Semileptonic Decay of $\Omega_c^0 \rightarrow \Xi^- l^+ \nu_l$ From Light-Cone Sum Rules

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We calculate the form factors of the weak decay of $\Omega_c$ to $\Xi$ in the method of QCD light-cone sum rule. With the form factors obtained, we also calculate the decay width of $\Omega_c^0 \rightarrow \Xi^- l^+ \nu_l$ and its decay branching ratio. To the twist-6 distribution amplitudes, we give the form factors $f_1 = -0.168$, $f_2 = 0.175$, $g_1 = -0.0078$ and $g_2 = 0.176$ at zero recoil point. The result of the semileptonic decay width of $\Omega_c^0 \rightarrow \Xi^- l^+ \nu_l$ is $\Gamma = (1.64 \pm 0.05) \times 10^{-16} \text{GeV}$, and the prediction of the decay branching ratio $Br(\Omega_c^0 \rightarrow \Xi^- l^+ \nu_l) = 6.68 \times 10^{-5}$.

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I. INTRODUCTION.

In recent years, many new experimental results of $\Omega_c^0$ baryon have been developed. The lifetime has been updated by LHCb and a new value $\tau(\Omega_c^0) = (268 \pm 24 \pm 10 \pm 2) \times 10^{-15} \text{s}$ [1] is given, about five times larger than the old measurements [2–4]. And also, five new narrow $\Omega_c^0$ states are reported by LHCb in 2017 [5], and confirmed by $e^+e^-$ collisions on Belle [6]. These new discoveries enrich the nature of $\Omega_c^0$ baryon greatly. Two of the important properties of $\Omega_c^0$ baryon are its decay properties, strong and weak decays, but there is no strong decay observed in experiment until now. The weak decay channels are the main decay channels of $\Omega_c^0$. The PDG listed fourteen Cabibbo-favored weak decay channels up to now [7], among them there are thirteen non-leptonic and only one semileptonic weak decay channel observed. For the investigation of the transition from $\Omega_c^0$ baryon to other lighter baryons, the simplicity object is the semileptonic weak decay.

One of the established semileptonic weak decay mode is the channel $\Omega_c^0 \rightarrow \Omega^- e^+ \nu_e$ [8]. In this channel, the decay mode is from charm quark decay to strange quark and radiative positron and neutrino. For charm quark, the other possible decay channel $c \rightarrow d l^+ \nu_l$ is not forbidden in the standard model, which was observed in $D^+ \rightarrow \eta\mu^+ \nu_\mu$ semileptonic decay by BESIII collaboration recently and also gives a new value of CKM element $|V_{cd}| = 0.242$ [9]. Therefore, the study of semileptonic weak decay of charm baryon provides an additional source of information for determining the CKM matrix elements of charm quark as well as exploring the internal dynamics of systems containing heavy-light quarks. In the $\Omega_c^0$ baryon case, the $\Omega_c^0 \rightarrow \Xi^- l^+ \nu_l$ decay channel gives the way to research.

The transition from $\Omega_c^0$ baryon to $\Xi$ baryon has been studied with many theoretical approaches, such as non-relativistic quark model [10], heavy quark effective theory [11, 12], light-front quark model [13], and MIT bag model [14], in these articles the form factors were calculated. With the development of experimental results of the mass, lifetime, and other parameters, the new calculation and prediction are necessary. In our work, we will study this transition with the method of light-cone sum rules [15–20], which is based on QCD sum rules [21–23]. We will evaluate our work from baryon and weak currents and estimate the semileptonic weak decay width with the new result of $\Omega_c^0$ baryon lifetime. This method has been used to study the strong decay properties of excited $\Omega_c$ baryons [24, 25].

After the introduction in Sec. I, we give the details of the light-cone sum rules derivation of the semileptonic weak decay of $\Omega_c^0 \rightarrow \Xi^- l^+ \nu_l$, and the sum rules of the form factors are given in Sec. II. Sec. III is the numerical analysis of the four form factors of transition matrix element of $\Omega_c^0 \rightarrow \Xi$. Conclusions are given in Sec. IV. In the Appendix, we give the explicit expressions of the $\Xi$ baryon distribution amplitudes.

II. LIGHT-CONE SUM RULES OF SEMILEPTONIC WEAK DECAY $\Omega_c^0 \rightarrow \Xi^- l^+ \nu_l$.

Weak decay dynamics can be investigated by the weak decay effective Hamiltonian [26], and the semileptonic decay can be processed as the rare decay of B mesons [27]. For the $c \rightarrow d l^+ \nu_l$ decay mode, we can write the Hamiltonian with the form [28]:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} |V_{cd}| O_1 O_2,$$

where $G_F$ is fermi constant. $|V_{ud}|$ and $|V_{cd}|$ are the CKM matrix elements. $O_1 = \bar{d}_\gamma^\mu (1 - \gamma_5)c$, $O_2 = \bar{l}_\gamma^\mu (1 - \gamma_5)\nu_l$ is quark and lepton current respectively.

The light-cone sum rule starts from the current algebra structure of hadrons, and is evaluated with analytical methods, then gives the hadron transition matrix element with form factors in theoretical expressions. This method uses two kinds of representations, hadrons and
QCD theoretical representations. On the one hand, one can write the hadrons transition correlation function with the hadrons quantum numbers, and on the other hand, one can calculate the hadrons properties with QCD quark currents on the light-cone by hadrons light-cone distribution amplitudes (LCDAs).

Decay properties of heavy baryons calculations need the decay matrix element of heavy baryon decay to light baryon. The decay matrix element of \( \Omega_c \to \Xi \) can be parameterized to six form factors as follows.

\[
\langle \Omega_c (p-q) | j_{\mu} | \Xi(p) \rangle = \bar{u}_{\Omega_c} (p-q) [f_1 \gamma_{\mu} - \frac{f_2}{M_{\Omega_c}} \sigma_{\mu \nu} q^\nu - \frac{f_3}{M_{\Omega_c}} q_\nu - (g_1 \gamma_{\nu} + \frac{g_2}{M_{\Omega_c}} \sigma_{\mu \nu} q^\mu - \frac{g_3}{M_{\Omega_c}} q_{\nu}) \gamma_5] u_{\Xi}(p) \]

where \( f_i(g_i), i = 1, 2, 3 \) are the weak decay form factors, \( M_{\Omega_c} \) the mass of \( \Omega_c \) baryon, \( u_{\Omega_c} \) and \( u_{\Xi} \) the spinor of \( \Omega_c \) and \( \Xi \) respectively.

In order to obtain the light-cone sum rules of these form factors, we begin with the two point correlation function sandwiched between vacuum and final baryon state

\[
T_{\nu}(p,q) = i \int d^4x e^{i q x} \left( \frac{1}{T} \right) \langle j_{\Omega_c}(0), j_{\nu}(x) \rangle \Xi(p) \]

(3)

where the \( j_{\Omega_c}(0) \) and \( j_{\nu}(x) \) are heavy baryon \( \Omega_c \) current and weak decay current respectively. In our study, we choose Ioffe-type current for our computation

\[
j_{\Omega_c}(x) = \epsilon_{ijk}(s^T(x) C \gamma_{\mu} s^i(x)) \gamma_{\mu} \gamma_5 e^k(x) \]

(4)

and the weak decay current

\[
j_{\nu}(x) = \bar{c}(x) \gamma_{\nu}(1 - \gamma_5) d(x) \]

(5)

We introduce a light-cone vector \( z^\nu \) satisfy the condition \( z^2 = 0 \) to simplify our next calculations, which gives

\[
z^\nu T_{\nu}(p,q) = iz^\mu \int d^4x e^{i q x} \langle 0 | T \{ j_{\Omega_c}(0), j_{\nu}(x) \} | \Xi(p) \rangle \]

(6)

We insert a complete set of baryon states with the same quantum numbers of \( \Omega_c \) baryon into Eq. (6), and the definition of decay constant of the baryon \( \Omega_c \)

\[
\langle 0 | j_{\Omega_c} | \Omega_c(p-q) \rangle = M_{\Omega_c} f_{\Omega_c} u_{\Omega_c}(p-q) \]

(7)

By using the completeness relation of \( \Omega_c \) baryon, we obtain the hadron representation

\[
z^\nu T_{\nu}(p,q) = \frac{f_0 M_{\Omega_c}}{M_{\Omega_c}^2 - (p-q)^2} \{ 2p \cdot z f_1 + (M_{\Omega_c} - M_{\Xi}) f_2 z f_2 - 2p \cdot z g_1 \gamma_5 \}

\]

(8)

In the equation above, the relation \( \sum_{n} u_{\Omega_n} (p-q) \bar{u}_{\Omega_n} = ([p - q] + M_{\Omega_n}) \) is used, also set \( q \cdot z = 0 \) because the small value on the light cone of the transfer momentum. If we only consider the light leptons \( e^\pm \) and \( \mu^\pm \). Their masses are very small in our system. In the zero masses limit, with the relation \( q \cdot \bar{c}(x) \gamma_5(1 - \gamma_5) d(x) = 0 \), the \( f_3 \) and \( g_3 \) terms do not contribute. We omit them and the corresponding terms in our following calculations [19].

Our next work is to derive the other way to describe the correlation function. In our work, we use the light-cone sum rule to express the QCD theoretical formalism. For this purpose, we expand the correlation function on the light-cone with the light-cone distribution amplitudes of \( \Xi \) baryon [20]. Substituted the heavy baryon current (4) and the weak decay current (5) into the correlation function (6), after contract the heavy quark, we obtain the correlation function

\[
z^\nu T_{\nu} = \int d^4x e^{i q x} \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik x} (C \gamma_\sigma) \alpha \beta [\gamma_\sigma \gamma_5 (k + m_c) \gamma_1 (1 - \gamma_5) \gamma_1] |0 \rangle \epsilon_{ijk} s^i T(0) s^j \gamma_k x | \Xi(p) \rangle \} \]

(9)

In these calculations, we use the LCDAs transformation relations, and the LCDAs of \( \Xi \) has been given in [20]. We will also give the LCDAs and other formulas we need in our calculations in Appendix A, where we only display the parameters we need, and the completeness form can be found in [20, 29]. After the standard procedure of light-cone sum rule calculations and performing Borel transformations both on the two sides of hadron and theoretical representation, we get the final light-cone sum rules of these form factors \( f_i(g_i)(i = 1, 2) \) as follows:
\[
\frac{1}{2\alpha_3 M_B^2} B_4(\alpha_3)(M_\Xi^2 m_c + 2\alpha_3 M_\Xi^3) + \frac{2}{\alpha_3 M_B^2} B_5(\alpha_3) M_\Xi^2 + \\
\frac{1}{\alpha_3 M_B} B_3(\alpha_3)(\alpha_3^2 M_\Xi^5 + \alpha_3 M_\Xi^2 m_c - M_\Xi^2 Q^2) e^{-s/M_B^2} + \\
\frac{1}{\alpha_3^3 M_B^2 + Q^2 + m_c^2} \left\{ B_1(\alpha_3)M_\Xi^2 m_c - M_\Xi Q^2 - \alpha_3 M_\Xi(M_\Xi^2 - Q^2 - M_{\Omega_c}^2) \right\} - \\
\frac{1}{2} B_3(\alpha_3)\alpha_3 M_\Xi^2 m_c + \frac{1}{2} B_4(\alpha_3)(\alpha_3 M_\Xi^2 m_c + 2\alpha_3 M_\Xi^3) + 2 B_5(\alpha_3)\alpha_3 M_\Xi^3 + \\
\frac{1}{M_B^2} B_5(\alpha_3)\alpha_3 M_\Xi^2 m_c - M_\Xi^2 Q^2) e^{-s_0/M_B^2} - \\
\frac{1}{\alpha_3 M_B^2 + Q^2 + m_c^2} \left\{ \frac{d}{d\alpha_3} \frac{1}{\alpha_3} B_3(\alpha_3)(\alpha_3^2 M_\Xi^6 + \alpha_3 M_\Xi^4 m_c - M_\Xi^2 Q^2) \right\} e^{-s_0/M_B^2}, \quad (10)
\]

\[
M_{\Omega_c} f_{\Omega_c} \left[ \frac{M_\Xi^2 - M_{\Omega_c}^2}{M_{\Omega_c}} f_2 + (M_{\Omega_c} - M_\Xi) f_1 e^{-M_{\Omega_c}^2/M_B^2} = \int_{\alpha_3}^1 d\alpha_3 \left[ \frac{1}{\alpha_3} B_0(\alpha_3)\alpha_3 M_\Xi^2 m_c - (M_\Xi^2 - Q^2 - M_{\Omega_c}^2) + \\
M_\Xi m_c - \frac{1}{\alpha_3} B_3(\alpha_3) M_\Xi^2 - \frac{1}{\alpha_3^3 M_B^2} B_1(\alpha_3) \alpha_3^2 M_\Xi^4 + \alpha_3 M_\Xi^2 m_c \right] - \\
M_\Xi m_c (M_\Xi^2 - Q^2 - M_{\Omega_c}^2) - M_\Xi^2 Q^2 - \alpha_3 M_\Xi^2 (M_\Xi^2 - Q^2 - M_{\Omega_c}^2) \right] - \\
\frac{1}{\alpha_3} B_3(\alpha_3) M_\Xi^2 m_c + \frac{1}{\alpha_3} B_4(\alpha_3) M_\Xi^3 + \frac{1}{2\alpha_3^2 M_B^2} B_3(\alpha_3) \alpha_3^2 M_\Xi^4 + \alpha_3 M_\Xi^2 m_c \right] - \\
M_\Xi^2 Q^2 - \alpha_3 M_\Xi^2 (M_\Xi^2 - Q^2 - M_{\Omega_c}^2) \right] - \frac{1}{2\alpha_3^2 M_B^2} B_4(\alpha_3) \alpha_3^2 M_\Xi^4 - M_\Xi^2 Q^2 \right] - \\
\left[ \alpha_3 (M_\Xi^2 - Q^2 - M_{\Omega_c}^2) \right] e^{-s/M_B^2} - \frac{1}{\alpha_3^3 M_B^2 + Q^2 + m_c^2} \left\{ B_1(\alpha_3) \alpha_3^2 M_\Xi^4 + \\
+ \alpha_3 M_\Xi^2 m_c - \alpha_3 M_\Xi (M_\Xi^2 - Q^2 - M_{\Omega_c}^2) - M_\Xi^2 Q^2 - \alpha_3 M_\Xi^2 (M_\Xi^2 - Q^2 - M_{\Omega_c}^2) \right\} - \\
- \frac{1}{2} B_3(\alpha_3) \alpha_3^2 M_\Xi^4 + \alpha_3 M_\Xi^2 m_c - \alpha_3 M_\Xi^2 (M_\Xi^2 - Q^2 - M_{\Omega_c}^2) \right\} + \\
\frac{1}{2} B_3(\alpha_3) \alpha_3^2 M_\Xi^4 - M_\Xi^2 Q^2 - \alpha_3 M_\Xi^2 m_c - \alpha_3 M_\Xi^2 (M_\Xi^2 - Q^2 - M_{\Omega_c}^2) \right\} + \\
\frac{2}{M_B^2} B_5(\alpha_3) \alpha_3^2 M_\Xi^4 \left[ \alpha_3^2 M_\Xi^4 - Q^2 - \alpha_3 (M_\Xi^2 - Q^2 - M_{\Omega_c}^2) \right] e^{-s_0/M_B^2} + \\
\frac{2\alpha_{30}^2}{\alpha_3 M_B^2 + Q^2 + m_c^2} \left\{ \frac{d}{d\alpha_3} \frac{1}{\alpha_3} B_3(\alpha_3) \alpha_3^2 M_\Xi^4 - Q^2 - \alpha_3 (M_\Xi^2 - Q^2 - M_{\Omega_c}^2) \right\} e^{-s_0/M_B^2}, \quad (11)
\]

\[
-M_{\Omega_c} f_{\Omega_c} g_1 e^{-M_{\Omega_c}^2/M_B^2} = \int_{\alpha_3}^1 d\alpha_3 \left[ \frac{1}{\alpha_3} B_0(\alpha_3) m_c - \frac{1}{\alpha_3^3 M_B^2} B_1(\alpha_3) \alpha_3 M_\Xi^2 m_c + M_\Xi Q^2 - B_2(\alpha_3) M_\Xi \right] + \\
+ \frac{1}{2\alpha_3 M_B^2} B_3(\alpha_3) M_\Xi^2 m_c - \frac{1}{2\alpha_3^2 M_B^2} B_4(\alpha_3) (M_\Xi^2 m_c - 2\alpha_3 M_\Xi^3) + \\
\frac{2}{\alpha_3^3 M_B^2} B_5(\alpha_3) M_\Xi^3 + \frac{1}{\alpha_3^2 M_B^2} B_3(\alpha_3) \alpha_3^2 M_\Xi^4 - \alpha_3 M_\Xi^2 (M_\Xi^2 - Q^2 - M_{\Omega_c}^2) \right\} e^{-s/M_B^2} - \\
\frac{1}{\alpha_3^3 M_B^2 + Q^2 + m_c^2} \left\{ B_1(\alpha_3) \alpha_3^2 M_\Xi^4 + \\
M_\Xi^2 Q^2 - \frac{1}{2} B_3(\alpha_3) \alpha_3 M_\Xi^2 m_c + \frac{1}{2} B_4(\alpha_3) (\alpha_3 M_\Xi^2 m_c - 2\alpha_3 M_\Xi^3) - \\
2 B_5(\alpha_3) \alpha_3 M_\Xi^3 - \frac{1}{M_B^2} B_5(\alpha_3) \alpha_3^2 M_\Xi^4 - \alpha_3 M_\Xi^2 (M_\Xi^2 - Q^2 - M_{\Omega_c}^2) \right\} + \\
\frac{1}{\alpha_3 M_B^2 + Q^2 + m_c^2} \left\{ \frac{d}{d\alpha_3} \frac{1}{\alpha_3} B_3(\alpha_3) \alpha_3^2 M_\Xi^4 - \alpha_3 M_\Xi^2 m_c - \alpha_3 M_\Xi^2 (M_\Xi^2 - Q^2 - M_{\Omega_c}^2) \right\} e^{-s_0/M_B^2}, \quad (11)
\]
\[
\left. \frac{d}{d\alpha_3} \frac{B_5(\alpha_3) \left[ \alpha_3^2 M_{\Xi}^2 + \alpha_3 M_{\Xi}^2 (M_{\Xi}^2 - Q^2 - M_{\Xi}^2) \right] - \alpha_3 M_{\Xi}^4 m_c - M_{\Xi}^2 Q^2}{\alpha_3^2 M_{\Xi}^2 + Q^2 + m_c^2} \right|_{\alpha_3} = \frac{1}{\alpha_3} B_0(\alpha_3) [\alpha_3 M_{\Xi}^2 - (M_{\Xi}^2 - Q^2 - M_{\Xi}^2)] e^{-s_0/M_{\Xi}^2}, \tag{12}
\]

\[-M_{\Xi} f_{\Xi}[\langle M_{\Xi} + M_{\Xi} \rangle g_1 + \frac{M_{\Xi}^2 - M_{\Xi}^2}{M_{\Xi}^2} g_2] e^{-s_0/M_{\Xi}^2} = \int_0^1 d\alpha_3 \left\{ -\frac{1}{\alpha_3} B_0(\alpha_3) [\alpha_3 M_{\Xi}^2 - (M_{\Xi}^2 - Q^2 - M_{\Xi}^2)]
\right. - M_{\Xi} m_c, + \frac{2}{\alpha_3^2} B_1(\alpha_3) M_{\Xi}^2 - \frac{1}{\alpha_3^2} B_1(\alpha_3) [\alpha_3^2 M_{\Xi}^2 + 2\alpha_3 M_{\Xi}^2 m_c - M_{\Xi} m_c (M_{\Xi}^2 - Q^2 - M_{\Xi}^2)]
\right. - \alpha_3 M_{\Xi}^2 m_c + M_{\Xi}^2 Q^2 + \alpha_3 M_{\Xi}^4 (M_{\Xi}^2 - Q^2 - M_{\Xi}^2) \right\} - \frac{1}{\alpha_3^2} B_2(\alpha_3) M_{\Xi} m_c
\]

\[-\frac{1}{2\alpha_3^2} B_3(\alpha_3) [\alpha_3^2 M_{\Xi}^4 - \alpha_3^2 M_{\Xi}^2 m_c - M_{\Xi}^2 Q^2 - \alpha_3 M_{\Xi}^4 (M_{\Xi}^2 - Q^2 - M_{\Xi}^2)] - \frac{1}{\alpha_3^2} B_3(\alpha_3) M_{\Xi}^2
\]

\[+ \frac{1}{2\alpha_3^2} B_4(\alpha_3) [\alpha_3^2 M_{\Xi}^4 - M_{\Xi}^2 Q^2 - \alpha_3 M_{\Xi}^4 (M_{\Xi}^2 - Q^2 - M_{\Xi}^2)] + \frac{2}{\alpha_3^2} B_5(\alpha_3) M_{\Xi}^4 [\alpha_3^2 M_{\Xi}^2 - Q^2 - \alpha_3 (M_{\Xi}^2 - Q^2 - M_{\Xi}^2)] e^{-s_0/M_{\Xi}^2} - \frac{1}{\alpha_3^2} M_{\Xi}^2
\]

\[\frac{1}{\alpha_3^2} M_{\Xi}^2 + Q^2 + m_c^2 \{ B_1(\alpha_3) [\alpha_3^2 M_{\Xi}^2 + 2\alpha_3 M_{\Xi}^2 m_c - M_{\Xi} m_c (M_{\Xi}^2 - Q^2 - M_{\Xi}^2)] - \alpha_3 M_{\Xi}^2 m_c
\]

\[+ M_{\Xi}^2 Q^2 + \alpha_3 M_{\Xi}^4 (M_{\Xi}^2 - Q^2 - M_{\Xi}^2) \right\} + \frac{1}{2} B_3(\alpha_3) [\alpha_3^2 M_{\Xi}^4 - \alpha_3 M_{\Xi}^4 (M_{\Xi}^2 - Q^2 - M_{\Xi}^2)]
\]

\[-\alpha_3 M_{\Xi}^2 (M_{\Xi}^2 - Q^2 - M_{\Xi}^2)] - \frac{1}{2} B_4(\alpha_3) [\alpha_3^2 M_{\Xi}^4 - \alpha_3 M_{\Xi}^4 (M_{\Xi}^2 - Q^2 - M_{\Xi}^2)]
\]

\[-\frac{2}{M_{\Xi}^2} B_5(\alpha_3) M_{\Xi}^4 [\alpha_3^2 M_{\Xi}^2 - Q^2 - \alpha_3 (M_{\Xi}^2 - Q^2 - M_{\Xi}^2)] e^{-s_0/M_{\Xi}^2} - \frac{2}{M_{\Xi}^2}
\]

\[\frac{2}{\alpha_3^2} \{ \frac{d}{d\alpha_3} B_5(\alpha_3) M_{\Xi}^4 [\alpha_3^2 M_{\Xi}^2 - Q^2 - \alpha_3 (M_{\Xi}^2 - Q^2 - M_{\Xi}^2)] \} e^{-s_0/M_{\Xi}^2}, \tag{13}
\]

where \(Q^2 = -q^2\) and the \(\alpha_{30}\) relates to the threshold \(s_0\) is

\[\alpha_{30} = \frac{-(Q^2 + s_0 - M_{\Xi}^2)}{2M_{\Xi}^2} + \sqrt{(Q^2 + s_0 - M_{\Xi}^2)^2 + 4M_{\Xi}^2 Q^2 (Q^2 + m_c^2)} \tag{14}\]

The Borel transform parameters is

\[s = (1 - \alpha_3) M_{\Xi}^2 + \frac{1 - \alpha_3}{\alpha_3} Q^2 + \frac{m_c^2}{\alpha_3} \tag{15}\]

The signus \(B_i(\alpha_3)\) and \(G_i(\alpha_3)\) we used above are defined in the following expressions

\[B_0(\alpha_3) = \int_0^{1-\alpha_3} d\alpha_1 V_1(\alpha), \tag{16}\]

\[B_1(\alpha_3) = \int_0^{1-\alpha_3} \int_0^{1-\alpha_3'} d\alpha_1 (V_1 - V_2 - V_3)(\alpha'), \tag{17}\]

\[B_2(\alpha_3) = \int_0^{1-\alpha_3} d\alpha_1 V_3(\alpha), \tag{18}\]

\[B_3(\alpha_3) = \int_0^{1-\alpha_3} \int_0^{1-\alpha_3'} d\alpha_1 (-2V_1 + V_3 + V_4 + 2V_5)(\alpha'), \tag{19}\]

\[B_4(\alpha_3) = \int_0^{1-\alpha_3} \int_0^{1-\alpha_3'} d\alpha_1 (V_4 - V_3)(\alpha'), \tag{20}\]

\[B_5(\alpha_3) = \int_0^{\alpha_3} \int_0^{\alpha_3'} \int_0^{1-\alpha_3''} d\alpha_1 (-V_1 + V_2 + V_3 + V_4 + V_5 - V_6) (\alpha''), \tag{21}\]

where \(\alpha = (\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3), (\alpha_3') = (\alpha_1, 1 - \alpha_1 - \alpha_3', \alpha_3'), (\alpha_3'') = (\alpha_1, 1 - \alpha_1 - \alpha_3'', \alpha_3'').\]

### III. NUMERICAL ANALYSIS.

In order to know the form factors and give the value properties of the semileptonic decay of \(\Omega_c \to \Xi^{-1}\tau^-\nu_l\), we should set the numerical value of the parameters in the formulas of form factors. In our analysis, we adopt the standard center values of charm quark mass and baryons masses from PDG [7], which gives the numbers \(m_c = 1.27 GeV, M_{\Xi^-} = 1.3217 GeV\), and the \(\Xi_c\) baryon mass \(M_{\Xi_c} = 2.6952 GeV\). The physical region of momentum transfer square \(q^2\) varies in the region \(0 < q^2 < (M_{\Xi_c} - M_{\Xi}^2)^2\). For the numerical analysis, we also need to know the nonperturbative parameters of
the $\Omega_c$ baryon decay constants and the $\Xi$ baryon decay constants $f_\Xi$ and $\lambda_1$. The nonperturbative parameters distribution amplitudes $V_i (i = 1, \ldots, 8)$ are dependent on the $\Omega_c$ baryon decay constants, and their forms have been derived in [20] where we list them in the appendix.

In our calculations, we adopt the decay constant $f_{\Omega_c}$ from reference [24] $M_{\Omega_c} f_{\Omega_c} = \sqrt{2} \times 0.0438 \text{GeV}^3$, and the decay constant of $\Xi$ can be seen in [20], the values $f_\Xi = (9.9 \pm 0.4) \times 10^{-3} \text{GeV}^3$, $\lambda_1 = -(2.8 \pm 0.1) \times 10^{-2} \text{GeV}^2$.

The other parameters we should determine are the Borel region. The chosen principle of Borel parameters and the pictures of form factors fitted are listed in Table I where we list them in the appendix.

| $f_i$ | our work | [10] | [11] | [13] | [14] |
|-------|----------|------|------|------|------|
| $f_1(0)$ | -0.168 | -0.23 | -0.34 | 0.653 | 0.34 |
| $f_2(0)$ | 0.175 | 0.21 | 0.35 | 0.620 | - |
| $g_1(0)$ | -0.0078 | 0.14 | 0.10 | -0.182 | -0.15 |
| $g_2(0)$ | 0.176 | -0.019 | -0.020 | 0.002 | - |

The pictures of curve are given in Fig. 1, and the pictures of form factors $f_i(g_i)(i = 1, 2)$ are plotted in Fig. 2.

Because the arriving of the decay width of the semileptonic process should be investigated on the whole physical region, we extrapolate these form factors by the three-parameter dipole formula Eq. (22) on the whole kinematical region. Another benefit of using this fitting formula is that we can simplify the procedure of our calculation.

$$f_i(q^2) = \frac{f_i(0)}{a(q^2/M_{\Omega_c}^2) + b(q^2/M_{\Omega_c}^2) + 1}. \tag{22}$$

The form factors at point $q^2 = 0 \text{GeV}^2$ are given in Table I, and the results obtained from other approaches are also listed in this table. The coefficients $a$ and $b$ we fitted are listed in Table II. Due to the smallness of $g_1$, we do not consider the $g_1$ term. We use the decay width of semileptonic weak decay as in [19, 20]. The differential decay rate dependent on $q^2$ is

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{cd}|^2}{192\pi^3 M_{\Omega_c}} q^2 \sqrt{q^2_m q^2_\Xi} \{-6 f_1 f_2 M_{\Omega_c} m_- q^2_\Xi +$$

$$+ 2(q^2 + 2 M_{\Omega_c} M_{\Xi}) + q^2 f_1 M_{\Omega_c} (\frac{m^2_2 m^2_\Xi}{q^2} + m^2_\Xi -$$

$$- 2(q^2 - 2 M_{\Omega_c} M_{\Xi}) - f^2_2 [-2 m^2_2 m^2_\Xi + m^2 q^2$$

$$+ q^2 (q^2 + 4 M_{\Omega_c} M_{\Xi}) - q^2_2 [-2 m^2_2 m^2_\Xi + m^2 q^2$$

$$+ q^2 (q^2 - 4 M_{\Omega_c} M_{\Xi})/392]} \}. \tag{23}$$

Where $m_\pm = M_{\Omega_c} \pm M_{\Xi}$ and $q^2_\Xi = q^2 - m^2_\Xi$. The fermi constant is $G_F = 1.166 \times 10^{-5} \text{GeV}^{-2}$, and CKM matrix element is $|V_{cd}| = 0.221$. Because the form factor $g_1$ is more than one order smaller compared with other form factors, we omit the term which contains $g_1$. Substituting these constants into the differential decay formula and integrating it in the dynamical region $0 < q^2 < (M_{\Omega_c} - M_{\Xi})^2$, we obtain the decay width of the weak semileptonic decay $\Omega^0_c \rightarrow \Xi^{-}\ell^+\nu_\ell$, that is $\Gamma = (1.64 \pm 0.05) \times 10^{-16} \text{GeV}$. The errors come from the Borel region which we choose. The picture of the differential decay width is plotted in Fig 3. With the mean lifetime of $\Omega^0_c$ in PDG [1, 7], which gives the value
$\tau = 268 \times 10^{-15}\text{s}$, the branching ratio of decay which we estimate by the dipole formula is $Br(\Omega_0^0 \rightarrow \Xi^- l^+ \nu_l) = 6.68 \times 10^{-5}$. We also list the results calculated by other methods compared with ours in Table III.

### IV. CONCLUSIONS

In this work we calculate the form factors of the semileptonic weak decay from heavy baryon $\Omega_c$ to light baryon $\Xi$ and two leptons with light-cone sum rule approach. The explicit expressions of the sum rules of form factors are given. By using the numerical value in PDG the form factors are calculated and are given in Table I. The form factors obtained by other approaches are compatible with ours in Table I too. The decay width of $\Omega_0^0 \rightarrow \Xi^- l^+ \nu_l$ is evaluated to the number $\Gamma = (1.64 \pm 0.05) \times 10^{-16}\text{GeV}$. The branching ratio of it is given by $Br(\Omega_0^0 \rightarrow \Xi^- l^+ \nu_l) = 6.68 \times 10^{-5}$. But there is no absolute branching of $\Omega_0^0$ decay which has been discovered in experiments, and the only reference of it is to set the $\Omega_0^0 \rightarrow \Omega^- \pi^+$ decay channel to one, and the other decay channels are all relative to it [30]. Our calculation results may be tested in the future experiments.

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Appendix

In this appendix we give the distribution amplitudes of Ξ baryon [20].

\[ \langle 0 | e_{ij} x_T^{ij} (0) s_0^j (0) d^3(x) \Xi (p) \rangle = V_1 (p C)_{\alpha \beta} (\gamma_5 \Xi)_\gamma + V_2 M_2 (p C)_{\alpha \beta} (\gamma_5 \Xi)_\gamma + V_4 M_4 (p C)_{\alpha \beta} (i \sigma^{\mu \nu} x_\mu \gamma_5 \Xi)_\gamma + V_6 M_6 (p C)_{\alpha \beta} (\gamma_5 \Xi)_\gamma \]

(A.1)

The distribution amplitudes \( V_i \) (\( i = 1, \ldots, 6 \)) did not have definite twist, in order to get the distribution amplitudes with the definite twist we should make the following transformation to get the amplitudes with definite twist

\[ V_1 = V_1, \quad 2p \cdot x V_2 = V_1 - V_2 - V_3, \]
\[ 2V_3 = V_3, \quad 4p \cdot x V_4 = -2V_1 + V_3 + V_4 + 2V_5, \]
\[ 4p \cdot x V_6 = V_4 - V_3, \]
\[ (2p \cdot x)^2 V_6 = -V_1 + V_2 + V_3 + V_4 + V_5 - V_6. \]

(A.2)

These distribution amplitudes \( V_i \) are functions of \( x \cdot p \), but we need the variable relevant to the longitude momentum fraction of quarks in the baryon, so we should make the following transformation formula.

\[ F(\alpha, p \cdot z) = \int Dx e^{-ipx} \sum_i x_i a_i F(x_i), \]

(A.3)

The integration measure \( \int Dx \) is

\[ \int Dx = \int_0^1 dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3 - 1). \]

(A.4)

The distribution amplitudes of \( \Xi \) with twist-3 is

\[ V_1 (x_i) = 120x_1 x_2 x_3 \phi_3^0 . \]

(A.5)

Twist-4 distribution amplitudes

\[ V_2 (x_i) = 24x_1 x_2 \phi_3^0 , \quad V_5 (x_i) = 12x_3 (1 - x_3) \phi_3^0. \]

(A.6)

Twist-5 distribution amplitudes

\[ V_4 (x_i) = 3(1 - x_3) \phi_3^0 , \quad V_6 (x_i) = 6x_3 \phi_3^0. \]

(A.7)

And twist-6 distribution amplitudes

\[ V_6 (x_i) = 2 \phi_3^0 . \]

(A.8)

In the leading order conformal spin accuracy, the coefficients of (A.5)-(A.8) can be expressed as

\[ \phi_3^0 = \phi_6^0 = f_\Xi, \]

(A.9)

\[ \phi_4^0 = \phi_5^0 = \frac{1}{2} (f_\Xi + \lambda_1), \]

(A.10)

\[ \psi_4^0 = \psi_5^0 = \frac{1}{2} (f_\Xi - \lambda_1). \]

(A.11)

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