Research Article

Approximation by Szász-Jakimovski-Leviatan-Type Operators via Aid of Appell Polynomials

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The main purpose of the present article is to construct a newly Szász-Jakimovski-Leviatan-type positive linear operators in the Dunkl analogue by the aid of Appell polynomials. In order to investigate the approximation properties of these operators, first we estimate the moments and obtain the basic results. Further, we study the approximation by the use of modulus of continuity in the spaces of the Lipschitz functions, Peetre’s K-functional, and weighted modulus of continuity. Moreover, we study A-statistical convergence of operators and approximation properties of the bivariate case.

1. Introduction

In 1969, Jakimovski and Leviatan introduced a sequence of positive linear operators \( \{ L_n \}_{n \geq 1} \) [1], by using Appell polynomials [2] \( F(v)e^{vy} = \sum_{i=0}^{\infty} P_i(y)v^i \) and defined as

\[
L_n(h; y) = \frac{e^{-y}}{F(1)} \sum_{i=0}^{\infty} P_i(ny)h\left( \frac{s}{n} \right),
\]

(1)

where \( F(1) \neq 0 \), \( F(v) = \sum_{i=0}^{\infty} c_i v^i \), and \( P_i(y) = \sum_{s=0}^{\infty} c_i(y^{s+i}(s-i)) \). For all \( n \in \mathbb{N} \) and \( c_i/F(1) \geq 0 \), the positive linear operators \( L_n \) are defined on \([0, 1]\) given by Wood in [3]. If we take \( h \in \mathcal{E}(0, \infty) \), then an analogue of Szász operators was proved by Jakimovski and Leviatan, where \( \mathcal{E}(0, \infty) \) denotes the set of functions on \([0, \infty)\) such that \( |h(y)| \leq ae^{\nu y} \), where \( a, \kappa \) are positive constants. They established \( \lim_{n \to \infty} L_n(h; y) = h(y) \) uniformly on each compact subset of \([0, 1]\) (see [1, 4]). Precisely, for \( F(1) = 1 \) in (1), the well-known classical Szász operators [5] were obtained defined in 1950 such that

\[
S_n(h; y) = e^{-y} \sum_{s=0}^{\infty} \frac{(ny)^s}{s!} h\left( \frac{s}{n} \right).
\]

(2)

Recently, Szász-Mirakyan operators have been obtained by researchers via the Dunkl generalization in approximation process; for instance, we refer the readers to [6–12]. For more details, related results relevant to the present article in different functional spaces are seen in [13–19] and [20–23]. Sucu [24] introduced Szász-Mirakyan operators by using the new exponential function given in [25] as

\[
epsilon\lambda(y) = \sum_{s=0}^{\infty} \frac{y^s}{\gamma\lambda(s)},
\]

\[
g\lambda(2p) = \frac{2\beta p\lambda! \Gamma(p + \lambda + 1/2)}{\Gamma(\lambda + 1/2)},
\]

(3)

\[
g\lambda(2p + 1) = \frac{2\beta p + 1\lambda! \Gamma(p + \lambda + 3/2)}{\Gamma(\lambda + 1/2)}.
\]

For \( p = 0, 1, 2, \cdots \) a recursion of \( g\lambda \) is given as

\[
g\lambda(p + 1) = \frac{g\lambda(p + 1)}{p + 2\lambda \theta(p + 1)} = g\lambda(p),
\]

(4)

\[
\theta_p = \begin{cases} 0 & \text{if } p = 2s, \ s \in \mathbb{N}, \\ 1 & \text{if } p = 2s + 1, \ s \in \mathbb{N}. \end{cases}
\]
These types of generalizations gave rise to exponential function and generalization of Hermite-type polynomials, expressed in the form of the confluent hypergeometric function (see [25]).

2. Construction of Operators and Estimation of Moments

For every $h \in C_{\infty}[0, \infty) = \{ h \in C[0, \infty) : h(s) = O(s^\theta) \}$ as $s \to \infty$, and all $y \in [0, \infty)$, $\theta > n$, $n \in \mathbb{N}$, $F(1) \neq 0$, $\lambda \geq 0$, we define

$$J_{n\lambda}^*(h : y) = \frac{1}{F(1)e_\lambda(ny)} \sum_{i=0}^{\infty} P_i(ny)h\left(\frac{s + 2i\theta}{n}\right). \quad (5)$$

**Lemma 1.** For all $y \in [0, \infty)$, $P_i(y) \geq 0$, $\lambda \geq 0$, and $F(1) \neq 0$, if we define

$$F(1)e_\lambda(ny) = \sum_{i=0}^{\infty} P_i(ny),$$

$$\sum_{i=0}^{\infty} P_i(ny) = \left(F'(1) + nyF(1)\right)e_\lambda(ny),$$

$$\sum_{i=0}^{\infty} i^2P_i(ny) = \left(F''(1) + (2ny + 1)F'(1) + ny(ny + 1)F(1)\right)e_\lambda(ny),$$

$$\sum_{i=0}^{\infty} i^3P_i(ny) = \left(F'''(1) + 3(ny + 1)F''(1) + (3n^2y^2 + 6ny + 2)F'(1) + ny(n^2y^2 + 3ny + 2)F(1)\right)e_\lambda(ny),$$

$$\sum_{i=0}^{\infty} i^4P_i(ny) = \left(F''''(1) + (4ny + 6)F'''(1) + (6n^2y^2 + 18ny + 11)F''(1) + (4n^3y^3 + 18n^2y^2 + 22ny + 6)F'(1) + ny(n^3y^3 + 6n^2y^2 + 11ny + 6)F(1)\right)e_\lambda(ny). \quad (7)$$

**Proof.**

**Lemma 2.** Let $\lambda \in [0, \infty)$, $F(1) \neq 0$ and take $\phi_r = s^r$ for $r = 0, 1, 2, 3, 4$.

Then, for operators $J_{n\lambda}^*(\cdot; \cdot)$ by (5), we have the following estimates:

$$J_{n\lambda}^*(\phi_0; y) = 1,$$

$$J_{n\lambda}^*(\phi_1; y) = y + \frac{1}{n}\left(F'(1) + 2\lambda\right),$$

$$J_{n\lambda}^*(\phi_2; y) = y^2 + \frac{1}{n}\left(2F'(1) + 4\lambda + 1\right)y + \frac{1}{n^2}\left(3F''(1) + (1 + 4\lambda)F'(1) + 4\lambda^2\right),$$

$$J_{n\lambda}^*(\phi_3; y) = y^3 + \frac{3}{n}\left(F'(1) + 2\lambda + 1\right)y^2 + \frac{1}{n^2}\left(3F''(1) + 6(1 + 2\lambda)F'(1) + 2 + 6\lambda\right)y + \frac{1}{n^3}\left(3F''(1) + 3(1 + 2\lambda)F''(1) + 2\right),$$

$$J_{n\lambda}^*(\phi_4; y) = y^4 + \frac{1}{n}\left(F'(1) + 8\lambda + 6\right)y^3 + \frac{1}{n^2}\left(6F''(1) + 8(1 + 3\lambda)F'(1) + 11 + 24\lambda + 24\lambda^2\right)y^2 + \frac{1}{n^3}\left(6F''(1) + 8(1 + 3\lambda)F''(1) + 11 + 24\lambda + 24\lambda^2\right)y + \frac{1}{n^4}\left(F''(1) + 2(1 + 3\lambda + 6\lambda^2)\right). \quad (8)$$
(1) Take $h = \phi_0$, then
\[
J_{n,\lambda}^*(\phi_0 ; y) = \frac{1}{F(1)e_\lambda(ny)} \sum_{s=0}^{\infty} P_s(ny) = 1.
\]

(2) For $h = \phi_1$
\[
J_{n,\lambda}^*(\phi_1 ; y) = \frac{1}{F(1)e_\lambda(ny)} \sum_{s=0}^{\infty} P_s(ny) \left( \frac{s + 2\lambda \theta_s}{n} \right)
= \frac{1}{nF(1)e_\lambda(ny)} \sum_{s=0}^{\infty} sP_s(ny)
+ \frac{2\lambda}{nF(1)e_\lambda(ny)} \sum_{s=0}^{\infty} \theta_s P_s(ny) \text{ for } k = 0, 1, 2, 3, \ldots
\]
\[
= 0, 1, 2, 3, \ldots = \frac{1}{nF(1)e_\lambda(ny)} \cdot \left( F'(1) + nyF(1) \right) e_\lambda(ny) + \frac{2\lambda}{n}.
\]

(3) For $h = \phi_2$
\[
J_{n,\lambda}^*(\phi_2 ; y) = \frac{1}{F(1)e_\lambda(ny)} \sum_{s=0}^{\infty} P_s(ny) \left( \frac{s + 2\lambda \theta_s}{n} \right)^2
= \frac{1}{n^2F(1)e_\lambda(ny)} \sum_{s=0}^{\infty} s^2P_s(ny) + \frac{4\lambda}{n^2F(1)e_\lambda(ny)}
\cdot \sum_{s=0}^{\infty} sP_s(ny) \theta_s \text{ for } k = 0, 1, 2, 3, \ldots
+ \frac{4\lambda^2}{n^2F(1)e_\lambda(ny)} \sum_{s=0}^{\infty} P_s(ny) \theta_{s}^2 \text{ for } k
\]
\[
= 0, 1, 2, 3, \ldots = \frac{1}{F(1)e_\lambda(ny)} \left( F''(1) \right)
+ (2ny + 1)F'(1) + ny(ny + 1)F(1) \right) e_\lambda(ny)
+ \frac{4\lambda^2}{n^2F(1)e_\lambda(ny)} \left( F'(1) + nyF(1) \right) e_\lambda(ny)
+ \frac{4\lambda^2}{n^2}
+ \frac{4\lambda^2}{n^2F(1)e_\lambda(ny)}
+ \frac{4\lambda^2}{n^2}.
\]

Similarly, we can prove easily (4) and (5).

**Lemma 3.** Let $\psi_j = (\phi_j - y)^j$ for $j = 1, 2, 3$, be the central moments, then
\[
J_{n,\lambda}^*(\psi_j ; y) = \frac{1}{n} \left( F'(1) + 2\lambda \right)
\]
\[
J_{n,\lambda}^*(\psi_2 ; y) = \frac{y + \frac{1}{n^2} \left( F''(1) + (1 + 4\lambda) F'(1) + 4\lambda^2 \right)
\]
\[
J_{n,\lambda}^*(\psi_3 ; y) = \frac{1}{n} \left( 4F'(1) + 8\lambda \right) y^3 + \frac{1}{n^2} \left( -10 F'(1) + 3 \right) y^3
+ \frac{1}{n^3} \left( -4F''(1) + 2(7 + 12\lambda) F'(1) + 16\lambda^2 \right)
+ (2 + 3y + 12\lambda^2) y^2 + \frac{1}{n^2} \left( F''(1) \right)
+ (2 + 3 + 12\lambda) F'(1) + (11 + 24\lambda + 24\lambda^2) F''(1) + 16\lambda^4
\]
\[
+ \frac{1}{n} \left( -4F'(1) \right) - 8\lambda
\]

### 3. Global Approximation

In the present section, we follow Gadžiev [11] and recall the weighted spaces of the functions on $[0, \infty)$, as well as additional conditions under which the analogous theorem of P.P. Korovkin holds for such a kind of functions. Take $y \rightarrow \phi(y)$ be continuous and strictly increasing function with $\sigma(y) = 1 + \phi'(y)$ and $\lim_{y \rightarrow \infty} \sigma(y) = \infty$. Let $B_o[0, \infty)$ be a set of functions defined on $[0, \infty)$, verifying the results
\[
B_o[0, \infty) = \{ h(y) : |h(y)| \leq K_o \sigma(y) \},
\]
where $K_o$ is a constant and depending only on function $h$ and $B_o[0, \infty)$ is space of all continuous as well as bounded functions on $[0, \infty)$. Let the set of all continuous functions on $[0, \infty)$ will be denoted by $C_o[0, \infty)$ and $B_o[0, \infty) \subset C_o[0, \infty)$ equipped with the norm $||h||_{o} = \sup_{y \in [0, \infty)} |h(y)/\sigma(y)|$.

Let us denote
\[
C^{\infty}_{o}[0, \infty) = \left\{ h \in C_o : \lim_{y \rightarrow \infty} \frac{h(y)}{\sigma(y)} = k, k \text{ is positive constant } \right\}.
\]

It is well known that (see [26]) the sequence of linear positive operators $\{L_n\}_{n=1}^\infty$ maps $C_o[0, \infty)$ into $B_o[0, \infty)$ if and only if
Def. 4. For all \( h \in C[0, \infty) \), the modulus of continuity for a uniformly continuous function \( h \) defined by

\[
\omega(h; \delta) = \sup_{|s_1 - s_2| \leq \delta} |h(s_1) - h(s_2)|, \quad s_1, s_2 \in [0, \infty). \tag{16}
\]

For every \( \delta > 0 \) and uniformly continuous function \( h \in C[0, \infty) \), we suppose

\[
|h(s_1) - h(s_2)| \leq \left( 1 + \frac{|s_1 - s_2|}{\delta} \right) \omega(h; \delta). \tag{17}
\]

**Theorem 5.** For all \( h \in [0, \infty) \cap \{ h : y \geq 0, h(y) / \sigma(y) \text{ is convergent as } y \to \infty \} \), operators \( \mathcal{F}_{n,\lambda}^* \) defined in (5) satisfy \( \mathcal{F}_{n,\lambda}^* h \) on each compact subset of \([0, \infty)\), with \( \sigma \) stands for uniform convergence.

**Proof.** From the well-known Korovkin’s theorem (see [27]), for all \( r = 0, 1, 2 \), it is sufficient to see that

\[
\mathcal{F}_{n,\lambda}^* (\phi_r ; y) \to y^r. \tag{18}
\]

In the view of Lemma 2, it is obvious that \( \mathcal{F}_{n,\lambda}^* (\phi_r ; y) \to y^r \) as \( n \to \infty, r = 0, 1, 2 \), which completes Theorem 5.

**Theorem 6.** Let \( \mathcal{F}_{n,\lambda}^* : C^m[0, \infty) \to B_c[0, \infty) \). Then for every \( h \in C^m[0, \infty) \), we have

\[
\lim_{n \to \infty} \| \mathcal{F}_{n,\lambda}^* (h ; y) - h \|_\sigma = 0. \tag{19}
\]

**Proof.** We prove this theorem by applying Korovkin’s theorem so it is sufficient to show that

\[
\lim_{n \to \infty} \| \mathcal{F}_{n,\lambda}^* (\phi_j ; y) - y^j \|_\sigma = 0, \quad \text{for } j = 0, 1, 2. \tag{20}
\]

From Lemma 2, we easily see that

\[
\| \mathcal{F}_{n,\lambda}^* (\phi_0 ; y) - y^0 \|_\sigma = \sup_{y \in [0, \infty)} \left| \mathcal{F}_{n,\lambda}^* (1 ; y) - 1 \right| / \sigma(y) = 0 \text{ for } j = 0. \tag{21}
\]

Similarly, for

\[
\| \mathcal{F}_{n,\lambda}^* (\phi_1 ; y) - y^1 \|_\sigma = \frac{1}{n} \left( \frac{F'(1)}{F(1)} + 2\lambda \right) \sup_{y \in [0, \infty)} \frac{1}{\sigma(y)}. \tag{22}
\]

which imply that \( \| \mathcal{F}_{n,\lambda}^* (\phi_1 ; y) - y \|_\sigma \to 0 \) as \( n \to \infty \). For \( j = 2 \)

\[
\| \mathcal{F}_{n,\lambda}^* (\phi_2 ; y) - y^2 \|_\sigma = \sup_{y \in [0, \infty)} \left| \mathcal{F}_{n,\lambda}^* (\phi_2 ; y) - y^2 \right| / \sigma(y) = \frac{1}{n^2} \left( \frac{2F'(1)}{F(1)} + 4\lambda + 1 \right) \sup_{y \in [0, \infty)} \frac{1}{\sigma(y)}.
\]

which clearly shows that \( \| \mathcal{F}_{n,\lambda}^* (\phi_2 ; y) - y^2 \|_\sigma \to 0 \), whenever \( n \to \infty \).

**Theorem 7.** For all \( h \in C_B[0, \infty) \), operators given by (5) satisfy

\[
\| \mathcal{F}_{n,\lambda}^* (\psi_2 ; y) - y^2 \|_\sigma \to 0, \quad \text{as } n \to \infty.
\]

Proof. We prove Theorem 7 by using the well-known Cauchy-Schwarz inequality and modulus of continuity. Thus, we see that

\[
\| \mathcal{F}_{n,\lambda}^* (\psi_2 ; y) - y^2 \|_\sigma \leq \frac{1}{F(1)} \sup_{y \in [0, \infty)} \left| \frac{\psi'(y)}{\sigma(y)} \right| \sup_{y \in [0, \infty)} \frac{1}{\sigma(y)}. \tag{23}
\]

If we take \( \delta = \delta_n = \sqrt{\mathcal{F}_{n,\lambda}^* (\psi_2 ; y)} \), we get the required result asserted by Theorem 7.

4. Some Direct Results of \( \mathcal{F}_{n,\lambda}^* \)

The present section gives some direct approximation results in the space of \( K \)-functional and in the Lipschitz spaces. We suppose the following.

**Definition 8.** For every \( \delta > 0 \) and \( h \in C[0, \infty) \), we define

\[
K_2(h; \delta) = \inf \left\{ \frac{\| h - \psi \|_{C_\lambda[0, \infty)} + \delta \| \psi' \|_{C_\lambda[0, \infty)}}{\psi, \psi' \in C_\lambda[0, \infty)} \right\}. \tag{26}
\]
where \( C^k_B(0, \infty) \) is defined by

\[
C^k_B(0, \infty) = \left\{ h : h \in C_B(0, \infty), k \in \mathbb{N}; \right. \\
\left. \text{such that } \lim_{y \to \infty} \frac{h(y)}{\sigma(y)} = k < \infty \right\}.
\] (27)

Now, there exists an absolute constant \( C > 0 \) such that

\[
K(h, \delta) < C \left\{ \omega_2 \left( h; \sqrt{\delta} \right) + \min \left( 1, \delta \right) \|h\|_{C^2_B(0, \infty)} \right\},
\] (28)

where \( \omega_2(h; \delta) \) is the second-order modulus of continuity given by

\[
\omega_2(h; \delta) = \sup_{0 < \epsilon < \delta; y \in [0, \infty)} |h(y + 2\eta) - 2h(y + \eta) + h(y)|.
\] (29)

Moreover, the modulus of continuity of order one is

\[
\omega(h; \delta) = \sup_{0 < \epsilon < \delta; y \in [0, \infty)} \left| h(y + \eta) - h(y) \right|.
\] (30)

**Theorem 9.** Let \( h \in C^2_B[0, \infty) \), we define an auxiliary operators \( \mathcal{K}^*_n \) such that

\[
\mathcal{K}^*_n(h; \delta) = \mathcal{J}^*_n(h; \delta) + \lambda(h) - \lambda\left\{ \frac{1}{n} \left( \frac{F'(1)}{F(1)} + 2\lambda \right) \right\}.
\] (31)

Then, for every \( \psi \in C^2_B[0, \infty) \), operators \( \mathcal{K}^*_n \) satisfy

\[
\left| \mathcal{K}^*_n(\psi; y) - \psi(\cdot) \right| \leq \Theta_n(\psi) ||\psi''||,
\] (32)

where \( \Theta_n(\psi) = (\delta_n(\psi))^2 + (1/n^2((F'(1)/F(1)) + 2\lambda)^2
\)

and \( \delta_n(\psi) \) are defined in Theorem 7.

**Proof.** Take \( \psi \in C^2_B[0, \infty) \); then, we easily conclude that

\[
\mathcal{K}^*_n(\phi_0; y) = 1
\]

and

\[
\mathcal{K}^*_n(\phi_1; y) = \mathcal{J}^*_n(\phi_1; y) + \psi(\cdot) + \frac{1}{n} \left( \frac{F'(1)}{F(1)} + 2\lambda \right).
\] (33)

We also know easily

\[
\left\| \mathcal{J}^*_n(h; \delta) \right\| \leq \|h\|.
\] (34)

Therefore,

\[
\left| \mathcal{K}^*_n(h; \delta) \right| \leq \left| \mathcal{J}^*_n(h; \delta) \right| + \|h\|
\]

\[
+ \left\{ \frac{1}{n} \left( \frac{F'(1)}{F(1)} + 2\lambda \right) \right\} \leq 3\|h\|.
\] (35)

From the Taylor series we see

\[
\psi(s) = \psi(y) + (s - y)\psi'(y) + \int_y^s (s - \mu)\psi''(\mu) \, d\mu.
\] (36)

Applying \( \mathcal{K}^*_n \), we have

\[
\left| \mathcal{K}^*_n(\psi; y) - \psi(y) \right| \leq \left| \mathcal{J}^*_n(\psi; y) \right| + ||\psi''||
\]

\[
+ \left| \int_y^s \left( \frac{F'(1)}{F(1)} + 2\lambda \right) \psi''(\mu) \, d\mu \right|.
\] (37)

Since we know

\[
\left| \int_y^s (s - \mu)\psi''(\mu) \, d\mu \right| \leq (s - y)^2 ||\psi''||,
\]

\[
\left| \int_y^s \left( \frac{F'(1)}{F(1)} + 2\lambda \right) \psi''(\mu) \, d\mu \right| \leq \left( \frac{F'(1)}{F(1)} + 2\lambda \right) ||\psi''||.
\] (38)

Therefore, we get

\[
\left| \mathcal{K}^*_n(\psi; y) - \psi(y) \right| \leq \left( \frac{F'(1)}{F(1)} + 2\lambda \right) ||\psi''||.
\] (39)

This gives the complete proof.
Theorem 10. Let \( h \in C_{\beta}[0, \infty) \) and any \( \psi \in C_{\beta}^2[0, \infty) \). Then, there exists a constant \( \Theta > 0 \) such that

\[
|J_{n,\lambda}^*(h; y) - h(y)| \leq \Theta \left( \frac{\sqrt{\Theta_n(y)}}{2} \right) + \min \left( 1, \frac{\Theta_n(y)}{4} \right) \| h \|_{C_{\beta}[0, \infty)} + \omega \left( h; \frac{1}{n} \left( \frac{F'(1)}{F(1)} + 2\lambda \right) \right),
\]

(40)

where \( \Theta_n(y) \) is defined by Theorem 9.

Proof. We prove the result asserted by Theorem 10 in the light of Theorem 9. Therefore, for all \( h \in C_{\beta}[0, \infty) \) and \( \psi \in C_{\beta}^2[0, \infty) \), we get

\[
|J_{n,\lambda}^*(h; y) - h(y)|
\]

\[
= |J_{n,\lambda}^*(h; y) - h(y) + h\left( y + \frac{1}{n} \left( \frac{F'(1)}{F(1)} + 2\lambda \right) \right) - h(y)|
\]

\[
\leq |J_{n,\lambda}^*(h - \psi; y)| + |J_{n,\lambda}^*(\psi; y) - \psi(y)| + |\psi(y) - h(y)|
\]

\[
+ \left| h\left( y + \frac{1}{n} \left( \frac{F'(1)}{F(1)} + 2\lambda \right) \right) - h(y) \right|
\]

\[
\leq 4\| h - \psi \| + \Theta_n(y)\| \psi'' \| + \omega \left( h; \frac{1}{n} \left( \frac{F'(1)}{F(1)} + 2\lambda \right) \right) \right). \tag{41}
\]

Taking infimum over all \( \psi \in C_{\beta}^2[0, \infty) \) and using (26), we get

\[
|J_{n,\lambda}^*(h; y) - h(y)|
\]

\[
\leq 4K_2 \left( h; \frac{\Theta_n(y)}{4} \right) \omega \left( h; \frac{1}{n} \left( \frac{F'(1)}{F(1)} + 2\lambda \right) \right) \omega \left( h; \frac{1}{n} \left( \frac{F'(1)}{F(1)} + 2\lambda \right) \right)
\]

\[
\leq \Theta \left\{ \omega \left( h; \frac{\sqrt{\Theta_n(y)}}{4} \right) \omega \left( h; \frac{\Theta_n(y)}{4} \right) \right\} + \min \left( 1, \frac{\Theta_n(y)}{4} \right) \| h \|_{C_{\beta}[0, \infty)}
\]

\[
+ \omega \left( h; \frac{1}{n} \left( \frac{F'(1)}{F(1)} + 2\lambda \right) \right) \right). \tag{42}
\]

Theorem 11. Let \( 0 < \theta \leq 1 \), then for all \( h \in C_{\beta}[0, \infty) \), operators \( J_{n,\lambda}^* \) satisfy

\[
|J_{n,\lambda}^*(h; y) - h(y)| \leq \omega(h; y)(\delta_n(y))^\theta, \tag{44}
\]

where \( \omega(h; y) \) is the Lipschitz maximal function defined by (43) and \( \delta_n(y) \) by Theorem 7.

Proof. To prove Theorem 11, we use the well-known Hölder inequality by applying (43)

\[
|J_{n,\lambda}^*(h; y) - h(y)|
\]

\[
\leq J_{n,\lambda}^*(|h(s) - h(y)| ; y) \leq \omega(h; y)|J_{n,\lambda}^*\left( |s - y|^\theta ; y \right)
\]

\[
\leq \omega(h; y)\left( J_{n,\lambda}^*\left( \phi_1 |y_1 - y|^\theta ; y \right) \right)^{\frac{2}{\theta}} \left( J_{n,\lambda}^*\left( |y_1 - y|^\theta ; y \right) \right)^{\frac{\theta}{\theta}}
\]

\[
= \omega(h; y)\left( J_{n,\lambda}^*\left( \psi_1 |y| ; y \right) \right)^{\frac{2}{\theta}}. \tag{45}
\]

The proof is complete.

From [28] for an arbitrary \( h \in C_{\beta}[0, \infty) \), the weighted modulus of continuity is introduced such that

\[
\Omega(h; \delta) = \sup_{y \in [0, \infty), \| h \| \leq \delta} \frac{|h(y + \eta) - h(y)|}{(1 + \eta^2)(1 + y^2)}. \tag{46}
\]

The two main properties of this modulus of continuity are \( \lim_{\delta \rightarrow 0} \Omega(h; \delta) = 0 \) and

\[
|h(s) - h(y)| \leq 2 \left( 1 + \frac{|s - y|}{\delta} \right)(1 + \delta^2)(1 + y^2) \cdot (1 + (s - y)^2) \Omega(h; \delta), \tag{47}
\]

where \( s, y \in [0, \infty) \).

Theorem 12. Let the operators \( J_{n,\lambda}^* \) be defined by (5); then for every \( h \in C_{\beta}[0, \infty) \), there exists a constant \( C > 0 \) such that

\[
\sup_{y \in (0, \infty)} \frac{|J_{n,\lambda}^*(h; y) - h(y)|}{\sigma(y)} \leq C \left( 1 + O \left( \frac{1}{n} \right) \right) \Omega \left( h; \sqrt{\frac{1}{n}} \right), \tag{48}
\]

where \( \sigma(y) = 1 + y^2 \) and \( C = (2 + C_1 + 2C_2) \) with \( C_1 > 0, C_2 > 0 \).
Proof. In light of (46), (47), and Cauchy-Schwarz inequality, we prove this theorem. Thus, we see
\[
\left| J_{n,\lambda}^*(h ; y) - h (y) \right|
\leq 2 \left( 1 + \delta^2 \right) \left( 1 + y^2 \right) O(h ; \delta) \left( 1 + J_{n,\lambda}^* \left( (\phi_1 - y)^2 ; y \right) \right.
\]
\[+ J_{n,\lambda}^* \left( \left( 1 + (\phi_1 - y)^2 \right) \frac{\phi_1 - y}{\delta} ; y \right) \left. \right) \right)
\]
(49)
\[
J_{n,\lambda}^* \left( \left( 1 + (\phi_1 - y)^2 \right) \frac{\phi_1 - y}{\delta} ; y \right)
\leq 1 + 2 \left( J_{n,\lambda}^* \left( (\phi_1 - y)^4 ; y \right) \right)^{\frac{1}{2}} \left( J_{n,\lambda}^* \left( \left( \frac{\phi_1 - y}{\delta^2} \right) ; y \right) \right)^{\frac{1}{2}}.
\]
(50)

From Lemma 3, we easily conclude that for any positive $C_1$ and $C_2$
\[
J_{n,\lambda}^* \left( (\phi_1 - y)^2 ; y \right) = O \left( \frac{1}{n} \right) (y + 1)^2 \leq C_1 (y + 1)^2 \text{ as } n \to \infty,
\]
(51)
\[
J_{n,\lambda}^* \left( (\phi_1 - y)^4 ; y \right) = O \left( \frac{1}{n} \right) (y + 1)^4 \leq C_2 (y + 1)^4 \text{ as } n \to \infty.
\]
(52)

Therefore,
\[
\left( J_{n,\lambda}^* \left( \frac{(\phi_1 - y)^2}{\delta^2} ; y \right) \right)^{\frac{1}{2}} = \frac{1}{\delta} \sqrt{O \left( \frac{1}{n} \right) (1 + y)}.
\]
(53)
\[
\left( J_{n,\lambda}^* \left( (\phi_1 - y)^4 ; y \right) \right)^{\frac{1}{2}} \leq C_2 (1 + y)^2.
\]
(54)

Hence, in light of (49), (50), (51), (52), (53) and (54) and choosing $\delta \sqrt{O(1/n)}$, if we take the supremum $y \in [0, O(1/n)]$, we get the result.

5. A-Statistical Convergence

Here, we obtain the A-statistical convergence for the operators $J_{n,\lambda}$ by (5). From [29], we recall the needed notations and notions for A-statistical convergence. Take $G = (D_{nk})$ be a nonnegative infinite summability matrix. For a given sequence $y = (y_k)$, the A-transform of $y$ is denoted by $Gy : (Gy)_n$, where the series converges for each $n$ and defined by
\[
(Gy)_n = \sum_{k=1}^{\infty} y_k D_{nk}.
\]
(55)

The matrix $G$ is said to be regular if $\lim(Gy)_n = L$ whenever $\lim x = L$ and $y = (y_n)$ are said to be a A-statistical convergently to $L$, i.e., $st_G \lim_{n} y_n = L$ if for each $\varepsilon > 0$
\[
\lim_{n} \sup_{|y-L|>\varepsilon} D_{nk} = 0.
\]

The recent work on statistical convergence and statistical approximation, we refer to [30–37].

Theorem 13. Let operators $J_{n,\lambda}$ be defined by 1 and a nonnegative regular summability matrix be $G = (D_{nk})$, then, for every $h \in C_0^\infty(0, \infty)$
\[
st_G \lim_{n} \left\| J_{n,\lambda}^* (h ; y) - h \right\|_{\sigma} = 0.
\]
(56)

Proof. It is enough to show that
\[
st_G \lim_{n} \left\| J_{n,\lambda}^* \left( \phi_j ; y \right) - y^j \right\|_{\sigma} = 0, \text{ for } j = 0, 1, 2.
\]
(57)

From Lemma 2, we conclude that
\[
\left\| J_{n,\lambda}^* \left( \phi_j ; y \right) - y^j \right\|_{\sigma} = \frac{1}{n} \left( F'(1) + 2\lambda \right) \sup_{y \in (0,\infty)} \frac{1}{\sigma(y)}
\]
(58)
\[
\leq \frac{1}{n} \left( F'(1) + 2\lambda \right).
\]

which implies that
\[
st_G \lim_{n} \left( \frac{1}{n} \left( F'(1) + 2\lambda \right) \right) = 0.
\]
(59)

Similarly for $j = 2$
\[
\left\| J_{n,\lambda}^* \left( \phi_2 ; y \right) - y^2 \right\|_{\sigma}
\]
\[
= \frac{1}{n} \left( 2F'(1) + 4\lambda + 1 \right) \sup_{y \in (0,\infty)} \frac{y}{\sigma(y)}
\]
\[+ \frac{1}{n^2} \left( F'(1) + 1 + 4\lambda \right) \frac{F'(1) + 4\lambda}{F(1)} \sup_{y \in (0,\infty)} \frac{1}{\sigma(y)}
\]
\[
\leq \frac{1}{2} \left( \frac{2F'(1) + 4\lambda + 1}{F(1)} \right) + \frac{1}{n} \left( F'(1) + 2\lambda \right).
\]
(60)
which shows that

$$\begin{align*}
st_G - \lim_{n \to \infty} \frac{1}{n} \left( \frac{2F'(1)}{F(1)} + 4\lambda + 1 \right)
&= st_G - \lim_{n \to \infty} \frac{1}{n} \left( \frac{F'(1)}{F(1)} + 2\lambda \right)
&= 0.
\end{align*}$$

Lemma 14 as follows:

For all $y, y_1 \in \mathbb{R}_+^2$ and sufficiently large $n, m \in \mathbb{N}$, we have

$$\mathcal{F}_{n,m}^*(s-y)^2; y, y_1) = O\left(\frac{1}{n} (y + 1)^2\right)$$

$$\leq M(y + 1)^2 as n, m \to \infty,$$

$$\mathcal{F}_{n,m}^*(t-y_1)^2; y, y_1) = O\left(\frac{1}{m} (y_1 + 1)^2\right)$$

$$\leq M(y_1 + 1)^2 as n, m \to \infty,$$

$$\mathcal{F}_{n,m}^*(s-y)^2; y, y_1) = O\left(\frac{1}{n} (y + 1)^4\right)$$

$$\leq M(y + 1)^4 as n, m \to \infty,$$

$$\mathcal{F}_{n,m}^*(t-y_1)^4; y, y_1) = O\left(\frac{1}{m} (y_1 + 1)^4\right)$$

$$\leq M(y_1 + 1)^4 as n, m \to \infty.$$  \hfill \text{(66)}

Lemma 15. If we let

$$\mathcal{H}_{n,\lambda}(g; y, y_1) = \frac{1}{F(1)e_{\lambda}(ny)} \sum_{i=0}^{\infty} P_i(ny) g\left(\frac{s + 2\lambda\theta_i}{n}, y_1\right),$$

$$\mathcal{L}_{m,\eta}(g; y, y_1) = \frac{1}{G(1)e_{\eta}(my)} \sum_{i=0}^{\infty} P_i(my) g\left(y, \frac{t + 2\eta\theta_i}{m}\right).$$ \hfill \text{(67)}

Then, it follows that

$$\mathcal{F}_{n,m}(g; y, y_1) = \mathcal{H}_{n,\lambda}(\mathcal{L}_{m,\eta}(g; y, y_1))$$

$$= \mathcal{L}_{m,\eta}(\mathcal{H}_{n,\lambda}(g; y, y_1)).$$ \hfill \text{(68)}

Proof. We easily see that

$$\mathcal{H}_{n,\lambda}(\mathcal{L}_{m,\eta}(g; y, y_1))$$

$$= \mathcal{H}_{n,\lambda}\left(\frac{1}{G(1)e_{\eta}(ny)} \sum_{i=0}^{\infty} P_i(my) g\left(y, \frac{t + 2\eta\theta_i}{m}\right)\right)$$

$$= \frac{1}{G(1)e_{\eta}(ny)} \sum_{i=0}^{\infty} P_i(my) g\left(y, \frac{t + 2\eta\theta_i}{m}\right) P_i(my_1)$$

$$= \frac{1}{F(1)e_{\lambda}(ny)} G(1)e_{\lambda}(my) \sum_{i=0}^{\infty} P_i(ny) P_i(my) g\left(\frac{s + 2\lambda\theta_i}{n}, \frac{t + 2\eta\theta_i}{m}\right) = \mathcal{F}_{n,m}^*(g; y, y_1).$$ \hfill \text{(69)}
Similarly, we can see \( \mathcal{L}^*_m (\mathcal{H}^*_{n;\lambda} (g; y, y_1)) = \mathcal{L}^*_m (g; y, y_1) \).

Let the weighted function \( \rho \) be \( \rho (y, y_1) = 1 + y^2 + y_1^2 \). Take \( B_\rho (\mathbb{R}^2) = \{ g : |g(y, y_1)| \leq M \rho(y, y_1), \quad M > 0 \} \). We denote the set of \( k \)-times continuously differentiable functions on \( \mathbb{R}^2 = \{ (y, y_1) \in \mathbb{R}^2 : y, y_1 \in [0, \infty) \} \) by \( C^k (\mathbb{R}^2) \). We also denote the class of functions such that

\[
C_\rho (\mathbb{R}^2) = \{ g : g \in B_\rho \cap C_\rho (\mathbb{R}^2) \},
\]

\[
C^k_\rho (\mathbb{R}^2) = \left\{ g : g \in C^k_\rho (\mathbb{R}^2) : \lim_{(y,y_1) \to (\infty, \infty)} \frac{g(y, y_1)}{\rho(y, y_1)} = k, \quad k < \infty \right\},
\]

\[
C^0_\rho (\mathbb{R}^2) = \left\{ g : g \in C^0_\rho (\mathbb{R}^2) : \lim_{(y,y_1) \to (\infty, \infty)} \frac{g(y, y_1)}{\rho(y, y_1)} = 0 \right\}. \tag{70}
\]

Let the norm on \( B_\rho \) be defined as \( \| g \|_\rho = \sup_{(y,y_1) \in \mathbb{R}^2} |g(y, y_1)|/\rho(y, y_1) \).

For all \( g \in C^0_\rho (\mathbb{R}^2) \) and \( \delta_1, \delta_2 > 0 \), the weighted modulus of continuity is given as

\[
\omega_\rho (g; \delta_1, \delta_2) = \sup_{(y,y_1) \in [0,\infty) \times [0,\infty]} \sup_{\delta_1 \leq \delta \leq \delta_2} |g(y + \alpha, y_1 + \beta) - g(y, y_1)|/\rho(y, y_1), \tag{71}
\]

and for any \( r_1, r_2 > 0 \) satisfying the inequality

\[
\omega_\rho (g; r_1 \delta_1, r_2 \delta_2) \leq 4 (1 + r_1) (1 + r_2) (1 + \delta_1^2) (1 + \delta_2^2) \omega_\rho (g; \delta_1, \delta_2), \tag{72}
\]

it also follows that

\[
|g(s, t) - g(y, y_1)| \leq \rho(y, y_1) \rho(|s - y|, |t - y_1|) \omega_\rho (g; |s - y|, |t - y_1|) \leq (1 + y^2 + y_1^2) (1 + (s - y)^2) (1 + (t - y_1)^2) \omega_\rho (g; |s - y|, |t - y_1|). \tag{73}
\]

**Theorem 16.** For all \( g \in C^0_\rho (\mathbb{R}^2) \) and sufficiently large \( n, m \in \mathbb{N} \),

\[
\left| \mathcal{F}^*_n (g; y, y_1) - g(y, y_1) \right| \leq \frac{\xi_{y, y_1}}{\rho(y, y_1)} \left( 1 + O(n^{-1}) \right) \left( 1 + O(m^{-1}) \right) \omega_\rho \tag{74}
\]

where \( \xi_{y, y_1} = (1 + (y + 1) + M_1 (y + 1)^2 + \sqrt{M_1} (y + 1)^3) (1 + (y_1 + 1) + M_2 (y_1 + 1)^2 + \sqrt{M_2} (y_1 + 1)^3) \), and \( M_1, M_2, M_3, M_4 > 0 \).

**Proof.** In view of the above explanation for all \( \delta_1, \delta_2 > 0 \), we see that

\[
|g(s, t) - g(y, y_1)| \leq 4 (1 + y^2 + y_1^2) (1 + (s - y)^2) (1 + (t - y_1)^2) \times \left( 1 + \frac{|s - y|}{\delta_n} \right) \left( 1 + \frac{|t - y_1|}{\delta_m} \right) \times \left( 1 + \frac{|s - y|}{\delta_n} + (s - y)^2 + \frac{|s - y|}{\delta_n} + (s - y)^2 \right) \times \left( 1 + \frac{|t - y_1|}{\delta_m} + (t - y_1)^2 + \frac{|t - y_1|}{\delta_m} + (t - y_1)^2 \right) \omega_\rho. \tag{75}
\]

On applying the operators \( \mathcal{F}^*_n \), we get

\[
\left| \mathcal{F}^*_n (g; y, y_1) - g(y, y_1) \right| \leq \mathcal{F}^*_n (|g(s, t) - g(y, y_1)|; y, y_1) 4 (1 + y^2 + y_1^2) \times \mathcal{F}^*_n \left( 1 + \frac{|s - y|}{\delta_n} + (s - y)^2 + \frac{|s - y|}{\delta_n} + (s - y)^2 ; y, y_1 \right) \times \mathcal{F}^*_n \left( 1 + \frac{|t - y_1|}{\delta_m} + (t - y_1)^2 + \frac{|t - y_1|}{\delta_m} + (t - y_1)^2 ; y, y_1 \right) \cdot \left( t - y_1 \right)^2 y_1 \cdot \left( 1 + \delta_n^2 \delta_m^2 \omega_\rho(g; \delta_n, \delta_m) \right) \cdot \left( t - y_1 \right)^2 y_1 \times \left( 1 + \delta_n^2 \delta_m^2 \omega_\rho(g; \delta_n, \delta_m) \right) \cdot \mathcal{F}^*_n (|s - y|; y, y_1) + \mathcal{F}^*_n (|s - y|; y, y_1) \times \mathcal{F}^*_n \left( 1 + \frac{|s - y|}{\delta_n} + (s - y)^2 ; y, y_1 \right) \times \mathcal{F}^*_n \left( 1 + \frac{|t - y_1|}{\delta_m} + (t - y_1)^2 ; y, y_1 \right) \cdot \mathcal{F}^*_n ((t - y_1)^2 ; y, y_1) \cdot \mathcal{F}^*_n ((t - y_1)^2 ; y, y_1). \tag{76}
\]
From Cauchy–Schwarz inequality, we see

\[
|J_{n,m}^*(g; y, y_1) - g(y, y_1)| \\
\leq 4 \left(1 + y^2 + y_1^2\right) \left(1 + \delta_n^2\right) \left(1 + \delta_m^2\right) \omega_p(g; \delta_n, \delta_m) \\
\times \left[1 + \frac{1}{\delta_n} \sqrt{J_{n,m}^* (y - y_1)^2; y, y_1} + J_{n,m}^* (y - y_1)^2; y, y_1 \right] \\
+ \frac{1}{\delta_n} \sqrt{J_{n,m}^* ((y - y_1)^2; y, y_1) \sqrt{J_{n,m}^* ((y - y_1)^4; y, y_1)}} \\
+ 1 \frac{1}{\delta_m} \sqrt{J_{n,m}^* ((y - y_1)^2; y, y_1) + 1} \\
+ 1 \frac{1}{\delta_m} \sqrt{J_{n,m}^* ((y - y_1)^2; y, y_1)} + 1 \\
+ \sqrt{J_{n,m}^* ((y - y_1)^4; y, y_1)} \right].
\]

(77)

From Lemma 14 we get

\[
|J_{n,m}^*(g; y, y_1) - g(y, y_1)| \\
\leq 4 \left(1 + y^2 + y_1^2\right) \left(1 + \delta_n^2\right) \left(1 + \delta_m^2\right) \omega_p(g; \delta_n, \delta_m) \\
\times \left[1 + \frac{1}{\delta_n} \sqrt{O\left(\frac{1}{n}\right)} (y + 1) + M_1(y + 1)^2 \\
+ \frac{1}{\delta_n} \sqrt{O\left(\frac{1}{n}\right)} \sqrt{M_3(y + 1)} \right] \\
\times \left[1 + \frac{1}{\delta_m} \sqrt{O\left(\frac{1}{m}\right)} (y_1 + 1) + M_2(y_1 + 1)^2 \\
+ \frac{1}{\delta_m} \sqrt{O\left(\frac{1}{m}\right)} \sqrt{M_4(y_1 + 1)} \right].
\]

(78)

By choosing \( \delta_n = O(n^{-1/2}) \) and \( \delta_m = O(m^{-1/2}) \), we arrived to our desired results.

7. Conclusion

The motivation of this present article is to provide the generalized error estimation of convergence rather than the classical Dunkl–Szász–Mirakyan operators. Here, we have defined Szász–Jakimovski–Leviatan operators by using the Appel polynomials with the aid of a new parameter \( \lambda \in [0,\infty) \). These types of approximation are able to give the generalized results and error estimation in comparison to earlier study demonstrations. We have obtained the approximations via the well-known weighted Korovkin’s spaces and investigated approximations in Peetre’s K-functional and Lipschitz spaces with the aid of modulus of continuity. Further, we have also obtained the approximation in \( A \)-statistical convergence. Lastly, we have studied the approximation properties for the bivariate case.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors are grateful to this manuscript and declare that have no competing interest.

Authors’ Contributions

All contents of this research article are checked and agreed to the integrity and accuracy of this manuscript.

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