MELOSH ROTATION AND THE NUCLEON TENSOR CHARGE
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Abstract

By making use of the effect of the Melosh rotation, we show that one can estimate, in a simple way, the nucleon tensor charge in a relativistic quark model formulated on the light-cone. We discuss the physical significance of our results and compare them with those recently obtained in different phenomenological models.

Key-Words: tensor charge, quark model.

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In high-energy processes, the nucleon structure is described by a set of parton distributions, some of which are fairly well known and best measured in deep inelastic scattering (DIS). In particular from unpolarized DIS, one extracts the quark distributions \( q(x) \), for different flavors \( q = u, d, s, \text{etc...} \), which are related to the forward nucleon matrix elements of the corresponding vector quark currents \( \bar{q} \gamma^\mu q \), and likewise for antiquarks. Similarly from longitudinally polarized DIS, one obtains the quark helicity distributions \( \Delta q(x) = q_+(x) - q_-(x) \), where \( q_+(x) \) and \( q_-(x) \) are the quark distributions with helicity parallel and antiparallel to the proton helicity. Clearly the spin-independent quark distribution \( q(x) \) is \( q(x) = q_+(x) + q_-(x) \). We recall that for each flavor the axial charge is defined as the first moment of \( \Delta q(x) + \Delta \bar{q}(x) \) namely,

\[
\Delta q = \int_0^1 dx \left[ \Delta q(x) + \Delta \bar{q}(x) \right] \quad (1)
\]

and in terms of the matrix elements of the axial quark current \( \bar{q} \gamma^\mu i \gamma^5 q \), it can be written in the form

\[
2 \Delta q s^\mu = < p, s | \bar{q} \gamma^\mu i \gamma^5 q | p, s > , \quad (2)
\]

where \( p \) is the nucleon four-momentum and \( s^\mu \) its polarization vector. In addition to \( q(x) \) and \( \Delta q(x) \), for each quark flavor, there is another spin-dependent distribution, called the transversity distribution \( h_1^q(x) \) related to the matrix elements of the tensor quark current \( \bar{q} \sigma^{\mu\nu} i \gamma^5 q \). The \( h_1 \) distribution measures the difference of the number of quarks with transverse polarization parallel and antiparallel to the proton transverse polarization. One also defines the tensor charge as the first moment

\[
\delta q = \int_0^1 dx \left[ h_1^q(x) - h_1^\bar{q}(x) \right] . \quad (3)
\]

The existence of \( h_1^q(x) \) was first observed in a systematic study of the Drell-Yan process with polarized beams [1] and some of its relevant properties were discussed later in various papers [2, 3, 4]. We recall that \( q(x) \), \( \Delta q(x) \) and \( h_1^q(x) \), which are of fundamental importance for our understanding of the nucleon structure, are all leading-twist distributions, but one can also consider a more complete set including several higher-twist distributions [4]. Due to scaling violations, these quark distributions have a \( Q^2 \)-dependence.
governed by QCD evolution equations, which are different in the three cases. On the experimental side, a vast programme of measurements in unpolarized DIS has been going on for more than twenty five years, and has yielded an accurate determination of the $x$ and $Q^2$ dependence of $q$ (and $\bar{q}$) for various flavors. From several fixed-targets experiments operating now at CERN, SLAC and DESY, we also begin having a good insight into the different quark helicity distributions $\Delta q$, although the present available $x$ and $Q^2$ ranges are rather limited. Concerning $h_1^q$ (or $h_1^{\bar{q}}$), they are not simply accessible in DIS because they are in fact chiral-odd distributions and they can be best extracted in polarized Drell-Yan processes or in $Z$ production with two transversely polarized proton beams. Such experiments will be undertaken with the polarized $pp$ collider at RHIC [5], but so far we have no experimental information on the shape and magnitude of these quark transversity distributions. However there are several different theoretical determinations of $h_1^q$ using either the MIT bag model [4] or QCD sum rules [6], and also based on either a chiral chromodielectric model [7] or a chiral quark-soliton model [8, 9].

At this point let’s mention the following positivity constraint

$$q(x) + \Delta q(x) \geq 2|h_1^q(x)|$$

which is exact in the parton model and valid for each flavor, likewise for antiquarks [10]. Although some doubts have been expressed on its validity in perturbative QCD [11], it has been recently demonstrated, from the two loop $Q^2$ evolution [12], that if the inequality is satisfied for a certain value of $Q^2$, it remains valid for higher $Q^2$. As we will see, this inequality is very useful, given the present poor knowledge we have on $h_1^q$.

In the non-relativistic quark model, transversely polarized quarks are in transverse spin states, which by rotational invariance implies that the axial charge and the tensor charge must be equal. For example by using the $SU(6)$ proton wavefunction one finds the well known valence contributions

$$\Delta u = \delta u = 4/3 \ , \ \Delta d = \delta d = -1/3 \ \text{and} \ \Delta s = \delta s = 0 .$$

So in this case the sum of spin of quarks (and antiquarks) are equal to the proton spin at rest since from eq. (5) we have

$$\Delta \Sigma \equiv \Delta u + \Delta d + \Delta s = 1 .$$
Of course in DIS one is probing the proton spin in the infinite momentum frame and the above result might not be longer true. In the light-cone formalism, which is suitable to describe the relativistic many-body problem, we have to transform the instant quark states \( q_{NR,i}^{\lambda} \) into the light-cone quark states \( q_{LC,i}^{\lambda} \) \((i = 1, 2, 3)\). The two sets of states are related by a general Melosh rotation \( [13] \), according to

\[
\begin{align*}
q_{LC,+}^{i} &= \frac{1}{\sqrt{\text{det}}} \left[ (m_q + x_i \mathcal{M}) q_{NR,+}^i + k_i^R q_{NR,-}^i \right], \\
q_{LC,-}^{i} &= \frac{1}{\sqrt{\text{det}}} \left[ -k_i^L q_{NR,+}^i + (m_q + x_i \mathcal{M}) q_{NR,-}^i \right],
\end{align*}
\]

where \( k_i^{R,L} = k_i^1 \pm ik_i^2 \), \( \text{det} = (m_q + x_i \mathcal{M})^2 + \vec{k}_{iT}^2 \) and the invariant mass \( \mathcal{M} \) is given by \( \mathcal{M}^2 = \sum_{i=1}^{3}(k_{iT}^2 + m_i^2)/x_i \). Here we are using light-cone momentum fractions \( x_i = p_i^+ / P^+ \), where \( P \) and \( p_i \) are the nucleon and quark momenta respectively \((p_i^+ = p_i^0 + p_i^3 \text{ and } P^+ = P^0 + P^3)\), and the internal momentum variables \( \vec{k}_{iT} \) are given by \( \vec{k}_{iT} = \vec{p}_{iT} - x_i \vec{P}_T \) with the constraints \( \sum_{i=1}^{3} \vec{k}_{iT} = 0 \) and \( \sum_{i=1}^{3} x_i = 1 \). In the zero binding limit \( x_i \mathcal{M} \to k_i^+ = k_i^0 + k_i^3 \), but this cannot be a justified approximation for QCD bound states.

We notice that the helicity states get mixed as long as the internal transverse momentum \( k_T \) is non-zero, which makes the Melosh rotation non-trivial. Actually, one can show that for light-cone states only the positive component of the axial current contributes, so eq. (2) reads also as

\[
2\Delta_q^{LC} = \langle p, s | \bar{q}_{LC,\lambda} \gamma^+ \gamma^5 q_{LC,\lambda} | p, s \rangle
\]

with \( \lambda = + \) or \( - \).

By using eq. (7) one sees that the light-cone axial charge \( \Delta_q^{LC} \) is related to the non-relativistic axial charge \( \Delta_q^{NR} \) as follows \([14]\)

\[
\Delta_q^{LC} = \langle M_q > \Delta_q^{NR},
\]

where

\[
M_q = \frac{(m_q + x_3 \mathcal{M})^2 - k_{3T}^2}{(m_q + x_3 \mathcal{M})^2 + k_{3T}^2}
\]

(10)
and \(< M_q >\) is its expectation value

\[
< M_q > = \int d^3k \, M_q \, |\Psi(k)|^2 ,
\]

where \(\Psi(k)\) is a simple normalized momentum wavefunction. By choosing two different reasonable wavefunctions, e.g. the harmonic oscillator and the power-law fall off, the calculation \([13]\) gave \(< M_q > = 0.75\) \((q = u, d\) if we assume \(m_u = m_d\)\) which leads to a reduction of \(\Delta \Sigma\) (see eq. (6)) from 1, in the naive quark model, to 0.75. From polarized DIS, one obtains the singlet axial charge of the proton \(a_0(Q^2_0) = 0.28 \pm 0.16\) at \(Q^2_0 = 10\) GeV\(^2\) \([14]\), which is related to \(\Delta \Sigma\) in a scheme dependent way, and from the value of the gluon polarization \(\Delta g \sim 2\), it implies \(\Delta \Sigma \sim 0.5\). This shows that although relativistic effects do not provide the correct result, they are responsible for a substantial shift in the right direction.

Let us now turn to the tensor charge. Like for the axial charge, it can be shown that for light-cone states only the positive component of the tensor quark current, which involves a spin-flip, contributes so we have

\[
2\delta q_{LC} = \langle p, s | \bar{q}_{LC,\lambda} \gamma^+ \gamma^\perp q_{LC,-\lambda} | p, s \rangle ,
\]

with \(\lambda = +\) or \(-\) and \(\gamma^\perp = \gamma^1 + i\gamma^2\). By using eq. (7) are easily finds how the light-cone tensor charge is related to the non-relativistic one, namely

\[
\delta q_{LC} = \langle \widetilde{M}_q \rangle > \delta q_{NR} ,
\]

where

\[
\widetilde{M}_q = \frac{(m_q + x_3 M)^2}{(m_q + x_3 M)^2 + k^2_{3T}}
\]

and \(\langle \widetilde{M}_q \rangle \) is its expectation value, as before for \(M_q\). At this point we note that in the non-relativistic case, which corresponds to the limit \(k_T = 0\), one has \(M_q = \widetilde{M}_q = 1\) and they both decrease under the relativistic effects. In addition it is interesting to remark that one has

\[
1 + M_q = 2\widetilde{M}_q ,
\]

which means that there is saturation of eq. (4). By taking the expectation value of eq. (15) and knowing that \(< M_q > = 3/4\), as indicated above, one finds immediately \(< \widetilde{M}_q > = 7/8\). By using eq. (5) it leads to

\[
\delta u_{LC} = 4/3 \times 7/8 = 7/6 \quad \text{and} \quad \delta d_{LC} = -1/3 \times 7/8 = -7/24 .
\]
which remarkably are exactly the values obtained in the MIT bag model [17].

It is worth recalling that the MIT bag model produces quark distributions which also saturate eq. (4) and this is also the case for the toy model proposed in ref. [2]. These values are perfectly compatible with the positivity bounds derived in ref. [10], namely

$$|\delta u| \leq \frac{3}{2} \quad \text{and} \quad |\delta d| \leq \frac{1}{3}.$$  \hspace{1cm} (17)

However in ref. [8] they obtain

$$\delta u = 1.12 \quad \text{and} \quad \delta d = -0.42$$ \hspace{1cm} (18)

but the large $N_c$ behavior is expected to generate in this model, large theoretical uncertainties, mainly for the $d$ quark.

Unlike the axial charge which is $Q^2$-independent, $\delta q$ has the following $Q^2$ evolution [2]

$$\delta q(Q^2) = \delta q(Q_0^2) \left[ \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right]^{\frac{4}{3N_f-2N_f}}$$ \hspace{1cm} (19)

where $N_f$ is the number of flavors. So $\delta q$ decreases for increasing $Q^2$ and in the model of ref. [4] where the initial scale of the nucleon is taken to be $Q_0^2 = 0.16 \text{ GeV}^2$, they obtain at $Q^2 = 25 \text{ GeV}^2$ from eq. (19)

$$\delta u = 0.969 \quad \text{and} \quad \delta d = -0.250,$$ \hspace{1cm} (20)

also consistent with the positivity bounds eq. (17).

The relativistic light-cone quark model then predicts the following values for the nucleon’s isovector and isoscalar tensor charges, respectively

$$\delta u - \delta d = \frac{35}{24} = 1.458 \quad \text{and} \quad \delta u + \delta d = \frac{7}{8} = 0.875.$$ \hspace{1cm} (21)

Let us now consider the non-relativistic and the ultra-relativistic limits in this formalism. One simple way [15] to obtain both limits is by varying the dimensionless quantity $M_p R_1$, where $M_p$ is the proton mass and $R_1$ is the proton radius ($R_1 = \sqrt{-6 \frac{dF_1(Q^2)/dQ^2}{|Q^2=0}}$, where $F_1(Q^2)$ is the Dirac form factor). The non-relativistic limit corresponds to $R_1 \rightarrow \infty$ with a fixed mass. Using eq. (15) and the fact that $< M_q > \rightarrow 1$ in the NR limit, as it should from eq. (14), we obtain that in this limit $< \tilde{M}_q > \rightarrow 1$ also, which is
then consistent. The ultra-relativistic limit is obtained by taking $M_p R_1 \to 0$, which then corresponds to a point-like particle. Again we use eq. (15), but now $< M_q > \to 0$, which means that $< \tilde{M}_q > \to 1/2$ in the UR limit. Thus the point-like values for the tensor charges are predicted to be

$$\delta u \to 2/3 \quad \text{and} \quad \delta d \to -1/6 .$$

(22)

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References

[1] J.P. RALSTON and D.E. SOPER, Nucl. Phys., B152 (1979) 109.

[2] X. ARTRU and M. MEKHFI, Z. Phys., C45 (1990) 669.

[3] J.L. CORTES, B. PIRE and D.E. J.P. RALSTON, Z. Phys., C55 (1992) 409.

[4] R.L. JAFFE and X.JI, Phys. Rev. Lett., 67 (1991) 552 ; Nucl. Phys., B375 (1992) 527.

[5] Y. MAKDISI, Proceedings of the 12th Int. Conference on High Energy Spin Physics, Amsterdam, Sept. 1996.

[6] B.L. IOFFE and A. KHODJAMIRIAN, Phys. Rev., D51 (1995) 33.

[7] V. BARONE, T. CALARCO and A. DRAGO, Phys. Lett., B390 (1997) 287.

[8] H.-C. KIM, M.V. POLYAKOV and K. GOEKE, Phys. Lett., B387 (1996) 577.
[9] P.V. POBYLITSA and M.V. POLYAKOV, Phys. Lett., B389 (1996) 350.

[10] J. SOFFER, Phys. Rev. Lett., 74 (1995) 1292.

[11] G.R. GOLDSTEIN, R.L. JAFFE and X. JI, Phys. Rev., D52 (1995) 5006.

[12] W. VOGELSANG, Contribution to the Ringberg Workshop on High Energy Polarization Phenomena, Feb. 24-28 (1997).

[13] H.J. MELOSH, Phys. Rev., D9 (1974) 1095.

[14] B.-Q. MA, J. Phys., G17 (1991) L53 ; B.-Q. MA and Q.-R. ZHANG Z. Phys., C58 (1993) 479.

[15] S.J. BRODSKY and F. SCHLUMPF, Phys. Lett., B329 (1994) 111.

[16] D. ADAMS et al. (Spin Muon Collaboration) hep-ex/9702003.

[17] H. HE and X. JI, Phys. Rev., D52 (1995) 2960.