Cooling fins to limit the hot-electron effect in dc SQUIDs

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Abstract. The generally accepted noise theory of the dc SQUID predicts that the energy resolution scales as the electron temperature in the Josephson junction shunt resistors. As in metals at low temperature the electron-phonon coupling becomes very weak, the electron gas of the thin film shunt resistors undergoes a Joule heating due to the bias current and its temperature can be significantly higher than that of the thermal bath. This heating, the hot-electron effect, causes a deviation from the linear behaviour of noise versus temperature and a saturation of the SQUID noise, typically at temperatures of about 200 mK. This effect can be reduced considerably by increasing the effective volume available for the electron-phonon interaction by attaching “large” cooling fins to the shunt resistors. Our measurements have been performed on two thin film devices made with the same design of a dc SQUID but without the Josephson junctions: one device with standard shunt resistors, the other with shunt resistors with cooling fins. From these measurements one can expect for the SQUID with cooling fins an improvement of the noise saturation temperature of at most a factor 2, from 200 mK to about 100 mK.

1. Introduction

The noise characteristics of the low-\(T_c\) dc SQUID are well-explained by the resistively- and capacitively-shunted junction (RCSJ) model [1] in which a Nyquist current noise source is associated to each of the two shunt resistances \(R\) that are necessary to avoid hysteresis in the I-V characteristic of the Josephson junctions. This model predicts [2] for a dc SQUID under optimum conditions an energy resolution \(\varepsilon = S_f 2L = 9k_BT/LR = 16k_BT(LC)^{1/2}\) where \(S_f\) is the flux noise spectral density, \(L\) is the SQUID loop inductance and \(C\) is the capacitance of each junction. The white noise of a large number of SQUIDs has been found in good agreement with these predictions. Although quoting this white noise (and the associated energy resolution) is not sufficient to completely characterize the SQUID noise (in fact one has to consider also the \(1/f\) noise and the back action current noise in the SQUID loop) it is usually taken as an indication of the sensitivity of the device.

In most experiments in which the SQUID is employed as the front-end amplifier, its noise is one of the parameters that limit the sensitivity of the measurement (see, for example [3]). Besides optimizing the geometrical parameters of the SQUID (low SQUID loop inductance and low junction capacitance)
the only method to lower the SQUID noise is to reduce the its operating temperature. In principle this method works as long as the thermal fluctuations in the shunt resistances become comparable to the zero point fluctuations where the energy resolution is roughly \( h \) (as in any quantum-limited amplifier) and for this reason it is common practice to quote the energy resolution in units of \( h \). Up to now a small number of SQUIDs have approached the quantum limit [4] but, in principle, this performance could be achieved by many modern SQUIDs if these were operated at temperatures of about 50 mK (achievable in a dilution refrigerator) and in a two-stage configuration [5] in order to render the noise contribution of the room temperature electronics negligible.

Unfortunately, there is an additional mechanism which will limit the noise performance of the SQUID at low temperatures: the hot-electron effect [6] in the shunt resistors. The Joule heating of the conduction electrons due to the decoupling of the electrons from the phonons at low temperatures causes a saturation of the SQUID noise. Given the typical dimensions of the shunt resistances of a thin film dc SQUID and the typical bias currents, the temperature of the electron gas will not drop to below 200-300 mK.

There are a number of models to describe the hot-electron effect. The simplest predicts an electron-phonon (e-p) interaction based on a clean three-dimensional free-electron model [6][7]. In this case the relation between the power \( P \) dissipated in the metallic film and the electron gas temperature \( T_e \) is

\[
P = \Sigma \Omega (T_e^5 - T_p^5)
\]

where \( T_p \) is the phonon temperature, \( \Omega \) is the volume of the film and \( \Sigma \) is a parameter that depends on the material used. Other models [8], in particular those that describe the metal in the dirty limit \( ql<1 \), where \( q \) is the wave vector of the dominant thermal phonons and \( l \) the electron mean free path, predict the same qualitative behavior described by equation (1), but with exponents different from 5 and other prefactors \( \Sigma \). It is clear, from equation (1), that a method to reduce the overheating due to the hot-electron effect consists in connecting cooling fins to the shunt resistances in order to increase the effective volume available for the e-p interaction, without changing too much their resistance [6].

This paper describes the effect of cooling fins on the electron gas temperature in palladium thin film shunt resistances.

\[\text{Figure 1. Schematic drawings of the devices without (a) and with (b) cooling fins}\]
2. Experimental procedure

A method to evaluate the effect of cooling fins could consist in preparing two SQUIDs, one with and the other without cooling fins and then measure their noise at low temperatures. This method, however, at least in this first phase of the development of low noise SQUIDs, could produce results affected by spurious effects. The SQUID behaviour is in fact subject to many disturbances that are difficult to discriminate. For this reason we chose to realize two thin-film niobium devices with the same design of our standard dc SQUIDs but without the Josephson junctions: one device with standard shunt resistors, the other with shunt resistors with cooling fins. These devices behave, for all practical purposes, like resistances and have the advantage to be as similar as possible to the final design of working dc SQUIDs.

In figure 1 the schematic drawings of the two devices are shown. In the device without cooling fins each of the two parallel palladium thin-film shunt resistors has a length of 25 \( \mu \text{m} \), a width of 15 \( \mu \text{m} \), and a thickness of 25 nm. The palladium resistors were prepared on an oxidized silicon substrate by rf magnetron sputtering in an argon atmosphere of 6 mTorr, with a sputtering rate of 12 nm/min. The resistors were connected to niobium traces on the substrate, to which thin aluminium wires were bonded for electrical contact. The measured resistance of the two parallel resistors at \( T=4.18 \) K is \( R_s=8.72 \) \( \Omega \), leading to a nominal resistivity \( \rho=2.6\times10^{-7} \) \( \Omega \text{m} \). The corresponding elastic mean free path is estimated as \( l \approx 3.3 \) nm using palladium electronic parameters reported in literature [9]. Thus, \( l \) is not substantially limited by the film thickness. The wave vector of the dominant thermal phonon \( q=2k_B T/\hbar v_s \) (\( v_s \) is the longitudinal sound velocity \( 5.5\times10^3 \) m/s) is about \( 3\times10^6 \) m\(^{-1}\) at the lowest operating bath temperature (\( T_0=59 \) mK), leading to \( ql \approx 0.01 \). The palladium of the shunt resistances is therefore in the dirty limit.

In the second device cooling fins, also made of palladium with a thickness of 100 nm, are connected to the shunt resistors. The areas of the two fins are 1.78 mm\(^2\) and 2.06 mm\(^2\). Thus, the overall volume of the cooling fins is \( 3.8\times10^{-13} \) m\(^3\). This volume has to be compared to that of the parallel resistors which is \( 1.9\times10^{-17} \) m\(^3\).

The measurement technique is similar to that described in reference [6] and is shown in figure 2.

![Schematic circuit diagram of the SQUID-based noise thermometry measurement.](image)

A constant power is dissipated in the film by Joule heating, and a SQUID-based noise thermometer directly measures the steady-state electron temperature. The film resistors are placed in parallel to a surface mount device (SMD) resistor \( R_m=2.21 \) \( \Omega \). The current noise of the \( R_m-R_s \) loop is measured by a commercial dc SQUID with input mutual inductance \( M_i=10.71 \) nH, operated in standard flux-locked loop configuration. The film resistors and the SMD resistor are mounted on a fiberglass holder, inserted in a niobium shield. The readout dc SQUID is inserted in a second niobium shield. The connection between resistors and SQUID is provided by a twisted pair of copper wire, of negligible
resistance, placed in a lead tube. An inductance \( L_m = 15 \, \mu \text{H} \), of negligible resistance, is placed in series with the SQUID input coil \( L_s = 1.6 \, \mu \text{H} \) to act as radiofrequency filter.

Measurements were performed at two thermal bath temperatures: 1.25 K and 59 mK. At 1.25 K the SQUID thermometer, the resistors holders and a germanium thermometer are immersed in a superfluid liquid helium bath. At 59 mK both SQUID and resistor holder are placed inside a copper box, in good thermal contact with the mixing chamber of a dilution refrigerator [10]. The box is filled with 1 atm of \(^4\text{He} \) gas at room temperature to improve thermalization of the internal elements. The box temperature \( T_0 \) is monitored by a germanium thermometer. Therefore the thermal impedances that connect the lattice phonon gas of the film resistors to the thermal bath in the two experimental configurations are very different.

The resistances \( R_s \) and \( R_m \) are biased in parallel with a constant current \( I \) from a battery supply through a room temperature resistor \( R_b = 100 \, \text{k} \Omega \). The total powers dissipated in the resistances \( R_s \) and \( R_m \) are calculated as \( P_s = \frac{I^2 R_s}{R_s + R_m} \) and \( P_m = \frac{I^2 R_m}{R_s + R_m} \). The low-frequency spectral density of the current noise, measured by the SQUID, is given by

\[
S_I = 4k_B \frac{T R_s + T R_m}{(R_s + R_m)^2} + S_{I0} \tag{2}
\]

where \( T_s \) and \( T_m \) are, respectively, the electron temperature of \( R_s \) and \( R_m \). The first term in the right-hand side of equation (2) represents the thermal noise of the resistances \( R_s \) and \( R_m \). \( S_{I0} \) represents the sum of all other noise terms, which include the thermal noise from \( R_b \), the noise from the current supply, and the noise of the room-temperature SQUID electronics. All these terms are expected to be independent of temperature or negligible. To calculate the temperature of the electrons in the film resistors \( T_s \) from a measurement of the total noise \( S_I \), \( S_{I0} \), the thermal noise of \( R_m \) and an accurate estimate of \( R_s \) and \( R_m \) have to be known. \( R_s \) and \( R_m \) have been evaluated in separate measurements and found independent of temperature down to 60 mK.

We have performed a set of noise measurements as a function of temperature at zero bias current in order to verify the linear dependence of the noise on the bath temperature \( T_0 \) and to calibrate the noise thermometer. The constant term \( S_{I0} \) has been estimated as the intercept of a linear fit to the data.

To verify if the thermal noise of the SMD resistor \( R_m \) can be estimated by assuming \( T_m = T_0 \) (negligible overheating of the SMD resistor), we have performed separate tests, both in vacuum at \( T_0 = 60 \, \text{mK} \) and in a liquid helium bath at \( T_0 = 1.25 \, \text{K} \), in which \( R_s \) was replaced with a SMD resistor identical to \( R_m \). In these tests, we have observed a small but not negligible overheating of the SMD resistors that we have taken into account for the calculation of the thermal noise of the film resistor \( R_s \).

By knowing \( S_{I0} \) and \( T_m \) one can evaluate the electron gas temperature \( T_s \) of the film resistor as a function of the measured total noise \( S_I \) at different bias power levels and then fit \( T_s \) vs \( P_s \), with the function

\[
T_s = \left( \frac{P_s + P_0}{\Sigma \Omega} + T_0^n \right)^{1/n} \tag{3}
\]

where \( T_0 \) is the bath temperature, as measured by the thermometer, and \( P_0 \), \( n \), and \( \Sigma \Omega \) are the fitting parameters. Equation (3) is another form of equation (1) in which \( T_s = T_m \) and \( T_0 = T_0 \) that is the phonon gas is assumed to be well thermalized to the bath temperature. The total power \( P \) is expressed by \( P_s + P_0 \), where \( P_s \) is the power dissipated in the film resistors by the bias current and \( P_0 \) is a constant positive term, which accounts for excess power dissipated in the resistor. For example, this excess power could be produced by rf interference, generating rf currents in the wiring.
3. Results and discussion

Figure 3 shows the measurements of the electron gas temperature for the device without cooling fins as a function of the dissipated power at the operating bath temperatures $T_0=59\ mK$ and $T_0=1.25K$.

![Figure 3](image)

**Figure 3.** Electron gas temperature for the device without cooling fins as a function of the dissipated power at the operating bath temperatures $T_0=59\ mK$ (circles) and $T_0=1.25K$ (squares).

The data at $T_0=59\ mK$ are in good agreement with equation (3) and the fit yields $P_0=1.4\pm0.2\ pW$, $n=5.01\pm0.05$, and $\Sigma\Omega=(2.2\pm0.7)\times10^{-8}\ W/K^5$. The magnitude of the measured excess power $P_0$ is comparable to what one expects from rf interference in a typical laboratory environment, especially as we did not use any rf or microwave filters in our wiring. As the typical intrinsic power dissipation of a dc SQUID is much larger, this excess power is not important.

The data at $T_0=1.25K$ are also in agreement with equation (3) with $n=5.1\pm0.1$

The data above the low power knee, obtained at 1.25K, with a completely different thermal contact to the thermal bath, are very close to the extrapolation of the fit of the data at 59 mK. This fact together with the exponent $n$ very close to 5, suggest that the overheating we have measured in the device without cooling fins is due to the hot-electron effect. In fact, the measured thermal impedance cannot be explained by other more conventional mechanisms because of the expected exponent and magnitude.

On the other hand, the exponent $n=5$ is in contrast with the predictions for a metal in the dirty limit: in this case, as well in others, the exponent $n=5$ suggests that this and other model are inappropriate. For a more detailed discussion see reference [11].

The electron gas temperatures as a function of the dissipated power at 59 mK for both the devices with and without cooling fins are reported in figure 4.

![Figure 4](image)

**Figure 4.** Electron gas temperatures as a function of the dissipated power at 59 mK for both the devices with (squares) and without (circles) cooling fins.

The effect of the cooling fins that allow the electrons to diffuse in a larger region, thus increasing the effective volume available for electron-phonon energy relaxation, is to significantly reduce the electron temperature at a given power. For a SQUID with shunt resistors similar to those tested in this
experiment, at power dissipation of the order of 10 pW, the saturation temperature would improve, from about 200 mK to 100 mK.

Contrary to the data of the device without the cooling fins, the data obtained with the device with cooling fins are not in agreement with equation (3). In particular, the slope is found to be slightly variable from about $n=2.5$ at low power to about $n=3.5$ at high power. The change in slope is qualitatively predicted by the hot-electron effect model, if one takes into account the diffusion of the hot electrons. In fact, only a limited portion of the total volume of the fin is available for the e-p thermalization, because the hot electrons generated in the resistor diffuse in the fin only over a distance comparable with the temperature-dependent diffusion length that increases with decreasing temperature. A course estimate yields for the diffusion length about 1 mm at $T_0=59$ mK. The following behavior is thus expected: at high temperatures, the hot electrons generated in the resistor cannot diffuse significantly into the fin, and one has the standard curve with $n=5$, as expected in the absence of cooling fins. At intermediate temperatures (roughly <1 K in our case), the effective volume increases with decreasing temperature. This is equivalent to an effective reduction of the exponent $n$ in equation (3), which in turn depends on the size, shape, and dimensionality of the fins. At sufficiently low temperatures, not achieved in our experiment, the entire volume of the fin will be available, and the slope of the curve is expected to return to $n=5$, but at a temperature level a factor ($\Omega'/\Omega)^{1/n}$ lower than at high temperature, where $\Omega$ and $\Omega'$ are, respectively, the volume of the resistor and that of the fin.

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