Reaction mechanism in odd-even staggering of reaction cross sections

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Introduction. Interaction cross section $\sigma_1$ and reaction cross section $\sigma_R$ are an important tool of determining radii of unstable nuclei. Actually, the halo structure as an exotic property was reported for unstable nuclei like $^{11}\text{Li}$ through analyses of measured $\sigma_1$ [1,2]. Very recently, $\sigma_1$ was measured for Ne isotopes [3] and it is suggested by the analyses [4,5] that $^{31}\text{Ne}$ is a halo nucleus with large deformation.

The difference between $\sigma_1$ and $\sigma_R$ is considered to be small for scattering of unstable nuclei at intermediate energies [6]. The reaction cross section is nearly proportional to a radius of projectile; for example, see Ref. [6] for detailed analyses. Meanwhile, it is well known that pairing correlations are important particularly in even-N nuclei. The correlations become essential in weakly bound nuclei, since they are not bound without the correlations. Effects of the correlations on nuclear radii of unstable nuclei were investigated by the Hartree-Fock Bogoliubov (HFB) method [7]. In the mean-field picture, the correlations make the quasi-particle energy larger and hence reduce the root-mean-square radius of the HFB density. Obviously, this effect is conspicuous for unstable nuclei with the separation energy smaller than the gap energy. Thus, the pairing correlation suppresses the growth of halo structure for even-even unstable nuclei. This is now called the pairing anti-halo effect.

The pairing anti-halo effect is an interesting phenomenon, but any clear evidence is not shown for the effect yet. Very recently, however, Hagino and Sagawa suggested that observed odd-even staggerings of $\sigma_R$ are possible evidence of the effect [8-10]. They introduced the staggering parameter [10]

$$\gamma_3 = -\frac{\sigma_R(A) - 2\sigma_R(A + 1) + \sigma_R(A + 2)}{2},$$

(1)

where the mass number $A$ of projectile is assumed to be even. In Ref. [8], the staggering was analyzed with the HFB method for $^{30,31,32}\text{Ne}+^{12}\text{C}$ scattering at 240 MeV/A [3] and with the three-body model for $^{14,15,16}\text{C}+^{12}\text{C}$ scattering at 83 MeV/A [11]. The analyses are successful in reproducing observed staggerings [3,11], although the reaction calculations are based on the Glauber model.

In this paper, we reanalyze not $^{30,31,32}\text{Ne}$ but $^{14,15,16}\text{C}$ scattering in order to focus our attention on the reaction mechanism, since $^{15}\text{C}$ has a simpler structure than $^{31}\text{Ne}$ in the sense that the first excited energy of $^{14}\text{C}$ as a core nucleus is much larger than that of $^{30}\text{Ne}$. For $^{14,15,16}\text{C}$, $\gamma_3$ is $163 \pm 52$ mb and about 10% of $\sigma_R(15\text{C}) = 1319 \pm 40$ mb [11]. Thus, the pairing anti-halo effect may be comparable with the projectile-breakup and nuclear-medium effects that are not taken into account in the previous analysis. Therefore, we investigate these effects on the staggering, using the continuum-discretized coupled-channels method (CDCC) [12-14]. CDCC for two-body (three-body) projectiles is often called three-body (four-body) CDCC; in the naming the target degree of freedom is taken into account. This is the first application of four-body CDCC to $^{16}\text{C}$.

Theoretical framework. Following Ref. [8], we assume the $n + ^{14}\text{C}$ two-body model for $^{15}\text{C}$ and the $n + n + ^{14}\text{C}$ three-body model for $^{16}\text{C}$. The three-body model of $^{16}\text{C}$ is a simple model for treating pairing correlations between extra two neutrons. In the present calculation, breakup reactions of $^{15}\text{C}$ and $^{16}\text{C}$ on $^{12}\text{C}$ are described by the $n + ^{14}\text{C} + ^{12}\text{C}$ three-body model and the $n + n + ^{14}\text{C} + ^{12}\text{C}$ four-body model, respectively. The Schrödinger equation is defined as

$$(H - E)\Psi = 0$$

(2)

for the total wave function $\Psi$, where $E$ is an energy of the total system. The total Hamiltonian $H$ is defined by

$$H = K_R + U + h,$$

(3)

where $h$ denotes the internal Hamiltonian of $^{15}\text{C}$ or $^{16}\text{C}$, $R$ is the center-of-mass coordinate of the projectile relative to a $^{12}\text{C}$ target. The kinetic energy operator associated with $R$ is
represented by $K_{R}$, and $U$ is the sum of interaction between the constituents in the projectile and the target defined as

$$U = U_{n}(R_{n}) + U_{14C}(R_{14C}) + \frac{e^{2}Z_{P}Z_{T}}{R},$$

(4)

for $^{15}$C and

$$U = U_{n_{1}}(R_{n_{1}}) + U_{n_{2}}(R_{n_{2}}) + U_{14C}(R_{14C}) + \frac{e^{2}Z_{P}Z_{T}}{R},$$

(5)

for $^{16}$C, where $U_{x}$ is the nuclear part of the optical potential between $x$ and $^{12}$C as a function of the relative coordinate $R_{x}$.

The optical potential $U_{x}$ is constructed microscopically by folding the Melbourne $g$-matrix nucleon-nucleon interaction [15] with densities of $x$ and $^{12}$C. For $^{12}$C, the proton density is obtained phenomenologically from the electron scattering [16], and the neutron density is assumed to be the same as the proton one, since the proton root-mean-squared (RMS) radius deviates from the neutron one only by less than 1% in the HFB calculation. For $^{15}$C, the matter density is determined by the HFB calculation with the Gogny-D1S interaction [17], where the center-of-mass correction is made in the standard manner [6]. As shown latter, the total reaction cross section calculated with the folding $^{14}$C-$^{12}$C potential $U_{14C}$ is good agreement with the experimental data for the $^{14}$C + $^{12}$C scattering at 83 MeV/A. The Melbourne $g$-matrix folding method is successful in reproducing nucleon-nucleus and nucleus-nucleus elastic scattering systematically [16-14]. The folding potentials thus obtained include the nuclear-medium effect. CDCC with these microscopic potentials is the microscopic version of CDCC.

In the present system, Coulomb breakup is quite small, since the projectile ($P$) and the target ($T$) are light nuclei and hence the Coulomb barrier energy between $P$ and $T$ is much smaller than the incident energy considered here. We then neglect Coulomb breakup, as shown in Eq. (5), where $Z_{P}$ and $Z_{T}$ are the atomic numbers of nuclei $P$ and $T$, respectively.

The $n-^{14}$C interaction in $h$ of $^{15}$C is taken as the same interaction as in Ref. [3], which well reproduces the properties of ground and 1st-excited states in $^{15}$C. For the $n + n + ^{14}$C system, we use the Bonn-A interaction [19] between two neutrons and take the same $n-^{14}$C interaction as mentioned above. Furthermore the effective three-body interaction is introduced to reproduce the measured binding energy of $^{16}$C. Eigenstates of $h$ are obtained with numerical techniques of Ref. [22], that is, the orthogonality condition is imposed. Now we introduce the dimensionless staggering parameter $\Gamma_{rds}$ for the RMS radii $\bar{r}$ of $P$ and $T$:

$$\Gamma_{rds} = \frac{\bar{R}^{2}(A + 1) - [\bar{R}^{2}(A) + \bar{R}^{2}(A + 2)]/2}{[\bar{R}^{2}(A + 2) - \bar{R}^{2}(A)]/2},$$

(6)

with

$$\bar{R}(A) = \bar{r}(A) + \bar{r}(T).$$

(7)

Here $\Gamma_{rds} \geq 1$ when $\bar{r}(A + 1) \geq \bar{r}(A + 2)$. Matter radii of $^{14,15,16}$C are summarized in Table I. The present two- and three-model yields $\Gamma_{rds} = 1.3$ for $^{14,15,16}$C.

In CDCC, eigenstates of $h$ consist of finite number of discrete states with negative energies and discretized-continuum states with positive energies. The Schrödinger equation (2) is solved in a modelspace $P$ spanned by the discrete and discretized-continuum states:

$$P(H - E)\Psi_{CDCC} = 0.$$  

(8)

Following Ref. [22], we obtain the discrete and discretized continuum states by diagonalizing $h$ in a space spanned by the Gaussian basis functions. This discretization is called the pseudo-state method. The elastic and discrete breakup $S$-matrix elements are obtained by solving the CDCC equation (8) under the standard asymptotic boundary condition [12, 23]. In actual calculations, we neglect the projectile spin, since the effect on $\sigma_{R}$ is small. We take the angular momentum between $n$ and $^{14}$C for breakup states of $^{15}$C up to $g$-wave, and $0^{+}$ and $2^{+}$ breakup states of $^{16}$C.

Now we define the dimensionless staggering parameter also for $\sigma_{R}$:

$$\Gamma_{R} = \frac{\sigma_{\gamma_{3}}}{|\sigma_{R}(A + 2) - \sigma_{R}(A)|/2},$$

(9)

where $\Gamma_{R} = 0$ when $\sigma_{R}(A + 1) = |\sigma_{R}(A + 2) + \sigma_{R}(A)|/2$ and $\Gamma_{R} = 1$ when $\sigma_{R}(A + 1) = \sigma_{R}(A + 2)$. When the absolute value of the elastic $S$-matrix element, $|S_{\gamma_{3}}(L)|$, is 0 for orbital angular momenta $L$ corresponding to the nuclear interior and 1 for those to the nuclear exterior, it is satisfied that $\sigma_{R}(A) \propto \bar{R}^{2}(A)$ [18]. In the black-sphere scattering, Eq. (9) is reduced to $\Gamma_{R} = \Gamma_{rds}$. Once this condition is satisfied, $\Gamma_{R}$ does not depend on an incident energy $E_{in}$. Three types of models are considered to investigate the nuclear-medium and projectile-breakup effects on $\sigma_{R}$.

- **Model I** is the $T$-matrix single-folding model that has no nuclear-medium and projectile-breakup effects. The $U_{x}$ are constructed from the Melbourne $g$-matrix nucleon-nucleon interaction at zero density. The single-channel calculation is done in [8].

- **Model II** is the $g$-matrix single-folding model that has the nuclear-medium effect but not the projectile-breakup effect. This is the same as Model I, but the density dependence of the Melbourne $g$-matrix is properly taken.

- **Model III** is the model that has both the nuclear-medium and the projectile-breakup effect. CDCC calculations

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
 & $\bar{r}^{(14}{C)}$ [fm] & $\bar{r}^{(15}{C)}$ [fm] & $\bar{r}^{(16}{C)}$ [fm] \\
\hline
Calc. & 2.51$^{a}$ & 2.87$^{a}$ & 2.83$^{a}$ \\
Exp. & 2.53$^{b}$ & 2.90$^{b}$ & 2.81$^{b}$ \\
\hline
\end{tabular}
\caption{Matter radii of $^{14,15,16}$C.}
\end{table}
are done for $^{15,16}$C scattering, but the $g$-matrix single-folding model is taken for $^{14}$C scattering, since $^{14}$C is a tightly-bound system.

**Results.** Figure 1 shows $\sigma_R$ for $^{14,15,16}$C$^{+12}$C scattering at 83 MeV/A. Triangle, circle and square symbols stand for the results of Model I, II and III, respectively. Model III well reproduces the experimental data [11], whereas Model I largely overestimates them; here the data are plotted with 2-$\sigma$ error (95.4% certainty). The nuclear-medium and projectile-breakup effects are thus important for $\sigma_R$. Model III yields $\Gamma_R = 0.77$ that is deviated from $\Gamma_{\text{rels}} = 1.3$. When the breakup effect is switched off from Model III, $\sigma_R$ is reduced from squares to circles. This reduction is most significant for $^{15}$C, so that $\Gamma_R$ is reduced from 0.77 to 0.56. Furthermore, when the medium effect is switched off from Model II, the $\sigma_R$ are enhanced by about 10% from circles to triangles for all the cases of $^{14,15,16}$C. More precisely, the enhancement is 13% for $^{14,16}$C but 15% for $^{15}$C, and consequently, $\Gamma_R$ increases from 0.56 to 0.83 by neglecting the medium effect. Thus the breakup and medium effects are nearly canceled for $\Gamma_R$. The resultant value $\Gamma_R = 0.83$ is still considerably deviated from $\Gamma_{\text{rels}} = 1.3$. This means that the present scattering are not the black-sphere scattering (BSS) exactly. This effect is referred to as “non-BSS effect” in this paper and is explicitly investigated below.

![Figure 1](image1.png)

**FIG. 1:** (Color online) Reaction cross sections $\sigma_R$ for $^{14,15,16}$C$^{+12}$C scattering at 83 MeV/A. Triangle, circle and square symbols stand for results of Model I, II and III, respectively. The experimental data are taken from Ref. [11].

Figure 2 shows the absorption probability $P(L) = 1 - |S_0(L)|^2$ and the partial reaction cross section $\sigma_R(L) = (2L + 1)P(L)\pi/K^2$ as a function of $L$, where $iK$ is an initial momentum of the elastic scattering. Here Model I is taken. For all the $^{14,15,16}$C scattering, $P(L)$ behaves as not a step function but a logistic function. Thus the scattering are not the BSS exactly. Furthermore, $L$ dependences of the $P(L)$ are different among the three projectiles at $60 \lesssim L \lesssim 150$ corresponding to the peripheral region of a $^{12}$C target. As a consequence of the difference, $\sigma_R$ is not proportional to $R^2$ properly. In fact, $^{15}$C has a larger RMS radius than $^{16}$C, but $^{15}$C scattering has a smaller $\sigma_R(L)$ than $^{16}$C one at $70 \lesssim L \lesssim 120$ because of the fact that the volume integral of the imaginary part of the single-folding potential $\langle \varphi_0 | U | \varphi_0 \rangle$ is smaller for $^{15}$C projectile than for $^{16}$C projectile; here $\varphi_0$ is the projectile ground-state wave function.

![Figure 2](image2.png)

**FIG. 2:** (Color online) $L$ dependence of (a) the absorption probability $P(L)$ and (b) the partial reaction cross section for $^{14,15,16}$C$^{+12}$C scattering at 83 MeV/A. Model I is taken.

Figure 3 shows $E_{\text{in}}$-dependence of $\Gamma_R$. Triangle, circle and square symbols correspond to the results of Model I, II and III, respectively, whereas the solid straight line denotes $\Gamma_{\text{rels}}$. The deviation of triangles from the solid straight line shows the non-BSS effect, the deviation of circles from triangles does the nuclear-medium effect, and the deviation of squares from circles comes from the projectile-breakup effect. As $E_{\text{in}}$ goes up, the breakup effect decreases rapidly, but the non-BSS effect increases. The nuclear-medium effect also decreases but very slowly. Thus the non-BSS and medium effects are important for $\Gamma_R$ at higher $E_{\text{in}}$ around 250 MeV/A. At lower $E_{\text{in}}$ from 50 to 80 MeV/A, meanwhile, the medium and breakup effects are nearly canceled, so that the non-BSS effect becomes most significant for $\Gamma_R$. Since the non-BSS effect is smaller at lower $E_{\text{in}}$, we can conclude that lower-incident energy scattering are a good probe of evaluating $\Gamma_{\text{rels}}$ from $\sigma_R$.

As mentioned above, the non-BSS effect becomes large as $E_{\text{in}}$. This can be understood as follows. In the high $E_{\text{in}}$ where the eikonal approximation is valid, $\sigma_R$ is proportional to the volume integral of the imaginary part $\langle \varphi_0 | W | \varphi_0 \rangle$ of
where $\langle \varphi_0 | U | \varphi_0 \rangle$ \cite{14,24}, since

$$
\sigma_R = \int d^2b [1 - |\langle \varphi_0 | S | \varphi_0 \rangle|^2] 
= -\frac{2}{\hbar v_0} \int d^3R \langle \varphi_0 | W | \varphi_0 \rangle
$$

(10)

with

$$
S = \exp \left[ -\frac{i}{\hbar v_0} \int_{-\infty}^{\infty} dZU \right],
$$

(11)

where $v_0$ is the incident velocity of P and $R = (b, Z)$. Equation (10) shows that $\sigma_R(A + 2) - \sigma_R(A) = 2(\sigma_R(A + 1) - \sigma_R(A))$ and hence $\Gamma_R = 0$.

**Summary.** The present microscopic version of three- and four-body CDCC calculations reproduces $\sigma_R$ for $^{14,15,16}$C$^{4,12}$C scattering at 83 MeV/A. The projectile-breakup effect is significant for $^{15}$C scattering and appreciable for $^{16}$C scattering, whereas the nuclear-medium effect is sizable for all the $^{14,15,16}$C scattering. In general, the $\sigma_R$-staggering $\Gamma_R$ is deviated from the radius-staggering $\Gamma_{\text{rds}}$ by the non-BSS, nuclear-medium and projectile-breakup effects. At lower $E_{\text{in}}$ from 50 to 80 MeV/A, the breakup and medium effects are nearly canceled and the remaining non-BSS effect is rather small for $\Gamma_R$. Therefore, the lower-$E_{\text{in}}$ scattering are a good probe of evaluating $\Gamma_{\text{rds}}$ from $\sigma_R$. At high $E_{\text{in}}$, meanwhile, the non-BSS effect is significant, whereas the nuclear-medium and projectile-breakup effects are small or negligible. The non-BSS effect largely reduces $\Gamma_R$ from $\Gamma_{\text{rds}}$. Thus the radius-staggering $\Gamma_{\text{rds}}$ is masked by the non-BSS effect at high $E_{\text{in}}$. This means that if experimental data show a large value of $\Gamma_R$, the corresponding radius-staggering $\Gamma_{\text{rds}}$ is even large. A good example is the $\sigma_R$-staggering for $^{30,31,32}$Ne scattering at 250 MeV/A. Thus $\Gamma_R$ is a good quantity to find exotic properties of unstable nuclei.

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