Dimensionality Reduction and Classification Feature Using Mutual Information Applied to Hyperspectral Images: A Filter Strategy Based Algorithm

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Abstract

Hyperspectral images (HIS) classification is a high technical remote sensing tool. The goal is to reproduce a thematic map that will be compared with a reference ground truth map (GT), constructed by expecting the region. The HIS contains more than a hundred bidirectional measures, called bands (or simply images), of the same region. They are taken at juxtaposed frequencies. Unfortunately, some bands contain redundant information, others are affected by the noise, and the high dimensionality of features made the accuracy of classification lower. The problematic is how to find the good bands to classify the pixels of regions. Some methods use Mutual Information (MI) and threshold, to select relevant bands, without treatment of redundancy. Others control and eliminate redundancy by selecting the band top ranking the MI, and if its neighbors have sensibly the same MI with the GT, they will be considered redundant and so discarded. This is the most inconvenient of this method, because this avoids the advantage of hyperspectral images: some precious information can be discarded. In this paper well accept the useful redundancy. A band contains useful redundancy if it contributes to produce an estimated reference map that has higher MI with the GT. To control redundancy, we introduce a complementary threshold added to last value of MI. This process is a Filter strategy; it gets a better performance of classification accuracy and not expensive, but less preferment than Wrapper strategy.

Keywords: Hyperspectral images, classification, feature selection, mutual information, redundancy
1 Introduction

In the feature classification domain, the choice of data affects widely the results. For the Hyperspectral image, the bands don't all contain the information; some bands are irrelevant like those affected by various atmospheric effects, see Figure 4, and decrease the classification accuracy. And there exist redundant bands to complicate the learning system and produce incorrect prediction [14]. Even the bands contain enough information about the scene they may not predict the classes correctly if the dimension of space images, see Figure 3, is so large that needs many cases to detect the relationship between the bands and the scene (Hughes phenomenon) [10]. We can reduce the dimensionality of hyperspectral images by selecting only the relevant bands (feature selection or subset selection methodology), or extracting, from the original bands, new bands containing the maximal information about the classes, using any functions, logical or numerical (feature extraction methodology) [11][9]. Here we focus on the feature selection using mutual information. Hyperspectral images have three advantages regarding the multispectral images [6], see Figure 1.

First: the hyperspectral image contains more than a hundred images but the multispectral contains three at ten images.

Second: hyperspectral image has a spectral resolution (the central wavelength divided by the width of spectral band) about a hundred, but that of multispectral is about ten.

Third: the bands of a hyperspectral image is regularly spaced, those of multispectral image is large and irregularly spaced.

Assertion: when we reduce hyperspectral images dimensionality, any method used must save the precision and high discrimination of substances given by hyperspectral image.

![Figure 1: Precision and discrimination added by hyperspectral images](image)

In this paper we use the Hyperspectral image AVIRIS 92AV3C (Airborne...
Visible Infrared Imaging Spectrometer). [2]. It contains 220 images taken on the region “Indiana Pine” at “north-western Indiana”, USA [1]. The 220 called bands are taken between 0.4m and 2.5m. Each band has 145 lines and 145 columns. The ground truth map is also provided, but only 10366 pixels are labeled fro 1 to 16. Each label indicates one from 16 classes. The zeros indicate pixels how are not classified yet, Figure.2.

The hyperspectral image AVIRIS 92AV3C contains numbers between 955 and 9406. Each pixel of the ground truth map has a set of 220 numbers (measures) along the hyperspectral image. This numbers (measures) represent the reflectance of the pixel in each band. So the pixel is shown as vector off 220 components. Figure 3.

![Figure 2: The Ground Truth map of AVIRIS 92AV3C and the 16 classes](image)

The hyperspectral image AVIRIS 92AV3C contains numbers between 955 and 9406. Each pixel of the ground truth map has a set of 220 numbers (measures) along the hyperspectral image. This numbers (measures) represent the reflectance of the pixel in each band. So the pixel is shown as vector off 220 components. Figure 3 shows the vector pixels notion [7]. So reducing dimensionality means selecting only the dimensions caring a lot of information regarding the classes.

![Figure 3: The notion of pixel vector](image)
We can also note that not all classes are carrier of information. In Figure 4, for example, we can show the effects of atmospheric affects on bands: 155, 220 and other bands. This hyperspectral image presents the problematic of dimensionality reduction.

2 Mutual Information based feature selection

2.1 Definition of mutual information

This is a measure of exchanged information between two ensembles of random variables \( A \) and \( B \):

\[
I(A, B) = \sum \log_2 p(A, B) \frac{p(A, B)}{p(A)p(B)}
\]

Considering the ground truth map, and bands as ensembles of random variables, we calculate their interdependence. Fano [14] has demonstrated that as soon as mutual information of already selected feature has high value, the error probability of classification is decreasing, according to the formula below:

\[
\frac{H(C/X) - 1}{\log_2(N_c)} \leq P_e \leq \frac{H(C/X)}{\log_2}
\]

with:

\[
\frac{H(C/X) - 1}{\log_2(N_c)} = \frac{H(C) - I(C; X) - 1}{\log_2(N_c)}
\]

and:

\[
P_e \leq \frac{H(C) - I(C; X)}{\log_2} = \frac{H(C/X)}{\log_2}
\]

The expression of conditional entropy \( H(C/X) \) is calculated between the ground truth map (i.e. the classes \( C \)) and the subset of bands candidates \( X \). \( N_c \) is the number of classes. So when the features \( X \) have a higher value of mutual information with the ground truth map, (is more near to the ground truth map), the error probability will be lower. But its difficult to compute this conjoint mutual information \( I(C; X) \), regarding the high dimensionality [14].

Geo [3] uses also the average of bands 170 to 210, to product an estimated ground truth map, and use it instead of the real truth map. Their curves are similar. This is shown at Figure 4.
3 The principle of proposed algorithm

3.1 Case of synthetic bands

- Band A contains only the class number 11 (Soybeans-min).
- Band B contains only the class number 14 (Woods).
- Band C contains the bands number 11 and 14.

Now we calculate the mutual information between each of them and the GT. We compute also the MI between the GT and the superposition of C and B. The results shown at Table.1

Table 1: MI of GT with synthetic bands and the accuracy of classification in each case

| Bands Synthetics | A     | B     | C     | A,C   | B,C   | A,B   | A,B,C |
|------------------|-------|-------|-------|-------|-------|-------|-------|
| MI               | 0.52  | 0.33  | 0.84  | 0.84  | 0.84  | 0.84  | 0.84  |
| Accuracy(%)      | 24.6  | 12.6  | 36.6  | 36.6  | 36.6  | 36.6  | 36.6  |
3.2 Comments

Table I allows us to comment on two cases:

- **Case 1:** The band A and B are superposed to produce band C: The MI of the estimated reference map C and GT, increases. We have information added.

- **Case 2:** The band A and C are superposed to produce an estimated reference map. We compute its MI with GT. The MI value is not change: the band A added redundant information. It's the same when we superpose B and C. It's also the same when we superpose more redundant bands: A, B, and C: no information added.

3.3 Partial conclusion

**First:** There is an important observation: the superposition of bands A and B to construct C, can be interpreted as construction of an estimated reference map by averaging the latest one and the band candidate to be selected.

**Second:** We can emit this roll: a band is relevant to classification if it contributes to produce an estimated reference map, that has the mutual information with the ground truth map increasing, if else it must be discarded.

4 Proposed algorithm

Our idea is based on this observation: the band that has higher value of Mutual Information with the ground truth map can be a good approximation of it. So we note that the subset of selected bands are the good ones, if they can
generate an estimated reference map, sensibly equal to the ground truth map. We generate the estimated reference map by averaging the latest one and the band candidate. So its a Filter approach [16] [13].

Our process of band selection will be equivalent to following steps: we order the bands according to value of its mutual information with the ground truth map. Then we initialize the selected bands ensemble with the band that has highest value of MI. At a point of process, we build an approximated reference map \( C_{\text{est}} \) by averaging the latest one and the band candidate, and we compute the MI\((C_{\text{est}}, GT)\). The latest band added (at those already selected) must make MI\((C_{\text{est}}, GT)\) increasing, if else it will be discarded from the ensemble retained. Then we introduce a complementary threshold Th to control redundancy. So the band to be selected must make MI increasing by a step equal to Th. The algorithm following shows more detail of the process:

**Algorithm 1:** Let SS be the ensemble of bands already selected and S the band candidates to be selected. SS is initially empty; R the ensemble of bands candidate, it contains initially all bands (1..220). MI is initialized with a value MI\(^*\), X the number of bands to be selected an Th the threshold controlling redundancy:

1) Select the first band to initialize \( C_{\text{est}} \):
   
   Select band index \( S = \arg \max_s \ MI(s) ; \)
   
   \( SS \leftarrow S ; \)
   
   \( R \leftarrow R \setminus S ; \)
   
   \( C_{\text{est}0} = \text{Band}(S) ; \)

2) Selection process:
   
   while \( |SS| < X \) do
     
     Select band index \( S = \arg \max_s \ MI(s) \) and \( R \leftarrow R \setminus S ; \)
     
     \[ C_{\text{est}} = \frac{C_{\text{est}0} + \text{Band}(S)}{2} ; // C_{\text{est}} = \text{Build,stimated} \]

   \( MI = \text{Mutual Information}(GT, C_{\text{est}}) \)

   if \( MI > MI^* + \text{Threshold} \) then
     
     \( MI^* = MI ; \)
     
     \( C_{\text{est}0} = C_{\text{est}} ; \)
     
     \( SS \leftarrow SS \cup S ; \)
   
   end if

end while
5 Results and Discussion

We apply this algorithm on the hyperspectral image AVIRIS 92AV3C [1], the labeled pixels are randomly chosen and used in training; and the other used for testing classification [3]. The classifier used is the SVM [5] [12] [4]. We had to choice negatives values of Th. It means that is impossible to increase accuracy of classification without allowing redundancy.

5.1 Results

Table 2 shows the results obtained for several thresholds. We can see the effectiveness selection bands of our algorithm, and the important effect of avoiding redundancy.

Table 2: Results illustrate elimination of Redundancy using algorithm proposed, for thresholds ($Th$)

| Bands retained | -0.02 | -0.01 | -0.005 | -0.004 | -0.0035 | 0 |
|----------------|-------|-------|--------|--------|---------|---|
| 2              | 47.44 | 47.44 | 47.44  | 47.44  | 47.44   | 47.44 |
| 3              | 47.87 | 47.87 | 47.87  | 47.87  | 47.87   | 48.92 |
| 4              | 49.31 | 49.31 | 49.31  | 49.31  | 49.31   | -   |
| 12             | 56.30 | 56.30 | 56.30  | 56.30  | 60.76   | -   |
| 14             | 57.00 | 57.00 | 57.00  | 57.00  | 61.80   | -   |
| 18             | 59.09 | 59.09 | 59.09  | 59.09  | 63.00   | -   |
| 20             | 63.08 | 63.08 | 63.08  | 63.53  | -       | -   |
| 25             | 66.12 | 64.89 | 64.89  | 65.38  | -       | -   |
| 30             | 73.54 | 70.72 | 70.72  | 67.68  | -       | -   |
| 33             | 73.72 | 74.79 | 75.65  | -      | -       | -   |
| 35             | 76.06 | 74.72 | 75.59  | -      | -       | -   |
| 36             | 76.49 | 76.60 | 76.19  | -      | -       | -   |
| 40             | 78.96 | 79.29 | -      | -      | -       | -   |
| 45             | 80.85 | 81.01 | -      | -      | -       | -   |
| 50             | 81.63 | 81.12 | -      | -      | -       | -   |
| 53             | 82.27 | 86.03 | -      | -      | -       | -   |
| 60             | 82.74 | 85.08 | -      | -      | -       | -   |
| 70             | 86.95 | -     | -      | -      | -       | -   |
| 75             | 86.81 | -     | -      | -      | -       | -   |
| 80             | 87.28 | -     | -      | -      | -       | -   |
| 83             | 88.14 | -     | -      | -      | -       | -   |
5.2 Analysis and Discussion

**Important:** When we apply our algorithm on the real data, here AVIRIS 92AV3C, we note that the cant increase the accuracy without allowing redundancy by negative thresholds. But the idea is good: we note that the algorithm is selective and the threshold control effectiveness the redundancy:

- **First:** For the relatively highest threshold values (-0.0035,-0.001,0,+) we obtain a hard selection: a few more informative bands are selected.
- **Second:** For the medium threshold values (-0.01, -0.005, -0.004), some redundancy is allowed, even if its harmful (negative values of thresholds), in order to made increasing the classification accuracy.
- **Tired:** As soon as the threshold value is more negative (-0.02), the redundancy allowed becomes useless, we have the same accuracy with more bands.
- **Finally:** for the more negative thresholds (for example -4), we allow all bands to be selected, and we have no action of the algorithm. This corresponds at selecting bay ordering bands on mutual information for numerous thresholds. The performance is low.

We can not here that Hui Wang [15] uses two axioms to characterize feature selection. Sufficiency axiom: the subset selected feature must be able to reproduce the training simples without losing information. The necessity axiom "simplest among different alternatives is preferred for prediction". In the algorithm proposed, reducing error probability between the truth map and the estimated minimize the information loosed for the samples training and also the predicate ones.

We not also that we can use the number of features selected like condition to stop the search. [16].

**Partial conclusion:** The algorithm proposed effectively reduces dimensionality of hyperspectral images.

6 Conclusion

In this paper we presented the necessity to reduce the number of bands, in classification of Hyperspectral images. Then we carried out the effectiveness of mutual information to select bands able to classify the pixels of ground truth. And also we insisted on saving the propriety of hyperspectral images regarding the multispectral images, when we reduce dimensionality. We introduce an algorithm based on mutual information. To choice a band, it must contribute to reproduce an estimated ground truth map more closed to the reference map. A complementary threshold is added to avoid redundancy. So each band retained has to reproduce an estimated ground truth map more closed to the reference map by a step equal to threshold even if it caries a redundant information. But the method used her to estimate the reference
map, play an important role: here with averaging bands; we are constraint to use negative value of threshold; so we allow more redundancy. We can tell that we conserve the useful redundancy by adjusting the complementary threshold. This algorithm is a feature selection methodology, and its a Filter approach. Its less expensive. It can be implemented in real time applications. This scheme is very interesting to investigate and improve, considering its performance.

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