Pattern formation in non-equilibrium spinor polariton condensates

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Abstract. Non-equilibrium polariton condensates entangle properties of lasers, atomic Bose-Einstein condensates (BEC) and semiconductor physics. They provide a great variety of physical phenomena while maintaining a simple theoretical description. Among those phenomena are nonlinear excitations such as solitons or spontaneous spin bifurcations. In this paper I first present a short overview on the theoretical basis of polaritons. Then starting from a scenario of excitation generation in equilibrium BEC I turn to corresponding phenomena in polariton condensates such as dark soliton formation. Later the spin sensitive phenomena such as non-equilibrium bright solitons and half-bright solitons in semiconductor microcavities are discussed. Theoretically all the considered scenarios are described by partial differential equations (PDEs) and coupled systems thereof. The system of PDEs defines a so called condensate wave function, which completely describes the experimental relevant aspects of the physical system in a certain parameter regime where condensation occurs. The developed theories enables us particularly to make a variety of statements about excitations such as solitons and half-solitons in spinor systems forming within a non-equilibrium condensate. It turns out that by those means we can elucidate in particular the experimental implementation of a coherent superposition analog in a spin sensitive setting forming a macroscopic QUBIT within the semiconductor microcavity at temperatures in the Kelvin range.

1. Introduction

Coherent quantum matter has intriguing properties such as BEC, a finite speed of sound and nonlinear excitations [1] like dark solitons [2]. Polaritons are quasiparticles consisting of excitons and cavity photons within semiconductor micro-cavities which obey Bose-Einstein statistics [3, 4] and thus the potential to condense into a single particle mode [5]. Excitons are coupled electron-hole pairs of oppositely charged spin-half particles in a semiconductor held together by an effective Coulomb force between them [4] as the energy to form a pair is lower than a free electron and a free hole. States of excitons interact with light fields [6] and can form polaritons in the so called strong coupling regime when confined to a micro-cavity [7]. These polaritons possess integer spin and for dilute systems condense and because polaritons are \( 10^9 \) times lighter than rubidium atoms [5], this condensation is observed in CdTe/CdMgTe micro-cavities [5, 3] and even at room temperatures in flexible polymer-filled micro-cavities [8]. This stands in contrast to atomic BEC observed below 200mK [9, 10, 11]. The Hamiltonian describing the interaction between the light modes and excitons is [4, 3]

\[
H = \sum_k E_c(k)\phi_k^\dagger\phi_k + \sum_k E_x(k)\xi_k^\dagger\xi_k + \hbar \Omega_R \sum_k E_c(k) \left( \phi_k^\dagger \xi_k + \xi_k^\dagger \phi_k \right). \tag{1}
\]
Here $\phi_+^\dagger_k$ is the creation and $\phi_k$ the annihilation operator of a photon, while $\xi_+^\dagger$ and $\xi_k$ the corresponding operators for the exciton. By diagonalizing the Hamiltonian one gets two energy eigenvalues (lower and upper branch of the polariton dispersion) $E_{L,U}(k) = \frac{1}{2} \left( E_c(k) + E_x(k) \mp \sqrt{(E_c(k) - E_x(k))^2 + 4\hbar^2\Omega_R^2} \right)$ that are functions of the in-plane wave vector $k$ and corresponding two basis vectors $\psi_{L,U}(k) = (h_{2,k}, -h_{1,k})$ and $\psi_{L,U}(k) = (h_{1,k}, h_{2,k})$, where the coefficients, the so called Hopfield factors [6], are given by $h_{1,k} = \sqrt{\left(1 - \frac{\delta(k)}{\sqrt{\delta(k)^2 + 4\hbar^2\Omega_R^2}}\right)}$ and $h_{2,k} = \sqrt{\left(1 + \frac{\delta(k)}{\sqrt{\delta(k)^2 + 4\hbar^2\Omega_R^2}}\right)}$. Generally the quantities $|h_{1,k}|^2$ and $|h_{2,k}|^2$ correspond to the fraction of photon and exciton within a given polariton. The dispersion of the cavity photon is $E_c(k) = \frac{\hbar c}{n_c} \sqrt{k_x^2 + k_y^2}$ and the exciton energy $E_x(p) = \varepsilon + p^2/(2m_x)$ where $m_x$ is the exciton mass and $\varepsilon$ the energy offset. $n_c$ is the refraction index between the cavity mirrors, $c$ the speed of light. Although not considered by the stated Hamiltonian polaritons can be generated at will and decay usually between 1–100 ps afterwards depending on the quality of confinement within the micro-cavity. When macroscopic coherent occupation of a single mode has been established the finite lifetime makes it essentially a non-equilibrium BEC with intriguing novel physical aspects and phenomena. The following section summarizes findings of the research paper [12], which discusses excitation generation in equilibrium BEC.

2. Quantum Piston

First, let us consider a scenario of equilibrium BEC. The many-body system can be described by a single giant matter wave when considering a dilute and weakly interacting atom cloud below the critical temperature [13]. In contrast to open systems the condensate wave function $\psi$ is normalized to a fixed value for all times, $\|\psi\|_{L^2(\mathbb{R}^d)} = N$. Here $d$ is the spatial dimension of the condensate. This models the scenario where particle numbers $N$ remain the same and we assume the particles to be stable, i.e. non-decaying and there is no exchange of particles. The time evolution of the condensate wave function is given by the so-called Gross-Pitaevskii equation [14, 15],

$$i\hbar \partial_t \psi(\vec{r}, t) = \left(-\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}, t) + g(z)\psi^2 \right) \psi(\vec{r}, t). \tag{2}$$

To generate excitations within the condensate wave we utilize the possibility of changing interactions between the condensed particles (e.g. atoms) locally via Feshbach resonances. The basic idea is presented in Fig. 1. Here we assume a cold atom cloud trapped by the potential $V(\vec{r}, t)$ to obey a 3D cigar shaped geometry. Once the self-interactions $g$ are changed at one half of the atom cloud, particles move from highly interacting areas to less interactive areas of the cloud. Mathematically this change of self-interactions can be modeled by letting the initially constant interactions $g$ become a step like function $g(z)$. This flow might carry topologically stable excitations within the superfluid. The excitations particularly depend on the dimension of the BEC. In Fig. 2 we see numerical results for 3D and 1D settings based on (2). The 1D setting becomes feasible for very tight trapping in the radial directions. The numerical results were generated by a 4th order Runge Kutta and 4th order finite differences method. All plots in Fig. 2 show the density of the superfluid. In a 3D scenario (Fig. 2 (a)) we observe the emergence of vortex rings propagating from the self-interaction step. In 1D we see the generation of so called dark soliton trains (Fig. 2 (b)). Those are regular arrays of dark solitons moving from
Figure 1. Schematic illustration of the quantum piston concept [12]. The area of change in self-interactions is illustrated by the hatched lines. Dots represent the emerging vortex rings as the self-interactions \( g \rightarrow g(z) \) become a step function.

Figure 2. BEC density \( n = |\psi|^2 \) for 3D showing vortex rings (a) and in 1D dark-solitons (b). (c) shows dependence of the frequency on \( g(z) \).

3. Controlled soliton generation in polariton BEC

So far we have considered the scenario of excitation generation in equilibrium BEC (atoms, molecules, closed systems), where particle numbers are conserved. Now let us turn to non-equilibrium BEC of polaritons observed in semiconductor microcavities and pattern formation within the generalized condensate wave function as discussed in the research paper [17]. Polaritons are quasiparticles formed as superpositions of electrons and light fields that can be created at a laser pump spot and decay after 1-100ps. To theoretically model many observed phenomena usually a complex Ginzburg-Landau (cGL)-type equation incorporating energy relaxation and a stationary reservoir is utilized [3, 16]

\[
i\partial_t \psi = \left(1 - i\eta\right) \left[-\frac{\hbar \nabla^2}{2m} + \alpha_1 |\psi|^2 + V(\vec{x}, t) + \frac{i}{2} (\delta_R N(\vec{x}, t) - \gamma_C)\right] \psi
\]  

with \( \partial_t N = -(\gamma_R + \beta |\psi|^2)N + P \), where \( N \) is the magnitude of pre condensed polaritons. This set of equations models the scenario of incoherent pumping [3]. \( \eta \) denotes energy relaxation, \( \alpha_1 \)
the self-interaction strength, $\delta_R$ the scattering rate into the condensate, $\gamma_C$ the condensate decay rate, $\gamma_R$ is the constant reservoir decay rate, $\beta$ the magnitude of scattering into the condensate and $P$ the spatially dependent pumping function. Physically hot excitons are formed by external illumination, which by a variety of scattering processes then macroscopically occupy the lowest energy state on the polariton dispersion, i.e. a giant matter wave similar as in the atomic BEC case discussed before. To generate excitations within the corresponding condensate wave function we proceed as follows and as discussed in [17]. First we pump polaritons at a spatially localized area, which approximately obeys the form of a Gaussian. Subsequently we apply a potential step, which is feasible by adding a local electrical field. As new polaritons are formed at the pump spot the potential step induces a perturbation to the matter wave. The main physical quantities for soliton generation are the local speed of sound $c_s(x) = \sqrt{\mu(x)/m}$ of the superfluid, where $\mu(x) = \alpha_1 n(x)$ is the chemical potential, and the superfluid velocity, which is proportional to the gradient of the phase of the condensate wave function. Once the speed of sound is exceeded by the superfluid dark solitons emerge regularly. By changing the step size or the pumping strength, i.e. the amount of new polaritons entering the condensate, the period of dark solitons can be changed on demand. Results are presented in Fig. 3 for different pumping strengths. In (d) we see a Sinusoidal pumping signal.

![Figure 3. BEC density $n = |\psi|^2$ for 1D showing dark-solitons.](image)

![Figure 4. Snapshots of velocity related quantities taken at $t_s = 120$ ps. The blue envelope displays $\pm c_s(x, t_s)$ (speed of sound) and the red line shows the condensate velocity $v(x, t_s)$. The panel (a) shows results from a pump amplitude $600\Gamma_R$ where no soliton develop. The panel (b) shows the development of the train for a lower amplitude of $375\Gamma_R$ due to the local supersonic regions at the step.](image)

The essential criterion for dark soliton generation is the breaking the speed of sound as Fig. 4 shows. Once the speed of sound is surpassed solitons are generated within the superfluid’s stream. We observe that higher pump rates imply higher speed of sound, because the superfluid density increases. Therefore the number of generated dark solitons becomes smaller. On the
other hand lower pump rates imply the emergence of more dark solitons in the same interval of time. In this setting one can further change the emergence of dark solitons on the fly by changing the step size or the pumping strength during the experiment.

Figure 5. (Color online) Control over the polarization of the trains. The colormap shows the degree of circular polarization $\rho_c = (n_+ - n_-)/(n_+ + n_-)$. (a) $\delta_2^R/\delta_1^R = 0.90$, (b) $\delta_2^R/\delta_1^R = 1.11$ and (c) $\delta_2^R/\delta_1^R = 0.99$.

Polaritons and polariton condensates have the property of spin, this is due the coupling of excitons of spin $\pm 1$ with circularly polarized light. By this coupling either spin up or spin down polaritons and condensates thereof form. The physical system can be described mathematically by two coupled complex Ginzburg-Landau equations each representing one spin component of the spinor. The coupling between the two components depends on the particular regime, but for condensates away from $k = 0$ the so called longitudinal transversal splitting (TMTE splitting) becomes relevant, which introduces an energy separation of the two spin component wave functions. Physically the concept to generate spin sensitive phenomena can be to generate different populations of spin up and spin down polaritons by slightly generating more polaritons in one of the spin components. Then via the mechanism of TMTE splitting the two components distinguish themselves even further. Once a dark soliton train emerges in the spinor wave function, i.e. when the speed of sound is broken in a 1D scenario the dark soliton trains separate for differently populated components within each component. Interestingly each train is generated at the same time on the same breaking point of the potential step, but both evolve differently particularly due to the different speed of sounds in each component as Fig. 5 shows. Here the circular polarization is plotted for different ratios of growth rates $\delta_1^R$ and $\delta_2^R$, for spin up and spin down polaritons respectively, modeling a slight imbalance in pumping strengths into each spin component.

4. Spontaneous spin bifurcation

Next we turn to the system of spinor polariton condensates far away from the pumping spots as discussed in [19]. In Fig 6 (a) and (b) the schematics of the four pump spot setting is provided. The results are based on the experimental finding that the condensate forms away from the areas where they are created by external illumination through laser beams that act as repulsive potentials [18]. This is possible as polaritons have a relative long lifetime compared to excitons. As polaritons move away from the 4 pump spots generating them they get trapped in-between and the bulk of polaritons forms mainly there. In contrast to all previous experiments we observe a spontaneous emergence of either a spin up or spin down component, which
mathematically corresponds to vanishing $L^2$ norm for one of the spin components, while having a fully populated complementary spin component. This holds although there is no pumping imbalance corresponding to a linearly polarized laser, i.e. both components are equally fed and initially should have the same population up to very small perturbations. The spinor polariton condensate wave function’s (i.e. $\Psi = (\psi_+, \psi_-)$) time evolution for this particular setting is given by: $i\partial_t \psi_\sigma = (1 - i\eta) \frac{\hbar}{\xi} \psi_\sigma$ with $\sigma \in \{+, -\}$ and

$$\frac{\mathcal{H}_\pm}{\hbar} \psi_\pm = \left( -\frac{\hbar \nabla^2}{2m^*} + aN_\pm + \alpha_1 |\psi_\pm|^2 + \alpha_2 |\psi_\mp|^2 \right) \psi_\pm + \frac{i}{2} \left( bN_\pm - \gamma C \right) \psi_\pm + iJ \psi_\mp$$

with complex Josephson coupling $J = \varepsilon + i\gamma \in \mathbb{C}$ between the two spin components. The radiative dissipative part $\gamma$ is due to interference of the light from different components and real Josephson coupling models tunneling between states $\varepsilon$. Here $\alpha_1$ denotes self-interaction strength and $\alpha_2 = -0.1\alpha_1$ the cross-interaction strength. $m^*$ is the effective mass of the polaritons of spin $\sigma$. The reservoir is approximately given by $N_\pm = P/\gamma R \left( 1 - \frac{\beta}{\gamma R} \left( |\psi_\pm|^2 + |\psi_\mp|^2 \right) \right)$ and pumping of polaritons into the reservoir is $P(\vec{r}) = \sum_i A \cdot \exp \left[ -\theta \left( (x - x_i)^2 + (y - y_i)^2 \right) \right]$.  

Fig 6 (b) shows the geometry in real space, while (c) shows the energy dispersions of the polariton, exciton and cavity photon as discussed in the introduction. Experimentally condensation happens at $k = 0$ and the width of the condensate peak is $\delta k = 0.4 \mu m^{-1}$. Interestingly we note that such a setting can be described by a mean-field theory over several minutes [18] without taking into account the emergence of decoherence. Mathematically in addition to the usual terms for incoherently pumped polaritons with a spin up and down component we add a complex Josephson coupling modeling the interference between both spin components and the tunneling i.e. interconversion between spin up and down particles, which can be understood as a transformation from spin up and down particles and vice versa. It turns out that this term is essential for the observed pattern formation. Figure 7 (a) shows the spontaneous realization of either a spin up or a spin down state. (b) clearly shows the stability of the spontaneously selected spin state as measured over time. Remember that the polariton lifetimes are between $1 - 100$ps in current settings and this figure shows stability over seconds. The degree of polarization is about 0.95 measured in experiments. Once a spin has been selected spontaneously, applying a constant gain of polaritons with opposite spin generates Bloch oscillations as indicated experimentally in second row of the Fig. 7 (a) and predicted theoretically in (b) in the same line. These findings indicate the existence of a macroscopic superposition between spin up and spin down states.

Acknowledgements.– I acknowledge support through my Schrödinger Fellowship.
5. References

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