Mixed Convection in a Differentially Heated Cavity with Local Flow Modulation via Rotating Flat Plates

Md. Azizul Hakim, Atiqul Islam Ahad, Abrar Ul Karim, and Mohammad Nasim Hasan

Department of Mechanical Engineering, Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh

Abstract. Mixed Convection inside a cavity resulting from thermal buoyancy force under local modulation via rotating flat plate has been investigated. The present model consists of a square cavity with the left and right vertical walls fixed at constant high and low temperatures respectively while the top and bottom walls are supposed to be adiabatic. Two clockwise rotating flat plates, having negligible thickness in comparison to their lengths, acting as flow modulators have been placed vertically along the centerline of the cavity. The moving boundary problem due to plate motion in this study has been solved by implementing Arbitrary Lagrangian Eulerian (ALE) finite element formulation with triangular discretization scheme. Simulations are conducted for air ($Pr = 0.71$) at different Rayleigh numbers ($10^2 \leq Ra \leq 10^6$). Rotational Reynolds number based on plate dynamic condition has been considered to be constant at 430. Numerical results identify critical Rayleigh number $Ra_{cr} = 4.41 \times 10^6$ beyond which two smaller flow modulators are more effective than a single larger modulator. Thermal oscillating frequency was observed to be insensitive to Rayleigh number for the case of double modulators.

INTRODUCTION

Investigations of mixed convection heat transfer have a great number of empirical applications in lubrication technologies, oil pipelines, electronic compound cooling, etc. These studies have significant importance in the designs of electronic devices to dissipate heat from them to avoid thermal damages. Fu et al. [2], Kimura et al. [4] and Ghad-dar and Thiele [3] took the primary steps to investigate the heat transfer in a square cavity owing to thermal buoyancy force under the effect of rotating cylinder. Lewis [1] numerically investigated steady flow in a square cavity with a rotating cylinder for $1 \leq Re \leq 1400$. Khanafer and Aithal [5] numerically studied mixed convection heat transfer in a lid-driven cavity having a rotating cylinder for various system parameters for example: Richardson number and dynamic condition of the cylinder. Their findings revealed that both the thermal and flow field inside the cavity depended heavily on the speed and direction of rotation of cylinder. Billah et al. [6] investigated mixed convection heat transfer in a channel with active flow modulation via rotating cylinder. They found that type of configuration and direction of cylinder rotation strongly influenced the heat transfer. The heat transfer performance as represented by average Nusselt number over the heat source had been found to show an increasing trend with Reynolds and Grashoff numbers but at higher values of Reynolds and Grashoff numbers, the influence of local flow modulation became weak. The experimental work of Kimura et al. [7] used a rotating blade in a square enclosure as a heat transfer augmentor. It had been realized that the rotating blade is more effective than rotating cylinder in strengthening the heat transfer. Lee et al. [8] led similar investigation numerically and reported that thermal oscillation appeared when Rayleigh number went beyond a critical value.

As delineated in the literature survey outlined above, a numerical study of laminar mixed convection heat transfer in a differentially heated cavity with single flow modulator had been carried out [8] but effects of multiple smaller flow modulators in a differential heated cavity has not been investigated yet. The present study aims to explore the effect of Rayleigh ($Ra$) number on the flow pattern and heat transfer in a differentially heated cavity with multiple flow modulators. Results obtained in this study are discussed in terms of streamlines, isothermal contours, heatlines, spatially averaged Nusselt number and time averaged Nusselt number at the heated wall of the cavity.

PROBLEM DESCRIPTION

The configuration under study, shown in Fig. 1, is a square cavity of dimension $L$ with two rotors along the centerline of the cavity having lengths $d (= 0.3L)$ and negligible thickness. The left wall is maintained at a constant high temperature ($T_L$) and the right wall is kept at low temperature ($T_L$) while the upper and lower walls are assumed to be insulated. Thermal resistance across the modulators is negligible as the thickness of the rigid rotors are assumed to be...
negligible. The cavity is completely filled with air and assumed to be a Newtonian fluid. The modulators are assumed to rotate at a constant frequency in clockwise direction to match the sense of natural convection.

**FIGURE 1:** Schematic of the present problem.

Within the cavity, the flow has been assumed to be laminar, incompressible and two-dimensional. The Boussinesq approximation is used to define the thermal buoyancy force. All other thermophysical fluid properties are assumed to be constant. In the energy equation, the viscous dissipation term is neglected. Taking into consideration the assumptions stated above, the non-dimensional conservation equations of momentum, mass and energy for mixed convection in 2-D Cartesian coordinate system can be expressed as:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)
\]

\[
\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (2)
\]

\[
\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{Ra}{Re^2 Pr} \theta \quad (3)
\]

\[
\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Re Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (4)
\]

Equations (1)-(4) have been normalized using the following dimensionless scales:

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{u_c}, \quad V = \frac{v}{u_c}, \quad P = \frac{p}{p_{ref}}, \quad \theta = \frac{T - T_c}{\Delta T}, \quad \tau = \frac{t}{f^{-1}}
\]

Here, \( x \) is the distance along horizontal direction and \( y \) is that along the vertical direction; \( u \) and \( v \) are the horizontal and vertical components of velocity respectively; \( p \) is pressure, \( T \) is temperature. The reference quantities are length \( L \), velocity \( u_c = fL \) where \( f \) is the rotational frequency, reference pressure \( p_{ref} = \rho u_c^2 \) and temperature difference \( \Delta T = T_h - T_c \). \( \alpha, \beta, \rho, v \) are thermal diffusivity, coefficient of volumetric expansion, fluid density and kinematic
viscosity respectively. In the above equations, the governing parameters Prandtl number ($Pr$), Grashof number ($Gr$), rotational Reynolds number ($Re$) and Rayleigh number ($Ra$) have been defined as follows:

$$Gr = \frac{g\beta(T_h - T_c)H^3}{v^2}, \quad Pr = \frac{v}{\alpha}, \quad Re = \frac{u_cH}{v}, \quad Ra = GrPr$$

The associated boundary conditions are as follows:

$$U(0,Y) = 0, V(0,Y) = 0, \theta(0,Y) = 1$$

$$U(1,Y) = 0, V(1,Y) = 0, \theta(1,Y) = 0$$

$$U(X,0) = 0, V(X,0) = 0, \frac{\partial \theta(X,0)}{\partial Y} = 0$$

$$U(X,1) = 0, V(X,1) = 0, \frac{\partial \theta(X,1)}{\partial Y} = 0$$

No-slip condition has also been implied on the blade surfaces. Heat flow can be visualized better in terms of the heat function ($\Pi$) obtained from conductive heat fluxes as well as from convective fluxes ($U \theta, V \theta$) as defined below:

$$\frac{\partial \Pi}{\partial Y} = U \theta - \frac{1}{RePr} \frac{\partial \theta}{\partial X}$$

$$- \frac{\partial \Pi}{\partial X} = V \theta - \frac{1}{RePr} \frac{\partial \theta}{\partial Y}$$

Local Nusselt number is defined as:

$$Nu(X, \tau) = - \left( \frac{\partial \theta}{\partial X} \right)_{X=0}$$

Spatial averaged Nusselt number is obtained after integrating the local Nusselt number along the left hot wall of the cavity:

$$Nu(\tau) = \frac{1}{L} \int_0^1 Nu(Y, \tau) dY$$

Both time and spatial averaged Nusselt number is calculated for one period of oscillation:

$$Nu_{avg} = \int_0^{1/ft} Nu(\tau) d\tau$$

For heat function ($\Pi$), at adiabatic top and bottom walls, Dirichlet boundary condition $\Pi = 0$ applies and for isothermal sidewalls a Neumann boundary condition ($n . \Delta \Pi = 0$) applies.

**NUMERICAL METHOD**

The numerical method that has been implemented to solve the governing equations (1)-(4) of the present moving mesh problem is based on Arbitrary Lagrangian Eulerian (ALE) finite element formulation with non-uniform triangular discretization scheme. The non-dimensional partial differential equations of fluid field (1)-(4) along with respective boundary conditions are discretized by Galerkin finite element method. In order to achieve better convergence of the calculations in minimum time, an variable time step method has been implemented. Moving boundary has been solved on fixed non-staggered Cartesian grids. Glowinski [9] and Hu [10] have studied various applications of this procedure in solving fluid-solid systems of moving rigid bodies. We used non-linear parametric solution technique to solve the governing equations because it helps to converge rapidly. Several numerical runs at high $Ra = 10^6$ and $Pr = 0.71$ for double modulator with $d = 0.3L$ have been performed to obtain optimum grid distribution with accurate results.
and minimal computational time. Table 1 summarizes the comparison of average Nusselt numbers with various grid sizes. Approximately 43538 elements ensure that the solution can be taken as mesh independent. In our study, at regions near the walls and the blade surfaces, mesh has been refined to investigate the swift changes in dependent variables. The present numerical procedure of moving mesh has been ascertained against the existing results of Lee et al. [8]. Fig. 2 represents the comparison between the variation of Nusselt number with dimensionless time for single modulator $d = 0.6L$ and $Re = 430$ for four representative Rayleigh numbers ($Ra$).

**FIGURE 2:** Validation of the present model against Lee et al. [8] in terms of transient average Nusselt number variation.

**TABLE I:** Grid independency check in terms of time averaged Nusselt number at Left Wall.

| Elements  | 22618 | 35158 | 40104 | 43538 | 47824 | 56082 |
|-----------|-------|-------|-------|-------|-------|-------|
| $Nu_{avg}$ | 7.88  | 8.28  | 8.17  | **8.26** | 8.25  | 8.30  |

**RESULTS AND DISCUSSION**

Present numerical computation has been initiated from a fictional initial condition. When periodic steady state is reached, the time is set as $\tau = 0$ and computed 10 more cycles ($0 < \tau < 10$). The prime attention has been paid to the effects of the number of modulators present and Rayleigh number ($Ra$) on flow characteristics and heat transfer within the computational domain. Dynamic condition based on rotation of blades ($Re = 430$) is kept fixed and Rayleigh number ($Ra$) is varied from $10^2 \leq Ra \leq 10^6$. Flow and thermal fields are visualized using streamlines, isotherms and heat lines. Fast Fourier Transform (FFT) is computed to find out the thermal oscillating frequency. The heat transfer performance within the computational domain is characterized by time and spatially averaged Nusselt number ($Nu(\tau)$) along the left heated wall.

**Heat Transfer Performance Analysis**

Effect of modulation on the relation between time average Nusselt number ($Nu_{avg}$) and Rayleigh number ($Ra$) is presented in Fig. 3. It is observed that heat transfer with no modulation is greater beyond the point $Ra = 0.14 \times 10^6$ for
FIGURE 3: Average Nusselt number($N_{\text{uavg}}$) variation with Rayleigh number for different flow modulations.

single modulation; below $Ra = 4.2 \times 10^3$ values of $N_{\text{uavg}}$ for double modulation is greater than that of no modulation. At $Ra > 0.41 \times 10^6$, double modulation increases $N_{\text{uavg}}$ more than single modulation case due to greater forced convection effect produced by two rotors rather than single one. However, these values of $N_{\text{uavg}}$ are lower than that of no modulation. This occurs because they can also act as an obstruction in mixing of fluid in the zone between the rotors, causing a decrease in natural convection. Temporal variation of spatially average Nusselt number for different modulation is presented in Fig. 4(a) and (b). From the figure it is clear that thermal oscillation is prominent at higher Rayleigh number.
Flow Field and Heat Flow Visualization

Flow field, isotherms and heatlines for $Ra = 0.0008 \times 10^6 (< Ra = 4.2 \times 10^3)$ and $Ra = 0.6 \times 10^6 (> Ra = 0.41 \times 10^6)$ are presented at $\tau = 10$ in Fig. 5 and Fig. 6. This particular dimensionless time is when the flat plates are in horizontal position. From the observation of streamlines for cavity without flow modulation no vortex is formed. But due to the presence of single modulation four vortices can be noticed at the four corners. For two flow modulator several vortices can be noticed but they are weaker compared to single modulator. For no modulation, parallel and uniform isotherms are noticed near the side walls. Upon introducing flow modulation, isotherms become more clustered towards side walls due to the forced convection induced by rotation in the direction of natural convection. Isotherms for double modulator are noticed to be aligned parallelly near the vertical walls. From heatlines for no modulation, the lines emanating from left hot wall are perpendicular to the isotherms, resulting in conduction heat transfer near the side walls. At the upper side of the cavity, heat lines are nearly parallel and less dense; hence the lesser flow of heat. For single flow modulation the heatlines near the walls and inside the cavity tend to bend hence enhancing the energy flow. For $Ra = 0.6 \times 10^6$ in Fig. 6 streamlines for no modulation involve two vortices. For single flow modulation streamlines involve only one vortex near the center. For the case of double flow modulation several vortices can be noticed near modulators including their centers. For no modulation, isotherms near the sidewalls are observed to be more clustered. Thick thermal boundary layer for no flow modulation results in decreasing Nusselt number (about 10.8% decrease). Two modulators tend to push the isotherm toward the left wall hence Nusselt number increases (about 6% increase) compared to single flow modulation.
FIGURE 5: Streamlines, Isotherms and Heatlines for different flow modulations at $Ra = 0.0008 \times 10^6$ at $\tau = 10$
FIGURE 6: Streamlines, Isotherms and Heatlines variation for different flow modulations at $Ra = 0.6 \times 10^6$ at $\tau = 10$

**Spectral Analysis of Spatially Average Nusselt Number ($Nu(\tau)$)**

Rotation of the plates changes the fluid flow pattern in a periodic manner. In Fig. 7 FFT (Fast Fourier Transform) has been implemented to quantify frequency of thermal oscillation. It has been observed that at low Rayleigh number ($Ra = 0.0008 \times 10^6$) thermal oscillating frequency for single flow modulation $f_{th}$ is 2 whereas for double modulation it has been found to be 0.984. This is because due to one full rotation of single larger modulator causes the flow patterns and isotherms to complete two full periods. But for double modulator $f_{th}$ is nearly equal to the rotational frequency due to the opposite effects caused by the rotation of each modulator restricts the natural convection. At $Ra = 0.6 \times 10^6$ for single modulation $f_{th}$ becomes 0.7115 which is lower than the frequency of modulator. This occurs due to instability occurring on the side wall boundary layer as reported by Liao & Lin [11]. At $Ra = 0.6 \times 10^6$
for double modulation $f_{th}$ is 1 which is same as in the case for lower Rayleigh number.

Single Modulation, $\frac{f}{L} = 0.6$

Double Modulation, $\frac{f}{L} = 0.3$

![Power Spectrum](image)

**FIGURE 7:** One Sided Fast Fourier Transform (FFT) Power Spectrum of Spatially Average Nusselt number for single/double Modulators for different Rayleigh numbers ($Ra$).

**CONCLUSIONS**

In the present study, effect of flow modulation (via rotating flat plates) and thermal buoyancy in a differentially heated cavity has been investigated thoroughly using ALE finite element formulation. Results for different parametric conditions were graphically presented and discussed systematically. Based on the numerical results for different flow modulation condition such as no modulation, single modulation and double modulation (having same dynamic effect of a single modulator characterised by rotor Reynolds number) following conclusions are drawn:
• Beyond Rayleigh number $Ra_r = 0.41 \times 10^6$ double flow modulators enhance heat transfer compared to single flow modulators. At $Ra = 0.6 \times 10^6$ enhancement is 5.28% and at $Ra = 10^6$ it is 7.14%.

• Thermal oscillating frequency of double flat plate is insensitive to Rayleigh number.

• At high Rayleigh numbers ($> 0.41 \times 10^6$) thermal oscillation is prominent for single modulator.

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