Bottomonium at Non-zero Temperature from Lattice
Non-relativistic QCD

Gert Aarts*, Chris Allton*, Seyong Kim†, Maria Paola Lombardo**,
Mehmet B. Oktay‡, Sinead M. Ryan§, D. K. Sinclair¶ and Jon-Ivar Skullerud∥

*Department of Physics, Swansea University, Swansea SA2 8PP, United Kingdom
†Department of Physics, Sejong University, Seoul 143-747, Korea
**INFN-LNF, I-00044, Frascati (RM) Italy, Humboldt-Universität zu Berlin, 12489 Berlin, Germany
‡Physics Department, University of Utah, Salt Lake City, Utah, USA
§School of Mathematics, Trinity College, Dublin 2, Ireland
¶HEP Division, Argonne National Laboratory, Argonne, IL 60439, USA
∥Department of Mathematical Physics, National University of Ireland, Maynooth, County Kildare, Ireland

Abstract. The temperature dependence of bottomonium states at temperatures above and below $T_c$ is presented, using non-relativistic dynamics for the bottom quark and full relativistic lattice QCD simulations for two light flavors on a highly anisotropic lattice. We find that the S-waves ($\Upsilon$ and $\eta_b$) show little temperature dependence in this range while the P wave propagators show a crossover from the exponential decay characterizing the hadronic phase to a power-law behavior consistent with nearly-free dynamics at approximately twice the critical temperature.

Introduction

Heavy quark states can be an important probe of the dynamics of the Quark Gluon Plasma (QGP). While charmonium suppression [1] has been observed at a range of energies, a number of sometimes competing effects complicate the interpretation of these patterns. Many of these difficulties are not present for bottomonium making suppression patterns observed in that regime in principle more straightforward to interpret. Intriguing early results from experiments at CMS [2] and STAR [3] demonstrating suppression in the Upsilon system suggest a lattice calculation is timely and may help to shed light on the link to the spectrum of bound states.

In this paper we use dynamical anisotropic lattice gauge configurations and treat the bottom quarks using non-relativistic QCD (NRQCD) at a range of temperatures, $0.42T_c < T < 2.09T_c$. NRQCD has been used extensively to study heavy quark physics both on and off the lattice. At finite temperature this formalism offers a number of technical advantages over the relativistic theory. Firstly, the temperature dependence in NRQCD correlators is due only to the thermal medium, with no contribution from thermal boundary conditions. Secondly, there is no nontrivial spectral weight at zero energy, which would yield a constant time-independent contribution to the correlators complicating the analysis [4] and casting doubt on the results for melting or survival of charmonium at high temperatures [5].

The non-relativistic theory is also the first effective theory to be obtained when integrating out the UV degrees of freedom [6, 7] and requires only that the heavy quark mass, $M \gg T$ which is reasonable for $b$-quarks at the temperatures in our simulations, up to $2T_c \sim 400$ MeV. A more detailed discussion is in Ref. [8] and references therein where some results shown here have also appeared. A complete MEM analysis will be presented in a forthcoming paper.

Theoretical formalism and results

In this paper we present results from a study of the temperature dependence of S and P waves (in the $\Upsilon$ and $\chi_{b1}$ channels respectively). From Ref. [7] the correlators for S and P states, in continuum NRQCD with $E_p = p^2/2M$ and in the absence of interactions, are

$$G_S(\tau) \sim \int \frac{d^3p}{(2\pi)^3} \exp(-2E_p \tau) \sim \tau^{-3/2}, \quad \text{and} \quad G_P(\tau) \sim \int \frac{d^3p}{(2\pi)^3} p^2 \exp(-2E_p \tau) \sim \tau^{-5/2},$$

(1)
indicating a power-law decay at large euclidean time. Although in a lattice simulation this behaviour can be modified by interactions and lattice artifacts it provides a useful guide for what to expect.

In this paper we also present preliminary results from a maximum entropy method (MEM) [9] analysis of bottomonium S waves. The dataset used in the MEM analysis is modified slightly - including more configurations and a larger range of temperatures as well as a more accurate nonperturbative tuning of the bare anisotropy, $\xi = a_\times/a_\tau$ in the action. The gauge configurations are those used in Ref. [11] with $\xi = a_\times/a_\tau = 6$. NRQCD bottomonium (point) propagators, using an action including terms up to $O(\Lambda^4)$, were computed and analysed for a number of S and P wave channels. The analysis presented in [8] and described briefly below was based on a subset of the temperatures now available (as indicated in the table). Full details of the nonperturbative tuning of the anisotropy are in Refs. [12, 13].

**TABLE 1.** Some lattice details for this work. The lattice spacing is $a_\times^{-1} \approx 7.35$ GeV, set using the 1P-1S spin-averaged splitting in charmonium [12].

| $N_t$ | $N_f$ | $T$(MeV) [in Ref [8]] | $T/T_c$ | No. of Configurations | No. of Configurations used in Ref [8] |
|-------|-------|------------------------|---------|-----------------------|---------------------------------------|
| 12    | 80    | 0.42                   | 250     | 74                    |                                       |
| 12    | 32    | 1.05                   | 1000    | 500                   |                                       |
| 12    | 28    | 1.20                   | 1000    | 500                   |                                       |
| 12    | 24    | 1.40                   | 500     | 500                   |                                       |
| 12    | 20    | 1.68                   | 1000    | 500                   |                                       |
| 12    | 18    | 1.86                   | 1000    | 500                   |                                       |
| 12    | 16    | 2.09                   | 1000    | 500                   |                                       |
| 12    | 8     | 3.00                   | 1000    | 500                   |                                       |
| 12    | 4     | 4.00                   | 1000    | 500                   |                                       |

To distinguish bound and free states we consider the effective mass, $m_{\text{eff}}(\tau) = -\log \frac{G(\tau)}{G(\tau-a_\tau)}$ and define a new quantity: the effective power, $\gamma_{\text{eff}}(\tau) = -\frac{T}{\text{eff}} \frac{G(\tau+a_\tau)}{G(\tau)}$. For states in the hadronic phase we expect to see the exponential decay characteristic of bound states, with $m_{\text{eff}}(\tau)$ becoming a constant at large $\tau$. When quarks are unbound we expect the correlators will show a power-law behavior and that $\gamma_{\text{eff}}(\tau)$ will tend towards a constant. These quantities are shown in Fig 1. We note that the P wave behaviour at $T > T_c$ rules out pure exponential decay, which we interpret as a medium modification at temperatures above $T_c$. In addition, we see a tendency of the P wave to flatten out and approach the free continuum behavior at large $T \sim 2T_c$.

**FIGURE 1.** The left panel shows the effective mass for the S and P wave states. The right panel is the effective power for the same states. Each pane shows a range of temperatures.

**Maximum entropy analysis.** To study in detail the temperature dependence of the bottomonium correlators we calculate the spectral functions with MEM using the relation, $G(\tau) = \sum_{M} d\omega' \frac{1}{2\pi} e^{-\omega'\tau} p(\omega')$, where $\omega = 2M + \omega'$ and dropping terms suppressed when $M \gg T$ [7]. For the MEM to give reliable results a sufficient number of points in the euclidean time direction is required: $\theta(10)$ independent lattice points is a good guide. At temperatures close to $2T_c$ this would necessitate a lattice spacing $a \sim 0.025$ fm, which on an isotropic lattice is prohibitively costly. The anisotropic

$$G(\tau) = \sum_{M} d\omega' \frac{1}{2\pi} e^{-\omega'\tau} p(\omega')$$
lattice provides an excellent solution: with $a_t \ll a$, a sufficient number of points for MEM to work can be simulated for reasonable numerical cost. The MEM analysis is performed using Bryan’s algorithm [10] and quadruple numerical precision. In the results shown here we have focused on the S waves. We begin by looking at the zero temperature spectral functions in the $\Upsilon$ (vector) channel. Fig. 2 (left panel) shows the spectral function at the lowest temperature as a function of the energy. The dotted lines indicate the ground- and first excited state determined using exponential fits to correlators. We observe that there is very good agreement with the MEM spectral function and that the ground- and first excited states are clearly determined. The right panel in Fig. 2 shows the temperature dependence in the $\Upsilon$ channel with each pane showing two neighboring temperatures. The ground state appears to survive to the highest temperature in our simulations ($\sim 2T_c$), although the height of the peak is diminished. The first excited state is gradually being suppressed and is no longer discernible at $T/T_c \sim 1.68$ indicating a melting temperature below $1.68T_c$.

Summary

In this paper we describe recent results from a study of bottomonium at non-zero temperature from lattice QCD. The NRQCD formalism offers a number of advantages at finite temperature making it a promising line of investigation. We find evidence of strong temperature dependence in the $P$ waves at $T > T_c$ and indications that while the $1S$ states survive up to $2T_c$ the $2S$ states are affected much more and appear to have melted at temperatures above $1.4T_c$.

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REFERENCES

1. T. Matsui and H. Satz, Phys. Lett. B 178 (1986) 416.
2. S. Chatrchyan et al. [CMS Collaboration], Phys. Rev. Lett. 107 (2011) 052302 [arXiv:1105.4894 [nucl-ex]].
3. STAR Collaboration, talk by Rosi Reed at Quark Matter, Annecy, France, 22-28 May 2011.
4. G. Aarts, J. M. Martinez Resco, JHEP 0204 (2002) 053. [hep-ph/0203177].
5. T. Umeda, Phys. Rev. D 75 (2007) 094502; P. Petreczky, Eur. Phys. J. C 62 (2009) 85.
6. N. Brambilla, M. A. Escobedo, J. Ghiglieri, J. Soto, A. Vairo, JHEP 1009 (2010) 038. [arXiv:1007.4156 [hep-ph]].
7. Y. Burnier, M. Laine and M. Vepsalainen, JHEP 0801 (2008) 043 [arXiv:0711.1743 [hep-ph]].
8. G. Aarts, et al, Phys. Rev. Lett. 106 (2011) 061602 [arXiv:1010.3725].
9. M. Asakawa, T. Hatsuda, Y. Nakahara, Prog. Part. Nucl. Phys. 46 (2001) 459-508. [hep-lat/0011040].
10. R. K. Bryan, Eur. Biophys. J 18, 165 (1990).
11. M. B. Oktay and J. I. Skullerud, arXiv:1005.1209.
12. G. Aarts, C. Allton, M. B. Oktay, M. Peardon and J. I. Skullerud, Phys. Rev. D 76 (2007) 094513 [arXiv:0705.2198 [hep-lat]].
13. R. Morrin, A. O. Cais, M. Peardon, S. M. Ryan and J. I. Skullerud, Phys. Rev. D 74 (2006) 014505 [arXiv:hep-lat/0604021].