Holography of Charged Black Holes with $RF^2$ Corrections

Da-Wei Pang$^1$ $^2$ $^3$

$^1$Center for Quantum Spacetime(CQUeST), Sogang University, Korea
$^2$Institute of Theoretical Physics, Chinese Academy of Sciences
$^3$CENTRA, Lisbon, Portugal

Based on ongoing work.
Talk given at ITP, CAS, 04.06.2011
Outline

1. Introduction
   - Gauge/gravity duality and condensed matter physics
   - A brief review of 1010.0443[hep-th]

2. The perturbative solution
   - The set-up
   - The perturbative solution
   - Thermodynamics

3. DC conductivity
   - The effective action approach
   - Our case

4. Shear viscosity, thermal conductivity and relevant ratios
   - Shear viscosity
   - Thermal conductivity

5. Summary and discussion
What is Gauge/gravity duality?

- A holographic duality between a weakly-coupled theory of gravity in certain spacetime and a strongly-coupled field theory living on the boundary of that spacetime.
- A powerful new tool for investigating dynamics of strongly-coupled field theories in the dual gravity side.
- A new window on understanding real-world physics: QCD, CMT, etc.
- Two complementary approaches: bottom-up and top-down.
Two complementary approaches:

**Bottom-up**
- Toy-models coming from simple gravity theory;
- Basic ingredients: $g_{\mu\nu}, A_\mu, \psi$ and/or dilaton $\phi$;
- Advantage(s): simplicity and universality;
- Disadvantage(s): the dual field theory is unclear.

**Top-down**
- Configurations originated from string/M theory;
- Exact solutions of SUGRA or Dp/Dq-branes;
- Advantage(s): good understanding on the field theory;
- Disadvantage(s): complexity.
The main results of 1010.0443[hep-th]

By R. C. Myers, S. Sachdev and A. Singh

- Charge transport near 2+1-dim strongly interacting quantum critical points;
- Background: Schwarzschild-AdS$_4$;
- Effective action for $A_\mu$

$$I_{\text{vec}} = \frac{1}{g_4^2} \int d^4 x \sqrt{-g}\left[-\frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd}\right], \quad (1)$$

- The DC conductivity

$$\sigma_{\text{DC}} = \frac{1}{g_4^2}(1 + 4\gamma). \quad (2)$$
A brief review of 1010.0443[hep-th]

An alternative form of the corrections

$$I'_{\text{vec}} = \frac{1}{\tilde{g}_4^2} \int d^4x \sqrt{-g} \left[ -\frac{1}{4} F_{ab} F^{ab} + \alpha L^2 (R_{abcd} F^{ab} F^{cd} ight.$$ \begin{align*} &- 4 R_{ab} F^{ac} F^{b}_c + RF^{ab} F_{ab}) \right], 
\end{align*} \tag{3}

arising from KK reduction of 5D Gauss-Bonnet gravity.

In neutral background $R_{ab} = -3/L^2 g_{ab}$, using the definition of the Weyl tensor, the action (3) becomes

$$I'_{\text{vec}} = \frac{1 + 8\alpha}{\tilde{g}_4^2} \int d^4x \sqrt{-g} \left[ -\frac{1}{4} F_{ab} F^{ab} + \frac{\alpha}{1 + 8\alpha} L^2 C_{abcd} F^{ab} F^{cd} \right]. \tag{4}$$
It is equivalent to (1) with the following identifications

\[ g_4^2 = \frac{\tilde{g}_4^2}{1 + 8\alpha}, \quad \gamma = \frac{\alpha}{1 + 8\alpha}. \tag{5} \]

Thus the charge transport properties are identical. In particular,

\[ \sigma_{\text{DC}} = \frac{1 + 12\alpha}{\tilde{g}_4^2}. \tag{6} \]

**QUESTION:** How about the case with a non-vanishing chemical potential?
The set-up

The starting point

Leading order solution: \( RN-AdS_4 \). The action

\[
S_0 = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [R + \frac{6}{L^2} - \frac{L^2}{g_F^2} F_{ab} F^{ab}]. \tag{7}
\]

The metric

\[
ds_0^2 = \frac{r^2}{L^2} [-f_0(r) dt^2 + dx^2 + dy^2] + \frac{L^2}{r^2} \frac{dr^2}{f_0(r)}, \tag{8}
\]

where

\[
f_0(r) = 1 - \frac{M}{r^3} + \frac{Q^2}{r^4}, \tag{9}
\]
The gauge field

\[ A_t^{(0)} = \mu_0 (1 - \frac{r_0}{r}). \]  \hspace{1cm} (10)

The horizon \( r_0 \) satisfies \( f_0(r_0) = 0 \), \( \Rightarrow M = r_0^3 + Q^2/r_0 \).

The chemical potential \( \mu_0 \), charge density \( \rho_0 \), energy density \( \epsilon_0 \) and entropy density \( s_0 \)

\[ \mu_0 = \frac{g_F Q}{L^2 r_0}, \ \ \rho_0 = \frac{2Q}{\kappa^2 L^2 g_F}, \]
\[ \epsilon_0 = \frac{M}{\kappa^2 L^4}, \ \ \ s_0 = \frac{2\pi r_0^2}{\kappa^2 L^2}. \]  \hspace{1cm} (11)
The set-up

The starting point  Cont’d

The temperature

\[ T_0 = \frac{3r_0}{4\pi L^2} \left(1 - \frac{Q^2}{3r_0^4}\right), \]  \hspace{1cm} (12)

The extremal limit

\[ T_0 = 0, \quad \Rightarrow \quad Q^2 = 3r_0^4. \]  \hspace{1cm} (13)

One can verify that the first law of thermodynamics holds

\[ d\epsilon_0 = T_0 ds_0 + \mu_0 d\rho_0. \]  \hspace{1cm} (14)
The set-up

The equations of motion

The full action including higher order corrections

\[
S \equiv S_0 + S_1 = \frac{1}{2 \kappa^2} \int d^4 x \sqrt{-g} \left[ R + \frac{6}{L^2} - \frac{L^2}{g_F^2} F_{ab} F^{ab} \right. \\
\left. + \frac{\alpha L^4}{g_F^2} (R_{abcd} F^{ab} F^{cd} - 4 R_{ab} F^{ac} F^{b}{}_{c} + R F^{ab} F_{ab}) \right].
\] (15)

\( \alpha \) - a dimensionless constant.

The modified Maxwell equation

\[
\nabla_a [F^{ab} - \alpha L^2 (R^{ab}{}_{cd} F^{cd} - 2 R^{ac} F^b{}_{c} + 2 R^{bc} F^a{}_{c} + R F^{ab})] = 0. \] (16)
The set-up

The equations of motion  Cont’d

The Einstein equation

\[
R_{ab} - \frac{1}{2} R g_{ab} = \frac{3}{L^2} + \frac{2L^2}{g_F^2} F_{ac} F_b^c - \frac{L^2}{2g_F^2} g_{ab} F^2
\]

\[+ \frac{\alpha L^4}{2g_F^2} \left( \frac{1}{2} g_{ab} R_{cdf} F^{cd} F^{ef} - 2R_{abcdefgh} F^{d} F^{ef} - 2R_{bdeq} F^{d} F^{ef} \right) \]

\[+ 2 \nabla^d \nabla^f F_{da} F_{bf} \right) + \frac{\alpha L^4}{2g_F^2} \left( -2 g_{ab} R_{cd} F^{ce} F^{d} F_{e} - 2 \nabla_d \nabla_a F_{bf} F^{df} \right) \]

\[+ 2 \nabla_d \nabla_b F_{af} F^{df} + 2 \Box F_{af} F^{bf} + 2 g_{ab} \nabla_c \nabla_d F^{cf} F^{df} \]

\[+ 4 R_{ac} F_{bf} F^{cf} + 4 R_{bc} F_{af} F^{cf} + 2 R_{cd} F_{a} F^{d} F_{b} + 2 R_{cd} F^{c} F_{b} F^{d} \]

\[+ \frac{\alpha L^4}{2g_F^2} \left( \frac{1}{2} g_{ab} RF^2 - R_{ab} F^2 + \nabla_a \nabla_b F^2 + g_{ab} \Box F^2 + 2RF_{ac} F_b^c \right). \]
The perturbative solution

The method for obtaining the perturbative solution

The ansatz for the perturbative solution

\[
  ds^2 = \frac{r^2}{L^2}[-f(r)dt^2 + dx^2 + dy^2] + \frac{L^2}{r^2} \frac{dr^2}{g(r)},
\]

\[
  A_t(r) = A_t^{(0)}(r) + H(r),
\]

where

\[
  f(r) = f_0(r)(1 + F(r)), \quad g(r) = f_0(r)(1 + F(r) + G(r)),
\]

The main steps proposed in R. C. Myers, M. F. Paulos and A. Sinha, JHEP 0906, 006 (2009) [arXiv:0903.2834 [hep-th]].
The perturbative solution

The method for obtaining the perturbative solution  Cont’d

1. Considering the combination $G_t^t - f/g G_r^r$, where $G_{ab}$ denotes the Einstein tensor, one finds a first-order linear ODE for $G(r)$, which is solvable.

2. Given $G(r)$, the modified Maxwell equation is easily solved.

3. With the above two perturbative solutions, $F(r)$ can be determined by solving the first-order linear ODE coming from the $rr$-component of the Einstein equation.

Step 1 gives

$$rf_0(r)\partial_r G(r) = 0, \quad \Rightarrow \quad \partial_r G(r) = 0, \quad G(r) = \text{const.} \quad (19)$$
Without loss of generality

\[ G(r) = 0, \quad \Rightarrow \quad f(r) = g(r) = f_0(r)(1 + F(r)). \quad (20) \]

Step 2 leads to

\[ H(r) = h_0 + \frac{h_1}{r} + \frac{2\alpha\mu_0 r_0}{2r^4} - \frac{\alpha\mu_0 Q^2}{2r^4} + \frac{2\alpha\mu_0 r_0 Q^2}{5r^5}. \quad (21) \]

Step 3 gives

\[ Y(r) \equiv f_0(r)F(r) = \frac{y_0}{r^3} - \frac{\alpha Q^2 r_0^3}{2r^7} - \frac{\alpha Q^4}{2r_0 r^7} + \frac{8\alpha Q^4}{5r^8}. \quad (22) \]
The perturbative solution

The explicit form of the perturbative solution  Cont’d

Several constraints (following 0903.2834[hep-th])

- $r = r_0$ is still the horizon.

\[ Y(r_0) = 0, \quad \Rightarrow \quad y_0 = \frac{\alpha Q^2}{2r_0} - \frac{11\alpha Q^4}{10r_0^5}. \]  

(23)

- The charge density remains invariant. Thus the additional terms in the modified Maxwell equation do not contribute

\[ \lim_{r \to \infty} \left[ \sqrt{-g} \alpha L^2 \left( 2 R_{rt} r^t F_{rt} - 2 R^r_r F_{rt} + 2 R^t_t F_{tr} + RF_{rt} \right) \right] = 0, \]

which leads to

\[ h_1 = 0. \]  

(24)
The perturbative solution

The explicit form of the perturbative solution  Cont’d

The gauge potential $A_t(r)$ vanishes at the horizon,

$$ H(r_0) = 0, \quad \Rightarrow \quad h_0 = \frac{\alpha \mu_0 Q^2}{10 r_0^4} - \frac{3}{2} \alpha \mu_0. \quad (25) $$

Thermodynamics:

The temperature

$$ T = \frac{1}{4\pi} \frac{1}{\sqrt{-g_{tt}g_{rr}}} \frac{d}{dr} g_{tt} \bigg|_{r=r_0} $$

$$ = \frac{1}{4\pi L^2 r_0^2} \left[ (3M - \frac{4Q^2}{r_0^3}) + \frac{2\alpha Q^2}{r_0^3} (1 - \frac{3Q^2}{r_0^4}) \right]. \quad (26) $$
The chemical potential and the entropy density

The chemical potential

\[ \mu = A_t(r \to \infty) = \mu_0 - \alpha \mu_0 \left( \frac{3}{2} - \frac{Q^2}{10r_0^4} \right). \] \hspace{1cm} (27)

The entropy density given by Wald formula

\[ s = -2\pi \frac{\partial L}{\partial R_{abcd}} \epsilon_{ab} \epsilon_{cd} = \frac{2\pi r_0^2}{\kappa^2 L^2} + \frac{2\pi \alpha Q^2}{\kappa^2 L^2 r_0^2}. \] \hspace{1cm} (28)

Calculating other thermodynamic quantities: the background subtraction method. Working in the grand canonical ensemble, fixed chemical potential. The reference background: pure AdS$_4$. 
The energy density and the charge density

The energy density

\[
\epsilon = \left( \frac{\partial I_E}{\partial \beta} \right)_\mu - \frac{\mu}{\beta} \left( \frac{\partial I_E}{\partial \mu} \right)_\beta = \frac{M}{\kappa^2 L^4} - \alpha \frac{29Q^4 + 5Q^2r_0^4}{5\kappa^2 L^4 r_0^5}. \tag{29}
\]

The charge density

\[
\rho = -\frac{1}{\beta} \left( \frac{\partial I_E}{\partial \mu} \right)_\beta = \frac{2Q}{g_F\kappa^2 L^2} + \frac{2\alpha(-29Q^5 + Q^3r_0^4)}{5g_F\kappa^2 L^2 r_0^4(Q^2 + 3r_0^4)}. \tag{30}
\]

Quantities characterizing the local stability: the specific heat \( C_\mu \) and the electrical permittivity \( \epsilon_T \).
The specific heat and the electrical permittivity

The specific heat

\[
C_\mu = T \left( \frac{\partial s}{\partial T} \right)_\mu \frac{4\pi r_0^2(3r_0^4 - Q^2)}{\kappa^2 L^2(Q^2 + 3r_0^4)} + \alpha \frac{4\pi Q^2(Q^6 - 527Q^4r_0^4 + 567Q^2r_0^8 + 135r_0^{12})}{5\kappa^2 L^2 r_0^2(Q^2 + 3r_0^4)^3}.
\]  \(31\)

The electrical permittivity

\[
\epsilon_T = \left( \frac{\partial Q}{\partial \mu} \right)_T = \frac{6r_0(Q^2 + r_0^4)}{g_F^2 \kappa^2(Q^2 + 3r_0^4)} + \alpha \frac{6(-39Q^6 + 247Q^4r_0^4 + 11Q^2r_0^8 + 45r_0^{12})}{10g_F^2 \kappa^2 r_0^3(Q^2 + 3r_0^4)^2}.
\]  \(32\)

\[T \geq 0 \rightarrow Q^2 \leq 3r_0^4.\] at leading order \(C_\mu \geq 0, \epsilon_T > 0,\) locally stable. \(\alpha\) corrections. numerical plots.
The Kubo formula

\[ G^{R}_{xx}(\omega, \vec{k} = 0) = -i \int dt d\vec{x} e^{i\omega t} \theta(t) \langle [J_{x}(x), J_{x}(0)] \rangle, \tag{33} \]

\( J_{a} \)-CFT current dual to the bulk gauge field \( A_{a} \).

The DC conductivity

\[ \sigma_{DC} = - \lim_{\omega \to 0} \frac{1}{\omega} \text{Im} G^{R}_{xx}(\omega, \vec{k} = 0), \tag{34} \]

One subtle point: since \( A_{t} \neq 0 \), the perturbation \( A_{x} \) can couple to the metric perturbations \( h_{xi} \).
Strategy: Gauge invariance imposes a relation between the two sets of perturbations which we use to integrate out the $h_{xi}$ and obtain an action that involves only the $A_x$ fluctuation. Introducing a new radial coordinate $u = r_0/r$, horizon $u = u_0$, the fluctuations of metric components and gauge field

\[
\begin{align*}
  h_t^x &= \int \frac{d^3k}{(2\pi)^3} t_k(u) e^{-i\omega t + iky}, \\
  h_u^x &= \int \frac{d^3k}{(2\pi)^3} h_k(u) e^{-i\omega t + iky}, \\
  A_x &= \int \frac{d^3k}{(2\pi)^3} a_k(u) e^{-i\omega t + iky},
\end{align*}
\] (35)
The effective action approach

The approach

The simplest method: considering the quadratic effective action

\[ I_a^{(2)} = \frac{1}{2\kappa^2} \int \frac{d^3k}{(2\pi)^3} du (N(u) a'_k a'_{-k} + M(u) a_k a_{-k}), \]  

(36)

where we have eliminated the contributions from \( t_k(u) \) by using the corresponding Einstein equation and imposing \( h_{ux} = 0 \).

The equation of motion

\[ \partial_u j_k(u) = \frac{1}{\kappa^2} M(u) a_k(u), \]  

(37)

where
The effective action approach

The approach  Cont’d

\( j_k(u) \equiv \frac{\delta I_a^{(2)}}{\delta a'_{-k}} = \frac{1}{\kappa^2} N(u) a'_k(u), \quad (38) \)

Requiring regularity at the horizon (N. Iqbal and H. Liu, Phys. Rev. D 79, 025023 (2009) [arXiv:0809.3808 [hep-th]].)

\[ j_k(u_0) = -i \omega \lim_{u \to u_0} \frac{N(u)}{\kappa^2} \sqrt{\frac{g_{uu}}{-g_{tt}}} a_k(u) + O(\omega^2), \quad (39) \]

The flux factor

\[ 2 \mathcal{F}_k = j_k(u) a_{-k}(u), \quad (40) \]
According to (34), the conductivity is given by

$$\sigma = \lim_{u, \omega \to 0} \frac{1}{\omega} \text{Im} \left[ \frac{2F_k}{a_k(u) a_{-k}(u)} \right]_{k=0} = \lim_{u, \omega \to 0} \text{Im} \left[ \frac{j_k(u) a_{-k}(u)}{\omega a_k(u) a_{-k}(u)} \right]_{k=0},$$

(41)

Note that

$$\frac{d}{du} \text{Im}[j_k(u) a_{-k}(u)] = \text{Im}(f_1(u) a_k(u) a_{-k}(u) + f_2(u) j_k(u) j_{-k}(u)) = 0,$$

(42)

as the two terms are real. Thus it is conserved and can be evaluated at the horizon.
Then the DC conductivity

\[
\sigma_{\text{DC}} = \frac{1}{\kappa^2} K_A^2(u_0) \frac{\mathcal{N}(u_0)}{\mathcal{N}(0)} \bigg|_{k=0},
\]

(43)

where

\[
K_A^2(u) = -N(u) \sqrt{\frac{g_{uu}}{-g_{tt}}}, \quad \mathcal{N}(u) = a_k(u) a_{-k}(u),
\]

(44)

Note that \(\mathcal{N}(u)\) is real and independent of \(\omega\) up to \(O(\omega^2)\). So it is regular at the horizon and is sufficient to set \(\omega = 0\) in the equation of motion for \(a_k\).
For our particular case,

\[ ds^2 = -\frac{r_0^2}{L^2 u^2} f(u)dt^2 + \frac{L^2 du^2}{u^2 f(u)} + \frac{r_0^2}{L^2 u^2} (dx^2 + dy^2), \]  

(45)

where

\[ f(u) = (1 - u)[F(u) + \alpha G(u)], \quad F(u) = 1 + u + u^2 - \frac{Q^2 u^3}{r_0^4}, \]

\[ G(u) = \frac{Q^2 u^3}{10r_0^8} [5r_0^4(1 + u + u^2 + u^3) \]

\[ - Q^2(11 + 11u + 11u^2 + 11u^3 + 16u^4)], \]

(46)
Consider the leading order solution

\[ f(u) = f_0(u) = (1 - u)F(u), \quad (47) \]

according to (34), it is sufficient to set \( k = 0 \).

The constraint for \( t_k \)

\[ t'_k = - 4 \frac{L^4 u^2}{g_F^2 r_0^2} A'_t a_k, \quad (48) \]

Therefore

\[ N(u) = - \frac{r_0}{g_F^2} f_0(u), \quad M(u) = \frac{L^4 \omega^2}{r_0 g_F^2 f_0(u)} - \frac{4 L^4 u^2}{r_0 g_F^4} A'_t^2, \quad (49) \]
The solution for $a_k(u)$

$$a_k(u) = a_0 \left(1 - \frac{4Q^2}{3(r_0^4 + Q^2)}u\right), \quad (50)$$

The DC conductivity

$$\sigma_{DC} = \frac{L^2}{\kappa^2 g_F^2} \frac{(3r_0^4 - Q^2)^2}{9(r_0^4 + Q^2)^2}, \quad (51)$$

which agrees with previous result (e.g. X. H. Ge, K. Jo and S. J. Sin, [arXiv:1012.2515 [hep-th]].).
Including corrections: the steps are more or less the same, but the equation becomes more complicated. Keeping the solution to first order of $\alpha$ and $Q^2$,

$$a_k(u) = a_0 + a_1 u + \alpha[(a_2 - 2a_1)u - \frac{1}{4}a_1 u^4$$

$$-\left(\frac{1}{3}a_0 + \frac{1}{4}a_1\right)\frac{Q^2 u^4}{r_0^4}\right], \quad a_1 = -\frac{4a_0 Q^2}{3(r_0^4 + Q^2)},$$

(52)

$a_2$-integration constant. The conductivity

$$\sigma_{\text{DC}} = \frac{L^2}{\kappa^2 g_F^2} \left[\left(\frac{3r_0^4 - Q^2}{9(r_0^4 + Q^2)}\right)^2 + 2\alpha(a_2 + \frac{8 - 4a_2 Q^2}{3r_0^4})\right],$$

(53)
In the limit of $Q = 0$,

\[
\sigma_{\text{DC}} = \frac{L^2}{\kappa^2 g_F^2} (1 + 2\alpha a_2),
\]  

(54)

one can reproduce the result in arXiv: 1010.0443[hep-th] by suitably choosing $a_2$. 

Our case  Cont’d
The retarded Green’s function

\[ G_{xy,xy}^R(\omega, \vec{k} = 0) = -i \int dt d\vec{x} e^{i\omega t} \theta(t) \langle [T_{xy}(x), T_{xy}(0)] \rangle, \tag{55} \]

The shear viscosity is given by

\[ \eta = - \lim_{\omega \to 0} \frac{1}{\omega} \text{Im} G_{xy,xy}^R(\omega, \vec{k} = 0), \tag{56} \]

One can still apply the effective action approach.
Shear viscosity

The approach

Consider the metric perturbation

$$h_{x}^{y}(t, u) = \int \frac{d^{3}k}{(2\pi)^{3}} \phi(u) e^{-i\omega t}, \quad (57)$$

and expand the action to quadratic order in $\phi$,

$$I_{\phi}^{(2)} = \frac{1}{2\kappa^{2}} \int \frac{d^{3}k}{(2\pi)^{3}} du [A(u)\phi''\phi + B(u)\phi'\phi' + C(u)\phi'\phi'

+ D(u)\phi\phi + E(u)\phi''\phi'' + F\phi''\phi' + K_{GH}], \quad (58)$$

$K_{GH}$-contributions from the Gibbons-Hawking terms.
Shear viscosity

The approach  Cont’d

After making use of the equation of motion and integrating by parts

\[ \tilde{i}^{(2)}_{\phi} = \frac{1}{2\kappa^2} \int \frac{d^3k}{(2\pi)^3} du [(B - A - \frac{F'}{2})\phi'\phi' + E(u)\phi''\phi''] \]

\[ + (D - \frac{(C - A')'}{2})\phi\phi] + \tilde{K}_{GH}. \] (59)

The canonical momentum is given by

\[ \Pi(u) \equiv \frac{\delta \tilde{i}^{(2)}_{\phi}}{\delta \phi'} = \frac{1}{\kappa^2} [(B - A - \frac{F'}{2}) - (E(u)\phi'')'] \] (60)
According to arXiv: 0809.3808[hep-th],

\[
\eta = \lim_{u, \omega \to 0} \frac{\Pi(u)}{i \omega \phi(u)}.
\]  

(61)

In the low frequency limit $\partial_u \Pi(u) = 0$, so we can evaluate $\Pi(u)$ at the horizon. Imposing the regularity condition,

\[
\eta = \frac{1}{\kappa^2} (K_\phi^2(u_0) + K_\phi^4(u_0)),
\]  

(62)

where

\[
K_{\phi}^{(2)}(u) = \sqrt{\frac{g_{uu}}{-g_{tt}}} (A - B + \frac{F'}{2}), \quad K_{\phi}^{(4)}(u) = \left[ E(u) \left( \sqrt{\frac{g_{uu}}{-g_{tt}}} \right)' \right]' .
\]  

(63)
for our particular background, the nonvanishing functions in $\tilde{I}_\phi^{(2)}$, 

$$
A(u) = \frac{2r_0^4 f(u)}{L^4 u^2}, \quad B(u) = \frac{3r_0^3 f(u)}{2L^4 u^2} - \frac{\alpha u^2 Q^2 f(u)}{L^4 r_0}, \\
C(u) = -\frac{6r_0^3 f(u)}{L^4 u^3} + \frac{2r_0^3 f(u)'}{L^4 u^2} - \frac{4\alpha uQ^2 f(u)}{L^4 r_0},
$$

(64)

therefore

$$
K_\phi^2(u) = \frac{r_0^2}{2u^2 L^2} + \frac{\alpha u^2 Q^2}{L^2 r_0^2}, \quad K_\phi^4(u) = 0,
$$

(65)

which leads to

$$
\eta = \frac{1}{\kappa^2} \left( \frac{r_0^2}{2L^2} + \frac{\alpha Q^2}{L^2 r_0^2} \right).
$$

(66)
The thermal conductivity determines the response of the heat flow to temperature gradients, $T^t_i = -\kappa_T \partial_i T$.

The expression (D. T. Son and A. O. Starinets, JHEP 0603, 052 (2006), hep-th/0601157)

$$\kappa_T = \left( \frac{S}{\rho} + \frac{\mu}{T} \right)^2 T \sigma,$$

One can easily obtain $\kappa_T$ by substituting previous results into this expression.
\( \frac{\eta}{s} \) and \( \frac{\kappa_T \mu^2}{(\eta T)} \)

One interesting ratio \( \frac{\eta}{s} \),

\[
\frac{\eta}{s} = \frac{1}{4\pi} \left(1 + \alpha \frac{Q^2}{r_0^4}\right). \tag{68}
\]

When \( Q = 0 \), it reproduces the well-known bound \( 1/4\pi \). It might be violated in the presence of a chemical potential. Another ratio

\[
\frac{\kappa_T \mu^2}{\eta T} = 2\pi^2 g_F^2 + \alpha \pi^2 g_F^2 [(4a_2 - 10) + \frac{422 + 80a_2}{15} \frac{Q^2}{r_0^4}], \tag{69}
\]

The bound in hep-th/0601157: \( 8\pi^2 \) can also be violated.
We consider $RF^2$ corrections to $RN – AdS_4$ black holes.

The perturbative solutions are calculated and the thermodynamic properties are discussed.

The DC conductivity is obtained via the effective action approach, which can reproduce the result in 1010.0443[hep-th] in certain limit.

The shear viscosity and the thermal conductivity are evaluated.

Two interesting ratios $\eta/s$ and $\kappa T \mu^2 / (\eta T)$ are obtained. The corresponding bounds can be violated.
Hydrodynamic quantities in extremal background, M. F. Paulos, arXiv:0910.4602[hep-th]. In particular, $\sigma \sim \omega^2$.

The full correlation functions in the presence of $RF^2$ corrections.

Holographic optics(1006.5714[hep-th]).

...
Thank you!