Structure of a free convective flow over a horizontal heated surface under conditions of conjugate heat transfer

E F Khrapunov, I V Potechin, Y S Chumakov
Peter the Great St. Petersburg Polytechnic University
Russia, 195251 St. Petersburg, Polytechnic st., 29
Email: hrapunov.evgenii@yandex.ru

Abstract. The paper presents the results of an experimental and numerical study of the structure of a plume formed over a heated disk under conditions of conjugate heat transfer in a wide range of Grashof number. The data on the temperature field in the plume is compared with the visualization patterns. Numerical simulation was performed using the Ansys Fluent code, where the problem of the flow of a compressible gas in conditions of conjugate heat transfer in an unsteady formulation was solved under the assumption of laminar flow regime. As a result of a comparison of the results of physical and numerical experiments, there is a conclusion about a good qualitative concurrence of the flow structure. The heat transfer characteristics were studied experimentally and numerically: the local heat transfer coefficient, the heat flow from the heated disk, and the local and average Nusselt numbers. It is shown that the establishment of a periodic flow regime leads to a violation of the monotonicity of the distribution of the local heat transfer coefficient. In general, the data is in satisfactory agreement with the results of known works.

1. Introduction
Natural convection is the determining type of motion in many applied problems, for example, meteorological, cooling and ventilating tasks, and the problems of creating a microclimate. An analysis of the known to the authors works shows that the main successes were achieved in the study of flows in closed volumes and along vertical heated surfaces. Heating of the surface leads to the appearance of a freely convective motion, and, as a consequence, to the formation of a natural convective boundary layer on this surface, the laws of development of which are similar to the laws of development of a well-studied forced convective boundary layer. In particular, it is possible to allocate sections of laminar, transient, and developed turbulent flow.

The situation is different when the heated surface is perpendicular to the acceleration vector of free fall. Formed in such configurations, the near-wall flow is destroyed under the influence of Archimedes' force, a rising free convective jet - a natural convective plume - appears. Heat transfer in this case is complicated due to the interaction between the near-wall and the rising flows.

The description of the presented flows is a complicated task that is why in some papers there are considered simplified models. For example, it is possible to consider the limiting situation when the heating surface is so small that it can be replaced by a point source of heat. Within this assumption, one of the first papers was performed [1], in which the balance relations for mass, momentum and buoyancy were analytically solved under the supposition of a completely turbulent self-similar free-convective plume. As a result, average temperature and velocity profiles were obtained, as well as data on the jet
boundary change. Subsequent studies (theoretical [2], [3], experimental [4], [5], and later also numerical [6], [7]) confirmed the results already obtained in the case of plates of finite dimensions. On the other hand, the results of numerous experiments are the accumulation of a large amount of data that are poorly consistent, and sometimes even contradictory.

As for free convection heat exchange, there is a large number of mostly experimental works (for example, [8], [9]), during which there was obtained the dependence of heat on the surface, and also the dependence of the Nusselt number on the Grashof number - the determining criteria in the problem. Separately it is worth mentioning the paper [10], in which interferometry is used to obtain information on the qualitative development of a flow in space.

Nowadays the technique of diagnosing the described currents has changed significantly. In a physical experiment, the flow rate measurement is often performed using systems such as LDA and PIV [11], in the course of numerical simulation there are used either LES-based vortex-simulating approaches [12] or direct numerical simulation (DNS) [13]. During the last 10 years, a qualitative change in the character of the flow was detected with increasing Grashof numbers. This phenomenon is called "puffing" and consists in the periodic destruction of the wall layer at a certain distance to the axis of the plume. Noteworthy is the fact that the stream can remain laminar, but it can have a complex vortex structure.

In this paper, we present the results of an experimental and numerical investigation of the flow for small Grashof numbers over a horizontal heated disk, and the effect of this flow on the characteristics of the heat transfer.

2. Experimental stand and computational model
The scheme of the experimental stand is shown in Figure 1. The main element of the stand is a heated brass disk with a diameter of 190 mm and a thickness of 8 mm. The lower surface of the disk contacts the heater, and to improve the thermal contact, a paste with a high coefficient of thermal conductivity is used. Into the layer of this paste a thermocouple is placed to control the temperature of the lower surface of the disk. The temperature mode of the disk is controlled by the controller, which switches on / off the heating when the set temperature is reached. So, on the lower surface of the disk there are realized conditions close to the conditions of constant temperature, the temperature of the upper surface is determined by the conditions of the conjugate heat exchange between brass and air. The thermal localization of the disk is carried out by using the water heat exchanger that maintains the temperature of the horizontal near-disk surface at room temperature. To eliminate the influence of external disturbances around the disk, a grid is installed at a distance of two diameters, and the whole system is placed in a leaky chamber whose walls are made of Plexiglas, which makes it possible to conduct the visualization of the flow. Measurements of the temperature field are carried out using a resistance thermometer. To visualize the flow, the entire measuring volume is seeded with particles that are illuminated with a laser plane.

One of the purposes of this work is to determine the characteristics of heat exchange between a heated disk and air. For this purpose, has been developed a technique based on the assumption of a thin, wall-mounted heat-conducting layer, distribution of the mean temperature in which is linear. On the basis of this assumption, it is possible to calculate the derivative of the temperature along the normal to the disk, and then the heat flux, the heat transfer coefficient, and local Nusselt numbers.
Figure 1. The scheme of the experimental stand
1 - heated disk, 2 - heat exchanger element,
3 - protective grid, 4 - coordinate device,
5 - resistance thermometer

Figure 2. Calculation mesh for numerical simulation

Numerical simulation was performed using the ANSYS Fluent code and consists in solving the Navier-Stokes equations in the Boussinesq approximation and assuming laminar flow in a three-dimensional region whose size correspond to the size of the experimental stand and is close to the size of the computational domains described in the other authors’ works [12]. The problem is solved both in steady and unsteady formulations. The air parameters are set as follows: specific heat
\[ c_p = 1006.43 \, \frac{J}{kgK} \],
thermal conductivity \( \lambda = 0.0242 \, \frac{W}{mK} \), thermal expansion coefficient \( \beta = 0.00366 \, \frac{1}{K} \).

The constructed quasistructured mesh (Figure 2) contains \( N_{cell} \approx 10^6 \) elements and has a thickening to the expected boundaries of the plume and to wall horizontal surfaces. On the lateral and upper surfaces, the outlet boundary conditions are set, as well as the conditions of the existence of only a normal component of the velocity for the backflow. The lower horizontal surface is a solid wall, maintained at room temperature (20 °C). On the lower surface of the disk, temperature condition of the first type is set, corresponding to the temperature in the experiment. The temperature of the upper surface is determined by the conditions of the conjugate heat transfer.

3. The results analysis
As it was noted, the Grashof number based on the characteristic temperature difference between the temperature of the lower surface of the disk \( T_w \) (controlled temperature in the experiment and given as a boundary condition for numerical simulation) and room temperature \( T_a \) is the defining parameter in the problem. The radius of the disk \( R \) is selected as the linear scale.
\[ Gr = \frac{g \beta (T_w - T_a) R^3}{\nu^2} \] (1)

The investigated range of Grashof numbers \( Gr = [1.35 \ldots 9.45] \times 10^6 \). In a physical experiment, it is established that the critical Grashof number at which the transition from steady flow to unsteady flow takes place is \( Gr = 8.10 \times 10^6 \), and in the numerical experiment \( Gr = 9.45 \times 10^6 \). At Grashof numbers less than the critical value, the streamlines of the plume are parallel to the axis of symmetry of the flow, the streamlines of the ejected flow are horizontal (Figure 3). It is worth noting that the plume is formed over a relatively small portion of the disk, the rest is occupied by a near-wall flow. Similar results were obtained as a result of numerical simulation (Figure 4).
As a result of computational modeling, it is established that there is a stationary axisymmetric solution of the problem for subcritical Grashof numbers. In this case, the characteristics of the plume coincide with the general representations [1], so the temperature distribution is described by a Gaussian dependence of the form

\[ \frac{\Delta T}{T_{\text{max}}} = \exp\left(-a \left(\frac{r}{b_r}\right)^2\right), \]

where \( T_{\text{max}} \) is the temperature radius of the plume (the radial coordinate in which the temperature is half of the maximum in this section), and \( a \) is the determined constant. It should be noted that, starting from a height of 1.5D above the surface of the disk, the dimensionless temperature profiles coincide, which indicates the phenomenon of self-similarity (Figure 5). As a self-similar coordinate, the ratio of the actual coordinate to the corresponding temperature radius of the plume is chosen.

Figure 6 shows the change in the temperature radius of the plume, depending on the height above the disk for different Grashof numbers. In particular, as the Grashof number increases, the radius of the plume decreases. The region of narrowing of the stream ("necking") near the surface of the disk, and also the region of linear growth of the thickness of the flow along the height can be singled out. The slope coefficients for the temperature radiiuses that are dimensioned on the disk diameter take values from 0.02 to 0.005. It should be noted that the velocity radius of the jet exceeds the temperature radius for any Grashof numbers, but its slope is less than for a turbulent jet [12].

Let’s consider the characteristics of heat transfer, namely the heat transfer coefficients and the Nusselt numbers. As was already noted, the experimental determination of these characteristics is based on the assumption of the existence of a wall-mounted heat conducting layer, the temperature change across which is linear. When processing the calculation results, the temperature field is processed by the experimental method, and the results of such processing (Exp. Method) are compared with the results...
of numerical and physical experiments. Figure 7 shows the temperature profile along the normal to the disk surface at a distance of \( \frac{R}{2} \) from the axis, and also the linear approximation (OLS). Figure 8 shows a comparison of the heat transfer coefficient, determined directly as a result of numerical simulation, according to the described method and experimentally. It can be seen that the values correspond well to each other.

Figure 7. Profile of the dimensionless temperature along the normal to the disk surface at \( Gr = 5.40 \times 10^6 \) at a distance \( r/R = 0.5 \)

Figure 8. The coefficient of heat transfer, determined numerically and by the experimental method at \( Gr = 5.40 \times 10^6 \)

Figure 9 shows the change in the local Nusselt numbers along the radius of the disk, determined by (2), for various Grashof numbers. The radial coordinate is measured from the center of the disk. The Nusselt number takes the minimum value near the symmetry axis, after which it increases to the periphery, where the heat exchange is most intense

\[
Nu_r = \frac{ar}{\lambda} = \frac{r}{(T_a - T_w)} \left( \frac{\partial T}{\partial n} \right)_{w}
\]  

(2)

\[
Nu = \frac{2\pi}{S} \int_0^R Nu_r r dr
\]  

(3)

Figure 10 shows the change in the integral Nusselt number determined by (3), depending on the Grashof number, where \( S \) is the surface area of the disk. The criterion dependence of the Nusselt number on the Grashof number is usually represented as an exponential function \( Nu = C(Gr Pr)^m = CRA^m \). The constants \( C \) and \( m \) are determined by the results of the experiment. Figure 11 shows the data processed as in [10]. It can be concluded that there is a good agreement between the known dependencies and the data obtained during this study.

Figure 9. Changing the local Nusselt numbers by the radius of the disk at \( Gr = 5.40 \times 10^6 \)

Figure 10. Change in the Nusselt integral number as a function of the Rayleigh number

Figure 11. Change in the integral Nusselt number (technique [10]) as a function of the Rayleigh number
When the Grashof numbers are higher than the critical values, the flow pattern changes qualitatively - the steady regime is replaced by an unsteady one, characterized by periodic destruction of the wall layer and the emergence of superheated air masses near the axis of the plume. This change is recorded both during visualization and during numerical simulation (Figure 11). In numerical simulation, the unsteady solution began with a steady non-converging field, and after calculating 200s of physical time, averaging was performed during the 60s physical time. The mean temperature and velocity fields satisfactorily agree with the results of [13]. Figure 12 shows the change in the average axial velocity along the flow axis. The figure also shows a power-law dependence from [13], which is characteristic of a fully developed turbulent flow.

As shown in Figure 12, the maximum value of the average axial velocity corresponds to the coordinate $z = 0.38m \left( \frac{z}{D} = 2 \right)$, which agrees well with the data of [13]. Nevertheless, due to the laminar flow regime, a power law is not satisfied.

The periodic nature of the flow significantly affects the characteristics of heat transfer. So, the local maximum appears in the distribution of the heat transfer coefficient (Figure 14). This behavior is found both in the results of numerical simulation, and as a result of a physical experiment.

The profiles of the average temperature also deform, at a certain distance from the axis local maxima appear, while the average flow does not differ much from the axisymmetric one.

4. Conclusion
As a result of experimental and numerical simulation of a natural convective plume, the boundaries of existence of a steady flow are established, in which the flow is axisymmetric, the temperature and velocity profiles are self-similar, the heat transfer coefficient and the local Nusselt number vary monotonically from the periphery to the center of the disk. If the critical Grashof number is exceeded, the flow pattern changes qualitatively - the monotonic nature of both the profiles of the average temperature over the heated disk and the heat exchange characteristics is disturbed.

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