Higher bottom and bottom-strange mesons

Yuan Sun$^{1,2}$, Qin-Tao Song$^{1,3,6}$, Dian-Yong Chen$^{1,3}$, Xiang Liu$^{1,2,*}$ and Shi-Lin Zhu$^{4,5,**}$

$^1$Research Center for Hadron and CSR Physics, Lanzhou University & Institute of Modern Physics of CAS, Lanzhou 730000, China
$^2$School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China
$^3$Nuclear Theory Group, Institute of Modern Physics of CAS, Lanzhou 730000, China
$^4$Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
$^5$Collaborative Innovation Center of Quantum Matter, Beijing 100871, China
$^6$University of Chinese Academy of Sciences, Beijing 100049, China

Motivated by the recent observation of the orbital excitation $B(5970)$ by the CDF Collaboration, we have performed a systematic study of the mass spectrum and strong decay patterns of the higher $B$ and $B_s$ mesons. Hopefully the present investigation may provide valuable clues to further experimental exploration of these intriguing excited heavy mesons.

PACS numbers: 14.40.Nd, 12.38.Lg, 13.25.Hw

I. INTRODUCTION

The past decade has witnessed the discovery of many charmed and charmed-strange states such as $D_{sJ}(2317)$ [1–3], $D_s(2460)$ [2,3], $D_{sJ}(2632)$ [4], $D_sJ(2860)$ [5], $D_{sJ}(2715)$ [6], $D_{sJ}(3040)$ [7], $D(2550)$ [8]/$D_{sJ}(2580)$ [9], $D^*(2600)$ [8]/$D_{sJ}(2650)$ [9], $D(2750)$ [8]/$D_s(2740)$ [9], $D^*(2760)$ [8]/$D_{sJ}(2760)$ [9], and $D_{sJ}^+(3000)$ [9], which have not only enriched the family of the charmed and charmed-strange states, but also stimulated extensive discussions of their properties (see Ref. [10] for a mini review of the research status of these newly observed charmed and charmed-strange states).

The present situation of the experimental exploration of bottom and bottom-strange states is strikingly similar to that of charmed and charmed-strange states in 2003; i.e., some candidates for the P-wave bottom and bottom-strange meson were announced [11–19] while the radially excited states seem within reach [20]. Very recently, the CDF Collaboration studied the orbitally excited $B$ mesons, and reported that the new $B(5790)$ state could be the radially excited state in the bottom meson family [20]. There are several theoretical studies of the bottom and bottom-strange mesons before [21, 22] and after [23, 24] the experimental observation of these states.

Now is a good time to carry out a comprehensive theoretical study on higher bottom and bottom-strange mesons. In this work, we will calculate the mass spectrum of higher bottom and bottom-strange mesons and the corresponding two-body strong decay behavior. We hope the present investigation may not only shed light on the properties of the observed bottom and bottom-strange states, but also provide valuable clues to further experimental exploration of the radially and orbitally excited bottom and bottom-strange states.

This paper is organized as follows. After the Introduction, we present the analysis of the mass spectrum of the bottom and bottom-strange meson family in comparison with the available experimental data and other theoretical results. In Sec. III, we discuss the two-body strong decay behavior of the higher bottom and bottom-strange mesons. The last section is devoted to the discussion and Conclusion.

II. THE MASS SPECTRUM

We first calculate the mass spectrum of the higher bottom and bottom-strange mesons in the framework of the relativistic quark model [25], where the total Hamiltonian $\hat{H}_1$ describes the interaction between quark and anti-quark in the meson

$$\hat{H}_1 = \left(p^2 + m_1^2\right)^{1/2} + \left(p^2 + m_2^2\right)^{1/2} + \hat{H}_{12}^{\text{conf}} + \hat{H}_{12}^{\text{so}} + \hat{H}_{12}^{\text{byp}},$$

(1)

where $\hat{H}_{12}^{\text{conf}}$ denotes the confinement term and $\hat{H}_{12}^{\text{so}}$ is the spin-orbit term which can be decomposed into the symmetric part $\hat{H}_{12}^{\text{so}}$ and the antisymmetric part $\hat{H}_{12}^{\text{so}}$. In addition, $\hat{H}_{12}^{\text{byp}}$ is the sum of the tensor and contact terms, i.e.,

$$\hat{H}_{12}^{\text{byp}} = \hat{H}_{12}^{\text{tens}} + \hat{H}_{12}^{\text{c}},$$

(2)

The concrete forms of these terms can be found in Appendix A of Ref. [25].

In the bases $|nL_J\rangle$, the antisymmetric part of the spin-orbit term $\hat{H}_{12}^{\text{so}}$ and the tensor term $\hat{H}_{12}^{\text{tens}}$ have nonvanishing off-diagonal elements, which result in the mixing of the states with quantum numbers $^3L_J$ and $^1L_J$ or with $^3(L \pm 2)$J. Thus, the total Hamiltonian $\hat{H}_1$ can be divided into two parts in this bases, which include the diagonal part $H_{\text{diag}}$ and off-diagonal part $H_{\text{off}}$ with the form

$$H_{\text{off}} = \hat{H}_{12}^{\text{so}} + \hat{H}_{12}^{\text{tens}}.$$

(3)

where $\hat{H}_{12}^{\text{tens}}$ denotes the off-diagonal part of $\hat{H}_{12}^{\text{tens}}$. In the following, we first diagonalize $H_{\text{diag}}$ in the simple harmonic oscillator bases and obtain the eigenvalues and eigenvectors corresponding to the wave function of the meson. One also needs to diagonalize the off-diagonal part $H_{\text{off}}$ in the bases $|nL_J\rangle$, which is treated as the perturbative term. We neglect the perturbative term $H_{\text{off}}$ in the present calculation.
The free parameters of the adopted relativistic quark model are listed in Table II of Ref. [25], which include the quark masses and the coefficients in the effective potential. Here, we list the quark masses

\[ m_u = m_d = 220 \text{ MeV}, \quad m_b = 4977 \text{ MeV}, \quad m_s = 419 \text{ MeV}, \quad (4) \]

which are also applied in the following two-body strong decay calculation.

With the above preparation, we obtain the mass spectra of the bottom and bottom-strange meson families as shown in Fig. 1, where the masses of the 1S, 2S, 3S, 1P, 2P, 1D, 2D, and 1F states are given. Godfrey et al. calculated the mass spectrum of some of the bottom and bottom-strange mesons long ago [25]. We also notice that there exist predictions of the mass spectra of the bottom and bottom-strange mesons using other theoretical models. For example, Ebert et al. adopted the relativistic quark model based on the quasipotential approach [26] while the authors of Ref. [27] used the relativistic quark model with the Dirac Hamiltonian potential, where the correction of 1/m_b in the potential is also included. In addition, in Refs. [28, 29] the masses of the radial excitations of \( D^-/D_s^-/B/B_s^- \) were predicted, which are comparable with the corresponding results listed in Table I.

In Fig. 1 and Table I, we list our results of the mass spectra of the bottom and bottom-strange meson families, the results from other groups, and compare them with the experimental data.

Until now, there has been only some limited experimental information on the low-lying bottom and bottom-strange mesons, which are shown in the fifth and ninth columns. The theoretical results can reproduce these experimental data well. When comparing our results with those given by Refs. [26, 27], we notice that the discrepancy of the values of the mass from different models is about 50 MeV for the 2S, 3S, and 1D states, while the mass difference for the 1F, 2P and 2D states from different model calculations may reach up to about 100 ~ 200 MeV. Thus, we will consider the mass dependence of the strong decay width of the higher bottom and bottom-strange mesons, which will be discussed in the next section. In the following, we will employ our numerical results to study the two-body strong decay behavior of these higher bottom and bottom-strange mesons.

### III. TWO-BODY STRONG DECAY BEHAVIOR

In this work, we will study the two-body strong decay of the higher bottom and bottom-strange mesons allowed by the Okubo-Zweig-Iizuka (OZI) rule, where the quark pair creation (QPC) model is adopted in our calculation.

The QPC model [31–37] is an effective approach to study the strong decay of hadrons, which has been widely applied to study the decay behaviors of the newly observed hadron states [38–45]. In the QPC model, the meson OZI-allowed decay occurs via the creation of a quark-antiquark pair from the vacuum. The created quark-antiquark pair is a flavor and color singlet with the vacuum quantum number \( J^{PC} = 0^{++} \).

![FIG. 1: (color online). The comparison of the mass spectrum of bottom and bottom-strange meson calculated by the relativistic quark model (see Table I for the concrete values) with the experimental data [11–20].](image)

The transition operator \( T \) is written as [46]

\[
T = -3 \gamma \sum_m \langle 1m; 1 - m|00 \rangle \int d\mathbf{k}_3 d\mathbf{k}_4 \delta^3(\mathbf{k}_3 + \mathbf{k}_4) \\
\times \mathcal{Y}_{lm}(k_3 - k_4) \chi_{1,m}^{34} \phi_9^{34} \omega_9^{34} \delta_{ij} \delta_{ij}(k_3) b_{ij}(k_4), \quad (5)
\]

where the flavor and color wave functions have the forms \( \phi_9^{34} = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3} \) and \( \omega_9^{34} = \delta_{ij}/\sqrt{3} \), respectively. The color indices are denoted by \( i \) and \( j \). The solid harmonic polynomial is \( \mathcal{Y}_{lm}(k) = |k| Y_{lm}(k) \). The spin wave function is \( \chi_{1,m}^{34} \) with an angular momentum quantum number \( (1,-m) \). \( \gamma \) denotes the model parameter, which describes the strength of the quark-antiquark pair creation from the vacuum. In our calculation, the \( \gamma \) value is chosen to be 6.3 and 6.3/ \( \sqrt{3} \) for the creations of \( u/d \) quark and \( s \) quark [41], respectively.

The helicity amplitude \( \mathcal{M}^{M_{1j}, M_{2j}, M_{3l}}(\mathbf{k}) \) of the \( A \rightarrow B + C \) decay is defined as

\[
\langle BC|T|A \rangle = \delta^3(\mathbf{k}_B + \mathbf{k}_C - \mathbf{k}_A) \mathcal{M}^{M_{1j}, M_{2j}, M_{3l}}(\mathbf{k}), \quad (6)
\]

which can be further expressed as the product of the flavor, spin and spatial matrix elements, where the calculation of the
spatial matrix element is the most crucial part in the whole QPC calculation (see [46] for more details). In order to obtain the spatial matrix element, we adopt the mock state to describe the mesons [47]. In our calculation, the spatial radial wave function of the meson is represented by a SHO wave function [46] with a parameter $\beta$. The $\beta$ values in the SHO wave functions are determined by reproducing the rms radius of the corresponding states calculated in the relativistic quark model. In Table I, we list these adopted $\beta$ values for the bottom and bottom-strange mesons discussed.

Converting the helicity amplitude $M^{M_A M_B M_C}(K)$ to the partial wave amplitude $M_{L S}(K)$ through the Jacob-Wick formula [48], we get

$$M_{L S}(K) = \frac{\sqrt{2L+1}}{2J_A+1} \sum_{M_B M_C} \langle L0 S M_J K \rangle \langle J_A M_J S M_J \rangle M^{M_A M_B M_C}(K). \quad (7)$$

Finally, the decay width relevant to the partial wave amplitude is

$$\Gamma = \pi^2 |K| M_A^2 \sum_{LS} |M_{L S}(K)|^2, \quad (8)$$

where $M_A$ is the mass of the initial meson $A$. We take the experimental mass as input if the corresponding decay involves the observed state. Otherwise, we take the predicted mass from the relativistic quark mode in Table I when studying the strong decay of the bottom and bottom-strange mesons.

In the following subsections, we illustrate the OZI-allowed strong decay behaviors of the bottom and bottom-strange mesons in detail. We use $B$ and $B_s$ to represent bottom and bottom-strange mesons, respectively.
### A. Bottom meson

#### 1. $1P$ and $2P$ states

| Channels | $2P_0^0$ | $2P_2^0$ | $2P(1^+)/2P'(1^-)$ |
|----------|---------|---------|-------------------|
| $\pi B$  | $47$    | $9.7 \times 10^{-2}$ | $-$ |
| $\pi B'$ | $-$     | $1.4$    | $-$ |
| $\pi B(1^+P_0)$ | $-$ | $-$ | $-$ |
| $\pi B(1^+P_2)$ | $-$ | $4.6$    | $-$ |
| $\pi B(1^P)$    | $10$    | $16$    | $-$ |
| $\pi B(1^P')$   | $51$    | $1.3$    | $-$ |
| $\eta B$        | $5.6$   | $0.11$   | $-$ |
| $\eta B'$       | $-$     | $0.45$   | $-$ |
| $\rho B$        | $-$     | $4.3$    | $-$ |
| $\rho B'$       | $27$    | $26$    | $-$ |
| $\omega B$      | $-$     | $1.3$    | $-$ |
| $\omega B'$     | $8.8$   | $9.3$    | $-$ |
| $KB_1$          | $3.6$   | $1.3 \times 10^{-2}$ | $-$ |
| $KB_1'$         | $-$     | $0.11$   | $-$ |
| $K^*B_1$        | $2.7 \times 10^{-2}$ | $-$ | $-$ |
| **Total**       | $154$   | $64$    | $-$ |

Although the predicted mass of the $1^3P_0$ bottom meson is above both $B\pi$ and $B^\ast\pi$ thresholds, $B(1^3P_0)$ can only decay into $B\pi$. Its $B^\ast\pi$ decay is forbidden due to the parity conservation. The partial decay width of $B(1^3P_0) \rightarrow B\pi$ can reach up to 225 MeV. This state is very broad, which provides a natural explanation of why $B(1^3P_0)$ is still missing experimentally. There were other theoretical calculations of the decay width of the $B(1^3P_0)$ state. For example, the authors used the chiral quark model and obtained $\Gamma(B(1^3P_0) \rightarrow B\pi) = 272$ MeV [23], which is consistent with our result.

The two $J^P = 1^+$ states are the mixture of the $1^1P_1$ and $1^3P_1$ states

$$\begin{pmatrix} 1P(1^+) \\ 1P'(1^+) \end{pmatrix} = \begin{pmatrix} \cos \theta_{1P} & \sin \theta_{1P} \\ -\sin \theta_{1P} & \cos \theta_{1P} \end{pmatrix} \begin{pmatrix} 1^1P_1 \\ 1^3P_1 \end{pmatrix}, \tag{9}$$

where $\theta_{1P}$ denotes the mixing angle. In general, in the heavy quark limit one has $\theta_{1P} = -54.7^\circ$[49]. However, the obtained $\theta_{1P}$ value deviates from this ideal value when the heavy quark mass is finite in the case of the bottom and bottom-strange mesons. We will discuss this issue later.

In heavy quark effective theory, the heavy-light meson system can be categorized into different doublets by the angular momentum of the light component $j_t$, which is a good quantum number in the limit of $m_Q \rightarrow \infty$. There are two $1^+$ states in the $(0^+, 1^-)$ and $(1^+, 2^-)$ P-wave doublets, which correspond to $j_t = 1/2^+$ and $j_t = 3/2^+$, respectively. We can easily distinguish these two $1^+$ states; i.e., the $1^+$ state in the $(0^+, 1^-)$ doublet is broad while the other $1^+$ state in the $(1^+, 2^-)$ doublet is narrow. In Eq. (9), the $1P(1^+)$ and $1P'(1^+)$ states correspond the $1^+$ states in the $(0^+, 1^-)$ and $(1^+, 2^-)$ doublets, respectively. The following quantitative study of the decay behaviors will confirm the above qualitative observation.

In 2007, D0 Collaboration reported the observation of $B_1(5721)^0$ in the $B^+\pi^-$ channel, where its measured mass is $5720.6 \pm 2.4 \pm 1.4$ MeV [15]. Later, CDF confirmed $B_1(5721)^0$ with the mass $5725.3^{+1.6+1.4}_{-2.2-1.5}$ MeV [16]. Very recently, CDF [20] studied the orbitally excited $B$ mesons by using the full CDF Run II data sample. Besides confirming $B_1(5721)$, CDF reported the width of $B_1(5721)$ for the first time, where the measured width is $20 \pm 2 \pm 5$ MeV and $42 \pm 11 \pm 13$ MeV for the neutral and charged $B_1(5721)$ states [20], respectively. Since $B_1(5721)$ has a narrow width, $B_1(5721)$ is a good candidate of the $1P'(1^+)$ state in the $B$ meson family.

![FIG. 2: (color online). (a) The dependence of the total decay widths of $B(1P(1^+))$ (dashed curve) and $B(1P'(1^+))$ (solid curve) on the mixing angle $\theta_{1P}$. (b) The dependence of the total width of $B(1P'(1^+))$ on the mixing angle $\theta_{1P} = (-80 \sim -30)^\circ$ and the comparison with the CDF data [20]. Here, the vertical dashed line in (a) corresponds to the ideal mixing angle $\theta_{1P} = -54.7^\circ$.](image)

For $B_1(5721)$, its dominant decay is $B^\ast\pi$. With the QPC model, we calculate $B_1(5721)^0 \rightarrow B^\ast\pi$. In Fig. 2, the dependence of the calculated width of $B_1(5721)^0$ on the mixing angle $\theta_{1P}$ is given. If taking the ideal value $\theta_{1P} = -54.7^\circ$, the corresponding width of $B_1(5721)^0$ is 1.8 MeV, which is far smaller than the experimental data. We find that the calculated width of $B_1(5721)^0$ overlaps with the experimental value when $\theta_{1P}$ is in the range of $(-77^\circ \sim -70^\circ)$ or $(-40^\circ \sim -33^\circ)$. We also predict the width of $B(1P(1^+))$ state, where is $\Gamma(B(1P(1^+))) = 170 \sim 197$ MeV corresponding to $\theta_{1P} = (-77^\circ \sim -70^\circ)$ or $(-40^\circ \sim -33^\circ)$, which confirms $B(1P(1^+))$ is a broad state.

$B_1(5747)$ was first observed by D0 in its $B^+\pi^-$ and $B^+\pi^-$ channels [15]. Other experimental information of $B_1(5747)^0$ given by D0 includes: $M(B_1(5747)^0) = 5746.8 \pm 2.4 \pm 1.7$.
MeV and $\Gamma(B_s'(5747)^0 \to B^{*+}\pi^-)/\Gamma(B_s'(5747)^0 \to B_s^{(*)}\pi^-) = 0.475 \pm 0.095 \pm 0.069$ [15]. In 2009, CDF announced the confirmation of $B_s'(5747)^0$ with mass $M = 5740.2^{+1.8-0.2}$ MeV and width $\Gamma = 22.7^{+3.8+3.2}_{-3.2-10.0}$ MeV [16]. In Ref. [20], CDF again carried out the study of $B_s'(5747)$. The mass and width are $M = 5736.6 \pm 1.2 \pm 1.2 \pm 0.2/5731.1 \pm 1.1 \pm 0.9 \pm 0.2$ MeV and $\Gamma = 26 \pm 3 / 3 / 7 / 6 \pm 8$ MeV, respectively for the neutral/charged $B_s'(5747)$ states [20].

As a $J^P = 2^+ B$ meson, $B_s'(5747)$ decays into the $B\pi$ and $B\pi$ channels. In the QPC model, the total decay width of $B_s'(5747)^0$ is around 3.7 MeV, which is composed of $\Gamma(B_s'(5747)^0 \to B\pi) = 1.9$ MeV and $\Gamma(B_s'(5747)^0 \to B_s^{*}\pi) = 1.8$ MeV. We notice that our result of the total decay width of $B_s'(5747)^0$ is smaller than the experimental central value [16, 20]. As shown in Refs. [16, 20], there exist large experimental errors for the measured width of $B_s'(5747)$. Thus, further experimental measurement of the resonance parameter of $B_s'(5747)$ will be helpful to clarify this difference. In addition, we also obtain the ratio

$$\frac{B(B_s'(5747) \to B^{*}\pi^-)}{B(B_s'(5747) \to B_s^{*}\pi^-)} = 0.95,$$

which is consistent with the experimental value $1.10 \pm 0.42 \pm 0.31$ [15].

For the $2P$ states in the $B$ meson family, more decay channels are open. However, there has been no experimental observation up to now. We list our predictions of the partial and total decay widths in Table II.

For the $B(2^3P_0)$ meson, the dominant decay modes are $\pi B(1P), \pi B*, \rho B*$. In addition, $B(2^3P_0)$ can decay into $B\omega, K\pi, B\pi$, and $\pi B(1P)$. The sum of all partial decay widths of $B(2^3P_0)$ reaches up to 154 MeV, which indicates $B(2^3P_0)$ is a broad state. Among the dominant decay channels of $B(2^3P_0), \pi B$ is the most suitable channel for searching $B(2^3P_0)$.

$B(2^3P_2)$ mainly decays into $\rho B*, \pi B(1P), \omega B*$, and $\pi B(1P)$, where the partial width of $B(2^3P_2) \to \rho B*$ is the largest since $B(2^3P_2) \to \rho B* \text{ occurs via } S\text{-wave}$. In contrast, the D-wave decay modes $B(2^3P_2) \to \pi B, K\pi, K^*B$, can be neglected. The width of $B(2^3P_2)$ is 64 MeV.

As the mixture of the $2^3P_1$ and $2^3P_1$ states, the two $1^+$ states $B(2P(1^+))$ and $B(2P'(1^+))$ satisfy the following relation

$$\begin{pmatrix} 2P(1^+) \\ 2P'(1^+) \end{pmatrix} = \begin{pmatrix} \cos \theta_{2P} & \sin \theta_{2P} \\ -\sin \theta_{2P} & \cos \theta_{2P} \end{pmatrix} \begin{pmatrix} 2P_1 \\ 2P'_1 \end{pmatrix},$$

which $\theta_{2P}$ denotes the mixing angle. The allowed decay modes of $B(2P(1^+))$ and $B(2P'(1^+))$ are shown in Table II.

We obtain the total and partial decay widths of $B(2P(1^+))$ and $B(2P'(1^+))$, which depend on the mixing angle $\theta_{2P}$ (see Fig. 3 for details). Since the experimental information of $B(2P(1^+))$ and $B(2P'(1^+))$ is absent, in the following discussion we take the typical value $\theta_{2P} = -54.7^\circ$. We find that the total decay width of $B(2P(1^+)) (\Gamma(B(2P(1^+))) = 153$ MeV) is larger than that of $B(2P'(1^+)) (\Gamma(B(2P'(1^+))) = 70$ MeV), which is consistent with the estimate under the heavy quark effective theory. For $B(2P(1^+))$ and $B(2P'(1^+))$, its main decay channels include $\pi B*, \pi B(1^P), \pi B(1^P), \rho B*$ and $\eta B$.

![FIG. 3: (color online). The dependence of the decay behavior of $B(2P(1^+))$ (the first column) and $B(2P'(1^+))$ (the second column) on the mixing angle $\theta_{2P}$.](image)

Very recently, the CDF Collaboration reported the evidence of a new resonance $B(5970)$ in analyzing both $B^0\pi^+$ and $B_s^{*}\pi^-$ mass distributions [20]. The mass and width of $B(5970)$ are $(M, \Gamma)_{B(5970)^0} = (5978 \pm 5 \pm 12, 70 \pm 18 \pm 31)$ MeV, $(M, \Gamma)_{B(5970)^+} = (5961 \pm 5 \pm 12, 60 \pm 20 \pm 40)$ MeV, which correspond to the neutral and charged $B(5970)$, respectively. The comparison of the mass of $B(5970)$ and the mass spectrum in Table I indicates that the mass of $B(5970)$ is close to the estimated masses of the $2^3S_0$ and $2^3S_1$ states of the $B$ meson family. Since $B(5970)$ can decay into $B\pi$, we can exclude the $B(2S_0^0)$ assignment of $B(5970)$.

The obtained total width of $B(2^3S_1)$ is 47 MeV, which is in agreement with the experimental width of $B(5970)$ if one considers the experimental error. The calculation of the partial decay widths of $B(2^3S_1)$ indicates that $\pi B*, \pi B$ and $\pi B(1P_1^+)$ are its main decay channels. The results of the other partial decay widths of $B(2^3S_1)$ are listed in Table III. At present, $B(5970)$ was only observed in the $\pi B$ channel. Our study indicates that the $B(5970)$ is very probably $B(2^3S_1)$. We also
suggest the experimental search for $B(9570)$ via its $\pi B^*$ decay.

As the partner of $B(2^3S_1)$, $B(2^1S_0)$ dominantly decays into $\pi B^*$. In addition, there exists a considerable partial width of $B(2^1S_0) \rightarrow \pi B(1^3P_0)$. Compared with the $\pi B^*$ and $\pi B(1^3P_0)$, the remaining two decay channels $\eta B^*$ and $\pi B(1^3P_2)$ can be ignored. Thus, $\pi B^*$ is the ideal channel to search for $B(2^1S_0)$ in future experiments.

The masses of the two 3S states $B(3^1S_0)$ and $B(3^3S_1)$ are 6334 MeV and 6355 MeV, respectively. As shown in Table III, more decay channels are open for $B(3^1S_0)$ and $B(3^3S_1)$. The main decay modes of $B(3^3S_0)$ are $\pi B(1^3P_2)$, $\rho B^*$, $\pi B^*$, $\omega B^*$. For $B(3^3S_1)$, mainly $\rho B^*$, $\pi B^*$, $\pi B(1^3P_2)$, $\pi B(1^P(1^*)$), $\pi B$, $\omega B^*$, and $\rho B$ contribute to the total decay width. The partial widths of the modes $\eta B(1^3P_2)$, $\omega B$, $KB_s(1^3P_2)$ and $KB_s(1^P(1^*))$ are quite small.

### Table III: The partial and total decay widths (in units of MeV) of 2S and 3S states in the B meson family. Here, we adopt – to denote the forbidden decay channels.

| Channels | $2^1S_0$ | $2^3S_1$ | $3^1S_0$ | $3^3S_1$ |
|----------|----------|----------|----------|----------|
| $\pi B$  | –        | 9.1      | –        | 2.6      |
| $\pi B^*$| 33       | 23       | 8.8      | 6.1      |
| $\pi B(1^3P_0)$ | 3.9  | –        | 3.7      | –        |
| $\pi B(1^3P_2)$ | $2.0 \times 10^{-3}$ | $1.0 \times 10^{-2}$ | 11.2    | 4.9      |
| $\pi B(1P(1^*))$ | –        | 10       | –        | 3.9      |
| $\eta B$ | –        | 0.2      | –        | 2.8      |
| $\eta B^*$ | 0.62     | 2.0      | 1.0      | 0.87     |
| $\eta B(1^3P_0)$ | –        | –        | 0.16     | –        |
| $\eta B(1^3P_2)$ | –        | –        | $7.2 \times 10^{-2}$ | 8.8 $\times 10^{-2}$ |
| $\eta B(1P(1^*))$ | –        | –        | –        | 0.28     |
| $\rho B$ | –        | –        | 1.3      | 1.3      |
| $\rho B^*$ | –        | –        | 9.4      | 7.0      |
| $\omega B$ | –        | –        | $0.49 \times 10^{-2}$ | 5.7 $\times 10^{-2}$ |
| $\omega B^*$ | –        | –        | 3.3      | 1.7      |
| $KB_s$ | –        | 0.43     | –        | 0.24     |
| $KB_s^*$ | –        | 0.58     | 0.45     | 0.41     |
| $K^* B_s$ | –        | –        | 0.46     | 0.27     |
| $K^* B_s^*$ | –        | –        | 0.49     | 0.90     |
| $KB_s(1^3P_0)$ | –        | –        | $4.0 \times 10^{-2}$ | –        |
| $KB_s(1^3P_2)$ | –        | –        | $2.8 \times 10^{-3}$ | –        |
| $KB_s(1P(1^*))$ | –        | –        | –        | 0.10     |
| Total    | 38       | 47       | 41       | 33       |

### Table IV: The partial and total decay widths (in units of MeV) of 1D and 2D states in the B meson family. The forbidden decay channels are marked by –. For the $1D(2^2)/1D'(2^2)$ and $2D(2^2)/2D'(2^2)$ states, we use $\xi$ to mark the allowed decay channels. Here, the value $a \times 10^{-8}$ is abbreviated as $a[b]$.

| Channels | $1^1D_1$ | $1^1D_3$ | $2^1D_1$ | $2^1D_3$ | $1D'(2^2)$ | $2D'(2^2)$ |
|----------|----------|----------|----------|----------|-----------|-----------|
| $\pi B$  | 69       | 4.9      | 27       | 1.2      | –         | –         |
| $\pi B^*$| 34       | 6.2      | 12       | 0.49     | –         | –         |
| $\pi B(1^3P_0)$ | –        | –        | –        | –        | –        | –         |
| $\pi B(1^3P_2)$ | 1.6      | 0.74     | 5.3      | 0.21     | –         | –         |
| $\pi B(1P(1^*))$ | 6.8[2]  | 9.0[2]  | 8.1      | 6.4      | –         | –         |
| $\pi B(1P(1^*))$ | 147      | 0.17     | 28       | 7.5[2]  | –         | –         |
| $\eta B$ | 1.2      | 0.21     | 4.3      | 5.8[2]  | –         | –         |
| $\eta B^*$ | 5.6      | 0.20     | 1.6      | 1.2[3]  | –         | –         |
| $\eta B(1^3P_0)$ | –        | –        | –        | –        | –         | –         |
| $\eta B(1^3P_2)$ | –        | –        | 0.62     | 0.19     | –         | –         |
| $\eta B(1P(1^*))$ | –        | –        | 0.74     | 0.57     | –         | –         |
| $\eta B(1P(1^*))$ | –        | –        | 0.65     | 2.1[2]  | –         | –         |
| $\rho B$ | 8.3      | 1.8[2]  | 0.12     | 2.0      | –         | –         |
| $\rho B^*$ | 0.93     | 1.3      | 23       | 6.1      | –         | –         |
| $\omega B$ | 2.4      | 3.7[3]  | 3.2[2]  | 0.67     | –         | –         |
| $\omega B^*$ | 0.10     | –        | 7.5      | 2.0      | –         | –         |
| $KB_s$ | 7.4      | 5.4[2]  | 2.5      | 4.4[2]  | –         | –         |
| $KB_s^*$ | 3.3      | 4.5[2]  | 0.86     | 8.6[3]  | –         | –         |
| $K^* B_s$ | –        | 0.25     | 1.9[3]  | –        | –         | –         |
| $K^* B_s^*$ | –        | 0.65     | 0.75     | –        | –         | –         |
| $KB_s(1^3P_0)$ | –        | –        | –        | –        | –         | –         |
| $KB_s(1^3P_2)$ | –        | –        | 0.20     | 4.5[2]  | –         | –         |
| $KB_s(1P(1^*))$ | –        | –        | 0.19     | 0.12     | –         | –         |
| $KB_s(1P(1^*))$ | –        | 0.14     | 4.2[3]  | –        | –         | –         |

Total 294 14 122 23 – –

3. 1D and 2D states

The decay behaviors of 1D and 2D states are given in Table IV, where we first list the allowed decay channels for the $1D(2^2)/1D'(2^2)$ and $2D(2^2)/2D'(2^2)$ states.

$B(1^3D_1)$ is very broad and its total width reaches up to 294 MeV. $B(1^3D_1)$ dominantly decays into $\pi B(1P(1^*))$, $\pi B$, and $\pi B^*$. These channels are suitable to search for $B(1^3D_1)$ in future experiments. $B(2^1D_1)$ is the first radial excitation of $B(1^1D_1)$. We notice that $\pi B(1P(1^*))$, $\pi B$, $\rho B^*$, and $\pi B^*$ are the main decay modes of $B(2^1D_1)$, which renders $B(2^1D_1)$ broad. Different from the broad $B(1^3D_1)$ and $B(2^3D_1)$, $B(1^3D_1)$ and $B(2^3D_1)$ are rather narrow states since the total decay widths of $B(1^3D_1)$ and $B(2^3D_1)$ are 14 and 23 MeV, respectively. For $B(1^3D_1)$, there exist two dominant decay channels, $\pi B$ and $\pi B^*$. Although more decay modes are included for $B(2^3D_1)$, its total decay width is not obviously enhanced. The $\pi B(1P(1^*))$, $\rho B^*$, $\rho B$, $\omega B^*$ and $\pi B$ are the main
We notice that the total decay width of $\eta_B$ is obtained with the typical value $\Gamma_{\eta_B} = 112$ MeV, which is shown in Fig. 4. The dependence of the partial and total decay widths of $B(2D(2^-))$ (the first column) and $B(2D'(2^-))$ (the second column) on the mixing angle $\theta_{ID}$.

There are four $1F$ states. Their decay pattern is listed in Table VII. We notice that the total decay width of $B(1F(3^+))$ is quite broad, and can reach up to 254 MeV. Among all decays of $B(1F(3^+), \pi B(1P'(1^+))$ is the most important channel. The other main decay modes of $B(1F(3^+), \pi B(1P'(1^+))$ is $\pi B(1P'(1^+))$, $\rho B$, and $\omega B$. For $B(1F(3^+))$, its total width is estimated as 103 MeV, where $\rho B$ and $\omega B$ are its dominant decay mode.

In the following, we focus on $B(1F(3^+))$ and $B(1F'(3^+))$, which are the mixing of the $1F_{3}$ and $1F_{3}'$ states, 

$$
\begin{align*}
\left\{ \begin{array}{c} 
F(3^+) \\
F'(3^+) 
\end{array} \right\} & = \left( \begin{array}{cc}
\cos \theta_{IF} & \sin \theta_{IF} \\
-\sin \theta_{IF} & \cos \theta_{IF} 
\end{array} \right) \left\{ \begin{array}{c} 
1F_{3} \\
1F_{3}' 
\end{array} \right\}.
\end{align*}
$$

with $\theta_{IF} = -49.1^\circ$ determined in heavy quark limit [26]. In Fig. 6, the $\theta_{IF}$ dependence of the partial and total decay widths of $B(1F(3^+))$ and $B(1F'(3^+))$ is presented. If we take the typical value $\theta_{IF} = -49.1^\circ$, we notice that $B(1F(3^+))$ is a quite broad resonance. From Fig. 6, we can easily distinguish the main decay mode among all allowed decay channels, and we can distinguish which channel is valuable to further experimental search of the $1F$ state in the $B$ meson family.
TABLE V: The partial and total decay widths (in units of MeV) of \(1F\) states in \(B\) meson family. The forbidden decay channels are marked by \(\square\). For \(1F(3^+)/1F'(3^+\)) states, we use \(\square\) to mark the allowed decay channels.

| Channels | \(1^3F_2\) | \(1^3F_4\) | \(1F(3^+)/1F'(3^+)\) |
|----------|--------------|--------------|------------------------|
| \(\pi B\) | 30           | 3.7          | –                      |
| \(\pi B'\) | 21           | 5.3          | \(\square\)          |
| \(\pi B(1^3P_0)\) | –            | –            | \(\square\)          |
| \(\pi B(1^3P_2)\) | 14           | 2.5          | \(\square\)          |
| \(\pi B(1P(1^+))\) | 0.5          | 1.2          | \(\square\)          |
| \(\pi B(1P'(1^+))\) | 121          | 0.48         | \(\square\)          |
| \(\eta B\) | 5.5          | 0.31         | –                      |
| \(\eta B'\) | 3.9          | 0.41         | \(\square\)          |
| \(\eta B(1^3P_0)\) | –            | –            | \(\square\)          |
| \(\eta B(1^3P_2)\) | 0.10         | 4.9 \times 10^{-3} | – |
| \(\eta B(1P(1^+))\) | 1.3 \times 10^{-3} | 7.4 \times 10^{-3} | \(\square\) |
| \(\eta B(1P'(1^+))\) | 11           | 2.4 \times 10^{-3} | \(\square\) |
| \(\rho B\) | 15           | 1.1          | \(\square\)          |
| \(\rho B'\) | 15           | 56           | \(\square\)          |
| \(\omega B\) | 4.7          | 0.33         | \(\square\)          |
| \(\omega B'\) | 2.2          | 31           | \(\square\)          |
| \(KB_B\) | 2.8          | 7.2 \times 10^{-2} | – |
| \(KB_B'\) | 2.0          | 9.2 \times 10^{-2} | \(\square\) |
| \(K^* B_B\) | 0.75         | 1.3 \times 10^{-2} | \(\square\) |
| \(K^* B_B'\) | 8.8 \times 10^{-2} | 0.24         | –                      |
| \(KB_B(1^3P_0)\) | –            | –            | \(\square\)          |
| \(KB_B(1^3P_2)\) | 1.1 \times 10^{-3} | 1.2 \times 10^{-4} | – |
| \(KB_B(1P(1^+))\) | 5.4 \times 10^{-4} | 2.0 \times 10^{-4} | \(\square\) |
| \(KB_B(1P'(1^+))\) | 3.3          | 4.9 \times 10^{-3} | \(\square\) |
| Total       | 254          | 103          | –                      |

B. Bottom-strange meson

1. \(1P\) and \(2P\) states

The decay modes of \(B_s(1^3P_0)\) is \(BK\). The partial width of \(B_s(1^3P_0) \rightarrow BK\) obtained from the QPC model is 225 MeV, which is almost the same as that from the chiral quark model in Ref. [23] (\(\Gamma = 227 \text{ MeV}\)). \(B_s(1^3P_0)\) is also a broad state, very similar to \(B(1^3P_0)\). Generally speaking, it is difficult to detect a very broad state experimentally.

The observed \(B_s(5830)\) and \(B_{s2}^*(5840)\) are good candidates of \(B_s(1P'(1^+))\) and \(B_s(1^3P_2)\), respectively. The experimental information on \(B_s(5830)\) and \(B_{s2}^*(5840)\) from the CDF, D0, and LHCb Collaborations [17-20] is collected in Table VI.

If we take the mass of \(B_{s2}^*(5840)\) as input, \(B_s(1^3P_2)\) can decay into \(BK\) and \(B^* K\). With the QPC model, the total decay width of \(B_s(1^3P_2)\) is around \(\Gamma(B_s(1^3P_2)) = 0.26 \text{ MeV}\) and slightly smaller than the experimental central value which has a large error [19, 20]. However, the obtained ratio

\[
\frac{\Gamma(B_{s2}^* \rightarrow B^* K)}{\Gamma(B_{s2} \rightarrow B^* K)} = 0.088
\]

is in good agreement with the experimental measurement.

FIG. 6: (color online). The dependence of the partial and total decay widths of \(B(1F(3^+))\) (the first column) and \(B(1F'(3^+))\) (the second column) on the mixing angle \(\theta_{1F}\).

FIG. 7: (color online). (a) The dependence of the total decay widths of \(B_s(1P'(1^+))\) (dashed curve) and \(B_s(1P(1^+))\) (solid curve) on the mixing angle \(\theta_{1F}\). (b) The variation of the total width of \(B_s(1P(1^+))\) with the mixing angle \(\theta_{1F} = (-64 \sim -46)\) and the comparison with the CDF data [20]. Here, the vertical dashed line in (a) corresponds to the ideal mixing angle \(\theta_{1F} = -54.7^\circ\).
leads to the total decay width around 142 MeV. Thus, for the allowed decay channels, we notice that there exists a difference between the realistic and ideal value of the mixing angle $\theta_{1P}$, which is similar to the situation of $B_s(5721)$. If adopting $\theta_{1P} = -60.5^\circ \sim -57.5^\circ$ or $-52.0^\circ \sim -49.0^\circ$, the total decay width of $B_s(1P(1^+))$ overlaps the observed width of $B_{s1}(5830)$ given by CDF [20]. We notice that there exists a difference between the realistic and ideal value of the mixing angle $\theta_{1P}$, which is similar to the situation of $B_s(5721)$. If adopting $\theta_{1P} = -60.5^\circ \sim -57.5^\circ$ or $-52.0^\circ \sim -49.0^\circ$, the total decay width of $B_s(1P(1^+))$ is about 110 MeV. Hence, $B_s(1P(1^+))$ is a broad resonance, consistent with the rough estimate in heavy quark limit.

For $2P(1^+)/2P(1^+)$ states, we use 3 to mark the allowed decay channels.

| Channels | $2P0$ | $2P1$ | $2P(1^+)/2P(1^+)$ |
|----------|-------|-------|---------------------|
| $KB$     | 65    | 2.6   | –                   |
| $KB'$    | –     | 7.3   | □                   |
| $K' B$   | –     | 1.6   | □                   |
| $K' B'$  | 36    | 69    | □                   |
| $KB(1P0)$ | –     | –     | □                   |
| $KB(1P(1^+))$ | 5.3 | 0.70 | □               |
| $KB(2P(1^+))$ | 32 | $3.5 \times 10^{-3}$ | □          |
| $\eta B_s$ | 1.7  | $5.3 \times 10^{-2}$ | –         |
| $\eta B'_s$ | –     | 0.14  | □               |
| Total    | 142   | 82    | –                   |

The decay behavior of the four $2P$ states are listed in Table VII, where $B_s(2P(1^+))$ and $B_s(2P'(1^+))$ are the mixed states satisfying Eq. (11). The three main decay channels for $B_s(2P0)$ are $KB$, $K'B'$, and $KB(1P(1^+))$. The sum of all partial decay widths listed in the second column of Fig. 8 leads to the total decay width around 142 MeV. Thus, $B_s(2P0)$ is a broad $B_s$ meson. For $B_s(2P2)$, there is only one dominant decay channel $K^*B'$, where the branching ratio of $B_s(2P2) \rightarrow K^*B'$ is 0.84. At present, $B_s(2P0)$ and $B_s(2P2)$ are still missing. Thus, the obtained main decay modes of $B_s(2P0)$ and $B_s(2P2)$ may be useful to the experimental search of $B_s(2P0)$ and $B_s(2P2)$.

The dependence of the total and partial decay widths of $B_s(2P(1^+))$ and $B_s(2P'(1^+))$ on $\theta_{2P}$ are presented in Fig. 8. If $\theta_{2P}$ takes the typical value, the main decay modes are $B_s(2P(1^+))/B_s(2P'(1^+)) \rightarrow K^*B'$ (see Fig. 8 for detailed information). We conclude that both $B_s(2P(1^+))$ and $B_s(2P'(1^+))$ are broad $B_s$ states.

### C. $2S$ and $3S$ states

In the following, we illustrate the decay behavior of $B_s(2S0)$, $B_s(2S1)$, $B_s(3S0)$, and $B_s(3S1)$ (see Table VIII).

---

**TABLE VI:** The summary of experimental information of $B_{s1}(5830)$ and $B_{s2}^*(5840)$.

| State        | Collaboration | Mass             | Width            | Observed decays | $\Gamma(B_{s1} \rightarrow B^{*+}K^-)/\Gamma(B_{s2}^* \rightarrow B^{*+}K^-)$ |
|--------------|---------------|------------------|------------------|----------------|--------------------------------------------------------------------------------|
| $B_{s1}(5830)$ | CDF [17]      | 5829.4 ± 0.7 MeV | –                | $B^{*+}K^-$    | –                                                                               |
| $B_{s2}^*(5840)$ | LHCb [19]    | 5828.40 ± 0.04 ± 0.41 | –                | $B^{*+}K^-$    | –                                                                               |
| CDF [20]     | 5828.3 ± 0.1 ± 0.1 ± 0.4 | 0.7 ± 0.3 ± 0.3 | $B^{*+}K^-$    | –                                                                               |
| $B_{s2}^*(5840)$ | LHCb [19]    | 5839.96 ± 0.1 ± 0.17 | 1.56 ± 0.13 ± 0.47 | $B^{*+}K^-$, $B^{*+}K^-$ | 0.093 ± 0.013 ± 0.012 |
| CDF [20]     | 5839.7 ± 0.1 ± 0.1 ± 0.2 | 2.0 ± 0.4 ± 0.2 | $B^{*+}K^-$, $B^{*+}K^-$ | 0.11 ± 0.03 |

**TABLE VII:** The partial and total decay widths (in units of MeV) of $2P$ states in the $B_s$ meson family. The forbidden decay channels are marked by □. For $2P(1^+)/2P(1^+)$ states, we use 3 to mark the allowed decay channels.
TABLE VIII: The partial and total decay widths (in units of MeV) of 2S and 3S states in the B_s meson family. Here, we adopt – to denote the forbidden decay channels.

| Channels | 2S 2S | 2S 3S | 3S 2S | 3S 3S |
|----------|-------|-------|-------|-------|
| KB′     | –     | 17    | –     | 5.8   |
| KB      | 44    | 34    | 15    | 12    |
| K′B     | –     | –     | 3.0   | 0.41  |
| K′B′    | –     | –     | 21    | 12    |
| KB(1S0) | –     | –     | 9.8 × 10⁻⁴ | – |
| KB(1P2) | –     | –     | 14    | 7.5   |
| KB(1P1) | –     | –     | –     | 0.95  |
| KB(1P′1) | –     | –     | –     | 6.0   |
| ηB_s   | –     | 0.29  | –     | 0.17  |
| ηB_s*  | 0.14  | 0.30  | 0.28  | 0.26  |
| ηB_s(1P0) | –     | –     | 0.11  | –     |
| ηB_s(1P2) | –     | –     | 5.4 × 10⁻⁴ | 1.2 × 10⁻² |
| ηB_s(1P1) | –     | –     | –     | 0.15  |
| ηB_s(1P′1) | –     | –     | –     | 1.3 × 10⁻² |
| φB_s   | –     | –     | 0.26  | 0.31  |
| Total   | 44    | 51    | 54    | 46    |

According to the numerical results in Table VIII, we conclude:

- \( B_s(2S_0) \) dominantly decays into \( KB' \). The contribution from \( ηB_s(0^+) \) is negligible.
- \( KB' \) and \( KB \) are the two main decays of \( B_s(2S_1) \). The contribution from both \( ηB'_s \) and \( ηB_s(0^+) \) decay modes is quite small.
- As the radial excitation of \( B_s(2S_0) \), the total decay width of \( B_s(3S_0) \) is \( Γ(B_s(3S_0)) = 54 \text{ MeV} \). Here, the branching ratios of \( B_s(3S_0) \rightarrow KB', K'B', KB(1P2) \) are 0.28, 0.39, and 0.26, respectively.
- The main decay modes of \( B_s(3S_1) \) include \( KB', K'B', KB, KB(1P2) \) and \( KB(1P'1) \).

D. 1D and 2D states

In Table IX, we list the numerical results of \( B_s(1D_1) \), \( B_s(1D_2) \), \( B_s(2D_1) \) and \( B_s(2D_2) \) and the allowed decay channels of \( B_s(1D(2^-))/B_s(1D'(2^-)) \) and \( B_s(2D(2^-))/B_s(2D'(2^-)) \).

Our calculation shows

- Both \( B_s(1D_1) \) and its radial excitation \( B_s(2D_1) \) are very broad, while both \( B_s(1D_2) \) and \( B_s(2D_2) \) are quite narrow.
- The dominant decay modes of \( B_s(1D_1) \) are \( KB \) and \( KB' \) with the branching ratios 0.65 and 0.30, respectively.

TABLE IX: The partial and total decay widths (in units of MeV) of 1D and 2D states in the B_s meson family. The forbidden decay channels are marked by –. For the 1D(2^-)/1D'(2^-) and 2D(2^-)/2D'(2^-) states, we use ⊙ to mark the allowed decay channels. Here, the value \( α \times 10^{-6} \) is abbreviated as \( α[b] \).

| Channels | 1D 1D | 1D 2D | 2D 2D | 2D 2D |
|----------|-------|-------|-------|-------|
| KB       | 125   | 5.2   | 49    | 0.61  |
| KB′      | 58    | 5.7   | 21    | 1.4[2]| ⊙ ⊙ |
| K′B      | 1.1   | 3.0[5]| 0.99  | 5.5   | ⊙ ⊙ |
| K′B′     | –     | –     | 41    | 13    | – ⊙ |
| KB(1P0)  | –     | –     | –     | –     | ⊙ ⊙ |
| KB(1P2)  | –     | –     | 9.7   | 4.3   | – ⊙ |
| KB(1P1)  | –     | –     | 12    | 11    | – ⊙ |
| KB(1P′1) | –     | –     | 33    | 0.38  | – ⊙ |
| ηB_s     | 4.8   | 5.3[2]| 1.7   | 1.2[2]| – ⊙ |
| ηB_s*    | 2.0   | 4.5[2]| 0.60  | 2.2[5]| ⊙ ⊙ |
| ηB_s(1P0)| –     | –     | –     | –     | ⊙ ⊙ |
| ηB_s(1P2)| –     | –     | 0.16  | 5.8[2]| – ⊙ |
| ηB_s(1P1)| –     | –     | 0.16  | 0.14  | – ⊙ |
| ηB_s(1P′1)| –     | –     | 0.13  | 6.3[3]| – ⊙ |
| φB_s     | –     | –     | 0.38  | 6.7[2]| – ⊙ |
| φB_s*    | –     | –     | 0.70  | 1.7   | – ⊙ |
| Total    | 191   | 11    | 171   | 37    | – ⊙ |

- \( B_s(2S_1) \) mainly decays into \( KB, K'B', KB(1P'(1^+)) \), \( KB', KB(1P'(1^+)) \), and \( KB(1P_2) \).
- For \( B_s(1D_2) \), there are two dominant decay modes, \( KB \) and \( KB' \). For \( B_s(2D_2) \), \( K'B', KB(1P'(1^+)), K'B, \) and \( KB(1P_2) \) are its main decay channels, while the remaining decay channels have small partial decay widths (see Table IX for more details).

As the mixed states \( B_s(1D(2^-))/B_s(1D'(2^-)) \) and \( B_s(2D(2^-))/B_s(2D'(2^-)) \) are similar to \( B(1D(2^-))/B(1D'(2^-)) \) and \( B(2D(2^-))/B(2D'(2^-)) \) in Eq. (12), in Figs. 9 and 10 we thus show the variation of their partial and total decay widths with \( θ_{1D}/θ_{2D} \). With the typical \( θ_{1D} = θ_{2D} = –50.8^∞ \), we have the following observations:

- For \( B_s(1D(2^-))/B_s(1D'(2^-)) \), its \( KB' \) mode is very important. The variation of the partial width \( B_s(1D(2^-))/B_s(1D'(2^-)) \rightarrow KB' \) with the mixing angle is very similar to that of the total decay width (see Fig. 9).

- \( B_s(2D(2^-)) \) is broad and mainly decays into \( KB', KB(1P_2), K'B', \) and \( K'B \). As the part of \( B_s(2D(2^-)) \), the total decay width of \( B_s(2D(2^-)) \) is smaller than that of \( B_s(2D(2^-)) \). The main decay mode of \( B_s(2D'(2^-)) \) is \( K'B' \). While the sum of the partial decay widths of the decay modes \( KB(1P(1^+)), KB(1P_0), KB(1P'(1^+)), K'B \) is compara-
The dependence of the partial and total decay widths of $B_s(2D(2^-))$ (the first column) and $B_s(2D'(2^-))$ (the second column) on the mixing angle $\theta_{1D}$.

FIG. 10: (color online). The dependence of the partial and total decay behavior of $B_s(1D(2^-))$ (the first column) and $B_s(1D'(2^-))$ (the second column) on the mixing angle $\theta_{1D}$.

**E. 1F states**

The decay behavior of the four 1F states is presented in Table X and Fig. 11.

- $B_s(1^3F_2)$ is a very broad resonance. Its total decay width can reach up to 319 MeV. The branching ratio of the $KB(1P(1^+))$ mode is around 50%. The other important decay modes are $Ks\pi\pi$ and $KB(1^3P_2)$.

- $B_s(1^3F_4)$ dominates the total decay width of $B_s(1^3F_4)$.

- The total and partial decay widths of $B_s(1F(3^+))$ and $B_s(1F'(3^+))$ are dependent on the mixing angle $\theta_{1F}$ in Eq. (13). In Fig. 11, the main and subordinate decay modes are given for $B_s(1F(3^+))$ and $B_s(1F'(3^+))$.

**TABLE X:** The partial and total decay widths (in units of MeV) of 1F states in the $B_s$ meson family. The forbidden decay channels are marked by –. For $1F(3^+)/1F'(3^+)$ states, we use □ to mark the allowed decay channels.

| Channels       | $1^3F_2$ | $1^3F_4$ | $1F(3^+)$ |
|----------------|----------|----------|-----------|
| $KB$           | 55       | 5.7      | –         |
| $KB'$          | 38       | 7.4      | □         |
| $K^* B$        | 28       | 1.7      | □         |
| $K^* B'$       | 26       | 100      | □         |
| $KB(1P_0)$     | –        | –        | □         |
| $KB(1P_2)$     | 17       | 0.94     | □         |
| $KB(1P(1^+))$  | 0.34     | 0.55     | □         |
| $KB(1P'(1^+))$ | 149      | 0.27     | □         |
| $\eta B_s$     | 1.9      | $7.3 \times 10^{-2}$ | – |
| $\eta B'_s$    | 1.4      | $8.9 \times 10^{-2}$ | □ |
| $\eta B_s(1P_0)$| –        | –        | □         |
| $\eta B_s(1P_2)$| 0.24     | $3.0 \times 10^{-4}$ | □ |
| $\eta B_s(1P(1^+))$| $5.4 \times 10^{-3}$ | $3.8 \times 10^{-4}$ | □ |
| $\eta B_s(1P'(1^+))$| 2.4      | $9.8 \times 10^{-5}$ | □ |
| $\phi B_s$     | 0.14     | 7.1 \times 10^{-5} | □ |
| $\phi B'_s$    | 2.3 \times 10^{-3} | – | □ |
| Total          | 319      | 116      | –         |

In the above discussions, we take the predicted mass values of these higher $B$ and $B_s$ meson families as input. However, the uncertainty of the calculated masses sometimes is around (50 ~ 200) MeV from various theoretical models. Therefore, when predicting the decay behavior of these higher $B$ and $B_s$ mesons, we also study the variations of their decay behavior with the mass of the parent state, which is illustrated in Fig. 12.

**IV. SUMMARY**

Inspired by the recent experimental observation of the orbital excitation $B(5970)$ for the first time by CDF Collaboration [20], we have carried out a systematic study of the higher $B$ and $B_s$ mesons. We have calculated both the masses of the higher $B$ and $B_s$ mesons and their OZI-allowed two-body strong decay patterns.
At present, the status of studying $B$ and $B_s$ mesons is very similar to that of the $D$ and $D_s$ mesons in 2003. In the past several years, CDF, D0, and LHCb Collaborations have played a very important role in the study of the radial and orbital excitations of the $B$ and $B_s$ meson families. The higher radial and orbital excitations of the $B$ and $B_s$ mesons begin to emerge in experiment. In the coming years, LHCb has the potential to discover more and more excited $B$ and $B_s$ mesons. Hopefully our present investigation will be helpful to future experimental searches for these interesting heavy mesons.

Acknowledgements

This project is supported by the National Natural Science Foundation of China under Grants No. 11222547, No. 11175073, No. 11035006, No. 11375240 and No. 11261130311, the Ministry of Education of China (FANEDD under Grant No. 200924, SRFDP under Grant No. 201202111000, and NCET), and the Fok Ying Tung Education Foundation (Grant No. 131006).

References

[1] B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. 90, 242001 (2003) [hep-ex/0304021].
[2] D. Besson et al. [CLEO Collaboration], Phys. Rev. D 68, 032002 (2003) [Erratum-ibid. D 75, 119908 (2007)] [hep-ex/0305100].
[3] P. Krokovny et al. [Belle Collaboration], Phys. Rev. Lett. 91, 262002 (2003) [hep-ex/0308019].
[4] A. V. Evdokimov et al. [SELEX Collaboration], Phys. Rev. Lett. 93, 242001 (2004) [hep-ex/0406045].
[5] B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. 97, 222001 (2006) [hep-ex/0607082].
[6] K. Abe et al. [Belle Collaboration], hep-ex/0608031.
[7] B. Aubert et al. [BaBar Collaboration], Phys. Rev. D 80, 092003 (2009) [arXiv:0908.0806 [hep-ex]].
[8] F. del Amo Sanchez et al. [BaBar Collaboration], Phys. Rev. D 82, 111101 (2010) [arXiv:1009.2076 [hep-ex]].
[9] RAAij et al. [LHCb Collaboration], JHEP 1309, 145 (2013) [arXiv:1307.4556 [hep-ex]].
[10] X. Liu, Int. J. Mod. Phys. Conf. Ser. 2, 147 (2011).
[11] P. Abreu et al. [DELPHI Collaboration], Phys. Lett. B 345, 598 (1995).
[12] R. Akers et al. [OPAL Collaboration], Z. Phys. C 66, 19 (1995).
[13] D. Buskulic et al. [ALEPH Collaboration], Z. Phys. C 69, 393 (1996).
[14] R. Barate et al. [ALEPH Collaboration], Phys. Lett. B 425, 215 (1998).
[15] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 99, 172001 (2007) [arXiv:0705.3229 [hep-ex]].
[16] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 102, 102003 (2009) [arXiv:0809.5007 [hep-ex]].
[17] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 100, 082001 (2008) [arXiv:0710.4199 [hep-ex]].
[18] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 100, 082002 (2008) [arXiv:0711.0319 [hep-ex]].
[19] RAAij et al. [LHCb Collaboration], Phys. Rev. Lett. 110, 151803 (2013) [arXiv:1211.5994 [hep-ex]].
[20] T. A. Aaltonen et al. [CDF Collaboration], arXiv:1309.5961 [hep-ex].
[21] A. F. Falk and T. Mehen, Phys. Rev. D 53, 231 (1996) [hep-ph/9507311].
[22] A. H. Orsland and H. Hogaasen, Eur. Phys. J. C 9, 503 (1999) [hep-ph/9812347].
[23] X. -H. Zhong and Q. Zhao, Phys. Rev. D 78, 014029 (2008) [arXiv:0803.2102 [hep-ph]].
[24] Z. -G. Luo, X. -L. Chen and X. Liu, Phys. Rev. D 79, 074020 (2009) [arXiv:0901.0505 [hep-ph]].
[25] S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).
[26] D. Ebert, R. N. Faustov and V. O. Galkin, Eur. Phys. J. C 66, 197 (2010) [arXiv:0910.5612 [hep-ph]].
[27] M. Di Pierro and E. Eichten, Phys. Rev. D 64, 114004 (2001) [hep-ph/0104208].
[28] T. Matsuki, T. Morii and K. Sudoh, Eur. Phys. J. A 31, 701 (2007) [hep-ph/0610186].
[29] T. Matsuki and K. Seo, Phys. Rev. D 85, 014036 (2012) [arXiv:1111.0857 [hep-ph]].
[30] J. Beringer et al. [Particle Data Group Collaboration], Phys.
FIG. 12: The variation of the decay widths (in units of MeV) of the higher $B$ and $B_s$ mesons with their masses.

Rev. D 86, 010001 (2012).
[31] L. Micu, Nucl. Phys. B 10, 521 (1969).
[32] A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D 8, 2223 (1973); Phys. Rev. D 9, 1415 (1974); Phys. Rev. D 11, 1272 (1975); Phys. Lett. B 72, 57 (1977); Phys. Lett. B 71, 397 (1977).
[33] A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, NEW YORK, USA: GORDON AND BREACH (1988) 311p.
[34] E. van Beveren, C. Dullemond and G. Rupp, Phys. Rev. D 21, 772 (1980) [Erratum-ibid. D 22, 787 (1980)].
[35] E. van Beveren, G. Rupp, T. A. Rijken and C. Dullemond, Phys. Rev. D 27, 1527 (1983).
[36] R. Bonnaz, B. Silvestre-Brac and C. Gignoux, Eur. Phys. J. A 13, 363 (2002) [hep-ph/0111121].
[37] W. Roberts, B. Silvestre-Brac, Few Body Syst. 11, 171 (1992).
[38] B. Zhang, X. Liu, W. -Z. Deng and S. -L. Zhu, Eur. Phys. J. C 50, 617 (2007) [hep-ph/0609013].
[39] X. Liu, Z. -G. Luo and Z. -F. Sun, Phys. Rev. Lett. 104, 122001 (2010) [arXiv:0911.3694 [hep-ph]].
[40] Z. -F. Sun, J. -S. Yu, X. Liu and T. Matsuki, Phys. Rev. D 82, 111501 (2010) [arXiv:1008.3120 [hep-ph]].
[41] Z. -F. Sun and X. Liu, Phys. Rev. D 80, 074037 (2009) [arXiv:0909.1658 [hep-ph]].
[42] J. -S. Yu, Z. -F. Sun, X. Liu and Q. Zhao, Phys. Rev. D 83, 114007 (2011) [arXiv:1104.3064 [hep-ph]].
[43] X. Wang, Z. -F. Sun, D. -Y. Chen, X. Liu and T. Matsuki, Phys. Rev. D 85, 074024 (2012) [arXiv:1202.4139 [hep-ph]].
[44] Z. -C. Ye, X. Wang, X. Liu and Q. Zhao, Phys. Rev. D 86, 054025 (2012) [arXiv:1206.0097 [hep-ph]].
[45] L. -P. He, X. Wang and X. Liu, Phys. Rev. D 88, 034008 (2013) [arXiv:1306.5562 [hep-ph]].
[46] H. G. Blundell, hep-ph/9608473.
[47] C. Hayne and N. Isgur, Phys. Rev. D 25, 1944 (1982).
[48] M. Jacob and G. C. Wick, Annals Phys. 7, 404 (1959) [Annals Phys. 281, 774 (2000)].
[49] F. E. Close and E. S. Swanson, Phys. Rev. D 72, 094004 (2005) [hep-ph/0505206].