MHD stagnation point flow towards a quadratically stretching/shrinking surface

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Abstract. A quadratically stretching/shrinking surface of two dimensional magnetohydrodynamics stagnation point flow is investigated numerically. The velocity of the surface is assumed in quadratic form and subject to a linear mass flux. The influences of the governing parameters namely stretching/shrinking parameter $\lambda$, suction/injection parameter $S$, fluid temperature index $m$, and magnetic parameter $M$ on the flow and thermal fields are studied. The model of nonlinear ordinary differential equations is obtained by reducing the partial differential equations of the boundary layer using an suitable similarity transformation. The equations are then solved numerically by boundary value problem solver, $bvp4c$ built in MATLAB software. The numerical results are verified by comparing them with previously reported results. The characteristics of the flow and heat transfer characteristics are shown graphically and analyzed for distinct values of the governing parameters. It is found that both magnetic and temperature index parameters reduce the velocity flow while the magnetic parameter enhances the heat transfer rate. There exist dual solutions for certain range of $\lambda$. A stability analysis is performed to determine which of these solutions are stable and which are not.

1. Introduction

In recent years, the viscous fluid movement near a solid surface stagnation region has been further studied by many researchers. The stagnation point occurs when the fluid flow is blocked from flowing on the solid surfaces. Stagnation point flows have lots of physical significance as they are used to compute velocity differences and heat and mass transfer rates near the stagnation area of frames in high velocity flows, transpiration cooling, rustproof bearing designs and much more. There are various different real applications for stagnation point flow such as electronic device cooling, nuclear reactor cooling, polymer extrusion, and wire drawing. Initially, [1] assessed the stagnation point flow in 1911. He was the one who found a precise solution for the two dimensional stagnation point flow on a stagnant surface. Since then, many researchers have been exploring stagnation point flow such as [2], [3] and [4]. The latest study by [5] examined the changes in the effects of stratification and cross diffusion on the porous medium stagnation point flow of magneto-convection. The findings showed that the parameter of thermal stratification helps to increase the rate of heat transfer at the stagnation point flow.
In contrast,[3] discovered that in solving the stagnation point flow of single and multi-wall carbon nanotubes, the nanoparticle volume fraction and the magnetic parameter decreased skin friction. The study of magnetohydrodynamics (MHD) has motivated many researchers to explore mainly because MHD theoretically prevents changes in fluid flow and heat transfer characteristics. This phenomenon occurs when Lorentz forces are produced by the fluid flow which is heated by a strong magnetic field (see [6]). Initially this study was initiated by [7] who worked on the steady MHD flow past a stretching surface. Other researchers such as [8], [9] and [10] investigate the effectiveness of MHD with different perspectives in delaying the separation of the boundary layer. It is true that the magnetic parameter can delay the detachment of the boundary layer from the shrinking surface on the basis of findings from [9].

However, in literature, there are a few researchers which looked into quadratically shrinking sheet such as [12] and [13]. Recently, [11] have investigated a quadratic shrinking sheet with suction effect. Therefore, the present paper will fill in this gap and extend the work done by [11] by emphasizing the problem to MHD stagnation point flow towards quadratically shrinking sheet. This problem will be solved with bvp4c, a boundary value problem solver in MATLAB software, and to determine which solution is stable, a stability assessment will be conducted.

2. Basic Equations
Consider a steady MHD two-dimensional stagnation-point boundary layer flow of a viscous and incompressible fluid past a permeable quadratically stretching/shrinking surface with the velocity \( \tilde{u}_w(x) = a\lambda x + b\lambda x^2 \) where \( a \) and \( b \) are a positive constants, \( \lambda \) is the parameter for stretching/shrinking, with \( \lambda < 0 \) for shrinking and \( \lambda > 0 \) for stretching (see Fig. 1), and \( x \) is a measured coordinate along the stretch/shrink surface. The flow is restricted to \( y \geq 0 \), as \( y \) measured the normal to the surface. The free stream velocity is also presumed to be \( \tilde{u}_e(x) = ax + bx^2 \) and the velocity of mass transfer is \( \tilde{v}_w(x) = \nu_c + cx \). Note that surface temperature is \( T_w = T_\infty + dx^m \) and ambient fluid temperature is \( T_\infty \).

\[ \text{Figure 1. A flow geometry model and coordinate system of a) stretching and b) shrinking surface.} \]

The governing boundary layer equation for the flow under a constant transverse magnetic field by using these assumptions are,

\[ \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0 \]  
\[ \tilde{u} \frac{\partial \tilde{u}}{\partial x} + \tilde{v} \frac{\partial \tilde{u}}{\partial y} = \tilde{u}_e \frac{d\tilde{u}_e}{dx} + \tilde{v} \frac{\partial^2 \tilde{u}}{\partial y^2} - \frac{\sigma B_0^2}{\rho} (\tilde{u}_e - \tilde{u}) \]  
\[ \tilde{u} \frac{\partial T}{\partial x} + \tilde{v} \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \]
and the appropriate boundary conditions given as

\[
\tilde{u} = \tilde{u}_w(x), \quad \tilde{v} = \tilde{v}_w(x), \quad T = T_w(x) \text{ at } y = 0 \\
\tilde{u} \to \tilde{u}_e(x), \quad T \to T_\infty \text{ as } y \to \infty
\]

(4)

where \(\tilde{u}\) and \(\tilde{v}\) are elements of the velocity along the \(x\) and \(y\) axes, \(T\) is the fluid temperature, provided that the thermal diffusivity is \(\alpha\), kinematic viscosity is \(\nu\), the applied constant magnetic field is known as \(B_0\), \(\sigma\) is the electrical conductivity, \(\rho\) is the density, and \(a, b, c, d, v_c\) and \(m\) are constants.

Introducing the dimensionless variables (see [11]):

\[
X = x \sqrt{\frac{a}{\nu}}, \quad Y = y \sqrt{\frac{a}{\nu}}, \quad U = \frac{\tilde{u}}{\sqrt{\alpha \nu}}, \quad U_e = \frac{\tilde{u}_e}{\sqrt{\alpha \nu}}, \quad V = \frac{\tilde{v}}{\sqrt{\alpha \nu}},
\]

\[
\theta = \frac{T - T_\infty}{d \tilde{\nu}} \frac{1}{m}, \quad M = \frac{\sigma B^2_0 \rho}{\nu a}, \quad Pr = \frac{\alpha}{\nu} \frac{a}{\nu}
\]

(5)

Then, Eqs. (1)-(4) become

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6)
\]

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = U_e \frac{dU}{dX} + \frac{\partial^2 U}{\partial Y^2} - M(U_e - U) \quad (7)
\]

\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} \quad (8)
\]

along with the boundary conditions

\[
U = U_w(X) = \lambda X + \lambda \beta X^2, \quad V = V_w(X) = \nu_0 + 2 \delta X, \quad \theta = \theta_w(X) = X^m \text{ at } Y = 0 \\
U \to U_e(X) = X + \beta X^2, \quad \theta \to 0 \text{ as } Y \to \infty
\]

(9)

It is noticeable that there are two cases arise. The first case is when \(\beta = 0\) and \(\delta = 0\) is the linear stretching/shrinking with constant cross flow. The second case is when \(\beta \neq 0\) and \(\delta \neq 0\) are equivalent to quadratic stretching/shrinking and linear cross flow.

3. Solution

Looking for a dimensionless stream function for Eqs. (6)-(9),

\[
\Psi = \frac{\psi}{\nu} = X f(Y) + \beta X^2 g(Y), \quad \theta = X^m h(Y)
\]

(10)

The dimensionless velocity components are known as \(U = -\frac{\partial \psi}{\partial Y}\) and \(V = -\frac{\partial \psi}{\partial X}\) and the continuity equation (6) is fulfill. Hence, it can be expressed in the form of the following equation

\[
U = X f'(Y) + \beta X^2 g'(Y), \quad V = -f(Y) - 2\beta X g(Y)
\]

(11)

where \(\prime\) is the differentiation with respect to \(Y\).

After substitute equation (11) into equations (7)-(8) and equating the coefficients of \(X, X^2\) and \(X^3\), yield the following system of nonlinear ordinary differential equations

\[
f''' + ff'' + 1 - f'^2 + M(1 - f') = 0 \quad (12)
\]

\[
g''' + fg'' - 3f'g' + 2f''g + 3 + M(1 - g') = 0 \quad (13)
\]
\[ h'' + \text{Pr}(fh' - mf' h) = 0 \] (14)

and the boundary conditions given by

\[
\begin{align*}
  f(0) &= S, \quad f'(0) = \lambda, \quad g(0) = S, \quad g'(0) = -\lambda, \quad h(0) = 1 \\
  f'(Y) &\to 1, \quad g'(Y) \to 1, \quad h(Y) \to 0 \quad \text{as} \quad Y \to \infty
\end{align*}
\] (15)

where \( S = -\nu_0 \).

The skin friction coefficient \( C_f \) and the local Nusselt number \( N_u \) are the quantities of physical significance provided by

\[
C_f = \frac{\tau_w}{\rho(\alpha x)^2}, \quad N_u = \frac{xq_w}{\varsigma(T_w - T_\infty)}
\] (16)

where \( \tau_w \) is the surface skin friction or the shear stress, and \( q_w \) is the surface heat flux that is given as

\[
\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -\varsigma \left( \frac{\partial T}{\partial y} \right)_{y=0}
\] (17)

Using (10) into (16) and (17), will produced

\[
\text{Re}_{x}^{1/2} C_f = f''(0) + \beta X g''(0), \quad \text{Re}_{x}^{-1/2} N_u = -h'(0)
\] (18)

where \( \text{Re}_{x} = (\alpha x)x/\nu \) is the local Reynolds number.

4. Stability Analysis

Stability analysis is essential to determine the solution that is physically viable. Introducing new dimensionless time variable \( \tau \),

\[
U = X \frac{\partial f}{\partial \eta}(\eta, \tau) + \beta X^2 \frac{\partial g}{\partial \eta}(\eta, \tau), \quad V = -f(\eta, \tau) - 2\beta X g(\eta, \tau), \quad \theta = h(\eta, \tau), \quad \tau = at
\] (19)

along the unsteady case for equation (7)-(8),

\[
\begin{align*}
  \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} &= U \frac{\partial U_e}{\partial X} + \frac{\partial^2 U}{\partial Y^2} - M(U_e - U) \\
  \frac{\partial T}{\partial t} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} &= \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial Y^2}
\end{align*}
\] (20)

as proposed by [14], the equation (20) will becomes,

\[
\begin{align*}
  \frac{\partial^3 f}{\partial \eta^3} + f \frac{\partial^2 f}{\partial \eta^2} + 1 - \left( \frac{\partial f}{\partial \eta} \right)^2 + M \left( \frac{\partial f}{\partial \eta} - 1 \right) - \frac{\partial^2 f}{\partial \eta \partial \tau} &= 0 \\
  \frac{\partial^3 g}{\partial \eta^3} + f \frac{\partial^2 g}{\partial \eta^2} - 3 \frac{\partial f}{\partial \eta} \frac{\partial g}{\partial \eta} + 2 \frac{\partial^2 f}{\partial \eta^2} g + 3 - M \left( \frac{\partial g}{\partial \eta} - 1 \right) - \frac{\partial^2 g}{\partial \eta \partial \tau} &= 0 \\
  \frac{\partial^2 h}{\partial \eta^2} + \text{Pr} \left( f \frac{\partial h}{\partial \eta} - m \frac{\partial f}{\partial \eta} h - \frac{\partial h}{\partial \tau} \right) &= 0
\end{align*}
\] (21)

subject to boundary conditions

\[
\begin{align*}
  f &= S, \quad \frac{\partial f}{\partial \eta} = \lambda, \quad g = S, \quad \frac{\partial g}{\partial \eta} = -\lambda, \quad h = 1, \quad \text{as} \quad \eta = 0 \\
  \frac{\partial f}{\partial \eta} &\to 1, \quad \frac{\partial g}{\partial \eta} \to 1, \quad h \to 0 \quad \text{as} \quad \eta \to \infty
\end{align*}
\] (22)
The behaviour of the equations (21) can be indicated by disrupting the fundamental flow $f = f_0$, $g = g_0$ and $h = h_0$ with the disruptions (see [15])

$$f = f_0 + e^{-\gamma \tau} F, \ g = g_0 + e^{-\gamma \tau} G, \ h = h_0 + e^{-\gamma \tau} H$$  (23)

with $F(\eta)$, $G(\eta)$ and $H(\eta)$ are small contrasted with the steady state solutions $f_0(\eta)$, $g_0(\eta)$ and $h_0(\eta)$ and $\gamma$ is unknown eigenvalue. The following equations can be solved numerically after replacing (23) into (21):

$$F''' + f_0 F'' - (2f'_0 + M - \gamma) F' + f''_0 F = 0$$
$$G''' + f_0 G'' - (3f'_0 + M - \gamma) G' - 3g'_0 F' + 2g_0 F'' + 2f''_0 G + g''_0 F = 0$$
$$H'' + Pr (f_0 H' + h_0 F - mf'_0 H - mh_0 F' + \gamma H) = 0$$  (24)

with boundary conditions given by

$$F = 0, \ F' = 0, \ G = 0, \ G' = 0, \ H = 0, \ as \ \eta = 0$$
$$F' \rightarrow 0, \ G' \rightarrow 0, \ H \rightarrow 0 \ as \ \eta \rightarrow \infty$$  (25)

By taking $M$, $Pr$, $S$, and $m$ fixed values, the stability of the systems will be determined by the smallest eigenvalue $\gamma_1$. The boundary condition $F'(\eta) \rightarrow 0$ will be replaced with $F''(0) = 1$ without affected the basic equations in order to obtained the possible eigenvalues (see [16]).

5. Results and Discussion

In this section, the behaviour of the physical quantities will be discussed corresponds to variety of physical parameters such that $M$, $S$, $\lambda$, and $m$. The governing equations (12)-(14) with (15) as boundary conditions are solved numerically using MATLAB software. The numerical outcomes are then compared to prior published outcomes in the literature for validation purposes. The comparisons are tabulated in Table 1 and the results show in a good agreement.

| Table 1. Comparison of $f''(0)$ for various values of $\lambda$ with $S = M = 0$. |
|----------------|----------------|----------------|
| $\lambda$ | Present results | Nasir et al.[11] | Ismail et al.[17] |
|             | First solution | First solution | First solution |
|             | Second solution | Second solution | Second solution |
| -0.25       | 1.40224078     | -              | 1.4022411      |
| -0.75       | 1.48929820     | -              | 1.4892982      |
| -1.00       | 1.32881685     | 0.0            | 1.3288179      |
| -1.15       | 1.8223113      | 0.11670214     | 1.0822311      |
| -1.20       | 0.93247331     | 0.23364793     | 0.9324735      |
| -1.2465     | 0.58428116     | 0.55429619     | 0.5842826      |

The findings presented have shown that there are two alternatives solution available for shrinking sheet. Table 2 represents the results of stability analysis with the smallest eigenvalues for some values of $\lambda$. It is noted that the stable solution will produce positive smallest eigenvalues for each of $\lambda$ and the unstable solution will give negative smallest eigenvalues. Based on this fact, from Table 2, it is obviously stated that the first solution is stable while the second solution is unstable. The discussion will therefore only be bound to the first solution in this section.
Table 2. Lowest eigenvalue, $\gamma_1$ for some values of $\lambda$ at $M = 1$, $m = 1$, and $S = 2.5$.

| $\lambda$ | First solution | Second solution |
|-----------|----------------|-----------------|
| -4.4884   | 0.0075         | -0.0002         |
| -4.488    | 0.8355         | -0.8041         |
| -4.4      | 1.1841         | -1.1219         |
| -4.0      | 1.7978         | -1.6563         |

Figure 2. Variation $f''(0)$ over $\lambda$ for certain values $S$ at $M = 1.0$ and $m = 1.0$.

Figure 3. Variation $g''(0)$ over $\lambda$ for certain values $S$ at $M = 1.0$ and $m = 1.0$.

Figure 4. Variation $-h''(0)$ over $\lambda$ for certain values $S$ at $M = 1.0$ and $m = 1.0$.

Figure 5. Variation $f''(0)$ over $\lambda$ for certain values $M$ at $S = 2.5$ and $m = 1.0$.

For simplicity reasons, the value of the Prandtl number is set to 0.025, known as mercury (see [18]). According to [19], mercury is one of the most promising liquid metals that can be used for converting solar energy and industrial heat sources with a source temperature of approximately $200^\circ C - 300^\circ C$. As we can see from Fig. 2, when the suction strength increases, the solution range for $|\lambda|$ is also increases. The value of $f''(0)$ increases when the surface is shrunk. The same phenomenon was observed for Fig. 3. The value for the local Nusselt number managed to improve when $S$ is increased, as shown in Fig. 4. It is observed that the suction effect from Figs. 2-4 is capable of delaying the boundary separation from the surface and increasing the solution range for $\lambda$.

Figs. 5 and 6 depict the increment of the $f''(0)$ and $g''(0)$ when the magnetic parameter is escalated. The Lorentz force is formed from the magnetic field in MHD fluid flow act as a
Figure 6. Variation $g''(0)$ over $\lambda$ for certain values $M$ at $S = 2.5$ and $m = 1.0$.

Figure 7. Variation $-h''(0)$ over $\lambda$ for certain values $M$ at $S = 2.5$ and $m = 1.0$.

Figure 8. Effects of $M$ towards velocity profiles $f'(0)$, at $S = 2.5, \lambda = -3.5$ and $m = 1.0$.

Figure 9. Effects of $M$ towards velocity profiles $g'(0)$, at $S = 2.5, \lambda = -3.5$ and $m = 1.0$.

Figure 10. Effects of $M$ towards temperature profiles $h'(0)$, at $S = 2.5, \lambda = -3.5$, and $m = 1.0$.

Figure 11. Effects of $m$ towards temperature profiles $h'(0)$, at $S = 2.5, \lambda = -3.5$, and $M = 1.0$.

retarded force to slow down the fluid flow. As shown in Fig 8 and Fig. 9, the thickness of the boundary layer decreases when the strength of $M$ rises. Hence, contributes to the increase of $f''(0)$ and $g''(0)$. Fig 7 shows the same trend that conveys the increased values of the local Nusselt number when $M$ is enhanced. It is observed that in Fig 10, the Lorentz force appears as $M$ increases in the flow, and gradually decreases the thermal boundary layer thickness as the flow velocity decreases. It is discovered that the rate of heat transfer rises when the flow velocity
and the temperature decreases.

Fig 11 reveals that the surface temperature gradient steepen with $m$ increasing due to the reduced thermal boundary layer thickness. It is noticed that the increases of the temperature index $m$, successfully reduces the thermal boundary layer thickness. As the temperature index increase, the heat transfer rate will also increase.

6. Conclusions
This paper theoretically investigated the effects of the magnetic and temperature index parameter on the fluid flow and heat transfer physical features of the MHD stagnation point flow towards a quadratically stretching/shrinking surface. The dual solutions are found to exist when the sheet shrinks and the first solution is physically feasible from the stability analysis and the second solution is not feasible. It is shown that increased suction strength, magnetic parameter and temperature index, will increased the skin friction coefficient and the rate of heat transfer. The suction, magnetic and temperature index parameters are therefore capable of maintaining the flow from separated from the surface.

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