Yang-Mills instantons on 7-dimensional manifold of $G_2$ holonomy

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Abstract

We investigate Yang-Mills instantons on a 7-dimensional manifold of $G_2$ holonomy. By proposing a spherically symmetric ansatz for the Yang-Mills connection, we have ordinary differential equations as the reduced instanton equation, and give some explicit and numerical solutions.

1 Introduction

Recent progress in string duality has stimulated a great deal of interest in Yang-Mills instantons on higher dimensional manifolds of special holonomy. The higher dimensional Yang-Mills instanton equations on such manifolds have been given in several papers. It is known that some Yang-Mills instantons on the manifolds can be extended to solitonic solutions of the low energy effective theory of the heterotic string preserving $\frac{1}{16}$ or $\frac{1}{8}$ of the 10-dimensional $N = 1$ supersymmetry.

The manifolds of special holonomy admit covariantly constant spinors and define the supersymmetric cycles (calibrated submanifolds). Solitons and instantons of superstring theory or M-theory can be considered to be wrapping branes around a supersymmetric cycle. The manifolds also play an important role in generalizations of the Donaldson-Floer-Witten theory to higher dimensions. Baulieu et al. found a topological action on a 8-dimensional manifold of Spin(7) holonomy; their covariant gauge fixing conditions lead to Yang-Mills instanton equations on the manifold. Compared with the 4-dimensional case, there are a few explicit Yang-Mills instantons on 7- and 8-dimensional manifolds of $G_2$ and Spin(7) holonomy. Recently Yang-Mills instantons on a 8-dimensional manifold of Spin(7) holonomy have been examined in detail.

In this paper we investigate Yang-Mills instantons on a 7-dimensional manifold of $G_2$ holonomy. Our paper is organized as follows: in section 2 and section 3 we briefly review the octonionic algebra and the explicit metric of a manifold of $G_2$ holonomy given by
Gibbons et al. in section 4 we express Yang-Mills instantons on 7-dimensional manifolds of $G_2$ holonomy; in section 5, under a spherically symmetric ansatz, we solve the Yang-Mills instanton equations on the manifold described in section 3, and present some new explicit solutions obtained from the Riccati equation and numerical solutions.

2 Octonionic Algebra

In this section we briefly review the octonionic algebra following the papers. The exceptional Lie group $G_2$ which is a subgroup of $Spin(7)$ is the automorphism group of the octonionic algebra. The generators of $Spin(7)$ are given by

$$\Gamma^{ab} = \frac{1}{2}(\Gamma^a \Gamma^b - \Gamma^b \Gamma^a),$$

(2.1)

where $\Gamma^a (a = 1, \cdots, 7)$ are 7-dimensional $\gamma$ matrices defined by

$$\{\Gamma^a, \Gamma^b\} = 2\delta^{ab}.$$  

(2.2)

The generators of the octonionic algebra $\{1, e_a\}, (a = 1, \cdots, 7)$ obey the relations

$$e_a \cdot e_b = -\delta_{ab} 1 + C_{abc} e_c,$$

(2.3)

where $C_{abc}$ is totally antisymmetric tensor with non zero components

$$C_{123} = C_{516} = C_{624} = C_{435} = C_{471} = C_{673} = C_{572} = +1.$$  

(2.4)

If the dual tensor $C_{abcd}$ is defined by

$$C_{abcd} = \frac{1}{3!} \delta_{abcd}^{ijk} C_{ijk},$$

(2.5)

the following relations are satisfied

$$C_{abpq} C_{cdpq} = 4(\delta^a_c \delta^b_d - \delta^a_d \delta^b_c) - 2C_{ab}^{cd},$$

(2.6)

$$C_{apq} C_{bcpq} = -4C_{abc}^{cd}.$$  

(2.7)

By the use of the tensor $C_{abcd}$, one can decompose the 21-dimensional Lie algebra of $Spin(7)$ into the direct sum of the 14-dimensional Lie algebra of $G_2$ and the 7-dimensional orthogonal subspace $\mathcal{P}$. Indeed, the projector $P_1^{ab}_{cd}$ onto the Lie algebra of $G_2$ and the projector $P_2^{ab}_{cd}$ onto $\mathcal{P}$ are given by

$$P_1^{ab}_{cd} = \frac{2}{3} \left( \frac{1}{2}(\delta^a_c \delta^b_d - \delta^a_d \delta^b_c) + \frac{1}{4} C_{ab}^{cd} \right),$$

(2.8)

$$P_2^{ab}_{cd} = \frac{1}{3} \left( \frac{1}{2}(\delta^a_c \delta^b_d - \delta^a_d \delta^b_c) - \frac{1}{2} C_{ab}^{cd} \right).$$

(2.9)

Thus the generators $G^{mn}$ of $G_2$ can be realized as

$$G^{ab} = \frac{3}{4} P_1^{ab}_{cd} \Gamma^{cd} = \frac{1}{2}(\Gamma^{ab} + \frac{1}{4} C^{ab}_{cd} \Gamma^{cd}).$$

(2.10)
3 Explicit metric on 7-dimensional manifold of $G_2$ holonomy

In this section we describe the explicit metric on a 7-dimensional manifold of $G_2$ holonomy. It has been first given as a metric on the $R^4$ bundle over $S^3$ by solving the Einstein equation:

$$ds^2 = \alpha^2 dr^2 + \beta^2 (\sigma^i - A^i)^2 + \gamma^2 \Sigma^i \Sigma^i,$$

(3.1)

where $i = 1, 2, 3$ and $\alpha, \beta, \gamma$ are functions solely of the radial coordinate $r$. The symbols $\Sigma^i$ denote the left invariant 1-forms on the $S^3$ base manifold and the symbols $\sigma^i$ denote the left invariant 1-forms on the fiber $R^4$. They satisfy

$$d\Sigma^i = -\frac{1}{2} \epsilon_{ijk} \Sigma^j \wedge \Sigma^k, \quad d\sigma^i = -\frac{1}{2} \epsilon_{ijk} \sigma^j \wedge \sigma^k.$$  

(3.2)

Then $SU(2)$ connection $A^i$ is given by $A^i = \frac{1}{2} \Sigma^i$ and the explicit forms of the functions are

$$\alpha^2 = \left(1 - \left(\frac{a}{r}\right)^3\right)^{-1}, \quad \beta^2 = \frac{1}{9} r^2 \left(1 - \left(\frac{a}{r}\right)^3\right), \quad \gamma^2 = \frac{r^2}{12}$$

(3.3)

where $a$ is an arbitrary constant.

We can also construct the metric (3.1) by solving the first order equation for the spin connection,

$$\omega_{ab} = \frac{1}{2} C_{ab}^{\ cd} \omega_{cd}. $$

(3.4)

Under the spherically symmetric ansatz (3.1), the orthonormal basis is

$$e^7 = \alpha dr, \quad e^i = \gamma \Sigma^i, \quad \hat{e}^i = \beta (\sigma^i - A^i)$$

(3.5)

where $\hat{i} = 1, 2, 3 = 4, 5, 6$, and the spin connection $\omega_{ab}$ is obtained by the equations $de^a = \omega^a_{\ b} \wedge e^b$ and $\omega_{ab} = -\omega_{ba}$:

$$\omega_{\hat{i}i} = -\frac{\gamma}{\alpha \gamma} e^i,$$

(3.6)

$$\omega_{\hat{i}i} = -\frac{\beta'}{\alpha \beta} e^i,$$

(3.7)

$$\omega_{ij} = \frac{\beta}{8 \gamma^2} \epsilon_{ijk} e^k,$$

(3.8)

$$\omega_{ij} = -\epsilon_{ijk} \left(\frac{1}{2 \gamma} e^k + \frac{\beta}{8 \gamma^2} e^k\right),$$

(3.9)

$$\omega_{ij} = -\epsilon_{ijk} \left(\frac{1}{2 \gamma} e^k + \frac{1}{2 \beta} e^k\right).$$

(3.10)

Eq. (3.4) then reduces to the following nonlinear ordinary equations:

$$\beta' = \frac{\alpha \beta^2}{8 \gamma^2} - \frac{\alpha}{2}, \quad \gamma' = -\frac{\alpha \beta}{4 \gamma},$$

(3.11)

which reproduce the solution (3.3).
4 Yang-Mills instanton on 7-dimensional manifold of $G_2$ holonomy

Here we define a Yang-Mills instanton equation on the 7-dimensional manifold of $G_2$ holonomy in terms of a special closed 3-form $\Omega$:

$$\Omega \wedge F = \ast F.$$  \hspace{1cm} (4.1)

Using the antisymmetric tensor $C_{abc}$, the 3-form $\Omega$ can be written as

$$\Omega = \frac{1}{3!} C_{abc} e^a \wedge e^b \wedge e^c,$$  \hspace{1cm} (4.2)

and then Eq. (4.1) becomes

$$F_{ab} = \frac{1}{2} C_{ab}^{cd} F_{cd}.$$  \hspace{1cm} (4.3)

The seven components of Eq. (4.3) are as follows:

$$
\begin{align*}
F_{71} &= F_{26} + F_{53}, \\
F_{72} &= F_{61} + F_{34}, \\
F_{73} &= F_{42} + F_{15}, \\
F_{74} &= F_{23} + F_{65}, \\
F_{75} &= F_{46} + F_{31}, \\
F_{76} &= F_{12} + F_{54}, \\
F_{63} &= F_{25} + F_{14}.
\end{align*}
$$  \hspace{1cm} (4.4)

The curvatures (4.3) provide the solutions of the Yang-Mills equation on the manifold, $D_a F^{ab} = 0$, as a consequence of the Bianchi identity. We shall call solutions of (4.3) Yang-Mills instantons. The Yang-Mills instantons can be embedded into supersymmetric theories as solutions to the equation of motion. It is known that there is a spinor $\eta$ on the manifold of $G_2$ holonomy which satisfies the conditions:

$$\nabla \eta = 0, \quad G^{ab} \eta = 0,$$  \hspace{1cm} (4.5)

where $\nabla$ denotes the covariant derivative of the Levi-Civita connection on the manifold. Then, one can use the covariantly constant spinor as a global supersymmetric parameter defined on the manifold. The supersymmetric transformation of a spinor field $\chi$, a super partner of Yang-Mills connection,

$$
\delta \chi = \frac{1}{2} F_{ab} \Gamma^{ab} \eta = \frac{1}{2} (P_1^{ab} + P_2^{ab}) F_{ab} \Gamma^{cd} \eta = \frac{2}{3} F_{ab} G^{ab} \eta + \frac{1}{6} (F_{ab} - \frac{1}{2} C_{ab}^{cd} F_{cd}) \Gamma^{ab} \eta
$$

is zero according to the Eqs. (4.3) and (4.5). So the Yang-Mills instanton becomes a supersymmetric solution preserving the spinor $\eta$.

Note that the spin connection leads to a $G_2$ Yang-Mills connection:

$$\mathcal{A} = \frac{1}{3} G^{ab} \omega_{ab}.$$  \hspace{1cm} (4.7)

Using (3.4) and the symmetry between the first and second pair of indices of the Riemann curvature $R_{abcd}$, we can see that the curvature of $\mathcal{A}$ satisfies the Yang-Mills instanton equation (4.3).
5 Ansatz for $G_2$ Yang-Mills connection

Now we construct Yang-Mills instantons on the 7-dimensional manifold of $G_2$ holonomy described in section 3. Referring to the spin connection \((3.6)-(3.10)\), we propose a spherically symmetric ansatz for the $G_2$ Yang-Mills connection $A = \frac{1}{3} G^{ab} A_{ab}$:

\begin{align}
A_{\bar{7}i} &= 2A e^i, \\
A_{\bar{7}i} &= (B - C)e^i, \\
A_{ij} &= \epsilon_{ijk} A e^k, \\
A_{ij} &= \epsilon_{ijk} (D e^k + B e^k), \\
A_{ij} &= \epsilon_{ijk} (D e^k + C e^k),
\end{align}

where $A, B, C, D$ are the functions of the radial coordinate $r$. We can see that the above connection satisfies

\[ A_{ij} = \frac{1}{2} C_{ij}^{kl} A_{kl} \]

and so does their curvature, i.e., they are just projected onto the Lie algebra of $G_2$. Note that the indices denote those of the Lie algebra of $G_2$, not of the differential forms. This connection reproduces Eq. \((4.7)\) by putting

\[ A = \frac{\beta}{8 \gamma^2}, \quad B = -\frac{\beta}{8 \gamma^2}, \quad C = -\frac{1}{2 \beta}, \quad D = -\frac{1}{2 \gamma}. \]

The components of the curvature $\mathcal{F}(= \frac{1}{3} G^{ab} F_{ab})$ are explicitly given by

\begin{align}
F_{\bar{7}i} &= \frac{2}{\alpha} (A' + \frac{\gamma}{\gamma} A)e^\gamma \wedge e^i - \epsilon_{ijk} A (\frac{1}{\gamma} + 2D)e^j \wedge e^k - \epsilon_{ijk} A (3B - C)e^j \wedge e^k, \\
F_{\bar{7}i} &= \frac{1}{\alpha} ((B - C)' + \frac{\beta}{\beta} (B - C))e^\gamma \wedge e^i - \epsilon_{ijk} (B - C) (\frac{1}{2 \beta} + C)e^j \wedge e^k - \epsilon_{ijk} (B - C) (\frac{1}{8 \gamma^2} (B - C) - 2A^2)e^j \wedge e^k, \quad (5.8) \\
F_{ij} &= \epsilon_{ijk} \frac{1}{\alpha} (A' + \frac{\gamma}{\gamma} A)e^\gamma \wedge e^k - A (\frac{1}{\gamma} + 2D)e^j \wedge e^i - A (3B - 2C)e^j \wedge e^i \\
&\quad - AC e^j \wedge e^k + \delta_{ij} A (B - C) e^l \wedge e^l, \\
F_{ij} &= \epsilon_{ijk} \frac{1}{\alpha} (D' + \frac{\gamma}{\gamma} D)e^\gamma \wedge e^k + \epsilon_{ijk} \frac{1}{\alpha} (B' + \frac{\beta}{\beta} B)e^\gamma \wedge e^i \\
&\quad - (\frac{D}{\gamma} - \frac{\beta}{4 \gamma^2} (B + 5A^2 + D^2))e^\gamma \wedge e^i - B (\frac{1}{\beta} + B)e^j \wedge e^j \\
&\quad - B (\frac{1}{2 \gamma} + 2\gamma^2) e^i \wedge e^j + B (\frac{1}{2 \gamma} + D)e^j \wedge e^i, \quad (5.11)
\end{align}
\[
F_{ij} = \epsilon_{ijk} \frac{1}{\alpha} (D' + \gamma' D) e^i \wedge e^j + \epsilon_{ijk} \frac{1}{\beta} (C' + \frac{\beta'}{\beta} C) e^i \wedge e^k \\
-(\frac{D}{\gamma} - \frac{\beta}{4\gamma^2} C + A^2 + D^2) e^i \wedge e^j - (\frac{C}{\beta} + (B - C)^2 + C^2) e^i \wedge e^j \\
-C(\frac{1}{2\gamma} + D) e^i \wedge e^j + C(\frac{1}{2\gamma} + D) e^i \wedge e^j. 
\]

(5.12)

Let us consider the Yang-Mills instanton equation (4.3) for the curvature \( F_{ab} = \frac{1}{2} F_{abij} e^i \wedge e^j \):

\[
F_{abij} = \frac{1}{2} C_{ijkl} F_{abkl}. 
\]

(5.13)

After some calculations, we obtain a set of ordinary differential equations for \( A, B, C, D \):

\[
(B - C)(\frac{1}{2\gamma} + D) = 0,
\]

(5.14)

\[
D' + \gamma' D = -2B\alpha(\frac{1}{2\gamma} + D), \quad D' + \gamma' D = -2C\alpha(\frac{1}{2\gamma} + D),
\]

(5.15)

\[
A' + \gamma' A = -A\alpha(3B - C), \quad A(\frac{1}{\gamma} + 2D) = 0,
\]

(5.16)

\[
(B - C)' + \frac{\beta'}{\beta} (B - C) = \alpha(\frac{\beta}{4\gamma^2} (B - C) - 4A^2) + 2\alpha(B - C)(\frac{1}{2\beta} + C),
\]

(5.17)

\[
- \frac{1}{\alpha}(B' + \frac{\beta'}{\beta} B) = \frac{D}{\gamma} - \frac{\beta B}{4\gamma^2} + 5A^2 + D^2 - \frac{B}{\beta} - B^2,
\]

(5.18)

\[
- \frac{1}{\alpha}(C' + \frac{\beta'}{\beta} C) = \frac{D}{\gamma} - \frac{\beta C}{4\gamma^2} + A^2 + D^2 - (\frac{C}{\beta} + (B - C)^2 + C^2).
\]

(5.19)

Eq. (5.14) can be satisfied in three cases (i) \( D = -\frac{1}{2\gamma}, B \neq C \), (ii) \( D = -\frac{1}{2\gamma}, B = C \), (iii) \( D \neq -\frac{1}{2\gamma}, B = C \), so we study these cases separately.

In case (i), we have two solutions:

\[
(A, B, C, D) = \left( \frac{\beta}{8\gamma^2}, -\frac{\beta}{8\gamma^2}, -\frac{1}{2\beta}, -\frac{1}{2\gamma} \right), \quad \left( -\frac{\beta}{8\gamma^2}, -\frac{\beta}{8\gamma^2}, -\frac{1}{2\beta}, -\frac{1}{2\gamma} \right).
\]

(5.20)

The first one is nothing but the connection (4.7), while the second one is a new solution. Although their difference is only the signature of \( A \), these two solutions lead to different curvatures.

In cases (ii) and (iii), \( A \) is zero from (5.18) so that the Yang-Mills instanton equation (4.3) gives two nonlinear ordinary differential equations for \( B \) and \( D \):

\[
B' = \alpha \left( -\frac{1 + 9\alpha^2}{2\alpha r} B - \frac{2\sqrt{3}}{r} D - D^2 + B^2 \right),
\]

(5.21)

\[
D' = \alpha \left( -\frac{D}{\alpha r} - \frac{2\sqrt{3}}{r} B - 2BD \right),
\]

(5.22)
where we have used Eqs. (3.11) and (3.3). Let us introduce new variables \( X \) and \( Y \),

\[
B = \frac{X + 1}{r}, \quad D = \frac{Y - \sqrt{3}}{r}
\]

and a new parameter \( s = \ln(\frac{r}{a}) \). Then Eqs. (5.21) and (5.22) become

\[
\frac{dX}{ds} = 1 - \frac{9\alpha^2}{2}(X + 1) + 3\alpha - \alpha Y^2 + \alpha(X + 1)^2, \quad (5.24)
\]

\[
\frac{dY}{ds} = -2\alpha(X + 1)Y. \quad (5.25)
\]

In case (ii), the Eq. (5.25) is trivial as \( Y = 0 (D = -\frac{1}{2}) \) and Eq. (5.24) becomes the Riccati equation:

\[
\frac{dX}{ds} = 1 - \frac{9\alpha^2}{2}(X + 1) + 3\alpha + \alpha(X + 1)^2. \quad (5.26)
\]

We can find a general solution of Eq. (5.26):

\[
X = 3\alpha - 1 + \frac{1}{\alpha(-\frac{1}{2} + \frac{c}{r^2})}, \quad (5.27)
\]

where \( c \) is an arbitrary integration constant. Taking the limit \( c \to \infty \), we have a special solution \( X = 3\alpha - 1 \). For the explicit solution (5.27) the nonzero components of the Yang-Mills connection and the curvature are \( \{A_{ij}, A_{\hat{i} \hat{j}} = A_{ij}\} \) and \( \{F_{ij}, F_{\hat{i} \hat{j}} = F_{ij}\} \). In this case the curvature becomes

\[
\mathcal{F} = \frac{1}{3} G^{ab} F_{ab} = \frac{1}{3} (G^{ij} + G^{\hat{i} \hat{j}}) F_{ij} = \frac{1}{3} T^{ij} F_{ij}, \quad (5.28)
\]

where \( T^{ij} \) are \( SU(2) \) generators; the gauge group reduces to \( SU(2) \) from \( G_2 \). The explicit components of the curvature become

\[
F_{ij\hat{k}} = F_{\hat{i}\hat{j}\hat{k}} = \epsilon_{ijk} \frac{1}{r^2} \left( \frac{2 + (\frac{c}{r^2})^3}{\frac{3}{2} + \frac{c}{r^2}} + \frac{1}{(-\frac{1}{2} + \frac{c}{r^2})^2\alpha^2} \right), \quad (5.29)
\]

\[
F_{ijij} = F_{\hat{i}\hat{j}\hat{i}\hat{j}} = -\frac{1}{r^2} \left( \frac{1}{\frac{3}{2} + \frac{c}{r^2}} \right)\alpha^2, \quad (5.30)
\]

\[
F_{ijij} = F_{\hat{i}\hat{j}\hat{i}\hat{j}} = -\frac{1}{r^2} \left( \frac{3}{\frac{1}{2} + \frac{c}{r^2}} + \frac{1}{(-\frac{1}{2} + \frac{c}{r^2})^2\alpha^2} \right). \quad (5.31)
\]

It is known that there are no Yang-Mills instantons with finite action on the \( n \)-dimensional flat space (or sphere) unless \( n = 4 \). Unfortunately, the action for our solution diverges too. However the second Chern class evaluated on the fiber \( R^4 \) has a finite value:

\[
c_2 = \frac{1}{\text{vol}_{SU(2)}} \int_{R^4} Tr \mathcal{F} \wedge \mathcal{F} = -\frac{4}{9} \text{vol}_{SU(2)} \int_0^\infty \frac{d}{dr} \left( \frac{1}{3\alpha^6}(-\frac{1}{2} + \frac{c}{r^2})^{-3} + \frac{3}{2\alpha^4}(-\frac{1}{2} + \frac{c}{r^2})^{-2} \right) dr = -\frac{40}{27} \text{vol}_{SU(2)}, \quad (5.32)
\]
where \( Tr \) refers to the adjoint representation of \( SU(2) \) and \( \text{vol}_{SU(2)} \) is the volume of \( SU(2) \). Note that the value comes from the infinite surface according to \( \frac{1}{\alpha} = 0 \) at \( r = a \). The meaning of the finite second Chern class remains an open problem here.

Now we proceed to case (iii). We first note that the two points on the \((X, Y)\)-plane, \( L = (-1, \sqrt{3}) \), \( M = (-1, -\sqrt{3}) \), are fixed points, i.e., the stationary solutions of Eqs. (5.24) and (5.25). We calculate other solutions numerically and show the flows of appropriate numerical solutions in Figure 1.

![Figure 1: Flows of the numerical solutions in the case of (iii). The arrows show the growing direction of \( r \).](image)

We can see general features of the solutions if we rewrite Eqs. (5.24) and (5.25) as

\[
\frac{dX}{ds} = - \frac{\partial W_\alpha}{\partial X}, \quad \frac{dY}{ds} = - \frac{\partial W_\alpha}{\partial Y},
\]

\[
W_\alpha = - \left( \frac{1 - 9\alpha^2}{4} (X + 1)^2 + \alpha(3 - Y^2)X + \frac{\alpha}{3}(X + 1)^3 - \alpha Y^2 \right).
\]

Note that the external field \( \alpha \) coming from the background metric is included in the equations. If we consider the asymptotic region (large radial coordinate region), we can regard Eqs. (5.33) and (5.34) as gradient flow equations since \( \alpha \) approaches a constant. Then two fixed points, \( O = (0, 0), N = (2, 0) \), appear, adding to the points \( L \) and \( M \) described above. By evaluating the \( 2 \times 2 \) Hessian matrix \( H \)

\[
H_{ij} = \frac{\partial^2 W_1}{\partial X^i \partial X^j} \quad (X^1 = X, X^2 = Y)
\]

at the four fixed points, we can see that \( O \) is a stable point and \( L, M, N \) are saddle points (the Morse index is 1). This analysis provides a qualitative explanation for the behavior of the flows in Figure 1.
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