Cryogenic power sensor enabling broad-band and traceable measurements

J.-P. Girard\textsuperscript{1, †, *}, W. Liu\textsuperscript{1,2, †, *}, R. Kokkonemi\textsuperscript{1,2†}, E. Visakorpi\textsuperscript{1}, J. Govenius\textsuperscript{1,3}, and M. Möttönen\textsuperscript{1,3}

\textsuperscript{1}QCD Labs, QTF Centre of Excellence, Department of Applied Physics, Aalto University, P.O. Box 13500, FIN-00076 Aalto, Finland.
\textsuperscript{2}IQM, Keilaranta 19, FI-02150 Espoo, Finland
\textsuperscript{3}VTT Technical Research Centre of Finland Ltd. & QTF Centre of Excellence, P.O. Box 1000, 02044 VTT, Finland.

\textsuperscript{†}These authors contributed equally.

\textsuperscript{*}Corresponding author email: jean-philippe.girard@aalto.fi; wei@meetiqm.com

Recently, great progress has been made in the field of ultrasensitive microwave detectors, reaching even the threshold for utilization in circuit quantum electrodynamics (cQED). However, these cryogenic sensors lack the ability to perform broad-band metrologically traceable power absorption measurements, which limits their scope of applications. Here, we demonstrate such measurements using an ultralow-noise nanobolometer supplemented by an additional direct-current (dc) input. The tracing of the absorbed power relies on comparing the response of the bolometer between radio frequency (rf) and dc heating powers traced through the Josephson voltage and quantum Hall resistance. To illustrate this technique, we demonstrate a fast calibration process of an attenuated input line over more than nine octaves of bandwidth with an rf heating power of \(-114\) dBm and uncertainty down to 0.33 dB.
Historically, the radio frequency (rf) power has been defined by a substitution to electrical dc standards, the Josephson voltage standard and the quantum Hall resistance standard, and therefore relies on calorimetric techniques. The International System of Units (SI) was redefined in 2018 through the fundamental physical constants, which has inspired traceable measurements of rf power using radiation pressure and Rydberg atom sensors. According to this redefinition, also the above-mentioned dc substitution provides a traceable path to the SI. It has been a common method for measuring microwave power in room temperature bolometers. It has also been utilized to calibrate low-temperature bolometers, for example, for X-ray detection and astronomical instruments. In astronomy, detectors may be calibrated using atmospheric models and reference black-body sources such as Mars. In the optical regime where single-photon detection is relatively standard due to the high photon energy, the detectors can be calibrated using, for example, heralded photons. Bolometers used in particle detectors are typically calibrated using radioactive sources. However, the traceable calibration technique we use in this manuscript is, in principle, applicable in all of the above cases.

Traceability to the SI at low power, typically below $-70$ dBm, may be challenging, as it falls below the operating power range of typical commercial power sensors. Inconveniently, calibration of rf power in a cryostat with heavily attenuated lines is not straightforward because the high attenuation calls for characterization in segments, yet the components are physically inaccessible when the cryostat is cold. In addition, if one has only a room temperature power meter, the signal has to be taken back from cryogenic temperatures to the room temperature. Since the lines are typically heavily attenuated, this requires amplification and calibration of the...
gain of the amplification chain, which adds uncertainties and noise to the measurement. For example for this reason, we recently introduced a method of calibrating the gain of cryogenic amplification chains\cite{15}, but yet a cryogenic power sensor seems more appealing for measuring rf power at low temperatures.

Devices used in circuit quantum electrodynamics\cite{15,17} (cQED) call for accurate characterization of the ambient radiation and their microwave properties at ultralow power levels. Experiments in cQED are based on superconducting circuits that use nonlinear elements, i.e., qubits, which operate mainly between 4 and 20 GHz\cite{18} and at a low signal level to prevent unwanted artifacts such as alternating-current (ac) Stark shifts\cite{19}. In this context, there is a strong incentive to develop efficient and practical detectors operating in the microwave range. Although some detectors for itinerant microwave photons recently managed to reach single-photon sensitivity with efficiencies up to 96\%\cite{20,22}, they rely on discrete qubit transitions or on cavity-confined photons to facilitate detection\cite{23,24}. This limits signal amplitude calibration to a narrow relative bandwidth\cite{25} with the possibility of extending it to 1 GHz by observing a high-level ac Stark shift in a multilevel quantum system with a large frequency detuning\cite{25,27}, but this extension comes at the cost of reduced energy sensitivity. Another method\cite{28} enables absolute calibration of power over a gigahertz-wide frequency range by measuring the spectra of scattered radiation from a two-level system in a transmission line. In addition, in-situ characterization of qubit control lines in the megahertz regime has been implemented\cite{29} using a transmon qubit coupled to a readout resonator. Overall, it is especially challenging to characterize ultralow microwave power at unknown frequencies.

Thus, despite the great progress sped up by the race for a useful quantum computer, experiments in cQED usually lack tools to achieve reliable traceable power absorption measurements over a broad relative frequency band, which hinders the accuracy of other measurements and
hence introduces limitations for the operation characteristics of the corresponding devices. This state of the art justifies the development of new tools to fill the gap in implementing the standard for ultralow rf power. In this work, we introduce a traceable power absorption measurement based on dc substitution, and thus enable high-precision measurements and furthermore provide a simple way to calibrate microwave power in a cryogenic environment. For this purpose, we deem bolometers to be a promising candidate because of their minute heat capacity and negligible radiative losses. They can even achieve remarkable sensitivity, since recent studies suggest that ultralow-noise bolometers based on the Josephson effect may enable direct measurement of itinerant microwave photons in the framework of cQED\textsuperscript{30,31}.

Here, we present a low-noise microwave bolometer with a temperature readout based on proximity Josephson junctions supplemented by a dc heater. The electrical power substitution capability enables traceability to the SI (see Figure 1). In our bolometer, a diffusive normal-metal–gold-palladium (Au\textsubscript{x}Pd\textsubscript{1−x}) nanowire acts as the element absorbing incident microwave radiation. The microwave heater signal is provided by a room temperature signal generator through 50-Ω coaxial transmission lines with filters and attenuators on the way down to the base temperature, 10 mK, of the cryostat.

Once the signal reaches the sample, it is absorbed by the nanowire, increasing its temperature. Based on an experimental study of metallic wires with similar dimensions and at similar temperatures\textsuperscript{32}, we expect the quasiparticles to quickly relax and reach thermal quasiequilibrium at least within a nanosecond time scale. The absorber and the thermometer are strongly thermally coupled, so that they reach similar temperatures, but we note that an equal temperature for them is not even necessary to operate the device; the correct operation requires only
an identical response at the thermometer from the two different heating types.

The thermometer is based on superconductor–normal-metal–superconductor (SNS) junctions. The junctions, together with the on-chip shunt capacitor $C_1$, form an $LC$ oscillator with a temperature-dependent resonance frequency. The inductance arising from the SNS junctions provides a high temperature responsivity. A probe signal reflected from the $LC$ oscillator out of the gate capacitor $C_g$ is used to determine the resonance frequency. In addition, we galvanically connected two dc-coupled resistors to the vicinity of the rf absorber. This allows us to heat the absorber with an accurately applied direct current. Comparing the response of the thermometer to rf heating with that to dc heating allows us to trace the unknown incident rf power to the SI through the Josephson voltage and quantum Hall resistance standards that are used to calibrate our dc setup.

To measure microwave power at room temperature, a typical experiment using a low-temperature detector requires an accurate calibration of the attenuation of the rf line, through which the heater signal is applied. Figure 2 shows a detailed diagram of our experimental setup. Multiple attenuators, amplifiers, filters, and directional couplers are placed between the sample and the instruments to enhance the signal-to-noise ratio. The specifications provided for the commercial devices gives a rough estimate of the attenuation, but a more careful investigation is needed since many of the components are specified to work at or near room temperature. Attenuations can be measured with a vector network analyzer (VNA), but this can be difficult with heavily attenuated rf lines. Furthermore, VNA measurement necessarily includes attenuation, or gain, of the return path. This can be circumvented to some extent by measuring the gain of the return path with the so-called $Y$-factor technique. Unfortunately, such
Figure 1: Sample and high-level measurement setup. (a) False-color optical image of the bolometer with a dc heater. The electrical current is applied to the nanowires through a dc-heater line (red), and the sample is grounded through aluminum leads (blue). (b) Circuit diagram of the measurement setup indicating the shunt capacitor $C_1$ and gate capacitor $C_g$ also shown in (a) in purple. (c) False-color scanning-electron-microscope image of the detector. The long junctions operating as resistive power absorbers are 1-µm long, and the short junctions operating as the thermometer are 300-nm long. The SNS junctions operating as the thermometer are formed by a AuPd nanowire galvanically connected to superconducting Al islands (green) and leads. The absorber is either heated by the microwave heater signal or by the dc heater. (d) Fraction of the probe power reflected at the gate capacitor as a function of probe frequency for heater powers of $-80$ dBm (blue) and $-83$ dBm (orange) at the absorber. An increase in the heater power induces a redshift of the resonance frequency.
a result is valid only for the specific setup and thus a typical uncertainty of 1 dB (26%) occurs at a power level of $-130$ dBm (see supplemental material of ref. 35). Furthermore, since thermal cycles may affect the components, this calibration needs to be carried out at each cooldown, regardless of whether the setup was modified or not.

The presence of a dc heater at the bolometer enables us to bypass the issue discussed above. Instead of a full setup calibration, we only need to determine the response of the bolometer as a function of the dc heater power. Note that this method can be applied without changing the cryostat base temperature. Thus, the result serves as a reference to determine the amount of microwave power absorbed.

In the following, we present in detail our calibration method. First, we measure the transmission coefficient $S_{21}$ of the probe signal, from source to digitization, with finite dc heating such that the resonance shifts by approximately 1 MHz with respect to the zero-bias case. Next, we measure the transmission coefficient over a range of probe frequencies $f_p$ with considerable output power of the rf heater signal generator and calculate the sum of the squared differences to the dc heating transmission, i.e., the residual as

$$R_0 = \sum_{f_p} \left\{ (\text{Re}(S_{21}^{dc}(f_p))) - \text{Re}(S_{21}^{rf}(f_p)) \right\}^2 + \left\{ \text{Im}(S_{21}^{dc}(f_p)) - \text{Im}(S_{21}^{rf}(f_p)) \right\}^2$$

A small residual indicates nearly equal absorbed power induced by the rf and dc heating sources. In Figure 3, we show the residual as a function of the applied rf power to the heater line. We find the rf power corresponding to the dc-heater power by fitting a third-order polynomial to the data near the minimum. We repeat such measurements many times to estimate the statistical uncertainty of the measurement.

We find strong frequency dependence in the attenuation of the heater line due to the
Figure 2: Main experimental setup for a traceable power absorption measurement with a bolometer equipped with a dc heater. The sample is placed inside a radiation shield to mitigate external noise. Thermalization of the lines going down to the base temperature of the cryostat is ensured by the presence of additional cable length (spiral shape) between each temperature stage.
Figure 3: Comparison of transmission curves used to deduce the equivalence between dc and absorbed heating power. (a) Example of the sum of squared differences of the complex-valued transmission coefficients $S_{21}$ for an applied dc heater power of $-114.03$ dBm and as a function of the rf heater power generated by a signal generator at a 7-GHz frequency. The minimum of the fitted third-order polynomial function is achieved at 8.77 dBm. Thus, the attenuation at 7 GHz is $-122.80$ dB, of which 0.55 dB arises from the imperfect rf absorber impedance matching to the 50-Ω transmission line. Inset: Phase of the $S_{21}$ curves for dc and rf heating for the smallest value of the sum of the squared difference of $S_{21}$. (b) Attenuation of the rf-heater line as a function of heater signal frequency. The strong frequency dependence is due to a Thermocoax cable used in the setup. The error bars are smaller than the marker size.
presence of a 25-cm Thermocoax cable as shown in Figure 3a. In addition, we have frequency-dependent attenuation arising from other coaxial cables in the heater line. However, at low frequencies at which the cable losses become small, we find that the attenuation approaches 80 dB, which corresponds to the cumulative nominal value of the attenuation present in the setup. The $1\sigma$ confidence interval of the type A uncertainty of the measured attenuation is below 0.1 dB for approximately 10 min of measurement time.

In addition to the noise in the measurement, we have systematic error arising from the calibration of our instruments. The current bias is provided by a Stanford Research Systems SIM 928 voltage source with nominal 0.5-GΩ resistors in both dc lines. The total measured resistance is 0.9955 GΩ ± 0.6 MΩ. The voltage source has an uncertainty of 160 ppm. The dc lines and the on-chip dc heater resistors have resistances well below 1 kΩ. Thus, their effect on bias current can be neglected without loss of accuracy.

We measured the resistance of the dc-heater resistors in a four-probe configuration using an Agilent digital multimeter model 34410A as a voltmeter. The voltage was amplified with a Femtoamp DLPVA-100 voltage amplifier, and the nominal 60-dB gain was measured to be 60.02 dB ± 0.0013 dB. The resistance of the heater $R_{\text{heater}}$ determined from the slope of the measured current-voltage curve yields 48.8 ± 0.2 Ω. Thus, the dc power reaching the bolometer has an uncertainty of approximately 0.04 dB, which is limited by noise and the uncertainty of the dc-heater resistance.

By a large margin, the dominant error in the measured attenuation arises from the signal generator used for rf heating. Its output power has an uncertainty of 0.33 dB. There is also a sharp drop of roughly 2 dB in the output power for frequencies above 100 MHz. We have
taken this into account in the calibration of the signal generator. However, we note that this uncertainty only affects the attenuation measurement, not the measurement of the absorbed rf power at cryogenic temperatures. Calibration details are provided in the Methods.

Alternatively, we can calibrate the rf heater power by measuring the resonance frequency with multiple dc heater currents and output powers of the rf-heater signal generator as shown in Figure 4. Finding the offset between the dc and rf heater powers that minimizes the difference between the resonance frequencies measured with the two heating methods yields the attenuation of the heater line. A similar method can be applied to quantities other than the resonance frequency. For example, we can measure the transmission coefficient at a given frequency as a function of dc and rf heater powers. In this case, the attenuation of the rf line is the offset power that minimizes the difference between the two transmission coefficients. However, we deem these alternative methods to be less robust since they do not utilize the full information on the resonance. Nevertheless, such a method could be greatly faster than the one we use above, and remains an appealing line of future research.

To characterize our cryogenic power sensor in more detail, we define and measure the noise equivalent power (NEP) as in our previous work by first measuring the noise spectrum of the output and then dividing it by the quasistatic responsivity of the detector. We further take into account the frequency dependence of the responsivity by dividing the quasistatic responsivity by a factor $\sqrt{1 + (2\pi f_n \tau)^2}$, where $\tau$ is the measured time constant of the detector, and $f_n$ is the noise frequency. This procedure yields the noise of the bolometer output in units of the input power, which we take as the definition of NEP. The experimental data shown in
**Figure 4**: Alternative method to deduce the attenuation of the heater line. (a) Phase of the reflection coefficient $\Gamma$ as a function of probe power and rf heater power and (b) of the dc heater power. The position of the resonance frequency is depicted by the red line. (c) Comparison of the position of the resonance frequency between the dc and rf heater power sources at $f_h = 5$ GHz. In this example, the best match between the two curves is obtained when considering $-114.3$ dB attenuation along the rf-heater line.

Figure 5 indicates that the lowest measured NEP is roughly $2 \text{ aW}/\sqrt{\text{Hz}}$, together with a 20 µs thermal time constant. For power sensing, the thermal time constant suggests a maximum repetition rate on the order of ten kilohertz. To measure, for example, an incoming power of 10 fW at an uncertainty of 1%, one needs an integration time of 0.2 ms according to the measured NEP, and is thus not limited by the thermal time constant.

Using the method we presented, one needs to carry out several measurements of the reflection coefficient to measure the attenuation of cabling. In our current experiment we sweep the rf heater power and compare it to a single dc heater power. Alternatively, to determine the power of an unknown signal, one can use calibration data for the reflection coefficient at different dc heater powers. Note that if one has obtained beforehand such calibration data, then the total measurement time is limited to the acquisition of a single trace of the transmission
Figure 5: (a) Spectral density of voltage noise measured for the bolometer probe signal. (b) Time trace of the in-phase (blue) and quadrature (red) voltage of the measured probe signal for the heater power turned on at the time $t = 2$ ms and turned off at $t = 7$ ms. The green line denotes a single-exponential fit of the in-phase part. (c) Noise equivalent power (NEP) of the bolometer as a function of noise frequency.

The measured NEP is roughly two orders of magnitude greater than that previously reported\textsuperscript{37} on similar sample without a dc heater. This difference can be explained by the larger volume of the metal in the dc bolometer and, perhaps more importantly, by the presence of two dc lines inducing additional noise in the sample.

Conclusion

In summary, we developed a sensitive bolometer device for traceable microwave power absorption measurements relying on dc substitution. The device operates at low temperatures, exhibits a broad input bandwidth and is suitable for characterization of devices operating in the framework of cQED. As an illustration of the utility of the introduced microwave sensor, we
demonstrated the calibration of a heavily attenuated rf line including several microwave components at low temperatures. We achieved this by comparing the response of the bolometer to a heater signal applied through the rf line and to heating applied through a dc line. From this comparison, we accurately determined the frequency-dependent attenuation of the rf line with an uncertainty of 0.33 dB for an input power of $-114$ dBm. We note that this method potentially leads to a substantial time reduction in setup characterization compared with the so-called Y-factor method that requires one to change the temperature of a resistor and consequently to wait for potentially slow saturation of the thermal relaxation. This work aims to facilitate the implementation of bolometers in experiments on cryogenic electronics as traceable power sensors and to enable highly accurate power measurements. From the noise performance of the device, we observed that the sample would benefit from improved shielding from dc line noise, and in future experiments, we plan to use such a bolometer for microwave signal calibration in qubit experiments.

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Methods

Calibration of instruments

The voltmeter and the amplifier were calibrated against a Fluke 5440B voltage calibrator, which in turn was recently calibrated by VTT MIKES and is traceable to the Josephson voltage standard. Next, a calibrated voltmeter was used to verify the output voltage of the SIM 928 voltage source. The resistances of the bias resistors were measured with an Agilent 34410A multimeter using a four-probe configuration. The multimeter resistance reading was calibrated against the Measurements International model 4310H resistance standard. The output power of the Gigatronics 2550B signal generator used as the rf heater source was calibrated using an Agilent N1913A power meter, which in turn was recently calibrated at VTT MIKES.

Uncertainty analysis

The uncertainty estimates of the different instrument calibrations and of the derived quantities are given in Table 1. We assume uncertainties to be independent, and hence we added them in squares for derived quantities. Furthermore, we transformed between the linear and logarithmic uncertainties using equation

\[
\text{Logarithmic uncertainty} = 10 \log_{10}(e) \times \text{Linear relative uncertainty},
\]

where \(e\) is the Euler’s number.
| Instrument or derived quantity | Linear uncertainty (%) | Logarithmic uncertainty (dB) |
|-------------------------------|------------------------|-----------------------------|
| SIM 928                       | 0.016                  | 0.0007                      |
| Voltmeter                     | 0.003                  | 0.0001                      |
| Amplifier gain @ 60 dB        | 0.4                    | 0.02                        |
| 1-GΩ reference resistor       | 0.04                   | 0.0017                      |
| 1-GΩ resistor                 | 0.06                   | 0.0026                      |
| Gigatronics 2550B             | 7.6                    | 0.33                        |
| dc heater power               | 0.4                    | 0.02                        |
| Attenuation                   | 7.6                    | 0.33                        |

Table 1: Uncertainty estimates of the different instrument calibrations

**Author Contributions**

R.K., J.-P. G. and W.L. conducted the measurements and analysis. W.L. designed the mask and fabricated the samples. The original bolometer design was made by J.G and the resistor was added to the design by R.K. and W. L. E. V. performed initial characterizations together with R. K. M.M supervised the work.

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Supporting Information

Sample fabrication A high-resistivity ($\rho > 10$ k$\Omega$ cm) 4 inch silicon wafer (100) was covered by a 300 nm thermal oxide SiO$_x$ and sputtered with a 200 nm of pure Nb. First, the waveguide was insolated in a Karl Suss MA-6 mask aligner using AZ5214E photoresist in positive mode with hard contact. After development, the sample was etched with Plasmalab 80Plus Oxford Instruments reactive ion etching (RIE) system. The plasma works with a gas flow of SF$_6$/O$_2$ at 40 sccm/20 sccm with a rf-field power of 100 W. After etching, the resist residuals were cleaned in an ultrasonic machine with acetone and IPA and dried with a nitrogen gun. Next, a thin film dielectric layer Al$_2$O$_3$ (45nm) was grown by atomic layer deposition (ALD) in a Beneq TFS-500 system. The dc heater dielectric layer was insolated using AZ5214E and wet-etched with an ammonium fluoride – hydrofluoric acid mixture. Then, the 4 inch wafer was cleaved into a $2 \times 2$ cm$^2$ chip by Disco DADdy. Second, the nanowire was patterned by EPBG5000pES electron beam lithography (EBL) with a bilayer of MMA/PMMA resist on a single chip. The 30 nm thick AuPd layer was deposited in e-beam evaporator at a rate of 0.5 Å/s. After liftoff in acetone overnight, the dc-heater and island were patterned by EBL and then deposited with 100 nm Al at a rate of 5 Å/s. Finally, each pixel ($5 \times 5$ mm$^2$) was cleaved by a laser micromachining
Supplementary Fig. S1: Voltage drop across the dc heater consisting of two AuPd nanowires as a function of applied current at 10 mK.

system and packaged with Al bonding wires.

Nanowire resistance We measured the resistance of both dc nanowires at 10 mK with a 4-point probe method (see Figure S1) with 2 s integration time and 5 averages. We obtained a resistance of 48.8 Ω.

EDX analysis We evaluated the chemical composition of the AuPd nanowires by performing energy dispersive X-ray (EDX) analysis. We found the nanowire composition to be Au\textsubscript{x}Pd\textsubscript{1−x} with \(x \approx 0.58\). The study was carried out on identical samples fabricated in a different batch. Figure S2 shows the spectrum X-ray energy versus counts. The peaks at 2.121 keV and 2.838 keV correspond to Au and Pd, respectively and the electron beam energy was set to 5 keV during analysis.

Thermal conductance Similar to ref\textsuperscript{38}, one can estimate the thermal conductance in the SNS junctions \(G_{\text{SNS}}\) and compare it to the thermal conductance to the cryostat phonon bath \(G_b\). To quantify this heat transfer, we considered the ratio between the superconducting aluminum is-
Supplementary Fig. S2: Energy-dispersive X-ray spectroscopy of the AuPd nanowires.

Land length $L_s$ and the superconducting coherence length $\xi_0 = \sqrt{\hbar D_s/\Delta_0}$. We obtained a ratio of $l_s \approx 1$ by considering the Al bulk energy gap at zero temperature $\Delta_0 = 200 \mu$eV, the diffusion constant of the superconductor $D_s = 50 \text{ cm}^2/\text{s}$, $L_s = 300 \text{ nm}$ and $\hbar$ the reduced Planck’s constant. According to ref[38], under these conditions $G_{\text{SNS}}$ can be approximated by the Wiedemann–Franz value $L_0 G_N T$, with $L_0$ the Lorentz number and $G_N$ the normal state electrical conductance. This yielded a thermal conductance of $G_{\text{SNS}} = 0.4 \text{ nW/K}$, which is approximately 6 orders of magnitude larger than the thermal conductance to the cryostat phonon bath that we previously reported on an otherwise similar device without a dc heater[39]. Therefore we expect the chain of SNS junctions to be strongly thermally coupled to the rf absorber and dc heater.