QCD CRITICAL POINT: WHAT IT TAKES TO DISCOVER

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This report summarizes the results of the work done in collaboration with K. Rajagopal and E. Shuryak. We analyze the physics behind the event-by-event fluctuations in heavy ion collisions. Using thermodynamic description of the ensemble of events we analyze and quantify various effects that are sensitive to the proximity of the critical point on the phase diagram of QCD.

1 Introduction

The phase diagram of QCD in the temperature – baryon chemical potential plane has been a subject of intensified theoretical interest recently. On the experimental front, with the advent of large acceptance detectors such as NA49 and WA98 at CERN SPS, we are now able to measure average event-by-event quantities which carry information about thermodynamic properties of the system at freeze-out. Our goal is to understand what we can learn about the phase diagram of QCD from this newly available and future data.

The main focus of our analysis is on providing tools for locating the critical point E on the phase diagram of QCD (Fig. 1) and studying its properties. The possible existence of such a point, as an endpoint of the first order transition separating quark-gluon plasma from hadron matter, and its universal critical properties have been pointed out recently in [2, 3]. In a previous letter, we have laid out the basic ideas for finding this endpoint in heavy ion collision experiments. The signatures proposed in [4] are based on the fact that such a point is a genuine thermodynamic singularity at which susceptibilities diverge and the order parameter fluctuates on long wavelengths. The resulting signatures all share one common property: they are nonmonotonic as a function of an experimentally varied parameter such as the collision energy, centrality, rapidity or ion size.

2 Thermodynamic Fluctuations in an Ideal Bose Gas

Most of our analysis is applied to the fluctuations of the observables characterizing the multiplicity and momenta of the charged pions in the final state of a heavy ion collision. We begin building our tools by re-analyzing a text-book
example of an ideal Bose gas. The basic fact is that every quantum state of such a system is completely characterized by a set of occupation numbers, \( n_p \). All observables are functions of these numbers and thus all we need to know is the fluctuations of \( n_p \) from one member of the ensemble (one event) to another:

\[
\langle n_p \rangle = \frac{1}{e^{\epsilon_p/T} - 1}, \quad \langle \Delta n_p \Delta n_k \rangle = \langle n_p \rangle (1 + \langle n_p \rangle) \delta_{pk} \equiv v^2_{pk} \delta_{pk} .
\]  

The correlator \( \langle \Delta n_p \Delta n_k \rangle \) is the central quantity which we calculate repeatedly, as we proceed beyond the ideal Bose gas approximation.

The fluctuations of various extensive observables are given in terms of the "master correlator" (1):

\[
\langle (\Delta Q)^2 \rangle = \sum_{pk} q_p q_k \langle \Delta n_p \Delta n_k \rangle = \sum_p q^2_p v^2_{p} \quad \text{for} \quad Q = \sum_p q_p n_p .
\]  

More interestingly, the fluctuations of an intensive, or average, quantity, such as energy or transverse momentum per particle, \( q = Q/N \), are given by:

\[
\langle (\Delta q)^2 \rangle = \frac{1}{(Q/N)^2} \sum_p (q_p - \langle q \rangle)^2 v^2_{p} .
\]  

Denoting by \( \langle n_p \rangle_{inc} \) the average over inclusive distribution \( \langle n_p \rangle \) we see that the ensemble average coincides with the inclusive one: \( \langle q \rangle = \langle Q/N \rangle = q_{inc} \).
This is not true for the fluctuation, however:

\[ \langle (\Delta q)^2 \rangle = \frac{1}{\langle N \rangle} \langle q_p - \langle q \rangle \rangle^2 (1 + \langle n_p \rangle)^{inc} \].

(4)

We see that the event-by-event variance is larger than the suitably rescaled (by \(1/\langle N \rangle\)) inclusive variance because of the Bose enhancement factor \((1 + \langle n_p \rangle)\). This order several percent effect is very sensitive to the over-population of the pion phase space characterized by the pion chemical potential \(\mu_\pi\).

Another interesting quantity is the correlation between the fluctuations of an average quantity and the total multiplicity \(N\):

\[ \langle \Delta q \Delta N \rangle = \frac{1}{\langle N \rangle} \sum_p v_p^2 (q_p - \langle q \rangle) = \frac{1}{\langle N \rangle} \sum_p (n_p)^2 (q_p - \langle q \rangle) \].

(5)

Its value is totally due to the Bose effect, i.e., it would vanish in the ideal classical gas limit \(v_p^2 = \langle n_p \rangle\). For example, for the ideal Bose gas of pions at a temperature \(T = 120\) MeV the value of \(\langle \Delta p_T \Delta N \rangle/[(\langle \Delta p_T \rangle^2)/(\langle \Delta N \rangle^2)]^{1/2}\) is of the order of a few percent and is negative. In general, such correlations, though small, are very sensitive to non-trivial effects, such as Bose enhancement, as we have just seen, or to the effects which we consider below such as the energy conservation and thermal contact or the interactions with the sigma field.

3 Noncritical Thermodynamic Fluctuations in Heavy Ion Collisions

The next step in our characterization of the thermodynamic fluctuations in heavy ion collisions is inclusion of pions from resonance decays. The hadronic matter produced in a heavy ion collision is not simply an ideal gas of pions. A number of approaches to heavy ion collisions have successfully treated the matter at freeze-out as a resonance gas in thermal equilibrium. The pions observed in the data are then a sum of (i) “direct pions” which were pions at freeze-out and (ii) “resonance pions” produced from the decay of resonances after freeze-out.

Our simulation of a resonance gas model shows that more than half of all observed pions come from resonance decays. The resonances also have a dramatic effect on the size of the multiplicity fluctuations. For an ideal classical gas the ratio \((\Delta N)^2/N\) is equal to 1 and is only slightly enhanced by the Bose effects. If some of the pions are produced in bunches from resonances,
which themselves follow Poisson statistics, this ratio increases. We find:

\[
\frac{\langle (\Delta N)^2 \rangle}{\langle N \rangle} \approx 1.5. \tag{6}
\]

The experimental value from NA49 of this ratio is 2.0. This is much larger than the ideal gas value of 1. The contribution of resonances is important to bring this number up. However, there is still room for non-thermodynamic fluctuations, such as fluctuations of impact parameter. Their effect can be studied and separated by varying the centrality cut using the zero degree calorimeter.

The \( p_T \) spectrum of the resonance pions is close to the spectrum of the direct ones. As a result resonance pions do not affect much the shape of the spectrum, and in particular the width of the inclusive distribution, which determines most of the event-by-event fluctuation of the average \( p_T \). The resonances, however, dilute the Bose enhancement effect by about a factor of two.

In order to compare the results with the experimental data one has to take into account the effect of hydrodynamic flow. This effect is not important for the multiplicity fluctuations. However, it distorts the \( p_T \) spectrum shifting it to larger \( p_T \). In the simplest approximation this can be treated as a “blue shift” of the spectrum. Essentially we assume that the effects of the flow largely cancel in the ratio \( \frac{v_{\text{inc}}(p_T)}{\langle p_T \rangle} \). This ratio in our simulation is equal 0.66. The direct contribution from the fluctuations of the flow velocity are small, order 2% or so. With the Bose enhancement included we obtain:

\[
\frac{v_{\text{inc}}(p_T)}{\langle p_T \rangle} = 0.68. \tag{7}
\]

The experimental value obtained from NA49 data is 0.75. We see that the major part of the observed fluctuation in \( p_T \) is accounted for by the thermodynamic fluctuations. A large potential source of the discrepancy is the “blue shift” approximation we used. This approximation can be improved on in the future study.

Another very important feature in the data is the value of the ratio of the scaled event-by-event variation to the variance of the inclusive distribution:

\[
F = \frac{\langle N \rangle v_{\text{b.e.}}^2(p_T)}{v_{\text{inc}}^2(p_T)} = 1.004 \pm 0.004. \tag{8}
\]

This is a remarkable fact, since the contribution of the Bose enhancement (see Section 3) to this ratio is almost an order of magnitude larger than the statistical uncertainty. Some mechanism must compensate for the Bose
enhancement. In the next section we find a possible origin of this effect: anti-correlations due to energy conservation and thermal contact between the observed pions and the rest of the system at freeze-out.

4 Thermal Contact and Energy Conservation

In this Section, we take a first step towards understanding how the physics characteristic of the vicinity of the critical point affects the event-by-event fluctuations. Along the way, we quantify the effects of energy conservation on the $p_T$-fluctuations.

We call the gas of direct pions “system B” and the rest of the system, which includes the neutral pions, the resonances, the pions not in the experimental acceptance and, if the freeze-out occurs near critical point, the order parameter or sigma field, — “system A”. We observe system B, which is our “thermometer”. The thermal contact of B with A and energy conservation affects the “master correlator” $\langle \Delta n_p, \Delta n_k \rangle$. For example, $n_p$ cannot fluctuate completely independently if the heat capacity $C_A$ of the system A is small. There is a constraint on the total energy $E_B = \sum p \epsilon_p n_p$ which gets stronger at small $C_A$. The result for the master correlator we find is:

$$\langle \Delta n_p, \Delta n_k \rangle = v_p^2 \delta_{pk} - \frac{v_p^2 \epsilon_p v_k^2 \epsilon_k}{T^2 (C_A + C_B)}.$$  

(9)

Using this expression of the correlator we can now calculate the effect of thermal contact and energy conservation on fluctuations of various observables, such as mean $p_T$, for example. In particular, we find that the anti-correlation introduced by this effect reduces the value of the ratio $F$ defined in (8) by:

$$\Delta F_T \approx -\frac{0.12}{C_A/C_B + 1}.$$  

(10)

If we take $C_A/C_B \sim 3$ for orientation, we find $\Delta F_T$ of the order of $-3\%$, before taking into account the dilution by non-direct pions. This effect is comparable in magnitude to the Bose enhancement, and acts in the opposite direction.

This effect can be distinguished from other effects, e.g., finite two-track resolution, also countering the Bose enhancement, by the specific form of the microscopic correlator (9). The effect of energy conservation and thermal contact introduces an off-diagonal (in $pk$ space, and also in isospin space) anti-correlation. Some amount of such anti-correlation is indeed observed in the NA49 data.
Another important point of (9) is that as the freeze-out approaches the critical point and $C_A$ becomes very large the anti-correlation due to energy conservation disappears.

5 Pions Near the Critical Point: Interaction with the Sigma Field

In this section, unlike the previous sections, we shall consider the situation in which the freeze-out occurs very close to the critical point. This point is characterized by large long-wavelength fluctuations of the sigma field (chiral condensate). We must take into account the effect of the $G\sigma\pi\pi$ interaction between the pions and such a fluctuating field. We do that by calculating the contribution of this effect to the “master correlator”. We find:

$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{1}{m_\sigma^2} \frac{G^2 v_p^2 v_k^2}{T \omega_p \omega_k}.$$  \hspace{1cm} (11)

We see that exchange of soft sigma field leads to a dramatic off-diagonal correlation, the size of which grows as we approach the critical point and $m_\sigma$ decreases. This correlation takes over the off-diagonal anti-correlation discussed in the previous section.

To quantify the effect of this correlation we computed the contribution to the ratio $F$ from (11). We find:

$$\Delta F_\sigma = 0.14 \left( \frac{G_{\text{freeze-out}}}{300 \text{ MeV}} \right)^2 \left( \frac{\xi_{\text{freeze-out}}}{6 \text{ fm}} \right)^2 \quad \text{for } \mu_\pi = 0 , \hspace{1cm} (12)$$

This effect, similarly to the Bose enhancement, is sensitive to over-population of the pion phase space characterized by $\mu_\pi$ and increases by a factor 2.5 for $\mu_\pi = 60$ MeV. We estimate the size of the coupling $G$ to be around 300 MeV near point E, and the mass $m_\sigma$, bound by finite size effects, to be less than 6 fm. The effect (12) can easily exceed the present statistical uncertainty in the data (8) by 1-2 orders of magnitude.

It is important to note that we have calculated the effect of critical fluctuations on $F$ because this ratio is being measured in experiments, such as NA49. This observable is not optimized for detection of critical fluctuations. It is easy to understand that observables which are more sensitive to small $p_T$ than $F$, and/or observables which are sensitive to off-diagonal correlations in $pk$ space would show even larger effect as the critical point is approached.
6  Pions From Sigma Decay

Near the critical endpoint, the excitations (quasiparticles) of the sigma field are nearly massless and are therefore numerous. Because the pions are massive at the critical point, these $\sigma$'s cannot immediately decay into two pions and persist as the system expands after freeze-out when it occurs near the critical point. During the expansion, the in-medium $\sigma$ mass rises towards its vacuum value and eventually exceeds the two pion threshold. At this point the $\sigma$'s decay quickly, yielding a non-thermal population of soft pions.

We estimate the mean momentum of these soft pions to be around $0.6m_\pi$ and the total number to be of the order of the number of direct pions (i.e., they should constitute up to a third of total observed pions near the critical point). The multiplicity fluctuations of these pions: $\left\langle \left( \Delta N \right)^2 \right\rangle / \left\langle N \right\rangle \approx 2.7$, are significantly larger than that of the rest of the pions [6].

7  Conclusions

In summary, our understanding of the thermodynamics of QCD will be greatly enhanced by the detailed study of event-by-event fluctuations in heavy ion collisions. We have estimated the influence of a number of different physical effects on the master correlator $\left\langle \Delta n_p \Delta n_k \right\rangle$. This is itself measurable, but we have in addition used it to make predictions for the fluctuations of observables which have been measured at present, such as $\left\langle (\Delta p_T)^2 \right\rangle$ and $\left\langle (\Delta N)^2 \right\rangle$ and also for the cross correlation $\left\langle \Delta N \Delta p_T \right\rangle$.

The signatures we analyze allow experiments to map out distinctive features of the QCD phase diagram. The striking example which we have considered in detail is the effect of a second order critical end point. The nonmonotonic appearance and then disappearance of any one of the signatures of the critical fluctuations which we have described would be strong evidence for the critical point. Simultaneous detection of the effects of the critical fluctuations on different observables would turn strong evidence into a decisive discovery.

References

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