Meson Mixing in Pion Superfluid

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We investigate meson mixing and meson coupling constants in pion superfluid in the framework of two flavor NJL model at finite isospin density. The mixing strength develops fast with increasing isospin chemical potential, and the coupling constants in normal phase and in the pion superfluid phase behave very differently.

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I. INTRODUCTION

Recently, the study on the phase diagram of Quantum Chromodynamics (QCD) is extended from finite temperature and baryon chemical potential to finite isospin chemical potential. The physical motivation to study pion superfluid formed at high isospin density is related to the investigation of neutron stars, isospin asymmetric nuclear matter and heavy ion collisions at intermediate energy.

Different approaches, such as chiral perturbation theory\[1\],\[2\],\[3\],\[4\], Nambu-Jona-Lasinio (NJL) model\[5\],\[6\],\[7\],\[8\],\[9\],\[10\],\[11\], random matrix method\[12\], lattice QCD\[13\], ladder QCD\[14\] and strong coupling lattice QCD\[15\], have been used to investigate the QCD phase structure at finite isospin density. In this section we introduce the NJL model in mean field approximation of this model.

The Letter is organized as follows. In Section II we review the NJL model in mean field approximation for quarks and random phase approximation (RPA) for mesons in the normal phase at finite isospin chemical potential. In Section III we focus on the meson mixing and meson couplings to quarks in the pion superfluid. We summarize in Section IV.

II. MESONS IN NORMAL PHASE

The mesons in the normal phase at finite isospin chemical potential are controlled by chiral dynamics and explicit isospin symmetry breaking. In this section we review the quark propagator in mean field approximation and meson polarizations in RPA in the NJL model at finite temperature and baryon and isospin chemical potentials. The Lagrangian density of the two flavor NJL model at quark level is defined as

\[ \mathcal{L} = \bar{\psi} (i \gamma ^\mu \partial _\mu - m_0 + \mu \gamma _5) \psi + G \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma _5 \tau _i \psi)^2 \right] \]

with scalar and pseudoscalar interactions corresponding to \( \sigma \) and \( \pi \) excitations, where \( m_0 \) is the current quark mass, \( G \) is the four-quark coupling constant with dimension \( (\text{GeV}^2)^{-2} \), \( \tau _i \) \((i = 1, 2, 3)\) are the Pauli matrices in flavor space, and \( \mu = \text{diag}(\mu_u, \mu_d) = \text{diag}(\mu_B/3 + \mu_0/2, \mu_B/3 - \mu_0/2) \) is the quark chemical potential matrix with \( \mu_u \) and \( \mu_d \) being the \( u \)- and \( d \)-quark chemical potentials and \( \mu_B \) and \( \mu_0 \) being the baryon and isospin chemical potentials.

At zero isospin potential, the Lagrangian density has the symmetry of \( U_B(1) \otimes SU_l(2) \otimes SU_A(2) \) corresponding to baryon number symmetry, isospin sym-
metry and chiral symmetry. At finite isospin chemical potential, the isospin symmetry $SU_f(2)$ and chiral symmetry $SU_A(2)$ are explicitly broken to $U_f(1)$ and $U_A(1)$, respectively. Therefore, the chiral symmetry restoration at finite isospin chemical potential means only degeneracy of $\sigma$ and $\pi_0$ mesons, the charged $\pi_+$ and $\pi_-$ behave still differently.

Introducing the chiral and pion condensates

$$\sigma = \langle \bar{\psi} \psi \rangle = \sigma_u + \sigma_d,$$
$$\sigma_u = \langle \bar{u} u \rangle, \quad \sigma_d = \langle \bar{d} d \rangle,$$
$$\pi = \sqrt{2} \langle \bar{\psi} i \gamma_5 \tau_+ \psi \rangle = \sqrt{2} \langle \bar{\psi} i \gamma_5 \tau_- \psi \rangle$$  \hspace{1cm} (2)

with $\tau_\pm = (\tau_1 \pm i \tau_2)/\sqrt{2}$ and taking all the condensates to be real, the quark propagator in mean field approximation can be expressed as a matrix in the flavor space

$$S(p) = \begin{pmatrix} S_{uu}(p) & S_{ud}(p) \\ S_{du}(p) & S_{dd}(p) \end{pmatrix}$$  \hspace{1cm} (3)

with the four elements

$$S_{uu} = \frac{(p_0 + E + \mu_d) \Lambda_+ + \gamma_0}{(p_0 + E + \mu_d) \Lambda_+ + \gamma_0},$$
$$S_{dd} = \frac{(p_0 - E + \mu_u) \Lambda_- + \gamma_0}{(p_0 - E + \mu_u) \Lambda_- + \gamma_0},$$
$$S_{ud} = \frac{(p_0 - E - \mu_u) \Lambda_- + \gamma_0}{(p_0 - E - \mu_u) \Lambda_- + \gamma_0},$$
$$S_{du} = \frac{2i G \pi \Lambda_+ + \gamma_5}{(p_0 - E - \mu_u) \Lambda_- + \gamma_0}.$$  \hspace{1cm} (4)

where $E_{\pm} = E \mp \mu_B/3$ are the energies of four quasi-particles with $E = \sqrt{(E \pm \mu_I/2)^2 + 4G^2\pi^2}$, $E = \sqrt{|p|^2 + M_q^2}$ and effective quark mass $M_q = m_0 - 2 G \sigma$, and $\Lambda_\pm = [1 \pm \gamma_0 \cdot p + M_q]/E$ are the energy projectors. The quark propagator (3) is the background of our following calculations for quarks and mesons. From the definitions of the chiral and pion condensates (2), it is easy to express them in terms of the matrix elements of the quark propagator,

$$\sigma_u = - \sum_p Tr [i S_{uu}(p)],$$
$$\sigma_d = - \sum_p Tr [i S_{dd}(p)],$$
$$\pi = \sum_p Tr [(S_{ud}(p) + S_{du}(p)) \gamma_5].$$  \hspace{1cm} (5)

where the trace $Tr = Tr_c Tr_D$ is taken in color and Dirac spaces, and the four momentum integration is defined as $\sum_p = i T \sum_n \int d^3p/(2\pi)^3$ with $p_0 = i \omega_n = i(2n + 1)\pi T$ ($n = 0, \pm 1, \pm 2, \cdots$) at finite temperature $T$. Note that, for $\mu_B = 0$ or $\mu_I = 0$ there is always $\sigma_u = \sigma_d$, since in this case the chemical potential difference between $\bar{u}$ and $u$ is the same as that between $d$ and $d$. The coupled set of gap equations (5) determines self-consistently the three condensates. The isospin chemical potential dependence of the effective quark mass $M_q$ and pion condensate $\pi$ is explicitly shown in (5). The phase transition from the normal phase to the pion superfluid is of second order.

In the NJL model, the meson modes are regarded as quantum fluctuations above the mean field. The two quark scattering via meson exchange can be effectively expressed at quark level in terms of quark bubble summation in RPA [20]. The quark bubbles, namely the meson polarization functions are defined as [21]

$$\Pi_{nm}(k) = i \sum_p Tr (\Gamma_n S(p+k) \Gamma_m S(p))$$  \hspace{1cm} (6)

with the trace $Tr = Tr_c Tr_F Tr_D$ taken in color, flavor and Dirac spaces and the meson vertexes

$$\Gamma_m = \begin{cases} 1 & m = \sigma \\ i \tau_+ \gamma_5 & m = \pi_+ \\ i \tau_- \gamma_5 & m = \pi_- \\ i \gamma_5 & m = \pi_0 \end{cases}, \quad \Gamma_m^* = \begin{cases} 1 & m = \sigma \\ i \tau_- \gamma_5 & m = \pi_+ \\ i \tau_+ \gamma_5 & m = \pi_- \\ i \gamma_5 & m = \pi_0 \end{cases}.$$  \hspace{1cm} (7)

The explicit $T, \mu_B$ and $\mu_I$ dependence of the meson polarization functions [21] used in the following discussion can be found in Appendix B of [8].

At $\mu_I \leq \mu_I^c$ where $\mu_I^c$ is the critical isospin chemical potential for the pion condensate, the system is in the normal phase with diagonal quark propagator, and the bubble summation in the effective interaction in RPA selects its specific isospin channel by choosing at each stage the same proper polarization [20]. Therefore, the meson masses $M_m(m = \sigma, \pi_+, \pi_-, \pi_0)$ which are determined by poles of the meson propagators at $k_0 = M_m$ and $k = 0$ are related only to their own polarization functions [20],

$$1 - 2G \Pi_{mm}(k_0)|_{k_0 = M_m} = 0.$$  \hspace{1cm} (8)

From the comparison of these mass equations with the gap equation for the pion condensate $\pi$ at $T = \mu_B = 0$ but finite $\mu_I$, the critical isospin chemical potential where the normal phase ends and the pion superfluid starts is exactly the pion mass in the vacuum [8] [13], $\mu_I^c = m_\pi$, and the $\mu_I$ dependence of the meson masses is simple [8], $M_\sigma(\mu_I) = m_\sigma$, $M_{\pi_+}(\mu_I) = m_\pi$ and $M_{\pi_+}(\mu_I) = m_\pi \mp \mu_I$, where $m_\sigma$ and $m_\pi$ are the $\sigma$ and $\pi$ masses at $T = \mu_B = \mu_I = 0$. The isospin neutral mesons keep their vacuum masses, while the isospin charged mesons change their masses linearly. These relations hold until the pion condensate starts.

The meson couplings to quarks, $g_{mq\bar{q}}$, are related to the residues at the corresponding poles of the meson propagators [20],

$$g_{mq\bar{q}}^2 = \left[ \frac{\partial \Pi_{nm}(k_0)}{\partial k_0^2} \right]^{-1} |_{k_0 = M_m}.$$  \hspace{1cm} (9)
III. MESONS IN PION SUPERFLUID

In the phase with finite pion condensate which leads to spontaneous isospin symmetry breaking, the quark propagator contains off-diagonal elements, we must consider all possible isospin channels in the bubble summation in RPA [20]. While there is no mixing between \( \pi_0 \) and other mesons [8], \( \Pi_{\pi_0\sigma} (k) = \Pi_{\pi_0\pi_+} (k) = \Pi_{\pi_0\pi_-} (k) = 0 \), the other three mesons are coupled together, and the effective interaction in two quark scattering via exchanging these mesons in RPA becomes a summation in the meson space,

\[
U(k) = \Gamma^*_m \mathcal{M}_{mn}(k) \Gamma_n, \quad m, n = \sigma, \pi_+, \pi_-
\]  

(10)

with the meson matrix \( \mathcal{M}(k) \) defined by

\[
\mathcal{M}(k) = \frac{2G}{1 - 2G \Pi(k)} = \frac{2G}{D(k)} \mathcal{M}(k),
\]

(11)

where \( 1 - 2G \Pi(k) \) is the meson polarization matrix [8]

\[
1 - 2G \Pi(k) = \begin{pmatrix}
1 - 2G \Pi_{\sigma\sigma} & -2G \Pi_{\sigma\pi_+} & -2G \Pi_{\sigma\pi_-} \\
-2G \Pi_{\pi_+\sigma} & 1 - 2G \Pi_{\pi_+\pi_+} & -2G \Pi_{\pi_+\pi_-} \\
-2G \Pi_{\pi_-\sigma} & -2G \Pi_{\pi_-\pi_+} & 1 - 2G \Pi_{\pi_-\pi_-}
\end{pmatrix},
\]

(12)

\( D(k) \) is its determinant,

\[
D(k) = \det (1 - 2G \Pi(k)),
\]

(13)

and \( \mathcal{M}(k) \) is defined as \( \mathcal{M}(k) = D(k) / (1 - 2G \Pi(k)) \). In this case, \( \sigma, \pi_+ \) and \( \pi_- \) are no longer the eigenmodes of the Hamiltonian, the new eigenmodes are linear combinations of them. In the following we call these new eigenmodes in the pion superfluid phase as \( \sigma_0, \pi_+ \) and \( \pi_- \).

The \( \pi_0 \) mass and coupling constant are still controlled by its own polarization function at \( k = 0 \),

\[
1 - 2G \Pi_{\pi_0\pi_0}(k_0)|_{k_0 = M_{\pi_0}} = 0,
\]

(14)

\[
g^2_{\pi_0\pi_0} = \left[ \partial \Pi_{\pi_0\pi_0}(k_0) / \partial k_0^2 \right]^{-1} |_{k_0 = M_{\pi_0}},
\]

since it is independent of the other mesons. At \( T = \mu_B = 0 \), the \( \pi_0 \) mass is exactly equal to the isoscalar chemical potential [8], \( M_{\pi_0}(\mu_B) = \mu_I \). At the critical point \( \mu_I^* = m_\pi \), it is continuous with the solution \( M_{\pi_0} = m_\pi \) in the normal phase.

The masses of the new eigenmodes \( \sigma_0, \pi_+ \) and \( \pi_- \) are determined by the pole of the effective interaction at \( k = 0 \),

\[
D(k_0)|_{k_0 = M_\theta} = 0, \quad \theta = \sigma, \pi_+, \pi_-.
\]

(15)

It can be proven analytically that there is always a zero solution which guarantees the Goldstone mode, \( M_{\pi_0} = 0 \), corresponding to the spontaneous isospin symmetry breaking [8].

In order to derive the coupling constants for the new modes, we first expand the effective interaction \( U \) around the meson mass \( k_0^2 = M^2_\theta \) at \( k = 0 \),

\[
U(k) \simeq \frac{2G}{(dD(k_0)/dk_0^2)|_{k_0 = M_\theta} \frac{\Gamma^*_m \mathcal{M}_{mn}(M_\theta) \Gamma_n}{k_0^2 - M^2_\theta}},
\]

(16)

and then make a transformation from the coupled meson space \( (\sigma, \pi_+, \pi_-) \) to the diagonalized meson space \( (\sigma_0, \pi_+, \pi_-) \). With the help of the pole equation \( \mathcal{M} \) for the mass \( M_\theta \), we can derive the relations between the diagonal and off-diagonal elements of the matrix \( \mathcal{M} \),

\[
\mathcal{M}_{mn}(M_\theta) = \mathcal{M}_{mm}(M_\theta) = \mathcal{M}_{mm}(M_\theta) - \mathcal{M}_{mn}(M_\theta) = (1 - 2G \Pi_{\rho}) D = 0,
\]

(17)

\( l, m, n = \sigma, \pi_+, \pi_- \), \( l \neq m \neq n \),

\[
U(k_0) \simeq \frac{2G \mathcal{M}(M_\theta)}{(dD(k_0)/dk_0^2)|_{k_0 = M_\theta}} \frac{\Gamma^*_m \mathcal{M}_{mn}(M_\theta) \Gamma_n}{k_0^2 - M^2_\theta},
\]

(19)

in terms of the new meson vertex

\[
\Gamma^*_m = \sum_m \sqrt{\mathcal{M}_{mm}(M_\theta)} \Gamma_m / \sqrt{\mathcal{M}(M_\theta)},
\]

(20)

\[
\Gamma^*_m = \sum_m \sqrt{\mathcal{M}_{mm}(M_\theta)} \Gamma_m / \sqrt{\mathcal{M}(M_\theta)},
\]

(21)

To explicitly describe the meson mixing in the pion superfluid phase, we can introduce mixing angles between two mesons, for instance, the angels \( \alpha \) between \( \pi_+ \) and \( \sigma \), \( \beta \) between \( \pi_- \) and \( \sigma \) and \( \gamma \) between \( \pi_+ \) and \( \pi_- \) in the \( \pi \)-meson channel,

\[
\tan \alpha = \sqrt{\mathcal{M}_{\pi_+\pi_+}(M_\pi)} / \sqrt{\mathcal{M}_{\sigma\sigma}(M_\pi)};
\]

\[
\tan \beta = \sqrt{\mathcal{M}_{\pi_-\pi_-}(M_\pi)} / \sqrt{\mathcal{M}_{\sigma\sigma}(M_\pi)};
\]

\[
\tan \gamma = \sqrt{\mathcal{M}_{\pi_+\pi_-}(M_\pi)} / \sqrt{\mathcal{M}_{\pi_-\pi_-}(M_\pi)} = \tan \alpha / \tan \beta.
\]

(22)

It is easy to see that only \( \alpha \) and \( \beta \) are independent. The mixing angels in the \( \pi_+ \) and \( \pi_- \) channels can be defined in a similar way.
Since the NJL model is non-renormalizable, we can employ a hard three momentum cutoff $\Lambda$ to regularize the gap equations for quarks and pole equations for mesons. In the following numerical calculations, we take the current quark mass $m_0 = 5$ MeV, the coupling constant $G = 4.93 \text{ GeV}^{-2}$ and the cutoff $\Lambda = 653$ MeV [21].

This group of parameters corresponds to the pion mass $m_\pi = 134$ MeV, the pion decay constant $f_\pi = 93$ MeV and the effective quark mass $M_q = 310$ MeV in the vacuum.

The phase diagram in the $\mu_I - \mu_B$ plane at fixed temperature $T$ is shown in Fig. 1. The system is in the normal phase at low $\mu_I$ or high $\mu_B$ and in the pion superfluid at high $\mu_I$ and low $\mu_B$. At zero temperature, the critical isospin chemical potential $\mu^*_I$ does not change with $\mu_B$ till

$$\mu_B = 3(M_q - m_\pi/2) \sim 730 \text{ MeV} \quad (23)$$

which is determined by the comparison of the gap equation for the pion condensate at $\pi = 0$ and $T = 0$ with the pion mass equation in the vacuum. With increasing temperature, the critical value $\mu^*_I$ increases and the pion superfluid region decreases, due to the melting of pion condensate in hot mediums.

The mixing angles $\alpha$, $\beta$, and $\gamma$ in the $\pi$-meson channel are shown in the upper panel of Fig 2 as functions of $\mu_I$ at $T = \mu_B = 0$. In the normal phase with $\mu_I < \mu^*_I$, $\sigma$ and $\pi$ themselves are the collective excitation modes of the system, and there is no mixing among them. In the pion superfluid phase with $\mu_I > \mu^*_I$, $\alpha$ and $\beta$ indicate the $\pi_+ - \sigma$ and $\pi_- - \sigma$ mixing strengths in the $\pi$-meson channel, and $\gamma$ reflects the relative strength between them. While in the very beginning of the superfluid any mixing is weak, it develops fast. For $\mu_I > 200$ MeV which corresponds to $\alpha = \pi/4$, the mixing is already so strong that the contribution from $\pi_+$ to $\pi$ is larger than that from $\sigma$. Similarly, the $\pi_-$-component in $\pi$ becomes more important than the $\sigma$-component itself even for $\mu_I > 150$ MeV. Therefore, at not very high isospin chemical potential the $\pi_-$ and $\pi_+$-components start to dominate the $\pi$ mesons. While the $\pi_- - \sigma$ mixing is always stronger than the $\pi_+ - \sigma$ mixing, namely $\beta > \alpha$, the relative strength shown by $\gamma$ decreases with increasing isospin density.

At zero temperature, since the phase structure of the pion superfluid does not change till the baryon chemical potential $\mu_B = 730$ MeV, see the vertical straight line in Fig 1, the mixing angles at finite baryon chemical potential $\mu_B < 730$ MeV are exactly the same as the ones shown in the upper panel of Fig 2. Since the pion superfluid happens only at low temperature, we calculated the $\mu_I$ dependence of the meson mixing angles at $T = 0.1 \text{ GeV}$ and $\mu_B = 0.4 \text{ GeV}$, shown in the lower panel of Fig 2. From the comparison with the upper panel, the shape of the mixing angles at finite temperature and baryon chemical potential is almost the same as that at $T = \mu_B = 0$, the only remarkable change is the critical value $\mu^*_I$. It increases from 134 MeV at $T = \mu_B = 0$ to 153 MeV at $T = 0.1 \text{ GeV}$ and $\mu_B = 0.4 \text{ GeV}$.

From our numerical calculation, the mixing angles in the $\pi$-meson channel behave similarly. However, the case is significantly changed for the Goldstone mode $\pi_+$. The vertex for $\pi_+$ can be greatly simplified as

$$\Gamma_{\pi_+} = \left(\Gamma_{\pi_+} - \Gamma_{\pi_-}\right)/\sqrt{2} \quad (24)$$

at any temperature and baryon and isospin chemical potentials. The Goldstone mode contains only $\pi_+$ and $\pi_-$ components, and the fractions for the two components are exactly the same.
The meson couplings to quarks are shown in Fig.3 as functions of $\mu_I$ at $T = \mu_B = 0$. In the normal phase with $\mu_I < \mu_I^c$, the coupling constants are calculated through the diagonal polarization functions $\Pi_{mm}$. At $T = \mu_B = 0$, $\Pi_{mm}$ depends only on the quark mass $M_q$, meson mass $M_m$ and isospin chemical potential $\mu_I$. Since $M_m$ for isospin neutral mesons $\sigma$ and $\pi_0$ and $M_q$ are constants, the coupling constants $g_{\sigma q\bar{q}}$ and $g_{\pi_0 q\bar{q}}$ are $\mu_I$ independent. In the pion superfluid phase, the coupling constants, determined by the diagonal and off-diagonal polarization functions $\Pi_{mn}$, behave very differently. From the mass relation $M_{\pi^-} \rightarrow M_{\pi_0}$ at $\mu_I \rightarrow \infty$, the couplings $g_{\pi^- q\bar{q}}$ and $g_{\pi_0 q\bar{q}}$ approach each other at large enough $\mu_I$. Note that $g_{\pi_0 q\bar{q}}$ is no longer a constant but changes slowly in the pion superfluid. Since we did not consider the meson widths in the pole equation (15), the condition for in the pion superfluid. Since we did not consider the meson widths in the pole equation (15), the condition for

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FIG. 3: The coupling constants for $\sigma$, $\pi_0$, $\pi^+$, $\pi^-$ in the normal phase and $\tau$, $\pi_0$, $\pi^+$, $\pi^-$ in the pion superfluid phase as functions of $\mu_I$ at $T = \mu_B = 0$.
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The meson mixing angles are only slightly changed by finite temperature $T$ and baryon chemical potential $\mu_B$, when $T$ and $\mu_B$ are in a reasonable region. Therefore, it can be expected that the meson coupling constants will not be significantly changed by finite $T$ and $\mu_B$, except a remarkable shift of the critical isospin chemical potential $\mu_I^c$ from 134 MeV to a higher value.

IV. SUMMARY

We have investigated the meson mixing and meson coupling constants in the NJL model at finite isospin chemical potential. In the pion superfluid phase, the normal mesons are no longer the collective excitation modes of the system, and the mixing among them becomes important. For the Goldstone mode, it contains only charged pions and their fractions are exactly the same in the whole superfluid region. For the other new eigenmodes, the meson mixing starts to control the system at $\mu_I \gtrsim 150$ MeV which is only a little bit higher than the critical value $\mu_I^c = m_{\pi} = 134$ MeV at $T = \mu = 0$. The coupling constants for the conventional mesons in the normal phase and for the new eigenmodes in the pion superfluid phase behave very differently. The splitting of meson mass and coupling constant due to explicit and spontaneous isospin symmetry breaking can be used to further calculate the $\pi\pi$ scattering and the number ratio $\pi^+ / \pi^-$ in isospin asymmetric matter, which can help us to measure the meson properties at finite isospin density.

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