Non-perturbative effects of geometry in wide-angle redshift distortions

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ABSTRACT

We use the formalism of Szapudi to derive full explicit expressions for the linear two-point correlation function, including redshift-space distortions and large angle effects. We take into account a non-perturbative geometric term in the Jacobian, which is still linear in terms of the dynamics. This term had been identified previously, but has been neglected in all subsequent explicit calculations of the linear redshift-space two-point correlation function. Our results represent a significant correction to previous explicit expressions and are in excellent agreement with our measurements in the Hubble volume simulation.

Key words: methods: analytical – large-scale structure of Universe.

1 INTRODUCTION

Redshift distortions represent a curse disguised as a blessing for high-precision cosmological applications. Radial coordinates of redshift surveys contain limited phase space information, which, in principle, can be used to constrain theories more than configuration information alone; moreover, velocities are sensitive to structure outside of the survey boundaries which potentially translates into a larger ‘effective volume’. On the other hand, redshift distortions are plagued with non-linearities, both on large and small scales, therefore in worst case they could amount to poorly understood contamination of the configuration space data. Our aim is to extend the theory of linear redshift distortions such that large angle information could be successfully extracted from galaxy surveys.

The work of Davis & Peebles (1983) and Peebles (1980) showing that redshift distortions affect the power spectrum spawned a lot of activity. The all-important linear, plane-parallel limit was first calculated by Kaiser (1987), showing that the effect on the power spectrum corresponds to ‘squashing’. The other well-known ‘fingers of God’ effect dominates small scales, and is irrelevant for our study. The Kaiser formula has been generalized for real space soon after (e.g. Hamilton 1993; Cole, Fisher & Weinberg 1995). These theories have been used to analyse surveys such as the Point Source Catalogue Redshift (PSCz) Survey (Tadros et al. 1999), the 2dF Galaxy Redshift Survey (2dFGRS; Peacock et al. 2001; Tegmark, Hamilton & Xu 2002; Hawkins et al. 2003) and the Sloan Digital Sky Survey (SDSS; Zehavi et al. 2002).

The distant observer approximation only holds if pairs are separated by a small angle. This means that a large fraction of pairs needs to be thrown away from modern wide-angle redshift surveys when they are analysed in this limit. These pairs are typically fewer and noisier than close pairs, but if our aim is to extract as much information as possible from a given survey, it would be desirable to add them in. Hamilton & Culhane (1996) related the ‘ω-transform’, a complexified Mellin-like transform, of the two-point correlation function to that of redshifted ω–space correlation function. The resulting spherical ωlm expansion is approximately orthogonal to redshift-space distortions. This expansion truncated at an appropriate mode was used in Tegmark et al. (2002) to analyse data in this transform space. The first explicit perturbation theory calculation in coordinate space was performed in Szalay, Matsubara & Landy (1998). The result is a simple-to-use finite expression, but only in coordinate space: in Fourier space, an infinite series will result for the redshift distorted analogue of the power spectrum. These formulae were later further generalized to include high-β effects in various cosmologies by Matsubara (2004). These calculations provide essential input for the pixel based Karhunen–Loève (KL) or quadratic likelihood analyses (e.g. Vogele & Szalay 1996; Tegmark, Taylor & Heavens 1997), since distant observer approximation is not valid for modern wide-angle galaxy surveys. The theory has been applied in several subsequent analyses of wide-angle redshift surveys, such as Pope et al. (2004) and Okumura et al. (2008). Despite its elegance, the theory did not agree well with dark matter simulations. Scoccimarro (2004) pointed out that this might be due to non-perturbative effects.

Szapudi (2004) reanalysed the redshift distortion problem from group theoretical point of view showing that tripolar spherical harmonics provide an excellent basis for expansion, and result in especially compact formulae. In addition it provided specific coordinate systems, one of which recovers the Legendre expansion of Szalay et al. (1998), while the other represents the same information in an even simpler two-dimensional Fourier mode expansion. We use this formalism to take into account a term in the Jacobian, previously neglected in all explicit calculations, to derive the full linear redshift distorted correlation function.

Kaiser’s original work starts with the full linear Jacobian. It contains a term negligible for small angles, that is linear in terms of the small fluctuations, and is essentially non-linear from the point of view of geometry: it contains a 1/r prefactor. Moreover, this term, if

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expanded in bipolar spherical harmonics (or any other way), would contribute infinite coefficients. Because of the presumed subdominance due to the prefactor, and complexity of the calculation, this term was neglected in all previous coordinate space expressions, although it is represented in the $c$-space expansion of Hamilton & Culhane (1996). In this paper, we introduce a hybrid approach, where we leave the essentially non-perturbative terms in the expansion intact; our tripolar expansion coefficients will still contain angular variables in a specific way. As we show later, this hybrid procedure results in a finite number of terms, and it provides significant corrections and improvement in the agreement with simulations. In retrospect, the omission of this term, while intuitively reasonable, is not justified, as its contribution can become important on the most interesting scales of tens of h$^{-1}$ Mpc’s.

In the next Section 2, we present the theory of linear redshift distortions including results from the geometric term in the Jacobian. We follow closely the formalism of Szapudi (2004), mainly focusing on the new aspects of this calculation. For reference, we print the full result, which has about twice as many terms as previously. In Section 3, we compare our results with preliminary measurements in the Hubble volume simulations, and present our conclusions.

## 2 REDSHIFT DISTORTION OF THE TWO-POINT CORRELATION FUNCTION

We use linear perturbation theory to predict the redshift distorted two-point correlation function in terms of the underlying power spectrum. Our calculation is based directly on the tripolar expansion formalism of Szapudi (2004), therefore our focus will be on the additional terms arising from the Jacobian.

The exact mapping between real and redshift space is $s_i = x_i - f v_i \hat{x}_i \hat{x}_j$, where the ‘hat’ denotes the proper unit vector, $f = \frac{\Omega_m}{\Omega_0}$, and the velocity has units which provides that its divergence is equal to the density up to linear order. From this, one can calculate the derivative of this matrix: $\delta s_i/\delta x_k = \delta_i^k + O_m$, where $O$ is linear in $v$. This results in a linear Jacobian $J = 1 + TrO = 1 - f \hat{x}_i \hat{x}_j \hat{v}_j - 2 f \frac{\hat{x}_i}{\hat{x}}\hat{x}_j$. The last term in the previous expression is usually omitted due to the fact that it scales with $1/x$, i.e. it would tend to zero for large distances, which loosely correspond to large angles as well. Closer examination of this term shows that it is of the same order as the previous term, not only in perturbation expansion (linear), but also of the order of magnitude. Our goal is to propagate this new term through the full calculation.

The linear density contrast and the two-point function can be expressed in the usual fashion:

$$\delta_1(x) = \int \frac{d^3k}{(2\pi)^3} e^{i (\hat{k} \cdot x)} \left[ 1 + f (\hat{\xi}_1 \cdot \hat{k})^2 - i 2 f \frac{\hat{\xi}_1 \cdot \hat{k}}{x_k} \right] \delta(k),$$

$$\langle \delta_1(x) \delta_1^*(x)^* \rangle = \int \frac{d^3k}{(2\pi)^3} P(k) e^{i (\hat{k} \cdot x)} \left[ 1 + \frac{f}{3} + 2 f \frac{P_1(\hat{\xi}_1 \cdot \hat{k})}{x_k} - i 2 f \frac{P_1(\hat{\xi}_1 \cdot \hat{k})}{x_k} \right] \frac{\hat{\xi}_1 \cdot \hat{x}_k}{x_k},$$

where $P_1$ and $P_2$ are Legendre polynomials, and $P(k)$ is the linear power spectrum. The third term in each of the brackets corresponds to the extension of the previous results; these would tend to zero in the plane-parallel limit. At wide angles, the separation between the galaxies, and the distance between a galaxy and the observer are of the same order, therefore $x_k$ is of the order of unity. This shows explicitly that the order of this term can be as large as the previous, and the detailed calculation confirms this.

Next we express the angular dependence of the correlation function with tripolar spherical harmonics:

$$S_{i,j}(\hat{\xi}_1, \hat{\xi}_2, \hat{\xi}) = \sum_{m_1, m_2, m} \left( \frac{l_1}{m_1} \frac{l_2}{m_2} \right) C_{l,m_1} C_{l,m_2} C_{l,m}(\hat{\xi}).$$

We use $x$ for denoting $x_1 - x_2$. On the right-hand side, one can find the Wigner $3j$ symbols and we define the normalized spherical functions as $C_{lm} = \sqrt{4\pi/2l + 1} Y_{lm}$; these latter result in simpler expressions.

Equation (2) has become more complex with the additions, $x_1$ and $x_2$ appear in the denominator resulting in the following angular dependence:

$$x_1 = g_1 x = \frac{\sin(\phi_2)}{\sin(\phi_2 - \phi_1)} x,$$

$$x_2 = g_2 x = \frac{\sin(\phi_1)}{\sin(\phi_2 - \phi_1)} x.$$

Expanding these terms into tripolar spherical harmonics would yield infinite terms, but simplification arises from the fact that they can be factored out of the integrals. All the rest can be expanded as in Szapudi (2004), resulting in finite expressions. We introduce $\phi_1$ to denote the angle between $\hat{x}_1$ and $\hat{\xi}$, and $\phi_2$ for the angle between $\hat{x}_2$ and $\hat{\xi}$. We emphasize that the coefficients of this (quasi-)tripolar expansion still has an angular dependence in the form of $g_1$ and $g_2$:

$$\xi_i = \sum_{l,j} B^{l;j}(x, \phi_1, \phi_2) S_{l,m}(\hat{\xi}_1, \hat{\xi}_2, \hat{\xi}).$$

After performing the expansions, only a finite number of coefficients survive. For reference, the ones from Szapudi (2004) are:

$$B^{00}(x) = \left( 1 + \frac{1}{3} f \right)^2 \xi_0^2(x),$$

$$B^{20}(x) = \frac{4}{9 \sqrt{5}} f^2 \xi_0^2(x),$$

$$B^{22}(x) = B^{20}(x) = \left( \frac{2}{3} f + \frac{2}{9} f^2 \right) \sqrt{5} \xi_2^2(x),$$

$$B^{22}(x) = \frac{4 \sqrt{10}}{9 \sqrt{7}} f^2 \xi_2^2(x),$$

$$B^{22}(x) = \frac{4 \sqrt{7}}{9 \sqrt{5}} f^2 \xi_2^2(x),$$

and the new terms, the main result of this paper, are:

$$B^{10}(x, \phi_1, \phi_2) = - \left( 2 f + \frac{2}{3} f^2 \right) \frac{\sqrt{3}}{g_1 \xi_1^2(x)},$$

$$B^{01}(x, \phi_1, \phi_2) = \left( 2 f + \frac{2}{3} f^2 \right) \frac{\sqrt{3}}{g_1 \xi_1^2(x)},$$

$$B^{11}(x, \phi_1, \phi_2) = \frac{4 \sqrt{7}}{\sqrt{15}} f^2 \frac{1}{g_2 \xi_1^2(x)},$$

$$B^{21}(x, \phi_1, \phi_2) = - \frac{4 \sqrt{7}}{\sqrt{15}} f^2 \frac{1}{g_2 \xi_1^2(x)},$$
$B^{123}(x, \phi_1, \phi_2) = \frac{4 \sqrt{3} f^2}{\sqrt{15} g_{1x}} \xi^n_1(x)$.

$B^{213}(x, \phi_1, \phi_2) = -\frac{4 \sqrt{3} f^2}{\sqrt{15} g_{2x}} \xi^n_1(x)$.

$B^{110}(x, \phi_1, \phi_2) = -\frac{4 f^2}{\sqrt{3} \xi g_{1x}^2} \xi^0_1(x)$.

$B^{112}(x, \phi_1, \phi_2) = -\frac{4 \sqrt{10} f^2}{\sqrt{3} \xi g_{1x}^2} \xi^0_2(x)$,

where $\xi^n_1(x) = \int dk/2\pi^2 k^n j(k) P(k)$ with $j$ being the spherical Bessel function.

For further elaboration, we choose coordinate system (a) from Szapudi (2004). This corresponds to our previous choice of angles with $\theta_1, \theta_2$, with which the $S_{l_{ij}}((\hat{x}_1, \hat{x}_2, \hat{x})) = S_{l_{ij}}(\pi/2, \phi_1, \pi/2, \phi_2, \pi/2, 0)$ functions can be expressed using cosines and sines only. Using the same notation as Szapudi (2004)

$\xi_i(\phi_1, \phi_2, x) = \sum_{n_1, n_2=0, 1, 2} a_{n_1 n_2} \cos(n_1 \phi_1) \cos(n_2 \phi_2) + b_{n_1 n_2} \sin(n_1 \phi_1) \sin(n_2 \phi_2)$.

Again, for reference, the previously calculated coefficients are

$\alpha_{00} = \left(1 + \frac{2 f}{3} + \frac{2 f^2}{15}\right) \xi^n_0(x)$

$\alpha_{02} = \frac{-f + \frac{2 f^2}{21}}{\xi^n_2(x) + \frac{3 f^2}{140} \xi^n_1(x)}$,

$\alpha_{22} = -\frac{f}{15} \xi^n_2(x) + \frac{19 f^2}{140} \xi^n_1(x)$,

$\beta_{22} = -\frac{f}{15} \xi^n_2(x) - \frac{4 f^2}{35} \xi^n_1(x)$

and the new expressions of this work correspond to

$\alpha_{10} = \frac{\tilde{a}_{10}}{g_1} = \left(2 f + \frac{4 f^2}{5}\right) \frac{1}{g_1} \xi^n_1(x)$

$\alpha_{01} = \frac{\tilde{a}_{01}}{g_2} = \left(2 f + \frac{4 f^2}{5}\right) \frac{1}{g_2} \xi^n_1(x)$,

$\alpha_{11} = \frac{\tilde{a}_{11}}{g_1 g_2} = \frac{4 f^2}{5 g_{1x} x} \xi^n_1(x) - \frac{8 f^2}{3 g_1 x^2} \xi^0_0$,

$\alpha_{21} = \frac{\tilde{a}_{21}}{g_2} = -\frac{f}{15} \xi^n_2(x) + \frac{3 f^2}{5 g_2 x} \xi^n_1(x)$,

$\alpha_{12} = \frac{\tilde{a}_{12}}{g_1} = \frac{2 f^2}{5 g_{1x} x} \xi^n_1(x) - \frac{3 f^2}{5 g_1 x} \xi^n_0$,

$\beta_{11} = \frac{\tilde{b}_{11}}{g_1 g_2} = \frac{4 f^2}{5 g_{1x} x} \xi^n_1(x) + \frac{4 f^2}{3 g_1 x^2} \xi^0_0$,

$\beta_{21} = \frac{\tilde{b}_{21}}{g_2} = \frac{2 f^2}{5 g_{2x} x} \xi^n_1(x) - \frac{2 f^2}{5 g_2 x} \xi^n_0$,

$\beta_{12} = \frac{\tilde{b}_{12}}{g_1} = \frac{2 f^2}{5 g_{1x} x} + \frac{2 f^2}{5 g_{1x}^2}

\xi^n_1(x)$.

(11)

The left-hand panel of Fig. 1 shows the measured and the theoretical two-point functions without redshift distortions. The theory agrees with the measurements only after a shift by a constant. This is due to the ‘integral constraint’ problem (e.g. Peebles 1980), possibly compounded with slight non-linear effects. This constant represents a bias which is approximately equal to the average of the two-point correlation function over the survey area. It can be determined several ways (see discussion below).

Next, an observer was placed at the centre of each subvolume and the mapping between real and redshift space was performed using the velocities recorded in the simulation. The correlation function was then measured using brute force counting of pairs in high-resolution bins matching our choice of coordinate system described earlier. The right-hand panel of Fig. 1 presents wide-angle redshift distortion theory both with and without non-perturbative geometric corrections. The latter cannot be made to agree with the measurements even using a constant offset due to the integral constraint. In contrast, the theory presented in this paper provides excellent agreement with the measurements if the effects of integral constraint are taken into account. Note that this shift corresponding to the latter is expected to be larger with redshift distortions included simply because the two-point function is enhanced on large scales.

3 DISCUSSION AND SUMMARY

To understand our results, we expanded our formulae to identify leading order corrections to the Kaiser limit.

The leading order corrections to the distant observer approximation are second order. Using the notation $\frac{1}{2}(\phi_1 + \phi_2) = \phi$ and $\frac{1}{2}(\phi_2 - \phi_1) = \Delta \phi$, and keeping leading order terms in $\Delta \phi$ results in

$\hat{\xi}_i(\phi, \Delta \phi, x) = a_{00} + 2a_{02} \cos(2\phi) + a_{22} \cos^2(2\phi) + b_{22} \sin^2(2\phi)$

$+ \left[ -4a_{02} \cos(2\phi) - 4a_{22} - 4b_{22} \right] \Delta \phi^2$

$+ \left[ -4a_{10} \cot^2(\phi) + 4a_{11} \cot^2(\phi) \right.

$- 4a_{12} \cos^2(\phi) \cos(2\phi) + 4b_{11} - 8b_{12} \cos^2(\phi) \Delta \phi^2 +$

$O(\Delta \phi^3)$.

The first line of equation (12) corresponds to the Kaiser formula ($\Delta \phi = 0$). The next line contains leading order corrections corresponding to previous work only, and the third line collects leading order corrections from the geometric term in the Jacobian. These are all of the same order, reassuring the need of keeping the geometric non-perturbative terms. We conjecture that the terms containing the $\cot^2(\phi)$ could be responsible for the reported failure of the linear theory for small angles along the line of sight (Okumura et al. 2008).

As a preliminary test of the validity of our calculations, we measured correlation functions in the Hubble volume simulation (Evrard et al. 2002), using cosmological parameters $\sigma_8 = 0.9$, $n_s = 1$, $\Omega_m = 0.3$, $\Omega = 0.7$, $h = 0.7$, $\Omega_b h^2 = 0.0196$ and a volume of $(3000 h^{-1} \text{Mpc})^3$, with and without redshift distortions. The volume of the simulation was divided into $9^3$ subvolumes to obtain the errorbars.

It is worth to emphasize again that the angular dependence $g_1$ and $g_2$ is suppressed for clarity in the above formulae, but it is obviously carried through according to the definition of these functions. If the equivalence of the configurations $(\phi_1, \phi_2) \rightarrow (\pi - \phi_2, \pi - \phi_1)$ is taken into account (same pairs can be counted twice), the number of independent new coefficients is five, i.e. the number of terms approximately doubled. Next, we explore the relevance of these calculations, and compare the theoretical predictions with measurements in dark matter only $N$-body simulations.
correlation length and motion of the local group can be transformed if we only deal with pairs further away from the observer than the 2008 The Authors. Journal compilation C⃝

With linear theory (dashed and solid lines). The errorbars were estimated from 93 subvolumes of the Hubble volume. Shifting the theory by 0.000 81 downward, motivated by the integral constraint, provides an excellent fit to the data. (Right-hand panel) Redshift distorted correlation function of the Hubble volume simulation (symbols) at constant opening angle (0.71 rad) and while the ratio of the distances of the particles in the pair is kept fixed (at 1.57). The errorbars were estimated as before. The lines indicate the linear theories with (this paper) and without the geometric terms. The solid line is the corrected theory with a downshift of 0.0016. The integral constraint correction is expected to be larger since the average of the two-point function is larger.

While one can simply fit this constant shift, corresponding to throwing away a constant from the two-point correlation function Fisher et al. (1993), we have estimated it in two more ways: Monte Carlo integrating the theoretical expression for the correlation function, and empirically measuring the variance of the average density on the scales of the subsamples. All three methods are consistent with each other; Fig. 1 uses the empirical variance over subsamples. Note that in applications, the first method, i.e. discarding a constant from the theory, is the most prudent procedure to follow, since fluctuations on the scale of the full survey are not measurable.

While these measurements are preliminary in the sense that we did not try to span the full parameter space of wide-angle redshift distortions, the results presented in Fig. 1 appear to be typical: any other configurations we measured showed similar improvement. Scanning the full parameter space with our present brute force two-point correlation function code would be impractical, since we need a very large number of pairs in each bin to beat down the errorbars enough that the difference between the two theories can be reliably measured. Although we developed a fast grid based code as well, we found that at these small values of the correlation function the pixel window function effects become important. These are more complex for the redshift distorted correlation function depending on three variables than in real space. Such effect should be modelled very accurately before one could fully span the available parameter space.

A few simple extensions and modifications of our theory are needed for practical applications when measuring the two-point function Okumura et al. (2008), or when using our results to estimate a theoretical covariance matrix for a KL analysis, (see Pope et al. 2004, for details). If the sample is not volume limited, the redshift-space density contrast is defined through the redshift-space selection function \[ \Phi_1(r) \]. The effect of this can be taken into account by \[ g \rightarrow 2g/\alpha \], where \[ \alpha = \frac{\text{deg} (r/\text{Mpc})}{\text{deg} (\text{Mpc})} \]. The local bias can be neglected if we only deal with pairs further away from the observer than the correlation length and motion of the local group can be transformed out by using the frame of the cosmic microwave background. These problems have been discussed in detail by Hamilton & Culhane (1996), and the solutions are exactly analogous in our case.

Note that the integral constraint problem does not appear in KL analysis where only modes orthogonal to the average density are used. This is more elegant than the simple treatment we have given here, but the essence of it is the same: regarding the constant in the two-point correlation function as a nuisance parameter accomplishes the same for direct applications of our formulae.

With these caveats we conclude that our theory of wide-angle redshift distortions yielded simple-to-use explicit formulae, which agree with simulations. The corrections to previous formulae represent significant improvement at modest cost in complexity. Possible generalizations along the lines of Matsubara (2004) are left for future research.

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