Enhanced heat transfer in H$_2$O inspired by Al$_2$O$_3$ and $\gamma$Al$_2$O$_3$ nanomaterials and effective nanofluid models

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Abstract
Currently, thermal improvement in the nanofluids over a curved Riga sheet is a topic of interest and attained popularity among the researchers. Therefore, the colloidal suspension of water suspended by Al$_2$O$_3$ and $\gamma$Al$_2$O$_3$ over a curved Riga surface is modeled for the heat transfer analysis. The nondimensionalization of the model is accomplished via invertible variables. On the basis of dynamic viscosities and thermal conductivities of Al$_2$O$_3$ and $\gamma$Al$_2$O$_3$ nanoparticles, two nanofluid models developed over a semi-infinite region. Then, the models solved numerically and found graphical results for the flow characteristics, thermophysical properties and local thermal performance rate by altering the pertinent flow parameters. It is examined that the fluid motion rapidly decreases for $\gamma$Al$_2$O$_3$ – H$_2$O and momentum boundary layer region decreases. The squeezed and curvature parameters lead to reduce in the nanofluid velocity. The temperature of more magnetized enhances significantly. Thermophysical characteristics of the nanofluids enhance for higher volumetric fraction factor. More heat transfer at the Riga surface for higher M and R.

Keywords
Colloidal suspension, curved Riga surface, Al$_2$O$_3$ and $\gamma$ Al$_2$O$_3$ nanoparticles, heat transfer, slip effects, numerical scheme

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Introduction
The heat transfer analysis is significant from engineering, industrial and technological aspects. Factually, to accomplish the process of many productions in the industries need remarkable rate of heat transfer. However, no such fluids available in the list of regular fluids which provide considerable heat transfer amount to accomplish the processes. To reduce these issues, a new list of fluids required which have extra thermal performance characteristics in comparison with regular liquids. Therefore, researchers, engineers and scientists focused their efforts toward the development of new list of fluids. The idea of thermal enhancement in the base
liquids came up in late 18th century. It was thought that the thermal performance in host liquids could be improved by dispersing the tiny particles of various metals Ag, Cu, CuO, SWCNTs, MWCNTs, Al2O3, γAl2O3, Fe3O4 in the host liquids water, propylene glycol, kerosene oil, and engine oil. It is assumed that the composition of metals particles and host liquids are in thermal equilibrium and the particles continuously suspended. This, newly developed list of fluids is entitled as Nanofluids and extensive applications in biotechnology, electronics, electrical engineering, and computer chips, etc.

In the nanofluids, thermal conductance of the nanomaterials is one of the significant ingredients which makes the thermal performance characteristics more efficient as compared to that base liquids. Therefore, Maxwell1 proposed a theoretical thermal conductivity model by considering volume fraction of nanoparticles as a key parameter. Later on, Choi2 inspired by the work and efforts of Maxwell1 extended the idea of Maxwell and named such fluids as Nanofluids. Thermal conductivity model that deals the influences of nanoparticles shape in the heat transfer characteristics was developed by Hamilton.3 This correlation handles different sort of nanoparticles like cylindrical, platelets, brick, and blades. A theoretical thermal conductivity model for spherical shaped particles of nanosized at high volume fraction was proposed by Bruggeman4 and Wasp5 extended the Hamilton Crossers model by considering the nanoparticles shaped factor $n = 3$. Koo and Kleinstreuer6,7 proposed thermal conductivities models for oil and ethylene glycol composed by Copper oxide (CuO) nanoparticles. They ingrained the influences of temperature in the proposed model.

Li and Peterson8 developed a correlation for water suspended by Al2O3. To enhance thermal conductivity, the effects fraction factor and temperature emerged in the model. In 2010, Patel et al.9 construct unique thermal conductance correlation which is applicable for oxides and metallic nanoparticles. In ordered to enrich thermal conductivity, the impacts of nanoparticles diameter and temperature are introduced in the model. A reliable thermal conductivity correlation for Ag/H2O nanofluid developed by Godson et al.10. Corcione11 introduced a thermal conductivity model for Al2O3/H2O nanofluid. For fascinating results of the proposed model, they incorporated the influences of freezing temperature in the correlation and found significant results.

The proposed thermal conductivity models became much popular. They used the models for various problems in the presence of different flow conditions and discussed significant alterations in the fluid characteristics. In 2017, Ahmed et al.12 developed a nanofluid model for the flow which squeezed between the plates rotating in the coordinate system. They studied the model for two host liquids composed by γAl2O3 nanoparticles and assumed that the mixture is thermally in an equilibrium. They found remarkable heat transfer characteristics for the nanofluids and also explored the results for the velocity, temperature and coefficient of skin friction and explained comprehensively. In 2016, Sheikholeslami et al.13 presented the heat transfer for nanofluid by implementing KKL model in the energy equation.

Carbon nanotubes is another solid material having high thermal conductivity characteristics. Due to unique heat transfer and mechanical properties, carbon nanotubes attained huge interest of the scientists and researchers. In 2005, Xu14 developed a nanofluid flow model. They used another material known as Carbon nanotubes in the base liquid. In 2017, Nadeem et al.15 discussed the thermal performance analysis in the nanofluid over oscillating channel. In 2018, Saba et al.16 presented a novel study on the flow of nanofluid over a curved shaped geometry. Khan et al.17 explored the heat transfer in oblique channel by mixing the carbon nanotubes in the host liquid. Influences of thermal radiation on the flow of carbon nanotubes composed nanofluid between Riga plates reported in Ahmed et al.18

The analysis of the nanofluids by considering various nanofluid effective models and thermophysical characteristics achieved much popularity of the scientists. Therefore, researchers started to analyze the flow regimes in the nanofluids. Reddy et al.19 reported the analysis of nanofluid over a curved sheet. They modeled the flow regimes for heat and mass transport under the impacts of nonlinear thermal radiations. They found enhanced heat and mass transport in the nanofluid over a curved surface comparative to linear surface. The behavior of sisko nanofluid by taking the influences of curvature is reported in Ahmad and Khan20. They analyzed that the temperature of the nanofluid could be enhanced by increasing the brownian motion effects. Another significant analysis over a curved which is capable to stretching/shrinking is presented by Usama et al.21 They analyzed the effective nanofluid models for Cu-H2O nanofluids and found the excellent contributions of Cu nanomaterial in the thermal transport performance. The unsteady nature of micropolar nanofluid flow over stretching/shrinking curved sheet was examined by Saleh et al.22

The effects of thermal slip on the flow behavior of micropolar nanofluid over a curved Riga surface was addressed by Abbas et al.23 A recently developed effective nanofluid correlation for γAl2O3 was implemented in Khan et al.24 by taking H2O and C2H4O2 as host liquids. They conducted the analysis for 3D squeezed flow and observed an excellent heat transport characteristics due to used effective nanofluid correlation. Another significant thermal transport investigation is examined in Ahmed et al.25 For novelty of the study, they plugged the cross-diffusion phenomenon in the
governing model. Recently, Abbas and Magdy investigated the heat and mass transport in the nanofluids for various sort of nanomaterials know as Cu, Al2O3, and TiO2. For thermal improvement of the nanofluid, they used Hamilton and Crosser’s model which deal with multiple shape effects of the nanomaterial. They observed the substantial role of particles shapes for thermal enhancement in the nanofluids. Moreover, they concluded that the spherical shape particles have high thermal performance capability.

A very recent study on the heat transport in the nanofluid is reported in Berrehal and Sowmya. They used the nanofluid prepared by Cu and Ag nanoparticles and the host liquid H2O. Further, they analyzed the thermal performance of the nanofluid by altering the flow parameters. The investigation in the nanofluid using Green method is conducted Narayanan Rakesh discussed the thermal performance in the nanofluid and their stability. They prepared the nanofluid by using Green method and found fascinating characteristics of the nanofluid. The preparation of TiO2 nanoparticles and its characteristics are comprehensively reported in Ali et al. They discussed various techniques like single and two step method for the preparation of nanoparticles. The significant study regarding to the nanomaterial’s preparation and their characteristics are discussed comprehensively in Yang and Hu and Jama et al., respectively.

The investigation of heat transfer in Al2O3 and γAl2O3 by taking H2O as a host liquid is significant due to their superior thermal conductance characteristics which is a substantial research topic in the filed of engineering. From the literature study, it is pointed that aforementioned study is not conducted for curved Riga sheet so far. Therefore, the analysis is made to explore the thermal performance in Al2O3–H2O and γAl2O3–H2O nanofluids under the influence of various physical flow parameters.

**Model formulation**

The flow of water suspended by Al2O3 and γAl2O3 nanomaterials is under consideration over a curved Riga surface in curvilinear coordinate system. It is assumed that the composition is thermally in as equilibrium. The surface has the property of exponentially stretching. Further, radius of the curve is R and S, r are the curvilinear coordinates. Figure 1 portraying the flow situation.

The dimensional flow model over a curved Riga surface for the nanofluid is defined as under:

\[
\frac{\partial}{\partial r}(V^*(r + R^*)) + R^* \frac{\partial U^*}{\partial S} = 0, \quad (1)
\]

\[
\frac{\partial}{\partial r}(H_1^*(r + R^*)) + R^* \frac{\partial H_2^*}{\partial S} = 0, \quad (2)
\]

\[
\frac{U^*}{r + R^*} - \frac{1}{\rho_{nf}} \frac{\partial p^*}{\partial s} = 0, \quad (3)
\]

\[
V^* \frac{\partial U^*}{\partial r} + R^* U^* \frac{\partial U^*}{\partial S} + V^* U^* + \frac{1}{\rho_{nf}} \left( \frac{1}{r + R^*} \right) \frac{\partial p^*}{\partial r} = \frac{1}{\rho_{nf}} (1 + K^*_1) \left( \frac{\partial^2 U^*}{\partial r^2} - \frac{U^*}{(r + R^*)^2} + \frac{1}{r + R^*} \frac{\partial U^*}{\partial r} \right) - \frac{K^*_1}{\rho_{nf}} \frac{\partial N^*}{\partial r} + \frac{\pi I_0^2 M_0^2}{8 \rho_{nf}} e^{-\frac{r}{R^*}} + \frac{\mu^*_e}{4 \pi \rho_e} \left( \frac{R^* H_1^*}{r + R^*} \right) \frac{\partial H_1^*}{\partial S} + H_2^* \frac{\partial H_1^*}{\partial r} + H_1^* H_2^* \right), \quad (4)
\]

\[
\left( \frac{R^*}{r + R^*} \right) \left( U^* \frac{\partial H_1^*}{\partial S} \right) + \frac{H_1^* H_2^*}{r + R^*} + V^* \frac{\partial H_1^*}{\partial r} = \left( \frac{R^*}{r + R^*} \right) \left( \frac{\partial^2 H_1^*}{\partial r^2} - \frac{H_1^*}{(r + R^*)^2} + \frac{1}{r + R^*} \frac{\partial H_1^*}{\partial r} \right), \quad (5)
\]

\[
V^* \frac{\partial N^*}{\partial r} + R^* U^* \frac{\partial N^*}{\partial S} = \frac{1}{\rho_{nf}} \left( \frac{\mu^*_e + K^*_1}{2} \right) \left( \frac{1}{r + R^*} \right) \frac{\partial N^*}{\partial r} + \frac{\partial^2 N^*}{\partial r^2} - \frac{K^*_1}{2 \rho_{nf}} \left( \frac{\partial N^*}{\partial r} + 2 N^* + \frac{U^*}{r + R^*} \right), \quad (6)
\]
\[ R^* U^* \frac{\partial T^*}{r + R^* \partial s} + V^* \frac{\partial T^*}{\partial r} = \frac{k_n}{(p c_p)_{nf}} \left( \frac{1}{r + R^* \partial r} + \frac{\partial^2 T^*}{\partial r^2} \right). \]

(7)

Associated boundary conditions in the presence of velocity slip and thermal jump defined as:

At curved Riga surface \( r \to 0 \)

\[
\begin{align*}
V^* &= 0 \\
U^* &= \exp(\bar{z}) + L \left( k n^* + \frac{U^*}{r + R^*} + \frac{\partial U^*}{\partial r} \right) \\
T^* &= T_w^* + \frac{k n^*}{k n^*} \frac{U^*}{r + R^*} \\
N^* &= H_2^* = 0 \\
N^* &= -n \frac{\partial U^*}{\partial r}
\end{align*}
\]

Away from the Riga surface \( r \to \infty \)

\[
\begin{align*}
H_1^* &\to H(s) = H_0^* \exp(\bar{z}) \\
T^* &\to T_w^* \\
N^* &\to 0
\end{align*}
\]

The suitable invertible transformations for the model are defined as:

\[
\begin{align*}
T^* &= T_w^* + c B(\eta) \\
\eta &= \sqrt{\frac{r}{r + R^*}} \\
U^* &= a c \bar{z} F(\eta) \\
V^* &= -\left( \frac{F'}{r + R^*} \right) \sqrt{\frac{r}{r + R^*}} \exp(\bar{z}) F'(\eta) \\
N^* &= \sqrt{\bar{z} a} \exp(\bar{z}) H(\eta) \\
P^* &= \rho^* a^* \bar{z} \rho(\eta) \\
H_1^* &= H_0^* \exp(\bar{z}) G'(\eta) \\
H_2^* &= -H_0^* \left( \frac{F'}{r + R^*} \right) \sqrt{\frac{r}{r + R^*}} G(\eta)
\end{align*}
\]

(9)

In order to enhance the thermophysical properties of the nanofluids, the effective models described in Khan et al.\textsuperscript{24} are implemented.

After successful dimensional analysis and implementing the invertible variables and supporting boundary conditions the following models are gained for Al\(_2\)O\(_3\) and γAl\(_2\)O\(_3\) nanoparticles:

**Al\(_2\)O\(_3\) – H\(_2\)O Model**

\[
\begin{align*}
\frac{1}{(1 - \phi)} &\left( (123\phi^2 + 7.3\phi + 1) + K_1^* \right) \\
&\left( F'' + \frac{2F''}{(k + \eta)} + \frac{F'}{(k + \eta)^2} - \frac{F''}{(k + \eta)^2} \right) + \\
&\frac{R_1 k}{(k + \eta)} \frac{F''}{(k + \eta)} - F' F'' + \frac{R_1 k}{(k + \eta)^2} (F' F'' - F'^2) \\
&- \frac{R_1 k}{(k + \eta)^3} F' F''
\end{align*}
\]

(13)

**γAl\(_2\)O\(_3\) – H\(_2\)O Model**

\[
\begin{align*}
\frac{1}{(1 - \phi)} &\left( (123\phi^2 + 7.3\phi + 1) + K_1^* \right) \\
&\left( F'' + \frac{2F''}{(k + \eta)} + \frac{F'}{(k + \eta)^2} - \frac{F''}{(k + \eta)^2} \right) + \\
&\frac{R_1 k}{(k + \eta)} \frac{F''}{(k + \eta)} - F' F'' + \frac{R_1 k}{(k + \eta)^2} (F' F'' - F'^2) \\
&- \frac{R_1 k}{(k + \eta)^3} F' F'' - \omega^* \Theta \exp(-\omega^* \eta) + \frac{B^*}{(k + \eta)} \frac{G G' + G' G + G G' + G G'}{(k + \eta)^2} = 0.
\end{align*}
\]

(14)
\[ \lambda^L G' + \frac{G'}{(k + \eta)} - \frac{G'}{(k + \eta)^2} + R_1 \left( \frac{k}{(k + \eta)} G' - \frac{1}{(k + \eta)} FF' - \frac{k}{(k + \eta)} G' + \frac{1}{(k + \eta)} GG' - \frac{k}{(k + \eta)} FFG' \right) = 0. \]
\[ (1 - \phi + \frac{\phi}{\rho_f}) \left( \frac{1}{2} (123\phi^2 + 7.3\phi + 1) + K_1^2 \right) \]
\[ R_1 \left( \frac{1}{k + \eta} H + \frac{1}{k + \eta} H' F \right) - \frac{1}{2} \left( 2H + \frac{F'}{(k + \eta)} \right) = 0. \]
\[ (4.97\phi^2 + 2.72\phi + 1) \left( \frac{1}{1 - \phi + \frac{\phi}{\rho_f}} (\beta' + \frac{1}{(k + \eta)} \beta') + \frac{kr_1}{k + \eta} \beta F - \frac{kr_1}{k + \eta} \beta = 0, \]

The invertible variables reduce the boundary conditions in the following form:

At the curved Riga surface \( \eta = 0 \)
\[ F'(\eta) = 1 + \frac{1}{1 - n} \left( \frac{F'(\eta)}{1 + \frac{1}{1 - n}} \right) \]
\[ H(\eta) = F'(\eta)(1 - n) \]
\[ G'(\eta) = 0 \]
\[ \beta(\eta) = 1 + \frac{M_{\text{sat}}}{h_v} \beta(\eta) \]

Far from the Riga surface \( \eta \rightarrow \infty \)
\[ F'(\eta) = 0 \]
\[ F'(\eta) = 0 \]
\[ H(\eta) = 0 \]
\[ G'(\eta) = 1 \]
\[ \beta(\eta) = 0 \]

The expression for Nusselt number is as follow:
\[ Nu = \frac{q_w s}{\kappa_{\text{eff}}(T'_w - T_w)}. \]

The heat flux in \( s \) direction defined in the following way:
\[ q_w = -\kappa_{\text{eff}} \frac{\partial T'}{\partial r}. \]

After incorporating the heat flux from equation (20) in equation (19), the following dimensionless form is attained:

For \( Al_2O_3 - H_2O \) nanofluid
\[ \sqrt{Re} \nu_s = \frac{-k_2 + \phi (k_2 - k_1) }{k_2 + \phi (k_2 - k_1) } \beta(0) \]

Mathematical analysis

The systems for nanofluid models enumerated in equations (10)–(17) are highly nonlinear and coupled in nature. For such nanofluid models, exact solutions are incredible. Therefore, numerical solutions are reliable approach under this situation. For under consideration models, RK scheme with shooting technique is used for the solution purpose. The following substitutions are made to initiate the technique:

\[ b_1 = F, b_2 = F', b_3 = F'', b_4 = F''' \]
\[ b_5 = G, b_6 = G', b_7 = G'' \]
\[ b_8 = H, b_9 = H' \]
\[ b_{10} = \beta, b_{11} = \beta' \]

By substituting the transformations defined in equation (22), in equations (10)–(17), a system of first order is obtained which then solved by aforementioned numerical technique.

Graphical results and discussion

The embedded flow parameters significantly affect the flow characteristics of the nanofluids. These are the velocity, temperature, local thermal performance rate analysis and the flow pattern. The results for aforementioned flow characteristics are plotted and a comprehensive discussion is provided against each result.

Velocity and temperature distribution

The motion of \( Al_2O_3 - H_2O \) and \( \gamma Al_2O_3 - H_2O \) for increasing parameters \( \gamma \) and \( R_1 \) are depicting in Figure 2. The velocity of the nanofluid against \( H \) drops rapidly near the Riga surface for both \( Al_2O_3 - H_2O \) and \( \gamma Al_2O_3 - H_2O \). However, the motion of \( Al_2O_3 - H_2O \) drops abruptly. As the density of aluminum oxide is greater than the density of \( \gamma \)-aluminum oxide, therefore the nanofluid composed by aluminum oxide becomes denser. Physically, due to higher density of \( Al_2O_3 - H_2O \), intermolecular forces become stronger and the collision between the particles drops, consequently the fluid movement drops. Another physical reason behind the slow movement of the nanofluid near the surface is the curvature. Due to curved surface, the particles could not move freely which ultimately cause the declines of the fluid velocity. The velocity of the nanofluids will undergo its asymptotic
behavior beyond the region $\eta>2$. These effects are sketched in Figure 2(a) over the feasible domain.

The effects of $R_1$ on the nanofluid velocity $F'\eta$ are depicted in Figure 2(b) over the desired domain. It is examined that the velocity drops for higher values of $R_1$ but, decrement in the fluid motion is quite slow comparative to the fluid motion depicted in Figure 2(a). The prominent decrement in the fluid motion is observed in the vicinity of the Riga surface and the fluid motion almost shows inconsequential decreasing behavior far from the region of interest and asymptotically vanishes beyond the region $\eta>8$.

The influences of squeezed and curvature parameters on the nondimensional velocity $F'\eta$ are pictured in Figure 3(a) and (b), respectively. It is noted that the squeezed parameter reduces the nanofluid velocity. The decreasing behavior of velocity for $\gamma\text{Al}_2\text{O}_3 - \text{H}_2\text{O}$ nanofluid is quite abrupt than that of $\gamma\text{Al}_2\text{O}_3 - \text{H}_2\text{O}$. It is investigated that the velocity of both nanofluids gradually drops far from the Riga surface and near the surface these changes are very abrupt. Similarly, in Figure 3(b), the velocity is decreasing function of curvature parameter. Physically, the larger curvature did not provide essential flow area for the fluid due to which the fluid velocity drops abruptly near the surface and under goes its asymptotic nature far from the surface.

The behavior of dimensionless temperature $\beta(\eta)$ against $M$ and the curvature parameter $k$ is sketched in Figure 4. It is examined that the temperature rises significantly near the surface against growing values of $M$. Physically, near the surface the effects of magnetic field are stronger which play significant role in the temperature enhancement. As we move far from it, the increment in the temperature gradually slowdown and undergo its asymptotic behavior beyond $\eta>2.5$. These variations in $\beta(\eta)$ are elucidated in Figure 4(a). The temperature in $\gamma\text{Al}_2\text{O}_3 - \text{H}_2\text{O}$ intensifies quickly comparative to $\gamma\text{Al}_2\text{O}_3 - \text{H}_2\text{O}$ due to their superior thermophysical characteristics. The thermal transport due to varying curvature of the Riga surface is elaborated.

Figure 2. The velocity profile for varying: (a) $\gamma$ and (b) $R_1$ over curved Riga surface.

Figure 3. The velocity profile for varying: (a) $S$ and (b) $k$ over curved Riga surface.
in Figure 4(b). It is detected that the temperature intensifies slowly over a more curved surface. Physically, for more curved surface, the fluid motion drops which causes the slow collision between the fluid particles. Due to low collision, the temperature rises slowly in the region of interest.

**Streamlines**

The flow pattern of Al\(_2\)O\(_3\) – H\(_2\)O and γAl\(_2\)O\(_3\) – H\(_2\)O for varying parameters \(k\) and \(R_1\) are portrayed in Figures 5 to 12. Three-dimensional view of the streamlines is also pictured. It is investigated that the streamlines are like almost of parabolic shape and for higher curvature of curved Riga surface it stretched. By decreasing the parameter \(R_1\), the streamlines compressed toward the center of Riga surface.

**Thermophysical characteristics and Nusselt number**

The variations in effective density, thermal conductivity, dynamic viscosity and electrical conductivity due to varying volumetric fraction \(\phi\) of the nanomaterials are sketched in this subsection. Further, the local thermal performance rate against the physical parameters is also described. The dynamic viscosity of Al\(_2\)O\(_3\) – H\(_2\)O significantly rises by increasing the volumetric fraction of the nanomaterials. This dominating behavior of dynamic viscosity against volumetric fraction plying significant role in the fluid characteristics like the
velocity and thermal behavior. These effects are captured in Figure 13(a). The effective electrical conductivity due to high volumetric fraction $\phi$ is elucidated in Figure 13(b). It is also investigated that the thermal conductance of $\gamma\text{Al}_2\text{O}_3 - \text{H}_2\text{O}$ is low comparative to $\text{Al}_2\text{O}_3 - \text{H}_2\text{O}$. The high thermal conductivity of $\text{Al}_2\text{O}_3 - \text{H}_2\text{O}$ is better for thermal transport. Further, the effective density and electrical conductivity against volume fraction are plotted in Figure 14(a) and (b), respectively. These thermophysical parameters enhances due to higher volume fraction.

The analysis of local thermal performance in the nanofluids against multiple physical parameters is significant from industrial point of view. Therefore, the local thermal performance for $M$ and $R$ are captured in Figure 15(a) and (b), respectively. It is examined that more heat transfer at the Riga surface by increasing the strength of magnetic field. The superior thermophysical characteristics of $\text{Al}_2\text{O}_3 - \text{H}_2\text{O}$ over $\gamma\text{Al}_2\text{O}_3 - \text{H}_2\text{O}$ showing the rapid thermal transport. These are portrayed in Figure 15(a) for feasible domain. On the other hand, the coefficient of heat transport for $R$ is demonstrated in Figure 15(b) and found slow increment as compared to Figure 15(a).

Isotherms

Figures 16 to 19 pictured the pattern of isotherms for varying curvature parameter $k$. It can be seen that increasing the curvature of Riga surface, isotherms
Figure 8. Three-dimensional view of streamlines pattern for (a) Al₂O₃ − H₂O \( k = 0.8 \) and (b) γAl₂O₃ − H₂O \( k = 0.8 \) nanofluid.

Figure 9. Streamlines pattern for (a) Al₂O₃ − H₂O \( R = 0.3 \) and (b) γAl₂O₃ − H₂O \( R = 0.3 \) nanofluid.

Figure 10. Three dimensional view of streamlines pattern for (a) Al₂O₃ − H₂O \( R = 0.3 \) and (b) γAl₂O₃ − H₂O \( R = 0.3 \) nanofluid.
Figure 11. Streamlines pattern for (a) Al₂O₃ – H₂O (R = 0.8) and (b) γAl₂O₃ – H₂O (R = 0.8) nanofluid.

Figure 12. Three dimensional view of streamlines pattern for (a) Al₂O₃ – H₂O, R = 0.8 and (b) γAl₂O₃ – H₂O, R = 0.8 nanofluid.

Figure 13. Influence of volume fraction on (a) dynamic viscosity and (b) thermal conductivity.
Figure 14. Influence of volume fraction on (a) density and (b) electrical conductivity.

Figure 15. Influence of (a) $M$ and (b) $R$ on local Nusselt number.

Figure 16. Isotherms pattern for (a) $\text{Al}_2\text{O}_3 - \text{H}_2\text{O}$, $k = 0.3$ and (b) $\gamma\text{Al}_2\text{O}_3 - \text{H}_2\text{O}$, $k = 0.3$. 
Figure 17. Three dimensional view of Isotherms for (a) Al$_2$O$_3$ – H$_2$O, $k = 0.3$ and (b) γAl$_2$O$_3$ – H$_2$O, $k = 0.3$.

Figure 18. Isotherms pattern for (a) Al$_2$O$_3$ – H$_2$O, $k = 0.8$ and (b) γAl$_2$O$_3$ – H$_2$O, $k = 0.8$.

Figure 19. Three dimensional view of Isotherms for (a) Al$_2$O$_3$ – H$_2$O, $k = 0.8$ and (b) γAl$_2$O$_3$ – H$_2$O, $k = 0.8$. 
become more curved and near the origin these become almost straight. Three-dimensional scenario of the isotherms is also portrayed.

**Validation of the study**

The presented nanofluid flow model is more generic and under certain assumptions, our model reduced into the particular model reported in the literature. Therefore, by taking $\beta = 0$, $R_1 = 1$, $\Theta = 0$, $\phi = 0$, $\omega = 0$, $\gamma = 0$, $S = 0$, the results for $-Re_0 C_F$ are computed and examined that the results are aligned with existing literature. This proves the authenticity of the study and implemented mathematical technique. These results are elaborated in Table 1.

**Major findings**

The investigation of heat transport in the nanofluids prepared by aluminum oxides nanoparticles and water is presented over a curved Riga sheet. After the nondimensionalization of the governing model, two different nanofluid models were attained on the basis of effective nanofluid correlations. Then the models are treated numerically and portrayed the results against the physical parameters. From the comprehensive analysis of the results, it is examined that the velocity of $\gamma Al_2O_3 - H_2O$ declines quickly on the basis of their thermophysical characteristics. The boundary layer region enhances as the curvature of the sheet increases. Further, the increment in the temperature intensifies significantly due to strong magnetic field strength. The dominating behavior of local thermal performance rate is observed for the parameters $M$ and $R$, respectively. Finally, a comparison is made by imposing certain assumptions on the models which showing the authenticity of the analysis.

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### Table 1. Authenticity of the study.

| $k$ | $\text{Sajid et al.}^{32}$ | Present $Re_0 C_F$ |
|-----|----------------|------------------|
| 5   | 0.75763 | 0.757631 |
| 10  | 0.87349 | 0.873489 |
| 20  | 0.93561 | 0.93561 |
| 30  | 0.95686 | 0.970198 |

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