Study of Michel parameters in leptonic $\tau$ decays at Belle

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Abstract

We present a study of Michel parameters in leptonic $\tau$ decays using experimental information collected at the Belle detector. Michel parameters are extracted in the unbinned maximum likelihood fit of the $(\tau^\pm \rightarrow \ell^\pm \nu \nu, \tau^\pm \rightarrow \pi^\pm \pi^0 \nu)$ events in the full nine-dimensional phase space. We exploit the spin-spin correlation of tau leptons to extract $\xi\rho\xi$ and $\xi\rho\xi\delta$ in addition to the $\rho$ and $\eta$ parameters.

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INTRODUCTION

In the Standard Model (SM), the charged weak interaction is described by the exchange of a $W^\pm$ boson with a pure vector coupling to only left-chirality fermions. Therefore, in the low-energy four-fermion framework, the Lorentz structure of the matrix element is predicted to be of the “V-A⊗V-A” type. Deviations from this behavior would indicate new physics and might be caused either by changes in the $W$-boson couplings or through interactions mediated by new gauge bosons. Leptonic decays such as $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau$ ($\ell = e, \mu$) (unless specified otherwise, charge-conjugated decays are implied throughout the paper) are the only ones in which the electroweak couplings can be probed without disturbance from the strong interaction. This makes them an ideal system to study the Lorentz structure of the charged weak current.

The most general, Lorentz invariant, derivative-free and lepton-number-conserving four-lepton point interaction matrix element for this decay can be written as [1]:

$$\mathcal{M} = \frac{4G}{\sqrt{2}} \sum_{N=S,V,T} \sum_{i,j=L,R} g_{ij}^N \left[ \bar{u}_i(l^-) \Gamma_N \nu_n(\bar{v}_l) \right] \left[ \bar{u}_m(\nu_{\tau}) \Gamma_N u_j(\tau^-) \right],$$

\(1\)

where $\Gamma_N$ are normalized as:

$$\Gamma^S = 1, \quad \Gamma^V = \gamma^\mu, \quad \Gamma^T = \frac{1}{\sqrt{2}} \sigma^{\mu\nu} \equiv \frac{i}{2\sqrt{2}}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

\(2\)

The $\Gamma_N$ matrices define the properties of the two currents under a Lorentz transformation with $N = S, V, T$ for scalar, vector and tensor interactions, respectively. The indices $i$ and $j$ label the right- or left-handedness (R, L) of the charged leptons. For a given $i, j$ and $N$, the handedness of the neutrinos $(n, m)$ is fixed. Ten non-trivial terms are characterized by ten complex coupling constants $g_{ij}^N$, those with $g_{RR}^T$ and $g_{LL}^T$ are identically zero. In the SM, the only non-zero coupling constant is $g_{LL}^T = 1$. As the couplings can be complex, with arbitrary overall phase, there are 19 independent parameters. The total strength of the weak interaction (charged weak current sector) is determined by the Fermi constant $G$, hence, the $g_{ij}^N$ are normalized as:

$$3 \left( |g_{LR}^T|^2 + |g_{RL}^T|^2 \right) + \left( |g_{LL}^S|^2 + |g_{LR}^S|^2 + |g_{RL}^S|^2 + |g_{RR}^S|^2 \right) +$$

$$+ \frac{1}{4} \left( |g_{LL}^L|^2 + |g_{LR}^L|^2 + |g_{RL}^L|^2 + |g_{RR}^L|^2 \right) \equiv 1$$

\(3\)

This constrains the coupling constants to be $|g_{ij}^S| \leq 2$, $|g_{ij}^V| \leq 1$ and $|g_{ij}^T| \leq 1/\sqrt{3}$.

In the case where neutrinos are not detected and the spin of the outgoing charged lepton is not determined, only four Michel parameters (MP) $\rho, \eta, \xi$ and $\delta$ are experimentally accessible. They are bilinear combinations of the $g_{ij}^N$ coupling constants [2]:

$$\rho = \frac{3}{4} - \frac{3}{4} \left( \frac{|g_{LR}^V|^2 + |g_{RL}^V|^2 + 2|g_{RL}^T|^2 + 2|g_{LR}^T|^2 + 8\Re(g_{LR}^S g_{LR}^T + g_{RL}^S g_{RL}^T)}{g_{LR}^S g_{LR}^T + g_{RL}^S g_{RL}^T} \right)$$

\(4\)

$$\eta = \frac{1}{2} \Re \left( 6g_{RL}^T g_{LR}^* + 6g_{LR}^T g_{RL}^* + 2g_{RR}^S g_{LL}^* + 2g_{LL}^S g_{RR}^* + 2g_{LR}^S g_{RL}^* + 3g_{RR}^S g_{RR}^* \right)$$

\(5\)

$$\xi = 4\Re(g_{LR}^S g_{LR}^T) - 4\Re(g_{RL}^S g_{RL}^T) + |g_{LL}^S|^2 + 3|g_{LR}^S|^2 - 3|g_{RL}^S|^2 - |g_{RR}^S|^2$$
\[
+ 5|g_{LR}|^2 - 5|g_{RL}|^2 + \frac{1}{4}|g_{LL}|^2 - \frac{1}{4}|g_{LR}|^2 + \frac{1}{4}|g_{RL}|^2 - \frac{1}{4}|g_{RR}|^2
\]

\[
\xi \delta = \frac{3}{16} |g_{LL}|^2 - \frac{3}{16} |g_{LR}|^2 + \frac{3}{16} |g_{RL}|^2 - \frac{3}{16} |g_{RR}|^2 - \frac{3}{4} |g_{LR}|^2 + \frac{3}{4} |g_{RL}|^2
\]

\[
+ \frac{3}{4} |g_{LL}|^2 - \frac{3}{4} |g_{LR}|^2 + \frac{3}{4} \Re(g_{LR}^* g_{LL})^* - \frac{3}{4} \Re(g_{RL}^* g_{RL})^*
\]

and appear in the predicted energy spectrum of the charged lepton.

In the \( \tau \) rest frame, neglecting radiative corrections, this spectrum is given by [3]:

\[
\frac{d\Gamma(\tau^\mp)}{d\Omega dx} = \frac{4G^2 M_\tau E_\max^4}{(2\pi)^4} \sqrt{x^2 - x_0^2}\left(x(1 - x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1 - x)\right)
\]

\[
\mp \frac{1}{3} P_\tau \cos \theta_\ell \xi \sqrt{x^2 - x_0}\left[1 - x + \frac{2}{3}\delta(4x - 4 + \sqrt{1 - x_0^2})\right),
\]

\[
x = \frac{E_\ell}{E_\max}, \quad E_\max = \frac{M_\tau}{2}(1 + \frac{m_\ell^2}{M_\tau^2}), \quad x_0 = \frac{m_\ell}{E_\max},
\]

where \( P_\tau \) is \( \tau \) polarization, and \( \theta_\ell \) is the angle between the \( \tau \) spin and the lepton momentum.

In the SM, the “V-A” charged weak current is characterized by \( \rho = 3/4, \eta = 0, \xi = 1 \) and \( \delta = 3/4 \).

**METHOD**

Measurement of \( \xi \) and \( \delta \) requires knowledge of the \( \tau \) spin direction. In experiments at \( e^+e^- \) colliders with unpolarized \( e^\pm \) beams, the average polarization of a single \( \tau \) is zero. However, spin-spin correlations between the \( \tau^+ \) and \( \tau^- \) produced in the reaction \( e^+e^- \to \tau^+\tau^- \) can be exploited [4]. The main idea of our method is to consider events where both taus decay to selected final states. One (signal) tau decays leptonically \( (\tau^- \to \ell^- \nu_\tau \bar{\nu}_\ell, \ell = e, \mu) \) while the opposite tau, which decays via \( \tau^+ \to \pi^+\pi^0\bar{\nu}_\tau \), serves as a spin analyser. We choose the \( \tau^+ \to \pi^+\pi^0\bar{\nu}_\tau \) decay mode because it has the largest branching fraction as well as properly studied dynamics [3]. To write the total differential cross section for \( (\tau^- \to \ell^- \nu_\tau \bar{\nu}_\ell, \tau^+ \to \pi^+\pi^0\bar{\nu}_\tau) \) (or, briefly, \( \ell - \rho \) events, we follow the approach developed in Refs. [6][8]. The differential cross section of the \( e^+e^- \to \tau^+\tau^- (\zeta^+\zeta^-) \) reaction in the center-of-mass system (c.m.s.) is given by their formula [4]:

\[
\frac{d\sigma(\zeta^+, \zeta^-)}{d\Omega} = \frac{\alpha^2}{64E_\tau^2} \beta_\tau (D_0 + D_{ij} \zeta_i^+ \zeta_j^-),
\]

where \( D_0 = 1 + \cos^2 \theta + \frac{1}{1 + \sin^2 \theta} \), \( D_{ij} \) is the spin-spin correlation tensor, and \( \zeta^+ \) is the polarization vector of the \( \tau^+ \) in the \( \tau^+ \) rest frame (unit vector along the \( \tau^+ \) spin direction). The asterisk denotes a parameter measured in the associated \( \tau \) rest frame. The differential decay width of the signal is written in the form (with the total normalization constant \( \kappa_\ell \) that is unimportant in this context):

\[
\frac{d\Gamma(\tau^+(\zeta^+) \to \ell^+\nu\bar{\nu})}{dx^*d\Omega_\ell^*} = \kappa_\ell(A(x^*) \mp \xi \bar{n}_\ell \zeta^{*+}B(x^*)],
\]
\[ A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*), \quad B(x^*) = B_1(x^*) + \delta B_2(x^*), \]  
where the form factors \( A_0, A_1, A_2, B_1 \) and \( B_2 \) can be extracted from Eq. [8]. The \( \tau^+ (\bar{\tau}^*) \rightarrow \rho^+ (K^*) \nu(q^*) \rightarrow \pi^+(p_1^*) \pi^0 (p_2^*) \nu(q^*) \) decay width reads (with the total normalization constant \( \kappa_\rho \)):  
\[
\frac{d\Gamma(\tau^+ \rightarrow \pi^+ \pi^0 \nu)}{dm_{\pi^+}^2 d\Omega_\rho d\Omega_\pi} = \kappa_\rho A'(1 + B\bar{\tau}^*) W(m_{\pi^+}^2),
\]
\[
A' = 2(q, Q)Q_0^* - Q^2 q_0^*, \quad B = Q^2 K^2 - 2(q, Q)\bar{Q}^*, \quad Q^* = p_1^* - p_2^*, \quad K^* = p_1^* + p_2^*,
\]
\[
W(m_{\pi^+}^2) = |F_\pi(m_{\pi^+}^2)|^2 \frac{2p^{*\pi}_{\rho}(m_{\pi^+}^2)\bar{p}_\pi(m_{\pi^+}^2)}{M_\pi m_{\pi^+}}, \quad m_{\pi^+}^2 = K^{*2}, \quad p^{*\pi}_\rho = M_\pi \left(1 - \frac{m_{\pi^+}^2}{M_\pi^2}\right),
\]
\[
\bar{p}_\pi = \sqrt{(m_{\pi^+}^2 - (m_\pi + m_{n_0})^2)(m_{\pi^+}^2 - (m_\pi - m_{n_0})^2)} / 2m_{\pi^+},
\]
where \( p_\ell^* \) and \( \Omega_\rho^* \) are the momentum and solid angle of the \( \rho \) meson in the \( \tau \) rest frame; \( \bar{p}_\pi \) and \( \Omega_\pi^0 \) are the momentum and solid angle of the charged pion in the \( \rho \) rest frame; and \( F_\pi(m_{\pi^+}^2) \) is the pion form factor taken from Ref. [5]. The total differential cross section for \( \ell - \rho \) events is:
\[
\frac{d\sigma(\ell^+, \rho^+)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi^+}^2 d\Omega_\pi} = \kappa_\ell \kappa_\rho \frac{\alpha^2 \beta_\tau}{64 E_\ell^2} \left(D_0 A' + \xi_\rho \xi D_{ij} n_i^* B_j^* B\bar{\tau}^* W(m_{\pi^+}^2)\right),
\]
Experimentally, we measure particle parameters in the c.m.s.; hence, the visible differential cross section is given by [7]:
\[
\frac{d\sigma(\ell^+, \rho^+)}{dp_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi^+}^2 d\Omega_\pi} = \int_{\Phi_1}^{\Phi_2} \left( \frac{d\sigma(\ell^+, \rho^+)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi^+}^2 d\Omega_\pi} \right) \frac{\partial(E_\ell^*, \Omega_\ell^*, \Omega_\rho^*, \Omega_\pi)}{\partial(p_\ell^*, \Omega_\ell^*, \Omega_\rho^*, \Omega_\pi)} \, dp_\ell^* \, d\Omega_\ell^* \, d\Omega_\rho^* \, d\Omega_\pi \, d\Omega_\pi,
\]
where the integration is performed over the unknown \( \tau \) direction, which is constrained by the \( (\Phi_1, \Phi_2) \) arc. Both \( \Phi_1 \) and \( \Phi_2 \) are calculated using parameters measured in the experiment. The differential cross section is used to construct the probability density function (p.d.f.) for the measurement vector \( z = (p_\ell^*, \cos \theta_\ell^*, \phi_\ell^*, \cos \theta_\rho^*, \phi_\rho, m_{\pi^+}, \cos \theta_\pi, \bar{\phi}_\pi) \):
\[
P(z) = \frac{F(z)}{\int F(z) \, dz}, \quad F(z) = \frac{d\sigma(\ell^+, \rho^+)}{dp_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi^+}^2 d\Omega_\pi} = F_0 + F_1 \rho + F_2 \eta + F_3 \xi_\rho \xi + F_4 \xi_\pi \xi_\delta, \]
\[
N = \int F(z) \, dz = N_0 + N_1 \rho + N_2 \eta + N_3 \xi_\rho \xi + N_4 \xi_\pi \xi_\delta, \quad N_i = \int F_i(z) \, dz, \quad i = 0...4,
\]
\[
P(z) = \frac{F_0(z) + F_1(z) \rho + F_2(z) \eta + F_3(z) \xi_\rho \xi + F_4(z) \xi_\pi \xi_\delta}{N_0 + N_1 \rho + N_2 \eta + N_3 \xi_\rho \xi + N_4 \xi_\pi \xi_\delta},
\]
where the form factors \( F_i \) are calculated for each event and the five normalisation constants \( N_i \) are evaluated using a Monte Carlo (MC) simulated sample. There are several corrections that must be incorporated in the procedure to take into account the real experimental situation. Physics corrections include electroweak higher-order corrections to the \( e^+ e^- \rightarrow \tau^+ \tau^- \) cross section [3,15], the effect of the radiative leptonic decay \( \tau^- \rightarrow \ell^- \nu_\ell \gamma \) [16,18], and the effect of the radiative hadronic decay \( \tau^- \rightarrow \pi^- \pi^0 \nu_\tau \gamma \) [19,20]. Apparatus corrections
include the effect of the finite detection efficiency and resolution, the effect of the external bremsstrahlung for $e^-\rho$ events, and the $e^\pm$ beam energy spread.

The method described is used for a precise measurement of Michel parameters in $\ell^-\rho$ events. This analysis is based on a 485 fb$^{-1}$ data sample that contains 446 $\times 10^6 \tau^+\tau^-$ pairs, collected with the Belle detector at the KEKB energy-asymmetric $e^+e^-$ (3.5 on 8 GeV) collider [21] operating at the $\Upsilon(4S)$ resonance.

THE BELLE DETECTOR

The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter (ECL) comprised of CsI(Tl) crystals located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside the coil is instrumented to detect $K^0_L$ mesons and to identify muons (KLM). Two inner detector configurations are used in this analysis. A beampipe with a radius of 2.0 cm and a 3-layer silicon vertex detector are used for the first sample of $124 \times 10^6 \tau^+\tau^-$ pairs, while a 1.5 cm beampipe, a 4-layer silicon detector and a small-cell inner drift chamber are used to record the remaining $322 \times 10^6 \tau^+\tau^-$ pairs [22]. The detector is described in detail elsewhere [23].

SELECTION OF $\ell^-\rho$ EVENTS, BACKGROUND

This analysis is based on events with one $\tau$ decaying to leptons $\tau^- \rightarrow \ell^-\bar{\nu}_\ell\nu_\tau$ and the other decaying via the hadronic channel $\tau^+ \rightarrow \pi^+\pi^0\bar{\nu}_\tau$.

The selection process, which is designed to suppress background while retaining a high efficiency for the decays under study, proceeds in two stages.

1) The first-stage criteria suppress beam background to a negligible level and reject most of the background from other physical processes:

- There should be exactly two tracks extrapolated to the interaction point within $\pm 0.5$ cm in the transverse direction and $\pm 2.5$ cm along the beam and having a transverse momentum in the c.m.s. $|\vec{P}_{\perp}^{CM}| > 0.1$ GeV/$c$ and a net charge of zero.
- The sum of the track absolute momenta in the c.m.s. must satisfy $P^{CM} < 9$ GeV/$c$.
- The maximum value of the transverse momentum for all tracks in the laboratory frame should satisfy $|\vec{P}_{\perp}^{LAB}| > 0.5$ GeV/$c$.
- The maximum opening angle $\psi$ between tracks should exceed 20°.
- The number of photons $N_\gamma$ with c.m.s. energy $E^{CM}_\gamma > 80$ MeV should be five or fewer.
- The total ECL energy deposition in the laboratory frame should satisfy $\sum_{i=1}^{N_{clusters}} E^{LAB}_i$(ECL) < 9 GeV.
- The total energy of additional photons in the laboratory frame should be $\sum E^{LAB}_{rest,\gamma} < 0.2$ GeV.
functions is evaluated from two variables: the difference between the range calculated from the extrapolated track \[25\].

$dE/dx$ likelihoods from the ACC response, the flight-time measurement, and form a likelihood ratio. an 88% efficiency for electrons is 93% while keeping the pion fake rate at the 6% level. The detection efficiencies for signal events are 93% while keeping the pion fake rate at the 6% level.

To separate pions from kaons, we determine for each track the pion \((L_\pi)\) and kaon \((L_K)\) likelihoods from the ACC response, the \(dE/dx\) measurement in the CDC, the transverse ECL shower shape, the light yield in the ACC [24]. The efficiency of this cut for electrons is 93.1%.

To select muons, the likelihood ratio cut \(P_\mu = L_\mu/(L_\mu + L_\pi + L_K) > 0.8\) is applied. It has an 88.0% efficiency for muons. Each of the muon \((L_\mu)\), pion \((L_\pi)\) and kaon \((L_K)\) likelihood functions is evaluated from two variables: the difference between the range calculated from the momentum of the particle and the range measured by KLM and the \(\chi^2\) of the KLM hits with respect to the extrapolated track [25].

To separate pions from kaons, we determine for each track the pion \((L_\pi)\) and kaon \((L_K)\) likelihoods from the ACC response, the \(dE/dx\) measurement in the CDC and the TOF flight-time measurement, and form a likelihood ratio \(P_{K/\pi} = L_K/(L_\pi + L_K)\) to separate pions and kaons. For pions, we require \(P_{K/\pi} < 0.6\), which provides a pion identification efficiency of about 98% while keeping the pion fake rate at the 6% level.

Finally, we select events with only one lepton \(\ell^\pm (\ell = e, \mu)\), one charged pion \(\pi^\pm\) and one \(\pi^0\) candidate. A \(\pi^0\) meson is reconstructed from a pair of gammas with an energy in the laboratory frame of \(E_{\gamma}^{LAB} > 80\) MeV and the \(\gamma\gamma\) invariant mass in the range 115 MeV/c\(^2\) < \(M_{\gamma\gamma}\) < 150 MeV/c\(^2\). The absolute value of the \(\pi^0\) momentum in the c.m.s. must satisfy \(P_{\pi^0}^{CMS} > 0.3\) GeV/c. The invariant mass of the \(\pi^\pm\pi^0\) system must lie in the range 0.3 GeV/c\(^2\) < \(M_{\pi^\pm\pi^0}\) < 1.8 GeV/c\(^2\). The opening angles between a lepton and charged pion and between a lepton and \(\pi^0\) should exceed 90°. To avoid the uncertainty due to the simulation of low energy fake ECL clusters, we allow additional photons in an event with the total energy in the laboratory frame of \(E_{\gamma}^{LAB} < 0.2\) GeV.

To evaluate the background and calculate efficiencies, a Monte Carlo sample of 2.87 × 10\(^9\) \(\tau^+\tau^-\) pairs is produced with the KKMC/TAUOLA generators [26, 27]. The detector response is simulated by a GEANT3-based program [28].

The detection efficiencies for signal events are \(\varepsilon_{det}(e - \rho) = (11.53 \pm 0.01)\%\) and \(\varepsilon_{det}(\mu - \rho) = (12.43\pm0.01)\%\). It is found that the dominant background comes from other \(\tau\) decays; a contribution from non-\(\tau\tau\) processes is very small – less than 0.1%. The dominant background arises from \((\tau^- \rightarrow \ell^- \nu_\ell \nu_\tau, \tau^+ \rightarrow \pi^+ \pi^0 \nu_\tau)\) (or, briefly, \(\ell - 3\pi\)) events, where the second \(\pi^0\) is lost. Its contribution is \(\lambda_{3\pi} = 10.0\%\) for the \(e - \rho\) and \(\lambda_{3\pi} = 8.1\%\) for the \(\mu - \rho\) events. For the \(\mu - \rho\) events, an additional background at the level of \(\lambda_\pi = 1.4\%\) originates from \((\tau^- \rightarrow \pi^- \nu_\tau, \tau^+ \rightarrow \pi^+ \pi^0 \nu_\tau)\) (or, briefly, \(\pi - \rho\)) events, where a pion is misidentified as a muon. The remaining background comes from other \(\tau\) decays; its contribution is \(\lambda_{other} = 2.0\%\) for \(e - \rho\) events and \(\lambda_{other} = 2.5\%\) for \(\mu - \rho\) events.

The main background processes, \(\ell - 3\pi\) and \(\pi - \rho\), are included in the p.d.f. analytically while the remaining background is taken into account using the MC-based approach [29].
The total p.d.f. is written as:

\[
P(\vec{z}) = \frac{\varepsilon(\vec{z})}{\varepsilon} \left( (1 - \lambda_{3\pi} - \lambda_\pi - \lambda_{\text{other}}) \frac{S(\vec{z})}{\int \varepsilon(\vec{z}) S(\vec{z}) d\vec{z}} + \lambda_{3\pi} \frac{\tilde{B}_{3\pi}(\vec{z})}{\int \varepsilon(\vec{z}) \tilde{B}_{3\pi}(\vec{z}) d\vec{z}} + \lambda_\pi \frac{\tilde{B}_{\pi}(\vec{z})}{\int \varepsilon(\vec{z}) \tilde{B}_{\pi}(\vec{z}) d\vec{z}} + \lambda_{\text{other}} \frac{B^{\text{MC}}_{\text{other}}(\vec{z})}{\int \varepsilon(\vec{z}) B^{\text{MC}}_{\text{other}}(\vec{z}) d\vec{z}} \right),
\]  

where \( S(\vec{z}) \), \( \tilde{B}_{3\pi}(\vec{z}) \) and \( \tilde{B}_{\pi}(\vec{z}) \) are the cross sections for the \( \ell - \rho \), \( \ell - 3\pi \) and \( \pi - \rho \) events, respectively; \( \varepsilon(\vec{z}) \) is the detection efficiency for signal events in the nine-dimensional phase space; and \( \varepsilon = \int \varepsilon(\vec{z}) S(\vec{z}) d\vec{z}/\int S(\vec{z}) d\vec{z} \) is the average signal detection efficiency.

**ANALYSIS OF EXPERIMENTAL DATA**

After all selections, about 5.5 million events in all four configurations \( ((e^+, \pi^-\pi^0), (e^-, \pi^+\pi^0), (\mu^+, \pi^-\pi^0), (\mu^-, \pi^+\pi^0)) \) are selected for the fit. Figure 1 shows the distributions of the selected \( (e^+, \pi^-\pi^0) \) events: \( e^+ \) momentum (upper left) and polar angle (upper right) in the c.m.s., \( \pi^-\pi^0 \) invariant mass (lower left), extra gamma energy in laboratory frame (lower right). Open histograms - signal MC simulation, yellow shaded histograms - the main background components from the \( (e^+, \pi^-\pi^0) \) events, green shaded histograms - the remaining background, points with errors - experimental data. Blue arrows show applied selections. MC and experimental histograms are normalized to the same number of events.
butions of selected kinematical parameters for \((e^+, \pi^0)\) events. Clearly, the experimental electron momentum spectrum is shifted markedly to higher momenta in comparison with the MC one. This is an artifact of the strong nonuniformity of the experimental trigger efficiency, which is not properly simulated. A special procedure has been developed to evaluate the trigger efficiency corrections, \(e_{\text{corr}}^{\text{TRG}} = \varepsilon_{\text{EXP}} / \varepsilon_{\text{MC}}\); see Fig. 2. The trigger efficiency

correction as well as the lepton identification efficiency correction, \(e_{\text{ID}}^{\text{corr}}\), are incorporated in the fitter by modifying the detection efficiency in Eq. 16: \(\varepsilon(z) \rightarrow \varepsilon(z) e_{\text{corr}}^{\text{TRG}}(p^\ell_{\text{LAB}}) e_{\text{corr}}^{\text{ID}}(p^\rho_{\text{LAB}})\).

The result of the fit of the \((e^+, \rho^-)\) experimental data is illustrated in Fig. 3. Reasonable agreement can be observed for the whole energy range, although the relative difference between these spectra indicates a remaining systematic effect of about a few percent. The distribution of the \(\tau\) helicity sensitive variable \(\omega\) is also shown in Fig. 3. A spin-spin correlation of tau leptons is clearly demonstrated in Fig. 4 for \((e^+, \rho^-)\); the \(e^+\) energy spectrum shape changes notably as \(\omega\) varies from \(-1\) to \(+1\).

It is confirmed that the uncertainties arising from the physical and apparatus corrections to the p.d.f. are well below 1\%; see Table II. The statistical uncertainties of the normalisation coefficients are kept as small as possible. The contribution to the systematic uncertainties of the Michel parameters due to the finite accuracy of the normalisation coefficients shown in Table II are evaluated with the entire available generic \(\tau^+\tau^-\) MC sample; they provide the dominant contributions. We observe a correlation of about 92\% between the \(\rho\) and \(\eta\) parameters. The slope of the corresponding error ellipse exhibits an approximate dependence of \(\Delta \eta \approx 4 \Delta \rho\), which is incorporated as an inflated uncertainty of the \(\eta\) parameter in Table II.

However, we still observe a systematic bias of the order of a few percent, especially in the \(\xi_{\ell\xi}\) and \(\xi_{\rho\xi\delta}\) Michel parameters. This bias originates from the remaining in accuracies in the description of the \(\ell - 3\pi\) background.
SUMMARY

We present a study of Michel parameters in leptonic \( \tau \) decays using a 485 fb\(^{-1} \) data sample collected at Belle. Michel parameters are extracted in the unbinned maximum likelihood fit of the \( \ell - \rho \) events in the full nine-dimensional phase space. We exploit the spin-spin correlation of tau leptons to extract \( \xi_\rho \xi_\xi \) and \( \xi_\rho \xi_\delta \) in addition to the \( \rho \) and \( \eta \) Michel parameters. Although systematic uncertainties coming from the physical and apparatus corrections as well as from the normalisation are below 1\%, currently we still have a relatively large systematic bias in the \( \xi_\rho \xi_\xi \) and \( \xi_\rho \xi_\delta \) parameters, which originates from the inaccurate description of the dominant \( \ell - 3\pi \) background.

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FIG. 4: Result of the fit of \((e^+, \rho^-)\) experimental events. Three \(e^+\) energy spectra are shown for different ranges in \(\omega\): \(\omega < -0.35\) (left), \(-0.35 < \omega < 0.35\) (middle), \(\omega > 0.35\) (right). Points with errors show experimental data, histogram - result of the fit. Open histograms show signal events, shaded histograms - background contributions.

TABLE I: Systematic uncertainties of Michel parameters related to physical and apparatus corrections, and accuracy of the normalisation coefficients \(N_i\). Values are shown in units of percent (i.e. absolute deviation of the Michel parameter is multiplied by 100%).

| Source         | \(\sigma(\rho), \%\) | \(\sigma(\eta), \%\) | \(\sigma(\xi_\rho\xi), \%\) | \(\sigma(\xi_\rho\xi\delta), \%\) |
|----------------|------------------------|------------------------|-----------------------------|-----------------------------|
| Physical corrections |                        |                        |                             |                             |
| ISR+\(O(\alpha^3)\) | 0.10                   | 0.30                   | 0.20                        | 0.15                        |
| \(\tau \to \ell \nu \nu' \gamma\) | 0.03                   | 0.10                   | 0.09                        | 0.08                        |
| \(\tau \to \rho \nu \gamma\) | 0.06                   | 0.16                   | 0.11                        | 0.02                        |
| Apparatus corrections |                        |                        |                             |                             |
| Resolution + brems. | 0.10                   | 0.33                   | 0.11                        | 0.19                        |
| \(\sigma(E_{beam})\) | 0.07                   | 0.25                   | 0.03                        | 0.15                        |
| Normalisation          |                        |                        |                             |                             |
| \(\Delta N_1\) | 0.21                   | 0.60                   | 0.14                        | 0.12                        |
| \(\Delta N_3\) | < 0.01                 | < 0.01                 | 0.35                        | 0.03                        |
| \(\Delta N_4\) | 0.02                   | 0.03                   | 0.05                        | 0.23                        |
| Total                  | 0.27                   | 0.81                   | 0.47                        | 0.40                        |

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