ON THE IMPACT OF RADIATION PRESSURE ON THE DYNAMICS AND INNER STRUCTURE OF DUSTY WIND-DRIVEN SHELLS

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ABSTRACT

Massive young stellar clusters are strong sources of radiation and mechanical energy. Their powerful winds and radiation pressure sweep up interstellar gas into thin expanding shells that trap the ionizing radiation produced by the central clusters affecting the dynamics and the distribution of their ionized gas. Here we continue our comparison of the star cluster winds and radiation pressure effects on the dynamics of shells around young massive clusters. We calculate the impact that radiation pressure has on the distribution of matter and thermal pressure within such shells, as well as on the density-weighted ionization parameter $U_w$, and put our results on the diagnostic diagram, which allows one to discriminate between the wind-dominated and radiation-dominated regimes. We found that model-predicted values of the ionization parameter agree well with typical values found in local starburst galaxies. Radiation pressure may affect the inner structure and the dynamics of wind-driven shells, but only during the earliest stages of evolution (before $\sim 3$ Myr) or if a major fraction of the star cluster mechanical luminosity is dissipated or radiated away within the star cluster volume and thus the star cluster mechanical energy output is significantly smaller than star cluster synthetic models predict. However, even in these cases radiation dominates over the wind dynamical pressure only if the exciting cluster is embedded into a high-density ambient medium.

Key words: galaxies: star clusters: general – H$\textsc{ii}$ regions – hydrodynamics – ISM: bubbles – ISM: kinematics and dynamics

1. INTRODUCTION

H$\textsc{ii}$ regions are fundamental to our understanding of young stellar cluster radiative and mechanical feedback on the interstellar medium (ISM). They are strong sources of emission-line radiation and thus serve as a powerful diagnostic tool to study star formation and the chemical composition of nearby and distant galaxies (Capriotti & Kozminski 2001; Dopita et al. 2005, 2006; Yeh & Matzner 2012). They have even been used as tracers of the Hubble expansion (Chávez et al. 2012). The idealized (Strömgren 1939) model for spherical static H$\textsc{ii}$ regions with a homogeneous density distribution was a revolutionary step forward in the study of photoionized nebulae. However, the consideration of a number of physical effects has led to a much more robust paradigm. Winds produced by the exciting clusters (Capriotti & Kozminski 2001; Arthur 2012; Silich & Tenorio-Tagle 2013) and the impact that radiation pressure provides on the swept-up interstellar gas (Elmegreen & Chiang 1982; Capriotti & Kozminski 2001; Matzner 2002; Krumholz & Matzner 2009; Nath & Silk 2009; Sharma & Nath 2012) are among such major physical effects. As recently shown by Draine (2011), the absorption of photons emerging from an exciting cluster by either dust grains or recombinating atoms leads to a nonhomogeneous density distribution even within static or pressure-confined H$\textsc{ii}$ regions, and under certain conditions, radiation pressure may pile up the ionized gas into a thin outer shell, as assumed by Krumholz & Matzner (2009). The action of cluster winds, as well as the strong evolution that the ionizing photon flux and the star cluster bolometric luminosity suffer after the first supernova explosion, makes the situation even more intricate (Silich & Tenorio-Tagle 2013).

The thermalization of the stellar winds and supernovae mechanical energy through nearby random collisions leads to a high central overpressure that forms a strong shock that moves supersonically and sweeps the ambient ionized gas into a thin, wind-driven shell. This shell cools down in a short timescale and begins to absorb ionizing photons, causing the ionization front to move back toward the cluster and finally become trapped within the shell. The size and density distribution of such ionized shells have little to do with the original Strömgren model. Their evolution depends not only on the ambient gas density distribution and the available Lyman continuum, but also on the mechanical power of the exciting cluster. Silich & Tenorio-Tagle (2013, hereafter ST13) discussed the impact that radiation pressure has on the dynamics of wind-driven shells powered by young star clusters and found radiation pressure not to be a dominant factor. They, however, did not consider the detailed impact that radiation pressure has on the inner shell structure. They also assumed that shells absorb all photons escaping from the central cluster and thus found an upper limit to the radiative feedback from the central cluster on the dynamics of the swept-up shell. Here we extend the analysis provided in ST13 and discuss how radiation pressure affects the distribution of density and thermal pressure within a shell and thus how it may affect the velocity of the outer shock and the dynamics of the ionized gas around young stellar clusters.

The paper is organized as follows: we first present in Section 2 the major equations formulated by Draine (2011) for static spherically symmetric H$\textsc{ii}$ regions and discuss how the inner and outer boundary conditions affect the solution. In Section 3 we discuss different hydrodynamic regimes and also show how Draine’s (2011) equations may be applied to the whole shell, including the outer, non-ionized segments. The results of the calculations are presented and discussed in Section 4, where we compare different hydrodynamical models (standard energy and momentum dominated, leaky and low star clusters’ heating efficiency), calculate the model-predicted values of the ionization parameter, and compare them to typical values found in local starburst galaxies. Our results are also placed onto a diagnostic diagram that allows one to discriminate between the radiation pressure and wind pressure (thermal or ram) dominated regimes. The summary of our major results is given in Section 5.
2. RADIATION PRESSURE IN STATIC, DUSTY H II REGIONS

Let us first consider the idealized model of a static spherically symmetric H II region ionized by a central star cluster and confined by the thermal pressure of the ambient ISM. Following Draine (2011, hereafter Dr11), we assume that the outward force provided by radiation pressure is balanced by the inward directed thermal pressure gradient. The set of equations describing such H II regions in the presence of dust grains is (see Dr11)

\[
\frac{d}{dr} \left( \frac{\mu_r}{\mu_a} n k T_i \right) = n \sigma_d \left[ \frac{L_i e^{-\tau} + L_a \phi}{4 \pi r^2 c} \right] + n^2 \beta_2 \left( \frac{h\nu}{c} \right),
\]

(1)

\[
S_0 \frac{d\phi}{dr} = - \beta_2 n^2 - n \sigma_d S_0 \phi,
\]

(2)

\[
\frac{d\tau}{dr} = n \sigma_d,
\]

(3)

where \( L_i \) and \( L_a \) are the luminosities in ionizing and non-ionizing photons, respectively (\( L_i + L_a = L_{bol} \), where \( L_{bol} \) is the bolometric luminosity of the cluster), \( n(r) \) is the ionized gas density, \( \phi(r) \) is the fraction of the ionizing photons that reaches a surface with radius \( r \), \( S_0 = Q_0/4 \pi r^2 \), with \( Q_0 \) being the number of ionizing photons emitted by the star cluster per second, \( \langle h\nu \rangle_i / Q_0 \) is the mean energy of the ionizing photons, \( \tau(r) \) is the dust absorption optical depth, \( \sigma_d \) is the effective dust absorption cross section per hydrogen atom, \( \beta_2 = 2.59 \times 10^{-15} \text{ cm}^2 \text{ s}^{-1} \) is the recombination coefficient to all but the ground level (Osterbrock 1989), \( k \) and \( c \) are the Boltzmann constant and the speed of light, respectively, and \( T_i \) is the ionized gas temperature. It is assumed that the gas in the H II region is completely ionized and has a normal chemical composition with one helium atom per every 10 hydrogen atoms.

The mean mass per particle and the mean mass per ion then are \( \mu_r = 14/23 m_H \) and \( \mu_a = 14/11 m_H \), respectively, where \( m_H \) is the proton mass. We set the value of the dust absorption cross section per hydrogen atom to \( \sigma_d = 10^{-21} \text{ cm}^2 \) (Dr11) and assumed that the temperature of the ionized gas is constant and equal to \( T_i = 10^4 \text{ K} \) in all our calculations. The first and the second terms on the right-hand side of Equation (1) correspond to the photon momentum absorbed by dust grains and by the gas, respectively. The right-hand terms in Equation (2) are the rates of absorption of ionizing photons in a thin spherical shell with radius \( r \) and thickness \( dr \) by recombination and by dust grains, respectively.

In order to select a unique solution of Equations (1)–(3), one has to adopt a set of initial or boundary conditions. For example, Draine (2011) selected solutions by choosing the initial value of density at some fixed radius \( r \). We use similar initial conditions in the case of the wind-driven shell (see next section), but prefer to select the static solution from the two boundary conditions, which are the values of the confining pressure at the inner and outer edges of the H II region. Here we assume that the H II region is static and that the radiation field from the central cluster is strong enough to clean up the central region with a radius \( R_i \), as it seems appropriate to many galactic and extragalactic H II regions, which are better fitted with models containing an empty central zone in the ionized gas distribution (see Mathews 1967, 1969; Kewley & Dopita 2002; Dopita et al. 2003). In such a case, the conditions at the inner edge of the H II region are \( \phi(R_i) = 1 \), \( \tau(R_i) = 0 \), and \( n(R_i) \rightarrow 0 \) (see Dr11), whereas the value of the initial radius \( R_i \) is selected by the outer boundary condition, which requires the thermal pressure at the outer edge of the H II region \( R_{Hi} \) to be equal to that in the ambient ISM. We use in the calculations a value of \( n(R_i) = 10^{-10} \text{ cm}^{-3} \) and stop the integration when all ionizing photons are trapped and thus the function \( \phi \) becomes equal to zero: \( \phi(R_{Hi}) = 0 \).

The input parameters \( (Q_0, L_a, \text{ and } L_i) \) for our calculations were taken from the Starburst99 synthesis model (Leitherer et al. 1999) and are summarized in Table 1. Models A, B, and C correspond to a 10\(^6\) \( M_\odot \) coeval stellar cluster with a standard Kroupa initial mass function with upper and lower cutoff mass of 100 \( M_\odot \) and 0.1 \( M_\odot \) respectively, and a turnoff mass at 0.5 \( M_\odot \), metallicity \( Z = 0.4 Z_\odot \), age \( t \sim 1 \text{ Myr} \), and Padova evolutionary tracks with asymptotic giant branch stars, embedded into an interstellar gas with number density 1 cm\(^{-3}\), 10\(^3\) cm\(^{-3}\) and 10\(^6\) cm\(^{-3}\), respectively. Models D, E, and F correspond to a two order of magnitude less massive cluster of the same age located within the same environments.

The calculated density distributions for static H II regions with a central cavity are shown in Figure 1. The density grows always rapidly in a very narrow inner zone and then presents an almost even or flat distribution in the rest of the volume if the density of the ambient ISM is not very large (models A, B, D, and E). Only when the exciting clusters are embedded into a very high density ambient medium (\( n_{\text{ISM}} = 10^6 \text{ cm}^{-3} \), models C and F) does the density of the ionized gas grow continuously across the whole H II region. However, such H II regions are very compact (see left-hand panels in Figure 1). The size of the H II region, \( R_{Hi} \), and the radius of the inner empty cavity, \( R_i \), are both functions of the interstellar ambient density. Both radii grow rapidly as one considers a lower ambient density (see the left-hand side panels in Figure 1, where the steps in the gas density distribution mark the edge of the H II regions and result from the condition that the thermal pressure at the H II region edge ought to be equal to that of the ambient neutral gas with a two order of magnitude lower temperature (\( T_{\text{ISM}} = 100 \text{ K} \)). This, however, is not evident when distances are normalized to the radius of the H II region and densities to their rms values as in Figure 2 of Dr11 (see right-hand panels in Figure 1). Thus, dimensionless plots do not allow one to realize that static models with a low ambient density are unrealistic, as in these cases the required time for the ionized gas re-distribution (the sound crossing time) highly exceeds the characteristic lifetime of the H II region, \( t_{\text{Hi}} \sim 10 \text{ Myr} \).

3. RADIATION PRESSURE IN DUSTY WIND-DRIVEN SHELLS

Given the continuous supply of photons and their instantaneous re-processing by the surrounding gas, here we use Dr11’s equations to calculate the impact that radiation pressure has on the structure and on the dynamics of evolving wind-driven shells. We consider a constant-density ISM and a set of evolving star cluster parameters to evaluate at consecutive times the impact of radiative pressure on the evolving shells and thus neglect all effects dealing with a plane-stratified density distribution.
Figure 1. Static H II regions with a central cavity. The upper left-hand panel presents the gas number density distribution as a function of radius for models A (dotted line), B (dashed line), and C (solid line) in a log–log scale. The upper right-hand panel shows the same distributions when all distances are normalized to the radius of the H II region and densities to their rms values. The bottom panels present similar density distributions for models D (dotted line), E (dashed line), and F (solid line), respectively.

Figure 2. Input parameters as a function of time. The left-hand panel shows the evolution of the bolometric (solid line), non-ionizing (dotted line), ionizing (dashed line), and mechanical (dash-dotted line) luminosities. The horizontal dash-dotted line displays the value of mechanical luminosity that has been used in Weaver et al. (1977) analytic relations. The right-hand panel shows the number of ionizing photons produced by a $10^6 M_\odot$ cluster per unit time. The vertical lines in both panels mark the onset of supernova explosions.
distribution in galactic disks, gas shear, and gravity, which were thoroughly discussed in our previous papers (see, for example, Tenorio-Tagle & Palous 1987; Silich 1992; Silich et al. 1996) and do not present the major aim of this paper. We also do not consider the impact that the ambient pressure has on the shell dynamics, as it is only significant when the shell expansion velocity approaches the sound speed value in the ambient ISM. The distribution of the ionized gas then becomes quasi-static and is defined by the values of thermal pressure at the inner edge of the shell and in the ambient ISM, as was discussed in the previous section. In the supersonic regime, which is the case in all our calculations (see Mach number values in the captions to Figures 3 and 4, calculated under the assumption that the sound speeds in the ionized and neutral ISM are 15 km s$^{-1}$ and 1.04 km s$^{-1}$, respectively), the rate of mass accumulation by the expanding shell depends on the speed of the leading shock, $V_s \sim (P_{\text{edge}}/\rho_{\text{ISM}})^{1/2}$, where $P_{\text{edge}}$ is the thermal pressure value immediately behind the leading shock and $\rho_{\text{ISM}}$ is the gas density in the ambient ISM. The impact of the external pressure on the shell dynamics is thus negligible in this case. When a star cluster wind impacts a constant-density ISM, a four-zone structure is established: there is a central free wind zone, surrounded by a shocked wind region. The latter is separated by a contact discontinuity from the matter swept up by the leading shock, which evolves into the constant-density ISM (see Weaver et al. 1977; Mac Low & McCray 1988; Koo & McKee 1992). In the wind-blown bubble case, the central zones are hot and thus transparent to the ionizing flux, as is also the case in the static H$\text{ii}$ regions with a central cavity considered in the previous section. However, the density at the inner edge of the ionized shell is not arbitrarily small, but must be selected from the condition that $P_{\text{H}^\text{ii}}(R_s) = P_1$, where $P_{\text{H}^\text{ii}}(R_s)$ and $P_1$ are the thermal pressures at the inner edge of the ionized shell and in the shocked wind region, respectively, and $R_s$ is the radius of the contact discontinuity (the inner radius of the ionized shell). The swept-up shell is also hot at first ($T \gtrsim 10^6$ K) and thus transparent to the ionizing radiation from the star cluster. However, it cools down in a short timescale due to strong radiative cooling. If the density and metallicity of the ambient medium are $n_{\text{ISM}}$ and $Z_{\text{ISM}}$, respectively, and the star cluster mechanical...
Figure 4. Wind-blown shell structure for a high-density environment. Two cases are presented: a high density standard model (HDS) and a high density low heating efficiency model (HDE). The left-hand column shows the results for models HDSa (top panel), HDSb (middle panel), and HDSc (bottom panel). The right-hand column displays the results for models HDEa (top panel), HDEb (middle panel), and HDEc (bottom panel). Solid lines correspond to the radial density distribution (left axis) for the free wind, shocked wind, ionized shell, neutral shell, and ambient ISM. Long-dashed and dotted lines depict the radial thermal and ram pressure distributions (right axis), respectively. The Mach number for the HDS models a, b, and c is 23.9, 8.2, and 5.8, respectively, while for the HDE models a, b, and c it is 12.5, 3.9, and 3.0.

![Diagram of shell structure for high-density environments](image)

It was assumed in all calculations that the temperature in the outer, neutral part of the shell is constant and equal to $T_\text{e} = 100$ K. It was also assumed that the shell is thin and thus the total mass of the shell is $M_{sh} = 4\pi\rho_{\text{ISM}}R_s^3/3$.

Thus, the initial conditions that allow one to select a unique solution of Equations (1)–(3) in the case of the wind-blown shell are very similar to those used in the previous section: $\phi(R_s) = 1$, $\tau(R_s) = 0$, and the value of the thermal pressure in the shocked wind zone, which depends on the dynamical time $t$. However, the inner radius of the ionized shell $R_i$ and the pressure $P_i$ at the inner edge of the shell at different evolutionary times $t$ are calculated from the Weaver et al. (1977) wind-blown bubble model, and the integration stops when the total mass of the ionized and neutral segments reaches $M_{sh}$. We then compare the values of the thermal pressures at the outer ($P_{\text{edge}}$) and inner ($P_i$) edges of the swept-up shell obtained from the calculations in order to check whether radiation pressure may affect the shell dynamics significantly.
In the energy-dominated regime, \( R_s \) and \( P_s \) are (Bisnovatyi-Kogan & Silich 1995)

\[
R_s(t) = \left[ \frac{375(\gamma - 1) L_{\text{mech}}}{28(9\gamma - 4)\pi \rho_{\text{ISM}}} \right]^{1/5} t^{3/5},
\]

\[
P_s(t) = 7^{1/3} \rho_{\text{ISM}}^{1/3} \left[ \frac{3(\gamma - 1) L_{\text{mech}}}{28(9\gamma - 4)\pi \rho_s^2} \right]^{2/3},
\]

where \( \rho_{\text{ISM}} \) is the interstellar gas density and \( \gamma = 5/3 \) is the ratio of specific heats. At this stage the free wind occupies only a small fraction of the bubble volume, and the value of thermal pressure \( P_s \) is defined by the amount of thermal energy accumulated in the shocked wind region and the bubble volume and thus does not depend on the wind terminal speed (see for more details Bisnovatyi-Kogan & Silich 1995).

The ion number density at the inner edge of the ionized shell then is

\[
n_i(t) = \frac{\mu_a P_s}{\mu_i k T_i}. \tag{9}
\]

However, evaporation of the swept-up shell into the hot shocked wind region may cause strong radiative cooling and lead to the end of the energy-dominated regime. If the star cluster is embedded into an ambient ISM with density \( n_{\text{ISM}} \), this occurs at (Mac Low & McCray 1988)

\[
\tau_{\text{tran}} = (1.6 \times 10^7) Z_{\text{ISM}}^{-35/22} n_{\text{ISM}}^{-8/11} \left( \frac{L_{\text{mech}}}{10^{38} \text{ erg s}^{-1}} \right)^{3/11} \text{ yr}. \tag{10}
\]

After this time, the free wind impacts directly on the shell and the thermal pressure at the inner edge of the swept-up shell is equal to the wind ram pressure \( P_{\text{ram}} = \rho_w V_w^2 \), where \( \rho_w = M_{\text{SC}}/4\pi R_s^2 V_\infty \), \( M_{\text{SC}} \) is the star cluster mass deposition rate, and \( V_\infty = (2M_{\text{mech}}/M_{\text{SC}})^{1/2} \) is the adiabatic wind terminal speed. The shell further expands in the momentum-dominated regime as (see ST13)

\[
R_s(t) = R_{\text{tran}} \left[ \frac{3L_{\text{mech}}(t^2 + \tau_{\text{tran}}^2)}{2\pi V_\infty \rho_{\text{ISM}} R_{\text{tran}}^4} \right]^{1/5} + \left( \frac{12}{5} - \frac{6L_{\text{mech}} \tau_{\text{tran}}^2}{\pi V_\infty \rho_{\text{ISM}} R_{\text{tran}}^2} \right) t^{7/3} \tau_{\text{tran}} - \frac{7}{5} t^{7/4}, \tag{11}
\]

\[
P_s(t) = \frac{L_{\text{mech}}}{2\pi V_\infty^2 R_s^2}. \tag{12}
\]

The radius of the shell at the time of the transition, \( R_{\text{tran}} \), must be calculated by means of Equation (7) at \( t = \tau_{\text{tran}} \). The above equations do not include the momentum of starlight. However, as shown below, this does not make a major difference in the evolution of the wind-blown shells except in the case of a low heating efficiency. Our calculations thus allow one to realize when the standard model assumptions may break down and radiation pressure may affect the dynamics of the wind-driven shells significantly.

Table 2

| \( N \) | Models \( a, b, c \) | \( L_{SC} \) (erg s\(^{-1}\)) | \( n_{\text{ISM}} \) (cm\(^{-3}\)) | \( Z_{\text{ISM}} \) (Z\(_{\odot}\)) | \( t \) (Myr) | Regime |
|---|---|---|---|---|---|---|
| 1 | LDS a, b, c | \( 10^{40} \) | 1 | 0.4 | 1, 3, 3, 5 | Low-density energy dominated |
| 2 | LDL a, b, c | \( 10^{40} \) | 1 | 0.4 | 1, 3, 3, 5 | Low-density with gas leakage |
| 3 | HDS a, b, c | \( 10^{40} \) | \( 10^3 \) | 0.4 | 1, 3, 3, 5 | High-density energy/momentum dominated |
| 4 | HDE a, b, c | \( 10^{40} \) | \( 10^3 \) | 0.4 | 1, 3, 3, 5 | High-density with low heating efficiency |

Note, however, that if thermal conduction and mass evaporation of the outer shell are inhibited by magnetic fields, the radiative losses of energy from the shocked wind region remain negligible and the wind-driven bubble expands in the energy-dominated regime during the whole evolution of the \( \text{H}\)\textsc{ii} region (see Silich & Tenorio-Tagle 2013).

4. RESULTS AND DISCUSSION

4.1. Shells Evolving in a Low-density ISM

We first explore the impact that radiation has on the wind-driven shells expanding into a low-density ambient medium (Table 2, LDS model). Models LDSa, LDSb, and LDSc present different evolutionary stages of the “standard bubble model.” In this case the wind-driven shell expands into a low-density (1 cm\(^{-3}\)) ISM in the energy-dominated regime. In all cases the mass of the driving cluster is \( 10^6 \text{ M}_\odot \) and the selected times allow one to see how the ionization structure of the shell changes with time due to the bubble and radiation field evolution.

Figure 3 displays the density (solid lines), thermal pressure (dashed lines), and ram pressure (dotted lines) distributions within and at both sides of the expanding shell, while this is exposed to the radiation from the central cluster. The sudden density jumps at the inner edge of the ionized shell result from the fact that the thermal pressure there must be equal to the thermal pressure of the hot thermalized cluster wind (Equation (9)), while the temperature in the ionized gas is \( 10^4 \text{ K} \). As shown in Figure 3, in the case of model LDSa (\( t = 1 \text{ Myr} \))...
the swept-up shell has already cooled down and is completely photoionized by the Lyman continuum from the young central cluster. Furthermore, a fraction of the ionizing photons still escapes from the shell into the ambient ISM, keeping it also at \( T = 10^4 \) K. Model LDSb presents the shell structure at the trapping time, \( \tau_{\text{trap}} = 3.3 \) Myr. At this time the shell absorbs all ionizing photons, and the mass of the ionized matter is exactly that of the swept-up shell: \( M_{\text{ion}} = M_{\text{ab}} \). The thermal pressure outside of the shell then falls by two orders of magnitude, as it is assumed that the temperature of the ambient neutral gas in this case is 100 K (see the left-hand middle and bottom panels in Figure 3). The first supernova explosion also occurs at this time, and thus the number of ionizing photons emerging from the central cluster begins to decay rapidly afterward. Model LDSc presents the shell structure at a later time, \( t = 5 \) Myr, when all ionizing photons are absorbed in the inner segments of the shell and thus the outer skin of the shell remains neutral.

The conditions for model LDL assume a leaky bubble model (e.g., Matzner 2002; Harper-Clark & Murray 2009). In this case, the thermal pressure inside the wind-driven bubble drops below the Weaver et al. (1977) model predictions due to the escape of hot shocked-wind plasma through holes in the wind-driven shell. In this case individual bow shocks around the shell fragments should merge to create a coherent reverse shock near the contact discontinuity, or the inner side of the broken shell (see Tenorio-Tagle et al. 2006; Rogers & Pittard 2013). We thus assume that the minimum driving force on the shell in the leaky bubble model is determined by the cluster wind ram pressure at the shell location and can never fall below such a value (ST13). Hereafter we will assume that in the leaky case the transition from energy- to momentum-dominated regimes occurs at 0.13 Myr, just after the shell cools down and begins to absorb ionizing photons. Equations (7)–(9) are replaced with Equations (11)–(13) at this time. Certainly, this time is arbitrary, but warrants the maximum possible effect of radiation pressure.

The density and thermal pressure distributions within and at both sides of the shell in the leaky case are shown in the right-hand panels of Figure 3. Here the top, middle, and bottom panels correspond to models LDLa, LDLb, and LDLc and thus present the density, thermal pressure, and ram pressure profiles at the same evolutionary times \( t = 1, 3.3, \) and 5 Myr, respectively. The size of the leaky shell is smaller and its thickness larger than those predicted by the standard bubble model (model LDS), and the difference grows with time (compare the right- and left-hand panels in Figure 3). Note also that the leaky shell is not able to trap all ionizing photons and form an outer neutral skin for a much longer time (in this case \( \tau_{\text{trap}} \approx 5 \) Myr). This is because in the leaky bubble model the driving pressure and thus the ionized gas density at the inner edge of the shell are much smaller than those in the standard case (LDS).

The expectations resulting from calculations of the ionized gas distribution in static configurations with low-pressure central cavities (Section 2) had been that radiation pressure would lead to a nonhomogeneous thermal pressure and density distributions inside the wind-driven shell. Both density and thermal pressure should grow from a low value at the inner edge of the shell to a maximum value at the outer edge, as in the high-density static models (see Section 2). The calculations, however, do not show such large enhancements in density and in the leading shock driving pressure relative to that at the inner edge of the shell. The density enhancement is about \( \sim 1.04 \) and \( \sim 1.09 \) at 1 Myr, \( \sim 1.07 \) and \( \sim 1.18 \) at 3.3 Myr, and \( \sim 1.04 \) and \( \sim 1.12 \) at 5 Myr in the standard and the leaky bubble model, respectively (see the left-hand and right-hand panels of Figure 3). We then provided similar calculations for an order of magnitude less massive cluster (10^5 \( M_\odot \)) and did not find significant difference with the above results. In all calculations with a 10^5 \( M_\odot \) cluster the density enhancement does not exceed \( \sim 1.1 \), despite that radii of the shells differ significantly from those obtained in the more energetic models LDS and LDL. These results demonstrate how significantly the inner boundary condition (the value of thermal pressure at the inner edge of the \( \text{H} \text{ii} \) region) may change the ionized gas density distribution. They also imply that the impact from radiation pressure on the dynamics of shells formed by massive young stellar clusters embedded into a low-density ambient medium is not significant throughout their evolution even if all of the hot plasma leaks out from the bubble interior into the surrounding medium. Consequently, we allow for the use of Equations (7)–(8) and (11)–(12), ignoring the impact of the starlight momentum.

4.2. Shells Evolving in a High-density ISM

The high-density models (Table 2, models HDS and HDE) are evaluated at the same dynamical times: \( t = 1, 3.3, \) and 5 Myr and are displayed in Figure 4. In these cases the model predicts that the transition from the energy-dominated to the momentum-dominated regime occurs at much earlier times (see Equation (10)). For example, in the case of model HDS, \( \tau_{\text{tran}} \approx 1.58 \) Myr. Thus, models HDSb and HDSc correspond to a shell expanding in the momentum-dominated regime. The size of the shell in this case is much smaller than when it expands into a low-density ISM; however, the shell is much denser and thus recombines faster. Therefore, in the high-density cases the ionizing radiation is not able to photoionize the whole shell from the very early stages of the bubble evolution (see the top left-hand panel in Figure 4). The density in the ionized shell drops when the transition to the momentum-dominated regime occurs. This allows the central cluster to photoionize a larger fraction of the swept-up material. Therefore, the relative thickness of the ionized shell increases between 1 Myr and 3.3 Myr (compare panels HDSa and HDSb in Figure 4). After 3.3 Myr, the number of ionizing photons decreases rapidly (Figure 2), and the relative thickness of the ionized shell becomes smaller again despite the drop in driving pressure and the consequent drop in the ionized gas density (see panel for HDSc). The density gradient also reaches the maximum value at 3.3 Myr and then drops at later times. The density (and thermal pressure) gradient across the ionized shell in the high-density models is larger than in the low-density cases. For example, the enhancement of density relative to that at the inner edge of the shell in model HDSa is \( \sim 1.14 \), in model HDSb is \( \sim 1.67 \), and in model HDSc is \( \sim 1.25 \) (see the left-hand panels in Figure 4). This is because the inner radius of the ionized shell in the high-density case is smaller and thus the impact that radiation pressure has on the shell is larger.

The right-hand panels in Figure 4 present the results of the calculations when the driving cluster has a low heating efficiency (models HDEa, HDEb, and HDEC). These calculations were motivated by the discrepancy between the Weaver et al. (1977) model predictions and the observed sizes and expansion velocities of the wind-blown bubbles known as “the growth-rate discrepancy” (Oey 1996) or “the missing wind problem” (Freyer et al. 2006; Dopita et al. 2005; Smith et al. 2006; Silich et al. 2007, 2009) and by the fact that at the initial stages of the bubble evolution the star cluster mechanical luminosity still
does not reach the average value adopted in our calculations (see Figure 2). The heating efficiency may also be small if the kinetic energy of stellar winds is converted to turbulence and radiated away in young stellar clusters (Bruhweiler et al. 2010). At later stages of evolution a low heating efficiency may be physically justified by assuming mass loading of the matter left over from star formation, as in Wünsch et al. (2011), or an oversolar metallicity of the supernova ejecta that enhances the cooling rate, as in Tenorio-Tagle et al. (2005). More recently, a low heating efficiency has been shown to also arise from the consideration of a continuous presence of dust within the cluster volume, dust produced within the ejecta of the multiple core-collapsed supernovae expected in young clusters (see Tenorio-Tagle et al. 2013). In this case, we keep the values of $L_{\text{bol}}$, $L_{\text{mech}}$, and $Q_0$ equal to those predicted by the Starburst99 synthetic model for a $10^4 M_\odot$ cluster, but instead of using $L_{\text{mech}} = 10^{40} \text{ erg s}^{-1}$, as in our models HDSa–HDSc, we use an order of magnitude smaller mechanical luminosity: $L_{\text{mech}} = 10^{39} \text{ erg s}^{-1}$. The transition to the momentum-dominated regime in this case occurs at $\approx 0.84 \text{ Myr}$. The relative thickness of the ionized shell is much larger than that in model HDS, as the size of the shell is about two times smaller and thus the flux of the ionizing radiation is about four times larger than in model HDS (compare the left-hand and right-hand panels in Figure 4). This leads to the largest calculated density enhancement in the shell density (and thus thermal pressure) relative to that at the inner edge of the shell, which is $\approx 6.41$ at $t = 1 \text{ Myr}$, $\approx 7.26$ at $3.3 \text{ Myr}$, and $\approx 3.47$ at $5 \text{ Myr}$. These results imply that radiation pressure must be taken into consideration in calculations with low heating efficiency and that the Weaver et al. (1977) model (Equations (7)–(8) and (11)–(12)) must be corrected in this case. The radiation pressure may also contribute to the shell dynamics at very early stages (before 3 Myr) of the wind-blown bubble evolution (see also Figure 3 in ST13). Similar results were obtained for the less massive ($10^3 M_\odot$) clusters. In this case the maximum enhancement of density is $\approx 1.43$ in the standard (HDS) case and $\approx 4.68$ in the low heating efficiency (HDE) model, respectively.

The time evolution of the thermal pressure excess, $P_{\text{edge}}/P_s$, where $P_{\text{edge}}$ and $P_s$ are the values of the thermal pressure behind the leading shock and at the inner edges of the wind-driven shell, respectively, is shown in Figure 5. In the high-density models (dashed and dash-dotted lines) this ratio decreases first, as the flux of ionizing energy at the inner edge of the shell drops faster (as $R_s^{-2}$) than thermal pressure in the shocked wind region, which drops as $R_s^{-4/3}$ (see Equation (8)). It then grows to a larger value when the hydrodynamic regime changes from the energy- to a momentum-dominated expansion and the wind pressure at the inner edge of the shell drops abruptly. After this time, both the flux of radiation energy and the wind ram pressure at the inner edge of the shell drop as $R_s^{-2}$. The $P_{\text{edge}}/P_s$ ratio then grows slowly as the number of non-ionizing photons absorbed by the outer neutral shell increases with time. The slow increase of the $P_{\text{edge}}/P_s$ ratio continues until the number of ionizing photons begins to drop after the first supernova explosion at 3.3 Myr, when the $P_{\text{edge}}/P_s$ ratio reaches 1.67 ($\log P_{\text{edge}}/P_s \approx 0.22$) in the case of model HDSb and 7.26 ($\log P_{\text{edge}}/P_s \approx 0.86$) in the case of model HDEb.

In the low-density cases (solid and dotted lines) the swept-up shell is not able to absorb all ionizing photons until it grows thick enough, and therefore the number of ionizing photons trapped inside the completely ionized shell grows continuously until the first supernova explosion at 3.3 Myr. This compensates the $R_s^{-2}$ drop of the ionizing energy flux and leads to a continuously growing $P_{\text{edge}}/P_s$ ratio at this stage. However, in the standard (solid and dashed lines) case and leaky (dotted line) bubble model this ratio remains always smaller than $\approx 1.7$. In the low-density models LDS and LDL it is even smaller (less than 1.2) and is below the upper limit obtained in ST13. This is because in the low-density cases wind-driven shells absorb only a fraction of the star cluster bolometric luminosity.

The fraction of the star cluster bolometric luminosity trapped within a shell as a function of time in models LDS, LDL, HDS, and HDE is shown in Figure 6 by solid, dotted, dashed, and dash-dotted lines, respectively. Note that dashed and dash-dotted lines overlap into a single horizontal line $L_{\text{abs}}/L_{\text{bol}} = 1$ at the earliest stages of the shell evolution.
radiation pressure by the multiple re-emitted IR photons, which is \( \sim \tau_{IR} L_{bol}/c \) (see Hopkins et al. 2011; Krumholz & Thompson 2012), thus remains less than unity. In all our calculations, the amplification factor never exceeds two, even if one uses a larger dust opacity, \( \kappa_d = 5 \text{ cm}^2 \text{ g}^{-1} \) adopted by Hopkins et al. (2011). This implies that the star cluster wind-driven shells expand in the radiation momentum rather than in the radiation-energy-dominated regime (see Fall et al. 2010; Krumholz & Thompson 2012, for the detailed discussion of the two limiting cases).

We also computed how radiation pressure affects the density and thermal pressure distribution in the case when the exciting cluster is embedded into a low-density (\( n_{ISM} = 1 \text{ cm}^{-3} \)) ISM and has a low heating efficiency and in the case of a leaky shell moving into a high-density (\( n_{ISM} = 1000 \text{ cm}^{-3} \)) medium. We found little difference between these calculations and models LDL and HDS, respectively. For example, the enhancement of density from the inner to the outer edge of the shell in the low-density calculations with a 10% heating efficiency is about 1.13, 1.2, and 1.11 at 1 Myr, 3.3 Myr, and 5 Myr, whereas in the leaky bubble model LDL it is \( \sim 1.1, \sim 1.19, \) and \( \sim 1.12 \), respectively. In the case when a leaky shell expands into a high-density medium, the enhancement of density is \( \sim 1.61 \) at 1 Myr, \( \sim 1.67 \) at 3.3 Myr, and \( \sim 1.25 \) at 5 Myr, whereas in model HDS it is \( \sim 1.14, \sim 1.67, \) and \( \sim 1.25 \), respectively, and thus the only difference between the last two models is that the transition from energy- to momentum-dominated regimes occurs at different times. Therefore, we do not present the detailed description of these calculations in our further discussion.

4.3. Comparison to Other Models and Observations

Having the exciting cluster parameters and the distribution of the ionized gas density in the surrounding shell, one can obtain the model-predicted values for diagnostic parameters often used in observations and compare them to the typically observed ones. In this section we first calculate the values of the ionization parameter and then put our results onto a diagnostic diagram proposed by Yeh & Matzner (2012), which allows one to conclude whether radiation or the wind dynamical pressure dominates the dynamics of the ionized gas around young stellar clusters.

The ionization parameter \( U \) is defined as the flux of ionizing photons per hydrogen atom. It is directly related to the state of ionization and to the radiation pressure over gas thermal pressure ratio and is usually calculated at the inner edge of the ionized medium (e.g., Dopita et al. 2005):

\[
U = \frac{Q_{0}}{4\pi n R^2 c} = \frac{\mu_i}{\mu_a} \frac{kT_i}{\hbar v} P_{rad} / P_{HII}. \tag{14}
\]

The ionization parameter may be measured observationally from the emission-line ratios (e.g., Rigby & Rieke 2004; Snijders et al. 2007; Yeh & Matzner 2012, and references therein) and thus is a powerful tool to measure the relative significance of the radiation and gas thermal pressure around young stellar clusters. However, the number of ionizing photons varies radially within \( H_{II} \) regions, and therefore the measured values of \( U \) are weighted by the density distribution in the ionized nebula. This led Yeh & Matzner (2012) to propose as a relevant model parameter

\[
U_w = \frac{\int 4\pi r^2 n^2 U(r) dr}{\int 4\pi r^2 n^2 dr}, \tag{15}
\]

where the integrals are evaluated from the inner to the outer edge of the \( H_{II} \) region. In our approach, we have neglected the presence of any neutral gas and dust able to deplete the radiation field in the free and hot shocked wind regions and thus assumed that all the photons produced by the star cluster are able to impact the shell. The integrals in Equation (15) thus were evaluated with the lower and upper limits \( R_s \) and \( R_{HII} \), respectively. Here we make use of our models to obtain the ionized gas density distribution within wind-driven shells expanding into different interstellar media and calculate the ionization parameter \( U_w \) at different times \( t \). The results of the calculations are presented in Figure 8. One can note that the time evolution of the ionization parameter \( U_w \) in the wind-driven bubble model is complicated as it depends not only on the varying incident radiation, but also on the hydrodynamics of the wind-driven shell. In all cases the value of \( U_w \) drops first as the wind-driven shell expands and the photon flux at the inner edge of the shell drops accordingly. In the standard case (LDS, solid line) the value of \( U_w \) drops continuously but starts to decrease faster after the first supernova explosion, as since that time the flux of incident photons per unit area drops not only because of the shell expansion, but...
also because of the reduced value of $Q_0$. In the high-density model HDS (dashed line) the value of the ionization parameter increases by about an order of magnitude after the transition to the momentum-dominated regime, as when the transition occurs, the wind pressure and the ionized gas density at the inner edge of the shell drop, which enhances the value of $U_w$ significantly (see Equations (14) and (15)). The value of the ionization parameter then remains almost constant until the first supernova explodes at about 3.3 Myr, as at this stage both the flux of ionizing photons and the ram pressure of the wind at the supernova explodes at about 3.3 Myr, as at this stage both the flux of ionizing photons and the ram pressure of the wind at the outer and the inner edge of the ionized shell and the values of $\Omega$ increase rapidly. The behavior of $U_w$ in the leaky (model LDL, dotted line) and low heating efficiency (model HDE, dash-dotted line) cases is very similar to that in the high-density case HDS. The only difference is that the transition to the momentum-dominated regime in these cases occurs at earlier times and the maximum values of the ionization parameter are larger than that in model HDS. One can also note that the ionization parameter reaches the maximum possible value, $\log \Psi \approx -1.5$, in the low heating efficiency model HDE and that the model-predicted values of the ionization parameter fall into the range of typical values found in local starburst galaxies: $-3 \lesssim \log U_w \lesssim -1.5$, (see Figure 10 in Rigby & Rieke 2004). The larger values of the ionization parameter (e.g., Snijders et al. 2007) require either a lower heating efficiency, as was also claimed in Dopita et al. (2005), or a more complicated physical model than a single ionized shell formed by a young stellar cluster (see the discussion in Snijders et al. 2007).

Finally, we put our results onto a diagnostic diagram proposed by Yeh & Matzner (2012) in order to show where physically motivated models are located in this diagram. For example, their model with more than an order of magnitude increasing density (see Figure 7 in their paper), $L_i = 10^{52}$ erg s$^{-1}$, $\log \Phi = -1.09$, and $\log \Omega = -1.56$, corresponds, according to our calculations, to a very compact ($R_{\text{HII}}$ less than 3 pc) and very dense ($n_e$ is a few hundred particles per cm$^3$) shell at the age of 2 Myr, which implies that the H$\alpha$ region is quasi-static and requires a very low star cluster heating efficiency and a large confining (thermal/turbulent) pressure in the ambient ISM (see Smith et al. 2006; Silich et al. 2007, 2009). Two-dimensional parameter space introduced by Yeh & Matzner (2012) is related to the compactness of the H$\alpha$ region (parameter $\Psi$) and to the relative strength of different driving forces (parameter $\Omega$). Parameter $\Psi$ is defined as the $R_{\text{HII}}/R_{\text{ch}}$ ratio, where $R_{\text{HII}}$ is the radius of the ionization front (in our case this is the radius of the outer edge of the ionized shell) and $R_{\text{ch}}$ is the radius of a uniform density Str"омgren sphere whose thermal pressure is equal to the maximum possible unattenuated radiation pressure at the edge of the H$\alpha$ region $P_{\text{rad}} = L_{\text{bol}}/4\pi c R_{\text{at}}^2$:

$$R_{\text{ch}} = \frac{\beta_{\text{ch}} L_{\text{bol}}^2}{12\pi c^2 (kT_{\text{ic}})^2 Q_0}. \quad (16)$$

Parameter $\Omega$ is related to the volume between the ionization front and the inner edge of the H$\alpha$ region and to the values of thermal pressure at its inner and outer edges:

$$\Omega = \frac{P_i R_i^3}{P_{\text{edge}} R_{\text{edge}}^3 - P_i R_i^3}. \quad (17)$$

We obtain parameter $\Omega$ by calculating the volume between the outer and the inner edge of the ionized shell and the values of thermal pressure $P_i$ and $P_{\text{edge}}$ even at earlier stages of models LDS and LDL when the ionized shell is still embedded into an extended diffuse H$\alpha$ region. As long as the ionized shell is thin, parameter $\Omega$ is

$$\Omega \approx 4\pi c R_i^2 P_i / L_{\text{bol}}, \quad (18)$$

and thus measures the wind dynamical over the radiation pressure ratio (the shell moves in the radiation-dominated regime if $\log \Omega < 0$ and in the wind-dominated regime if $\log \Omega > 0$). In all static models discussed in Section 2 the parameter $\Omega$ is very small ($\log \Omega \sim -15$), which implies that radiation pressure controls the ionized gas distribution in all static configurations with low-pressure central cavities. In the wind-blown cases the parameter $\Psi$ is a function of time as both radii, $R_{\text{HII}}$ and $R_{\text{ch}}$, change with time. Therefore, it is instructive to show first how parameter $\Omega$ changes with time. This is shown in Figure 9(a). Panel (b) in this figure displays the evolutionary tracks of our models in the $R_{\text{HII}}/R_{\text{ch}}$ parameter space.
space. The initial points for models LDS, LDL, HDS, and HDE were calculated at the star cluster age of 0.13 Myr. The initial values of the normalization radius $R_{\text{ch}}$ then are $\sim 72$ pc in model LDL and $\sim 70$ pc in models LDS, HDS, and HDE, respectively. As both star cluster parameters, $L_{\text{bol}}$ and $Q_{\Omega}$, change with time, the value of $R_{\text{ch}}$ also changes with time significantly and by 10 Myr reaches $\sim 720$ pc. In cases LDS and LDL parameter $\Omega$ declines until the shell grows thick enough to absorb all ionizing photons and then grows continuously (see panel (a), solid and dotted lines). In the high-density cases parameter $\Omega$ drops drastically when the transition occurs to the momentum-dominated regime, then slightly declines and increases again after the first supernova explosion as the number of the ionizing photons then drops rapidly. The strong time evolution of $R_{\text{ch}}$ leads to the intricate tracks of the ionized shells in the log $\Omega$–log $\Psi$ diagram (see panel (b)). In the low-density models LDS and LDL the tracks go to the left and up because the normalization radius $R_{\text{ch}}$ grows with time faster than the radius of the shell and thus the ionization front radius $R_{\text{HII}}$. In the high-density cases HDS and HDE the tracks are more intricate. They first go to the right, then drop down when the transition to the momentum-dominated regime occurs, make a loop, and finally go back to the left and up.

Thus, in the low-density cases the impact of radiation pressure on the shell dynamics is always negligible and declines with time. In the high-density model HDS the contribution of radiation pressure to the shell dynamics becomes more significant when the shell makes a transition from the energy to the momentum-dominated regime. However, in this case parameter log $\Omega$ also remains positive, and thus in all models with 100% heating efficiency the shells expand in the wind-dominated regime. Parameter log $\Omega$ falls below a zero value only in the low heating efficiency case HDE. Thus, only in this case may radiation pressure dominate the shell dynamics. The radiation-dominated phase lasts from the beginning of the momentum-dominated regime at $\sim 0.85$ Myr until $\sim 7.36$ Myr (see panel (a)). This implies that radiation pressure may dominate the dynamics of the gas around young stellar clusters either at early stages of evolution (before $\sim 3$ Myr) or if the major fraction of the star cluster mechanical luminosity is dissipated or radiated away within the star cluster volume, and thus the star cluster mechanical energy output is much smaller than star cluster synthetic models predict. However, even in these cases radiation effects may be significant only if the exciting cluster is embedded into a high-density ambient medium.

4. The impact that radiation pressure provides on the dynamics and inner structure of the wind-driven shell is always negligible during the advanced stages of evolution, as the radiation energy flux declines rapidly after the first supernova explosion, whereas the mechanical power of the cluster does not.

5. The calculated values of the density-weighted ionization parameter $U_w$ fall into the range of typical values found in nearby starburst galaxies ($-3 \leq \log U_w \leq -1.5$). The larger values of the ionization parameter sometimes detected around very young stellar clusters require either a lower heating efficiency or a more complicated than a single ionized shell physical model.

6. The model location in the log $\Omega$–log $\Psi$ diagnostic diagram proposed by Yeh & Matzner (2012) strongly depends on the evolutionary time $t$, which leads to intricate evolutionary track patterns. The standard wind-driven and leaky bubble models are located in the upper segments in this diagram, where $\text{H II}$ regions evolving in the thermal-pressure-dominated regime settle in. The only evolutionary track that temporarily passes through the lower left corner, where radiation-pressure-dominated $\text{H II}$ regions are located, is that resulting from calculations with a low heating efficiency.

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