Hadronic Dijet Imbalance and Transverse-Momentum Dependent Parton Distributions

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Abstract

We compare several recent theoretical studies of the single transverse spin asymmetry in dijet-correlations at hadron colliders. We show that the results of these studies are all consistent. To establish this, we investigate in particular the two-gluon exchange contributions to the relevant initial and final state interactions in the context of a simplifying model. Overall, the results confirm that the dijet imbalance obeys at best a non-standard or “generalized” transverse-momentum-dependent factorization.

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1. Introduction. Recently, there has been tremendous interest in the “imbalance” of two jets that are produced nearly back-to-back in azimuthal angle in a hadronic reaction. Especially the single transverse spin asymmetry (SSA) for this process has received a lot of attention in theoretical work [1, 2, 3, 4, 5, 6, 7, 8, 9], while experimental studies have begun at the Relativistic-Heavy-Ion-Collider (RHIC) at Brookhaven National Laboratory (BNL) [10]. In Ref. [1], it was proposed to use dijet-correlations at RHIC to learn about the transverse-spin and transverse momentum dependent (TMD) Sivers functions [11]. A transversely polarized proton (with momentum $P_A$ and polarization vector $S_\perp$) is scattered off an unpolarized proton with momentum $P_B$, producing two jets with momenta $P_1$ and $P_2$:

$$A(P_A, S_\perp) + B(P_B) \rightarrow J_1(P_1) + J_2(P_2) + X .$$  

(1)

For convenience, two additional momentum vectors, $P = (P_1 + P_2)/2$ and $q = P_1 - P_2$, are introduced. The transverse momentum components of $P$ and $q$ are relevant for our discussions: $P_\perp$ is the average of the transverse momenta of the two jets and $q_\perp$ represents their imbalance. The most interesting kinematic region arises when the imbalance between the two jets is small: $q_\perp \ll P_\perp$, so that there are two very separate momentum scales in the problem.

Much of the very recent theoretical work has centered on the question if cross sections for the process may be factorized in terms of TMD functions in this kinematic regime [2, 3, 6, 7, 8]. This question has been addressed by three groups, in the following ways:

(1) in Refs. [2, 3], the gauge-link structure of the TMD parton correlators appearing in the dijet process was investigated. It had been known beforehand [12, 13, 14, 15] that Sivers-type SSAs in semi-inclusive deep inelastic scattering (SIDIS) or the Drell-Yan process may be generated by final-state or initial-state interactions, respectively, of the involved partons from the polarized hadron. These interactions sum to a simple gauge link in both cases, which carries the color charge of that parton, albeit – famously – with an opposite sign between the two processes. As was discussed in [2, 3], the SSA for dijet (or dihadron) production is associated with both initial-state and final-state interactions of the partons involved in the hard-scattering. It was found that if all interactions are summed up, the resulting gauge link for the TMD parton distributions takes a much more complicated form. It does not involve just the color charge of the
relevant parton, but has in general knowledge about the full hard-scattering process and its color structure. As such, different correlators were found to appear in different partonic channels, even if the parton type entering from the polarized proton is the same. This observation makes the non-universality of the TMD parton distributions much more dramatic than previously indicated by their sign difference between the SIDIS and Drell-Yan processes. It was also found in [2, 3] that a certain weighted moment of the spin-dependent cross section reduces the expression to a more standard form akin to collinear factorization, involving the twist-three matrix elements of [16, 17] and special hard-scattering factors.

(2) In Refs. [6, 7], a different approach was taken. The SSA for the dijet imbalance was examined at first order in perturbation theory, starting from collinear factorization in the intermediate transverse momentum region \( \Lambda_{\text{QCD}} \ll q_\perp \ll P_\perp \), and carefully examining the one-gluon radiation contributions. It was found that, at this order, the cross sections do factorize into TMD parton distributions that follow their definitions in either the SIDIS or the Drell-Yan process, while all the initial-state and final-state interaction effects can be absorbed into the hard-scattering factors. These leading-order hard factors can be calculated either in a model-inspired fashion or in a partonic scattering picture [7]. They coincide with the ones found in [2, 3] for the weighted spin-dependent cross section mentioned above.

(3) Ref. [8] stresses that the non-trivial gauge link structure found in [2, 8] implies that a “standard” factorization in terms of TMD parton distributions cannot hold for this process. A particularly transparent example is given in terms of a simple abelian model in which the two scattering partons are assumed to have different and unrelated charges, \( g_1 \) and \( g_2 \). By a first-order calculation it is suggested that indeed the gauge link to be associated with the parton of charge \( g_1 \) will in general depend on the charge \( g_2 \) as well, in violation of standard factorization. From a phenomenological point of view, TMD correlators extracted from dijet-correlations would have no connection with those from the SIDIS and Drell-Yan processes, because their definitions are different. It is argued that if a standard definition of parton distributions is kept, there will be uncancelled singularities at higher order of perturbation theory. This happens for the spin-dependent as well as the spin-averaged cross section.
The goal of the present paper is to put the results of Refs. [2, 3, 6, 7, 8] into context\(^1\). It is clear from the above that there is not necessarily a contradiction among the results. (1) and (3) agree in their assessment that universality, and hence standard factorization, of the TMD distributions is broken for this process. (2) is based on a first-order calculation, and it is conceivable that deviations from the factorized structure found in Refs. [6, 7] become apparent only at higher orders. In order to shed more light on this, we perform the following studies: first, we compare the results in (1) and (2) by expanding the gauge links given in [2, 3] to first order in the strong coupling. This should lead to the results of (2), if there is mutual consistency. Furthermore, we will consider the two-gluon contributions to the initial and final-state interactions. This will unambiguously show if the gauge link exponentials have the complicated and process-dependent form presented in [2, 3] and [8], which would demonstrate that indeed standard factorization breaks down beyond the order considered in (2). It will also address the consistency between (1) and (3). For simplicity, we will perform the calculation in the context of the model considered in Ref. [2, 8], and only for one underlying partonic process, \(qq' \rightarrow qq'\).

2. First-order expansion of the gauge-links in Refs. [2, 3].

Ref. [2, 3] finds the following gauge link in the correlator for the partonic channel \(qq' \rightarrow qq'\) contributing to the SSA for the dijet imbalance:

\[
U_{qq'} = \frac{1}{N_c^2 - 1} \left[ (N_c^2 + 1) \frac{\text{Tr} \left( U'^{\square} \right)}{N_c} U^{[+] - 2U'^{\square} U^{[+]}} \right],
\]

which is normalized to the correlator for unpolarized scattering. Replacing the factor \(1/(N_c^2 - 1)\) with \(1/(4N_c^2)\) will recover the normalization used in [6, 7]. In the first-order perturbative expansion, we will find the following contributions from the various gauge links in the above equation,

\[
U^{[+]} \rightarrow (-ig) \frac{1}{-k^+ + i\epsilon}, \quad U'^{\square} \rightarrow (-ig) \left[ \frac{1}{-k^+ + i\epsilon} + \frac{1}{k^+ + i\epsilon} \right].
\]

Here we have suppressed the color matrices. Since the latter are traceless, one has \(\text{Tr} \left( U'^{\square} \right) = 1\) to this order. Using these expansions, we get the following first-order expansion of the

\(^1\) We will from now on use for simplicity the labels “(1),(2),(3)” to refer to the papers [2, 3, 6, 7, 8], respectively, following the list given above.
gauge link $U_{qq'}$:

$$U_{qq'} \rightarrow ( -ig) \frac{1}{N_c^2 - 1} \left[ \left( N_c^2 - 3 \right) \frac{1}{k^+ + i \epsilon} + \left( -2 \right) \frac{1}{k^+ + i \epsilon} \right]. \tag{4}$$

Comparing with the results in [6, 7], we find that the first term corresponds to the sum of the two final-state interaction factors of [6, 7], and the second term to the initial-state interaction factor there. More precisely, the coefficient of $(-ig)/(k^+ + i \epsilon)$ coincides with the factor $(C_F1 + C_F2)/C_u$ found in column (1) of Table I of [7], while the coefficient of $(-ig)/(k^+ + i \epsilon)$ is identical to $C_I/C_u$. This comparison can be extended to the diagrams for the other partonic channels, and agreement between the results of [2, 3] and [6, 7] is found in each case. In addition to Eq. (3), one then also needs $U_{qq'} \rightarrow (ig)/(k^+ + i \epsilon)$.

3. Two-gluon exchange contribution.

In this section we study the two-gluon exchange contribution to the initial and final-state interaction effects in the dijet imbalance. As we stated earlier, we will for simplicity use the abelian model of [2, 8] in which the two scattering partons are assumed to have different and unrelated charges, $g_1$ and $g_2$. We also follow [8] to consider only the case of an underlying $qq' \rightarrow qq'$ hard process; however, we take the $q$ and $q'$ as fermions and not as scalars as in [8].

To set the stage, we write the differential cross sections for the dijet-correlation in this model in the factorized forms assumed in [6, 7]:

$$\frac{d\Delta\sigma(S_{\perp})}{dy_1 dy_2 dP_\perp^2 d^2 q_{\perp}} = \frac{e^{\alpha\beta} S_{q\perp}^\alpha q_{\perp}^\beta}{q_{\perp}^2} \int d^2k_{1\perp} d^2k_{2\perp} d^2\lambda_{\perp}$$

$$\times \frac{\vec{k}_{1\perp} \cdot \vec{q}_{\perp}}{M_P} x_a q_{T\alpha}^a (x_a, k_{1\perp}) x_b q_{T\beta}^b (x_b, k_{2\perp})$$

$$\times \left[ S_{qq' \rightarrow qq'}(\lambda_{\perp}) H_{qq' \rightarrow qq'}(P_{\perp}^2) \right]_c \delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{\lambda}_{\perp} - \vec{q}_{\perp}), \tag{5}$$

for the transverse-spin dependent case, and

$$\frac{d\sigma^{uu}}{dy_1 dy_2 dP_\perp^2 d^2 q_{\perp}} = \int d^2k_{1\perp} d^2k_{2\perp} d^2\lambda_{\perp} x_a q_{a\perp}^{SIDIS}(x_a, k_{a\perp}) x_b q_{b\perp}^{SIDIS}(x_b, k_{b\perp})$$

$$\times \left[ S_{qq' \rightarrow qq'}(\lambda_{\perp}) H_{qq' \rightarrow qq'}(P_{\perp}^2) \right]_c \delta^{(2)}(\vec{k}_{a\perp} + \vec{k}_{b\perp} + \vec{\lambda}_{\perp} - \vec{q}_{\perp}) \tag{6},$$

for the spin-averaged case. $H_{qq' \rightarrow qq'}$ and $S_{qq' \rightarrow qq'}$ are partonic hard and soft factors, respectively, and the $[ \ ]_c$ represents a trace in color space between the hard and soft factors due to the color flow into the jets [18, 19]. The hard factors in Eqs. (5), (6) only depend on the single hard scale $P_\perp$ in terms of partonic Mandelstam variables of the reaction $qq' \rightarrow qq'$.
and \( x_a = \frac{P_a}{\sqrt{s}} (e^{-y_1} + e^{-y_2}) \), \( x_b = \frac{P_b}{\sqrt{s}} (e^{-y_1} + e^{-y_2}) \) with \( y_1 \) and \( y_2 \) the rapidities of the two jets. In Eq. [5], \( q^{\text{SIDIS}}_a \) and \( q^{\text{SIDIS}}_b \) denote the transverse-spin dependent Sivers quark distribution for hadron \( A \) and the unpolarized TMD quark distribution for hadron \( B \), respectively; these TMD parton distributions were chosen to follow their definitions in the SIDIS process.

In the present model where the two initial partons have separate charges, the associated parton distributions will depend on these charges. For example, for a polarized hadron with momentum \( P_A = (P^+_A, 0^-, 0^\perp) \) with \( P^+_A = \frac{1}{\sqrt{2}} (P^0_A \pm P^3_A) \) and transverse spin vector \( \vec{S}^\perp \), the TMD distribution for quark flavor \( a \) with charge \( g_1 \) can be defined through the decomposition of the following matrix element,

\[
\mathcal{M}_a = \int \frac{P^+ d\xi^-}{\pi} \frac{d^2 \xi^\perp}{(2\pi)^2} e^{-ix\xi^- P^+ + i\xi^\perp \cdot k^\perp} \langle P_A S | \psi_a(\xi) \mathcal{L}_{\nu_a}^\dagger (g_1; \infty; \xi) \mathcal{L}_{\nu_a}^\dagger (g_1; \infty; 0) | P_A S \rangle \\
= \frac{1}{2} \left[ q^{\text{SIDIS}}_a (x, k^\perp) \gamma_\mu P^\mu + \frac{1}{M_P} q^{\text{SIDIS}}_a (x, k^\perp) \epsilon_{\mu\nu\alpha\beta} \gamma_\mu P^\nu k^\alpha \mathcal{S}^\beta + \ldots \right],
\]

where the gauge link \( \mathcal{L} \) is defined in a covariant gauge as

\[
\mathcal{L}_{\nu_a} (g_1; \infty; \xi) \equiv \exp \left( -ig_1 \int_0^\infty d\lambda v_a \cdot A(\lambda v + \xi) \right),
\]

with the path extended to \( +\infty \). \( v_a \) is a vector conjugate to the momentum vector \( P_A \). Since we will work in a covariant gauge throughout this paper, the vector \( v_a \) could be chosen to be a light-cone vector with \( v_a^2 = 0 \) and \( v_a \cdot P_A = 1 \). If we work in a singular gauge, like the light-cone gauge, an additional gauge link at spatial infinity \( (\xi = +\infty) \) will have to be included in order to ensure the gauge invariance of the above definitions \[14\].

Similarly, the quark distribution for the unpolarized hadron \( B \) can be defined as

\[
\mathcal{M}_b = \int \frac{P_B^- d\xi^+}{\pi} \frac{d^2 \xi^\perp}{(2\pi)^2} e^{-ix\xi^+ P^- + i\xi^\perp \cdot k^\perp} \langle P_B | \psi_b(\xi) \mathcal{L}_{\nu_b}^\dagger (g_2; \infty; \xi) \mathcal{L}_{\nu_b}^\dagger (g_2; \infty; 0) | P_B \rangle \\
= \frac{1}{2} \left[ q^{\text{SIDIS}}_b (x, k^\perp) \gamma_\mu P^\mu + \ldots \right],
\]

where the gauge link is \( \mathcal{L}_{\nu_b} (g_2; \infty; \xi) \equiv \exp \left( -ig_2 \int_0^\infty d\lambda v_b \cdot A(\lambda v + \xi) \right) \), with \( v_b \) a vector conjugate to \( P_B \) satisfying \( v_b^2 = 0 \) and \( v_b \cdot P_B = 1 \). Notice that in the above definition the gauge link is associated with the coupling \( g_2 \).

At the lowest order, in the one-gluon contribution case, the relevant hard factors in this model read:

\[
H^{u\bar{u}}_{qq'\rightarrow qq'} = \frac{g_1^2 g_2^2}{16\pi \hat{s}^2} \frac{2(\hat{s}^2 + \hat{u}^2)}{\hat{t}^2},
\]

\[
H^{\text{Sivers}}_{qq'\rightarrow qq'} = \frac{g_1^2 g_2^2}{16\pi \hat{s}^2} \frac{g_1 + 2g_2}{g_1} \frac{2(\hat{s}^2 + \hat{u}^2)}{\hat{t}^2},
\]

\( \hat{s}^2 = (p_1 + p_2)^2 \), \( \hat{u}^2 = (p_1 - p_2)^2 \), \( \hat{t}^2 = \hat{s}^2 - \hat{u}^2 \).
FIG. 1: Two-gluon exchange contributions proportional to $g_1^2$.

where $\hat{s}$, $\hat{t}$, $\hat{u}$ are the partonic Mandelstam variables. The above results can be directly obtained from our previous result in [7] by replacing there $C_u$ by 1, $C_I$ and $C_{F2}$ by $g_2/g_1$, and $C_{F1}$ by 1. The expressions for the hard-scattering factors for other partonic channels may be found in a similar way. We note that the above result for $H_{Sivers}^{qq'}$ also agrees with that given in [8].

Before we move on to the two-gluon exchange contributions, we note that we have in fact also adapted our previous calculations of [6, 7] (see also [20]) to this model case. As in [6, 7], we have started from a collinear-factorized framework and considered the one-gluon contributions in the intermediate transverse momentum region $\Lambda_{QCD} \ll q_\perp \ll P_\perp$. We have found that indeed the factorized structure of Eqs. (5), (6) emerges, with perturbatively defined TMD distributions that depend on the charges $g_1$ and $g_2$ as given in Eqs. (7), (9), and with the hard-scattering factors in (10). We refrain from giving the details of these calculations in this paper, but stress that the one-loop TMD factorization for the dijet-imbalance found in [6, 7] therefore extends to the model of [8] we consider here.

We now turn to the two-gluon exchange diagrams with the spectator of hadron $A$, and their contributions to the SSA and the spin-averaged cross section. They can be grouped into three parts: one proportional to $g_1^2$ as shown in Fig. 1(a) where both gluons attach to the outgoing quark line with charge $g_1$; one proportional to $g_1 g_2$ as shown in Fig. 2 where one gluon attaches to $g_1$ quark line and another gluon to the incoming or outgoing $g_2$ quark line; and one proportional to $g_2^2$ as shown in Fig. 3 where both gluons attach to the $g_2$ quark line.

Let us first discuss the contributions from the diagrams in Fig. 1. In the following calculations, we only consider the dominant contributions from the exchange of gluons collinear to the polarized hadron $A$, which survive at the leading power of $q_\perp/P_\perp$. We can then utilize the eikonal approximation [2, 3, 7, 8]. The contribution from the diagram in Fig. 1(a) will
FIG. 2: Two-gluon exchange contributions proportional to $g_1 g_2$.

FIG. 3: Two-gluon exchange contributions proportional to $g_2^2$.

then be proportional to

$$
g_1^2 \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{1}{(P_A - k' - k_1 - k_2)^2 - m^2 + i\epsilon (k' + k_1)^2 - \mu^2 + i\epsilon (k' + k_1 + k_2)^2 - \mu^2 + i\epsilon} \times \frac{1}{k_1^2 - \lambda^2 + i\epsilon k_2^2 - \lambda^2 + i\epsilon} \left( \frac{1}{-k_1^2 + i\epsilon} \right) \left( \frac{1}{-k_1^2 + k_2^2 + i\epsilon} \right), \tag{11}
$$

where $k_1$ and $k_2$ are the momenta of the two exchanged gluons, and where we have introduced the masses $m, \mu, \lambda$ for the scattering quarks and for the gluon, respectively. The last two
factors in (11) come from the eikonal approximations for the two gluon attachments to the outgoing quark line \( g_1 \). A similar contribution is obtained from diagram (b), except for a difference in the eikonal propagators, for which we now have

\[
\left( \frac{1}{-k_2^+ + i\epsilon} \right) \left( \frac{1}{-k_1^+ - k_2^+ + i\epsilon} \right) .
\]

Thus, the total contribution by these two diagrams will depend on the following expression for the eikonal propagators,

\[
A = g_1^2 \left[ \left( \frac{1}{-k_1^+ + i\epsilon} \right) \left( \frac{1}{-k_1^+ - k_2^+ + i\epsilon} \right) \right] + i\pi g_1^2 \left[ \frac{\delta(k_1^+)}{k_2^+} + \frac{\delta(k_2^+)}{k_1^+} \right] ,
\]

and the rest of the expression is identical for these two diagrams. The imaginary part of the above expression contributes to the SSA, whereas the real part contributes to the unpolarized cross section. We can further simplify the above term as

\[
A = g_1^2 \left[ \frac{1}{k_1^+ k_2^+} + (-i\pi)^2 \delta(k_1^+) \delta(k_2^+) \right] + i\pi g_1^2 \left[ \frac{\delta(k_1^+)}{k_2^+} + \frac{\delta(k_2^+)}{k_1^+} \right] ,
\]

where partial cancellations between the diagrams have occurred in the final result. The contributions from diagrams 3 (a,b,d,e,g,h) will depend on the factor

\[
B = g_1 g_2 \left[ 2 \left( \frac{1}{-k_1^+ + i\epsilon} \right) \left( \frac{1}{-k_2^+ + i\epsilon} \right) + \left( \frac{1}{k_1^+ + i\epsilon} \right) \left( \frac{1}{k_2^+ + i\epsilon} \right) \right] + i2\pi g_1 g_2 \left[ \frac{\delta(k_1^+)}{k_2^+} + \frac{\delta(k_2^+)}{k_1^+} \right] ,
\]

where partial cancellations between the diagrams have occurred in the final result. The contributions from diagrams 3 (a,b,d,e,g,h) will depend on the factor

\[
C = g_2^2 \left[ \left( \frac{1}{-k_1^+ + i\epsilon} \right) \left( k_2^+ + i\epsilon \right) + \left( k_1^+ + i\epsilon \right) \left( -k_2^+ + i\epsilon \right) \right] + \left( \frac{1}{k_1^+ + i\epsilon} \right) \left( \frac{1}{k_2^+ + i\epsilon} \right) \left( \frac{1}{k_1^+ + k_2^+ + i\epsilon} \right) + \left( \frac{1}{-k_1^+ + i\epsilon} \right) \left( -k_2^+ + i\epsilon \right) \left( -k_1^+ - k_2^+ + i\epsilon \right) \right] \left[ 4(-i\pi)^2 \delta(k_1^+) \delta(k_2^+) \right] .
\]

Again, there are cancellations between the diagrams, which in this case leave us with only a real part contribution.
The total contribution from the diagrams discussed above thus becomes

$$A + B + C = g_1^2 \left[ \frac{1}{k_1^+ k_2^+} + (-i\pi)^2 \delta(k_1^+) \delta(k_2^+) \right] + g_1(g_1 + 2g_2)(i\pi) \left[ \frac{\delta(k_1^+)}{k_1^+} + \frac{\delta(k_2^+)}{k_2^+} \right]$$

$$+ 4 \left( g_1 g_2 + g_2^2 \right) (-i\pi)^2 \delta(k_1^+) \delta(k_2^+) . \quad (17)$$

As we mentioned above, the first and third terms will contribute to the unpolarized cross section, whereas the second term will contribute to the SSA. Clearly, the first term can be factorized into the unpolarized TMD quark distribution, multiplied by the leading-order hard factor $H_{uu}^{qq'} \rightarrow qq'$ in Eq. (10). The second term can also be factorized into the quark Sivers function for hadron $A$, multiplied by the leading-order hard factor $H_{Sivers}^{qq'} \rightarrow qq'$ in Eq. (10). The third term, however, cannot be factorized into the unpolarized quark distribution defined in Eq. (7), multiplied by the leading hard factor $H_{uu}^{qq'} \rightarrow qq'$. In order to account for this uncancelled contribution, we have to modify the gauge link definition in Eq. (8):

$$\mathcal{L}_{v_a}^\prime (g_1; g_2; \xi) \equiv \mathcal{P} \lim_{\xi \rightarrow \infty} \left[ \frac{1}{v_a} \int_{0}^{\infty} d\lambda v_a \cdot A(\xi + \lambda v_a) \right] \times \mathcal{P} \lim_{\xi \rightarrow \infty} \left[ \frac{1}{v_a} \int_{0}^{\infty} d\lambda v_a \cdot A(\xi + \lambda v_a) \right] \times \mathcal{P} \lim_{\xi \rightarrow \infty} \left[ \frac{1}{v_a} \int_{0}^{\infty} d\lambda v_a \cdot A(\xi + \lambda v_a) \right] , \quad (18)$$

which corresponds to what has been proposed in [2, 3]. The first term takes into account the contributions due to final-state interactions of the outgoing quark with charge $g_1$, the second one those of the outgoing quark with charge $g_2$, and the third one the initial-state interactions of the incoming quark with charge $g_2$. With this modification of the gauge link in the definition of the TMD quark distributions, the above results can all be reproduced by the two-gluon exchange diagram contributions, multiplied by the leading order hard factor. This is true for both the unpolarized and the spin-dependent (Sivers) case, even though for the latter, as seen from Eq. (17), the complications that require the redefinition of the gauge link are not yet visible at this order but should first occur at the next order of perturbation theory.

A similar analysis can be performed for the diagrams in Fig. 1(c), Fig. 2(c,f), and Fig. 3(c,f,i). All their contributions are reproduced by the above modified TMD quark distribution, multiplied by the leading-order hard factor. It is important to note that these
diagrams do not contribute to the SSA. Moreover, their contributions, along with that in the last term of Eq. (17), will lead to an infrared-finite contribution to the differential unpolarized cross section. It will be interesting to investigate further how this will affect the dijet-correlation at hadron colliders.

In the QED-like model of Ref. [8], further study shows that we can simplify the gauge link in Eq. (18) to

\[
\mathcal{L}'_{v_a}(\xi) = \mathcal{P} \exp \left( -ig_1 \int_0^\infty d\lambda v_a \cdot A(\xi + \lambda v_a) \right) \times \mathcal{P} \exp \left( -ig_2 \int_{-\infty}^\infty d\lambda v_a \cdot A(\xi_\perp + \lambda v_a) \right). \tag{19}
\]

We notice that the last factor will reduce to a pure phase when integrated over the transverse momentum, because in that case \(\xi_\perp\) is set to zero and there is no other dependence on \(\xi\). In this case, a collinear factorization approach is appropriate, and the gauge link will be defined in a standard way [21]. In particular, the last term of Eq. (17) will be cancelled out by diagrams (c) and (c,f) of Figs. 2 and 3, respectively.

4. Conclusions. In this paper, we have compared three recent theoretical studies [2, 3, 6, 7, 8] of the factorization properties of dijet-correlations in a polarized hadronic reaction when the two jets are produced nearly back-to-back in azimuthal angle. We have shown that the first-order perturbative results of [6, 7] are reproduced when the gauge links derived in [2, 3] are expanded to that order. Within the abelian model considered in [2, 8], we have also calculated the two-gluon contributions to the initial-state and final-state interactions and verified that the results are consistent with the expected gauge-link structure. We therefore conclude that the results of the various studies are mutually consistent. They show that beyond the first order, the TMD quark distributions need to be modified in a process-dependent way, in order for them to correctly take into account collinear gluon contributions. This is a departure from standard factorization, which on the other hand still does not exclude that the dijet observable obeys a more generalized factorization in terms of more complicated TMD correlators. Further studies will be needed here.

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