STOCHASTIC GRAVITATIONAL WAVE BACKGROUND FROM NEUTRON STAR r-MODE INSTABILITY REVISITED

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ABSTRACT

We revisit the possibility and detectability of a stochastic gravitational wave (GW) background produced by a cosmological population of newborn neutron stars (NSs) with r-mode instabilities. The NS formation rate is derived from both observational and simulated cosmic star formation rates (CSFRs). We show that the resultant GW background is insensitive to the choice of CSFR models, but depends strongly on the evolving behavior of CSFR at low redshifts. Nonlinear effects such as differential rotation, suggested to be an unavoidable feature which greatly influences the saturation amplitude of the r-mode, are considered to account for GW emission from individual sources. Our results show that the dimensionless energy density \(\Omega_{GW}\) could have a peak amplitude of \((1-3.5) \times 10^{-8}\) in the frequency range \((200-1000)\) Hz, if the smallest amount of differential rotation corresponding to a saturation amplitude of order unity is assumed. However, such a high-mode amplitude is unrealistic as it is known that the maximum value is much smaller and at most \(10^{-2}\). A realistic estimate of \(\Omega_{GW}\) should be at least four orders of magnitude lower \((\sim 10^{-12})\), which leads to a pessimistic outlook for the detection of the r-mode background. We consider different pairs of terrestrial interferometers (IFOs) and compare two approaches to combine multiple IFOs in order to evaluate the detectability of this GW background. Constraints on the total emitted GW energy associated with this mechanism to produce a detectable stochastic background (a signal-to-noise ratio of 2.56 with 3 year cross-correlation) are \(\sim 10^{-3} M_{\odot} c^2\) for two co-located advanced LIGO detectors, and \(2 \times 10^{-5} M_{\odot} c^2\) for two Einstein telescopes. These constraints may also be applicable to alternative GW emission mechanisms related to oscillations or instabilities in NSs depending on the frequency band where most GWs are emitted.

Key words: cosmology: miscellaneous – gravitational waves – stars: formation – stars: neutron

Online-only material: color figures

1. INTRODUCTION

A stochastic gravitational wave background (SGWB) is a target for gravitational wave (GW) interferometers (IFOs). It could have two very different origins. It may result from a cosmological population of newborn neutron stars (NSs) with r-mode instabilities. The NS formation rate is derived from both observational and simulated cosmic star formation rates (CSFRs). We show that the resultant GW background is insensitive to the choice of CSFR models, but depends strongly on the evolving behavior of CSFR at low redshifts. Nonlinear effects such as differential rotation, suggested to be an unavoidable feature which greatly influences the saturation amplitude of the r-mode, are considered to account for GW emission from individual sources. Our results show that the dimensionless energy density \(\Omega_{GW}\) could have a peak amplitude of \((1-3.5) \times 10^{-8}\) in the frequency range \((200-1000)\) Hz, if the smallest amount of differential rotation corresponding to a saturation amplitude of order unity is assumed. However, such a high-mode amplitude is unrealistic as it is known that the maximum value is much smaller and at most \(10^{-2}\). A realistic estimate of \(\Omega_{GW}\) should be at least four orders of magnitude lower \((\sim 10^{-12})\), which leads to a pessimistic outlook for the detection of the r-mode background. We consider different pairs of terrestrial interferometers (IFOs) and compare two approaches to combine multiple IFOs in order to evaluate the detectability of this GW background. Constraints on the total emitted GW energy associated with this mechanism to produce a detectable stochastic background (a signal-to-noise ratio of 2.56 with 3 year cross-correlation) are \(\sim 10^{-3} M_{\odot} c^2\) for two co-located advanced LIGO detectors, and \(2 \times 10^{-5} M_{\odot} c^2\) for two Einstein telescopes. These constraints may also be applicable to alternative GW emission mechanisms related to oscillations or instabilities in NSs depending on the frequency band where most GWs are emitted.

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1. INTRODUCTION

A stochastic gravitational wave background (SGWB) is a target for gravitational wave (GW) interferometers (IFOs). It could have two very different origins. It may result from a large variety of cosmological processes developed in the very early universe, such as amplification of quantum vacuum fluctuations, phase transitions, cosmic strings, etc. (see, e.g., Maggiore 2000; Buonanno 2003 for reviews). Additionally, an astrophysical GW background (AGWB) is expected to be produced by the superposition of a large number of unresolved sources since the beginning of star formation (Schneider et al. 2000; Regimbau & Mandic 2008). There have been a host of literatures dedicated to the studies of various AGWB sources, such as core collapse supernovae (CCSNs; Blair & Ju 1996) leading to the formation of neutron stars (NSs; Coward et al. 2001; Howell et al. 2004; Buonanno et al. 2005) or black holes (BHs; Ferrari et al. 1999a; de Araujo et al. 2004; Pereira & Miranda 2009), phase transitions in NSs (Sigl 2006; de Araujo & Marranghello 2009), coalescing compact binaries consisting of NSs and/or BHs (Schneider et al. 2001; Farmer &phinney 2002; Regimbau & de Freitas Pacheco 2006a; Regimbau & Chauvineau 2007), magnetars (Regimbau & de Freitas Pacheco 2006b), and Population III stars (Sandick et al. 2006; Suwa et al. 2007; Marassi et al. 2009) among others.

In this paper, we revisit the possibility that the r-mode instabilities in newly born NSs could form an SGWB. NSs, having long been considered to be likely observational sources for GW detection, emit gravitational radiation in a number of ways, for example, through CCSNe, inspiralling compact binaries, rotating deformed stars, oscillations, and instabilities (Andersson et al. 2010). First postulated more than 10 years ago (Andersson 1998; Friedman & Marsink 1998), the r-mode instability has been attracting increased attention due to the fact that it is driven unstably by GW emission and it can be active for a wide range of core temperatures and angular velocities (Lindblom et al. 1998; Andersson & Kokkotas 2001; Andersson 2003). Early estimate indicated that an energy equivalent to roughly 1% of a solar mass is radiated in GWs (Andersson et al. 1999) as an initially rapidly rotating star spins down. This led to the expectation of an SGWB produced by a cosmological population of young rapidly rotating NSs with closure density \(h^2\Omega_{GW}\) peaking at \(\sim 10^{-8}\) of the present-day critical energy density of the universe (Owen et al. 1998; Ferrari et al. 1999b).

The most important aspect of the r-mode instability is the largest amplitude (often called the saturation amplitude \(\alpha\)) that the perturbation can grow to. This maximum amplitude determines how fast the NS spins down and whether the associated GW emission will be detectable (either in terms of single event or a stochastic background). In Owen et al. (1998) and then in Ferrari et al. (1999b), it was taken to be of order unity (there were no estimates of this maximum at that time). Later, more detailed (both analytical and numerical) studies seriously questioned the potential of the instability as well as the efficiency of GW emission (Rezzolla et al. 2000; Ho & Lai 2000; Lindblom et al. 2000; Rezzolla et al. 2001a, 2001b; Lindblom & Owen 2002). On the one hand, it was suggested that energy transfer to other stellar inertial modes can significantly reduce the saturation amplitude of the r-mode (Schenk et al. 2002; Marsink 2002; Brink et al. 2004). Arras et al. (2003) tested a
nonlinear coupling between stellar inertial modes and revealed much lower values of saturation amplitude ($\alpha \sim 10^{-4}$ to $10^{-5}$). Then, a specific resonant three-mode coupling between the $r$-mode and the pair of fluid modes was identified to be responsible for the catastrophic decay of large-amplitude $r$-modes, and a perturbative analysis of the decay rate suggested a maximum dimensionless saturation amplitude $\alpha_{\text{max}} < 10^{-2}$ to $10^{-2}$ (Lin & Suen 2006). More recently, Bondarescu et al. (2009) examined the three-mode coupling between the $r$-mode and two other inertial modes and showed that the $r$-mode evolution can progress in a number of different directions depending on unknown properties of the viscosity, leading to very complex consequent mode evolution and the associated GW signal.

On the other hand, differential rotation, first suggested by Rezzolla et al. (2000, 2001a, 2001b), is an unavoidable feature of nonlinear $r$-modes (Stergioulas & Font 2001; Lindblom et al. 2001; Sá 2004). The small values of $\alpha$ mentioned above are also supported by studies on the role of differential rotation, causing large-scale drift of fluid elements, in the nonlinear evolution of $r$-modes (Sá & Tomé 2005). In particular, they parameterize the initial amount of differential rotation by $K$ and then relate the parameter $K$ to the largest amplitude that the $r$-mode can grow to. In this paper, we will use the characteristic GW amplitude given by Sá & Tomé (2006) to account for the average source spectrum. The adopted GW amplitude, parameterized by parameter $K$, in fact scales with the saturation amplitude $\alpha$. Below we will discuss the influence of this quantity on the $r$-mode background.

In addition to the average source spectra, the properties of AGWB also depend on GW source formation rate. Studies of cosmic star formation rate (CSFR) allow the estimation of the birth rate of NSs. In the last decade, our knowledge of cosmic star formation has been greatly improved due to advances in astronomical observation and hydrodynamic simulation. Here we take into account both the observational and simulation-based CSFR models to obtain the NS formation rate and discuss their effects on the resultant GW background. In particular, we will investigate the role of the maximal redshift of different CSFR models in our results.

The high-frequency window of GW spectrum ($10 \, \text{Hz} \leq f \leq 10^9 \, \text{kHz}$) is open today through pioneering efforts of the first-generation terrestrial IFOs, such as the Laser Interferometer Gravitational Wave Observatory (LIGO; Abramovici et al. 1992) in Livingston (LIGO) and in Hanford (LIGOH), Virgo (Caron et al. 1997) near Pisa, GEO600 (Lück et al. 1997) in Hannover, and TAMA300 (Ando et al. 2001) at Tokyo. Although GWs were not detected, an observational upper limit ($\Omega_{\text{GW}} < 6.9 \times 10^{-6}$) was placed on the energy density of the SGWB at around 100 Hz, exceeding previous indirect limits from the big bang nucleosynthesis and the cosmic microwave background (Abbott et al. 2009). In the future, the SGWB from NS $r$-mode instability, among others, may offer an important detection target for the proposed second- and third-generation detectors represented by advanced LIGO\(^5\) (or advanced Virgo\(^6\)) and the Einstein Telescope (ET\(^6\)), respectively.

Detectors throughout the world can act as a network in order to improve the ability to detect the SGWB. Two approaches for combining 2N IFOs are proposed in Allen & Romano (1999): (1) correlating the outputs of a pair of IFOs, then combining the multiple pairs, and (2) directly combining the outputs of 2N IFOs. For any given real IFOs, it is necessary to compare these two optimal approaches for detecting the SGWB. Cella et al. (2007) have shown that the approach of combining multiple pairs of IFOs using Virgo, LIGO, and GEO can improve the ability to detect the SGWB by simulating an isotropic GW background with an astrophysically motivated spectral shape. Fan & Zhu (2008) compared the detection ability of the two approaches for stochastic GWs from string cosmology.

Detectability of the $r$-mode background is demonstrated here by calculating signal-to-noise ratios (S/Ns) for pairs of currently operating IFOs and advanced detectors at their design sensitivities. We also consider two approaches of combining four real IFOs to examine how many improvements can be obtained. The organization of this paper is as follows. In Section 2, we review works on the determination of CSFR and present five CSFR models. In Section 3, we derive the NS formation rate as a function of redshift using the adopted CSFR models. Then, by combining the source formation rate from Section 3 and the characteristic GW amplitude of individual events, spectral properties of the $r$-mode stochastic background are investigated in Section 4. We will discuss the detectability of $r$-mode background in Section 5, and finally Section 6 is devoted to our conclusions.

Throughout the paper, the so-called $\Lambda$CDM cosmology is assumed with $H_0 = 100h \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}$ with $h = 0.7$ and $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ (e.g., Komatsu et al. 2009).

2. COSMIC STAR FORMATION RATE

The CSFR, which has a tight connection with the GW event rate, is of intense interest to many fields of astrophysics. For many years, effort has gone into studying the cosmic star formation history (see, e.g., Madau et al. 1996; Hopkins 2004; Wilkins et al. 2008). Since CSFR is not a directly observable quantity, usually the rest-frame ultraviolet (UV) light is considered to be an indicator (see Calzetti 2008 and references therein for details about CSFR indicators) of star formation because it is mainly radiated by short-lived massive stars. With the help of the Hubble Space Telescope and other large telescopes, the galaxy luminosity density of rest-frame UV radiation is studied, and then converted into CSFR density through the adoption of a universal stellar initial mass function (IMF) to calculate the conversion factor. Many authors have developed parameterized fits to the expected evolution of the CSFR with redshift. First, following Porciani & Madau (2001), we adopt three different forms which model the CSFR density for redshifts up to $z \approx 4$:

$$\dot{\rho}_* (z) = 1.67 C_i h_{65} F(z) G_i(z) M_\odot \text{yr}^{-1} \text{Mpc}^{-3},$$

with $i = 1, 2, 3$ denoting the different models, $C_i$ a constant, $h_{65} = h/0.65$, $G_i(z)$ a function of $z$, and $F(z) = (\Omega_m (1+z)^3 + \Omega_\Lambda)^{1/2} / (1+z)^{3/2}$. A constant factor of $1.67$ is applied to account for the conversion of a Salpeter IMF with a lower cutoff from 0.5 $M_\odot$ to 0.1 $M_\odot$ (the one we will use below). $F(z)$ convert the assumed cosmology from an Einstein–de Sitter universe to the $\Lambda$CDM cosmology (Porciani & Madau 2001). The first fit (hereafter SFR1) is given by Madau & Pozzetti (2000),

\(^5\) http://www.ligo.caltech.edu/advLIGO/
\(^6\) http://www.et-gw.eu/
with $C_1 = 0.3$ and $G_1(z) = e^{3.4z}/(e^{3.8z} + 45)$, where the CSFR increases rapidly from $z = 0$ to reach a peak at around $z = 1.5$ and then gradually declines at higher redshifts. The second one (SFR2) is from Steidel et al. (1999) with $C_2 = 0.15$ and $G_2(z) = e^{3.4z}/(e^{3.4z} + 22)$, where the CSFR remains roughly constant at $z \geq 2$. The third model (SFR3) from Blain et al. (1999) has $C_3 = 0.2$ and $G_3(z) = e^{3.05z - 0.4}/(e^{2.95z} + 15)$, where CSFR increases at higher redshifts to account for effects of dust extinction.

With the improvement in measurements of galaxy luminosity functions at a broad range of wavelengths, star formation history can be traced to higher redshifts. Here we consider the work by Hopkins & Beacom (2006), who refined the previous models up to redshift $z \sim 6$ from new measurements of the galaxy luminosity function in the UV (Sloan Digital Sky Survey, Galaxy Evolution Explorer, and COMBO17) and far-infrared (FIR) wavelengths (Spitzer Space Telescope). A parametric fit (hereafter HB06) is given by

$$\rho_s(z) = \frac{0.017 + 0.13} {1 + (z/3.3)^{5.3}} \, M_\odot \, \text{yr}^{-1} \, \text{Mpc}^{-3},$$

assuming 737 cosmology and a modified Salpeter A IMF (Baldry & Glazebrook 2003). Although the IMF used to derive HB06 is different from the standard Salpeter, this will not introduce considerable errors to our results because the evolution of the CCSN rate based on the CSFR and on an assumed universal IMF is largely independent of the choice of the IMF (Madau 1998).

Many authors have addressed the issue of calibrating the high-$z$ CSFR through long-duration gamma-ray bursts (GRBs; see, e.g., Yüksel et al. 2008 and Kistler et al. 2009). Recently, Wang & Dai (2009) use the latest GRBs data to constrain the CSFR up to $z = 8.3$. Meanwhile, there are other methods for determining the high-$z$ CSFR, such as observations of color-selected Lyman break galaxies (Bouwens et al. 2008) and Lyα emitters (Ota et al. 2008). However, such calibrations cannot reach considerable agreements except for an overall decline at $z \geq 4$ (see Figure 1 of Yüksel et al. 2008; Kistler et al. 2009; Wang & Dai 2009). Due to huge uncertainties and the incompleteness of data sets, we will not include them here.

On the other hand, Springel & Hernquist (2003) derive the CSFR from hydrodynamic simulation of structure formation in ΛCDM cosmology. They study the history of cosmic star formation from the “dark ages,” at redshift $z = 20$, to the present. The CSFR obtained in their study is broadly consistent with measurements given observational uncertainty and can be remarkably well fitted by the following form (hereafter SH03):

$$\rho_s(z) = \rho_m \frac{\beta \exp[\alpha(z - z_m)]}{\beta - \alpha + \alpha \cdot \exp[\beta(z - z_m)]},$$

where $\alpha = 3/5$, $\beta = 14/15$, $z_m = 5.4$ marks a break redshift, and $\rho_m = 0.15 \, M_\odot \, \text{yr}^{-1} \, \text{Mpc}^{-3}$ fixes the overall normalization. It is worth mentioning that they consider a ΛCDM model with the same parameters with our assumed 737 cosmology.

In Figure 1, we plot the CSFR predicted in the above five models as a function of redshift. SFR1, SFR2, and SFR3 show distinguishable features at $z \geq 2$. SH03 peaks at a much higher redshift, between $z = 5$ and 6, than observation-based models (around $z \approx 2$). The cutoff of each curve in Figure 1 corresponds to maximum redshifts of CSFR models: $z_c = 4$ for SFR1–3, $z_c = 6$ for HB06, and $z_c = 20$ for SH03. What Figure 1 illustrates is our poor understanding of star formation history at high redshifts from astronomical observations.

Note that there are some other CSFR models, similar to or different from the five models adopted here, that are not included since our aim is not to make a complete survey of this issue but to phenomenologically investigate its influences on the SGWB from an ensemble of astrophysical sources. We refer readers to Calura & Matteucci (2003), Daigne et al. (2004), Bromm & Loeb (2006), Nagamine et al. (2006), and Fardal et al. (2007) for details of other studies on the determination of CSFR. In the following sections, we will investigate how different CSFRs affect the rate of NS formation and spectral properties of AGWB.

3. NEUTRON STAR FORMATION RATE

Since the evolving rate of CCSNe closely tracks the star formation rate, using the CSFR models presented in Section 2 we can estimate the number of NSs formed per unit time within the comoving volume out to redshift $z$ (Ferrari et al. 1999a):}

$$R_{\text{NS}}(z) = \int_0^z \rho_s(z') \frac{dV}{dz'} dz' \int_{m_{\text{max}}}^{m_{\text{min}}} \Phi(m) dm,$$

where $\rho_s(z)$ is the CSFR density, $dV/dz$ is the comoving volume element, and $\Phi(m)$ is the IMF. Here we assume that each CCSN results in either an NS or a BH and take an NS progenitor mass range of $8$–$25 \, M_\odot$. In order to compare with Ferrari et al. (1999b), we also consider a lower upper limit for NS progenitor masses $m_{\text{max}} = 20 \, M_\odot$ as indicated from core collapse simulations by Fryer (1999). However, according to Belczynski & Taam (2008), the mass of the NS progenitor might be greater than $40 \, M_\odot$ for stars in a binary system. So we will also include a higher limit of $m_{\text{max}} = 40 \, M_\odot$ for our calculations of the NS formation rate. (In Sigl 2006, the progenitor mass to form an NS ranges from $10 \, M_\odot$ to $40 \, M_\odot$.)

Note that in some studies (e.g., Coward et al. 2001; de Araujo et al. 2004; Regimbau & Mandic 2008) with respect to AGWB, there is an additional $(1 + z)$ term in Equation (4) dividing the CSFR to account for the time dilatation of CSFR due to cosmic expansion. Here, we do not include such a term according to de Araujo & Miranda (2005) who argue that the inclusion of this additional term is inadequate.

To integrate through Equation (4), one still needs to know the forms of $dV/dz$ and $\Phi(m)$. Following Regimbau & Mandic
(2008), the comoving volume element is related to \( z \) through

\[
\frac{dV}{dz} = 4\pi c \frac{r(z)^2}{H_0 E(\Omega, z)}, \tag{5}
\]

where \( H_0 \) is the Hubble constant, \( E(\Omega, z) = \sqrt{\Omega_\Lambda + \Omega_m(1 + z)^3} \), and \( r(z) \) is the comoving distance related to the luminosity distance by \( d_L = r_c(1 + z) \).

We consider the standard Salpeter IMF: \( \Phi(m) = A m^{-1.35} \) with \( A \) is a normalization constant, obtained through the relation \( \int_{m_{\min}}^{m_{\max}} m \Phi(m) dm = 1 \) with \( m_1 = 0.1 M_\odot \) and \( m_u = 125 M_\odot \). Then we plot the NS formation rate, \( R_{\text{NS}}(z) \), defined in Equation (4) for the CSFR models presented in Section 2 with a modest value of \( m_{\max} = 25 M_\odot \) in Figure 2. Note that SFR1, SFR2, and SFR3 show quite different behaviors for \( z > 2.5 \) and observation-based models give rise to more NS formation than SH03 up to respective redshift limits, but SH03 predicts a much higher cumulative NS formation rate for \( z \geq 10 \).

In Table 1, we present the total number (per unit time) of CCSN explosions leaving behind an NS out to corresponding redshift limits for the five CSFR models and for three values of \( m_{\max} : 20 M_\odot, 25 M_\odot, \) and \( 40 M_\odot \). We compare the results obtained here with those in Ferrari et al. (1999b), and find a factor of \( \sim 2–3 \) enhancement for the total NS formation rate, which is mainly due to differences of CSFR models and cosmology terms (e.g., different forms for the comoving volume element).

4. SPECTRAL PROPERTIES OF THE SGWB FROM NS \( r \)-MODE INSTABILITY

In this section, we will evaluate the spectral properties of the stochastic background produced by an ensemble of newly born NSs with nonlinear \( r \)-mode instabilities. Initially, let us review the formalism used to characterize the AGWB.

It is useful to characterize the spectral properties of an SGWB by specifying how the energy is distributed in the frequency domain. Explicitly, one introduces a dimensionless quantity, \( \Omega_{\text{GW}} \), given by

\[
\Omega_{\text{GW}}(\nu_{\text{obs}}) = \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d\ln \nu_{\text{obs}}}, \tag{6}
\]

where \( \rho_{\text{GW}} \) is the GW energy density, \( \nu_{\text{obs}} \) is the frequency in the observer frame, and \( \rho_c = 3H_0^2/8\pi G \) is the critical energy density required to close the universe today. For a stochastic background of astrophysical origin, the energy density is given by

\[
\Omega_{\text{GW}}(\nu_{\text{obs}}) = \frac{\nu_{\text{obs}}}{c^3 \rho_c} F_j(\nu_{\text{obs}}), \tag{7}
\]

where the spectral density of the flux at the observed frequency \( \nu_{\text{obs}} \) is defined as

\[
F_j = \int f_j(\nu_{\text{obs}}) dR, \tag{8}
\]

where \( f_j(\nu_{\text{obs}}) \) is the energy flux per unit frequency (in erg cm\(^{-2}\) Hz\(^{-1}\)) produced by a single source and \( dR \) is the differential GW event rate.

The energy flux per unit frequency \( f_j(\nu_{\text{obs}}) \) can be written as follows (Carr 1980):

\[
f_j(\nu_{\text{obs}}) = \frac{\pi c^3}{2G} h_c^2, \tag{9}
\]

where \( h_c \) is the dimensionless amplitude produced by an event that generates a signal with observed frequency \( \nu_{\text{obs}} \).

In order to obtain the spectral properties (e.g., the values of \( \Omega_{\text{GW}} \) as a function of \( \nu_{\text{obs}} \)) of the \( r \)-mode stochastic background, we have the differential rate of source formation \( dR_{\text{NS}}(z) \) through Equation (4) and still need to know the energy flux emitted by a single source. It has been shown that differential rotation can significantly influence the detectability of GWs emitted by a spinning-down newborn NS due to \( r \)-mode instability (Sá & Tomé 2006). Studies of the mode–mode coupling in rotating stars also indicate that the maximum amplitude that the \( r \)-mode can grow to is much smaller than previously estimated (Arras et al. 2003).

Here, we use the characteristic GW amplitude given by Sá & Tomé (2006)

\[
h_c(\nu) = \frac{5.5 \times 10^{-22}}{\sqrt{K + 2}} \sqrt{\frac{\nu}{\nu_{\text{max}}} \left( \frac{20 \text{ Mpc}}{d_L} \right)}, \tag{10}
\]

where \( K \) is a constant giving the initial amount of differential rotation associated with the \( r \)-mode and lies in the interval \(-5/4 \leq K \leq 10^3\) (see Sá & Tomé 2006 for details), \( \nu = \nu_{\text{obs}}(1 + z) \) is the frequency in the source frame, \( \nu_{\text{max}} \) is the maximum frequency of emitted GWs given by \( 2\Omega_K/3\pi \), where \( \Omega_K = 5612 \) Hz is the Keplerian frequency at which the star starts shedding mass at the equator, assumed to be the initial value of the angular velocity of the star, and \( d_L \) is the luminosity distance to the source. Note that the saturation amplitude \( \alpha \propto (K + 2)^{-1/2} \) (Sá & Tomé 2006), which means the GW amplitude is proportional to \( \alpha \), same as those given in Bondarescu et al. (2009). In the following calculations, we should keep in mind that the parameter \( K \) used in this paper

| Model (Redshift Limit) | \( m_{\max} = 20 M_\odot \) | \( 25 M_\odot \) | \( 40 M_\odot \) |
|-----------------------|-----------------|-----------------|-----------------|
| SFR1 \((z_\ast = 4)\) | 30.0            | 33.2            | 37.4            |
| SFR2 \((z_\ast = 4)\) | 39.3            | 44.5            | 49.1            |
| SFR3 \((z_\ast = 4)\) | 47.2            | 52.2            | 58.9            |
| HB06 \((z_\ast = 6)\) | 47.5            | 52.6            | 59.3            |
| SH03 \((z_\ast = 20)\) | 62.0            | 68.6            | 77.4            |
is equivalent to the saturation amplitude $\alpha$ of the NS $r$-mode instability.

From the above equations, we can obtain the dimensionless energy density

$$\Omega_{GW}(\nu_{\text{obs}}) = \frac{4\pi^2(1.1 \times 10^{-20})^2 \nu_{\text{obs}}^2}{3H_0^2(K+2)\nu_{\text{max}}} \times \left[ \int_{z_{\text{min}}}^{z_{\text{max}}} \int_{m_{\text{min}}}^{m_{\text{max}}} \rho_*(z)(1+z)^4 \left( \frac{1}{d_E} \right)^2 dV(m)dm dz \right].$$

(11)

Thus, by setting a value for $K$, one can calculate $\Omega_{GW}$ numerically through Equation (11) combined with the corresponding equations for CSFR, comoving volume element, and IMF. Here, we set $m_{\text{min}} = 8 M_\odot$ and $m_{\text{max}} = 25 M_\odot$, while $z_{\text{min}}$ and $z_{\text{max}}$ can be determined in such a way: since frequencies of emitted GWs in the source frame range from $v_{\text{min}} = 77$–$80$ Hz to $v_{\text{max}} = 2\Omega_{k}/3\pi = 1191$ Hz, where the minimum frequency corresponds to the final angular velocity of the star—$0.065\Omega_k$ for $K = -5/4$ and $0.067\Omega_k$ if $K \geq 1$, we have $v_{\text{min}}/(1+z) \leq \nu_{\text{obs}} \leq v_{\text{max}}/(1+z)$, which means sources with different redshifts that produce a signal at the same frequency $\nu_{\text{obs}}$ should meet the condition: $v_{\text{min}}/v_{\text{obs}} - 1 \leq z \leq v_{\text{max}}/v_{\text{obs}} - 1$. Besides, we consider signals emitted at early epochs up to the present ($z > 0$) and take into account the maximal redshift ($z_*$) of the CSFR model. Then, we obtain $z_{\text{min}} = \max(0, v_{\text{min}}/v_{\text{obs}} - 1)$, $z_{\text{max}} = \min(z_*, v_{\text{max}}/v_{\text{obs}} - 1)$, which is similar to that of Owen et al. (1998), where $z_* = 4$ is considered to be the maximum redshift where there was significant star formation.

In Figure 3, we plot the dimensionless energy density $\Omega_{GW}$ calculated for the five CSFR models presented in Section 2 by setting $K$ at its minimal value: $K = -5/4$ corresponding to the smallest amount of differential rotation at the time when the $r$-mode instability becomes active. However, as emphasized by Sá & Tomé (2005), if $K$ is small, namely, $K \approx 0$, it is necessary to consider other nonlinear effects like mode–mode couplings in the calculation of $\alpha$, which will again limit the maximum $r$-mode amplitude to values much smaller than unity (Arras et al. 2003). In this respect, we choose a minimum $K$ results in an unrealistically high upper limit for the $r$-mode background.

It is worth noting from Figure 3 that no obvious differences are recorded for the three curves of SFR1, SFR2, and SFR3, and observation-based CSFR models give rise to stochastic backgrounds about two times stronger than that of SH03 over a broad frequency band, although SH03 leads to a much higher NS formation rate. The sharp contrast between Figures 2 and 3 indicates that the main contribution to the GW background comes from low-redshift sources because those events that happened at higher redshifts have minor influences due to the inverse squared luminosity distance dependence of the single event energy flux. For the same reason, poor observational understanding of high-$z$ star formation history (see Figure 1) is not severe to studies about AGWB here.

To assess the role of $z_*$ in our results, we choose SFR2 (since it remains constant for $z \geq 2$), set three values for $z_*$ ($z_* = 4, 10$, and 20), and then plot the $\Omega_{GW}$ in Figure 4. It is surprising that the three curves exhibit almost the same pattern in the frequency range $\nu_{\text{obs}} > 100$ Hz, and extending the redshift limit from 4 to 20 results in a growth of the lower-frequency background. Increasing $z_*$ will enhance the background at lower frequencies and even enable some formerly “unavailable” low-frequency signals to emerge. This low-frequency GW “tail” can be accounted for by the contribution from high-redshift sources. However, if CSFR is much lower at high-$z$, this effect will be negligible. Thus, Figure 4 further supports the conclusions from Figure 3 and indicates that the most significant contribution to an AGWB comes from GW events occurring at redshifts $z \leq 4$.

From Equation (11), we find that $\Omega_{GW}$ depends on the values of $v_{\text{max}} \sim \Omega_k$ and $K (\Omega_{GW} \propto \frac{1}{k^{3/2}} \sim \alpha^{-2})$. As suggested by Ferrari et al. (1999b), the Keplerian velocity $\Omega_k$ may have a broad distribution due to the different masses and radii of rotating NSs. Here, we arbitrarily set $v_{\text{max}}$ ranging from 1000 Hz to 2000 Hz. For $K$, we set $-5/4, 100$, and $10^4$ corresponding to $\alpha = 1, 0.1$, and $10^{-2}$, respectively. Then we adopt HB06 as the CSFR model (below we will use only HB06) and plot $\Omega_{GW}$ as a function of the observed frequency for different values of $\alpha$ and $v_{\text{max}}$ in Figure 5. We can see a higher peak for $\Omega_{GW}$ when we increase the maximum emitting frequency, while the $r$-mode background for smaller $v_{\text{max}}$ is slightly enhanced at lower frequencies ($\leq 400$ Hz). On the other hand, increasing the amount of differential rotation significantly reduces the closure density and then affects the detectability of the $r$-mode background as we will discuss later. Considering a
maximum value of $10^{-2}$ for $\alpha$, a realistic estimate of the $r$-mode background should have an energy density at most $\sim 10^{-12}$ like the lowest curve shown in Figure 5.

Another important quantity of the AGWB is the so-called duty cycle, which classifies the stochastic backgrounds in terms of continuous background, popcorn noise, and short noise (Coward & Regimbau 2006):

$$D = \int_0^\infty \tilde{r}(1 + z) dR_{NS}(z),$$  \hspace{1cm} (12)

where $\tilde{r}$ is the average time duration of the GW emission from a single source at the source frame, which dilates to $\tilde{r}(1 + z)$ due to the cosmic expansion, and $dR_{NS}(z)$ is the differential rate of NS formation in Equation (4). It has been suggested that differential rotation can remarkably influence the long-term spin and thermal evolution of NSs by prolonging the duration of the $r$-modes (Yu et al. 2009). In view of the prolonged $r$-mode, the spinning-down phase can last even longer than one year, which indicates a duty cycle $> 10^9$. In the next section, we will discuss the detectability of this continuous GW background.

5. DETECTABILITY

5.1. Detecting the $r$-Mode Background with a Network of IFOs

We cannot reach sufficient sensitivity to detect an SGWB with a single terrestrial IFO since the output of a detector is dominated by the noise rather than by the signal due to the stochastic background itself (Allen 1996a; Maggiore 2000). The optimal strategy for searching for an SGWB is to cross-correlate measurements of two or more detectors. It has been shown that after correlating signals of two detectors for an (1) isotropic, (2) unpolarized, (3) stationary, and (4) Gaussian stochastic background (see Allen & Romano 1999 for discussions of these assumptions), the optimal S/N during an integration time $T$ (here, we assume $T = 3$ yr $\simeq 10^8$ s) is given by an integral over frequency $f$:

$$\left( \frac{S}{N} \right)^2 = \frac{9 H_0^4}{50 \pi^4 T} \int_0^\infty df \frac{\gamma^2(f) M_{GW}(f)}{f^6 P_1(f) P_2(f)}$$  \hspace{1cm} (13)

where $P_1(f)$ and $P_2(f)$ are the power spectral noise densities of the two detectors and $\gamma(f)$ is the so-called overlap reduction function, first calculated by Flanagan (1993). This is a dimensionless function of frequency and determined by the relative locations and orientations of two detectors. For $\gamma(f)$, we refer readers to Flanagan (1993), Allen & Romano (1999), and Maggiore (2000) for more details.

To assess the detectability of the $r$-mode background, we will calculate the S/Ns for several pairs of detectors for $\Omega_{GW}$ computed with the HB06 CSR model, $K = -5/4$ and $f_{\text{max}} = 1191$ Hz (unless otherwise stated we use these parameters in Section 5). Here we consider the four IFOs which are in routine operations—LIGOH (4 km), LIGOL (4 km), Virgo (3 km), and GEO (600 m), as well as the second-generation detectors— advanced LIGO (adL) and advanced Virgo (adV).

Design sensitivity curves of these detectors are shown in Figure 6. It is worth mentioning that the first-generation GW interferometric detectors have taken data at, or close to, their design sensitivities (Fairhurst et al. 2010). In particular, the design sensitivity curve for initial LIGO was almost attained by its S5 run. In the following calculations, we will use real $\gamma(f)$ for different pairs of IFOs unless otherwise stated.

Two approaches of combining $2N$ detectors to improve the detection ability to the SGWBs are proposed in Allen & Romano (1999): (1) correlating the outputs of a pair of detectors, then combining multiple pairs (combining pairs, "c–p") and (2) directly combining the outputs of $2N$ detectors (directly combining, “d–c”). For the first approach, the squared S/N is given by

$$\left( \frac{S}{N} \right)_{\text{optI}}^2 = \sum_{\text{pair}} \left( \frac{S}{N} \right)_{\text{pair}}^2, \hspace{1cm} (14)$$

and for the second one

$$\left( \frac{S}{N} \right)_{\text{optII}}^2 \approx (12) \left( \frac{S}{N} \right)_{\text{optI}}^2 \left( \frac{S}{N} \right)_{\text{optII}}^2 \ldots \left( \frac{S}{N} \right)_{\text{optII}}^2 \ldots \left( \frac{S}{N} \right)_{\text{optII}}^2 \text{, all possible permutations}, \hspace{1cm} (15)$$
advanced LIGO and advanced Virgo detectors.

Figure 7. \(\text{S/N as a function of } K \text{ for the H–L, H–V, and L–V pairs when we set } v_{\text{max}} = 1191 \text{ Hz.} \) We assume a factor of 10-fold improvement in sensitivity for advanced LIGO and advanced Virgo detectors.

(A color version of this figure is available in the online journal.)

Table 2

| Case   | H–L   | L–V   | H–V   | V–G   |
|--------|-------|-------|-------|-------|
| 1      | 1.8 × 10^{-3} | 1.0 × 10^{-3} | 8.5 × 10^{-4} | 2.6 × 10^{-4} |
| 2      | 0.40  | 0.21  | 0.17  | 0.25  |

| Case   | L–G   | H–G   | c–p   | d–c   |
|--------|-------|-------|-------|-------|
| 1      | 1.4 × 10^{-4} | 9.7 × 10^{-5} | 2.3 × 10^{-3} | 5.0 × 10^{-7} |
| 2      | 0.16  | 0.11  | 0.58  | 0.11  |

Note. Case 1 corresponds to first-generation detectors and Case 2 adapts sensitivities of advanced LIGO and advanced Virgo, while G refers to an assumed interferometer with the same sensitivity as advanced Virgo at the GEO site.

We show in Table 2 the S/Ns calculated for different pairs of LIGO H, LIGOL L, Virgo V, and GEO (G) and Two Approaches of Combining the Four IFOs—Combining Pairs (c–p) and Directly Combining (d–c).

Figure 8. Same as Figure 7. \(\text{S/N as a function of } v_{\text{max}} \text{ for the H–L, H–V, and L–V pairs when we set } K = -5/4. \)

(A color version of this figure is available in the online journal.)

ET can reach a sensitivity roughly an order of magnitude better than that of adL (Hild et al. 2008).

In Figure 7, we plot the S/N as a function of \(K\) for H–L, H–V, and L–V pairs. As a natural result from Figure 5, the detectability of the SGWB from \(r\)-mode instability is drastically reduced to 0 as \(K\) approaches 10. The higher S/N of the H–L pair reflects the lower noise level of adL. Due to the similarity of the overlap reduction functions (see Figure 2 of Fan & Zhu 2008), no significant difference is shown between L–V and H–V pairs.

Figure 8 shows the S/N evolution with \(v_{\text{max}}\) for the H–L, H–V, and L–V pairs. In contrast to Figure 5, in which a higher peak value of \(\Omega_{GW}\) was obtained for larger \(v_{\text{max}}\), there are no identical features for S/N evolutions here. For the H–L pair, we note that S/N varies inversely with the increase of \(v_{\text{max}}\). This can be explained that the low-frequency GW background is enhanced for smaller \(v_{\text{max}}\) while the growth of high-frequency background due to larger \(v_{\text{max}}\) is suppressed by the \(1/f^n\) term in Equation (13). On the other hand, we can reach higher S/Ns for the H–V and L–V pairs for larger \(v_{\text{max}} \geq 1600 \text{ Hz}\). This unique feature can be attributed to particular evolving behaviors of \(\gamma(f)\) for the two pairs (see again Figure 2 of Fan & Zhu 2008) and we will find that \(\gamma(f)\) is extremely close to zero at high frequencies for H–L, while it still fluctuates above and below zero up to 1000 Hz for the H–V and L–V pairs). This is strongly supported by three curves in Figure 9, where we plot the S/N as a function of \(v_{\text{max}}\) by assuming \(\gamma(f) = 1\), and show exactly the same evolving pattern as the curve of H–L in Figure 8.

For a detection rate of 90% and a false alarm rate of 10%, the total optimal S/N threshold should be 2.56. In order to evaluate the promise of detecting the \(r\)-mode background, we present in Figure 10 the regions in the \((v_{\text{max}}, K)\) plane where S/N could be higher than 2.56 for multiple detector pairs. It is shown that the detectable parameter space is quite limited even for a real network of third-generation IFOs. In particular, a strong constraint \((K < 0\) or equivalently \(x \sim 1\)) is obtained. Meanwhile, we find that the two approaches of combining multiple IFOs can effectively improve\(^9\) the detection ability as compared to the H–L pair, and the “c–p” method performs

\(^9\) In fact, such an improvement is negligible considering that S/N is extremely sensitive to the saturation amplitude of the \(r\)-mode (\(x \alpha^2\)).
better than the “d–c” approach in terms of detecting the r-mode background.

5.2. The Detection Prospects of Future Detectors

In this section, we discuss further the detection prospects of an SGWB from NS r-mode instability by networks of third-generation instruments.

Figure 10 indicates that only when the initial amount of differential rotation is near its minimum value ($\Omega \sim 1$), could the r-mode background be detectable by a real network of third-generation GW detectors. However, as emphasized by Sá & Tomé (2005), if $K$ is close to its minimum value, other nonlinear effects like mode–mode couplings should be included in the calculation of the saturation amplitude $\alpha$. Still, in this case, $\alpha$ will be limited to values much smaller than unity (Arras et al. 2003). In this respect, the physically reasonable values of the parameter $K$ introduced in Sá & Tomé (2005) should be larger than $10^3$ in order to be consistent with a maximum saturation amplitude $\alpha < 10^{-2}$ (Lin & Suen 2006).

It seems most probable that the r-mode background is not going to be detectable. Given the relation $S/N \sim \Omega GW \sim \alpha^2$, a $10^{-2}$ saturation amplitude (compared with a value of order unity) will reduce the $S/N$ by four orders of magnitude for any measurements of SGWB from the NS r-mode instability. In particular, to reach the same $S/N$ requires a 10,000 times improvement in the sensitivities of the two detectors.

If we put the small $\alpha$ issue (the dominant aspect) aside, the outlook is still quite pessimistic for detecting the r-mode background as seen from Table 2 and Figure 10. This is due to the following facts: (1) the closure density of this background peaks at much higher frequencies than the most sensitive frequency band of ground-based detectors; (2) the minimum frequency of the r-mode GW signal, $\nu_{\text{min}} = 80$ Hz, just misses the frequency band (1–60 Hz) where the overlap reduction functions of real detector pairs are significant; and (3) we should not forget that we assumed all NSs are born with angular velocities near their maximal value $\Omega_K$. This is not necessarily always the case since it would make more sense to consider that some fraction of NSs are born with rapid spins (Owen et al. 1998). Actually, it has been suggested that most NSs are born with very small rotation rates (Spruit & Phinney 1998). Current population synthesis studies favor spin periods of NSs at birth in the range from tens to hundreds of milliseconds (Ott et al. 2006; Perna et al. 2008).

Realistic overlap reduction functions are adopted here for multiple ground-based IFOs. Recall that two co-located adL detectors ($\gamma(f) = 1$, $S/N = 11$) perform even better than combining four IFOs with 10-fold better sensitivities ($S/N = 5.8$). With this in mind, we also evaluate the detectability of the r-mode background with ET, assuming two detectors located in Cascina, of triangular shape (60° between the two arms), and separated by an angle of 120° (Howell et al. 2010). The $\gamma(f)$ benefits a lot from this configuration, nearly linearly decreasing from $-0.372$ at 1000 Hz to $-0.375$ at 1 Hz. We adopt the ET-B sensitivity from Hild et al. (2008). For a three year integration, an $S/N$ of 2.56 requires $K \lesssim 150$ corresponding to $\alpha \sim 0.1$.

Then, we convert the constrains on $K$ or $\alpha$ to absolute numbers: the total emitted GW energy associated with the NS r-mode instability that enables the stochastic background to be detectable by future detectors. The energy flux of a single event can also be written as

$$f(\nu_{\text{obs}}, z) = \frac{1}{4\pi d_L(z)^2} \frac{dE_{GW}}{d\nu}(1 + z),$$

where $dE_{GW}/d\nu$ is the gravitational spectral energy. Combined with Equations (9) and (10), we can obtain the spectral energy density and thus the total GW energy emitted by individual sources that contribute to this background. We give the results (required energy level in order to obtain an $S/N$ of 2.56 by three year integration) as follows (in $M_{\odot}c^2$): (1) $1.8 \times 10^{-5}$ for a real network of “third-generation” IFOs; (2) $9.4 \times 10^{-4}$ for two co-located adL detectors; and (3) $2 \times 10^{-5}$ for two detectors with ET–B sensitivity.

6. CONCLUSIONS

We revisit the possibility and detectability of an SGWB produced by a cosmological population of young NSs with
While a minimum initial amount of differential rotation ($z$-redshifts of astrophysical origin comes from GW events occurring at redshifts $z \leq 2$). This is in good agreement with that in Howell et al. (2004). But here we further investigate the effect of the maximal redshift of CSFR models on the AGWB and find that high-redshift ($z > 4$) sources could form a GW “tail” in the low-frequency side ($\leq 30$ Hz, this number depends on the particular source spectrum and thus only applies to the $r$-mode background) and have no effect on the high-frequency background. Such an effect will be negligible if the high-$z$ CSFR is much smaller. Our Figures 3 and 4 indicate that the most significant contribution to the GW background of astrophysical origin comes from GW events occurring at redshifts $z \leq 4$.

The characteristic GW amplitude parameterized with the initial amount of differential rotation ($K$) during $r$-mode evolution is adopted as the average GW signal for individual sources. While a minimum $K$ corresponding to a saturation mode amplitude $\alpha \sim 1$ is assumed, the energy density $\Omega_{GW}$ has a maximum amplitude around $3 \times 10^{-3}$, agreeing well with those of Owen et al. (1998) and Ferrari et al. (1999b). However, since we know that the maximum amplitude $\alpha$ that the $r$-mode can grow to is at most $10^{-3}$ to $10^{-2}$, this means that the physically reasonable values of parameter $K$ are at least $10^4$. Consequently, a realistic estimate of $\Omega_{GW}$ for the $r$-mode background should be at most $\sim 10^{-12}$.

We further consider multiple IFOs, including the first-generation ones that are in routine operations and upgraded counterparts, for the detection of the $r$-mode background. We also illustrate how a network of ground-based IFOs could improve the ability to detect this GW background. Since the detectability is dominated by the square of saturation amplitude ($S/N \propto a^2$), it is likely that the $r$-mode background will not be detectable for any future detectors. Still we give the constraints on the total emitted GW energy associated with this mechanism to enable a detection with a $90\%$ detection rate ($S/N = 2.56$) by three year cross-correlation as $\sim 10^{-3} M_{\odot} c^2$ for two co-located adL detectors and $2 \times 10^{-5}$ for two IFOs with ET sensitivity. Considering the relatively certain NS formation rate, these constraints might be applicable to alternative emission mechanisms associated with NS oscillations and instabilities. This requires further investigation. The requirement for the GW energy level could be lower if more signals are emitted near the most sensitive frequency band of ground-based detectors ($\sim 40$–200 Hz). Through reasonable assumptions of average source spectra, the lowest detectable (in terms of stochastic background) GW energy for an individual source will be $10^{-7} M_{\odot} c^2$ for third-generation detectors like ET (Zhu et al. 2010). Overall, for the detection of an SGWB from NS instabilities, more efficient emitters are required (see, e.g., Andersson et al. 2010 for reviews of GW emission from NSs and Kastaun et al. 2010 for details of a possible more efficient mechanism—$f$-mode).

While the SGWB from NS $r$-mode instability is difficult to detect, the associated GW signal is still detectable in terms of single events (Owen 2010), although this possibility also depends strongly on the saturation amplitude. For instance, it was initially believed that GWs from $r$-mode instability in a newborn NS could be detected by adL out to a distance of 20 Mpc (Owen & Lindblom 2002). However, even for the most optimistic case in Bondarescu et al. (2009), the detectable distance for adL is only 1 Mpc. The LIGO Scientific Collaboration and Virgo Collaboration have already performed many searches for periodic GWs from rapidly rotating NSs including the first search targeting the youngest known NS Cassiopeia A (Abadie et al. 2010). Although no GWs have been detected, direct upper limits on GW emission from known pulsars like the Crab pulsar have beaten down the indirect spin-down limits (Abbott et al. 2008).

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