A Nodal Analysis Method for Laterally-Driven Electrothermal Microactuators

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Abstract. This paper introduces a novel method for the simulation and verification of the laterally-driven electrothermal microactuator. A nodal analysis model of the electrothermal microactuator was built without meshing. This model may simulate the static and dynamic behaviour of the electrothermal microactuators with variational thermal conductivity, electrical resistivity and thermal expansion coefficient. There are only three stages of ODEs in the model, but its static relative error is below 5% in comparison with the results of the nonlinear model of ANSYS with the nonlinear geometrical options turned on. The equivalent circuit was then built to present the capability of co-simulating with the control and feedback circuits.

1. Introduction

With the increasing of the complexity of the MEMS devices, the design of these devices depends highly on the MEMS-specific modeling methodologies and simulation tools. The system-level simulation is demanded when several physical domains incorporated in the MEMS device should be taken into account. Hierarchical design and simulation methods such as nodal analysis method were developed in the past years[1~3]. The reusability of the parametric models in the hierarchical design and the interoperability with the electronic circuit of the circuit-level modeling makes the system-level simulation possible.

The electrothermal microactuator is widely used for its large output force and large deflection. Several laterally-driven electrothermal microactuators were developed with surface micromachined process. It is important to model these microactuators to improve the efficiency of the design and optimization. The modeling of these microactuators is focused on FEM, FDM[4] and lumped parameter[5]. The model order reduction (MOR) is also utilized due to the large scale of data produced by meshing in FEM and FDM[6]. However, FEM and FDM are not suitable in the system-level simulation because they could not co-simulate with electronic circuit and was computationally expensive.

The electrothermal microactuators mentioned above are mainly composed of beams and anchors. When the power consumption of the electrothermal microactuator is not too large, the temperature of the anchor could be assumed identical with the ambient approximately. Therefore, the model of the anchor is simplified as an ideal heat sink herein. This paper focuses on the modeling of the beam element in the electrothermal microactuators. A nonlinear lumped parameter model for the beam elements in the surface micromachined electrothermal microactuators was presented in this paper without meshing. Three types of the electrothermal microactuators are finally simulated with the nodal
analysis method by SPICE. The results are confirmed by ANSYS simulation with geometrical nonlinear options on.

2. Modeling of the beam

The electrical, thermal and mechanical domains in the electrothermal microactuators interact with each other. The electrical current causes the Joule heating and the temperature redistribution causes the change of the electrical resistivity. The temperature redistribution also causes the mechanical movements. However, the mechanical movement and the electrical performance would hardly influence each other in the surface micromachined electrothermal microactuators. The mechanical movements would hardly cause the change of the temperature in the micron scaled electrothermal microactuators. Thus, the complex coupled electro-thermal-mechanical problem is simplified significantly to indirectly coupled electrothermal and thermomechanical problem. So the beam element in this paper would be modeled in two steps, one is electrothermal model and another is thermomechanical model. The electrothermal model would be built from the governing equation of the temperature and the thermomechanical model be constructed then.

2.1. Governing equation of the temperature in the beam element

The governing equation of the transient heat transfer problem is so called heat equation (Fourier’s equation)

$$\rho c \frac{\partial T(r,t)}{\partial t} - \nabla \cdot [\kappa \nabla T(r,t)] = g(r,t)$$

where $T(r,t)$ is the temperature distribution, $g(r,t)$ is the heat generation per unit volume at position $r$ and time $t$, $\alpha ( \alpha = \frac{\kappa}{\rho c} )$ is the heat diffusivity, $\rho$ is the density, $c$ the specific heat, and $\kappa$ the thermal conductivity, $R$ is the region of the MEMS structure. The boundary conditions are applied at the boundary of the region, $R$.

The beam in Fig. 1 is the schematic view of the basic element of the thermal microactuator. To model the beam using the nodal analysis method, the equation should be simplified by the assumptions listed below:

a. The heat loss by the radiation is negligible when the electrothermal microactuators are working\[5\].

b. The temperature distributes in one dimension in the beam elements. The theory of heat transfer proves that it’s true in most part of the electrothermal microactuators.

c. The ambient temperature, which is identical with the temperature of the substrate, is constant when the electrothermal microactuators are working.

Based on the assumptions, the governing equation in the beam element could simplified significantly in the coordination of Fig. 1 as

$$\rho c\omega \frac{\partial T_w(x,t)}{\partial t} - \omega \frac{\partial}{\partial x}\left[ \kappa \frac{\partial T_w(x,t)}{\partial x} \right] = -\left( h w \omega + \frac{w S}{R_i} \right) [T_w(x,t) - T_a] + \frac{\bar{\rho}_t(t)}{w_b}$$

(2)
where \( T_b(x,t) \) is the temperature distribution of the beam, \( T_s \) is the temperature of the substrate, \( \kappa \) is the thermal conductivity of the beam, \( h \) is the convection coefficients, \( \rho_e \) is the electrical resistivity of the material of the beam, \( i(t) \) is the electric current through the beam, \( w \) and \( b \) are the width and thickness of the beam, respectively, \( S \) is the shape factor which accounts for the impact of the shape of the element on heat conduction to the substrate. \( R_T \) would be the thermal resistance between the beam and the substrate if the beam is wide enough. \( R_T \) is given by the following equation [5]

\[
R_T = \frac{t_x}{\kappa_{air} \kappa_N} + \frac{t_y}{\kappa_{air} \kappa_O} + \frac{t_z}{\kappa_{air} \kappa_{SiO_2}}
\]

where \( \kappa_N \) and \( \kappa_O \) is the thermal conductivity of Si3N4 and SiO2 respectively, and \( t_N, t_O \), the thickness of the Si3N4 and SiO2 respectively, as shown in Fig. 1. The following equation gives the shape factor for heat conduction [5]

\[
S = \frac{b}{w} \left( \frac{2t_w}{b} + 1 \right) + 1
\]

2.2. Coupling electrothermal beam model

The relative temperature distribution was used below:

\[
\bar{T}(x,t) = T_b(x,t) - T_s
\]

where \( \bar{T}(x,t) \) means the temperature relative to the ambient. To simulate the dependence of the material properties on the temperature, the thermal conduction and electrical resistivity are assumed to be linear functions of the temperature herein as

\[
\kappa = \kappa_0 + \kappa_1 T, \quad \rho_e = \rho_{e0} (1 + \kappa T)
\]

By the assumption

\[
\xi = \frac{b}{\kappa_0 \kappa_1 RT}, \quad m = \frac{\rho_{e0}}{\kappa_0 w^2 b^2}, \quad \phi = \frac{\rho_e \xi}{\kappa_0 w^2 b^2}
\]

the governing equation could be written as

\[
\frac{1}{\alpha} \frac{\partial^2 \bar{T}(x,t)}{\partial t^2} \left( \frac{\kappa_0}{\kappa_0} \bar{T}(x,t) + 1 \right) \frac{\partial \bar{T}(x,t)}{\partial x} = -\bar{T}(x,t) + \phi \bar{T}(x,t) + \nu m
\]

The boundary conditions of this equation is

\[
\left\{ \begin{array}{l}
\bar{T}(x,t) \big|_{x=0} = \bar{T}_1(t), \quad \bar{T}(x,t) \big|_{x=L} = \bar{T}_2(t) \\
\bar{T}(x,t) \big|_{x=0} = \eta_1(t), \quad \bar{T}(x,t) \big|_{x=L} = \eta_2(t)
\end{array} \right.
\]

By fitting by Hermite polynomial at the two ends of the beam and offsetting the residue by Fourier series, the temperature distribution expressed below is approximated to discretize the governing PDE into ODEs.

\[
\sum_{n=0}^{N} C_n(t) = 0, \quad \sum_{n=0}^{N} C_n(t) = 0
\]

The accuracy was high enough when only the first three order of Fourier series were picked. Substituting Eq. (10) into Eq. (8) and performing Fourier transformation, the PDE is discretized into ODEs. And only the first three stages of equations were picked due to the high enough accuracy. The ODEs was in the form as

\[
\bar{T}_1 + \bar{T}_2 + \sum_{n=0}^{N} = 0, \quad n \in \{0,1,2\}
\]

where \( \bar{T}_1 \) was the item which was independent of temperature, \( \bar{T}_2 \) and \( \bar{T}_3 \) the items that introduced by temperature dependent thermal conductivity and electrical resistivity, respectively. The heat flux out of the 2 nodes of the beam was
The resistance of the beam could be written as

\[ R = \int_{b}^{a} \left( 1 + \frac{\zeta T(x,t)}{wb} \right) \rho_{ct} \, dx = \frac{\rho_{ct} l}{wb} + \frac{\zeta \rho_{ct}}{wb} \left[ T_{1}(t) + T_{2}(t) \right] \frac{[\eta_{1}(t) - \eta_{2}(t)]}{12} + C_{o}(t) \]  

(14)

2.3. Coupling thermomechanical beam model

By considering the important effect in the electrothermal microactuators, the coupling thermomechanical model is built on the basis of the mechanical model of the beam. Based on the electromechanical model of the beam in the nodal analysis method, the thermal expansion caused by Joule heating was equivalent to an axial force applied at the two ends of the beam.

\[ F_{eq} = \frac{EA}{l} \left( \int \varepsilon_{r} \, dx - \Delta l \right) \]  

(15)

where \( \varepsilon_{r} \) is the axial thermal strain and \( \Delta l \) is the change of the axial length introduced by lateral deflection. The change of the axial length would release the thermal strain significantly. So it should be considered in the modelling of the beam. The change could be written as[7]

\[ \Delta l = l' - l = \int \left[ 1 + \frac{1}{2} \left( \frac{du}{dx} \right)^{2} \right] \, dx - l \approx \int \left[ 1 + \frac{1}{2} \left( \frac{du}{dx} \right)^{2} \right] \, dx - l = \frac{1}{2} \left( \frac{du}{dx} \right)^{2} \]  

(16)

where \( u \) is the lateral displacement of the beam; \( l' \) is the effective axial length after bending without axial force applied externally; \( l \) is the original axial length.

The laterally-driven electrothermal microactuators are actuated by the axial force. Due to the large axial stress in the beam of the electrothermal microactuators, the effect of the large axial stress should be considered. The axial force \( N \) in the beam is

\[ N = \frac{EA}{l} \left( x_{2} - x_{1} + \Delta l - \int \varepsilon_{r} \, dx \right) \]  

(17)

where \( E \) is the Young’s modulus, \( A \) the section area of the beam. \( x_{2} \) and \( x_{1} \) are the axial movements of the right and left ends, respectively.

The temperature dependent thermal expansion coefficient in this paper was assumed to be a linear function of the temperature as

\[ \alpha_{w} = \alpha_{w0} + \alpha_{w1} T(x,t) \]  

(18)

where \( \alpha_{w0} \) is the expansion coefficient of the ambient temperature and \( \alpha_{w1} \) is the first-order coefficient.

3. Representation of the equivalent circuit of the beam

The circuit simulation is an ODE solver essentially. So the ODE is equivalent to some format of circuit. By assuming the temperature as the across quality and the heat flux as the through quality, the ODEs of the electrothermal model and the thermomechanical model could be represented in the circuit simulation tools.

The equivalent circuit of the ODEs in Eq. (11) could be represented with KCL and KVL law. The derivative of an across quality is represented by the current through a capacitor with this across quality applied across it. All the nonlinear functions in the ODEs are represented by the nonlinear controlled sources.

The coefficient matrices of the thermomechanical model of the beam are positive definite matrices. So the equivalent circuit of the model could be modeled by some passive circuit components, such as
resistors and capacitors[8]. However, the nonlinear effects such as the large axial stress and the change of the axial length should be represented by nonlinear controlled sources.

4. Simulation and verification
The three representative laterally-driven electrothermal microactuators, V-shaped, U-shaped and long-short beam electrothermal microactuators, were simulated with this model and verified with ANSYS. The elements used in ANSYS are sequentially coupled SOLID69 and SOLID45 with the command ‘NLGEOM, ON’ to present the ability of simulating geometrical nonlinearity. The mechanical, electrical and thermal parameters of the actuator in the simulations are listed in Table 1.

| Property | \( \rho \) | \( h \) | \( c \) | \( \kappa_0 \) | \( \kappa_0 \) | \( \rho_{e0} \) | \( \zeta \) | \( \alpha_{m0} \) | \( \alpha_{m1} \) |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Value    | 2330     | 1E4      | 700      | 61.7     | -0.0658  | 2.97E-5  | 0.0021   | 2.7E-6   | -5.4E-6  |
| Unit     | kg m\(^{-3}\) | W m\(^{-1}\) K\(^{-1}\) | J Kg\(^{-1}\) K\(^{-1}\) | W m\(^{-1}\) K\(^{-1}\) | W m\(^{-1}\) K\(^{-1}\) | \( \Omega \) m\(^{-2}\) | K\(^{-1}\) | K\(^{-1}\) | K\(^{-2}\) |

The simulated temperature data in this paper are all relative to the ambient temperature.

4.1. V-shaped thermal microactuator
The V-shaped thermal microactuator is composed of two beams and two anchors. Its dimensions are listed in Table 2.

| Geometrical dimensions | Length of arm | Width of arm | Thickness of arm | Tilt angle |
|------------------------|---------------|--------------|------------------|------------|
| Value                  | 200 \( \mu \) m | 2 \( \mu \) m | 2 \( \mu \) m | 4 DEG |

When the voltage applied increased from 0.1V to 6V with step of 0.1V, the effect of the change of the axial length is illustrated in Fig. 2. When the tilt angle of the actuator increased from 0.2 degree to 3 degree, the effect of large axial stress is illustrated in Fig. 3.

4.2. U-shaped thermal microactuator
The U-shaped thermal microactuator is composed of four beams and two anchors. Its dimensions are listed in Table 3.

| Geometrical dimensions | Length of hot arm | Width of hot arm | Length of cold arm | Width of cold arm | Gap | Length of flexure | Width of flexure | Thickness |
|------------------------|-------------------|------------------|--------------------|-------------------|-----|------------------|-----------------|----------|
| Value(\( \mu \)m)     | 240               | 2                | 200                | 10                | 2   | 40               | 2              | 2        |

A step stimulus voltage was applied across the anchors of the actuators. The deflection of each node is shown in Fig. 4.

4.3. Long-short beam thermal microactuator
The long-short beam thermal microactuator is composed of three beams and two anchors. Its dimensions are listed in Table 4.

| Geometrical dimensions | Length of long arm | Length of short arm | Gap | Width of the arm | Thickness of the arm |
|------------------------|--------------------|---------------------|-----|-----------------|---------------------|
| Value(\( \mu \)m)     | 300                | 150                 | 7   | 2               | 2                   |

A pulse stimulus with the width of 1ms was applied across the anchors of the actuators. The dynamic of the actuator in 2ms is illustrated in Fig.5.
Fig. 2. Comparison of simulation results of the tip stroke of the V-shaped microactuator (A) with the change of the effective axial length (B) without the change of the effective axial length.

Fig. 3. Comparison of the simulation results of the tip stroke of the V-shaped microactuator with and without the large axial stress (A) with the large axial stress (B) without the large axial stress.

Fig. 4. Simulation results of the nodes deflection response of the U-shaped microactuator with an ideal step stimulus of the applied voltage.

Fig. 5. Simulation results of the nodes temperature of the long-short beam microactuator with the pulse voltage stimulus.

5. Conclusion
This paper introduces a nodal analysis model of the electrothermal microactuators. It could simulate the static and dynamic characteristic of these actuators in system level with high accuracy and efficiency.

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