NEW DESIGN OF QUATERNARY LCZ AND ZCZ SEQUENCE SET FROM BINARY LCZ AND ZCZ SEQUENCE SET

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(Communicated by Cunsheng Ding)

Abstract. In this paper, we suggest two construction methods of quaternary low and zero correlation zone (LCZ and ZCZ) sequence set. The new construction methods use a binary LCZ/ZCZ sequence set and the Gray mapping to produce new quaternary LCZ/ZCZ sequence sets. The parameters of the generated quaternary LCZ/ZCZ sequence set are the same as those of the employed binary LCZ/ZCZ sequence set. That means, an optimal quaternary LCZ/ZCZ sequence set can be constructed from an optimal binary LCZ/ZCZ sequence set.

1. Introduction

High-frequency band provides wider spectrum for high speed data transmission. However, since electromagnetic attenuation is very high in high-frequency range of several tens GHz, the base station in a cellular network can serve only very small cell area such as a microcell or a picocell. In such a small cell, propagation delay is very small and thus it is possible to maintain the time delay in reverse link within a few chips. Such environment gives us more flexibility in designing the wireless communication system. In 1992, Gaudenzi, Elia, and Vilola [4] proposed the quasi-synchronous code-division multiple-access (QS-CDMA) systems, in which only small time delay among different users is allowed. In order to reduce the inter-user interference in QS-CDMA systems [13, 21], the only necessary condition for spreading codes is to have small magnitude of maximum correlation value within limited delays around the origin. This condition is achieved by introducing a new type of sequence sets called low correlation zone (LCZ) sequence sets which have low correlation values around the origin. Therefore, it is a fundamental issue in

2000 Mathematics Subject Classification: Primary: 58F15, 58F17; Secondary: 53C35.
Key words and phrases: CDMA, Gray mapping, low correlation zone (LCZ), LCZ sequence sets, quasi-synchronous (QS) CDMA, quaternary sequences, sequences, zero correlation zone (ZCZ), ZCZ sequence sets.

The first author is supported by the Korea Research Foundation Grant funded by the Korean Government KRF 2008-357-D00170.

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QS-CDMA systems to find a good LCZ sequence set which has large cardinality of the set and long LCZ length at the same time.

Let $S$ be a set of $M$ sequences of period $N$. If the magnitude of correlation function between any two sequences in $S$ takes the values less than or equal to $\epsilon$ within the range, $-L < \tau < L$, of the offset $\tau$, then $S$ is called an LCZ sequence set with parameters $(N, M, L, \epsilon)$. In several studies, various LCZ sequence sets have been constructed. Long, Zhang, and Hu [13] proposed a binary LCZ sequence sets, the first one, by using Gordon-Mills-Welch (GMW) sequences [15] and Tang and Fan [16] extended Long, Zhang, and Hu’s work to build $p$-ary LCZ sequence sets. Kim, Jang, No, and Chung [10] proposed optimal quaternary LCZ sequence sets using binary sequence with ideal autocorrelation and it’s generalized version, $p^2$-ary LCZ sequence set using unified sequence, is constructed by Jang, No, and Chung [8]. And Jang, No, Chung, and Tang [9] constructed optimal $p$-ary LCZ sequence sets. All these constructions generate LCZ sequence sets with fixed LCZ length and cardinality.

There is another type of LCZ sequence sets with flexible parameters. Kim, Jang, No, and Chung [11] first constructed a new type of LCZ sequence sets with flexible LCZ and cardinality of the set and Zhou, Tang, and Gong [22] generalized Kim, Jang, No, and Chung’s work as a method based on the interleaving technique. Chung and Yang [1] proposed new quaternary LCZ sequence sets by applying the Gray mapping to a binary sequence and its cyclically shifted sequence. Lately, Jang and Kim [7] constructed new optimal quaternary LCZ sequence sets from the Gray mapping of binary sequences.

As a special case of LCZ sequence, there are zero correlation zone (ZCZ) sequence sets that have correlation value $\epsilon = 0$ within the correlation zone. A number of studies on ZCZ sequence sets have been published. In this paper, we pay attention only to the binary ZCZ sequence sets to construct quaternary ZCZ sequence sets. For example, Matsufuji, Suehiro, Kuroyanagi, Fan, and Takatsukasa [14] constructed binary ZCZ sequence pairs derived from complementary pairs. Fan, Suehiro, Kuroyanagi, and Deng proposed a binary ZCZ sequence set [3].

Although it is desired to increase both the cardinality of the set and the LCZ/ZCZ length as large as possible, it cannot be achieved since there is a tradeoff between these two parameters. Thus, one of the the most significant problems in the design of LCZ/ZCZ sequence sets is to construct an optimal set which has the maximum cardinality given the LCZ/ZCZ length.

In this paper, we suggest two construction methods of quaternary LCZ and ZCZ sequence set. The new construction methods use a binary LCZ/ZCZ sequence set and the Gray mapping to produce new quaternary LCZ/ZCZ sequence sets. The parameters of the generated quaternary LCZ/ZCZ sequence set are the same as those of employed binary LCZ/ZCZ sequence set. That means, an optimal quaternary LCZ/ZCZ sequence set can be constructed from an optimal binary LCZ/ZCZ sequence set.

2. Preliminaries

In this section, we introduce some definitions and notations.

The correlation function $R_{s_1, s_2}(\tau)$ of $q$-ary sequences $s_1(t)$ and $s_2(t)$ of period $N$ is defined as

$$R_{s_1, s_2}(\tau) = \sum_{t=0}^{N-1} \omega_q^{s_1(t) - s_2(t+\tau)}$$
where \( \omega_q \) is a complex primitive \( q \)th root of unity. The autocorrelation function of a sequence \( s(t) \) is readily denoted by \( R_s(\tau) \).

Generalizing Welch’s bound, Tang, Fan, Matsufuji proposed a widely used criterion on the optimal relation of the parameters \((N, M, L, \epsilon)\) of LCZ/ZCZ sequence sets as in the following theorem.

**Theorem 2.1** (Tang, Fan, and Matsufuji [17]). Let \( S \) be an LCZ/ZCZ sequence set with parameters \((N, M, L, \epsilon)\). Then we have

\[
ML - 1 \leq \frac{N - 1}{1 - \epsilon^2 / N}.
\]

Let \( \phi[a, b] \) be the Gray mapping defined by

\[
\phi[a, b] = \begin{cases} 0, & \text{if } (a, b) = (0, 0) \\ 1, & \text{if } (a, b) = (0, 1) \\ 2, & \text{if } (a, b) = (1, 1) \\ 3, & \text{if } (a, b) = (1, 0). \end{cases}
\]

Let \( N \) be a positive integer, and \( a(t) \) and \( b(t) \) binary sequences with period \( N \). The quaternary sequence \( g(t) \) is defined as \( g(t) = \phi[a(t), b(t)] \). Then \( g(t) \) can be represented by using the binary sequences, \( a(t) \) and \( b(t) \) [12]

\[
\omega_4^{g(t)} = \frac{1 + \omega_4}{2}(-1)^{a(t)} + \frac{1 - \omega_4}{2}(-1)^{b(t)}.
\]

Krone and Sarwate derived the relation between the correlations of binary sequences and quaternary sequences in (3) as follows.

**Lemma 2.2** (Krone and Sarwate [12]). Let \( a(t), b(t), c(t), \) and \( d(t) \) be binary sequences with the same period. Let \( g_1(t) \) and \( g_2(t) \) be quaternary sequences defined by \( g_1(t) = \phi[a(t), b(t)] \) and \( g_2(t) = \phi[c(t), d(t)] \), respectively. Then cross-correlation function \( R_{g_1, g_2}(\tau) \) between \( g_1(t) \) and \( g_2(t) \) is given as

\[
R_{g_1, g_2}(\tau) = \frac{1}{2} \left\{ R_{a, c}(\tau) + R_{b, d}(\tau) + \omega_4(R_{a, d}(\tau) - R_{b, c}(\tau)) \right\}
\]

where \( R_{a, b}(\tau) \) is the cross-correlation function between \( a(t) \) and \( b(t) \).

3. New constructions of quaternary LCZ/ZCZ sequence set

In this section, new construction methods of quaternary LCZ/ZCZ sequence sets from binary LCZ/ZCZ sequence sets are proposed. We generate two quaternary LCZ/ZCZ sequence sets by applying the Gray mapping to sequences in a binary LCZ/ZCZ sequence set. The parameters of new quaternary LCZ/ZCZ sequence sets are identical to those of binary LCZ/ZCZ sequence set used in the construction.

Using the Gray mapping and Lemma 2.2, we have the following lemma on the correlation property of the pair of quaternary sequences generated from a pair of binary sequences which belong to a binary LCZ sequence set.

**Lemma 3.1.** Let \( N, M, L, \) and \( \epsilon \) be positive integers and \( B = \{ b_i(t) \mid 0 \leq i < M \} \) a binary LCZ sequence set with parameters \((N, M, L, \epsilon)\). Assume that correlation function between all sequences in \( B \) have the same correlation value within the LCZ
except for in-phase autocorrelation. Using two distinct sequences $b_i(t)$ and $b_k(t)$, $i \neq k$, define $u(t)$ and $v(t)$ as

\[
\begin{align*}
u(t) &= \phi[b_i(t), b_k(t)] \\
v(t) &= \phi[b_k(t), b_i(t)].
\end{align*}
\]

Then the correlation function $R_{u,v}(\tau)$ between $u(t)$ and $v(t)$ within the LCZ, $|\tau| < L$, is given as

\[
R_{u,v}(\tau) = \frac{1}{2}\{R_{b_i,b_k}(\tau) + R_{b_k,b_i}(\tau)\} = \epsilon
\]

where $R_{b_i,b_k}(\tau)$ is the correlation function between binary sequences $b_i(t)$ and $b_k(t)$.

**Proof.** By Lemma 2.2, $R_{u,v}(\tau)$ can be represented as

\[
R_{u,v}(\tau) = \frac{1}{2}\{R_{b_i,b_k}(\tau) + R_{b_k,b_i}(\tau)\} + \frac{\omega_4}{2}\{R_{b_i,b_i}(\tau) - R_{b_k,b_k}(\tau)\}.
\]

Since all sequences in $B$ have the same correlation value within the LCZ, $R_{u,v}(\tau)$ can be rewritten as

\[
R_{u,v}(\tau) = \frac{1}{2}\{R_{b_i,b_k}(\tau) + R_{b_k,b_i}(\tau)\} = \epsilon
\]

where $|\tau| < L$. \hfill \Box

Note that ZCZ sequence set is a special case of the above lemma with $\epsilon = 0$. From Lemmas 2.2 and 3.1, a quaternary LCZ sequence set can be constructed from a binary LCZ sequence set as in the following theorem.

**Theorem 3.2.** Let $N$, $M$, $L$, and $\epsilon$ be positive integers and $B = \{b_i(t) \mid 0 \leq i < M\}$ a binary LCZ sequence set with parameters $(N, M, L, \epsilon)$. Assume that correlation function between all sequences in $B$ has the same correlation value within the LCZ except for in-phase autocorrelation. For even $M$, define the quaternary sequence $g_i(t)$ as

\[
g_i(t) = \begin{cases} 
\phi[b_{2i}(t), b_{2i+1}(t)], & \text{for } 0 \leq i < \frac{M}{2} \\
\phi[b_{2(i-M/2)+1}(t), b_{2(i-M/2)}(t)], & \text{for } \frac{M}{2} \leq i < M.
\end{cases}
\]

And for odd $M$, define the quaternary sequence $g_i(t)$ as

\[
g_i(t) = \begin{cases} 
\phi[b_{2i}(t), b_{2i+1}(t)], & \text{for } 0 \leq i < \frac{M-1}{2} \\
\phi[b_{2(i-(M-1)/2)+1}(t), b_{2(i-(M-1)/2)}(t)], & \text{for } \frac{M-1}{2} \leq i < M - 1 \\
\phi[b_{M-1}(t), b_{M-1}(t)], & \text{for } i = M - 1.
\end{cases}
\]

Let $G$ be a sequence set defined by

\[
G = \{g_i(t) \mid 0 \leq i < M\}.
\]

Then $G$ is a quaternary LCZ sequence set with parameters $(N, M, L, \epsilon)$.

**Proof.** It is clear that every sequence in $G$ has period $N$ and $G$ has $M$ sequences. Therefore, what has to be shown is that the magnitude of correlation value $R_{g_i,g_k}(\tau)$ between any two sequences $g_i(t), g_k(t) \in G$ does not exceed $\epsilon$. Let’s think about the case that $M \equiv 0 \mod 2$. In this case, three subcases should be considered.

**Case 1)** $M \equiv 0 \mod 2$ and $0 \leq i, k < M/2$

By Lemma 2.2, $R_{g_i,g_k}(\tau)$ can be rewritten as

\[
R_{g_i,g_k}(\tau) = \frac{1}{2}\{R_{b_{2i},b_{2k}}(\tau) + R_{b_{2i+1},b_{2k+1}}(\tau)\} + \frac{\omega_4}{2}\{R_{b_{2i},b_{2k+1}}(\tau) - R_{b_{2i+1},b_{2k}}(\tau)\}.
\]
Since all correlation values between any sequence pairs in $B$ are the same within the LCZ, we have $R_{b_{2i},b_{2k+1}}(\tau) - R_{b_{2i+1},b_{2k}}(\tau) = 0$. Therefore, for $\tau \not\equiv 0 \mod N$, it is clear that

$$|R_{g_i,g_k}(\tau)| = \left| \frac{1}{2} \left( R_{b_{2i},b_{2k}}(\tau) + R_{b_{2i+1},b_{2k+1}}(\tau) \right) \right| = \epsilon$$

where $|\tau| < L$ and $\tau \not\equiv 0$. For the case of $\tau \equiv 0 \mod N$, we have

$$|R_{g_i,g_k}(\tau)| = \begin{cases} N, & \text{if } i = k \\ \epsilon, & \text{otherwise}. \end{cases}$$

**Case 2)** $M \equiv 0 \mod 2$ and $M/2 \leq i, k < M$

By the similar way to case 1), for $\tau \not\equiv 0 \mod N$, we have

$$|R_{g_i,g_k}(\tau)| = \left| \frac{1}{2} \left( R_{b_{2i},b_{2k}}(\tau) + R_{b_{2i+1},b_{2k+1}}(\tau) \right) \right| = \epsilon$$

where $|\tau| < L$ and $\tau \not\equiv 0$. For the case of $\tau \equiv 0 \mod N$, we have

$$|R_{g_i,g_k}(\tau)| = \begin{cases} N, & \text{if } i = k \\ \epsilon, & \text{otherwise}. \end{cases}$$

**Case 3)** $M \equiv 0 \mod 2$, $0 \leq i < M/2$, and $M/2 \leq k < M$

Using Lemma 2.2, we have

$$|R_{g_i,g_k}(\tau)| = \left| \frac{1}{2} \left( R_{b_{2i},b_{2k}}(\tau) + R_{b_{2i+1},b_{2k+1}}(\tau) \right) \right| = \epsilon$$

where $|\tau| < L$.

Now, let’s think about the case that $M \equiv 1 \mod 2$. In this case, we also should consider the following three subcases.

**Case 4)** $M \equiv 1 \mod 2$, $i \not\equiv M - 1$ and $k \not\equiv M - 1$

By the similar way to cases 1)-3) with the result of Lemma 3.1, we have

$$|R_{g_i,g_k}(\tau)| = \epsilon$$

where $|\tau| < L$ and $\tau \not\equiv 0$. For the case of $\tau \equiv 0 \mod N$, we have

$$|R_{g_i,g_k}(\tau)| = \begin{cases} N, & \text{if } i = k \\ \epsilon, & \text{otherwise}. \end{cases}$$

**Case 5)** $M \equiv 1 \mod 2$ and $i = k = M - 1$

Using Lemma 2.2, $|R_{g_i,g_k}(\tau)|$ is rewritten as

$$|R_{g_i,g_k}(\tau)| = |R_{b_{M-1},b_{M-1}}(\tau)|.$$

Since $b_{M-1}(t)$ is an element of $B$, it is clear that $|R_{g_i,g_k}(\tau)| = N$ for $\tau \equiv 0 \mod N$, and $|R_{g_i,g_k}(\tau)| = \epsilon$, $|\tau| < L$ and $\tau \not\equiv 0$, otherwise.

**Case 6)** $M \equiv 1 \mod 2$, $i \not\equiv M - 1$ and $k = M - 1$

Applying Lemma 2.2, we have

$$R_{g_i,g_k}(\tau) = \begin{cases} \{ R_{b_{M-1},b_{2k}}(\tau) + R_{b_{M-1},b_{2k+1}}(\tau) \}/2, & \text{for } 0 \leq k < (M - 1)/2 \\ \{ R_{b_{M-1},b_{2k-((M-1)/2)+1}}(\tau) + R_{b_{M-1},b_{2k-((M-1)/2)}}(\tau) \}/2, & \text{for } (M - 1)/2 \leq k < M - 1. \end{cases}$$
Table 1. A list of binary LCZ/ZCZ sequence sets with constant correlation values in LCZ/ZCZ.

| Authors | N  | M  | \(L\) | \(\epsilon\) |
|---------|----|----|--------|--------|
| Long, Zhang, and Hu [13] | \(2^n - 1\) | \(< 2^n - 1\) | \((2^n - 1)/(2^n - 1)\) | 1 |
| Jang, No, Chung, and Tang [9] | \(2^n - 1\) | \(2^n - 1\) | \((2^n - 1)/(2^n - 1)\) | 1 |
| Tang and Udaya [19] | \(2^n - 1\) | \(2^n - 1\) | \((2^n - 1)/(2^n - 1)\) | 1 |
| Hayashi [6] | \(2Lk(2n + h)\) | \(2n\) | \(2kL - 1\) | 0 |
| Deng and Fan 1 [2] | \(4^nL_0M_0\) | \(2^nM_0\) | \(2^nL_0\) | 0 |
| Deng and Fan 2 [2] | \(2^{2n-1}L_0M_0\) | \(2^nM_0\) | \(2^nL_0\) | 0 |
| Deng and Fan 3 [4] | \(2L_M1\) | \(M_1\) | \(L_1\) | 0 |
| Hayashi [5] | \(2^{2n+3}mn\) | \(4mn\) | \(2^{n+1}\) | 0 |

From the results of case 1 – case 6), \(\mathcal{G}\) is a LCZ sequence set with parameters \((N, M, L, \epsilon)\). 

It is possible to construct another quaternary LCZ sequence set by using binary LCZ sequences in a LCZ sequence set \(\mathcal{B}\) and their complementary sequences as in the following theorem.

**Theorem 3.3.** Let \(N, M, L, \text{ and } \epsilon\) be positive integers and \(\mathcal{B} = \{b_i(t) \mid 0 \leq i < M\}\) a binary LCZ sequence set with parameters \((N, M, L, \epsilon)\). Assume that correlation function between all sequences in \(\mathcal{B}\) have the same correlation value within the LCZ except for in-phase autocorrelation. In the case of even \(M\), define the quaternary sequence \(g_i(t)\) as follows

\[
\text{if } t < \frac{M}{2} \quad g_i(t) = \left\{ \begin{array}{ll}
\phi[b_2(i, t), b_{2i+1}(t) + 1], & \text{for } 0 \leq i < M/2 \\
\phi[b_2(i-M/2, t), b_{2i-M/2}(t) + 1], & \text{for } M/2 \leq i < M.
\end{array} \right.
\]

And in the case of odd \(M\), define the quaternary sequence \(g_i(t)\) as follows

\[
\text{if } t < \frac{M-1}{2} \quad g_i(t) = \left\{ \begin{array}{ll}
\phi[b_2(i, t), b_{2i+1}(t) + 1], & \text{for } 0 \leq i < (M-1)/2 \\
\phi[b_2(i-(M-1)/2, t), b_{2i-(M-1)/2}(t) + 1], & \text{for } (M-1)/2 \leq i < M - 1 \\
\phi[b_{M-1}(t), b_{M-1}(t) + 1], & \text{for } i = M - 1.
\end{array} \right.
\]

Let \(\mathcal{G}\) be a sequence set defined by

\(\mathcal{G} = \{g_i(t) \mid 0 \leq i < M\}\).

Then \(\mathcal{G}\) is a quaternary LCZ sequence set with parameters \((N, M, L, \epsilon)\).

The proof of the above theorem is straightforward from the following lemma.

**Lemma 3.4.** Let \(N\) be an even integer and \(b(t)\) a sequence with period \(N\). Let define quaternary sequences \(\overline{g}(t)\) as follows

\[
\overline{g}(t) = \phi[a(t), b(t) \oplus 1]
\]

where \(\oplus\) is addition modulo 2. Then autocorrelation function \(R_{\overline{g}}(\tau)\) is given as

\[
R_{\overline{g}}(\tau) = \frac{1}{2} \left\{ R_a(\tau) + R_b(\tau) \right\} + \frac{\omega_4}{2} \left\{ R_{b,a}(\tau) - R_{a,b}(\tau) \right\}.
\]

**Proof.** From Lemma 2.2, it is clear that

\[
R_{\overline{g}}(\tau) = \frac{1}{2} \left\{ R_a(\tau) + R_{b\oplus 1}(\tau) + \omega_4 R_{a,b\oplus 1}(\tau) - R_{b\oplus 1,a}(\tau) \right\}.
\]
And $R_{b\oplus 1}(\tau)$, $R_{a,b\oplus 1}(\tau)$, and $R_{b\oplus 1,a}(\tau)$ can be rewritten as

$$R_{b\oplus 1}(\tau) = \sum_{t=0}^{N-1} (-1)^b(t)\oplus 1 + b(t+\tau)\oplus 1 = \sum_{t=0}^{N-1} (-1)^{b(t)+b(t+\tau)} = R_b(\tau)$$

$$R_{a,b\oplus 1}(\tau) = \sum_{t=0}^{N-1} (-1)^a(t)+b(t+\tau)\oplus 1 = -\sum_{t=0}^{N-1} (-1)^a(t)+b(t+\tau) = -R_{a,b}(\tau)$$

$$R_{b\oplus 1,a}(\tau) = \sum_{t=0}^{N-1} (-1)^b(t)\oplus 1 + a(t+\tau) = \sum_{t=0}^{N-1} (-1)^{b(t)+a(t+\tau)} = -R_{b,a}(\tau).$$

Therefore, $R_\pi(\tau)$ can be rewritten as

$$R_\pi(\tau) = \frac{1}{2} \left( R_a(\tau) + R_b(\tau) \right) + \frac{\omega_4}{2} \left( R_{b,a}(\tau) - R_{a,b}(\tau) \right).$$

Table 1 shows a list of binary LCZ sequence sets which satisfy the constraint of constant correlation values in Theorems 3.2 and 3.3.

**Remark 1.** It is clear that a quaternary ZCZ sequence set can be constructed from a binary ZCZ sequence set using Theorems 3.2 and 3.3 since the correlation values of ZCZ sequences set is the same as zero in ZCZ by definition. Some examples of binary ZCZ sequence sets are also included in Table 1.

As in Table 1, the parameters of the constructed binary ZCZ sequence sets satisfy the relation $2ML \leq N$ instead of $ML \leq N$ and thus it is not possible to obtain optimal quaternary ZCZ sequence sets from them. Also note that there are a few construction method of almost optimal quaternary ZCZ sequence sets [18], [20].

However, there is no reason to assume that an optimal ZCZ sequence set does not exist. Even though there is no example of optimal binary ZCZ sequence set to the best of our knowledge, it is able to construct an optimal quaternary ZCZ sequence set once an optimal binary ZCZ sequence set is given.

Since the parameters of new proposed quaternary LCZ/ZCZ sequence sets are exactly the same as those of the binary LCZ/ZCZ sequence sets, the optimality of the new quaternary LCZ/ZCZ sequence sets is directly determined by the employed binary LCZ/ZCZ sequence set.

**Example 1.** From [9], we can construct an optimal binary LCZ sequence set $\mathcal{B} = \{b_i(t) \mid 0 \leq i \leq 6\}$ with parameters $(63, 7, 9, 1)$ where

- $b_0(t) = 01000101101110100100100001100001$,
- $b_1(t) = 00110001000011110011100110000010$,
- $b_2(t) = 0111001011001101000001$,
- $b_3(t) = 00111100010000110011100110000010$,
- $b_4(t) = 010010100101000000011011100000011$,
- $b_5(t) = 00101001001100000001101110000011$,
- $b_6(t) = 011101101100000010110000011100111011.$
Using Theorem 3.2, we have an optimal quaternary LCZ sequence set \( Q_1 = \{ g_i(t) | 0 \leq i \leq 6 \} \) with parameters (63, 7, 9, 1) where
\[
\begin{align*}
g_0(t) &= 03110302303321213013121310211303320022323100201112201223201 \\
g_1(t) &= 0322121030230302203222123303331321011122031001132310020013112 \\
g_2(t) &= 030021213013121010211030320322323130332112201223230100113320 \\
g_3(t) &= 01330102101112323103313231023101200221213002033322032221203 \\
g_4(t) &= 012232010211010221223211031113123033320213003121300313320 \\
g_5(t) &= 01002323103113230302330101201212131021133220322013003311120 \\
g_6(t) &= 02220222200220200000022000002200200022002002220202202222.
\end{align*}
\]

As an instance, we represent the cross-correlation between \( g_0(t) \) and \( g_1(t) \) as
\[
R_{g_0 g_1}(\tau) = \begin{cases} 
31 + j16, & \text{if } \tau \equiv 9 \mod 63 \\
-1 - j16, & \text{if } \tau \equiv 18 \mod 63 \\
-1 + j16, & \text{if } \tau \equiv 27 \mod 63 \\
-1 - j32, & \text{if } \tau \equiv 36 \mod 63 \\
15 + j16, & \text{if } \tau \equiv 45 \mod 63 \\
15, & \text{if } \tau \equiv 54 \mod 63 \\
-1, & \text{otherwise.}
\end{cases}
\]

Using Theorem 3.3, we have an optimal quaternary LCZ sequence set \( Q_2 = \{ \overline{g}_i(t) | 0 \leq i \leq 6 \} \) with parameters (63, 7, 9, 1) where
\[
\begin{align*}
\overline{g}_0(t) &= 1200121321222303021022030201300210133232011311003310332310 \\
\overline{g}_1(t) &= 1233030121321231321333032210222030100303132011002231 \\
\overline{g}_2(t) &= 1211303021022030101300121231233232021322000331033312011002231 \\
\overline{g}_3(t) &= 102210103000323202012320100311330302113112223312330312 \\
\overline{g}_4(t) &= 103323210130011313033332300120002032122231302112203021131120223 \\
\overline{g}_5(t) &= 101132320120232121321013033030201300222312331302112200031 \\
\overline{g}_6(t) &= 13333133313313311111313113113113133133113311331133133133.
\end{align*}
\]

Similarly, we can represent the cross-correlation between \( \overline{g}_0(t) \) and \( \overline{g}_1(t) \) as
\[
R_{\overline{g}_0 \overline{g}_1}(\tau) = \begin{cases} 
31 - j16, & \text{if } \tau \equiv 9 \mod 63 \\
-1 + j16, & \text{if } \tau \equiv 18 \mod 63 \\
-1 - j16, & \text{if } \tau \equiv 27 \mod 63 \\
-1 + j32, & \text{if } \tau \equiv 36 \mod 63 \\
15 - j16, & \text{if } \tau \equiv 45 \mod 63 \\
15, & \text{if } \tau \equiv 54 \mod 63 \\
-1, & \text{otherwise.}
\end{cases}
\]

4. CONCLUDING REMARKS

In this paper, we propose two construction methods of quaternary LCZ/ZCZ sequence sets by using binary LCZ/ZCZ sequence sets. An interesting aspect of this paper is that the parameters of binary LCZ/ZCZ sets are handed down to
the generated quaternary LCZ sequences. Therefore, if we have an optimal binary LCZ/ZCZ sequence set, then we can also get an optimal quaternary LCZ/ZCZ sequence set. An obvious application of the proposed quaternary LCZ sequence is signature codes for QS-CDMA systems which adopt QPSK modulation. Although it is possible to use two binary sequences to form a quaternary sequence via the Gray mapping, the properties of correlation function of the binary sequences do not preserve. Therefore, it can be considered that the contribution of this paper is to provide a clever way of utilizing given binary sequences in quaternary applications.

ACKNOWLEDGEMENTS

The authors would like to thank the referees very much for their valuable comments and suggestions.

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Received November 2008; revised March 2009.

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