The large–scale modulation of the density distribution in standard axionic CDM and its cosmological and physical impact *

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Abstract

It is shown, that the energy density of coherent axion field oscillations in the cosmology of standard invisible axion should be distributed in the Universe in the form of archioles, being nonlinear inhomogeneous structure, reflecting the large scale distribution of Brownian structure of axion strings in the very early Universe. Spectrum of inhomogeneities, generated by archioles, is obtained and their effects in the spectrum and quadrupole anisotropy of relic radiation are considered. The axionic–string–decay–model–independent restriction on the scale of axion interaction is obtained.

1 Introduction

The modern theory of large-scale structure of the Universe is based on the assumption that this structure is formed as the result of development of gravitational instability from small initial perturbations of density or gravitational potential. As a rule, these perturbations are gaussian, but some version of nongaussian perturbations have also been considered.

In this paper we analyze the problem, inherent to practically all the cosmological cold dark matter models of invisible axion, that concerns primordial inhomogeneity in the distribution of the energy of coherent oscillations of the axion field. This problem, referred to as the problem of archioles, invokes nongaussian component in the initial perturbations for axionic

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cold dark matter. Archioles are the formation that represents a replica of the percolation Brownian vacuum structure of axionic walls bounded by strings, which is fixed in the strongly inhomogeneous primeval distribution of cold dark matter. They can give rise to interesting alternative scenarios of structure formation that relate the mechanism responsible for structure formation to inhomogeneities of the type of topological defects.

The analysis of observable effects associated with archioles leads to a new model-independent constraint on the mass of invisible axion. Such analysis is also very useful for further development of full cosmological theories, based on the model of horizontal unification (MHU), which has been proposed in Ref. [1] as the minimal phenomenology of everything, including the physics of inflation, bariosynthesis, and dark matter. In particular, the combination of archioles effect with the consideration of nonthermal symmetry restoration in the horizontal phase transitions on inflation stage, puts the upper limit on the scale of family symmetry breaking (the main parameter of MHU) and consequently severely reduces the set of possible realizations of dark matter scenarios in this model.

2 Formation of the archiole structure in the early Universe

In the standard invisible axion scenario Ref. [2] the breaking of the Peccei- Quinn symmetry is induced by the complex $SU(3) \otimes SU(2) \otimes U(1)$ – singlet Higgs field $\phi$ with a ”Mexican hat” potential

$$V(\phi) = \frac{\lambda}{2} (\phi^\dagger \phi - F_a^2)^2$$

(1)

Such field can be represented as $\phi = F_a \exp(i\theta)$, where $\theta = a/F_a$ and $a$ is the angular Goldstone mode-axion. QCD instanton effects remove the vacuum degeneracy and induce effective potential for $\theta$

$$V(\theta) = \Lambda_{QCD}^4 (1 - \cos(\theta N))$$

(2)

Below we will simply assume for standard axion that $N = 1$. In the context of standard big bang scenario it is usually assumed that the phase transition with $U(1)$ – symmetry breaking occurs when the Universe cools below the temperature $T \approx F_a$. Thus, in the standard case the crucial assumption is that from the moment of the PQ phase transition and all the way down to the temperatures $T \approx \Lambda_{QCD}$, the bottom of the potential Eq. (1) is exactly flat and there is no preferred value of $a$ during this period (the term given by Eq. (2) vanishes). Consequently, at the moment of the QCD phase transition, when the instanton effects remove vacuum degeneracy, $a$ rolls to the minimum and starts coherent oscillations (CO) about it with energy density Ref. [2]

$$\rho_a(T, \theta) = 19.57 \left( \frac{T^2 m_a}{M_P} \right) \left( \frac{T}{T_1} \right)^3 T^2 F_a^2$$

(3)

The coherent axion field oscillations turn on at the moment $\tilde{t} \approx 8.8 \cdot 10^{-7}$s.

It is generally assumed, that PQ transition takes place after inflation and the axion field starts oscillations with different phase in each region causally connected at $T \approx F_a$, so one has the average over all the values to obtain the modern axion density. Thus in the standard
cosmology of invisible axion, it is usually assumed that the energy density of coherent oscillations is distributed uniformly and that it corresponds to the averaged phase value of $\bar{\theta} = 1$ ($\bar{\rho}_a = \rho(\bar{\theta})$). However, the local value of the energy density of coherent oscillations depends on the local phase $\theta$ that determines the local amplitude of these coherent oscillations. It was first found in Ref. [3] that the initial large-scale (LS) inhomogeneity of the distribution of $\theta$ must be reflected in the distribution of the energy density of coherent oscillations of the axion field. Such LS modulation of the distribution of the phase $\theta$ and consequently of the energy density of CO appears when we take into account the vacuum structures leading to the system of axion topological defects.

As soon as the temperature of Universe becomes less than $F_a$, the field $\phi$ acquires the vacuum expectation value (VEV) $\langle \phi \rangle = F_a \exp(i\theta)$, where $\theta$ varies smoothly at the scale $F_a^{-1}$. The existence of noncontractable closed loops that change the phase by $2\pi n$ leads to emergence of axion strings. These strings can be infinite or closed. The numerical simulation of global string formation Ref. [4] revealed that about 80% of the length of strings corresponds to infinite Brownian lines. The remaining 20% of this length is contributed by closed loops. Infinite strings form a random Brownian network with the step $L(t) \approx t$. After string formation when the temperature becomes as low as $T \approx \Lambda_{QCD}$, the Eq. (2) makes a significant contribution to the total potential so that the minimum of energy corresponds to a vacuum with $\theta = 2\pi k$, where $k$ is an integer – for example, $k = 0$. However, the vacuum value of the $\theta$ cannot be zero everywhere, since the phase must change by $\Delta \theta = 2\pi$ upon a loop around a string. Hence, we come from the vacuum with $\theta = 0$ to the vacuum with $\theta = 2\pi$ as the result of such circumvention. The vacuum value of $\theta$ is fixed at all points with the exception of the point $\theta = \pi$. At this point, a transition from one vacuum to another occurs, and the vacuum axion wall is formed simultaneously with CO turning on. The width of such wall bounded by string is $\delta \approx m_a^{-1}$. Thus, the initial value of $\theta$ must be close to $\pi$ near the wall, and the amplitude of CO in Eq. (3) is determined by the difference of the initial local phase $\theta(x)$ and the vacuum value, which is different from the one of the true vacuum only in a narrow region within the wall of thickness $\delta \approx m_a^{-1}$. Therefore in this region we can write $\theta(x) = \pi - \varepsilon(x)$, where Ref. [3] $\varepsilon(x) = 2\tan^{-1}(\exp(m_a x))$ and $x \approx m_a^{-1}$. Thereby the energy density of CO in such regions is given by

$$\rho_A \approx \pi^2 \rho_a$$

And so we obtain, that the distribution of CO of axion field is modulated by nonlinear inhomogeneities in which relative density contrasts are $\delta \rho / \rho > 1$. Such inhomogeneities were called archioles. In the other words archioles are a formation that represents a replica of the percolational Brownian vacuum structure of axionic walls bounded by strings an which is fixed in the strongly inhomogeneous initial distribution of axionic CDM. The scale of this modulation of density distribution exceeds the cosmological horizon because of the presence of 80% infinite component in the structure of axionic wall bounded by strings system. The superweakness of the axion field selfinteraction results in the separation of archioles and of the vacuum structure of axionic walls – bounded – by – strings. So these two structures evolve independently. The structure of walls bounded by strings disappears rapidly due to disintegration into separate fragments and further axion emission. The structure of archioles remains frozen at the RD stage. On the large scales, the structure of archioles is an initially nonlinear formation, a
Brownian network of quasi-one-dimensional filaments of dustlike matter with the step

\[ L^A(t) = \lambda \tilde{t} \]  

(5)

(where \( \lambda \approx 1 \)). At the moment of creation \( \tilde{t} \), the linear density of this quasilinear filamentary formations given by

\[ \mu_A = \pi^2 \rho_{\tilde{A}} \tilde{t} \delta \]  

(6)

In accordance with this, the cosmological evolution of archioles in the expanding Universe is reduced to the extension of lines along only one direction.

We have studied in Ref. [5] the spectrum of inhomogeneities that the density develops in response to the large-scale Brownian modulation of the distribution of CO of axion field. Density perturbations, associated with Brownian network of archioles, may be described in the terms of a two-point autocorrelation function Ref. [5]. To obtain such autocorrelation function, it is necessary to perform averaging of energy density of infinite Brownian lines over all lines and over the Winner measure, which corresponds to the position along of Brownian line (see Ref. [5]).

The two-point autocorrelation function in the Fourier representation has the form

\[ \langle \frac{\delta \rho}{\rho_0}(\vec{k}) \frac{\delta \rho}{\rho_0}(\vec{k}') \rangle = 12 \rho_A \mu_A k^{-2} \delta(\vec{k} + \vec{k}') \tilde{t}^{-1} f^{-2} G^2 k^4 \]  

(7)

where \( \rho_0 \) is background density, \( f_{MD} = 3/(32\pi) \) for dustlike stage, \( f_{RD} = (6\pi)^{-1} \) for RD stage, \( G \) is the gravitational constant, \( \rho_A \) is the total energy density of the Brownian lines. The mean-square fluctuation of the mass is given by

\[ \left( \frac{\delta M}{M} \right)^2(k, t) = 12 \rho_A \mu_A \tilde{t}^{-1} f^{-2} G^2 k t^4 \]  

(8)

3 Cosmological impact of archioles

Let us consider a region characterized at instant \( t \) by a size \( l \) and a density fluctuation \( \Delta \). For anisotropy of relic radiation we then obtain

\[ \frac{\delta T}{T} \approx -\delta \left( \frac{l}{t} \right)^2 \]  

(9)

If \( l = t \), we have \( |\delta T/T| \approx |\Delta| \); that is, the anisotropy of relic radiation is equal to the density contrast calculated at the instant when the size of the region is equal to the size of the horizon (Sachs–Wolf effect). To estimate the quadrupole anisotropy that is induced in relic radiation by the structure of archioles, we must find the amplitude of perturbations on the scale of the modern horizon

\[ \left( \frac{\delta M}{M} \right)^2 = 2.1 \cdot 10^{-25} \left( \frac{F_a}{10^{10} GeV} \right)^4 \left( \frac{t_{RD}}{1s} \right)^{2/3} \left( \frac{t_{pres}}{1s} \right) \left( \frac{k_{hor}}{k_{pres}} \right)^{1/3} \]  

(10)
Thus Sachs-Wolf quadrupole anisotropy of relic radiation induced by archioles will be
\[
\frac{\delta T}{T} \simeq 2.3 \cdot 10^{-6} \left( \frac{F_a}{10^{10} GeV} \right)^2
\]  
(11)

According to COBE data (see for example Ref. [6]), the measured quadrupole anisotropy of relic radiation is at the level of
\[
\frac{\delta T}{T} \approx 5 \cdot 10^{-6}
\]  
(12)

If we take into account the uncertainties of our consideration such as the uncertainties in correlation length scale of Brownian network (\( \lambda \approx 1 \div 13 \)) and in temperature dependence of axion mass, we can obtain a constraint on the scale of symmetry breaking in the model of an invisible axion
\[
F_a \leq 1.5 \cdot 10^{10} GeV \div 4 \cdot 10^9 GeV; \quad m_a \geq 410 \mu eV \div 1500 \mu eV
\]  
(13)

This upper limit for \( F_a \) is close to the strongest upper limits in Refs. [7, 8, 9], obtained by comparing the density of axions from decays of axionic strings with the critical density, but has an essentially different character.

The point is that the density of axions formed in decays of axionic strings depends critically on the assumption about the spectrum of such axions (see Refs. [7, 8]) and on the model of axion radiation from the strings (see Ref. [9]). For example, Davis Ref. [7] assumed that radiated axions have a maximum wavelength of \( \omega(t) \approx t^{-1} \) while Harari and Sikivie Ref. [8] have argued that the motion of global strings was overdamped, leading to an axion spectrum emitted from infinite strings or loops with a flat frequency spectrum \( \propto k^{-1} \). This leads to an uncertainty factor of \( \approx 100 \) in the estimate of the density of axions from strings and to the corresponding uncertainty in the estimated upper limit on \( F_a \)
\[
F_a \leq 2 \cdot 10^{10} \zeta GeV; \quad m_a \geq 300/\zeta \mu eV
\]  
(14)

Here, \( \zeta = 1 \) for the spectrum from Davis Ref. [7], and \( \zeta \approx 70 \) for the spectrum from Harari and Sikivie Ref. [8].

In their treatment of axion radiation from global strings, Battye and Shellard Ref. [9] found that the dominant source of axion radiation are string loops rather than long strings, contrary to what was assumed by Davis Ref. [7]. This leads to the estimations
\[
F_a \leq 6 \cdot 10^{10} GeV \div 1.9 \cdot 10^{11} GeV; \quad m_a \geq 31 \mu eV \div 100 \mu eV
\]  
(15)

Arguments that lead to the constraint Eq. (13) are free from these uncertainties, since they have a global string decay model – independent character.

At the smallest scales, corresponding to the horizon in the period \( \tilde{t} \), evolution of archioles just in the beginning of axionic CDM dominancy in the Universe (at redshifts \( z_{MD} \approx 4 \cdot 10^4 \)) should lead to formation of the smallest gravitationally bound axionic objects with the minimal mass \( M \simeq \rho_a \tilde{t}^3 \simeq 10^{-6} M_\odot \) and of typical minimal size \( \tilde{t}(1 + z_A)/(1 + z_{MD}) \approx 10^{13} \text{cm} \). One can expect the mass distribution of axionic objects at small scale to peak around the minimal mass, so that the existence of halo objects with the mass \( (10^{-6} M_\odot \div 10^{-1} M_\odot) \) and size \( 10^{13} \div 10^{15} \text{cm} \) is rather probable, what may have interesting application to the theoretical interpretation of MACHOs microlensing events.
4 Physical impact of archioles

The inclusion of obtained restriction into the full cosmoparticle analysis provides detailed quantitative definition of the cosmological scenario, based on the respective particle physics model. Consider, for example, a simple variant of gauge theory of broken family symmetry (TBFS) Ref. [10], which is based on the standard model of electroweak interactions and QCD, supplemented by spontaneously broken local $SU(3)_H$ symmetry for quark–lepton families. This theory provides natural inclusion of Peccei–Quinn symmetry $U(1)_H \equiv U(1)_{PQ}$, being associated with heavy ”horizontal” Higgs fields and it gives natural solution for QCD CP – violation problem. The global $U(1)_H$ symmetry breaking results in the existence of axion–like Goldstone boson – $a$. TBFS turns to be a simplest version of the unified theoretical physical quantitative description of all main types of dark matter (HDM–massive neutrinos, axionic CDM and UDM in the form of unstable neutrinos Refs. [10, 11]) and the dominant form of the dark matter is basically determined by the scale of the ”horizontal” symmetry breaking $V_H$, being the new fundamental energy scale of the particle theory. For given value of $V_H$ the model defines the relative contribution of hot, cold and unstable dark matter into the total density. Since in the TBFS the scale of horizontal symmetry breaking $V_H$ is associated with $F_a$, we have, from Eq. (13), the same upper limit on $V_H$. However, this limit assumes, that the considered inflationary model permits topological defects and hence archioles formation due to the sufficiently high reheating temperature $T_{RH} \geq V_H$. In the inflationary model, which occurs in TBFS, we can achieve $T_{RH} \sim 10^{10}$GeV. The ”horizontal” phase transitions on inflationary stage lead to the appearance of a characteristic spikes in the spectrum of initial density perturbations. These spike–like perturbations, on scales that cross the horizon $(60–1)$ e–folds before the end of inflation reenter the horizon during the radiation or dust like era and could in principle collapse to form primordial black holes. The minimal interaction of ”horizontal” scalars of TBFS $\xi^{(0)}$, $\xi^{(1)}$, $\xi^{(2)}$ with inflaton allows us to include them in the effective inflationary potential Ref. [12]:

$$V(\phi, \xi^{(0)}, \xi^{(1)}, \xi^{(2)}) = -\frac{m^2_\phi}{2} \phi^2 + \frac{\lambda_\phi}{4} \phi^4 - \sum_{i=0}^{2} \frac{m^2_i}{2} \left(\xi^{(i)}\right)^2 + \sum_{i=0}^{2} \frac{\lambda_i}{4} \left(\xi^{(i)}\right)^4 + \sum_{i=0}^{2} \frac{\nu_i^2}{2} \phi^2 \left(\xi^{(i)}\right)^2$$ (16)

The analysis of processes of primordial black holes formation from density fluctuations, which can be generated by ”horizontal” phase transitions at the inflationary stage gives rise to an upper limit on the scale of horizontal symmetry breaking Ref. [12]:

$$V_H \leq 1.4 \cdot 10^{13}GeV$$ (17)

Therefore the range between the two upper limit Eq. (13) and Eq. (17) turns to be not closed, and the following values seem to be possible

$$10^{11}GeV \leq V_H \leq 10^{13}GeV$$ (18)

The indicated range corresponds to the case when all the horizontal phase transitions take place on the dust like stage and $\phi_{c_2} \ll m_{Pl}$. In this case the inflationary field $\phi$ oscillates with initial amplitude $\sim m_{Pl}$. According to Refs. [13, 12] it means that any time the amplitude of the field becomes smaller then $\phi_{c_2} \ll m_{Pl}$, the last (axion $\xi^{(2)}$) phase transition with symmetry
breaking occurs, and topological defects are produced. Then the amplitude of the oscillating field $\phi$ becomes greater than $\phi_c^2$, and the symmetry is restored again. However, this regime does not continue too long. Within a few oscillations, quantum fluctuations of the field $\xi^{(2)}$ will be generated with the dispersion $\langle (\xi^{(2)})^2 \rangle \simeq \nu^{-1}_x \lambda_\phi^{1/2} \ln^{-2} 1/\nu_x^2$ Ref. [13]. For
\begin{equation}
m_2^2 \leq \nu^{-1}_x \lambda_\phi^{1/2} \lambda_\xi m^2_{Pl} \ln^{-2} 1/\nu_x^2 \tag{19}\end{equation}
these fluctuations will keep the symmetry restored. The symmetry breaking will be finally completed when $\langle (\xi^{(2)})^2 \rangle$ will become small enough. Thus such phase transition leads to formation of topological defects and archioles without any need for high – temperature effects. Substituting the typical values for potential Eq. (16) such as $m^2_2 \approx 10^{-3} V_H^2$, $\lambda_\xi \simeq 10^{-3}$, $\nu_x \simeq 10^{-10}$ (see Ref. [12]) we will obtain that the condition Eq. (19) means that for the scales
\begin{equation}V_H \leq 10^{-3} m_{Pl} \tag{20}\end{equation}
the phenomenon of non – thermal symmetry restoration takes place in simplest inflationary scenario based on TBFS. Owing to this phenomenon oscillations of the field $\xi^{(2)}$ do not suppress the topological defects and archioles production for the range Eq. (18). So the range Eq. (18) turns to be closed by comparison of BBBR quadrupole anisotropy, induced by archioles, with the COBE data. As a result, the upper limit on the scale of horizontal symmetry breaking will be given by Eq. (13).

Note, in conclusion, that the axion emission can influence the time scale and energetics of neutrino flux from collapsing stars. Analysis of this effect for SN1987A excludes the interval $3 \cdot 10^6 GeV \leq V_H \leq 3 \cdot 10^9 GeV$ (see Ref. [14]) and establishes the lower limit on the high energetic branch of TBFS. Thus putting together all these limits we can extract narrow window in the high energetic branch of the so called model of horizontal unification (MHU) Ref. [1]:
\begin{equation}3 \cdot 10^9 GeV \leq V_H \leq 1.5 \cdot 10^{10} GeV \tag{21}\end{equation}

On the base of this choice for the main parameter of MHU we can build a quantitatively definite dark matter scenario, which associates the formation of the large–scale structure in the Universe with a mixture of axions and massive neutrinos, since in this interval the total density equal to the critical one makes in the most cases the contribution of massive neutrinos necessary.

References

[1] M.Yu.Khlopov and A.S.Sakharov, *Phys. Atom. Nucl.* 57, 651 (1994); M.Yu.Khlopov and A.S.Sakharov, in: "Particle astrophysics, atomic physics and gravitation. Eds. J.Tran Thanh Van, G.Fontaine, E.Hinds. Editions Frontieres, 197 (1994); M.Yu.Khlopov and A.S.Sakharov, in: Cosmion’94, Proc. 1 International conference on cosmoparticle physics Eds. M.Yu.Khlopov, M.E.Prokhorov, A.A.Starobinsky, J.Tran Thanh Van, Editions Frontieres, 273 (1996)

[2] J.E.Kim, *Phys. Rep.* 150, 1 (1987)
[3] M.Yu.Khlopov and A.S.Sakharov, *Phys. Atom. Nucl.* **57**, 485 (1994)
[4] T.Vachaspati and A.Vilenkin, *Phys. Rev. D* **30**, 2036 (1984)
[5] M.Yu.Khlopov, D.D.Sokoloff and A.S.Sakharov, *Phys. Atom. Nucl.* **59**, 1005 (1996)
[6] C.L.Bennett et al., *Astrophys. J. Lett.* **464**, L1 (1996)
[7] R.Davis, *Phys. Lett. B* **180**, 277 (1986)
[8] D.Harari and P.Sikivie, *Phys. Lett. B* **195**, 361 (1987)
[9] R.A.Battye and E.D.S.Shellard, *Phys. Rev. Lett.* **73**, 2954, (1994)
[10] Z.G.Berezhiani and M.Yu.Khlopov, *Z. Phys. C* **49**, 73 (1991)
[11] Z.G.Berezhiani and M.Yu.Khlopov, *Sov. J. Nucl. Phys.* **52**, 60 (1990); M.Yu.Khlopov and A.S.Sakharov, *Gravitation & Cosmology* **3**, 219 (1995)
[12] M.Yu.Khlopov and A.S.Sakharov, *Phys. Atom. Nucl.* **56**, 412 (1993)
[13] L.Kofman, A.Linde and A.A.Starobinsky, *Phys. Rev. Lett.* **76**, 1011 (1996)
[14] M.S.Turner, *Phys. Rep.* **197**, 67 (1990)