Dark states of a moving mirror in the single-photon strong-coupling regime

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Abstract

We investigate an optomechanical system in which a cavity with a moving mirror is driven by two external fields. When the field frequencies match resonance conditions, we show that there exists a class of dark states of the moving mirror in the single-photon strong-coupling regime. These dark states, which cause the cavity to be decoupled from the external fields, is a manifestation of quantum coherence associated with the mirror’s mechanical degrees of freedom. We discuss the properties of the dark states and indicate how they can be generated by optical pumping due to the decay of cavity field.

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I. INTRODUCTION

Quantum effects of a moving mirror interacting with electromagnetic fields in a cavity via radiation pressure has been a subject of considerable research interest recently [1–5]. The motivation of this research is not only because the optomechanical coupling can provide novel applications in quantum information processing, such as the storage of optical information as a mechanical excitation [6] and optomechanical transducers for long-distance quantum communication [7], but also because the system could be a platform to explore fundamental quantum phenomena at macroscopic scales. These include, for example, quantum entanglement [8–13], Schrödinger cat states [14–17], and the modification of uncertainty relations due to quantum gravity [18].

In this paper we show how a harmonically bounded mirror can evolve into a dark state when the cavity is driven by two laser fields at certain resonance frequencies. Dark states are well known in a Λ–type three-level atom, in which a coherent superposition of two atomic ground levels can suppress light absorption completely. Our dark states reported in this paper share a similar mechanism, i.e., by quantum interference an optomechanical cavity cannot absorb photons from the external lasers.

We note that interference effects of an optomechanical cavity driven by two light fields have been discussed in literature [19–24]. By linearizing the system equations, an optomechanically induced transparency (OIT) effect, which is an analogy of electromagnetically induced transparency, has been studied theoretically [19–21]. Recently OIT has been observed in experiments [22–24]. Different from previous theoretical work, here we focus on the single-photon strong-coupling regime in which the radiation pressure from a single photon can displace the mirror with a displacement comparable to the zero-point fluctuations. In such a regime, the usual linearized photon-phonon theory becomes inadequate because of the significant quantum fluctuations of the fields and the mirror. To study the physics in the strong coupling regime, one needs to solve the full quantum dynamics beyond the linear approximation, and some authors have reported interesting features, such as photon blockade effect [25], multiple mechanical sidebands [26], single-photon scattering [27] and cooling [28].

It should be noted that in previous studies of OIT, the transparency refers to the non-absorption of a probe field, while the cavity contains a large number of photons due to the
presence of a control field \[19–24\]. Here we show that by exploiting the single-photon strong coupling, the driven cavity can have zero photon when the mirror is in the dark state. In this paper we discuss how such dark states of the mirror exist under a certain rotating wave approximation, and we indicate how they can be prepared by an optical pumping effect.

II. THE MODEL

We consider an optomechanical cavity formed by a harmonically bounded movable end-mirror and a fixed end-mirror [Fig.1(a)], in which the cavity field and the movable end-mirror are coupled with each other via radiation pressure. The optomechanical cavity is driven by two lasers with frequencies \(\omega_1\) and \(\omega_2\). The Hamiltonian of the system is given by

\[
H = \omega_c a^\dagger a + \omega_M b^\dagger b - ga^\dagger a (b^\dagger + b) + \left[ (\Omega_1 e^{-i\omega_1 t} + \Omega_2 e^{-i\omega_2 t}) a^\dagger + h.c. \right]
\]

where \(a\) (\(b\)) and \(\omega_c\) (\(\omega_M\)) are respectively the annihilation operator and resonant frequency of the cavity field (mechanical) modes. The \(\Omega_1\) and \(\Omega_2\) are proportional to the amplitudes of the external fields. The parameter \(g\) is the single-photon coupling strength between the intracavity photon and the mirror caused by radiation pressure. In the frame rotating at the frequency \(\omega_c\), the transformed Hamiltonian is:

\[
H_r = \omega_M b^\dagger b - ga^\dagger a (b^\dagger + b) + \left[ (\Omega_1 e^{-i\Delta_1 t} + \Omega_2 e^{-i\Delta_2 t}) a^\dagger + h.c. \right]
\]

where the detunings \(\Delta_1 = \omega_1 - \omega_c\) and \(\Delta_2 = \omega_2 - \omega_c\) are defined.

The first two terms of \(H_r\) corresponds to the Hamiltonian \(H_0\) without driving, and it can be diagonalized by introducing a displaced oscillator basis as:

\[
H_0 = \omega_M b^\dagger b - ga^\dagger a (b^\dagger + b) = \sum_{n,p} \varepsilon_{n,p} |\psi_{n,p}\rangle \langle \psi_{n,p}|.
\]

Here the eigenvectors \(|\psi_{n,p}\rangle = |n\rangle_c \otimes D(n g / \omega_M) |p\rangle_M = |n\rangle_c \otimes |\tilde{p}(n)\rangle_M\), with \(n(p)\) being the cavity photon (phonon) number. The \(|\tilde{p}(n)\rangle_M\) denotes the \(n\)-photon displaced Fock state of the mirror via the displacement operator \(D(n g / \omega_M) = \exp[\frac{ng}{\omega_m} (b^\dagger - b)]\). The energy eigenvalues of \(H_0\) are \(\varepsilon_{n,p} = p \omega_M - \frac{n^2 g^2}{\omega_M}\), which depend nonlinearly on photon number \(n\), and linearly on phonon number \(p\).

By using the eigenbasis of \(H_0\), \(H_r\) becomes

\[
H_r = \sum_{n,p} \varepsilon_{n,p} |\psi_{n,p}\rangle \langle \psi_{n,p}| + \sum_{n,p,p'} A_{p,p'}^{(n)} (\Omega_1 e^{-i\Delta_1 t} + \Omega_2 e^{-i\Delta_2 t}) |\psi_{n,p}\rangle \langle \psi_{n-1,p'}| + h.c. \]

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FIG. 1: (Color online) (a) Schematic diagram of an optomechanical system consisting of a fixed end mirror and a movable end mirror with two driving fields. (b) The coupling scheme between energy levels of the optomechanical system (only zero- and one-photon states are shown). Here each laser is used to establish a set of resonant transitions, and $|p\rangle_M = |\tilde{p}(0)\rangle_M$ and $|\tilde{p}\rangle_M = |\tilde{p}(1)\rangle_M$ for simplicity. By choosing $g = g_N$, there is no transition between $|0\rangle_c|N\rangle_M$ and $|1\rangle_c|\tilde{N}\rangle_M$.

where we have expressed the annihilation operator $a$ in the eigenbasis as

$$a = \sum_{n,p} \sum_{n',p'} |\psi_{n,p}\rangle \langle \psi_{n',p'}| a |\psi_{n',p'}\rangle \langle \psi_{n,p}| = \sum_{n,p,p'} A^{(n)}_{p,p'} |\psi_{n-1,p}\rangle \langle \psi_{n,p'}|$$

with the coefficients $A^{(n)}_{p,p'} = \sqrt{n} \langle p | D^\dagger [(n-1) g/\omega_m] D (n g/\omega_M) |p'\rangle = \sqrt{n} \langle p | D (g/\omega_M) |p'\rangle$. Specifically, the coefficients are given by

$$A^{(n)}_{p,p'} = \begin{cases} \sqrt{n} \sqrt{\frac{p+1}{p}} e^{-\xi^2/2} (-\xi)^{p'-p} L^p_{p'} (\xi^2), & p \leq p' \\ \sqrt{n} \sqrt{\frac{p'}{p}} e^{-\xi^2/2} (\xi)^{p-p'} L^{p-p'}_{p'} (\xi^2), & p > p' \end{cases}$$

where $\xi = g/\omega_M$ and $L^s_r(x)$ are associated Laguerre polynomials.

III. EFFECTIVE RESONANT HAMILTONIAN WITH FINITE DIMENSIONS

In this section, we show how evolution of the system state can be confined to a finite dimensional subspace involving only the zero and one cavity photon number and $N$ displaced
phonon number states, assuming the initial state is the ground state. This is achieved by exploiting resonances and specific values of optomechanical coupling strength \( g \). First of all, we choose the frequencies \( \omega_1 \) and \( \omega_2 \) of the driving fields such that the detunings \( \Delta_1 \) and \( \Delta_2 \) satisfy the resonance conditions:

\[
\Delta_1 = \varepsilon_{1,p} - \varepsilon_{0,p} = -g^2/\omega_M
\]

\[
\Delta_2 = \varepsilon_{1,p} - \varepsilon_{0,p+1} = -\omega_M - g^2/\omega_M
\]

In this way the driving field \( \Omega_1 \) can resonantly couple the states \( |\psi_{0,p}\rangle \) and \( |\psi_{1,p}\rangle \) as illustrated by vertical arrows in Fig. 1b. Similarly, the driving field \( \Omega_2 \) can resonantly couple the states \( |\psi_{0,p+1}\rangle \) and \( |\psi_{1,p}\rangle \).

It is important to note that \( \varepsilon_{n,p} \) depends nonlinearly on the photon number \( n \), therefore the above resonance relations do not hold for states with photon numbers larger than 1. In fact, if the optomechanical coupling strength \( g \) is sufficiently strong, the conditions (7) and (8) correspond to far off resonance for transitions from 1-photon states to 2-photon states, and this leads to the photon blockade effect [25]. Hence if the driving fields are sufficiently weak, it is justified to ignore the off-resonant transitions from 1-photon states to 2-photon states. Specifically, we require (for \( i = 1, 2 \)),

\[
\Omega_i \ll \left| 2g^2/\omega_M - K\omega_M \right|
\]

where \( K \) is the nearest integer to \( 2(g/\omega_M)^2 \). For example, if \( g < \omega_M/2 \) then \( K = 0 \). The inequality (9) means that \( \Omega_i \) should be much less than the smallest detuning between 2-photon manifold to 1-photon manifold.

With the conditions (7-9), if the initial cavity photon number is zero, the system can be effectively confined to the subspace of zero and one photon. Furthermore, we will keep only the co-rotating terms in (4) as a rotating wave approximation. Under such an approximation, \( \Omega_i \) can drive the corresponding resonant transitions only, and the Hamiltonian (4) in the interaction picture [i.e., the first term in (4) is removed by a rotating frame] becomes,

\[
H'_r = \sum_{p=0}^{N-1} \left[ (A_{p,p}^{(1)} \Omega_1 |\psi_{0,p}\rangle \langle \psi_{0,p}| + A_{p+1,p}^{(1)} \Omega_2 |\psi_{1,p}\rangle \langle \psi_{0,p+1}|) + h.c. \right].
\]

Here the upper limit of phonon number \( N \) can be infinite in general. However, a finite value of \( N \) is possible if the transition between \( |\psi_{0,N}\rangle \) and \( |\psi_{1,N}\rangle \) can be completely suppressed.
FIG. 2: (Color online) Solution of $g_N$ satisfying Eq. (11) as a function of $N$. For each $N$, there are multiple roots, and only the smallest positive root is shown.

(Fig. 1b). This can be achieved by a specific value of $g = g_N$ such that

$$A^{(1)}_{N,N} = \exp\left(-\frac{g_N^2}{2\omega_M^2}\right)L_N^0\left(g_N^2/\omega_M^2\right) = 0.$$  \hspace{1cm} (11)

In other words, one can design the Hamiltonian $H'_r$ with a prescribed $N$ by using the optomechanical coupling strength $g_N$. In Fig. 2, we illustrate the roots $g_N$ as a function of $N$. We see that the value of $g_N$ decreases when $N$ becomes larger and larger. For example, $g_N \approx 0.12\omega_M$ for $N = 100$.

IV. DARK STATES

The Hamiltonian $H'_r$ has an eigenvector $|D\rangle$ with a zero eigenvalue:

$$|D\rangle = C \sum_{p=0}^{N} \beta_p |p\rangle_M \otimes |0\rangle_c \hspace{1cm} (12)$$

where $\beta_0 = 1$, and for $p > 0$,

$$\beta_p = (-1)^p \left(\frac{\Omega_1}{\Omega_2}\right)^p \prod_{i=0}^{p-1} \frac{A^{(1)}_{i,i}}{A^{(1)}_{i+1,i}} \hspace{1cm} (13)$$

and $C$ is a normalization constant. Such an eigenvector is a coherent superposition of phonon states and the cavity contains no photon. It is a dark state induced by quantum coherence of the mirror, and the interference forbids any excitation of cavity field even though the cavity is constantly driven by the two external fields.
The phonon number distribution of the dark states is complicated by the Laguerre functions in Eq. (13). While the details form of $|\beta_p|^2$ requires a numerical evaluation of Eq. (13), we find that $|\beta_p|^2$ is mainly controlled by the ratio of the strengths of driving fields. Such a feature is illustrated in Fig. 3 for various $\Omega_2/\Omega_1$. For example, when $\Omega_2/\Omega_1 = 3$, the probability decreases quickly with the increase of phonon number $m$. In the case $\Omega_2/\Omega_1 = 1$, there is a peak appears in the probability distribution. If the ratio is further decreased to $\Omega_2/\Omega_1 = 1/3$, the peak is shifted towards higher phonon numbers.

It is worth noting that phonon number distributions of dark states exhibit a sub-Poissonian statistics. This is illustrated in Fig. 4 in which the ratio $\langle(\Delta n)^2\rangle/\langle n\rangle$ as a function of $\Omega_1/\Omega_2$ is plotted. We see that $\langle(\Delta n)^2\rangle/\langle n\rangle$ is always less than 1 (i.e., sub-Poisson distribution) except at the small region $\Omega_1/\Omega_2$ near zero. In particular, the $\langle(\Delta n)^2\rangle/\langle n\rangle$ de-
creases with $\Omega_1/\Omega_2$. In Fig. 4, we also find that the curves are quite insensitive to the value of $g_N$ used.

V. PREPARATION OF DARK STATES BY CAVITY-FIELD DAMPING

In this section we discuss how the system can be optically pumped into the dark state by cavity-field damping. The evolution of the dissipative system is governed by the master equation:

$$\frac{d\rho}{dt} = -i [H_r, \rho] - \frac{\gamma_c}{2} \left( a^\dagger a \rho - 2a \rho a^\dagger + \rho a^\dagger a \right)$$

(14)

where $\rho$ is the density matrix of the photon-mirror system, and $\gamma_c$ is the cavity-field decay rate. Here we have assumed that the mechanical motion of the mirror has the damping rate $\gamma_M$ that is much smaller than $\gamma_c$. In addition, we will focus on a finite time interval $1/\gamma_c \ll t \ll 1/\gamma_M$ during which the optically pumping is complete while the decay of mirror’s motion remains negligible, and so we only include the decay effect of the cavity field in the above master equation. Note that the original Hamiltonian $H_r$ defined in Eq. (2) is used without employing the rotating wave approximation in Eq. (10). If $H_r$ is simply replaced by $H'_r$, then $\rho = |D\rangle \langle D|$ is already a steady state solution of the master equation Eq. (14), because the cavity field damping term has no effect on $|D\rangle$ (which has zero photon).

We have solved the master equation (14) numerically with an initial ground state of the system. Specifically, we are interested in the fidelity $F$ defined by

$$F = Tr \left( |D\rangle \langle D| \rho(t) \right),$$

(15)

which measures the probability of the system in the dark state $|D\rangle$. Some examples are given in Fig. 5 in which $F$ increases with time and approaches a steady value close to 1 in a finite time. For the three cases shown in Fig. 5, the fidelities can reach $F \approx 0.99$ with $g_N/\omega_M = 0.37 \ (N = 10)$ at the time $T \approx 8000\omega_M^{-1}$. To ensure that the mechanical decoherence is negligible, we need $\gamma_M < 10^{-4}\omega_M$ for the parameters used in Fig. 5. Indeed, we have tested numerically the performance by including mechanical damping in the master equation, and found that $F \approx 0.99$ when $\gamma_M = 10^{-5}\omega_M$ and $F \approx 0.93$ for a larger $\gamma_M = 10^{-4}\omega_M$. We remark that the time $T$ required to generate dark states is shorter for smaller $N$. For example, when $N = 3$, our numerical calculations indicate that $T \approx 2000\omega_M^{-1}$.  

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The increase of $F$ is understood as an optical pumping effect. This is because when a photon leaks out of the cavity, the mirror can have a non-zero probability making a transition to the dark state. Since the dark state is decoupled from the driving fields, it can no longer be excited, and hence the occupation of the dark state accumulates as time increases. In our system, the loss is mainly due to a leakage of phonon population beyond the phonon number $N$, because the cavity field decay also causes the mirror to make transitions to states other than the dark state. Note that the transition amplitudes from $|1\rangle_c|\tilde{p}\rangle_M$ to $|0\rangle_c|\tilde{q}\rangle_M$ due to the transmission of a photon out of the cavity is proportional to the Frank-Condon factor $\langle \tilde{p} | \tilde{q} \rangle_M$, the loss can be reduced by choosing a sufficiently high $N$ and $\Omega_2 > \Omega_1$. This is because dark states with $\Omega_2 > \Omega_1$ concentrate on lower phonon numbers (Fig. 3) and hence the Frank-Condon factors for transitions to states of phonon number higher than $N$ is smaller.

VI. CONCLUSION REMARKS

To conclude, we have addressed a quantum interference effect in the single-photon strong-coupling regime of optomechanics. In such a regime we discover a class of dark states of a moving mirror under the conditions (7-9). These dark states make the cavity decoupled from two external driving fields, and the decoupling is derived without employing the linearization treatment. We provide an analytical expression of the dark states which indicate the dependence of the ratio $\Omega_2/\Omega_1$ of the driving fields and the optomechanical coupling strength $g$. 

FIG. 5: (Color online) Time evolution of the fidelity $F$ for various $\Omega_2/\Omega_1$ ratios. The parameters are: $N = 10$, $g/\omega_M = 0.37$, $\gamma_c/\omega_M = 0.05$, $\Omega_2/\omega_M = 0.01$, $\Delta_1/\omega_M = -0.14$, $\Delta_2/\omega_M = -1.14$. 

\[ \Omega_2/\Omega_1 = 3 \]
\[ \Omega_2/\Omega_1 = 2 \]
\[ \Omega_2/\Omega_1 = 1 \]
With the help of cavity damping, dark states can be prepared by optical pumping.

In this paper specific optomechanical coupling strengths $g_N$ have been used in order to ‘trap’ the mirror state in the space of finite phonon numbers. It is important to ask how a slight deviation of $g$ away from $g_N$ would affect the dark state generation, because the trapping effect would become imperfect when $g \neq g_N$. To address the issue, we have tested the sensitivity of the fidelity to small variations of $g$ values. For example, with $\Omega_2 > \Omega_1$ and parameters in Fig. 5, by introducing a 3% deviation of the $g$ value we found that the fidelities can still reach about $F \approx 0.99$. This can be understood by Eq. (13). Strictly speaking, when $g$ is not exactly equal to $g_N$, the upper limit $N$ in Eq. (12) should be extended to infinity because $A_{NN}^{(1)}$ is no longer zero. However, if $\Omega_1/\Omega_2$ is less than one, then $\beta_p$ appearing in Eq. (13) can quickly converge to zero, yielding a significant population concentrated at phonon numbers much lower than $N$ (Fig. 3). In this case the leakage of phonon numbers beyond $N$ would be negligible and hence a slight deviation of $g$ would not affect the dark state generation appreciably. On the other hand, we find that the system with $\Omega_2 < \Omega_1$ is quite sensitive to $g$. This is because if $\Omega_1/\Omega_2 > 1$, $\beta_p$ in Eq. (13) tends to shift to higher phonon numbers (Fig. 3), making the leakage beyond $N$ significant. Therefore it would be more favorable to employ $\Omega_2 > \Omega_1$ for experimental realizations of our scheme. Finally, we remark that the realization of single-photon strong-coupling regime is still a challenge for current experiments, but some recent progress have been made in various optomechanical systems in order to reach the strong coupling regime [29–31].

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