Asymmetric Neutrino Emission from Magnetized Proto-Neutron Star Matter including Hyperons in Relativistic Mean Field Theory

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We calculate asymmetric neutrino absorption and scattering cross sections on hot and dense magnetized neutron-star matter including hyperons in fully relativistic mean field theory. The absorption/scattering cross sections are suppressed/enhanced incoherently in the direction of the magnetic field \( \mathbf{B} = B \hat{z} \). The asymmetry is 2–4% at the matter density \( \rho_0 \leq \rho_B \leq 3 \rho_0 \) and temperature \( T \leq 40 \text{MeV} \) for \( B = 2 \times 10^{17} \text{G} \). This asymmetry is comparable to the effects owing to parity violation or asymmetric magnetic field topology proposed for the origin of pulsar kicks.

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The magnetic field is presumed to play an important role in many astrophysical phenomena. Especially after the discovery of asymmetry in supernova (SN) remnants, pulsar kicks [1], and magnetars [2, 3], strong magnetic field seems to hold the key to a still unresolved mechanism of non-spherical SN explosion and also to an unknown origin of required natal velocity [4] that the proto-neutron star receives at its birth. Although several post-collapse instabilities were studied as a possible source to trigger a non-spherical explosion leading eventually to the pulsar kicks, the unknown origin of global initial asymmetric perturbations and the uncertainties folded in the numerical simulations make this possibility unclear [5, 6]. Another viable candidate is an asymmetric neutrino emission which emerges from parity violation in weak interaction [7, 8] or an asymmetric distribution of the magnetic field [9] in strongly magnetized proto-neutron stars. Recent theoretical calculations [10, 11] have suggested that even \( \sim 1\% \) asymmetry in neutrino emission out of total neutrino luminosity \( \sim 10^{53} \) ergs might be enough to explain the observed pulsar kick velocity.
However, their underlying neutron star model is too primitive although both static and dynamic properties of the neutron-star matter at high temperature and high density have been studied very precisely [12–14] by including, for example, exotic phase of strangeness condensation [15–17], nucleon superfluidity [18], rotation-powered thermal evolution [19], quark-hadron phase transition [20], etc. Reddy et al. [21] studied also the neutrino propagation in the proto-neutron star matter including hyperons. Unfortunately, however, these sophisticated theoretical models of high-density hadronic matter did not include the effects of strong magnetic field. Although Lai and his collaborators [10, 11] studied the effects of magnetic field on the asymmetric neutrino emission, neutrino-nucleon scattering processes were calculated in non-relativistic framework [10].

We here report for the first time our calculated results of the neutrino scattering and absorption cross sections on hot and dense magnetized neutron-star matter including hyperons in fully relativistic mean field (RMF) theory [22]. In the present RMF framework we take account of the Fermi motions of baryons and electrons, their recoil effects, distortion effects of Fermi spheres made by the magnetic field, and effects of the energy difference of the mean field between the initial and final baryons. We then discuss implications of the present result for the pulsar kicks.

Let us now consider the system including nucleons, lambda particles, electrons and electron-type neutrinos by setting up the Lagrangian density which consists of four parts, \( \mathcal{L} = \mathcal{L}_{RMF} + \mathcal{L}_{Lept} + \mathcal{L}_{Mag} + \mathcal{L}_{W} \). The first, second, third and fourth terms are respectively the RMF, lepton, magnetic field and weak interaction Lagrangian densities. The RMF Lagrangian density is chosen to be the same as that in Refs. [23].

In the present work we assume a uniform dipole magnetic field along \( z \)-direction, \( \mathbf{B} = B \hat{z} \). The strength of magnetic filed inside the neutron star \( B = 10^{14} \sim 10^{17} \)G of astrophysical interest is still weaker than the energy scale of strong interaction, \( \mu_b B \ll \varepsilon_b \), where \( \mu_b \) and \( \varepsilon_b \) are the magnetic moment and chemical potential of baryon \( b \). In this case we can estimate the effects of magnetic field in a perturbative way by ignoring the contribution from convection current and considering only the spin-interaction term. The Lagrangian density for the interaction between baryons and the magnetic field is thus written as

\[
\mathcal{L}_{Mag} = \sum_b \mu_b \bar{\psi}_b \gamma_5 \sigma_{\mu\nu} \psi_b (\partial^\mu A^\nu - \partial^\nu A^\mu) = \sum_b \mu_b \bar{\psi}_b \sigma_z \psi_b B,
\]

where \( \psi_b \) is the field operator of baryon \( b \), \( A^\mu \) is the electromagnetic field, \( \sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2i \) with the Dirac matrix \( \gamma_\mu \), and \( \sigma_3 \) is the Pauli matrix. We obtain the Dirac spinor \( u_b(p,s) \) with momentum \( p \) and spin \( s \) in the magnetic field by solving the Dirac equation, \( [\gamma_\mu p^\mu - M^*_b - U_0(b)\gamma_0 - \mu_b B\sigma_z] u_b(p,s) = 0 \), where \( M^*_b = M_b - U_s(b) \) is the effective mass, and
$U_s(b)$ and $U_0(b)$ are the scalar mean-field and the time-component of the vector mean-field. These mean fields are calculated in the RMF theory \[22\].

When $|\mu_b B| \ll E^*_b(p) \equiv \sqrt{p^2 + M^*_b}$, the single particle energy of baryon $b$ becomes

$$
e_b(p, s) \approx E^*_b(p) + U_0(b) + \frac{\sqrt{p_T^2 + M^*_b}}{E^*_b(p)} \mu_B s,$$

and the Dirac spinor satisfies

$$u_b(p, s)\bar{u}_b(p, s) = \frac{1}{4E^*_b(p)}[E^*_b(p)\gamma_0 - p \cdot \vec{\gamma} + M^*_b](1 + s\gamma_5 a_\mu(p))$$

with $a_z(p) = \frac{E^*_b(p)}{\sqrt{p_T^2 + M^*_b}}$, $a_T = 0$, and $a_0(p) = \frac{p_s}{\sqrt{p_T^2 + M^*_b}}$, where the suffix $T$ indicates the transverse component of the vector which is perpendicular to the magnetic field. This spin vector $a_\mu$ is the same as that maximizing $|u^\dagger \beta \sigma_z u|$ \[24\]. From Eq. (2) the Fermi-Dirac distribution function is given by

$$n_b(e_b, s) \approx n_0(E^*_b + U_0(b)) + \frac{\frac{\partial n_b}{\partial e_b}(E^*_b + U_0(b))}{E^*_b(p)} \frac{\sqrt{p_T^2 + M^*_b}}{E^*_b(p)} \mu_B s.$$ 

Let us remark on the proton current here. The proton current includes both Dirac and anomalous currents and its appropriate treatment is complicated \[25\]. Since the proton fraction inside the neutron star is not very large, we take an approximation of substituting observed magnetic moment in rhs of Eq. (1) in the same manner as for the other neutral baryons.

As for the electrons in strong magnetic field, their energy levels are quantized in the Landau level. When $\sqrt{2|eB|} \ll \varepsilon_e$, where $\varepsilon_e$ is the chemical potential of electron, the sum of the Landau levels can be carried out approximately by integration over smoothed energy, and only the spin current remains. In this limit of weak magnetic field, the wave function becomes plane wave and the single particle energy is given by $e_e(p, s) = \sqrt{p^2 + m^2_e + eBs}/\sqrt{p^2 + m^2_e}$, where $m_e$ is the electron mass. The upper component of the electron wave function is an eigenvector of $eB\sigma_z$. Then the spin vector is $(0, 0, 0, 1)$ at the rest frame of the electron.

We now consider the neutrino scattering ($\nu_e \rightarrow \nu_e$) and absorption ($\nu_e \rightarrow e^-$) on the neutron-star matter, where the initial and final momenta of leptons are denoted by $k_i$ and $k_f$. Substituting Eqs. (3) and (4) into the standard cross section formula in Ref. \[21\] and taking the terms up to the first order of the magnetic field $B$, we can calculate the cross section: $\frac{d^3\sigma}{dk^3_f} = \frac{d^3\sigma_0}{dk^3_f} + \frac{d^3\Delta\sigma}{dk^3_f}$, where $\sigma_0$ is independent of $B$, and $\Delta\sigma$ is proportional to $B$. The $\Delta\sigma$ term apparently arises from the distortion of the Fermi-Dirac distribution caused by the magnetic field and shows an angular dependence made by the spin vector which is shown below Eq. (3). We note that the spherical Fermi-Dirac
distribution breaks down explicitly in the momentum space in the relativistic framework under the magnetic field. Details of the theoretical framework is discussed in forthcoming paper.

In the numerical calculations we use the parameter set of PM1-L1 [23] for the RMF. The $\sigma - \Lambda$ and $\omega - \Lambda$ couplings are taken to be $2/3$ of those of nucleon, i.e. $g_{\sigma,\omega}^\Lambda = 2/3 g_{\sigma,\omega}$ by taking account of quark degrees of freedom. Note that since Reddy et al. [21] used different couplings $g_{\sigma,\omega}^\Lambda = g_{\sigma,\omega}$, their calculated hyperon fraction is smaller than ours. We set $B = 2 \times 10^{17}$G as a representative field strength inside the neutron star. This value corresponds to $\mu_N B = 0.63$ MeV satisfying $|\mu_b B| \ll \varepsilon_\nu \ll E_b^*(p) \equiv \sqrt{p^2 + M_b^*}$.

The initial momentum here is taken to be the chemical potential $|\mathbf{k}_i| = \varepsilon_\nu$.}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Upper panels (a) and (c): Density dependence of the total energy per baryon $E_T/A$ of the neutron-star matter for $T = 20$ MeV (a) and 40 MeV (c). The red solid and blue long-dashed lines represent the calculated results in the systems with and without $\Lambda$. Lower panels (b) and (d): Number fractions of proton $x_p$, lambda particle $x_\Lambda$, and neutrino $x_\nu$ for $T = 20$ MeV (b) and 40 MeV (d). The red solid, dot-long-dashed, and short-dashed lines represent the calculated proton, lambda particle, and neutrino number fractions, respectively, in the system including $p, n$, and $\Lambda$. The blue long-dashed and dotted lines represent the calculated proton and neutrino number fractions in the system without $\Lambda$. We use the parameter-set PM1-L1 [23] for the RMF and the lepton fraction is set to be $Y_L = 0.4$ in the present calculation.}
\end{figure}

Figure 1 shows the calculated total energy per baryon $E_T/A$ and number fractions of several constituent particles in the system. If the neutron-star matter consists of only protons and neutrons without $\Lambda$ (see the blue lines), the proton fraction and therefore the electron fraction increase
gradually as the density increases, while the neutrino fraction decreases. However, when one allows the system to include hyperons (see the red lines), Λ appears at the density above $\rho_B \approx 2\rho_0$ and its fraction $x_\Lambda$ increases progressively as the density becomes larger. The proton fractions gradually increase up to about $2.2\rho_0$ where Λs do not exist abundantly, and then turn to decrease in higher density region, while the neutrino fractions describe completely opposite behavior. As such, the appearance of Λ takes appreciable baryon number and then suppresses the proton fraction at $2.2\rho_0 \leq \rho_B$, which results in softening the EOS of the neutron-star matter (see the upper panels of Fig. 1). This is almost independent of the temperature.

The calculated differential cross sections per baryon $d\sigma/d\Omega/A$ at $\rho_B = 3\rho_0$ for the neutrino scattering and absorption are shown in Fig. 2 where the initial neutrino angle is fixed to be $\theta_i = 0^\circ$. One prominent feature is that the neutrino scattering cross sections describe forward peak, but the absorption cross sections are strongly quenched in forward angles and show a small peak around $\theta_f \approx 30^\circ$ when Λ is included in the system (see the red lines). This quenching in absorption cross sections is made by the Pauli blocking effect on the final electrons. There are several interesting facts on the effects of hyperons and magnetic field. First, the magnetic field strongly suppresses the absorption cross sections in forward directions and slightly enhances them in backward directions in both cases with and without Λ. Particularly, at the forward angle $\theta_f \approx 0^\circ$ the suppression due to the magnetic field amounts to $20-30\%$, which is as large as the effect made by including Λ in the system. Second, the neutrino scattering cross sections are in contrast almost independent of the magnetic field. In fact, the contributions from constituent protons and neutrons cancel each other although each contribution is not small for the magnetic field $B = 2 \times 10^{17}$. For these reasons we hereafter discuss only the absorption cross sections.

Fig. 3 shows the magnetic part of the total absorption cross sections $\Delta \sigma$ normalized to $\sigma_0$, $\Delta \sigma/\sigma_0$, as a function of $\theta_i$ for various matter densities $\rho_0 \leq \rho_B \leq 7\rho_0$. In this calculation we set $|k_f| = \varepsilon\nu$. Deviation of this quantity from zero indicates an asymmetric neutrino cross sections. Asymmetry becomes smaller as the density becomes larger at any angles $\theta_i$ except for $\theta_i = 90^\circ$, and hyperons make a slight change at higher densities $2.2\rho_0 \lesssim \rho_B$ where Λ emerges abundantly as shown by the red dot-dashed lines in Figs. 1(b) and (d). This density dependence arises from the fact that $\Delta \sigma$ is approximately proportional to the fractional area of distorted Fermi surface caused by the magnetic field so that the relative strength $\Delta \sigma/\sigma_0$ gets smaller with increasing density.

Although the asymmetry $|\Delta \sigma/\sigma_0| \approx 0.044 \sim 0.022$ at $\theta_i \approx 0^\circ$ for $\rho_B = (1 \sim 3)\rho_0$ and $B = 2.0 \times 10^{17}G$ seems to be a small effect, we expect that it reaches 10–20 % for stronger magnetic field $B = 10^{18}G$ because this quantity is linearly proportional to $B$. Such a strong magnetic field
FIG. 2: Upper panels (a): Differential cross sections per baryon \( d\sigma/d\Omega/A \) in units of \( 10^{-16} \text{ fm}^2 \) for the neutrino scattering \( (\nu_e \rightarrow \nu_e) \) on neutron-star matter at \( \rho_B = 3\rho_0 \) for \( T = 20 \text{ MeV} \) (a). Lower panels (b): The same as the upper panels but for the neutrino absorption \( (\nu_e \rightarrow e^-) \). The initial momentum and angle of incident neutrino are taken to be \( |\vec{k}_i| = \varepsilon_{\nu} \) and \( \theta_i = 0^\circ \). The red solid and short-dashed lines represent the calculated results including \( \Lambda \) in the neutron star associated with and without magnetic field \( B = 2 \times 10^{17} \text{G} \), respectively. The blue dot-dashed and dotted lines represent the results without \( \Lambda \).

is not an unnatural hypothesis because the recent observations of magnetars suggest \( 10^{14} - 10^{15} \text{G} \) at the surface of the neutron stars, which leads to a stronger magnetic field \( (3 \sim 4) \times 10^{18} \text{G} \) at high densities inside the neutron star according to the scalar virial theorem. We presume that our perturbative approach is still valid for such strong magnetic field \( B \lesssim 10^{18} \text{G} \).

We comment on the neutrino scattering cross sections. When we integrate differential cross section over the initial angle \( \theta_i \), \( \Delta\sigma \) exhibits an excess by about 1.2\% at \( \theta_f = 0^\circ \) for \( B = 2 \times 10^{17} \text{G} \) at \( T = 20 \text{ MeV} \). When the magnetic field is as strong as \( 10^{18} \text{G} \), this contribution becomes significantly large although density dependence is small. It enhances the scattering cross sections in the direction along \( \vec{B} \) around \( \theta_f = 0^\circ \) and suppresses in the opposite direction. This effect therefore enlarges the asymmetry in the net neutrino emission incoherently with the effect on the neutrino absorption as discussed above.

Let us discuss how important implications these findings would have for the neutrino transport in strongly magnetized proto-neutron star. Lai and his collaborators proposed that an effect of parity violation in weak interaction would make asymmetric neutrino emission due to multiple neutrino-
FIG. 3: Normalized magnetic part of the total absorption cross sections, \( \Delta \sigma/\sigma_0 \), as a function of incident neutrino angle \( \theta_i \) for systems without hyperons at \( T = 20 \) MeV (a) and \( T = 40 \) MeV (b), and with hyperons at \( T = 20 \) MeV (c) and \( T = 40 \) MeV (d). In each panel the dotted, solid, dash-dotted and dashed lines represent the results at \( \rho_B = \rho_0 \), \( 3\rho_0 \), \( 5\rho_0 \) and \( 7\rho_0 \), respectively. Magnetic field of \( B = 2 \times 10^{17} \) G is used in this calculation.

neutrino scatterings [10, 11]. It is also proposed that an asymmetric magnetic field topology would induce asymmetric neutrino absorption [11]. In their calculations of semi-analytic 1D neutrino transport, both effects are claimed to result in fractional asymmetry in the neutrino emission at the level of 1% for a total neutrino luminosity \( \sim 10^{53} \) ergs in supernovae, which is enough to account for the observed pulsar kick velocity.

In our full RMF theory including hyperons, we find that the net neutrino emission increases by a few % in the forward direction along \( \mathbf{B} \) (for the magnetic field \( B = 2.0 \times 10^{17} \) G) and decreases in the opposite direction at any matter densities \( (\rho_0 \leq \rho_B \leq 7\rho_0) \). Our result is consistent with those estimated previously [10] in non-relativistic kinematics without hyperons. We, however, find for the first time that this asymmetry survives when we include \( \Lambda \) as presumed to emerge inside the neutron star as discussed in literature. Although we have not calculated neutrino transport as refs. [11], we expect that the asymmetry contributes to an aligned drift flux in the direction \( \mathbf{B} \) and dominates over diffusive flux, leading to an origin of pulsar kicks. It is very likely because fractional asymmetry in neutrino emission is dominated by the density region \( \rho_B \lesssim 3\rho_0 \) where neutrino opacity changes drastically, while the higher density region is dominated by diffusive flux
which smears the expected asymmetry in neutrino scattering and absorption cross sections. The hyperon effect is still important in the density region $2\rho_0 \lesssim \rho_B \lesssim 3\rho_0$ (see Fig. 1). Extensive study of the neutrino transport is now underway.

In summary, we study the neutrino scattering and absorption on magnetized proto-neutron star matter. We adopt full RMF theory for the description of the EOS of hot and dense hadronic matter including hyperons. We find that the strong magnetic field $B \sim 10^{17}$ G enhances the neutrino scattering cross sections and suppresses the absorption cross sections in the direction parallel to the magnetic field $B$, which could be an origin of the pulsar kicks. In the present study we did not take account of the neutrino-flavor conversion caused by the MSW effect [26], neutrino self-interaction [27], or resonant spin-flavor conversion [28], which would be another source of variation in the asymmetric neutrino emission. Phase transition to quark matter [29] or hyper-nuclear matter [30] under the strong magnetic field is another possibility to affect neutrino asymmetry. These are open questions for future studies.

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