Influential User Subscription on Time-Decaying Social Streams

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ABSTRACT
Influence maximization which asks for k-size seed set from a social network such that maximizing the influence over all other users (called influence spread) has widely attracted attention due to its significant applications in viral marketing and rumor control. In real world scenarios, people are interested in the most influential users in particular topics, and want to subscribe the topics-of-interests over social networks. In this paper, we formulate the problem of influential users subscription on time-decaying social stream, which asks for maintaining the k-size influential users sets for each topic-aware subscription queries. We first analyze the widely adopted sliding window model and propose a newly time-decaying influence model to overcome the shortages when calculating the influence over social stream. Developed from sieve based streaming algorithm, we propose an efficient algorithm to support the calculation of time-decaying influence over dynamically updating social networks. Using information among subscriptions, we then construct the Prefix Tree Structure to allow us minimizing the times of calculating influence of each update and easily maintained. Pruning techniques are also applied to the Prefix Tree to optimize the performance of social stream update. Our approach ensures a $1 - \epsilon$ approximation ratio. Experimental results show that our approach significantly outperforms the baseline approaches in efficiency and result quality.

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1 INTRODUCTION
Influence maximization, which asks for k-size set of users in a social network maximizing the influence spread over all users. Online social networks, like Facebook and Weibo, have boosted researches on the influence maximization problem due to its potential commercial value, such as viral marketing [7], rumor control, and information monitoring [7].

In real world social networks, users have topics or keywords indicating their fields of interests, e.g., hashtags of Twitter, subreddits of reddit, etc. A user related to certain keywords or topics will more possibly influence and be influenced by other users. For example, a user who is interested in basketball will participate in discussion of subreddits such as sport, basketball and MBA. The related keywords of this user can be represented as {sport, basketball, MBA}. In this case, recording numbers of subscription queries, people can subscribe the most influential users in particular areas of interests. For example, one can subscribe $q = \{\text{neural learning, machine learning}\}$ to keep track of the users who are most influential in the area of machine learning over time. In this paper, We formulate problem of influential user subscription on time-decaying social stream (or PSIM problem for short), which asks for k-size seed set of every subscription queries on dynamic social stream.

The topic-aware influence maximization can be applied to many real world scenarios. For example, advertiser who has limited budget hope that its advertisement will be push to the most influential users on some particular topics. The advertiser will hope the users who receive the information will be interested in some particular topics, who will most likely repost the advertisement. Therefore, the social network companies will want the PSIM subscriptions for certain topics or keywords, and locate these users for the advertisers.

Our formulation is different from the traditional online influence maximization in three ways. First, as the influence possibility between two users on social networks decays over time, i.e., a more recent response action (e.g., repost, comment and cite) between a pair of users infers stronger influence between the two user than a more previous action, we propose time-decaying influence model in this paper to meet this intuition. Second, we take into consideration keywords or topics of users. The most influential users are constrained by both influence and related keywords. Third, we aim at solving the PSIM problem of hundreds of thousands of subscription queries, which means our algorithm should be efficient enough to support the online queries. However, the PSIM problem is NP-hard. We develop a approximating algorithm from sieve based streaming algorithm to meet the requirement of both online subscription pushing and high quality.

To achieve the efficiency and quality requirement for answer the PSIM problem, we first develop the naive sieve based streaming algorithm to support the fast calculation of time-decaying influence model over evolving social networks. In order to improve the performance, we propose a Prefix Tree Structure which meets the the following properties: (1) every candidate sets are stored only once, thus the marginal influence of same candidate sets will not be calculated repeatedly, (2) downward closure property can be applied to the Prefix Tree to minimizing the times of calculating marginal influence, and (3) can be easily updated as the actions of social stream arrives in sequence. We propose an efficient streaming update algorithm based on the Prefix Tree Structure and design three pruning conditions which are applicable on the Prefix
Tree to avoid unnecessary marginal influence calculation of each update. Our approach ensures a $\frac{1}{2} - \epsilon$ approximation for PSIM problem, where $\epsilon$ is the approximation ratio subject to $\epsilon \in (0, 0.5)$.

To summarize, we make the following contributions.

- We propose a time-decaying influence model, which outperforms sliding window models in both quality and efficiency.
- We propose Prefix Tree Structure which can be easily updated for minimizing the times of calculation of marginal influence of each update. We also propose the pruning techniques over Prefix Tree to avoid unnecessary influence calculation. We develop algorithm based on the Prefix Tree Structure which returns the results for PSIM problem when an approximation ratio $\frac{1}{2} - \epsilon$.
- Experimental results on real datasets show our algorithm significantly outperforms state-of-the-art approaches.

The rest of this paper is organized as follows. Section 2 formulate the PSIM problem. We then discuss the time-decaying model and develop the sieve based streaming algorithm to support this influence model, which will be discussed in Section 3.1. In the following section, i.e., Section 4, we propose the Prefix Tree Structure and discuss the streaming update over the Prefix Tree. The pruning techniques will be also discussed in this section. Section 5 reports the experiment results. We conclude our work and result in ??.

2 PROBLEM DEFINITION

2.1 Time-Decaying Social Streams

A Social Network is a graph reveals some kind of influence relation between the users on an online society, e.g., Twitter, DBLP, etc. A social graph can be formulated as $\langle V, E \rangle$, denoted as SN, where $V$ is the vertex set and $E$ is the edge set. For each edge $e \in E$, we define the tail end user $u_e$ as influencer, and the head end user $u_r$ as influencee.

Social Stream is the sequence of actions generated by the users from a beginning time to the current time $t_0$, which is denoted by $S = \{a_1, a_2, \ldots, a_m\}$. In particular, each $a_i = \langle u_e, t_e, u_r, t_r \rangle$ represents that a user $u_e$ performs an action at time point $t_e$ to another user $u_r$. Social activity. In different sequential, the response refers to different kind of social actions, e.g., a citation of a former research paper in bibliography networks, or post on Twitter. The action represents one time influence from $u_e$ to $u_r$, therefore, the action performer $u_e$ and $u_r$ are called influencee and influencer, respectively. We consider there is a influence relation from the influencer and the influencee.

Next, we provide some examples to illustrate the intuitions of the above-defined action and influence.

- In an online social network, such as Twitter and Reddit, users either create posts (e.g., tweets) or respond to others’ posts (e.g., retweet, comment, etc.). In such a scenario, the influence relation can be extracted from the poster $u_e$ and the respondent $u_r$. Each post can be formalized as an action $a$ with influencee $u_e$, influencer $u_r$, the response time $t_r$ and the responded performance time $t_r$.
- A bibliography database, such as DBLP [1, ], maintains citations among research papers. In such a database, each paper can be considered as a set of action $\{a_i\}$ consisting of the influence relation from the authors of the cited paper to the authors of the cited paper. In this scenario, the influence time and influencee time are the publish time of the research papers.

From the aforementioned examples, we can see that social action influence relations are general formalizations that capture many real-world scenarios.

Each action is given a time-decaying weight based on the influencer time and influencee time of the action, denoted as $\Xi(a)$. The intuition of the weight is that recent actions are more significant than old ones to capture the strength of the influence between two vertices. The time-decaying weight will be further discussed in the following part.

2.2 User Influence Model

As mentioned in previous part, social stream is time-sensitive. The strength between the influencer and influencee revealed from an influence relation is depend on the influencer time $t_r$ and influencee time $t_e$: decreases by the time elapse from $t_r$ to $t_e$ to current time $t_0$.

Sliding window models are the most widely-adopted model to handle the time sensitivity of streaming data. There are two kinds of sliding window models: sequence-based and time-based [? ]. The sequence-based method maintains a size-$N$ sliding window over the latest items in the social stream; while the time-based method maintains a flexible length window with fix-length of time. For the sequence-based time sliding window, the length of time will be changing when the window slides, however, in real-world streaming scenarios, the number of actions per timestamp can be largely dispersed. Therefore, this method cannot control the length of time window flexibly. Time-based sliding window can maintain a fixed length of time window of interest, however, it regards the actions in the time window with same weight. Therefore, it cannot explicitly reveals the importance of the actions in the sliding window. A common shortage of both of these sliding window models is that they throws away some information of the previous actions, which can leads to bias of the of the calculation of influence of the social network.

In this paper, we assign each action a time-decaying weight, which is non-decrease with the influencer time and the influencee time of the action. In this paper, we focus on the the commonly used exponential decay function, i.e.,

$$\Xi(a) = e^{-\lambda \cdot \frac{t_r - t_0}{(t_r - t_0) + \epsilon}},$$

(1)

where $t_0$ is the current time, and $\lambda > 0$ is a parameter for controlling the decay speed (aka. decay constant). Here we can see that the weight of a action increases as the influencer time $t_r$ or influencee time $t_e$ is getting closer to current time. It is corresponded to the sensitivity of social streams.

There can be multiple influence relations between a pair of users $u_r$ and $u_e$, the collection of these actions is denoted as $A_{u_r \rightarrow u_e}$. For each action $a \in A_{u_r \rightarrow u_e}$, there is a corresponded weight $\Xi(a)$.

We consider the influence between $u_r$ and $u_e$ as the largest weight among these corresponded weights, i.e.,

$$f(u_r \rightarrow u_e) = \max_{a \in A_{u_r \rightarrow u_e}} \Xi(a).$$

(2)
within these keywords. In this part, we will first discuss the user profile, with determine the keywords related to users. We then define the influence maximization subscriptions. Finally, we will formulate the PSIM problem.

**User Profile.** In real world social networks, users will have topic keywords suggesting its topic-of-interests. These keywords, e.g., tags, can extracted from either the post’s content or catagory. For example, A reddit user whose is interested in basketball will participate in numbers of discussion in basketball, sport, MBA subreddits, we can therefore extract the user’s set of keywords from the subreddits he most frequently participates in, i.e. \{basketball, sport, MBA\}. This set of keywords are called the profile of a user, which can be denoted as \( P_u = \{ T_1, T_2, \cdots , T_{P_u} \} \), where \( T_i \) are the extracted keywords of user \( u \).

**Influential User Subscription.** A subscription can be several keywords-of-interests of which people want to know the most influencers. A subscription query can be denoted as \( q = \{ T_1, T_2, \cdots , T_{Q_q} \} \). A query will be corelated to numbers users in the the social network. We determine the corelation to subscription query \( q \) and user \( u \) according to the keywords sets subscription, i.e. a user \( u \) is related to query \( q \) if \( q \subset P_u \). The intuition is that . . .

Now, we are ready to define our problem of influential user subscription on time-decaying social stream (or the PSIM problem for short) as follows.

**Definition 2.1. (The PSIM Problem)** Given a set of subscription queries \( Q = \{ q_1, q_2, \cdots , q_m \} \) and s social stream \( S \), the problem maintains a size-k user set having the maximum influence with respect to each subscription query \( q \in Q \), i.e., \( R_q = \arg \max_{S, |S| \leq k} f(S) \) as stream \( S \) continuous updates in time order.

3 TIME-DECAYING INFLUENCE MAXIMIZATION

3.1 Time-Decaying SieveStreaming

As far as we know, state-of-the-art influence maximization algorithms can’t support time-decay model, instead, most of them are based on sliding window models. In this part, we focus on developing a sieve based algorithm SieveStreaming to support the time-decaying influence model.

SieveStreaming is a set streaming algorithm which finds the most coverage set by just scanning through set stream only one time. A \((\frac{1}{e} - \epsilon)\)-approximation is ensured by this method. The former application of SieveStreaming in Influence Maximization is based on sliding window model \([7]\). In this section, we will improve it to support time-decaying influence model.

3.2 Niave Time-Decaying SieveStreaming

The psudocode for SieveStreaming is shown in Algorithm 1. The basic idea is to generate a series of estimations \( E = \{ \epsilon : \epsilon = (1 + \epsilon)^i, s.t. m < \epsilon \leq 2km, i \in \mathbb{Z} \} \), where \( \epsilon \) is the approximation ratio, \( k \) is the maximum seed set size, and \( m \) is the largest set so far in the process of scanning the set stream. The optimal result of the set streaming problem lays in the arrange of \((m, 2km)\), for the optimal result will not smaller than \( m \), which is the influence of a single set, while it can not exceed \( 2km \), which can only happen when there are \( k \) disjoint set with influence \( m \). Each of the estimation is
Algorithm 1: SieveStream

1. $E \leftarrow \{ e = (1 + \epsilon)^j : i \in \mathbb{Z} \}$;
2. For each $e \in E$, $S_e \leftarrow \emptyset$;
3. $m \leftarrow 0$;
4. For $i = 1$ to $n$ do
   5. $m \leftarrow \max(m, f(s))$;
   6. $E_i \leftarrow \{ e = (1 + \epsilon)^j : m \leq (1 + \epsilon)^j \leq 2km, i \in \mathbb{Z} \}$;
   7. Delete all $S_e$ such that $e \notin E_i$;
   8. For $e \in E_i$ do
      9. if $\Delta(s|S_e) \geq \frac{\epsilon - f(S)}{k - |S|}$ then
         10. $S_e \leftarrow S_e \cup \{ s \}$
5. return $\arg \min_{e \in E} f(S_e)$;

corelated to a candidate collection of sets, denoted as $S$, which is originally emptysets. When a new set is being scanned in the set stream, the algorithm will calculate the marginal coverage of the current set $s$ w.r.t. each candidate collection $S$, and compare the value to the sieve condition, which is

$$\Delta(s|S) \geq \frac{\epsilon - f(S)}{k - |S|}$$

where $\Delta(s|S)$ is the marginal coverage of $s$ w.r.t. $S$, and $f(S)$ is the coverage of candidate collection $S$. In the next step, the newcoming set $s$ will be inserted to the candidate collections which satisfies the sieve condition, while nothing will be done for those do not satisfy the condition. The algorithm ends by selecting the collection with maximum coverage among the candidates and return it as the answer to the most coverage problem.

At each iteration, SieveStream will update the maximal set and adjust the range of its estimations.

The SieveStream can be translated when applied to Online Influence Maximization Problem, presented by [7]. In this translation, the coverage of a set is replaced by a influence of a user $f(u)$, the marginal coverage $\Delta(s|S)$ is converted to marginal influence $\Delta(u|S)$, the candidate collections of sets therefore be converted to candidate user set.

To improve SieveStream to allow it to support time-decaying influence model, a straightforward idea is to record the future estimations in advance, i.e., $(1 + \epsilon)^j \cdot e^{2\lambda \Delta t}$, where $\lambda t$ is the difference between the future time and current time. In this way, we can calculate the future candidate sets for future estimations. And when the base time to changes, the current estimations will be expired and erased out, the rest of the estimations will decay by the decaying factor, which leads the estimations of new time to be exacly the estimations of current time. In this way, we can modify SieveStream to support the time-decaying influence model.

But too many estimations will slow down the streaming process, so we therefore should determine which is the minimum number of future estimations we should store in advance. By SieveStream, we know that the optimal result to a subscription $q$ should be in the range of $(m, 2km)$, where $m = \max u \epsilon f(u)$. The estimations smaller than $m$ is not necessarily stored, as we know that there is at least one user will influence of $m$, while the estimations larger than $2km$ will all be empty sets, for there’s no such user the influence of which is larger than $\frac{2km}{2k} = m$. Therefore, the estimations-in-advance should be in the range of $(m, 2km)$, i.e., $m < (1 + \epsilon)^j \cdot e^{2\lambda \Delta t} < 2km$. As the influences decay by time, the estimations of far future will decay dramatically and hard to detect. Therefore, we have a minimum detecting threshold $\tau_d$, a set with influence smaller that $\tau_d$ can be regarded as zero. Therefore, the future estimations should subject to $\tau_d \cdot e^{2\lambda \Delta t} < (1 + \epsilon)^j \cdot e^{2\lambda \Delta t} < 2km$. Therefore, $\Delta t$ is subject to $\tau_d \cdot e^{2\lambda \Delta t} < 2km$, and equally,

$$\Delta t < \frac{1}{2\lambda} \log \frac{2km}{\tau_d},$$

which is the upper bound of $\Delta t$.

When the timestamp changes, i.e., $\tau_{cur} > \tau_0$, we can erase the timestamps of current time and their related candidate, and decay the rest estimations and the influence of their correalted candidate sets by decay factor $e^{-2\lambda(t_{cur}-t_0)}$. The resulting estimations are those of new timestamp.

3.3 Estimation Shift and Lazy Time Decaying

The naive implementation of time-decaying SieveStream will be time-consuming for it generates many times more estimations than the original algorithm.

Therefore, we hope to make use of the estimations of a former timestamp to generate the estimations of current time. The basic idea of this improvement is to convert the estimations of previous timestamp, and make sure they cover the entire range of possible optimal result for the PSIM problem. In the following part, we proved a theorem which can help us easily generate estimations from the those of previous timestamp.

On the other hand, in frequent update social networks, in which the current timestamp changes much more faster than other social networks, the time-decay will be called frequently and therefore be time-consuming. However, we can’t use exponential increased influence to replace the exponential decay influence because of the limited data range. We therefore propose lazy time decaying strategy to cut down the overhead costs. The basic idea is to set a threshold based on data range of different systems. The time decay process will be call only when the maximum influence exceeds the pre-set threshold. It can be shown in the section 5 that the lazy time decay strategy exceeds the naive time decay process for two orders of magnitude.

3.3.1 Estimation Shift. Instead of keeping track of every estimations of optimal result of $(1 + \epsilon)^j$, we record of estimations with value of $b(1 + \epsilon)^j$, where $b$ is a random base parameter smaller than $\max u \epsilon f(u)$. We prove the following theorem to guarantee the correctness of our change.

**Theorem 3.1.** Given subscription $q$, for any $b < \max u \epsilon f(u)$, if we denote $\max u \epsilon f(u)$ as $m$ and generate the set of estimations as $E = \{ e = b(1 + \epsilon)^j, m \leq e \leq 2km, i \in \mathbb{Z} \}$, the SieveStream returns the $\left( \frac{b}{2} - \epsilon \right)$ approximation of the optimal result for $q$.

In this way, we can shift the original estimations with a coefficient and just make sure the set of estimations covers the whole possible range of optimal result. Therefore, instead of keep the $(1 + \epsilon)^j$ estimations in advance, we just shift all the estimations for a coefficient of $b = e^j - 2\lambda \Delta t$ and generate some new estimations for optimal result of value $b(1 + \epsilon)^j$. In frequent update social
Algorithm 2: TimeDecay (t_{cur})

1 // called when max_{u \in q} f(u) \geq \tau_{inf}
2 d \leftarrow e^{-2\lambda(t_{cur} - t_0)}
3 b \leftarrow bd
4 j \leftarrow \text{integer that let } b(1 + e)^j \text{ closest to } 1
5 b \leftarrow b(1 + e)^j
6 \text{for every } e \text{ of every subscription } q \text{ do}
7 m \leftarrow md
8 Adjust estimations to cover the range of } (m, 2km)
9 \text{for every estimation set } S \text{ do}
10 \text{Decay } S \text{ inf by } b \text{ }

In frequent update social networks, we may temporarily decay to fits the correct influence is just before subscription pushing. In sum, we can define the time to calculate for each action update. However, some candidates contain same users, and will cause a same candidate set to calculate multiple times. For example, within an action a target user is v, for each same candidate sets S, we should calculate the marginal influence for each candidate sets; but as the marginal influence of a user \( v \) w.r.t. to candidate set \( S \) will not change, we can just calculate the marginal influence only once. To accelerate action update, we hope there is a data structure that maintains these same candidate sets only once for each, in order to minimize the times of marginal influence computation.

Candidate sets also have downward closure property, which means that some of the conditions which a subset satisfies can be also applied to any of its supersets. Some of the properties are usefull in pruning out some of the unnecessary marginal influence computation. This properties are discussed in the following subsection 3.3.3.

When scanning through the action stream, new candidate sets will be generated, while some of the existing candidate sets will be erased. To erase a candidate set, it is required to search for a certain set in a collection of sets efficiently. This means our data structure should support efficient search for set for a large collection of sets.

We therefore propose a Prefix Tree Structure to manage every candidate sets. In this structure, each candidate set will be stored only once. The subsumption relationship of different candidates are also maintained, in order to support pruning over the collection of candidate sets using downward closure property. The insertion and the erasion of candidates on the Prefix Tree is also efficient, which allows us to manage the collection of sets dynamically.

In the following of this section, we will first explain how the Prefix Tree can help us manage candidate sets and solve the PSIM problem. Then we will discuss the downward closure properties and the pruning conditions of marginal influence computation, which can help us further minimize the times to calculate marginal influence. In the end of this section, we will discuss how to maintain the structure dynamically.

4.1 Prefix Tree Structure and Action Update Process

In this part, we will first introduce the Prefix Tree Structure. Then we will discuss how we can use this structure to help us solve PSIM problem.
The Prefix Tree Structure is shown in fig 2, it has 2 parts: the first part is a User Index; the second part is the prefix tree constructed by elements in candidate sets. The User Index is a list, and each item of it is the user id and the pointer to the location in the prefix tree where the user first appears. The location it points to will further points to where the user appears next time. The linked list will continue until it contains all locations of the user. Each node on the Prefix Tree is a user id, whose locations satisfy the following condition: The user id from root to leaves is in increasing order. A path from the root to solid points represents a candidate set, a solid point, therefore, corresponded to a candidate set. Thus, we also refer a candidate set \( S \) as a path on the Prefix Tree. The hollow points is not responded to any candidate sets, but is an element in the paths it belongs to. Each solid point restore the essential information related to the corresponded candidate set. It has 5 main fields: \( f(S) \) stores the influence of the path, \( \Delta(e|S) \) contains the calculated marginal influence of user \( v \) w.r.t. current candidate set, \( S \) contains all element on the path, and \( Q \) and \( E \) is the related set of subscriptions and estimations, respectively.

The Prefix Tree contains all the essential information we need to solve PSIM problem. Here we will discuss how the structure deal with an action update and push results to every subscriptions.

When an action \( a = (u_r, t_e, u_r, t_r) \) arrives, the following steps will be taken in sequence:

- **Influence update.** Calculate the influence of action \( a \) as \( f(u_r \rightarrow u_e) \), the influence of edge \( e = (u_r, u_e) \) will be also updated in social graph. The following update is to update the influence of candidate sets which contains \( u_r \).

- **Calculate marginal influence.** We will calculate the marginal influence of target user \( u_r \) w.r.t. to the correponded candidate sets, i.e. paths on the Prefix Tree. These candidates are all related to the subscriptions to which target user \( u_r \) belongs to, as target user \( u_r \) will never be inserted to those candidates which has no shared subscriptions with it. As the correlded set of subscriptions of a path \( S \) is the subset of the set of subscriptions of its prefix paths, and the root element (which is empty set), is related to every subscriptions. We can calculate the marginal influence of each paths in Depth First Search (DFS) style. Once a path shares no common subscriptions with user \( u_e \), the we can prune out the subtree of the current path. As a result, the marginal influence of \( u_r \) w.r.t. every related candidate sets will be calculated only once.

- **Judge sieve condition and update Prefix Tree.** After all the marginal influence is calculated, we further decide the candidate sets into which the target user can be inserted. For each candidate sets calculated in the previous step, we check the sieve condition for each of the corresponded estimations of the set. If there’s a estimation \( e \) satisfies the sieve condition in candidate \( S \), the new path \( S' = S \cup \{u_r\} \) will be created and inserted into the Prefix Tree, by which we will be further discussed in the following parts of this section. If the path \( S' \) is already exist, the only thing we do it to move the estimation \( e \) from its current path to the new path; otherwise, the new path will be created in the Prefix Tree, and the same estimation movement will be performed. After a new path for a current path is generated, whether a existing path or a newly inserted path, the current path will point to the new path, in case of other following estimations can be moved quickly and without search for the path for multiple times. The information of the new path can be retrived from the current path, which will be further discussed in the following parts in this section. The erasion of the paths will be performed after the insertion. In this procedure, we check if there is still estimation linked to the candidate set. If no single estimation found, the path will be erased from the Prefix Tree. Instead of checking all paths on the Prefix Tree, we can only check those calculate in the previous step, as all the estimation removal is preformed within those candidates.

- **Push results to subscriptions.** In this step, we push the current influence of each path on the Prefix Tree to every subscriptions. For each path on the Prefix Tree, we push the influence of the path to all its related subscriptions. Within all the influence pushed to subscriptions, we pick out the largest influence among them as the result of the subscription. The result will be stored in the subscription for user to visit.

The overall procedure is summarized in Fig 3.

We calculate all the marginal influence in one step and update the candidate sets and Prefix Tree Structure in another step. And all the information is calculated on the Prefix Tree, which is finally pushed to subscriptions.

In the following sections, we will further discuss how we continue minimizing the times of marginal influence in step 2 and how we efficiently update the Prefix Tree in step 3.

![Figure 3: Update and time decay process](image-url)
4.2 Pruning Conditions

Even we calculate the related candidates only once in every update, the times of influence marginal calculation will grows dramatically when the number of subscriptions grows. Moreover, the calculation method of marginal influence under different influence model can be very different. We therefore aim at minimize the times we calculate marginal influence. In this part, we will discuss 3 downward closure properties of Prefix tree, and propose three pruning conditions based on these properties to prun out some branches of the Prefix Tree.

If an element v is in one path S, for all paths S' which satisfy S' ⊆ S, it is clear that v is also contained in S'. On the other hand, for each node on the Prefix Tree u, its related path is contained in the related paths of nodes of its subtree, called super-path of path S. Therefore, we can propose the first downward closure property:

**Definition 4.1.** If one path of the Prefix Tree S contains element u, all the super-path of this path also contains element u.

With this property, we can prun out the subtree rooted to the target user ur, as the the paths related to the nodes on ur’s subtree contains ur, and therefore don’t need to decide whether the user can be inserted into the path. We therefore reach our first pruning condition:

**Definition 4.2. First Pruning Condition.** In the DFS process marginal influence computation for target influencer ur using DFS, we can prun down the subtree rooted to current tree node u if ur = u.

This pruning condtian can help us prun out the nodes that are unnecessary to calculate marginal influence. The second downward closure property which reveals the relationship of paths can help us prun out the irrelated paths.

Noted that the related set of subscriptions of a give path is the intersection of all the related subscriptions of all users in the path, that is,

$$Q_S = \cap_{u \in S} Q_u$$  \hfill (9)

where Qs and Qu are the set of related subscription of path S and u, respectively. The intuition is that, as a subscription is equivalent to a subset of user set of a social network. Therefore, if a set S belongs to subscription q, it means that all the elements in set S will be also included in the q, otherwise the set can’t be included in q. As a result, we can easily find out the second downward closure property:

**Definition 4.3.** Given path S, for each its super-path S′, we have QS ⊇ QS′.

According to the relationship between a candidate set and its responded subscriptions, we can know that, if the target user ur and a set S shares no common subscription, i.e., Qu, ∩ Qs = ∅, the set S′ = S ∪ {ur} will not belongs to any subscriptions. Therefore, we have our second pruning condition:

**Definition 4.4. Second Pruning Condition.** If path S and target user ur have no common subcription, i.e., Qs ∩ Qu = ∅, super-path of S can be pruned.

In this pruning process, to overcome the overhead computing large subscription sets intersection, we keep track of the related subscriptions of target user ur. After calculating the intersection of the Q′ ur = Qu ∩ Qs, we keep track of Q′ ur, for further subscription sets computation.

If the root of each subtree u knows the minimal related estimation of the subtree, denoted as e_u min, we have the following downward closure property:

**Definition 4.5.** For two nodes u and v, and v correlated to a super-path of u’s correlated path, then e_u min ≤ e_v.

According to SieveStreaming, every node u which can be inserted into the a candidate set S should be satisfy the following condition:

$$f(u) ≥ \frac{e_u - f(S)}{k - |S|}$$  \hfill (10)

Therefore, if for the minimal estimation of u e_min, we have

$$f(u) < \frac{e_{min} - f(S)}{k - |S|},$$

we can skip the computation of the marginal influence of this candidate set. Assuming that the subtree node knows the minimal estimation among the its subtree, we have:

**Definition 4.6. Third Pruning Condition.** For Prefix Tree node u and the minimal estimation among the subtree rooted to u e_min. We can prun out the subtree rooted to u if

$$\Delta(u|S') < \frac{e_u - f(S)}{k - |S|}$$  \hfill (11)

where S’ is the nearest sub-path of related path of u.

4.3 Prefix Tree Maintainance

The insertion of user into candidate sets and the erasion of candidate sets will cause the insertion of a new path and the delete of a existing path. In this part, we will discuss how we update the Prefix Tree according to the marginal influence.

Path insertion happens when a new vertex satisfies the sieve condition and inserted into the candidate set. In this case, we should search through the Prefix Tree to find if the path already exists. If such path has been found, the only thing to do is to rellink the estimation to a new path that related to it; otherwise a new path should be inserted into the right place of the Prefix Tree, while the relink should be done following the insertion. Algorithm 3 shows the process of finding path and rellink the estimation e and its new path. When a new path is generated, the influence of the path will be retrieved by adding the marginal influence calculate in previous step to the influence of the original candidate set. The related subscriptions of the new path is the intersection of the subscriptions of original path and the related subscriptions of target user ur.

In the FindPath process, the algorithms searches the path related to the candidate set S and returns the path. The algorithm begins from the root node, where it finds if there is a child c of current node which is equal to the first element of S. If found, it iteratively search the rest of the S begins from c, until if reaches the end of S. Otherwise, the algorithm inserts the rest of S as a branch from current node.

The clearance of a path happens when an candidate set has no related estimations. The paths which should probably be erased is
Algorithm 3: MODIFY \((u, S, e)\)

1. \(S' \leftarrow S \cap \{u\} ; / S'\) is in sorted order
2. if \(\text{new path haven't been generated}\) then
3. \(S' \leftarrow \text{FINDPATH(} \text{root}, S')\);
4. else
5. \(S' \leftarrow \text{thenewlygeneratedpath}\);
6. \(f(S') = f(S) + \Delta(u, |S|)\);
7. \(Q_{S'} \leftarrow Q_S \cap Q_u\);
8. Remove the link of \(e\) from \(S\) and relink it to \(S'\);

Algorithm 4: FINDPATH (node, \(S\))

1. \(u\leftarrow\)the first element of \(S\);
2. \(S \leftarrow S \setminus \{u\} ;
3. \text{if one of the child } c \text{ of node } s.t. c = u\) then
4. \(\text{return path from root to } c ;
5. \text{return FINDPATH(} \text{c, } S)\);
6. else
7. Insert \(S\) sequentially as a branch of \(node\) ;
8. Insert new position of each newly insert node into vertex index ;
9. return the new path ;

Algorithm 5: CLEAR \((P)\)

1. for each path whose marginal influence calculated do
2. if the last element of \(P\) isn't leaf then return;
3. while the last element has no related estimations or has less than 2 children do
4. \(\text{Erase the last element of } P \text{ from Prefix Tree} ;
5. \text{Erase the related position linkage in the vertex index} ;

restrained to those the marginal influence of which has been computed. Candidate sets which satisfies the sieve condition will generate a new path in the process of Algorithm 3 and move the related estimations to their newly related path. Therefore, the links between estimations and candidate sets changes dynamically. When the links to a candidate set, i.e., a path on the Prefix Tree, it should be erased out from the Tree. The algorithm of clearing a estimation set is shown in Algorithm 5. The algorithm first check whether the path is the longest path in its branch, i.e., the last node of the path is leaf. If so, it will continue to erase the path starting from the leaf node, until it finds first node which has more than one child or has related estimations to it; otherwise, the the algorithm doesn’t do anything, as the path is related to other candidate sets.

5 EXPERIMENT

5.1 Experimental Setup

Datasets. We collect data of research papers from DBLP, real world social network Reddit and Twitter. The timestamp of DBLP dataset is based on the publish year of research paper, it thus updates very slow, each timestamp contains millions of actions. While Reddit and Twitter frequent update social networks. There are only tens of actions per timestamp.

- DBLP: DBLP is a datasets of research papers retrived from DBLP and other sources [7]. For each paper item, the dataset contains 7 fields of information of the paper: \(\text{paper id, title, authors, venue, year, references and abstract.}\) The dataset contains 2,503,993 valid research papers from 1,422,578 authors. Keyword are extracted from the the title, venue and abstract fields of dataset based on TF-IDF.

- Reddit: Reddit is an online social forum with large number of users. The dataset is divided into posts and comments. The data collects the essential information about the response relationship and topics from which we can generate subscriptions. The dataset contains 1,995,836 users, 30,744,232 streaming actions and 2,554,003 subreddits, which is regarded as topic keywords.

- Twitter:

| Dataset  | Vertex  | Edges | Avg. Deg. | Avg. Keywords per User | Stream |
|----------|---------|-------|-----------|------------------------|--------|
| DBLP     | 1,422,578 | 131,878,718 | 92.70 | 9.49 | 240,940,225 |
| Reddit   | 1,955,836 | 24,537,116  | 12.29 | 3.82 | 30,744,232 |
| Twitter  | 0       | 0     | 0         | 0                      | 0      |

The statistics of the abovementioned dataset are shown in Table 1.

The subscriptions are generated by randomly choose a sample from keywords set of the social networks. In real world subscription queries, users are usually interested in a small area of topics compared to the total topic sets of the social networks. On the other hand, using TF-IDF, we generate user subscriptions based on the informaton provided by each keyword. We generate subscription sets that each user will be related to 1.2? subscriptions in average.

Approaches.

- IC: IC is a state-of-the-art dynamic IM algorithm. We assign an index to each of the actions sequentially in the social stream, and reconstruct according to the trigger model which is used by IC.

- SIC: SIC is also proposed by [? ]. The parameter \(\beta\), which controls the trade-off between quality and efficiency, is set to the proposed default value, i.e., \(\beta = 0.2\).

- PSIM: The PSIM proposed in section 4. We set the approximating parameter in SieveStreaming \(\epsilon\) to 0.1.

(How to compare multi-topic settings and non-topic setting??)

Quality Metric. IC and SIC retrieve the most influential users based on cardinalities functions, while our approach is based on one-hop coverage influence model described in section 2. In order to verify the quality of our solution, we adopt well-reganized IC influence model in evaluation the influence spread of the results retriwed by each approach. For sliding window based approaches, we use the default window size of 100K actions length. The solutions are retrived by every timestamp and evaluated by IC influence model. Finally, we use the average influence spread of all results as the quality metric.

Performance Metric. We use throughput as our performance metric. The throughput is measured by measuring the CPU time elapse of returning \(L\) results, and divide \(L\) by the elapse time.

Parameters. The parameters examined in our experiments: (1) \(k\) is the size of seed set. (2) \(\lambda\) is the time decaying constant controlling
the weights decay of actions over time elapse. (3) \( |Q| \) is the number of subscription queries to answer. The summary of the parameters is listed in Table 2 with default values in bold.

### Table 2: Parameters in experiments

| Parameter | Values          |
|-----------|-----------------|
| \( k \)   | 5, 25, 50, 75, 100 |
| \( \lambda \) | 0.01, 0.05, 0.1, 0.3, 0.5 |
| \(|Q|\)    | 200K, 400K, 600K, 800K, 1M |

**Experiment Environment.** All experiments are conducted on a server machine running Ubuntu 14.04 with (CPU) and 250 GB memory. IC and SIC is implemented in Java 8, while the implementation of PSIM is in C++.

### 5.2 Performance of Time-Decaying

**SieveStreaming**

We first test the efficiency and the performance of the time-decaying model, comparing to the results of sliding window. In this part, we run naive SieveStreaming over a window size of 500K and PSIM over DBLP dataset. In the case study of Neural Networks, we search for the \( k \)-influencers in the area of Neural Networks, and compare the influencers extracted by the two influence model. We also compare the influence spread of users extracted by the two models. Then we test the efficiency of the naive time-decaying SieveStreaming and sparse and lazy time-decaying SieveStreaming when varying the time decaying constant \( \lambda \).

#### 5.2.1 Result Comparison between Time-Decaying and Sliding Window Models.

In this part, we will compare the performance of the time-decaying model and sliding window models.

**Case Study on Neural Networks:** The result of the case study of extracting \( k \)-influencers under the topic of Neural Networks is shown in Table 3. It is shown that the time-decaying model can extract more well-known influencers of the area of Neural Networks than sliding window do. It is caused by the fact that sliding window throws the information of previous actions. At the time of returning result for the most influencers, just few actions performed by influencers are in the window. On the other hand, as there are increasing number of research papers published each year, the sliding window of of fixed length in different year includes actions diversified length of time, it therefore cannot reveals the true influencers over time.

**Quality** (Influence spread of users extracted by the two models)

#### 5.2.2 Varying Decaying Constant \( \lambda \)

In this part, we compare the efficiency of the naive SieveStreaming and sparse and lazy time-decaying SieveStreaming.

**Throughput**

### Table 3: Well-known influencer of Neural Networks

| Year | Time-decaying | Sliding window |
|------|---------------|----------------|
| 2000 | Michael I. Jordan | ... |
| 2010 | ... | ... |