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Abstract. In this paper, an optimization methodology for turbulent air jet impingement cooling system is reported. The continuity, Reynolds-Averaged Navier-Stokes, turbulent kinetic energy ($k$), turbulent dissipation rate ($\epsilon$) and energy equations are solved using the Streamline Upwind/Petrov-Galerkin (SUPG) Finite Element (FE) Method. In this study, Reynolds number ($Re_j$) and channel height ($H/L$) are selected as design parameters. The influence of design parameters on turbulent flow, heat transfer and entropy generation distribution for the system are presented and discussed. As $Re_j$ increases and $H/L$ decreases, the magnitude of overall Nusselt number and global total entropy generation increases. Subsequently, the Multi-Objective Genetic Algorithm (MOGA) was introduced to achieve optimal impingement system configurations with maximum heat transfer and minimum entropy generation.

1. Introduction
Heat transfer enhancement through impinging jets has gained significant interest due to various engineering applications viz. thermal drying, cryosurgery, aircraft wing anti-icing, electronics cooling and gas turbine blade cooling etc. [1, 2, 3]. Over the last few decades, several experimental, theoretical and computational studies were reported on turbulent impinging jets [4, 5, 6, 7]. Afroz and Sharif [8] has recently studied turbulent fluid flow and thermal transport for both round as well as annular impinging jet flow using realizable $k$-$\epsilon$ model. It was reported that, in comparison to the round impinging jet, annular impinging jets have produced an improved heat transfer and distribute heat more evenly over the impingement surface.

In all the above reported studies, jet impingement heat transfer over a continuous hot surface is addressed. However, in certain applications, such as data center cooling, micro-sized electronic cooling equipment and micro-electro-mechanical systems (MEMS), thermal loads are imposed as independent heating elements with intermittent thermal boundary layer [9, 10]. Forster and Weigand [11] performed an experimental and computational analysis to predict thermal transport on the heated concave surface through an array of impinging jets with reference to cooling of gas turbine blade’s leading edge.
Bejan [12] was first to report thermodynamic analyses of numerous convection heat transfer systems. Over the last few years, numerous studies have been published on the generation of entropy in jet configurations impinging [13, 14, 15]. Esmailpour et al. [16] have recently investigated the thermodynamic performance of pulsatile jet impingement flow over flat plate. Maximum viscous and thermal entropy generation has been shown to occur at low nozzle-impingement plate height. To the best of authors’ knowledge, in spite of its significance in thermal control of electronic applications, turbulent heat transport and entropy generation minimization in impinging jet system through multi-objective design optimization has not yet been documented, which form the motivation to this research work.

The main purpose of this work is to report momentum transport, thermal energy transport and thermodynamic performance of a turbulent air jet impinging system at wide ranges of design parameters viz. Reynolds number and channel height. Furthermore, as objective functions, $\overline{Nu}$ and $S_{\text{tot,}\Omega}$ are chosen. Subsequently, MOGA design optimization procedure is employed to achieve optimal solutions with maximal heat transport and minimal entropy generation.

2. Methodology

As illustrated in Fig. 1, the air jet impingement system is a channel with a series of discrete heated elements, mounted along the impinging substrate, identical to Lam and Prakash [17].

![Figure 1. Physical Domain of interest](image)

The non-dimensional equations governing turbulent forced convection are as follows [17]:

**Continuity Equation:**

$$\frac{\partial u_{n,i}}{\partial x_i} = 0$$

**Momentum Equations:**

$$\frac{\partial u_{n,i}}{\partial t} + u_{n,j} \frac{\partial u_{n,i}}{\partial x_j} = -\frac{\partial p_n}{\partial x_i} + \frac{1}{Re_j} (1 + \nu_{t,n}) \left\{ \frac{\partial}{\partial x_i} \left( \frac{\partial u_{n,i}}{\partial x_j} + \frac{\partial u_{n,j}}{\partial x_i} \right) \right\}$$

Here, the turbulent viscosity ($\nu_{t,n} = c_{\mu} Re_j \frac{k_n^2}{\varepsilon_n}$) is estimated through Standard $k - \varepsilon$ turbulence model. Further, near wall dynamics are estimated through standard wall functions [18].

The equations governing turbulent kinetic energy ($k_n$) and turbulent dissipation ($\varepsilon_n$) are as follows:
Turbulent Kinetic Energy ($k_n$):

$$\frac{\partial k_n}{\partial \tau} + \left[ u_j \frac{\partial k_n}{\partial x_j} \right] = \frac{1}{Re_j} \left[ \frac{\partial}{\partial x_j} \left( \frac{\nu_{t,n} \partial k_n}{\sigma_k \partial x_j} \right) \right] + G_n - \epsilon_n \tag{3}$$

Turbulent dissipation ($\epsilon_n$):

$$\frac{\partial \epsilon_n}{\partial \tau} + \left[ u_j \frac{\partial \epsilon_n}{\partial x_j} \right] = \frac{1}{Re_j} \left[ \frac{\partial}{\partial x_j} \left( \frac{\nu_{t,n} \partial \epsilon_n}{\sigma_\epsilon \partial x_j} \right) \right] + \frac{\epsilon}{k_n} (C_1 \epsilon G_n - C_2 \epsilon) \tag{4}$$

Energy Equation:

$$\frac{\partial \theta_n}{\partial \tau} + u_j \frac{\partial \theta_n}{\partial x_j} = \frac{1}{Re_j Pr} (1 + \alpha_{t,n}) \left( \frac{\partial^2 \theta_n}{\partial x_j x_j} \right) \tag{5}$$

Here, $G_n$, $\nu_{t,n}$ and $\alpha_{t,n}$ are stated as

$$G_n = \frac{\nu_{t,n}}{Re_j} \left[ 2 \left( \frac{\partial u_{n,j}}{\partial x_j} \right)^2 + \left( \frac{\partial u_{n,i}}{\partial x_j} + \frac{\partial u_{n,j}}{\partial x_i} \right)^2 \right]; \quad \nu_{t,n} = C_\mu Re_j \frac{k_n^2}{\epsilon_n} \quad ; \quad \alpha_{t,n} = C_\mu Re_j \frac{Pr k_n^2}{\sigma_t \epsilon_n} \tag{6}$$

Also, the model coefficients are:

$$C_\mu = 0.09, \quad \sigma_k = 1.0, \quad \sigma_\epsilon = 1.3, \quad \sigma_t = 0.9, \quad C_{1\epsilon} = 1.44, \quad C_{2\epsilon} = 1.92 \tag{7}$$

The local entropy generation is defined as follows [19]:

$$S_{tot} = S_\theta + S_\psi = \left[ \left( \frac{\partial \theta_n}{\partial x} \right)^2 + \left( \frac{\partial \theta_n}{\partial y} \right)^2 \right]^2 + \chi_n \left[ 2 \left( \frac{\partial u_n}{\partial x} \right)^2 + \left( \frac{\partial v_n}{\partial y} \right)^2 \right] + \left( \frac{\partial v_n}{\partial x} + \frac{\partial u_n}{\partial y} \right)^2 \tag{8}$$

In Equation 8, $S_\theta$ is contribution of entropy generation from heat transfer irreversibility, and $S_\psi$ is from fluid friction irreversibility.

The pertinent non-dimensional parameters are defined as, Prandtl number ($Pr$) = $\frac{\nu}{\alpha}$, Reynolds number ($Re_j$) = $\frac{UW}{\nu}$, local Nusselt number ($Nu = \frac{L}{W} \frac{\partial \theta}{\partial n}$) and local Bejan number ($Be = \frac{S_\theta}{S_\theta + S_\psi}$). Also, irreversibility distribution ratio ($\chi_n$) is assumed to be unity [19].

The boundary conditions are as follows:

a. Jet inlet:

$$u_n = 0.0; \quad v_n = -1.0; \quad \frac{\partial p_n}{\partial n} = 0; \quad \theta = 0; \quad k_n = 1.5 \quad I^2; \quad \epsilon_n(x) = (k_n^{1.5} C_\mu^{0.75}) / \chi \quad x \tag{9}$$

Here, Turbulent intensity ($I$) = 10%, Von-Karman constant ($\chi$) = 0.42, $\lambda = 0.09$ and $E = 9.743$.

b. Solid surfaces:

$$u_n = v_n = 0; \quad \frac{\partial p_n}{\partial y} = 0; \quad \theta = 1.0; \tag{10}$$

c. Outlet:

$$p_n = 0, \quad \frac{\partial f}{\partial x} = 0; \quad f = (u_n, v_n, \theta_n, k_n, \epsilon_n) \tag{11}$$
The equations governing turbulent forced convection are solved through an in-house algorithm based on SUPG-FE method [20]. The detailed formulation is presented in Arul Prakash et al. [21]. Further, the validation of in-house algorithm and grid resolution study are presented in Lam and Prakash [17]. A detailed mesh resolution study was performed with three grids such as $456 \times 60$, $560 \times 100$ and $684 \times 140$ along $x$ and $y$ directions respectively. The deviation of $\frac{N_{u_{ov}}}{S_{tot,\Omega}}$ between grids, $560 \times 100$ and $684 \times 140$ is observed to be marginal and equal $3.75\%$ and $4.89\%$ respectively. Hence, a grid with $560 \times 100$ elements is selected for FE simulations.

3. Results and discussion

In this section, momentum transport, thermal energy transport and thermodynamic performance of turbulent air jet impingement system is reported at wide ranges of $Re_j$ and $H/L$. Further, $N_{u_{ov}}$ and $S_{tot,\Omega}$ are chosen as objective functions and MOGA has been implemented for achieving optimal configuration with a simultaneous heat transfer enhancement and minimal entropy generation.

Figure 2. Effect of $Re_j$ on Streamlines at $H/L = 0.75$ for (a) $Re_j = 5000$, (b) $Re_j = 10000$, (c) $Re_j = 15000$, (d) $Re_j = 20000$ and (e) $Re_j = 25000$

Figure 2 illustrates streamlines at $H/L = 0.75$ for different Reynolds number ($Re_j = 5000$ - 25000) values. It is noticed that, in cavities between the heated components, tiny vortices are developed. Also, a stagnation area is developed when fluid from the jet inlet impinges on the first heated element, allowing pressure to build up, resulting fluid to convect into adjacent heated elements. A primary recirculation bubble is noticed near the jet inlet and confinement surface. This is mainly due to the shear flow caused by jet on surrounding fluid. Finally, as
**Re** increases, a small rise in maximum velocity magnitude value ($V_{M_{\text{max}}}$) is observed due to increased convection, as seen in Fig. 2.

![Figure 3](image-url)

**Figure 3.** Effect of $Re_j$ on Isotherms at $H/L = 0.75$ for (a) $Re_j = 5000$, (b) $Re_j = 10000$, (c) $Re_j = 15000$, (d) $Re_j = 20000$ and (e) $Re_j = 25000$

The associated isotherms at $Re_j = 5000, 10000, 15000, 20000, 25000$ in the turbulent air jet impingement system are presented in the Fig. 3. The thermal boundary layer thickness is nominal along the top surface of the first heated element (stagnation zone), and starts to grow in streamwise direction over the neighboring heated elements. Also, at both leading ($0.045 \leq X_P \leq 0.05$) and trailing ($0.045 \leq X_P \leq 0.05$) edges of all the heated elements, thermal boundary layer thickness is high as illustrated in Fig. 3. In addition, as the number of Reynolds increases, along all the individual heated components, the respective thermal gradient are observed to rise due to increased convection, which is in agreement with observed streamlines.

Turbulent Kinetic Energy (TKE) distribution for turbulent jet impingement system is demonstrated in Fig. 4. TKE is defined through root-mean-square (RMS) of velocity fluctuations and is governed by a production term ($G_n$), turbulent dissipation ($\epsilon_n$), mean flow convection ($\frac{D_k}{D_t}$), and turbulent transport ($\Delta T'$). From Fig. 4, TKE is high in the zones of shear-driven interaction caused by jet on surrounding fluid. This may be due to the domination of turbulence production ($G_n$). The magnitude of TKE is also substantial in the stagnation region. In addition, TKE approaches to zero downstream the fifth heated component due to fluid flow deceleration. This may be due to negligible velocity fluctuations and improved TKE dissipation to the ambient fluid. Finally, as expected with increase in $Re_j$, the maximal values of TKE ($k_{\text{max}}$) is observed to increase.

Figure 5 illustrates the turbulent viscosity contours and observed a maximal value at the same locations where high TKE is noticed. Also, as expected, with increase in $Re_j$, the maximal value of turbulent viscosity ($\nu_{t,\text{max}}$) is found to increase as demonstrated in Fig. 5.

From Fig. 6, it is found that, the magnitude of $S_\theta$ is significant over the upper surface of first heated element (stagnation zone), caused by higher temperature gradients, which is in
Figure 4. Effect of $Re_j$ on turbulent kinetic energy at $H/L = 0.75$ for (a) $Re_j = 5000$, (b) $Re_j = 10000$, (c) $Re_j = 15000$, (d) $Re_j = 20000$ and (e) $Re_j = 25000$

Figure 5. Effect of $Re_j$ on turbulent viscosity at $H/L = 0.75$ for (a) $Re_j = 5000$, (b) $Re_j = 10000$, (c) $Re_j = 15000$, (d) $Re_j = 20000$ and (e) $Re_j = 25000$
accordance with observed streamlines and isotherms. It is noticed that, as $Re_j$ increases, the zones of prominent $S_\theta$ values also decreases. Further, as $Re_j$ increases, a significant increase in $S_{\theta,\max}$ observed.

![Figure 6](image.png)

**Figure 6.** Effect of $Re_j$ on $S_\theta\Omega$ at $H/L = 0.75$ for (a) $Re_j = 5000$, (b) $Re_j = 10000$, (c) $Re_j = 15000$, (d) $Re_j = 20000$ and (e) $Re_j = 25000$

From Fig. 7, it is noticed that, in the zones of shear-driven flow caused by jet on ambient fluid, the magnitude of $S_\psi$ is significant due to increased velocity gradients. Also, in stagnation zone, $S_\psi$ is observed to be maximum due to improved viscous dissipation. Also, in the proximity of all the solid surfaces, $S_\psi$ is significantly is fould to be higher. Finally, the magnitude of $S_{\psi,\max}$ is noticed to raise with increase in $Re_j$. This could be attribute to increase in convection caused by an enhanced velocity magnitude and velocity gradients with increasing Reynolds number.

Figure 8 demonstrates corresponding contour maps of local total entropy generation ($S_{tot} = S_\theta + S_\psi$) at different values of Reynolds number ($Re_j = 5000 - 25000$). From Fig. 8, it is noticed that, at the locations of high velocity and temperature gradients, $S_{tot}$ is significant due to contributions from $S_\theta$ and $S_\psi$. Furthermore, as $Re_j$ increases, $S_{tot,\max}$ is also found to increase.

Figure 9 depicts the variation of local Nusselt number ($Nu$) distribution on the periphery of heated elements at $H/L = 0.75$ for $Re_j = 5000 - 25000$. From Fig. 9, as $Re_j$ increases, a noticable augmentation in $Nu$ is observed on the top sources of all heated elements. Also, as $Re_j$ increases, $Nu$ along side (left and right) surfaces of heated element increases. This is attributed to enhanced shear-driven convection of air trapped in the cavities between heated elements.

Figures 10 (a) - 10 (e) show the influence of $Re_j$ and $H/L$ on $\bar{Nu}$ for all heated elements. As $Re_j$ increases from $Re_j = 5000$ to $25000$, $\bar{Nu}$ for all the heated elements due to improved
Figure 7. Effect of $R_e_j$ on $S_{\psi, \Omega}$ at $H/L = 0.75$ for (a) $R_e_j = 5000$, (b) $R_e_j = 10000$, (c) $R_e_j = 15000$, (d) $R_e_j = 20000$ and (e) $R_e_j = 25000$

Figure 8. Effect of $R_e_j$ on $S_{\text{tot}, \Omega}$ at $H/L = 0.75$ for (a) $R_e_j = 5000$, (b) $R_e_j = 10000$, (c) $R_e_j = 15000$, (d) $R_e_j = 20000$ and (e) $R_e_j = 25000$
Figure 9. Effect of $Re_j$ on local $Nu$ distribution on the periphery of (a) First, (b) Second, (c) Third, (d) Fourth and (e) Fifth heated element

Convection. Further, the effect of $H/L$ on $Nu$ for all heated elements is noticed to be negligible.

Figures 10 (g) - 10 (i) illustrate the effect of $Re_j$ and $H/L$ on global entropy generation. It is noticed that, as $Re_j$ increases from $Re_j = 5000$ - 25000, the magnitude of $S_{\theta,\Omega}$ and $S_{\psi,\Omega}$ increases. This may be attributed to enhanced thermal and velocity gradients. Finally, for all the values of $Re_j$, it is found that, as $H/L$ increases, the magnitude of $S_{\theta,\Omega}$, $S_{\psi,\Omega}$ and $S_{tot,\Omega}$ decreases.

4. Optimization

In order to achieve optimal designs for turbulent air jet impingement system, $Nu_{ov}$ and $S_{tot,\Omega}$ are chosen as objective functions such that heat transfer is maximized at minimal entropy generation. For this purpose, response surfaces are derived for $Nu_{ov}$ and $S_{tot,\Omega}$ with the design parameters ($Re_j$ and $H/L$). The surrogate models defining the response surfaces for $Nu_{ov}$ and $S_{tot,\Omega}$ are as follows:

$$Nu_{ov} = 57.43 + 0.0064 \times Re - 39.64 \times H/L + 1.588 \times 10^{-4} \times Re \times (H/L) - 1.128 \times 10^{-7} \times Re^2 + 13.22 \times (H/L)^2$$  \(13\)

$$S_{tot,\Omega} = 1991.14 + 0.081 \times Re - 4254.102 \times H/L - 0.0153 \times Re \times (H/L) - 1.094 \times 10^{-6} \times Re^2 + 2357.36 \times (H/L)^2$$  \(14\)

The $R^2$ values for $Nu_{ov}$ and $S_{tot,\Omega}$ are 0.9978 and 0.9952, respectively. This suggests that, at 95% confidence limit, the derived surrogate models could be used in lieu of finite element simulations.

Furthermore, the accuracy of derived surrogate models (Equations 13 and 14) are examined through a pairity plot as shown in Fig. 11. It is noticed that, from Figs. 11 (a) and 11 (b) the
Figure 10. Effect of design parameters ($Re_j$, $H/L$) on Average Nusselt number and global entropy generation: (a) $Nu_1$, (b) $Nu_2$, (c) $Nu_3$, (d) $Nu_4$, (e) $Nu_5$, (f) $Nu_{ov}$, (g) $S_{\theta,\Omega}$, (h) $S_{\Psi,\Omega}$, (i) $S_{tot,\Omega}$

deviations of surrogate models ($Nu_{ov}$ and $S_{tot,\Omega}$) are within a ±15% deviation. Hence, the surrogate models for $Nu_{ov}$ and $S_{tot,\Omega}$ are comparable with the results from FE computations. Therefore, to predict optimal designs through Multi-Objective Genetic Algorithms (MOGA), the derived surrogate models could be employed instead of FE Computations.

The multi-objective optimization problem is formulated as:

Maximize : $Nu_{ov} = f(Re_j, H/L)$  ;  Minimize : $S_{tot,\Omega} = f(Re_j, H/L)$

Subjected to : $5000 \leq Re_j \leq 25000$ and $0.5 \leq H/L \leq 1.0$.

In order to run the optimization problem, the population size is assumed to be 1000 with number of generations is equal to 1000 and crossover fraction = 0.7. Then, other variables such as function tolerance and Pareto-front population fraction are equal to $10^{-5}$ and 0.8 respectively.

Figure 12 shows the Pareto-optimal frontier in functional space featuring optimal designs with maximal heat transfer and minimal entropy generation. In the Pareto front, 80 optimal configurations are found trade-off between $Nu_{ov}$ and $S_{tot,\Omega}$. It is noted that, a significant improvement in $Nu_{ov}$ is always followed by a proportional rise in $S_{tot,\Omega}$ and vice versa on the Pareto optimum frontier. Further, in Fig. 12, it is seen that, the simulated designs through
Figure 11. Comparison of results from FE simulation and Response Surfaces: (a) $\overline{Nu_{ov}}$ and (b) $S_{tot,\Omega}$

Figure 12. Pareto frontier for objective functions ($\overline{Nu_{ov}}$ and $S_{tot,\Omega}$)

in-house algorithm are scattered above the pareto-optimal frontier and no one is found below the same. It is also to be noticed that, none of the configurations in the frontier are preferable to others, as each design is a global Pareto-optimal solution, where both $\overline{Nu_{ov}}$ and $S_{tot,\Omega}$ fulfilled simultaneously.

For this reason, for six sets of representative Pareto-optimal designs ($Re_j$ and $H/L$), FE simulations are performed as shown in the Table. 1. From the table, it is observed that, both $\overline{Nu_{ov}}$ and $Stot,\Omega$ from FE simulations and Pareto frontier of MOGA (as shown in Fig. 12) are comparable, suggesting excellent accuracy of RSA and MOGA.

5. Conclusions
In this work, an optimization methodology was presented to obtain optimum designs for turbulent air jet impingement system for maximal heat transfer and minimal entropy generation. An in-house algorithm based on Streamline Upwind/Petrov-Galerkin (SUPG) Finite Element (FE) Method was employed to solve continuity, Reynolds-Averaged Navier-Stokes, turbulent
Table 1. Comparison of the results ($\overline{Nu}_{ov}$, $S_{tot, \Omega}$) from MOGA and FE Simulation for six representative Designs in Pareto-optimal Frontier (DPOF)

| DPOF | $Re_j$ | $H/L$ | $\overline{Nu}_{ov}$ | $S_{tot, \Omega}$ |
|------|--------|-------|----------------------|------------------|
|      | MOGA   | FEM   | MOGA                | FEM              |
| 1    | 25000.00 | 0.50 | 132.40              | 1604.23          |
| 2    | 25000.00 | 0.59 | 130.39              | 1409.33          |
| 3    | 24185.65 | 0.87 | 125.08              | 1071.57          |
| 4    | 16275.95 | 0.89 | 109.23              | 880.67           |
| 5    | 10038.85 | 0.91 | 86.58               | 635.52           |
| 6    | 5000.00  | 0.92 | 62.08               | 380.70           |

kinetic energy ($k$), turbulent dissipation rate ($\epsilon$) and energy equations were solved to predict momentum transport, thermal energy transport and thermodynamic performance at different values of $Re_j$ and $H/L$. It was noticed that, as $Re_j$ increases and $H/L$ decreases, the magnitude of $\overline{Nu}_{ov}$ and $S_{tot, \Omega}$ increases. Subsequently, $\overline{Nu}_{ov}$ and $S_{tot, \Omega}$ were chosen as objective functions and surrogate models were derived through the Response Surface Approximation. Then, the Multi-Objective Genetic Algorithm (MOGA) was introduced to obtain optimal designs of impingement system with maximal heat transfer and minimal entropy generation. Finally, six representative pareto-optimal designs ($Re_j$ and $H/L$) were chosen and FE simulations were performed for those representative designs. It was found that, the performance parameters, $\overline{Nu}_{ov}$ and $S_{tot, \Omega}$ from FE simulations and Pareto frontier of MOGA are comparable, suggesting excellent accuracy of RSA and MOGA.

References

[1] Zuckerman N and Lior N 2006 Advances in Heat Transfer 39 565–631
[2] Dewan A, Dutta R and Srinivasan B 2012 Heat Transfer Engineering 33 447–460
[3] Bode F, Patrascu C and Nastase I 2020 Thermal Science 227–227
[4] Gardon R and Akfirat J C 1965 International Journal of Heat and Mass Transfer 8 1261–1272
[5] Park T, Choi H, Yoo J and Kim S 2003 International Journal of Heat and Mass Transfer 46 251 – 262
[6] Cañiero G, Discetti S and Asta R T 2014 International Journal of Heat and Mass Transfer 75 173 – 183
[7] Nebuchinov A S, Lozhkin Y A, Bilsky A V and Markovich D M 2017 Experimental Thermal and Fluid Science 80 139–146
[8] Afroz F and Sharif M A 2018 Applied Thermal Engineering 138 154–172 ISSN 1359-4311
[9] Lam P A K and Prakash K A 2015 Energy Conversion and Management 89 626 – 643
[10] Lam P A K and Prakash K A 2016 Energy Conversion and Management 111 38 – 56
[11] Forster M and Weigand B 2021 International Journal of Thermal Sciences 164 106862 ISSN 1290-0729
[12] Bejan A Entropy generation through heat and fluid flow ISBN 9780471094388
[13] Ruocco G 1997 International Communications in Heat and Mass Transfer 24 201 – 210
[14] Shuja S, Yiibas B and Budilair M 2007 International Journal of Numerical Methods for Heat and Fluid Flow 17 677–691
[15] Lam P A K and Prakash K A 2017 International Journal of Heat and Mass Transfer 108, Part A 880 – 900
[16] Esmaeilpour K, Bozorgmehr B, Hoseinialipour S M, Mujumdar A s and Lewis R 2015 International Journal of Numerical Methods for Heat & Fluid Flow 25
[17] Lam P A K and Prakash K A 2017 Thermal and thermodynamic analyses of impingement cooling system with turbulent jet arrays Proceedings of the 24th National and 2nd International ISHMT-ASTFE Heat and Mass Transfer Conference (IHMTC-2017) (Begel House Inc.)
[18] Launder B and Spalding D 1974 Computer Methods in Applied Mechanics and Engineering 3 269 – 289
[19] Lam P A K and Prakash K A 2014 International Journal of Heat and Mass Transfer 69 390 – 407
[20] Brooks A N and Hughes T J R 1982 Computer Methods in Applied Mechanics and Engineering 32 199–259
[21] Arul Prakash K, Biswas G and Rathish Kumar B V 2006 International Journal of Heat Mass Transfer 49 4633–4652