Torque Ripple Suppression of Permanent Magnet Synchronous Motor Based On Robust Current Injection

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Abstract. In order to improve the performance of permanent magnet synchronous motor (PMSM) servo system and suppress the influence of torque ripple on the control system, a robust current injection control method combining active disturbance rejection controller (ADRC) and current injection is proposed. We designed a current injection module to suppress the torque ripple of the motor and proposed an ADRC to improve the anti-disturbance performance of the system, which ensures the strong robustness and rapidity of the system. The simulation results show that compared with the conventional PI control and PI superimposed current injection, the proposed method can more effectively suppress the torque ripple, while improving the robustness and dynamic response of the system.

1. Introduction

PMSM has been widely used in different fields such as robot, high precision numerical control machine tool, photoelectric turntable, aerospace and so on because of its advantages of high performance, high integration, high air gap magnetic density, high power density, high torque inertia ratio and high efficiency [1]. The high-precision servo drive system requires the motor to have stable and fast speed regulation performance and good stability. However, the torque ripple caused by cogging torque, magnetic flux harmonic and current detection error affects the application of PMSM in high-precision servo control system [2, 3]. Moreover, it will further cause speed pulsation, which will cause vibration and noise during motor operation [4], reduce the speed tracking performance of PMSM, and even damage motor components in severe cases.

There are two types of methods to suppress torque ripple: one is to improve the design of the motor body, and the other is to design and improve the control method. Among them, there are mainly two types of improved motor body design: changing the air gap magnetic density of the permanent magnet and changing the relative air gap magnetic permeability. Such as adopting chute or fractional slot, improving stator winding distribution, improving stator and rotor magnetic circuit, and so on [5]. This method can effectively suppress the torque ripple, especially the cogging torque, but this method not only increases the machining complexity of the motor, but also increases the control cost. The second kind of method focuses on the design and improvement of the controller, without additional hardware,
and has a wide range of applications. Early research scholars proposed a control method for open-loop compensation. By pre-storing the set current value, a lookup table was used to compensate at a specific position of the rotor [6]. However, this method is too dependent on the parameters of the motor. Fluctuations or uncertainties of the parameters will affect the compensation effect. Afterwards, a closed-loop controller is designed in the speed loop or the current loop to compensate [7, 8]. For example, an observer is designed to estimate and compensate the ripple torque online. But these observers can only estimate the torque ripple generated by the electrical part, and cannot observe or suppress the tooth slot torque and load jitter. In addition, the ILC has a good suppression effect on the periodic torque ripple of the motor. In [9], two schemes for minimizing the periodic torque ripple based on ILC were proposed, and had a good suppression effect. But this method is more sensitive to disturbances of non-periodic systems. In [10], an adaptive linear network algorithm was used to calculate the optimal current to minimize torque ripple. But it is complicated to calculate and tedious to implement. In addition, for the measurement error of the sensor, it is mainly compensated from the improvement of accuracy and indirect control. In [11], the disturbance frequency caused by the deviation error of the current sensor is embedded in the controller design based on model prediction to reduce the speed fluctuation.

It can be seen that the engineering implementation of the above method is relatively complicated and it is difficult to be widely applied. We hope to inject a current compensation signal on the basis of motor vector control to suppress torque ripple. It is easy to implement and easier to synthesize. At this point, the robustness of the system is poor when there are parameter disturbances and external load disturbances. ADRC is widely used because of its quick response and strong robustness. Besides it is simple and has a good engineering effect [12]. In this paper, we hope to combine ADRC with current injection to obtain a simple structured controller that is easy to implement in engineering. It can suppress the torque ripple and improve the anti-disturbance capability of the system.

This paper is organized as following sequence: The mathematical model of PMSM and the analysis of torque ripple is introduced in Section 2. The design of robust current injection controller is expanded in Section 3. The simulation results are shown in Section 4. The conclusion of this paper is made in Section 5.

2. Motor model and torque ripple analysis

2.1. Motor model

Assuming that the magnetic circuit of the motor is not saturated and the influence of hysteresis and eddy current loss is ignored, the state space equation of a non-salient PMSM in the $d-q$ coordinate system rotating with the rotor is as follows:

$$
\begin{bmatrix}
i_d \\
i_q \\
i_{\omega}
\end{bmatrix} = 
\begin{bmatrix}
-R_s/L & n_p \omega & 0 \\
n_p \omega & -R_s/L - n_p \psi_f/L & -B/J \\
0 & n_p \psi_f/J & -L/J
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q \\
i_{\omega}
\end{bmatrix} + 
\begin{bmatrix}
u_d/L \\
u_q/L \\
T_l/J
\end{bmatrix}
$$

(1)

Where $i_d$ and $i_q$ are the $d-q$ axis components of the stator current; $u_d$ and $u_q$ are the $d-q$ axis components of the stator voltage; $\omega$ is the rotor angular velocity; $R_s$ and $L$ are the stator resistance and inductance; $\psi_f$ is the permanent magnet flux; $J$ is the moment of inertia; $T_l$ is the load torque; $n_p$ is the number of pole pairs; and $B$ is the coefficient of viscous friction.
According to the above state space equation, it can be seen that PMSM is a nonlinear system with a coupling term. There is a coupling relationship between the currents, which cannot be adjusted independently. Using the control strategy of $i_d = 0$, the expected value of the DC current of the stator armature is always zero, so the original state space equation can be reduced to:

$$\begin{bmatrix}
  i_q \\
  \dot{\omega}
\end{bmatrix} = \begin{bmatrix}
  -\frac{R_s}{L} & -\frac{n_p\psi_f}{L} \\
  \frac{n_p\psi_f}{J} & -\frac{B}{J}
\end{bmatrix} \begin{bmatrix}
  i_q \\
  \dot{\omega}
\end{bmatrix} + \begin{bmatrix}
  \frac{u_q}{L} \\
  -\frac{T_L}{J}
\end{bmatrix}$$  \hspace{1cm} (2)

In this way, the system decoupling of PMSM is realized, and only the q-axis is injected during current injection, which simplifies the control flow.

At this time, the electromagnetic torque equation of PMSM is as follows:

$$T_e = \frac{3}{2} n_p\psi_f i_q = K_i i_q$$  \hspace{1cm} (3)

Where $K_i$ is a constant torque.

The equation of motion of PMSM is as follows:

$$J \frac{d\omega_m}{dt} = T_e - T_L - B\omega_m = K_i i_q - T_L - B\omega_m$$  \hspace{1cm} (4)

Where $\omega_m$ is the mechanical angular velocity.

2.2. Torque ripple analysis

Torque ripple is the main factor affecting the high-precision control performance of PMSM. Analysis of the influence factors of torque ripple is the basis for suppressing torque ripple. The main factors that cause PMSM torque ripple are cogging, magnetic flux harmonics, and current detection errors. The electromagnetic torque is expressed as the fundamental part ($T_{eo}$) and the harmonic part ($\Delta T_e$):

$$T_e = T_{eo} + \Delta T_e$$  \hspace{1cm} (5)

The cogging effect is an inherent characteristic of the motor, which is caused by the stator cogging harmonic magnetic field, completely depends on the mechanical structure of the motor, and has nothing to do with whether the stator is applying current or not. The literature [13] shows that the cogging torque changes periodically, and the changing period depends on the number of cogging, the number of magnetic poles, and the size of the mechanical angular velocity. Cogging torque has a small effect when the motor is running at high speed, and the pulsation amplitude increases when the motor is running at low speed, which has a serious impact.

Another influencing factor of torque ripple is the periodic magnetic flux caused by the non-sinusoidal magnetic flux density distribution in the air gap. The corresponding torque harmonics appear at the 6th, 12th and other multiples [9], which are expressed as:

$$\Delta T_{eo} = \sum_{i=1}^{k} A_{6i} \cos(6i\theta_e)$$  \hspace{1cm} (6)

Where $A_{6i}$ is the amplitude of the harmonic component; $\theta_e$ is the electrical angle.
Torque ripple is also caused by offset errors and gain mismatches in current sensors. DC bias in the stator current measurement will cause ripple torque oscillation at the fundamental frequency [9]. The gain mismatch error is caused by the difference in the gain of the current sensor. It causes the torque to oscillate at a frequency twice the fundamental frequency [14].

In summary, the torque ripple of the motor exists at a frequency higher than the fundamental frequency, including the 1st, 2nd, 6th, and 12th harmonic torques. We design a robust current injection controller to suppress the high-frequency torque pulsation mentioned above.

2.3. Speed pulsation
The transfer function between the mechanical angular velocity of the motor and the electromagnetic torque is as follows:

$$\omega_m(s) = \frac{T_e(s) - T_L(s)}{Js + B}$$

(7)

It can be seen that the speed and torque pulsate at the same frequency, and the speed pulsation reflects the situation of torque pulsation to a certain extent. And because the electromagnetic torque is not easy to measure, the torque ripple is reflected here by the speed pulsation, which avoids measuring the torque or estimating the torque.

3. Design of robust current injection controller
Robust current injection control combines the advantages of ADRC and current injection. ADRC can control the uncertainty of motor model and external load disturbance to ensure the robustness and quick response of the system. The high frequency noise is extracted by current injection to eliminate the torque ripple of the motor.

3.1. ADRC
The design of ADRC includes the following three steps: design of tracking differentiator (TD), design of extended state observer (ESO) and design of nonlinear state error feedback control law (NSEF). Equation (4) is converted into the following form:

$$\dot{\omega}_m = \frac{K_i}{J} i_q - \frac{B}{J} \omega_m - \frac{T_L}{J}$$

(8)

3.1.1. TD. In equation (8), there is no differential of q-axis current, so only the first-order ADRC can be designed without a TD. We design a transition process to speed up the convergence speed and avoid overshoot, which can be expressed as:

$$v = v - \alpha (v - \omega^r_m)$$

(9)

Where \(v\) is the transition process of the reference speed; \(\omega^r_m\) is the reference speed; \(\alpha\) is a adjustable parameter.

3.1.2. ESO. Simplify equation (8) to the following:

$$\dot{\omega}_m = a + bu$$

(10)
Where $a = -\frac{B}{J} \omega_m - \frac{T_L}{J}$ is total disturbance; $b = \frac{K}{J}$. At this time, a second-order extended state observer can be designed as follows:

$$\begin{align*}
e &= z_1 - \omega_m \\
\dot{z}_1 &= z_2 - \beta_1 e + bu \\
\dot{z}_2 &= -\beta_2 e
\end{align*}$$

(11)

Where $z_1$ is the observed value of the speed, and perform a certain noise reduction process and track the speed; $z_2$ is an estimate of the total disturbance including friction, model error and load disturbance; $\beta_1, \beta_2$ are the adjustable parameter of the extended observer.

### 3.1.3. NSEF.

Since the controlled object is a first-order system, a linear error control law is used in this paper. The control law is designed through the transition process of the reference speed and the error of the observation speed. Finally, the control input is obtained by compensating the observed value of the total disturbance of the system. It can be described as:

$$u = K (v - z_i) - \frac{z_2}{b}$$

(12)

Where $K$ is the proportional gain of ADRC.

The above is the design process of the PMSM speed controller, which is summarized as follows:

$$\begin{align*}
v &= v - \alpha (v - \omega_m^{\text{ref}}) \\
e &= z_1 - \omega_m \\
\dot{z}_1 &= z_2 - \beta_1 e + bu \\
\dot{z}_2 &= -\beta_2 e \\
\dot{u} &= K (v - z_i) - \frac{z_2}{b}
\end{align*}$$

(13)

Fig. 1 shows the detailed structure of the ADRC speed controller for PMSM.

Fig. 1. The ADRC speed controller for PMSM.
3.2. Current injection module

The control strategy of $i_d = 0$ is adopted in this paper. It can be seen from equation (3) that, assuming $K_q$ is constant, the electromagnetic torque is proportional to the q-axis current. Therefore, the torque ripple can be suppressed by suppressing the q-axis current ripple. The principle of current injection is to obtain the high-frequency component of the current through a high-pass filter and superimpose it with the feedback current component, so as to eliminate the high-frequency noise of the current and achieve the purpose of suppressing torque ripple. The detailed structure is shown in Fig. 2.

![Fig. 2. The Current injection module for PMSM.](image)

The compensation current $i_{qc}$ provided by the compensator and the reference current $i_q^{ref}$ provided by the speed loop are superimposed to modify the main reference current to obtain the modified reference current $i_q^{ref}$, which is expressed as follows:

$$i_q^{ref} = i_q^{ref} - i_{qc}$$ (14)

The design of the compensator uses a high pass filter and a gain link to generate a compensation signal, which is expressed as follows:

$$i_{qc} = K_{qc} \frac{s}{s + \omega_F} i_q$$ (15)

Where $K_{qc}$ is a compensator gain to modify the size of the compensation signal and $\omega_F$ is the filter cutoff angular frequency. By setting a pair of reasonable values to make the motor generate a proper compensation signal to achieve the effect of suppressing the pulsation.

3.3. Robust current injection controller

When the ADRC and the current injection compensation module are designed respectively, they are combined to obtain the robust current injection controller. Fig. 3 shows the detailed structure of the robust current injection controller for PMSM.
4. Simulation results
In order to verify the effectiveness of the robust current injection controller, we carried out simulation experiments on MATLAB SIMULINK platform, and analysed the torque ripple suppression performance and anti-disturbance performance respectively. When the motor is running at high speed, the torque ripple will be filtered by mechanical filtering to a certain extent, and the torque ripple phenomenon is not obvious, so we only analyse the control performance of the motor at low speed. The parameters of the PMSM used in the simulation are shown in Table 1.

### Table 1. Parameters of the PMSM.

| PMSM Parameters            | Value       |
|----------------------------|-------------|
| Stator phase resistance    | 0.901 Ω     |
| Inductances                | 6.552 mH    |
| Flux linkage               | 0.076855 Wb |
| Moment of inertia          | 0.00774 kg·m² |
| Pole pairs                 | 4           |
| Viscous friction coefficient| 0.0001 N·m/s |
| Torque constant            | 0.046113 N·m/A |

4.1. Analysis of torque ripple suppression performance
In order to verify the effectiveness of the robust current injection controller on the suppression of motor torque ripple, a comparison and analysis with the simulation results of PI and PI-Current Injection was performed. The effectiveness of the control method has been verified under low speed (100 r/min) and lower speed (30 r/min) operating conditions. Considering the influence of magnetic flux harmonics, equation (6) is applied for simulation, and the amplitudes of the 6th and 12th harmonics are set as 8% and 2% of the fundamental components, respectively.

The effectiveness of the speed pulsation suppression method is evaluated by the speed ripple factor (SRF), which is set as a percentage of the peak-to-peak value to the reference value [9], expressed as:

\[
SRF = \frac{\omega_p}{\omega_{ref}} \times 100(\%)
\]

where \(\omega_p\) is the peak-to-peak speed pulsations and \(\omega_{ref}\) is the reference speed.

The parameters of PI controller of speed loop are: \(K_{sp} = 2\), \(K_{s} = 1\); the parameters of PI controller of current loop are: \(K_{sp} = 100\), \(K_{i} = 10\). The parameters of PI-Current Injection are: the same PI
parameters are used, $K_{qc} = -0.7$, $\omega_p = 10 \text{ rad} / s$. The parameters of ADRC are: $\alpha = 0.9$, $\beta_1 = 600$, $\beta_2 = 90000$, $K = 3$.

Under the running condition of speed of 100 r/min, the speed response of the three control methods is shown in Fig. 4. Both PI-Current Injection and the proposed method can reduce the torque ripple phenomenon, but the proposed method has better suppression performance. By this time, the SRF rates are decreased from 7.18% with the PI to 4.13% with the PI-Current Injection and to 2.93% with the proposed method.

Under the running condition of speed of 30 r/min, the speed response of the three control methods is shown in Fig. 5. Under lower speed running conditions, the torque ripple phenomenon is more obvious, the suppression amplitude of the proposed method is larger, and the suppression effect is more obvious. At this point, the SRF rates are decreased from 45.77% with the PI to 17.03% with the PI-Current Injection and to 6.67% with the proposed method.

Fig. 4. Comparison of speed response at 100 r/min.
The above results show that the lower the running speed of the motor, the greater the amplitude of the pulsation. Compared with other methods, the proposed method has better suppression performance, and the effect of suppression is more obvious at lower speeds.

4.2. Analysis of anti-disturbance performance

In order to verify the good anti-disturbance performance of the robust current injection controller, in the presence of torque ripple, the motor is suddenly loaded through the load side to achieve external load disturbance. The simulation results of PI-Current Injection and the proposed method are compared and analyzed.

Under the running condition of speed of 100 r/min, a load disturbance torque of 2 N·m is suddenly applied at 0.5s, the load lasts for 0.02s, and then the load torque is removed. The simulation results of the speed response are shown in Fig. 6. Under the running condition of speed of 30 r/min, external load disturbance is applied in the same way. The simulation results of the speed response are shown in Fig. 7.
Under the running condition of speed of 100 r/min, the maximum value of the speed fluctuation is decreased from 9.43 r/min with the PI-Current Injection to 2.98 r/min with the proposed method, and it returns to the steady state after a minimum time of 0.038s with the proposed method. Under the running condition of speed of 30 r/min, the maximum value of the speed fluctuation is decreased from 8.93 r/min with the PI-Current Injection to 4.38 r/min with the proposed method, and it returns to the steady state after a minimum time of 0.048s with the proposed method. The above results show that compared with PI-Current Injection control, the proposed method has a better anti-disturbance performance at low speed.
5. Conclusion

In this paper, a robust current injection control method combining ADRC and current injection is designed for the torque ripple phenomenon of PMSM servo system. Current injection has a good ability to suppress torque ripple, while ADRC further improves the anti-disturbance performance and dynamic response performance of the system. The simulation results show that compared with PI control and PI superimposed current injection, the proposed method is more effective in suppressing the system torque ripple under low-speed operating conditions. At the same time, it can ensure that the system has a good dynamic response performance and robustness, which improves the speed control performance of the system. The proposed controller has a simple structure and does not require additional calculation costs. It can be easily applied to high-precision servo drive systems.

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