A big data based method for pass rates optimization in mathematics university lower division courses

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Abstract

In this paper an algorithm designed for large databases is introduced for the enhancement of pass rates in mathematical university lower division courses with several sections. Using integer programming techniques, the algorithm finds the optimal pairing of students and lecturers in order to maximize the success chances of the students’ body. The students-lecturer success probability is computed according to their corresponding profiles stored in the data bases.

Keywords: big data, optimization, probabilistic modeling.

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1. Introduction

It is well-known that improving pass rates in mathematics courses is of paramount importance for academic institutions all over the world. This objective becomes even more critical for public universities as they subside partially or totally the education of its enrolled undergraduates; subject to the country’s legislation and the student’s economic stratification.

The general consensus is that creating better conditions for the students will improve students’ success. Hence, most of the work done in order to address this challenge has two principal directions:

(i) The traditional pedagogical approach which, essentially aims to improve the presentation of the course contents on two fronts: presentation of mathematical discourse i.e., curricula reform, development of course materials and improvement the lecturer’s instructional practice. Part of the latter are the teaching evaluations’ open questions, giving feedback to the instructor about how people felt during his/her classes.

(ii) The uses of technology in the learning of mathematics. On one hand, the collection of data and the measurement of the resource impact in the cognitive process: development of LMS platforms and real-time feedback interfaces \cite{18}. On the other hand, the use of the aforementioned harvested information to improve the learning process \cite{7}: targeted problem sets and training tests \cite{2}, identification of favorable pedagogical approaches and learning patterns/styles \cite{14}, identification of fortitudes and weaknesses, assessment of study exercise vs skills building \cite{14}, problem solving approaches, platforms for interaction between students through the learning process \cite{10}. There is also the use of big data to asses learning rather than improve instructional techniques, such early detection of students at risk \cite{15}.

The present work fits in the second category, in this case, the use of big data to define policies enhancing the higher education production \cite{13} and without raising the costs. More specifically the algorithm will suggest an optimal design of student body/composition to maximize the pass rate chances. The design is driven by favorable...
teacher-student partnership, rather than peer diversity or a peer interaction criterion (see\cite{8}). A second aspect of the method is that it is based on the computation of expectations (conditioned to the students’ segmentation) and not on statistical regression (linear or not) as it pursues the construction of a probabilistic model rather than the construction of a production function (see\cite{12}). In addition, the database (which is the algorithm’s input), is updated from one academic term to the next, therefore, it seems more adequate to recompute the conditional expectations term-wisely instead of pursuing rigid regression models. Moreover, given the current computational tools and possibilities, once the algorithm is coded, the academic term update proposed approach will come at zero cost increase.

The algorithm starts from databases of a public university for 15 semesters between February 2010 and July 2017 containing the information of academic performance and demographics of its population. A first stage of descriptive statistics allows to identify the success factors by correlation. A second stage revisits the historical performance of the sections (15 semesters in total), it defines profile quartiles and then computes the historical efficiency for each of the involved instructors, conditioned to the quartiles of segmentation and uses integer programming to find the optimal matching of students-instructors in two different ways. A third stage of the algorithm randomizes the involved factors, namely the profile of the group taking the class, the number of Tenured Track instructors, the number of sections and such. This is aimed to produce Monte Carlo simulations and find the expected values of improvement in the long run. The factors are regarded as random variables with probabilistic distributions computed from the empirical knowledge recorded in the database and each Monte Carlo simulation arises from one random realization of the algorithm.

The whole algorithm is implemented in Python and the databases are handled with pandas (Python Data Analysis Library).

We close this section introducing the mathematical notation. For any natural number \( N \in \mathbb{N} \), the symbol \([N] = \{1, 2, \ldots, N\}\) indicates the set/window of the first \( N \) natural numbers. For any set \( E \) we denote by \(|E|\) its cardinal and \( \wp(A) \) its power set. We understand \( \Omega \) as a generic finite probability space \((\Omega, \wp(\Omega), \mathbb{P})\) in which all outcomes are equally likely, i.e. the event probability function satisfies \( \mathbb{P}(\{\omega\}) = |\Omega|^{-1} \) for all \( \omega \in \Omega \). In particular for any event \( E \subseteq \Omega \) it holds that

\[
\mathbb{P}(E) = \frac{|E|}{|\Omega|} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}. \tag{1}
\]

A particularly important probability space is \( \mathcal{S}_N \), where \( \mathcal{S}_N \) denotes the set of all permutations in \([N]\), its elements will be usually denoted by \( \pi, \sigma, \tau, \ldots \) etc. Random variables will be represented with upright capital letters, namely \( X, Y, Z, \ldots \), expectation and variance of such variables with \( \mathbb{E}(X) \) and \( \text{Var}(X) \) respectively. Vectors are indicated with bold letters, namely \( \mathbf{p}, \mathbf{g}, \ldots \) etc. Deterministic matrices are represented with capital letters i.e., \( A, G, T \).

2. The study case and its databases

In this work our study case is the performance of lower division mathematics courses at Universidad Nacional de Colombia, Sede Medellín (National University of Colombia at Medellín). The Institution is a branch of the National University of Colombia, the best ranked higher education Institution in Colombia; it is divided in five colleges: Architecture, Science, Humanities & Economical Sciences, Agriculture and Engineering (Facultad de Minas). The colleges are divided in departments. The University offers 26 undergraduate programs and 85 graduate programs divided in Specializations, MS and PhD levels, depending on the department. Each semester, it has an average enrollment of 10000 undergraduates and 2000 graduates with graduation rates of 1240 and 900 respectively. The College of Engineering is the most numerous, consequently, the mathematics lower division courses are highly demanded and have a profound impact in the campus’ life; its teaching and evaluation is in charge of the School of Mathematics.

The School of Mathematics is part of the College of Science, it teaches two types of courses: specialization (advanced undergraduate and graduate courses in mathematics) and service (lower division). The latter are: Basic Mathematics (college algebra), Differential Calculus (DC), Integral Calculus (IC), Vector Calculus (VC), Differential Equations (ODE), Vector & Analytic Geometry (VAG), Linear Algebra (LA), Numerical Methods (NM), Discrete Mathematics and Applied Mathematics. The total demand of these courses amounts to an
average of 7200 enrollment registrations per semester. The last two courses, *Discrete Mathematics* and *Applied Mathematics* have very low an unstable enrollment, therefore, their data are not suitable for statistical analysis and they will be omitted in the following. On the other hand, the remaining courses due to its massive nature are ideally suited for big data analysis; see Table 1 below. On a typical semester these courses are divided in sections

| Year | Semester | DC  | IC  | VC  | VAG | LA  | ODE | BM  | NM  | Total |
|------|----------|-----|-----|-----|-----|-----|-----|-----|-----|-------|
| 2010 | 1        | 1631| 782 | 381 | 1089| 983 | 668 | 848 | 142 | 6524  |
| 2010 | 2        | 1299| 1150| 427 | 1003| 1007| 562 | 792 | 261 | 6501  |
| 2011 | 1        | 1271| 1136| 512 | 1078| 900 | 663 | 1139| 269 | 6968  |
| 2011 | 2        | 951 | 850 | 513 | 652 | 812 | 1170| 920 | 289 | 6157  |
| 2012 | 1        | 1619| 1096| 559 | 1110| 1116| 752 | 1122| 366 | 7740  |
| 2012 | 2        | 1486| 1190| 601 | 1076| 1144| 825 | 962 | 356 | 7640  |
| 2013 | 1        | 1476| 1044| 604 | 1231| 1037| 902 | 916 | 319 | 7529  |
| 2013 | 2        | 1446| 1212| 549 | 1187| 1103| 786 | 846 | 326 | 7455  |
| 2014 | 1        | 1460| 1184| 676 | 1192| 1000| 890 | 729 | 295 | 7426  |
| 2014 | 2        | 1399| 1126| 564 | 1198| 1012| 695 | 1015| 234 | 7243  |
| 2015 | 1        | 1097| 925 | 565 | 1076| 793 | 601 | 724 | 201 | 5982  |
| 2015 | 2        | 1797| 1214| 605 | 1314| 1099| 808 | 824 | 274 | 7935  |
| 2016 | 1        | 1675| 1323| 582 | 1549| 1017| 950 | 867 | 263 | 8226  |
| 2016 | 2        | 1569| 1296| 594 | 1355| 1009| 1019| 1111| 284 | 8237  |
| 2017 | 1        | 1513| 1315| 515 | 1088| 798 | 736 | 766 | 134 | 6865  |

| Mean | Does not apply | 1445.9 | 1122.9 | 549.8 | 1146.5 | 988.7 | 801.8 | 905.4 | 267.5 | 7228.5 |

(between 8 to 22, depending on the enrollment) of sizes ranging from 80 to 140 (because of classroom physical capacities). There is no graded homework but students have problem sheets as well as optional recitation classes. As for the grading scale 5.0 is the maximum and 3.0 is the minimum pass grade and grades contain only one decimal. The evaluation consists in three exams which the students take simultaneously; the activity is executed with the software SiDEx-Ω which manages the logistics of the evaluation activities, including the organization of the grading stage. More specifically, for fairness and consistency of the grading process a particular problem is graded by one single grader for all the students i.e., it is a centralized grading process. As a consequence of the institutional policies described above, the data are statistically comparable. Moreover, the paper-based tests administrator SiDEx-Ω introduces high levels of fraud control, due to its students seating assignment algorithm; this increases even more the reliability of the data.

### 2.1. The Databases

The University allowed limited access to its data bases for the production of this work. The information was delivered in five separate tables which were merged in one single database using Pandas: the file *Assembled_Data.csv* which contains 108940 rows, each of them with the following fields

(i) Student’s Personal Information: • Year of Birth • e-mail • ID Number • Last Name and Names • Gender

(ii) Student’s General Academic Information: • University Entrance Year (AA, Academic Age) • Career • Academic Average (GPA)
(iii) Student's Academic Information Relative to the Course: • Course • Course Code • Academic Year • Academic Semester • Grade • Completed vs Canceled Course • Number of Attempts • Number of Cancellations

(iv) Student's Administrative Information Relative to the Course: • Section Number • Schedule • Section Capacity • Number of Enrolled Students • Instructor's ID Number • Instructor's Name • Tenured vs Adjoint Instructor

It is understood that one registration corresponds to one row, for instance if a particular student registers for DC and LA one row is created for each registration repeating all the information listed in items (i) and (ii) above. The same holds when an individual needs to repeat a course because of previous failure or cancellation.

3. The Method

In the present work we postulate the Lecturer as one of the most important factors of success, more specifically the aim is to attain the optimal Instructor-Student partnership; in this we differ from [9, 8] where it's proposed that the class composition should be driven by the peers interaction. To that end, it becomes necessary to profile the students' population according to its relevant features.

3.1. Determination of the Segmentation Factor

Computing the correlation matrix of the quantitative factors considered in the Assembled Data.csv; in the Table 2 we display the correlation matrix for the Differential Calculus course. From the Grade row it is clear that the most significant factor in the Grade is the Academic Average (GPA) followed by the Academic Age (AA) and the Age. However, the impact of the GPA is about four times the impact of AA and the same holds for the Age factor. Moreover, from the GPA row, it is clear that the most significant factor after the Grade are precisely the Academic Age and the Age (the younger the student, the higher the GPA). In addition, for the remaining courses, similar correlation matrices are observed. Hence, we keep the GPA as the only significant quantitative factor in the Grade variable.

Remark 1. It is important to mention that the impact of the class size in the students' performance has been subject of extensive discussion without consensus. While [1, 12] report a significant advantage in reducing class sizes, [11] finds no effect. In our particular case Table 2 shows that the Section Capacity is uncorrelated not only to the Grade variable, but also to the GPA variable. Moreover the Section Capacity is uncorrelated with the Cancellations (drop out) variable.

Table 2: Quantitative Factors Correlations Table, Course: Differential Calculus

| FACTOR       | Section Capacity | Age     | AA       | # Enrolled Students | Grade    | # Cancellations | # Attempts | GPA    |
|--------------|------------------|---------|----------|--------------------|----------|-----------------|------------|--------|
| Section Capacity | 1.0000           | -0.0108 | 0.0570   | 0.8334             | 0.0074   | 0.0003          | 0.0393     | 0.0180 |
| Age          | -0.0108          | 1.0000  | 0.3069   | -0.0168            | -0.2031  | 0.0775          | 0.1384     | -0.2082|
| AA           | 0.0570           | 0.3069  | 1.0000   | 0.0416             | -0.2164  | 0.1668          | 0.4294     | -0.1667|
| # Enrolled Students | 0.8334          | -0.0168 | 0.0416   | 1.0000             | 0.0252   | 0.0131          | -0.0041    | 0.0325 |
| Grade        | 0.0074           | -0.2031 | -0.2164  | 0.0252             | 1.0000   | -0.1247         | -0.0241    | 0.8207 |
| # Cancellations | 0.0003          | 0.0775  | 0.1668   | 0.0131             | -0.1247  | 1.0000          | 0.3101     | -0.0686|
| # Attempts   | 0.0393           | 0.1384  | 0.4294   | -0.0041            | -0.0241  | 0.3101          | 1.0000     | -0.0401|
| GPA          | 0.0180           | -0.2082 | -0.1667  | 0.0325             | 0.8207   | -0.0686         | -0.0401    | 1.0000 |

Two binary variables remain to be analyzed namely Pass/Fail (PF) and Gender. If we generically denote by X the binary variables and by Y a variable of interest, the point-biserial correlation coefficient is given by

\[ r_{pb} = \frac{M_1 - M_0}{\sigma} \sqrt{\frac{N_1 N_0}{N^2}}. \]
Here, the indexes 0, 1 are the values of the binary variable $X$. For $i = 0, 1$, $M_i$ is the mean value of the variable $Y$ for the data points in the group $X = i$, $N$ denotes the population of each group $X = i$, $N = N_1 + N_0$ stands for the total population and $\sigma$ indicates the standard deviation of the variable $Y$.

The correlation analysis between the binary $Pass/Fail$ (PF) variable vs the quantitative factors is displayed in Table 3. As in the $Grade$ variable analysis, the most significant factor in the $Pass/Fail$ variable, is the $Academic Average$ (GPA) followed by the $Academic Age$ (AA) and the Age. In this case however, the impact of the GPA is only three times the impact of AA as well as the Age factor. Again, we keep the GPA as the only significant quantitative factor in the $Pass/Fail$ variable.

| FACTOR                  | COURSE          | DC    | IC    | VC    | VAG   | LA    | ODE   | BM    | NM    |
|-------------------------|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Section Capacity        |                 | 0.0058| 0.0139| -0.0296| -0.0986| -0.0147| 0.0448| -0.0451| 0.0840|
| Age                     |                 | -0.1757| -0.2554| -0.3048| -0.1879| -0.2444| -0.2555| -0.0892| -0.3333|
| AA                      |                 | -0.1783| -0.2855| -0.3012| -0.1550| -0.2230| -0.3841| -0.0209| -0.2731|
| # Enrolled Students     |                 | 0.0070| 0.0276| 0.0172| -0.0410| 0.0602| 0.1258| -0.0374| 0.1916|
| Grade                   |                 | 0.8072| 0.7987| 0.7864| 0.8145| 0.7959| 0.7999| 0.7988| 0.7922|
| # Cancellations         |                 | -0.1129| -0.1397| -0.1420| -0.1299| -0.1178| -0.1647| -0.0096| -0.1489|
| # Attempts              |                 | -0.0988| -0.1635| -0.1683| -0.1374| -0.1399| -0.1577| -0.0054| -0.2065|
| GPA                     |                 | 0.6062| 0.5892| 0.5884| 0.6445| 0.6063| 0.5341| 0.6125| 0.5828|

Next, the correlation analysis $Gender$ variable vs the Academic Performance Variables, i.e., $Grade$, GPA and $Pass/Fail$ (PF) is summarized in the Table 4 below. Clearly, the $Gender$ variable has negligible incidence in the Academic Performance Variables, with the exception of the GPE for the Basic Mathematics (BM) course case, where females do slightly better. Since this unique correlation phenomenon is not present in the remaining courses, the $Gender$ variable will be neglected from now on. Finally, it is important to stress that given the binary nature of the $Gender$ and the $Pass/Fail$ (PF) variables, all the correlation coefficients agree i.e., point-biserial, Pearson and Spearman and Kendall.

| FACTOR                  | COURSE          | DC    | IC    | VC    | VAG   | LA    | ODE   | BM    | NM    |
|-------------------------|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Grade                   |                 | 0.0545| 0.0584| 0.0240| -0.0485| 0.0154| 0.0164| -0.0740| -0.0405|
| GPA                     |                 | -0.0425| -0.0441| -0.0419| -0.0766| -0.0343| -0.0824| -0.1159| -0.0663|
| Pass/Fail (PF)          |                 | 0.0509| 0.0382| 0.0166| -0.0373| 0.0103| 0.0085| -0.0535| -0.0226|

From the previous discussion, it is clear that out of the analyzed variables the GPA is the only one with significant incidence on the academic performance variables $Grade$ and $Pass/Fail$. Consequently, this will be used as the unique criterion for the segmentation of students' population. From now on, our analysis will be focused on the $Average$ and the $Pass/Fail$ variables as measures of success, while the GPA will be used for segmentation purposes discussed in Section 3.2. In Table 5 the global averages (from 2010-1 to 2017-1) of these variables for all the service courses are displayed; the variable $Number of Tries$ has also been included because later on, it will illustrate better the benefits of the proposed method.
Table 5: Academic Performance Variables Average Values, Course: All

| VARIABLE      | COURSE | DC       | IC       | VC       | VAG      | LA       | ODE      | BM       | NM       |
|---------------|--------|----------|----------|----------|----------|----------|----------|----------|----------|
| Grade         |        | 2.6849   | 2.7829   | 3.2198   | 2.8616   | 3.0233   | 3.1170   | 2.8308   | 3.1893   |
| GPA           |        | 3.2213   | 3.3527   | 3.4969   | 3.2386   | 3.3201   | 3.4548   | 3.2696   | 3.5330   |
| Pass/Fail     |        | 0.5010   | 0.5339   | 0.7151   | 0.5901   | 0.6398   | 0.6846   | 0.5441   | 0.6924   |
| Number of Tries |      | 1.7382   | 1.9140   | 1.4996   | 1.5205   | 1.5495   | 1.7710   | 1.0549   | 1.4115   |

3.2 Segmentation Process

The profiling of students’ population is to be made course-wise. For each a group taking a course, the algorithm computes a partition of the interval $[0, 5]$ of ten numerical GPA intervals $(I_\ell : \ell \in [10])$, so that approximately ten percent of the population is contained in $I_\ell$ for all $\ell \in [10]$. Equivalently, if a histogram of relative frequencies is drawn, as in Figure 1, the area between the curve and any interval should be around 0.1. Hence, if $f_{GPA}$ is the relative frequency of the $GPA$ variable then $\int_{I_\ell} f_{GPA} \, dx \sim 0.1$ for all $\ell \in [10]$. The process described above is summarized in the pseudocode below.

![Example DC. GPA Histogram, Semesters from 2010-1 to 2017-1.](image_a)

![Example DC. GPA Histogram, Semesters 2010-1 and 2015-1.](image_b)

Figure 1: Differential Calculus GPA Normalized Histogram. Figure (a) Shows the normalized frequencies histograms for all the semesters available in the database Assembled_Data.csv. Figure (b) displays the normalized histogram of only two semesters for optical purposes. Observe that in both cases the area beneath the normalized histogram is exactly equal to one.
Data: Database: Assembled\_Data.csv.
Year (y) and Semester (s).
Analyzed Course: DC, IC, ..., NM.
Result: Extremes of the GPA Segmentation Intervals ($I_\ell : \ell \in [L]$); extremes = [0, n\_1, n\_2, ..., 5 ]
initialization;
List\_GPA ← sorted (hash GPA variable from Assembled\_Data.csv[(Course = Analyzed Course) & (Year = y) & (Semester = y ) ] ) ;
extremes\_0 ← 0;
for $i \in [10]$ do
  extremes\_j ← $\lfloor i \times \text{length of List\_GPA} \rfloor$;
end
if list extremes contains repetitions then
  extremes ← remove repetitions from extremes
end

Algorithm 1: Segmentation of Students

Remark 2. Observe that Algorithm 1 is aimed to produce ten segmentation intervals, however the last instruction considers removing some points out of the eleven extremes previously defined, in case of repetition. Such situation could arise when a particular GPA value is too frequent as it can be seen in Figure 1 (b) for the case of Semester 2011-1 which has a peak at GPA = 3. Similar peaks can be observed in other semesters as Figure 1 (a) shows.

3.3. Computation of the Lecturer Performance

The treatment of the lecturer as a success factor is completely tailored to the case of study and it cannot be considered as a general method, the expected (average) performance will be computed for the Grade and the Pass/Fail variables. For the computation of instructors’ performance first a segmentation process ($I_\ell : \ell \in [L]$) (as described in Subsection 3.2) has to be done. Next, the computation is subject to the following two principles

(i) Adjunct and Tenured (Track or not) lecturers are separated in different groups.

(ii) If the experience of a particular instructor (the full personal teaching log inside the database Assembled\_Data.csv) within a segment $I_\ell$ of analysis, accumulates less than 30 individuals, his/her performance within such $I_\ell$ is replaced by the average performance of the group he/she belongs to (Adjunct or Tenured) in such segment i.e., the conditional expectation of the Academic Performance Variable (APV) subject to the segment of analysis: $E(APV|\text{Instructor} = x)$, see [16, 4].

Remark 3. The separation of Adjunct and Tenured instructors is done because the working conditions, expectations, as well as the results, are significantly different from one group to the other within the Institution of analysis. In particular, the adjunct instructors are not stable nor full-time personnel. Consequently, these two groups are hardly comparable. On the other hand, there is an internal policy of rotating teaching faculty through the lower division courses, according to the needs of the School of Mathematics. Hence, due to the hiring and teaching-rotation policies, an Adjunct instructor rarely accumulates 30 or more students of experience in a profile segment $I_\ell$. 
The performance computation is described in the following pseudocode:

**Data:** Database: Assembled_Data.csv.

Analyzed Course: DC, IC, ..., NM.

Academic Performance Variable (APV): Grade, Pass/Fail

Group Segmentation \( \{l_\ell : \ell \in [L]\} \)

**Result:** Hash tables of performance for the analyzed course, conditioned to each segmentation interval \( \{l_\ell : \ell \in [L]\} \) and for the chose Academic Performance Variable (APV):

- APV_Performance_Tenured[\ell], APV_Performance_Adjoint[\ell], APV_Performance_Instructor[\ell], \ell \in [L]

initialization;

Instructors_List ← hash lecturer list from Assembled_Data.csv[Course = Analyzed Course];

Tenured_List ← choose from Instructors_List the Tenured lecturers;

AdjInstructor_List ← Instructors_List − Tenured_List;

for \( \ell \in [L] \) do
  X = hash APV field from table:
  Assembled_Data.csv[(Course = Analyzed Course) & (GPA \in l_\ell) & (Instructor \in Tenured_List)];
  APV_Performance_Tenured[\ell] ← E(X);
  X = hash APV field from table:
  Assembled_Data.csv[(Course = Analyzed Course) & (GPA \in l_\ell) & (Instructor \in AdjInstructor_List)];
  APV_Performance_Adjoint[\ell] ← E(X).
end

for \( \ell \in [L] \) do
  for instructor ∈ Instructors_List do
    X = hash APV field from table:
    Assembled_Data.csv[(Course = Analyzed Course) & (GPA \in l_\ell) & (Instructor = instructor)];
    if length of X \geq 30 then
      APV_Performance_Instructor[\ell] ← E(X);
    else
      if instructor ∈ Tenured_List then
        APV_Performance_Instructor[\ell] ← APV_Performance_Tenured[\ell]
      else
        APV_Performance_Instructor[\ell] ← APV_Performance_Adjoint[\ell]
      end
    end
  end
end

**Algorithm 2:** Computation of Instructors’ Performance

4. Core Optimization Algorithm and Historical Assessment

In this section we describe the core optimization algorithm. Essentially, it is the integration of the previous algorithms with an integer programming module whose objective function is to maximize the Expected Academic Performance Variables (Grade and Pass/Fail) according to the big data analysis described in Section 3.3. Two methods are implemented for each course and semester recorded in the database.

I. **Instructors Assignment (IA).** Assuming the groups of students are already decided, assign the instructors pursuing the optimal partnership Instructor-Conformed Group expected performance. This is known in integer programming as the Job Assignment Problem.

II. **Students Assignment (SA).** Assuming the sections (with a given capacity) and their corresponding lecturers are fixed, assign the students to the available sections in order to optimize the Student-Instructor partnership expected performance. This is the integer programming version of the Production Problem in linear optimization.
In order to properly model the integer programs we first introduce some notation

**Definition 1.** Let \( N, L, J \in \mathbb{N} \) be respectively the total number of students, \( L \) the total number of segmentation profiles and \( J \) the total number of sections. Let \( \mathbf{p} = (p_{\ell} : \ell \in [L]) \in \mathbb{N}^L \), \( \mathbf{g} = (g_j : j \in [J]) \in \mathbb{N}^J \) be respectively the population of students in each profiling segment and the capacities of each section, in particular the following sum condition holds.

\[
\sum_{\ell=1}^{L} p_{\ell} = \sum_{j=1}^{J} g_j = N. \tag{3}
\]

**Remark 4.** Observe that the condition \( \sum_{j=1}^{J} g_j = N \) implies there are no slack variables for the capacity of the sections. This is due to the study case unlike other Universities where slack capacities can be afforded.

**Definition 2.** Let \( N, L, J \in \mathbb{N} \), \( \mathbf{p} \in \mathbb{N}^L \), \( \mathbf{g} \in \mathbb{N}^J \) be as in Definition 1

(i) We say a matrix \( G \in \mathbb{R}^{L \times J} \) is a **group assignment matrix** if all its entries are non-negative integers and

\[
\sum_{j \in [J]} G(\ell, j) = p_{\ell}, \forall \ell \in [L], \sum_{\ell \in [L]} G(\ell, j) = g_j, \forall j \in [J]. \tag{4}
\]

Furthermore, define the **group assignment space** by \( G \overset{\text{def}}{=} \{ G : G \text{ is a group assignment matrix} \} \).

(ii) Let \( \{ t_j : j \in [J] \} \) be the teachers assigned to the course. For a fixed \( APV \in \{ \text{Grade, Pass/Fail} \} \) define the **expected performance matrix** \( T_{APV} \in \mathbb{R}^{J \times L} \) as the matrix whose entries \( T_{APV}(j, \ell) \) are the APV variable performance, corresponding to the instructor \( t_j \) within the segmentation interval \( l_\ell \).

(iii) Given a chosen group assignment matrix \( G \) and a chosen faculty team \( \{ t_j : j \in J \} \), define the **choice performance matrix** \( C_{APV} \) by

\[
C_{APV} \overset{\text{def}}{=} T_{APV} G. \tag{5}
\]

**Remark 5.** Notice that \( \{ G(\ell, j) : j \in [J] \} \) is a weak composition of \( p_{\ell} \) for every \( \ell \in [L] \), and \( \{ G(\ell, j) : j \in [L] \} \) is a weak composition of \( g_j \) for every \( j \in [J] \).

Recall that the expected performance matrix \( T_{APV} \) is recovered from the file \( APV, Performance, Instructor \) hash table constructed in Algorithm 2, Section 3.3.

Next we introduce the integer problems

**Problem 1 (Instructors Assignment Method).** Let \( N, L, J \in \mathbb{N} \) be as in Definition 1 let \( \xi = (\xi(i,j) : i \in [I], j \in [J]) \in \{0,1\}^{L \times J} \) and let \( C_{APV} \) be as in Definition 2 for chosen group assignment matrix \( G \), faculty team \( \{ t_j : j \in J \} \). Then, the instructors assignment problem is given by

\[
v_{\text{IA}} \overset{\text{def}}{=} \max_{\xi \in \{0,1\}^{L \times J}} \sum_{i=1}^{I} \sum_{j=1}^{J} C_{APV}(i,j) \xi(i,j), \tag{6a}
\]

subject to:

\[
\sum_{j=1}^{J} \xi(i,j) = 1, \forall j \in [J], \sum_{i=1}^{I} \xi(i,j) = 1, \forall i \in [I]. \tag{6b}
\]

**Problem 2 (Students Assignment Problem).** With the notation introduced in Definition 1 and a chosen faculty team \( \{ t_j : j \in J \} \), let \( \pi \) be a permutation in \( S_J \) such that \( t_{\pi(j)} \) is the instructor of section \( j \) for all \( j \in [J] \) i.e., a chosen assignment of lecturers to the sections. Then, the students assignment problem is given by

\[
v_{\text{SA}} = \max_{G \in \mathcal{G}} \sum_{j=1}^{J} (T_{APV} G)(j, \pi(j)) = \max_{G \in \mathcal{G}} \sum_{j=1}^{J} C_{APV}(j, \pi(j)). \tag{7}
\]
Remark 6.  

(i) Observe that the constraints of the Problem 2 are only those of Equation (4); these are fully contained in the condition $G \in G$.

(ii) Notice that although the search space of Problem 2 is significantly bigger than the search space of Problem 1 the optimum of the former need not be bigger or equal than the optimum of the latter. However, in practice in the numerical results below we observe this is the case, not because of search spaces inclusion, but due to the overwhelming difference of sizes.

In order to assess the enhancement introduced by the method, it is necessary to compute rates of optimal performance over the historical one i.e., if $G_h \in G$, $\pi_h \in S_J$ are respectively the historical group composition and instructors assignation for a given semester $h$ then, relative enhancement $\rho_m$, due to a method $m$ is given by

$$
\rho_{mt} \overset{\text{def}}{=} 100 \frac{\sum_{j=1}^{J} (T_{APV} G_h)(j, \pi_h(j))}{\sum_{j=1}^{J} (T_{APV} G_h)(j, \pi_h(j))}, \quad mt \in \{IA, SA\}. \tag{8}
$$

Finally, we describe below the optimization algorithm

**Data:** Database: Assembled Data.csv.  
Year ($y$) and Semester ($s$).  
Analyzed Course: DC, IC, ..., NM.  
Academic Performance Variable (APV): Grade, Pass/Fail.  
Group Segmentation ($I_\ell : \ell \in [L]$).  
APV_Performance_Instructor[$\ell$, $\ell \in [L]$.  
Optimization Method: $mt \in \{IA, SA\}$.  

**Result:** Relative enhancement value $\rho_{mt}$ for method $mt$, in chosen course, year and semester.

**Initialization:**

Course_Table ← hash Assembled Data.csv[(Course = Analyzed Course) & (Year = $y$) & (Semester = $s$)];

Instructors_List ← hash lecturer list from Course_Table;

Instructors_Performance ← hash APV_Performance_Instructor[Instructor ∈ Instructors_List];

Section_List ← hash section list from Course_Table

**for** $\ell \in [L]$ **do**

**for** $j \in J$ **do**

$T_{APV}(\ell, j) \leftarrow$ hash APV_Instructors_Performance[(Instructor = Instructors_List($j$)) & (Segmentation = $I_\ell$)];

$G_h(\ell, j) \leftarrow$ lengths(hash Course_Table[(Section = $j$) & (Segmentation = $I_\ell$)])

**end**

**if** $m = IA$ **then**

$C_{APV} \leftarrow T_{APV} G_h$;

$v_{IA} \leftarrow$ solve Problem 1, input: $C_{APV}$.  

else

$p \leftarrow \left( \sum_{j=1}^{J} G(\ell, j) : \ell \in [L] \right)$;

$g \leftarrow \left( \sum_{i=1}^{J} G(\ell, i) : jl \in [J] \right)$;

$\pi \leftarrow$ Section_List;

$v_{SA} \leftarrow$ solve Problem 2, input: ($T_{APV}$, $p$, $g$, $\pi$).

**end**

$\pi_h \leftarrow$ Section_List;

$\rho_{mt} \leftarrow$ compute Equation (8), input: ($T_{APV}$, $G_h$, $\pi_h$, $v_{mt}$).

**Algorithm 3:** Optimization Algorithm
4.1. Historical Assessment

In order to assess the enhancement of the proposed method with respect to the expected historical results. To that end, we merely integrate Algorithms 1, 2 and 3 in a master algorithm going through a time loop to evaluate the performance of each semester and then store the results in a table, this is done in Algorithm 4. It is important to observe that except for the database, all the remaining parameters must be by the user.

The numerical results for the Differential Calculus course are summarized in Table 6 and illustrated in Figure 2. It is clear that the Students Assignment method (SA) yields better results than the Instructor Assignment method (IA), which holds for both Academic Performance Variables: Pass/Fail and Average. Such difference happens not only for the mean value, but on every instance of the method, this is due to the difference of size between search spaces for the problems and as discussed in Remark 6. On the other hand, it can be observed that the Pass/Fail variable is considerably more sensitive to the optimization process than the Average variable.

Again, the phenomenon takes place not only for the enhancement’s mean value, the former around three times the latter, but the domination occurs for every semester analyzed by the algorithm. For an improvement on the Average variable to occur, a general improvement in the students grades should take place while the improvement of the pass rate is not as demanding.

The results of the optimization methods yield similar behavior for all the remaining lower division courses. Consequently, in the following we will only be concerned with the analysis of the Pass/Fail variable, which gives the title to the present paper. The two optimization methods will be kept for further analysis, not because of efficiencies (clearly SA yields better results), but because of the administrative limitations a Higher Education Institution could face when implementing the solution. Clearly it is way easier to implement IA than SA, which implicitly brings the scheduling problem for a large number of students.

It is also important to mention, that although enhancements of 1.4 or 7 percent may not appear significant at first sight, the benefit is substantial considering the typical enrollments displayed in Table 1 and the average Number of Tries a student needs to pass the course displayed in Table 5. In addition, the fact that Latin American public universities heavily subsidize its students despite having serious budgeting limitations, as in our study case, gives more relevance to the method’s results.

Data: Database: Assembled_Data.csv.

Analysis Course: DC, IC, ..., NM.

Academic Performance Variable (APV): Grade, Pass/Fail.

Optimization Method: \( mt \in \{ \text{IA, SA} \} \).

Result: Table of Relative Enhancement Values \( \rho_{mt} \) for chosen method, course and academic performance variable.

\[
\text{Algorithm 4: Historical Assessment Algorithm}
\]

5. Randomization and Predictive Assessment

So far, the method has been assessed with respect to the historical log i.e., comparing its optimization outcomes with those of 15 recorded semesters. The aim of the present section is to perform Monte Carlo simulations on the efficiency of the method and apply the Law of Large Numbers to estimate the expected enhancement of the algorithm; which we present below for the sake of completeness, its proof and details can be found in [4].
Figure 2: Example: Differential Calculus course. Figure (a) shows the relative enhancement results $\rho_{mt}$ for the Instructor Assignment (IA) and Students Assignment (SA) optimization methods when considering the Pass/Fail variable. Figure (b) shows the relative enhancement results $\rho_{mt}$ for the Instructor Assignment (IA) and Students Assignment (SA) optimization methods when considering the Average variable.

Table 6: Historical Relative Enhancements, $\rho_{mt}$, Course: Differential Calculus

| Academic Semester | $APV = Pass/Fail$ | $APV = Average$ |
|-------------------|------------------|-----------------|
|                   | $mt = IA$ | $mt = SA$ | $mt = IA$ | $mt = SA$ |
| 2010-1            | 2.0482   | 5.8891    | 0.7868   | 1.8393   |
| 2010-3            | 1.4164   | 6.4276    | 0.3392   | 1.4353   |
| 2011-1            | 1.4990   | 8.6794    | 0.7126   | 2.7954   |
| 2011-3            | 0.3495   | 4.9708    | 0.3484   | 2.2110   |
| 2012-1            | 0.8575   | 6.4689    | 0.5355   | 2.2256   |
| 2012-3            | 0.9127   | 6.7779    | 0.5217   | 2.1541   |
| 2013-1            | 0.9090   | 5.8924    | 0.4230   | 2.0870   |
| 2013-3            | 1.7720   | 7.2932    | 0.5094   | 2.4379   |
| 2014-1            | 1.7127   | 8.3662    | 0.7740   | 2.5629   |
| 2014-3            | 1.0070   | 6.1735    | 0.5282   | 2.0759   |
| 2015-1            | 0.8617   | 5.4224    | 0.2977   | 1.6277   |
| 2015-3            | 1.4383   | 7.0959    | 0.6095   | 2.0165   |
| 2016-1            | 2.0939   | 8.2952    | 0.4391   | 2.1050   |
| 2016-3            | 1.8961   | 9.6391    | 0.7828   | 2.4718   |
| 2017-1            | 1.9424   | 8.2568    | 0.6440   | 2.3300   |
| Mean              | 1.3811   | 7.0432    | 0.5501   | 2.1584   |

Theorem 1 (Law of Large Numbers). Let $(Z^{(n)} : n \in \mathbb{N})$ be a sequence of independent, identically distributed
random variables with expectation $\mu = \mathbb{E}(Z^{(1)})$, then

$$\Pr \left\{ \frac{Z^{(1)} + Z^{(2)} + \ldots + Z^{(n)}}{n} - \mu > 0 \right\} \xrightarrow{n \to \infty} 0,$$

i.e., the sequence $(Z^{(n)} : n \in \mathbb{N})$ converges to $\mu$ in the Cesàro sense.

In order to achieve Monte Carlo simulations, we first randomize in Section 5.1 several factors/variables which define the setting of a semester for each course. Next we discuss in Section 5.2 normalization criteria, to make the enhancement simulations comparable. Finally, we present in Section 5.3 the Monte Carlo simulations results for both the random variable simulating the benefits of the method $(Z^{(n)} : n \in \mathbb{N})$ as well as the evolution of its Cesàro means $(\frac{Z^{(1)} + Z^{(2)} + \ldots + Z^{(n)}}{n})$ in Theorem 1 to determine the asymptotic performance of the proposed algorithm.

Throughout this section we adopt a notational convention, the label RandInputAlgorithm will refer to the random versions of the respective algorithm developed in the previous sections. For instance RandInputAlgorithm input: (Group Assignment Matrix $G$, List of Lecturers $\mathcal{L}$, Analyzed Course, APV, Group Segmentation ($\ell : \ell \in [L]$, $mt$) refers to Algorithm 3 above, but with a different set of input data; for clarity the randomly generated input data are underlined. This notation is introduced for exposition brevity: avoiding to write an algorithm whose logic is basically identical to its deterministic version.

5.1. Randomization of Variables

Four factors will be randomized in the same fashion: Number of Tenured Lecturers, Number of Enrolled Students, List of Students’ GPA and Number of Groups. First we randomize the scalar integer-valued statistical variables using by merely computing 95 percent confidence intervals from the empirical data and then assuming the impact of the factor can be modeled by a random variable uniformly distributed on such confidence interval, see [16].

Definition 3. (i) Let $x$ be a real-valued statistical variable with mean $\bar{x}$, standard deviation $\sigma$ and $n$ its sample size then, its 95 percent confidence interval is given by

$$I_x \overset{\text{def}}{=} \left[ \bar{x} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}} \right].$$

(ii) Let $x$ be a scalar integer-valued statistical variable with mean $\bar{x}$, standard deviation $\sigma$ and $n$ its sample size. Then, its 95 percent confidence interval is given by

$$I_x \overset{\text{def}}{=} \left[ \left\lfloor \bar{x} - 1.96 \frac{\sigma}{\sqrt{n}} \right\rfloor, \left\lceil \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}} \right\rceil \right] \cap \mathbb{Z},$$

where $\lfloor \cdot \rceil, \lceil \cdot \rceil : \mathbb{R} \to \mathbb{R}$ denote the floor and ceiling functions respectively.

The randomization of the statistical variables listed above heavily uses the empirical distributions mined from the database.

Hypothesis 1. (i) Let $x$ be a scalar statistical variable, then its associated random variable $X$ is uniformly distributed on its confidence interval $I_x$, i.e. $X \sim \text{Unif}(I_x)$, with the confidence interval is defined by (11) or (10) depending on whether the variable $x$ is integer or real valued.

(ii) Let $x = (x_1, \ldots, x_d) \in \mathbb{R}^d$ be a vector statistical variable, then its associated random variable is given by $X = (X_1, \ldots, X_d)$, where $X_i$ is the random variable associated to $x_i$ for all $i \in [d]$ as defined above.

From here, it is not hard to compute the confidence intervals (or ranges) of the random variables as it is shown in Tables 7 and 8. In contrast, the Sections and the GPA variables will need further considerations in its treatment.

The Sections variable is a list of several sections with different capacities. Statistical scan of the data shows that this list is a most unpredictable statistical variable where section capacities range from 15 to 150 with very low relative frequencies in each of its values. Consequently, it was decided to group the section capacities in integer intervals
Table 7: Random Variable: Number of Tenured Instructors NT, Course: All

| PARAMETERS | DC | IC | VC | VAG | LA | ODE | BM | NM |
|------------|----|----|----|-----|----|-----|----|----|
| Upper Extreme | 8  | 5  | 3  | 7   | 5  | 4   | 4  | 2  |
| Lower Extreme | 6  | 3  | 2  | 4   | 3  | 3   | 2  | 1  |
| Average     | 7.2667 | 4.0667 | 2.6000 | 5.1333 | 3.7333 | 3.5333 | 3.3333 | 1.6667 |
| Standard Deviation | 0.7037 | 1.1629 | 0.7368 | 1.9952 | 1.2228 | 0.9155 | 1.0465 | 0.6172 |

Table 8: Random Variable: Number of Enrolled Students NE, Course: All

| PARAMETERS | DC | IC | VC | VAG | LA | ODE | BM | NM |
|------------|----|----|----|-----|----|-----|----|----|
| Upper Extreme | 1554 | 1203 | 586 | 1243 | 1045 | 882 | 974 | 301 |
| Lower Extreme | 1337 | 1043 | 513 | 1050 | 932 | 721 | 837 | 234 |
| Average     | 1445.9333 | 1122.8667 | 549.8000 | 1146.5333 | 988.6667 | 801.8000 | 905.4000 | 267.5333 |
| Standard Deviation | 213.4446 | 156.8642 | 70.9969 | 188.8304 | 110.7229 | 158.4433 | 134.2971 | 65.4847 |

Definition 4. Given the list of integer intervals
\[ I \] defined as
\[ \{ [15, 30], [31, 45], [46, 60], [61, 75], [76, 90], [91, 105], [106, 120], [121, 135], [136, 150] \}, \quad (12) \]
for each semester and for each course, the sections frequency variable is given by
\[ sf = \left( \frac{\sum ns_i^k}{\sum ns_k} : K \in I \right), \quad (13) \]
where \( ns_i^k \) is the number of sections whose capacity belongs to the interval \( I \in I \).

Hypothesis 2. The Sections variable \( S \) is completely defined by the number of groups variable \( NS \) in the following way
\[ S = \lceil NS \overline{sf} \rceil. \quad (14) \]
Here \( \overline{sf} \) is the average vector of \( sf \) introduced in Equation \( (13) \) and it is understood that the ceiling function \( \lceil \cdot \rceil \) applies to each component of the vector.

Finally, the GPA variable is treated as follows

Hypothesis 3. For each semester define \( \overline{x} = \left( x_i : i \in [50] \right) \), where \( x_i \) is the relative frequency of registering students whose GPA is equal to \( i \). In particular, \( \sum_{i \in [50]} x_i = 1 \). Let \( X_{GPA} \) be the associated random variable to the list of relative frequencies \( x \), as introduced in Hypothesis \( (1) \). Then, the GPA random variable \( GPA \) is given by
\[ GPA = \lceil NE \overline{X_{GPA}} \rceil, \quad (15) \]
where \( NE \) is the number of enrolled students random variable and it is understood that the ceiling function \( \lceil \cdot \rceil \) applies to each component of the vector.
Remark 7. Notice that both random variables $S$ and GPA are the product of a scalar and a vector. However, for $S$ the scalar is a random variable $NS$ and the vector $sf$ is deterministic, while in the case of GPA the scalar $NE$ and the vector $X_{GPA}$ are random variables. There relies the difference in the randomization of the vector variables.

So far $S$ is producing a list of sections whose capacity lies within the ranges declared in $I$ (12), this introduces a set of slacks which will be used later on to match the number of enrolled students $NE$ with the total sections capacity. The match will be done in several steps, once the equality $\sum_{K \in I} s_K = NE$ is attained, a group assignment matrix $G$ (as in Definition 8) will be generated randomly.

I. Solve the following Data Fitting Problem (see [3] for its solution)

**Problem 3.** Given two realizations of $S = (s^{(1)}_K : K \in I)$ and $NE$, consider the integer problem

$$df^{(1)} = \min \left\{ \left| \sum_{K \in I} \sum_{i = 1}^{s^{(1)}_K} x^{(1)}_{K,i} - NE \right| : x^{(1)}_{K,i} \in K, \text{for all } K \in I \right\}. \quad (16)$$

II. If $\min_{K \in I} s^{(1)}_K \leq df^{(1)} \leq \max_{K \in I} s^{(1)}_K$, increase or decrease number of sections $s^{(1)}_K \mapsto s^{(2)}_K$ using a Greedy Algorithm from the large sections to the small ones and get

$$df^{(2)} \equiv \left| \sum_{K \in I} \sum_{i = 1}^{s^{(2)}_K} x^{(1)}_{K,i} - NE \right| < df^{(1)}. \quad (17)$$

III. If $0 < df^{(2)} < \min_{K \in I} s^{(i)}_K$ ($i = 1$ or $i = 2$ depending on Greedy Algorithm III is applied or not), apply increase/decrease capacities Algorithm (breaking the constraints (??)), firstly changing $x^{(1)}_{K,i} \mapsto x^{(2)}_{K,i}$ as evenly as possible, secondly distributing the reminder in randomly chosen sections $x^{(2)}_{K,i} \mapsto x^{(3)}_{K,i}$ and get

$$df^{(3)} \equiv \left| \sum_{K \in I} \sum_{i = 1}^{s^{(3)}_K} x^{(3)}_{K,i} - NE \right| \equiv 0 < df^{(2)}. \quad (18)$$

IV. Generate randomly a group assignment matrix $G$. 

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The random setting described above is summarized in the following pseudocode:

**Data:** NE random variable distribution, \( X_{GPA} \) random variable distribution
NS random variable distribution, average sections frequency \( sf \) variable
Analyzed Course: DC, IC, ..., NM.
List of Tenured Lecturers

**Result:** Random Group Assignment Matrix \( G \).

**Initialization:**
compute a realization of \( NE \) and a realization for \( X_{GPA} \);
compute \( GPA \), input: \((NE, X_{GPA})\);
call RandInputAlgorithm input: \((GPA\ list)\);
compute \( S \), input: \((NS, sf, Analyzed\ Course)\);
solve Problem input: \((S, NE)\);
if \( df^{(1)} > 0 \) then
    run the Greedy Algorithm;
    if \( df^{(2)} > 0 \) then
        run the increase/decrease capacities algorithm:
        \( S \leftarrow (x_{K,i}^{(3)} : i \in [s_{K}^{(2)}], K \in I) \);
        return \( S \);
    else
        \( S \leftarrow (x_{K,i}^{(1)} : i \in [s_{K}^{(2)}], K \in I) \);
        return \( S \);
    end
else
    \( S \leftarrow (x_{K,i}^{(1)} : i \in [s_{K}^{(1)}], K \in I) \);
end
compute a random group matrix assignment \( G \), input: \((S, GPA)\);

**Algorithm 5:** Random Setting Algorithm

We close this section displaying three tables. Table 9 contains the confidence intervals for the number of sections random variable NS. The capacities distribution vector \( sf \), as well as the confidence intervals of \( X_{GPA} \) are shown in Table 10 for the course of Differential Calculus only; due to the \( GPA \) range length the table has been split in five rows to fit the page format. Finally, Table 12 presents an example of a group assignment matrix \( G \) produced by Algorithm 5.

### 5.2. Normalization of the method and probabilistic spaces

In order to measure the proposed method’s enhancement now there is need to normalize the results as in the case of the historical assessment of the algorithm, Section 4, Equation (8) where the improvement in the academic performance variable was divided over the historical performance of the semester at hand. In the case of Monte Carlo simulations, the concept of “historical performance” simply does not apply as the assignment of students and/or lecturers actually did not happen. We approach this fact in two different ways:

(a) **Random Normalization.** Normalize against a random assignation of instructors or students, depending on the method IA or SA respectively.

(b) **Expected Normalization.** Normalize against the *expected assignation* of instructors or students, depending on the method IA or SA respectively.

In the first case, it is straightforward to compute the normalization, in the second case, the concept of *expected assignation* needs to be stated in neater terms. To that end, we need to present some intermediate mathematical results and definitions.
Table 9: Random Variable Number of Sections NS and Capacities Distribution Vector $\vec{r}$, Course: All

| Parameters | DC | IC | VC | VAG | LA | ODE | BM | NM |
|------------|----|----|----|-----|----|-----|----|----|
| Upper Extreme | 20 | 11 | 5  | 17  | 10 | 7   | 17 | 3  |
| Lower Extreme | 16 | 9  | 3  | 15  | 8  | 5   | 13 | 1  |

Table 10: Random Variable: $X_{\text{GPA}}$, Course: Differential Calculus

| INTERVALS GPA | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Upper Extreme | 0.0021 | 0.0027 | 0.0031 | 0.0031 | 0.0035 | 0.0031 | 0.0044 | 0.0040 | 0.0047 | 0.0046 |
| Lower Extreme | 0.0012 | 0.0013 | 0.0017 | 0.0024 | 0.0016 | 0.0031 | 0.0021 | 0.0020 | 0.0028 | |

Theorem 2. Let $K \in \mathbb{N}$ be fixed and $(T(i, k) : i, k \in [K])$ be a matrix. Define the Random Variable

$$X_{IA} : S(K) \rightarrow \mathbb{R}, \quad X_{IA}(\sigma) \overset{\text{def}}{=} \sum_{k \in [K]} T(k, \sigma(k)).$$
Then,
\[
\mathbb{E}(X_{IA}) = \frac{1}{K} \sum_{i,k \in [K]} T(j, k) = \frac{1}{K} \text{sum}(T),
\] (19)

where \( \text{sum}(T) \) is defined as
\[
\sum_{(k,i) \in [K] \times [K]} T(k,i) = \sum_{k \in [K]} \sum_{i \in [K]} T(k,i).
\]

\textbf{Proof.} Consider the following calculation
\[
\mathbb{E}(X_{IA}) = \frac{1}{K!} \sum_{\sigma \in S(K)} X_{IA}(\sigma) = \frac{1}{K!} \sum_{\sigma \in S(K)} \sum_{k \in [K]} T(k, \sigma(k))
\]
\[
= \frac{1}{K!} \sum_{k \in [K]} \sum_{\sigma \in S(k)} T(k, \sigma(k))
\]
\[
= \frac{1}{K!} \sum_{k \in [K]} \sum_{i \in [K]} \sum_{\sigma \in S(J)} T(k, \sigma(k))
\]
\[
= \frac{1}{K!} \sum_{k \in [K]} \sum_{i \in [K]} T(k, i) \sum_{\sigma \in S(n)} 1
\]
\[
= \frac{(K - 1)!}{K!} \sum_{k \in [K]} \sum_{i \in [K]} T(k, i).
\]

From here, Equation (19) follows trivially.

\textbf{Remark 8.} Notice that in Proposition the following holds

(i) It is understood that the probabilistic is \( \Omega \equiv S_K \) where all the outcomes are equally likely.

(ii) Assuming the setting of a semester is given, namely: number of sections with capacities, conformation of sections and a set of instructors teaching the course. Then \( T = C_{APV} \) and \( J \) is the number of sections \( \mathbb{E}(X_{IA}) \) is the expected performance when assigning instructors randomly to the available sections with defined students i.e., the IA method.

Our next step is to be able to compute the expected performance of a group when assigning students randomly to available sections with defined instructors. This task is far more complicated due to the richness of the search space. We begin introducing some notation.

\textbf{Definition 5.} Let \( N, L, J \in \mathbb{N}, p \in \mathbb{N}^L, g \in \mathbb{N}^J \) be as in Definition 1.

(i) Let \( c : [N] \to [L] \) be the classification function of each student i.e., for each student \( n \in [N] \) it assigns the label \( c(n) \in [L] \) describing the profile to which he/she belongs to.

(ii) Define the \textbf{student assignation} probabilistic space by
\[
\Omega = \left\{ \omega : [N] \to [J] : |\omega^{-1}(j)| = g_j, \text{ for all } j \in [J] \right\}.
\]

(iii) For a fixed element \( \omega \in \Omega \), define the matrix \( G^\omega \in \mathbb{R}^{L \times J} \) whose entries are given by
\[
G^\omega(\ell, j) = \left| \left\{ n \in [N] : c(n) = p_\ell, \omega(n) = g_j \right\} \right| = |c^{-1}(\ell) \cap \omega^{-1}(j)|,
\]
\( \forall \ell \in [L], j \in [J] \).

\textbf{Remark 9.} In Definition notice the following
(i) The student classification function satisfies that $p_\ell \overset{\text{def}}{=} \left| c^{-1}(\ell) \right|$ for all $\ell \in [L]$.

(ii) An element $\omega$ of the student assignation space is such that every individual is assigned to a section and every section is full (recall the sum condition (3)).

(iii) In our study, a list of $N$ enrolled students is completely characterized a classification function $c : [N] \rightarrow [L]$ and a section assignment function $\omega \in \Omega$

\[ \begin{array}{cccc}
1, & 2, & \ldots, & N, \\
c(1), & c(2), & \ldots, & c(N), \\
\omega(1), & \omega(2), & \ldots, & \omega(N).
\end{array} \]

The first row represents identity and the second indicates profile classification, therefore, only the third row is subject to decision or randomization as it is done in this model.

(iv) For every $\omega \in \Omega$ the matrix $G^\omega$ is clearly a group assignment matrix as introduced in Definition 2.

**Proposition 3.** Let $N, L, J \in \mathbb{N}^\ell, p \in \mathbb{N}^J, g \in \mathbb{N}^J$ be as in Definition 1. Then

\[ \text{trace} \left( T_{APV} G^\omega \right) = \sum_{j=1}^{J} (T_{APV} G^\omega)(j,j) = \sum_{n \in [N]} T_{APV}(\omega(n), c(n)) \] 

for all $\omega \in \Omega$. (21)

**Proof.** Consider the following identities

\[ \sum_{n \in [N]} T_{APV}(\omega(n), c(n)) = \sum_{(\ell, j) \in [L] \times [J]} \sum_{c(n) = \ell, \omega(n) = j}^{N} T_{APV}(\omega(n), c(n)) = \sum_{j \in [J]} \sum_{\ell \in [L]} T_{APV}(j, \ell) \sum_{c(n) = \ell, \omega(n) = j}^{N} 1 \]

\[ = \sum_{j \in [J]} \sum_{\ell \in [L]} T_{APV}(j, \ell) G^\omega(\ell, j) \]

\[ = \sum_{j \in [J]} (T_{APV} G^\omega)(j,j). \]

i.e., the result holds.

**Remark 10.** Observe that if it is assumed that the instructor $t_j$ is assigned to the section $j$ for all $j \in J$, i.e., the instructor assignment function $\pi \in S_J$ of Problem 2 is the identity then, the previous result states that

\[ \text{trace} \left( T_{APV} G^\omega \right) = \sum_{j=1}^{J} (T_{APV} G^\omega)(j, \pi(j)) = \sum_{n \in [N]} T_{APV}(\omega(n), c(n)), \] 

for each $\omega \in \Omega$. Since the expression measures the middle is the global performance of the group, so does the right hand side. Hence, it makes sense to declare the left hand side in the expression above as a random variable.

**Definition 6.** Let $N, L, J \in \mathbb{N}^\ell, p \in \mathbb{N}^J, g \in \mathbb{N}^J$ be as in Definition 1 and let $T \in \mathbb{R}^{J \times L} = (T(j, \ell) : k \in [J], \ell \in [L]),$ be a fixed matrix. Define the **student assignment performance** random variable

\[ X_{SA} : \Omega \rightarrow \mathbb{R}, \quad X_{SA}(\omega) \overset{\text{def}}{=} \sum_{n \in [N]} T(\omega(n), c(n)). \]
Before computing the expectation of the random variable $X$ some previous results from combinatorics are needed.

**Lemma 4.**  
(i) The cardinal of the student assignment space is given by

$$|\Omega| = N! \prod_{j=1}^{J} \frac{1}{g_j!}. \quad (23)$$

(ii) Let $n \in [N]$, $j \in [J]$ be fixed, and define the set

$$\Omega_{n,j} \overset{\text{def}}{=} \{ \omega \in \Omega : \omega(n) = j \}.$$ 

Then $|\Omega_{n,j}| = \frac{(N-1)!}{(g_j - 1)!} \prod_{i \in [K] \setminus \{j\}} \frac{1}{g_i!}$.

**Proof.**  
(i) Let $\omega$ be an element of $\Omega$ and write it in the extended way, i.e,

$$\omega(1), \omega(2), \ldots, \omega(N), 1, 2, \ldots, N.$$ 

Clearly, $\omega$ is a permutation of the multiset

$$\{1, 1, \ldots, 1, 2, 2, \ldots, 2, \ldots, J, J, \ldots, J\} = \{1 \cdot g_1, 2 \cdot g_2, \ldots, J \cdot g_J\}. \quad (24)$$

From elementary combinatorics, it is known that the number of permutations of the multiset $\overset{\text{def}}{=} \{1 \cdot g_1, 2 \cdot g_2, \ldots, J \cdot g_J\}$ is given by the expression $\overset{\text{def}}{=} \{1 \cdot g_1, 2 \cdot g_2, \ldots, J \cdot g_J\}$, see Theorem 3.5 in [5].

(ii) First we consider the set $\Omega_{N,j}$. Recalling the expression $\overset{\text{def}}{=} \{ \omega : [N] \rightarrow [J] : |\omega^{-1}(i)| = g_i, \omega(N) = j, \text{for all } i \in [J] \}$, it is direct to see there is bijection with the set

$$\tilde{\Omega} \overset{\text{def}}{=} \left\{ \omega : [N - 1] \rightarrow [J] : |\omega^{-1}(i)| = \tilde{g}_i, \text{for all } i \in [J] \right\} \text{ where } \tilde{g}_i \text{ is defined as follows}.$$ 

$$\tilde{g}_i \overset{\text{def}}{=} \begin{cases} g_i, & i \neq j, \\ g_i - 1, & j = i. \end{cases}$$

Applying the previous part on the set $\tilde{\Omega}$ it follows that $\Omega_{N,j}$ satisfies the result. For the general case $\Omega_{n,j}$ take the permutation $\sigma \in S_N$ defined by

$$\sigma(k) \overset{\text{def}}{=} \begin{cases} N, & k = n, \\ n, & k = N, \\ k, & \text{otherwise.} \end{cases}$$

Observe that the map $\varphi : \Omega_{n,j} \rightarrow \Omega_{N,j}$ defined by $\varphi(\omega) \overset{\text{def}}{=} \omega \circ \sigma$ is clearly a bijection. Consequently, $|\Omega_{n,j}| = |\Omega_{N,j}|$ and the proof is complete.

**Theorem 5.** The expectation of the random variable $X_{SA}$ is given by

$$\mathbb{E}(X_{SA}) = \frac{1}{N} g^T p. \quad (25)$$
Proof. By definition

\[|\Omega| \mathbb{E}(X_{\text{SA}}) = \sum_{\omega \in \Omega} \sum_{n=1}^{N} T(\omega(n), c(n)) = \sum_{\omega \in \Omega} \sum_{n=1}^{N} T(\omega(n), c(n)) = \sum_{\omega \in \Omega} \sum_{n=1}^{N} \sum_{j=1}^{J} T(\omega(n), c(n)) = \sum_{\omega \in \Omega} \sum_{n=1}^{N} \sum_{j=1}^{J} 1 \]

Recalling Lemma 4 (ii), it follows

\[\mathbb{E}(X_{\text{SA}}) = \frac{1}{|\Omega|} \sum_{n=1}^{N} \sum_{j=1}^{J} T(j, c(n)) = \frac{1}{N} \sum_{j=1}^{J} \sum_{n=1}^{N} g_j T(j, c(n)) = \frac{1}{N} \sum_{j=1}^{J} \sum_{n=1}^{N} \sum_{\ell=1}^{L} T(j, \ell) \rho_{\ell j} = \frac{1}{N} \sum_{\ell=1}^{L} \sum_{j=1}^{J} T(j, \ell) g_j.\]

From here, the result follows trivially.

Remark 11. Let \(\pi \in S_J\) be a permutation and \(A^\pi\) its associated permutation matrix

\[A^\pi = [\hat{e}_{\pi(1)}, \hat{e}_{\pi(2)}, \ldots, \hat{e}_{\pi(J)}],\]

where \((\hat{e}_j : j \in [J])\) is the canonical basis of \(\mathbb{R}^J\). Then, if the instructors \(\{t_j : j \in [J]\}\) are assigned to their corresponding sections by a permutation \(\pi \in S_J\) other than the identity by taking

\[T \overset{\text{def}}{=} T_{\text{APV}} A^\pi,\]

\(X(\omega)\) computes the global performance of the group for each \(\omega \in \Omega\), as in Remark 10. Therefore, without loss of generality, it can be assumed that \(\pi \in S_J\) is the identity.

Finally we define

Definition 7. The random version of Algorithm 3 will have two methods.

(i) The Random Normalization method described in (i) defined in Equation (8). However, it is important to observe that this time \(v_{mt}, \rho_{mt}\) and \(X_{mt} \overset{\text{def}}{=} \sum_{j=1}^{J} (T_{\text{APV}} G_h)(j, \pi_n(j))\) are all random variables.

(ii) Second, the Expected Normalization method introduced in (ii) which is computed using

\[\gamma_{mt} \overset{\text{def}}{=} \frac{100 \cdot v_{mt}}{\mathbb{E}(X_{mt})}, \quad m_t \in \{IA, SA\}.\]

Here \(\mathbb{E}(X_{mt})\) is given by Theorem 3 if \(mt = IA\) and by Theorem 5 if \(mt = SA\). Again, \(v_{mt}\) and \(\gamma_{mt}\) are both random variables.
Remark 12. (i) It is understood that for the application of the Law of Large Numbers in the numerical experiments, the random variables above will be considered as sequences of independent identically distributed i.e. \((v^{(n)}_{mt} : n \in \mathbb{N}), (X^{(n)}_{mt} : n \in \mathbb{N}), (\rho^{(n)}_{mt} : n \in \mathbb{N})\) and \((\gamma^{(n)}_{mt} : n \in \mathbb{N})\), where the index \(n\) indicates an iteration of the Monte Carlo simulation.

(ii) It is direct to see that \((\gamma^{(n)}_{mt} : n \in \mathbb{N})\) converges in the Cesaro sense to \(\mathbb{E}(v^{(1)}_{mt}) \left( \mathbb{E}(X^{(n)}_{mt}) \right)^{-1} - 1\).

(iii) If we define \(Z^{(n)}_{mt} \equiv \frac{1}{X^{(n)}_{mt}}\), since \((v^{(n)}_{mt} : n \in \mathbb{N})\) and \((X^{(n)}_{mt} : n \in \mathbb{N})\) are independent, it holds that

\[
\rho^{(n)}_{mt} = \frac{V^{(n)}_{mt} - X^{(n)}_{mt}}{X^{(n)}_{mt}} = \frac{v^{(n)}_{mt} - X^{(n)}_{mt}}{X^{(n)}_{mt}} - 1 = v^{(n)}_{mt} X^{(n)}_{mt} - 1 \xrightarrow{\text{Cesaro}} \mathbb{E}(v^{(1)}_{mt}) \mathbb{E}(Z^{(n)}_{mt}) - 1 = \mathbb{E}(v^{(1)}_{mt}) \mathbb{E}\left(\frac{1}{X^{(n)}_{mt}}\right) - 1. \quad (29)
\]

The right hand side of the expression above has the reciprocal of the harmonic mean of the variable \((X^{(n)}_{mt} : n \in \mathbb{N})\). Clearly, \((\gamma^{(n)}_{mt} : n \in \mathbb{N})\) and \((\rho^{(n)}_{mt} : n \in \mathbb{N})\) converge (in the Cesaro sense) to different limits. Unfortunately, the harmonic mean has no simple expression equivalent to that of Equation (25) for the arithmetic mean. Consequently it can be handled only numerically in the next section.

5.3. The Monte Carlo Simulation Algorithm and Numerical Results

The randomization of the variables as well as its normalization discussed in the sections 5.1 and 5.2 respectively are summarized in the pseudocode below. A particular example of the Monte Carlo simulation results are depicted in Figure 3; the corresponding body/composition of enrolled students displayed presented in Table 12.

The results of several simulations for the Differential Calculus course are summarized in Table 11, out several experiments it is observed that a reasonable level of convergence of the Cesaro means is attained above 800 iterations. Given that we are simulating the behavior of a highly complex random process, it is clear that no convergence rate can be actually concluded, the threshold for which the Cesaro mean stabilizes shifts significantly from one experiment to the other. This is because every experiment defines a number of sections NS, an enrollment body/composition of students as in Table 12, group matrix assignment G and a number of tenured lecturers NT, from here, the iteration process begins as it is shown in Algorithm 6. Therefore, the starting triple (NS, G, NT) changes greatly between simulations as it can be seen in Table 11 not to mention what happens when shifting from one course to the other as it is the case of Table 13, reporting the algorithm’s performance for all the remaining seven service courses.

It is also important to observe that difference between Random vs. Expected normalization methods is not significant in the simulation of the method’s performance (see Figure 3 (a) and (b)) and it is negligible in the behavior of their corresponding Cesaro means i.e., regardless of the chosen normalization method (see figure Figure 3 (c) and (d)) the asymptotic behavior difference negligible at least from the numerical point of view. The latter can be also observed on the Tables 11 and 13.
Table 11: Monte Carlo Simulations Summary: 10 Experiments, 800 iterations each, Course: Differential Calculus

| Experiment Number | Random Normalization, $\rho_{mt}$ | Expected Normalization, $\gamma_{mt}$ | Enrollments | Sections | Lecturers |
|-------------------|-----------------------------------|--------------------------------------|--------------|-----------|-----------|
|                   | $100 \times \frac{V_{IA}}{\rho_{IA}}$ | $100 \times \frac{V_{SA}}{\rho_{SA}}$ | $100 \times \frac{V_{IA}}{\gamma_{IA}}$ | $100 \times \frac{V_{SA}}{\gamma_{SA}}$ | $\sum_{K \in I} s_{K}$ | NT |
| 1                 | 0.3097                            | 2.9059                               | 0.3056       | 2.9044    | 1355      | 15  | 6  |
| 2                 | 0.4595                            | 3.2880                               | 0.4588       | 3.2854    | 1445      | 14  | 8  |
| 3                 | 0.4373                            | 3.2655                               | 0.4414       | 3.2653    | 1225      | 14  | 7  |
| 4                 | 0.4158                            | 3.2689                               | 0.4130       | 3.2663    | 1456      | 15  | 7  |
| 5                 | 0.4357                            | 2.9680                               | 0.4315       | 2.9651    | 1296      | 14  | 6  |
| 6                 | 0.4943                            | 3.1690                               | 0.5053       | 3.1697    | 1547      | 15  | 8  |
| 7                 | 0.5099                            | 3.4486                               | 0.5008       | 3.4439    | 1556      | 16  | 8  |
| 8                 | 0.4937                            | 3.3720                               | 0.4841       | 3.3666    | 1532      | 16  | 8  |
| 9                 | 0.4080                            | 3.1254                               | 0.4009       | 3.1301    | 1444      | 15  | 7  |
| 10                | 0.4843                            | 3.4498                               | 0.4807       | 3.4454    | 1546      | 16  | 8  |
| Mean              | 0.4448                            | 3.2261                               | 0.4422       | 3.2242    | 1440.2    | 15.0| 7.3|

**Data:** Database: `Assembled_Data.csv` Analyzed Course: DC, IC, ..., NM.
Optimization Method: $mt \in \{IA, SA\}$.
NT random variable distribution
Numer of Iterations: $NI$

**Result:** Table of Relative Enhancement Values $\rho_{mt}$, $\gamma_{mt}$ for chosen method, course and academic performance variable.

initialization;
call Algorithm 5;
$nt \leftarrow \text{compute a realization of NT}$;
call Algorithm 2 input: (Assembled Data.csv, Analyzed Course, APV, Group Segmentation ($I_{\ell}$ : $\ell \in [L]$));
for $iteration \in [NI]$ do
  list $\leftarrow \text{compute a random list of nt-lecturers}$;
call RandInputAlgorithm 3 input: (Group Assignment Matrix $G$, List of Lecturers $L_{\text{list}}$, Analyzed Course, APV, Group Segmentation ($I_{\ell}$ : $\ell \in [L]$), $mt$);
  $APV_{mt, \text{Assessment}[iteration]} \leftarrow [\rho_{mt}, \gamma_{mt}]$.
end

Algorithm 6: Monte Carlo Simulation Algorithm

6. Conclusions and Future Work

The present work delivers several conclusions.

(i) A big data based algorithm has been implemented aimed to increase the academic performance for massive university lower division courses in mathematics. It is based on integer programming and big data analysis to compute the associated cost functions, while the constraints (such as number of sections and corresponding capacities) are defined by administrative sources. The integer programs come from two mechanisms: assign instructors optimally (IA method) or assign students optimally (SA method).

(ii) The academic performance is explored using two measures; Pass Rate and Grade. After correlation analysis of the data, it is determined that the one relevant factor, known at the time when the semester begins
and incident on these statistical variables is the GPA. Consequently the profiling of students as well as the student body composition is defined in terms of the GPA (see Table 6).

(iii) The historical assessment of the algorithm yield poor relative enhancements for the Grade variable due to its typical statistical robustness. However, the Pass Rate yields more satisfactory results; good enough to pursue a deeper analysis such as the algorithm’s randomization and its asymptotic assessment, presented in Section 5.

(iv) The asymptotic analysis of the algorithm is done by randomizing the enrollment population and the administrative factors, statistically based on the empirical observations reported in the database Assembled_Data.csv. The Monte Carlo experiments establish that the method does not deliver a fixed value of relative enhancement, it depends on the starting parameters (NS, G, NT) whose remarkable randomness inherit uncertainty to the algorithm’s output values.

(v) Computing a weighted average across the courses by crossing the Tables 8 and 13 gives a rough estimate of 3.3 percent full scale benefit, if the students assignment method (IA) is implemented. This is approximately
Table 12: Example, random realization of Algorithm 5, Course: Differential Calculus

| SECTION | [0.2, 2.2] | (2.2, 2.7] | (2.7, 3.0] | (3.0, 3.1] | (3.1, 3.3] | (3.3, 3.5] | (3.5, 3.7] | (3.7, 3.8] | (3.8, 4.1] | (4.1, 5.0] | Total |
|---------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------|
| 1       | 6          | 1          | 15         | 5          | 13         | 8          | 8          | 4          | 8          | 6          | 74    |
| 2       | 10         | 6          | 14         | 3          | 5          | 7          | 11         | 5          | 8          | 5          | 74    |
| 3       | 9          | 9          | 9          | 8          | 6          | 3          | 9          | 1          | 15         | 5          | 74    |
| 4       | 11         | 12         | 13         | 1          | 5          | 3          | 8          | 7          | 8          | 6          | 74    |
| 5       | 5          | 9          | 12         | 7          | 11         | 6          | 9          | 2          | 5          | 9          | 75    |
| 6       | 5          | 9          | 7          | 6          | 3          | 13         | 11         | 11         | 1          | 7          | 6     |
| 7       | 12         | 5          | 7          | 2          | 7          | 10         | 12         | 6          | 21         | 7          | 89    |
| 8       | 7          | 11         | 11         | 2          | 6          | 12         | 14         | 8          | 9          | 9          | 89    |
| 9       | 11         | 18         | 14         | 7          | 6          | 12         | 10         | 7          | 10         | 9          | 104   |
| 10      | 14         | 8          | 20         | 4          | 15         | 17         | 14         | 0          | 13         | 14         | 119   |
| 11      | 7          | 11         | 14         | 6          | 13         | 21         | 14         | 4          | 18         | 11         | 119   |
| 12      | 15         | 15         | 9          | 15         | 16         | 11         | 3          | 11         | 9          | 119     |
| 13      | 14         | 14         | 20         | 5          | 17         | 15         | 12         | 4          | 9          | 9          | 119   |
| 14      | 15         | 13         | 25         | 6          | 15         | 16         | 10         | 9          | 17         | 8          | 134   |
| Total   | 152        | 148        | 210        | 75         | 161        | 171        | 169        | 69         | 167        | 119        | 1441  |

Table 13: Monte Carlo Simulations Summary, 800 iterations each, Course: All

| Course | Random Normalization, $\rho_{\text{mt}}$ | Expected Normalization, $\gamma_{\text{mt}}$ | Enrollment | Sections | Lecturers |
|--------|-----------------------------------------|---------------------------------------------|------------|----------|-----------|
|        | $100 \times \frac{\nu_{IA}}{\rho_{IA}}$ | $100 \times \frac{\nu_{SA}}{\gamma_{IA}}$ | $100 \times \frac{\nu_{IA}}{\gamma_{IA}}$ | $100 \times \frac{\nu_{SA}}{\gamma_{SA}}$ | $\sum_{K \in I} s_K$ | NT |
| DC     | 0.4448                                  | 3.2261                                      | 0.4422     | 3.2242   | 1440.2    | 15.0 | 7.3 |
| IC     | 0.3267                                  | 2.8094                                      | 0.3196     | 2.8104   | 1068.0    | 8.2  | 5.1 |
| VC     | 0.1684                                  | 1.9797                                      | 0.1800     | 1.9923   | 586.6     | 4.1  | 2.4 |
| VAG    | 0.4070                                  | 3.1366                                      | 0.4079     | 3.1474   | 1080.8    | 14.5 | 6.2 |
| LA     | 0.2131                                  | 3.2906                                      | 0.2009     | 3.2788   | 1078.2    | 8.0  | 4.2 |
| ODE    | 0.4323                                  | 5.6270                                      | 0.4269     | 5.6332   | 798.2     | 6.4  | 3.1 |
| BM     | 0.5706                                  | 3.0775                                      | 0.5909     | 3.0791   | 910.9     | 11.1 | 3.0 |
| NM     | 0.3750                                  | 3.3825                                      | 0.3405     | 3.3920   | 263.1     | 2.3  | 1.7 |

240 extra students per semester passing their respective courses which, in the long run represent a significant gain for the Institution.

(vi) A 3.3 % enhancement for the method’s benefit may seem low at first sight. However, it is important to stress this enhancement corresponds to a detailed treatment of the tenured lecturers only while the adjunct lecturers are treated in general terms because of insufficient data as they are unstable personnel. Tenured lecturers represent a fraction of less than 50 percent from the involved faculty team as Table 13 shows. Consequently, should the stable personnel fraction increase, the algorithm would deliver more accurate and perhaps more optimistic results.

(vii) The algorithms 4 and 6 could have been adjusted keep only the sections with tenured lecturers. However the Authors chose to discard this artificial setting because it is biased with respect to the study case.
(viii) It is the perception of the Authors that no general conclusions can be derived for the method’s enhancement level. On one hand it is sufficiently general and flexible to be implemented at any Institution with massive courses and therefore big databases available. On the other hand, the experiments performed in the present work suggest that its effectiveness needs to be evaluated on a case-wise basis.

(ix) Considering age as a factor is also possible by merely applying the segmentation process described in Section 3.2 (Algorithm 1 with an adequate number of segmentation intervals \(I_{\ell} : \ell \in [\tilde{L}]\)), then computing the lecturers performance conditioned to the Age variable as in section 3.3 (Algorithm 2, output \(T_{\text{Age}}\)), and finally weighting its impact according to the correlation values, namely the costs table in Equation (5) can be modified

\[
C = \frac{4}{5}T_{\text{APV}}G + \frac{1}{5}T_{\text{Age}}\tilde{G}.
\]

Here it is understood that the group matrix \(\tilde{G}\) is constructed according to the Age variable segmentation \((\tilde{I}_{\ell} : \ell \in [\tilde{L}]\)). The weighting coefficients were proposed according to the correlation with the Grade variable reported in Table 2: Age 0.2, GPA 0.8 i.e., the second is 4 times the first one, see see [9] for further discussion on these type of models. Yet again, the flexibility of the method allows to introduce in the same fashion any number of variables fitting to the case at hand.

(x) This paper has worked two methods, assign instructors while keeping the students fixed (IA) and assign students while keeping the instructors fixed (SA) both of them come down to a linear optimization problem, \(1\) and \(2\) respectively. However, moving both instructors and students is no longer a linear but a bilinear optimization question (see [17, 6]). This view will be further explored in future works.

(xi) So far the present worked assumed that allocating students and/or students is a centralized decision by the administrative departments of the analyzed Institution. However, in our study case, student location is decided differently, using a GPA competition-based mechanism to assign priority starting from the highest to the lower scorers. This scenario is better modeled using game theory which will be explored in future works.

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