Understanding the classes better with class-specific and rule-specific feature selection, and redundancy control in a fuzzy rule based framework

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Abstract. Recently, several studies have claimed that using class-specific feature subsets provides certain advantages over using a single feature subset for representing the data for a classification problem. Unlike traditional feature selection methods, the class-specific feature selection methods select an optimal feature subset for each class. Typically, class-specific feature selection (CSFS) methods use one-versus-all split of the data set that leads to issues such as class imbalance, decision aggregation, and high computational overhead. We propose a class-specific feature selection method embedded in a fuzzy rule-based classifier, which is free from the drawbacks associated with most existing class-specific methods. Additionally, our method can be adapted to control the level of redundancy in the class-specific feature subsets by adding a suitable regularizer to the learning objective. Our method results in class-specific rules involving class-specific subsets. We also propose an extension where different rules of a particular class are defined by different feature subsets to model different substructures within the class. The effectiveness of the proposed method has been validated through experiments on three synthetic data sets.

Keywords: Class-specific Feature selection · Rule-specific Feature Selection · Redundancy Control · Fuzzy rule-based Classifiers · Within-class substructures.

1 Introduction

Feature selection is an important step for many machine learning tasks. The main motto of the feature selection methods is to reject unnecessary and derogatory features and select the features that benefit the intended task. Traditional feature selection methods choose a single subset of the features as the “optimal” subset for the entire dataset. Apart from the commonly used global approach, some studies have used class-specific approaches for selecting features, where for each class, a unique subset of the original features is
selected. If there are $C$ classes, in the class-specific approach, $C$ subsets are chosen. In a traditional feature selection method, the selected feature subset is chosen based on the global characteristics of the data. It does not take into account any class-specific or local characteristics of the data which may be present. For example, there may exist a group of features that follows a distinct distribution for a specific class but varies randomly over the remaining classes. Such a group of features plays a significant role in distinguishing the specific class from other classes. However, such a group may not be a very useful feature subset for the $C$-class problem as a whole. Class-specific characteristics may also exist in the form of class-specific redundancy. Different sets of features could be redundant for different classes. The class-specific feature selection (CSFS) works in [8,6,9,10,11,12,13,14] have proposed suitable frameworks that exploit class-specific feature subsets to solve classification problems. They have shown that the classifiers built with the subsets chosen by class-specific methods performed better than or comparable to the classifiers built with subsets chosen by the traditional global feature selection methods. The CSFS may also enhance the transparency/explainability of the classification process associated with it [6].

The majority of the CSFS methods [8,6,10,11,13,14] follow the one-versus-all (OVA) strategy to decompose a $C$-class classification problem into $C$ binary classification problems. They choose $C$ class-specific feature subsets optimal for the $C$ binary classification problems, OVA strategy-based class-specific feature selection methods have certain drawbacks. Generally, it leads to class imbalance. The performance of an OVA-based method would depend on how efficiently the class imbalance problem is handled. The OVA strategy-based methods is computationally intensive and complex as we need to design $C$ classifiers and for testing we need an aggregation mechanism.

Here, we propose a CSFS scheme embedded in a fuzzy rule-based classifier (FRBC) that does not use the OVA strategy. Our method selects class-specific feature subsets by learning a single FRBC and hence avoid the issues introduced in the OVA-based approaches. Moreover, we extend our framework to deal with- (i) CSFC with redundancy control, and (ii) rule-specific feature selection (RSFS) that can exploit presence of substructure within a class. The rules provided by the FRBC are generally interpretable and more specific. Exploiting the class-specific local features, the proposed FRBC enjoys more transparency and interpretability than a standard classifier exploiting class-specific feature subsets. Our contributions are summarised as follows.

1. We propose a class-specific feature selection method that is not based on the OVA strategy like most of the existing class-specific feature selection schemes. Thus, our method is free from the weaknesses of the OVA strategy.
2. Our method can monitor the level of redundancy in the selected features.
3. We also propose a general version of our class-specific feature selection method that not only chooses different subsets for different classes but also, chooses different subsets within a class if different substructures are present.
2 Proposed Method

We want to develop a method for CSFS based on a training dataset. Let, the input data be \(X = \{x^i = (x^i_1, x^i_2, \cdots, x^i_P) \in \mathbb{R}^P : i \in \{1, 2, \cdots, n\}\}\). The collection of class labels of \(X\) be \(y = \{y^i \in \{1, 2, \cdots, C\} : i \in \{1, 2, \cdots, n\}\}\), where, \(y^i\) is the class label corresponding to \(x^i\). For our purposes we represent the class label of \(x^i\) as \(v\). Then translate into \(P\) fuzzy sets,

\[
\mu_k(x^i) = \prod_{l=1}^{P} \mu_{j,kl}(x^i_l),
\]

where, \(\mu_{j,kl}\) is the membership to the fuzzy set \(A_{j,kl}\) defined on the \(j\)th feature for the \(k\)th class, \(R_{kl}\) is a linguistic value (fuzzy set) defined on the \(j\)th feature for the \(k\)th rule of the \(j\)th class. Let, \(\alpha_{kl}\) be the firing strength of the rule \(R_{kl}\). The rule firing strength is computed using a T-norm \([7]\) over the fuzzy sets \(A_{1,kl}, A_{2,kl}, \cdots, A_{P,kl}\). We use the product T-norm. Let the membership to the fuzzy set \(A_{j,kl}\) be \(\mu_{j,kl}\). So, \(\alpha_{kl}\) is given by, \(\alpha_{kl} = \prod_{j=1}^{C} \mu_{j,kl}\). The final output of the FRBC is of the form \(o = (o_1, o_2, \cdots, o_C)\), where, \(o_k\) is the support for \(k\)th class, computed as \(o_k = \max(\alpha_{k1}, \alpha_{k2}, \cdots, \alpha_{kN_k})\). To learn an efficient classifier from the initial fuzzy rule-based system, the parameters defining the fuzzy sets \(A_{j,kl}\) can be tuned by minimizing the classification error,

\[
E_{cl} = \sum_{i=1}^{n} \sum_{k=1}^{C} (o^i_k - t^i_k)^2.
\]

To extract the \(l\)th rule of the \(k\)th class, \(R_{kl}\) we need to define the fuzzy sets \(A_{j,kl}\). Following \([4,5]\), we cluster the training data of the \(k\)th class into \(N_k\) clusters. We note here that the \(k\)th class may not have \(n_k\) clusters in the pattern recognition sense. By clustering we just group the nearby points and then define a rule for each group. Let the centroid of the \(l\)th cluster of the \(k\)th class be represented as \(v_{kl} = (v_{1,kl}, v_{2,kl}, \cdots, v_{P,kl})\). The cluster centroid \(v_{kl}\) is then translated into \(P\) fuzzy sets, \(A_{j,kl} = \text{“close to” } v_{j,kl} \forall j \in \{1, 2, \cdots, P\}\). The fuzzy set ‘ “close to” \(v_{j,kl}\’ is modeled by a Gaussian membership function with mean \(v_{j,kl}\). Although the membership parameters can be tuned to refine the fuzzy rules, in this study, we have not done that. We have used fixed rules defined by the obtained cluster centers.
2.2 Feature Selection

Following [2345], we use feature modulators which stop the derogatory features and promote useful features to take part in the rules of the FRBC. We choose the modulator function same as used in [5]. For each feature, there is an associated modulator of the form \( M(\lambda_j) = \exp(-\lambda_j^2) \), where \( j \in \{1, 2, \ldots, P\} \). To select or reject a feature using the modulator function, the membership values associated with the \( j \)th feature are modified as

\[
\hat{\mu}_{j,kl} = \hat{\mu}_{j,kl}^{M(\lambda_j)} = \mu_{j,kl}^{\exp(-\lambda_j^2)} \quad \forall k, l
\]  

(3)

Note that, \( \lambda_j \approx 0 \) makes \( \hat{\mu}_{j,k} \approx \mu_{j,k} \). Similarly, when \( \lambda_j \) is high (say, \( \lambda_j \geq 2 \)), \( \hat{\mu}_{j,k} \approx 1 \). The rule firing strength is now calculated as \( \alpha_{kl} = \prod_{j=1}^{P} \hat{\mu}_{j,kl} \). So, when \( \hat{\mu}_{j,kl} \approx \mu_{j,kl} \), \( j \)th feature influences the rule firing strength \( \alpha_{kl} \) and in turn influences the classification process, whereas, if \( \hat{\mu}_{j,kl} \approx 1 \) then the \( j \)th feature has no influence on the firing strength and hence on the predictions by the FRBC. This would be true for any T-Norm as \( T(x, 1) = x, x \in [0, 1] \). Thus, for useful features, \( \lambda_j \)s should be made close to zero and for derogatory features \( \lambda_j \)s should be made high. The desirable values of \( \lambda_j \)s are obtained by minimizing \( E_{cl} \) defined in [2] with respect to \( \lambda_j \)s. The training begins with \( \lambda_j = 2+ \) Gaussian noise. \( M(\lambda_j) \approx 0 \) indicates a strong rejection of \( x_j \), while \( M(\lambda_j) \approx 1 \) suggests a strong acceptance of \( x_j \). However, training may lead \( \lambda_j \)s such that \( M(\lambda_j) \) takes a value in between 0 and 1. This implies that the corresponding feature influences the classification partially. This is not desirable in our case, as our primary goal is to select or reject features. To facilitate this, we add a regularizer term \( E_{select} \) to \( E_{cl} \) such that \( E_{select} \) adds penalty if any \( \lambda_j \) allows the corresponding feature partially. In [5], \( E_{select} \) is set as follows,

\[
E_{select} = \left(\frac{1}{P}\right) \sum_{j=1}^{P} \exp(-\lambda_j^2)(1 - \exp(-\lambda_j^2))
\]  

(4)

So, the overall loss function for learning suitable \( \lambda_j \)s becomes

\[
E = E_{cl} + c_1 E_{select}.
\]  

(5)

Class-specific feature selection So far we have not considered selection of class-specific features. In the class-specific scenario, for each class, a different set of \( P \) modulators is engaged. So, a total of \( C \times P \) feature modulators are employed. Consequently, for each class a different set of features, if appropriate, can be selected. Here, we represent the feature modulator for the \( j \)th feature of the \( k \)th class as \( M(\lambda_{j,k}) = \exp(-\lambda_{j,k}^2) \), where \( j \in \{1, 2, \ldots, P\}; k \in \{1, 2, \ldots, C\} \). The modulator value \( M(\lambda_{j,k}) \) modify the membership values corresponding to the \( j \)th feature of the \( k \)th class as following:

\[
\hat{\mu}_{j,kl} = \hat{\mu}_{j,kl}^{M(\lambda_{j,k})} = \mu_{j,kl}^{\exp(-\lambda_{j,k}^2)} \forall l
\]  

(6)

For this problem, the \( E_{select} \) is changed to

\[
E_{select} = \left(\frac{1}{CP}\right) \sum_{k=1}^{C} \sum_{j=1}^{P} \exp(-\lambda_{j,k}^2)(1 - \exp(-\lambda_{j,k}^2))
\]  

(7)

We now Minimize [5] with respect to \( \lambda_{j,k} \)s to find the optimal \( \lambda_{j,k} \)s.
2.3 Monitoring Redundancy

Suppose a data set has three useful features say $x_1, x_2, x_3$ such that each of $x_2$ and $x_3$ is strongly dependent on (say correlated with) $x_1$ then all the three features carry the same information and only one of them is enough. These three form a redundant set of features. However, if we just use one of them and there is some error in measuring that feature, the system may fail to do the desired job. Therefore, instead of minimizing redundancy, a controlled use of redundant features is desirable. For the global feature selection framework, redundancy control has been realized using the feature modulators \[2, 3, 5\] by adding the regularizer (8) to (5):

$$E_r = \frac{1}{P(P-1)} \sum_{j=1}^{P} \sum_{m=1, m \neq j}^{P} \sqrt{\exp(-\lambda_j^2) \exp(-\lambda_m^2) \rho^2(x_j, x_m)}$$ \hspace{1cm} (8)

Here, $\rho()$ is the Pearson’s correlation coefficient, which is a measure of dependency between two features. When $x_j$ and $x_m$ are highly correlated, $\rho^2(x_j, x_m)$ is close to one (its highest value). In this case, to reduce the penalty $E_r$, the training process will adapt $\lambda_j$ and $\lambda_m$ in such a way that one of $\exp(-\lambda_j^2)$ and $\exp(-\lambda_m^2)$ is close to 0 and the other is close to 1. Note the (8) is not suitable for class-specific scenario. Next we change (8) for class-specific redundancy.

**Class-specific redundancy** For class-specific redundancy, we need to compute class-specific dependency of a feature pair. So we compute $\rho_k(x_j, x_m)$ between features $x_j$ and $x_m$ considering only instances of the $k$th class. In the class-specific case, for each class, we have $P$ feature modulators, $M(\lambda_{j,k}) = \exp(-\lambda_{j,k}^2)$, where $j \in \{1, 2, \cdots, P\}; k \in \{1, 2, \cdots, C\}$. So, (8) is modified as following.

$$E_{rc} = \frac{1}{CP(C-1)} \sum_{k=1}^{C} \sum_{j=1}^{P} \sum_{m=1, m \neq j}^{P} \sqrt{\exp(-\lambda_{j,k}^2) \exp(-\lambda_{m,k}^2) \rho^2_k(x_j, x_m)}$$ \hspace{1cm} (9)

Considering the class-specific redundancy, our new loss function for learning the system becomes:

$$E_{tot} = E_{cl} + c_1 E_{select} + c_2 E_{rc}.$$ \hspace{1cm} (10)

2.4 Exploiting Substructures Within a Class.

For some real world problems, the data corresponding to a class may have distinct clusters and some of the clusters may lie in different sub-spaces. For example, in a multi-cancer gene expression data set, each cancer may have several sub-types, where each sub-type is characterized by a different set of genes/features. This generalizes the concept of class-specific feature selection further. To exploit such local substructures within a class while extracting rules, we need to use rule-specific feature modulators. Each rule of the $k$th class is assumed to represent a local structure or cluster present in the $k$th class. So, for the $k$th class there are $n_k \times P$ feature modulators. For the overall system there are $n_{rule} \times P$ feature modulators.
modulators where, \( n_{\text{rule}} (= \sum_{k=1}^{C} n_k) \) is the total number of rules. A modulator function is now represented by \( M(\lambda_{j,k}) \) and the corresponding modulated membership is the following.

\[
\hat{\mu}_{j,kl} = \mu_{j,kl} M(\lambda_{j,kl}) = \hat{\mu}_{j,kl} \exp(-\lambda_{j,kl}^2)
\]

(11)

The regularizer, \( E_{\text{select}} \) is now modified as

\[
E_{\text{select}} = \left(\frac{1}{CP}\right) \sum_{k=1}^{C} \left(\frac{1}{n_k}\right) \sum_{l=1}^{P} \sum_{j=1}^{P} \exp(-\lambda_{j,kl}^2) (1 - \exp(-\lambda_{j,kl}^2))
\]

(12)

In this framework, we do not consider redundancy. Using (11) for \( E_{\text{cl}} \) and (12) for \( E_{\text{select}} \) we define the loss function \( E = E_{\text{cl}} + c_1 E_{\text{select}} \) for discovering rule-specific feature subset.

3 Experimentation

We do three experiments to validate three main contributions of our proposed framework. In Experiment 1, we show effectiveness of the proposed class-specific feature selection over the usual global feature selection using a FRBC. In Experiment 2, we demonstrate the significance of class-specific redundancy control using our approach. In Experiment 3, a data set having multiple sub-structures in different sub-spaces within a class is considered to show the utility of our method. We do not tune the rule base parameters of the FRBC and only tune the feature modulators to select/reject features. For clustering, we use the K-means algorithm. To minimize the error functions using stochastic gradient descent, we use the optimizer, \texttt{train.GradientDescentOptimizer} from TensorFlow [1]. For all experiments, the learning rate is set to 0.2. As mentioned in sec 2 we denote the class-specific feature subset for class 1 as \( s_1 \), for class 2 as \( s_2 \) and so on.

3.1 Experiment 1

For the first experiment, we have considered a three class synthetic data set Synthetic1 with six features having distributions as described in Table 1. Here,

Table 1. Description of the dataset Synthetic1

| Instances | Features | Class |
|-----------|---------|------|
| \( x_1 \cdots x_{100} \) | \( \mathcal{N}(0,0.5) \) \( \mathcal{N}(0,0.5) \) \( \mathcal{U}(-10,10) \) \( \mathcal{U}(-10,10) \) \( \mathcal{U}(-10,10) \) | 1 |
| \( x_{101} \cdots x_{200} \) | \( \mathcal{U}(-10,10) \) \( \mathcal{U}(-10,10) \) \( \mathcal{N}(0,0.5) \) \( \mathcal{N}(0,0.5) \) \( \mathcal{U}(-10,10) \) | 2 |
| \( x_{201} \cdots x_{300} \) | \( \mathcal{U}(-10,10) \) \( \mathcal{U}(-10,10) \) \( \mathcal{U}(-10,10) \) \( \mathcal{N}(0,0.5) \) \( \mathcal{N}(0,0.5) \) | 3 |

\( \mathcal{N}(m,s) \) represents a normal distribution with mean, \( m \) and standard deviation,
s; \( \mathcal{U}(a, b) \) represents a uniform distribution over the interval \((a, b)\). Without loss, we have assigned the first 100 points to class 1, next 100 points to class 2, and last 100 points to class 3. From Table 1 we can see that class 2 and class 3 are uniformly distributed over a given interval for features \(x_1\) and \(x_2\). On the other hand, class 1 is clustered around \((0, 0)\) in the feature space formed by \(x_1\) and \(x_2\). Hence the feature space formed by \(x_1\) and \(x_2\) discriminate class 1 from the other two classes. Similarly, the feature spaces formed of \((x_3, x_4)\) and \((x_5, x_6)\) discriminate class 2 and class 3 respectively, from the corresponding remaining classes. To understand the importance of CSFS, we perform both global feature selection (GFS) and CSFS, and compare their performances. We have also computed the performance of the FRBC with all features. Number of rules considered per class is one. We have conducted 5 runs for each of the FRBC. We observe from Table 2 that in class-specific feature selection, in all five runs, for each class its characteristic features (i.e. \(x_1, x_2\) for class 1 and so on) are selected. The FRBC with the class-specific selected features has achieved an average accuracy of 98.7\% - in fact, each run achieved the same accuracy. Whereas, in global feature selection, the selected subset is \(x_3, x_6\). The FRBC using globally selected feature subset has achieved an accuracy of \(34.7\%\) in each of the five runs. One can argue that the class-specific model uses all six features, hence performs better than the global model which uses two features. But, when we learn the FRBC rules using all six features it has achieved an average accuracy of \(62.08\%\) over the five runs. Importance of class-specific feature selection is clearly established through this experiment.

### 3.2 Experiment 2

For Experiment 2, we have considered another synthetic dataset Synthetic2 which is produced by appending two additional features \(x_7\) and \(x_8\) to Synthetic1 data set. For class 1, \(x_7\) and \(x_8\) are generated as \(x_1 + \mathcal{N}(0, 0.1)\) and \(x_2 + \mathcal{N}(0, 0.1)\), respectively. For the other two classes, \(x_7\) and \(x_8\) are generated from \(\mathcal{U}(-10, 10)\) and \(\mathcal{U}(-10, 10)\), respectively. We observe that \(x_7\) is dependent on \(x_1\) and \(x_8\) is dependent on \(x_2\) for class 1 but the remaining two classes are indiscernible among themselves considering a feature space with \(x_7\) and \(x_8\). Clearly, the features \(x_7, x_8\) are also discriminatory for class 1. However, do \(x_7, x_8\) add any information over \(x_1, x_2\) for class 1? The answer is no, as for class 1, \(x_7\) and \(x_8\) are noisy versions of \(x_1\) and \(x_2\), respectively. This feature redundancy is specific to class 1. In Table 3 we have described the performances of the FRBCs in the CSFS framework without and with class-specific redundancy control. Here also, we have set the

| Run | Features selected | Avg. accuracy of FRBC (%) |
|-----|------------------|-------------------------|
| 1-5 | \(s_1 : x_1, x_2; s_2 : x_3, x_4; s_3 : x_5, x_6\) | 98.7 | 34.7 | 62.08 |
number of fuzzy rules per class as one and repeated the experiments five times with each model. The term ‘Acc.’ mentioned in Table 3 refers to accuracy of the FRBC in percentage. Table 3 confirms the effectiveness of using class-specific redundancy control. For class 1, features $x_1$ and $x_7$ are heavily dependent. Hence, to avoid redundancy only one of them should be selected. The same argument is true for features $x_2$ and $x_8$. Using an objective function (10) which considers a regularizer on class-specific redundancy associated penalty (9), the FRBC has successfully chosen only one from $x_1$ and $x_7$ and one from $x_2$ and $x_8$ to include in $S_1$ in all five runs. On the other hand, we observe that without any redundancy control, the class-specific feature selection framework selects all the four discriminating features to include in $S_1$ in four runs. The best accuracy achieved by the CSFS framework without any redundancy control and that of CSFS with class-specific redundancy control are same and equal to 99.3% although the later selects only two features. This experiment establishes the benefit of class-specific redundancy control.

3.3 Experiment 3

In experiment 3, we validate our proposed framework for handling the presence of different clusters or structures in different sub-spaces within a class. We have synthesized, Synthetic3, a two class data having four features where each class is composed of two distinct clusters lying in two different sub-spaces. The data set Synthetic3 is described in Table 4. Without loss, we have assigned the first

| Run | Without redundancy control | With class-specific redundancy control |
|-----|-----------------------------|----------------------------------------|
|     | Selected features           | Acc. | Selected features           | Acc. |
| 1   | $s_1; x_1, x_2, x_7, x_8; s_2; x_3, x_4; s_3; x_5, x_6$ | 99.3 | $s_1; x_2, x_7; s_2; x_3, x_4; s_3; x_5, x_6$ | 99.3 |
| 2   | $s_1; x_1, x_2, x_7; s_2; x_3, x_4; s_3; x_5, x_6$ | 99.3 | $s_1; x_1, x_2; s_2; x_3, x_4; s_3; x_5, x_6$ | 99.3 |
| 3   | $s_1; x_1, x_2, x_7, x_8; s_2; x_3, x_4; s_3; x_5, x_6$ | 99.3 | $s_1; x_1, x_2; s_2; x_3, x_4; s_3; x_5, x_6$ | 99.3 |
| 4   | $s_1; x_1, x_2, x_7, x_8; s_2; x_3, x_4; s_3; x_5, x_6$ | 99.3 | $s_1; x_1, x_2; s_2; x_3, x_4; s_3; x_5, x_6$ | 99.3 |
| 5   | $s_1; x_1, x_2, x_7, x_8; s_2; x_3, x_4; s_3; x_5, x_6$ | 99.3 | $s_1; x_1, x_2; s_2; x_3, x_4; s_3; x_5, x_6$ | 99.3 |

Table 4. Description of the dataset Synthetic3

| Instances | Features | Class |
|-----------|----------|-------|
| $x_1 \cdots x_{100}$ | $N(0,0.5)$, $N(-5,0.5)$ | 1 |
| $x_{101} \cdots x_{200}$ | $U(-10,10)$, $U(-10,10)$ | 1 |
| $x_{201} \cdots x_{300}$ | $U(-10,10)$, $N(0,0.5)$ | 2 |
| $x_{301} \cdots x_{400}$ | $N(5,0.5)$, $U(-10,10)$ | 2 |
200 points to class 1, and the last 200 points to class 2. For class 1, instances 1 to 100 create a distinct cluster around \((0, -5)\) in the feature space formed of \(x_1, x_2\) and instances 101 to 200 create a distinct cluster around \((0, 0)\) in the feature space formed of \(x_3, x_4\). Similarly, class 2 is also composed of two groups of points creating two distinct clusters in the feature spaces formed of \(x_2, x_3\) and \(x_1, x_4\) respectively. To handle a dataset like Synthetic3 having within-class substructures we employ our proposed rule-specific approach implemented using (11), and (12). As observed from Table 5, the rule-specific feature selection is successful in identifying the two important sub-spaces i.e. \(x_1, x_2\) and \(x_3, x_4\) for class 1 and \(x_2, x_3\) and \(x_1, x_4\) for class 2. It is noteworthy that for both the classes the selected rule-specific subsets interchange between rule 1 and 2. This is natural as the cluster number assignment to different groups of points for a class varies.

We also note from Table 5, using the class-specific feature selection method, in different runs, \(s_1\) comprises of \(x_1\) or \(x_2\) and \(s_2\) comprises of \(x_2\) or \(x_1, x_2\). These subsets obviously do not characterize the classes correctly. The average accuracy of the FRBC with using feature subsets selected by rule-specific, class-specific feature selection and using all features are 100%, 77.4%, and 87.5%, respectively. This demonstrates the usefulness of our proposed RSFS framework in data sets having multiple subspace-based structures or clusters within a class.

### Table 5. Features subsets selected for synthetic3

| Run | Rule-specific | Class-specific |
|-----|---------------|---------------|
| 1   | rule 1: \(x_1, x_2\); rule 2: \(x_3, x_4\) | rule 1: \(x_2, x_3\); rule 2: \(x_1, x_4\) |
| 2   | rule 1: \(x_1, x_2\); rule 2: \(x_3, x_4\) | rule 1: \(x_1, x_4\); rule 2: \(x_2, x_3\) |
| 3   | rule 1: \(x_3, x_4\); rule 2: \(x_1, x_2\) | rule 1: \(x_1, x_4\); rule 2: \(x_2, x_3\) |
| 4   | rule 1: \(x_3, x_4\); rule 2: \(x_1, x_2\) | rule 1: \(x_1, x_4\); rule 2: \(x_2, x_3\) |
| 5   | rule 1: \(x_3, x_4\); rule 2: \(x_1, x_2\) | rule 1: \(x_2, x_3\); rule 2: \(x_1, x_4\) |


4 Conclusion

In this work, first, we have proposed a class-specific feature selection scheme using feature modulators embedded in a fuzzy rule-based classifier. The feature modulators can allow or stop the features from participating in the classification process by modifying their parameters. The feature modulator parameters are tuned by minimizing a loss function comprising of classification error and a regularizer to make the modulators completely select or reject features. This framework is used in [11] for selecting globally useful features. We modified it to make it suitable for CSFS. Our proposed class-specific feature selection method does not employ OVA strategy like most of the existing class-specific feature selection works and hence free from the enhanced computational overload and hazards associated with the existing OVA based methods. We have two more
contributions. First, we have extended the CSFS scheme so that it can monitor class-specific redundancy by adding a suitable regularizer. Second, our CSFS framework is generalized to a rule-specific feature selection framework to handle the presence of multiple sub-space based structures or clusters within a class. All three approaches are validated through three experiments on appropriate synthetic data sets.

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