Bose-Einstein Condensates as a Probe for Lorentz Violation

Don Colladay and Patrick McDonald

New College of Florida

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The effects of small Lorentz-violating terms on Bose-Einstein condensates are analyzed. We find that there are changes to the phase and shape of the ground-state wave function that vary with the orientation of the trap. In addition, spin-couplings can act as a source for spontaneous symmetry breaking in ferromagnetic condensates making them sensitive probes for fundamental symmetry violation.

I. INTRODUCTION

Recently there has been much interest in searching for miniscule violations of Lorentz symmetry in nature. These effects may arise from more fundamental theories that underly the standard model. Bose-Einstein condensates have provided a rich testing ground for large, coherent quantum mechanical systems. By now, condensates have been successfully produced using a number of atomic species in different spin states using a variety of trapping techniques. It is the goal of this paper to analyze the general effects of small Lorentz-breaking terms on these condensates.

Tests of Lorentz symmetry have been performed in a wide variety of physical systems. For example, various experiments utilizing mesons, baryons, electrons, photons, and muons have reached a precision that probes Lorentz-violating parameters at Planck-suppressed scales. Recent analysis has also been extended to the neutrino sector, instantons, supersymmetric models, and the gravitational sector.

A framework for including Lorentz-breaking effects into low-energy field theory is provided by the Standard Model Extension (SME). The SME uses the concept of spontaneous symmetry breaking to generate couplings between standard model fields and vacuum expectation values of tensor fields that parameterize the symmetry violations. In this paper, a more tractable version of the SME is used that includes various restrictions on the possible couplings that preserve gauge invariance and power counting renormalizability. This restricted theory has been shown to preserve microcausality in concordant frames where the Lorentz-violating terms are reasonably small.

Previous related work involves an analysis of statistical mechanics in the presence of Lorentz violation and provides the formal thermodynamic techniques used in the current paper. One important modification is the explicit inclusion of the confining potential, necessary for describing a condensate wave function.

The paper is organized as follows: In section II, the simplified case of spin-0 bosons in a condensate is studied. This gives the basic effects on the spin-independent part of the wave function. Section III generalizes the system to spin-polarized hydrogen (in the absence of interactions), a relatively simple physical boson used in experiments. Section IV includes the effects of interactions and the more complex atoms used in the majority of experiments. Section V generalizes to optically-trapped condensates with multiple possible spin components.

II. NONINTERACTING SPIN-0 BOSONS

As a first step, we consider the case of noninteracting spin-0 bosons in a harmonic trap (interactions and spin effects are discussed in later sections). A free spin-0 boson gas in the presence of Lorentz violation may be modelled using the effective hamiltonian:

\[ H = \frac{p^2}{2m} + A + C_j \frac{p_j p_j}{m} + F_{jk} \frac{p_j p_k}{2m} + V_{\text{trap}}, \]

where \( V_{\text{trap}} \) is the trapping potential. The parameters \( A, C, \) and \( F \) are effective Lorentz violation parameters for the boson. They are expressible in terms of fundamental violation parameters of the electrons, protons, and neutrons, if the explicit wave function for the boson is known.

In a previous paper, a calculation of the properties of a low-temperature Bose gas was performed and the trapping potential was taken as a standard particle in a box. When a condensate is present, detailed knowledge of the trapping potential is required to calculated the ground state wave-function. For a large class of magnetic and optical traps, the trapping potential takes the form of a harmonic oscillator:

\[ V_{\text{trap}} = \frac{1}{2} m \sum_i (\omega_i x_i)^2. \]

The ground state wave function plays a special role, therefore it is useful to first study some of its properties. Without Lorentz violation, the unperturbed ground state takes the standard form:

\[ \psi_{\text{GS}}^0 = \left( \frac{m \omega_{\text{trap}}}{\pi \hbar} \right)^{3/4} \exp \left( -\frac{m}{2 \hbar} \sum_i \omega_i x_i^2 \right). \]
where \( \omega_{ho} = (\omega_1 \omega_2 \omega_3)^{1/3} \). The unperturbed energy for the ground state is

\[
E_{GS}^0 = \frac{1}{2} \hbar (\omega_1 + \omega_2 + \omega_3) \quad .
\]  

The first order correction to the energy can easily be found using standard first-order perturbation theory as

\[
E_{GS}^1 = \langle \psi_{GS}^0 | H' | \psi_{GS}^0 \rangle = A + \frac{1}{4} \hbar \sum_i F_i \omega_i \quad .
\]  

Similarly, the first-order correction to the ground-state wave function is found using

\[
\psi_{GS}^1 = \sum_{n \neq 0} \frac{\langle \psi_{GS}^0 | H' | \psi_{GS}^0 \rangle \psi_{n}^0}{E_{GS}^0 - E_{n}} \quad ,
\]  

where \( \psi_{n}^0 \) are the standard unperturbed states that can be written in terms of appropriate Hermite polynomials and exponentials. Only a few of the matrix elements in the sum are nonzero. The resulting corrections are calculated first for the \( C \)-terms as

\[
\psi_{GS}^{(C)} = -i \left( \frac{1}{2 m \hbar} \right)^{1/2} \left[ C_1 \omega_1^{-1/2} \psi_{100}^0 + C_2 \omega_2^{-1/2} \psi_{010}^0 + C_3 \omega_3^{-1/2} \psi_{001}^0 \right]
= \frac{i}{\hbar} \vec{C} \cdot \vec{x} \psi_{GS}^0 \quad .
\]  

In fact, the exact solution for this case is given by

\[
\psi_{GS}^{(C)} = \exp \left( - \frac{i}{\hbar} \vec{C} \cdot \vec{x} \right) \psi_{GS}^0 \quad ,
\]  

indicating that the sole effect is to introduce a position-dependent phase shift into the ground state. Such a term could contribute to the pattern observed in condensate interference experiments.

The corrections due to the \( F \)-terms may be handled using the same procedure. The resulting first-order correction for the ground state is

\[
\psi_{GS}^{(F)} = \frac{1}{2} \left[ \left( \frac{1}{2 \sqrt{2}} (F_{11} \psi_{200}^0 + F_{22} \psi_{020}^0 + F_{33} \psi_{002}^0) \right)
+ \frac{(\omega_1 \omega_2)^{1/2}}{\omega_1 + \omega_2} F_{12} \psi_{110}^0 + \frac{(\omega_2 \omega_3)^{1/2}}{\omega_2 + \omega_3} F_{23} \psi_{011}^0
+ \frac{(\omega_1 \omega_3)^{1/2}}{\omega_1 + \omega_3} F_{13} \psi_{011}^0 \right] .
\]  

Substitution of the unperturbed states yields the explicit form

\[
\psi_{GS}^{(F)} = \frac{m}{2 \hbar} \sum_{i,j} \frac{\omega_i \omega_j}{\omega_i + \omega_j} F_{ij} x_i x_j \psi_{GS}^0 \quad .
\]  

This shows that the condensate shape takes the form of a perturbed ellipsoid.

The Fourier transform gives the momentum-space wave function as

\[
\phi_{GS}(\vec{p}) = \left( \frac{1}{\pi \hbar m \omega_{ho}} \right)^{3/4} \left( 1 + \frac{1}{8} Tr(F) \right) \otimes 
\exp \left( - \frac{1}{2m \hbar} \left[ \sum_i \frac{p_i^2}{\omega_i} + \sum_{i,j} F_{ij} \frac{p_i p_j}{\omega_i + \omega_j} \right] \right) .
\]  

This formula provides the momentum distribution of the particles in the condensate. If the trapping potential is suddenly turned off, the velocity distribution can be measured and compared with the above formula. Unfortunately, current shape sensitivity is only at the 1% level and is unlikely to yield interesting bounds on Lorentz violation parameters in the near future. We now turn to the case of finite temperature and analyze the particle distribution.

Employing the notation of [27] the associated grand partition function is

\[
\ln Z_G = - \sum_n \ln(1 - e^{-\alpha e^{-\beta E_n}})
- \ln(1 - e^{-\alpha e^{-\beta E_{GS}}}) \quad ,
\]  

where \( \alpha = - \beta \mu \) is defined in terms of the chemical potential, and the ground state has been separated out to allow for Bose-Einstein condensation at low temperatures. In order for a large number of atoms to condense into the ground state, the chemical potential must be very close to the ground state energy. It is therefore convenient to define \( \mu = E_{GS} - \epsilon \), where \( \epsilon \) is a small parameter.

Approximating the sum as an integral and taking the limit as \( \epsilon \) gets small gives the result

\[
\ln Z_G = (1 - \frac{1}{2} Tr(F)) \left( \frac{k T}{\hbar \omega_{ho}} \right)^3 I_4(\epsilon) + \ln(1 - e^{-\beta \epsilon}) \quad ,
\]  

where

\[
I_\nu(\epsilon) = \frac{1}{\Gamma(\nu)} \int_0^\infty dx \frac{x^{\nu-1}}{e^{-\beta \epsilon + x} - 1} \quad ,
\]  

is an integral that reduces to the Riemann Zeta function \( I_\nu(\epsilon) \rightarrow \zeta(\nu) \) in the limit \( \epsilon \rightarrow 0 \). Note that the thermally distributed particles are only affected by the rotationally invariant parameter \( Tr(F) \) as is expected from equipartition of energy.

The expected number of particles in excited states can be found by differentiation of the first term in the partition function with respect to the parameter \( \alpha = - \beta \mu \). The result is

\[
\langle N \rangle - \langle N_0 \rangle = (1 - \frac{1}{2} Tr(F)) \left( \frac{k T}{\hbar \omega_{ho}} \right)^3 I_3(\epsilon) \quad .
\]  

The number of particles in the ground state can be found by differentiation of the second term in the partition function. The result is

\[
\langle N_0 \rangle = \frac{1}{e^{\beta \epsilon} - 1} \quad .
\]
When the condensate is present, \( \langle N_0 \rangle \approx kT/\epsilon \) must be large.

The critical temperature for condensation can be found by setting the number of particles in excited states equal to the total number of particles in the limit \( \epsilon \to 0 \) yielding

\[
kT_c = \hbar \omega_{ho}(1 + \frac{1}{6} Tr(F)) N^{1/3} \zeta^{-1/3}(3). \tag{17}
\]

Combining the above results gives the relation

\[
\frac{N_0}{N} = 1 - (T/T_c)^3, \tag{18}
\]

showing that the fraction of atoms in the condensate (expressed in terms of \( T_c \)) is independent of the Lorentz-violating parameters. Note that systems with \( T \ll T_c \) are in an almost pure condensate state. These are the systems focused on in the remainder of the paper.

This completes the description of spin-0 condensates in the presence of Lorentz violation. Most actual experiments involve atoms with nontrivial total spin, significant interactions, or both. As a next step, the previous case is generalized to spin-polarized hydrogen.

### III. SPIN-POLARIZED HYDROGEN

Hydrogen provides a theoretically simple example of a physical Bose-Einstein condensate. Interactions are still neglected; they are in fact important and will be discussed in the next section. The Lorentz-violating terms in the hamiltonian for the system can be taken as a simple sum of the electron and proton terms. The momentum terms for the electron and proton may be written in terms of the total momentum \( \vec{p} \) and the relative momentum \( p_r \) in the standard way

\[
\vec{p}_e = \frac{\mu_e}{m_e} \vec{p} + \vec{p}_r, \quad \vec{p}_p = \frac{\mu_p}{m_p} \vec{p} - \vec{p}_r, \tag{19}
\]

where \( \mu_r \) is the reduced mass. The part of the hamiltonian that is relevant for condensate corrections is

\[
H_{LV} = A^{(e)} + A^{(p)} + B_j^{(e)} \sigma_j^{(e)} + B_j^{(p)} \sigma_j^{(p)} + \left(C_j^{(e)} + C_j^{(p)} + D_j^{(e)} \sigma_j^{(e)} + D_j^{(p)} \sigma_j^{(p)} \right) \frac{P_j}{M} + \left(F_{jk}^{(e)} + F_{jk}^{(p)} + G_{jk}^{(e)} \sigma_j^{(e)} + G_{jk}^{(p)} \sigma_j^{(p)} \right) \frac{P_j P_k}{2M} \tag{20}
\]

consisting of the expressions that couple to the total atomic momentum. In this expression, \( M \) is the mass of the atom and the superscripts \( (e) \) and \( (p) \) denote the parameters for the electron and proton. The SME parameters for Lorentz violation \([31]\) have been collected as

\[
A = (a_0 - m c_{00} - m c_0), \tag{21}
\]

\[
B_j = (-b_j + m d_{j0} - \frac{1}{2} m c_{jkl} g_{k0} + \frac{1}{2} \epsilon_{jkl} H_{kl}), \tag{22}
\]

\[
C_j = [a_j - m(c_{0j} + c_{j0}) - m c_j], \tag{23}
\]

\[
D_{jk} = \left[ -b_0 \delta_{jk} + m(d_{kj} + d_{jk}) + m c_{klm} (\frac{1}{2} g_{mij} + g_{m00} \delta_{ij}) + \epsilon_{jkl} H_{kl} \right], \tag{24}
\]

\[
F_{jk} = -2 \left[ (c_{j} + \frac{1}{2} \epsilon_{000} \delta_{jk}) \right], \tag{25}
\]

\[
G_{jkl} = \left[ (d_{ij} + d_{ji}) - \frac{1}{2} (b_j/m + d_{j0} + \frac{1}{2} \epsilon_{jmn}(g_{m0n} + H_{mn}/m)) \right] \delta_{kl} + \frac{1}{2} (b_i/m + 2 \epsilon_{jmn} g_{mn0}) \delta_{jk} - \epsilon_{jlm} (g_{m0k} + g_{mn0}) \right]. \tag{26}
\]

Note that if the trap selects out the singlet configuration, the system would be equivalent to the spin-0 case discussed in the previous section.

In order to trap hydrogen magnetically, it must be in the triplet spin configuration. For example, suppose the corrections can be expressed in terms of fundamental SME parameters. This calculation neglects interactions that play a significant role in this system. These are discussed in the next section.

### IV. MORE COMPLEX ATOMS AND INTERACTIONS

Most traps use more complicated atoms, such as \(^7\)Li, \(^{23}\)Na, and \(^{87}\)Rb. The hamiltonian given in Eq. (20) can be extended by formally performing a sum over all constituent particles \([32]\). In practice, the resulting hamiltonian is unwieldy and certain approximations must be made to obtain tractable results. These three commonly utilized atoms have a nuclear spin of 3/2 with a single valence electron. It is therefore possible to magnetically trap them in a spin-1 or spin-2 state. The detailed contribution of the various particle types to the ground-state corrections will depend on the specific nuclear model used. One approach is to adopt a Schmidt model in which all of the nuclear spin is attributed to a single unpaired nucleon. For the atoms listed above, the
unpaired nucleon is a proton, indicating that these atoms are particularly sensitive to proton violation parameters (this will be discussed further in the section regarding optical traps where spin-couplings are important). In addition to more complicated hamiltonians, these atoms have significant interactions that we will now discuss.

In the conventional case with no Lorentz violation, at low energies and densities relevant to the condensate, the two-body interactions may be incorporated using a single parameter $a$, called the scattering length. The arguments leading to the above conclusion do not depend on the specific details of the potential between the atoms. This can be seen by looking at the Born approximation for the scattering amplitude at low energies (called the scattering length)

$$ a \simeq -\frac{m}{4\pi\hbar^2} \int V(\vec{r})d^3\vec{r}. \quad (28) $$

As a result, any Lorentz-violating effects in the interaction potential will be absorbed into the definition of the scattering length.

The second-quantized hamiltonian may be written as

$$ \hat{H} = \int d^3\vec{r} \psi^\dagger(\vec{r}) \left[ -\frac{\hbar^2}{2m} \nabla^2 + H_{\text{LV}} + V_{\text{trap}}(\vec{r}) \right] \psi(\vec{r}) + \frac{i}{2} \int d^3\vec{r} d^3\vec{r}' \psi^\dagger(\vec{r}) \psi^\dagger(\vec{r}') V(\vec{r} - \vec{r}') \psi(\vec{r}) \psi(\vec{r}'), \quad (29) $$

where $H_{\text{LV}}$ is the Lorentz-violating piece of the hamiltonian and $V(\vec{r})$ is the interatomic potential. This potential may be replaced by the effective interaction

$$ V(\vec{r}) = \frac{4\pi\hbar^2a}{m} \delta^3(\vec{r}), \quad (30) $$

because it produces the same scattering behavior as the full potential at low energies and densities. The bosonic field operators may be expanded about the condensate wave function $\Phi$ as $\Psi(\vec{r},t) = \Phi(\vec{r},t) + \Psi'(\vec{r},t)$, yielding the modified Gross-Pitaevskii equation for the condensate

$$ i\hbar \frac{\partial}{\partial t} \Phi = \left[ -\frac{\hbar^2}{2m} \nabla^2 + H_{\text{LV}} + V_{\text{trap}} + \frac{4\pi\hbar^2a}{m} |\Phi|^2 \right] \Phi. \quad (31) $$

The time dependent piece in the context of mean field theory is given by $\Phi(\vec{r},t) = \phi(\vec{r}) \exp(-i\mu t/\hbar)$ in terms of the chemical potential $\mu$ as a result of Anderson’s equations $30$. This equation is nonlinear and generally must be solved numerically, however, the Thomas-Fermi limit is relevant for most experiments for which the interaction energy is much larger than the kinetic energy over the bulk of the condensate. In this limit, the kinetic terms are neglected and the only unsuppressed contribution from the Lorentz-violating terms comes from the momentum independent spin couplings. In the case of a strong external magnetic trapping field, the condensate will consist of a single spin-component, and the density is given by

$$ n(\vec{r}) = \phi^2(\vec{r}) = \frac{m}{4\pi\hbar^2a}(\mu - E_{\text{LV}} - V_{\text{trap}}(\vec{r})), \quad (32) $$

where $E_{\text{LV}} = \langle \phi | H_{\text{LV}}^{\text{indep}} | \phi \rangle$ is the expectation value of the momentum-independent terms in the Lorentz-violating hamiltonian. The field $\phi$ is normalized to the total number of particles in the condensate such that $\int d^3 r \phi^2 = N_0$, implying that

$$ \mu - E_{\text{LV}} = \frac{\hbar^2a_{\text{ho}}}{2} \left( \frac{15N_0}{a_{\text{ho}}} \right)^{2/5}, \quad (33) $$

where $a_{\text{ho}} = (\hbar/m\omega_{\text{ho}})^{1/2}$ corresponds to the average width of the free-particle condensate solution. This means that the Lorentz-violating terms may be effectively absorbed into the chemical potential and therefore do not affect the bulk properties of the condensate.

V. OPTICAL TRAPS

More interesting are the nontrivial spin-states that are found in optical traps where a superposition of various spin projections in the condensate are possible. The trapping potential is produced using the electric field of an optical beam. Depending on the scattering lengths for the different spin channels, the condensate may be ferromagnetic or polar. In these traps, the Lorentz violation terms coupling to spin can mimic external magnetic fields. The common case of $f = 1$ bosons is considered here.

The Gross-Pitaevskii equation is modified to include spin-dependent scattering lengths (in the Thomas-Fermi limit) by writing the energy functional

$$ K = \int d^3 \vec{r} \nu[V_{\text{trap}} + \frac{c_0 n}{2} + \frac{c_1 n}{2} (\vec{F})^2 + E_{\text{Ze}}], \quad (34) $$

where $c_0$ and $c_2$ are appropriate linear combinations of scattering lengths for the total spin-0 and total spin-2 scattering states $34$, and $E_{\text{Ze}}$ is the Zeeman energy contributed by any external magnetic field that may be present. This expression assumes that angular momentum can be exchanged between the condensate and the environment. A Lagrange multiplier can be included to incorporate total spin conservation when necessary. For our purposes, it suffices to set the external magnetic fields to zero, eliminating the Zeeman contributions to the energy. In addition, we have neglected any magnetostatic interactions between the boson magnetic moments.

The Lorentz-violating terms may then be included using the Schmidt model for the nuclei. Assuming that the momentum-dependent terms are suppressed, the only relevant terms are $\vec{B}(c) \cdot \vec{\sigma}^{(c)}$ and $\vec{B}(p) \cdot \vec{\sigma}^{(p)}$. Calculating the matrix elements of these operators in the basis of states used in the above energy functional yields the
effective Lorentz-violating correction to the hamiltonian for a single atom

\[ H_{LV} = -\frac{1}{2} \vec{B}^{(e)} \cdot \vec{F} + \frac{5}{6} \vec{B}^{(p)} \cdot \vec{F} . \]  

(35)

The proton orbital angular momentum has been set to \( l = 1 \) for definiteness. The other choice of \( l = 2 \) simply alters the coefficient on the proton term to \(-1/2\). The spin-dependent part of the energy functional to be minimized is therefore

\[ K_s = \frac{c_2 n}{2} \langle \vec{F} \rangle^2 + \left( \frac{5}{6} \vec{B}^{(p)} - \frac{1}{2} \vec{B}^{(e)} \right) \cdot \langle \vec{F} \rangle . \]  

(36)

The type of condensate formed depends on the sign of \( c_2 \). If \( c_2 > 0 \), \( \langle \vec{F} \rangle = 0 \) minimizes the energy and the state is called polar. If \( c_2 < 0 \), \( \langle \vec{F} \rangle \neq 0 \) such that \( \langle \vec{F} \rangle^2 = 1 \) minimizes the energy. In this case, the state is called ferromagnetic due to the collective polarization of the system. The above expression demonstrates that the effect of the Lorentz-violating terms is to mimic a constant external magnetic field. This could have a significant effect in a well-shielded optical condensate in which the Lorentz-violating terms may provide a source for symmetry breaking in the system. Note that the total energy correction due to Lorentz violation grows linearly with the total number of particles in the condensate indicating that improved bounds should come with larger condensates. In order to observe an effect, the effects of Lorentz breaking terms will have to be clearly distinguished from an actual stray magnetic field. This should be possible as the Lorentz-violating terms have constant direction through time as the apparatus is rotated, while stray magnetic fields will tend to rotate with the apparatus. Even a minuscule field may provide the necessary symmetry breaking for an observable effect. As an estimate of the experimental sensitivity, the torque applied to the condensate cloud by the Lorentz-violating background fields is compared to the inertia at some reasonable angular acceleration that should be observable. The spin expectation value is taken orthogonal to \( \vec{B} = \frac{1}{2} \vec{B}^{(e)} - \frac{1}{4} \vec{B}^{(p)} \) for the maximum effect. This estimate yields an estimated bound of \( |\vec{B}| \sim m R^2 \alpha \) where \( m \) is the mass of a single atom in the condensate, \( R \) is the radius of the condensate, and \( \alpha \) is the angular acceleration of the cloud. Taking \( R \sim a_{\text{ho}} \sim 1 \mu m \) for a typical condensate with \( m \sim 100 \text{GeV} \) and \( \alpha \sim 10^{-2} \text{s}^{-2} = 10^{-19} \text{m}^{-2} \) yields an estimated sensitivity at the \( |\vec{B}| \sim 10^{-22} \text{GeV} \) level, comparable to the best CPT and Lorentz tests to date. Interactions will typically increase the radius of the cloud by one or two orders of magnitude with a corresponding decrease in sensitivity of two to four orders of magnitude.

VI. CONCLUSION

Many of the thermodynamic properties of low-temperature boson gases remain unaffected by Lorentz violation. However, when a condensate is present, the specific form of the ground state wave function becomes important and small symmetry breaking terms can induce collective effects on the shape of ground state. In particular, the main effect on a noninteracting gas is to perturb the ellipsoid slightly. Given current estimates of the experimental resolution of the wave function shape at approximately the 1% level, such a direct test is unlikely to yield useful bounds on Lorentz-violation parameters. When interactions are dominant, as is the case for most physical condensates, the kinetic energy terms can often be neglected and the chemical potential absorbs the effects of the Lorentz-breaking terms leaving the density distribution the same as in the conventional case. The more interesting effects occur when the condensate has multiple spin components. The spin couplings act as an effective constant external magnetic field that can act on the condensate. These terms have negligible effects on polar condensates for which the expectation of the spin vanishes. On the other hand, ferromagnetic condensates couple collectively to the Lorentz breaking field making it particularly sensitive to the spin couplings.

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