Generalized $\mu$-Terms from Orbifolds and M-Theory

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Abstract

We consider solutions to the $\mu$-term problem originating in the effective low energy theories, of $N = 1$ $Z_N$ orbifold compactifications of the heterotic string, after supersymmetry breaking. They are consistent with the invariance of the one loop corrected effective action in the linear representation for the dilaton. The proposed $\mu$-terms naturally generalize solutions proposed previously, in the literature, in the context of minimal low-energy supergravity models. They emanate from the connection of the non-perturbative superpotential to the determinant of the mass matrix of the chiral compactification modes. Within this approach we discuss the lifting of our solutions to their M-theory compactification counterparts.

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1. introduction

One of the necessary ingredients of the minimal supersymmetric extension of the standard model of electroweak interactions is the existence in its matter superpotential of a mixing term between the two Higgs doublets, namely $W_{\text{tree}} = \mu H_1 H_2$. The coupling of this bilinear term is the so called $\mu$-term. Its presence introduces a hierarchy problem in the theory. Clearly the presence of such a term at the electroweak scalar potential of the theory is essential in order to avoid the breaking of the Peccei-Quinn symmetry and the appearance of the unwanted axion and to give masses to the d-type quarks and e-type leptons which otherwise will remain massless. In addition the presence of the $\mu$-term is necessary for the correct electroweak symmetry breaking. However during the latter process the low energy parameter $\mu$, of the electroweak scale, is identified with a parameter of order of the Planck scale something unacceptable. An explanation of the origin of $\mu$ term generates the $\mu$-problem and several scenaria have appeared in the literature providing a solution.

Mechanisms for the generation of the $\mu$ term make use of gaugino condensation to induce an effective $\mu$ term [6] or the presence of mixing $H_{IJ}$ terms in the Kähler potential [7, 6, 9], which induce after supersymmetry breaking an effective $\mu$ term of order $O(m_3/2)$. In the context of supergravity models coming from string vacua, with spontaneously broken supersymmetries, the first examples of a $\mu$-term generated by the Kähler potential appeared in [18]. A general discussion of the generation of the $\mu$ and B-terms in the context of perturbative and non-perturbative supersymmetry breaking appeared in [19]. Another solution, applicable to supergravity models, makes use of non-renormalizable terms (fourth or higher order) in the superpotential. They have the form $M_{Pl}^{1-n} A^n H_1 H_2$ and generate a contribution [8] to the $\mu$ term of order $\sim O(M_{Pl}^{1-n} M_{\text{hidden}})$ after the hidden fields A acquire a vacuum expectation value. Superpotential generation of $\mu$-terms in the context of freely acting orbifold compactifications of the heterotic string on the $K_3 \times T^2$, appeared in [8]. The latter was examined in F-theory compactification on Calabi-Yau 4-folds in [9].

In this paper we explore the origin of $\mu$ terms in $(2,2)$ orbifold compactifications of the heterotic string. We discuss particular solutions to the $\mu$ problem related to the generation of the mixing terms between Higgs fields behaving as neutral scalar moduli in the Kähler potential. They are related to the presence of duality symmetries originating from
subgroups of the modular group \(PSL(2, Z)\). The latter appears in non-decomposable (2,2) symmetric \(Z_N\) orbifold compactifications of the heterotic string in four dimensions and in certain orbifold limits of \(K_3 \times T^2\). Previous solutions in the literature of generating \(\mu\) for the \(SL(2, Z)\) case appeared in \([1, 39]\).

Let us explain the origin of such mixing terms in superstring theory. We assume that our effective theory of the massless modes after compactification is that of the heterotic string preserving \(N = 1\) supersymmetry while our effective theory is described by the usual \(N = 1\) two derivative supergravity. Let us fix the notation \([10]\) first. We are labeling the 27, \(\bar{27}\) with letters from the beginning (middle) of the Greek alphabet while moduli are associated with latin characters. The gauge group is \(E_6 \times E_8\), the matter fields are transforming under the 27, \(\bar{27}\) representations of the \(E_6\), 27’s are related to the \((1, 1)\) moduli while \((\bar{27})\)’s are related to the \((2, 1)\) moduli in the usual one to one correspondence. The Kähler potential is given by

\[
K = B^\alpha B^\bar{\alpha} Z^{(1,1)}_{\alpha\bar{\alpha}} + C^\nu C^{\bar{\nu}} Z^{(2,1)}_{\nu\bar{\nu}} + (B^\alpha C^{\nu} H_{\alpha\nu} + c.c) + \ldots
\]

(1.1)

with the B and C corresponding to the 27’s and \(\bar{27}\)’s respectively. Appropriate expansion of the low energy supergravity observable scalar potential, after supersymmetry breaking, generates a general contribution in the form

\[
\mu_{IJ} B^I C^J, \quad \mu_{IJ} = m_{3/2} H_{1J} - F^j \bar{\partial}_j H_{1J} + \bar{\mu}_{1J}, \quad H_{1J} = \frac{1}{(T + \bar{T})(U + \bar{U})}.
\]

(1.2)

where \(F\) is the auxiliary field of the \((1, 1)\) or \((2, 2)\) moduli and have assumed that matter fields and moduli come from the same complex plane. In this work, as the contributions to the \(\mu\)-term coming from the first two terms in (1.2) are standard, we will examine the origin of the additional \(\bar{\mu}\)-term. Our study requires an expansion of the superpotential, confirmed posteriori\(^1\), in the form

\[
\mathcal{W} = \mathcal{W}_o + \mathcal{W}_{BC} BC.
\]

(1.3)

In this case the \(\mu\) term receives an additional contribution in the form

\[
\bar{\mu}_{BC} = e^{G/2} \mathcal{W}_{BC}.
\]

(1.4)

\(^1\)The solution to the \(\mu\)-problem proposed in [3] in the context of a general minimal supergravity, required the same general expansion of \(W\).
The superpotential of the theory in the form \( \text{[13]} \) comes from non-perturbative effects since terms in this form don’t arise in perturbation theory, due to non-renormalization theorems \([8, 1]\). Furthermore, because supersymmetry cannot be broken by any continous parameter\([8]\), the origin of such terms may not come from a spontaneous breaking version of supersymmetry but necessarily its origin must be non-perturbative.

In the beginning of section 2, we will exhibit the method of calculating superpotentials that receive contributions from the integration of massive modes. For this reason we will use initially the non-decomposable orbifold \( \mathbb{Z}_6 - II - b \) \([13]\). In this case the expansion of the superpotentials into the form \( W = W_o + W_{BC} BC \), is consistent with the invariance of the one-loop corrected effective action, in the linear representation of the dilaton, under tree level \( \Gamma_o(3) \) transformations,

\[
T \Gamma_o(3) T \rightarrow aT - ib\frac{icT + d}{c \equiv 0 \mod 3},
\]

which leave the tree level Kähler potential

\[
K = -\log[(T + \bar{T})(U + \bar{U}) - (B + \bar{C})(C + \bar{B})]
\]

invariant, only if \( W \rightarrow (icT + d)^{-1}W \) and

\[
W_o \rightarrow (icT + d)^{-1}W_o, \text{ and } W_{BC} \rightarrow (icT + d)^{-1}W_{BC} + icW_o.
\]

The paper is organized as follows. In sect. 2 we analyse the different modular orbits appearing in the moduli space for the non-decomposable orbifolds. Next, we describe the identification of the non-perturbative superpotential \( \mathcal{W} \) with the mass matrix of the chiral masses of the compactification modes. We perform the sum over modular orbits, integrating the \( N = 2 \) massive untwisted states of the compactification. In this respect, we calculate \( \mathcal{W} \) in four dimensions by taking into account contributions from general non-decomposable \( N = 1 (2, 2) \) symmetric Coxeter orbifolds. In addition, we calculate the dilaton dependence of the non-perturbative superpotentials either by using the BPS sums or by the use of the gaugino condensates. We identify the \( W^{\text{non-pert}} \) for all classes of non-decomposable Coxeter orbifolds. In sect. 3 we identify the relevant orbifolds appearing in the classification list of \([14]\) and which can be characterized as generalized Coxeter orbifolds and calculate \( \mathcal{W} \). In sect. 4 we describe the contributions to the \( \tilde{\mu} \) term coming from sections 2 and 3. As we
will see, this analysis allows to describe a variety of possible phenomenological scenarios. In sect. 5 we explain the promotion of the soft terms, and in particular the B soft-term arising through the \( \mu \)-terms of section four, to their counterparts coming from M-theory compactifications to four dimensions. In sect. 5 we summarize our conclusions.

2. Non-perturbative Superpotentials from Modular Orbits of Coxeter Orbifolds

Let us consider first the generic case of an orbifold where the internal torus factorizes into the orthogonal sum \( T_6 = T_2 \oplus T_4 \) with the \( Z_2 \) twist acting on the 2-dimensional torus lattice. We will be interested in the mass formula of the untwisted subspace associated with the \( T_2 \) torus lattice. In this case, the momentum operator factorises into the orthogonal components of the sublattices with \( (p_L; p_R) \subset \Gamma_{q+2;2} \) and \( (P_L; P_R) \subset \Gamma_{20-q;4} \). And as a result the mass operator \( M^2 \) factorises as below while the spin \( S \) for the \( \Gamma_{q+2;2} \) sublattice becomes

\[
\frac{\alpha'}{2} M^2 = p_R^2 + P_R^2 + 2N_R, \quad p_L^2 - p_R^2 = 2(N_R + 1 - N_L) + \frac{1}{2} P_R^2 - \frac{1}{2} P_L^2 = 2n^T m + b^T C b, \tag{2.1}
\]

where \( C \) is the Cartran metric operator for the invariant directions of the sublattice \( \Gamma_q \) of the \( \Gamma_{16} \) even self-dual lattice\(^2\). The above formula involves perturbative BPS states which preserve 1/2 of the supersymmetries, which belong to short multiplet representations of the supersymmetry algebra.

Let us now consider the \( Z_6 - II - b \) orbifold. This orbifold is non-decomposable in the sense that the action of the lattice twist does not decompose in the orthogonal sum \( T_6 = T_2 \oplus T_4 \) with the fixed plane lying in \( T_2 \). Its complex twist is \( \Theta = (2, 1, -3)(2\pi i)/6 \). The orbifold twists \( \Theta^2 \) and \( \Theta^4 \), leave the second complex plane unrotated. The lattice in which the twists \( \Theta^2 \) and \( \Theta^4 \) act as an lattice automorphism is the \( SO(8) \). In addition there is a fixed plane which lies in the \( SU(3) \) lattice and is associated with the \( \Theta^3 \) twist.

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\(^2\)Compatibility of the untwisted moduli with the twist action on the gauge coordinates comes from the non-trivial action of the twist in the gauge lattice. This means that that the untwisted moduli of the orbifold have to the equation \( M A = A Q \), where \( Q, A \) and \( M \) represent the internal, Wilson lines and gauge twist respectively.
The action of the internal twist can be made to act as $-I_2$ on a $T_2$ by appropriate parametrization of the momentum quantum numbers. Consider now a $k$-twisted sector of a six-dimensional orbifold of the the heterotic string associated with the twist $\theta^k$. If this sector has an invariant complex plane then its twisted sector quantum numbers have to satisfy

$$Q^k n = n, \quad Q^{*k} m = m, \quad M^k l = l,$$

(2.2)

where $Q$ defines the action of the twist on the internal lattice and $M$ defines the action of the gauge twist on the $E_8 \times E_8$ lattice. This means that with

$$n = \begin{pmatrix} \bar{n}_1 \\ \bar{n}_2 \end{pmatrix}, \quad m = \begin{pmatrix} \bar{m}_1 \\ \bar{m}_2 \end{pmatrix}, \quad l = \begin{pmatrix} \bar{l}_1 \\ \bar{l}_2 \end{pmatrix}$$

(2.3)

and with $E_a$, $a=1,2$ a set of basis vectors in the fixed directions of the orbifolds, $\tilde{E}_\mu$, $\mu = 1, \ldots, d$ the set of basis vectors in the fixed directions of the gauge lattice, the momenta take the form

$$P'_L = \left( \rho \frac{\bar{m}}{2} + \left( G_\perp - B_\perp - \frac{1}{4} A_\perp^t C_\perp A_\perp \right) \bar{n} - \frac{1}{2} A_\perp^t C_\perp \bar{l}, \bar{l} + A_\perp \bar{n} \right)$$

$$P'_R = \left( \rho \frac{\bar{m}}{2} + \left( G_\perp - B_\perp + \frac{1}{4} A_\perp^t C_\perp A_\perp \right) \bar{n} - \frac{1}{2} A_\perp^t C_\perp \bar{l}, 0 \right),$$

(2.4) (2.5)

\( \rho_{ab} \overset{\text{def}}{=} E_a \cdot \tilde{E}_b. \) Here $\rho$ is the $\rho_{ab}$ matrix, $C_\perp$ is the Cartran matrix for the fixed directions and $A_\perp^I$ is the matrix for the continuous Wilson lines in the invariant directions $i = 1, 2, I = 1, \ldots, d$. $G_\perp$ and $B_\perp$ are $2 \times 2$ matrices and $\bar{n}$, $\bar{m}$, $\bar{l}$ are the quantum numbers in the invariant directions.

For orbifold compactifications, where the underlying internal torus does not decompose into a $T_6 = T_2 \oplus T_4$, the $Z_2$ twist associated with the reflection $-I_2$ does not put any additional constraints on the moduli $U$ and $T$. As a consequence the moduli space of the untwisted subspace is the same as in toroidal compactifications. For the $Z_6 - II - b$ orbifold, $\rho = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$. The mass formula [13] for the $\Theta^2$ subspace reads

$$m^2 = \sum_{n_1, n_2} \frac{2}{Y} \left| -TU'n^2 + iTn^1 - iU'm_1 + 3m_2 \right|^2 \mid_{U'=U-2i} = |M_{\text{pert}}|^2/(Y/2),$$

(2.6)
The quantity $Y$ is connected to the Kähler potential, $K = -\log Y$. The target space duality group is $\Gamma^0(3)_T \times \Gamma^0(3)_U$ where $U' = U - 2i$. Let us now connect $W$ to the target space partition function $Z$ coming from the integration of the massive chiral compactification modes

$$Z = e^{-F_{\text{fermionic}}} = -\det((\mathcal{M}^{\text{pert}})^\dagger \mathcal{M}^{\text{pert}}) = -\frac{|W|^2}{Y},$$

(2.8)

We define the free energy as the one coming from the integration of the massive compactification modes, i.e. Kaluza-Klein and winding modes. Because non-compactification modes like massive oscillator modes are excluded from the sum the free energy can be characterized as topological. From (2.8) we can deduce that

$$W = \det \mathcal{M}^{\text{pert}}. \quad (2.9)$$

Here $\mathcal{M}$ represents the fermionic mass matrix, $F$ the topological free energy and $W$ the perturbative superpotential. Working in this way, we define the free energy as the one coming from the integration of the massive compactification modes, i.e. Kaluza-Klein and winding modes. Because non-compactification modes like massive oscillator modes are excluded from the sum the free energy can be characterized as topological. From (2.8) we can deduce that

$$W = \det \mathcal{M}^{\text{pert}}. \quad (2.9)$$

Here $\mathcal{M}^{\text{pert}}$ is the mass matrix of the chiral hypermultiplet masses of the massive compactification modes. Note that in order to identify (2.9) as a non-perturbative superpotential in order to exhibit an $e^{-S}$ dependence, we will connect $M$ to special geometry of a vector multiplet sector. According to the latter the $N=2$ non-perturbative mass matrix is identified with the BPS short multiplet expression

$$\mathcal{M}^{\text{non-pert}}_{\text{BPS}} = 2e^K \mathcal{M} = 2e^K |M_I X^I + iN_I F^I|^2, \quad (2.10)$$

$$\text{where } I = 1, \ldots, n_V \text{ counts the number of vector multiplets and } F \text{ denotes the } N = 2 \text{ prepotential} \quad (2.10)$$

Here $K$ is the Kähler potential and $M_I, N_I$ are the “electric” and “magnetic” quantum number analogs of the $N = 4$ supersymmetry in heterotic string compactifications [22]. When $N_I$ is equal to zero then the classical spectrum of “electric” states can be shown to agree with the spectrum of momentum and winding numbers in (2.6). The $N = 2$ Kähler potential is defined in terms of $N = 2$ special geometry coordinates $(X^I, iF_I)$ as

$$K = \log(X^I \tilde{F}_I + \bar{X}^I F_I) = \log(-i\Omega^I \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Omega), \quad F_I = \frac{\partial F}{\partial X^I}, \quad (2.11)$$
where the "period" $\Omega = (X', iF_t)$ defines a holomorphic section. At the moment we will concentrate our efforts to exhibit the dependence of the mass operator, in the perturbative vector multiplet $T, U$ moduli, as it originates from the classical mass formula (2.6). The dependence of the "mass" formula on its "non-perturbative" $S$-part will be exhibited later. Especially, for the case where the calculation of the free energy is that of the moduli space of the manifold $\frac{SO(2,2)}{SO(2) \times SO(2)}$, in a factorizable 2-torus $T_2$, the topological bosonic free energy is exactly the same as the one, coming from the string one loop calculation in [21]. The total contribution to the non-perturbative superpotential, coming from perturbative modular orbits associated with the presence of massless particles of elementary string states, is connected with the existence of the following orbits $^{3, 5}$

\begin{align*}
\Delta_0 &= \sum_{2n^T m + l^T Cl = 2} \log \mathcal{M}|_{reg}, \\
\Delta_1 &= \sum_{2n^T m + q^T Cq = 0} \log \mathcal{M}|_{reg}.
\end{align*}

In the previous expressions, a regularization procedure is assumed that takes place, which renders the final expressions finite, as infinite sums are included in their definitions. The regularization is responsible for the subtraction $^4$ of a moduli independent quantity from the infinite sum e.g $\sum_{n, m \in \text{orbit}} \log \mathcal{M}$. We demand that the regularization procedure for $\exp[\Delta]$ has to respect both modular invariance and holomorphicity.

In eqn. (2.12), $\Delta_0$ is the orbit relevant for the stringy Higgs effect. This orbit is associated with the quantity $2n^T m + l^T Cl = 2$ where $n^T m = m_1 n^1 + 3m_2 n^2$ for the $Z_6$-II-b orbifold. We should note, that the term "non-decomposable" will be used in a loose sense, in section four, referring to the effective supergravity theories coming from both the non-decomposable $N = 1$ orbifolds and the freely acting orbifold limits, Enriques involutions, of $K_3$ where the same modular subgroups of $PSL(2, Z)$, do appear $^{[16]}$. We note that non-perturbative superpotentials invariant under $SL(2, Z)_T \times SL(2, Z)_U$ were formulated, in terms of the genus two modular form $C_{12}$, in $^{[17]}$. The total contribution $^5$ from the previously mentioned orbit is

\[ \Delta_0 \propto \sum_{2n^T m + l^T Cl = 2} \log \mathcal{M} = \sum_{n^T m = 1, l^T Cl = 0} \log \mathcal{M} + \sum_{n^T m = 0, l^T Cl = 2} \log \mathcal{M} + \]

$^3$We calculate only the $\sum \log \mathcal{M}$ since the $\sum \log A_i^*$ quantity will give only the complex conjugate.

$^4$More details of this procedure can be found in [20].

$^5$We use a general embedding of the gauge twist in the gauge degrees of freedom.
\[
\sum_{n^T m = -1, l^T Cl = 4} \log \mathcal{M} + \ldots \tag{2.13}
\]

We must notice here that we have written the sum over the states associated with the \(SO(4, 2)\) invariant orbit \(2n^T m + l^T Cl = 2\) in terms of a sum over \(\Gamma^0(3)\) invariant orbits \(n^T m = \text{constant}\). The precise parametrization of the Wilson lines in the invariant directions is not important since it is of no interest to us in the calculation of the modular orbits. Only the momentum and winding number dependent part of the spin operator is enough for our purposes. Remember that initially in (2.1), we discussed the level matching condition in the case of a \(T_6\) orbifold admitting an orthogonal decomposition. Mixing of these equations for the non-decomposable orbifolds gives us the following equation

\[
p_L^2 - \frac{\alpha'}{2} M^2 = 2(1 - N_L - \frac{1}{2} P^2_L) = 2n^T m + l^T Cl. \tag{2.14}
\]

The previous equation can be written in the form described in (2.5) and (2.4). In particular, it may give us a number of different orbits invariant under \(SO(q+2, 2; Z)\) transformations. Namely, \(i\) the untwisted orbit with \(2n^T m + l^T Cl = 2\). In this orbit, \(N_L = 0, \ P^2_L = 0\). When \(M^2 = 0\), this orbit is associated with the "stringy Higgs effect". The "stringy Higgs effect appears as a special solution of the equation (2.14) at the point where \(p_L^2 = 2\), where additional massless particles may appear. Or, \(ii\) the untwisted orbit where \(2n^T m + l^T Cl = 0\). Here \(2N_L + P^2_L = 2\). This is the orbit relevant to the calculation of threshold corrections to the gauge couplings, without the need of enhanced gauge symmetry points, as may happen in the orbit \(i\). Or, \(iii\) The massive untwisted orbit with \(2N_L + P^2_L \geq 4\).

In this work we will always have \(M^2 \geq 0\). In eqn. (2.12), \(\Delta_0\) is the orbit relevant for the stringy Higgs effect. This orbit is associated with the quantity \(2n^T m + l^T Cl = 2\) where \(n^T m = m_1n^1 + 3m_2n^2\) for the \(Z_6\)-II-b orbifold.

We will be first consider the contribution from the orbit \(2n^T m + l^T Cl = 0\). We will be working in analogy with calculations associated with topological free energy considerations \[15\]. From the second equation in (2.12), considering in general the \(SO(4, 2)\) coset, we get for example that

\[
\Delta_1 \propto \sum_{n^T m + l^T Cl = 0} \log \mathcal{M} = \sum_{n^T m = 0, l^T Cl = 0} \log \mathcal{M} + \sum_{n^T m = -1, l^T Cl = 1} \log \mathcal{M} + \ldots \tag{2.15}
\]

Consider in the beginning the term \(\sum_{n^T m = 0, l^T Cl = 0} \log \mathcal{M}\). We are summing up initially
the orbit with $n^Tm = 0; n, m \neq 0$, i.e \( \Delta_1 \)

\[ M = 3m_2 - im_1U' + in_1T + n^2(-U'T + BC) + l \text{ dependent terms.} \]  \tag{2.16}

We calculate the sum over the modular orbit $n^Tm + l^TCl = 0$. As in \[39\] we calculate the sum over massive compactification states with $l^TCl = 0$ and $(n, m) \neq 0$. Namely, the orbit

\[
\sum_{n^Tm=0, l^TCl=0} \log M = \sum_{(n,m)\neq(0,0)} \log(3m_2 - im_1U' + in_1T + n_2(-U'T)) \\
+ BC \sum_{(n,m)\neq(0,0)} \frac{n_2}{(3m_2 - im_1U' + in_1T - n_2U'T)} + O((BC)^2). \tag{2.17}
\]

The sum in relation (2.17) is topological(it excludes oscillator excitations) and is subject to the constraint $3m_2n_1^2 + m_1n_1^2 = 0$. Its solution receives contributions from the following sets of integers:

\[ m_2 = r_1r_2 , \ n_2 = s_1s_2 , \ m_1 = -3r_2s_1 , \ n_1 = r_1s_2 \] \tag{2.18}

and

\[ m_2 = r_1r_2 , \ n_2 = s_1s_2 , \ m_1 = -r_2s_1 , \ n_1 = 3r_1s_2. \] \tag{2.19}

So the sum becomes,

\[
\sum_{n^Tm=0} \log (3m_2 - im_1U' + in_1T - n_2U'T) = \sum_{(r_1,s_1)\neq(0,0)} \log (3(r_1 + is_1U')) \times \\
\sum_{(r_2,s_2)\neq(0,0)} \log (r_2 + is_2T) + \sum_{(r_1,s_1)\neq(0,0)} \log 3(r_1 + is_1U') \sum_{(r_2,s_2)\neq(0,0)} \log (r_2 + is_2T) \tag{2.20}
\]

Substituting explicitly in eqn.(2.17), the values for the orbits in equations (2.18) and (2.19) together with eqn.(2.20), we obtain

\[
\sum_{n^Tm=0; l^TCl=0} \log M = \log \left( \frac{1}{3} \eta^{-2}(U')\eta^{-2}\left(\frac{T}{3}\right) \right) + \log \left( \frac{1}{3} \eta^{-2}\left(\frac{U'}{3}\right)\eta^{-2}(T) \right) + \\
\left( BC \sum_{(r_1,s_1)\neq(0,0)} \frac{s_1}{r_1 + is_1U'} \right) \left( \sum_{(r_2,s_2)\neq(0,0)} \frac{s_2}{3(r_2 + is_2T)} \right) + \\
\left( BC \sum_{(r_1,s_1)\neq(0,0)} \frac{s_1}{3(r_1 + is_1U')^2} \right) \left( \sum_{(r_2,s_2)\neq(0,0)} \frac{s_2}{r_2 + is_2T} \right) + O((BC)^2). \tag{2.21}
\]
Notice that we used the relation

$$
\sum_{(r_1,s_1) \neq (0,0)} \log 3 = \log \frac{1}{3}
$$

(2.22)

with $$\sum_{(r_1,s_1) \neq (0,0)} = -1$$. We substitute $$\sum_{(r_1,s_1) \neq (0,0)} \overset{def}{=} \sum'$$ and $$\sum_{(r_2,s_2) \neq (0,0)} \overset{def}{=} \sum''$$. Remember that $$\sum' \log(t_1 + it_2 T) = \log \eta^{-2}(T)$$, with $$\eta(T) = \exp \frac{-\pi T}{12} \prod_{n>0} (1 - \exp^{-2\pi n T})$$ This means that eqn. (2.21) can be rewritten as

$$
\sum_{nTm=0; q=0} \log \mathcal{M} = \log \left( \eta^{-2}(U') \eta^{-2} \left( \frac{T}{3} \right) \left( \frac{1}{3} \right) \right) + \log \left( \frac{1}{3} \eta^{-2} \left( \frac{U'}{3} \right) \eta^{-2}(T) \right) +
$$

$$
-BC \left( \left( \partial_{U'} \sum r_1 + is_1 U' \right) \left( \partial_T \sum \log(r_2 + is_2 \frac{T}{3}) \right) \right)
$$

$$
-BC \left( \left( \partial_{U'} \sum \log(r_1 + is_1 \frac{U'}{3}) \right) \left( \partial_T \sum \log(r_2 + is_2 \frac{T}{3}) \right) \right) + \mathcal{O}((BC)^2). \quad (2.23)
$$

Finally

$$
\sum_{nTm=0; q=0} \log \mathcal{M} = \log \left( \eta^{-2}(U') \eta^{-2} \left( \frac{T}{3} \right) \left( \frac{1}{3} \right) \right) + \log \left( \frac{1}{3} \eta^{-2} \left( \frac{U'}{3} \right) \eta^{-2}(T) \right) -
$$

$$
-BC \left( \left( \partial_T \log \eta^{-2}(T) \right) \left( \partial_{U'} \log \eta^{-2} \left( \frac{U'}{3} \right) \right) + \left( \partial_T \log \eta^{-2} \left( \frac{T}{3} \right) \right) \left( \partial_{U'} \log \eta^{-2}(U') \right) \right) +
$$

$$
\mathcal{O}((BC)^2).
$$

(2.24)

So

$$
\sum_{nTm=0; q=0} \log \mathcal{M} = \log \left[ \eta^{-2}(T) \frac{1}{3} \eta^{-2} \left( \frac{U'}{3} \right) \left( 1 - BC \left( \partial_T \log \eta^2(T) \right) \right) \right. 
$$

$$
\left( \partial_{U'} \log \eta^2 \left( \frac{U'}{3} \right) \right] + \log \left[ \left( \eta^{-2} \left( \frac{U'}{3} \right) \eta^{-2} \left( \frac{T}{3} \right) \right) \left( 1 - BC \left( \partial_T \log \eta^2 \left( \frac{T}{3} \right) \right) \right) \right] + \mathcal{O}((BC)^2)
$$

(2.25)

or

$$
\sum_{nTm=0; q=0} \log \mathcal{M} = \log \left[ \eta^{-2}(T) \frac{1}{3} \eta^{-2} \left( \frac{U'}{3} \right) \left( 1 - 4 BC \left( \partial_T \log \eta(T) \right) \right) \right.
$$

$$
\left( \partial_{U'} \log \eta \left( \frac{U'}{3} \right) \right] + \mathcal{O}((BC)^2)
$$
\[(\partial_U' \log \eta(\frac{U'}{3})) + \log[\frac{1}{3}((\eta^{-2}(U'))\eta^{-2}(\frac{T}{3}))(1 - 4BC(\partial_T \log \eta(\frac{U'}{3}))))] + O((BC)^2)\] 

(2.26)

The last expression provides us with the part of the (0,2) non-perturbative \([21, 39]\) generated superpotential \(\mathcal{W}_{\text{non-pert}}\) that comes from the direct integration of the string massive modes. The non-perturbative part that depends on the dilaton will be included later either by use of gaugino condensates or by associating it to BPS sums. We should say that (2.26) may be of non-perturbative origin only after an \(e^{-S}\) dependence is included. The derivation of the superpotential from the sum over modular orbits in the case that the effective string action of decomposable orbifolds is invariant under the target space duality group \(SL(2, Z)\times SL(2, U)\) was found in \([39]\) to be the same as the expression argued to exist in \([4]\), for the non-perturbative superpotential. The latter was obtained \([4]\) through gaugino condensation and the requirement that the one loop effective action in the linear formulation for the dilaton be invariant \([\mathcal{H}]\) under the full \(SL(2, Z)\) symmetry up to quadratic order in the matter fields. In exact analogy, we expect our expression in (2.25), to represent the non-perturbative superpotential of the \(Z_6 - II - b\).

Let us now exhibit the derivation of the \(e^{-S}\) dependence from BPS and gaugino condensation generated superpotentials we have just comment. We will first discuss the BPS derivation. Consider again (2.10). We will attempt to generate the non-perturbative \(S\) dependence in the case of the orbifold \(Z_6 - II - b\) in (2.6). Let us consider transforming the period vector to the weakly coupled basis \([28, 29, 30]\), where all gauge couplings become weak at the strongly coupled limit. We transform the section, or period vector, according to the identification

\[(x^I, iF_I) \rightarrow (Y^I, iL_I),\] 

(2.27)

where

\[Y^1 = iF_1, \quad L_1 = iX^1, \quad Y^i = X^i, \quad L_i = F_i, \quad \text{for} \quad i = 0, 2, 3.\] 

(2.28)

\[^6\] The linear formulation is naturally present in string theory since the dilaton is in the same supermultiplet with the antisymmetric tensor.
That means

$$\Omega^T = (1, TU - BC, iT, iU, iS(TU - BC), iS, -SU, -ST)$$  \hspace{1cm} (2.29)$$

and

$$M_0 = 3sm_2, \ M_1 = -sn_2, \ M_2 = sn_1, \ M_3 = -sm_1$$

$$N_0 = -pn_2, \ N_1 = 3pm_2, \ N_2 = -pm_1, \ N_3 = pn_1.$$ \hspace{1cm} (2.30)

In eqn. (2.29, 2.30) we assumed that the tree level $N = 2$ prepotential $F^{\text{tree}}$ in the presence of the Wilson lines for the "non-factorized" $T^2$ torus is given by

$$F^{\text{tree}} = -S(TU - BC),$$ \hspace{1cm} (2.31)

where (2.29) has come from the identifications $S = iX^1/X^0$, $T = -iX^2/X^0$, and $U = -iX^3/X^0$ with the graviphoton $X^0 = 1$.

Under the identifications (2.30) the contribution of the BPS orbit that includes the perturbative orbit (2.18) factorizes and (2.10) gives

$$\log M = \sum_{s,p,m,n} \log[(s + ip)(3m_2 - im_1U + in_1T - n_2(TU - BC))].$$ \hspace{1cm} (2.32)

Treating in the same way the other orbit in (2.19) we get that the exact form of the non-perturbative effective superpotential for the $Z_6$-II-b orbifold is given by

- $Z_6 - II - b \xrightarrow{SU(3) \times SO(8)} W^{\text{non-pert}}_{\text{BPS}} \eta^2(S) = \eta^{-2}(T)(\frac{1}{3})\eta^{-2}(\frac{U'}{3})(1 - BC (\partial_T \log \eta^2(T)) \times$

  $$(\partial_U \log \eta^2(\frac{U'}{3}))\tilde{W} + [ (\eta^{-2}(U')(\eta^{-2}(\frac{T'}{3}))\frac{1}{3})(1 - BC ((\partial_T \log \eta^2(\frac{T}{3}))$$

  $$\times (\partial_U \log \eta^2(\frac{U'}{3}))\tilde{W} + O((BC)^2).$$ \hspace{1cm} (2.33)

The superpotential in (2.33) transforms with modular weight $-1$ under $SL(2, Z)_S$ S-duality transformations

$$S \rightarrow \frac{aS - ib}{icS + d}, \quad ad - bc = 1,$$ \hspace{1cm} (2.34)

where $a,b,c,d$ are integers. That means that the BPS calculation "communicates" to the low energy heterotic string action $SL(2, Z)$ S-duality something which is known not to be
true as far as perturbation theory is concerned. The latter can be easily recalled from
gauge kinetic function arguments. However, because (2.33) is of non-perturbative nature
such an expression can be compatible with perturbation theory only if we assume that, at
the weak coupling limit, it matches the superpotentials that are calculated from gaugino
condensation. So let as an example consider a class of models\(^7\) that have gauge group
\(SU(N)\) in the "hidden" sector and involve matter fields with \(M\) families of quarks. If a
gauge singlet \(A\) is present that gives masses to all quarks then the effective superpotential
arising from gaugino condensation can be written as \([52]\)

\[
W = \tilde{h} e^{\frac{2\pi i S}{3N-M}},
\]

(2.35)

where \(\tilde{h}\) depends on moduli other than \(S\). Promoting the BPS expression (2.33) into the
gaugino condensation one (2.35) is then equivalent for the following condition to hold

\[
\frac{3}{3N-M} = -\frac{1}{12}.
\]

(2.36)

A similar condition was found in a different context in \([53]\) where general forms of S-
duality invariant superpotentials were matched with corresponding expressions coming from
gaugino condensation.

Given that at the moment there is not a non-perturbative heterotic string formulation
the issue of existence of BPS non-perturbative generated superpotentials like in (2.33) has
to decided when non-perturbative heterotic calculations are available.

We will now discuss an alternative form for the non-perturbative effective superpotential
where the appearance of the non-perturbative dilaton factor is of field theoretical origin.
Its derivation originates from the use of the effective theory of gaugino condensates. In
this case the non-perturbative generated superpotential exhibits only the leading dilaton
dependence as it originates from the the perturbative gauge kinetic function \(f\) that we have
effectively calculated through eqn.’s (2.17-2.26). The appearance of the non-perturbative
superpotential for gaugino condensation comes after integrating the contribution of the
gaugino composite field out of the effective action and its form will be used in section 5 to
generate contributions to B-terms. It reads

\[
\bullet \quad Z_b - II - b^{SU(3) \times SO(8)} W^{gau} e^{-3S/2b} = \eta^{-2}(T)\left(\frac{1}{3}\right)\eta^{-2}(\frac{U'}{3})(1 - BC (\partial_T \log \eta^2(T)) \times
\]

\(^7\) such issues will be clarified in section four
\[
\left( \partial_U \log \eta^2 \left( \frac{U'}{3} \right) \right) \tilde{W} + \left[ (\eta^{-2}(U')(\eta^{-2}(T)) \frac{1}{3}) \right] (1 - BC \left( (\partial_T \log \eta^2 \left( \frac{T}{3} \right) \right) \times \\
\left( \partial_U \log \eta^2 \left( \frac{U'}{3} \right) \right) \tilde{W} + O((BC)^2),
\] 

(2.37)

where \( S \) is the dilaton and \( b \) the \( \beta \)-function of the condensing gauge group, e.g. for an \( E_8 \) hidden group \( b = -90 \), and \( \tilde{W} \) depends on the moduli of the other planes, e.g. the third invariant complex plane, when there is no cancellation of anomalies by the Green-Schwarz mechanism. For example for the \( Z_6-II-b \) orbifold, if \( T_3 \) is the moduli of the third complex plane then

\[
\tilde{W} = \frac{1}{\eta^2(T_3)}.
\] 

(2.38)

The previous discussion was restricted to small values of the Wilson lines where our \((0,2)\) orbifold goes into \((2,2)\). The grouping of terms in the form presented in (2.26) is our natural choice. In this form \( \mathcal{W} \) the two additive factors in eqn.(2.37) have separately the invariances of the Kähler potential in the linear representation of the dilaton. In other words, candidates for \( \mathcal{W} \) for the \( Z_6-II-b \), or some other heterotic string compactification having the same duality symmetry group, are either the sum of the individual factors in eqn.(2.37) or its factor separately. Specifically, grouping together the first with the third term and the second with the forth term we get the result (2.26). On the other hand, any other regrouping of terms in (2.24) is excluded as a candidate for \( \mathcal{W} \) at it does not have the correct modular weight. Similar results hold for the other orbifolds.

For the \( Z_4-a \) orbifold defined by the action of the complex twist \( \Theta = (i, i, -1) \) on the lattice \( SU(4) \times SU(4) \), the mass operator for the \( \Theta^2 \) subspace is

\[
m^2 = \frac{1}{2T_2U_2} [TU^n^2 + Tn^1 - 2Um_1 + 2m_2]^2.
\] 

(2.39)

and it is invariant under the \( \Gamma_o(2)_T \times SL(2, Z)_U \) target space duality modular group. The spin is

\[
n^T m = 2m_1n^1 + 2m_2n^2.
\] 

(2.40)

\(^8\)Remember, that we have changed the notation from \( U' \) to \( U \).
Considering, as before, a general embedding of the Wilson lines in the gauge degrees of freedom we get

\[
\log \mathcal{M} = \sum_{n^T m = 0, q = 0} \log (2m_2 - 2im_1 U' + in_1 T + n_2 (-U'T)) + BC \sum_{(n,m) \neq (0,0)} \frac{n_2}{(2m_2 - 2im_1 U' + in_1 T - n_2 U'T)} + O((BC)^2).
\] (2.41)

The topological sum constraint \(2m_2n^2 + 2m_1n^1 = 0\) receives contributions from the following sets of integers:

\[
m_2 = r_1 r_2, n_2 = s_1 s_2, m_1 = -r_2 s_1, n_1 = r_1 s_2.
\] (2.42)

Expanding,

\[
\log \mathcal{M} = \sum_{n^T m = 0, q = 0} \log \left( (r_1 + is_1 U)2(r_2 + \frac{T}{2}s_2) \right) + BC \left( \sum' \frac{s_1}{r_1 + is_1 U} \right) \left( \sum'' \frac{s_2}{2(s_2 + i\frac{T}{2}s_2)} \right) + O((BC)^2).
\] (2.43)

Explicitly,

\[
\log \mathcal{M} = \log(\eta^{-2}(U)\frac{1}{2}\eta^{-2}(\frac{T}{2})) - BC \sum' (\partial_U \log(r_1 + is_1 U)) \sum'' (\partial_T \log((r_2 + \frac{T}{2}s_2)) + O((BC)^2),
\] (2.44)

and

\[
\log \mathcal{M} = \log(\eta^{-2}(U)\frac{1}{2}\eta^{-2}(\frac{T}{2})) - BC \left( (\partial_U \log \eta^{-2}(U))(\partial_T \log \eta^{-2}(\frac{T}{2})) \right) + O((BC)^2).
\] (2.45)

Finally, \(\mathcal{W}_\text{pert}\) becomes

- \(Z_4 - a \to SU(4) \times SU(4)\)

\[
\mathcal{W}_\text{pert} = (\eta^{-2}(U)\frac{1}{2}\eta^{-2}(\frac{T}{2}))(1 - 4BC(\partial_U \log \eta(U))(\partial_T \log \eta(\frac{T}{2})) + O((BC)^2)
\] (2.46)

For the \(Z_8 - II - a\) orbifold defined by the action of the complex twist \(\Theta = \exp[2\pi i(1, 3, -4)/8]\) on the torus lattice \(SU(2) \times SO(10)\), the mass operator for the \(\Theta^2\) subspace is given by

\[
m^2 = \frac{1}{2T_2 U_2} |(TU - BC)n^2 + Tn_1 - 2Um_1 + m_2 + l \text{ dep. terms}|^2
\] (2.47)
and it is invariant under the modular group $\Gamma_o(2)_T \times \Gamma_o(2)_U$ when $B = C = l = 0$. The superpotential $\mathcal{W}$ receives contributions from the orbit

$$n^T m = 2m_1n_1 + m_2n_2, \ l = 0$$

(2.48)

The latter can be solved by decomposing it in the two inequivalent orbits

$$m_2 = 2r_1r_2, \ r_2 = s_1s_2, \ m_1 = -r_2s_1, \ n_1 = r_1s_2$$

$m_2 = r_1r_2, \ n_2 = 2s_1s_2, \ m_1 = r_2s_1, \ n_1 = -r_1s_2.$

(2.49)

Summing over the orbits we get that $\mathcal{W}$ is

- $Z_8 - II - a \ {SU(2) \times SO(10) \rightarrow} \mathcal{W} = \left( \frac{1}{2} \eta^{-2}(T) \eta^{-2}(U) \right) \left( 1 - 4BC(\partial_T \log \eta(\frac{T}{2})) \right) \times (\partial_U \log \eta(U)) + \left( \eta^{-2}(T) \eta^{-2}(2U) \right) \left( 1 - 4BC(\partial_T \log \eta^{-2}(T))(\partial_U \log \eta^{-2}(2U)) \right)$

(2.50)

while the non-perturbative generated superpotential coming from gaugino condensation is given by

$$\mathcal{W}^{\text{non-pert}} = e^{3S/2b} \mathcal{W}|_{Z_8-III-a}$$

(2.51)

The same form for $\mathcal{W}$ as in (2.50) can be shown to hold for the non-decomposable $Z_4 - b$ orbifold defined by the action of the complex twist $\Theta = (1, 1, -4)/2$ on the six dimensional torus lattice $SU(4) \times SO(5) \times SU(2)$. The latter has the same modular symmetry group as the $Z_8 - II - a$ when $B = C = 0$.

For the orbifold, listed as $Z_6 - II - c$, and defined by the action of the complex twist $\Theta = [(2, 1, -3)/6]$, on the lattice $SU(3) \times SO(7) \times SU(2)$, we derive $\mathcal{W}$ as

- $Z_6 - II - c \ {SU(3) \times SO(7) \times SU(2) \rightarrow} \mathcal{W} = \left( \frac{1}{3} \eta^{-2}(\frac{T}{3}) \eta^{-2}(U) \right) \left[ 1 - 4BC(\partial_T \log \eta(\frac{T}{3})) \right] \times (\partial_U \log \eta(U)) + \left( \eta^{-2}(T) \eta^{-2}(3U) \right) \left( 1 - 4BC(\partial_T \log \eta(T))(\partial_U \log \eta(3U)) \right).$

(2.52)

The modular group for the latter orbifold is $\Gamma_o(3)_T \times \Gamma_o(3)_U$ when $B = C = 0$. 
3. Non-perturbative Superpotentials from Generalized Coxeter Orbifolds

We will end the discussion of non-perturbative superpotentials by calculating $W$ for generalized Coxeter orbifolds (GCO). We will present the discussion for the CGO $Z_8$ orbifolds, defined\[ by the Coxeter twist $(e^{\pi i/4}, e^{3\pi i/4}, -1)$ on the root lattice of $A_3 \times A_3$. We remind that this orbifold is non-decomposable in the sense that the action on the lattice twist on the $T_6$ torus does not decompose into the orthogonal sum $T_6 = T_2 \oplus T_4$ with the fixed plane lying on $T_2$ torus. For this orbifold, the twist can be equivalently be realized through the generalized Coxeter automorphism $S_1 S_2 S_3 P_{35} P_{36} P_{45}$. In general, the GCO is defined as a product of the Weyl reflections $S_i$ of the simple roots and the outer automorphisms represented by the transposition of the roots. An outer automorphism represented by a transposition which exchange the roots $i \leftrightarrow j$, is denoted by $P_{ij}$ and is a symmetry of the Dynkin diagram. The realization of the point group is generated by

\[
Q = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & -1 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}.
\]

Therefore the metric defined by $g_{ij} = \langle e_i | e_j \rangle$ has three and the antisymmetric tensor $b$ another three deformations. The threshold corrections, topological free energy and effectively the $W$ will depend on the moduli of the unrotated complex plane. That is if the action of the generator of the point group leaves some complex plane unrotated the equations

\[
gQ = Qg, \quad bQ = Qb
\]

This orbifold was included in the classification list of [14]. However, neither its moduli dependent gauge couplings nor its topological free energy were studied in [12]. Here, we calculate its free energy. The moduli dependence of the gauge couplings were treated in [20].
determine the background fields in terms of the independent deformation parameters. Solv-
ing these equations one obtains for the metric

\[
G = \begin{pmatrix}
R^2 & u & v & -u & -2u - R^2 & -u \\
u & R^2 & u & v & -u & -2v - R^2 \\
v & u & R^2 & u & v & -u \\
-2u - R^2 & -u & v & u & R^2 & u \\
-u & -2v - R^2 & -u & v & u & R^2 \\
\end{pmatrix}, \quad (3.3)
\]

with \( R, u, v \in \mathcal{R} \) and the antisymmetric tensor field:

\[
B = \begin{pmatrix}
0 & x & z & y & 0 & -y \\
-x & 0 & x & z & y & 0 \\
-z & -x & 0 & x & z & y \\
-y & -z & -x & 0 & x & z \\
0 & -y & -z & -x & 0 & x \\
y & 0 & -y & -z & -x & 0 \\
\end{pmatrix}. \quad (3.4)
\]

The \( N = 2 \) orbit for these sectors will contain completely unrotated planes, \( \mathcal{O} = (1, \Theta^4), (\theta^4, 1), (\theta^4, \theta^4) \). Consider now the usual parametrization of the \( T^2 \) torus with the \((1,1)\) \( \text{T-modulus} \) and the \((2,1)\) \( \text{U-modulus} \) as,

\[
T = T_1 + iT_2 = 2(b + i \sqrt{\det g_{\perp}}) \\
U = U_1 + iU_2 = \frac{1}{g_{\perp 11}}(g_{\perp 12} + i \sqrt{\det g_{\perp}}). \quad (3.5)
\]

Here, \( g_{\perp} \) is uniquely determined by \( w^4 Gw = (n^1 n^2) g_{\perp} \left( \begin{array}{c} n^1 \\ n^2 \end{array} \right) \). Here, \( b \) is the value of the \( B_{12} \) element of the two-dimensional matrix of the antisymmetric field \( B \). This way one gets

\[
T = 4(x - y) + i8v, \quad (3.6)
\]

\[
U = i. \quad (3.7)
\]

The mass operator in the \((1, \Theta^4)\) untwisted subspace takes the form

\[
m^2 = \frac{1}{2T_2U_2} - TUn_2 + iTn^1 - 2Um_1 + 2m_2|^2 \quad (3.8)
\]
but with the values of $T, U$ given by eqn's (3.6, 3.7). Because, $n^T m = 2(m_1 n_1 + m_2 n_2) + l – \text{terms}$, $W$ takes exactly the same form as in eqn. (2.46) implemented by the exponential dilaton $\propto e^{-3S/2b}$ factor.

Here we can make a comment related to the contribution from the first equation in (2.12) which is relevant to the stringy Higgs effect. Take for example the expansion (2.13). Let’s examine the first orbit corresponding to the sum $\Delta_{0,0} = \sum_{n^T m=1, q=0} \log M$. This orbit is the orbit for which some of the previously massive states, now become massless. At these points $\Delta_{0,0}$ has to exhibit the logarithmic singularity. In principle we could predict, in the simplest case when the Wilson lines have been switched off the form of $\Delta_{0,0}$. The exact form, when it will be calculated has to respect that that the quantity $e^{\Delta_{0,0}}$ has modular weight $-1$ and reflects exactly the presence of the physical singularities of the theory. We will not attempt to calculate this orbit as its sum is largely unknown.

4. Solutions to the $\mu$ Problem

In the previous section, we have calculated contributions to the non-perturbative superpotentials that are coming from the sum over modular orbits of massive untwisted states present in non-decomposable orbifold compactifications. The latter vacua represent $N=1$ $(0, 1)$ heterotic compactifications based on the orbifolds $T^6/Z_\nu$. The non-perturbative superpotentials take the form of eqn. (1.3). The fields $B, C$ represent matter fields. If we identify the matter fields $B, C$ associated with the untwisted complex plane, with the Higgs fields of the minimal supersymmetric standard model the $\mu$-terms are exactly given by eqn. (1.2) without any contributions from higher weight interactions.

The contributions from the first two terms in (1.2) are the standard contributions present in effective supergravity theories irrespectively of the origin of supergravity theory. The tree level expression for the function $H$ in eqn. (1.2) is given by

$$H_{BC} = \frac{1}{(T + \bar{T})(U + \bar{U})}.$$  \hfill (4.1)

Later we will provide the $\tilde{\mu}$ contributions of the third term in (1.2) as its contribution

---

\footnote{This point was not explained in [39] but it is obvious that it corresponds to the superpotential and thus transforming with modular weight $-1$.}
can be read from the non-perturbative superpotential. The \( \tilde{\mu} \) contributions to the \( \mu \)-term can be determined once \( W_{BC} \) is singled out. Let us discuss how the form of the non-perturbative superpotentials calculated on the previous section solves the \( \mu \) problem. The central element of our "proof" will be to justify to presence of an effective \( \mu_{BC}BC \) non-zero mass term after supersymmetry breaking, at low energies, in the observable sector associated with the presence of the \( \tilde{\mu} \)-term in (1.2). Lets us for the moment suppose that \( \tilde{\mu} = 0 \). Let me first take the contributions of the first two terms in eqn. (1.2). Their presence has been discussed in \([7, 6]\). Their origin resides on the presence of the mixing \( H_{BC} \) term in the Kähler potential as in (1.1) e.g
\[
\hat{K} = \ldots + H_{BC}BC; \quad W = W_o.
\]
(4.2)

The effect of this term in the effective theory can be shown to be equivalent to the effect that is coming from the Kähler potential \( K' \) and superpotential \( W' \), in the lowest order of expansion in the matter fields, given by

\[
K' = \hat{K} - H_{BC}BC; \quad W' = W_o e^{H_{BC}BC} \approx W_o (1 + H_{BC}BC)|_{\text{lowest order}},
\]
(4.3)
\[
G = K' + \ln |W'|^2.
\]
(4.4)

That means that the effective theories coming from (4.4) and (1.2) have the same G-function. Especially to the lowest order of expansion in the matter fields the theory with the Higgs mixing term in the Kähler potential, eqn. (1.2) becomes equivalent to the one coming from the expansion of the superpotential in (4.4). If we assume vanishing of the cosmological constant, (4.4) gives rise to the generation of mass terms in the low energy potential that look like as
\[11\]
\[
V \propto m_3^2/2(1 + \kappa)^2 |B|^2 + m_3^2/2(1 + \kappa)^2 |C|^2 + 2m_3^2\kappa(BC + h.c) \propto -\tilde{\mu}^2(BC + h.c),
\]
(4.5)
\[
m_3^2/2 = e^K W_o, \quad \tilde{\mu} = 2m_3^2 e^{K/2} \mu.
\]
(4.6)

The previous potential gives the usual Higgs potential of the supersymmetric standard model when it rewritten in the form
\[
V = \ldots \mu_1^2 |B|^2 + \mu_2^2 |C|^2 + B_\mu m_3^2/2 = e^{K/2} \mu.
\]
(4.7)

\[11\]see for example \([8]\).
That means that the effective $\mu$-term is created in the observable superpotential, that includes MSSM model couplings, from the following expression

$$\mu_{BC}, \quad \mu = <W_o> \equiv H_{BC} = W_{BC}. \quad (4.8)$$

We conclude that the presence of the mixing term $W_{BC}$ in the superpotential of the theory has the same effect in the low energy spectrum of the observable theory as the one coming from the presence of the $H_{BC}$ term in the Kähler potential. They both generate dynamically the same $\mu$-term presence in the effective theory.

Take now for example $W$ found for the $Z_4$ orbifold in eqn. (2.46). Now $W$ takes the form

$$W|_{Z_4 \times Z_4} = W_o(T, U) + \kappa(T, U) W_o(T, U) BC + \mathcal{O}((BC)^2) \quad (4.9)$$

with

$$W_o(T, U) = \frac{1}{2} \eta^{-2}(U) \eta^{-2}(T), \quad \kappa(T, U) = -4(\partial_U \log \eta(U))(\partial_T \log \eta(T)). \quad (4.10)$$

By comparison of (2.46), (1.6) and (4.10) the following identifications follow

$$W = W_o + \kappa W_o BC, \quad \mu \equiv \kappa <W_o>, \quad W = W_o + \mu BC. \quad (4.11)$$

In other words, the form of the string theory superpotential in (4.11) generates naturally the form of the effective superpotential that is needed in order to produce the mixing effect of an effective $\mu$-term in the low energy superpotential of the theory. As we have already said the form of the string theory generated superpotential is consistent with the invariance of the one loop corrected effective action, in the linear representation of the dilaton, under the target space duality transformations that leave the tree level Kähler potential (1.6) invariant.

We should always remember at this point that the condensation superpotential may be used to break supersymmetry at a scale smaller than $M_{Plank}$ and to generate masses to quark fields in the presence of the Higgs fields B, C. Lets us now justify the connection of the non-perturbative superpotentials calculated previously to solutions of the $\mu$-problem. Lets us consider for this reason that the effective theory of light modes is coming from a superstring vacuum which has a YM gauge group $SU(N)$ and in addition $MSU(N)$
"quarkslike" $Q_a$, $a = 1, \ldots, M$ fields that are given mass by the same $SU(N)$ singlet field $A$. The mass term in this case reads

$$W^{pert} = -\sum_{a=1}^{M} AQ_a\bar{Q}_a. \quad (4.12)$$

After applying a standard procedure [57] the non-perturbative superpotential from gaugino condensates (NPS) appears in the form

$$W^{non-pert} \propto (det \Pi)^{1/N} e^{3S/2b}, \quad (4.13)$$

with $\Pi$ a mass matrix defined as

$$det \Pi = A^M. \quad (4.14)$$

Assume now that the coupling $ABCQ_a\bar{Q}_a$ is allowed then the superpotential (4.12) may be promoted to the form [53, 57]

$$W^{pert} = -\sum_{a=1}^{M} A(1 + \kappa' BC)Q_a\bar{Q}_a, \quad (4.15)$$

Comparing (4.13, 4.14, 4.15) we derive

$$W^{non-pert} \propto (det \Pi')^{1/N} e^{3S/2b}, \quad (4.16)$$

where the mass matrix $\Pi'$ is given by

$$det \Pi' = A^M(1 + \kappa' BC)^M. \quad (4.17)$$

The usefulness of the non-perturbative superpotential found for the $Z_4-b N = 1$ orbifold relies on the fact that when for a particular vacuum the coupling $ABCQ_a\bar{Q}_a$ is allowed, signalling the presence of a non-renormalizable term, then we can identify

$$W^{non-pert} \approx A^N(1 + \kappa' M/N BC)e^{3S/2b} \equiv W^{non-pert}|_{Z_4-b}. \quad (4.18)$$

Comparison between (4.18) and (4.10) for the $Z_4-b$ suggests the identification

$$A^N = W_o(T, U), \quad \kappa' M/N = \kappa(T, U), \quad (4.19)$$

so that the "hidden" field $A$ is a function of the moduli fields $T$, $U$. 
We also note that the results \((4.18, 4.19)\) presuppose in their derivation the fact that at the limit \(S \to \infty\) and \(M_{\text{Planck}} \to \infty\) the NPS superpotentials fix the dilaton at its acceptable weak coupling value and are given by a truncation procedure that identifies them with the superpotentials appearing in the globally supersymmetric QCD in the form \((4.13)\).

For the orbifold \(Z_6 - II - b\) from \((2.37)\) the \(\tilde{\mu}\) contributions to the \(\mu\) term read

\[
\begin{align*}
\bullet Z_6 - II - b^{SU(3) \times SO(8)} & \to \tilde{\mu} e^{-G/2} e^{-3S/2b} = -\frac{1}{2} \eta^{-2}(\frac{T}{2}) \eta^{-2}(U)(\partial_T \log \eta^2(T)) (\partial_U \log \eta^2(U)) , \\
& \quad \times \left[ \eta^2 \left( \frac{U}{3} \right) \right] \tilde{W} + \left[ \eta^{-2}(U) \eta^{-2} \left( \frac{T}{3} \right) \right] \left( (\partial_T \log \eta^2(T)) \times \right.
& \left. (\partial_U \log \eta^2 \left( \frac{U}{3} \right)) \right] \tilde{W} + \mathcal{O}((BC)^2).
\end{align*}
\]

As we have said each of the factors in eqn. \((4.20)\) can be used as a possible \(\tilde{\mu}\)-term contribution. The latter forms exclude the ansatz for the \(\tilde{\mu}\)-term used in \([42]\) in the context of CP violation.

The expressions of the \(\tilde{\mu}\) terms for the classes of non-decomposable orbifolds of the orbifolds appearing in sections 2 and 3 may be summarized as

\[
\begin{align*}
\bullet Z_8 | A_3 \times A_3 & \to \tilde{\mu} e^{-3S/2b} e^{-G/2} = -\frac{1}{2} \eta^{-2}(\frac{T}{2}) \eta^{-2}(U) \left( \partial_T \log \eta^2(\frac{T}{2}) \right) (\partial_U \log \eta^2(U)) , \\
& \quad \times \left[ \eta^2 \left( \frac{U}{3} \right) \right] \tilde{W} + \left[ \eta^{-2}(U) \eta^{-2} \left( \frac{T}{3} \right) \right] \left( (\partial_T \log \eta^2(T)) \times \right.
& \left. (\partial_U \log \eta^2 \left( \frac{U}{3} \right)) \right] \tilde{W} + \mathcal{O}((BC)^2).
\end{align*}
\]

\[
\begin{align*}
\bullet Z_4 - a^{SU(4) \times SU(4)} & \text{ the same value of } \tilde{\mu}\text{-term as the } Z_8 | A_3 \times A_3 \\
\bullet Z_4 - b^{SU(4) \times SO(5) \times SU(2)} & \to \tilde{\mu} e^{-3S/2b} e^{-G/2} = -\frac{1}{2} \eta^{-2} \left( \frac{T}{2} \right) \times \eta^{-2}(U) \left( \partial_T \log \eta^2 \left( \frac{T}{2} \right) \right) \\
& \quad \times \left( \partial_U \log \eta^2(U) \right) - \eta^{-2}(T) \eta^{-2}(2U) \left( (\partial_T \log \eta^2(T)) (\partial_U \log \eta^2(2U)) \right) , \\
& \quad \times \left[ \eta^2 \left( \frac{U}{3} \right) \right] \tilde{W} + \left[ \eta^{-2}(U) \eta^{-2} \left( \frac{T}{3} \right) \right] \left( (\partial_T \log \eta^2(T)) \times \right.
& \left. (\partial_U \log \eta^2 \left( \frac{U}{3} \right)) \right] \tilde{W} + \mathcal{O}((BC)^2).
\end{align*}
\]

\[
\begin{align*}
\bullet Z_8 - II - a^{SU(2) \times SO(10)}; \text{ the same value of } \tilde{\mu}\text{-term as the } Z_4 - b^{SU(4) \times SO(5) \times SU(2)} \\
\bullet Z_8 - II - c^{SU(3) \times SO(7) \times SU(2)} \to \tilde{\mu} e^{-3S/2b} e^{-G/2} = -\frac{1}{2} \eta^{-2} \left( \frac{T}{3} \right) \eta^{-2}(U) \left( \partial_T \log \eta^2 \left( \frac{T}{3} \right) \right) \\
& \quad \times \left( \partial_U \log \eta^2(U) \right) - \eta^{-2}(T) \eta^{-2}(3U) \left( (\partial_T \log \eta^2(T)) (\partial_U \log \eta^2(3U)) \right) , \\
& \quad \times \left[ \eta^2 \left( \frac{U}{3} \right) \right] \tilde{W} + \left[ \eta^{-2}(U) \eta^{-2} \left( \frac{T}{3} \right) \right] \left( (\partial_T \log \eta^2(T)) \times \right.
& \left. (\partial_U \log \eta^2 \left( \frac{U}{3} \right)) \right] \tilde{W} + \mathcal{O}((BC)^2).
\end{align*}
\]
5. The $\mu$-terms in M-theory

- Different scenarios

The purpose of this section is to examine some phenomenological consequences of our approach and in particular to show in which way it is possible to convert our $\mu$-terms solutions of the previous section to their M-theory equivalents. In the context of M-theory Horava and Witten\cite{HoravaWitten} proposed that the strong coupling limit of the $E_8 \times E_8$ heterotic string is described by eleven dimensional supergravity on a manifold with a boundary. The two $E_8$'s, "considered" as the observable and hidden sectors of the early supergravity theories, live on the opposite ends of the squashed $K_3$ surface, the line segment $S^1/Z_2$. Further compactification of M-theory to four dimensions revealed a number of interesting features including computation of Kähler potential \cite{KahlerPotential}, and supersymmetry breaking terms \cite{SupersymmetryBreaking}. The low energy supergravity which describes the theory in four dimensions has been discussed in several works \cite{LowEnergySupergravity}, while soft supersymmetry breaking terms like gaugino masses $M_{1/2}$, scalar masses $m_0$, and A-terms have been calculated in \cite{SoftSupersymmetryBreaking}. We are particularly interested in the B-soft supersymmetry breaking term. The latter depends on the details of the mechanism that generates the $\mu$-terms. Because of the non-perturbative nature of our solutions we will further assume, initially, that the B-soft term is the result of non-perturbative $\mu$-term generation.

- M-theory B-terms flowing to $N = 1$ orbifolds

We are interested to calculate the value of the M-theory B-term that its weakly coupled limit flows to the limit of B-terms associated with the $N = 1$ orbifolds \cite{MtheoryBTerm}. In this case the M-theory B-term reads\footnote{Note that a different form of the B-soft term was appeared in \cite{DifferentFormBSoft}. We disagree with the form of the B-term presented in this work.}

$$B_\mu = \left( -3 \tilde{C} \cos \theta e^{-i \alpha_S} - \sqrt{3} \tilde{C} \sin \theta + \frac{6 \tilde{C} \cos \theta (S + \bar{S})}{3(S + S) + \alpha(T + \bar{T})} + \frac{2 \sqrt{3} \tilde{C} \sin \theta \alpha(T + \bar{T})}{3(S + S) + \alpha(T + \bar{T})} - 1 \right) \frac{\partial S}{\partial \mu_M} + \frac{\partial T}{\partial \mu_M},$$

(5.1)
where

\[ F^S = \sqrt{3} m_{3/2} \bar{C} (S + \bar{S}) \sin \theta, \quad F^T = m_{3/2} \bar{C} (T + \bar{T}) \cos \theta, \quad \bar{C} = 1 + \frac{V_o}{3m_{3/2}}, \quad (5.2) \]

\( V_o \) the value of the cosmological constant and \( F^S, F^T \) the dilaton and moduli auxiliary fields. In addition, we have used the Kähler function \( K \) of M-theory

\[ K = - \ln (S + \bar{S}) - 3 \ln (T + \bar{T}) + \left( \frac{3}{T + T} + \frac{\alpha}{S + S} \right) |C|^2 \quad (5.3) \]

\[ f_{E_6} = S + \alpha T, \quad f_{E_8} = S - \alpha T, \quad W = d_{pq} C^p C^q C^r, \quad (5.4) \]

the gauge kinetic functions \( f \) for the observable and hidden sector \((2,2)\) model, and finally the perturbative superpotential \( W \). The latter is a function of the matter fields \( C \). In the derivation of eqn. (5.1) we have used the relation

\[ B^{M-theory}_\mu = m_{3/2} \left[ -1 + \bar{C} \sqrt{3} \sin \theta (K_o^S)^{-1/2} \left( K_o^S + \frac{\mu S}{\mu} - \frac{\tilde{K}_o^S}{K_C} - \frac{\tilde{K}_o^S}{K_C} \right) \right. \]
\[ \left. \bar{C} \sqrt{3} \cos \theta (K_o^T)^{-1/2} \left( K_o^T + \frac{\mu T}{\mu} - \frac{\tilde{K}_o^T}{K_C} - \frac{\tilde{K}_o^T}{K_C} \right) \right], \quad (5.5) \]

which is a generalization of the standard relations existing for the B-terms of \( N = 1 \) four dimensional orbifolds [33]. In this way, even if the relation between B-term and \( \mu \)-term is fixed in supergravity it is made compatible to include changes in the moduli metrics. Note that by \( K_o \) we denote that part of the Kähler potential containing the moduli metrics while \( \tilde{K} \) is the part involving matter fields.

The B-soft term involving the \( F^i \partial_i \ln \mu \) term has modular weight \(-1\) as it should. In this form it can be easily proved that the M-theory B-soft term eqn.(5.1) flows into its weakly coupled heterotic limit, e.g by taking appropriate limits in eqn.(2.19) of [33], when \( \alpha(T + T) <\ll (S + S) \). At the latter limit the B and the rest of the soft terms flow to their large Calabi-Yau limit equivalent to the blow up of twisted moduli fields of abelian \((2,2)\) orbifold compactifications counterparts [33]. The form of the M-theory \( \mu \)-term in (5.1) may be replaced by the heterotic \( \mu \)-terms, of sect. 4, only when

\[ \text{Re}(S) >> \left[ 4\pi^2 \text{Re}(T) \right]^3, \quad \text{Re}(T) >> \frac{1}{4\pi^2} \quad (5.6) \]

\[ ^{13} \text{we see in the following that this form of B-term is compatible with the M-theory limit of certain decomposable orbifold compactifications of the heterotic string} \]
When (5.6) holds we are in the heterotic limit so that starting with our heterotic string \( \mu \)-term solution of sect. 4 and by varying \( \text{Re}(S) \) while keeping \( \text{Re}(T) \) fixed we can extrapolate smoothly from heterotic string to M-theory [44]. We should note that retaining the weak perturbative properties of our theory demands in addition that 
\[
(S + \bar{S}) + \alpha(T + \bar{T}) \approx 4 \quad \text{or} \quad 0 < \alpha(T + \bar{T}) \leq 2.
\]

We shall make a comment at this point regarding the validity of the effective theory coming from the specific compactification of the M-theory [40] that gives us the quantities in (5.4) and its compatibility with the existing \( N = 1 \) orbifold compactifications of the \( N = 1 \) heterotic string [24]. Remember that the three \( T_i, \ i = 1, 2, 3 \) moduli are all considered in the same footing in (5.4) i.e \( T_1 = T_2 = T_3 = T \). Because the definition of the \( \mu \)-term in (5.1) involves at least one \( U \) moduli it is valid for those B-terms that flow, at their weakly coupled limit, to the \( N = 1 \) orbifolds having at least one \( U \)-moduli in the e.g third complex plane. The latter orbifolds include the \( Z_N \) one’s based on \( Z_4, Z_6, Z_8, Z_{12}' \) and the \( Z_N \times Z_M \) one’s based on \( Z_2 \times Z_4 \) and \( Z_2 \times Z_6 \).

Take for example the \( Z_2 \times Z_6 \) orbifold. This orbifold has three \( N = 2 \) \( T_1, T_2, T_3 \) and one \( U_3 \) moduli. The non-perturbative superpotential associated with the \( T_3, U_3 \) moduli is given by

\[
W_{Z_2 \times Z_6}^{\text{non-pert}} = e^{-3S/2b} \eta^{-2}(U_3) \eta^{-2}(T_3) \left( 1 - AB(\partial_{T_3} \ln(\eta^2(T_3))) \right) \left( \partial_{U_3} \ln(\eta^2(U_3)) \right).
\] (5.7)

Lifting this \( \mu \)-term, where both \( T, U \) moduli are involved, to the one coming from M-theory compactification might look unnatural as the compactification limit of M-theory \( \mu \)-term in eqn. (5.1) involves only the \( T \)-moduli. However, at the large \( T \) limit, and assuming that the contribution of the auxiliary field \( U \) to supersymmetry breaking is very small, the contributions of the \( T, U \) moduli are expected to be decoupled so that we can safely use the value of (5.7) in (5.4). In this case the Kähler potential in eqn. (5.4) may be safely completed by the addition of the \(- \ln(U + \bar{U})\) term. In addition because of the special nature of solutions for the Kähler potential [46] we must take the \( B \to C \) limit in (5.7).

When the last considerations are taken into account the solutions for the \( \mu \)-terms in the previous section may be lifted to its M-theory counterparts. If the \( T \)-moduli prove to be associated only to four dimensional \( N = 2 \) orbifold moduli then the B-terms correspond to \( Z_2 \times Z_4 \) and \( Z_2 \times Z_6 \) \( N = 1 \) orbifolds.
The \(\mu\)-term dependence of the B-term in (5.1) can be expressed in a general form in terms of \(F^T\) and \(F^S\) as

\[
\left( -3\tilde{C}\cos\theta e^{-i\alpha} - \sqrt{3}\tilde{C}\sin\theta + \frac{6\tilde{C}\cos(S + \bar{S})}{3(S + \bar{S}) + \alpha(T + \bar{T})} + \frac{2\sqrt{3}\tilde{C}\sin\theta\alpha(T + \bar{T})}{3(S + \bar{S}) + \alpha(T + \bar{T})} - 1 \right) \\
+ F^S\left(\frac{3}{2b}\right) + \frac{1}{2}\left(\frac{1}{(S + \bar{S})^2m_{3/2}}\right)F^2 + \frac{1}{2}\left(\frac{1}{(S + \bar{S})^2m_{3/2}}\right)F^2_T + F_T m_{3/2} e^{\frac{3\epsilon}{2} \partial_T \mu(T, U)} \right),
\]

(5.8)

where we have parametrized the \(\mu\)-term solutions of the previous section as

\[
\mu = e^{G/2} e^{\frac{3\epsilon}{2} \hat{\mu}(T, U)} = e^{G/2} e^{\frac{3\epsilon}{2} \mathcal{W}_{BC}}
\]

(5.9)

and have assumed (5.2) and the following parametrization of the dilaton and T-moduli auxiliary fields

\[
F^S = e^{G/2} G_{S}^{-1}, \quad F^T = e^{G/2} G_{TT}^{-1}.
\]

(5.10)

For example for the M-theory compactifications flowing to the non-decomposable \(Z_8\) -IIa \(N = 1\) orbifold the previous discussion applies with

\[
\hat{\mu}(T, U) = -\eta'(T) \eta(T)^{-2}(U)^{-2}(T)\eta^{-2}(U)(\partial_T \log \eta^2(T))\partial_U \log \eta^2(U)
\]

\[
-\frac{1}{2}\eta^{-2}(U)^2(\partial_T^2 \log \eta^2(T))\partial_U \log \eta^2(U)
\]

\[
+ 2\eta'(T) \eta(T)^{-2}(2U)(\partial_T \log \eta^2(T))\partial_U \log \eta^2(2U)
\]

\[
- [\eta^{-2}(T)\eta^{-2}(2U)](\partial_T^2 \log \eta^2(T))\partial_U \log \eta^2(2U).
\]

(5.11)

6. Conclusions

In the context of heterotic string theory we calculated 4-dimensional non-perturbative superpotentials \(\mathcal{W}\). The dependence of the vector T, U moduli arises from integrating out the massive compactification modes by summing over their modular orbits. Our calculation used the relation between the topological free energy and non-perturbative superpotentials [20] to perform this task. The inclusion of the non-perturbative factor \(e^S\) for the dilaton was performed either from using the gaugino condensation [55, 56, 57, 58] in the hidden \(E_8\) sector or using the BPS formula. In the literature [39] the same calculation that utilizes
the sum over the perturbative modular orbits have been performed for the orbifold models invariant under the target space duality group $SL(2, Z)_T \times SL(2, Z)_U$. We extended the previous result, by applying the technique of performing the sum over modular orbits of the chiral compactification modes, to the more general classes of non-decomposable $N = 1$ orbifolds. The latter string vacua exhibit target space duality groups which are subgroups of the modular group $PSL(2, Z)$. These solutions were later used to calculate solutions to the $\mu$-term problem. The form of the $\mu$ term that we have proposed can be used to test observable CP violation effects in non-decomposable orbifold compactifications of the heterotic string in the spirit suggested in [40, 41]. The form of the $\mu$-terms associated with the non-decomposable orbifolds was used to examine the value of the B-soft parameter term obtained from compactifications of M-theory to four dimensions. Here we impose the condition that at the weakly coupled limit the B-theory M-terms flow to their 4D orbifold counterparts.

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