AUTOMATIC CALCULATION OF COMPLETE $O(\alpha)$ CORRECTIONS TO $e^+e^- \rightarrow W^+\mu\bar{\nu}_\mu$

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Abstract

Using the automatic system GRACE-LOOP, the full $O(\alpha)$ electroweak corrections has been calculated for the process $e^+e^- \rightarrow W^+\mu\bar{\nu}_\mu$. The total correction to the cross section is found to be typically $-6.4\%$ at $\sqrt{s} = 190$ GeV with $10^\circ$ cut on the muon angle from the beam, including the correction from the hard photon emission. It is observed that the correction is rather sensitive to the physical parameters. With the same conditions the correction corresponding to the real $W$-boson pair is $-4.1\%$ and hence the deviation is not negligible.

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1 Introduction

To study the $W$-boson properties in LEP2 experiments one needs a generator which can produce 4-fermion final states, since one can observe only these states but not the $W$-boson pair directly. This brings some technical difficulties in the calculation; there are so many (76) 4-fermion processes induced by $e^+e^-$ collisions and the number of Feynman diagrams in each process is also big, for instance 144 for the case of $e^+e^-e^+e^-$ even in the tree level. In performing such a large scale computation it is desirable to have an automatic calculating system. Such a system GRACE [1] has been developed for the tree level calculation and utilized to create a generator in the most efficient way [2].

Including the loop diagrams several works [3] have shown that the $\mathcal{O}(\alpha)$ full electroweak corrections to the on-shell $W$-pair production become sizeable depending on the physical parameters, such as the mass of top-quark. Then it is interesting to see what is the $\mathcal{O}(\alpha)$ corrections to the 4-fermion state and to compare them with those of $W$-pair. This is a study also indispensable for a precise data analysis. Till now, unfortunately, there exists no complete calculation of the corrections except for those from the hard photon emission or from a very limited number of loops [4]. The reason hard to carry out the full calculation is clearly a huge number of Feynman diagrams involved. For instance there are about 1,000 diagrams for $e^+e^-\rightarrow \mu\bar{\nu}_\mu u\bar{d}$, and it amounts more than 9,000 for the case of two $e^+e^-$ pairs.

As an intermediate step and also as an exercise toward various 4-fermion processes, we report here the full $\mathcal{O}(\alpha)$ corrections to $e^+e^-\rightarrow W^+\mu\bar{\nu}_\mu$. This is unsatisfactory in regard with the final goal, the 4-fermion case, but certainly goes one step further from the real $W$-pair approximation.

2 One-loop Corrections to $e^+e^-\rightarrow W^+\mu\bar{\nu}_\mu$

There are 6 tree diagrams and 287 loops which contribute to the $O(\alpha)$ corrections. Feynman diagrams and corresponding matrix elements were generated by the automatic calculation system GRACE-LOOP [5]. In addition the radiative process $e^+e^-\rightarrow W^+\mu\bar{\nu}_\mu\gamma$ with a hard photon $(k > k_c)$ has been calculated with GRACE, which is also necessary to complete the total correction. We assume the Feynman gauge throughout the calculation of the one-loop diagrams.

First we give the result of this work. Some technical aspects will be described later. In the estimation we have restricted the scattering angle of the final muon to $|\cos \theta_\mu| < \cos 10^\circ$, to avoid a numerical instability which happened in a few loop diagrams. The unpolarized total cross section with full $O(\alpha)$ corrections is then $1.720 \pm 0.008$ pb at $\sqrt{s} = 190$ GeV for the following physical parameters: $M_Z = 91.187$ GeV, $M_W = 80.37$ GeV, $\Gamma_W = 2.05$ GeV, $M_{Higgs} = 300$ GeV and $m_{top} = 180$ GeV. Here $-1.413$ pb comes from the one-loop diagrams and the soft radiation with the photon energy cut $k_c = 100$ MeV, and $1.295$ pb from the hard radiation. The corresponding tree cross section is $1.838$ pb. Hence the total radiative correction to the lowest order is $-6.4 \pm 0.4\%$. Figure 1(a) and (b) depict the scattering angle distribution of $W^+$ and $\mu^-$, respectively. Angles $\theta_W$ and $\theta_\mu$ are defined from the initial electron. The histogram represents the distribution of the tree process and the black squares indicate that corrected to $O(\alpha)$. The deviation seen around the forward muon angle (backward $W^+$) is similar to that for $W$-pair, which is due to the hard radiation.

Let us compare this result with the case of the real $W$-boson pair production, $e^+e^-\rightarrow W^+W^-$. This process has 3 tree and 140 one-loop diagrams. With the same set of physical parameters but without any cut on $W^\pm$ we found the total correction of $-4.1\%$ with the tree
cross section of 18.04 pb. It is interesting that the corrections are sensitive to the top-quark
mass, which appears in the quantities relevant to the two-point functions. This is common
to both processes; for example $-1.4 \%(W_{\mu\nu})$ and almost $0\%(W$-pair) for $M_W = 80.22$ GeV
and $m_{\text{top}} = 130$ GeV.

One can see that the difference between these two processes is not negligible. Several
sources could contribute to this. For $W_{\mu\nu}$ there remain 147 diagrams that cannot be reduced
to the double $W$-resonant process. Off-shellness of $W^-$ also gives some portion. A detailed
analysis of the difference is now under investigation.

Next we briefly touch the technicality of the calculation, particularly how it was checked.
First of all the system has been successfully applied to several $2 \to 2$ processes to repro-
duce the known results[5]. Hence we are convinced that every part of the system works
correctly, i.e., the amplitude generation, the loop integrations for 3- and 4-point functions,
the renormalization constants and the renormalized self-energies.

However, since the process interested here is $2 \to 3$-body, the following new aspects
appear which are absent in any $2 \to 2$-body process: The multi-dimensional integration in
the phase space and an evaluation of 5-point functions. The 3-body kinematics is already
equipped in the library of the automatic system. The Feynman parameters are numerically
integrated by a Monte Carlo method together with the kinematical variables in the phase
space[6]. We would like to emphasize that this way of integration provides enough accurate
results, with an uncertainty of less than 0.1%. The 5-point function is estimated by a known
reduction formula which relates the former to a sum of five 4-point functions[7].

We have performed the following checks as the evident self-test of the calculation: (1)
Renormalization. $1/\epsilon$ from UV-divergence is kept as a variable in the program. The result
was independent of this variable, as it should be. (2) Infra-red divergence. This divergence
appearing in some loop diagrams is regularized by a fictitious photon mass $\lambda$. This is also
kept in the program and it was checked whether the dependence is compensated by the soft
radiation($k < k_c$)[5]. (3) Soft-photon cut independence. Independence of the unphysical soft
photon energy cut parameter $k_c$ was observed, which separates the soft radiation(analytic
formula) and the hard one(Monte Carlo).
3 Summary

By using GRACE-LOOP, designed to automate the evaluation of the one-loop diagrams in the standard model, a numerical estimation has been carried out, for the first time, to get the full $O(\alpha)$ corrections to $e^+e^- \to W^+\mu\bar{\nu}_\mu$. This process is still not a realistic one because the $W^+$ is treated as a real particle. Although our estimation has such a limitation, we found that the total correction shows a deviation from that of $W$-pair case. The difference is not small to be ignored completely, though these two corrections are of course comparable in magnitude. From the view point of the precise measurement it is very interesting and urgent to look how much the corrections are for various 4-fermion final states, such as $e^+e^- \to \mu\bar{\nu}_\mu ud$.

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