Updated Combination of CDF and D0 Results for the Mass of the $W$ Boson

The Tevatron Electroweak Working Group* for the CDF and D0 Collaborations

Abstract

We summarize and combine the results on the direct measurements of the mass of the $W$ boson in data collected by the Tevatron experiments CDF and D0 at Fermilab. Results from CDF Run-0 (1988-1889) and Run-I (1992-1995) have been combined with D0 results from Run-I, the CDF 200 pb$^{-1}$ published results from the first period of Run-II (2001-2004) and the recent 1 fb$^{-1}$ result in the electron channel from D0 (2002-2006). The results are corrected for any inconsistencies in parton distribution functions and assumptions about electroweak parameters used in the different analyses. The resulting Tevatron average for the mass of the $W$ boson is $M_W = 80,420 \pm 31$ MeV.

*The Tevatron Electroweak Working group can be contacted at tev-ewwg@fnal.gov. More information is available at http://tevewwg.fnal.gov.
1 Introduction

The CDF and D0 experiments at the Tevatron proton-antiproton collider located at the Fermi National Accelerator Laboratory have made several direct measurements of the width, $\Gamma_W$, and mass, $M_W$, of the $W$ boson. These measurements use both the $e\nu$ and $\mu\nu$ decay modes of the $W$.

Measurements of $M_W$ have been reported by CDF from the data of Run-0 [1], Run-I [2, 3] and recently Run-II [4] and by D0 from Run-I [5, 6, 7]. This document adds a new D0 measurement from Run-II [8]. There are no new measurements of the width of the $W$ since the previous average in July 2008 [10] and it will not be discussed further.

This note reports the combination of the mass measurements, and takes into account statistical and systematic uncertainties as well as correlations among systematic uncertainties. It supercedes the previous summaries [9, 10]. The measurements are combined using the analytic BLUE method [11, 12] which is mathematically equivalent to the methods used in previous combinations [9, 10], but, in addition, yields the decomposition of the uncertainty on the average in terms of categories specified in the input measurements [12]. These changes are described in more detail in Ref. [10].

As in the July 2008 analysis [10], there are three significant changes relative to the pre-2008 averages:

- The individual $M_W$ measurement channels for CDF Run-0, Run-Ia and Run-Ib are now combined for each run period using the BLUE method to achieve a consistent statistical treatment across all results. The Run-I D0 and Run-II CDF measurements were already combined using the BLUE method.

- The central values of the mass measurements made with very old PDF sets are corrected to use the same parton distribution functions (PDFs) from CTEQ6M [13] with uncertainty estimates from the CTEQ6M, CTEQ6.1M [14] and MRST2003 [15] PDF sets.

The new D0 Run-II measurement uses CTEQ6.1M while the CDF Run II measurement used CTEQ6M. The difference in the mass extracted using these PDF sets was found to be less than $1 \pm 4$ MeV, where 4 MeV is the statistical uncertainty on the estimated difference. No correction is applied for this difference.

- The CDF Run-0 and Run-Ia results were obtained from very old PDF sets (MRS-B [16] and MRS-D' [17] respectively) that did not utilize the $W$ charge asymmetry results and so provide somewhat offset predictions from the more modern PDF sets used in the later analyses. The predictions based on the more modern MRS [15] and CTEQ [13] sets used in Run-Ib and Run-II analyses have a variance smaller than the common PDF errors assumed in these analyses i.e. $\approx 10$ MeV. We therefore only apply PDF corrections to the CDF Run-0 and Run-Ia results since the shifts for these data are larger than 10 MeV. We retain the PDF uncertainties of 60 MeV and 50 MeV quoted in the original publications. We note that these corrections were also applied in the Run-I CDF combination presented in [3].
The mass values are also corrected to the same assumed W boson width value in order to achieve consistency across all results. The value of $\Gamma_W$ quoted here corresponds to a definition based on a Breit-Wigner propagator in the “running-width scheme”, $1/(M^2 - M_W^2 + iM^2\Gamma_W/M_W)$, with a width parameter, $\Gamma_W = 2093.2 \pm 2.2$ MeV predicted by the standard model [18] using the 2008 world average W boson mass of $80,398 \pm 25$ MeV. All measured masses are corrected to this value using $\Delta M_W = -(0.15 \pm 0.05)\Delta \Gamma_W$ as in Ref. [10]. The W boson mass uncertainty arising from an uncertainty in the W boson width is now consistently treated across all measurements.

The shifts due to these corrections are shown in Table 1 below, which summarizes all of the inputs to the combination. ²

|                      | CDF 0 | CDF Ia | CDF Ib | D0 I | CDF II | D0 II |
|----------------------|-------|--------|--------|------|--------|-------|
| $M_W$ published      | 79,910| 80,410 | 80,470 | 80,483| 80,413 | 80,400.7|
| Total uncertainty published | 390  | 180    | 89     | 84   | 47.9   | 43    |
| $\Gamma_W$ used in publication | 2,100| 2,064  | 2,096  | 2,062| 2,094  | 2,099.6|
| **Corrections**      |       |        |        |      |        |       |
| $\Delta \Gamma_W$ correction applied | 1.1  | -4.4   | 0.5    | -4.7 | 0.2    | 1.0   |
| PDF correction applied | 20    | -25    | 0      | 0    | 0      | 0     |
| BLUE correction applied | -3.5 | -3.5   | -0.1   | 0    | 0      | 0     |
| Total correction     | 17.6  | -32.9  | 0.4    | -4.7 | 0.2    | 1.0   |
| $M_W$ corrected      | 79,927.6 | 80,377.1 | 80,470.4 | 80,478.3 | 80,413.2 | 80,401.7|
| **Uncertainties**    |       |        |        |      |        |       |
| Total BLUE uncertainty | 390.9 | 181.0  | 89.3   | 83.4 | 47.9   | 43.3  |
| Uncorrelated uncertainty | 386.1 | 172.8  | 87.9   | 82.1 | 44.7   | 41.3  |
| PDFs                 | 60    | 50     | 15     | 8.1  | 12.6   | 10.4  |
| Radiative corrections | 10    | 20     | 5      | 12   | 11.6   | 7.5   |
| $\Gamma_W$ (published) | 0    | 20     | 0      | 10   | 0      | 0.5   |
| $\Gamma_W$ (this analysis) | 0.5  | 1.5    | 0.5    | 1.5  | 0.5    | 0.5   |

Table 1: Table 1 of the 2008 summary [10], updated with the D0 result from Run-II. All entries are in MeV.

²As described in [10], the use of the BLUE method in internal combinations caused the early Run-0, Run-Ia and Run-Ib CDF $M_W$ values to change by $-3.5$ MeV, $-3.5$ MeV and $+0.1$ MeV respectively. These corrections are also listed in Table 1. When these new values are combined using the BLUE method, the Run-0/I CDF combination is changed from $80,433 \pm 79$ MeV quoted in [3] and used in previous combinations [9], to $80,436 \pm 81$ MeV.
2 New data from D0 on $M_W$

The $W$ boson mass determined by D0 in Run-II [8], using $W$ decays into electrons and neutrinos from 1 fb$^{-1}$ of data at $\sqrt{s} = 1960$ GeV, derives from 3 observables: the transverse mass $M_T$, the electron transverse momentum $p_T^e$ and the transverse missing momentum, which yield a combined $W$ boson mass of $80.401 \pm 21(stat.) \pm 38(syst.)$ MeV. The individual contributions to the uncertainty are summarized in Table 2.

| Source                        | Uncertainty in MeV | Correlation coefficient with other experiments |
|-------------------------------|--------------------|-----------------------------------------------|
| **Experimental uncertainties**|                    |                                               |
| W Statistics                  | 21.0               | 0                                             |
| Electron energy calibration   | 33.4               | 0                                             |
| Electron resolution model     | 2.2                | 0                                             |
| Electron energy offset        | 5.2                | 0                                             |
| Electron energy loss model    | 4.0                | 0                                             |
| Recoil model                  | 7.8                | 0                                             |
| Electron efficiencies         | 5.2                | 0                                             |
| Backgrounds                   | 3.2                | 0                                             |
| **Production uncertainties**  |                    |                                               |
| PDFs                          | 10.4               | 1.0                                           |
| EWK radiative corrections     | 7.5                | 1.0                                           |
| Boson $p_T$                   | 2.7                | 0                                             |
| $\Gamma_W$                   | 0.5                | 1.0                                           |

Table 2: Contributions (in MeV) to the uncertainty for the D0 Run-II $W$ boson mass result.

3 Correlation of the D0 Run II result with other measurements

The experimental systematic uncertainties on the new D0 measurement are dominated by the energy scale for electron candidates and are almost purely statistical, as they are mainly derived from the limited sample of $Z^0$ decays. All of the experimental uncertainties are assumed to be uncorrelated with previous measurements.

Three systematic uncertainties due to the production of $W$ and $Z$ bosons are assumed to be fully correlated between all Tevatron measurements, namely (1) the parton distribution functions (PDFs), (2) the width of the $W$ boson ($\Gamma_W$) and (3) the electroweak radiative corrections.
The D0 measurement also includes an uncertainty in the boson $p_T$ distribution parameterization which is derived from a global fit to deep-inelastic scattering and hadron collider data [19]. In previous analyses, this source of uncertainty is treated differently, and it is therefore regarded as uncorrelated with the earlier measurements.

Current estimates of the uncertainties due to radiative corrections include a significant statistical component. The WGRAD/ZGRAD [20] and PHOTOS [21] models are used in the different measurements and yield results consistent within statistical uncertainties. We assume that the effects of radiative corrections are 100% correlated between measurements because the models used are quite similar, but we anticipate that both the uncertainties and the correlations have the potential to be reduced in the future using better models and higher statistics in simulations.

4 Combination of Tevatron $M_W$ measurements

The six measurements of $M_W$ to be combined are given in Table 1. The CDF Run-0, Run-Ia and Run-Ib values correspond to averages of two measurements in different channels where internal correlated systematic uncertainties e.g. momentum scale, are accounted for in the averaging. The Run-I D0 measurement combines 10 measurements using the BLUE method. The Run-II CDF measurement combines a total of six individual measurements in the muon and electron decay channels.

Tables 1 and 2 of our previous summary [10] are now extended with the combined D0 Run-II result. The combined Tevatron result is calculated using BLUE with input from Table 1 which includes the new D0 Run-II result.

The combined Tevatron result for the mass of the $W$ is:

$$M_W = 80,420 \pm 31 \text{ MeV}.$$  \hspace{1cm} (1)

The $\chi^2$ for the combined result of 2.69 for 5 degrees of freedom, corresponds to a probability of 74.8%. Table 3 shows the weight of each measurement entering the combination.

The total uncertainty of 31 MeV on the Tevatron average is split up into an uncorrelated uncertainty of 26.9 MeV, and systematic uncertainties due to assumptions about the production of $W$ and $Z$ bosons of 11.6 (PDF), 9.3 MeV (radiative corrections), and 0.8 MeV ($\Gamma_W$). The global correlation matrix for the 6 measurements is shown in Table 4.
5 Conclusion

The new direct measurement of the mass of the $W$ by the D0 experiment has been combined with the previous CDF and D0 measurements. The new Tevatron result for the $W$ boson mass is:

$$M_W = 80,420 \pm 31 \text{ MeV}.$$  \hspace{1cm} (2)

For the first time the total uncertainty of 31 MeV from the Tevatron is smaller than that of 33 MeV from LEP II [23].

The combination of the new Tevatron result with the LEP II preliminary result, assuming no correlations, yields the world average:

$$M_W = 80,399 \pm 23 \text{ MeV}.$$ \hspace{1cm} (3)

Figure 1 shows an update of the Figure 1 of the TEVEWWG note [10] displaying the new results.

|        | Relative Weights in % |
|--------|------------------------|
| CDF 0  | 0.10                   |
| CDF Ia | 0.60                   |
| CDF Ib | 9.39                   |
| D0 I   | 10.98                  |
| CDF II | 34.64                  |
| D0 II  | 44.28                  |

Table 3: Relative weights of the contributions in %.
Table 4: Correlation coefficients between the different experiments using the method of Ref. [10].
Figure 1: Summary of the measurements of the $W$ boson mass and their average as of July 2009. The result from the Tevatron corresponds to the values in this note (see Table 1) which include corrections to the same $W$ boson width and PDFs. The LEP II result is from Ref. [23]. An estimate of the world average of the Tevatron and LEP results assuming no correlations between the Tevatron and LEP is included.
References

[1] The CDF Collaboration, F. Abe et al., “A Measurement of the W Boson Mass in 1.8 TeV pp Collisions”, Phys. Rev. D43, 2070 (1991).

[2] The CDF Collaboration, F. Abe et al., “Measurement of the W Boson Mass”, Phys. Rev. D52, 4784 (1995).

[3] The CDF Collaboration, T. Affolder et al., “Measurement of the W boson mass with the Collider Detector at Fermilab”, Phys. Rev. D64, 052001 (2001).

[4] The CDF Collaboration, T. Aaltonen et al., “First Measurement of the W Boson Mass in Run II of the Tevatron”, Phys. Rev. Lett. 99, 151801 (2007); Phys. Rev. D77, 112001 (2008).

[5] The DØ Collaboration, B. Abbott et al., “Determination of the mass of the W boson using the D0 detector at the Tevatron”, Phys. Rev. D58, 012002 (1998).

[6] The DØ Collaboration, B. Abbott et al., “A measurement of the W boson mass using large rapidity electrons”, Phys. Rev. D62, 092006 (2000).

[7] The DØ Collaboration, V. Abazov et al., “Improved W boson mass measurement with the D0 detector”, Phys. Rev. D66, 012001 (2002).

[8] The DØ Collaboration, V.M. Abazov et al., “Measurement of the W boson mass”, submitted to Physical Review Letters, Fermilab-Pub-09/388-E, arXiv:0908.0766 [hep-ex] (2009).

[9] The CDF Collaboration, the D0 Collaboration and the Tevatron Electroweak Working Group, “Combination of CDF and DØ results on W boson mass and width”, Phys. Rev. D70, 092008 (2004).

[10] The CDF Collaboration, the D0 Collaboration and the Tevatron Electroweak Working Group, “Combination of CDF and D0 results on the W boson mass and width”, arXiv:0808.0147 [hep-ex] (2008).

[11] L. Lyons, D. Gibaut, and P. Clifford, “How to combine correlated estimates of a single physical quantity”, Nucl. Instrum. Meth. A270, 110 (1988).

[12] A. Valassi, “Combining correlated measurements of several different physical quantities”, Nucl. Instrum. Meth. A500, 391 (2003).

[13] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. M. Nadolsky and W. K. Tung, “New generation of parton distributions with uncertainties from global QCD analysis”, JHEP 0207, 012 (2002).
[14] D. Stump, J. Huston, J. Pumplin, W. K. Tung, H. L. Lai, S. Kuhlmann and J. F. Owens, “Inclusive jet production, parton distributions, and the search for new physics”, JHEP 0310, 046 (2003).

[15] A. D. Martin, R. G. Roberts and W. J. Stirling, Phys. Rev. D50, 6734 (1994); E. W. N. Glover, A. D. Martin, R. G. Roberts and W. J. Stirling, Phys. Lett. B381, 353 (1996); A. D. Martin, R. G. Roberts and W. J. Stirling, Phys. Lett. B387, 419 (1996); A. D. Martin, R. G. Roberts, W. J. Stirling and R. S Thorne, Eur. Phys. J. C4, 463 (1998).

[16] A. D. Martin, R. G. Roberts and W. J. Stirling, “Structure Function Analysis and psi, Jet, W, Z Production: Pinning Down the Gluon,” Phys. Rev. D 37, 1161 (1988).

[17] A. D. Martin, W. J. Stirling and R. G. Roberts, “Parton distributions updated”, Phys. Lett. B 306, 145 (1993).

[18] P. Renton, “Updated SM calculation of $\sigma_W/\sigma_Z$ and the W boson width”, arXiv:0804.4779 [hep-ph] (2008).

[19] F. Landry, R. Brock, P. Nadolsky, and C. P. Yuan, “Tevatron Run-1 Z boson data and Collins-Soper-Sterman resummation formalism”, Phys. Rev. D67, 073016 (2003).

[20] U. Baur, S. Keller and D. Wackeroth, “Electroweak radiative corrections to W boson production in hadronic collisions,” Phys. Rev. D59, 013002 (1999)

[21] E. Barbiero and Z. Was, PHOTOS: A Universal Monte Carlo for QED radiative corrections. Version 2.0 Comput. Phys. Commun 79, 291 (1994).

[22] G. A. Ladinsky and C. P. Yuan, “The Nonperturbative regime in QCD resummation for gauge boson production at hadron colliders,” Phys. Rev. D50, 4239 (1994), F. Landry, R. Brock, P. M. Nadolsky and C. P. Yuan, “Tevatron Run-1 Z boson data and Collins-Soper-Sterman resummation formalism,” Phys. Rev. D67, 073016 (2003).

[23] The LEP Collaborations: ALEPH, DELPHI, L3, OPAL and the LEP Electroweak Working Group, “A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model”, arXiv:hep-ex/0612034 (2006).