Modelling and optimizing sensor wireless network systems

I Ya Lvovich¹, Ya E Lvovich², A P Preobrazhenskiy¹, Yu P Preobrazhenskiy¹ and O N Choporov²

¹Information Systems And Technologies Department, Voronezh institute of high technologies, 73a, Lenina st, Voronezh, 394043, Russia
²Information Security Department, Voronezh state technical university, 14, Moscow dist., Voronezh, 394026, Russia

E-mail: komkvvivt@yandex.ru

Abstract. This paper is connected with the necessity of problem modelling and optimizing sensor wireless network systems. The general scheme of a multi-step process of making the optimal decision in sensor wireless network systems is given. The main steps of the computational scheme of the dynamic programming method are given. It is shown how to increase the efficiency of the wireless sensor network. All possible variants of optimal strategies are demonstrated that ensure obtaining the maximum value of a generalized performance indicator. The results of the paper can be useful in the development of wireless sensor networks with high performance indicators.

1. Introduction
With the development of network infrastructure as the basis for the activities of modern enterprises and with the increasing complexity of applications used in the network, the requirements for bandwidth, reliability and protection of the network, its manageability, and lower operating costs are increasing [1]. To meet today's network infrastructure requirements, it must support the following network applications and services: integrated transmission of voice, video and digital data; creation of virtual local and private networks; rule-based network management; using agreements on the level of services provided; accounting of the resources used; user management; transmission of multicast traffic; construction of Internet, Intranet, Extranet networks.

Thus, an intelligent network [2] should be built, the administrator of which has the ability to transform the requirements of the enterprise's business processes into specific rules that connect the enterprise's activity with the requirements for the network, such as the provision of quality of service, security and access control. The intelligent network provides a wide range of services and mechanisms for ensuring the operation of business applications through the implementation of a directory service, a variety of network services [3] (switching, virtual local and private networks, access rules, protection, quality of service, resource accounting).

The network infrastructure must support the structure of management tools and switched networks, creating a high-performance, scalable, reliable infrastructure that implements traffic transfer and priority, bandwidth management, and resource accounting [4].

To deliver voice and video data, it is necessary to ensure the guaranteed quality of their transmission (service). The nature of requests to the network has become more complex - a large number of users are accessing various information resources located on different platforms.
The aim of the paper is to consider an approach based on which it is possible to increase the efficiency of wireless communication networks.

2. Performance characteristics of the wireless sensor network
All the set of the most commonly used criteria for network performance can be divided into two groups. One group characterizes the network performance, the second - reliability.

Network performance is measured using two types of metrics — time metrics, which measure the latency introduced by the network when exchanging data, and throughput metrics, which measure the amount of information transmitted by the network per unit of time [5, 6].

These two types of indicators are mutually inverse, and knowing one of them, you can calculate the other. Typically, a metric such as response time is used as a temporal characteristic of network performance. The term "reaction time" can be used in a very broad sense, therefore, in each specific case, it is necessary to clarify what is meant by this term. In general, response time is defined as the time interval between the occurrence of a user’s request for a network service and the receipt of a response to this request. Obviously, the meaning and value of this indicator depend on the type of service that the user is accessing, on which user and which server is accessing, as well as on the current state of other network elements - the load of the segments through which the request passes, the load of the server, etc. For the end user in this way, a certain response time is an understandable and most natural indicator of network performance (the file size, which introduces some uncertainty in this indicator, can be fixed by estimating the response time when transferring, for example, one megabyte of data). However, a network specialist is primarily interested in the performance of the network itself [7], therefore, for a more accurate assessment of it, it is advisable to isolate from the reaction time the components corresponding to the stages of non-network data processing - searching for the necessary information on disk, writing it to disk, etc.

The resulting time savings can be considered another definition of network response time at the application layer.

3. General scheme of a multistep process for making an optimal decision
In general, the mathematical model for making an optimal decision can be formulated as a nonlinear programming problem:

$$\min_{\mathbf{x} \in D} Q(\mathbf{x})$$

(1)

where \(\mathbf{x} = (x_1, x_2, ..., x_n)\) is a vector of controlled variables; \(D\) is the area of admissible solutions [8].

The efficiency of the numerical solution of the optimization problem (1) is largely determined by its dimension. In a one-step process of making an optimal decision using the appropriate methods, the choice of numerical values for all \(n\) components of the vector of optimized variables \(\mathbf{x}\) is carried out simultaneously [8]. This, with a large dimension of problem (1), can significantly complicate the search for the optimal solution \(x^*\). One of the approaches to overcoming the noted difficulty is the organization of the decision-making process in the form of an \(N\)-step process for making an optimal decision (multistep PMOD). In it, the vector of optimized variables \(\mathbf{x} = (x_1, ..., x_n)\) is split into \(N\) unconnected parts \(q_k = (x_{j1}, x_{jm}) \in D, k = 1, N\). They are called the vector of controlled parameters, each of which is selected at the corresponding step of the multistep PMOD. In this case, the controlled parameters \((q_1, ..., q_N)\) satisfy the following two conditions:

1. the set of all vectors \(q_k, k = 1, N\), including specific components \(x_j\), form the original vector \(x\)

$$\bigcup_{k=1}^{N} q_k = (x_1, ..., x_n)$$

(2)

2. each component \(x_j\) of the original vector \(x\) can enter only one of the vectors \(q_k\):

$$q_k \cap q_i = \emptyset, k, i = 1, N, k \neq i.$$  

(3)
4. Optimality principle and its implementation in the form of a system of functional equations

The mathematical model of the multistep process for making the optimal decision (multistep PMOD) can be written as the following optimization problem:

\[ f_N(p_N + 1)R(q_1^*, \ldots, q_N^*, p_{N+1}) = \min_{(q_1, \ldots, q_N) \in D_N} \left\{ \sum_{k=1}^{N} R_k(p_k + 1, q_k) \right\}, \quad (4) \]

where \( f_N(p_N + 1) \) - minimum value of the generalized performance criterion \( N \)-step PMOD. Its initial state is characterized by a vector of variables \( p_{N+1}; (q_1^*, \ldots, q_N^*) \) - the optimal admissible strategy to obtain the value \( f_N(p_N + 1)R(q_1^*, \ldots, q_N^*, p_{N+1}) \).

We consider a general approach to solving the optimization problem (4). For this, we construct a family of extremal problems in which the number of steps \( k \) is an arbitrary non-negative integer \((0 \leq k \leq N)\). The initial state of \( p_{k+1} \) \( k \)-step confinal subprocess is characterized by a non-negative parameter \( p(0 \leq p \leq p_0) \):

\[ f_k(p) = \min_{(x_1, \ldots, x_k)} \left\{ \sum_{i=1}^{k} R_i(x_i) \right\}. \quad (5) \]

in this case, the condition is fulfilled:

\[ \sum_{i=1}^{k} x_i \leq p; \ x_i \geq 0, i = 1, k. \quad (6) \]

In three specific cases, the family of problems (5) - (6) takes a simple form.

Case 1. The amount of the allocated resource is equal to zero:

\[ p = 0, f_k(0) = 0 \text{ for all } k = 1, N. \quad (7) \]

This ratio is a consequence of the assumption that for any \( k \)-th step, if there is no resource, then the efficiency indicator of the \( k \)-th step \( R_k(0) = 0 \).

Case 2. The decision-making process is over: \( k = 0 \).

\[ f_0(p) = 0 \text{ for any } p > 0. \quad (8) \]

Relation (8) means that for any positive amount of resources, a positive value of the efficiency indicator of the \( k \)-th step cannot be obtained if all decision-making steps have been exhausted.

Case 3. The optimal decision making process consists of one step: \( k=1 \).

\[ f_1(p) = \min R_1(x_1), \quad (9) \]

in this case, the condition is fulfilled:

\[ x_1 \geq 0; \ x_1 \geq p. \quad (10) \]

In this case, the minimum value \( f_1(p) \) coincides with the minimum value of the performance indicator \( R_1(x_1) \). The recurrence relation connecting the generalized indicator of the \( k \)-step confinal subprocess \( f_k(p_{k+1}) \) with the minimum values of the generalized indicator \((k-1)\)-the step confinal subprocess \( f_{k-1}(p_k) \) has the form:

\[ f_k(p_{k+1}) = \min_{0 \leq x_k \leq p_{k+1}} \{ R_k(x_k) + f_k - 1(p_{k+1} - x_k) \}. \quad (11) \]

for all \( k = 1, N \).

The resulting recurrence relation (11) forms a system of functional equations. They allow, taking into account case 3 (optimization problem (9) - (10)), to find a sequence of minimum values \( f_k(p), k = 1, N \), starting from the end of the \( N \)-th PMOD (step with the index “1”). Then we go to its beginning (step with index “N”) by sequentially solving one-dimensional optimization problems:
The set of functional equations (12) implements the dynamic programming method. It allows you to get rid of the difficulties associated with the problem of large dimensions, since at each step of the multistep PMOD it is required to solve a one-dimensional optimization problem. But you have to pay for this with the additional amount of memory required to store the values \( f_k(p) \) for a set of discrete values \( p \in [0, p_0] \). This is due to the fact that it is not known which value of \( p \) is the best at the \( k \)-th step, except for the \( N \)-th step, where the best value of \( p \) is \( p_{N+1} = p_0 \). The constructed system of functional equations (12) reflects the principle of optimality. Its main idea is as follows. It does not matter how, using the optimal strategy \( (x'_N, ..., x'_{k+1}) \) or just an admissible strategy \( (x_N, ..., x_{k+1}) \in D_q \), we obtained for further resource allocation \( p_{k+1} \), the strategy \( (x_k, ..., x_1) \) at the remaining \( k \) steps of the confinal subprocess must be optimal with respect to the state \( p_{k+1} \). Then it should provide the minimum value of not only the efficiency indicator of the \( k \)-th step \( R_k(x_k) \), but also the minimum value of the generalized indicator \((k-1)\)-th step confinal subprocess \( f_{k-1}(p_k) \). That is, when choosing the optimal value of the controlled parameter \( x'_k \), we must think about its consequences at the next \((k-l)\) steps of the multistep PMOD, because the mistake in its choice made at the \( k \)-th step cannot be corrected at subsequent steps with the indices "\(k-l\)"; "\(k-2\)"; ..., "\(1\)".

5. Computational scheme of the dynamic programming method

We describe the structure of a computational circuit that implements the dynamic programming method using the example of solving a simple distribution problem of the following form:

\[
f_N(p_0) = \min_{x_1, ..., x_N} \left\{ \sum_{i=1}^{N} R_i(x_i) \right\}
\]

the following condition is considered to be satisfied.

The computational scheme consists of two stages. At the first stage, making assumptions about the values of the variables \( p_{k+1}, k = 1, N \) using the system of recurrence relations (12), the values of the functions \( f_k(p_{k+1}), k = 1, N \) are tabulated. There is also a tabulation of the corresponding optimal controlled parameters \( q_k = x_k(p_{k+1}) \) [9, 10]. The beginning goes from the end of the multistep PMOD (step with the index "1") and further we go to its beginning (the step with the index "N"). Then the sequence of steps involved in making a decision is the reverse of their sequence in time. Assuming \( k=1 \), a one-dimensional optimization problem is solved:

\[
f_1(p) \min_{x_1=0,1,...,\left[\frac{p_1}{a_1}\right]} R_1(x_1) = \min \left( R_1(0), R_1(1), ..., R_1(p) \right)
\]

for all integer values \( p = 0, 1, ..., p_0 \). The result of this procedure is table 1, which consists of three rows that contain the values \( p, f_1(p) \) and \( x_1(p) \).

Then, assuming \( k=1 \), a one-dimensional optimization problem is solved:

\[
f_2(p) \min_{x_2=0,1,...,\left[\frac{p_2}{a_2}\right]} = \min \left\{ R_2(0) + f_1(p); R_2(1) + f_1(p - a_2); ... R_2(p) + f_1(p - a_2) \right\}
\]

for all integer values \( p = 0, 1, ..., p_0 \). Here, the values \( f_1(p - a_2x_2) \) are determined from table 1. To do this, you just need to take the number at the intersection of the line \( f_1(p) \) and the column containing the number \( (p - a_2x_2) \). The result of the performed procedure is table 2, which consists of...
three rows that contain the values \( p, f_2(p), x_2(p) \). Calculations similar to those in the construction of table 2 are made for all steps with indices \( k = 2,3, ..., N - 1 \). This makes it possible to construct a set of tables with the values \( f_k(p) \) and \( x_k(p) \), \( k = 1, N - 1 \), tabulated by the parameter \( p \). The function \( f_N(p) \) need not be tabulated, since we need to know its value only for \( p = p_0 \). Therefore, to determine the minimum value of the generalized indicator of the efficiency \([11, 12]\) of a multistep PMOD, a one-dimensional optimization problem is solved:

\[
\min_{x_N = 0,1, ..., N} \{ R_N(x_N) + f_N - 1(p_0 - a_N x_N) \}. \tag{16}
\]

At the second stage, using the obtained tabulated solutions (table 1, table 2, table N-1), an optimal distribution strategy \( (x^*_1, ..., x^*_N) \) is constructed. It provides the \( f_N(p_0) \) value.

At this stage, all calculations are carried out in the forward direction in time \([13, 14]\), starting from the first step (the step with the index "N") and going to the end of the multistep PMOD (the step with the index "1"). According to the optimal controlled parameter \( x^*_N = x_N(p_0) \) obtained at the first stage of solving the optimization problem (15), according to the equation of state, the value of the variable in this case, the condition is fulfilled:

\[
p_N = p_{N+1} - a_N x^*_N = p_0 - a_N x_N(p_0). \tag{17}
\]

Then the value of the controlled parameter \( x^*_{N-1} \) is determined from the \( (N - 1) \)-th tabular table. To do this, we just need to take the number at the intersection of the row \( x_N(p) \) and the column that contains the number \( p_N \). Calculations similar to those described above are done for all \( k = N - 2, N - 3, ..., 1 \). This makes it possible to determine the optimal values \([15]\) of all controlled parameters by the corresponding tabulated ones:

\[
x^*_{N-i} = x_{N-i} \left( p_0 - \sum_{k=0}^{i-1} a_N - k^*_{N-k} \right)
\]

for all \( i = 2,3, ..., N - 1 \).

### 6. Results

Let the following optimization problem be the mathematical model of the performance control in this case, the condition is fulfilled:

\[
\max_{(x_1, x_2, x_3)} \{ \sum_{i=1}^{3} R_i(x_i) \} \text{ under such condition}, \sum_{i=1}^{3} x_i = 6. \tag{19}
\]

where \( x_i, i = 1,3 \) – non-negative integers. Components \( x_j \) are the number of network nodes, the mathematical expectation of the data delivery time from the corresponding node to the required station, and the system uptime. The form of performance indicators for each \( k \)-th step, which are \( s \) – functions, is shown in Figure 1.

\[
f_1(p_2) = \max_{0 \leq x_1 \leq p_2} \{ R_1(x_1) \} \tag{20}
\]

for specific values \( p_2 = 0,1,2, ..., 6 \) are given in table 1.

| \( p_2 \) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| \( f_1(p_2) \) | 0 | 0.5 | 1 | 3 | 6 | 6.5 | 7 |
| \( x_1(p_2) \) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

At the second step (step with index "2"), a set of one-dimensional optimization problems is solved, that we can see in the mathematical expression:
\[
f_2(p_3) = \max_{0 \leq x_2 \leq p_3} \{R_2(x_2) + f_1(p_3 - x_2)\}, \tag{21}
\]

Figure 1. S-shaped functions estimating the efficiency \(R_k(x_k)\) of decision making at the 1 (curve 1), 2 (curve 2) (a) and 2 (b) steps of a 3-step PMOD.

whose optimal solutions for specific values \(p_2 = 0, 1, 2, ..., 6\) during solving the considered problem are shown in Table 2.

| \(p_2\) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---------|---|---|---|---|---|---|---|
| \(f_2(p_3)\) | 0 | 0.5 | 1 | 3 | 6 | 6.5 | 7 |
| \(x_2(p_3)\) | 0 | 1 | 0.1 | 0 | 0 | 0.1 | 0.1 |

Table 2 shows that the fixed values \(p_3 = 1, 2, 5\) and 6, the optimal value of the control parameter \(x_2(p_3)\) has a non-unique value \((x_{2,1}(p_3) = 0\) and \(x_{2,2}(p_3) = 1\)).

At the last step (the step with the index "3") for a fixed value \(p_4 = p_0 = 6\), a one-dimensional optimization problem is solved:

\[
f_3(p_4) = \max_{0 \leq x_3 \leq p_4} \{R_3(x_3) + f_2(p_4 - x_3)\}. \tag{22}
\]

The function \(f_3(p_4)\) is a multi-extremal function with the largest value equal to 7, with the following values of the controlled parameter \(x_3(p_4)\):

\[
x_{3,1}(6) = 0; x_{3,2}(6) = 1; x_{3,3}(6) = 2; x_{3,4}(6) = 4; x_{3,5}(6) = 5; x_{3,6}(6) = 6. \tag{23}
\]

Any of the six obtained values \(x_{3,4}(6), k = 1, 6\) can be chosen as the optimal controlled parameter \(x_3^*\).

\[
x_3^* = x_{3,4}(6) = 0. \tag{24}
\]

Then the state variable

\[
p_3 = p_4 - x_3^* = 6 - 0 = 6. \tag{25}
\]

Therefore, from Table 2 for a fixed value \(p_3 = 6\) we obtain that \(f_2(p_3) = 7\) for \(x_2^* = x_{2,4}(p_3) = 0\) or \(x_2^* = x_{2,2}(p_3) = 0\). This ambiguity in the choice of the controlled parameter \(x_2^*\) leads to the need to build a tree (Figure 2), in which for each of the values \(x_{2,1}(6)\) and \(x_{2,2}(6)\), its own value of the phase variable \(p_2\) is calculated:

\[
p_2^d = p_3 - x_{2,1}(p_3) = 6 - 0 = 6, p_2^d = p_3 - x_{2,2}(p_3) = 6 - 1 = 5. \tag{26}
\]
by which the values $x_1^*$ are determined from Table 1:

$$x_1^* = x_1\left(p_2^2\right) = 6; \quad x_1^* = x_1\left(p^//\right) = 5.$$  \hspace{1cm} (27)

Thus, the maximum value of the generalized performance indicator $f_3(6) = 7$ can be obtained using one of the following optimal strategies: $(x_1^*, x_2^*, x_3^*) = (6, 0, 0)$ or $(x_1^*, x_2^*, x_3^*) = (5, 1, 0)$.

![Figure 2. A fragment of the tree of optimal strategies in the case of non-uniqueness of the optimal value of the controlled parameter $x_2(p_3)(x_{2,1}^* \neq x_{2,2}^*)$.](image)

All possible variants of optimal strategies $(x_1^*, x_2^*, x_3^*)$, providing the maximum value $f_3(6) = 7$ during solving the considered problem, are shown in Table 3.

| $x_3^*$ | $x_2^*$ | $x_1^*$ | $R_1(x_1^*)$ | $R_2(x_3^*)$ | $R_3(x_3^*)$ | No of optimal strategy |
|---------|---------|---------|--------------|--------------|--------------|------------------------|
| 0       | 0       | 6       | 7            | 0            | 0            | 1                      |
| 1       | 0       | 5       | 6.5          | 0.5          | 0            | 2                      |
| 1       | 1       | 4       | 6            | 0.5          | 0.5          | 4                      |
| 2       | 0       | 4       | 6            | 0            | 1            | 5                      |
| 4       | 0       | 2       | 1            | 0            | 6            | 6                      |
| 1       | 1       | 1       | 0.5          | 0.5          | 6            | 7                      |
| 5       | 0       | 1       | 0.5          | 0            | 6.5          | 8                      |
| 1       | 0       | 0       | 0.5          | 6.5          | 7            | 9                      |
| 6       | 0       | 0       | 0            | 0            | 7            | 10                     |

7. Conclusion
In the paper modelling and optimizing of sensor wireless network systems is carried out. The performance characteristics of the wireless sensor network are shown. The general scheme of a multistep process for making an optimal decision is demonstrated. Computational scheme of the dynamic programming method is shown. Some results are obtained that shown optimal strategy during solving the problem.
References

[1] Lutakamale A S, Kaijage S 2017 Wildfire Monitoring and Detection System Using Wireless Sensor Network: A Case Study of Tanzania Wireless Sensor Network 9 274-89

[2] Minerva R, Biru A, Rotondi D 2015 Towards a definition of the Internet of Things (IoT). IEEE Internet Initiative, Torino, Italy Retrieved from: https://iot.ieee.org/images/files/pdf/IEEE_IoT_Towards_Definition_Internet_of_Things_Revision1_27MAY15.pdf

[3] Stankovic J A 2014 Research directions for the internet of things IEEE Internet of Things Journal 1 3-9

[4] Broadband Internet Technical Advisory Group. Internet of Things (IoT) Security and Privacy Recommendations 2016 Retrieved from: www.bitag.org/documents/BITAG_Report_-_Internet_of_Things_(IoT)_Security_and_Privacy_Recommendations.pdf

[5] Odu G O and Charles-Owaba O E 2013 Review of Multi-criteria Optimization Methods Theory and Applications 3 1-14

[6] Shah A and Ghaahramani Z 2015 Parallel predictive entropy search for batch global optimization of expensive objective functions Advances in Neural Information Processing Systems 3330-8

[7] Neittaanmki P, Repin S and Tuovinen T (Eds.) 2016 Mathematical Modeling and Optimization of Complex Structures; Series: Computational Methods in Applied Sciences Springer International Publishing AG, Switzerland

[8] Rios L M and Sahinidis N V 2013 Derivative-free optimization: a review of algorithms and comparison of software implementations Journal of Global Optimization 54 1247-93

[9] Morais H, Kádár P, Faria P, Vale Z A and Khodr H M 2010 Optimal Scheduling of a Renewable Micro-Grid in an Isolated Load Area Using Mixed-Integer Linear Programming Elsevier Editorial System(tm) for Renewable Energy Magazine 35(1) 151-6

[10] Orlova D E 2018 Stability of solutions in ensuring the functioning of organizational and technical systems Modeling, Optimization and Information Technologies 6(1) 325-36

[11] Talluri S, Kim M K and Schoenherr T 2013 The relationship between operating efficiency and service quality: are they compatible? Int. J Prod Res 51 548-2567

[12] Groefsema H, Beest N R T P 2015 Design-time compliance of service compositions in dynamic service environments Int. Conf. on Service Oriented Computing & Applications 108-15

[13] Yao Y and Chen J 2010 Global optimization of a central air-conditioning system using decomposition-coordination method Energy and Buildings 5 570-83

[14] Lvovich I, Preobrazhenskiy A, Preobrazhenskiy Y, Lvovich Y, Choporov O 2019 Managing developing internet of things systems based on models and algorithms of multi-alternative aggregation Int. Seminar on Electron Devices Design and Production, SED-2019 - Proceedings 8798413

[15] Lvovich I Ya, Lvovich Ya E, Preobrazhenskiy A P, Preobrazhenskiy Yu P, Choporov O N 2019 Modelling of information systems with increased efficiency with application of optimization-expert evaluation Journal of Physics: Conf. Ser. Krasnoyarsk Science and Technology City Hall of the Russian Union of Scientific and Engineering Associations; Polytechnical Institute of Siberian Federal University. Bristol, United Kingdom 33079