\[ \alpha \text{-decay chains of } ^{289}_{175} \text{ and } ^{293}_{175} \text{ in the relativistic mean-field model} \]

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A comparison of calculated and experimental \(Q_a\) values of superheavy even-even nuclei and a few selected odd-\(N\) nuclei is presented in the framework of the relativistic mean-field model with the parameterization NL-Z2. Blocking effects are found to be important for a proper description of \(Q_a\) of odd nuclei. The model gives a good overall description of the available experimental data. The mass and charge assignment of the recently measured decay chains from Dubna and Berkeley is in agreement with the predictions of the model. The analysis of the new data does not allow a final conclusion about the location of the expected island of spherical doubly-magic superheavy nuclei.

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In the last few years, the synthesis of superheavy nuclei with \(Z = 110–112\) at GSI (Darmstadt) and JINR Dubna – see [1] and references therein – has renewed the interest in the properties of superheavy nuclei. These are by definition the nuclei with \(Z > 100\) which have a negligible fission barrier and are stabilized by shell effects only. The ultimate goal is to reach an expected `island of stability' located around the next spherical doubly-magic nucleus which was predicted to be \(^{298}_{184}^{114}\) thirty years ago [2].

Recently the discovery of new rather neutron-rich isotopes of the elements \(Z = 112\) [3] and \(Z = 114\) [4] was reported from JINR Dubna, while an experiment at Berkeley led to the synthesis of even heavier nuclei with \(Z = 118\) [5]. While earlier superheavy nuclei could be unambiguously identified by their \(\alpha\)-decay chains leading to already known nuclei, the decay chains of the new-found superheavy nuclei cannot be linked to any nuclei known so far. Their identification relies on comparison with theoretical models.

The experimental progress is accompanied by a large-scale investigation of superheavy nuclei with the latest nuclear mean-field models. While refined macroscopic-microscopic (MM) models like the Yukawa-plus-exponential model with Woods-Saxon single-particle potentials (YPE+WS) [6] or the finite-range droplet model with Folded-Yukawa single-particle potentials (FRDM+FY) [7] confirm the older prediction of \(^{298}_{184}^{114}\) for the next spherical doubly-magic nucleus, most self-consistent models shift that property to the more proton-rich \(^{298}_{172}^{120}\) or even \(^{310}_{184}^{126}\) [8,9]. The conflicting predictions have several reasons. MM models have very restricted degrees of freedom of the radial density distribution and the shape of the single-particle potentials, which becomes visible in superheavy nuclei where this hinders the occurrence of proton shells at \(Z = 120\) or \(Z = 126\) [10,11]. Another source for different extrapolation to superheavy nuclei among the models is the spin-orbit interaction. While the self-consistent relativistic mean-field model (RMF) naturally incorporates the nuclear spin-orbit interaction (which is a purely relativistic effect), it has to be put in by hand into all non-relativistic models, self-consistent ones using Skyrme (SHF) interactions and MM models. Surprisingly all non-relativistic models – which have one or several additional parameters to explicitly adjust the experimental spin-orbit splittings – perform not as good in this respect as the RMF which has no parameters adjusted to single-particle data at all. The non-relativistic models overestimate spin-orbit splittings with increasing mass number which might have some impact on their actual predictions for shell closures in the superheavy region [11]. Superheavy nuclei with their large density of single-particle states are a sensitive probe for models of nuclear shell structure and can be used to discriminate among models which describe stable nuclei with comparable quality.

Macroscopic-microscopic models – which have been the favorite tool to describe superheavy nuclei for a long time – provide a very good description of masses throughout the chart of nuclei, but they rely on preconceived knowledge about the densities and single-particle potentials which fades away when going towards the limits of stability. Although self-consistent models have not yet reached the overall performance of MM models, they incorporate polarization effects on the density distribution which might be crucial for a proper description of superheavy nuclei where the strong Coulomb field pushes the protons to the nuclear surface [12,13].

Recently Cwik et al. have reported a detailed comparison of the new experimental data with SHF calculations employing the interaction SLy4 [14]. Although the calculated \(Q_a\) values show some deviations from the measured values, the analysis confirms the assignment of the mass and charge number of the new nuclides. SLy4 shifts the island of stability towards very high charge numbers around \(^{310}_{184}^{126}\) [15]. It is the aim of this contribution to compare the experimental data with predictions of the
The experimental values for even-even nuclei (filled diamonds) where available and odd-N isotopes (open diamonds) of the heaviest even \(Z\) elements. Solid lines connect nuclei with the same proton number \(Z\), dotted lines nuclei in \(\alpha\)-decay chains. Including data for nuclei with \(Z > 116\) leads to overlapping curves, therefore those are omitted in the plot.

RMF model \([13]\) obtained with the recent parameterization NL-Z2 by P.-G. Reinhard \([14]\). This particular force provides the best overall description of binding energies of superheavy nuclei among the current parameterizations of the RMF \([13,18]\) and with \(N=120\) gives an alternative prediction for the next doubly magic nucleus which is quite close to the upper end of the new \(Z = 118\) chain.

The calculations are performed on an axially symmetric grid assuming reflection symmetry. Nuclei with odd mass number are calculated in a self-consistent blocking approximation taking into account time-odd contributions to the Dirac equation as described in \([19]\). Pairing correlations are treated within the BCS scheme using a delta pairing force, see \([14]\) for details. There remains a small uncertainty of the calculated \(Q_\alpha\) values on the order of a few 100 keV from a possible variation of pairing recipes (i.e. particle-number projection and choice of pairing interaction), especially in odd nuclei. The pairing strength is fitted to pairing gaps calculated in spherical blocking approximation. Taking polarization effects in odd nuclei into account during the fit gives larger pairing strengths \([20,21]\), but this affects mainly the odd-even staggering, while the \(Q_\alpha\) of heavy nuclei are rather robust. The correction for spurious center-of-mass motion is performed a posteriori as done in the original fit of NL-Z2, while corrections for spurious rotational and vibrational motions are neglected (as it is done in all other studies of superheavy nuclei) which might affect \(Q_\alpha\) values in some cases to the order of 1 MeV and tend to diminish shell effects \([22]\).

A first test of the reliability of the RMF to describe the new data is to check its performance for known \(Q_\alpha\) values of even-even nuclei, see Figure 1. Owing to the lack of data for even-even nuclei the \(Q_\alpha\) of odd-N nuclei are drawn beyond \(Z = 104\). They have to be handled very carefully, some of these \(Q_\alpha\) values might correspond to transitions involving excited states, and due to the blocking effects the ground-state-to-ground state \(Q_\alpha\) values might deviate on the order of 500 keV from the systematics of the \(Q_\alpha\) values of even-even nuclei. NL-Z2 gives a good overall description of the data except for some nuclei around \(N_{104}\) where NL-Z2 overestimates a deformed proton sub-shell closure while the deformed \(N=152\) shell is underestimated. The latter is a problem from which virtually all self-consistent models suffer \([13,18]\). The particular Skyrme interaction SLy4 used in \([10]\) performs better in the region around \(Z = 104\), but it is to be noted that this interaction has small errors in the isotopic and isotonic mass systematics of these nuclei \([13,15]\), which cancel when calculating \(Q_\alpha\) values.

To get an impression of the shell structure (as predicted by NL-Z2) of nuclei in the new-found region, Fig. 2 shows the single-particle spectra of the even-even nuclei with one neutron less than those in the decay chain of \(^{293}\text{Hs}\)\(^{118}\). The nuclei close to the \(Z = 120\) shell closure come out spherical including \(^{292}\text{Hs}\)^{118}, while all lighter nuclei have deformed ground states. In addition to the spherical \(Z = 120\) shell, the proton spectra show several deformed shell closures indicated in the plot. In contrast to MM models and some SHF interactions, however, the proton spectra show no preferred (deformed) shell closure for the light nuclei in this chain, but a region of small level density between \(Z = 104\) and \(Z = 110\). The single-particle spectra of the neutrons are much denser with a spherical \(N = 172\) shell and deformed shell closures at \(N = 174\) and \(N = 162\). The latter ones are also predicted by other models \([16]\) and \(N = 162\) is already confirmed experimentally \([15]\). The single-particle spectra of nuclei close to the decay chain of \(^{293}\text{Hs}\)^{114} look similar except for the fact that all nuclei there are deformed.

Three \(\alpha\) decay chains of odd-mass nuclei are discussed in the following: the one starting with \(^{277}\text{Bi}\)^{112} measured at GSI in 1994 \([23]\) serving as a testing ground for the performance of NL-Z2 when describing odd-mass number nuclei, and the new data assigned to \(^{293}\text{Hs}\)^{114} and \(^{293}\text{Hs}\)^{118}. The experimental data were obtained from two, one and three events respectively. Fig. 3 compares calculated and measured \(Q_\alpha\) values of these selected chains. Values calculated with the FRDM+FY model \([9]\), the YPE+WS model \([16]\) and the SHF+SLy4 model \([16]\) are given for comparison. As can be seen from Fig. 3 the quasiparticle spectra of odd-\(N\) nuclei in this region are very dense. Each single-quasiparticle state is the band head of a rotational band with transition energies from the first excited state to the band head below 50 keV which further increases the level density significantly. It cannot be expected that the synthesis of these nuclei and their decay chains lead always to ground states but very often to excited states. Experimental evidence for that was found in the decay of \(^{277}\text{Bi}\)^{112}, each of the two nuclei identified so far \([23]\) has decayed through a different state in \(^{293}\text{Hs}\)^{110}. The branch with large \(Q_\alpha\) in \(^{293}\text{Hs}\)^{110} presumably connects low-
lying states in both nuclei while the branch with the low $Q_\alpha$ goes through a highly excited state in that nucleus. This demonstrates a fundamental problem when comparing measured and calculated $Q_\alpha$ of decay chains with a few identified events only: the low statistics does not allow to identify the whole spectrum of possible $\alpha$ transitions. Detailed information on the $\alpha$-decay fine structure of superheavy nuclei is available for a few nuclides up to $Z = 104$ only \cite{1}. Some guidance when transitions to or between excited states might be favored is given by the fact that among transitions with similar $Q_\alpha$ values those between states with the same quantum numbers are favored.

In view of the remaining uncertainties NL-Z2 gives a very good description of the heavy nuclei in the decay chain of $^{293}_{114}112$ above $N > 157$ and reproduces the large jump in $Q_\alpha$ between $^{273}_{112}110$ and $^{269}_{112}108$ caused by the deformed $N = 162$ shell closure, which cannot be seen in the predictions of the FRDM+FY model. For small neutron numbers the $Q_\alpha$ calculated with NL-Z2 are too small as in the case of the even-even nuclei discussed above. NL-Z2 gives also a rather good description of the $\alpha$-decay chain of $^{289}_{112}114$. The missing kink in the calculated values can be explained assuming that the decay of $^{293}_{114}114$ leads to one of the numerous low-lying excited states in $^{289}_{112}112$.

The decay chain of $^{293}_{112}118$ leads through a region with seemingly constant total shell effects, while most mean-field models predict several spherical or deformed shell closures in this region. The FRDM+FY data show a pronounced shell effect for $^{285}_{112}114$, while the SHF+SLy4 results show one for $^{289}_{110}116$ which is related to deformed shells at $Z = 116$ and $N = 174$ \cite{10}. NL-Z2 predicts a deformed $N = 174$ shell as well but a deformed $Z = 114$ proton shell (see Fig. 2) that leads to a broad plateau at smaller atomic numbers. The predictions of the YPE+WS model of \cite{10} follow qualitatively the experimental data for the heavy nuclei in this chain but show a shell effect for $^{273}_{112}110$. In contrast to the other models discussed here the YPE+WS model predicts the nuclei at the upper end of this decay chain to be oblate deformed \cite{9}.

The mass and charge assignment of the Dubna and Berkeley data are based on theoretical models. While

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Single-particle spectrum of the protons (upper panel), neutrons (middle panel) and the relative quadrupole and hexadecapole moments given by $\beta_2 = 4\pi<r^2Y_{00}>/(3AR^4)$ with $R = 1.2A^{1/3}$ fm (lower panel) for the $\alpha$-decay chain $^{289}_{110}120 \rightarrow ^{293}_{112}118 \rightarrow \cdots \rightarrow ^{285}_{112}114$ of even-even nuclei adjacent to the $\alpha$-decay chain of $^{277}_{112}118$. For the deformed nuclei at small neutron number the states are labeled with angular-momentum projection and parity, while for the spherical nuclei at large neutron number the total angular momentum and parity are given. The full (dotted) lines in the upper and middle panels denote positive (negative) parity states, the dashed lines the Fermi energies.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Comparison of experimental and calculated $Q_\alpha$ values for the decay chains of $^{273}_{112}112$, $^{289}_{112}114$, and $^{293}_{112}118$, in the latter two cases following the mass and charge assignment of the experimental groups. In the $^{293}_{112}112$ chain two distinct branches leading through different states of the intermediate nuclei are known. The calculated values from RMF+NL–Z2 and SHF+SLy4 connect the lowest states with positive parity in all cases (in the new chains only $^{285}_{112}114$ and $^{289}_{110}118$ are predicted by NL-Z2 to have ground states with negative parity in agreement with \cite{9}), while the FRDM+FY and YPE+WS data are ground state to ground state values.}
\end{figure}
excluded. The spherical shell closures at

\[ Z \]

shell, restricted to higher neutron numbers, cannot be

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the vicinity of the (potential) doubly magic

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ues and corresponding long half-lives are caused by
defined feature of the NL-Z2 results is that

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values calculated between the lowest negative-parity quasiparticle

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states starting with

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nuclei can be expected to decay to states in rotational

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the adjacent even-even nuclei. Since the 2\( n \) bands build on lower-lying quasiparticle states with pos-

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and higher neutron number

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of odd nuclei at the upper end of the

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predicted by the RMF are relatively weak and do not significantly change the systematics of

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values of odd-\( N \) nuclei at the upper end of the \( Z = 118 \) chain measured in Berkeley.

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the proton number is rather certain in both cases, the neutron number \( N \) might also be smaller. Fig shows the \( Q \) values measured in Berkeley in comparison with calculated \( Q \) values for decay chains with varied neutron number of the initial nucleus. The heavy nuclei in all decay chains have very similar \( Q \) values, while there are visible differences for the lower end of the decay chain. It can be clearly seen that \( Q \) values of odd nuclei do not necessarily follow exactly the trend of the \( Q \) of the adjacent even-even nuclei. Since the 2\( n \) and higher reaction channels are energetically forbidden and statistically suppressed, an interpretation of the Berkeley data in terms of the \( \alpha \)-decay chain of \( ^{177}_{291} \) is in agreement with predictions of the RMF with NL-Z2. An interesting feature of the NL-Z2 results is that \( Q \) values calculated between the lowest negative-parity quasiparticle states starting with \( ^{177}_{291} \) or \( ^{175}_{291} \) follow very closely the experimental data, but several of the intermediate nuclei can be expected to \( \gamma \) decay to states in rotational bands build on lower-lying quasiparticle states with positive parity which prevents a long \( \alpha \)-decay chain between negative-parity states (see also [16]).

To summarize: The RMF with the parameterization NL-Z2 gives a reasonable description of the \( Q \) values of known superheavy nuclei with an overall quality comparable to other mean-field models that predict different spherical magic numbers. On the basis of the available experimental data, a decision about the location of the 'island of stability' cannot be made. The analysis of the new data from Dubna and Berkeley in terms of the RMF model suggests that the relatively small \( Q \) values and corresponding long half-lives are caused by deformed \( Z = 114 \) and \( N = 174 \) shell closures rather than the vicinity of the (potential) doubly magic \( ^{288}_{184} \) predicted by some other models, but a spherical \( Z = 114 \) shell, restricted to higher neutron numbers, cannot be excluded. The spherical shell closures at \( Z = 120 \) and

\[ N \]

172 predicted by the RMF are relatively weak and do not significantly change the systematics of \( Q \) values of odd-\( N \) nuclei at the upper end of the \( Z = 118 \) chain measured in Berkeley.

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