Reconstructing the Universe

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ABSTRACT
We test the mutual consistency between the baryon acoustic oscillation measurements from the eBOSS SDSS final release, as well as the Pantheon supernova compilation in a model independent fashion using Gaussian process regression. We also test their joint consistency with the ΛCDM model, also in a model independent fashion. We also use Gaussian process regression to reconstruct the expansion history that is preferred by these two datasets. While this methodology finds no significant preference for model flexibility beyond ΛCDM, we are able to generate a number of reconstructed expansion histories that fit the data better than the best-fit ΛCDM model. These example expansion histories may point the way towards modifications to ΛCDM. We also constrain the parameters $\Omega_\Lambda$ and $H_0 r_d$ both with ΛCDM and with Gaussian process regression. We find that $H_0 r_d = 10030 \pm 130$ km/s and $\Omega_\Lambda = 0.05 \pm 0.10$ for ΛCDM and that $H_0 r_d = 10040 \pm 140$ km/s and $\Omega_\Lambda = 0.02 \pm 0.20$ for the Gaussian process case.

Key words: keyword1 – keyword2 – keyword3

1 INTRODUCTION
ΛCDM (Λ for a cosmological constant and CDM for cold dark matter) has emerged as the concordance model of cosmology. This model explains a number of datasets well, at least individually. In broad strokes, the ΛCDM model explains well the anisotropies in the cosmic microwave background, how those anisotropies cluster and grow into the observed large scale structure of the Universe, and how the expansion of the Universe accelerates at late times.

However, there have emerged a number of tensions in the ΛCDM parameters inferred by different datasets. Most notably is the so-called “$H_0$ tension”, which is a 4.4σ discrepancy between the present-day expansion rate directly observed from low-redshift distances, including Cepheid anchoring of supernova distances (SN) (Riess et al. 2019), strong lens time delay distances (Wong et al. 2020; Liao et al. 2020), Tip of the Red Giant Branch (Freedman et al. 2020), and that rate inferred from Planck measurements of the cosmic microwave background (Planck Collaboration et al. 2018). There are a number of other tensions involving the growth of structure (Hildebrandt et al. 2017; Heymans et al. 2020), and the inferred curvature (Di Valentino et al. 2020). Taken together, these may point towards a discrepancy between high and low-redshift physics (Keeley et al. 2019).

The $H_0$ tension is primarily about the absolute scale of the lower end of the distance-redshift relation but the shape of this relation can point towards possible extension to ΛCDM that may explain the $H_0$ tension. Two tracers of the shape of the distance-redshift relation are SN distances and the baryon acoustic oscillations (BAO) feature in the clustering of galaxies. Jointly, the SN and BAO datasets are particularly relevant for this $H_0$ tension because they are anchored by the two datasets in question (Cepheids and CMB). Indeed, if the $H_0$ from the Cepheids is taken to anchor the SN and $r_d$, the size of the Universe at the drag epoch, is taken to anchor the BAO, then the distances inferred from the two datasets are discrepant regardless of any cosmological interpretation of those distances (Knox & Millea 2020). Thus the two datasets, even on their own and unanchored, can point the way towards what new physics might be needed to explain the $H_0$ tension.

While there have emerged a large number of explanations that have reconciled the Planck “TT” dataset and the Cepheid dataset, most have not been able to jointly explain every cosmological dataset. Notably, explanations tend to fail explaining the polarization datasets (Bernal et al. 2016; Agrawal et al. 2019; Poulin et al. 2019; Hill et al. 2020), or the large scale structure including the BAO (Keeley et al. 2019; Li & Shafieloo 2020, 2019), or the SN distances (Li & Shafieloo 2020, 2019). Rather than iterate through a possibly infinite number of nested extensions to ΛCDM or discrete alternative models, it can be more fruitful to use model independent
methods. That is, it is better to use data driven techniques to reconstruct the distance-redshift relation from the data directly. With the reconstructions, one can then build a model around what the data are trying to say.

In this paper, we first (Sec. 2) seek to use model independent methods to test that the SN and BAO distances are, in fact, consistent with each other and that they are jointly consistent with the ΛCDM model. In Sec. 3, we then reconstruct the expansion history of the Universe (h(z)) implied from these two datasets, as well as additional diagnostics that test the consistency with ΛCDM (ωm(z) and q(z)). We continue in Sec. 4 where we use our model independent methods to constrain the relative anchoring of the two SN and BAO distance datasets (H₀d(z)) as well as constrain the curvature (Ω_k).

We compare the model independent results with the tension. If the two datasets are consistent with each other and that they are jointly consistent with the ΛCDM model, then we can choose it as the mean function for our hyperfunctions, for instance, the expansion history characterized by a covariance function (Rasmussen & Williams 2006). A GP is an infinite collection of correlated random variables characterized by a covariance function about which the random fluctuations of the GP varies. Therefore, the draws of a GP can be thought of as hyperfunctions with a GP as a sampling method but instead of sampling over a parameter, i.e. a function to be marginalized over, thus making any outcome of the test more robust.

2 CONSISTENCY TESTS

In this section, we use Gaussian process (GP) regression, to perform model independent tests of the mutual consistency of the Pantheon (Scolnic et al. 2018) and SDSS BAO (Blanton et al. 2017) eBOSS (Dawson et al. 2016; eBOSS Collaboration et al. 2020) BAO datasets and their joint consistency with the ΛCDM model. It is important to perform consistency tests to answer whether there are any systematics which might hinder the ability to interpret these distances accurately. Further, if the datasets have some certain systematic between them, any attempt to derive cosmological parameters from a joint inference of the two datasets will yield inaccurate and artificially precise results. The inferred posteriors would be meaningless. Additionally, it is important to perform these kinds of tests in a model independent manner so that we can avoid making assumptions that we would want to test, thus making any outcome of the test more robust.

2.1 Datasets

Both the Pantheon (Scolnic et al. 2018) and SDSS BAO (eBOSS Collaboration et al. 2020) datasets are unanchored so it is trivial to get the absolute scale of the datasets to agree. Thus we are effectively only testing if the shape of the inferred expansion histories are consistent. In other words, these two datasets, on their own, will not be able to adjudicate which value of H₀ is correct, but it will be able to adjudicate the kinds of beyond-ΛCDM modifications can explain the tension. If the two datasets are consistent with each other and with ΛCDM, then we can be more sure that the modification must occur outside the redshift range of the datasets.

The Pantheon SN dataset is composed of 1048 Type Ia SN between z = 0.01 and z = 2.3. SN are able to constrain cosmological distances because they are empirically assumed to be standardizable candles. That is, SN with the same light-curves, modolo the color and stretch of the SN and properties of the host galaxy, are thought to have the same intrinsic luminosity. This intrinsic luminosity, parameterized by M₀, is unknown, however, and degenerate with H₀. Thus measuring the brightness of a SN can yield information about the relative distances of the SN but not their absolute distances. SN constrain the shape of the expansion history but they are unanchored.

The SDSS baryon acoustic oscillation (BAO) dataset measures the correlation function of galaxies. This correlation function contains a “BAO feature” which is an overdensity of power at the drag scale r_d, which is the size of the Universe at the drag epoch. This feature arises from sound waves in the plasma of the early Universe. This scale evolves along with the expansion of the Universe and so is encoded in the clustering of galaxies. Overdensities of galaxies are more likely to be located at a distance of r_d apart.

In a similar case to the SN, since r_d is unknown the BAO datasets cannot be used alone to constrain the low redshift distances (or conversely, since the distances to these clustering galaxies is unknown, BAO datasets cannot be used alone to measure r_d), thus making the BAO datasets unanchored. Like M₀, r_d is another nuisance parameter to be marginalized over when comparing the SN and BAO distances.

Because the distances to these clustered galaxies is unknown, the BAO datasets on their own to not constrain an absolute scale for the expansion history of the Universe. As such, the BAO can constrain either the drag scale in dimensionless units of h⁻¹Mpc or the low-redshift distances relative to the drag scale D_M,H/r_d. Equivalently, the BAO datasets can be anchored either at high or low redshift with r_d or H₀ respectively.

The SDSS eBOSS final release measures the BAO feature in a variety of tracers over a variety of redshifts, with the Main Galaxy Sample at z = 0.15 (Ross et al. 2015; Howlett et al. 2015) the BOSS Luminous Red Galaxy sample at redshifts z = 0.38 and 0.51 (Beutler et al. 2017; Alam et al. 2017) (the z = 0.61 bin from this sample is merged into the eBOSS sample), the eBOSS Luminous Red Galaxy sample at redshift z = 0.70 (eBOSS Collaboration et al. 2020), the eBOSS Emission-line Galaxy sample at redshifts z = 0.85 (eBOSS Collaboration et al. 2020), the eBOSS quasar sample at redshift z = 1.48 (eBOSS Collaboration et al. 2020), and the BOSS/eBOSS Lyman forest and quasar sample at z = 2.33 (du Mas des Bourboux et al. 2017, 2020).

2.2 Gaussian Process

A GP is an infinite collection of correlated random variables characterized by a covariance function (Rasmussen & Williams 2006). Where a Gaussian distribution draws a single number, a GP generalizes this concept and draws a function. So, in a sense, GP can be thought of as a sampling method but instead of sampling over a finite dimensional parameter space as in Markov chain Monte Carlo (MCMC), GP samples an infinite dimensional function space.

GPs generally can take an input called a mean function, a function about which the random fluctuations of the GP varies. Therefore, the draws of a GP can be thought of as hyperfunctions (not the holomorphic variety, just a generalization of a hyperparameter, i.e. a function to be marginalized over) and thus GP can be used as a method to look for deviations away from this mean function. To test ΛCDM, then we can choose it as the mean function for our GP inferences.

Using GPs to perform a regression utilizes both these understandings of GP. We essentially take a GP as the prior in a Bayesian analysis. We essentially sample the prior for a Bayesian analysis using a GP. When we want to sample the prior for a Bayesian analysis, we use a GP as that prior. In proper Bayesian fashion, we marginalize over this family of hyperfunctions drawn from the GP prior. Each of these hyperfunction samples is weighted by their likelihood, by how well they fit the data. Histogramming each of these weighted hyperfunctions then allows us to calculate the posterior, not in terms of a set of parameters, but directly in terms of the reconstructed cosmological functions, for instance, the expansion history H(z).

We can summarize the methodology up to this point with the equation via Bayes’ theorem,

$$P(H(z)|D) = \int d\phi GP L(D|H(z, \phi GP)) P(\phi GP)/P(D),$$

where
Figure 1. Hyperparameter posteriors for three different cases of consistency checks. The solid, dashed, and dotted white lines correspond to the 1,2,3-\sigma contours of the posterior, respectively. The top-left case corresponds to a GP inference of the SDSS data with a mean function taken from a best-fit GP sample to the SN data, the top-right is the same but switched: a GP inference of SN data with a mean function taken from a best-fit GP sample to the SDSS data. The bottom case corresponds to a GP inference of both the SN and SDSS data with a mean function taken from the best-fit $\Lambda$CDM fit to the two datasets.

where $\phi_{GP}$ is the family of hyperfunctions from the GP, $D$ is the data, $P(H(z)|D)$ is the posterior, $L(D|H(z, \phi_{GP}))$ is the likelihood, $P(\phi_{GP})$ is the prior and $P(D)$ is the evidence. The hyperfunctions are related to the expansion history by the following formula,

$$H(z) = H_{mf}(z) \exp(\phi(z)).$$

where $H_{mf}(z)$ is the expansion history of the mean function. From the posterior, we can calculate quantities like the 68% and 95% confidence levels (CL) for the value of $H(z)$ at any particular redshift. Joining the CLs for various redshifts allows us to generate the “band” plots, which if the data significantly prefer some amount of evolution in the expansion history, relative to the mean function, then it will show up in these plots.

To be concrete about some of the details of GP, as mentioned previously, a GP is characterized by a covariance function. This covariance function can be quite general so long as it satisfies some general properties like being symmetric and positive semidefinite. We specifically use a squared-exponential covariance matrix with the following form,

$$\langle \phi(s_1)\phi(s_2)\rangle = \sigma_f^2 e^{-\frac{(s_1-s_2)^2}{2\ell^2}},$$

where our evolution variable is $s(z) = \log(1+z)/\log(1+z_{\text{max}})$. We take $z_{\text{max}} = 3$. Importantly, the covariance function is characterized by two hyperparameters. $\sigma_f$ determines the heights of the random fluctuations of the GP, i.e. the scale of the deviations away from the mean function. If the data prefer additional information or flexibility beyond the input mean function, they will pick out a value for $\sigma_f$ above zero. $\ell$ which determines the length of the random fluctuations. In simple terms, $1/\ell$ is roughly the number of independent random fluctuations in the range. So GP samples with large $\ell$ and large $\sigma_f$ would have a few large deviations while small $\ell$ and small $\sigma_f$ would have many smaller deviations. Because these hyperparameters encode information about the inferred expansion histories, the hyperparameters must be fit for and cannot be assumed.

One might be concerned that this sort of analysis is prone to overfitting. If $\ell$ is small, say around $\ell \sim 0.001$, that is, in effect, $\sim 1000$ degrees of freedom and the worry is that this analysis could easily achieve an arbitrarily good $\chi^2$ value. It is possible for GP
achieved simply by fitting two datasets are jointly consistent with the function of a GP is consistent with the data, we can test if the mean function to be a GP reconstruction of one of the datasets to serve as the mean function for a GP reconstruction of the SDSS BOSS and Pantheon datasets are prefer no additional information beyond the mean function. The \( \Lambda \)CDM model as the mean function. Thus if posterior \( \sigma_f \) is consistent with 0, then \( \Lambda \)CDM is assumed. The notable feature here is that the \( \Lambda \)CDM expansion history to the SDSS BAO dataset, for example, which we use as a model independent method to constrain this quantity will necessarily give uncertain and noisy results.

The results of these reconstructions are shown in Fig. 2. The lighter blue shaded regions correspond to the 68% and 95% CLs of the GP reconstruction. These quantities are calculated by taking each GP reconstruction, calculating \( h(z), om(z), \) and \( q(z) \) for each, then, for each redshift, grab the quantity at that redshift, and then make a histogram over all the different reconstructions, weighted by their likelihoods. We use these histograms to calculate the 68% and 95% CLs.

In orange, we show the corresponding CLs but for the case where \( \Lambda \)CDM is assumed. The notable feature here is that the \( \Lambda \)CDM case yields tighter constraints on these functions of the expansion history. This feature is expected, since, by construction, the GP case is more agnostic about the expansion history.

On top of these bands, we plot specific GP reconstructions that have a better \( \chi^2 \) than the best-fit \( \Lambda \)CDM. The reconstructions of \( om(z) \) have some noticeable, though rare, evolution towards low redshift, thought this is the region where the data is least constraining on this quantity.

From these reconstructions, we see that the Pantheon and SDSS BAO datasets prefer no significant evolution with respect to \( \Lambda \)CDM model, and show a non-exhaustive set of example expansion histories that happen to fit these datasets better then the best-fit \( \Lambda \)CDM.

However, there do exist a number of GP reconstructions that fit the data better than the best-fit \( \Lambda \)CDM model. An examination of the GP hyperparameters that generated these reconstructions can give insight into what features of the data the reconstructions are fitting better than \( \Lambda \)CDM. Most of these better than \( \Lambda \)CDM reconstructions have large \( \ell \) values while only having a better fit by \( \Delta \chi^2 \approx 0.1 - 1.0. \) The GP reconstructions that have the best fit to the data \( (\Delta \chi^2 = 6) \) also have the smallest values of \( \ell \), which, in some sense, translates to a large number of degrees of freedom, so these reconstructions have rapidly varying \( h(z) \). The vary in such a way that \( h(z) \) is not monotonically increasing with redshift, indicating for that reconstruction, the inferred dark energy density would have to be negative at some point. It might be reasonable to reject any

2.3 Testing \( \Lambda \)CDM

Now that we have a general methodology to test whether the mean function of a GP is consistent with the data, we can test if the two datasets are jointly consistent with the \( \Lambda \)CDM model. This is achieved simply by fitting \( \Lambda \)CDM to the two datasets, finding which parameters fit them best, and then using the expansion history of this best-fit \( \Lambda \)CDM model as the mean function. Thus if posterior of \( \sigma_f \) is consistent with 0, then \( \Lambda \)CDM is consistent with the two datasets.

3 RECONSTRUCTIONS

In this section, we present reconstructions of various parameters of the Universe’s expansion history that are independent of the absolute scale of the expansion history, including \( h(z) = H(z)/H_0 \), the deceleration parameter \( q(z) \), and the om diagnostic \( om(z) \). The deceleration parameter is given by the following formula,

\[
q(z) = \frac{\ddot{a}}{a} = -1 + \frac{d \log H(z)}{d \log(1+z)},
\]

and the om diagnostic is given by

\[
om(z) = \frac{h(z)^2 - 1}{(1+z)^3 - 1}.
\]

Each of these parameters are potentially useful for testing whether the GP reconstructions are consistent with \( \Lambda \)CDM or if some evolution is preferred. The dimensionless expansion history, \( h(z) \), is the most obvious and direct quantity to reconstruct from the data, however, its interpretation is more uncertain because the variation within \( \Lambda \)CDM can look categorically similar to the variation within models of evolving dark energy. Specific functions of \( h(z) \), however, can make more robust tests of \( \Lambda \)CDM or not \( \Lambda \)CDM.

For instance, \( om(z) \) should be constant and equal to the matter density \( \Omega_m \) if \( \Lambda \)CDM correctly described the low-redshift distances. Any evolution in the expansion history, for example from an evolving dark energy, would show up as an evolution in this parameter.

Similarly, for \( \Lambda \)CDM \( 1 + q(z) = \frac{\Omega_m(1+z)}{\Omega_m + \Omega_k} \), so at large redshifts it is equal to 3/2 and then transitions to 3/2\( \Omega_m \) by \( z = 0 \). Since this parameter is, effectively, the second derivative of the data, using a model independent method to constrain this quantity will necessarily give uncertain and noisy results.

The results of these reconstructions are shown in Fig. 2. The lighter blue shaded regions correspond to the 68% and 95% CLs of the GP regression. These quantities are calculated by taking each GP reconstruction, calculating \( h(z), om(z), \) and \( q(z) \) for each, then, for each redshift, grab the quantity at that redshift, and then make a histogram over all the different reconstructions, weighted by their likelihoods. We use these histograms to calculate the 68% and 95% CLs.

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reconstruction that would have a negative dark energy density at any point on purely a priori grounds, but it is still interesting to see what sort of expansion history is needed to fit the data better than ΛCDM. Taken together, each of these discussed features point towards the conclusion that these reconstructions that fit the data better than ΛCDM are merely over-fitting the noise in the data.

You can, of course, hack your way to a significant model by taking these reconstructions and contriving a model that generically predicts the reconstructions over the entirety of its parameter space. This feature of model-fitting methods demonstrates another aspect of why model-independent methods are useful; you cannot hack your way to a significant result.

4 ANCHORS AND CURVATURE

In this section, we use the SDSS BAO measurements alongside the Pantheon SN distances to constrain $H_0 r_d$. This parameter is the relative anchor of these two unanchored datasets. The SN constrain unanchored distances, or similarly, they constrain $E(z) = H(z)/H_0$. The individual BAO constraints measure $H(z)r_d$ or $D_M/r_d$ at various redshifts. However, to get $H_0 r_d$, one still needs to make assumptions to project down to $z = 0$. This is where the SN enter, since it is in this region where they have the greatest constraining power. Thus combining the two dataset can yield robust and tight constraints on $H_0 r_d$, even with model-independent methods.

It is trivial to calculate this in the ΛCDM case. The low redshift expansion history and distances are simply given by the typical parameters $H_0, \Omega_m$ and $\Omega_k$. We also fit for $M_B$ and $r_d$ to calibrate the distances from the Pantheon and SDSS BAO datasets. Where normally, within ΛCDM, $r_d$ is a derived parameter which depends on the other parameters of the background model ($H_0, \Omega_B, \Omega_{cdm}, \Omega_k$). In this analysis, because we are only fitting low-redshift distances, we seek to remain agnostic about any potential beyond-ΛCDM modifications which might affect $r_d$. Therefore, we treat $r_d$ as an independent parameter and fit for it independently of the other parameters. In summary, we vary the five mentioned parameters ($H_0, \Omega_m, \Omega_k, M_B, r_d$) and use MCMC to calculate the posterior of these parameters. Thus it is relatively trivial to express the constraint in terms of $H_0 r_d$, the samples of $H_0 r_d$ are simply the multiplication of the samples of $H_0$ and $r_d$.

Each sample from the GP is a randomly generated $H(z)$, so $H_0$ in this case is simply $H(z = 0)$. For the GP case, we also treat $M_B$ and $r_d$ as nuisance parameters, so we also fit for these parameters alongside the expansion histories generated from the GP. Thus, again, the samples of $H_0 r_d$ are simply the multiplication
of the samples of \( H(z) = 0 \) and \( r_d \). Since the BAO dataset constrains both the distances \( D_M(z) \) and the expansion rate \( H(z) \), the GP can constrain the curvature, since

\[
D_M(z) = \Omega_k^{1/2} \sinh \left( \Omega_k^{1/2} \int_0^z \frac{dz'}{H(z')} \right). \tag{6}
\]

The GP case cannot constrain the matter density in a similar way since for any \( \Omega_m \), I can then choose \( w(z) \) to get the \( H(z) \) needed for the GP reconstruction.

For the \( \Lambda \)CDM case, we find that \( H_0r_d = 10030 \pm 130 \) km/s, \( \Omega_m = 0.28 \pm 0.04 \), and \( \Omega_k = 0.05 \pm 0.10 \) and for the GP case, we find that \( H_0r_d = 10040 \pm 140 \) km/s and \( \Omega_k = 0.02 \pm 0.20 \).

5 DISCUSSION AND CONCLUSIONS

These results can be used to constrain the low redshift expansion history and so SN and BAO can adjudicate low-redshift explanations for the H0 tension, e.g. curvature or evolving dark energy.

These results can also be used to constrain any potential evolution of the SN. It has been claimed (Kim et al. 2019) that SN might not show any evidence for an accelerating Universe or for dark energy. The claim instead is that SN evolve with redshift, that either the light-curve calibration with SALT-2 or the Tripp formula would need extra freedom to account for properties of the host galaxy. Our methodology implicitly constrains these ideas. Since we found that both the reconstructed expansion histories from the SN and from the BAO are consistent with each other and with \( \Lambda \)CDM, any astrophysical evolution would break this consistency and thus be disfavored.

This shows that the inference of dark energy is dependent on not just the SN dataset; the BAO dataset confirms this as well.

In this paper, we use GP regression to show that the SN and BAO datasets are consistent with each other and with \( \Lambda \)CDM. Further, we reconstruct dimensionless functions of the expansion history of the Universe, \( h(z), q(z), om(z) \). This allows us to visually inspect the consistency between these datasets and the \( \Lambda \)CDM model and to demonstrate that there is still some flexibility allowed at low redshifts. Finally, we calculate \( \Lambda \)CDM posteriors from the two datasets finding that \( H_0r_d = 10030 \pm 130 \) km/s, \( \Omega_m = 0.28 \pm 0.04 \), and \( \Omega_k = 0.05 \pm 0.10 \). We also constrain \( H_0r_d \) and \( \Omega_k \) using our non-parametric GP method finding that \( H_0r_d = 10040 \pm 140 \) km/s and \( \Omega_k = 0.02 \pm 0.20 \).

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