Engineering Entanglement between two cavity modes

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Abstract

We present scheme for generation of entanglement between different modes of radiation field inside high-Q superconducting cavities. Our scheme is based on the interaction of a three-level atom with the cavity field for pre-calculated interaction times with each mode. This work enables us to generate complete set of Bell basis states and GHZ state.

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Quantum information theory implements quantum mechanics into classical information theory. The EPR-Bell correlations, and quantum entanglement \[1,2\], in general, form the essential new ingredients, which distinguish quantum information theory from its classical counterpart. An entangled state of two or more quantum systems is a state which cannot be factorized \[3\]. The most familiar example of an entangled state is the Bohm state \((|\uparrow_A, \downarrow_B\rangle + |\downarrow_A, \uparrow_B\rangle) / \sqrt{2}\), which, represents the state of two spin-1/2 particles decaying from a spin zero parent \[4\]. The two particles are correlated, their spins are always antiparallel, and they remain that way no matter what is the separation between them. Hence entangled systems demonstrate non-local quantum effects.

Entanglement is one of the main pillars of quantum computation \[5\], quantum cryptography \[6\], quantum teleportation \[7\] and many other application of quantum information technology \[8,13\], therefore, generation of entangled states and its further applications are of immense importance. Several schemes have been proposed for the generation of entangled states in atoms, ions, and photons \[14-22\].

The first ever evidence of entangled state generation of two cavity fields is seen in the teleportation procedure presented by Davidovich et.al. \[23\]. This entangled state of two cavity field of same mode occurs as an intermediate step in the teleportation procedure and it requires presence of one photon in either of the two cavities. The generation of GHZ state in two cavities has been suggested in a similar manner \[24\]. An entangled state in which number of photons between two cavities is fixed has also been proposed \[25\]. All these schemes provide entanglement between radiation fields of same mode in two cavities, recently, Ref. \[26\] has reported the existence of Bohm entangled state between different modes of radiation field.

In this paper we propose schemes for the generation of entanglement between different modes of electromagnetic field and engineer EPR-Bell basis and GHZ entangled state \[27\]. We propagate a three level atom through a cavity which contains initially field modes in vacuum. We may express the three levels as \(|a\rangle\), \(|b\rangle\) and \(|c\rangle\) with their eigen energies as \(E_a\), \(E_b\) and \(E_c\), as shown in Fig. 1. The dipole transition between the upper two levels, \(|a\rangle\)
and \(|b\rangle\), of the atom is forbidden, whereas transitions from the two upper levels to lower level, \(|c\rangle\) are allowed. We consider that frequencies, \(\omega_A\) and \(\omega_B\), of the two modes, \(A\) and \(B\), respectively, of cavity field are in resonance with the transition frequencies, such that 
\[
\omega_A = (E_a - E_c)/\hbar \quad \text{and} \quad \omega_B = (E_b - E_c)/\hbar.
\]
With the help of a Ramsey field we prepare the upper two levels of the atom in linear superposition before it enters the cavity field. We may express the initial state of the system as,
\[
|\phi(0)\rangle = \frac{1}{\sqrt{2}} \left[ |a\rangle + e^{i\phi} |b\rangle \right] |0_A, 0_B\rangle,
\]
where \(\phi\) is the relative phase between two atomic states.

We write the interaction picture Hamiltonian in the dipole and rotating wave approximation as
\[
H = \hbar g_1 (a \langle a | + a^\dagger \langle c | c \rangle) + \hbar g_2 (b \langle b | + b^\dagger \langle c | c \rangle)
\]
where \(g_1\) and \(g_2\) are vacuum Rabi frequencies of the two modes while \(a(a^\dagger)\) and \(b(b^\dagger)\) are the annihilation(creation) operators of the two cavity modes \(A\) and \(B\), respectively. The atom-field state vector can be written as
\[
|\psi(t)_{A,B}\rangle = C_{a,0,0} |a, 0, 0\rangle + C_{b,0,0} |b, 0, 0\rangle + C_{c,1,0} |c, 1, 0\rangle + C_{c,0,1} |c, 0, 1\rangle
\]
where \(C_{a,n,m}\), \(C_{b,n,m}\), and \(C_{c,n,m}\) represent the time dependent probability amplitudes for the atom to be in the states \(|a\rangle\), \(|b\rangle\), and \(|c\rangle\), respectively, with \(n\) number of photons in mode \(A\) and \(m\) photons in mode \(B\). The rate equations of these probability amplitudes can be obtained by the Schrödinger equation as
\[
\frac{d}{dt}C_{a,0,0} = -i g_1 C_{c,1,0},
\]
\[
\frac{d}{dt}C_{c,1,0} = -i g_1 C_{a,0,0},
\]
\[
\frac{d}{dt}C_{b,0,0} = -i g_2 C_{c,0,1},
\]
\[
\frac{d}{dt}C_{c,0,1} = -i g_2 C_{b,0,0}.
\]
Solving these differential equations in presence of the initial condition mentioned in Eq. (1), we find the atom field entangled state as
\[ |\psi^{(t)}(A, B)\rangle = \frac{1}{\sqrt{2}} \left[ \cos g_1 t |a, 0, 0\rangle - i \sin g_1 t |c, 1, 0\rangle + e^{i\phi} \cos g_2 t |b, 0, 0\rangle - ie^{i\phi} \sin g_2 t |c, 0, 1\rangle \right] \]

(8)

For the generation of maximally entangled field state between two cavities such that if one cavity has one photon then the other will be in vacuum, the atom after its interaction with the cavity fields, is required to be detected in ground state \(|c\rangle\). This leads to the condition that probability amplitudes of the states \(|c, 1, 0\rangle\) and \(|c, 0, 1\rangle\) are equal, i.e.,

\[ \sin g_1 t = \sin g_2 t. \]

(9)

The total probability of detecting the atom in ground state is determined as

\[ P_c = \frac{1}{2} \left( \sin^2 g_1 t + \sin^2 g_2 t \right). \]

(10)

This probability becomes maximum when the time of interaction of atom with mode \(A\) and mode \(B\) is \(m\pi/2g_1\) and \(n\pi/2g_2\), respectively. Here, \(m\) and \(n\) are odd integer numbers. Hence, in order to generate two mode entanglement the time of interaction of the atom with the cavity is odd integer multiple of half of the Rabi cycle. This ensures that the cavity will obtain one photon in either of the two modes when atom is detected in ground state after its propagation through the cavity.

The interaction times of the atom with the two modes of the cavity field would be different because of the different coupling constants of each mode of radiation field. These interaction times of atom in the cavity can be controlled by using a velocity selector before the cavity and then applying Stark field adjustment so that atom becomes resonant with the cavity field modes only for the suggested amount of time in each mode of the cavity field [23].

Hence the atom passing in the superposition of levels \(|a\rangle\) and \(|b\rangle\) interacts with the two cavity modes \(A\) and \(B\) for \(m\pi/2g_1\) and \(n\pi/2g_2\) interaction times, respectively. As a result the atom leaves the cavity in ground state and develops an entangled state between the two cavity modes, viz.,
\[ |\psi(A, B)\rangle = \frac{-i}{\sqrt{2}} \left[ |0_A, 1_B\rangle + e^{i\phi} |1_A, 0_B\rangle \right]. \]  

In order to generate the other two Bell bases we control the interaction time of the atoms with the cavities, hence we find

\[ |\psi(A, B)\rangle = \frac{-i}{\sqrt{2}} \left[ |0_A, 1_B\rangle - e^{i\phi} |1_A, 0_B\rangle \right]. \]  

By using similar experimental setup we may prepare GHZ entangled states \[27\], between two different modes of the radiation field. We again consider a three level atom in V configuration, initially prepared in level \( |a\rangle \). We pass the atom through a system of two cavities which are prepared initially in vacuum state. The transition from level \( |a\rangle \) to \( |c\rangle \) is again in resonance with cavity mode \( A \), whereas the transition from \( |b\rangle \) to \( |c\rangle \) is in resonance with cavity mode \( B \).

We adjust the interaction time of the atom with first cavity field such that it sees a \( \pi/2 \) pulse. Hence, there occurs equal probability of finding the atom in ground state \( |c\rangle \), after contributing one photon in the cavity mode \( A \), and of finding the atom in the excited state \( |a\rangle \), leaving the cavity mode in vacuum state. As a result we find an atom-field entanglement, such that,

\[ |\psi_{at}(A, B)\rangle = \frac{1}{\sqrt{2}} \left[ |a, 0_A\rangle + |c, 1_A\rangle \right] \otimes |0_B\rangle. \]  

Before the atom enters in the next cavity, we apply a laser field resonant to atomic transition \( |b\rangle \) to \( |c\rangle \). The width of the beam is adjusted such that the exiting atom from the first cavity field in the ground state \( |c\rangle \), is pumped to excited state \( |b\rangle \) with unit probability. However, if the exiting atom is in excited state \( |a\rangle \) after interacting with the cavity mode \( A \), the laser field will provide no excitation to the atom.

After passing through the laser field, the atom interacts with cavity mode \( B \), which is initially in vacuum state. The interaction time of the atom with the field is adjusted such that the atom in the excited state \( |b\rangle \) will be detected in ground state \( |c\rangle \) with unit probability adding a photon in the cavity mode \( B \). However, if the atom enters the cavity
in the excited state $|a\rangle$, it will contribute no photon and will exit in the same atomic state, leaving the cavity mode $B$ in the vacuum state. Therefore, we find the entanglement of the two modes of radiation field with the atomic states as

$$|\psi_{at}(A, B)\rangle = \frac{1}{\sqrt{2}} [ |a, 0_A, 0_B\rangle + |c, 1_A, 1_B\rangle ]. \quad (14)$$

By taking projection over the atomic states, we may find the entangled state between two cavity modes as

$$|\psi(A, B)\rangle = \frac{1}{\sqrt{2}} [ |0_A, 0_B\rangle + |1_A, 1_B\rangle ]. \quad (15)$$

The interaction times of the atoms with the cavity field mode $A$, laser field, and cavity field mode $B$ are found as $m\pi/4g_1$, $\pi/\Omega$, and $n\pi/2g_2$, respectively, where $m$ and $n$ are odd integers. Here $\Omega$ is the Rabi frequency of the laser field which interacts with the atom between the two cavities. If the relative difference of interaction times of atoms with the two cavities is taken to be $\pi$, we may generate the entangled state

$$|\psi(A, B)\rangle = \frac{1}{\sqrt{2}} [ |0_A, 0_B\rangle - |1_A, 1_B\rangle ]. \quad (16)$$

Hence, we can obtain the complete set of Bell basis by controlling the interaction times of the atom with the cavities in both the schemes.

In order to generate a multi-mode entangled state we may repeat the same process again as suggested above. We provide another laser pulse which is in resonant to the transition from level $|c\rangle$ to another higher level $|b_1\rangle$. The atomic interaction with the laser pulse occurs for a time $\pi/2\Omega_1$, where $\Omega_1$ is the Rabi frequency of laser pulse. This pulse causes the atom to be in the excited state $|b_1\rangle$ with unit probability. The atom, in the excited state $|b_1\rangle$ then interacts with a cavity mode $B_1$, initially in vacuum state, for an interaction time $m\pi/2g_3$, where $g_3$ is the vacuum Rabi frequency of $B_1$ mode of the cavity field. Hence if the atom was in ground state $|c\rangle$ after interacting with the second cavity then it will contribute one photon to the cavity mode $B_1$ after this interaction, whereas if it was in excited state $|a\rangle$ after second cavity then the field will not interact with it because of the detuning. By
repeating the process using various different excited states we may develop GHZ entangled state as

$$|\psi(A, B, ..., N)\rangle = \frac{1}{\sqrt{2}} \left[ |0_A, 0_B, ..., 0_N\rangle + |1_A, 1_B, ..., 1_N\rangle \right], \quad (17)$$

which indicates entanglement between N-number of modes.

In order to measure any component of the Bell basis we may introduce a standing wave field normal to the axis of the cavities providing entangled state. A super cooled atom passing through the standing wave field and, thus interacting with the entangled state would manifest interference pattern unique to each of the Bell basis. A comparison of the interference pattern with already stored patterns would reveal the information of entangled states [28].

In order to realize our suggested scheme in laboratory experiment within microwave region, we may consider slow Rb atoms in higher Rydberg states which have life time of the order of few milli seconds [23]. These slow atoms, initially pumped to high Rydberg state, pass through a high-\(Q\) superconducting cavity of dimension of a few centimeters with a velocity of around 400 m/s [15,23,26]. The interaction times of atom with cavities come out to be of the order of few tens of microseconds which is far less than the cavity life time. The high-\(Q\) cavities of life time of the order of millisecond are being used in recent experiments [26]. The interaction time of the atom with different cavities can be controlled by using a velocity selector and applying Stark field adjustment in different cavities in order to make the atom resonant with the field for right amount of time [23]. The atomic decay rates, interaction times, and cavity life time ensure that atom does not decay spontaneously. As this entanglement remains only for the cavity life time period so any application regarding this entangled state should be accomplished during this period.
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Figure Captions

Fig. 1. Schematic diagram for the generation of entangled state between two cavity modes: We prepare three-level atom in superposition of upper two levels by Ramsey field and let it pass through the cavity carrying two different modes of radiation.