The effect of deconfinement phase transition on rotochemical deviations in stars containing mixed phase matter

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ABSTRACT
As a neutron star spins down, its core density increase, changing the relative equilibrium concentration, and causing deconfinement phase transition as well. Hadron matter are converted into quark matter in the interior, which enhances the deviation of chemical equilibrium state. We study such deviations and its chemical energy release. Applying to the simulation of cooling neutron stars, we find the surface effective temperature of neutron stars is promoted obviously. This implies that the deconfinement phase transition is able to raise the chemical heating efficiency.

Key words: dense matter – nuclear reactions, nucleosynthesis, abundances – stars: neutron – stars: interiors

1 INTRODUCTION
Neutron star cooling is an important tool to probe its inner structure. The comparison of cooling models with observation of thermal emission provide insight into the EOS of dense matter, and the signatures of exotic particles (Page et al. 2006) (Yakovlev & Pethick 2004). A newly formed neutron star loses the thermal energy through neutrino emission initially, and is taken over by the surface photon radiation about $\sim 10^5$yr after birth. However, heating sources inside neutron stars can keep neutron stars hot beyond the standard cooling timescale $\sim 10^7$yr. Some mechanisms have been extensively discussed. These include the mutual friction between superfluid and normal component of the star (Shibazaki & Lamb 1989), heat released through deconfinement phase transition (Kang & Zheng 2007) and release of strain energy stored by the solid crust due to spin-down deformation (Cheng et al. 1992).

Another of these mechanism is rotochemical heating, firstly proposed by Reisenegger (Reisenegger 1995) and applied to quark stars by Cheng (Cheng & Dai 1996). Then Reisenegger improved it, in the framework of general relativity, by considering the internal structure via realistic EOSs (Fernandez & Reisenegger 2003) and superfluid nucleons core (Petrovich & Reisenegger 2010). All studies above agree that rotochemical heating is important for old neutron stars.

Rotochemical heating origins in the deviation from weak reaction equilibrium due to spin-down compression. As the star spins down, the decrease of the centrifugal force leads to compression, increasing the star’s internal density. This compression results in a displacement of the equilibrium concentration of each particle species and changes the chemical equilibrium state. Reactions that change the chemical composition drive the system to the new equilibrium configuration. If the reaction rate is slower than the change of the equilibrium concentrations due to spin-down compression, the star is never exactly in chemical equilibrium, so energy is stored. The excess of energy is dissipated by enhanced neutrino emission and heat generation.

It has long been hypothesized that some compact stars might actually be "hybrid stars" containing cores of quark matter (Baym & Chin 1976). Unlike neutron stars with pure hadronic matter and strange stars with pure strange matter, those hybrid stars probably contain the mixed phase in light of Glendenning’s phase transition theory (Glendenning 2000). Of course, the compression of a hybrid star due to spin-down causes deconfinement phase transition (hereafter DPT), besides nonequilibrium Urca processes (Steiner et al. 2009). The spin-down compression increases the interior density gradually, which results in the transformation of hadron matter into quark matter in the interior little by little. The deconfinement reactions change relative concentrations of the particles, neutrons, protons, and $u, d, s$ quarks, and result in the displacement of equilibrium state as well. Because neutrons and protons are converted into two flavor quarks, and it takes a finite amount of time that two flavor quarks decay into three. Thus the deconfinement con-
drive the system deviating to a nonequilibrium state 2, \( \Delta N \) protons are converted into neutrons, which was treated in equilibrium condition from equilibrium state 1 to equilibrium state 3 in previous study, \( \Delta N_\nu \) and change of nucleon baryon number density \( \Delta n_b \). Since neutrons and protons are converted into \( u \) and \( d \) quarks, without \( s \) quarks, we have
\[
\Delta n_q = \Delta n_u + \Delta n_d, \quad \Delta n_h = \Delta n_u + \Delta n_p.
\]
(4)

The reactions change the concentration of particles and drive the system deviating from an equilibrium state (state 1) to a nonequilibrium state (state 2). Then, the system tries to recover to a new equilibrium state (state 3) through reactions \( \beta \)-equilibrium and weak reactions, the system is never exactly in chemical equilibrium due to deconfinement behavior.

For quarks, the non-leptonic reaction and Durca reactions are all involved in the evolution of chemical potential, among which only two processes are needed in the calculation of the chemical departure, resulting in rotochemical heating. In the later calculation, we find that the contribution of the nucleon dominates hybrid star thermal evolution and the chemical evolution of nucleon is independent of reaction \( \beta \). So we don’t distinguish the difference between the non-leptonic reaction and Durca reactions, and choose the Durca reactions to calculate the chemical potential difference.

To express the chemical deviation quantities, under consideration of deconfinement reactions, we define the chemical potential of particle \( i \) in nonequilibrium state (state 2) as \( \mu_i' \) and \( \mu_i'' \) in new equilibrium state (state 3). The chemical potential differences of three channels in reactions \( \beta \) are:
\[
\delta \mu_d = \delta \mu_d - \delta \mu_u - \delta \mu_e, \\
\delta \mu_s = \delta \mu_s - \delta \mu_u - \delta \mu_e, \\
\delta \mu_N = \delta \mu_n - \delta \mu_p - \delta \mu_e.
\]
where reaction channels denoted by capitals, particles denoted by small letters.

We assume \( n_m = a n_h, n_p = (1 - \alpha) n_u \), under beta equilibrium, then \( \Delta n_h = \alpha \Delta n_h, \Delta n_p = (1 - \alpha) \Delta n_h \). The chemical potential of particles in each state for density change and density of a given initial state.

\[
\Delta n_d = \frac{\alpha + 1}{2 - \alpha} \Delta n_u.
\]
(5)

Substitute \ref{2} into \ref{2}, we obtain
\[
\Delta n_u = \frac{2 - \alpha}{3} \Delta n_q, \quad \Delta n_d = \frac{1 + \alpha}{3} \Delta n_q.
\]
At zero temperature, chemical potential of Fermions satisfies
\[
\mu_i^2 = m_i^2 + \left(3 \pi^2 n_i r_i \right)^2, \quad \text{then we determine chemical potential of particles in each state for density change and density of a given initial state.}
\]

\[
\mu_u = (3 \pi^2 n_u r_u)^2 \left(3 + \frac{6 \mu_e}{\mu_u} - \frac{3m_u^2}{2\mu_u}\right)^{-\frac{1}{2}}, \\
\mu_d = \mu_s = \mu_u + \mu_e, \\
\mu_p = (3 \pi^2 n_p r_p)^2 \left(2 + \frac{3 \mu_e}{\mu_p} - \frac{3m_u^2 + 3m_d^2}{2\mu_p}\right)^{-\frac{1}{2}}, \\
\mu_n = \mu_p + \mu_e.
\]
To leading order, we obtain
\[ \frac{d\delta \mu}{dt} = \frac{2}{3} \frac{\Delta n_\mu}{n_\mu} \left( \mu_u + \mu_d + \mu_s \right), \]

\[ \mu_u = \mu_d(1 + \frac{\Delta n_\mu}{3n_\mu}), \]

\[ \mu_s = \mu_s, \]

\[ \mu_p = \mu_p(1 + \frac{\mu_p^2 - m_\mu^2 + \Delta n_\mu}{3n_p}). \]

\[ \frac{d\delta N}{dt} = \frac{2}{3} \frac{\alpha_s}{n_\mu} \left( \mu_u + \mu_d + \mu_s \right), \]

\[ \mu_u' = \mu_d', \]

\[ \mu_p' = \mu_p + \mu_e'. \]

State 2 and 3 can be expressed in light of state 1 and density change, we immediately write
\[ \delta \mu_u = \mu_u' - \mu_u, \]

\[ \delta \mu_s = \mu_s' - \mu_s, \]

\[ \delta \mu_d = \mu_d' - \mu_d, \]

\[ \delta \mu_p = \mu_p - \mu_p', \]

\[ \delta \mu_e = \mu_e' - \mu_e. \]

since state 3 has been defined as equilibrium state, the \( \beta \)-equilibrium condition should enter to simplify our calculations. So
\[ \delta \mu_d = \mu_d - \mu_d', \]

\[ \delta \mu_s = \mu_s - \mu_s', \]

\[ \delta \mu_p = \mu_p - \mu_p', \]

\[ \delta \mu_e = \mu_e - \mu_e'. \]

3 EQUATIONS OF CHEMICAL AND THERMAL EVOLUTIONS

According to the discussion above, We first calculate the departure rate from \( \beta \) equilibrium in channel \( d \rightarrow u + e^- + \nu_e \)
\[ \frac{d\mu_d}{dt} = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \left[ \frac{\mu_u - \mu_u'}{3n_\mu} \right]. \]

To leading order, we obtain
\[ \frac{d\mu_d}{dt} = \frac{2}{3} \frac{\mu_u - \mu_u'}{n_\mu} + 0. \]

In the similar way, we give
\[ \frac{d\mu_s}{dt} = \frac{2}{3} \frac{\mu_s - \mu_s'}{n_\mu} + 0. \]

\[ \frac{d\mu_p}{dt} = \frac{2}{3} \frac{\mu_p - \mu_p'}{n_p} + 0. \]

\[ \frac{d\mu_e}{dt} = \frac{2}{3} \frac{\mu_e - \mu_e'}{n_e} + 0. \]

On the other hand, the departure is preferred to recover its equilibrium through reactions \( \beta \). Therefore, we have
\[ \frac{d\delta \mu}{dt} = -\Gamma_\mu E_{xx}(\mu), \]

\[ \frac{d\delta N}{dt} = \Gamma_N E_{xx}(\mu), \]

where \( x \) indicates \( d, s, n \) fraction of baryon density, \( E_{xx} \) is partial derivative of chemical energy per baryon (Cheng & Dai 1996). In the hadron-quark mixed phase, hadron and quark undergo direct Urca process (Kang & Zheng 2007). \( \Gamma_d, \Gamma_s, \) and \( \Gamma_n \) are the reaction rates per baryon (Dai et al 1993) (Haensel 1992). Combin Departure and corresponding restore, we immediately get the time evolution equations of the chemical potential differences.
\[ \frac{d\delta \mu_d}{dt} = \frac{2}{3} \frac{\mu_u - \mu_u'}{n_\mu} - \Gamma_d E_{xx}(\mu), \]

\[ \frac{d\delta \mu_s}{dt} = -\frac{2}{3} \frac{\mu_s - \mu_s'}{n_\mu} + \Gamma_s E_{xx}(\mu), \]

\[ \frac{d\delta \mu_p}{dt} = \frac{2}{3} \frac{\mu_p - \mu_p'}{n_p} - \Gamma_n E_{xx}(\mu), \]

\[ \frac{d\delta \mu_e}{dt} = \frac{2}{3} \frac{\mu_e - \mu_e'}{n_e} - \Gamma_n E_{xx}(\mu). \]

Equation (6) and (7) show that \( D \) and \( S \) channels obey their evolution equations respectively, differing from Cheng & Dai (Cheng & Dai 1996) discussion of strange quark matter. Inclusion of deconfinement effect causes such serious difference. To solve these equations, thermal equation is necessary, reads
\[ c_v \frac{dT}{dt} = (\Gamma_d \mu_d + \Gamma_s \mu_s + \Gamma_n \mu_n) \]

\[ - (\epsilon_d + \epsilon_s + \epsilon_n) - E_\gamma, \]

where the three terms represent the chemical energy released by the reactions, the energy radiated by neutrino and antineutrino, and the energy in photons from the surface of the star. \( c_v \) is the specific heat of dense matter \( c_v = \chi c_0 + (1 - \chi)c_3 \). Now we need to treat \( \mu_d, \mu_s, \mu_p \) as the function of total baryon number density. In 1992, Glendenning first presented the existence of first-order phase transition with Gibbs construction. He applied the relativistic mean field theory for hadronic matter and MIT for quark matter to construct the phase diagram. Baryon densities of each phase in mixed phase are depicted in figure 7. We here consider a uniform star model for simulation and take baryon density \( n = 1.0 fm^{-3} \). We approximate, around the baryon density, \( n_\mu \approx 0.3431 + 0.8228n_b, \)

\[ n_b \approx 0.7143 + 0.7428n_b, \]

\[ \mu_d \approx \frac{1}{3} n_b \approx (4.826 + 1.44n)_b. \]

From Reisenegger (Reisenegger 1993), the total baryon number density changes with the spin-down by
\[ \frac{1}{n_b} \frac{dn_b}{dt} = \frac{\Omega}{G \rho_c} \frac{d\Omega}{dt}, \]

where \( \rho_c \) is central density of the stars.

4 RESULTS

We give an initial rotation period \( P_I = 1 ms \) and \( \alpha = 0.7 \), the latter corresponding to the threshold for direct Urca process to be allowed (Lattimer et al 1994). After giving the strong coupling constant, \( s \)-quark mass, and the surface magnetic
field strengths, we integrate the coupling evolution equations (13–16).

Figure 2 shows the chemical evolution in hadron-quark mixed phase. The solid line, dashed line and dotted line are chemical evolution for $S$ channel, $D$ channel and nucleon channel, respectively. Since neutrons and protons are converted into $u$ and $d$ quarks, without $s$ quarks, the chemical potential departures of $S$ channel is higher than that of $D$ channel. The four panels correspond to four different magnetic field strengths: (a) $B = 10^8 G$, (b) $B = 10^9 G$, (c) $B = 10^{10} G$, (d) $B = 10^{11} G$. The stronger the magnetic field strength, the more the kinetic energy of rotation converting into chemical energy, the larger the chemical potential departure, the earlier the appearance of heating effect. The platform indicates a relative quasi-equilibrium process. And the rapid dropping of chemical departure weaken the rotochemical heating at later times for strong magnetic. But rotochemical heating is still substantial for $B = 10^8 G$ at later times. Figure 3 gives the chemical potential departure of nucleon channel with magnetic field strengths $B = 10^8 G$ and $B = 10^{10} G$, to compare deconfinement and spin-down situations. Obviously, the deconfinement reactions enlarge the chemical potential departure.

The chemical evolution influence the cooling of neutron stars, when the chemical heating mechanism is considered. The DPT enhance the chemical potential departure of particles, and effect the thermal evolution of stars containing mixed phase matter as well. Figure 4 shows the evolution of the effective surface temperature for uniform hybrid star with rotochemical heating under consideration of DPT, and the traditional case. Although we use a toy model in which several approximations are made, the effective surface temperature of the star is about 40 percent higher than the traditional one, and the heating effect appears earlier.

We also analysis the contributions of quarks and nucleons by comparing temperature magnitudes. Figure 5 gives the results with $B = 10^8 G$. Since nucleon’s curve is close to full one, the contribution of the nucleon dominates hybrid star thermal evolution.

Figure 6 shows the evolution of the effective surface temperature with different magnetic field strengths $B = 10^8 G$ (dashed line), $10^9 G$ (dotted line), $10^{10} G$ (dashed-dotted line), $10^{11} G$ (dashed-dotted-dotted line), and in the absence of chemical heating. One sees that the effect of rotochemical heating with DPT can be substantial at later times for stars with weak fields and still important at earlier times for stars with strong fields.
Figure 5. Effective surface temperature as a function of time, with magnetic field strengths $B = 10^8 G$, for only considering quark’s chemical deviation and emission (dashed line). only considering nucleon’s (dotted line) and considering both (solid line).

Figure 6. Effective surface temperature as a function of time, with no heating (solid lines) or rotochemical heating with magnetic field strengths $B = 10^8 G$ (dashed line), $10^9 G$ (dotted line), $10^{10} G$ (dashed-dotted line), and $10^{11} G$ (dashed-dotted-dotted line).

5 CONCLUSIONS

Besides the spin-down, an adjoint of DPT indeed causes the departure from $\beta$-equilibrium in mixed phase matter. This mechanism is investigated in this paper. The new mechanism makes a substantial promotion in effective surface temperature. We usually believe hybrid stars cool down rapidly following standard cooling theory. Our results show that a hybrid star should have much higher temperature when the chemical heating with DPT is considered. Although we consider a uniform density model to facilitate our understanding of the mechanism, the results are very close to the hybrid stars containing large mixed phase core if their mean density equals the assumed uniform density.

In our calculations, we set $\alpha$ as a constant and only consider the contribution of the DPT in chemical deviation.

The new mechanism makes a greater contribution than the spin-down one. So the direct contribution of spin-down compression is not important for rotochemical deviations in stars containing mixed phase matter and can be ignored in calculations.

This paper focus on studying a new mechanism in heating generation inside hybrid stars, our consideration of initial period $Pi = 1 ms$ is reasonable for millisecond pulsars, but is too short for classical pulsars with higher magnetic field strengths. So the more reasonable initial period is needed in the accurate calculations comparing with the observations.

Accurate calculations are worth being expected. Our future works would study the thermal evolution of hybrid star including DPT, taking the structure of star into account in the frame of general relativity, using realistic EOSs of dense matter. The results thus would compare with the observations.

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