Galaxy clusters at $0.3 < z < 0.4$ and the value of $\Omega_0$

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ABSTRACT

The observed evolution of the galaxy cluster X-ray integral temperature distribution function between $z = 0.05$ and $z = 0.32$ is used in an attempt to constrain the value of the density parameter, $\Omega_0$, for both open and spatially-flat universes. We estimate the overall uncertainty in the determination of both the observed and the predicted galaxy cluster X-ray integral temperature distribution functions at $z = 0.32$ by carrying out Monte Carlo simulations, where we take into careful consideration all the most important sources of possible error. We include the effect of the formation epoch on the relation between virial mass and X-ray temperature, improving on the assumption that clusters form at the observed redshift which leads to an overestimate of $\Omega_0$. We conclude that at present both the observational data and the theoretical modelling carry sufficiently large associated uncertainties to prevent an unambiguous determination of $\Omega_0$. We find that values of $\Omega_0$ around 0.75 are most favoured, with $\Omega_0 < 0.3$ excluded with at least 90 per cent confidence. In particular, the $\Omega_0 = 1$ hypothesis is found to be still viable as far as this dataset is concerned. As a by-product, we also use the revised data on the abundance of galaxy clusters at $z = 0.05$ to update the constraint on $\sigma_8$ given by Viana & Liddle (1996), finding slightly lower values than before.

Key words: galaxies: clusters – cosmology: theory.

1 INTRODUCTION

The number density of rich clusters of galaxies at the present epoch has been used to constrain the amplitude of mass density fluctuations on a scale of $8 h^{-1}$ Mpc (Evrard 1989; Henry & Arnaud 1991; White, Efstathiou & Frenk 1993a; Viana & Liddle 1996, henceforth VL; Eke, Cole & Frenk 1996; Hattori & Matsuzawa 1995; Oukbir & Blanchard 1997). This is usually referred to as $\sigma_8$, where $h$ is the present value of the Hubble parameter, $H_0$, in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. However, the derived value of $\sigma_8$ depends to a great extent on the present matter density in the Universe, parameterized by $\Omega_0$, and more weakly on the presence of a cosmological constant, $\Lambda$. The cleanest way of breaking this degeneracy is to include information on the change in the number density of rich galaxy clusters with redshift (Frenk et al. 1990), the use of X-ray clusters for this purpose having been proposed by Oukbir & Blanchard (1992) and subsequently further investigated (Hattori & Matsuzawa 1995; Oukbir & Blanchard 1997). Several attempts have been made recently, with wildly differing results (Henry 1997; Fan, Bahcall & Cen 1997; Gross et al. 1997; Blanchard & Bartlett 1997; Eke et al. 1998; Reichart et al. 1998).

The best method to find clusters of galaxies is through their X-ray emission, which is much less prone to projection effects than optical identification. Further, the X-ray temperature of a galaxy cluster is at present the most reliable estimator of its virial mass. This can then be used to relate the cluster mass function at different redshifts, calculated for example within the Press–Schechter framework (Press & Schechter 1974; Bond et al. 1991), to the observed cluster X-ray temperature function. We can therefore compare the evolution in the number density of galaxy clusters seen in the data with the theoretical expectation for large-scale structure formation models, which depends significantly only on the assumed values of $\Omega_0$ and $\lambda_0 \equiv \Lambda/3H_0^2$, the latter being the contribution of $\Lambda$ to the total present energy density in the Universe.

However, the X-ray temperature of a cluster of galaxies is not an easily measurable quantity, as compared to its X-ray luminosity. A minimum flux is required, so that there is a high enough number of photons for the statistical errors in the temperature determination to be reasonably small. Because of this, although estimates of the present-day cluster X-ray temperature function have been available since the
early 90's [Edge et al. 1990; Henry & Arnaud 1991], the change in the cluster X-ray temperature function as we look further into the past has been much more difficult to determine. Estimates for the X-ray temperatures of individual clusters with redshifts as high as 0.3 have been available for some years (e.g. see David et al. 1993), but only with the advent of the ASCA satellite has it been possible to measure X-ray temperatures for clusters of galaxies around that redshift in a systematic way, and to go to even higher redshifts.

The evolution of the cluster X-ray luminosity function with redshift, though easier to determine, provides much weaker constraints on $\Omega_0$ and $\lambda_0$, due to the fact that the X-ray luminosity of a galaxy cluster is not expected to be a reliable estimator of its virial mass (e.g. Hanami 1993). Though it could in principle provide some indication of the change of the cluster X-ray temperature function with redshift, the problem is that not only is there considerable scatter in the present-day cluster X-ray temperature versus luminosity relation (David et al. 1993; Fabian et al. 1994), but it is also not known how the relation may change with redshift, though recently it has been argued that at least up to $z = 0.4$ it does not seem to evolve (Mushotzky & Schartel 1997; Allen & Fabian 1998; Reichart et al. 1998; Sadat, Branchard & Oukbir 1998).

The deepest complete X-ray sample of galaxy clusters presently available is the one obtained from the *Einsteins Medium Sensitivity Survey* (EMSS) (Gioia et al. 1990; Henry et al. 1992). This sample is restricted to objects with declination larger than $-40^\circ$ and is flux-limited, with $F_{\text{obs}} \geq 1.33 \times 10^{-13}$ erg cm$^{-2}$ s$^{-1}$, where $F_{\text{obs}}$ is the cluster flux in the 0.3 to 3.5 keV band which falls in a 2'4 × 2'4 EMSS detect cell. It presently contains 90 clusters of galaxies, after a few misidentifications were recently removed (Gioia & Luppino 1994; Nichol et al. 1997). This is the only complete galaxy cluster catalogue beyond a redshift of 0.3, and as such unique in providing the means to distinguish between different possible values for $\Omega_0$ and $\lambda_0$. However, until the recent effort by Henry (1997), very few X-ray temperatures were known for those galaxy clusters in the EMSS sample with redshifts exceeding 0.15 (see Sadat et al. 1998 for a recent compilation). Henry (1997) used ASCA to observe all galaxy clusters in the EMSS cluster sample with $0.3 \leq z \leq 0.4$ and $F_{\text{obs}} \geq 2.5 \times 10^{-13}$ erg cm$^{-2}$ s$^{-1}$. The resulting sub-sample of 10 clusters has a median redshift of 0.32, and the data obtained for each cluster, the X-ray flux, luminosity and temperature, can be found in Table 1 of Henry (1997).

We will use this data together with the present-day (median redshift 0.05) cluster X-ray temperature function. We work within the extended Press–Schechter formalism proposed by Lacey & Cole (1993, 1994), which allows an estimation of the formation times of dark matter halos. We will assume the dark matter to be cold, and consider the cases of an open universe, where the cosmological constant is zero, and a spatially-flat universe, such that $\lambda_0 = 1 - \Omega_0$.

2 THEORETICAL MASS AND TEMPERATURE FUNCTIONS

The usual means by which the mass function of virialized structures can be determined analytically is through the Press–Schechter approach, which has been found to reproduce well the mass functions recovered from various N-body simulations (Lacey & Cole 1994; Eke et al. 1996; Tormen 1998). The comoving number density of galaxy clusters in a mass interval $dM$ about $M$ at a redshift $z$ is given by

$$n(M, z) dM = \frac{\sqrt{2}}{\pi M} \frac{\delta_c}{\sigma^3(R, z)} \frac{d\sigma(R, z)}{dM} \exp \left( -\frac{\delta_c^2}{2\sigma^2(R, z)} \right) dM,$$

where $\rho_c$ is the comoving matter density, $\sigma(R, z)$ is the dispersion of the linearly-evolved density field smoothed by a top-hat window function on the comoving scale $R$, such that $R^2 = 3M/4\pi\rho_c$, and $\delta_c$ is the linear density threshold associated with the collapse and subsequent virialization of the galaxy clusters. This last quantity depends to some extent on the geometry of the collapsing structures (Monaco 1993), but since rich galaxy clusters are relatively rare, it would seem to be a fair assumption to take their collapse to be close to spherical ($\lambda = 0$). The analytical calculation would then yield $\delta_c = 1.69$ in the case of an Einstein–de Sitter universe, with a decrease at most of 5 per cent when one goes to a universe with sub-critical density, as long as $\Omega_0 > 0.1$, whether or not a cosmological constant is present (Eke et al. 1997). This is supported by the results from N-body simulations, which prefer $\delta_c = 1.7 \pm 0.2$ (Lacey & Cole 1994; Eke et al. 1996; Tormen 1998). As we want to be conservative, we will allow for this margin of variation in the value of $\delta_c$, assuming it to be equivalent to a 95 per cent confidence interval.

If we have in mind a particular shape for the power spectrum of density perturbations, we can further simplify equation (1) by writing the derivative in terms of $\lambda = 3\Omega_0^2/(8\pi G) - 1$. As we will be assuming all the dark matter to be cold, the value of $\sigma(R, z)$ in the vicinity of $8h^{-1}$ Mpc can be obtained to a good approximation through

$$\sigma(R, z) = \sigma_s(z) \left( \frac{R}{8h^{-1} \text{ Mpc}} \right)^{-\gamma(R)},$$

where

$$\gamma(R) = (0.3\Gamma + 0.2) \left[ 2.92 + \log_{10} \left( \frac{R}{8h^{-1} \text{ Mpc}} \right) \right],$$

and $\Gamma$ is the usual shape parameter of the cold dark matter (CDM) transfer function.

Note that $\gamma(R)$ is independent of $z$, reflecting the fact that the shape of the power spectrum of density perturbations in the case of CDM models does not change after the epoch of matter–radiation equality. We will assume that $\Gamma = 0.23^{\pm 0.042}_{\pm 0.034}$ at the 95 per cent confidence level, for which a good fit to the observed present shape of the galaxy and cluster correlation functions in the vicinity of $8h^{-1}$ Mpc is obtained (Peacock & Dodds 1994; Viana & Liddle 1996). It should be mentioned however that when the abundance of X-ray clusters over a range of temperatures is used, the preferred value for $\Gamma$ seems to be closer to 0.10 (Eke et al. 1996; Eke et al. 1998). Though in this case $\Gamma$ is not as well constrained, there is clearly a significant disagreement between the values obtained through the two methods. The source of this discrepancy needs to be investigated, and may lie ultimately in a failure of models whose power spectrum can be

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Galaxy clusters at $0.3 < z < 0.4$ and the value of $\Omega_0$ \footnote{Carried out in an Einstein-de Sitter universe. They imply that a galaxy cluster with an X-ray temperature of 7.5 keV has a virial mass of $M_v = (1.23 \pm 0.32) \times 10^{15} h^{-1} M_\odot$, for an estimated virialization redshift of $z_v \sim 0.05 \pm 0.05$ (Metzler & Evrard 1994; Navarro et al. 1995). This corresponds to a turnaround redshift of $z_m \sim 0.67 \pm 0.08$, since $z_c$ and $z_m$ are related by the fact that the time of collapse and subsequent virialization, $t_c$, is twice the time of maximum expansion, $t_m$, and in an Einstein-de Sitter universe $t = \frac{2}{3} H_0^{-1} (1 + z)^{-3/2}$.}

parametrized through $\Gamma$ to reproduce the observed perturbation spectrum (see e.g. Peacock 1997).

The quantity $\sigma_s(z)$ is related with its present value $\sigma_s(0)$ via the perturbation growth law

$$\sigma_s(z) = \sigma_s(0) \frac{g(\Omega(z))}{g(\Omega_0)} \left(1 + z\right),$$

where the appropriate formulae for $g(\Omega)$ and $\Omega(z)$, depending on whether the universe is open or spatially-flat, can be found in VL [equations (8) and (10), and (9) and (11), respectively].

Using expression (2), we can now calculate the derivative appearing in equation (4), and substituting we end up with

$$n(M, z) \Delta M = \sqrt{\frac{2}{\pi}} \frac{\rho_s}{M^2} \frac{1}{3 \sigma(R, z)} \left(0.3 \Gamma + 0.2\right) \times$$

$$\left[2.92 + \log_{10} \left(\frac{R}{8h^{-1} \text{Mpc}}\right) \exp \left(-\frac{\eta^2}{2 \sigma^2(R, z)}\right)\right]$$

$$dM.$$

In order to transform this mass function into the cluster X-ray temperature function, one needs to relate the virial mass of a galaxy cluster to its X-ray temperature. Such a relation has been obtained analytically by Lilje (1992) (see also Hanami 1993), and confirmed through hydrodynamical N-body simulations (Navarro et al. 1995; Bryan & Norman 1998).

$$M_v \propto \Omega_{0}^{-1/2} \chi^{-1/2} \left(\frac{r_v}{r_m}\right)^{3/2} \times$$

$$\left[1 - \eta \left(\frac{r_v}{r_m}\right)^{3/2} \left(1 + z_m\right)^{-3/2} \left(k_B T\right)^{3/2} h^{-1}\right],$$

where $z_m$ is the redshift of cluster turnaround, and

$$\chi = \left(\frac{4}{3 \pi} \right)^2 \xi,$$

$$\eta = 2 \left(\frac{4}{3 \pi} \right)^2 \left(\frac{\lambda_0}{\Omega_0}\right)^{-1} \chi^{-1} (1 + z_m)^{-3},$$

with $\xi$ the ratio between the cluster and background densities at turnaround. This quantity was calculated numerically in VL, depending only on $\Omega \equiv \Omega(z_m)$ via $\chi = \Omega^{-b(\Omega)}$, where $b(\Omega) = 0.76 - 0.25 \Omega$ in the case of an open universe, and $b(\Omega) = 0.73 - 0.23 \Omega$ for a spatially-flat universe.

The radii of turnaround and virialization, respectively $r_m$ and $r_v$, are related through

$$\frac{r_v}{r_m} = \frac{1 - \eta/2}{2 - \eta/2},$$

in the case when a galaxy cluster is assumed to be an ideal virialized system collapsed from a top-hat perturbation. The presence of significant substructure during collapse would lead to dynamical relaxation thus making the clusters more compact, decreasing the ratio $r_v/r_m$.

The proportionality constant in expression (6) can be obtained either through analytical derivation assuming hydrostatic equilibrium (e.g. see Bryan & Norman 1998), or by using results from hydrodynamical N-body simulations. We choose the latter option, as it provides an estimate of the uncertainties involved, allowing for the possibility of deviation from hydrostatic equilibrium due for example to bulk gas motions and turbulence (Norman & Bryan 1998).

The hydrodynamical N-body simulations we use are those of White et al. (1993), carried out in an Einstein-de Sitter universe. They imply that a galaxy cluster with an X-ray temperature of 7.5 keV has a virial mass of $M_v = (1.23 \pm 0.32) \times 10^{15} h^{-1} M_\odot$, for an estimated virialization redshift of $z_v \sim 0.05 \pm 0.05$ (Metzler & Evrard 1994; Navarro et al. 1995). This corresponds to a turnaround redshift of $z_m \sim 0.67 \pm 0.08$, since $z_c$ and $z_m$ are related by the fact that the time of collapse and subsequent virialization, $t_c$, is twice the time of maximum expansion, $t_m$, and in an Einstein-de Sitter universe $t = \frac{2}{3} H_0^{-1} (1 + z)^{-3/2}$.

Putting all this together we are now able to normalize expression (6),

$$M_v = (1.23 \pm 0.33) \times 10^{15} \Omega_0^{-1/2} \chi^{-1/2} \left(\frac{r_v}{r_m}\right)^{3/2} \times$$

$$\left[1 - \eta \left(\frac{r_v}{r_m}\right)^{3/2} \left(1 + z_m\right)^{-3/2} \left(k_B T\right)^{3/2} h^{-1}\right],$$

where the error is 1-sigma. This normalization of the virial mass—X-ray temperature relation agrees very well with that obtained by Bryan & Norman (1998), who used the largest set of hydrodynamical N-body simulations ever assembled.

We now have the problem that even after the background cosmology is chosen, by fixing $\Omega_0$ and $\lambda_0$, expression (11) depends on the redshift of cluster turnaround, $z_m$. As this can be determined from the virialization redshift, $z_c$, using the fact that $t_m = 2t_c$ [for expressions of $t$ as a function of $z$ in open and spatially-flat universes see VL, equations (26) and (27)], we simply need a means to estimate the distribution of the redshifts of cluster virialization at each given virial mass. The most well justified method which provides this was put forward by Lacey & Cole (1993, 1994), though it may slightly underestimate $z_c$ (Norman 1998).

Lacey & Cole constructed a merging history for dark matter halos based on the random walk trajectories technique, and derived an analytical expression for the probability that a galaxy cluster with present virial mass $M$ would have formed at some redshift $z$,

$$p(z) = p(w(z)) \frac{dw(z)}{dz},$$

where

$$p(w(z)) = 2 w(z) \left(f^{-1} - 1\right) \exp\left(\frac{w(z)}{\sqrt{2}}\right) -$$

$$\sqrt{\frac{2}{\pi}} \left(f^{-1} - 2\right) \exp\left(-\frac{w^2(z)}{2}\right),$$

and

$$w(z) = \frac{\delta_c (\sigma(M, z) - 1)}{\sqrt{\sigma^2(fM, 0) - \sigma^2(M, 0)},}$$

with $f$ the fraction of the cluster mass assembled by redshift $z$. Independently of background cosmology we will assume $f = 0.75 \pm 0.15$, as after this mass fraction has been assembled it is expected that the X-ray temperature of a cluster of galaxies will not change significantly (Navarro et al. 1995). We will consider the uncertainty in the value of $f$ to roughly correspond to a 95 per cent confidence interval. Although the expression for the formation probability
given above was obtained for a power-law spectrum of matter density fluctuations with index $n = 0$, while at the cluster scale $n$ is expected to be close to $-2$ (Henry et al. 1992; Hanami 1993; Dvorkin, Bartlett & Blanchard 1999; Markelvitch 1999), numerical results show that $p(w(z))$ depends only very weakly on $n$ (Lacey & Cole 1993).

The present comoving number density of galaxy clusters per temperature interval $d(k_B T)$ with a mean X-ray temperature of $k_B T$, that formed at each redshift $z$, can now be calculated using the chain rule and equation \[ n = \frac{M}{2k_B T} n(M, z = 0) p(z) dM \, dz. \] (15)

The present cluster X-ray temperature function at $k_B T$ is obtained by integrating this expression from redshift zero up to infinity, with the virial mass $M$ obtained through expression \[ n = \frac{M}{2k_B T} n(M, z = 0) p(z) dM \, dz. \] (16) taking into account the assumed $k_B T$ and the value of the integration variable $z = z_e$.

The cluster X-ray temperature function at any redshift $z$ for some temperature $k_B T$, can be obtained by taking the point of view of someone placed at that redshift, i.e. transferring the conditions prevalent at that redshift to the present epoch. For example, one takes $\Omega_m = \Omega(z)$, and changes the normalization of expression \[ n = \frac{M}{2k_B T} n(M, z = 0) p(z) dM \, dz. \] (15) so that for $z = 0$ one obtains the virial mass—X-ray temperature relation that applies at the redshift of interest.

In order to compare with the available data, we will actually need to calculate the cumulative or integral cluster X-ray temperature function, $N(> k_B T, z)$, i.e. the comoving number density of galaxy clusters with an X-ray temperature exceeding $k_B T$ at redshift $z$. This is obtained from the differential cluster X-ray temperature function, $n(k_B T, z) d(k_B T)$, by integrating it from the minimum X-ray temperature required up to infinity.

3 COMPARING OBSERVATIONAL DATA WITH THEORETICAL EXPECTATIONS

We use two pieces of data, the integral cluster X-ray temperature functions at $z = 0.05$ and $z = 0.32$, where these are the median redshifts of the X-ray cluster samples we are considering.

3.1 The low-redshift data

The integral cluster X-ray temperature function at $z = 0.05$ can be determined using the dataset presented in Henry & Arnaud (1999). We actually use an updated version of it, with more accurate X-ray flux and temperature determinations, which has been kindly provided to us by Pat Henry. Without taking into consideration the temperature measurement errors, we derived for the number density of galaxy clusters at $z = 0.05$ with X-ray temperature exceeding $6.2$ keV

$$N(> 6.2 \text{ keV}, 0.05) = 1.80 \times 10^{-7} h^3 \text{ Mpc}^{-3}, \quad \text{(16)}$$

which as expected agrees with the value obtained by Eke et al. (1996,1998) and Henry (1997). It is also in good agreement with the results of Edge et al. (1996) and Markevitch (1998).

However, what not many people take into account is that the existence of temperature measurement errors makes the above value a biased estimator of the real value of $N(> 6.2 \text{ keV}, 0.05)$ in the Universe. This can be easily seen by constructing a large number of datasets with the same clusters in each as in the $z = 0.05$ dataset, but where the X-ray temperature for each cluster is randomly drawn from a Gaussian distribution with mean and dispersion as observed for that cluster. This procedure simulates the repetition of the temperature measurements, assuming the cluster X-ray temperatures originally measured are the actual ones. If one now determines $N(> 6.2 \text{ keV}, 0.05)$ for each cluster dataset thus obtained, the mean of the distribution turns out to be $2.12 \times 10^{-7} h^3 \text{ Mpc}^{-3}$. Hence, the existence of measurement errors in the X-ray temperature determinations leads to an overestimation of the real value for $N(> 6.2 \text{ keV}, 0.05)$, if one uses the actual measured temperatures. This can be easily understood if one remembers that as the X-ray temperature goes up, the cluster number density decreases. Given that to a first approximation it is as probable for a cluster with actual X-ray temperature below $6.2$ keV to have a measured temperature above that value, as it is for a cluster with $k_B T > 6.2$ keV to have a smaller measured temperature, the net effect will therefore be an apparent increase in the number density of galaxy clusters with X-ray temperature above $6.2$ keV. We expect $1.80 \times 10^{-7} h^3 \text{ Mpc}^{-3}$ to be an overestimation of the real value for $N(> 6.2 \text{ keV}, 0.05)$ in the same proportion as $2.12 \times 10^{-7} h^3 \text{ Mpc}^{-3}$ exceeds the assumed correct value $1.80 \times 10^{-7} h^3 \text{ Mpc}^{-3}$ used in constructing the artificial datasets above. Therefore the corrected best estimate for the number density of galaxy clusters at $z = 0.05$ with X-ray temperature exceeding $6.2$ keV is

$$N(> 6.2 \text{ keV}, 0.05) = 1.53 \times 10^{-2.16} \times 10^{-7} h^3 \text{ Mpc}^{-3}, \quad \text{(17)}$$

where the errors represent 1-sigma confidence levels. They were obtained through a bootstrap procedure, which allows an estimation of the uncertainty associated with the sampling variance, where we constructed $10^4$ cluster datasets by randomly selecting, with replacement, from the original list of 25 X-ray clusters in Henry & Arnaud (1999). The number of clusters in each sample is drawn from a Poisson distribution with mean 25, in order to include the counting error. Each time a cluster is selected its X-ray temperature is estimated by randomly drawing from a Gaussian distribution with the mean and dispersion observed for the cluster. In this way the temperature measurement errors also contribute to the total uncertainty.

In Eke et al. (1998), that X-ray temperature measurement errors lead to an overestimate of the real value for $N(> k_B T, z)$ is dealt with through a Gaussian smoothing of the temperature distribution function predicted in each $(\Omega_m, \Omega_\Lambda)$ cosmology, which is then integrated to give $N(> k_B T, z)$.

There are several reasons why we chose to concentrate on galaxy clusters with X-ray temperature exceeding $6.2$ keV. The first is that $N(> 6.2 \text{ keV})$ best represents the mean curve going through the observational points for $N(> k_B T)$, both at $z = 0.05$ and $z = 0.32$, as can be seen in Figure 2 of

† We are very much indebted to Alain Blanchard for pointing this out to us.

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Henry (1997). Also, the Press-Schechter framework should work best on the largest scales, i.e. for the highest masses and X-ray temperatures, as in hierarchical cosmologies these are the ones for which the density field has evolved least, therefore keeping its gaussianity to a greater extent. Related to this is the problem of shear, which starts becoming an important factor in the collapse of density perturbations as the density field develops, leading to deviations from the idealized spherical collapse situation. Another reason is that in the normalization of the relation between virial mass and X-ray temperature for galaxy clusters, we used hydrodynamical $N$-body simulations which do not take into account a possible (pre-)heating of the intracluster medium due to starbursts and supernovae in the galaxies. This effect is still quite difficult to model realistically, but the few attempts that have been made seem to show that it becomes more important as the cluster virial mass decreases. For a galaxy cluster whose X-ray temperature would otherwise be 5 keV, the heating may increase the cluster X-ray temperature by as much as 15 per cent (Metzler & Evrard 1994; Navarro et al. 1995).

3.2 The high-redshift data

The comoving number density of galaxy clusters with X-ray temperature exceeding 6.2 keV at $z = 0.32$ can be calculated using the EMSS sub-sample of 10 clusters with redshifts between 0.3 and 0.4 and fluxes above $2.5 \times 10^{-13}$ erg cm$^{-2}$ s$^{-1}$, for which Henry (1997) obtained the mean X-ray temperatures through ASCA. We used the data in Table 1 of Henry (1997), regarding the X-ray flux and temperature for each cluster, to estimate the integral cluster X-ray temperature function at $z = 0.32$. We did this both for open and spatially-flat universes, using the estimator

$$N(> k_B T, z) = \sum_{i=1}^{N_{\text{MAX}}(z)} \frac{1}{V_{\text{MAX},i}},$$

where the sum is over all clusters with $k_B T_i > k_B T$, and $V_{\text{MAX},i}$ is the maximum volume in which cluster $i$ could have been detected at the 4$\sigma$ level in the EMSS within the redshift shell under consideration (0.3 to 0.4 in our case).

The steps which need to be taken in order to calculate these volumes are described in detail in Henry et al. (1992). They involve the determination of the maximum redshift at which each galaxy cluster could have been detected as a function of its observed flux, using equations (1) and (2) of Henry et al. (1992). In this calculation one has to compensate for the fact that clusters of galaxies are extended objects, and thus some of their flux will be outside the EMSS detect cell. One therefore needs to estimate the typical core radius, from which most of the flux originates, of the galaxy clusters in the EMSS sub-sample from Henry (1997). In the absence of data specific to this sub-sample, we use the ratio between the extended and detect cell fluxes estimated in Henry et al. (1992) for a sample of 4 galaxy clusters extended within the EMSS with a mean redshift of 0.29. They find this ratio to be equal to $2.10 \pm 0.19$, where we will assume the error to be $1\sigma$. This implies a typical core radius around 0.15 h$^{-1}$ Mpc, depending on the chosen values for $\Omega_0$ and $\lambda_0$, which we will assume to remain the same for $0.3 < z < 0.4$.

The detection volume for a given galaxy cluster is then obtained by summing, over all limiting fluxes of the EMSS (see Table 3 of Henry et al. 1992) starting at $2.5 \times 10^{-13}$ erg cm$^{-2}$ s$^{-1}$, the volumes lying between a redshift of 0.3 and whichever is the lesser of 0.4 and the maximum detection redshift for the cluster. The errors affecting the calculation of the detection volumes are thus those associated with the flux measured for each galaxy cluster and the compensation for the extended nature of galaxy clusters.

As in the lower redshift case, the calculation of $N(> 6.2 \text{keV}, 0.32)$ using the X-ray temperatures measured for the galaxy clusters found between $z = 0.3$ and $z = 0.4$ would lead to the overestimation of $N(> 6.2 \text{keV}, 0.32)$ due to the presence of errors in the X-ray temperature determinations. Again, we correct for this by simulating the repetition of the temperature measurements. The ratio between the mean value obtained for $N(> 6.2 \text{keV}, 0.32)$ from all the simulated datasets, and the value one gets for $N(> 6.2 \text{keV}, 0.32)$ using the original dataset, provides an estimate for the expected ratio between the latter and the real value for $N(> 6.2 \text{keV}, 0.32)$ in the Universe. After performing this correction, the best estimate for the number density of galaxy clusters at $z = 0.32$ with X-ray temperature exceeding 6.2 keV becomes

$$N(> 6.2 \text{keV}, 0.32) = 3.98 \Omega_0^{B(\Omega_0)} \times 10^{-8} h^3 \text{Mpc}^{-3},$$

where $B(\Omega_0) = 0.09 + 0.38 \Omega_0 - 0.29 \Omega_0^2$ if the Universe is open and $B(\Omega_0) = 0.25 + 0.94 \Omega_0 - 0.78 \Omega_0^2$ if the Universe is spatially-flat. These fitting functions have an associated error of less than one per cent and are valid for $\Omega_0$ between 0.1 and 1.

The bootstrap method is the simplest way of simulating the procedure involved in the extraction of a sample from a given distribution. In our case this distribution is simply the population of galaxy clusters in the Universe in the redshift bin $0.3 < z < 0.4$ with EMSS X-ray fluxes exceeding $2.5 \times 10^{-13}$ erg cm$^{-2}$ s$^{-1}$. The bootstrap method therefore allows the estimation of the dispersion one would expect to obtain in the values measured for some quantity related to that population, for example $N(> 6.2 \text{keV}, 0.32)$, if the type of sampling that led to the dataset in Henry (1997) was repeated a large number of times across the sky.

3.3 The method of comparison

Let us now assume that in our Universe $N(> 6.2 \text{keV}, 0.32)$ takes some particular overall value. We would then expect this value to be the mean of the distribution function assembled with the values that would be measured for $N(> 6.2 \text{keV}, 0.32)$ if the type of sampling that led to the dataset in Henry (1997) was repeated a large number of times across the sky. On the other hand, we would expect that the shape of this distribution would be that obtained through the bootstrap procedure mentioned at the end of the previous subsection. We are therefore now in a position to ask the following question. If $N(> 6.2 \text{keV}, 0.32)$ took some overall value in the Universe, how probable would it be to measure the value for $N(> 6.2 \text{keV}, 0.32)$ given by the dataset in Henry (1997), after correcting it for the displacement due to errors in the X-ray temperature measurements? We can then attach, for each value of $\Omega_0$, a probability of the value for $N(> 6.2 \text{keV}, 0.32)$ given by the dataset in
Henry [1997] being actually measured. The *exclusion level* on each value of $\Omega$ is obtained simply by subtracting this probability from one.

In summary the following steps were taken, so that an exclusion level can be associated with each $\Omega$ based on the X-ray cluster datasets for $z = 0.05$ and $z = 0.32$:

(i) The Universe was assumed to be either open or spatially-flat, with $\Omega$ taking a value between 0.1 and 1.

(ii) The best estimate for $N(> 6.2\,\text{keV}, 0.32)$ in the Universe, given the dataset in Henry [1997], was calculated. This is the result shown in equation (19), after correcting for the effect of the X-ray temperature measurement errors.

(iii) A bootstrap procedure analogous to that described for the $z = 0.05$ data was performed in order to determine the expected *shape* for the distribution function of $N(> 6.2\,\text{keV}, 0.32)$, if the type of sampling that led to the dataset in Henry [1997] was repeated a large number of times across the sky. The number of clusters in each sample is now drawn from a Poisson distribution with mean 10, and the input observational errors (in the typical ratio of extended to EMSS detect cell fluxes at $z = 0.29$, and in the ASCA X-ray fluxes and temperatures) are modelled as Gaussian distributed.

(iv) Using the method described in the section 2, the theoretically-expected overall value for $N(> 6.2\,\text{keV}, 0.32)$ given the assumed $\Omega$ was calculated. The normalization $\sigma_0$ of the spectrum was fixed by the low-redshift data using equation (17).

(v) The distribution function for $N(> 6.2\,\text{keV}, 0.32)$, determined through the bootstrap procedure, was modified by dividing the values obtained for $N(> 6.2\,\text{keV}, 0.32)$ by their mean and multiplying them by the value determined in (iv), so that this value becomes the new mean and the relative shape of the distribution is maintained.

(vi) We calculated the probability of obtaining a value as high, or as low, as that determined in (ii), given the distribution constructed in (v). The exclusion level on the assumed $\Omega$ equals one minus this probability.

It should be noted that in step (iv), the calculation of the theoretically-expected overall value for $N(> 6.2\,\text{keV}, 0.32)$ for each assumed $\Omega$, has some theoretical uncertainty associated with it. This results from the uncertainties associated with the values one should use for the normalization of the cluster X-ray temperature to virial mass relation, $\delta$, $f$ and $\Gamma$. There is also the uncertainty associated with the observed value for $N(> 6.2\,\text{keV}, 0.05)$. The total uncertainty in the theoretically-expected overall value for $N(> 6.2\,\text{keV}, 0.32)$, for each assumed $\Omega$, will be calculated through Monte Carlo simulations described in section 4, which will also explain the procedure used to incorporate this uncertainty into the calculation of the final exclusion level on each assumed $\Omega$.

In general, it is incorrect to calculate the exclusion level associated with some theoretically-expected overall value by directly using the distribution obtained through a bootstrap procedure, and asking how probable it would be to obtain such a high (or low) value given that distribution. In practice, however, if the bootstrap distribution is symmetric, the exclusion level calculated this way, and in the more correct way described above in points (v) and (vi), is nearly the same. Nevertheless, in the case of the high-redshift cluster data, the distribution for $N(> 6.2\,\text{keV}, 0.32)$ obtained through the bootstrap procedure is highly asymmetric with a long right-sided tail. Given that high $\Omega$ models tend to predict an overall value for $N(> 6.2\,\text{keV}, 0.32)$ which is smaller than the value determined through the dataset in Henry [1997], identifying the mean of the bootstrap distribution with the latter rather than with the former would lead to an exaggerated difficulty for higher $\Omega$ models to reproduce the observations, and thus to a higher exclusion level. On the contrary, lower $\Omega$ models, which tend to predict the inverse, would benefit from incorrectly calculating the exclusion level. In the next section we will estimate this effect.

In the case of the low-redshift data, though the bootstrap procedure yields a distribution for $N(> 6.2\,\text{keV}, 0.05)$ which is best characterized as a lognormal, in practice the deviation from symmetry is sufficiently small for it to be preferable, given the much simpler calculations involved, to associate the bootstrap uncertainty with the corrected value for $N(> 6.2\,\text{keV}, 0.05)$ determined from the dataset in Henry & Arnaud [1993], rather than with some theoretically-expected overall value.

In Figure 1 we show the binned probability distributions obtained through the bootstrap method for $N(> 6.2\,\text{keV}, 0.32)$, in the cases of $\Omega = 1$ and 0.3, where the Universe is assumed either open or spatially-flat. The distributions were altered so that their mean now coincides with the theoretically-expected overall value in those cases. It also shows the corrected value for $N(> 6.2\,\text{keV}, 0.32)$ determined from the dataset in Henry & Arnaud [1993]. The peaks in the distributions are not an artifact of the chosen binning, but correspond to different numbers of galaxy clusters in each bootstrap sample having an X-ray temperature in excess of our chosen threshold 6.2 keV.

The calculations we have just described assume that there is no scatter in the relation between cluster X-ray temperature and luminosity. This is not correct, as mentioned previously, and can lead to an *increase* in the estimated value for $N(> k_{\beta}T, z)$. This incompleteness problem worsens as the threshold X-ray temperature $k_{\beta}T$ is lowered, as one starts considering clusters with X-ray fluxes dangerously close to the flux detection limit. For the same threshold X-ray temperature, the problem is also potentially much more serious in the case of the $z = 0.32$ data than for the $z = 0.05$ data. The reason is simply that for the same flux detection limit, the faintest clusters that can be detected nearby have X-ray luminosities (and thus temperatures) which are considerably smaller than those of the faintest clusters further away.

The effect of the scatter in the X-ray cluster temperature–luminosity relation in the calculation of $N(> 6.2\,\text{keV}, 0.05)$ is negligible, as the $z = 0.05$ dataset in Henry & Arnaud [1993] is claimed to be nearly complete down to at least 3 keV.

In the case of the $z = 0.32$ data, it is not clear whether the presence of scatter in the X-ray cluster temperature–luminosity relation may affect the determination of $N(> 6.2\,\text{keV}, 0.32)$. Due to the scatter, there is a finite probability that some of the 5 EMSS galaxy clusters with X-ray flux below $2.5 \times 10^{-13} \text{erg cm}^{-2} \text{s}^{-1}$, that were found in the redshift range from 0.3 to 0.4 [Henry et al. 1992], may not only have an X-ray temperature in excess of the lowest X-ray
temperature present in the sub-sample of 10 clusters from Henry (1997), 3.8 keV for MS1512.4, but also in excess of the threshold temperature. For example, at 4 keV the effect is already expected to be significant (Eke et al. 1998).

We calculated the expected increase in the corrected value of $N(> 6.2 \text{ keV}, 0.32)$ given the dataset in Henry (1997), as a result of the existence of the 5 EMSS galaxy clusters mentioned above, by doing 1000 Monte Carlo simulations where the X-ray temperatures for those clusters were estimated via the X-ray cluster temperature–luminosity relation determined in Eke et al. (1998) using the more recent data for the galaxy clusters in Henry & Arnaud (1991). The X-ray temperatures for those 5 clusters were obtained through the power-law relation

$$k_0 T = \left( \frac{10^4 \, L_{X}}{h} \right)^{1/b},$$

(20)

where in each Monte Carlo simulation the values of the parameters $a$ and $b$ were drawn from Gaussian distributions with respectively mean 2.53 and dispersion 0.69, and mean 3.54 and dispersion 0.47. The X-ray luminosities in the 0.3 to 3.5 keV band for the 5 clusters in question in units of $10^{44} \, h^{-2} \, \text{erg s}^{-1}$, $L_{X}$, were determined using the X-ray fluxes given in Henry et al. (1992), with a K-correction of 15 per cent included independently of the assumed cosmology (Henry et al. 1992, Henry 1997). The maximum volumes within which the 5 clusters could have been observed were calculated in the same way as those for the 10 higher flux clusters, using the EMSS data provided in Henry et al. (1992). The final exclusion level for each value of $\Omega_0$ was calculated simply by taking the mean of the exclusion levels associated with the new higher estimated values for $N(> 6.2 \text{ keV}, 0.32)$ in the Universe, resulting from each Monte Carlo simulation.

By using the X-ray cluster temperature–luminosity relation observed for $z = 0.05$ to estimate X-ray temperatures for galaxy clusters where $z$ is within 0.3 to 0.4, we are implicitly assuming that this relation does not evolve much between a redshift of 0.4 and the present. This is supported by the recent analyses of Mushotzky & Scharf (1997) and Allen & Fabian (1998) (see also Sadat, Blanchard and Oukbir 1998). However, the actual dataset in Henry (1997) does not support this assumption, as a chi-square fit of a power-law to the data prefers the situation where the X-ray luminosity is practically independent of the X-ray temperature. This is driven by the relatively low X-ray temperature measured for the galaxy cluster MS2137.3, which, although it is by far the brightest cluster in the dataset, has only the 7th highest X-ray temperature. Removing this cluster from the dataset, the best-fit power-law for the X-ray temperature–luminosity relation becomes compatible with the $z = 0.05$ one. It was this very strong dependence of the best-fit X-ray cluster temperature–luminosity relation on the inclusion or not of a single galaxy cluster which led us to choose not to estimate the X-ray temperatures for the 5 clusters with the lowest fluxes using the data for the 10 clusters that make up the dataset in Henry (1997).

In the end, we found that allowing for the presence of scatter in the X-ray cluster temperature–luminosity relation when calculating $N(> 6.2 \text{ keV}, 0.32)$ has only a small effect, at the few per cent level, on the exclusion levels obtained for different $\Omega_0$, and does not alter our conclusions.

4 RESULTS

The calculation of the theoretically-predicted overall value for $N(> 6.2 \text{ keV}, 0.32)$ was performed as described in Section 2, with $\sigma_8$ obtained from the observed value for $N(> 6.2 \text{ keV}, 0.05)$, and the uncertainty calculated via a Monte Carlo procedure.

Our first result is the value required for $\sigma_8$, as a function of both $\Omega_0$ and $\lambda_0$, so that the observed value for $N(> 6.2 \text{ keV}, 0.05)$ is reproduced. This supersedes the result obtained in VL. We find that the best-fitting value is given by

$$\sigma_8 = \begin{cases} 
0.56 \, \Omega_0^{0.34} & \text{Open} \\
0.56 \, \Omega_0^{0.47} & \text{Flat}.
\end{cases}$$

This is accurate within 3 per cent for $\Omega_0$ between 0.1 and 1.

The most important reason why this value is smaller than that quoted in VL is the decrease in the assumed number density of galaxy clusters at $z = 0.05$. This results from
the revision of the Henry & Arnaud dataset and from the correction due to the existence of X-ray temperature measurement errors, which had not been taken into consideration in VL. Also, the cluster X-ray temperature function obtained in Henry & Arnaud ([99]) had been slightly overestimated due to a calculational error ([ke et al. 1996]).

The overall uncertainty in the value of $\sigma_8$ was calculated in the same way as in VL, through a Monte Carlo procedure where the sources of error, namely the normalization of the cluster X-ray temperature to virial mass relation, the value of $\delta_c$ and the value of $f$, are modelled as being Gaussian distributed, and $\Gamma$ and $N$ (> 6.2 keV, 0.05) as having a lognormal distribution. As in VL, we find that for each $\Omega_0$ between 0.1 and 1 the distribution of $\sigma_8$ can be approximated by a lognormal. For open models, the 95 per cent confidence limits are roughly given by $+20\Omega_0^{0.32} \Omega_0^{1056} 10^{-5}$ per cent and $-18\Omega_0^{0.32} \Omega_0^{1056} 10^{-5}$ per cent, while for flat models we have $+20\Omega_0^{0.32} \Omega_0^{1056} 10^{-5}$ per cent and $-18\Omega_0^{0.32} \Omega_0^{1056} 10^{-5}$.

The calculation of the uncertainty in the theoretically-predicted overall value for $N$ (> 6.2 keV, 0.32), for each assumed $\Omega_0$, was made using the Monte Carlo simulations performed in order to calculate the uncertainty in the value of $\sigma_8$, described above.

We find that for $\Omega_0$ between 0.1 and 1, the distribution of $N$ (> 6.2 keV, 0.32) is close to lognormal and, with an associated error of less than 4 per cent, its mean is fitted by

$$N (> 6.2 \text{ keV}, 0.32) = 2.67\Omega_0^{-G(\Omega_0)} \times 10^{-8} \text{ h}^3 \text{ Mpc}^{-3},$$

where $G(\Omega_0) = 0.57 + 2.69 \Omega_0 - 1.87 \Omega_0^2$ if the Universe is assumed open and $G(\Omega_0) = 0.45 + 2.56 \Omega_0 - 2.12 \Omega_0^2$ if it is assumed spatially-flat. The 95 per cent confidence intervals are to a fair approximation given by $+170\Omega_0^{0.17+0.31} \Omega_0^{1056} 10^{-5}$ per cent and $-62\Omega_0^{0.17+0.31} \Omega_0^{1056} 10^{-5}$ per cent in the open case, and $+170\Omega_0^{0.06+0.24} \Omega_0^{1056} 10^{-5}$ per cent and $-64\Omega_0^{0.06+0.24} \Omega_0^{1056} 10^{-5}$ per cent in the flat case.

As described in subsection 3.3, we can now build the distribution function one would expect to recover if $\Omega_0$ took a certain value in the Universe and $N$ (> 6.2 keV, 0.32) was measured a large number of times across the sky under the same type of sampling that led to the dataset in Henry ([99]). Its mean is the theoretically-expected overall value for $N$ (> 6.2 keV, 0.32) given in expression [21] for the $\Omega_0$ under consideration, and the shape of the distribution is that obtained through the bootstrap procedure described in subsections 3.2 and 3.3. The exclusion level on the assumed $\Omega_0$ is then given by one minus the probability of measuring a value for $N$ (> 6.2 keV, 0.32) as high (or as low) as that implied by the dataset in Henry ([99]), calculated in expression [21], given the expected distribution.

However, due to the uncertainties in the estimation of the theoretically-expected overall value for $N$ (> 6.2 keV, 0.32), the actual calculation of the exclusion level for each $\Omega_0$ is not as simple. So that we can obtain it, we need to integrate over all possible values for the theoretically-expected overall value $N$ (> 6.2 keV, 0.32), which we will denote $u$. The overall exclusion is the product of the probability, $P(u, \Omega_0)$, of each $u$ being the correct overall value one would expect for $N$ (> 6.2 keV, 0.32) in the Universe (given the assumed $\Omega_0$), and the exclusion level $\text{Ex}(u)$ calculated as described above for each assumed $u$, i.e.

Exclusion probability of $\Omega_0 = \int_{-\infty}^{+\infty} P(u, \Omega_0) \text{Ex}(u) du$ \hspace{1cm} (22)

As mentioned above, the $P(u, \Omega_0)$ are lognormal distributions with mean given by expression [21]. The dispersion can be calculated from the 95 per cent confidence limits.

In Figure 2 we show the exclusion levels for $\Omega_0$ obtained in this way. Even for the values of $\Omega_0$ for which it is easiest to reproduce the observations, from 0.7 to about 0.8, the exclusion level is quite high, around 70 per cent. The reason lies with the large uncertainty in the theoretically-expected overall value for $N$ (> 6.2 keV, 0.32). Because of it, most theoretically-expected overall values end up far away from the value for $N$ (> 6.2 keV, 0.32) which is expected in the Universe given the dataset in Henry ([99]). A large uncertainty in the theoretical prediction is clearly no basis to discard models. However, for the high and low $\Omega_0$ we are aiming to constrain, this effect becomes much less important; the high exclusion levels are caused by most of the distribution for the theoretically-expected overall values for $N$ (> 6.2 keV, 0.32) being higher (for low $\Omega_0$), or lower (for high $\Omega_0$), than the observations. Note that the exclusion levels are absolute, and not relative as one would obtain from the calculation of a likelihood function.

In the calculation of the theoretically-expected overall value for $N$ (> 6.2 keV, $z = 0.32$), the parameter $f$ represents the fraction of the cluster mass at the redshift of cluster observation, $z_{\text{clus}}$, that had been assembled by the time the cluster virialized, $z_c$. We considered this parameter to be equal to 0.75, though we allowed for the possibility that it could be as low as 0.60 or as high as 0.90, corresponding to something like a 95 per cent confidence interval. We modelled this uncertainty by assuming in the Monte Carlo simulations that $f$ was Gaussian distributed with mean 0.75 and dispersion 0.075.

In order to estimate the effect of changing the assumed value for $f$ in the determination of $N$ (> 6.2 keV, $z = 0.32$), we also performed calculations where we treated $f$ as having no associated uncertainty. We considered the cases where $f$ was equal either to 0.60, 0.75 or 0.90. We found that changing $f$ from 0.75 to 0.60 decreased the estimated value for $N$ (> 6.2 keV, $z = 0.32$) by about 10 per cent, while changing $f$ from 0.75 to 0.90 increased $N$ (> 6.2 keV, $z = 0.32$) by

![Figure 2](image-url)
Galaxy clusters at $0.3 < z < 0.4$ and the value of $\Omega_0$

5 DISCUSSION

From the above analysis, we conclude that at present it is not possible to reliably exclude any interesting value for $\Omega_0$ on the basis of X-ray cluster number density evolution alone, due to the limited statistical significance of the available observational data and to uncertainties in the theoretical modelling of cluster formation and evolution. However, we do find that values of $\Omega_0$ below 0.3 are excluded at least at the 90 per cent confidence level. Values of $\Omega_0$ between 0.7 to 0.8 are those most favoured, though not strongly. These results are basically independent of the presence or not of a cosmological constant.

Our conclusions support those of Colafrancesco, Mazzotta & Vittorio (1997), who tried to estimate the uncertainty involved in the estimation of the cluster X-ray temperature distribution function at different redshifts based on its present-day value. They found this uncertainty, given the still relatively poor quality of the data, to be sufficiently large to preclude the imposition of reliable limits on the value of $\Omega_0$.

Our results disagree with those of Henry (1997) and Eke et al. (1998), as they found the preferred $\Omega_0$ to lie between 0.4 to 0.5, with the $\Omega_0 = 1$ hypothesis strongly excluded. This disagreement is mainly the consequence of our focus on the threshold X-ray temperature of 6.2 keV, while they draw their conclusions based on the analysis of the results obtained for several threshold X-ray temperatures. Further below we will repeat our calculations assuming a threshold X-ray temperature of 4.8 keV, and we will find that when we calculate the joint probability of some value for $\Omega_0$, being excluded on the basis of the results concerning either one or both threshold X-ray temperatures of 6.2 keV and 4.8 keV, the favoured value for $\Omega_0$ decreases to around 0.55. Some of the reasons for our choice of deriving the conclusions solely based on the results obtained for the 6.2 keV threshold were mentioned at the end of subsection 3.1 and others will be detailed below.

Other less important contributions to the difference between our results and those presented by Henry (1997) and Eke et al. (1998) are the different assumed normalization for the virial mass to X-ray temperature relation, and the corrections in the expected values in the Universe for both $N(> 6.2 \text{ keV, } 0.05)$ and $N(> 6.2 \text{ keV, } 0.32)$ due to the uncertainties in the X-ray cluster temperature measurements. Note that changing the mean of the bootstrap distribution obtained for $N(> 6.2 \text{ keV, } 0.32)$ to its theoretically-expected overall value in some $\Omega_0$ universe and then calculating the exclusion level on the estimated value for $N(> 6.2 \text{ keV, } 0.32)$ in the Universe given the dataset in Henry (1997), rather than just using the original bootstrap distribution to in-

Figure 3. The expected redshift evolution of $N(> 6.2 \text{ keV, } z)$ for $\Omega_0 = 1$ and 0.3. The solid lines show the result obtained using the Lacey & Cole method for estimating $z_c$, and the dashed ones the result obtained assuming $z_c = z_{\text{obs}}$. Each curve is normalized to reproduce the observed value for $N(> 6.2 \text{ keV, } 0.05)$. Note that the divergence at high $z$ is caused by this renormalization; the absolute correction is largest at the lowest redshift, where $\Omega(z)$ is smallest.

around 11 per cent. The impact of removing the uncertainty in the value of $f$ from the estimation of the overall uncertainty in $N(> 6.2 \text{ keV, } z = 0.32)$ is therefore negligible.

In all previous uses of the Press-Schechter framework to calculate the evolution of the number density of rich galaxy clusters with redshift (Oukbir & Blanchard 1992; Eke et al. 1996; Eke et al. 1998; Markevitch 1998; Reichart et al. 1998), it has been assumed that the redshift of cluster virialization, $z_c$, coincides with that at which the galaxy cluster is observed, $z_{\text{obs}}$. In Figure 3 we compare the value of $N(> 6.2 \text{ keV, } z)$ obtained using the Lacey & Cole (1993, 1994) prescription for the estimation of $z_c$ with the result of the assumption that $z_c = z_{\text{obs}}$. We always require that the observed value for $N(> 6.2 \text{ keV, } 0.05)$ is recovered.

As expected, the difference in the theoretically-predicted overall value of $N(> 6.2 \text{ keV, } z)$ resulting from the two distinct assumptions regarding $z_c$ becomes larger for $\Omega_0 = 0.3$, reflecting the fact that as $\Omega_0$ goes down galaxy clusters tend to form increasingly at an earlier epoch than that at which they are observed. We find that neglecting the fact that some clusters of galaxies virialize prior to the epoch at which they are observed leads to an underestimation of the predicted degree of evolution in the value of $N(> k_B T, z)$ for $z > z_{\text{norm}}$, where $z_{\text{norm}}$ is the redshift at which $N(> k_B T, z)$ is normalized through observations, e.g. in our case $z_{\text{norm}} = 0.65$. Taking into account the possibility that $z_c$ may be larger than $z_{\text{obs}}$, therefore requires lower values for $\Omega_0$ in order for the high-redshift data on $N(> k_B T, z)$ to be reproduced.

Allowing for $z_c > z_{\text{obs}}$ means that some galaxy clusters that otherwise would not be massive enough to reach a given threshold temperature $k_B T$ can now be counted when calculating $N(> k_B T, z)$. In principle this would have the effect of increasing the expected value of $N(> k_B T, z)$ for any z. However, at the normalization redshift 0.05 the higher value for $N(> k_B T, 0.05)$ means that a less well developed density field at $z = 0.05$ is required, i.e. a lower value of $\sigma_8$ results from introducing the possibility that $z_c > z_{\text{obs}}$. As the number density of virialized objects evolves faster for the same relative change in the value of the dispersion of the density field the smaller this value is, the decrease in the required value for $\sigma_8$ has the effect of enhancing the decrease in the value of $N(> k_B T, z)$ as $z$ gets larger. This effect turns out to be more important than the expected increase in the value of $N(> k_B T, z)$ due to higher cluster X-ray temperatures at fixed cluster mass resulting from the possibility of $z_c > z_{\text{obs}}$. 
pose an exclusion level on the theoretically-expected overall value for \( N > 6.2 \text{ keV}, 0.32 \) in that \( \Omega_0 \) universe, does not seem to make much difference. This is a reflection of the fact that the bootstrap distributions recovered do not have a strongly asymmetric shape.

Our disagreement with Eke et al. (1998) on the level of exclusion of the \( \Omega_0 = 1 \) hypothesis is also due to our much larger assumed uncertainty in the theoretically-expected overall value for \( N > 6.2 \text{ keV}, 0.32 \).

For the \( \Omega_0 = 1 \) hypothesis to be favoured, one requires the lowest possible observed value for \( N > 6.2 \text{ keV}, 0.32 \). This is best achieved if, for the sample of 10 galaxy clusters used in its calculation, the X-ray temperatures turn out to be on average lower than the assumed mean, and the X-ray fluxes higher. A higher ratio between the extended and detect cell fluxes for the EMSS at \( z = 0.32 \) would also help. On the theoretical side, the higher one decides the expected value for \( N > 6.2 \text{ keV}, 0.32 \) is, the more compatible with the data the \( \Omega_0 = 1 \) hypothesis becomes. This can be best achieved if, in decreasing order of importance, the cluster virial mass at fixed X-ray temperature is being underestimated, \( \delta_c \) is lower than the canonical value 1.7 and \( f \), the assembled fraction of a cluster virial mass after which the X-ray temperature does not change significantly, is assumed greater than 0.75. However, the single most important factor in determining the theoretically-expected overall value for \( N > 6.2 \text{ keV}, 0.32 \) is the present-day normalization for the dispersion of the density field, \( \sigma_8 \), which in turn results from the observational value for the present density \( N > 6.2 \text{ keV}, 0.05 \).

Although we worked with all X-ray clusters that make up the dataset in Henry (1997), and even estimated the effect of also considering the 5 clusters with lower X-ray fluxes present in the EMSS in the redshift bin from 0.3 to 0.4, in fact we only used the abundance of clusters with X-ray temperatures in excess of 6.2 keV to constrain \( \Omega_0 \). We mentioned some of the reasons for this choice in Section 3. Nevertheless, we decided to repeat the same calculations for a threshold X-ray temperature of 4.8 keV. This value also well represents the mean curve going through the observed cumulative X-ray temperature distribution function at both \( z = 0.05 \) and \( z = 0.32 \).

The results regarding the best-fit value for \( \Omega_0 \), presented in Figure 4, are somewhat different from those we obtained when the threshold X-ray temperature was assumed to be 6.2 keV. This is particularly true if the correction for the possibility of any of the 5 clusters with the lowest X-ray fluxes in the 0.3 < \( z < 0.4 \) EMSS sub-sample having X-ray temperatures in excess of 4.8 keV is included, as can be seen in Figure 5 for the open case. While the standard analysis without these 5 X-ray clusters prefers a value for \( \Omega_0 \) between 0.4 to 0.5, when the correction for the scatter in the relation between the cluster X-ray temperature and luminosity is included, in the way described in subsection 3.3, the preferred value for \( \Omega_0 \) decreases to about 0.3. Now the \( \Omega_0 = 1 \) hypothesis is excluded at more than the 95 per cent confidence level, with or without the correction. At the 90 per cent confidence level, one finds that \( \Omega_0 > 0.8 \) is excluded without the correction, being this limit lowered to 0.7 when the correction is included.

One can also estimate the joint probability of some \( \Omega_0 \) value being excluded on the basis of the results relative to either one or both X-ray temperature thresholds. Assuming the data used in the calculations for the two thresholds is independent, the results then imply that the favoured value for \( \Omega_0 \) is close to 0.55 (0.50 if the incompleteness correction is included) and the \( \Omega_0 = 1 \) hypothesis is excluded at the 99 per cent level. This agrees very well with the results of Henry (1997) and Eke et al. (1998), leading us to believe that the main difference between our analysis and theirs is our decision to draw our conclusions solely based on the exclusion levels obtained for the X-ray temperature threshold of 6.2 keV.

A further potential problem one must consider when working with clusters whose observed X-ray temperature is as low as 4.8 keV is the possibility that the energy in the intracluster gas has increased as a result of (pre-)heating by supernovae and starbursts in the cluster galaxies. In fact this is the leading hypothesis (e.g. Navarro, Frenk & White 1995; Markevitch 1998) put forward to explain the discrepancy between the observed slope of the X-ray temperature–luminosity relation, close to 0.3, and the expected value of 0.5 if clusters evolve in a self-similar way (Kaiser 1986).

Following Eke et al. (1998), we assume that in a cluster whose observed X-ray temperature is 4.8 keV, 17 per cent of its energy, that is 0.8 keV per intracluster gas particle, was due to (pre-)heating produced by processes occurring inside the cluster galaxies. This is approximately the amount of energy that gets injected into the intracluster gas particles in the simulation of Metzler & Evrard (1994), where a galaxy cluster’s X-ray temperature, which would otherwise be 5.6 keV, increased to 6.4 keV. Note however that in the scheme proposed by Eke et al. (1998) a cluster this large would not be (pre-)heated to the extent simulated by Metzler & Evrard (1994), as in their proposal Eke et al. (1998) assume that the energy gained by each intracluster gas particle due to (pre-)heating decreases as a galaxy cluster becomes larger, being close to zero for galaxy clusters with X-ray temperatures exceeding 6.2 keV.

The above assumption means that the observed values for \( N > 4.8 \text{ keV}, z \), when \( z = 0.05 \) and \( z = 0.32 \), should now be compared with the theoretically-expected values for \( N > 4.0 \text{ keV}, z \) at those redshifts. The resulting exclusion levels on the value of \( \Omega_0 \) can be seen in Figure 5 for the open case. There is little difference compared to the results.
Figure 5. The absolute exclusion levels for different values of Ω0 only for the open case, when the threshold X-ray temperature of 4.8 keV is used. The full curve includes a correction (FC) for the possibility of the 5 clusters with lowest X-ray temperatures in excess of 4.8 keV. The dashed curve includes a correction (HC) for the possibility of (pre-)heating of the intracluster medium due to processes within the cluster galaxies. The dotted curve includes both corrections.

in Figure 4 that follow from the standard no-heating calculation. The lower value for σc, required to match theory and observations at z = 0.05, more than compensates for the expected increase in the number of galaxy clusters with X-ray temperatures in excess of 4.8 keV at z = 0.05, in effect bringing this number down. In fact, the standard no-heating calculation for a threshold X-ray temperature of 4.8 keV requires a value for σc(Ω0), so that the observed value for N(>4.8 keV, 0.05) is reproduced, that is less than 3 per cent below that required by the > 6.2 keV data, quoted in equation (1). On the other hand, including the (pre-)heating correction, the required σc(Ω0) value drops to 19 per cent below that preferred by the > 6.2 keV data. Though the coincidence between the σc values obtained for the two X-ray temperature thresholds 4.8 keV and 6.2 keV under the no-heating assumption may be incidental, it could indicate that (pre-)heating was relatively unimportant at least for the galaxy clusters observed at z = 0.05 with X-ray temperatures exceeding 4 keV. If (pre-)heating was more important in the past than today, then the required σc(Ω0) value would be that obtained through the standard no-heating hypothesis, but the comparison at z = 0.32 would include the (pre-)heating correction. This would push the theoretically-expected value for N(>4.8 keV, 0.32) up, favouring higher values for Ω0. This is not as far-fetched as it may seem, given that it is well known that the star-formation rate peaks before z = 1 (e.g. Madau, Ferguson & Dickinson 1998; Baugh et al. 1998), and consequently so does the rate of supernovae Type II (the rate of supernovae Type Ia peaks a few Gyr later) and the probability of starbursts.

The results for the 4.8 keV threshold X-ray temperature are close to those found by Eke et al. (1998), leading us to believe that their exclusion levels for Ω0 are dominated by the information associated with the threshold X-ray temperatures 4.0 keV and 5.0 keV. In our view, the analysis for these X-ray temperature thresholds carries with it a sufficient number of uncertainties, due to the problems men-
tioned above, so as to render the constraints imposed on Ω0 not very trustworthy. Only the data regarding clusters with X-ray temperatures in excess of about 6 keV seems sufficiently free of modelling problems so as to be potentially useful in constraining Ω0.

Another possible complication has arisen from recent work by Blanchard, Bartlett and Sadat (1998), who use a sample of 50 galaxy clusters with mean redshift of 0.05, which were identified through the ROSAT satellite, to estimate the cumulative X-ray temperature distribution function at z = 0.05. They claim the number density of galaxy clusters at z = 0.05 with X-ray temperatures exceeding 4 keV is being underestimated when the Henry & Arnaud cluster sample is used. Through the X-ray cumulative temperature distribution function at z = 0.05 they obtain, they then estimate Ω0 using the EMSS cluster abundance in the redshift bin 0.3 < z < 0.4 and the X-ray temperature data gathered in Henry (1997). They find the favoured value for Ω0 to be 0.75, while 0.3 is excluded at more than the 95 per cent level. These results coincide very well with ours when only the 6.2 keV threshold X-ray temperature is considered, thus perhaps implying that the discrepancy between the favoured value for Ω0 found when different X-ray temperature thresholds are considered may arise from a underestimation of the cumulative distribution function at z = 0.05 for X-ray temperatures below about 6 keV.

Unfortunately, due to uncertainties associated both with the observational measurements and the theoretical modelling of cluster evolution, the presently-available data on galaxy clusters with X-ray temperatures exceeding about 6 keV is not able to strongly discriminate between cosmologies with different values for Ω0. And in any case, the data available is probably not yet statistically significant. More is needed to support or disclaim the preliminary conclusions that can be obtained from it. In particular there are some oddities with the sub-sample of EMSS galaxy clusters observed by Henry, such as the strange redshift distribution, strongly clustered around 0.32, and the unexpectedly low X-ray temperature of MS2137.3, that makes one have some doubts about how representative this dataset is of the Universe.

Within the next few years, with the launch of the XMM satellite, possibly in late 1999, a significant increase in the quantity and quality of the available data is expected to occur (Romer 1998). It should then be possible to place stronger constraints on Ω0 on the basis of the evolution of the galaxy cluster X-ray temperature function. This would be helped by improvements in the theoretical modelling of cluster evolution, perhaps based on the high-resolution hydrodynamical N-body simulations on cosmological scales expected in the near future.

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REFERENCES

Allen S. W., Fabian A. C., 1998, MNRAS, 297, L57
Baugh C.M., Cole S., Frenk C.S., Lacey C.G., 1998, ApJ, 498, 504
Bernardeau F., 1994, ApJ, 427, 51
Blanchard A., Bartlett J. G., 1997, A&A, 332, L49
Blanchard A., Bartlett J. G., Sadat R., 1998, to appear in Les Comptes Rendus de l’Academie des Sciences
Bond J. R., Cole S., Efstathiou G., Kaiser N., 1991, ApJ, 379, 440
Bryan G. L., Norman M. L., 1998, ApJ, 495, 80
Colafrancesco S., Mazzotta P., Vittorio N., 1997, ApJ, 488, 566
David L. P., Slyz A., Jones C., Forman W., Vrtilek S. D., Arnaud K. A., 1993, ApJ, 412, 479
Edge A. C., Stewart G. C., Crawford C. S., Mushotzky R. F., 1994, MNRAS, 267, 779
Fan X., Bahcall N. A., Cen R., 1997, ApJ 490, L123
Frenk C. S., White S. D. M., Efstathiou G., David M., 1990, ApJ, 351, 10
Gioia I. M., Luppino G. A., 1994, ApJS, 94, 583
Gioia I. M., Henry J. P., Maccacaro T., Morris S. L., Stocke J. T., Wolter A., 1990, ApJ, 356, L35
Gross M. A. K., Somerville R. S., Primack J. R., Borgani S, Girardi M., 1997, Santa Cruz preprint astro-ph/9711037
Hanami H., 1993, ApJ, 415, 42
Hattori M., Matsuzawa H., 1995, A&A 300, 637
Henry J. P., 1997, ApJ, 489, L1
Henry J. P., Arnaud K. A., 1991, ApJ, 372, 410
Henry J. P., Gioia I. M., Maccacaro T., Stocke J. T., Wolter A., 1992, ApJ, 386, 408
Kaiser N., 1986, MNRAS, 222, 232
Kitayama T., Suto Y., 1997, ApJ, 490, 557
Lacey C., Cole S., 1993, MNRAS, 262, 627
Lacey C., Cole S., 1994, MNRAS, 271, 676
Lilje P. B., 1992, ApJ, 386, L33
Madau P., Ferguson H.C., Dickinson M., 1998, ApJ, 498, 106
Markevitch M., 1998, CFA preprint astro-ph/9802059
Metzler C. A., Evrard A. E., 1994, ApJ, 437, 564
Monaco P., 1995, ApJ, 447, 23
Mushotzky R. F., Scharf C. A., 1997, ApJ, 482, L13
Navarro J. F., Frenk C. S., White S. D. M., 1995, MNRAS, 275, 720
Nichol R. C., Holden B. P., Romer A. K., Ulmer M. P., Burke D. J., Collins C. A., 1997, ApJ, 481, 444
Norman M. L., Bryan G. L., 1998, to appear in the proceedings of the Ringberg Workshop on M87, eds. Meisenheimer K., Roerer H-J., Springer Verlag, astro-ph/9802535
Oukbir J., Bartlett J. G., Blanchard A., 1997, A&A, 320, 365
Oukbir J., Blanchard A., 1992, A&A, 262, L21
Oukbir J., Blanchard A., 1997, A&A, 317, 1
Peacock J. A., 1997, MNRAS, 284, 85
Peacock J. A., Dodds S. J., 1994, MNRAS, 267, 1020
Press W. H., Schechter P., 1974, ApJ, 187, 452
Press W. H., Teukolsky S. A., Vetterling W. T., Flannery B. P., 1992, Numerical Recipes, 2nd edition, Cambridge University Press

Reichart D. E., Nichol R. C., Castander F. J., Burke D. J., Romer A. K., Holden B. P., Collins C. A., Ulmer M. P., 1998, Chicago preprint astro-ph/9802153
Romer A. M., 1998, to appear in the proceedings of the 14th IAP meeting, “Wide Field Surveys in Cosmology”, eds. Mellier Y., Colombi S., astro-ph/9809198
Sadat R., Blanchard A., Oukbir J., 1998, A&A, 329, 21
Tormen G., 1998, MNRAS, 297, 648
Viana P. T. P., Liddle A. R., 1996, MNRAS, 281, 323 [VL]
White S. D. M., Efstathiou G., Frenk C. S., 1993a, MNRAS, 262, 1023
White S. D. M., Navarro J. F., Evrard A. E., Frenk C. S., 1993b, Nat, 366, 429

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