Pion Interferometry for a Granular Source of Quark-Gluon Plasma Droplets

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We examine the two-pion interferometry for a granular source of quark-gluon plasma droplets. The evolution of the droplets is described by relativistic hydrodynamics with an equation of state suggested by lattice gauge results. Pions are assumed to be emitted thermally from the droplets at the freeze-out configuration characterized by a freeze-out temperature $T_f$. We find that the HBT radius $R_{\text{out}}$ decreases if the initial size of the droplets decreases. On the other hand, $R_{\text{side}}$ depends on the droplet spatial distribution and is relatively independent of the droplet size. It increases with an increase in the width of the spatial distribution and the collective-expansion velocity of the droplets. As a result, the value of $R_{\text{out}}$ can lie close to $R_{\text{side}}$ for a granular quark-gluon plasma source. The granular model of the emitting source may provide an explanation to the RHIC HBT puzzle and may lead to a new insight into the dynamics of the quark-gluon plasma phase transition.

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Recent experimental pion HBT measurements at RHIC give the ratio of $R_{\text{out}}/R_{\text{side}} \approx 1$ \cite{1, 2, 3, 4, 5, 6, 7, 8}. This RHIC HBT puzzle hints that the pion emitting time may be very short \cite{9, 10}. Various models have been put forth to explain the HBT puzzle \cite{11, 12, 13, 14, 15, 16}.

We shall make the approximation that the hydrodynamical solution for many independent droplets can be obtained by superposing the hydrodynamical solution of a single droplet. It suffices to focus attention on the hydrodynamics of a single droplet. Knowing the entropy density $s(T)$, one can get the pressure $p$, energy density $\epsilon$, and the velocity of sound $c_s$ in the droplet with the following equations as in Ref. \cite{3, 28}.

$$p = \int_0^T dT' s(T'), \quad \epsilon = Ts - p, \quad c_s^2 = \frac{dp}{d\epsilon}.$$  \hspace{1cm} (2)

The energy momentum tensor of a thermalized fluid cell in the center-of-mass frame of the droplet is \cite{3, 28, 32, 33}.

$$T^{\mu\nu}(x) = [\epsilon(x) + p(x)]u^\mu(x)u^\nu(x) - p(x)g^{\mu\nu},$$  \hspace{1cm} (3)

where $x$ is the space-time coordinate, $u^\mu = \gamma(1, v)$ is the 4-velocity of the cell, and $g^{\mu\nu}$ is the metric tensor. With the local conservation of energy and momentum, one can follow Rischke and Gyulassy and get the equations for spherical geometry as \cite{3, 28}.

$$\partial_\tau E + \partial_r [(E + p)v] = -F,$$  \hspace{1cm} (4)

$$\partial_\tau M + \partial_r (Mv + p) = -G,$$  \hspace{1cm} (5)

where $E \equiv T^{00}$, $M \equiv T^{0r}$,

$$F = \frac{2v}{r}(E + p), \quad G = \frac{2v}{r}M.$$  \hspace{1cm} (6)

We assume the initial conditions as \cite{3, 28}.

$$\epsilon(0, r) = \begin{cases} \epsilon_0, & r < r_d, \\ 0, & r > r_d, \end{cases}, \quad v(0, r) = \begin{cases} 0, & r < r_d, \\ 1, & r > r_d, \end{cases}$$  \hspace{1cm} (7)

where $\epsilon_0 = 1.875T_c s_c$ \cite{3, 28} is the initial energy density of the droplets, and $r_d$ is the initial droplet radius. Using the Harten-Lax-van Leer-Einfeldt (HLLE) scheme.
and the relation of \( p = p(\epsilon) \) obtained from Eqs. 1 and 2, one can get the solution of the hydrodynamical equations for \( F = G = 0 \). One then obtains the solution for Eqs. 3 and 4 by using the Sod’s operator splitting method 5, 28, 35. The grid spacing for the HLLE scheme is taken to be \( \Delta x = 0.01 r_d \), and the time step for the HLLE scheme and Sod’s method corrector is \( \Delta t = 0.99 \Delta x \). Figure 1(a) and 1(b) show the temperature and velocity profiles of the droplet. Figure 1(c) gives the isotherms for the droplet.

The two-particle Bose-Einstein correlation function is defined as the ratio of the two-particle momentum distribution \( P(k_1, k_2) \) to the product of the single-particle momentum distribution \( P(k_1)P(k_2) \). For a chaotic pion-emitting source, \( P(k_i) \) \((i = 1, 2)\) and \( P(k_1, k_2) \) can be expressed as

\[
P(k_i) = \sum_{X_i} A^2(k_i, X_i),
\]

\[
P(k_1, k_2) = \sum_{X_1, X_2} \Phi(k_1, k_2; X_1, X_2)^2,
\]

where \( A(k_i, X_i) \) is the magnitude of the amplitude for emitting a pion with 4-momentum \( k_i = (k_i, E_i) \) in the laboratory frame at \( X_i \) and is given by the Bose-Einstein distribution in the local rest frame of the source point. \( \Phi(k_1, k_2; X_1, X_2) \) is the two-pion wave function. Neglecting the absorption of the emitted pions by other droplets, \( \Phi(k_1, k_2; X_1, X_2) \) is simply

\[
\Phi(k_1, k_2; X_1, X_2) = \frac{1}{\sqrt{2}} \left[ A(k_1, X_1)A(k_2, X_2)e^{ik_1 \cdot X_1 + ik_2 \cdot X_2} + A(k_1, X_2)A(k_2, X_1)e^{ik_1 \cdot X_2 + ik_2 \cdot X_1} \right].
\]

Using the components of “out” and “side” 5, 28, 35 of the relative momentum of the two pions, \( q = |k_1 - k_2| \), as variables, we can construct the correlation function \( C(q_{out}, q_{side}) \) from \( P(k_1, k_2) \) and \( P(k_1)P(k_2) \) by summing over \( k_1 \) and \( k_2 \) for each \( (q_{out}, q_{side}) \) bin. The HBT radius \( R_{out} \) and \( R_{side} \) can then be extracted by fitting the calculated correlation function \( C(q_{out}, q_{side}) \) with the following parametrized correlation function

\[
C(q_{out}, q_{side}) = 1 + \lambda e^{-q^2_{out} R_{out}^2 - q^2_{side} R_{side}^2}.
\]

The explicit procedure for calculating the two-pion correlation function is as follows.

Step 1: select the emission points of the two pions randomly on the space-time freeze-out surfaces of the droplets, and get their space-time coordinate \( X_1 \) and \( X_2 \) in the laboratory frame.

Step 2: generate the momenta \( k'_1 \) and \( k'_2 \) of the two pions in local frame according the Bose-Einstein distribution characterized by the temperature \( T_f \), and obtain their momenta \( k_1 \) and \( k_2 \) in the laboratory frame by Lorentz transforms.

Step 3: calculate \( [E'_1/E_1][E'_2/E_2] \) for \( P(k_1)P(k_2) \) and \( [E'_1/E_1][E'_2/E_2] \cos[(k_1 - k_2)(X_1 - X_2)] \) for \( P(k_1, k_2) \), and accumulate them in the corresponding \( (q_{out}, q_{side}) \) bin.

Step 4: repeat steps 1 through 4 many times to get the correlation function within a certain accuracy.

We first examine the two-pion correlation function for a singlet droplet source. By fitting the two-dimension correlation function \( C(q_{out}, q_{side}) \) obtained with the above steps with Eq. 15, we get the parameters \( R_{out} \), \( R_{side} \), and \( \lambda \) simultaneously. In our calculations, the transverse momenta of the pions are integrated over. The average transverse momenta of the pions in our fitting samples are 307, 329, and 386 MeV for \( T_f = 0.8T_c, 0.65T_c \), and \( 0.5T_c \) respectively. The reason for a larger average

FIG. 1: (a) Temperature profile and (b) velocity profile for the droplet at \( t_s = 3n\lambda r_d, \lambda = 0.99 \). (c) Isotherms for the droplet.
transverse momentum to associate with a smaller freeze-out temperature is due to the larger average expansion velocity in the case of a smaller freeze-out temperature. Figure 2(a), 2(b), 2(c), and 2(d) show the HBT results $R_{\text{out}}$, $R_{\text{side}}$, $R_{\text{out}}/R_{\text{side}}$, and $\lambda$ as a function of the initial radius $r_d$ of the droplet, for the freeze-out temperatures $T_f = 0.80T_c$, $T_f = 0.65T_c$, and $T_f = 0.50T_c$, respectively. The symbols $\circ$, $\bullet$, and $\ast$ are for the freeze-out temperatures $T_f$ = 0.80$T_c$, $T_f$ = 0.65$T_c$, and $T_f$ = 0.50$T_c$ (symbol $\ast$). It can be seen that the HBT radii $R_{\text{out}}$ and $R_{\text{side}}$ increase linearly with $r_d$, but the ratio $R_{\text{out}}/R_{\text{side}}$ is about 3 within the errors. The radius $R_{\text{side}}$ reflects the spatial size of the source and the radius $R_{\text{out}}$ is related to the lifetime of the source.

From the hydrodynamical solution in figure 1(c), both the average freeze-out time and freeze-out radial distance increase with $r_d$ for different $T_f$. As a consequence, $R_{\text{out}}/R_{\text{side}}$ is insensitive to the values $r_d$ and $T_f$. The value of $R_{\text{out}}/R_{\text{side}} \sim 3$ for a single droplet is however much larger than the observed values.

In our calculations, we did not including resonances in the hadronic phase. If we take the hypothetical case of $d\sigma/d\Omega = 3$ to include a resonance gas in the hadronic phase, as discussed by Rischke and Gyulassy, we find that the ratio of $R_{\text{out}}/R_{\text{side}}$ is about 2.75. It is still much larger than the observed values. In Fig. 1(d), the values of $\lambda$ for $r_d = 4$ fm are larger than unity. This is a non-Gaussian correlation function effect, and the effect is larger for a wider correlation function, corresponding to a smaller source.

As the average freeze-out time is proportional to the initial radius of the droplet, the freeze-out time and $R_{\text{out}}$ decreases if the initial radius of the droplet decreases. On the other hand, $R_{\text{side}}$ increases if the width of the droplet spatial distribution increases. A variation of the droplet size and the width of droplet spatial distribution can result in $R_{\text{out}}$ nearly equal to $R_{\text{side}}$. Accordingly, we calculate next the two-pion correlation function for a Gaussian distribution source of $N_d$ droplets. The spatial center-of-mass coordinates $X_d$ of the droplets are assumed to obey a static Gaussian distribution $\exp(-X_d^2/2R^2)$.

Figure 3(a), 3(b), 3(c), and 3(d) give the HBT $R_{\text{out}}$, $R_{\text{side}}$, $R_{\text{out}}/R_{\text{side}}$, and $\lambda$ as a function of the number of droplets $N_d$ for different values of $r_d$. In this calculation, we take $T_f = 0.65T_c$ and $R_c = 5.0$ fm. The symbols $\circ$, $\bullet$, and $\ast$ correspond to $r_d = 2.0$ fm, $r_d = 1.5$ fm, and $r_d = 1.0$ fm, respectively. It can be seen that the radii $R_{\text{out}}$ and $R_{\text{side}}$ have a slow increasing tendency as $N_d$ increases but their ratio $R_{\text{out}}/R_{\text{side}}$ is almost independent of $N_d$. $R_{\text{out}}$ decreases as $r_d$ decreases but $R_{\text{side}}$ is relatively independent of $r_d$. Consequently, the ratio $R_{\text{out}}/R_{\text{side}}$ decreases when $r_d$ decreases. The ratio $R_{\text{out}}/R_{\text{side}}$ for $r_d = 1.5$ fm is about 1.15 which is much smaller than the result of about 3 for the single droplet source.

Finally, to study the effect of additional collective expansion of the droplets, we calculate the two-pion correlation function for an expanding source. The initial distribution of the droplets is the same Gaussian distribution as the above granular source, but the droplets are assumed to expand collectively with a constant radial velocity $v_r$ after the initial time, in addition to their hydrodynamical expansion. Figure 4(a), 4(b), 4(c), and 4(d) give the HBT $R_{\text{out}}$, $R_{\text{side}}$, $R_{\text{out}}/R_{\text{side}}$, and $\lambda$ for different values of $v_r$. In this calculation, we take the initial radius of the droplets to be $r_d = 1.5$ fm, the other parameters are the same as the above calculations for a static granular source. The symbols $\circ$, $\bullet$, and $\ast$ correspond to $v_r = 0$, 0.3, and 0.6, respectively. The results in Figs. 4(a) and 4(b) show that $R_{\text{side}}$ increases more rapidly with the droplet collective expansion velocity $v_r$ than $R_{\text{out}}$. A radial expansion will increase the transverse size of the granular source and $R_{\text{side}}$. On the other hand, $R_{\text{out}}$ mea-
FIG. 4: Two-pion HBT results for the granular source with a radial collective expansion of the droplets. The symbols $\circ$, $\bullet$, and $\ast$ are for the expansion velocity $v_d = 0$, 0.3, and 0.6, respectively.

Examines the source life time and the spatial extension where the two pions are emitted with nearly parallel and equal momenta, and the additional radial boost modifies only slightly the spatial separation of these points for most cases. As a result, $R_{out}$ does not increase as rapidly as $R_{side}$ and $R_{out}/R_{side}$ is smaller at large $v_d$ than at zero $v_d$ (see Fig 1(c)). The ratio $R_{side}/R_{out}$ is of order 1 which is close to the observed value $[1,2]$.

In summary, we propose a simple model of granular source of quark-gluon plasma droplets to examine the HBT interferometry data. The droplets evolve hydrodynamically and pions are emitted thermally from the droplets at the freeze-out configuration characterized by a freeze-out temperature $T_f$. As the average freeze-out time is proportional to the radius of the droplet, smaller droplet size allows pions to be emitted within a shorter time and the life-time of the source decreases, leading to a smaller HBT radius $R_{out}$. On the other hand, the HBT radius $R_{side}$ depends on the width of the spatial distribution of the droplets and is insensitive to the initial size of the droplets. The ratio of $R_{out}$ to $R_{side}$ decreases significantly if the emitted source is granular in nature. Furthermore, $R_{side}$ increases with the collective-expansion velocity of the droplets more rapidly than $R_{out}$. The ratio $R_{out}/R_{side}$ is about 1.15—0.88 for the collective-expansion velocity of the droplets from zero to 0.6, for the granular source with a Gaussian initial radius 5 fm and a droplet initial radius 1.5 fm. This $R_{out}/R_{side}$ ratio is close to the experimental values $[1,2]$. The granular model of quark-gluon plasma may provide a possible explanation to the RHIC HBT puzzle. It may also lead to a new insight into the dynamics of the quark-gluon plasma phase transition as the formation of a granular structure is expected to occur in a first-order QCD phase transition $[22,24,25,26,27]$.

In order to bring out the most important features, we have neglected the multiple scattering effects on HBT interferometry $[31,40,41]$, and have not considered how the granular nature of the plasma may arise from detailed phase transition dynamics $[22,24,27]$. The sizes of the droplets in a collision can also have a distribution. Future refinements of the present model to take into account these effects on $R_{out}/R_{side}$ will be of great interest.

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