More about Electroweak Baryogenesis
in the Minimal Supersymmetric Standard Model

Antonio Riotto

Department of Physics, Theoretical Physics, University of Oxford
1 Keble Road, Oxford OX1 3NP, United Kingdom
(September 1997)

Abstract

We compute the baryon asymmetry generated at the electroweak phase transition by the Higgs scalar sector of the minimal supersymmetric standard model. Because of large enhancement effects from low momentum modes, Higgs particles may be responsible for the observed baryon asymmetry even though CP-violation in the Higgs sector only appears at the one-loop level. We also discuss the approximations made in the analysis and suggest possible improvements.

¶ Advanced PPARC Fellow. From December 1997 through November 1999 on leave of absence at the CERN Theory group as CERN Fellow.
The Standard Model (SM) fulfills all the requirements for a successful generation of the baryon number at the electroweak scale [1] due to the presence of baryon number violating processes which also impose severe constraints on models where the baryon asymmetry is created at very high energy scales [2]. Unfortunately, the electroweak phase transition is at best weakly first order in the SM [3] meaning that the baryon asymmetry generated during the transition would be subsequently erased by unsuppressed sphaleron transitions in the broken phase. Therefore, if one is willing to pursue the idea of electroweak baryogenesis, some new physics at the weak scale must be called for. The most promising and well-motivated candidate seems to be supersymmetry (SUSY). Electroweak baryogenesis in the framework of the Minimal Supersymmetric Standard Model (MSSM) has attracted much attention in the past years with particular emphasis on the strength of the phase transition [4–6] and the mechanism of baryon number generation [7–9].

Recent analytical [10] and lattice computations [11] have revealed that the phase transition can be sufficiently strongly first order if the lightest stop is not much heavier than the top quark, the ratio of the vacuum expectation values of the two neutral Higgses $\tan\beta$ is smaller than $\sim 4$ and the lightest Higgs is lighter than about 85 GeV. Moreover, the MSSM has the bonus that additional sources of CP-violation may be present besides the CKM matrix phase. These new phases are essential for the generation of the baryon number since large CP-violating sources may be locally induced by the passage of the bubble wall separating the broken from the unbroken phase. Baryogenesis is fueled when transport properties allow the CP-violating charges to efficiently diffuse in front of the advancing bubble wall where anomalous electroweak baryon violating processes are not suppressed. The new phases appear in the soft supersymmetry breaking parameters associated to the stop mixing angle and to the gaugino and neutralino mass matrices; large values of the stop mixing angle are, however, strongly restricted in order to preserve a sufficiently strong first order electroweak phase transition. Therefore, an acceptable baryon asymmetry from the stop sector may only be generated through a delicate balance between the values of the different soft supersymmetry breaking parameters contributing to the stop mixing parameter, and their associated CP-violating phases [8]. On the other hand, charginos and neutralinos may be responsible for the observed baryon asymmetry if the phase of the parameter $\mu$ is larger than about 0.1 [8].

Since in the case in which the bubble wall is thick, the Higgs background carries a very low momentum (of order of the inverse of the bubble wall width $L_w$), the production
of the baryon asymmetry is enhanced if the degrees of freedom in the gaugino/neutralino sectors are nearly degenerate in mass [8]: values of the phase of $\mu$ lower than 0.1 are only consistent with the observed baryon asymmetry for values of $|\mu|$ of order of the gaugino mass parameters. This is due to a large enhancement of the computed baryon asymmetry for these values of the parameters.

The fact that CP-violating sources are most easily built up by the transmission of low momentum particles over a distance $L_w$ [7,8] is an indication that particles with masses much smaller than the temperature $T$ may be relevant in the process of quantum interference leading to CP-violating currents in the bubble wall. It is therefore fair to guess that the nearly massless neutral and charged modes present in the scalar Higgs sector during the electroweak phase transition may play a significant role in supersymmetric electroweak baryogenesis. This is analogous to what happens in the SM where the barrier in the effective potential separating the two minima at finite temperature is originated by infrared effects.

CP-violating Higgs currents are generated by the interactions with the bubble wall only if some CP-violation is present in the scalar potential. This does not happen at the tree-level because the form of the scalar potential is dictated by supersymmetry and CP-violating operators are forbidden. However, once supersymmetry is broken and if CP-violation is present in the stop and in the -ino sector, CP-violating Higgs operators are induced at finite temperature by one-loop diagrams involving stops, charginos and neutralinos [12]. CP-violating scatterings of the Higgs modes with the advancing bubble wall are therefore expected to give rise to some baryon asymmetry.

The goal of this Letter is to compute the final baryon asymmetry generated by the Higgs scalar sector. Because of the large enhancement from the infrared region, Higgs particles may be responsible for the observed baryon asymmetry even though the CP-violation in the Higgs sector only appears as a one-loop effect. The details of the computation are contained in the next section, while section 3 is devoted to our conclusions and comments.

2. To compute the CP-violating Higgs current we make use of the method proposed in Ref. [13] and recently adopted by Carena et al. [8] to compute the stop and the -ino contribution to the baryon asymmetry. This method is entirely based on a nonequilibrium quantum field theory diagrammatic approach. It may be applied for all wall shapes and sizes of the bubble wall and it naturally incorporates the effects of the incoherent nature of plasma physics on CP-violating observables. What we need to compute is the temporal evolution of a classical order parameter, namely the CP-violating Higgs current, with definite
initial conditions. In this respect, the ordinary equilibrium quantum field theory at finite temperature may not be applied, since it mainly deals with transition amplitudes in particle reactions. Therefore, the closed-time path formalism is used, which is a powerful Green’s function formulation for describing nonequilibrium phenomena in field theory [14].

Following [7,8], we are interested in the generation of some charge which is approximately conserved in the symmetric phase, so that it can efficiently diffuse in front of the bubble where baryon number violation is fast, and non-orthogonal to baryon number, so that the generation of a non-zero baryon charge is energetically favoured. A charge with these characteristics is the Higgs charge density \( \langle J^0_H \rangle = \langle J^0_{H_1} - J^0_{H_2} \rangle \), where \( J^0_{H_i}(z) = i \left( H^\dagger_i \partial^0 H_i - \partial^0 H^\dagger_i H_i \right) \) with \( i = 1, 2 \) and \( H_1 \) and \( H_2 \) denote the two Higgs doublets. The CP-violating sources \( \gamma_H(z) \) (per unit volume and unit time) of the Higgs charge density \( J^0_H \) associated to the current \( J^0_H \) and accumulated by the moving wall at a point \( z^\mu \) of the plasma can then be constructed from \( J^0_H(z) \) using the relation \( \gamma_H(z) = \partial_0 \langle J^0_H(z) \rangle \). This is appropriate to describe the damping effects originated by the plasma interactions, but does not incorporate any relaxation time scale arising when diffusion and particle changing interactions are included. However, one can leave aside diffusion and particle changing interactions and account for them independently in the rate equations. This is a good approximation (at least for small bubble wall velocity) since, for instance, the typical diffusion time \( t_D \sim D/v_w^2 \), where \( D \) is the typical diffusion constant, is much larger than any other time scales [7,8].

The CP-violating part of the MSSM Higgs scalar potential reads

\[
V_{\text{CP}} = \lambda_5 (H_1 H_2)^2 + \lambda_6 |H_1|^2 H_1 H_2 + \lambda_7 |H_2|^2 H_1 H_2 + \text{h.c.}
\]  

As mentioned in the introduction, the coefficients \( \lambda_{5,6,7} \) are zero at the tree-level, but are nonvanishing at the one-loop level at finite temperature when interactions of the Higgs fields with charginos and neutralinos are taken into account [12]. Since these coefficients appear only once supersymmetry is broken, their values are proportional to the gaugino masses \( M_{1,2} \), and to the supersymmetric parameter \( \mu \). The typical value for \( \lambda_{6,7} \) is about \( 10^{-3} \), while \( \lambda_5 \) is at most \( 10^{-4} \) [12]. What is relevant for us is that the \( \lambda \)'s are complex if the parameter \( \mu \) carries a phase \( \phi_\mu \), \( \lambda_5 = |\lambda_5| e^{i2\phi_\mu} \), \( \lambda_{6,7} = |\lambda_{6,7}| e^{i\phi_\mu} \). It is the presence of this phase that gives origin to the baryon asymmetry.

\(^1\)The stop contribution is suppressed when the left-handed stop mass is much larger than the temperature. This condition is necessary for the phase transition to be strong enough [10].
The interesting dynamics for baryogenesis takes place in a region close to or inside the bubble wall and we approximate it with an infinite plane traveling at a constant speed $v_w$ along the $z$-axis.

The computation of the CP-violating Higgs current is done by expanding $\langle J_H^\mu \rangle$ around the symmetric phase defined by $\langle H_1^0 \rangle \equiv v_1(T) = 0$ and $\langle H_2^0 \rangle \equiv v_2(T) = 0$. This is equivalent to a diagrammatic approach where $\langle J_H^\mu \rangle$ is expressed as a sum of different Feynman diagrams with increasing powers of external Higgs insertions. The external Higgs background has to be identified with the bubble wall configuration and the expansion is only justified in a region of the parameter space for which the mean free time $\tau$ is smaller than the scale of variation of the masses, i.e. the wall thickness $L_w$. The reader is referred to refs. [13,8] for more details. Since the coefficients of the expansion are computed in the symmetric phase, we must deal with the resummation of the propagators of the Higgs fields in order to deal with infrared divergencies [15]. Indeed, to account for the interactions with the surrounding particles of the thermal bath, particles must be substituted by quasiparticles and dressed propagators are to be adopted. Self-energy corrections at one- or two-loops to the propagator modify the dispersion relations and introduce a nontrivial damping rate $\Gamma$ due to the imaginary contributions to the self-energy. In what follows we shall adopt dressed propagators to compute the thermal averages of the composite Higgs operator, which allows us us to naturally and self-consistently include the effects of the incoherent nature of plasma physics.

In the unbroken phase, the Higgs spectrum contains two complex electrically neutral fields and two charged ones. At the tree-level, the squared masses of one of the neutral states and one of the charged ones are negative, since the origin of the field space becomes a minimum of the effective potential only after inclusion of the finite temperature corrections. The resummation can be achieved by considering the propagators for the eigenstates of the thermal mass matrix, which has positive eigenvalues given by

$$m^2_{h,H} = \frac{m^2_1(T) + m^2_2(T) \mp \sqrt{(m^2_1(T) - m^2_2(T))^2 + 4 m^4_3(T)}}{2},$$

where the $m^2_i(T)$ are the thermal corrected mass parameters of the effective potential, $m^2_1(T) \simeq m^2_1 + (3/8)g^2T^2$, $m^2_2(T) \simeq m^2_2 + (1/2)h^2_t T^2$, while $m^2_3(T) = m^2_3$ (+ logarithmic corrections in $T$). Here $m^2_{1,2,3}$ are the $T = 0$ coefficients of the operator $|H_1|^2$, $|H_2|^2$ and $(H_1 H_2)$ respectively and $g$ and $h_t$ are the $SU(2)_L$ gauge coupling and the top Yukawa coupling. Correspondingly, the neutral complex eigenstates are given by
\[
\begin{aligned}
\begin{cases}
    h = c_\theta H_0^* + s_\theta H_2, \\
    H = -s_\theta H_1^* + c_\theta H_2,
\end{cases}
\end{aligned}
\]  

(3)

where \(c_\theta = \cos \theta\), \(s_\theta = \sin \theta\) and \(\theta\) is the critical angle identified by the flat direction of the effective potential around \(v_1 = v_2 = 0\) at the critical temperature. Analogous formulae hold for the charged eigenstates.

Following refs. [13, 8], it is not difficult to show that \(\langle J_H^0(z) \rangle\) gets contributions from several one-loop Feynman diagrams which are obtained by assigning in all the possible ways the space-time points \(z\) and \(x\) (which is the point where the external Higgs configurations are attached to) on the positive or negative time branches typical of the nonequilibrium approach [14]. Disregarding the \(T = 0\) piece, the result is (in the plasma frame)

\[
\langle J_H^0(z) \rangle = 8 \cos^2 \frac{2 \theta}{2} \text{Im}[\dot{f}(z)] I(\beta, m_h, \Gamma),
\]

\[
I(\beta, m_h, \Gamma) = \int_0^\infty du \int \frac{d^3k}{(2\pi)^3} \frac{e^{-2\Gamma u}}{2\omega_k^2} \sin^2 \omega_k u \sin \beta \Gamma.
\]

\[
f(z) = \left\{2(\lambda_6 v_1^2 + \lambda_7 s_\theta^2) v_1(z) v_2(z) + 2c_\theta s_\theta \left[\lambda_6 v_1^2(z) + \lambda_7 v_2^2(z)\right]\right\}.
\]  

(4)

In the expression above we have only included the dominant contribution coming from the diagrams where only nearly Higgs massless modes at the critical temperature propagate; \(\omega_k^2 = k^2 + m_h^2\), where \(m_h\) denote the mass of the degrees of freedom propagating in the loop whose decay rate in the plasma is \(\Gamma\); \(\beta = T^{-1}\). It is easy to show that the contribution proportional to \(\lambda_5\) is suppressed because \(|\lambda_5| \ll |\lambda_{6,7}|\) and because heavy states must propagate in the relative one-loop diagrams. The integral over the “time” variable \(u\) makes evident the causality inherent to the nonequilibrium approach: only the processes taking places at times smaller than \(t_z\) may give rise to a nonvanishing current.

Performing the integrals in \(I(\beta, m_h, \Gamma)\) and taking the limit \(\Gamma \ll T\), we obtain the final expression

\[
\langle J_H^0(z) \rangle \simeq \frac{1}{\pi} \cos^2 \frac{2 \theta}{2} \text{Im}[\dot{f}(z)] \frac{T}{\sqrt{m_h^2 + \frac{T^2}{4}}}. 
\]  

(5)

Eqs. (4) and (5) warrant some comments. First, we notice that the momentum integration is infrared dominated: quasiparticles with long wavelengths and momentum perpendicular to the wall give a large contribution to \(\gamma_H(z)\) and a classical approximation is not adequate to describe the quantum interference nature of \(CP\)-violation. Secondly, the final result has the same infrared singularity responsible for the failure of the perturbative expansion [10] for the values of the parameters such that \(m_h = 0\) (here the singular behaviour is smeared.
out by the presence of the additional "mass" term $\sim \tilde{\Gamma}/2$). However, the theory remains perturbative as far as $\alpha_h \equiv (g^2/2\pi)(T^2/m_h^2) \lesssim 1$, which gives $m_h \gtrsim 0.3T$. An estimate of the Higgs damping rate in the SM was obtained in Ref. [17] in the low momentum limit and can be used here only to give very crude estimate of the Higgs coherent time, $\tau \sim (10 - 10^2)/T$. With such values, our derivative expansion is barely justified since the wall thickness can certainly span the same range $(10 - 10^2)/T$. We will discuss the validity of our approximation in the next section.

The next step amounts to solving the set of coupled differential equations describing the effects of diffusion, particle number changing reactions and CP-violating source terms. We suppose that, among the supersymmetric particles, charginos, neutralinos and the right-handed stops as well as the Higgs degrees of freedom, are in equilibrium in the thermal bath. Under this hypothesis, strong sphalerons do not drive the asymmetry to zero [18].

Closely following the approach taken in Ref. [7,8] we can estimate the final baryon asymmetry generated by the Higgs sector to be

$$\left(\frac{n_B}{s}\right)_H = -g(k_i)\frac{A\bar{D}\Gamma_{ws}}{v_w^2 s}, \quad (6)$$

where $s = 2\pi^2 g_\ast T^3/45$ is the entropy density ($g_\ast$ being the effective number of relativistic degrees of freedom); $g(k_i)$ is a numerical coefficient depending upon the light degrees of freedom present in the thermal bath, $\bar{D}$ is the effective diffusion constant, $\Gamma_{ws} = 6\kappa\alpha_W^4 T$ is the weak sphaleron rate ($\kappa \simeq 1$) [19] and

$$\mathcal{A} = B_+ \left(1 - \frac{\lambda_+}{\lambda_-}\right) = B_- \left(\frac{\lambda_+}{\lambda_-} - 1\right) = \frac{1}{B_-} \int_0^\infty du \tilde{\gamma}_H(u)e^{-\lambda_- u}, \quad (7)$$

where

$$\lambda_\pm = v_w \pm \sqrt{v_w^2 + 4\bar{D}\Gamma}. \quad (8)$$

The quantity $\bar{\Gamma}$ is the effective decay constant and $\tilde{\gamma}_H(z) = v_w \partial_z J_H^0(z)f(k_i)$ is now defined in the bubble wall frame, $f(k_i)$ being a coefficient depending on the number of degrees of freedom present in the thermal bath and related to the definition of the effective source [4]. Since for relatively low values of the pseudoscalar mass $m_A$ ($m_A \gtrsim m_Z$) the variation of the ratio of the vacuum expectation values of the Higgs fields along the bubble wall is small

---

\[2\] The correct value of $\kappa$ is at present the subject of debate, see, for instance, Ref. [20].
we can take $\theta \sim \beta$. For typical values $\tan \beta \sim 2$, $\Gamma \simeq 5 \times 10^{-2}T$, $m_h \simeq 0.4T$ and 
$\sqrt{v_1^2(T) + v_2^2(T)}/T \simeq 1.2$, we find

$$\left( \frac{n_B}{s} \right)_H \simeq -6 \times 10^{-11} \sin \phi_\mu,$$

(9)

where the dependence upon $v_w$ is very weak.

The Higg scalar sector may account for the baryon asymmetry for large values of the phase $\phi_\mu$. Such large values may be tolerated in view of the experimental limits on the neutron electric dipole moment of the neutron if the squarks of the first and second generation have masses of the order of a few TeV.

3. One should not claim victory too soon, though. Let us look back at the approximations we have made and discuss to which extent our estimate might be altered by performing a more refined analysis:

a) Our result is rigously valid only for $L_w \Gamma \gtrsim 1$. Given our poor knowledge of both parameters, this condition does not seem unreasonable. Moreover, for $L_w \Gamma \ll 1$ one may expect that the dependence of the final result upon $\Gamma$ disappears [4] so that $I(\beta, m_h, \Gamma)$ will saturate at the value $\Gamma \sim L_w^{-1}$. Our results seem to confirm this expectation.

b) We have seen that quasiparticles with long wavelengths give the largest contribution to the source. This means that the classical approximation is not adequate to describe the quantum interference nature of $CP$-violation and a quantum approach must be adopted to compute the $CP$-violating source. This was done in the present paper. However, in the second stage of the computation, we have made use of classical Boltzmann equations. For low momentum particles, the validity of the classical Boltzmann equation starts to break down. It is indisputable that the ultimate answer can be provided only by a complete nonequilibrium quantum field theory approach. Kinetic theory and classical Boltzmann equations have been used to describe the dynamics of particles treated as classical with a defined position, energy and momentum. This requires that, in particular, the mean free path must be large compared to the Compton wavelength of the underlying particle in order for the classical picture to be valid, which is not guaranteed for particles with a small momentum perpendicular to the wall. Distribution functions obeying the quantum Boltzmann equations are the only correct functions to describe particles in an interacting, many-particle environment.

Solving the quantum Boltzmann equations represents an Herculean task. However, recent investigations have revealed that the quantum Wigner distributions posses strong memory
effects and that their relaxation time is typically longer than the one obtained in the classical limit [22]. This slowdown of the relaxation processes may keep the system out of equilibrium for longer times and therefore enhance the final baryon asymmetry.

\textit{c}) When computing the Higgs source, we have been assuming the loopwise expansion being reliable, in other words we have computed only the lowest order loop diagrams. However, there is an infinite class of loop diagrams with more interaction vertices, the so-called “ladder” diagrams [23], which may contribute at the leading order even though full thermal corrections have been used in the evaluation of the lowest level diagrams. This is because there are additional infrared divergencies at finite temperature when the ladder-diagrams contain nearly massless Higgs modes and the infrared cut-off is provided by the mass $m_h$. The physical reason for this large correction has to be identified with the fact that the lowest loop diagram takes into account only the effects of particles which have undergo only a few collisions in the plasma, after which particles are not fully thermalized yet. Summing up all the leading contributions is a nontrivial problem (in fact, it has been recently claimed that the ladder graphs may be canceled by other types of terms in the loop expansion [24]). A reasonable estimate of the relative size of of the contribution of the ladder-diagrams with respect to the lowest diagram computed in this paper is given by (loop suppression factor) $\times (g^2 T^2 / m_h^2) \ln(T / m_h)$. Numerically this contribution may be sizeable and might change the final estimate of the baryon asymmetry by some factor $\mathcal{O}(1)$.

\underline{Acknowledgements:}

It is a pleasure to thank my collaborators M. Carena, M. Quiros, I. Vilja and C.E.M. Wagner for useful discussions.
REFERENCES

[1] For recent reviews, see: A.G. Cohen, D.B. Kaplan and A.E. Nelson, Annu. Rev. Nucl. Part. Sci. 43 (1993) 27; M. Quirós, Helv. Phys. Act. 67, 451 (1994); V.A. Rubakov and M.E. Shaposhnikov, preprint CERN-TH/96-13 [hep-ph/9603208].

[2] G. t’Hooft, Phys. Rev. Lett. 37 (1976) 8; Phys. Rev. D14, 3432 (1976); P. Arnold and L. McLerran, Phys. Rev. D36, 581 (1987); and D37 (1988) 1020; S.Yu Khlebnikov and M.E. Shaposhnikov, Nucl. Phys. B308, 885 (1988); F.R. Klinkhamer and N.S. Manton, Phys. Rev. D30, 2212 (1984); B. Kastening, R.D. Peccei and X. Zhang, Phys. Lett. B266, 413 (1991); L. Carson, Xu Li, L. McLerran and R.-T. Wang, Phys. Rev. D42, 2127 (1990); M. Dine, P. Huet and R. Singleton Jr., Nucl. Phys. B375, 625 (1992).

[3] M. Shaposhnikov, JETP Lett. 44 (1986) 465; Nucl. Phys. B287, 757 (1987) and B299 (1988) 797; M.E. Carrington, Phys. Rev. D45, 2933 (1992); M. Dine, R.G. Leigh, P. Huet, A. Linde and D. Linde, Phys. Lett. B283, 319 (1992); Phys. Rev. D46, 550 (1992); P. Arnold, Phys. Rev. D46, 2628 (1992); J.R. Espinosa, M. Quirós and F. Zwirner, Phys. Lett. B314, 206 (1993); W. Buchmüller, Z. Fodor, T. Helbig and D. Walliser, Ann. Phys. 234 (1994) 260; J. Bagnasco and M. Dine, Phys. Lett. B303, 308 (1993); P. Arnold and O. Espinosa, Phys. Rev. D47, 3546 (1993); Z. Fodor and A. Hebecker, Nucl. Phys. B432, 127 (1994); K. Kajantie, K. Rummukainen and M.E. Shaposhnikov, Nucl. Phys. B407, 356 (1993); Z. Fodor, J. Hein, K. Jansen, A. Jaster and I. Montvay, Nucl. Phys. B439, 147 (1995); K. Kajantie, M. Laine, K. Rummukainen and M.E. Shaposhnikov, Nucl. Phys. B466, 189 (1996).

[4] G.F. Giudice, Phys. Rev. D45, 3177 (1992).

[5] S. Myint, Phys. Lett. B287, 325 (1992).

[6] J.R. Espinosa, M. Quirós and F. Zwirner, Phys. Lett. B307, 106 (1993); A. Brignole, J.R. Espinosa, M. Quirós and F. Zwirner, Phys. Lett. B324, 181 (1994).

[7] P. Huet and A.E. Nelson, Phys. Rev. D53, 4578 (1996).

[8] M. Carena, M. Quiros, A. Riotto, I. Vilja and C.E.M. Wagner, CERN-TH-96-242 preprint [hep-ph/9702409], to appear in Nucl. Phys. B.

[9] J.M. Cline, M. Joyce and K. Kainulainen, McGill 97-26 preprint [hep-ph/9708393].

[10] M. Carena, M. Quiros and C.E.M. Wagner, Phys. Lett. B380, 81 (1996); D. Delepine,
J.M. Gerard, R. Gonzalez Felipe and J. Weyers, Phys. Lett. **B386**, 183 (1996); J.R. Espinosa, Nucl. Phys. **B475**, 273 (1996); B. de Carlos and J.R. Espinosa, UPR-0737-T preprint, [hep-ph/9703317].

[11] M. Laine, Nucl. Phys. **B481**, 43 (1996); J.M. Cline and K. Kainulainen, Nucl. Phys. **B482**, 73 (1996).

[12] D. Comelli and M. Pietroni, Phys. Lett. **B306**, 67 (1993); D. Comelli and M. Pietroni and A. Riotto, Nucl. Phys. **B412**, 441 (1994); Phys. Rev. **D50**, 7703 (1994); Phys. Lett. **343**, 207 (1995); J.R. Espinosa, J.M. Moreno, M. Quirós, Phys. Lett. **B319**, 505 (1993).

[13] A. Riotto, Phys. Rev. **D53**, 5834 (1996).

[14] See, for instance: K. Chou, Z. Su, B. Hao and L. Yu, Phys. Rep. **118**, 1 (1985) and references therein.

[15] P. Arnold and O. Espinosa, Phys. Rev. **D47** (1993) 3546.

[16] L. Dolan and R. Jackiw, Phys. Rev. **D9**, 3320 (1974); S. Weinberg, Phys. Rev. **D9**, 3357 (1975).

[17] P. Elmfors, K. Enqvist and I. Vilja, Nucl. Phys. **B412**, 459 (1992).

[18] G.F. Giudice and M. Shaposhnikov, Phys. Lett. **B326**, 118 (1994).

[19] J. Ambjorn and A. Krasnitz, Phys. Lett. **B362**, 97 (1995).

[20] P. Arnold, D. Son and L.G. Yaffe, Phys. Rev. **D55**, 6264 (1997).

[21] T. Multamaki and I. Vilja, prepint TURKU-FL-P26-97 preprint, [hep-ph/9705469].

[22] P.A. H Henning, Phys. Essays **9**, 569 (1996).

[23] S. Jeon, Phys. Rev. **D47**, 4586 (1993).

[24] M. Carrington, R. Kobes and E. Petitgirard, WIN-97-12 preprint, [hep-ph/9708412].