Title
Quantum Mechanics, Spacetime Locality, and Gravity

Permalink
https://escholarship.org/uc/item/0nt6v1xn

Journal
FOUNDATIONS OF PHYSICS, 43(8)

ISSN
0015-9018

Author
Nomura, Yasunori

Publication Date
2013-08-01

DOI
10.1007/s10701-013-9729-1

Peer reviewed
Quantum Mechanics, Spacetime Locality, and Gravity

Yasunori Nomura

Berkeley Center for Theoretical Physics, Department of Physics, University of California, Berkeley, CA 94720, USA
Theoretical Physics Group, Lawrence Berkeley National Laboratory, CA 94720, USA

Abstract

Quantum mechanics introduces the concept of probability to physics at the fundamental level, yielding the measurement problem. On the other hand, recent progress in cosmology has led to the “multiverse” picture, in which our observed universe is only one of the many, bringing an apparent arbitrariness in defining probabilities, called the measure problem.

In this paper, we discuss how these two problems are intimately related with each other, developing a complete picture for quantum measurement and cosmological histories in the quantum mechanical universe. On one hand, quantum mechanics eliminates the arbitrariness of defining probabilities in the multiverse, as discussed previously in arXiv:1104.2324. On the other hand, the multiverse allows for understanding why we observe an ordered world obeying consistent laws of physics, by providing an infinite-dimensional Hilbert space. This results in the irreversibility of quantum measurement, despite the fact that the evolution of the multiverse state is unitary.

In order to describe the cosmological dynamics correctly, we need to identify the structure of the Hilbert space for a system with gravity. We argue that in order to keep spacetime locality in the description, which plays a crucial role in understanding quantum measurement, the Hilbert space for dynamical spacetime must be defined only in restricted spacetime regions: in and on the (stretched) apparent horizon as viewed from a fixed reference frame. This requirement arises from fixing/eliminating all the redundancies and overcountings in a general relativistic, global spacetime description of nature. It is responsible for horizon complementarity as well as the “observer dependence” of horizons/spacetime—these phenomena arise because changes of the reference frame are represented in the Hilbert space defined on restricted spacetime regions. This can be viewed as an extension of the Lorentz/Poincaré transformation in the quantum gravitational context, as the Lorentz transformation is viewed as an extension of the Galilean transformation.

Given an initial condition, the evolution of the multiverse state obeys the laws of quantum mechanics—it evolves deterministically and unitarily, asymptoting to the “heat death” consisting of supersymmetric Minkowski and singularity states. The beginning of the multiverse, however, is still an open issue.
Contents

1 Introduction—The Basic Picture 1

2 Probabilistic Interpretation of Quantum Mechanics 6
   2.1 Quantum measurement a la von Neumann 6
   2.2 The problem of the preferred basis 7

3 Physical Predictions and Spacetime Locality 9
   3.1 Physical information is in matrix elements 9
   3.2 Spacetime locality and the basis in Hilbert space 11
   3.3 What physical questions may one ask? 13

4 Classical Reality in Quantum Mechanical Systems 14
   4.1 A double-slit experiment in a large system 14
   4.2 Dynamical selection of a measurement basis—spacetime locality 16
   4.3 Ordered observations require infinitely large Hilbert space 20

5 Spacetime Locality in Theories with Gravity 22
   5.1 Black hole complementarity 22
   5.2 Quantum states are defined in restricted spacetime regions 24

6 Importance of Fixing a Reference Frame 27
   6.1 Fixing a gauge—physics should be described in a single reference frame . . . 27
   6.2 Hilbert space for dynamical spacetime—analogy with Fock space 28
   6.3 “Reference frame dependence” of the concept of spacetime 31

7 Hilbert Space for Quantum Gravity 33
   7.1 Meaning of spacetime singularities 33
   7.2 The heat death of the multiverse 35

8 Probabilities in the Quantum Multiverse 36
   8.1 The (extended) Born rule 36
   8.2 Unification of the eternally inflating multiverse and many worlds in quantum mechanics 38

9 Summary 40

A Quantum Measurement: Dissipation of Coherence into (Infinitely) Large Hilbert Space 41
1 Introduction—The Basic Picture

This paper discusses two subjects: quantum mechanics and gravity, especially in the context of cosmology. Quantum mechanics introduced the concept of probability to physics at the fundamental level. This has led to the issue of the quantum-to-classical transition, in particular the measurement problem. Despite much progress, a complete and satisfactory picture, particularly the one including the entire universe, still seems missing.

Recent progress in cosmology has led to the “multiverse” picture—our observed universe may be one of the many in which low energy physical laws take different forms. This view is suggested by both observation and theory: it provides a successful understanding of the order of magnitude of the observed dark energy [1], and arises naturally as a result of eternal inflation [2] and the string landscape [3]. This elegant picture, however, suffers from the issue of predictivity—in the multiverse, any event that can happen will happen in infinitely many times (due to eternally expanding spacetime), making any definition of probabilities extremely subtle [4]. Many proposals have been put forward to regulate these infinities, but they seem to be arbitrary, without relying on a solid fundamental principle. This arbitrariness of defining probabilities in the multiverse is called the measure problem, and has been a focus of much recent research.

More recently, it has been proposed that the above two issues—both connected to probabilities—are in fact related [5]. In particular, the probabilities in the eternally inflating multiverse must be defined based on the principles of quantum mechanics, which eliminates the ambiguity for the definition (as well as the problems and paradoxes plaguing some of the earlier measures). The probability formula given in Ref. [5] takes the form

\[ P(B|A) = \frac{\int dt \langle \Psi(t)| O_{A \cap B} | \Psi(t) \rangle}{\int dt \langle \Psi(t)| O_A | \Psi(t) \rangle}, \quad (1) \]

where \( |\Psi(t)\rangle \) is the state representing the entire multiverse, while \( O_A \) and \( O_{A \cap B} \) are projection operators implementing physical questions one would ask. This is (essentially) the Born rule. Indeed, the formula of Eq. (1) can be used to answer questions both regarding global properties of the universe and outcomes of particular experiments, providing complete unification of the eternally inflating multiverse and many worlds in quantum mechanics.

In Ref. [5], it was argued that the state \( |\Psi(t)\rangle \) must be defined only in the restricted spacetime regions—in and on the (stretched) apparent horizons—consistently with what we learned about quantum gravity in the past two decades: the holographic principle [6] and black hole complementarity [7]. In the cosmological context, however, the locations of horizons are “observer dependent.” What does this really mean? Moreover, Ref. [5] also discussed the meaning of spacetime singularities from the low energy viewpoint, and argued that it implies that the multiverse evolves asymptotically into a supersymmetric Minkowski world. Do these results have any implications
for the problem of quantum measurement?

In this paper, we study these issues, developing a complete picture for quantum measurement and cosmological histories in the quantum mechanical universe. A crucial ingredient for our discussion is the structure of the Hilbert space corresponding to semi-classical spacetime, which is identified in Ref. [5] (and will be suitably refined here):

$$\mathcal{H} = \bigoplus_{\mathcal{M}} \mathcal{H}_{\mathcal{M}}, \quad \mathcal{H}_{\mathcal{M}} = \mathcal{H}_{\mathcal{M},\text{bulk}} \otimes \mathcal{H}_{\mathcal{M},\text{horizon}},$$

where $\mathcal{H}_{\mathcal{M}}$ is the Hilbert (sub)space for a set of fixed semi-classical geometries $\mathcal{M}$ that have the same (stretched) apparent horizon $\partial \mathcal{M}$, as viewed from a local Lorentz frame of a fixed spatial point $p$. It consists of the parts corresponding to the regions in and on the horizon, $\mathcal{H}_{\mathcal{M},\text{bulk}}$ and $\mathcal{H}_{\mathcal{M},\text{horizon}}$, both of which have the dimension of $\exp(A_{\partial \mathcal{M}}/4)$, where $A_{\partial \mathcal{M}}$ is the area of the horizon in Planck units. (We will see that the complete Hilbert space for quantum gravity also has “intrinsically quantum mechanical” elements associated with spacetime singularities, but they are irrelevant for physical predictions.) We argue that, as quantum mechanics has helped the measure problem in eternal inflation, the multiverse helps the measurement problem in quantum mechanics, which demonstrates another “cooperation” between quantum mechanics and cosmology. In particular, the fact that we observe an ordered, classical world is explained by a combination of spacetime locality and the fact that the multiverse ultimately evolves into a Minkowski (or singularity) world, which has an infinite-dimensional Hilbert space

$$\dim \mathcal{H}_{\mathcal{M}} = \infty \quad \text{for} \quad \mathcal{M} = \text{Minkowski}. \quad (3)$$

This results in the irreversibility of quantum measurement, despite the fact that the evolution of the multiverse state is unitary.

We also elucidate the meaning of the Hilbert space structure in Eq. (2), which provides a better understanding of quantum gravity. It is well known that to do Hamiltonian quantum mechanics, all the gauge redundancies must be fixed—and a theory of gravity has huge redundancies associated with general coordinate transformations. Defining a state in Eq. (2) provides a simple way to fix these redundancies and to extract causal relations among events, which are physical (coordinate reparameterization invariant). In other words, we need to fix a reference frame when we describe a system with gravity quantum mechanically—this is the real meaning of the phrase: “physics must be described from the viewpoint of a single observer” in Ref. [5]. In particular, the location of a physical object/observer (with respect to “the origin of the coordinates” $p$) has physical meaning, so it needs to be included as a part of specification in condition $A$ when applying Eq. (1).

Since the Hilbert space $\mathcal{H}$ in Eq. (2) is defined on restricted spacetime regions, changes of the reference frame represented in $\mathcal{H}$ in general mix elements of different $\mathcal{H}_{\mathcal{M}}$ as well as the degrees of freedom associated with $\mathcal{H}_{\mathcal{M},\text{bulk}}$ and $\mathcal{H}_{\mathcal{M},\text{horizon}}$. (More generally, changing the reference frame
can also mix elements of Eq. (2) with intrinsically quantum mechanical states associated with singularities.) This is the origin of horizon complementarity (mixture between different $\mathcal{H}_M$) and of the “observer dependence” of cosmic horizons (mixture between the bulk and horizon degrees of freedom)! This general transformation can be viewed as an extension of the Lorentz/Poincaré transformation in the quantum gravitational context. It introduces more “relativeness” in physical descriptions—it makes even the concept of spacetime relative, as it mixes the bulk and horizon degrees of freedom in general.

Two key aspects of our picture of quantum measurement are dynamical evolution and the infinite dimensionality of the Hilbert space, given (partly) by Eqs. (2, 3). We argue that spacetime locality—a special property of the time evolution operator—plays a crucial role in the evolution of a state. It leads to “amplification” of classical information in a single component of the multiverse state. Schematically,

$$|\uparrow\rangle \rightarrow |\uparrow\rangle\ket{\Theta} \rightarrow |\uparrow\rangle\ket{\Theta}\ket{\mu\mu \ldots} \rightarrow \ldots,$$

which shows that the classical information (i.e. that the spin is up) is amplified in a detector pointer and brain state of an observer (which can be further amplified, e.g., in a note and a paper on the experiment, brain states of the people who have read the paper, and so on). Note that since the faithful duplication of quantum information is prohibited, only classical information can be amplified, whose content is much smaller than the full quantum information. At the same time, the dynamical evolution also leads to “branching”: the state splits into many different components having well-defined classical configurations. For example, the initial $e^+e^-$ state becomes a superposition of various components having well-defined particle configurations:

$$|e^+e^-\rangle \rightarrow |e^+e^-\rangle + |\mu^+\mu^-\rangle + \cdots + |e^+e^-e^+e^-\rangle + \cdots \rightarrow \ldots,$$

where we have omitted the coefficients for various components as well as momentum and spin indices. Note that through this process, the same quantum information can be distributed into multiple components over time; what the no-cloning theorem prohibits is the duplication of quantum information in a single component.

The evolution of the multiverse state experiences both these effects as it evolves in the full quantum gravitational Hilbert space. Schematically,

$$|\Sigma\rangle \rightarrow |A\rangle + |B\rangle \rightarrow |aa\rangle + |bb\rangle + |cc\rangle + |dd\rangle \rightarrow |\alpha\alpha \cdot \cdot \cdot \alpha\rangle + |\beta\beta \cdot \cdot \cdot \beta\rangle + \cdots,$$

where the various letters indicate classical information. This evolution is deterministic and unitary, i.e., obeys the basic laws of quantum mechanics. The amplification generically occurs from a

---

1. This does not mean that we cannot observe a superposition of classically different configurations. It just says that the statement “the system was in a superposition state” is already classical, and it is this information that is actually amplified.
smaller system to larger systems. At the early stage of this process, the basis of the amplification is determined by the details of the system, as the standard analysis of decoherence shows [11]. On the other hand, at later stages, where the relevant systems are large, the amplification occurs in the basis corresponding to states having well-defined classical configurations, as a result of spacetime locality. Various components of the state will then correspond to different macroscopic worlds, which will eventually evolve into different supersymmetric Minkowski (or singularity) states. Since the Hilbert space dimension of Minkowski space is infinite, these worlds do not recohere—they really branch into different worlds!

The above picture provides a complete account for the process of quantum measurement in the eternally inflating multiverse. While not all the aspects of the dynamics described above are fully proven, the basic picture is strongly supported by recent progress on understanding the quantum-to-classical transition (e.g. [8, 11]). What are the implications of this in calculating physical probabilities in Eq. (1)? Physical information we can handle is only the “robust” kind, i.e. the one that can appear multiple times in physical systems (e.g. as data stored in some “memories,” including someone’s brain state). It therefore only makes sense to ask questions about information that is amplified in some component of the state. This corresponds to choosing projection operators $O_A$ and $O_{A\cap B}$ to extract only such information; in particular, it corresponds to projecting onto classically well-defined configurations when we ask questions about macroscopic systems.

The framework described here provides a solid theoretical ground for asking any physical questions in the quantum universe. However, to make actual predictions in the context of the multiverse, e.g. of the value of a physical parameter we observe, we still need to know the explicit form of the time evolution operator as well as the initial condition for the multiverse state (except for a few special cases, including that for calculating the distribution of the cosmological constant [12]). In particular, knowing the complete evolution of the state requires understanding of the dynamics of the horizon degrees of freedom as well as the full string landscape. The former can be bypassed if we adopt the semi-classical approximation based on the “bulk density matrix” $\rho_{\text{bulk}}(t) = \text{Tr}_{\text{horizon}} |\Psi(t)\rangle \langle \Psi(t)|$, while the latter needs further progress in string theory. The initial condition for the multiverse state must be given by some external theory. Some (speculative) possibilities are presented in Ref. [5], but here we leave this issue aside and simply assume that an appropriate initial state is provided by some theory of initial conditions.

The organization of this paper is as follows. In the first half of the paper, Sections 2 – 4, we discuss quantum measurement without taking into account the effect of gravity; readers who are only interested in quantum gravitational aspects might proceed directly to Section 5. In Section 2 we provide a pedagogical introduction to the problem of quantum measurement, especially the problem of the preferred basis. In Section 3 we discuss carefully how spacetime locality selects a basis in the Hilbert space, a necessary ingredient to discuss basis selection for quantum measurement.
Section 4 we analyze quantum measurement in the context of applying quantum mechanics to the whole universe. We argue that the preferred basis for measurement is determined purely by the dynamics; in particular, the ultimate openness of the system is not required. Spacetime locality plays a crucial role in the appearance of an approximately classical world. (We discuss the example of spin measurements from this perspective in the appendix.) We also argue that the dimension of the Hilbert space for the entire universe must be infinite in order to be consistent with the fact that we observe an ordered world obeying the laws of physics. This provides an important constraint on the structure of the fundamental theory.

In the second half of the paper, Sections 5–8, we discuss quantum mechanics in a system with gravity. This part is largely elucidation of the results obtained in Ref. [5] from a clearer perspective of giving quantum mechanical description of a general relativistic system. There are, however, some important refinements, e.g. on the precise definition of $\mathcal{M}$ in Eq. (2), the treatment of spacetime singularities, and a useful probability formula applying in many practical cases. In Section 6 we argue that spacetime locality, which plays a crucial role in understanding quantum measurement, is preserved only if we define quantum states in appropriately restricted spacetime regions. In Section 6 we determine the Hilbert space of Eq. (2) from the viewpoint of fixing/eliminating all the redundancies and overcountings in a general relativistic, global spacetime description of nature. We argue that complementarity as well as the observer dependence of horizons can be understood in a unified manner from the fact that changes of the reference frame are represented in the Hilbert space defined in restricted spacetime regions. In Section 7 we reproduce the argument of Ref. [5], which says that the multiverse state evolves asymptotically into a supersymmetric Minkowski world, with an important modification associated with the treatment of spacetime singularities. This determines the complete Hilbert space structure for quantum gravity, and provides the required infinite dimensionality to explain our ordered observation. In Section 8 we discuss probabilities in the quantum mechanical multiverse. We restate that the eternally inflating multiverse and many worlds in quantum mechanics are the same. Finally, in Section 9 we briefly summarize the whole paper.

Relations between quantum mechanics and the multiverse have been discussed in other work as well. Reference [13] considered the issue of basis selection in the context of the multiverse, although the resulting picture is crucially different from the one here, especially about unitarity of quantum mechanics. Earlier considerations of quantum mechanics in the multiverse/universe can be found in Ref. [14]. Reference [15] uses supersymmetric Minkowski space as an important ingredient of their proposal, although in a different way than in ours. The picture of the multiverse from a local viewpoint, which arises here as a consequence of quantum mechanics, has been promoted in the context of geometric cutoff measures; see Ref. [16] for example.
2 Probabilistic Interpretation of Quantum Mechanics

In this and the next two sections, we discuss how the probabilistic interpretation of quantum mechanics arises in a complete quantum mechanical system that includes a physical observer who actually measures physical quantities. Note that in order to provide a full account of the measurement process, both the observer and an experimental apparatus must be a part of the description. Having a precise understanding of this process is crucial to apply quantum mechanics to the entire universe (or the eternally inflating multiverse).

2.1 Quantum measurement a la von Neumann

Let us begin our discussion with a simple nonrelativistic state in two dimensional Hilbert space, ignoring the experimental apparatus and observer for the moment. For definiteness, we take this to be a spin-1/2 system:

\[ |\Psi_{\text{sys}}\rangle = c_{\uparrow} |\uparrow\rangle + c_{\downarrow} |\downarrow\rangle, \] (7)

where \(|c_{\uparrow}|^2 + |c_{\downarrow}|^2 = 1\). The conventional Copenhagen interpretation says that if we measure the spin of this system at some time \(t = t_{m}\), then we find it up or down with the probabilities \(P_{\uparrow} = |c_{\uparrow}|^2\) and \(P_{\downarrow} = |c_{\downarrow}|^2\), respectively. We may write this as

\[ P_\alpha = \langle \Psi_{\text{sys}} | O_{\text{spin},\alpha} |\Psi_{\text{sys}}\rangle \quad (\alpha = \uparrow, \downarrow), \] (8)

where \(O_{\text{spin},\alpha}\) is the operator that acts on \(|\Psi_{\text{sys}}\rangle\) and projects onto the state with a definite spin \(\alpha\), e.g. \(O_{\text{spin},\uparrow} |\uparrow\rangle = 1\) and \(O_{\text{spin},\uparrow} |\downarrow\rangle = 0\). After we measure a definite outcome, e.g. spin up, the wavefunction \(|\Psi_{\text{sys}}\rangle\) of the system “collapses”

\[ |\Psi_{\text{sys}}\rangle \xrightarrow{t=t_{m}} |\uparrow\rangle, \] (9)

so that subsequent measurements will always find the spin pointing up.

In a modern viewpoint, a physical measurement is treated as interactions between the measured system (the spin-1/2 system under the case considered) and an experimental apparatus, as discussed originally by von Neumann [17]. The process described above then corresponds to the following situation. For \(t \ll t_{m}\), the combined state of the spin and the apparatus is

\[ |\Psi_{\text{sys+app}}(t \ll t_{m})\rangle = (c_{\uparrow} |\uparrow\rangle + c_{\downarrow} |\downarrow\rangle) \otimes |\bigcirc\rangle, \] (10)

where \(|\bigcirc\rangle\) represents the apparatus being in a “ready” state. (We take the Schrödinger picture throughout.) After \(t \approx t_{m}\), the full state becomes

\[ |\Psi_{\text{sys+app}}(t \gg t_{m})\rangle = c_{\uparrow} |\uparrow\rangle \otimes |\bigcirc\rangle + c_{\downarrow} |\downarrow\rangle \otimes |\bigcirc\rangle, \] (11)

\[ For\ simplicity,\ here\ we\ focus\ only\ on\ a\ single\ direction\ of\ the\ spin.\ Including\ other\ directions\ does\ not\ affect\ the\ argument\ below.\]
due to the *standard time evolution* of the state: \( |\Psi_{\text{sys+app}}(t_1)\rangle = e^{-iH(t_1-t_2)}|\Psi_{\text{sys+app}}(t_2)\rangle \). Here, \( |\uparrow\rangle \) and \( |\downarrow\rangle \) represent configurations of the apparatus showing that it has measured spin up and down, respectively, and \( H \) is the Hamiltonian for the combined system of the apparatus and spin. This particular process, making the state of the apparatus entangled with that of the measured system, is called decoherence in the narrow sense [11]. A striking fact is that the dynamical evolution from Eq. (10) to Eq. (11) occurs very quickly; namely, a microscopic system can affect a macroscopic system drastically in a rather short timescale, in a way that they can no longer be considered independent, separate systems. This is a crucial aspect of quantum mechanics that makes it hard to grasp using classical intuition.

### 2.2 The problem of the preferred basis

It is tempting to interpret Eq. (11) to show that the apparatus always measures either spin up or down, and not their superpositions.\(^3\) In fact, if the measured system is not a single spin, but a macroscopic object such as a chair, then the above discussion seems to explain the fact that we never observe superpositions of a macroscopic object in our everyday experience. Equation (11) alone, however, is not enough to show this because it can also be written in an arbitrary basis as [18]

\[
|\Psi_{\text{sys+app}}(t \gg t_m)\rangle = |1\rangle \otimes |\uparrow\rangle + |2\rangle \otimes |\downarrow\rangle.
\]

(12)

Here,

\[
|\uparrow\rangle = \sum_{\alpha=\uparrow,\downarrow} U_{i\alpha} |\alpha\rangle \quad (i = 1, 2)
\]

(13)

is a basis for the apparatus, and

\[
|i\rangle = \sum_{\alpha=\uparrow,\downarrow} c_{i\alpha} (U^{-1})_{\alpha\alpha} |\alpha\rangle
\]

(14)

the corresponding states for the measured system, where \( U \) is an arbitrary \( 2 \times 2 \) unitary matrix. In particular, if \( |c_\uparrow| = |c_\downarrow| \), then the states \( |1\rangle \) and \( |2\rangle \) form an orthogonal basis, so that they can be eigenstates of some Hermitian operator. How can one then say that the apparatus has measured the system in the \{\( |\uparrow\rangle, |\downarrow\rangle \}\) basis, not in the \{\( |1\rangle, |2\rangle \}\) basis?

The ambiguity of the basis described above has been confusing some fundamental physicists. A standard answer to this question is *environmental decoherence* [18, 19], whose implementation in the present context goes as follows. We first regard the apparatus and spin as open quantum systems, interacting with some "environment" \( |E_0\rangle \). We can then define the _preferred states_ for
the combined apparatus-spin system as the states that are least sensitive to the interaction with the environment, i.e. those that are least entangled with the environment by dynamical evolution. For instance, if the interaction between the apparatus and environment is such that

\[
\begin{align*}
|\uparrow\rangle \otimes |E_0\rangle &\rightarrow |\uparrow\rangle \otimes |E_1\rangle \\
|\downarrow\rangle \otimes |E_0\rangle &\rightarrow |\downarrow\rangle \otimes |E_2\rangle,
\end{align*}
\]

with \(\langle E_1|E_2\rangle \rightarrow 0\), then the preferred states are the two terms in the right-hand side of Eq. (11) because each of them will not get entangled with the environment according to Eq. (15). (Here, we have ignored the interaction between the spin and environment.) The measurement is then claimed to be performed in this preferred state basis.

In this picture of environment-induced basis selection, the openness of quantum systems plays a crucial role in understanding measurement processes. In fact, such a picture is appropriate for the purpose of discussing consequences of quantum measurement performed in terrestrial experiments, which are indeed open. At the fundamental level, however, this raises the following question: what if we include the environment in the description of our quantum state? One might say that there is always some environment for any system in practice, but here we are talking about the fundamental issue. This question becomes particularly acute if we try to apply quantum mechanics to describe the entire universe, since then it is not even clear what one can take as an environment for the entire universe.

A line of reasoning like this has recently led the authors of Ref. [13] to claim that quantum mechanics is operationally well defined only under the existence of intrinsically inaccessible degrees of freedom, which they took to be those escaping a cosmic horizon in the eternally inflating multiverse. In this picture, quantum mechanical evolution is intrinsically irreversible—to obtain probabilistic interpretation of quantum mechanics, degrees of freedom outside the horizon must be traced out. Here we will argue differently—we need not introduce such irreversibility at the fundamental level. We argue that, as discussed in Ref. [5], the principles of quantum mechanics, including deterministic unitary evolution of the states, are fully respected if one describes physics as viewed from a single reference frame.

The effective irreversibility of the quantum-to-classical transition appears simply because the dimension of Hilbert space is infinite. The ambiguity of the basis is fixed by a feature in the dynamics, specifically spacetime locality as encoded in the algebra of (low energy) operators. We now see in detail what the implications of spacetime locality are in our context.

---

4In Ref. [5], the word “observer” was used to denote a reference frame in which the quantum multiverse is described. The issue of the reference frame in a quantum mechanical system with gravity will be discussed in detail in Section 6, where we will see that quantum mechanics forces us to describe the system from the viewpoint of a single “observer.”
3 Physical Predictions and Spacetime Locality

To follow what a measurement actually means in a realistic context, here we consider a more elaborate model of the setup discussed in the previous section. We will see that all the physical information is encoded in matrix elements, and that the ultimate answer to the basis question should lie in the algebra of quantum operators and the dynamics associated with it.

3.1 Physical information is in matrix elements

Suppose that at early times \( t \ll t_m \), the detector apparatus has not yet interacted with the spin, as in Eq. (10). We assume that the detector is located on a desk, which we also include in our description. Moreover, we also consider an observer who has not initially been looking at the detector. The initial state of this entire system is then

\[
|\Psi(t \ll t_m)\rangle = (c_\uparrow |\uparrow\rangle \otimes |\uparrow\rangle \otimes |\text{desk}\rangle \otimes |\text{observer}\rangle).
\]

(16)

Here, we do not necessarily consider that \( c_\uparrow \) and \( c_\downarrow \) are normalized as \( |c_\uparrow|^2 + |c_\downarrow|^2 = 1 \) (anticipating that the state in general may have terms additional to the ones shown above). At \( t \approx t_m \), the detector interacts with the spin, but we assume that the observer does not look at it until a later time \( t_{\text{obs}} > t_m \), so

\[
|\Psi(t_m \ll t \ll t_{\text{obs}})\rangle = \left(c_\uparrow |\uparrow\rangle \otimes |\text{desk}\rangle \otimes |\text{observer}\rangle + c_\downarrow |\downarrow\rangle \otimes |\text{desk}\rangle \otimes |\text{observer}\rangle\right) \otimes |\text{spin}\rangle \otimes |\text{detector}\rangle.
\]

(17)

Finally, at \( t = t_{\text{obs}} \), the observer reads what the detector shows, and his/her brain state reacts accordingly:

\[
|\Psi(t \gg t_{\text{obs}})\rangle = c_\uparrow |\uparrow\rangle \otimes |\text{desk}\rangle \otimes |\text{observer}\rangle \otimes |\text{spin}\rangle \otimes |\text{detector}\rangle + c_\downarrow |\downarrow\rangle \otimes |\text{desk}\rangle \otimes |\text{observer}\rangle \otimes |\text{spin}\rangle \otimes |\text{detector}\rangle.
\]

(18)

Note that the time evolution of the state, Eq. (16) \( \rightarrow \) Eq. (17) \( \rightarrow \) Eq. (18), is caused by the standard, deterministic quantum evolution: \( |\Psi(t_1)\rangle = e^{-iH(t_1-t_2)} |\Psi(t_2)\rangle \), where \( H \) is the Hamiltonian for the combined system of the spin, apparatus, desk, and the observer.

According to the standard rule of quantum mechanics, we expect that the probabilities for the observer to measure spin up and down should respectively be

\[
P_\alpha = \frac{|c_\alpha|^2}{|c_\uparrow|^2 + |c_\downarrow|^2} \quad (\alpha = \uparrow, \downarrow),
\]

(19)

ignoring the issue of the basis ambiguity. What is the precise meaning of this equation? Note that the question we are asking here is actually the following: assuming that the observer learns...
the result of the experiment by reading the apparatus, what does he/she find? This conditional probability is given, according to the standard Born rule, by

$$P(\alpha|\text{obs}) = \frac{\langle \Psi(t \gg t_{\text{obs}}) | O_{\text{obs},\alpha} | \Psi(t \gg t_{\text{obs}}) \rangle}{\langle \Psi(t \gg t_{\text{obs}}) | O_{\text{obs}} | \Psi(t \gg t_{\text{obs}}) \rangle} \quad (\alpha = \uparrow, \downarrow),$$

(20)

where $O_{\text{obs},\alpha}$ is the operator projecting onto the state in which the observer learns the result to be $\alpha$

$$O_{\text{obs},\alpha} = 1 \otimes 1 \otimes 1 \otimes \left\{ | \begin{array}{c} \uparrow \downarrow \alpha \\ \downarrow \uparrow \alpha \end{array} \rangle \langle \begin{array}{c} \downarrow \uparrow \alpha \\ \uparrow \downarrow \alpha \end{array} | \right\},$$

(21)

and $O_{\text{obs}}$ onto the one in which he/she learns some result, whatever it is:

$$O_{\text{obs}} = \sum_{\alpha} O_{\text{obs},\alpha}.$$  

(22)

Here, the states appearing in Eq. (20) have been chosen simply as $| \Psi(t \gg t_{\text{obs}}) \rangle$, since $| \Psi(t) \rangle$ is assumed to be (approximately) time independent for $t \gg t_{\text{obs}}$. The case in which a state has general time dependence, leading to Eq. (1) (or its generalization, Eq. (84)), will be discussed carefully later (in Section 8).

The simple analysis above highlights some of the important features of our present description of quantum measurement. First, expressed in the form of conditional probabilities as in Eq. (20), physical predictions do not depend on how the state is written, including in what basis it is expanded. This property is obvious in Eq. (20), since the matrix elements do not depend on the basis used to expand the state, but it is obscured if one focuses only on the state and its expansion coefficients, as in Eq. (19). Second, our (more fundamental) formula of Eq. (20) reproduces Eq. (19) even if the state $| \Psi(t) \rangle$ contains additional terms that are not selected by the projection operator $O_{\text{obs}}$. In fact, given an initial condition, the state at late times might contain a term representing a possibility that is not listed in Eq. (18); for example, the apparatus might break before the observer reads it, or the observer might change his/her mind and never look at the apparatus. (Note that, once the initial condition is given, the future state $| \Psi(t) \rangle$ is uniquely determined according to the deterministic quantum evolution specified by $H$, i.e., the choice is not left to us to eliminate these “unwanted” possibilities.) Because of the way we asked the question, however, our answer always satisfies

$$\sum_{\alpha} P(\alpha|\text{obs}) = 1.$$  

(23)

Namely, the possible additional terms in the state $| \Psi(t) \rangle$ (or additional “components” in the wavefunction) are irrelevant for the question we are asking. As discussed in detail in Ref. [5], any physical question can be phrased in the form of a conditional probability; in the simplest setup, we can phrase it as: “Given what we know about our past light cone, $A$, what is the probability of that light cone to have properties $B$ as well?” This eliminates the question of “What is the right basis to expand the state?” The answer is that “It doesn’t
matter.” Once the question is phrased in this way using the appropriate projection operators \( O_A \) and \( O_{A \cap B} \) (e.g. \( O_{\text{obs}} \) and \( O_{\text{obs},\alpha} \) in the above example), the desired probability \( P(B|A) \) is defined unambiguously. Is this sufficient to eliminate the basis ambiguity for quantum measurement, discussed in Section 2.2? The answer is no—we need to discuss quantum operators, especially \( O_A \) and \( O_{A \cap B} \), to see if there is any ambiguity there.

### 3.2 Spacetime locality and the basis in Hilbert space

We have seen that once a physical question is phrased in terms of \( O_A \) and \( O_{A \cap B} \), the answer is unambiguously given by the probability formula, such as Eq. (20). But, how can these operators be constructed? In particular, is there any ambiguity in writing these operators (and if so, wouldn’t that just be trading the basis ambiguity of states for that of operators)?

First of all, we note that it is appropriate to discuss the issue of basis in terms of operators, rather than states, as we will do here. This is because Hilbert space by itself does not carry any physical information other than its dimensionality—any (complex) Hilbert spaces having the same dimension are identical with each other. *All the information about a physical system (except for its dimensionality) is encoded in quantum operators and the algebra they satisfy.* Of course, being operators acting on a vector space, these quantum operators may also be written in an arbitrary basis. However, we now have dynamical structures that may distinguish some basis over the others. In particular, there can be a special basis in which algebraic relations among operators look particularly simple.

Consider a (special) relativistic system. At length scales much larger than the possible quantum gravity scale (and the entropy density much lower than that of a black hole), such a system is described by quantum field theory [20]. Suppose there is only a single species of particles, represented by a quantum field \( \phi(x) \) with \( x \) being spatial coordinates. The field \( \phi(x) \) at different \( x \) should be regarded as different operators, which satisfy

\[
[\phi(x), \pi(x')]_\mp = i \delta_{\text{x,x'},}\quad [\phi(x), \phi(x')]_\mp = [\pi(x), \pi(x')]_\mp = 0,
\]

where \( \pi(x) \) is a conjugate momentum of \( \phi(x) \), and \( [\cdot, \cdot]_\mp \) represents a commutator and anti-commutator if the particle is a boson and fermion, respectively. Here, we have discretized spatial coordinates \( x \) for presentation purposes.\(^5\) An important point is that in this “local field” basis, the time evolution operator \( U(t_1, t_2) = e^{-iH(t_1-t_2)} \) takes a particularly simple form

\[
H = \sum_x H_x (\phi(x + \epsilon_0), \phi(x + \epsilon_1), \cdots; \pi(x + \epsilon_0), \pi(x + \epsilon_1), \cdots),
\]

\(^5\)Since we consider low energy physics, some sort of discretization, e.g. a finite (effective) ultraviolet cutoff, is appropriate. Note that here we use the Schrödinger picture, which is unconventional in quantum field theory.
where \( \epsilon_i \) \((i = 0, 1, 2, \cdots)\) runs only over a very small subset of the coordinates around \( \epsilon_0 \equiv 0 \) (typically “nearest neighbors”). This is not the case if we use an arbitrary basis

\[
\phi(z) = \sum_x c_{zx} \phi(x) \quad (c_{zx} \neq \delta_{zx}).
\]  

(26)

Namely, if we write \( H \) in terms of \( \phi(z) \) and \( \pi(z) \) in the form of Eq. (25), then \( \epsilon_i \) need not run only over a very small subset of the coordinates for generic \( c_{zx} \).

The existence of a special basis satisfying Eq. (25) is exactly what we call spacetime locality. This is a property of nature whose origin is not yet fully understood—it is simply an empirical fact that there is such a basis at length scales that have been probed experimentally so far.\(^6\)

This property, however, is crucial in selecting a particular basis in Hilbert space in which a simple description of physics is obtained. Specifically, consider a set of (time-independent) states \( |\kappa_m\rangle \) that are eigenstates of the particle-number operators \( N_x \) for all \( x \) (not \( z \)):

\[
N_x(\phi(x), \pi(x)) |\kappa_m\rangle = n^{(m)}_x |\kappa_m\rangle.
\]  

(27)

These states are special in that they have well-defined configurations in physical space \( x \). Furthermore, since the set of states in Eq. (27) spans Fock space, it can form an orthonormal basis of Hilbert space; namely, an arbitrary state \( |\Psi(t)\rangle \) may be written as a superposition

\[
|\Psi(t)\rangle = \sum_m c_m(t) |\kappa_m\rangle,
\]  

(28)

where \( c_m(t) \) are complex functions and \( \langle \kappa_m | \kappa_n \rangle = \delta_{mn} \). Note that the particular basis here, \( |\kappa_m\rangle \), has been chosen such that an algebraic relation between operators—specifically the form of \( H \) in terms of \( \phi(x) \) and \( \pi(x) \)—takes a simple form in that basis. In fact, the very concept of “configurations in space” arises as a result of the special property in Eq. (25); without that, \( x \) could not even be interpreted as spatial coordinates.

We should emphasize that the choice of the Hilbert space basis discussed here does not by itself address the issue of basis selection for quantum measurement described in Section 2.2, although the former is needed for the discussion of the latter. Indeed, the choice described here is, in some sense, “a matter of convenience,” in that we can also describe physics using the \( \phi(z) \) basis in principle (because the matrix elements, appearing in the probability formula, do not depend on the basis). In this basis, however, the time evolution operator has an extremely complicated form, which completely obscures the fact that the dynamics respects spacetime locality.\(^7\)

\(^6\)It is important to realize that spacetime locality is the property of the operator algebra, and not states. In fact, a state can be easily (and, indeed, is generically) non-local, e.g., as the Bell state appearing in the Einstein-Podolsky-Rosen experiment.

\(^7\)As an analogy, one can imagine describing high energy behaviors of QCD in the gravitational picture, using the gauge/gravity duality. In that picture, any high energy scattering will have contributions from complicated high-curvature stringy effects, which completely obscures the fact that it is given by a simple, perturbative gluon exchange amplitude.
practice one always needs to choose a Hilbert space basis associated with locality: either $|\kappa_m\rangle$ in Eq. (27) or a basis that has a simple relation to it (such as the momentum basis).

### 3.3 What physical questions may one ask?

Let us choose the “locality basis” $|\kappa_m\rangle$, given in Eq. (27). Then there is no ambiguity in expanding states as in Eq. (28). The question, however, still remains: how can we choose the “correct form” for projection operators $O_A$ and $O_{A\cap B}$ appearing in the probability formula? Experience says that all the information we can explicitly handle (in the sense that it can be duplicated in physical systems) is given in the form of, e.g., Eqs. (20 – 22)—i.e. by operators projecting onto states that have well-defined macroscopic configurations in the phase space (up to some uncertainties). Why is that?

In general, states having well-defined macroscopic configurations, e.g. $|\bigcirc\rangle$, $|\bigtriangledown\rangle$, $|\bigwedge\rangle$, and $|\bigcirc\bigtriangledown\rangle$ in previous sections, are obtained as superpositions of $|\kappa_m\rangle$ that have “similar” spatial configurations. For each macroscopic configuration $i$, we have a set of $n_i$ corresponding microstates

$$|\psi^{(i)}_a\rangle = \sum_{m} f^{(i)}_{a,m} |\kappa_m\rangle \quad (a = 1, \cdots, n_i),$$

which we collectively call $|\alpha_i\rangle$. Here, $f^{(i)}_{a,m}$ for each $(i,a)$ play the role of a smearing function in position space, ensuring that the configuration has a well-defined momentum at the macroscopic level. The projection operator onto macroscopic configuration $i$ can then be defined as

$$|\alpha_i\rangle \langle \alpha_i| \equiv \sum_{a=1}^{n_i} |\psi^{(i)}_a\rangle \langle \psi^{(i)}_a|,$$

where we have taken $\langle \psi^{(i)}_a | \psi^{(i)}_b \rangle = \delta_{ab}$. Since $\langle \psi^{(i)}_a | \psi^{(i)}_b \rangle \approx 0$ for different macroscopic configurations $i \neq j$, these projection operators satisfy $P_i P_j \approx P_j P_i \approx 0$ for $i \neq j$, where $P_i = |\alpha_i\rangle \langle \alpha_i|$. The issue is why physical questions we ask are always phrased in terms of $O_A$ and $O_{A\cap B}$ that take the form

$$O_A = \sum_{i \in A} |\alpha_i\rangle \langle \alpha_i|$$

(and similarly for $O_{A\cap B}$), where $i \in A$ implies that the sum is taken for the configurations that satisfy condition $A$. In particular, what is wrong with using $|\alpha_i\rangle$ corresponding to a superposition of macroscopically different configurations, i.e. microstates $|\psi^{(i)}_a\rangle$ in which the expansion coefficients $f^{(i)}_{a,m}$ have significant supports from macroscopically different configurations $m$? This strong restriction on possible questions we may ask is the essence of basis selection for quantum measurement, and composes what we call the quantum-to-classical transition. Its origin is in the dynamics, specifically spacetime locality as encoded in Eq. (25), as we will discuss in the next section.
4 Classical Reality in Quantum Mechanical Systems

In this section, we discuss the origin of the following basic observational fact: we perceive our macroscopic world to be classical, having a well-defined configuration in phase space. In fact, this statement consists of at least two elements, which are respectively to do with “basis selection” and “wavefunction collapse” of quantum measurement:

(i) Probabilistic processes in quantum mechanics are well described by density matrices that are diagonal in the “classical state basis” $|\alpha_i\rangle$, at least for macroscopic systems.

(ii) A measurement selects an outcome; namely, we can ignore other possible outcomes after a measurement is actually performed.

In the standard treatment of these problems, the openness of a system is emphasized [11]. Here we ask if the openness is really necessary at the fundamental level to account for these features. We will argue that the answer is no—the quantum-to-classical transition may occur consistently with observation even in a closed quantum mechanical system.

On the other hand, we will also argue that cosmology based on a closed system fails to explain another basic element necessary to explain the observational fact:

(iii) We observe an ordered world, i.e., we perceive a world that obeys consistent laws of physics.

This argument will force us to consider that the Hilbert space for the entire universe (multiverse) is infinitely large: $\dim \mathcal{H} = \infty$, unless we abandon unitarity of quantum mechanical evolution. In the actual universe, this requirement is satisfied because it asymptotically becomes a supersymmetric Minkowski (or singularity) world, which we will see in more detail in Section [7].

4.1 A double-slit experiment in a large system

We begin by a standard analysis of the double-slit experiment, which sets the stage for later discussions. For the moment, we can be agnostic about whether the entire system is open or closed. Our analysis applies as long as the dynamical timescale is much shorter than the thermalization timescale $t_{th}$, which is indeed the case for a sufficiently large system ($t_{th} = \infty$ for an open system).

The setup of the experiment is such that an electron, initially prepared at slits as

$$|\psi_{e,\text{init}}\rangle = \frac{1}{\sqrt{2}} (|1\rangle + |2\rangle)$$

(32)

evolves according to

$$|1\rangle \to \int dx \ \psi_1(x) |x\rangle, \quad |2\rangle \to \int dx \ \psi_2(x) |x\rangle.$$

(33)
Figure 1: The double-slit experiment with an electron. The electron at the two slits are represented by $|1\rangle$ and $|2\rangle$, respectively, while that at the screen at position $x$ by $|x\rangle$.

Here, $|1\rangle$ and $|2\rangle$ represent the electron localized at slits 1 and 2, respectively, while $|x\rangle$ represents the electron localized at position $x$ on the screen (see Fig. 1). We consider sending only a single electron for the sake of simplicity.

The entire system consists of the electron, detector apparatus, and the rest of the world, which is initially in the state

$$|\Psi_{\text{init}}\rangle = \left\{ \frac{1}{\sqrt{2}} (|1\rangle + |2\rangle) \right\} \otimes |\bigcirc\rangle \otimes |R\rangle,$$

(34)

where $|\bigcirc\rangle$ represents the detector in a ready state and $|R\rangle$ the degrees of freedom that are not included in the electron or detector state. Assuming that there is no interaction between the experimental apparatus and the rest of the world, this state evolves into

$$|\Psi_{\text{fin}}\rangle = \frac{1}{\sqrt{2}} \left( \int dx \, \psi_1(x) |0\rangle \otimes |\bigcirc\rangle + \int dx \, \psi_2(x) |0\rangle \otimes |\bigcirc\rangle \right) \otimes |R\rangle.$$

(35)

Here, we have assumed that the combined electron and detector system evolves as

$$|x\rangle \otimes |\bigcirc\rangle \rightarrow |0\rangle \otimes |\bigcirc\rangle,$$

(36)

where $|0\rangle$ implies that the electron has absorbed into the apparatus, and $|\bigcirc\rangle$ represents the status of the detector showing that the electron has arrived at $x$ on the screen. The probability density

\[8\text{Here we consider } |\bigcirc\rangle \text{ and } |R\rangle \text{ to be one of the microstates having these macroscopic configurations. The projection operators corresponding to macroscopic configurations, e.g. the one in Eq. (38) below, should then be interpreted as defined in Section 3.3. This is appropriate since we are interested in the fundamental theory here. In practice, we do not know which microstate a given macroscopic configuration is in, so we may consider a density matrix in which all the microscopic information is coarse-grained. Our formalism is straightforwardly extended to this case; see e.g. Eq. (37) in Section 8.1.}\]
of finding the electron at $x$ in this experiment is then given by

$$P(x|\text{exp}) = \frac{\langle \Psi_{\text{fin}} | O_x | \Psi_{\text{fin}} \rangle}{\langle \Psi_{\text{fin}} | \int O_x dx | \Psi_{\text{fin}} \rangle},$$

(37)

where

$$O_x = 1 \otimes \left\{ | x \rangle \langle x | \right\} \otimes 1$$

(38)

is the projection operator extracting the result of the experiment. Plugging Eq. (35) into Eq. (37), we obtain the standard result:

$$P(x|\text{exp}) \propto |\psi_1(x)|^2 + |\psi_2(x)|^2 + 2 \text{Re}\{\psi_1(x)\psi_2^*(x)\}.$$

(39)

Here, we have used $\langle x | y \rangle = \delta(x - y)$, and the last term represents the interference effect.

Let us now consider the situation that the path of the electron is “monitored” by interactions of the electron with other degrees of freedom, e.g. a detector located close to the slits. Including these degrees of freedom in $| R \rangle$, the final state of the evolution is now given by

$$| \Psi_{\text{fin}} \rangle = \frac{1}{\sqrt{2}} \left( \int dx \: \psi_1(x) | 0 \rangle \otimes | x \rangle \otimes | R_1 \rangle + \int dx \: \psi_2(x) | 0 \rangle \otimes | x \rangle \otimes | R_2 \rangle \right),$$

(40)

instead of Eq. (35). Here, $| R_1 \rangle$ and $| R_2 \rangle$ represent the status of the degrees of freedom after the electron passes slits 1 and 2, respectively. Applying Eq. (37), the probability density of finding the electron at $x$ is now

$$P(x|\text{exp}) \propto |\psi_1(x)|^2 + |\psi_2(x)|^2 + 2 \text{Re}\{\psi_1(x)\psi_2^*(x)| R_2 \rangle \langle R_1 | \}.$$

(41)

Therefore, once the interactions of the electron at the slits lead to distinct configurations for the surroundings depending on which slit it passes, i.e. $\langle R_2 | R_1 \rangle \ll 1$, the interference term disappears. This is the well-known result obtained by Wootters and Zurek in Ref. [21].

The question, again, is why the result of the experiment is described using the operator in Eq. (38). In other words, why do we perceive the world in such a way that a macroscopic quantum system decoheres in the classical state basis—in this case, the location of the pointer of the detector apparatus? In the standard explanation due to environment-induced basis selection, the openness of a system plays a crucial role. Below, we will obtain (essentially) the same result without invoking an openness of the whole system, which elucidates the real origin of the basis selection in quantum measurement.

### 4.2 Dynamical selection of a measurement basis—spacetime locality

Let us keep following the state of the system after the double-slit experiment was performed. For simplicity, here we focus on the case where the electron path was not monitored, but the
same argument applies to the monitored case as well. Denoting the evolution after the double-slit measurement by
\[ |\psi_x(t = 0)\rangle \equiv |0\rangle \otimes |\varnothing\rangle \otimes |R\rangle \rightarrow |\psi_x(t)\rangle , \] (42)
the state of the entire system is given by
\[ |\Psi(t)\rangle = \int dx \, c_x \, |\psi_x(t)\rangle , \] (43)
where \( t \) is the time passed after the measurement, and \( c_x \equiv (\psi_1(x) + \psi_2(x))/(|\int dy |\psi_1(y) + \psi_2(y)|^2)^{1/2} \).

Now, let us imagine that some other experiment is performed in this system at a late time \( t_{\text{exp}} \), which is still earlier than \( t_{\text{th}} \). This experiment will involve only a very small subset of the degrees of freedom in \( |\Psi(t)\rangle \), as we consider that the entire system is very large. We isolate these degrees of freedom by writing \( |\psi_x(t)\rangle = |\phi_x(t)\rangle \otimes |r_x(t)\rangle \):
\[ |\Psi(t)\rangle = \int dx \, c_x \, |\phi_x(t)\rangle \otimes |r_x(t)\rangle , \] (44)
where the first factor represents degrees of freedom associated with the experiment while the second the rest. The outcome of this experiment can be calculated, using the probability formula based on matrix elements as
\[ P(A|\text{exp}) = \frac{\langle \Psi(t_{\text{exp}}) | O_A | \Psi(t_{\text{exp}}) \rangle}{\langle \Psi(t_{\text{exp}}) | \sum_A O_A | \Psi(t_{\text{exp}}) \rangle} \]
\[ = \frac{\int dx dy \, c_x^* c_y \langle r_x(t_{\text{exp}}) | r_y(t_{\text{exp}}) \rangle \langle \phi_x(t_{\text{exp}}) | O_A | \phi_y(t_{\text{exp}}) \rangle}{\int dx dy \, c_x^* c_y \langle r_x(t_{\text{exp}}) | r_y(t_{\text{exp}}) \rangle \langle \phi_x(t_{\text{exp}}) | \sum_A O_A | \phi_y(t_{\text{exp}}) \rangle} , \] (45)
where \( O_A \) is the projection operator acting on \( |\phi_x(t)\rangle \) selecting the situation where the experiment is performed with a definite outcome \( A \).

The expression of Eq. (45) contains terms representing interference between different outcomes of the first, double-slit experiment, i.e. \( x \) and \( y \) with \( x \neq y \). These terms, however, disappear if \( \langle r_x(t_{\text{exp}}) | r_y(t_{\text{exp}}) \rangle \ll 1 \) for \( x \neq y \), in which case
\[ P(A|\text{exp}) = \frac{\int dx |c_x|^2 \langle \phi_x(t_{\text{exp}}) | O_A | \phi_x(t_{\text{exp}}) \rangle}{\int dx |c_x|^2 \langle \phi_x(t_{\text{exp}}) | \sum_A O_A | \phi_x(t_{\text{exp}}) \rangle} \]
\[ = \int dx |c_x|^2 \langle \phi_x(t_{\text{exp}}) | O_A | \phi_x(t_{\text{exp}}) \rangle . \] (46)

Here, we have assumed \( \langle \phi_x(t_{\text{exp}}) | \sum_A O_A | \phi_x(t_{\text{exp}}) \rangle = 1 \), i.e. the second experiment occurs no matter what the outcome of the double-slit experiment. The probabilities for the outcomes of the two experiments now follow what we expect classically:
\[ P(A|\text{exp}) = \int dx \, P(x \mid \text{double-slit exp}) \, P_x(A \mid \text{second exp}) . \] (47)
Note that the result of the second experiment may depend on that of the first, as indicated by the subscript $x$ on the second probability factor.

In fact, the condition used above to obtain Eq. (47), $\langle r_x(t_{\text{exp}})|r_y(t_{\text{exp}}) \rangle \ll 1$ for $x \neq y$, is exactly what we expect. Since the detector states $|\varnothing \rangle$ for different $x$ have different macroscopic configurations and since the Hamiltonian of the system is local, i.e. has the form of Eq. (25), the future states corresponding to different $|\varnothing \rangle$ are almost orthogonal:

$$\langle \psi_x(t)|\psi_y(t) \rangle \sim \delta(x - y).$$

(For example, photons scattered from the detector will have different configurations for different $x$ [22].) This implies that $\langle r_x(t)|r_y(t) \rangle \sim \delta(x - y)$ because $|\phi_x(t)\rangle$ is only a very small subset of the entire degrees of freedom in $|\psi_x(t)\rangle$, leading to Eq. (47).

The argument given above, however, is not by itself sufficient to explain selection of the measurement basis, because the same analysis applies to any (not necessarily classical state) basis $z = \int dx r_{zx} \varnothing$ satisfying $\int dx r_{zx} r_{z'x} = \delta(z - z')$. The selection happens because “amplification” occurs (only) in a particular basis (called quantum Darwinism) [8]. Schematically,

$$|\psi_x(0)\rangle \to |\psi_x(t)\rangle \equiv |x\rangle |X\rangle |X\rangle \cdots,$$

where the last expression implies that the same classical information, i.e. “the double-slit experiment measured the electron at $x$,” is available (independently) to many subcomponents of the system. For example, for a superposition of two outcomes after the double-slit experiment, this gives

$$|\Psi(0)\rangle = c_x |\psi_x(0)\rangle + c_y |\psi_y(0)\rangle \to c_x (|x\rangle |X\rangle |X\rangle \cdots) + c_y (|y\rangle |Y\rangle |Y\rangle \cdots).$$

Note that this is different from the cloning of quantum information, which would yield

$$|\Psi(0)\rangle \to (c_x |x\rangle + c_y |y\rangle) (c_x |X\rangle + c_y |Y\rangle) (c_x |X\rangle + c_y |Y\rangle) \cdots.$$  

Namely, only selected information, that corresponding to classical states $|\alpha_i\rangle$, can be amplified. The origin of the particular evolution in Eq. (50) is the special form of the evolution operator, Eq. (25)—the classical state basis is selected as a dynamical consequence of spacetime locality. While this is yet to be proven in the general case, analyses of simple quantum mechanical systems [8] strongly suggest it to be the case.

\[9\]Note that this would not generally be the case if the relevant object were microscopic, i.e. described by a Hilbert space with small dimensions. In that case, two different states could significantly overlap (recohere) in the future, as the states $|1\rangle$ and $|2\rangle$ in Eq. (33). Indeed, quantum interference effects arise precisely because of such recoherence, which may occur easily in a system with small Hilbert space dimensions.
The properties of Eqs. (48) and (50) for $|\alpha_i\rangle$ imply that if $n-1$ successive experiments are performed after the double-slit experiment, we obtain

$$P(A^{(n)}|\text{exp}) = \sum_{A^{(n-1)}} \cdots \sum_{A^{(2)}} \int dx \, P(x|\text{double-slit exp}) \times P_x(A^{(2)}|\text{second exp}) \cdots P_xA^{(2)}\cdots A^{(n-1)}(A^{(n)}|n\text{-th exp})$$

(52)

(up to negligible corrections, e.g., from $\langle \psi_x(t)|\psi_y(t)\rangle \ll 1$ for $x \neq y$). This is the classical probability formula. The subscripts in $P$'s on the right-hand side indicate that the results of earlier experiments, e.g. $x$ of the double-slit experiment, are available independently to many successive experiments, just by accessing small subsets of the entire system. This forms a crucial ingredient for classical objectivity [8]. We emphasize that the basis of decomposition in Eq. (52) is determined dynamically. In fact, the very existence of a special basis in which the classical formula of Eq. (52) is true is a dynamical consequence of spacetime locality.

The absence of the interference term in Eq. (52) implies that we can describe the system using “decohered” density matrix

$$\rho = \frac{1}{\int dy |\psi_1(y) + \psi_2(y)|^2} \int dx |\psi_1(x) + \psi_2(x)|^2 \left\{ |0\rangle \langle 0| \right\} \otimes \left\{ |\alpha\rangle \langle \alpha| \right\} \otimes \left\{ |R\rangle \langle R| \right\}$$

(53)

after the electron is measured in the double-slit experiment. In particular, if we are interested only in physics after the electron is measured at a particular point $x$, then we need only keep the term containing $|\alpha\rangle$, which is equivalent to using the “collapsed” state

$$|\Psi_{\text{col}}\rangle = \frac{\psi_1(x) + \psi_2(x)}{\sqrt{|\psi_1(x) + \psi_2(x)|}} |0\rangle \otimes |\alpha\rangle \otimes |R\rangle.$$  

(54)

The argument presented here is the heart of the basis selection in describing any experimental result in a way that our classical intuition is manifest. It also provides a real rationale behind Eq. (31), which was chosen to be diagonal in the classical state basis $|\alpha_i\rangle$. The information about classical configuration $|\alpha_i\rangle$ is that which can be amplified by the dynamical evolution—this is not very surprising given that $|\alpha_i\rangle$’s are deeply related to the locality basis states, which are determined by the form of the time evolution operator. Questions we ask are about information that can be objectively accessed by multiple physical processes (including what is stored in memory states of our brains), hence the form of Eq. (31). Note that by integrating out $|R\rangle$ as well as all the histories after $t = 0$, this reproduces the usual einselection criterion in the decoherence paradigm. The argument here, however, makes it clear that the basis selection has nothing to do with the ultimate openness of the system—indeed, the present argument still applies even if the entire system is closed, being subject to thermalization and recurrences at later times. The origin of the basis selection lies entirely in the dynamics—more specifically, the fact that the time evolution operator takes a special form of Eq. (25) in the locality basis.
4.3 Ordered observations require infinitely large Hilbert space

Does the preceding argument ensure that the two features listed at the beginning of this section, (i) and (ii), are valid in any quantum system described by a local theory? In other words, can we always consider that a sufficiently macroscopic measurement collapses the wavefunction to one of the possible states having a well-defined classical configuration?

The answer is yes, as long as we are interested only in dynamics at timescales much shorter than $t_{th}$. However, if the system is closed, and we are interested in arbitrarily long timescales, then the answer is nontrivial. In this case the entire system thermalizes at $\sim t_{th}$, after which it occasionally experiences rare fluctuations producing low entropy regions, and eventually comes back to a state that is arbitrarily close to the original state at timescale $t_s \sim e^{S_{th}}$, where $S_{th}$ is the thermal entropy of the system. This picture applies regardless of the details of the system, as long as quantum mechanical evolution is unitary and the initial condition is generic, which we assume throughout. Since the process of producing low entropy fluctuations generically involves interferences between macroscopically different terms, replacing $|\Psi_{\text{fin}}\rangle$ by the collapsed state $|\Psi_{\text{col}}\rangle$ might seem to give an obviously wrong answer. This conclusion, however, is too naive.

Consider a process in which a thermal state having a large coarse-grained entropy, $\sim S_{th}$, fluctuates into a state with a low coarse-grained entropy and then evolves back to another thermal state:

$$
|\Psi_{\sim S_{th}}\rangle \rightarrow |\Psi'_{S_{\text{low}}}\rangle \rightarrow |\Psi''_{\sim S_{th}}\rangle,
$$

where we have denoted the coarse-grained entropy of a state by the subscript ($S_{\text{low}} \ll S_{th}$). One might think that the first part of this process, $|\Psi_{\sim S_{th}}\rangle \rightarrow |\Psi'_{S_{\text{low}}}\rangle$, looks like the "classical-to-quantum transition," as it involves recoherence of macroscopically different configurations, thus contradicting observation. This is, however, not true. Because of the reversibility of quantum evolution, the process is given, with high probability, by the time reversal of a usual entropy increasing process [23]:

$$
|\Psi_{\sim S_{th}}\rangle \rightarrow |\Psi'_{S_{\text{low}}}\rangle \underset{\text{time reversal}}{\leftrightarrow} |\Psi'_{S_{\text{low}}}\rangle \rightarrow |\bar{\Psi}_{\sim S_{th}}\rangle,
$$

where a state with a bar is the CPT conjugate of the corresponding state without. Since any physical observer is necessarily a part of the system, the two processes in Eq. (56) look identical to him/her—these are really time reversals of each other including memory states of the observer. In particular, they both look like a standard thermalization process, involving the quantum-to-classical transition.\footnote{This statement seems evident to the author, given that the dynamics is such that macroscopic correlations between subsystems increase as entropy increases, and that “perception” is nothing but (classical) memory states of physical computational devices, including our brains (see e.g. [24]). If it does not hold, however, it would only strengthen our conclusion that the entire universe cannot be a finite, closed system.}
Figure 2: Entropy fluctuations in a closed quantum system can provide "observations," indicated by (both solid and dashed) arrows, with each fluctuation corresponding to two "regular worlds" obeying the second law of thermodynamics (as well as the standard rules of Copenhagen quantum mechanics). However, the vast majority of these fluctuations correspond to random, irregular observations (dashed arrows), as opposed to ordinary, ordered observations, which compose only a tiny fraction of the fluctuations (solid arrows).

In fact, if we follow the system for an arbitrarily long time, replacing $|\Psi_{\text{fin}}\rangle$ by $|\Psi_{\text{col}}\rangle$ at one time will lead only to a negligible error on final probabilities, obtained by a formula based on matrix elements (e.g. Eq. (1), which will be discussed in detail in Section 8). In this case, the probability for a fluctuation such as Eq. (55) to occur is simply proportional to the Boltzmann factor, multiplied by the number of microstates: $e^{-M_{\text{fluc}}/T+S_{\text{low}}}$ where $M_{\text{fluc}}$ is the mass associated with the fluctuation and $T$ the temperature of the system. For an "observer," who can arise only as a part of such a fluctuation, the world always appears to obey properties (i) and (ii): the standard rules of Copenhagen quantum mechanics (as well as the second law of thermodynamics), as illustrated by two (red) arrows in Fig. 2 labeled by “World I” and “World II,” respectively.

What is wrong, then, with the picture that the universe is a closed quantum mechanical system, with the history repeating with the period of order $t_*$? The problem is that a fluctuation, which for internal observers appears as two "regular worlds" related by $CPT$, generically has "initial conditions" (i.e. $|\Psi'_{S_{\text{low}}}\rangle$ and $|\tilde{\Psi}'_{S_{\text{low}}}\rangle$ in Eq. (56)) that are not expected to be obtained by evolving the system from a state having a smaller coarse-grained entropy. Indeed, according to standard equilibrium thermodynamics, the probability distribution of the “initial conditions” should follow $\propto e^{-F/T}$, where $F = M_{\text{fluc}} - T S_{\text{low}}$ is the free energy associated with the “initial configurations” (i.e. the configurations at the bottom of entropy dips). This implies that if we consider processes...
that involve any “perceptions,” they will be overwhelmingly dominated by those observing random, irregular worlds, as opposed to a regular world obeying consistent laws of physics (because such random perceptions/consciousnesses arise much more easily from fluctuations; see Fig. 2 for a schematic depiction), contradicting what we actually observe (assuming, of course, that we are “typical” perceptions). This is nothing but the well-known Boltzmann brain problem [25] as applied to a general closed system.

Hence a description of the universe consistent with our observation, i.e. item (iii) listed at the beginning of this section, is obtained only if the thermalization timescale of the system, $t_{\text{th}}$, is (much) larger than the timescale of interest $t$. Since $t_{\ast} \sim e^{S_{\text{th}}} \gg t_{\text{th}}$ and $S_{\text{th}} < \ln[\dim \mathcal{H}]$, where $\dim \mathcal{H}$ is the Hilbert space dimension of the entire system, this implies

$$t < t_{\text{th}} \ll t_{\ast} \sim e^{S_{\text{th}}} < \dim \mathcal{H}. \quad (57)$$

(Note that the unit of time does not matter in the last inequality because $e^{S_{\text{th}}}$ is double-exponentially large, with $S_{\text{th}}$ being macroscopic.) In particular, this implies that if we want to describe the entire history of the universe ($t \to \infty$), which we must do if quantum mechanical evolution is fundamentally unitary, then we need to take

$$\dim \mathcal{H} = \infty, \quad (58)$$

i.e. the Hilbert space describing the quantum universe must be infinitely large.

As we will see in Section 7, the condition of Eq. (58) is satisfied in the eternally inflating multiverse because the multiverse evolves asymptotically to a supersymmetric Minkowski (or singularity) world, which contains an infinite number of states. This completes an ultimate picture for quantum measurement—a quantum measurement is a process in which a coherence existing in a (microscopic) system is dissipated into larger systems, ultimately into states in a supersymmetric Minkowski (or singularity) world. During this process, classical information about the measurement result (e.g. $x$ of the double-slit experiment) is amplified in each component of the multiverse state. Because of an infinitely large coarse-grained entropy of Minkowski (and singularity) space, recoherence of macroscopically different worlds does not occur. The ordered, classical world we see is a consequence of the fact that the universe (multiverse) is in an infinite-dimensional representation of the time evolution operator. The evolution itself, however, still obeys the laws of quantum mechanics; in particular, it is deterministic and unitary.

5 Spacetime Locality in Theories with Gravity

We have seen that spacetime locality plays a crucial role in quantum measurement processes. In theories with gravity, however, this property—i.e. that the time evolution operator takes a special form of Eq. (25)—is not automatically guaranteed. In particular, if we take wrong hypersurfaces to
quantize the system, theories are not local *even at distances much larger than the quantum gravity scale*. Historically, this issue appeared first in the study of black holes, but it is much more general and provides an important constraint on how to define quantum states in theories with gravity.

In the rest of the paper, we derive the structure of the Hilbert space describing the entire quantum universe, starting with the well-known discussion on quantum mechanics of black holes. A key ingredient is to fix all the redundancies and to eliminate all the overcountings of a general relativistic description of nature. In this broader context, black hole complementarity arises as a part of the general transformation of the Hilbert space associated with a change of the “reference frame.” This transformation can be regarded as an extension of the Lorentz/Poincaré transformation in the quantum gravitational context. With this Hilbert space, the probabilities are defined following the argument presented in previous sections, with an important extension to make physics invariant under time reparameterization (as required in theories with gravity).

The basic results presented in the following, such as the structure of the Hilbert space and the probability formula, were obtained mostly in Ref. [5]. There are, however, some important refinements here, including the treatment of spacetime singularities and a useful probability formula that applies in many practical cases. We also provide a clearer argument leading to the results, and discuss their meaning especially in the context of physical measurement processes. As emphasized in Ref. [5], these measurements can be either regarding global properties of the universe or outcomes of particular experiments. This, therefore, provides complete unification of the eternally inflating multiverse and the many worlds interpretation of quantum mechanics.

5.1 Black hole complementarity

Here we review black hole complementarity [7 26], which we assume gives the correct description of black hole physics. We focus particularly on aspects relevant to our later discussions.

Consider a traveler falling into a black hole, carrying some information. From the point of view of a distant observer, this traveler is absorbed into the horizon, which has an extremely high local temperature. (The temperature of radiation arriving at the distant observer is enormously redshifted, and is given by the standard formula $1/8\pi G_N M_{\text{BH}}$.) Assuming that quantum mechanics is valid, the original information carried by the traveler must be stored at the horizon, which will eventually come out as Hawking radiation. In the limit that the black hole is very large, i.e. a static black hole, this implies that the evolution of the system is unitary on the Hilbert space

$$\mathcal{H}_{\text{BH}}^{(\text{distant})} = \mathcal{H}_{\text{horizon}} \otimes \mathcal{H}_{\text{outside}},$$

where $\mathcal{H}_{\text{horizon}}$ and $\mathcal{H}_{\text{outside}}$ represent the Hilbert space factors associated with the degrees of freedom on and outside the horizon, respectively. (Strictly speaking, the horizon here means the stretched horizon, which is $\sim l_s$ away from the mathematical horizon, where $l_s$ is the string length.)
On the other hand, from the point of view of the falling traveler, there is nothing special about the horizon, and physics is described in the Hilbert space

$$\mathcal{H}_{\text{BH}}^{(\text{falling})} = \mathcal{H}_{\text{inside}} \otimes \mathcal{H}_{\text{outside}},$$  \hspace{1cm} (60)

where $\mathcal{H}_{\text{inside}}$ is the Hilbert space associated with the degrees of freedom inside the horizon. Again, assuming the validity of quantum mechanics, the evolution of the system is unitary on the above Hilbert space (until the singularity is hit).

Now, let us consider the fate of the information originally carried by the traveler. From the distant observer’s viewpoint, elements of Eq. (59) will be mapped, after the back hole evaporates, into those of the Hilbert space associated with spacetime without the black hole:

$$\mathcal{H}_{\text{horizon}} \otimes \mathcal{H}_{\text{outside}} \mapsto \mathcal{H}_{\text{after evaporation}}.$$  \hspace{1cm} (61)

(A more complete treatment of the Hilbert space for dynamically evolving spacetime will be given in Section 6.) The information is then first in $\mathcal{H}_{\text{horizon}}$, and later in $\mathcal{H}_{\text{after evaporation}}$ as subtle quantum correlations in Hawking radiation. From the falling traveler’s viewpoint, on the other hand, this information is in $\mathcal{H}_{\text{inside}}$. A problem arises when we mix these two viewpoints in the global spacetime picture. In this picture, we can draw spacelike hypersurfaces—often called nice slices—on which the information exists both in Hawking radiation and inside spacetime. From a general relativistic point of view, there is nothing wrong with defining states on such hypersurfaces. This, however, leads to contradiction with the laws of quantum mechanics, specifically the no-cloning theorem [9], which says that quantum information cannot be faithfully duplicated.

Black hole complementarity asserts that the problem arises because we have taken the global viewpoint that does not have any operational meaning. Indeed, because of the existence of the horizon, no physical observer can obtain the same information from inside region and Hawking radiation [27]. This implies that if quantum mechanics is defined on equal-time hypersurfaces that pass through both the inside and outside information, e.g. on nice slices, then the low energy theory (not just states) must be non-local in such a way that these spatially separated degrees of freedom are not independent. (For suggestive calculations in string theory consistent with this picture, see Refs. [28, 29].) Alternatively, if we want to keep locality in our low energy description of nature (which we do), then the Hilbert space should be restricted to the one associated with appropriate spacetime regions, e.g. Eq. (59) or (60). Namely, including both the inside spacetime region and Hawking radiation in a single description is overcounting—they cannot coexist as independent degrees of freedom in a single component of a quantum state.

### 5.2 Quantum states are defined in restricted spacetime regions

How should we then define quantum states without sacrificing locality of the theory at distances larger than the quantum gravity scale? In Ref. [5], it is proposed that:
The states are defined on the past light cone bounded by the (stretched) apparent horizon. Here, the apparent horizon is defined as a surface on which at least one pair of orthogonal null congruences have zero expansion; in particular, the local expansion of past directed light rays emitted from the tip to form the past light cone turns from positive to negative there. The degrees of freedom exist both inside and on the horizon, and the system is described as viewed from a local Lorentz frame at the tip of the light cone.

The definition above provides the simplest way of avoiding the overcounting of the type described in Section 5.1, making the time evolution operator local at distances larger than the quantum gravity scale. The evolution of a quantum state is deterministic and unitary in this Hilbert space (until spacetime singularities are hit; see Section 7.1).

The use of the apparent horizon in the above definition implies that we are selecting a “reference frame” from which we describe physics—indeed, the light cone by definition must be associated with some point in spacetime. What does that mean? We will address this question in the next section, where we will also discuss the evolution of a quantum state in general dynamical spacetime. Here we simply focus on the issue of unitarity in fixed background geometries.

Let us consider de Sitter space. Following the above definition, the states on this background are given on the past light cone of some fixed center $p$. The structure of the Hilbert space, of which these states are elements, is thus

$$\mathcal{H}_{dS} = \mathcal{H}_{dS,\text{horizon}} \otimes \mathcal{H}_{dS,\text{inside}},$$

(62)

where $\mathcal{H}_{dS,\text{horizon}}$ and $\mathcal{H}_{dS,\text{inside}}$ are the Hilbert space factors associated with the degrees of freedom on and inside the stretched horizon, respectively. On a fixed de Sitter background, the evolution of a state is unitary in the space of Eq. (62). In particular, information leaving the horizon of $p$ in the global spacetime picture is regarded as being stored in $\mathcal{H}_{dS,\text{horizon}}$, which can later be sent back to $\mathcal{H}_{dS,\text{inside}}$ in the form of subtle quantum correlations in Hawking radiation. Note that, in this description, a physical observer need not be at $p$, which simply plays the role of “the origin of the coordinates.” Indeed, since the information in Hawking radiation spreads non-locally over the space, gathering it requires physical processes at locations other than the origin (and therefore, these processes feel local temperatures higher than the Gibbons-Hawking temperature $H/2\pi$, where $H$ is the Hubble parameter).

A nontrivial consistency check of this picture was given in Appendix C of Ref. [5], which we briefly reproduce here. Let us consider a semi-classical geometry in which a Minkowski bubble

---

11Defining states on the past light cone is not absolutely necessary, although it provides the simplest formulation of the framework. An alternative possibility is to use spacelike hypersurfaces foliating the causal patch and define states on them in the region inside and on the (stretched) apparent horizon. Because the numbers of degrees of freedom on the past light cone and on a spacelike hypersurface bounded by the same horizon are the same in our setup (due to the spacelike projection theorem [30]), the Hilbert space dimensions in both cases are identical.
forms in the future of a meta-stable de Sitter vacuum. We assume that the reference point \( p \) enters into the Minkowski bubble at a time \( t_{\text{nucl}} \) in the flat coordinates of the de Sitter space. Since the information retrieval time from the de Sitter horizon is \( t_{\text{ret}} \simeq H^{-1} \ln(H^{-1}/l_P) \), where \( l_P \) is the Planck length, we need to have

\[
t_{\text{nucl}} \gtrsim t_{\text{ret}} \simeq \frac{1}{H} \ln \left( \frac{1}{l_P H} \right),
\]

in order to retrieve any information from Hawking radiation that was “carried away” by a traveler who left the horizon at time \( t = 0 \). On the other hand, we may consider a process in which an observer (who sits within \( p \)’s horizon) obtains information directly from the traveler after he/she enters the Minkowski bubble, by receiving some signal; see Fig. 3. It turns out that this succeeds only if the bubble nucleation occurs early enough:

\[
t_{\text{nucl}} \lesssim \frac{1}{H} \ln \left( \frac{1}{l_P H} \right),
\]

given that the traveler needs a Planck time to send a bit of information. We find that Eq. (63) and Eq. (64) are (barely) incompatible—no physical observer can receive duplicate quantum information from Hawking radiation and from the direct signal. Note that a physical observer can obtain information from Hawking radiation only before entering the Minkowski bubble, since the information is spread non-locally throughout the de Sitter space. Although some of the Hawking radiation quanta originated in the parent de Sitter vacuum could reach the Minkowski bubble across the bubble wall, this would not be enough to transmit the information to the observer.

The examples considered above clearly demonstrate that the restriction of spacetime regions is crucial to keep locality of the low energy theory while being consistent with quantum mechanics. Assuming that information absorbed into the de Sitter horizon can be retrieved later (which is necessary for stable de Sitter space to be regarded as a closed system, as suggested by the holographic principle), quantum states cannot be defined on hypersurfaces that pass through both Hawking radiation and outside spacetime containing the same information. (Such a description would require a low energy theory to be non-local at distances larger than the quantum gravity scale.) This situation is analogous to that in the black hole case. The only difference is that the de Sitter horizon is “observer dependent”: its location changes depending on from whose point of view the system is described.

In Ref. [5], the definition of quantum states considered above was stated as: the system is described from the viewpoint of a single “observer” (geodesic). Here we phrase the same thing as: physics should be described using a single reference frame. This might capture the essential physics better. We have seen that choosing a reference frame involves a restriction of spacetime regions in which quantum states are defined. In the next section, we see the procedure of defining states in more detail from this perspective.
Figure 3: In the situation where a Minkowski bubble forms in a meta-stable de Sitter vacuum, there are two possible ways to retrieve information carried away from the horizon of $p$ by some traveler: from Hawking radiation and from a direct signal. The conditions for successful information retrieval from these sources, however, are mutually incompatible, prohibiting faithful duplication of quantum information.

6 Importance of Fixing a Reference Frame

In this section we determine the structure of the Hilbert space for dynamical spacetime, Eq. (2). (The Hilbert space for full quantum gravity, including spacetime singularities, will be discussed in the next section.) We emphasize the importance of fixing a reference frame. We also consider the effect of changing the reference frame, and see that it leads to a transformation in the Hilbert space that can be viewed as an extension of the Lorentz/Poincaré transformation in the context of quantum gravity.

6.1 Fixing a gauge—physics should be described in a single reference frame

What are observables in physical theories? They should be “gauge invariant,” i.e. quantities that do not depend on arbitrary parameterizations of the system corresponding to the redundancy of the description. In theories with gravity, the coordinatization of spacetime is precisely one such parameterization, so it might be thought that only observables are certain global quantities, e.g. the ones associated with the topology of spacetime. This is not true—causal relations among events are invariant under general coordinate transformations, and thus are physically observable.

It is well known that to do Hamiltonian quantum mechanics, all the gauge redundancies must
be fixed. A theory of gravity has huge redundancies associated with general coordinate transformations: general covariance. Indeed, without gauge fixing, the Hamiltonian one would obtain is zero, reflecting invariance under local time translations. The definition of the states described in the previous section provides a simple way to fix these redundancies and extract causal relations among the events. The origin of a special point \( p \) (i.e. the tip of the past light cone used to define states) is now clear—it arises from the fact that the theory is invariant under local spatial translations and that we need to fix the resulting redundancies. By choosing a local Lorentz frame with the origin at \( p \), all the redundancies associated with \( p \) are fixed. While this prescription by itself does not completely determine the gauge for general covariance, fixing the residual ones, coming from coordinate transformations on the past light cone, is simple conceptually and gives only minor effects on the overall picture.

Together with the restriction of spacetime within the (stretched) apparent horizons discussed in Section 5.2, this comprises the statement in Ref. [5] that “physics is described from the viewpoint of a single observer.” The choice of the local Lorentz frame at \( p \) implies that the tip of the light cone follows a geodesic at the semi-classical level. The overcounting of the type encountered before does not arise, and the time evolution operator is local at large distances. Here we summarize all these as: physics should be described in a single reference frame if we want to keep locality of the theory at distances larger than the quantum gravity scale.

### 6.2 Hilbert space for dynamical spacetime—analogy with Fock space

We now construct the Hilbert space for dynamical spacetime, following the discussion so far. To do so, it is instructive to draw a close analogy with the construction of the Hilbert space in usual (non-gravitational, non-conformal) quantum field theory.

Consider a non-gravitational quantum field theory in which asymptotic states contain a single species of particles described by creation/annihilation operators \( a_{p,s}^\dagger, a_{p,s} \), where \( p \) and \( s \) are the momentum and spin indices, respectively. (An extension to the case of multiple species is straightforward.) The Hilbert space of the theory is then (isomorphic to) the Fock space

\[
\mathcal{H} = \bigoplus_{n=0}^\infty \mathcal{H}_{1P}^\otimes n,
\]

where \( \mathcal{H}_{1P}^\otimes n \) represents the \( n \)-particle Hilbert space given by

\[
\mathcal{H}_{1P}^{\otimes 0} = |0\rangle, \quad \mathcal{H}_{1P}^{\otimes 1} = \{ a_{p,s}^\dagger \, |0\rangle \}, \quad \mathcal{H}_{1P}^{\otimes 2} = \{ a_{p_1,s_1}^\dagger a_{p_2,s_2}^\dagger \, |0\rangle \}, \quad \ldots \ .
\]

---

\(^{12}\)Strictly speaking, there are apparent non-localities associated with gauge fixing and with the fact that we quantize the system on null hypersurfaces. These non-localities, however, are different from the “real” non-locality discussed in Section 5. For example, the former disappears in physical quantities, and the latter is associated with propagation of massless particles, a process that is local in spacetime (and so it does not arise if we instead use spacelike hypersurfaces; see footnote 11).
With generic interactions, a state in $\mathcal{H}$ in Eq. (65) evolves across different components $\mathcal{H}_{1P}^\otimes n$, i.e., the time evolution operator allows for a process changing the particle number. For example, in the Standard Model, collision of an electron and a positron having well-defined momenta/spins at $t = -\infty$ leads to

$$\Psi(t = -\infty) = |e^+e^-\rangle \rightarrow \Psi(t = +\infty) = c_e|e^+e^-\rangle + c_\mu|\mu^+\mu^-\rangle + \cdots,$$

(67)

where we have expanded the state $\Psi(t)$ in terms of the Fock-space states, which is appropriate for $t \to \pm\infty$ when interactions are weak. (We have suppressed momentum and spin indices.) Note that Eq. (67) should not be interpreted that the initial $e^+e^-$ state evolves probabilistically into different states $|e^+e^-\rangle$, $|\mu^+\mu^-\rangle$, and so on. According to the laws of quantum mechanics, the evolution of the state $\Psi(t)$ is deterministic—it simply evolves into a unique state $\Psi(t = +\infty)$ which contains components $|e^+e^-\rangle$, $|\mu^+\mu^-\rangle$, $\cdots$ when expanded in Fock-space states.

The situation in quantum gravity is analogous. We first need to fix the Hilbert space basis to discuss states unambiguously. We assume that, with a fixed local Lorentz frame associated with a fixed reference point $p$, we have a set of local operators at low energies; specifically, we have a set of quantum fields $\phi_i(x)$ defined on the past light cone of $p$. (These fields do not depend on time as we take the Schrödinger picture.) This can provide “meaning” to the states according to the responses to these field operators, and we can now construct states using the language of, e.g., spacetime points, which are already fixed by the operator algebra (see the discussion in Section 3.2).

Let us consider Hilbert space $\mathcal{H}_M$ corresponding to a set of fixed semi-classical geometries $\mathcal{M} = \{\mathcal{M}_i\}$, which are defined on the past light cone of $p$ and have the same (stretched) apparent horizon $\partial \mathcal{M}$ (in the sense that they lead to the same internal geometry of $\partial \mathcal{M}$). Note that if $\partial \mathcal{M}$ is $d - 2$ dimensional, the corresponding $\mathcal{M}_i$’s (which are then $d - 1$ dimensional) represent $d$ dimensional spacetime. As discussed before, the states are defined on the past light cone bounded by the horizon; specifically, the states on these geometries form Hilbert space

$$\mathcal{H}_M = \mathcal{H}_{M,\text{bulk}} \otimes \mathcal{H}_{M,\text{horizon}},$$

(68)

where $\mathcal{H}_{M,\text{bulk}}$ and $\mathcal{H}_{M,\text{horizon}}$ represent Hilbert space factors associated with the degrees of freedom inside and on $\partial \mathcal{M}$, respectively. The product space structure is dictated by locality. What do we know about $\mathcal{H}_{M,\text{bulk}}$ and $\mathcal{H}_{M,\text{horizon}}$? The covariant entropy bound [30] suggests that the dimension of $\mathcal{H}_{M,\text{bulk}}$ is $\exp(A_{\partial M}/4)$, where $A_{\partial M}$ is the area of the horizon $\partial \mathcal{M}$ in Planck units, and the standard horizon entropy implies that the dimension of $\mathcal{H}_{M,\text{horizon}}$ is the same, so

$$\dim \mathcal{H}_M = \dim \mathcal{H}_{M,\text{bulk}} \times \dim \mathcal{H}_{M,\text{horizon}} = \exp \left( \frac{A_{\partial M}}{2} \right).$$

(69)

The fact that the maximum number of degrees of freedom (i.e. the logarithm of the dimension of the Hilbert space) scales with the area, rather than the volume, is a manifestation of the holographic principle.
Analogously to the case of non-gravitational quantum field theory, Eq. \((65)\), the full Hilbert space of dynamical spacetime is (isomorphic to) the direct sum of the Hilbert spaces for different \(\mathcal{M}'s:\)

\[
\mathcal{H} = \bigoplus_{\mathcal{M}} \mathcal{H}_{\mathcal{M}},
\]

so that

\[
\dim \mathcal{H} = \sum_{\mathcal{M}} \dim \mathcal{H}_{\mathcal{M}} = \sum_{\mathcal{M}} \exp \left( \frac{A_{\partial \mathcal{M}}}{2} \right).
\]

Since \(\dim \mathcal{H}_{\mathcal{M}}\) are integers, \(A_{\partial \mathcal{M}}\) are quantized. This suggests that \(\sum_{\mathcal{M}}\) in the above equation may indeed be a discrete sum in the full quantum theory, although the argument itself does not prohibit a continuous degeneracy for a fixed \(A_{\partial \mathcal{M}}\). Note that the dimension in Eq. \((71)\) includes that of “matter” degrees of freedom, i.e. excitations associated with the quantum fields \(\phi_i(x)\).

We emphasize that defining the states in \(\mathcal{H}_{\mathcal{M},\text{bulk}}\) need not require a fixed background space \(a \text{ priori}\); rather, in a more complete framework, spacetime interpretation of the states would arise as a result of the algebra of quantum field operators, \(\phi_i(x)\), and the responses of the states to these operators. There is a good feature in our framework in pursuing such a construction: since our states represent regions within the apparent horizon, on which the local expansion of light rays forming the past light cone of \(p\) turns from positive to negative, the cross sectional area for a set of past directed light rays emanating from \(p\) always increases as they move away from \(p\). This implies that these light rays do not form caustics, allowing for an unambiguous coordinatization. More work, however, is needed to make this construction more precise.

The general evolution of a state in dynamical spacetime is unitary in the full Hilbert space \(\mathcal{H}\) in Eq. \((70)\), but not in each \(\mathcal{H}_{\mathcal{M}}\). The evolution, therefore, generically leads to the multiverse (or quantum many worlds) picture:

\[
\Psi(t = t_0) = |\Sigma\rangle \rightarrow \Psi(t) = \sum_i c_i(t) |(\text{cosmic configuration } i)\rangle,
\]

where \(|\Sigma\rangle\) is an initial state at \(t = t_0\), e.g. an eternally inflating (metastable de Sitter) state in \(\mathcal{H}_{\mathcal{M}=\text{dS}}\), while the sum in the final state \(\Psi(t)\) runs over states in different \(\mathcal{H}_{\mathcal{M}}\), giving a superposition of macroscopically different worlds (universes). Quantum field theory on a fixed gravitational background corresponds to a very special case in which transitions between (some of the) \(\mathcal{H}_{\mathcal{M}}\) are prohibited. This is analogous to the nonrelativistic limit of usual quantum field theory, in which transitions between different \(\mathcal{H}_{\text{ip}}\) do not occur. The underlying dynamics for gravity, however, is much more general—the time evolution operator allows for “hopping” between different components \(\mathcal{H}_{\mathcal{M}}\) in Eq. \((70)\). The semi-classical evolution in which the area of the apparent horizon changes is precisely a succession of such processes.
6.3 “Reference frame dependence” of the concept of spacetime

What happens if we change the reference frame, e.g. by a spatial translation or boost? As in any symmetry transformation, this operation must be represented by a unitary transformation in Hilbert space. In particular, if we focus on histories before any component of the state hitting spacetime singularities (the effect of which will be discussed in Section 7.1), then it must be represented entirely in the Hilbert space $\mathcal{H}$ in Eq. (70), but not necessarily in each component $\mathcal{H}_M$. Namely, the transformation can mix elements in different $\mathcal{H}_M$. Moreover, even if the transformation maps all the elements in $\mathcal{H}_M$ onto themselves for some $M$, there is no reason that it should not mix the degrees of freedom associated with $\mathcal{H}_M$, bulk and $\mathcal{H}_M$, horizon.

Let us consider a state $|\Psi(t)\rangle$ that represents the entire quantum universe, which we call the multiverse state. Suppose that at some time $t$, the multiverse state is expanded in terms of the locality basis states $|\kappa_m\rangle$ that are elements of $\mathcal{H}$ in Eq. (70):

$$|\Psi(t)\rangle = \sum_{m=1}^{\dim \mathcal{H}} c_m(t) |\kappa_m\rangle,$$

(73)

where $\langle \kappa_m | \kappa_n \rangle = \delta_{mn}$. The parameter $t$ here is introduced to describe the evolution of $|\Psi(t)\rangle$: $|\Psi(t_1)\rangle = e^{-iH(t_1-t_2)} |\Psi(t_2)\rangle$, where $H$ is the time evolution operator (i.e. gauge-fixed Hamiltonian) for the entire quantum universe. Since the introduction of $t$ is arbitrary, the final physical predictions should not depend on this particular parameterization. (This is ensured by general covariance, which includes invariance under time reparameterization; see Section 8.1 for more detail.) A useful choice for $t$ is the proper time at $p$, although any other monotonic parameterization works as well at the cost of (potentially) making the explicit form of $H$ complicated.

In $d$ spacetime dimensions, a change of the reference frame can be specified by $d(d+1)/2$ parameters $\{r_i, \eta_i, \theta_{[ij]}, t\}$: $d - 1$ spatial translations $r_i$, $d - 1$ boosts $\eta_i$, and $(d - 1)(d - 2)/2$ rotations $\theta_{[ij]}$, performed at time $t$, where $i = 1, \cdots, d - 1$. These correspond to the freedom in electing a local inertial frame in spacetime, from which we view the system. Let us consider the Hilbert (sub)space corresponding to de Sitter space: Eq. (62) where $\mathcal{H}_{dS, \text{inside}} \equiv \mathcal{H}_{M=dS, \text{bulk}}$ and $\mathcal{H}_{dS, \text{horizon}} \equiv \mathcal{H}_{M=dS, \text{horizon}}$. Consider a state in this space represented as

$$|\psi\rangle = |\text{excited}\rangle_{\text{inside}} \otimes |0\rangle_{\text{horizon}}$$

(74)

at time $t$, where $|\text{excited}\rangle_{\text{inside}}$ implies that there is an object within the horizon, and $|0\rangle_{\text{horizon}}$ is (one of) the ground state(s) for the horizon degrees of freedom. If we change the reference frame by performing a large spatial translation (larger than the Hubble radius) at this time, then the state transforms as

$$|\psi\rangle \xrightarrow{\text{translation} \atop T \text{ at } t} U_T |\psi\rangle = |0\rangle_{\text{inside}} \otimes |\text{excited}\rangle_{\text{horizon}},$$

(75)
where $|0\rangle_{\text{inside}}$ represents (one of) the vacuum state(s) within the horizon, and $|\text{excited}\rangle_{\text{horizon}}$ an excited state for the horizon degrees of freedom. This provides a simple example in which degrees of freedom in the bulk and on the horizon are mixed under a change of the reference frame.

A more drastic situation may occur when there is a black hole. Consider a state $|\phi\rangle$ in $\mathcal{H}_M$ at time $t$, where $\mathcal{M}$ is a spacetime containing a black hole but with the reference point $p$ staying outside the horizon, i.e. $\mathcal{H}_M \subset \mathcal{H}_{\text{BH}}^{(\text{distant})}$ in Eq. (59):

$$|\phi\rangle \in \mathcal{H}_{\text{BH}}^{(\text{distant})}. \quad (76)$$

Now, suppose we evolve the state $|\phi\rangle$ back in time to an early time $t_0$, perform a boost so that $p$ crosses the horizon at some time between $t_0$ and $t$, and then evolve $|\phi\rangle$ forward in time to the same $t$. Under this reference frame change, $|\phi\rangle$ is transformed into a state that is not in $\mathcal{H}_{\text{BH}}^{(\text{distant})}$ but in $\mathcal{H}_{\text{BH}}^{(\text{falling})}$ in Eq. (60):

$$|\phi\rangle \overset{\text{boost } B \text{ at } t_0}{\longrightarrow} U_B |\phi\rangle \in \mathcal{H}_{\text{BH}}^{(\text{falling})}, \quad (77)$$

so that the degrees of freedom associated with the horizon, $\mathcal{H}_{\text{horizon}}$, are mapped into those with the inside spacetime, $\mathcal{H}_{\text{inside}}$, which did not exist in the $\mathcal{H}_M$ containing $|\phi\rangle$ before the transformation. This mapping should be one-to-one if $U_B$ is unitary, which is possible because the holographic principle ensures $\dim \mathcal{H}_{\text{horizon}} = \dim \mathcal{H}_{\text{inside}}$. (Here, we have assumed that the time $t$ is before $p$ hits the singularity in the boosted frame.) Note that both $\mathcal{H}_{\text{BH}}^{(\text{distant})}$ and $\mathcal{H}_{\text{BH}}^{(\text{falling})}$ are contained in the full Hilbert space $\mathcal{H}$ as separate components:

$$\mathcal{H} = \cdots \oplus \mathcal{H}_{\text{BH}}^{(\text{distant})} \oplus \mathcal{H}_{\text{BH}}^{(\text{falling})} \oplus \cdots, \quad (78)$$

since $\mathcal{H}$ contains all the possible semi-classical geometries as viewed from a local Lorentz frame of $p$. Thus the transformation at $t$ corresponding to the reference frame change $B$ at $t_0$ is represented within $\mathcal{H}$, as it should be.

The statement in Eq. (77) is nothing but black hole complementarity. This, therefore, leads to the following picture. 

**Black hole complementarity (or more generally, horizon complementarity) arises because changes of the reference frame are represented in the Hilbert space of Eq. (70), which contains components $\mathcal{H}_M$ that are defined only in restricted spacetime regions because of the existence of horizons.** These changes in general transform degrees of freedom associated with spacetime to those with a horizon, or vice versa—the concept of spacetime depends on the reference frame.

The transformation discussed here can be viewed as an extension of the Lorentz/Poincaré transformation in the quantum gravitational context, as the Lorentz transformation is viewed as an extension of the Galilean transformation. For a given $t$, the transformation here is specified by $(d-1)(d+2)/2$ parameters: $r_i, \eta_i, \theta_{ij}$. In the limit $G_N \to 0$, where relative accelerations between
all families of geodesics vanish, the transformation is reduced to an element of the \((d - 1)(d + 2)/2\)
parameter subset of standard Poincaré transformations consisting of spatial translations, rotations
and boosts. Time translation also arises from invariance under a shift of the origin of \(t\) in the proper
time parameterization. The set of these transformations, therefore, is reduced to the standard
Poincaré transformations in the limit \(G_N \to 0\).

This is very much analogous to the fact that the standard Lorentz transformation is reduced
to the Galilean transformation in the limit \(c \to \infty\), where \(c\) is the speed of light. In the Galilean
transformation a change of the reference frame leads only to a constant shift of all the velocities,
while in the Lorentz transformation it also alters temporal and spatial lengths (time dilation and
Lorentz contraction) and makes the concept of simultaneity relative. With gravity, a change of the
reference frame makes even the concept of spacetime relative. The trend is consistent—as we “turn
on” fundamental constants in nature (\(c = \infty \to \text{finite}\) and \(G_N = 0 \to \text{finite}\)), physics becomes
more and more “relative,” i.e. the description of the same physical system from different reference
frames differ more. The transformations described here (together with time translation) provide
the extension of the Galilean group with \(c, G_N\), and \(\hbar\) all finite.

7 Hilbert Space for Quantum Gravity

Here we discuss the full Hilbert space for quantum gravity. We argue that it contains an infinite
number of “intrinsically quantum mechanical” states associated with spacetime singularities, which
do not admit any classical interpretation. The evolution of the multiverse state is unitary in this
full Hilbert space, and an arbitrary change of the reference frame is represented as a unitary
transformation in that space. The ultimate fate of the multiverse state is a superposition of
supersymmetric Minkowski and singularity states, composing the “heat death of the multiverse.”
This implies that the Hilbert space dimension of the quantum universe is infinite, explaining the
fact that we observe an ordered world obeying consistent laws of physics.

7.1 Meaning of spacetime singularities

What happens if a component of Eq. \((73)\) hits a spacetime singularity at some time \(t_s\)? We
conjecture that it should be dropped from physical considerations after time \(t_s\). This is motivated
by the following independent arguments \([5]\):

- The covariant entropy bound does not count the degrees of freedom that have hit a singularity.
  Imagine sending a light sheet inwards from a black hole horizon \(H\). The degrees of freedom
  swiped by the light sheet are then bounded by the area of the horizon \(A_H/4\). The entropy
  bound, however, does not limit the amount of information that hits the singularity before
being swiped by the light sheet. The simplest interpretation of this fact is that degrees of freedom that hit the singularity disappear from the spacetime.

- **Super-Planckian physics does not have degrees of freedom as suggested by field theory.** Consider a black hole (or de Sitter) horizon. Because of the blueshift, the local temperature at the mathematical horizon formally diverges. In fact, the physical horizon, from which Hawking radiation arises, is a Planckian distance away from the mathematical horizon, as suggested by the fact that Bekenstein-Hawking (Gibbons-Hawking) entropy is saturated by thermal entropy outside (inside) this “stretched” horizon. This suggests that super-Planckian physics does not have degrees of freedom as indicated by field theory. And spacetime singularities are precisely the regions where the curvature is super-Planckian.

What does dropping from consideration mean? In Ref. [5], it was postulated that a component that hits a singularity at time $t_s$ is simply eliminated from the state $|\Psi(t)\rangle$ at that time, $t = t_s$. This, however, leads to the following paradoxical situations. What if $|\Psi(t)\rangle$ contains only components that hit singularities in the future? This must violate unitarity, and therefore so does any state containing them. Furthermore, even if we accepted unitarity violation, systems having only de Sitter and anti de Sitter vacua would allow for (a constant fraction of) $|\Psi(t)\rangle$ to stay in a de Sitter phase at $t \to \infty$, since components tunneled into the anti de Sitter vacuum keep disappearing. This is extremely counter-intuitive.

We are therefore led to the following picture. The components that hit spacetime singularities keep existing in $|\Psi(t)\rangle$, but they are no longer extracted by projection operators $O_A$ or $O_{A \cap B}$ appearing in the probability formula because these states cannot be interpreted as those associated with (semi-)classical spacetime. This implies that the full Hilbert space for quantum gravity must contain these “intrinsically quantum mechanical” states, associated with singularities, in addition to those discussed in Section 6.2:

$$H_{\text{QG}} = H \oplus H_{\text{sing}},$$  \hspace{1cm} (79)

where $H$ is the component that allows for spacetime interpretation, Eq. (70), while $H_{\text{sing}}$ is the one that does not. Note that this treatment of “dropping” is different from simply eliminating the relevant components. For example, a de Sitter phase can now disappear by tunneling purely into an anti de Sitter vacuum.

We assume that the evolution of the multiverse state $|\Psi(t)\rangle$ is unitary in $H_{\text{QG}}$, and that any change of the reference frame at an arbitrary time $t$ is represented by a unitary transformation in this Hilbert space. According to the current understanding of string theory, a full quantum gravitational theory possesses many de Sitter, anti de Sitter, and Minkowski vacua. In particular, it possesses exactly supersymmetric Minkowski vacua, which are absolutely stable due to the

---

13 This picture has been developed after discussions with Alan Guth. I thank him for his insightful suggestions.
positive energy theorem \[31\]. This implies that the dimension of $\mathcal{H}$ is infinite, since the Hilbert space dimension of (stable) Minkowski space is infinite:

$$\dim(\mathcal{H}_{\text{Minkowski}}) = \infty \implies \dim(\mathcal{H}) = \infty.$$ \hspace{1cm} (80)

What about $\mathcal{H}_{\text{sing}}$? Consider a set of states that hit singularities at some late time. In the eternally inflating multiverse, these states can be mapped into those that evolve into stable supersymmetric Minkowski states, by appropriate boost transformations. This suggests that

$$\dim(\mathcal{H}_{\text{sing}}) = \infty,$$ \hspace{1cm} (81)

which makes it possible that components that hit singularities do never return to states in $\mathcal{H}$, associated with spacetime. Namely, stable Minkowski and anti de Sitter vacua can act as “sinks” in the landscape, despite the fact that the evolution of the multiverse state is unitary (at least after the “birth”).

### 7.2 The heat death of the multiverse

What is the ultimate fate of the multiverse state? Starting from a generic eternally inflating state, the coefficients of any components in unstable vacua will eventually decay. In the string landscape, we expect that all the de Sitter as well as non-supersymmetric Minkowski vacua decay into some lower energy vacua, given that there is a huge number of possible decay channels. The multiverse state, therefore, will asymptotically become a superposition of supersymmetric Minkowski and singularity states:

$$|\Psi(t)\rangle \xrightarrow{t \to \infty} \sum_i a_i(t) |\text{supersymmetric Minkowski world } i\rangle + \sum_j b_j(t) |\text{singularity world } j\rangle,$$ \hspace{1cm} (82)

where the first sum runs over components with varying matter content, spatial dimensions, and the amount of supersymmetries, while the second sum contains terms associated with black hole and big crunch singularities. In fact, this is simply a consequence of the second law of thermodynamics, given that the Hilbert space dimensions of Minkowski and singularity worlds are infinite, Eqs. \[80\] \[81\]. In other words, the coarse-grained entropy of the multiverse diverges in the asymptotic future:

$$S_{\text{final}} \equiv S_{\text{multiverse}}(t \to \infty) = \infty.$$ \hspace{1cm} (83)

The initial state of the multiverse must be given by some theory external to the current framework. We do not address this issue further here, although some possibilities were discussed in Ref. \[5\]. We may naturally speculate that the coarse-grained entropy of the initial state is small, e.g. $S_{\text{init}} \sim O(1)$. This is, however, not required by observation \[32\].

35
In any event, the fact that $S_{\text{final}} = \infty$, i.e. the final state Hilbert space has an infinite dimensionality, plays a crucial role in explaining the basic fact that we perceive an ordered, regular world obeying the laws of physics, as discussed in Section 4.3. Indeed, the ultimate future of the multiverse is the “heat death,” as represented in Eq. (82).

8 Probabilities in the Quantum Multiverse

We now discuss the probability formula in the eternally inflating multiverse. This provides a device through which we can relate the “whole reality” in the multiverse state $|\Psi(t)\rangle$ with our own experience, i.e. what we—physical objects within $|\Psi(t)\rangle$—may observe. The resulting formula can be applied to answering questions both regarding global properties of the universe and outcomes of particular experiments; in particular, it reduces to the standard Born rule in a setup corresponding to a usual terrestrial experiment. This, therefore, provides complete unification of the two concepts: the eternally inflating multiverse and many worlds in quantum mechanics [5].

8.1 The (extended) Born rule

Having understood how the state $|\Psi(t)\rangle$ is defined, the probability formula can be obtained following the earlier discussions in Sections 2–4. An important new point here is that the “time” $t$ in quantum gravity is simply an auxiliary parameter introduced to describe the “evolution” of the state, exactly like a variable $t$ used in a parametric representation of a curve on a plane, $(x(t), y(t))$. The physical information is only in correlations between events, like correlations between $x$ and $y$ in the case of a curve on a plane [33]. Specifically, time evolution of a physical quantity $X$ is nothing more than a correlation between $X$ and a quantity that can play the role of time, such as the location of the hands of a clock or the average temperature of the cosmic microwave background in our universe.

Any physical question can then be phrased as: given condition $A$ we specify, what is the probability for an event $B$ to occur? For example, one can specify a certain “premeasurement” situation $A_{\text{pre}}$ (e.g. the configuration of an experimental apparatus and the state of an experimenter before measurement) as well as a “postmeasurement” situation $A_{\text{post}}$ (e.g. those after the measurement but without specifying outcome) as $A = \{ A_{\text{pre}}, A_{\text{post}} \}$, and then ask the probability of a particular result $B$ (specified, e.g., by a physical configuration of the pointer of the apparatus in $A_{\text{post}}$) to be obtained. The information about real, physical time is included in conditions $A$ and $B$ through specifications of any non-static observables. The relevant probability $P(B|A)$ is then given by

$$
P(B|A) = \frac{\iiint dt_1 dt_2 \langle \Psi(0) | U(0,t_1) O_{A_{\text{pre}}} U(t_1,t_2) O_{A_{\text{post}} \cap B} U(t_2,t_1) O_{A_{\text{pre}}} U(t_1,0) | \Psi(0) \rangle}{\iiint dt_1 dt_2 \langle \Psi(0) | U(0,t_1) O_{A_{\text{pre}}} U(t_1,t_2) O_{A_{\text{post}}} U(t_2,t_1) O_{A_{\text{pre}}} U(t_1,0) | \Psi(0) \rangle}.
$$

(84)
Here, \( U(t_1, t_2) = e^{-iH(t_1-t_2)} \) is the “time evolution” operator with \( H \) being the Hamiltonian for the entire system, and \( O_X \) is the operator projecting onto states consistent with condition \( X \):

\[
O_X = \sum_{i \in X} |\alpha_i\rangle \langle \alpha_i|,
\]

where \( |\alpha_i\rangle \) are the classical states discussed in Section 3.33 and \( i \in X \) implies that the sum is taken for the configurations that satisfy condition \( X \).

Note that since we have already fixed a reference frame, conditions \( A_{\text{pre}} \) and \( A_{\text{post}} \) in general must involve specifications of ranges of location and velocity for physical objects with respect to the origin of the coordinates \( p \) (in addition to those of physical times made through configurations of non-static objects, e.g. the hands of a clock or the status of an experimenter). This is important for the uniqueness of the framework, eliminating the ambiguity associated with how these objects must be counted. Of course, there is still a freedom in specifying in what state the objects must be in \( A_{\text{pre}} \); for example, we could put them at \( p \) or some other point at rest, or could specify a phase space region in which they must be. But this is the freedom of questions one may ask, and not that of the framework itself. (And the final answer does not depend on the location/velocity of reference point \( p \), i.e. the overall relative location/velocity between \( p \) and the specified configurations in \( A_{\text{pre}} \) and \( A_{\text{post}} \), if the multiverse state \( |\Psi(t)\rangle \) as a whole is invariant under the corresponding reference frame changes.)

Equation (84) is our final formula for the probabilities. The integrations over “time” \( t \) are taken from \( t = t_0 \), where the initial condition for \( |\Psi(t)\rangle \) is specified, to \( t = \infty \), which arise because conditions \( A_{\text{pre}} \) and \( A_{\text{post}} \) may be satisfied at any values of \( t \) (denoted by \( t_1 \) and \( t_2 \) in the equation). This, together with appropriate transformations of \( H \), ensures that \( P(B|A) \) is invariant under reparameterization of \( t \), as required by general covariance. If conditions \( A_{\text{pre}} \) and \( A_{\text{post}} \) are about configurations at the same “time,” i.e. if we ask the question like “given what we know about our past light cone, \( A \), what is the probability of that same light cone to have properties \( B \) as well?,” then

\[
O_{A_{\text{pre}}} U(t_1, t_2) O_{A_{\text{post}}} U(t_2, t_3) O_{A_{\text{pre}}} \xrightarrow{t_1=t_2} O_{A_{\text{pre}}} O_{A_{\text{post}}} O_{A_{\text{pre}}} = O_A,
\]

so that Eq. (84) is reduced to the formula of Eq. (1). (One can see this explicitly by inserting \( \delta(t_1 - t_2) \) both in the numerator and denominator and using \( |\Psi(t)\rangle = U(t, 0) |\Psi(0)\rangle \).)\(^{14}\)

In many practical situations, we do not know the exact (multiverse) state, so that the system is described by a density matrix: \( \rho(t) \equiv \sum_i p_i |\Psi_i(t)\rangle \langle \Psi_i(t)| \) with \( \sum_i p_i = 1 \). In this case, Eq. (84)

\(^{14}\)The formula in Eq. (84) treats the state between \( t_1 \) and \( t_2 \) as that evolved from \( O_{A_{\text{pre}}} |\Psi(t_1)\rangle \), not \( |\Psi(t_1)\rangle \). This is justified under the current setup, i.e. a generic initial state \( |\Psi(t_0)\rangle \) evolving as in Eq. (82), but not in the case where the possibility of recoherence cannot be ignored even for macroscopic objects. In such a case, the formula in Eq. (1) must be used, which applies in any quantum mechanical system (and allows for answering any physical questions, although it requires some elaboration to apply to questions like the ones asked in Eq. (84).)
is straightforwardly extended to

$$P(B|A) = \frac{\int dt_1 dt_2 \text{Tr} \left[ \rho(0) U(0,t_1) O_{A_{\text{pre}}} U(t_1,t_2) O_{A_{\text{post}}} U(t_2,t_1) O_{A_{\text{pre}}} U(t_1,0) \right]}{\int dt_1 dt_2 \text{Tr} \left[ \rho(0) U(0,t_1) O_{A_{\text{pre}}} U(t_1,t_2) O_{A_{\text{post}}} U(t_2,t_1) O_{A_{\text{pre}}} U(t_1,0) \right]}.$$  (87)

This formula is also applicable for a reduced density matrix, obtained by tracing out some of the degrees of freedom, with the understanding that $O_X$ act on degrees of freedom that are not traced out. An important example includes the bulk density matrix, which is obtained by tracing out horizon (and possibly, singularity) degrees of freedom

$$\rho_{\text{bulk}}(t) = \text{Tr}_{\text{horizon}} \rho(t).$$  (88)

Since the evolution of $\rho_{\text{bulk}}(t)$ can be determined by semi-classical calculations, this allows us to make predictions/postdictions in the multiverse without knowing full quantum gravity (at the cost of unitarity in processes involving horizons/singularities).

### 8.2 Unification of the eternally inflating multiverse and many worlds in quantum mechanics

The probability formula of Eq. (84), or Eq. (87), can be applied to any physical questions, from the smallest possible (Planck length) to the largest possible (multiverse) scales.

Under the usual situation of a terrestrial experiment, the formula is reduced to the standard Born rule. This can be seen by isolating the degrees of freedom relevant to the experiment (which may involve the apparatus and/or experimenter, in addition to the system to be measured). Suppose $O_{A_{\text{pre}}}$ acts on these degrees of freedom and selects a particular premeasurement situation $A_{\text{pre}}$, which is realized in components of $|\Psi(t)\rangle$ at multiple values of $t = \hat{t}_i$ ($i = 1, 2, \cdots$):

$$O_{A_{\text{pre}}} U(t_1,0) |\Psi(0)\rangle = \sum_i c_i |\phi(\hat{t}_i)\rangle \otimes |\hat{\Psi}_i(\hat{t}_i)\rangle \delta(t_1 - \hat{t}_i),$$  (89)

where $|\phi(\hat{t}_i)\rangle$ represents the degrees of freedom relevant to the experiment in a configuration selected by $A_{\text{pre}}$, and $|\hat{\Psi}_i(\hat{t}_i)\rangle$ the other degrees of freedom in the relevant component of $|\Psi(t)\rangle$ at $t = \hat{t}_i$. In the limit that $A_{\text{pre}}$ selects the initial experimental setup infinitely accurately, which we are considering here for simplicity, the initial state for the experiment is

$$|\phi(\hat{t}_1)\rangle = |\phi(\hat{t}_2)\rangle = \cdots \equiv |\phi(t_{\text{before}})\rangle.$$  (90)

Then, assuming that $O_{A_{\text{post}}}$, which selects a particular postmeasurement situation, acts on the same degrees of freedom as $O_{A_{\text{pre}}}$, and that the effect of the other degrees of freedom on the experimental system is negligible, we can write as

$$O_{A_{\text{post}}} \hat{U}(t_2, t_{\text{before}}) |\phi(t_{\text{before}})\rangle = c |\phi(t_{\text{after}})\rangle \delta(t_2 - t_{\text{after}}) + \cdots,$$  (91)
where $\hat{U}(t_1, t_2)$ is a factor in $U(t_1, t_2)$ acting only on the experimental degrees of freedom, and $t_{\text{after}}$ is the smallest value of $t_2$ consistent with the projection $O_{A_{\text{post}}}$, which completely dominates the right-hand side. (In the usual setup of a terrestrial experiment, the coefficients of the terms in $\cdots$ are exponentially smaller than $c$.) Rewriting $O_{A_{\text{post}}}$ and $O_{A_{\text{post}} \cap B}$ as $O_{\text{obs}}$ and $O_{\text{obs} \cap \alpha}$, respectively, Eq. (84) then gives the probability of obtaining outcome $\alpha$

$$P(\alpha|\text{obs}) = \frac{\langle \phi(t_{\text{after}})| O_{\text{obs} \cap \alpha} |\phi(t_{\text{after}}) \rangle}{\langle \phi(t_{\text{after}})| O_{\text{obs}} |\phi(t_{\text{after}}) \rangle}.$$  \hspace{1cm} (92)

(All the other factors cancel between the numerator and the denominator.) This is nothing but the usual Born rule.

At scales of our everyday life, the formulae of Eqs. (84, 87) should (approximately) reproduce Newtonian dynamics. To see how the standard picture of time evolution arises, consider following the motion of a baseball (with position $\mathbf{x}$ and velocity $\mathbf{v}$) as a function of the hands of a clock $\tau$, in the absence of any force acting on it. In this case, we should take

$$O_{A_{\text{pre}}} = |\mathbf{x}, \mathbf{v} \rangle \langle \mathbf{x}, \mathbf{v} | \otimes |\tau \rangle \langle \tau |,$$  \hspace{1cm} (93)

$$O_{A_{\text{post}}} = 1 \otimes |\tau + d\tau \rangle \langle \tau + d\tau |,$$  \hspace{1cm} (94)

$$O_{A_{\text{post}} \cap B} = |\mathbf{x}', \mathbf{v}' \rangle \langle \mathbf{x}', \mathbf{v}' | \otimes |\tau + d\tau \rangle \langle \tau + d\tau |,$$  \hspace{1cm} (95)

(with some widths around these values), and the probability $P(B|A)$ will then be peaked at

$$\mathbf{x}' = \mathbf{x} + \mathbf{v} d\tau,$$  \hspace{1cm} (96)

$$\mathbf{v}' = \mathbf{v},$$  \hspace{1cm} (97)

as indicated by the Newtonian mechanics. Note that $\tau$ here represents the physical location of the hands of the clock, and not the “time” variable $t$, which is integrated out to obtain the probability.

The formulae of Eqs. (84, 87) can also be used to answer questions regarding global properties of our universe. To predict/postdict physical parameters $x$, for example, we need to choose $A$ to select the situation of making a measurement of $x$. We can then use various different values (ranges) of $x$ for $B$, to obtain the probability distribution $P(x)$. Despite the fact that the $t$ integrals run to $\infty$, the resulting $P(B|A)$ is well-defined, since $|\Psi(t)\rangle$ is continually “diluted” into supersymmetric Minkowski and singularity states [5]. The procedure to make predictions/postdictions in this way was discussed in Ref. [12], where the probability distribution of the vacuum energy, $x = \rho_A$, was computed and shown to agree with observation at an order of magnitude level.

It is striking that the simple, basic formalism developed here applies to physics at all scales. In particular, the single probability formula Eq. (84) (or Eq. (87)) can be used to answer any physical questions, given a state $|\Psi(t)\rangle$ (or $\rho(t)$). This, therefore, provides complete unification of the eternally inflating multiverse and many worlds in quantum mechanics. These two are really the same thing—they simply refer to the same phenomenon occurring at (vastly) different scales.
9 Summary

An essential feature of quantum mechanics is that information is fragile—as is well known, a generic measurement destroys the state of the system after the measurement is performed. Indeed, quantum information cannot be faithfully duplicated: the exact identification of a single state is not possible without having any prior knowledge of the state. Moreover, quantum information transmitted through physical processes will in general become non-local, encoded in the entanglement structure of a resulting quantum state.

Despite this intrinsically non-local nature of quantum states, however, the dynamics is local. Specifically, the time evolution operator takes a special form such that the concept of locality can be defined in spacetime. This feature allows for a limited set of information (among the full quantum information) to be copiously duplicated, i.e. “amplified,” and it is (only) this information that we can meaningfully store, compare, and handle. Because of the structure of the evolution operator, the relevant information is associated with well-defined classical configurations in phase space, at least for macroscopic systems.

In a system with gravity, the whole picture is more subtle, since if we choose wrong quantization hypersurfaces, then spacetime locality is not manifest even at distances much larger than the quantum gravity scale. We argued, however, that spacetime locality can be preserved if we define quantum states in restricted spacetime regions: in and on (stretched) apparent horizons viewed from a local Lorentz frame of a fixed spatial point $p$. This can be viewed as a “unitary gauge for quantum gravity,” on which our intuition should be based. By appropriately limiting the dimensions of the Hilbert subspaces corresponding to a fixed semi-classical geometry, all the redundancies/overcountings associated with a general relativistic, global spacetime description of nature are fixed/eliminated. These include general covariance, global overcounting related to complementarity, and local overcounting implied by the holographic principle.

The need for fixing a reference frame in describing the gravitational system quantum mechanically was emphasized. We identified the transformation associated with changes of the reference frame, which is specified by $d(d+1)/2$ parameters in $d$ spacetime dimensions. This transformation is the origin of horizon complementarity as well as the “observer dependence” of horizons and spacetime. The transformation is reduced to the standard Poincaré transformation in the limit $G_N \to 0$, so it can naturally be regarded as an extension of the Poincaré transformation in the quantum gravitational context. This is much like that the standard Lorentz transformation is regarded as an extension of the Galilean transformation, where the former is reduced to the latter in the limit $c \to \infty$.

It is remarkable that the simple framework described in this paper is applicable to physics at all scales, from the smallest (Planck length) to the largest (multiverse). Indeed, it is quite striking that quantum mechanics does not need any modification to be applied to phenomena at such
vastly different scales. In the 20th century, we have witnessed the tremendous success of quantum mechanics, following its birth at the beginning. In the early 21st century, quantum mechanics seems still giving us an opportunity to explore deep facts about nature, such as spacetime and gravity. Does quantum mechanics break down at some point? We don’t know. But perhaps, the beginning of the multiverse might provide one.

Acknowledgments

I am grateful for useful discussions with Alan Guth, Grant Larsen, and I-Sheng Yang. This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, of the US Department of Energy under Contract DE-AC02-05CH11231, and in part by the National Science Foundation under grant PHY-0855653.

A Quantum Measurement: Dissipation of Coherence into (Infinitely) Large Hilbert Space

The picture of quantum measurement presented in this paper involves several stages of “dissipation of coherence” into larger Hilbert spaces: from microscopic to macroscopic, eventually to a supersymmetric Minkowski or singularity world. At each stage, a superposition of the smaller system in some basis is translated into a superposition of the larger system. In the early stages of this process, i.e. when the systems involved are small, the selected bases depend on the details of the setup, as the standard analysis of decoherence shows. However, at the later stages, where the relevant systems become larger, the appropriate bases quickly approach the classical state basis, as discussed in Section 4. In those stages, information associated with the selected basis of the original system is tremendously amplified in each term, producing an effective classical world. These classical worlds, represented by various terms in the full multiverse state, then evolve independently and eventually become different supersymmetric Minkowski (or singularity) worlds. Since the Hilbert space dimension of Minkowski states is infinite, the resulting worlds do not recohere.

The picture described above provides an ultimate answer to the question of basis selection in quantum measurement. Here we focus only on one classic example: why the apparatus \( \alpha \) measures the spin in the \( z \) direction and not in the \( x \) direction. The question is typically phrased in the following form. Suppose the apparatus interacted with the spin \( (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2} \). Then the combined apparatus-spin system after the measurement can be written either as

\[
|\Psi\rangle = \frac{1}{\sqrt{2}} |\uparrow\rangle |\oplus\rangle + \frac{1}{\sqrt{2}} |\downarrow\rangle |\ominus\rangle
\]

(98)
or
\[ |\Psi\rangle = \frac{1}{2\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)(|\uparrow\rangle + |\downarrow\rangle) + \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)(|\uparrow\rangle - |\downarrow\rangle), \] where we have omitted the symbol \( \otimes \) for notational simplicity. This seems to indicate that there is an ambiguity associated with the basis in which the measurement has been performed: along the z axis (Eq. (98)) or the x axis (Eq. (99)).

Since the apparatus is macroscopic, however, further dissipation/amplification of coherence occurs in the classical state basis, which is more or less a “smeared locality basis.” For example, in a complete treatment of the measurement including the observer and the desk, the state after the measurement is given by Eq. (18) with \( c_\uparrow = c_\downarrow = 1/\sqrt{2}:
\[ |\Psi(t \gg t_{\text{obs}})\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle |\uparrow\rangle |\uparrow\rangle |\uparrow\rangle + |\downarrow\rangle |\downarrow\rangle |\downarrow\rangle |\downarrow\rangle \cdot |\uparrow\rangle + |\downarrow\rangle |\downarrow\rangle |\downarrow\rangle |\downarrow\rangle \cdot |\downarrow\rangle\] + \frac{1}{2\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)(|\uparrow\rangle |\uparrow\rangle |\uparrow\rangle |\uparrow\rangle - |\downarrow\rangle |\downarrow\rangle |\downarrow\rangle |\downarrow\rangle \cdot |\downarrow\rangle \cdot |\downarrow\rangle\] (100)

This is enough to conclude that the experiment measures the spin in the z direction, i.e. the direction perpendicular to the surface of the desk, because the two terms will evolve (eventually) into two different supersymmetric Minkowski (or singularity) states, which will never recohere. It is clear that writing the state as in Eq. (99),
\[ |\Psi(t \gg t_{\text{obs}})\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)(|\uparrow\rangle + |\downarrow\rangle)\] (101)
does not affect physical predictions.

Of course, we could alternatively measure the spin in the x direction, instead of the z direction. To do so, however, we need to prepare a different apparatus (or arrange a different configuration of the detector). Representing such an apparatus by a square, the dynamical evolution is now
\[ \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)|\square\rangle \rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)|\square\rangle\] (102)
or in a more complete description
\[ |\Psi(t \ll t_{\text{m}})\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)|\square\rangle |\uparrow\rangle |\uparrow\rangle \cdot |\square\rangle \rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)|\square\rangle |\uparrow\rangle |\uparrow\rangle \cdot |\square\rangle, \] (103)

or
\[ |\Psi(t \gg t_{\text{obs}})\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)|\square\rangle |\uparrow\rangle |\uparrow\rangle \cdot |\square\rangle \cdot |\uparrow\rangle \cdot |\uparrow\rangle \cdot |\square\rangle, \] (104)

In this case, Eq. (104) itself is the selected, classical state basis, corresponding to the outcome of the experiment.

In practice, once coherence is promoted to a sufficiently macroscopic level, we can regard components of a state that have different phase space configurations as different worlds, since the
probability for them to recohere is infinitesimally small. As we have seen, this is a consequence of spacetime locality—the fact that the time evolution operator takes a special form of Eq. (25)—as well as the infinite dimensionality of the Hilbert space of the quantum universe. Ultimately, it is these features that are responsible for the appearance of (apparently classical, and ordered) many worlds in the quantum mechanical universe.

References

[1] S. Weinberg, Phys. Rev. Lett. 59, 2607 (1987).

[2] A. H. Guth and E. J. Weinberg, Nucl. Phys. B 212, 321 (1983); A. Vilenkin, Phys. Rev. D 27, 2848 (1983); A. D. Linde, Phys. Lett. B 175, 395 (1986); Mod. Phys. Lett. A 1, 81 (1986).

[3] R. Bousso and J. Polchinski, JHEP 06, 006 (2000) [arXiv:hep-th/0004134]; S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D 68, 046005 (2003) [arXiv:hep-th/0301240]; L. Susskind, [arXiv:hep-th/0302219] M. R. Douglas, JHEP 05, 046 (2003) [arXiv:hep-th/0303194].

[4] For reviews, see e.g. A. H. Guth, Phys. Rept. 333, 555 (2000) [arXiv:astro-ph/0002156]; A. Vilenkin, J. Phys. A 40, 6777 (2007) [arXiv:hep-th/0609193]; S. Winitzki, Lect. Notes Phys. 738, 157 (2008) [arXiv:gr-qc/0612164]; A. Linde, Lect. Notes Phys. 738, 1 (2008) [arXiv:0705.0164 [hep-th]].

[5] Y. Nomura, JHEP 11, 063 (2011) [arXiv:1104.2324 [hep-th]].

[6] G. ’t Hooft, [arXiv:gr-qc/9310026] L. Susskind, J. Math. Phys. 36, 6377 (1995) [arXiv:hep-th/9409089]; R. Bousso, JHEP 07, 004 (1999) [arXiv:hep-th/9905177].

[7] L. Susskind, L. Thorlacius and J. Uglum, Phys. Rev. D 48, 3743 (1993) [arXiv:hep-th/9306069]; L. Susskind, Phys. Rev. Lett. 71, 2367 (1993) [arXiv:hep-th/9307168]; C. R. Stephens, G. ’t Hooft and B. F. Whiting, Class. Quant. Grav. 11, 621 (1994) [arXiv:gr-qc/9310006].

[8] H. Ollivier, D. Poulin and W. H. Zurek, Phys. Rev. Lett. 93, 220401 (2004) [arXiv:quant-ph/0307229]; Phys. Rev. A 72, 042113 (2005) [arXiv:quant-ph/0408125]; R. Blume-Kohout and W. H. Zurek, Found. Phys. 35, 1857 (2005) [arXiv:quant-ph/0408147]; Phys. Rev. A 73, 062310 (2006) [arXiv:quant-ph/0505031].

[9] W. K. Wootters and W. H. Zurek, Nature 299, 802 (1982).

[10] H. Everett, III, Rev. Mod. Phys. 29, 454 (1957).

[11] See, e.g., M. Schlosshauer, _Decoherence and the Quantum-to-Classical Transition_ (Springer, Berlin, 2007), and references therein.
[12] G. Larsen, Y. Nomura and H. L. L. Roberts, arXiv:1107.3556 [hep-th].
[13] R. Bousso and L. Susskind, arXiv:1105.3796 [hep-th].
[14] A. Aguirre, M. Tegmark and D. Layzer, arXiv:1008.1066 [quant-ph]; see also M. Tegmark, Found. Phys. Lett. 9, 25 (1996) arXiv:quant-ph/9603008.
[15] B. Freivogel, Y. Sekino, L. Susskind and C.-P. Yeh, Phys. Rev. D 74, 086003 (2006) arXiv:hep-th/0606204; L. Susskind, arXiv:0710.1129 [hep-th]; Y. Sekino and L. Susskind, Phys. Rev. D 80, 083531 (2009) arXiv:0908.3844 [hep-th]; R. Bousso and L. Susskind, in Ref. [13].
[16] R. Bousso, Phys. Rev. Lett. 97, 191302 (2006) arXiv:hep-th/0605263; R. Bousso, B. Freivogel and I.-S. Yang, Phys. Rev. D 74, 103516 (2006) arXiv:hep-th/0606114; Phys. Rev. D 79, 063513 (2009) arXiv:0808.3770 [hep-th].
[17] J. von Neumann, Mathematische Grundlagen der Quantenmechanik (Springer, Berlin, 1932).
[18] W. H. Zurek, Phys. Rev. D 24, 1516 (1981).
[19] W. H. Zurek, Phys. Rev. D 26, 1862 (1982); see also H. D. Zeh, Found. Phys. 1, 69 (1970).
[20] See, e.g., S. Weinberg, The Quantum Theory of Fields Volume I, (Cambridge University Press, Cambridge, UK, 1995); M. E. Peskin and D. V. Schroeder, An Introduction to Quantum Field Theory, (Westview Press, 1995).
[21] W. K. Wootters and W. H. Zurek, Phys. Rev. D 19, 473 (1979).
[22] E. Joos and H. D. Zeh, Z. Phys. B 59, 223 (1985).
[23] A. Aguirre, S. M. Carroll and M. C. Johnson, arXiv:1108.0417 [hep-th].
[24] L. S. Schulman, Entropy 7, 221 (2005).
[25] D. N. Page, Phys. Rev. D 78, 063535 (2008) arXiv:hep-th/0610079; see also L. Dyson, M. Kleban and L. Susskind, JHEP 10, 011 (2002) arXiv:hep-th/0208013.
[26] See, e.g., L. Susskind and J. Lindesay, An Introduction to Black Holes, Information and the String Theory Revolution: The Holographic Universe (World Scientific, Singapore, 2005).
[27] L. Susskind and L. Thorlacius, Phys. Rev. D 49, 966 (1994) arXiv:hep-th/9308100; P. Hayden and J. Preskill, JHEP 09, 120 (2007) arXiv:0708.4025 [hep-th].
[28] D. A. Lowe, J. Polchinski, L. Susskind, L. Thorlacius and J. Uglum, Phys. Rev. D 52, 6997 (1995) arXiv:hep-th/9506138.
[29] O. Lunin and S. D. Mathur, Nucl. Phys. B 623, 342 (2002) arXiv:hep-th/0109154.
[30] R. Bousso, in Ref. [6]; JHEP 06, 028 (1999) arXiv:hep-th/9906022; Rev. Mod. Phys. 74, 825 (2002) arXiv:hep-th/0203101.
[31] S. Weinberg, Phys. Rev. Lett. 48, 1776 (1982); S. Deser and C. Teitelboim, Phys. Rev. Lett. 39, 249 (1977).

[32] R. Bousso, talk given at the FQXi conference, Bergen, Norway and Copenhagen, Denmark, August–September 2011, [http://fqxi.org/conference/talks/2011](http://fqxi.org/conference/talks/2011)

[33] B. S. DeWitt, Phys. Rev. 160, 1113 (1967).