Thermodynamics with 3 and 2+1 Flavors of Improved Staggered Quarks *

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We present preliminary results [1] from exploring the phase diagram of finite temperature QCD with three degenerate flavors and with two light flavors and the mass of the third held approximately at the strange quark mass. We use an order $\alpha_s a^2, a^4$ Symanzik improved gauge action and an order $\alpha_s a^2, a^4$ improved staggered quark action. The improved staggered action leads to a dispersion relation with diminished lattice artifacts, and hence better thermodynamic properties. It decreases the flavor symmetry breaking of staggered quarks substantially, and we estimate that at the transition temperature for an $N_t = 8$ to $N_t = 10$ lattice all pions will be lighter than the lightest kaon. Preliminary results on lattices with $N_t = 4, 6$ and 8 are presented.

With the Relativistic Heavy Ion Collider (RHIC) now producing data, it has become even more important to understand the phase diagram of QCD at finite temperature, and to determine properties of the high temperature quark-gluon-plasma phase with confidence, i.e. with controlled lattice spacing errors.

It is fairly well established that QCD with two flavors of massless quarks has a second order finite temperature, chiral symmetry restoring phase transition. This transition is washed out as soon as the quarks become massive. QCD with three flavors of massless quarks has a first order finite temperature, chiral symmetry restoring phase transition, which is stable for small quark masses. Not well known is how large the quark masses can be until the phase transition turns second order and then into a crossover, both for degenerate quarks and especially for the physically relevant case of two light and one heavier strange quark.

In previous studies, the second question is particularly badly answered due to the flavor symmetry breaking in Kogut-Susskind quarks, usually used for this purpose: how can one study the influence of the strange quark when most of the (non-Goldstone) pions are heavier than the (Goldstone) kaon?

Adding a few terms to the conventional Kogut-Susskind action, namely three-link, five-link and seven-link staples and a third-neighbor coupling, removes all tree-level $O(a^2)$ errors [2,3]. This “Asqtad” action shows improved flavor and rotational symmetry [4,5], and, at least in quenched QCD, good scaling properties [6].

Based on our zero temperature simulations [5]...
Figure 1. Real part of the Polyakov line for three flavors with $m_q \approx m_s$.

and the estimate of $T_c \sim 150 - 170$ MeV we deduce that for $N_t = 8 - 10$ the kaon will be heavier than the heaviest non-Goldstone pion at the finite temperature transition.

We have zero temperature results, in particular, the value of the (bare) strange quark mass, at fixed lattice spacing $a \sim 0.13$ fm and $a \sim 0.2$ fm. Since we want to keep the physical quark masses approximately constant when we vary the temperature (which, at fixed $N_t = 1/(aT)$, means varying the gauge coupling $\beta$), we interpolated (extrapolated) between the values at the two lattice spacings.

We show in Fig. 1 the real part of the Polyakov line and in Fig. 2 the condensate as function of the temperature for our simulations with three degenerate quarks of mass $m_q \approx m_s$. To set the scale we used the distance $r_1$, defined in terms of the static QQ potential by $r_1^2 F_{Q\bar{Q}_{\text{static}}}(r_1) = 1$, i.e. $r_1 \sim 0.35$ fm and interpolated $a/r_1$, with the form advocated by Allton, between the two lattice spacings $a \sim 0.13$ fm and $a \sim 0.2$ fm.

The Polyakov line shows a crossover behavior, but the condensate appears to decrease, with increasing temperature, rather smoothly. However, the chiral susceptibility, shown in Fig. 3 for $N_t = 6$ and in Fig. 4 for $N_t = 8$, exhibits a peak. The peak height appears to be independent of the spatial volume, indicative of a smooth crossover.

Our other results look similar. Fig. 5 shows the real part of the Polyakov line for our three-flavor simulations with $m_q \approx 0.6 m_s$, and Fig. 6 for our 2+1 flavor simulations with $m_u,d = 0.6 m_s$ and $0.4 m_s$.

For the masses considered so far, we have not seen any sign of a genuine phase transition. This result is compatible with other recent simulations which found phase transitions only at quark masses lighter than those studied by us so far. The temperature at the crossover, with the scale set by $r_1$, $T_c \sim 190 - 200$ MeV, is a little higher than expected from previous determinations. This is presumably also due to our quark masses still being rather high.

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REFERENCES

1. For a more extended version, see C. Bernard et al. (The MILC collaboration), hep-lat/0110030.
2. S. Naik, Nucl. Phys. B316 (1989) 238.
3. K. Orginos, D. Toussaint and R.L. Sugar,
Figure 3. The $\bar{\psi}\psi$ susceptibility for three flavors with $m_q \approx m_s$ for lattices with $N_t = 6$.

Figure 4. The $\bar{\psi}\psi$ susceptibility for three flavors with $m_q \approx m_s$ for lattices with $N_t = 8$.

Figure 5. Real part of the Polyakov line for three flavors with $m_q \approx 0.6m_s$.

Figure 6. Real part of the Polyakov line for 2+1 flavors.

Phys. Rev. D 60 (1999) 054503; Nucl. Phys. (Proc. Suppl.) 83 (2000) 878.

4. G.P. Lepage, Phys. Rev. D 59 (1999) 074501.

5. C. Bernard et al. (The MILC collaboration), Phys. Rev. D 64, (2001) 054506.

6. C. Bernard et al. (The MILC collaboration), Phys. Rev. D 61, (2000) 111502.

7. See, e.g., S. Ejiri, Nucl. Phys. B (Proc. Suppl) 94, (2001) 19; F. Karsch, hep-lat/0106019.

8. C. Bernard et al. (The MILC collaboration), Phys. Rev. D 62, (2000) 034503.

9. C. Allton, hep-lat/9610016.

10. S. Aoki et al. (JLQCD), Nucl. Phys. B (Proc. Suppl) 73, (1999) 459; F. Karsch, E. Laermann and Ch. Schmidt, hep-lat/0107020.