Exploring the Di-\(J/\psi\) Resonances around 6.9 GeV Based on \textit{ab initio} Perturbative QCD

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We propose an \textit{ab initio} method to explore the nature of the newly discovered particle \(X(6900)\). We find that there should exist another state near the resonance at around 6.9 GeV, and the ratio of production cross sections of \(X(6900)\) to the undiscovered state is very sensitive to the nature of \(X(6900)\), whereas is almost independent of the transverse momentum and rapidity. This behavior is unlikely changed by higher order corrections. Therefore, the nature of \(X(6900)\) can be uncovered by experimental facts in the near future. If there is another state near \(X(6900)\) with cross section larger than half of that of \(X(6900)\), \(X(6900)\) should be a tetraquark state. Otherwise, it should be a molecule-like state.

\textbf{Introduction.} — Although tetraquarks and pentaquarks \cite{1} were predicted along with the quark model in 1964, more than half a century passed, multiquark-like states observed in experiment are still rare. Among these states, the structure of hardly any was identified; whether they are multiquarks, molecules of hadrons, or other possible species of resonances are still under debate (for a review, see e.g. \cite{2, 3}). Most theoretical interpretations of such states are not based on first-principle theories, therefore suffer from large parameter uncertainties or / and model dependencies. Lattice QCD, which is a first-principle method, becomes extremely complicated when it is applied to multiquark states.

Recently, a new resonance of \(J/\psi\) pair around 6.9 GeV (we call it \(X(6900)\)) is observed by LHCb \cite{4}, which provides a golden opportunity to identify such an exotic state and has drawn a lot of attentions in the field \cite{5–18}. We also refer the reader to early theoretical studies of the fully-heavy multiquark states \cite{19–44}.

Although a broader di-\(J/\psi\) enhancement is observed between 6.2 GeV (di-\(J/\psi\) threshold) and 6.9 GeV (mass of \(X(6900)\)), it is more likely a “threshold enhancement” \cite{4}, i.e., only one genuine resonance of \(J/\psi\) pair is confirmed. It is natural to consider it as the ground state of a series of possible structures, otherwise, the production rate of the ground state should be too small to observe.

Assuming \(X(6900)\) as a standard-model particle, it could be a tetraquark (\(cc\bar{c}\bar{c}\)) state or a state consisting of two hidden-charm mesons bound with screened strong forces. We call the latter a molecule-like state for convenience, regardless of the true binding mechanism of its constituent hadrons. If the two hidden-charm mesons are not \(J/\psi\), when the molecule-like state decays into a \(J/\psi\) pair, additional products will also be emitted, which will take away most of the residual energies. As a consequence, the di-\(J/\psi\) invariant energy will be close to twice of the \(J/\psi\) mass and smeared. This picture is inconsistent with the narrow resonances observed at 6.9 GeV, a much higher energy above the di-\(J/\psi\) threshold.

### Table I: Possible angular momentum configurations of the constituents in \(X(6900)\).

| Tetraquark Configuration | Angular Momentum |
|--------------------------|------------------|
| \(T_{cc\bar{c}\bar{c}}\)   | \(^1S_0\)         |
| \(M_{J/\psi-J/\psi}\)    | \(^3S_1\)        |

\(T\) and \(M\) are given in the following: (\(T_{cc\bar{c}\bar{c}}\), the first, second, and third angular momentum configurations are for the \(cc\bar{c}\bar{c}\), \(c-c\), and \(\bar{c}-\bar{c}\) systems, respectively, while for molecule-like states (denoted as \(M_{J/\psi-J/\psi}\)), the angular momenta of the \(J/\psi-J/\psi\) system are listed in the table.

It is important to explore the nature of \(X(6900)\) and answer the following questions. Is it a tetraquark or a molecule-like state? If it is a tetraquark state, what is the mixing angle between \(T(\,^{1}\!S_{0}\,)\) and \(T(\,^{3}\!S_{1}\,)\)? This Letter will be devoted to the study of these questions based on the well-established NRQCD factorization \cite{45}, in this framework, the production cross section of a state consisting of fully-heavy quarks is double expansion in the strong coupling, \(\alpha_s\), and the typical velocity, \(v\), of the constituents inside the state. Up to leading order in \(v\), only one nonperturbative overall factor, the square of the wave function at the origin, is involved. Such a factor can be systematically calculated, as we will show.
On the other hand, even without the knowledge of this factor, the perturbative calculation can also provide essential information to identify the nature of the newly discovered resonance.

**NRQCD factorization.** — In the framework of NRQCD, the production cross section of a bound state \((H)\) consisting only of heavy quarks can be factorized as \([40]\)

\[
\mathrm{d}\sigma(H) = \sum_n \mathrm{d}\sigma_n \langle O^H(n) \rangle,
\]

where \(\sigma_n\) is perturbatively calculable short-distance coefficient, which is independent of the species of the produced hadron \(H\), \(\langle O^H(n) \rangle\) are long-distance matrix elements, which are independent of the short-distance process producing the \(c\) and \(\bar{c}\) quarks, and the summation runs over all the allowed intermediate states with quantum numbers \(n\). Up to leading order in \(v\), the intermediate states have the same components and quantum numbers as \(H\), and the corresponding \(\langle O^H(n) \rangle\) can be simply related to normalized wave function of tunum numbers \(\varepsilon\).

Then the cross section for \(J/\psi\) production can be evaluated in the squared amplitude with the following equation

\[
\sum_s \varepsilon_{2s+} \varepsilon_{2s-} = \frac{1}{2} (\Pi^{\alpha\beta} \Pi^{\alpha\beta} + \Pi^{\alpha\beta} \Pi^{\alpha\beta}) - \frac{1}{3} \Pi^{\mu\nu} \Pi^{\mu\nu}. \quad (7)
\]

The calculation of tetraquark production is more complicated due to the antisymmetry of the amplitudes while exchanging two quarks (antiquarks). Since only S-wave states are considered, the spin and color wave functions for the \(c\)-\(\bar{c}\) \((c\bar{c})\) system should be antisymmetric with respect to exchange of the quarks (antiquarks). Having the tensor decomposition of the fundamental representation of the \(SU(3)\) group, \(3 \otimes 3 = 6 \oplus 3\), the \(cc\) with angular momentum \(1S_0\) should be color-singlet, while the \(cc\) with angular momentum \(3S_1\) should be color-triplet.

To produce a \(J/\psi\), the intermediate \(cc\) pair should be a color-singlet \(3S_1\) state, and the cross section can be obtained by replacing the spinors product \(v_i(p_a) \pi_j(p_c)\), in the amplitude for a \(cc\) pair production, by the projector

\[
\Pi_{J/\psi} = \frac{1}{2\sqrt{2}} \delta_{ij} \delta(p + m),
\]

and at the same time replacing the product of the phase spaces for \(p_a\) and \(p_c\) by

\[
\frac{\mathrm{d}\Phi_{J/\psi}}{2\sqrt{2}} = \frac{\mathrm{d}^3p}{2p_0} \frac{2}{m} |\psi_{J/\psi}(0)|^2,
\]

where \(p_a\) (\(p_c\)) and \(i\) (\(j\)) are respectively the momentum and color of \(c\) (\(\bar{c}\)), and \(p, m, c, \) and \(\psi_{J/\psi}(0)\) are respectively the momentum, mass, vector polarization, and normalized wave function at the origin of the \(J/\psi\) meson. At leading order in \(v\), \(p_a = p_c = p/2\).

Then the cross section for \(M_{J/\psi,J/\psi}\) production can be obtained through that for double \(J/\psi\) production in the same way as described above, with trivial phase space,

\[
\frac{\mathrm{d}\Phi_M}{2\sqrt{2}} = \frac{\mathrm{d}^3P}{(2\pi)^32P_0} \frac{2}{M} |\psi_M(0)|^2.
\]

Here, \(P, M,\) and \(\psi_M(0)\) are the momentum, mass, and normalized wave function at the origin of \(M_{J/\psi,J/\psi}\). Summing over the spin of \(J/\psi\) leads to the following replacement

\[
\varepsilon_{ss'} \rightarrow \varepsilon_{ss'}^{\mu\nu},
\]

where \(s=0\) or \(2\) is the spin of \(M_{J/\psi,J/\psi}\). \(\varepsilon_{00}\) has a compact form, namely,

\[
\varepsilon_{00}^{\mu\nu} = \frac{1}{3} (-g^{\mu\nu} + \frac{p^\mu p^\nu}{m^2}) = \frac{1}{3} \Pi^{\mu\nu},\quad (6)
\]

while \(\varepsilon_{2s+}\) can be evaluated in the squared amplitude with the following equation

\[
\sum_s \varepsilon_{2s+} \varepsilon_{2s-} = \frac{1}{2} (\Pi^{\alpha\beta} \Pi^{\alpha\beta} + \Pi^{\alpha\beta} \Pi^{\alpha\beta}) - \frac{1}{3} \Pi^{\mu\nu} \Pi^{\mu\nu}. \quad (7)
\]
It is much more difficult to solve the tetraquark wave function, which is a four-body quantum mechanics problem. The success of the nuclear shell model and the application of Hartree-Fock equation therein provides a heuristic example, which demonstrates that the Hartree-Fock method is available even though 1) the interaction is strong and complicated, and 2) there is no preference center like a nucleus in atom. The success of the Hartree-Fock method in describing Helium nucleus [49] is quite inspiring and enables us to perform the same calculation on a fully-heavy tetraquark. Unfortunately, the wave function is extremely sensitive to the input parameters, which can be seen from the following relation,

$$\Psi_T(0) \equiv C^0_F/\psi_T(0) \propto C^0_F/\alpha_s^{1/2}m_c^{1/2},$$

(12)

where $C_F$ counts the overall interaction factor acted on a c-quark by all the other three quark-antiquarks. We have $C_F = 7/6$ ($C_F = 5/6$) for color-sextuplet (color-triplet) c-c configurations. Note that $\psi_T(0)$ is independent of the color configurations of the constituent quarks. The large powers in Equation (12) defeats any hope of finding a small-error solution of the wave function with only one state $X(6900)$ as input. As experimental data accumulates, the parameters, $\alpha_s$ and $m_c$ in Equation (12), can be fixed through global fits.

Even without the wave functions, we can also obtain useful information helping to identify the observed resonance. In the following, we will compute the normalized differential cross sections for the hadroproduction of the five states listed in Table I. They are defined by $d\sigma_N \equiv d\sigma/|\psi_{JM}(0)|^4/|\psi_{M}(0)|^2$ for molecule-like states, and $d\sigma_N \equiv d\sigma/|\psi_T(0)|^2$ for tetraquarks. As we will see, comparison of theoretical results with further experimental data can distinguish the nature of $X(6900)$.

**Numerical results.** — To reduce soft gluon emission effects, we choose $4m_c \approx 2m \approx M = 6.9$ GeV in the spirit of Ref. [50]. We will only present our result with c.m. energy $\sqrt{s} = 13$ TeV, because our main conclusions are found to be independent of the colliding energies. CTEQ6L1 [51] is employed as the input of parton distribution functions in our calculation, with the collinear factorization scale chosen as $\mu_f = \sqrt{M^2 + p_t^2}$. The renormalization scale, $\mu_r$, is set to be the same value as $\mu_f$.

With the above parameter choice, we can compute the integrated cross sections for the states listed in Table I in the kinematic region, $p_t > 6$ GeV and $2 < y < 5$, according to the experimental cuts, where $p_t$ and $y$ are the transverse momentum and rapidity of $X(6900)$. Their values are obtained as

$$d\sigma_N(M(1S_0)) = 0.30 \mu b/GeV^9, \quad d\sigma_N(M(3S_0)) = 4.9 \mu b/GeV^9, \quad d\sigma_N(T(1S_0)s) = 5.9 \mu b/GeV^9, \quad d\sigma_N(T(1S_0)v) = 0.22 \mu b/GeV^9, \quad d\sigma_N(T(1S_0)t) = -1.1 \mu b/GeV^9, \quad d\sigma_N(T(5S_2)) = 2.9 \mu b/GeV^9,$$

(13)

where $d\sigma_N(T(1S_0)_T)$ denotes the interference contribution between $M(1S_0)s$ and $M(1S_0)v$. The physical spin-0 tetraquark states should be two orthogonal mixtures of the two different angular momentum configurations, and can be described by a mixing angle $\theta$, in terms of which the two physical states can be decomposed as

$$T(1S_0)_1 = T(1S_0)_s \cos \theta + T(1S_0)_v \sin \theta, \quad T(1S_0)_2 = -T(1S_0)_s \sin \theta + T(1S_0)_v \cos \theta.$$  

(14)

Then the normalized production cross sections for $1S_0$ tetraquark production can be evaluated according to

$$\sigma_N(T(1S_0)_1) = \sigma_N(T(1S_0)s) \cos^2 \theta + \sigma_N(T(1S_0)v) \sin^2 \theta + 2\sigma_N(T(1S_0)_T) \cos \theta \sin \theta, \quad \sigma_N(T(1S_0)_2) = \sigma_N(T(1S_0)s) \sin^2 \theta - \sigma_N(T(1S_0)_v) \cos \theta \sin \theta.$$ (15)

We denote the larger one as $\sigma_N(T(1S_0)m)$, which is more easily observed in experiment. A short calculation shows that $\sigma_N(T(1S_0)m)$ reaches its maximum (minimum) value at $\theta = -0.19$ ($\theta = 0.59$), which leads to

$$3.1 \mu b/GeV^9 \leq \sigma_N(T(1S_0)m) \leq 6.1 \mu b/GeV^9.$$ (16)

The transverse momentum and rapidity distributions of $\sigma_N$ are presented in Figure I where the cutoffs, $2 < y < 5$ and $p_t > 6$ GeV, are respectively imposed. We can see that the $p_t$ and $y$ behavior for all these states are almost the same. As a matter of fact, they are produced mainly via double parton fragmentation [52, 53], which

FIG. 1: The $p_t$ and $y$ distribution of $X(6900)$ production at $\sqrt{s} = 13$ TeV.
results in a next-to-leading power $p_t$ behavior, or equivalently, $d\sigma/dp_t \sim p_t^{-5}$ at partonic level. This behavior is similar to double $J/\psi$ production at next-to-leading order \textsuperscript{54, 55} because they share similar Feynman diagrams. As a result, it is very difficult to distinguish tetraquark from molecule-like state through the slope of the $\log(d\sigma/dp_t) - \log(p_t)$ curve. An interesting observation is that, for molecule-like configurations, the cross section for the spin-2 state is much larger than that for the spin-0 one, while for tetraquarks, the cross section for the spin-2 state is smaller than that for the spin-0 one. Since the mass deviation of the two spin configurations is at the order of $m_v^4$, approximately 100 MeV, it is expected that at least two resonances appear in a small energy interval around 6.9 GeV. However, only one narrow resonance is observed. A possible explanation is that the number of events of the other one is too small and escaped the experimental resolution. Thus, the observed one should have the largest production cross section among all the existing states. Note that 100 MeV is comparable with the width of $X(6900)$, which implies that other potential state(s) might hide inside the observed peak or just inside the background. As the data accumulates, it (they) might be recognized. If $X(6900)$ is a molecule-like state, the spin of the observed state should be 2 and a much lower peak should exist slightly below 6.9 GeV. As a tetraquark, the spin of $X(6900)$ should be 0 and also another peak should exist near 6.9 GeV, but with a slightly higher mass and small but comparable magnitude.

Denoting the ratio of the cross sections for the observed resonance to the latent one as $r$, we find that $r_M \equiv \sigma_N(M(3S_2))/\sigma_N(M(1S_0))$ and $r_T \equiv \sigma(T(3S_0)_m)/\sigma(T(3S_2))$ are quite different from each other, hence we need to look at them in more detail. The $p_t$ and $y$ distributions for $r_M$ and $r_T$ are presented in Figure 2. $r_T$ ranges from about 1 to 2, depending on the mixing angle, while $r_M$ is about 16. These two ratios are unlikely affected by higher order corrections, since all states of a specific structure (tetraquark and molecule-like state) share the same Feynman diagrams, consequently share similar perturbative correction factors. For the above reasons, the value of $r$ provides a clear signal to distinguish different structures. Further, if the value of $r$ is measured to be between 1 and 2, Equation \textsuperscript{15} provides a feasible way to determine the mixing angle. Another important feature we can see in Figure 2 is that the values of $r$ is almost a constant in all the kinematic regions, which implies that when measuring this ratio the systematic error can be essentially reduced.

All our discussions are based on an assumption that the resonance should be an $S$-wave ground state. We need also to address the possibility that it may be not. In this case, the ground state has to be below the di-$J/\psi$ threshold, otherwise, it should also decay into a $J/\psi$ pair with a larger production cross section, which is detectable in experiment. Regardless of whether it is a $^3P_J$ state or a radially higher excited state \textsuperscript{1}, our conclusion stays unchanged, i.e. at least one other resonance exists very close to the observed one, and the first-principle perturbative calculation in this work can distinguish the nature of $X(6900)$ through the cross section ratio of the states.

\textbf{Summary.} — To summarize, we proposed a method based on the first principle QCD to explore the nature of the newly discovered particle $X(6900)$. With a thorough analysis, we concluded that there should exist another resonance close to the mass of $X(6900)$. We found the ratio of production cross sections of these states is very sensitive to the nature of $X(6900)$, but almost independent of transverse momentum and rapidity of the produced particles. If it is a tetraquark, the cross section of the other state is smaller by a factor of 1 to 2 and thus can be discovered soon with higher luminosity. But if it is a molecule-like state, the cross section of the other state is smaller by a factor of 16 and thus should be hard to discover. Our conclusion will not be changed by higher order corrections. Therefore, the nature of $X(6900)$ can be uncovered by experimental facts in the near future. We argued that, even if $X(6900)$ is not a ground state, our method is still valid. We also discussed the way to calculate nonperturbative parameters,

\textsuperscript{1} In the latter case, there should exist several $^3P_J$ states below 6.9 GeV, which might relate to the broader resonance observed in experiment.

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The ratio of the cross sections for spin-2 states to those for spin-0 ones as functions of $p_t$ and $y$.}
\end{figure}
the wave functions at the origin, which can be used to obtain absolute cross section of $X(6900)$.

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