Stability of metal cylindrical grid shell structures

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Abstract. Based on the fundamental thin shell theory applied to thin continuous cylindrical shells, a method for calculating the stability of the corresponding grid shell structures is developed. Mathematical modeling is performed and factors of critical state of structures are determined. Critical deformation zones are identified and the patterns of changes in shell geometry under actual loads are established. The dependencies of strength and shape-forming parameters are obtained depending on the material and the number of half-waves of the stability loss shape. The critical load is determined taking into account the possible rigidity of multi-element grids, and the results of numerical studies are presented. A noticeable difference in the values of critical load is shown and the admissible limits of applicability of geometric parameters are found. A significant sensitivity of shells with a rhombic mesh are noted.

Keywords: cylindrical grid shell, critical load, buckling analysis, continual model, discrete model

1. Introduction

The problem of the stability of cylindrical grid shell structures now is a special part of design of reliable roofs for buildings and structures. There are studies by Zhonghao Zhang et al. [1], devoted to systems with reinforcing elements. A detailed analysis of the stability of similar structures was carried out. This study focused on the tension member installation and examined the effect of tension member on buckling load and strength of single layer two-way grid shell, taking account of the units of grid, load distribution, initial imperfection and the initial axial force. However, the main attention is paid to single-layered surfaces. According to the results of recent studies (Raghavan Ramalingam and S. Arul Jayachandran [2]) the stability loss of particular zones influenced by actual loads may occur. Based on the collected data and the design experience, Huihuan Ma et al. [3] conducted experimental and numerical studies of the cylindrical grid shell structures. The complete load–deflection response curve and the buckling mode of the test structure were obtained. FEA models were also established to carry out numerical analysis. Through comparison of the FEA results and the corresponding test results, it is shown that the proposed FEA model can be used to effectively predict the mechanical behavior of the reticulated shell including its buckling load. Obviously, the mesh, type of external loads, and the boundary conditions have a significant effect on the structures behavior [4-9], and the need to follow economic principles further complicates the problem. There are known discrete-continuous methods of calculation using iterative procedures and the results of experimental tests of continuous shells [10]. The developed algorithms contain analytical dependencies and are based on a discrete or continual representation of the surface. The solutions obtained are applied to grid shells of arbitrary geometry and have a unified approach to determining the value of the critical [12]. The shape of the shells is often specified in the first approximation and is remote from the actual operation of the structures. The conducted studies showed a number of factors which had not been taken into account before and considerable margin of bearing capacity [12]. However, cylindrical grids, in comparison with other curvilinear multi-element structures [13], are subjected to substantial curvature of surfaces, have different schemes of geometry variation and the smallest strength parameters of stability. However, in
past research using numerical models, shell elements have been widely used to simulate bridges, beams, columns and structural components [14-15], concrete slabs, and steel plates [16]. Therefore, the goal of this paper is to develop a mathematical model to determine the critical load and to investigate the stability of the cylindrical grid shell structures. The finite element method and Lira-SAPR 2018 software are used. The analytical study is included in order to calculate the critical load.

2. Scheme of the model

To determine the behavior and the features of grids deformation, a continuum method is used [17]. In accordance with it, geometric and strength parameters are given.

The shape of the surface is described by the radius of curvature $R$, the angle $\alpha$ and the thickness $h$, the position of each point being determined by the $x$ and $y$ coordinates along the generator and along the arc direction of the circle, respectively [12] (see figure 1).

The displacements are represented in the form of deflections $w$. The deviations of the span points of the surface along the external normal to the center of curvature are assumed positive.

The material is taken into account by Young modulus $E$. Flexural cylindrical rigidity and subcritical stress are given by the force parameters $D$ and $p_y = qR/h$, respectively. The load $q$ is uniformly distributed over the surface and applied radially along the outer normal.

3. Continual model analysis

The differential equation of stability [17] is:

$$D\nabla^8 w + \frac{Eh}{R^2} \frac{\partial^4 w}{\partial x^4} + qR \nabla^4 \left( \frac{\partial^2 w}{\partial y^2} \right) = 0$$

Equation (1) includes bending $D$ and axial $Eh$ rigidities. The grid surface of the shell is defined from longitudinal, transverse and diagonal rods with the formation of a diagonal grating (figure 2).

Thin shell theory can be applied to the grid shell structure analysis for the first preliminary calculation. Such analytical model, in which grid reticular structure can be substituted by equivalent continuous shell was developed in [17].

Using the differential equation for continuous shells, the equivalent rigidity relations are written [17]:

$$Eh \frac{EA}{a} ; \quad D - \frac{EI}{a},$$

where $I$ is moment of inertia, $A$ is cross-sectional area of the structural elements. Being combined with the elastic modulus $E$ these geometrical parameters I and A form the flexural EI and the membrane EA rigidities of the elements respectively.

Then, equation (1) can be written as [17]:

$$\frac{EI}{a} \nabla^8 w + \frac{EA}{aR^2} \frac{\partial^4 w}{\partial x^4} + qR \nabla^4 \left( \frac{\partial^2 w}{\partial y^2} \right) = 0$$

Taking into account the hinged support of the grid on the contour, the boundary conditions are represented in the form:
\[
\begin{align*}
\left\{ \begin{array}{l}
w|_{x=0} = w|_{x=L} = 0; \\
\frac{\partial^2 w}{\partial x^2} |_{x=0} = \frac{\partial^2 w}{\partial x^2} |_{x=L} = 0 \\
w|_{y=0} = w|_{y=R} = 0; \\
\frac{\partial^2 w}{\partial y^2} |_{y=0} = \frac{\partial^2 w}{\partial y^2} |_{y=R} = 0
\end{array} \right.
\end{align*}
\]

where \( L \) is the structure length along the generator; \( \alpha R \) – is the arc length.

According to the proposed conditions (5), the deflection \( w \) is defined as a function of:

\[
\left( \sin \frac{m\pi x}{L}, \sin \frac{n\pi y}{aR} \right)
\]

Here, \( m \) is the number of half-waves of the curved surface along the generator; \( n \) is the number of half-waves in the direction of the arc. Substituting into the stability equation (2), it is found:

\[
q_{cr} = \frac{EA}{aR} \left[ \frac{I}{A} \left( \frac{\pi \alpha R}{nL^2} + \frac{n\pi}{\alpha R} \right)^2 + \left( \frac{\alpha R}{R^2 \alpha^2 \pi^2} \right)^2 \right] \left( 1 + \frac{nL^2}{(\alpha R)^2} \right)^2
\]

The shell changes its shape along the generator always in the form of one half-wave. Therefore, when determining the critical load \( q_{cr} \) in Eq. (4) the quantitative parameter \( m \) is omitted and is not used in the calculations.

To carry out analytical preliminary calculation, it is introduced the value of the dimensionless parameter

\[
\hat{q} = \frac{aR}{q_{cr} EA}
\]

The geometric and strength characteristics are set and the necessary calculations are done.

4. Discrete model (FEA)

The finite element method and building a model in Lira SAPR 2018 software are used. A rectangular grid type with a rod that divides the cell region into two identical triangles is chosen. The geometric stability of the structure is ensured by the hinged support conditions, what is more, at one corner point there is the complete prohibition of displacements (X, Y, Z axes), and at the other three corner points we impose bonds along the axes X, Z. The other contour support nodes are hinged vertically (Z axis).

Mechanical properties are given in Table1.

Table 1. Geometric parameters and material characteristics

| B (m) | f (m) | R (m) | a (m) | E (kN/m²) | Steel |
|-------|-------|-------|-------|-----------|-------|
| 36    | 9     | 22.5  | 3.0   | 2.06×10⁸  | S235  |

The stiffness characteristics of tubular profile rods are assigned based on two sizes of cross-sections of the software assortment (see Table 2).

Table 2. Cross-section parameters of tubular profile rods

| System | D (m)    | t (m)    | A (m²)    | I (m⁴)    |
|--------|----------|----------|-----------|-----------|
| 1      | 168×10⁻³ | 5.5×10⁻³ | 28.1×10⁻⁴ | 92.9×10⁻⁷ |
| 2      | 140×10⁻³ | 4.5×10⁻³ | 19.2×10⁻⁴ | 44.2×10⁻⁷ |

The influence of the length \( L \) on the magnitude \( \hat{q} \) and number of half-wave \( n \) of the structural stability loss form is investigated with the \( L/a \) ratio varying along the generator in the range from 12 to 48. The correlations of shell deformation under critical load are revealed.
Finally the correlation of dimensionless critical pressure $q$ and L/a ratio can be presented graphically. The curves show differential equation solution got by numeric method, the lower eigenvalue critical pressure is obtained. So, these curves present the results of the calculation, based on continual hypothesis.

The points, marked by circles, present the results of FEA calculations of special cases, realized by Lira SAPR2018. The discrete model is realized, the rods are modeled by rod FE. This calculation is provided for verification of the continual model analysis.

It is obvious that the second system is more vulnerable to the loss of stability than the first one. The rods of the second system have a cross section of $140 \times 4.5$ mm, and the critical load got using computer analysis and the obtained by analytical dependence shows almost the same value. For a design with any of the accepted cross-sections of the rods, a decrease in the parameter $q$ and a decrease in the number of half-wave $n$ of the shape of the loss of stability are noticed with an increase in the number of cells along the generator. That is, increasing the length in the direction of the generator led to reduction in the critical load. The deformation of the system occurs in the form of one half-wave $m$ along the length of the structure and different half-wave $n$ along the direction of the arc. The forms of stability loss of the most vulnerable shell with three values of the shape-forming parameter are shown in figures 4(a)-(c).

The increase in the length of the shell is seen to adversely affect the stability of the structure and reduce the magnitude of the critical load. As shown by the calculations using Lira SAPR 2018, software based on FEM, the maximally compressed rod bears a higher critical load. A comparison of the corresponding parameter of the most compressed element with critical loading ($q_{cr,b}$) and the structure as a unit ($q_{cr}$) for the three shells with the smallest cross-section of the rods is shown in Table 3.

| $D$ (m) | $t$ (m) | $L/a=12$ | $L/a=30$ | $L/a=48$ |
|---------|---------|----------|----------|----------|
| $140 \times 10^{-3}$ | $4.5 \times 10^{-3}$ | $22.97/3.08$ | $10.98/1.36$ | $4.25/0.73$ |

![Figure 3. The curves of $q$ - L/a ratio dependence (obtained by continual method) and control points of FE calculation](image)

![Figure 4 (a). $L/a=12$](image) ![Figure 4 (b). $L/a=30$](image) ![Figure 4 (c). $L/a=48$](image)
Thus, the larger value of the parameter $q_{cr,b}$ (see Table 3) makes it necessary to use in calculations the value of the lower critical load, at which the shell is already losing its stability.

5. Comparison of grids

The rods arrangement in the cylindrical surface is important. The detailed descriptions and some recommendations for calculation of systems with square and rhombic grids are known. However, various software based on the finite element method is widely used to analyze critical load values. That is why it was decided to create the similar mesh models (see Table 4) and examine them to verify results.

| $B$ (m) | $f$ (m) | $R$ (m) | $E$ (kN/m$^2$) | Steel |
|---------|---------|---------|---------------|-------|
| 41.72   | 14.07   | 22.50   | 2.06 $\times$ 108 | S235  |

The different shape of the intersection of the rods allows to obtain the values of the length of the construction with orthorhombic and triangular mesh with the appropriate sizes and cell numbers (see Table 5).

| $L_r$ (m) | $a_r$ (m) | $L_t/a_t$ | $L_t$ (m) | $a_t$ (m) | $L_t/a_t$ |
|-----------|-----------|-----------|-----------|-----------|-----------|
| 63.90     | 4.26      | 15        | 90.0      | 3.0       | 30        |

Subscript: $r$ – orthorhombic; $t$ – triangular

Here $L$ is the length of the shell; $a$ is the size of the grid (figure 2). Subscript $r$ and $t$ indicate the shape of the cell: $r$ - rhombic; $t$ - triangular Stiffness characteristics are assumed and chosen from the software library (see Table 6).

| $D$ (m) | $t$ (m) | $A$ (m$^2$) | $I$ (m$^4$) |
|---------|---------|-------------|-------------|
| $140\times10^{-3}$ | $4.5\times10^{-3}$ | $19.2\times10^{-4}$ | $44\times10^{-7}$ |

Modeling is performed for a system with a rhombic and triangular grid. The corresponding shell calculations with different parameters along the length have been made. The values of the critical loads and the loss of stability forms are shown in figures 6(a) and 6(b).

The general condition of the structure with two schemes of rods distribution on a cylindrical surface is revealed. In the absence of longitudinal and transverse ribs, the shell with a rhombic grid is more sensitive and vulnerable. The significant difference in the values of the critical loads $q_{cr}$ and the number of half-wave $n$ of the loss-of-stability form confirms this. It can be explained by the fact that the rhombic cell does not have the required rigidity in its plane and is almost geometrically unstable system. The largest energy reserve is characterized by a triangular grid, which has both bending and membrane stiffness at the same time.

Figure 5 (a). $L_r/a_r=15$

The forms of stability loss of shells with possible nets

Figure 5 (b). $L_t/a_t=30$
6. Conclusions
Based on thin shell theory, a mathematical model of cylindrical grid shell structure is built and a technique for the stability analysis of this structure is developed. The analysis of the stability of the cylindrical grid shell structures is performed, and zones of the geometry change are determined depending on the geometrical parameters and the number of half-wave n in the direction of the arc of the circle. The necessary components of multi-element grids rigidity are identified, and the possibility to determine the critical load q_{cr} taking into account the exact ratios of the force and shape-forming characteristics. The paper shows the acceptable agreement between the values of the finite element and analytical analysis. A significant decrease in performance and a decrease in the number of half-wave n of the structural stability loss form with an increase in the number of cells along the generator from 12 to 48 is revealed.

The elimination of the maximally compressed rod from the grid system is investigated, and by comparing the parameters q_{cr}/q_{dl}, the necessity to perform the shell calculation taking into account the lower critical load is revealed. Limit values of B x L dimensions of the grid surface horizontal projection are determined depending on the magnitude of the applied load and different stiffness characteristics of the rods. The range of permissible values of parameters and possible design of the structure is shown. For a given length L and different values of width B, the number of half-waves n varied from 2 to 4. It is found that at B = 35.91 m, the number of half-waves n of stability loss form has reached its limiting value. Based on the results of introducing a fixed length L and changing the width B of the system, the overall dimension condition L>B is recommended. The design of shells with a rhombic mesh is undesirable because of the increased vulnerability and the strong sensitivity of the structure.

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