QCD with many fermions and QCD topology

Edward Shuryak
Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY 11794 USA
E-mail: Edward.Shuryak@stonybrook.edu

Abstract. Major nonperturbative phenomena in QCD – confinement and chiral symmetry breaking – are known to be related with certain topological objects. Recent lattice advances into the domain of many \( N_f = O(10) \) fermion flavors have shown that both phase transitions had shifted in this case to much stronger coupling. We discuss confinement in terms of monopole Bose condensation, and discuss how it is affected by fermions “riding” on the monopoles, ending with the \( N_f \) dependence of the critical line. Chiral symmetry breaking is discussed in terms of the (anti)selfdual dyons, the instanton constituents. The fermionic zero modes of those have a different meaning and lead to strong interaction between dyons and antidyons. We report some qualitative consequences of this theory and also some information about our first direct numerical study of the dyonic ensemble, in respect to both chiral symmetry breaking and confinement (via back reaction to the holonomy potential).

1. Lattice advances into the domain of multi-flavor QCD

Let us start by reviewing recent advances in lattice data using our own version of the phase diagram, see Fig.1. We plot the “critical lattice coupling at scale \( T_c \)” \( \beta_c(T_c) = 2N_c/g^2(T_c) \) as a function of \( N_f \). The “bare” coupling values reported in lattice works (defined at the lattice UV scale \( a \)) are evolved (by the two-loop beta function) by a factor \( N_t \) (the number of points in the time direction) to the physical scale \( N_t a = 1/T_c \). The near overlapping points in the figure are from different lattice \( N_t \): their spread is a measure of the inaccuracy of the two-loop beta function used.

The lines at the right show trajectory of the infrared fixed point in various perturbative approximations: but since the bottom of the plot corresponds to strong coupling, the real curve there can deviate from the perturbative trend.

The open boxes correspond to lattice data from Ref. [1]: they show the critical line of the chiral restoration. For completeness we’ve also displayed (by red diamonds) results from few other lattice groups: a spread is due to different lattice actions used. The qualitative trend with \( N_f \) is clear: the curve is moving downward with \( N_f \), which means the phase transitions is happening at stronger gauge coupling. Thus going to \( N_f \sim 10 \) produces “the most strongly coupled QGP”, which is by itself a very interesting phenomenon. Why is it happening?

Let us add that the situation at \( N_f = 12 \) is special. Ref. [2] argued that chiral symmetry remains unbroken. More recent paper [3] have studied the region of even stronger coupling \( 6/g^2 \sim 1 \), and perhaps clarified the picture. They do observe two distinct transitions, with chiral condensate disappearing at stronger coupling than deconfinement, with a novel intermediate phase in between. Furthermore, those phases are separated by a bulk transition from weaker coupling domain, in which there seem to exist a conformal (infrared fixed point) behavior. If so,
the two phase transitions must be below (on the other side of) the true trajectory of the fixed point.

2. Confinement and unusual confinement

Pioneering works by 't Hooft and Mandelstam proposed the so called “dual superconductor” model, which attributed it to Bose-Einstein condensation (BEC) of color magnetic monopoles. Seiberg and Witten [4] had shown how it works in the $\mathcal{N}=2$ supersymmetric theories. Monopoles in certain gauges were identified on the lattice, and the so called “Pisa operator” inserting them into vacuum has been shown to have a nonzero VEV at $T < T_c$ [5]. Approaching $T_c$ from above, one had observed $k$-clusters of identical monopoles exchanging places [6], as all bosons do near a BEC transition. Thus there is little doubt in my mind that the “dual superconductor” picture is correct. (As a digression let me mention here that even up to $T \sim 2 - 3 T_c$ studies in heavy ion collisions at RHIC/LHC monopoles remain important component of the plasma [7], even significantly contributing to its record-small viscosity [8].)

The main issue to be addressed below is why fermions apparently affect the deconfinement transition line so strongly. But before we go into it in more detail, let me also mention another general question I raised in a note [9], if “unusual confinement phases” are possible, in which BEC of magnetic objects with quantum numbers different from that of a single monopole can be formed instead. (In such case one would of course expect the phase transitions between different confining phases, if their order parameters do not have the same quantum numbers.)

In fact such examples are known in supersymmetric QCD literature. Seiberg-Witten $\mathcal{N}=2$ QCD with $N_f = 3$ had found two possible confinements: one for (i) a quartet of states with magnetic charge $n_m = 1$ and electric charge $n_e = 0$; and (ii) a singlet with charges $n_m = 2, n_e = 1$. The former automatically leads to confinement with the chiral symmetry breaking, and the latter has confinement without it. Dynamics of the latter case is especially interesting, as it is related to BEC of a two-monopole composite bound by fermions. Note further

![Figure 1](image.png)

**Figure 1.** Dependence of the critical lattice coupling $\beta_c$ at scale $T_c$ versus the number of fundamental quark flavors $N_f$ in QCD-like theories. Blue boxes are from [1] with near-coincident boxes being lattice data for the same $N_f$ with different $N_\tau$ which demonstrate lattice spacing consistency. Red diamonds are from various other literature. The thick blue line is the fitting curve, extended as dashed blue line beyond $N_f = 12$. The black/purple/red dashed curves on the right are lines for vanishing beta function at 1,2,3-loop levels.
that \( N_f = 3 \) is the last value before the conformal window (starting in this case at \( N_f = 4 \)): perhaps it is a hint that something like that should be looked at right before the conformal window also in theories without supersymmetry?

3. Monopoles with fermions

How can the light fermions affect the monopoles? It is known that the light fermions can become attached to them. The so called “fermionic zero modes” are specific bound states in which positive kinetic energy of localization exactly cancels the magnetic-moment-field interaction. These zero modes are known explicitly for ’t Hooft-Polyakov monopoles which are present in gauge theories with adjoint scalars, such as in \( \mathcal{N} = 2 \) supersymmetric gauge theories. Since flipping the charge and the spin leads to the same Dirac equation, antiquarks also have the same zero modes. Furthermore, for each of these zero mode states, it can be either populated or not, so the number of totally available states grows exponentially \( \sim 2^{2N_fN_M} \). Spectroscopy of those states in the supersymmetric setting was developed in 1990’s.\(^1\) There is one zero mode for the fundamental while two for the adjoint (Dirac) fermions: so these monopole-single-fermion states case into spin 0 and \( \frac{1}{2} \) objects, respectively. While in the static case zero mode states are degenerate with the pure monopole, it is not so for the dynamical lattice monopoles with non-static paths.

Now back to confinement/BEC transition. The new idea, from Ref[11], is that if fermions can “ride” on the monopoles, they are giving them flavor indices and thus are making them “distinguishable”, unsuitable for Bose-Einstein Condensation! Such dilution of “unoccupied” monopoles need to be compensated somehow for BEC to happen, and this can only be done by a shift stronger coupling that makes monopoles lighter and more numerous.

A very interesting BEC criterion for an interacting boson ensemble was proposed by Feynman in 1950’s (originally for liquid \( ^4\)He) and was recently revived/generalized to interacting systems by Cristoforetti and Shuryak [10]. In the finite-\( T \) description with periodic paths there appear “\( k \)-clusters” of bosons interchanging their initial (at Matsubara time 0) and final (at Matsubara time \( \beta = 1/T \)) positions. Those clusters are “Feynman polygons” with \( k \) points. Their probability depends on two competing factors, suffering from suppression due to the extra action \( \exp (-kS_{ex}) \) while benefiting from large combinatory number of \( k \)-polygons. The balance point marks the onset of condensation by divergence of the sum over the \( k \)-clusters. According to Feynman, quantitative BEC condition is a un\( \text{universal} \) critical value of the extra action per particle, which in three spatial dimensions is given by \( S_{ex} \leq S_c \approx 1.655 \). Upon fulfilling it, infinitely long sequences of “hopping” bosons will occur, creating macroscopic “supercurrents”. Let us now apply the above criteria to the monopole condensation in QCD-like theories. The minimal exchange action \( S_{ex} \) for two nearest-neighbor bosons that are separated by a typical distance \( d = n^{-1/3} \) (with \( n \) the number density) during the Matsubara time from \( \tau = 0 \) to \( \tau = \beta = 1/T \) could be estimated as

\[
S_{ex} = m^* \sqrt{\beta^2 + d^2} - m^* \beta + S_V
\]

with an explicitly written kinetic term, containing an effective mass \( m^* \), and the implicit potential term \( S_V \). When close to condensation, the monopoles are very dense, with typical spatial separation \( d \) comparable or smaller than the inverse temperature \( \beta \), therefore justifying a further approximation of the above expression: \( S_{ex} \approx \frac{1}{2} m^* Td^2 + S_V = \frac{1}{2} \left( \frac{m^*}{T} \right) \left( \frac{n}{n^*} \right)^{-\frac{2}{3}} + S_V \).

\(^1\) “Magnetic supermultiplets” have been constructed and for two famous conformal theories, the \( \mathcal{N}=4 \) SYM and the \( \mathcal{N}=2 \) SQCD with \( N_f = 4 \) one finds exactly the same set of spins/multiplicities as that in the original electric one. Since those theories are electric-magnetic self-dual, with couplings related by \( g \leftrightarrow 1/g \), their coupling cannot run at all!
The term \( S_V = \int_0^{1/T} V(r(\tau))d\tau \) is basically the ratio of the inter-monopole interaction potential and \( T \), also known as classical plasma coupling \( \Gamma_M \sim <V>/T \sim S_V \). As shown in Fig.3b of [7], at high \( T \) where the “magnetic scaling” \( d \sim 1/(g^2T) \) and \( g_{\text{magnetic}} \sim 1/g \) works, this ratio \( \Gamma_M \) does not depend on the coupling or \( T \) and is a constant \( \approx 5 \), while close to condensation it decreases to a value of about 1 as \( T \to T_c \) for \( N_f = 0 \). The following onset condition will then be applied for the monopole condensation in QCD-like theories for the rest of our analysis:

\[
\left( \frac{m^*}{T} \right) \left( \frac{n}{T^3} \right)^{-\frac{2}{3}} \leq \tilde{S}_c \equiv 2(S_c - S_V)
\]  

(2)

The constant \( \tilde{S}_c \) is of order one and its precise value is not needed as long as its \( N_f \)-dependence is negligible. Now, how would the transition get affected by adding light fermions? As already pointed out, the fermions can be attached to some of the monopoles via zero modes and effectively reduce the number of identical monopoles. Consider a monopole with one flavor of light quark added: for each of its allowed zero modes there is a probability for it to be occupied by a fermion or not. Let us assume the ratio of the probabilities (occupied/not-occupied) to be \( f \) (a kind of zero-mode fugacity), we then see that the overall probability for a monopole (with \( N_M \) number of zero modes for each fermion flavor) to stay as a “pure” monopole is simply \( 1/(1+f)^{2N_M} \) (with the factor 2 accounting for both quark and anti-quark contributions for Dirac fermions). So effectively the available density for BEC condensation will be \( n/(1+f)^{2N_fN_M} \). Combined with the BEC condition in Eq.(2) we obtain

\[
\left( \frac{m^*}{T} \right) \left( \frac{n}{T^3} \right) \left( \frac{1 + f}{(n/T^3)^{2/3}} \right)^{4N_fN_M/3} \leq \tilde{S}_c
\]  

(3)

This implies that with increasing \( N_f \), the density \( n \) has to increase and mass \( m^* \) to decrease correspondingly so as to reach the same condensation condition. This pushes the transition to stronger coupling, therefore explaining the \( N_f \)-dependence of the critical coupling in Fig.1.

To make a semi-quantitative estimate, we use the following magnetic scaling relations that connect the monopole mass and density with gauge coupling: \( m^*/T \sim 1/g \) and \( n^{1/3}/T \sim g^2 \) [7]. Combined with the above condition, we obtain the critical gauge coupling for monopole condensation to be: \( \beta(N_f) = \beta_0(1 + f)^{4N_fN_M/15} \) where \( \beta_0 \) is the corresponding critical coupling for pure gauge case. This can be further converted to the lattice coupling \( \beta = 2N_c/g^2 \):

\[
\beta_c(N_f) = \beta_0(1 + f)^{-8N_fN_M/15}
\]  

(4)

For fundamental fermions (with \( N_M = 1 \)) the critical (lattice) gauge coupling, \( \beta_c \equiv \beta(T = T_c) = 2N_c/g^2(T_c) \) as a function of \( N_f \), the resulting curve is shown in fig.1. We’ve used the above Eq.(4) to make a fit for these data (blue boxes) and obtained the optimal value \( f \approx 0.154 \); the fitting curve is shown as the thick blue line in Fig.1. Our model formula in Eq.(4) with one parameter nicely describes all the data points in [1] from \( N_f = 0 \) to \( N_f = 12 \)! The suppression factor \( f \) of monopole-fermion as compared with pure monopole may be understood as follows: the monopole-fermion has color-electric charge, and in the near-\( T_c \) plasma it was known from previous studies of the “electric mass” on the lattice, telling us that the electric particles are heavier than the magnetic particles by roughly \( \Delta M \sim 2T_c \), thus leading to a suppression factor \( \sim e^{-\Delta M/T} \sim 0.135 \) which is fairly close to \( f \approx 0.154 \).

It is well known that the \( N_c = 2 \) theory is a very special case, with extra symmetry between quarks-antiquarks and mesons-diquarks. It also allows the finite density lattice simulations without the “sign problem”. Lattice study of this theory was recently extended to the low-\( T \) finite-\( \mu \) region with \( N_f = 2, 4 \) quarks by Hands, et al [12]. The quark density (per flavor) shown in their Fig.3 displays (i) a structure at \( \mu \approx m_\pi/2 \) due to rotation from the \( \bar{\psi}\psi \) to
diquark condensate; (ii) the usual quark Fermi sphere at higher \( \mu \); and (iii) an unexpected growth of quark density at still higher \( \mu \). The deconfinement as per the Polyakov loop appears concurrent with (iii). We now propose that this extra quark density (iii) appears due to the condensate of the monopole-quark states. At low \( T \) of this simulation the dominant monopole-single-quark objects, being bosonic, would appear mostly as a condensate, like the diquarks. Assuming standard effective potential with a repulsive binary interaction \( V_{\text{int}} = \lambda n^2 / 4 \), one gets the condensate density growing linearly with \( \mu \), \( n_{\text{BEC}} \sim (\mu - m) / \lambda \) at \( \mu > m \), which is consistent with observations. Furthermore, a similar density per flavor for both \( N_f = 2 \) and \( N_f = 4 \) is consistent with our view that these objects are dominantly states with only a single quark per monopole. This proposal should and can be checked in many ways.

4. Chiral symmetry breaking and Zero Mode Zone

Spontaneous breaking of \( SU(N_f) \) chiral symmetry has been modeled by a hypothetical Nambu-Jona-Lasinio 4-fermion interaction since 1960’s, and attributed to instanton-induced ’t Hooft interactions in 1980’s. Its detailed studies in 1990’s the context of the so called Instanton Liquid Models (ILM) are reviewed in [13]. Chiral restoration at \( T > T_c \) in this approach can be viewed as a structural phase transition in the instanton ensemble, from a random plasma at low \( T \) into a gas of strongly correlated \( II \) instanton-antiinstanton pairs [19]. The pairing mechanism is due to the fermion exchange, thus it gets stronger as \( N_f \) grows. Therefore studies covered in [13] were not able to confirm existence of chiral symmetry breaking for \( N_f > 5 \).

The nonzero holonomy (VEV of Polyakov line different from 1) becomes important at \( T < 2T_c \) and, in particular, it modifies the instantons. Classical solutions with nonvanishing fields at spatial infinity has been constructed \([14, 16]\): they revealed “instanton quarks”, or (anti)self dual constituent dyons, which in a set of \( N_c \) form an instanton. The names and quantum numbers of those (for the simplest \( SU(2) \) gauge group we will discuss below) are given in the Table 1. The redesign of the Instanton Liquid in terms of these dyons is the main topic of the rest of the talk.

| name | E | M | mass |
|------|---|---|------|
| \( \bar{M} \) | + | + | \( v \) |
| \( M \)  | + | - | \( v \) |
| \( L \)  | - | - | \( 2\pi T - v \) |
| \( \bar{L} \) | - | + | \( 2\pi T - v \) |

Table 1. The charges and the mass (in units of \( 8\pi^2 / g^2 T \)) for 4 \( SU(2) \) dyons.

The main consequence of the ILM is the formation of the so called “Zero Mode Zone” (ZMZ) of fermionic Dirac eigenvalues inside some narrow strip

\[
| \lambda | < \sigma \sim< T_{IA} >\sim \frac{\rho^2}{R^3} \sim 20 \text{ MeV} \tag{5}
\]

where we used the typical instanton size and separation \( \rho \approx 1/3 \text{ fm}, R \approx 1 \text{ fm} \) [20]. Alternatively, one can estimate this width from requirements that their total number in a give lattice volume is about the number of instantons/antiinstantons \( V_4 / R^4 \) since those are supposed to be made of their collectivized zero modes. ZMZ states are qualitatively different from perturbative fermionic plane waves and concentrate in them the chiral symmetry breaking. ILM studies had shown that the lowest hadrons, pions especially, are “made of” the ZMZ states. This has also been confirmed in lattice studies in which only the ZMZ contribution has been kept in quark propagators. Recent lattice work [21] had demonstrated what happens if one removes
Figure 2. The schematic picture of the dyonic molecule, for 2 colors and large $N_f$.

the ZMZ states from propagators: the hadronic spectroscopy changes drastically, with chiral partners ($A_1$ for $\rho$, $N \ast (1/2 \gamma)$ for the nucleon) getting much lighter and near-degenerate with their partners. Thus a tiny fraction $\sim 10^{-3}$ of all states inflict $O(1)$ changes of the hadronic masses!

Quasizero or ZMZ fermionic states are also responsible for main fluctuations in the Dirac matrix inversion and bad convergence of chiral perturbation theory on the lattice, as fermionic masses used are not small compared to its width: so even from practical perspective of effective usage of computer time one should study and understand them better than other states. Looking at the eigenvectors $\psi_\lambda(x)$ one tried to understand their nature. Early studies which used a bit “cooled” lattice configurations had identified instantons, with ILM size and density. Yet uncooled configurations had shown structures more consistent with lower dimensional topology [22, 23]. How well the “dyonic vacuum” we study now can explain those observations remains to be studied.

5. Statistical mechanics of the selfdual dyons

Here I report two papers in which this issue is addressed. The first [24] deals with a number of qualitative observations made in various lattice works, which can be understood directly from the main properties of the dyonic zero modes. For example, one can see why there is such huge difference between periodic or anti periodic fermions, or between different $Z_N$ sectors (fermions couple to different dyons of different mass).

Due to a very limited space, we only discuss one effect, related with large $N_f$ behavior of the chiral symmetry boundary. Depending which kind of fermions the theory has, their zero modes fall on different dyons. For physical antiperiodic fermions those mode exist for (twisted) or $L, \bar{L}$ dyons. As the number of fundamental fermions $N_f$ in the theory increases, they bind this dyon-antidyon pair into tight $L\bar{L}$ “clusters”, which play a role of the nucleus of these molecules, see Fig.5. Chiral symmetry is unbroken and the lowest Dirac eigenstates “at the gap” are due to independent $L\bar{L}$ clusters.

The standard Abelian electric charges of both $L$ and $\bar{L}$ are equal to -1, so the cluster nuclei of such molecule has a charge -2, so it looks like anti-He, with $M, \bar{M}$ as “positrons” around
it. (This sign of a charge does not violate C parity, of course, because the Abelian projected
type of the individual instantons/antiinstantons in the ILM. One writes the Dirac operator as a matrix
S geometry, and use the matrix G it. The obvious condition is that the simulation is restricted to positive positive eigenvalues of
known as the famous Atiyah-Hitchin manifold: and we need to simulate the anybody version of
behavior at higher densities is non-trivial. For two identical monopoles the shape/metric is
these expressions reduce to Coulombic classical plasma, with electric and magnetic charges. Its
This as well as the Coulomb terms require simulation of only charge-neutral systems: this forces
the partition function is the fermionic determinant

\[ \det F \] associated with the nonzero bosonic modes. The third essential ingredient of
moduli space metric is separated into the so called

The second paper [25] formulates the partition function and perform the first Monte-Carlo
simulations of the ensemble. We are not presenting the simulation results at this point yet,
but outline the structure of the partition function. The bosonic interactions of the dyons
is separated into the so called moduli space metric, associated with bosonic zero-modes, and
electric screening associated with the nonzero bosonic modes. The third essential ingredient of
the partition function is the fermionic determinant, so schematically it is written as

\[ dZ \sim \{dX_i\} e^{-S_{\text{cl}}} \det G \frac{\det F_{znm} \det' F_{mzm}}{\sqrt{\det B}} \] (6)

where the first factor is the product of the differentials of all collective variables, the exponent
contains the classical actions of all dyons, two subsequent determinants of matrices G and
F_{znm} are related to bosonic and fermionic zero-modes, respectively. Two remaining (primed)
determinants correspond to nonzero-modes of the one-loop bosonic and fermionic operators
describing small linearized perturbations of the classical solution.

The moduli spaces metric is described by the matrix G in the approximation proposed by
Diakonov [18]. For arbitrary SU(Nc) group it is

\[ [G]_{mn} = \delta_{mn} (4\pi \nu_m + \frac{1}{T|x_m - x_{m-1}|} + \frac{1}{T|x_m - x_{m+1}|}) - \frac{\delta_{mn-1}}{T|x_m - x_{m-1}|} - \frac{\delta_{mn+1}}{T|x_m - x_{m+1}|} \]

where the indexes m, n run over the different types of dyons. When the system is dilute,
these expressions reduce to Coulombic classical plasma, with electric and magnetic charges. Its
behavior at higher densities is non-trivial. For two identical monopoles the shape/metric is
known as the famous Atiyah-Hitchin manifold: and we need to simulate the anybody version of
it. The obvious condition is that the simulation is restricted to positive positive eigenvalues of
the matrix G. We also found that Coulombic terms in it need to be regulated at small distances.

The so called screening term (from non-zero mode determinant) produces effective linear
potential between dyons, with sign-altering coefficient proportional to the Debye mass.
This as well as the Coulomb terms require simulation of only charge-neutral systems: this forces
us to have equal number of all L, M, L, M dyons. We also decided not to use transitional 4-torus
geometry, and use S³ sphere instead. Current study is done with 64 dyons.

The fermionic zero modes of the self dual (antiseudual) dyons are treated as the zero modes
of individual instantons/antinstantons in the ILM. One writes the Dirac operator as a matrix
of the type

\[ i\mathcal{D} = \begin{pmatrix} \text{im} & T_{SA} \\ T_{AS} & \text{im} \end{pmatrix}, \] (7)
with sermonic masses on the diagonal and the overlap sub-matrices $T_{SA}$ in the other two blocks: those are defined by

$$T_{SA} = \int d^4x \psi^\dagger_{0,\text{Selfdual}}(x - z_I) i D \psi_{0,\text{Antiselfdual}}(x - z_A)$$

(8)

Here $S, A$ are understood as indices which run over all selfdual and antiselfdual dyons, and $\psi_0$ are their fermionic zero modes. The individual matrix elements have the meaning of a hopping amplitude for a quark from one pseudo-particle to another, and the determinant of this matrix is nothing else but the sum over the loop diagrams in which quarks visit each relevant dyon once.

At high temperature – low dyon density – the approximate factorization of the Dirac matrix into independent $2 \times 2$ boxes (separate dyon-antidyon clusters) explains disappearance of the near-zero eigenvalues and the so-called spectral gap. As $T$ goes down and effective coupling constant $g$ grows, the dyons become lighter and thus more numerous: eventually they form dense enough ensemble to develop ZMZ with a nonzero density at $\lambda = 0$, breaking spontaneously the chiral symmetry. We had studied about two orders of magnitude variation of the dyon density, with the number of light fermions $N_f = 0, 1, 2, 4$.

The simulation determines correlation functions of the dyons and the chiral phase transition line, as a function of the dyon density. We also calculate the free energy of the system, relative to that of noninteracting dyon gas. Its magnitude explains the strength of the back reaction, the dyonic contribution to the effective holonomy potential. Where this potential becomes comparable to perturbative one, it cancels it and widely fluctuating holonomy matrix leads to confinement.

At the moment we are not calculating the fermionic propagators and hadron correlation functions, but it can be done in a way similar to that in ILM. Another set of questions for future studies is whether the resulting ZMZ states do or do not agree with those seen on the lattice.

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