We present predictions for the Z-boson $p_t$-spectrum at Tevatron within the framework of unintegrated distributions evolved according to evolution equations recently proposed by us. We discuss the dependence of the results on the choice of non-perturbative parameters, the coupling constant and the impact of soft gluon resummation.

Keywords: TMD distributions, Drell-Yan, pQCD, evolution equations, transverse momentum resummation

PACS Nos.: 12.38.Bx,12.38.Cy,13.60.-r,13.85.Ni

1. Introduction

Transverse momentum dependent, or equivalently, unintegrated distributions are currently object of intense research activity. The motivation for such an interest relies on their wide range of applicability, from spin physics to jet observables in high energy collisions at LHC. Their correct formalization in quantum field theory, in the present case in quantum chromodynamics, is however far from being trivial. In order to investigate various properties of unintegrated distributions, detailed calculations in various gauges have been performed. The structure of additional rapidity divergences is understood and an ad hoc subtraction scheme has been proposed. Their factorization properties in hard processes have been also investigated and a factorization theorem has been given. A point which, to date, escapes a rigorous answer is how these distributions behave as long as the scale which characterizes the hard process is varied. If this behaviour were known we would be able to relate to each other results coming from different experiments, possibly at different energies. The latter possibility is therefore of great phenomenological importance. Although a definitive answer to this question, especially in the light of new developments in the field, is absent in the literature, there have been however some attempts. In particular evolution equations for unintegrated distributions were proposed in the unpolarized time-like case and very recently extended to space-like kinematics. In a subsequent phenomenological study, performed in the context of semi-inclusive
deep inelastic scattering, it was shown that a reasonable description of data could be obtained once unintegrated evolution equations were solved with suitable, but motivated, initial conditions and assuming factorization for the cross-sections of interest. This result has stimulated us to apply the same formalism to Drell-Yan type process in hadronic collisions. The $p_t$-spectrum of the gauge boson has, in fact, a rich structure and manifests many perturbative and non-perturbative features of the underlying theory. In particular techniques for the resummation of the perturbative series in the multiple soft gluon emission limit were first developed for this prototype observable.

2. **Unpolarized evolution**

We briefly summarize the basic ingredients of $k_t$-evolution equations. Let us consider parton emissions off a active, space-like, parton line in ladder approximation. In the collinear limit, at each branching, the active parton increases its virtuality and acquires a small relative transverse momentum with respect to the parent. These iterated emissions generate therefore an appreciable transverse momentum, up to the order of the hard scale in the process, which adds to the non-perturbative one due to Fermi motion of the parton in the parent hadron. Collinear emissions give however leading logarithmic corrections to cross-sections when the transverse momenta are ordered along the ladder and can be resummed to all orders by using DGLAP evolution equations. In the unintegrated case, unlike DGLAP case, the integration on relative transverse momenta at each branching are left undone.

\[
Q^2 \frac{\partial F^\rho_i(x_B, Q^2, \mathbf{k}_\perp)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{x_B}^1 \frac{du}{u^3} P_{ji}(u, \alpha_s(Q^2)) \cdot \int \frac{d^2l_\perp}{\pi} \delta((1-u)Q^2 - l_\perp^2) F^\rho_j \left( \frac{x_B}{u}, Q^2, \frac{k_\perp - l_\perp}{u} \right),
\]

(1)

as indicated by the additional $d^2l_\perp$ integration. In the splitting $k \to k + p$ depicted in Fig. 1, the parton $k$ carries a fraction $u$ of the fractional momentum of $\tilde{k}$. With this notations the following mass-invariant constraint can be derivered:

\[
l_\perp^2 = -(1-u)k^2 + u(1-u)\tilde{k}^2 - up^2.
\]

(2)

If one assumes that the virtualities increase along the ladder, $k^2 \gg \tilde{k}^2$, and on-shell partons are emitted, $p^2 = 0$, the last two terms in eq. (2) can be disregarded. In these limits, setting $-k^2 = Q^2$ one obtains $l_\perp^2 = (1-u)Q^2$, which can be found in the $\delta$-function in eq. (1). The transverse arguments of $F^\rho_j$ on r.h.s. of eq. (1) are derived by taking into account the Lorentz boost of transverse momenta from the emitting parton $\tilde{k}$ reference frame to the interacting $k$ parton one. In particular, the transverse momentum $\bar{k}_\perp$ of the parton which undergoes the splitting can be expressed as follows:

\[
\bar{k}_\perp = (k_\perp - l_\perp)/u.
\]

(3)
Fig. 1. The incoming proton on the bottom left is assigned to have unitary momentum and defines the reference axis with respect to which the transverse momentum \( k_\perp \) and \( k_\perp' \) are defined. The small blob represents the iteration of emissions in the parton ladder of whose only the last, \( k \to k + p \), is explicitly shown. \( l_\perp \) is the relative transverse momentum between parton \( k \) and \( p \). The interacting parton \( k \) enters the hard scattering vertex indicated by the blob on top of the diagram.

Unintegrated parton distribution functions \( F_i^p(x_B, Q^2, k_\perp) \) in eq. (1) give the probability to find, at a given scale \( Q^2 \), a parton \( i \) with longitudinal momentum fraction \( x_B \) and transverse momentum \( k_\perp \) relative to the parent hadron, see Fig. (1). \( P_{ji}(u) \) are the space-like splitting functions. The unintegrated distributions fulfil the normalization condition:

\[
\int d^2 k_\perp F_i^p(x_B, Q^2, k_\perp) = f_i^p(x_B, Q^2),
\]

where \( f_i^p \) are ordinary parton distributions. It is important to remark that, given eq. (4), performing the \( d^2 k_\perp \) integration on both side of eq. (1), we recover the integrated evolution equations \( \ref{eq:evolution} \) for \( f_i^p \).

3. The \( p_t \)-spectrum of \( Z \) boson at Tevatron

The analysis described in the following is inspired to a similar one \( \ref{13} \), in which however a different kind of evolution for of unintegrated distributions is assumed. The differential cross-sections for \( Z \)-production at rapidity \( y \) and transverse momentum \( p_t \) is given by

\[
\frac{d^3 \sigma}{dy d^2 p_t} = \sigma_0 \sum_q w_q^2 \int d^2 k_{\perp,1} \int d^2 k_{\perp,2} \delta^{(2)}(k_{\perp,1} + k_{\perp,2} - p_t) \cdot \left[ F_q(x_1, \mu^2, k_{\perp,1}) F_{\bar{q}}(x_2, \mu^2, k_{\perp,2}) + (1 \leftrightarrow 2) \right].
\]

This formula can be understood as the counterpart of the standard factorization formula for Drell-Yan type processes \( \ref{13} \). The weak charges are denoted by \( w_q \), the parton momentum fractions are evaluated through \( x_{1,2} = m_T / \sqrt{s} \exp(\pm y) \) with \( m_T = \sqrt{M^2 + p_T^2} \) being the transverse mass of the gauge boson. The hadronic collision energy is \( \sqrt{s} = 1.8 \) TeV and \( \sigma_0 = \frac{\pi \alpha^2}{4 s} \). Parton distributions are evaluated at the scale \( \mu^2 = m_Z^2 \). The differential cross-section, eq. (5), is then
integrated according to the experimental cuts\cite{16} and accounted for the Z branching ratio into electrons, \(BR(e^+e^-) = 0.033632\). The initial conditions for \(k_t\)-evolution equation at the minimum scale \(Q_0^2 = 5 \text{ GeV}^2\) are chosen as a product of longitudinal parton densities\cite{14} times a gaussian transverse factor with \(x\)-independent and flavour independent width\cite{7}:

\[
F_i(x_B, Q_0^2, k_{t,i}) = f_P(x_B, Q_0^2) \frac{e^{-k_{t,i}^2}}{\pi k_{t,i}^2_i} \quad i = q, \bar{q}, g. \tag{6}
\]

We evolve light flavours only\cite{14} and effects of heavy flavours are included in the running of the strong coupling evaluated in leading logarithmic approximation. We tune the parameters appearing in the intial conditions, eq. (6), as well as the strong coupling at the Z-boson mass, \(\alpha_s(M_Z^2)\), to data. During this procedure we have observed two peculiar features: setting the quark intrinsic momentum \(\langle k_{t,q}^2 \rangle\) to larger values shifts the position of the maximum towards higher \(p_t\) while lowering the value of the coupling, \(\alpha_s(M_Z^2)\), do overestimate its height and viceversa. The tuning procedure gives for the quarks (and antiquarks) an intrinsic momentum \(\langle k_{t,q}^2 \rangle = 4 \text{ GeV}^2\). Quite interestingly the same amount of intrinsic transverse momentum is also required by Monte Carlo programs\cite{15} when used to predict the same process. We will discuss this large value, well above the one expected by Fermi motion, after the discussion of soft gluon resummed results. Predictions are, as expected, almost insensitive to the gluon intrinsic transverse momentum so that we fix it at \(\langle k_{t,g}^2 \rangle = 1 \text{ GeV}^2\). In order to have a reasonable agreement with data, a large value of the coupling is required, namely \(\alpha_s(M_Z^2) = 0.150\). The high value for \(\alpha_s(M_Z^2)\) indicates that a more rapid evolution is necessary, especially in the low \(p_t\) region. In order to have a better description of the \(p_t\)-distribution tail, we use the full invariant mass-constraint at the branching vertex, eq. (2), still however considering emission
of massless partons, $p_t^2 = 0$. The predictions within these settings are shown, along with experimental data, in Fig. (2). The main features already observed in the analysis of semi-inclusive deep inelastic scattering appear here again: the use of $k_t$-dependent distributions allows one to describe gauge boson $p_t$-spectrum without resorting to any artificial procedure to match higher order pQCD corrections at large $p_t$ and non-perturbative predictions at small $p_t$, possibly corrected for soft gluon emissions. This quite interesting feature is the result of taking in full account the transverse kinematics in the proposed evolution equations. As is well known, the low $p_t$ part of the spectrum is sensitive not only to pure non-perturbative effects but also to effects coming from multiple soft gluon emissions. In this case the $k_t$-evolution equations can be slightly modified in order to resum logarithms of soft nature. In the non-singlet channel the resummed evolution equation reads

$$Q^2 \frac{dF_p(x_B, Q^2, k_{\perp})}{dQ^2} = \int_{x_B}^{1} \frac{du}{u^3} \left[ \frac{\alpha_s(Q^2(1-u))}{2\pi} \hat{P}_{qq}(u) \right] + \int \frac{d^2l_{\perp}}{\pi} \delta \left( (1-u)Q^2 - l_{\perp}^2 \right) \mathcal{F}_p \left( \frac{x_B}{u}, Q^2, \frac{k_{\perp} - l_{\perp}}{u} \right).$$

In the previous equation $\hat{P}_{qq}(u)$ denotes the unregularized splitting function. The resummation of leading soft logarithms is performed by changing the argument of the running coupling from the virtuality to the relative transverse momentum of partons at each branching, $\alpha_s(Q^2) \rightarrow \alpha_s(l_{\perp}^2)$. We wish to note that eq. (5) can be easily recast in impact parameter space. Taking advantage of the solution of the $k_t$-evolution equations in the soft limit, it is then possible to recover the well known results for the perturbative form factor in double logarithmic approximation. The off-diagonal term in the Altarelli-Parisi splitting matrix are assigned to have the standard coupling, $\alpha_s(Q^2)$, since no soft enhancement is present in the $q \rightarrow g(\bar{g})$ and $g \rightarrow q(\bar{q})$. The soft gluon resummation can be also performed in the gluon figure.
and the whole formalism extended up to next-to-leading logarithmic accuracy\textsuperscript{17,18}. The latter improvements however are not still implemented. Despite for \( u \to 1 \) virtual and real contributions exactly cancel, as guaranteed by the plus prescription, the rescaled coupling can be sensitive to how the infrared limit is approached. We adopt here the simplest model\textsuperscript{10}, e.g. a freezed coupling \( \alpha_s(k^2_{\perp} + \langle g^2_{\perp} \rangle) \) being \( g^2_{\perp} = 0.5 \text{ GeV}^2 \) the freezing scale. We stress that more refined prescription for the infrared behaviour of the strong coupling could be used\textsuperscript{19}. In Fig. (3) we show the predictions which include soft gluon resummation as we have described above. The tuning procedure gives an intrinsic momentum \( \langle k^2_{\perp,q} \rangle = 3 \text{ GeV}^2 \), and a much lower value for the coupling, \( \alpha_s(M_Z^2) = 0.120 \). Moreover the insensitivity to gluon parameters persists so that we still fix the gluon intrinsic momentum to \( \langle k^2_{\perp,g} \rangle = 1 \text{ GeV}^2 \). The inclusion of higher order terms in the perturbative calculation provided by the resummation has therefore the effect of strongly reduce the coupling and slightly reduce the quark intrinsic transverse momentum, not still in the range of what expected by Fermi motion. In the resummed case, the sensitivity to the quark intrinsic momentum is shown in Fig. (4). The fact that a large amount of intrinsic momentum is required in the description of the Z-boson data is known in the literature and it persists even when soft resummation is pushed to next-to-leading logarithmic accuracy\textsuperscript{20}. There are however attempts to solve this problem, either using \( x \)-dependent (i.e. energy dependent) non-perturbative form factor\textsuperscript{21} or introducing modifications to the coupling constant\textsuperscript{15}. The former approach is particularly suitable when combined analysis of low energy Drell-Yan and \( Z \) or \( W \) data are performed. A possible energy dependence of the non-perturbative form factor used in that works, and here embodied in the transverse part of the initial conditions, eq. (6), is therefore of great importance in view of LHC Drell-Yan physics program. In order to asses the reliability of all these results a careful anal-
ysis of the uncertainty due to scales and parameters variation is presently under way. Furthermore an analysis of low energy Drell-Yan is also planned as well as the implementation of soft gluon resummation in the gluon channel.

Conclusions

We have found that the predictions based on the proposed evolution equations for the unintegrated distributions are able to reproduce Tevatron data on the $p_T$-spectrum of the $Z$-boson. However the unresummed results are characterized by large values of the quark intrinsic transverse momentum and coupling constant. The inclusion of soft gluon resummation strongly reduces the value of the coupling and brings it more close to world average. On the other hand it only slightly reduces the large values of the quark intrinsic transverse momentum with respect to unresummed predictions, leaving therefore open the problem of the nature of the latter. Owing to these results and within the evolution equations scheme adopted in this analysis, we conclude that non-perturbative parameters differ significantly from the one used in our semi-inclusive DIS data analysis.

Acknowledgments

The author would like to thank the Organizers of the Workshop Recent Advances in Perturbative QCD and Hadronic Physics, ECT*, Trento (Italy), for the invitation and would like to express his best wishes to Professor A. V. Efremov. The author also would like to thank Luca Trentadue, Nicos Stefanis, Francesco Hautmann, Barbara Pasquini, Ugo Aglietti and Massimiliano Grazzini for valuable correspondence or discussions. A special thank goes to Oleg Teryaev for his warm interest on the subject.

References

1. J. C. Collins, D. E. Soper, Nucl. Phys. B193 (1981) 381, Erratum-ibid. B213 (1983) 545.
2. I. O. Cherednikov, N. G. Stefanis, Phys. Rev. D80 (2009) 054008; Nucl. Phys. B802 (2008) 146; Phys. Rev. D77 (2008) 094001.
3. J. C. Collins, F. Hautmann, JHEP 0103 (2001) 016; F. Hautmann, Phys. Lett. B655 (2007) 26.
4. X. Ji, J. Ma, F. Yuan, Phys. Rev. D71 (2005) 034005.
5. A. Bassetto, M. Ciafaloni, G. Marchesini, Nucl. Phys. B163 (1980) 477.
6. F. A. Ceccopieri, L. Trentadue, Phys. Lett. B636 (2006) 310.
7. F. A. Ceccopieri, L. Trentadue, Phys. Lett. B660 (2008) 43.
8. Y. L. Dokshitzer, D. Diakonov, S. I. Troian, Phys. Rept. 58 (1980) 269.
9. G. Parisi, R. Petronzio, Nucl. Phys. B154 (1979) 427.
10. J. C. Collins, D. E. Soper, G. Sterman, Nucl. Phys. B250 (1985) 199.
11. V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 438; L. N. Lipatov, Sov. J. Nucl. Phys. 20 (1975) 94; G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298; Y. L. Dokshitzer Sov. Phys. JETP 46 (1977) 641.
12. S. Gieseke, P. Stephens and B. Webber, *JHEP* **12** (2003) 045.
13. J. Kwicinski, A. Szczurek, *Nucl. Phys.* **B680** (2004) 164.
14. M. Gluck, E. Reya, and A. Vogt, *Z. Phys.* **C67** (1995) 433.
15. S. Gieseke, M. H. Seymour, A. Siodmok, *JHEP* **06** (2008) 001.
16. D0 Collaboration (B. Abbott & al.) *Phys. Rev. Lett.* **84** (2000) 2792.
17. J. Kodaira, L. Trentadue, *Phys. Lett.* **B112** (1982) 66.
18. S. Catani, E. D’Emilio, L. Trentadue, *Phys. Lett.* **B211** (1988) 335.
19. D. V. Shirkov, I. L. Solovtsov, *Phys. Rev. Lett.* **79** (1997) 1209.
20. G. Bozzi, S. Catani, G. Ferrera, D. de Florian, M. Grazzini, *Nucl. Phys.* **B815** (2009) 174.
21. F. Landry, R. Brock, P. M. Nadolsky, C. P. Yuan, *Phys. Rev.* **D67** (2003) 073016.