Mixing the Strong and E-W Higgs Sectors.*

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Abstract

After noting the well known similarity of the minimal electro-weak Higgs sector with the linear sigma model for the pion and the sigma, it is found that a small mixing term between the two models generates a pion mass. Although the custodial $SU(2)_L \times SU(2)_R$, and the gauged $SU(2)_L \times U(1)$ symmetry for the whole model remains intact, the mixing breaks the relative chiral symmetry between the two sectors. The mixing should be calculable from light quark masses as a quantum correction. This simple mechanism of "relative symmetry breaking" is believed to have applications for other forms of symmetry breaking.

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Spontaneous symmetry breaking in the vacuum is certainly a very beautiful and important concept in many areas of physics. The prototype for this mechanism in particle physics is given by the linear sigma model (L$\sigma$M)[1], (which can be looked upon as an effective theory at sufficiently low energy of a more fundamental theory as QCD). The minimal electro-weak scalar sector was essentially copied from the L$\sigma$M, except for a much higher value for the vacuum expectation value, $v = 246$ GeV, instead of the 92.6 MeV in the L$\sigma$M, which we here denote by $\hat{v}$. The latter value, $\hat{v}$, is also the pion decay constant ($f_\pi$), and is proportional to the $q\overline{q}$ condensate of QCD. Thus the vacuum values are orders of magnitudes different, $v/\hat{v}$ is about 2656.

The analogy is more evident if we represent the conventional complex Higgs-doublet composed of $\phi^+$ and $\phi^0$ by a seemingly redundant matrix form (We follow the notations of Willenbrock[2])

$$
\begin{pmatrix}
\phi^+ \\
\phi^0
\end{pmatrix}
\rightarrow
\frac{1}{\sqrt{2}}
\begin{pmatrix}
\phi^0 & \phi^+ \\
-\phi^- & \phi^0
\end{pmatrix}
= \Phi.
$$

(1)

The Higgs Lagrangian then takes the form

$$
L_{\text{Higgs}}(\Phi) = \text{Tr}[(D_\mu \Phi)^\dagger D_\mu \Phi] + \mu^2 \text{Tr}[\Phi^\dagger \Phi] - \lambda \text{Tr}[\Phi^\dagger \Phi]^2,
$$

(2)

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from which (at the tree level) the vacuum value is fixed by $\mu^2$ (which is assumed to have the right sign for spontaneous symmetry breaking) and $\lambda$, $v = [\mu^2/(2\lambda)]^{1/2} = 246$ GeV, and where the covariant derivative is

$$D_\mu \Phi = (\partial_\mu \Phi + ig_2 \tau \cdot W_\mu \Phi - ig'_2 B_\mu \Phi \tau_3).$$

(3)

(The $\tau_3$ matrix shows explicitly in this matrix representation that the right handed gauge group is only $U(1)$). The vacuum value of the $\Phi$ field is

$$<\Phi> = \frac{1}{\sqrt{2}} \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}.$$ 

(4)

The Lagrangian is, as well known, invariant under the gauge symmetry $SU(2)_L \times U(1)_Y$ ($\Phi \to L \Phi$ and $\Phi \to \Phi e^{-i\sigma_3\rho}$), which is broken spontaneously down to $U(1)_{EM}$ by the $v$ of eq.(4). But, in the limit of $g' \to 0$, i.e. $\theta_W \to 0$, (or if one disregards the gauging) there is, in fact, also a global $SU(2)_L \times SU(2)_R (\Phi \to L \Phi e^{-i\pi_3\rho})$. Thus in this limit the electro-weak Higgs sector has a global $SU(2)_L \times SU(2)_R$ symmetry, which is spontaneously broken by $v$ to $SU(2)_{L+R} = SU(2)_V$ with $W$ and $Z$ degenerate. This symmetry is usually called the custodial symmetry. In our discussion we treat this as an exact global symmetry, as the g' term (or the isospin breaking) is not essential here.

This is just like the simplest $L\sigma M$ (without isospin breaking) for the pion and $\sigma$. The analogue of $\Phi$ is

$$\hat{\Phi} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma + i\pi^0 \\ -i(\pi_1 - i\pi_2) \end{pmatrix} = \frac{1}{\sqrt{2}} (\sigma \cdot 1 + i\pi \cdot \bar{\tau}).$$

(5)

Thus $\phi^0$ corresponds to $(\sigma - i\pi^0)$ and $\phi^+ \mapsto i(\pi_1 + i\pi_2) = i\sqrt{2}\pi^+$ in conventional notation. (The generalization of eq.(5) to full scalar and pseudoscalar nonets, $s_k, p_k, k = 0$ to 8, is $\hat{\Phi} = \sum_k (s_k + ip_k) \lambda_k$, where $\lambda_k$ are Gell-Mann matrices.) The gauged $L\sigma M$ is apart from different constants identical to eq.(2)

$$L_{L\sigma M}(\hat{\Phi}) = \text{Tr}[(D_\mu \hat{\Phi})^\dagger D_\mu \hat{\Phi}] + \hat{\mu}^2 \text{Tr}[\hat{\Phi}^\dagger \hat{\Phi}] - \hat{\lambda} \text{Tr}[\hat{\Phi}^\dagger \hat{\Phi}]^2,$$

(6)

from which $\hat{v}$ is given by $\hat{v} = [\hat{\mu}^2/(2\hat{\lambda})]^{1/2} = 92.6$ MeV. (With a $\sigma$ mass of $\approx 600$ MeV $\hat{\lambda} = [m_\sigma/(2\hat{v})]^2 \approx 10$, but we actually do not need that number here).

Now add the two Lagrangians with a small mixing term between the $\Phi$ and the $\hat{\Phi}$ proportional to $\epsilon^2$:

$$L = L_{Higgs}(\Phi) + L_{L\sigma M}(\hat{\Phi}) + \epsilon^2 [\text{Tr}(\hat{\Phi}^\dagger \hat{\Phi}) + h.c.]/2.$$ 

(7)

This is, in fact, similar to the Two-Higgs-Doublet Model, although the application here is different and much more down to earth, i.e., not the usual supersymmetric or technicolor extensions of the standard model.\footnote{A similar effective Lagrangian as in eq.(7) was also suggested previously, but with a quite different phenomenological application in mind, which involved two light scalar meson nonets and only strong interactions.}
The \( c^2 \) term breaks a relative (global) \( SU(2)_L \times SU(2)_R \) in the sense that if only one of the two \( \Phi \)'s is transformed by, say, a left handed rotation (\( L\Phi \) or \( \hat{L}\phi \)), the relative symmetry is broken. One could, of course, write down many other terms\(^3\), which similarly break another relative symmetry\(^2\). But, for our discussion here the simplest possible choice as in eq. (7) is sufficient.

Neglecting for a moment the gauging, and with our choice of the \( \epsilon \) term the Lagrangian (7) has an overall global \( SU(2)_L \times SU(2)_R \) symmetry (i.e. when \( \Phi \to L\Phi R \) is transformed simultaneously as \( \hat{\Phi} \to L\hat{\Phi} R \) with the same \( L \) and \( R \)). Also the overall \( SU(2)_L \times U(1) \) gauge symmetry is left intact. Let us first discuss how the mixing term breaks the relative global symmetry, neglecting the gauging.

For \( \epsilon = 0 \) and \( v \neq 0 \), \( \hat{v} \neq 0 \), there is a triplet of Goldstone (or would-be Goldstone) bosons in each sector. With \( \epsilon \neq 0 \) one of these triplets, (the pion) gets mass proportional to \( \epsilon \), since the relative symmetry is broken. The pseudoscalar mass matrix \( m^2(0^-) \) gets contributions in two ways 1) from the fact that the vacuum values are disturbed, and 2) directly from the mixing term. The corrections to \( v \) and \( \hat{v} \) obey the relations:

\[
v^2(\epsilon) = v^2(0) + \epsilon^2 \frac{\hat{v}(\epsilon)}{2v(\epsilon) \lambda}, \quad \hat{v}^2(\epsilon) = \hat{v}^2(0) + \epsilon^2 \frac{v(\epsilon)}{2\hat{v}(\epsilon) \lambda}.
\]

The pseudoscalar squared mass matrix becomes

\[
m^2(0^-) = \begin{pmatrix} 2\lambda v^2(\epsilon) - \mu^2 & -\epsilon^2 \\ -\epsilon^2 & 2\lambda \hat{v}^2(\epsilon) - \hat{\mu}^2 \end{pmatrix} = +\epsilon^2 \begin{pmatrix} \frac{\hat{v}(\epsilon)}{v(\epsilon)} & -1 \\ \frac{v(\epsilon)}{\hat{v}(\epsilon)} & 1 \end{pmatrix},
\]

which is diagonalized by a small rotation \( \theta = \frac{1}{2} \arctan[2\hat{v}v/(v^2 + \hat{v}^2)] \approx \hat{v}/v = 3.764 \cdot 10^{-4} \) (we leave out from now on the small dependence on \( \epsilon \) in \( v \) and \( \hat{v} \)). Thus the eigenvalues are 0 and \( \epsilon^2(v/\hat{v} + \hat{v}/v) \approx \epsilon^2 v/\hat{v} \):

\[
\begin{pmatrix} c & s \\ -s & c \end{pmatrix} m^2(0^-) \begin{pmatrix} c & -s \\ s & c \end{pmatrix} = \epsilon^2 \begin{pmatrix} v^2 + \hat{v}^2/v^2 \\ 0 \end{pmatrix} \approx \begin{pmatrix} 0 & 0 \\ 0 & \epsilon^2 \end{pmatrix}.
\]

(Here \( s = \sin \theta \) and \( c = \cos \theta \).) Note that the mass matrix only depends on \( v, \hat{v} \) and \( \epsilon \), not on \( \lambda \) nor \( \hat{\lambda} \), which are not well known from experiment as they are related to the Higgs or \( \sigma \) (600) masses. Note also that the pseudoscalar which gets mass is the one which is related to the pion in the \( \sigma \) sector, (i.e. it is not the "would-be Goldstone" of the E-W sector. To get the right pion mass \( \epsilon \) should be about 2.70 MeV.) On the other hand the Higgs and \( \sigma \) bare masses and mixings are only very little affected since the corrections are proportional to the very small number \( 3\epsilon^2 \).

As the whole Lagrangian\(^7\) still has the full \( SU(2)_L \times U(1) \) gauge symmetry the remaining Goldstone is turned, in the usual way, into the longitudinal
components of the $W$ and $Z$ bosons. But, these masses get a small contribution also from the $\sigma M$ sector

$$M_W^2 = \frac{g^2(\nu^2 + \bar{\nu}^2)}{4}, \quad M_Z^2 = \frac{(g^2 + g'^2)(\nu^2 + \bar{\nu}^2)}{4}. \quad (11)$$

One can also easily see that the $\epsilon^2$ term mixes the longitudinal $W$ and the pion in the expected way by the same angle $\theta$.

Thus the pion gets mass from a small breaking of a relative symmetry between the E-W and strong interactions, and through a small mixing with the longitudinal $W$, described by $\theta = 3.764 \cdot 10^{-4}$. Still, the symmetry remains intact, although spontaneously broken by the vacuum, for the combined strong plus electro-weak Higgs sectors.

Our result is in no way in conflict with the usual reference to light quark masses as the source of pion mass. In fact, it is natural to assume that the $\epsilon^2$ term arises because of quantum corrections involving virtual quark loops ($\hat{\Phi} \rightarrow q\bar{q} \rightarrow \Phi$ or even $\hat{\Phi} \rightarrow q\bar{q} \rightarrow VV \rightarrow \Phi$, which would involve ABJ anomaly graphs). In principle, such terms should be calculable from QCD+EW gauge theory.

A few concluding final remarks are in order. I discussed a way of how to look at a certain kind of symmetry breaking, which I named "relative symmetry breaking". By mixing two models with the same symmetry group $SU(2)_L \times SU(2)_R$, one for the scalar sector of strong interactions, and the other for the Higgs sector of electro-weak interactions, a relative symmetry is broken giving in our example rise to a pion mass, although for the whole model the chiral symmetry is still exact and unbroken in the Lagrangian, except for spontaneous symmetry breaking in the vacuum. I believe that this as a general idea has not received proper attention in the literature. It need not be limited to the application discussed here.

For example, it can be applied to the perhaps simplest spontaneous symmetry breaking one can think of, - that of O(2) or $U(1)$ symmetry models. One needs two $U(1)$ models, each described by a "Mexican hat" potential, which are coupled to each other by a mixing term. In this system of "coupled spontaneous symmetry breaking" the coupled potentials are "tilted" compared to each other. Therefore, because of the mixing, one combination of the two would-be Goldstones gets mass. The whole system would still have one massless Goldstone, which is a linear combination of the original massless states. Then, if one gauges the whole system also this massless scalar is "eaten" by the vector boson, and no massless states remain.

In this simple $U(1)$ example, our relative symmetry breaking may seem trivial, almost self-evident, but its possible generalization within more complicated structures is often missed in current literature. Our main conjecture is thus that a symmetry of the standard model for strong and electro-weak interactions combined can remain exact in the total Lagrangian, although the symmetry looks "as if it were broken explicitly by one of the interactions". If this conjecture turns out to be true it should have consequences for a better understanding of, say, PCAC, chiral symmetry and isospin breaking.
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