Light Singlino Dark Matter at the LHC

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Light Dark matter

Different evidences from astrophysics and cosmology \( \Rightarrow \) DM exists..

\[
\Omega h^2 = 0.12 \pm 0.001 \quad \Rightarrow \quad \text{Natural for WIMP} \quad \Rightarrow \quad \text{strong exclusion from different experiments}
\]

Focus on 2 – 20 GeV range of DM mass

- Not MSSM
  - Rough **Lower mass bound** on MSSM neutralino DM is \( \sim 34 \text{ GeV} \)
    
    (Phys. Rev. D 95, 095018 (2017))

- What then?
  - Maybe DM is SM **singlet**, inert to SM fermions or gauge bosons
  - This is natural in NMSSM when DM is **singlino-like neutralino**
  - The **singlino-like LSP** can be **very light** in the allowed parameter space of NMSSM

Different evidences from astrophysics and cosmology \( \Rightarrow \) DM exists..
NMSSM, Physical States and relevant parameters

NMSSM Superpotential

\[ W_{\text{MSSM}}(\mu = 0) + \lambda S H_u H_d + \frac{1}{3} \kappa S^3 \]

- **Seven Physical Higgs States**
  - \( H_1, H_2, H_3, A_1, A_2, H^\pm \)
    - 3 neutral scalars
    - 2 neutral pseudo-scalars
    - Two charged Higgs

- **Important parameters**
  - \( \lambda, \kappa, \tan\beta, A_\lambda, A_\kappa, \mu_{\text{eff}} \)

- **Five neutralino states** comes from linear combination of:
  - \( \tilde{B}, \tilde{W}, \tilde{H}_u, \tilde{H}_d, \tilde{S} \)

- **LSP**:
  \[ \chi_1^0 = N_{11} \tilde{B} + N_{12} \tilde{W} + N_{13} \tilde{H}_u + N_{14} \tilde{H}_d + N_{15} \tilde{S} \]

- **Important parameters**
  - \( M_1, M_2, \mu_{\text{eff}}, \lambda, \kappa, \tan\beta \)

- **Singlino-like**
  - \( \tilde{\chi}_1^0 \rightarrow |2\kappa v_s| << \mu_{\text{eff}}, M_1, M_2 \)
  - \( M_{\tilde{\chi}_1} \simeq 2\kappa v_s = 2\frac{\kappa}{\lambda} \mu_{\text{eff}} \)
We need **singlet-like light Higgs boson** for DM annihilation:

\[ m_{A_1/H_1} \sim 2m_{\chi_1} \]

**Approximate coupling:**

\[ g_{\chi_1 \chi_1 H_1} \sim \sqrt{2} S_{13} (\lambda N_{13} N_{14} - \kappa N_{15}^2) \sim -\sqrt{2} S_{13} \kappa N_{15}^2 \]

\[ g_{\chi_1 \chi_1 A_1} \sim -\sqrt{2} P_{12} \kappa N_{15}^2 \]

**kappa plays a important role to satisfy DD bounds**

For a **low mass DM-compatible** scenario

\[ |\kappa| \sim 10^{-3} - 10^{-2}, \quad \lambda \sim 10^{-1}, \]

\[ |A_\kappa| \sim 10 - 100 \text{ GeV}, \quad A_\lambda \sim 1 \text{ TeV} \]

\[ \kappa A_\kappa < 0 \]
Find allowed parameter spaces

- Scanned all the parameters using **NMSSMTools-5.5.0**

\[ 0.1 \leq \lambda \leq 0.65, \quad -0.01 \leq \kappa \leq 0.01, \quad 1.5 \leq \tan \beta \leq 20, \quad 100 \text{ GeV} \leq \mu_{\text{eff}} \leq 1000 \text{ GeV}, \]
\[ 500 \text{ GeV} \leq A_\lambda \leq 3500 \text{ GeV}, \quad -100 \text{ GeV} \leq A_\kappa \leq 100 \text{ GeV}. \]

Ranges comes out to be consistent with our arguments

Selected BPs consistent with all relevant constraints

- **Features of Branching Ratios**
  
  - \( \text{BR}(H_{SM} \to H_1 H_1/A_1 A_1) \) is at most 11-12%
  
  - Light Higgs bosons primarily decays to \( b\bar{b}/\tau\tau \) and \( \tilde{\chi}_1^0\tilde{\chi}_1^0 \)
Ranges of important parameters
How to search at the LHC

- Singlino DM \textbf{indirectly} produced via production of \textbf{light singlet Higgs bosons}

- \textbf{Light singlet Higgs bosons} act as \textbf{portal} between visible and dark sector

- Features of the \textbf{fermions} and the \textbf{extra jet} helps to characterise the MET as coming from DM

\[
\Delta R(f, \bar{f}) \simeq \frac{m_{A_1}/H_1}{z(z-1)p_T}
\]

- \(m_{H_1/A_1} \leq 10 \text{ GeV}\) \hspace{1cm} \text{(Low mass region)}

- \(10 \text{ GeV} \leq m_{H_1/A_1} \leq 30 \text{ GeV}\) \hspace{1cm} \text{(Moderate mass region)}

- \(30 \text{ GeV} \leq m_{H_1/A_1} \leq 60 \text{ GeV}\) \hspace{1cm} \text{(High mass region)}
Decay products from these light Higgs bosons emerge as a **single fat jet**

Signal & background (high and moderate mass region)

\[ J_{bb} + E_T + \geq 1j \]

- a) with \( m_{J_{bb}} < 30 \) GeV for lower mass range
- b) with \( 30 < m_{J_{bb}} < 60 \) GeV for high mass range

**Signal**

- \( t\bar{t}, Wb\bar{b} + \text{jets}, Zb\bar{b} + \text{jets} \)

**Dominating backgrounds**

Also checked \( WH_{SM} + \text{jets}, ZH_{SM} + \text{jets}, WZ + \text{jets}, ZZ + \text{jets} \)

Generated using

- MADGRAPH +
- PYTHIA +
- Delphes
Kinematic variables and other selections

Mass of Higgs-jet

\[ m_{J_{b\bar{b}}} < 30 \text{ GeV} \]

\[ 30 \text{ GeV} < m_{J_{b\bar{b}}} < 60 \text{ GeV} \]

Other requirements:

➢ Missing pT
➢ lepton veto
➢ Exactly one Higgs-jet
➢ At least one other jet

Transverse mass of Higgs-jet and MET

\[ m_T(J_{b\bar{b}}, \slashed{E_T}) \leq 140 \text{ GeV} \]

\[ R(n_j^{\text{min}}) = \frac{\sum_{i=1}^{n_j^{\text{min}}}}{H_T} \]

\[ R > 0.5 \]
Low mass region

- **ττ decay mode** of light Higgs get enhanced
- At this very low pT, hadronic decay mode of τ suffers from large background of QCD
- We consider **leptonic decay mode of τ**

**Signal**

\[ \ell^+ \ell^- + \not{E}_T + \geq 1 \text{jets} \]

**Dominating backgrounds**

Drell – Yan, t\bar{t}, W + jets, WW + jets, WZ + jets

- We also checked Υ and J/ψ, ZZ + jets, ZH_{SM} + jets backgrounds, having negligible contributions
**Summary**

- NMSSM can provide this light DM solution **allowed by all relevant theoretical and experimental constraints** if the DM is singlino-like.

- Light singlino-like DM comes along with **singlet-like light Higgs bosons** which are also allowed in this scenario.

- **DM annihilation** takes place through these singlet-like Higgs bosons.

- Devised a process by which singlino DM and the singlet Higgs bosons can be searched at the LHC.

- We observed **reasonable signal sensitivity for high luminosity options** at LHC.

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|          | BP1 | BP2 | BP3 | BP4 | BP5 | BP6 |
|----------|-----|-----|-----|-----|-----|-----|
| $\frac{S}{\sqrt{B}} (\mathcal{L} = 300 \text{ fb}^{-1})$ | 6   | 11  | 14  | 8   | 7   | 3.5 |
| $\frac{S}{\sqrt{B}} (\mathcal{L} = 3000 \text{ fb}^{-1})$ | 19  | 35  | 44  | 25  | 22  | 11  |
Backup Slides
### Flow of cuts (moderate mass region)

|                | BP2 | BP3 | bbZ + jets | bbW + jets | t\(\bar{t}\) |
|----------------|-----|-----|------------|------------|-------------|
| \(\sigma (pb)\) | 12.4| 12.4| 152.8      | 139.8      | 597.9       |
| \(\sigma \times \epsilon_{BR}\) | 0.7 | 0.9 | 152.8      | 139.8      | 597.9       |
| lepton veto    | 0.6 | 0.8 | 108.5      | 97.6       | 298.2       |
| \(n_j \geq 1\) | 0.5 | 0.7 | 107.4      | 96.3       | 297.7       |
| \(E_T > 40.0\ GeV\) | 0.3 | 0.4 | 32.8       | 24.4       | 109.4       |
| No. of J\(b\bar{b}\) = 1 | 0.05 | 0.06 | 1.8        | 3.0        | 4.9         |
| \(m_{J\bar{b}} < 30.0\ GeV\) | 0.05 | 0.05 | 0.3        | 1.0        | 1.3         |
| \(m_T(J\bar{b}, E_T) \leq 140\ GeV\) | 0.04 | 0.04 | 0.2        | 0.8        | 0.9         |
| \(R > 0.5\) | 0.034 | 0.04 | 0.08       | 0.6        | 0.4         |
| \(\sigma \times \text{K-factor} \times \epsilon^2_b\) | 0.018 | 0.022 | 0.04       | 0.47       | 0.24        |

### K-factors:
- signal \(\rightarrow 1.8\)
- bbZ + jets \(\rightarrow 1.7\)
- bbW + jets \(\rightarrow 2.6\)
- t\(\bar{t}\) \(\rightarrow 1.4\)

Used MCFM

### Additional b-tagging efficiency
\[
\epsilon_b = \begin{cases} 
0.66 & \text{for } t\bar{t} \\
0.55 & \text{otherwise}
\end{cases}
\]

(CMS Collaboration, JINST 13 no. 05, (2018) P05011)
## Flow of cuts (High mass region)

| Cut Description                        | BP4  | BP5  | BP6  | bbZ + jets | bbW + jets | tt   |
|----------------------------------------|------|------|------|------------|------------|------|
| $\sigma$ (pb)                          | 12.4 | 12.4 | 12.4 | 152.8      | 139.8      | 597.9|
| $\sigma \times \epsilon_{BR}$         | 1.3  | 1.2  | 1.0  | 152.4      | 139.8      | 597.9|
| Lepton veto                            | 1.3  | 1.1  | 0.9  | 108.6      | 97.6       | 298.2|
| $n_j \geq 1$                           | 1.2  | 1.0  | 0.9  | 108.0      | 97.3       | 297.8|
| $E_T > 35.0$ GeV                       | 0.9  | 0.6  | 0.4  | 39.4       | 30.4       | 127.9|
| No. of $J_{b\bar{b}} = 1$              | 0.05 | 0.04 | 0.03 | 3.0        | 2.9        | 7.8  |
| $30.0 < m_{J_{b\bar{b}}} < 60.0$ GeV   | 0.03 | 0.03 | 0.01 | 0.6        | 0.8        | 1.8  |
| $m_T(J_{b\bar{b}}, E_T) \leq 140$ GeV | 0.03 | 0.03 | 0.01 | 0.5        | 0.5        | 1.2  |
| $R > 0.5$                              | 0.024| 0.02 | 0.01 | 0.26       | 0.4        | 0.5  |
| $\sigma \times K$-factor$\times \epsilon_b^2$ | 0.013| 0.011| 0.0055| 0.13       | 0.3        | 0.3  |
Flow of cuts (Low mass region)

|                | BP1 | tt  | DY + jets | W+jets | WW+jets | WZ+jets |
|----------------|-----|-----|-----------|--------|---------|---------|
| $\sigma \times \epsilon_{BR}$ (pb) | 1.2 | 598 | 4242      | $5 \times 10^4$ | 116  | 51      |
| $E_T > 30$ GeV | 0.8 | 371.7 | 314.2 | 10771 | 46.8 | 23.7 |
| $n_j \geq 1$   | 0.74 | 371.1 | 301.7 | 10516 | 45.2 | 23.3 |
| $N(\text{lepton}) = 2$ | 0.005 | 15.2 | 16.5 | 0.2 | 1.1 | 0.4 |
| $M_{\ell \ell} < 10$ GeV | 0.0032 | 0.08 | 0.11 | 0.07 | 0.01 | 0.001 |
| b-veto        | 0.0032 | 0.024 | 0.11 | 0.07 | 0.01 | 0.001 |
| $\sigma \times$ K-factor | 0.006 | 0.034 | 0.14 | 0.1 | 0.02 | 0.002 |

**K-factors:**
- $\text{DY + jets} \rightarrow 1.3$ (arXiv:2001.11377)
- $\text{W + jets} \rightarrow 1.4$ Phys. Rev. Lett. 115 no. 6, (2015) 062002
- $\text{WW + jets} \rightarrow 1.8$ Phys. Rev. Lett. 113 no. 21, (2014) 212001
- $\text{WZ + jets} \rightarrow 2.07$ Phys. Lett. B 761 (2016) 179–183
Next-to Minimal Supersymmetric Standard Model

- **NMSSM Superpotential**
  \[ W_{\text{MSSM}}(\mu = 0) + \lambda S H_u H_d + \frac{1}{3} \kappa S^3 \]

  \( S \) gets a VEV:  
  \[ v_s = \langle S \rangle \]

  We get an effective \( \mu \)-term:  
  \[ \lambda v_s H_u H_d \]
  with
  \[ \mu_{\text{eff}} = \lambda v_s \rightarrow \text{Solves } \mu\text{-problem} \]

- Also in NMSSM, SM-Higgs mass comes out more naturally than MSSM without requirement of much fine tuning

  \[ m^2_H \sim M_Z^2 \cos^2 2\beta + \lambda^2 v_s^2 \sin^2 2\beta + \Delta m^2_H \]

  Nucl. Phys. B860 (2012) 207–244
Light singlino LSP

\[ M_N = \begin{pmatrix}
M_1 & 0 & -g_1 v s_\beta & g_1 v c_\beta & 0 \\
0 & M_2 & \frac{g_2 v s_\beta}{\sqrt{2}} & \frac{-g_2 v c_\beta}{\sqrt{2}} & 0 \\
\frac{-g_1 v s_\beta}{\sqrt{2}} & \frac{g_2 v s_\beta}{\sqrt{2}} & 0 & -\mu_{\text{eff}} & -\lambda v c_\beta \\
\frac{g_1 v c_\beta}{\sqrt{2}} & \frac{-g_2 v c_\beta}{\sqrt{2}} & -\mu_{\text{eff}} & 0 & -\lambda v s_\beta \\
0 & 0 & -\lambda v s_\beta & -\lambda v c_\beta & 2\kappa v_s
\end{pmatrix} \]

- \( \tilde{\chi}_1^0 \) becomes more singlino-like, as: \(|2\kappa v_s| \ll \mu_{\text{eff}}, M_1, M_2\)

- In singlino limit: \( M_{-0}^{\tilde{\chi}_1} \approx 2\kappa v_s = 2\frac{\kappa}{\lambda} \mu_{\text{eff}} \)

- For very light singlino: \( |\frac{\kappa}{\lambda}| \sim 10^{-2} - 10^{-1}, \text{ for } \mu_{\text{eff}} > 100 \text{GeV} \) (Due to LEP limit on chargino mass)
Implication to Higgs sector

- Need a light singlet-like Higgs for DM annihilation
- Pseudoscalar Mass Matrix

\[
M_{P,11}^2 = \frac{2\mu_{\text{eff}}}{\sin 2\beta} (A_\lambda + \kappa \nu_s), \\
M_{P,22}^2 = \lambda^2 \nu_s^2 \frac{\sin 2\beta}{2\mu_{\text{eff}}} (A_\lambda + 4\kappa \nu_s) - 3A_\kappa \kappa \nu_s, \\
M_{P,12}^2 = \lambda \nu (A_\lambda - 2\kappa \nu_s)
\]

Decoupling type of scenario

\[M_{P,11}^2 \gg M_{P,12}^2, M_{P,22}^2\]

- \(\mu_{\text{eff}} = \lambda \nu_s\) can not be very large to keep singlino light
- \(A_\lambda\) has to be very large
Implication to Higgs sector (contd.)

- Lighter CP odd state \( A_1 \rightarrow \) singlet-like

\[
m_{A_1}^2 \simeq -3 A_\kappa \kappa \nu_s
\]

\( A_\kappa \) should not be very large in order to accommodate a light Higgs

- A nice sum rule

\[
m_{H_1}^2 + m_{H_2}^2 \sim M_Z^2 + \frac{1}{2} \kappa \nu_s (4 \kappa \nu_s + \sqrt{2} A_\kappa)
\]

When \( H_2 \) becomes SM-like

\( H_1 \) can be very light for moderate \( A_\kappa \)

- We get a light CP-even Higgs boson, which can be shown to be singlet-like for large \( A_\lambda \)
Constraints

- $M_{H_2} = 125 \pm 3$ GeV
- $0.108 < \Omega h^2 < 0.132 \ (\Omega h^2 (\text{exp.}) = 0.12)$
- Direct and Indirect detection constraints
- Constraints from collider experiments like LEP, LHC, Tevatron
- Flavour Physics constraints, etc.
Higgs Fat Jet and MDT

- Decay products from these light Higgs bosons emerge as a single fat jet

Use Mass Drop Tagger and find two subjets

- Undo the last step of clustering to find 2 subjets
- Take them as final subjets if,

\[
\frac{\max(m_{j_1}, m_{j_2})}{m_j} < \mu \quad \text{and} \quad \frac{\min(p_{T}^2(j_1), p_{T}^2(j_2))}{m_j^2} \Delta R_{j_1,j_2}^2 > \gamma_{\text{cut}}
\]

- Else define j1 to j and repeat
**Illustrative Benchmark Points**

|       | BP1   | BP2   | BP3   | BP4   | BP5   | BP6   |
|-------|-------|-------|-------|-------|-------|-------|
| $\lambda$ | 0.34195 | 0.17783 | 0.22140 | 0.24670 | 0.24980 | 0.29853 |
| $\kappa$ | 0.00080 | 0.00241 | -0.00564 | 0.00520 | -0.00690 | 0.00438 |
| $\tan\beta$ | 8.46 | 5.99 | 4.79 | 5.85 | 4.96 | 4.63 |
| $A_\lambda$ | 3114.53 | 793.52 | 1201.50 | 1654.39 | 1968.95 | 1528.60 |
| $A_\kappa$ | -46.48 | -29.91 | 36.66 | -57.21 | 69.65 | -60.15 |
| $\mu_{\text{eff}}$ | 340.39 | 150.68 | 232.94 | 290.40 | 378.55 | 364.86 |
| $m_{H_2}$ | 123 | 126 | 126 | 126 | 123 | 127 |
| $m_{H_1}$ | 43 | 14 | 28 | 36 | 44 | 56 |
| $m_{A_1}$ | 8 | 12 | 24 | 31 | 47 | 30 |
| $m_{\tilde{\chi}_1^0}$ | 3 | 5 | 10 | 14 | 20 | 13 |
| $\Omega h^2$ | 0.1115 | 0.1188 | 0.1188 | 0.1255 | 0.1180 | 0.1098 |

| BR Process | BP1   | BP2   | BP3   | BP4   | BP5   | BP6   |
|------------|-------|-------|-------|-------|-------|-------|
| $\text{BR}(H_2 \rightarrow H_1H_1)$ | 0.0001 | 0.06 | 0.01 | 0.11 | 0.08 | 0.07 |
| $\text{BR}(H_2 \rightarrow A_1A_1)$ | 0.10 | 0.004 | 0.06 | 0.001 | 0.02 | 0.01 |
| $\text{BR}(H_1 \rightarrow bb)$ | 0.81 | 0.57 | 0.75 | 0.22 | 0.50 | 0.50 |
| $\text{BR}(H_1 \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0)$ | 0.07 | 0.31 | 0.18 | 0.75 | 0.45 | 0.44 |
| $\text{BR}(H_1 \rightarrow \tau\tau)$ | 0.07 | 0.08 | 0.06 | 0.02 | 0.04 | 0.05 |
| $\text{BR}(A_1 \rightarrow bb)$ | - | 0.35 | 0.32 | 0.55 | 0.18 | 0.73 |
| $\text{BR}(A_1 \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0)$ | 0.22 | 0.13 | 0.64 | 0.40 | 0.80 | 0.19 |
| $\text{BR}(A_1 \rightarrow \tau\tau)$ | 0.69 | 0.42 | 0.03 | 0.05 | 0.01 | 0.06 |
Event generation and selection of different objects

- Matrix elements are generated in Madgraph5aMC@NLO-2.6.4
- Showering and Hadronizations are done using PYTHIA8
- Detector effects are taken into account using Delphes-3.4.2

**Missing pT**

\[ \vec{p}_T = - \sum \vec{p}_T^i \]

where i runs over all constructed collection from the Detector

- Delphes stores of each events taking into account detector detector effects

**Leptons** : Used Delphes leptons with

\[ p_T^\ell > 10 \text{ GeV} \quad |\eta| < 2.5 \]

Needed to veto out events with any leptons
Selection of Jets

**Fat jet reconstruction**
- Used Fastjet3.3.2 and Delphes e-flow objects
- CA algorithm, $R=1.0$ (moderate mass region) and $R=1.6$ (High mass region)
- $p_T(\text{jet}) > 40$ GeV, $|\eta| < 4.0$

**Identify Higgs Fat jet**
- Use MDTagger to get tagged fat jet with two subjets
- Subjets are matched with b-quarks of the event
- Used $\Delta R < 0.3$, $|\eta| < 2.5$
  - Call it “Higgs Jet” (HJ) if matching is successful

**“Ordinary” jets**
- If a fat jet is not HJ, get back all its constituents
- Recluster using Anti-$kT$ algorithm, with $R=0.5$
- $p_T(\text{jet}) > 20$ GeV, $|\eta| < 4.0$
Exclusion Plots
F, D and soft terms

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \quad S.$$ 

$$V_F = |\lambda S|^2(|H_u|^2 + |H_d|^2) + |\lambda H_u H_d + \kappa S^2|^2,$$

$$V_D = \frac{1}{8}g^2(|H_d|^2 - |H_u|^2)^2 + \frac{1}{2}g^2|H_u^+H_d|^2,$$

$$V_{\text{soft}} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + [\lambda A \lambda S H_u H_d + \frac{1}{3} \kappa A \kappa S^3 + \text{h.c.}]$$

$$\bar{g} = \sqrt{g^2 + g'^2}, \quad \text{where } g \text{ and } g' \text{ are gauge couplings of } SU(2)_{L} \text{ and } U(1) \text{ interactions}$$

MSSM Superpotential

$$W_{\text{MSSM}} = \bar{u}y_u Q H_u - \bar{d}y_d Q H_d - \bar{e}y_e L H_d + \mu H_u H_d .$$
Couplings to fermions

\[ g_{t\bar{t}H_1/c\bar{c}H_1} = -\frac{m_{t/c} S_{12}}{\sqrt{2} v \sin \beta}, \]

\[ g_{b\bar{b}H_1/\tau\tau H_1} = \frac{m_{b/\tau} S_{11}}{\sqrt{2} v \cos \beta} \]

\[ g_{b\bar{b}A_1/\tau\tau A_1} = \frac{\text{im}_{b/\tau} P_{11}}{\sqrt{2} v \cos \beta} \]
Branching Ratios
CP-even mass matrix

\[ M^2_{S,11} = M_Z^2 \sin^2 \beta + \mu_{\text{eff}} \cot \beta (A_\lambda + \kappa v_s), \]
\[ M^2_{S,22} = M_Z^2 \cos^2 \beta + \mu_{\text{eff}} \tan \beta (A_\lambda + \kappa v_s), \]
\[ M^2_{S,33} = \frac{\lambda^2 v^2 A_\lambda \sin 2\beta}{2\mu_{\text{eff}}} + \kappa v_s (A_\kappa + 4\kappa v_s), \]
\[ M^2_{S,12} = (\lambda^2 v^2 - \frac{M_Z^2}{2}) \sin 2\beta - \mu_{\text{eff}} (A_\lambda + \kappa v_s), \]
\[ M^2_{S,13} = \lambda v (2\mu_{\text{eff}} \sin \beta - (A_\lambda + 2\kappa v_s) \cos \beta), \]
\[ M^2_{S,23} = \lambda v (2\mu_{\text{eff}} \cos \beta - (A_\lambda + 2\kappa v_s) \sin \beta). \]
Ycut and Lepton pT