Flavor changing Z-decays from scalar interactions at a Giga-Z Linear Collider

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The flavor changing decay \( Z \to d_I \bar{d}_J \) is investigated with special emphasis on the \( b \bar{s} \) final state. Various models for flavor violation are considered: two Higgs doublet models (2HDM’s), supersymmetry (SUSY) with flavor violation in the up and down-type squark mass matrices and SUSY with flavor violation mediated by R-parity-violating interaction. We find that, within the SUSY scenarios for flavor violation, the branching ratio for the decay \( Z \to b \bar{s} \) can reach \( 10^{-6} \) for large tan\( \beta \) values, while the typical size for this branching ratio in the 2HDM’s considered is about two orders of magnitudes smaller at best. Thus, flavor changing SUSY signatures in radiative \( Z \) decays such as \( Z \to b \bar{s} \) may be accessible to future “\( Z \) factories” such as a Giga-Z version of the TESLA design.

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I. INTRODUCTION

Rare processes involving various particles have always been a gold mine for extracting interesting physics \([1,2]\). For example, the smallness of Flavor Changing Neutral Currents (FCNC) in the \( K \) system prompted the introduction of the GIM mechanism and subsequently to the prediction \( m_c \approx 1.5 \) GeV and the discovery of \( J/\Psi \) and \( D \)'s. \( B \bar{B} \) mixing was a precursor to a heavy top quark, as confirmed by experiment. FCNC rare top decays, for which there are only weak upper bounds, will hopefully be discovered in future experiments, thus serving as direct indications for deviations from the Standard Model (SM), since the latter leads to branching ratios which are smaller than \( 10^{-10} \).

The situation in rare \( Z \) decays, which is the subject of this paper, bears some similarities to rare \( t \) decays. In both cases the SM results from the loop induced FCNC decays are very small, beyond reach, at least for \( t \), in the foreseeable future. Therefore, any significant detection of a rare decay at the level higher than \( 10^{-10} \) or \( 10^{-8} \) for \( t \) or \( Z \), respectively, would serve as an indisputable proof for physics beyond the SM. If new physics is “around the corner”, i.e. at \( \approx 1 \) TeV, the \( Z \) boson and the top being so close to that scale, are expected to be the particles most affected by new physics.

In this paper we study the rare decays \( Z \to d_I \bar{d}_J \), where \( I, J \) indicate the generation index of a charge \(-1/3\) quark, in various models. In the SM it was found that \( \sum_{q=d,s} \text{Br}(Z \to b \bar{q}) \approx 5 \times 10^{-6} \), we do not repeat the SM calculation here. Three of the models we discuss, have already been considered in connection with \( Z \to b \bar{s} \), namely the 2 Higgs Doublets Model type II (2HDMII) \([3]\), Supersymmetry (SUSY) \([4]\) and SUSY with R-Parity Violation (RPV) \([5]\). Therefore we comment, wherever it is relevant in the coming sections, about differences and similarities with previous works. Note that in addition one can find in the literature discussion of FCNC hadronic \( Z \) decays, in models not covered by us in the present article \([6]\).

Experimentally, the attention devoted to FCNC in \( Z \) decays at LEP and SLD has been close to nil. The best upper limit is \( \sum_{q=d,s} \text{Br}(Z \to b \bar{q}) \leq 1.8 \times 10^{-3} \) at 90% CL .

\[ \sum_{q=d,s} \text{Br}(Z \to b \bar{q}) \leq 1.8 \times 10^{-3} \text{ at 90\% CL} . \tag{1} \]

This preliminary result is based on about \( 3.5 \times 10^6 \) hadronic decays, and we used \( \text{Br}(Z \to \text{hadrons}) = 0.7 \). We urge our experimental colleagues to sift through their LEP data to improve the current, rather loose, limit.

In the future, there will be at least two venues in which \( Z \) bosons will be produced in much larger quantities than their number in LEP. In a high luminosity LHC with an integrated luminosity of \( 100 \) fb\(^{-1} \), one expects \( 5.5 \times 10^9 \) \( Z \) bosons to be produced \([7]\). A cleaner environment for the processes at hand, will be provided by a future...
$e^+e^-$ linear collider. In particular, there is a viable possibility to lower the TESLA center of mass energy down to $\sqrt{s} = m_Z$, the so called “Giga-Z” option. With integrated luminosity of 30 fb$^{-1}$, it is possible to produce more than $10^9$ Z bosons [1], about 2 orders of magnitudes larger than in LEP. To grasp the improvement in going from LEP to Giga-Z option of TESLA, we note that while the sensitivity of LEP to $Z \rightarrow \tau \mu$ was $\approx 10^{-5}$ [9], it is expected to be $\approx 10^{-8}$ in Giga-Z TESLA [11].

Beyond LHC and the $e^+e^-$ linear collider, there is also considerable interest in the community for a high energy muon collider [12]. If this ever becomes a reality, it would also afford another very good opportunity for studying rare flavor changing decays and interactions [13].

The paper is organized as follows: In Section 2 we present a generic calculation of the $Zd_Id_J$ vertex at the one loop level. This result will assist us to evaluate the branching ratio for the FCNC $Z$ decays in any particular model. In Section 3, the results of two variants of the Two Higgs Doublets Model (2HDM), namely the so called type II 2HDM (2HDHII) and the Top-Higgs Two Doublets Model (T2HDM), will be reported. In 2HDM, T2HDM we get the disappointing results $\text{Br}(Z \rightarrow b\bar{s} + bs) \sim 10^{-10}, 10^{-8}$, respectively. We then move on to Section 4, where our results in Supersymmetry (SUSY) with squark mixing, are displayed. Again, two options are presented, the first one with $b-\bar{s}$ mixing and the second one with $\bar{t}-\bar{c}$ mixing. In the first case the branching ratio can reach a respectable $\text{Br}(Z \rightarrow b\bar{s} + bs) \sim 10^{-6}$ while the second case yields a branching ratio of $O(10^{-8})$.

In Section 5 we turn to SUSY with R-parity violation (RPV), where the effects of $\lambda'$ trilinear coupling terms in the RPV superpotential and of $b$ terms ($b$ is the coefficient of the soft breaking RPV bilinear term), are considered. Two categories of RPV are considered: Those which lead to a branching ratio $\propto \lambda' \times \lambda'$ and those with a branching ratio $\propto b\lambda'$. For the first category we get typically branching ratios at the level of $10^{-10}$, while for the second type of RPV, we find an encouraging possibility of $\text{Br}(Z \rightarrow b\bar{s} + bs) \sim 10^{-8}$. Finally, in Section 6 we summarize our results.

II. GENERIC SCALAR CALCULATION

In this section we outline the generic framework for calculating the radiative one-loop flavor changing interaction vertex $Vd_Id_J$ with $I \neq J$ and $V = Z$ or $\gamma$. We define the one-loop amplitude for $V \rightarrow d_Jd_J$ in terms of form factors which are calculated for the complete set of one-loop diagrams that can potentially contribute to $V \rightarrow d_Jd_J$ in the presence of flavor violating interactions between neutral scalars and fermions as well as non-diagonal vertices of charged scalars with fermions of different generations.

The diagrams that modify (at one loop) the $Vd_Jd_J$ coupling due to charged or neutral scalar exchanges are depicted in Fig. 1.

In what follows, we will denote the internal scalars ($S$) in the loops by Greek letters and fermions ($f$) will be given Latin indices $i,j$.

From Fig. 1 it is evident that we have only three types of interaction vertices to consider. These are defined as follows:

1. $V_{\mu} - f_i - \bar{f}_j$ interaction

$$V_{\mu} - f_i - \bar{f}_j$$

where $L(R) = (1-(+\gamma_5))/2$. For the case of the SM couplings of a vector boson $V$ to a pair of fermions, i.e., $f = u$ (up-quark) or $f = d$ (down-quark), there are only diagonal $Vff$ couplings. In this case we have:

$$V_{\mu} \gamma_\mu \left( a^{ij}_{L(V)} + a^{ij}_{R(V)} \right)$$
\[ a_{L,R(Vf)}^{ij}(i = j) \equiv a_{L,R(Vf)} \]  

where

\[ a_{L,R(Zf)} = g_Z \left( T_{L,R}^{3(f)} - s_W^2 Q_f \right), \]

\[ a_{L,R(f)} = a_{R(\gamma f)} = -g_\gamma Q_f, \]

with \( T_{L}^{3(a)} = 1/2 \) and \( T_{L}^{3(d)} = -1/2 \) for an up and down-quark, respectively, and \( T_{R}^{3(f)} = 0 \). Also, \( Q_f \) is the charge of \( f \) and

\[ g_Z = \frac{e}{s_W c_W}, \quad g_\gamma = e. \]

2. \( V_\mu - S_\alpha - S_\beta \) interaction

\[ V_\mu, S_\beta, S_\alpha \]

where \( S_\alpha \) and \( S_\beta \) are charged or neutral spin 0 par-

\[ M_k^{ij} \equiv \frac{i}{16\pi^2} \epsilon^\nu(q) \tilde{d}_i(p_t) \{ \gamma^\mu \left( A_{L,k}^{ij} L + A_{R,k}^{ij} R \right) + (B_{L,k}^{ij} L + B_{R,k}^{ij} R) p_\mu \} v_{d_j}(p_J), \]

where \( \epsilon^\nu(q) \) is the polarization vector of \( V, q \) is its 4-momenta and \( \tilde{d}_i \) (\( v_{d_j} \)) is the Dirac spinor of the outgoing \( d_i \) with 4-momenta \( p_t \) (\( \tilde{d}_j \)) with 4-momenta \( p_J \) such that \( q = p_t + p_J \). Also, \( A_{L,k}^{ij}, A_{R,k}^{ij}, B_{L,k}^{ij}, B_{R,k}^{ij} \) are momentum dependent form factors.

Using the Feynman rules in [\( \square \), [\( \square \) and [\( \square \)], these form factors can be readily calculated for each diagram. Neglecting terms of \( O(m_b/\sqrt{q^2}) \) we get:

\[ A_{L,1}^{ij} = -2 \sum_{\alpha,\beta,i} g_\nu \left( b_{L(\alpha)}^{ij} b_{L(\beta)}^{ij} C_{24} \right), \]

\[ B_{L,1}^{ij} = 2 \sum_{\alpha,\beta,i} \hat{P}_i m_f g_\nu \left( b_{R(\alpha)}^{ij} b_{L(\beta)}^{ij} \right) \left( C_1^1 + C_{11}^1 \right), \]

where \( \hat{P}_i = -1 \) if the internal fermion in the loop is a charged conjugate state \( f_i^- \) or else \( \hat{P}_i = 1. \)

Combining the contribution of the two self energy diagrams, i.e., \( M_3 + M_4 \equiv M_{34} \), and similarly for the form factors, e.g., \( A_{L,3}^{ij} + A_{L,4}^{ij} \equiv A_{L,34}^{ij} \) etc., we have:

3. \( S_\alpha - \tilde{f}_i - d_j \) interaction

\[ \begin{array}{c}
S_\alpha \\
\downarrow \quad \searrow \quad \nearrow \quad \downarrow \\
F_i \\
\downarrow \\
S_\beta \\
\downarrow \quad \searrow \quad \nearrow \quad \downarrow \\
D_j \\
\end{array} \]

where \( d \) is a down-quark.

The one-loop amplitudes \( M_k \) \((k = 1, 2, 3, 4 \) corresponding to diagrams 1, 2, 3, 4 in Fig. [\( \square \), respectively), for the decay \( V \rightarrow d_i d_j \) can be parametrized generically as follows:
\[ A_{L,34}^{ij} = a_{L(Vd)} \sum_{\alpha,i,j} b_{k_{(\alpha)}}^{ij} b_{k_{(\alpha)}}^{ij} B_1^3, \]  
\[ B_{L,34}^{ij} = 0. \]  

The right-handed form factors, \( A_{R,k}^{ij} \) and \( B_{R,k}^{ij} \), are obtained from the corresponding left handed ones, \( A_{L,k}^{ij} \) and \( B_{L,k}^{ij} \), respectively, by interchanging \( L \rightarrow R \) and \( R \rightarrow L \) in all the couplings which appear in \( (10)-(13) \).

The three-point one-loop form factors \( C_x \) with \( x \in 0,1,2,3 \), correspond to the loop integrals of diagrams \( k = 1-4 \) and are given by:

\[ C_x^1 = C_x \begin{pmatrix} m_f^2, m_{S_\alpha^2}, m_{S_\beta^2}, m_{d_{ij}}, q^2, m_{d_{ij}} \end{pmatrix}, \]  
\[ C_x^2 = C_x \begin{pmatrix} m_{S_\alpha^2}, m_{d_{ij}}, m_{d_{ij}}, m_{d_{ij}}, q^2, m_{d_{ij}} \end{pmatrix}, \]  
\[ B_x^3 = B_x \begin{pmatrix} m_f^2, m_{S_\alpha^2}, m_{d_{ij}} \end{pmatrix}. \]

In terms of the above form factors, the partial width for the decay \( Z \rightarrow d_i \bar{d}_j \) is:

\[ \Gamma(Z \rightarrow d_i \bar{d}_j) = \frac{N_C}{3} \left( \frac{1}{16\pi^2} \right)^2 \frac{M_Z}{16\pi} \times \left[ 2 \left( |A_{L}^T|^2 + |A_{R}^T|^2 \right) + \frac{M_Z^2}{4} \left( |B_{L}^T|^2 + |B_{R}^T|^2 \right) \right], \]  

where \( N_C = 3 \) is the color factor and

\[ A_{P}^T = A_{P,1}^{ij} + A_{P,2}^{ij} + A_{P,34}^{ij}, \]  
\[ B_{P}^T = B_{P,1}^{ij} + B_{P,2}^{ij} + B_{P,34}^{ij}, \]

for \( P = L \) and \( R \).

### III. TWO HIGGS DOUBLETS MODELS

In a 2HDM with flavor diagonal couplings of the neutral Higgs to down-quarks, the flavor changing decay \( Z \rightarrow d_i \bar{d}_j \) proceeds through the one-loop diagrams in Fig. 3.

The interaction vertices required for the calculation of the form factors defined in \( (19) \) in such models are:

\[ V_{\mu} f_i \bar{f}_j \rightarrow Z_{\mu} u_i \bar{u}_j, \]
\[ V_{\mu} S_{\alpha} S_{\beta} \rightarrow Z_{\mu} H^+ H^-, \]
\[ S_{\alpha} f_i \bar{d}_j \rightarrow H^+ \bar{u}_i d_j, \]  

where the \( Z_{\mu} u_i \bar{u}_j \) coupling is the SM one as given in \( (19) \), \( S_1 = H^+ \) is the only charged Higgs present in this type of models and \( f_i = u_i, i = 1,2,3 \) for the three up-type quarks \( u_1 = u, u_2 = c, u_3 = t \). The \( Z^\mu H^+ H^- \) coupling is obtained from the scalar kinetic term \( (D^\mu \Phi_1)^\dagger (D^\mu \Phi_1) \), where \( \Phi_1, \Phi_2 \) are the two SU(2) Higgs doublets. This coupling is, therefore, generic to any version of a 2HDM.

The coupling \( H^+ \bar{u}_i d_j \) is obtained from the Yukawa potential. The most general Yukawa interaction of a 2HDM can be written as (see e.g., \( [4] \)):

\[ U_{ij}^1 \rightarrow G_{ij} (\delta_{j1} + \delta_{j2}), \]
At around $\tan \beta \sim 13$ there is a “turning point” at which the $BR(Z \to b\bar{s} + b\bar{s})$ starts to increase with $\tan \beta$. At this point the contributions from the top and charm-quark loop exchanges become comparable, since the charm-quark effect being $\propto m_{c}^{2} m_{b}^{2} \tan^{4} \beta (V_{CKM}^{*})^{2} (V_{CKM}^{ch})^{2}$ (for $\tan \beta \gtrsim 13$) the charm-quark exchange is dominated by the Yukawa couplings $b_{L}^{2}$ and $b_{R}^{2}$, see Table 2; equals that of the top-quark. As $\tan \beta$ is further increased ($\tan \beta > 13$) both the top and charm-quark loop exchanges are dominated by the right-handed down-quark Yukawa couplings $b_{L}^{2}$ in (8) (which is $\propto \tan \beta$) and are, therefore, comparable and increasing with $\tan \beta$.

Note that the curves in Fig. 2 for the 2HDMII scenario (the left side) pass through unrealistic values in the $\tan \beta - m_{H^{+}}$ plane. In particular, the decay $b \to s \gamma$ imposes strong constraints on the $\tan \beta - m_{H^{+}}$ plane [3]: $m_{H^{+}} \gtrsim 400$ GeV independent of $\tan \beta$. Thus, if $\tan \beta = 1$, then the largest allowed value for the $BR(Z \to b\bar{s} + b\bar{s})$ is $10^{-10}$ if $m_{H^{+}}$ lies close to its lower bound from $b \to s \gamma$. For even smaller values of $\tan \beta$, say $\tan \beta \lesssim 0.5$, the $b \to s \gamma$ constraint requires $m_{H^{+}} \gtrsim 500$ GeV for which the $BR(Z \to b\bar{s} + b\bar{s})$ is again $\lesssim 10^{-10}$ in the 2HDMII.

B. $Z \to b\bar{s}$ in T2HDM

The main difference between the T2HDM and the 2HDMII charged Higgs sectors lies in the structure of the $d_{i}H^{+}$ Yukawa interactions ($d_{i} = d_{i}$, $s$, $b$ for $i = 1, 2, 3$ respectively). In particular, while in both models the top Yukawa coupling to down-quarks, i.e., the $t_{R}d_{L}H^{+}$ coupling, is $\propto m_{t}V_{CKM}^{td}$, the charm-quark Yukawa coupling is completely altered by the presence of the matrix $\Sigma$ in (26). Specifically, the $c_{R}b_{L}H^{+}$ coupling is $\propto m_{c}^{2}V_{CKM}^{cb}$, $\tan \beta$, and the $c_{R}s_{L}H^{+}$ coupling is $\propto m_{s}^{2}V_{CKM}^{cs}$, $\tan \beta$. These couplings are, therefore, a factor of $\tan^{2} \beta \times (V_{CKM}^{tb} / V_{CKM}^{ch})$ and $\tan^{2} \beta$, respectively, larger than in the 2HDMII if $\xi$ is of $O(1)$.

As can be seen from Fig. 2 (the right side), in the range $\tan \beta \lesssim 5$ the $BR(Z \to b\bar{s} + b\bar{s})$ is practically identical in both the T2HDM and the 2HDMII; in this range it is dominated by the top-quark loop and, therefore, by the top-quark Yukawa couplings to the $b$ and $s$-quarks which are essentially the same in these two versions of a 2HDM. On the other hand, for larger values of $\tan \beta$, in contrast to the case of a 2HDMII, in the T2HDM the charm-quark loop starts to dominate due to the enhancement in the $c_{R}b_{L}H^{+}$ and $c_{R}s_{L}H^{+}$ Yukawa couplings (see discussion above). In fact, the $c_{R}b_{L}H^{+}$ coupling is doubly enhanced in the T2HDM; first by the $\tan \beta$ factor and second by a factor of $V_{CKM}^{tb} / V_{CKM}^{ch}$, i.e., in this model this coupling does not suffer from the CKM suppression factor $V_{CKM}^{ch}$.

It should be emphasized that a large $\tan \beta$, e.g., $\tan \beta \gtrsim O(10)$, is the “working assumption” of the T2HDM. In particular, the T2HDM loses its attractiveness in the small $\tan \beta$ regime, since in this range it fails to explain the large top-quark mass - this being
the main motivation behind this version of a 2HDM. At the same time, taking into account low-energy experimental data from $K - \bar{K}$ mixing, $\epsilon_K$ and $b \rightarrow s\gamma$, the $\tan \beta - m_{H^+}$ plane is also constrained in the T2HDM [17, 19], especially in the large $\tan \beta$ region in which this model differs from the usual 2HDMII. For example, for $\xi = 1$ and taking the SM best fit value for the Wolfenstein parameters $\rho$ and $\eta$, then the $\epsilon_K$ constraint implies $m_{H^+} \gtrsim 500$ GeV for $\tan \beta \sim 20$ and $m_{H^+} \gtrsim 4$ TeV for $\tan \beta \sim 50$ [19]. Imposing these bounds we find that $BR(Z \rightarrow b\bar{s} + \bar{b}s) \sim O(10^{-8})$ is the best case value in this model assuming a large $\tan \beta$. 

| Scalar ($S_{\alpha=1}$) | 2HDMII | T2HDM |
|------------------------|--------|-------|
| $H^+$                  | $H^+$  | $H^+$ |
| $u_i$, $i = 1,2,3$    | $a_L(Zu)$ | $a_L(Zu)$ |
| $a_{ij}^{L(\alpha)}$  | $a_R(Zu)$ | $a_R(Zu)$ |
| $b_{L(\alpha=1)}^{ij}Y_{ij} \times \frac{1}{\tan \beta} \frac{m_{\mu}}{m_{t}}$ | $Y_{ij} \times \frac{1}{\tan \beta} \frac{m_{\mu}}{m_{t}}$ | $Y_{ij} \times \frac{1}{\tan \beta} \frac{m_{\mu}}{m_{t}}$ |
| $b_{R(\alpha=1)}^{ij}Y_{ij} \times \frac{m_{\mu}}{m_{t}}$ | $-e^{1-2\tan \beta}$ | $-e^{1-2\tan \beta}$ |
| $g_{Z}^{\alpha=1=1} \frac{1}{\tan \beta} \frac{m_{\mu}}{m_{t}}$ | $\frac{1}{\tan \beta} \frac{m_{\mu}}{m_{t}}$ | $\frac{1}{\tan \beta} \frac{m_{\mu}}{m_{t}}$ |

TABLE I: The couplings required for the calculation of $\Gamma(Z \rightarrow d_i\bar{d}_j)$ in a 2HDMII and a T2HDM. The couplings follow the notation used in the Feynman rules of (2), (7) and (8). Also, $a_R(Zu)$, $a_L(Zu)$ are given in (3)-(6). 

FIG. 2: $BR(Z \rightarrow b\bar{s} + \bar{b}s)$ as a function of $\tan \beta$, for $m_{H^+} = 100$, 400 and 600 GeV, in a 2HDMII (left side) and a T2HDM with $\xi = 1$ (right side). 

$BR(Z \rightarrow b\bar{s} + \bar{b}s) \sim O(10^{-8})$ is the best case value in this model assuming a large $\tan \beta$. 

$m_{H^+} \gtrsim 500$ GeV for $\tan \beta \sim 20$ and $m_{H^+} \gtrsim 4$ TeV for $\tan \beta \sim 50$ [19]. Imposing these bounds we find that $BR(Z \rightarrow b\bar{s} + \bar{b}s) \sim O(10^{-8})$ is the best case value in this model assuming a large $\tan \beta$. 

The couplings follow the notation used in the Feynman rules of (2), (7) and (8). Also, $a_R(Zu)$, $a_L(Zu)$ are given in (3)-(6). 

$g_{Z}^{\alpha=1=1} \frac{1}{\tan \beta} \frac{m_{\mu}}{m_{t}}$ | $\frac{1}{\tan \beta} \frac{m_{\mu}}{m_{t}}$ | $\frac{1}{\tan \beta} \frac{m_{\mu}}{m_{t}}$ |
IV. SUPERSYMMETRY WITH SQUARK MIXINGS

In SUSY flavor changing phenomena can emanate from mixings of squarks of different generations through the soft breaking Lagrangian terms in the squark sector:

\[
\mathcal{L}_{\text{squark}}^{\text{soft}} = - \tilde{Q}_i (M_{\tilde{Q}}^2)_{ij} \tilde{Q}_j - \tilde{U}_i (M_{\tilde{U}}^2)_{ij} \tilde{U}_j - \tilde{D}_i (M_{\tilde{D}}^2)_{ij} \tilde{D}_j + A_u^i \tilde{Q}_i H_u \tilde{U}_j + A_d^i \tilde{Q}_i H_d \tilde{D}_j,
\]

where \( M_{\tilde{Q}}^2 \), \( M_{\tilde{U}}^2 \), and \( M_{\tilde{D}}^2 \) are 3 \( \times \) 3 matrices. As in [21] we adopt the following simplified Ansatz:

\[
V_L^U A_u^i V_R^U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & x_u \\ 0 & y_u & 1 \end{pmatrix} A,
\]

and

\[
V_L^D A_d^i V_R^D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & x_d \\ 0 & y_d & 1 \end{pmatrix} A,
\]

such that \( A \) is a common trilinear soft breaking parameter for both up and down-squarks and the parameters \( x_u, y_u \) and \( x_d, y_d \) represent flavor mixing effects in the \( \tilde{t} - \tilde{c} \) and \( \tilde{b} - \tilde{s} \) sectors, respectively. It then follows that \( \delta_{LR}^{(U)} \), \( \delta_{LR}^{(D)} \) and \( \delta_{LR}^{(U)} \), \( \delta_{LR}^{(D)} \) are associated with non-diagonal (flavor changing) entries in the trilinear soft breaking terms \( M_{\tilde{Q}}^2, M_{\tilde{U}}^2 \) and \( M_{\tilde{D}}^2 \) in (27). Similarly, \( \delta_{LR}^{(U)} \) are associated with non-diagonal (flavor changing) entries in the trilinear soft breaking terms M.

\[
\delta_{LR}^{(U)} = -x_u \frac{\sin\beta}{\sqrt{2}} \times \frac{vA}{m_0}, \quad \delta_{LR}^{(D)} = -y_u \frac{\sin\beta}{\sqrt{2}} \times \frac{vA}{m_0},
\]
Within this mixing scenario, in which flavor changing effects in the squark sector are present only in the second and third generations, the $6 \times 6$ mass matrices in the up and down-squark sectors reduce to $4 \times 4$ matrices.

For the $t\rightarrow c$ sector, in the basis $\Phi_U^0 = (\tilde{c}_L, \tilde{c}_R, \tilde{t}_L, \tilde{t}_R)$, we then have:

$$M^e_{cl} = \begin{pmatrix}
1 & 0 & \delta_{LL}^{(23)} & \delta_{LR}^{(23)} \\
0 & 1 & \delta_{LR}^{(23)} & -X_t/m_0^2 \\
\delta_{LR}^{(23)} & \delta_{LR}^{(23)} & 1 & -X_t/m_0^2 \\
\delta_{LR}^{(23)} & -X_t/m_0^2 & 1 & 1
\end{pmatrix} m_0^2 ,$$

and similarly for the $b\rightarrow s$ sector, in the basis $\Phi_b^0 = (\tilde{s}_L, \tilde{s}_R, \tilde{b}_L, \tilde{b}_R)$, we have:

$$M^e_{sb} = \begin{pmatrix}
1 & 0 & \delta_{LL}^{(32)} & \delta_{LR}^{(32)} \\
0 & 1 & \delta_{LR}^{(32)} & -X_b/m_0^2 \\
\delta_{LR}^{(32)} & \delta_{LR}^{(32)} & 1 & -X_b/m_0^2 \\
\delta_{LR}^{(32)} & -X_b/m_0^2 & 1 & 1
\end{pmatrix} m_0^2 .$$

A. $b\rightarrow s$ mixing

Here the flavor changing decay $Z \rightarrow b\bar{s}$ is generated by one-loop exchanges of the $b\rightarrow s$ admixture states, $\Phi_D$, and gluinos, $\tilde{g}$. We thus have $S_\alpha = \Phi_D, \alpha$ with $\alpha = 1-4$, and $f = \tilde{g}$ in the diagrams of Fig. 7. Note that diagram (2) in Fig. 7 which requires the $V^f f$ coupling, does not contribute since the $Z$-boson does not couple to gluinos at tree-level.

The one-loop $b\rightarrow s$ mixing effect on the decay $Z \rightarrow b\bar{s}$ was considered more than a decade ago in [1], where it was assumed that flavor violation in the down-squark sector is controlled by radiative corrections to the down-squark mass matrix induced by off-diagonal CKM elements. Instead, as described above, we assume here that the flavor violation is rooted with non-diagonal entries in the soft SUSY breaking sector.

The following interaction vertices are required for the calculation of $\Gamma(Z \rightarrow b\bar{s})$ in the $b\rightarrow s$ mixing scenario:

$$V_{\mu} S_\alpha S_\beta \rightarrow Z_\mu \Phi_D, \alpha \Phi_D, \beta ,$$

$$S_\alpha \tilde{f}_i d_j \rightarrow \Phi_D, \alpha \tilde{g}_i d_j .$$

These are derived from [2]:

[1] Note also that [1] used unrealistic top-quark and squark masses.
\[ \mathcal{L}(V_\mu \bar{d}d) = -i \left[ \frac{1}{2} \frac{e}{s_W} A_\mu^3 + \frac{e}{6 c_W} B_\mu \right] \bar{d}_{L,\ell} \gamma^{\mu \nu} \partial_\nu \Phi_{L,\ell} + \bar{d}_{L,\ell} \partial^\mu \Phi_{L,\ell}, \]

\[ \mathcal{L}(\bar{d}g d) = g_s \sqrt{2} T^a \bar{d}_L \gamma^\mu \left( -\bar{d}_{L,R} \gamma^\nu \partial_\nu R \right) d_J + h.c. , \]

where \( g_s \) is the SU(3) coupling constant and \( T^a \) are the SU(3) group generators.

Using the above interaction Lagrangian terms, the couplings required for the calculation of \( \Gamma(Z \to b\bar{s}) \) in the form defined in (7) and (8) are obtained by rotating the SU(3) group generators.

II. The SUSY parameter space needed to evaluate \( \Gamma(Z \to b\bar{s}) \) in this scenario is: \( m_0, \mu, A, \tan \beta, m_{3/2} \) and \( \delta^D \). The low-energy values of these six parameters fully determine the \( b \sim \bar{s} \) scalar spectrum (i.e., masses and mixing matrices) and the gluino mass \( (m_3) \), from which all the couplings in Table II are obtained. These six parameters will be varied subject to the requirement that squark masses as well as the gluino mass are heavier than 150 GeV.

In Figs. 3, 5 and 7 we plot \( BR(Z \to b\bar{s} + \bar{b}s) \) as a function of \( A, \tan \beta, m_{3/2} \) and \( \delta^D \). The rest of the parameters are varied subject to the above criteria (23). In order to better understand the dependence of \( BR(Z \to b\bar{s} + \bar{b}s) \) on the physical squark mass spectrum, we accompany Figs. 3, 5 and 7 by Figs. 8 and 9, respectively, in which we depict the masses of the four physical squarks \( m_{1/2} \) and \( m_{3/2} \) that correspond to the same choices of the SUSY parameter space as in Figs. 3, 5 and 7.

Let us summarize the results shown in Figs. 3, 5 and 7.

- The branching ratio of the decay \( Z \to b\bar{s} \) is enhanced dramatically with the increase of the mass splittings between the four physical squarks. This is due to a GIM-like cancellation which is operational in the limit of degenerate squark masses as a result of the unitarity of the rotation matrix \( R_D \).

Thus for example, a typical mass spectrum that can drive the branching ratio to the \( 10^{-6} \) level is when the lightest down squark, \( \tilde{b}_1 \), has a mass below 250 GeV, while the rest of the squarks have masses at the 1-3 TeV range.

- As expected, \( BR(Z \to b\bar{s}) \) drops sharply as \( \delta^D \) is decreased. Clearly, this is traced to the fact that the mixing among the bottom and strange type scalars diminishes in the limit \( \delta^D \to 0 \), see Figs. 3, 5 and 7.

[2] The term “low-energy” refers to the electroweak (or collider energies) scale and is used in order to distinguish it from the scale in which the soft breaking couplings are generated (e.g., the GUT scale).

[3] The bounds on the different delta’s reported in [2], [3] were obtained using the mass insertion approximation, while we perform an exact diagonalization of the squark mass matrices. Therefore, in the cases where \( O(1) \) delta’s are allowed (e.g., for \( \delta^{D(32)} \)) these bounds may only serve as indicative for their expected size, since the mass insertion approximation necessarily assumes small delta’s.

[4] The unitarity of \( R_D \) also ensures that the infinities that appear in the individual diagrams of Fig. 4 cancel.
For a sufficiently large $\tan \beta$, $BR(Z \to b \bar{s} + \bar{b}s)$ is almost insensitive to the value of the common trilinear soft breaking parameter $A$ as long as $\mu$ is large enough to drive the desired mass splittings between the squarks. This behavior is due to the dominance of the $\mu$ term in the quantity $X_b$ defined in (38) for large $t_\beta$ (recall that $X_b$ is responsible for the mass splitting between the two bottom-type scalars). On the other hand, for $t_\beta \sim \mathcal{O}(1)$, the term $\propto A$ in $X_b$ (see the r.h.s. of (38)) becomes important when
\( m_\tilde{g} = 200 \text{ GeV} \quad \text{m}_\tilde{g} = 600 \text{ GeV} \)

\[ m_{\tilde{t}_R} = 50, A = 1500 \text{ GeV} \]

\( \delta^D \)

\( BR(Z \rightarrow b\bar{s}) \) as a function of the flavor mixing parameter \( \delta^D \), for combinations of \( m_0 = 1000, 1600, 2200 \text{ GeV} \) with \( \mu = -1000, -2000, -3000 \text{ GeV} \), for \( t_\beta = 50, A = 1500 \text{ GeV} \) and for \( m_\tilde{g} = 200 \text{ GeV} \) (left plots) and \( m_\tilde{g} = 600 \text{ GeV} \) (right plots).

\[ A \sim \mu. \] This feature can be seen in Fig. 7.

- For the reason outlined above, \( BR(Z \rightarrow b\bar{s}) \) is symmetric about \( \mu = 0 \) for large \( t_\beta \), in which case the term \( \propto \mu \) in \( X_b \) dominates and the effect of the \( A \) term is negligible.

- For \( \mu^2/A^2 >> 1 \), \( BR(Z \rightarrow b\bar{s}) \) is increased with \( t_\beta \). Again, this is related to the dominance of the \( \mu \) term in \( X_b \) for large \( t_\beta \).
\( BR(Z \to b\bar{s} + b\bar{s}) \) drops with \( m_{\tilde{g}} \).

To conclude this section, we have shown that \( BR(Z \to b\bar{s}) \sim O(10^{-6}) \) can be achieved in the \( b\bar{s} \) mixing scenario provided that the gluino mass and one of the third generation down-type squark masses are at the TeV range, i.e., a large mass splitting between the lightest and rest of the down-type squarks is needed. Such a mass hierarchy in the squark sector requires a typical "heavy" SUSY mass scale with soft breaking parameters at the level of a few TeV. This scenario is somewhat motivated by the non-observability of SUSY particles in past and present high energy colliders.

B. \( \tilde{t} - \tilde{c} \) mixing

In the stop-scharm mixing scenario the flavor changing decay \( Z \to b\bar{s} \) proceeds through one-loop exchanges of the \( \tilde{t} - \tilde{c} \) admixture states, \( \Phi_U \), and charginos, \( \chi \). More specifically, we have \( S_\alpha = \Phi_{U,\alpha} \) with \( \alpha = 1 - 4 \), and \( f_i = \chi^c_i \) with \( i = 1, 2 \) for the two charginos (we find it convenient to calculate the exchanges of the charged conjugate chargino states \( \chi^\alpha \)).

Thus, in the \( \tilde{t} - \tilde{c} \) mixing scenario the following interaction vertices are required:

\[ V_\mu f_i \bar{f}_j \rightarrow Z_\mu \chi^c_i \bar{\chi}_j^c, \]
\[ V_\mu S_\alpha S_\beta \rightarrow Z_\mu \Phi^*_{U,\alpha} \Phi_{U,\beta}, \]
\[ S_\alpha \bar{d}_j \rightarrow \Phi^*_{U,\alpha} \bar{\chi}_j^c \chi^c. \]

These vertices are taken from [22].

\[ \mathcal{L}(V_\mu \tilde{t}_i \tilde{c}_j) = -\frac{e}{2 s_W c_W} \tilde{t}_i c_j \gamma_\mu \left( a_{L(VR)}^{ij} \mathcal{L} + a_{R(V)}^{ij} R \right) \chi^c_i V_\mu, \]
\[ \mathcal{L}(V_\mu \tilde{t}_i \tilde{t}_j) = -i \left( \frac{1}{2} e A_\mu^a + \frac{e}{6} e B_\mu \right) \tilde{u}_L,\ell \tilde{d}_j \tilde{d}_j \tilde{d}_j \tilde{u}_L,\ell \tilde{d}_j \tilde{d}_j, \]
\[ \mathcal{L}(\tilde{t}_i \tilde{c}_j) = \tilde{u}_L,\ell \tilde{d}_j \left( f_1^{L(\ell i j)} L + f_2^{R(\ell i j)} R \right) \chi^c_i \chi^c_j \tilde{C}_{KR} + \tilde{u}_R,\ell \tilde{d}_j \left( f_1^{R(\ell i j)} L + f_2^{R(\ell i j)} R \right) \chi^c_i \chi^c_j \tilde{C}_{KR} + h.c., \]

where

\[ a_{L(Z\chi^c)}^{ij} = (Z_{11}^{ij} Z_{12}^{ij} + \cos 2 \theta_W \delta_{ij}), \]
SUSY with \( \tilde{t} - \tilde{c} \) mixing

| \( \Phi_{U,\alpha} \), \( \alpha = 1, 2, 3, 4 \) | \( \chi_i^\pm \), \( i = 1, 2 \) |
|---|---|
| \( a^{ij}_{L(Zf)} \) | \( \frac{\epsilon}{2 s_W c_W} a^{ij}_{L(ZfX^c)} \) |
| \( a^{ij}_{R(Zf)} \) | \( \frac{\epsilon}{2 s_W c_W} a^{ij}_{R(ZfX^c)} \) |
| \( b^{L(\alpha)}_{i(\alpha)} \) | \( f^{R(\alpha i)}_{L(\alpha)} \) |
| \( f_{L(\alpha i)}^{R(\alpha i)}(R_{U,1\alpha}V_{CKM}^{2j} + R_{U,3\alpha}V_{CKM}^{3j}) + \frac{f_{R(\alpha i)}}{m_{\alpha i}}(m_u R_{U,2\alpha}V_{CKM}^{2j} + m_t R_{U,4\alpha}V_{CKM}^{3j}) \) | \( m_{\alpha i} \) |
| \( b^{R(\alpha)}_{i(\alpha)} \) | \( \frac{s_W}{c_W} f_{L(\alpha i)}^{R(\alpha i)}(R_{U,1\alpha}V_{CKM}^{2j} + R_{U,3\alpha}V_{CKM}^{3j}) \) |
| \( \frac{s_W}{c_W} (R_{U,1\alpha} R_{U,1\beta} + R_{U,3\alpha} R_{U,3\beta} - \frac{1}{2} s_W^2 \delta_{\alpha\beta} \) | \( m_2 \) |

TABLE III: The couplings required for the calculation of \( \Gamma(Z \to b\bar{s}) \) in the \( \tilde{t} - \tilde{c} \) mixing scenario. The couplings follow the notation in [3], [5] and [8]. Also, \( a^{ij}_{L(ZX^c)} \) and \( a^{ij}_{R(ZX^c)} \) are given in [13] and [49]. \( f_{L(\alpha i)}^{R(\alpha i)} \) and \( f_{R(\alpha i)} \) are defined in [13]. \( R_U \) is the rotation matrix defined in [3].

FIG. 8: \( BR(Z \to b\bar{s} + \bar{b}s) \) as a function of \( \tan \beta \), for combinations of \( m_0 = 1000, 1600, 2200 \) GeV with \( m_2 = 200, 600, 1000 \) GeV, for \( A = 1000 \) GeV, \( \mu = -2000 \) GeV and for \( \delta_M = 0.9, \delta_V = 0 \) (left plots) and \( \delta_M = 0, \delta_V = 0.9 \) (right plots).

\[
d^{ij}_{R(ZX^c)} = (Z_{1i}^{+}Z_{1j}^{+} + \cos 2\theta_W \delta_{ij}) \quad \text{and} \quad Z^{\pm} \text{ are the chargino mixings.} \quad \text{(49)}
\]

\[
f^{(ti)}_{L} = \frac{e}{\sqrt{2 s_W} M_W} \frac{m_{d_{i}}}{\cos \beta} Z_{2i}^{-}, \quad \text{and} \quad Z^{\pm} \text{ are the chargino mixings.} \quad \text{(50)}
\]

\[
f^{(ti)}_{R} = \frac{e}{s_W} Z_{1i}^{+},
\]

\[
f^{L(\tilde{t}i)} = 0, \quad f^{R(\tilde{t}i)} = \frac{e}{\sqrt{2 s_W} M_W \sin \beta} Z_{2i}^{+}.
\]
up-type scalars.

Here also, the couplings needed for the calculation of \( \Gamma(Z \rightarrow b \bar{s}) \) in the form defined in \([5]\), \([6]\) and \([7]\) are obtained from the Lagrangian terms in \((45)-(50)\) by rotating the weak states, \( \Phi_U \), to the physical states, \( \Phi_U \), according to \([59]\). These couplings are summarized in Table III.

The contribution of the \( \tilde{t} - \tilde{c} \) mixed states to the one-loop diagrams in Fig. 1 are characterized as follows:

I. The quantities that mediate the flavor changing transition \( b \rightarrow s \) in the \( \tilde{t} - \tilde{c} \) mixed scenario are: \( \delta_M^{U(23)} \), \( \delta_L^{U(23)} \), \( \delta_R^{U(23)} \), \( \delta_L^{U(32) \prime} \), \( \delta_R^{U(32) \prime} \), \( \delta_{LL}^{U(23)} \), \( \delta_{RR}^{U(23)} \), \( \delta_{LL}^{U(32) \prime} \), \( \delta_{RR}^{U(32) \prime} \), \( \delta_{LL}^{U(32)} \), \( \delta_{RR}^{U(32)} \), \( \delta_{LL}^{U(32) \prime} \), \( \delta_{RR}^{U(32) \prime} \), \( \delta_{LL}^{U(32) \prime} \), \( \delta_{RR}^{U(32) \prime} \), \( \delta_{LL}^{U(32) \prime} \), \( \delta_{RR}^{U(32) \prime} \), \( \delta_{LL}^{U(32) \prime} \), \( \delta_{RR}^{U(32) \prime} \), \( \delta_{LL}^{U(32) \prime} \), \( \delta_{RR}^{U(32) \prime} \), \( \delta_{LL}^{U(32) \prime} \), \( \delta_{RR}^{U(32) \prime} \). Recall that the \( LL \) and \( RR \) delta’s originate from the bilinear soft terms in \((27)\), while the \( LR \) delta’s are associated with the trilinear soft breaking SUSY terms. Thus, we will separate these two types of flavor violating sources in our numerical analysis. In particular, we define \( \delta_M^{U} = \delta_M^{U(23)} = \delta_M^{U(32)} = \delta_M^{U(23)} = \delta_M^{U(32)} = \delta_M^{U(32)} \), \( \delta_L^{U} = \delta_L^{U(23)} = \delta_L^{U(32)} = \delta_L^{U(23)} = \delta_L^{U(32)} = \delta_L^{U(32)} \), and we vary either \( \delta_M^{U} \) or \( \delta_L^{U} \) in the range \([0,1]\). Note that an \( \mathcal{O}(1) \) value for either \( \delta_M^{U} \) or \( \delta_L^{U} \) is consistent with all experimental data \([20, 23]\).

II. The required SUSY parameter space is: \( m_0, \mu, A_t, \tan \beta, m_2, \delta_M^{U} \), \( \delta_A^{U} \), where \( m_2 \) is the SU(2) gaugino mass parameter. The low-energy values of these six parameters fix the \( \tilde{t} - \tilde{c} \) scalar spectrum (i.e., masses and mixing matrices) and the chargino masses and mixing matrices from which all couplings in Table III are derived.

As in the \( b - s \) mixing case, these parameters will be varied subject to the requirement that the squark masses are heavier than 150 GeV and, in addition, that the charginos are heavier than 100 GeV \([24]\).

Taking maximal flavor violation in the \( \tilde{t} - \tilde{c} \) mixing scenario, i.e., \( \delta_M^{U} \sim \mathcal{O}(1) \) or \( \delta_A^{U} \sim \mathcal{O}(1) \), and varying the rest of the SUSY parameters involved subject to the above criteria, we find that \( BR(Z \rightarrow b \bar{s} + b \bar{s}) \) can reach few \( \times 10^{-8} \) at best. Here also, the \( BR(Z \rightarrow b \bar{s} + b \bar{s}) \) is significantly enhanced when large mass splittings between the four up-type squarks, \( m_{\tilde{t}_{1,2}} \) and \( m_{\tilde{t}_{3,4}} \), are present. Such a hierarchy in the up-type squark mass spectrum makes the GIM-like cancellation mentioned earlier less effective.

Indeed a two orders of magnitude difference between the \( \tilde{t} - \tilde{c} \) and \( b - s \) mixing cases is expected due to an \( (\alpha_s/\alpha)^2 \) enhancement factor in the \( b - s \) scenario (compared to the \( \tilde{t} - \tilde{c} \) mixing case) which arises from the gluino QCD coupling.

In Figs. 8 we plot \( BR(Z \rightarrow b \bar{s} + b \bar{s}) \) as a function of \( \tan \beta \), for combinations of \( m_0 = 1000, 1600 \) and 2200 GeV with \( m_2 \) = 200, 600 and 1000 GeV and for either \( \{ \delta_M^{U} = 0.9, \delta_A^{U} = 0 \} \) or \( \{ \delta_M^{U} = 0, \delta_A^{U} = 0.9 \} \). [5] For illus-

---

[5] Note that the physical up-squark masses have the same dependence on \( \delta_M^{U} \) or \( \delta_A^{U} \) when one of the two delta’s is set to zero.
At the one-loop level the $\lambda$ following soft SUSY breaking bilinear term is relevant to the ones in (51) and (52). For our purpose, only the set of $\tilde{\lambda}$ type RPV terms of the superpotential, i.e., extended by new trilinear and bilinear soft terms which are generation indices and $a, b$ are SU(2) singlet down-type quark supermultiplet. Also, $\sigma$ is the SU(2) singlet doublet quark and lepton supermultiplets, respectively, and $\tilde{D}$ is the SU(2) singlet down-type quark supermultiplet.

where $\tilde{\lambda}$ are generation indices and $a, b$ are SU(2) indices.

The RPVB operator is:

$$W_{RPVB} = -\epsilon_{ab} \lambda_{ijk} \tilde{L}_i^a \tilde{Q}_j^b \tilde{D}_k^c,$$  \hspace{0.5cm} (52)

where $\tilde{H}_u$ is the up-type Higgs supermultiplet and $i = 1, 2, 3$ labels the lepton generation.

In addition, if one does not impose $R_P$, then the usual set of $R_P$ conserving (RPC) soft SUSY breaking terms is extended by new trilinear and bilinear soft terms which correspond to the RPV terms of the superpotential, i.e., to the ones in (51) and (52). For our purpose, only the following soft SUSY breaking bilinear term is relevant:

$$V_{RPVB} = \epsilon_{ab} h_{ij} \tilde{L}_i^a \tilde{H}_u^b,$$  \hspace{0.5cm} (53)

where $\tilde{L}$ and $\tilde{H}_u$ are the scalar components of $\tilde{L}$ and $\tilde{H}_u$, respectively.

The RPVT operator ($\propto \lambda'$) in (51) gives rise to the following scalar-fermion-fermion RPV interactions:

$$\mathcal{L} = \lambda'_{ijk} \left\{ \tilde{\nu}_i^L \tilde{d}_R^h \tilde{u}_L^j + \tilde{d}_R^h \tilde{d}_R^h \tilde{e}_L^j + \left( \tilde{d}_R^h \right)^* (\tilde{\nu}_i^L)^c \tilde{d}_L^j \right\},$$

where $d(u)$ is a down(up)-quark, $e(\nu)$ is a charged-lepton(neutrino) and scalars are denoted with a tilde.

The RPVB operator ($\propto \mu_i$) in (52) gives rise to mixings among charged leptons and charginos as well as between neutrinos and neutralinos. However, low energy flavor changing processes [29], flavor changing lepton Z-decays [30] and neutrino masses [26, 30, 31, 32] suggest that the $\mu_i$ are expected to be vanishingly small.

We will, therefore, neglect its contribution to the decay $Z \rightarrow d_j d_j$.[7] On the other hand, the soft breaking RPVB term ($\propto b_i$) in (53) gives rise to mixings between sleptons and Higgs-bosons which may be exchanged in the loops of the diagrams shown in Fig. 4.

Let us now categorize the different types of RPV interactions that contribute at one-loop to the flavor changing decay of $Z \rightarrow d_j d_j$. Since the decay $Z \rightarrow d_j d_j$ conserves $R_P$, there should be two insertions of RPV vertices in the one-loop diagrams of Fig. 4. We can thus divide the various types of RPV one-loop exchanges into two categories, type A and type B, according to the pair of RPV couplings involved:

**Type A:** The RPV contributions that are proportional to the product $\lambda' \lambda'$, i.e., $\Gamma(Z \rightarrow d_j d_j) \propto \lambda' \lambda'$, where $\lambda'$ is defined in (51).

**Type B:** The RPV contributions that are proportional to the product $b \lambda'$, i.e., $\Gamma(Z \rightarrow d_j d_j) \propto b \lambda'$, where $b$ is the soft breaking RPV bilinear coupling defined in (53).

### A. Type A RPV effect

The Type A RPV contribution to $Z \rightarrow d_j d_j$ emanates from the first five RPV Yukawa-like interaction vertices in (54). In this case we assume that $b_i \rightarrow 0$ such that mixing effects between sleptons and the Higgs fields are absent.

We can further sub-divide the type A contributions into 6 types according to the type of scalar ($S$) and type of fermion ($f$) that are being exchanged in the loops:

- **type A1:** $S_\alpha = \tilde{e}_{L,\alpha}, \ f_i = u_i$
- **type A2:** $S_\alpha = \tilde{d}_{L,\alpha}, \ f_i = u_i$

- [6] Note that at the one-loop level the $\lambda$ type couplings do not contribute to the decay $Z \rightarrow d_j d_j$.
- [7] The one-loop exchanges of possible lepton-chargino and neutrino-neutralino admixture states in $Z \rightarrow d_j d_j$ will be controlled by the square of the RPV couplings product $\mu_i \times \lambda'$. 

type A3: \( S_\alpha = \tilde{d}_{R,\alpha} \), \( f_i = \nu^c_i \)

\( S_\alpha = \tilde{\nu}_{L,\alpha} \), \( f_i = d_i \)

\( S_\alpha = \tilde{\nu}^c_{L,\alpha} \), \( f_i = d_i \)

\( S_\alpha = \tilde{u}_{L,\alpha} \), \( f_i = e_i \), \( (59) \)

where \( \alpha = 1, 2, 3 \) and \( i = 1, 2, 3 \).

For each of the type A RPV exchanges above, the generic couplings defined in (4), (3) and (8) are summarized in Table IV. In particular, for a given \( f \), the \( Z f f \) couplings of (8) are given by (3). The \( S_d f \) couplings are taken from the Yukawa like interactions (54), while the \( ZS_s \) couplings are extracted from \( \mathcal{L}(V_\mu \tilde{u} u) \) in (48), from \( \mathcal{L}(V_\mu \tilde{d} d) \) in (44) and from the \( \tilde{V} \tilde{L} \tilde{L} \) interaction Lagrangian:

\[
\mathcal{L}(V_\mu \tilde{L} \tilde{L}) = -i \frac{1}{2} \tilde{L} \left[ \frac{e}{s_w} \tau^3 A_\mu - \frac{e}{c_w} B_\mu \right] \tilde{\ell} \tilde{\ell}, \quad (56)
\]

where \( A_\mu \) and \( B_\mu \) are the SU(2) and U(1) gauge fields, respectively, \( \tilde{L} = \left( \tilde{u}_L \right. \) and \( \tau^3 = \left( \begin{array}{c} 1 \\ 0 \\ 0 \\ -1 \end{array} \right) \).

Given the couplings in Table IV and the structure of the form factors in (10)-(15) it is evident that there are only two types of \( \lambda \chi \) product combinations which enter the type A RPV contribution to the decay \( Z \to d f \bar{d} f \):

1. The product \( \lambda^*_{mnJ} \lambda_{nnJ} \). Types A1, A2, A5 and A6 are proportional to this couplings product.

2. The product \( \lambda^*_{mnJ} \lambda_{nnJ} \). Types A3 and A4 are proportional to this couplings product.

Furthermore, since none of the scalars have both a left and a right handed RPVT coupling to fermions in the type A scenario, i.e., in the notation of (8) either \( b^i_{L(\alpha)} = 0 \) or \( b^i_{R(\alpha)} = 0 \) (see Table IV), the form factors \( B^i_{L,k} \) and \( B^i_{R,k} \) in the amplitude (9) (which requires a none-zero value for both the left and the right handed scalar-fermion-down quark couplings) vanish. Also, since \( b^i_{L(\alpha)} = 0 \) for the RPV contributions of types A1, A2, A5 and A6, they contribute only to the right-handed vector-like form factor \( A^i_{R,k} \). Similarly, the RPV contributions of types A3 and A4 have \( b^i_{R(\alpha)} = 0 \), therefore, contributing only to \( A^i_{L,k} \).

It should be noted that for any one of the type A RPV exchanges, if the scalars of different flavors that are being exchanged in the loops are degenerate and upon neglecting all fermion masses except for the top-quark, then there remain only three distinct types of contributions of the \( \lambda \) products in the type A RPV scenario. That is, under this assumption \( BR(Z \to d_f \bar{d}_f) \) can have only three different values which we denote by \( BR_1^{1J} \), \( BR_2^{1J} \) and \( BR_3^{1J} \) as follows:

\[
\begin{align*}
BR_1^{1J} &= BR(Z \to d_f \bar{d}_f) \text{ when } \left( \lambda_{11}^* \times \lambda_{1j}^* \right)^2 \neq 0 : j \neq 3, i = 1, 2, 3 \quad (57) \\
BR_2^{1J} &= BR(Z \to d_f \bar{d}_f) \text{ when } \left( \lambda_{31}^* \times \lambda_{1j}^* \right)^2 \neq 0 : i = 1, 2, 3 \quad (58) \\
BR_3^{1J} &= BR(Z \to d_f \bar{d}_f) \text{ when } \left( \lambda_{1j}^* \times \lambda_{1j}^* \right)^2 \neq 0 : i, j = 1, 2, 3 \quad (59)
\end{align*}
\]

such that \( BR(Z \to b\bar{s}) = BR_1^{32}, BR_2^{32} \) or \( BR_3^{32} \) depending on which of the three \( \lambda \chi \) product combinations is non-zero.

In Table IV we give a sample of our numerical results for the three BR’s in (57)-(59) scaled by the square of the appropriate \( \lambda \chi \) product. The results presented in Table IV correspond to the case of a single non-zero \( \lambda \chi \) product (one index combination) contributing to each of the BR’s \( BR_1^{32}, BR_2^{32} \) and \( BR_3^{32} \). In addition, the masses of the squark and slepton being exchange in the loop (for a given index combination of the corresponding \( \lambda \chi \) product) are set to either \( m_q = 500 \) GeV with \( m_\ell = 200 \) GeV or \( m_S = 1000 \) GeV with \( m_\ell = 500 \) GeV.

The existing limits on the \( \lambda \chi \) coupling products in (57)-(59) seem to indicate that the typical allowed values of \( \lambda \times \lambda' \) for any of the index combinations in (57)-(59) is at the level of \( \sim \) few \( \times 10^{-2} \) \( [13] \). It should be noted, however, that the limits reported in [8] assume 100 GeV scalar masses. These limits scale with the scalar masses (typically as \( m_\ell/100 \) GeV\(^2 \), where \( m_\ell \) is the scalar mass) and are, therefore, relaxed as the scalars become heavier.[8]

Using \( \lambda \chi \sim \mathcal{O}(10^{-2}) \) in conjunction with the results presented in Table IV, we see that the expected branching ratio for \( Z \to b\bar{s} \) in the type A RPV scenario investigated in this section lies in the range \( BR(Z \to d_f \bar{d}_f) \sim 10^{-11} - 10^{-10} \).

This type A RPV one-loop effect in \( BR(Z \to d_f \bar{d}_f) \) was also investigated in [4]. Although [4] evaluated some distinct limiting cases of the type A RPV contributions,
TABLE IV: The couplings required for the calculation of $\Gamma(Z \to d_l d_j)$ in the type A RPV scenario. The couplings follow the notation in (2)-(8).

|   | type A1 | type A2 | type A3 |
|---|--------|--------|--------|
| scalar ($S_α$) | $\tilde{e}_{L,α}$, $α = 1, 2$ | $\tilde{e}_{L,α}$, $α = 1, 2$ | $\tilde{e}_{L,α}$, $α = 1, 2$ |
| fermion ($f_i$) | $\tilde{u}_{i}$, $i = 1, 2, 3$ | $\tilde{d}_{L,α}$, $α = 1, 2$ | $\tilde{d}_{L,α}$, $α = 1, 2$ |
|   | $a_{ij}^{L(Zf)}$ | $a_{ij}^{L(Zf)}$ | $a_{ij}^{L(Zf)}$ |
|   | $a_{ij}^{R(Zf)}$ | $a_{ij}^{R(Zf)}$ | $a_{ij}^{R(Zf)}$ |
|   | $b_{ij}^{L(α)}$ | $b_{ij}^{L(α)}$ | $b_{ij}^{L(α)}$ |
|   | $b_{ij}^{R(α)}$ | $b_{ij}^{R(α)}$ | $b_{ij}^{R(α)}$ |
| $g_{Z}^{αβ}$ | $-\frac{e^2_{λα} - e^2_{λβ}}{2s_{W}c_{W}}\delta_{αβ}$ | $-\frac{e^2_{λα} - e^2_{λβ}}{2s_{W}c_{W}}\delta_{αβ}$ | $-\frac{e^2_{λα} - e^2_{λβ}}{2s_{W}c_{W}}\delta_{αβ}$ |

TABLE V: Results for the three types of branching ratios $BR^{12}$, $BR^{22}$, $BR^{32}$ as defined in (57)-(59), each scaled by its appropriate $λ λ'$ coupling product. Results are given for two sets of squark and slepton masses as indicated.

|   | $BR^{12}$ | $BR^{22}$ | $BR^{32}$ |
|---|-----------|-----------|-----------|
|   | $\frac{BR^{12}(λ_{1β}λ'_{2β})}{(λ_{1β}λ'_{1β})}$, $j \neq 3$ | $\frac{BR^{22}(λ_{3β}λ'_{3β})}{(λ_{3β}λ'_{3β})}$ | $\frac{BR^{32}(λ_{1β}λ'_{2β})}{(λ_{1β}λ'_{1β})}$ |
| $m_q = 500 \text{ GeV}$, $m_ℓ = 200 \text{ GeV}$ | $4.2 \times 10^{-7}$ | $2.4 \times 10^{-6}$ | $3.4 \times 10^{-6}$ |
| $m_q = 1000 \text{ GeV}$, $m_ℓ = 500 \text{ GeV}$ | $3.9 \times 10^{-7}$ | $6.4 \times 10^{-8}$ | $3.0 \times 10^{-6}$ |

Our results agree with the highlights of their analysis, i.e., that the typical $BR(Z \to d_l d_j)$ is expected to be at the level of $10^{-11} - 10^{-10}$ if $λ λ' \sim 0(10^{-2})$.

Thus, the type A RPV scenario is expected to yield a BR smaller even from the SM one. We, therefore, proceed below to the second RPV type B scenario which seems to give a much larger $BR(Z \to b s)$ within the experimentally allowed range of values for its relevant RPV parameter space.
B. Type B RPV effect

The type B RPV effect arises when a Higgs particle that is being exchanged in the loops mixes with a slepton through the RPVB operator in (53) and then couples to the external down quark via a $\lambda^\prime$ type coupling.

For simplicity we will assume that $b_i \neq 0$ only for $i = 3$ in (53), thus, considering only the mixing between the third generation sleptons ($\tilde{L}_3$) and the Higgs scalar fields ($H_d$ and $H_u$).[9] It should be noted that $b_3 \neq 0$ leads in general to a non-vanishing tau-sneutrino VEV, $v_3$. However, since lepton number is not a conserved quantum number in this scenario, the $H_d$ and $\tilde{L}_3$ superfields lose their identity and can be rotated to a particular basis ($H', \tilde{L}'_3$) in which either $\mu_3$ or $v_3$ are set to zero.[24, 28, 32, 35]. In what follows, we find it convenient to choose the “no VEV” basis, $v_3 = 0$, which simplifies our analysis below.

Let us define the SU(2) components of the up-type Higgs, down-type Higgs and $\tilde{L}_3$ scalar doublet fields (setting $v_3 = 0$):

$$H_u = \begin{pmatrix} \frac{h_u^+}{\sqrt{2}} \\ \frac{c_\tau + vy + i\phi_0^\tau}{\sqrt{2}} \end{pmatrix},$$

$$H_d = \begin{pmatrix} \frac{c_d^0 + vy + i\phi_0^d}{\sqrt{2}} \\ \frac{h_d^-}{\sqrt{2}} \end{pmatrix},$$

$$\tilde{L}_3 = \begin{pmatrix} \frac{i\phi_0^\tau + i\tilde{v}_\tau^0}{\sqrt{2}} \\ \frac{\tilde{c}_3^-}{\sqrt{b_3}} \end{pmatrix},$$

where $\tilde{v}_\tau^0$, $\tilde{v}_-^0$ and $\tilde{c}_3^-$ are the SU(2) CP-even, CP-odd tau-sneutrino and left handed stau fields, respectively.

When $b_3 \neq 0$ the 3rd generation slepton SU(2) fields in (54) mix with the Higgs fields. In particular, in the basis $\Phi_C = (h_u^+, h_d^-, \tilde{c}_3^-)$, the squared mass matrix in the charged scalar sector becomes:[10]

$$M_C^2 = \begin{pmatrix} m_{\tilde{c}_3^0}^2 + m_{\tilde{A}_3^0}^2 & m_{\tilde{c}_3}^0 & (m_{\tilde{A}_3^0})^2 \\ m_{\tilde{c}_3}^0 & m_{\tilde{A}_3^0}^2 & m_{\tilde{c}_3}^0 \\ (m_{\tilde{A}_3^0})^2 & m_{\tilde{c}_3}^0 & m_{\tilde{A}_3^0}^2 \end{pmatrix}.$$  (61)

Similarly, in the basis $\Phi_E = (\xi_u^0, \xi_d^0, \tilde{v}_\tau^0)$, the CP-even neutral scalar squared mass matrix becomes (at tree-level):[11]

$$M_E^2 = \begin{pmatrix} m_{\xi_u^0}^2 + m_{\xi_d^0}^2 & m_{\xi_u^0} & m_{\xi_d^0} \\ m_{\xi_u^0} & m_{\xi_d^0} + m_{\tilde{A}_3^0}^2 & m_{\xi_u^0} \\ m_{\xi_d^0} & m_{\xi_u^0} & m_{\xi_d^0} + m_{\tilde{A}_3^0}^2 \end{pmatrix}.$$  (62)

Finally, in the CP-odd neutral scalar sector and in the basis $\Phi_O = (\phi_u^0, \phi_d^0, \phi_v^0)$ one obtains:

$$M_O^2 = \begin{pmatrix} (m_{\phi_u^0})^2 + (m_{\phi_d^0})^2 & m_{\phi_u^0} & m_{\phi_d^0} \\ m_{\phi_u^0} & (m_{\phi_d^0})^2 & m_{\phi_u^0} \\ m_{\phi_d^0} & m_{\phi_u^0} & (m_{\phi_d^0} + m_{\tilde{A}_3^0}^2) \end{pmatrix}.$$  (63)

The new charged scalar and CP-even and CP-odd neutral scalar mass-eigenstates (i.e., the physical states) are then derived by diagonalizing $M_C^2$, $M_E^2$ and $M_O^2$, respectively. Let us denote the physical states by:

$$\Phi_C = \begin{pmatrix} H^+ \\ \tilde{h}^+ \\ \tilde{\tau}^+ \end{pmatrix}, \quad \Phi_E = \begin{pmatrix} H \\ h \\ \tilde{\nu}_\tau^+ \end{pmatrix}, \quad \Phi_O = \begin{pmatrix} A \\ G \\ \tilde{\nu}_\nu^- \end{pmatrix}.$$  (64)

[9] The consequences of $b_1 \neq 0$ and/or $b_2 \neq 0$ is to introduce additional mixings among sleptons of different generations and mixings between the selectron and/or smuon scalar doublets with the Higgs fields. These extra mixing effects are not crucial for the main outcome of this section.

[10] We neglect the mixing between the right-handed SU(2) stau singlet and the charged Higgs fields which is proportional to the tau mass.

[11] The one loop corrections to the $2 \times 2$ Higgs block in $M_E^2$ can cause a significant deviation to the tree-level mass of the light Higgs. This effect will be discussed below.
such that, for a small RPVB in the scalar potential, the new physical states in (64) are the states dominated by what would be the corresponding physical states in the RPC limit, $b_3 = 0$, for which the Higgs sector decouples from the snepton sector in (61), (62) and (63). In particular, if $b_3 \rightarrow 0$, then $H$, $h$, $A$ and $H^+$ become the usual RPC MSSM’s CP-even heavy Higgs, CP-even light Higgs, CP-odd pseudo-scalar Higgs and charged Higgs states, respectively. Similarly, in this limit $\tilde{\nu}_i^+ and $\tilde{\nu}_i^-$ become the two mass-degenerate CP-even and CP-odd sneutrino states with a common mass $m_{\tilde{\nu}} = m_{\tilde{\nu}} = m_0$, while $\tilde{\tau}^+$ is the usual pure left handed stau field with a mass $m_{\tilde{\tau}}^+ = \sqrt{(m_0^2)^2 - m_W^2 \cos 2\beta}$. Note also that $G$ and $G^+$ are the neutral and charged Goldstone bosons that are absorbed by the $Z$ and $W$-bosons and are, therefore, the states with a zero eigenvalue in $M^2_{\Phi}$ and $M^2_{\Phi}$, respectively.

The physical states $\Phi_C$, $\Phi_E$ and $\Phi_O$ are related to the weak states $\Phi^0_C$, $\Phi^0_E$ and $\Phi^0_O$ via:

$$\Phi^0_{C,i} = R_{C,ik} \Phi_{C,k} ,$$
$$\Phi^0_{E,i} = R_{E,ik} \Phi_{E,k} ,$$
$$\Phi^0_{O,i} = R_{O,ik} \Phi_{O,k} ,$$

where $R_{C}$, $R_{E}$ and $R_{O}$ are the rotation matrices that diagonalize $M^2_{C}$, $M^2_{E}$ and $M^2_{O}$, respectively.

Notice that the mass matrices $M^2_{C}$, $M^2_{E}$ and $M^2_{O}$ depend only on four SUSY parameters: $m_{A}^0$, $m_{\tilde{\nu}}^0$, $b_3$ and $\tan\beta$. These parameters, therefore, completely fix the rotation matrices $R_{C}$, $R_{O}$ and $R_{E}$ from which the CP-even and CP-odd neutral scalar spectrum as well as the charged scalar spectrum is completely determined.

Clearly then, the type B RPV contributions involve the 3rd generation sleptons that can mix with the Higgs fields through a $b_3$ bilinear RPV coupling which enters the slepton-Higgs mixed mass matrices in (61)-(63). Here also, we can further sub-divide the type B RPV effects according to the type of scalar ($S$) and type of fermion ($f$) that are being exchanged in the loops:

**type B1**

- $S_\alpha = \Phi_{C,\alpha}$; $f_i = u_i$

with $\alpha = 1, 3$, $i = 1, 2, 3$ and

- $S_\alpha = \Phi_{E,\alpha}$; $f_i = d_i$

with $\alpha = 1, 2, 3$, $\beta = 1, 3$, $i = 1, 2, 3$.

Note that we have omitted virtual exchanges of the two Goldstone bosons $G$ and $G^+$ since the one-loop amplitudes are being calculated in the unitary gauge.

The two RPV effects, of types B1 and B2 above, are driven by the Higgs-slepton scalar admixtures $\Phi_C$, $\Phi_E$, $\Phi_O$ which couple to quarks through a combination of $\lambda^\prime$ and Higgs Yukawa coupling. Hence, the Higgs-like components in $\Phi_C$, $\Phi_E$ and $\Phi_O$ will couple through the Higgs Yukawa terms, while the slepton-like component interact with the quarks via the $\lambda$-type RPV couplings in (64).

For the type B1 RPV contribution in (66) the form factors defined in (61) are calculated following the prescription described in section II. The generic couplings defined in (4), (6) and (8) are summarized in Table VI for the type B1 RPV exchanges. In particular, the $Sd_f$ couplings (for $S = \Phi_C$ and $f = u$) are a combination of the Yukawa-like trilinear RPV interactions in (65), (66) (those with the third generation snepton indices) and the charged Higgs Yukawa couplings which are the same as in the 2HDM of type II (given in section III).

The $ZSS$ couplings (for $S = \Phi_C$) in Table VI are derived from $\mathcal{L}(V_u L_3 L_3)$ in (64) and from the following $\mathcal{L}(V_u H_d H_d)$ and $\mathcal{L}(V_u H_u H_u)$ pieces (68):

$$\mathcal{L}(V_u H_d H_d) = -i \frac{1}{2} H_d^* \left[ \frac{e}{sw} t^3 A_3^\prime - \frac{e}{cw} B_3 \right] \partial_H^{\mu} H_d ,$$

(68)

$$\mathcal{L}(V_u H_u H_u) = -i \frac{1}{2} H_u^* \left[ \frac{e}{sw} t^3 A_3^\prime + \frac{e}{cw} B_3 \right] \partial_H^{\mu} H_u ,$$

(69)

where the SU(2) scalar doublet fields $\tilde{L}_3$, $H_d$ and $H_u$ are defined in (60).

**TABLE VI:** The couplings required for the calculation of $\Gamma(Z \rightarrow d_i d_j)$ in the type B1 RPV scenario. The couplings follow the notation in (4), (6) and (8). The couplings $a_{L,R}(\tau u)$ are given in (61).
where we have combined the contribution of the self energy diagrams 5+6 and 7+8: $M_5 + M_6 \equiv M_{56}$ and $M_7 + M_8 \equiv M_{78}$, which leads accordingly to $A_{L,5}^{I J} + A_{L,6}^{I J} \equiv A_{L,56}^{I J}$ and $A_{L,7}^{I J} + A_{L,8}^{I J} \equiv A_{L,78}^{I J}$. Also,

$$B_{L,k}^{I J} = 0 \quad \text{for} \quad k = 1 - 8$$

$$B_{L,9}^{I J} = a_{R(Zd)} \sum_{\alpha,i,j} g_Z \phi_{R,L(\alpha)} b_{L(\alpha)}^{I J} \left[ 2 \left( C_0^\alpha - C_1^\alpha \right) \right] + \frac{1}{m_Z^2} \left( C_{10}^0 + C_{11}^0 \right) \right] \quad \text{(78)}$$

Here also the right-handed form factors, $A_{R,k}^{I J}$ and $B_{R,k}^{I J}$, are obtained from the corresponding left handed ones by interchanging $L \rightarrow R$ and $R \rightarrow L$ in all the couplings in (70)-(73).

The two-point and three-point loop form factors $B_k^I$ with $k = 5, 7, C_x^k$ with $x \in 1, 2, 3, 4, 5$ and $k = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ and $C_x^k$ with $x \in 0, 11, 12$ and $k = 9, 10$, which appear in (70)-(73) are given by:

$$B_5^I = B_1 \left( m_{d_4}, m_{\tilde{d}_d}, m_{\tilde{d}_d} \right) \quad \text{(80)}$$

$$B_7^I = B_1 \left( m_{d_4}, 2 m_{\tilde{d}_d}, m_{\tilde{d}_d} \right) \quad \text{(81)}$$

and

$$C_x^1 = C_x \left( m_{d_4}, m_{\tilde{d}_d}, m_{\tilde{d}_d}, m_{d_4}, q^2, m_{d_4} \right) \quad \text{(82)}$$

$$C_x^2 = C_x \left( m_{d_4}, m_{\tilde{d}_d}, m_{\tilde{d}_d}, m_{d_4}, q^2, m_{d_4} \right) \quad \text{(83)}$$

$$C_x^3 = C_x \left( m_{d_4}, m_{\tilde{d}_d}, m_{\tilde{d}_d}, m_{d_4}, q^2, m_{d_4} \right) \quad \text{(84)}$$

$$C_x^4 = C_x \left( m_{d_4}, m_{\tilde{d}_d}, m_{\tilde{d}_d}, m_{d_4}, q^2, m_{d_4} \right) \quad \text{(85)}$$

FIG. 10: One-loop diagrams that contribute to the flavor changing decays $Z \rightarrow d_j \bar{d}_j$ in the type B2 RPV scenario.
\[ C'^{9}_{x}; \quad \tilde{C}'^{9}_{x} = C_{x}; \quad \tilde{C}_{x} \left( m_{d_{i}}^{2}, m_{Z}^{2}, m_{\Phi_{E},\alpha}, m_{d_{i}}, q^{2}_{i}, m_{d_{i}}^{2} \right), \quad (86) \]

\[ C^{10}_{x}; \quad \tilde{C}'^{10}_{x} = C_{x}; \quad \tilde{C}_{x} \left( m_{d_{i}}^{2}, m_{Z}^{2}, m_{\Phi_{E},\alpha}, m_{d_{i}}, q^{2}_{i}, m_{d_{i}}^{2} \right), \quad (87) \]

where \( B_{1}(m_{d_{i}}^{2}, m_{Z}^{2}, p^{2}), \quad C_{x}(m_{d_{i}}^{2}, m_{Z}^{2}, p^{2}, p_{Z}^{2}, p_{3}^{2}) \) and \( \tilde{C}_{x}(m_{d_{i}}^{2}, m_{Z}^{2}, p^{2}, p_{Z}^{2}, p_{3}^{2}) \) are defined in the appendix.

The couplings \( a_{L,(Z)d_{i}} \) and \( a_{R,(Z)d_{i}} \) follow the notation of the generic Sdf vertex in \( (8) \); for \( S = \Phi_{E} \) or \( \Phi_{O} \) and \( f = d \):

\[ \Lambda(\Phi_{E,\alpha} \tilde{d}_{i} d_{j}) = i \left( h_{L}^{(E)}(\alpha, i) + h_{R}^{(E)}(\alpha, i) \right), \quad (88) \]

\[ \Lambda(\Phi_{O,\alpha} \tilde{d}_{i} d_{j}) = i \left( h_{L}^{(O)}(\alpha, i) + h_{R}^{(O)}(\alpha, i) \right). \quad (89) \]

These couplings emanate from both the Yukawa-like trilinear RPV interactions in \( (\text{14}) \) and the neutral Higgs Yukawa vertices of a 2HDM of type II in \( (\text{15}) \).

The couplings \( g_{Z}^{\alpha \beta} \) of a \( Z \)-boson to a \( \Phi_{E,\alpha} \Phi_{O,\beta} \) pair follow our generic definition of the VSS vertex in \( (7) \). It is derived from the Lagrangian terms in \( (5d, 68) \) and \( (5f) \).

The coupling \( g_{ZZ\Phi_{E}}^{\alpha} \) of a \( Z \)-boson to a pair of \( Z \)-bosons is defined as:

\[ \Lambda(Z_{\mu}Z_{\nu}^{\alpha}) = ig_{ZZ\Phi_{E}}^{\alpha} g_{\mu\nu}, \quad (90) \]

and is obtained from the following \( ZZ\Phi_{E}^{0} \) and \( ZZ\Phi_{E}^{0} \) interaction terms (recall that \( \phi_{L}^{0} \) are the CP-even SU(2) components of \( H_{d,u} \) as defined in \( (\text{80}) \) \( \text{22} \):

\[ \mathcal{L}(ZZ\Phi_{E}^{0}) = \frac{\epsilon^{2}}{(2swcW)}Z_{\mu}Z_{\nu} \left( v_{d_{i}}^{0} + v_{u}^{0} \right), \quad (91) \]

where \( v_{d} \) and \( v_{u} \) are the VEV’s of the down-type and up-type Higgs doublets, respectively.

Before presenting our numerical results for the type B RPV contribution let us discuss some of its salient features and outline the main assumptions and notations regarding the relevant parameter space involved:

I. The pseudo-scalar “bare” mass (i.e., its mass in the RPC limit of \( b_{3} \to 0 \)) can be approximated from the tree-level relation which, for \( t_{3} \gg 1 \), gives \( (m_{A}^{0})^{2} \sim b_{0} t_{3}^{2} \), where \( b_{0} \) is the usual RPC soft-breaking bilinear Higgs term in the scalar potential, i.e., \( V_{RPC} \supset b_{0} H_{d} H_{u} \).

\[
\begin{array}{|c|c|}
\hline
\text{type B2} & \text{fermion (} f_{i} \text{)} \\
\hline
\Phi_{E,\alpha}, \quad \alpha = 1, 2, 3 & d_{i}, \quad i = 1, 2, 3 \\
\hline
\Phi_{O,\alpha}, \quad \alpha = 1, 3 & a_{L(Z)d} \\
\hline
\Phi_{O,\alpha}, \quad \alpha = 1, 3 & a_{R(Z)d} \\
\hline
\end{array}
\]

TABLE VII: The couplings required for the calculation of \( \Gamma(Z \to d_{i} d_{j}) \) in the type B2 RPV scenario. The couplings follow from the Feynman rules in \( (\text{3}) \), \( (\text{4}) \), \( (\text{5}) \), \( (\text{6}) \) and \( (\text{15}) \).

Thus, without loss of generality, we trade the bilinear RPV parameter, \( \varepsilon \), as follows:

\[ b_{3} \equiv \varepsilon (m_{A}^{0})^{2} \cot \beta, \quad (92) \]

such that \( \varepsilon \sim \frac{b_{0}}{m_{A}^{0}} \) parametrizes the relative amount of RPV in the scalar potential. In particular, \( \varepsilon \ll 1 \) for small bilinear RPV and \( \varepsilon \sim 1 \) if RPV/RPC \( \sim 1 \) in the SUSY scalar sector.

The existing experimental limits on \( \varepsilon \) come from:

1. A non-vanishing \( b_{3} \) can generate a radiative (one-loop) tau-neutrino mass. The laboratory limit on the tau-neutrino mass allows, however, the quantity \( \frac{b_{3}}{m_{A}^{0}} \sim \varepsilon \) to be of \( \sim O(1) \) \( \text{28} \).
2. The parameter \( b_{3} \), or equivalently the quantity \( \varepsilon \sim \frac{b_{3}}{m_{A}^{0}} \), can have important consequences on the CP-even and CP-odd Higgs-like scalar mass spectrum, see \( \text{36, 37} \). In particular, \( \varepsilon \) can drive the mass of the physical CP-even light Higgs below its present LEP2 lower bound which, for \( m_{A} \gtrsim 200 \text{ GeV} \), is roughly
\(m_h \gtrsim 110 \text{ GeV}\) irrespective of \(\tan \beta\) \cite{[12]}. Also, a non-zero \(\varepsilon\) can give rise to negative eigenvalues (i.e., to the physical square masses) for the CP-even and CP-odd mass matrices \(M_E^2\) and \(M_S^2\) in \cite{[62]} and \cite{[63]}, depending on the values of the rest of the type B parameter space, i.e., on \(m_{A_0}, m_{S^0}\) and \(\tan \beta\).

Therefore, in what follows, we will vary the parameters \(\{m_{A_0}, m_{S^0}, \tan \beta, \varepsilon\}\) subject to the existing LEP2 lower bound on the light Higgs mass and to the requirement that \(m_{\tilde{u}_R^0}, m_{\tilde{d}_R^0}\) and \(m_{\tilde{s}^+}\) are \(> 150 \text{ GeV}\).

Since the light Higgs mass is very sensitive to higher order corrections to the \(2 \times 2\) Higgs block in \(M_E^2\), as in \cite{[34], [35]} in order to derive realistic exclusion regions for the parameter space \(\{m_{A_0}, m_{S^0}, \tan \beta, \varepsilon\}\) through the requirement \(m_h \gtrsim 110 \text{ GeV}\), we include the dominant higher order corrections (coming from the \(t\) \(- \tilde{t}\) sector) to the \((\tilde{t}_R^0, \tilde{c}_R^0)\) block in \(M_E^2\), following the approximate formulae given in \cite{[34]} and taking the maximal mixing scenario with a typical squark mass of \(m_{\tilde{q}} = 1 \text{ TeV}\).

II. Since \(b_3\) is not a flavor changing parameter, the transition between down-quarks of different generations, i.e., between the external down-quarks \(d_I \rightarrow d_J\), is necessarily driven by a \(\lambda^d\) coupling with the appropriate non-diagonal indices (disregarding flavor changing transitions due to small non-diagonal CKM elements). Thus, the type B RPV one-loop effect in \(Z \rightarrow d_I \tilde{d}_J\) is necessarily proportional to either \(b_3 \lambda^d_{11J}\) or \(b_3 \lambda^d_{31J}\).

In particular, for \(Z \rightarrow b\bar{s}\) we find that the dominant contribution is attributed to the type B1 exchanges of the charged scalars and it arises when \(\lambda_3^{132} \neq 0\). The only other possible index combination for \(Z \rightarrow b\bar{s}\), which is \(\lambda_3^{132} \neq 0\), yields a much smaller branching ratio. This enhancement for the \((332)\) index combination can be traced to the fact that, for this particular combination, the charged scalar amplitude involves also a top-quark exchange, thus gaining a factor of \(m_{t}/m_c\) compared to the \(\lambda_3^{132} \neq 0\) case (which involves a charm-quark exchange in the

3. A non-vanishing \(\varepsilon\) can also alter the cross-section for \(ZZ\) and \(WW\) pair production through s-channel exchanges of the CP-even scalars \(\Phi_E\) \cite{[36], [37]}. The measured \(ZZ\) and \(WW\) cross-sections in LEP2 can thus be used to place limits on \(\varepsilon\) as a function of \(\{m_{A_0}, m_{S^0}, \tan \beta\}\). These limits, however, can be evaded if the \(\tilde{e}_R^+ e^-\) trilinear RPV coupling \(\lambda_{131}\) is assumed small enough (see \cite{[34], [35]}).

We will, therefore, not consider such limits below.

\[\text{FIG. 11: BR}(Z \rightarrow b\bar{s} + \bar{b}b)\) as a function of the “bare” sneutrino mass parameter \(m_{\tilde{s}^0}\) (see text), for some combinations of values of \(m_{\tilde{q}^0}\) and \(\varepsilon\) (as indicated in the figure) and for \(\tan \beta = 3\) (left plots) and \(\tan \beta = 50\) (right plots). \(\lambda_3^{132} = 1\) is used and \(\varepsilon\) is defined in \cite{[12]}.
FIG. 12: Physical masses of the heavy CP-even Higgs ($m_H$), CP-odd Higgs ($m_A$), charged Higgs ($m_{H^±}$), CP-even tau-sneutrino ($m_{ν^τ}$), CP-odd tau-sneutrino ($m_{ν^τ}$) and the stau ($m_{τ}$), as a function of the “bare” tau-sneutrino mass ($m_{ν^τ}$), for $t_β = 50$, for $m_A = 300, 600$ or $900$ GeV and for $ε = 0.4$ (left figures) and $ε = 0.9$ (right figures). $ε$ is defined in (12).

III. In the limit $ε → 0$ the type B2 effect vanishes. However, since $ε → 0$ causes the charged Higgs sector to decouple from the stau sector and since the RPC MSSM Higgs sector is similar to the 2HDM of type II, the type B1 contribution approaches that of the type II 2HDM in this limit. Thus, for $ε → 0$, the type B1 RPV effect will be proportional to the off-diagonal CKM elements as in the case of the type II 2HDM discussed in section III.

IV. In the numerical analysis below we will set $λ'_{332} = 1$, while all other lambda’s with different index combinations are set to zero. The experimental limit on this coupling, derived from $R_ℓ = Γ(Z → ℓℓ)/Γ(Z → hadrons)$ in the presence of $ε$, is (at the $2σ$ level) $λ'_{332} = 0.45$ for squark masses of $∼ 500$ GeV, while $λ'_{332} = 1$ is allowed for squark masses $≥ 500$ GeV. The perturbativity bound on this coupling is $λ'_{332} = 1.04$ (41). Thus, we will assume that the squarks are heavy enough to allow $λ'_{332}$ to lie near its perturbativity limit (recall that no squarks are involved in the type B RPV contribution to $Z → bs$).

In Fig. 11 we show $BR(Z → bs + b̅s)$ as a function of the “bare” tau-sneutrino mass $m_{ν^τ}$ (i.e., what would be its mass in the RPC limit), for various possible values of $m_A$, $ε$ and for $t_β = 3$ (left side) and $t_β = 50$ (right side) loops).

Evidently, $BR(Z → bs + b̅s)$ is much larger in the high tan $β$ scenario and it drops with $m_{ν^τ}$.

The masses of the heavy CP-even Higgs, CP-odd Higgs and charged Higgs as well as the CP-even, CP-odd tau-sneutrino and the stau particles are depicted in Fig. 12, for $t_β = 50$ and for the same combinations of $ε$ and $m_A$ that are used in Fig. 11. We note that in the limit $(m_A^0)^2 ≥ m_Z^2$ (applicable to the values of $m_A^0$ in Figs. 1 and 2), one has $m_H ∼ m_A ∼ m_{H^±}$ and if in addition $(m_{ν^τ})^2 ≥ m_Z^2$, then also the CP-even, CP-odd tau-sneutrinos and the stau are roughly degenerate. Thus, only two curves are shown in each plot in Figs. 12, which are sufficient to approximately describe all these six scalar masses.

Fig. 12 shows that at some instances, the Higgs-like and slepton-like scalar masses exhibit a discontinuous jump, at which point they “switch” identities. This phenomena is caused by the particular dependence of the physical scalar masses on the “bare” masses $m_A^0$ and $m_{ν^τ}$ in the presence of $ε ≠ 0$. In particular, the corrections to the “bare” scalar masses due to a non-vanishing $b_3$ term are proportional to factors of $[(m_A^0) − (m_{ν^τ}^0)]$ (for more details see [56, 57]), thereby changing sign at the turning points. Moreover, the off-diagonal elements of the rotation matrices $R_E$, $R_O$ and $R_C$, which are responsible for the slepton-Higgs mixings, are also inversely proportional to factors of $[(m_A^0) − (m_{ν^τ}^0)]$, therefore, enhancing the type B RPV effect as $m_A$ approaches $m_{ν^τ}$ as can be seen in Figs. 1.
| model            | scalars in the loops         | $BR(Z \rightarrow b\bar{s})$ |
|------------------|------------------------------|-------------------------------|
| SM               | W-boson (no scalars)         | $10^{-8}$                     |
| 2HDMII           | charged Higgs                | $10^{-10}$                    |
| T2HDM            | charged Higgs                | $10^{-8}$                     |
| SUSY with $\tilde{t} - \tilde{c}$ mixing | $\tilde{t} - \tilde{c}$ admixtures | $10^{-8}$ |
| SUSY with $\tilde{b} - \tilde{s}$ mixing | $\tilde{b} - \tilde{s}$ admixtures | $10^{-6}$ |
| SUSY with trilinear $RPV$-violation | squarks and sleptons | $10^{-10}$ |
| SUSY with bilinear $RPV$-violation | slepton-Higgs admixtures | $10^{-6}$ |

TABLE VIII: The best case values for the branching ratio of $Z \rightarrow b\bar{s}$ for each of the six models considered in this paper upon imposing the available experimental limits on the relevant parameter space of each of them. The SM prediction is also given.

To summarize this section, with a large tan $\beta$, a $BR(Z \rightarrow b\bar{s} + b\bar{s}) \sim O(10^{-6})$ is possible within the type B scenario, e.g., for 40% lepton number violation in the SUSY scalar potential ($\epsilon = 0.4$) and if the sleptons masses lie around $\sim 200$ GeV. For a heavier slepton spectrum a larger $\epsilon$ is required in order to push the branching ratio to the $10^{-6}$ level.

It should also be emphasized that since $BR(Z \rightarrow b\bar{s} + b\bar{s})$ is dominated by the $\lambda_{B}^{\prime}$, the decay $Z \rightarrow b\bar{s}$ is an efficient probe of this specific flavor changing trilinear RPV coupling.

VI. EXPERIMENTAL FEASIBILITY

In this section we will very briefly comment about the feasibility of observing (or achieving a limit) a signal of $Z \rightarrow b\bar{s}$ with a branching ratio of order $10^{-6}$, at a Linear Collider producing $10^{9}$ Z-bosons.

Such a signal should appear in the detector as an event with one b-jet and one light-jet (assuming no distinction is made between light = d, u or s-quarks). In the spirit of the analysis made with the 1993 and 1994 LEP data on $Z \rightarrow b\bar{s}$, one defines $\epsilon_{B}^{d}$ and $\epsilon_{L}^{d}$ to be the efficiencies that a quark (or anti-quark) of flavor $q$ is tagged as a b-jet ($B$) and light-jet ($L$), respectively. Thus, the key efficiency parameters for the detection of $Z \rightarrow b\bar{s}$ are $\epsilon_{B}^{d}$, $\epsilon_{L}^{d}$ and $\epsilon_{B}^{\bar{d}}$, where the latter represent the probability that a b-jet is identified as a light-jet and is important for controlling the dominant background (to the $Z \rightarrow B + L$ signal) coming from $Z \rightarrow bb$. Note that due to the expected smallness of the "Purity" parameters $\epsilon_{L}^{d}$ and $\epsilon_{B}^{d}$ (see [8]), the background to $Z \rightarrow B + L$ caused by the SM $Z \rightarrow d\bar{d}$, $s\bar{s}$, $u\bar{u}$, $c\bar{c}$ decays will be sub-dominant.

With $10^{9}$ Z-bosons, the expected number of events coming from $Z \rightarrow b\bar{s}$ (i.e., from new physics) and identified as $Z \rightarrow B + L$, is:

$$S \sim 10^{9} \times \epsilon_{B}^{d} \epsilon_{L}^{d} BR(Z \rightarrow b\bar{s}) .$$  \hspace{1cm} (93)

Similarly, the expected number of background $Z \rightarrow B + L$ events coming from the SM decay $Z \rightarrow bb$ is:

$$B \sim 10^{9} \times \epsilon_{B}^{d} \epsilon_{B}^{\bar{d}} BR(Z \rightarrow b\bar{b}) .$$  \hspace{1cm} (94)

Using (93) and (94), the expected statistical significance, $S/\sqrt{B}$, of the new physics signal $Z \rightarrow b\bar{s}$, with a branching ratio of order $10^{-6}$, can reach beyond the 3-sigma level for $\epsilon_{B}^{d} \sim 0.6 - 0.8$, $\epsilon_{L}^{d} \sim 0.3 - 0.5$ and $\epsilon_{B}^{\bar{d}} \sim O(10^{-4})$. These values require an improvement to the 1993 and 1994 analysis [8], by a factor of 2-3 for $\epsilon_{B}^{d}$ and $\epsilon_{L}^{d}$ and by an order of magnitude for $\epsilon_{B}^{\bar{d}}$. With the expected advancement in the jet-tagging methods, in particular, for two-body decays of the Z-boson, these required values for the efficiency parameters above should be well within the reach of the future Linear Collider.

We can also get a clue about how low one can go in the value (or limit) of $BR(Z \rightarrow b\bar{s})$ with $10^{9}$ Z-bosons, from the fact that the LEP preliminary results [9] achieved $BR(Z \rightarrow b\bar{s}) < O(10^{-3})$ with $O(10^{6})$ Z-bosons. Scaling this limit, especially with the expected advance in b-tagging and identification of non-b jets methods, an $O(10^{-6})$ branching ratios should be easily attained at a Giga-Z factory.

VII. SUMMARY

We have re-examined the flavor changing radiative decays of a Z-boson to a pair of down-quarks, $Z \rightarrow d\bar{d}$, with $I \neq J$. These Z-decay channels may prove useful in searching for new flavor physics beyond the SM at the
in the scalar sector:
in six beyond the SM model scenarios for flavor-violation branching ratios as small as $BR(Z \rightarrow b\bar{s}) \sim 10^{-7} - 10^{-6}$.

The $d_I \rightarrow d_J$ transition was assumed to be generated at one-loop through flavor violation in interactions between scalars and fermions.

A complete analytical derivation of the width $\Gamma(Z \rightarrow d_I d_J)$ is presented using the form factor approach for the $Zd_I d_J$ interaction vertex. These form factors are evaluated for the complete set of scalar-fermion one-loop exchanges with generic scalar-fermion flavor-violating couplings.

This prescription is then applied to the decay $Z \rightarrow b\bar{s}$ in six beyond the SM model scenarios for flavor-violation in the scalar sector:

1. Two Higgs doublet models with non-standard charged-Higgs couplings to quarks:
   - A two Higgs doublet model of type II (2HDM-II).
   - A two Higgs doublet model "for the top-quark" (T2HDM).

2. Supersymmetry with flavor-violation in the squark sector:
   - Supersymmetry with stop-scharm mixing.
   - Supersymmetry with sbottom-sstrange mixing.

3. Supersymmetry with flavor-violation from R-parity violating interactions:
   - Supersymmetry with trilinear R-parity violation.
   - Supersymmetry with trilinear and bilinear R-parity violation.

Folding in the existing experimental limits on the relevant parameter space of each of these models, we calculated the branching ratio for the decay $Z \rightarrow b\bar{s}$. The highlights of our results are summarized in Table VII. In particular, we find that two Higgs doublet models with flavor violation originating from charged scalar interactions with fermions are expected to yield an extremely small $BR(Z \rightarrow b\bar{s})$; smaller than the SM prediction and smaller than the reach of a Giga-Z $\ell^+\ell^-$ collider. Thus, a signal of $Z \rightarrow b\bar{s}$ in TESLA will be inconsistent with the underlying mechanisms for flavor violation in these two Higgs doublets model and will, therefore, rule out these options.

The same conclusions can be drawn in the stop-scharm mixing and the trilinear R-parity violation SUSY scenarios. On the other hand, SUSY with mixings between the bottom and strange-type squarks and/or mixings between sleptons and Higgs fields (bilinear R-parity violation) both of which may originate from the soft SUSY breaking sector, can drive the $BR(Z \rightarrow b\bar{s})$ to the $10^{-6}$ level for large $\tan\beta$ values. This enhancement is typical to these two flavor-violating SUSY scenarios if there are large mass-splittings between the scalars exchanged in the loops due to a GIM-like cancellation which is operational in the scalar mass-matrices and is, therefore, less effective as the scalar masses depart from degeneracy.

A $Z \rightarrow b\bar{s}$ signal in a Giga-Z TESLA or any other collider may, therefore, be a good indication for the underlying dynamics of these two flavor-violating SUSY scenarios and, if interpreted in that way, will provide for evidence of an hierarchical structure in the mass spectrum of the SUSY scalar sector.

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APPENDIX A: ONE-LOOP FORM FACTORS

In this appendix we give the two-point and three-point one-loop form factors which are defined by the one-loop momentum integrals as follows [12]:
\[ \tilde{C}_0; \tilde{C}_\mu (m_1^2, m_2^2, m_3^2, p_1^2, p_2^2, p_3^2) = \int \frac{d^4q}{i\pi^2} \frac{q^2; q^2q_\mu}{(q^2 - m_1^2)((q + p_1)^2 - m_2^2)((q - p_3)^2 - m_3^2)}, \]  
(A2)

\[ B_0; B_\mu (m_1^2, m_2^2, p^2) = \int \frac{d^4q}{i\pi^2} \frac{1; q_\mu}{(q^2 - m_1^2)((q + p)^2 - m_2^2)}, \]  
(A3)

where \( \sum_i p_i = 0 \) is to be understood above.

The coefficients \( B_x \) with \( x \in 0, 1 \), \( C_x \) with \( x \in 0, 11, 12, 21, 22, 23, 24 \) and \( \tilde{C}_x \) with \( x \in 0, 11, 12 \) are then defined through the following relations [43]:

\[ B_\mu = p_\mu B_1, \]  
(A4)

\[ C_\mu = p_\mu C_{11} + p_2 \mu C_{12}, \]  
(A5)

\[ \tilde{C}_\mu = p_1 \mu \tilde{C}_{11} + p_2 \mu \tilde{C}_{12}, \]  
(A6)

\[ \tilde{C}_\mu = p_1 \mu \tilde{C}_{11} + p_2 \mu \tilde{C}_{12}, \]  
(A6)

and

\[ C_{\mu\nu} = p_1 \mu p_1 \nu C_{21} + p_2 \mu p_2 \nu C_{22} + \{p_1 p_2\}_{\mu\nu} C_{23} + g_{\mu\nu} C_{24}, \]  
(A7)

where \( \{ab\}_{\mu\nu} \equiv a_\mu b_\nu + a_\nu b_\mu \).