Correlations and spectral functions in asymmetric nuclear matter

Kh.S.A. Hassaneen and H. M"uther
Institut f"ur Theoretische Physik,
Universit"at T"ubingen, D-72076 T"ubingen, Germany

The self-energy of nucleons in asymmetric nuclear matter is evaluated employing different realistic models for the nucleon-nucleon interaction. Starting from the Brueckner Hartree Fock approximation without the usual angle-average in the two-nucleon propagator the effects of the hole-hole contributions are investigated within the self-consistent Green’s function approach. Special attention is paid to the isospin-dependence of correlations, which can be deduced from the spectral functions of nucleons in asymmetric matter. The strong components of the proton-neutron interaction lead in neutron-rich matter to a larger depletion for the occupation probability of proton states below the Fermi momentum.

PACS numbers: 21.65.+f, 21.30.-x, 26.60.+c

I. INTRODUCTION

The development of the nuclear shell model has been one of the keys for a microscopic understanding of nuclear structure. Therefore a lot of effort has been made to deduce the properties of the mean field of nucleons in nuclei from realistic models of the nucleon-nucleon (NN) interaction. The simple Hartree-Fock approach fails badly if one employs realistic NN interactions, i.e. interactions which are adjusted to describe the NN scattering data: such Hartree Fock calculations typically yield unbound nuclei and no binding energy for nuclear matter at saturation density. This demonstrates the necessity to account for the effects of correlations, which are induced from the strong short-range components and the tensor components of realistic NN forces. Therefore various methods for treating strongly interacting many Fermion systems have been developed and applied to account for these correlation effects (see e.g. [1] and [2] for a review on this topic). One of the most popular approaches in nuclear physics has been the hole-line expansion and in particular the lowest order approach in this expansion, the Brueckner-Hartree-Fock (BHF) approximation.

Like the ordinary Hartree-Fock approach also the BHF approximation assumes a Slater-determinant for the nuclear wave function and the effects of correlations are taken into account by means of an effective interaction, the so-called $G$ matrix. This $G$ matrix is obtained from solving the Bethe-Goldstone equation, which in some sense corresponds to the NN scattering $T$ matrix. In contrast to the Lippmann-Schwinger equation, leading to $T$, the Bethe-Goldstone equation accounts for effects of the nuclear medium: The propagator for the intermediate two-particle states is restricted to particle states, i.e. to single-particle states with energies above the Fermi energy $\varepsilon_F$, and is defined in terms of the single-particle energies for the nucleons in the medium. Therefore the solution of the Bethe-Goldstone equation, the $G$ matrix accounts for particle-particle ladders. The single-particle energies are determined from the Hartree-Fock approximation for the nucleon self-energy or single-particle potential, however, replacing the bare NN interaction by the effective interaction, the $G$ matrix. This $G$ matrix is also used to determine the total energy of the system.

The BHF approximation may also be considered as a first step towards a many-body approach, which is based on a a self-consistent evaluation of Green’s function $G$. Within the scheme of self-consistent Green’s function, however, the evaluation of the self-energy is done employing a scattering matrix for two nucleons in the medium, which accounts for particle-particle as well as hole-hole ladders. In nuclear physics typical values for the Fermi momentum are small compared to the single-particle momenta which can be reached by NN scattering processes employing realistic NN forces. Therefore the contributions of the particle-particle ladder terms are more important than those of the hole-hole terms, which provides a justification for the approximation used in the hole line expansion.

Therefore the main advantage of the self-consistent Green’s function approach is not the inclusion of hole-hole terms in the effective NN scattering matrix but that it yields a consistent result for the single-particle Green function, which can be analyzed to determine the spectral functions of nucleons in the nuclear medium. These spectral functions directly reflect the effects of the NN correlations and can be explored by the analysis of nucleon knock-out experiments like $(e,e'p)$.

Various attempts have been made to perform self-consistent calculations of the Green’s function for nucleons in nuclear matter using realistic NN forces. A serious problem of those investigations is the appearance of the so-called pairing instability $\frac{1}{2}$. Such pairing effects can be taken into account by means of the BCS approach $\frac{3}{2}$. At the empirical saturation density of symmetric nuclear matter the solution of the gap equation in the $^3S_1 - ^3D_1$ partial wave leads to an energy gap of around 10 MeV. Another approach is to consider an evaluation of the generalized ladder diagrams with “dressed” single-particle propagators. This means that the single-particle Green’s functions are
not approximated by a mean-field approach but consider single-particle strength distributed over all energies. Various attempts have been made in this direction, considering a parameterization of the single-particle Green’s function in terms of various poles \[10\], employing simplified (separable) interaction models \[6\] or considering the case of finite temperature \[11\]. It is worth noting that the same instabilities have also been observed in studies of finite nuclei \[12\], leading to divergent contributions to the binding energy from the generalized ring diagrams. These contributions remain finite if the single-particle propagator are dressed in a self-consistent way.

An approximative scheme for the self-consistent evaluation of the single-particle Green’s function has been developed and applied to symmetric nuclear matter in \[13\]. In this approach the \( G \) matrix is evaluated avoiding the angle-average approximation for two-nucleon propagator which is usually considered. The effects of the hole-hole scattering terms are taken into account by means of a perturbative approach and the single-particle propagator are approximated by a single-pole approach, deriving the energy of the pole from the energy weighted spectral distribution. It has been shown that this approach yields results for the spectral function, which are in good agreement with a determination in a complete self-consistent Green’s function approach. However, it underestimates the hole strength at very high momenta and missing energies \[14\].

In this manuscript we present an approximation scheme close to the one developed in \[13\] and apply it to asymmetric nuclear matter. The main aim of this study is to explore the dependence of the spectral functions for protons and neutrons on the nuclear asymmetry and relate this to the correlations induced by the proton-proton (neutron-neutron) and proton-neutron interaction. Since the tensor components of the NN interaction are suppressed in the proton-proton and neutron-neutron interaction, this analysis shall also allow us to distinguish the correlation effects originating from the tensor components from those induced by the central short-range components of the NN interaction.

The dependence of these results on the model for the NN interaction is explored by comparing the results derived from two modern NN interactions, which fit NN phase shifts with high accuracy. One of them, the so-called Nijm2 interaction, is a localized version of the Nijmegen interaction \[13\]. The other one, the CD-Bonn potential, is defined in momentum space and no attempt is made to localize it \[16\].

After this introduction we will shortly review the scheme to evaluate the spectral function within the Green’s function approach. Results will be presented in section 3 including a discussion of the usual angle-average approximations for the NN propagator in the scattering equation. The main conclusions are summarized in section 4.

**II. SELF-ENERGY AND SINGLE-PARTICLE GREEN’S FUNCTION**

The single-particle potential or self-energy of a nucleon with isospin \( \tau \), momentum \( \vec{k} \) and energy \( \omega \) in asymmetric nuclear matter is defined in the Brueckner-Hartree-Fock (BHF) approximation by

\[
\Sigma_{\tau}^{\text{BHF}}(\vec{k}, \omega) = \sum_{\tau'} \int d^3q < \vec{k}q|G(\Omega)|\vec{q}q' >_{\tau'\tau} n_{\tau'}^0(q') .
\]  

(1)

Note that here and in the following we suppress the spin quantum numbers in order to simplify the notation. In this equation \( n_{\tau'}^0(q') \) refers to the occupation probability of a free Fermi gas of protons (\( \tau = \pi \)) and neutrons (\( \tau = \nu \)). This means for asymmetric matter with a total density \( \rho \) and asymmetry \( \alpha \)

\[
\rho = \rho_{\pi} + \rho_{\nu} \\
\alpha = \frac{\rho_{\nu} - \rho_{\pi}}{\rho} ,
\]  

(2)

this occupation probability is defined by

\[
n_{\tau'}^0(q') = \begin{cases} 
1 & \text{for } |q'| \leq k_{F\tau} \\
0 & \text{for } |q'| > k_{F\tau} 
\end{cases}
\]  

(3)

with Fermi momenta for protons (\( k_{F\pi} \)) and neutrons (\( k_{F\nu} \)) which are related to the corresponding densities by

\[
\rho_{\tau} = \frac{1}{3\pi^2} k_{F\tau}^3 .
\]  

(4)

The matrix elements in \[11\] refer to the anti-symmetrized \( G \) matrix elements, which for a given NN interaction \( V \) are obtained by solving the Bethe-Goldstone equation

\[
< \vec{k}q|G(\Omega)|\vec{q}q' >_{\tau\tau'} = < \vec{k}q|V|\vec{p}p' > + \int d^3p_1 d^3p_2 < \vec{k}q|V|\vec{p}p_2 >_{\tau\tau'} \\
\times \frac{Q(p_1, p_2, \Omega)}{\Omega - (\varepsilon_{\pi, \tau} + \varepsilon_{\nu, \tau'}) + i\eta} < \vec{p}p_2|G(\Omega)|\vec{q}q' >_{\tau\tau'} .
\]  

(5)
The single-particle energies $\varepsilon_{pr}$ should be identified with the BHF single-particle energies which are defined in terms of the real part of the BHF self-energy of $\Sigma_{\tau}^{BHF}$ by

$$\varepsilon_{kr} = \frac{k^2}{2m} + \text{Re} \left[ \Sigma_{\tau}^{BHF}(\vec{k}, \omega = \varepsilon_{kr}) \right],$$

(6)

with a value for the starting energy parameter $\Omega$ in the Bethe-Goldstone equation $\Sigma_{\tau}$ of

$$\Omega = \omega + \varepsilon_{qr} = \varepsilon_{kr} + \varepsilon_{qr}.'$$

(7)

The Pauli operator $Q(p_1 \tau, p_2 \tau')$ in the Bethe Goldstone Eq. (4) is used to restrict the intermediate states to particle-particle states, i.e. to states in which the momenta of nucleons with isospin $\tau$ are above the corresponding Fermi momentum. This Pauli operator as well as the single-particle energies for the intermediate states are defined in terms of the single-particle momenta $p_1$ and $p_2$. Often, however, one uses a parameterization of the single-particle spectra in terms of an effective mass in the form

$$\varepsilon_{kr} \approx \frac{k^2}{2m_{\tau}} + C_{\tau}.$$  

(8)

This parameterization allows the definition of a so-called angle-averaged propagator, which reduces the Bethe-Goldstone equation to an integral equation in one dimension, to be solved for various partial waves. In our calculations we avoid this angle average procedure and solve the Bethe Goldstone equation with the exact propagator using the techniques discussed in [17].

In a next step we extend the definition of the self-energy and include the effects of hole-hole scattering terms in a kind of perturbative way [18].

$$\Delta \Sigma_{\tau}^{2h1p}(k, \omega) = \sum_{\tau'} \int_{k_{F,\tau'}}^{\infty} d^3p \int_{0}^{k_{F,\tau'}} d^3p_1 \int_{0}^{k_{F,\tau'}} d^3p_2 \frac{<k, p|G|h_1, h_2 >^2_{\tau,\tau'}}{\omega + \varepsilon_{pr} - \varepsilon_{h1 \tau} - \varepsilon_{h2 \tau'} - i\eta}.$$  

(9)

The comparison of the contributions from particle-particle ladders, contained in $\Sigma_{\tau}^{BHF}$, and those originating from the hole-hole ladders for symmetric nuclear matter in [18] justifies the perturbative treatment of $\Delta \Sigma_{\tau}^{2h1p}$. Also for this extended definition of the self-energy we can define a quasi-particle energy by

$$\varepsilon_{k\tau}^{QP} = \frac{k^2}{2m} + \text{Re} \left[ \Sigma_{\tau}^{BHF}(\vec{k}, \omega = \varepsilon_{k\tau}^{QP}) + \Delta \Sigma_{\tau}^{2h1p}(\vec{k}, \omega = \varepsilon_{k\tau}^{QP}) \right].$$

(10)

The quasi-particle energy for the momentum $k = k_{F,\tau}$ defines the Fermi energy for protons and neutrons, respectively

$$\varepsilon_{F,\tau} = \varepsilon_{k_{F,\tau}}.$$  

(11)

The real part (Re) and imaginary part (Im) of the self-energy, $\Sigma = \Sigma_{\tau}^{BHF} + \Delta \Sigma_{\tau}^{2h1p}$ can also be used to determine the spectral functions $S_{\tau}^{h}(k, \omega)$ and $S_{\tau}^{p}(k, \omega)$ for hole and particle strength, respectively

$$S_{\tau}^{h}(k, \omega) = \pm \frac{1}{\pi} \frac{\text{Im}\Sigma_{\tau}(k, \omega)}{(\omega - k^2/2m - \text{Re}\Sigma_{\tau}(k, \omega))^2 + (\text{Im}\Sigma_{\tau}(k, \omega))^2}, \text{ for } \omega < \varepsilon_{F,\tau} (\omega > \varepsilon_{F,\tau}).$$

(12)

The hole spectral function represents the probability that a particle with isospin $\tau$, momentum $k$ and energy $\omega$ can be removed from the ground state of the system, leaving the residual nucleus with $(A-1)$ nucleons in an eigenstate of the Hamiltonian with an energy $E_{A-1} = E_{A0} - \omega$, where $E_{A0}$ refers to the ground-state energy of the original system. Integrating the spectral distribution of the hole states yields the occupation probability

$$n_{\tau}(k) = \int_{-\infty}^{\varepsilon_{F,\tau}} d\omega S_{\tau}^{h}(k, \omega).$$

(13)

The mean energy for the distribution of the hole strength is obtained by

$$\langle \varepsilon_{k\tau}(k) \rangle = \frac{\int_{-\infty}^{\varepsilon_{F,\tau}} d\omega \omega S_{\tau}^{h}(k, \omega)}{n_{\tau}(k)},$$

(14)

and the corresponding energy for the distribution of the particle strength can be calculated as

$$\langle \varepsilon_{p\tau}(k) \rangle = \frac{\int_{-\infty}^{\varepsilon_{F,\tau}} d\omega \omega S_{\tau}^{p}(k, \omega)}{1 - n_{\tau}(k)}.$$  

(15)
Our approximation to a self-consistent Green’s function (GF) calculation can now be defined by identifying the single-particle to be used in the energy denominators of the Bethe-Goldstone equation as well as in the calculation of the 2h1p correction term in by

$$
\varepsilon_{k\tau} = \begin{cases} 
\langle \varepsilon_{h\tau}(k) \rangle & \text{for } k < k_F \tau \\
\langle \varepsilon_{p\tau}(k) \rangle & \text{for } k > k_F \tau 
\end{cases}.
$$

(16)

With this definition we employ a single-particle Green’s function, which is defined for each momentum by just one pole at the energy \( \omega = \varepsilon_{k\tau} \). This means that in the calculation of the self-energy we account for the fact that the spectral strength is distributed over a wide range of energies and consider the mean value of the distribution. This leads to an energy spectrum exhibiting a small gap at \( k = k_F \). This gap is of similar size as the pairing gap obtained in BCS calculations. With this choice we circumvent the problems of the pairing instabilities. However, in calculating the terms of the self-energy we do not consider a depletion of the occupation for momenta \( k \) below the Fermi momentum and partial occupation of states with momenta \( k \) larger than \( k_F \).

This partial depletion and occupation is taken into account, when the total energy per nucleon is determined by

$$
\frac{E}{A} = \frac{\sum_{\tau} \int d^3k \int_{-\infty}^{\varepsilon_{k\tau}} d\omega \; S^h(k,\omega) \frac{1}{2m} \left( \frac{k^2}{2m} + \omega \right)}{\sum_{\tau} \int d^3k \; n_{\tau}(k)}.
$$

(17)

### III. RESULTS AND DISCUSSION

As a first step we would like to discuss the self-energies or single-particle potentials as a function of the asymmetry of nuclear matter. For that purpose we consider nuclear matter at the empirical value for the saturation density of symmetric nuclear matter (\( \rho = 0.17 \text{ fm}^{-1} \)) and consider the self-energies for various values of the asymmetry parameter \( \alpha \).

Results for the BHF approximation of the self-energy for protons and neutrons are displayed in Fig. 1 for values of the asymmetry parameter \( \alpha \) ranging from 0.05 to 0.75. The CD-Bonn interaction has been used to generate the results for this figure.

One finds that the single-particle potentials for protons are more attractive than those for neutrons and that the depth of the potential for the protons, the value of the self-energy at \( k = 0 \) is more attractive with increasing asymmetry. This reflects the fact that the effective interaction is more attractive between protons and neutrons than between nucleons of the same isospin. For the CD-Bonn interaction this feature can be observed to some extent already on the level of the Hartree-Fock (HF) approximation. The HF self-energies, however, are less attractive and the dependence...
FIG. 2: (Color online) Effective $k$-masses (solid lines) and $E$-masses (dashed lines) derived from the BHF self-energy. Results were obtained using the CD-Bonn interaction at density $\rho = 0.17 \text{ fm}^{-1}$ for an asymmetry $\alpha=0.5$.

of the depth of the self-energies is weaker. For $\alpha = 0.75$ the values for the HF self-energies at $k = 0$ range from -52 MeV to -42 MeV for protons and neutrons, respectively. Using the local interaction Nijm2 of the Nijmegen group one obtains results in the BHF approximation, which are rather similar to those displayed in Fig. 2. Using the HF approximation, however, the Nijm2 interaction yields much less attractive self-energies, ranging from 52 MeV to -8 MeV and in this case the self-energy is more attractive for the neutrons than for the protons.

This is already a first indication for the differences between the two interactions considered: The local interaction Nijm2 is stiffer than the non-local CD-Bonn potential. Therefore a larger part of the attraction in the effective interaction originates from the particle-particle ladder contributions to the $G$-matrix. This is true in particular for the proton-neutron interaction, which can be traced back to the correlations in the $^{3}S_{1}-^{3}D_{1}$ channel of the interaction of nucleons with total isospin $T=0$.

From Fig. 2 one can also see that the BHF self-energies do not have a simple parabolic shape as a function of the momentum. There is a characteristic dip in the self-energy always occurring at momenta slightly above the Fermi momentum of the kind of nucleons under consideration. The momentum dependence of the single-particle potential in a homogeneous infinite system is a sign of the non-locality of the single-particle potential and is typically characterized in terms of an effective mass $m^*$, which can be used to parameterize the momentum dependence of the single-particle energies according Eq. (8). This non-locality of the single-particle potential can be due to a non-locality in space, which is characterized by the so-called $k$-mass,

$$
\frac{m_k(k)}{m} = \left[ 1 + \frac{m}{k} \frac{\partial \Sigma(k, \omega)}{\partial k} \right]^{-1},
$$

or due to a non-locality in time, which is expressed in terms of the $E$-mass

$$
\frac{m_E(\omega)}{m} = \left[ 1 - \frac{\partial \Sigma(k, \omega)}{\partial \omega} \right],
$$

so that the total effective mass is given by

$$
\frac{m^*(k)}{m} = \frac{m_k(k)}{m} \frac{m_E(\omega = \varepsilon_k)}{m}.
$$

Fig. 2 shows typical examples for the effective $k$-mass (solid lines) and the $E$-mass (dashed lines) derived from the BHF self-energy for protons and neutrons at asymmetry $\alpha = 0.5$ using the CD-Bonn interaction. The effective $k$-mass shows a rather smooth dependence on the momentum $k$ with typical values ranging from 0.6 m up to 0.9 m for larger $k$. The effective $k$ mass is hardly effected by the particle-particle ladder contributions. It is larger for the neutrons.
than for the protons. The effective $E$-masses on the other hand exhibit a strong momentum or energy dependence (note that the energy variable of the self-energy is related to the momentum by the self-consistency requirement (6)). It reaches a maximum just above the Fermi momentum of the nucleon type under consideration. These maximal values for the $E$-mass lead to an effective mass $m^*$ close to 1 at momenta around the Fermi momentum, which corresponds to the characteristic momentum dependence of the self-energy displayed in Fig. 2.

The energy dependence of the BHF self-energy is due to the particle-particle ladder terms in the $G$-matrix. The values for the effective $E$-mass are larger for the protons than for the neutrons (at positive values for the asymmetry parameter), which is due the fact that particle-particle correlations are of particular importance for the proton-neutron interaction. The larger $E$-mass combined with a smaller $k$-mass for the protons leads to effective masses, which similar for protons and neutrons[21, 22, 23].

In the next step we consider the quasi-particle energies defined in (10) within the Green’s function approach (GF). In order to enhance the momentum dependence, we subtract the contribution for the kinetic energy and compare in Fig. 3 the resulting values for the self-energy with those determined from the BHF approach.

The hole-hole ladder contribution leads to less attractive single-particle energies in the GF as compared to the BHF approach. The difference tends to be larger for the neutrons than for the protons (at positive values for the asymmetry parameter) and decreases with increasing momentum. As a consequence we find that repulsive effect of the hole-hole ladder term is essentially identical for protons and neutrons at the corresponding Fermi momenta.

This can be seen from Fig. 4 which displays the Fermi energies derived from the BHF and GF single-particle energies for protons and neutrons. These Fermi energies for protons (neutrons) decrease (increase) linearly with the asymmetry parameter $\alpha$. The slope is identical for BHF and GF approach and essentially the same for CD-Bonn and Nijm2 interaction. The only difference is that the GF Fermi energies are shifted by a value of around 7 MeV. This implies that the conditions for $\beta$-equilibrium are not affected by the inclusion of the hole-hole scattering terms. In order to investigate the features of the nucleon self-energy more in detail we display in Figs. 5 and 6 the self-energies for protons and neutrons in asymmetric nuclear matter ($\alpha = 0.5$) using the GF approach for the CD-Bonn and the Nijm2 interaction. The lower panels of these figures show the absolute value of the imaginary part of these self-energies for a fixed momentum as a function of the energy variable $\omega$. The contributions at energies below the Fermi energy $\varepsilon_F$ originate form the $2h1p$ part of the self-energy, while those at energies above $\varepsilon_F$ are due to the BHF part. It is rather obvious that the energy integrated $2h1p$ contribution is significantly smaller than the BHF part of the self-energy. This reflects the fact that for realistic NN interactions and nuclear densities around the saturation density the particle-particle ladder contributions are much larger than those of the hole-hole type. This is again a justification of the hole-line expansion or our perturbative treatment of the $2h1p$ contribution.

One also sees rather clearly that the imaginary part calculated from the Nijm2 interaction is larger than the corre-
FIG. 4: (Color online) Fermi energies for protons and neutrons derived from the BHF single-particle energy (6) and from the quasiparticle energy of the Green’s function (GF) approach of (10). Results were obtained using the CD-Bonn at density $\rho = 0.17\,\text{fm}^{-1}$. Results are displayed as a function of the asymmetry parameter $\alpha$.

FIG. 5: (Color online) Real (upper panel) and the absolute value of the imaginary part (lower panel) of the self-energy for protons with $k=0.4\,k_F$ as a function of energy $\omega$. The self-energy has been evaluated in the Green’s function approach including $\Sigma^{\text{BHF}}$ and $\Delta\Sigma^{2h1p}$ for asymmetric nuclear matter at density $\rho = 0.17\,\text{fm}^{-1}$ for an asymmetry $\alpha=0.5$ using the CD-Bonn (solid line) and the Nijm2 (dashed line) interaction.

The real part of the self-energy originates from the energy-independent Hartree-Fock contribution plus the energy-dependent corrections, which are related to the imaginary part of the self-energy by a dispersion relation. The gross-structure of the energy-dependence of the real part of $\Sigma$ is due to the BHF contribution. The larger imaginary part obtained with the Nijm2 interaction leads to a more negative slope of the self-energy in the region $\omega - \varepsilon_F < -100\,\text{MeV}$. The fine-structure in the real part of $\Sigma$ at energies $-100\,\text{MeV} < \omega - \varepsilon_F < 50\,\text{MeV}$ has its origin in the $2h1p$ term.
FIG. 6: (Color online) The self-energy for neutrons with $k = 0.4 \, k_F$ as a function of energy $\omega$. Further details see caption of Fig. 5.

FIG. 7: (Color online) Spectral functions for nucleons with $k = 0.4 \, k_F$ in the upper part and $k = 1.5 \, k_F$ in the lower panel as a function of energy $\omega$. Further details see caption of Fig. 5.

and is absent in the BHF approximation.

Note, that for the asymmetries considered, the $2h1p$ contribution is significantly larger for the neutron- than for the proton-self-energy. This indicates, that the imaginary part in $\Sigma^{2h1p}$ increases with the density of the kind of nucleon under consideration. On the other hand, the imaginary part of the BHF contribution is larger for the protons than for the neutrons. This is related to the fact that the particle-particle correlations described by the BHF term are to a large extent due to the strong tensor force in the proton-neutron interaction. Since the neutron density is increasing with the asymmetry one can expect a larger imaginary part in $\Sigma^{BHF}$ for the protons as compared to the neutrons. The upper part of Fig. 7 displays the spectral functions for protons and neutrons, considering the same asymmetry and momenta as in Figs. 5 and 6. While the spectral function for the proton exhibits a pronounced quasiparticle peak, the spectral function for the neutron shows a rather broad distribution. This is partly due to the fact that
with this choice of the momentum ($k_\tau = 0.4 k_F \tau$) the quasiparticle energy minus the corresponding Fermi energy is slightly lower for the neutrons than for the protons. The main reason for this broader distribution of the hole strength, however, is the larger imaginary part in $\Delta \Sigma_{2h1p}$ for the neutrons as compared to the protons, as we discussed above. The larger imaginary part in $\Sigma_{BHF}$ for the protons, on the other hand, leads to larger values for the proton spectral functions at positive energies than for the neutrons, if we consider nucleons with momenta $k < k_F$. Consequently, the spectral strength for nucleons with $k > k_F$ at energies below $\varepsilon_F$ is larger for protons than for the neutrons (see lower panel of Fig. 8).

This means that the occupation numbers, $n_\tau(k)$ (see Eq. (13)), for nucleons with momenta $k$ below the corresponding Fermi momentum are, at positive values of $\alpha$, smaller for protons than for neutrons. Because of the strong proton-neutron interaction, an increase of the neutron density with increasing asymmetry $\alpha$ yields a larger depletion of the proton hole-states than of the neutron hole states. This is shown in Fig. 7 where occupation probabilities for protons and neutrons, averaged over all momenta below the corresponding Fermi momenta, are presented as a function of asymmetry. One observes an almost linear dependence of the occupation probabilities as a function of asymmetry. The occupation probabilities are slightly larger for the CD-Bonn interaction as compared to the stiffer Nijm2 interaction.

The spectral distribution of hole-strength can be explored in nucleon knock-out experiments on neutron rich nuclei. Our results would predict that the proton distribution functions should exhibit more pronounced quasi-particle peaks in states, which are occupied in the independent particle model, than the neutrons. Nevertheless, the energy integrated strength in these partial waves, the occupation numbers, should be larger for the neutrons.

Finally a few words about the total energy per nucleon as a function of asymmetry. The calculated energies are rather well approximated by a function of the form

$$\frac{E}{A} = E_0 + a_S \alpha^2,$$

with the coefficient $a_S$ for the symmetry energy. Results for $E_0$ the binding energy per nucleon of symmetric nuclear matter at saturation density and the asymmetry coefficient are listed in Table I and compared to the empirical values. Note, however, that for these calculations we do not determine the saturation density but just calculate at the empirical value of the saturation density $\rho_0$.

As we discussed already before, the Nijm2 interaction is a stiffer interaction and predicts less binding energy per nucleon than the softer CD-Bonn potential. The inclusion of $2h1p$ terms in the self-energy leads to smaller binding energies per nucleon. The effect is small in the case of the CD-Bonn interaction but considerably larger for the Nijm2 model. The inclusion of the $2h1p$ term has only little effect on the symmetry energy. Also the influence of the interaction is not very significant.
TABLE I: Energy per nucleon, $E_0$ and symmetry energy, $a_S$ determined from BHF and GF calculations using the CD-Bonn and the Nijm2 interaction

|         | $E_0$ [MeV] | $a_S$ [MeV] |
|---------|-------------|-------------|
| CD-Bonn | -18.8       | 31.5        |
| GF      | -18.0       | 33.3        |
| Nijm2   | -16.7       | 29.3        |
| GF      | -12.3       | 34.0        |
| Exp.    | -15.7       | 30±2        |

IV. CONCLUSIONS

An extension of the Brueckner Hartree Fock (BHF) approximation to a self-consistent Green’s function method (GF) has been used to study properties of asymmetric nuclear matter. We investigate in particular the spectral distribution of single-particle strength for such asymmetric systems.

It is observed that the imaginary part of the self-energy at energies below the Fermi energy increases with the density of the kind of nucleons under consideration. Since this part is responsible for the spreading of the hole strength, we obtain more pronounced quasiparticle states for protons than for neutrons in neutron-rich matter.

On the other hand, the occupation probabilities of single-particle states, which are occupied in the independent particle model, is significantly smaller for protons than for neutrons in such neutron-rich systems. This can be traced back to the strong components of proton-neutron interaction, which is the main source for the imaginary part of the self-energy at momenta below the Fermi momentum and energies above $\varepsilon_F$.

The influence of the 2h1p contributions to the self-energy on total energy, symmetry energy and Fermi energies is not very significant for the soft CD-Bonn interaction. It leads to considerable effects for the local Nijm2 interaction: reducing the calculated energy and increasing the coefficient for the symmetry energy. Therefore a more systematic study of asymmetric nuclear systems should be useful, which goes beyond the present approach and treats the hole-hole scattering terms in a non-perturbative way [6, 11, 24].

Various discussions with Tobias Frick are gratefully acknowledged. This work has been supported by the European Graduate School “Hadron in Vacuum, Nuclei and Stars” (Basel - Tübingen).

[1] M. Baldo, *Nuclear Methods and the Nuclear Equation of State*, Int. Rev. of Nucl. Physics, Vol. 9 (World-Scientific Publ. Comp., Singapore 1999).

[2] H. Mütter and A. Polls, *Prog. in Part. and Nucl. Phys.* 45, 243 (2000).

[3] W.H. Dickhoff and C. Barbieri *Prog. in Part. and Nucl. Phys.* 52, 377 (2004).

[4] B.E. Vonderfecht, W.H. Dickhoff, A. Polls, and A. Ramos, *Nucl. Phys. A* 555, 1 (1993).

[5] T. Alm, G. Röpke, A. Schnell, N.H. Kwong, and H.S. Köhler, *Phys. Rev. C* 53, 2181 (1996).

[6] P. Bożek, *Nucl. Phys. A* 657, 187 (1999).

[7] M. Baldo, I. Bombaci, and U. Lombardo, *Phys. Lett. B* 283, 8 (1992).

[8] T. Alm, B.L. Friman, G. Röpke, and H. Schulz, *Nucl. Phys. A* 551, 45 (1993).

[9] O. Elgarøy, L. Engvik, M. Hjorth-Jensen, and E. Osnes, *Phys. Rev. C* 57, R1069 (1998).

[10] Y. Dewulf, D. Van Neck, and M. Waroquier, *Phys. Lett. B* 510, 89 (2001).

[11] T. Frick and H. Mütter, *Phys. Rev. C* 68, 034310 (2003).

[12] E. Heinz, H. Mütter, and H.A. Mavromatis, *Nucl. Phys. A* 587, 77 (1995).

[13] T. Frick, Kh. Gad, H. Mütter, and P. Czerski, *Phys. Rev. C* 65, 034321 (2002).

[14] T. Frick, Kh.S.A. Hassanein, D. Rohe, and H. Mütter, Phys. Rev. C, in print, preprint nucl-th/0406010.

[15] V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen, and J.J. de Swart, *Phys. Rev. C* 49, 2950 (1994).

[16] R. Machleidt, F. Sammarruca, and Y. Song, *Phys. Rev. C* 53, R1483 (1996).

[17] E. Schiller, H. Mütter, and P. Czerski, *Phys. Rev. C* 59, 2934 (1999); erratum *Phys. Rev. C* 60, 059901 (1999).

[18] P. Grange, J. Cugnon, and A. Lejeune, *Nucl. Phys. A* 473, 365 (1987).

[19] H. Mütter and A. Polls, *Phys. Rev. C* 61, 014304 (2000).

[20] C. Mahaux and R. Sartor, *Adv. Nucl. Phys.* 20, 1 (1991).

[21] O. Elgarøy, L. Engvik, E. Osnes, F.V. De Blasio, M. Hjorth-Jensen, and G. Lazzari, *Phys. Rev. Lett.* 76, 1994 (1996).

[22] L. Engvik, M. Hjorth-Jensen, E. Osnes, G. Bao, and E. Ostgaard, *Astrophysical J.* 469, 794 (1996).

[23] D. Alonso and F. Sammarruca, nucl-th/0301032.

[24] P. Bożek, *Phys. Lett. B* 586, 239 (2004).