Ground state Pseudo-scalar Meson masses in the Separation Geometry model.

Abstract

In this paper the techniques of discretised QCD developed in [physics/0109024] are extended to a study of ground-state pseudo-scalar mesons involving higher generation quarks. The correspondence with empirical values is good with the exception of the charmed-bottom meson with the predicted value of 6.911GeV varying considerably from the empirical value of $6.4 \pm 0.39 \pm 0.13$ although it is still within two sigma and provides an avenue for further testing of the theory.

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1 Introduction

In a previous paper a calculation methodology was developed to perform discretised QCD mass calculations. Calculation of the nucleon masses was performed as an example of the technique.

The purpose of this paper is to extend the calculation technique developed in the previous paper to cover quarks of generation higher than the first. In this paper we will examine a selection of $J = 0$ ground state mesons. To do so requires additional hypotheses above and beyond those presented in the previous paper; the validity of which once again depends upon the empirical success (or failure!) of the methodology. We begin by re-examining some of the basic geometric concepts from the previous paper in an abbreviated form.

2 Basic ideas.

Consider the symmetry of a coloured cube;
where the cube has been given some colour; two opposite faces red, two green and two blue. In this example the observed colour of the cube is a function of the position of the cube (or equivalently of the observer!); in the language of contemporary physics it is ‘gauge dependent’ (although ‘phase dependent’ would, of course, be a better description). Indeed, if we imagined that the rotational symmetries of the cube manifested as continuous rotations in the embedding space then the observed colour would indeed be a function of the local phase of the generators of the rotations. Alternatively we could consider that the colour axes are somehow fixed in an ‘internal’ space defined by geometry generators and that as the cube rotates there is a continuous change in the spectrum of colours seen by the observer. The colour itself would then be described by a continuous local phase symmetry.

This is the basic idea behind discretised QCD; that quark ‘colour’ may be represented by a special kind of ‘gauged’ cubic symmetry. In standard theory the ‘colour’ of a quark is not an observable. The ‘colour’ of a quark is like phase in electromagnetism theory; local gauge invariance means that the colour is not an observable just as the local phase of an electron wave-function is not an observable.

In standard QED a phase factor that varies locally (i.e. varies with spatial position) is not invariant in the absence of a gauge field because;

\[
\partial_x(e^{i\theta(x)}\psi) = i\partial_x e^{i\theta(x)}\psi + e^{i\theta(x)} \partial_x \psi
\]

and so, because the first derivative of the field enters into the Lagrangian, the Lagrangian will not be invariant under a local phase transformation. In QED if we add the photon gauge field, however, local phase invariance of the Lagrangian is restored.

In the case of quarks the local phase information is more complex than a simple \( U(1) \) \( e^{i\theta(x)} \) and is governed by \( SU(3)_c \) symmetry but the principle is the same. However, the gauge particles of QCD, the gluons, unlike the photon of QED, carry the charge of the underlying field; the gluon carry ‘colour’ charge. Moreover, instead of the single photon the gluons come in eight ‘colour’ varieties. There difference are a consequence of the non-abelian nature of \( SU(3) \) and the fact that there are eight generators instead of a single \( U(1) \) generator.

In discretised QCD the ‘local phase’ information for \( SU(3)_c \) is carried by a three-dimensional ‘cube’ embedded in a four-dimensional space-time; which is to say that the time dimension is \textit{undefinable} inside the boundary of the cube. The discrete phase of the cube - the spatial
position of its faces, edges and vertices, and the length of the edges, area of square faces and three-dimensional volume of the cube - represents colour (and other local gauge) information which is rendered *unobservable* by the gauge fields; the geometry of the colour gauge fields is postulated to be in the form of two-dimensional ‘flat-square’ geometries analogous to the square faces of the cube. Thus the geometry of a gluon in this schema is that of a ‘flat square’ and it contains colour information relative to the surface of the cube from which it is emitted and the surface to which it is absorbed in the time-containing embedding space. There are fundamental theorems in the theory which tell us that any two-dimensional geometry will always have zero rest-mass so that the ‘flat-square’ geometries will always propagate at the speed of light. Conversely, a three-dimensional geometry such a ‘cubic’ quark will always have non-zero rest mass.

With local gauge invariance no geometric feature of the ‘cube’ structure itself will be directly observable as such - we might consider it a snapshot ‘image’ of a quark at an instant in time but to observe it over an interval of time t=0 requires, by the time-energy uncertainty relation, an infinite energy probe which of course is not possible. Can we indirectly infer the existence of such an underlying symmetry for quark structure?

Well, we might try and see if some of the physical properties of quarks emerge from a study of such discrete symmetry. The property which seems to correlate is particle mass and in a previous paper [3] this idea was explored. Discrete subgroups of SU(3) can be extracted which can be mapped onto the cubic symmetry - rather like a collection of discrete ‘snapshot’ images of a continuously rotating cube and it is found that the order of these groups (i.e. the number of elements in the group) are related to the rest mass. These discrete sub-groups have the same basic symmetry properties of the parent group but in a discrete form - so, for example, we might have three fixed discrete colours for our cube rather than a continuous spectrum as the generators of the geometry act. In the previous paper the issue of how an extended object such as a ‘cube’ may be a representation for an object, such as a quark, which empirically behaves like a point was discussed so that issue will not be revisited here.

A brief summary of the ideas from [3] is as follows.

Material particles such as quarks and leptons are assumed to be represented by a special kind of geometry termed ‘affine-set’ geometry. The nature of affine-sets in this schema is related to a decomposition of the structure of the continuum of space-time. Local phase factors - U(1), S.U.(2) or S.U.(3) - are assumed to be due to this geometry and local gauge invariance is assumed to be the mechanism whereby this local phase is rendered unobservable. Geometrically this becomes equivalent to making the underlying decomposition of the continuum - which is present on the boundary of the geometry - unobservable. The gauge fields themselves also relate to the geometry of the boundary.

The mass of any fundamental object is then a relic of an otherwise empirically unobservable geometric foundation to physical structure. The fundamental symmetry of an affine-set geometry is a permutation symmetry and it is the number of matrix elements representing this symmetry (always discrete) - minus any massless generators and modulo any radiative corrections from the gauge fields - which in general defines the mass; although the situation becomes quite complex with hadrons.
Conventionally to calculate a hadron mass, such as a spin-zero meson, one would postulate a form for the potential, insert assumed current quark masses and then try and determine the (quantum) ground state energy to define the mass. Now whilst this approach may be completely valid, and may be necessary for calculating excited states, in discretised QCD we assume that the discrete symmetry dictates the form and value that the potential and current quark masses can take in the ground state by dictating the total energy, current quark masses and all the symmetry features that the potential must satisfy. To calculate the mass one then determines the appropriate components of energy from the symmetry and then sums them to obtain a mass; which is a much simpler approach than working from an assumed potential - particularly if the current quark mass is significantly different to the constituent mass implying that the quarks in the hadron are very relativistic even in the ground state. Constituent and current quark mass components are readily calculated in the discretised QCD approach without the requirement for knowledge about the velocity of the quarks or relativistic corrections. The energy of the quarks is simply dictated by the symmetry and can be written down from the matrix representation.

The calculation of the mass of a ground state meson is performed by summing the mass/energy of several different component parts. These parts are:

1; constituent quark energy - this is equivalent to the kinetic energy of the quarks; always taken ‘on-shell’ at $\alpha_s = 1$ (‘on-shell’ here means a particular energy scale, called the $T_r$ scale, which is dictated by the symmetry),

2; constituent gluon energy - due to the energy of the gluons exchanged by the quarks,

3; current quark mass -which is the self-energy of individual quarks and consists of the following three sub-components:

a; energy equivalence of current quark operator content,
b; Higgs scalar components and
c; potential energy of separation of current quarks.

Each of these parts must be separately calculated and, after application of radiative corrections, summed to obtain a final mass. We begin by revising the matrix representations for the different components.

3 Matrix representations 1; the $T_r$ group

$T_r$ is the unique sub-group of SU(2) with 24 elements and is the S.U.(2) analogue of the tetrahedral point symmetry (spinor) three-dimensional group $T_d$. $T_r$ has the following fundamental representation;
\[ \begin{array}{cccc}
1 & 2 & 3 & 4 \\
\alpha : & (1 \ 0) & (-1 \ 0) & (0 \ i) & (0 \ -i) \\
\beta : & (i \ 0) & (-i \ 0) & (0 \ -1) & (0 \ 1) \\
\gamma : & (ia \ -b) & (-ia \ b) & (-ia \ -b) & (ia \ b) \\
\delta : & (b \ ia) & (-b \ ia) & (b \ -ia) & (-b \ ia) \\
\epsilon : & (a \ ib) & (-a \ -ib) & (-a \ ib) & (a \ -ib) \\
\phi : & (ib \ a) & (-ib \ -a) & (ib \ -a) & (-ib \ a) \\
\end{array} \]

where \( a = \frac{1}{2} \) and \( b = \frac{\sqrt{3}}{2} \). The elements of these matrices are built up from the roots of unity; square roots, fourth roots and sixth roots. These elements close to form a group with 24 elements which we shall designate \( T_r \). The generators of the group may be taken as the \( \gamma_4 \) and \( \delta_4 \) matrices. Note that, although it is a sub-group of the continuous version \( SU(2) \), \( T_r \) has only two, instead of three, generators.

\( T_r \) group is a kind of discrete spinor group. It is analogous to the geometric group \( T_d \) familiar to physical chemists which is the group of the tetrahedron in three-dimensions with 24 elements and 2 generators. \( T_d \) is a point space group in three dimensions familiar to physical chemists which is spinorial but is not isomorphic to \( T_r \).

The mass-equivalence of \( T_r \) is defined as \( R.(4! - 2) \) where \( (4! - 2) \) is the number of non-generator elements in the group and \( R \) incorporates any radiative corrections due to the elevation of the cardinality of the boundary of the geometry to the continuum by the gauge fields. In the transition to a field theory analogue the discrete pair of \( T_r \) generators are assumed to manifest as continuous (massless) intrinsic spin generators and this is the source of the \(-2\) in the above definition; \( 4! \) is of course simply the order of the \( T_r \) group.

4 Matrix representations 2; embedding \( T_r \) in \( SU(3) \)

Quarks are assumed to have a cubic symmetry in affine-set geometry in discretised QCD. Just as with the analogy between \( T_d \) and \( T_r \) we wish to find a way of embedding a point-symmetry representation of a cube in a complex space.
We can parallel the root structure of SU(3) by embedding three copies of SU(2) in SU(3). We can embed three copies of the $T_r \{\lambda_{ij}\}$ group as follows to form a discrete sub-group of SU(3);

$$r = \begin{pmatrix}
1 & 0 & 0 \\
0 & \lambda_{11} & \lambda_{12} \\
0 & \lambda_{21} & \lambda_{22}
\end{pmatrix};
\quad g = \begin{pmatrix}
\lambda_{11} & 0 & \lambda_{12} \\
0 & 1 & 0 \\
\lambda_{21} & 0 & \lambda_{22}
\end{pmatrix};
\quad b = \begin{pmatrix}
\lambda_{11} & \lambda_{12} & 0 \\
\lambda_{21} & \lambda_{22} & 0 \\
0 & 0 & 1
\end{pmatrix}$$

and we also form the following ‘colour-dual’ embeddings;

$$\tilde{r} = \begin{pmatrix}
-1 & 0 & 0 \\
0 & i\lambda_{11} & i\lambda_{12} \\
0 & i\lambda_{21} & i\lambda_{22}
\end{pmatrix};
\quad \tilde{g} = \begin{pmatrix}
i\lambda_{11} & 0 & i\lambda_{12} \\
0 & -1 & 0 \\
i\lambda_{21} & 0 & i\lambda_{22}
\end{pmatrix};
\quad \tilde{b} = \begin{pmatrix}
i\lambda_{11} & i\lambda_{12} & 0 \\
i\lambda_{21} & i\lambda_{22} & 0 \\
0 & 0 & -1
\end{pmatrix}$$

The three ‘colour’ matrices $r, g$ and $b$ plus the ‘colour-dual’ basis $\tilde{r}, \tilde{g}$ and $\tilde{b}$ together form a discrete version of the group of SU(3)$_c$. Also notice that each set of colour matrices forms a group with 24 elements. The ‘dual’ or ‘colour-bar’ matrices alone do not form a group since the product of any two colour-bar matrices is a colour matrix. However, the combination of $c_i + \tilde{c}_i$ (for colour index $i$) forms a group with 48 elements and two generators located in the colour-bar set of elements. Let us call this group $T_c$. It is also a ‘discrete spinor’ analogous to the $T_r$ group;

$$(i\gamma_4, i\delta_4)^2 = -I_2$$

Note that the colour-dual is not the same as the Hermitian conjugate matrix.

We might propose $T_c$ as a model for a Skyrmion field of charge 4; i.e. one with cubic symmetry. A corresponding point symmetry (spinor) discrete group would be $O_h$ which has 48 elements and two generators but, again like the $T_d \to T_r$ analogy, is not isomorphic to $T_c$. We will see later how the geometric analogy works but to obtain a picture think of the three colours as represented by the three orthogonal pairs of opposite faces of the cube; one pair of faces representing each of the three possible $T_{ci}$ groups with the four vertices of each of the two square faces representing each tetrahedral sub-group. To represent the total geometry we form a ‘particle vector’ which is composed of three components. First form a matrix representation of $T_{ci}$ as follows;

$$R = \begin{pmatrix}
\tilde{r} & 0 \\
0 & \tilde{r}
\end{pmatrix},
G = \begin{pmatrix}
g & 0 \\
0 & g
\end{pmatrix},
B = \begin{pmatrix}
b & 0 \\
0 & \tilde{b}
\end{pmatrix}$$

and then form the three-component particle vector $\vec{P} = (R, G, B)$. (Of course the order of the components here is arbitrary).

Now define the following ‘current-quark’ operators;

$$q_r = \begin{pmatrix}
-1 & 0 & 0 \\
0 & i & 0 \\
0 & 0 & i
\end{pmatrix},
q_g = \begin{pmatrix}
i & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & i
\end{pmatrix},
q_b = \begin{pmatrix}
i & 0 & 0 \\
0 & i & 0 \\
0 & 0 & -1
\end{pmatrix}$$
Lastly we require what is referred to in the text as a ‘script identity’ $\mathcal{I}$. It carries a colour index:

\[
\mathcal{I}_r = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \mathcal{I}_g = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \mathcal{I}_b = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\] (8)

The purpose of these constructions will become apparent as follows. We form operators from the $q_i$’s and $\mathcal{I}$’s as follows (the following examples are red format but the other colours follow suit):

\[
D_r = U_r = \begin{pmatrix} \mathcal{I}_r & 0 \\ 0 & I_3 \end{pmatrix} \begin{pmatrix} q^*_r \\ 0 \end{pmatrix}, \quad
D_g = U_g = \begin{pmatrix} q^*_g \\ 0 \end{pmatrix} \begin{pmatrix} \mathcal{I}_g & 0 \\ 0 & I_3 \end{pmatrix}, \quad
D_b = U_b = \begin{pmatrix} q^*_b \\ 0 \end{pmatrix} \begin{pmatrix} \mathcal{I}_b & 0 \\ 0 & I_3 \end{pmatrix}
\] (9)

where each operator has three components represented above in column format.

Two things need mentioning at this stage; 1; the quark operators $q_i$ and $\mathcal{I}_i$ are assumed confined to their respective colour space i; that is, the $q_r$ operators only operate on the red particle vector matrix etc, and 2; the order of the application of the operators is of no significance because they are diagonal matrices.

These ‘operators’ will represent the current quark content of a hadron and the ‘particle vector’ will determine the constituent mass/energy.

5 Matrix representations 3; glue

Discretised gluon operators are represented as follows;

\[
b \begin{pmatrix} I_3 \\ 0 \end{pmatrix} q^* g^* \begin{pmatrix} g \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ \bar{g} \end{pmatrix} = \begin{pmatrix} b \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ \bar{b} \end{pmatrix}
\] (10)

where * represents the complex conjugate. Similar upper product operators are defined for example;

\[
r \begin{pmatrix} q^* q^*_g \\ 0 \end{pmatrix} \begin{pmatrix} g \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ \bar{g} \end{pmatrix} = \begin{pmatrix} \bar{r} \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ r \end{pmatrix}
\] (11)

These are then combined to form operators which operate on the particle vector $\vec{P}$ for
example;

\[
\begin{pmatrix}
g \left( I_3 \quad 0 \quad q_g q_r^* \right) r^* \\
r \left( I_3 \quad 0 \quad q_r q_g^* \right) g^* \\
\left( I_3 \quad 0 \quad 0 \right)q
\end{pmatrix}
\]

(12)

when applied to the particle vector \( \vec{P} = (R, G, B) \) produces \((G, R, B)\) i.e. this is a R-G gluon.

6 Matrix representations 4; current Quarks.

The current quarks are built up from the quark operators used previously to define the particle types. The current quark masses are the corresponding masses of these operators plus an extra piece whose origin may be the Higgs field but which can be interpreted in terms of a potential of separation between current quarks.

We can split up the operators into two pieces; which is convenient for calculations. Each current quark operator consists of a vector composed of 3 blocks \( 6 \times 6 \) matrices. Each \( 6 \times 6 \) block is composed of two \( 3 \times 3 \) blocks on the diagonal. We separate out the upper \( 3 \times 3 \) blocks as one set of matrices and the lower \( 3 \times 3 \) set of blocks as another and represent them as follows (the representation below is for the proton as the reader can confirm by applying the operators to the 3-quark (first generation only) column particle vector \( \vec{P} = (R, G, B) \)) remembering that the red operators operate only on the red \( P \) components etc.;

\[
\text{strong component } = \begin{cases} 
\text{red } & \left( \begin{array}{ccc} I & q^* & I \\
q^* & I & I \\
q^* & q^* & q^* \end{array} \right) \\
\text{green } & \left( \begin{array}{ccc} I & q^* & I \\
q^* & I & I \\
q^* & q^* & q \end{array} \right) \\
\text{blue } & \left( \begin{array}{ccc} I & q^* & I \\
q^* & I & I \\
q^* & q^* & q \end{array} \right) 
\end{cases}
\]

(13)

The rows of these matrices are the three colours (order red, green and blue in the notation used in this paper from top down) and the columns are the type of quark; the left hand and central column of each matrix above is an up-type quark and the right hand column is a down-type quark. The up-type quarks have their colour defined by the colour of the single identity they carry. Thus for example the first column of these matrices represents an up red quark. The single \( q \) or \( q^* \) in the down defines its colour so that the last column above is a blue down quark in each case.

The strong components couples to the \( C_i \) colour matrix of the particle vector which does not contain any generators. The E.M. component couples to the \( \bar{C}_i \) matrix with two
generators. (This coupling is required to properly define the charge characteristic of the particle in the given representation). The order of a $C$ matrix is 24 elements. The order of a $\bar{C}$ matrix is $(4!-2)$ elements where, as in the case of the $T_r$ group, the generators are assumed to be massless intrinsic spin generators in the transition to a continuum field theory.

The masses of the current quark operators are defined in terms of the matrices to which they couple. The exception to the rule applies to the identities in the E.M. component which are spinless and of order $4!$ (i.e. the generators of $\bar{C}$ are massive under the identity components of the current quarks - the I or the $I$ components). The identities in the strong component act like ‘strong charges’ and carry a relative mass sign. For example, in the proton mass listed the two identities cancel the two script identities leaving only the net 5 $q^*$’s to contribute to the ‘strong’ matrix mass.

7 The Higgs sector

This is the least satisfactory part of the theory but some progress can be made with a combination of a few simple suppositions and comparison with empirical data. There are some interesting hints of new physics in the discretised model of the Higgs sector and in what follows we shall mention these in passing. We divert initially to study the vector gauge boson masses because these provide some support for what follows with the fermions.

There seems to be a general rule in affine-set geometry relating the the maximal massless subgroup of a geometry to possible unitary symmetries in the transition to a field theory although we have no formal mathematical proof of the connection. Let us look at this in terms of the $T_r$ tetrahedral group. This is the group which in the previous paper was used to underpin lepton (and indirectly viz the $T_c$ analogue also quark) structure. The permutation group corresponding to $T_r$ is $S_4$ - the characteristic symmetry of a four-point affine-set geometry.

If we form a mental picture of a tetrahedron we note that it has four 3-vertex subsets forming four triangular faces. To see that there is an orbit of the $T_r$ group with three elements note that;

$$(\epsilon_2)^3 = I_2$$

where $\epsilon_2$ is one specific matrix listed in the table of $T_r$ group matrices. Here $\epsilon_2$ is the analogue of a $C_3$ rotation in a three-dimensional tetrahedron described by the group $T_d$ familiar to physical chemists. The continuous analogue of this will be a U(1) rotation in a two-dimensional plane. The stabiliser of this orbit is the single remaining point of the set of four defining the geometry and this sets the orientation of the associated U(1) symmetry axis. The affine-set tetrahedron in the previous paper was associated with charged lepton masses and the three-point sub-group associated with the photon geometry. That is, the photon geometry is the maximal massless subgroup of the $S_4$ tetrahedral group and the single stabiliser point dictates the associated symmetry as U(1).

Without proof we may seek to generalise this result to scalar geometries. Scalar affine-set geometries have the peculiar property that any sub-set of points always defines a permutation...
group. There will always be a maximal massless set which will form an orbit with the remaining points forming the associated stabiliser. We postulate that this stabiliser, when part of a scalar set of points, will always define a unitary symmetry as a function of the number of points in the stabiliser. We express this more precisely as follows;

Hypothesis: if a scalar affine-set geometry of order $S_n$ exists then its’ corresponding unitary symmetry group is at most $SU(n-m)$ where the index $(n-m)$ is the number of points of the affine-set geometry present in the stabiliser of the orbit of the maximal massless subgroup containing $m$ points and $(n-m)$ is a prime number; or, if $(n-m)$ is not a prime number, then the geometry defines a product of unitary groups of prime index whose sum of indices is $(n-m)$.

For $S_8$ (fermionic!)cubic symmetry it is interesting to note that, although we have previously seen that it is a spinor geometry not a scalar, in the case of the strong interaction in colour space the gluon geometry is a ‘flat’ (two-dimensional and hence massless; see [8] for more detail) 4-point geometry which can represent a single square face of the cube. In colour space this is the maximal massless subgroup of degree $S_4$. Hence $m=4$, $n=8$ and the decomposition is either to $SU(2)xSU(2)$ or $SU(3)xU(1)$; the latter of these two prevails empirically - presumably because the $U(1)$ subgroup is specified by the requirement that the quarks are electro-magnetically charged.

Now, for the standard model we have $SU(3)xSU(2)xU(1)$ so that $m=6$. What is the appropriate value of $n$? In affine-set theory all massive fermions decompose to $T_r$ ($S_4$) equivalent units (for example, in calculating the mass of a cubic quark in previous sections the cube was decomposed into $T_r$ units which were then summed). In the absence of the Higgs field, standard model fermions are massless so the obvious choice is to take $n-m=4$; that is, setting the fermions as massless and representing the maximal massless subgroup - so that $n=10$ or an $S_{10}$ scalar affine-set geometry with 10! elements represents the standard model prior to spontaneous symmetry breaking. Now, when we give the fermions mass there will be a shift in the maximal massless sub-group from $n-m=4$ to $n-m=3$ where the triangle $S_3$ symmetry is the photon group. (Note that the persistence of a massless object of order $S_4$ - gluons - is unique to colour space and not a $T_r$ (spinor) representation; the issue of the masses of neutrinos is more complex - in the case at hand we are only concerned with the usual electro-weak symmetry breaking problem of ensuring that the photon is massless but the standard model charged fermions gain mass). This shift in maximal massless subgroup may be the affine-set theory analogue of spontaneous symmetry breaking and can be analysed in discrete theory in the following way.

A factorial number has a decomposition into factorial sub-groups;

$$n! = \sum_{k=1}^{n-1} k.k! + 0!$$  \hspace{1cm} (14)

Symmetry breaking will demand a decomposition of the $S_{10}$ symmetry to sub-groups of order no more than $S_9$. (This is because the standard model has $n=6$ and for the photon...
m=3). Now, a scalar affine set can be decomposed in the pattern of (14) because, in the absence of any spin generators, every subset of points defines a subgroup which is always a permutation group. The symmetry breaking can be effected by discarding the massless pieces of order $S_3$ (or less), which gives a minimal decomposition into exactly six massive components (remember we have discarded the massless pieces);

$$S_{10} \rightarrow 9.S_9 + 8.S_8 + 7.S_7 + 6.S_6 + 5.S_5 + 4.S_4$$ (15)

The photon is now the maximal massless geometry of order $S_3$ and the possible unitary symmetries by hypothesis 9 are (neglecting the possibility of multiple U(1) subgroups and allowing only groups of prime index);

A :

$$S_9 \rightarrow SU(3) \times SU(3) \text{ or } SU(3) \times SU(2) \times U(1) \text{ or } SU(2) \times SU(2) \times SU(2)$$

B :

$$S_8 \rightarrow SU(5)$$

C :

$$S_7 \rightarrow SU(2) \times SU(2) \text{ or } SU(3) \times U(1)$$

D :

$$S_6 \rightarrow SU(3)$$

E :

$$S_5 \rightarrow SU(2)$$

F :

$$S_4 \rightarrow U(1)$$

It appears that a decomposition into a product of unitary groups requires a gauged symmetry and that the scalar Higgs components are not gaugable as such in their couplings to the fermions. The physical explanation for this is that the scalar Higgs components for the fermions ‘live’ in the interior space of the geometry (e.g. ‘inside’ the cube for a quark) and that this space does not contain the time dimension and cannot be gauged for this reason. Comparison with empirical data shows that, for the Higgs field - fermion mass couplings, possibilities A and C do not occur whilst B,D,E and F appear to manifest physically. (Possibility A seems to contains the standard model decomposition for the non-Higgs gauged sector but this was by construction; see above). It is particularly interesting that, as we shall see, the empirical correlation with possibility B is good which may be the first empirical suggestion of the existence of an $SU(5)$ symmetry in nature.

The situation is complicated because of the requirement for a propagating Higgs field to render the electro-weak theory unitary. This requirement is specifically in relation to the vector gauge bosons which require the Higgs field in their radiative corrections to preserve unitarity. This is not possible if the Higgs components are ‘buried outside time’ as it were in the ‘interior’ timeless space of three-dimensional affine geometries representing the massive vector gauge bosons! Indeed, as we shall see, comparison with empirical data does indeed show that the Higgs propagates in relation to the vector gauge bosons which we will now study in some detail.

We know that, for completeness and consistency, we require a model for the massive vector gauge bosons. $T_r$ is a spinor group and unsuitable but there is a three-dimensional point symmetry group $T$ which has only vector generators. $T$ has 10 non-generator elements and 2 generators but 12 elements do not represent an $S_n$ symmetry of an affine set ($S_3$ for example has 6 elements whilst $S_4$ has 24 elements). However, if we discard the massless photon orbit from the scalar $S_{10}$ the $S_4$ scalar subgroup which results will acquire mass
through a decomposition of the Higgs field. This decomposition is an inevitable consequence of affine sets. This could be used to couple to a four-point \textit{bosonic} \( \mathcal{T} \) geometry to give it an affine symmetry (i.e. re-define its' elements as a permutation group). Thus we might then try to couple this scalar quantity to a \( \mathcal{T} \) geometry to define it by chopping it in half to mirror the fact that \( \mathcal{T} \) has 12 instead of the \( S_4 = 4! \) elements of the affine set of a four-point (spinor) geometry. This seems to work. The other mass components we must assume couple to the geometry rather like 'lumps of lead' and are not distinctly related to the \( \mathcal{T} \) morphology - although comparison with empirical data shows that this does not apply to the (scalar not fermionic!) \( S_8 \) sub-component which is also chopped in half. One may account for this as relating to a coupling to a '\( \mathcal{T} \)'bosonic-version of the quark \( T_c \) geometry. Empirically this gives a very good fit to the data i.e. with the singular assumption that there is mixing, which is maximal, at the \( S_8 \) (quark) level and also at the \( S_4 \) (lepton) level; which intuitively is physically sensible. The way we do it is, given that the number of elements of \( \mathcal{T} \) is half that of \( T_r \), we assume \( \frac{1}{2} \cdot 4 \cdot S_4 \) may be regarded as defining 4 units of (bosonic) \( \mathcal{T} \) as distinct from 2 units of (fermionic) \( T_r \). Similarly, we may postulate a bosonic point symmetry analogue of the fermionic \( T_c \) group with half the number of elements in each \( T_r \) sub-group analogue (i.e. \( C_i \) and \( \bar{C}_i \)) and representing the point group \( O \) of the cube with vector generators. Comparison with empirical data gives the mixing represented as;

\[
S_{10} \rightarrow 9 \cdot S_9 + 8 \cdot S_8 + 7 \cdot S_7 + 6 \cdot S_6 + 5 \cdot S_5 + 4 \cdot S_4
\]

| Boson | \( 9 \cdot S_9 \) | \( 8 \cdot S_8 \) | \( 7 \cdot S_7 \) | \( 6 \cdot S_6 \) | \( 5 \cdot S_5 \) | \( 4 \cdot S_4 \) |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( W^+ \) | 1               | \( \frac{1}{2} \) | 1               | 1               | 1               | \( \frac{1}{2} \) |
| \( W^- \) | 1               | \( \frac{1}{2} \) | 1               | 1               | 1               | \( \frac{1}{2} \) |
| \( Z^0 \) | 1               | 2               | 1               | 1               | 1               | 2               |

Here each boson is represented by an \( S_{10} \) permutation symmetry based on a permutation of the ten non-generator elements of the discrete (bosonic) group \( \mathcal{T} \). The decomposition results from rendering the photon massless which pulls out the \( S_3 \) subgroups leaving only the massive pieces of order \( S_4 \) or greater. Notice how mass components have shifted from the \( W^\pm \) to the \( Z^0 \) with 50% of the charged boson \( S_8 \) component shifting across from the \( W^\pm \) to the \( Z^0 \). The resulting electro-weak mixing angle can be precisely calculated but it is interesting to note that, in spite of the value of the angle, it this theory it results from maximal mixing! We proceed with a calculation of the predicted masses of the bosons from the above table.

The \( W \) bosons acquire radiative corrections to their mass. The standard affine-set geometry correction is applied; it is non-perturbative (see \cite{3}); \( \mathcal{R} = 1 + \alpha (q^2 - M^2_W) \) and the shifted (mixed into the \( Z^0 \)) components keep their radiative corrections. Here the Higgs components acquire radiative corrections; unlike the case for the fermions the Higgs field propagates in space-time in the case of the mass components of the \( W \) bosons (necessary for
unitarity) and those components mixed from the $W$’s into the $Z$. Comparison with physical data shows that the Higgs components native to the $Z^0$ do not propagate in the theory and do not acquire radiative corrections! The physical implications of this are quite astonishing. (It implies, for example, that spin statistics and space-time structure are not independent and that electro-magnetism has an interpretation in terms of space-time structure - this arises because of the double whammy implied by the data correspondence; to a high degree of precision the data correspondence with theory shows that the Higgs components do not propagate with respect to the fermions (they are buried inside the geometry and outside the time dimension). This is represented by the lack of radiative correction to the mass components of the fermions derived directly from the Higgs sector. Only for the charged fundamental (i.e. no fermionic sub-structure) bosons does the Higgs appear to propagate in space-time; but only in respect to the fundamental boson carrying electro-magnetic charge. We must speculate that the mixed component from the $W$’s to the $Z$ are sufficient to confer unitarity on the $Z$. The radical interpretation, which I favour, is that the Higgs changes the statistics of space-time; that is, it acts as a supersymmetry operator on space-time structure (as distinct from supersymmetry in the particle spectrum) and generates electro-magnetic charge in the process. If this is true we would not expect to find a ‘free’ Higgs floating around in a particle accelerator because its’ existence would violate charge conservation even though it is electrically neutral!

Now, $\alpha_{(q^2=M_W^2)} \approx 128^{-1}$ and the calculation proceeds by summing the number of matrix elements of the groups in question (the matrix order will be equal to the number of group elements). Calculation from the above table gives the $W$ mass as;

$$W_M^\pm = (1 + \alpha_{q=M_W^2}).3467400$$

and the $Z^0$ mass as;

$$Z_M^0 = 3628776 + (1 + \alpha_{q=M_W^2}).322656$$

where the numbers are obtained by summing over the number of elements in the respective groups. (For example, $9S_9$ has $9.9!\approx3265920$ elements). These numbers are converted to MeV by using the mass equivalence of the $T_r$ unit which is exactly equal to the electron rest mass; $T_r = (1 + \alpha_{q^2=m_e^2} + G_{f})(4! - 2) \approx 0.511MeV$ and $G_{f}$ is a dimensionless number which reflects the relative strength of the weak interaction at low energy; we may take $(1 + \alpha_{q^2=m_e^2} + G_{f}) \approx 1.0073115$. One then readily obtains;

$$M_W = 80.578GeV(80.41(0.10))$$

and

$$M_Z = 91.173GeV(91.187(0.007))$$

and

$$sin\theta_w =$$

This provides empirical support for the claim that the breakdown components of the $S_{10}$ symmetry listed previously do indeed represent the Higgs field.
If a 3 ‘units’ of $S_{10}$ Higgs couples to the vector gauge bosons which represent the adjoint rep. of S.U.(2) then we might expect a 2 units of ‘Higgs’ to specify the fermion masses - at least this is the supposition we will make. As before requiring a vacuum structure with massless photons (and gravitons ; also ignoring for the moment the existence of the massless gluon which we must treat as somehow separate to spontaneous symmetry breaking) irreversibly breaks down the $S_{10}$ scalar into six components. If we start with two $S_{10}$ units this means that in the quark sector a maximum of six objects can acquire mass and in the lepton sector another six objects can acquire mass; the six masses in each case will have a hierarchy based on eq.(15). In the situation of simplest symmetry the tau neutrino has the same mass as the top-quark etc. as per the following table;

| Fermion | 9.$S_9$ | 8.$S_8$ | 7.$S_7$ | 6.$S_6$ | 5.$S_5$ | 4.$S_4$ |
|---------|--------|--------|--------|--------|--------|--------|
| top     | 1      | 0      | 0      | 0      | 0      | 0      |
| bottom  | 0      | 1      | 0      | 0      | 0      | 0      |
| charm   | 0      | 0      | 1      | 0      | 0      | 0      |
| strange | 0      | 0      | 0      | 1      | 0      | 0      |
| up      | 0      | 0      | 0      | 0      | 1      | 0      |
| down    | 0      | 0      | 0      | 0      | 0      | 1      |
| $\nu_\tau$ | 1      | 0      | 0      | 0      | 0      | 0      |
| $\tau$  | 0      | 1      | 0      | 0      | 0      | 0      |
| $\nu_\mu$ | 0      | 0      | 1      | 0      | 0      | 0      |
| $\mu$   | 0      | 0      | 0      | 1      | 0      | 0      |
| $\nu_e$ | 0      | 0      | 0      | 0      | 1      | 0      |
| $e$     | 0      | 0      | 0      | 0      | 0      | 1      |

where one ‘Higgs’ unit of mass has been used to generate the quark masses and one to generate the lepton masses as per eq.(14). (Of course we would need another two ‘Higgs’ if we wanted to also give mass to equivalent sets of anti-particles). However, the geometry of the neutrinos in the theory is ‘flat’ which means they can only have masses generated radiatively; they cannot acquire ‘volume’ masses which are characteristic of the the Higgs components (a scalar affine-set geometry of four or more points will always define at least a three-dimensional space. Thus the above table is badly broken. We may assume that mass components cannot be shifted ‘horizontally’ between columns because to do so would require another round of symmetry breaking to strip-off more massless sub-units. Clearly all the mass from the listed $\nu_\tau$ will appear in the top (it has nowhere else to go). Using a number of different inputs including theoretical and empirical (for example the known lepton masses have been used to help construct the following table) it is possible to determine the correct form of the quark-lepton mass mixing matrix;
The first thing to notice is that, no sooner had we argued that components cannot be shifted ‘horizontally’ than a $5. S_5$ disappears from the up/$\nu_e$ column entries. We will modify the accounting later to bring the ‘books back into balance’. We might have expected a ‘1’ at the up on-diagonal entry; and indeed in the charged pions appear to acquire an $S_5$ mass component as an SU(2) singlet (see the calculation of the charged pion mass given in subsequent sections) but the mixing of the components of the first generation quarks and leptons is a little complicated and not well understood but we will look at it more closely later. For the moment simply regard the above table as a semi-empirical analysis.

The Higgs components are lopsided towards the quark sector precisely because of the masslessness (with respect to the Higgs mechanism) of the neutrinos. One might have expected a 9.9! for the $\nu_\tau$ but if the neutrinos are massless (or have masses only generated radiatively) then this component is picked up by the top quark which has a 2 in the first column. The 8.8! has broken up in the S.U.(5) multiplets $(1+5+10) . 8! = 2 . 8 . 8!$ in descending order of mass from the top. This decomposition is in keeping with hypothesis 9 which specifies SU(5) as the appropriate symmetry for the stabiliser of the $S_8$ terms under a massless photon. In fact $(1+5+10)$ is the only breaking possible here if S.U.(5) dictates the allowed components (there are only three available ‘slots’ if the neutrino is massless and the sum of terms must be 16). This symmetry is apparently not a gauged symmetry and appears quite distinct to familiar local-gauge symmetries. The tau gets a singlet S.U.(5) rep of mass and there is mixing of mass components between the top and the bottom based on the S.U.(5) multiplets 5 and 10. No Higgs mass shifts between columns but there is effective quark mass mixing because the Top acquires a component from the column diagonal for the bottom. The list gives the top quark mass as $\approx 160 \text{GeV}$. and the bottom mass as $\approx 4.65 \text{GeV}$. These are the dominant, but not the only, components of the ‘current-quark’ mass of these quarks.

The $S_7$ symmetry is unbroken. The neutrinos have no Higgs components of mass in this theory and the lepton sector 7.7! units of elements shifts to the tau; the only available slot. The third tau term is interpreted as the sum of two singlets. The other entries follow the
appropriate multiplets of the unitary groups appropriate for the corresponding decomposition given by hypothesis 9.

Note that there is inter-generational mixing of mass components in the lepton sector with the tau extending into the muon column. This inter-generational mass-mixing, seen also at the level of the strange quark, could be significant for neutrino oscillations if the radiatively generated neutrino masses follow a pattern similar to the massive leptons.

The up and down quarks and the electron have no Higgs components at all. The scalar Higgs components (a scalar component has no massless generators) are associated with unstable particles and the stripping of these components from the up and down quark will also be required if proton stability is to be ensured. Where the expected 5.5! for the up quark goes we shall shortly see.

No components appear in the 4.4!=U(1) column. The two 4.4! components and the up-quark 5.5! component get rearranged as the following which represent a mixture of S.U.(2) and U(1) components:

| quark type | strong-charge | generation multiplier | strong component   |
|------------|---------------|-----------------------|-------------------|
| top        | +             | 3                     | 2.(4!-2)          |
| bottom     | -             | 3                     | 4.(4!-2)          |
| charm      | +             | 2                     | 2.(4!-2)          |
| strange    | -             | 2                     | 4.(4!-2)          |
| up         | +             | 1                     | 2.(4!-2)          |
| down       | -             | 1                     | 4.(4!-2)          |

The total mass represented by the above table is exactly 5.5!+2.4.4! as required so that every part of the Higgs mass is accounted for except the components 3.3! + ... the masslessness of which induced the breakdown of the initial Higgs unit of mass 10!≈83GeV in the first place. These are the exotic U(1) components we have sought to complete our calculation of proton and neutron masses. They represent a truly remarkable piece of structure of the Higgs field because with them the scalar components have been transmuted into tetrahedral spinor units of mass with clear mixing implied between quark generations. ‘Higgs’ units, such as the 2.9.S_9 associated with the top appear with no gauge field corrections to them in mass calculations (a common feature of all Higgs components) which is an indicator that they are ‘buried outside time’; they are situated effectively ‘inside’ the timeless space demarcated by the boundary of tetrahedral units (one can think of them as occupying the inner surface of the boundary of the geometry whereas the gauge fields occupy the outer surface - the time dimension does not exist within the boundary of any affine-set geometry embedded in space-time). This is discussed in more detail in [3]. In that paper we also saw that the current-quark U(1) components could be interpreted in terms of a strong-interaction potential between current quarks [3]. In the case of the up quark four out of the six surfaces of the cube (each quark is a ‘cube’ in this theory) are electro-magnetically charged (i.e. 2/3rds). The remaining
two surfaces of the cube are uncharged electro-magnetically but acquire a ‘Higgs-charge’, or perhaps more appropriately a ‘strong-charge’ because it can be interpreted as a strong interaction potential; in the case of the up the two units represented in the above chart (vis-a-vis 4 for the down) are the complement of the four (res.2) electro-magnetically charged square surfaces of the cube; accounting for the total of six square surface sub-geometries for each cube. One can easily see this structure in the quark operators given in the previous section. Presumably this means that, for example in the case of the up, only four of the six surfaces of the cube are being presented to space-time; the other two surfaces are outside time (?folded in).

The ‘Higgs-charge’ concept arose because the above U(1) mass units in question have been given massless generators and in space-time this would mean electro-magnetic charge due the the spinning triangular sub-geometries. The Higgs charge seems to be an analogue of electromagnetism but in a timeless colour space (it might lend support to the idea of instantons?). From a phenomenological point of view the Higgs charges are difficult to understand (will they effect the results of deep inelastic scattering at ultra-high energy in spite of the fact that they are located in a timeless 3-space? Can they be ‘pushed’ into space-time at high energy leading to deviation from the standard model predictions at high energy?) but the methodology required to use them in calculations is known precisely. To calculate the energy associated with current quark U(1) components one averages the absolute value of the sum of the respective ‘Higgs charges’. For example, in the case of two up quarks the value is;

\[
\text{up-up current quark Higgs energy} = \frac{1}{2}| + 2.(4! - 2) + 2.(4! - 2)| = 2.(4! - 2) \quad (16)
\]

and so-on for the other components. One sees that the choice of charge is arbitrary except that the up-type quarks and down-type quarks must have opposite sign just as they do for conventional electro-magnetic charge. The logical choice is to give the same Higgs charge sign as the E.M. charge for each quark. Thus for the up-quark four of the six surfaces have positive electro-magnetic charge (2/3rds of the surface of the cube) and two of the six surfaces (1/3rd of the surface) have a positive Higgs charge. It is these terms which define the additional masses for the proton and neutron expressions set earlier in the paper. The proton we have three kinds of couplings; one up-up and two up-downs so the sum \( \Sigma = 4(4! - 2) \) and for the neutron we have two up-downs and one down-down \( \Sigma = 6.(4! - 2) \).

Notice that, from the point of view of the Higgs field, it really is only at the level of the U(1) Higgs charges that the nucleons can be considered to have any component of the strange or other higher-generation quark content since these Higgs energies may alternatively be interpreted as being generated fractionally by the first generation component of the higher generation quarks - which are identical to that of the up and down (the pattern is repeated identically for each generation so that, for example, a top-quark has a +2(4!-2) corresponding to a first, second and third generation representation - although what you get depends upon the actual quark content of a given hadron). We will later see however that the remaining components of the current up and down quarks (developed in the preceding section) have
a similar interpretation in terms of mixing with higher generations not obvious from the previous calculations performed for the nucleons [3].

8 Ground state meson masses

Now we start on the main purpose of this paper; a study of ground state meson masses in order to gain insight into the mass structure of higher generation quarks.

The nucleon masses and the electron mass are particularly simple which might be expected for the fundamental stable matter constituents of the universe. They were examined closely in [3]. However we can use the same techniques of analysis to determine ground state meson masses. The techniques are a generalisation of the methods already developed for the nucleons with slight modifications because of the presence of only two effective quarks in a meson.

For the nucleons we had only first generation quarks to consider and we developed a three-colour particle vector on which the current quark representations acted. For mesons the situation is slightly different because in general only one colour is active in the particle vector. For example, we may consider that a $\pi^+$ meson consists of the quark content $U_r \bar{D}_r$ where $r$ is a single colour. In calculating the corresponding constituent mass this means that only one of the possible three colour components of the particle vector becomes massive and the remaining two are treated as massless. However, we then sum over all possible permutations of discrete colour which gives a multiplier of 3. (By contrast with the nucleons we had three massive components and hence a multiplier of 3! for the colour permutations).

For a first generation particle vector representation we thus have only one massive component but in two possible parity states over which we must sum;

\[
\begin{pmatrix}
  c_i & 0 \\
  0 & \bar{c}_i \\
\end{pmatrix} 
\begin{pmatrix}
  \bar{c}_i & 0 \\
  0 & c_i \\
\end{pmatrix}
\]

The matrix order of $\bar{C}_i$ is $(4! - 2)$ and the matrix order of $C_i$ is simply 4! (the generators of the $T_c$ group are in the barred matrix and assumed to be massless intrinsic spin generators in the transition to a field theory). Current quark coupling to both possible parity combinations introduces a multiplier of 2. Thus the total constituent order for a meson with two first generation quarks becomes;
The order of the constituent gluons is similar. Typical gluon formats are:

\[
C_i \left( \begin{array}{cc}
q_i q_j^* & 0 \\
0 & I_3
\end{array} \right) C_j^* \quad ; \quad C_i \left( \begin{array}{cc}
q_i q_j^* & 0 \\
0 & I_k
\end{array} \right) C_j^*
\]

\[
C_i \left( \begin{array}{cc}
q_i I_k^* q_j & 0 \\
0 & I_3
\end{array} \right) C_j^* \quad ; \quad C_i \left( \begin{array}{cc}
q_i I_k^* q_j & 0 \\
0 & I_k
\end{array} \right) C_j^*
\]

and the order of any gluon operator is the same as the order of the product \( C_i C_j^* \) which is \( 4!^2 \) (note that * indicates the conjugate transpose matrix which is not the same as the barred matrix). With discrete colour there are two possible transitions for any given colour state i.e.;

\[
C_i \rightarrow C_j \quad \text{and} \quad C_i \rightarrow C_k
\]

where \( \{i,j,k\} \) are the three possible discrete colours. This doubles the gluon matrix order. There is a further doubling of the order due to coupling to either of the two possible parity states of the particle vector which gives the glue matrix order for a ground state meson with only one pair of first generation quarks as \( 2 \times 2 \times (4!)^2 \).

When dealing with higher generation quarks the situation is similar but more complex; generally the rule holds that second generation quarks have double representations and third generation quarks have triple representations. Notice that the U(1) Higgs components have exactly this structure.

The rules for meson calculation are as follows;

1; From the possible matrix representations of a particle vector choose a representation capable of expressing the quark generation content of the meson. This is the G or generation number \( 1 \leq G \leq 3 \) where G is the highest generation present. Thus for example a bottom containing meson will have a G=3. A \( K^0 \) meson will have G=2 etc.

2; Calculate the P or parity number which represents the number of parity formats present in the particle vector. (For example, in the case of the first generation we only have
two formats; left and right given by eq.(17); in higher generation representations there can be more than two!). Parity ‘formats’ are not always the same as conventional parity; only for first-generation mesons is there a true correspondence. The meaning of this will become clear as we proceed through examples.

3; Calculate the number S. S is the number of distinct states seen by the gluons and is a multiplier for the gluon energy. S stands for ‘summed quantum numbers’ and is sometimes represented as $\Sigma$ in the text.

4; Calculate the representation number R which is the total number of representations needed to represent the quark content of the meson. Up-type quarks have two representations in the second and third generation components. These additional representations eliminate spinorial identities which otherwise appear in the charm or top in the higher generation content. We will discuss this in more detail as we come to it. These additional representations do not contribute to the generation number G. However, these additional representations induce additional permutations of ‘parity format’ and do change the P number. Such permutations are called superparity in the text because of the induced interchange of scalar and spinorial components.

5; Calculate the ‘bare’ constituent quark mass = $G.P.22.24.3$ for three possible colours.

6; Calculate the constituent ‘bare’ gluon mass = $R.S.24.24.2$ for interchange of one pair of colours.

7; Sum the terms for the charged current quark mass = C using the standard format of current quark representations.

8; Add the universal radiative correction multiplier $\mathcal{R}$ for quark-lepton unification;

$$= \mathcal{R} (G.P.22.24.3 + R.S.24.2 + C)$$

9; Add on the appropriate scalar and U(1) Higgs field components for the quark content of the meson; these have no radiative correction.

This technique should work for any meson. We have already seen that it works for the nucleons; modulo changes in the formula for the presence of three discrete colours ‘simultaneously’ in the particle vector with the corresponding glue order changes for three colours of quark simultaneously present. We will now examine some specific examples to illustrate the above principles. We will see that we get quite good results except for the charmed-bottom meson where there is a marked deviation away from the empirical result.

9 The pions

The charged pion consists of an up/antidown or down/anti-up combination. The calculation is the same for both. We calculate first the constituent mass which consists of both quark and gluon energy. These values are found in the theory by summing over the matrix order of the representation of the particle vector and glue. Note that the particle vector does not specify the quantum state of the object but sums over all possible discrete quantum states. The specific quantum numbers of the state are specified by the current quark components which are calculated separately.

For the pions $G=1$, $P=2$ so the ‘bare’ constituent quark order is $2.3.22.24=3168$. 

20
S is best calculated using a table.

| Parity combinations | ± | ± |
|---------------------|---|---|
| First generation parity | -1 | +1 |
| Superparity | 0 | 0 |
| Summed numbers | -1 | +1 |

A word of explanation here. The two parity combinations are the same as in eq. (17). For example we may regard the ± as the diag. \{C, C\} combination and the ± as diag. \{\bar{C}, C\} matrix. It makes no difference to the mass calculation which we regard as the left-handed part of the quark particle vector and which the right-handed but we routinely adhere to the pattern ± = -1 and ± = +1 for first generation parity. The superparity of the two states represented here is identical and both have been set at zero (superparity is only formally definable for mesons with 2nd and 3rd generation valance quarks; we will discuss this when we reach it). The summed numbers are simply the sum of the superparity and parity numbers in the case at hand (but with higher generation quarks includes other quantum numbers). S is the number of different summed numbers \(\Sigma\) and represents the number of distinct states coupled to by gluons. For example, in the case of the pions the possible summed numbers are +1 and -1 so that S=2 (physically this is equivalent to the statement that the strong interaction is independent of the parity of the state and couples to both right and left-handed states doubling the gluon matrix order).

We only need one representation for the pions so R=1. (R is the number of ±’s and or ±’s per box in the parity combinations section of the table - for the pions each box has only one such symbol).

Thus the gluon order is \(R.S.24^2 = 2304\).

To calculate the current quark mass we need a standard representation of the quark content. The notation is identical to that used for the nucleons and we create an anti-down (or anti-up) by taking the complex-conjugate representation. For a meson we have colour and anticolour combinations of quarks in the current-quark content. The red-antired \(\pi^+\) content is:

| Charged Pion Current Quarks. |
|-----------------------------|
| strong component | E.M. component |

| red \(\rightarrow\) | \(I\) | \(q\) | \(I\) | \(q^*\) |
| green \(\rightarrow\) | \(q^*\) | \(I\) | \(q^*\) | \(I\) |
| blue \(\rightarrow\) | \(q^*\) | \(I\) | \(q^*\) | \(I\) |
| \(\uparrow\) anti-down | \(\uparrow\) anti-down | \(\uparrow\) anti-down |
Notice that the definition of unit charge differs with a bosonic meson to a fermion. The application of the above current quarks to the standard particle vector interchanges the $C$ and $\bar{C}$ matrices but adds a + sign to one and a - sign to the other which is the definition of unit charge for the boson. (Incidentally this notation means that only one neutral pion occurs in the representation along with the two charged states; for this reason we will call this method of defining unit charge the ‘adjoint’ representation as distinct to the ‘fundamental’ representation used for the fermionic nucleons). Note, however, that the bosonic nature of the object is represented in its current-quark content not in the particle vector.

The calculation of current-quark mass proceeds as for the case of the nucleons. We need only sum one colour combination of current quarks since the particle vector includes all possible colour permutations. The masses are read off the chart. The three $q^*$’s in the E.M. component generate, with the doubling due to the coupling to the second parity state (as per the nucleons), $6.(4!-2)$ units and, with the two cancellations in the strong component (the $q^*$ cancels the $q$ and the $I$ cancels the $I$ in the strong matrix), the residual $q^*$ and 4 identities (one from the strong component and three from the EM part) generate, again with parity doubling, $5.2.4!$. Summing all these components and applying rule 8 we obtain, with some reorganisation, $R\{7! + 6! + 4.4! - 12\}$ for the total mass. To this we must add the Higgs U(1) components. This is quite simple in the case of the charged pion and was previously calculated as $(4!-2)$ for an up-down type combination with no radiative correction. However the resulting mass is about 136MeV whereas the empirical mass is 139.5679MeV. Comparison with empirical data shows that the deficiency is almost exactly a scalar Higgs of value $5! + 4!$ which suggests that there is a component of strange quark present (which has an identical first-generation current-quark representation to the down except for the Higgs scalar component). The effective component of strange quark present is given by the ratio;

$$\frac{\text{Higgs scalar in pion}}{\text{Higgs scalar in strange}} = \frac{5! + 4!}{6! + 3.5!}$$

(19)

where the strange Higgs scalar component has been taken from the Higgs component table. Thus if we use a mixture of down and strange to form the charged pion we obtain the mass $\pi^\pm = 139.5679$MeV, which is close to the correct value of 139.57018(35)MeV.

The calculation for the neutral pion proceeds in an analogous fashion with the appropriate current-quark content substituted. The Higgs U(1) component is most accurately given by averaging one up-up, one up-down and one down-down component from the Higgs U(1) table and the result of the calculation is $\pi_0 \approx 134.9728$MeV. which is the correct value. This value, however, does not include any component of Higgs scalar for the strange quark which appears to be absent in the neutral pion. Why a component of strange appears explicitly in the $\pi^\pm$ and not in the $\pi^0$ is unclear. It is possible that the strange-down mix (Cabibbo-type) is an incorrect interpretation and that the presence of singlet SU(2) and U(1) Higgs components in the charged pions represents an incomplete formation of the Higgs U(1) components with the $\pi^\pm$’s; that is to say, that the up-quark has a singlet SU(2) and U(1) scalar piece in the charged pion. An answer will require a more sophisticated version of the theory of the Higgs field. Interestingly the charged B meson seems to have identical
Higgs SU(2) and U(1) singlets associated with the up quark component (see subsequent calculation) which are also missing in the neutral B meson.

10 Higher generation quarks.

For the up and down species of quarks we employed a block diagonal representation of both the particle vector and the current quark operators that act on the particle vector. Thus for the $T_c$ group each colour $6 \times 6$ block has the format:

\[
\begin{pmatrix}
C_i & 0 \\
0 & \bar{C}_i
\end{pmatrix} \quad \text{or} \quad \begin{pmatrix}
\bar{C}_i & 0 \\
0 & C_i
\end{pmatrix}
\]

where $C_i$ is a $T_c$ colour matrix of colour $i$. This is not the only way that a consistent representation can be formed capable of coupling to a given current quark operator representation however. We will call this arrangement, when the $q_i$'s of the current quark operators and the $C_i$'s of the particle vector are in the same format as 'in phase'. It only applies to the situation where a hadron has first generation quarks. Consider the following matrix:

\[
\begin{pmatrix}
\lambda_{11} & 0 & 0 & 0 & \lambda_{12} & 0 \\
0 & \bar{\lambda}_{11} & 0 & \bar{\lambda}_{12} & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 \\
0 & \bar{\lambda}_{21} & 0 & \bar{\lambda}_{22} & 0 & 0 \\
\lambda_{21} & 0 & 0 & 0 & \lambda_{22} & 0 \\
0 & 0 & 0 & 0 & 0 & +1
\end{pmatrix}
\]

where the $\lambda_{ij}$ are the individual elements of a given $C$ matrix. The above matrix is a higher generation representation; it is blue format following the standard used in this paper and the colour is dictated by the position of the identities on the main diagonal. We call this a ‘phase-shifted’ representation. Now if we apply a similarly ‘phase-shifted’ blue quark operator the physical results are unaltered. However if we operate on the phase-shifted particle rep. with the original $q_b$ $6 \times 6$ matrix (in this example a blue down quark q operator);

\[
\begin{pmatrix}
i & 0 & 0 & 0 & 0 & 0 \\
0 & i & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & -i & 0 & 0 \\
0 & 0 & 0 & 0 & -i & 0 \\
0 & 0 & 0 & 0 & 0 & -1
\end{pmatrix}
\]

we get the following;
which is the same as the original matrix in terms of the positions of the $C$ and $\bar{C}$ matrices (note that the actual entries $C_{ij}$ or $\bar{C}_{kl}$ will have changed in value - we are only interested in whether the barred and unbarred matrices mix or not) but the sign on the identities on the main diagonal has changed. To maintain consistency we modify the down blue $q$ operator by changing sign on the operator identities (compare with previous);

$$
\begin{pmatrix}
\lambda_{11} & 0 & 0 & 0 & \lambda_{12} & 0 \\
0 & \lambda_{11} & 0 & \lambda_{12} & 0 & 0 \\
0 & 0 & +1 & 0 & 0 & 0 \\
0 & \lambda_{21} & 0 & \lambda_{22} & 0 & 0 \\
\lambda_{21} & 0 & 0 & 0 & \lambda_{22} & 0 \\
0 & 0 & 0 & 0 & 0 & -1
\end{pmatrix}
$$

This sign change has three effects;

1. it distinguishes the first and higher generation quark operators,
2. it causes the higher generation q operator to act like a script identity operator which un-couples it from the $T_c$ generators; it behaves like a scalar rather than a spinor operator. As we shall see, this actually has an effect on particle mass calculation which may represent the first evidence of supersymmetry in nature and
3. it is required to maintain charge conservation.

Remarkably, as can be checked, the (non phase-shifted) script identity operator requires a similar change in the sign of its' component which couples to the diagonal identity of the $C$ or $\bar{C}$ matrix which causes it to act as a discrete spinor generator! Thus under the altered phase representation of the particle vector the non-phase shifted current quark operators undergo an interchange of scalar and fermionic pieces!

These changes apply to all second and third generation representations. The altered phase couplings can be pictorially represented;
In the above diagram the diagonal entries in a q operator (here blue) have been listed in the vertical column of entries. The first generation couplings are the in-phase ones and the second and third generation couplings are out of phase. The ‘coupling’ lines in this diagram represent the relative position in the 6x6 matrix occupied by the $C_i$ and $\bar{C}_i$ with respect to the discrete current-quark operator on the left. There is an apparent ambiguity between what we call second and what we call the third generation representation; it makes no difference in practice since the third generation is found by summing over all three possibilities. The first generation is always fixed in phase to that of the current quark operators; in the notation used here the ‘phase’ of the operators stays fixed and that of the particle vector changes between different generation representations. It might be possible to represent the symmetry using fixed particle vector and changing the phase of the operators but the physics will be the same.

Note two interesting things;

1; the operator structure undergoes a supersymmetry transformation in the higher generation representations and

2; the representation theory limits the number of possible generations to three; this seems to be a unique feature of the discretised form of QCD since the continuum theory does not demand only three generations.

As a general interpretation of the higher generation representations probably the simplest way to consider them is as additional possible ways in which the symmetry can be expressed geometrically. The ‘in-phase’ representation dominates the first generation representation because this is the representation in which electro-magnetic charge is defined in a consistent way.
11 Kaons

In this section we calculate the mass of the spin-0 $K^0$ and $K^\pm$ using the principles outlined in the previous sections.

The basic principles are as with the pions. A particle vector is established capable of expression all permutations of particle type and the actual quark content is carried in the current quarks. Thus the ‘stangeness’ quantum number, which will be $\pm 1$, is carried by the current quark content but the particle vector carries the potential for either strangeness quantum number. The $G$ number is clearly 2 since the highest generation number in a Kaon corresponds to the strange quark. The particle vector contains a representation for the first generation and a second one for the second generation; it is duplicated. The tabulation for the structure of the particle vector is as follows:

| $C/C$ combinations (1st,2nd) | $\pm$ | $\mp$ | $\mp$ | $\mp$ | $\pm$ |
|-----------------------------|--------|--------|--------|--------|--------|
| (First generation) parity   | -1     | -1     | +1     | +1     |        |
| (Second generation) strangeness | +1    | -1     | +1     | -1     |        |
| Superparity                 | +2     | -2     | -2     | +2     |        |
| Summed Numbers              | +2     | -4     | 0      | +2     |        |

From which we can read from the first row that $R=2$ (the number of sets of $\pm$ signs per box in the first row) and $P=4$. $P$ is the number of different boxes in the first row; in this case the ‘parity’ of the second generation component, which is the strangeness quantum number, contributes to the total number of different ‘parity’ states - the ‘stangeness quantum number’ is literally the parity of the second component of the particle vector; more specifically it is related to the corresponding part of the current quark that couples to the second generation component of the particle vector. Strangeness is actually defined in the opposite way to the ‘parity’ in the first generation; for example here $\pm$ has been chosen as odd parity in the first generation component whilst $\mp$ has been chosen as even for the strangeness. The ‘superparity’ depends on whether the combination of parity and strangeness assignment is odd or even. Thus, for example $\pm\pm$ is even with value +2, whilst $\pm\mp$ is odd with value -2. The $S$ number is the number of different summed numbers and represents the number of distinct states recognised by gluons. A summed number is formed by adding all the entries above it in a given column. In the present example $S=3$.

We can immediately write down the particle vector order as $G.P.3.22.24 = 12672$. The gluon order is $R.S.2.24.24 = 6912$.

The components we have just calculated represent the constituent quark content of the Kaons; they are common to the charged and neutral versions and represent a sum over all possible discrete states that the quark pair can exist in. We must now determine the current quark mass of the different Kaons. From the Higgs mass chart we know that the strange quark has Higgs scalar contributions $6!+3.5!$ with no gauge field radiative corrections. To
calculate the remaining components of mass we need a chart of current quark components. In the second generation representation only the strange quark is represented;

### Neutral kaon; Current Quarks (First generation).

| strong component | E.M. component |
|------------------|----------------|
| red $\rightarrow$ | $q \ast$       |
| green $\rightarrow$ | $I$           |
| blue $\rightarrow$ | $I$           |

* $q \ast$ = down anti-strange

### Neutral kaon; Current Quarks (Second generation).

| strong component | E.M. component |
|------------------|----------------|
| red $\rightarrow$ | $q$           |
| green $\rightarrow$ | $I$           |
| blue $\rightarrow$ | $I$           |

* $q$ = anti-strange

The q’s cancel the q*’s in the first generation representation of the current quarks leaving the eight identities which, with the parity doubling (coupling to both the right and left handed components of the particle vector) we obtain a mass of 16.24.

For the second generation representation there is the analogue of a supersymmetry transformation on the components with the identities in the E.M. component coupling to generators (i.e. (4!-2) unit which is explicitly spinorial) and the q’s and q*’s coupling to the scalar value 4!; this is the reverse of the pattern seen in the first generation components. This kind of transformation is essential to eliminate any electro-magnetic charge generation component of the second generation representation; since $I^\dagger = I$ the identities coupled to the generators generate no charge. (It is interesting to note that the existence of this kind of structure may be the first evidence of supersymmetry in nature - it is a form of concealed supersymmetry and, of course, in the present example discretised and not continuous). The q therefore cancels the q* in the second generation representation and the mass of the second generation representation, again with doubling for coupling to both parity components, is 4.4!+4.(4!-2).

Finally we must calculate the Higgs U(1) components from the table. The strange has a component of 4(4!-2) for each of its two generation components. There is an ‘interaction’
between the down and each of the two components of the strange leading to a doubling of the down-down U(1) order to 8(4!-2).

Summing all these components and applying the universal radiative correction $\mathcal{R}$ we obtain:

\[
K^0 \text{mass} = \mathcal{R}\{\text{Particle order + gluon order + current quarks}\} + \text{Higgs sector} \\
= \mathcal{R}\{12672 + 6912 + 568\} + 6! + 3.5! + 8.(4! - 2)
\]

which translates into a mass of $K^0 = 497.0368\text{MeV}$ using the electron rest mass expression which compares with the empirical value of 497.672(31)MeV.

For the charged Kaon we must alter the current quark content to accommodate the up or anti-up quark. The appropriate chart is:

| Charged kaon; Current Quarks (First generation). |
|-----------------------------------------------|
| **strong component** | E.M. component |
| red→ | $I$ | $q$ | $I$ | $q^*$ |
| green→ | $q^*$ | $I$ | $q^*$ | $I$ |
| blue→ | $q^*$ | $I$ | $q^*$ | $I$ |

\[\uparrow \text{up anti-strange} \quad \uparrow \text{anti-strange}\]

| Neutral kaon; Current Quarks (Second generation). |
|-----------------------------------------------|
| **strong component** | E.M. component |
| red→ | $q$ | $q^*$ |
| green→ | $I$ | $I$ |
| blue→ | $I$ | $I$ |

\[\uparrow \text{anti-strange} \quad \uparrow \text{anti-strange}\]

The calculation proceeds as before. The particle vector and gluon calculations are identical. The mass of the first generation chart can be read off as \(2\{3.(4!-2)+5.4!\}\). The second generation component is identical to the neutral Kaon and is \(2\{2.4!+2.(4!-2)\}\). The Higgs U(1) component is twice the up-down value and is therefore \(2.(4!-2)\). Thus the mass is;

\[
K^{\pm} \text{mass} = \mathcal{R}\{12672 + 6912 + 556\} + 6! + 3.5! + 2.(4! - 2) = 493.7143\text{MeV}
\]

which compares favourably with the empirical value of 493.677(13)MeV[2].
12 D mesons

In this next example we will study the simplest mesons containing charm to see how the second generation structure of the strange is extended.

Charm quarks, as for the case of the strange quark, have a second generation representation but it is dual. The reason for this seems to be the presence of the script identity. Since \( \mathcal{I}^\dagger \neq \mathcal{I} \) a supersymmetry transformation from the first to second generation component produces an effective additional charge. This is eliminated by duplicating the second generation representation in the particle vector. A working chart for the particle vector and gluon contributions to \( D^\pm \) and \( D^0 \) is;

| \( C/\bar{C} \) (1st,2nd) | \( \mp\mp\mp \) | \( \mp\pm\mp \) | \( \pm\mp\mp \) | \( \mp\mp\pm \) | \( \pm\mp\mp \) | \( \pm\pm\mp \) | \( \pm\pm\pm \) |
|--------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| parity                   | +1              | +1              | +1              | -1              | -1              | -1              | +1              |
| charm                    | +1              | -1              | +1              | +1              | -1              | +1              | -2              |
| Sup.parity               | +2              | -1              | -2              | -2              | +1              | -1              | +2              |
| \( \Sigma \)             | +4              | -1              | 0               | -2              | -1              | +2              | +2              |

From which we can read that \( G=2 \) (contains a second generation quark), \( P=8 \) (total number of permutations of the plus/minus signs), \( R=3 \) (the number of plus/minus signs per combination) and \( S=5 \) (the number of different summed numbers). Plugging these into the mass formula gives a total order for the particle vector plus glue as 42624.

The next step is to evaluate the current quark mass. Let us consider the neutral D meson first.

\( D^0 \); Current Quarks (First generation).

| strong component | E.M. component |
|------------------|----------------|
| red→             | \( \mathcal{I} \) | \( \mathcal{I} \) | \( I \) | \( I \) |
| green→           | \( q^* \) | \( q \) | \( q^* \) | \( q \) |
| blue→            | \( q^* \) | \( q \) | \( q^* \) | \( q \) |
| \^ up anti-charm  | \^ up anti-charm |
In evaluating the mass it is assumed that the script identity generates a mass component with the opposite sign to that of the identity in the second generation representation. With the ‘supersymmetry’ transformation the q’s in the second generation representation couple to a Tr unit of 4! with no massless generators. A cancellation occurs in the first generation component between the q’s and the q*’s. The coupled script identities in the first generation are considered equivalent to the coupled I’s and don’t cancel. The scalar Higgs components from the table for the charm quark are $7.7!+3.6!$ and an explicit calculation of the U(1) Higgs components gives a value of $4.(4!-2)$. The calculation gives a mass $D^0=1864.39\text{MeV}$. (empirical value 1864.6(0.5). A similar calculation of the $D^\pm$ mass gives a value of 1867.56MeV (the empirical value is 1869.3(0.5).

13 Charmed strange

Again this meson contains only second generation quarks at most so $G = 2$. The strange quark requires only one second generation representation and the charm requires two so $R = 3$. The superparity numbers are the same as for the D mesons;
In the above diagram the left hand sign-pair of each triplet is due to a single component in the second generation strange representation and the remaining two sign pairs are due to the charm. The quantum numbers which the particle vector must represent can be calculated from this diagram and are as follows;

| C/S meson Structure; Particle Vector |
|-------------------------------------|
| $C/C$ ‘parity’S 2C                 |
| $\pm \pm \pm$                      |
| $\pm \pm \mp$                      |
| $\pm \mp \pm$                      |
| $\pm \mp \mp$                      |
| $\mp \pm \pm$                      |
| $\mp \pm \mp$                      |
| $\mp \mp \pm$                      |
| $\mp \mp \mp$                      |

| Strangeness | -1 | -1 | -1 | +1 | +1 | +1 |
|-------------|----|----|----|----|----|----|
| charm       | +1 | -1 | +1 | +1 | -1 | +1 |
| Superparity | +2 | -1 | -2 | -2 | -1 | +2 |
| $\Sigma$    | +2 | -3 | -2 | 0  | -1 | +4 |

Points to note;

1; The strangeness assignment is given by the first sign pair of any triplet; $\pm = +1$ and $\mp = -1$.

2; The charm assignment is a function of the remaining two sets of sign-pairs; $\pm \pm$ or $\mp \mp$ is an even permutation of charm $+1$ and $\mp \pm$ or $\pm \mp$ is an odd permutation and defined as charm of -1.

3; The superparity of an odd charm pair (charm=-1) is zero. The superparity assignment of the strange is 1 with the sign depending on whether it is an odd or even in sign with respect to the charm. The superparity assignment of the even charm permutation pair (charm $= +1$) is 1. Thus an even combination (such as $\pm \pm \pm$) has superparity +2 whilst an odd combination (such as $\mp \pm \pm$) has superparity -2. The superparity is found by summing over the two quarks. Where the absolute value of the superparity of the charm is zero (i.e. for odd charm pair permutations charm $= -1$) then the single strange is odd by definition and has a superparity of -1.

4; The $\Sigma$ value is the sum of the strangeness assignment, charm and superparity for a given triplet of signs. $S$ is the number of different values of $\Sigma$ so in this case $S=6$.

5; $P$ is the number of different sign triplets; here $P=8$. $R$ is the number of entries in the sign blocks; here we have triplets so $R=3$. Clearly for only second generation quarks $G=2$.

Plugging in the values for the constituent quark and gluon energy;

$$G.P.3.(4! - 2).4! + R.S.2.(4!)^2 = 46080$$

which is the matrix order which may be interpreted as the number of possible distinct discrete states that are possible particle vector representations.

Now we must calculate the current quark mass.
The first and second generation representations are identical although for the second generation rep. it is the identities in the E.M. matrix which couple to the $\bar{C}$ generators whilst the $q^*$ in the E.M. matrix are un-coupled from the generators and are acting as scalars. Unit E.M. charge is defined by coupling to the G=1 part of the particle vector. We need, however, to calculate the mass of only one set of representations of the current quark operators since only one set is needed to couple to both the G=1 and G=2 parts of the particle vector.

As usual the masses are read off the chart. The triplet of $q^*$’s couples to a $T_r$ equivalent unit of order $(4!-2)$ and all other components couple to order $4!$. One $q$ cancels a $q^*$ and one $I$ cancels an $I$ in the strong matrix. There is a doubling of the order to couple to reverse particle vector formats giving a total matrix order of:

$$10.4! + 6.(4! - 2)$$

which, along with the 46080 matrices of the constituent mass will be given a standard non-perturbative radiative correction $\mathcal{R} = 1 + \alpha_{em} + \alpha_{GF} \approx 1.0073115$.

To this the appropriate Higgs components for the two current quarks must be added. These are read off the Higgs chart as a matrix equivalent order. The energy of potential separation of the two quarks is also read off the standard chart; both the charm and the strong have values -2 for the second generation representation. There is no first generation representation of the current quarks so the appropriate matrix order is $| -2 - 2 |(4! - 2)$ with no radiative correction ($\alpha_s = 1$ at the $T_c$ or $T_r$ scale). Thus the C-S ground state meson has the composition;

$$\begin{align*}
\text{matrix order of current quark operators} & \quad \text{Quark sep. potential} \\
R & (40680 + 372) + (7.7! + 3.6!) + (6! + 3.5!) + 4(4! - 2)
\end{align*}$$

constituent matrix order. Scalar higgs part of strange

Scalar higgs part of charm

\[32\]
Which gives the charmed-strange meson mass as $1969.12\text{MeV (1968.6(0.6)}^{2}$] which is within the empirical boundaries.

14 B mesons

In this section we will evaluate a B meson mass as an example of bottom containing calculations.

The combination of an up or down quark with a bottom quark will require a generation number of $G=3$ to define a particle vector. To create a bottom quark we duplicate the second generation representation and keep it in phase with the third representation in terms of parity and superparity assignments. The following tabulation will illustrate the ideas involved:

| B meson Structure; Particle Vector |
|-----------------------------------|
| 1st, 2nd & 3rd gen. | + | + | + | + | ± | ± | ± | ± | ± | ± | ± | ± |
| C/ ¯C comb. |  | | | | | | | | | | | | |
| (1st gen.) parity | +1 | +1 | -1 | -1 | | | | | | | | |
| B. ness = Σ 2nd & 3rd. | -2 | +2 | +2 | -2 | | | | | | | | |
| Superparity | -3 | -3 | +3 | +3 | | | | | | | | |
| Summed Numbers | -4 | 0 | +4 | 0 | | | | | | | | |

The Bottomness here is the sum of the ‘parity’ of the second and third generation components and is rather like an extension of strangeness as it has the same sign configuration. Notice that for the Bottom quark the second and third generation components are always ‘in phase’.

The superparity is related to the total number of $±$ or $∓$’s and is odd or even according to the product e.g.; $± + ±=even=+3$. The summed numbers are self explanatory.

From the above table we can read off $P=4$ and $S=3$. R in this case is 2 not 3 because the representations of the second and third generation components are identical in each case.

Thus the particle order is:

$$R.S.2.24^2 + G.P.3.22.24 = 25920$$

for two colour-exchange equivalent gluons and three possible colours of particle vector.

The first-generation current quark mass chart is identical to that for the Kaons apart from the scalar pieces. This was given as $R(10.4! + 6.\{4! − 2\})$ + scalar part for the $B^±$.

The second generation and third generation components of the B meson are identical duplicates of the second generation component for the Kaons so the corresponding mass is tripled (the third generation component is double valued with respect to the second). Thus the sum of the second and third generation pieces for the charged B contributes $R(12.4! + 12.\{4! − 2\})$.

From the tables of Higgs scalar components the B quark has components $5.8!$ plus $U(1)$ contributions. The $U(1)$ contribution is assumed to be four times the up-down current
and thus = 4(4!-2); i.e. doubled on the coupling to the third generation representation. The charged B mass is then 5276.1MeV (the empirical value is 5278.9(1.8)MeV). A similar calculation gives the $B^0$ mass as 5280.5MeV (5279.2(1.8)MeV empirical). Note that we seem to have an identical situation to that with the pions; the up quark appears to have scalar Higgs pieces single SU(2) and single U(1) in the charged B meson. If these are included the predicted charged B mass would increase to 5279.4MeV.

## 15 Bottom-strange meson

As for the case of the charmed-strange the first generation component of the current quarks of the strange-bottom will not acquire mass because of the absence of any massive up or down current-quark content. However the particle vector continues to reflect the summing over generation representations and the G number will be 3. A strange and a bottom can be represented with R=2. The SQN’s are found from the following chart of quantum numbers:

| B-S meson Structure; Particle Vector |
|-------------------------------------|
| 2nd, 2nd & 3rd gen. | ± | ± | ± | ± | ± | ± |
| C/C comb. |                           |
| S. ness | -1 | -1 | +1 | +1 |
| B. ness = Σ 2nd and 3rd | -2 | +2 | +2 | -2 |
| Superparity | -3 | -3 | +3 | +3 |
| Summed Q Numbers | -6 | -2 | +6 | +2 |

Thus G=3, R=2 (the two bottom sets of ± signs - the two on the right of each box - are always ‘in phase’ and are treated as a single rep.), P=4 (the number of different boxes of sets of ± signs) and S=4 because there are 4 different SQN’s.

Hence the constituent particle vector order is 3.4.3.(4!-2).4! and the corresponding glue order is 4.2.2.2.(4!)².

For the current quarks we note that the bottom will sum over both higher generation reps. whilst the strange will only have a single higher generation rep. Thus for a second generation representation of the current quark content of the neutral S-B we will have;

| SB meson; Current Quarks (first/second generation). |
|--------------------------------------------------|
| strong component | E.M. component |
| red→ | $q^*$ | $q$ | $q$ | $q^*$ |
| green→ | $I$ | $I$ | $I$ | $I$ |
| blue→ | $I$ | $I$ | $I$ | $I$ |
| ↑ | strange anti-bottom |
| ↑ | strange anti-bottom |
The $q$ cancels the $q^*$ and the mass of the identities is 8.4! in the first generation and 4.4!+4.(4!-2) in the second generation (because of the super-symmetry transform the identities couple to the spinor generators in the second generation rep.). These values then double for the coupling to $CC/\bar{C}c$ pairs in the particle vector.

The bottom alone has a third generation rep.;

\[ S\bar{B} \text{ meson; Current Quarks (3rd. generation)}. \]

| strong component | E.M. component |
|------------------|----------------|
| red $\rightarrow$ | $q$ | $q^*$ |
| green $\rightarrow$ | $I$ | $I$ |
| blue $\rightarrow$ | $I$ | $I$ |

\[ \uparrow \text{ strange} \quad \uparrow \text{ anti-bottom} \quad \uparrow \text{ strange} \quad \uparrow \text{ anti-bottom} \]

The mass equivalence of the above table is of double value again with respect to the previous table and is thus 8.4! + 8.(4!-2) because the $q$ again cancels the $q^*$ and the identities on the right couple to the spinor generators.

From the Higgs table the scalar mass of the bottom is again 5.8! and for the charm 6!+3.5! whilst the potential energy of separation of the current quarks (the U(1) terms) are based on bond between second to second and second to third generation reps. The second to second bond for the BS will be double the down-down. (The d-d is 4.(4!-2)). The second to third bond is double again giving net \( \{16+8\}.(4!-2) \).

With the radiative correction we have a total mass matrix order;

\[ \mathcal{R}.(29344) + \text{current quark mass} \]

Which gives a value 5.367GeV using the given values for the current quarks and the Higgs components for the bottom and the strange from the chart. The empirical value is 5.369(24)\[2\].

16 Charmed bottom.

| C/B,2C | $\uparrow \uparrow \uparrow$ | $\uparrow \pm \uparrow$ | $\uparrow \uparrow \pm$ | $\uparrow \uparrow \uparrow$ | $\uparrow \pm \pm$ | $\uparrow \pm \pm$ | $\pm \pm \pm$ | $\pm \pm \pm$ |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| B.ness | -2 | -2 | -2 | +2 | +2 | +2 | +2 | +2 |
| charm  | +1 | -1 | +1 | +1 | -1 | +1 | +1 | +1 |
| Sp.parity | -3 | +2 | +3 | -3 | +2 | +3 | +3 | +3 |
| $\Sigma$ | -4 | -1 | +2 | 0 | +3 | +6 | +6 | +6 |
Notes; 1; The first of every three sets of ± signs is due to a double valued bottom. It represents the relative orientation of the \(C/\bar{C}\) in the combined second and third generations which are always in sync. The other pair of ± signs is due to the second generation representation of the charm quark.

2; The bottomness quantum number is defined from the first of each three sets of signs so that \(\mp \ast \ast\) is the particle vector potential to express bottomness -2 (this quantum number is double valued in discretised QCD as mentioned - it does NOT mean a bottomness of 2 in conventional theory when the number is normalised to unity) and \(\pm \ast \ast\) is bottomness +2.

3; as previously the charm quantum number is defined in terms of odd or even permutations of its second generation reps. so that \(\ast \pm \pm\) or \(\ast \mp \mp\) is +1 unit of charm and \(\ast \pm \mp\) or \(\ast \mp \pm\) is -1 unit of charm.

4; The superparity number of the negative charm quantum number is 0; that is the charm product \(\pm \mp\) or \(\mp \pm\) has zero superparity. The charm combination \(\pm \pm\) has superparity +1 and the combination \(\mp \mp\) has superparity -1. The bottomness quantum number has a superparity \(|2|\) value with the sign following the quantum number bottomness. The given superparity states are then the corresponding sums of superparity.

5; The \(\Sigma\) is then the sum of the bottomness, charm and superparity quantum numbers. here \(S = 6\) is the number of different \(\Sigma\) values and represents the number of distinct matrix representations seen by the gluons.

In the case of the ground state C-B meson \(G = 3\) is the generation number. The \(P\) number is the number of distinct sets of sign triplets and here \(P = 8\). \(R\) is the representation number and parallels the quantum numbers \(R = 1\) for the charm and \(R = 2\) for the bottom in this chart so that the total is \(R = 3\).

Thus the constituent matrix order (number of distinct matrices in the particle vector and gluon representations);

\[
\text{Constituent quarks} = G.P.3.4!.\left(4! - 2\right) = 38016
\]

\[
\text{Constituent glue} = R.S.2.\left(4!\right)^2 = 20736
\]

The current quark reps. look identical to those of the Kaon with the C replacing the up and the B replacing the strange but remember that this rep. is duplicated for the second generation representation with the appropriate supersymmetry transformation of the operators;
C-B; Current Quarks (2nd generation rep.).

| strong component | E.M. component |
|------------------|----------------|
| red→             | I             | q         |
| q*               | I             | q*        |
| green→           | q*            | I         |
| blue→            | q*            | I         |
| charm anti-bottom| ♦             | ♦         |
| ♦                | charm         | anti-bottom|

The matrix order here is $2\{10.4! + 6.(4! − 2)\}$ where the multiplier 2 arises because we are summing over first and second generation representations. The bottom will additionally have a third generation rep. which will have a mass of $2(4.4! + 4\{4! − 2\})$

The ‘static’ purely strong energy of separation of the quarks using the table:

| chart of static strong potentials |
|-----------------------------------|
|                                  |
|                                up | down | charm | strange | top | bottom |
| First generation                | +1   | -2    | +1      | -2  | +2     | -4     |
| Second generation               | 0    | 0     | -2      | -2  | 0      | 0      |
| Third generation                | 0    | 0     | 0       | 0   | ?+4    | ?-4    |

Using the above table we have potential first generation rep. to first for the CB as $|+1−4|(4! − 2)$ and first to third an identical value while second to third potential $|-2−4|(4! − 2)$ with an identical second to first giving a net $18(4! − 2)$. The Higgs values from the charts are

charm = $6! + 3.5!$

and

bottom = $5.8!$

These last values i.e. the $18.(4! − 2) + 5.8! + 6! + 3.5!$ have no radiative corrections. The remaining mass is given a $T_r$ scale radiative correction $\mathcal{R} = 1 + \alpha_{em} + G_f$ and the result of the calculation is

Mass C-B = 6.9116GeV

which unfortunately is well above the empirical value of about 6.4GeV.[2]

17 Top-antidown and Top-antiup meson ground states.

We saw in the case of the B mesons that the second generation component of the B quark is a ‘dummy’ duplicate; it doesn’t contribute to the R representation number. Thus we can
use the model of the charm-containing meson to produce a calculation of the T mesons by extending the the D meson calculations just as an extension of the K calculations gives the B meson masses. The top quark mass however is overwhelmingly dominated by the Higgs scalar components of the top quark which, from the table, are 2.9.9! + 10.8! \approx 159.9\text{GeV}. With discretised Q.C.D. we can do a precision calculation of the ground state mass containing a top quark but the result varies little from the top mass for combination with light quarks. Such combinations are unlikely to be stable. Nevertheless the calculation can be done as an exercise.

Clearly G=3 for a top/anti-up or top/anti-down combination. To get the R,P and S values we need a table of possible quantum states (only the first entry of the $C/\bar{C}$ is given; the reader can fill in the remaining values if interested!);

| $C/\bar{C}$ | $\mp$ | $\pm$ | $\mp$ | $\mp$ | $\pm$ | $\mp$ | $\pm$ |
|-------------|------|------|------|------|------|------|------|
| (1st Gen)   | +1   | +1   | +1   | -1   | -1   | -1   | -1   |
| Top         | +2   | -2   | +2   | +2   | -2   | +2   |
| Supp.      | -3   | -1   | -3   | +3   | +1   | +3   |
| $\Sigma$   | 0    | -2   | 0    | +4   | -2   | +4   |

Because of the dummy duplication of the second generation representation we take R=3. Obviously P=8 and S=3. Thus the particle and glue order is 48384 (to be multiplied by $R$). The topness number is defined in much the same way as charm but this in not the top quantum number as such; this is carried in the current quark representation. (The particle vector sums over possible states and that is why it includes top quantum numbers with opposite signs; just as for bottomness, charm and stangeness in the other particle vectors studied). We can use the D meson current quark mass calculations to determine the current quark mass by doubling the value of the second generation component for the D to get the top containing masses. The charged $T\bar{D}$ mass is then $\approx 161.038\text{GeV}$ and $T\bar{U} \approx 161.037\text{GeV}$.

We will leave the calculation of other top containing meson masses to the reader as an exercise.

18 Conclusion

We have seen that the discretised system, as outlined in previous papers and extended here, is capable of providing a good method of representing particle masses and defining symmetries involved in the observed spectrum of masses. The model has all the overall symmetry features of the standard model and is predictive. It is interesting that the predicted mass of the charmed bottom meson varies significantly from the early measurements although it is still within two standard deviations and it will be interesting to see if the empirical value moves toward the predicted mass as measurements of the rest mass of the C-B ground state improve.
The weakest area of the theory is the Higgs sector but this is also the most interesting as it seems to provide the first empirical evidence that $SU(5)$ is a real symmetry of nature; albeit of a most unusual kind!

There is quite a lot of scope for further research in this topic with numerous unsolved problems and a number of hints in the correspondence with empirical data of new physics beyond the standard model. In particular it would be interesting to address the question of the relationship between the mass calculation, and the mixing patterns present, and the CKM matrix entries.

References

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