Note on a noncritical holographic model with a magnetic field

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Abstract

We consider a noncritical holographic model constructed from an intersecting brane configuration $D_4/D_4\bar{D}_4$ with an external magnetic field. We investigate the influences of this magnetic field on strongly coupled dynamics by the gauge/gravity correspondence.
1 Introduction

The AdS/CFT correspondence [1]-[5] is a useful method to study strongly coupled dynamics in gauge theory. In string theory, some effective QCD-like theories can be constructed through intersecting brane configurations. Then strongly coupled physics in gauge theories can be investigated by the supergravity approximation. Recently, there are many studies, such as [7]-[16]. For reviews, one can see [6].

Through studying some holographic models in critical string theory, we get some better understandings on strongly coupled physics in the QCD-like effective theories. But there still exists many faults for the critical holographic models. An important one is that color brane backgrounds are ten-dimensional, so some part of such backgrounds need to be compactified on some compact manifolds. It will produce some Kaluza-Klein(KK) tower modes. However, in real QCD theory, there doesn’t exist such KK modes. Also some KK modes are at the same order as hadronic modes in the QCD-like effective theory. So it is difficult to distinguish hadronic modes from these KK modes. In order to overcome this point, one can consider some intersecting brane configurations in noncritical string theory. The reason is now gravity backgrounds lie at low dimension. In the Refs. [17]-[21], such noncritical holographic models were investigated, for example, the D4/D4-D̄4 brane configuration. However, the D-brane gravity backgrounds in noncritical string theory have some shortcomings. The string coupling constants of these gravity backgrounds are proportional to $1/N_c$. It means small string coupling constant corresponds to large color number $N_c$. In the large $N_c$ limit, the 't Hooft coupling constant $g_{YM}^2 N_c$ is order one. The scalar curvature of gravity background is also order one. Thus, the gauge/gravity correspondence is not very reliable in noncritical string theory. But noncritical string models are still deserved to study. It will deepen our understandings on some universal properties of general holographic models.

In [20], the authors consider an intersecting brane configuration, which is composed of D4 and anti-D4 brane in six-dimensional noncritical string theory. The color brane is D4, which extends along the directions $t, x_1, \cdots, x_4$. The worldvolume coordinates of $N_f$ flavor D4-D̄4 brane are $t, x_1, \cdots, x_3$ and $u$. Under the quenched approximation $N_c \gg N_f$, the backreaction of the flavor D4-D̄4 on the color gravity background can be omitted. Just like the Sakai-Sugimoto (SS) model [10], we choose the coordinate $x_4$ to be periodic, then the adjoint fermion on the color D4 brane satisfies an anti-periodic condition on the $x_4$ circle. At low energy, they get mass and are decoupled. So the final low energy
effective theory on this intersecting brane configuration is a four-dimensional QCD-like effective theory with a global chiral symmetry $U(N_f)_L \times U(N_f)_R$ induced by $N_f$ D4-D4 flavor brane pairs.

From the Refs. \cite{17}-\cite{20}, the near-horizon gravity background of D4 branes with a periodic coordinate $x_4$ at low temperature is

$$ds^2 = \left( \frac{u}{R} \right)^2 \left( dt_E^2 + dx_i dx_i + f(u) dx_4^2 \right) + \left( \frac{R}{u} \right)^2 \frac{1}{f(u)} du^2,$$

$$F_6 = Q_c \left( \frac{u}{R} \right)^4 dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge du \wedge dx_4,$$

$$e^\phi = \frac{2\sqrt{2}}{\sqrt{3}Q_c}, \quad R^2 = 15/2, \quad f(u) = 1 - \left( \frac{u_{KK}}{u} \right)^5,$$  

where $i, j = 1, \cdots, 3$ and the parameter $Q_c$ is proportional to the color brane number $N_c$. The Euclidean time is periodic $t_E \sim t_E + \beta$. Since $\beta$ is arbitrary, the temperature $1/\beta$ of this background is undetermined. In order to void a singularity, the coordinate $x_4$ needs to satisfy a periodic condition

$$x_4 \sim x_4 + \delta x_4 = x_4 + \frac{4\pi R^2}{5u_{KK}}. \quad \text{(1.2)}$$

It corresponds to a KK mass scale

$$m_{KK} = \frac{2\pi}{\delta x_4} = \frac{5u_{KK}}{2R^2}. \quad \text{(1.3)}$$

By a double wick rotation, we get a black hole solution. It reads

$$ds^2 = \left( \frac{u}{R} \right)^2 \left( f(u) dt_E^2 + dx_i dx_i + dx_4^2 \right) + \left( \frac{R}{u} \right)^2 \frac{1}{f(u)} du^2,$$

$$f(u) = 1 - \left( \frac{u_T}{u} \right)^5,$$  

where the Euclidean time satisfies a periodic condition

$$t_E \sim t_E + \delta t_E = t_E + \frac{4\pi R^2}{5u_T}, \quad \text{(1.5)}$$

and now the radius of the coordinate $x_4$ is arbitrary. By comparing the free energy between the gravity background (1.1) and (1.4), we find there exists a first order Hawking-Page phase transition (corresponding to the confinement/deconfinement phase transition in the boundary theory) at critical temperature $\beta = \delta x_4$. Below this temperature, the background (1.1) is dominated. Otherwise, the background (1.4) will be dominated.
These results are similar to the cases [11] in the Sakai-Sugimoto model. From the gravity backgrounds (1.1) and (1.4), it is clear that the ’t Hooft coupling constant is order one, and the curvature scalar is also order one. Thus, it is not very reliable to use AdS/CFT correspondence to study some strong coupled physics in this holographic model. In the following, we ignore this point and perform some investigations by using the usual method.

In this paper, we consider to turn on an external magnetic field on the flavor D4-D4 branes just like the critical string cases [22]-[26], and study its influences on strongly coupled dynamics. Some effects of external magnetic field on the dynamics of QCD theory were extensively studied in references, for example [27]. Here we investigate its influences in a noncritical string model by the gauge/gravity correspondence. We choose this magnetic field along the directions $x_2$ and $x_3$ on the worldvolume of D4 brane as follows

$$2\pi\alpha' F_{23} = B. \quad (1.6)$$

Since there exists gauge invariance on the flavor D4-brane worldvolume, this magnetic field is equivalent to a constant Neveu-Schwarz–Neveu-Schwarz (NS-NS) field. So to investigate the D4-brane classical dynamics with an external magnetic field becomes to study the flavor D4-brane dynamics in the gravity background with a constant NS-NS field.

The organizations of this paper is as follows. In section two and three, we investigate the flavor D4-brane dynamics in the low temperature background (1.1) and high temperature phase (1.4), respectively. In section four, we study a spinning fundamental string in the gravity background (1.4) and calculate the Regge trajectory behaviors. The last section is a summary.

## 2 Low temperature

In the low temperature phase, the gravity background is the equation (1.1). And we assume the worldvolume coordinate $u$ of flavor D4-D4 brane is depended on the background coordinate $x_4$. Then the induced metric on the worldvolume of flavor D4 brane is

$$ds^2 = \frac{u^2}{R^2}(dt_E^2 + \sum_{i=1}^{3} dx_i dx_i) + \frac{u^2}{R^2} \left( f(u)(\partial_u x_4)^2 + \frac{R^4}{f(u)u^4} \right) du^2. \quad (2.1)$$
With the magnetic field \( B \), the DBI action for the flavor D4 brane is

\[
S \sim \int \frac{du}{R^3} u^3 \sqrt{\frac{u^4}{R^4} + B^2} \left( f(u)(\partial_u x_4)^2 + \frac{R^4}{f(u) u^4} \right). \tag{2.2}
\]

So the equation of motion is derived as

\[
\frac{\partial}{\partial x_4} \left( \frac{u^5}{R^5} f(u) \frac{1 + B^2 R^4}{u^8} \right) = 0. \tag{2.3}
\]

We choose a boundary condition as \( u' = 0 \) at \( u = u_0 \) (where \( ' = \partial_{x_4} \)). It means \( u_0 \) is a connected point between the flavor D4 and anti-D4 branes. After an integration, the equation (2.3) becomes

\[
\frac{u^5}{R^5} f(u) \frac{1 + B^2 R^4}{u^8} = \frac{u_0^5}{R^5} f(u_0)(1 + B^2 \frac{R^4}{u_0^4}). \tag{2.4}
\]

Define \( y \equiv \frac{u}{u_0}, y_{KK} \equiv \frac{u_{KK}}{u_0} \) and \( f(y) = 1 - \frac{y_{KK}}{y^5} \), we get

\[
u' = \frac{u^2 f(y)}{R^2} \frac{\sqrt{f(y)(1 + B^2 \frac{R^4}{u_0^4})y^{10}}}{(1 + B^2 \frac{R^4}{u_0^4} f(1) - 1}. \tag{2.5}
\]

Then the asymptotic distance between the D4 and anti-D4 brane is

\[
L = 2 \int_{u_0}^{\infty} \frac{d u}{u'} = \frac{2R^2}{5u_0} \int_0^1 dz \frac{z^{1/5}}{(1 - y_{KK} z)^{4/5}(1 - y_{KK} z) - z^2(1 - y_{KK})^2(1 + B^2 \frac{R^4}{u_0^4})} \tag{2.6}
\]

where \( z = y^{-5} \). So the connected point \( u_0 \) satisfies the equation

\[
u_0 = \frac{2R^2}{5L} \int_0^1 dz \frac{z^{1/5}}{(1 - y_{KK} z)^{4/5}(1 - y_{KK} z) - z^2(1 - y_{KK})^2(1 + B^2 \frac{R^4}{u_0^4})} \tag{2.7}
\]

So \( u_0 \) depends on the parameters \( B \) and \( L \). Its dependence is plotted in Fig. [1]. It shows that the joint point increases with increasing the magnetic field \( B \), and decreases as the distance \( L \) increases.

\footnote{Follow the arguments in [20], here we don’t consider the contribution of the Chern-Simons (CS) term.}
Figure 1: The connected point $u_0$ varies with the magnetic field $B$ at $L = 1$, 0.5 and 0.2 (from bottom to top). Here we choose $R = 1$ and $u_{KK} = 0.4$.

By inserting the equation (2.5) to the action (2.2), the on-shell action of the connected solution is

$$S_{\text{connected}} \sim \int_1^\infty dy \frac{y^3(1 + B^2 R_4^4 y^{-4})}{\sqrt{(1 + B^2 R_4^4 y^{-4}) f(y) - (1 + B^2 R_4^4) f(1) y^{-10}}}$$

$$\sim \int_0^1 dz \frac{1}{z^{9/5}} \frac{1 + B^2 R_4^4 z^{4/5}}{(1 + B^2 R_4^4 z^{4/5})(1 - y_{KK}^5 z) - z^2(1 - y_{KK}^5)(1 + B^2 R_4^4) z^{4/5}}$$

(2.8)

In the gravity background (1.1), there doesn’t exist separated flavor D4 and $\overline{\text{D4}}$ solution. The reason is the flavor branes don’t have any place to end in this background. If the coordinate $x^4$ is not periodic, and there is not $f(u)$ factor in this background, then the separated flavor solution will be existed [16]. Thus, the global chiral symmetry $U(N_f)_L \times U(N_f)_R$ is always broken to its diagonal part at low temperature.

This connected solution corresponds to the chiral symmetry breaking phase in the gauge theory side. It means there exists a quark condensation. Its energy scale corresponds to the length of a fundamental string connected between $u_{KK}$ and $u_0$ in the background (1.1). It reads

$$M_q = \frac{1}{2\pi\alpha'} \int_{u_{KK}}^{u_0} du \sqrt{g_{uu}} = \frac{1}{2\pi\alpha'} \int_{u_{KK}}^{u_0} \frac{du}{\sqrt{f(u)}} = \frac{u_0}{2\pi\alpha'} \int_{y_{KK}}^{1} \frac{dy}{\sqrt{f(y)}}$$

(2.9)

By inserting the equation (2.7) for $u_0$ into the equation (2.9), we get

$$M_q = \frac{2R^2}{10L\alpha'} \int_{y_{KK}}^{1} \frac{dy}{\sqrt{f(y)}}$$
\[ \int_0^1 \frac{dz}{(1 - y_{KK}^5 z^5) \sqrt{(1 + B^2 R^4 u^4 z^4)(1 - y_{KK}^5 z)^2 - z^2 (1 - y_{KK}^5)(1 + B^2 R^4 u^4)}} \]  

(2.10)

For a fixed asymptotic distance \( L \), we plot the Fig. 2 by using the same numerical way in [24]. From this figure, the condensation energy scale increases with increasing the magnetic field \( B \). Its behavior almost grows like \( B^{1/6} \). And the scale of chiral symmetry breaking becomes large with increasing the magnetic field \( B \).

3 high temperature

In the high temperature background (1.4), by using the same embedding ansatz as the low temperature case, the induced metric on the flavor D4-brane is

\[ ds^2 = \frac{u^2}{R^2} (f(u) dt^2 + \sum_{i=1}^{3} dx_i^2) + \frac{u^2}{R^2} \left( \frac{\partial x_4}{\partial u} \right)^2 + \frac{R^4}{u^4 f(u)} du^2. \]  

(3.1)

Then the D4-brane effective action is

\[ S \sim \int dx_4 \frac{u^5}{R^5} \sqrt{(1 + B^2 R^4 u^4)(f(u) + \frac{R^4}{u^4} u^2)}. \]  

(3.2)

And the equation of motion is

\[ \frac{\partial}{\partial x_4} \left( \frac{u^5}{R^5} \frac{f(u)}{f(u) + \frac{R^4}{u^4} u^2} \right) = 0. \]  

(3.3)
Like as the low temperature case, we choose a boundary condition \( u' = 0 \) at \( u = u_0 \). Then we get a first derivative equation of motion

\[
\frac{u^5 f(u)}{R^5} \left( \sqrt{1 + B^2 \frac{R^4}{u^2}} \right) = \frac{u_0^5}{R^5} \sqrt{f(u_0)(1 + B^2 \frac{R^4}{u_0^2})}.
\] (3.4)

With the definition \( y \equiv \frac{u}{u_0} \), the above equation becomes

\[
y' = u_0 \frac{y^2}{R^2} \sqrt{f(y)} \sqrt{\frac{(1 + B^2 \frac{R^4}{u_0^2})y^{10} f(y)}{(1 + B^2 \frac{R^4}{u_0^2}) f(1)}} - 1.
\] (3.5)

Similarly, the asymptotic distance \( L \) between the D4 and anti-D4 brane reads

\[
L = 2 \int_{u_0}^{\infty} \frac{du}{u' \sqrt{f(y)}} = \frac{u_0}{R^2} \int_1^{\infty} \frac{dy}{y^2 \sqrt{f(y)}} \sqrt{(1 + B^2 \frac{R^4}{u_0^2}) f(1)}
\] (3.6)

It can be written as

\[
u_0 = \frac{2R^2}{5L} \sqrt{(1 + B^2 \frac{R^4}{u_0^2})(1 - y_T^5)} \cdot \int_0^1 dz \sqrt{(1 - y_T^5) \left( 1 + B^2 \frac{R^4}{u_0^2} z^{4/5} \right) \left( 1 - y_T^{5} z \right) - z^2 \left( 1 + B^2 \frac{R^4}{u_0^2} \right) \left( 1 - y_T^5 \right) ^-},
\] (3.7)

Some numerical results of \( u_0(B, L) \) are shown in Fig. 3. It is clear that the point \( u_0 \) decreases by increasing the asymptotic distance \( L \). And this point increases as the magnetic field \( B \) increases. These results are similar to the cases at zero temperature.

Figure 3: The joint point \( u_0 \) depends on the magnetic field \( B \) at \( L = 1, 0.5 \) and 0.2 (from bottom to top). We set \( u_T = 0.3 \) and \( R = 1 \).
After substituting the equation (3.4) into the action (3.2), we get the on-shell energy of the connected D4-D4 brane solution as follows

\[ S_{\text{connected}} \sim \int_1^\infty dy \frac{y^3(1 + B^2 R^4 u_0^4 y^{-4}) \sqrt{f(y)}}{\sqrt{(1 + B^2 R^4 u_0^4 y^{-4}) f(y) - (1 + B^2 R^4 u_0^4) f(1)y^{-10}}} \]. \quad (3.8)

Now the gravity background is a black hole background (1.1), so there exists a separated D4 and anti-D4 brane solution \( u' \to \infty \). Its on-shell energy is

\[ S_{\text{separated}} \sim \int_0^\infty dy y^3 \sqrt{1 + B^2 R^4 u_0^4 y^{-4}}. \quad (3.9) \]

Then the energy difference between the connected and separated solution is

\[ \delta S \sim \int_1^\infty dy \left( \frac{y^3(1 + B^2 R^4 u_0^4 y^{-4}) \sqrt{f(y)}}{\sqrt{(1 + B^2 R^4 u_0^4 y^{-4}) f(y) - (1 + B^2 R^4 u_0^4) f(1)y^{-10}}} - y^3 \sqrt{1 + B^2 R^4 u_0^4 y^{-4}} \right) \]

\[ - \int_{y_T}^1 dy y^3 \sqrt{1 + B^2 R^4 u_0^4 y^{-4}}. \quad (3.10) \]

Its numerical result is shown in Fig. 4. The energy difference has two branches. Below some critical temperatures, the difference is negative. Now the connected solution is dominated, and means that the chiral symmetry in the gauge theory is broken. Above this critical temperature, the energy difference is positive, the separated solution is dominated and the chiral symmetry is restored. Also the critical point \( y_T \) decreases with increasing

Figure 4: The energy difference depends on \( y_T \) at different values \( B = 0, 1, 3, 8 \) and 12 (from red to blue, or the length of dashed line segment is increased).
Figure 5: The critical temperature of chiral phase transition depends on the magnetic field $B$.

the magnetic field $B$. In the unit of $1/L$, we draw the Fig. 5 which shows how the critical temperature to vary with the magnetic field $B$. Above this curve, it denotes the chiral restoration phase. Below it, this is the chiral symmetry breaking phase. The critical temperature of chiral symmetry restoration increases as the magnetic field $B$ increases. Here these results are also similar to some results in [24] and [26]. It is also consist with some already known results in field theory with a magnetic background field [27].

In the chiral symmetry broken phase, there exists a quark condensation in gauge theory side. The condensation energy scale corresponds to the string length between the connected point $u_0$ and the horizon of black hole. By using the equation (3.7), the energy scale of quark condensation is derived as

$$M_q = \frac{1}{2\pi\alpha'} \int_{u_T}^{u_0} du = \frac{u_0}{2\pi\alpha'} (1 - y_T).$$

(3.11)

Its dependence on the magnetic field $B$ is plotted in Fig. 6. Just like the low temperature cases, now the condensation also increases with increasing the magnetic field $B$. When the magnetic field is located around the region $(0, 0.5)$, its dependence is almost linear. But above this region, the dependence is similar to the low temperature case. From the equation (3.11), this quark condensation vanishes at $y_T = 1$. It corresponds to a chiral phase transition point.

4 Quark and Meson in hot QGP

In the introduction, we already discussed that turn on a magnetic field on the flavor D4-brane is equivalent to add a NS-NS field into the gravity backgrounds (1.1) and (1.4).
Figure 6: The quark condensation $M_q$ varies with the magnetic field $B$ at $u_T = 0.3$.

From the supergravity action of noncritical string action [17], this new background is still a solution. In the following, we mainly consider a fundamental string in this new gravity background with a NS-NS field (1.6).

Firstly, we consider a quark moving into the hot plasma. The quark corresponds to one endpoint of fundamental string on the flavor brane. By using the method in [28], we parametrize the world-sheet coordinates of this fundamental string as $\tau = t$ and $\sigma = u$, and assume the endpoint (quark) on the flavor brane moving along the direction $x_2$ with

$$x_2 = vt + \xi(u), \quad (4.1)$$

where $v$ is the velocity. Because of a rotational symmetry in $x_2$ and $x_3$ plane, it is equivalent to let quark move along the direction $x_3$. It is easy to see the Wess-Zumino term in the string action vanishes. So this NS-NS field (1.6) doesn’t produce any contributions on the drag force and energy loss for quark moving through this hot plasma. But in the non-trivial NS-NS background, the influence of the NS-NS background field is investigated in [26] and [29]. Usually, a NS-NS background field will decrease the drag force and energy loss of a quark moving through the hot-QGP (quark-gluon-plasma).

Now we turn to consider high spin mesons. We mainly focus on the chiral symmetry breaking phase at high temperature. Now the flavor D4 and $\overline{D4}$ are connected each other through a wormhole. The bound state of two endpoints of a spinning fundamental string on the flavor D4-$\overline{D4}$ brane pairs corresponds to a high spin meson in the boundary effective theory. Define $\rho^2 = x_2^2 + x_3^2$, we rewrite the background (1.4) as

$$ds^2 = \left(\frac{u}{R}\right)^2 (f(u)dt_E^2 + dx_1^2 + d\rho^2 + \rho^2 d\phi^2 + dx_4^2) + \left(\frac{R}{u}\right)^2 \frac{1}{f(u)} du^2,$$
\[ f(u) = 1 - \left( \frac{u_T}{u} \right)^5, \quad (4.2) \]

and the NS-NS background field is

\[ Bdx_2 \wedge dx_3 = B\rho d\rho \wedge d\phi. \quad (4.3) \]

We choose the string worldsheet coordinates as

\[ \tau = t_E, \quad \sigma = \rho, \quad u(\sigma), \quad \phi = \omega \tau. \quad (4.4) \]

So the Nambu-Goto action of this spinning fundamental string is

\[ S_{NG} = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \left( \frac{u}{R} \right)^2 \sqrt{(f(u) - \rho^2\omega^2) \left( 1 + \frac{1}{f(u)} \left( \frac{R}{u} \right)^4 u'^2 \right)} + \frac{1}{2\pi\alpha'} \int d\tau d\sigma B\rho \omega, \quad (4.5) \]

where \( \rho' = \partial_{\rho} \). Then we can obtain the equations of motion for \( u \). For simplicity, we don’t show those equations here. Set the boundary conditions as \( u' \to \infty \) at the boundary and \( u' = 0 \) at \( u = u_0 \), we plot the shape of this spinning string in the Fig. 7 and Fig. 8. (Here we only plot the zero-node cases. \( \because \) It shows the asymptotic distance for two endpoints of fundamental string decreases as the angular velocity \( \omega \) increases. And the turning point of fundamental string becomes large with increasing this angular velocity. But the influences of the NS-NS field on the string shape are not very sensitive.

From the string action (4.5), it is clear there exists two conserved quantities for this spinning string. One is the energy \( E \), the other one is the angular momentum \( J \). Their expressions are derived as

\[ E = \frac{1}{2\pi\alpha'} \int d\tau d\sigma \left( \frac{u}{R} \right)^2 \frac{f(u)\sqrt{1 + \frac{1}{f(u)} \left( \frac{R}{u} \right)^4 u'^2}}{\sqrt{f(u) - \rho^2\omega^2}}, \quad (4.6) \]

\[ J = \frac{1}{2\pi\alpha'} \int d\tau d\sigma \omega \rho^2 \left( \frac{u}{R} \right)^2 \frac{\sqrt{1 + \frac{1}{f(u)} \left( \frac{R}{u} \right)^4 u'^2}}{\sqrt{f(u) - \rho^2\omega^2}} + \frac{1}{2\pi\alpha'} \int d\tau d\sigma B\rho. \quad (4.7) \]

By doing some numerical calculations, we plot some figures to show the relations \( E^2(\omega), \ J(\omega) \) and \( E^2(J) \). The energy \( E^2 \) decreases as the angular velocity \( \omega \) increases, and it is not sensitive to different NS-NS background field value \( B \). For all the different magnetic field \( B \), the Fig. 9(a) shows that the energy of spinning string has a similar dependence on the angular velocity \( \omega \). From the Fig. 9(b), the angular momentum \( J \)

\[ ^2 \text{In plotting all the following figures, we choose } u_0 = 20 \text{ and } R = u_T = 1. \]
Figure 7: It is the string shape at $B = 0$ and $w = 1, 1.5$ and 3 (red, blue and green) from left to right.

Figure 8: It is the string shape at $w = 1, 1.5$ and 3 (red, blue and green) from left to right. (a) $B = 3$; (b) $B = 5$.

have maximums at some particular points $\omega$. As the angular velocity increases, the angular momentum $J$ also increases. However, after maximum points, it will decreases. For a larger NS-NS field $B$, the angular momentum $J$ becomes large. In Fig. 10 it is the Regge trajectory behavior $E^2(J)$, which has two branches relative to the angular momentum $J$. With increasing the angular momentum (increasing the angular velocity), the energy square $E^2$ decreases. Beyond maximums, the value $E^2$ also decreases as the angular momentum $J$ (still increasing the angular velocity) decreases. These behaviors have some differences with some results in some critical string holographic models [15], [23] and [26].
Figure 9: (a) $E^2$ varies with the angular velocity $\omega$ at $B = 0$, 3 and 5. These three curves are totally overlapped; (b) angular momentum $J$ varies with the angular velocity $\omega$ at $B = 0$, 3 and 5 (red, purple, blue) from bottom to top.

Figure 10: The energy $E^2$ varies with the angular momentum $J$ at $B = 0$, 3 and 5 from left to right (red, purple, blue).

5 Summaries

In this paper, we mainly consider a noncritical string holographic model with an external magnetic background field. We investigate the influences of this magnetic field on the underlying dynamics by using the gauge/gravity correspondence. In section two and three, we mainly consider the chiral symmetry breaking in low temperature and high temperature phase. The scale of the chiral symmetry breaking increases as the magnetic field increases. At high temperature, the critical temperature of the chiral phase transition is different at a different magnetic field. In the unit of $1/L$, this phase transition temperature increases as the magnetic field increases. Finally, we investigate high spin mesons in the chiral symmetry broken phase at high temperature. Our results here, except for
the Regge behavior, are similar to the cases in some other critical string holographic model. It confirms some universal properties of holographic models constructed through intersecting brane configurations in string theory.

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