Study of problem solution quality for detection of discrete failure parameter by complex adaptive data processing algorithms for mobile ground object navigation systems

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Abstract. The study of the possibility of detecting radio signals of global satellite navigation systems in the mobile ground object navigation systems, developed on complex adaptive algorithms of data processing is conducted. Quality assessment of the discrete parameter detection characterizing the presence or absence of radio signals is performed by the methods of statistical computer modeling. The probability of the false alarm and probability of a miss dependences on the threshold level, and the total probability error dependence on the signal-noise ratio at the receiving channel input by the maximum-correctness criteria are obtained.

Keywords: Identification, adaptive estimation, discrete-continuous process, Markov theory of stochastic processes estimation

1. Introduction
Most of modern navigation systems for mobile ground objects include the inertial navigation system (INS) and the receiver for global satellite navigation system (GNSS) radio signals. In spite of its high accuracy in determining the object location coordinates (SD from 1.5 m to 5 m) [1, 2], mobile ground object navigation systems are not able to provide the user with reliable data, if, for example, at any time there is no signal at the GNSS receiver input. The input signal absence, even if it is a short-term one, adversely impacts the navigation data reliability, so it is necessary to monitor the presence of signals at the GNSS receiver input. In the radio signal absence from at least one navigation spacecraft of the selected working constellation, it is necessary to reconfigure the GNSS working constellation.

Complementary to monitoring the radio signals presence, it is also important to control the reliability of the received navigation data. Even with a radio signal at the GNSS receiver input it is not always possible to obtain reliable data. This is due to the fact that the navigation signal is exposed to a large number of natural and artificial origin noises. Besides, object-orientated variation of the navigation data in the navigation spacecrafts is possible. Therefore, to provide the user with reliable data, autonomous navigation data integrity monitoring is required, which involves the possibility of simultaneous determining the navigation spacecrafts failure and transmission of inaccurate navigation data by GNSS navigation spacecrafts [3].

Currently a common drawback of the existing mobile ground object navigation system with internal autonomous monitoring of the GNSS navigation data integrity is that the control methods used in the GNSS receiver are based on secondary information processing [4-7]. Secondary information processing is the processing of output signals from measurers to determine navigation data performed in specialized computer. Methods of secondary information processing do not allow solving the
problem of determining the radio signals presence at the GNSS receiver input, since they are not used for processing radio signals. In case of no radio signal at the GNSS receiver input, noises can be perceived as a radio signal, this signal will be processed, and as a result inaccurate navigation data can be obtained.

In view of the above stated, the mobile ground object navigation system should be supplemented with autonomous integrity control systems that allow one to solve the problem of detection, identification and adaptive signals estimation and use primary information processing methods. Primary information processing is the search, detection, selection, transformation and amplification of the measurers input signals to determine the navigation data. The detection problem solution answers the signal presence question at the GNSS receiver input, and the adaptive estimation problem solution, involving model identification of the object by the measurement signals, allows one to answer the navigation information reliability question, transmitted by the GNSS signals.

This problem was solved in [8, 9] using the Markov theory of optimal nonlinear filtering [10, 11]. As a result, algorithms were synthesized that are optimal by the maximum a posteriori probability estimate, and a multichannel information processing diagram was developed. The diagram is shown in figure 1.

![Information processing diagram](image)

**Figure 1.** Information processing diagram

The results of the multichannel data processing scheme are:
- mobile ground object coordinates and motion parameters estimates included in the vector of a continuous process \( \mathbf{X}(t_{k+1}) \);
- the presence or absence GNSS signals assessment and the transmitted service information correctness evaluation in GNSS signals, combined into a two-component discrete vector process \( \Omega(t_{k+1} - 0) \);
- the transmitted navigation data reliability parameter \( \mu_i(t_k) \) estimation, a constant error of the accelerometer \( \Delta a^c_i(t_k) \) being used as this parameter.

In papers [7, 8] there was no research conducted on the problem solution quality for GNSS radio signals detection.

The aim of the work is to estimate the signal detection problem solution quality by means of statistical computer simulation of algorithms obtained in [7, 8] for signal detection, identification and adaptive estimation.
2. Problem statement

To assess the detection problem solution quality in the tracking mode, a computer experiment was performed by the statistical testing method. The aim of the experiment was to determine detection characteristics for the synthesized algorithms of joint detection, identification and adaptive estimation, namely: the probability of a false alarm, the probability of a miss and the total probability error [12].

We assume that at the \( \ell \)-th, \( \ell = 1,4 \) GNSS receiver radio signals input at the time interval \( t \in [t_k, t_k + T] \) the process implementation in the following form is observed

\[
\xi_{\ell k}(t) = \lambda_{\ell k} S_{\ell k}[\tau, x, \theta_{\ell k}] + n_{\ell}(t), \ t \in [t_k, t_k + T],
\]

in which the signal has the form:

\[
S_{\ell k}[\tau, x, \theta_{\ell k}] = A_{\ell k} g(t - \tau) \cos[\omega_{\ell k} t + \theta_{\ell k}], \ t \in [t_k, t_k + T],
\]

where \( A_{\ell k} \) is the amplitude of the signal; \( g(\tau) \) is the pseudorandom sequences with a known law of formation; \( x = x_1 / c \) is the delay of the signal relative to a given time scale; \( \lambda_{\ell k} \) is the two-valued discrete parameter (detection parameter); \( \theta_{\ell k} \) is the discrete parameter used to transmit service information.

We denote the true parameter value in the implementation of observation (1) through \( \tau_{\ell} \). In this case, the expressions describing the signal at the correlation receiver output, for example, for the case \( \theta_{\ell k} = 1 \), are similar to [10] in the form:

\[
\Phi_{\ell}(t_{k+1} - t_k, x(t_k), \lambda, \theta, t_k + 0) = 0, \theta_{\ell k} = 0 = \left\{ \begin{array}{l}
\frac{t_{k+1}}{2} F_{\ell}(t, x, \lambda, \theta, t_k + 0) = 0, \theta_{\ell k} = 0 = 1
\end{array} \right\} dt =
\]

\[
= \exp[\Phi_{\ell}(t_{k+1} - t_k, x(t_k))] + \Phi_{\ell}(t_{k+1} - t_k, x(t_k)),
\]

in the signal absence

\[
\Phi_{\ell}(t_{k+1} - t_k, x(t_k), \lambda, \theta, t_k + 0) = 0, \theta_{\ell k} = 0 = \left\{ \begin{array}{l}
\frac{t_{k+1}}{2} F_{\ell}(t, x, \lambda, \theta, t_k + 0) = 0, \theta_{\ell k} = 0 = 1
\end{array} \right\} dt =
\]

\[
= \exp[\Phi_{\ell}(t_{k+1} - t_k, x(t_k))],
\]

where signal and noise functions are determined by the expressions:

\[
\Phi_{\ell}(t_{k+1} - t_k, x(t_k)) = \frac{2A_{\ell k}^2 t_{k+1}}{N_0} g(\tau - x_\ell) g(\tau - x) dt
\]

\[
\Phi_{\ell}(t_{k+1} - t_k, x(t_k)) = \frac{2A_{\ell k}^2 t_{k+1}}{N_0} n_{\ell}(\tau) g(\tau - x) dt
\]

Suppose that the pseudorandom sequences delay uncertainty does not exceed the pseudorandom sequences unit interval period. In this case, the likelihood functional \( \Phi_{\ell} \) when approximating the pseudorandom sequences by rectangular pulses sequence with duration \( \tau_p \) take the form:

\[
\Phi_{\ell}(M \tau_p, x) = 2(qM / \tau_p) \left[ \frac{2A_{\ell k}^2 \tau_p^2}{2N_0} g(x) + \sqrt{2qM / \tau_p} \gamma(x) \right].
\]

In the expressions (3), (4) \( q = A_{\ell k}^2 \tau_p / 2N_0 \) is the signal-noise ratio per one the pseudorandom sequences element, \( M \) is the total pseudorandom sequences received elements number, \( M_1 = M / 2 \) is the pseudorandom sequences number updates for the time \( T = t_{k+1} - t_k = M \tau_p \) determined by the law of its
formation, $\Theta(x)$ is the standard Wiener process, its changes in the coordinate $x$ being described by the equation

$$\frac{d\Theta(x)}{dx} = n_g(x), \quad \Theta(0) = 0,$$

where $n_g(x)$ is the standard forming white Gaussian noise.

The computer experiment functional scheme is shown in figure 2.

![Functional diagram of the computer experiment](image)

Figure 2. Functional diagram of the computer experiment

Signal and noise function simulators generate signals in accordance with expressions (3) and (4), respectively. In the processing unit, statistical processing is performed to calculate the detection characteristics. Given that the discrete failure parameter a priori probabilities at a time $t_j$ are known, we use the maximum-correctness criteria (ideal observer criterion) to solve the detection problem [12, 13].

3. Theory

According to the maximum-correctness criteria it is necessary to calculate the probability of false alarm dependence and the probability of a miss on the threshold. The threshold is then set to minimize the total probability error. Thus, the optimal character of an ideal observer is that it minimizes the total probability error or, otherwise, maximizes the probability of detection [14].

4. Experimental

To obtain the probability of false alarm dependence on the threshold value, a noise function simulator was connected to the processing device input (figure 2). $N_{op}$ experiments were conducted. In each experiment, the probability of false alarm $P_F(j), j = 1, N_{op}$ was estimated by the signal exceedances number ratio at the output of the noise function simulator of the set detection threshold to the total number of tests $K$:

$$P_F(j) = \frac{1}{K} \sum_{i=1}^{K} d_{ij}(j),$$

where the value $d_{ij}(j)$ being equal to 0 or 1, is an the threshold indicator when no signal is applied. $d_{ij}(j) = 1$ is when the threshold is exceeded. $d_{ij}(j) = 0$ is when the threshold is not exceeded. Note, that to calculate the threshold fixing the probability of false alarm, $P_F(j) = 10^{-r}, (r = 1, 2)$ at least $K = 10^{r+1}$ tests are required. After $N_{op}$ experiments the probability of false alarm mean was determined

$$P_F(j) = \frac{1}{N_{op}} \sum_{i=1}^{N_{op}} P_F(i).$$

By calculating different threshold values, one can obtain a dependency $P_F$ on the threshold level.

To calculate the probability of a miss dependence on the threshold level, the signal function simulator was connected to the adder and the observed process was subjected to the same transformations. In doing so, the value was taken as the probability of target missing in the $j$ experiment.
\[ \beta(j) = \frac{1}{K} \sum_{i=1}^{K} d_{0i}(j), \]

where \( d_{0i}(j) \) is threshold not exceeding indicator in the process at the signal function simulator output. \( d_{0i}(j) = 0 \) is when the threshold is exceeded, \( d_{0i}(j) = 1 \) is when the threshold is not exceeded.

After calculating the probability of a false alarm and the probability of a miss dependencies, the threshold with minimal total probability error is determined and this value is fixed. By doing these calculations for different signal-noise ratio values, we can obtain the dependence of a total probability error on this ratio.

5. Research results

The calculations were carried out with the following initial data:

\( \tau_p = 1 \times 10^{-6} \text{c}, M = 20000, K = 10^4, N_{op} = 10^2. \) The value of the useful signal delay error \( x - x_p \) relative to the given time scale was chosen to be equal to the root mean square (RMS) value of a posterior estimation error.

The results of calculations for different signal-noise ratios are shown in figure 3 – figure 7. Figure 3– Figure 6 show the probability of a false alarm \( P_F \), the probability of a miss \( \beta \) and the total probability error \( P_e \) dependences on the normalized threshold relative to the maximum value

\[ \delta_h = \frac{h}{h_{max}}, \quad h_{max} = \Phi_{h,i}(M\tau_p,x = x_p), \] for the signal-noise ratio \( q = 10^{-3}, q = 10^{-4}, q = 10^{-5} \) and \( q = 6 \times 10^{-3}. \)

![Figure 3](image-url)

**Figure 3.** The probability of a false alarm \( P_F \), the probability of a miss \( \beta \) and total probability error \( P_e \) dependences on the normalized threshold relative to the maximum value \( \delta_h = \frac{h}{h_{max}}, h_{max} = \Phi_{h,i}(M\tau_p,x = x_p) \), for the signal-noise ratio \( q = 10^{-3} \).
Figure 4. The probability of a false alarm $P_F$, the probability of a miss $\beta$ and total probability error $P_e$ dependences on the normalized threshold relative to the maximum value $\delta_h = \frac{h}{h_{\text{max}}}$, $h_{\text{max}} = \Phi_{1}(M \tau_{p}, x = x_{pe})$, for the signal-noise ratio $q = 10^{-4}$.

Figure 5. The probability of a false alarm $P_F$, the probability of a miss $\beta$ and total probability error $P_e$ dependences on the normalized threshold relative to the maximum value $\delta_h = \frac{h}{h_{\text{max}}}$, $h_{\text{max}} = \Phi_{1}(M \tau_{p}, x = x_{pe})$, for the signal-noise ratio $q = 10^{-5}$.
Figure 6. The probability of a false alarm $P_F$, the probability of a miss $P_M$ and total probability error $P_e$ dependences on the normalized threshold relative to the maximum value $\delta_h = \frac{h}{h_{\text{max}}}$, $h_{\text{max}} = \Phi_{\text{h}}(Mx_P, x = x_P)$, for the signal-noise ratio $\eta = 6 \times 10^{-3}$.

Figure 7 shows the total probability error $P_e$ minimum value dependence on the signal-noise ratio.

Figure 7. The total probability error $P_e$ minimum value dependence on the signal-noise ratio

6. Results and discussion
The calculations show that there are thresholds $\delta_{h_0}$ at which the total probability error minimum value is achieved. With the signal-noise ratio equal to $\eta = 10^{-4}$, the total probability error minimum value is $P_e = 0.39189$, and with ratio equal to $\eta = 10^{-5}$ is $P_e = 0.45544$. The threshold levels $\delta_{h_0}$, at which the total probability error minimum value are reached, become lower with increasing signal-noise ratios. So, with $\eta = 10^{-5}$ the threshold value is $\delta_{h_0} = 0.5\delta_h$, with $\eta = 10^{-3}$ it is $\delta_{h_0} = 0.2\delta_h$. 
On increasing the signal-noise ratio, the total probability error minimum value decreases. For example, with the signal-noise ratio equal to \( q = 10^{-5} \), it has the value \( P_e = 0.45544 \), and if the ratio is equal to \( q = 6 \times 10^{-3} \), its value is \( P_e = 0.13825 \).

7. Conclusion

Thus, the GNSS signals detection quality estimation by the synthesized algorithms of detection, identification and adaptive estimation is carried out by methods of statistical computer modeling. The probability of a false alarm and the probability of a miss dependences on the threshold level, as well as the maximum-correctness criteria, the total probability error dependence on the signal-noise ratio at the GNSS receiver input were obtained. It is shown that the synthesized complex optimal adaptive information processing algorithms in navigation system based on GNSS have a high signal detection probability at the receiving channel input. So with the signal-noise ratio equal to \( q = 10^{-3} \) the total probability error does not exceed the value \( P_e = 0.23984 \). With the growth of signal-noise ratio, the total probability error is reduced and does not exceed the value \( P_e = 0.13825 \) at \( q = 6 \times 10^{-3} \).

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