The signal of $Z^{\pm}(4430)$ in nucleon-antinucleon scattering

Hong-Wei Ke and Xiang Liu

1Department of Physics, Nankai University, Tianjin 300071, China
2Centro de Física Computacional, Departamento de Física, Universidade de Coimbra, P-3004-516 Coimbra, Portugal
(Dated: October 2, 2008)

We study the production of $Z^{\pm}(4430)$ at a nucleon-antinucleon scattering experiment. Considering the PANDA experiment to be an ideal platform to explore the production of the charmonium and charmonium-like states, we suggest the forthcoming PANDA experiment to pay attention to the production of $Z^{\pm}(4430)$.

PACS numbers: 13.75.Cs

I. INTRODUCTION

Recently the Belle Collaboration announced the observation of $Z^{\pm}(4430)$ at the invariant mass spectrum of $\psi'\pi^{\pm}$ of $B \to K\psi'\pi^{\pm}$. As an excellent candidate of the exotic state, $Z^{\pm}(4430)$ possesses $m = 4433 \pm 4(\text{stat}) \pm 1(\text{syst})$ MeV and $\Gamma = 44^{+15}_{-11}(\text{stat})^{+30}_{-11}(\text{syst})$ MeV [1]. Up to now, theorists have paid extensive attention to $Z^{\pm}(4430)$ [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. $Z^{\pm}(4430)$ was explained as an S-wave threshold effect [2], a $D_1D^*/D*D^*$ molecular state [2, 3, 4, 5, 6, 7, 14], a tetraquark state [3, 4, 5, 6, 15], a cusp effect [11], a $\Lambda_c\Sigma_c$ bound state [12] and the radially excited state of $D^*_1$ [17]. Besides exploring the structure of $Z^{\pm}(4430)$, the hidden and open charm decays were studied in Ref. [3, 10].

Up to now, $Z^{\pm}(4430)$ was only observed by the Belle experiment. Since searching $Z^{\pm}(4430)$ in other experiments will be helpful to establish $Z^{\pm}(4430)$ finally, the theoretical study of the production of $Z^{\pm}(4430)$ is an interesting topic. For example, recently the author of Ref. [15] studied the production of $Z^{\pm}(4430)$ by the photoproduction experiment.

Exploring the charmonium and charmonium-like states above the open charm threshold is one of the most important physics aims of the forthcoming PANDA (AntiProton Annihilations at Darmstadt) experiment at FAIR [18]. Recently some groups performed the phenomenological studies of the productions of the charmonium and charmonium-like states around the PANDA experiment. Barnes and Li calculated the differential and total cross sections of the near-threshold associated charmonium production processes $pp \to \pi^{\pm}\Psi$ with $\Psi = \eta_c, J/\psi, \psi', \chi_{c0}, \chi_{c1}$ using a simple initial-state light meson emission model [19]. In Ref. [20], the authors studied the production of $X(3872)$ at the PANDA experiment assuming $X(3872)$ to be the molecular state $DD^*$ or the charmonium state $\chi_{c1}(2P)$. These instructive explorations not only further guide the PANDA experiment to find charmonium or charmonium-like state but also stimulate more theorists to focus on this field.

The PANDA experiment could be an ideal platform to explore the production of $Z^{\pm}(4430)$. Thus, in this work, we first carry out a phenomenological study of $Z^{+}(4430)$ production at antiproton-proton scattering experiment. Meanwhile we also tentatively investigate $Z^{-}(4430)$ production by the antiproton-neutron annihilation process.

This work is organized as follows. After the introduction, we present the formula of the production of $Z^{+}(4430)$ by the $pp$ interaction. Next, we formulates the expressions of $Z^{-}(4430)$ production by the $\bar{p}n$ annihilation in Sec. III. Section IV is for numerical results and our discussion.

II. THE PRODUCTION OF $Z^{\pm}(4430)$ BY THE SCATTERING OF PROTON AND ANTIPROTON

In 1987, Gaillard, Maiani and Petronzio for the first time applied the hadron-level pole diagram to calculate the production of charmonium from the $pp$ states with fixed quantum number [21]. Later Lundborg, Barnes and Wiedner developed this method to do in-depth study of $pp \to \pi^{0}\Psi$ ($\Psi = \eta_c, J/\psi, \psi', \chi_{c0}, \chi_{c1}$) [22]. The hadron-level pole diagram is a suitable approach to study the charmonium or charmonium-like state in the low-energy $pp$ scattering process.

As an excellent candidate for the exotic state, the component of $Z^{\pm}(4430)$ should be $\bar{c}\bar{c}q\bar{q}$ for explanations in terms of the $D_1D^*$ molecular state [2, 3, 4, 5] or the tetraquark state [6]. Thus the $Z^{\pm}(4430)$ production mechanism for the $pp$ scattering process is similar to $pp \to \pi^{0}\Psi$. In this work, we use the so called hadron-level pole diagram to depict the production of $Z^{\pm}(4430)$ at the proton and antiproton scattering process, which is depicted in Fig. 1. By exchanging neutron between proton and antiproton, the process of $pp \to Z^{\pm}(4430)\pi^{-}$ occurs.

In this work, we consider three possible quantum numbers for $Z^{\pm}(4430)$, i.e. $J^{P} = 0^{-}, 1^{-}, 2^{-}$ [4]. The effective Lagrangians describing the interaction of $Z^{\pm}(4430)$ with...
The value of the coupling constants $g_{\gamma pZ}^{(0)}, g_{\gamma pZ}^{(1)}, g_{\gamma pZ}^{(2)}$ will be given in the section on the numerical results. The Lagrangian relevant to the coupling of $\pi$ and the nucleons is well established [23]:

$$\mathcal{L}_{N_N\pi} = -i \mathcal{G} \bar{N} \gamma_5 \tau \cdot \pi N$$

$$= -i \sqrt{2} \mathcal{G} \left[ (\bar{p}\gamma_5 \rho + \bar{n}\gamma_5 \pi) \gamma_5 n^++ \bar{n}\gamma_5 p^+ \right] \frac{1}{\sqrt{2}},$$

where $\bar{N} = (\bar{p}, \bar{n})$. The coupling constant $\mathcal{G}$ is taken to be 13.5.

Using the above Lagrangian, we write out the amplitudes of the $pp \rightarrow Z^+(4430)\pi^-$ process depicted in Fig. 1:

$$\mathcal{M}^{J=0} = \sqrt{2} \mathcal{G} g_{\gamma pZ}^{(0)} \bar{p} \gamma_5 \gamma_5 1 \frac{1}{\bar{p}_2 - \bar{b}_n - m_n \gamma_5 u_p},$$

$$\mathcal{M}^{J=1} = \sqrt{2} \mathcal{G} g_{\gamma pZ}^{(1)} \bar{p} \gamma_5 \gamma_5 \frac{1}{\rho_{\pi} \bar{p}_2 - \bar{b}_n - m_n \gamma_5 u_p},$$

$$\mathcal{M}^{J=2} = -\sqrt{2} \mathcal{G} g_{\gamma pZ}^{(2)} \bar{p} \gamma_5 \gamma_5 \frac{1}{\bar{p}_2 - \bar{b}_n - m_n \gamma_5 u_p},$$

respectively, corresponding to the production of $Z^+(4430)$ with $J^P = 0^-, 1^-, 2^-$. Here $p_a$, $p_b$, $p_1$ and $p_2$ denote the four-momentum of $p$, $\bar{p}$, $Z^+(4430)$ and $\pi^-$ respectively.

Since $Z^+(4430)$ is observed in the invariant mass spectrum of $\psi'\pi^+$; thus, we further explore $pp \rightarrow Z^+(4430)\pi^- \rightarrow \psi'\pi^+\pi^-$ process, depicted in Fig. 2:

The interactions of $Z^+(4430)$ with $\pi^-$ and $\psi'$ are

$$\mathcal{L}_{Z^+\pi^-\psi'} = \left\{ \begin{array}{ll} ig_{\gamma Z\pi}^{(0)} \gamma_5 (\pi^- \partial_\mu Z - \partial_\mu \pi^- Z) \psi'^\mu & \text{for } J^P = 0^- \\ ig_{\gamma Z\pi}^{(1)} \epsilon_{\mu\nu\alpha\beta} \partial_\mu \psi'^\nu \partial_\alpha Z_\beta \pi^- + h.c. & \text{for } J^P = 1^- \\ ig_{\gamma Z\pi}^{(2)} \psi'^\nu Z_\nu \partial_\mu \pi^- & \text{for } J^P = 2^- \end{array} \right.$$
and $\pi^-$, can be obtained by integrating over the phase space [24]

$$
\sigma(p\bar{p} \to \psi' \pi^+ \pi^-) = \int_{a_1}^{a_2} dp_2 \int_{b_1}^{b_2} dp_3 \int_{0}^{2\pi} d\eta \int_{-1}^{1} d(\cos \theta) d\sigma,
$$

where $a_1, a_2, b_1$ and $b_2$ are defined as respectively

$$
\begin{align*}
  a_1 &= m_2, \quad a_2 = \frac{\sqrt{s}}{2} - \frac{(m_1 + m_3)^2 - m_2^2}{2 \sqrt{s}}, \\
  b_1 &= \frac{1}{2} \left[ \rho(\tau + m_+ m_-) - |p_2| (\tau - m_+^2) \right], \\
  b_2 &= \frac{1}{2} \left[ \rho(\tau + m_+ m_-) + |p_2| (\tau - m_+^2) \right], \\
  \rho &= \sqrt{s} - p^0_2, \quad \tau = \rho^2 - |p_2|^2, \quad m_\pm = m_3 \pm m_1.
\end{align*}
$$

The definition of $d\sigma$ reads as

$$
d\sigma = \frac{1}{32(2\pi)^4 |p_\pi^\prime| \sqrt{s}} |\mathcal{M}|^2.
$$

Here $m_{1,2,3}(p_{1,2,3})$ denote the masses (four-momentum) of $\psi'$, $\pi^+$ and $\pi^-$, respectively. $\mathcal{M}$ represents the amplitude shown in (10).

### III. THE PRODUCTION OF Z\(^+(4430)\) IN THE ANTIPROTON-NEUTRON ANNIHILATION PROCESS

In this section, we study the process that $\bar{p}n \to Z^-(4430) \to \psi' + \pi^-$, which is depicted in Fig. 3. Figure 3 describes the background $\bar{p}n \to \psi' \pi^-$ relevant to the production of $Z^+(4430)$.

Thus, we can directly obtain the amplitudes for the production of $Z^+(4430)$ at antiproton-neutron annihilation, which are represented as

$$
\mathcal{M}^{J=0} = i g_{\pi \pi}^{(1)} g_{\bar{p}\pi}^{(0)} \bar{p} \gamma_5 u_n
\times \frac{1}{(q^2 - m_Z^2 + i m_Z \Gamma_Z)} (p_1 + 2p_2) \cdot \varepsilon,
$$

$$
\mathcal{M}^{J=1} = i g_{\pi \pi}^{(1)} g_{\bar{p}\pi}^{(1)} \bar{p} \gamma_5 u_n
\times \frac{q_{\mu \alpha} - q_{\nu \beta} \varepsilon^\mu \varepsilon^\nu}{(q^2 - m_Z^2 + i m_Z \Gamma_Z)} p_\alpha p_\beta \varepsilon^\mu \varepsilon^\nu,
$$

$$
\mathcal{M}^{J=2} = -\frac{g_{\pi \pi}^{(2)} g_{\bar{p}\pi}^{(2)}}{2} \bar{p} \gamma_5 u_n
\times \frac{1}{(q^2 - m_Z^2 + i m_Z \Gamma_Z)} p_\alpha p_\beta \varepsilon^\mu \varepsilon^\nu
\times \left( q_{\mu \alpha} q_{\nu \beta} - q_{\mu \beta} q_{\nu \alpha} \right)
\times \frac{1}{3} q_{\mu \alpha} q_{\nu \beta}.
$$

respectively corresponding to the production of $Z^+(4430)$ with $J^P = 0^-, 1^-, 2^-$. Here $q = q_1 + q_2$ and $q_{\mu \nu} = -g_{\mu \nu} + q_{\mu} q_{\nu}/q^2$.

For obtaining the amplitude of the background $\bar{p}n \to \psi' \pi^-$ shown in Fig. 4, we need the effective Lagrangian of the strong interaction between $\psi'$ and the nucleons

$$
\mathcal{L}_{NN\psi'} = i g_{\bar{p}\psi} p^{\mu} \bar{p} \psi^{\nu} + i g_{n\bar{n}\psi'} n^{\mu} \bar{n} \psi^{\nu}.
$$

Since the partial decay width of $\psi' \to p\bar{p}$ is given experimentally, the coupling constant $g_{\bar{p}\psi\psi'}$ can be obtained. In Ref. [19], Barnes and Li obtained $g_{\bar{p}\psi\psi'} = (0.97 \pm 0.04) \times 10^{-3}$ by the experimental branching ratio $B(\psi' \to p\bar{p}) = (2.65 \pm 0.22) \times 10^{-4}$. Due to SU(2) symmetry, one can estimate the relation of $g_{n\bar{n}\psi'}$ and $g_{n\bar{n}\psi}$ by $g_{n\bar{n}\psi} \approx g_{n\bar{n}\psi'}$.

The amplitudes of the process $\bar{p}n \to \psi' \pi^-$ read as

$$
\mathcal{M}_a = \sqrt{2} i g_{\pi \psi} \bar{p} \psi^{\nu} \gamma_5 \frac{1}{p_\alpha - p_\beta - m_n} \varepsilon u_n,
$$

$$
\mathcal{M}_b = \sqrt{2} i g_{\pi \psi} \bar{p} \psi^{\nu} \gamma_5 \frac{1}{p_\alpha - p_\beta - m_p} \varepsilon u_n.
$$

### IV. NUMERICAL RESULT AND DISCUSSION

The input parameters include $m_{\psi'} = 3686.1$ MeV, $m_{\pi^-} = 139.6$ MeV, $m_p = 938.3$ MeV, $m_n = 939.6$ MeV [23]; $m_Z = 4433$ MeV, $\Gamma_Z = 44$ MeV [1].
In [3], Meng and Chao calculated the decay width of \( Z^+(4430) \to \psi' \pi^+ \) by rescattering mechanism assuming \( Z^+(4430) \) to be an S-wave \( D_1 D^* \) molecular state, and their result indicates that the order of magnitude of \( \Gamma_{Z\psi'\pi^+} \) is several MeV, which can be applied to estimate the coupling constants \( g_{Z\psi'\pi^+}^{(0,1,2)} \). Thus, in this work we adopt the below coupling constants
\[
g_{Z\psi'\pi^+}^{(0)} = 0.08, \quad g_{Z\psi'\pi^+}^{(1)} = 0.05 \text{ GeV}^{-1}, \quad g_{Z\psi'\pi^+}^{(2)} = 0.33,
\]
which are determined by taking \( \Gamma_{Z\psi'\pi^+} = 1 \text{ MeV} \).

It is difficult to obtain the value of the coupling constant \( g_{Z^+p\bar{n}} \) by a reliable dynamics calculation. On taking the total width of \( Z^+(4430) \) as the maximum of the \( \Gamma_{Z^+p\bar{n}} \), one gets the maximum of coupling constant of \( Z^+p\bar{n} \)
\[
g_{Z^+p\bar{n}}^{(0)} = 0.52, \quad g_{Z^+p\bar{n}}^{(1)} = 1.27, \quad g_{Z^+p\bar{n}}^{(2)} = 0.36 \text{ GeV}^{-2}.
\]
Using the above parameters, we obtain the dependence of the cross section of \( p\bar{p} \to Z^+(4430)\pi^- \) on the center-of-mass energy \( \sqrt{s} \), which is shown in Fig. 5.

The design luminosity of PANDA is \( 2 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1} \) for the collision of the antiproton and the proton target. Thus, the integrated luminosity is able to reach approximately 1.5 fb\(^{-1}\)/year if the machine is of 50% efficiency and in half a year of data taking [20]. There exist sizable events for the production of \( Z^+(4430)\pi^- \) in \( p\bar{p} \) scattering, i.e. the maximum events with \( \sqrt{s} = 5 \text{ GeV} \) can be up to \( 8.9 \times 10^{10}, 1.4 \times 10^{12} \) and \( 2.9 \times 10^{12} \), corresponding to \( Z^+(4430) \) productions with \( J^P = 0^-, 1^-, 2^- \), respectively. However, one needs to note that the above observation is only valid on taking the total width of \( Z^+(4430) \) as the decay width of \( Z^+(4430) \to p\bar{n} \). If the order of magnitude of \( \Gamma_{Z^+p\bar{n}} \) is about 1 MeV, the above estimated events is suppressed by a factor \((1/44)^2 \sim 0.015\).
$5 \times 10^{-4}$, still enough to comply with the needs of the experimentalist. Even though $\Gamma_{Z^+p\bar{n}}$ is at the order of keV, there also exists $10 \sim 10^2$ events of $Z^+(4430)$ production.

In Fig. 3 we present the variation of the cross section of $pp \rightarrow Z^+(4430)\pi^- \rightarrow \psi'\pi^+\pi^-$ to $\sqrt{s}$. Due to $p\bar{p} \rightarrow \psi'\pi^+\pi^-$ with intermediate state $Z^+(4430)$ being a $2 \rightarrow 3$ process, the cross section is $3 \sim 4$ order smaller than that of $pp \rightarrow Z^+(4430)\pi^-$. The line shape of the cross section of $pp \rightarrow \pi^+Z^+(4430) \rightarrow \psi'\pi^+\pi^-$ is similar to that of $p\bar{p} \rightarrow Z^+(4430)\pi^-$. Being a low-energy collision experiment of the antiproton with fixed target, the PANDA experiment has unique advantages to search for $Z^\pm(4430)$ and shed light on the nature of $Z^\pm(4430)$ further.

Figure 7 shows the cross sections of $\bar{p}n \rightarrow Z^-(4430) \rightarrow \psi'\pi^-$ as the function of $\sqrt{s}$, which includes the contribution from the background. Here we present the different results by taking several values of the coupling constant $g_{Zpn}$, which correspond to the $\Gamma_{Zpn} = 44, 22, 4.4$ and 1 MeV respectively. The line shape of the cross section of $\bar{p}n \rightarrow Z^-(4430) \rightarrow \psi'\pi^-$ shows the enhancement at energy $\sqrt{s} = 4.433$ GeV, which is different from the case of $Z^+(4430)$ production at $p\bar{p}$ scattering process.

In terms of our calculation, we find that the largest production rate of $Z^+(4430)$ mainly comes from the $p\bar{n}$ annihilation process. Unfortunately, the PANDA experiment mainly refers to the collision of proton and antiproton. Thus, it is difficult to test our proposal of $Z^+(4430)$ production by $p\bar{n}$ annihilation based on present high-energy experiments. One of the difficulties is that it is not easy to provide and control the source of the neutron, which is waiting to be solved by clever experimentalists.

![Graphs showing cross sections of $Z^-(4430)$ production](image)

FIG. 7: The cross section of the production of $Z^-(4430)$ in $\bar{p}n$ annihilation process.

**Acknowledgments**

One of us (X.L.) would like to thank Prof. Eef van Beveren for useful discussion and fruitful comments. This work was supported by the Fundação para a Ciência e Tecnologia of the Ministério da Ciência, Tecnologia e Ensino Superior of Portugal (SFRH/BPD/34819/2007) and the National Natural Science Foundation of China under Grant 10705001.
[1] Belle Collaboration, K. Abe et al., Phys. Rev. Lett. 100, 142001 (2008).
[2] J.L. Rosner, Phys. Rev. D 76, 114002 (2007).
[3] C. Meng and K.T. Chao, arXiv:0708.4222 [hep-ph].
[4] X. Liu, Y.R. Liu, W.Z. Deng and S.L. Zhu, Phys. Rev. D 77, 034003 (2008), arXiv:0711.0494.
[5] X. Liu, Y.R. Liu, W.Z. Deng, S.L. Zhu, Phys. Rev. D 77, 094015 (2008), arXiv:0803.1295 [hep-ph].
[6] G.J. Ding, arXiv:0711.1485 [hep-ph]; G.J. Ding, W. Huang, J.F. Liu and M.L. Yan, arXiv:0805.3822 [hep-ph].
[7] S.H. Lee, A. Mihara, F.S. Navarre and M. Nielsen, arXiv:0710.1029 [hep-ph].
[8] L. Maiani, A.D. Polosa and V. Riquer, arXiv:0708.3997 [hep-ph].
[9] K. Cheung, W.Y. Keung and T.C. Yuan, Phys. Rev. D 76, 117501 (2007).
[10] S.S. Gershtein, A.K. Likhoded and G.P. Pronko, arXiv:0709.2058 [hep-ph].
[11] D.V. Bugg, arXiv:0802.0934 [hep-ph].
[12] C.F. Qiao, arXiv:0709.4066 [hep-ph].
[13] E. Braaten and M. Lu, arXiv:0712.3885 [hep-ph].
[14] Y. Li, C.D. Lu and W. Wang, Phys. Rev. D 77, 054001 (2008).
[15] X.H. Liu, Q. Zhao and F.E. Close, arXiv:0802.2648 [hep-ph].
[16] X. Liu, B. Zhang and S.L. Zhu, Phys. Rev. D 77, 114021 (2008), arXiv:0803.3270 [hep-ph].
[17] T. Matsuki, T. Morii and K. Sudoh, arXiv:0805.2442 [hep-ph].
[18] PANDA Collaboration, Technical Progress Report for PANDA, http://www-panda.gsi.de.
[19] T. Barnes and X. Li, Phys. Rev. D 75, 054018 (2007).
[20] G.Y. Chen and J.P. Ma, arXiv:0802.2982 [hep-ph].
[21] M.K. Gaillard, L. Maiani and R. Petronzio, Phys. Lett. B 110, 489 (1982).
[22] A. Lundborg, T. Barnes and U. Wiedner, Phys. Rev. D 73, 096003 (2006).
[23] Z. Lin, C.M. Ko and B. Zhang, Phys. Rev. C 61, 024904 (2000); B. Holzenkamp et al. Nucl.Phys. A 500, 485 (1989).
[24] T. Hahm, FormCalc 5 Users Guide, http://www.feynarts.de/formcalc.
[25] W.M. Yao et al., Particle Data Group, J. Phys. G 33, 1 (2006).