Optimal spin-quantization axes
for quarkonium with large transverse momentum

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Abstract

The gluon collision process that creates a heavy-quark-antiquark pair with small relative momentum and large transverse momentum predicts at leading-order in the QCD coupling constant that the transverse polarization of the pair should increase with its transverse momentum. Measurements at the Fermilab Tevatron of the polarization of charmonium and bottomonium states with respect to a particular spin-quantization axis are inconsistent with this prediction. However the predicted rate of approach to complete transverse polarization depends on the choice of spin-quantization axis. We introduce axes that maximize and minimize the transverse polarization from the leading-order gluon collision process. They are determined by the direction of the jet that provides most of the balancing transverse momentum.

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Important information about the fundamental interactions between elementary particles is encoded in the spins of particles produced in high-energy collisions. It is difficult to access that information, because the spin cannot be measured directly. The measurement of the spin state of a stable particle is usually not possible in a high-energy collider experiment. Information about the spin state of an unstable particle can be inferred from the angular distribution of its decay products. That angular distribution can be strongly correlated with the momenta of other particles in the final state, and if they are not measured, information about the spin is diluted. Hadron collisions have the additional complication that the fundamental interactions involve collisions of partons with longitudinal momenta that must be integrated over, which further dilutes the information about the spin.

The accessible information about the spin of the unstable particle could in principle be obtained by measuring the complete angular distribution of its decay products for all possible values of the other relevant final-state variables. In practice, there is usually not enough data to make this feasible. The challenge is to find simple measurements that extract as much information as possible about the spin. In a two-body decay, the simplest measurement is the distribution of a polar angle $\theta$ with respect to a specified axis. If that axis is used as the spin-quantization axis (SQA) for the unstable particle, the $\theta$ distribution gives the probabilities for the projection of the spin along that axis. Additional information about the spin could be obtained by also measuring the polar angle distribution about a second SQA. These SQA’s can be chosen as functions of other final-state variables to maximize the information about the spin.

An interesting unsolved problem in high-energy physics is the spin dependence of the production rates for heavy quarkonia, whose constituents are a heavy (charm or bottom) quark and its antiquark. The nonrelativistic QCD (NRQCD) factorization framework exploits the large mass of the heavy quark to express a cross section for inclusive quarkonium production as the sum of products of parton cross sections that can be calculated using perturbative QCD and NRQCD matrix elements that can be measured in other experiments [1]. This approach leads to unambiguous predictions for the spin dependence of the cross sections. Measurements of the spin dependence of charmonium and bottomonium states produced at the Tevatron $p\bar{p}$ collider [2, 3, 4] are inconsistent with predictions based on leading-order parton cross sections [5, 6, 7, 8]. The resolution of the discrepancy may lie in large perturbative corrections to the parton cross sections [9, 10].
Most theoretical predictions and the experimental measurements of quarkonium polarization have been carried out for a single SQA. In this paper, we point out that more detailed information about the spin can be extracted by measuring the polarization for a pair of SQA’s for which the predicted polarizations are as different as possible. We construct a pair of optimal SQA’s for spin-triplet $S$-wave quarkonium states with large transverse momentum under the assumption that the dominant parton process is a gluon-gluon collision.

To be specific, we consider the production of the spin-1 charmonium state $J/\psi$ in hadron collisions. There is a simple argument that a direct $J/\psi$, i.e. one that does not come from the decay of a heavier particle, should be increasingly transversely polarized as its transverse momentum $Q_T$ increases $[11]$. Of the leading-order parton processes that produce a $c\bar{c}$ pair with large $Q_T$, the gluon collision process $gg \rightarrow (c\bar{c})g$ has the largest cross section. At asymptotically large $Q_T$, this cross section is dominated by gluon fragmentation, the production of an almost on-shell gluon by $gg \rightarrow gg$ followed by the splitting of that virtual gluon into a collinear $c\bar{c}$ pair. The $c\bar{c}$ pair is created in a color-octet state and is predominantly transversely polarized for almost any choice of SQA. Heavy-quark spin symmetry implies that the binding of this $c\bar{c}$ pair into a $J/\psi$ is unlikely to change the spin states of the heavy quarks. Thus, the $J/\psi$ should be increasingly transversely polarized as $Q_T$ increases. These qualitative arguments are supported by quantitative calculations using the NRQCD factorization formulas with leading-order parton cross sections $[5, 6, 7]$. However, measurements by the CDF Collaboration seem to be completely incompatible with these predictions $[2]$.

The longitudinal polarization 4-vector $\epsilon_L$ for a $J/\psi$ with 4-momentum $Q$ must satisfy $Q \cdot \epsilon_L = 0$ and $\epsilon_L^2 = -1$. The most general 4-vector satisfying these conditions can be written in the form

$$\epsilon_L^\mu = \tilde{X}^\mu / \sqrt{-\tilde{X}^2}, \quad \tilde{X}^\mu = (-g^{\mu\nu} + Q^{\mu}Q^{\nu}/Q^2)X^\nu,$$

where $X$ is a 4-vector. The physical interpretation of $X$ is that in a $J/\psi$ rest frame, the direction of the 3-vector $-X$ is the SQA of the $J/\psi$. If $X$ is timelike, the projection of the spin along the SQA coincides with the helicity in the rest frame of $X$. If the $J/\psi$ is produced in the collision of two hadrons with 4-momenta $P_1$ and $P_2$, the SQA is generally chosen to lie in the production plane defined by the momenta of the colliding hadrons in the $J/\psi$ rest frame. Thus, $X$ in Eq. (1) has the form

$$X^\mu = aP_1^\mu + bP_2^\mu,$$
where \(a\) and \(b\) are scalar functions. A simple example is the c.m. helicity axis specified by \(X_{\text{cmh}} = P_1 + P_2\). The projection of the spin of the \(J/\psi\) along this axis is its helicity in the center-of-momentum (c.m.) frame of the colliding hadrons. In the \(J/\psi\) rest frame, the SQA specified by Eq. \(2\) is antiparallel to the unit vector \(\hat{X} = (aP_1 + bP_2)/(|aP_1 + bP_2|)\), so it is determined by the ratio \(a/b\). If the \(J/\psi\) is the only particle in the final state whose momentum is measured, then \(a/b\) can only depend on the 4-vectors \(P_1\), \(P_2\), and \(Q\). If additional information about the final state is measured, \(a/b\) can also depend on this information.

If a cross section is dominated by a specific parton process, a natural prescription for a pair of optimal SQA’s is that they maximize and minimize the transverse cross section from that parton subprocess at leading order in the QCD coupling constant \(\alpha_s\). As a simple illustration of a pair of optimal SQA’s, we consider dilepton production by the Drell-Yan mechanism \(q\bar{q} \rightarrow \mu^+\mu^-\) in the parton model with intrinsic transverse momentum \([12, 13]\), which can be regarded as a model for the effects of soft gluon radiation from the colliding partons. The cross section for a dilepton with longitudinal polarization vector given by Eqs. \(1\) and \(2\) is

\[
\hat{\sigma}_L = \frac{8\pi^2 e_q^2 a\langle k_1^2 \rangle (a^2 x_2^2 + b^2 x_1^2)}{3Q^2(a x_2 - b x_1)^2} \delta(x_1 x_2 s - Q^2),
\]

where \(x_1\) and \(x_2\) are the longitudinal momentum fractions of the colliding partons, \(e_q\) is the electric charge of the quark, and \(\langle k_1^2 \rangle\) is the mean-square transverse momentum of the colliding partons. The cross section has been expanded to second order in the intrinsic transverse momentum vectors and then averaged over them. The longitudinal cross section in Eq. \(3\) depends on the ratio \(a/b\). It is minimized by choosing \(a/b = -x_1/x_2\). It can be maximized by choosing \(a/b = +x_1/x_2\), so that the denominator vanishes. The values \(a/b = \mp x_1/x_2\) that maximize and minimize the transverse cross section depend on the parton momentum fractions only through the ratio \(x_1/x_2\). Under the assumption that the cross section is dominated by \(q\bar{q} \rightarrow \mu^+\mu^-\), we can derive an expression for \(x_1/x_2\) in terms of variables that can be directly measured. At leading order in the intrinsic transverse momentum vectors, the energy-momentum conservation condition \(Q = x_1 P_1 + x_2 P_2\) implies \(x_1/x_2 = Q \cdot P_2/Q \cdot P_1\). The 4-vectors \(X\) in Eq. \(2\) associated with the maximal and minimal SQA’s can then be expressed as

\[
X_{\text{CS/\perp h}}^\mu = \frac{P_1^\mu}{Q \cdot P_1} \mp \frac{P_2^\mu}{Q \cdot P_2}.
\]
The upper and lower signs correspond to the *Collins-Soper axis* \[12\] and the *perpendicular helicity axis* introduced in Ref. \[14\], respectively. The perpendicular helicity axis is so named because the projection of the total spin of the dilepton along this SQA coincides with its helicity in the frame obtained from the hadron c.m. frame by a longitudinal boost that makes the dilepton momentum perpendicular to the beam direction. Lam and Tung pointed out that the Collins-Soper axis maximizes the transverse polarization in the parton model with intrinsic transverse momentum \[13\]. That the perpendicular helicity axis minimizes the transverse polarization in this model does not seem to have been pointed out previously.

We now consider the direct production of a $J/\psi$ with large transverse momentum $Q_T \gg \Lambda_{\text{QCD}}$. Since the mass $m_c$ of the charm quark is large compared to the scale $\Lambda_{\text{QCD}}$ of nonperturbative effects in QCD, the inclusive cross section can be calculated using NRQCD factorization formulas \[1\]. The leading-order parton processes that create a $c\bar{c}$ pair with small relative momentum are $q\bar{q} \rightarrow (c\bar{c})g$, $gg \rightarrow (c\bar{c})q$, $\bar{q}g \rightarrow (c\bar{c})\bar{q}$, and $gg \rightarrow (c\bar{c})g$. The parton process with the largest cross section is $gg \rightarrow (c\bar{c})g$. This process is further enhanced by the growth of the gluon distribution at small values of the parton momentum fraction $x$ as the momentum scale increases. We therefore focus on the parton process $gg \rightarrow (c\bar{c})g$.

The color-octet $^3S_1$ state of the $c\bar{c}$ pair [$c\bar{c}\delta(^3S_1)$] dominates at asymptotic $Q_T$, because it has a fragmentation contribution. At leading order in $\alpha_s$, the dependence of the longitudinal differential cross section on the scalar functions $a$ and $b$ defined by Eq. \[2\] is

$$Q_0 d\hat{\sigma}_L \propto \frac{a^2 x_2 \hat{u}^2 + b^2 x_1 \hat{t}^2 + (ax_2 - bx_1)^2 \hat{s}^2}{(aw_1 + bw_2)^2 - abQ^2 s}, \quad (5)$$

where $w_1 = Q \cdot P_1$, $w_2 = Q \cdot P_2$, $s$ is the c.m. energy of the colliding hadrons, and $\hat{s} = x_1 x_2 s$, $\hat{t} = 4m_c^2 - 2x_1 w_1$, and $\hat{u} = 4m_c^2 - 2x_2 w_2$ are the parton Mandelstam variables. There is a delta function constraint on these variables:

$$2(x_1 w_1 + x_2 w_2) = x_1 x_2 s + 4m_c^2. \quad (6)$$

Minimizing and maximizing Eq. \[5\] with respect to $a/b$, we get

$$\left.\frac{a}{b}\right|_{gg,\text{max}} = \left.\frac{x_1 (\hat{s} - \hat{t})}{x_2 (\hat{s} - \hat{u})}\right|_{gg,\text{max}}, \quad (7)$$

$$\left.\frac{a}{b}\right|_{gg,\text{min}} = \left.\frac{x_1 [\hat{s}^2 (\hat{t} - \hat{u}) + \hat{t}^2 (\hat{s} - \hat{u})]}{x_2 [\hat{s}^2 (\hat{t} - \hat{u}) + \hat{u}^2 (\hat{t} - \hat{s})]}\right|_{gg,\text{min}}. \quad (8)$$
At asymptotic transverse momentum $Q_T \gg 2m_c$, the corresponding 4-vectors reduce to

\[
X_{gg,\text{max}}^\mu \rightarrow \frac{x_1 P_1^\mu}{x_1 w_1 + 2x_2 w_2} + \frac{x_2 P_2^\mu}{x_2 w_2 + 2x_1 w_1},
\]

(9)

and

\[
X_{gg,\text{min}}^\mu \rightarrow \frac{P_1^\mu}{w_1} - \frac{P_2^\mu}{w_2} = X_{CS}^\mu.
\]

(10)

Note that the minimal $gg$ axis reduces in this limit to the Collins-Soper axis.

Our optimality criteria were based on the assumption that the parton process $gg \rightarrow c\bar{c}8(^3S_1) + g$ dominates. If that parton process implies values for $x_1$ and $x_2$ that are determined by a measurable property of the final state, we can insert those values into Eqs. (7) and (8) to obtain optimal SQA’s that are experimentally useful. The large transverse momentum $Q_T$ of the $c\bar{c}$ pair is balanced by that of the recoiling gluon, which produces a jet of hadrons with nearly collinear momenta. The polar angle of the recoiling gluon in the hadron c.m. frame is approximately equal to the polar angle $\theta_{\text{jet}}$ of the jet. The ratio $x_1/x_2$ can be expressed as a function of $\theta_{\text{jet}}$ and the transverse and longitudinal momenta $Q_T$ and $Q_L$ of the $J/\psi$ in the hadron c.m. frame:

\[
\frac{x_1}{x_2} = \frac{(Q_0 + Q_L) \sin \theta_{\text{jet}} + Q_T (1 + \cos \theta_{\text{jet}})}{(Q_0 - Q_L) \sin \theta_{\text{jet}} + Q_T (1 - \cos \theta_{\text{jet}})},
\]

(11)

where $Q_0 = (Q_T^2 + Q_L^2 + M_{J/\psi}^2)^{1/2}$. Given the value of $x_1/x_2$, the delta function constraint in Eq. (6) can be solved for $x_1$ and $x_2$. The two-fold ambiguity associated with the root of a quadratic polynomial is resolved by choosing the smaller values of $x_1$ and $x_2$. Inserting them into Eqs. (7) and (8), we obtain expressions for $a/b$ that depend on quantities that can be directly measured. We will refer to the corresponding SQA’s as the maximal and minimal $gg$ axes, respectively. The prescriptions for these optimal SQA’s can be extended beyond leading order in $\alpha_s$ by choosing $\theta_{\text{jet}}$ in Eq. (11) to be the angle in the hadron c.m. frame of the jet with the largest transverse energy. Since the direction of the jet is insensitive to soft gluon radiation and to the splitting of a parton into collinear partons, QCD radiative corrections to the polar angle distributions defined by our optimal SQA’s can be calculated systematically using perturbative QCD.

To illustrate the use of pairs of SQA’s, we consider direct $J/\psi$ production at the Tevatron, which is a $p\bar{p}$ collider with c.m. energy 1.96 TeV. We use NRQCD factorization formulas with leading-order parton cross sections to calculate the transverse and longitudinal cross sections for $J/\psi$. For the NRQCD matrix elements, we use central CTEQ5L values from Ref. [7] with $x = 1/2$: $\langle O_1^{J/\psi}(^3S_1) \rangle = 1.4$ GeV$^3$, $\langle O_8^{J/\psi}(^3S_1) \rangle = 0.0039$ GeV$^3$, $\langle O_8^{J/\psi}(^1S_0) \rangle = 0.033$ GeV$^3$. 

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FIG. 1: Polarization variable $\alpha$ for various spin-quantization axes as functions of $Q_T/M_{J/\psi}$ for direct $J/\psi$ at the Tevatron (left panel) and direct $\Upsilon(1S)$ at the LHC (right panel). The SQA’s are the maximal and minimal $gg$ axes (upper and lower solid lines), the perpendicular helicity and Collins-Soper axes (upper and lower dashed lines at large $Q_T$), and the c.m. helicity axis (dotted line).

and $\langle O_8^{J/\psi}(3P_0)\rangle/m_c^2 = 0.0097$ GeV$^3$. In the parton cross sections, we set $m_c = M_{J/\psi}/2 = 1.5$ GeV. We use the CTEQ6L parton distributions [15] with 3 flavors of quarks and the next-to-leading-order formula for $\alpha_s(\mu)$ with 4 flavors of quarks and $\Lambda_{QCD} = 326$ MeV. The factorization and renormalization scales are set to $\mu = (Q^2 + Q_T^2)^{1/2}$. We impose a rapidity cut $|y| < 1$ on the $J/\psi$ momentum.

A convenient polarization variable for $J/\psi$ is $\alpha = (\sigma_T - 2\sigma_L)/\sigma_T$, whose range is $-1 \leq \alpha \leq +1$. We consider five SQA’s: the maximal and minimal $gg$ axes defined by Eqs. (7) and (8) together with Eqs. (6) and (11), the perpendicular helicity and Collins-Soper axes defined by $X_{\perp h}$ and $X_{CS}$ in Eq. (4), and the c.m. helicity axis, which has been used in most previous work on this problem. The leading-order predictions for $\alpha$ for the five SQA’s are shown as functions of $Q_T/M_{J/\psi}$ in the left panel of Fig. 1. For the c.m. helicity and perpendicular helicity axes, $\alpha$ increases with $Q_T$ and asymptotically approaches 1. The two axes are essentially identical at large transverse momentum. For the maximal $gg$ axis, it approaches 1 much more rapidly. For the Collins-Soper axis, $\alpha$ decreases with $Q_T$, reaching $-0.33$ at $Q_T = 10 M_{J/\psi}$. For the minimal $gg$ axis, $\alpha$ approaches the same asymptotic value at large $Q_T$.\[\frac{\langle O_8^{J/\psi}(3P_0)\rangle}{m_c^2} = 0.0097 \text{ GeV}^3.\]
much more rapidly.

We also consider direct $\Upsilon(1S)$ production at the CERN LHC, which is a $pp$ collider with c.m. energy 14 TeV. For the NRQCD matrix elements, we use central CTEQ5L values from Ref. [16]:

\[
\langle O^{T}(3S_1) \rangle = 10.9 \text{ GeV}^3, \quad \langle O^{T}(1S_0) \rangle = 0.025 \text{ GeV}^3, \quad \langle O^{T}(3P_0) \rangle = 0.068 \text{ GeV}^3, \quad \langle O^{T}(3P_1) \rangle = 0.014 \text{ GeV}^3.
\]

We have used the averages of the values for the $\langle O^{T}(1S_0) \rangle = 0$ and $\langle O^{T}(3P_0) \rangle = 0$ cases of Ref. [16]. In the parton cross sections, we set $m_b = M_\Upsilon/2 = 4.7$ GeV. We imposed a rapidity cut $|y| < 3$ on the $\Upsilon$. The leading-order predictions for $\alpha$ for the five SQA’s are shown as functions of $Q_T/M_\Upsilon$ in the right panel of Fig. 1.

Our optimal SQA’s are not useful for fixed-target experiments, because the recoiling jet is usually not observed. However, the angle $\theta_{\text{jet}}$ of the recoiling jet can be measured relatively easily in high-energy hadron colliders. To provide some idea of how much the data sample will be decreased by the requirement that $\theta_{\text{jet}}$ be measured, we impose a cut on the pseudorapidity $\eta_{\text{jet}} = \ln \tan(\theta_{\text{jet}}/2)$ of the jet. For $J/\psi$ at the Tevatron, the fraction of events satisfying the $J/\psi$ rapidity cut $|y| < 1$ that also survive a jet pseudorapidity cut $|\eta_{\text{jet}}| < 1$ is greater than 0.21 for $Q_T > M_{J/\psi}$. For $\Upsilon(1S)$ at the LHC, the fraction of events satisfying the $\Upsilon$ rapidity cut that also survive a jet pseudorapidity cut $|\eta_{\text{jet}}| < 3$ is greater than 0.72 for $Q_T > M_\Upsilon$. These fractions are large enough that measuring $\theta_{\text{jet}}$ should not dramatically decrease the size of the data sample.

The CDF and D0 Collaborations have measured the polarization as a function of $Q_T$ for charmonium and bottomonium mesons produced at the Tevatron [2, 3, 4]. For $J/\psi$, the variable $\alpha$ for the c.m. helicity axis was measured for $Q_T/M_{J/\psi}$ as high as 9.7. For $\Upsilon(1S)$, $\alpha$ for the c.m. helicity axis was measured for $Q_T/M_\Upsilon$ as high as 2.1. At the LHC, it should be possible to measure $\alpha$ for these and other heavy quarkonium mesons out to much larger values of $Q_T$.

The measurement of $\alpha$ for two different SQA’s will provide more information about the spin if there is a large difference in the prediction of the transverse polarization with respect to the two axes. Our leading-order calculations predict a large difference in $\alpha$ for the perpendicular helicity and Collins-Soper axes for $Q_T \gg M$. One advantage of these two axes is that $X_{CS}$ and $X_{\perp h}$ in Eq. (4) are invariant under independent longitudinal boosts of the two colliding hadrons. This implies that $\alpha$ for these axes is insensitive to collinear radiation from the colliding partons, so radiative corrections may be smaller than for the c.m. helicity axis. Our leading-order calculations predict an even larger difference in $\alpha$ for the maximal
and minimal $gg$ axes. These optimal axes require the measurement of the direction of a recoiling jet, but we have shown that this should not dramatically decrease the size of the data sample.

Similar methods could be used to derive optimal SQA’s for the production of heavy elementary particles in the standard model, such as the weak bosons $W^\pm$ and $Z^0$ and top quark. There should be a large difference in the polarization between the perpendicular helicity and Collins-Soper axes, but an even larger difference between the appropriate maximal and minimal SQA’s. If predictions for the polarization of these particles with respect to optimal SQA’s can be verified, then optimal SQA’s will also provide a new window into the spins of new particles created at the LHC.

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