Baryonic and Gluonic Correlators in Hot QCD

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ABSTRACT

We extend our earlier work on static color singlet correlators in high T QCD (DeTar correlators) to baryonic and gluonic sources, and estimate the corresponding screening masses using the dimensionally reduced theory. We discuss spin and polarization dependence of meson and baryon masses in the $T \to \infty$ limit, and possible nonperturbative effects at non-asymptotic temperatures.

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1. Introduction

Recent lattice measurements of correlation functions of the type

\[ C_\alpha(x_3) = \left< \int_0^\beta d\tau \int d^2 r J_\alpha(\vec{r}, x_3, \tau) \int_0^\beta d\tau' J_\alpha(\vec{0}, 0, \tau') \right> \] (1.1)

(where \( \beta = 1/T \), \( \vec{r} = (x_1, x_2) \) and \( J_\alpha \) a color-singlet operator)\footnote{For baryonic sources the average over imaginary time, \( \tau \), is weightened by \( \cos \omega_0 \tau \) where \( \omega_0 \) is the lowest Matsubara frequency.} show that the hadronic screening lengths above the QCD finite temperature (chiral) phase transition point fall into chiral multiplets. The screening masses are approximately \( 2\pi T \) for mesons and \( 3\pi T \) for baryons, with the notable exceptions being the pion and its scalar partner, which are considerably lower \footnote{For baryonic sources the average over imaginary time, \( \tau \), is weightened by \( \cos \omega_0 \tau \) where \( \omega_0 \) is the lowest Matsubara frequency.}. It has also been known for some time that bulk thermodynamic quantities, like energy and entropy density, are roughly as expected from a gas of non-interacting quarks and gluons \footnote{For baryonic sources the average over imaginary time, \( \tau \), is weightened by \( \cos \omega_0 \tau \) where \( \omega_0 \) is the lowest Matsubara frequency.}. The exception here is the pressure, which is considerably lower than expected from the free gas picture. Finally, the fermionic susceptibility, \( \partial^2 \ln Z/\partial m^2 \) varies rapidly across the transition region from being small in the hadron phase, to being large in the quark phase \footnote{For baryonic sources the average over imaginary time, \( \tau \), is weightened by \( \cos \omega_0 \tau \) where \( \omega_0 \) is the lowest Matsubara frequency.}. All these results are suggestive of a high temperature phase of weakly interacting (Debye screened) quarks and gluons.

On the other hand, the Coulomb gauge lattice calculations of the spatial structure of the mesonic correlators via

\[ C_\alpha(\vec{r}, x_3) = \left< \int_0^\beta d\tau \int d^2 r_1 d^2 r_2 \psi^\dagger(\vec{r}_1, 0, \tau) \Gamma_\alpha \psi(\vec{r}_2, 0, \tau) \int_0^\beta d\tau' \psi^\dagger(\vec{R}, x_3, \tau') \Gamma_\alpha \psi(\vec{R} - \vec{r}, x_3, \tau') \right> \] (1.2)

show strong evidence for correlations in the transverse direction \footnote{For baryonic sources the average over imaginary time, \( \tau \), is weightened by \( \cos \omega_0 \tau \) where \( \omega_0 \) is the lowest Matsubara frequency.}. The presence of such correlations is not unexpected given that the the spatial Wilson loops obey an area law at all temperatures \footnote{For baryonic sources the average over imaginary time, \( \tau \), is weightened by \( \cos \omega_0 \tau \) where \( \omega_0 \) is the lowest Matsubara frequency.}.\footnote{For baryonic sources the average over imaginary time, \( \tau \), is weightened by \( \cos \omega_0 \tau \) where \( \omega_0 \) is the lowest Matsubara frequency.}

What causes the correlations in the spatial directions, and to what extent they are important for our understanding of the high temperature phase, is not clear. What is clear, however, is that any description of the high temperature phase has to account for the above results. In a recent paper \footnote{For baryonic sources the average over imaginary time, \( \tau \), is weightened by \( \cos \omega_0 \tau \) where \( \omega_0 \) is the lowest Matsubara frequency.}, hereafter referred to as I, two of us suggested that at very high temperature, the screening lengths and "wave functions" discussed above, can be understood from an analysis of the static part of the gluon field together with the lowest energy quark modes. This "dimensionally reduced" QCD is a YM-Higgs model with
heavy \((i.e. \, M = \pi T)\) quarks, which is believed to be confining. The screening masses and wave functions correspond to masses and wave functions of heavy quark states in the dimensionally reduced theory, and can be calculated as Coulomb bound states using usual charmonium-type methods. The masses, \(m_\alpha\), in the mesonic correlators are thus naturally \(m_\alpha \approx 2M = 2\pi T\), and the wave functions show exponential fall-off at large distances, both in qualitative agreement with the numerical calculations. It has also been shown, that in the case of 2+1 dimensional QCD at high \(T\), the Coulomb bound state picture leads to the correct description of the quark susceptibilities at asymptotic temperatures \([9],[10]\).

In this paper we extend our analysis in I to (color singlet) baryonic and gluonic sources and give a more detailed discussion of the spin and polarization dependence of the screening masses. For the baryons the calculations are very similar to the mesonic case and are on the same level of rigour. In the case of gluonic operators, our methods are more questionable since the size of the resulting bound state is larger than the Debye screening length, and we cannot rule out the possibility that the infrared divergences in the magnetic sector will change our predictions considerably. In section 2 we recapitulate the arguments that lead to the dimensional reduction scheme using a formalism that is somewhat different from the one used in I. We also discuss the polarization dependence of the various mesonic correlators. Our results for baryonic and gluonic correlators can be found in section 3 and 4 respectively. In the latter we also consider correlations between Wilson lines. In section 5, we discuss non-asymptotic effects that might explain the difference between the lattice data and our asymptotic prediction for the fine-structure of the screening mass spectrum. Section 6, finally, contains some summary comments and discussion. Some details about the calculations can be found in Appendix, A, B and C.

2. Dimensional Reduction and DeTar Correlators

We now briefly summarize the dimensional reduction scheme for the fermionic part of the high \(T\) QCD lagrangian. The starting point is the Euclidian action for free fermions,

\[
\mathcal{L}_4 = i\bar{\psi}^\dagger (-\hat{\gamma}_\mu \hat{D}_\mu + m)\psi ,
\]

where \(\hat{D}_\mu = \hat{\partial}_\mu - ig\hat{A}_\mu\), and where the (Hermitian) Euclidian gamma matrices

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\(^2\) This is based on the already mentioned fact that space-like Wilson loops obey an area law at all temperatures. Recent work by Kärkkäinen et al.\([3]\) shows that the string tension \(\sigma(T) \sim T^2\) at high temperature.
satisfy
\[ \{ \hat{\gamma}_\mu, \hat{\gamma}_\nu \} = 2\delta_{\mu\nu} \]  
(2.2)

Now, retain only the modes with energy \( \omega = \pm \pi/\beta = \pm \pi T \) and define \( \hat{\phi}_\pm \) by
\[
\begin{align*}
\sqrt{\beta} \hat{\psi} &= VU e^{i\pi T \hat{x}_4} \hat{\phi}_+ + VU e^{-i\pi T \hat{x}_4} \hat{\phi}_- \\
\sqrt{\beta} \hat{\psi}^\dagger &= \hat{\phi}_+^\dagger U^\dagger e^{-i\pi T \hat{x}_4} + \hat{\phi}_-^\dagger U^\dagger e^{i\pi T \hat{x}_4}
\end{align*}
\]  
(2.3)

where \( U_\pm = e^{\pm i \frac{\vartheta}{2} \gamma^3} \) with \( \cot \vartheta = \frac{m}{\pi T} \), and where \( V = e^{i\vartheta_3 \pi/4} \) with \( \hat{\sigma}_{\mu\nu} = \frac{i}{2} [\hat{\gamma}_\mu, \hat{\gamma}_\nu] \), rotates an angle \( \pi/2 \) in the (\( \mu\nu \)) plane. In particular \( V^\dagger \hat{\gamma}_4 V = \hat{\gamma}_4 \) and \( V^\dagger \hat{\gamma}_4 V = -\hat{\gamma}_3 \). Expressed in these variables the dimensionally reduced Lagrangian becomes
\[
L_3 = \sum_{a=\pm} -i\hat{\phi}_a^\dagger [\hat{\gamma}_4 (\hat{\partial}_3 - ig \hat{A}_3) + \hat{\gamma}_i (\hat{\partial}_i - ig \hat{A}_i) - M] + ig \hat{A}_4 (\cos \vartheta \hat{\gamma}_3 - ai \sin \vartheta) \hat{\phi}_a
\]  
(2.4)

where \( M^2 = (\pi T)^2 + m^2 \). The last step is now to rotate to a fictitious 2+1 dimensional Minkowski space (from now on greek indices run from 0 to 2 and roman from 1 to 2) by \( \hat{\phi} = \phi, \hat{\phi}^\dagger = i\bar{\phi}, (\hat{x_1, \hat{x_3}) = (x^i, ix^0), (\hat{\gamma}_i, \hat{\gamma}_3, \hat{\gamma}_4) = (-i\gamma_i, -i\gamma_3, -\gamma_0) \) and \( (\hat{A}_i, \hat{A}_3, \hat{A}_4) = \sqrt{T}(A_i, -iA_0, H) \), where \( H \) is the Higgs field, \( g_3 = \sqrt{T}g \) the 3d gauge coupling constant and where we use the metric (+−−) in Minkowski space. The resulting Lagrangian reads
\[
L_{2+1} = \sum_{a=\pm} \bar{\phi}_a \left[ i \gamma^\mu (\partial_\mu - ig_3 A_\mu) - M + g_3(a \sin \vartheta - \cos \vartheta \gamma^3) H \right] \phi_a
\]  
(2.5)

In the dimensionally reduced theory the quarks are heavy with mass \( M \), the adjoint Higgs scalar has the electric mass \( m_E = \sqrt{4/3}gT \) (for two light fermions) and the gluons are expected to acquire a magnetic mass \( \sim g^2 T \). The quark and Higgs masses are easily understood in perturbation theory as a consequence of the absence of fermionic zero modes and Debye screening respectively. In contrast, the magnetic mass is not calculable in perturbation theory and the predictions made on the basis of the dimensionally reduced theory are only reliable if they are insensitive to distance scales \( \geq 1/g^2 T \). A more detailed discussion of these points can be found in I.

\footnote{In I we used a different reduction scheme based on two-spinors. The lagrangian given there was not correct due to a subtlety in one of the transformations. However, the conclusions of the paper are not affected. We give the correct two-spinor lagrangian in Appendix A.}
Note that our conventions are slightly unusual in that we use 4 dimensional matrices to represent the 2+1 dimensional Clifford algebra. In Appendix A we give a formulation in terms of two different two-spinors that however have non-diagonal mass terms. For practical calculation the Lagrangian (2.5) is more useful.

For \( m = 0 \) the original Lagrangian (2.1) is invariant under the chiral transformation \( \hat{\psi} \to e^{i\alpha \hat{\gamma}_5} \hat{\psi}, \hat{\psi}^\dagger \to \hat{\psi}^\dagger e^{i\alpha \hat{\gamma}_5} \) where \( \hat{\gamma}_5 = \gamma_5 = -\hat{\gamma}_1 \hat{\gamma}_2 \hat{\gamma}_3 \hat{\gamma}_4 \). After the variable change, this corresponds to the Lagrangian (2.5) being invariant under \( \phi_\pm \to e^{\pm i\alpha \gamma_3 \gamma_5} \phi_\pm \). Also note that in spite of the mass term \( \mathcal{L}_{2+1} \) must be invariant under parity transformations since the original \( \mathcal{L}_4 \) is. It is easy to verify that the relevant transformation is \( x_1 \to -x_1, \phi_\pm \to -i\gamma_5 \gamma_1 \phi_\pm \).

Under the variable change (2.3) the expressions for the sources are also changed and we give a translation table for the most important currents in the chiral limit \( m = 0 \). Others are easily arrived at using (2.3):

\[
\begin{align*}
\hat{\psi}^\dagger \hat{\psi} & \to \mp i \phi_\pm \gamma_3 \phi_\pm \\
\hat{\psi}^\dagger \hat{\gamma}_5 \hat{\psi} & \to \mp i \phi_\pm \gamma_5 \phi_\pm \\
\hat{\psi}^\dagger \hat{\gamma}_4 \hat{\psi} & \to \mp \phi_\pm \phi_\pm \\
\hat{\psi}^\dagger \hat{\gamma}_3 \hat{\psi} & \to -i \phi_\pm \gamma_0 \phi_\pm \\
\hat{\psi}^\dagger \hat{\gamma}_i \hat{\psi} & \to \mp \phi_\pm \gamma_i \phi_\pm
\end{align*}
\]

As explained in I, the screening lengths and wave functions for mesonic and baryonic operators, can be identified with the masses and wave functions of the non-relativistic bound states corresponding to (2.5). The relevant non-relativistic Hamiltonian is obtained by Breit reduction. Keeping only the leading Coulomb interaction we have

\[
H = \sum_i \frac{\vec{p}_i^2}{2M} + \sum_{i<j} \frac{e^2}{2\pi} \left( \frac{1}{2} + \ln(M|\vec{r}_i - \vec{r}_j|) \right)
\]

where \( e^2 = g_3^2 C_F \) (quark-antiquark) and \( e^2 = g_3^2 C_F / 2 \) (quark-quark) with \( C_F \) the Casimir operator in the fundamental representation. To arrive at this expression one has to cancel the infrared singularities between self-energy and exchange contributions as explained in I.

We now give a formula for the spin-spin interaction in non-relativistic bound states by performing a Breit reduction of the Hamiltonian corresponding to (2.5). The easiest way to proceed is to notice that the expressions for the currents are exactly as in the standard (3+1) dimensional case, except that the momenta in the 3-direction are identically zero. Since, in the nonrelativistic
approximation, the spin-dependent part of the current is
\[ \chi^\dagger(p')i\vec{\sigma} \times (p' - \vec{p})\chi(\vec{p}) \]
the Pauli interaction only depend on the spin component in the 3-direction.
A simple calculation yields,
\[ H_{ss} = \sum_{i<j} \frac{e^2}{4M^2} (1 + 4S_3^iS_3^j) \delta^2(\vec{r}_i - \vec{r}_j) \]  
(2.8)

\( H_{ss} \) is subleading in the temperature and, as usual, it will only be considered
as a perturbation. In the case of mesons (2.8) simplifies to
\[ H_{ss} = \frac{e^2}{2M^2} S_3^2 \delta^2(\vec{r}_1 - \vec{r}_2) \]  
(2.9)

where \( S_3 \) is the total spin of the meson in the 3-direction. As mentioned earlier,
is only one difference, namely that the different polarization components
of the rho acquire different screening masses. Of the four components of
the interpolating massive vector field \( \rho_\mu = \hat{\psi}_\mu \hat{\psi} \), the components 1, 2 and 4
are measured on the lattice. Using the translation table (2.6) we see that
these correspond to \( \phi \gamma_4 \phi \) and \( \phi \phi \). The first has spin-projection \( \pm 1 \), while the
second is not an eigenstate of \( S_3^2 \), but a mixture of 1 and 0. The component \( \rho_3 \), which has not been measured on the lattice, corresponds to the current
\( \phi \gamma_0 \phi \), and is a \( S_3^2 = 0 \) eigenstate.

Using (2.9) we predict \( m_\pi \simeq 2\pi T < m_{\rho_4} < m_{\rho_i} \) and  
\[ m_{\rho_i} - m_\pi = \frac{e^2}{M^2}|\Psi(0)|^2 \]  
(2.10)

The lattice measurements at temperatures not much above \( T_c \) do not exhibit
this pattern but rather \( \frac{2}{3}m_\pi \simeq m_{\rho_i} \simeq 2\pi T < m_{\rho_4}. \) One should remember,
however, that our predictions are good at asymptotically high temperatures,
while the lattice calculations are performed not far above the transition temperature.
The consequences of this will be discussed further in section 6.

At this point it is also pertinent to make a few comments on the flavour
assignments in the lattice simulations of the mesonic screening lengths that we
have referred to. These were carried out using staggered fermions, and in this
scheme the space-time and flavour symmetries are intermingled on the lattice

\footnote{In \cite{[1]} \( \rho_1 \) and \( \rho_4 \) are referred to as \( \nu \bar{v}1 \) and \( \nu \bar{v}0 \) respectively, where the 0 and 1 denote
the "helicity" which is the component of the angular momentum about the 3-direction. We
do not agree with this helicity assignment for \( \rho_4 \), as discussed in the text.}

\footnote{Similar results have also been obtained by Brown \textit{et.al} using somewhat different
methods\cite{[2]}}
and are retained only in the continuum limit. The pion and sigma quoted originally by Detar and Kogut are related to an exact $U(1) \otimes U(1)$ symmetry of the free lattice fermion theory. The generators corresponding to sigma and pi are $1 \otimes 1$ and $\gamma_5 \otimes \gamma_5$ respectively, and the near degeneracy of their screening masses above $T_c$ is interpreted as a restoration of the symmetry. However this may be subject to some doubt, since, as first pointed out by Shuryak [13], only the connected part in Fig. 2a was simulated on the lattice. The disconnected part in Fig. 2b remains to be calculated.

Since there is no lattice measurements of any other member in the pseudoscalar multiplet we still do not know whether the whole continuum $SU(4)_R \otimes SU(4)_L$ is restored, or only the above mentioned subgroup. There is also no information on whether or not the axial $U_A(1)$ symmetry (not to be confused with the lattice symmetry mentioned above) is restored. This question, which is directly related to the strength of the anomaly at finite temperature is important in order to correctly identify the symmetry group of the high temperature phase, and is thus crucial for arguments of the type recently advanced by Rajagopal and Wilczek [14]. Clearly it would be very useful to perform a direct finite temperature lattice calculation of the $U_A(1)$-types of order parameters such as

$$\det \left( \psi^T \left( \frac{1}{2}(1 \pm \gamma_5) \psi \right) \right).$$

as originally suggested by t’Hooft [15].

3. Baryonic Correlators

The DeTar correlator in the baryonic channels can be calculated completely analogously to the mesonic ones. The currents appropriate for the nucleon and the delta isobar, omitting color, flavour and space indices, are

$$J^N = \left( \bar{\psi} \tilde{C} \gamma_5 \psi \right) \hat{\psi} + \kappa \left( \bar{\psi} \tilde{C} \gamma_5 \psi \right) \hat{\gamma}_5 \hat{\psi}$$

$$J^\Delta_\mu = \left( \bar{\psi} \tilde{C} \gamma_\mu \gamma_5 \psi \right) \hat{\psi}$$

where $\kappa$ is an arbitrary mixing factor and $\tilde{C} = \gamma_2 \gamma_4$ the charge conjugation matrix. The existence of the two nucleon currents (hence $\kappa$) was first noted by Ioffe [13]. To use the dimensional reduction method we should transcribe (3.1) and (3.2) in terms of the fields $\phi_\pm$, as shown in Appendix B. Using these currents, the asymptotic form of the correlator (1.1) gives the screening mass $m_\alpha$ in the baryonic channels. To leading order $m_\alpha = 3M + E_\alpha$ is just the mass of three heavy quarks in a color singlet configuration, interacting via Coulomb forces. In the ground state, the three quarks are in a symmetric
spatial configuration, in which case (2.8) simplifies to

\[ H_{ss} = \frac{e^2}{2M^2} \left( S_3^2 + \frac{3}{4} \right) \delta^2(\vec{r}_{12}) , \]  

where \( S_3 \) again is the total spin along the 3-direction and \( \vec{r}_{12} \) the relative two body separation in the three body system. This is to be compared with the spin-spin interaction (2.9) in the mesonic channel.

The ground state baryon wavefunctions are antisymmetric in color and symmetric in spin, flavour and space, and are solution to the Faddeev equations. For simplicity, however, we will only make variational estimates. In the center of mass frame we use the following Gaussian variational ansätze for the ground state wavefunction (we also give results for high \( T \) QCD in 2+1 dimensions\(^6\)),

\[ \psi^{QCD_{2+1}}(\rho, \eta) = \sqrt{\frac{8\alpha}{\pi}} e^{-\alpha(\rho^2 + \eta^2)} , \]  

\[ \psi^{QCD_{3+1}}(\rho, \eta) = \frac{2\alpha}{\pi} e^{-\alpha(\rho^2 + \eta^2)} , \]

where \( \vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \) and \( \vec{\eta} = \sqrt{\frac{3}{8}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) \). Minimizing the expectation value of (2.7) in (3.4) and (3.5) with respect to \( \alpha \) yields for \( QCD_{2+1} \), \( \alpha = \frac{1}{4} \left( \frac{9e^4M^2}{\pi} \right)^{\frac{1}{4}} \) and for \( QCD_{3+1} \), \( \alpha = \frac{9Me^2}{16\pi} \). The corresponding baryonic screening lengths are

\[ \psi^{QCD_{2+1}}(\rho, \eta) = \sqrt{\frac{8\alpha}{\pi}} e^{-\alpha(\rho^2 + \eta^2)} , \]  

\[ \psi^{QCD_{3+1}}(\rho, \eta) = \frac{2\alpha}{\pi} e^{-\alpha(\rho^2 + \eta^2)} , \]

\[ QCD_{2+1} : \quad m_\alpha = 3M + \frac{9}{4} \left( \frac{3e^2}{M\pi} \right)^{\frac{1}{4}} , \]  

\[ QCD_{3+1} : \quad m_\alpha = 3M + \frac{9e^2}{8\pi} \left( 2 - C - \ln\left( \frac{e^2}{16\pi M} \right) \right) , \]

where \( C \approx 0.577 \) is Euler’s constant. The spatial distribution of the baryonic correlators is Gaussian in both \( \vec{\rho} \) and \( \vec{\eta} \), and have widths \( \sim 4\pi/e\sqrt{M} \). Another simple alternative to a full solution to the Faddeev equations is to use a quark-diquark model as shown in Appendix C.

Like in the mesonic case, the degeneracy of the nucleon and delta screening masses will be lifted by spin dependent effects. At very high temperatures we expect the splitting to be due to the spin-spin interaction (3.3). This gives an upward shift to both the nucleon and the delta. The removal of the degeneracy

\(^6\) As discussed in I, this theory, while of no direct physical interest is easier to handle theoretically and perfectly amenable to lattice calculations.
is, however, only partial as the $\Delta(3/2, \pm 1/2)$ states remain degenerate with the $N(1/2, \pm 1/2)$ states. We get

$$m_N^{\pm 1/2} - m_\alpha = \frac{1}{3} (m_\Delta^{\pm 3/2} - m_\alpha)$$  \hspace{1cm} (3.8)$$

and the splitting is $\sim e^2/4M^2$. As already mentioned, the different polarization's of the vector and axial vector mesons, are split so the $j = \pm 1$ states are pushed up, while the $j = 0$ state is unaffected along with the pion and its scalar partner the $\sigma$. At very high temperature where this effect should be dominant, we thus predict

$$m_\Delta^{\pm 3/2} - m_\Delta^{\pm 1/2} = m_\Delta^{\pm 3/2} - m_N^{\pm 1/2}$$  \hspace{1cm} (3.9)$$

of the same order of magnitude and direction as the meson splitting

$$m_\rho^{\pm 1} - m_\rho^0 = m_\rho^{\pm 1} - m_\pi$$  \hspace{1cm} (3.10)$$

Since chiral symmetry is manifest at high temperature, these relations also hold for the chiral partners.

4. Gluonic Correlators

In this section we discuss correlators of local operators containing gluon fields. Color singlets are easily constructed from the field strength tensor $G_{\mu\nu}$, and its covariant derivatives. We shall only consider the operators $E^2$ and $B^2$, which are the lowest dimensional operators that couple to the two gluon channel, and the trace of the Polyakov loop,

$$L(x) = Pe^{ig \int_0^{1/T} d\tau A_0(\tau, x)}$$

where $P$ denotes path-ordering, which has been extensively studied by lattice calculations.

At finite temperature, $E^2$ and $B^2$ are distinct operators since Lorentz invariance is broken. In the dimensional reduction scheme, this is manifest by the presence of both a $(2+1)$ dimensional vector gluon field, $A_\mu$, and a scalar adjoint Higgs field, $H$, in the lagrangian (2.5). The operator $E^2$ couples to the two-particle channel $(HH)$ and $(A_\mu A_\mu)$ with strength 1 and $g^2$ respectively, while $B^2$ only couples to $(A_\mu A_\mu)$, see fig. 1. Since $g \to 0$ at high $T$, we would thus naively expect the screening mass $2m_E$ in the $(E^2 E^2)$ correlator and a power-law fall off in the $(B^2 B^2)$ one. Of course we know that this is oversimplified. As already discussed there are lots of evidence that the dimensionally
reduced theory is confining so it makes no sense to treat the magnetic sector, i.e. the $A_\mu$ field, in perturbation theory. We might hope, however, that the $(HH)$ channel can be treated in the same way as the $\bar{q}q$ one. In that case the above asymptotic (i.e. $T \to \infty$) prediction

$$\langle E^2(x^i, x^3)E^2(0, 0) \rangle \sim K_1(m_{E^2} x^3)$$ (4.1)

where $m_{E^2} = 2m_E = \frac{4}{\sqrt{3}}gT$, and $K_1$ a modified Bessel function, should be good at least for some moderately large values of $x_3$. (For really large distances there will always be an uncontrollable contamination by the magnetic sector since the small coupling strength will be compensated for by a larger exponential factor. This is not the case in 3d QCD at high temperature where we expect the above result to hold with the substitution $K_1 \to K_0$). It is important to stress that if it were not for the correlations in the transverse directions, (4.1) would decay as $K_1^2(m_{E^2} x^3)$, which has the same exponential fall-off as (4.1). It is the pre-exponent factor that distinguishes between bound and free electric gluons at asymptotic temperatures.

The binding energy is easily calculated using formulae similar to (2.7) and (2.8) modified for the particles having octet color charge and being scalars. A simple test of the self consistency of the estimate (4.1) is that the radius of the corresponding bound state is small enough for the magnetic effects to be ignored. Using the same variational estimates as in the mesonic case yields bound state radii $\sim 1/g^{3/2}T$, which is in between the electric and magnetic length scales $1/gT$ and $1/g^2T$ respectively. We consider it an open question whether the method based on dimensional reduction is reliable in this case. We should also mention that $E^2$ also couples to three and four particle channels, but these contributions are again suppressed by exponential’s and/or powers of $g$.

We already concluded that the operator $B^2$, is beyond the range of applicability of our method. If, however, the main effect of the infrared divergences in the magnetic sector is the generation of a magnetic mass $m_M \sim g^2T$, we have the qualitative prediction that asymptotically the $B^2$ screening length is larger than the $E^2$, since the magnetic mass is smaller than the electric. It is clearly of interest to simulate both these correlators on the lattice.

We now turn to correlators of Polyakov loops (or Wilson lines),

$$\langle \text{Tr} L(\vec{x})\text{Tr} L(\vec{0}) \rangle \to N_c^2 e^{-V_1(T,x)/T}$$ (4.2)

corresponding to inserting two static quarks in a heat bath of gluons. $V_1$ in (4.2) is the singlet potential. In the confined phase the singlet potential is
believed to behave as $\sigma(T)x$ for large $x = |\vec{x}|$, where $\sigma(T)$ is the (temperature dependent) string tension. In the deconfined high temperature phase the potential is usually believed to be screened,

$$V_1(T, \vec{x}) = 2F_Q(T) + V_{1,0}(T)e^{-2m_Ex}.$$  \hspace{1cm} (4.3)

The exponential fall off is again governed by the two (electric) gluon channel just as for the $E^2$ correlator. As pointed out by Nadkarni [17], also in this case we expect contamination from the magnetic sector at large enough distances. However, using the same arguments as for $E^2$ we suggest that the screening length should be calculable in the dimensionally reduced theory as an $(HH)$ bound state. Clearly, one would need very precise measurements to test whether the non-relativistic bound state interpretation of the gluonic correlators is correct. In particular it would be interesting to simulate the above correlator on the lattice and compare the results with the Wilson like loop at finite temperature.

Recently there have been lattice simulations aimed at probing the light fermion distributions ($\bar{\psi}\psi$ and $\psi^\dagger\psi$) [18]. Specifically, it has been observed that the fermion number distribution around a heavy quark given by

$$\langle \text{TrL}(\bar{0})\psi^\dagger\psi(\vec{x}) \rangle$$  \hspace{1cm} (4.4)

is localized around the heavy source (with a weight of minus one) at low temperature but spread out at high temperature. We expect that this spread is related to the screened two-gluon (dipole) exchange with a range of the order of $1/2m_E$ as discussed above. The correlation function (4.4) is consistent with Gauss’ law in the following sense. In the presence of light fermions the sampled gauge configurations on the lattice have an excess or a deficiency of color charge (in the sense of a grand canonical colored ensemble). When introducing a heavy triplet source, the configurations with an anti-triplet or a sextet of color are sampled out so that on the average Gauss law is enforced. The sampled configurations together with the heavy source carry zero triality.

5. Nonasymptotic Effects

As already mentioned, our estimate of the fine structure of the screening mass spectrum is only reliable at asymptotically high temperature. Close to the transition region, where the available lattice data are collected, many other effects are possible. Examples are Higgs interactions, effects induced by a dilute gas of monopoles, a residual rarefied gas of instantons and anti-instantons, nonvanishing magnetic condensates... Such effects are unfortunately

\footnote{We thank Carleton DeTar for a discussion on this point.}
much harder to calculate in a controlled manner, but we shall nevertheless attempt some estimates.

At zero temperature, the presence of gluon condensates modifies the perturbative spin-spin interaction and gives a contribution to the spin-spin splitting $\sim \langle B^2 \rangle$ [19, 20]. These effects will presumably persist even at finite temperature, but since we lack information about the condensates they are hard to estimate. However, the symmetry structure of these terms would be the same as for the perturbative spin-spin interaction (2.8).

The effects due to Higgs exchange is rather straightforward to estimate. First we consider the $m = 0$ case where the Higgs couple only to the scalar density $\bar{\phi}\phi$. To lowest order, we just have to consider the exchange of a scalar octet particle with mass $m_E$. The coupling is only to the color charge, so this will not cause any level splitting. Rather than trying to solve the Schrödinger equation in a potential including the Yukawa potential from the Higgs exchange, we consider the latter as a perturbation. If we furthermore neglect the propagation of the Higgs particle, i.e. make the local approximation $1/m_E^2$ for the propagator, we get the following local potential,

$$V_{\text{Higgs}} = \frac{e^2}{m_E} \delta^2(\vec{r}) = \frac{1}{T} \delta^2(\vec{r}), \quad (5.1)$$

where the last equality follows for two light quarks. This interaction will shift the meson screening masses by

$$\Delta m = \frac{2}{T} |\psi(0)|^2 \quad (5.2)$$

and comparing with the $\pi - \rho$ mass-shift due to the spin-spin splitting

$$m_{\rho}^\pm - m_{\pi} = \frac{2}{3} \frac{g^2}{\pi^2} \frac{1}{T} |\psi(0)|^2 \quad (5.3)$$

we see that the Higgs effect is an order of magnitude larger at $\alpha_s = 0.1$. Similar estimates of the level-shifts due to Higgs exchange can be made for the baryon and gluon correlators.

In order to split the rho from the pi without splitting the polarization components of the rho, we need a coupling to $\bar{\phi}\gamma_5\phi$. A possible, and even plausible, scenario is that the effective lagrangian for the quarks includes an effective four fermi interaction of the Nambu Jona-Lassinio type. Below the transition temperature such an interaction is known to induce a spontaneous breaking of chiral symmetry (at the mean field level) and generate a constituent quark mass. It is possible that the same interaction could be responsible for
the large observed splitting of the pi and rho screening masses. Neglecting
flavour (which is anyhow not correctly represented in the lattice calculations)
we can take
\[
\mathcal{L}_{NL} = \lambda \left[ (\bar{\phi} \gamma_5 \phi) - (\bar{\phi} \gamma_3 \phi)^2 \right] \tag{5.4}
\]
which is chirally invariant. The corresponding spin-spin interaction between
a quark and an antiquark is of the form \( \sim \lambda S_i^1 S_2^i \) which is appropriate for
pushing down the pi and leaving the rho untouched. The interaction is however
too singular to be treated in perturbation theory. A correct treatment,
which would amount to resolving the Schrödinger equation with a pointlike
interaction, will not be attempted here.

It is also instructive to consider the limit \( M \gg T \) (heavy quark limit).
In this case the coupling to the Higgs is entirely due to the density \( \bar{\phi} \gamma_3 \phi \),
which reduces to \( 1/2M \chi^i (\epsilon^{ij} q_j^i) \chi \) in the nonrelativistic limit, where \( \chi \) is a
two-spinor and \( \vec{q} \) the momentum transfer. The resulting spin-spin interaction
due to the Higgs is of the form
\[
H_{ss,H} = \sum_{i<j} e^2 \left( S_1^i S_1^j + S_2^i S_2^j \right) \left( \delta^2(\vec{r}_i - \vec{r}_j) - \frac{m^2}{2\pi} K_0(m E |\vec{r}_i - \vec{r}_j|) \right) \tag{5.5}
\]
The local part in (6.5) combines with the local part of (2.8) to give
\[
H_{ss} = \sum_{i<j} \frac{e^2}{4 M^2} \left( 1 + 4 S_i^i \cdot S_j^j \right) \delta^2(\vec{r}_i - \vec{r}_j) \tag{5.6}
\]
So, as expected, for heavy quarks, we recover the conventional polarization
independent spin-spin interaction. This result is also supported by the lattice
results. For mesonic systems we have \( H_{ss} = (2S^2 - 1)e^2/4M^2 \), while for
baryonic systems we have \( H_{ss} = (2S^2 - 3/2)e^2/8M^2 \). This spin-spin interaction
pushes the rho up, the pion down, leaves the nucleon unchanged and pushes
the delta up. Specifically, \( m_\rho - m_\pi = e^2/M^2 \) and \( m_\Delta - m_N = 3e^2/4M^2 \). In
this regime we expect
\[
m_\Delta - m_N = \frac{3}{4}(m_\rho - m_\pi) \tag{5.7}
\]
It is not ruled out, that the lattice results for the sources quoted are carried out
with still large quark masses, in which case the present discussion is pertinent.

Finally we give a rough estimate of the effect of instantons assuming that as
chiral symmetry is restored, the instanton-antiinstanton medium gets rarefied
single particles, molecules, small clusters). The effects would be to push down
the nucleon but leave the delta unaffected. Since the instanton contribution is
strongest in the quark-antiquark or quark-quark spin-isospin zero channel, we
would expect a delta-nucleon splitting in the screening lengths in comparison
to the $\rho - \pi$ splitting of the order of

$$m_\Delta - m_N \sim \frac{3}{2}(m_\rho - m_\pi). \quad (5.8)$$

Above the transition temperature this splitting is entirely due to the nonva-
nishing of the strange quark mass. The splitting vanishes exponentially at
asymptotic temperatures. This splitting is comparable in magnitude and di-
rection to the spin-spin splitting discussed above for heavy quarks.

6. Discussion

We have presented simple estimates of the baryonic screening lengths and
spatial extension using arguments on dimensional reduction. These arguments
are straightforward extensions of our previous arguments for mesons. We
have also provided some estimates for the gluonic correlation functions, and
suggested new correlation functions to be evaluated on the lattice.

While the dimensional reduction scheme is only valid at asymptotic tem-
peratures, we have also used it to estimate nonasymptotic effects caused by
both perturbative gluons (spin-spin interaction) and nonperturbative gluons
(instantons). It would be interesting to see how these predictions compare
with lattice simulations for the hadronic correlation functions. We stress that
our results are only suggestive since the calculations may be sensitive to the
magnetic length scale. If, however, future lattice calculations will confirm, or
at least support, our calculations, our calculations of nonasymptotic effects
could be much improved. Since the problem is essentially that of bound states
in 2+1 dimensional QCD, we can imagine to use many of the phenomenologi-
cal methods, like sum rules, bags etc., that have been developed to calculate
bound states in 3+1 dimensions. It remains to be seen whether such a ”high
$T$ bound state phenomenology” will become feasible.

There are several places for improvement in our calculations. Most note-
worthy: if our estimates of sizes and binding energy are correct, they are
midway between the electric and magnetic scale. Thus, it is important to do a
more complete calculation including hard thermal loops to better assess these
effects. Since all the correlators discussed in this work are gauge invariant, the

---

We recall that the splitting is proportional to the instanton density which is proportional
to $e^{-4/3\pi^2}T^2$ where $\rho$ is the instanton size. For temperatures of the order of $T_c$, the density
drops by an order of magnitude $[21, 22]$. 
tricky issue of gauge dependence related to the subdivision into hard \((g^2T)\) and soft \((gT)\) does not arise. This exercise is worth pursuing in both four and three dimensional QCD. Again, lattice measurements are crucial to settle these questions.

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We thank Carleton DeTar and Bengt Petterson for explaining their staggered lattice fermion calculations to us.

**Appendix A. Two-component formulation**

In this appendix we shall show how to formulate the dimensionally reduced theory in a two-component formalism. We again start from the Lagrangian (2.1) (for simplicity we only consider the lowest positive node), but we now rotate to Minkowski space by the transformations

\[
\hat{\psi}^\dagger \rightarrow i\vec{\psi} = i\psi^\dagger \gamma^0
\]  

where \(\gamma^0 = -\hat{\gamma}_3\). We also introduce the transverse Minkowski gamma matrices by \(\hat{\gamma}_i = i\gamma^i\) to get

\[
\mathcal{L} = \overline{\psi}(i\gamma^0\partial_0 + i\gamma^i\partial_i - i\hat{\gamma}^0 M)\psi
\]  

After introducing the two two-spinors \(\psi_1\) and \(\psi_2\) via

\[
\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}
\]

the Lagrangian takes the form

\[
\mathcal{L} = \overline{\psi}_1 i\gamma^0_3 \partial_1 \psi_1 + \overline{\psi}_2 i\gamma^0_3 \partial_1 \psi_2 - M(\overline{\psi}_2 \psi_2 + \overline{\psi}_1 \psi_1)
\]

where the 3-d gamma matrices are defined by \((\gamma^0_3, \gamma^1_3, \gamma^2_3) = (\sigma^3, i\sigma^1, i\sigma^2)\), and \(\overline{\psi}_i = \psi^\dagger_i \gamma^0_3\). We leave it as an exercise to put in the gluons and a mass for the quark. (A.4) has the form of a pair of standard 2+1 dimensional Dirac Lagrangian’s, but with an off-diagonal mass term. This means that in order to obtain a propagator one must diagonalize a four-dimensional matrix anyway, and it is more convenient to use the representation given in section 2.

**Appendix B. Quantum numbers of baryonic currents**

There are two inequivalent local sources for spin \(\frac{1}{2}\) baryons of opposite parity
\[ B_1^+ = (\hat{\psi}^T \hat{C}_5 \hat{\gamma}_5 \hat{\psi}) \hat{\psi} \]
\[ B_2^+ = (\hat{\psi}^T \hat{C}_5 \hat{\gamma}_3 \hat{\gamma}_1 \hat{\gamma}_5 \hat{\psi}) \hat{\gamma}_5 \hat{\psi} \]  \hspace{1cm} (B.1)

\[ B_1^- = (\hat{\psi}^T \hat{C}_5 \hat{\gamma}_3 \hat{\gamma}_1 \hat{\gamma}_5 \hat{\psi}) \hat{\gamma}_5 \hat{\psi} \]
\[ B_2^- = (\hat{\psi}^T \hat{C}_5 \hat{\gamma}_3 \hat{\gamma}_1 \hat{\gamma}_5 \hat{\psi}) \hat{\gamma}_5 \hat{\psi} \]  \hspace{1cm} (B.2)

For convenience, we will throughout omit reference to color, flavour and space indices. Under parity (\( \hat{\psi} \rightarrow \hat{\gamma}_4 \hat{\psi} \)) : \( B^\pm \rightarrow \pm \hat{\gamma}_4 B^\pm \). \( B^+ \) and \( B^- \) refer to the octet \( \frac{3}{2}^+ \) and \( \frac{1}{2}^- \) baryons respectively. The nucleon is an arbitrary linear combination of \( B_1^+ \) and \( B_2^+ \), i.e. \( J^N = B_1^+ + \kappa B_2^+ \).

In the chiral basis the combinations \( B_1^\pm \pm B_2^\pm \) are more appropriate to use. In this basis the nucleon can be viewed either as \((RRR) - (LLL)\) or \((RRL) - (LLR)\) or any linear combination of the two. The odd parity partners of the nucleon are \((RRR) + (LLL)\) or \((RRL) + (LLR)\) or again any linear combination of the two. This representation is suited to the two component formulation discussed in Appendix A.

To exhibit the structure of the currents in the four-component formulation discussed in the text, we first derive the reduced forms for the diquark constituents. Specifically

\[ \beta \hat{\psi}^T \hat{C}_5 \hat{\gamma}_5 \hat{\psi} = \hat{\phi}_-^T \hat{\gamma}_1 \hat{\gamma}_3 U_2^\phi \hat{\phi}_- e^{-i2\pi T\hat{x}_4} - \hat{\phi}_+^T \hat{\gamma}_1 \hat{\gamma}_3 U_2^\phi \hat{\phi}_+ e^{i2\pi T\hat{x}_4} \]  \hspace{1cm} (B.3)
\[ \beta \hat{\psi}^T \hat{C}_5 \hat{\psi} = \hat{\phi}_-^T \hat{\gamma}_2 \hat{\gamma}_4 \hat{\phi}_- e^{-i2\pi T\hat{x}_4} + \hat{\phi}_+^T \hat{\gamma}_2 \hat{\gamma}_4 \hat{\phi}_+ e^{i2\pi T\hat{x}_4} \]  \hspace{1cm} (B.4)

The unaveraged sources for the baryons follow from these diquark structures and the reduced fermionic fields (2.3). The resulting expressions involve terms with the \( \tau \)-dependence \( e^{\pm i3\pi T\tau} \) and \( e^{\pm i\pi T\tau} \). Static sources can be constructed by averaging over \( \pi T \) or \( 3\pi T \). We choose the latter and define

\[ \overline{B}^\pm_{1,2} = T \int_0^\frac{1}{T} \hat{d}\hat{x}_4 \cos(3\pi T\hat{x}_4) B^\pm_{1,2} \]  \hspace{1cm} (B.5)

the lattice calculations have been carried out by averaging over the lowest Matsubara frequency \( \pi T \). Using (B.3), the reduced diquark sources (B.3) and
(B.4), and the fermionic fields (2.3), we obtain

\[ 2\beta_2^0 B_1^+ = -i \left( \hat{\phi}_T^+ \gamma_1 \hat{\phi}_- \right) VU_- \hat{\phi}_- + i \left( \hat{\phi}_T^+ \gamma_1 \hat{\phi}_+ \right) VU_+ \hat{\phi}_+ \]

\[ 2\beta_2^0 B_2^+ = (\hat{\phi}_T^+ \gamma_2 \gamma_4 \hat{\phi}_-) \tilde{\gamma}_5 VU_- \hat{\phi}_- + (\hat{\phi}_T^+ \gamma_2 \gamma_4 \hat{\phi}_+) \tilde{\gamma}_5 VU_+ \hat{\phi}_+ \]  

(B.6)

\[ 2\beta_2^0 B_1^- = \left( \hat{\phi}_T^+ \gamma_2 \gamma_4 \hat{\phi}_- \right) VU_- \hat{\phi}_- + \left( \hat{\phi}_T^+ \gamma_2 \gamma_4 \hat{\phi}_+ \right) VU_+ \hat{\phi}_+ \]

\[ 2\beta_2^0 B_2^- = -i \left( \hat{\phi}_T^+ \gamma_1 \hat{\phi}_- \right) \tilde{\gamma}_5 VU_- \hat{\phi}_- + i \left( \hat{\phi}_T^+ \gamma_1 \hat{\phi}_+ \right) \tilde{\gamma}_5 VU_+ \hat{\phi}_+ \]  

(B.7)

Our static sources do not mix the \( \phi_+ \) and \( \phi_- \) fields while the lattice sources averaged over \( \cos(\pi T \hat{x}_4) \) do. It would be interesting to find out how (if at all) this affects the structure of the resulting correlation functions.

In the case of the isobar current, there is a unique local current \( (\hat{\psi}^T \hat{C} \hat{\gamma}_\mu \hat{\psi}) \hat{\psi} \). Its form in the four component formalism can be obtained without difficulty using techniques similar to the ones above, and will not be given here. We note, however, that the isobar carries a polarization. The isobar diquark field is just \( \hat{\psi}^T \hat{C} \hat{\gamma}_\mu \hat{\psi} \). This means that the sources for the transverse and longitudinal fields are distinct at high temperature and are subject to the same remarks developed in the text for the vector meson fields. In particular, we expect the subleading spin-spin interactions to affect the longitudinal and transverse parts of the isobar differently.

**Appendix C. Quark-diquark model for baryons**

If we were to use the quark-diquark picture for baryons than the screening lengths and wavefunctions are readily obtained from the mesonic screening lengths and wavefunctions. In the case of a free diquark of mass \( M_* = 2M \) with antitriplet charge, a variational estimation shows that the screening mass is,

\[ QCD_{2+1} : \quad m_\alpha = 3M + \frac{3}{2} \left( \frac{3e^4}{16\pi M} \right)^{\frac{1}{3}} \]  

(C.1)

\[ QCD_{3+1} : \quad m_\alpha = 3M + \frac{e^2}{8\pi} \left( 2 - C - \ln \left( \frac{e^2}{12M\pi} \right) \right) \]  

(C.2)

In the antitriple channel, the diquark mass is,

\[ QCD_{2+1} : \quad M_* = 2M + \frac{3}{2} \left( \frac{e^4}{4\pi M} \right)^{\frac{1}{3}} \]  

(C.3)

\[ QCD_{3+1} : \quad M_* = 2M + \frac{e^2}{8\pi} \left( 2 - C - \ln \left( \frac{e^2}{8\pi M} \right) \right) \]  

(C.4)
The baryonic screening lengths in this case are,

\[ QCD_{2+1} : \quad m_\alpha = M + M_* + \frac{3}{2} \left( \frac{e^4 (M + M_*)}{8\pi M M_*} \right)^\frac{1}{2} \]  
\[ (C.5) \]

\[ QCD_{3+1} : \quad m_\alpha = M + M_* + \frac{e^2}{8\pi} \left( 2 - C - \ln \left( \frac{e^2}{2\pi (M + M_*)} \right) \right) \]  
\[ (C.6) \]

In the diquark picture, the baryonic wavefunctions are similar to the mesonic wavefunctions.

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Figure Captions

Fig. 1: Typical couplings to $E^2$ (a) and $B^2$ (b) to the electric and magnetic gluons.

Fig. 2: (a) Direct contribution to the sigma-sigma correlator; (b) exchange contribution to the sigma-sigma correlator.
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