Properties of a protoneutron star in the Effective Field Theory

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The complete form of the equation of state of strangeness rich proto-neutron and neutron star matter has been obtained. The currently obtained lower value of the $U(Λ)$ potential at the level of 5 MeV permits the existence of additional parameter set which reproduces the weaker ΛΛ interaction. The effects of the strength of hyperon-hyperon interactions on the equations of state constructed for the chosen parameter set have been analyzed. It has been shown that replacing the strong $Y − Y$ interaction model by the weak one introduces large differences in the composition of a proto-neutron star matter both in the strange and non-strange sectors. Also concentrations of neutrinos have been significantly altered in proto-neutron star interiors. The performed calculations have indicated that the change of the hyperon-hyperon coupling constants affects the value of the proto-neutron star maximum mass.

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I. INTRODUCTION

The mass of a star is a key factor in determining the stellar evolution. Low-mass stars developing a degenerate core can evolve to the white dwarf stage. Evolutionary models of massive stars, with the main sequence mass exceeding $8M_⊙$ show that their evolution proceeds through sequences of consecutive stages of nuclear burning until the iron core is formed. The formation of $^{56}Fe$ nucleus, with the maximum binding energy per nucleus, indicates the beginning of the end of massive star’s life as a normal star. The interior composition of a massive star in a very advanced stage of evolution prior to core collapse reveals a shell structure. Each shell has different chemical composition of gradually heavier elements. After the nuclear fuel is exhausted in a star, nuclear fusion in the central part of the star can no longer supply enough energy to sustain a high thermal pressure. The obtained theoretical models of the evolved massive stars indicate that they develop central iron cores of mass $\sim 1.5M_⊙$ with the electron gas as the dominant source of pressure. Both thermal pressure and the pressure of degenerate electron gas attempt to support the core and determine the following equation of state of the innermost region of the star

$$\frac{P}{\rho} \simeq \frac{Y_e k_B T}{m_B} + K_Γ Y_e T^Γ \rho^{Γ−1}. \quad (1.1)$$

In this equation $Y_e = n_e/n_b$ is the electron concentration, $\rho$ is the density, $k_B$ is Boltzmann’s constant and $Γ$ is the adiabatic exponent. Owing to the high temperature of the core the adiabatic exponent is close to the critical value 4/3 and the core become dynamically unstable.

The collapse of the core is accelerated by the combination of two mechanisms: reactions of electron capture and photodisintegration of iron-peak nuclei into $α$ particles $^{56}Fe \rightarrow 13α + 4n$. Iron dissociation is an endothermic reaction, the energy cost equals 124.4 MeV, which comes
from the kinetic energy of nuclei and electrons and decreases their thermal pressure. The resulting pressure deficit is compensated by a further contraction of the core. Electron capture occurs on nuclei and on free protons, diminishes the electron fraction $Y_e$ and also leads to pressure deficit. As a consequence the core collapse sets in and the star undergoes supernova explosion, ejecting a large part of its mass. Dynamical conditions in the central region of the star split the core into two separate parts: homologously collapsing inner core and the free falling outer one. After the inner core reaches the density comparable with the nuclear density, the equation of state stiffens, nuclear forces are expected to resist compression and stop the collapse. The bounced inner core settles into a hydrostatic configuration in the dynamical time scale of milliseconds. At the boundary between the inner core and the supersonically falling outer core a pressure wave, which originates from the center, become a shock wave. The shock wave travels into the outer core through the material that is falling towards the center. This outgoing shock dissociates the iron nuclei into protons and neutrons. The shocked region attains high temperature at comparatively low densities. Under these conditions electron degeneracy is not high and relativistic positrons are thermally created. The thermal energy is carried out by neutrinos which are produced in the following reactions $e^+ + e^- \rightarrow \nu + \bar{\nu}$, $e^+ + n \rightarrow \bar{\nu} + p$. Within the shock and in the region behind the shock the velocity of the infalling matter is significantly reduced and the falling matter eventually settles onto the newly formed protoneutron star. Electron neutrinos produced in $e^- + p \rightarrow \nu_e + n$ carry out electron-type lepton number. Deleptonization in the outer envelope proceeds faster than that in the inner core. After stellar collapse the electron lepton number is trapped inside the matter. The collapse of an iron core of a massive star leads to the formation of a core residue, namely a proto-neutron star which can be considered as an intermediate stage before the formation of a cold compact neutron star. After the core collapse the electron lepton number is trapped inside the matter. This leads to large electron neutrino $\nu_e$ chemical potential. Evolution of a proto-neutron star which proceeds owing to the neutrino and photon emission causes the star change itself from a hot, bloated object into a cold compact neutron star and can be described in terms of separate phases starting from the moment when the star becomes gravitationally decoupled from the expanding ejection. Through the events which lead up to the formation of a neutron star, two limiting cases can be distinguished: the very beginning stage characterized by the low entropy density $s = 1$ (in units of the Boltzmann’s constant), unshocked core with trapped neutrinos $Y_L = 0.4$ and low density, high entropy outer layer. The final stage is identified with a cold deleptonized object ($Y_L = 0, s = 0$). These two distinct stages are separated by the period of deleptonization when the neutrino fraction decreases from the nonzero initial value ($Y_\nu \neq 0$) which is established by the requirement of the fixed total electron lepton number at $Y_L = 0.4$, to the final one characterized by $Y_\nu = 0$. 
II. PROTO-NEUTRON STAR MODEL

The presented model of a nascent neutron star is constructed under the assumption that the star can be divided into two main parts: the dense core and the outer layer. As it was stated above, at the very beginning of a proto-neutron star evolution neutrinos are trapped on the dynamical time scale within the matter both in the core and in the outer layer. The estimated electron lepton fraction \( Y_L = Y_e + Y_\mu + Y_{\nu_e} \) at trapping, with the value of \( \approx 0.4 \) and the assumption of constant entropy, allows one to specify the star characteristics at the conditions prevailing in the star interior. Assuming the evolutionary scenarios presented in the paper by Prakash et al.\[3\] the following stages in the life of a proto-neutron star have been distinguished:

- stage 1: the post bounce phase - the bounced inner core settles into a hydrostatic configuration. Under the assumption of very low kinetic energy of the matter behind the shock also the structure of the outer envelope can be approximated by hydrostatic equilibrium. Thus a proto-neutron star model immediately after the collapse can be constructed basing on the following physical conditions: the low entropy core \( s = 1 \) with trapped neutrinos \( Y_L = 0.4 \) is surrounded by high entropy \( s = 2 - 5 \) outer layer also with trapped neutrinos. The energy is carried out from a proto-neutron star by neutrinos and antineutrinos of all flavors whereas lepton number is lost by the emission of electron neutrinos which are produced in the \( \beta \) process \( e^- + p \rightarrow \nu_e + n \). Deleptonization in the outer envelope proceeds faster than that in the inner core.

- stage 2: deleptonization of the outer layer is completed. The physical conditions that characterized the whole system are as follows: the core with the entropy \( s = 2 \) and \( Y_L = 0.4 \) and the deleptonized outer envelop with \( s = 2 \) and \( Y_{\nu_e} = 0 \).

- stage 3: during this stage the deleptonization of the core takes place, after which the core has neutrino-free, high entropy \( s = 2 \) matter. Thermally produced neutrino pairs of all flavors are abundant, and they are emitted with very similar luminosities from the thermal bath of the core.

- stage 4: cold, catalyzed object.

III. THE EQUATION OF STATE AND THE EQUILIBRIUM CONDITIONS

The solution of the hydrostatic equilibrium equation of Tolman, Oppenheimer and Volkov which allows one to construct a theoretical model of a proto-neutron star and to specify and analyze its properties demands the specification of the equation of state. The mean field approach to the relativistic theory of hadrons has been used extensively in order to describe properties of nuclear matter and finite nuclei. The degrees of freedom relevant to this theory are nucleons interacting through the exchange of isoscalar-scalar \( \sigma \), isoscalar-vector \( \omega \), isovector-vector \( \rho \) and the pseudo-scalar \( \pi \) mesons. In the relativistic mean...
field approach nucleons are considered as Dirac quasiparticles moving in classical meson fields. The contribution of the \( \pi \) meson vanishes at the mean field level. The chiral effective Lagrangian proposed by Furnstahl, Serot and Tang constructed on the basis of effective field theory and density functional theory for hadrons gave in the result the extension of the standard relativistic mean field theory and introduced additional non-linear scalar-vector and vector-vector self-interactions. This Lagrangian in general includes all non-renormalizable couplings consistent with the underlying symmetries of QCD and can be considered as the one of the effective field theory of low energy QCD. Applying the dimensional analysis of Georgi and Manohar [6], [7] and the concept of naturalness one can expand the nonlinear Lagrangian and organize it in increasing powers of the fields and their derivatives and truncated at given level of accuracy [8].

In the high density regime in neutron star interiors when the Fermi energy of nucleons exceeds the hyperon masses additional hadronic states are produced [4], [5], [9], [10], [11]. Thus the considered model involves the full octet of baryons interacting through the exchange of \( \sigma \) mesons which produce the medium range attraction and the exchange of \( \omega \) mesons responsible for the short range repulsion. The model also includes the isovector mesons \( \rho \).

In order to reproduce attractive hyperon-hyperon interaction two additional hidden-strangeness mesons, which do not couple to nucleons, have been introduced, namely the scalar meson \( f_0(975) \) (denoted as \( \sigma^* \)) and the vector meson \( \phi(1020) \) [12]. If the truncated Lagrangian includes terms up to the forth order it can be written in the following form:

\[
\mathcal{L} = \sum_B \bar{\Psi}_B (i\gamma^\mu D_\mu - m_B + g_{\sigma B}\sigma + g_{\sigma^* B}\sigma^*)\Psi_B
\]

\[
+ \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2 \left( 1 + \frac{\kappa_3g_{\sigma B}\sigma}{3!M} + \frac{\kappa_4g_{\sigma^* B}\sigma^2}{4!M^2} \right) + \frac{1}{2}\partial_\mu\sigma^*\partial^\mu\sigma^* - \frac{1}{2}m_{\sigma^*}^2\sigma^{*2} + \frac{1}{2}\partial_\mu\phi^\mu - \frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}(1 + \eta_1\frac{g_{\sigma B}\sigma}{M} + \frac{\eta_2g_{\sigma^* B}\sigma^2}{2M^2})m_\omega^2\omega^\mu\omega_\mu + \frac{1}{2}m_\rho^2\rho^\mu\rho_\mu + \frac{1}{4}m_\omega^2\omega^\mu\omega_\mu + \frac{1}{4}m_\rho^2\rho^\mu\rho_\mu + \frac{1}{2}\zeta_m g_{\omega B}\sigma + \frac{1}{4}\zeta_m g_{\omega B}\sigma^2)(\omega_\mu\omega^\mu)^2.
\]

where \( \Psi_B^T = (\psi_N, \psi_\Lambda, \psi_\Sigma, \psi_\Xi) \). The covariant derivative \( D_\mu \) is defined as

\[
D_\mu = \partial_\mu + ig_\omega B\omega_\mu + ig_\phi B\phi_\mu + ig_\rho B I_3 B\tau_3^a\rho_\mu^a
\]

whereas \( R^{a}_{\mu\nu}, \Omega_{\mu\nu} \) and \( \phi_{\mu\nu} \) are the field tensors

\[
R^{a}_{\mu\nu} = \partial_\mu \rho^a_\nu - \partial_\nu \rho^a_\mu + g_\rho \varepsilon_{abc} \rho^b_\mu \rho^c_\nu,
\]

\[
\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \quad \phi_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu,
\]
$m_B$ denotes baryon mass whereas $m_i$ ($i = \sigma, \omega, \rho, \sigma^*, \phi$) are masses assigned to the meson fields, $M$ is the nucleon mass. There are two parameter sets presented in the original paper by Furnstahl et al [13], [15] G1 and G2 which have been determined by calculating nuclear properties such as binding energies, charge distribution and spin-orbit splitting for a selected set of nuclei [16]. For the purposes of this paper the G2 parameter set has been chosen. Calculations performed with the use of this parameter set properly reproduce properties of finite nuclei and make it possible to compare the obtained results with the Dirac-Brueckner-Hartree-Fock (DBHF) calculations for nuclear and neutron matter above the saturation density. The DBHF method results in a rather soft equation of state in the vicinity of the saturation point and for higher densities. Calculations performed on the basis of the effective FST Lagrangian with the use of the G2 parameter sets predict similar, soft equation of state. Due to the fact that the expectation value of the $\rho$ meson field is an order of magnitude smaller than that of $\omega$ meson field, the Lagrangian function (3.2) does not include the quartic $\rho$ meson term. In addition, as this paper deals with the problem of infinite nuclear matter the terms in the Lagrangian function in the paper [15] involving tensor couplings and meson field gradients have been excluded. The derived equations of motion constitute a set of coupled equations which have been solved in the mean field approximation. In this approximation meson fields are separated into classical mean field values and quantum fluctuations which are not included in the ground state

\[
\sigma = \bar{\sigma} + s_0 \quad \sigma^* = \bar{\sigma}^* + s_0^* \\
\phi_\mu = \bar{\phi}_\mu + f_0 \delta_\mu_0 \quad \omega_\mu = \bar{\omega}_\mu + w_0 \delta_\mu_0 \quad \rho_\mu^a = \bar{\rho}_\mu^a + r_0 \delta_\mu_0 \delta^3 \frac{a}{a}.
\]

As it was stated above the derivative terms are neglected and only time-like components of the vector mesons will survive if one assumes homogenous and isotropic infinite matter. The field equations derived from the Lagrange function at the mean field level are

\[
m_\sigma^2(s_0 + \frac{g_{\sigma B} \kappa_3}{2M} s_0 + \frac{g_{\omega B} \kappa_4}{6M^2} s_0^3) - \frac{1}{2} m_\omega^2 (\eta_1 \frac{g_{\sigma B}}{M} + \eta_2 \frac{g_{\omega B}}{M^2} s_0) w_0^2 - \frac{1}{2} m_\rho^2 \eta_\rho \frac{g_{\rho B}}{M} r_0^2 = \sum_B g_{\sigma B} m_{\text{eff},B}^2 S(m_{\text{eff},B})
\]

(3.6)

\[
m_\omega^2 (1 + \frac{\eta_1 g_\sigma}{M} s_0 + \frac{\eta_2 g_\omega^2}{2M^2} s_0^3) w_0 + \frac{1}{6} \zeta_0 g_{\omega B} w_0^3 = \sum_B g_{\omega B} n_B
\]

(3.7)

\[
m_\rho^2 (1 + \frac{\eta_1 g_\sigma}{M} s_0 + \frac{g_{\rho B} \eta_\rho}{M} s_0) r_0 = \sum_B g_{\rho B} I_{3B} n_B
\]

(3.8)

\[
m_{\sigma^*}^2 s_0^* = \sum_B g_{\sigma^* B} m_{\text{eff},B}^2 S(m_{\text{eff},B})
\]

(3.9)

\[
m_\phi^2 f_0 = \sum_B g_{\phi B} n_B.
\]

(3.10)
The function $S(m_{\text{eff},B})$ is expressed with the use of the integral

$$S(m_{\text{eff},B}) = \frac{2J_B + 1}{2\pi^2} \int_0^\infty \frac{k^2 dk}{E(k, m_{\text{eff},B})} (f_B - f_{\overline{B}})$$ (3.11)

where $J_B$ and $I_{3B}$ are the spin and isospin projection of baryon $B$, $n_B$ is given by

$$n_B = \frac{2J_B + 1}{2\pi^2} \int_0^\infty k^2 dk (f_B - f_{\overline{B}})$$ (3.12)

The functions $f_B$ and $f_{\overline{B}}$ are the Fermi-Dirac distribution for particles and anti-particles, respectively

$$f_{B,\overline{B}} = \frac{1}{1 + e^{E(k, m_{\text{eff},B})/\mu_B}/k_B T}.$$ (3.13)

where $\mu_B$ denotes the baryon chemical potential defined as

$$\mu_B = \nu_B + g_{\omega B} w_0 + g_{\rho B} I_{3B} r_0 + g_{\phi B} f_0 = \sqrt{k^2 + m_{\text{eff},B}^2 + g_{\omega B} w_0 + g_{\rho B} I_{3B} r_0 + g_{\phi B} f_0}$$ (3.14)

The obtained Dirac equation for baryons has the following form

$$(i\gamma^\mu \partial_\mu - m_{\text{eff},B} - g_{\omega B} \gamma^0 \omega_0 - g_{\phi B} \gamma^0 f_0)\psi_B = 0$$ (3.15)

with $m_{\text{eff},B}$ being the effective baryon mass generated by the baryon and scalar fields interaction and defined as

$$m_{\text{eff},B} = m_B - (g_{\sigma B} s_0 + g_{\sigma^* B} s_0^*).$$ (3.16)

In the case of a proto-neutron star matter which includes hyperon degrees of freedom the considered parameterization has to be supplemented by the parameter set related to the strength of the hyperon-nucleon and hyperon-hyperon interactions. The scalar meson coupling to hyperons can be calculated from the potential depth of a hyperon in the saturated nuclear matter \[17\]

$$U_N Y = -g_{\sigma Y} s_0 + g_{\omega Y} \omega_0$$ (3.17)

The considered model does not include $\Sigma$ hyperons due to the remaining uncertainty of the form of their potential in nuclear matter at saturation density \[18, 19, 20, 21\]. In the scalar sector the scalar coupling of the $\Lambda$ and $\Xi$ hyperons requires constraining in order to reproduce the estimated values of the potential felt by a single $\Lambda$ and a single $\Xi$ in saturated nuclear matter ($\rho_0 \sim 2.5 \times 10^{14} g/cm^3$)

$$U_\Lambda^{(N)}(\rho_0) = g_{\sigma \Lambda} s_0(\rho_0) - g_{\omega \Lambda} w_0(\rho_0) \simeq 27 - 28 MeV$$ (3.18)

$$U_\Xi^{(N)}(\rho_0) = g_{\sigma \Xi} s_0(\rho_0) - g_{\omega \Xi} w_0(\rho_0) \simeq 18 - 20 MeV.$$
Table I: Chosen parameter sets.

| $m_\sigma$ [MeV] | $g_\sigma$ | $g_\omega$ | $g_\rho$ | $\eta_1$ | $\eta_2$ | $\eta_0$ | $\zeta_0$ |
|------------------|-----------|-----------|----------|---------|---------|---------|-------|
| G2               | 520.254   | 10.4957   | 12.7624  | 9.48346 | 0.64992 | 0.10975 | 0.3901 | 2.6416 |

Table II: Strange scalar sector parameters.

|       | $g_{\sigma \Lambda}$ | $g_{\sigma \Xi}$ | $g_{\sigma^* \Lambda}$ | $g_{\sigma^* \Xi}$ |
|-------|----------------------|------------------|------------------------|---------------------|
| G2    | 6.41007              | 3.33730          | 3.89005                | 11.64347            |
| weak  |                      |                  |                        |                     |
| strong|                      |                  |                        |                     |

Assuming the SU(6) symmetry for the vector coupling constants and determining the scalar coupling constants from the potential depths, the hyperon-meson couplings can be fixed.

The strength of hyperon coupling to strange meson $\sigma^*$ is restricted through the following relation \[11\]

$$U_\Xi^{(\Xi)} \approx U_\Lambda^{(\Xi)} \approx 2U_\Xi^{(\Lambda)} \approx 2U_\Lambda^{(\Lambda)}. \tag{3.19}$$

which together with the estimated value of hyperon potential depths in hyperon matter provides effective constraints on scalar coupling constants to the $\sigma^*$ meson. The currently obtained value of the $U_\Lambda^{(\Lambda)}$ potential at the level of 5 MeV \[25\] permits the existence of additional parameter set \[26\] which reproduces this weaker $\Lambda\Lambda$ interaction. In the text this parameter set is marked as weak, whereas strong denotes the stronger $\Lambda\Lambda$ interaction for $U_\Lambda^{(\Lambda)} \approx 20$ MeV \[12\]. The vector coupling constants for hyperons which are determined from $SU(6)$ symmetry constraints \[24\] remain unchanged. They are of primary importance in determining the high density range of repulsive baryon potentials \[24\] as

$$\frac{1}{2}g_{\omega \Lambda} = \frac{1}{2}g_{\omega \Sigma} = g_{\omega \Xi} = \frac{1}{3}g_{\omega N} \tag{3.20}$$

Since the dynamical time scale of neutron star evolution is much longer than the weak decay time scale of baryons (including strangeness violating processes) neutron star matter is considered as a system with conserved baryon number $n_b = \sum_B n_B$ ($B = n, p, \Lambda, \Xi^-, \Xi^0$) and electric charge being in chemical equilibrium with respect to weak decays. The chemical equilibrium is characterized by the correlations between chemical potentials which in the case of the process $p + e^- \leftrightarrow n + \nu_e$ leads to the following relation

$$\mu_{asym} \equiv \mu_n - \mu_p = \mu_e - \mu_{\nu_e}. \tag{3.21}$$
The similar weak process \( \mu + \nu_e \leftrightarrow e + \nu_\mu \) leads to

\[
\mu_\mu = \mu_e - \mu_{\nu_e} + \mu_{\nu_\mu} = \mu_{\text{asym}} + \mu_{\nu_e}.
\]  

(3.22)

Assuming that the muon neutrinos are not trapped inside the protoneutron star (\( \mu_{\nu_\mu} = 0 \)) these relations involve three independent chemical potentials \( \mu_n, \mu_e \) and \( \mu_{\nu_e} \) corresponding to baryon number, electric charge and lepton number conservation. In general, weak processes for baryons can be written in the following form

\[ B_1 + l \leftrightarrow B_2 + \nu_l \]  

(3.23)

where \( B_1 \) and \( B_2 \) are baryons, \( l \) and \( \nu_l \) denote lepton and neutrino of the corresponding flavor, respectively. Provided that the weak processes stated above take place in thermodynamic equilibrium the following relation between chemical potentials can be established

\[ \mu_B = q_B \mu_n - q_{e_B} \left( \mu_e - \mu_{\nu_e} \right). \]

(3.24)

In this equation \( \mu_B \) denotes chemical potential of baryon \( B \) with the baryon number \( q_B \) and the electric charge \( q_{e_B} \).

Using the relation (3.24) for \( \Lambda, \Sigma \) and \( \Xi \) hyperons the following results can be obtained

\[
\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n, \quad \mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + (\mu_e - \mu_{\nu_e}), \quad \mu_p = \mu_{\Sigma^+} = \mu_n - (\mu_e - \mu_{\nu_e}).
\]  

(3.25)

The presented constrains of charge neutrality and \( \beta \)-equilibrium imply the presence of leptons which are introduced as free particles. Thus the Lagrangian of free leptons has to be added to the Lagrangian function (3.2)

\[
L_L = \sum_{i=e,\mu} \bar{\psi}_i (i\gamma^\mu \partial_{\mu} - m_i) \psi_i + \sum_{i=e,\mu} \bar{\psi}_{\nu_i} (i\gamma^\mu \partial_{\mu}) \psi_{\nu_i}.
\]  

(3.26)

Muons start to appear in neutron star matter in the process \( e^- \leftrightarrow \mu^- + \bar{\nu}_\mu + \nu_e \) after \( \mu_\mu \) has reached the value equal to the muon mass. The appearance of muons not only reduces the number of electrons but also affects the proton fraction. Numerical models of the collapsing inner core have shown that there are no muon neutrinos \( (\mu_{\nu_\mu} = 0) \) and only electron neutrinos are trapped in the matter \[ 3 \]. After the deleptonization of a proto-neutron star matter there are no trapped neutrinos either and \( \mu_{\nu_e} = 0 \). In this case the number of independent chemical potentials reduces to two \( (\mu_n \) and \( \mu_e \)).

IV. RESULTS

More realistic description of a neutron star requires taking into consideration not only the interior region of a neutron star but also the remaining layers, namely the
inner and outer crust and surface layers. The composite equation of state has been constructed by adding Bonn and Negele-Vautherin (NV) equations of state (describing the inner crust) to the one determined with the use of the G2 parameterization of the FST model ([13], [14]) allows to calculate the neutron star structure for the entire neutron star density span. The proto-neutron star models have been constructed on the assumption that they include a core and an outer layer. The outer region of a proto-neutron star is characterized by the following conditions: it is less dense and less massive than the core, the assumptions of nearly complete disintegration of nuclei into free nucleons and \( \beta \) equilibrium have been made. Chemical potentials of the constituents of the matter are evaluated by the requirement of charge neutrality and \( \beta \) equilibrium. At sufficiently low density, in the absence of interactions which alter the nucleon effective mass nucleons are considered as non-relativistic.

The pressure in the outermost layer of a proto-neutron star is determined by nucleons, electrons and neutrinos. The theoretical model of the dense inner core can be constructed under the following assumptions: the matter includes the full octet of baryons interacting through the exchange of meson fields, the composition is determined by the requirements of charge neutrality and generalized \( \beta \) equilibrium. The loss of lepton number from the collapsed star proceeds in separated stages. First, very fast, carried on in a very short time in comparison with the Kelvin-Helmholtz neutrino cooling time, the deleptonization of the surface layer takes place [3]. After that the deleptonization of the core proceeds by the emission of electron neutrinos which are produced efficiently via the \( \beta \)-process

\[
e^{-} + p \rightarrow n + \nu_{e}. \tag{4.1}
\]

The events between the two distinguished moments depend on details of the considered models, especially on the equation of state. The initial moment \( t = 0 \) characterizes the stage with neutrinos trapped (\( Y_{L} = 0.4 \)) both in the core and in the outer envelope. Introducing a parameter \( \alpha \), which determines the degree of deleptonization of the proto-neutron star matter, the moment \( t = 0 \) can be identified with \( \alpha = 1 \) whereas the final completely deleptonized stage (\( Y_{\nu} = 0 \)) is described by \( \alpha = 0 \). The density at the core-outer layer interface is time dependent. This dependance can be presented by comparison of the interface between the inner core equation of state and the equations of state which have been obtained for different values of the parameter \( \alpha \) in the outer layer. Results are shown in Fig. [1]. The deleptonization leads to the substantial reduction in the extension of the proto-neutron star envelope. The obtained form of the equation of state for the strong and weak \( Y - Y \) interactions are presented in Fig. [2]. Both models have yielded identical results up to a certain value of density: the hot, neutrino-trapped matter is described by the stiffest equation of state whereas the hot, deleptonized and cold (\( T = 0 \)) proto-neutron star models result in softer
Figure 1: The density dependance of the interface between the inner core equation of state and the equations of state which have been obtained for different values of the parameter $\alpha$ in the outer layer for the the strong (left panel) and weak (right panel) $Y-Y$ interaction models.

Figure 2: The equation of state of the hyperon star matter for the G2 parameter set. In the left and right panels the results for the strong and weak $Y-Y$ interactions are presented.

equations of state. However, the strong $Y-Y$ interaction model gives for the hot, deleptonized and for the cold $T=0$ proto-neutron star matter equations of state stiffer than that for the weak model.

The composition of hyperon star matter as well as the the threshold density for hyperons are altered when the strength of the hyperon-hyperon interaction is changed. Fig. 3 presents fractions of particular strange baryon species $Y_B$ as a function of baryon number density $n_b$ for the weak and strong model of the G2 parameterization, for the three selected evolutionary phases. All calculations have been done on the assumption that the repulsive $\Sigma$ interaction shifts the onset points of $\Sigma$ hyperons to very high densities and they do not appear in the considered proto-neutron
Figure 3: Relative concentrations of Λ and Ξ hyperons in hyperon star matter for the G2 parameter set. In the left and right panels the results for the strong and weak Y-Y interaction are presented.

and neutron star models. The onset of Ξ⁰ hyperons takes place at high densities and they are not presented in Fig. 3. In the case of strong and weak Y − Y interactions the onset of Λ hyperons is followed by the onset of Ξ⁻ hyperons. In general, when Λ hyperons appear they replace protons and, in the consequence, lower the energy of the system. This fact, through the requirement of charge neutrality of neutron star matter, results in diminished electron chemical potential. The populations of Λ hyperons obtained in the weak models in the considered evolutionary stages are reduced in comparison with those calculated for the strong Y − Y interaction models. Thus the weak model with the reduced Λ hyperon population leads to the increased value of electron and proton concentrations. Results are presented in Fig. 4 where relative concentrations of electrons and protons are depicted. One can also compare the concentrations of neutrinos in the hot, neutrino trapped matter. In Fig. 5 neutrino concentrations for the two different models are shown. The weak model leads to lower concentrations of neutrinos. For comparison results for non-strange proto-neutron star matter is included.

Once the equations of state have been calculated for each evolutionary phase and for the strong and weak models the corresponding hydrostatic models of proto-neutron stars have been obtained. Solutions of the Oppenheimer-Tolman-Volkov equation for the considered parameter sets are presented in Fig. 6. In agreement with the introduced evolutionary stages of a proto-neutron star the obtained mass-radius relations include a sequence of models calculated for a given equation of state characteristic for a specific phase of a proto-neutron star evolution. In the left and right panel of Fig. 6 results for six equations of state have been plotted. The four mass-radius relations are constructed for the strangeness-rich baryonic matter and they represent the evolution of a proto-neutron star starting from the
moment when the proto-neutron star can be modelled as a low entropy core surrounded by a high entropy outer layer. Neutrinos are trapped both in the core and in the outer layer. This very beginning phase of a proto-neutron star is followed by two subsequent stages connected with the process of deleptonization of a proto-neutron star. The first during which the deleptonization of the outer layer takes place whereas in the core neutrinos are still trapped and the second one representing a hot deleptonized object with thermally produced neutrino pairs of all flavor abundant in the core and in the outer layer. The final case is exemplified by a solution obtained for cold neutron star matter which includes hyperons. For comparison the mass-radius relations for two limiting cases of proto-neutron star evolution.
Figure 6: The mass-radius relations for subsequent stages of the proto-neutron star evolution. The left panel presents results for G2 parameterization with the strong $Y^{-} - Y$ interaction, the right panel for the weak interaction model. Dotted lines represent solutions obtained for non-strange baryonic matter.

Table III: Proto-neutron and neutron star parameters obtained for the maximum mass configurations for the G2 parameterization with the strong $Y^{-} - Y$ interactions.

| $\rho / \rho_0$ | R (km) | $M (M_\odot)$ | $M_B (M_\odot)$ |
|----------------|--------|----------------|-----------------|
| T=0 A          | 5.00   | 12.73          | 1.298           |
|                | 1.407  |                |                 |
| T=0 B          | 21.76  | 8.22           | 1.320           |
|                | 1.634  |                |                 |
| s=2            | 8.76   | 13.528         | 1.670           |
|                | 1.859  |                |                 |

for nonstrange baryonic matter have been presented. A similar sequence of the mass-radius relations have been constructed for the G2 parameterization supplemented by the weak $Y^{-} - Y$ parameter set for the strange sector. There are qualitative changes in proto-neutron star parameters which occur for a given evolutionary stage. The G2-strong parameterization leads to proto-neutron neutron stars with the reduced value of the maximum mass. For the strong hyperon-hyperon interaction strength in the case of the cold neutron star model besides the ordinary neutron star branch there exists additional stable branch of solutions which are characterized by a similar value of the mass but with a significantly reduced radius. For the purpose of this paper A denotes the maximum mass configuration of the ordinary neutron star branch and B the additional maximum. Proto-neutron and neutron star parameters are summarized in Tables III and IV. The presented results have been obtained for maximum mass configurations. The analysis of the existence and stability of the additional branch in the mass-radius relation will be the subject of further investigations.

Neutron stars are purely gravitationally bound object. The gravitational binding energy of a relativistic star is defined
Table IV: Proto-neutron and neutron star parameters obtained for the maximum mass configurations for the G2 parameterization with the weak $Y-Y$ interactions.

| $\rho/\rho_0$ | $R$ (km) | $M (M_\odot)$ | $M_B (M_\odot)$ |
|--------------|----------|----------------|-----------------|
| $T=0$        | 8.00     | 11.63          | 1.527           | 1.749           |
| $s=2$        | 9.12     | 13.14          | 1.754           | 1.916           |

Figure 7: The neutron star mass versus the baryon mass $M_B = M c^2 N_B$. In the left and right panels the results for the strong and weak $Y-Y$ interaction, respectively, are presented.

as the difference between its gravitational and baryonic masses.

$$E_{b,g} = (M - M_B)c^2$$ (4.2)

where

$$M = 4\pi \int_0^R dr r^2 (1 - \frac{2Gm(r)}{c^2r}) \frac{1}{4} \rho(r)$$ (4.3)

and $M_B = n_b m_B$ is given by the total baryon number $n_b$. The numerical solution of the above equation has been found for the selected equations of state for the strong and weak interactions. The results are shown in Fig. 7 which also includes for comparison solutions for non-strange proto-neutron and neutron star models. The obtained mass-radius relations for the non-strange baryonic matter confirm the well-known fact that in this case the maximum mass of a proto-neutron star with trapped neutrinos is lower than that of cold deleptonized matter. This has a consequence for the possibility of a black hole formation during the relatively long lasting phase of deleptonization. In the case of non-strange matter a proto-neutron star is not able to achieve the unstable configuration due to deleptonization. In general, neutrino trapping increases the value of the maximum mass for the strangeness rich baryonic matter. In all
evolutionary stages there are configurations which due to deleptonization go to the unstable branch of proto-neutron and neutron stars.

The gained solutions of the structure equations allows as to carry out a similar analysis of the onset point, abundance and distributions of the individual baryon and of lepton species but now as functions of the star radius $R$. The analysis includes results obtained for two extreme evolutionary phases: the very beginning hot, neutrino-trapped matter stage and the cold, deleptonized one. Two characteristic configurations have been considered for the $T = 0$ solution. Namely, the one connected with the maximum mass configurations marked as A and the other for the B maximum mass configuration of the G2 strong parameterization. The compact hyperon core which emerges in the interior of the maximum mass configuration consist of $\Lambda$, $\Xi^-$ and $\Xi^0$ hyperons. The hyperon population is reduced to $\Lambda$ and $\Xi^-$ in the case of hot neutrino trapped matter for the weak model. The process of deleptonization and cooling diminishes the concentration of $\Lambda$ hyperons and increases concentrations of $\Xi^-$, $\Xi^0$ hyperons. The reduction of $\Lambda$ hyperon population in the weak model is larger then that obtained for the strong $Y - Y$ interaction model. The innermost hyperon core has larger radius and contains much more negatively charged $\Xi$ hyperons in the weak model.

The relative fractions of hyperons in the core of the maximum mass configurations are presented in Fig. 8 and Fig. 9. Horizontal lines in the left panels of Fig. 8 and Fig. 9 correspond to the threshold density of individual hyperons in the considered model. The configuration marked as A does not contain hyperons. The appearance of $\Xi^-$ hyperons through the condition of charge neutrality affects the lepton fraction and causes a drop in their contents. Charge neutrality tends to be guaranteed with the reduced lepton contribution. Results are shown in Fig. 10. The relative hadron-lepton composition in this model can be also analyzed through the density dependence of the asymmetry parameter $fa$ which describes the neutron excess in the system and the parameter $fs = (N_\Lambda + 2N_{\Xi^-} + 2N_{\Xi^0})/n_b$ which is connected with the strangeness contents. Fig. 11 and 12 present both parameters as functions of baryon number density and the stellar radius $R$. As the density increases, the asymmetry of the matter decreases and the parameter $fs$ increases for all the considered cases. Deleptonization and cooling leads to stellar matter which is more asymmetric and possesses substantially enhanced strangeness contents.

V. CONCLUSIONS

In this paper the complete form of the equation of state of strangeness rich proto-neutron and neutron star matter has been obtained. The considered models are constructed on the assumption that the proto-neutron star consists of two main parts: the core and the outer layer. The pressure in the outer envelope is determined by free nucleons, electrons and neutrinos in $\beta$ equilibrium. The theoretical model of the dense inner core is described by the Lagrangian which includes the full
Figure 8: The fraction of species $i$, $Y_i$ in the maximum mass configuration as a function of star radius for the G2 strong parameter set ($k_F$ means the effective Fermi momentum which specifies the value of $\nu_b$ in equation (3.14)).

eoctet of baryons interacting through the exchange of meson fields. The chosen parameter set based on the effective field theory includes nonlinear scalar-vector and vector-vector interaction terms which leads to rather soft equation of state. This is in agreement with the DBHF results. The meson sector has been extended not only by nonlinear terms but also by two additional hidden-strangeness mesons which reproduce attractive hyperon-hyperon interactions. Thus the chosen parameter set G2 has been supplemented by the parameter set related to the strength of the hyperon-nucleon and hyperon-hyperon interactions. The currently obtained lower value of the $U_\Lambda^{(A)}$ potential at the level of 5 MeV permits the existence of additional parameter set which reproduces this weaker $\Lambda\Lambda$ interaction. The main goal of this paper was to study the influence of the strength of hyperon-hyperon interactions on the properties of the proto-neutron star matter and through this on a proto-neutron star structure during selected phases of its evolution. The presence of hyperons in general leads to the softening of the equation of state. The behavior of the equation of state is directly connected with the value of the maximum star mass. Equilibrium conditions, namely charge neutrality and $\beta$-equilibrium, determine the composition of the star. It has been shown that replacing the strong $Y - Y$ interaction model by the weak one introduces large differences in the composition of a proto-neutron star matter both in the strange and non-strange sectors. There is a con-
Figure 9: The fraction of species $i$, $Y_i$, in the maximum mass configuration as a function of star radius for the G2 weak parameter set.

Figure 10: Relative concentrations of $\Lambda$ and $\Xi$ hyperons in hyperon star matter for the G2 parameter set. In the left and right panels the results for the strong and weak $Y-Y$ interaction, respectively, are presented.

A considerable reduction of $\Lambda$ hyperon concentration whereas the concentrations of $\Xi^-$ and $\Xi^0$ hyperons are enhanced during the deleptonization. In addition, the population of $\Lambda$ hyperons obtained in the weak models in the considered evolutionary stages is lower in comparison with those calculated for the strong $Y-Y$ interaction models.
Thus the weak model with the reduced $\Lambda$ hyperon population permits larger fractions of protons and electrons and leads to lower concentrations of neutrinos. The G2-strong parameterization leads to proto-neutron neutron stars with the reduced value of the maximum mass. For the strong hyperon-hyperon interaction strength in the case of the cold neutron star model besides the ordinary neutron star branch there exists additional stable branch of solutions which are characterized by a similar value of the mass but with significantly reduced radius. This
may be connected with the existence of the third family of stable compact stars \cite{28}. The analysis of baryon and lepton concentrations as a function of stellar radius $R$ shows that for the chosen parameter set there is no strange baryons in the maximum mass configuration $A$. This star resembles an ordinary neutron star. The additional stable configurations of neutron stars which appear on the mass-radius diagram include large fraction of hyperons (hyperstar). Analysis of the concentrations of leptons (electrons and muons) in the compact star $B$ as a stellar radius $R$ leads to the conclusion in the leptons are gathered in the outer layer of the star. Thus, the star has a lepton rich outer part and almost completely deleptonized inner core. This analysis shows that there exists a very strong correlation between the value of the maximum neutron star mass and the strength of hyperon coupling constants. The inclusion of additional nonlinear meson interaction terms which modify the high density behavior of the equation of state together with the strong hyperon-hyperon interaction lead to the existence of additional stable stellar configurations with similar masses and smaller radii than an ordinary neutron star. The reduction in radius is of the order of 2.5 km. The internal composition of this additional neutron star configurations is almost completely free of leptons. Transmutation similar to the phase transition from the ordinary neutron star to the more compact hyperstar may be the main origin of the short gamma ray burst \cite{29}.

Employing the data concerning the estimated value of the $\Lambda$ well depth $U_{\Lambda\Lambda} \simeq 5$ MeV the existence of the additional stable branch of very compact stars have not been confirmed.

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