Isospin-violating strong decays of scalar single-heavy tetraquarks

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Abstract

We present a study of the isospin-violating one-pion strong decays of single heavy tetraquarks \(X_Q = X(sq\bar{q}\bar{Q}), Q = c, b,\) and \(q = u, d\) with spin-parity \(J^P = 0^+\). We assume that the tetraquarks have the configuration of a color diquark and an antidiquark. Three mechanisms of isospin violation can contribute to the decay rate: (1) mixing of the \(X(su\bar{u}\bar{Q})\) and \(X(sd\bar{d}\bar{Q})\) tetraquark currents, (2) an explicit \(m_d - m_u\) quark mass difference in the quark diagrams describing the corresponding decay transitions, and (3) \(\pi^0 - \eta\) mixing in the final state. Our main results are as follows: (1) It is quite likely that the investigated tetraquark states are isosinglet states with a small admixture of an isotriplet component; (2) The first isospin-breaking mechanism affects the decay rather more significantly than the others; (3) Our calculations contain a size parameter \(\Lambda_{X_Q}\), characterizing the distribution of the quarks in the tetraquark state \(X_Q\). Absolute decay rates depend very much on the choice of \(\Lambda_{X_Q}\), varied from 1 to 2 GeV, reflecting the compactness of the multiquark system.

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I. INTRODUCTION

The past years have been marked by extensive experimental and theoretical studies of the $X$, $Y$, and $Z$ states or of heavy mesons containing at least one heavy $c$ or $b$ quark. Many of these reported resonances cannot easily be explained as quark-antiquark configurations; alternative structure interpretations involve, for example, hadron molecules or compact tetraquark states among others.

In the present study we focus on the scalar resonance $D_{s0}^*(2317)$ and its possible partner state $B_{s0}^*$. We use a tetraquark picture to analyze the isospin-violating one-pion strong decays of the $D_{s0}^*(2317)$ and its bottom companion $B_{s0}^*$. Assuming tetraquark configurations these states contain a single heavy quark ($c$ or $b$), a strange quark and a pair of nonstrange quarks.

The $D_{s0}^*(2317)$ and $B_{s0}^*$ states have been studied before in detail using different theoretical approaches, including the hadronic molecular approach (see overview and references in [1]-[4]), where these states are considered to be bound states of the $D$ and $K$, $B$ and $K$, respectively. In most of the approaches, the $D_{s0}^*(2317)$ and $B_{s0}^*$ states have been considered to be isosinglet states, which is consistent with recent results by the Belle Collaboration [5]. The Belle Collaboration reported that possible isotriplet partners of the $D_{s0}^*(2317)$ have not been found. In Ref. [1] it was proposed that in the framework of the hadronic molecular picture there are two mechanisms for the pion emission in the reaction $D_{s0}^* \to D_s + \pi^0$: (1) a direct mechanism due to the emission from the $(DK)$ loop; (2) the $\eta - \pi^0$ mixing transition. It was shown that the direct transition dominates over the $\eta - \pi^0$ mixing transition in the $D_{s0}^* \to D_s \pi^0$ decay. In Ref. [3] the formalism proposed in Ref. [1] has been extended to the case of the $B_{s0}^*$ (bound state of $B$ and $K$ mesons) — bottom partner of the $D_{s0}^*$ state. In calculations performed in Refs. [1,3] the distribution of hadronic constituents in $D_{s0}^*$ and $B_{s0}^*$ was described by a scale parameter $\Lambda$, which was found to be of order of 1 GeV. In particular, in Refs. [1,3] it was calculated that $\Gamma(D_{s0}^* \to D_s + \pi^0) = 46.7 - 111.9$ keV and $\Gamma(B_{s0}^* \to B_s + \pi^0) = 55.2 - 89.9$ keV when the scale parameter describing the distribution of constituents in the hadronic molecule was varied from 1 to 2 GeV.

In the present manuscript we return to the problem of the isospin-violating decays $D_{s0}^* \to D_s \pi^0$ and $B_{s0}^* \to B_s \pi^0$. A new feature in our study is that we apply the covariant confined multiquark approach proposed and developed in Refs. [6]. This method is an extension of the covariant relativistic quark model [7] devised for a unified description of bound state
structures of hadrons and exotic states. As in the hadronic molecular approach one has a free scale parameter $\Lambda$, which is fixed from the description of the decay rates $\Gamma(D_{s0}^* \to D_s + \pi^0)$ and $\Gamma(B_{s0}^* \to B_s + \pi^0)$. We find that our $\Lambda \sim 1$ GeV is compatible with the scale parameter found in the framework of the hadronic molecular picture [1]-[3].

II. FORMALISM

Our starting point is that $D_{s0}^*$ and $B_{s0}^*$ states are bound states of four quarks having the configuration of a color diquark and an antidiquark. In our phenomenological Lagrangian formalism such a configuration is encoded in the tetraquark interpolating currents discussed in detail in the literature (see e.g. recent review [8]) and, in particular, in the context of the covariant tetraquark confinement model [6].

For our specific cases of the $D_{s0}^{*+}$ and $B_{s0}^{*0}$ tetraquark states we construct the currents in the form of the mixed isosinglet $J^S$ and isotriplet $J^T$ (third component) currents

$$J_{D_{s0}^{*+}} = \cos \delta J^S_{D_{s0}^{*+}} + \sin \delta J^T_{D_{s0}^{*+}},$$
$$J_{B_{s0}^{*0}} = \cos \delta J^S_{B_{s0}^{*0}} + \sin \delta J^T_{B_{s0}^{*0}}. \quad (1)$$

The singlet-octet mixing angle $\delta$ parametrizes the isospin violation in the structure of the $D_{s0}^*$ or $B_{s0}^*$ states. We keep $\delta$ as a free parameter. The isosinglet $J^S$ and isotriplet $J^T$ currents are defined as

$$J^S/T_{D_{s0}^{*+}} = \frac{1}{\sqrt{2}} \left[ J(cu\bar{u}s) \pm J(cd\bar{d}s) \right],$$  \hspace{1cm} (2)
$$J^S/T_{B_{s0}^{*0}} = \frac{1}{\sqrt{2}} \left[ J(su\bar{u}b) \pm J(sd\bar{d}b) \right]. \quad (3)$$

The color structure of a generic tetraquark current $J(q_1q_2\bar{q}_3\bar{q}_4)$ has the configuration of a color diquark-antidiquark with

$$J(q_1q_2\bar{q}_3\bar{q}_4) = D_{12}^c \cdot D_{34}^\dagger = \varepsilon^{abc} \varepsilon^{dec} \left[ q_1^a C \Gamma_1 q_2^b \right] \left[ \bar{q}_3^d \Gamma_2 C \bar{q}_4^e \right]. \quad (4)$$

where

$$D_{12}^c = \varepsilon^{abc} \left[ q_1^a C \Gamma_1 q_2^b \right]. \quad (5)$$
The indices \(a, b, c, d, e\) refer to color, \(C = \gamma^0\gamma^2\) is the charge conjugation matrix, and \(\Gamma_1\) and \(\Gamma_2\) are the Dirac spin matrices resulting in zero total angular momentum and positive \(P\) parity for the interpolating tetraquark current. In particular, the following combinations of the \((\Gamma_1, \Gamma_2)\) matrices (without involving derivatives) are possible:

\[
\begin{align*}
P &= \Gamma_1 \otimes \Gamma_2 = \gamma^5 \otimes \gamma^5, \\
S &= \Gamma_1 \otimes \Gamma_2 = I \otimes I, \\
A &= \Gamma_1 \otimes \Gamma_2 = \gamma^5\gamma^\mu \otimes \gamma^\mu \gamma^5, \\
V &= \Gamma_1 \otimes \Gamma_2 = \gamma^\mu \otimes \gamma^\mu, \\
T &= \Gamma_1 \otimes \Gamma_2 = \frac{1}{2}\sigma^{\mu\nu}\gamma_5 \otimes \sigma_{\mu\nu}\gamma_5.
\end{align*}
\]

When we take the heavy quark limit for the \(b\) constituent, which is equivalent to the non-relativistic limit, the \(S\) current vanishes, the \(P\) and \(A\) currents are degenerate resulting in the spin structure \(\sigma^2 \otimes \sigma^2\), and the \(V\) and \(T\) currents are also degenerate producing the spin structure \(\sigma^2\sigma^i \otimes \sigma^i\sigma^2\). In this paper, for simplicity, we work with the simplest \(P\) current. From our experience based on analysis of single heavy baryons (see Ref. [9]) the observables are not so sensitive to a choice of the interpolating current. Therefore, we do not expect that the use of \(V(T)\) or mixing of two possible currents \(P(A)\) and \(V(T)\) for scalar tetraquarks with an extra free parameter should drastically change the description of physical properties of these exotic states. However, such an analysis could be done in our future study. After having specified the color, spin and flavor structure of our tetraquark currents we are in the position to implement the coordinate (or space-time) part and construct phenomenological Lagrangians describing the interaction of the tetraquark states \(H = D_{s0}^+, B_{s0}^0\) with their constituents. We proceed in complete analogy to the original work on the \(X(3872)\) treated as a tetraquark state (see details in Refs. [6]). The interaction Lagrangian of the tetraquark states \(H = D_{s0}^+, B_{s0}^0\) with their constituents is constructed as

\[
\mathcal{L}_H(x) = g_H H(x) J_H(x) + \text{H.c.}
\]

\(J_H(x)\) is the nonlocal confined tetraquark current including the appropriate spin, flavor and color structure discussed before. For example, a generic current \(J_H\) for \(H \equiv X(sq\bar{q}\bar{Q})\) corresponding to the coupling of the color diquark \(D_c^1 = \varepsilon^{abc} q^a C\gamma_5 s^b\) and the antidiquark
\[ D_2^\dagger = \varepsilon^{dec} \left[ q^d \gamma^5 C \bar{Q}^e \right] \] has the form

\[ J_{X(sqq\bar{Q})}(x) = \int d^4 x_1 \cdots \int d^4 x_4 \delta \left( x - \sum_{i=1}^4 w_i x_i \right) \]
\[ \times \Phi \left( \sum_{i<j} (x_i - x_j)^2 \right) \varepsilon^{abc} \varepsilon^{dec} \left[ q_a(x_1) C\gamma^5 s_b(x_2) \right] \left[ \bar{q}_d(x_3) \gamma^5 C \bar{Q}_e(x_4) \right] \]

(8)

where \( \Phi \) is the correlation function of the \( X(sqq\bar{Q}) \) state providing for the ultraviolet finiteness of all matrix elements.

The coupling constant \( g_H \) is determined through the compositeness condition \( Z_H = 1 - g_H^2 \Pi'(M_H^2) = 0 \), where \( \Pi' \) is the derivative of the tetraquark mass operator (the relevant diagram for the mass operator is displayed in Fig.1). The compositeness condition sets the wave function renormalization constant to zero, which means that the tetraquark state is a dressed bound state of four valence quarks.

![FIG. 1: Mass operator of the tetraquark \( X(sqq\bar{Q}) \)](image)

![FIG. 2: Strong decay of the tetraquark \( X(sqq\bar{Q}) \) into two mesons \( M(s\bar{Q}) \) and \( M(q\bar{q}) \)](image)

The explicit expression for the derivative of the mass operator \( \Pi'(p^2) \) corresponding to Fig. 1 reads

\[ \Pi'(p^2) = \frac{1}{2p^2} p^\alpha \frac{\partial}{\partial p^\alpha} \Pi(p^2) = \frac{4N_c}{2p^2} F(m_Q, m_s, m_q) \]

(9)

where \( 4N_c = 12 \) is the appropriate color factor and the structure integral \( F(m_Q, m_s, m_q) \) is
given by

\[
F = \prod_{i=1}^{3} \int \frac{d^4k_i}{(2\pi)^4} \Phi^2(-K^2) \\
\times \left[ -w_s tr[S_{s}^{[12]} \gamma^5 S_{q}^{[2]} \gamma^5] tr[\gamma^5 S_{q}^{[13]} \gamma^5] \\
- w_Q tr[S_{s}^{[13]} \gamma^5 S_{q}^{[2]} \gamma^5] tr[\gamma^5 S_{q}^{[13]} \gamma^5] \\
+ w_q tr[S_{s}^{[12]} \gamma^5 S_{q}^{[2]} \gamma^5] tr[\gamma^5 S_{q}^{[13]} \gamma^5] \\
+ w_q tr[S_{s}^{[12]} \gamma^5 S_{q}^{[2]} \gamma^5] tr[\gamma^5 S_{q}^{[13]} \gamma^5] \right].
\]

(10)

The free constituent quark propagators \(1/(m_q - \vec{k})\) are denoted by \(S_{q}^{[-]}\) with the specific momentum dependence

\[
S_{s}^{[12]} = S_s(k_1 + k_2 - w_sp), \quad S_{s}^{[3]} = S_s(k_3 - w_Qp),
\]
\[
S_{q}^{[2]} = S_q(k_2 + w_u p), \quad S_{q}^{[13]} = S_q(k_1 + k_3 + w_d p),
\]

(11)

where \(w_i = m_i/(2m_q + m_s + m_Q)\) is the fractional quark mass. The distribution of the constituent quarks in the bound state is modeled by the correlation function \(\Phi(-K^2) = \exp(K^2/\Lambda^2)\) with the momentum dependence

\[
K^2 = \frac{1}{8}(k_1 + 2k_2)^2 + \frac{1}{8}(k_1 + 2k_3)^2 + \frac{1}{4}k_1^2
\]

(12)

where \(\Lambda\) is a free parameter.

We are now in the position to discuss the calculation of the isospin-violating one-pion strong decay of the tetraquark states summarized by the Feynman diagram of Fig. 2. As partially emphasized in the Introduction and at the beginning of this section there are several mechanisms that contribute to the amplitude of this isospin-violating decay process. The first mechanism (I) refers to the isospin violation in the structure of the \(D_{s0}^*\) and \(B_{s0}^*\) states. It is already parametrized by the singlet-octet mixing angle present in the tetraquark currents of Eq. (11). The second mechanism (II) includes isospin breaking corrections based on the \(u\) and \(d\) quark mass difference. We work with the \(u\) and \(d\) quark masses,

\[
\tilde{m}_u = m - \frac{\Delta}{2}, \quad \tilde{m}_d = m + \frac{\Delta}{2},
\]

(13)

where \(m = m_u = m_d\) is their value in the isospin limit and \(\Delta = \tilde{m}_d - \tilde{m}_u\) is the \(d - u\) quark mass difference. For an estimate of \(\Delta\) we take a value given by the difference \(M_d - M_u\) of
the $d$ and $u$ current quark masses as listed by the PDG \[4\],

$$\Delta = \tilde{m}_d - \tilde{m}_u = M_d - M_u = 2.5 \text{ MeV},$$

(14)

To implement the third mechanism (III) we take into account the $\pi^0 - \eta$ mixing in the interaction Lagrangians of these mesons with their constituent quarks. As shown in Ref. \[10\] the $\pi^0$ and $\eta$ meson fields are modified by a unitary transformation given by

$$\pi^0 \rightarrow \pi^0 \cos \varepsilon - \eta \sin \varepsilon, \quad \eta \rightarrow \pi^0 \sin \varepsilon + \eta \cos \varepsilon$$

(15)

where $\varepsilon$ is the $\pi^0 - \eta$ mixing angle fixed by \[10\]:

$$\tan 2\varepsilon = \frac{\sqrt{3}}{2} \frac{M_d - M_u}{M_s - \frac{M_u + M_d}{2}} \approx 0.024$$

(16)

and where $M_q$ are the current quark masses. For the above estimate of $\tan 2\varepsilon$ we use the central values for the current quark masses from the PDG \[4\]:

$$M_d - M_u = 2.5 \text{ MeV}, \quad M_u + M_d = 7.6 \text{ MeV}, \quad M_s = 95 \text{ MeV}.$$ 

(17)

As a result of the unitary transformation (15) the neutral pion couples to both isosinglet $\bar{u}u + \bar{d}d$ and isotriplet $\bar{u}u - \bar{d}d$ currents

$$\mathcal{L}_\pi(x) = \frac{1}{\sqrt{2}} \pi^0(x) \int d^4y \Phi_\pi(y^2) \left[ g_\pi \cos \varepsilon J_{\pi^-}(x, y) + \frac{g_\eta}{\sqrt{3}} \sin \varepsilon J_{\pi^0}^+(x, y) \right]$$

(18)

where

$$J_{\pi^0}^\pm(x, y) = \bar{u}(x + y/2)i\gamma^5u(x - y/2)$$

$$\pm \bar{d}(x + y/2)i\gamma^5d(x - y/2).$$

(19)

Combining all three isospin breaking contributions we obtain the following expression for the amplitude of strong $D_s^* \rightarrow D^\pm_s + \pi^0$ and $B_s^0(\bar{B}_s^0) \rightarrow B_s^0(\bar{B}_s^0) + \pi^0$ decays:

$$\mathcal{B} = \frac{B_{u\bar{u}}}{2} \left( 1 + \sin \delta + \frac{g_\eta}{g_\pi \sqrt{3}} \sin \varepsilon \right)$$

$$+ \frac{B_{d\bar{d}}}{2} \left( -1 + \sin \delta + \frac{g_\eta}{g_\pi \sqrt{3}} \sin \varepsilon \right)$$

$$= \frac{B_{u\bar{u}} - B_{d\bar{d}}}{2} + \left( \sin \delta + \sin \varepsilon \frac{g_\eta}{g_\pi \sqrt{3}} \right) \frac{B_{u\bar{u}} + B_{d\bar{d}}}{2}.$$ 

(20)
The quantity $B_{q\bar{q}}$ is the amplitude of the one-pion strong isospin-violating decay of the tetraquark with two specific light nonstrange quarks $u\bar{u}$ or $d\bar{d}$,

$$B_{q\bar{q}} = 6g_X s_{q\bar{q}} g_{M(s\bar{Q})} g_{\pi} G(m_Q, m_s, m_q, m_{\bar{q}}).$$  \hspace{1cm} (21)

The last equation corresponds to the amplitude of the transition $X(sq\bar{q}\bar{Q}) \rightarrow M(s\bar{Q}) + M(q\bar{q})$ and is defined as the product of a color factor $N_c = 6$, constants describing the respective couplings of the tetraquark $X(sq\bar{q}\bar{Q})$, meson $M(s\bar{Q})$, and pion $g_{\pi}$ with the constituent quarks and the mass-dependent structure integral $G(m_Q, m_s, m_{\bar{q}})$. The latter is given by

$$G = \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \Phi(-L^2) \Phi_1(-L_1^2) \Phi_2(-L_2^2) \times \text{tr}[\gamma^5 S_u(k_1)\gamma^5 S_Q(k_1 + q_1)\gamma^5 S_u(k_2)\gamma^5 S_d(k_2 + q_2)],$$

where

$$L^2 = \frac{1}{8} [2k_1 + q_1(1 + w_Q - w_s) + q_2(w_Q - w_s)]^2 + \frac{1}{8} [2k_2 + q_2]^2 + \frac{1}{4} [2q_1 w_Q - q_2(w_Q - w_s)]^2,$$

$$L_1^2 = [k_1 + \tilde{w}_Q]^2, \quad L_2^2 = [k_2 + 1/2]^2$$  \hspace{1cm} (22)

and $\tilde{w}_Q = m_Q/(m_Q + m_s)$. Here $\Phi_1(-L_1^2) = \exp(L_1^2/\Lambda_{M_1}^2)$ and $\Phi_2(-L_2^2) = \exp(L_2^2/\Lambda_{M_2}^2)$ are the correlation functions of the mesons $M(s\bar{Q})$ and $M(q\bar{q})$, respectively.

The strong decay width of the scalar tetraquark state $H$ into two pseudoscalar mesons $M_1$ and $M_2$ is evaluated according to the expression

$$\Gamma(H \rightarrow M_1 + M_2) = \frac{\lambda^{1/2}(M_H^2, M_1^2, M_2^2)}{16\pi M_X^3} |B|^2.$$  \hspace{1cm} (23)

Most of the model parameters have been fixed in previous calculations: the constituent quark masses $m_u = m_d = 241$ MeV, $m_s = 428$ MeV, $m_c = 1.672$ GeV, $m_b = 5.046$ GeV, the scale parameters of the interaction vertex with $\Lambda = 0.871$ GeV, $\Lambda_\eta = 1$ GeV, $\Lambda_{D_s} = 1.81$ GeV, $\Lambda_{B_s} = 2.05$ GeV, and the infrared confinement scale parameter $\lambda = 0.181$ GeV. The scale parameters of the tetraquark states are free parameters. The coupling constants of the pion $g_{\pi}$, $\eta$ meson $g_{\eta}$, and $D_s$ and $B_s$ mesons $g_{D_s}$ and $g_{B_s}$ have also been evaluated in previous calculations (see, e.g., Ref. [6]): $g_{\pi} = 5.18, g_{\eta} = 4.15, g_{D_s} = 3.76, g_{B_s} = 4.97$.

Finally, we remind reader that three scenarios are specified by the following choice of isospin-breaking parameters [see definitions in Eqs. (1), (4), (15), and (16)] and strong amplitude of one-pion transition $B$: 
Scenario I

\[
\sin \delta = 0.012, \quad \Delta \equiv 0, \quad \sin \varepsilon \equiv 0,
\]

\[
B = \sin \delta \frac{B_{u\bar{u}} + B_{d\bar{d}}}{2}, \quad (24)
\]

Scenario II

\[
\sin \delta \equiv 0, \quad \Delta = 2.5 \text{ MeV}, \quad \sin \varepsilon \equiv 0,
\]

\[
B = \frac{B_{u\bar{u}} - B_{d\bar{d}}}{2}, \quad (25)
\]

Scenario III

\[
\sin \delta \equiv 0, \quad \Delta \equiv 0, \quad \sin \varepsilon = 0.012,
\]

\[
B = \sin \varepsilon \frac{g_{\eta}}{g_{\pi}\sqrt{3}} \left( \frac{B_{u\bar{u}} + B_{d\bar{d}}}{2} \right), \quad (26)
\]

Full result including all mechanisms of isospin breaking I+II+III

\[
\sin \delta = \sin \varepsilon = 0.012, \quad \Delta = 2.5 \text{ MeV},
\]

\[
B = \frac{B_{u\bar{u}} - B_{d\bar{d}}}{2} + \left( \sin \delta + \sin \varepsilon \frac{g_{\eta}}{g_{\pi}\sqrt{3}} \right) \frac{B_{u\bar{u}} + B_{d\bar{d}}}{2}. \quad (27)
\]

One can see that in the numerical analysis we use approximation \( \sin \delta \approx \sin \varepsilon = 0.012 \).

III. RESULTS

Our numerical results for the isospin-violating decay rates of the scalar single-heavy tetraquarks are shown in Tables I - IV. We present our results for different scenarios; i.e., we first restrict ourselves to a specific isospin breaking mechanism I, II, or III and then calculate the full result, including all mechanisms I+II+III. From the results it is obvious that the mixing of the isosinglet and isotriplet tetraquark currents has the largest effect on the decay rate rather than the pure \( d-u \) quark mass difference or the \( \pi^0-\eta \) mixing.

We furthermore vary the scale parameter \( \Lambda \) within a reasonable range of values from 1 to 2 GeV. This follows estimates done for the scale parameter \( \Lambda_{X_b} \sim 1.4 – 2 \text{ GeV} \) of the hypothesized single-heavy tetraquark \( X(5568) \) as performed in Ref. [11]. A larger value of \( \Lambda \) would correspond to a compact tetraquark configuration, and a \( \Lambda \) close to 1 GeV is also reflected in configurations with a larger extent like hadronic molecules. In the case of the
$B_{s0}^*$ state its mass is varied from 5.725 to 6.3 GeV, and for the $D_{s0}^*$ the mass is fixed at 2.317 GeV. With $\Lambda \approx 1$ GeV the one-pion decay rates of the isosinglet tetraquark states with a small admixture of the isotriplet tetraquark component are actually compatible with results in the hadronic molecular picture [1]-[3] giving predictions for the isospin-breaking decay rate of the order of 100 keV. In particular, in Refs. [1]-[3] we found that $\Gamma(D_{s0}^* \to D_s + \pi^0) = 46.7 - 111.9$ keV and $\Gamma(B_{s0}^* \to B_s + \pi^0) = 55.2 - 89.9$ keV when the scale parameter describing the distribution of constituents in the hadronic molecule is varied from 1 to 2 GeV.

As we mentioned before, we present numerical results using approximation $\sin \delta \simeq \sin \varepsilon$ or $R = \frac{\sin \delta}{\sin \varepsilon} \simeq 1$. For completeness we also give our predictions for arbitrary values of the ratio parameter $R$ for two limiting values of the scale parameter $\Lambda_H = 1$ and 2 GeV, respectively,

$$\Gamma(D_{s0}^* \to D_s + \pi^0) = 6.17 (1 + 1.82 R)^2 \text{ keV},$$
$$\Gamma(B_{s0}^* \to B_s + \pi^0) = 5.15 (1 + 1.83 R)^2 \text{ keV at } M_{B_{s0}^*} = 5.725 \text{ GeV},$$
$$\Gamma(B_{s0}^* \to B_s + \pi^0) = 6.36 (1 + 1.77 R)^2 \text{ keV at } M_{B_{s0}^*} = 6.1 \text{ GeV},$$
$$\Gamma(B_{s0}^* \to B_s + \pi^0) = 5.49 (1 + 1.73 R)^2 \text{ keV at } M_{B_{s0}^*} = 6.3 \text{ GeV} \quad (28)$$

and

$$\Gamma(D_{s0}^* \to D_s + \pi^0) = 0.46 (1 + 1.83 R)^2 \text{ keV},$$
$$\Gamma(B_{s0}^* \to B_s + \pi^0) = 0.36 (1 + 1.84 R)^2 \text{ keV at } M_{B_{s0}^*} = 5.725 \text{ GeV},$$
$$\Gamma(B_{s0}^* \to B_s + \pi^0) = 0.63 (1 + 1.87 R)^2 \text{ keV at } M_{B_{s0}^*} = 6.1 \text{ GeV},$$
$$\Gamma(B_{s0}^* \to B_s + \pi^0) = 0.71 (1 + 1.88 R)^2 \text{ keV at } M_{B_{s0}^*} = 6.3 \text{ GeV}. \quad (29)$$

The present calculation gives predictions for the one-pion decay rates of about $3 - 5$ keV when the configuration is chosen to be more compact. Absolute rates clearly depend on the size of the tetraquark configuration, because of the lack of data especially for the $D_{s0}^*(2317)$ the two scenarios, a rather compact or an extended configuration, cannot be distinguished.

IV. SUMMARY AND CONCLUSIONS

Let us summarize the main results of our paper. We have considered the isospin-violating decays of the isosinglet states with a small admixture of the isotriplet component. We found that for values of the scale parameter of the order of 1 GeV the one-pion decay rates of
these tetraquark states are compatible with results found within the hadronic molecular approach. On the other hand, an increase in the scale parameter up to 2 GeV leads to a sizable decrease of the decay rates. Therefore, forthcoming data on the absolute rates of the isospin-violating decays could shed light on the nature of these states: either compact tetraquark states with a scale parameter of the order of 2 GeV or more extended objects, which could be viewed as hadronic molecules.

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TABLE I: Decay rate $\Gamma(B_s^{*0} \to B_s^0 + \pi^0)$ in keV at $M_{B_s^{*0}} = 5.725$ GeV.

| $\Lambda_{B_s^{*0}}$ [GeV] | I  | II | III | Full |
|---------------------------|----|----|-----|------|
| 1.00                      | 17.3 | 0.12 | 3.7 | 41.3 |
| 1.25                      | 8.7  | 0.06 | 1.9 | 20.7 |
| 1.50                      | 4.4  | 0.03 | 0.9 | 10.4 |
| 1.75                      | 2.3  | 0.01 | 0.5 | 5.4  |
| 2.00                      | 1.2  | 0.01 | 0.3 | 2.9  |

TABLE II: Decay rate $\Gamma(B_s^{*0} \to B_s^0 + \pi^0)$ in keV at $M_{B_s^{*0}} = 6.1$ GeV.

| $\Lambda_{B_s^{*0}}$ [GeV] | I  | II | III | Full |
|---------------------------|----|----|-----|------|
| 1.00                      | 19.9 | 0.21 | 4.3 | 48.8 |
| 1.25                      | 12.2 | 0.09 | 2.6 | 29.3 |
| 1.50                      | 6.9  | 0.04 | 1.5 | 16.5 |
| 1.75                      | 3.9  | 0.02 | 0.8 | 9.2  |
| 2.00                      | 2.2  | 0.01 | 0.5 | 5.2  |

TABLE III: Decay rate $\Gamma(B_s^{*0} \to B_s^0 + \pi^0)$ in keV at $M_{B_s^{*0}} = 6.3$ GeV.

| $\Lambda_{B_s^{*0}}$ [GeV] | I  | II | III | Full |
|---------------------------|----|----|-----|------|
| 1.00                      | 16.4 | 0.22 | 3.5 | 40.9 |
| 1.25                      | 11.6 | 0.10 | 2.5 | 28.0 |
| 1.50                      | 7.2  | 0.05 | 1.5 | 17.1 |
| 1.75                      | 4.3  | 0.02 | 0.9 | 10.1 |
| 2.00                      | 2.5  | 0.01 | 0.5 | 5.9  |

TABLE IV: Decay rate $\Gamma(D_s^{*+} \to D_s^+ + \pi^0)$ in keV at $M_{D_s^{*+}} = 2.317$ GeV.

| $\Lambda_{D_s^{*0}}$ [GeV] | I  | II | III | Full |
|---------------------------|----|----|-----|------|
| 1.00                      | 20.5 | 0.15 | 4.4 | 49.1 |
| 1.25                      | 10.5 | 0.08 | 2.2 | 25.1 |
| 1.50                      | 5.4  | 0.04 | 1.1 | 12.8 |
| 1.75                      | 2.8  | 0.02 | 0.6 | 6.7  |
| 2.00                      | 1.5  | 0.01 | 0.3 | 3.7  |
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