Ionized Plasma and Neutral Gas Coupling in the Sun’s Chromosphere and Earth’s Ionosphere/Thermosphere

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Abstract We review physical processes of ionized plasma and neutral gas coupling in the weakly ionized, stratified, electromagnetically-permeated regions of the Sun’s chromosphere and Earth’s ionosphere/thermosphere. Using representative models for each environment we derive fundamental descriptions of the coupling of the constituent parts to each other and to the electric and magnetic fields, and we examine the variation in magnetization of the components. Using these descriptions we compare related phenomena in the two environments, and discuss electric currents, energy transfer and dissipation. We present examples of physical processes that occur in both atmospheres, the descriptions of which have previously been conducted in contrasting paradigms, that serve as examples of how the chromospheric and ionospheric communities can further collaborate. We also suggest future collaborative studies that will help improve our understanding of these two different atmospheres which while sharing many similarities, also exhibit large disparities in key quantities.

1 Introduction

In a universe where partially ionized gases abound, interactions between plasma components and neutral gases play a critical role in planetary and stellar atmospheres, including those of the Earth and the Sun. They also are important at the heliopause, where the solar wind meets the interstellar medium, and in other astrophysical contexts. Plasma-neutral interactions modulate momentum and energy exchange among the neutral gas, electrons, and ions, and between the ionized plasma and electromagnetic fields. The physics of plasma-neutral coupling adds another layer of complexity to problems that previously have been addressed by assuming a fully ionized plasma or some other single-fluid approximation. The importance of these transitional layers in the inner heliosphere – in particular, the solar chromosphere and terrestrial ionosphere/thermosphere – lies in their impact on space weather processes that can profoundly affect Earth and society. This motivates our attention to the underlying physics of plasma-neutral coupling.

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The solar chromosphere is the highly dynamic, complex region above the relatively cool visible surface of the Sun and beneath the very hot corona. It is characterized by several transitions that occur with increasing altitude: from predominantly neutral to ionized hydrogen; from essentially unmagnetized to strongly magnetized charged particles; from collisional to collision-less behavior; and from gas-dominated to magnetic field-dominated dynamics. These transitions vary in space and time as the chromosphere is driven continually from below by convective motions and magnetic evolution. The chromosphere modulates the flow of mass and energy into the corona. In addition, recent observations suggest that waves present in the chromosphere have the power to drive the solar wind (De Pontieu et al. 2007a), though this has yet to be convincingly demonstrated. The chromosphere’s complexity is increased further by its state of thermodynamic and ionization non-equilibrium, which makes understanding its observed emission and absorption spectra very challenging.

The ionosphere/thermosphere (hereafter I/T) is a similarly transitional region in the Earth’s upper atmosphere, in which the gas is ionized by incident solar radiation. It encompasses the same physical transitions as those occurring in the chromosphere. In this paper we use the term I/T to denote that region of the Earth’s upper atmosphere between about 80-600 km altitudes. The word “thermosphere” technically denotes a distinct region based on the temperature profile of the neutral component of the upper atmosphere. The word “ionosphere” refers to the ionized component of the gas in the upper atmosphere, usually in the same 80-600 km altitude region. Thus, I/T includes both the neutral and ionized constituent parts of the weakly ionized mixture. The I/T is continually driven, from below by the mesosphere, and from above by the magnetosphere. Understanding how the many various forcing mechanisms interact to cause variability in the I/T system remains a major challenge.

Previous authors have discussed similarities and differences between the Sun’s chromosphere and the Earth’s ionosphere (Haerendel 2006; Fuller-Rowell and Schrijver 2009). These authors emphasized the strong collisional coupling of the minority plasma constituents to the majority neutral species in both atmospheres; a collisional coupling of the neutrals to the plasma that is very substantial in the solar chromosphere but relatively weak in the terrestrial I/T; and the impact of these interactions on the highly anisotropic electrical conductivities along and across ambient magnetic fields in the two environments. Haerendel (2006) further presented analogies between density fluctuations in solar spicules and sporadic E layers, atmospheric heating by Alfvén waves in chromospheric plages and auroral arcs, and plasma erosion driven by currents aligned with the magnetic fields in solar flares and auroral ion outflows. Fuller-Rowell and Schrijver (2009) provided a detailed summary of processes occurring in the ionosphere, followed by a survey of phenomena in the Sun’s chromosphere and comparisons between the two. They discuss the similar variability of the charged particle magnetizations in the two atmospheres and the role of convective overshoot from the photosphere/mesosphere below the chromosphere/ionsphere, and highlight major contrasts in the dynamic/static character of the magnetic fields and the resultant dominance of magnetohydrodynamics/electrodynamics in describing macroscopically the evolution of those fields.

One of the goals of this review is to highlight the commonalities and differences between the chromosphere and I/T in order to develop cross-disciplinary collaboration between the two communities, which typically use different approaches to the same fundamental physics. In doing so, we hope to identify important questions concerning the transition from weakly ionized dense mixtures to fully ionized tenuous plasmas linked by electromagnetic fields, and present methods by which we can enhance our physical understanding of such systems using improved analytical and numerical modeling of plasma-neutral coupling in the chro-
The paper is structured as follows. In §2 we examine representative time and space averaged models of the chromosphere and I/T, and compare the two environments in terms of their fundamental neutral, plasma, and magnetic properties as well as some key dimensionless ratios. In §3 we present the governing equations for a weakly ionized reacting plasma-neutral mixture. In §4 we investigate in detail the physics and the equations that govern the coupling of the plasma components and neutral gas to each other and to the electromagnetic field. We compare the magnetization and mobility of the ionized component of the two environments and relate them to the evolution of electric currents. In §5 we discuss processes which are examples of such coupling, and consider the contrasting approaches that the I/T and chromosphere communities use to describe essentially the same phenomena. In §6 we address the transfer and dissipation of energy, first focusing on the state of the field’s knowledge. Then we discuss the importance of the conversion of electromagnetic energy into thermal and kinetic energy, and look at the efficiency of plasma-neutral coupling in this energy transfer. In §7 we present an illustrative analytical and numerical case study of the Rayleigh-Taylor instability, which is common to the chromosphere and I/T yet also highlights the contrasting conceptual and mathematical approaches employed by the two communities. We conclude in §8 with some parting thoughts about current challenges to our understanding of plasma-neutral coupling on the Sun and at the Earth.

2 Basic Properties

In this section, basic properties of the Sun’s chromosphere and Earth’s I/T are presented and compared. As we will show quantitatively, there are both significant similarities and substantial differences between these environments. From fundamental principles, we deduce qualitative implications about how the majority neutral and minority plasma constituents couple hydrodynamically in both atmospheres. In §4 we will discuss how they couple principally magnetohydrodynamically in the chromosphere (where conductivity is generally high) but electrodynamically in the I/T (where the conductivity is low). Several of the following general introductory considerations are elaborated on in more detail in the later sections of the paper, which deal with the governing multi-fluid equations, generalized Ohm’s law and its low-frequency limit, the mobility of plasma, electromagnetic energy transfer, and common plasma processes occurring in both atmospheres.

2.1 Models for the Chromosphere and Ionosphere/Thermosphere

The chromosphere is represented here by the semi-empirical quiet-Sun model “C7” developed and tabulated by Avrett and Loeser (2008), hereafter referred to as the ALC7 model (see also Vernazza et al. 1981; Fontenla et al. 1993, 2006). In this model, simulated emissions are matched to the observed spectra to obtain estimates of the total density, ionization level and temperature in the chromosphere. This model takes into account that in the lower chromosphere where hydrogen is mostly neutral, heavy ions such as Fe, Ca, and Mg are more abundant than hydrogen ions and contribute almost all the free electrons. The transition region above is modeled by assuming an energy balance between the downward total energy flow from the overlying hot corona and local radiative loss rates. Although this one-dimensional model certainly does not capture all variations, either temporally or in three dimensions, it is useful for characterizing the generic structure of the chromosphere.
The I/T is represented here by the NCAR Thermosphere-Ionosphere-Mesosphere-Electrodynamic General Circulation Model (NCAR TIMEGCM). TIMEGCM is a time-dependent, three-dimensional model that solves the fully coupled, nonlinear, hydrodynamic, thermodynamic, and continuity equations of the neutral gas along with the ion energy, momentum, and continuity equations from the upper stratosphere through the ionosphere and thermosphere (Roble et al. 1988; Richmond et al. 1992; Roble and Ridley 1994). TIMEGCM predicts global neutral winds, neutral temperatures, major and minor species composition, electron and ion densities and temperatures, and the ionospheric dynamo electric field. The input parameters are solar EUV and UV spectral fluxes, parameterized by the F10.7 cm index, plus auroral particle precipitation, an imposed magnetospheric electric field, and the amplitudes and phases of tides from the lower atmosphere specified by the Global Scale Wave Model (Hagan et al. 1999). Many features of the model, such as increased electron temperature in the E and F layers, have been validated against observations (e.g., Lei et al. 2007). For the atmospheric profiles used in this paper, TIMEGCM was run under equinox conditions with a F10.7 value of 150, a Kp index of 2, and tidal forcing. The vertical profile was taken from 47.5° N latitude at 12:00 local time. Thus like the model chromospheric profile described above, we represent the I/T structure by a single 1-D, time-independent profile.

In displays of quantities provided by, or derived from, these models for the chromosphere and I/T, we use as the primary (left) ordinate axis the normalized total gas pressure \( P/P_0 \), where \( P_0 \) is defined to be the pressure at a selected reference height in the domain. The approximate corresponding altitude is shown as the secondary (right) ordinate axis. For the chromosphere, we chose the Sun’s visible surface, the photosphere, as the reference height. The pressure there is \( 1.23 \times 10^4 \) Pa in the ALC7 model. For the Earth’s atmosphere, we selected a reference height of 30 km, even though it is in the stratosphere and outside the I/T region. This choice of lower boundary is to allow comparison of electrodynamics between the two atmospheres later in this review. We set the top of the chromosphere where the pressure has decreased from its base value by six orders of magnitude; this occurs at an altitude of 1989 km, above which the temperature rises steeply in the transition region. Ten orders of magnitude of pressure reduction were included in the I/T, which extends about 640 km in altitude.

2.2 Neutral Gas, Plasma, and Magnetic Field

Figure 1 depicts the temperature profiles in the ALC7 chromosphere and the TIMEGCM I/T. Note that we do not show the region above 2000 km in the chromosphere model, where the temperature rapidly increases in the transition region. The physical description of the chromosphere we will present later in terms of transitions (in magnetization, conductivity, etc) does not require this upper region. Overall, the chromosphere is about one order of magnitude hotter (4,400–6,700 K) than the I/T (200–2,800 K). All species temperatures are assumed to be equal in the model chromosphere (left), whereas the neutral, electron and ion temperatures are allowed to differ in the model I/T (right). We also note that the I/T values are plotted down to the reference pressure level near 30 km (stratosphere), for comparison with the chromospheric profile, even though the I/T altitudes are at 80-600 km. Both the solar and Earth profiles show a decline with increasing altitude in the lower atmosphere to a minimum value, beyond which the temperature begins to rise in the upper atmosphere. In the chromosphere, the temperature increase is due to both local heating, the nature of which is not well understood, and downward heat conduction from the overlying, much hotter solar corona. The increasing ionization fraction of the chromosphere and transition region above
with height is a direct consequence of this temperature increase. In the I/T, on the other hand, the neutral gas is photo-ionized by incident UV radiation from the Sun. The excess kinetic energy of the liberated photoelectrons is thermalized by electron-electron collisions and is transferred to the ions and neutrals by collisional thermal equilibration, accompanied by additional electron liberation due to impact ionization. Additional heating processes contribute at high latitudes, including frictional heating and energetic particle precipitation in the polar caps. Because the equilibration rate is much higher between ions and neutrals than with electrons, the ion and neutral temperatures are essentially equal to each other and equal to (at low altitudes) or below (at mid altitudes) the temperature of the electrons. Above about 300 km altitude in the thermosphere, Coulomb coupling between electrons and ions becomes increasingly important, and the three temperatures increasingly separate, with electrons the hottest and neutrals the coolest. We speculate that a fully comprehensive model of the chromosphere would show similar qualitative trends in the coupling of the neutral and plasma temperatures, but the details of the temperature profiles of the species would depend sensitively on how the unknown heating mechanisms partition thermal energy among the particles.

The approximate altitude ranges of the ionospheric D, E, and F-layers are shown in the right-hand Panel of Figure 1. The lowest, D, layer of the ionosphere extends from about 60 to 90 km in altitude. Its dominant neutral is molecular nitrogen (N$_2$), while its dominant ion is nitric oxide (NO$^+$) photo-ionized by penetrating Ly $\alpha$ radiation at $\lambda$ 121.5 nm. Water cluster ions can also be significant in the D-layer. The middle, E-, layer extends upward from 90 km to about 150 km altitude. N$_2$ remains the dominant neutral species, but at this height solar soft X-ray and far UV radiation, together with chemical reactions, add molecular oxygen ions (O$_2^+$) to the otherwise NO$^+$ plasma. The highest, F-, layer ranges from about 180 km to well over 500 km in altitude. At these heights, due to molecular dissociation at the elevated temperatures and ionization by extreme ultraviolet radiation, atomic oxygen is dominant in both its neutral (O) and ionized (O$^+$) states.

In contrast to the rich compositional structure of the Earth’s I/T, the composition of the chromosphere is relatively uniform with altitude, consisting primarily of hydrogen (H, H$^+$), secondarily of helium (He, He$^+$) with a number density $\sim$ 10% that of hydrogen, and there-
after a smattering of minority neutrals and ions up to iron at much smaller concentrations. However, the radiation in the chromosphere is dominated by emission from minor ions in the lower chromosphere, such as calcium, magnesium and iron. Plasma-neutral coupling in the cool material of solar prominences (see \S 7) has been shown to lead to species separation and preferential draining of He relative to H (Gilbert et al. 2002, 2007). This occurs due to the very strong charge-exchange collisional coupling of H to H$^+$, which retains the majority hydrogen atoms while the minority helium atoms leak out much more freely. In the solar chromosphere, the constantly churning convection driven from below maintains the roughly uniform composition through turbulent mixing. In contrast, in the Earth’s atmosphere, above about 100 km (the turbopause) turbulent mixing is too weak to homogenize the atmosphere and maintain a uniform composition, so the atmospheric composition becomes stratified under gravitational attraction according to the species molecular weights.

The number densities of the neutral and plasma constituents of the atmospheres are shown in Figure 2. The neutral densities are nearly equal in the ALC7 chromosphere (left) and TIMEGCM I/T (right) over their common range of normalized pressures, falling from a value of about $10^{23}$ m$^{-3}$ where the base pressure was chosen in each atmosphere. In contrast, the plasma densities differ by several orders of magnitude between the chromosphere and the I/T, due to the combination of their order-of-magnitude temperature difference and to the disparate ionization processes that dominate in the two atmospheres. Thus, the neutral and plasma densities become nearly equal at the top of the Sun’s chromosphere, whereas in the Earth’s I/T region, the plasma density is much smaller than the neutral density throughout the displayed altitude range. Equality of the plasma and neutral densities in the I/T occurs only at much higher altitudes than those shown in our graphs. The dotted line below 70 km in Figure 2 denotes a region below the ionosphere proper, where the plasma density is small and poorly characterized in the TIMEGCM so we have elected to hold it fixed at its value at 70 km altitude.

These very similar neutral densities, but radically different plasma densities, in the chromosphere and I/T have important consequences for the roles of plasma-neutral coupling in
the dynamics of the mixture. The collision frequencies of ions on neutrals ($\nu_{in}$) and of neutrals on ions ($\nu_{ni}$) are shown in Figure 3. Clearly, the profile shapes mostly reflect those of the neutral and plasma densities, respectively. A dependence on the thermal speed of the colliding particles introduces variations directly proportional to the square root of the temperature, and inversely proportional to the square root of the average particle mass; the ion-neutral collision frequency $\nu_{in}$ therefore is about an order of magnitude higher in the hydrogen-dominated chromosphere than in the nitrogen and oxygen-dominated I/T. The frequency $\nu_{in}$ reaches 1 GHz at the base of the ALC7 chromosphere (left) and about 1 MHz at the base of the TIMEGCM D layer at an altitude of about 70 km (right). Therefore the ions respond very strongly to the neutrals at the base of both the chromosphere and ionospheric D-layer and respond well above. In contrast, the neutral-ion collision frequency $\nu_{ni}$ is relatively high ($\sim 1$ MHz) at the base of the chromosphere, but is miniscule ($\sim 1$ $\mu$Hz) at the base of the D layer; thus, the response of the neutrals to the ions is strong in the chromosphere but extremely weak in the I/T. As a result, it is generally a reasonable approximation to treat the neutral density and wind velocity as fixed in studies of the lower I/T, while feedback on the neutral gas from the ensuing plasma dynamics is ignored. However, this is a poor approximation in Earth’s polar regions during geomagnetic storms, where the ions can strongly influence the neutral gas, and it is not well justified in the chromosphere, particularly for disturbances occurring at long scale lengths and low frequencies.

An explicit example of how these considerations apply to important chromospheric and ionospheric phenomena can be found in incompressible motions of the atmospheres. As will be shown by a linear analysis of the multifluid equations in §7, the motions are characterized by the Brunt-Väisälä frequency $N$ (Brunn 1927, Väisälä 1925),

$$N^2 = \frac{g}{L},$$

where $g$ is the gravitational acceleration and $L$ is the local scale height of the particle density, neutral or plasma. If the associated density is stably stratified (i.e., decreasing with height),
then the motions are purely oscillatory. If, on the other hand, the density is unstably stratified (increasing with height), then the motions consist of one exponentially damped and one exponentially growing mode, the Rayleigh-Taylor instability \cite{Rayleigh1882, Taylor1950}. A glance at Figure\ref{fig:2} indicates that the I/T must be susceptible to Rayleigh-Taylor instability, due to the high altitude peak in the plasma number density, in the $F$ layer. The plasma number density also increases with altitude for a small region of the model chromosphere.

Figure\ref{fig:3} shows the Brunt-Väisälä frequencies for the neutral gas and for the plasma, calculated from their density profiles in the ALC7 chromosphere (left) and the TIMEGCM I/T (right). The frequencies for the plasma are similar in magnitude to, but much more variable than, those of the neutral gas. The variability is due to changes in the slopes of the density profiles with height. The frequencies for the neutral gas turn out to be roughly equal in the chromosphere and I/T (between 0.01 and 0.05 Hz). This occurs because the Sun’s much stronger gravity is compensated for by the much higher thermal speed of its neutrals: the neutral frequency $N_n$ is essentially the ratio of those quantities, after using the pressure scale height to approximate the neutral-density scale height $L_n$. In both environments, the frequency $N_p$ set by $L_p$ is smaller than the ion-neutral collision frequency, $N_p/\nu_{in} < 1$, so that the Brunt-Väisälä oscillations (or Rayleigh-Taylor instabilities) of the plasma are affected by coupling to the neutrals. In the chromosphere, the oscillations and instabilities of the neutrals are similarly affected by coupling to the plasma, since there we have $N_n/\nu_{in} < 1$. However, the opposite is true in the I/T, where $N_n/\nu_{in} > 1$: the oscillations of the stably stratified neutral gas are unaffected by the plasma, and the neutral motion is essentially undisturbed by the evolution of the unstably stratified plasma. These contrasting consequences of the plasma-neutral coupling will be borne out by the analysis and numerical simulations shown in §7.

Both the Sun’s chromosphere and the Earth’s I/T are permeated by magnetic fields. The Lorentz force on charged particles acts in the direction perpendicular to the field, so that when the field is sufficiently strong, the properties of the plasma become highly anisotropic although, as will be shown later, anisotropy also depends on magnetization, the ratio of gyrofrequency to collision frequency, and so also depends on plasma temperatures. Some consequences of this anisotropy will be discussed in subsequent sections of the paper. For the general considerations presented here, we will assume that the magnetic field of the I/T is locally uniform with a field strength of $5.15 \times 10^{-5}$ T, which is a good approximation as the I/T magnetic field is basically a dipole. Combined with the 1D snapshot taken from the TIMEGCM model described above, this 1D, time-independent model for the I/T is simple but reasonable. The chromosphere’s magnetic field, by contrast, has a temporally and spatially varying magnetic field. For example, the magnetic field is known to decay with height, but the field value at the surface is different above quiet Sun regions compared to active regions. To capture some of this variance we adopt an approach similar to Goodman\cite{Goodman2000, Goodman2004a} and use a height-dependent 1D magnetic field model:

$$B(z) = B_0 \exp\left(-\gamma z \int_0^z \frac{dz'}{2L(z')}\right)$$

where $L(z) = \frac{k_BT}{m_e\mu_0}$ is the local scale height, and $\gamma = 0.75$. Furthermore we choose a range of values for $B_0, 10 - 1000$ G.

A key parameter governing the coupling of the fluid to the magnetic field is the so-called plasma $\beta$, the ratio of the thermal pressure ($P$) to the magnetic pressure ($B^2/2\mu_0$) appearing in the equations of motion,

$$\beta = \frac{2\mu_0P}{B^2}.$$
where $\mu_0$ is the magnetic permeability of free space. Values of $\beta$ derived from both the plasma and neutral pressures are displayed in Figure 4 for the ALC7 chromosphere (left) and the TIMEGCM I/T (right). This dimensionless number measures the relative strength of the pressure and magnetic forces exerted on the fluid. It also measures the local amplification of the ambient magnetic field that can be accomplished by stagnation-point flows that compress the field in the perpendicular direction and evacuate the thermal pressure in the parallel direction. Total pressure balance between the pre- ($B_0$, $P_0$) and post-compression ($B = B_0 + \delta B$, $P = 0$) states implies a fractional amplification of the field strength given by

$$\frac{\delta B}{B_0} = (1 + \beta_0)^{1/2} - 1,$$

with only the solution $\delta B > 0$ considered. Direct dynamical compression of the field by plasma motions not coupled to the neutrals can occur only up to strengths set by $\beta_p$ associated with the plasma pressure $P_p$. For the chromosphere, $\beta_p$ has an approximate maximum of 100 (Figure 4) and the resulting maximum amplification is $\sim 9$, while in the I/T $\beta_p$ has a maximum of $10^{-4}$ and so the maximum amplification is $\sim 5 \times 10^{-5}$. In principle, plasma motions strongly coupled by collisions to flows of the neutral gas could compress the field up to strengths set by $\beta_n$ associated with the neutral pressure $P_n$. Using maximum values of $\beta_n$ from Figure 4 of $2 \times 10^4$ for the chromosphere and $2 \times 10^5$ for the I/T, would allow amplification factors as large as 140 and 450, respectively. However, this process requires the neutral gas to couple sufficiently strongly to the magnetic field to maintain total force balance through the intermediary of fast neutral-ion collisions. This coupling in the I/T is far too weak to maintain such a balance; therefore, such strong terrestrial magnetic-field fluctuations are never observed. The coupling in the chromosphere is much stronger, and likely is responsible for at least some of the much larger magnetic-field fluctuations observed in the lower solar atmosphere. As already noted, the strongest chromospheric magnetic fields originate above sunspots, which form in the higher-pressure photosphere and convection
zone below the chromosphere. Typical neutral (and plasma) flows can range from km/s in convective cells to as large as 100 of km/s in chromospheric jets. In the lower I/T typical neutral flows are 100 m/s, but at higher latitudes during storms these can increase to 500 m/s or more.

2.3 Summary

The basic atmospheric profiles, and quantities derived from them, that have been discussed in this section clearly show both quantitative similarities and differences between the solar chromosphere and the terrestrial I/T. The neutral number densities, neutral beta, ion-neutral collision frequencies and Brunt-Väisälä frequencies of both neutrals and plasma track reasonably closely over the common range of normalized pressures in the two atmospheres. On the other hand, the ionized plasma number densities, plasma beta, and neutral-ion collision frequencies are very different, due to the much higher ionization fraction on the Sun vs. the Earth. The plasma is driven strongly by the neutral gas in both atmospheres, as a result, while the neutrals are driven by the ions fairly strongly in the chromosphere but relatively weakly in the I/T. Relative magnetic field fluctuations of order unity are ubiquitous on the Sun, but the fluctuations are far smaller in magnitude on the Earth. Both atmospheres exhibit regions of instability driven by a combination of gravity and convection where upward-increasing particle number densities occur, modulated by the frequency of collisions between the ions and neutrals. In the remainder of the paper, further implications of this rich combination of similarities and differences between the Sun’s chromosphere and Earth’s I/T will be elucidated.

3 Governing Equations

When determining the correct physical model to use for both the chromosphere and ionosphere, one must consider collision frequencies both among particles of a particular species \( \nu_{aa} \), and also between different species \( \nu_{ab} \), relative to the inverse timescales (or frequencies) of the system. At one extreme, where the system has high frequencies (relative to all collision frequencies), all species must be treated with a kinetic approach. At the other (low frequency) extreme, the entire mixture of species can be treated with a single fluid model, where the average fluid variables can be evolved. We assume that both the chromosphere and ionosphere/thermosphere can be approximated by an intermediate multi-fluid model. This model assumes that the system frequency is smaller than the collision frequency among particles of each individual species \( \nu_{aa} \), i.e., each species can be treated as a fluid since each species is collision dominated, in which case the single particle distribution functions are linear perturbations of Maxwellians, and the transport is classical (e.g., Braginskii 1965). However, we do not assume that the system frequency is smaller than the collision frequencies between species \( \nu_{ab} \), which is borne out by the right Panel in Figure 3. Hence we do not assume that a single-fluid model can be used, and instead use a fluid model for each species, with the collisional transfer of number, momentum and energy between each species explicitly included in the equations. We point out that the assumption that each species can be treated as a fluid may not always be valid. There is an intermediate regime in which the system frequency can exceed collision frequencies among species \( \nu_{aa} \) by some amount for some time, but in which classical transport is still dominant, and the corresponding multi-fluid model remains valid (Braginskii 1965).
Our multi-fluid model solves the continuity, momentum and energy equation for all three components: ions, electrons and neutrals. However, in some cases we can subsequently combine the ion and electron equations to create a two-fluid model which solves the continuity, momentum and energy equation separately for the neutral fluid and the ionized fluid. In doing so we neglect electron inertia in the momentum equations and electron-ion drift is captured by the Hall term in the generalized Ohm’s law (see below). This multi-fluid model is especially relevant when magnetic fields are present, as they directly affect the ionized component of the mixture but not the neutral component. The details of the model’s application in the chromosphere can be found in Leake et al. (2012) and Leake et al. (2013), where it is shown that the ion and neutral fluids can decouple as current sheets form and thin in the chromosphere, and hence a multi-fluid model is vital. Other applications include high frequency waves from flares propagating down into the chromosphere (e.g., Voitenko and Goossens 2002, Kigure et al. 2010, Edmondson et al. 2011, Russell and Fletcher 2013), and waves interacting non-linearly to create flows and currents on smaller and smaller scales (see review by Narain and Ulmschneider 1990). Due to a low ionization level in the I/T compared to the chromosphere, neutral-ion collisions can be much less frequent than in the chromosphere, and phenomena that occur on timescales of minutes require a multi-fluid model (Roble et al. 1988, Richmond et al. 1992, Roble and Ridley 1994, Fuller-Rowell et al. 1996, Millward et al. 1996). The multi-fluid approach in the I/T system has been discussed in detail, where the individual average drift velocity for ions, electrons and neutrals are used to define the reference frame for their respective transport equations (e.g., Schunk 1975, 1977, Conrad and Schunk 1979, St. Maurice and Schunk 1981, Schunk and Sojka 1982). The observed significant differences in velocity and temperature of these various species warrant such an approach (St. Maurice and Hanson 1982, 1984).

In the multi-fluid model, the three fluids, ions \(i\), electrons \(e\), and neutrals \(n\), can undergo recombination and ionization interactions. The ions are assumed to be singly ionized which is a good approximation for the dominant species in both atmospheres. The rate of loss of ions/electrons (or gain of neutrals) due to recombination is \(\Gamma_{\text{rec}}\), and the rate of gain of ions/electrons (or loss of neutrals) due to ionization is \(\Gamma_{\text{ion}}\).

### 3.1 Continuity

Assuming charge quasi-neutrality \(n_i = n_e = n\), where \(n_\alpha\) is the number density of species \(\alpha\), the ion, electron, and neutral continuity equations are:

\[
\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i V_i) = \Gamma_{\text{ion}} - \Gamma_{\text{rec}},
\]

\[
\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e V_e) = \Gamma_{\text{ion}} - \Gamma_{\text{rec}},
\]

\[
\frac{\partial n_n}{\partial t} + \nabla \cdot (n_n V_n) = -\Gamma_{\text{ion}} + \Gamma_{\text{rec}}.
\]

Subtracting Equation (6) from (5) then yields

\[
\nabla \cdot (n_i [V_i - V_e]) = 0,
\]

which is simply a statement of current conservation in a quasi-neutral plasma (see also Equation (3.4) below).
3.2 Momentum

The ion, electron and neutral momentum equations are shown below:

\[
\frac{\partial}{\partial t}(m_n n_i \mathbf{V}_i) + \nabla \cdot (m_n n_i \mathbf{V}_i \mathbf{V}_i + \mathbf{P}_i) = en(\mathbf{E} + \mathbf{V}_i \times \mathbf{B}) + m_n n_i \mathbf{g} + \mathbf{R}_{\text{en}}^i + \mathbf{R}_{\text{in}}^i + \Gamma_{\text{ion}} m_i \mathbf{V}_n - \Gamma_{\text{rec}} m_i \mathbf{V}_i,
\]

\[
\frac{\partial}{\partial t}(m_e n_e \mathbf{V}_e) + \nabla \cdot (m_e n_e \mathbf{V}_e \mathbf{V}_e + \mathbf{P}_e) = -en(\mathbf{E} + \mathbf{V}_e \times \mathbf{B}) + m_e n_e \mathbf{g} + \mathbf{R}_{\text{en}}^e + \mathbf{R}_{\text{in}}^e + \Gamma_{\text{ion}} m_e \mathbf{V}_n - \Gamma_{\text{rec}} m_e \mathbf{V}_e,
\]

\[
\frac{\partial}{\partial t}(m_n n_n \mathbf{V}_n) + \nabla \cdot (m_n n_n \mathbf{V}_n \mathbf{V}_n + \mathbf{P}_n) = m_n n_e \mathbf{g} + \mathbf{R}_{\text{en}}^n + \mathbf{R}_{\text{in}}^n + \Gamma_{\text{ion}} m_i \mathbf{V}_n + \Gamma_{\text{rec}} (m_i \mathbf{V}_i + m_e \mathbf{V}_e).
\]

The velocity and mass of species \( \alpha \) are denoted \( \mathbf{V}_\alpha \) and \( m_\alpha \), respectively. The electric and magnetic field are denoted \( \mathbf{E} \) and \( \mathbf{B} \), respectively. The pressure tensor is \( \mathbf{P}_\alpha = P_\alpha \mathbf{I} + \pi_\alpha \mathbf{I} \) where \( P_\alpha \) is the scalar pressure and \( \pi_\alpha \) is the viscous stress tensor. For the neutral fluid this is isotropic, but for electrons and ions this has elements that are functions of the particle magnetization (the magnetization is the ratio of the gyrofrequency to the collision frequency), and is anisotropic. The reader can find derivations of these viscous stress tensors in [Braginskii](1965). The Coriolis force has been omitted for simplicity in further derivations, although its effects can be important in the I/T (e.g., Fuller-Rowell et al. 1984). On the Sun, only long time-scale phenomena are affected by this force, for example, the rotation of sunspot groups and the large-scale solar dynamo.

The transfer of momentum to species \( \alpha \) due to a combination of identity-preserving collisions and charge-exchange collisions with species \( \beta \) is given by

\[
\mathbf{R}^{\alpha \beta}_\alpha \equiv m_{\alpha \beta} n_\alpha v_{\alpha \beta}(\mathbf{V}_\beta - \mathbf{V}_\alpha),
\]

where \( m_{\alpha \beta} = m_\alpha m_\beta / (m_\alpha + m_\beta) \) and \( R^{\alpha \beta}_\alpha = -R^{\beta \alpha}_\beta \). The relative importance of charge-exchange collisions and identity-preserving collisions varies in the solar atmosphere and the I/T, but for the purposes of a comparison of the two plasma environments we combine the two types of collisions into one general “interaction”. The collision frequency \( v_{\alpha \beta} \) is then defined using a solid body approximation with a relevant choice of cross-section (e.g., Leake et al. 2013). The above equations are for a single “average” species of ions of mass \( m_i \) and neutrals of mass \( m_n \).

The full three-fluid system of a partially, singly, ionized fluid contains the three momentum equations described above. Typically the ions and electron equations are combined into a plasma equation, and some assumptions are made with respect to the ratio of the electron to ion mass ratio. Adding the ion and electron equations, and assuming quasi-neutrality \( (n_e = n_i = n) \), gives

\[
\frac{\partial}{\partial t}(\rho_p \mathbf{V}_p) + \nabla \cdot (\rho_p \mathbf{V}_p \mathbf{V}_p + \mathbf{P}_p) = \mathbf{J} \times \mathbf{B} + \mathbf{R}_{\text{en}}^e + \mathbf{R}_{\text{in}}^e + \Gamma_{\text{ion}} \rho_p \mathbf{V}_n - \Gamma_{\text{rec}} \rho_p \mathbf{V}_p,
\]

where \( \rho_p = \rho_i + \rho_e = n(m_i + m_e) \) is the plasma density, \( \mathbf{P}_p = \mathbf{P}_i + \mathbf{P}_e \) is the plasma pressure, and

\[
\mathbf{V}_p = \frac{(\rho_i \mathbf{V}_i + \rho_e \mathbf{V}_e)}{\rho_p} = \mathbf{V}_i - \frac{m_e}{m_i} \mathbf{J} / \rho_p
\]
is the plasma velocity. For small electron-ion drifts relative to the ion velocity, we can make the assumption \( V_p = V_i \), but in locations where the electrons undergo different dynamics than the ions, this is not the case. A two-fluid model makes this assumption, solving one equation for the plasma, and one for the neutrals. To complete the full model, however, the electron equation of motion is kept, with significant simplifications, as will be shown later in the Ohm’s law section.

3.3 Energy

The full derivation of the energy equations for all three components can be found in [Meier and Shumlak 2012]. We denote the individual thermal energy \( \varepsilon_\alpha = \rho_\alpha k_B T_\alpha / (\gamma_\alpha - 1) \) where \( T_\alpha \) is the temperature, and the internal energy \( \Psi_\alpha = \rho_\alpha V_\alpha^2 / 2 + P_\alpha / (\gamma_\alpha - 1) \) of species \( \alpha \). Then the individual conservation equation for \( \Psi_\alpha \), neglecting ionization and recombination, is

\[
\frac{\partial \Psi_\alpha}{\partial t} + \nabla \cdot (\Psi_\alpha V_\alpha + \Psi_\alpha \cdot \nabla \alpha + h_\alpha) = V_\alpha \cdot \left( q_\alpha n_\alpha E + \sum_{\beta \neq \alpha} R_\alpha^{\alpha \beta} + m_\alpha \nu_\alpha \right) + \sum_{\beta \neq \alpha} Q_\alpha^{\alpha \beta} + S_\alpha + U_\alpha
\]

where \( \rho_\alpha = m_\alpha n_\alpha \), \( \gamma_\alpha \) is the ratio of specific heats, \( q_\alpha \) is the charge, \( Q_\alpha^{\alpha \beta} = \frac{1}{2} R_\alpha^{\alpha \beta} \cdot (V_\beta - V_\alpha) + \frac{m_\alpha}{m_\alpha} n_\alpha \nu_\alpha (T_\beta - T_\alpha) \)

is the heating of species \( \alpha \) due to collisions with species \( \beta \), \( S_\alpha \) and \( U_\alpha \) are radiative and chemical process contributions, respectively, and \( h_\alpha \) is the heat flux. This heat flux involves a thermal conductivity tensor that depends on the magnetization of the species, see [Braginskii 1965].

3.4 Maxwell’s Equations

The equations relating changes in the electric and magnetic field are

\[
\nabla \times E = -\frac{\partial B}{\partial t},
\]

\[
\nabla \times B = \varepsilon_0 \mu_0 \frac{\partial E}{\partial t} + \mu_0 J,
\]

\[
\nabla \cdot B = 0,
\]

\[
\nabla \cdot E = \sigma / \varepsilon_0.
\]

Here \( \sigma \) is the charge density, and \( \varepsilon_0, \mu_0 \) are the permittivity and permeability of free space. The second of these Maxwell equations can be written as an equation for \( J \):

\[
J = \frac{\nabla \times B}{\mu_0} - \frac{\varepsilon_0}{\varepsilon_0} \frac{\partial E}{\partial t},
\]

the second term being the displacement current. Taking the ratio of the magnitude of the two terms on the right-hand side gives

\[
\frac{|\nabla \times B|}{|\varepsilon_0 \frac{\partial E}{\partial t}|} \sim \frac{B_0 \sigma_0}{L_0 \mu_0 \varepsilon_0} \sim \frac{\sigma_0}{c^2 L_0^2}
\]
where \( c = (\mu_0\varepsilon_0)^{-1/2} \) is the speed of light, subscripts “0” indicate representative values for each variable, and \( B_0/t_0 \sim E_0/L_0 \) is used from the first Maxwell equation. From Equation (22), the displacement current can be ignored in the equation for \( J \) if the system time scale \( t_0 \) is longer than the time it takes light to travel across the system \( L_0/c \). This is a reasonable assumption in the Sun and the Earth’s atmospheres, and under this assumption the evolution of the electric and magnetic fields is given by the two equations:

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad \mathbf{J} = \frac{\nabla \times \mathbf{B}}{\mu_0}
\]

and an equation relating \( \mathbf{E} \) and \( \mathbf{J} \). For reasons discussed in detail later, in ionospheric literature, the current conservation equation \( \nabla \cdot \mathbf{J} = 0 \) is often invoked with the \( \mathbf{B} \) field represented as a conservative field. We note that this relation follows immediately from \( \mathbf{J} = \frac{\nabla \times \mathbf{B}}{\mu_0} \) and the identity \( \nabla \cdot (\nabla \times \mathbf{B}) = 0 \), and is equivalent to Equation (8).

As we shall see later, the relationship between \( \mathbf{E} \) and \( \mathbf{J} \) is often given by the generalized Ohm’s law, obtained from the electron momentum equation, and for low-frequency phenomena has the form

\[
\mathbf{E} = -(\mathbf{V} \times \mathbf{B}) + \eta \mathbf{J}
\]

where \( \eta \) is the resistivity tensor, and \( \mathbf{V} \) is a reference bulk flow. Ionospheric physicists prefer the use of the conductivity formulation \( \mathbf{J} = \sigma \cdot \mathbf{E} \), where \( \mathbf{E} \) is the electric field in the bulk flow reference frame, since electric fields can persist due to the generally low conductivity of the plasma, while solar physicists generally use the resistivity formulation \( \mathbf{E} = \frac{\eta}{\sigma} \mathbf{J} \). As we shall see, the chromosphere is highly conductive, and so the resistivity is low. This means that \((\mathbf{V} \times \mathbf{B})\) dominates in the equation for \( \mathbf{E} \), which when inserted into Equation (17) tells us that the magnetic field moves with the bulk flow \( \mathbf{V} \) and the electric field, when considered in the rest frame of the bulk flow \( \mathbf{E} + (\mathbf{V} \times \mathbf{B}) \), is negligible. We will also show that the I/T, by contrast, is not very conductive, and so \( \eta \mathbf{J} \) dominates, so there can be persistent electric fields in that system. Thus the chromosphere is magnetohydrodynamic in nature, while the I/T is electrodynamic.

In ionospheric studies, the assumption that \( \nabla \times \mathbf{E} = 0 \) is often made (electric field is related to a gradient in the potential), which is equivalent to \( \partial \mathbf{B}/\partial t = 0 \) via (17). This may seem trivial to justify, by noting that the magnetic field is mostly static \( \mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B} \) where \(|\delta \mathbf{B}|/|\mathbf{B}_0| \ll 1 \). However, given that the only two terms in Equation (17) are \( \nabla \times \mathbf{E} \) and \( \partial \mathbf{B}/\partial t \), there are no quantities to compare them to and no obvious criteria to neglect them. In addition, Vasyliunas [2012] showed that even if \(|\delta \mathbf{B}|/|\mathbf{B}_0| \ll 1 \), this does not imply that the ratio of the magnitudes of the non-potential electric field \( \delta \mathbf{E} \) to the total electric field \( \mathbf{E} \), \(|\delta \mathbf{E}|/|\mathbf{E}| \), also has to be small. However we can estimate \(|\delta \mathbf{E}| \) relative to \(|\mathbf{E}| \). From \( \partial \mathbf{B}/\partial t = -\nabla \times \mathbf{E} \) we have \(|\delta \mathbf{E}| \sim (|\delta \mathbf{B}|/|\mathbf{B}_0|)L_E \) where \( L_E \) is the gradient length scale of \( \mathbf{E} \) and \( t_B \) is the time scale of magnetic perturbations. The Earth’s auroral oval has the largest field variations, and here \( L_E \sim 10^6 \) m and \((\delta \mathbf{B}/|\mathbf{B}_0|) \sim 10^{-9} \) T/s which gives an inductive electric field \(|\delta \mathbf{E}| \) of 1 mV/m, which is much less than the static electric field \(|\mathbf{E}| \) (10-100) mV/m, and so in this case the assumption is not unreasonable. However, the approximation that \( \partial \mathbf{B}/\partial t = 0 \) has limitations when considering the coupled magnetosphere-I/T system, particularly during substorms when enhanced signals observed on the ground and in the ionosphere are partially due to the reflection of the magnetospheric waves which cannot be described within the long timescale approximation.

The system of momentum equations and Maxwell’s equations contain time derivatives for all four variables \( \mathbf{B}, \mathbf{V}, \mathbf{E}, \) and \( \mathbf{J} \). As will be shown in this section, under certain assumptions (e.g. low-frequency or long timescale), we can ignore \( \partial \mathbf{J}/\partial t \) and rewrite the Ohm’s law
as an equation which relates the current density \( J \) to the electric field \( E \) in a specific rest frame using either the conductivity \( \sigma \) or resistivity \( \eta \) tensor. The time derivative \( \partial E / \partial t \) can also be dropped for timescales longer than the light crossing time of the system (see above). This leaves just the time derivatives for \( B \) and \( V \); we will discuss this MHD approach in the following two sections.

The fact that the chromosphere can in general be approximated by ideal MHD (\( V \times B \) dominates), and that the I/T is electrodynamic (\( \eta J \) dominates), as well as the fact that for slow processes in the ionosphere the electrodynamics are assumed to not affect the static magnetic field, leads to two different approaches when describing the chromosphere and I/T. Parker (2007) refers to these approaches as the B,V and E,J paradigms, used for the chromosphere and I/T, respectively. In the B,V paradigm, the magnetic field and velocity are primary variables, and each has an evolution equation, while the electric field and current are secondary variables derived from the primary variables. In the E,J paradigm, as Parker (2007) describes it, the reverse is assumed, the electric field and current are the primary variables. An example is the discussion later in this review of neutral wind dynamos. However, as will be discussed later in more detail, both paradigms have been used to explain the same phenomena, and can sometimes, if the correct criteria are met, arrive at the same answer for global or averaged quantities. Parker (2007) and Vasyliunas (2012) argue that the B,V paradigm is actually the only tractable approach, and the E,J paradigm leads to misunderstanding of the physics. We will discuss the validity of the E,J paradigm in later sections.

3.5 Ohm’s Law

3.5.1 Derivation of Generalized Ohm’s law

As mentioned above, the two-fluid model includes two momentum equations for \( \partial V_i / \partial t \) and \( \partial V_n / \partial t \), having made the assumption that \( V_i = V_p \), based on \( m_e \ll m_i \). The electron equation of motion must be kept to describe the entire system, and this equation is often called the generalized Ohm’s law, because ignoring the electron inertia term in this equation allows one to write a relationship between the electric field in the rest frame of the electrons (or plasma, or indeed any other reference frame) and current density. In the literature this is sometimes represented as an equation for \( \partial J / \partial t \) (Vasyliunas 2012), which along with similar equations for the time derivatives for \( V, E, \) and \( B \) closes the complete system for magnetohydrodynamics. Obtaining an equation for \( \partial J / \partial t \) involves linear combinations of the three full momentum equations. However, here we present a derivation, valid for the chromosphere and I/T, that starts with just the electron equation of motion, and thus contains a term proportional to \( \partial V_e / \partial t \).

The electron equation of motion \( (10) \), in the limit that ionization and recombination rates are faster than the system frequencies, is

\[
\frac{\partial}{\partial t} (m_e n V_e) + \nabla \cdot (m_e n V_e V_e + P_e) = -e n (E + V_e \times B) + m_e n g + R_e^{ei} + R_e^{en},
\]

which, using the definitions of \( R_{ab}^{ij} \), \( J = e n (V_i - V_e) \), and \( W = V_i - V_n \), and using the electron continuity equation in the fast ionization/recombination limit, can be written as

\[
E V_e \equiv E + (V_e \times B) = \left( \frac{m_e V_{en} + m_e V_{en}}{e n} \right) J - \left( \frac{m_e}{e} \frac{d}{dt} (V_e) + g \right) \frac{V_e \cdot P_e}{e n} - \frac{m_e V_{en}}{e} W
\]

(26)
where \( \frac{d}{dt} \) is the Lagrangian derivative \( [\frac{\partial}{\partial t} + \mathbf{V}_e \cdot \nabla] \). The LHS of this equation is the electric field in the rest frame of the electron fluid \( \mathbf{E}^V \). One can arbitrarily redefine this equation using \( \mathbf{E}^V \) for any velocity \( \mathbf{V} \) as long as one can relate \( \mathbf{V} \) to \( \mathbf{V}_e \). For a fully ionized collisional plasma, the rest frame of the electrons is not the most practical of frames. More practical would be the plasma rest frame (see Equation 14). For a very weakly ionized plasma, such as the lower chromosphere, and most of the I-T, the neutral rest frame using \( \mathbf{V}_n \) is more appropriate, while for the upper chromosphere the center of mass frame, defined using \( \mathbf{V}_{CM} = (\rho_i \mathbf{V}_i + \rho_e \mathbf{V}_e + \rho_n \mathbf{V}_n)/(\rho_i + \rho_e + \rho_n) \) is the better choice. Before redefining the generalized Ohm’s law for these rest frames, at this point we rewrite the above equation in terms of the magnetizations (the reduced-mass gyrofrequencies \( \Omega_{\alpha \beta} = eB/m_{\alpha \beta} \) divided by the collision frequencies \( \nu_{\alpha \beta} \)):

\[
\begin{align*}
    k_{in} &= \frac{eB}{m_{in} v_{in}}, \\
    k_{en} &= \frac{eB}{m_{en} v_{en}}, \\
    k_{ei} &= \frac{eB}{m_{en} v_{ei}}, \\
    k_e &= \frac{1}{k_{en}} + \frac{1}{k_{ei}}.
\end{align*}
\]

The magnetizations are measures of the ability of the ions and electrons to freely gyrate around the magnetic field. For example, if \( k_e \gg 1 \), then the gyration of the electrons around the magnetic field is largely unaffected by collisions with ions and neutrals. When \( k_e \ll 1 \), then the collisions dominate over the gyration. The same situation applies to the ions. Note that ion-electron collisions hardly affect the gyration of the ions, so \( k_i \), defined in the same way as \( k_e \), is approximately \( k_{in} \), and from hereon, we use \( k_{in} \) instead of \( k_i \).

Equation (26) can be rewritten as:

\[
\mathbf{E}^V = \left( \frac{1}{k_{ei}} + \frac{1}{k_{en}} \right) \mathbf{B} \mathbf{J} - \left( \frac{m_e}{e} \frac{d}{dt} (\mathbf{V}_e) + \mathbf{g} \right) - \frac{\nabla \cdot \mathbf{P}_e}{en} - \frac{B}{k_{en}} \mathbf{W}.
\] (31)

An expression for \( \mathbf{W} \) is needed to close this equation. This can be done by a linear combination of the plasma and neutral momentum equations. Subtracting the neutral momentum equation, divided by \( \rho_n \), from the plasma momentum equation, divided by \( \rho_p \), rearranging to make \( \mathbf{W} \) the subject, and using the magnetization definitions above, gives:

\[
\begin{align*}
    \mathbf{W} &= \frac{k_{in}}{k_{en} + k_{in}} \left[ \mathbf{J} + k_{en} \left( \frac{1}{en} \mathbf{J} + k_{en} \left[ \xi_n \mathbf{J} \times \mathbf{B} + \xi_p \nabla \cdot \mathbf{P}_p - \xi_n \nabla \cdot \mathbf{P}_n - \xi_n \rho_p \frac{d}{dt} (\mathbf{V}_p - \mathbf{V}_n) \right] \right] eBn \\
    \xi_n &= \frac{\rho_n}{(\rho_p + \rho_n)}, \quad \xi_p = 1 - \xi_n.
\end{align*}
\] (32)

Inserting Equation (32) into Equation (31) gives

\[
\begin{align*}
    \mathbf{E}^V &= \left( \frac{1}{k_{ei}} + \frac{1}{k_{en} + k_{in}} \right) \mathbf{B} \mathbf{J} - \left( \frac{m_e}{e} \frac{d}{dt} (\mathbf{V}_e) - \mathbf{g} \right) - \frac{\nabla \cdot \mathbf{P}_e}{en} \\
    &\quad - \frac{k_{in}}{k_{en} + k_{in}} \left( \xi_n \mathbf{J} \times \mathbf{B} + \xi_p \nabla \cdot \mathbf{P}_p - \xi_n \nabla \cdot \mathbf{P}_n - \xi_p \rho_p \frac{d}{dt} (\mathbf{V}_p - \mathbf{V}_n) \right) eBn \\
    \xi_n &= \frac{\rho_n}{(\rho_p + \rho_n)}, \quad \xi_p = 1 - \xi_n.
\end{align*}
\] (33)
3.5.2 Low Frequency Limit of Ohm’s Law

Equation (33) is the generalized Ohm’s law, derived from the momentum equations, using only the assumption that the ionization and recombination rates are much faster than the system frequency. Now we examine this equation to see under what circumstances it can be simplified. We can non-dimensionalize Equation (33) by expressing each variable $A$ as $A = A_0 \bar{A}$ where $\bar{A}$ has no dimensions. The dimensional constants $A_0$ are related to each other. For example, if $L_0$ is the system length, $B_0$ is the system magnetic field strength, and $n_0$ is the system number density of plasma, then $v_0 = B_0 / \sqrt{\mu_0 m_e n_0}$ is the system velocity, $J_0 = B_0 / \mu_0 L_0$ is the system current density, and $f_0 = v_0 / L_0$ is the system frequency (or inverse time scale). We can then remove the dimensions from Equation (33) by dividing it by $v_0 B_0$. Using the following definitions of electron gyrofrequency ($\Omega_e$), ion inertial scale length ($d_i$), ion plasma frequency ($\omega_{pi,i}$), electron inertial scale length ($d_e$), and electron plasma frequency ($\omega_{pe,e}$):

$$\Omega_e = e B_0 / m_e, \quad d_i = \frac{c}{\omega_{pi,i}}, \quad \omega_{pi,i}^2 = \frac{e^2 n_0}{m_i e_0}, \quad d_e = \frac{c}{\omega_{pe,e}}, \quad \omega_{pe,e}^2 = \frac{e^2 n_0}{m_e e_0}$$

(34)

and noting that $k_{en} > k_{in}$ due to $m_e < m_i$, yields the dimensionless equation

$$\mathbf{E} \cdot \mathbf{v}_e = \left( \frac{v_e}{f_0} \right) \left( \frac{d_e}{L_0} \right)^2 \frac{\mathbf{j}}{n} - \left( \frac{d_e}{L_0} \right)^2 \frac{d}{dt} (\mathbf{v}_e) - \left( \frac{f_0}{\Omega_e} \right) \mathbf{g} - \left( \frac{d_i}{L_0} \right) \left( \frac{\nabla \cdot \mathbf{v}_e}{n} \right)$$

$$- \frac{k_{in}}{k_{en} + k_{in}} \left( \frac{d_i}{L_0} \right) \left( \frac{\xi_e \mathbf{j} \times \mathbf{B} + \xi_p \mathbf{V}_n \mathbf{V}_p - \xi_e \mathbf{j} \nabla \cdot \mathbf{V}_n - \xi_p \mathbf{j} \nabla \cdot \mathbf{V}_p - \left( \frac{k_e}{v_{en}} \right) \xi_n \frac{d}{dt} (\mathbf{V}_p - \mathbf{V}_n) }{n} \right)$$

(35)

where $\mathbf{g} = 274$, the dimensionless gravitational acceleration at the surface of the Sun. Let us assume that that $\mathbf{E} + (\nabla \times \mathbf{B})$ is of order 1. Then we must compare the pre-factors for the terms on the RHS to see which can be neglected. For length scales longer than the electron inertial scale $L_0 \gg d_e$, the $\partial \mathbf{V}_e / \partial t$ term can be neglected. For the chromosphere, using the smallest density from Figure 2 of $10^{17}$ m$^{-3}$ gives a maximum for $d_e$ of 1.7 cm. For the ionosphere, using the smallest density from Figure 2 of $10^9$ m$^{-3}$ gives 170 m. Hence, it is safe to neglect the $\partial \mathbf{V}_e / \partial t$ term in the chromosphere, and appropriate in the I/T for lengths larger than a km. The first term in Equation (35) has a pre-factor $\left( \frac{v_e}{f_0} \right) \left( \frac{d_e}{L_0} \right)^2$. For the high frequency chromosphere, where $v_e$ is larger than $10^7$ Hz (see Figure 5), and in the lower I/T, the pre-factor $\left( \frac{v_e}{f_0} \right) \left( \frac{d_e}{L_0} \right)^2$ can be non-negligible, and this term is retained. The last two terms on the RHS of Equation (35) have a pre-factor $d_i / L_0$. The ion inertial scale $d_i$ has a maximum value of about 1 m in the chromosphere. For the I/T, the ion inertial scale can be as large as 2 km. Hence this term can also be non-negligible for short length scale phenomena. Hence these terms are retained. However the $\mathbf{g}$ term and the very last term on the RHS, proportional to $\frac{d}{dt} (\mathbf{V}_p - \mathbf{V}_n)$ have pre-factors $f_0 / \Omega_e$ and $f_0 / v_{en}$, respectively, and so for low frequency phenomena compared to the ion-neutral collisional frequency and electron gyro-frequency, they can be ignored. At the base of both atmospheres this is satisfied for frequencies much less than 10 MHz, or for periods longer than $10^{-7}$ seconds or longer. However, as the collision frequencies decrease exponentially with height, this is harder to satisfy in the upper I/T and chromosphere. In the chromosphere, only for frequencies much less than $10^2$ Hz (i.e periods larger than $10^{-2}$ s) is the low frequency limit satisfied, while for the I/T, it is for frequencies much less than $10^{-2}$ Hz (i.e periods larger than $10^2$ s) that
this is the case. Therefore in the upper I/T, changes of the order of minutes can violate the low frequency approximation.

Under these approximations, the low frequency Ohm’s law, in the rest frame of the electrons, is

$$E^e = \left( \frac{1}{k_{ei}} + \frac{1}{k_{en} + k_{in}} \right) \frac{B}{en} J - \left( \frac{\bar{\xi}_{in} k_{in}}{k_{en} + k_{in}} \right) \frac{B}{en} J \times \hat{b} - \frac{\nabla \cdot \Pi}{en} - G$$ \hspace{1cm} (36)

where

$$G = \left( \frac{k_{in}}{k_{en} + k_{in}} \right) \left( \frac{\bar{\xi}_p \nabla \cdot \Pi_n - \bar{\xi}_n \nabla \cdot \Pi}{en} \right).$$ \hspace{1cm} (37)

### 3.5.3 Ohm’s Law in Different Fluid Frames

As stated above, the electron frame is not a practical choice of frame for the chromosphere and I/T, as in general the observed motions are those of the bulk plasma or neutrals. One can transform the Ohm’s law (36) to the bulk plasma frame via $V_e = V_p - \frac{m_i}{m_i + m_e} J / en$, and noting that $m_i + m_e \approx m_i$ to give

$$E^\rho \equiv E + (V_p \times B) = \left( \frac{1}{k_{ei}} + \frac{1}{k_{en} + k_{in}} \right) \frac{B}{en} J + \left( 1 - \bar{\xi}_{in} k_{in} \right) \frac{B}{en} J \times \hat{b} - G$$ \hspace{1cm} (38)

The second term is the so-called Hall term and is important at smaller scales when the electron and ion velocities can diverge.

Alternatively, one could use the center of mass frame, given by $V_{CM} \equiv (\rho_i V_i + \rho_e V_e + \rho_n V_n) / (\rho_i + \rho_e + \rho_n)$. We can rewrite this as $V_e = V_{CM} - \bar{\xi}_n W - J / en$. In the low frequency limit we applied to Equation (32) we find that Equation (32) reduces to

$$W = \frac{k_{in}}{k_{en} + k_{in}} \left\{ \frac{J}{en} + k_{en} \left( \frac{\bar{\xi}_n J \times \hat{b}}{en} \right) \right\} + k_{en} G.$$ \hspace{1cm} (39)

and so

$$E^{V_{CM}} \equiv E + (V_{CM} \times B) = \left( \frac{1}{k_{ei}} + \frac{1}{k_{en} + k_{in}} \right) \frac{B}{en} J + \left( 1 - \frac{2\bar{\xi}_n k_{in}}{k_{en} + k_{in}} \right) \frac{B}{en} J \times \hat{b} + \left( \frac{\bar{\xi}_n k_{en} k_{in}}{k_{en} + k_{in}} \right) \frac{B}{en} J \perp - G - \bar{\xi}_n k_{en} G \times B$$ \hspace{1cm} (40)

where an additional $J \times \hat{b} \times \hat{b}$ term appears from $W \times B$. The quantity

$$J_\perp = \hat{b} \times (J \times \hat{b}) = J - J \parallel = J - (J \cdot \hat{b}) \hat{b}.$$ \hspace{1cm} (41)

Finally, it is also common to transform to the neutral frame of reference, $V_n$, via $V_e = V_n + W - J / en$ to give:

$$E^{V_n} \equiv E + (V_n \times B) = \left( \frac{1}{k_{ei}} + \frac{1}{k_{en} + k_{in}} \right) \frac{B}{en} J + \left( \frac{k_{en} - \bar{\xi}_n k_{in}}{k_{en} + k_{in}} \right) \frac{B}{en} J \times \hat{b} + \left( \frac{\bar{\xi}_n k_{en} k_{in}}{k_{en} + k_{in}} \right) \frac{B}{en} J \perp - G - k_{en} G \times B.$$ \hspace{1cm} (42)

It is important to note the difference in the terms in the two equations (40) and (42). Both are the low frequency Ohm’s law for a partially ionized mixture, but the former is cast in the rest frame of the center of mass velocity, and the latter is cast in the rest frame of the neutrals. The two equations are equivalent, but end up with different pre-factors on the right.
hand side. The pre-factor in front of the $J_\perp$ term, which will be discussed later in terms of the Pedersen conductivity or Pedersen resistivity, contains $\xi_n$ for the neutral frame equation and a $\xi^2_n$ for the center of mass equation, which is purely a result of the choice of frame. To allow for a collaborative discussion of the chromosphere and I/T in the context of the generalized Ohm’s law, we will use the neutral frame equation (42) from hereon, but will refer to other frames where necessary. As will be seen in §6.2 changing the reference frame leads to different terms appearing in the thermal energy equations for the individual species’ temperatures, but cannot change the final values for these temperatures which determine the ionization faction.

3.5.4 Cold Plasma Approximation

Under the cold plasma approximation, we can drop pressure gradient terms relative to Lorentz forces, For a partially ionized mixture this involves ignoring the terms $\xi_p \nabla \cdot P_p$, $\xi_n \nabla \cdot P_p$, and $\nabla \cdot P_e$ relative to $J \times B$. This approximation is valid if both the plasma ionization ($\xi_p \sim n/n_e$) and plasma beta ($\beta_p \sim \mu_0 P_p/B^2$) are low at all locations. Looking at Figures 2 and 4 this is a reasonable approximation for the I/T where the background magnetic field is strong (relative to the strength of the plasma pressure). However, in the relatively denser lower chromosphere $\beta_p$ is not <$1$ and so the $\xi_n \nabla \cdot P_p$ and $\nabla \cdot P_e$ term are not always negligible relative to the $J \times B$ term. To facilitate a cross-disciplinary discussion of the two environments, we use the cold plasma approximation, but note that for self-consistent studies of the chromosphere, one must determine whether or not the pressure gradient terms in the Ohm’s law affect the evolution of the EM fields.

We now define the following resistivities:

$$\eta_\parallel = \frac{B}{en} \left( \frac{1}{k_{ei}} + \frac{1}{k_{en} + k_{in}} \right),$$

$$\eta_C = \frac{B}{en} \left( \xi_n k_{en} k_{in} \right),$$

$$\eta_P = \eta_\parallel + \eta_C,$$

$$\eta_H = \frac{B}{en} \left( \frac{k_{en} - \xi_n k_{in}}{k_{en} + k_{in}} \right).$$

The low-frequency, cold plasma, limit of Ohm’s law is then given by

$$E^V_n = \eta_\parallel J + \eta_H J \times \hat{b} + \eta_C J_\perp,$$

or

$$J_\parallel = \eta_\parallel J + \eta_H J \times \hat{b} + \eta_C J_\perp.$$  (47)

The quantities $\eta_P$ and $\eta_C = \eta_P - \eta_\parallel$ are also referred to as the Pedersen and Cowling resistivities (Cowling 1956). We can invert Equation (47) to give $J = \sigma \cdot E^V_n$ in terms of the parallel, Hall and Pedersen conductivities:

$$J = \sigma_\parallel E^V_n + \sigma_P E^V_\perp - \sigma_H E^V_\perp \times \hat{b}$$  (48)

where

$$\sigma_\parallel = \frac{1}{\eta_\parallel},$$

$$\sigma_P = \frac{\eta_P}{\eta_\parallel + \eta_H},$$

$$\sigma_H = \frac{\eta_H}{\eta_\parallel + \eta_H}.$$  (51)
Ohm’s law, whether expressed in the form $E^V = \eta \cdot J$ or $J = \sigma \cdot E^V$, can be derived in a number of different ways (e.g., Song et al. [2001], Vasyliūnas [2012]). Here, we have derived it from the first-principles governing equations of motion for the electrons, ions, and neutrals, under suitable approximations based on length and time scales. Song et al. (2005) and Vasyliūnas (2012) state that the current given by the ”conventional ionospheric Ohm’s law” (Equation (48) above) is a stress-balance current, determined by a balance between Lorentz force and plasma-neutral collisional friction. This is the assumption made in the cold plasma low-frequency limit of the Equation for $W$ which is used in the Ohm’s law. Note that in the limit of vanishing collisions, the resistive terms in the generalized Ohm’s law (26) tend to zero, but that in the low-frequency Ohm’s law (47) the Pedersen resistivity tends to infinity. This behavior originates from our method of obtaining an Equation for $W$, Equation (32). By inverting the linear combination of individual species momentum equations to make $W$ the subject, we implicitly assume that collisions cannot become zero.

The use of the cold plasma, low-frequency Ohm’s law when describing a time-dependent system has its limitations. The macroscopic behavior of the EM fields may be well described by the low frequency Ohm’s law, but for fast or transient changes that alter these EM fields the dynamics cannot be self-consistently captured. As long as one is careful to ensure that the frequencies of interest are smaller than the collision frequencies and gyro frequencies, the approximations made are valid and applicable.

4 Magnetization, Mobility, and Resistive Properties

4.1 Magnetization Domains

Having derived Ohm’s law in terms of the plasma magnetizations, it is useful to now compare and contrast these fundamental parameters in the two environments, as was originally done by Goodman (2000, 2004a). Figure 5 shows the contributing collision frequencies ($\nu_{ei}$, $\nu_{en}$, $\nu_e \equiv \nu_{en} + \nu_{ei}$, $\nu_{in}$) and the electron and ion gyrofrequencies ($\Omega_e$, $\Omega_i$) as functions of height in the chromosphere and I/T.

The frequencies of collisions of charged particles with neutrals are similar at the base of both environments, and are very high, in the range $10^8$–$10^{10}$ Hz. They decline to about 1 kHz at the top of the chromosphere, and much farther, to below 1 Hz, at the top of the I/T. The charged and neutral fluids may be considered to be strongly coupled in response to waves and transient phenomena whose frequencies fall well below these collision frequencies at any particular height. Independent motions of the charged particles and neutrals can occur at frequencies well above those thresholds, hence a single-fluid model is not sufficient, and a multi-fluid model such as the one presented in this paper must be used. One must also be careful at high frequencies as single particle distribution functions might become highly non-Maxwellian, in which case the transport is by definition anomalous, and the multi-fluid approach becomes more complex. Some consequences of this frequency-varying coupling on the propagation of Alfvén waves, and the relative motions of plasma and magnetic field generally, are considered in more detail below in §5.1.

In the I/T (right Panel), the horizontal lines mark altitudes at which $\nu_e = \Omega_e$ (black line) and $\nu_{in} = \Omega_i$ (red line). These locations are where $k_e = 1$ and $k_{in} = 1$, respectively, and represent transitions from unmagnetized to magnetized. In the chromosphere model (left Panel), these respective heights do not have one single value, but a representative line is shown for these transitions. The magnetizations, defined in Equations (27–30), associated with these
collision frequencies and gyrofrequencies are shown in Figures 6 and 7 for both the chromosphere and I/T. This analysis was originally conducted by Goodman (2000, 2004a) using a different semi-empirical model for the chromosphere, and the profiles obtained in this review are quantitatively comparable to those of Goodman (2000, 2004a) and those obtained in a more recent analysis by Vranjes and Krstic (2013).

From Figure 7, one can define three distinct regions in the I/T:

- **M1 domain** both ions and electrons are unmagnetized \( k_{in}, k_e < 1 \)
- **M2 domain** ions are unmagnetized; electrons are magnetized \( k_{in} < 1, k_e > 1 \)
- **M3 domain** both ions and electrons are magnetized \( k_{in}, k_e > 1 \)

We point out that the magnetization domains are not directly related to the D, E, and F layers of the I/T, which are characterized by the dominant ion and neutral chemistry that occurs in the corresponding altitude bands. Our M1, M2, and M3 domains instead highlight commonalities in the charged-particle dynamics within the chromosphere and I/T. For the chromosphere, where there is a range of locations where these transitions occur, these three magnetization regions do not have specific heights. For example, for strong field strengths, e.g., above sunspots, the electrons can be magnetized \( k_e > 1 \) at all heights (thick black line in Figure 6), but for low field strengths (thin black line) they can be magnetized above about 500 km and unmagnetized beneath. For ions in the chromosphere, there is a range between about 600 and 1600 km where they can transition from being unmagnetized \( k_{in} < 1 \) to magnetized \( k_{in} > 1 \), depending on the field strength.

In general, both the chromosphere and I/T are stratified, partially ionized mixtures in which the charged particles undergo a transition from completely unmagnetized (lower M1)
to completely magnetized (upper M3), with a central region (M2) in which ions are unmagnetized and electrons are magnetized. Figure [7] also shows the function $\Gamma \equiv \xi_n k_e k_{in}$ (green line) which is equal to unity near the center of the M2 domain. As will be discussed below, the transition from $\Gamma < 1$ to $\Gamma > 1$ is a transition from isotropic transport processes to anisotropic. This transition can be important for both the electrodynamics and magnetohydrodynamics, and the heating of the chromosphere (if not the I/T). In the chromosphere, the locations of the M1, M2, and M3 domains change as the magnetic field strength changes, and so can be quite different over active regions compared to quiet regions of the Sun. In the I/T, in contrast, the magnetic field is nearly static and the collision frequencies are dictated by the vertical distribution of the neutral gas. Thus, the locations of the M1, M2 and M3 domains in the I/T change only slowly with respect to altitude and latitude.

![Figure 6](image)

**Fig. 6** Individual magnetizations in the chromosphere (left) and I/T (right). The magnetizations are shown as blue lines ($k_{en}$), red lines ($k_{in}$) and black lines ($k_{ei}$). For the chromosphere (left Panel), where magnetizations depend on magnetic field strength, the lines for the two extreme values of $B_0 = 10$ G (thin lines) and $B_0 = 1000$ G (thick lines) in Equation (2) are shown, with a shaded area between them.

Figure 5 shows that electron-ion collisions are much less frequent than electron-neutral collisions in the M1 and M2 domains of the I/T, but not so in the chromosphere. This is due to the much larger ionization fraction in the chromosphere relative to the I/T. In the M3 domains of both the chromosphere and I/T, on the other hand, electron-ion collisions dominate as the neutral density falls off. Nevertheless, a common assumption in ionospheric physics is that electron-ion collisions do not play a part in the electrodynamics (Song et al. 2001). We will discuss this assumption in relation to the mobility of charged particles below. When electron-ion collisions are negligible, then $k_{ei}$ becomes very large (as is evident in Figure 6 in the lower I/T) and $k_e \approx k_{en}$ in Equations (27–30). Also, the neutral fraction $\xi_n$ is very close to 1, which is valid for all of the I/T and all but the highest altitudes in the chromosphere. In this limit, the resistivities (43–46), simplify to

\[
\eta_{\parallel} \approx \frac{B}{en k_{en} + k_{in}},
\]

\[
\eta_C \approx \frac{B}{en k_{en} k_{in}},
\]

where

\[
\eta_{\parallel} \approx \frac{B}{en k_{en} + k_{in}},
\]

\[
\eta_C \approx \frac{B}{en k_{en} k_{in}},
\]
\[ \eta_p \approx \frac{B}{en} \frac{1 + k_{en}k_{in}}{k_{en} + k_{in}}, \quad (54) \]
\[ \eta_H \approx \frac{B}{en} \frac{k_{en} - k_{in}}{k_{en} + k_{in}}. \quad (55) \]

Substituting these expressions into (49–51) and combining terms yields the corresponding conductivities

\[ \sigma_{\parallel} \approx \frac{en}{B} (k_{en} + k_{in}), \quad (56) \]
\[ \sigma_P \approx \frac{en}{B} \left( \frac{k_{in}}{1 + k_{in}^2} + \frac{k_{en}}{1 + k_{en}^2} \right), \quad (57) \]
\[ \sigma_H \approx \frac{en}{B} \left( \frac{1}{1 + k_{in}^2} - \frac{1}{1 + k_{en}^2} \right). \quad (58) \]

These expressions (56–58) are identical to those in Equations (27–29) of Song et al. (2001), who derived them under the same assumptions: \( \nu_{ei} \to 0 \) and \( \xi_n \to 1 \).

Fig. 7 Combined magnetizations in the chromosphere (left) and I/T (right). The magnetizations are shown as black lines \( (k_e) \), red lines \( (k_{in}) \) and green lines \( (\xi_n k_{in} k_e) \). For the chromosphere (left Panel), where magnetizations depend on magnetic field strength, the lines for the two extreme values of \( B_0 = 10 \text{ G} \) (thin lines) and \( B_0 = 1000 \text{ G} \) (thick lines) in Equation (2) are shown, with a shaded area between them. In the I/T (right Panel), the horizontal lines mark altitudes at which \( k_e = 1 \) (black line), \( k_{in} = 1 \) (red line), and \( \xi_n k_{in} k_e = 1 \) (green line).

In the chromosphere model (left Panel), these heights do not have one single value, but a representative line is shown for these transitions.

4.2 Charged Particle Mobilities and Electrical Currents

We can extract the common pre-factor \((en/B)\) from the conductivities (56–58): 

\[ \mu_{\parallel} \equiv \frac{\sigma_{\parallel}}{en} \approx k_{en} + k_{in}, \quad (59) \]
\[
\mu_P \equiv \frac{\sigma_P}{en} \approx \frac{k_{en}}{1 + k_{en}^2} + \frac{k_{in}}{1 + k_{in}^2}, \quad (60)
\]

\[
\mu_H \equiv \frac{\sigma_H}{en} \approx \frac{1}{1 + k_{in}^2} - \frac{1}{1 + k_{en}^2}. \quad (61)
\]

These definitions also were introduced by Song et al. (2001). The mobilities \(\mu_P\) \((59–61)\) are explicit functions of the magnetizations \(k\) \((27–30)\) that show the relative contributions of electrons and ions to the electric current.

Figure 8 shows the mobilities \(\mu_P\) and \(\mu_H\) for the chromosphere and I/T. The plots for the full mobilities and those for the approximations assuming \(v_{ei} = 0\) and \(\xi_n = 1\) \((59–61)\) overlay each other, validating the assumption that electron-ion collisions do not contribute to the electrodynamics. The overall shape of the mobility curves in the chromosphere and

![Figure 8](image_url)

**Fig. 8** Pedersen (\(\mu_P\); red) and Hall (\(\mu_H\); green) mobilities in the chromosphere (left) and I/T (right). For the chromosphere, the solid, dotted and dashed lines represent the profiles using the three different values of \(B_0 = 10, 100, 1000\) G, respectively, in Equation (2) for the magnetic field.

in the I/T is very similar, though the vertical extent of these curves in the chromosphere depends on the magnetic field model used. Three curves for each mobility are shown in the left Panel of Figure 8, one for each of the magnetic field surface strengths of 10, 100, 1000 G in Equation (2). In the chromosphere and I/T, the Pedersen mobility \(\mu_P\) (red line in Figure 8) is double peaked, with maxima very near the transitions from domains M1 to M2 (where \(k_e = 1\)) and from M2 to M3 (where \(k_in = 1\)). The Hall mobility \(\mu_H\) (green line) is single peaked, with its maximum between the two peaks in the Pedersen mobility. The lower peak for \(\mu_P\) for the strongest chromospheric magnetic field model is missing from the plot as it appears below the lower boundary of the atmosphere. We shall discuss the mobility of the ions and electrons and their contribution to currents below.

The conductivities (including the electron density and representative magnetic field strength for the chromosphere and I/T) are shown in Figure 9. The calculations using the full expressions are shown as solid lines, while those neglecting electron-ion collisions are shown as dashed lines (only the curves for \(\sigma_P\) differ in this case). Convoluting the electron density with the relative mobilities causes some of the altitude variation in Figure 8 to be less obvious in Figure 9. In particular, for the I/T, the lower Pedersen conductivity peak is
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Fig. 9 Parallel ($\sigma_\parallel$; black), Pedersen ($\sigma_P$; red), and Hall ($\sigma_H$; green) conductivities (in Siemens/m) in the chromosphere (left) and I/T (right). Dashed lines show the corresponding values when electron-ion collisions are ignored. For the chromosphere (left panel), where the Pedersen and Hall conductivity depend on magnetic field strength, the lines for the two extreme values of $B_0 = 10$ G (thin lines) and $B_0 = 1000$ G (thick lines) in Equation (2) are shown, with a shaded area between them. The parallel conductivity is the only conductivity that appears to be affected by the assumption that electron-ion collisions can be ignored.

significantly reduced due to the strongly diminished electron density at those altitudes. The figure also reveals that the entire I/T above 70 km is subject to anisotropic electrodynamics ($\sigma_P \neq \sigma_\parallel$). In the chromosphere, for weak fields, $\sigma_P$ can be equal to $\sigma_\parallel$ up to about 500 km before anisotropy develops, but for strong fields, the anisotropy can be evident for the entire chromosphere. An important role of electron-ion collisions in limiting the parallel conductivity ($\sigma_\parallel$) is obvious at high altitudes (compare the solid and dashed black lines), throughout the upper half of the chromosphere and also above 250 km in the ionosphere, as can be seen in Figure 9. All three conductivities, especially the Pedersen and Hall conductivities, are far larger in the chromosphere than in the I/T.

It is worth further study of the contribution of electrons and ions to the Pedersen and Hall currents responding to a generic electric field in the rest frame of the neutrals $E^{V_n}$, via the Pedersen and Hall mobilities. Examination of the curves for the I/T allows a relatively simple discussion, but one must note that the chromosphere has a variable magnetic field, and so the transitions discussed here, such as unmagnetized to magnetized plasma, will vary with height depending on the field strength.

First recall the cold plasma, low frequency limit of Ohm’s law:

$$ J = \sigma_\parallel E_\parallel + \sigma_P E^{V_n}_\perp - \sigma_H E^{V_n}_\perp \times \hat{b} $$

(62)

and so the current perpendicular to the magnetic field vector ($J_\perp$) is a combination of Hall ($-\sigma_H E^{V_n}_\perp \times \hat{b}$) and Pedersen ($\sigma_P E^{V_n}_\perp$) currents. We shall see later how the relative contribution of these two currents affects the efficiency of heating due to plasma-neutral collisions. Let us first consider the variation with altitude of these mobilities, conductivities, and currents. A complete description would involve examining the momentum equations for ions and electrons, and solving for velocity, and then using $J = en(V_i - V_e)$ to define the current. For brevity, we discuss the motions of the ions and electrons and do not present such a derivation, though the results are consistent with such a method. It is also worth noting that
the magnitudes of the mobilities mainly determine the direction of the current, while the current magnitude is mainly dependent on the magnetic field strength and number density.

- At the lowest altitudes in the I/T, and near the solar surface in weak field regions, collisions with neutrals are high, and the mobility of electrons and ions are low and they are unmagnetized, \( k_{en}, k_{in} \ll 1 \) (see Figure 8).
- With increasing altitude, just below the lower peak in the Pedersen mobility (Figure 8), electrons become more mobile \( k_{en} > k_{in} \) and carry a Pedersen current in the \( E \cdot n \) direction. Here \( \mu_P > \mu_H \) and \( \sigma_P \sim \sigma_\parallel > \sigma_H \) (see Figure 2).
- There is a height at which \( k_{en} \approx 1 \), while \( k_{in} \ll 1 \). This is the lower peak in the Pedersen mobility in Figure 8. Here \( \mu_P \approx \mu_H \approx 1/2 \) and the perpendicular current is equally contributed to by the Hall and Pedersen currents. At this point, there is still isotropy in the conductivity: \( \sigma_P \approx \sigma_H \approx \sigma_\parallel \), as seen in Figure 7.
- At higher altitudes, the electrons become completely magnetized, but the ions remain unmagnetized. Here \( k_{en} \gg 1 \) while \( k_{in} \ll 1 \), \( \mu_H \approx 1 \), \( \mu_P < \mu_H \) and \( \sigma_P < \sigma_H < \sigma_\parallel \). This is the peak in the Hall mobility in Figure 8. \( J_\perp \) is essentially an electron Hall current, \( -\sigma_H E \times b \).
- The next altitude of interest is where ions start to become mobile and \( k_{en} \approx 1 \) and \( k_{in} \gg 1 \). This is the higher peak in the Pedersen mobility, where \( \mu_P = \mu_H = 1/2 \), and the perpendicular current is equally Pedersen and Hall current. However, unlike at the lower peak in the Pedersen mobility, at this upper peak \( \sigma_P \approx \sigma_H \ll \sigma_\parallel \) and there is significant anisotropy in the conductivities (this will be important for plasma-neutral heating, as discussed in later sections).
- Just above the upper Pedersen mobility peak, \( \mu_P > \mu_H \) and \( \sigma_P > \sigma_H \), and the perpendicular current is mainly the Pedersen current in the \( E \cdot n \) direction and \( \sigma_H < \sigma_P \ll \sigma_\parallel \).
- Finally, at the highest altitudes where collisions with neutrals have fallen sufficiently, the electrons and ions are completely magnetized, and \( k_{en}, k_{in} \gg 1 \). Here \( J_\perp \) is almost entirely an ion Pedersen current, and \( \sigma_H < \sigma_P < \sigma_\parallel \).

This discussion of currents not only highlights the similarities between the two atmospheres, but allows for a discussion of the conversion of electromagnetic energy into particle kinetic energy, via the \( E \cdot J \) term, as will be discussed in a later section on energy transfer.

4.3 Perpendicular vs. Parallel Current Resistivity

As we have discussed in some detail, the magnetization of charged particles introduces a substantial anisotropy in the resistivities and conductivities of a partially ionized mixture. At high altitudes of the chromosphere and I/T, the parallel and Pedersen conductivities increasingly diverge from one another, with \( \sigma_P < \sigma_\parallel \), or equivalently \( \eta_P < \eta_\parallel \). This reveals, for a given general current density \( \mathbf{J} = J_\perp \mathbf{J}_\perp + J_\parallel \mathbf{J}_\parallel \), a preference in both atmospheres toward preferential dissipation of currents that are directed perpendicular to the magnetic field (\( J_\perp \)) and so contribute to magnetic forces (\( \mathbf{J} \times \mathbf{B} \)). On the Sun, during the emergence of magnetic flux from below the photosphere into the overlying atmosphere, the anisotropic resistivities in the chromosphere reshape the current profile and transition the magnetic field to the essentially force-free configuration (\( J_\perp / J_\parallel \ll 1 \)) that it must assume in the very low-beta corona (Leake and Arber 2006; Arber et al. 2007, 2009; Leake and Linton 2013). Goodman (2000, 2004a) discusses the relationship between anisotropy and the force-free nature of the field for current dissipation driven by wave motions in the chromosphere. At Earth, the anisotropic conductivities in the I/T enable magnetic-field-aligned currents to neutralize any
parallel electric field \( \mathbf{E}_\parallel = \mathbf{b} \cdot \mathbf{E} \approx 0 \) while perpendicular currents and electric fields can be sustained when plasma is driven by, for example, cross-field neutral winds. As mentioned earlier, a slowly evolving electric field can be written as the gradient of a scalar potential, \( \mathbf{E} = -\nabla \Phi \). Then \( \mathbf{E}_\parallel \approx 0 \Rightarrow \mathbf{b} \cdot \nabla \Phi \approx 0 \), and magnetic field lines are equipotential paths through the I/T. However, when considering the coupled magnetosphere and I/T system, it is possible that this assumption may not hold.

Another way of expressing this anisotropy in the resistivities is to use the magnetizations. In general, one would expect that \( m_e V_{en} \ll m_i V_{in} \), since \( m_i \alpha \nu \approx \sqrt{m_i T_\alpha} \). Assuming that \( m_e V_{en} \ll m_i V_{in} \), and noting that this implies \( k_{in} \ll k_{en} \), the resistivities (43–46) in Ohm’s law (47) become

\[
\eta_\parallel \approx \frac{B}{en k_e}, \quad (63)
\]
\[
\eta_C \approx \frac{B}{en} \xi_{en} k_{in}, \quad (64)
\]
\[
\eta_P \approx \frac{B}{en} \left( 1 + \frac{\xi_{en} k_e k_{in}}{k_e} \right), \quad (65)
\]
\[
\eta_H \approx \frac{B}{en}. \quad (66)
\]

The corresponding expressions for \( \sigma_\parallel, \sigma_P \) and \( \sigma_H \), after using Equations (49–51), are exactly the same as Equations (23–25) in Song et al. (2001), derived under the same assumptions. The parallel, Pedersen, and Hall resistivities for the chromosphere and I/T are shown in Figure 10. The plots calculated under the assumption \( m_e V_{en} \ll m_i V_{in} \) overlay the full calculations, verifying the assumption.

Fig. 10 Parallel (black), Pedersen (red), and Hall (green) resistivities in the chromosphere (left) and I/T (right). For the chromosphere (left Panel), where the resistivities depend on magnetic field strength, the lines for the two extreme values of \( B_0 = 10 \text{ G} \) (thin lines) and \( B_0 = 1000 \text{ G} \) (thick lines) in Equation (2) are shown, with a shaded area between them.
An important quantity that sets the anisotropy of the resistivities is the ratio of the Cowling to parallel resistivity:

\[ \frac{\eta_C}{\eta_{||}} \approx \xi n_k e_k \in_k. \] (67)

Note that this was denoted \( \Gamma \) in [Goodman (2004a)], where the Ohm’s law was cast in the center of mass frame and thus had the form \( \xi n_k e_k \in_k \). Figure 7 shows the product \( \xi n_k e_k \in_k \) in the chromosphere and I/T. Increasing with height from the bottom, the electrons become magnetized first \( (k_e > 1) \), then the ions do so \( (k_{in} > 1) \). Between these two points, \( \xi n_k e_k \in_k = 1 \) within the M2 domain. At lesser heights, the current density is isotropic, \( \eta_e / \eta_{||} \ll 1 \), or \( \eta_{||} \approx \eta_p \) and \( \mathbf{E} \cdot \mathbf{V} n \approx \eta_{||} \mathbf{J} + \eta_{in} \mathbf{J} \times \hat{b} \), while at greater heights it is anisotropic, \( \eta_p \gg \eta_{||} \) and \( \mathbf{E} \cdot \mathbf{V} n = \eta_{||} \mathbf{J} \parallel + \eta_p \mathbf{J} \perp + \eta_{in} \mathbf{J} \times \hat{b} \). As will be discussed later in this paper, this transition is believed to be important with respect to the heating of the chromosphere.

4.4 Summary

Despite a large disparity in electron number density between the chromosphere and I/T, the governing physics of the two regions display some remarkable similarities. These similarities are mainly related to the transition from unmagnetized to magnetized plasma, and the variation of the electron and ion mobilities with altitude. The decrease in collisional frequency with height in both environments causes a change in magnetization with altitude, such that a central region is created where the ions are unmagnetized and electrons are magnetized. This is characterized by the region \( \Gamma = \xi n_k e_k \in_k \approx 1 \). As the ions become magnetized with increasing altitude, \( \Gamma \) increases, and \( \mathbf{J} \perp \) increasingly becomes an ion Pedersen current. This dissipative current is perpendicular to the field, and is a possible source of heating via the \( \mathbf{E} \cdot \mathbf{V} n \cdot \mathbf{J} \) term, which as we will see later is composed of Ohmic heating due to electron-ion collisions, and heating due to ion-neutral collisions.

However, a large electron number density difference between the two regions leads to a large difference in the magnitudes of the conductivities and resistivities. As we shall see in the next section this leads to the I/T being resistively dominated, while the chromosphere is only resistive for scales less than about a km. Also, the nature of the drivers in the two environments is also very different. The drivers of the low frequency (relative to the collision frequency) electric fields in the I/T are mainly neutral winds colliding with ions and electrons, and externally imposed electric fields from the magnetosphere. In the chromosphere, the drivers exist on a large range of time scales, as long as weeks (sunspot and prominence formation and persistence) and as short as seconds (magnetic reconnection events such as jets and flares, and high-frequency waves).

5 Magnetohydrodynamic and Electrodynamic Processes

5.1 Frozen-In vs. Resistive-Slip Evolution

A key parameter governing the coupling of a fully ionized plasma to the magnetic field is the Lundquist number ([Lundquist 1952]),

\[ S \equiv \frac{\mu_0^{1/2} B_0 \ell}{\rho_0^{1/2} \eta_{||}} = \frac{\mu_0 V_{A0} \ell}{\eta_{||}} , \] (68)
where $\rho_0 = m_i n$ is the mass density, $\eta_\parallel$ is the resistivity, $\ell$ is a characteristic length scale of variation, and $V_{A0} = B_0 / (\mu_0 \rho_0)^{1/2}$ is the Alfvén speed. This dimensionless ratio measures the relative importance of convection and resistive diffusion in the evolution of the magnetic field. Where it is large, convection dominates, and the plasma moves with the magnetic field so that the field lines are “frozen” into the fluid (Alfvén and Falthammar 1963); where it is small, resistivity dominates, and the plasma slips through the magnetic field lines. A derivation of the Lundquist number demonstrating this fundamental property is instructive and guides its generalization to multi-fluid situations with plasma-neutral coupling.

The motion of the fully ionized plasma induced by fluctuations in the magnetic field is estimated by balancing inertia against Lorentz forces in the plasma equation of motion,

$$\rho_0 \frac{\partial V_p}{\partial t} \approx \frac{1}{\mu_0} (\nabla \times \delta B) \times B_0,$$

whence

$$\omega \rho_0 V_p \approx \frac{B_0 \delta B}{\mu_0 \ell},$$

where $\omega$ is the frequency associated with the length $\ell$. The evolution of the magnetic field $B$ is governed by Faraday’s law, which requires Ohm’s law. In the limit of a fully ionized plasma, and for low frequency ($\omega \ll \nu_{in}, \Omega_i$), long wavelength ($L \gg d_e, d_i$) allowing the Hall term to be omitted) phenomena the Ohm’s law is

$$E + (V_p \times B) = \eta_\parallel J = \eta_\parallel \mu_0 \nabla \times B,$$

consistent with Alfvén and Falthammar (1963).

Assuming that $\eta_\parallel$ is spatially independent, Faraday’s law then yields the standard MHD induction equation:

$$\frac{\partial B}{\partial t} = \nabla \times (V_p \times B) + \frac{\eta_\parallel}{\mu_0} \nabla^2 B.$$

Balancing the time derivative of the fluctuating field $\delta B$ against the convection of the ambient field $B_0$, which is assumed to dominate the effects of resistivity acting on the fluctuating field, we find

$$\omega \delta B \approx \frac{1}{\ell} V_p B_0 \gg \frac{\eta_\parallel}{\mu_0 \ell^2} \delta B.$$

Using Equation (73) to eliminate $\omega$ from Equation (70) and solving for $V_p$ gives

$$V_p^2 \approx \frac{\delta B^2}{\mu_0 \rho_0},$$

so that the velocity $V_p$ is just the Alfvén speed evaluated at the perturbed field strength $\delta B$. This relationship is also known as the Walen relation (e.g. Sonnerup et al. 1981). The ratio of the retained convection term to the neglected resistivity term in Equation (73) then becomes

$$\frac{\mu_0 \ell B_0}{\eta_\parallel} \frac{V_p}{\delta B} = \frac{\mu_0^{1/2} B_0 \ell}{\rho_0^{1/2} \eta_\parallel} = S,$$

the Lundquist number. Thus, as claimed, convection dominates resistivity if $S \gg 1$. Reversing the inequality in Equation (73) and carrying through the rest of the analysis, the
unimportance of the convection term relative to the resistivity term in that case is measured by the ratio
\[ \frac{\mu_0 B_0 V_p}{\eta_\parallel \delta B / \rho_0 \eta_\parallel} = S^2. \] (76)

Therefore, again as claimed, resistivity dominates convection if \( S \ll 1 \).

Now we generalize these considerations to a partially ionized mixture. First we note that the Lundquist number is directly proportional to the characteristic length \( \ell \) of the variations in the magnetic and velocity fields. By combining Equations (70) and (73), we find in the convection-dominated case that \( \ell \) is just the reciprocal wavenumber of an Alfvén wave at frequency \( \omega \),
\[ \ell = \frac{B_0}{\mu_0 V_{A0}^2 / \rho_0^2 \omega} = \frac{V_{A0}}{\omega}. \] (77)

In a fully ionized plasma, the waves span a continuum of frequencies and wavenumbers whose character changes from frozen-in, oscillatory motions at large \( S \) (large \( \ell \), small \( \omega \)) to resistive-slip, damped motions at small \( S \) (small \( \ell \), large \( \omega \)). In a partially ionized mixture, the collisions between plasma particles and neutrals modify the response of the gas to the magnetic field and also raise the resistivity acting on perpendicular currents \( J_\perp \) from the parallel value \( \eta_\parallel \) to the Pedersen value \( \eta_P \), as in Equation (47). Two limiting frequency ranges are particularly illustrative. At high frequencies \( \omega > v_{in} \), the plasma-neutral coupling is weak, and the waves propagate at the Alfvén speed \( V_{A0} \) determined by the plasma mass density \( \rho_{p0} \) alone. We note here that the low frequency Ohm’s law may not exactly apply in this high frequency situation, and there will be a term related to \( \frac{d}{dt}(V_p - V_n) \) that contributes to the electric field in the rest frame of the plasma, as in Equation (33). As discussed in §3.5.2 and by Equation (35) the \( \frac{d}{dt}(V_p - V_n) \) term can be neglected if
\[ \left( \frac{\omega}{v_{in}} \right) \left( \frac{d}{dt} \right) \left( \frac{k_{in}}{k_{en} + k_{in}} \right) \sim \left( \frac{\omega}{v_{in}} \right) \left( \frac{\omega}{\Omega_i} \right) \left( \frac{k_{in}}{k_{en} + k_{in}} \right) \] (78)
is much less than unity. The first term of this product is large in this high frequency regime, and the last of which is always small \( (k_{in} \ll k_{en}) \). Choosing a magnetized regime where \( v_{in} < \Omega_i \), we can have \( \omega > v_{in} \) and be able to apply the low frequency limit of the Ohm’s law (47). Using the transition frequency \( \omega = v_{in} \) in Equation (77) to set \( \ell \), the Lundquist number in Equation (68) then takes the value
\[ S'_{in} = \frac{\mu_0 V_{A0}^2 \rho_{p0}}{\eta_P v_{in}}. \] (79)

where \( V_{A0} \) is determined by the plasma mass density alone. At low frequencies \( \omega < v_{ni} \), on the other hand, the plasma-neutral coupling is strong, and the waves propagate at the Alfven speed \( V_{A0} \) determined by the total (plasma+neutral) mass density \( \rho_0 = \rho_{p0} + \rho_{n0} \) (e.g., Song et al. 2005). Using the transition frequency \( \omega = v_{ni} \) for these waves, we obtain the Lundquist number
\[ S'_{ni} = \frac{\mu_0 V_{A0}^2 \rho_{p0}}{\eta_P v_{ni}}. \] (80)
The ratio of the convection to the resistivity terms in the plasma-neutral-modified MHD induction equation at the transition frequency \( v_{ni} \) (\( v_{ni} \)) is \( S'_{ni} (S'_{ni}) \). Thus, the motions of the plasma in response to magnetic field fluctuations at that frequency are frozen-in or resistive-slip, respectively, according to whether \( S' \gg 1 \) or \( S' \ll 1 \). We point out that the values of
Values of the Lundquist number $S'_{ni}$, Equation (80), for Alfvén waves at the neutral/ion collision frequency $v_{ni}$, and of the Lundquist scale (black line) $\ell_S$, in m, Equation (81), using the Pedersen resistivities in the ALC7 chromosphere (left) and the TIMEGCM I/T (right). For the chromosphere (left Panel), where the values depend on magnetic field strength, the lines for the two extreme values of $B_0 = 10$ G (thin lines) and $B_0 = 1000$ G (thick lines) in Equation (2) are shown, with a shaded area between them.

The values of the Lundquist number $S'_{ni}$ at the lower transition frequency $v_{ni}$ are shown (red curves) for the ALC7 chromosphere (left) and TIMEGCM I/T (right) in Figure 11. For the chromosphere (left Panel), where the values depend on magnetic field strength, the lines for the two extreme values of $B_0 = 10$ G (thin lines) and $B_0 = 1000$ G (thick lines) in Equation (2) are shown, with a shaded area between them. The numbers are very small at low altitudes and approach, but do not quite attain, unity at high altitudes, in both atmospheres. Consequently, the field lines resistively slip through the gas very readily at low altitudes, and they are not well frozen-in to the gas motions anywhere in the two atmospheres, for frequencies at or above the neutral/ion collision frequency $v_{ni}$. Flux-freezing will occur only at still lower frequencies $\omega \leq v_{ni}S'_{ni}$, where the condition $S \geq 1$ can, in principle, be met. This is mostly likely to happen at high altitudes.

The condition $S = 1$ is met at the Lundquist scale $\ell_S$, defined using Equations 68 and then 80 as

$$\ell_S \equiv \frac{\eta P}{\mu_0 V_{A0}} = \frac{V_{A0}}{v_{ni} S'_{ni}}. \quad (81)$$

The values of the Lundquist scale $\ell_S$ (in m) are also shown (black solid curves) for the ALC7 chromosphere (left) and TIMEGCM I/T (right) in Figure 11. We emphasize the very sharp contrast in the Lundquist scales for the two environments. In the chromosphere, in quiet average conditions, $\ell_S$ is everywhere below 1 km; only shorter-wavelength disturbances are resistivity-dominated, whereas an extended range of longer-wavelength disturbances on the Sun is convection-dominated, with increasingly frozen-in motions of gas and magnetic field. Thus the system is magnetohydrodynamic. In the I/T, on the other hand, $\ell_S$ everywhere exceeds $9 \times 10^4$ km, or $15 R_E$; therefore, all Alfvénic disturbances at Earth are resistivity-dominated, with easy slippage of the partially ionized mixture relative to the magnetic field. The I/T system is therefore electrodynamic. This implies that any fluctuations in the geo-
magnetic field of significant amplitude must be driven by some external agent, rather than from within the I/T itself. Thus, excepting disturbances incident from the overlying magnetosphere, Earth’s magnetic field is approximately static. Meanwhile, both the photosphere and chromosphere serve as very active sources of strong magnetic-field fluctuations on the Sun. This confluence of circumstances – the disparate small/large Lundquist scales and the dynamic/static character of the chromospheric/ionospheric magnetic field – is responsible for the prevalence of magnetohydrodynamics in conceptualizing and quantifying processes in the chromosphere, on the one hand, and electrodynamics in the I/T, on the other.

As mentioned previously, the MHD nature of the chromosphere and the electrodynamic nature of the I/T, among other factors, has led to two different approaches being used by the two communities when describing physical processes. In chromospheric physics, the primary variables are the velocity and magnetic field (B,V paradigm). While in I/T physics, they are the electric field and current density (E,J paradigm). Parker (2007) and Vasyliunas (2012) have recently criticized the E,J paradigm as not being the correct time-dependent description of the physical system, and capable of introducing misunderstanding in the community. In the next section we will consider a specific example where the two different approaches are used to discuss the same process which occurs in the chromosphere and I/T, the neutral wind driven dynamo. We will then go on to discuss the validity and appropriateness of the two paradigms. In a later section we will look at another example, where we also apply analytic and numerical analysis to study the Rayleigh-Taylor instability.

The estimates in this section suggest that a numerical resolution $\leq 1$ km is necessary to resolve resistive processes in the chromosphere. Similar estimates for the characteristic length scales below which viscosity, thermal conduction, and thermoelectric effects, which are all anisotropic, are important, are not considered here.

5.2 Neutral-Wind Driven Dynamos: An example of the E,J and V,B paradigms

In this section we discuss the phenomena of neutral-wind driven dynamos, and use it as an example of how the I/T and chromospheric communities have previously used two different paradigms to explain the same phenomena. Another example will be given in §7.

In Earth’s thermosphere, hydrodynamic forcing of the neutral gas through pressure-gradients forces generated by differential solar radiative heating, Coriolis forces, viscous forces, nonlinear advection, and ion-drag forces create a global circulation of neutral winds. Modification of these forces on the neutral gas by vertically propagating waves from the lower atmosphere and influences from the magnetosphere complicate the circulation. This neutral circulation is impressed upon the ionospheric plasma through collisions, causing the ions and electrons to undergo differential motion leading to the production of currents and electric fields (e.g., Richmond and Thayer 2000).

Neutral winds on the Sun that, in principle, might drive I/T-like dynamo action through collisional coupling to the plasma originate in (1) the randomly shifting near-surface convection cells in the neutral-dominated photosphere and (2) the global atmospheric oscillations produced by acoustic and gravity waves. Hence, both the chromosphere and I/T may be subject to dynamo action in which inhomogeneous flows of neutrals couple to the plasma and drive persistent electric currents.

For clarity, we point out that the usual usage of the term “dynamo” in solar physics refers to amplification of seed magnetic fields to greater strengths by convective turnover, twisting, and differential shearing of the plasma alone. Such dynamo action is widely accepted to occur deep in the Sun’s fully ionized convective zone, producing the sunspot cycle of strong
magnetic fields (e.g., Charbonneau 2010), and also may occur within the near-surface convection layers to produce flux concentrations at much smaller scales (e.g., Cattaneo 1999; Cattaneo and Hughes 2001; Cattaneo and Emonet and Weiss 2003; Stein 2012). Neither of these dynamos relies upon plasma-neutral coupling to generate magnetic fields; indeed, neutrals generally are not considered in these models, especially in the deep Sun.

Below is a discussion of the neutral wind driven dynamo action from two points of view, dynamic MHD and static electrodynamics, which highlights the differing approaches of the chromospheric and I/T communities to common phenomena.

Proceeding from the full time-dependent equations of electromagnetics and plasma physics, Vasyliunas (2012) describes the sequence of events that establishes the distributed neutral-wind driven dynamo as follows:

1. Plasma motions $V_p$ are induced locally by collisions with the neutral wind $V_n$;
2. The resultant bulk plasma flow $V_p$ produces a persistent electric field, $E_\perp = -V_p \times B_0$;
3. Gradients in $E_\perp$ with altitude and across $B_0$ generate magnetic perturbations $\delta B$;
4. Electric currents $J$ arise due to gradients in the magnetic perturbations $\delta B$;
5. Lorentz forces $J \times B_0$ drive MHD waves propagating away from the dynamo region, propagating flows and currents;

From the perspective of solar chromospheric physics, this chain of processes is intuitively appealing and non-controversial, although the details undoubtedly would be debated and additional study would be warranted to confirm its essential correctness. The chromosphere exhibits unceasing vigorous flows and strong magnetic variability. Therefore, explaining any chromospheric phenomenon based on a neutral-wind driven dynamo demands consistency with the full time-dependent equations and their implications from the outset. Exploratory work in this direction has been done on the quasi-static structuring of so-called network magnetic fields in the lanes of chromospheric convection cells by Henoux and Somov (1991), who call this process a “photospheric dynamo” (see also Henoux and Somov 1997). In addition, Kropotkin (2011) has proposed that the highly intermittent, collimated chromospheric upflows known as spicules, which are ubiquitous in the inter-cellular lanes, are powered by Alfvén waves driven by neutral-wind dynamo action. One of the necessary conditions for this “photospheric dynamo” is that the electrons are magnetized but the ions are not. This condition is realized in the 1D static chromosphere model used here at about 500 km above the solar surface; for stronger background fields, this critical height is reduced due to the increase in the electron gyrofrequency. Krasnoselskikh et al. (2010) recently suggested that intense currents can be generated when magnetized electrons drift under the action of electric and magnetic fields induced in the reference frame of ions moving with the neutral gas, and that the resistive dissipation of these currents may be important for chromospheric heating.

From the perspective of ionospheric electrodynamics, the low frequency Ohm’s law works very well on time scales longer than the ion-neutral collision time, which for the ionospheric M2 and M3 domains where primary dynamo activity occurs is only a few hundredths of a second to a few seconds. For periods exceeding this time scale, and just considering the I/T (no coupling to the magnetosphere) steady-state electrodynamic behavior results and electrostatic electric fields and divergence-free current densities are assumed. Consequently, a static approach is often applied to describing the I/T’s neutral wind dynamo process; where adjustments in currents and electric fields occur on short time scales in order to maintain potential electric fields and divergence-free currents. The analysis by Vasyliunas (2012) serves as a reminder that these established relationships lack any time-dependency and thus cannot describe the causal structure of the dynamo process. Nonetheless, descrip-
tions of evolution in the neutral dynamo process have been based on these relationships and described heuristically as:

1. Collisions with neutrals create plasma motions $V_p$ along $V_n$. At the same time, oppositely directed $R_p^\text{en} \times B$ and $R_e^\text{en} \times B$ ion and electron drifts are created;
2. Electric currents $J$ resulting from the drifts drive charge separation between the ions and electrons;
3. An electric field $E$ is established in the dynamo region due to the charge separation;
4. Potential mapping along magnetic field lines due to rapid electron motions along $B_0$ extends the electric field to increasingly remote regions;
5. $E \times B$ drifts create bulk plasma flows $V_p$ outside the dynamo region;

The first sequence above asserts that bulk plasma flows drive the electric field and distribute the dynamo along magnetic field lines through the intermediary of magnetic fluctuations having associated currents; thus, $B$ and $V$ have primary roles, while $E$ and $J$ are secondary. The second sequence, on the other hand, states that current-generated electric fields distribute the dynamo along magnetic field lines and drive the bulk plasma flows; thus, $E$ and $J$ have primary roles, while $B$ and $V$ are secondary. As previously mentioned, the first approach is known as the $B,V$ paradigm, and the second is known as the $E,J$ paradigm. For the application to wind driven dynamos discussed here, we find that the difference in perspectives between the solar and ionospheric approach is driven by a difference in the plasma parameters. The chromosphere is ideal (non-resistive) on length scales much larger than a km, and so the electric field in the rest frame of the neutrals (or any frame for that matter) $E + (V_n \times B)$ is small enough that using $E = -V_n \times B$ in Faraday’s law gives the correct magnetic field evolution. This leads to a magnetic perturbation and wave-propagation interpretation of the dynamo. In the resistive I/T however, there is an electric field in the rest frame of the neutrals created by charge separation due to ion electron drifts. Thus the dynamics are often interpreted as locally generated electric field propagating electrostatically along the field leading to a different interpretation of the phenomena.

Vasyliunas (2011, 2012) criticizes the $E,J$ interpretation as originating from extending intuition developed from the simple ”steady-state” relations among the four variables $- E, J, B, V$ – into time-varying situations, without due regard for the causal relationships implied by the dynamical equations. At scales long compared to the electron plasma oscillation period and large compared to the electron Debye length, the displacement current can be neglected in Ampère’s law and the time derivative of the particle current can be neglected in the generalized Ohm’s law, so there are no dynamical equations for $\partial E/\partial t$ and $\partial J/\partial t$ to be solved (Vasyliunas 2005a). Instead, $J$ is related to $B$ through Ampère’s law, and $E$ is related to $B, V$, and $J$ through Ohm’s law. The dynamical equations that remain then contain only $\partial B/\partial t$ and $\partial V/\partial t$ among the four variables. These dynamical issues also have been addressed more broadly and in several specific applications by Parker (1996a,b, 2007). He emphasizes that the converse procedure, eliminating $B$ and $V$ from the dynamical relations in favor of $E$ and $J$, culminates in nonlinear integrodifferential equations that are nearly intractable to solve and obscure the underlying physical processes. Parker (2007) concludes that applying insights from laboratory experimental configurations – in which electric fields drive currents and generate magnetic fields – to anything beyond symmetric, static situations in the solar and terrestrial atmospheres is fraught with difficulty. The result has been much misunderstanding, and even misdirection of effort, in the two communities, according to Parker (2007).

The discussion here highlights that even though the end result can be the same for the two paradigms, the description of the time-dependent evolution is different. Furthermore,
for cases where the steady-state approximations made in the EJ paradigm are not valid, it is 
possible to lose important physics. Another example of the disparate perspectives imparted 
by these frameworks is provided in \[\text{§7}\] on the Rayleigh-Taylor instability, which we analyze 
from both the B,V (chromospheric) and E,J (ionospheric) perspectives.

5.3 Farley-Buneman Instability

As mentioned above, in the M2 domains of the chromosphere and I/T neutral flows per-
pendicular to the magnetic field can drive ions along with them but not electrons, and this 
produces currents. In the I/T, these currents comprise the electrojet current systems that oc-
cur in equatorial and auroral regions \cite{Schunk2000, Kelley2009}. The equatorial current system is driven by tidal E-layer neutral winds generated by daytime solar 
heating \cite{Schunk2000}. It peaks near the magnetic 
equator as a consequence of the nearly horizontal field lines. The auroral current system is associated with field-aligned, high-latitude currents driven by the solar-wind/magnetosphere 
interaction, as well as by precipitating energetic particles \cite{Kamide1982}.

The M2 domains in both the I/T and chromosphere environments are an ideal place for 
the occurrence of the Farley-Buneman instability \cite{Dimant1995, Fontenla2005, Otani2006, Fontenla2008, Oppenheimer2013, Madsen2013}, which is a two-stream kinetic instability in which the ions are unmagnetized 
while the electrons are tied to the magnetic field \cite{Farley1963, Buneman1963}. Its ther-

dynamic consequences may not be very important compared to Joule heating, especially 
in the chromosphere where the spatial scales associated with the currents are so small that 
Joule heating outweighs the instability-related heating \cite{Gogoberidze2009}. However, 
there may be transient situations, such as the exhaust regions of magnetic reconnection in 
the chromosphere, where electron and ion fluids separate on small scales and the Farley-
Buneman instability could be thermodynamically important. In the Earth’s electrojet cur-
rent systems, the plasma waves generated by the Farley-Buneman instability can provide an 
anomalous resistivity, which in turn modifies the ambient current systems and electric fields 
\cite{Hamza1995}. When plasma waves are excited, the low frequency 
Ohm’s law is incorrect, and as in the work of \cite{Hamza1995}, terms such as 
the electron and ion inertia should be included in the model.

5.4 Summary

Factors such as the resistive nature and the static magnetic field of the I/T have lead to a 
common approach where the electric field \((\mathbf{E})\) and electric current \((\mathbf{J})\) are the primary vari-
ables, and these are often used to describe how the I/T is driven. An example was performed 
in our previous discussion of mobility of ions and electrons in an electric field. However, 
only slow changes in the I/T can be accommodated by an electrostatic field. Furthermore, 
while for long time scales we can remove the dynamical equations for \(\partial \mathbf{E} / \partial t\) and \(\partial \mathbf{J} / \partial t\) 
and have \(\mathbf{B}\) and \(\mathbf{V}\) be the primary variables \(\text{(V-B paradigm)}\), the converse is not true. At the 
same time, using the EJ paradigm may allow one to arrive at an equivalent result as the 
V-B paradigm. This was achieved in a general sense for wind-driven dynamos, though the 
interpretation of the time-dependent physics was different, and will be shown to be possible 
for the Rayleigh-Taylor instability in a following section.
6 Energy Transfer

In this section we first discuss the major contributions to the energy balance in both the chromosphere and I/T, then go on to discuss the general process of conversion of electromagnetic energy to thermal and kinetic energy. Then we present ideas on the flux and dissipation of electromagnetic energy in the two environments, and the role of plasma-neutral collisions in this process.

6.1 Major Contributions to Energy Balance

6.1.1 Chromosphere

The bulk of the chromosphere is a few thousand degrees hotter than the underlying photosphere. It is also far cooler than the corona above, but is so much denser that it requires roughly ten times more heat input than the corona (when measured as a height-integrated net radiative loss rate) to maintain its elevated temperature. Typical estimated net radiative loss measurements for the chromosphere are $\sim 10^7 \text{ergs cm}^{-2} \text{s}^{-1}$. Hydrogen ionization in the chromosphere acts to balance heating even as the density drops with height. This is because the large abundance and ionization energy of hydrogen allows ionization to absorb energy, which creates free electrons that excite heavier species such as iron, magnesium and calcium, resulting in steady radiative cooling. The heating of the chromosphere, which must account for these radiative losses, comes from a combination of collisional effects (Joule and viscous heating) and compressional heating. The Joule and viscous heating mechanisms become relatively more important on progressively smaller scales, such as those associated with current sheets, shocks, or wave motions.

A large class of potential heating mechanisms for the chromosphere is derived from the fact that the turbulent convection zone below is capable of supplying a flux of wave energy into the chromosphere. Previously suggested heating mechanisms for the chromosphere have included acoustic wave dissipation, though it is unclear if acoustic waves can propagate high enough to deposit their energy in the chromosphere (Biermann 1946; Schwarzschild 1948; Ulmschneider 1990; Fossum and Carlsson 2005a, b; 2006; Kalkofen 2007). MHD waves have more recently been investigated, particularly Alfvén waves as they can propagate upwards along magnetic fieldlines into the upper chromosphere (e.g. De Pontieu et al. 2007b; Tomczyk et al. 2007). However, although plasma-neutral collisions and viscous effects can potentially dissipate wave energy in the chromosphere, it is not fully understood whether the dissipation and heating of the waves provided by the convection zone is sufficient to counter the radiative losses (De Pontieu 1999; Leake et al. 2005; De Pontieu et al. 2007a; Hasan and van Ballegooijen 2008; Goodman 2011; Song and Vasyliunas 2011). Recent theoretical and numerical work by Goodman (2011) and Tu and Song (2013) show that Joule (electron-ion collisional) damping of waves in the lower chromosphere (below 800 km) is large and controls the Poynting flux to the upper chromosphere, and as Goodman (2011) notes, also places an upper limit on the chromospheric heating rate by wave damping of $\sim 6 \times 10^7 \text{ergs cm}^{-2} \text{s}^{-1}$.

Recently, the dissipation of Alfvénic wave energy by other mechanisms has been considered, such as non-linear interactions, mode conversion, and resonant heating (Narain and Ulmschneider 1990) present a more comprehensive literature review of these types of investigations. Low frequency mechanisms, where the system frequencies (evaluated using $1/t_0$ where $t_0$ is the timescale for changes in flows) are typically much less than 1 Hz, have
also been investigated. These include the dissipation of currents perpendicular to the magnetic field by Pedersen resistivity (Goodman 2004b; Leake and Arber 2006; Arber et al. 2007, 2009; Khomenko and Collados 2012; Martínez-Sykora et al. 2012; Leake and Linton 2013), magnetic reconnection (Parker 1983, 1988; Dahlburg et al. 2003; Dahlburg et al. 2005; Klimchuk 2006; Goodman and Judge 2012), and neutral-wind dynamos (Krasnoselskikh et al. 2010).

Clearly, there is a “zoo” of possible chromospheric heating mechanisms, and a complete review of the chromospheric heating problem is one which is outside the scope of this review. What is also clear is that this heating is occurring in a region of highly coupled plasma components and neutral gas, and that plasma-neutral interactions are vital to the heating of the chromosphere by bulk motions.

6.1.2 Ionosphere/Thermosphere

The contributions to the energy transfer in the ionosphere/thermosphere are better understood than those in the chromosphere. However, the large variability in time and space of these mechanisms and their relative contributions to the overall heating and cooling remain challenges to understanding the thermal properties of the system. The lower amount of plasma-neutral and electron-ion coupling in the I/T (based on the collision frequencies), compared to the chromosphere, means that the individual heating of the constituents and their thermal coupling must be considered. The dominant heating term for the neutral gas is absorption of UV/EUV radiation. The UV/EUV energy flux from the Sun is \( \sim 4 \) orders of magnitude less than at visible wavelengths (e.g., Tobiska et al. 2000), but is of sufficient energy to form the ionosphere through photoionization (by contrast, ionization is predominantly collisional excitation in the chromosphere), produce secondary ionization, energize gas emissions in airglow, and raise neutral and plasma temperatures to more than one thousand degrees. The UV/EUV flux is nearly completely absorbed between 80 and 200 km altitude. Heating of the neutral gas by collisions with ions can be as significant as solar heating during geomagnetic storms and represents the most variable source of energy to quantify in the I/T energy equation. Other internal processes within the weakly ionized mixture, such as thermal exchange between the plasma and neutral gas, lead to differing thermal structure with height for the plasma and neutral gas, as indicated in Figure 1.

It is worth examining high latitude heating further as it is central to the aims of this paper. It has been recognized for some time that the global thermal structure of the I/T is only adequately represented when energy resulting from solar wind interactions with the Earth’s magnetosphere is included in the I/T energy budget. This energy input to the I/T is manifested in the form of Poynting flux (see below) and particle kinetic energy flux. The partitioning of auroral kinetic energy (KE) flux in the 80-200 km altitude range is roughly 50% heating, 45% ionization, and 5% optical production (e.g., Thayer and Semeter 2004). Typically the energy flux by particles is less than that due to electromagnetic energy processes (Knipp et al. 2004). Electromagnetic fields transfer energy from the plasma to the neutral gas on the neutral-ion collisional timescale. Differential velocities between ions and neutrals lead to frictional heating of both species. Owing to the lower ion density relative to neutral density, ion temperatures can become quite high in the I/T M2 region at high latitudes due to ion frictional heating. The neutral gas is also subject to frictional heating but due to its greater mass density this results in only modest changes in temperature. The resultant differential temperature between the hotter ions and cooler neutrals leads to heat transfer to the neutral gas. Thus, ultimately the neutral gas serves as the depository of electromagnetic energy transfer in the I/T system. The electrons within the M2 region are also susceptible
to heating by electrodynamic processes but in a different manifestation. Farley-Buneman (F-B) instability in the M2 region is a common occurrence in the I/T. At high latitudes, where strong electric fields of 20 mV/m are present [Dimant and Oppenheim 2011], the F-B instability can lead to very efficient plasma wave heating of electrons in the M2 region with temperatures exceeding 2000 K. However, this does not lead to any significant change in the ion and neutral temperature as they have much greater thermal heat capacity. Electrons are also susceptible to frictional heating in the lower M2 and upper M1 domains in the I/T (e.g., Brower et al. 2009) but, rather than being cooled by elastic collisions with neutrals, inelastic collisions with neutrals dominate in this region.

6.2 Electromagnetic Energy to Thermal Energy Conversion

The partially ionized, highly collisional domains of the chromosphere and the I/T lead to significant mechanical and thermal coupling between the plasma and neutral gas. The degree of coupling is strongly dependent on the relative concentration of plasma to neutrals. As discussed and demonstrated in §2, the greater ionization fraction in the chromosphere results in stronger plasma-neutral coupling than in the I/T. This has justified in the past the use of single-fluid approaches in describing the evolution of chromosphere macroscopic properties, while in the I/T system separate fluid equations for electrons, ions and neutrals were required to account for each species having a different temperature and velocity. In the multi-fluid approach, electrons, ions and neutrals are treated as coexisting fluids with terms in their respective conservation equations accounting for the relative interactions with each other. For example, to account for energy transfer processes between species, a thermal energy equation for each species defined by its own Maxwellian velocity distribution can be derived in the I/T using the 13th moment approximation to the Boltzmann equation (Schunk 1977). The coupling terms between species in this form of the thermal energy equation consist of a heat exchange rate term, accounting for the heat transfer rate due to differential temperatures between species, and a frictional heating rate, accounting for energy generation due to differential motion between species.

A total thermal energy conservation equation for a N-fluid, collision-dominated plasma can be expressed, as described by Braginskii (1965), Mitchner and Kruger (1973), Balescu (1988), in the following form:

$$\frac{\partial \varepsilon_{CM}}{\partial t} + \nabla \cdot (\varepsilon_{CM} \mathbf{V}) + \nabla \cdot \mathbf{h}_{CM} + P_{CM} : \nabla \mathbf{V}_{CM} = (\mathbf{E} + \mathbf{V}_{CM} \times \mathbf{B}) \cdot \mathbf{J} = E^{CM} \cdot \mathbf{J}$$

(82)

where $\mathbf{h}_{CM}$, $P_{CM}$, and $\varepsilon_{CM}$ are simple summations of the heat flux, pressure tensor and thermal energy, over the N fluids using a center of mass reference frame.

The $E^{CM} \cdot \mathbf{J}$ term is commonly referred to as the Joule heating rate and, in general, accounts for the conversion of electromagnetic energy into thermal energy. However, its physical description depends on the makeup of the conducting mixture and the defining reference frame (see below). Poyntings theorem, derived directly from Maxwell’s equations, relates the electromagnetic (EM) energy with the particle kinetic energy of the total fluid. One form of Poyntings theorem is

$$\frac{\partial}{\partial t} \left( \frac{B^2 + E^2/\epsilon^2}{2\mu_0} \right) + \nabla \cdot \left( \frac{\mathbf{E} \times \mathbf{B}}{\mu_0} \right) + \mathbf{E} \cdot \mathbf{J} = 0.$$  

(83)

In words, Poyntings theorem is a conservation equation of electromagnetic energy describing the time rate of change in EM energy density caused by EM energy divergence.
(Poyntings flux) in a defined volume and EM energy conversion to (or from) particle kinetic energy. In a fluid model, the particle kinetic energy is divided among the thermal energy (e) and the bulk kinetic energy of the fluid. $E \cdot J$ is the rate at which EM energy is converted to thermal energy and the bulk kinetic energy of the fluid. It is this electromagnetic exchange between fields and the plasma-neutral mixture that is of interest in contrasting the chromosphere and the I/T.

The term,
\[ E_{CM} \cdot J = [E + (V_{CM} \times B)] \cdot J \]  
(84)

is then the rate at which EM energy is converted into thermal energy in the center-of-mass reference frame, and
\[ E \cdot J - E_{CM} \cdot J = -(V_{CM} \times B) \cdot J = (J \times B) \cdot V_{CM} \]  
(85)

is the rate at which EM energy is exchanged with the bulk kinetic energy of the CM fluid. Sometimes this is also referred to as the amount of work done on or by the CM fluid (e.g. Lu et al. 1995; Thayer et al. 1995; Fujji et al. 1999; Thayer 2000; Richmond and Thayer 2000; Goodman 2000, 2004a,b; Vasyliūnas and Song 2005). Note that we can relate the Joule heating rate in the CM fluid $E_{CM} \cdot J$ to the Joule heating in the neutral gas reference frame, $E_{n} \cdot J$, where $E_{n} = E + (V_{n} \times B)$ as
\[ E_{n} \cdot J = E_{CM} \cdot J + [(V_{n} - V_{CM}) \times B] \cdot J = E_{CM} \cdot J + (J \times B) \cdot (V_{CM} - V_{n}). \]  
(86)

Given that in the I/T, $V_{CM} \approx V_{n}$, the term $E_{n} \cdot J$ is typically called the Joule heating rate in the I/T literature where the EM energy dissipated in the plasma-neutral mixture ends up as thermal energy in the neutral gas. The relationship of the neutral Joule heating rate to the frictional heating rate of the neutral gas has been demonstrated in descriptions put forward by St. Maurice and Schunk (1981), and more recently by Vasyliūnas and Song (2005).

Using the generalized Ohm’s law in the rest frame of the neutrals (47) and applying the low-frequency approximation gives
\[ E_{n} \cdot J = [E + (V_{n} \times B)] \cdot J = \eta_{||} J^2 + \eta_{\perp} J_{\perp}^2. \]  
(87)

It should be noted that care must be taken to ensure that the assumptions made to obtain the low frequency limit are not violated when discussing Joule heating by rapidly changing EM wave phenomena, such as Alfvén waves.

As pointed out in the work of Vasyliūnas and Song (2005), $E_{n} \cdot J$ is often called the Joule heating rate in the I/T community but this usage is not traditionally the description of Ohmic heating. Ohmic heating is typically defined in the rest frame of the plasma leading to a description of Joule heating in the plasma frame as
\[ E_{p} \cdot J = [E + (V_{p} \times B)] \cdot J = \eta_{||} J^2 \]  
(88)

and is then related to the neutral Joule heating rate $E_{n} \cdot J$ by
\[ E_{n} \cdot J = E_{p} \cdot J - [(V_{p} - V_{n}) \times B] \cdot J = E_{p} \cdot J + (J \times B) \cdot (V_{p} - V_{n}) \approx E_{p} \cdot J + \rho_{i} v_{in} |V_{p} - V_{n}|^2, \]  
(89)

where the approximation assumes that Lorentz and plasma-neutral drag forces dominate the plasma (ion and electron) equation of motion $(J \times B \sim R_{in})$ and inertial, pressure and gravity terms are neglected, as discussed in Vasyliūnas and Song (2005). Hence, the neutral
Joule heating rate can be described as the heating rate in the rest frame of the plasma plus the frictional heating rate due to differential plasma neutral velocities. The plasma Joule heating rate is Ohmic Joule heating in the sense that the electric field in the rest frame of the plasma is linearly proportional to the current by electron collisions with ions and neutrals.

Figure 10 shows that $\eta_C > \eta_{\parallel}$ for reasonable proportions of the chromosphere and I/T. Using this fact, and noting that $|E_{\parallel}| < |E_{\perp}|$ in the chromosphere and ionosphere except for inductive cases such as solar flares and at high I/T latitudes on auroral fieldlines, we have that $\eta_{\parallel}J_{\parallel}^2 < \eta_C J_{\perp}^2$ (Kelley and Hellis 2009). Comparing Equations (88) and (87) tells us that for the upper chromosphere and I/T, the Joule heating $E_V \cdot J$ has only a tiny contribution from the Ohmic heating $E_V \cdot J$, and is dominated by the transfer of heat due to the frictional exchange between ions and neutrals. However, theoretical and numerical work by Goodman (2011) and Tu and Song (2013) suggests that for wave processes, Ohmic Joule heating is stronger in the lower chromosphere.

6.3 The Role of Plasma-neutral Coupling in Energy Transfer

As mentioned above, Joule heating $Q \equiv E_V \cdot J$ where $E_V$ is the electric field in the neutral or center of mass frame of the chromosphere and I/T contains a component due to ion-neutral collisions. This component can dominate over Ohmic Joule heating when $\eta_C > \eta_{\parallel}$, or $\sigma_C < \sigma_{\parallel}$. Hence plasma-neutral collisions play a major role in the conversion of electromagnetic energy into thermal energy.

We can discuss the efficiency of this heating without referring to a particular mechanism for the generation of currents. In §4.2 we presented a description of the altitude variation of electrical currents of a given electric field. In particular, we looked at how the contributions to the current perpendicular to the magnetic field $J_{\perp}$ by Pedersen ($J_P \equiv \sigma_P E_{\perp}$) and Hall ($J_H \equiv -\sigma_H E_{\perp} \times \hat{b}$) currents varied as the mobilities of the electrons and ions varied. Let us now look at the contributions to the neutral Joule heating ($E_V \cdot J$). Goodman (2004a) showed that when the heating is written as

$$Q \equiv E_V \cdot J = \frac{J_{\parallel}^2}{\sigma_{\parallel}} + \frac{\sigma_P J_{\perp}^2}{\sigma_{\parallel}^2 + \sigma_{\parallel}^2} \equiv Q_{\parallel} + Q_P$$

(90)

then the efficiency of Pedersen heating $Q_P$, the ratio of Pedersen heating to its maximal value, obtained when the perpendicular current $J_{\perp}$ is all Pedersen current and no Hall current, can be expressed as

$$R_Q \equiv Q_P/Q_{P,max} = \frac{\sigma_P^2}{\sigma_{\parallel}^2 + \sigma_{\parallel}^2}.$$  

(91)

This tells us how efficiently the mechanism that generates $E_{\perp}$ heats the atmosphere. We discuss the general nature of $R_Q$ here and return to examples of such mechanisms later in this section.

Figure 12 shows this efficiency for both chromosphere and I/T, including the results for the three different magnetic field models for the chromosphere. Also shown is the temperature (we show the neutral temperature only for the I/T).

The above definition of $R_Q$ is independent of $J_{\perp}$, and represents the efficiency of heating for any given $J_{\perp}$. However, we can say that $R_Q$ is close to 1 when the perpendicular current is predominantly a Pedersen current. Recall from §4.2 that this occurs just below the lower peak, and just above the upper peak, in the Pedersen mobility. Near the lower peak, the
conductivity (and resistivity) is isotropic and Ohmic Joule heating and Pedersen heating are the same. Near the upper peak, $\eta_P > \eta_\|$, and Pedersen heating dominates over Ohmic Joule heating. Between these two regions of $R_Q \approx 1$ is a region where the Hall current dominates the perpendicular current and so $R_Q$ is minimal. Thus one transition of interest is from the minimal efficiency near the Hall mobility peak up to the region above the upper peak in the Pedersen resistivity. This transition altitude occurs somewhere around the temperature minimum. In fact, the efficiency increases from its minimum to 1 with altitude as the term $\Gamma = \xi_k k_e k_i n$ increases from below to above 1. This can be observed by comparing Figure 12 and Figure 7.

Goodman \citeyearpar{2000, 2004a,b} proposed that the increase in Pedersen heating as $\Gamma$ undergoes the transition from much less than 1 to much more than 1 may explain the source of chromospheric heating that has so puzzled solar physicists. Moreover, theoretical work by Goodman \citeyearpar{2011} and Song and Vasyliunas \citeyearpar{2011} and numerical simulations by Tu and Song \citeyearpar{2013} have found that given an atmospheric profile similar to the one used here, the heating due to damping of propagating Alfven waves is predominantly Ohmic Joule heating below the temperature minimum, and ion-neutral collisional heating (or Pedersen) heating above. These recent simulations suggest a possible explanation for the presence of the temperature minimum. However, such simulations, which contain no energy equation and thus no self-consistent heating, do not self-consistently produce the observed temperature structure of the chromosphere, but use an initial condition which is designed to look like observationally inferred 1D profiles. The physics of thermal conduction from the hot corona, as well as radiative, ionization, viscous, and thermoelectric processes, must be included for the simulations to obtain self-consistent heating rates. Future simulations that investigate heating mechanisms must be able to create the correct temperature profile self-consistently (for example of attempts to do so see the papers of Carlsson and Leenaars \citeyearpar{2012} and Abell \citeyearpar{2007}) as well as including well-resolved physical mechanisms (such as Alfven wave wave heating). This is the largest obstacle to identifying chromospheric heating mechanisms through numerical and theoretical investigations.

Pedersen current dissipation is a general mechanism for dissipating the energy in electric currents which are orthogonal to the magnetic field. Therefore any process which drives such currents is damped by this mechanism. To actually estimate $Q_P$ and not just $R_Q$, a process must first be identified and a physical model of the process must then be created. The degree of the damping of the process which drives perpendicular currents depends on two things: the first is the reservoir of EM energy to drive the currents, and the second is the rate at which this EM energy is used to generate and maintain electric fields to support the currents, and thus be thermalized. In the chromosphere, we do not fully understand the importance of Pedersen heating. For the solar atmosphere, transient electric fields are created by general time-dependent flows, such as convection zone flows, wave motions, and magnetic reconnection sites.

The chromosphere and I/T are atmosphere regions commonly characterized as weakly ionized mixtures permeated by magnetic field lines. These regions are subject to electromagnetic energy flux from neighboring regions of electrical energy generation. On the Sun, the turbulent convection zone beneath the chromosphere creates a spectrum of oscillations, in the range 0.5 to 1000 minutes, or $10^{-5}$ to 0.03 Hz (e.g., Cranmer and van Ballegooijen \citeyearpar{2005}). In the high plasma $\beta_p$, highly ionized convection zone there are three types of fluid (non-kinetic) waves: Alfven waves, isotropic sound waves, and magneto-acoustic waves that are guided by the magnetic field \cite{Zaqarashvili2011}. The last two of these types are compressional and most likely do not propagate into the upper chromosphere. However, the changing conditions with height change the nature of these waves as they propagate into
the chromosphere. Alfvén waves from the convection zone cause the oscillation of weakly ionized plasma in the presence of a magnetic field which provides a mechanism for the conversion of the kinetic energy of convection into electrical energy in a manner equivalent to a magnetohydrodynamic (MHD) electrical generator. In this case, $E \cdot J$ is negative and particle kinetic energy is converted to electromagnetic energy which propagates with the waves into the chromosphere. Thus there is an influx of electromagnetic energy from the convection zone into the chromosphere. Much work has been done on the propagation and dissipation of Alfvén waves into the chromosphere from the convection zone (examples of recent work include De Pontieu 1999; Leake et al. 2005; De Pontieu et al. 2007a; Hasan and van Ballegooijen 2008; Goodman 2011; Song and Vasyliūnas 2011; Tu and Song 2013).

A similar situation occurs above the I/T. The Earth’s magnetosphere undergoes convection due to the interaction of the solar wind with the magnetosphere. There is a net downflow of energetic particles and Alfvén waves. Thus the Earth’s magnetosphere can also be considered an MHD electrical generator converting solar wind mechanical energy to electrical energy that is transferred to the I/T along highly conducting magnetic field lines (Thayer and Semeter 2004; Song and Vasyliūnas 2011; Tu et al. 2011).

Both atmospheres therefore exhibit an inflow of electromagnetic energy in the form of waves from outside regions. The chromosphere experiences a large drop in density with altitude, and this implies that for a given field strength, the Alfvén speed rapidly increases. If the length scale of the increase is comparable to the wavelength of an upwardly propagating wave then reflection can occur, similar to the situation in the Ionospheric Alfvén Resonator (see Poliakov and Rapoport 1981 and references therein). Thus the chromosphere also has downwardly propagating reflected waves. High frequency (1-100 Hz) Alfvén waves generated from coronal reconnection sites (e.g. Voitenko and Goossens 2002; Kigure et al. 2010; Edmondson et al. 2011) may also propagate downward into the chromosphere, as well as leaking coronal loop oscillations caused by disturbances in the corona (e.g. Nakariakov et al. 1999; Ofman 2002). In the I/T, Alfvén waves and quasi-static fields imposed by the magnetosphere are manifested in the form of auroral arcs, field-aligned currents and electric fields
that constitute the electromagnetic energy flux into the region (e.g. Erlandson et al. 1994; Tung et al. 2001; Keiling et al. 2003; Drob et al. 2013).

The chromosphere and Earth’s I/T are conductors (the M2-domain in particular) and can convert the electrical energy of the waves to thermal and mechanical energy of the entire mixture ($\mathbf{E} \cdot \mathbf{J} > 0$). As shown previously, ion-neutral collisions dominate over electron Ohmic Joule heating, and must play a vital role in the dissipation of EM wave energy in both environments. For a finite superposition of waves with commensurate frequencies, one can use Poynting’s theorem, Equation (83), averaged over a wave period, to give

$$\langle \nabla \cdot \mathbf{S}_p \rangle = \langle \mathbf{E} \cdot \mathbf{J} \rangle.$$  \hspace{1cm} (92)

Integrating this over a volume can give us a maximum possible Joule heating rate for a proposed heating mechanism. Recall that $\mathbf{E} \cdot \mathbf{J}$ is the rate at which EM energy is converted to particle kinetic energy, and the Joule heating $\mathbf{E}^{VCM} \cdot \mathbf{J}$ cannot be greater than this. Examples of this have been performed in polar regions of the Earth’s atmosphere (e.g. Kelley et al. 1991; Thayer and Semeter 2004) and for simple models of the Sun’s chromosphere (Leake et al. 2005; Goodman 2011; Song and Vasyliunas 2011; Tu and Song 2013).

Taking the volume integral of the total thermal energy equation (82) and using Gauss’ divergence theorem for volume integrals gives

$$\frac{\partial}{\partial t} \int_V \varepsilon_{CM} dV = - \int_S \varepsilon \mathbf{V}_{CM} \cdot d\mathbf{S} + \int_S \mathbf{h}_{CM} \cdot d\mathbf{S} + \int_S \mathbf{F}_{CM} \cdot \mathbf{V}_{CM} \cdot d\mathbf{S} \ - \int_V (\mathbf{F}_{CM} \cdot \mathbf{V}_{CM}) dV + \int_V (\mathbf{E}^{VCM} \cdot \mathbf{J}) dV$$  \hspace{1cm} (93)

where $\mathbf{F}_{CM} = -\nabla \cdot \mathbf{P}_{CM}$ is the total pressure force per unit volume. Equation (93) shows that the rate of change of thermal energy in a volume consists of a number of terms in addition to the Joule heating in the volume ($\int_V (\mathbf{E}^{VCM} \cdot \mathbf{J}) dV$). The first two terms on the RHS of (93) are the convective and diffusive flux of thermal energy through the surface of the volume. The third term is the rate at which thermal energy is converted (or vice-versa) into CM kinetic energy (KE) that flows through the surface due to action of the pressure force $\mathbf{F}_{CM}$ on the CM at the surface. The fourth term is the rate at which thermal energy is converted into CM KE (or vice-versa) within the volume by the total pressure force $\mathbf{F}_{CM}$ acting on the CM. There are both pressure gradient and viscous forces related to $\mathbf{F}_{CM}$. The various forcing and dissipation mechanisms and the scales on which they operate present a significant challenge for the use of convergence of the Poynting flux to understand the heating of the two environments. Even if ion and electron collisions with neutrals are not a main contributor to the heating in the chromosphere and I/T, they significantly affect the propagation of waves (e.g. Song et al. 2005; Zaqarashvili et al. 2011), and thus must be included in theoretical and numerical investigations into wave heating.

7 A Case Study in Contrasting Paradigms: The Rayleigh-Taylor Instability

Here we present an illustrative analytical and numerical case study of the Rayleigh-Taylor instability, which highlights the contrasting conceptual and mathematical approaches employed by the two communities. Other examples in this review are the neutral wind dynamo and the Farley-Buneman Instability.
7.1 Occurrence in the Sun's and Earth's atmospheres

The unsteady transfer of energy and mass in the dynamic atmospheres of both the Earth and the Sun often creates configurations in which dense matter overlies tenuous matter. In the presence of gravity, such configurations can be unstable to disturbances that exchange the overlying heavy fluid with the light fluid below. This evolution produces falling spikes of the former and rising bubbles of the latter, and the atmosphere evolves toward a state of lower gravitational potential energy. The linear stability of such unstable hydrostatic equilibria has been examined both for broadly distributed layers of continuously upward-increasing mass density, in which the characteristic wavelength of the disturbances is comparable to or smaller than the layer thickness (Rayleigh 1882), and for very narrow layers of effectively discontinuous upward jumps in mass density, in which the characteristic wavelength of the disturbances is much larger than the thickness of the layer (Taylor 1950).

At Earth, a continuously distributed negative gradient in density of charged plasma is created by the nonuniform production and loss of ions. The result is a plasma density that peaks at an intermediate height of about 300 km (see Figure 2), and at altitudes below that it can be unstable to the Rayleigh-Taylor instability. Typically this occurs after sunset. Disturbances generate large-scale density depletions in the lower I/T that can rise to high altitudes, over 1000 km, (“spread-F”, see review by Woodman 2009).

On the Sun, negative density gradients can be produced by the plasma pressure deficit present in regions of strong magnetic field, which compensates for the magnetic pressure enhancement there and maintains an overall balance of the total pressure force across the magnetic region. The magnetic Rayleigh-Taylor instability creates filamentary structures in both newly emerged and established flux regions (Isobe et al. 2005, 2006; Arber et al. 2007; Berger et al. 2010; Hillier et al. 2011, 2012a,b). In observations and models of solar prominences, which are cold dense neutral structures in the background hot, tenuous corona, bubbles of low-density, high-temperature coronal plasma well up from below and intrude into the high-density, low-temperature body of the prominence above.

![Fig. 13 Left: Rayleigh-Taylor instabilities in the heliosphere. Airglow measurements showing evidence of the Rayleigh-Taylor instability in the terrestrial I/T. The dark shapes in the gray color-scale are expanding bubbles of plasma due to the instability. Colored tracks (red/green/blue) are paths of Global Positioning System (GPS) satellites. Figure is reprinted courtesy of J. Makela and Annales Geophysicae. Right: Filamentary structure in a quiescent solar prominence, a possible consequence of Rayleigh-Taylor instability in the corona. Figure is reprinted courtesy of Y. Lin and Solar Physics.](image)

The qualitative similarities in these phenomena at the Earth and Sun are illustrated by the images in Figure 13. The left Panel in Figure 13 shows data from airglow measurements...
made at Haleakala, Hawaii during the interval 29-30 September 2002 of spread-F bubbles in the I/T (Makela et al. 2004). Colored tracks (red/green/blue) are paths of Global Positioning System (GPS) satellites. Notice the intricate fingering of the dark bubbles, which are regions of very low electron density. This image is taken from a movie (Sep29_30_GPS_7774.mov) that illustrates more completely the dynamics of these features. The right Panel in Figure 13 shows data from the Hinode Solar Optical Telescope on 30 November 2006 of a solar prominence at the solar limb (Berger et al. 2010, Figure 1). The dashed white box outlines a region of a dark bubble forming, rising, fingering, and fragmenting among the shimmering strands of the prominence gas. Note that this measured emission originates in the Balmer series of neutral hydrogen atoms, marking cool material; the dark bubble is hot material in which the hydrogen is fully ionized and so does not radiate in this line (Berger et al. 2011). The inset image at the upper right shows the prominence as a dark feature on the otherwise bright solar disk three days earlier. The Hinode image is part of a movie (Berger et al. 2010, ApJ330604F1.mov) that shows the full evolution of this bubble and many others. Both the terrestrial and solar movies are included in this paper as supplementary material.

7.2 Stability Analysis in Two Different Paradigms

The occurrence of Rayleigh-Taylor instabilities is important in both the chromospheric and ionospheric contexts and has a common physical origin in the negative density stratification of a fluid in the presence of gravity, as discussed above. However, the mathematical manipulations performed and the language used to describe the underlying physics are quite different in the two communities of investigators. In the following subsections, we present simple derivations of the dispersion relation for unstable Rayleigh-Taylor disturbances within the contexts of the B,V paradigm (§7.2.1) and E,J paradigm (§7.2.2). As previously mentioned, the first uses B and V as primary variables, while the second uses E and J. In the end, however, the approaches are complementary, and the same dispersion relation is obtained irrespective of how the analysis is framed, although the assumptions used in the E,J paradigm are not universally acceptable. In the concluding subsection (§7.4), we simplify the general dispersion equation for regimes in which the plasma is strongly coupled to the neutrals by collisions. This is the case in the solar chromosphere and also in prominences, and in the I/T. This example illustrates an important physical process that is common to the two environments, but is conceptualized and analyzed in very different ways, therefore, in our experience, impeding mutual understanding across disciplines.

7.2.1 B,V (Chromospheric) Context

The basic equations used in the analysis are those for continuity and momentum of the neutral gas and the one-fluid plasma (electrons+ions). The plasma equations are the mass-weighted sum of the ion and electron continuity equations (5 and 6) and the sum of the momentum equations for the ions and electrons (9 and 10). Ionization and recombination terms are ignored, as are electron inertia effects; in particular, we assume that $m_e n_e \ll m_i n_i$ (see §4.3). Consider a simple neutral-plasma configuration in a slab geometry, $B = B_0 \hat{e}_z$, $g = g \hat{e}_x$, $V_{n0} = V_{p0} = 0$, $n_n(x) = m_n n_n(x)$, and $n_p(x) = m_i n_i(x)$ (assuming $m_e \ll m_i$), where the subscript 0 indicates an equilibrium value. Perturbed variables $\tilde{q}$ are assumed to vary as $\tilde{q} \exp(iky - i\omega t)$, where $\omega = \omega_r + i\gamma$ and the disturbances propagate in the $y$ direction. The neutral and plasma flows are assumed to be incompressible, $\nabla \cdot \mathbf{V} = 0$. For the uniform equilibrium magnetic field assumed, the ideal MHD induction equation, Equation
with $\eta = 0$, then yields $\vec{B} = 0$, so the magnetic field remains undisturbed. Since we neglect the $x$ dependence of the perturbed quantities, incompressibility requires $\vec{V}_y = \vec{V}_p = 0$. Physically, the time scale associated with the instability is required to be much longer than those associated with compressive acoustic or magnetosonic waves.

The linearized continuity equation (7) and the $x$ component of the linearized momentum equation (11) for the neutrals then take the forms

$$-i\omega \tilde{n}_n = -\tilde{V}_{nx} \frac{d n_0}{dx},$$

$$-i\omega n_0 m_n \tilde{V}_{nx} = +\tilde{n}_n m_n g - n_0 m_n v_{ni} (\tilde{V}_{nx} - \tilde{V}_{px}).$$

The corresponding equations for the plasma, from (5–6) and (9–10), are

$$-i\omega \tilde{n} = -\tilde{V}_{px} \frac{d n_0}{dx},$$

$$-i\omega n_0 m_i \tilde{V}_{px} = +\tilde{n}_m g - n_0 m_i v_{ni} (\tilde{V}_{px} - \tilde{V}_{nx}).$$

Solving for $\tilde{n}_n$ and $\tilde{n}$ in Equations (94) and (96) and substituting into Equations (95) and (97), respectively, yields, after some rearrangement,

$$\left(\omega^2 + i\nu v_{ni} - \frac{g}{L_n}\right) \tilde{V}_{nx} = i\nu v_{ni} \tilde{V}_{px},$$

$$\left(\omega^2 + i\nu v_{in} - \frac{g}{L_p}\right) \tilde{V}_{px} = i\nu v_{in} \tilde{V}_{nx},$$

where $L_n^{-1} = d \ln n_0/\text{dx}$ and $L_p^{-1} = d \ln n_0/\text{dx}$ are the local neutral and plasma density scale heights. Combining Equations (98) and (99), we arrive at the dispersion equation

$$\left(\omega^2 + i\nu v_{ni} - \frac{g}{L_n}\right) \left(\omega^2 + i\nu v_{in} - \frac{g}{L_p}\right) + \omega^2 v_{ni} v_{in} = 0,$$

which can be expanded to read

$$\omega^4 + i\nu^3 (v_{in} + v_{ni}) - \omega^2 \left(\frac{g}{L_p} + \frac{g}{L_n}\right) - i\nu \left(\frac{v_{ni} g}{L_p} + \frac{v_{in} g}{L_n}\right) + \frac{g^2}{L_n L_p} = 0.$$

Rewritten in terms of the Brunt-Väisälä frequencies $N_n$ and $N_p$, Equation (1) becomes

$$\omega^4 + i\nu^3 (v_{in} + v_{ni}) = \omega^2 (N_n^2 + N_p^2) - i\nu (v_{ni} N_p^2 + v_{in} N_n^2) + N_n^2 N_p^2 = 0.$$

Approximate solutions to this equation in the strong-coupling limit reveal the instability growth rates, as discussed in §7.4 below.
7.2.2 \( E,J \) (Ionospheric) Context

The set of three-fluid equations used to analyze the Rayleigh-Taylor instability in the F layer of the Earth’s I/T consists of those for electron continuity and current conservation, electron and ion momentum, and neutral continuity and momentum (Ossakow 1981). The equilibrium state has \( n_{e0}(x) = n_{i0}(x) = n_{0}(x) \), \( E_0 = 0 \), \( B = B_0 \hat{e}_x \), and \( \nabla v_i = 0 \). The equilibrium is perturbed such that all disturbances are proportional to \( \exp(iky - i\omega t) \). We invoke the local approximation that \( \partial / \partial x \ll k \) with regard to the perturbed variables. In conjunction with Faraday’s law, this implies that \( \vec{E}_x = 0 \) in order for \( \vec{B} = 0 \) to be maintained.

Neglecting inertia, gravity, and collisional coupling due to the small electron mass, the perturbed electron velocity from the linearized Equation (10) is simply the \( \vec{E} \times \vec{B} \) drift due to the instability,

\[
\vec{V}_{ex} = \frac{E_y}{B_0}, \quad \vec{V}_{ey} = 0.
\]  
(103)

Applying the local approximation to the current conservation constraint, Equation (3.4), we find that it must be the case that \( J_y = 0 \). Hence,

\[
\vec{V}_{ix} = \vec{V}_{ey} = 0,
\]  
(104)

where the second equality follows from Equation (103). In the \( y \) component of the ion momentum equation (9), only the drag term due to collisions with neutrals now remains. Thus, we also must have that

\[
\vec{V}_{iy} = \vec{V}_{iy} = 0.
\]  
(105)

The three conditions (103–105) imply that \( \nabla \cdot \vec{V}_e = \nabla \cdot \vec{V}_i = \nabla \cdot \vec{V}_n = 0 \), i.e., the flows of all three fluids are incompressible in this approximation. This demonstration justifies a posterno-riori the incompressibility assumption that we made at the outset of 7.2.1.

A second deduction that follows from the current conservation constraint and \( J_y = 0 \) is that \( J_x \) must be uniform along \( x \),

\[
\frac{\partial J_x}{\partial x} = (\vec{V}_{ix} - \vec{V}_{ex}) e \frac{dn_e}{dx} = 0.
\]  
(106)

Consequently, we also must have that

\[
\vec{V}_{ix} = \vec{V}_{ex} = \frac{E_y}{B_0}.
\]  
(107)

The bulk flows of the electrons and ions due to the instability are, therefore, identical, \( \vec{V}_e = \vec{V}_i = \vec{E} \times \vec{B} \). This is consistent with a vanishing electric field in the frame of the electrons, \( \vec{E} + \vec{V}_e \times \vec{B}_0 = 0 \), which was tacitly assumed in the magnetohydrodynamic analysis in 7.2.1 where \( \vec{E} \) plays no explicit role. In addition, the total perturbed current vanishes, \( J = 0 \), consistent with \( \vec{B} = 0 \) and with the earlier analysis.

From the linearized electron continuity equation (6), the perturbed electron density satisfies

\[
- i \omega \vec{n}_e = \frac{dn_{e0}}{dx} \vec{V}_{ex} \frac{dn_{e0}}{dx} \vec{V}_{ix} = - i \omega \vec{n}_i,
\]  
(108)

after using first \( n_{i0} = n_{e0} \) and \( \vec{V}_{ix} = \vec{V}_{ex} \), and then the ion continuity equation (5). Substituting the definition \( L_p = d\ln n_{i0}/dx = d\ln n_{e0}/dx \), and the \( \vec{E} \times \vec{B} \) drift velocity from Equation (107), we obtain the (equal) electron and ion density perturbations

\[
\frac{\vec{n}_e}{n_0} = \frac{\vec{n}_i}{n_0} = \frac{i}{\omega} \frac{1}{L_p} \vec{V}_{ex} = \frac{i}{\omega} \frac{1}{L_p} \vec{V}_{ix} = \frac{-i}{\omega} \frac{1}{L_p} \frac{E_y}{B_0}.
\]  
(109)
Equations (108) and (109) are equivalent to Equation (96) in §7.2.1, since \( n_e = n_i = n \) and \( V_e = V_i = V_p \). Finally, after recalling that \( \tilde{E}_x = 0 \) and \( \tilde{V}_{iy} = 0 \), the \( x \) component of the linearized ion momentum equation (9) yields

\[
\nu_{ni} \tilde{V}_{nx} = ( -i \omega + \nu_{in}) \tilde{V}_{ix} - g \frac{\tilde{n}_i}{n_{i0}}
\]

(110)

\[
= \left( -i \omega + \nu_{in} + \frac{i g}{\omega L_p} \right) \tilde{V}_{ix}
\]

(111)

\[
= \left( -i \omega + \nu_{in} + \frac{i g}{\omega L_p} \right) \frac{\tilde{E}_y}{B_0}.
\]

(112)

The linearized continuity and momentum equations for the neutrals are combined as in §7.2.1 to obtain (cf. Equation 98)

\[
\left( -i \omega + \nu_{ni} + \frac{i g}{\omega L_n} \right) \tilde{V}_{nx} = \nu_{ni} \tilde{V}_{ix} = \nu_{ni} \frac{\tilde{E}_y}{B_0}.
\]

(113)

Eliminating \( \tilde{V}_{nx} \) from Equations (112) and (113) leads to the desired dispersion equation,

\[
(\omega^2 + i \omega \nu_{ni} - \frac{g}{L_n}) \left( \omega^2 + i \omega \nu_{in} - \frac{g}{L_p} \right) + \omega^2 \nu_{ni} \nu_{in} = 0.
\]

(114)

This is identical to the magnetohydrodynamic result, Equation (100), and will be analyzed further in the next section.

7.3 Comparing the Two Paradigms

Our derivations of the dispersion equation in the B,V and E,J contexts culminate in the same result, albeit by following different paths. The principal difference is the role assigned to the electric field, which is primary in the ionospheric context and in the literature of the I/T community, but is all but invisible in the chromospheric context and in much of the literature of the solar community (excepting situations of rapidly changing magnetic fields associated with flares and other transient behavior). As the derivations highlight, it is commonly said about the I/T that the electric field gives rise to \( E \times B \) drifts, implying that \( E \) drives \( V \). In contrast, the same is rarely, if ever, said about the chromosphere or corona, where \( E \) is principally a consequence of plasma flow \( V \) across the magnetic field \( B \), so \( V \) drives \( E \). The normal mode analysis performed here does not distinguish between these two perspectives; only an initial value analysis can do that. As discussed previously in 5.2, Vasyltunias [2001, 2011, 2012] has argued persuasively for the view that flows drive electric fields, but electric fields do not drive flows, in magnetized plasmas occurring in nature, and that the conventional \( E \times B \) drift cannot describe the complete time-dependent physics.

For completeness, we note that this analysis of the Rayleigh-Taylor instability in the ionospheric context is predicated on a local evaluation of plasma and neutral variables, i.e., it applies in a restricted region in space. In this limit, as we have seen, the dispersion equation is the same as in the solar context. However, in the I/T, as discussed in 3.4, the magnetic field lines are assumed to be equipotentials and the electric field generated by the instability extends along the entire magnetic flux tube. Thus, the instability is affected by the plasma on the flux tube that encompasses both the E and F layers of the I/T and a “flux-tube integrated” analysis of the instability is required. A discussion of this type of analysis as it applies to
Rayleigh-Taylor instability in the I/T is given by Sultan (1996). The result is a growth rate averaged all along an equipotential field line, which in order of magnitude has a growth time of about 15 min. In contrast, the Alfvén travel time is only about 10 s. Thus, as argued by Vasyliunas (2012) (see our §5.2), the non-potential component of the electric field is small, and the electrostatic approximation is quite good. A similar conclusion follows from a more complete electromagnetic analysis of the Rayleigh-Taylor instability, including Alfvén waves and the Pedersen resistivity of the I/T, performed by Basu (2005). The result found is that the magnetic field perturbations are very small due to resistive slip between the plasma and magnetic field (see our §5.1), so that, again, the electrostatic approximation is well justified.

7.4 Strong-coupling Limit of the Rayleigh-Taylor Instability

The general dispersion equation (100) or (114) trivially factors into distinct neutral and plasma modes in the limit of weak collisions, $\nu_{\alpha\beta} \to 0$:

$$\omega^2 = N^2 = \frac{g}{L_n}; \quad \omega^2 = N^2 = \frac{g}{L_p}.$$

(115)

In §2 Figure 3, on the other hand, we noted that the characteristic Brunt-Väisälä frequencies $N \equiv (g/L)^{1/2}$ in the solar and terrestrial atmospheres are much smaller than the ion-neutral collision frequencies $\nu_{in}$. We can scale these quantities by setting $\nu_{in} = O(1)$, and $N_p, N_n = O(\varepsilon)$ where $\varepsilon \ll 1$. We also have $\nu_{in} = O(\varepsilon)$ in a weakly ionized plasma. We can now look for strongly-coupled solutions to the general quartic dispersion relation given in Equation (102), i.e. $|\omega| \gg N_p, N_n$, of which $\omega = 0$, $\gamma = O(1)$ is one. Balancing the largest terms in the dispersion relation (102) for this scaling, the quartic and cubic terms, yields Equation (116) below. Next we consider intermediate solutions, where $|\omega| \sim N_p, N_n$, of which $\omega = O(\varepsilon)$, $\gamma = 0$ is one. Then the largest terms in the dispersion relation (102) are the cubic and linear terms, and balancing these yields Equation (117) below. Finally we consider low-frequency solutions, where $|\omega| \ll N_p, N_n$, of which $\omega = 0$, $\gamma = O(\varepsilon^2)$ is one. Then the largest terms in the dispersion relation (102) are the linear and constant terms, and balancing these yields Equation (118) below:

$$\omega \approx -i(V_{in} + V_{ni}); \quad (116)$$

$$\omega^2 \approx \left(\frac{V_{in}N_n^2 + V_{ni}N_p^2}{V_{in} + V_{ni}}\right); \quad (117)$$

$$\omega \approx -iN_n^2N_p^2 / \left(V_{in}N_n^2 + V_{ni}N_p^2\right). \quad (118)$$

The solution in Equation (116) is a purely damped inter-penetrating mode in which the two fluid velocities are $180^\circ$ out of phase. It is always stable.

The pair of intermediate-frequency solutions in Equation (117) are a weighted average of the classical single-fluid Rayleigh-Taylor growth rates (or Brunt-Väisälä frequencies) of the neutral gas and plasma. Due to the preponderance of neutrals over ions (i.e., $n_n \gg n_i$, hence $V_{in} \gg V_{ni}$; see Fig. 3) these solutions simplify to

$$\omega^2 \approx N_n^2 = \frac{g}{L_n}. \quad (119)$$
They are oscillatory in the chromosphere and the I/T where the neutrals are stably stratified, \( gL_n > 0 \). On the other hand, in a cool, dense, partially neutral prominence suspended within the ionized solar corona, \( gL_n < 0 \), and one of these modes is a growing Rayleigh-Taylor instability. A two-fluid (electrically neutral ionized plasma plus neutral gas) numerical simulation of such an unstable configuration is presented below in §7.5.1. Due to the strong collisional coupling of the neutral gas and plasma, their velocities are essentially equal,

\[
V_n \approx V_p. \tag{120}
\]

This can be deduced readily from Equations (98) and (99).

A second potentially unstable solution is the low-frequency mode in Equation (118). It also is a weighted average of the plasma and neutral contributions, and as \( v_{in} \gg v_{ni} \), simplifies to

\[
\omega \approx -i \frac{N_p^2}{v_{in}} = -i \frac{g}{v_{in} L_p}. \tag{121}
\]

This is the classical collision-dominated Rayleigh-Taylor instability in the I/T (e.g., Os- sakow 1981), driven by the upward-increasing plasma density, \( gL_p < 0 \). A similar instability should occur in the ALC7 solar chromosphere where the ionization fraction increases sufficiently rapidly with height above the surface (cf. Fig. 2). A multi-fluid simulation of this low-frequency mode is presented below in §7.5.2. In this case, due to the approximate balance between gravity and collisional coupling to neutrals on the part of the ions, Equation (99), and to the very low frequency and the weak collisional coupling to ions on the part of the neutrals, Equation (98), the velocities satisfy

\[
|V_p| \gg |V_n|. \tag{122}
\]

All of these features represented by Equations (119–122) are evident in the simulation results described next.

### 7.5 Simulation Study

We now present a simulation study applicable to both the solar and ionospheric case performed within the HiFi spectral-element multi-fluid modeling framework (Lukin 2008). The calculations use an implementation of the partial differential equations describing self-consistent, nonlinear evolution of a partially ionized, three-fluid hydrogen mixture of ions, electrons and neutrals (Leake et al. 2013, and references therein). The set of equations solved are (5-15), but neglecting the electron momentum, and with the low frequency, cold plasma, approximation to the Ohm’s law, cast in the plasma velocity frame (as we can follow the evolution of the plasma directly). The electron pressure term is kept however, because, in some cases, when there are small variations of out-of-plane B-field from the uniform \( \mathcal{E}(1) \) guide field, as is the case for the I/T simulation, it isn’t quite so clear that this term can always be ignored. Hence the Ohm’s law used is:

\[
E^p + (V_p \times B) = \left[ \frac{1}{k_{ei}} + \frac{1}{k_{en} + k_{in}} \right] \frac{B}{en} J + \left[ 1 - \frac{\xi_{en} k_{in}}{k_{en} + k_{in}} \right] \frac{B}{en} J \times \hat{b} - \frac{V \cdot \hat{p}_e}{en} \tag{123}
\]

We point out that using a hydrogen fluid is not truly appropriate for the F-layer I/T, which is dominated by oxygen ions. However, the significance of our study is that we can capture the essential physics of both situations within the framework of a single model, using parameters
appropriate to the two different environments. The boundary conditions are periodic on the sides, and are closed and reflecting on the top and bottom, respectively.

In the subsections below, we describe the basic parameters and show the key figures for the two cases studied. Some mathematical details are relegated to the Appendix to streamline the presentation of the essential results.

**Fig. 14** Two-fluid numerical simulation of Rayleigh-Taylor instability in a solar prominence: (a) magnetic field $[B_z - B_0]/B_0$; (b) electron or ion density $n_e/n_0 = n_i/n_0$; (c) neutral density $n_n/n_0$; (d) neutral temperature $T_n/T_0$. Vector velocities for the neutrals ($V_n$) and ions ($V_i$), and their difference ($V_i - V_n$), are shown in Panels (c), (b), and (a) respectively.

### 7.5.1 Solar Prominence

Our first case is a model for the Rayleigh-Taylor instability in a solar prominence. We adopt a two-dimensional slab equilibrium for the prominence structure, showing the result of the simulation at a time when the instability is already well-developed in Figure 14. In the initial equilibrium, the electron (and ion) density $n_e$ of the background corona (Fig. 14b) is exponentially stratified, attaining a value $n_0 = 1 \times 10^{15} \text{ m}^{-3}$ at height $x = 0$ in the Cartesian coordinate system scaled to a typical prominence size, $L_0 = 1 \times 10^6 \text{ m}$. The neutral density $n_n(x)$ within the prominence slab (Fig. 14c) is an order of magnitude larger, reaching $10n_0 = 1 \times 10^{16} \text{ m}^{-3}$ at its central height, $x = 0.5$. The characteristic temperatures of the fluids are $T = 2 \times 10^5 \text{ K}$ in the corona and $T = 1 \times 10^4 \text{ K}$ in the prominence; the neutral temperature is shown in Figure 14d, scaled to a normalization temperature $T_0 = 5.8 \times 10^7 \text{ K}$. The electron and ion temperatures are very similar, due to the fast thermal exchange between the fluids. Finally, the deviation of the out-of-plane magnetic field $B_z(x)$ from a uniform value $B_0 = 1 \times 10^{-3} \text{ T}$ is displayed in Figure 14. As detailed in the Appendix, $B_z$ is vertically stratified to balance the initial pressure and gravity forces in magnetohydrostatic equilibrium. Because beta is low for both the plasma and neutral gas, the associated magnetic-field deviations are relatively small.
This unstable neutral-plasma system is initialized with small-amplitude neutral-density perturbations centered at $x = 0$, on the bottom side of the prominence slab. Velocity vectors are shown in Figure 14 Panels a, b, and c that correspond to the differential ion-neutral velocity, ion velocity, and neutral velocity, respectively. We observe that all contours have the same shape, and the plasma and neutrals track each other quite closely. The neutral gas is unstable and, due to the collisional coupling to the plasma, the plasma follows the neutrals. We determined the e-folding growth time $\tau$ of the flow velocity and the resulting growth rate is $\tau^{-1} \sim 2 \times 10^{-2} \text{s}^{-1}$. For solar gravity and the chosen neutral density profile, the Brunt-Väisälä frequency, which is the analytic growth rate for the strong-coupling limit (119), is $N_n \approx 2.6 \times 10^{-2} \text{s}^{-1}$, which is in very good agreement with the numerically determined growth rate, given the simplicity of the derivation compared to the complexity of the simulation model. An evaluation of the collision frequencies at $x = 0$ gives $\nu_{in} = 2.2 \times 10^2 \text{s}^{-1}$ and $\nu_{ni} = 5.2 \times 10^1 \text{s}^{-1}$. Thus, this calculation lies in the strong-coupling regime of §7.4 and Equations (119) and (120) apply.

7.5.2 Terrestrial Ionosphere

Our second case is a model for the Rayleigh-Taylor instability in the I/T. The well-developed instability in the simulation initialized with a two-dimensional slab equilibrium is shown in Figure 15. In this case, the neutral density $n_n(x)$ of the background atmosphere (Figure 15c) is exponentially stratified, attaining a value $n_0 = 1 \times 10^{16} \text{m}^{-3}$ at height $x = 0$ in the Cartesian coordinate system scaled to a typical F-layer size, $L_0 = 2 \times 10^4 \text{m}$. The electron (and ion) density $n_e(x)$ within the I/T (Figure 15b) is four orders of magnitude smaller,
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reaching $10^{-4} n_0 = 1 \times 10^{12} \text{ m}^{-3}$ at its central height, $x = 0.5$. The temperatures of the fluids are all uniform and equal, at $T_n = 1.2 \times 10^3 \text{ K}$; the deviation of the neutral temperature from this value is shown in Figure [15], scaled to a normalization temperature $T_0 = 5.3 \times 10^3 \text{ K}$. Finally, the deviation of the out-of-plane magnetic field $B_z(x)$ from a uniform value $B_0 = 3 \times 10^{-5} \text{ T}$ is displayed in Figure [15]. Here, $B_z$ is vertically stratified to balance the plasma pressure and gravity forces in magnetohydrostatic equilibrium. Because the plasma beta is far smaller in the I/T than in the chromosphere and corona, the resulting ionospheric magnetic-field deviations are similarly smaller, by comparison.

This unstable neutral-plasma system is initialized with small-amplitude electron-density perturbations centered at $x = 0$, on the bottom side of the ionization layer. Velocity vectors again are shown in Panels a, b, and c that correspond to the differential ion-neutral velocity, ion velocity, and neutral velocity, respectively. In this case, we observe that the plasma and neutrals do not track each other very well. The plasma is unstable and develops a low-density bubble that rises through the plasma layer, as is observed in the I/T (Fig. 13). The contours of perturbed magnetic field closely resemble those of the plasma density. The formation of the bubble and the generation of the ion flows are strongly affected by drag exerted by the neutrals. Because the collisional coupling to the neutrals by the plasma is very weak, however, the neutrals move only slowly and somewhat independently, as shown by the neutral density, velocity, and temperature. The numerically calculated growth rate is $\tau^{-1} \sim 1.3 \times 10^{-5} \text{ s}^{-1}$. For terrestrial gravity and the chosen plasma density profile, the Brunt-Väisälä frequency is $N_p = 3.7 \times 10^{-2} \text{ s}^{-1}$, which is much smaller than the ion-neutral collision frequency $v_{in} = 1.1 \times 10^2 \text{ s}^{-1}$. This calculation also lies in the strong-coupling regime of §7.4, but here Equations (121) and (122) are the relevant solutions. The low-frequency growth rate calculated from Equation (121) is $\gamma \approx 1.3 \times 10^{-5} \text{ s}^{-1}$, in excellent agreement with the numerically determined rate. We point out that the growth rate for an O$^+\,$ plasma in the real I/T would be about ten times greater than the rate for our simulated H$^+\,$ plasma, with a resulting e-folding time of about 2 hrs, in reasonable agreement with observations.

### 7.6 Summary

These simulations show how a common framework has been used to describe the Rayleigh-Taylor instability in both the chromosphere and I/T, even though historically, the phenomena has been approached in two very different ways. There are many other common phenomena between the chromosphere and I/T, and much knowledge can be gained from applying this universal approach. However, as already pointed out, the use of the E,J paradigm to describe the physics is based on a number of assumptions (steady-state, electrostatic electric fields) that cannot be justified for all I/T processes, and that when these assumptions are invalid, the EJ paradigm could potentially miss the time-dependent physics.

### 8 Conclusions

In this paper we have compared the Sun’s chromosphere and Earth’s ionosphere/thermosphere (I/T). Both are weakly ionized, stratified mixtures of plasma and neutral gas with an increasing ionization fraction with height (altitude). Both have typical plasma $\beta$ less than one, and a neutral (or total) $\beta$ which transitions from above to less than one with increasing altitude. Thus plasma motions alone are not capable of creating large perturbations in the magnetic field, but if the coupling between plasma and neutrals is strong enough, the average motions
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of neutrals and plasma can potentially create large perturbations in the field. For the chromosphere, where the neutral-ion collision frequency is larger than $10^3$ Hz, then for phenomena with timescales longer than $10^{-3}$ s, the coupling is strong enough. For the I/T the neutral-ion collision frequency is smaller than $10^{-3}$ Hz, so most timescales of interest (e.g. minutes to hours) the coupling is not sufficient. This difference is brought about by the much lower ionization level in the I/T compared to the chromosphere, something which creates many important differences between the two atmospheres, such as in the electrical conductivity. It also affects fluid instabilities, as demonstrated by our discussion of the Brunt-Väisälä frequency.

In both environments the Brunt-Väisälä oscillations (or Rayleigh-Taylor instabilities) of the plasma are affected by coupling to the neutrals. In the chromosphere, the oscillations and instabilities of the neutrals are strongly affected by coupling to the plasma, since there the neutral Brunt-Väisälä frequency is less than the neutral-ion collision frequency. However, the opposite is true in the I/T, and so the oscillations of the stably stratified neutral gas are unaffected by the plasma, and neutral motion is essentially undisturbed by the evolution of unstably stratified plasma.

Both environments exhibit a variation of the magnetization of the ions and electrons with altitude, with magnetization being defined by the ratio of gyrofrequency to collision frequency. The magnetization is also a measure of how mobile the ions and electrons are in the presence of neutrals. As the magnetization depends on magnetic field, we found a range of behavior in the chromosphere, but for certain magnetic field cases we found a similar behavior of the magnetization and mobility in both the chromosphere and the I/T. In general, at low heights, the electrons and ions are unmagnetized due to high collision rates with neutrals. With increasing altitude, the electrons become mobile first, and the perpendicular current is a combination of Pedersen ($J_P$) and Hall ($J_H$) current. Higher up still, the ions become magnetized and Pedersen current dominates with the Pedersen current heating efficiency $\Gamma \rightarrow 1$. We found that for the 1D I/T model, there are three regions, representing unmagnetized plasma (M1), magnetized electrons and unmagnetized ions (M2), and magnetized plasma (M3). In the middle of the M2 region is a height at which the conductivity became anisotropic (this is where $\Gamma = \xi n k e k_i$ become larger than unity, and is also the height at which heating by Pedersen current becomes important see below). Similar transitions were seen in the chromosphere, with a variation of the altitude of these transitions for different magnetic field models. This anisotropy can also be seen in the relative contribution of current dissipation by perpendicular currents and parallel currents, and is a factor in the location at which the field becomes force-free.

The large disparity in plasma density between the chromosphere and I/T also creates differences in the modeling and analysis of phenomena in the two atmospheres. We showed that the chromosphere is ideal (non-resistive) on length scales much larger than a km, such that the evolution of the magnetic field is dominated by advection by coupled plasma-neutral flows, but resistive below these lengths, where the field becomes decoupled from the average flow. However, because the conductivity is so much lower in the I/T, the Earth’s atmosphere is essentially resistive such that the magnetic field is always decoupled from the average flow and coupled to electron flow. This difference, combined with the assumption that $E$ is a potential field in the I/T for long enough timescales, leads to the treatment of I/T phenomena where $E$ and $J$ are considered primary variables, while in the chromosphere $B$ and $V$ are the primary variables. An example of this was applied to the phenomena of wind driven dynamos in §5.2. Parker (2007), Vasyliūnas (2001), Vasyliūnas (2011), and Vasyliūnas (2012) criticize the EJ paradigm, stating that for long time scales we can remove the dynamical equations for $\partial E/\partial t$ and $\partial J/\partial t$ and have $B$ and $V$ be the primary variables (V-B paradigm), but the converse is not true, and the E,J paradigm is not tractable. We find
that for localized phenomena not coupled to the magnetosphere, using the EJ paradigm may allow one to arrive at the same result as the V-B paradigm, as was achieved in a general sense for wind-driven dynamos and the Rayleigh-Taylor instability in this review. However, in general this may not always be the case, especially when magnetosphere-I/T coupling is considered or processes of short time and spatial scales are under consideration.

The two atmospheres of the Earth and the Sun have different mechanisms that contribute to I/T and chromosphere heating. We know more about the heating mechanisms in the I/T compared to the chromosphere. In the I/T the dominant heating term for the neutral gas is absorption of UV/EUV radiation, which forms the ionosphere through photoionization and raises neutral and plasma temperatures to more than one thousand degrees. However, heating of the neutral gas by collisions with ions can be as significant as solar heating during geomagnetic storms and represents the most variable source of energy to quantify in the I/T energy equation. Other processes, such as ion-neutral frictional heating and electron heating by Farley-Buneman instability can also contribute to energy transfer in the I/T system. In the chromosphere, the presence of a temperature minimum and temperature gradient reversal is a major challenge for solar physics. The observed average chromospheric temperature profile is created by a balance of a few major processes. Radiation in the chromosphere is both optically thick and optically thin, and can be formed in non-LTE. The downward conduction of energy from the much hotter corona (the heating of which is also a major open question in solar physics) is important in the upper chromosphere, but in the lower chromosphere, the radiation must be balanced by some heating mechanism. In fact the chromosphere requires roughly ten times more heat input than the corona to maintain its elevated temperature, due to a much larger density (Narain and Ulmschneider 1990).

In §6.2, we showed how electromagnetic energy is converted into thermal energy of the plasma-neutral gas mix, via the term $E_{V_{CM}} \cdot J$. As discussed above, this form of Joule heating includes the Ohmic term due to electron collisions with ions and neutrals, and a plasma-neutral frictional term, the latter of which dominates when the ions depart from neutral motion, i.e. when they start to become magnetized. We showed that when this occurs the currents perpendicular to the magnetic field become dominated by ion Pedersen currents. As shown in Goodman (2000, 2004a), the efficiency $R_Q$ of the Pedersen current heating rate, given by $(E_\perp + V_{CM} \times B) \cdot J_\perp = \eta P_J$ increases as $\Gamma k_{eB} \eta$ starts to become large, which is when the conductivity (and resistivity) tensor becomes anisotropic. Below this region, the heating is mainly Ohmic Joule heating (electron-ion collisions). This analysis applies to a general dissipation mechanism, and to derive the actual heating term one must model the generation of such currents in the atmosphere. Goodman (2000, 2001, 2011), Song and Vasilyunas (2011), and Tu and Song (2013) considered the propagation of MHD waves into the chromosphere. The center of mass flow due to the wave motion has a component perpendicular to B which drives a center of mass electric field, which in turn drives the Pedersen current. Future simulations with self-consistent thermodynamics and resolved non-linear wave interactions and reflections are key to confirming the hypothesis that wave damping by Pedersen current dissipation is responsible for the required chromospheric heating.

In reviewing the two atmospheres in terms of plasma-neutral coupling, we have brought to light some of the many open questions and issues. We review here some of those issues, and suggest what future studies should address, and what are the main challenges.

As mentioned, one of the main unanswered questions in the chromosphere is the mechanism which maintains its elevated temperature. Over the last 60 or so years there have been many proposed mechanisms, and reviews of such mechanisms can be found in Narain and Ulmschneider (1990). These include dissipation of acoustic, magneto acoustic and Alfvén wave which originate at the turbulent convection zone, which is capable of creating a spec-
trum of waves. Other mechanisms include magnetic reconnection, the Farley-Buneman instability, and low frequency current dissipation. The majority of these proposed mechanisms are only efficient at small scales, which creates one of the main challenges in modeling them. A numerical model must be able to sufficiently resolve the scales on which processes such as magnetic reconnection and wave dissipation operate (down to 10m or smaller), but it must also cover the spatial scales of interest, namely the extent of the chromosphere which is $\sim 2$ Mm. The other major problem when modeling proposed heating mechanisms is that to self-consistently model the heating, the model must include the complicated radiation in the chromosphere. Recent advances in modeling the chromosphere have included the coupling of the 3D non-LTE radiative physics to the MHD physics (see e.g., Carlsson and Leenarts 2012 and references therein) on regions of small extent. The approach of using such detailed simulations to parameterize the radiation physics in terms of MHD variables (total density, temperature) and model chromospheric proposed heating mechanisms, is a possibly fruitful approach to solving the chromospheric heating problem. However, care must be taken to ensure the correct mechanisms are resolved sufficiently, and that on the length and time scales of interest, the correct equations are used, particularly when it comes to the generalized Ohm’s law. The problems in understanding the thermodynamic structure of the chromosphere also apply to prominences, which as mentioned are structures of chromospheric material suspended in the corona. The cause of fine structure in prominences is also an open issue, as well as fine structure in the chromosphere such as fibrils, jets, and surges. In addition to explaining the emission in the average, or quiet chromosphere, the increase of emission during flares is also an interesting issue, and has been newly investigated with simulations (e.g., Russell and Fletcher 2013). As with the generic chromospheric heating problem, this issue is an example of the need for well-resolved self-consistent simulations with all of the MHD and radiative physics and with the ability too reproduce high fidelity observations of the atmosphere.

The relatively lesser amount of knowledge of the chromosphere is surely due to the fact that we can only remotely sample the plasma in the chromosphere, and this part of the solar atmosphere is optically thick in some lines, which makes for a difficult interpretation of the spectra obtained. The recently launched Interface Region Imaging Spectrograph (IRIS, De Pontieu et al. 2013) will be valuable in the effort to identify chromospheric heating mechanisms, using relatively high resolution (0.33 arcsec) observations of plasma emission at temperatures between 5,000 K and 10 MK. These observations will also be used to constrain improved chromospheric simulations which couple the non-LTE radiation that occurs in the chromosphere to the MHD evolution of the plasma (e.g., Carlsson and Leenarts 2012). These simulations are not currently able to resolve all chromospheric physics, but are valuable in the ongoing effort to test proposed heating mechanisms.

The Earth’s I/T system is better understood than the chromosphere because it is easier to make a wide range of measurements in the former domain. Despite this there are many challenges remaining to I/T science. Rishbeth (2007) outlined a number of them including: semiannual variations, and the annual asymmetry in both the ionosphere and thermosphere; why the I/T survives at night; day-to-day atmospheric and ionospheric variability, its forcing mechanisms and the I/Ts apparent predilection for certain time scales; and ionospheric memory and preconditioning. Other challenges include the solar cycle change in the winter anomaly (Torr and Torr 1973) and the way that high latitude forcing apparently drives changes in the low latitude I/T (Wang et al. 2008). Understanding these phenomena is difficult because the I/T system is driven from both below and above. The Earth’s ionosphere is affected by both tropospheric/stratospheric dynamic through modification of the neutral composition/temperature/winds, and the magnetosphere through high-latitude currents and
heating, so one must have a good understanding of these other regions as well as the ability to incorporate these effects self-consistently into an ionosphere model. Moreover, ionospheric dynamics spans an enormous range of spatial and temporal scales and different physical processes are important in different regions. It is difficult to bring all of this together in a single, coherent model (this is also a generic issue with most geospace and solar regions).

Virtually all ionosphere models assume the magnetic field lines are equipotentials. This reduces the potential equation to 2D and is readily solvable. However, it is clear that this is an approximation and should be relaxed. Recently, Aveiro and Hysell (2010, 2012); Aveiro and Huba (2013) developed a 3D electrostatic model of the ionosphere and applied it to the development of equatorial spread F. Although, in a general sense, the results are similar to the 2D results there are differences which could be important. Huba and Joyce (2013) have been able to embed a very high resolution grid (.06 degrees over a 60 degree sector) within the context of a global model. They were able to simulate for the first time the onset and evolution of equatorial bubbles (scale sizes ~ 10 km) in a global model. Such coupled simulations are one possible solution to deal with the multi-scale problem of I/T physics. This approach could also be applied to traveling ionospheric disturbances and gravity waves.

The recently selected NASA I/T missions ICON (Ionospheric Connection) and GOLD (Global-scale Observations of the Limb and Disk) will also address some of the key I/T issues. In particular, ICON will obtain the baseline characterization of the internally driven non-linear coupling between the neutral atmospheric drivers of winds, composition changes and the ionospheric responses of plasma densities and ion drifts. During periods of enhanced solar and geomagnetic activity ICON will determine how these parameters deviate from their baseline and will relate them to the strength of the solar wind electrical forcing that is externally applied to the global ionosphere-magnetosphere system. The GOLD mission will investigate the significance of atmospheric waves and tides propagating from below on the temperature structure of the thermosphere, and it will resolve how the structure of the equatorial ionosphere influences the formation and evolution of equatorial plasma density irregularities.

One of the goals of this review paper is to highlight how considering the commonalities of two different atmospheres can shed light on what can be learnt from one about the other. We have found a lot of commonalities, and have been able to talk about the two atmospheres within a common framework, but the differences between the two atmospheres create barriers to common studies. One main difference is the plasma density, which affects the neutral-ion collision frequency and the coupling of the two ionized and neutral fluids. It also affects conductivity, and whether the evolution of the magnetic field is dominated by diffusion by collisions or advection by the bulk flow. However, the Rayleigh-Taylor instability is one particular phenomena that we were able to simulate in both atmospheres using the same equations and model (§7). Another major difference is not only the magnitude of the magnetic field, which affects plasma-β, magnetizations and electrical conductivities, but also its geometry and time dependence. Rather than search for common phenomena that exist in both atmospheres, the key to future collaboration is to identify common fundamental partially ionized plasma physics problems such as the Rayleigh-Taylor instability. Other common fundamental partially ionized plasma physics studies include the development of two stream instabilities in regions where ions are unmagnetized but electrons are magnetized, and the subsequent generation of turbulence (e.g. the Farley-Buneman instability). Also the neutral wind-driven dynamo is a common problem that can occur in the chromosphere and I/T. The use of the V-B paradigm to describe I/T phenomena that were previously described in the E,J paradigm, as was done for the neutral wind dynamo by Vasyliunas (2012), will help shed light on the fundamental physics. The use of detailed sim-
ulations based on these fundamental studies is also a possible route to better understanding. Previous examples include the work by (e.g., Tu and Song 2008) on driving by electric fields. Simple experiments could use a two-fluid model (neutrals and charge-balanced plasma) to determine whether charge separation in the electrodynamic description truly is required to generate the dynamo or, instead, is transient and merely incidental to the process. However at some point, we must also address the effects of anisotropic, multi-species thermal conduction, viscosity, and thermoelectric effects in such studies.

A Appendix

The two-dimensional simulation results shown in Figures 14 and 15 were performed with the HiFi spectral-element multi-fluid model (Lukin 2008). Effective grid sizes of $480 \times 1920$ and $180 \times 720$ were used in the solar-prominence and ionosphere cases, respectively, along the horizontal ($y$) and vertical ($x$) directions. Periodic conditions were applied at the side boundaries ($y$), while closed, reflecting, free-slip, perfect-conductor conditions were applied at the top and bottom boundaries ($x$), which were placed sufficiently far from the unstable layer to have negligible effect on the Rayleigh-Taylor evolution. Details of the plasma and neutral profiles and parameters used in the simulations are given below.

A.1 Prominence

Normalization constants for this case are number density $n_0 = 1 \times 10^{15}$ m$^{-3}$, length scale $L_0 = 1 \times 10^6$ m, and magnetic field $B_0 = 1 \times 10^{-3}$ T. Using these in a hydrogen plasma, normalization values for the time $t_0 = 1.45$ s and temperature $T_b = 5.76 \times 10^7$ K can be derived. The ion inertial scale is so much smaller than any scale of interest that, in this case, its value has been set explicitly to zero, $d_i = (c/\omega_{pe0})/L_0 = 0$. This is equivalent to neglecting the Hall term in the Ohm’s law. Using the ratio of Hall term to Pedersen in the center of mass Ohm’s law [47], and the simplifications used in [4.3] this is equivalent to $\xi^n b_{\text{in}} \gg 1$, which is valid for these simulations.

The electron and ion density profiles are given by atmospheric stratification,

$$n_e(x) = n_i(x) = n_0 \exp \left( -\frac{x}{x_0} \right), \quad (124)$$

The scale height $x_0$ is set by the solar gravitational acceleration, $g_S = 2.74 \times 10^2$ m s$^{-2}$, and the assumed background temperature of the corona, $T_b = 3.5 \times 10^3$ K, scaled to the normalization length $L_0$; its value is $x_0 = 12.2$. The density profile of the neutral fluid that constitutes the prominence is given by a prescribed function of $x$ plus a very low uniform background value,

$$n_0(x) = n_{0b} \text{sech}^2 (2x - 1) + n_{nb}. \quad (125)$$

The peak neutral number density enhancement is taken to be $n_{nb} = 1 \times 10^{16}$ m$^{-3} = 10n_0$, while $n_{nb} = 3.5 \times 10^{-7}n_0$, corresponding to the neutral fraction obtained in the HiFi ionization/recombination equilibrium at the background temperature $T_b$. We chose an artificially low value of $T_b$ (compared to a typical coronal temperature of about $2 \times 10^6$ K) in order to prevent the background neutral density $n_{nb}$ from being far smaller still.

The electron, ion, and neutral temperatures are all assumed to be equal to each other initially. The temperature profile is given by a prescribed function $f(x)$,

$$T(x) = T_b f(x) = T_b \frac{\cosh^2(x - 0.5)}{\cosh^2(x - 0.5) + \lambda}, \quad (126)$$

which approaches $T_b$ away from the prominence and attains a minimum value $T_p = T_b/(1 + \lambda)$ within the prominence. To obtain a temperature approximately corresponding to that observed on the Sun, with an associated low ionization fraction $(n_{0b}/(n_{0b} + n_{0})) = 0.091$, we set the parameter $\lambda = 20$. The resulting prominence temperature $T_p = 9.60 \times 10^5$ K.
The magnetic field is initialized to lie in the out-of-plane direction \( \hat{e}_z \), so that it is perpendicular to both gravity in the \( -\hat{e}_z \) direction and the instability wavenumber \( k \) in the \( \hat{e}_y \) direction. It is given by
\[
B = B_0 \hat{e}_z \left[ 1 + \beta \left( \frac{n_e(x)}{n_0} [1 - f(x)] - \frac{n_n(x)}{2n_0} f(x) - \frac{1}{x_0} \frac{n_n(x)}{2n_0} [\tanh(2x - 1) - 1] \right) \right]^{1/2},
\]
where \( \beta = 1.4 \times 10^{-2} \) is the plasma beta evaluated using the background plasma pressure at \( x = 0 \) and \( B_0 \).

The magnetic field profile so constructed accommodates (1) the plasma pressure change from the isothermal hydrostatic profile (2) the neutral pressure, and (3) the gravitational force exerted on the bulk of the neutral fluid (neglecting the small background contribution \( n_{eb0} \)) throughout the atmosphere.

This initial condition is not an exact solution to the multi-fluid model which includes ionization, recombination, and viscous forces, but would be if only gravity, pressure gradients, and Lorentz forces were considered. It is close enough to the full solution, however, that any flows such as those created by pressure gradients driven by ionization/recombination of the initial condition are small compared to the flows initiated by the instability. The instability is initiated by introducing a small neutral density perturbation localized in \( x \) on the bottom side of the prominence,
\[
\Delta n_n(x,y) = \delta n_n(x) \exp[-4x^2] \left[ \frac{1}{2} \sum_{j=1}^{5} \sin(j\pi y) \right],
\]
where we chose \( \delta = 10^{-2} \), and \( y \) is the normalized distance along the gravitational equipotential surface.

### A.2 Ionosphere

The normalization constants are number density \( n_0 = 1 \times 10^{16} \text{m}^{-3} \), length scale \( L_0 = 2 \times 10^4 \text{m} \), and magnetic field \( B_0 = 3 \times 10^{-5} \text{T} \). Using these in a hydrogen plasma, normalization values for the time \( t_0 = 3.06 \text{s} \), temperature \( T_0 = 5.19 \times 10^3 \text{K} \), and ion inertial scale length \( d_i = (e/\rho_0 m)/L_0 = 1.14 \times 10^{-4} \) can be derived.

The neutral density profile is given by atmospheric stratification,
\[
n_n(x) = n_0 e^{-x/x_0}.
\]
The scale height \( x_0 \) is set by Earth’s gravitational acceleration, \( g_E = 9.81 \text{ m s}^{-2} \), and the assumed background temperature of the ionosphere, \( T_b = 0.225T_0 = 1.17 \times 10^3 \text{K} \), scaled to the normalization length \( L_0 \); its value is \( x_0 = 49.1 \). The electron/ion density profile of the plasma is given by a prescribed function of \( x \) plus a low uniform background value,
\[
n_e(x) = n_i(x) = n_{eb} \text{sech}^2(2x - 1) + n_{eb}.
\]
The peak electron number density is taken to be \( n_{eb0} = 1 \times 10^{12} \text{ m}^{-3} = 10^{-4}n_0 \), while \( n_{eb} = 0.05n_{eb0} \). The electron, ion, and neutral temperatures are all assumed to be equal and uniform initially, at \( T_b = 1.17 \times 10^3 \text{K} \).

As before, the magnetic field is initialized to lie in the out-of-plane direction. It is given by
\[
B = B_0 \hat{e}_z \left[ 1 - \beta \left( \frac{2n_e(x)}{n_0} + \frac{1}{2x_0} [\tanh(2x - 1) - 1] \right) \right]^{1/2},
\]
where \( \beta = 0.450 \) is the neutral beta calculated using the background neutral pressure at \( x = 0 \) and \( B_0 \).

The magnetic field profile so constructed accommodates (1) the plasma pressure and (2) the gravitational force exerted on the bulk of the plasma (neglecting the small background contribution \( n_{eb0} \)) throughout the atmosphere.

The instability is initiated with a small electron density perturbation localized in \( x \) on the bottom side of the ionization layer,
\[
\Delta n_e(x,y) = -\delta n_e(x) \exp[-4x^2] \left[ \frac{1}{2} \sum_{j=1}^{5} \sin(j\pi y) \right],
\]
where again we chose \( \delta = 10^{-2} \), and \( y \) is the normalized distance along the gravitational equipotential surface.

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