Discriminating different models of luminosity–redshift distribution

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Abstract

The beginning of the cosmological phase bearing the direct kinematic imprints of supernovae (SNe) dimming may significantly vary within different models of late-time cosmology, even if such models are able to fit present SNe data at a comparable level of statistical accuracy. This effect—useful in principle to discriminate among different physical interpretations of the luminosity–redshift relation—is illustrated here with a pedagogical example based on the Lemaître–Tolman–Bondi geometry.

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(Some figures may appear in colour only in the online journal)
The associated effect of the redshift drift [10]. Other possibilities of testing LTB models are provided by studies of scalar perturbations [11], small-scale cosmic microwave background (CMB) effects [12], the cosmic age parameter [13] and baryon acoustic oscillation data [14].

The main purpose of this paper is to point out another possible difference between inhomogeneous and more conventional interpretations of the SNe data, not yet discussed in the literature; such a difference is based on the value of the redshift parameter \( z_{\text{acc}} \) (to be defined below, see after equation (14)), marking the beginning of the regime directly characterized by the kinematic imprints of SNe dimming. The value of such parameter can be largely different even within models able to fit the presently observed luminosity–redshift distributions at a comparable level of accuracy (see e.g. [15] for earlier studies on the beginning of the accelerated regime in the context of a homogeneous geometry).

This suggests two possible experimental ways of discriminating among models of the luminosity–redshift relation: first, direct observations able to extend our present knowledge of the Hubble diagram up to values of \( z \) higher than those allowed by present SNe data, for instance, gamma-ray burst (as discussed in [16]), or even gravitational waves observations, through an analysis of the luminosity distance of the so-called standard sirens [17] and second, indirect observations which are sensitive to the time dependence of the so-called transfer function [18], which controls the evolution of the primordial perturbation spectrum inside the horizon down to the present epoch, and which is crucially affected by the kinematics of the cosmological background (see e.g. [19]).

The possible relevance of the parameter \( z_{\text{acc}} \) will be illustrated in this paper by a simple exercise, in which the SNe data of the recent Union2 compilation [20] are fitted using an inhomogeneous, matter-dominated LTB model, and such a fit is compared with the standard one performed in the context of the flat concordance \( \Lambda \)-cold dark matter (LCDM) model. We stress that our aim is not to provide a realistic alternative to the successful concordance cosmology, but only to discuss how to distinguish, at least in principle, different fits of SNe data based on different geometric schemes. The proposed diagnostic may be added to other general methods aiming at discriminating the expansion history of competing models, like—in particular—dark-energy-based diagnostics for homogeneous models [21], a Friedmann equation diagnostic for homogeneous versus inhomogeneous models [22, 23] and the already mentioned test of the redshift drift [10, 23, 24].

The cosmological configuration we will consider refers to a late-time (in particular, post-reionization) Universe, characterized by a stochastic distribution of many overdense and underdense regions, of various possible sizes and shapes, possibly even incoherently superimposed among each other. Let us suppose that in such a context, and up to a given scale \( r_V \) (to be specified below), the effective (averaged) large-scale geometry can be locally described by a model of the LTB type. Such a model is characterized in general by three arbitrary functions of the radial coordinate (see e.g. [26]). For the illustrative purpose of this paper, however, it will be enough to consider a simple example where the contribution of the spatial curvature is negligible and the gravitational sources are dominated by an isotropic CDM distribution (but the model could be easily generalized by the addition of an arbitrary cosmological constant).

We will assume that the large-scale geometry around a given observer is described—in polar coordinates and in the synchronous gauge—by the following metric:

\[
\text{d}s^2 = \text{d}t^2 - A^2 (r, t) \text{d}r^2 - A^2 (r, t) (\text{d}\theta^2 + \sin^2 \theta \, \text{d}\phi^2), \tag{1}
\]

3 Such a configuration is in principle different from that of a typical ‘Swiss cheese’ scenario, where the void regions are more or less regularly distributed and well disconnected (see e.g. [25]).
where a prime denotes partial derivatives with respect to $r$ and a dot with respect to $t$. In the limit $A(r, t) = r a(t)$ one recovers the well-known, spatially flat, Friedman–Lemaître–Robertson–Walker (FLRW) metric. In general, the unknown function $A(r, t)$ is to be determined by the Einstein equations, which in our case reduce to

$$H^2 + 2HF = 8\pi G \rho, \quad 2H + 3H^2 = 0,$$

(2)

where $H(r, t) = \dot{A}/A$ and $F(r, t) = \dot{A}'/A'$. The density profile of the CDM distribution around a central observer, $\rho = \rho(r, t)$, satisfies the covariant conservation equation

$$\dot{\rho} + (2H + F) \rho = 0,$$

(3)

while all the other Einstein equations are identically satisfied by the metric (1) (see e.g. [27]).

The above cosmological equations can be integrated exactly, and in this paper we will adopt the particular exact solution

$$A(r, t) = r \left[1 + \frac{1}{2} t H_0(r)\right]^{2/3},$$

(4)

normalized in such a way that $A = r$ at $t = 0$. The arbitrary function $H_0(r)$ depends only on the radial coordinate, and the usual matter-dominated FLRW solution is exactly recovered in the limit $H_0 = \text{const.}$ We will use, in particular, the parametrization

$$H_0(r) = \frac{H}{r} + \Delta H e^{-r/r_V},$$

(5)

already suggested in [27] for a similar LTB scenario. (A brief discussion of other possible choices for the phenomenological profile $H_0(r)$ will be given in the final part of this paper.)

For the chosen profile, the combination of parameters $\Delta H = H_0(0)$ corresponds to the locally measured value of the Hubble constant, while the distance $r_V$ represents the typical distance scale above which inhomogeneity effects become rapidly negligible.

To make contact with more general forms of the LTB metric appearing in the literature, and expressed in terms of three functions $M(r)$, $t_B(r)$, $E(r)$, (we are following the notations of [26]), it may be useful to report here the values of those functions for the model we are using. The effective gravitational mass with a comoving radius $r$, for our solution, is given by

$$M(r) = \left(\frac{1}{2}\right) r^3 H^2_0(r).$$

It can be easily checked that this function grows like $r^3$ for $r \ll r_V$ and $r \gg r_V$, while, in the transition regime $r \sim r_V$, it is characterized by a fractal index $D = 0.4$, i.e. $M(r) \sim r^{3-D} \sim r^{2.6}$. The timescale $t_B$—i.e. the local ‘big-bang time’ at which $A(r, t) = 0$—in our case is given by $t_B(r) = -(2/3) H_0^{-1}(r)$.

Finally, it is important to stress that the obtained solution is consistent with our assumption of vanishing spatial curvature, i.e. with the choice $E(r) = 0$. Perturbing the solution with the addition of scalar curvature (and assuming that $E(r) \sim r^2$ as in the large-scale FLRW limit), we have checked indeed that the curvature contribution to the total energy density may have a variation which is at most of the order of 0.05% over length scales of order $r_V$ and timescales of order $H_0^{-1}$. Hence, if initially small but nonzero, it keeps small over the whole spatial and temporal range of interest for this paper.

Let us now compute the luminosity distance $d_L$ of a source emitting light at a cosmic time $t$ and a radial distance $r$ from the origin. We will assume, for the moment, that the observer is also located at the origin (the consequences of a possible off-centre position will be discussed later). The angular distance (or area distance) of the source, for the metric (1), is then given by

$$d_A = A(r, t),$$

and the luminosity–distance, according to the so-called reciprocity law [28], reduces to

$$d_L = (1 + z)^2 A(r, t),$$

where $z$ is the redshift parameter evaluated along a null radial geodesic connecting the source to the origin.

Calling $u^a$ the static (time-like) geodesic vector field tangent to the worldlines of source and observer and $k^a$ the null vector tangent to the null radial geodesic, we find in our metric
\[ u^i = \frac{dt^i}{dr} = \left(1, 0, 0, 0\right) \text{ and } k^i = \left((A')^{-1}, -(A')^{-2}, 0, 0\right). \] Hence, for light emitted at time \( t \), radial position \( r \), and observed at the origin at \( t = t_0 \),

\[
1 + z = \left(\frac{k^iu_i}{(k^iu_i)_0}\right)_t = \frac{A'_0}{A'(r, t)},
\]

where \( A'_0 = A'(0, t_0) = \text{const.} \)

For the phenomenological applications of this paper we need to express \( d_L \) completely in terms of the redshift, namely we need to invert equation (6) to determine \( r(z) \) and \( t(z) \). We may consider, to this purpose, the differential variation of \( z \) with respect to the proper time interval \( dt \) separating two different instants of light emission, at fixed observation coordinates: \( \frac{dz}{dt} = u^i \delta_{i2}z = -(1 + z)A'/A'. \) It follows that, along a null radial geodesic (where \( dt = -A'dr \)),

\[
\frac{dr}{dz} = -\frac{A'}{(1 + z)A'}, \quad \frac{dt}{dz} = -\frac{1}{A'} \frac{dr}{dz} = -\frac{1}{A'}.
\]

For the model of equation (4), in particular, we obtain the differential equations

\[
\frac{dr}{dz} = -\frac{1}{1 + z} \left[2 + 3H_0(r)\right] \left[2 + 3H_0(r) + 2rtH_0'(r)\right],
\]

\[
\frac{dt}{dz} = \frac{1}{2^{1/3}(1 + z)\left[2 + 3H_0(r)\right]^{1/3}} \left[2 + 3H_0(r)\right] + 2rH_0'(r) + 2H_0(r)\left[1 + rtH_0'(r)\right].
\]

Solving the above equations for \( t(z) \) and \( r(z) \), and inserting the solutions into the explicit definition of \( d_L \),

\[
d_L(z) = \left(1 + z\right)^2 A(r, t) = \left(1 + z\right)^2 r(z)\left[1 + \frac{1}{2}t(z)H_0(r(z))\right]^{2/3}.
\]

we are now in the position of comparing the predictions of our model with the observational data (as well as with the predictions of the standard \( \Lambda \)CDM scenario).

Let us first recall that the Union2 compilation of the Supernova Cosmology Project [20] concerns redshift-magnitude measurements of 557 SNe of type Ia and provides, for each supernova, the observed distance modulus (with relative error) \( \mu_{\text{obs}}(z_i) \pm \Delta \mu(z_i) \), \( i = 1, \ldots, 557 \), for the redshift values ranging from \( z_1 = 0.015 \) to \( z_{557} = 1.4 \). The distance modulus \( \mu(z) \) controls the difference between apparent and absolute magnitude, and it is related to the luminosity distance \( d_L(z) \) by

\[
\mu(z) = 5\log_{10} \left[ \frac{d_L(z)}{1\text{ Mpc}} \right] + 25.
\]

Here \( d_L \) is given in units of Mpc, and the constant number 25 is determined by the conventional reference scale assumed for the absolute magnitude.

The luminosity distance of equation (9), with \( H_0(r) \) given by equation (5), is characterized in principle by three independent parameters, and can be applied to fit the experimental data by allowing free variations of \( \Omega, \Delta H \) and \( r_V \). We have performed that exercise and found that the resulting best fit provides for \( H_0(0) \equiv \bar{H} + \Delta H \) a value very close to 70 km s\(^{-1}\) Mpc\(^{-1}\). We have thus chosen to concentrate the present discussion on a simpler, two-parameter fit of the data—which, in any case, is sufficiently accurate for the illustrative purpose of this paper—by imposing on our model the \textit{a priori} constraint \( \bar{H} + \Delta H = 70 \text{ km s}^{-1}\text{ Mpc}^{-1}. \) In this way we can eliminate, for instance, \( \bar{H} \), and we can fit the experimental points \( \mu_{\text{obs}}(z_i) \pm \Delta \mu(z_i) \) by performing a standard \( \chi^2 \) analysis with

\[
\chi^2 = \sum_{i=1}^{557} \left[ \frac{\mu_{\text{obs}}(z_i) - \mu(z_i, r_V, \Delta H)}{\Delta \mu(z_i)} \right]^2.
\]
The theoretical values $\mu(z_i, r_V, \Delta H)$ can be determined, for each value of $z_i$, by numerically integrating the two equations (8), and computing the corresponding $d_L(z_i)$ as a function of the two parameters $r_V$ and $\Delta H$. By minimizing the above $\chi^2$ expression, we have found the best-fit values

$$r_V = 3000 \pm 497 \text{ Mpc}, \quad \Delta H = 26.6 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1},$$

at a confidence level (CL) of 95%, and with a goodness of fit $\chi^2/{\text{d.o.f.}} = 0.99$. The minimization has been performed using the MINUIT package from CERNLIB [29]. The result of the fit is graphically illustrated by the red curve plotted in the left panel of figure 1, superimposed to the full set of Union2 data (reported with the error bars).

Consider now, for comparison, a fit of the same data performed in the context of a spatially flat FLRW geometry, with perfect fluid sources representing CDM and a cosmological constant $\Lambda$. Denoting with $\Omega_m$ and $\Omega_\Lambda$ the present critical fraction of dark matter and dark energy, we can express the luminosity distance in the usual integral form as

$$d_L(z) = \frac{1+z}{H_0} \int_0^z dx \left[ \Omega_m (1 + x)^3 + \Omega_\Lambda \right]^{-1/2},$$

(13)

(see e.g. [30]). Proceeding as in the previous case, we will reduce the number of parameters from 3 to 2 by imposing the same phenomenological constraint as before, which in this case amounts to the condition $H_0 (\Omega_m + \Omega_\Lambda)^{1/2} = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Using equation (13) to compute $\mu(z_i, \Omega_m, \Omega_\Lambda)$, and minimizing the corresponding $\chi^2$ expression, we obtain the best-fit values $\Omega_m = 0.27 \pm 0.01$, $\Omega_\Lambda = 0.71 \pm 0.03$, at a CL of 95%, with $\chi^2/{\text{d.o.f.}} = 0.98$. The result of the fit is illustrated by the blue curve in the right panel of figure 1.

The luminosity–redshift relations of the two models of figure 1 are in good agreement with the data, and in both cases the data points are fitted at a comparable level of statistical accuracy. However, we can disclose an important physical difference between the two fits if we subtract the distance modulus $\mu_{\text{Milne}}(z)$ of a linearly expanding (but globally flat) homogeneous Milne geometry (see e.g. [31]) from the distance modulus of the two models, namely if we consider the quantity

$$\Delta(z) = \mu(z) - \mu_{\text{Milne}}(z) = 5 \log_{10} \left[ \frac{d_L(z)}{1 \text{ Mpc}} \right] - 5 \log_{10} \left[ \frac{z(2+z)}{2H_0 \text{ Mpc}} \right].$$

(14)
where $H_0$ is given in units of $\text{Mpc}^{-1}$. It is clear that positive or negative values of $\Delta$ correspond to luminosity distances which are—at a given fixed $z$—respectively larger or smaller than the reference values of the Milne model.

The case $\Delta < 0$ is typical of a decelerated Universe like that described by the standard cosmological scenario, where, at the same fixed $z$, the distances are smaller (or the received fluxes of radiation, i.e. the apparent magnitudes, are larger) than predicted by a linearly expanding model. The case $\Delta > 0$, in contrast, corresponds at the same $z$ to larger distances (or smaller radiation fluxes) than predicted by a linear expansion, and is only possible if the model undergoes a period of ‘effective’ accelerated expansion. In this last case, the transition across the value $\Delta = 0$ defines an epoch—characterized by the parameter $z_{\text{acc}}$ such that $\Delta(z_{\text{acc}}) = 0$—marking the beginning of the cosmological phase directly imprinted by the kinematic effects of the acceleration.

The plot of $\Delta(z)$ is presented in figure 2 for three cases: the standard CDM-dominated (always decelerated) model and the two best-fit models of figure 1 (corresponding to our example of the inhomogeneous geometry and to a typical example of the homogeneous concordance cosmology). In the last two cases we have plotted the central values of the fit (solid curves), as well as the corresponding error bands\textsuperscript{4} at the 95% level of confidence (bounded by the dotted curves).

We can see from figure 2 that $\Delta(z)$ is always negative for the CDM model, as expected. For the other two models, instead, we have $\Delta(z) > 0$ in the redshift range $z < z_{\text{acc}}$ (Because, as expected, a successful fit of the SNe data requires the presence of a phase describing—or mimicking—accelerated expansion.) However, the values of $z_{\text{acc}}$ defined by the condition $\Delta(z_{\text{acc}}) = 0$ are largely different in the two models. We find, in particular,

$$z_{\text{acc}}^{\text{LTB}} = 1.07 \pm 0.06, \quad z_{\text{acc}}^{\Lambda \text{CDM}} = 1.43 \pm 0.10,$$

\textsuperscript{4} The error bands have been numerically computed by varying the fit parameters within the range determined by the corresponding estimated errors. All possible fitting curves lying inside the given error region satisfy the constraint $\chi^2 \leq \tilde{\chi}^2 + \delta$, where $\tilde{\chi}^2$ is the value obtained by minimizing equation (11) and the constant $\delta$ depends on the number of parameters and on the CL of the fit determined by the MINUIT package [29]. In our case, in particular, $\delta = 5.99$ for a CL of 95%.
and this difference falls outside the error bands illustrated in figure 2. (It is also much larger than the experimental uncertainty affecting present redshift measurements.) This suggests that a precise (near-future?) determination of this parameter could provide a clear physical discrimination among different models implementing successful (and statistically equivalent) fits of SNe data.

It should be mentioned, at this point, that in the computations of the error bands we have neglected the dispersion of data due to the possible presence of a cosmic background of stochastic perturbations: indeed, such a background may induce large errors at very small \( z \), but in the range \( z \sim 1 \) (typical of \( z_{\text{acc}} \)) the induced errors typically lie in the few-percent range [7], hence are not expected to have a crucial impact on the results illustrated in figure 2. The same is expected to be true for the systematic errors—possibly slightly bigger than the previous ones, but in any case \( \leq 10\% \)—induced on \( z_{\text{acc}}^{\Lambda \text{CDM}} \) by the methods of SNe data reduction based on the assumption of standard homogeneous cosmology (and used in particular for the Union2 catalogue, see e.g. [32]). Finally, we should note that a value of \( z_{\text{acc}}^{\Lambda \text{CDM}} \) compatible with that of the inhomogeneous model considered here could be reproduced also in a homogeneous \( \Lambda \text{CDM} \) context, with realistic values of \( \Omega_{m}^{\Lambda \text{CDM}} \) and \( \Omega_{\Lambda}^{\Lambda \text{CDM}} \), but only at the price of introducing a large enough negative spatial curvature, with \( \Omega_{k} \sim 0.1 \). (For instance, a model with \( \Omega_{m}^{\Lambda \text{CDM}} = 0.3, \Omega_{\Lambda}^{\Lambda \text{CDM}} = 0.6, \Omega_{k} = 0.1 \) gives \( z_{\text{acc}}^{\Lambda \text{CDM}} = 1.087 \).)

In order to stress the importance of the parameter \( z_{\text{acc}} \), let us now consider another possible form of the phenomenological profile \( H_{0}(r) \) appearing in the LTB solution (4), for instance the profile\(^5\)

\[
H_{0}(r) = \tilde{H} + \Delta H \tanh \left( \frac{r_{0} - r}{2\Delta r} \right).
\]

We can then explicitly check that different models are characterized by largely different values of \( z_{\text{acc}} \) even within the same class of inhomogeneous geometries. By imposing, as before, the phenomenological constraint \( H_{0}(0) = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \) (in order to eliminate \( \tilde{H} \)), we find that the new profile (16) provides indeed a satisfactory three-parameter fit of the Union2 data (see figure 3, left panel), with best-fit values \( r_{0} = 2500 \pm 322 \text{ Mpc}, \Delta r = 2387 \pm 170 \text{ Mpc}, \Delta H = 37.5 \pm 2.8 \text{ km s}^{-1} \text{ Mpc}^{-1} \), at a CL of 95%, with \( \chi^{2}/\text{d.o.f.} = 1.31 \). However, the corresponding value of \( z_{\text{acc}} \) for this model (called LTB\(_{1} \) in figure 3) is significantly different

\(^{5}\) We thank an anonymous referee for this suggestion. See also [33] for other similar profiles.
from that of the previous LTB model, and, most important, the behaviour of $\Delta (z)$ is exactly the opposite of the standard one, for the range of $z$ of our interest (see figure 3, right panel). We have checked that the value of $\Delta (z)$, for LTB$_1$, turns back to the standard negative range only for $z \gtrsim 50$.

Let us finally comment on the possibility that an off-centre position of the observer embedded in a spherically symmetric LTB geometry may significantly affect the determination of $z_{\text{LTB acc}}$, thus providing obstructions to a precise discrimination between LTB-based and a more conventional (homogeneous) fit of the SNe data. Indeed, if the observer is located at a distance $r_0 \neq 0$ from the centre of a spherically symmetry geometry, then the corresponding luminosity distance $d_L$ (referred to the position $r_0$) is no longer isotropic but acquires an angular dependence, and this in turn induces an angular dispersion of the value of $z_{\text{acc}}$ which depends on $r_0$, and which obviously grows (in modulo) with the increase of $r_0$.

The luminosity distance of a source for off-centre observers in an LTB geometry has been computed in [4] (see also [34]) as a function of $z$, of the distance $r_0$ from the centre, and of the polar observation angle $\gamma$ (referred to as $r_0$). We have applied the results of [4] to compute the directional variation of $z_{\text{acc}}$, at the fixed values of $r_0$. We have considered, in particular, possible displacements from the centre in the range $r_0 \lesssim 10^{-2} r_V$, because—as discussed in [4]—higher values of $r_0$ would induce a dipole anisotropy too high to be compatible with the present CMB observations.

The results of our exercise are illustrated in figure 4, where we have plotted the fractional variation

\[
\frac{\Delta z_{\text{acc}}}{z_{\text{acc}}} \equiv \left[ z_{\text{acc}}(r_0, \gamma) - z_{\text{acc}}(0) \right] / z_{\text{acc}}(0),
\]

for different values of $r_0$ up to $10^{-2} r_V$, for the LTB model characterized by the parameter $z_{\text{LTB acc}}$ of equation (15). For the normalization of $\mu_{\text{Milne}}$ we have consistently used $H_0(r_0)$, but we have checked that using the fixed value $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ simply rescales the zero of the difference $\Delta z_{\text{acc}}$, without affecting the overall amplitude of the dispersion. As shown in figure 4, the angular variation of $z_{\text{acc}}$ induced by $r_0 \neq 0$ is bounded to be at most at the 1% level, and has thus a negligible impact on the results of figure 2.

In conclusion, we would like to stress again that the inhomogeneous model discussed in this paper should not be intended as a realistic alternative to the successful concordance cosmology, but only as a pedagogical example to learn how to distinguish different fits of SNe data based on different geometrical schemes. For this purpose, we have shown, in particular, that in the model of this paper the Universe enters the regime directly affected the accelerated kinematics later than predicted by the \Lambda CDM scenario, i.e. $z_{\text{LTB acc}} < z_{\text{acc}}^{\Lambda \text{CDM}}$. Hence, a precise
determination of the transition epoch $z_{acc}$ (possibly through future extensions of the Hubble diagram to higher values of $z$ or through indirect studies of the transfer function of primordial perturbations [35]) could help us to physically discriminate among statistically equivalent fits.

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