CHARM MIXING IN THE STANDARD MODEL AND BEYOND

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The motivation most often cited in searches for $D^0 - \bar{D}^0$ mixing and CP-violation in charm system lies with the possibility of observing a signal from New Physics which dominates that from the Standard Model. We review recent theoretical predictions and experimental constraints on $D^0 - \bar{D}^0$ mixing parameters, concentrating on possible effects of New Physics.

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1. Introduction

Quantum mechanical meson-antimeson oscillations are sensitive to heavy degrees of freedom which propagate in the underlying mixing amplitudes. Comparing observed meson mixing with predictions of the Standard Model (SM), modern experimental studies would be able to constrain models of New Physics (NP). Yet, extensive precision data from B-factories and the Tevatron collider show that the large SM mixing successfully describes all available experimental data in $B_d$ and $B_s$ oscillations. The only flavor oscillation not yet observed is that of the charmed meson $D^0$, where SM mixing is very small and the NP component can stand out. This situation is an exact opposite to what happens in the $B$ system, where $B^0 - \bar{B}^0$ mixing measurements are used to constrain top quark couplings.

Together with the one loop Standard Model effects,\textsuperscript{1} NP effects can contribute to $\Delta C = 1$ (decays) or $\Delta C = 2$ (mixing) transitions. In the case of $D^0 - \bar{D}^0$ mixing these operators generate contributions to the effective operators that change $D^0$ state into $\bar{D}^0$ state, leading to the mass eigenstates

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle,$$

where the complex parameters $p$ and $q$ are obtained from diagonalizing the $D^0 - \bar{D}^0$ mass matrix. Note that $|p|^2 + |q|^2 = 1$. If CP-violation in mixing is neglected, $p$ becomes equal to $q$, so $|D_{1,2}\rangle$ become $CP$ eigenstates, $CP|D_{\pm}\rangle = \pm |D_{\pm}\rangle$. The mass and width splittings between these eigenstates are given by

$$x \equiv \frac{m_2 - m_1}{\Gamma}, \quad y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma}.$$

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2. Phenomenology of $D^0 - \bar{D}^0$ mixing

**Semileptonic decays.** The most natural way to search for charm mixing is to employ semileptonic decays. It is also not the most sensitive way, as it is only sensitive to $R_D = (x^2 + y^2)/2$, a quadratic function of $x$ and $y$. Use of the $D^0$ semileptonic decays for the mixing search involves the measurement of the time-dependent or time-integrated rate for the wrong-sign (WS) decays of $D^0$, where $c \rightarrow (\bar{c} \text{ via mixing}) \rightarrow s\ell^-\bar{\nu}$, relative to the right-sign (RS) decay rate, $c \rightarrow s\ell^+\nu$.

**Nonleptonic decays to non-CP eigenstates.** Currently the most stringent limits on the $D^0$ mixing parameters arise from the decay time dependent $D^0 \rightarrow K^+\pi^-$ measurements. Time-dependent studies allow separation of the direct doubly-Cabbibo suppressed (DCS) $D^0 \rightarrow K^+\pi^-$ amplitude from the mixing contribution $D^0 \rightarrow \bar{D}^0 \rightarrow K^+\pi^-$,

$$\Gamma_{D^0 \rightarrow K^+\pi^-} = e^{-\Gamma t} |A_{K^+\pi^-}|^2 \left[ R + \sqrt{R} R_m(y' \cos \phi - x' \sin \phi) \Gamma t + R_m^2 R^2_D(\Gamma t)^2 \right]$$

(3)

where $R$ is the ratio of DCS and Cabibbo favored (CF) decay rates. Since $x$ and $y$ are small, the best constraint comes from the linear terms in $t$ that are also linear in $x$ and $y$. A direct extraction of $x$ and $y$ from Eq. (3) is not possible due to unknown relative strong phase $\delta_D$ of DCS and CF amplitudes, as $x' = x \cos \delta_D + y \sin \delta_D$, $y' = y \cos \delta_D - x \sin \delta_D$. This phase can be measured independently. The corresponding formula can also be written for $\bar{D}^0$ decay with $x' \rightarrow -x'$ and $R_m \rightarrow R_m^{-1}$.

**Nonleptonic decays to CP eigenstates.** $D^0$ mixing can be measured by comparing the lifetimes extracted from the analysis of $D$ decays into the CP-even and CP-odd final states. In practice, lifetime measured in $D$ decays into CP-even final state $f_{CP}$, such as $K^+K^-, \pi^+\pi^-$, $\phi K_S$, etc., is compared to the one obtained from a measurement of decays to a non-CP eigenstate, such as $K^-\pi^+$. This analysis is also sensitive to a linear function of $y$ via

$$y_{CP} = \frac{\tau(D \rightarrow K^-\pi^+)}{\tau(D \rightarrow K^+K^-)} - 1 = y \cos \phi - x \sin \phi \left[ \frac{R_m^2 - 1}{2} \right],$$

(4)

where $\phi$ is a CP-violating phase. In the limit of vanishing CP violation $y_{CP} = y$. This measurement requires precise determination of lifetimes.

**Quantum-correlated final states.** The construction of tau-charm factories introduces new time-independent methods that are sensitive to a linear function of $y$. One can use the fact that heavy meson pairs produced in the decays of heavy quarkonium resonances have the useful property that the two mesons are in the CP-correlated states. For instance, by tagging one of the mesons as a CP eigenstate, a lifetime difference may be determined by measuring the leptonic branching ratio of the other meson. The final states reachable by neutral charmed mesons.
are determined by a set of selection rules according to the initial virtual photon quantum numbers $J^{PC} = 1^{−−}$. Since we know whether this $D(k_2)$ state is tagged as a (CP-eigenstate) $D_{±}$ from the decay of $D(k_1)$ to a final state $S_σ$ of definite CP-parity $σ = ±$, we can easily determine $y$ in terms of the semileptonic branching ratios of $D_{±}$, which we denote $B_{±}^L$. Neglecting small CP-violating effects,

$$y = \frac{1}{4} \left( \frac{B_{+}^L(D)}{B_{-}^L(D)} - \frac{B_{-}^L(D)}{B_{+}^L(D)} \right).$$

A more sophisticated version of this formula as well as studies of feasibility of this method can be found in Ref. [3].

The current experimental bounds on $y$ and $x$ are

$$y < 0.008 \pm 0.005, \quad x < 0.029 \quad (95\% \, C.L.) \ .$$

3. Charm mixing predictions in the Standard Model

The current experimental upper bounds on $x$ and $y$ are on the order of a few times $10^{-3}$, and are expected to improve in the coming years. To regard a future discovery of nonzero $x$ or $y$ as a signal for new physics, we would need high confidence that the Standard Model predictions lie well below the present limits. As was shown in [3], in the Standard Model, $x$ and $y$ are generated only at second order in SU(3)$_F$ breaking,

$$x, \ y \sim \sin^2 \theta_C \times [SU(3) \, \text{breaking}]^2,$$

where $θ_C$ is the Cabibbo angle. Therefore, predicting the Standard Model values of $x$ and $y$ depends crucially on estimating the size of SU(3)$_F$ breaking.

Theoretical predictions of $x$ and $y$ within the Standard Model span several orders of magnitude$^4$ (see Fig. 1). Roughly, there are two approaches, neither of which give

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$^a$Horizontal line references are tabulated in Table 5 of Ref. [3].
very reliable results because $m_c$ is in some sense intermediate between heavy and light. The “inclusive” approach is based on the operator product expansion (OPE). In the $m_c \gg \Lambda$ limit, where $\Lambda$ is a scale characteristic of the strong interactions, $\Delta M$ and $\Delta \Gamma$ can be expanded in terms of matrix elements of local operators. Such calculations typically yield $x, y < 10^{-3}$. The use of the OPE relies on local quark-hadron duality, and on $\Lambda/m_c$ being small enough to allow a truncation of the series after the first few terms. The charm mass may not be large enough for these to be good approximations, especially for nonleptonic $D$ decays. An observation of $y$ of order $10^{-2}$ could be ascribed to a breakdown of the OPE or of duality, but such a large value of $y$ is certainly not a generic prediction of OPE analyses. The “exclusive” approach sums over intermediate hadronic states, which may be modeled or fit to experimental data. Since there are cancellations between states within a given $SU(3)$ multiplet, one needs to know the contribution of each state with high precision. However, the $D$ is not light enough that its decays are dominated by a few final states. In the absence of sufficiently precise data on many decay rates and on strong phases, one is forced to use some assumptions.

The above discussion shows that, contrary to $B$ and $K$ systems, theoretical calculations of $x$ and $y$ in the Standard Model are very uncertain, from a percent to orders of magnitude smaller. Thus, New Physics (NP) contributions can easily stand out.

### 4. New Physics contribution to $D^0 - \bar{D}^0$ mixing

As one can see from the previous discussion, mixing in the charm system is very small. As it turns out, theoretical predictions of $x$ and $y$ in the Standard Model are very uncertain, from a percent to orders of magnitude smaller. Thus, New Physics (NP) contributions can easily stand out.

In order to see how NP might affect the mixing amplitude, it is instructive to consider off-diagonal terms in the neutral D mass matrix,

$$2M_D \left( M - \frac{i}{2} \Gamma \right)_{12} = \langle D^0 | H^{\Delta C = -2} | D^0 \rangle + \sum_n \frac{\langle D^0 | H^{\Delta C = -1}_w | n \rangle \langle n | H^{\Delta C = -1}_w | D^0 \rangle}{M_D - E_n + i\epsilon}$$

where $H^{\Delta C = -1}_w$ is the effective $|\Delta C| = 1$ hamiltonian.

**New Physics in $|\Delta C| = 2$ interactions.** Since all new physics particles are much heavier than the Standard Model ones, the most natural place for NP to affect mixing amplitudes is in the $|\Delta C| = 2$ piece, which corresponds to a local interaction at the charm quark mass scale. Integrating out NP degrees of freedom at some scale $\Lambda$, we are left with an effective Hamiltonian written in the form of series of operators...
of increasing dimension. Realizing this, it is not hard to write the complete basis of those effective operators, which most conveniently can be done in terms of left- and right-handed quark fields,
\[
\mathcal{H}_{NP}^{\Delta C=2} = \sum_{i=1}^{C_{NP}} C_i(\mu) Q_i(\mu),
\]
where \(C_i\) are the Wilson coefficients, and \(Q_i\) are the effective operators,
\[
\begin{align*}
Q_1 &= \bar{u}_L \gamma_\mu c_L \bar{u}_L \gamma_\mu c_L, \\
Q_2 &= \bar{u}_R \gamma_\mu c_R \bar{u}_L \gamma_\mu c_L, \\
Q_3 &= \bar{u}_L c_R \bar{u}_R c_L, \\
Q_4 &= \bar{u}_R c_L \bar{u}_R c_L, \\
Q_5 &= \bar{u}_R \sigma_{\mu\nu} c_R \bar{u}_R \sigma^{\mu\nu} c_L, \\
Q_6 &= \bar{u}_R \gamma_\mu c_R \bar{u}_R \gamma_\mu c_R, \\
Q_7 &= \bar{u}_L c_R \bar{u}_L c_R, \\
Q_8 &= \bar{u}_L \sigma_{\mu\nu} c_R \bar{u}_L \sigma^{\mu\nu} c_R.
\end{align*}
\]
These operators exhaust the list of possible contributions to \(\Delta C = 2\) transitions. Since these operators are generated at the scale \(\mu = \Lambda\) (at which new physics is integrated out), a non-trivial operator mixing can occur if we take into account renormalization group running of these operators between \(\mu = \Lambda\) and \(\mu \approx m_c\) scales. This running can be accounted for by solving RG equations obeyed by the Wilson coefficient functions,
\[
\frac{d}{d \log \mu} \tilde{C}(\mu) = \tilde{\gamma}^T(\mu)\tilde{C}(\mu),
\]
where \(\tilde{\gamma}^T(\mu)\) represents the matrix of anomalous dimensions of operators of Eq. (9). A prediction for a mixing parameter \(x\) in a particular model of new physics is then obtained by computing \(C_i(\Lambda)\) for a set of \(Q_i(\Lambda)\) generated by a given model, running the RG equations of Eq. (10) and computing matrix elements \(\langle D^0 | \bar{Q_i}(m_c) | D^0 \rangle \). As can be seen from Fig. (2) predictions for \(x\) vary by orders of magnitude for different

\textbf{Horizontal line references are tabulated in Table 5 of Ref. [6].}
models. It is interesting to note that some models require large signals in the charm system if mixing and FCNCs in the strange and beauty systems are to be small (e.g. the SUSY alignment model).

**New Physics in $|\Delta C| = 1$ interactions.** The local $|\Delta C| = 2$ interaction cannot, however, affect $\Delta \Gamma_D$ because it does not have an absorptive part. Thus, naively, NP cannot affect lifetime difference $y$. This is, however, not quite correct. Consider a $D^0$ decay amplitude which includes a small NP contribution, $A[D^0 \to n] = A_n^{(\text{SM})} + A_n^{(\text{NP})}$. Here, $A_n^{(\text{NP})}$ is assumed to be smaller than the current experimental uncertainties on those decay rates. Then it is a good approximation to write $y$ as

$$y \simeq \sum_n \frac{\rho_n}{\Gamma_D} A_n^{(\text{SM})} \bar{A}_n^{(\text{SM})} + 2 \sum_n \frac{\rho_n}{\Gamma_D} A_n^{(\text{NP})} \bar{A}_n^{(\text{SM})}. \quad (11)$$

The SM contribution to $y$ is known to vanish in the limit of exact flavor $SU(3)$. Moreover, the first order correction is also absent, so the SM contribution arises only as a second order effect. Thus, those NP contributions which do not vanish in the flavor $SU(3)$ limit must determine the lifetime difference there, even if their contributions are tiny in the individual decay amplitudes. A simple calculation reveals that NP contribution to $y$ can be as large as several percent in R-parity-violating SUSY models or as small as $\sim 10^{-10}$ in the models with interactions mediated by charged Higgs particles.

Consider, for example, models that do not vanish in the $SU(3)$ limit. The two most common scenarios involve models whose low energy $|\Delta C| = 1$ effective hamiltonian involves (V-A)$\otimes$(V-A) and (S-P)$\otimes$(S+P) couplings. For instance, SUSY models without R-parity would fit the bill. In these models, there are flavor-changing interactions of sleptons that can be obtained from the lagrangian

$$\mathcal{L}_R = \lambda_{ijk} L_i Q_j D_k^c, \quad (12)$$

The slepton-mediated interaction is not suppressed in the flavor $SU(3)$ limit and leads to

$$y_R = \frac{C}{M_T^2} \left[ (C_2 - 2C_1) \langle Q' \rangle + (C_1 - 2C_2) \langle \bar{Q}' \rangle \right]. \quad (13)$$

where $C' = -G_F m_t^2/(6\sqrt{2} \pi M_T \Gamma_T)$, $M_T$ is a slepton mass, $\lambda$ is given by $\bar{\lambda} = \lambda_{sd} - \lambda(\lambda_{dd} - \lambda_{ss}) - \lambda^2 (\lambda_{ds} + \lambda_{sd})$ with $\lambda_{sd} = \lambda'_{12} \lambda'_{21} \leq 1 \times 10^{-9}$, $\lambda_{ss} = \lambda'_{11} \lambda'_{21} \leq 5 \times 10^{-5}$, $\lambda_{dd} = \lambda'_{21} \lambda'_{21} \leq 5 \times 10^{-5}$, $\lambda_{ds} = \lambda'_{11} \lambda'_{22} \leq 5 \times 10^{-2}$ (see [13]), and $\langle Q' \rangle$ is

$$\langle Q' \rangle = \langle D^0 | \bar{\nu}_i \gamma_\mu P_L c_i \bar{\nu}_j \gamma_\rho P_R c_j | D^0 \rangle. \quad (14)$$

Operators with a tilde are obtained by swapping color indices in the charm quark operators. Using factorization to estimate matrix elements of the above operators and taking for definiteness $M_T = 100$ GeV, we arrive at $y_R \simeq -3.7\%$.

On the other hand, there are also several reasons that some NP models vanish in the flavor $SU(3)$ limit. First, the structure of the NP interaction might simply
mimic the one of the SM. Effects like that can occur in some models with extra space dimensions. Second, the chiral structure of a low-energy effective lagrangian in a particular NP model could be such that the leading, mass-independent contribution vanishes exactly, as in a left-right model (LRM). Third, the NP coupling might explicitly depend on the quark mass, as in a model with multiple Higgs doublets. However, most of these models feature second order SU(3)-breaking already at leading order in the 1/m_c expansion. This should be contrasted with the SM, where the leading order is suppressed by six powers of m_s and the second order only appears as a 1/m^6_c-order correction.

For instance, LRM provide new tree-level contributions mediated by right-handed (W(R)) bosons. The relevant effective lagrangian is

$$\mathcal{L}_{LR} = -\frac{g_R}{\sqrt{2}} V_{ab}^{(R)} \bar{u}_{a,i} \gamma^\mu P_R d_{b,i} W_{\mu}^{(R)} + \text{h.c.},$$

(15)

where V^{(R)}_{ik} are the coefficients of the right-handed CKM matrix. Since current experimental limits allow W^{(R)}_masses as low as a TeV, a sizable contribution to y is quite possible. We obtain

$$y_{LR} = -C_{LR} V_{cs}^{(R)} V_{us}^{(R)*} \left[ C_1 \langle Q' \rangle + C_2 \langle \bar{Q}' \rangle \right],$$

(16)

where C_{LR} = \lambda G_F^{(R)} G_F m_s^2 x_s/\sqrt{\pi} M_D \Gamma_D, G_F^{(R)} / \sqrt{2} = g_R^2/8 M^2_{W_R}, C_{1,2} are the SM Wilson coefficients and the operators appearing in Eq. (16) are given in Eq. (14). Using [4], we obtain numerical values for two possible realizations: (i) Manifest LR (V^{(L)} = V^{(R)}) gives y_{LR} = -4.8 \cdot 10^{-6} with M_{W_R} = 1.6 \text{ TeV} and (ii) Nonmanifest LR (V^{(R)}_{ij} \sim 1) gives y_{LR} = -8.8 \cdot 10^{-5} with M_{W_R} = 0.8 \text{ TeV} [12].

As one can see, small NP contributions to |\Delta C| = 1 processes produce substantial effects in the D^0\overline{D}^0 lifetime difference, even if such contributions are currently undetectable in the experimental analyses of charmed meson decays. Coupled with a known difficulty in computing SM contributions to D-meson decay amplitudes, it might be advantageous to use experimental constraints on y in order to test various NP scenarios due to better theoretical control over the NP contribution and SU(3) suppression of the SM amplitude.

New Physics and CP-violation in D^0\overline{D}^0 mixing. Another possible manifestation of new physics interactions in the charm system is associated with the observation of (large) CP-violation. This is due to the fact that all quarks that build up the hadronic states in weak decays of charm mesons belong to the first two generations. Since 2 \times 2 Cabbibo quark mixing matrix is real, no CP-violation is possible in the dominant tree-level diagrams which describe the decay amplitudes. CP-violating amplitudes can be introduced in the Standard Model by including penguin or box operators induced by virtual b-quarks. However, their contributions are strongly suppressed by the small combination of CKM matrix elements V_{cb} V_{us}^*.

It is thus widely believed that the observation of (large) CP violation in charm decays or mixing would be an unambiguous sign for new physics.
5. Summary

Charm physics provides new and unique opportunities for indirect searches for New Physics. NP can affect charm mixing in a variety of ways, mainly affecting both $x$ and $y$, as well as providing CP-violating asymmetries. Expected large statistical samples of charm data will allow new sensitive measurements of charm mixing and CP-violating parameters.

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