ANISOTROPY OF POISSON'S RATIO OF DENTIN AND ENAMEL

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Abstract. In this paper, the orientation dependence of the Poisson's ratio of teeth's dentin and enamel on the basis of matrices of elastic constants and the compliance coefficients of hexagonal crystals, such as crystals of dentine hydroxyapatite, was obtained for the first time. The results of calculating the Poisson's ratios of dentin and enamel as a crystalline system with a hexagonal structure are presented in the form of tables and diagrams in the polar and Cartesian coordinate systems. It is shown that the maximum value of the Poisson's ratio of dentin (0.54) is greater than the upper limit for the Poisson's ratio of isotropic materials, including known restoration materials, which in some cases may reduce the quality of restorations. The effective elastic characteristics of dentine and enamel, including Poisson's ratio using different averaging methods, are calculated.

Key words: enamel, dentin, Poisson's ratio, crystals of hydroxyapatite.

1. Introduction
Elastic properties of hard tissues of tooth and hydroxyapatite, as their mineral component, quite often cause interest among researchers [4-7, 12-15, etc.]. However, a number of aspects of this subject, in particular the value of the Poisson's ratio (coefficient of transverse deformation) of dentine and enamel and as anisotropic media with hexagonal symmetry, remain relevant to the end. The study was conducted on the basis of two different methods of calculation [2, 16].

According to modern concepts, the range of possible values of the Poisson's ratio [1, 3] can be negative $\mu<0$ (auxetics), and more than 0.5. For dentin and enamel the "technical" coefficient according to the literature data lies in the range of 0.29-0.33 [9], for example, according to the results of ultrasonic measurements for dentin, it is equal to 0.32, in enamel – 0.28 [15].

2. The study of anisotropy Poisson ratio of dentin and enamel
It is believed that dentin is a biocomposite and consists of approximately 45-70 % of inorganic material [10, 14] in the form of hydroxyapatite crystals. Crystals of hydroxyapatite are located between the collagen fibers and the class of symmetry belong to hexagonal crystal structure. The mineral base of enamel is represented by hexagonal crystals of hydroxide, carbonate, chlorine, fluorapatite, and enamel prisms are the main structural formation. Therefore, there is every reason to consider dentine and enamel as anisotropic media, which means that their elastic properties are described by the matrix of elastic constants $c_{ij}$ or compliance coefficients $s_{ij}$.

The Poisson ratios of such a medium in the general form can be defined as
\[ \mu_{kl} = -\frac{e_{ll}}{e_{kk}}, \]

where \( e_{kl} \) – components of the strain tensor. As a result, hexagonal systems, dentin and enamel are also described by two Poisson’s ratios \( \mu_{31} \) and \( \mu_{32} \) [16]. According to the literature data, the values of the elastic constants of dentin and enamel are well known [12, 13], so knowing them, we can calculate the coefficients of compliance and the Poisson’s ratio of dentin and enamel by the corresponding formulas [8, 16] for hexagonal crystals.

\[ c_{11} + c_{12} = \frac{s_{33}}{s}, \quad c_{11} - c_{12} = \frac{1}{s_{11} - s_{12}}, \quad c_{13} = -\frac{s_{13}}{s}, \]

\[ c_{33} = \frac{s_{33} + s_{12}}{s}, \quad c_{44} = \frac{1}{s_{44}}, \quad c_{66} = \frac{1}{s_{66}}, \quad s = s_{33}(s_{11} + s_{12}) - 2s_{13}^2, \]

\[ \mu_{31} = -[\sin^2 \psi \sin^2 \theta \cos^2 \theta (s_{11} + s_{33} - s_{44}) + (\cos^2 \theta - \sin^2 \theta \cos 2\theta \sin^2 \psi)s_{11} + \sin^2 \theta \cos^2 \psi s_{12}] / s_{33}, \]

\[ \mu_{32} = -[\cos^2 \psi \sin^2 \theta \cos^2 \theta (s_{11} + s_{33} - s_{44}) + (\cos^2 \theta - \sin^2 \theta \cos 2\theta \cos^2 \psi)s_{13} + \sin^2 \theta \sin^2 \psi s_{12}] / s_{33}, \]

where

\[ s_{33} = \cos^4 \theta s_{33} + \sin^2 \theta \cos^2 \theta (2s_{13} + s_{44}) + \sin^4 \theta s_{11}, \]

\( \theta \) и \( \psi \) – Eulerian angles (describing the rotation of an absolutely rigid body in a three-dimensional Euclidean space). Orthogonal system \( x_i \) has the \( O x_3 \) axis parallel to the \( <c>-\)axis of the crystal.

Equation (4) gives Poisson’s ratio for an extension along \( O x_3 \) and a contraction along \( O x_2 \), equation (3) gives Poisson’s ratio for a contraction in the perpendicular direction \( O x_1 \).

For an isotropic solid \( s_{11} = s_{22} = s_{33}, s_{44} = s_{55} = s_{66} = 2(s_{11} - s_{12}), s_{12} = s_{13} = s_{23} \) and equation (3), (4) and (5) are reduced to \( s_{33} = s_{11}, \mu_{32} = \mu_{31} = -s_{12} / s_{11} \), so that Poisson’s ratio is independent of orientation, what we wanted to prove.

Values of elastic constants \( c_{ij} \) and compliance coefficients \( s_{ij} \) of dentin and enamel are presented in Table 1 and Table 2.

**Table 1. Elastic constants of dentin and enamel (GPa)**

| Material | \( c_{11} \) | \( c_{12} \) | \( c_{13} \) | \( c_{33} \) | \( c_{44} \) |
|----------|-------------|-------------|-------------|-------------|-------------|
| Dentin   | 37.00       | 16.60       | 8.70        | 39.00       | 5.70        |
| Enamel   | 115.00      | 42.40       | 30.00       | 125.00      | 22.80       |

**Table 2. Compliance coefficients of dentin and enamel (GPa\(^{-1}\))**

| Material | \( s_{11} \) | \( s_{12} \) | \( s_{13} \) | \( s_{33} \) | \( s_{44} \) |
|----------|-------------|-------------|-------------|-------------|-------------|
| Dentin   | 0.0346      | -0.0145     | -0.0045     | 0.0276      | 0.1754      |
| Enamel   | 0.0104      | -0.0034     | -0.0017     | 0.0088      | 0.0439      |

The results of calculating the Poisson coefficients (directional dependence) of the dentin and enamel as crystal systems with a hexagonal structure are shown in Figure 1 and Figure 2.
\(\mu_{31}(\theta)\) (a-g: \(\psi = 0, 0.25, 0.5, 0.75, 1, 1.25, 1.5\))

\(\mu_{32}(\psi)\) (a-g: \(\theta = 0, 0.25, 0.5, 0.75, 1, 1.25, 1.5\))
\[ \mu_{\theta} (\psi) \text{ (a-g: } \theta = 0, 0.25, 0.5, 0.75, 1, 1.25, 1.5) \]

**Figure 1.** The Poisson’s ratio \( \mu_{31} \) (1-4) and \( \mu_{32} \) (5-8) of dentin for different directions \( \theta \) and \( \psi \).

**Cartesian coordinates**

\[ \mu_{31}(\theta) \text{ (a-g: } \psi = 0, 0.25, 0.5, 0.75, 1, 1.25, 1.5) \]

**Polar coordinates**

\[ \mu_{31}(\theta) \text{ (a-g: } \psi = 0, 0.25, 0.5, 0.75, 1, 1.25, 1.5) \]

\[ \mu_{31}(\psi) \text{ (a-g: } \theta = 0, 0.25, 0.5, 0.75, 1, 1.25, 1.5) \]

\[ \mu_{32}(\psi) \text{ (a-g: } \theta = 0, 0.25, 0.5, 0.75, 1, 1.25, 1.5) \]
Worthy of attention approach based on calculation of extreme values of the Poisson ratio of crystalline media and create isosurfaces of the coefficient in space [2, 11]. The results of calculating the coefficients of the dentin and enamel Poisson as a crystal hexagonal structure are presented in Table 3 and Figure 3.

Table. 3. Extreme values of Poisson’s ratio (μ_{min}, μ_{max}) for dentine and enamel, values of Poisson’s ratio in particular orientations and “anisotropy coefficient” μ_{max} / μ_{min}.

| Material | μ_{max} | μ_{min} | μ_{(2\overline{1}0)(0001)} | μ_{(0\overline{1}0)(2\overline{1}00)} | μ_{(0001)(2\overline{1}00)} | μ_{max} / μ_{min} |
|----------|---------|---------|---------------------------|---------------------------|-----------------------------|------------------|
| Dentin   | 0.54    | 0.13    | 0.16                      | 0.13                      | 0.42                        | 4.15             |
| Enamel   | 0.47    | 0.16    | 0.19                      | 0.16                      | 0.33                        | 2.94             |
Figure 3. Variability of the Poisson's ratio of dentin (a) and enamel (b) in space.

All graphics (Figure 1-3) clearly show the pronounced anisotropic behavior of the Poisson's ratio of dentin and enamel crystals. Its values, depending on the direction in space, vary within a very wide range (4.15 times in dentin and 2.94 times in enamel). This number of times can be called the anisotropy coefficient for the Poisson ratio.

Note the anomaly high value of the maximum value of the Poisson's ratio (0.534-0.54) along a number of directions, which is extremely unusual for materials. This means that the compression of the local areas of the dentin along these directions will increase their volume, and when stretched – on the contrary, decrease. The elastic behavior under load of the restoration material is fundamentally different (Poisson's ratio of the filling material is significantly less than 0.5, for example, according to [17] it is between the values of 0.24 to 0.35 for dental composite materials), so its volume decreases when compressed and increases when stretched. This discrepancy in the deformation behavior at the boundary of heterogeneous media of the filling material and dentin can lead to the formation of overstress domains at this boundary, to the weakening of adhesion of the restoration material with dentin and, as a negative result, the degradation of fixation and the often encountered situation of destruction of direct and sometimes indirect restoration, especially composite materials.

3. The average of the Poisson's ratio of dentine and enamel

The average value of the Poisson's ratio of dentine and enamel can be obtained by applying the formulas (3-5) and the expression [16]:

$$<\mu_{ij}>=\frac{1}{2\pi}\int_{0}^{\pi}\int_{0}^{\pi}\mu_{ij}(\theta,\psi)\sin\theta d\theta d\psi . \quad (6)$$

Numerical calculations of this integral in Mathcad software give the results: 0.312 for dentine and 0.286 for enamel, which is very well consistent with the data of ultrasonic measurements [15].

4. Calculation of the effective value of the Poisson's ratio of dentine and enamel on the basis of their elastic constants and compliance coefficients
On the basis of data obtained on single crystals we can predict the elastic properties of dentin and enamel as microinhomogeneous materials [18].

According to the Voigt averaging, the module of all-round compression and the shear modulus for hexagonal crystals are equal respectively

\[
K_V = \frac{2c_{11} + c_{33} + 2(c_{12} + 2c_{13})}{9}, \\
G_V = \frac{7c_{11} + 2c_{33} - 5c_{12} - 4c_{13} + 12c_{44}}{30}.
\]  

(7)

In the approach proposed by Reuss, the averaging of the compliance tensors is performed, as a result of which for the hexagonal lattice we obtain

\[
\frac{1}{K_R} = 2s_{11} + s_{33} + 2(s_{12} + 2s_{13}), \\
\frac{1}{G_R} = \frac{2(7s_{11} + 2s_{33} + 3s_{44} - 5s_{12} - 42s_{13})}{15}.
\]  

(8)

In many cases, a good agreement with the experimental data gives the proposed by Hill arithmetic mean value, found by the averaging for Voigt and Reuss, namely

\[
K_H = \frac{1}{2}(K_V + K_R), \quad G_H = \frac{1}{2}(G_V + G_R).
\]  

(9)

Keeping in mind that two modules are independent in the isotropic approximation, we can write expressions for \(E_H\) and \(\mu_H\) as functions \(K_H\) and \(G_H\):

\[
\langle E_H \rangle = \frac{9K_H G_H}{3K_H + G_H}, \quad \langle \mu_H \rangle = \frac{3K_H - 2G_H}{6K_H + 2G_H}.
\]  

(10)

Table 4 shows the values of the averaged elastic modules and the Poisson's ratio of dentine and enamel, calculated from the values of elastic constants and compliance coefficients using the described averaging methods used for single-phase hexagonal polycrystals.

| Material  | \(K_V\)  | \(G_V\)  | \(K_R\)  | \(G_R\)  | \(K_H\)  | \(G_H\)  | \(E_H\)  | \(\mu_H\)  |
|-----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Dentin    | 20,11    | 9,59     | 20,03    | 8,21     | 20,07    | 8,90     | 23,26    | 0,31     |
| Enamel    | 62,20    | 33,22    | 62,20    | 30,54    | 62,20    | 31,88    | 81,69    | 0,28     |

Note that the calculated values of \(\mu_H\) are in good agreement with the known experimental data [15] and results of section 3 for \(\mu_H^{ij}\).

**Conclusion**

1. Dentin and tooth enamel are not isotropic media due to the symmetry of their mineral component – hydroxyapatite crystals.
2. The expressed anisotropy of the Poisson's ratio of dentine and enamel is established on the basis of calculations by formulas of elastic constants and compliance coefficients for hexagonal symmetry. The maximum value of the Poisson's ratio of dentine (0.534-0.54) is more than 4 times (4.15) higher than the minimum (0.13).
3. The maximum value of the Poisson's ratio of dentine is higher than the upper limit for the Poisson's ratio of isotropic, including restoration materials used in dentistry, which can locally affect the
quality of restorations. In this context, it is suggested that the established elastic anisotropy of the dentin model with crystal hexagonal symmetry is a clinically undesirable factor.

4. A more careful analysis of the elastic anisotropy of the hard tissues of the tooth as a mineral-organic complex and a microinhomogeneous heterophase system is possible with the involvement of the theory of anisotropic media with a crystallographic texture (despite the fact that all the prisms of minerals in the dentine and enamel have the same or similar crystal structure, they differ in the mutual orientation of the crystallographic axes). Further analysis is based on the knowledge of the spatial distribution of the crystallographic axes of individual mineral prisms.

5. The study of the anisotropy of dentine and enamel as an anisotropic inhomogeneous medium is of practical importance in the study of the strength of tooth tissues and the quality of restorations.

6. The values of the Poisson's ratio of enamel are not fundamentally different from those of dentin. They are still quite large and indicate anisotropy of elastic properties of tooth enamel, although to a lesser extent: \( \mu_{\text{max}} / \mu_{\text{min}} = 2.94 \).

7. Calculation of effective Poisson's ratios of hard tooth tissues based on elastic constants and compliance coefficients and different averaging methods gives results that are in good agreement with empirical data.

References
[1] Belomestnykh V N, Tesleva E P 2003 Poisson’s ratio and the Grüneisen parameter of solids Bulletin of the Tomsk Polytechnic University 306(5) 8-12
[2] Goldstein R V, Gorodtsov A, Lisovenko D S 2011 Variability of elastic properties of hexagonal auxetics DAN 441(4) 468-471
[3] Ivanov G P, Lebedev T A 1964 On the physical sense of the Poisson’s ratio Proceedings of Leningr. polytechnic. in-ta named M.I. Kalinina 236 38-46
[4] Lebedenko I Yu, Arutyunov S D, Muslov S A, Useinov A S 2009 Nanohardness and Young’s modulus of tooth enamel Vestnik RUDN, series Medicine 4 637-638
[5] Lebedenko I Yu, Arutyunov S D, Muslov S A, Useinov A S 2009-2010, Winter Study of nanomechanical properties of tooth enamel Chair 32 24-28
[6] Lebedenko I Yu, Arutyunov S D, Muslov S A, Bolataev Z B 2010 Elastic anisotropy of the hard tissues of the tooth Proceedings of the Eurasian Congress of medical physics and engineering “MEDICAL PHYSICS-2010”, June 21-25, 2010. Moscow State University 1 279-280
[7] Lebedenko I Yu, Arutyunov S D, Useinov A S, Muslov S A, Brandt N N, Apresyan S V Study of the effect of ultralow (4.2 K) temperatures on the nanomechanical properties of tooth enamel 2010 Proceedings of the Eurasian Congress of medical physics and engineering “MEDICAL PHYSICS-2010”, June 21-25, 2010. Moscow State University 4 Part I 323-324
[8] Nye D 1960 Physical properties of crystals M: IL 385 p
[9] Pertsov S S, Stureva G M, Muslov S A, Sinitzyn A A, Korneev A A, Zaitseva N V 2017 The basics of biomechanics for the dentists M: MSMSU 115 p
[10] Khystiktuyev B S, Khystiktueva N 2004 Biochemistry of the oral cavity Chita 84 p
[11] Goldstein R V, GorodtsovV A, Komarov A M, Lisovenko D S 2017 Extreme values of the shear modulus for hexagonal crystals Scripta Mater. 140 55-58 Doi: 10.1016 / j.scriptamat.2017.07.002
[12] Katz J L, Ukraincik K 1971 J. Biomech. 4 221. Doi: 10.1016 / 0021-9290 (71)90007-8
[13] Katz J L 1971 J. Biomech. 4 455. Doi: 10.1016 / 0021-9290 (71)90064-9
[14] Lees S, Davidson C L 1977 The role of collagen in the elastic properties of calcified tissues J. Biomechanics 10(8) 473-86
[15] Lees S et al 1972 Anisotropy in hard dental tissues J. Biomech. 5(6) 557-64
[16] Povolo F et al 1983 Poisson’s ratio in zirconium single crystals. J. Nuclear Materials 118(1) 78-82
[17] Sew Meng Chung et al 2004 Measurement of Poisson’s ratio of dental composite restorative materials. Biomaterials 25 2455–2460
[18] Shermergor T D 1977 *Theory of elasticity of microinhomogeneous mediums* Moscow Science 399 p