Probing the high-density behavior of nuclear symmetry energy with high-energy radioactive beams

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Central collisions induced by high energy radioactive beams can be used as a novel means to obtain crucial information about the high density (HD) behavior of nuclear symmetry energy. This information is critical for understanding several key issues in astrophysics. Within an isospin-dependent hadronic transport model using phenomenological equations of state (EOS) for dense neutron-rich matter, we investigate several experimental probes of the HD behavior of nuclear symmetry energy, such as, the $\pi^-$ to $\pi^+$ ratio, neutron-proton differential flow and its excitation function. Measurements of these observables will provide the first terrestrial data to constrain stringently the HD behavior of nuclear symmetry energy and thus also the EOS of dense neutron-rich matter.

1. INTRODUCTION

Nuclear reactions using rare isotopes has opened up several new frontiers in nuclear sciences\cite{1-3}. In particular, high energy heavy rare isotopes to be available at the future Rare Isotope Accelerator (RIA) and the new GSI accelerator facility provide a unique opportunity to explore novel properties of dense neutron-rich matter that was not in reach in terrestrial laboratories before. This exploration will reveal crucial information about the EOS of neutron-rich matter. To understand the latter and its astrophysical implications, such as, the origin of elements, structure of rare isotopes and properties of neutron stars are presently among the most important goals of nuclear sciences. The EOS of neutron-rich matter of isospin asymmetry $\delta \equiv (\rho_n - \rho_p)/(\rho_n + \rho_p)$ can be written as

$$e(\rho, \delta) = e(\rho, 0) + E_{\text{sym}}(\rho)\delta^2$$  \hspace{1cm} (1)

within the parabolic approximation (see e.g., \cite{4}), where $e(\rho, 0)$ is the energy per nucleon in isospin symmetric nuclear matter and $E_{\text{sym}}(\rho)$ is the symmetry energy at density $\rho$. To study the $E_{\text{sym}}(\rho)$ has been a longstanding goal of extensive research with various microscopic and/or phenomenological models over the last few decades. The predicted results on the density dependence, especially at high densities, are extremely diverse and often contradictory. Theoretical results can be roughly classified into two groups, i.e., a group where the $E_{\text{sym}}(\rho)$ rises monotonously and one in which it falls with the increasing

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density above about twice the normal nuclear matter density. Above $3\rho_0$, even the sign of the symmetry energy is unclear. The fundamental cause of the extremely uncertain HD behaviour of $E_{\text{sym}}(\rho)$ is the complete lack of terrestrial laboratory data to constrain directly the model predictions. Recently, several promising observables were proposed to extract the HD behavior of the symmetry energy using high energy radioactive beams[5].

![Graph showing isospin asymmetry-density correlations](image)

Figure 1. Upper window: the isospin asymmetry-density correlations at t=20 fm/c and $E_{\text{beam}}/A = 400$ MeV in the central $^{132}\text{Sn} + ^{124}\text{Sn}$ reaction with the nuclear symmetry energy $E_{\text{sym}}^a$ and $E_{\text{sym}}^b$, respectively. Lower window: the same correlation as in the upper window but at 10 fm/c and $E_{\text{beam}}/A = 2$ GeV/nucleon. The corresponding correlation in neutron stars is shown in the insert.

2. CORRELATION BETWEEN DENSITY AND ISOSPIN ASYMMETRY IN NEUTRON STARS AND HEAVY-ION COLLISIONS

In a neutron star, several of its main properties are affected significantly by the correlation between the density and the proton fraction $x_\beta$ at $\beta$ equilibrium. The latter is determined by[6]

$$hc(3\pi^2 p x_\beta)^{1/3} = 4E_{\text{sym}}(\rho)(1 - 2x_\beta).$$

(2)

The equilibrium proton fraction is therefore entirely determined by the $E_{\text{sym}}(\rho)$. We use the following two representatives of the symmetry energy

$$E_{\text{sym}}^a(\rho) \equiv E_{\text{sym}}(\rho_0)u \quad \text{and} \quad E_{\text{sym}}^b(\rho) \equiv E_{\text{sym}}(\rho_0)u \cdot \frac{3 - u}{2},$$

(3)
where $u \equiv \rho/\rho_0$. The values of $\delta_\beta = 1 - 2x_\beta$ corresponding to these two forms of symmetry energy are shown in the insert of Fig. 1. With the $E_{\text{sym}}^a(\rho)$, the $\delta_\beta$ is 1 for $\rho/\rho_0 \geq 3$, indicating that the neutron star has become a pure neutron matter at these high densities. On the contrary, with the $E_{\text{sym}}^b(\rho)$, the neutron star becomes more proton-rich as the density increases. To see the connection between the physics governing properties of both neutron stars and heavy-ion collisions, we show in Fig. 1 the correlation between density $\rho$ and the isospin asymmetry $\delta_{\text{like}}$ (effective isospin asymmetry taking into account different charge states of $\Delta(1232)$ resonances) over the entire reaction volume at the time of about the maximum compression in the central $^{132}\text{Sn} + ^{124}\text{Sn}$ reactions with $E/A=400$ (upper window) and 2000 (bottom window) MeV/nucleon, respectively. These results were obtained using an isospin-dependent hadronic transport model[7]. The overall rise of $\delta$ at low densities is mainly due to the neutron skins of the colliding nuclei and the distillated neutrons. Effects due to the different symmetry energies are more clearly distillated neutrons. Of course, this is no surprise since the same underlying nuclear EOS is at work in both cases. The decreasing $E_{\text{sym}}^b(\rho)$ above $1.5\rho_0$ makes it more energetically favorable to have the denser region more neutron-rich in both neutron stars and heavy-ion collisions.

3. $\pi^-/\pi^+$ PROBE OF THE $E_{\text{sym}}(\rho)$

It is well known that the $\pi^-/\pi^+$ ratio in heavy-ion collisions depends strongly on the isospin asymmetry of the reaction system. It is also qualitatively easy to understand why this dependence can be used to extract crucial information about the EOS of neutron-rich matter. On one hand, within the $\Delta$ resonance model for pion production from first-chance independent nucleon-nucleon collisions[15], the primordial $\pi^-/\pi^+$ ratio is $(5N^2 + NZ)/(5Z^2 + NZ) \approx (N/Z)^2$. It is thus a direct measure of the isospin asymmetry $(N/Z)_{\text{dense}}$ of the dense matter in the participant region of heavy-ion collisions. It was shown that the $(N/Z)_{\text{dense}}$ is uniquely determined by the high density behaviour of the nuclear symmetry energy[5]. Therefore, the $\pi^-/\pi^+$ ratio can be used to probe sensitively the EOS of neutron-rich matter. On the other hand, within the statistical model for pion production[16], the $\pi^-/\pi^+$ ratio is proportional to $\exp \left[ (\mu_n - \mu_p)/T \right]$, where $T$ is the temperature, $\mu_n$ and $\mu_p$ are the chemical potentials of neutrons and protons, respectively. At modestly high temperatures ($T \geq 4$ MeV), the difference in the neutron and proton chemical potentials can be written as[17]

$$\mu_n - \mu_p = V_{\text{asy}}^n - V_{\text{asy}}^p - V_{\text{Coulomb}} + T \left[ \ln \frac{\rho_n}{\rho_p} + \sum_m \frac{m+1}{m} b_m (\frac{\lambda_\pi}{2})^m (\rho_n^m - \rho_p^m) \right],$$

where $V_{\text{Coulomb}}$ is the Coulomb potential for protons, $\lambda_\pi$ is the thermal wavelength of a nucleon and $b'_m$'s are the inversion coefficients of the Fermi distribution function. The difference in neutron and proton symmetry potentials $V_{\text{asy}}^n - V_{\text{asy}}^p = 2v_{\text{asy}}(\rho)\delta$, where the function $v_{\text{asy}}(\rho)$ is completely determined by the density-dependence of the symmetry energy[5]. It is seen that the kinetic part of the difference $\mu_n - \mu_p$ relates directly to the
isospin asymmetry $\rho_n/\rho_p$ or $\rho_n - \rho_p$. Thus within the statistical model too, the $\pi^-/\pi^+$ ratio is sensitive to the $(N/Z)_{\text{dense}}$. Moreover, the value of $\pi^-/\pi^+$ ratio is affected by the competition of the symmetry and Coulomb potentials which all depend on the isospin asymmetry of the reaction system. At relatively low temperatures there is a good chance to extract crucial information about the symmetry potential.

![Graph](image)

**Figure 2.** The evolution of neutron/proton ratio of dense region (left) and $\pi^-/\pi^+$ ratio (right) in the central $^{132}\text{Sn} + ^{124}\text{Sn}$ reactions at $E_{\text{beam}}/A =$ 200, 400 and 1000 MeV.

Shown in Fig. 2 are the isospin asymmetry of the high density region $(n/p)_{\rho \geq \rho_0}$ (left) and the $(\pi^-/\pi^+)_\text{like}$ ratio (right)

$$(\pi^-/\pi^+)_\text{like} \equiv \frac{\pi^- + \Delta^- + \frac{1}{3}\Delta^0}{\pi^+ + \Delta^+ + \frac{1}{3}\Delta^+}$$

(5)

as a function of time for the central $^{132}\text{Sn} + ^{124}\text{Sn}$ reaction at beam energies from 200 to 1000 MeV/nucleon. This ratio naturally becomes the final $\pi^-/\pi^+$ ratio at the freeze-out when the reaction time $t$ is much longer than the lifetime of the delta resonance $\tau_\Delta$. The $(\pi^-/\pi^+)_\text{like}$ ratio is rather high in the early stage of the reaction because of the large numbers of neutron-neutron scatterings near the surfaces where the neutron skins of the colliding nuclei overlap. It is seen that a variation of about 30% in the $(n/p)_{\rho \geq \rho_0}$ due to the different $E_{\text{sym}}(\rho)$ results in about 15% change in the final $\pi^-/\pi^+$ ratio. It has thus an appreciable response factor of about 0.5 to the variation of HD n/p ratio and is approximately beam energy independent. Therefore, the $(\pi^-/\pi^+)_\text{like}$ ratio is a sensitive probe of the HD behaviour of nuclear symmetry energy.
Figure 3. The neutron-proton differential collective flow in the mid-central $^{132}$Sn+$^{124}$Sn reactions at $E_{beam}/A = 400$ MeV (upper window) and the excitation function of flow parameter (lower window) with the nuclear symmetry energy $E^{a}_{sym}$ and $E^{b}_{sym}$, respectively.

4. NEUTRON-PROTON DIFFERENTIAL FLOW PROBE OF THE $E_{sym}(\rho)$

The neutron-proton differential collective flow is measured by\cite{21}

$$F_{np}(y) \equiv \frac{1}{N(y)} \sum_{i=1}^{N(y)} p_{x_i} \tau_i,$$ \hspace{1cm} (6)

where $N(y)$ is the total number of free nucleons at the rapidity $y$, $p_{x_i}$ is the transverse momentum of particle $i$ in the reaction plane, and $\tau_i$ is $+1$ and $-1$ for neutrons and protons, respectively. The free nucleons are identified as those having local nucleon densities less than $1/8\rho_0$. The $F_{np}(y)$ combines constructively the in-plane transverse momenta generated by the isovector potentials while reducing significantly influences of the isoscalar potentials of both neutrons and protons. Thus, it can reveal more directly the HD behaviour of $E_{sym}(\rho)$ in high energy heavy-ion collisions. A typical analysis for mid-central $^{132}$Sn+$^{124}$Sn reactions at 400 MeV/nucleon is made in the upper window of Fig. 3. A clear signature of the HD behaviour of $E_{sym}(\rho)$ appears at both forward and backward rapidities. The slope $dF_{np}/d(y_{cm}/y_{beam})$ at mid-rapidity is different by about a factor of 2 in reactions at $E_{beam} \geq 200$ MeV/nucleon as shown in the lower window of Fig. 3. This large effect can be easily observed by using available detectors at several heavy-ion facilities in the world.
5. SUMMARY

In summary, the HD behaviour of nuclear symmetry energy has been puzzling physicists for decades. In this work, central collisions induced by high energy radioactive beams are proposed as a novel means to solve this longstanding problem. Within an isospin dependent hadronic transport model using two representative density dependent symmetry energy functions predicted by many body theories, several experimental probes for the HD symmetry energy are studied. Among them, the $\pi^-/\pi^+$ ratio, neutron-proton differential collective flow and its excitation function are found most promising. Future comparisons between the experimental data and model calculations will constrain stringently the HD behaviour of nuclear symmetry energy.

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