Metastable strange matter and compact quark stars

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Abstract. Strange quark matter in beta equilibrium at high densities is studied in a quark confinement model. Two equations of state are dynamically generated for the same set of model parameters used to describe the nucleon: one corresponds to a chiral restored phase with almost massless quarks and the other to a chiral broken phase. The chiral symmetric phase saturates at around five times the nuclear matter density. Using the equation of state for this phase, compact bare quark stars are obtained with radii and masses in the ranges $R \sim 5 - 8$ km and $M \sim M_\odot$. The energy per baryon number decreases very slowly from the center of the star to the periphery, remaining above the corresponding values for the iron or the nuclear matter, even at the edge. Our results point out that strange quark matter at very high densities may not be absolutely stable and the existence of an energy barrier between the two phases may prevent the compact quarks stars to decay to hybrid stars.

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1. Introduction

The recent discoveries of X-ray sources, whose origin may be attributed to strange stars [1, 2, 3], is stimulating theoretical research on the structure, composition, dynamics and evolution of such objects. It is believed that the innermost region of compact stars, such as neutron stars, can be made out of hyperons, meson condensates or quark matter [1, 4, 6, 7, 8], whose thermodynamical behavior can be expressed by means of a relativistic equation of state (EOS) [1, 8, 9].

Various effective models for the nucleon, using quarks as fundamental fields, have been used to provide EOS’s for quark matter, which are subsequently applied to investigate the structure of compact stars [8, 10, 11]. In this vein, we use the chromodielectric model (CDM) [12, 13] to obtain EOS’s for dense quark matter, and report, in this letter, the properties of the resulting quark stars. The chiral CDM provides a reasonable phenomenology for the nucleon [14, 15] and N–N* transition amplitudes [16]. Baryons appear as solitons with three quarks dynamically confined by a scalar field, $\chi$, whose quanta can be assigned to $0^{++}$ glueballs. In the quark matter sector, the model yields a relatively soft EOS at large densities [17, 18]. Drago et al. applied the CDM with a quadratic potential to the structure of compact stars [19, 20]. They connected the EOS for quark matter, as provided by the CDM, with an EOS for hadron matter, obtaining stars with masses in the range $1 - 2 M_\odot$ and radii of the order $10$ km with a hadron crust of $2$ km.

In the present work we consider an extension of the model used in Ref. [19], now taking quartic instead of quadratic potentials. For such potentials two qualitatively different EOS’s are dynamically generated for the same set of model parameters (the same used to describe the nucleon). They correspond to two distinct phases: in one phase the $\chi$ field is large and the quarks are massless (chiral symmetric phase). In the other one, $\chi$ is small and quarks are massive (chiral broken phase). The EOS’s are degenerate at very high densities but with an energy barrier between them. As the density decreases, they separate and saturate at very different densities, yielding different energies per baryon number. The EOS for the chiral symmetric matter saturates at densities much higher than the nuclear matter equilibrium density and with an energy per baryon number above that quantity in iron or in nuclear matter at low densities. The compact stars obtained using this EOS are made out of quarks only, since the density at the edge is much above the nuclear matter saturation density, and hadronization processes do not take place.

The prediction of smaller and denser objects in comparison with the neutron stars is quite exciting in view of the recent discovery of X-ray sources, by the Hubble and Chandra telescopes, which increased the plausibility that these sources might be strange quark stars [11, 2, 3]. In particular, the isolated compact object RX J1856.5-3574, not showing evidence for spectral lines or edge features [21, 22], reinforced the conjecture for the existence of strange matter stars. However, the first results indicating a small radius for that compact object were based on the reported parallax of Ref. [23], which was not
taking into account the camera geometrical distortion. Considering these corrections
the parallax was reduced by roughly a factor of two \cite{24, 25} and the distance has
been revised from 60 to 117 pc approximately. The revised radius ($R = 11.4 \pm 2$ km)
and mass ($M/M_\odot = 1.7 \pm 0.4$) make that isolated compact object reproducible by
many EOS with and without strange matter \cite{24, 26}. Hence, it is not guaranteed that
RX J1856.5-3574 might only be described as a quark star.

2. Quark matter

The CDM Lagrangian can be written as \cite{12, 13, 14}
\begin{equation}
\mathcal{L} = \mathcal{L}_q + \mathcal{L}_{\sigma,\pi} + \mathcal{L}_{q-\text{meson}} + \mathcal{L}_\chi,
\end{equation}
where
\begin{align}
\mathcal{L}_q &= i \bar{\psi} \gamma^\mu \partial_\mu \psi, \\
\mathcal{L}_{\sigma,\pi} &= \frac{1}{2} \partial_\mu \hat{\sigma} \partial^\mu \hat{\sigma} + \frac{1}{2} \partial_\mu \hat{\pi} \cdot \partial^\mu \hat{\pi} - W(\hat{\pi}, \hat{\sigma}), \\
W(\hat{\pi}, \hat{\sigma}) &= \text{the usual Mexican hat potential for the chiral mesons sigma and pi.}
\end{align}

The quark-meson interaction, assuming two flavours, is
\begin{equation}
\mathcal{L}_{q-\text{meson}} = \frac{g}{\chi} \bar{\psi} (\hat{\sigma} + i \hat{\pi} \gamma_5) \psi.
\end{equation}
The last term in (1) contains the kinetic and the potential piece for the $\chi$-field:
\begin{equation}
\mathcal{L}_\chi = \frac{1}{2} \partial_\mu \hat{\chi} \partial^\mu \hat{\chi} - U(\hat{\chi}).
\end{equation}

We consider a quartic potential
\begin{equation}
U(\chi) = \frac{1}{2} M^2 \chi^2 \left[ 1 + \frac{8 \eta^4}{\gamma^2} - 2 \right] \frac{\chi}{\gamma M} + \left( 1 - \frac{6 \eta^4}{\gamma^2} \right) \frac{\chi^2}{(\gamma M)^2},
\end{equation}
where $M$ is the $\chi$ mass. It has a global minimum at $\chi = 0$ and a local one at $\chi = \gamma M$.
The height of the local minimum, $B = U(\gamma M) = (\eta M)^4$, may be interpreted as a “bag
pressure” \cite{27}, as in the MIT bag model, and that will be used to fix parameters in
$U(\chi)$.

In the soliton sector of the model, a good description of the nucleon is obtained for
$\chi$ close to zero, a region of the potential \cite{15} where the cubic and quartic terms play no
role. All dependences go into just one parameter, namely $G = \sqrt{g M}$, and best nucleon
properties are obtained for $G \sim 0.2$ GeV (e.g. $M = 1.7$ GeV and $g = 0.023$ GeV \cite{14}).
In non-strange homogeneous quark matter the dependence on the single parameter $G$
is even exact for the quadratic model \cite{28}. In our study we keep that value for $G$. In the
quark matter sector of the model with double minimum potential there are solutions
with small $\chi$ and solutions with large $\chi$, i.e. in the region of the local minimum of the
potential, as discussed below in more detail. From $B = \eta^4 M^4$ and assuming the wide
range $0.150 \leq B^{1/4} \leq 0.250$ GeV, one has $0.08 \leq \eta \leq 0.15$, using $M = 1.7$ GeV.
The parameter $\gamma$, which does not affect the phenomenology of the homogeneous matter, is
not a totally free parameter: $\gamma^2 \geq 6 \eta^4$ since the quartic term in (5) must be positive.
The extension of the model to include the strange quark, requires that one more term be added to the interaction Lagrangian \( \mathcal{L} \), accounting for the coupling between the strange quark and the \( \chi \) field. We take it in the simplest form \[ (6) \]

\[ L_{s-meson} = \frac{g_s}{\chi} \bar{\psi}_s \psi_s. \]

In addition, for the sake of beta equilibrium, an electron gas must also be included. The mean-field energy per unit volume for a homogeneous system of \( u, d \) and \( s \) quarks, interacting with \( \chi \) and \( \sigma \), plus electrons, is

\[ \varepsilon = \alpha \sum_{f=u,d} \int_{k_f}^{k_u} \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m_f(\sigma, \chi)^2} \]

\[ + \alpha \int_{0}^{k_s} \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m_s(\chi)^2} + 2 \int_{0}^{k_e} \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m_e^2} \]

\[ + U(\chi) + \frac{m_{\sigma}^2}{8 f_\pi^2} (\sigma^2 - f_\pi^2)^2, \]

\[ f_\pi = 93 \text{ MeV} \] and we always use \( m_{\sigma} = 1.2 \text{ GeV} \) in this paper, though the results are not sensitive to the sigma mass]. The first two terms refer to quarks, and the third one to the electrons, all described by plane waves. The degeneracy factor is \( \alpha = 6 \) (for spin and color). The Fermi momentum for each type of particle, \( k_i \), is related to the corresponding density, \( \rho_i \), through \( \rho_i = \alpha k_i^3 / (6\pi^2) \). The quark masses in \[ (8) \]

\[ m_{u,d} = g_{u,d} \frac{\sigma}{f_\pi}, \quad m_s = \frac{g_s}{\chi}, \]

with the coupling constants for each flavour given by \( g_u = g (f_\pi + \xi_3) \), \( g_d = g (f_\pi - \xi_3) \) and \( g_s = g (2f_k - f_\pi) \) \[ \xi_3 = -0.75 \text{ MeV}, \, f_K = 113 \text{ MeV} \]. Since the vacuum expectation value of the confining field is zero, the quark masses raise up to infinity for densities approaching zero.

A variational principle applied to the energy density, Eq. \[ (7) \], leads to two gap equations for \( \sigma \) and \( \chi \). In the interior of a compact star the matter should satisfy both the electrical charge neutrality and chemical equilibrium. These conditions together with the gap equations, form a system of six algebraic equations solved at each baryon density \( \rho = \frac{1}{3} (\rho_u + \rho_d + \rho_s) \). For a given \( \rho \) one obtains a self-consistent set \( \sigma, \chi, k_u, k_d, k_s, k_e \) and, with such solution for each density, the energy density [Eq. \[ (7) \]] is readily evaluated as well as the energy per baryon number, \( \varepsilon / \rho \), the pressure, etc.

For the same set of parameters we found two distinct stable solutions, hereafter denoted by I and II. For both solutions, \( \sigma \) remains always close to \( f_\pi \) irrespective of the density. In solution I, the \( \chi \) field, close to zero, is a slowly increasing function of the density. For small \( \chi \), the quartic potential \[ (5) \] is indistinguishable from \( U = \frac{1}{2} M^2 \chi^2 \), thus, in practice, our solution I is the one obtained and used by Drago et al. \[ (19) \] in the framework of the quadratic potential. Due to the smallness of the \( \chi \) field, quark masses are large [see Eq. \[ (8) \]] and the system is in a chiral broken phase. On the other hand, for solution II, the confining field is large, \( \chi \sim \gamma M \) (local minimum of \( U \)),
almost independent of the density. The resulting quark masses are similar for the three flavours and very close to zero (chiral restored phase). Because the chemical potentials are dominated by the Fermi momentum, one has $\mu_u \simeq \mu_d \simeq \mu_s$, and therefore $\mu_e \simeq 0$, i.e. in solution II there are almost no electrons. Besides solutions I and II, there is an additional unstable solution corresponding to $\chi \sim \gamma M/2$ [local maximum of $U(\chi)$].

For each solution we obtained the corresponding energy per baryon number as a function of the baryon density (EOS) (see Fig. 1). EOS-I is not sensitive to $\gamma$ and $\eta$ (since $\chi$ is small), just depends on $G$, and it is rather similar to the one used in Ref. [19]. The saturation density occurs at a low density, slightly higher than the nuclear matter equilibrium density, $\rho_0$. Its shape, at intermediate densities, is similar to hadronic EOS’s (see Ref. [17] for the two flavours sector).

![Figure 1](image_url)

**Figure 1.** Energy per baryon number versus density for solutions I (solid line, small $\chi$) and II (dashed line, $\chi \sim \gamma M$) and various parameters $\eta$ (other model parameters: $M = 1.7$ GeV, $g = 0.023$ GeV and $\gamma = 0.2$). The dotted line corresponds to the unstable solution with $\chi \sim \gamma M/2$.

The EOS-II is also insensitive to $\gamma$, but does depend on $\eta$ [in fact, the dependence is on $(\eta M)^4$, as we have already discussed]: the energy per baryon number increases with $\eta$ and so does the saturation density. Depending on $\eta$ the minimal energy per baryon number of solution II can be either below or above solution I. For $\eta \sim 0.12$ the saturation occurs at $\rho \sim 5\rho_0$ and the energy per baryon number is some 230 MeV higher than for solution I at its saturation density. The two stable solutions are almost degenerated at high densities in the narrow range $0.1 \leq \eta \leq 0.12$. In Fig. 1 the dotted lines refer to the EOS for the unstable solution of the gap equations (supplemented by electric neutrality and beta equilibrium conditions), for which the $\chi$ is at the local maximum.
of the potential \[5\]. In order to undergo a transition from I to II, the system has to go through the energy barrier represented by the dotted EOS. The barrier gets higher at small densities and, for \(\eta = 0.12\) (third panel in Fig.1), at \(\rho \sim 5\rho_0\), \(\epsilon/\rho \sim 2.7\) GeV for the unstable solution. Therefore a transition from one regime to the other is not likely to occur and both minima in the EOS I and II are stable. In a 3D plot of the energy per baryon number versus \((\rho, \chi)\) the stable solutions correspond to two distinct “valleys”, and the unstable solution corresponds to the top of the barrier between these two valleys as it is shown in Fig. 2. Our results point out that the metastability of strange quark matter only occurs at high densities for solution II.

![3D plot of the energy per baryon number versus density and \(\chi\) field. Solutions I lies in the valley for small \(\chi\), the unstable solution is at the maximum and solution II lies in the valley at high \(\chi\). The parameters are as in Fig. 1.](image)

Interesting enough, our results with two distinct EOS, are consistent with the results from a recent calculation in perturbative QCD \[30\]. We checked that, for \(\eta \sim 0.12\) the CDM reproduces accurately the EOS’s obtained in Ref. \[30\]. However, in our case, there is no parameter fit to get two equations of state: they are dynamically generated for the same Lagrangian parameters. Also in Ref. \[31\], using an extension to finite chemical potential of lattice QCD data for the equation of state, similar results were obtained. Therefore, there seems to be some model independence, which is worth to point out here.

To investigate the structure of stars we solved the Tolman-Oppenheimer-Volkoff equation. Since EOS-I is identical to the one using a quadratic potential, it leads to stars that have the same phenomenology as the hybrid stars obtained by Drago et al. \[19\]: \(R \sim 10 - 12\) km, a hadron crust and a mass \(M \sim 1 - 2M_\odot\).
Since EOS-II saturates at a high density and, in addition, the system is not likely to undergo a transition to solution I, one should not perform any connection to the hadronic sector: the EOS-II alone generates a new family of strange quark stars. In Fig. 3 it is shown the mass-radius relation for different values of $\eta$. These quark stars are smaller and denser in comparison with those resulting from EOS-I. For $\eta \sim 0.115$ (and $M = 1.7$ GeV, yielding $B^{1/4} \sim 0.195$ GeV) one obtains a maximum radius $R \sim 6$ km (and a corresponding mass $M \sim 0.9M_\odot$). According to our calculation, such star has a central density of $10\rho_0$ ($\rho_0$ is the nuclear matter density) and a central energy density $\epsilon \sim 3 \times 10^{15}$ g/cm$^3$. At the edge, the density drops to $5\rho_0$ and $\epsilon \sim 1.35 \times 10^{15}$ g/cm$^3$. The ratio $\epsilon/\rho$ remains approximately constant inside the star and the minimum period of the star, computed using the expression given in Ref. [32], is $\sim 0.4$ ms.

Figure 3. Mass versus radius for the pure quark stars (solution II) in the CDM model. For the other model parameters see caption of Fig. 1.

A possible mechanism to explain the formation of such bare strange stars could be a supercooling effect in the early universe, when a significant amount of quark matter was frozen into quark stars. This mechanism needs an effective potential with two minima, as discussed in [33], resembling the quartic $\chi$ potential. As we see from Fig. 3, the mass-radius relation for these strange small stars mainly depends on the height of the local minimum of the $\chi$ potential. Finally, it is also worth mentioning that our results for the maximum radius and maximum mass for the stars from solution II are in agreement with Ref. [34], where it was shown that the Chandrasekhar limit for quarks stars depends on $B$.

Let us summarize our results. We have considered the CDM with a quartic potential, with parameters fixed in the nucleon sector and to yield a reasonable bag constant. Using a mean-field variational method we obtained two solutions for homogeneous strange matter in beta equilibrium, one similar to the already known solution for quadratic potentials (with massive quarks) and a new one with massless quarks. The pure quark stars emerging from the chiral symmetric solution are small and dense compact objects. To get so a small stars, the saturation density should be high enough, and this is achieved with a corresponding energy per baryon number always
above the energy per nucleon in the iron at low density or the energy per nucleon in nuclear matter at its saturation density.

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