx 0\) and \(R_b \approx iv\) if the real part of the surface wave dispersion function \((21)\) is equal to zero, i.e.

\[ 1 + \frac{\kappa_b \varepsilon_{a0} \mu_a}{\kappa_a \varepsilon_b} + 2 \frac{(1 - k_x^2 z_b^2/\kappa_b^2)}{(1 + k_x^2 z_b^2/\kappa_b^2)} e^{-2\kappa_b z_0} \approx 0, \quad (22)\]

and if

\[ \frac{|\varepsilon_{a1}| \omega^2}{8 \kappa_a^2 c^2} = \frac{k_x z_b}{\kappa_b (1 + k_x^2 z_b^2/\kappa_b^2)} e^{-2\kappa_b z_0}. \quad (23)\]

When \(R_a \approx 0\) it also follows that \(|R_b| \approx 1\). Equation \((22)\) means that the incident wave resonantly excites a leaking surface wave, in which most of the energy is concentrated near the interface at \(z = z_0\). Equation \((23)\) then defines the condition for the backward energy flux to be equal to the incident flux. We note that the relation \((22)\) only can be fulfilled if \(\varepsilon_{a0} \mu_a/\varepsilon_b < 0\). For example, for a metamaterial in region \(a\) \((\varepsilon_{a0} < 0, \mu_a < 0)\) we must have negative dielectric permittivity also in region \(b\), whereas for a nonmagnetic material the dielectric permittivity must change sign between the regions \(a\) and \(b\). In general, to be able to study the present phenomenon, it is necessary to consider configurations with suitable properties so that weakly damped leaking waves can exist \([14]\). Specifically in the absence of the boundary region \((i.e. if z_0 = 0, or if \varepsilon_b \rightarrow 1),\) there are no such waves. In our model, it is possible to investigate leaking waves, however \([14]\). Similar studies can naturally be performed for other bounded media, but the mathematics would then be much more involved.

**Summary** In the present paper, we have considered the electromagnetic reflection properties of an arbitrary dielectric and magnetic material covered by a dielectric layer. For homogeneous media, it was shown that leaking surface waves cannot be excited by an incident wave, and we have thus the familiar specular reflection. For small periodic variations of the dielectric permittivity \([20]\), however, the situation is radically different, and the coupling to leaking surface waves can then strongly influence the reflective properties. In particular, we have found that specular reflection is absent \((R_a = 0)\) if the frequency of the incident wave satisfies the dispersion relation \((22)\), and if the width \(z_0\) of the dielectric layer is related to the amplitude \(\varepsilon_{a1}\) of the permittivity modulation by means of the relation \((23)\). All the incident wave energy is then reflected in the backward direction. These properties make it possible to pick out a specific angle and a specific frequency for a wave which is backward reflected, while still keeping most of the signal in the ordinary reflected signal. Obviously, such a peculiar mirror effect is of experimental interest.

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