Tunneling into strongly biased Tomonaga-Luttinger liquid

M. Trushin\textsuperscript{1,2} and A. L. Chudnovskiy\textsuperscript{1(a)}

\textsuperscript{1} Institut f"ur Theoretische Physik, Universit"at Hamburg - Jungiusstr 9, D-20355 Hamburg, Germany, EU
\textsuperscript{2} Institut f"ur Theoretische Physik, Universit"at Regensburg - D-93040 Regensburg, Germany, EU

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Abstract – We calculate the tunneling density of states for a Tomonaga-Luttinger liquid placed under a strong bias voltage. For the tunneling through a side-coupled point contact, one can observe the power law singularities in the tunneling density of states separately for the right- and left-movers despite the point-like tunnel contact. The calculated nonequilibrium tunneling exponents differ strongly from the equilibrium case.

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Introduction. – Power law suppression of the tunneling density of states into the Tomonaga-Luttinger liquid is one of the most profound manifestations of interactions in one-dimensional (1D) electron systems [1]. It paves the way to get the information about interactions in 1D systems experimentally [2,3]. Exact calculation of the tunneling density of states is possible in the framework of the Tomonaga-Luttinger liquid (TLL) model using the bosonization method [4,5]. The basis of the TLL model is the linearization of one particle spectrum around the right and left Fermi points. At the same time, a proper account for the finite curvature of the one-particle dispersion is necessary for the theoretical description of high-energy excitations in 1D electron systems, and particularly in strongly nonequilibrium 1D systems [6]. The problem considered in this letter also relates to the rectification in conductors having asymmetric $I-V$ curves such as Luttinger liquids in the presence of an asymmetric potential (the ratchet effect) [7].

In this letter we consider tunneling from the Fermi-liquid reservoir into the nonequilibrium TLL through a point tunnel contact (see fig. 1). The nonequilibrium conditions are created by a strong transport voltage $V_{sd}$ applied to a TLL channel. In the equilibrium the whole system is filled by electrons up to the Fermi energy $E_F$. Finite source-drain voltage results in the shift of the chemical potentials for the right- and left-movers to the quasi-Fermi energies $E_F + eV_{sd}/2$ and $E_F - eV_{sd}/2$, respectively. At strong enough voltages, the nonlinearity of the electronic dispersion leads to different Fermi velocities of the right- and left-movers $v_{FR,L} = \sqrt{(2E_F \pm eV_{sd})/m^*}$, as depicted in fig. 1. (Here, $m^*$ is the effective electron mass.) In turn, the tunneling densities of states for the left- and right-moving spectral branches differ. Furthermore, since the directions of partial tunneling currents into the left branch and out of the right branch are opposite, these two tunnel currents do not compensate any more even at zero voltage $V_{pc}$ at the point contact (see fig. 1). Therefore, a finite tunnel current flows between the nonequilibrium TLL and the reservoir. This current depends as a power law both on the source-drain voltage in TLL $V_{sd}$ and on the voltage on the point contact $V_{pc}$, with the exponent reflecting the interaction strength in TLL. The nonequilibrium tunneling exponent differs from the one in...
dependence of the current through the point contact on 
the equilibrium TLL, growing with the bias voltage. The 
M. Trushin and A. L. Chudnovskiy 
influence of the rapidly oscillating terms is already very 
for the left- and right-movers. At those energies, the 
trons and holes lie therefore close to the quasi-Fermi levels 
The corresponding tunneling exponents are determined by 
low-energy excitations. The energies of the excited elec-
trons and holes lie therefore close to the quasi-Fermi levels 
for the left- and right-movers. At those energy scales, the 
influence of the rapidly oscillating terms is already very 
small and can be neglected.

For the description of the tunneling it proves conve-
nient to perform a gauge transformation \( \psi_R(L) (x) \rightarrow \hat{U} (x) \psi_R(L) (x) \hat{U} (x) \). The matrix \( \hat{U} \) can be represented as \( \hat{U} (x) = \hat{U}_R(x) \hat{U}_L(x) \) with 
\[
\hat{U}_R(L)(x) = \exp \left[ - \frac{i e V_{sd}}{h v_{F,R}} \int dx' \psi_{R(L)}^\dagger (x') \psi_{R(L)} (x') \right].
\]

This gauge transformation absorbs the quasi-Fermi en-
ergies into the spatial dependence of the transformed 
fields.

Using the bosonization identity, the total Hamil-
tonian (1) can be written in a bosonized form often 
encountered in the literature, namely 
\[
H = \frac{h v_{F,R}}{2} \int \frac{d x}{2 \pi} \partial_x \phi^T \left( \begin{array}{cc} 1 + g_4 & g_2 \\ g_2 & g + g_4 \end{array} \right) \partial_x \phi,
\]

where \( \phi^T (\phi_R(x), \phi_L(x)) \) is the bosonic field corre-
sponding to the fermions in TLL. Here the terms with 

\begin{align}
\text{Diagonalization of the Hamiltonian.} \quad & \text{The conventional way to diagonalize the Hamiltonian of any interacting system in the framework of the Tomonaga-Luttinger model is to introduce so-called \textit{“dual fields”}. The standard couple of \textit{“dual fields”} is defined as a difference and a sum between boson fields with opposite chiralities [5]. However, because of the different Fermi velocities for right- and left-moving electrons, the intro-
duction of dual fields cannot be made in that direct manner for the problem at hand. Rather, we introduce adi-
tional fictitious bosonic fields \( \phi_R (x), \phi_L (x) \) in a way which does not change the dynamics of the system. Then, in terms of the four-component bosonic fields \( B = (\phi_R, \phi_L, \phi_R', \phi_L')^T \), the Hamiltonian (4) can be rewritten as 
\[
H = \frac{h v_{F,R}}{2} \int \frac{d x}{2 \pi} (\partial_x \Phi)^T \left( \begin{array}{cc} 1 + g_4 & g_2 \\ g_2 & g + g_4 \end{array} \right) (\partial_x \Phi).
\]

Note that there is no coupling between the original and fictitious fields, which guarantees that the dynamics of the original fields remains unchanged. Using the fact of the chiral symmetry between the original and fictitious branches we form the two couples of dual fields out of \( \phi_{R,L} \) and \( \phi_{R,L}' \) which read 
\[
\Phi_1 = \frac{1}{\sqrt{2}} (\phi_L' + \phi_R), \quad \Theta_1 = \frac{1}{\sqrt{2}} (\phi_L' - \phi_R);
\]
\[
\Phi_2 = \frac{1}{\sqrt{2}} (\phi_L + \phi_R'), \quad \Theta_2 = \frac{1}{\sqrt{2}} (\phi_L - \phi_R').
\]

Substituting (6), (7) into the Hamiltonian (5) we obtain 
\[
H = \frac{h v_{F,R}}{2} \int \frac{d x}{2 \pi} \left[ (\partial_x \Phi)^T M_\Phi (\partial_x \Phi) + (\partial_x \Theta)^T M_\Theta (\partial_x \Theta) \right],
\]

where 
\[
M_{\phi, \Theta} = \left( \begin{array}{cc} 1 + g_4 & \pm g_2 \\ \pm g_2 & g + g_4 \end{array} \right),
\]

the upper and lower signs corresponding to \( M_\Phi \) and \( M_\Theta \), respectively, and \( \Phi = (\Phi_1, \Phi_2)^T, \Theta = (\Theta_1, \Theta_2)^T \).

Hamiltonian (5) can be brought to the canonical form 
applying a composition of unitary rotations and rescalings 
that preserve conformal invariance. We first diagonalize 
the matrix \( M_\Phi = P_\Phi A_\Phi P_\Phi^{-1} \) by a unitary rotation of the
fields with a matrix $P_\Phi$. Then we rescale the fields while preserving duality relations $(P_\Theta \Phi_i) \rightarrow (P_\Theta \Phi_i) / \sqrt{\lambda_\Phi}$, $(P_\Phi \Theta_j) \rightarrow \sqrt{\lambda_\Phi} (P_\Phi \Theta_j)$, so that after the rescaling the fields $\Phi_i$ are coupled by the unity matrix, and the Hamiltonian acquires the form

$$H = \frac{\hbar v_R}{2} \int \frac{dx}{2\pi} \left[ (\partial_x \Phi^T) \mathbf{1} (\partial_x \Phi) + (\partial_x \Theta^T) \tilde{M}_\Theta (\partial_x \Theta) \right],$$

(10)

where $\tilde{M}_\Theta = \sqrt{\lambda_\Phi} P_\Phi^{-1} M_\Theta P_\Phi \sqrt{\lambda_\Phi}$.

Further we diagonalize the quadratic form with the fields $\Theta$ by a unitary rotation with a matrix $P_\Theta$, i.e. $P_\Theta^{-1} \tilde{M}_\Theta P_\Theta = \Lambda_\Theta$. The unity matrix that couples the one-particle Green’s function of an electron in the TLL.

The tunneling matrix element describing the point contact, and $\nu$ is the Fermi distribution at zero chemical potential. The electron density of states in a 2DEG $\nu_{2DEG}$ is just an energy-independent constant. The tunneling density of states $\nu_{R(L)}(e)$ is calculated from the one-particle Green’s function of an electron in the TLL.

Using the bosonization identity $\psi_{R(L)}(x) \sim e^{i\phi_{R(L)}}$, eqs. (6), (7), and the relations between the initial and canonical bosonic fields given by eqs. (12), we obtain the imaginary-time one-particle Green’s function in the form

$$G_{R(L)}(x, \tau; x, 0) = \langle \hat{T} \psi_{R(L)}(x, \tau) \psi^+_{R(L)}(x, 0) \rangle = \left( \frac{\nu_0}{(1/2) [a/(u_1 \tau)]^{b_{11}(21)} [a/(u_2 \tau)]^{b_{12}(22)}} \right)^{1/2},$$

(16)

where $b_{ij} = \frac{1}{2} \left( \langle \Delta^2 \rangle_{\nu} + \langle \Delta^2 \rangle_{\nu} \right)$. Performing analytical continuation to real time and Fourier transformation [10], we finally obtain the chiral tunneling density of states in the form

$$\nu_{R(L)}(\omega) = \frac{1}{2\pi^2 \hbar} \left( \frac{\nu_0}{(1/2) [a/(u_1 \tau)]^{b_{11}(21)} [a/(u_2 \tau)]^{b_{12}(22)}} \right)^{1/2} \times \left( \sqrt{\frac{\pi}{b_{11}(21)}} + \sqrt{\frac{\pi}{b_{12}(22)}} \right) \Gamma (1 - b_{11}(21) - b_{12}(22)),$$

(17)

where $\omega$ is the short-length cutoff, and $\omega = \varepsilon / \hbar$.

The nonequilibrium tunneling density of states can be best seen in the measurements of the differential conductances $\partial I_{pc} / \partial V_{pc}$ at small voltages $V_{pc}$ on the point contact. The expression for the differential conductance can be obtained straightforwardly from eqs. (14) and (15). The result at zero temperature can be written in the form

$$\frac{\partial I_{pc}}{\partial V_{pc}} = \frac{4\pi e^2 t^2}{\hbar} \nu_{2DEG} \times \left[ \nu \left( \frac{eV_{sd}}{2} + eV_{pc} \right) \pm \nu (eV_{pc} - \frac{eV_{sd}}{2}) \right],$$

(18)

where $\nu = 1/2$ for $\partial I_{pc} / \partial V_{sd}$ and $\zeta = 1$ for $\partial I_{pc} / \partial V_{pc}$. In general, the dependence of the differential conductance on $V_{sd}$ and on $V_{pc}$ is smooth and is determined not only by the power law singularity in (17) but also by the dependence of $g$ on $V_{sd}$. The latter makes the powers $b_{ij}$ and the velocities $u_{ij}$ dependent on the voltage, in accordance with eq. (13). The power law singularities of nonequilibrium chiral densities of states can still be seen in the differential conductance at $V_{pc} = \pm V_{sd}/2$, as it follows from (18). At these voltages, the Fermi level in the Fermi liquid reservoir coincides with the quasi-Fermi energy for the left- or the right-moving fermions in TLL, and the tunneling density of states in the corresponding channel is suppressed. These singularities are illustrated in fig. 2 for $\partial I_{pc} / \partial V_{pc}$ at different values of $g_2$ which characterizes the screening of electron-electron interactions. Similar anomalies in the density of states can also be seen in the two-terminal transport through TLL [11]. At strong screening, when $V(q = 0) \geq V(q = k_B \omega - k_F \omega)$ and hence $g_2 \ll g_4$, the differential conductance exhibits a sharp dip close to $V_{pc} = \pm V_{sd}/2$. At weaker screening ($g_2$ closer to $g_4$) the conductance is getting smaller, and the dip is essentially broadened.
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The application of such high bias voltages lies within the range of typical GaAs-based electron gases: m' = 0.067m_e, a = 5.65 Å, E_F = 30 meV. Note that the absolute value of the necessary bias voltage can be strongly diminished by lowering the Fermi energy.

Fig. 2: (Color online) Differential conductance ∂I_pc/∂V_pc in units of 8πe² |t|^2 ν_{2DEG} ν_{1D} / πh (ν_{1D} being the density of states at the Fermi level of the unbiased noninteracting 1D channel) as a function of voltage at the point contact for different values of g_2 at a given g_1. The conductance dependences ∂I_pc/∂V_{sd}(v_{pc}) exhibit singularities at V_{pc} = ±V_{sd}/2 (see eq. (18)). The bias voltage V_{sd} is taken equal to 0.1E_F, and the other parameters are relevant for typical GaAs-based electron gases: m' = 0.067m_e, a = 5.65 Å, E_F = 30 meV. Note that the absolute value of the necessary bias voltage can be strongly diminished by lowering the Fermi energy.

Furthermore, since b_{11} = b_{22} and b_{21} = b_{12} for any parameters g and g_{2,4}, the energy dependences of ν_{1R} and ν_{1L} are the same. There is however a difference in the prefactors that depend on the powers of plasmon velocities v_{1} and v_{2}. This asymmetry is maximal at b_{12(21)} = 0 (i.e. when g_2 = 0). In that case the only effect of interactions consists of the renormalization of the plasmon velocities, while the singularity in the tunneling density of states disappears. At larger g_2 (i.e. b_{12(21)} ≠ 0) the electrons with opposite chiralities interact with each other which leads to the alignment of the chiral tunneling densities of states ν_{1R} and ν_{1L}, as one can see from eq. (17). The dependence of the tunneling exponent α = b_{11} + b_{12} - 1 = b_{21} + b_{22} - 1 on the asymmetry of the Fermi velocities is shown in fig. 3. The power α increases with the growth of asymmetry (smaller g) from its equilibrium value at g = 1. Therefore, the singularity in the tunneling density of states becomes stronger with the increase of the bias voltage V_{sd}. Interestingly, the finite size of the TLL also leads to the increase of the exponent of a power in the tunneling density of states [12].

Another interesting feature of the system considered is the finite current through the point contact even at vanishing V_{pc}. This is again due to the chiral asymmetry of the plasmon spectrum subject to the bias voltage V_{sd}. The existence of a finite current follows directly from eq. (18), but in order to ease the understanding of that fact, we rewrite ∂I_{pc}/∂V_{sd} in the limiting case of V_{pc} = 0 and strong screening when g_2 = 0. The latter leads to

b_{12} = b_{21} = 0, b_{11} = b_{22} = 1, w_1 = ν_{1R}(1 + g_4), w_2 = ν_{1L} + ν_{1R} g_4, and the differential conductance assumes the form

\frac{∂I_{pc}}{∂V_{sd}} = \frac{e^2 |t|^2 ν_{2DEG}(ν_{1R} - ν_{1L})}{ℏ^2 (ν_{1L} + ν_{1R} g_4)(ν_{1R} + ν_{1R} g_4)}. \tag{19}

From eq. (19) it is clear that, on the one hand, the tunneling current into the TLL can be suppressed due to the electron-electron interactions. On the other hand, one can facilitate the tunneling applying the bias voltage V_{sd} to the TLL. We emphasize that one does not need to change V_{pc}. Since high voltages at the point contact are not always possible in the linear response measurements, the biasing of the TLL might be a powerful tool to study its transport properties.

Conclusions. – In conclusion, we showed that in the experiment on the tunneling into a strongly biased TLL through a point contact, the power law singularities in the tunneling densities of states can be seen separately for the right- and left-movers. We obtained analytical expressions for the chiral tunneling densities of states that turn out to be different from the equilibrium case. The predicted behavior can be observed in experiments with GaAs quantum wires of nominal width ~14 nm, where the application of a strong bias voltage does not cause the population of the next one-dimensional subband. The fabrication of such wires (as well as the application of such high bias voltages) lies within the range of current experiments [3,13]. We also developed a method of diagonalization of the TLL Hamiltonian with different

Fig. 3: (Color online) The dependence of the tunneling exponent \( α = b_{11} + b_{12} - 1 = b_{21} + b_{22} - 1 \) on the asymmetry of the Fermi velocities \( ν_{1R} \) and \( ν_{1L} \) expressed through the parameter \( g = ν_{1L}/ν_{1R} \). The unbiased TLL corresponds to \( g = 1 \). The asymmetry increases (g diminishes) increasing the bias voltage \( V_{sd} \). The suppression of the tunneling density of states grows with bias voltage. Inset: differential conductance \( ∂I_{pc}/∂V_{pc} \) as a function of voltage at the point contact for different values of \( V_{sd} \) at a given \( g_2 = g_4 = 0.5 \). The other parameters are taken the same as for fig. 2.
Fermi velocities for the right- and left-movers. The method can be useful for a number of problems which involve chiral asymmetry of the density of states such as a TLL wire in an external magnetic field.

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