Neutrino Mixing and Leptogenesis in Type II Seesaw Mechanism

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Abstract

In the framework of type II seesaw mechanism we propose two simple but instructive ansätze for neutrino mixing and leptogenesis. In each ansatz, the effective Majorana neutrino mass matrix is composed of two parts — the part with $Z_2$ symmetry arises from the ordinary type I seesaw mechanism and the part with $S_3$ symmetry arises from an additional Higgs triplet vacuum expectation value. The two ansätze can simultaneously account for the current neutrino oscillation data and the cosmological baryon number asymmetry via leptogenesis.

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I. INTRODUCTION

In the minimal standard model of electroweak interactions, the lepton number conservation is assumed and neutrinos are exactly massless Weyl particles. However, recent Super-Kamiokande [1], SNO [2], KamLAND [3] and K2K [4] neutrino oscillation experiments have provided us with very strong evidence that neutrinos are actually massive and lepton flavor mixing does exist. The ordinary (Type I) seesaw mechanism [5] gives a very simple and appealing explanation of the smallness of left-handed neutrino masses – it is attributed to the largeness of right-handed neutrino masses. Furthermore, the lepton-number-violating and out-of-equilibrium decays of heavy right-handed neutrinos may lead to the cosmological baryon asymmetry via leptogenesis [6]. The latter makes the seesaw mechanism more attractive. On the other hand, if there is an additional SU(2)\(_L\) Higgs triplet, it may also contribute to the neutrino masses and the cosmological baryon asymmetry. In this scenario, the Lagrangian relevant for neutrino masses reads:

\[
-\mathcal{L} = \frac{1}{2} N_R^c M_R N_R + M_\Delta^2 \text{Tr}(\Delta_L^\dagger \Delta_L) + \bar{\psi}_L Y_\nu N_R H \\
+ \bar{\psi}_L Y_\Delta i?2 \Delta_L \psi_L - \mu H^T i?2 \Delta_L H + h.c.,
\]

where \(\psi_L = (\nu_L, l_L)^T\) and \(H = (H^0, H^-)^T\) denote the left-handed lepton doublet and the Higgs-boson weak isodoublet respectively, \(N_R\) stands for the right-handed Majorana neutrino singlets, and

\[
\Delta_L = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Delta^+ & \Delta^{++} \\
\Delta^0 & -\frac{1}{\sqrt{2}} \Delta^+ 
\end{pmatrix}
\]

is the SU(2)\(_L\) Higgs triplet. After spontaneous gauge symmetry breaking, one may arrive at the so-called type II seesaw formula [7]

\[
M_\nu = M_L - M_D M_R^{-1} M_D^T = M_\nu^{II} + M_\nu^{I},
\]

where \(M_D \equiv Y_\nu \langle H \rangle\) with \(\langle H \rangle = v \simeq 174\text{GeV}\) and \(M_L \equiv 2 Y_\Delta \langle \Delta^0 \rangle\) with \(\langle \Delta^0 \rangle \simeq \mu^* v^2 / M_\Delta^2\). \(M_\nu^{I} = -M_D M_R^{-1} M_D^T\) is the ordinary type I seesaw mass term, and the type II seesaw mass term \(M_\nu^{II} = M_L\) arises from the additional Higgs triplet vacuum expectation value. One advantage for considering the type II seesaw is that it can naturally explain the degenerate neutrino masses [8].

Recently, some works have been done in the framework of type II seesaw mechanism. The decay asymmetries of right-handed neutrinos, including the Higgs triplet contribution, have been calculated in the standard model and minimal supersymmetric standard model [9–11]. The bounds on the lightest right-handed neutrino mass are also discussed. The deviations from the bimaximal neutrino mixing can be explained through the type II seesaw [12]. The large atmospheric neutrino mixing angle requires \(b - \tau\) unification in the minimal renormalizable SO(10) theory [13].

So far the connections between low-energy neutrino masses and high-energy leptogenesis have not been detailedly investigated in the framework of type II seesaw mechanism. In this paper, we propose two simple but instructive ansätze. They can reproduce the bi-large
neutrino mixing pattern and give rise to the cosmological baryon asymmetry via leptogenesis. In section II we first calculate the exact neutrino masses and lepton flavor mixing angles in the basis where the charged lepton mass matrix is diagonal, then discuss the corrections of an off-diagonal charged lepton mass matrix to the lepton flavor mixing matrix. In section III, we figure out twelve patterns of the Dirac neutrino mass matrices $M_D$ with the help of the type I seesaw formula, and investigate the leptogenesis in the two ansätze. Finally the summary and comments are given in section IV.

II. NEUTRINO MASSES AND MIXING

The light (left-handed) Majorana neutrino mass matrix $M_\nu$ can be diagonalized by one unitary matrix $U_\nu$:

$$U_{\nu}^\dagger M_{\nu} U_{\nu}^\dagger = \text{Diag}\{m_1, m_2, m_3\}. \quad (4)$$

The charged lepton mass matrix $M_l$ is in general non-Hermitian, hence the diagonalization of $M_l$ needs a bi-unitary transformation:

$$U_{\nu}^\dagger M_{l} U_{\nu} = \text{Diag}\{m_e, m_\mu, m_\tau\}. \quad (5)$$

The Maki-Nakagawa-Sakata (MNS) lepton flavor mixing matrix [14] $V = U_{\nu}^\dagger U_{l}$ can in general be parametrized as follows:

$$V = \left( \begin{array}{ccc} c_x c_z & s_x c_z & s_z \\ -c_x s_y s_z - s_x c_y e^{-i\delta} - s_x s_y s_z + c_x c_y e^{-i\delta} & \pm s_y c_z & \mp s_y c_z \\ -c_x c_y s_z + s_x s_y e^{-i\delta} - s_x c_y s_z - c_x s_y e^{-i\delta} & \mp s_y c_z & \pm s_y c_z \end{array} \right) \left( \begin{array}{ccc} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{array} \right) \quad (6)$$

with $s_x = \sin \theta_x$, $c_x = \cos \theta_x$, and so on. Note that three mixing angles of $V$ can directly be given in terms of the mixing angles of solar, atmospheric and reactor [15] neutrino oscillations. Namely, $\theta_x \approx \theta_{\text{sun}}$, $\theta_y \approx \theta_{\text{atm}}$ and $\theta_z \approx \theta_{\text{chz}}$ hold as a good approximation. In view of the current experimental data, we have $\theta_x \approx 33^\circ$ and $\theta_y \approx 46^\circ$ (best-fit values [16]) as well as $\theta_z < 12^\circ$. The mass-squared differences of solar and atmospheric neutrino oscillations are defined respectively as [16]

$$\Delta m_{\text{sun}}^2 \equiv m_2^2 - m_1^2 \approx 6.9 \times 10^{-5} \text{ eV}^2;$$
$$\Delta m_{\text{atm}}^2 \equiv |m_3^2 - m_2^2| \approx 2.3 \times 10^{-3} \text{ eV}^2. \quad (7)$$

We first assume $M_l$ is diagonal (i.e., $U_l = 1$ being a unity matrix), then consider the corrections of an off-diagonal charged lepton mass matrix to the lepton flavor mixing matrix. In this paper, we take $U_{\nu}$ to has two large mixing angles; namely $\theta_x \sim 30^\circ$, $\theta_y = 45^\circ$, $\theta_z = 0^\circ$. Without loss of generality, $U_{\nu}$ can be expressed by

$$U_{\nu} = \left( \begin{array}{ccc} c_x & s_x & 0 \\ -\frac{\sqrt{2}}{2} c_y & \frac{\sqrt{2}}{2} & \sqrt{\frac{\rho}{2}} \\ -\frac{\sqrt{2}}{2} s_y & \frac{\sqrt{2}}{2} & -\sqrt{\frac{\rho}{2}} \end{array} \right) \left( \begin{array}{ccc} e^{i\lambda_1} & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{array} \right). \quad (8)$$

In the following part of this section, we shall discuss two simple but instructive ansätze, from which neutrino masses and lepton flavor mixing angles can be specified when the charged lepton mass matrix is assumed to be diagonal. Then we discuss the corrections of an off-diagonal charged lepton mass matrix to the lepton flavor mixing matrix.
A. Ansatz I

In ansatz I, the neutrino mass matrix \( M_\nu = M^I_\nu + M^{II}_\nu \) is given by

\[
\begin{align*}
M^I_\nu & = - \begin{pmatrix} a & 0 & 0 \\ 0 & b & -b \\ 0 & -b & b \end{pmatrix}, \\
M^{II}_\nu & = \begin{pmatrix} a & a & a \\ a & a & a \\ a & a & a \end{pmatrix},
\end{align*}
\]

(9)

with \( a = |a|e^{i\phi} \) and \( b = |b|e^{i\psi} \). In Eq.(9), we have assumed \( M^{I}_{\nu 11} = -M^{II}_{\nu 11} \), which implies \( M_{\nu 11} = 0 \). Here \( M^I_\nu \) and \( M^{II}_\nu \) (and those of Ansatz II) have a flavor \( 2 \leftrightarrow 3 \) (\( Z_2 \)) symmetry \(^1\) and a permutation \( S_3 \) symmetry \(^2\), respectively. Similar textures of the neutrino mass matrix have been phenomenologically investigated at low energies \(^{[17]}\). In Ref. \(^{[17]}\), the same texture as \( M^{II}_\nu \) is introduced as the perturbation term. However, in this paper, \( M^{II}_\nu \) arises from the additional Higgs triplet vacuum expectation value. Moreover, our case includes the CP-violating phases.

The neutrino mass matrix \( M_\nu \) in Eq.(9) can be diagonalized by the unitary matrix \( U_\nu \) of Eq.(8), with

\[
\lambda_1 = \frac{\pi + \phi}{2}, \quad \lambda_2 = \frac{\phi}{2}, \quad \lambda_3 = \frac{\pi + \psi}{2}.
\]

(10)

The eigenvalues of \( M_\nu \) are

\[
\begin{align*}
m_1 & = (\sqrt{3} - 1)|a|, \\
m_2 & = (\sqrt{3} + 1)|a|, \\
m_3 & = 2|b|.
\end{align*}
\]

(11)

The mixing angle \( \theta_x \) is related to the neutrino masses \( m_1 \) and \( m_2 \):

\[
\sin \theta_x = \sqrt{\frac{m_1}{m_1 + m_2}}.
\]

(12)

With the help of Eqs.(7), (11) and (12) we can derive

\[
\begin{align*}
\theta_x & = \arcsin \sqrt{\frac{3 - \sqrt{3}}{6}} \approx 27.4^\circ, \\
|a| & \approx 3.2 \times 10^{-3} \text{ eV}, \\
|b| & \approx 2.4 \times 10^{-2} \text{ eV}.
\end{align*}
\]

(13)

\(^1\)From Eq.(3) we know that \( M^I_\nu \) and \( M^{II}_\nu \) are symmetric matrices. In Eqs.(9) and (14), \( M^I_{\nu 12} = M^{II}_{\nu 13} \) and \( M^I_{\nu 22} = M^{II}_{\nu 33} \) can be determined by \( 2 \leftrightarrow 3 \) (\( Z_2 \)) symmetry. In this paper, we have assumed \( M^I_{\nu 12} = M^{II}_{\nu 13} = 0 \) and \( M^I_{\nu 22} = -M^{II}_{\nu 23} \) in both ansatz I and ansatz II.

\(^2\)A permutation \( S_3 \) symmetry implies \( M^{II}_{\nu 11} = M^{II}_{\nu 22} = M^{II}_{\nu 33} \) and \( M^{II}_{\nu 12} = M^{II}_{\nu 13} = M^{II}_{\nu 23} \). For simplicity, we assume \( M^{II}_{\nu 11} = M^{II}_{\nu 12} \) in ansatz I and \( M^{II}_{\nu 11} = 0 \) in ansatz II. \( M^{II}_\nu \) of Eq.(9) (often called the “democratic” mass matrix ) is a unique representation of the \( S_3(L) \times S_3(R) \) symmetric matrix.
It is clear that we have known all the mixing angels and neutrino masses \((m_1 \approx 2.3 \times 10^{-3} \text{ eV}, m_2 \approx 8.7 \times 10^{-3} \text{ eV} \text{ and } m_3 \approx 4.8 \times 10^{-2} \text{ eV})\) in this ansatz, but we don’t know the phases \(\lambda_i\).

### B. Ansatz II

In this case, the neutrino mass matrix \(M_\nu = M_\nu^I + M_\nu^{II}\) is written as

\[
M_\nu^I = \begin{pmatrix} a & 0 & 0 \\ 0 & b & -b \\ 0 & -b & b \end{pmatrix}, \quad M_\nu^{II} = \begin{pmatrix} 0 & d & d \\ d & 0 & d \\ d & d & 0 \end{pmatrix},
\]

where \(|a| \ll |b|, |d|\), the phases of \(a, b\) and \(d\) are defined to be \(\phi, \psi\) and \(\varphi\) respectively. \(M_\nu\) can also be diagonalized by the unitary matrix \(U_\nu\) of Eq.(8). Here we have neglected \(a\) of Eq.(14). The phases \(\lambda_i\) can be expressed as

\[
\lambda_1 = \frac{\pi + \varphi}{2}, \quad \lambda_2 = \frac{\varphi}{2}, \quad \lambda_3 = \frac{\pi}{2} + \frac{\text{arg}(2b + d)}{2},
\]

and the eigenvalues of \(M_\nu\) are

\[
m_1 = |d|, \\
m_2 = 2|d|, \\
m_3 = |2b + d|.
\]

The mixing angle \(\theta_\times\) is also related to the neutrino masses \(m_1\) and \(m_2\) as Eq.(12). Using Eqs.(7), (12) and (16), we can get

\[
\theta_\times = \arcsin \frac{\sqrt{3}}{3} \approx 35.3^\circ,
\]

\[
|d| \approx 4.8 \times 10^{-3} \text{ eV},
\]

\[
|b| \approx (2.2 \sim 2.7) \times 10^{-2} \text{ eV}.
\]

The neutrino masses read \(m_1 = m_2/2 \approx 4.8 \times 10^{-3} \text{ eV}\) and \(m_3 \approx 4.9 \times 10^{-2} \text{ eV}\). At present, the unitary matrix \(U_\nu\) is

\[
U_\nu = \begin{pmatrix} \frac{\sqrt{2}}{3} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{\sqrt{2}}{2} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{2} \end{pmatrix}, \quad \begin{pmatrix} e^{i\lambda_1} & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix},
\]

which implies the tri-bimaximal neutrino mixing. It has been extensively investigated by several authors [17,18].
C. Neutrino Mixing Corrected by the Charged Lepton Mass Matrix

In the above paragraphs, we have assumed $M_l$ to be diagonal and have thus determined all the neutrino mixing angels: $\theta_x \approx 27.4^\circ (35.3^\circ)$, $\theta_y = 45^\circ$ and $\theta_z = 0^\circ$ in ansatz I (ansatz II). Now we consider the corrections of an off-diagonal charged lepton mass matrix to the lepton flavor mixing matrix. For simplicity and illustration, we take $M_l$ to be

$$
M_l = \begin{pmatrix}
0 & C_l & 0 \\
C_l^* & B_l & 0 \\
0 & 0 & A_l
\end{pmatrix}, \quad (19)
$$

where $A_l = m_\tau$, $B_l = m_\mu - m_e$, and $C_l = \sqrt{m_em_\mu} e^{i\xi}$ with the inputs $m_e = 0.511$ MeV, $m_\mu = 105.658$ MeV, and $m_\tau = 1.777$ GeV [20]. Because $M_l$ in Eq.(19) has been assumed to be Hermitian, it can in general be diagonalized by a unitary matrix

$$
U_l = \begin{pmatrix}
\cos \theta & \sin \theta e^{i\xi} & 0 \\
-\sin \theta e^{-i\xi} & \cos \theta & 0 \\
0 & 0 & 1
\end{pmatrix}. \quad (20)
$$

The mixing angle $\theta$ in $U_l$ turns out to be

$$
\tan 2\theta = \frac{2 \sqrt{m_em_\mu}}{m_\mu - m_e}. \quad (21)
$$

Taking into account the hierarchy of charged lepton masses (i.e., $m_e \ll m_\mu \ll m_\tau$), one obtains $\sin \theta \approx \sqrt{m_e/m_\mu}$ to a good degree of accuracy. Using $V = U_l^\dagger U_\nu$, we can derive $\theta_{\text{sun}} \approx 30.1^\circ$, $\theta_{\text{atm}} \approx 44.9^\circ$ and $\theta_{\text{chz}} \approx 2.8^\circ$ in ansatz I by assuming $\xi = \pi$. The value of $\theta_{\text{sun}}$ agrees with the current experimental data in $2\sigma$ confidence level [16]. In ansatz II, $\theta_{\text{sun}} \approx 33.1^\circ$, $\theta_{\text{atm}} \approx 44.9^\circ$ and $\theta_{\text{chz}} \approx 2.8^\circ$ can be obtained by assuming $\xi = \pi/4$. The result of $\theta_{\text{sun}} \approx 33.1^\circ$ is consistent very well with the best fit value $\theta_{\text{sun}} \approx 33.2^\circ$ [16]. Our numerical analysis show the two ansätze are compatible with the current experimental data. The nonvanishing angle $\theta_{\text{chz}}$ implies that there are probably CP-violating effects in neutrino oscillations.

III. LEPTOGENESIS

The decays of heavy Majorana neutrinos, $N_i \rightarrow l + H^\dagger$ and $N_i \rightarrow l^c + H$, violate both the lepton number conservation and the CP symmetry. A CP-violating asymmetry $\varepsilon_i^N$ can be generated through the interference between the tree-level and one-loop decay amplitudes. In the type II seesaw mechanism, because of the presence of a Higgs triplet, there are other two decay processes $N_i \rightarrow l + H^\dagger$ (contribution from exchanging a virtual Higgs triplet) and $\Delta_L \rightarrow l + l$, which can generate the CP asymmetries $\varepsilon_i^\Delta$ and $\varepsilon_\Delta$, respectively. For simplicity, in the following parts of this paper, we consider the case that three heavy Majorana neutrinos $N_i$ have a hierarchical mass spectrum ($M_1 \ll M_2 \ll M_3$) and the triplet mass $M_\Delta$ is much larger than $M_1$ ($M_1 \ll M_\Delta$). Then the interactions of $N_i$ can be in thermal equilibrium when $N_2, N_3$ and $\Delta_L$ decay. The CP-violating asymmetries produced in the decays of $N_2, N_3$
and $\Delta L$ can be erased before $N_i$ decays. Then only the asymmetries $\varepsilon_1^N$ and $\varepsilon_1^\Delta$ produced by out-of-equilibrium decay of $N_1$ survives. In a flavor diagonal basis for the heavy right-handed neutrinos one has [9–11]

$$
\varepsilon_1^N \approx -\frac{3}{16\pi v^2} \frac{M_1}{(M_D^* M_D)_{11}} \text{Im}[(M_D^T (m_\nu^I)^* M_D)_{11}],
$$

(22)

and

$$
\varepsilon_1^\Delta \approx -\frac{3}{16\pi v^2} \frac{M_1}{(M_D^* M_D)_{11}} \text{Im}[(M_D^T (m_\nu^H)^* M_D)_{11}].
$$

(23)

The $\varepsilon_1^N$ and $\varepsilon_1^\Delta$ can result in a net lepton number asymmetry

$$
Y_L \equiv \frac{n_L - n_{\bar{L}}}{s} = \frac{\kappa}{g_*} \varepsilon_1,
$$

(24)

where $\varepsilon_1 = \varepsilon_1^N + \varepsilon_1^\Delta$, $g_* = 106.75$ is an effective number characterizing the relativistic degrees of freedom which contribute to the entropy $s$ of the early universe, and $\kappa$ accounts for the dilution effects induced by the lepton-number-violating wash-out processes. The dilution factor $\kappa$ can be determined by solving the full Boltzmann equations [21]. Because the Higgs triplet $\Delta_L$ does not couple directly to the lightest right-handed neutrino $N_1$, the scatterings with a virtual triplet can not change the number density of $N_1$ in the Boltzmann equations. On the other hand, the additional scattering $l + l \rightarrow \Delta_L \rightarrow H + H$ is largely suppressed [9], so we neglect the contribution to dilution effects from the scatterings involving the additional Higgs triplet. We take the ordinary dilution factor formulas of type I seesaw [22]:

$$
\kappa = 0.3 \left( \frac{10^{-3} \text{eV}}{\tilde{m}_1} \right) \left[ \ln \left( \frac{\tilde{m}_1}{10^{-3} \text{eV}} \right) \right]^{-0.6}, \quad 10^{-2} \text{eV} \lesssim \tilde{m}_1 \lesssim 10^3 \text{eV}
$$

$$
\kappa = \frac{1}{2\sqrt{\left( \frac{\tilde{m}_1}{10^{-3} \text{eV}} \right)^2 + 9}}, \quad \tilde{m}_1 \lesssim 10^{-2} \text{eV}
$$

(25)

with $\tilde{m}_1 = (M_D^* M_D)_{11}/M_1$ [23]. The lepton number asymmetry $Y_L$ is eventually converted into a net baryon number asymmetry $Y_B$ via the nonperturbative sphaleron processes [24]: $Y_B \approx -0.55Y_L$. A generous range $0.7 \times 10^{-10} \lesssim Y_B \lesssim 1.0 \times 10^{-10}$ has been drawn from the recent WMAP observational data [25].

### A. Determining the Structure of $M_D$

From Eqs.(9) and (14) we know

3In this note, we take the result of Ref. [10] for $\varepsilon_1^\Delta$. 

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7
\[ M'_\nu = -M_D M_R^{-1} M_D^T = - \begin{pmatrix} a & 0 & 0 \\ 0 & b & -b \\ 0 & -b & b \end{pmatrix}, \] (26)

where \( M_R \) is assumed to be diagonal. It is worth mentioning that \( M'_\nu \) has the two-zero texture [26]. The following conditions can be derived from Eq.(26):

\[ \begin{align*}
M'_{\nu_{12}} &= M'_{\nu_{13}} = 0, \\
M'_{\nu_{22}} &= M'_{\nu_{33}} = -M'_{\nu_{23}} \neq 0.
\end{align*} \] (27)

In terms of the above conditions, we can obtain some constraints on \( M_D \). In order to fix the structure of \( M_D \), we exclude the possible cancellations among the elements of \( M_D \) and \( M_R \), and require that the above conditions should be from the zeros and relations of the elements in \( M_D \). We find that there are twelve patterns of \( M_D \) which can produce the texture of \( M'_\nu \) through the type I seesaw formula. We have listed all of them in Table 1 and have classified them into four categories according to their structures.

B. Leptogenesis for Ansatz I

We find that only patterns A1 and A2 in Table 1 can generate nonvanishing \( \varepsilon_1 \). It is worth remarking that patterns C1 and D1 give rise to \( \varepsilon_1^N = -\varepsilon_1^\Delta \neq 0 \), which means \( \varepsilon_1 = 0 \). Patterns C3 and D3 lead to \( (M_D^\dagger M_D)_{11} = 0 \) for the two given ansätze, implying that \( N_1 \) does not decay. So we neglect these two textures in this paper. Because A1 and A2 have similar physical consequences, we shall only focus on pattern A1.

In pattern A1, the Dirac neutrino mass matrix \( M_D \) is

\[ M_D = \begin{pmatrix} 0 & 0 & C \\ A & B & 0 \\ -A & -B & 0 \end{pmatrix}. \] (28)

It is then straightforward to obtain \( \tilde{m}_1 = 2 |A^2|/M_1 \). With the help of Eqs.(22), (23) and (26), we have

\[ \begin{align*}
\varepsilon_1^N &= -\frac{3}{8\pi v^2} M_1 \left| \frac{B^2}{M_2} \right| \sin 2(\beta - \alpha), \\
\varepsilon_1^\Delta &= 0.
\end{align*} \] (29)

Using Eqs.(13) and (26), we can derive

\[ \begin{align*}
|a| &= \left| \frac{C^2}{M_3} \right| \approx 3.2 \times 10^{-3} \text{ eV}, \\
|b| &= \left| \frac{A^2}{M_1} + \frac{B^2}{M_2} \right| \approx 2.4 \times 10^{-2} \text{ eV}.
\end{align*} \] (30)

The relations between \( |A^2|/M_1 \) and \( |B^2|/M_2 \) shown in Eq.(30) are very important, because they affect the dilution factor \( \kappa \) (i.e., \( \tilde{m}_1 \)) and \( \varepsilon_1 \). For illustration, we discuss two special cases:
(1) In the $|A^2|/M_1 \gg |B^2|/M_2$ case, $|A^2|/M_1 \approx 2.4 \times 10^{-2}$ eV and $|B^2|/M_2 \ll 2.4 \times 10^{-2}$ eV. Therefore we use the first formula of Eq.(25) to calculate the dilution factor. With the help of Eqs.(24), (25) and (30), we have

$$Y_B \approx -0.55 \frac{\kappa}{g_*} \epsilon_1 \approx 1.7 \times 10^{-6} \frac{M_1}{v^2} \frac{B^2}{M_2} \sin 2(\beta - \alpha).$$

(31)

For example, when we take $|B^2|/M_2 = 10^{-3}$ eV, the successful leptogenesis requires $1.2 \times 10^{12}$ GeV $\lesssim M_1 \lesssim 1.8 \times 10^{12}$ GeV by assuming $\sin 2(\beta - \alpha) = 1$.

(2) In the $|A^2|/M_1 \ll |B^2|/M_2$ case, $|B^2|/M_2 \approx 2.4 \times 10^{-2}$ eV and $|A^2|/M_1 \ll 2.4 \times 10^{-2}$ eV. Using the second formula of Eq.(25) we find $0.048 < \kappa \leq 0.167$. In this case the baryon asymmetry $Y_B$ is

$$Y_B \approx 8.5 \times 10^{-17} \frac{M_1}{v^2} \sin 2(\beta - \alpha).$$

(32)

Here we take $\sin 2(\beta - \alpha) = 1$, $M_1 = 10^9$ GeV and $\kappa = 0.16$, only for illustration. The baryon asymmetry is found to be $Y_B \approx 0.78 \times 10^{-10}$, which is compatible with the recent WMAP observational data. So this case can produce the cosmological baryon asymmetry via leptogenesis.

C. Leptogenesis for Ansatz II

Through a global analysis we find that seven patterns (A1, A2, B1, B2, B3, C1 and D1) in Table 1 can generate the cosmological baryon asymmetry via leptogenesis. It is worth remarking that A1 and A2 (B1, B2 and B3; C1 and D1) have similar physical consequences, therefore we concentrate on patterns A1, B1 and C1 as three typical examples for numerical illustration.

For pattern A1, $\bar{m}_1 = 2|A^2|/M_1$ and

$$\epsilon_1^N = -\frac{3}{8\pi v^2} M_1 \frac{B^2}{M_2} \sin 2(\beta - \alpha),$$

$$\epsilon_1^A = -\frac{3}{16\pi v^2} M_1 |d| \sin(\varphi - 2\alpha),$$

(33)

with

$$|a| = \frac{|C^2|}{M_3} \ll |d| \approx 4.8 \times 10^{-3} \text{ eV},$$

$$|b| = \frac{|A^2|}{M_1} + \frac{B^2}{M_2} \approx 2.5 \times 10^{-2} \text{ eV}.$$  

(34)

For simplicity, we have taken the average values of $|b|$ in Eq.(17). If $\sin(\varphi - 2\alpha) = 0$ or $|d| \sin(\varphi - 2\alpha) \ll |B^2|/M_2 \sin 2(\alpha - \beta)$ (i.e., $\epsilon_1^N \gg \epsilon_1^A$) holds, we shall obtain similar results as shown in pattern A1 of ansatz I. If we assume $\sin 2(\beta - \alpha) = \sin(\varphi - 2\alpha) = 1$ and then consider the $|d| \gg |B^2|/M_2$ case (thus we neglect the contribution of $\epsilon_1^N$), the baryon
asymmetry is found to be \( Y_B \approx 2.2 \times 10^{-20} \frac{M_1}{v} \). It is clear that the successful leptogenesis requires \( 5.5 \times 10^{11} \text{ GeV} \lesssim M_1 \lesssim 7.9 \times 10^{11} \text{ GeV} \).

Now let us focus on pattern B1

\[
M_D = \begin{pmatrix} 0 & B & C \\ A & 0 & 0 \\ -A & 0 & 0 \end{pmatrix} .
\]

(35)

We have \( \bar{m}_1 = 2 |A^2| / M_1 \), and

\[
\begin{align*}
\varepsilon_1^N &= 0 , \\
\varepsilon_1^\Delta &= -\frac{3}{16\pi v^2} M_1 |d| \sin(\varphi - 2\alpha) , \\
\end{align*}
\]

(36)

with

\[
\begin{align*}
|a| &= \left| \frac{B^2}{M_2} + \frac{C^2}{M_3} \right| \ll |d| \approx 4.8 \times 10^{-3} \text{ eV} , \\
|b| &= \frac{A^2}{M_1} \approx 2.5 \times 10^{-2} \text{ eV} .
\end{align*}
\]

(37)

In this pattern we find the baryon asymmetry \( Y_B \approx 2.2 \times 10^{-20} \frac{M_1}{v} \sin(\varphi - 2\alpha) \). The recent observational data of \( Y_B \) requires \( M_1 \geq 5.5 \times 10^{11} \text{ GeV} \).

In pattern C1, the Dirac neutrino mass matrix \( M_D \) is

\[
M_D = \begin{pmatrix} B & 0 & C \\ 0 & A & 0 \\ 0 & -A & 0 \end{pmatrix} .
\]

(38)

In this pattern \( \bar{m}_1 = |B^2| / M_1 \). The CP asymmetries \( \varepsilon_1^N \) and \( \varepsilon_1^\Delta \) are

\[
\begin{align*}
\varepsilon_1^N &= -\frac{3}{16\pi v^2} M_1 \left| \frac{C^2}{M_3} \right| \sin 2(\gamma - \beta) , \\
\varepsilon_1^\Delta &= 0 .
\end{align*}
\]

(39)

The constraints from Eqs.(17) and (26) are

\[
\begin{align*}
|a| &= \left| \frac{B^2}{M_1} + \frac{C^2}{M_3} \right| \ll |d| \approx 4.8 \times 10^{-3} \text{ eV} , \\
|b| &= \frac{A^2}{M_2} \approx 2.5 \times 10^{-2} \text{ eV} .
\end{align*}
\]

(40)

For illustration, here we assume \( |B^2| / M_1 = |C^2| / M_3 = 10^{-4} \text{ eV} \) and \( \sin 2(\gamma - \beta) = 1 \). This produces a baryon asymmetry \( Y_B \approx 2.9 \times 10^{-20} \frac{M_1}{v} \), which in turn requires \( 4.2 \times 10^{11} \text{ GeV} \lesssim M_1 \lesssim 6.0 \times 10^{11} \text{ GeV} \).

Because the three light neutrinos have a clear mass hierarchy in ansatz I and ansatz II, we have neglected possible renormalization-group running effects of neutrino masses and lepton flavor mixing parameters between the scales \( v \) and \( M_1 \) [27].
Finally, let us comment on the relation between leptogenesis and the CP violation at low energies. Because of the corrections from the off-diagonal charged lepton mass matrix $M_l$, we do not find a direct link between $\varepsilon_1$ and the low-energy CP violation in neutrino oscillations, but we can derive some relations among the CP-violating phases. For example, we have $\psi = 2\alpha$ in pattern B1 of ansatz II and $\psi = \arg(|A|^2/M_1 e^{2i\alpha} + |B|^2/M_2 e^{2i\beta})$ in pattern A1 of ansatz I, and so on. If the charged lepton mass matrix $M_l$ is real (i.e., $\xi = 0$), then there is no CP violation in neutrino oscillations, because the Dirac CP-violating phase $\delta = 0$ holds.

**IV. SUMMARY AND COMMENTS**

We have proposed two simple but instructive ansätze of neutrino mass matrix in type II seesaw mechanism. The explicit neutrino masses and bi-large neutrino mixing can be calculated when the charged lepton mass matrix is assumed to be diagonal. We have also discussed the possible corrections of an off-diagonal charged lepton mass matrix to the lepton flavor mixing matrix, and the results are compatible with the current neutrino oscillation data. On the other hand, we have figured out twelve patterns of the Dirac neutrino mass matrix $M_D$ with the help of the type I seesaw formula, and have investigated the leptogenesis in the two ansätze. Two patterns of $M_D$ in ansatz I and seven patterns of $M_D$ in ansatz II can generate the desired cosmological baryon asymmetry. We have briefly discussed the relation between leptogenesis and the CP violation at low energies.

It is worth mentioning that one can carry out a similar analysis of $Y_B$ in the framework of supersymmetric type II seesaw and leptogenesis models. If the future KATRIN [28] and WMAP experiments can pin down the absolute neutrino masses [29], this will be very helpful to examine the two ansätze. Our results will be very useful for model building, in order to understand why neutrino masses are so tiny and why two of the lepton flavor mixing angles are so large.

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TABLE I. Twelve patterns of the Dirac neutrino mass matrix $M_D$ derived from Eq.(26). They may be favored or disfavored by the current cosmological baryon asymmetry $Y_B$. In this paper, $A$, $B$ and $C$ are complex, and their phases are $\alpha$, $\beta$ and $\gamma$ respectively.

| Pattern | By current data | Pattern | By current data |
|---------|----------------|---------|----------------|
| $A_1$ : $\begin{pmatrix} 0 & 0 & C \\ A & B & 0 \\ -A & -B & 0 \end{pmatrix}$ | Ansatz I favored | $C_1$ : $\begin{pmatrix} B & 0 & C \\ 0 & A & 0 \\ 0 & -A & 0 \end{pmatrix}$ | Ansatz I disfavored |
| $A_2$ : $\begin{pmatrix} 0 & C & 0 \\ A & 0 & B \\ -A & 0 & -B \end{pmatrix}$ | Ansatz I favored | $C_2$ : $\begin{pmatrix} B & 0 & 0 \\ 0 & A & 0 \\ 0 & -A & 0 \end{pmatrix}$ | Ansatz I disfavored |
| $A_3$ : $\begin{pmatrix} C & 0 & 0 \\ 0 & A & B \\ 0 & -A & -B \end{pmatrix}$ | Ansatz I disfavored | $C_3$ : $\begin{pmatrix} 0 & 0 & C \\ 0 & A & 0 \\ 0 & -A & 0 \end{pmatrix}$ | Ansatz I disfavored |
| $B_1$ : $\begin{pmatrix} 0 & B & C \\ A & 0 & 0 \\ -A & 0 & 0 \end{pmatrix}$ | Ansatz I disfavored | $D_1$ : $\begin{pmatrix} B & C & 0 \\ 0 & 0 & A \\ 0 & 0 & -A \end{pmatrix}$ | Ansatz I disfavored |
| $B_2$ : $\begin{pmatrix} 0 & B & 0 \\ A & 0 & 0 \\ -A & 0 & 0 \end{pmatrix}$ | Ansatz I disfavored | $D_2$ : $\begin{pmatrix} B & 0 & 0 \\ 0 & 0 & A \\ 0 & 0 & -A \end{pmatrix}$ | Ansatz I disfavored |
| $B_3$ : $\begin{pmatrix} 0 & 0 & C \\ A & 0 & 0 \\ -A & 0 & 0 \end{pmatrix}$ | Ansatz I disfavored | $D_3$ : $\begin{pmatrix} 0 & C & 0 \\ 0 & 0 & A \\ 0 & 0 & -A \end{pmatrix}$ | Ansatz I disfavored |