Solar Models with Convective Overshoot, Solar-wind Mass Loss, and PMS Disk Accretion: Helioseismic Quantities, Li Depletion, and Neutrino Fluxes

Qian-Sheng Zhang1,2,3,4, Yan Li1,2,3,4, and Jørgen Christensen-Dalsgaard5,6

1 Yunnan Observatories, Chinese Academy of Sciences, 396 Yangfangwang, Guanhu District, Kunming 650216, People’s Republic of China; zqs@ynao.ac.cn(QSZ)
2 Center for Astronomical Mega-Science, Chinese Academy of Sciences, 20A Datun Road, Chaoyang District, Beijing 100020, People’s Republic of China
3 Key Laboratory for the Structure and Evolution of Celestial Objects, Chinese Academy of Sciences, 396 Yangfangwang, Guanhu District, Kunming 650216, People’s Republic of China
4 University of Chinese Academy of Sciences, Beijing 100049, People’s Republic of China
5 Stellar Astrophysics Centre and Department of Physics and Astronomy, Aarhus University, DK-8000 Aarhus C, Denmark
6 Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106-4030, USA

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Abstract

Helioseismic observations have revealed many properties of the Sun: the depth and helium abundance of the convection zone, the sound speed, and the density profiles in the solar interior. Those constraints have been used to judge the stellar evolution theory. With the old solar composition (e.g., GS98), the solar standard model is in reasonable agreement with the helioseismic constraints. However, a solar model with a revised composition (e.g., AGSS09) with a low abundance Z of heavy elements cannot be consistent with those constraints. This is the so-called “solar abundance problem,” standing for more than 10 yr even with the recent upward revised Ne abundance. Many mechanisms have been proposed to mitigate the problem. However, there is still no low-Z solar model satisfying all helioseismic constraints. In this paper, we report a possible solution to the solar abundance problem. With some extra physical processes that are not included in the standard model, solar models can be significantly improved. Our new solar models with convective overshoot, the solar wind, and early mass accretion show consistency with helioseismic constraints, the solar Li abundance, and observations of solar neutrino fluxes.

Key words: convection – Sun: abundances – Sun: helioseismology – Sun: interior

1. Introduction

Observations have revealed many properties of the Sun with high accuracy. Element abundances can be determined from the absorption line analyses of the solar atmosphere. The information of the solar interior can be extracted from helioseismology. The properties of the solar core can be probed from observations of the solar neutrino fluxes. Therefore, the Sun is the best target to benchmark the stellar evolutionary theory in detail.

1.1. The Solar Abundance Problem

The standard solar models (SSMs) based on the old solar composition—e.g., with Z/X = 0.0245 for Grevesse & Noels (1993; hereafter GN93) or 0.0229 for Grevesse & Sauval (1998; hereafter GS98)—are in reasonable agreement with the helioseismic inferences on the solar sound-speed profile, the location of the base of the convection zone (BCZ) RBCZ, and the helium abundance in the convection zone Ys (e.g., Christensen-Dalsgaard et al. 1993, 1996; Basu et al. 1997; Bahcall et al. 2001). However, by using three-dimensional hydrodynamic model atmospheres (Stein & Nordlund 1998; Asplund et al. 2000; Freytag et al. 2002) and relaxing the assumption of local thermodynamic equilibrium in the spectral line formation (Asplund et al. 2004), the resulting value of the solar metallicity has been significantly revised downward; for example, the Asplund et al. (2009; hereafter AGSS09) composition shows Z/X = 0.0181. Compared with the SSMs with GN93 or GS98 compositions, the SSM with AGSS09 compositions shows significant deviations from helioseismic inferences (e.g., Bahcall et al. 2005; Asplund et al. 2009; Serenelli et al. 2009). For example, the properties of the SSM with the AGSS09 composition (Model SSM09) are shown in Table 1. Comparing with the GS98 SSM (Model SSM98), the BCZ is shallower, and ȲS is lower. The sound-speed and density deviations resulting from helioseismic inversions are shown in Figure 1. The deviations of the sound speed of Model SSM98 are less than 0.2% in most parts of the solar interior and significant (but also less than 0.5%) only in the region 0.6 < r/R⊙ < 0.7 below the BCZ. However, Model SSM09 shows overall significant deviations of sound speed in the solar interior. The density profiles also show that Model SSM09 is worse than Model SSM98. Those significant deviations on RBCZ, ȲS, and the sound-speed and density profiles caused by the low-Z composition in SSM are called the “solar abundance problem.” More comprehensive comparisons between solar models with GS98 and AGSS09 compositions by using a Monte Carlo method taking into account the observations of solar neutrino fluxes and uncertainties of model physics confirmed the existence of the problem that the AGSS09 composition is excluded at a high confidence level when used in an SSM (e.g., Villante et al. 2014; Vinyoles et al. 2017; Song et al. 2018).

1.2. Some Attempts to Adjust Solar Models

Serenelli et al. (2009) pointed out that an opacity enhancement of 12%–15% at the BCZ to 2%–5% in the solar core can improve the sound-speed profile of the AGSS09 SSM to the level of the GS98 SSM. Similarly, an opacity enhancement of about 20% from the BCZ to about 2% in the solar core is required to improve the solar model to the level of Model S (Christensen-Dalsgaard & Houdek 2010). Comparing with the widely used OPAL or OP (Seaton 2005) opacity tables, a...
recently revised opacity table, OPAS (Mondelet et al. 2015), shows an opacity enhancement of about 6% at the BCZ. Although solar models with AGSS09 composition and an OPAS opacity table show some improvements in the sound-speed profile and \( R_{\text{bc}} \), the discrepancies cannot be efficiently removed (Le Pennec et al. 2015). A main result of the OPAS opacity table is that the individual contribution of iron to opacity is 40% higher than that in OP tables. This tendency is consistent with the measurements of iron opacity at physical conditions similar to the base of the solar convection zone (Bailey et al. 2015), which yield values 30%–400% higher than calculations. It is urgent to extend the effects of the revision on opacity to a larger range of temperature and density and more kinds of elements, as well as to understand the physical origin of the differences (e.g., Pradhan & Nahar 2018). Even so, it would be a remarkable coincidence if the errors in the opacity calculations exactly compensate the variation of solar composition.

An enhancement in the solar neon abundance could enlarge the opacity and improve solar models. It was found that the required enhancement of neon abundance is about 200%–300% (Antia & Basu 2005; Delahaye & Pinsonneault 2006; Zautri et al. 2007).

However, a higher neon abundance enlarges the discrepancy in the adiabatic exponent in the region (0.75–0.9)\( R_c \) (Lin et al. 2007). Investigation of low-activity stars has shown a correlation between their Ne/O ratio and stellar activity and suggested that a significantly enhanced solar Ne/O ratio seems unlikely (Robrade et al. 2008). Recently, Young (2018) inferred the Ne abundance in the solar photosphere by using the Mg/Ne and Ne/O ratios in the transition region of the quiet Sun and found a Ne abundance enhancement of \( \sim 40\% \), which is significantly less than the level of enhancement required to restore the solar model.

Molecular diffusion leads to heavy-element settling, which increases the opacity below the BCZ. Asplund et al. (2004) suggested that the sound speed of solar models with low-Z could be improved by enhancement of molecular diffusion. The effects of enhanced diffusion in solar models were explored (Basu & Antia 2004; Montalbán et al. 2004; Guzik et al. 2005; Yang & Bi 2007; Yang 2016, 2019). It is found that \( R_{\text{bc}} \) and the sound-speed profile can be significantly improved when the diffusion is enhanced by a factor of 1.5–2.5. However, there is no justification for such a large enhancement. And, even though \( R_{\text{bc}} \) and the sound-speed profile are improved, the helium abundance in the
convection zone $Y_S$ and the $^7\text{Be}$ and $^8\text{B}$ neutrino fluxes become worse.

Guzik & Mussack (2010) investigated low-Z solar models with mass loss. They found that sound speed, $Y_S$, and $R_{bc}$ can be improved simultaneously, and the sound-speed profile can be almost restored for a 0.3$M_\odot$ mass-loss solar model. However, $Y_S$ and $R_{bc}$ are still not in their acceptable ranges for that model. Also, such extensive mass loss strongly blows away the stellar envelope, exposing on the surface regions of the interior where lithium has been destroyed.

Guzik et al. (2005) noted that the solar envelope may be diluted by pre-main-sequence (PMS) low-Z accretion. In this case, the bulk metallicity of the Sun is higher than that of the envelope. A possible justification for the low-Z accretion is that planet formation locks high-Z elements in planets (Meléndez et al. 2009; Serenelli et al. 2011). Castro et al. (2007) and Guzik & Mussack (2010) calculated solar models with low-Z accretion near the zero-age main sequence (ZAMS). They found that the sound speed in the high-Z interior and $Y_S$ is improved, but the sound-speed deviations near the BCZ remain, and $R_{bc}$ is not significantly improved. A more detailed analysis of solar models with PMS accretion was carried out by Serenelli et al. (2011), who calculated solar models with different masses, metallicities, starting times, and durations of accretion in the PMS stage. Surprisingly, they found that some models with high-Z accretion show good agreements on sound-speed profiles and $R_{bc}$. However, those models have worse $Y_S$, and their neutrino fluxes of $^7\text{Be}$ and $^8\text{B}$ are too low due to the low-Z cores.

Montalbán et al. (2006) and Guzik & Mussack (2010) investigated the effect of convective overshoot below the BCZ on low-Z solar models. They did not find significant improvements on the sound-speed profile below the BCZ. Bi et al. (2011) and Yang (2016) calculated the low-Z solar models with rotational mixing and magnetic field and found that $Y_S$ can be improved.

Recently, von Steiger & Zurbuchen (2016) published a new solar metallicity, $Z_0 = 0.0196 \pm 0.0014$, derived from in situ measurements of the solar-wind composition (vSZ16), which is much higher than the AGSS09 composition. Serenelli et al. (2016) and Vagnozzi et al. (2017) calculated solar models with vSZ16 metallicity. They found that, although the solar models have correct $R_{bc}$, they have excessively modified sound-speed profiles, very high $Y_S$, and neutrino fluxes of $^7\text{Be}$ and $^8\text{B}$. Serenelli et al. (2016) argued that the vSZ16 metallicity based on the solar-wind measurement cannot be trusted as representative of the photosphere or the bulk Sun because of the first ionization potential (FIP) effect.

### 1.3. The Contradiction of the Structure of the Solar Convective Envelope

Zhang (2014) investigated the solar convective envelope models with AGSS09 composition. Because the gravitational energy release can be ignored for the Sun and the abundances are determined by a given $Y_S$ and $(Z/X)_S$, the structure of the solar convective envelope can be directly determined by integrating the stellar structure equations from the solar surface with a given radius and luminosity downward to the BCZ without calculations of the solar evolutionary models. For the standard model of the solar convection envelope with the old GN93 composition, the density profile, $Y_S$, and $R_{bc}$ are all in good agreement with helioseismic inferences. However, for the corresponding model with the AGSS09 composition, the density profile, $Y_S$, and $R_{bc}$ cannot be consistent with helioseismic inferences simultaneously. This is an inherent contradiction of the standard model of the solar convective envelope with AGSS09 composition, which could be part of the reason for the solar abundance problem. The profile of density $\rho$ determines the profile of pressure $P$ in the solar interior by integrating the hydrostatic equation and hence largely determines the sound speed $c_s$ with

$$c^2_s = \frac{\Gamma_1 P}{\rho},$$

given that $\Gamma_1$ is nearly constant in most of the solar interior; here $\Gamma_1 = (\partial \ln P/\partial \ln \rho)_S$ is the adiabatic index, the derivative being at constant specific entropy $S$. This contradiction could explain that the sound-speed profile, $Y_S$, and $R_{bc}$ cannot be improved simultaneously for solar models with extra physics that does not affect the input physics of the convective
envelope, e.g., enhanced molecular diffusion, mass loss, accretion, and rotational mixing.

A successful AGSS09 solar model must eliminate this contradiction. In order to do that, besides opacity enhancement, a probable mechanism has to be taken into account, i.e., the turbulent kinetic energy flux \( F_K \) below the BCZ caused by convective overshoot. The turbulent kinetic energy flux below the BCZ is negative; therefore, the equilibrium of the total flux requires a higher temperature gradient. The effect is similar to an opacity enhancement. Even if the contradiction is not completely caused by missing the turbulent kinetic energy flux, it would reduce the opacity enhancement required to bring the solar models into accordance with the observations.

For the standard model of the solar convection envelope with vSZ16 metallicity, in order to obtain the required density profile, \( Y_S \), and \( R_{bc} \), we require a positive \( F_K \), which is physically unacceptable, or a reduction of opacity near the BCZ, which is excluded by the opacity measurement (e.g., Bailey et al. 2015) and calculations (e.g., Mondet et al. 2015).

### 1.4. About This Paper

In this paper, we focus on the solar abundance problem and attempt to find a possible solution. We investigate solar models with convective overshoot, solar wind, and PMS accretion. The details of those physical process are introduced and discussed in Section 2. The resulting solar models are described in Section 3. A discussion of the results is presented in Section 4, while Section 5 provides a summary and conclusions.

### 2. Input Physics in Solar Models

Solar models are calculated using the YNEV code (Zhang 2015) with a revision to calculate accretion/mass loss with a specific composition for the accreted/lost mass. The adopted element abundances (except Ne) are based on the AGSS09 (Asplund et al. 2009) solar photosphere composition. The latest solar photosphere abundances of elements heavier than Ne are available (Grevesse et al. 2015; Scott et al. 2015a, 2015b), but the revisions are generally slight. Neon abundance in the AGSS09 (Asplund et al. 2009) solar photosphere composition is not directly measured and is based on the Ne/O ratio of the quiet Sun measured by Young (2005). Recently, the Ne/O ratio of the quiet Sun has been upward revised \( \sim 40\% \) due to the updated atomic data (Young 2018). Therefore, the AGSS09 metal composition with the revised Ne abundance was assumed as the solar composition. This composition is denoted as AGSS09Ne in this paper. The abundances of the main heavy elements and the resulting ratio of metallicity to hydrogen at the solar surface are listed in Table 2. The equation of state is interpolated from the OPAL equation-of-state tables (Rogers & Nayfonov 2002). The opacities are interpolated from the OPAL tables (Iglesias & Rogers 1996) and low-temperature opacity tables (Ferguson et al. 2005). The OPAL tables are used in the high-temperature region with \( \log T > 4.5 \), and the low-temperature opacity tables are used in the low-temperature region with \( \log T \leq 4.3 \). In the region with \( 4.3 < \log T < 4.5 \), the opacity is smoothly interpolated from the two tables by using the following formula:

\[
\log \kappa = f_\kappa \log \kappa_1 + (1 - f_\kappa) \log \kappa_2, \\
 f_\kappa = \frac{1}{2} \left\{ 1 + \sin \left( \frac{2}{\pi} \left( \frac{\log T - 4.3}{0.2} - \frac{1}{2} \right) \right) \right\},
\]

where \( \kappa_1 \) is the opacity given by the OPAL tables and \( \kappa_2 \) is given by the low-temperature tables. In this case, \( \kappa \) in the superadiabatic convective envelope of solar models is from the low-temperature opacity tables. It should be pointed out here that the interpolation scheme between high- and low-temperature opacity tables is not unique, and it may be different in different stellar evolutionary codes. Because the integral of the temperature gradient in the solar superadiabatic convective envelope is restricted by the calibration of the model radius, and the temperature gradient profile in that region is determined by the opacity profile and the value of \( \alpha_{\text{MLT}} \), the interpolation of the opacity from two tables could affect the value of the mixing-length parameter \( \alpha_{\text{MLT}} \) for solar models. Therefore, different interpolation schemes between high- and low-temperature opacity tables in different codes should lead to a variation of \( \alpha_{\text{MLT}} \). Nuclear reaction rates are adopted from SFiII (Adelberger et al. 2011), enhanced by using a weak screening (Salpeter 1954). Molecular diffusion is taken into account by solving Burgers’ equation with resistance coefficients in the screening case (Zhang 2017). The convective heat flux is calculated by using the standard mixing-length theory. The Krishna Swamy (K–S) relation between temperature and optical depth (Krishna Swamy 1966) in the solar atmosphere is adopted.

Table 2

| Element | Abundance Index |
|---------|----------------|
| C       | 8.43 ± 0.05    |
| N       | 7.83 ± 0.05    |
| O       | 8.69 ± 0.05    |
| Ne      | 8.08 ± 0.09    |
| Na      | 6.24 ± 0.04    |
| Mg      | 7.60 ± 0.04    |
| Al      | 6.45 ± 0.03    |
| Si      | 7.51 ± 0.03    |
| S       | 7.12 ± 0.03    |
| Ar      | 6.40 ± 0.13    |
| Ca      | 6.34 ± 0.04    |
| Cr      | 5.64 ± 0.04    |
| Mn      | 5.43 ± 0.04    |
| Fe      | 7.50 ± 0.04    |
| Ni      | 6.22 ± 0.04    |

Note. Abundance index is defined as \( A(W) = \log_{10}(\rho_N/\mu) + 12 \) for element W. Values of abundance indices for all elements (except Ne) come from Asplund et al. (2009), and the abundance index of Ne comes from Young (2018). The resulting \((Z/X)_S\) is derived using a Monte Carlo simulation with 1,000,000 samples with element abundances based on the suggested values and standard deviations.
2.1. The Problem Remains: The SSM with AGSS09Ne Composition

The information for SSMs with three different compositions (Model SSM09Ne for AGSS09Ne, Model SSM09 for the original AGSS09, and Model SSM98 for GS98) is listed in Table 1. The sound-speed and density deviations derived from helioseismic inversions are shown in Figure 1. It is found that, although the revised Ne abundance leads to overall improvements on the solar model, the solar abundance problem remains. The depth and helium abundance of the convection zone and the sound-speed and density profiles are still not in agreement with helioseismic inferences. This is not surprising, since the required Ne enhancement for solving the solar abundance problem is 200%–300% (Antia & Basu 2005; Delahaye & Pinsoneault 2006; Zaatri et al. 2007), but the actual enhancement of the revised Ne abundance is only about 40%.

In order to try to alleviate the solar abundance problem, we take into account some extra physical processes missed in the SSM: the convective overshoot (leads to mixing and turbulent kinetic energy flux) below the BCZ, the inhomogeneous mass loss caused by the solar wind, and PMS accretion with inhomogeneous materials. The motivation for including these processes is as follows. Although it has been shown that the turbulent kinetic energy flux could be a possible mechanism to eliminate the contradiction of the structure of the solar convective envelope (Zhang 2014), its effects on the whole solar interior model have not been investigated. Since that contradiction could be a part of the reason causing the solar abundance problem, it is necessary to investigate the effects of turbulent kinetic energy flux in solar evolutionary models. Observations have shown that the composition of the solar wind is not the same as that of the solar photosphere. Although the solar wind is currently weak, it may have affected the evolution of the solar photospheric composition, which, to our knowledge, has not been taken into account in solar evolutionary models. Here we do take into account such effects. The effects of inhomogeneous PMS accretion on solar models have been investigated (e.g., Castro et al. 2007; Guzik & Mussack 2010; Serenelli et al. 2011). They have mainly concerned the varied metallicity and have not found a satisfactory solar model. Serenelli et al. (2011) pointed out that PMS accretion with varied helium abundance could improve some properties of solar models. However, there has been no detailed investigation of that. Although PMS accretion cannot solve the solar abundance problem in solar because it cannot eliminate the contradiction of the structure of the solar convective envelope, those investigations of PMS accretion (e.g., Castro et al. 2007; Guzik & Mussack 2010; Serenelli et al. 2011) have shown that PMS accretion significantly affects the structure of the solar models. Therefore, it should be taken into account.

2.2. The Convective Overshoot Below the Solar Convection Zone

Convective overshoot leads to a turbulent kinetic energy flux $F_k$, which may contribute to resolving the contradiction that the standard solar convective envelope structure is not consistent with helioseismic inferences (Zhang 2014), and overshoot mixing below the BCZ, which is a possible mechanism for the solar Li depletion. Partial turbulent mixing caused by convective overshoot may also play a role in eliminating the bump in the sound-speed difference found for some models (see Figure 1) just below the convection zone (Christensen-Dalsgaard et al. 2018). The basic theory of convective overshoot is generally complicated. Although there are nonlocal mixing-length models for convective overshoot (e.g., Shaviv & Salpeter 1973; Maeder 1975; Bressan et al. 1981), they are excessively simplified and have been excluded by helioseismic inferences (Christensen-Dalsgaard et al. 2011). These helioseismic inferences show that the temperature gradient $\nabla$ in the solar convective overshoot region smoothly changes between $\nabla_r$ and $\nabla_m$. Nonlocal mixing-length overshoot models show nearly adiabatic $\nabla$ in the overshoot region, with $\nabla$ changing abruptly to $\nabla_m$ at the boundary of the region. At present, the required smooth profile of $\nabla$ can be obtained only by using statistical turbulent convection models (e.g., Xiong 1981, 1985, 1989; Deng et al. 2006; Li & Yang 2007). However, the closure models in those models introduce many parameters that cannot be determined by first principle. Although they are more reasonable than nonlocal mixing-length models, statistical turbulent convection models still cannot perfectly reproduce the numerical simulations. Here we introduce a simple model to deal with the convective overshoot below the base of the solar convection zone.

2.2.1. The Turbulent Kinetic Energy Flux

Because buoyancy opposes fluid motion outside the convectively unstable region (i.e., the convection zone defined by the Schwarzschild criterion), the source of the energy to support convection in the overshoot region is the kinetic energy flux that transports kinetic energy from the convective unstable region to the overshoot region. Therefore, the equation to describe the convective overshoot is the equation of transport of turbulent kinetic energy (e.g., Xiong 1981, 1985; Li & Yang 2007; Meakin & Arnett 2010),

$$\frac{\partial L_k}{\partial m_r} = -\frac{g}{\rho} \frac{\partial u^2}{\rho} - \varepsilon_{\text{turb}} = \frac{\delta g F_C}{\rho c_p T} - \varepsilon_{\text{turb}},$$

(3)

where $L_k = 4\pi r^2 F_k$ is the turbulent kinetic energy luminosity, $g = G m_r / r^2$ is the gravitational acceleration, $G$ is the gravitational constant, $m_r$ is the enclosed mass within radius $r$, $\delta = -(\partial \ln \rho / \partial \ln T)_p$, and $\varepsilon_{\text{turb}}$ is the dissipation rate of turbulent kinetic energy. The term $-g u^2 / \rho$ is the buoyancy work. The last equal sign holds because the Boussinesq approximation $\rho' / \rho = -\delta T / T$ is adopted. The physical meaning of Equation (3) in the overshoot region is obvious. The buoyancy work term is negative in the overshoot region, and $\varepsilon_{\text{turb}}$ is always positive. Therefore, the right-hand side of the equation, which is the local net turbulent kinetic energy generation rate, is always negative in the overshoot region. Integrating Equation (3) in the whole overshoot region shows a negative $L_{K,bc}$ or $F_{K,bc}$, which is the input energy flux to maintain the convective overshoot. There are three turbulent variables in Equation (3): $F_K, F_C$, and $\varepsilon_{\text{turb}}$, thus, the equation is not closed. We need two extra equations to make it possible to solve Equation (3).

Statistical turbulent convection models (Xiong 1989; Zhang & Li 2012a) have shown that $-g F_C / (\rho c_p T) \approx \eta \varepsilon_{\text{turb}}$ and $\eta$ in most of the overshoot region is basically a constant. We adopt
this property, and Equation (3) becomes
\[
\frac{\partial L_K}{\partial m_r} = -(1 + \eta) \varepsilon_{\text{turb}}. \tag{4}
\]

Analyses of turbulent convection models (Xiong 1989; Zhang & Li 2012a) have shown that the turbulent variables (e.g., k, F_K, and \( \varepsilon_{\text{turb}} \)) are basically exponentially decreasing in the overshoot region. Although these turbulent convection models may result in too simple a representation of \( F_K \), numerical simulations (Freytag et al. 1996) have also obtained exponentially decreasing turbulent variables. Therefore, we set \( L_K \) as an exponentially decreasing function as follows:
\[
L_K = L_{K,\text{bc}} \exp(x) \quad \text{for} \quad x \leq 0, \tag{5}
\]
where
\[
x = \frac{r - R_{\text{bc}}}{\theta H_P}, \tag{6}
\]
\( L_{K,\text{bc}} \) is the turbulent kinetic energy luminosity at the BCZ, \( \theta \) is a parameter, and \( \theta H_P \) is the e-folding length of \( L_K \) in the overshoot region (i.e., the scale height of \( L_K \)). Here \( \theta \) and \( L_{K,\text{bc}} \) need to be determined.

The parameter \( \theta \) can be estimated as follows. Christensen-Dalsgaard et al. (2011) showed that the length of the overshoot region with modification in \( \nabla T \) is \( L_{\text{c}} \approx 0.03 R_\odot \). Theoretical analysis of the turbulent convection model has shown that \( L_{\text{c}} \approx H_K \), where \( H_K = |dr/d \ln k| \) is the turbulent kinetic energy scale height (Zhang & Li 2012b). This property does not depend on model parameters and can be validated from other turbulent models (Marik & Petrovay 2002) and numerical simulations (Meakin & Arnett 2010), so it can be taken as a general property of convective overshoot. Equations (4) and (5) show that \( \varepsilon_{\text{turb}} \) should also be an exponentially decreasing function with the same e-folding length as \( L_{K,\text{bc}} \); thus, \( L_{K,\text{bc}} \propto \varepsilon_{\text{turb}} \). It is well known that turbulence theory shows \( \varepsilon_{\text{turb}} \propto k^{3/2}/l \); therefore, \( L_{K,\text{bc}} \propto k^{3/2} \) when the characteristic length \( l \) is a slowly varying function. Finally, we can estimate \( \theta \) as follows:
\[
\theta = \frac{1}{H_P} \frac{dr}{d \ln [L_K]} = \frac{1}{H_P} \frac{2}{3} \frac{dr}{d \ln k} = \frac{2}{3} \frac{H_K}{H_P} \approx 0.2. \tag{7}
\]
Here \( L_{K,\text{bc}} \) is a key parameter in this model. The local convection theory (i.e., mixing-length theory) predicts \( L_{K,\text{bc}} = 0 \) because it ignores the turbulent transport of turbulent kinetic energy. However, as has been discussed, the existence of overshoot requires a negative \( L_{K,\text{bc}} \) to support the energy of fluid moving in the overshoot region. At present, statistical turbulent convection models and numerical simulations show significant differences on \( L_{K,\text{bc}} \). In Xiong’s (1981) and Li & Yang’s (2007) models, the typical value of the turbulent kinetic energy flux at the bases of thick convective envelopes (for solar models or red giant models) is of order \( L_{K,\text{bc}} = -10^{-3} - 10^{-2} L_{\text{total}} \). In contrast, numerical simulations (e.g., Singh et al. 1995; Tian et al. 2009; Hotta et al. 2014; Käpylä et al. 2017) show significant turbulent kinetic energy in a convective envelope, and the \( L_{K,\text{bc}} / L_{\text{total}} \) could be as significant as \( \sim -40\% \) (e.g., Singh et al. 1995). However, the gradient type approximations adopted to model the third-order correlations in those statistical turbulent convection models could be invalid near the BCZ (Tian et al. 2009). On the other hand, these numerical simulations do not reproduce the conditions, in particular the thermal timescale, at the base of the solar convection zone and hence likely exaggerate the kinetic energy flux. At present, it is difficult to determine \( L_{K,\text{bc}} \) from the hydrodynamic equations directly. In this case, we estimate \( L_{K,\text{bc}} \) by using an indirect method to calibrate the structure of the solar convective envelope (Zhang 2014): for the given composition in the solar convection zone, we can adjust the values of \( L_{K,\text{bc}} \) and the parameter \( \alpha_{\text{MLT}} \) for the mixing-length theory to obtain a solar convective envelope that has correct \( R_{\text{bc}} \) and a density profile with the best agreement with helioseismic inferences. It is shown in Zhang (2014) that, for the original AGSS09 composition, the required ratio \( L_{K,\text{bc}} / L_{\odot} \) is about \( -0.13 \). The required \( L_{K,\text{bc}} \) for AGSS09Ne should be a little weaker, since the upward revised Ne abundance enhances the opacity near the BCZ. We have tested and found that \( L_{K,\text{bc}} = -0.13 L_{\odot} \) is suitable for the AGSS09Ne composition. Now the turbulent kinetic energy flux below the BCZ has been determined.

It is also required to investigate the turbulent kinetic energy flux in the convection zone. Due to the shortcomings of the third-order correlation models in statistical turbulent convection models and the required huge amount of numerical calculation, it is difficult to determine \( L_K \) in the solar convection zone from the analyses or simulations of hydrodynamic equations. However, \( L_K \) should not significantly affect the structure of the convection zone for the following reasons. In the deep convection zone with \( T > 10^5 \) K, convection is very efficient, so the temperature gradient is nearly adiabatic and insensitive to \( L_K \). In this region, \( L_K \) has little effect on the stellar structure because the possible variation of \( F_K \) should be exactly compensated by \( F_C \) to ensure an \( F_K \) determined by the nearly adiabatic temperature gradient. Therefore, setting \( L_K \) in the deep convection zone (\( T > 10^5 \) K) is essentially arbitrary. In the uppermost part of the convection zone, with \( T < 10^5 \) K, convection is not sufficiently efficient to ensure an adiabatic stratification, so the temperature gradient is determined by the balance between total flux, \( F_K, F_C, \) and \( F_K \), depending on \( F_K \). The stellar radius is sensitive to the temperature gradient in this thin envelope; thus, the integrated effects of the temperature gradient should be calibrated such that the radius of the solar models is consistent with observations. Therefore, variations in \( F_K \) should be compensated by a change of \( F_C \) to maintain a required integral of \( F_K \). For example, when we calculate \( F_C \) by using the mixing-length theory, a negative \( F_K \) in the uppermost part of the convection zone will lead to a larger \( \alpha_{\text{MLT}} \) parameter than the SSM. Because of the calibration on the stellar radius, the integral of the temperature gradient in the uppermost part of the convection zone should also be insensitive to the variations on \( F_K \). For this reason, we could ignore \( F_K \) in the uppermost part of the convection zone, and it should not significantly affect the stellar structure. However, it should be remembered that the modifications of \( F_K \) will affect the profile of the temperature gradient and the structure of the uppermost part of the convection zone. Based on this analysis, there is an arbitrariness in the definition of \( L_K \) within the convection zone.

Because \( L_K \) satisfies a differential equation (i.e., Equation (3)), \( L_K \) should be smooth, which means that \( L_K \) and \( dL_K / dr \) must be continuous at the BCZ. Based on the smooth nature and the arbitrariness in the definition of \( L_K \) within the convection zone, we use the following formula for
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Figure 2. Turbulent kinetic energy luminosity and overshoot mixing diffusion coefficient profiles of solar Model TWA. We set \( L_{K,0} = -0.13L_\odot \). The overshoot mixing diffusion coefficient is based on Equation (10) with \( C_X = 5 \times 10^{-4} \). The location of the BCZ is shown by the vertical dashed line.

\[ L_K \text{ within the convection zone:} \]

\[ L_K = L_{K,0} \exp \left( \frac{r}{\sqrt{x^2 + 1}} \right) \left( \log T, 5.7, 6.2 \right) \text{ for } x > 0, \tag{8} \]

where \( f \) is a smooth decaying function for the arbitrary independent variables \( y, a, \) and \( b \) as

\[ f(y, a, b) = \begin{cases} 1, & y > b \\ \frac{1}{2} + \frac{1}{2} \sin \left[ \left( \frac{y-a}{b-a} \right) - \frac{1}{2} \pi \right], & a \leq y \leq b. \end{cases} \tag{9} \]

This formula ensures that \( L_K \) is smooth at the BCZ and \( L_K \) will not increase exponentially without limit in the convection zone because \( x/\sqrt{x^2 + 1} < 1 \). The decaying function \( f \) ensures that \( L_K = 0 \) in the uppermost part of the convection zone so that the resulting \( \alpha_{MLT} \) is not affected by \( L_K \) and comparable with that in the SSM.

In the evolution of solar models, \( L_K \) in the overshoot region is calculated, based on Equation (5), as \( L_K(r) = -0.13L(r)P(r)/P_{BCZ}^{1/10} \), where \( L_r \) and \( P_r \) are the total luminosity and pressure at radius \( r \), and \( P_{BCZ} \) is the pressure at the BCZ. In the convection zone, \( L_K \) is calculated using Equation (8) with \( L_{K,0} = -0.13L_{BCZ} \), where \( L_{BCZ} \) is the total luminosity at the BCZ. The profile of \( L_K/L_\odot \) of a typical solar model is shown in Figure 2.

2.2.2. The Overshoot Mixing

Another important effect of convective overshoot is the overshoot mixing. The overshoot mixing is usually assumed to be very efficient to keep the overshoot region always homogeneous and with the same composition as the adjacent convection zone, as was tested by Zhang (2012). However, this assumption is questionable. It implies that a 0.37\( H_p \) overshoot region (Christensen-Dalsgaard et al. 2011) below the BCZ results in \( A(Li) < -4 \), which is much lower than observed (e.g., Asplund et al. 2009 shows \( A(Li) = 1.05 \pm 0.1 \)). Another viewpoint is that the overshoot mixing can be assumed to be a diffusion process, and the diffusion coefficient exponentially decreases in the overshoot region (e.g., Herwig 2000; Zhang 2013). A model for convective overshoot mixing based on the statistical turbulent convection model has been developed by Zhang (2013). It shows that the diffusion coefficient is related to the turbulent dissipation and the effect of buoyancy as

\[ D_{OV} = C_X \frac{\varepsilon_{\text{turb}}}{N_i^2} = \frac{C_X g L_K}{4\theta(1 + \eta)\pi r^2 P N_i^2}, \tag{10} \]

where \( C_X \) is a parameter much less than unity (Zhang 2013) and

\[ N_i^2 = -\frac{\delta g}{H_p}(\nabla - \nabla_{ad} - \chi \nabla_{\mu}) \quad \tag{11} \]

is the effective squared buoyancy frequency for overshoot mixing, where

\[ \nabla = \sum_i \left( \frac{\partial \ln T}{\partial X_i} \right)^2 \frac{dx_i}{x_i} \frac{d\ln P}{\ln P}; \tag{12} \]

here \( X_i \) is the abundance of the \( i \)-th species and \( \chi \) is a positive parameter of order unity based on turbulent convection models (Zhang 2013). If \( \chi = 1, N_i^2 = N_i^2 \) is exactly the squared Brunt–Väisälä frequency. We will adopt Equation (10) to calculate the overshoot mixing below the BCZ. The value of \( \chi \) depends on turbulent convection models and is difficult to determine from first principles. However, in the solar overshoot region in the solar evolution models, \( \nabla_{\mu} \) in the overshoot region is of order less than 0.01, while \( \nabla_{ad} - \nabla \) is of order 0.1. Therefore, the contribution of the \( \chi \nabla_{\mu} \) term to \( N_i^2 \) is not significant when \( \chi \) is of order unity. In this case, we ignore the contribution of the \( \nabla_{\mu} \) term, i.e., assuming \( \chi = 0 \). The effect of \( C_X \) and \( \eta \) is determined only by their combination \( C_X/(1 + \eta) \); thus, \( \eta \) is set to 0.1, which is a typical result shown in a statistical turbulent convection model (Zhang & Li 2012b).

The typical profile of \( D_{OV} \) below the BCZ is shown in Figure 2. From the BCZ downward to the solar center, \( D_{OV} \) quickly decreases near the BCZ because \( N_i^2 \) is zero at the BCZ and increases as \( r \) decreases. After a short distance away from the BCZ, \( D_{OV} \) exponentially decreases because the dissipation rate \( \varepsilon_{\text{turb}} \) exponentially decreases and \( N_i^2 \) changes much more slowly than \( \varepsilon_{\text{turb}} \). We set the parameter \( C_X = 5 \times 10^{-4} \) in order to approximately reproduce the observation of the solar Li abundance in solar models. When \( \chi \) is set to be nonzero or \( \eta \) is enlarged, \( C_X \) should be slightly enlarged to compensate for their variations.

2.3. A Helium-poor Mass Loss: The Solar Wind

The solar wind, which is not included in the SSMs, may result in an effect detectable by comparing solar models with helioseismic inferences. Wood et al. (2002) investigated mass-loss rates of solar-like stars and found that \( dM/dt \propto t^{-2.00\pm0.52} \) and that the mass-loss rate should be roughly constant before ~0.1 Gyr, corresponding to a saturation of the stellar X-ray flux; this implies that the solar wind may have been 10^3 times more massive in the early stage. Based on the present solar-wind mass-loss rate \( dM/dt = -2 \times 10^{-14}M_\odot \text{ yr}^{-1} \) (Feldman et al. 1977), the estimated typical value of the total mass loss is of the
order of magnitude of $10^{-3}$–$10^{-2}M_\odot$, which is comparable with the mass of the solar convection zone. Observations have revealed an important property of the solar wind: the composition of the solar wind is different from the photosphere, and in particular, the helium abundance in the solar wind is only about half of $Y_\odot$ (e.g., Bame et al. 1975; Reames 1995; von Steiger et al. 2000, 2010). This is called the FIP effect; i.e., the abundances of elements with a FIP less than 10 eV (e.g., Mg, Si, and Fe) are enhanced in the corona with respect to photospheric values, and high-FIP elements (e.g., O, Ne, and He) have much smaller abundance enhancements or even abundance depletions in the corona (Laming 2015). It is commonly interpreted as caused by the ponderomotive force resulting from the propagation or reflection of magnetohydrodynamic waves in the chromosphere (Laming 2015). We assume that this helium-poor property is always satisfied in the evolution of the solar wind. The typical value of the solar-wind mass loss and its helium-poor property show that taking the solar wind into account could enhance $Y_\odot$ at a level of $\sim$0.01, which is about three times the uncertainty of the helioseismically inferred $Y_\odot$. Therefore, the solar wind should be included in solar models.

We take into account the helium-poor mass loss caused by the solar wind in solar models. The abundances of the lost mass are set as

$$
(X_L, Y_L, Z_L) = \frac{(X_S, Y_S, Z_S)}{X_S + \lambda_Y Y_S + \lambda_Z Z_S},
$$

where $\lambda_Y$ is the relative efficiency of helium mass loss relative to hydrogen, and $\lambda_Z$ is the relative efficiency of heavy-element mass loss. Here $\lambda_Y = 0.5$ is adopted based on observations, $\lambda_Z = 1$ is adopted as the default, and the effects of varied $\lambda_Z$ will also be investigated. Variations of $\lambda_Z$ correspond to the possible FIP or reversed FIP effects on heavy elements escaping in the solar wind.

Since the mass-loss rate should be saturated before $\sim$0.1 Gyr (Wood et al. 2002) and the main effect of mass loss occurs in the MS stage, the adopted time-dependent mass loss starts from 0.1 Gyr, and the mass-loss rate is calculated as

$$
\frac{dM}{dt} = -Ct^\gamma.
$$

Integrating the above equation shows a total mass loss of

$$
M_L = \frac{C}{\gamma + 1} \left( t_1^{\gamma+1} - t_0^{\gamma+1} \right),
$$

where $t_0 = 0.1$ Gyr and $t_1 = 4.57$ Gyr. The total mass loss $M_L$ is a free parameter. Here $C$ and $\gamma$ are adjusted to satisfy Equation (15) and reproduce the present solar-wind mass-loss rate $Ct_1^\gamma = 2 \times 10^{-14} M_\odot$ yr$^{-1}$ (e.g., Feldman et al. 1977).

2.4. Inhomogeneous Disk Accretion in the Solar PMS Stage

Because of angular momentum conservation, circumstellar disks are an unavoidable consequence during stellar formation through gravitational collapse. Much observational evidence has shown that accretion disks are commonly found to surround T Tauri stars, and the stars are accreting from the disks (Beckwith et al. 1990; Skrutskie et al. 1990; Beckwith & Sargent 1991; Hartmann et al. 2016). The relations of mass accretion rates of T Tauri stars to stellar mass and age can be measured by using the excess hot continuum emission caused by accretion onto stars (Gullbring et al. 1997). For the solar-mass stars, the typical accretion duration is of the order of magnitude of $1$–$10$ Myr, and the typical accretion rate is $10^{-8}$–$10^{-9} M_\odot$ yr$^{-1}$ (e.g., Hartmann et al. 1998, 2016). Those lead to a typical total accreted mass of about $10^{-2}$–$10^{-1} M_\odot$ for a solar-mass star. For the Sun, the disk should exist because the minimum-mass solar nebula, i.e., the lowest mass of the disk that formed planets in the solar system, can be estimated from the planetary composition as 0.01–0.07$M_\odot$ (Weidenschilling 1977). Therefore, it is reasonable to assume that the Sun experienced disk accretion at an age less than some 10 Myr in the PMS stage with a typical accretion rate of $10^{-9}$–$10^{-8} M_\odot$ yr$^{-1}$. The adopted PMS accretion rate in solar models is a combination of the relations for accretion rate versus stellar mass and stellar age (both are given by Hartmann et al. 2016).

$$
\log \left( \frac{d(M/M_\odot)}{d(t/yr)} \right) = -1.32 - 1.07 \log(t/yr) + 2.1 \log \left( \frac{M/M_\odot}{0.7} \right),
$$

with a dispersion of 0.5 dex.

The solar PMS accretion with varied metallicity has been investigated (e.g., Serenelli et al. 2011), and no satisfactory solar model has been found. They suggested investigating PMS accretion with varied helium abundance. We now mainly consider the variation on helium abundance of the accreted material in the solar PMS stage. The effects of the variation on metallicity will also be investigated. Possible justifications for the inhomogeneous accretion may be related to planet formation and details of the accretion process. Planet formation, including the condensation of heavy elements, may lead to varied composition of the accreted material if the timescales of the accretion and planet formation processes are comparable (Meléndez et al. 2009; Serenelli et al. 2011). On the other hand, the physical processes in the accretion process may also lead to varied composition of the accreted material. Since the mechanism of PMS accretion from a protoplanetary disk is believed to be in the scenario of magnetosphere accretion (Hartmann et al. 1994, 2016) driven by the magnetorotational instability (Balbus & Hawley 1991), this mechanism requires coupling between the gas and the magnetic field. Since the temperature in the disk is low, the bulk of the disk is only weakly ionized. The coupling mainly occurs in the surface layers of the disk, where the ionization fraction is enhanced by nonthermal ionization processes such as cosmic rays and X-rays (Gammie 1996). Because these ionization processes are affected by the FIP of each element, it is possible that the element abundances in ions in the coupling region are not the same as the element abundances in neutrals. The ions could be accelerated and separated from neutrals by the ponderomotive force due to magnetohydrodynamic waves, and this process is also affected by the FIPs of elements leading to FIP or inverse FIP effects (Laming 2015). If the separation is sufficient, the accreted material should be ion-dominated, and it is possible that its composition is different from the bulk of the disk.

We take into account inhomogeneous PMS accretion in solar models. Since we mainly investigate the effects of a varied helium abundance of accreted material, we set the accreted helium abundance $Y_{acc}$ as a free parameter. The metallicity of the accreted mass is set to $Z_{acc} = 0.015$ as a default, but variations in $Z_{acc}$ will also be investigated. In the fully convective phase in the early PMS stage, the accretion is in the form of freefall so
that the composition of the accreted mass should be same as the protostar. Even if the composition of the accreted mass is different from the protostar, the protostar is also homogeneous before it develops a radiative core due to the complete convective mixing. Therefore, we let the accretion start from 2 Myr when the Sun is developing its radiative core. In this case, the “initial” abundances in our solar models may be a little different from the primordial abundances because the possible inhomogeneous accretion in the fully convective phase may change the composition of the star. The accretion duration $\tau_{\text{acc}}$ is a free parameter. A 0.5 dex dispersion for the accretion rate (Equation (16)) will be considered.

3. Results

The properties of solar models with convective overshoot, inhomogeneous mass loss caused by solar wind, and inhomogeneous PMS accretion will be investigated in this section. At first, a solar model with only convective overshoot will be discussed, then the properties of several typically improved solar models with all three extra physical processes will be described, and finally, we will investigate the effects of the parameters of those extra physical processes. Because there are six free parameters (i.e., $\lambda_Z$ and $M_L$ for solar-wind mass loss and $Y_{\text{acc}}$, $Z_{\text{acc}}$, $\tau_{\text{acc}}$, and the dispersion of the accretion rate for the inhomogeneous PMS accretion), the required number of solar models is tremendous if those parameters completely cover their ranges independently, i.e., $N^6$, where $N \sim 10$ is the typical number of sampling points for each parameter. In order to reduce the amount of numerical calculation, we classify those parameters into two sets. The primary parameters ($M_L$ and $Y_{\text{acc}}$) are varied over the full ranges. The effects of variations on $\lambda_Z$, $Z_{\text{acc}}$, $\tau_{\text{acc}}$, and the strength of the accretion rate will be investigated in turn. It reduces the number of solar models to be on the order of a magnitude of $10^3$–$10^4$, which is sustainable.

The main factor in the evaluation of solar models is the sound-speed profile. The comparisons of sound-speed and density profiles between solar models and the helioseismic inferences are done in two ways. For some representative solar models (e.g., SSMs, Model OV09Ne, and some typical improved models, e.g., Model TWA; see below), we carried out inversions for the relative differences $\delta c_s/c_s$ and $\delta \rho/\rho$ in sound speed and density between the Sun and the model, with the technique of optimally localized averages (e.g., Gough & Thompson 1991; Rabello-Soares et al. 1999) and using the so-called “best” set of observed frequencies introduced by Basu et al. (1997). For other solar models, since the number of them is huge, we compare the sound-speed or density profile between solar models and a reference solar sound-speed or density profile derived from the helioseismic inversion on Model TWA. The reference profile slightly depends on the solar model used as a reference in the inversion, but the variations are small in most of the solar interior, except in a shallow envelope with $r > 0.96 R_\odot$, when different solar models are used as reference. Therefore, the second way is a reasonable alternative that significantly reduces the time of numerical calculation. In the second way, the sound speed and density of the models are compared with the reference sound-speed and density profiles only in the solar interior, $r < 0.96 R_\odot$.

3.1. Effects of the Convective Overshoot

The basic properties of Model OV09Ne are listed in Table 1 and shown in Figure 1. The only distinction between Model OV09Ne and the standard Model SSM09Ne is that the helioseismically based convective overshoot model introduced in Section 2.2 is applied in OV09Ne. The location of the BCZ $R_{bc}$ of Model OV09Ne is 0.7155, which is significantly improved and close to the value of Model SSM98. The lithium abundance of Model OV09Ne is significant depleted to close to the observations. The surface helium abundance $Y_S$ is also improved in Model OV09Ne. All of those improvements directly result from the convective overshoot: the negative turbulent kinetic energy moves the BCZ downward, overshoot mixing contributes to lithium depletion, and mixing counters helium settling near the BCZ, thus leading to a higher $Y_S$ as shown in Figure 3.

It is no surprise that these improvements in the properties of the convection zone are found in Model OV09Ne, because the parameters ($L_{K, bc}$, $\theta$, and $C_X$) of the adopted overshoot model are actually based on the requirements for improvements on helioseismic properties and the lithium abundance. However, the helioseismically based overshoot model cannot solve the solar abundance problem in solo, since it cannot improve the sound-speed profile in the solar radiative interior; i.e., the sound-speed deviations of Model OV09Ne in the region of $r < 0.6 R_\odot$ are more significant than for the standard Model SSM09Ne, even though $R_{bc}$ is significantly improved. The reason is that, since $(Z/X)_S$ is calibrated, the overshoot mixing counters the settling of heavy elements, hence leading to a lower metallicity in the solar radiative interior as shown in Figure 3. A lower metallicity results in a lower opacity, thus making the sound-speed profile worse. On the other hand, because the lower metallicity leads to a lower $T_C$ in Model OV09Ne, its $^8$B neutrino flux is lower than that of Model SSM09Ne and close to the lower limit of the $1\sigma$ range of the $^8$B neutrino flux taking into account both the observational and theoretical uncertainties.

3.2. Improved Solar Models

We have calculated about 900 solar models with $\lambda_Z = 1$, $Z_{\text{acc}} = 0.015$, and different $\tau_{\text{acc}}$, $M_L$, and $Y_{\text{acc}}$. We have tested and
found that helium-poor accretion could improve the helioseismic properties of the solar model and vice versa. Therefore, we mainly consider helium-poor accretion so that $Y_{\text{acc}}$ varies in the range from 0.00 to 0.26 with a step of 0.02 and $M_{\text{t}}$ varies in the range from 0.00 to 0.01 with a step of 0.001. Six sample points of $\tau_{\text{acc}}$ are 8, 10, 12, 15, 20, and 30 Myr, since observations show that accretion occurs for stars with ages in the order of magnitude of 1–10 Myr. For models with accretion duration $\tau_{\text{acc}} < 8$ Myr, their properties cannot be well improved simultaneously. The total accreted masses are mainly determined by the accretion duration and slightly varying with total mass loss: 0.0462–0.0472$M_{\odot}$ for $\tau_{\text{acc}} = 8$ Myr, 0.0529–0.0540$M_{\odot}$ for $\tau_{\text{acc}} = 10$ Myr, 0.0582–0.0594$M_{\odot}$ for $\tau_{\text{acc}} = 12$ Myr, 0.0645–0.0658$M_{\odot}$ for $\tau_{\text{acc}} = 15$ Myr, 0.0723–0.0738$M_{\odot}$ for $\tau_{\text{acc}} = 20$ Myr, and 0.0829–0.0846$M_{\odot}$ for $\tau_{\text{acc}} = 30$ Myr.

Some main properties related to observations for those solar models (sound-speed deviations, $^8$B neutrino flux, location of the BCZ, and surface helium abundance) are shown in Figure 4. The abscissa are the total mass loss, and the ordinates are the helium abundance of the accreted material. The color and the black contours show the rms sound-speed deviations multiplied by 1000, the white contours show the surface helium abundance, the blue contours show the $^8$B neutrino flux multiplied by $10^{-6}$, and the purple contours show the location of BCZ $R_{\text{bc}}/R_{\odot}$. In the following, we discuss the relations between those model properties and the parameters ($Y_{\text{acc}}, M_{\text{t}},$ and $\tau_{\text{acc}}$) and offer explanations for these relations.

The surface helium abundance is positively correlated with both $Y_{\text{acc}}$ and $M_{\text{t}}$. The former is obvious. The latter arises because the helium-poor mass loss concentrates helium in the convection zone (CZ). Here $R_{\text{bc}}$ is also positively correlated with both $Y_{\text{acc}}$ and $M_{\text{t}}$. The main reason could be that, at the BCZ, opacity is anticorrelated with $Y_S$ because both hydrogen and helium are fully ionized and hydrogen is more efficient in contributing electron scattering opacity than helium. The rms sound-speed deviation is correlated with $R_{\text{bc}}$ for $R_{\text{bc}} > 0.71R_{\odot}$ and anticorrelated with $R_{\text{bc}}$ for $R_{\text{bc}} < 0.71R_{\odot}$. This is because the rms sound-speed deviation is mainly contributed by the sound-speed deviation in about $0.6 < r/R < 0.7$ below the BCZ, and the deviation in that region is strongly affected by $R_{\text{bc}}$. It is shown that, for all solar models, models with $R_{\text{bc}} = 0.71R_{\odot}$ show the best sound-speed deviation. This is a little deeper than the helioseismic inference $R_{\text{bc}} = 0.71R_{\odot}$ (Christensen-Dalsgaard et al. 1991; Basu & Antia 1997). The model $R_{\text{bc}}$ should be improved if the temperature gradient modification caused by $F_C$ overshoot is taken into account. When the negative turbulent heat flux in the overshoot is taken into account, it will enhance the temperature gradient below the BCZ (e.g., Xiong & Deng 2001; Zhang & Li 2012a). Therefore, in order to keep a sound-speed profile, the location of the BCZ should move upward.

The $^8$B neutrino flux is anticorrelated with $Y_{\text{acc}}$ and weakly correlated with $M_{\text{t}}$. A lower $Y_{\text{acc}}$ requires a lower $X_S$ for the calibrations of the solar model, which leads to a higher central temperature because of the lower central hydrogen abundance and the calibration of model luminosity. A higher central temperature leads to a higher $^8$B neutrino flux. The correlation with $M_{\text{t}}$ arises because the helium-poor mass loss slightly depletes heavy elements in the CZ (i.e., see Equation (13); $Z_C < Z_S$ for the case of $X_Y = 0.5$ and $X_Z = 1$), thus leading to a slightly higher $Z_C$ because of the calibration of $(Z/X)_S$ and hence to a slightly higher $^8$B neutrino flux.

Comparing each panel in Figure 4, it is found that, for models with a longer $\tau_{\text{acc}}$, the effects of $Y_{\text{acc}}$ are more significant. The main effect of the helium-poor PMS accretion is to lead to a helium-abundance gradient and a helium-poor envelope for the ZAMS model. For a model with a longer $\tau_{\text{acc}}$, the helium-abundance gradient occurs in a more extended region owing to the continued retreat of the convective envelope, which also reduces the mass of the convective envelope and hence amplifies the effect of the helium-poor accretion, enhancing the reduction in the helium abundance in the envelope. Therefore, a longer $\tau_{\text{acc}}$ amplifies the effects of the helium-poor accretion.

Since the solar sound-speed profile strongly constrains the solar models, we are mainly concerned with the sound-speed deviations. For those models with helium-poor accretion, the sound-speed profiles can be significantly improved. For the solar models with improved sound-speed profiles with rms deviations less than 0.1%, their $^8$B neutrino fluxes are consistent with observations in the $1\sigma$ observational and theoretical uncertainty. Helioseismic inference on surface helium abundance suggested that $Y_S = 0.245–0.252$ (Basu & Antia 2004). For each $\tau_{\text{acc}}$, solar models with the best sound-speed profiles and in the required range of $Y_S$ have been obtained by adjusting the model parameters $M_{\text{t}}$ and $Y_{\text{acc}}$ along the contours of $Y_S = 0.245$ in Figure 4. The information on the best model for each $\tau_{\text{acc}}$ is shown in Table 3, and their sound-speed and density deviations are shown in Figure 5. The sound-speed and density profiles are significantly improved. The sound-speed deviations of those models with $\tau_{\text{acc}} \geq 10$ Myr are basically less than 0.1%. We therefore take Model TWA (with $\tau_{\text{acc}} = 12$ Myr) as an example to investigate the reason for the improvements in the sound-speed profile.

### 3.2.1. Evolution of the Helium-abundance Profile

The difference between Models TWA and OV09Ne is the inclusion of inhomogeneous accretion and mass loss. The main effect of both processes is on the evolution of abundance profiles. Therefore, it is natural to start the investigation of the properties of Model TWA from its evolution of abundance profiles. The evolution of the helium-abundance profile in the interior of solar Model TWA is shown in Figure 6(a). The Sun is fully convective before 2 Myr, and the helium abundance is homogeneous, as shown with the dashed line. From 2 Myr, the Sun is developing its radiative core, and the convective envelope is retreating. The helium-poor accretion and retreating convective envelope result in a negative helium-abundance gradient $dY/dr < 0$ in the interior, shown as the purple (4 Myr) and cyan (12 Myr) lines. After the helium-poor accretion stops (at 12 Myr) and before the solar wind becomes active (at 0.1 Gyr), the effect of molecular diffusion is not significant, owing to its long characteristic timescale; thus, the mantle, which we define roughly as the radiative region with $0.3 < r/R_{\odot} < 0.7$, and the convective envelope stay homogeneous, shown as the blue line for an age of 0.1 Gyr. From 0.1 Gyr, the helium-poor mass loss caused by the solar wind is active; thus, helium is concentrated in the convective envelope, and the helium abundance in the convective envelope is higher than that in the mantle, shown as the green line for an age of 0.4 Gyr. After about 0.33 Gyr, the mass loss decays, and the helium settling caused by the molecular diffusion dominates the helium-abundance profile in the solar mantle and envelope, shown as red and black lines.
Figure 4. Properties of solar models with different duration of accretion $\tau_{acc}$, total mass loss $M_L$, and helium abundance of accreted material $Y_{acc}$. Colors and black contours represent rms deviations of squared sound speed $\sqrt{(\Delta c_s/c_{ss,ref})^2}$ in $r < 0.96 R_e$ multiplied by 1000, where $\Delta c_s = c_{ss,ref} - c_s$ and $c_{ss,ref}$ is the reference solar sound speed inferred from helioseismic inversion based on Model TWA. Purple contours represent the location of the BCZ $R_{bc}/R_e$. Blue contours represent neutrino fluxes $F_B/10^6$. White contours represent $Y_S$. The Astrophysical Journal, 881:103 (26pp), 2019 August 20 Zhang, Li, & Christensen-Dalsgaard.
It is shown that there is a helium-abundance bump during the evolution of the helium-abundance profile (e.g., at about \( r = 0.6 \bar{R}_0 \) for the 2 Gyr case). It is caused by the combined effects of mass loss, molecular diffusion, and convective overshoot mixing. Helium settling caused by molecular diffusion forces the region with \( dY/dr > 0 \) in the helium profile, which is caused by helium-poor mass loss, to move downward. Therefore, the strength of overshoot mixing, which decays with depth, in that region becomes weaker and weaker and cannot completely remove the positive gradient on the helium profile. Although helium settling can reduce the helium gradient, the effect is slow because its timescale is too long. Therefore, the positive gradient in the helium profile remains on a timescale of \( \sim 1 \) Gyr. On the other hand, the molecular diffusion causes a negative gradient in the helium profile below the convective overshoot region. Those lead to a helium-abundance bump at about \( 0.6 \bar{R}_0 \). The helium-abundance bump leads to a region with \( \nabla \mu < 0 \), where \( \nabla \mu \) is defined in Equation (12), which results in thermohaline instability. Thermohaline mixing is not taken into account in the TWA solar model. Its effect will be discussed later.

The evolution of the helium abundance \( Y_S \) at the surface of Model TWA is shown in Figure 6(b). At the helium-poor accretion stage, \( Y_S \) is decreasing due to the accumulation of the helium-poor material in the convective envelope. After the end of the accretion and before the solar wind gets active, \( Y_S \) is slightly reduced by helium settling. From 0.1 Gyr, the helium-poor mass loss concentrates helium in the convective envelope so that \( Y_S \) is increasing. However, the solar wind significantly decreases with age. At about 0.33 Gyr, \( Y_S \) reaches its maximum and then decreases due to helium settling.

### 3.2.2. Sound-speed Profile

Assuming as an approximation an ideal gas, it follows from Equation (1) that the sound speed depends on temperature and abundances as

\[
\frac{c_s^2}{\mu} \approx \Gamma_1 \frac{k_B T}{m_a \mu},
\]

where \( k_B \) is Boltzmann’s constant, \( m_a \) is the atomic mass unit, and \( \mu \) is the mean molecular weight. The differences of sound speed, temperature, and mean molecular weight between Model TWA and Model SSM09Ne are shown in Figure 7, which helps to understand how the sound speed of Model TWA is improved.

In the convection zone, the \( Y_S \) of Model TWA is 0.245, higher than that for Model SSM09Ne. This leads to a higher mean molecular weight, shown as the dashed line. Accordingly, the temperature in the convection zone of Model TWA is also higher than that of Model SSM09Ne because the sound speed in the convection zone is well defined by the hydrostatic equation and polytropic relation (e.g., Christensen-Dalsgaard 1986); thus, Model TWA requires a higher temperature to compensate for its higher mean molecular weight. The direct reason for the higher temperature in the convection zone of Model TWA is as follows. The higher mean molecular weight leads to a higher density to maintain the pressure; thus, the pressure scale height \( H_P \) is smaller than that for Model

### Table 3

| Model | Best08 | Best10 | TWA | Best15 | Best20 | Best30 |
|-------|--------|--------|-----|--------|--------|--------|
| \( \alpha_{\text{MLT}} \) | 2.3560 | 2.3711 | 2.3708 | 2.3697 | 2.3707 | 2.3627 |
| \( X_0 \) | 0.7087 | 0.7082 | 0.7096 | 0.7114 | 0.7130 | 0.7134 |
| \( Z_0 \) | 0.01469 | 0.01471 | 0.01472 | 0.01472 | 0.01474 | 0.01488 |
| \( X_C \) | 0.3485 | 0.3472 | 0.3485 | 0.3503 | 0.3519 | 0.3522 |
| \( Z_C \) | 0.01569 | 0.01571 | 0.01572 | 0.01572 | 0.01574 | 0.01589 |
| log \( T_C \) | 7.1924 | 7.1928 | 7.1925 | 7.1922 | 7.1919 | 7.1919 |
| log \( \alpha \) | 2.1845 | 2.1870 | 2.1857 | 2.1839 | 2.1823 | 2.1813 |

Neutrino fluxes

| Reaction | \( (10^{10}) \) | \( (10^9) \) | \( (10^8) \) | \( (10^7) \) | \( (10^6) \) |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \nu_e \) | 5.97 | 5.97 | 5.98 | 5.98 | 5.98 |
| \( \nu_x \) | 1.47 | 1.47 | 1.47 | 1.47 | 1.47 |
| \( \nu_x \) | 8.14 | 8.12 | 8.13 | 8.14 | 8.15 |
| \( \nu_B \) | 4.83 | 4.87 | 4.84 | 4.80 | 4.77 |
| \( \nu_B \) | 5.10 | 5.18 | 5.13 | 5.05 | 4.99 |
| \( \nu_B \) | 2.18 | 2.20 | 2.19 | 2.17 | 2.16 |
| \( \nu_B \) | 1.62 | 1.65 | 1.63 | 1.61 | 1.60 |
| \( \nu_B \) | 3.57 | 3.62 | 3.59 | 3.54 | 3.50 |

Note. See notes for Table 1.
SSM09Ne. This leads to a higher $|d \ln T/dr|$ in Model TWA because $|d \ln T/dr| = \nabla / H_p$.

In the solar radiative mantle, the mean molecular weight in Model TWA is lower than that in Model SSM09Ne because the PMS helium-poor accretion leaves a helium-poor solar mantle. Below the BCZ, the bumps of the temperature difference $\delta \ln T$ around $0.6 < r/R_\odot < 0.7$ are caused by the negative turbulent kinetic energy flux enhancing the temperature gradient. Going downward, the differences in the temperature decrease. The lower mean molecular weight and higher temperature in Model TWA result in a higher sound speed in the solar radiative mantle than in Model SSM09Ne.

In the solar core, the main contribution to $\delta \ln c_s^2$ is $\delta \ln \mu$. The helium abundance in the core in Model TWA is higher than that in Model SSM09Ne due to the helium-poor accretion leaving a negative helium-abundance gradient. Thus, the mean molecular weight is higher than that in Model SSM09Ne, resulting in a lower sound speed.

The overall effect is that the sound speed in the mantle of Model TWA is higher than that in Model SSM09Ne, and the sound speed in the core of Model TWA is lower than that in Model SSM09Ne. This almost exactly compensates for the differences of sound speed between Model SSM09Ne and helioseismic inferences. As shown in Figure 5(a), the maximum deviation of the sound speed in Model TWA is only about 0.1% in $r < 0.95 R_\odot$. Comparing the temperature and mean molecular weight modifications with the sound-speed modification, it can be found that the modification of the mean

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**Figure 5.** Sound-speed (a) and density (b) deviations from helioseismic inversions (similar to Figure 1) of some improved solar models.

**Figure 6.** Evolution of helium abundance. (a) Helium profiles in Model TWA at various ages. (b) Surface helium abundance $Y_S$ of Model TWA as a function of age. In panel (b), the stages of the $Y_S$ evolution are split by the vertical lines at age 12 Myr, 0.1 Gyr, and 0.3258 Gyr. The mechanism dominating the evolution of $Y_S$ in each phase is indicated.

**Figure 7.** Differences in squared sound speed, temperature, and mean molecular weight between solar models TWA and SSM09Ne.
molecular weight in the solar radiative interior is the main factor for the improvements in the model sound-speed profile, and the temperature modification is significant only in the overshoot region.

Once the density profile is given, the mass profile can be determined by integrating the density profile, then the pressure profile can be determined by integrating the hydrostatic equation. Since \( \Gamma_1 \) is almost constant in most of the solar interior, according to Equation (1), the sound-speed profile is also determined, and hence the sound-speed profile is related to the density profile. Therefore, the density profile of Model TWA is also in good agreement with helioseismic inferences at a level of less than 1\%, as shown in Figure 5(b).

Differences in sound speed and mean molecular weight between other improved models with different \( \tau_{\text{acc}} \) (Best10, Best15, Best20, and Best30) and Model TWA are shown in Figure 8. It is found that the main factor contributing to the improvements in their sound-speed profiles is also the mean molecular weight, as for Model TWA. Since the differences in mean molecular weight represent the differences in the model helium-abundance profiles, the result that the sound-speed profiles of those improved models are consistent with helioseismic inferences could help to investigate the solar helium-abundance profile.

The helium-abundance profiles of improved solar models and Models OV09Ne and SSM09Ne at the present solar age are shown in Figure 9(a). The improved models have steeper abundance profiles in the solar core and a helium-reduced solar mantle. Both distinctions result from the inhomogeneous PMS accretion. Since the accretion stops before the ZAMS, and other mechanisms to affect element abundances (i.e., nuclear reactions and diffusion) have little effect before the ZAMS, the effects of the inhomogeneous PMS accretion will be clear when we investigate the abundance profile of the ZAMS models. The helium-abundance profiles of the improved solar models (Best10, TWA, Best15, Best20, and Best30) and Models OV09Ne and SSM09Ne are shown in Figure 9(b). At the ZAMS, Models OV09Ne and SSM09Ne have nearly homogeneous helium-abundance profiles, and each improved model has a helium-poor envelope and a helium-abundance gradient region between the convective core and the envelope, resulting from the helium-poor accretion and the retreat of the convective envelope. Because the accretion rates of those models are the same, the helium-abundance gradient is determined by \( Y_{\text{acc}} \), such that a lower \( Y_{\text{acc}} \) leads to a steeper gradient. The boundary of the helium-abundance gradient for each ZAMS model is the location of the BCZ when the accretion stops, denoted by triangles in Figure 9(a).

The differences in the helium-abundance profiles among the improved models are not significant (less than 0.003 in most parts of the solar mantle). A characteristic of those models is that their helium abundance in most of the mantle (for \( 0.3 < r/R_\odot < 0.65 \)) is about 0.01 less than that for Model SSM09Ne. Since those models are selected from many models with varied parameters, it implies that a solar model with this characteristic could have an improved sound-speed profile.
3.2.3. Neutrino Fluxes

Besides helioseismology, observations of the solar neutrino fluxes place constraints on the properties of the solar core. As shown in Table 1, when the uncertainties of the observations and models are taken into account, the $pp$ chain neutrino fluxes of Models SSM09Ne and OV09Ne and the improved models are all in acceptable ranges compared with observations. Since $^7\text{Be}$ and $^8\text{B}$ neutrino fluxes more strongly depend on temperature, and the luminosity of solar models is calibrated, higher $T_C$ leads to higher $^7\text{Be}$ and $^8\text{B}$ neutrino fluxes and lower $pp$ and $\text{pep}$ neutrino fluxes (see, e.g., Bahcall & Ulmer 1996; Serenelli et al. 2011), which is consistent with the relation between $T_C$ and the neutrino fluxes shown in Tables 1 and 3.

3.2.4. Lithium Abundance

The observed solar Li abundance provides another constraint on the evolution and structure of solar models. The solar Li depletion is thought to be related to mixing below the solar convection zone (e.g., Schlattl & Weiss 1999; Xiong & Deng 2002, 2009; Zhang 2012). The Li abundances of the improved models show significant depletion compared with Model SSM09Ne and are close to the observation, as shown in Table 1. This is because we set an appropriate value for the overshoot mixing parameter $C_X$. The evolution of the Li abundance of the SSM and improved models is shown in Figure 10. There are two main differences between the improved models and the SSM: the improved models show more Li depletion in the PMS stage (i.e., at an age less than about 0.01 Gyr), as well as Li depletion in the MS stage, but the SSM does not. Both effects are due to convective overshoot: the former is because the negative turbulent kinetic energy flux makes the BCZ deeper (Zhang 2014), and the latter is because of the overshoot mixing.

Figure 10 shows that the Li abundance is increasing at the later stage of the PMS accretion. Lithium is not depleted in the accreted material, and the convective envelope is retrograding so that the mass of the envelope is decreasing. Therefore, the contribution to the envelope of the accreted material with no Li depletion is increasing and results in Li enrichment in the convective envelope. A model with larger $\tau_{\text{acc}}$ shows more Li enrichment in the envelope because its mass of the envelope at the later stage of accretion is less due to the retreat of the convective envelope; thus, this Li-enrichment mechanism is more efficient.

3.3. Effects of Parameters of Accretion and Mass Loss

3.3.1. The Strength of the Accretion Rate

We have calculated about 2000 solar models to investigate the effects of the strength of the accretion rate. This is varied by multiplying the accretion rate given in Equation (16) by a factor $10^n$, corresponding to the 0.5 dex dispersion of the formula; $\eta$ varies in the range from $-0.5$ to 0.5 with a step of 0.1, and $Y_{\text{acc}}$ varies in the range from 0.00 to 0.26 with a step of 0.02. Seven sample points of $M_L$ are considered: 0, 0.004, 0.007, 0.010, 0.015, 0.020, and 0.030 $M_\odot$. The default values $\lambda_Z = 1$ and $Z_{\text{acc}} = 0.015$ are adopted. Two cases of $\tau_{\text{acc}}$, i.e., 12 and 20 Myr, have been investigated.

We introduce a “lacking helium mass,” defined as

$$m_{Y,\text{lack}} = \frac{1 - X_0 - Z_0 - Y_{\text{acc}}}{X_0 + Z_0 + Y_{\text{acc}}} m_{\text{acc}}(t),$$  

which represents the lack of the mass of helium at age $t$ during the helium-poor accretion compared with a homogeneous accretion. Here $m_{\text{acc}}(t)$ is the accreted mass at age $t$. The total accreted mass $M_{\text{acc}} = m_{\text{acc}}(\tau_{\text{acc}})$ satisfies with $\tau_{\text{acc}}$ and $t$ in Myr,

$$M_{\text{acc}} = M_L - \left[ \left( 1 + \frac{M_L}{M_\odot} \right)^{1.1} - 0.607 \times 10^9 \left( \frac{1}{\tau_{\text{acc}}} - \frac{1}{\tau_0} \right) \right]^{1/\nu},$$

which is derived from the adopted accretion rate (Equation (16)), the presence of the solar-wind mass loss, and the final mass equaling the solar mass. The total lacking helium mass is defined by $M_{Y,\text{lack}} = m_{Y,\text{lack}}(t = \tau_{\text{acc}})$. Because $M \propto t^{-1.07}$, we obtain

$$m_{\text{acc}} = \frac{M_{\text{acc}}}{\tau_{\text{acc}}^{-1.07} - 2^{-0.07}}$$

and

$$M_{Y,\text{lack}}^{-1} \frac{dm_{Y,\text{lack}}}{dt} = M_{\text{acc}}^{-1} \frac{dm_{\text{acc}}}{dt} = -0.07 \tau_{\text{acc}}^{-1.07} - 2^{-0.07}. \tag{21}$$

For those solar models with given $\tau_{\text{acc}}$ and $M_L$, there are two free parameters: $\eta$ and $Y_{\text{acc}}$. Therefore, the properties of each of those models should be determined by the values of the two parameters. However, as shown by the symbols in each color in Figure 11, if the total lacking helium mass is adopted as the independent variable, the properties of the solar models show little dispersion. Therefore, the effect of $\eta$ and $Y_{\text{acc}}$ can be combined in $M_{Y,\text{lack}}^{-1}$. This can be explained as follows. For a ZAMS stellar model, nuclear fusion dominates the energy release; thus, the gravitational and thermal energy release, which is the only time-dependent term in the stellar structure equations, is negligible. Therefore, the structure of the ZAMS stellar model is mainly determined by its interior abundance profiles, and it basically cannot remember the details of its PMS stage. For a given $\tau_{\text{acc}}$, Equation (21) shows that the evolution of the lack of helium mass $dm_{Y,\text{lack}}/dt$ is determined.
only by $M_{Y,\text{lack}}$. Because the accreted material is mixed with the material in the convective envelope, the variation $\Delta Y_S$ of the helium abundance in the convective envelope during a time step $\Delta t$ is determined by the adding of lacking helium mass in the time step such that $\Delta Y_S/\Delta t \approx -(dM_{Y,\text{lack}}/dt)/m_{\text{CZ}}$. Therefore, the evolution of $Y_S$ in the accretion stage is determined mainly by $M_{Y,\text{lack}}$, since $dM_{Y,\text{lack}}/dt$ is determined only by $M_{Y,\text{lack}}$ and $m_{\text{CZ}}$ is insensitive to the accretion. Since the convective envelope retreats in the PMS stage, the variation of helium abundance in the convective envelope will determine the helium-abundance profile in the radiative region below the convective envelope. Therefore, the structure of the ZAMS models is mainly determined by $M_{Y,\text{lack}}$ for a given $\tau_{\text{acc}}$.

An exceptional model property for which the effects of $\eta$ and $Y_{\text{acc}}$ cannot be combined into $M_{Y,\text{lack}}$ is the surface lithium abundance of solar models. This is because lithium in the convective envelope is significantly depleted in the PMS stage due to the high temperature at the BCZ. Since accretion refreshes the lithium in the convective envelope, the evolution of lithium abundance is directly affected by the strength of the accretion rate described by $\eta$ but not by $M_{Y,\text{lack}}$.

For example, three solar models with the same $M_{Y,\text{lack}}$, $M_*$, and $\tau_{\text{acc}}$ as Model TWA but different $\eta$ and $Y_{\text{acc}}$ are compared with Model TWA. The information in those models is listed in Table 4, and their sound-speed deviations and lithium-abundance evolution are shown in Figure 12. Their sound-speed deviations are very close to that of Model TWA, and the differences in the deviations between those models and Model TWA are less than 0.03%, implying that their interior structures are very close to that of Model TWA. On the other hand, the lithium abundances of those models show significant differences. For a model with a stronger accretion rate (a larger $\eta$), the effect of the accretion of material with no Li depletion refreshing the lithium abundance is more significant.

Since the parameters $\eta$ and $Y_{\text{acc}}$ affect most of the model properties only by their combination function $M_{Y,\text{lack}}$, an enhancement of the strength of the accretion rate can be compensated for by an enhancement of the accreted helium.
Table 4  
Basic Information of Some Solar Models with Different \( \eta \) Compared with Model TWA

| \( \rho \text{M} \) | \( X_0 \) | \( Z_0 \) | \( X_C \) | \( Z_C \) | \( \log T_C \) | \( \log \rho_C \) | \( \tau_{\text{acc}}/\text{Myr} \) | \( \eta \) | \( M_{\text{acc}}/M_\odot \) | \( Y_{\text{acc}} \) | \( M_{\text{acc}}/M_\odot \) | \( Y_{\text{acc}} \) | \( Z/\text{X}_\odot \) | \( R_{\text{eff}}/R_\odot \) | \( A(\text{Li}) \) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 | TWA | E2 | E3 | E1 | TWA | E2 | E3 | E1 | TWA | E2 | E3 | E1 | TWA | E2 | E3 | E1 | TWA | E2 | E3 |
| 2.3651 | 2.3708 | 2.3739 | 2.3692 | 0.7105 | 0.7096 | 0.7088 | 0.7080 | 0.01472 | 0.01472 | 0.01471 | 0.01466 | 0.3493 | 0.3485 | 0.3482 | 0.3487 | 0.01572 | 0.01572 | 0.01571 | 0.01567 | 7.1924 | 7.1925 | 7.1926 | 7.1924 | 2.1849 | 2.1857 | 2.1861 | 2.1857 |

Note. The accretion rate has been multiplied by a factor of \( 10^\eta \) (see Equation (19)). See notes for Table 1.

abundance, keeping a fixed \( M_{\text{f, acc}} \). Therefore, we can eliminate a free parameter and then reduce the number of required solar models by setting \( \eta = 0 \) without loss of generality. It should be mentioned that setting a fixed \( \eta \) may reduce the range of \( M_{\text{f, acc}} \) because of \( Y_{\text{acc}} \geq 0 \).

3.3.2. The Metallicity of the Accreted Material

In order to investigate the effects of varied \( Z_{\text{acc}} \), we have calculated about 1800 solar models with \( \eta = 0 \), \( \lambda_2 = 1 \), \( \tau_{\text{acc}} = 12 \text{ Myr} \), and different \( Z_{\text{acc}} \), \( M_{\text{L}} \), and \( Y_{\text{acc}} \). Here \( Y_{\text{acc}} \) varies in the range from 0.00 to 0.26 with a step of 0.02, \( M_{\text{L}} \) varies in the range from 0.00 to 0.01 with a step of 0.001, and \( Z_{\text{acc}} \) varies in the range from 0.005 to 0.025 with a step of 0.002. The cases of \( Z_{\text{acc}} = 0.010 \) and 0.020 are also calculated.

The properties (sound-speed deviations, \(^{8}\text{B} \) neutrino flux, location of the BCZ, and surface helium abundance) of those solar models with \( Z_{\text{acc}} = 0.005, 0.010, 0.020, \) and 0.025 are shown in Figure 13. The case of \( Z_{\text{acc}} = 0.015 \) is shown in Figure 4(c). Solar models with other \( Z_{\text{acc}} \) are not shown. The properties of those solar models are continuously varied from \( Z_{\text{acc}} = 0.005 \) to 0.025. Comparing Figure 13 with Figure 4(c), it is found that there are two main effects of varied \( Z_{\text{acc}} \). One is that a higher \( Z_{\text{acc}} \) significantly reduces the \(^{8}\text{B} \) neutrino flux of solar models. The second is that the required degree of helium depletion in PMS accretion for improving the helioseismic quantities of solar models is reduced when a higher \( Z_{\text{acc}} \) is used. Now we investigate the reasons for these effects.

Solar models (ZA05, ZA10, ZA20, and ZA25) similar to Model TWA but with different \( Z_{\text{acc}} \) are compared with Model TWA to investigate the effects of varied \( Z_{\text{acc}} \). Information on those models is shown in Table 5, and the sound-speed deviations are shown in Figure 14. The ZA models are the selected best models with the minimum rms sound-speed deviation and \( Y_{\text{S}} \) in the helioseismically inferred range. The main differences between ZA models and Model TWA are the abundance profiles shown in Figure 15. Because the \( (Z/X)_\odot \) of the models is calibrated to 0.0188 and the efficient mixing in the convective envelope, a reduction of the initial metallicity \( Z_0 \) is required to compensate for an enhanced \( Z_{\text{acc}} \), as shown by \( Z_0 \) in Table 5 and the metallicity in the core of the ZAMS models in Figure 15. Between the convective core boundary and the mass layer, which is the BCZ at age \( t = \tau_{\text{acc}} \), there is a range with a gradient of metallicity, and the gradient is determined by \( Z_{\text{acc}} \) such that \( Z_{\text{acc}} > 0.015 \) leads to \( dZ/dr > 0 \) and vice versa. Although the metallicity in the core will increase during the evolution because of molecular diffusion and CNO cycle nuclear reactions, the differences of the core metallicity among those ZAMS models remain in the models with the present solar age. Therefore, the \( Z_{\text{C}} \) of those solar models is anticorrelated with \( Z_{\text{acc}} \). A higher \( Z_{\text{C}} \) leads to a higher temperature gradient and then
a higher temperature in the core. Therefore, it leads to higher $^7{\text{Be}}$ and $^8{\text{B}}$ neutrino fluxes. A consequence of the higher $T_C$ is a lower $X_C$ and then a lower $X_0$ and a higher $Y_0$. Therefore, the required $Y_{\text{acc}}$ is lower for improving the sound-speed profile. In a word, a lower $Z_{\text{acc}}$ leads to a higher $Z_C$; thus, the $^8{\text{B}}$ neutrino flux is higher, and the required $Y_{\text{acc}}$ is lower. This explains the properties shown in Figure 13.

We recall that the interior structure of a main-sequence (MS) stellar model is completely determined by the abundance profiles. On the other hand, for a solar model, there are four conditions at the stellar surface, i.e., luminosity, radius, temperature, and density. The number of conditions equals the number of stellar structure equations. In this case, the interior structure of a solar model can be integrated from the surface to the center when its abundance profiles are given. This means that the envelope structure of a solar model is completely determined by the abundance profiles in the envelope, even if the abundance profiles in the core are unknown. Thus, if the sound-speed profile of a solar envelope model is constrained by the helioseismic inferences, the helium-abundance profile and the metallicity profile in the envelope should be in one-to-one correspondence, such that fixing one of the profiles essentially determines the other. For the ZA solar models, the abundance profiles in $r > 0.3R$ are basically identical to that of Model TWA because they have the same abundances at the surface (i.e., $X_S \approx 0.245$ and $(Z/X)_S = 0.0188$). As shown in Figure 14, the sound-speed profiles of ZA models and Model TWA are in very good agreement with the helioseismic inferences, implying that the $Y$ and $Z$ profiles of Model TWA are suitable for each other, given the helioseismic constraint on sound speed. It should be pointed out here that the correspondence between the $Y$ and $Z$ profiles is affected by the detailed input physics involved in the structure equations of the envelope, e.g., opacity, equation of state, and the model of convection overshoot that affects the profiles of $L_K$ and $L_C$.

As shown in Figure 14, the differences between the sound-speed deviations of those solar models are quite small even in the core, where the abundance profiles of those models are quite different. The possible reason is the anticorrelation between $X_C$ and $T_C$ leading to a compensation between temperature and $\mu$ so that the variation of the sound-speed profile is slight.

3.3.3. The Loss Rate of Metallicity in the Solar Wind

About 600 solar models with $\eta = 0$, $Z_{\text{acc}} = 0.015$, $\tau_{\text{acc}} = 12$ Myr, and different $\lambda_Z$, $M_L$, and $Y_{\text{acc}}$ have been calculated to
investigate the effects of varied $\lambda_Z$, which represents the relative escape speed of heavy elements in the solar wind. Here $Y_{\text{acc}}$ varies in the range from 0.00 to 0.26 with a step of 0.02, $M_L$ varies in the range from 0.00 to 0.01 with a step of 0.001, and $\lambda_Z$ varies in the range from 0.6 to 1.4 with a step of 0.2.

Sound-speed deviations, $^8$B neutrino fluxes, locations of the BCZ, and surface helium abundances of those solar models with $\lambda_Z = 0.6, 0.8, 1.2,$ and 1.4 are shown in Figure 16. The case of $\lambda_Z = 1.0$ is shown in Figure 4(c). There are two main effects of varied $\lambda_Z$, as shown in Figure 16 and Figure 4(c): the $^8$B neutrino fluxes of the solar models are anticorrelated with $M_L$ for $\lambda_Z < 1$ and positively correlated with $M_L$ for $\lambda_Z > 1$, and the required mass loss for improving helioseismic quantities of a solar model is anticorrelated with $\lambda_Z$. We now investigate the reasons for these relations.

### Table 5

| TWA | ZA05 | ZA10 | ZA20 | ZA25 | ZM06 | ZM08 | ZM12 | ZM14 |
|-----|------|------|------|------|------|------|------|------|
| $\lambda_Z$ | 1.0  | 1.0  | 1.0  | 1.0  | 0.6  | 0.8  | 1.2  | 1.4  |
| $Y_{\text{acc}}$ | 0.070 | 0.005 | 0.010 | 0.020 | 0.025 | 0.015 | 0.015 | 0.015 |
| $\lambda_Z$ | 1.0  | 1.0  | 1.0  | 1.0  | 0.6  | 0.8  | 1.2  | 1.4  |
| $\gamma$ | $- 1.868$ | $- 1.855$ | $- 1.868$ | $- 1.868$ | $- 2.069$ | $- 1.956$ | $- 1.797$ | $- 1.746$ |
| $Y_S$ | 0.250 | 0.245 | 0.245 | 0.244 | 0.247 | 0.250 | 0.245 | 0.245 |
| $R_{\text{BCZ}}/R_E$ | 0.710 | 0.710 | 0.710 | 0.710 | 0.710 | 0.710 | 0.710 | 0.710 |
| $\alpha_{(Li)}$ | 0.82  | 0.77  | 0.78  | 0.86  | 0.86  | 0.78  | 0.77  | 0.77  |
| $\delta e/e_c$ | -0.018 | -0.018 | -0.018 | -0.018 | -0.018 | -0.018 | -0.018 | -0.018 |

Note. Here $\lambda_Z$ characterizes the relative efficiency of heavy-element mass loss (see Equation (13)), and $Z_{\text{acc}}$ is the heavy-element abundance of the accreted material during PMS evolution. See notes for Table 1.
Solar models (ZM04, ZM08, ZM12, and ZM14) similar to Model TWA but with different $\lambda_Z$ are compared with Model TWA to investigate the effects of varied $\lambda_Z$. The information from those models is shown in Table 5, and the sound-speed deviations are shown in Figure 17. The ZM models are the selected best models with the minimum rms sound-speed deviation and $Y_S$ in the helioseismically inferred range. As discussed above, the structure of a solar model is determined by its abundance profiles. Therefore, it is straightforward to compare the ZM solar models by comparing their abundance profiles, which are shown in Figure 18. Unlike the case of the ZA models, the metallicity profiles of the ZM models are significantly different from Model TWA such that a lower $\lambda_Z$ leads to a lower metallicity profile in the solar interior with $r < 0.6 R$. For $\lambda_Z < \lambda_Z,cr$, where $\lambda_Z,cr = (X_0 + \lambda_Y Y_0)/(1 - Z_0) \approx 0.9$ for $\lambda_Y = 0.5$, we obtain $Z_1 < Z_c$; thus, the mass loss concentrates the heavy elements in the convective envelope, just as $\lambda_Y = 0.5$ concentrates helium in the convective envelope. If $\lambda_Z > \lambda_Z,cr$, $Z_1 > Z_c$; thus, the mass loss depletes heavy elements in the convective envelope. The effect of varied $\lambda_Z$ on the metallicity profile of the solar model

![Figure 16](https://example.com/f16.png)

*Figure 16.* Similar to Figure 4(c) but for solar models with different relative efficiency $\lambda_Z$ of heavy-element mass loss (see Equation (13)). The $^8$B neutrino flux of the solar models is now shown as the red contours.

![Figure 17](https://example.com/f17.png)

*Figure 17.* Sound-speed deviations from helioseismic inversions (similar to Figure 1(a)) for some solar models (see Table 5) with different $\lambda_Z$ compared with Model TWA.
Figure 18. Helium-abundance and metallicity profile of some solar models (see Table 5) with different $\lambda_Z$ compared with Model TWA.

is similar to a modification of the strength of molecular diffusion near the BCZ; i.e., $\lambda_Z > \lambda_{Z,cr}$ is similar to an enhancement of diffusion and vice versa. Because $(Z/X)_S$ is calibrated, the effect of $\lambda_Z$ on the metallicity profile is represented by the part of the solar interior with $r < 0.6R$, as shown in Figure 17. The final effect of $\lambda_Z$ on the metallicity profile is that a lower $\lambda_Z$ leads to a lower metallicity profile in the solar interior with $r < 0.6R$ and vice versa. Therefore, the $Z_C$ is correlated (for $\lambda_Z > \lambda_{Z,cr}$) or anticorrelated (for $\lambda_Z < \lambda_{Z,cr}$) with the total mass loss. Because of the correlation between $Z_C$ and the $^7$Be and $^8$B neutrino fluxes, the correlation between the $^8$B neutrino flux and $M_L$ shown in Figure 16 is now explained.

Figure 17 shows that the sound-speed profiles of the ZM models are in very good agreement with helioseismic inferences, implying that the helium-abundance profile is suitable for the metallicity profile for each ZA model. As shown in Figure 18, a lower metallicity profile requires a lower helium-abundance profile to reproduce the helioseismically inferred sound-speed profile. This is because a lower metallicity leads to a lower temperature gradient and then a lower temperature in the solar mantle; thus, based on Equation (17), a lower helium abundance in the mantle is required to retain the sound speed. Since the $Y_S$ of all ZM models is 0.245, the helium-abundance profile in the mantle is determined by the total mass loss caused by the solar wind, which concentrates helium in the convective envelope. Therefore, for a lower metallicity profile, a massive mass loss is required to obtain a lower helium-abundance profile in order to retain the sound-speed profile. This explains that the required mass loss for improving the helioseismic quantities of a solar model is anticorrelated with $\lambda_Z$, as shown in Figure 16.

4. Discussion

The SSM based on AGSS09 composition is inconsistent with helioseismic inferences (of sound-speed and density profiles, helium abundance in the solar convection zone, and the location of the base of the solar convection zone) and observations of the solar Li abundance. This difficulty still stands even with the recent upward revised Ne abundance. We have investigated the possible mechanisms to improve the solar model with three extra physical processes, i.e., convective overshoot, which leads to the turbulent kinetic energy flux $F_K$ and overshoot mixing; inhomogeneous mass loss caused by the solar wind; and PMS accretion with inhomogeneous materials. The turbulent kinetic energy flux is necessary to resolve the problem of the structure of the solar convective envelope. Convective overshoot mixing is required for the solar Li depletion. The mass loss caused by the solar wind shows a deficiency in helium (e.g., Bame et al. 1975), and the total mass loss is about $(10^{-3} - 10^{-2})M_\odot$ (e.g., Wood et al. 2002). It is not difficult to estimate that the mass loss could increase $Y_S$ by a level of $\sim 0.01$, which is larger than its uncertainty inferred by helioseismology. The motivation for PMS accretion with inhomogeneous materials is to adjust the abundance profiles in the solar ZAMS models and thereby affect the sound speed of the solar models. The PMS accretion is commonly found in T Tauri stars, which may resemble the Sun at its early stage. It is believed that the mechanism of PMS accretion in T Tauri stars is magnetospheric accretion from the disks around stars. For this scenario, we propose that the accreted materials may be inhomogeneous because the nonthermal ionization processes in the surface of the protoplanetary disk could be affected by the different FIPs of the elements.

The convective overshoot below the solar convection zone is described as a simple model based on an exponentially decreasing $F_K$ and a given value $F_{K,0}$ of the turbulent kinetic energy flux at the BCZ. The parameters in the overshoot model are derived from helioseismic inferences and solar lithium abundance. The mass loss caused by the solar wind is modeled by using a decreasing power-law function of the stellar age. The composition of the lost material is assumed to be helium-poor, as revealed by observations. Effects of varied metallicities of the lost material are also investigated. The mass accretion rate of PMS accretion is based on observations, and its dispersion is also taken into account. The duration of the PMS accretion and the composition of the accreted material are free parameters for the solar models. We have analyzed the solar evolutionary models with those extra physical processes and found that, if the PMS accretion is helium-poor, there are solar models consistent with helioseismic inferences and the observation of the solar neutrino fluxes. A typical improved solar model is Model TWA, shown in Table 1 and Figure 1.

In this section, we carry out discussions on the following issues: the implications of solar Li and Be abundances on the structure of the solar interior, possible mechanisms to improve the sound-speed profile in solar models, and possible thermohaline mixing in the solar interior.

4.1. Fresh Insight on the Solar Li and Be Abundance

It is useful to consider the constraints on the structure of the solar interior provided by the solar Li and Be abundances. Due to their proton-capture reactions, Li and $^7$Be are fragile elements. The typical temperatures for those proton-capture reactions are $\log T \approx 6.4$ and 6.5 for $^7$Li and $^9$Be, respectively, very close to (or a little higher than) the temperature of the BCZ in stars with mass close to the solar mass. Observations of the Li abundance of low-mass stars $(0.8 < M/M_\odot < 1.2)$ in open clusters have shown more Li depletion than the predictions by the standard stellar models. The discrepancy is generally thought to be caused by some extra mixing missing in the standard stellar models, e.g., overshoot (Straus et al. 1976; Schlattl & Weiss 1999; Xiong & Deng 2002, 2009; Zhang 2012; Baraffe et al. 2017), rotational mixing (Pinsonneault et al. 1990, 1992; Charbonnel et al. 1992), internal
The solar surface Li abundance is significantly depleted by about 2 dex relative to the meteoritic abundance (e.g., Asplund et al. 2009). Figure 19 shows the Li abundances of the Sun and some open cluster stars with metallicity close to the Sun. The Li abundances of the solar-age open cluster M67 stars with effective temperatures similar to the Sun have shown the same Li depletion level as the Sun, indicating that the solar Li depletion should not be unique, which is also supported by investigations of Li abundances of solar twins (see, e.g., King et al. 1997; Meléndez & Ramírez 2007; Lubin et al. 2010; Castro et al. 2011). Since the effective temperature changes little for solar-mass stars during the MS stage, the Li abundances of low-mass stars in open clusters with different ages shown in Figure 19 validate that Li is gradually depleted during the MS stage. This evidence strongly indicates that the Sun gradually experienced Li depletion during its MS stage. The Li abundances of young open clusters (e.g., Pleiades and Praesepe) indicate that the solar-mass stars’ lithium depletion is about 0.5–1.0 dex in the PMS stage. Therefore, the solar Li depletion from ZAMS to the present age is about 1.0–1.5 dex; or, equivalently, the e-folding timescale of Li depletion is about $\tau_{\text{Li}, \odot} \sim 1$ Gyr in the solar MS stage.

In the MS stage, the structure of the stellar envelope is basically in a quasi-static state. The variation of the ratio of $R_{\odot}$ to the stellar radius $R$ is small. The variation in the envelope of the relation between $m_r/M$ and $r/R$ is also small. Therefore, we can analyze the issue of solar Li depletion in the MS stage in the quasi-static case as follows.

The Li depletion timescale $\tau_{\text{Li}, \odot}$ is determined by the competition between burning and mixing. At a radius $r$ below the BCZ, there is an e-folding time of Li burning,

$$\tau_{\text{b}}(r) = \frac{1}{\rho_Y \gamma(R)},$$

where $\gamma(R)$ is the rate of the $^7$Li proton-capture reaction, and $\rho_Y \sim 1$ is the hydrogen abundance in mol g$^{-1}$. There is also a characteristic timescale of the mixing below the BCZ, in the region between $r$ and $R_{bc}$,

$$\tau_{\text{mix}}(r) = \frac{(R_{bc} - r)^2}{D_{\text{mix}}},$$

(23)

where $D_{\text{mix}}$ is the typical mixing diffusion coefficient between $r$ and $R_{bc}$. Since the former decreases and the latter increases toward the solar center, there is a location $r_k$ where $\tau_{\text{mix}}(r_k) = \tau_{\text{b}}(r_k)$. For $r > r_k$, the mixing can efficiently mix materials below Li burning; thus, the mixing could transport material into a higher-temperature region, so that $\tau_{\text{Li}, \odot} \lesssim \tau_{\text{b}}(r)$. For $r < r_k$, the mixing cannot catch the Li burning; thus, the rate of Li depletion in the envelope is lower than the Li burning rate at $r$, i.e., $\tau_{\text{Li}, \odot} > \tau_{\text{b}}(r)$. The combination shows that $\tau_{\text{Li}, \odot} = \tau_{\text{b}}(r_k) = \tau_{\text{mix}}(r_k)$. Since $\tau_{\text{Li}, \odot} \sim 1$ Gyr, according to $\gamma(R)$ given by Angulo et al. (1999) and Adelberger et al. (2011), the corresponding temperature for $\tau_{\text{b}} \sim 1$ Gyr is $\log T \approx 6.5$. At $R_r \approx 0.68 R_\odot$, we can find that $\tau_{\text{mix}}(0.68 R_\odot) \sim 1$ Gyr. Since the age of the Sun is much greater than 1 Gyr, the solar envelope with $r > 0.68 R_\odot$ should be efficiently mixed and nearly homogeneous. The molecular diffusion has a characteristic timescale significantly longer than the solar age, so it cannot compete with the mixing.

A similar analysis can also be applied to the solar Be abundance. Because the solar Be abundance shows almost no depletion (e.g., Asplund et al. 2009), and the temperature resulting in a characteristic timescale of $^7$Be burning as the solar age is about $\log T \approx 6.5$ at $r_k \approx 0.68 R_\odot$, we can conclude that $\tau_{\text{mix}}(0.68 R_\odot) > 1$ Gyr. Since the age of the Sun is much greater than 1 Gyr, the solar envelope with $r > 0.68 R_\odot$ is not less than the solar age. Therefore, any kind of mixing below the base of the solar convection zone cannot show a considerable effect below $0.6 R_\odot$.

4.2. On Improving the Sound Speed of Solar Models

The main problem of the SSMs with low-Z composition (i.e., AGSS09 and AGSS09Ne) is that the sound-speed profile in the SSM is not consistent with the helioseismic inferences. It is shown in Figure 1(a) that, in the solar mantle, the sound speed in the solar models should be increased in order to be consistent with the helioseismic inferences. It is shown in Equation (17) that there are only two ways to increase $c_s$: increase the temperature $T$ and decrease the mean molecular weight $\mu$. The sound speed in the solar convection zone can be obtained from integrating the hydrostatic equation with the polytropic relation and is well defined (e.g., Christensen-Dalsgaard 1986). The composition in the solar convection zone is also well defined from the spectral analyses and the helioseismic determination of helium abundance. Therefore, the temperature in most of the solar convection zone is well defined. To increase $T$ in the region $0.3 < r/R_\odot < 0.7$ is equivalent to increasing the temperature gradient $dT/dr$ or $d\ln T/d\ln P$ in that region. The mean molecular weight $\mu$ is determined by the composition as $\mu^{-1} = 2 - 1.25Y - 1.5 Z$ (for the fully ionized case), where $Y$ is the helium abundance and $Z$ is the metallicity. Because the contribution of metallicity $Z$ to $\mu$ is only $\sim 1\%$ in the Sun and $Z$ should not change too much, $\mu$ is mainly determined by the helium abundance $Y$. Therefore, in the solar mantle, to decrease the helium abundance could improve the sound speed. Now we discuss the possible mechanisms for improving the sound-speed profile of the SSM.
Specifically, increasing the temperature gradient below the base of the solar convection zone can be achieved via increasing the opacity, enhancing the molecular diffusion, and taking into account extra energy inward transport mechanisms (e.g., inward energy fluxes caused by convective overshoot: convective heat flux, Zhang & Li 2012a; Zhang et al. 2012; turbulent kinetic energy, Zhang 2014; and internal gravity waves, Arnett et al. 2010). The modifications of the opacity and the molecular diffusion have been extensively investigated, as introduced in Section 1. The effects of turbulent kinetic energy caused by convective overshoot on the solar model are investigated in Section 3.1. With the overshoot parameters inferred by helioseismology, the solar Model OV09Ne does not show a satisfactory sound-speed profile. This means that the overshoot model with helioseismically inferred parameters is not sufficient to solve the problem. Since the effects of negative $\lambda_K$ are equivalent to an enhancement of opacity (Zhang 2014), the required modifications of the opacity for improving solar models (e.g., Serenelli et al. 2009; Christensen-Dalsgaard & Houdek 2010) indicate that it is possible to use the overshoot model to solve the solar abundance problem in solo if a longer $e$-folding length of $L_K$ (i.e., a larger $\theta$) is adopted. It can be estimated from Figure 13 in Christensen-Dalsgaard & Houdek (2010) that the required value of $\theta$ is about 5–7$H_P$, significantly larger than the helioseismic suggestion. However, a 5–7$H_P$ $e$-folding length of $L_K$ will show significant mixing in the solar mantle and leads to significant $^4$He depletion at the solar surface, which conflicts with observations. Dissipation of other possible negative kinetic energy fluxes below the BCZ (e.g., convective heat flux and internal gravity waves) should also lead to mixing. Therefore, taking them into account still cannot solve the solar abundance problem in solo.

Except for increasing the temperature gradient $\nabla_Y$, the only way to improve the sound speed of the solar model is to reduce the mean molecular weight $\mu$ or, specifically, the helium abundance in the solar mantle. An example of the effects of changed helium abundance in that region in the literature is the solar models with PMS accretion shown by Serenelli et al. (2011): for the metal-rich early-accretion scenario, the initial hydrogen abundance becomes higher than the SSM due to the luminosity calibration, and therefore the helium abundance is lower than the SSM; thus, solar models with metal-rich early accretion have improved sound-speed profiles and $R_{\infty}$ compared to the SSM. However, we have to notice that, in the AGSS09 or AGSS09Ne SSMs, $Y_S$ is already less than the helioseismic inferences. Therefore, in order to improve the sound speed and $Y_S$ of SSMs simultaneously, extra mechanisms must be added to increase $Y$ in the convective envelope and decrease $Y$ in the solar mantle. There are three possible mechanisms to achieve that: mixing below the BCZ, helium-poor mass accretion in the PMS stage, and helium-poor mass loss in the MS stage. All three mechanisms have been investigated in this paper. The mixing below the BCZ competes with the molecular diffusion and thus is helpful to achieve the required changes of the $Y$ profile. However, the strength of the mixing is restricted by the observed light-element depletion. As shown by the OV09Ne solar model, such mixing is not enough to restore the sound-speed profile. The helium-poor PMS accretion is the main factor to improve the sound-speed profiles of the improved models listed in Tables 3–5 because it leads to helium-poor envelopes in solar models. The effect of helium-poor mass loss in the MS stage is to concentrate helium in the convective envelope, which helps to solve the problem of low $Y_S$ in SSMs. As shown in this work, the combined effects of those processes could lead to suitable helium-abundance profiles to reproduce the helioseismically inferred sound-speed profile and $Y_S$ simultaneously.

An interesting issue is the effects on the solar models of the mass loss caused by the solar wind. As pointed out, varying $\lambda_\chi$ works like a modification of molecular diffusion of heavy elements near the BCZ, as does $\lambda_\gamma$ on helium. Therefore, the interior helium abundance and metallicity profiles can be adjusted by the variations of $\lambda_\chi$ and $\lambda_\gamma$. Since the sound-speed profile could be significantly improved if the helium-abundance profile is suitable for the metallicity profile, such as for Model TWA and the ZM models, one may wonder whether it is possible to obtain a solar model as good as Model TWA by only using the helioseismically based overshoot model and the mass loss with suitable values of $\lambda_\chi$ and $\lambda_\gamma$. In this case, the helium-poor PMS accretion is not necessary. In order to investigate that, we have calculated some solar models with $0 \leq \lambda_\chi \leq 2$ with a step of 0.5, $0 \leq \lambda_\gamma \leq 2$ with a step of 0.2, and $0 \leq M_{\ast}/M_\odot \leq 0.01$ with a step of 0.001 and without PMS accretion. However, no satisfactory model has been found.

Since the inhomogeneous PMS accretions are absent in the test, the helium-abundance profiles of ZAMS models in the test are homogeneous, which is the only difference between the models in the test and the improved models with all three extra processes. As mentioned above, the structure of a solar envelope model is completely determined by its $Y$ and $Z$ profile; thus, the $Y$ and $Z$ profile should be in one-to-one correspondence in order to keep its sound-speed profile consistent with helioseismic inferences. Because the relative escape speed of heavy elements in the test varies over a large range, leading to a large range of $Z$ profiles, so does the relative escape speed of helium. No satisfactory solar model being found indicates the unsuitability of solar models with a homogeneous helium-abundance profile at ZAMS. Therefore, it supports the necessity of the helium-poor PMS accretion and that an inhomogeneous ZAMS model with a helium-poor envelope is necessary to reproduce a solar model with a sound-speed profile consistent with helioseismic inferences.

Since the relative escape speed of each heavy element assumes the same value of $\lambda_\chi$ in this paper, more comprehensive effects of the inhomogeneous mass loss on solar models could be investigated if the relative escape speed of each heavy element is treated independently and the varied heavy-element abundances are considered in the interpolation of opacity in the solar interior.

4.3. On the Effects of Thermohaline Mixing

A main factor in improving the sound speed in the TWA solar models is that the helium abundance in the mantle is lower than the SSM. As shown in Figure 6(a), there is a layer of positive helium-abundance gradient below the convective envelope during the evolution of Model TWA, e.g., at an age between 0.4 and 2 Gyr. This positive helium-abundance gradient, which results from the helium-poor mass loss at the early MS stage, is required to appropriately compensate for the helium settling so that the sound-speed profile and $Y_S$ can be consistent with helioseismic inferences simultaneously. However, the positive helium-abundance gradient leads to thermohaline mixing, since $\nabla_Y < 0$ in this layer. Thermohaline mixing is not included in our solar models. It could reduce or...
Figure 20. Evolution of the helium abundance of Model TWAth, including thermohaline mixing, compared with Model TWA.

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remove the positive helium-abundance gradient and then affect the helium-abundance profile of the solar model at the present age. If thermohaline mixing is too efficient to form a positive helium-abundance gradient layer, the TWA solar models would no longer reproduce a sound-speed profile and \( Y_\text{S} \) consistent with helioseismic inferences simultaneously.

A widely used formula for the diffusion coefficient for thermohaline mixing is (Ulrich 1972; Kippenhahn et al. 1980)

\[
D_{\text{th}} = C_{\text{th}} \frac{\lambda}{\rho c_p} \frac{\nabla \mu}{\nabla B - \nabla} \quad \text{for} \quad \nabla \mu < 0, \tag{24}
\]

where \( C_{\text{th}} = 8(\pi \alpha_{\text{th}})^2/3 \) and \( \alpha_{\text{th}} \) is the aspect ratio of length to width of the fingers. By using the formula above, a value of \( C_{\text{th}} \approx 1000 \) is required to explain the abundance observations of red giant branch stars (Charbonnel & Zahn 2007; Cantiello & Lagarde 2010; Charbonnel & Lagarde 2010). We have tried to calculate a solar model with the same parameters as Model TWA and including thermohaline mixing with the diffusion coefficient defined by the above equation, setting \( C_{\text{th}} = 1000 \). However, the resulting solar model does not show similar improvements as does Model TWA.

On the other hand, numerical simulations have shown a much lower diffusion coefficient for thermohaline mixing (Denissenkov 2010; Traxler et al. 2011; Brown et al. 2013). Traxler et al. (2011) suggested a formula for the diffusion coefficient for thermohaline mixing based on their simulations (see Equation (24) in their paper). At the conditions of the base of the solar convection zone, it shows a typical value of the diffusion coefficient for thermohaline mixing \( D_{\text{th}} \sim 100 \text{ cm}^2 \text{s}^{-1} \). We have calculated a solar model, TWAth, with the same parameters as Model TWA and a diffusion coefficient of thermohaline mixing \( D_{\text{th}} = 100 \text{ cm}^2 \text{s}^{-1} \). The evolution of the helium-abundance profile is shown in Figure 20. Thermohaline mixing slightly extends the \( \nabla \mu < 0 \) region and makes it milder but cannot completely erase the \( \nabla \mu < 0 \) region in the timescale of \( \sim 1 \text{ Gyr} \). The seismic properties of Model TWAth are quite similar to those of Model TWA: the differences of the sound-speed profile are less than 0.03%, the differences of the density profile are less than 0.1%, \( X_\text{S} = 0.2448 \), and \( R_{\text{bc}} = 0.7109R_\odot \) for Model TWAth. The lithium abundance of Model TWAth is \( A(\text{Li}) = 0.76 \), a little lower than that of Model TWA due to thermohaline mixing enhancing the Li depletion in the early MS stage. The \(^7\text{Be} \) and \(^8\text{B} \) neutrino fluxes of Model TWAth are \( 4.84 \times 10^{8} \) and \( 5.12 \times 10^{8} \text{ cm}^{-2} \text{s}^{-1} \), respectively, a little less than those of Model TWA, possibly due to thermohaline mixing slightly offsetting the heavy-element settling and leading to a lower \( Z_\text{C} = 0.01570 \). It can be found that taking into account the diffusion coefficient of thermohaline mixing based on numerical simulations should not change the main results of the improved solar models.

5. Summary and Conclusions

In this paper, we have focused on the solar abundance problem that solar models with revised low-Z composition are not consistent with helioseismic inferences. We have proposed to take into account three extra physical processes missed in SSMs (i.e., convective overshoot, which leads to turbulent kinetic energy flux and mixing; the helium-poor mass loss caused by the solar wind; and inhomogeneous PMS accretion from the protoplanetary disk). Convective overshoot is predicted by many stellar convection models and shown by many numerical simulations of stellar convection. The helium-poor property of the solar-wind mass loss is confirmed by observations. And PMS accretion is a common property of low-mass stars revealed by abundant observations of T Tauri stars. Therefore, those extra processes are supported by theory or observations. A possible physical justification for the crucial assumption of the material of the PMS accretion being inhomogeneous is that the magnetospherical accretion could be ion-dominated; thus, the effect of the different FIP of each element could lead to a difference between its ion abundance and the neutral. Since Serenelli et al. (2011) tested PMS accretion with variations in metallicity and did not find an overall satisfactory solar model, they also suggested investigating PMS accretion with variations in helium abundance.

With the recent upward revised Ne abundance (Young 2018), the SSM shows a little improvement compared with the original AGSS09 SSM, but the solar abundance problem remains. An inherent reason for the solar abundance problem is the contradiction of the structure of the solar convective envelope revealed in Zhang (2014). In order to eliminate the contradiction, we have taken into account the convective overshoot in the solar model. The overshoot leads to negative turbulent kinetic energy, which result in a deeper \( R_{\text{bc}} \), so that it eliminates the contradiction, and overshoot mixing, which leads to significant lithium depletion. The main parameters in the overshoot model are derived from helioseismic inferences and the required solar lithium depletion. Although the resulting solar model shows good properties of the convective envelope (i.e., \( R_{\text{bc}} \) and \( Y_\text{S} \)), the sound-speed profile in the solar interior with \( r < 0.6R_\odot \) is worse than that of the SSM, indicating that the helioseismically based overshoot model is not sufficient to solve the solar abundance problem. Extra mechanisms must be taken into account to improve the sound-speed profile in the solar interior.

We then calculated solar models with the helioseismically based overshoot model and varied parameters of accretion and mass loss, i.e., duration of accretion, abundance of accreted material, and abundance of the solar wind. We found that significantly and overall improved solar models exist when the PMS accretion is helium-poor, such as Model TWA, which is a typical improved solar model. Their sound-speed profiles, density profiles, surface helium abundances \( Y_\text{S} \) and \( (Z/X)_\text{S} \),
be well mixed so that the solar envelope with the improved solar models. Fruitful discussions with Prof. Zhang should lead to a similar strength of the adopted overshoot mixing and should not change the main results on the improved solar models.

The best set of parameters of accretion and mass loss cannot be determined because there are many solar models with different parameters (e.g., models in Tables 3–5) showing similar improvements as Model TWA. A common property of those models is that they have helium-poor PMS accretion with lacking helium masses (see Equation (18)) about (1% ~ 2%) M⊙, indicating an inhomogeneous ZAMS solar interior in which the helium abundance in the envelope is lower than that in the core. The necessity of the helium-poor PMS accretion is indicated in the test that the sound-speed profile cannot be significantly improved by only using the helioseismically based overshoot model and varied abundances of both Y and Z in the solar wind.

The analysis of improving the sound-speed profile shows that there are only two ways to improve: enhance the opacity or reduce the helium abundance in the solar mantle. Different from the ad hoc enhancements of opacity, neon abundance, or molecular diffusion, which correspond to the former, this study shows that the latter is also a possible solution to the solar abundance problem.

The comparison of the solar Li abundance with the Li abundances of open cluster stars has indicated that there is a mixing process (very likely caused by convective overshoot) in the thin layer 0.68R⊙ < r < R⊙, and the characteristic timescale of the mixing in that region is about ~1 Gyr. Since the timescale is shorter than the present solar age, the layer should be well mixed so that the solar envelope with r > 0.68R⊙ should be almost homogeneous. The ~1 Gyr timescale of the mixing indicates that the overshoot mixing is a weak mixing process, which is consistent with Zhang (2013). The solar Be abundance does not show an obvious depletion, indicating that the mixing mixing cannot extend to r = 0.68R⊙. In this paper, overshoot mixing is the mechanism to deplete lithium, and the parameter C_X of the overshoot mixing is based on the required lithium depletion. If other mixing mechanisms are taken into account (e.g., rotational mixing, Bi et al. 2011; Yang 2016, 2019; internal wave mixing, Arnett et al. 2010), the value of C_X should be downward revised. However, any mixing mechanism below the BCZ should be restricted by the above results indicated by the solar Li and Be abundances. Therefore, they should lead to a similar strength of the adopted overshoot mixing and should not change the main results on the improved solar models.

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