Resilience-oriented Comprehensive Planning Strategy of Distributed Generator in Power Distribution System

Z.P. Li, K.C. Huang, T. Qian, W.W. Huang and W.H. Tang*

School of Electric Power Engineering, South China University of Technology, Guangzhou 510641, China

*epzepengli@mail.scut.edu.cn

Abstract. Power distribution system resilience is not considered in traditional planning of distributed generator (DG), which leads to severe power outages and economic losses during extreme weather event. To address this issue, this paper proposes a resilience-oriented comprehensive planning strategy of DG in power distribution system. The problem is formulated as a two-stage stochastic mixed-integer second-order cone programming (SMISOCP). The objective of the first stage is to determine the number, location and capacity of DG and obtain the economic cost of power distribution system. The second stage minimizes the resilience cost under uncertain failure scenarios. First, sufficient failure scenarios are generated by the Monte Carlo method. Then the failure scenarios are reduced to the most representative scenarios by using the K-means clustering algorithm to reduce computational burden. Finally, the two-stage SMISOCP is solved based on the reduced failure scenarios. The simulation results of the IEEE 33-bus test systems illustrate the effectiveness of the proposed two-stage strategy.

1. Introduction

In recent years, extreme weather events have caused severe economic losses and power outages to power distribution systems [1]. For example, the 2008 ice disaster in China resulted in economic losses over 15 billion dollars [2]. In 2017, the Hurricane Irma caused electric outages to nearly 15 million customers [3]. Therefore, it is significance to investigate the resilience of power distribution system.

Great progress has been made in optimal planning of DG [4]-[5]. The optimal location and capacity decisions are made in some research to minimize the economic cost of power distribution system. These papers mainly study the normal operating condition but ignore the effects caused by extreme weather events. Ref. [6]-[8] propose a tri-level robust optimization model for resilience-oriented planning of DG. The system planner deploys an optimal planning of DG in the first level. The load shedding is maximized in the second layer. While in the third level, the utilities minimize the load shedding by using an outage management strategy. Ref. [9] formulate a two-stage stochastic mixed-integer programming model to obtain the resilience-oriented planning of DG. However, the above methods have mainly considered a single economic cost in normal operating condition or post-disaster resilience cost. A comprehensive approach that considers the economic cost and resilience cost for DG planning is lacking. Therefore, a resilience-oriented comprehensive planning strategy of DG is proposed in this paper to obtain the minimum total cost composed of resilience cost and economic cost. The innovation is summarized as below:

- A two-stage SMISOCP is proposed to obtain the resilience-oriented comprehensive planning of DG with the minimum total cost.
Based on the probability and characteristics of each scenario, the Monte Carlo method and K-means clustering algorithm is used to obtain the reduced failure scenarios for DG planning by omitting some similar scenarios.

The rest of this paper is organized as follows. Section 2 demonstrates the problem formulation. Method of failure scenarios generation and reduction is studied in Section 3. Section 4 provides the numerical results. Finally, Section 5 concludes the paper.

2. Problem formulation

Traditional planning strategies of DG are based on economy, which is not applicable for power distribution systems in areas prone to extreme weather events. Thus, the resilience and economy are comprehensively considered in this paper for the planning of DG. The whole problem is formulated in a two-stage SMISOCP.

2.1. Objective

The objective function is formulated as follows to minimize the total cost of power distribution system.

\[
\min F_{\text{plan}} + F_{\text{loss}} + F_{\text{failure}}
\]

\[
F_{\text{plan}} = \sum_{i \in \Omega_N} \left( c^{DG,v} P_i^{DG} + c^{DG,f} \right)
\]

\[
F_{\text{loss}} = \sum_{v \in \mathcal{O}_L} \sum_{i \in \mathcal{O}_N} c^{\text{Loss}} TL_{i,v}
\]

\[
F_{\text{failure}} = \sum_{s \in S} P(s) \left( \sum_{v \in \mathcal{O}_L} c^{\text{PC}} PC_{i,v}^{s} + \sum_{v \in \mathcal{O}_L} c^{\text{Loss}} TL_{i,v}^{s} \right)
\]

where, \( F_{\text{plan}} \) is the annual average planning cost of DGs, \( F_{\text{loss}} \) indicates the annual transmission loss cost under normal operating condition, \( F_{\text{plan}} \) and \( F_{\text{loss}} \) represent the cost in the first stage. \( F_{\text{failure}} \) denotes that the resilience cost in the second stage, which is the annual weighted sum cost of all failure scenarios. \( \Omega_N \) denotes the set of buses, \( \Omega_L \) is the set of lines, \( T_i \) represents the set of annual average normal operation time, \( T_f \) is the set of annual average time of failure caused by extreme weather event, \( S \) is the set of scenarios. \( c^{DG,f} \) is the annual average fixed installation cost of DG, \( c^{DG,v} \) denotes the per-unit annual average variable cost of DG, \( c^{\text{Loss}} \) is the per-unit cost of transmission loss, \( c^{\text{PC}} \) denotes the per-unit payment of load shedding. \( P_{DG} \) is the planning capacity of DG at bus \( i \). \( TL_{i,v}^{s} \) indicates the transmission losses of line \( l \) at time \( t \), and \( TL_{i,v}^{s} \) indicates the transmission loss of line \( l \) at time \( t \) under scenario \( s \). \( P(s) \) is the probability of scenario \( s \). \( PC_{i,v}^{s} \) is the load shedding of bus \( i \) at time \( t \) under scenario \( s \).

2.2. First Stage Constraints

In the first stage, the number, location and capacity of DG are planned based on the random failure scenarios in the second stage. Constraints (5)-(18) represent the planning constraints in the first stage.

\[
\sum_{i \in \Omega_N} x_{i}^{DG} \leq N_{DG}^{\max}
\]

\[
x_{i}^{DG} P_{DG}^{\min} \leq P_{i}^{DG} \leq x_{i}^{DG} P_{DG}^{\max}
\]

where, \( x_{i}^{DG} \) is a binary variable indicating whether bus \( i \) is installed with DG (1 - installed, 0 - not installed). \( N_{DG}^{\max} \) is the maximal number of DGs to be planned. \( P_{DG}^{\max} \) and \( P_{DG}^{\min} \) are the maximum and
minimum planning capacity of DG. Constraint (5) limits the number of DG to be planned. The maximal and minimal planning capacity of DG are restricted in constraint (6).

The normal operation constraints of the distribution system in the first stage are described as follows.

\[
\sum_{j \in \Omega} pf_{j,i} - \sum_{j \in \Omega} TL_{j,i} + pg_{i,j} + p_{i}^{DG} = \sum_{j \in \Omega} pf_{j,i} + pd_{i,j}, \quad \forall i \in \Omega_N, \forall t
\]  
\[\sum_{j \in N} qf_{j,i} - \sum_{j \in N} TL_{j,i} + qg_{i,j} = \sum_{j \in N} qf_{j,i} + qd_{i,j}, \quad \forall i \in \Omega_N, \forall t
\]

\[V_{i,j} - V_{i,j} \leq M_1 (1-u_{i,j}) + 2 \left( r_q pf_{j,i} + x_q qf_{j,i} \right) + \left( r_q^2 + x_q^2 \right) I^2_{ij}, \quad \forall i, j, \forall t
\]

\[V_{i,j} - V_{i,j} \geq M_1 (u_{i,j} - 1) + 2 \left( r_q pf_{j,i} + x_q qf_{j,i} \right) + \left( r_q^2 + x_q^2 \right) I^2_{ij}, \quad \forall i, j, \forall t
\]

\[I^2_{ij} = \frac{pf_{ij}^2 + qf_{ij}^2}{V_{ij}}, \quad \forall i, j, \forall t
\]

\[TL_{j,i} = I^2_{i,j} r_q, \quad \forall i, j, \forall t
\]

\[V_{ij}^{\min} \leq V_{ij} \leq V_{ij}^{\max}, \quad \forall i, \forall t
\]

\[-u_{ij} \cdot P_{ij}^{\min} \leq pf_{ij} \leq u_{ij} \cdot P_{ij}^{\max}, \quad \forall i, j, \forall t
\]

\[-u_{ij} \cdot Q_{ij}^{\max} \leq qf_{ij} \leq u_{ij} \cdot Q_{ij}^{\max}, \quad \forall i, j, \forall t
\]

\[0 \leq pg_{i,j} \leq PG_{i}^{\max}, \quad \forall i, \forall t
\]

\[0 \leq qg_{i,j} \leq QG_{i}^{\max}, \quad \forall i, \forall t
\]

\[0 \leq p_{i}^{DG} \leq P_{i}^{DG}, \quad \forall i, \forall t
\]

where, \( pf_{j,i} \) and \( qf_{j,i} \) are the power flow from bus \( i \) to \( j \) at time \( t \). \( I^2_{ij} \) is the square of current magnitude from bus \( j \) to \( i \) at time \( t \). \( r_q \) and \( x_q \) are the resistance and reactance of line \( ij \). \( pg_{i,j} \) and \( qg_{i,j} \) denote the power output of substation at bus \( i \) at time \( t \). \( p_{i}^{DG} \) indicates the active power output of DG at bus \( i \) at time \( t \). \( pd_{i,j} \) and \( qd_{i,j} \) indicate the power demand supplied of bus \( i \) at time \( t \). \( V_{ij} \) represents squared voltage magnitude of bus \( i \) at time \( t \). \( M \) stands for a large enough positive number. \( u_{ij} \) is a binary variable indicating the state of line \( ij \) at time \( t \) (1 - intact, 0 - damaged). \( V_{ij}^{\max} \) and \( V_{ij}^{\min} \) represent the maximum and minimum squared voltage magnitude of bus \( i \). \( P_{ij}^{\max} \) and \( Q_{ij}^{\max} \) are the maximum power flow of line from bus \( i \) to \( j \). \( PG_{i}^{\max} \) and \( QG_{i}^{\max} \) denote the maximum active and reactive power output of substation at bus \( i \). Constraints (7)-(8) stand for the power balance at bus \( i \). According to the DistFlow method [10], constraints (9)-(10) denote the line voltage drop by using the big-M method. Equation (11) indicates the current calculation of line \( ij \). Equation (12) describes the transmission loss calculation of line \( ij \). Constraint (13) denotes the range of voltage magnitudes. The power flow in lines are limited in (14)-(15), respectively. Constraint (16)-(17) restrict the power of substation at bus \( i \). The power of DG at bus \( i \) is constrained in (18).

2.3. Second Stage Constrains
The objective of the second stage is to optimize the DG operation under random failure scenarios. The scenario-based constraints in the second stage are shown as follows.
\[
\sum_{j \in \Omega_k} p_{f,i,j}^s - \sum_{j \in \Omega_k} T_{L,i,j}^s + p_{g,i,j}^s + p_{DG,i,j}^s + p_{c,i,j}^s = \sum_{j \in \Omega_k} p_{f,i,j} + p_{d,i,j}, \forall i \in \Omega_N, \forall t, \forall s
\] (19)
\[
\sum_{j \in \Omega_k} q_{f,i,j}^s - \sum_{j \in \Omega_k} T_{L,i,j}^s + q_{g,i,j}^s + q_{c,i,j}^s = \sum_{j \in \Omega_k} q_{f,i,j} + q_{d,i,j}, \forall i \in \Omega_N, \forall t, \forall s
\] (20)

\[
0 \leq p_{c,i,j} \leq p_{d,i,j}, \forall i, t, s
\] (21)
\[
0 \leq q_{c,i,j} \leq q_{d,i,j}, \forall i, t, s
\] (22)
\[
(6) - (15)
\] (23)

where, \(p_{c,i,j}\) and \(q_{c,i,j}\) denote the load shedding of bus \(i\) at time \(t\) under scenario \(s\). Other parameters and variables are the same as in the previous stage. Constraints (19)-(20) indicate the balance at bus \(i\) under a random failure scenario \(s\). Constraints (21)-(22) represent the ranges of load shedding of bus \(i\) under scenario \(s\).

3. Failure scenario generation and reduction

The SMISOCP model in (1)-(23) is an optimization problem with all the random failure scenarios. The random failure scenarios in this paper indicate the randomness of failure locations caused by extreme weather event. There are \(2^{N_L}\) combinations of failure scenarios (\(N_L\) is the number of lines), which is not feasible to calculate all of them. Thus, the Monte Carlo method is used to generate numerous failure scenarios to obtain the result.

To further improve the computational efficiency, this paper uses the K-mean clustering algorithm to reduce the similar scenarios. The characteristic vector of the failure scenario \(s\) is defined as \(y_s = (\delta_s, \Delta E_s)\), where \(\delta_s\) denotes the number of failure lines in scenario \(s\) and \(\Delta E_s\) is formulated as follows to indicate the accumulative energy loss during the period of failure in scenario \(s\).

\[
\Delta E_s = \sum_{\forall i \in \Omega_k} \sum_{\forall j \in \Omega_N} p_{c,i,j}
\] (24)

There might be some similar stochastic scenarios generated by Monte Carlo methods. K-means clustering is an effective algorithm to merge these similar vectors [11]. The purpose of K-means clustering is to divide the \(N_s\) dimensional vectors \(\{y_1, y_2, ..., y_{N_s}\}\) into \(k\) sets \(\{\phi_1, \phi_2, ..., \phi_k\}\). The formulation of K-means clustering is shown below.

\[
dis(k) = \arg \min \sum_{i \in \Omega_k} \sum_{\forall y \in \phi_i} \|y - \mu\|^2
\] (25)

where, \(dis(k)\) represents the sum of distance in cluster \(k\). \(\mu_i\) denotes the mean of points in \(\phi_i\). Any scenario in set \(\phi_i\) can be selected to represent this set. The probability of this representative scenario is the sum of all scenarios in this cluster:

\[
Pr(s,i) = \sum_{i \in \phi_k} Pr(s) = \sum_{i \in \phi_k} \frac{1}{N_s}
\] (26)

Where, \(Pr(s,i)\) is the probability of scenario \(s\) in set \(\phi_i\). \(N_s\) denotes the number of failure scenarios.

4. Numerical simulation results and discussion

The proposed optimal planning is tested on an IEEE 33-bus test system [12]. The method is implemented in a computer with Intel i7-9700 processor and 8 GB of memory. The formulated SMISOCP model is solved using Gurobi solver in MATLAB/YALMIP interface.
4.1. System description

The IEEE 33-bus test system is shown in Figure 1, where bus 1 is connected to the substation and the rest of the buses can be installed with DG. The main parameters are listed in Table 1.

**Figure 1.** IEEE 33-bus test system.

| Scenario | Probability | Damaged line |
|----------|-------------|--------------|
| 1        | 0.126       | L5-6, L8-9, L10-11, L11-12, L2-19, L23-24, L19-20, L32-33 |
| 2        | 0.042       | L3-4, L10-11, L2-19, L23-24 |
| 3        | 0.078       | L7-8, L14-15, L2-19, L23-24, L26-27, L30-31 |
| 4        | 0.081       | L9-10, L6-26, L32-33 |
| 5        | 0.087       | L2-3, L6-7, L20-21, L27-28, L30-31, L31-32 |
| 6        | 0.105       | L7-8, L8-9, L11-12, L21-22, L27-28 |
| 7        | 0.049       | L17-18, L29-30 |
| 8        | 0.043       | L2-3, L14-15, L21-22 |
| 9        | 0.076       | L8-9, L6-7, L5-6, L4-5, L3-4, L14-15, L16-17, L3-23, L26-27, L28-29 |
| 10       | 0.039       | L2-3, L5-6, L2-19, L27-28, L29-30 |
| 11       | 0.045       | L6-7, L16-17, L3-23, L6-26, L29-30 |
| 12       | 0.086       | L8-9, L9-10, L13-14, L15-16, L23-24, L27-28 |
| 13       | 0.065       | L7-8, L9-10, L24-25, L26-27 |
| 14       | 0.026       | L9-10, L32-33, L31-32, L29-30, L28-29, L27-28 |
| 15       | 0.052       | L11-12, L16-17, L17-18, L28-29 |

Based on the Monte Carlo method, 1000 failure scenarios are generated. Then the 15 representative scenarios are generated by using the K-means clustering algorithm. The representative scenarios and their probabilities are shown in Table 2.

**Table 1.** Parameters of the IEEE 33-bus test system.

| Type | Parameter | Value |
|------|-----------|-------|
| Constraint | $N_{DG}^{min}$ | 5     |
| Cost | $P_{DG}^{max} / P_{DG}^{min}$ | 800kW/100kW |
| Cost | $c_{DG, fC}$ | $5350$ |
| Cost | $c_{DG, vC}$ | $9.4/kWh$ |
| Cost | $c_{Loss}$ | $0.4/kWh$ |
| Cost | $c_{PC}$ | $0.9/kWh$ |

4.2. DG planning result

Three cases are studied to show the effectiveness of the DG planning strategy.

- Case 1: DG is planned according to the strategy proposed in this paper.
- Case 2: DG is planned based on the traditional method of economic optimality without considering resilience.
Case 3: DG is planned according to the method of optimal post-disaster resilience without considering cost.

The result of DG planning of three cases are shown in Table 3. As illustrated in Table 3, the optimized total capacity of DG in Case 1 is 2910kW, which is 30kW and 1510kW higher than Case 2 and Case 3. The number of DGs in Case 1 is 5, which are located around the power distribution system. When multiple faults occur, DG can provide power to most of the buses to prevent widespread outages in power distribution system.

The cost in (1) of three cases are depicted in Figure 2. Based on the DG planning strategy proposed in this paper, the total cost in Case 1 is $447588, which is 7.4% and 28.5% lower than Case 2 and Case 3.

Case 2 has the lowest total investment in $F_{plan}$ and $F_{loss}$ but ignores the resilience cost $F_{failure}$ so that the total cost is higher compared to Case 1. The resilience cost $F_{failure}$ in Case 3 is minimum but the economic cost $F_{loss}$ in Case 3 is higher than Case 1 and Case 2. The reason is that the DGs with less capacity are planned at the end of power distribution system in Case 3, which results in smaller outage losses when the fault occurs but larger transmission losses during normal operation.

The results reveal the advantages of the proposed DG planning strategy to obtain the minimum total cost composed of resilience cost and economic cost. Appropriate increasing in planning investments in DG can effectively reduce the resilience cost during extreme weather event. It can provide a quantitative reference for utilities to perform DG planning with the minimum total cost in disaster-prone areas.

### Table 3. Optimal planning results of three cases.

| Case     | The number of DGs | Optimized total capacity of DGs (kW) | Optimized location and capacity of DGs (kW) |
|----------|-------------------|-------------------------------------|-------------------------------------------|
| Case 1   | 5                 | 2910                                | $P_{8}^{DG}$ = 740, $P_{14}^{DG}$ = 530, $P_{24}^{DG}$ = 490, $P_{25}^{DG}$ = 420, $P_{32}^{DG}$ = 720 |
| Case 2   | 4                 | 2880                                | $P_{7}^{DG}$ = 800, $P_{14}^{DG}$ = 660, $P_{25}^{DG}$ = 710, $P_{31}^{DG}$ = 710 |
| Case 3   | 5                 | 1400                                | $P_{24}^{DG}$ = 420, $P_{25}^{DG}$ = 420, $P_{30}^{DG}$ = 200, $P_{51}^{DG}$ = 150, $P_{32}^{DG}$ = 210 |

![Figure 2. Optimal planning cost of three cases.](image-url)
5. Conclusion
A resilience-oriented comprehensive planning strategy of DG is proposed in this paper to obtain the minimum total cost composed of resilience cost and economic cost. Based on the random failure scenarios, a two-stage SMISOCP is established to obtain the result. To reduce the computational burden, the K-means clustering algorithm is used to reduce the failure scenarios generated by the Monte Carlo method. The results show that when the number and total capacity of planning DG increase, although the economic cost increases, it can effectively reduce the resilience cost after extreme weather event. The strategy provides a reference for utilities to perform DG planning in disaster-prone areas.

In the future, the planning strategy of DG, energy storage system and remote-controlled switch will be integrally considered. A more practical integrated model will be studied to obtain the optimal planning strategy with the minimum total cost.

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