Reliability Analysis of Large Span Cable-stayed Bridges Based on Support Vector Machine

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Abstract. A method of combining the support vector machine and the intelligent algorithm to perform reliability analysis on large span cable-stayed bridges was proposed in this paper. The algorithm is verified by an example, which shows that the algorithm has higher precision and can be extended to the actual project. Therefore, this paper takes Jiangmen North Street Waterway Bridge as the engineering background, uses the finite element software ANSYS to establish the cable-stayed bridge model and MATLAB software to compile the intelligent algorithm and data processing. It mainly analyzes the reliability of the main beam mid-span deflection, the cable stress and main tower stress of the North Street Waterway Bridge. Finally, the general limit range method is used to estimate the reliability index of the North Street waterway bridge structure system. The results show that the cable stay failure has the greatest impact on the reliability of the entire structural system, so it is necessary to pay attention to the maintenance and monitoring of stay cables.

1. Introduction

In the reliability analysis of long-span cable-stayed bridges, because of the complex structure system of multiple materials, components and a variety of different connection mode, the performance function is often a highly nonlinear implicit function. The traditional solution method is complicated and computationally intensive.

In order to solve this problem, domestic and foreign researchers have done a lot of relevant research. Reference [1] refers to the Kriging model to replace the implicit function of the structure. In reference [2], in order to avoid direct derivation of the structural limit state equations that cannot be explicitly expressed, the structural limit state equation is simulated by the artificial neural network method.

In this paper, support vector machine [3] is applied to the field of bridge structure reliability analysis. By selecting appropriate kernel function, nonlinear map is used to transform sample points from low-dimensional space to high-dimensional space, so that in high-dimensional space the use of linear learning machines to solve highly nonlinear problems has been applied [4].

2. Example analysis of support vector machine

In order to illustrate the feasibility of combining support vector machine and intelligent algorithm for structural reliability analysis, this method is validated by classical examples.

Example 1: The limit state function of a structure is known as \( G(x_1, x_2, x_3) = x_1x_2 - x_3 \), where the variables \( x_1, x_2, x_3 \) are independent normal distributions. \( \mu_{x_1} = 0.5472, \sigma^2_{x_1} = 0.0274, \mu_{x_2} = 3.8, \sigma^2_{x_2} = 0.304, \mu_{x_3} = 1.3, \sigma^2_{x_3} = 0.91 \), to solve the reliability of this structure.
Example 2: The limit state function of a structure is \( G(x_1, x_2) = 1 + x_1x_2 - x_2 = 0 \), where the variables \( x_1, x_2 \) are independent normal distributions, \( \mu_{x_1} = 2, \sigma_{x_1}^2 = 0.4, \mu_{x_2} = 4, \sigma_{x_2}^2 = 0.8 \), to solve the reliability of this structure.

Due to space limitations, this paper only analyzes the case 1 in detail, and the method of the example 2 is the same. The solution process is as follows:

According to the 3σ criterion (\( f = 3 \) in the first step of calculating the function value, then \( f = 1 \) in each step), we can know that the sample area of each variable is \( [\mu - 3\sigma, \mu + 3\sigma] \), and \( x_1 \in [-0.0506, 1.0438], x_2 \in [2.1459, 5.4541], x_3 \in [-1.5618, 4.1618] \). Using the uniform experimental design method, 100 sets of data are selected in the sample regions of \( x_1, x_2, x_3 \), and substituted into the limit state function equation \( G(x_1, x_2, x_3) \) to obtain training samples.

The obtained sample number is combined with a support vector machine to perform a fitting performance function. The parameters of SVM (support vector machine) are optimized by CV (cross validation) and GA (genetic algorithm), and the best result is selected according to the principle of minimum mean square deviation. It can be seen from Fig 1 and Fig 2 that the result of GA is more ideal, so the GA optimization result is selected: \( c = 254.4508, g = 0.020027 \). After obtaining the performance function, it is necessary to judge the fitting precision. In this paper, 100 groups of data are randomly generated to verify the accuracy of the function, and finally the sample data and the fitting data are obtained. The result shows that the error between the two is small, and the fitting performance function has high precision.

The constructor equation for solving the optimization problem by combining the penalty function method is:

\[
\beta(Z) = \sqrt{Z_1^2 + Z_2^2 + Z_3^2 + C\| G(Z) \|}
\]

where \( Z_1, Z_2 \) are \( x_1, x_2 \) standard normalized random variables; \( G(Z) \) is a performance function.

The minimum value obtained by equation (1) is the required reliability index. The result is 3.7992, which is obtained using the particle swarm optimization algorithm, and the difference is small compared with 3.795 in the reference [5]. The solution process is shown in Figure 3. Therefore, the accuracy of this algorithm is high and can be applied to actual projects.

3. Reliability analysis of long-span cable-stayed bridge

3.1 Engineering situation
This paper takes North Street Waterway Bridge as the research object. The length of the bridge is 2579.3m, of which the main bridge is 800m long. It adopts half float system and the span is arranged \((60+150+380+150+60)\) m, as shown in Figure 4. It is a three-span double-tower concrete cable-stayed bridge, which adopts a central double cable plane arrangement. There are 116 pairs of stay cables in the whole bridge, which are symmetrically arranged on both sides of the center line of the bridge. The diagonal cables on both sides are 5m apart. The pylon is a single-column pylon with a main beam width of 40.8m.

3.2 Reliability Analysis of Displacement Failure of Long Span Cable-Stayed Bridges

3.2.1 Establishment of the finite element model of ANSYS. This paper uses ANSYS finite element analysis software to establish the model of the North Street Waterway Bridge, as shown in Figure 5. The BEAM44 and BEAM188 units are used for simulation according to the material, geometry and force characteristics of the main beam and the pylon. Considering the main span is 380m, the LINK10 rod unit is used for the stay cable simulation, and the Ernst formula is used to simulate the stay cable sag.

![Finite element model diagram of the North Street Waterway Bridge](image)

3.2.2 Reliability Analysis. The parameters of the random variables are determined according to the reference [6] and the “Unified Design Standard for Structural Reliability of Highway Engineering”, as shown in Table 1.

| Random variables                  | Mean    | Coefficient of variation | Distribution form |
|-----------------------------------|---------|-------------------------|-------------------|
| Main beam elastic modulus (GPa)   | 35.5    | 0.1                     | Normal            |
| Main tower elastic modulus (GPa)  | 34.5    | 0.1                     | Normal            |
| Stay cable elastic modulus (GPa)  | 204     | 0.1                     | Normal            |
| Main beam bulk density (kN/m³)    | 26      | 0.1                     | Normal            |
| Main tower bulk density (kN/m³)   | 25      | 0.1                     | Normal            |
According to the reference [7], the serviceability limit states based on deformation is as follows:

\[ Z = G(x) = \delta - u_{\text{max}}(E_1, E_2, E_3, \gamma_1, \gamma_2, \gamma_3, q_1, q_2, A_1, A_2, I_i) \]  

(2)

where \( \delta \) is the maximum vertical deformation of the main span, \( E_1 \) is the main beam elastic modulus, \( E_2 \) is the elastic modulus of the main tower, \( E_3 \) is the elastic modulus of the stay cable; \( \gamma_1 \) is the bulk density of the main beam, \( \gamma_2 \) is the bulk density of the main tower, \( \gamma_3 \) is the bulk density of the cable; \( q_1 \) is the secondary loads, \( q_2 \) is the vehicle load; \( A_1 \) is the main beam area, \( A_2 \) is the cable–stayed single cable area; \( I_i \) is the main beam inertia Moment.

Referring to the solution process of the example, the training samples are calculated based on the 11 random variables selected in the text. It is input into the finite element model of the North Street Waterway Bridge for deterministic analysis, and the deformation data of the main beam is obtained. The data is substituted into the formula (2) to calculate the output sample. The number of samples obtained is combined with the support vector machine to fit the function, and the parameters of SVM are optimized by CV and GA. The optimization results of GA were selected according to the principle of least mean square deviation. After verification, the obtained fitting function is highly accurate and meets the requirements.

The required reliability index is obtained by formula (1), and the particle swarm optimization algorithm is used for iterative solution. The reliability index value when convergence is 4.3139. The reference [8] propose that the target reliability index of the serviceability limit states should be between 0.675 and 1.645. It can be seen that the reliability of the North Street Waterway Bridge based on the deflection control has a high reliability index and meets the safety requirements.

3.3 Reliability Analysis of Stress Failure of Long Span Cable-Stayed Bridges

According to reference [7], the limit state equation based on compressive stress is:

\[ Z = G(x) = [\sigma] - \sigma(E_1, E_2, E_3, \gamma_1, \gamma_2, \gamma_3, q_1, q_2, A_1, A_2, I_i) \]  

(3)

where \([\sigma]\) is the allowable stress of the stay cable (the limit of cable bent tower stress value), and the meaning of other random variables is given in formula (2).

According to the reliability calculation method described above, the reliability index value of each stay cable stress and main tower unit stress is calculated according to formula (3). The results are shown in Fig. 6(a) and Fig. 6(b). The abscissa of the figure is the cable number (main tower unit number), and the ordinate is the reliability index value of the cable stress (main tower stress).

It can be seen from Fig 6 that the reliability index of the cable-stayed cable of the North Street Waterway Bridge is relatively uniform, and the reliability index of the 30# cable stayed at the bridge tower is the largest, with a value of 5.2508. The 55# stay cable near the middle of the main span is the
least reliable, with a value of 3.4024. According to the change of the reliability index of 58 cable stays, it can be seen that the reliability index of the cable stayed near the bridge tower is larger, and the reliability index away from the bridge tower is smaller.

![Figure 7. Reliability index of main tower units](image)

It can be seen from Fig 7 that the reliability of the 9# tower unit located at the upper position of the tower beam junction is the smallest, and its value is 4.4594. According to the change of the reliability index of 37 tower units, it can be seen that the reliability index of the main tower at the following part of the intersection of the tower and beam is larger, and the reliability index near the intersection of the tower and beam is the smallest. The farther away from the junction of the tower and the beam, the greater the reliability index of the tower unit. The reliability index at the top and bottom positions of the main tower are large.

According to the reference [8], the target reliability index of the serviceability limit states should be between 0.675 and 1.645. Thereby, the North Street Waterway Bridge based on the reliability of the stress control has a high reliability index and meets the safety requirements.

3.4 Reliability Analysis of Long Span Cable-Stayed Bridge System

In this paper, the general boundary range method [8] is used to calculate the structural system reliability of the North Street Waterway Bridge. The reliability index of the mid-span deflection is 4.3139, and the minimum reliability indexes of the cable stress and the main tower stress are: 3.4022, 4.4594. The failure probabilities of the mid-span displacement failure, the cable stress failure and the main tower stress failure are: $8.02 \times 10^6$, $3.32 \times 10^4$, $4.11 \times 10^6$.

$$\max\{P_{f_{\text{st}}}\} \leq P_f \leq 1 - \prod_{i=1}^{1\text{st}}(1 - P_{f_{\text{st}}})$$  \hspace{1cm} (4)

where $P_f$ is the failure probability of the structural system, and $P_{f_{\text{st}}}$ is the failure probability of the structural member.

Using the formula (4) to calculate the upper and lower bounds of the reliable indicators of the system:

$$3.3943 \leq P_f \leq 3.4042$$

It can be seen that the reliability index of the North Street waterway bridge system is not less than 3.3943. The reference [8] considers that the target reliability index $\beta$ of serviceability limit states should be between 0.675 and 1.645. Thereby, the North Street waterway bridge structure system has high reliability indicators to meet safety requirements.

4. Conclusion

After comparing the reliability of the main beam mid-span deflection, stay cable stress and main tower stress of the North Street Waterway Bridge, it can be concluded that the reliability of the stay cable is low, and its reliability is closest to the North Street waterway bridge structure system. Therefore, in the design, construction and later monitoring and maintenance, it is necessary to strengthen the emphasis on stay cables.
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