Exclusive processes with a leading neutron in $ep$ collisions

V.P. Gonçalves$^{1,2}$, F.S. Navarra$^3$ and D. Spiering$^3$

$^1$Department of Astronomy and Theoretical Physics, Lund University, SE-223 62 Lund, Sweden
$^2$High and Medium Energy Group, Instituto de Física e Matemática, Universidade Federal de Pelotas
Caixa Postal 354, 96010-900, Pelotas, RS, Brazil.
$^3$Instituto de Física, Universidade de São Paulo,
C.P. 66318, 05315-970 São Paulo, SP, Brazil.

In this paper we extend the color dipole formalism to the study of exclusive processes associated with a leading neutron in $ep$ collisions at high energies. The exclusive $\rho$, $\phi$ and $J/\Psi$ production, as well as the Deeply Virtual Compton Scattering, are analysed assuming a diffractive interaction between the color dipole and the pion emitted by the incident proton. We compare our predictions with the HERA data on $\rho$ production and estimate the magnitude of the absorption corrections. We show that the color dipole formalism is able to describe the current data. Finally, we present our estimate for the exclusive cross sections which can be studied at HERA and in future electron-proton colliders.

PACS numbers: 12.38.-t, 24.85.+p, 25.30.-c

Keywords: Quantum Chromodynamics, Leading Particle Production, Saturation effects.

I. INTRODUCTION

The study of electron - proton ($ep$) collisions at HERA has improved our understanding of the structure of the proton as well as about the dynamics of the strong interactions at high energies (For a review see e.g. Ref. [1]). In particular, the study of diffractive processes has been one of the most successful areas at HERA, with vector meson production and Deeply Virtual Compton Scattering (DVCS) in exclusive processes ($\gamma^*p \rightarrow Ep$, with $E = \rho, \phi, J/\Psi, \gamma$) being important probes of the transition between the soft and hard regimes of QCD. These processes have been the subject of intensive theoretical and experimental investigations, with one of the main motivations for these studies being the possibility to probe the QCD dynamics at high energies, driven by the gluon content of the proton which is strongly subject to non-linear effects (parton saturation) [2]. An important lesson from the analysis of the HERA data at small values of the Bjorken - $x$ variable is that the inclusive and diffractive processes can be satisfactorily described using a unified framework – the color dipole formalism. This approach was proposed many years ago in Ref. [3] and considers that the high energy photon can be described by a color quark - antiquark dipole and that the interaction of the dipole with the target can be described by the color dipole cross section $\sigma_{dt}(x,r)$, which is determined by the imaginary part of the forward amplitude of the scattering between a small dipole (a colorless quark-antiquark pair) and a dense hadron target, denoted by $\mathcal{N}(x,r,b)$, where the dipole has transverse size given by the vector $r = x - y$, with $x$ and $y$ being the transverse vectors of the quark and antiquark, respectively, and $b = (x + y)/2$ is the impact parameter. In the Color Glass Condensate (CGC) formalism [4, 5], $\mathcal{N}$ contains all the information about non-linear and quantum effects in the hadron wave function. It can be obtained by solving an appropriate evolution equation in the rapidity $Y \equiv \ln(1/x)$, which in its simplest form is the Balitsky-Kovchegov (BK) equation [5, 6]. Alternatively, the scattering amplitude can be obtained using phenomenological models based on saturation physics constructed taking into account the analytical solutions of the BK equation which are known in the low and high density regimes. As demonstrated in [7], the combination between the color dipole formalism and saturation physics are quite successful to describe the recent and very precise HERA data on the reduced inclusive cross section as well as the data on the exclusive processes in a large range of photon - proton center - of - mass energies $W$, photon virtualities $Q^2$ and $x$ values.

HERA has also provided high precision experimental data on semi - inclusive $e + p \rightarrow e + n + X$ processes, where the incident proton is converted into a neutron via a charge exchange [8]. Very recently the first measurements of
II. EXCLUSIVE PROCESSES ASSOCIATED WITH LEADING NEUTRON PRODUCTION IN THE COLOR DIPOLE FORMALISM

At high energies, the differential cross section for a given process (semi-inclusive or exclusive) associated with a leading neutron production can be expressed as follows:

\[
\frac{d^2\sigma(W,Q^2,x_L,t)}{dx_Ldt} = f_{\pi/p}(x_L,t)\sigma_{\gamma^*\pi}(W^2,Q^2)
\]

(1)

where \(Q^2\) is the virtuality of the exchanged photon, \(W\) is the center-of-mass energy of the virtual photon-proton system, \(x_L\) is the proton momentum fraction carried by the neutron and \(t\) is the square of the four-momentum of the exchanged pion. Moreover, \(f_{\pi/p}\) is the flux of virtual pions emitted by the proton and \(\sigma_{\gamma^*\pi}(W^2,Q^2)\) is the cross section of the interaction between the virtual-photon and the virtual-pion at center-of-mass energy \(W\), which is given by \(W^2 = (1 - x_L)W^2\). The pion flux \(f_{\pi/p}(x_L,t)\) (also called sometimes pion splitting function) is the virtual pion momentum distribution in a physical nucleon (the bare nucleon plus the “pion cloud”). In general, it is parametrized as follows [10–19]

\[
f_{\pi/p}(x_L,t) = \frac{1}{4\pi} \frac{2g_{\pi p\pi}^2}{(1 - m^2_n)^2} (1 - x_L)^{1-2\alpha(t)} |F(x_L,t)|^2
\]

(2)

where \(g_{\pi p\pi}^2/(4\pi) = 14.4\) is the \(\pi^0 pp\) coupling constant, \(m_n\) is the pion mass and \(\alpha(t)\) is the Regge trajectory of the pion. The form factor \(F(x_L,t)\) accounts for the finite size of the nucleon and of the pion and is model dependent. As in Ref. [20], we will consider the following parametrizations for the form factor:

\[
F_1(x_L,t) = \exp \left[ R^2 \frac{(t - m^2_n)}{(1 - x_L)} \right], \quad \alpha(t) = 0
\]

(3)
from Ref. [11], where \( R = 0.6 \text{ GeV}^{-1} \).

\[
F_2(x_L, t) = \frac{1}{\alpha(t)} \quad \alpha(t) = \alpha(t)_\pi
\]

(4)

from Ref. [10], where \( \alpha_\pi(t) \approx t \) (with \( t \) in GeV\(^2\)) is the Regge trajectory of the pion.

\[
F_3(x_L, t) = \exp \left[b(t - m_\pi^2)\right] \quad \alpha(t) = \alpha(t)_\pi
\]

(5)

from Ref. [12], where \( \alpha_\pi(t) \approx t \) (with \( t \) in GeV\(^2\)) and \( b = 0.3 \text{ GeV}^{-2} \).

\[
F_4(x_L, t) = \frac{\Lambda_m^2 - m_\pi^2}{\Lambda_m^2 - t} \quad \alpha(t) = 0
\]

(6)

from Ref. [13], where \( \Lambda_m = 0.74 \text{ GeV} \).

\[
F_5(x_L, t) = \left[\frac{\Lambda_m^2 - m_\pi^2}{\Lambda_q^2 - t}\right]^2 \quad \alpha(t) = 0
\]

(7)

also from Ref. [13], where \( \Lambda_q = 1.2 \text{ GeV} \). In what follows we will denote the corresponding pion flux associated to these different form factors by \( f_1, f_2, \ldots, f_5 \), respectively. Moreover, it is important to emphasize that in the case of the more familiar exponential [3], monopole [9] and dipole [7] forms, the cut-off parameters have been determined by fitting low energy data on nucleon and nuclear reactions and also data on deep inelastic scattering and structure functions [21].

In Ref. [20], we described the semi-inclusive leading neutron processes in the color dipole formalism. The basic idea is that at high energies, this process can be seen as a sequence of three factorizable subprocesses [See Fig. 1 (left panel)]: i) the photon fluctuates into a quark-antiquark pair (the color dipole), ii) the color dipole interacts with the pion, present in the wave function of the incident proton, and iii) the leading neutron is formed. Consequently, the photon - pion cross section can be factorized in terms of the photon wave functions \( \Psi \), which describes the photon splitting in a \( q \bar{q} \) pair, and the dipole-pion cross section \( \sigma_{d\pi} \). In the eikonal approximation the dipole-proton cross section \( \sigma_{d\pi} \) is given by:

\[
\sigma_{d\pi}(\hat{x}, r) = 2 \int \hat{d}^2bN^\pi(\hat{x}, r, b)
\]

(8)

where

\[
\hat{x} = \frac{Q^2 + m_\pi^2}{W^2 + Q^2} = \frac{Q^2 + m_\pi^2}{(1 - x_L)W^2 + Q^2}
\]

(9)

is the scaled Bjorken variable and \( N^\pi(x, r, b) \) is the imaginary part of the forward amplitude of the scattering between a small dipole (a colorless quark-antiquark pair) and a pion, at a given rapidity interval \( Y = \ln(1/\hat{x}) \). In Ref. [20] we proposed to relate \( N^\pi \) with the dipole-proton scattering amplitude \( N^p \), usually probed in the typical inclusive and exclusive processes at HERA, assuming that

\[
N^\pi(\hat{x}, r, b) = R_q \cdot N^p(\hat{x}, r, b)
\]

(10)

with \( R_q \) being a constant. In the additive quark model it is expected that \( R_q = 2/3 \), which is the ratio between the number of valence quarks in the target hadrons. This model was first applied to soft hadronic reactions [22] and, in particular, it predicted the following relation between the pion-proton and proton-proton total cross sections: \( \sigma_{\pi p} = 2/3 \sigma_{pp} \). This relation is observed experimentally. It refers to total cross sections and the only kinematical variable is the c.m.s energy \( \sqrt{s} \). In the low energy domain, where it was verified, the dependence on \( \sqrt{s} \) was very weak. As it was discussed in Ref. [23], in hard hadronic reactions, where a high energy scale is present, Eq. (10) may still be valid, although deviations from 2/3 are likely to be seen. The idea is that dipoles with \( Q^2 \geq 10 \text{ GeV}^2 \) can resolve the quarks in the target and interact with each of them independently. The cross section is then proportional to the number of quarks in the target. At increasing \( Q^2 \) and/or collision energies quantum fluctuations become more important, increasing the effective number of quarks. According to Ref. [23], this growth is stronger in the proton than in the pion and hence 2/3 \( \rightarrow 1/2 \) (or even 1/3). Here we consider a range of \( R_q \), going from 1/3 up to 2/3. As it will be seen, our results imply that if \( R_q = 2/3 \) the absorption factor \( K \) tends to be too small. In view of the existing calculations of \( K \), we would conclude that \( R_q = 2/3 \) is probably too large. Since the effective value of this quantity is still an open question [10, 17, 23, 24], we have considered in [20] that \( R_q \) could be in the range 1/3 \( \leq R_q \leq 2/3 \). With
this basic assumption we have estimated the dependence of the predictions on the description of the QCD dynamics at high energies as well as the contribution of gluon saturation effects to leading neutron production. Moreover, with the parameters constrained by other phenomenological information, we were able to reproduce the basic features of the H1 data on leading neutron spectra [3].

As mentioned in Ref. [21], one source of uncertainty in the study of inclusive leading neutron production (in Fig. 1 and on the left) is the fact that there are several processes which lead to the same final state. Apart from one pion emission we may have, for example, \( \rho \) emission. Even with pion emission we may have \( \Delta \) production with the subsequent decay \( \Delta \to n + \pi \). The strength of these contributions is highly model dependent and their existence prevents us from extracting more precise information on the photon-pion cross section or on the pion flux. In contrast, in \( \rho \) exclusive production with a leading neutron none of these processes contributes to the exclusive reaction shown in the right panel of Fig. 1. This feature makes the leading neutron spectrum measured in exclusive processes a better testing ground for both the determination of the photon-pion cross section and of the pion flux.

In what follows we will assume that the factorization given by Eq. (1) also is valid and that the photon-pion cross section is the effective charge of the vector meson, \( e^* \), and ultra peripheral hadronic collisions (See, e.g. Refs. [7, 32, 33]). In the DVCS case, as one has a real photon and a quark-antiquark dipole and the factor in the exponential arises when one takes into account non-forward corrections to the wave functions [25]. In what follows we will assume that the vector meson is predominantly a quark-antiquark state and that the spin and polarization structure is the same as in the photon [26–29] (for other approaches see, for example, Refs. [30]). As a consequence, the overlap between the photon and the vector meson wave function, for the exclusive final state \( E = \gamma \), in the \( \gamma^* \pi \to \pi \) process is given in the color dipole formalism by:

\[
\sigma(\gamma^* \pi \to \pi \gamma) = \sum_{i=L,T} \int_{-\infty}^{0} \frac{d\sigma_i}{dt} d\hat{t} = \frac{1}{16\pi} \sum_{i=L,T} \int_{-\infty}^{0} |A_i(\gamma^* \to \pi \gamma)|^2 d\hat{t}, \tag{11}
\]

with the scattering amplitude being given by

\[
A^{\gamma^* \pi \to \pi \gamma}_i(\hat{x}, \Delta) = i \int d\hat{z} d^2r d^2b e^{-i(b \cdot (1-z)r)} \Delta (\Psi^{E*\psi})_{T,L} 2N_\pi(\hat{x}, r, b) \tag{12}
\]

where \((\Psi^{E*\psi})_{T,L}\) denotes the overlap of the photon and exclusive final state wave functions. The variable \( z (1-2z) \) is the longitudinal momentum fractions of the quark (antiquark) and \( \Delta \) denotes the transverse momentum lost by the outgoing pion \( (t = -\Delta^2) \). The variable \( b \) is the transverse distance from the center of the target to the center of mass of the \( qq \) dipole and the factor in the exponential arises when one takes into account non-forward corrections to the wave functions [22]. In what follows we will assume that the vector meson is predominantly a quark-antiquark state and that the spin and polarization structure is the same as in the photon [26–29] (for other approaches see, for example, Ref. [30]). As a consequence, the overlap between the photon and the vector meson wave function, for the transversely and longitudinally polarized cases, is given by (For details see Ref. [31])

\[
\begin{align*}
(\Psi_{T}^{*} \phi_{T})_{L} &= \frac{\hat{e} \psi_{e} N_{e}}{4\pi} \frac{1}{\pi z(1-z)} \left\{ m_{V}^{2} K_{0}(er) \phi_{T}(r, z) - [z^2 + (1-z)^2] \epsilon K_{1}(er) \partial_{r} \phi_{T}(r, z) \right\}, \tag{13}

(\Psi_{L}^{*} \phi_{L})_{L} &= \frac{\hat{e} \psi_{e} N_{e}}{4\pi} 2Q z(1-z) K_{0}(er) \left[M_{V} \phi_{L}(r, z) + \delta \frac{m_{V}^{2} - \nabla_{r}^{2}}{M_{V} z(1-z)} \phi_{L}(r, z) \right]. \tag{14}
\end{align*}
\]

where \( \hat{e} \) is the effective charge of the vector meson, \( m_{V} \) is the quark mass, \( N_{e} = 3, \epsilon^2 = z(1-z)Q^2 + m_{V}^2 \) and \( \phi_{L}(r, z) \) define the scalar parts of the vector meson wave functions. We will consider the Boosted Gaussian and Gauss-LC models for \( \phi_{T}(r, z) \) and \( \phi_{L}(r, z) \), which are largely used in the literature. In the Boosted Gaussian model the functions \( \phi_{L}(r, z) \) are given by

\[
\phi_{T,L}(r, z) = C_{T,L} z(1-z) \exp \left[ -\frac{m_{V}^{2} R_{T,L}^{2}}{8z(1-z)} - \frac{2z(1-z)r^2}{R_{T,L}^{2}} + \frac{m_{V}^{2} R_{T,L}^{2}}{2} \right]. \tag{15}
\]

In contrast, in the Gauss-LC model, they are given by

\[
\begin{align*}
\phi_{T}(r, z) &= N_{T} z(1-z)^2 \exp \left( -r^2/2R_{T}^{2} \right), \tag{16}

\phi_{L}(r, z) &= N_{L} z(1-z) \exp \left( -r^2/2R_{L}^{2} \right). \tag{17}
\end{align*}
\]

The parameters \( C_{T,L} \), \( R_{T} \), \( N_{T} \), and \( R_{L} \) are determined by the normalization condition of the wave function and by the meson decay width. In Table I we present the value of these parameters for the vector meson wave functions. It is important to emphasize that predictions based on these models for the wave functions have been tested with success in ep and ultra peripheral hadronic collisions (See, e.g. Refs. [31, 32, 33]). In the DVCS case, as one has a real photon at the final state, only the transversely polarized overlap function contributes to the cross section. Summed over the quark helicities, for a given quark flavour \( f \) it is given by [31],

\[
\begin{align*}
\phi_{T}(r, z) &= N_{T} z(1-z)^2 \exp \left( -r^2/2R_{T}^{2} \right), \tag{16}

\phi_{L}(r, z) &= N_{L} z(1-z) \exp \left( -r^2/2R_{L}^{2} \right). \tag{17}
\end{align*}
\]
In this model, the proton saturation scale able to describe the vector meson production in ep \cite{31} and recently updated in Ref. \cite{7}, which is based on the CGC formalism and takes into account the impact 

where we have defined the quantities \( \varepsilon^2_{1,2} = z\bar{z}Q^2_{1,2} + m^2_f \) and \( \bar{z} = (1-z) \). Accordingly, the photon virtualities are \( Q^2_1 = Q^2 (\text{incoming virtual photon}) \) and \( Q^2_2 = 0 \) (outgoing real photon).

Finally, in order to estimate the photon - pion cross section, we must specify the dipole - pion scattering amplitude \( N^\pi \). As considered in Ref. \cite{20} for the semi - inclusive processes, we will assume the validity of the approximation \( \rho = \rho_0 (x_0) \), with \( \rho_0 \) being the LO BFKL characteristic function. The coefficients \( A \) and \( B \) are determined uniquely from the condition that \( N^p(\hat{x}, r, b) \), and its derivative with respect to \( r Q_s(b) \), are continuous at \( r Q_s(b) = 2 \).

In this model, the proton saturation scale \( Q_s(b) \) depends on the impact parameter:

\[
Q_s(b) = Q_s(\hat{x}) = \left( \frac{Q_0(b)}{\hat{x}} \right)^{\frac{1}{2}} \left[ \exp \left( -\frac{b^2}{2B_{CGC}} \right) \right]^{\frac{1}{2}}.
\]

The parameter \( B_{CGC} \) was adjusted to give a good description of the \( t \)-dependence of exclusive \( J/\psi \) photoproduction. Moreover, the factors \( N_0 \) and \( \gamma_s \) were taken to be free. The set of parameters which will be used here is the following: \( \gamma_s = 0.6599, \kappa = 9.9, B_{CGC} = 5.5 \text{ GeV}^{-2}, N_0 = 0.3358, x_0 = 0.00105 \) and \( \lambda = 0.2063 \). Moreover, in order to estimate the dependence of our predictions on the choice of the model for \( N^p \), we will also consider the IIMS \cite{33, 34} and GBW \cite{30} models, as well as the numerical solution of the BK equation obtained in Ref. \cite{37}. Such models were discussed in detail in Ref. \cite{20}. For these models, we assume \( N^p(\hat{x}, r, b) = N^p(\hat{x}, r) S(b) \) and \( \sigma_{dp}(\hat{x}, r) = \sigma_0 \cdot N^p(\hat{x}, r) \), with the normalization of the dipole cross section \( \sigma_0 \) being fitted to data, and that the \( \hat{t} \)-dependence of the photon - pion cross section can be approximated by an exponential ansatz, \( d\sigma/d|t| = d\sigma/d|t| (t = 0) \cdot e^{-B|t|} \), with the slope being given by \( B = \sigma_0/4\pi \). It is important to emphasize that the conclusions obtained in \cite{20} are not modified if the bCGC model is used as input in the calculations.

Before discussing our results, a comment is in order. As in the semi - inclusive case, our predictions for the exclusive processes associated with a leading neutron are essentially parameter free, depending only on the choices of the models for the pion flux and on the dipole scattering amplitude. The main uncertainties are associated with the choice of \( R_0 \) (in Eq. \cite{18}) and the magnitude of the absorption effects which can arise by soft rescatterings. These latter are difficult to calculate \cite{18, 19} but are expected to modify almost uniformly all the \( x_I \) spectrum of the leading neutrons. As in Ref. \cite{20}, in what follows we will assume that these effects can be mimicked by a factor \( K \), which multiplies the right side of Eq. \cite{11} changing the normalization of the spectra and which should be estimated from the analysis of experimental data. In spite of the efforts made in several studies of absorptive corrections in semi - inclusive processes \cite{16, 18, 22, 58}, the magnitude of these effects in exclusive processes remains an open question.

### III. RESULTS

Let us start our analysis considering the exclusive proton production associated with leading neutrons as analysed by the H1 Collaboration \cite{9}. In what follows we will assume that \( W = 60 \text{ GeV}, Q^2 = 0.04 \text{ GeV}^2 \) and that \( p_T < 0.2 \)
FIG. 2: Leading neutron spectra in exclusive $\rho$ photoproduction considering two different models for the vector meson wave function (Boosted Gaussian and Gauss-LC) and two different models for the pion flux ($f_2$ and $f_3$).

FIG. 3: Leading neutron spectra in exclusive $\rho$ photoproduction considering different models of the pion flux. Data from Ref. [9].

$\rho$ photoproduction considering two different models for the vector meson wave function (Boosted Gaussian and Gauss-LC) and two different models for the pion flux ($f_2$ and $f_3$). We present our results for two different models of the pion flux. We find that the predictions are similar, with the Gauss-LC results being a lower bound. This conclusion is also valid for other models of the pion flux and for the $\phi$ and $J/\Psi$ production. Consequently, in what follows we will consider only the Gauss-LC model for the vector meson wave functions. Let us now compare our predictions with the experimental data [9] considering different models for the pion flux. In order to constrain the value of the $K$ factor associated to absorptive corrections, our strategy will be following: for a given model of the pion flux, $R_q$ and dipole cross section, we will estimate the total cross section. The value of $K$ will be the value necessary to make our prediction consistent with the H1 data [9].

In Fig. 3 we present our predictions for the leading neutron spectra in exclusive $\rho$ photoproduction considering different models for the pion flux. The corresponding $K$ values are also presented. We obtain a reasonable agreement with the experimental data, with $K$ values in the range $0.134 < K < 0.210$. It is important to emphasize that these values of $K$ are strongly correlated with our choice for $R_q$. For example, if instead of $R_q = 2/3$ we assume $R_q = 1/3$, the corresponding $K$ values should be multiplied by 4, since the exclusive cross sections depend quadratically on the dipole scattering amplitude. If we assume a priori that the magnitude of the absorptive correction factor is of the order of 0.7 for exclusive processes, as predicted in [19] for semi-inclusive one, this implies a preference for the value $R_q = 1/3$. However, as the magnitude of these corrections for exclusive processes is still an open question, as well as the value of $R_q$, we refrain from drawing strong conclusions. Therefore, in what follows we will only present results for $R_q = 2/3$. GeV, where $p_T$ is the transverse momentum of the leading neutron. Moreover, we will assume initially that $R_q = 2/3$ and the bCGC dipole model. In Fig. 2 we analyse the dependence of our predictions on the choice of the vector meson wave function. We present our results for two different models of the pion flux. We find that the predictions are similar, with the Gauss-LC results being a lower bound. This conclusion is also valid for other models of the pion flux and for the $\phi$ and $J/\Psi$ production. Consequently, in what follows we will consider only the Gauss-LC model for the vector meson wave functions. Let us now compare our predictions with the experimental data [9] considering different models for the pion flux. In order to constrain the value of the $K$ factor associated to absorptive corrections, our strategy will be following: for a given model of the pion flux, $R_q$ and dipole cross section, we will estimate the total cross section. The value of $K$ will be the value necessary to make our prediction consistent with the H1 data [9].

In Fig. 3 we present our predictions for the leading neutron spectra in exclusive $\rho$ photoproduction considering different models for the pion flux. The corresponding $K$ values are also presented. We obtain a reasonable agreement with the experimental data, with $K$ values in the range $0.134 < K < 0.210$. It is important to emphasize that these values of $K$ are strongly correlated with our choice for $R_q$. For example, if instead of $R_q = 2/3$ we assume $R_q = 1/3$, the corresponding $K$ values should be multiplied by 4, since the exclusive cross sections depend quadratically on the dipole scattering amplitude. If we assume a priori that the magnitude of the absorptive correction factor is of the order of 0.7 for exclusive processes, as predicted in [19] for semi-inclusive one, this implies a preference for the value $R_q = 1/3$. However, as the magnitude of these corrections for exclusive processes is still an open question, as well as the value of $R_q$, we refrain from drawing strong conclusions. Therefore, in what follows we will only present results for $R_q = 2/3$. GeV, where $p_T$ is the transverse momentum of the leading neutron. Moreover, we will assume initially that $R_q = 2/3$ and the bCGC dipole model. In Fig. 2 we analyse the dependence of our predictions on the choice of the vector meson wave function. We present our results for two different models of the pion flux. We find that the predictions are similar, with the Gauss-LC results being a lower bound. This conclusion is also valid for other models of the pion flux and for the $\phi$ and $J/\Psi$ production. Consequently, in what follows we will consider only the Gauss-LC model for the vector meson wave functions. Let us now compare our predictions with the experimental data [9] considering different models for the pion flux. In order to constrain the value of the $K$ factor associated to absorptive corrections, our strategy will be following: for a given model of the pion flux, $R_q$ and dipole cross section, we will estimate the total cross section. The value of $K$ will be the value necessary to make our prediction consistent with the H1 data [9].

In Fig. 3 we present our predictions for the leading neutron spectra in exclusive $\rho$ photoproduction considering different models for the pion flux. The corresponding $K$ values are also presented. We obtain a reasonable agreement with the experimental data, with $K$ values in the range $0.134 < K < 0.210$. It is important to emphasize that these values of $K$ are strongly correlated with our choice for $R_q$. For example, if instead of $R_q = 2/3$ we assume $R_q = 1/3$, the corresponding $K$ values should be multiplied by 4, since the exclusive cross sections depend quadratically on the dipole scattering amplitude. If we assume a priori that the magnitude of the absorptive correction factor is of the order of 0.7 for exclusive processes, as predicted in [19] for semi-inclusive one, this implies a preference for the value $R_q = 1/3$. However, as the magnitude of these corrections for exclusive processes is still an open question, as well as the value of $R_q$, we refrain from drawing strong conclusions. Therefore, in what follows we will only present results for $R_q = 2/3$. GeV, where $p_T$ is the transverse momentum of the leading neutron. Moreover, we will assume initially that $R_q = 2/3$ and the bCGC dipole model. In Fig. 2 we analyse the dependence of our predictions on the choice of the vector meson wave function. We present our results for two different models of the pion flux. We find that the predictions are similar, with the Gauss-LC results being a lower bound. This conclusion is also valid for other models of the pion flux and for the $\phi$ and $J/\Psi$ production. Consequently, in what follows we will consider only the Gauss-LC model for the vector meson wave functions. Let us now compare our predictions with the experimental data [9] considering different models for the pion flux. In order to constrain the value of the $K$ factor associated to absorptive corrections, our strategy will be following: for a given model of the pion flux, $R_q$ and dipole cross section, we will estimate the total cross section. The value of $K$ will be the value necessary to make our prediction consistent with the H1 data [9].

In Fig. 3 we present our predictions for the leading neutron spectra in exclusive $\rho$ photoproduction considering different models for the pion flux. The corresponding $K$ values are also presented. We obtain a reasonable agreement with the experimental data, with $K$ values in the range $0.134 < K < 0.210$. It is important to emphasize that these values of $K$ are strongly correlated with our choice for $R_q$. For example, if instead of $R_q = 2/3$ we assume $R_q = 1/3$, the corresponding $K$ values should be multiplied by 4, since the exclusive cross sections depend quadratically on the dipole scattering amplitude. If we assume a priori that the magnitude of the absorptive correction factor is of the order of 0.7 for exclusive processes, as predicted in [19] for semi-inclusive one, this implies a preference for the value $R_q = 1/3$. However, as the magnitude of these corrections for exclusive processes is still an open question, as well as the value of $R_q$, we refrain from drawing strong conclusions. Therefore, in what follows we will only present results for $R_q = 2/3$. GeV, where $p_T$ is the transverse momentum of the leading neutron. Moreover, we will assume initially that $R_q = 2/3$ and the bCGC dipole model. In Fig. 2 we analyse the dependence of our predictions on the choice of the vector meson wave function. We present our results for two different models of the pion flux. We find that the predictions are similar, with the Gauss-LC results being a lower bound. This conclusion is also valid for other models of the pion flux and for the $\phi$ and $J/\Psi$ production. Consequently, in what follows we will consider only the Gauss-LC model for the vector meson wave functions. Let us now compare our predictions with the experimental data [9] considering different models for the pion flux. In order to constrain the value of the $K$ factor associated to absorptive corrections, our strategy will be following: for a given model of the pion flux, $R_q$ and dipole cross section, we will estimate the total cross section. The value of $K$ will be the value necessary to make our prediction consistent with the H1 data [9].

In Fig. 3 we present our predictions for the leading neutron spectra in exclusive $\rho$ photoproduction considering different models for the pion flux. The corresponding $K$ values are also presented. We obtain a reasonable agreement with the experimental data, with $K$ values in the range $0.134 < K < 0.210$. It is important to emphasize that these values of $K$ are strongly correlated with our choice for $R_q$. For example, if instead of $R_q = 2/3$ we assume $R_q = 1/3$, the corresponding $K$ values should be multiplied by 4, since the exclusive cross sections depend quadratically on the dipole scattering amplitude. If we assume a priori that the magnitude of the absorptive correction factor is of the order of 0.7 for exclusive processes, as predicted in [19] for semi-inclusive one, this implies a preference for the value $R_q = 1/3$. However, as the magnitude of these corrections for exclusive processes is still an open question, as well as the value of $R_q$, we refrain from drawing strong conclusions. Therefore, in what follows we will only present results for $R_q = 2/3$. GeV, where $p_T$ is the transverse momentum of the leading neutron. Moreover, we will assume initially that $R_q = 2/3$ and the bCGC dipole model. In Fig. 2 we analyse the dependence of our predictions on the choice of the vector meson wave function. We present our results for two different models of the pion flux. We find that the predictions are similar, with the Gauss-LC results being a lower bound. This conclusion is also valid for other models of the pion flux and for the $\phi$ and $J/\Psi$ production. Consequently, in what follows we will consider only the Gauss-LC model for the vector meson wave functions. Let us now compare our predictions with the experimental data [9] considering different models for the pion flux. In order to constrain the value of the $K$ factor associated to absorptive corrections, our strategy will be following: for a given model of the pion flux, $R_q$ and dipole cross section, we will estimate the total cross section. The value of $K$ will be the value necessary to make our prediction consistent with the H1 data [9].
Figs. 8 and 9 we show our results for the energy and photon virtualities, respectively. As expected, the leading range we predict $\sigma$ values needed to describe the $\rho$ our predictions for the kinematical range which may be probed in future $W$ case, we will assume $x$ the different models of the pion flux. As done before, we will constrain the value of $\mu$ GeV

These results indicate that the experimental data are better described using the pion flux $\phi$ spectra in exclusive $\phi$ production as well as in DVCS are presented in Fig. 7. For the HERA kinematical processes with leading neutron production, it may be possible to disentangle $F$ from $R_q$. It is interesting to notice that leading neutron production is dominated by pion emission from the proton, i.e. $p \to \pi + n$. In all existing theoretical approaches this pion is soft and takes only a small fraction of the incoming proton energy, leaving the neutron with most of it. This is the physical reason for the peak seen at $x_L \simeq 0.75$ in the $x_L$ spectrum of the leading neutron.

In Fig. 4 we analyse the dependence of our predictions on the choice of the dipole scattering amplitude for two different models of the pion flux. As done before, we will constrain the value of $K$ by adjusting the predictions of the different dipole models to the experimental value of the total cross section. We find that the different predictions for the $x_L$ spectra are very similar. However, the effective value of the absorptive correction $K$ depends on the model of the dipole scattering amplitude as expected, since they predict different values for the $B$ slope, which determines the normalization of the photon - pion cross section. Figs. 3 and 4 are in a sense complementary, since what changes in the former (the flux factor) is kept fixed in the latter (where the dipole model is changed) and vice-versa. In each curve the overall constant $K R_q^2$ is chosen so as to bring our calculations as close as possible to the experimental points.

Comparing the curves we can conclude that the shape of the leading neutron spectrum is much more sensitive to the flux factor than to the dipole scattering amplitude $N$. The normalization of the spectrum is hence determined by $K$ and $R_q$, since $N$ is fixed from the analysis of other data. The values used for $K$ are significantly smaller than those found in theoretical estimates. Larger values of $K$ would be more plausible implying a deviation from the valence quark scaling and the consequent change in the factor $2/3$. A comparison between the curves in Fig. 4 favors the choice of the pion flux $f_3$, which, unlike $f_2$, contains a $t$-dependent form factor. A similar preference was found in Ref. 33, where a combined analysis of E866 and HERA data was performed.

In what follows we will only consider the bCGC model, which successfully describes the HERA data on exclusive processes. In Fig. 5 we present our predictions taking into account the experimental uncertainty present in the H1 data for the total cross section [3], which in our analysis translates into a range of possible values for the $K$ factor. These results indicate that the experimental data are better described using the pion flux $f_3(y)$. As a cross check of our results, we can compare our predictions with other H1 data obtained assuming $p_T < 0.69 \cdot x_L$ GeV. Assuming the same range of values for $K$ obtained in Fig. 4 [3] we can see in Fig. 6 that our predictions describe these data quite well.

Considering that the main inputs of our calculations have been fixed by the experimental data on exclusive $\rho$ photoproduction we can extend our analysis to other exclusive final states. We will assume the Gauss-LC model for vector meson wave function, the bCGC dipole scattering amplitude, the $f_3$ model for the pion flux and the same $K$ values needed to describe the $\rho$ data. Initially, let us consider the kinematical range probed by HERA. As in the $\rho$ case, we will assume $W = 60$ GeV and $p_T < 0.2$ GeV. However, for $\phi$ and $J/\Psi$ production we assume $Q^2 = 0.04$ GeV$^2$, while for the DVCS we consider that $Q^2 = 10$ GeV$^2$. The corresponding predictions for the leading neutron spectra in exclusive $\phi$ and $J/\Psi$ production as well as in DVCS are presented in Fig. 7. For the HERA kinematical range we predict $\sigma(\gamma p \to \phi \pi n) = 25.47 \pm 3.70$ nb, $\sigma(\gamma p \to J/\Psi \pi n) = 0.22 \pm 0.03$ nb and $\sigma(\gamma^* p \to \gamma \pi n) = 0.008 \pm 0.001$ nb, where the uncertainty is estimated taking into account the range of possible values of $K$. Finally, let us present our predictions for the kinematical range which may be probed in future $ep$ colliders assuming $p_T < 0.2$ GeV. In Figs. 8 and 9 we show our results for the energy and photon virtualities, respectively. As expected, the leading

\[ \frac{d\sigma}{dx} \]

\[ f_3(y) \]
neutron spectra increases with the energy at fixed \( Q^2 \) and decreases with the virtuality at fixed \( W \). In particular, for \( W = 1 \text{ TeV} \) and \( Q^2 = 5 \text{ GeV}^2 \) we predict \( \sigma(\gamma^*p \rightarrow \rho \pi n) = 6.55 \pm 0.95 \text{ nb}, \sigma(\gamma^*p \rightarrow \phi \pi n) = 1.71 \pm 0.25 \text{ nb}, \sigma(\gamma^*p \rightarrow J/\Psi \pi n) = 1.20 \pm 0.17 \text{ nb} \) and \( \sigma(\gamma^*p \rightarrow \gamma \pi n) = 0.16 \pm 0.02 \text{ nb} \). We believe that for these values of total cross sections, the experimental analysis of the exclusive processes associated with a leading neutron is feasible in future \( ep \) colliders. In particular, as the cross sections strongly increase when \( Q^2 \rightarrow 0 \), the analysis of the vector meson photoproduction in \( ep \) collisions can be useful to understand leading neutron spectra, which are of crucial importance in particle production in cosmic ray physics. Another possibility is the study of this process in ultraperipheral hadronic collisions, with the leading neutron being a tag for exclusive production. In principle these processes can be studied in the future at the LHC. Such proposition will be discussed in detail in a forthcoming publication.

IV. SUMMARY

One of the important goals in particle physics is to understand the production of leading particles, i.e. the production of baryons which have large fractional longitudinal momentum (\( x_L \geq 0.3 \)) and the same valence quarks (or at least one of them) as the incoming particles. Recent measurements of leading neutron spectra in \( ep \) collisions at HERA have shed a new light on this subject. However, the description of the semi-inclusive and exclusive leading neutron processes remains without a satisfactory theoretical description. In a previous work [20], we proposed to study semi-inclusive leading neutron production using the color dipole formalism, which successfully describes both inclusive and diffractive HERA data, taking into account the QCD dynamics and its non-linear effects, which are expected

FIG. 5: Leading neutron spectra in exclusive \( \rho \) photoproduction obtained considering the possible range of values of the \( K \) factor and two models for the pion flux. Data from Ref. [9].

FIG. 6: Leading neutron spectra in exclusive \( \rho \) photoproduction obtained considering the possible range of values of the \( K \) factor fixed using the other set of experimental data and two models for the pion flux. H1 data [9] obtained assuming that \( p_T < 0.69 \cdot x_L \text{ GeV} \).
FIG. 7: Predictions for the leading neutron spectra in exclusive $\phi$, $J/\Psi$ and DVCS production in the HERA kinematical range: $W = 60$ GeV and $p_T < 0.2$ GeV.

FIG. 8: Energy dependence of the leading neutron spectra in exclusive $\rho$, $\phi$, $J/\Psi$ and DVCS production in the kinematical range of the future $ep$ colliders ($Q^2 = 5$ GeV$^2$).

to be present at high energies. Making use of very simple assumptions about the relation between the dipole - pion and the dipole - proton scattering amplitudes and about the absorptive corrections, we demonstrated that the semi-inclusive data can be described by the dipole formalism and that Feynman scaling is expected at high energies. In this paper we have extended our analysis to exclusive processes associated with a leading neutron. Considering the same assumptions used for the semi-inclusive case, we have analysed in detail the dependence of our predictions on the choices of the vector meson wave function, of the dipole model and of the pion flux. We demonstrated that the HERA data on the exclusive $\rho$ photoproduction associated with a leading neutron can be quite well described by the color dipole formalism. Assuming the validity of this approach, we have presented for the first time predictions for the exclusive $\phi$, $J/\Psi$ and $\gamma$ production in $ep$ collisions for the energies of HERA and future colliders. Our results indicate that the experimental analysis of these processes is feasible and that they can be used to understand this long standing problem in high energy physics.
Finally, it is important to emphasize that the current sources of uncertainties in the computation of leading neutron spectra are: i) the strength of the absorptive corrections represented by the factor $K$; ii) the validity of the additive quark model for the photon-pion cross section; iii) the strength of the contribution from direct fragmentation of the proton into neutrons; iv) the precise form of the pion flux; v) the precise form of the dipole cross section. With sufficient experimental information, we can rule out candidates of the pion flux and of the photon-pion cross section. We believe that it is possible to constrain the unknown numbers and assumptions with the help of more experimental data on other processes with tagged leading neutrons, such as those on $D^*$ production [40] and those with dijet production [41]. Work along this line is in progress.

Acknowledgments

This work was partially financed by the Brazilian funding agencies CNPq, CAPES, FAPERGS and FAPESP.

[1] P. Newman and M. Wing, Rev. Mod. Phys. 86, 1037 (2014)
[2] F. Gelis, E. Iancu, J. Jalilian-Marian and R. Venugopalan, Ann. Rev. Nucl. Part. Sci. 60, 463 (2010); E. Iancu and R. Venugopalan, arXiv:hep-ph/0303204; H. Weigert, Prog. Part. Nucl. Phys. 55, 461 (2005); J. Jalilian-Marian and Y. V. Kovchegov, Prog. Part. Nucl. Phys. 56, 104 (2006); J. L. Albacete and C. Marquet, Prog. Part. Nucl. Phys. 76, 1 (2014).
[3] N. N. Nikolaev, B. G. Zakharov, Phys. Lett. B 332, 181 (1994); Z. Phys. C 64, 631 (1994).
[4] J. Jalilian-Marian, A. Kovner, L. McLerran and H. Weigert, Phys. Rev. D 55, 5414 (1997); J. Jalilian-Marian, A. Kovner and H. Weigert, Phys. Rev. D 59, 014014 (1999), ibid. 59, 014015 (1999), ibid. 59 034007 (1999); A. Kovner, J. Guilherme
Milhano and H. Weigert, Phys. Rev. D 62, 114005 (2000); H. Weigert, Nucl. Phys. A703, 823 (2002); E. Iancu, A. Leonidov and L. McLerran, Nucl. Phys. A692, 583 (2001); E. Ferreiro, E. Iancu, A. Leonidov and L. McLerran, Nucl. Phys. A701, 489 (2002).

[5] I. I. Balitsky, Phys. Rev. Lett. 81, 2024 (1998); Phys. Lett. B 518, 235 (2001); I.I. Balitsky and A.V. Belitsky, Nucl. Phys. A703, 823 (2002); E. Iancu, A. Leonidov and L. McLerran, Nucl. Phys. A692, 583 (2001); E. Ferreiro, E. Iancu, A. Leonidov and L. McLerran, Nucl. Phys. A701, 489 (2002).

[6] Y.V. Kovchegov, Phys. Rev. D 60, 034008 (1999); Phys. Rev. D 61, 074018 (2000).

[7] A. H. Rezaeian and I. Schmidt, Phys. Rev. D 88, 074016 (2013).

[8] J. Olsson [H1 Collaboration], PoS DIS 2014, 156 (2014); V. Andreev et al. [H1 Collaboration], Eur. Phys. J. C 74, 2915 (2014); F. D. Aaron et al. [H1 Collaboration], Eur. Phys. J. C 68, 381 (2010).

[9] V. Andreev et al. [H1 Collaboration], arXiv:1508.03176 [hep-ex].

[10] M. Bishari, Phys. Lett. B 38, 510 (1972).

[11] H. Holtmann et al., Phys. Lett. B 338, 363 (1994).

[12] B. Kopeliovich, B. Povh and I. Potashnikova, Z. Phys. C 73, 125 (1996).

[13] S. Kumano, Phys. Rev. D 43, 59 (1991).

[14] N.N. Nikolaev et al., Phys. Rev. D 60, 014004 (1999).

[15] L.L. Frankfurt, L. Mankiewicz and M.I. Strikman, Z. Phys. A 334, 343 (1989); T.T. Chou and C.N. Yang, Phys. Rev. D 50, 590 (1994); M. Przybycien, A. Szczurek and G. Ingelman, Z. Phys. C 74, 509 (1997); W. Melnitchouk, J. Speth and A.W. Thomas, Phys. Rev. D 59, 014033 (1998); A. Szczurek, N.N. Nikolaev and J. Speth, Phys. Lett. B 428, 383 (1998).

[16] A.B. Kaidalov, V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. C 47, 385 (2006).

[17] V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. C 48, 797 (2006).

[18] N.N. Nikolaev, J. Speth and B.G. Zakharov, hep-ph/9708290.

[19] U. D'Alesio and H.J. Pirner, Eur. Phys. J. A 7, 109 (2000).

[20] F. Carvalho, V. P. Goncalves, D. Spiering and F. S. Navarra, Phys. Lett. B 752, 76 (2016).

[21] See for example: F. Carvalho, V. O. Duraes, F. S. Navarra, M. Nielsen, F. S. Navarra, F. O. Duras and L. L. Barz, Mod. Phys. Lett. A 13, 2715 (1998); W. Melnitchouk and A.W. Thomas, Phys. Lett. B 414, 134 (1997).

[22] E. M. Levin and L. L. Frankfurt, JETP Lett. 2, 65 (1965).

[23] B. Z. Kopeliovich, I. K. Potashnikova, B. Povh and I. Schmidt, Phys. Rev. D 85, 114025 (2012).

[24] N.N. Nikolaev, J. Speth and V.R. Zoller, Phys. Lett. B 473, 157 (2000).

[25] J. Bartels, K. Golec-Biernat and K. Peters, Acta Phys. Polon. B 34, 3051 (2003).

[26] H. G. Dosch, T. Gousset, G. Kulzinger and H. J. Pirner, Phys. Rev. D 55, 2602 (1997).

[27] J. Nemchik, N.N. Nikolaev, E. Predazzi and B. G. Zakharov, Z. Phys. C 75, 71 (1997).

[28] J. R. Forshaw, R. Sandapen and G. Shaw, Phys. Rev. D 69, 094013 (2004).

[29] H. Kowalski and D. Teaney, Phys. Rev. D 68, 114005 (2003).

[30] J. P. B. C. de Melo and T. Frederico, Phys. Rev. C 55, 2043 (1997).

[31] H. Kowalski, L. Motyka and G. Watt, Phys. Rev. D 74, 074016 (2006); G. Watt and H. Kowalski, Phys. Rev. D 78, 014016 (2008).

[32] V. P. Goncalves, M. V. T. Machado and A. R. Meneses, Eur. Phys. J. C 68, 133 (2010).

[33] V. P. Goncalves and M. V. T. Machado, Phys. Rev. C 84, 011902 (2011); V. P. Goncalves, B. D. Moreira and F. S. Navarra, Phys. Rev. C 90, 015203 (2014); V. P. Goncalves, B. D. Moreira and F. S. Navarra, Phys. Lett. B 742, 172 (2015).

[34] E. Iancu, K. Itakura, S. Munier, Phys. Lett. B 590, 199 (2004).

[35] G. Soyez, Phys. Lett. B 655, 52 (2007).

[36] J. L. Albacete, N. Armesto, J. G. Milhano and C. A. Salgado, Phys. Rev. D 80, 034031 (2009).

[37] B. Z. Kopeliovich, I. K. Potashnikova, I. Schmidt and J. Soffer, Phys. Rev. D 78, 014031 (2008).

[38] J. R. McKenney, N. Sato, W. Melnitchouk and C. R. Ji, arXiv:1512.04459 [hep-ph].

[39] S. Chekanov et al. [ZEUS Collaboration], Phys. Lett. B 590, 143 (2004).

[40] S. Chekanov et al. [ZEUS Collaboration], Nucl. Phys. B 827, 1 (2010).