Noise induced phenomena in point Josephson junctions

Anna V. Gordeeva\textsuperscript{1}, Andrey L. Pankratov\textsuperscript{1} and Bernardo Spagnolo\textsuperscript{2}

\textsuperscript{1} Institute for Physics of Microstructures of RAS, Nizhny Novgorod, 603950, Russia\textsuperscript{*}
\textsuperscript{2} Dipartimento di Fisica e Tecnologie Relative, Università di Palermo and CNISM-INFM, Unità di Palermo, Group of Interdisciplinary Physics\textsuperscript{†}, Viale delle Scienze, edificio 18, I-90128 Palermo\textsuperscript{‡}

(Dated: October 9, 2008)

We present the analysis of the mean switching time and its standard deviation of short overdamped Josephson junctions, driven by a direct current and a periodic signal. The effect of noise enhanced stability is investigated. It is shown that fluctuations may both decrease and increase the switching time.

Keywords: Fluctuation effects, Josephson junction, noise delayed switching, noise enhanced stability, metastability.

I. INTRODUCTION

The investigation of nonlinear properties of Josephson junctions (JJ) is very important due to their broad applications in logic devices. Recently, a lot of attention was paid to Josephson logic devices with high damping. In papers [Rylyakov & Likharev, 1999; Bunyk \textit{et al.}, 2001] a description and analysis of the entire system of single flux quantum logic elements are presented. The noise properties of systems consisting of many elements may be well understood from the noise properties of a single JJ. The processes occurring in such devices are based on a reproduction of quantum pulses due to $2\pi$ spasmodic change of the phase difference of the overdamped JJs.

The use of high-$T_c$ JJs creates many new problems, such as the thermally induced digital errors and the effect of spreading of the switching speed due to thermal fluctuations. The analysis of influence of thermal fluctuations on different characteristics of JJs, such as the current-voltage characteristic or the life time of superconductive state, has been made before on the basis of Langevin approach [Barone & Paternò, 1982; Likharev, 1986; Ortlepp & Uhlmann, 2004; Ortlepp & Uhlmann, 2005]. Recently experimental and theoretical work has been done on the mean switching time or mean escape time from the metastable state in periodically driven Josephson junctions [Yu & Han, 2003; Pankratov & Spagnolo, 2004; Peltonen \textit{et al.}, 2006; Sun \textit{et al.}, 2007].

The process of switching of a single JJ with high damping under periodic driving was considered in [Pankratov & Spagnolo, 2004]. In that paper the influence of the resonant activation (RA) and the noise enhanced stability (NES) phenomena in the accumulation or suppression of timing errors in rapid single flux quantum (RSFQ) devices was analyzed. Specifically an interval of frequencies was found where the switching time increases with increasing noise intensity. This effect is called noise enhanced stability [Man tegna & Spagnolo, 1996; Agudov & Spagnolo, 2001; Dubkov, Agudov & Spagnolo, 2004; Fiasconaro \textit{et al.}, 2005; Spagnolo \textit{et al.}, 2007]. This paper presents the analysis of the effects of thermal fluctuations on the switching time for a single Josephson element with...
high damping, within the well-known resistive model of a Josephson junction [Likharev, 1986]. The goal of this work is to investigate in detail how the NES phenomenon depends on the parameters of the periodical driving signal such as the amplitude and the frequency.

II. GENERAL EQUATIONS AND STATEMENT OF THE PROBLEM

The dynamics of a short overdamped JJ, under a current \(i(t)\) is given by the following Langevin equation

\[
\begin{align*}
    \omega_c^{-1} \frac{d\varphi(t)}{dt} & = -\frac{du(\varphi)}{d\varphi} - i_f(t), \\
    u(\varphi) & = 1 - \cos(\varphi) - i(t)\varphi,
\end{align*}
\]

where \(\varphi\) is the phase difference of the order parameter [Barone & Paternò, 1982], \(u(\varphi)\) is the dimensionless potential profile (see Fig. 1.), \(\omega_c = 2eR_NI_c/\hbar\) is the characteristic frequency of the JJ, \(I_c\) is the critical current, \(R_N^{-1} = G_N\) is the normal conductivity of the JJ, \(e\) is the electron charge and \(\hbar = h/2\pi\), with \(h\) the Planck constant. Here \(i(t) = i_0 + f(t)\) is the total current across the junction, \(i_0\) is the constant bias current, \(f(t)\) is the driving signal, \(i(t) = I(t)/I_c\), \(i_f(t) = I_f(t)/I_c\), and \(I_f(t)\) is the random component of the current. Because of thermal fluctuations, the random current may be represented by white Gaussian noise

\[
\langle i_f(t) \rangle = 0, \quad \langle i_f(t) i_f(t + \tau) \rangle = \frac{2\gamma}{\omega_c} \delta(\tau),
\]

where \(\gamma = 2ekT/\hbar I_c = I_T/I_c\) is the dimensionless intensity of fluctuations, \(T\) is the temperature and \(k\) is the Boltzmann constant.

Initially, the JJ is biased with a current across the junction smaller than the critical one, that is \(i_0 = (I_0/I_c) < 1\), so as the initial condition we take the location of the phase in a potential minimum, \(\varphi_0 = \arcsin(i_0)\). A current signal \(f(t)\), such that \(i(t) > 1\), switches therefore the junction into the resistive state. In Fig. 1 we show the periodical potential profile of the JJ and its extreme positions within which it varies in time. The switching occurs not immediately, but at the later time, which is called the switching time. Due to the noise the switching time is a random quantity. We investigate therefore the mean switching time (MST) \(\tau\) and its standard deviation (SD) \(\sigma\). As a driving signal we choose a sinusoidal signal \(f(t) = A\sin(\omega t)\), where \(\omega\) is the oscillation frequency and \(A\) is the signal amplitude.

According to the definition [Malakhov & Pankratov, 2002] the mean switching time \(\tau\) and its standard deviation \(\sigma\) are

\[
\begin{align*}
\tau & = \langle t \rangle = \frac{\int_0^\infty tw(t)dt}{\int_0^\infty w(t)dt}, \\
\sigma & = \sqrt{\langle t^2 \rangle - \langle t \rangle^2},
\end{align*}
\]

where \(w(t) = -\partial P(t)/\partial t\), \(P(t)\) is the probability to find \(\varphi\) within the interval \((-\pi, \pi)\).

\[
U(\varphi)
\]

\[
\begin{array}{c}
\varphi_0 \\
\varphi_2
\end{array}
\]

FIG. 1 The potential profile of the JJ: the extreme positions during the periodical variations \((i = -0.5\) and \(i = 1.5\)), and the intermediate configuration \((i = 0.5)\). The bias current and the signal amplitude are respectively: \(i_0 = 0.5\) and \(A = 1\). The height of the barrier in the middle configuration of the potential profile is approximately 0.7.

In Fig. 2 the MST is shown versus the signal frequency \(\omega\) for \(i_0 = 0.5\), \(A = 1\) and for different values of the noise intensity, namely \(\gamma = 0.05, 0.2, 0.5, 1\). A frequency range, from
0.2 to 0.4, where the MST increases by increasing the noise intensity is clearly visible. In all the figures the MST $\tau$ and the frequency $\omega$ are normalized to $1/\omega_c$ and $\omega_c$, respectively.

In both Figs. 2 and 3, the MST has a minimum as a function of the driving frequency. This is the signature of the resonant activation phenomenon, investigated in a JJ in Refs. [Yu & Han, 2003; Pankratov & Spagnolo, 2004; Sun et al., 2007].

III. RESULTS AND DISCUSSION

The enhancement of the switching time may be considered in detail by plotting the dimensionless time MST as a function of the noise intensity for different values of the signal frequencies. Different behaviors of MST, depending on the values of the signal frequency, occur for physical systems with metastable states [Agudov & Spagnolo, 2001; Dubkov, Agudov & Spagnolo, 2004]. Specifically a nonmonotonic behavior and a monotonic one may be observed.

Let us start our consideration from small to large frequencies for the case $i_0 = 0.5$ and $A = 1$, see Fig. 2. For frequencies smaller than 0.2 the character of the curves is similar: the MST decreases monotonically with increasing noise intensity. This case is presented in Fig. 4 by two values of frequency: $\omega = 0.05$ and 0.1. If the value of the noise intensity becomes greater or equal to the height of the barrier, which is approximately equal to 0.7 in the middle configuration (see Fig. 1), the particle does not see the fine structure of the potential. This is the reason why we restrict to 1 the values of the dimensionless noise intensity in the plots (Figs. 2-9). So, we can see that the MST decreases, with respect to the level of deterministic switching time (i.e. for $\gamma = 0$), by increasing the noise intensity. The explanation of this behavior is the following. For small noise intensity and very low frequencies, the particle lies in the minimum for a long enough time due to the slow variation of the potential profile. The switching from the superconductive state to the resistive one does not occur until the potential barriers disappear almost completely. By increasing the noise intensity, the proba-
bility of the thermal activated switching increases and as a result the MST decreases.

The next interval of frequencies which we consider is from 0.2 to, approximately, 0.52, because for such frequencies the behavior of the curves is similar. In this frequency range the NES effect appears, the MST(\(\gamma\)) curve behaviour becomes nonmonotonic in contrast with the previous case (\(\omega < 0.2\)). In Fig. 4 this behavior is shown for three frequency values, namely \(\omega = 0.5, 0.45, \text{ and } 0.4\). We note that in these three curves, after the nonmonotonic behavior, the MST increases again with the noise intensity \(\gamma\) (for \(\gamma > 1\)). Due to the increasing values of the signal frequency the escape process from the metastable state becomes more rapid for low noise intensities and the MST for \(\gamma \to 0\) is lower with respect to the previous frequency range (\(\omega < 0.2\)). Due to the periodic variation of the potential profile and for noise intensities smaller than the barrier height, the particle starting from the minimum reaches a position near the top of the barrier. Then there is an optimum range of the system parameters, including the driving parameters, for which the particle turns back at the metastable state because of the noise [Agudov & Spagnolo, 2001; Dubkov, Agudov & Spagnolo, 2004]. An enhancement of the lifetime of the metastable state produces an enhancement of the mean switching time. By increasing the noise intensity the thermally activated escape increases too and the MST decreases. For a further increase of the noise, for intensity values greater than the barrier height (\(\gamma > 1\)), the potential profile ”seen” by the particle is a linear one, which fluctuates between the extreme positions of Fig. 1. The particle can reach more easily positions near the state at \(\varphi = 0\) and the MST, therefore, restarts to increase again with the noise intensity.

The nonmonotonic behavior shown in Fig. 4 may be explained also by considering the competition of two factors in the above mentioned frequency range: the influence of the returning force from the lefthand side of the potential profile and the increase of the variance of the phase position with increasing temperature. This explanation is related to the static case when the switching event starts because the value of the bias current is larger than the critical one [Malakhov & Pankratov, 1996]. By increasing the noise intensity the phase variance increases too. When the phase variance is not so large, the main part of the phase probability distribution is located on the flat part near the point \(\varphi = \arcsin(i_0)\), where the influence of the returning force is low. Thus, the influence of variance dominates, and the mean switching time increases with increasing of the noise intensity, because the particle stays more near the metastable state. Further increase of the phase variance, due to the noise, produces a larger probability distribution of the particle, which reaches high slope positions of the potential profile. Then the returning force begins to affect from left to right because of the asymmetry induced by the potential profile and does not permit, in this frequency range, the probability distribution to move into the direction where the potential profile goes up quickly. Therefore, the probability distribution expands only into the right direction and the MST decreases because the particle escapes. Finally, the noise intensity

![FIG. 4](image-url) MST versus noise intensity for seven different values of driving frequencies, namely \(\omega = 0.05, 0.1, 0.2, 0.3, 0.4, 0.45, 0.5\). Here \(A = 1\) and \(i_0 = 0.5\).
becomes so large that it is possible for particle to move upstairs along the potential. The returning force is not able to hold the increasing of the variance anymore and the switching time increases again.

By further increasing of the signal frequency \( \omega > 0.5 \), for the parameter values used to obtain the curves in Fig. 4 a trapping phenomenon occurs. A threshold frequency \( \omega_{th} \) exists which does not allow the particle to move to the next valley during one period of the signal. This means that for driving frequency \( \omega > \omega_{th} \), the particle is trapped within one period of the potential profile and, as a consequence, the MST diverges (tends to an infinite value) without noise. The value of the threshold frequency increases with increasing the bias current and/or the maximal current across the junction [Agudov & Spagnolo, 2001; Dubkov, Agudov & Spagnolo, 2004]. The frequency dependence of the switching time, for zero noise intensity, is presented in the inset of Fig. 5. To be trapped the particle, starting at \( t = 0 \) from the position \( \varphi_0 \) (see Fig. 1) of the potential profile in the middle configuration \( (i = 0.5) \), should not reach the position at \( \varphi_2 \), in the lowest configuration of the potential profile \( (i = 1.5) \), within a quarter of the signal period. As a consequence the switching event from the superconductive state to the resistive one doesn’t occur. To estimate the threshold frequency we calculate, in absence of noise, the time \( t \) spent by the particle to go from the point \( \varphi_0 \) to the point \( \varphi_2 \), in the most favorable case, when the potential profile is fixed and is in the lowest configuration, with the current \( i = 1.5 \) (see Fig. 1).

This time \( t \), which is well approximated by one quarter of the threshold period \( T_{th} \), can be found from the solution of Eq. (11) without random current \( i_f \) and for a constant value of the total current \( i(t) \) exceeding the critical one [Malakhov & Pankratov, 1996]

\[
t = \frac{F(\varphi_2) - F(\varphi_0)}{\omega_c},
\]

From here the threshold frequency \( \omega_{th} = 2\pi/T_{th} \approx \pi/2t \). The values of \( \omega_{th} \) obtained from Eq. (1) are marked by the vertical lines in the inset of Fig. 5.

![FIG. 5 MST versus noise intensity for different values of the driving frequency, namely \( \omega = 0.55, 0.65, 0.7, 0.71, 0.72 \), which approach the threshold frequency \( \omega_{th} \approx 0.72 \) for a bias current \( i_0 = 0.8 \) and a signal amplitude \( A = 0.7 \). Inset: the MST versus the driving frequency, from numerical simulations of Eq. (11) in the absence of noise (\( \gamma = 0 \)), for three groups of values of \( A \) and \( i_0 \). The different values of \( \omega_{th} \), calculated by Eq. (1), are marked by vertical lines.](image)

Specifically we have: \( \omega_{th} = 0.48 \), for \( i_0 = 0.5 \) and \( A = 1 \); \( \omega_{th} = 0.24 \), for \( i_0 = 0.5 \) and \( A = 0.7 \); \( \omega_{th} = 0.69 \), for \( i_0 = 0.8 \) and \( A = 0.7 \). As it is seen from the inset of Fig. 5, these values are very close to the exact values calculated by numerical simulations of Eq. (11) in the absence of noise (\( i_f(t) = 0 \)). The most prominent NES effect is observed when the frequency of the periodic signal is near the threshold frequency (see Fig. 5). For \( i_0 = 0.8 \) and \( A = 0.7 \) the maximum MST is approximately three times greater than its value at \( \gamma = 10^{-4} \).

In the next Fig. 6 the curves of MST are shown vs the noise intensity for five frequency values near and larger than the threshold \( \omega_{th} \), that is for \( \omega \geq 0.5 \approx \omega_{th} \). Two distinct
transient dynamical regimes are visible. For $\omega=0.5$ we see a nonmonotonic behavior, with a finite value of $\tau$ for $\gamma \to 0$. For frequencies larger than the threshold value $\omega_{th}$, all the curves are characterized by a monotonic divergent behavior in the limit of small noise intensity ($\gamma \to 0$). In both dynamical regimes a minimum of MST is present for a noise intensity value of the order of the barrier height. By increasing the signal frequency the curves approach the behavior of the MST obtained with a fixed potential profile, corresponding to a constant total current $i=0.5$. For high frequency values, in fact, the fluctuations of the potential profile are so rapid that the potential "seen" by the particle is the average potential, that is the middle configuration of Fig. 1, with $i=0.5$. In Fig. 6 the bold curve corresponds to the case of fixed potential profile with constant total current $i=0.5$, and it was calculated by using the exact analytical expression of MST, obtained in [Malakhov & Pankratov, 1996].

In the regime of frequencies above the threshold one the conditions of stochastic resonance (SR) observation are fulfilled (see e.g. [Borromeo & Marchesoni, 2000]). SR is manifested when the MST does not exceed the period of signal and thus, the system response becomes periodic owing to noise (see Fig. 5, curve $\omega=0.72$ for which minimal MST is equal to $\approx 6$ and Fig. 6, curve $\omega=0.6$, minimal MST $\approx 10$).

In Fig. 7 the MST and its SD for four values of the signal frequency are shown. We note that, rising from low to high frequencies near the threshold $\omega_{th}$, monotonic and non-monotonic behavior of the MST correspond to the same behavior of the SD, respectively. Moreover, in the limit of small noise intensity, all the SD behaviors show lower values with respect to the corresponding MST values, but with different behavior depending on how much the frequency is close to the threshold $\omega_{th}$. For frequency $\omega=0.1$, in fact, the SD increases slowly in such a way that it takes the same value of the MST for large noise intensity ($\gamma=1$). But for greater frequency the noise intensity value, for which the two curves of MST and SD cross, decreases. Specifically we have the following cross point values: $\gamma=0.04$ for $\omega=0.45$ and $\gamma=0.002$ for $\omega=0.5$. This means that when the driving frequency is close to the threshold one the system becomes more unstable and even a small variation of the noise intensity has a strong influence on the
system. The curve, shown in Fig. 7 by the bold line, is calculated for the case of the total current \( i = 1.5 \) and demonstrates a behavior of square root of the noise intensity (\( \sim \sqrt{\gamma} \)). This curve was obtained from the asymptotic analytical expansion of the SD in the small noise limit \( \gamma \ll 1 \) and for a fixed potential (\( \omega = 0 \)) [Pankratov & Spagnolo, 2004; Gordeeva & Pankratov, 2006]. In the limit of \( \gamma = 0.15 \cdot 10^{-3} \). After the threshold frequency, we recover for the SD the same divergent behavior of the MST, but with larger values. This fact confirms that the switching event in this region of parameters is due to the noise. It should be noted also that for large noise intensities (\( \gamma > 1 \)) all graphs of the MST and SD collapse to the same curve, not depending on the signal frequency (see Fig. 5 and Fig. 8). This means that the new regime of switching is unaffected by the signal and it is only due to the noise.

\[
\begin{align*}
\sigma &= 10^0, 10^1, 10^2, 10^3, 10^4, 10^5, 10^6, 10^7, 10^8, 10^9, 10^{10}, 10^{11} \\
\tau &= 1, 10, 100, 1000, 10000, 100000, 1000000, 10000000, 100000000, 1000000000 \\
\end{align*}
\]

FIG. 8 SD versus noise intensity for five driving frequencies, namely \( \omega = 0.55, 0.65, 0.7, 0.71, 0.72 \). The bold straight line gives the asymptotic analytical behavior (\( \gamma \ll 1 \)) of SD for constant total current \( i = 1.5 \). Here \( A = 0.7, i_0 = 0.8 \).

\( \omega \to 0 \) the curves of the SD, obtained by our simulations, approach the asymptotic curve obtained for fixed potential. The behavior of the SD curve for \( \omega = 0.1 \), obtained by numerical simulations of Eq. (1), is very close to the theoretical asymptotic curve (\( i = 1.5 \)) not only in the low noise limit but also at larger values of \( \gamma (\gamma = 0.1 - 1) \). This feature of the SD(\( \gamma \)) - curve behavior is shown in more detail in the next logarithmic plot (Fig. 8), which demonstrates the large rising of the SD for small noise intensities with increasing frequency. In Fig. 8 the curves of the SD are shown for the same frequency values and parameters used in Fig. 5 for the MST behaviors. Again we note that the cross point between the MST and SD curves, for frequencies close to the threshold one (\( \omega \approx 0.72 \) in this case), is at very low noise intensities. For example for \( \omega = 0.71 \), the cross point is at

\[
\begin{align*}
\sigma &= 10^0, 10^1, 10^2, 10^3, 10^4, 10^5, 10^6, 10^7, 10^8, 10^9, 10^{10}, 10^{11} \\
\tau &= 1, 10, 100, 1000, 10000, 100000, 1000000, 10000000, 100000000, 1000000000 \\
\end{align*}
\]

FIG. 9 MST versus noise intensity for two cases: (i) fixed bias current, namely \( i_0 = 0.5 \) and \( A = 1.05, 1.1 \), and (ii) fixed signal amplitude, namely \( A = 1 \) and \( i_0 = 0.5, 0.55, 0.6 \). Here \( \omega = 0.5 \).

In Fig. 9 the curves of MST vs \( \gamma \) for different values of the bias current and signal amplitude are shown. We consider two cases: (i) the bias current is fixed, namely \( i_0 = 0.5 \) and \( A = 1.05, 1.1 \), and (ii) the signal amplitude is fixed, namely \( A = 1 \) and \( i_0 = 0.5, 0.55, 0.6 \). We choose \( \omega = 0.5 \) as a signal frequency for which the NES effect is more pronounced. We see that changes of the bias current modify the curves of MST more than changes of the signal amplitude.

IV. CONCLUSIONS

We analyzed the transient dynamics of a single overdamped Josephson junction driven by a periodic signal. The conditions of existence of the NES effect are investigated. We
find that the enhancement of the switching time is larger for frequencies of the periodic signal close to the threshold frequency. In the region of the noise intensity values in which the mean switching time begins to increase, the standard deviation increases too but with different behavior depending on the proximity of the signal frequency to the threshold \( \omega_{\text{th}} \). The interval of frequencies, in which the NES effect is observed, depends on the maximal value of the total current across the junction and on the value of the bias current. There are two ways to change the value of the total current: by changing the amplitude of the driving signal or the value of the bias current. We find that the switching time changes more significantly due to the bias current variation, than due to the signal amplitude. We note that for small but nonzero values of the McCumber-Stewart parameter the MST increases by several percent but qualitative character of the curves remains the same, as in the considered overdamped case.

References

Agudov, N.V. & Spagnolo, B. [2001] "Noise enhanced stability of periodically driven metastable states", \textit{Phys. Rev. E} \textbf{64}, 035102(R)(4).

Barone, A. & Paternò, G. [1982] \textit{Physics and Applications of the Josephson Effect} (Wiley, New York).

Borromeo, M. & Marchesoni, F. [2000] "AC driven jump distributions on a periodic substrate", \textit{Surf. Sci. Lett.} \textbf{465}, L771-L776.

Bunyk, P., Likharev, K. & Zinoviev, D. [2001] "RSFQ technology: physics and devices", \textit{Int. J. High Speed Electron. Syst.} \textbf{11}, 257 - 305.

Gordeeva, A. V. & Pankratov, A. L. [2006] "Minimization of timing errors in reproduction of single flux quantum pulses", \textit{Appl. Phys. Lett.} \textbf{88}, 022505.

Fiasconaro, A., Spagnolo, B. & Boccaletti, S. [2005] "Signatures of noise-enhanced stabil-

ity in metastable states", \textit{Phys. Rev. E} \textbf{72}, 061110(5).

Dubkov, A.A., Agudov, N.V. & Spagnolo, B. [2004] "Noise enhanced stability in fluctuating metastable states", \textit{Phys. Rev. E} \textbf{649}, 061103(7).

Likharev, K. K. [1986] \textit{Dynamics of Josephson junctions and Circuits} (Gordon and Breach, New York) pp. 45-48.

Malakhov, A.N. & Pankratov, A.L. [1996] "Influence of thermal fluctuations on time characteristics of single josephson element with high damping. Exact solution", \textit{Physica C} \textbf{269}, 46-54.

Malakhov, A.N. & Pankratov, A.L. [2002] "Evolution times of probability distributions and averages - Exact solutions of the Kramers’ problem". \textit{Adv. Chem. Phys.} \textbf{121}, 357-438.

Mantegna, R.N. & Spagnolo, B. [1996] "Noise enhanced stability in an unstable system", \textit{Phys. Rev. Lett.} \textbf{76}, 563-566.

Ortlepp, T. & Uhlmann, H. [2004] "Noise analysis for intrinsic and external shunted Josephson junctions", \textit{Supercond. Sci. Technol.} \textbf{17}, S112-116.

Ortlepp, T. & Uhlmann, H. [2005] "Noise induced timing jitter: a general restriction for high speed RSFQ devices", \textit{IEEE Trans. Appl. Supercond.} \textbf{16}, 344-347.

Pankratov, A. L. & Spagnolo, B. [2004] "Suppression of timing errors in short overdamped josephson junctions", \textit{Phys. Rev. Lett.} \textbf{93}, 177001.

Peltonen, J. T., Timofeev, A. V., Meschke, M. & Pekola, J. P. [2006] "Detecting current noise with a Josephson junction in the macroscopic quantum tunneling regime", \textit{Journal of Low Temp. Phys.} \textbf{146}, 135-159.

Rylyakov, A. V. & Likharev, K. K. [1999] "Pulse jitter and timing errors in RSFQ circuits", \textit{IEEE Trans. Appl. Supercond.} \textbf{1} (3), 3539-3544.

Spagnolo, B., Dubkov, A. A., Pankratov, A. L., Pankratova, E. V., Fiasconaro, A. & Ochab-Marcinek, A. [2007] "Lifetime of metastable states and suppression of noise in Inter-
disciplinary Physical Models”, *Acta Physica Polonica B* 38 (5), 1925-1950.

Sun, G. *et al.* [2007] "Thermal escape from a metastable state in periodically driven Josephson junctions”, *Phys. Rev. E* 75, 021107(4).

Yu, Y. & Han, S. [2003] "Resonant escape over an oscillating barrier in underdamped Josephson tunnel junctions”, *Phys. Rev. Lett.* 91, 127003.