Option Encoder: A Framework for Discovering a Policy Basis in Reinforcement Learning

Arjun Manoharan  
Department of Computer Science  
Indian Institute of Technology Madras  
arjunmanoharan2811@gmail.com

Rahul Ramesh  
Department of Computer Science  
Indian Institute of Technology Madras  
rahull3ramesh@gmail.com

Balaraman Ravindran  
Department of Computer Science  
Indian Institute of Technology Madras  
ravi@cse.iitm.ac.in

Abstract

Option discovery and skill acquisition frameworks are integral to the functioning of a Hierarchically organized Reinforcement learning agent. However, such techniques often yield a large number of options or skills, which can potentially be represented succinctly by filtering out any redundant information. Such a reduction can reduce the required computation while also improving the performance on a target task. In order to compress an array of option policies, we attempt to find a policy basis that accurately captures the set of all options. In this work, we propose Option Encoder, an auto-encoder based framework with intelligently constrained weights, that helps discover a collection of basis policies. The policy basis can be used as a proxy for the original set of skills in a suitable hierarchically organized framework. We demonstrate the efficacy of our method on a collection of grid-worlds and on the high-dimensional Fetch-Reach robotic manipulation task by evaluating the obtained policy basis on a set of downstream tasks.

1 Introduction

Reinforcement learning (RL) deals with solving sequential decision-making tasks and primarily operates through a trial-and-error paradigm for learning. The increased interest in Reinforcement learning can be attributed to the powerful function approximators from Deep learning. Deep Reinforcement learning (DRL) has managed to achieve competitive performances on some challenging high-dimensional domains. To scale to larger problems or reduce the training time drastically, one could attempt to structure the agent in a hierarchical fashion. The agent hence makes decisions based on abstract state and action spaces, which helps reduce the complexity of the problem.

However, when addressing a new task, it is possible that one discovers a large number of skills or options. Typically, option discovery methods yield a large number of options, and some works resort to heuristics that help prune this set. In such a scenario, a compression algorithm is of utility, since it would be wasteful to discard these options, but ineffective to use all of them simultaneously. Firstly, the computation expended for determining each option policy is higher, when compared to the smaller set basis policies. Secondly, we show that a reduced set of basis policies result in a better empirical performance on the target tasks. This can be attributed to the fact that the algorithm dedicates less time for determining the relevance of each option to the current task.
Figure 1: An overview of the Option Encoder framework. A collection of options are converted into a set of distilled policies. This reduced set of policies can be used as a substitute for the option policies for any hierarchical framework.

Resorting to an existing policy distillation or compression method is one possible alternative. However, these methods distill the options into a single network, resulting in a single policy that captures the behavior of all policies. However, one would like to distill multiple options into a minimal set of policies. In this work, we propose the Option Encoder, a framework that attempts to find a suitable collection of basis policies, based on the discovered set of options. We use an auto-encoder based model where an intermediate hidden layer is interpreted as a set of basis policies which we term as distilled policies. The options are mixed and are reconstructed back using attention weights. The intermediate hidden layers would hence be forced to capture the commonalities between the various options and potentially eliminate redundancies. The overview of this framework is summarized in Figure 1. The obtained distilled policies can be used to solve a new set of tasks and can be used as a proxy for the original set of options in a new algorithm. Since the extraction of the distilled policies relies on training an auto-encoder, the training time is almost insignificant in comparison to the time required to train the RL agent.

We show the efficacy of our method by evaluating the obtained basis, against the original set of options. In this work, we empirically demonstrate that distillation helps improve the performance on downstream tasks when compared to using the raw set of options. We tackle a few challenging grid-worlds and show results on the robotic control task of Fetch-Reach.

2 Preliminaries

RL deals with sequential decision making problems and addresses the interaction of an agent with an environment. It is traditionally modeled by a Markov Decision Process (MDP) defined by the tuple \((S, A, P, r, \gamma)\), where \(S\) defines the set of states, \(A\) the set of actions, \(T : S \times A \rightarrow \mathcal{P}(S)\) the transition function (that maps to a probability distribution over states), \(r : S \times S' \times A \rightarrow \mathbb{R}\) the reward function and \(\gamma\) the discount factor. In the context of optimal control, the objective is to learn a policy that maximizes the expected discounted return \(R_t = \sum_{i=t}^{T} \mathbb{E} [\gamma^{(i-t)} r(s_i, s_{i+1}, a_i)]\), where \(r(s_i, s_{i+1}, a_i)\) is the reward function.

**Options:** An Option formalizes the notion of a temporally extended sequence of actions and is denoted by the tuple \((I, \beta, \pi_o)\). The initiation set \(I \subseteq S\) denotes the set of states, \(\beta : \mathcal{S} \rightarrow [0, 1]\) is the probability that the option terminates in state \(s\) and \(\pi_o\) is the option policy. In this work, we assume that the initiation set is the set of all states.

**Policy Gradient Methods:** Policy-based optimal control techniques model the behaviour policy \(\pi(a|s)\) of the agent. Actor-critic algorithms are policy gradient methods which learn a critic using a TD-based update and an actor using a gradient direction determined by the critic. Consider a value function \(V_\theta(S)\) parameterized by \(\phi\) and a policy \(\pi_\theta(a|s)\) parameterized by \(\theta\). One version of the actor-critic algorithms use the following update for the actor:

\[
\theta_{t+1} \leftarrow \theta_t - \nabla_{\theta} \log (\pi(\alpha_t|s_t)) \left[ r_{t+1} + \gamma V_\phi(s_{t+1}) - V_\phi(s_t) \right]
\]

The value function is updated using the TD-error. The second multiplicand (in Equation 1) is typically known as the Advantage function \((A(s_t, a) = Q(s_t, a) - V(s_t) \approx V(s_{t+1}) + r_t - V(s_t))\). As a result, this implementation of the actor-critic algorithm is called the Advantage actor-critic (A2C).
Attend, Adapt and Transfer: The Attend, Adapt and Transfer architecture (A2T) \cite{16}, is a model for utilizing expert policies (or options) from \( N \) different source tasks in order to tackle a target task. Consider the policy, for each of the tasks to be represented by \( K_i(s) \). Apart from the expert networks, A2T also has a base network represented by \( K_B(s) \), which is used to learn in regions of the state space, where the set of experts do not suffice.

\[
K_T(s) = w_{N+1}(s)K_B(s) + \sum_{i=1}^{N} w_i(s)K_i(s)
\]  

(2)

The \( w_j(s) \) are attention weights and satisfy the constraint \( \sum_{i=1}^{N+1} w_j(s) = 1 \). The A2T framework is useful for transferring the solution to \( N \) source tasks, to a similar target task and is functional with policies.

In this work, we use a modified version of the A2T algorithm, where we use a hard-attention mechanism as an alternative to the soft-attention. Furthermore, we persist an expert (option) for \( T \) steps, which we term as the termination limit. The hard-attention weights are trained using the A2C algorithm since the network is no longer differentiable. Henceforth, any reference to A2T refers to this modified version.

3 Method

The Option Encoder framework attempts to find a collection of basis policies, that can accurately characterize a collection of option policies. Let the policy of an option \( j \), at a state \( s \) be denoted by \( \pi_j(s) \). Let the \( i^{th} \) policy of the distilled set (intermediate layer) at state \( s \) be represented by \( \pi_i^d(s) \). \( \pi_i^d \) is found by minimizing the objective given in Equation 3.

\[
\pi^d = \arg \min_{\pi_i^d} \min_W \sum_{s \in S} \sum_j L \left( \pi_j(s), \left( \sum_i w_i(s) \times \pi_i^d(s) \right) \right)
\]

(3)

\( W \) is a weight matrix such that \( \sum_i w_{ij} = 1 \ \forall j \), i.e the rows of the matrix are attention weights. \( W \) can be a function of the state \( s \). Note that each \( w_{ij} \) is a scalar such that \( 0 \leq w_{ij} \leq 1 \) and it indicates the contribution of distilled element \( \pi_i^d(s) \) to the reconstruction of option \( \pi_j(s) \). The function \( L \) is a distance measure between the two probability distributions (for example KL-divergence or Huber Loss). The objective states that a convex combination of the distilled policies should be capable of reconstructing the original set of options.

Equation 3 is realized using an auto-encoder. The encoder can consist of any suitable neural network architecture. The output of the encoder must consist of \( M \) different distilled policies. A problem with a discrete action space, would contain \( M \) different softmax functions while a continuous action space problem would contain \( M \) sets of policy parameters (for example, the mean and variance of a particular distribution). Since the distilled policies are combined linearly, a single layer for the decoder should suffice, since the addition of more layers will not add any more representational power.

The architecture is an auto-encoder with two key constraints (see Figure 2). The first restriction is that each distilled policy has a single shared weight, i.e., every action of a single policy will have the same weight. This makes the re-constructed expert policies to be convex combinations of the distilled policies.

The second restriction is that the set of weights responsible for reconstructing any option policy must sum to 1. These weights are attention weights and can be agnostic to the current state or be an arbitrary function of it. These restrictions are imposed to respect the objective specified in Equation 3. This constraint ensures that the distilled policies are coherent since a heavily parameterized auto-encoder will result in a non-interpretable set of distilled policies.

For example, consider a scenario in which all the option policies indicate that the action \( a \) has the highest preference in state \( s \). Let action \( a \) be assigned the least probability after passing the options through an encoder. If the weights are allowed to take arbitrary values on the decoder side, the distilled policies can be capable of reconstructing the options by assigning higher weights to action \( a \),
even though it has a low probability as per the distilled policies. Alternately, the decoder can make use of negative weights to flip the preference order over the actions dictated by the distilled policies.

Algorithm 1: Summary of the Option Encoder Framework

$L = \text{Number of steps for rollout} ;$
$N = \text{Number of Option policies} ;$
$M = \text{Number of Distilled Policies} ;$

Dataset = Empty list ;
VisitedStates = Empty set ;

for $j$ in $(1, \ldots, N)$ do
    env.reset() ;
    for $i$ in $(1, \ldots, L)$ do
         Get policies $(\pi_1(s), \pi_2(s), \ldots, \pi_N(s))$ of each Option ;
         Add $(s, (\pi_1(s), \pi_2(s), \ldots, \pi_N(s)))$ to Dataset ;
         $a = \text{Sample}(\pi_j(s)) ;$
         $s = \text{env.step}(a) ;$
         Add $s$ to VisitedStates
    end
end

DistillPolicies = train_auto-encoder(Dataset) ;

for $j$ in $(1, \ldots, M)$ do
    DistillDataset = None ;
    for $s$ in VisitedStates do
        $\pi = \text{DistillPolicies}(j, s) ;$
        add $\pi$ to DistillDataset ;
    end
    $\pi_j = \text{ActorMimic}(\text{DistillDataset}) ;$
end

Ideally, one would want the distilled policies to capture the fact that action $a$ is preferred in-state $s$ among all options. Hence, the proposed two restrictions ensure that this intended behavior is achieved. The second restriction also ensures that the output of the decoders are also valid policies since a weighted combination of the policies (with the weights summing to 1) will also result in another policy.

Since the entire distillation procedure is a supervised learning problem, the time taken to perform the distillation is often minimal, in comparison to the time required to run the reinforcement learning algorithm. However, the current framework would require all the option policies to be run in order to obtain the distilled policies. This would however defeat the entire purpose of the distillation procedure since we would like to discard the options after the distillation. Hence, the distilled policies can be considered as targets for a procedure like Actor-mimic [13]. Each distilled policy can be potentially

Figure 2: A visual depiction of the Option Encoder Framework. The red colored layers correspond to the original option policies, and the blue layer indicates the distilled policy. The decoder weights of the same color correspond to attention weights (sum to 1) and each distilled policy has a single weight that corresponds to the reconstruction of every option.
transferred to a smaller network using a supervised learning procedure. This can be used during
decision-time planning or for policy execution since the network is lightweight by design. The entire
procedure is summarized in Algorithm 1.

4 Experiments

In this section, we attempt to answer the following questions:

- How do the distilled policies compare against the option policies on a set of tasks?
- Is the performance gain solely because of a reduced number of policies?
- Does varying the number of distilled policies, have an effect on the performance?
- Does the length of temporal persistence in the hierarchical agent, affect the performance?

4.1 Task description

**Grid-world**: We consider the grid-worlds depicted in Figure 3. The grid-worlds are stochastic in
nature where the agent moves in the intended direction with probability 0.8 and takes a random action
(uniform probability) otherwise. The environment has 5 actions available from every state, which are
up, down, left and right and a terminate action. Each episode terminates upon taking the terminate
action or after 3000 environment steps have been completed. We consider a task where the agent gets
a reward of +1 on reaching the designated goal and a reward of 0 for every other transition. Fifty
options were learned using the Eigen-options framework [8] for each grid which were then used to
solve the task of reaching 100 randomly selected goals. These goals are denoted by the yellow dots
on the grid in Figure 3. Three different grids GW1, GW2, and GW3 (left to right in Figure 3) were
considered. GW1 and GW2 are of sizes 28x31 each and GW3 is of size 41x41.

![Figure 3: The 3 grid-world tasks tackled in this work. The yellow dots indicate the goals for a
collection of tasks that we attempt to solve for.](image)

**Fetch-Reach Task**: We consider the Fetch-Reach robotic manipulation
task [14]. The Fetch-Reach task (see Figure 4) has a continuous state
space which is used to describe a particular arm configuration. For the
Fetch domain, we use the same state and action spaces as dictated in
[14]. We however modify the environment, to have a random start state
and a fixed goal state. We consider 15 randomly sampled goals as the
target task that we attempt to solve for. The agent receives a reward of
+1 on reaching the goal and 0 for every other transition.

![Figure 4: The Fetch-Reach robotic manipulation task.](image)

4.2 Architecture

**Option Encoder**: The state in all grid-world experiments is represented
as an image of the grid (with depth 1) with all zeros, except at the
location of the agent. We impose the encoder to also have shared attention weights (each policy has a
single attention weight) like the decoder. This implies that the original set of options are combined
using attention weights to yield the distilled policies which are then combined using another set of
attention weights to yield the reconstructions.

The encoder and the decoder are comprised of attention weights which are functions of the current
state. The state-based attention network consists of two convolution layers (4 filters of size 5x5 and
stride 2 and 8 filters of size 3x3 and stride 1) and the fully connected layer with 32 units. These units
are used to yield \( M \) sets of attention weights (total of \( M \times N \) weights) for the encoder and \( N \) sets
of attention weights (total of $N \times M$ weights) for the decoder, where $M$ is the number of option policies and $N$ is the number of distilled policies. All the option policies corresponding to a given state are fed to the auto-encoder, with its reconstruction guided by the Huber-loss. RMSProp with a learning rate of $10^{-4}$ and batch size of 4 was used.

For the Fetch-Reach task, the attention networks in the Option Encoder contain 2 fully connected layers of size 10 each. The output then yields the attention weights as described above. Like in the above case, the encoder and decoder both have shared attention weights.

Hierarchical Agent: The A2C algorithm with the modified A2T framework (described in Section 2) was used to train the agent (referred to as the A2T + A2C agent). The agent has a base network and is augmented by a collection of option policies. The base network consists of 2 convolution layers (same configuration as earlier) followed by a fully connected layer of size 128 which is projected to yield the policy and the value function heads. The base network policy and the option policies are combined using an attention mechanism to yield the final policy.

The attention network has 2 convolution layers (identical to ones described above) which are followed by a fully connected layer of size 128. These 128 units are projected to obtain the attention weights over the original/distilled option policies and the base network policy. The agent follows a hard-attention mechanism where the selected option policy is persisted for $T=20$ steps (termination limit is 20). For the Fetch-Reach, a similar setup is used, where the base agent has 2 fully connected layers of size 64 (instead of the 2 convolution layers and 1 fully connected layer) which is projected to yield the policy and the value function. A2T + A2C attention network over the option/distilled policies uses 2 fully connected layers of size 20. The option policy is persisted for $T=15$ steps (termination limit is 15).

4.3 Evaluating on Grid-worlds

50 options were obtained using the Eigen-options framework where the eigen-vectors of the graph laplacian are used to define options. The policies corresponding to each eigen-vector is obtained using a vanilla A2C agent (architecture identical to an A2C+A2T agent barring the attention). This is followed by the Option Encoder framework, which distills the options policies obtained from all states in the grid-world, into 5 distilled policies. The A2C+A2T agent can make use of the original set of options or the distilled set, which we term as A2C + original and A2C + distilled respectively. We also evaluate a vanilla A2C agent. Actor-mimic is another baseline that we consider, where all the options are distilled to one policy which is used in the A2C + A2T setup (we refer to this as A2C+AMN). Finally, we consider the random average agent which consists of an A2T+A2C agent attending to 5 policies and a base network. Each of the policies are obtained by averaging the policies of 10 randomly chosen option policies.

Figure 6: The leftmost figure denotes the set of 16 expert policies while the other 4 figures visualize the visitation count of the rollouts of the distilled policy.
The agents are periodically evaluated every 500 environment steps and the performance curves are presented in Figure 5. The graphs are clearly indicative of the fact that the A2C+distilled agent outperforms all other baselines. Since we tackle 100 different target tasks, the effort required to obtain the distilled policies (or the options) is negligible when compared to solving the multi-task problem. Hence, the presented performance curves are comparable.

4.4 Understanding the distilled policies

To understand the Option Encoder framework, we consider 16 policies, each navigating to a specific goal as indicated in the left-most image in Figure 6. Each blue cell denotes a goal towards which the expert is navigates to optimally. These experts are distilled to 4 different policies. In order to visualize these policies, we develop a heatmap of the visitation counts for each policy. The heatmap is obtained by sampling from the respective distilled policy. The agent is executed for 50,000 steps and is reset to a random start state after 100 steps in the environment. Figure 6 demonstrates how the Option Encoder framework capture the commonalities between various policies.

4.5 Randomly sampling a set of options

This analysis on GW2 was conducted to verify that our technique does not derive an advantage, solely based on a reduced number of option policies. Ten sets of options five in each set were considered, and each option appears in any one of the ten sets. Figure 7a shows that a random sample of options can lead to vastly varying performances (based on the relevance of the options). However, the distilled policies outperform every displayed sample of options, hinting at the fact that all the options are useful on solving a new set of tasks.

4.6 Varying the number of Distilled policies

An experiment on GW2, was conducted (see Figure 7b) to identify the impact of the number of distilled policies on the final performance. The plots are indicative of the fact that the agent is not
highly sensitive to this parameter. Distilling to a single policy yield a poor performance curve since a single policy is incapable of capturing the variety in the various options.

4.7 Varying termination limit

We analyze the impact of termination limit \( T \) (defined in Section 2) on the obtained performance curves for GW2. When the termination limit is low, the agent fails to leverage the knowledge of a useful sequence of actions since it cycles between the various options. Hence, a higher termination limit results in a persistent strategy for an extended duration, thereby yielding better performance (see Figure 8a). However, beyond a certain value for the termination limit, the performance deteriorates, owing to the fact that the agent spends an excessive amount of time on a single option.

4.8 The Fetch-reach task

In this task, we attempt to solve for 15 randomly sampled goals. The \( \gamma \) is set to 0.99 for this task and the Proximal Policy Optimization (PPO) algorithm [18] was used. The mean and the standard deviations are the inputs to the Option Encoder framework and the distilled output of the network. The distillation is done in a fashion similar to previous sections with regards to the other details. Since we do not have a suitable option-discovery method for high-dimensional domains, we consider 15 expert policies which each navigate to a randomly selected goal (different sample from the one used for evaluation). These experts are a proxy for the options and are distilled down to 5 policies using the Option Encoder. The experts are also trained using the PPO algorithm. Figure 8b compares the performance curves (averaged over the 15 target goals) of the distilled options in the PPO + A2T framework against the original set of experts (options).

5 Related Work

Several works have attempted to address the policy distillation and compression scenario. Prior works like Actor-mimic [13] have attempted to compress a collection of policies. This framework can, however, only distill a collection of policies into one single policy. Our method, on the other hand, can distill the expert policies into multiple basis policies. Pathnet [3] utilizes a large network, with weights being frozen appropriately. However, like Actor-mimic, Pathnet only discovers a single policy which may not have sufficient representational power. Pathnet addresses the continual learning setup, where there is a sequence of tasks to be solved, which may not be an appropriate approach to option pruning. Overcoming catastrophic forgetting [4], address a scenario identical to that addressed in Pathnet [3] by modifying the gradients in an appropriate manner but suffers from same problems described for the other two methods.

Option pruning has not been addressed in the context of option-discovery in great detail. Option compression is not necessary for works like Option-critic [2]. Deep Feudal reinforcement learning [25] or a collection of other works that discover options relevant to the current task. On the other hand, task-agnostic option discovery methods often need a large number of options to capture diverse behaviors. McGovern and Barto use a collection of filters to eliminate redundant options and Machado et al. have observed an improved performance when around 128 options are used in a rather modest 4-room grid-world. Basis functions have been explored in the context of value functions [9, 6], where the structure of the graph is used to define features for every state. On the other hand, our work uses a set of options to define the basis and over policies. Our work shares similarities with the PG-ELLA [1] lifelong learning framework, in that both attempt to discover a shared latent space, from which a set of tasks are solved. In this work, we focus on multi-task learning, where the basis policies are utilized across a set of tasks.

6 Conclusion

In this work, we present the Option Basis framework, which attempts to derive a policy basis from a collection of option policies. The distilled policies can be used as a substitute for the original set of options. We demonstrate the utility of the distilled policies using an empirical evaluation on a collection of tasks. As future work, one could extend the framework to work with value functions. Another area worth exploring is the continual learning setup. This could be set up in a batch-like
manner, where the set of basic policies can be constantly refined. The robust learning setup is also an area worth pursuing. The various distilled policies could help generalize across multiple environment configurations. Lastly, this work looks to find the basis but doesn’t attempt to re-orient the action space when compressing policies.

References

[1] Haitham Bou Ammar, Eric Eaton, Paul Ruvolo, and Matthew Taylor. Online multi-task learning for policy gradient methods. In International Conference on Machine Learning, pages 1206–1214, 2014.

[2] Pierre-Luc Bacon, Jean Harb, and Doina Precup. The option-critic architecture. 2017.

[3] Chrisantha Fernando, Dylan Banarse, Charles Blundell, Yori Zwols, David Ha, Andrei A Rusu, Alexander Pritzel, and Daan Wierstra. Pathnet: Evolution channels gradient descent in super neural networks. arXiv preprint arXiv:1701.08734, 2017.

[4] James Kirkpatrick, Razvan Pascanu, Neil Rabinowitz, Joel Veness, Guillaume Desjardins, Andrei A Rusu, Kieran Milan, John Quan, Tiago Ramalho, Agnieszka Grabska-Barwinska, et al. Overcoming catastrophic forgetting in neural networks. Proceedings of the national academy of sciences, 114(13):3521–3526, 2017.

[5] Vijay R Konda and John N Tsitsiklis. Actor-critic algorithms. In Advances in neural information processing systems, pages 1008–1014, 2000.

[6] George Konidaris, Sarah Osentoski, and Philip Thomas. Value function approximation in reinforcement learning using the fourier basis. In Twenty-fifth AAAI conference on artificial intelligence, 2011.

[7] Timothy P Lillicrap, Jonathan J Hunt, Alexander Pritzel, Nicolas Heess, Tom Erez, Yuval Tassa, David Silver, and Daan Wierstra. Continuous control with deep reinforcement learning. arXiv preprint arXiv:1509.02971, 2015.

[8] Marlos C Machado, Marc G Bellemare, and Michael Bowling. A laplacian framework for option discovery in reinforcement learning. arXiv preprint arXiv:1703.00956, 2017.

[9] Sridhar Mahadevan and Mauro Maggioni. Proto-value functions: A laplacian framework for learning representation and control in markov decision processes. Journal of Machine Learning Research, 8(Oct):2169–2231, 2007.

[10] Amy McGovern and Andrew G Barto. Automatic discovery of subgoals in reinforcement learning using diverse density. 2001.

[11] Ishai Menache, Shie Mannor, and Nahum Shimkin. Q-cut—dynamic discovery of sub-goals in reinforcement learning. In European Conference on Machine Learning, pages 295–306. Springer, 2002.

[12] Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A Rusu, Joel Veness, Marc G Bellemare, Alex Graves, Martin Riedmiller, Andreas K Fidjeland, Georg Ostrovski, and others. Human-level control through deep reinforcement learning. Nature, 518(7540):529, 2015.

[13] Emilio Parisotto, Jimmy Lei Ba, and Ruslan Salakhutdinov. Actor-mimic: Deep multitask and transfer reinforcement learning. arXiv preprint arXiv:1511.06342, 2015.

[14] Matthias Plappert, Marcin Andrychowicz, Alex Ray, Bob McGrew, Bowen Baker, Glenn Powell, Jonas Schneider, Josh Tobin, Maciek Chociej, Peter Welinder, Vikash Kumar, and Wojciech Zaremba. Multi-goal reinforcement learning: Challenging robotics environments and request for research, 2018.

[15] Martin L Puterman. Markov decision processes: Discrete stochastic dynamic programming. 1994.

[16] Janarthanan Rajendran, Aravind S Lakshminarayanan, Mitesh M Khptra, P Prasanna, and Balaraman Ravindran. Attend, adapt and transfer: Attentive deep architecture for adaptive transfer from multiple sources in the same domain. arXiv preprint arXiv:1510.02879, 2015.

[17] John Schulman, Sergey Levine, Pieter Abbeel, Michael Jordan, and Philipp Moritz. Trust region policy optimization. In International Conference on Machine Learning, pages 1889–1897, 2015.

[18] John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal policy optimization algorithms. arXiv preprint arXiv:1707.06347, 2017.
[19] David Silver, Aja Huang, Chris J. Maddison, Arthur Guez, Laurent Sifre, George van den Driessche, Julian Schrittwieser, Ioannis Antonoglou, Veda Panneershelvam, Marc Lanctot, Sander Dieleman, Dominik Grewe, John Nham, Nal Kalchbrenner, Ilya Sutskever, Timothy Lillicrap, Madeleine Leach, Koray Kavukcuoglu, Thore Graepel, and Demis Hassabis. Mastering the game of Go with deep neural networks and tree search. Nature, 529(7587):484–489, January 2016.

[20] Özgür Şimşek and Andrew G Barto. Using relative novelty to identify useful temporal abstractions in reinforcement learning. In Proceedings of the twenty-first international conference on Machine learning, page 95. ACM, 2004.

[21] Özgür Şimşek, Alicia P. Wolfe, and Andrew G. Barto. Identifying useful subgoals in reinforcement learning by local graph partitioning. pages 816–823. ACM Press, 2005.

[22] Richard S Sutton and Andrew G Barto. Reinforcement learning: An introduction. MIT press, 1998.

[23] Richard S Sutton, Doina Precup, and Satinder Singh. Between MDPs and semi-MDPs: A framework for temporal abstraction in reinforcement learning. Artificial intelligence, 112(1-2):181–211, 1999.

[24] Richard S Sutton, David A McAllester, Satinder P Singh, and Yishay Mansour. Policy gradient methods for reinforcement learning with function approximation. In Advances in neural information processing systems, pages 1057–1063, 2000.

[25] Alexander Sasha Vezhnevets, Simon Osindero, Tom Schaul, Nicolas Heess, Max Jaderberg, David Silver, and Koray Kavukcuoglu. FeUdal Networks for Hierarchical Reinforcement Learning. arXiv:1703.01161 [cs], March 2017.