Early Classification of Time Series
Cost-based Optimization Criterion and Algorithms

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Received: 05-15-2020 / Accepted: date

Abstract An increasing number of applications require to recognize the class of an incoming time series as quickly as possible without unduly compromising the accuracy of the prediction. In this paper, we put forward a new optimization criterion which takes into account both the cost of misclassification and the cost of delaying the decision. Based on this optimization criterion, we derived a family of non-myopic algorithms which try to anticipate the expected future gain in information in balance with the cost of waiting. In one class of algorithms, unsupervised-based, the expectations use the clustering of time series, while in a second class, supervised-based, time series are grouped according to the confidence level of the classifier used to label them.

Extensive experiments carried out on real data sets using a large range of delay cost functions show that the presented algorithms are able to satisfactorily solving the earliness vs. accuracy trade-off, with the supervised-based approaches faring better than the unsupervised-based ones. In addition, all these methods perform better in a wide variety of conditions than a state of the art method based on a myopic strategy which is recognized as very competitive.

Keywords Early classification of time series · Cost estimation · Sequential decision making

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1 Introduction

In emergency wards of hospitals [16], in control rooms of national or international electrical power grids [6], in government councils assessing emergency situations, in all kinds of contexts, it is essential to make timely decisions in absence of complete knowledge of the true outcome (e.g. should the patient undergo a risky surgical operation?). The issue facing the decision makers is that, usually, the longer the decision is delayed, the clearer is the likely outcome (e.g. the critical or not critical state of the patient) but, also, the higher the cost that will be incurred if only because earlier decisions allow one to be better prepared. How to optimize online the tradeoff between the earliness and the accuracy of the decision is the object of the early classification of time series problem.

Formally, we suppose that measurements are made available over time in a time series which, at time $t$, is $x_t = \langle x_1, \ldots, x_t \rangle$ where $x_t$ is the current measurement and the $x_i(1 \leq i \leq t)$ belong to some input domain (e.g. temperature and blood pressure of a patient, instantaneous power consumption on a national electrical grid). We suppose furthermore that each time series can be ascribed to some class $y \in \mathcal{Y}$ (e.g. patient who needs a surgical operation or not). The task is to make a prediction about the class of an incoming time series as early as possible because a cost is incurred at the time of the decision, where the cost function increases with time.

If the measurements in a time series are supposed independently and identically distributed (i.i.d.) according to a distribution of unknown “parameter” $\theta$, then the relevant framework is the one of sequential decision making and optimal statistical decisions [8,4]. In this setting, the problem is to determine as soon as possible whether the measurements have been generated by a distribution of parameter $\theta_0$ (hypothesis $H_0$) or of parameter $\theta_1$ (hypothesis $H_1$) with $\theta_0 \neq \theta_1$. One technique especially has gained a wide exposition: Wald’s Sequential Probability Ratio Test [24]. The log-likelihood ratio $R_t = \log \frac{P(\langle x_1, \ldots, x_t \rangle | y = -1)}{P(\langle x_1, \ldots, x_t \rangle | y = +1)}$ is computed and compared with two thresholds that are set according to the required error of the first kind $\alpha$ (false positive error) and error of the second kind $\beta$ (false negative error). Extensions to non-stationary distributions have been put forward (see [15, 20]).

In the early classification of time series problem, however, the successive measurements are not supposed to be i.i.d. To compensate for this weaker assumption, it is assumed that a labeled training set exists made of time series of finite length $T$: $x_T^i = \langle x_1^i, \ldots, x_T^i \rangle$ together with their corresponding labels, $S = \{(x_T^i, y_i)\}_{1 \leq i \leq m}$. Each measurement $x_T^i$ can be multivariate.

In the test phase, the scenario goes as follows. At each time step $t < T$, a new measurement $x_t$ is collected and a decision has to be made as whether to make a prediction now or to defer the decision to some future time step. When $t = T$, a decision is forced.

Interestingly, the problem of deciding online whether a prediction, and the attendant actions, should be made, or if it should be delayed, can be cast in the LUPI (Learning Under Privileged Information) framework [23]. During the learning phase, the learner has access to the full knowledge about the training time series $x_T$, while at testing time, only a subsequence $x_t$ ($t < T$) is known. The question is how to take advantage of the additional knowledge available at learning.
time to learn better online decision rules that apply when only partial information about the incoming time series is known. In the following, we examine previous works on the early classification problem in the light of the LUPI framework.

Maybe the earliest paper explicitly mentioning “classification when only part of the series are presented to the classifier” is [1] which shows how the boosting method can be employed to the classification of incomplete time series.

For many researchers, the question to solve is can we classify an incomplete times series while ensuring some minimum probability threshold that the same decision would be made on the complete input? To answer this question several approaches have been put forward.

One is to assume that the time series are generated i.i.d. according to some probability distribution, and to estimate the parameters of the class distributions from the training set. Once \( p(x_T|x_t) \) the conditional probability of the entire time series \( x_T \) given an incomplete realization \( x_t \) is estimated, it becomes possible to derive guarantees of the form:

\[
p(h_T(X_T) = y | x_t) = \int_{x_T \text{ s.t. } h_T(x_T) = y} p(x_T | x_t) \, dx_T \geq \epsilon
\]

where \( X_T \) is a random variable associated with the complete times series, \( \epsilon \) is a confidence threshold, and \( h_T(\cdot) \) is a classifier learnt over the training set \( S \) of complete times series. At each time step \( t \), \( p(h_T(X_T) = y | x_t) \) is evaluated and the prediction is triggered if this term becomes greater than some predefined threshold.

[2,21] present this method and propose ways to make the required estimations, in particular the mean and the covariance of the complete training data, when the time series are generated by Gaussian processes. It so far applies only with linear and quadratic classifiers.

[25] do not make assumptions about the form of the underlying distributions on the time series. They propose to use a 1NN classifier that choses the nearest training time series \( x_1^t \in S \) to the incoming one \( x_t = \langle x_1, \ldots, x_t \rangle \) to make its prediction. To determine for which time step \( t \) it is appropriate to make the prediction, the method is based on the idea of the minimum prediction length (MPL) of a time series. For a time series \( x_1^t \), one finds the set of every training time series \( x_j^t \) that have \( x_1^t \) as their one nearest neighbor. The MPL of \( x_1^t \) is then defined as the smallest time index for which this set does not change when the rest of the time series \( x_1^t \) is revealed. In the test phase, at time step \( t \), it is deemed that \( x_t \) can be safely labeled if its 1NN = \( x_1^t \) for which the MPL is \( t \). The idea is that from this point on, the prediction about \( x_t \) should not change. The authors found experimentally that this procedure, called ECTS (Early Classification of Times Series), leads to too conservative estimations of the earliest safe time step for prediction. They therefore proposed heuristic means to lower the estimated values. The stability criterion acts in a way as a proxy for a measure of confidence in the prediction. Similarly, [17] proposes a method where the evolution of the accuracy of a set of probabilistic classifiers is monitored over time, which allows the identification of timestamps from whence it seems safe to make predictions.

Another line of research is concerned with finding good descriptors of the time series, especially on their starting subsequences, so that early predictions can be reliable because they would be based on relevant similarities on the time series. For instance, in the works of [26,10,13], the principle is to look for shapelets, subsequences of time series which can be used to distinguish time series of one class.
from another, so that it is possible to perform classification of time series as soon as possible.

By contrast, there are methods for early classification of time series that do not take advantage of the complete knowledge available in the training set. For instance, in [21][12][11], a model \( h_t(\cdot) \) is learnt for each early timestamp and various stopping rules are defined in order to decide whether, at time \( t \), a prediction should be made or not. The price to pay for being outside the LUPI framework is that decisions are made in a myopic fashion which may prevent one from seeing that a better trade-off between earliness and accuracy is achievable in the future.

This is also the case for the work presented in [18]. The merit of this paper is to recognize that the earliness vs. accuracy trade-off depends on each domain and dataset, and that it must therefore be expressed as a single optimization criterion. This criterion serves as a stopping rule which determines whether it is time to output a prediction or wait for more data. This contrasts with approaches whereby the decision is made solely on the basis of a given confidence threshold which should be attained. However, the optimization criterion put forward is heuristic, supposes that the cost of delaying a decision is linear in time, and involves a complex setup. Most importantly, again, it is a myopic procedure which does not consider the foreseeable future. For all these apparent shortcomings, this method has been found to be quite effective, beating most competing methods in extensive experiments. This is why it is used as a reference method for comparison in this paper, as is done also in [22] which compares several techniques for early classification of time series.

In [7], for the first time, the problem of early classification of time series is cast as the optimization of a loss function which combines the expected cost of misclassification at the time of decision plus the cost of having delayed the decision thus far. Besides the fact that this optimization criterion is well-founded, it permits also to apply the LUPI framework because the expected costs for an incoming subsequence \( x_t \) can be estimated for future time steps and thus a non-myopic decision procedure can be used. These expectations can indeed be learned from the training set of \( m \) complete time series \( S = \{(x^i_T, y^i)\}_{1 \leq i \leq m} \).

Our paper extends the framework presented in [7] by proposing new techniques to estimate the future expected costs of decision and by presenting the results of extensive experiments. The ECONOMY methods are also compared to the competing approach of [18].

The rest of the paper is organized as follows. Section 2 presents the optimisation criterion that is the basis of the approach. Then, Section 3 presents four different methods that allow the estimation of this criterion. These methods vary by the way they profit from the complete training set to estimate future costs of decision. This gives rise to a set of questions as what are the characteristics that most drive the performance up. Section 4 presents the experimental setup to answer these questions. And the results with analysis are reported in Section 5. Section 6 concludes by underlying the main findings of this research and by discussing directions for future works.
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2 Early classification as a cost optimization problem

The expected cost of a decision at time step \( t \), when \( x_t \) is the incoming time series, can be expressed as in [7]:

\[
f(x_t) = \sum_{y \in \mathcal{Y}} P_t(y|x_t) \sum_{\hat{y} \in \mathcal{Y}} P_t(\hat{y}|y, x_t) C_m(\hat{y}|y) + C_d(t)
\]  

(1)

where \( C_m(\hat{y}|y) : \mathcal{Y} \times \mathcal{Y} \rightarrow \mathbb{R} \) is the *misclassification cost function* that defines the cost of predicting \( \hat{y} \) when the true class is \( y \) and \( C_d(t) \) is the *delay cost function* which is supposed to be non-decreasing over time. Both of these costs are expressed in the same unit (e.g. in dollars) and convey the characteristics of the application domain and can be provided by experts. It can be noticed that this framework can handle the case where \( C_m(\hat{y}|y) \) depends on time.

The expectation comes both from the misclassification probability \( P_t(\hat{y}|y, x_t) \) which can be estimated by the confusion matrix of the classifier \( h_t(\cdot) \) applied at time \( t \), and the posterior probability of each class given the input incomplete time series estimate \( P_t(y|x_t) \).

If the input time series was fully observed, this cost could be computed for all time steps \( t \in \{1, ..., T\} \), and the optimal time \( t^* \) for triggering the classifier’s prediction would be:

\[
t^* = \arg\min_{t \in \{1, ..., T\}} f(x_t)
\]  

(2)

But of course, this would defeat the whole purpose of early classification, as one would have to observe the entire time series before knowing what would have been the optimal decision time! Then, instead of waiting until the entire time series is known, at each time \( t \), one could “look into the future” and guess what will be the best decision time. And if the estimated best decision time matches the current time step \( t \), then the decision must be made. For the incoming time series \( x_t \), the expected cost at \( \tau \) time steps in the future is:

\[
f_{\tau}(x_t) = \sum_{y \in \mathcal{Y}} P_t(y|x_t) \sum_{\hat{y} \in \mathcal{Y}} P_{t+\tau}(\hat{y}|y, x_{t+\tau}) C_m(\hat{y}|y) + C_d(t + \tau)
\]  

(3)

where \( x_{t+\tau} \) is the foreseen continuation of \( x_t \). Accordingly, the best expected decision time in the future becomes:

\[
\tau^* = \arg\min_{\tau \in \{0, ..., T-t\}} f_{\tau}(x_t)
\]  

(4)

and if \( \tau^* = 0 \) the decision is instantly requested, and \( \hat{t}^* = t \) denotes the trigger time. The problem now is *how to predict \( x_{t+\tau} \) from the knowledge of \( x_t \).* Can the LUPI framework help? Yes it can. Figure 1a provides an overview of the principle in the case of a univariate time series. The “envelope” of its foreseeable futures can be learned using the training data set of complete time series \( \mathcal{S} = \{(x_i, y_i)\}_{1 \leq i \leq m} \).

Importantly, the solution chosen to guess the “envelope” of the \( x_{t+\tau} \) will also provide a way to estimate the terms \( P_{t+\tau}(\hat{y}|y, x_{t+\tau}) \) because a confusion matrix can be learned on this envelope.

However, it might be difficult to estimate the likely continuations of the incoming time series because \( x_t \) was actually never observed before (see Figure 1a). Another approach then consists in learning typical groups of time series from the training set, and then in predicting the likely continuations of \( x_t \) with regard to these groups (see Figure 1b).
Fig. 1: (a) Given an incomplete time series \( x_t \), the objective is to try to guess the “envelope” of its foreseeable futures. Various methods can be used to do so. (b) The incoming time series \( x_t \) is viewed as a member of or close to some group(s) of times series, and this is used to guess the “envelope” of its foreseeable futures.

Let us note \( g_k \) the \( k \)-th typical groups of time series, Equation (1) then can be re-expressed as:

\[
f(x_t) = \sum_{g_k \in \mathcal{G}} P(g_k | x_t) \sum_{y \in \mathcal{Y}} P(y | g_k) \sum_{\hat{y} \in \mathcal{Y}} P_t(\hat{y} | y, g_k) C_m(\hat{y}) + C_d(t)
\]

And similarly, for Equation (3):

\[
f_{t}(x_t) = \sum_{g_k \in \mathcal{G}} P(g_k | x_t) \sum_{y \in \mathcal{Y}} P(y | g_k) \sum_{\hat{y} \in \mathcal{Y}} P_{t+\tau}(\hat{y} | y, g_k) C_m(\hat{y}) + C_d(t + \tau)
\]

Equation (6) can be easily interpreted by splitting it into two parts. The first term \( P(g_k | x_t) \) estimates the posterior probabilities of each group given \( x_t \). The next term expresses the expectations of the cost of misclassification over future possible continuations of \( x_t \). Namely, the second term \( P(y | g_k) \) corresponds to the prior probabilities of class values within each group. And the third term \( P_{t+\tau}(\hat{y} | y, g_k) \) estimates the probabilities of misclassification within each group, at time step \( t + \tau \). The terms \( C_m(\hat{y}) \) and \( C_d(t + \tau) \) are the cost functions expressing properties of the domain of application.

In this general framework, several choices can be made to implement this optimization criteria. Foremost is the determination of relevant groups \( g_k \) of time series from the complete training set \( \mathcal{S} \). In what follows, we propose four different alternatives to anticipate the expected future misclassification costs.

3 Anticipating the future: a key to the optimization criterion

In this section, we examine ways to anticipate the cost of postponing decision when given an incoming time series \( x_t \) (see Table 1). We first present two methods, called ECONOMY-K and ECONOMY-MULTI-K, that rely on the clustering of time series, irrespective of their label. Next, we describe two methods, ECONOMY-\( \gamma \)-LITE and ECONOMY-\( \gamma \), which group time series according to the confidence level of the classifier at time \( t \). All of these approaches have been implemented in open-source code available at: [https://lelab.orange.fr/coming_soon](https://lelab.orange.fr/coming_soon)
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| Approaches       | Partition type                   | Partitions number | Anticipation type                      |
|------------------|----------------------------------|-------------------|----------------------------------------|
| ECONOMY-K        | Unsupervised (K-means)           | 1 partition of full-length time series. | Weak model: the continuations of time series are used in the groups. |
| ECONOMY-MULTI-K  | Unsupervised (K-means)           | T partitions corresponding to the different time steps. | Weak model: the continuations of time series are used in the groups. |
| ECONOMY-γ-lite   | Supervised: quantiles of the classifiers’ confidence levels are used | T partitions corresponding to the different time steps. | Weak model: the continuations of time series are used in the groups. |
| ECONOMY-γ        | Supervised: quantiles of the classifiers’ confidence levels are used | T partitions corresponding to the different time steps. | Markov Chain technique is used to anticipate missing measurements. |

Table 1: Overview of the design choices of the different approaches: each approach differing from the previous one by only one design choice.

3.1 ECONOMY-K

ECONOMY-K has been introduced in [7]. The idea is to first identify groups $g_k$ of times series using a clustering algorithm, here K-means with Euclidian distance, over a training set of complete time series $x^t_i$. Then, given an incoming time series $x^t$, the memberships $P(x^t|g_k)$ are estimated using a logistic function of a distance between $x^t$ and the centers of the clusters $g_k$. In order to estimate the terms $P_t(\hat{y}|y, g_k)$ of the confusion matrix for each time step $t = 1, \ldots, T$, a collection of classifiers $\{h_t\}_{t \in \{1, \ldots, T\}}$ is learned using training sets $\{S^t\}_{t \in \{1, \ldots, T\}}$ of time series truncated to their first $t$ measurements.

As explained in Section 2 Equation (4) is used to estimate the cost of deciding for future time steps $t + \tau (0 \leq \tau \leq T - t)$, and if $\tau^*$ given by Equation (4) is equal to zero or $t = T$, then a decision is triggered, otherwise a new measurement $x^{t+1}$ is made, and the decision mechanism is called again.

3.2 ECONOMY-MULTI-K

Instead of grouping time series using their full-length descriptions, an alternative consists in computing the clusters $g^t_k$ for each time step $t$ using training sets $\{S^t\}_{t \in \{1, \ldots, T\}}$ of truncated time series from the training set $S$. Indeed, clustering time series on the fly, at each time step, may allow for a increased adaptiveness to the specifics of the the beginning of the time series. The term $P(g_k|x_t)$ in Equation 6 then becomes $P(g^t_k|x_t)$. The cost of potential future decisions is now estimated based on the terms $P_t(\hat{y}|y, g^t_k)$.

3.3 ECONOMY-γ-lite

In the previous approaches, the confusion matrix with the term $P_{t+\tau}(\hat{y}|y, g_k)$ in Equation 6, is computed using time series in $g_k$ and potentially aggregates all confidence levels of $h_{t+\tau}$, corresponding to all possible values of the conditional probability $p(y = 1|x_{t+\tau})$. If this confusion matrix was instead computed over time series that share approximately the same confidence level in their classification, the estimation of future decision costs could be much more precise. This is the motivation behind the algorithms ECONOMY-γ and ECONOMY-γ-lite.
In these methods, the groups $g_{tk}$ are obtained by stratifying the time series by confidence level of $h_t$. At each time step $t$, the confidence level $p(h_t(x_t) = 1)$ of the classifier can take a value in $[0, 1]$. Examining the confidence levels for all time series in the validation set $S_t$ truncated to the first $t$ observations, we can discretize the interval $[0, 1]$ into $K$ equal frequency intervals, denoted $\{I_1^t, \ldots, I_K^t\}$. For instance, if $K = 5$, and $|S_t| = 1000$, the intervals $I_1^t = [0, 0.30]$, $I_2^t = [0.30, 0.45]$, $I_3^t = [0.45, 0.58]$, $I_4^t = [0.58, 0.83]$, $I_5^t = [0.83, 1]$ could each correspond to 200 training time series. The discretization of confidence levels into equal frequency intervals corrects any bias in the calibration of $h_t$, in a similar way to isotonic calibration [9].

Then, given an incoming time series $x_t$, the classifier $h_t$ is used to get an estimate of $p(y = 1|x_t)$ and determine the group $g_{tk}$ to which $x_t$ belongs. The algorithm is then the same as ECONOMY-MULTI-K, only with the groups $g_{tk}$ obtained in a supervised way by leveraging the information about the membership to the classes.

One can notice that, in addition to the expected gain in performance due to a more informed grouping of time series than in the clustering-based approaches, this method as well as ECONOMY-$\gamma$, does not require (i) the choice of a distance function for K-means, nor (ii) the determination of another distance between an incomplete time series $x_t$ and a cluster of full-length time series, and finally (iii) neither the choice of a membership function in order to estimate $P(g_k|x_t)$. The approach is therefore much simpler to implement.

![Fig. 2: ECONOMY-$\gamma$, computing the probability distribution $p(\gamma_{t+t}|\gamma_t)$](image)

Here $h_t(x_t)$ falls in the second confidence level interval. Given a supposed learned transition matrix $M^t_{t+1}$, the next vector of confidence levels will be $(0.15, 0.3, 0.3, 0.2, 0.05)^\top$.

### 3.4 ECONOMY-$\gamma$

ECONOMY-$\gamma$ uses the ECONOMY-$\gamma$-LITE principle to assign an incoming time series $x_t$ to a given group $g_k$, but it tries to get better estimates of the future terms

\[ \gamma = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \gamma_{t+1} = \begin{pmatrix} 0.15 \\ 0.3 \\ 0.3 \\ 0.2 \\ 0.05 \end{pmatrix} \]

\[ p(h_t(x_t) = 1) \]

\[ x_t \]

\[ M^t_{t+1}, M^{t+2}_{t+1}, \ldots \]

\[ t, t+1, t+2, t+\tau-1, t+\tau \]

\[ 1 \] This restricts these methods to binary classification problems.
Like in Equation (6), the future expected costs of decision are estimated through:

\[ P_{t+\tau} (\hat{y} | y, g_k^t) \]

of the confusion matrices by replacing \( g_k^t \) by a projection \( g_k^{t+\tau} \) into the future as a probability distribution over the confidence intervals of \( h_{t+\tau} \).

Let us call \( \tilde{\gamma}^t \) = \( (\gamma^t_1, \ldots, \gamma^t_K) \)\(^T\) the real-value vector of \( K \) components \( \gamma^t_i \), where each of the components is the probability that \( p(h_t(x_t) \in I^t_i) \). For instance, in Figure 2, \( \tilde{\gamma}^t_1 = (0, 1, 0, 0, 0) \), where all components are zero except \( \gamma^t_1 = 1 \).

We would like to compute the vectors \( \bar{\gamma}_{t+\tau} \) (\( 0 < \tau \leq T-t \)) consisting of the components:

\[ \gamma^t_{i+\tau} = p(h_{t+\tau}(x_{t+\tau}) \in I^t_i) \]  

In ECONOMY-\( \gamma \), we propose to estimate \( \gamma^t_{i+\tau} \) by using the \( K \times K \) transitions matrices \( \{M_{t+1}^{t'}\} \) from \( \tilde{\gamma}^t_{t'} \) to \( \tilde{\gamma}_{t'+1} \), where each component of the matrix is estimated by:

\[ m_{i,j} = p(p(h_{t'+1}(x_{t'+1}) \in I^{t'+1}_j | p(h_{t'}(x_t) \in I^{t'}_i)) \]  

given a validation set of time series. At time step \( t \), and from \( \tilde{\gamma}^t_1 \) it then becomes possible to compute \( \bar{\gamma}_{t+\tau} \) by:

\[ \bar{\gamma}_{t+\tau}^t = \tilde{\gamma}^t \prod_{s=0}^{\tau-1} M_{t+s}^{t+1} \]  

Like in Equation (6), the future expected costs of decision are estimated through:

\[ f_{\tau}(x_t) = \sum_{j=1}^{K} \gamma^t_{i+\tau} \sum_{y \in Y} P(y | I^t_{i+\tau}) \sum_{\hat{y} \in Y} P_{\hat{y}+\tau}(\hat{y} | y, I_{t+\tau}^t) C_m(\hat{y}) C_d(\tau + \tau) \]  

(1): for all confidence intervals \( I^t_{i+\tau} \) of \( h_{t+\tau} \)

(2): probability of misclassification when \( h_{t+\tau}(x_{t+\tau}) \in I^t_{i+\tau} \)

Again, a decision is triggered at time \( \hat{t} = t, \) if \( \tau^* = \text{ArgMin}_{\tau \in \{0, \ldots, T-t\}} f_{\tau}(x_t) \) is found to be 0.

In the following section, we present extensive experiments to compare the different ECONOMY approaches presented above.

4 Description of the experiments

4.1 Goal of the experiments

The approaches presented all rely on the determination of the best decision time based on a cost-based criterion which expresses the expected misclassification cost for future time steps plus a delay cost. This allows for a non myopic strategy.

Accordingly, the first question is whether this departure from the perspective of other approaches in the state of the art brings a gain in performance? For the end user, the important factor is the average cost incurred when using the competing methods (see Section 4.2 for the definition of \( \text{AvgCost} \)). Can they be reduced? And how much? It is also interesting to examine how far is the cost incurred using one of the method from the ideal optimal cost if one had had the knowledge of the whole series and could determine a posteriori what would have been the
best decision time. This is akin to a regret for not having a perfect a posteriori knowledge. All early classification methods can be evaluated using this criteria. This paper compares the four variants of the Economy family of algorithms plus the algorithm presented in [18] which has state of the art performance, as confirmed by a recent paper [22].

A second set of questions concerns the impact of the various design choices that distinguish the Economy algorithms. First, with regard to the partitioning of time series, it is carried out in an unsupervised mode for Economy-K and Economy-Multi-K while being based on a supervised principle for Economy-γ-lite and Economy-γ. Is one approach better than the other? Second, on a finer grain, is it better to cluster series on their full-length descriptions (Economy-K) or on their truncated description at each time step t (Economy-multi-K)? And third, is it useful to try to have a more precise anticipation of the future of the incoming time series as is done in Economy-γ? The experiments are designed to answer these questions.

4.2 Evaluation criterion

In order to compare the methods, it is important to consider a criterion which expresses its worth for the final user. We define a new evaluation criterion used in our experiments both to optimize K and to evaluate the early classification approaches on the test sets. In actual use, ultimately, the value of employing an early classification method corresponds to the average cost that is incurred using it. For a given data set S, it is defined as follows:

\[
\text{AvgCost}(S) = \frac{1}{|S|} \sum_{(x_t, y) \in S} \left( C_m (h_{\hat{t}}(x_{\hat{t}}) | y) + C_d(\hat{t}) \right)
\]  

(11)

where \( \hat{t} \) is the decision time chosen by the method as the one optimizing the trade-off between earliness and accuracy. In our experiments, AvgCost is evaluated for each dataset and for each early classification method. Statistical tests allow us to detect significant difference in performance.

4.3 Datasets

All the datasets used in our experiments come from the UEA & UCR Time Series Classification Repository2. These datasets were prepared and selected according to the following steps:

Dataset selection: we removed potentially redundant datasets by identifying almost identical dataset names and sizes. For instance, the datasets “Ford A” and “Ford B” contain the same number of time series with the same length. In this case, we keep only one dataset chosen at random. It is important to select only independent data sets in order to use statistical tests. We also need to ensure that the classifiers can be trained on data sets large enough so that the various methods can be compared on their own merit: poor classifiers would cause all methods to perform equally badly. Accordingly, we have selected the datasets containing at

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2 Available at: [http://www.timeseriesclassification.com](http://www.timeseriesclassification.com)
least 800 examples. After having trained the classifiers for each dataset and for each possible length, we filter the datasets based on the evaluation of the classifiers in order to reach reasonable performance. We kept only datasets for which the learned classifiers reach a Cohen’s kappa score of at least 0.3 for the time series truncated at half-length and more for time series of longer lengths. Four datasets were discarded as a result, leaving 34 independent datasets. All these datasets and their description are available at [https://tinyurl.com/ycmbxurq](https://tinyurl.com/ycmbxurq).

**Dataset preparation:** first, the training and test sets were merged for each dataset to overcome the possibly unbalanced or biased split of the original data files. The remaining datasets were then transformed into binary classes since ECONOMY-γ and ECONOMY-γ-lite are limited to binary classification. This was done by retaining the majority class and merging all the others. In order to reduce the computation time of the experiments and to compare datasets with time series of different lengths, we trained a classifier every 5% of the total length of the time series, instead of one classifier per time step, as done in [18]. Furthermore, for each dataset and for each possible length (i.e. 5%, 10%, 15% ... of the total length), we extracted 60 features[^3] from the corresponding truncated time series in order to train the associated classifiers. To do this, we used the Time Series Feature Extraction Library [3], which automatically extracts features on the statistical, temporal and spectral domains.

### 4.4 Experimental protocol

The datasets were divided by uniformly selecting 70% of the examples for the training set and using the remaining 30% for the test set. During the learning phase of the ECONOMY approaches, the training sets were divided into three disjoint subsets as follows: (subset a) 40% for training the various classifiers \( \{h_t\}_{t \in \{1,...,T\}} \); (subset b) 40% for learning the meta parameters; (subset c) 20% to optimize the number of groups in \( G \).

(subset a) *Learning the collection of classifiers:* for each dataset, the classifiers corresponding to the possible lengths of the input time series (i.e. every 5% of the total length) were learned. The XGBoost Python library[^4] was used, keeping the default values for the hyper-parameters.

(subset b) *Learning the meta-parameters:* they are learned for each ECONOMY approach, except the parameter \( K \) that is optimized at (subset c). For instance meta-model learned by the ECONOMY-γ approach consist of: (i) the discretization into \( K \) intervals of the confidence level for each classifier (one for each possible length); (ii) the transition matrices between a time step to the next one (i.e. every 5% of the time series length).

(subset c) *Optimizing the number \( K \) of groups:* the ECONOMY algorithms were trained by varying the number of groups between 1 to 20 and evaluated by the \( \text{AvgCost}(.) \) criterion which represents the average cost actually paid by the user (see Equation 11). The value of \( K \) which minimizes the \( \text{AvgCost}(.) \) criterion has been kept.

[^3]: More details are available in: [https://docs.google.com/spreadsheets/d/13u7L_51x3xFu_q-SnbGzFIdQf postfixingUri/](https://docs.google.com/spreadsheets/d/13u7L_51x3xFu_q-SnbGzFIdQf postfixingUri/)
[^4]: XGBoost is available in: [https://xgboost.readthedocs.io](https://xgboost.readthedocs.io)
Costs setting: the misclassification cost is set in the same way for all datasets: 

\[ C_m(\hat{y}|y) = 1 \] if \( \hat{y} \neq y \), and = 0 otherwise. The delay cost \( C_d(t) \) is provided by the domain experts in actual use case. In the absence of this knowledge, we define it as a linear function of time, with coefficient, or slope, \( \alpha \):

\[ C_d(t) = \alpha \times \frac{t}{T} \]  

(12)

The higher the \( \alpha \) coefficient, the more costly it is to wait for more measurements in the incoming time series. The delay cost \( C_d(t) \) is obviously of paramount importance to control the best decision time. If \( \alpha \) is very low, it does not hurt to wait for the whole time series is known and \( t^* = T \). If, on the contrary, \( \alpha \) is very high, the gain in misclassification cost obtained while awaiting more observations cannot compensate for the increase of the delay cost, and it is better to make a decision right at the start of the observations. Our experiments were run over a three ranges of values of \( \alpha \): low time cost with \( \alpha \in [1e-04, 2e-04, 4e-04, 8e-04, 1e-03, 3e-03, 5e-03, 8e-03] \); medium time cost with \( \alpha \in [0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09] \); high time cost with \( \alpha \in [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0] \).

5 Results and analysis

5.1 Comparison of the ECONOMY approaches with a non adaptive baseline

As a first sanity test, it is interesting to see if the ECONOMY algorithms do indeed adapt the decision time to the incoming time series, or if they treat them all the same. In order to perform this test, each of the ECONOMY approaches is run once in its adaptive mode, and once made unable to adapt by forcing the number of groups \( K = 1 \) (there is thus no difference made between the series).

The ECONOMY approaches are trained on the 34 selected datasets by varying the value of \( \alpha \), and then, evaluated on the test sets using the \( AvgCost \) criterion. The Wilcoxon signed-rank test is used to assess whether the observed performance gap is significant. Figure 3 presents the results of the Wilcoxon signed-rank test for each ECONOMY approach, applied over the 34 datasets by varying the values of \( \alpha \).
In the range $\alpha \in [0.0001, 0.01]$, ECONOMY-MULTI-K is the only approach that succeed in adapting its trigger times. By contrast, in the range $\alpha \in [0.02, 0.1]$, it appears that the ECONOMY approaches actually succeed in adapting their trigger times, with the exception of ECONOMY-MULTI-K which fails this test one-third of the time and behaves rather erratically when $\alpha$ varies.

At the end, these approaches succeed in improving performance by adapting their trigger times, differing in their range of success.

5.2 Comparison of the ECONOMY approaches

(a) Comparison with respect to the average decision cost

The AvgCost criterion was evaluated on the 34 test sets, and $\alpha$ was adjusted for each dataset in order to reveal the greatest differences in performance between the best and worst approach (see Table 2 for more details). The Nemenyi test [19] was used to rank the different ECONOMY approaches in terms of average decision cost. The Nemenyi test consists of two successive steps. First, the Friedman test is applied to the average decision cost of competing approaches to determine whether their overall performance is similar. If not, the post-hoc test is applied to determine groups of approaches whose overall performance is significantly different from that of the other groups.

Fig. 4: Evaluation based on AvgCost: (a) Nemenyi test applied to the 34 datasets; (b) pairwise comparison using the Wilcoxon signed-rank test, with black squares identifying non-significant comparisons.

Figure 4a, reporting the results of the Nemenyi test, shows two groups of methods of which the performances are significantly different. Specifically, the ECONOMY-$\gamma$ and ECONOMY-$\gamma$-LITE methods exhibit much better average decision costs than ECONOMY-K and ECONOMY-MULTI-K.

Figure 4b shows pairwise comparison using the Wilcoxon signed-rank test between the approaches. The small black squares identify pairs of approaches that do not differ significantly in performance. It is thus apparent that ECONOMY-$\gamma$ performs significantly better than ECONOMY-$\gamma$-LITE.

(b) Comparison with respect to the earliness of the decision time

In the following, the earliness of early classification approaches is evaluated using
Fig. 5: Earliness (a, b) and predictive performance (c, d) comparison of the Economy approaches.

The median of the trigger times normalized by the length of the series, defined by 
\[ \text{earliness} = \frac{\text{med}\{\hat{t}^*\}}{T} \]  
(see Table 2). Figure 5a shows that Economy-γ, on average, triggers its decision earlier than the competing methods, followed by Economy-γ-lite. Furthermore, according to the Wilcoxon signed-rank test, this difference is significant compared to the other Economy approaches (Figure 5b).

(c) Comparison with respect to the predictive performance of the algorithms
The predictive performance is evaluated using the Cohen’s kappa score \([5]\) computed at \(\hat{t}^*\), since this criterion properly manages unbalanced datasets (see Table 2). Again, the Economy-γ and Economy-γ-lite dominate in terms of predictive performance, but here the difference is not statistically significant.

(d) Pareto curves when varying the \(\alpha\) coefficient controlling the delay cost
In Figure 6, the coordinates of each point are given by the average Kappa score and the average earliness obtained over the 34 datasets when the delay cost \(\alpha\) is chosen in the range \([10^{-4}, 1]\), and the Pareto curve is drawn for each of the approaches. The result is strikingly clear. For each value of \(\alpha\), Economy-γ dominates all others approaches, even if Economy-γ-lite is not far behind. The Economy-K and Economy-multi-K approaches yield much weaker results and are indistinguishable from each other.
Fig. 6: Average Earliness vs. Average Kappa score obtain over the 34 datasets by varying the slope of the time cost, such as $\alpha \in [10^{-4}, 1]$.

(e) Comparison with the best possible performance
The approaches presented are able to adapt their decision time $\hat{t}^*$ to the characteristics of the time series and to perform well in terms of average decision costs $\text{AvgCost}$, but to what extent these results differ from the optimal ones $\text{AvgCost}^*$ computable after the entire time series is known? For each dataset, $\Delta_{\text{cost}} = |\text{AvgCost} - \text{AvgCost}^*|$ was computed. Figure 7 shows that ECONOMY-$\gamma$ provides the best online decisions compared to the optimal ones, on average, followed by ECONOMY-$\gamma$-LITE. According to the Wilcoxon signed-rank test, this difference is significant compared to the other ECONOMY approaches (Figure 7b).

Fig. 7: Evaluation of the quality of online decisions based on $\Delta_{\text{cost}}$.

From all these results, several conclusions can be drawn.

1. The ECONOMY approaches that partition the time series using the learned classifiers (supervised-based methods) perform significantly better than those which exploit the K-means algorithm (unsupervised-based methods).
2. For the unsupervised approaches, partitioning the time series on full-length time series (ECONOMY-K), or on truncated ones (ECONOMY-MULTI-K) does not significantly affect the performances.

3. Regarding the supervised methods, using a more sophisticated anticipation mechanism of the incoming time series as done by ECONOMY-\(\gamma\) is profitable and allows it to beat the less sophisticated ECONOMY-\(\gamma\)-LITE method.

5.3 Comparison with a state of the art approach

An important question is whether it is worth considering explicitly, in a single optimization criterion, earliness and accuracy, as in the ECONOMY approaches, and furthermore to adopt a non-myopic strategy with the modeling and computational costs involved. To assess this, we compared the ECONOMY methods with a competing algorithm, called SR presented in [18] which is claimed to dominate all other algorithms over 45 benchmark data sets, of which 19 belong to the set of 34 datasets we used. The SR algorithm uses a trigger function to decide if the current prediction is reliable (output 1) or if it is preferable to wait for more data (output 0). Among several triggered functions, all of a heuristic nature, the most effective is:

\[
\text{Trigger} (h_t(x_t)) = \begin{cases} 
0 & \text{if } \gamma_1 p_1 + \gamma_2 p_2 + \gamma_3 \frac{t}{T} \leq 0 \\
1 & \text{otherwise}
\end{cases}
\]

where \(p_1\) is the largest posterior probability estimated by the classifier \(h_t\): \(p_1 = \text{ArgMax}_{y \in Y} (\hat{p}(y|x_t))\), \(p_2\) is the difference between the two largest posterior probabilities, defined as \(|\hat{p}(y = 1|x_t) - \hat{p}(y = 0|x_t)|\) in the case of binary classification problems, and where the last term \(\frac{t}{T}\) represents the proportion of the incoming time series that is visible at time \(t\).

The parameters \(\gamma_1, \gamma_2, \gamma_3\) are real values in \([-1, 1]\) to be optimized. In our experiments, these parameters were tuned using a grid-search over the set of values \([-1, -0.95, -0.90, ..., 0.05, ..., 0.90, 0.95, 1]\) in order to minimize the criterion \(\text{AvgCost}\). The optimization was carried out for all possible time cost functions with a slope \(\alpha \in [10^{-4}, 1]\).

![Fig. 8: SR vs. ECONOMY-\(\gamma\): evaluation based on \(\text{AvgCost}\).](image-url)
After training, the AvgCost criterion was evaluated on the 34 test sets, and $\alpha$ was adjusted for each dataset in order to find the most favorable setting for the SR algorithm, namely one maximizing $\text{AvgCost}_{\text{SR}} - \text{AvgCost}_{\text{Eco-}\gamma}$.

Figure 8a reports the results of the Nemenyi test and demonstrates that even in these situations favoring the SR algorithm, ECONOMY-$\gamma$ reaches significantly better performances. The Wilcoxon signed-rank test presented in Figure 8b reinforces this conclusion.

We also carried out the Wilcoxon signed-rank test to compare the SR approach with the four ECONOMY approaches, for each value of $\alpha \in [10^{-4}, 1]$. The results (see Figure 9) shows forcibly that the ECONOMY approaches perform significantly better than the SR approach, regardless of the value of $\alpha$; except for $\alpha = 10^{-4}$ where this difference is not significant for ECONOMY-K and ECONOMY-$\gamma$-LITE.

6 Conclusions

An increasing number of applications require the ability to recognize the class of an incoming time series as quickly as possible without unduly compromising the accuracy of the prediction. In this paper, we reformulated in generic way an optimization criterion put forward in [7] which takes into account both the cost of misclassification and the cost of delaying the decision.

This generic framework has been technically declined, leading to the design of three new “non-myopic” algorithms - i.e. able to anticipate the expected future gain in information in balance with the cost of waiting. In one class of algorithms, unsupervised-based, the expectations use the clustering of time series, while in a second class, supervised-based, time series are grouped according to the confidence level of the classifier used to label them.

We have defined a new evaluation criterion that represents the average cost incurred when the method is applied over a set of labelled time series. This criterion makes it possible to evaluate both earliness and predictive performance as a single objective, with respect to the ground truth. It offers a well-grounded framework widely applicable for the comparison of methods.

Extensive experiments carried out on real data sets using a large range of delay cost functions show that the presented algorithms are able to satisfactorily solving the earliness vs. accuracy trade-off, with the supervised-based approaches generally faring better than the unsupervised-based ones. In addition, all these
methods perform better in a wide variety of conditions than the state of the art competitive method of [18].

Given the merit of the proposed approach, we envision several extensions. One is to allow the supervised-based approaches, which use the confidence level of a binary classifier, to solve multi-classes problems. Another one is to use a supervised clustering technique to compute groups of times series (see [14]). Finally, we are working on the adaptation of these methods to the on-line detection of anomalies in a data stream.

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| Datasets                      | Nb groups: K | AvgCost | Median CI          | Kappa |
|-------------------------------|--------------|---------|--------------------|-------|
|                              | α             | E=1   | E=5   | K 1=E 1   | K 1=E 5 | E=1   | E=5   | K 1=E 1   | K 1=E 5 |
| CBF                           | 0.8           | 0.11  | 7     | 4     | 0.167 | 0.163 | 0.211 | 0.222 | 0.09 | 0.14 | 0.23 | 0.23 |
| ChlorineConcentration         | 0.4           | 0.14  | 11    | 20    | 0.252 | 0.302 | 0.303 | 0.422 | 0.10 | 0.14 | 0.05 | 0.14 |
| CinCECGTorso                  | 0.1           | 0.20  | 15    | 1     | 0.014 | 0.012 | 0.021 | 0.021 | 0.05 | 0.05 | 0.05 | 0.05 |
| Crop                          | 0.06          | 0.10  | 19    | 19    | 0.028 | 0.031 | 0.031 | 0.042 | 0.04 | 0.04 | 0.04 | 0.04 |
| ECG2500                       | 0.5           | 0.18  | 15    | 1     | 0.046 | 0.051 | 0.065 | 0.064 | 0.05 | 0.05 | 0.05 | 0.05 |
| ECGFiveDays                   | 0.3           | 0.5   | 7     | 14    | 0.115 | 0.097 | 0.163 | 0.125 | 0.09 | 0.13 | 0.19 | 0.18 |
| ElectricDevices               | 0.1           | 0.18  | 4     | 13    | 0.114 | 0.109 | 0.150 | 0.136 | 0.08 | 0.04 | 0.13 | 0.46 |
| FaceAll                       | 0.01          | 0.16  | 16    | 19    | 0.002 | 0.002 | 0.006 | 0.004 | 0.05 | 0.05 | 0.05 | 0.05 |
| FaceSUC                       | 0.5           | 0.11  | 2     | 13    | 0.093 | 0.111 | 0.123 | 0.133 | 0.05 | 0.05 | 0.14 | 0.14 |
| FiftyWords                    | 0.5           | 0.15  | 2     | 13    | 0.104 | 0.119 | 0.148 | 0.160 | 0.05 | 0.10 | 0.05 | 0.05 |
| FordA                         | 0.3           | 0.14  | 7     | 7     | 0.114 | 0.118 | 0.155 | 0.164 | 0.15 | 0.15 | 0.15 | 0.15 |
| FreezerRegularTrain           | 0.2           | 0.17  | 4     | 7     | 0.016 | 0.021 | 0.024 | 0.025 | 0.05 | 0.05 | 0.05 | 0.05 |
| HandOutlines                  | 0.01          | 0.17  | 4     | 10    | 0.109 | 0.113 | 0.124 | 0.160 | 0.25 | 0.40 | 0.50 | 1.00 |
| InsectWingbeatSound           | 1             | 0.19  | 1     | 1     | 0.151 | 0.130 | 0.130 | 0.130 | 0.05 | 0.05 | 0.05 | 0.05 |
| ItalyPowerDemand              | 0.5           | 0.14  | 9     | 1     | 0.265 | 0.302 | 0.349 | 0.349 | 0.04 | 0.08 | 0.33 | 0.33 |
| Malat                         | 0.08          | 0.09  | 12    | 10    | 0.016 | 0.019 | 0.034 | 0.028 | 0.05 | 0.05 | 0.20 | 0.15 |
| MedicalImages                 | 0.07          | 0.12  | 11    | 19    | 0.210 | 0.238 | 0.280 | 0.279 | 0.08 | 0.16 | 0.12 | 0.69 |
| MelbournePedestrian           | 0.8           | 0.12  | 3     | 1     | 0.062 | 0.111 | 0.134 | 0.122 | 0.04 | 0.04 | 0.04 | 0.04 |
| MixedShapesRegularTrain       | 0.1           | 0.20  | 5     | 3     | 0.030 | 0.020 | 0.047 | 0.044 | 0.05 | 0.10 | 0.15 | 0.10 |
| MotoStrain                    | 0.4           | 0.17  | 4     | 11    | 0.128 | 0.142 | 0.156 | 0.178 | 0.10 | 0.14 | 0.14 | 0.29 |
| NonInvasiveFetalECG2          | 0.04          | 0.05  | 9     | 7     | 0.011 | 0.010 | 0.014 | 0.012 | 0.05 | 0.05 | 0.05 | 0.05 |
| PhalangesOutlinesCorrect      | 0.2           | 0.5   | 16    | 4     | 0.297 | 0.348 | 0.320 | 0.316 | 0.15 | 0.25 | 0.15 | 0.15 |
| ProximalPhalangesOutlinesCo   | 0.5           | 0.6   | 4     | 7     | 0.267 | 0.300 | 0.300 | 0.306 | 0.05 | 0.10 | 0.05 | 0.20 |
| SongHandGenderChk2            | 0.3           | 0.4   | 8     | 1     | 0.176 | 0.176 | 0.174 | 0.265 | 0.15 | 0.20 | 0.20 | 0.40 |
| SongAIIBORobotSurFace2        | 0.8           | 0.10  | 7     | 15    | 0.176 | 0.210 | 0.228 | 0.208 | 0.09 | 0.18 | 0.18 | 0.18 |
| StarLightCurves               | 0.3           | 0.17  | 12    | 18    | 0.062 | 0.067 | 0.089 | 0.095 | 0.05 | 0.05 | 0.15 | 0.20 |
| Strawberry                    | 0.6           | 0.2   | 9     | 17    | 0.195 | 0.193 | 0.251 | 0.201 | 0.09 | 0.14 | 0.05 | 0.19 |
| Symbols                       | 0.2           | 0.15  | 15    | 15    | 0.034 | 0.034 | 0.057 | 0.042 | 0.05 | 0.05 | 0.05 | 0.05 |
| TwoLeadECG                    | 0.9           | 0.15  | 16    | 19    | 0.160 | 0.168 | 0.197 | 0.185 | 0.10 | 0.10 | 0.15 | 0.15 |
| TwoPatterns                   | 0.08          | 0.13  | 11    | 10    | 0.079 | 0.065 | 0.104 | 0.104 | 0.56 | 0.66 | 0.89 | 0.89 |
| UWGestureLibraryX             | 0.5           | 0.12  | 11    | 18    | 0.130 | 0.148 | 0.153 | 0.126 | 0.10 | 0.14 | 0.12 | 0.12 |
| Wafer                         | 0.2           | 0.18  | 15    | 6     | 0.010 | 0.010 | 0.011 | 0.010 | 0.05 | 0.05 | 0.05 | 0.05 |
| WordSynonyms                  | 0.6           | 0.19  | 20    | 20    | 0.177 | 0.218 | 0.209 | 0.307 | 0.10 | 0.10 | 0.10 | 0.14 |
| Yogas                         | 0.03          | 0.14  | 8     | 8     | 0.168 | 0.246 | 0.183 | 0.193 | 0.10 | 0.49 | 0.49 | 0.49 |

Table 2: Details of experimental results for each dataset.