Students’ relational understanding of the rectangle: a case study

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Abstract. Relational understanding refers to know both what to do and why. In other words, relational understanding is an understanding of all the parts, how they relate, and why they are applied in the manner they are. Previous researches showed that students got difficulties in solving mathematics problem using relational understanding, especially in geometry problems. The purpose of this study was to investigate students’ relational understanding of the rectangle. The subjects of this study were four 8th graders. The data were collected using the student’s worksheets and interviews. This study was conducted in four meetings for every student. Every meeting was held for 30 minutes. The students’ understanding was appraised during the first ten minutes using the rectangle problem followed by interview for twenty minutes. The result of this study showed that the students do not have a relational understanding of the rectangle concept, but only instrumental understanding. Additionally, none of the students understands the relationship between both measurement and square with the area and perimeter of the rectangle concept. Furthermore, the result indicated that the relational understanding of a concept would be not understood well if one of the parts of the concept was ignored.

1. Introduction

Conceptual understanding refers to the comprehending of the concepts, operations, symbols, diagrams, procedures, and the relations between the concepts in mathematics [1]. It relates to the integrated understanding of mathematics so that students can comprehend both the content and context of mathematics. It is one of the strands of mathematics proficiency. Furthermore, mathematics proficiency is the main purpose of the mathematics learning process, and the mathematics learning process should emphasize student conceptual understanding [2]. Therefore, students are required to have a conceptual understanding of all mathematics concepts.

Conceptual understanding is divided into instrumental and relational understandings [3]. Instrumental understanding is not a part of understanding but only using the rules without knowing the reasons of using the rules in solving a problem, while relational understanding refers to knowing both what to do and why [3]. In other words, relational understanding is an understanding of all the parts, how they relate, and why they are applied in the manner they are. The growth of relational understanding is unlimited and more complicated than instrumental understanding because the students have to relate ideas so that this type of understanding takes a longer time and should be the purpose of every learning process, including geometry.

Geometry is an essential subject in mathematics that relates to other subjects such as numbers and arithmetic [4]. Furthermore, it can help students to solve their problems, including their daily life
problems [5][6]. On the other hand, students face difficulties in solving geometry problems [7]–[10]. They face difficulties in understanding and comprehending the geometry problem, deciding the appropriate strategies and steps [7], [9], visualizing geometry concept [10], and transforming the daily geometry problem[8].

Several studies in mathematics education have begun focusing on students’ relational understanding of basic concepts in mathematics, such as limit [11], functions and graphs [12], derivative [13], and geometry [14]. The result of the study conducted by Kizito showed that the students could not relate between derivative and integral concepts [15]. Then Sahin, Yenmez, and Erbaz conducted research and stated that sophomore graduate students could not explain the relationship between limit, rate of change, and the slope of tangent using the concept of the derivative [11]. Furthermore, Anwar, As’ari & Rahmawati stated that secondary students with visual or symbolic representation were able to build their relational understanding of a rectangle well [14]. Although there were several studies that had been discussed about relational understanding, but no one of them focuses on assessing students’ relational understanding in geometry, especially rectangle. So this study was conducted to investigate students’ relational understanding of a rectangle.

2. Method
This study was a qualitative study with a case study design. The case study design is an investigation strategy where the researchers probe the programs, events, activities, and processes deeply on one or more cases [16]. The case in this study was about the relational understanding of the rectangle concept. There were four 8th graders participated in this study. All of the subjects have learned the rectangle concept before, especially during elementary school. The subjects were chosen based on their mathematics achievement. One of the students has high mathematics achievement, two of them have a moderate level, and another has low mathematics achievement. The data were collected using the student’s worksheets and interviews. The study was conducted in four meetings for every student. Every meeting was held for 30 minutes. The students' understanding was evaluated for ten minutes using the rectangle problem followed by interviews for twenty minutes. There were four problems concerned with the area and perimeter of a rectangle given in this study. They were adopted from the mathematics textbook for 8th grade of secondary school. Two of them only required instrumental understanding and the other two required both instrumental and relational understandings to solve it.

3. Result and Discussion
Four problems were assessed in this study. Two of them only need instrumental understanding, but the others needed both instrumental and relational understanding. The first and the second problems required the students to apply the rules or formula for solving the area and perimeter of a rectangle. Whereas, in solving the third problem, the students should recognize different units of measurement given and equalized it. Then, the students need to understand the plane figures and the relation among them for solving the fourth problem.

Problem #1
This problem presented a rectangle with length and width values. The students were asked to find the area and perimeter of the rectangle. In solving this problem, the students only needed instrumental understanding, applying the formula of the rectangle.

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30 cm

50 cm

Find the area and the perimeter of the rectangle!
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Subjects one, two, and four could solve this problem well. They found the area of the rectangle by substituting the length and width values to the formula, “Area = Length X Width”. Subjects two and four wrote several steps for finding the final result, as shown in figure 1, while Subject one only wrote the final result in the answer sheet. Based on the interview, Subject one actually used the same steps as his friends. The kind of problem given influences the student strategies, especially routine and non-routine problem. Students solve the routine problem by using no more than two strategies [17]. Meanwhile, non-routine problems encourage the implementation of multiple strategies in solving them [18]. On the other hand, Subject three only wrote the measurements that were stated in the problem without writing any solutions. He was able to indicate the length of the width of the rectangle but could not understand and comprehend it. Most of the students’ errors in solving the problem happen in the comprehension step [19], [20].

![Figure 1. Subject two answer sheet of problem #1.](image)

Furthermore, the answer sheet showed that the student solved the perimeter problem without writing the perimeter formula “Perimeter = 2 (Length + Width)”. They only subbed all the sides of the perimeter. From the interview, we knew that all of the students could not mention the perimeter formula, but they were able to solve the perimeter problem correctly. They defined the perimeter of the rectangle as the perimeter definition used in daily life. The students did not use formal definitions at all in solving mathematics problems [21]. On the other hand, most of the students’ misconceptions were established from their understanding of the concept using daily life experiences incorrectly [22].

**Problem #2**
A rectangle has a 200 cm$^2$ area and a 20 cm length. Find out the perimeter and the width of the rectangle.

This problem stated the area and length of the rectangle, and then the students were asked to find the perimeter and the width of the rectangle. This problem is similar to the first problem where the student only needs to apply the rules and formulas. From the answer sheet, it showed that Subjects one, two, and four were able to solve this problem. They identified the given measurement and solved the problem using the area and perimeter formula of the rectangle. Unfortunately, Subject three was unable to solve this problem. Based on the interview, he felt confused, even though he had been learned this concept before. He only knew the rectangle definition and was able to mention the example and contra example. There are three scenarios of learning; no learning, rote learning, and meaningful learning [23]. No learning means that the students cannot remember most of the subject, but only a little bit about the key terms and the facts [23].
Problem #3
A corn garden with a rectangle shape has 90 cm in length and 7 m in width. Find out the area and the perimeter of the corn garden.

The third problem was a mathematical problem that used a daily life situation. This problem mentioned the length and width but with different units. It asked students to convert the measurement to the same unit before counting the area and perimeter of the rectangle. All the subjects cannot solve the problem. They did not equalize the measurement given. Subjects one and two solved this problem by writing the final result with the units of cm². The answer sheet of Subject one can be seen in figure 1. Based on the interview, both of the students stated that they usually used cm² as the unit of an area. They used the unit of the area without understanding what the unit is and why they use it. Meanwhile, Subject four wrote the final result without writing any unit. Subject four also did not equate the units between the length and width, but only multiplied them. Based on the interview, Subject four did not realize the differences of the unit while reading and solving the problem but recognized it when he wrote the final result. However, Subject four felt confused to write down the unit and decided to ignore it at all.

| Original | Translation |
|----------|-------------|
| ต้นทุ่งจ้าว : 

\[
\text{Length} = 90 \text{ cm} \\
\text{Width} = 7 \text{ m}
\]

\[
\text{Area} = \text{Length} \times \text{Width} = 90 \times 7 = 630 \text{ cm}^2
\]

\[
\text{Perimeter} = 90 + 7 + 90 + 7 = 194
\] |
| Condition: length: 90 cm 

Width: 7 

Task: area = length x width 

\[
= 90 \times 7 = 630 \text{ cm}^2
\]

\[
\text{Perimeter} = 90 + 7 + 90 + 7 = 194
\] |

Figure 2. Subject one answer sheet of problem #3.

In finding the perimeter of the rectangle in this problem, Subjects one, two, and four solved this problem by ignoring the units used. They added up all of the sides without converting the numbers given into the same units. It showed that the student did not understand the relation between the measurement, the area, and the perimeter of the rectangle. Whereas Subject three could not solve this problem, both finding the area and perimeter of the rectangle. When a student cannot solve instrumental problems, he cannot solve relational problems either because the instrumental understanding is shallower than relational understanding [3].

Problem #4
The area of the rectangle is equal to the area of the square which whose side is 20 cm. If the area of the square is 100 cm² and the width of the rectangle is 5 cm, find out the length and perimeter of the rectangle.

In problem 4, it was known that the area of a rectangle is equal to the area of a square. Then the values of the square and the rectangle width were stated. Then the students were asked to find the length and the perimeter of the rectangle. From the answer sheet, we know that all subjects were unable to solve this problem. Based on the interview, the students did not understand the given problem. They faced difficulties in comprehending the problem. Furthermore, all the students had been learned about both rectangle and square before. They were able to state the square definition,
mentioned the example and contra example, and solved the square problem related to the area and perimeter. Unfortunately, they did not find a relation between the square and the rectangle, even though the relationship had been given in problem four explicitly. The pupils conceive many concepts but do not possess a relational understanding of them [11].

4. Conclusion
From this study, it can be concluded that the students did not possess the relational understanding to find the area and the perimeter of a rectangle. They only used instrumental understanding in solving the problem. None of the students was able to relate between the concept of measurement with the area and the perimeter of the rectangle. Then it also showed that none of the students could comprehend the relation between the square and rectangle concepts. Furthermore, the relational understanding of a concept would be not understood well if one of the parts of the concept was ignored. We suggest continuing the research about how to build a relational understanding of learning mathematics with more subjects.

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