QRPA and its extensions in a solvable model

QRPA y sus extensiones en un modelo soluble exactamente

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Abstract

An exactly solvable model is introduced, which is equivalent to the exact shell-model treatment of protons and neutrons in a single j-shell for Fermi-type excitations. Exact energies, quasiparticle numbers and double beta decay Fermi amplitudes are computed and compared with the results of both the standard quasiparticle random phase approximation (QRPA) and the renormalized one (RQRPA), and also with those corresponding to the hamiltonian in the quasiparticle basis (qp). A zero excitation energy state is found in the exact case, occurring at a value of the residual particle-particle interaction at which the QRPA collapse. The RQRPA and the qp solutions do not include this zero-energy eigenvalue in their spectra, probably due to spurious correlations.

Resumen Se presenta un modelo exactamente soluble que es equivalente al modelo de capas para protones y neutrones en una sola capa con momento angular j para el caso de excitaciones tipo Fermi. Se calculan los valores exactos de las energías y de las amplitudes del decaimiento beta doble tipo Fermi y se los compara con los resultados de la aproximación de fases al azar para cuasipartículas (QRPA) usual y con los de

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1 Introduction

In order to improve the reliability of the Quasiparticle Random Phase Approximation (QRPA) description of nuclear double beta decay transitions much work has been done recently related with its extensions, in particular with the Renormalized Quasiparticle Random Phase Approximation (RQRPA).

The matrix elements for ground state to ground state two-neutrino double-beta decay transitions ($\beta\beta_{2\nu}$) calculated in the QRPA are extremely sensitive to details of the nuclear two-body interaction [1, 2, 3, 4]. The inclusion of renormalized particle-particle correlations in the QRPA matrix amounts to a drastic suppression of the $\beta\beta_{2\nu}$-matrix elements. However, for some critical values of the model parameters the QRPA eigenvalue problem becomes complex. The most notorious example of this behavior, of the QRPA approach, is the calculation of the $\beta\beta_{2\nu}$ decay of $^{100}\text{Mo}$ [1, 2, 5, 6].

The renormalized version of the QRPA (RQRPA) [7, 8], which includes some corrections beyond the quasiboson approximation, has been recently reformulated [9] and applied to the $\beta\beta_{2\nu}$ decay problem [10]. Contrary to the QRPA, the RQRPA does not collapse for any value of the residual two-body interaction. Based on its properties, the RQRPA was presented as a cure for the instabilities of the QRPA and it was applied to calculations of the $\beta\beta_{2\nu}$ decay of $^{100}\text{Mo}$ [10]. Similar studies have been performed in the framework of the RQRPA and with the inclusion of proton-neutron pairing correlations in symmetry breaking hamiltonians [11].

In a recent paper [12] we have shown that the RQRPA violates the Ikeda sum rule and that this violation is indeed present in many extensions of the QRPA. The study was based on the schematic proton-neutron Lipkin model.

In the present paper we will review the general features of the exactly solvable model and its comparison with the QRPA, the RQRPA and the qp-hamiltonian [12, 13, 14]. The presence of a zero excitation energy state in the spectrum corresponding to the exact solution of the model hamiltonian is discussed. It will be shown that the RQRPA and the qp solutions, do not display the same feature, most likely due to the presence of spurious states caused by the mixing of orders, of the relevant interaction terms, in the expansion procedure.

The structure of the paper is the following: the model and its solutions are presented in Section 2, the quasiparticle version of the hamiltonian, its linear
representation in terms of pairs of unlike (proton-neutron) quasiparticle-pairs and its properties are introduced in Section 3. The QRPA and RQRPA treatments of the hamiltonian are discussed in Section 4. The matrix elements of double-beta-decay transitions, calculated in the framework of the different approximations introduced in the previous sections, are given in Section 5. Conclusions are drawn in Section 6.

2 The model

The model hamiltonian, which includes a single particle term, a pairing term for protons and neutrons and a schematic charge-dependent residual interaction with both particle-hole and particle-particle channels, is given by

\[ H = \epsilon_p N_p - G_p S_p^\dagger S_p + \epsilon_n N_n - G_n S_n^\dagger S_n + 2\chi \beta^- \cdot \beta^+ - 2\kappa P^- \cdot P^+, \] (1)

with

\[ N_i = \sum_{m_i} a_{m_i}^\dagger a_{m_i}, \quad S_i^\dagger = \sum_{m_i} a_{m_i}^\dagger a_{m_i}/2, \quad i = p, n \]

\[ \beta^- = \sum_{m_p = m_n} a_{m_p}^\dagger a_{m_n}, \quad P^- = \sum_{m_p = -m_n} a_{m_p}^\dagger a_{m_n}^\dagger, \] (2)

\[ a_{j_p}^\dagger = a_{j_n}^\dagger \] being the particle creation operator and \( a_{j_p}^\dagger = (-1)^{j_p - m_p} a_{j_n}^\dagger \) its time reversal. The parameters \( \chi \) and \( \kappa \) play the role of the renormalization factors \( g_{ph} \) and \( g_{pp} \) introduced in the literature [1, 2, 3, 4].

The relatively simple schematic force (1) can approximately describe the correlations induced by a more realistic interaction.

In a single-one-shell limit, for the model space \((j_p = j_n = j)\) and for monopole \((J = 0)\) excitations the hamiltonian (1) can be solved exactly. In spite of the fact that the solutions obtained in this restricted model space cannot be related to actual nuclear states, the excitation energies, single- and double-beta decay transition amplitudes and ground state correlations depend on the particle-particle strength parameter \( \kappa \) in the same way as they do in realistic calculations with many single particle levels and with more realistic interactions. We shall obtain the eigenstates of (1), by using different approximations, in order to built-up a comprehensive view about the validity of them and their predictive power.

The hamiltonian (1) can be expressed in terms of the generators of an SO(5) algebra [13]. The Hilbert space is constructed by using the eigenstates of the particle-number operator \( \mathcal{N} = N_p + N_n \), the isospin \( \mathcal{T} \) and its projection \( \mathcal{T}_z = (N_n - N_p)/2 \). The raising and lowering isospin operators are defined as \( \beta^\pm = \mathcal{T}^\pm \), where \( \mathcal{T}^- |n\rangle = |p\rangle \). With them we can construct the isospin scalar \( \mathcal{T}^2 \) and the second order SO(5) Casimir.

The hamiltonian (1) is diagonal in the \( \mathcal{N}, \mathcal{T}, \mathcal{T}_z \) basis if \( G = 4\kappa \). It can be reduced to an isospin scalar if its parameters are selected as
If $4\kappa \neq G$ the hamiltonian (3) will not be diagonal in this basis. The hamiltonian mixes states with different isospin $T$ while its eigenstates still have definite $N$ and $T_z$. The dynamical breaking of the isospin symmetry is an essential aspect of the model which is directly related to the nuclear structure mechanism responsible for the suppression of the matrix elements for double-beta-decay transitions.

### 2.1 The diagonal case $G = 4\kappa$

For the numerical examples we have selected $N_n > N_p$ and a large value of $j$ to simulate the realistic situation found in medium- and heavy-mass nuclei. To perform the calculations we have adopted the following set of parameters:

\[
\begin{align*}
  j &= 19/2, \\
  N &= 20, \\
  1 \leq T_z \leq 5, \\
  e_p &= 0.69 MeV, \\
  e_n &= 0.0 MeV, \\
  G_p &= G_n = 0.2 MeV, \\
  \chi &= 0 \text{ or } 0.025 MeV, \\
  0 \leq \kappa \leq 0.1
\end{align*}
\]

The dependence of the spectrum and transition matrix elements on the parameters $\chi$ and $\kappa$ is analyzed in the following paragraphs.

**Fig. 1**

The complete set of $0^+$ states, belonging to different isotopes, is shown in Fig. 1a for $G = 4\kappa$ and $\chi = 0$, as a function of the number of protons ($Z$). The states are labeled by the isospin quantum numbers ($T, T_z$). Ground states are shown by thicker lines. As shown in the figure the structure of the mass parabola is qualitatively reproduced.

The upper insert, case a), shows the full spectrum corresponding to $\chi = 0$. The lower one, case b), shows the results corresponding to $\chi = 0.025 MeV$. Obviously the particle-hole channel of the residual interaction stretches the spectra of all isotopes. It also increases the energy of the Isobaric Analog State (IAS).

Beta decay transitions of the Fermi type, mediated by the action of the operator $\beta^- = t^-$, are allowed between states belonging to the same isospin multiplet. The energy of each member of a given multiplet increases linearly with $Z$.

In this example the $0^+$ states belonging to each odd-odd-mass nuclei (N-1, Z+1, A) are the IAS constructed from the $0^+$ states of the even-even-mass nuclei with (N, Z, A) nucleons. Thus, Fermi transitions between them are allowed.

Since the isospin of the ground state of each of the even-even-mass nuclei differs, for different isotopes, Fermi-double-beta-decay transitions connecting them are forbidden in this diagonal limit $G = 4\kappa$. 

\[
e_p = e_n, \quad \chi = 0, \quad G = 4\kappa. \quad (3)
\]
2.2 Exact solutions

The hamiltonian (1) has a $\mathcal{T} = 2$ tensorial component which mixes states with different isospin, while particle number and isospin projection remain as good quantum numbers. The diagonalization of (1) is performed in the basis of states belonging to the SO(5) irrep $(\Omega, 0)$ \cite{14}.

$|N^\alpha T, z\rangle = \sum_T C^\alpha_N |N^TT_z\rangle$ \hfill (5)

**Fig. 2**

The energies of the ground-state ($0^+_g$) and of the first-excited state ($0^+_1$), as a function of the ratio $4\kappa/G$ for $N_n = 12$, $N_p = 8$ and $\chi = 0$ are shown if Fig. 2.

The most characteristic feature of the results is the barely avoided crossing of levels, due to the repulsive nature of the effective residual interaction between them. In the neighbourhood of the value $4\kappa/G \approx 1$ a major structural change in the wave functions will develop.

**Fig. 3**

The full-thin line of Fig. 3a (3b) represents the excitation energy $E_{exc}$ of the lowest $0^+$ state belonging to the double-odd-mass nucleus ($N_n = 13, N_p = 7$) with respect to the parent even-even-mass nucleus ($N_n = 14, N_p = 6$), as a function of the ratio $4\kappa/G$ for $\chi = 0$ (0.025). It is clear that when $4\kappa/G \approx 1.3$ attractive proton-neutron correlation dominates over proton-proton and neutron-neutron pairing correlations and the excitation energy goes to zero.

The vanishing of the energy of the first excited state, and the subsequent inversion of levels (or negative excitation energies) would indicate that the double-odd nucleus becomes more bound than their even-even neighbours, contradicting the main evidence for the dominance of like-nucleons pairing in medium- and heavy-mass nuclei. It would also completely suppress the double beta decay because the single beta decay from each "side" of the double-odd nucleus would be allowed.

These result simply emphasizes the fact that the hamiltonian (1) will not be the adequate one when attractive proton-neutron interactions are too large. In a realistic situation, obviously, the true hamiltonian includes other degrees of freedom, like quadrupole-quadrupole interactions, and permanent deformations of the single-particle mean-field can also be present. These additional degrees of freedom will prevent the complete crossing of levels which, of course, is not observed. However, in many cases the experimentally observed energy-shift of double-odd-mass nuclei, respect to their double-even-mass neighbours is very small. This finding reinforces the notion of an underlying dynamical-symmetry-restoration-effect.
3 The hamiltonian in the quasiparticle (qp) basis

By performing the transformation of the particle creation and annihilation operators of the hamiltonian (1) to the quasiparticle representation [14] we have obtained the qp-hamiltonian [13, 14].

The linearized version of the qp-hamiltonian is obtained by neglecting the scattering terms. The solutions of this truncated hamiltonian have been discussed in [12].

Finding the eigenvalues and eigenvectors of the qp-hamiltonian requires the use of the same algebraic techniques involved in solving the original hamiltonian. However, the complexity of the problem increases severely, due to the fact that neither the quasiparticle number or the quasiparticle isospin projection (or equivalently the number of proton and neutron quasiparticles) are good quantum numbers. It implies that the dimension of the basis will increase by two orders of magnitude [14].

Particle number is not a good quantum number, obviously, because it is broken spontaneously by the Bogolyubov transformation. Thus, zero-quasiparticle states belonging the even-even-mass nucleus have good average number of protons and neutrons while states with a non-vanishing number of quasiparticles show strong fluctuations in the particle number. Fluctuations in the particle number can induce, naturally, important effects on the observables. Moreover, the admixture of several quasiparticle-configurations in a given state, induced by residual particle-particle interactions, can also strongly influence the behavior of the observables. An example of this effect is given in [12], concerning the violation of the Ikeda Sum Rule produced by large values of the particle-particle strength \( \kappa \).

The spectrum of the qp-hamiltonian for odd-odd nuclei is shown in Figure 3. The curves shown by small-dotted-lines, in Figs. 3.a), 3.b), display the dependence of the excitation energy for the qp-hamiltonian upon the ratio \( \kappa/G \). The energies in this qp-approximation closely follow the exact ones up to the point where the last become negatives \( (4\kappa/G \approx 1.4 \sim 1.8 \) in the different cases). From this point on they vanish, rather than taking negative values, instead. The excitation energies for the linearized hamiltonian \( H_{22} + H_{04} \) are shown as thick lines in these figures. We can see that the linearized hamiltonian is able to reproduce qualitatively the behavior of the full-qp one, but in general it overestimates the values of the excitation energies.

As it is mentioned above, the results shown in Figures 3 have been obtained both with the complete qp-hamiltonian and with the truncated hamiltonian which includes only the product of pair-creation and annihilation-operators. In [12] the relevance of the scattering terms in the qp-hamiltonian was pointed out. From the present results it can be seen that the inclusion of these terms is indeed important if one looks after a better description of the qp-excitation
energies, up to the point where the exact excitation energies become negative. For larger values of \( \kappa \) even the eigenstates of the complete hamiltonian fail to describe negative excitation energies. This is a clear indication that other effects can play an important role, like, i.e; effects associated to the appearance of spurious states. This can be quantitatively illustrated by the following. There are four exact eigenstates for \( j = 19/2, N_n = 13, N_p = 7 \), as can be seen in Fig. 2.a), while the spectrum of the qp-hamiltonian has 220 eigenstates. It is well known that states with \( N_n = 14 \pm N_n, N_p = 6 \pm N_p \), where \( N_p \) and \( N_n \) are the number of quasiparticle protons and neutrons, respectively, are mixed with the two p-n quasiparticle state in the odd-odd nucleus and provide a large number of states belonging to other nuclei. When \( 4\kappa/G \ll 1 \) the spurious states remain largely un-mixed with the lower energy two-qp state. But when \( 4\kappa/G \approx 1 \) the mixing becomes important. This fact up-grades the relevance of particle-number violation effects in dealing with this case.

The full qp-treatment represents the best possible extension of the quasi-boson approximation, without performing a particle-number projection, in a single-j shell. It goes beyond any second extended RPA [17] and it includes explicitly all number of proton- and neutron-quasiparticles (\( N_p \) and \( N_n \)) in the eigenstates.

To analyze the effects associated to the number of quasiparticles in the ground state of double-even nuclei, and particularly the effects associated to the number of quasiprotons, we have calculated the average number of quasi-protons [12, 14].

**Fig 4**

In Figs. 4.a) and 4.b) the average number of proton-quasiparticles in the ground state of the even-even nucleus with \( N_p = 6, N_n = 14 \) is shown as a function of \( 4\kappa/G \), for \( j = 19/2, \chi = 0 \) and 0.04. The dashed-lines represent the results corresponding to the full qp-hamiltonian case while the large dots refer to the linearized \( H_{22} + H_{04} \) version of it. The difference between both approximations is evident. Using the linearized hamiltonian the states are composed only by proton-neutron-quasiparticle pairs [12], while the presence of the scattering terms introduces also the like quasiparticle pairs. The presence of these pairs, which for \( 4\kappa/G \approx 1 \) play a crucial role, increases notably the number of quasiparticles and yields excitation energies closer to the exact ones.

The average quasiparticle number shows a saturation in the full-qp case for \( 4\kappa/G \approx 1.8 \). At this value of the residual pn-interaction the ground state is far-away for the qp vacuum, and has a structure which can be described as full quasiparticle shell. Notice that at this point the exact and full-qp excitation energies depart for each other. A state with four proton and four neutron quasiparticles has very large number-fluctuations. Spurious states become strongly mixed with physical states. In this way the resulting excitation energies average to zero, a limit which differs from the exact value, which is negative.
4 QRPA and RQRPA

The QRPA hamiltonian $H_{QRPA}$ can be obtained from the linearized version of the qp-hamiltonian, by keeping only the bilinear-terms in the pair-creation and pair-annihilation operators. The pair-creation and pair-annihilation operators are defined by coupled pairs of fermions. The commutation relations between these pseudo-boson operators include number-like quasiparticle operators in addition to unity. By taking the limit $(2 j + 1) \to \infty$ these extra terms vanish and the commutation relations between pairs of fermions can be treated like exact commutation relations between bosons. This is the well-known quasi-boson approximation.

The QRPA states are generated by the action of the one-phonon operator on the correlated QRPA vacuum. If ground state correlations are too strong the first eigenvalue becomes purely imaginary. For this limit the backward-going amplitudes of the QRPA phonon-operator become dominant, thus invalidating the underlying assumption about the smallness of the quasi-boson vacuum-amplitudes. The QRPA excitation energies, obtained with the above introduced hamiltonian are shown in Figs. 3a, 3b. It can be seen that in the cases displayed in these figures the collapse of the QRPA values occurs near the point where the exact excitation energies become negative. This is a very important result because it means that that the QRPA description of the dynamics given by the hamiltonian (1) is able to reproduce exact results. At this point one can naturally ask the obvious question about the nature of the mechanism which produces such a collapse. The fact that the QRPA approximation is sensitive to it, together with the fact that the same behavior is shown by the exact solution, reinforces the idea about the onset of correlations which terminate the regime of validity of the pair-dominant picture. In order to identify such correlations we have calculated the expectation value of the number of quasi-fermions and bosons on the QRPA ground state [12, 14]. Figs. 4.a), 4.b) show the results corresponding to these occupation numbers. The QRPA results extend up to the value $4 \kappa/G \approx 1$, where the QRPA collapses. The sudden increase of the average quasiparticle number near the collapse of the QRPA is a clear evidence about a change in the structure of the QRPA ground state.

In the renormalized QRPA the structure of the ground state is included explicitly [8]. The quasi-boson approximation is not enforced explicitly. The renormalization procedure consists of retaining approximately the number of quasiparticle-like-terms of the commutators keeping them as a parameter to be determined. It produces the reduction of the residual interaction which is needed to avoid the collapse of the QRPA equations [10]. Due to this fact the RQRPA energy $E_{RQRPA}$ is always real. Its value can be obtained by solving simultaneously a set of non-linear equations [10].

RQRPA excitation energies are shown in Figs. 3a and 3b. The main finding of the present calculations is that the exact excitation energies are closer to the QRPA energies, rather than to the renormalized ones. In exact calculations
including the spin degrees of freedom a phase transition was found at the point where the QRPA collapses [18], thus reinforcing the present results.

The average number of quasiparticles in the RQRPA vacuum is shown in Fig. 4.a), 4.b). It is fairly obvious, from these results, that the RQRPA ground state correlations double in all the cases those of the complete solutions of the linearized hamiltonian. This is clearly an overestimation, and it is probably one of the most notorious difficulties confronting the use of the RQRPA.

It allows too much ground state correlations, and with them the particle number fluctuations are introducing spurious states which can dominate the low energy structure for large values of $\kappa$.

Near "collapse" the average number of quasiparticles given by the QRPA and the RQRPA are comparable. For the case of the QRPA the increase of the ground-state-correlations is determined by the change in the sign of the backward-going matrix relative to the forward-going one near collapse. From there on the QRPA cannot produce any physically acceptable result since one of the underlying conditions of the approximation, i.e: the positive definite character of either linear combination of the forward- and backward-going blocks of the QRPA matrix will not be fulfilled. This collapse is prevented in the RQRPA, by the use of the renormalization of the matrix elements, but the drawback of the approximation is the contribution coming from spurious states, which ought to be removed. By going beyond the leading order QRPA approximation, more terms have to be added to the diagrams which represent the transition amplitudes. It has been done for a pure seniority model in [19].

5 Double beta decay

In this section we shall briefly discuss some of the consequences of the previously presented approaches on the calculation of nuclear double-beta decay observables. In the following we shall focus our attention on the two-neutrino mode of the nuclear double-beta decay, since the matrix elements governing this decay mode are more sensitive to nuclear structure effects than neutrinoless mode. As said in the introduction we shall consider only double-Fermi transitions. The nuclear matrix elements of the two-neutrino double-beta-decay $M_{2\nu}$ are discussed in [12, 13, 14].

The results for the matrix elements $M_{2\nu}$, obtained with the exact wave functions are shown, as a function of the ratio $4\kappa/G$, in Figs. 5.a) and 5.b). These results have been obtained with the following set of parameters: $j = 19/2, (N_p = 6, N_n = 14) \rightarrow (N_p = 8, N_n = 12)$ and $\chi = 0$ and 0.025 MeV.

The exact value of $M_{2\nu}$ vanishes at the point $4\kappa/G = 1$. As mentioned above, this cancellation appears in the model due to the fact that for this value of $\kappa$ the isospin-symmetry is recovered and the ground states of the initial and final nuclei belong to different isospin multiplets, as it can be seen also from the
A similar mechanism, in the context of a solvable model possessing a SO(8) algebra including spin and isospin degrees of freedom was used a decade ago to shown that the cancellation of the $M_{2\nu}$ matrix elements for certain values of the particle-particle residual interaction was not an artifact of the QRPA description [2].

The results corresponding to the matrix elements $M_{2\nu}$, calculated with the different approximations discussed in the text are also shown in Fig. 5, as a function of the coupling constant $\kappa$. The values of $M_{2\nu}$ are very similar to those found in realistic calculations [1, 3, 4, 10], including its strong suppression for values of the coupling constant $\kappa$ near the value which produces the collapse of the QRPA description. Distinctively, the RQRPA results extends to values of $\kappa$ passing the "critical" value. However, the validity of this result can be questioned because, as we have shown above, the RQRPA missed the vanishing of the excitation energy. The $M_{2\nu}$ matrix elements, evaluated with the complete qp-hamiltonian, is quite similar to that of the RQRPA up to point where it vanishes. From this point-on the results of both the full-qp and the RQRPA approximations are different. Both matrix elements change their sign at a value of $\kappa$ which is larger than the one corresponding to the change of the sign of the matrix elements calculated with the exact wave function. The fact that the RQRPA results and the ones of the qp-approximation are similar, although these models differ drastically in the correlations which they actually include, suggest that a kind of balance is established between terms which are responsible for ground state correlations and those which produce the breaking of coherence in the wave functions. Obviously this mechanism must be related to the presence of scattering terms in the commutators as well as in the hamiltonian.

6 Conclusions

An exactly solvable model for the description of single- and double-beta- decay-processes of the Fermi-type was introduced. The model is equivalent to a complete shell model treatment in a single-j shell for the adopted hamiltonian. It reproduces the main features of the results obtained in realistic calculations, with many shell and effective residual interaction, like those used in the literature to describe the microscopic structure of the nuclei involved in double beta decay processes.

We have constructed the exact spectrum of the hamiltonian and discussed its properties. The results concerning the energy of the states belonging to the exact solution of the model show that, in spite of its very schematic structure, the hamiltonian is able to qualitative reproduce the nuclear mass parabola. The sequence of levels of the exact solution shows that the ground-state and the first-excited state, of the spectrum of double-even nuclei, approach a band-crossing situation for a critical value of the strength associated to attractive particle-
particle interactions. At the crossing these states interchange their quantum numbers. This behavior is connected with the description of "shape" transitions in similar theories, where the order parameter is clearly associated with multipole deformations in r-space. In the present model the "deformation" mechanism is related with the breaking of the isospin symmetry and the space-rotation correspond to a rotation in isospin-space which preserves the third-component of the isospin.

We have compared the exact values of the excitation energy and of the double-beta-decay matrix elements, for double-Fermi transitions, with those obtained by using the solutions of the approximate qp-hamiltonian, its linearized version and both the QRPA and RQRPA ones.

It was shown that the collapse of the QRPA correlates with the presence of an exact-eigenvalue at zero energy. The structure of the RQRPA solutions has been discussed and it was found that though finite they are not free from spurious contributions. The role of scattering-terms was discussed and they were shown to be relevant in getting excitation energies closer to the exact values. However they are not enough to generate the correlations which are needed to produce the band-crossing, or negative excitation energies, as it was found in the exact solution for large values of the coupling constant $\kappa$.

In order to correlate the break-up of the QRPA approximation with the onset of strong fluctuations in the particle number we have calculated the average number of quasiparticles in the different approximations discussed in the text.

It was shown that the solutions of the complete qp-hamiltonian display a strong change in the structure of the ground state when the particle-particle strength increases. The qp-content of the ground state varies from a nearly zero-value to an almost full qp-occupancy. The particle number fluctuations associated with states with a large number of quasiparticles were mentioned as a possible source of spurious states.

Double beta decay amplitudes were evaluated in the different formalisms. Their similitudes and differences were pointed out.

As a conclusion the need of additional work, to clarify the meaning of the different approximations posed by the RQRPA, was pointed out.

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Figure Captions

Figure 1.a (1.b): $0^+$ states of different isotopes are shown for $j = 19/2$, $4\kappa/G = 1$ and $\chi = 0$. (0.025)MeV. in an energy vs. $Z$ plot. States are labeled by $(T, T_z)$. The lowest energy state of each nucleus is shown by a thick-line.

Figure 2: Energy of the ground state $0^+_{gs}$ (full line) and first excited state $0^+_1$ (dotted line), as a function of the ratio $4\kappa/G$, for $j = 19/2$, $N_n = 12$, $N_p = 8$.

Figure 3.a (3.b): Excitation energy $E_{exc}$ of the lowest $0^+$ state in the odd-odd intermediate nucleus ($N_n = 13, N_p = 7$) with respect to the parent even-even nucleus ($N_n = 14, N_p = 6$) against $4\kappa/G$, for $j = 19/2$, $\chi = 0$ (0.025). Exact results are shown as thin-full-lines while those of the qp-hamiltonian are shown as small-dotted-lines. Results corresponding to the linearized qp-hamiltonian are shown as full-thick-lines and the results obtained with the QRPA and RQRPA methods as large-dotted- and dashed-lines, respectively.

Figure 4.a (4.b): Average number of proton-quasiparticles in the ground state of the even-even nucleus with $N_p = 6$, $N_n = 14$ as function of $4\kappa/G$, for $j = 19/2$, $\chi = 0$ (0.025)MeV. Results corresponding to the qp-hamiltonian are shown as dashed-lines. The ones corresponding to the linearized qp-hamiltonian are shown as large-dotted lines and those of the QRPA and RQRPA methods as full-lines and small-dotted-lines, respectively.

Figure 5a (5b): Matrix elements $M_{2\nu}$, for the double-Fermi two-neutrino double-beta decay mode, as functions of the ratio $4\kappa/G$ for $j = 19/2$, ($N_p = 6$, $N_n = 14$) → ($N_p = 8$, $N_n = 12$) and $\chi = 0$ (0.025)MeV Exact results are indicated by thin-full-lines. The results obtained with the qp-hamiltonian are shown as small-dotted-lines and the results of the QRPA and RQRPA methods as dashed-lines and large-dotted-lines, respectively.
Figure 3

(a) $\chi = 0.0$

(b) $\chi = 0.025$
\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{\textbf{Figure 4}}
\end{figure}

(a) $\chi = 0.0$, $N_n = 14, N_p = 6$

(b) $\chi = 0.025$, $N_n = 14, N_p = 6$
Figure 5

(a) $\chi = 0$

(b) $\chi = 0.025$