An arbitrary-operation gate with a SQUID qubit

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A novel approach is proposed for realizing an arbitrary-operation gate with a SQUID qubit via pulsed-microwave manipulation. In this approach, the two logical states of the qubit are represented by the two lowest levels of the SQUID and an intermediate level is utilized for the gate manipulation. The method does not involve population in the intermediate level or tunneling between the two logical qubit states during the gate operation. Moreover, we show that the gate can be much faster than the conventional two-level gate. In addition, to take the advantage of geometric quantum computing, we further show how the method can be extended to implement an arbitrary quantum logic operation in a SQUID qubit via geometric manipulation.

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In the growing field of quantum computing, superconducting devices including cooper pair boxes, Josephson junctions, and superconducting quantum interference devices (SQUIDs) [1-10] have appeared to be among the most promising candidates for quantum computing. Reasons for this are that these systems can be easily fabricated to implement a large-scale quantum computing and have been demonstrated to have relatively long decoherence time [11,12]. In the past few years, for SQUID systems, many methods for demonstrating macroscopic quantum coherence [7], creating a superposition state (i.e., a Shördinger cat state) [9], or completing population transfer between low-lying levels (i.e., a NOT gate) [10] have been presented. However, the key point for quantum computing is how to realize an arbitrary single-qubit operation plus a two-qubit controlled-NOT (or controlled-phase shift) operation, since this set of operations makes a universal quantum computing [13].

In this letter, we focus on how to achieve an arbitrary-operation gate with a SQUID qubit. The proposed method has several advantages: (a) during the gate operation, the intermediate level is not populated and thus decoherence induced by spontaneous emission from the intermediate level is greatly suppressed; (b) no tunneling between the SQUID-qubit levels |0⟩ and |1⟩ is required during the gate operation, therefore the decay from the qubit levels can be made negligibly small via increasing the potential barrier between the qubit levels; (c) more importantly, as shown below, the gate operation can be performed faster compared with the conventional gate operation. In addition, we show that the approach can be extended to perform an arbitrary operation on a SQUID qubit via geometric manipulation of phase.

Consider a SQUID driven by two classical microwave pulses I and II. In the following, the SQUID is treated quantum mechanically, while the microwave pulses are treated classically. The Hamiltonian $H$ for the coupled system can be written as

$$H = H_s + H_I^{(I)} + H_I^{(II)},$$

where $H_s$ is the Hamiltonian for the SQUID; and $H_I^{(I)}$ and $H_I^{(II)}$ are the interaction energies for the SQUID with the two pulses, respectively.

The SQUIDs considered throughout this letter are rf SQUIDs each consisting of a Josephson tunnel junction enclosed by a superconducting loop. The Hamiltonian for an rf SQUID (with junction capacitance $C$ and loop inductance $L$) can be written in the usual form

$$H_s = \frac{Q^2}{2C} + \frac{(\Phi - \Phi_0)^2}{2L} - E_J \cos \left(2\pi \frac{\Phi}{\Phi_0}\right),$$

where $\Phi$ is the magnetic flux threading the ring, $Q$ is the total charge on the capacitor, $\Phi_0$ is the static (or quasistatic) external flux applied to the ring, and $E_J \equiv I_c\Phi_0/2\pi$ is the Josephson coupling energy ($I_c$ is the critical current of the junction and $\Phi_0 = h/2e$ is the flux quantum). In practice, the single junction is often replaced by a dc SQUID with low inductance $l \ll L$ which is effectively a junction with its critical current tunable by varying the flux applied to the dc SQUID.

The SQUID is coupled inductively to the $k$th pulse ($k = I, II$) through the interaction energy $H_I^k = \lambda_k (\Phi - \Phi_x) \Phi_{kw}^k$, where $\lambda_k = -1/L$ is the coupling coefficient between the SQUID and the $k$th pulse; and $\Phi_{kw}^k$ is the magnetic flux generated by the magnetic component $B^k(t)$ of the $k$th pulse, which is given by $\Phi_{kw}^k \equiv \int_S B^k(t) \cdot dS$ ($S$ is any surface that is bounded by the ring). The expression of $B^k(t)$ takes the form $B^k(t) = B_0^k \cos (\omega_k t + \phi_k)$, where $B_0^k$, $\omega_k$ and $\phi_k$ are the amplitude, frequency and phase of the $k$th pulse, respectively.

Consider a Λ-type configuration formed by the two lowest levels and an intermediate level of the SQUID, denoted by |0⟩, |1⟩ and |a⟩ with energy eigenvalues $E_0$, $E_1$, and $E_a$, respectively [Fig. 1(a)]. Suppose that the

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coupling of \(|0\rangle, |1\rangle\) and \(|a\rangle\) with the other levels via the pulses is negligible, which can always be realized by adjusting pulse frequencies, or level spacings of the SQUID. Under this consideration, it is easy to find that (i) if no \(|0\rangle \leftrightarrow |1\rangle\) and \(|0\rangle \leftrightarrow |a\rangle\) transitions are induced by the first pulse, i.e., the first pulse is far-off resonant with the \(|0\rangle \leftrightarrow |1\rangle\) and \(|0\rangle \leftrightarrow |a\rangle\) transitions, which can be realized by setting \(|\omega_{01} - \omega_{1}| \gg \Omega_{01}^2\) and \(|\omega_{0a} - \omega_{0}| \gg \Omega_{0a}^2\); (ii) if the second pulse is far-off resonant with \(|0\rangle \leftrightarrow |1\rangle\) and \(|0\rangle \leftrightarrow |a\rangle\) transitions, i.e., \(|\omega_{01} - \omega_{1II}| \gg \Omega_{01}^2\) and \(|\omega_{0a} - \omega_{1II}| \gg \Omega_{0a}^2\); and (iii) if the detuning of the first pulse with the \(|0\rangle \leftrightarrow |a\rangle\) transition and the detuning of the second pulse with the \(|1\rangle \leftrightarrow |a\rangle\) transition are the same \(\omega_{0a} - \omega_{1} = \omega_{1a} - \omega_{1II} = \Delta\), which creates a two-photon Raman resonant between the levels \(|0\rangle\) and \(|1\rangle\) (a standard technique in quantum optics [14,15]) [Fig. 1(a)], the effective interaction Hamiltonian in the interaction picture with respect to \(H_{S} = \sum_{i=0,1,a} E_{i} |i\rangle \langle i|\) can be written as

\[
H_{I} = \hbar \Omega_{01} |0\rangle \langle 0| + \Omega_{1a} |1\rangle \langle a| + \hbar c.\]  (3)

In above, \(\omega_{ij} = (E_{j} - E_{i})/\hbar\) is the transition frequency between the levels \(|i\rangle\) and \(|j\rangle\), and \(\Omega_{ij} = \sum_{k=0}^{\infty} \langle \Phi_{k} | \Phi_{j}\rangle \Phi_{ijm}\) is the Rabi flopping frequencies between the levels \(|i\rangle\) and \(|j\rangle\) (on resonance) generated by the \(k\)th pulse here, \(i, j = 0, 1, a; i \neq j; \Phi_{ijm} = \int_{S} B_{k}^{m} \cdot dS;\) and \(k = I, II\).

To simplify our presentation, we now replace \(\Omega_{0a}^2\) and \(\Omega_{11}^2\) of Eq. (3) by \(\Omega_{I}\) and \(\Omega_{II}\), respectively. From Eq. (3), it is easy to show that when the condition \(\Delta \gg \Omega_{II}\) is satisfied (i.e., large detuning), the Hamiltonian (3) reduces to the following effective Hamiltonian after eliminating the intermediate level \(|a\rangle\)

\[
H_{eff} = -\hbar \left( \frac{\Omega_{01}^2}{\Delta} \sigma_{00} + \frac{\Omega_{11}^2}{\Delta} \sigma_{11} \right) + e^{i(\phi_{1} - \phi_{0})} \sigma_{-} + e^{-i(\phi_{1} - \phi_{0})} \sigma_{+} \]  (4)

in a rotating frame \(U = e^{-i\eta} \theta = -\Delta (\sigma_{00} + \sigma_{11}) t\), where the operators \(\sigma_{00} = |0\rangle \langle 0|\), \(\sigma_{11} = |1\rangle \langle 1|\), \(\sigma_{-} = |0\rangle \langle 1|\), and \(\sigma_{+} = |1\rangle \langle 0|\). In Eq. (4), the first two terms are ac-Stark shifts of the levels \(|0\rangle\) and \(|1\rangle\), which are induced by the two microwave pulses, respectively; while the last two terms indicate that the levels \(|0\rangle\) and \(|1\rangle\) are coupled to each other with an effective coupling constant \(g = \Omega_{I} \Omega_{II}/\Delta\), due to the two-photon Raman transition. It is clear that Eq. (4) can be rewritten as

\[
H_{eff} = \hbar \Omega_{01} \sigma_{z} - \hbar g e^{i(\phi_{1} - \phi_{0})} \sigma_{-} - \hbar g e^{-i(\phi_{1} - \phi_{0})} \sigma_{+}, \]  (5)

where \(\omega_{0} = (\Omega_{01}^2 - \Omega_{11}^2)/(2\Delta)\) and \(\sigma_{z} = \sigma_{11} - \sigma_{00}\).

In the following, the two logical states of a SQUID qubit are represented by the two lowest levels \(|0\rangle\) and \(|1\rangle\) of the SQUID. The dynamics of the qubit is governed by the Hamiltonian (4) or (5). Here, it should be mentioned that a term \(-\hbar \Delta (\sigma_{00} + \sigma_{11})\) in Eq. (4) and another term \(-\hbar \Omega_{01}^2/\Delta (\sigma_{00} + \sigma_{11})\) in Eq. (5) have been safely omitted, because these two terms are proportional to the identity \(I = |0\rangle \langle 0| + |1\rangle \langle 1|\) in the qubit Hilbert space, i.e., they bring a common phase factor to the states \(|0\rangle\) and \(|1\rangle\) during the time evolution.

By means of Eq. (5), one can easily obtain the time evolution of the logical states \(|0\rangle\) and \(|1\rangle\) as follows

\[
|0\rangle \rightarrow (\cos \sqrt{g^2 + \omega_{0}^2} t + i \frac{\varphi_{1}}{\sqrt{g^2 + \omega_{0}^2}} \sin \sqrt{g^2 + \omega_{0}^2} t) |0\rangle
+ i g e^{-i(\phi_{1} - \phi_{0})} \sin \sqrt{g^2 + \omega_{0}^2} t |1\rangle,
|1\rangle \rightarrow (\cos \sqrt{g^2 + \omega_{0}^2} t - i \frac{\varphi_{0}}{\sqrt{g^2 + \omega_{0}^2}} \sin \sqrt{g^2 + \omega_{0}^2} t) |0\rangle
+ (\cos \sqrt{g^2 + \omega_{0}^2} t - i \frac{\varphi_{0}}{\sqrt{g^2 + \omega_{0}^2}} \sin \sqrt{g^2 + \omega_{0}^2} t) |1\rangle. \]  (6)

In particular, for the case of \(\omega_{0} = 0\), i.e., \(\Omega_{I} = \Omega_{II}\), or \(\omega_{0} \ll g\), i.e., \(\delta\Omega \ll \Omega_{I} \Omega_{II}/\Omega\) (here, \(\delta\Omega = \Omega_{I} - \Omega_{II}\) and \(\Omega = \sqrt{\Omega_{I}^2 + \Omega_{II}^2}\)), the state rotation is given by

\[
|0\rangle \rightarrow \cos gt |0\rangle + ie^{-i(\phi_{1} - \phi_{0})} \sin gt |1\rangle,
|1\rangle \rightarrow ie^{i(\phi_{1} - \phi_{0})} \sin gt |0\rangle + \cos gt |1\rangle. \]  (7)

Eq. (6) or (7) shows that the intermediate level \(|a\rangle\) is not populated during the time evolution. This is due to the fact that the level \(|a\rangle\) is not involved in the above Hamiltonian (4) or (5).

Based on Eq. (7), one can see that (i) A flipping between \(|0\rangle\) and \(|1\rangle\) (i.e., \(\pi\)-rotation) can be implemented by setting \(t = \pi/(2g)\); (ii) The Hadamard transformation \(|0\rangle \rightarrow |(1) + |0\rangle) / \sqrt{2}\) and \(|1\rangle \rightarrow (|1\rangle - |0\rangle) / \sqrt{2}\) can be achieved with \(t = (\pi/2) / (2g)\) and \(\phi_{1} - \phi_{0} = \pi/2\); (iii) More importantly, the single-qubit phase shift gate \(|0\rangle \rightarrow e^{-i(\phi_{1} - \phi_{0})} |0\rangle\) and \(|1\rangle \rightarrow e^{i(\phi_{1} - \phi_{0})} |1\rangle\) can be realized by a two-step operation: setting the operation time \(t = \pi/(2g)\) with an arbitrary \(\phi_{1} - \phi_{0}\) for the first step and \(t = \pi/(2g)\) and \(\phi_{1} - \phi_{0} = \pi\) for the second step, respectively. Thus, a complete set of single-qubit arbitrary operations can be achieved by combining the qubit-state rotation (7) with a single-qubit phase shift [13].

It is interesting to note that the gate operations described above can be much faster than the conventional gate operations via resonant Rabi oscillations between the qubit levels \(|0\rangle\) and \(|1\rangle\) [Fig. 1(a)]. For the sake of concreteness, let’s choose a SQUID with the following parameters: \(\beta_{L} = 1.20, Z_{0} = 50\Omega, \omega_{LC} = 5 \times 10^{11}\text{rad/s}\) (i.e., \(C = 40 \text{fF}, L = 100 \text{pH}, I_{c} = 3.95 \mu\text{A}\), and \(\Phi_{0} = -0.501\Phi_{0}\), where \(\beta_{L} \equiv 2\pi L_{c}/\Phi_{0}, Z_{0} \equiv \sqrt{L/C},\) and \(\omega_{LC} = 1/\sqrt{LC}\) are, respectively, the SQUID’s potential shape parameter, characteristic impedance, and characteristic frequency. For simplicity, denote \(\phi_{ij} \equiv \langle i | \Phi_{j} \rangle / \Phi_{0}\). A simple numerical calculation shows \(\phi_{0a} = 8.4 \times 10^{-5}, \phi_{1a} = 5.4 \times 10^{-5},\) and \(\phi_{01} = 7.9 \times 10^{-7}\). Based
on the values of these coupling matrix elements, assuming the amplitudes of the microwave phases for the above three-level gate and the conventional two-level gate on the same order, we have $\Omega_I \simeq \Omega_{II} \sim 10^2 \Omega_{01}$. Thus, if we set the detuning $\Delta = 10$ max $\Omega_I, \Omega_{II}$, the effective Rabi-flopping frequency $\gamma$ in equation (7) and the Rabi-flopping frequency $\Omega_{01}$ in the conventional gate operation have the relationship $\gamma \simeq 10^2 \Omega_{01}$, i.e., the gate speed in the present scheme can be as about ten times as that in the conventional scheme. Here, we should point out that setting $\Delta = 10 \Omega_I$ or $10 \Omega_{II}$ is reasonable, because if $\Delta = 10 \Omega_I$, the maximum population of the level $|a\rangle$ would be about $\Omega^2_{II}/(\Omega^2_I + 100 \Omega^2_{01}) \simeq 0.01$ or 1% during the gate operations.

Some points may need to be addressed here. Firstly, since both the $\Omega_I$ and $\Omega_{II}$ couplings are far-off resonance, no transition between any two levels is induced by a single pulse. Secondly, in order to avoid two-photon Raman transitions between the levels $|0\rangle$ and $|a\rangle$ on the levels $|1\rangle$ and $|\phi\rangle$, the conditions $\omega_I \pm \Omega_{II} \neq \omega_{0a}, \omega_{1a}$ must be satisfied. Fortunately, the condition $\omega_I - \Omega_{II} \neq \omega_{0a}, \omega_{1a}$ is ensured because of $\omega_{0a}, \omega_{1a} \gg \omega_{01} = \omega_I - \Omega_{II}$. On the other hand, the condition $\omega_I + \Omega_{II} \neq \omega_{0a}, \omega_{1a}$ can be easily satisfied by adjusting the SQUID’s level structure [see Fig. 1(a)]. In fact, all of the conditions required can be readily achieved experimentally by adjusting microwave frequencies, or level spacings of the SQUID qubit.

Recently, geometric quantum computing has been paid much attention because apart from its fundamental interest, geometric phase is intrinsically fault-tolerant to certain types of computational errors due to its geometric property [3,16-20]. On one hand, several basic ideas of geometric computing based on Berry phase have been proposed by using NMR [16], superconducting electron box [17], or trapped ions [18]; and an experimental realization of the conditional adiabatic phase shift has been reported with NMR technique [16]. On the other hand, geometric computing based on Aharonov and Anandan (A-A) phase [21] is currently getting considerable attention and several proposals have been also presented recently [3,19,20]. In A-A phase geometric manipulation, if the external field is perpendicular to the evolution path, there is no dynamic phase accumulated in the whole process; thus no extra operation is required to eliminate the dynamic phase. In contrast, geometric manipulation based on Berry phase always involves the dynamic phase and an extra operation is needed to remove its adverse effect [3]. In the following, we show explicitly how to perform an arbitrary rotation and a phase shift for a SQUID qubit via the A-A phase geometric manipulation [22].

We start by rewriting the Hamiltonian (5) as

$$H_{eff} = \hbar \omega_0 \sigma_z - \hbar g \cos (\phi_I - \phi_{II}) \sigma_x - \hbar g \sin (\phi_I - \phi_{II}) \sigma_y$$

$$= \vec{B} \cdot \vec{\sigma},$$

where the fictitious field $\vec{B} = \{-h g \cos (\phi_I - \phi_{II}), -h g \sin (\phi_I - \phi_{II}), \hbar \omega_0\}$ and $\vec{\sigma}$ are the Pauli operators. To see how the logical states $|0\rangle$ and $|1\rangle$ are rotated via geometric means, let us first show how the eigenstates $|\pm\rangle$ of $\sigma_y$, defined by $\sigma_y |\pm\rangle = \pm |\pm\rangle$, will undergo a cyclic evolution and obtain a A-A phase under the following operations: (i) turn on the two microwave pulses $I$ and $II$, which have $\phi_I - \phi_{II} = (2n + 1) \pi$ ($n$ is an integer). After a time $t = \pi/(2\sqrt{\omega_0^2 + g^2})$, the state $|+\rangle$ rotates around the fictitious field $\vec{B} = \{h g, 0, \hbar \omega_0\}$, from $|+\rangle$ in the $\vec{e}_y$ direction to $|-\rangle$ in the $-\vec{e}_y$ direction along curve ABC on the Bloch sphere [see Fig. 1(b)]. (ii) Change the two-phase difference to $\phi_I - \phi_{II} = 2n \pi$. After another time $t = \pi/(2\sqrt{\omega_0^2 + g^2})$, the state $|-\rangle$ rotates back to $|+\rangle$ around $\vec{B} = \{-h g, 0, \hbar \omega_0\}$ along curve CDA on the Bloch sphere. In the above operations, the trace of the state vector encloses an area on the Bloch sphere. After the above cyclic evolution, the state $|+\rangle$ becomes $e^{-i2\theta} |+\rangle$ with a A-A geometric phase $-2\theta$, where $2\theta = 2 \arctan(\omega_0/g)$ is the solid angle subtended by the area of ABCDA. It should be mentioned that because the state vector is always perpendicular to the effective magnetic field during the above operations, no dynamical phase is accumulated during the evolution. In a similar way, the state $|-\rangle$ will become $e^{i2\theta} |-\rangle$ with an accumulated A-A phase of $2\theta$ after the above operations. Finally, based on $|0\rangle = -i(|+\rangle - |-\rangle) / \sqrt{2}$ and $|1\rangle = (|+\rangle + |-\rangle) / \sqrt{2}$, one can see that at the end of the above operation, the logical states $|0\rangle$ and $|1\rangle$ are rotated as

$$|0\rangle \to \cos \alpha |0\rangle - \sin \alpha |1\rangle,$$

$$|1\rangle \to \sin \alpha |0\rangle + \cos \alpha |1\rangle,$$

(9)

where $\alpha = 2\theta$ is within the range of $[0, 2\pi]$, i.e., (9) accomplishes a rotation with angle $\alpha$.

For the case of $\Omega_I = \Omega_{II}$, i.e., $\omega_0 = 0$, the Hamiltonian (8) reduces to $H_{eff} = B \cdot \vec{\sigma}$ with $\vec{B} = \{-h g \cos (\phi_I - \phi_{II}), -h g \sin (\phi_I - \phi_{II}), 0\}$. To implement a single-qubit phase shift gate, one can first apply two pulses with a certain phase difference $\delta \phi_0$. After a time $t = \Delta \pi/(2\Omega_I \Omega_{II})$, the logical states $|0\rangle$ and $|1\rangle$ rotate around the effective magnetic field $\{-h g \cos \delta \phi_0, -h g \sin \delta \phi_0, 0\}$ to $|1\rangle$ and $|0\rangle$, respectively. In the second step of the operation, one changes the phase difference to $-\delta \phi_0$. After another time $t = \Delta \pi/(2\Omega_I \Omega_{II})$, the logical states rotate around the effective magnetic field $\{-h g \cos \delta \phi_0, -h g \sin \delta \phi_0, 0\}$ and return to the original states $|0\rangle$ and $|1\rangle$. After this cyclic evolution, the logical states acquire geometric A-A phases as $|0\rangle \to e^{i\beta} |0\rangle$ and $|1\rangle \to e^{-i\beta} |1\rangle$, where $\beta = \pi - 2 \delta \phi_0$ is the solid angle subtended by the area of the loop. According to [13], combining this phase shift gate with the qubit state rotation (9) constitutes a complete set of single-qubit arbitrary operations.

One significant point needs to be made here. For the
rotation gate, it is not possible to change suddenly the two-pulse phase difference after the first step operation. Imagine that after the first step, the two pulses are turned off for a time interval $\delta t$ in order to adjust the phase difference to $-\delta \phi_0$. In this case, by a simple calculation, we find that after the whole operation, the two logical states evolve into

\[
|0\rangle \rightarrow \cos \alpha |0\rangle - e^{-i(E_1-E_0)\delta t/\hbar} \sin \alpha |1\rangle, \\
|1\rangle \rightarrow \sin \alpha |0\rangle + e^{-i(E_1-E_0)\delta t/\hbar} \cos \alpha |1\rangle, 
\]

where the relative phase $e^{-i(E_1-E_0)\delta t/\hbar}$ between $|0\rangle$ and $|1\rangle$ is induced due to the free evolution of the logical states during the period of adjusting the phase difference. However, from Eq. (10), one can see that if the time interval satisfies $\delta t = 2n\pi/(E_1-E_0)$ ($n$ is an integer), the qubit-state rotation described by Eq. (10) is the same as that given by Eq. (9) exactly.

In summary, we have explicitly shown how to perform an arbitrary-operation gate with a SQUID qubit. The scheme has three distinct advantages: (i) The intermediate level is unpopulated during the gate operation, thus decoherence due to energy relaxation from the level $|\alpha\rangle$ is minimized; (ii) No tunneling between the qubit levels $|0\rangle$ and $|1\rangle$ is required, therefore the decay from the level $|0\rangle$ can be made negligibly small via increasing the potential barrier between the qubit levels; (iii) For the 3-level gates described here, stronger microwave field can be used to achieve faster operation than the conventional 2-level gates. More interestingly, as shown above, an arbitrary quantum logic via global A-A phase geometric means can be achieved in a SQUID qubit, by extending the present method. In addition, the method can also be applied to any other type of qubits with a $\Lambda$-type level configuration. The present proposal provides new approaches to demonstrate a single-qubit arbitrary gate with superconducting devices, and we hope that the proposed approaches will stimulate further theoretical and experimental activities in this area.

Before we conclude, it should be noticed that recently, M. H. S. Amin et al. [23] have proposed a similar scheme for obtaining a complete set of one-qubit gate with a SQUID qubit. In their proposal, an arbitrary gate operation is achieved based on a kind of rotation that is performed over a half period of Rabi oscillation by applying microwave to induce transition to the intermediate level (a higher state). As addressed in Ref. [23], the probability of finding the system in the intermediate level is zero again after a half of Rabi period. However, under their assumption of small detuning, population in the intermediate level is finite during the evolution, resulting in higher probability of spontaneous decay. Finally, we should point out that qubit operations via adiabatic passage [24] in general are rather slow and require precise control over the amplitude of external fields.

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Figure Captions

FIG. 1. (a) Simplified level diagram of an rf SQUID with Λ-type three levels $|0\rangle$, $|1\rangle$ and $|a\rangle$. (b) The path evolution of the SQUID qubit initially in the state $|+\rangle$. 
Fig. 1
