Double exotic state productions in pion and kaon induced reactions

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Abstract In the present work, we investigate the pion/kaon induced $P_c/P_{c5}$ productions with $Z_c$ state off proton target with an effective Lagrangian approach. Our estimations indicate that the cross sections depend on the model parameters and beam energies, in particular, the cross sections for $\pi p \rightarrow Z_c(3900)P_c, (4312)$ and $\pi p \rightarrow Z_c(3900)P_c, (4440)$ are similar and can reach up to 10 nb, while those for $\pi p \rightarrow Z_c(3900)P_c, (4457)$ can be 30 nb. As for $Kp \rightarrow Z_{c5}(3985)P_c$, the cross sections are about 2.5 times smaller than those for $\pi p \rightarrow Z_c(3900)P_c$. The discussed processes in the present work, especially $\pi p \rightarrow Z_c(3900)P_c, (4457)$ process, may be accessible by the high-energy pion and kaon beams in the facilities of J-PARC and COMPASS.

1 Introduction

The investigations of exotic states beyond the conventional hadrons are one of the most intriguing topics in hadron physics in the past two decades. Since the observations of $X(3872)$ in 2003 [1], a series of new hadron states have been observed (see Refs. [2–11] for recent reviews of experimental and theoretical status), which turns a new page of searching exotic states. Among these new hadron states, some of them are particular interesting, such as the series of $Z_c/Z_{c5}$ [12–18] and $P_c/P_{c5}$ states [19–22], which are good candidates of tetraquark [9,23] and pentaquark states [24–26], respectively.

The tetraquark candidate $Z_c(3900)$ was first observed by the BESIII and Belle Collaboration in 2011 [12,13] in the $J/\psi \pi^\pm$ invariant mass distributions of $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ process. Later the CLEO-c confirmed the existence of $Z_c^0(3900)$ by using the data of $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ process at $\sqrt{s} = 4170$ MeV [27]. Moreover, the CLEO-c Collaboration also reported the neutral isospin partner of $Z_c^+ (3900)$, i.e., $Z_c^0 (3900)$ for the first time [27], which was further observed by the BESIII Collaboration [18]. Similar to $Z_c(3900)$, the BESIII Collaboration reported another tetraquark candidate $Z_c(4020)$ in the $h_c\pi$ invariant mass spectrum of $e^+e^- \rightarrow \pi^+\pi^- h_c$ [16] and $e^+e^- \rightarrow \pi^0\pi^0 h_c$ [22] processes. Besides the hidden charm channel, these two tetraquark candidates have also been observed in the open charm channels [14,17]. In the $D^+\bar{D}^*$ invariant mass spectrum of $e^+e^- \rightarrow (D^*\bar{D}^*)^{\pm}\pi^\mp$ process, a state named $Z_c(3885)$ was reported by the BESIII Collaboration in 2013 [14], while in the $D^*\bar{D}^*$ invariant mass spectrum of $e^+e^- \rightarrow D^*\bar{D}^*\pi$, one state, $Z_c(4015)$ was also reported by the BESIII Collaboration in the same year [17]. As the strange partner of $Z_c(3900)$, $Z_{c5}(3985)$ was observed in the $D_s^- D^{0\ast} + D_s^{-\ast} D^0$ invariant mass spectrum of $e^+e^- \rightarrow K^+ (D_s^- D^{0\ast} + D_s^{-\ast} D^0)$ process by BESIII Collaboration [28], which makes the charmonium-like tetraquark family abundant. More recently, the LHCb collaboration reported a structure, $Z_{c5}(4003)$, in the $J/\psi \phi$ invariant mass spectrum of $B^+ \rightarrow J/\psi \phi K^+$ [29] process, the mass of $Z_{c5}(4003)$ is consistent with that of $Z_{c5}(3985)$, but the width is much different. The measured resonance parameters of these tetraquark candidates are collected in Table 1. For comparison, we also present the thresholds near the observed masses of these tetraquark candidates.

As pentaquark candidates, $P_c(4380)$ and $P_c(4450)$ were first observed in the $J/\psi p$ invariant mass spectrum of $A_h \rightarrow K J/\psi p$ decay process by the LHCb Collaboration in 2015 [19]. The lower mass state is a broader one with a width $\Gamma = 205 \pm 18 \pm 86$ MeV, while the higher mass state is much narrower with a width to be 39 $\pm 5 \pm 19$ MeV [19]. In 2019, the LHCb Collaboration reanalyzed the same process with a data sample to be nine times more than the one used in Ref. [19] and reported a new narrow pentaquark state $P_c(4312)$ [20], and the $P_c(4450)$ pentaquark structure was observed to consist of two narrow overlapping peaks.
Table 1 A summary the experimental information of Zc/Zcs and Pc/Pcs states

| State         | Threshold | Mass (MeV) | Width (MeV) | Channel                  | Experiment |
|---------------|-----------|------------|-------------|--------------------------|------------|
| Zc⁺ (3900)    | D⁺ D̄     | 3899.0 ± 3.6 ± 4.9 | 46 ± 10 ± 20 | e⁺ e⁻ → π⁺ π⁻ J/ψ      | BESIII [12] |
|               |           | 3894.5 ± 6.6 ± 4.5 | 63 ± 24 ± 26 |                           | Belle [13]  |
|               |           | 3901 ± 4     | 58 ± 27     |                           | CLEO-c [27] |
| Zc⁰ (3900)    |           | 3886 ± 4 ± 2 | 37 ± 4 ± 8  | e⁺ e⁻ → π⁰ π⁺ J/ψ      | CLEO-c [27] |
| Zc⁺ (3885)    |           | 3883.9 ± 1.5 ± 4.2 | 24.8 ± 3.3 ± 11.0 | e⁺ e⁻ → (D⁺ D̄)± π⁺   | BESIII [14] |
| Zc⁺ (4020)    | D⁺ D̄*    | 4022.9 ± 0.8 ± 2.7 | 7.9 ± 2.7 ± 2.6 | e⁺ e⁻ → π⁺ π⁻ h_c      | BESIII [16] |
| Zc⁰ (4020)    |           | 4023.9 ± 2.2 ± 3.8 | -           | e⁺ e⁻ → π⁰ π⁻ h_c      | BESIII [22] |
| Zc⁺ (4025)    |           | 4026.3 ± 2.6 ± 3.7 | 24.8 ± 5.6 ± 7.7 | e⁺ e⁻ → (D⁺ D̄)± π⁺   | BESIII [17] |
| Zcs⁺ (3985)   | Dcs⁻ Dcs⁰/Dcs⁻ Dcs⁰ | 3982.5±18.2±6.1 | 12.8±5.3±4.4 | e⁺ e⁻ → K⁺ (Dcs⁻ Dcs⁰ + Dcs⁻ Dcs⁰) | BESIII [28] |
| Zcs⁰ (4003)   |           | 4003 ± 6.3±14 | 131 ± 15 ± 26 | B⁺ → J/ψ K⁻ p         | LHCb [29]  |
| Pc⁺ (4380)    | Σ⁺ D̄     | 4380 ± 8 ± 9 | 205 ± 18 ± 86 | Λc⁰ → J/ψ K⁻ p         | LHCb [19]  |
| Pc⁺ (4450)    | Σ⁺ D̄*    | 4457.3 ± 0.6±1.7 | 6.4 ± 2.0±1.9 |                           | LHCb [19]  |
| Pc⁺ (4312)    | Σ⁺ D̄     | 4311.9 ± 0.7±6.8 | 9.8 ± 2.7±3.7 |                           | LHCb [20]  |
| Pc⁺ (4440)    | Σ⁺ D̄*    | 4440±1.3±4.7 | 20.6 ± 4.9±8.7 |                           | LHCb [20]  |
| Pc⁺ (4457)    | Σ⁺ D̄*    | 4457.3 ± 0.6±1.7 | 6.4 ± 2.0±1.9 |                           | LHCb [20]  |
| Pc⁺ (4459)    | Σ⁺ D̄*    | 4458.8 ± 2.9±1.7 | 17.3 ± 6.5±8.0 |                           | LHCb [21]  |

Pc (4440) and Pc (4457) [20]. Later in 2020, the LHCb Collaboration reported an open-strange and hidden-charm pentaquark state, named \( P⁺_{cs} \) (4459) in \( J/ψ\Lambda \) invariant mass spectrum of \( Ξ⁻_b → K⁻ J/ψ\Lambda \) decay process [21]. The resonance parameters of these pentaquark states are also collected in Table 1.

These observations stimulated theorists great interests to investigate the nature and internal structure of these tetraquark and pentaquark candidates. The observed channels indicates that the most possible quark components of \( Zc/Zcs \) states should be \( ccq̅q̅/ccq̅s̅ \), while those of \( Pc/Pcs \) should be \( ccu̅d/u̅c̅d, s̅c̅u̅d/u̅c̅d, s̅q̅u̅d/u̅c̅d \), which means that these states can be naturally considered as multiquark states. In the compact tetraquark scenario, the mass spectrum [30–41] and decay properties [36,42–45] of \( Zc/Zcs \) have been investigate by various methods, such as QCD sum rule, constituent quark model. The estimations by QCD sum rule [46–48] and constituent quark model [39,49–52] supported the compact pentaquark interpretations of \( P⁺ /Pcs \).

As presented in Table 1, all these observed tetraquark and pentaquark candidates are close to the thresholds of a pair of hadrons. Then, all these states can be considered as molecular states. In the \( D⁺ D̄* / D̄ D̄* \) molecular scenario, the mass spectrum of \( Zc^{(*)} / Zcs \) were estimated by using QCD sum rule [37,47,48,53] and potential model with one-boson-exchange potential [54–57]. The decay [58,59] and production properties [60–66] were also investigated in an effective Lagrangian approach. As for \( P⁺ /Pcs \), their properties, including mass spectrum [67–70], decay properties [60,71–73] and production behaviors [74–76] have been investigated in the molecular scenarios. Besides the molecular interpretation, these tetraquark and pentaquark states were also considered as structures generated by some special kinematics mechanisms, such as cusp effect [77,78], initial single chiral partial emission mechanism [79,80], and triangle singularity [81].

As one of important aspects of evaluating the properties of the tetraquark and pentaquark candidates, the productions of these states have been widely investigated. For example, in Ref. [61], the production process \( Y (4260) → Zc (3900)π \) was investigated in a molecular scenario, where \( Y (4260) \) and \( Zc (3900) \) were considered as D̄ D̄* + h.c and D̄ D̄* + h.c molecular states, respectively. The productions of \( Zc^{(*)} \) states from \( B_c \) decays were predicted in Ref. [82]. Moreover, the productions of \( Zc \) in the \( πp \) [63], \( γp \) [35] process have also been investigated in an effective Lagrangian approach. As for \( P⁺ /Pcs \) states, the observed production process has been investigated in Refs. [75,83] and the productions of these pentaquark states from \( πp \) scattering [76] and bottom baryon decay processes [81,84–86] were predicted.

It worth to mention that in the high energy pion/kaon induced reaction, the contributions from \( s^- \) and \( u^- \) channels are much smaller than the \( t \)-channel contribution, which indicates that the \( s^- \) and \( u^- \) channels can usually be ignored. Considering the strong coupling between \( Zc \) and \( J/ψπ \), the process \( πp → Zc p \) could occur via a \( J/ψ \) exchange while the \( J/ψ \) couples to the proton via vector meson dominance as indicated in Ref. [63]. The cross sections were predicted to be of order of 1 nb when considering \( J/ψπ \) channel to be the
dominant one of $Z_c$. Actually, the exchanged $J/\psi$ and proton target can strongly couples to the pentaquark candidate $P_c$ states directly. Thus, one can construct a possible double exotic production process, i.e., $\pi p \rightarrow Z_c P_c$. In a same way, one can replace the pion beam by the kaon beam to construct a very similar double exotic production process $K p \rightarrow Z_c P_c$. In the present work, we adopt an effective Lagrangian approach to estimate the cross sections for these two kinds of double exotic production processes, which may be accessible by the high-energy pion and kaon beams in the facilities of J-PARC [87,88] and COMPASS [89].

This work is organized as follows. After the introduction, we present the effective Lagrangians used in the present work and the amplitudes in Sect. 2. The numerical results of the total cross section and the differential cross sections are present in Sect. 3 and we give a brief summary of this work in the last section.

2 Pion and kaon induced productions on a proton target

As indicated in Fig. 1, the double exotic production process $\pi p \rightarrow P_c Z_c$ occur via a $J/\psi$ exchange. These diagrams can be estimated in an effective Lagrangian approach, where the relevant effective Lagrangians read [74,90],

$$\mathcal{L}_{\pi \psi Z_c} = \frac{g_{\pi \psi Z_c}}{m_{Z_c}} (\partial_\mu \psi_\nu \partial^\mu \pi Z_c^\nu - \partial_\mu \psi_\nu \partial^\nu \pi Z_c^\mu),$$

$$\mathcal{L}_{K \psi Z_{cs}} = \frac{g_{K \psi Z_{cs}}}{m_{Z_{cs}}} (\partial_\mu \psi_\nu \partial^\mu \pi Z_{cs}^\nu - \partial_\mu \psi_\nu \partial^\nu \pi Z_{cs}^\mu),$$

$$\mathcal{L}^{1/2-}_{P_c \psi p} = \frac{g_{P_c \psi p}}{m_{P_c}} \bar{p}_f \gamma_5 \gamma_\mu P_c \psi_\mu + H.c.,$$

$$\mathcal{L}^{3/2-}_{P_c \psi p} = \frac{-ig_{P_c \psi p}}{2m_{P_c}} \bar{p}_f \gamma_\mu \psi_\mu \gamma_\nu p + H.c.,$$

where $\psi_{\mu \nu} = \partial_\mu \psi_\nu - \partial_\nu \psi_\mu$. The concrete values of the relevant coupling constants will be discussed in the next section.

With the above effective Lagrangians, one can obtain the amplitudes corresponding to the diagrams in Fig. 1, which are

$$\mathcal{M}^{1/2-} = \frac{g_{P_c \psi p}}{2m_{P_c}} \left[ \bar{u}(p_1, m_1) \gamma_\mu u(p_2, m_2) \right] \times g_{Z_c \pi \psi} \left[ (q^\mu p_1^\nu - q^\nu p_1^\mu) q^{\nu \rho} - (q^\mu q^\nu - m_\psi^2) q^{\rho \gamma} \right] F(q^2, m_\psi),$$

$$\mathcal{M}^{3/2-} = \frac{-ig_{P_c \psi p}}{2m_{P_c}} \left[ \bar{u}(p_1, m_1) \gamma_\mu u(p_2, m_2) \right] \times g_{Z_c \pi \psi} \left[ (q^\mu p_1^\nu - q^\nu p_1^\mu) q^{\nu \rho} - (q^\mu q^\nu - m_\psi^2) q^{\rho \gamma} \right] F(q^2, m_\psi).$$

3 Numerical results and discussion

3.1 Coupling constants

In the present work, an effective Lagrangian approach is adopt to estimate the cross sections for the double exotic states production process. The effective coupling constants in Eq. (1) can be determined by using the corresponding partial

\[ \frac{d\sigma}{d\cos \theta} = \frac{1}{32\pi s} \left| \frac{\bar{p}_f}{|\bar{p}_f|} \left( \frac{1}{2} |\mathcal{M}|^2 \right) \right|, \]

where $\mathcal{M}^{1/2-}$ and $\mathcal{M}^{3/2-}$ are the amplitudes of $\pi p \rightarrow Z_c P_c$ with the $J^P$ quantum numbers of $P_c$ state being $1^-\gamma$ and $3^-\gamma$, respectively. In particular, $\mathcal{M}^{1/2-}$ can be the amplitudes of $\pi p \rightarrow Z_c(3900) P_c(4312)$ and $\pi p \rightarrow Z_c(3900) P_c(4450)$, while the amplitude $\mathcal{M}^{3/2-}$ corresponds to the process $\pi p \rightarrow Z_c(3900) P_c(4457)$. In the amplitudes, a form factor, $F(q^2, m_\psi)$, is introduced to depict the internal structure and the off-shell effect of the exchanged $J/\psi$. In this work, we adopt the form factor in the dipole form, which is [91,92],

\[ F(q^2, m_\psi) = \frac{m_\psi^2 - \Lambda^2}{q^2 - \Lambda^2}. \]
widths. From the effective Lagrangians listed in Eq. (1), one can estimate the partial widths of \( P_c(p_0) \to J/\psi(p_1)p(p_2) \) and \( Z_c(p_0) \to J/\psi(p_1)\pi(p_2) \), which are,

\[
\Gamma_{K \to J/\psi p}^{1/2} = \frac{g_{K^+ J/\psi p}^2}{8\pi m_0^2 m_1^2} \left[ (m_2^2 + m_0^2 - m_1^2) \right], \\
\times (m_2^2 - m_0^2 + 2m_1^2), \\
\Gamma_{K \to J/\psi p}^{3/2} = \frac{g_{K^+ J/\psi p}^2}{192\pi m_0^2 m_1^2} \left[ (3m_0^2 - m_1^2(m_1^2 + 5m_2^2) + 12m_0^2m_2^2 + m_0^2(m_2^4 - m_1^4) - (m_2^4 - m_1^2)^2) \right], \\
\Gamma_{Zc \to J/\psi \pi} = \frac{g_{Zc J/\psi \pi}^2}{96\pi m_0^2} \left[ 2m_0^2 - m_0^4(3m_2^2 + 4m_2^4) + 2m_0^2(3m_2^2m_2^2 + m_2^4) + (m_1^2 - m_1^2m_2^2)^2 \right], \quad (5)
\]

respectively. Here \(|\vec{p}| = \lambda(m_0^2, m_1^2, m_2^2)/2m_0 \) and \( \lambda \) is the Källen function with the definition \( \lambda(x, y, z) \equiv \sqrt{x^2 + y^2 + z^2 - 2xy - 2xz - 2yz} \).

As for \( Z_c(3900) \), the partial width ratio of \( D^*D \) and \( J/\psi \pi \) channels is measured to be \( 14 \) [14].

\[
\Gamma(Z_c \to D^*D) / \Gamma(Z_c \to J/\psi \pi) = 6.2 \pm 1.1 \pm 2.7. \quad (6)
\]

With the assumption that \( Z_c(3900) \) dominantly decays into \( D^*D \) and \( J/\psi \pi \) and considering the width of \( Z_c(3900) \) [93], one can estimate the partial width of \( Z_c(3900) \to J/\psi \pi \) channel, and then the coupling constant \( g_{Zc J/\psi \pi} \) can be evaluated by using Eq. (5). As for \( Z_{cs}(3985) \), the experimental measurements are still not abundant. Here, we assume that the branching ratio of \( Z_{cs}(3985) \to J/\psi K \) is the same as the one of \( Z_c(3900) \to J/\psi \pi \) by considering the SU(3) symmetry [94].

As for \( P_c \) states, only the \( J/\psi p \) decay mode have been observed experimentally at present, which indicates that the \( J/\psi p \) should be one of important decay modes of \( P_c \) states. The branching ratios of \( P_c \to J/\psi p \) were estimated from several percents to several tens percents depending on different models [75]. In the present work, the branching ratios of \( P_c \to J/\psi p \) are assumed to be 10%. The the coupling constants \( g_{P_c J/\psi p} \) can be evaluated by using Eq. (5) with the measured widths of \( P_c \) states. All the estimated coupling constants have been collected in Table 2. The error of the coupling constants are resulted from the uncertainties of the measured widths.

### 3.2 Cross sections for double exotic states production processes

With the above preparation, we can estimate the cross sections for the double exotic states production processes. The cross sections for \( \pi p \to Z_c(3900)P_c \) are presented in Fig. 2, where diagrams (a), (b), and (c) correspond to \( P_c(4312) \), \( P_c(4440) \), and \( P_c(4457) \) productions, respectively. The band in the diagrams are the uncertainties of the cross sections resulted from the relevant coupling constants. For comparison, we take the same model parameter \( \Lambda \) as the one used in the estimation of \( \pi p \to Z_c(3900)P_c \) [63], and then we can compare the cross sections for double exotic production process with those for \( \pi p \to Z_c(3900)P_c \) [63]. Our estimations indicate that the cross sections for \( \pi p \to Z_c(3900)P_c \) increase very fast near the thresholds and then become rather flat at higher pion momentum region. The cross sections can be greater than 10 nb when the momentum of pion is 70 GeV with \( \Lambda = 1.0 \) GeV. In particular, when including the effect of model parameter and the uncertainties resulted from the relevant coupling constants, we find the cross sections for \( \pi p \to Z_c(3900)P_c(4312) \) is 1 \( \sim \) 13 nb when \( \Lambda \) varying from 1.0 to 2.0 GeV at \( p_\pi = 70 \) GeV. As for \( \pi p \to Z_c(3900)P_c(4440) \) and \( \pi p \to Z_c(3900)P_c(4457) \), the cross sections at \( p_\pi = 70 \) GeV are 2 \( \sim \) 20 nb and 6 \( \sim \) 72 nb, respectively. The cross sections for \( K p \to Z_{cs}(3985)P_c \) are presented in Fig. 3, where diagrams (a), (b), and (c) correspond to \( P_c(4312) \), \( P_c(4440) \), and \( P_c(4457) \) productions, respectively. The behaviors of the cross sections are very similar to those of \( \pi p \to Z_c(3900)P_c \), which show sharp increases near the threshold but the cross sections for \( K p \to Z_{cs}(3985)P_c \) are about 2.5 times smaller than those for \( \pi p \to Z_c(3900)P_c \) due to the coupling constants \( g_{Z_{cs} J/\psi K} \) is smaller than the one of \( g_{Z_c J/\psi \pi} \).

As shown in Figs. 2 and 3, the energy and model parameter dependences of the cross sections for \( \pi p \to Z_c(3900)P_c \) for
The same as Fig. 2 but for $Kp \rightarrow Z_{c1}(3985)P_c$.

Fig. 4 The ratios of cross sections depending on beam energy and model parameter $\Lambda$. Diagrams (a) and (b) present the beam energy dependences for processes $\pi p \rightarrow Z_c(3900)P_c$ and $Kp \rightarrow Z_{c1}(3985)P_c$ with $\Lambda = 2.0$, respectively. Diagram (c) and (d) present the model parameter dependences for $\pi p$ and $Kp$ reactions with beam energy to be 55 GeV, respectively. The bands are the uncertainties resulted from the error of $P_c$, $P_c$, $Z_{cs}$, and $P_{c}$ widths.

$P_c(4312), P_c(4440)$, and $P_c(4457)$ are very similar, then one can define cross sections ratio as following,

$$R_{10} = \frac{\sigma(\pi p \rightarrow Z_c(3900)P_c(4440))}{\sigma(\pi p \rightarrow Z_c(3900)P_c(4312))}$$

$$R_{20} = \frac{\sigma(\pi p \rightarrow Z_c(3900)P_c(4457))}{\sigma(\pi p \rightarrow Z_c(3900)P_c(4312))},$$

which are expected to be independent on the energy and the model parameter. In Fig. 4, the ratios of the cross sections for $\pi p \rightarrow Z_c(3900)P_c$ and $Kp \rightarrow Z_{c1}(3985)P_c$ depending on the model parameter and beam energy are presented. The bands in the diagrams indicate the uncertainties resulted from the error of $P_c$, $P_c$, $Z_{cs}$ widths, while the errors of $Z_c$, $Z_{cs}$ widths are cancelled. As shown in Fig. 4c, d, the ratios are entirely independent on the model parameter $\Lambda$. By using the center values of the $P_c$ width, our estimations indicate that the cross sections for $P_c(4457)$ productions from $\pi p$ and $Kp$ reactions are about 3 times larger than those of $P_c(4440)$ and $P_c(4312)$, while cross sections for $P_c(4440)$ and $P_c(4312)$ are almost the same.

Besides the cross sections, the differential cross sections depending on the scattering angle related to the outgoing $P_c$ and the incident pion/kaon beam are also estimated. In Fig. 5, we present the differential cross sections for $\pi p \rightarrow Z_c(3900)P_c$ depending on $\cos \theta$, where diagrams (a), (b), and (c) correspond to $P_c(4312), P_c(4440)$, and $P_c(4457)$ production, respectively. Here, the pion beam momentum is set to be 40, 50, and 60 GeV, respectively. Moreover, the cross sections uncertainties resulted from the relevant coupling constants will only affect the magnitude of differential cross sections, but have no influence on the relative distributions of $\cos \theta$, thus in Fig. 5, we only present the results estimated with the center values of the coupling constants. From the diagrams, one can find that the differential cross sections reach their maximum at the forward angle limit, i.e., $\cos \theta = 1$. For the case of $P_c = 40$ GeV, the beam energy is just a bit larger than the threshold of the reaction, thus the differential cross sections do not strongly depend on the scattering angle. With the $P_c$ increasing, more produced pentaquark or tetraquark states are concentrated in the forward angle area. Moreover, the $\cos \theta$ dependence of the differential cross sections are very similar for $P_c(4312), P_c(4440)$, and $P_c(4457)$ due to the similarity of these double exotic production process.

The differential cross sections for $Kp \rightarrow Z_{c1}(3985)P_c$ are presented in Fig. 6, where the kaon beam momentum are also set to be 40, 50, and 60 GeV, respectively. Our estimations indicate that the scattering angle dependences of the cross sections are almost the same as those of $\pi p \rightarrow Z_c(3900)P_c$ due to the similarity between these two kinds of double exotic production processes.
4 Summary

As the typical tetraquark and pentaquark candidates, $Z_c/Z_{cs}$ and $P_c/P_{cs}$, have stimulated theorists’ great interests, their properties have been investigated from various aspects, such as mass spectra, decay behaviors and production processes, which are expected to reveal the inner structure of these multiquark states.

In the present work, we evaluate the experimental potential of the double exotic production in the $\pi p$ and $Kp$ scattering processes. The cross sections for $\pi p \rightarrow Z_c(3900)P_c$ and $Kp \rightarrow Z_{cs}(3985)P_c$ are evaluated. Our estimations indicate that the cross sections depend on the model parameter and the beam energy. In particular, at $P_\pi = 70$ GeV the cross sections for $\pi p \rightarrow Z_c(3900)P_c$ can reach up to 13, 20, and 72 nb for $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$, at $\Lambda = 1.0$ GeV, respectively. Moreover, the beam energy and model parameter dependences of the cross sections are very similar for three $P_c$ states, thus the ratios of these cross section are almost independent on the beam energy and model parameter, which can be tested by further experimental measurements. In addition, the beam energy and model parameter dependences of the cross sections for $Kp \rightarrow Z_{cs}(3985)P_c$ are almost the same as those of $\pi p \rightarrow Z_c(3900)P_c$. Besides the cross sections, the differential cross sections for the considered processes are also evaluated and our estimations indicate that more produced pentaquark or tetraquark states are concentrated in the forward angle area.

At the end of the present work, we would like to discuss the experimental potential of these double exotic production processes by comparing the cross sections for $\pi p \rightarrow Z_c(3900)P_c$ with those for $\pi p \rightarrow Z_c(3900)p$ [63]. In Ref. [63], the cross section for $\pi p \rightarrow Z_c(3900)p$ at $P_\pi = 70$ GeV is of order of 100 nb with the assumption that $J/\psi \pi$ should be the dominant decay channel of $Z_c(3900)$. However, the experimental measurements indicated that the branching fraction of $Z_c(3900) \rightarrow J/\psi \pi$ should be of order 10% as indicated in Eq. (6), thus the cross sections for $\pi p \rightarrow Z_c(3900)p$ should be about 10 nb at $P_\pi = 70$ GeV, which is the same order as the cross sections for $\pi p \rightarrow Z_c(3900)P_c(4312)/P_c(4440)$ and about several times smaller than the one of $\pi p \rightarrow Z_c(3900)P_c(4457)$. Thus, if the experimental measurement in COMPASS or J-PARC could detect $Z_c(3900)$ in the $\pi p \rightarrow Z_c(3900)p$ process, the double exotic production process, especially $\pi p \rightarrow Z_cP_c(4457)$, should be more easily accessible in these facilities.

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