Towards the lattice study of M-theory (II)* †

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We present new results of the quenched simulations of the reduced $D=4$ supersymmetric Yang-Mills quantum mechanics for larger gauge groups $SU(N)$, $2 < N < 9$. The model, studied at finite temperature, reveals existence of the two distinct regions which may be precursors of a black hole and the elementary $D0$ branes phases of M-theory conjectured in the literature. Present results for higher groups confirm the picture found already for $N=2$. Similar behaviour is observed in the preliminary simulations for the $D=6$ and $D=10$ models.

1. SUPERSYMMETRIC YANG-MILLS QUANTUM MECHANICS

Supersymmetric Yang-Mills quantum mechanics (SYMQM) provides the quantitative model of M-theory \cite{1}. Even though much simpler than the original theory the model is not solved in spite of its long history\cite{2–4}. We have therefore decided to set up a systematic lattice survey of SYMQM beginning with the simplest case of $D=4$, $N=2$, $N_f=0$ (quenched)\cite{5} and gradually extending it as far as possible towards the BFSS limit i.e. $D=10$, $N \rightarrow \infty$ and $N_f=1$. In this talk I will report on the second step along this programme: the first results for higher $N$ will be presented.

The action of the SYMQM reads

$$S = \int dt \left( \frac{1}{2} \text{Tr} F_{\mu\nu}(t)^2 + \bar{\Psi}^a(t) D \Psi^a(t) \right).$$  \hfill (1)

where $\mu, \nu = 1 \ldots D$, and all fields are independent of the space coordinates $\vec{x}$. The supersymmetric fermionic partners belong to the adjoint representation of $SU(N)$. The discretized system is put on a $D$ dimensional hypercubic lattice $N_1 \times \ldots \times N_D$ which is reduced in all space directions to $N_i = 1$, $i = 1 \ldots D - 1$. The gauge part of the action has now the usual form

$$S_G = -\beta \sum_{m=1}^{N_t} \sum_{\mu>\nu} \frac{1}{N} \text{Re}(\text{Tr} U_{\mu\nu}(m)),$$

(2)

with

$$\beta = 2N/a^3g^2,$$

(3)

and $U_{\mu\nu}(m) = U_{\mu}^\dagger(m)U_{\nu}^\dagger(m+\nu)U_{\nu}(m+\mu)U_{\mu}(m)$, $U_{\mu}(m) = \exp\left(i\alpha A_{\mu}(am)\right)$, where $\alpha$ denotes the lattice constant and $g$ is the gauge coupling in one dimension. The integer time coordinate along the lattice is $m$. Periodic boundary conditions $U_{\mu}(m+\nu) = U_{\mu}(m)$, $\nu = 1 \ldots D - 1$, guarantee that Wilson plaquettes $U_{\mu\nu}$ tend, in the classical continuum limit, to the appropriate components $F_{\mu\nu}$ without the space derivatives.

2. RESULTS

Up to date we have addressed the two problems: 1) extracting the continuum limit from the lattice data, and 2) the search for the nontrivial phase structure. The first point is essential in any approach based on the discretization. In particular restoration of the continuum supersymmetry, of the full unquenched model, may crucially depend on the ability to control the continuum
limit. The second issue is connected to the problem of the Bekenstein-Hawking entropy which has an elegant solution in the framework of M-theory \[6\]. Namely the supersymmetric, extremal black holes found in the latter can be viewed as composed of the elementary D0 brane excitations, providing the statistical interpretation of the area of the "Schwarzschild" horizon which is known to behave as an entropy. In particular, the theory also predicts existence of the two phases in which the gravity and the elementary D0 branes provide good description respectively\[7\].

2.1. SU(2)

We have found in \[5\] that the continuum limit of the model can be readily extracted with the bare parameters scaling with canonical dimensions. This is expected for the one dimensional system. To search for the phase transition we have studied the distribution of the eigenvalues of the Polyakov line 

\[ L = \prod_{m=1}^{N_t} U_D(m), \]

which is a very sensitive determinant of the phase structure in gauge theories. It was found that, similarly to the large volume QCD, in the low temperature phase the eigenvalues are concentrated around zero, while at high temperature the distribution is peaked around ±1 which constitute the center of SU(2). In the space extended theories the \(Z_2\) symmetry is spontaneously broken in the infinite volume limit and only one direction is populated. In the present 0-volume system, this may happen only in the infinite-N limit, the Gross-Witten model being a known example of the critical behaviour emerging at large N.

2.2. SU(3 - 8)

For higher groups we find now the same behaviour, see Fig.1. Since \(\beta = 2N T^3\), the histograms correspond to the low and high temperature regions for a range of \(N\). Evidently they change from convex to concave at some critical value of \(\beta(= \beta_c)\) similarly to the \(N=2\) case. The nature of the transition is not resolved yet. Nevertheless, our data show unambiguously that the system behaves differently in both regions. For example, we have also measured the dependence of the size of the system, \(R^2 = g^2 \sum_a (A_a^T)^2\), on the temperature, and found that it is definitely different, and the change in the behaviour occurs at the same T where the distributions in Fig. 1 change their shapes. It is also possible, for the first time to confront the \(N\) dependence of the transition temperature with theoretical expectations. It follows from Eq.(3) and \(N_t = 1/T a\), that the 't Hooft scaling \(T_c \approx (g^2 N)^{1/3}\) implies that the lattice coupling \(\beta_c \approx N^2\) at fixed \(N_t\). Fig.2 shows the \(N\) dependence of \(\beta_c /N^2\) for available range of \(N\) together with the fit of the first 1/\(N^2\) correction. Indeed, the reduced critical coupling seems to saturate towards higher \(N\) and one can estimate that \(N = 8\) result is within \(\approx 15\%\) of the \(N = \infty\) one (a horizontal line).
3. NONCOMPACT FORMULATION

We have also studied the new, noncompact formulation of the model which has better numerical behaviour \[8\]. In this approach the \( D - 1 \) spatial degrees of freedom are noncompact \( X^i(m) = g A^i(m) \) and are defined at the discrete time intervals, while the temporal one remains compact \( U_D(m) = U(m+1,m) \). The action \( S = S_{\text{kin}} + S_{\text{pot}} \) reads

\[
S_{\text{pot}} = \frac{g^2}{2 a^2} \sum_m \text{Tr} (X'^i X'^k)^2, \quad S_{\text{kin}} = \frac{1}{a} \sum_m \text{Tr} (\Delta X'^i)^2.
\]

(4)

The covariant finite difference along the time direction

\[
\Delta X'^i(m+1) = X'^i(m+1) - U(m+1,m) X'(m) U(m,m+1),
\]

(5)
takes into account the parallel transport between adjacent lattice sites. This system has the same local gauge invariance as the compact version, Eq.(3). With the new action we have extended previous study to higher dimensions, \( D=6 \) and \( D=10 \). Preliminary simulations confirm results found for \( D=4 \). In particular the average size of the system \( R^2 \) shows a characteristic break in the temperature dependence at the position consistent with the 't Hooft scaling. Moreover, the \( R^2 \) decreases with \( D \) in agreement with the mean field results \[9\].

4. FUTURE PROSPECTS

Quantitative lattice study of the Yang-Mills quantum mechanics, and possibly the M-theory, have just begun. Quenched results are encouraging, but a lot remains to be done. Simulations work for all interesting values of the dimension \( D \) and are feasible for a range of \( N \). Recent results give us a rough idea how the large \( N \) limit is approached and where the asymptotics sets it. All quenched simulations performed up do date indicate existence of the two regions at finite temperature. This intriguing correspondence with the predictions of the M-theory should be further quantified. Of course, the next step is to include the dynamical fermions. This can be done by a brute force for \( D=4 \) and for the first few \( N \)'s at \( D=10 \). The one dimensional nature of the system should help considerably. For higher \( N \), at \( D=10 \), we face the problem of the complex pfaffian. An important insight into the whole subject may be gained by applying the full potential of the small volume approach \[10\].

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