Online transition matrix identification of the state evolution model for the extended Kalman filter in electrical impedance tomography

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Abstract. One of the electrical impedance tomography objectives is to estimate the electrical resistivity distribution in a domain based only on contour electrical potential measurements caused by an imposed electrical current distribution into the boundary. In biomedical applications, the random walk model is frequently used as evolution model and, under this conditions, it is observed poor tracking ability of the Extended Kalman Filter (EKF). An analytically developed evolution model is not feasible at this moment. The present work investigates the possibility of identifying the evolution model in parallel to the EKF and updating the evolution model with certain periodicity. The evolution model is identified using the history of resistivity distribution obtained by a sensitivity matrix based algorithm. To numerically identify the linear evolution model, it is used the Iterative Time Domain Method, normally used to identify the transition matrix on structural dynamics. The investigation was performed by numerical simulations of a time varying domain with the addition of noise. Numerical difficulties to compute the transition matrix were solved using a Tikhonov regularization. The EKF numerical simulations suggest that the tracking ability is significantly improved.

1. Introduction
Electrical Impedance Tomography (EIT) is a imaging method of the resistivity distribution within a domain. The images are estimated from a set of electrical potential measurements on the boundary of a domain. This set of electrical potentials are obtained by applying current through electrodes and measuring the resulting electrical potentials on electrodes.

Electrical Impedance Tomography have a wide range of applications [1], [2]. It may be used to monitor cardiac function, detect internal hemorrhage and breast cancer, for instance. Examples of non-clinical applications are the visualization of multiphase flow, detection of minerals in the soil, soil pollution monitoring and crack detection on mechanical components. The application in mind of the present work is lung monitoring, since lung conditions may change dramatically within few respiratory cycles [3]. Furthermore, inadequate lung ventilation in Intensive Care Units may cause barotrauma or hypoxia.

When EIT is applied to monitor the lungs, it estimates electrical properties distributions on the thorax. The problem can be seen as a state observation problem within the control theory framework. Among several absolute EIT algorithms, the Extended Kalman Filters (EKF) have been investigated due to its ability to track the state on time varying non-linear systems. The
EKF is a probabilistic estimation algorithm in the sense that it minimizes the variance of the estimation error.

The EKF requires a state evolution model. The state evolution model adopted in lung monitoring applications is the Random Walk ([4], [5] and [6]), but the tracking ability of the EKF is poor for lung monitoring [5] with this model. The present work investigates whether the tracking ability of EKF is improved if the evolution model is frequently estimated through a sensitivity matrix algorithm and updated in EKF.

2. The Extended Kalman Filter

The Extended Kalman Filter was developed in the early 60's. It is a predictor-corrector estimator, it minimizes the trace of the covariance matrix of the estimation error. The EKF takes into account modelling errors and measurement errors [7].

The discrete time EKF is described by equation (1) to equation (5),

\[ \dot{\rho}_k^{(-)} = \Phi \dot{\rho}_{k-1}^{(+)} \]  
\[ P_k^{(-)} = \Phi P_{k-1}^{(+)} \Phi^T + Q_{k-1} \]  
\[ G_k = P_k^{(-)} H_k^T (\dot{\rho}_k^{(-)} [H_k (\dot{\rho}_k^{(-)}) P_k^{(-)} H_k^T (\dot{\rho}_k^{(-)}) + R_k]^{-1} \]  
\[ \dot{\rho}_k^{(+)} = \dot{\rho}_k^{(-)} + G_k [v_k - H_k \dot{\rho}_k^{(-)}] \]  
\[ P_k^{(+)} = [I - G_k H_k \dot{\rho}_k^{(-)}] P_k^{(-)} \]

where (1) and (2) represent the propagation phase and (3) to (5) represent the update phase.

The vector \( \dot{\rho} \) is the state vector, \( \Phi \) is the transition matrix, \( P \) is the covariance matrix of the estimation error, \( Q \) is the covariance matrix of the state noise, \( H \) is the linear observation matrix, \( R \) is the covariance of the measurement noise and \( G \) is the Kalman gain matrix.

In order to use this set of equations, the observation vector must have dimension of number of electrodes \( e \) times the number of current patterns \( p \) [5]. Effectively there are \( ep \) different sensors, in the sense that, the observation model depends, also, on the current pattern. When this fact is not taken into account, there must be \( p \) different \( P \) matrices, one for each current pattern.

3. The Evolution Model

The EKF requires a transition matrix \( \Phi \), which is part of the state evolution model. The transition matrix is estimated in this paper through the Ibrahim Time Domain Method (ITD) [8], [9], [10]. The ITD requires two matrices \( \mathbf{X} \) and \( \mathbf{X}^+ \) filled with state vectors in its columns. Considering a finite elements mesh with \( n \) elements, matrices \( \mathbf{X} \in \mathbb{R}^{2n \times m} \) and \( \mathbf{X}^+ \in \mathbb{R}^{2n \times m} \) should be composed by \( m \) augmented state vectors \( \dot{\rho}(k) \in \mathbb{R}^{2n} \) (equation (8)) because is used, in this paper, a \( n \) degrees of freedom second order ordinary differential equation (o.d.e.) model for the state evolution, transformed in one \( 2n \) degrees of freedom first order o.d.e. equation [11].

The state vectors that compose \( \mathbf{X} \) are a time increasing sequence of state vectors, from discrete time \( k \) to discrete time \( k+m-1 \) and the state vectors that compose \( \mathbf{X}^+ \) are a time increasing sequence of state vectors, from discrete time \( k+1 \) to discrete time \( k+m \).

\[ \mathbf{X} = \begin{bmatrix} \dot{\rho}(k) & \dot{\rho}(k+1) & \cdots & \dot{\rho}(k+m-1) \end{bmatrix} \]  
\[ \mathbf{X}^+ = \begin{bmatrix} \dot{\rho}(k+1) & \dot{\rho}(k+2) & \cdots & \dot{\rho}(k+m) \end{bmatrix} \]  
\[ \dot{\rho}(k) = \begin{bmatrix} \rho(k-1) & \rho(k) \end{bmatrix} \]
It is good to have \( m > 2n \) augmented state vector to estimate \( \Phi \) to form a least square solution. For this solution, a pseudo-inverse of \( X \) is required and a Tikhonov regularization is also necessary [8], [12], [11]. The transition matrix \( \Phi \) is estimated according to

\[
\Phi = (X^+X^T)(XX^T + \alpha I)^{-1},
\]

where \( \alpha \) is the Tikhonov regularization parameter and \( I \) is the identity matrix.

For the sequence of absolute images \( \rho_{(k)} \), it was used a sensitivity matrix algorithm [13] and the absolute images are approximated through the addition of the resistivity distribution used for the Taylor series expansion. The sensitivity matrix images are under estimated, but the method is robust, gives good localization of the objects and, therefore, are used to estimate the transition matrix. The images provenient from EKF take a longer time to become informative.

With a certain periodicity, a new transition matrix can be estimated in parallel with the EKF estimation. The periodicity is problem dependent. In the present work, the transition matrix is estimated only once.

4. Numerical Simulations
A finite elements numerical phantom comprising a cylindrical domain of 300 mm of diameter, \( n = 514 \) linear triangular elements, \( e = 32 \) electrodes, \( p = 32 \) current patterns and an object of 40 mm of diameter was developed. The object, which has time varying resistivity, was placed in two positions: in the center of the domain and 126 mm distant from the center of the domain.

Two different functions were used to represent the time varying resistivity of the object. The first function is used to generate the sequence of voltages that will result in a sequences of images obtained by the sensitivity matrix algorithm, which will comprise data for ITD.

\[
\rho_{\text{object}}(k) = \frac{10}{50}k + 35.0 \Omega m, \quad \rho_{\text{basal}} = 35.0 \Omega m
\]

The second function is a step function in the object resistivity at \( k = 0 \), from 35 to 100 \( \Omega m \) to generate the sequence of voltages that will be used in EKF. Two different functions were used to avoid inverse crime.

The complete electrode model was used [14] and Gaussian noise was added to the electrical potential measurements.
5. Numerical Results
The results will be presented in two parts, the first part shows the estimation of the evolution model, the second part shows the state estimation using the EKF.

5.1. Part 1
The numerical phantom was used to generate electrical potential data when the object is in the center or off the center. The initial uniform resistivity was $\rho^0 = 35\Omega m$ and the current intensity was $I = 2mA$ p-p.

Figure (1) shows the time history of two representative elements from the basal region and from the object region, using the estimated transition matrix.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure1a.png}
\includegraphics[width=0.4\textwidth]{figure1b.png}
\caption{Time history. (a) Object at the center; (b) Object off the center.}
\end{figure}

Figure (2) shows the state at $k = 1495$ using the estimated transition matrix. The circle marks the correct object location in two different positions.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure2a.png}
\includegraphics[width=0.4\textwidth]{figure2b.png}
\caption{Difference Image at iteration $k = 1495$. (a) Object at the center; (b) Object off the center.}
\end{figure}

The transition matrix was calculated using $\alpha = 1.0$ to avoid instabilities in the state evolution. Smaller values of $\alpha$ generate instabilities.
5.2. Influence of the regularization parameter

Figure (3) shows the Nyquist plot of the eigenvalues of transition matrix for two different values of $\alpha$ when the object is off the center of the domain. Increasing $\alpha$ increase the damping on most eigenvalues. Similar results are observed when the object is located in the other position. The eigenvalues close to the unitary circle are not heavily influenced.

![Nyquist plots](image)

**Figure 3.** Nyquist plot of the eigenvalues of matrix $\Phi$, object off the center. (a) $\alpha = 10^0$; (b) $\alpha = 10^{-2}$.

The dominant modes have eigenvalues close to the point $(1.0, 0.0)$ in the unit circle and represent the position of the object according to figure (4) and figure (5). The correlation of the eigenvectors and the most recent image was computed using the Modal Assurance Criterion (MAC) [8] revealing that only the first three eigenvectors have high correlation, $MAC > 0.5$, with the image.

![Eigenvectors](image)

**Figure 4.** Dominant eigenvectors of matrix $\Phi$, ramp resistivity variation with $\alpha = 1.0$. (a) Eigenvector 1; (b) Eigenvector 2.
5.3. Part 2
In the second part is shown the stated estimation using the EKF with the estimated evolution model. The results are grouped according to the object localization.

5.3.1. Object off the center The procedure to adjust the EKF parameters was done adjusting one parameter at a time. Matrix $P_0$ was chosen to overestimate the quality of the initial image. The initial resistivity was chosen such that $\rho_0 = 60\Omega m$ and $P_0 = 10^4 I$. Matrix $Q_k$ was updated following

$$Q_k = \begin{cases} 
\lambda P_{k-1}, & k \leq 3 \\
Q_{k-1}, & k > 3 
\end{cases}$$

(11)
to keep the filter open to new information.

The Kalman gain matrix is influenced by errors in $H$ due to wrong linearization state. Therefore, it was used a relaxation factor in the Kalman Gain equation (equation (12))

$$G_k = \gamma_k (P_k^{-1}H_k^T (H_kP_k^{-1}H_k^T + R_k)^{-1})$$

(12)

where

$$\gamma_k = \begin{cases} 
0.1, & k < 5 \\
0.5, & k < 8 \\
1.0, & k \geq 8 
\end{cases}$$

(13)
to avoid large oscillations of the initial state updates.

Matrix $R$ adjustment

Although $R_k$ can be estimated from electronic noise, modeling errors in the observation equation effectively increases $R_k$ [4]. Therefore, $R_k$ was adjusted taking into consideration the observation residue history from a few EKF tests, keeping the observation residue inside the limit of three measurement noise standard deviation [5]. Table (1) shows the parameters used.
Table 1. Parameters used in EKF

| Test   | 2-01 | 2-02 | 2-03 | 2-04 |
|--------|------|------|------|------|
| $R_k$ (constant) | $10^{-2}I$ | $10^{-3}I$ | $10^{-4}I$ | $10^{-5}I$ |
| $P_0$   | $10^1I$ | $10^1I$ | $10^1I$ | $10^1I$ |
| $Q_0$ [Ωm] | $10^{-3}P_0$ | $10^{-3}P_0$ | $10^{-3}P_0$ | $10^{-3}P_0$ |
| $\rho_0$ | 60 | 60 | 60 | 60 |
| $\lambda$ | $10^{-3}$ | $10^{-3}$ | $10^{-3}$ | $10^{-3}$ |

Figure (6) shows the history of the observation residue for different values of $R_k$. An adequate value is $R_k = 10^{-4}I$, since the observation residue keeps inside the limit.

Figure 6. History of the state estimation error. (a) Test 2-01; (b) Test 2-02; (c) Test 2-03; (d) Test 2-04;
Matrix $Q$ adjustment

The $Q_k$ was adjusted taking in account the estimated state evolution and the quantity of artifacts on the images. The table (2) shows the parameter used.

| Test | 2-05 | 2-06 |
|------|------|------|
| $R_k$ (constant) | $10^{-4}I$ | $10^{-4}I$ |
| $P_0$ | $10^3I$ | $10^3I$ |
| $Q_0$ | $10^{-3}P_0$ | $10^{-5}P_0$ |
| $\rho_0$ [$\Omega$ m] | 60 | 60 |
| $\lambda$ | $10^{-3}$ | $10^{-5}$ |

Figure (7) shows the state history and the image on iteration $k = 300$. It suggests that a reduction of $Q_k$ causes a reduction of the tracking ability of EKF and increases the spatial resolution. Test 2-06 was adopted as the best set of parameters for EKF.

Figure 7. State history for two values of $Q_k$: (a) state history, test 2-05; (b) state on test 2-05 at $k = 300$; (c) state history, test 2-06; (b) state on test 2-06 at $k = 300$.
5.3.2. When the object is at the center of the domain  The set of parameters from test 2-06 were used in 2-07. Figure (8) shows the state history, the image at iteration $k = 300$, and the observation residue.

![Figure 8](image)

**Figure 8.** Test 2-07. (a) State history; (b) Observation residue; (c) State at $k = 300$.

It can be observed that the observation residue is lower in test 2-07 than in test 2-06. When the object is in the center, errors in the state estimation have low influence in the measurements. Therefore, a direct comparison between observation residues is not fair. Another set of parameters was proposed through the variation of the set of parameters from test 2-06 since the tracking ability can be improved. A smaller value of matrix $R_k$ was attempted. Table (3) shows the parameters used.

| Test | 2-07 | 2-08 |
|------|------|------|
| $R_k$ (constant) | $10^{-4}I$ | $10^{-6}I$ |
| $P_0$ | $10^{1}I$ | $10^{1}I$ |
| $Q_0$ | $10^{-5}P_0$ | $10^{-5}P_0$ |
| $\rho_0$ [Ωm] | 60 | 60 |
| $\lambda$ | $10^{-5}$ | $10^{-5}$ |
Figure (9) shows the state history, the image at iteration $k = 300$ and the observation residue.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure9}
\caption{Test 2-08. (a) State history; (b) Observation residue; (c) State at $k = 300$.}
\end{figure}

5.4. \textit{Comparison between Random Walk and ITD Estimated Evolution Model}

In this section a comparison between the tracking ability of EKF using the Random Walk Model and EKF using the ITD Estimated Evolution Model is performed.

Figure (10) shows the state history, the observation residue and the state at iteration $k = 800$ for the two different object positions. The object was not visible in the center although the observation residue lies within three standard deviation range. It can be observed that there are many artifacts in both images.
Figure 10. Results using the Random Walk model. (a) State history, object at the center; (b) State history, object off the center; (c) observation residue, object at the center; (d) observation residue, object off the center; (e) State at $k = 800$, object at the center; (f) State at $k = 800$, object off the center.
6. Final Comments
The results show that the Ibrahim Time Domain method can be used to estimate the evolution model based on a sequence of images obtained by a sensitivity matrix algorithm. Although the resistivity distribution obtained by the sensitivity matrix algorithm is underestimated, the tracking ability of the EKF is improved when compared to the use of the Random Walk model for the evolution model.

The computational effort to implement the direct identification of the transition matrix seems to be justified since the tracking ability improved approximately sixteen times when the object was in the center and five times when the object was near the boundary.

The use of observation vector of dimension 1024 instead of 32 caused each iteration to be longer but, on the other hand, the $P$ update can be performed according to the equation (5) and an oscillation with periodicity 32 was avoided.

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