Electro-Mechanical Resonant Magnetic Field Sensor

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Abstract

We describe a new type of magnetic field sensor which is termed an Electro-Mechanical Resonant Sensor (EMRS). The key part of this sensor is a small conductive elastic element with low damping rate and therefore a high Q fundamental mode of frequency $f_1$. An AC current is driven through the elastic element which, in the presence of a magnetic field, causes an AC force on the element. When the frequency of the AC current matches the resonant frequency of the element, maximum vibration of the element occurs and this can be measured precisely by optical means.

We have built and tested a model sensor of this type using for the elastic element a length of copper wire of diameter 0.030 mm formed into a loop shape. The wire motion was measured using a light emitting diode phototransistor assembly. This sensor demonstrated a sensitivity better than 0.001G for an applied magnetic field of $\sim 1$G and a good selectivity for the magnetic field direction. The sensitivity can be easily improved by a factor of $\sim 10 - 100$ by more sensitive measurement of the elastic element motion and by having the element in vacuum to reduce the drag force.

1 Introduction

There are several types of magnetic field sensors in common use. Included are compass needles, Hall probes, flux-gate magnetometers, and SQUIDs (superconductive quantum interference devices). The compass needle was discovered $\sim 2000$ years ago by the Chinese. Other types of sensors were developed relatively recently. The field of application of the different sensors clearly depends on the required accuracy, sensitivity, and expense. A compass needle is simple and does not require electric power or circuits but
it indicates only the field direction. Hall probes are more sophisticated devices which can measure fields over a large range of field strengths. They are simple to use, but have problems related to baseline drift and high sensitivity to ambient temperature changes, and they tend to have noisy signals. Their typical resolution is $\sim 10^{-3}$, but with special precautions it can reach $5 \times 10^{-4}$. Flux-gate magnetometers are versatile and sensitive but require sophisticated signal processing which tends to limit the frequency band of their response. The most advanced type of magnetic field sensors are SQUIDs (super-conductive quantum interference devices). These probes can measure magnetic fields with extremely high precision, but they require liquid nitrogen or liquid helium to operate and sophisticated electronics. That makes this type of probe expensive and limits its range of application.

In this paper we discuss a new type of inexpensive magnetic field sensor which is depicted in Figure 1. The sensor is a highly miniaturized version of the vibrating wire field measuring technique (Temnykh 1997). In §2 we briefly consider the theory and in §3 we give the results of measurements on a model EMR sensor. In §4 we give a summary.

2 Theory

Here we briefly consider properties of an elastic loop sketched in Figure 2. Assume that both ends of the loop, $A$ and $B$, are fixed. A torque $T$ applied to the loop will cause it to tilt through an angle $\alpha$ around horizontal $x-$axis, where

$$T = \frac{\pi S d^4}{16L} \alpha .$$

where $S$ is the shear modulus of the wire, $d$ the wire diameter, $L$ is the “arm” length indicated in Figure 2 (see for example Brekhovskikh & Goncharov 1993). For copper $S \approx 0.42 \times 10^{11} \text{Pa}$, with $\text{Pa} \equiv \text{N/m}^2$.

Consider now a DC current $I$ flowing through the loop. The Lorentz force between the current in the loop and the vertical magnetic field $B_y$ gives a torque about the $x-$axis approximately

$$T_I = B_y IR^2 \left( \pi + \frac{2H}{R} - 4 \right) ,$$

where $H$ and $R$ are defined in Figure 2. In the equilibrium, $T_I = (\pi S d^4/16L) \alpha$, so that

$$\alpha = \frac{16}{\pi} \left( \pi + \frac{2H}{R} - 4 \right) \frac{LR^2}{S d^4} B_y I_{dc} .$$
The displacement of the upper point of loop in $z-$direction is

$$ \delta z = \alpha H = \frac{16}{\pi} \left( \pi + \frac{2H}{R} - 4 \right) \frac{LHR^2}{Sd^4} B_y I_{dc} . $$ (4)

For the loop parameters indicated in the caption of Figure 2, and $B_y = 110$G (used in the tests described below), we obtain

$$ \frac{\delta z[\text{mm}]}{I[\text{A}]} \approx 0.294 . $$ (5)

The measured dependence discussed in §3.3 is $\delta z[\text{mm}]/I[\text{A}] \approx 0.35$. This is consistent with the analytic calculation if account is taken of the approximate equation (2) for the torque.

The lowest frequency of vibration $f_1$ of the loop can be estimated as follows. The moment of inertia of the loop about the $x-$axes is

$$ I = \frac{\pi}{4} \left[ (\pi - \frac{4}{3})R^3 - 2HR^2 + (\pi - 2)H^2R + \frac{2}{3}H^3 \right] \rho \ell d^2 . $$ (6)

where $\rho \ell$ is the density of the loop. Thus the equation of motion for the free vibrations of the loop is

$$ I \frac{d^2\alpha}{dt^2} = - \left( \pi \frac{Sd^4}{16L} \right) \alpha . $$ (7)

The frequency of vibration is thus

$$ f_1 = \frac{d}{4\pi} \sqrt{\frac{S}{\rho \ell L} \left[ (\pi - \frac{4}{3})R^3 - 2HR^2 + (\pi - 2)H^2R + \frac{2}{3}H^3 \right]^{-1/2}} . $$ (8)

For the loop dimensions shown in Figure 2 and for the copper wire used ($\rho \ell = 8.9 \times 10^3 \text{kg/m}^3$), equation (8) gives $f_1 \approx 280$Hz. This is consistent with the measured resonance frequency discussed in §3.2, $f_1 \approx 259$Hz.

The full equation of motion for the driven motion of the loop including the low Reynolds number air friction is

$$ I \frac{d^2\alpha}{dt^2} = - \left( \frac{\pi Sd^4}{16L} \right) \alpha - K \frac{d\alpha}{dt} + T_I(t) . $$ (9)

Here, $T_I(t)$ is given by equation (2) with the current $I(t)$ a function of time, and $K \approx (4\pi \eta/A)(2H^3/3 + 2RH^2)$, where $\eta \approx 1.8 \times 10^{-5} \text{ kg/(m s)}$ the
dynamic viscosity of air. Also, for a long cylinder, Λ = 1/2 − γ − ln(v_p d/8ν) (Landau & Lifshitz 1959), with γ ≈ 0.577 Euler’s constant, with v_p the peak velocity of the top of the loop, and with ν = η/ρ_{air} ≈ 1.5 × 10^{-5} m^2/s the kinematic viscosity of air. Equation (9) implies that the quality factor for the vibrations is \( Q = 2\pi f I/K \). For the displacement amplitudes discussed in §3.2, the Reynolds numbers \( Re = v_p d/\nu \) are indeed less than unity, and this formula gives the prediction \( Q \approx 297 \) which is larger than the measured value of 198 (§3.2). We believe that this difference is due to the approximations in the theory of the drag coefficient \( K \).

3 EMR Sensor Tests

We built a number of models of EMR sensors. One of these are shown schematically in Figures 1 and 2. The elastic element was fabricated from a 0.030mm diameter copper wire formed into a loop shape diagrammed in Figure 2. The fundamental mode of vibration corresponds to the top of the loop moving in the z-direction in Figure 2. To measure position of the loop we used a “Π” shaped opto-electronic assembly H21A1 (Newark Electronics) consisting of a light-emitting-diode (LED) on one leg of the assembly and a photo-transistor on other. The light flux detected by photo-transistor is very sensitive to loop position.

In the following we give the characteristics of the EMR sensor components and results of the sensor tests.

3.1 Calibration of Loop Position Sensor

The opto-electronic detector was calibrated by the moving the entire loop using a precise micro-screw and measuring the signal from the photo-transistor. The measured dependence is shown in Figure 3.

One can see that in the range 0.7 – 1.1mm the signal from phototransistor is proportional to the loop displacement.

\[
\frac{\delta U [mV]}{\delta z [mm]} = 421
\]  

\( \delta U \)[mV] is the change in the detector signal. \( \delta U = 1mV \) corresponds to a current \( \delta I = \delta U/R = 0.16\mu A \) through the external circuit resistor \( R = 6.35k\Omega \). In subsequent measurements, the loop position was adjusted to be in the middle of the range of linear dependence.
3.2 Elastic Element Resonance Response

Important characteristics of EMR probe are the fundamental resonance frequency $f_1$ and the quality factor $Q$. We measured these parameters by driving an AC current with various frequencies through the element and measuring amplitude of the AC signal from photo-transistor. In Figure 4 the measured amplitude (RMS of AC voltage) is plotted as a function of frequency $f$ of the AC current.

The data was fitted to the resonance formula

$$A = \frac{A_0}{\sqrt{(f^2 - f_1^2)^2 + f^2 f_1^2/Q^2}},$$

(11)

where, $f_1$ is the resonance frequency, $Q$ is the quality factor, and $A_0$ is a constant. This expression follows from equation (9). The fit gave: $f_1 = 259\text{Hz}$ and $Q = 198$. Note that according to calibration the maximum of 16mV RMS of AC signal seen at resonance on Figure 4 corresponds to $\pm 0.053\text{mm}$ of amplitude of vibration of the top of the loop. This small amplitude indicates that the optical detector was operating in the linear region of Figure 3. In this test the AC current amplitude through the element was $170\text{mA}$ and the magnetic field was $\sim 0.5\text{G}$.

3.3 Elastic Element Static Test

This test was done to measure the static properties of the elastic element. A triangular AC current of low frequency of 1Hz was driven through the element. A 110G vertical magnetic field was imposed at the EMR probe location by a permanent magnet. The Lorentz force between the magnetic field and current flowing through the probe caused the loop displacement.

Figure 5 shows the current through the element, $I_{el}$, and photo-transistor signal as a function of time. One can see the triangular wave current with 1sec period and photo-transistor signal with similar form. In Figure 6 the signal is plotted as function of current. The right-hand vertical scale shows the element displacement, $z$, calculated from the signal by using calibration. The data indicates a linear dependence of element displacement on current,

$$\frac{\delta z[\text{mm}]}{\delta I[\text{A}]} \approx 0.365,$$

(12)

for the magnetic field of 110G.
3.4 EMR Sensor Calibration and Comparison with a Hall Probe

In this test a Hall probe was placed very close to the EMR probe. The Hall probe orientation was accurately adjusted so that it sensed only the vertical component of the magnetic field, which is the component which causes the vibration of elastic element in EMR probe. A sinusoidal AC current of peak amplitude 85mA at the resonance frequency 259Hz was driven through the element. The imposed magnetic field was created by a small permanent magnet. The field strength was varied by accurately moving the permanent magnet relative to the fixed probes. Figure 7 shows the field measured by the Hall probe and the signal from the EMR sensor as a function of the test magnet position.

The data shown in Figure 7 was used to calibrate the EMR probe. Figure 8 shows a plot of the EMR probe signal (RMS of AC voltage) versus magnetic field strength measured with Hall probe and fitted it with linear dependence. This fit gives

$$\frac{\delta B[G]}{\delta U_{AC}[mV]} \approx 0.0518 .$$

That is, a 1mV change of the EMR probe signal indicates a 52mG change of the magnetic field strength.

Note that because the size of the test magnet was much smaller than the distance between probes and the magnet, the dependence of the magnetic field strength on the position $p$ can be approximated by

$$B(p) = C_1 + \frac{C_2}{(p - C_3)^3} .$$

Here, $C_1$ is a parameter which represents either the background field or the zero drift for the Hall probe. The parameter $C_2$ is proportional to the testing magnet magnetic moment, The parameter $C_3$ is set by the location of the probe in coordinate system used to define testing magnet position. The measurements with the Hall and EMR probes were fitted with the theoretical dependence 14 using $C_1$, $C_2$ and $C_3$ as a free parameters. The residual between measured data and theoretical fit for both probes is shown in Figure 4.

For the EMR probe the difference (measurement - fit) was converted into magnetic field strength using equation 13. Note that at each point, the signal from EMR probe was measured several times. Bars shown for the
EMR sensor data represent 1σ errors found from a statistical analysis of the measurements.

We can now compare the difference between measurement and theoretical fit for the EMR sensor and that for the Hall probe. For Hall probe RMS the residual between measured data and theoretical fit is $2.9 \times 10^{-3} \text{G}$, which is consistent with the probe specifications. For our EMR sensor the residual is $0.45 \times 10^{-3} \text{G}$. That is, the EMR sensor is 6 times better!

## 4 Discussion

In future refinements, an EMR sensor element optimized for sensitivity can be developed. The geometry may be similar to that discussed above but the wire can be of diameter $d = 0.010 \text{mm}$ and consist of 10 turns around the loop shown in Figures 1 and 2. Scaling of our test results using equations (3) and (8) gives a sensitivity $5 \times 10^{-6} \text{G}$, resonance frequency $\sim 25 \text{Hz}$, and a peak current amplitude $9.4 \text{mA}$ where the current was scaled $\propto d^2$. The elastic element can be vacuum encapsulated using nano-fabrication techniques. For vacuum conditions where the mean-free-path is longer than the dimensions of the loop, the friction coefficient $K$ in equation (9) is proportional to the gas density or pressure. We estimate that the quality factor of the resonant element can be increased to $Q \sim 10^3 - 10^4$. This increase in $Q$ will increase the sensitivity by a factor $\sim 5 - 50$. As a result the EMR sensor can have a sensitivity of $1 \times 10^{-6}$ to $10^{-7} \text{G}$. This is a few orders of magnitude more sensitivity than a Hall probe, but of course it is not as sensitive as SQUIDS. Note that the measurement times at the lowest field levels need to be $\sim 1 - 10 \text{min}$.

## References

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Figure Captions

Figure 1: Three dimensional view of sample EMR sensor. The wires marked (a) and (b) connect to the conducting loop in the center of the figure. The “Π” shaped assembly with light-emitting-diode (LED) on one leg and a photo-transistor on other is used as the loop position detector. The straight white segment above the loop represents the light beam of LED. The photo-transistor current indicates the position of the top of the loop.

Figure 2: Schematic view of elastic element used in tests. The element was made of copper wire of diameter $d = 0.030\text{mm}$. The other dimensions are $H = 4\text{mm}$, $R = 1\text{mm}$, and $L = 6.25\text{mm}$.

Figure 3: Signal from photo-transistor ($U$) as a function of the loop position. A least-squares fit of a straight line through the solid circles gives $U[\text{mV}] = m_0 + m_1 z[\text{mm}]$ with $m_0 = -110$ and $m_1 = 421$.

Figure 4: EMR sensor resonance characteristic. The curve represent a least-squares fit of equation (9) to the measured points. This fit gives $f_1 = 259\text{Hz}$ and $Q = 198$.

Figure 5: EMR sensor static test for an imposed magnetic field $B = 110\text{G}$. Current through the elastic element (dashed line) and signal from photo-transistor (solid line) as a function of time.

Figure 6: EMR sensor static test for an imposed magnetic field of $B = 110\text{G}$. Signal from photo-transistor (left-hand scale) as a function of current through the element. Right-hand scale is the loop deflection $z$ in mm. The linear fit of the data gives: $z[\text{mm}] = -3.96 \times 10^{-2} + 0.365 I[\text{A}]$.

Figure 7: Signal from EMR sensor (open circles) and magnetic field measured by the Hall probe (solid triangles) as function of test magnet position.

Figure 8: RMS of AC signal from EMR sensor ($U_{AC}$) as function of the magnetic field measured with Hall probe ($B$) at 170mA of peak-to-peak current driven through the loop. A least-squares fit gives $U_{AC}[\text{mV}] = 1.95 + 19.31 B[\text{G}]$ or $\delta B[\text{G}] \approx 0.0518 \delta U_{AC}[\text{mV}]$.

Figure 9: Residual of measured fit for EMR sensor (circles) and the Hall probe (diamonds). The Hall probe RMS = $2.9 \times 10^{-3} \text{G}$, whereas the EMRS RMS = $0.45 \times 10^{-3} \text{G}$.
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