Analysis of the absorption line profile at 21 cm for the hydrogen atom in the interstellar medium

D Solovyev

Department of Physics, St. Petersburg State University 198504, St. Petersburg, Russia

E-mail: solovyev.d@gmail.com

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Abstract
This paper analyzes the absorption line profile at 21 cm for the hydrogen atom in the interstellar medium (ISM). The hydrogen atom is treated as a three-level system illuminated by a powerful light source at neighboring resonances corresponding to the hyperfine splitting of the ground state and Lyman-α (Lyα) transition. The field acting upon the resonances gives rise to physical processes, which can be explained as interfering pathways between different transitions. The paper considers particular cases when the 21 cm line profile is substantially modified by the Lyα transition. A correction to the optical depth is introduced as a result of theory. It is shown that the correction can be considerable and should be taken into account when determining the column density of hydrogen atoms in the ISM. The paper also deals with the effects of non-Doppler broadening and frequency shift.

Keywords: atomic physics, hydrogen atom, interstellar medium, 21 cm line profile

(Some figures may appear in colour only in the online journal)

1. Introduction
Investigation of the interstellar medium (ISM) is of extreme importance for understanding the factors and mechanisms responsible for the formation of gas clouds and dust complexes and their part in the evolution of stars. Measurements dealing with radio-loud sources can provide information on the structure and physical conditions of galaxies. The observations of hydrogen clouds are the most probable candidate as they cover the greater part of the interstellar gas. The 21 cm absorption line in the hydrogen atom has a special meaning in investigations of this type, allowing the determination of the distribution and kinematical properties of neutral hydrogen (HI). Since direct imaging is restricted by large observing time requirements at all wavebands, optical imaging studies are complicated [1]. Inasmuch as the ISM is transparent for radio frequencies, this makes the HI 21 cm line the obvious target for these studies.

At the same time, damped Lyman-α (Lyα) systems are of particular interest as high atomic hydrogen HI column density absorbers. Lyα absorption can be used as a very sensitive probe of the HI column in clusters of galaxies [2]. The large cross-section for Lyα transition makes the technique the most sensitive method for detecting baryons at any redshift [3]. Lyα (whose profile itself is controlled by the kinetic temperature) provides a more effective coupling between the spin temperature and the kinetic temperature for high-density cloud illuminated by a powerful source of light [4]. It is possible to detect the damping wings of the Lorentzian component of the absorption profile from about the \( \sim 10^{19} \text{ cm}^{-2} \) column density, attaining their maximum in the damped Lyα systems [3]. At low densities of the ISM, collisions are inefficient for lowering the spin temperature. If Lyα radiation penetrates the HI without heating it, it can actually lower the spin temperature so that the 21 cm line becomes a stronger absorption feature. Thus, the 21 cm absorption and Lyα absorption line profiles provide two independent tools for the investigation of the ISM. Yet, investigation in the absorption 21 cm and Lyα lines cannot be considered separately. For example, the author of [5] has considered the populations of the magnetic sublevels for hyperfine splitting of the ground state 2\( \text{S}_1/2 \) in the hydrogen atom, whereas [5] investigated the atomic orientation depending on
the intensity, spectrum, angular distribution, and polarization of the incident optical and radio emission.

In view of the above, this paper focuses on the interstellar HI atom subjected to an external field at frequencies of hyperfine energy splitting of the ground state, 21 cm, and the Ly\textsubscript{α} transition. The description of such an atom-field system can be reduced to the examination of the three-level atom.

The main constraints in our analysis arise due to describing the hyperfine structure of the ground state without accounting for the fine and hyperfine structure of the excited 2p state. However, the approximation can be justified by the large value of the Ly\textsubscript{α} transition rate and approximate equivalence of the one-photon 1s \rightarrow 2p transition probabilities between the fine and hyperfine sublevels in the hydrogen atom.

The paper provides a detailed analysis of the absorption profile at the 21 cm line for the hydrogen atom in the ISM. To this end, the atom-field system is described within the framework of density matrix formalism [6]. The hydrogen atom is treated as a three-level cascade system: two hyperfine sublevels of the ground atomic state and the excited 2p state, see figure 1.

The unperturbed 21 cm line profile is shown to arise at a negligible field strength of the neighboring transition (Ω\textsubscript{c} \rightarrow 0). However, the profile can be modified significantly by the Ly\textsubscript{α} transition induced in the field of a powerful light source. In this case, the absorption process cannot be thought of as a one-photon transition between the hyperfine sublevels of the ground state in the hydrogen atom. The developed theory allows finding a correction to the optical depth. The correction correlates directly with the determination of the column density of the hydrogen atom in the ISM. The paper presents certain events where the correction is of significance, some of them just illustrating the importance of the analysis.

2. Correction to the optical depth via the evaluation of three-level Ξ scheme

Recently, the density matrix formalism was applied to study the effect of electromagnetically-induced transparency (EIT) in hydrogen atoms under conditions corresponding to the recombination era of the Universe [7, 8]. It was shown that the EIT phenomenon could lead to corrections in the order of 1% in the observed cosmic microwave background. To study the absorption line profile corresponding to the 21 cm transition in the hydrogen atom, the present work draws on the same formalism.

A theoretical description of an atom within the three-level cascade Ξ-scheme approximation can be found in [9, 10], which employed the density matrix formalism. The example of the rubidium atom was used to investigate the EIT phenomenon in [11], where the physical picture of the effect was rendered in terms of interfering multiphoton transition pathways. Based on the formalism given in [9–11], the influence of the EIT phenomenon on the optical depth determination for the interstellar hydrogen atom is investigated. The three-level Ξ scheme in hydrogen atoms is as follows: state [1] is the lower and state [2] is the upper hyperfine sublevel of the ground 1s state, with state [3] representing the excited 2p level. The absorber is assumed to be subjected to the probe and controlled fields with the corresponding frequencies ω\textsubscript{p} \rightarrow ω\textsubscript{21} \rightarrow [1] \rightarrow [2] and the Ly\textsubscript{α} transition ω\textsubscript{L} \rightarrow ω\textsubscript{32} \rightarrow [2] \rightarrow [3].

The set of equations for the ladder scheme is

\[ \rho_{21} = \frac{i}{2} \left( \frac{\Omega_p^* (\rho_{32} - \rho_{11}) - \Omega_p \rho_{31}}{\gamma_{31} - i\delta_p} \right), \]
\[ \rho_{32} = \frac{i}{2} \left( \frac{\Omega_p (\rho_{33} - \rho_{22}) + \Omega_p^* \rho_{32}}{\gamma_{32} - i\delta_p} \right), \]
\[ \rho_{31} = \frac{i}{2} \left( \frac{\Omega_p (\rho_{32}^* - \rho_{12}^*)}{\gamma_{31} - i(\delta_p + \delta_c)} \right), \]
\[ \rho_{22} = \frac{i}{2\Gamma_2} \left( \frac{\rho_{32}^* - \rho_{12}^*}{\gamma_{32} - i\delta_c} \right), \]
\[ \rho_{33} = \frac{i}{2\Gamma_3} \left( \frac{\rho_{32}^* - \rho_{12}^*}{\gamma_{31} - i\delta_c} \right), \]

(1)

where detunings for the probe and controlled fields are \( \delta_p = \omega_p - \omega_{21}, \delta_c = \omega_c - \omega_{32} \), respectively, and \( \omega_{21}, \omega_{32} \) represent the exact values of corresponding transitions. The Rabi frequencies are denoted with \( \Omega_p = 2d_{23} E_c / \hbar \) and \( \Omega_p^* = 2\mu_3 B_p / \hbar \). Since the transition between the hyperfine sublevels corresponds to the magnetic dipole M1 emission/absorption, the Rabi frequency \( \Omega_p^* \) is written in terms of the magnetic field strength \( B_p \) and magnetic moment \( \mu_3 \). \( E_c \) represents the electric field strength for Ly\textsubscript{α} transition and \( d_{ij} \) is the dipole matrix element. The wave functions for \[3\], \[2\], and \[1\] states can be taken as the solution of the Schrödinger equation. In the absence of collisions, \( \gamma_j = (\Gamma_j + \Gamma_i)/2 \), where \( \Gamma_i \) is the natural width of the \( i \)th level. The set of equations (1) is written in the steady-state and rotating wave approximations, see [9–11].

With the use of equation (1), the monochromatic absorption coefficient at frequency \( \omega_i \) can be defined as

\[ k = \frac{Nd_j^2 \omega_j}{2\pi \Omega_j} \text{Im} \{ \rho_j \}, \]

(2)

where \( z_0 \) is the vacuum permittivity, \( N \) is the number of atoms and \( \Omega_j \) is the corresponding Rabi frequency. Taking into account the relation \( k = k_0(\omega) \), where \( k \) is the integrated line

![Figure 1](image-url)
absorption coefficient and \( \phi(\omega) \) is the normalized line profile, the monochromatic optical depth is
\[
d\tau(\omega_i) = -\frac{\tilde{k}(\omega_i) dl}{l},
\]
where \( l \) is the distance along the ray
[12].

In the ordinary case the monochromatic optical depth corresponds to the one-photon resonant process that reduces to the evaluation of the two-level atomic system. Then the one-photon absorption process is described by the Lorentz line profile:
\[
\text{Im} \{ \rho_{21}^{(0)} \} = -\frac{\gamma_{21} \Omega_p^2}{\delta_p^2 + \gamma_{21}^2},
\]
where \( \delta_p \) can be considered as a variable. A more accurate solution of equation (1) corresponds to the accounting of the second field acting upon the adjacent resonance. Then, in the limit of the weak probe field [10], matrix element \( \rho_{21} \) in the first order of the probe field and in all orders of the control field is
\[
\rho_{21} = \frac{i \Omega_p / 2}{i \delta_p - \gamma_{21} + \gamma_1^2 / 4 + \frac{i}{\delta_p + \delta_c - \gamma_1}},
\]
Expression (5) depends on the field parameters \( \Omega_p \) and \( \delta_c \) and reduces to equation (4) in the limit \( \Omega_p \to 0 \), i.e. when the influence of the field on adjacent resonance is negligible. In this case, the corrections to the ‘ordinary’ determination (4) can be found via the series expansion in \( \Omega_p \) at zero detunings \( \delta_p \) and \( \delta_c \). Then the transition amplitudes associated with \( [1] \to [2] \) and \( [2] \to [3] \) pathways result in destructive interference and the reduction of the total probability that a probe photon will be absorbed [11].

However, the series expansion in Rabi frequencies cannot be employed in our case due to the smallness of level width \( \Gamma_2 \approx 2.85 \times 10^{-15} s^{-1} \). Nonetheless, the imaginary part of \( \rho_{21}^{(0)} \) can be separated out
\[
-\text{Im} \{ \rho_{21} \} \equiv -\text{Im} \{ \rho_{21}^{(1)} \} - \text{Im} \{ \rho_{21}^{(2)} \} = 
\]
\[
\frac{\gamma_{21} \Omega_p^2 / 2}{\left( \delta_p - \frac{\gamma_1 \Omega_p^2 / 4}{\delta_p + \delta_c \delta_c^2 + \gamma_1^2} \right)^2 + \left( \gamma_1 \Omega_p^2 / 4 \right)^2}
\]
\[
+ \frac{\gamma_3 \Omega_p^4 \Omega_2^2 / 8}{(\delta_p - \frac{\gamma_1 \Omega_p^2 / 4}{\delta_p + \delta_c \delta_c^2 + \gamma_1^2})^2 + \left( \gamma_1 \Omega_p^2 / 4 \right)^2}
\]
\[
\left( \delta_p - \delta_c \right)^2 + \gamma_1^2
\]
The first term here represents the one-photon \( [1] \to [2] \) (21 cm) absorption process, and the second term can be associated with the additional process \( [1] \to [2] \to [3] \to [2] \) [11]. In absence of the second field \( \Omega_c = 0 \), the second term in equation (6) vanishes, and the ordinary definition (4) can be found. The line profiles corresponding to equations (4) and (6) are given schematically in figures 2 and 3, respectively. In particular, figures 2 and 3 show that the contribution arising via the additional pathways leads to the distortion of the line profile in the vicinity of zero detuning \( \delta_c \).

Thus, the absorption coefficient and the optical depth, respectively, cannot be described by the single Lorentz contour equation (4) with subsequent transformation to the Voigt profile. The Voigt fitting, in this case, is overabundant and covers the physical processes occurring in the medium illuminated with the radiation from a powerful light source at the adjacent resonances. It can also be noted that the first term corresponding to the absorption at the 21 cm line \( [1] \to [2] \) transition) shows the line profile to be broadened and shifted \( a \) \( p\)riori.
3. Non-Doppler broadening and frequency shift

The section deals with the effects of absorption line broadening and frequency shift for an atom at rest and the assumption of \( \Omega \) smallness.

3.1. Non-Doppler broadening

The non-Doppler line broadening for the \([1] \rightarrow [2]\) transition follows from the denominator in the first term and is proportional to \( \gamma_c \). This broadening can be expressed as an additional term to natural width \( \gamma_{21} \):

\[
\gamma = \gamma_{21} + \frac{\gamma_{31} \Omega_c^2 / 4}{(\delta_p + \delta_i)^2 + \gamma_{31}^2} = \gamma_{21} + \gamma_{\text{broad}}.
\]

The maximum value of \( \gamma_{\text{broad}} \) is attained for the exact two-photon resonance \( \delta_p + \delta_i = 0 \):

\[
\gamma_{\text{broad}} = \frac{\Omega_c^2}{4 \gamma_{31}}.
\]

For a very powerful light source and small distances between the absorber and the source, it can be expected that the value of \( \gamma_{\text{broad}} \) is possibly larger than the natural level width \( \gamma_{21} \).

Taking into account the motion of the interstellar gas cloud, we can find that the resonant frequency should be shifted. This Doppler shift leads to \( \delta_i \rightarrow \delta_i + \frac{c}{\lambda} \omega_c \) [11], where \( c \) is the speed of light. The speed of hydrogen clouds can be of the order of a few hundred km s\(^{-1} \) [13, 14] and, in some cases, as high as 1000 km s\(^{-1} \) [15]. Then the sum of detunings \( \delta_p + \delta_i \) can be estimated as \( (10^{-3} - 10^{-2}) \omega_c \sim (10^4 - 10^5) \gamma_{31} \), where \( \gamma_{31} = \frac{1}{2} \Gamma_p = \frac{1}{2} \omega_c \cdot 10^{2} \). Thus, the Doppler shift leads to the suppression of \( \gamma_{\text{broad}} \). Nonetheless, since the emission spectrum of the source is of a continuum nature, the case of the exact two-photon resonance can always be singled out. It should be underscored that this discussion corresponds to \( \gamma_{\text{broad}} \) and does not cancel the Doppler broadening leading to the Voigt profile.

3.2. Frequency shift

Equation (6) also allows finding the frequency shift for the transition \([1] \rightarrow [2]\). To this end, detuning \( \delta_p \) can be considered as the ‘scanning’ parameter (variable). Then the resonance condition reads

\[
\delta_p = \frac{(\delta_p + \delta_i) \Omega_c^2 / 4}{(\delta_p + \delta_i)^2 + \gamma_{31}^2} = 0.
\]

Now, the frequency shift is zero for the exact two-photon resonance, \( \delta_p + \delta_i = 0 \). In the case when the detuning of the two-photon resonance \( \delta_p + \delta_i = \gamma_{31}/2 \), the frequency shift can be found to be

\[
\delta_{\text{shift}} = \frac{\Omega_c^2}{4 \gamma_{31}}.
\]

Here, level width \( \gamma_{31} \) acts as a natural parameter for the atomic resonant excitation.

Another result arises in the assumption that \( \delta_p \sim \gamma_{31} \ll \delta_i \sim \gamma_{31} \) (one-photon resonances). Then, neglecting \( \delta_p \) in the second term of equation (12), the frequency shift is

\[
\delta_{\text{shift}} = \frac{\delta_i \Omega_c^2}{4 \delta_i^2 + 4 \gamma_{31}^2}.
\]

Here, we can take into account the motion of the hydrogen cloud by using the parameter \( \beta \): \( \delta_i = \nu/c \cdot \omega_c \equiv \beta \gamma_{31} \). Therefore,

\[
\delta_{\text{shift}} = \frac{\Omega_c^2}{4 \gamma_{31}} \left( 1 + \beta^2 \right) \approx \frac{\Omega_c^2}{4 \gamma_{31}} \frac{1}{\beta}.
\]

The shift is negligibly small, and the maximum shift can be attained for the two-photon resonance with the detuning being \( \delta_p + \delta_i = \gamma_{31}/2 \), see equation (13).

4. Numerical results

To evaluate the contribution of non-Doppler broadening, frequency shift, and correction to the optical depth, see equations (11), (14), and (9), respectively, one needs to find the Rabi frequency \( \Omega_c \). This can be done via the flux density or luminosity of the light sources and the distance between the source and the absorber. To this end, the observation data of damped Ly\(\alpha \) systems at the 1216 Å line in hydrogen [15–31] were used. Finding the distance employed the following expression:

\[
r = \frac{c}{H_0} (z_{\text{em}} - z_{\text{abs}}),
\]

where \( H_0 = 2.3 \cdot 10^{-18} \text{ s}^{-1} \) is the Hubble constant, and \( z_{\text{em}} \) and \( z_{\text{abs}} \) are the redshifts of the source and the absorber, respectively. The radiation intensity at the absorber can be defined as

\[
I_{\text{abs}} = \frac{L_s}{4\pi(1 + z_{\text{em}} - z_{\text{abs}})^2 r^2}
\]

where \( L_s \) is the star luminosity (measured in units of W/Hz), which is independent of the distance. For the observed flux
Table 1. The first column contains the names of sources. The second and third columns represent the redshift of the star and the absorber, respectively, where the 21 cm absorption in conjunction with \( \text{Ly}_\alpha \) absorption were observed. The next column shows the density flux at 1.4 GHz frequency (hyperfine splitting of the ground state in the hydrogen atom, \( |1\rangle \leftrightarrow |2\rangle \) transition in the present calculations). The fifth column lists the values of density flux and luminosity at \( \text{Ly}_\alpha \) frequency. The values of hydrogen velocity at the 21 cm line are given in the sixth column. Finally, the optical depth values are given in the last column of the table. References to the data used are given in square brackets.

| Name   | \( z_{em} \) | \( z_{abs} \) | \( S_{Ly\alpha, Jy} \) | \( log(L_{\nu \alpha}, \text{W} \cdot \text{Hz}^{-1}) \) | \( v_\nu (\text{km} \cdot \text{s}^{-1}) \) | \( \tau_\nu \) |
|--------|--------------|--------------|-----------------|---------------------------------|-------------------|--------|
| 0235 + 164 | 0.94     | 0.523 869 | 1.7 [16] | \( log(\nu_\nu) \approx -12.5 \) [17] | 125 [18] | \( \left( \frac{1}{2} \right) \int \pi_\nu = 13 \pm 0.6 \) [18] |
| 3C 190 | 1.194 6    | 1.195 65 | 2.47 [19] | 0.17 Jy [19] | -37.1 [21] | 0.002 7 \( \pm 0.000 2 \) [21] a |
| 3C 216 | 0.668     | 0.63    | 3.4 [19] | 22.699 [20] | 102 [15, 22] | 0.38 [23] |
| J0414 + 0534 | 2.636 5   | 0.958 6  | 1.82 [24] | 22.188 [20] | 205 [25] | 0.021 [2(16)] [25] |
| J0414 + 0534 | 2.636 5   | 2.635 34 | 3.31 [26] | 22.188 [26] | -94 [26] | (0.015 \( \pm 0.002 \)) [26] |
| 0902 + 343 | 3.398     | 3.396 8  | 1.2 [27] | 22.422 [20] | 120 [27, 28] | - |
| 3C 49  | 0.621     | 0.620 7  | 7.28 [29] | 20.777 [20] | -138 [29] | 0.036 \( \pm 0.003 \) [29] b |
| 3C 286 | 0.849     | 0.692 153 | 14.7 [19] | 2.73 Jy [19] | 4.2 [31] | 0.280 \( \pm 0.004 \) [31] c |
| 0118–272 | 0.559   | 0.558    | 0.83 [19] | 0.95 Jy [19] | - | \( log[N_H] = 20.3 \) [19] |
| 0045–331 | 2.570    | 2.562    | 0.63 [32] | 0.56 Jy [32] | - | \( log[N_H] = 20.6 \) [32] |
| 0537–286 | 3.104    | 2.976    | 0.862 [19] | 0.90 Jy [19] | - | (0.41 \( \pm 0.22 \)) \( \cdot 10^{12} \) |
| 0957 + 561A | 1.413   | 1.391    | 0.59 [19] | 0.15 Jy [19] | 25 [33] | \( N_H = 7 \cdot 10^{13} \pm 30\% \) [34] |
| 0248 + 430 | 1.31    | 0.393 9  | 1.4 [19] | 1.5 Jy [19] | 40 [35] | \( log[N_H] = 21.25, \gamma < 0.2 \) [36] |
| 0336–017 | 3.197    | 3.061 9  | 0.60 [19] | 0.15 Jy [19] | 13 | \( log[N_H] = 21.3 \pm 0.1 \) [38] |
| 0528–250 | 2.813    | 2.811 0  | 1.16 [19] | 0.59 Jy [19] | 5 [37] | \( log[N_H] = 19.37 \pm 0.08, \tau_L \approx 150 \) [39] d |
| 2128–123 | 0.501    | 0.430    | 1.8 [19] | 0.7 Jy [19] | 75 [39] | \( log[N_H] = 19.37 \pm 0.08, \tau_L \approx 150 \) [39] d |

a The component with the poorest accuracy is taken from [21].
b Just one of the components from [29] is considered.
c Cloud 3 is treated in accordance with data [31].
d Optical depth at the Lyman limit [39].

5. Analysis of results

Typically, astrophysical investigations of absorption lines employ the one-photon profile. Within the framework of density matrix formalism, the one-photon absorption line profile can be derived in the two-level approximation, see equation (4), for an atom in an external field. According to the approximation, the monochromatic absorption coefficient and, hence, the optical depth can be defined via the imaginary part of the density matrix element, see equations (2) and (3). In this case, the definition of the optical depth and all the ensuing physical quantities leads to the results of the ordinary theory. However, section 2 demonstrated that this was the case of the zeroth approximation, when the one-photon absorption process is considered for an isolated transition in the atom. Accounting for the absorption/emission processes that occur at adjacent transitions leads to a substantial modification of the line profile. This paper employs the three-level approximation for the atom. Within the theory [7–11], the imaginary part of a density matrix element fails to correspond to the isolated transition and strongly depends on the field parameters defined for the adjacent resonance, see equation (5).

5.1. Correction to the optical depth

The absorption line profile of the \( |1\rangle \leftrightarrow |2\rangle \) transition is shown in the paper to be formed by two contributions: \( \text{Im} \{ \rho_{12}^{(1)} \} \) and \( \text{Im} \{ \rho_{12}^{(2)} \} \), see equation (6). The additional term in the line profile is proportional to \( \Omega_p \) and \( \Omega_u \). Rabi
Table 2. The first column contains the name of sources in correspondence to table 1. The second column gives the values evaluated with equation (11) for non-Doppler broadening. The frequency shift $\delta_{\text{shift}}$ equation (15) for the $|1\rangle \leftrightarrow |2\rangle$ transition is represented in the third column. The fourth column lists the relative contributions of $\delta \tau$ at detunings $\delta_{\nu}\ll\delta_{\nu} = \gamma_{21}$. Values of $\delta \tau$ are given in the last column, in which the second subrow contains the values of correction to the optical depth listed in table 1.

| Name       | $\gamma_{\text{broad}}$ in s$^{-1}$ equation (11) | $\delta_{\text{shift}}$ in s$^{-1}$ equation (15) | $\delta \tau$ at $\delta_{\nu} = \gamma_{21}$ | $\delta \tau_{0}$ | $\delta \tau_{0} : \tau_{0}$ |
|------------|---------------------------------------------------|-------------------------------------------------|---------------------------------------------|-----------------|--------------------------|
| 0235 + 164 | $9.28 \times 10^{-20}$                           | $3.87 \times 10^{-23}$                          | $1.53 \times 10^{-13}$                      | $6.51 \times 10^{-5}$ | $8.46 \times 10^{-4}$   |
| 3C 216     | $2.34 \times 10^{-17}$                           | $7.97 \times 10^{-21}$                          | $5.805 \times 10^{-11}$                     | $0.0164$        | $6.23 \times 10^{-3}$   |
| J0414 + 0534 | $4.93 \times 10^{-24}$                         | $3.37 \times 10^{-27}$                          | $3.02 \times 10^{-18}$                      | $3.46 \times 10^{-9}$ | $7.33 \times 10^{-11}$  |
| $\gamma_{\text{abs}} = 0.9586$ |                                        |                                                |                                             |                 |                          |
| J0414 + 0534 | $5.59 \times 10^{-16}$                         | $1.75 \times 10^{-19}$                          | $1.63 \times 10^{-9}$                       | $0.392$         | $5.88 \times 10^{-3}$   |
| $\gamma_{\text{abs}} = 2.63534$ |                                        |                                                |                                             |                 |                          |
| 0902 + 343 | $8.502 \times 10^{-16}$                         | $3.403 \times 10^{-19}$                         | $1.52 \times 10^{-9}$                       | $0.597$         |                          |
| 3C 49      | $3.07 \times 10^{-16}$                           | $1.41 \times 10^{-19}$                          | $4.15 \times 10^{-10}$                      | $0.215$         |                          |
| 0248 + 430 | $6.38 \times 10^{-16}$                           | $8.51 \times 10^{-20}$                          | $1.03 \times 10^{-8}$                       | $0.448$         |                          |
| 2128-123   | $1.32 \times 10^{-14}$                           | $3.31 \times 10^{-18}$                          | $6.07 \times 10^{-8}$                       | $0.108$         |                          |
| 3C 190     | $3.08 \times 10^{-11}$                           | $3.798 \times 10^{-15}$                         | $5.79 \times 10^{-4}$                       | $4.63 \times 10^{-5}$ | $1.25 \times 10^{-7}$ |
| 3C 286     | $8.398 \times 10^{-14}$                         | $1.18 \times 10^{-18}$                          | $1.23 \times 10^{-4}$                       | $0.0169$        |                          |
| 0118-272   | $1.72 \times 10^{-10}$                           |                                   |                                               | $0.00475$        | $8.29 \times 10^{-6}$ |
| 0405-331   | $8.95 \times 10^{-10}$                           |                                   |                                               | $1.59 \times 10^{-6}$ |                          |
| 0537-286   | $2.07 \times 10^{-11}$                           |                                   |                                               | $6.88 \times 10^{-5}$ |                          |
| 0957 + 561A | $1.89 \times 10^{-12}$                         | $1.58 \times 10^{-16}$                         | $7.81 \times 10^{-5}$                       | $7.53 \times 10^{-4}$ | $0.0753$ |
| 0336-017   | $3.69 \times 10^{-12}$                           | $1.60 \times 10^{-16}$                         | $5.63 \times 10^{-4}$                       | $3.86 \times 10^{-4}$ | $7.72 \times 10^{-5}$ |
| 0528-250   | $2.41 \times 10^{-8}$                           | $4.02 \times 10^{-13}$                         | $0.0402$                                    | $5.91 \times 10^{-8}$ |                          |

frequencies of the $|1\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ transitions, respectively. Its physical interpretation was given in (11) as the interfering emission/absorption pathways in an atom. Using such modification, the correction to the optical depth can be found as equation (9). The correction should be small for $\Omega_{\nu} \ll \gamma_{21}$. However, in view of the smallness of level width $\gamma_{21}$, the condition is met for a very distant source and absorber, while the opposite situation can be found for $\zeta_{\text{em}} \approx \zeta_{\text{abs}}$ and a very powerful source of light. In this case, the main contribution to the line profile comes from the second term $\text{Im} \{\rho_{21}^{(2)}\}$, and the correction to the optical depth should be taken as the reciprocal value of the former one. Numerical results for the correction to the optical depth at zero detunings in the case of $\Omega_{\nu} \ll \gamma_{21}$ and $\gamma_{21} \ll \Omega_{\nu}$ are collected in the first and second parts of table 2 as $\delta \tau_{0}$, respectively.

Although the case of zero detuning can always be singled out, since the light source emission is of a continuum nature, the velocity of clouds can be taken into account for the detailed description of the 21 cm absorption line profile $|1\rangle \leftrightarrow |2\rangle$ transition in the ISM. This can be rendered by the approximate equality $\delta_{\nu} \approx \frac{\text{Im} \{\rho_{21}^{(2)}\}}{\text{Im} \{\rho_{21}^{(1)}\}}$, where values of $\zeta$ are listed in table 1. Numerical results for $\delta \tau$ are also given in table 2.

In particular, it follows from table 2 that the contribution of $\delta \tau_{0}$ can be significant and exceed the accuracy of the experimental determination of $\tau_{0}$. Although our analysis is rather rough and does not include the Voigt profile fitting, the main conclusion is that the two-level approximation of the atom is insufficient. Already in the three-level approximation, the additional processes occurring in the atom should be taken into consideration in the appropriate fitting of the absorption profile. The parameters of the medium extracted from such fitting can be corrected with use of equations (6) and (9).

5.2. Non-Doppler broadening and frequency shift

In keeping with equation (6), the absorption line profile can be analyzed in terms of line broadening. Without regard to which contribution is dominant, $\text{Im} \{\rho_{21}^{(1)}\}$ or $\text{Im} \{\rho_{21}^{(2)}\}$, the
absorption line derived via the density matrix element is modified by width $\gamma_{\text{broad}}$, see equation (10). The maximum broadening can be estimated as $\Omega^2/4\gamma_31$, see equation (11). The values of $\gamma_{\text{broad}}$ at zero detunings are given in table 2. It is found that the broadening can be significant and exceed the natural line width by several orders of magnitude.

An aspect of interest in such investigations is the determination of frequency shifts and, therefore, refinement of the distances to the source of light and the sizes of the cloud. The accuracy of the redshift determination is on the level of $10^{-10}$ [45], and reaches $10^{-11}$ in some cases [31]. The procedure of the redshift definition can be reduced to finding the maximum of the corresponding line contour. In the same way, the frequency shift arises when the column density can be as high as 60%, see table 2. So, the uncertainty of the redshift, $\delta z_{\text{shift}}$, can be estimated via the frequency shift $\delta z_{\text{shift}}$:

$$\delta z_{\text{shift}} = \frac{1 + z_{\text{abs}}}{\nu_0} \delta z_{\text{shift}},$$

where $\nu_0$ is the transition frequency. The values given in table 2 show that this effect is quite negligible and can be excluded from the corresponding analysis.

6. Conclusions

This paper studied the 21 cm line profile for the hydrogen atom within the framework of the density matrix formalism. Application of the density matrix theory allows a detailed description of the emission/absorption processes when the atom is illuminated by a powerful source of light. The one-photon absorption line profile can be obtained in this case within the two-level approximation of the atomic system, which represents the zeroth approximation. However, the additional emission/absorption processes should be taken into account. These processes can be evaluated within the three-level approximation. The additional interfering transitions were shown to lead to a substantial modification of the corresponding line profile.

Corrections to the frequency and level width were found. The frequency shift can be attributed to the redshift. Although the frequency shift is negligibly small, the width of line profile can be several orders larger than the natural one. The most significant effect arises for the optical depth. In particular, uncertainty of the optical depth determination is about 13% in the case of the J0414 + 0534 source, see table 1, whereas correction equation (9) is of the order of 39%. The same result can be found for the 3C 49 light source: uncertainty and correction are about 8% and 20%, respectively. The magnitude of correction to the optical depth and, therefore, to the column density can be as high as 60%, see table 2. In particular, when $z \approx z_{\text{abs}}$, fitting of the observed line profile with the one-photon isolated resonant contour can lead to overestimation of the corresponding magnitudes.

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ORCID IDs

D Solovyev @ https://orcid.org/0000-0003-0634-0906

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