On the oscillations response prediction of a small liquid petroleum gas carrier in random oblique waves

I. Domnisoru
"Dunarea de Jos" University of Galati, Department of Naval Architecture,
Domneasca Street 111, 800201, Galati, Romania
E-mail: leonard.domnisoru@ugal.ro

Abstract. The research from this paper is focused on the dynamic study of a small liquid petroleum gas carrier, with mono-hull shape, length of 125.6 m and maximum speed 17 knots. As loading cases, full cargo and ballast conditions are considered. The random waves are modelled on short-term by the ITTC averaged power density spectrum, with the maximum significant wave height 12 m, corresponding to unrestricted navigation cases. The oblique random waves heading angle has 0-360 degrees values. The seakeeping analysis includes the main oscillation components, heave, roll and pitch, based on eigen program DYN (OSC). The numerical analyses include the oscillation transfer functions computation and the short-term evaluation in oblique random waves, for the prediction of the most probable statistical response. For the roll oscillation component, supplementary the influence of the vertical position of the small LPG gravity centre is analysed. The seakeeping criteria are formulated in terms of admissible oscillation statistical most probable amplitudes. The short-term dynamic responses of the small LPG, assessed by the seakeeping criteria, deliver the oscillations cumulative safety polar diagrams. The predicted seakeeping response of the small LPG leads to practical results for the ship’s navigation safety limits in oblique random waves.

1. Introduction
From the many ships’ design criteria [1, 2], the maritime ships also have to be assessed by the oscillations response limits in random oblique waves [3, 4], representing the short-term seakeeping operation capabilities safety prediction function to the environmental conditions [5, 6].

This study is focused on the seakeeping analysis of a small liquid petroleum gas carrier, with the maximum loading capacity of 10000 cbm [7] (section 3). The small LPG is mono-hull, with slender shapes at both ends and prismatic amidships (figure 1), having a transom stern and a bulbous bow.

For the small LPG dynamic analysis, a hydrodynamic model by linear strip theory [8, 9, 10] is developed, under regular waves conditions, being suitable for this study due to the slenderness of the ship’s body, having length over breadth ratio L/B = 6.127 > 5 (table 1). For random oblique waves, the dynamic analysis is extended by a short-term statistical method, based on the response amplitude operators by the hydrodynamic analysis in regular waves and the irregular waves power density spectrum [5, 11], implemented in eigen DYN (OSC) code [8] (section 2).

Applying the seakeeping limit criteria, formulated as the admissible statistical most probable amplitudes of the main ship’s oscillations components [3, 8], heave, roll and pitch, the safety statistical polar diagrams in random oblique waves for the navigation of the small LPG are obtained.
2. Bases of the ship’s oscillations response prediction in random oblique waves

The prediction of the ship’s dynamic response in random waves is based on a combination between deterministic and statistical approaches, according to the ITTC procedures [4], with the eigen DYN (OSC) code [8], including the next three main steps.

- The first analysis step of the small LPG is the oscillations response in regular waves computation.

For this purpose, a hydrodynamic model by a linear strip theory [8, 9] has been developed, under the ship’s slender body hypothesis (L/B ≥ 5). The small LPG offset-lines from figure 1 are parameterized by the Lewis conformal transformation approach [9, 10], that includes the free surface influence, the hydrodynamic added masses and damping terms are computed at each transversal station of the small LPG (figure 1). The mass distribution over the ship’s length (figure 3) results from the onboard mass data tables of the small LPG [7]. Initial the terms are computed at each transversal station of the small LPG (figure 1). The mass distribution over the frequency domain (2), resulting the response amplitude operators (RAO) functions on the main oscillation components.

For preliminary design, the ITTC [4] (figure 4) short-term power density spectrum of the random waves $\Phi_i(\omega_i)$ is considered, for unrestricted navigation conditions, with maximum significant wave height $H_s=12$ m, step $\delta H_s=0.05$ m.

The oscillations equation system (1) for regular wave excitation, model Airy [5], has the solution in the frequency domain (2), resulting the response amplitude operators $RAO_i(\omega_i)$ for ship motions [11].

\[
\begin{align*}
[A_\Phi(\omega_i)][Q(\omega_i)] + [B_\Phi(\omega_i)][Q'(\omega_i)] + [C_\Phi(\omega_i)][Q''(\omega_i)] &= [P_{\Phi_0}(\omega_i)] \cos \omega_i t + [P_{\Phi_i}(\omega_i)] \sin \omega_i t \\
Q(\omega_i) &= Q'_i(\omega_i) \cos \omega_i t + Q''_i(\omega_i) \sin \omega_i t; i = 1,6; \quad \omega_i = \omega - k \cdot v \cdot \cos \mu; \quad k = \omega^2/g
\end{align*}
\]

(1)

where: $\omega, \omega_i$, the regular wave circular-frequency and amplitude; $\{A_\Phi\} [B_\Phi] [C_\Phi] [P_{\Phi_0}] [P_{\Phi_i}]$ the radiation and diffraction hydrodynamic general matrix; $v$ ship speed; $g$ gravity acceleration; the main motion components heave ($i=3$), roll ($i=4$) and pitch ($i=5$).

The oscillations circular-frequency range is $\omega=0-3$ rad/s, with step $\delta \omega=0.001$ rad/s, ensuring the accuracy for the $RAO$ functions peak values computation for all the motion components.

For an extended dynamic analysis of the small LPG carrier, the oblique random waves heading angle is $\mu=0-180$ (360) deg, with step $\delta \mu=5$ deg, considering the ship hull CL symmetry (figure 1).

The small LPG has a maximum speed of 17 knots, so that the dynamic analysis includes also the 0, 5, 10, 15 knots intermediate speed values.

The hydrodynamic solution has been previous validate by several experimental models at our university towing tank [13, 14], proving that the numerical $RAO$ functions (3) are suitable for practical ship’s dynamic response analyses. Those studies have pointed out that the numerical analysis is delivering bigger dynamic response values in compare to the experimental results, being conservative from the practical point of view. The correction factors [14], averaged over the frequency domain as in equation (4), can be used for the $RAO$ functions on the main oscillation components.

\[
\begin{align*}
\text{heave } &K_3 = 0.8321; \quad \text{roll } K_4 = 0.8385; \quad \text{pitch } K_5 = 0.8768
\end{align*}
\]

(4)

- The second analysis step of the small LPG is the short-term statistical response in random waves.

For preliminary design, the ITTC [4] (figure 4) short-term power density spectrum of the random waves $\Phi_i(\omega_i)$ is considered, for unrestricted navigation conditions, with maximum significant wave height $H_s=12$ m, step $\delta H_s=0.05$ m.

Based on the $RAO_i(\omega_i)$ (3) functions and the ITTC wave spectrum (figure 4), the dynamic response power density spectra $\Phi_i(\omega_i)$ (5) and the spectra moments $m_0^i, m_4^i$ (6) are computed, resulting the short-term statistical most probable motions and accelerations $RMS_i, RMS_i^{\infty}$ (7) values, for each set of sea state $(H_s, \mu)$ and LPG speed $v$.

\[
\begin{align*}
\Phi_i(\omega_i) &= (RAO_i(\omega_i))^2 \Phi_i(\omega_i); \quad i = 1,6; \quad \Phi_i(\omega_i) = \Phi_i(\omega_i)\sqrt{-2v \cdot \omega / g \cdot \cos \mu}
\end{align*}
\]

(5)
The third analysis step of the small LPG is the assessment of the navigation capabilities.

By combining the seakeeping criteria (10, 11), results the safety polar diagrams in terms of limit significant height of random waves \(H_{\text{lim}}(\nu, \mu, \text{load})\) and Beaufort sea grade level \(B_{\text{lim}}(\nu, \mu, \text{load})\) [8, 14], function to the ship speed \(v\), waves heading angle \(\phi\) and loading case of the small LPG carrier.

The seakeeping limit criteria, defined for the short-term statistical most probable amplitudes of motions (10) and accelerations (11), on main oscillation components \((i=3,4,5)\), have the admissible values presented in table 1 for the small LPG carrier, based on equations (8) – (11) [8, 14].

\[
m_0(H, \mu, v) = \int_0^{\phi_{\text{max}}} \Phi_i(\phi) \, d\phi,
\]

\[
m_i(H, \mu, v) = \int_0^{\phi_{\text{max}}} \phi \Phi_i(\phi) \, d\phi ; \quad i = 1, 6
\]

\[
\text{RMS}_i(H, \mu, v) = \sqrt{m_0^2(H, \mu, v) + m_i^2(H, \mu, v)} ; \quad i = 1, 6
\]

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\[
\text{RMS}_{a, \text{mid, fore}} = \text{RMS}_3 + \text{RMS}_4 \cdot 0.50B + 0.25H_s + \text{RMS}_5 \cdot \left| x_{a, \text{mid, fore}} - x_F \right|
\]

\[
\text{RMS}_{a, \text{mid, fore}} = H_{a, \text{mid, fore}} - d_{a, \text{mid, fore}} - F_{\text{min}} ; \quad x_{a, \text{mid, fore}} = 0, L/2, L
\]

\[
\text{RMS}_{a, \text{mid, fore}} \leq \text{RMS}_{a, \text{mid, fore}} ; \quad \text{RMS}_3 \leq \text{RMS}_4 = 9 \text{ deg} ; \quad \text{RMS}_5 \leq \text{RMS}_5^\text{adm} = 3 \text{ deg}
\]

\[
\text{RMS}_3 \leq \text{RMS}_3^\text{adm} = 0.1g ; \quad \text{RMS}_4 \leq \text{RMS}_4^\text{adm} = 0.1g \cdot 2/B ; \quad \text{RMS}_5 \leq \text{RMS}_5^\text{adm} = 0.15g \cdot 2/L
\]

3. The small liquid petroleum gas carrier characteristics

For the first seakeeping analysis step it has been developed the numerical 3D-CAD model of the external hull shape from figure 1 of the small LPG 10000 cbm carrier. The transversal stations distribution density is between 0.3-0.5 m at aft and fore parts, where slender and bulbous shapes are designed, and 0.6-0.7 m amidships part, where a prismatic shape is designed, so that an appropriate 3D-CAD modelling all over the small LPG carrier length results. With 190 stations, having each 30 points, the resulting 3D-CAD model of the small LPG external hull shape (figure 1) satisfies the requirements for a hydrodynamic model by the linear strip theory formulation [9, 11].

Table 1 includes a synthesis of the main data [7], loading cases definition and seakeeping admissible limits [8, 14], for the small LPG carrier. Two loading cases are selected, full cargo and ballast (figure 3), initial both been balanced in still water condition [12]. In the ballast case, the small LPG has initially a longitudinal trim of -0.01978 rad, which is zero in the full cargo loading case.

Figure 2 presents the transversal static stability curves [12] for the small LPG, on both loading cases, with the influence of the vertical position of the ship’s gravity centre \(z_G\)=6.5-8.5 m. Up to 20 deg healing angle, for both loading cases, the stability curves are linear, so that a linearized roll motion model can be considered. The influence of the bilge keels is not considered in the numerical model.

Table 1. The small LPG 10000 cbm carrier characteristics [7].

| Main data |
|-----------|
| Loading cases data | \(T_s\) & \(\Gamma M_a\) (m) | Seakeeping adm. full & ballast |
|-------------|-----------------|-------------------|
| \(L (m)\) | 125.600 | Case | Full cargo | Ballast | \(z_G (m)\) | \(F_{\text{min}} (m)\) | \(0.300\) |
| \(B (m)\) | 20.500 | \(\Delta (t)\) | 13194.62 | 6.5 | 9.885 | 2.409 | 0.0957 |
| \(H_{\text{eff}} (m)\) | 15.500 | \(x_C (m)\) | 61.867 | 7.0 | 11.341 | 1.909 | 0.1571 |
| \(H_{\text{mid}} (m)\) | 13.000 | \(y_C (m)\) | 0 | 7.5 | 13.452 | 1.409 | 0.0524 |
| \(H_{\text{fore}} (m)\) | 16.500 | \(d_{\text{fore}} (m)\) | 7.640 | 8.0 | 17.066 | 0.909 | 0.0957 |
| \(\text{Points} 5700\) | \(d_{\text{fore}} (m)\) | 7.640 | 4.000 | 6.983 | 6.368 | 0.9810 |
| \(g (m/s^2)\) | 9.81 | \(x_C (m)\) | 59.197 | 6.227 | 6.229 | 0.0234 |
| \(\rho_{\text{water}} (t/m^3)\) | 1.025 | \(v\) (knots) | 0, 5, 10, 15, 17 | \(H_s (m)\) 0-12 | \(\mu\) (deg) 0-180(360) | \(\text{RMS}_{a, \text{adm}} (\text{rad})\) | 0.0234 |
Figure 1. The LPG 10000 cbm offset-lines 3D-CAD model, (a) general view and (b) front view [7].

Figure 2. The LPG 10000 cbm static stability curves $z_G=6.5-8.5$m, (a) full cargo and (b) ballast.

Figure 3. The LPG mass distribution $m_x$(t/m) [7].

Figure 4. The ITTC wave spectrum $\Phi_w(\omega)$ [4].

For the small LPG the natural oscillation periods on the main components (table 1) are: heave $T_3=6.368-6.983$ s, pitch $T_5=6.227-6.229$ s and roll $T_4=9.885-25.864$ s. The change of the vertical position of the gravity centre $z_G=6.5-8.5$m has a significant influence on roll dynamic behaviour, increasing the roll period and in the same time decreasing the transversal stability characteristic up to the rules limit $GM=0.409 - 2.845 > 0.150$ m [1, 2, 12]. An average roll condition is for $z_G=7.5$m.

The seakeeping admissible values (table 1) are different for the vertical combined motions, aft, middle, fore, criteria (8, 9), being more restrictive in the full cargo case in compare to the ballast case, due to an initial designed smaller free board for the first loading case.
Figure 3 presents the mass distributions over length $m$, for the full cargo and ballast cases (table 1), being similar at aft and fore parts. Figure 4 presents the ITTC [4] averaged power density spectrum $\Phi_\omega(\omega)$ for significant wave height $H_s=0\text{–}12$ m, describing on short-term the random waves.

4. The deterministic dynamic analysis of the small LPG in regular waves
The response amplitude operators $RAO$ (3), on heave, roll and pitch ($i=3,4,5$), are obtained as solution of the linear motions equations system (1) in frequency domain, for regular waves with $a_w=1$. For each loading case of the small LPG carrier (table 1), a number of 555000 navigation conditions ($v, \mu, \omega$) are analysed by DYN (OSC) [8], for LPG’s speed range $v=0, 5, 10, 15, 17$ knots, LPG-wave heading angle range $\mu=0\text{–}180$ deg, step 5 deg, and waves’ circular-frequency range $\omega=0\text{–}3$ rad/s, step 0.001 rad/s. The zero-speed case represents the damage condition, when the propulsion system has failed out.

4.1. The small LPG carrier oscillations response amplitude operators in full cargo loading case
This subsection presents a synthesis of the dynamic analysis in regular waves of the small LPG carrier in full cargo case (table 1), with the following results:
- Figures 5 and 6 present a selection of the heave motion response amplitude operators $RAO_3$ for reference speeds 0-17 knots, heading angles $\mu=0, 45, 90, 135, 180$ deg and full cargo case. The maximum heave $RAO_3$ is at beam sea condition ($\mu=90$ deg) for zero speed ($v=0$) and shifted at quarter-head sea condition ($\mu=135$ deg) for top speed ($v=17$ knots) (figure 5). At beam sea ($\mu=90$ deg) the influence of the LPG’s speed on heave $RAO_3$ is much reduced for circular-frequency $\omega=0.60\text{–}1.50$ rad/s and none for $\omega < 0.60$ rad/s (figure 6). At head sea ($\mu=180$ deg) the influence of the LPG’s speed on heave $RAO_3$ is very significant for circular-frequency $\omega=0.25\text{–}1.35$ rad/s, reduced for $\omega=1.35\text{–}1.50$ rad/s and none for $\omega < 0.25$ rad/s (figure 6). For any speed and heading angle the heave $RAO_3$ response becomes negligible for circular-frequency $\omega > 1.50$ rad/s.

![Figure 5. Small LPG, heave $RAO_3$ (m/m), $\mu=0\text{–}180$ deg, full cargo, (a) $v=0$ knots and (b) $v=17$ knots.](image1)

![Figure 6. Small LPG, heave $RAO_3$ (m/m), $v=0\text{–}17$ knots, full cargo, (a) $\mu=90$ deg and (b) $\mu=180$ deg.](image2)
- Figures 7 and 8 present a selection of the pitch motion response amplitude operators \( RAO_p \), for reference speeds 0-17 knots, heading angles \( \mu = 0, 45, 90, 135, 180 \) deg and full cargo case.

The maximum pitch \( RAO_p \) is at quarter sea condition (\( \mu = 45, 135 \) deg) for zero speed (\( v = 0 \)) and at head or quarter-head sea condition (\( \mu = 135 \) deg-180 deg) for top speed (\( v = 17 \) knots), having at beam sea, for any speed, the smallest values (figure 7). At follow sea (\( \mu = 0 \) deg) the influence of the LPG’s speed on pitch \( RAO_p \) is very significant for circular-frequency \( \omega = 0.35 \) - 1.75 rad/s and none for \( \omega < 0.35 \) rad/s (figure 8). At head sea (\( \mu = 180 \) deg) the influence of the LPG’s speed on pitch \( RAO_p \) is very significant for circular-frequency \( \omega = 0.40 \) - 1.40 rad/s, much reduced for \( \omega = 1.40 \) - 1.75 rad/s and none for \( \omega < 0.40 \) rad/s (figure 8). For any speed and heading angle, the pitch \( RAO_p \) response becomes negligible for circular-frequency \( \omega > 1.75 \) rad/s. At follow sea (\( \mu = 0 \) deg), for any speed, the pitch \( RAO_p \) response is smaller than at head sea (\( \mu = 180 \) deg) condition (figure 8).

![Figure 7. Small LPG, pitch \( RAO_p (\text{rad/m}) \), \( \mu = 0 \) - 180 deg, full cargo, (a) \( v = 0 \) knots and (b) \( v = 17 \) knots.](image1)

![Figure 8. Small LPG, pitch \( RAO_p (\text{rad/m}) \), \( v = 0 \) - 17 knots, full cargo, (a) \( \mu = 0 \) deg and (b) \( \mu = 180 \) deg.](image2)

- Figures 9 and 10 present a selection of the roll motion response amplitude operators \( RAO_r \), for reference speeds 0, 10, 17 knots, \( \mu = 70-110 \) deg, \( z_G = 6.5, 7.0, 8.0, 8.5 \) m and full cargo case.

In the range of the gravity centre vertical position \( z_G = 6.5-8.5 \) m the roll natural period is changing significantly \( T_4 = 9.885-25.864 \) s (0.243-0.636 rad/s), so that at beam sea (\( \mu = 90 \) deg) the minim roll \( RAO_r \) is for \( z_G = 8.5 \) m and maximum for \( z_G = 6.5 \) m (figure 9). When the vertical position of the gravity centre \( z_G \) is decreased, the natural roll frequency increases and the roll hydrodynamic damping drops down [9], so that the roll response amplitude operator records the changes from figure 9. The linear hydrodynamic model leads to zero \( RAO_r \) at follow (\( \mu = 0 \) deg) and head (\( \mu = 180 \) deg) waves conditions [10]. In the case of zero LPG speed \( v = 0 \) and \( z_G = 7.5 \) m (figure 9) the \( RAO_r \) has the maximum at beam sea and for other heading angle \( \mu = 70-110 \) deg their peak decreases, but remains at the same frequency equal with the natural roll frequency (table 1). Considering as reference the average vertical position of the gravity centre \( z_G = 7.5 \) m (figure 10), results that the LPG speed has significant influence on the roll \( RAO_r \) for the circular-frequency range \( \omega = 0.35 \) - 0.85 rad/s and the heading angle range \( \mu = 70-110 \) deg.
deg, which can be neglected for lower or higher frequencies. The roll is a narrow band motion in comparison to the heave and pitch oscillations. Taking as reference the beam sea condition ($\mu=90$ deg), in the case with LPG speeds $v=10, 17$ knots, for $\mu=70-80$ deg the roll $RAO_4$ peaks are decreasing and for $\mu=100-110$ deg are increasing. The peaks $RAO_4$ are higher for the top speed $v=17$ knots.

Figure 9. LPG, roll $RAO_4$, $v=0$ knots, full, (a) $z_G=6.5-8.5$ m, $\mu=90$ deg and (b) $z_G=7.5$ m, $\mu=70-110$ deg.

Figure 10. Small LPG, roll $RAO_4$ (rad/m), $z_G=7.5$ m, full cargo, (a) $v=10$ knots and (b) $v=17$ knots.

4.2. The small LPG carrier oscillations response amplitude operators in ballast loading case

For the oscillations analysis in regular waves of the small LPG carrier in ballast loading case (table 1), in this subsection a synthesise is presented.

The $RAO$ functions of the LPG for ballast case are having similar changes by $v$, $\mu$ parameters as in the full cargo case, so that only the next relevant results are included:

- Figure 11 presents the heave motion response amplitude operators $RAO_3$ selection for ballast case. In the ballast case, the maximum heave $RAO_3$ is at beam sea condition ($\mu=90$ deg) for top speed ($v=17$ knots) and the other speed values. At head sea ($\mu=180$ deg) the influence of the LPG speed on heave $RAO_3$ is recorded only for circular-frequency range $\omega=0.15-1.50$ rad/s. For $\omega>1.75$ rad/s the heave $RAO_3$ can be neglected (figure 11).

- Figure 12 presents the pitch motion response amplitude operators $RAO_5$ selection for ballast case. In the ballast case, the maximum pitch $RAO_5$ is at head sea condition ($\mu=90$ deg) at top speed ($v=17$ knots) and the other speed values. At head sea ($\mu=180$ deg) the influence of the LPG speed on pitch $RAO_5$ is recorded only for circular-frequency range $\omega=0.30-1.60$ rad/s. The pitch $RAO_5$ can be neglected for $\omega>1.75$ rad/s, for any speed and heading angle (figure 12).

- Figure 13 presents the roll motion response amplitude operators $RAO_4$ selection for ballast case. For the ballast case, the roll natural period is in the range $T_4=10.460-22.072$ s ($0.284-0.601$ rad/s), due to the vertical position of the gravity centre changes $z_G=6.5-8.5$ m, leading to a significant variation of the roll hydrodynamic damping [9], so that similar to full load case at beam sea ($\mu=90$ deg) the
maximum roll $RAO_4$ is for $z_G=6.5\text{m}$. The roll response is zero at head and follow conditions ($\mu=0,\ 180\ \text{deg}$). Taking the vertical position of the gravity centre reference $z_G=7.5\text{m}$, for the circular-frequency range $\omega=0.25-1.65\ \text{rad/s}$ the roll $RAO_4$ is significant influenced by the LPG speed in the range of heading angle $\mu=70-110\ \text{deg}$. In the case of ballast, the roll motion has a wider frequency band in compare to the full cargo case (figures 9, 10, 13).

5. The oscillations statistical short-term analysis of the small LPG in random waves

The statistical short-term most probable amplitudes $RMS$ (7) for heave, roll and pitch ($i=3,4,5$) motions and accelerations are obtained. Corresponding to each LPG loading condition, a number of 44400 cases ($v, \mu, H_s$) are analysed, speed $v=0,\ 5,\ 10,\ 15,\ 17\ \text{knots}$, heading angle $\mu=0-180\ \text{deg}$ (step 5 deg) and random waves’ statistical short-term significant height $H_s=0-12\ \text{m}$ (step 0.05 m).
The navigation capabilities in random oblique waves, based on the short-term dynamic response (7) are assessed by the seakeeping criteria (10, 11) for all the navigation cases \((\nu, \mu, H)\) combinations, resulting the safety seakeeping polar diagrams (figures 17, 18, 21, 22). For the small LPG 10000 cbm carrier the admissible seakeeping RMS adm values are in the last two columns of table 1.

The seakeeping RMS values and safety polar diagrams are obtained by DYN (OSC) \([8]\) program (5, 6, 7), using the heave, roll and pitch RAO functions of the small LPG carrier from section 4, combined with the short-term averaged power density random waves’ spectrum ITTC (figure 4).

5.1. The small LPG carrier oscillations short-term statistical response in full cargo loading case

This subsection includes, in synthesise, the short-term statistical analysis in random oblique waves of the small LPG carrier, in full cargo case (table 1), with the next results:

- Table 2 presents the maximum statistical most probable amplitudes (7) for the main oscillation components, heave, roll and pitch, motions RMS and accelerations RMS ac, for LPG speeds 0 and 17 knots, in full cargo loading case, vertical position of the gravity centre \(z_G = 6.5, 7.5, 8.5\) m.

- Figure 14 presents the statistical most probable roll motion \(\text{RAO}_1\) and acceleration \(\text{RAO}_2\) function to \(H_s = 0-12\) m and \(z_G = 6.5-8.5\) m, at beam sea \((\mu = 90\, \text{deg})\), \(v = 0\) knots and full cargo case.

Due to the roll natural frequency changes induced by the vertical position of the gravity centre (table 1) and the waves’ ITTC (figure 4) spectral energy distribution, the maximum roll motion is for \(z_G = 6.5-7.5\) m, decreasing significantly for \(z_G = 8.5\) m (figure 14). The roll acceleration is maximum for \(z_G = 6.5\) m and decreases up to the minimum for \(z_G = 8.5\) m (figure 14). The changes of the roll response are also significantly affecting the combined vertical motions response (8) (table 2).

- Figures 15 and 16 present the variation of the maximum statistical most probable response, vertical middle and fore, roll motions, heave acceleration, for \(\mu = 0-180\, \text{deg}, v = 0-17\) knots, \(z_G = 7.5\) m and full cargo case. It results that the most restrictive criteria (10, 11) are the vertical and roll motions.

- Table 3 presents the seakeeping safety limits of the small LPG in full cargo case, by criteria (10, 11), in terms of limit significant wave height \(H_{\text{slimit}}\) and Beaufort grade level \(B_{\text{limir}}\), for \(v = 0, 17\) knots, \(z_G = 6.5, 7.5, 8.5\) m. The dynamic response is analysed without and with the \(\text{RAO}\) correction factors (4).

- Figure 17 presents the safety polar diagrams by seakeeping criteria \(H_{\text{slimit}}\) of the small LPG at full cargo case, for \(v = 0-17\) knots and \(z_G = 6.5-8.5\) m, in the case without the \(\text{RAO}\) correction (4).

- Figure 18 presents the safety polar diagrams \(H_{\text{slimit}}\), \(B_{\text{limir}}\) in full cargo loading case, average \(z_G = 7.5\) m, with correction factors (4) for the heave, roll and pitch \(\text{RAO}\) response.

### Table 2. The maximum seakeeping RMS response for the small LPG 10000 cbm, full cargo case.

| mot. / acc. | \(z_G\) (m) | RMS \_zG \_zG | RMS \_slimit \_zG | RMS \_fore \_zG | RMS \_acc \_zG | RMS \_S | RMS \_acc | RMS \_ac | RMS \_ac |
|------------|--------------|----------------|----------------|---------------|---------------|--------|--------|--------|--------|
| adm \_rad | 6.5-8.5      |                |                 |               |               |        |        |        |        |
| \(v = 0\) knots \_max \_ (%) | 6.5          | 10.261         | 9.080           | 10.496        | 0.857         | 0.051  | 0.016  | 0.295  | 0.120  |
|           | 35.73%       | 79.45%         | 22.62%          | -12.64%       | -2.67%        | -31.62% | 87.78% | 25.39% |        |
|           | 10.848       | 9.531          | 11.088          | 0.857         | 0.051         | 0.016  | 0.339  | 0.076  |        |
|           | 43.49%       | 88.36%         | 29.53%          | -12.64%       | -2.67%        | -31.62% | 115.79%| -20.59%|        |
|           | 8.578        | 6.463          | 8.927           | 0.857         | 0.051         | 0.016  | 0.040  | 0.003  |        |
|           | 13.47%       | 27.73%         | 4.29%           | -12.64%       | -2.67%        | -31.62% | -74.54%| -96.87%|        |
| \(v = 17\) knots \_max \_ (%) | 6.5          | 10.961         | 9.192           | 11.302        | 1.309         | 0.062  | 0.026  | 0.306  | 0.124  |
|           | 44.99%       | 81.66%         | 32.03%          | 33.44%        | 18.32%        | 11.11% | 94.78% | 29.57% |        |
|           | 11.024       | 9.796          | 11.339          | 1.309         | 0.062         | 0.026  | 0.377  | 0.084  |        |
|           | 45.82%       | 93.60%         | 32.46%          | 33.44%        | 18.32%        | 11.11% | 139.97%| -12.23%|        |
|           | 9.617        | 6.992          | 10.066          | 1.309         | 0.062         | 0.026  | 0.152  | 0.010  |        |
|           | 27.21%       | 38.18%         | 17.59%          | 33.44%        | 18.32%        | 11.11% | -3.25% | -89.55%|        |
Figure 14. Small LPG, $v=0$ knots, $\mu=90$ deg, $H_s=0$-12 m, (a) $RMS_4$(rad) and (b) $RMS_{4\text{ac}}$(rad/s²), full.

Figure 15. Small LPG, $v=0$-17 knots, $z_G=7.5$ m, maximum (a) $RMS_{\text{mid}}$(m) and (b) $RMS_{3\text{ac}}$(m/s²), full.

Figure 16. Small LPG, $v=0$-17 knots, $z_G=7.5$ m, maximum (a) $RMS_4$(rad) and (b) $RMS_{4\text{ac}}$(rad/s²), full.

Table 3. The seakeeping safety limits $H_{\text{slimit}}$(m) and $B_{\text{limit}}$ for the small LPG 10000 cbm, full cargo.

| $v$ (knots) | $z_G$(m) | $H_{\text{slimit}}$(m) | $B_{\text{limit}}$ | Main criterion | $H_{\text{slimit}}$(m) | $B_{\text{limit}}$ | Main criterion |
|-------------|----------|------------------------|--------------------|----------------|------------------------|--------------------|----------------|
| 0           | 6.5      | 3.705-10.681           | 6.61-10.51         | Roll motion    | 4.307-11.823           | 7.11-10.93         | Roll motion     |
| 7.5         |          | 5.617-10.681           | 8.02-10.51         | Roll motion    | 6.209-11.823           | 8.39-10.93         | Roll motion     |
| 8.5         |          | 9.726-10.681           | 10.16-10.51        | Vertical mid.  | 10.551-11.823          | 10.47-10.93        | Vertical mid.   |
| 17          | 6.5      | 2.888-11.687           | 5.85-10.88         | Roll motion    | 3.557-12.000           | 6.48-11.00         | Roll motion     |
| 7.5         |          | 2.953-11.687           | 5.93-10.88         | Roll motion    | 3.662-12.000           | 6.58-11.00         | Roll motion     |
| 8.5         |          | 6.686-11.687           | 8.68-10.88         | Vertical mid.  | 9.460-12.000           | 10.06-11.00        | Vertical mid.   |
5.2. The small LPG carrier oscillations short-term statistical response in ballast loading case

This subsection presents a selection of the small LPG short-term statistical results in ballast (table 1).

- Table 4 presents the maximum statistical most probable amplitudes (7) for the main oscillation components, heave, roll and pitch, motions \(RMS\) and accelerations \(RMS_{ac}\), for LPG speeds 0 and 17 knots, in ballast loading case, vertical position of the gravity centre \(z_G=6.5, 7.5, 8.5\) m.

- Figure 19 presents the statistical most probable roll motion \(RAO_4\) and acceleration \(RAO_{4ac}\) function to \(H_s=0-12\) m and \(z_G=6.5-8.5\) m, at beam sea (\(\mu=90\) deg), \(v=0\) knots and ballast loading case.

- Figure 20 presents the variation of the maximum statistical most probable combined vertical aft and pitch motions, \(\mu=0-180\) deg, \(v=0-17\) knots, \(z_G=7.5\) m and ballast loading case.
- Table 5 presents the seakeeping safety limits of the small LPG in ballast loading case, by criteria (10, 11), $H_{\text{limit}}$ and $B_{\text{limit}}$, for $v=0, 17$ knots, $z_G=6.5, 7.5, 8.5$ m, without and with correction factors (4).
- Figure 21 presents the safety polar diagrams by seakeeping criteria $H_{\text{limit}}$ of the small LPG in ballast case, for $v=0-17$ knots and $z_G=6.5-8.5$ m, in the case without the RAO correction (4).
- Figure 22 presents the safety polar diagrams $H_{\text{limit}}, B_{\text{limit}}$, in ballast, $z_G=7.5$ m, with correction (4).

Table 5. The seakeeping safety limits of the small LPG in ballast loading case, by criteria (10, 11), $H_{\text{limit}}$ and $B_{\text{limit}}$, for $v=0, 17$ knots, $z_G=6.5, 7.5, 8.5$ m, without and with correction factors (4).

| Criteria | $H_{\text{limit}}$ | $B_{\text{limit}}$ |
|----------|-------------------|------------------|
| $v=0$ knots | $6.5$ | $0.056$ |
| $v=17$ knots | $10.256$ | $0.068$ |

Table 4. The maximum seakeeping RMS response for the small LPG 10000 cbm, ballast case.

| mot. / acc. | $z_G$ | $RMS_{\text{adm}}$ | $RMS_{\text{mid}}$ | $RMS_{\text{fwd}}$ | $RMS_{\text{RMS}}$ | $RMS_{\text{RMS}_{\text{acc}}}$ | 
|------------|------|------------------|-----------------|-----------------|------------------|-------------------| 
| units (m) | (m) | (m) | (m) | (m) | (m) | (rad) | 
| adm | 6.5-8.5 | 8.700 | 7.450 | 12.200 | 0.9810 | 0.0524 | 0.1571 | 0.0957 |

Figure 19. Small LPG, $v=0$ knots, $\mu=90$deg, $H_s=0-12$ m, (a) $RMS_4$ (rad) and (b) $RMS_{\text{acc}}$ (rad/s²), ballast.

Figure 20. Small LPG, $v=0-17$ knots, $z_G=7.5$ m, maximum (a) $RMS_{\text{adm}}$ (m) and (b) $RMS_4$ (rad), ballast.
Figure 21. Small LPG, ballast, (a) $H_{\text{slim}}(m)$ $z_G=6.5-8.5m$, $\nu=0$kn and (b) $H_{\text{slim}}(m)$ $z_G=7.5m$, $\nu=0-17$kn.

Table 5. The seakeeping safety limits $H_{\text{slim}} (m)$ and $B_{\text{limit}}$ for the small LPG 10000 cbm, ballast.

| $v$ (knots) $z_G$ (m) | $H_{\text{slim}} (m)$ | $B_{\text{limit}}$ | Main criterion | $H_{\text{slim}} (m)$ | $B_{\text{limit}}$ | Main criterion |
|-----------------------|------------------------|---------------------|----------------|------------------------|---------------------|----------------|
| 0                     | 6.5                    | 10.684-12.00        | 10.52-11.00    | Pitch motion           | 12.00              | 11.00          | No restriction |
| 8.5                   |                        |                     |                |                        |                     |                |                |
| 17                    | 6.5                    | 7.695-12.00         | 9.24-11.00     | Pitch motion           | 9.428-12.000       | 10.05-11.00    | Pitch motion   |
|                       | 7.5                    | 7.695-12.00         | 9.24-11.00     | Pitch motion           | 9.428-12.000       | 10.05-11.00    | Pitch motion   |
|                       | 8.5                    | 7.695-12.00         | 9.24-11.00     | Pitch motion           | 9.428-12.000       | 10.05-11.00    | Pitch motion   |

Figure 22. Small LPG, ballast, $z_G=7.5$ m, $\nu=0-17$ knots, corrected response (a) $H_{\text{slim}} (m)$ and (b) $B_{\text{limit}}$. 
6. Discussions
The results of the oscillation analyses in random waves of the small LPG 10000 cbm (section 3), by a linear hydrodynamic model and short-term statistical approach (section 2), are synthesized by the safety navigation polar diagrams from figures 17, 18, 21, 22 and table 6, for both loading cases.

Table 6. The small LPG 10000 cbm comparative safety navigation limits by seakeeping criteria.

| \(v\) (knots) | \(K_{set}(4)\) | \(H_s\) limit (m) | \(T_4\) limit (s) | \(T_2\) limit (s) |
|----------|---------------|-----------------|-----------------|-----------------|
| Ballast | no | 3.705 | 10.681 | 3.012 | 11.687 | 7.04 | 10.65 |
| yes  | 11.823 | 5.390 | 7.820 | 6.955 | 9.30 | 10.02 | 9.24 |
| Full cargo | no | 12.000 | 9.336 | 8.335 | 9.428 | 11.00 | 10.95 | 10.05 |
| yes | 12.000 | 11.855 | 10.417 | 9.611 | 11.00 | 10.95 | 10.42 |

From the analysis of heave RAO, (figures 5, 6, 11) results that the maximum response is mainly at beam sea condition (\(\mu=90\) deg), for both loading cases, with a shift around the quarter-head sea condition (\(\mu=135\) deg) at full cargo loading case, increasing for the top LPG speed. The ship’s gravity centre vertical position has no influence on the heave response. The ship’s speed influence on RAO is mainly recorded at quarter-head and head sea conditions (\(\mu=135-180\) deg), in the circular-frequency range \(\omega=0.25-1.35\) rad/s (full cargo) and \(\omega=0.15-1.50\) rad/s (ballast).

From the analysis of pitch RAO, (figures 7, 8 12) results that the maximum response is mainly at quarter-head and head sea conditions (\(\mu=135-180\) deg), having the smallest values at beam sea (\(\mu=90\) deg), for both loading cases. The ship’s gravity centre vertical position has no influence on the pitch response. The ship’s speed influence on RAO is mainly recorded for \(\mu=0-45\) and 135-180 deg, in the circular-frequency range \(\omega=0.35-1.50\) rad/s (full cargo) and \(\omega=0.30-1.60\) rad/s (ballast).

From the analysis of roll RAO, (figures 9, 10, 13) results that \(z_G\) has a significant influence on the roll response, inducing major changes of natural period \(T_c=9.885-25.864\) s (full cargo) and 10.460-22.072 s (ballast), in compare to heave and pitch (table 1). The roll response peak is increasing when \(z_G\) decreases, due to the hydrodynamic damping and inertial changes. The maximum roll response is at quarter-beam sea (\(\mu=70-110\) deg), where also the ship’s speed influence on RAO is recorded for \(\omega=0.35-0.85\) rad/s (full cargo) and \(\omega=0.30-1.60\) rad/s (ballast). Roll is zero at \(\mu=0\) and 180 deg.

From the short-term statistical analysis of the small LPG in full cargo case (tables 2, 3 and figures 14-18) results that the main seakeeping restrictions are from roll criterion for \(z_G=6.5-7.5\) m, with 87.78-139.97% over admissible limits and from combined middle vertical motion criterion for \(z_G=8.0-8.5\) m, with 27.73-38.18% (figure 14, tables 2, 3). Also, secondary restrictions are recorded for the other criteria (10, 11), function to the LPG speed and \(z_G\) position (figures 15, 16 and table 3). The main restrictions are recorded around beam sea (\(\mu=50-100\) and 260-310 deg) and quarter-head sea (\(\mu=150-210\) deg) conditions, becoming more severe as the small LPG carrier speed is increasing up to the top speed of 17 knots (figures 17, 18).

From the short-term statistical analysis of small LPG in ballast case (tables 4, 5 and figures 19-22) results that the main seakeeping restrictions are only from pitch criterion (10), with 6.87-29.77% over admissible limits, without influence from \(z_G\) and roll response (figure 19, tables 4, 5). The secondary restrictions are recorded for aft combined vertical motion, heave and pitch accelerations criteria (10, 11) (figure 20 and table 4). The main restrictions are recorded around quarter-head (\(\mu=110-250\) deg) conditions, with the most restrictive case for the LPG speed of 17 knots (figures 21, 22).

Comparing the safety polar diagrams by seakeeping criteria of the small LPG, for full cargo (figures 17, 18) and ballast (figures 21, 22), results that the restrictions in the first loading case are more severe than the second one (table 6). The extreme restrictions are \(H_s=2.888\) m (full) and 7.695 m (ballast).
m (ballast), being improved considering the corrections (4), \( H_{\text{small}} = 3.557 \) m (full) and 9.428 m (ballast), for the small LPG carrier speed \( v = 17 \) knots, recorded at quarter-beam sea (full) and head sea (ballast) conditions.

7. Conclusions

For the assessment of ship’s navigation safety by seakeeping criteria, in this study an own numerical code has been used, with previously validations by experimental tests [13, 14]. In order to increase the analysis accuracy, in this study the linear numerical hydrodynamic results have been corrected by the coefficients from equation (4), based on experimental trials. The sets of seakeeping criteria have been extended to combined oscillations motions, as in equations (8)-(11), so that the ship’s dynamic assessment is done in a realistic way.

As study case, the numeric model of a small LPG 10000 cbm carrier has been developed, based on a hydrodynamic strip theory formulation, with slender shapes at both extremities (section 3), combined with an extended parametric analysis.

Although, initial the wave circular frequency has been considered up to 3 rad/s, the response amplitude operators’ analyses have pointed out that for the small LPG carrier the oscillation response on main motion components, heave, pitch and roll, can be neglected above 1.75 rad/s (section 4). Thus, the numerical overall simulation time could be reduced.

Comparing the response amplitude operators for the two loading conditions (section 4), the motions particularities of the small LPG carrier are obtained, as the specific sensitivity of this ship to the waves’ frequencies, ship’s heading angle, speed and displacement parameters. For the heave motion, this ship has maximum sensitivity at beam-quarter-head waves’ conditions. For the pitch motion, the ship has maximum sensitivity at quarter-head waves’ conditions. The ship’s behaviour on roll motion is very different to the heave and pitch oscillations, with a high influence from the ship’s gravity centre vertical position and a maximum sensitivity at quarter-beam waves’ conditions. The peak values of the response amplitude operators are higher for the full cargo case in compare to the ballast case.

The short-term statistical response analysis delivers the navigation capabilities of the small LPG carrier, according to the safety seakeeping criteria (section 5). Significant differences concerning the small LPG carrier behaviour in random oblique waves for the two loading cases are obtained (table 6, figures 17, 18, 21, 22). In the full cargo loading case, the navigation restriction results higher in compare to ballast case, induced by the limits formulated on roll and middle vertical combined motions, mainly at beam-quarter-head waves’ conditions. In ballast case, the navigation restrictions are induced only by pitch motion limits, at quarter-head waves’ conditions. Appling the correction factors from equation (4), the navigation restriction for the small LPG carrier became more realistic predicted (table 6 and figures 18, 22).

In conclusion, the results from this study made possible to analyse the oscillations’ sensitivity of the small LPG 10000 cbm carrier function to the environmental parameters and to predict the ship’s operation limits for the navigation safety in random oblique waves.

Besides the loading cases included in this research, further studies shall continue with other displacement cases, specific to the small LPG carrier and requested by the international regulations [1, 2, 5], in order to obtain the whole range of the polar navigation safety diagrams, useful for ship’s class approval and exploitation. In addition, combined structural and seakeeping analysis shall be considered in further studies, in order to have a multi-criteria LPG’s operation capabilities evaluation.

For the seakeeping software improvements, further studies shall continue with a nonlinear hydrodynamic formulation module, analyses on other ship types and benchmarks tests, in order to extend the own code computation capabilities.

8. References

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