Holographic Foam, Dark Energy and Infinite Statistics

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Abstract

Quantum fluctuations of spacetime give rise to quantum foam, and black hole physics dictates that the foam is of holographic type. Applied to cosmology, the holographic model requires the existence of dark energy which, we argue, is composed of an enormous number of inert “particles” of extremely long wavelength. These “particles” necessarily obey infinite statistics in which all representations of the particle permutation group can occur. For every boson or fermion in the present observable universe there could be $\sim 10^{31}$ such “particles”. We also discuss the compatibility between the holographic principle and infinite statistics.

PACS numbers: 04.60.-m, 95.36.+x, 05.30.-d

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I. INTRODUCTION

According to folklore, there are two kinds of statistics: Fermi-Dirac statistics for identical particles of half-integral spin and Bose-Einstein statistics for identical particles of integral spin. (There are also generalizations of these statistics known as para-Fermi and para-Bose statistics. \[1\]) But it is far less well-known that there is a third kind of particle statistics, known as infinite statistics \[2, 3, 4\], that is consistent with the general principles of quantum field theory. A collection of particles obeying infinite statistics can be in any representation of the particle permutation group: compare this with the rule that fermions (bosons) can only be in a totally antisymmetric (symmetric) state. While there are plenty of examples of fermions and bosons, there is no empirical evidence for particles of infinite statistics — until now perhaps. In this Letter, we will argue that actually infinite statistics should be the most familiar kind of statistics. For every observed fermion or boson,\(^1\) there could be as many as \(\sim 10^{31}\) “particles” obeying infinite statistics in the observable universe in the present cosmic era.

We will apply our argument to the cosmology derived from the so-called holographic model of spacetime foam. The outline of this Letter is as follows. In Section II, partly for completeness, we explain the logic behind the holographic quantum foam. In Section III, we show that the holographic model, applied to cosmology, predicts that the cosmic energy is of critical density, and the cosmic entropy is the maximum allowed by the holographic principle. We also discuss the random-walk model which predicts a coarser spatial resolution in the mapping of spacetime geometry than the holographic model. Existing archived data on quasars from the Hubble Space Telescope can be used to rule out the random-walk model the demise of which, coupled with the fact (see below) that ordinary matter composed of fermions and/or bosons maps out spacetime only to the accuracy corresponding to the random-walk model, can be used to infer the existence of unconventional energy and/or matter (independent of recent cosmological observations). The main part of our argument is given in Section IV: there we show that, in the framework of holographic foam cosmology, positivity of entropy requires the “particles” (or bits) constituting dark energy to obey infinite statistics; we also discuss the compatibility between the holographic principle and

\(^1\) Not including the degrees of freedom behind black hole horizons, nor the entropy of gravitons and dark matter.
infinite statistics. We address some issues facing holographic foam cosmology and give a summary in the final section.

II. HOLOGRAPHIC QUANTUM FOAM

Conceivably spacetime, like everything else, is subject to quantum fluctuations. As a result, spacetime is “foamy” at small scales, giving rise to a microscopic structure of spacetime known as quantum foam, also known as spacetime foam, and entailing an intrinsic limitation $\delta l$ to the accuracy with which one can measure a distance $l$. In principle, $\delta l$ can depend on both $l$ and the Planck length $l_P = \sqrt{\hbar G/c^3}$, the intrinsic scale in quantum gravity, and hence can be written as $\delta l \gtrsim l^{1-\alpha}l_P^{\alpha}$, with $\alpha \sim 1$ parametrizing the various spacetime foam models. (For related effects of quantum fluctuations of spacetime geometry, see Ref. [6].) In what follows, we will advocate the so-called holographic model corresponding to $\alpha = 2/3$, but we will also consider the (random walk) model with $\alpha = 1/2$ for comparison.

Let us first derive the holographic model by using an argument based on quantum computation. Since quantum fluctuations of spacetime manifest themselves in the form of uncertainties in the geometry of spacetime, the structure of spacetime foam can be inferred from the accuracy with which we can measure that geometry. Let us consider a spherical volume of radius $l$ over the amount of time $T = 2l/c$ it takes light to cross the volume. One way to map out the geometry of this spacetime region is to fill the space with clocks, exchanging signals with other clocks and measuring the signals’ times of arrival. This process of mapping the geometry is a sort of computation; hence the total number of operations (the ticking of the clocks and the measurement of signals etc) is bounded by the Margolus-Levitin theorem in quantum computation, which stipulates that the rate of operations for any computer cannot exceed the amount of energy $E$ that is available for computation divided by $\pi\hbar/2$. A total mass $M$ of clocks then yields, via the Margolus-Levitin theorem, the bound on the total number of operations given by $(2Mc^2/\pi\hbar) \times 2l/c$. But to prevent the clocks from collapsing into a black hole, $M$ must be less than $lc^2/2G$. Together, these

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2 We find it reasonable and consistent to assume that $\delta l$ depends on only $l$ and $l_P$.

3 Later, we will extend this process of mapping the geometry to the entire universe. For the readers who find the idea of treating the universe as a computer unpalatable, we should mention that there are other ways to derive the holographic model; see Ref. [7].
two limits imply that the total number of operations that can occur in a spatial volume of radius \( l \) for a time period \( 2l/c \) is no greater than \( \sim (l/l_p)^2 \). (Here and henceforth we neglect numerical factors of order unity, set \( c = 1 = \hbar \) and will also set the Boltzmann constant equal to 1.) To maximize spatial resolution, each clock must tick only once during the entire time period. The operations partition the spacetime volume into “cells”, and, on the average, each cell occupies a spatial volume no less than \( \sim l^3/(l^2/l_p^2) = ll_p^2 \), yielding an average separation between neighboring cells no less than \( l^{1/3}l_p^{2/3} \). This spatial separation is a measure of the uncertainty in the geometry of the spacetime volume, and hence can be interpreted as yielding an average minimum uncertainty \(^4\) in the measurement of a distance \( l \) given by \( \delta l \gtrsim l^{1/3}l_p^{2/3} \).

Parenthetically we can now understand why this quantum foam model has come to be known as the holographic model. Since, on the average, each cell occupies a spatial volume of \( ll_p^2 \), a spatial region of size \( l \) can contain no more than \( l^3/(ll_p^2) = (l/l_p)^2 \) cells. Thus this model corresponds to the case of maximum number of bits of information \( l^2/l_p^2 \) in a spatial region of size \( l \), that is allowed by the holographic principle \(^1\). It will prove to be useful to compare the holographic model in the mapping of the geometry of spacetime with the one that corresponds to spreading the spacetime cells uniformly in both space and time. For the latter case, each cell has the size of \( (l^2l_p^2)^{1/4} = l^{1/2}l_p^{1/2} \) both spatially and temporally so that each clock ticks once in the time it takes to communicate with a neighboring clock. Since the dependence on \( l^{1/2} \) is the hallmark of a random-walk fluctuation, this quantum foam model corresponding to \( \delta l \gtrsim (ll_p)^{1/2} \) is called the random-walk model \(^2\). Compared to the holographic model, the random-walk model predicts a coarser spatial resolution, i.e., a larger distance fluctuation, in the mapping of spacetime geometry. It also yields a smaller bound on the information content in a spatial region, viz., \( (l/l_p)^2/(l/l_p)^{1/2} = (l^2/l_p^2)^{3/4} = (l/l_p)^{3/2} \) bits.

One remark is in order. The minimum \( \delta l \) just found for the holographic model corresponds to the case of maximum energy density \( \rho \sim (ll_p)^{-2} \) for the region not to collapse into a black hole. Hence the holographic model, in contrast to the random-walk model \(^5\) and other models, requires, for its consistency, the energy density to have the critical value.

\(^4\) Note that this result is not inconsistent with that found in \(^1\). \(^5\) The random-walk model (corresponding to \( \delta l \gtrsim (ll_p)^{1/2} \)) does not require the maximum energy density because the clocks can tick less frequently than once in the amount of time \( (ll_p)^{1/2} \).
III. DARK ENERGY/MATTER

The Planck length \( l_P \sim 10^{-33} \) cm is so short that we need an astronomical (even cosmological) distance \( l \) for its fluctuation \( \delta l \) to be detectable. Let us consider light (with wavelength \( \lambda \)) from distant quasars or bright active galactic nuclei. \([13, 14]\) Due to quantum fluctuations of spacetime, the wavefront, while planar, is itself “foamy”, having random fluctuations in phase \([14]\) given by \( \Delta \phi \sim 2\pi \delta l/\lambda \) as well as in the direction \([15]\) given by \( \Delta \phi/2\pi \). In effect, spacetime foam creates a “seeing disk” whose angular diameter is \( \sim \Delta \phi/2\pi \). For an interferometer with baseline length \( D \), this means that dispersion will be seen as a spread in the angular size of a distant point source, causing a reduction in the fringe visibility when \( \Delta \phi/2\pi \sim \lambda/D \).

Now we can use existing archived high-resolution data on quasars or ultra-bright active galactic nuclei from the Hubble Space Telescope to test the quantum foam models. \([15]\) Consider the case of PKS1413+135 \([16]\), an AGN for which the redshift is \( z = 0.2467 \). With \( l \approx 1.2 \) Gpc and \( \lambda = 1.6\mu m \), we \([14]\) find \( \Delta \phi \sim 10 \times 2\pi \) and \( 10^{-9} \times 2\pi \) for the random-walk model and the holographic model of spacetime foam respectively. With \( D = 2.4 \) m for HST, we expect to detect halos if \( \Delta \phi \sim 10^{-6} \times 2\pi \). Thus, the HST image only fails to test the holographic model by approximately 3 orders of magnitude.

However, the absence of a spacetime foam induced halo structure in the HST image of PKS1413+135 rules out convincingly the random-walk model. (In fact, the scaling relation discussed above indicates that all spacetime foam models with \( \alpha \lesssim 0.6 \) are ruled out by this HST observation.) This result has profound implications for cosmology. \([8, 15, 17]\) To wit, from the observed cosmic critical density in the present era (consistent with the prediction of the cosmology inspired by the holographic model of quantum foam) we deduce that \( \rho \sim H_0^2/G \sim (R_H l_P)^{-2} \), where \( H_0 \) and \( R_H \) are the present Hubble parameter and Hubble radius of the observable universe respectively. Treating the whole universe as a computer \([8, 18]\), one can apply the Margolus-Levitin theorem to conclude that the universe \(^7\) computes at a rate \( \nu \) up to \( \rho R_H^3 \sim R_H l_P^2 \) for a total of \( (R_H/l_P)^2 \) operations during its

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6 For this result, we assume comparable fluctuations in both the longitudinal and transverse components of the wave vector due to spatial isotropy.

7 Note that the total energy of the universe is increasing; this is due to the fact that total amount of energy/matter within the horizon is growing with time, as more energy/matter enter the horizon.
lifetime so far. If all the information of this huge computer is stored in ordinary matter, we can apply standard methods of statistical mechanics to find that the total number $I$ of bits is $(R_H^2/l_P^3)^{3/4} = (R_H/l_P)^{3/2} \sim 10^{92}$. Then each bit flips once in the amount of time given by $I/\nu \sim (R_Hl_P)^{1/2}$. On the other hand, the average separation of neighboring bits is $(R_H^3/I)^{1/3} \sim (R_Hl_P)^{1/2}$. Hence, the time to communicate with neighboring bits is equal to the time for each bit to flip once. It follows that the accuracy to which ordinary matter maps out the geometry of spacetime corresponds exactly to the case of events spread out uniformly in space and time discussed above for the case of the random-walk model of spacetime foam.

In other words, ordinary matter only contains an amount of information dense enough to map out spacetime at a level consistent with the random-walk model. Observationally ruling out the random-walk model suggests that there must be other kinds of matter/energy with which the universe can map out its spacetime geometry to a finer spatial accuracy than is possible with the use of conventional matter. This line of reasoning then strongly hints at the existence of dark energy/matter, independent of the evidence from recent cosmological observations.

Moreover, the fact that our universe is observed to be at or very close to its critical energy density $\rho \sim (H/l_P)^2 \sim (R_Hl_P)^{-2}$ must be taken as solid albeit indirect evidence in favor of the holographic model because, as aforementioned, this model is the only model that requires the energy density to have the critical value. The holographic model also predicts a huge number of degrees of freedom for the universe in the present era, with the cosmic entropy given by $I \sim (R_H/l_P)^2 \sim 10^{123}$. Hence the average energy carried by each bit or “particle” is $\rho R_H^3/\rho \sim R_H^{-1}$. It is now natural to identify these “particles” of unconventional energy/matter of extremely long wavelength as constituents of dark energy. Since altogether $\sim (R_H/l_P)^2$ operations have been performed with $\sim (R_H/l_P)^2$ bits, we note, for later discussion, that the overwhelming majority of the bits have had time to flip only of order one time over the course of cosmic history. In other words, each “particle” has had only of order one interaction. The inertness of these “particles” may explain why dark energy is dark.
IV. INFINITE STATISTICS

What is the overriding difference between conventional matter and unconventional energy/matter (i.e., dark energy and perhaps also dark matter)? To find that out, let us first consider a perfect gas of \( N \) particles obeying Boltzmann statistics (which, rigorously speaking, is not a physical statistics but is still a useful statistics to work with) at temperature \( T \) in a volume \( V \). For the problem 8 at hand, we take \( V \sim R_H^3 \), \( T \sim R_H^{-1} \), and very roughly \( N \sim (R_H/l_P)^2 \). A standard calculation (for the relativistic case) yields the partition function
\[
Z_N = (N!)^{-1}(V/\lambda^3)^N,
\]
where \( \lambda = (\pi^{2/3}/T) \). With the free energy given by \( F = -T\ln Z_N = -NT[\ln(V/N\lambda^3) + 1] \), we get, for the entropy of the system,
\[
S = -\left(\frac{\partial F}{\partial T}\right)_{V,N} = N[\ln(V/N\lambda^3) + 5/2]. \tag{1}
\]

For the non-relativistic case with the effective mass \( m \sim R_H^{-1} \) (coming from some sort of potential with which we are not going to concern ourselves), the only changes in the above expressions are given by the substitution \( \lambda \to (2\pi/mT)^{1/2} \). With \( m \sim T \sim R_H^{-1} \), there is no significant qualitative difference between the non-relativistic and relativistic cases.

The important point to note is that, since \( V \sim \lambda^3 \), the entropy \( S \) in Eq. (1) becomes nonsensically negative unless \( N \sim 1 \) which is equally nonsensical because \( N \) should not be too different from \((R_H/l_P)^2 \gg 1\). Intentionally we have calculated the entropy by employing the familiar Boltzmann statistics (with the correct Boltzmann counting factor), only to arrive at a contradictory result. But now the solution to this contradiction is pretty obvious: the \( N \) inside the log in Eq. (1) somehow must be absent. Then \( S \sim N \sim (R_H/l_P)^2 \) without \( N \) being small (of order 1) and \( S \) is non-negative as physically required. That is the case if the "particles" are distinguishable and nonidentical! For in that case, the Gibbs \( 1/N! \) factor is absent from the partition function \( Z_N \), and the entropy becomes
\[
S = N[\ln(V/\lambda^3) + 3/2]. \tag{2}
\]

We can add that, with or without the Gibbs factor, the internal energy is given by \( U = F + TS = (3/2)NT \).

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8 As the lowest-order approximation, let us neglect the contributions from matter to the cosmic energy density. Then it can be shown that the Friedmann equations for \( \rho \sim H^2/G \) are solved by \( H \propto 1/a \) and \( a \propto t \) with pressure \( p \sim -\rho/3 \), where \( a(t) \) is the cosmic scale factor.
Now the only known consistent statistics in greater than two space dimensions without the Gibbs factor \(^9\) is infinite statistics (sometimes called “quantum Boltzmann statistics”) \([2, 3, 4]\). Thus we have shown that the “particles” constituting dark energy obey infinite statistics, instead of the familiar Fermi or Bose statistics. What is infinite statistics? Succinctly, a Fock realization of infinite statistics is provided by a \(q\) deformation of the commutation relations of the oscillators:  
\[ a_k a_l^\dagger - qa_l^\dagger a_k = \delta_{kl} \]  
with \(q\) between -1 and 1 (the case \(q = \pm1\) corresponds to bosons or fermions). States are built by acting on a vacuum which is annihilated by \(a_k\). Two states obtained by acting with the \(N\) oscillators in different orders are orthogonal. It follows that the states may be in any representation of the permutation group. The statistical mechanics of particles obeying infinite statistics can be obtained in a way similar to Boltzmann statistics, with the crucial difference that the Gibbs \(1/N!\) factor is absent for the former. Infinite statistics can be thought of as corresponding to the statistics of identical particles with an infinite number of internal degrees of freedom, which is equivalent to the statistics of nonidentical particles since they are distinguishable by their internal states.

Infinite statistics appears to have one “defect”: a theory of particles obeying infinite statistics cannot be local \([4, 21]\). The expressions for the number operator, Hamiltonian, etc., are both nonlocal and nonpolynomial in the field operators. The lack of locality may make it difficult to formulate a relativistic version of the theory; but it appears that a non-relativistic theory can be developed. Lacking locality also means that the familiar spin-statistics relation is no longer valid for particles obeying infinite statistics; hence they can have any spin. Remarkably, the TCP theorem and cluster decomposition have been shown to hold despite the lack of locality. \([4]\)

Actually the lack of locality for theories of infinite statistics may have a silver lining. According to the holographic principle, the number of degrees of freedom in a region of space is bounded not by the volume but by the surrounding surface. This suggests that the physical degrees of freedom are not independent but, considered at the Planck scale, they must be infinitely correlated, with the result that the spacetime location of an event may lose its invariant significance. Since the holographic principle is believed to be an important ingredient in the formulation of quantum gravity, the lack of locality for theories of infinite

\(^9\) Recall that the Fermi statistics and Bose statistics give similar results as the conventional Boltzmann statistics at high temperature.
statistics may not be a defect; it can actually be a virtue. Perhaps it is this lack of locality that makes it easier to incorporate gravitational interactions in the theory. Quantum gravity and infinite statistics appear to fit together nicely. This may be the reason why (charged, extremal) black holes appear to obey infinite statistics. Indirectly this may also explain why the holographic foam model has use for infinite statistics as we have just shown.

V. DISCUSSION

We have considered a perfect gas consisting of “particles” of extremely long wavelength, obeying Boltzmann statistics (first in the conventional, then in the quantum version) in the Universe at temperature $T$. But we have seen that those “particles” have had interactions only of order one time on the average during the entire cosmic history. A question can be raised as to whether such an inert gas can come to thermal equilibrium at any well defined temperature. We do not have a good answer; but the fact that all these “particles”, though extremely inert, have a wavelength comparable to the observable Hubble radius may mean that they overlap significantly, and accordingly can perhaps share a common temperature.

Another question concerns the sign of the pressure for this gas and whether it is sufficiently negative to accelerate the expansion of the present Universe as has been observed. Indeed the pressure for such a gas can be easily shown to be $P = (2/3)U/V$ and is blatantly positive. But this calculation is based on the simplifying assumption that the gas is perfect. Such a treatment may be sufficient for estimating the entropy, but it is obviously inadequate to give the correct pressure. After all, as shown above, each “particle” has an energy comparable to $R_H^1$. Such long-wavelength bits or “particles” carry negligible kinetic energy. Since pressure (energy density) is given by kinetic energy minus (plus) potential energy, a negligible kinetic energy means that the pressure of the unconventional energy/matter is roughly minus its energy density, plausibly leading to accelerating cosmic expansion.

10 Thus these “particles” provide a spatially uniform energy density, like a time-dependent cosmological constant. But in a way, this type of models is preferrable to the cosmological constant because it may be easier to understand a zero cosmological constant (perhaps due to a certain not-yet-known symmetry) than an exceedingly small (but non-zero) cosmological constant. We also find it amusing to recall that earlier cosmic epochs are associated with $\rho \propto a^{-4}$ (radiation-dominated) and (followed by) $\rho \propto a^{-3}$ (matter-dominated). If the holographic foam cosmology is correct, these epochs are now succeeded by the dark-energy-dominated era with $\rho \propto a^{-2}$.

11 As noted above, for cosmic energy density $\rho \sim H^2/G$, the equation of state is given by $p = -\rho/3$. To have
very similar to that of quintessence [24], but it has its origin in local small scale physics – specifically, the holographic quantum foam!

Finally, is there any useful phenomenology that we can predict or use to explicitly check whether dark energy (and perhaps even dark matter) is composed of particles obeying infinite statistics? Since all those “particles” are so inert, we do not foresee any useful desktop experiments forthcoming soon that can shed light on the phenomenology of dark energy, a safer bet would be on cosmological observations (e.g., in connection with the scale-invariance of density fluctuations [25]). Further study is warranted.

In summary, according to holographic foam cosmology, (1) the cosmic energy density takes on the critical value, (2) dark energy/matter exists, and (3) the cosmic entropy is the maximum allowed by the holographic principle. This scenario may lead to cosmic accelerating expansion in the present cosmic era, and interestingly it suggests that dark energy is composed of $\sim 10^{123}$ extremely cold, inert, and long-wavelength “particles”. Furthermore we have shown that these “particles” necessarily obey infinite statistics. By a staggering factor of $\sim 10^{123-92} = 10^{31}$, these “particles” appear to far outnumber particles of the familiar Bose and Fermi statistics that we are all made of. Indeed we may be quite insignificant in the cosmic grand scheme. This is a most humbling realization.

Note added: After this work was posted on the arXiv (gr-qc/0703096), we learned of a recent paper [26] by V. Jejjala, M. Kavic and D. Minic on a similar subject. In the framework of M-theory, these authors argue that dark energy has a fine structure compatible with infinite statistics. We also learned of another recent paper [27] in which, in the framework of loop quantum gravity, R. Gambini and J. Pullin derive, from first principles, the fundamental limits on the measurements of space and time and the ultimate limits of computability, and they also show the consistency of these limits with holography. Their work lends support to

\[ p \sim -\rho, \]

one may need to take into account, for instance, the interactions between dark energy and matter. See, e.g., [23]. At this point, we simply assume that the full dynamics can generate a sufficiently negative pressure to yield accelerating cosmic expansion as observed. It is also possible that the thermodynamics of infinite statistics is more complicated than we realize. Further study is warranted.
Acknowledgments

This work was supported in part by the US Department of Energy and the Bahnson Fund of the University of North Carolina. I thank O. W. Greenberg for a brief but useful tutorial on infinite statistics during the 2005 Miami Conference, and T. W. Kephart, L. Mersimi, R. Rohm and M. Arzano for useful discussions.

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