Gluon topology and the spin structure of the constituent quark

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Gluon topology makes a potentially important contribution to the spin of the constituent quark.

1. INTRODUCTION

The small value of the flavour-singlet axial charge $g_A^{(0)}$ which is extracted from the first moment of $g_1$ (the nucleon’s first spin dependent structure function)

$$g_A^{(0)}|_{\text{pDIS}} = 0.2 - 0.35.$$  (1)

has inspired much theoretical and experimental effort to understand the internal spin structure of the nucleon \[1\]. A key issue \[2\] is the role of the axial anomaly in the transition from parton to constituent quark degrees of freedom in low energy QCD. In this paper I explain why some fraction of proton’s spin may be carried by gluon topology. The topological contribution has support only at Bjorken $x$ equal to zero.

In deep inelastic processes the internal structure of the nucleon is described by the QCD parton model \[3\]. The deep inelastic structure functions may be written as the sum over the convolution of “soft” quark and gluon parton distributions with “hard” photon-parton scattering coefficients. The (target dependent) parton distributions describe a flux of quark and gluon partons carrying some fraction $x = \frac{p_{+\text{parton}}}{p_{+\text{proton}}}$ of the proton’s momentum into the hard (target independent) photon-parton interaction which is described by the hard scattering coefficients.

In low energy processes the nucleon behaves like a colour neutral system of three massive constituent quark quasi-particles interacting self consistently with a cloud of virtual pions which is induced by spontaneous chiral symmetry breaking \[4, 5\].

One of the most challenging problems in particle physics is to understand the transition between the fundamental QCD “current” quarks and gluons and the constituent quarks of low-energy QCD. The fundamental building blocks are the local QCD quark and gluon fields together with the non-local structure \[6\] associated with gluon topology \[7\].

Relativistic constituent-quark pion coupling models predict $g_A^{(0)} \simeq 0.6$ — two standard deviations greater than the value of $g_A^{(0)}|_{\text{pDIS}}$ in Eq.(1). Can we reconcile these two values of $g_A^{(0)}$ without abandoning the constituent quark picture of the nucleon?

2. GLUON TOPOLOGY AND $g_A^{(0)}$

The flavour-singlet axial charge $g_A^{(0)}$ is measured by the proton forward matrix element of the gauge invariantly renormalised axial-vector current

$$J_{\mu 5}^{GI} = \langle p, s | J_{\mu 5}^{GI} | p, s \rangle_c$$

viz.

$$2m_s g_A^{(0)} = \langle p, s | J_{\mu 5}^{GI} | p, s \rangle_c$$

In QCD the axial anomaly \[8, 9\] induces various gluonic contributions to $g_A^{(0)}$. The flavour-singlet axial-vector current satisfies the anomalous divergence equation

$$\partial^\mu J_{\mu 5}^{GI} = 2f \partial^\mu K_{\mu} + \sum_{i=1}^{f} 2im_i \bar{q}_i \gamma_5 q_i$$

where

$$K_{\mu} = \frac{g^2}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} \left[ A_\mu \left( \partial^\nu A_\sigma - \frac{1}{3} g f_{abc} A_5^a A_5^b A_5^c \right) \right]$$
is a renormalised version of the Chern-Simons current and $f = 3$ is the number of light-flavours. Eq.(4) allows us to write
\[ J^{GI}_{\mu 5} = J^{con}_{\mu 5} + 2f K_{\mu} \]
where
\[ \partial^\mu K_{\mu} = \frac{g^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}. \]
and
\[ \partial^\mu J^{con}_{\mu 5} = \sum_{l=1}^{f} 2i m_l \bar{q}_l \gamma_5 q_l \]
The partially conserved axial-vector current $J^{con}_{\mu 5}$ and the Chern-Simons current $K_{\mu}$ are separately gauge dependent. Gauge transformations shuffle a scale invariant operator quantity between the two operators $J^{con}_{\mu 5}$ and $K_{\mu}$ whilst keeping $J^{GI}_{\mu 5}$ invariant.

One would like to isolate the gluonic contribution to $g_A^{(0)}$ associated with $K_{\mu}$ and thus write $g_A^{(0)}$ as the sum of “quark” and “gluonic” contributions. Here we have to be careful because of the gauge dependence of $K_{\mu}$.

Whilst $K_{\mu}$ is a gauge dependent operator, its forward matrix elements are invariant under the “small” gauge transformations which change the topological winding number \([13]\). The topological winding number is a non-local property of QCD. It is determined by the gluonic boundary conditions at “infinity” \([7]\) — a large surface with boundary which is spacelike with respect to the positions $z_k$ of any operators or fields in the physical problem — and is insensitive to any local deformations of the gluon field $A_{\mu}(z)$ or of the gauge transformation $U(z)$ — that is, perturbative QCD degrees of freedom. When we take the Fourier transform to momentum space the topological structure induces a light-cone zero-mode which has support only at $x = 0$ \([4]\). Hence, we are led to consider the possibility that there may be a term in $g_1$ which is proportional to $\delta(x)$.

It remains an open question whether the net non-perturbative quantity which is shuffled between the $J^{con}_{\mu 5}$ and $K_{\mu}$ contributions to $g_A^{(0)}$ under “large” gauge transformations is finite or not. If it is finite and, therefore, physical then we find a net topological contribution $C$ to $g_A^{(0)}$ \([3]\)
\[ g_A^{(0)} = \left( \sum_q \Delta q - f \frac{\alpha_s}{2\pi}\Delta g \right)_{\text{partons}} + C \]
The topological term $C$ has support only at $x = 0$. It is missed by polarised deep inelastic scattering experiments which measure $g_1(x, Q^2)$ between some small but finite value $x_{\text{min}}$ and an upper value $x_{\text{max}}$ which is close to one. As we decrease $x_{\text{min}} \to 0$ we measure the first moment
\[ \Gamma \equiv \lim_{x_{\text{min}} \to 0} \int_{x_{\text{min}}}^{1} dx \; g_1(x, Q^2). \]
This means that the singlet axial charge which is extracted from polarised deep inelastic scattering is the combination $g_A^{(0)} |_{\text{pDIS}} = (g_A^{(0)} - C)$. In contrast, elastic $Z^0$ exchange processes such as $\nu p$ elastic scattering \([13]\) and parity violation in light atoms \([14]\) measure the full $g_A^{(0)}$. One can, in principle, measure the topology term $C$ by comparing the flavour-singlet axial charges which are extracted from polarised deep inelastic and $\nu p$ elastic scattering experiments.
If some fraction of the spin of the constituent quark is carried by gluon topology in QCD, then the constituent quark model predictions for $g_A^{(0)}$ are not necessarily in contradiction with the small value of $g_A^{(0)}|_{pDIS}$ extracted from deep inelastic scattering experiments.

The presence or absence of topological $x = 0$ polarisation is intimately related to the dynamics of $U_A(1)$ symmetry breaking in QCD. A simple dynamical mechanism for producing topological $x = 0$ polarisation is provided by Crewther's theory of quark-instanton interactions [7]. There, any instanton induced suppression of $g_A^{(0)}|_{pDIS}$ is compensated by a net transfer of axial charge or “spin” from partons carrying finite momentum fraction $x > 0$ to the flavour-singlet topological term at $x = 0$.

A large positive $\Delta g_A (\sim +1.5$ at $Q^2 = 1$GeV$^2$) and topological $x = 0$ polarisation are two possible explanations for the small value of $g_A^{(0)}|_{pDIS}$. Measurements of both quantities are urgently needed!

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