MODELS OF DYNAMICAL SUPERSYMMETRY BREAKING AND QUINTESSENCE.

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We study several models of relevance for the dynamical breaking of supersymmetry which could provide a scalar component with equation of state \( p = w \rho, -1 < w < 0 \). Such models would provide a natural explanation for recent data on the cosmological parameters.

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INTRODUCTION

There are increasing indications that the energy density of matter in the Universe is smaller than the critical density [1]. If one sticks to the inflation prediction of \( \Omega_T = 1 \), then the natural question is the origin of the extra component providing the missing energy density. An obvious candidate is a cosmological constant, whose equation of state is \( p = -\rho \). This faces particle physics with the unpleasant task of explaining why the energy of the vacuum should be of order \((0.003 \text{ eV})^4\), a task possibly even harder than the one of explaining why the cosmological constant is zero. In particular, it seems to require new interactions with a typical scale much lower than the electroweak scale, long range interactions that would have remained undetected.

It has recently been proposed to consider instead a dynamical time-dependent and spatially inhomogeneous component, with an equation of state \( p = w \rho \), \(-1 < w < 0\). Such a component has been named “quintessence” by Caldwell, Dave and Steinhardt [2]. Indeed, present cosmological data seem to prefer [3], in the context of cold dark matter models, a value for \( w \) of order \(-0.6\). Several candidates have been proposed for this component: tangled cosmic strings [4], pseudo-Goldstone bosons [5]. Of particular relevance to some issues at stake in the search for a unified theory of fundamental interactions is a scalar field with a scalar potential decreasing to zero at infinity. This is usually considered as a drawback of spontaneous supersymmetry breaking models from the point of view of cosmology: in the standard approach, the potential has a stable ground state, where the potential is fine tuned to zero (in order to account for a vanishing cosmological constant); but the initial conditions and the subsequent cosmological evolution may lead to a situation where the field misses the ground state and evolves to infinite values.

Dynamical supersymmetry breaking is often favoured because it can more easily account for large mass scale hierarchies such as \( M_W/M_P \) through some powers of \( \Lambda/M_P \), where \( \Lambda \) is the dynamical scale of breaking. It is thus a natural question to ask whether the corresponding models may account for quintessence. Indeed, in this case, there is a fundamental reason why the scalar potential vanishes at infinity: this is related to the old result that global supersymmetry yields a vanishing ground state energy. And there may be reasons as to why once it dominates, the contribution of the scalar field to the energy density is very small (again through powers of \( \Lambda/M_P \)).

In the following, we will discuss two models of dynamical supersymmetry breaking which may be considered as representative of semi-realistic models for high energy physics. One is based on gaugino condensation coupled to the dynamics of a dilaton field, the other uses the condensation of \( N_f \) flavors in a \( SU(N_c) \) gauge theory.

MODELS WITH A DILATON

We start with a class of models, reminiscent of many superstring models, where supersymmetry is broken through gaugino condensation [6] along the flat direction corresponding to the dilaton field. Indeed, in many superstring models, the dilaton field \( s \) does not appear in the superpotential and thus corresponds to a flat direction in the scalar potential. It couples to the gauge fields in a model-independent way:

\[ w = \left( \frac{\lambda}{\Lambda} \right)^{\frac{1}{2}} \]

This means a vanishing gauge coupling and thus restoration of supersymmetry.

In some cases, the field value may be interpreted as the inverse coupling constant associated with the dynamics responsible for supersymmetry breaking. An infinite field value smoothly decreasing to zero at infinity. This is usually considered as a drawback of spontaneous supersymmetry breaking models from the point of view of cosmology: in the standard approach, the potential has a stable ground state, where the potential is fine tuned to zero (in order to account for a vanishing cosmological constant); but the initial conditions and the subsequent cosmological evolution may lead to a situation where the field misses the ground state and evolves to infinite values.

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\[ \mathcal{L} = \frac{1}{4} s F_{\mu\nu} F_{\mu\nu} \] (1)

where \( F_{\mu\nu} \) is the field strength corresponding to a generic gauge symmetry group \( G \) and, throughout this article, \( s \) is expressed in Planck mass units. Thus the vacuum expectation value \( < s > \) can be interpreted as the inverse of the gauge coupling \( 1/g^2 \) at the string scale. Indeed, it is directly related to the inverse of the string coupling constant (see below). The interaction corresponding to the gauge group \( G \) becomes strong at a scale:

\[ \Lambda = M_p e^{-1/2b g^2} = M_p e^{-s/2b_0} \] (2)

where \( b_0 \) is the one-loop beta function coefficient of the gauge group \( G \). The corresponding gaugino fields are expected to condense:

\[ < \lambda \lambda > = \Lambda^3 = M_p^3 e^{-3s/2b_0} \] (3)

and they lead to a potential energy, quadratic in the gaugino condensates, that scales like \( e^{-3s/b_0} \). In the limit of infinite \( s \), that is of vanishing gauge coupling, the dynamics is inoperative and one recovers the flat direction associated with the dilaton.

We have followed a very crude approach and there are, of course, many possible refinements: one may include supergravity corrections, the effect of other scalar fields such as moduli, as well as corrections which may be needed to stabilize the potential for small values of \( s \) (that is in the regime of strongly coupled string [8]).

For example, in a given model [9], the potential reads,

\[ \rho_B s^2 \left( \frac{e^{-s/2b}}{2s^2} + 3H \frac{\dot{s}}{2s^2} + \frac{dV}{ds} \right) = 0 \]

where \( \rho_B \) is the background energy density associated with matter \((w_B = 0)\) or radiation \((w_B = 1/3)\) and \( \rho_s = s^2/(4s^2) + V(s) \).

If we first consider that \( V_0(s) \) is a constant and solve these equations assuming that \( \rho_B \) dominates for some time, there exists a scaling solution with the following behaviour [10]: the field \( s \) evolves down the exponentially decreasing potential as \((t/t_1)^{-3w/2b} \) as long as \( s \) remains smaller than \( s_1 = \frac{2b_0}{3} \frac{1-w_B}{1-w_B} \) reached at \( t = t_1 \); for larger values, there exists a scaling solution [11-13] where the field evolves logarithmically as \( s = s_1 + (2b_0/3) \ln(t/t_1) \).

The ratio \( \rho_s/\rho_\text{tot} \) starts at \( 3(1-w_B)^2/16 \) for \( t \leq t_1 \) and from then on slopes down to zero as \( (b_0^2/6s^2)(1+w_B) \) for large values of \( s \).

Finally, if \( w_s = \rho_s/\rho_s \) starts at a value of 1 and decreases monotonically towards \( w_B \) as \( s \) increases. There is therefore no hope of using the dilaton for the dynamical component of quintessence since \( w_s \) never reaches a negative value. Power law corrections \((V_0(s) \propto s^\alpha)\) do not change this conclusion.

This might be in some sense a welcome conclusion since the vacuum expectation value \( < s > \) provides, after renormalisation down to low energy, the fine structure constant \( 1/\alpha \). A sliding dilaton would make the fine structure constant vary with time at an unacceptable rate [14].

Similar conclusions can be reached with other types of weakly coupled scalar particles, such as the moduli of string theories. For example, in a model with several gaugino condensates and a modulus field \( t \) describing the radius of the six dimensional compact manifold, the scalar potential scales for large values of \( t \) as [15]:

\[ V = \sum_a t^{-1+b_a} e^{-\pi^2 b_a/6} e^{-2<s>/b_a} \] (8)

where the sum runs over the different condensates (one for each group \( G_a \), with corresponding beta function coefficient \( b_a \)). We have fixed the dilaton field \( s \) at its ground state value. Let us note that, although the modulus \( t \) definitely cannot be used for quintessence (since, as above, the corresponding \( w_t \) reaches asymptotically \( w_B \)), a large value of \( < s > \) may contribute to giving a small contribution from \( t \) to the vacuum energy.

**A MODEL OF FERMION CONDENSATES**

We now turn to a model which yields inverse powers of fields in the potential, a welcome situation for
quintessence models \([11]\). It is based on the gauge group
\(SU(N_c)\) and has \(N_f \leq N_c\) flavors: quarks \(Q_i, i = 1 \cdots N_f\) in fundamentals of \(SU(N_c)\) and antiquarks \(\bar{Q}_i, i = 1 \cdots N_f\) in antifundamentals of \(SU(N_c)\).

Below the scale of dynamical breaking of the gauge symmetry \(\Lambda\), the effective degrees of freedom are the fermion condensate ("pion") fields \(\Pi^i_j \equiv Q^i Q_j\). The dynamically generated superpotential reads \([15]\):

\[
W = (N_c - N_f) \frac{\lambda_{3N_c-N_f}}{(det \Pi)^{N_c-N_f}}.
\]

(9)

Usually, one allows a term linear in \(\Pi\) in the superpotential in order to stabilize this field. We will instead assume here that a discrete symmetry ensures that no linear term is allowed by the abelian symmetry. Let us note that this symmetry cannot be a continuous gauge symmetry since this would yield in the scalar potential D-terms with positive powers of \(\Pi\) which would stabilize the field.

The effective Lagrangian reads:

\[
\mathcal{L} = -\frac{1}{2} \text{Tr} \left[ \left( \Pi^i_j \Pi^j_i \right)^{-1/2} \partial_\mu \Pi^i_j \partial_\nu \Pi^j_i \right] + 2 \text{Tr} \left[ \left( \Pi^i_j \Pi^j_i \right)^{-1/2} \left( \Lambda \Lambda^\dagger \right)^{3N_c-N_f} \right] (\text{Det} \Pi^i_j \Pi^j_i)^{N_c-N_f}.
\]

(10)

where the potential originates from the F-term for the field \(\Pi\). For simplicity, we will take \(\Pi^i_j\) to be diagonal and write \(\Pi^i_j \equiv \Phi^2 \delta^i_j\) with \(\Phi\) real. One obtains:

\[
\frac{1}{4N_f} \mathcal{L} = -\frac{1}{2} \partial_\mu \Phi \partial_\mu \Phi + V(\Phi)
\]

(11)

where

\[
V(\Phi) = \lambda \phi^{4+\alpha},
\]

(12)

with \(\mu = (\Lambda \Lambda^\dagger)^{1/2}\) and

\[
\alpha = 2 \frac{N_c + N_f}{N_c - N_f}.
\]

(13)

The corresponding potential has been studied in Ref. \([11]\) in the case where \(\rho_B\) dominates over the energy density \(\rho_\Phi\) of the \(\Phi\) field. One obtains

\[
\frac{\rho_\Phi}{\rho_B} = \left( \frac{a}{a_Q} \right) \frac{6(1 + w_B)}{(2 + \alpha)}.
\]

(14)

Hence \(\rho_\Phi\) decreases less rapidly than \(\rho_B\) until it dominates it for values of the cosmic scale factor larger than \(a_Q\). Throughout this period (which must obviously include nucleosynthesis), one has:

\[
\rho_\Phi = \frac{2(2+\alpha)}{4 + \alpha(1-w_B)} \left( 3 + w_B \right) \frac{\lambda_{\frac{4+\alpha}{\mu}}}{\Phi^\alpha (\frac{a}{a_Q})^{\frac{6(1+w_B)}{(2+\alpha)}}}.
\]

(15)

The equation of state for the \(\Phi\) field has \([11]\):

\[
w_\Phi = -1 + \frac{\alpha(1 + w_B)}{2 + \alpha}.
\]

(16)

Thus, in a matter-dominated universe \((w_B = 0)\), \(w_\Phi = -1/2 + 2N_f/N_c\) which is between \(-1/2\) and 0 for \(N_f \leq N_c\). This provides a candidate for the dynamics of quintessence.

Once \(\Phi/M_P\) has reached the value \(\sqrt{\frac{\alpha(2 + \alpha)}{3(1 + w_B)}}\), we enter a different regime where \(\rho_\Phi\) dominates the energy density. The field \(\Phi\) slows down and one may solve for it neglecting the terms \(\Phi\) in its equation of motion and \(\Phi^2/2\) in \(\rho_\Phi\). One obtains:

\[
\Phi = \Phi_0 \left[ 1 + \frac{1}{2\sqrt{3}} \alpha(4 + \alpha)V(\Phi_0)(t - t_0) \right]^{\frac{3}{4 + \alpha}},
\]

(17)

where \(\Phi_0\) is the present value for \(\Phi\), and one obtains

\[
w_\Phi \sim -1 + \frac{\alpha^2}{3\Phi^2}.
\]

(18)

If \(\rho_\Phi\) at \(a_Q\) is already close to the present value (this occurs typically for \(\mu \sim 10^{-12+30\beta/(4+\alpha)}\) GeV), this second period is short \((a_Q \sim a_0)\) and \(w_\Phi\) will be given approximately by \([16]\). For simplicity, we will suppose from now on that this is so. In this case, the value of \(w_\Phi\) might prove to be too small to account for the data \([16]\).

However, larger values for \(w_\Phi\) may be obtained by complicating slightly the model and introducing other fields. As an example, we will assume the presence of a dilaton field, much in the spirit of the models of the previous section (although the dilaton is this time not sliding but fixed at its ground state value). The dynamical scale \(\Lambda\) is expressed in terms of the dilaton through \([2]\) with \(b_0 = (3N_c - N_f)/(16\pi^2)\). This induces a new term in the scalar potential:

\[
\delta V = 4s^2|F_s|^2,
\]

(19)

with

\[
F_s = \frac{dW}{ds} = -8\pi^2 \frac{\lambda_{\frac{4+\alpha}{\mu}}}{(\text{Det} \Pi)^{N_c-N_f}}.
\]

(20)

that is an extra term of the form \(\mu^{4+\beta}/\Phi^3\) with

\[
\beta = \frac{4N_f}{N_c - N_f}.
\]

(21)

Since \(\beta < \alpha\), this term dominates for large values of the condensate \(\Phi\) and, for \(w_B = 0\),
\[ w_\Phi = -1 + \frac{2N_f}{N_c + N_f}, \quad (22) \]

which precisely lies between \(-1\) and 0: taking for example \(N_c = 5\) and \(N_f = 1\) yields \(w_\Phi = -2/3\).

There could be other contributions to the \(F\)-term auxiliary field for \(S\), say \(F_0\) (which will contribute to supersymmetry breaking). If so, the leading term in \(\delta V\) for large \(\Phi\) is 

\[ F^\dagger S F_0 + F_S F_0^\dagger, \]

in which case \(w_\Phi = -1 + N_f/(N_c - N_f)\). This time, one may even obtain \(w_\Phi = -2/3\) with \(N_c = 3\) \((N_f = 1)\).

Strictly speaking, the leading term is \(|F_0|^2\) and thus of the cosmological constant type. But this is an artifact of global supersymmetry and it is well-known that, by going to supergravity, we may cancel this cosmological constant term, while keeping a non-vanishing contribution \(F_0\) to the \(F\)-term of the \(S\) field. Such a study goes beyond the framework of this paper. This stresses however an important fact: even if we deal here with a dynamical component \(\Phi\) which may account for a cosmological constant type behaviour of the cosmological parameters, it is important that the \(\Phi\) energy density eventually dominates over all other forms and thus that these other components do not produce a significant cosmological constant of their own. Thus, the cosmological constant remains a problem for all other components.

Likewise, the amount of supersymmetry breaking due to the fact that \(\Phi\) has not reached an infinite value (and thus its \(F\)-term is not vanishing) is not sufficient to account for the amount of supersymmetry-breaking observed in nature. There must be other sources (e.g. \(F_0\) in our example) which may produce unwanted amounts of cosmological constant if care is not taken.

In other words, there is still a “cosmological constant problem” in the models studied here (that is to say, from the point of view of the quantum theory) but the interest of such models lies in the fact that they can successfully account for the recent cosmological data on supernovae of type Ia, if confirmed.

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