Fundamental Limits of Cloud and Cache-Aided Interference Management with Multi-Antenna Base Stations

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Abstract

In cellular systems, content delivery latency can be minimized by jointly optimizing edge caching, fronthaul transmission from a cloud processor with access to the content library, and wireless transmission. In this paper, this problem is studied from an information-theoretic viewpoint by making the following practically relevant assumptions: 1) the Edge Nodes (ENs) have multiple antennas; 2) the fronthaul links are used to send fractions of contents; and 3) the ENs are constrained to use linear precoding on the wireless channel. Assuming offline caching and focusing on a high signal-to-noise ratio (SNR) latency performance metric, the proposed caching and delivery policy is shown to be either exactly optimal or optimal within a multiplicative factor of $3/2$. The results bring insights into the optimal interplay between edge and cloud processing in fog-aided wireless networks as a function of system resources, including the number of antennas at the ENs, the ENs’ cache capacity and the fronthaul capacity.

Index Terms

Fog, cloud, edge caching, interference management.

I. INTRODUCTION

Content delivery is one of the most important use cases for mobile broadband services in 5G networks. A key technology that promises to help minimize delivery latency is edge caching, which relies on the storage of popular contents at the Edge Nodes (ENs), i.e., at the base stations or access points. The information-theoretic analysis of edge caching, which has been undertaken in the past few years starting with [1], has concentrated on the interplay between the cached content distributions across the ENs and the ENs’ capability to carry out interference management. To this end, this line of work has focused on the interference-limited regime of high Signal-to-Noise Ratios (SNRs).

In [1], [2], as well as in [3]–[5], this analysis was performed under the assumption that the overall cache capacity available in the system, including at the receivers, is sufficient to store the entire library of popular contents. When this assumption is violated, contents need to be retrieved from a content server by leveraging transport links that connect the ENs to the access or core network. This more general scenario was studied in [6], [7], as well as in the follow-up works [8], [9], in which a cloud processor is considered to be connected to the ENs via so called fronthaul links, as seen in Fig. 1 (see also [10], [11]).

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For the model of Fig. 1, which is referred to as Fog-Radio Access Network (F-RAN), the key design problem concerns the optimal use of fronthaul and wireless edge resources for caching and delivery. Assuming the standard offline caching scenario with static popular set, reference [6] identified optimal caching and delivery strategies within a multiplicative factor of two. Optimality is defined in the high-SNR regime as in [1]–[11]. The results in [6] hold under no constraints on the strategies allowed for use on fronthaul and wireless channels. In particular, the approximately optimal scheme in [6] relies on interference alignment, which is known to have significant performance losses under imperfect Channel State Information (CSI) [12], and on fronthaul quantization, which comes with stringent synchronization constraints [13].

In this work, we revisit the results in [6] by making the following practically relevant assumptions: 1) the ENs have multiple antennas; 2) the fronthaul links can only be used to send uncoded fractions of contents; and 3) the ENs are constrained to use linear precoding on the wireless channel. In light of the last assumption, the set-up studied extends the model in [2] by including cloud processing, fronthauling, and multi-antenna ENs, but, unlike [2], it excludes caching at the receivers’ sides.

**Related Work:** Assuming offline caching, cache-aided interference management was first studied in [1], in which transmitter-side caches are considered, and a delivery strategy is proposed by leveraging interference alignment, Zero-Forcing (ZF) precoding and interference cancellation. Extensions that account for caching at both transmitter and receiver sides can be found in [2]–[5], [14]. In [4], a novel strategy based on the separation of physical and network layers is investigated. Under the assumption of one-shot linear precoding, references [2], [14] reveal that the transmitters’ caches and receivers’ caches contribute equally to the high-SNR performance. As discussed, the joint design of cloud processing and edge caching for F-RANs was studied in references [6], [7] and then in [9], [10], [15], by focusing on the high-SNR latency performance metric known as Normalized Delivery Time (NDT) proposed in [6]. These works obtain approximately optimal values without considering assumptions 1)-3). References [10], [11] study a related scenario with coexisting macro- and small-cell base stations.

**Main contributions:** This paper investigates interference management in a cloud and cache-aided F-RAN illustrated in Fig. 1, with multiple antennas at the transmitters, under the assumptions of one-shot linear precoding and transmission of uncoded...
contents on the fronthaul links. We first derive an upper bound on the minimum NDT as a function of the cache storage capacity, the fronthaul rate and the number of ENs’ antennas. To this end, we propose a scheme that manages interference via ZF by means of the ENs’ cooperation as enabled by both fronthaul and edge caching resources. Then, an information-theoretic lower bound on the minimum NDT is derived. As a result, the minimum NDT is characterized exactly for a large subset of system parameters, and approximately within a multiplicative factor of 3/2 for any value of the parameters.

**Notation:** For any integer $K$, we define the set $[K] \triangleq \{1, 2, \cdots, K\}$. For a set $A$, $|A|$ represents the cardinality. We use the notation $\{f_n\}_{n=1}^{N} \triangleq \{f_1, \cdots, f_n, \cdots, f_N\}$. For function $g(n)$, the notation $f(n) = o(g(n))$ denotes a function $f(n)$ that satisfies the limit $\lim_{n \to \infty} (f(n)/g(n)) = 0$. The ceiling function $\lceil x \rceil$ maps $x$ to the least integer that is greater than or equal to $x$, and the floor function $\lfloor x \rfloor$ maps $x$ to the greatest integer that is less than or equal to $x$. We also have $(x)^+ \triangleq \max\{x, 0\}$.

II. SYSTEM MODEL AND PERFORMANCE METRIC

In this section, we present the model under study, which consists of an F-RAN system with multi-antenna ENs operating under uncoded fronthaul transfer and one-shot linear precoding on the wireless edge channel. We also adapt the NDT metric [6] to this model.

A. System Model

We consider the F-RAN model shown in Fig. 1 where $K_T$ ENs, each having $n_T$ antennas, are connected to $K_R$ single-antenna receivers through a shared wireless channel, as well as to a cloud processor (CP) via fronthaul links. The CP has access to a library of $N$ files $\{W_n\}_{n=1}^{N}$, of $L$ bits each. Any file $W_n$ contains $F$ packets $W_n = \{W_{nf}\}_{f=1}^{F}$, where each packet $W_{nf}$ is of size $L/F$ bits, and $F$ is an arbitrary parameter. Note that we refer to the set of packets $\{W_{nf}\}_{f=1}^{F}$ in file $W_n$ as $W_n$. Each fronthaul link has capacity $C_F$ bits per symbol, where a symbol refers to a channel use of the wireless channel, and each EN has a cache with capacity of $\mu N L$ bits, with $\mu \in [0, 1]$. Parameter $\mu$ is referred to as the fractional cache size.

In the pre-fetching phase, the caches of the ENs are pre-filled with content from the library under the cache capacity constraints. The content of the cache of each EN $i$ is described by the set $C_i = \{C_{i1}, \cdots, C_{in}, \cdots, C_{iN}\}$, where $C_{in} \subseteq W_n$ represents the subset of packets from file $W_n$ that are cached at EN $i$. Due to the cache capacity constraint, its size must satisfy the inequality

$$\frac{|C_{in}|}{F} \leq \mu. \tag{1}$$

Note that as in [2], the model at hand allows for no coding either within or across files.

In the delivery phase, each user $k$ requests a file $W_{dk}$, with $d_k \in [N]$, from the library. Given the request vector $d = \{d_1, \cdots, d_{K_R}\}$, the CP transmits information about the requested files $\{W_{d_1}, \cdots, W_{d_{K_R}}\}$ to the ENs via the fronthaul links. Specifically, on each fronthaul $i$, the set $F_i = \{F_{id_1}, \cdots, F_{id_{K_R}}\}$ of packets is sent, where $F_{id_k} \subseteq W_{dk}$ is a subset of packets from file $W_{dk}$. Note that, as mentioned, the described model assumes hard-transfer fronthauling. After the fronthaul transmission, any EN $i$ has access to the fronthaul information $F_i$, as well as to the cached content $C_i$. This information is used by the ENs to deliver the users’ requests $\{W_{d_1}, \cdots, W_{d_{K_R}}\}$ through the wireless channel.
To this end, we constrain the wireless transmission strategy to one-shot linear precoding by following [2]. Accordingly, wireless transmission takes place over $B$ blocks to deliver the $K_R F$ desired packets. In any block $b \in [B]$, the ENs send a subset of the requested packets, denoted by $D_b \subseteq \{W_{d_{1, f}}, \cdots, W_{d_{K, f}}\}_{f=1}^F$, to a subset $R_b$ of $K_T$ users, such that each user in $R_b$ can decode exactly one packet without interference at the end of the block. To this purpose, in any block $b$, each EN $i$ sends a linear combination of the subset of packets in $D_b$ that it has available in its cache or based on the fronthaul signal. For any given symbol within the block, the transmitted signal of EN $i$ is hence given as

$$x_i(b) = \sum_{W_{n, f} \in D_b \cap \{C_i \cup F_i\}} v_{i, n, f}(b) s_{n, f}(b),$$

where $s_{n, f}(b)$ is a coded symbol for file $W_{n, f}$, and $v_{i, n, f}(b) \in \mathbb{C}^{n_T \times 1}$ is the precoding vector for the same file. As we have described, each file $W_{n, f} \in D_b \cap \{C_i \cup F_i\}$ is intended for a single user in $R_b$. We impose the power constraint $\mathbb{E}[||x_i(b)||^2] \leq P$.

The received signals of each user $k \in R_b$ in block $b$ is given as

$$y_k(b) = \sum_{i=1}^{K_T} h_{k, i}(b) x_i(b) + z_k(b),$$

where $h_{k, i}(b) \in \mathbb{C}^{n_T \times 1}$ is the channel vector between EN $i$ and user $k$, and $z_k(b)$ is the zero-mean complex Gaussian noise with normalized unitary power. We assume that all the ENs and users have access to the full CSI $\{h_{k, i}(b)\}_{k \in [K_T], i \in [K_R]}$ as necessary. The delivery of the packets in the set $D_b$ in block $b$ is achievable if there exist precoding vectors $\{v_{i, n, f}(b)\}$, such that, with full CSI, each user $k \in R_b$ can decode without interference its intended file. More precisely, delivery is successful if the received signal $y_k(b)$ is directly proportional to the desired symbol $s_{n, f}(b)$ plus additive Gaussian noise with constant power, i.e., not scaling with the signal power $P$. The resulting point-to-point interference-free channel from the ENs to user $k$ supports transmission at rate $\log(P) + o(\log(P))$.

**B. Performance Metric: NDT**

Given the fronthaul messages defined by set $\{F_i\}_{i=1}^{K_T}$, the time required for fronthaul transmission can be computed as

$$T_F = \max_{i \in [K_T]} \frac{|F_i| L}{F} \frac{1}{C_F},$$

since $|F_i| L / F$ bits need to be delivered to EN $i$ over a fronthaul link of capacity $C_F$ and $T_F$ is the maximum among the $K_T$ fronthaul latencies. Furthermore, given the delivered packet set $\{D_b\}_{b=1}^B$, the total time needed for wireless edge transmission over $B$ blocks is

$$T_E = \frac{B L}{F} \frac{1}{\log(P) + o(\log(P))}.$$

This is because, in each of the $B$ blocks, one packet with $L / F$ bits is sent to each user in $R_b$ at rate $\log(P) + o(\log(P))$.

As in [6], we normalize the latency by the term $L / \log(P)$. This corresponds to the transmission latency, neglecting $o(\log(P))$ terms, for a reference system that transmits interference-free to all users at the maximum rate $\log(P)$. Moreover, as in [6], we evaluate the impact of the fronthaul capacity $C_F$ in the high-SNR regime by using the scaling $C_F = r \log(P)$, so that the
parameter \( r \) measures the ratio between the fronthaul capacity and the interference-free wireless channel capacity to any user. Accordingly, we define the fronthaul NDT of the given policy as

\[
\delta_F = \lim_{P \to \infty} \lim_{L \to \infty} \frac{T_F}{L/\log(P)} = \max_{i \in [K_T]} \frac{|F_i|}{F^{r_i}},
\]

and the edge NDT as

\[
\delta_E = \lim_{P \to \infty} \lim_{L \to \infty} \frac{T_E}{L/\log(P)} = B_F.
\]

The overall NDT is given as \( \delta = \delta_E + \delta_F \). For any pair \((\mu, r)\), the minimum NDT across all achievable policies \(\{F_i, C_i, \{v_{in}(b)\}_{b=1}^{B_i}\}_{i=1}^{K_T}\) is defined as

\[
\delta^*(\mu, r) = \inf\{\delta(\mu, r) : \delta(\mu, r) \text{ is achievable for some } F \geq 1\}.
\]

Note that in the definition \(\delta^*\), we allow for a partition of the files in an arbitrary number of \( F \) packets.

**Lemma 1:** The minimum NDT \( \delta^*(\mu, r) \), is convex function of \( \mu \), with \( 0 \leq \mu \leq 1 \), for any fixed \( r \geq 0 \).

**Proof:** The lemma follows by a standard time-sharing argument, which is detailed in [6, Lemma 1].

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### III. Normalized Delivery Time Analysis

In this section, we present an upper bound (Proposition 1) and a lower bound (Proposition 2) on the minimum NDT for the described F-RAN model. These bounds provide a characterization of the minimum NDT that is conclusive for a wide range of values of the system parameters (Proposition 3) and is generally within a multiplicative factor of \( 3/2 \) from optimality (Proposition 4). The main results offer insight into the optimal use of cloud and edge resources as a function of the fronthaul capacity, cache resources and number of ENs’ transmit antennas.

#### A. Achievable Scheme and Upper Bound on the Minimum NDT

In this subsection, we describe an achievable scheme and the corresponding achievable NDT. In the proposed scheme, in each block \( b \), a subset of ENs serve a given number \( u(\mu, r) \) of users by using cooperative ZF precoding on the wireless channel. Cooperation at the ENs via ZF is enabled by the availability of shared contents across the ENs as a result of both caching and fronthaul transmission. Therefore, the number \( u(\mu, r) \) is a non-decreasing function of both cache capacity \( \mu \) and fronthaul rate \( r \).

To quantify the availability of the requested files at the ENs, we define the multiplicity \( m(\mu, r) \leq K_T \) of any requested file as the number of times that the file appears across all the ENs after fronthaul transmission. This number hence accounts for both the pre-stored caching contents and the information received at the ENs from the fronthaul transmission. Given the multiplicity \( m(\mu, r) \), a number \( m(\mu, r) \) of ENs can transmit cooperatively in each block, creating up to \( m(\mu, r)n_T \) interference-free channel via ZF. Hence, the number of users that can be served via ZF in each block is given as

\[
u(\mu, r) = \min\{m(\mu, r)n_T, K_T\}.
\]
A multiplicity \( m(\mu, r) = \lfloor \mu K_T \rfloor \) can be ensured by the ENs’ cached contents. The multiplicity can be further increased by sending information on the requested files from the cloud, albeit at the cost of fronthaul transmission latency. Moreover, the multiplicity \( m(\mu, r) \) can be upper bounded without loss of optimality by

\[
    m_{\text{max}} = \min \left\{ K_T, \left\lceil \frac{K_R}{n_T} \right\rceil \right\}. \tag{10}
\]

This is because, when the multiplicity reaches \( m_{\text{max}} \), the ENs can cooperate in each block via ZF beamforming to completely eliminate inter-user interference for the maximum possible number of users, which, by (9), is given by \( \min\{K_T n_T, K_R\} \).

To formalize the main result, we first define the function \( m_{\text{min}}(r) \) as the piece-wise constant non-decreasing function illustrated in Fig. 2. This is the multiplicity selected by the proposed scheme when there is no caching, i.e., when \( \mu = 0 \). The function \( m_{\text{min}}(r) \) in Fig. 2 is defined mathematically as

\[
m_{\text{min}}(r) = \begin{cases} 
    m, & \text{for } r \in \left[ \frac{m(m-1)n_T}{K_T}, \frac{m(m+1)n_T}{K_T} \right) \text{ and } r < r_{\text{th}} \\
    m_{\text{max}}, & \text{for } r \geq r_{\text{th}},
\end{cases} \tag{11}
\]

where the first condition applies for \( m = 1, 2, \ldots, m_{\text{max}} \), and we have

\[
r_{\text{th}} = \frac{n_T}{K_T} m_{\text{max}}^2. \tag{12}
\]

Note that function \( m_{\text{min}}(r) \) is non-decreasing with the fronthaul rate \( r \). When \( \mu > 0 \), we can generally support a larger multiplicity \( m(\mu, r) \) which is given for the proposed scheme as

\[
m(\mu, r) = \begin{cases} 
    m_{\text{min}}(r), & \text{if } \mu K_T < m_{\text{min}}(r) \\
    \lfloor \mu K_T \rfloor, & \text{if } m_{\text{min}}(r) \leq \mu K_T \leq m_{\text{max}} \\
    m_{\text{max}}, & \text{if } \mu K_T > m_{\text{max}},
\end{cases} \tag{13}
\]

The achievable NDT is presented in the following proposition.

**Proposition 1:** For an F-RAN system with \( n_T \) antennas at each EN, we have the upper bound on the minimum NDT
\[ \delta^*(\mu, r) \leq \delta_{ach}(\mu, r), \] where \( \delta_{ach}(\mu, r) \) is the achievable NDT given as

\[
\delta_{ach}(\mu, r) = \begin{cases} 
\frac{K_R(u(\mu, r) - \mu K_T)}{K_T} + \frac{K_R R}{u(\mu, r)}, & \text{for } \mu K_T \leq m_{min}(r) \\
\alpha \frac{K_R R}{u(\mu, r)} + (1 - \alpha) \frac{K_R}{u(\mu, r)}, & \text{for } \mu K_T \geq m_{min}(r),
\end{cases}
\]

where \( \alpha = 1 + \lfloor \mu K_T \rfloor - \mu K_T, u(\mu, r) = \min\{\lfloor \mu K_T \rfloor n_T, K_R\}, \) and functions \( m(\mu, r) \) and \( u(\mu, r) \) are defined in (9)–(12).

**Proof:** The full proof is presented in Appendix A and a sketch of the proof follows.

According to (13), as illustrated in Fig. 2, the multiplicity \( m(\mu, r) \) of each requested file is obtained by comparing \( \mu K_T \), i.e., the multiplicity allowed by caching only, with the upper and lower bound \( m_{max} \) and \( m_{min}(r) \). We distinguish two cases: (i) **Edge-only transmission:** for \( \mu K_T \geq m_{min}(r) \), we have \( m(\mu, r) = \min\{\lfloor \mu K_T \rfloor, m_{max}\} \) and \( u(\mu, r) = \min\{m(\mu, r)n_T, K_R\} \), and hence the edge caches can support the selected multiplicity without the need for fronthaul transmission; and (ii) **Cloud and edge-aided transmission:** for \( \mu K_T < m_{min}(r) \), we have \( m(\mu, r) = m_{min}(r) > \mu K_T \), and hence fronthaul transmission is needed in order to support the multiplicity \( m(\mu, r) \) and serve \( \min\{m_{min}(r)n_T, K_R\} \) users simultaneously.

**Remark 1:** From the discussion above, whenever \( \mu K_T \leq m_{min}(r) \), the proposed policy uses both cloud and edge. From Fig. 2, this implies that, as the fronthaul rate \( r \) increases, it is advantageous to use cloud-to-edge communications even when the edge alone would be sufficient to deliver all requested contents, that is, even when we have \( \mu K_T \geq 1 \). This is because, in this regime, the cloud can send information cached at some ENs to other ENs in order to foster cooperation, at the cost of a fronthaul delay that does not offset the cooperation gains. However, when \( \mu K_T \geq m_{min}(r) \), the scheme only uses edge resources. Since \( m_{min}(r) \) in Fig. 2 is a decreasing function of the number \( n_T \) of each EN’s antenna, this suggests that, as \( n_T \) increases, edge processing becomes more effective, making cloud processing unnecessary for smaller values of the cache capacity \( \mu \). We illustrate this fact in Fig. 3 in which the region of values of the pair \((\mu, n_T)\) for which only edge transmission is used by the proposed scheme can be found above the plotted curves. Note also from the figure that an increased \( r \) enlarges the region of values \((\mu, n_T)\) for which fronthaul transmission is used.

**Sketch of proof:** Using Fig. 3 for illustration, we now provide more details on the proposed caching and delivery policy, starting with the case \( \mu K_T \geq m_{min}(r) \). Assume first that \( \mu K_T \) is an integer, so that the multiplicity in (13) is \( m(\mu, r) = \lfloor \mu K_T \rfloor \).
min\{\mu K_T, m_{max}\}. In the caching phase, each file $W_n$ is equally split into $\binom{K_T}{m(\mu, r)}$ disjoint segments $\{W_{n\tau}\}_{\tau \subseteq T}$, where each segment $W_{n\tau}$ is stored at all ENs in the subset $\tau \subseteq T$ with $T = \{\tau \subseteq [K_T] : \mid \tau \mid = m(\mu, r)\}$. Note that $\tau$ runs over all subsets of $m(\mu, r)$ ENs. This guarantees that each segment is stored at $m(\mu, r)$ ENs. As seen in Fig. 4(a), in the delivery phase, the delivery of all $K_T$ requested files relies only on edge transmission. In particular, for any subset $\tau \subseteq T$, since all the ENs in $\tau$ have access to the parts $\{W_{n\tau}\}_{n \in d}$ of the requested files, this information can sent to $m(\mu, r)n_T$ users simultaneously via ZF precoding.

For the case $\mu K_T < m_{min}(r)$, where $m(\mu, r) = m_{min}(r)$, as seen in Fig. 4(b), each of the $N$ popular files is divided into two fractions $W_n = \{W^1_n, W^2_n\}$, where $W^1_n$ contains $\beta L$ bits with $\beta = \mu K_T/m(\mu, r)$. Part $W^1_n$ is cached, while part $W^2_n$ is sent on the fronthaul if the file $W_n$ is requested. Specifically, both parts are divided into $\binom{K_T}{m(\mu, r)}$ equal segments $W^1_n = \{W^1_{n\tau}\}_{\tau \subseteq T}$ and $W^2_n = \{W^2_{n\tau}\}_{\tau \subseteq T}$, where subset $\tau$ is defined as above. During the caching phase, each segment $W^1_{n\tau}$ is stored at all ENs in subset $\tau$. In the delivery phase, for a demand vector $d$, each uncached segment $W^2_{n\tau}$, with $n \in d$, is sent to all the ENs in subset $\tau$ via the fronthaul link. As a result, all requested segments $W^1_{n\tau}$ and $W^2_{n\tau}$ are available at all ENs in subset $\tau$, which can serve cooperatively $m(\mu, r)n_T$ users at a time via ZF.

To complete the sketch of the achievable scheme, we should finally consider the case when $\mu K_T$ is not an integer and we have $\mu K_T \geq m_{min}(r)$. To make full use of the edge caches, the library files are split into two disjoint fractions, which are cached with two multiplicities $\lceil \mu K_T \rceil$ and $\lfloor \mu K_T \rfloor$, respectively, and time sharing is used for the delivery of the two fractions. The details can be found in Appendix A.

**B. Lower Bound on the Minimum NDT**

A lower bound on the minimum $\delta^*(\mu, r)$ is presented in the following proposition, where we define the function $m^*(r)$ as

$$m^*(r) = \begin{cases} \max \left\{ \sqrt{\frac{K_T}{n_T}}, 1 \right\}, & \text{for } r < r_{th} \\ m_{max}, & \text{for } r \geq r_{th}, \end{cases}$$  \hspace{1cm} (15)

with $r_{th}$ defined in (12).
Fig. 5. Achievable NDT $\delta_{\text{ach}}(\mu, r)$ in Proposition 1 (solid curve), and lower bound on the minimum NDT $\delta^*(\mu, r)$ in Proposition 2 (dashed line) versus $\mu$ for a given value of $r$. The figure highlights the two regimes of values of the cache capacity $\mu$ with which the achievable schemes use edge-only or both cloud and edge transmission.

### Proposition 2:
In an F-RAN with $n_T$ antennas at each transmitter, the minimum NDT $\delta^*(\mu, r)$ is lower bounded as

$$\delta^*(\mu, r) \geq \delta_{lb}(\mu, r) = \begin{cases} \max \left\{ \frac{K_R(m^*(r) - \mu K_T)}{K_T r}, 1 \right\}, & \text{for } \mu K_T < m^*(r) \\ \max \left\{ \frac{K_R}{\mu K_T n_T}, 1 \right\}, & \text{for } \mu K_T \geq m^*(r) \end{cases}$$

(16a)

(16b)

**Proof:** The proof is presented in Appendix B.

### C. Minimum NDT

The following Proposition characterizes the minimum NDT $\delta^*(\mu, r)$ for the regime of low cache and fronthaul capacities, i.e., when $\mu K_T \in [0, 1]$ and $r \in [0, n_T/K_R]$, as well as for any set-up with $\mu K_T$ integer or $\mu K_T \geq m_{\text{max}}$.

**Proposition 3:** For an F-RAN system with $n_T$ antennas at each EN, the minimum NDT $\delta^*(\mu, r)$ is given as

$$\delta^*(\mu, r) = \begin{cases} \max \left\{ \frac{K_R(1-\mu K_T)}{K_T r}, \frac{K_R}{n_T}, 1 \right\}, & \text{for } \mu K_T \in [0, 1] \text{ and } r \in [0, \frac{n_T}{K_R}] \\ \max \left\{ \frac{K_R}{\mu K_T n_T}, 1 \right\}, & \text{for } \mu K_T \in \{m_{\text{min}}(r) + 1, \ldots, m_{\text{max}}\} \cup (m_{\text{max}}, K_T]. \end{cases}$$

(17)

**Proof:** The result follows by the direct comparison of the bounds in Proposition 1 and Proposition 2.

More generally, the achievable NDT in Proposition 1 is within a factor of $3/2$ from the lower bound of Proposition 2 for any fractional caching size $\mu$ and fronthaul rate $r$.

**Proposition 4:** For an F-RAN system with $n_T$ antennas at each EN, and any value of $\mu \geq 0$ and $r \geq 0$, we have the inequality

$$\frac{\delta_{\text{ach}}(\mu, r)}{\delta^*(\mu, r)} \leq \frac{3}{2}.$$  

(18)

**Proof:** The proof is presented in Appendix D.
regime, the fronthaul NDT decreases linearly with $\mu K_T$, which leads to a linear decrease in the overall NDT $\delta_{ach}(\mu, r)$. Instead, in the second regime, the achievable NDT $\delta_{ach}(\mu, r)$ is piece-wise linear and decreasing. For this range of values of $\mu$, time-sharing between two successive multiplicities is carried out for delivery, unless $\mu K_T$ is an integer. By comparison with the lower bound, the figure also highlights the regimes, identified in Proposition 3, in which the scheme is exactly optimal.

Achievable NDT in Proposition 1 and lower bound in Proposition 2 are plotted in Fig. 6 as a function of $\mu$ for $K_T = 8$ and $K_R = 32$ and for different values of $r$ and $n_T$. We note that, as stated in Proposition 3, the achievable NDT is optimal when $\mu$ and $r$ are small enough, as well as when $\mu$ equals a multiple of $1/K_T = 1/8$ or is large enough. For values of $r$ close to zero, the NDT diverges as $\mu$ tends to $1/K_T = 1/8$, since requests cannot be supported based solely on edge transmission. For larger values of $r$ and/or $n_T$, the NDT decreases. In particular, when $\mu K_T \geq m_{max} = 4$ and $n_T = 8$, as discussed, we have the ideal NDT of one, since the maximum possible number 32 of users can be served.

A comparison of the bounds derived here under the assumptions of hard-transfer fronthauling (HF), i.e., the transmission of uncoded files on the fronthaul links, and of one-shot linear precoding (LP) are compared with those derived in [6] without such constraints, as illustrated in Fig. 7. The figure is obtained for $K_T = 8, n_T = 1, r = 4$ and different values of $K_R$. It is observed that the loss in performance caused by the practical constraints considered here is significant, and that it increases
with the number $K_R$ of users. This conclusion confirms the discussion in [6, Sec. IV-B].

IV. Conclusions

In fog-aided cellular systems, fronthaul resources enable a cloud processor with access to the content library to communicate uncached contents to the edge nodes. This information is not only necessary to enable content delivery when the overall system’s capacity is insufficient, but it also to facilitate cooperative interference management. In this paper, we have studied the resulting optimal trade-off between fronthaul latency overhead and overall delivery latency from an information-theoretic viewpoint under the assumption of multi-antenna edge nodes. The main result is the characterization, within a small multiplicative factor, of the minimum high-SNR latency as a function of system parameters such as fronthaul capacity, edge cache capacity and number of per-edge node antennas.

APPENDIX A

PROOF OF PROPOSITION 1

In this section, we present the proof of the Proposition 1, by distinguishing the two cases $\mu K_T \geq m_{\min}(r)$ and $\mu K_T < m_{\min}(r)$, following the discussion in Section IIA.

A. Achievability for $\mu K_T \geq m_{\min}(r)$

a) Caching policy: For any $\mu K_T \geq m_{\min}(r)$, as discussed in Section IIA, time sharing is used between the two multiplicities $[\mu K_T]$ and $[\mu K_T]$. To this end, each library file $W_n$ is divided into two disjoint fractions $\{W_{n1}^1, W_{n2}^2\}$, with $|W_{n1}^1| = \alpha L$ bits, and each fraction is cached with the associated multiplicity by file-splitting. In particular, the first fraction $W_{n1}^1$ is split into $\binom{K_T}{[\mu K_T]}$ equal segments $\{W_{n1\tau}^1\}_{\tau \subseteq \mathcal{T}}$, with $\mathcal{T} = \{\tau \subseteq [K_T] : |\tau| = [\mu K_T]\}$, where each segment $W_{n1\tau}^1$ is stored at all ENs in the subset $\tau$. Similarly, fraction $W_{n2}^2$ is divided into the segments $\{W_{n2\tau'}^2\}_{\tau' \subseteq \mathcal{T}'}$, with $\mathcal{T}' = \{\tau' \subseteq [K_T] : |\tau'| = [\mu K_T]\}$, where each segment $W_{n2\tau'}^2$ is stored at all ENs in the subset $\tau'$. As a result, each EN $i$ stores the fractions $\{W_{n1\tau}^1, W_{n2\tau'}^2\}$ of each file $W_n$ for all subsets $\tau$ and $\tau'$ that include $i$. Hence, the total size of the cached content of the two fractions is

$$\frac{\alpha L}{\binom{K_T}{[\mu K_T]}} (\frac{K_T - 1}{[\mu K_T] - 1}) + \frac{(1 - \alpha)L}{\binom{K_T}{[\mu K_T]}} (\frac{K_T - 1}{[\mu K_T] - 1}) = \mu L \text{ bits},$$

(19)

which satisfies the cache capacity constraints.

b) Delivery policy: In the delivery phase, for a demand vector $d$, the requested segments $\{W_{n\tau}^1\}$ and $\{W_{n\tau'}^2\}$, $\forall n \in d, \tau \in \mathcal{T}, \tau' \in \mathcal{T}'$, are sent sequentially by using ZF precoding. In particularly, to send the segments $\{W_{n\tau}^1\}$, a number $u(\mu, r) = \min\{[\mu K_T]|n_T, K_R\}$ of users can be served in each block, requiring a number of blocks $B = K_R \alpha F/u(\mu, r)$. Likewise, to send the segments $\{W_{n\tau'}^2\}$, a number $u'(\mu, r) = \min\{[\mu K_T]|n_T, K_R\}$ of users can be served in each block, requiring a number of blocks $B' = K_R (1 - \alpha) F/u'(\mu, r)$. By using [5] and [7], the achievable NDT, comprising only the edge NDT, is

$$\delta_{ach}(\mu, r) = \frac{B}{F} + \frac{B'}{F} = \frac{\alpha K_R}{u(\mu, r)} + \frac{(1 - \alpha)K_R}{u'(\mu, r)},$$

(20)

as reported in Proposition 1.
B. Achievability for $\mu K_T < m_{\min}(r)$

a) Caching policy: As described in Section II-A for each file $W_n$ in the library, fraction $W_n^1$ has size $\beta L$ bits, and from this fraction each segment $W_{n}\tau$ is stored in the caches of all $m(\mu, r)$ ENs in subset $\tau$. As a result, each EN $i$ stores the fractions $W_{n}\tau$ all subsets $\tau$ that include EN $i$. The total size of the cached content is

$$\frac{\mu K_T L}{m(\mu, r)} \frac{1}{K_T \choose m(\mu, r)} \left( \frac{K_T - 1}{m(\mu, r) - 1} \right) = \mu L \text{ bits,} \quad (21)$$

which satisfies the cache capacity constraints.

b) Fronthaul policy: For each requested file $W_n$ with $n \in d$, the uncached segments $W_{n,\tau}^2$, each with size $(1 - \beta)L/(K_T m(\mu, r))$ bits, are delivered to the ENs in subset $\tau$. Thus, the cloud sends a total of $K_R m(\mu, r) - \mu K_T L/K_T$ bits to each EN. Since the fronthaul link has capacity $C_F = r \log(P)$, the fronthaul latency in (4) is given as $T_F = K_R m(\mu, r) - \mu K_T L/(K_T r \log(P))$, and the fronthaul NDT in (7) as

$$\delta_F = K_R m(\mu, r) - \mu K_T \frac{1}{K_T r}. \quad (22)$$

c) Edge transmission policy: By using the ZF precoding, $u(\mu, r) = \min\{m(\mu, r)n_T, K_R\}$ users can be served simultaneously in each block. This gives an edge NDT of

$$\delta_E = \frac{K_R}{u(\mu, r)}. \quad (23)$$

Summing up the fronthaul NDT $\delta_F$ in (22) and the edge NDT $\delta_E$ in (23), we obtain the achievable overall NDT $\delta(\mu, r)$

$$\delta(\mu, r) = \frac{K_R m(\mu, r) - \mu K_T}{K_T r} + \frac{K_R}{u(\mu, r)}, \quad (24)$$

which equals (14) in Proposition 1.

APPENDIX B

PROOF FOR PROPOSITION 2

The proof follows [2, Section 5] with the important caveats that here we need to additionally consider the delivery latency due to fronthaul transmission, as well as the extension to the general case $n_T \geq 1$. To start, we consider an arbitrary split of each file into $2^{K_T} - 1$ parts, such that each part $W_{n}\tau$, indexed by a subset $\tau \subseteq [K_T]$, contains an integer number of packets, including possibly no packets. We recall that each packet contains $L/F$ bits. Part $W_{n}\tau$ is available at the ENs in the subset $\tau$, either from the edge caches or from the cloud after fronthaul transmission. Note that this partition comes with no loss of generality, since each packet $W_{n}\tau$ is available at all EN $i$ such that $W_{n}\tau \in C_i \cup F_i$ (see definitions in Section II-A).

To distinguish between the contributions of cache and fronthaul resources, we use $c_{n}\tau$ to denote the number of cached packets from file $W_n$ at the ENs in subset $\tau$; while $f_{n}\tau(d)$ is the number of packets of file $W_n$ sent on the fronthaul links of all ENs in subset $\tau$ for a given demand vector $d$. Hence, part $W_{n}\tau$ has $a_{n}\tau = c_{n}\tau + f_{n}\tau(d)$ packets in total. The variables $\{c_{n}\tau\}$ and $\{f_{n}\tau(d)\}$, for all $n \in [N], \tau \subseteq [K_T]$ and vectors $d$, fully specify the operation of the cache strategy $C_i$ and fronthaul policy $F_i$ defined in Section II-A.
Minimizing the NDT with respect to the caching strategy \( \{c_{nT}\}_{n \in [N]} \) and fronthaul policy \( \{f_{nT}(d)\}_{n \in d, T_\pi \in T} \) for all vectors \( d \) yields the following integer problem

\[
\begin{align*}
\text{minimize} & \quad \max_{d} \delta^*_E(\{c_{nT}\}, \{f_{nT}(d)\}, d) + \delta^*_f(d) \\
\text{s.t.} & \quad \sum_{i=1}^K \sum_{T_\pi \subseteq [K_T]} (c_{nT} + f_{nT}(d)) = F, \forall n \in d, \forall d \\
& \quad \sum_{n=1}^N \sum_{T_\pi \subseteq [K_T]} c_{nT} \leq \mu FN, \forall i \in [K_T] \\
& \quad \frac{1}{F_T} \sum_{n \in d} \sum_{T_\pi \subseteq [K_T]} f_{nT}(d) \leq \delta^*_f(d), \forall i \in [K_T], \forall d \\
& \quad c_{n,T} \geq 0, f_{nT}(d) \geq 0 \\
& \quad 0 \leq \delta^*_f(d) \leq \delta_{F_{\text{max}}}.
\end{align*}
\] (25a)

where \( \delta^*_E(\{c_{nT}\}, \{f_{nT}(d)\}, d) \) is the minimum edge NDT (7) for given cache and fronthaul policies when the request vector is \( d \). In (25b), the equality constraints enforce that all \( F \) packets of each requested file are available collectively at the ENs after the fronthaul transmission; inequalities (25c) come from the fact that the size of the cache content \( C_i \) of each EN \( i \), which is given as \( \sum_{n=1}^N \sum_{T_\pi \subseteq [K_T]} c_{nT} \), is constrained by the cache capacity \( \mu FN \) (see (1)); inequalities (25d) follow from the definition of fronthaul NDT (6), since the left-hand side is the number of packets sent to EN \( i \) on the fronthaul for request vector \( d \); and inequalities (25e) impose that the fronthaul NDT is no larger than \( \delta_{F_{\text{max}}} = \frac{K_R(m_{\text{max}} - \mu K_T)^+}{K_T} \). (26)

This is because, as discussed in Section III-A the multiplicity of the requested files can be upper bounded without loss of generality by \( m_{\text{max}} \), and the maximum overall number of bits that are needed from the cloud to ensure this multiplicity is given as \( K_R(m_{\text{max}} - \mu K_T)^+L \) bits.

The optimum value of optimization problem (25) is lower bounded by substituting the maximum over all the request vector \( d \) with an average. In particular, since the number of ways to request all the \( K_R \) distinct files out of \( N \) library files is \( \pi(N, K_R) = N!/(N - K_R)! \), the lower-bounding problem can be written as

\[
\begin{align*}
\text{minimize} & \quad \frac{1}{\pi(N, K_R)} \sum_{d} \delta^*_E(\{c_{nT}\}, \{f_{nT}(d)\}, d) + \delta^*_f(d) \\
\text{s.t.} & \quad (25b) - (25f).
\end{align*}
\] (27a)

We now obtain a lower bound on the optimal value of problem (27) and hence also of problem (25). To this end, we first bound the minimum edge NDT \( \delta^*_E(\{c_{nT}\}, \{f_{nT}(d)\}, d) \) in (27a) by studying the number of packets that can be served in each block as a function of the availability of files at the ENs.

**Lemma 2**: Consider a single edge transmission block \( b \) in which a set \( \{W_{n,b}\}_{n=1}^L \) of \( L \) packets are sent to \( L \) distinct users
in set $\mathcal{R}_b \subseteq [K_R]$. In order for each user in $\mathcal{R}_b$ to be able to decode the desired packet without interference at the end of the block, the number $L$ of packets must be upper bounded as

$$L \leq \min_{l \in [L]} |\tau_l| n_T,$$

(28)

where for any packet $W_{n,l}$, $\tau_l$ denotes the subset of ENs that have access to it, either as part of the pre-stored contents at the EN’s cache or of the fronthaul received signals, i.e., $W_{n,l} \in \{C_i \cup F_i\}_{i \in \tau_l}$.

**Proof:** The proof follows from [2, Lemma 3] with the following differences. For a block $b$, each EN $i$ sends

$$x_i(b) = \sum_{l: i \in \tau_l} v_{in_{lf}}(b)s_{n_{lf}}(b),$$

(29)

and the received signal at user $k \in \mathcal{R}_b$, is given as

$$y_k(b) = \sum_{i=1}^{K_T} h_{ki}^T(b)x_i(b) + z_k(b)$$

(30)

$$= \sum_{i=1}^{K_T} h_{ki}^T(b) \sum_{l: i \in \tau_l} v_{in_{lf}}(b)s_{n_{lf}}(b) + z_k(b)$$

(31)

$$= \sum_{l=1}^{L} \sum_{i \in \tau_l} h_{ki}^T(b)v_{in_{lf}}(b)s_{n_{lf}}(b) + z_k(b).$$

(32)

From (32), the channel can be considered as a multi-antenna broadcast channel with $L$ transmitters, each having $|\tau_l| n_T$ antennas, that are connected to $L$ single-antenna users. By following the same steps as in [2, Eq. (28)-(36)] the proof is completed. □

Each subset $\tau$ of ENs needs to deliver parts $\{W_{n,\tau}\}$, $n \in d$, which consists of a total of $\sum_j (c_{d_j,\tau} + f_{d_j,\tau})$ packets. From Lemma 2, the number of necessary blocks is at least $\sum_j (c_{d_j,\tau} + f_{d_j,\tau})/(|\tau| n_T)$. By summing over all subsets $\tau$ and applying (7), the minimum edge NDT $\delta^*_E(\{c_{n_{\tau}}\}, \{f_{n_{\tau}}(d)\}, d)$ can be lower bounded as

$$\delta^*_E(\{c_{n_{\tau}}\}, \{f_{n_{\tau}}(d)\}, d) \geq \frac{1}{F} \sum_{i=1}^{K_T} \sum_{j=1}^{K_R} \sum_{\tau \subseteq [K_T]: |\tau| = i} \frac{c_{d_j,\tau} + f_{d_j,\tau}}{in_T}.$$  

(33)

This bound is instrumental in proving the following lemma, which completes the proof upon combination with the trivial lower bound $K_R/\min\{K_T n_T, K_R\}$ on the edge NDT.

**Lemma 3:** The optimal value of the problem (27) is lower bounded by

$$f_{\min} = \begin{cases}  
\frac{K_R(m^*(r) - \mu K_T)}{K_T n_T} + \frac{K_R}{m^*(r) n_T} \mu K_T < m^*(r) \\
\frac{K_R}{\mu K_T n_T}, \quad \mu K_T \geq m^*(r),
\end{cases}$$

(34)

where $m^*(r)$ is defined in (15).

**Proof:** The proof is presented in Appendix C. □
We lower bound the two terms in (27a) separately by starting with the minimum average edge NDT

\[
\frac{1}{\pi(N, K)} \sum_d \delta_E\{\{c_{n\tau}\}, \{f_{n\tau}(d)\}, d\} \geq (35a)
\]

\[
\frac{1}{F\pi(N, K)} \sum_{i=1}^{K_T} \sum_{\tau \subseteq [K_T] \colon \mid \tau \mid = i} \left[ \sum_{j=1}^{K_R} c_{d_j, \tau} + f_{d_j, \tau}(d) \right] (35b)
\]

\[
\frac{1}{F\pi(N, K)} \sum_{i=1}^{K_T} \frac{1}{K_R} \sum_{\tau \subseteq [K_T] \colon \mid \tau \mid = i} \left[ \sum_{j=1}^{K_R} \pi(N-1, K-1) \sum_{n=1}^{N} (c_{n\tau} + \tilde{f}_{n\tau}) \right] (35c)
\]

\[
\frac{K_R}{NF_nT} \sum_{i=1}^{K_T} \frac{1}{K_T} \sum_{\tau \subseteq [K_T] \colon \mid \tau \mid = i} \sum_{n=1}^{N} (c_{n\tau} + \tilde{f}_{n\tau}) (35d)
\]

\[
\frac{K_R}{NF_nT} \sum_{i=1}^{K_T} \frac{1}{K_T} \sum_{\tau \subseteq [K_T] \colon \mid \tau \mid = i} \sum_{n=1}^{N} (c_{n\tau} + \tilde{f}_{n\tau}) (35e)
\]

\[
\frac{K_R}{NF_nT} \sum_{i=1}^{K_T} \frac{1}{K_T} \sum_{\tau \subseteq [K_T] \colon \mid \tau \mid = i} \sum_{n=1}^{N} (c_{n\tau} + \tilde{f}_{n\tau}) (35f)
\]

where inequality (a) follows from inequality (33); equality (b) holds because, for any library file \(W_n\), the number of different request vectors that include file \(W_n\) is \(K_R \pi(N-1, K-1)\), i.e., \(\sum_d \sum_{j=1}^{K_R} W_{d_j, \tau} = K_R \pi(N-1, K-1) \sum_{n=1}^{N} W_{n, \tau}\), and hence we have

\[
\sum_d \sum_{j=1}^{K_R} c_{d_j, \tau} = K_R \pi(N-1, K-1) \sum_{n=1}^{N} c_{n\tau} (36a)
\]

and

\[
\sum_d \sum_{j=1}^{K_R} f_{d_j, \tau}(d) = \sum_d \sum_{n=1}^{N} f_{n\tau}(d) = K_R \pi(N-1, K-1) \sum_{n=1}^{N} \tilde{f}_{n\tau}, (36b)
\]

where \(\tilde{f}_{n\tau} = \sum_{d, n \in d} f_{n\tau}(d) / (K_R \pi(N-1, K-1))\) represents the number of packets in part \(W_{n, \tau}\) for each user in \(\tau\) received from the cloud; equality (c) follows the definition

\[
b_i = \sum_{\tau \subseteq [K_T] \colon \mid \tau \mid = i} \sum_{n=1}^{N} (c_{n\tau} + \tilde{f}_{n\tau}); (37)
\]

and inequality (d) applies the Cauchy-Schwarz inequality \((\sum_{i=1}^{n} u_i v_i)^2 \leq (\sum_{i=1}^{n} u_i^2)(\sum_{i=1}^{n} v_i^2)\) by setting \(u_i = \sqrt{b_i/i}\) and \(v_i = \sqrt{i/b_i}\).
To compute the term $\sum_{i=1}^{K_T} b_i$ in (35f), we impose the constraint (25b), obtaining

$$
\pi(N, K_R) = K_{FR} \sum_{n \in d} \sum_{i=1}^{K_T} \sum_{\tau \subseteq [K_T]} \sum_{|\tau| = i} (c_{n\tau} + f_{n\tau}(d)) \tag{38a}
$$

$$
= K_{R} \pi(N - 1, K_{R} - 1) \sum_{i=1}^{K_T} \sum_{\tau \subseteq [K_T]} \sum_{n=1}^{N} f_{n\tau} \tag{38b}
$$

$$
= K_{R} \pi(N - 1, K_{R} - 1) \sum_{i=1}^{K_T} b_i, \tag{38c}
$$

where equality (a) holds by summing up the constraints in (25b) for all $\pi(N, K_R)$ request vectors and for all $K_R$ files in each vector $d$; and equalities (b) and (c) follow from the equalities in (36) and the definition of $b_i$ in (37), respectively. From (38), we have the equality $\sum_{i=1}^{K_T} b_i = NF$.

We move on to lower bound the second term in (27a), i.e., the minimum fronthaul NDT $\delta_F(d)$. We start by bounding the size of the cached content. From (25c), we have

$$
\mu FN_K \geq \sum_{i=1}^{K_T} \sum_{n=1}^{N} c_{n\tau} = \sum_{n=1}^{N} \sum_{i=1}^{K_T} \sum_{\tau \subseteq [K_T]} c_{n\tau} \tag{39a}
$$

where inequality (a) holds by summing the inequalities in (25c) for all the $K_T$ ENs; and equality (b) comes from the fact that the size of the cached content of a file $W_n$ across the ENs is given as $\sum_{i=1}^{K_T} \sum_{\tau \subseteq [K_T]} c_{n\tau} = \sum_{i=1}^{K_T} i \sum_{\tau \subseteq [K_T]} c_{n\tau}$.

With the above inequality, the minimum fronthaul NDT can be bounded as

$$
\frac{1}{\pi(N, K_R)} \sum_{n \in d} \delta_F(d) \geq \frac{1}{\pi(N, K_R)} \sum_{n \in d} \sum_{i=1}^{K_T} \frac{1}{K_T} \sum_{\tau \subseteq [K_T]} \sum_{|\tau| = i} f_{n\tau} \tag{40a}
$$

$$
= \frac{1}{\pi(N, K_R)} \frac{1}{K_T} \sum_{n \in d} \sum_{i=1}^{K_T} \sum_{\tau \subseteq [K_T]} \sum_{|\tau| = i} f_{n\tau} \tag{40b}
$$

$$
= \frac{K_R}{NK_T} \sum_{i=1}^{K_T} \sum_{\tau \subseteq [K_T]} \sum_{|\tau| = i} f_{n\tau} \tag{40c}
$$

$$
= \frac{K_R}{NK_T} \sum_{i=1}^{K_T} \sum_{\tau \subseteq [K_T]} \sum_{|\tau| = i} f_{n\tau} \tag{40d}
$$

$$
\geq \frac{K_R}{K_T} \left( \frac{1}{NF} \sum_{i=1}^{K_T} \frac{K_T}{b_i - \mu K_T} \right), \tag{40e}
$$

where inequality (a) holds by averaging the constraints in (25d); equality (b) follows in a manner similar to equality (b) in (39a); equalities (c) and (d) follow the equality in (26b) and the definition of $b_i$ in (37), respectively; and inequality (e) holds by using (39a).
Now we can bound the minimum NDT by using (35f), (38) and (40a) as

\[
\frac{1}{\pi(N, K_R)} \sum_{\mathbf{d}} \delta_E'\{\{c_{\mathbf{n}}\}, \{f_{\mathbf{n}}(\mathbf{d})\}, \mathbf{d}\} + \delta_F'(\mathbf{d})
\]

(41a)

\[
\geq \frac{K_R}{NF} \frac{(NF)^2}{\sum_{i=1}^{K_T} ib_i + K_R \sum_{i=1}^{K_T} (ib_i - \mu K_T)}
\]

(41b)

\[
= \frac{K_R(x - \mu K_T)}{K_T r} + \frac{K_R}{n_T} x
\]

(41c)

where in the last step, we have defined the variable \(x = \sum_{i=1}^{K_T} ib_i/(NF)\). Since, by (37), the expression \(\sum_{i=1}^{K_T} ib_i\) is the overall number of packets of all library files that are available upon fronthaul transmission at subsets of ENs of any size \(i\), the variable \(x\) can be interpreted as the average multiplicity of each file at the ENs after fronthaul transmission.

From (41c), we define the function

\[
f(x) = \frac{K_R(x - \mu K_T)}{K_T r} + \frac{K_R}{n_T} x.
\]

(42)

To complete the proof, we now minimize \(f(x)\) in (42) over \(x\). To this end, we first focus on defining the domain of \(x\). From (25f) and (40a), we have the bounds \(K_R(x - \mu K_T)/(K_T r) \leq \delta_F'(\mathbf{d}) \leq \delta_{Fmax}\), yielding the upper bound \(x \leq (m_{max} - \mu K)^+ + \mu K_T = \max\{m_{max}, \mu K_T\}\). We also have the inequality \(x \geq \mu K_T\) due to the bound \(\delta_F'(\mathbf{d}) \geq 0\).

Furthermore, from (38), we have the inequality \(\sum_{i=1}^{K_T} b_i/NF \geq 1\), yielding \(x = \sum_{i=1}^{K_T} b_i/NF \geq 1\). In summary, variable \(x\) needs to lie in the interval \(\max\{1, \mu K_T\} = x_{min} \leq x \leq x_{max} = \max\{m_{max}, \mu K_T\}\). We then turn to minimizing the function \(f(x)\) in the interval \(x \in [x_{min}, x_{max}]\). Function \(f(x)\) is convex for \(x > 0\), and the only stationary point is \(x = \sqrt{K_T r/n_T}\), i.e., \(f'\left(\sqrt{K_T r/n_T}\right) = 0\). Therefore, the desired minimum \(f_{min}\) is given as

\[
f_{min} = \begin{cases} 
    f\left(\sqrt{K_T r/n_T}\right), & \text{if } x_{min} \leq \sqrt{K_T r/n_T} \leq x_{max} \\
    \min\{f(x_{min}), f(x_{max})\}, & \text{otherwise},
\end{cases}
\]

(43)

which is as reported in (34).

**APPENDIX D**

**PROOF OF PROPOSITION 4**

To prove Proposition 4 we first derive a lower bound \(\delta_{lb}(\mu, r)\), which is looser than the lower bound \(\delta_{lb}(\mu, r)\) in Proposition 2 but more tractable. The bound leverages Proposition 2, Proposition 3 and the convexity of the minimum NDT \(\delta^*(\mu, r)\) as stated in Lemma 1. The lower bounds \(\delta_{lb}(\mu, r)\) and \(\delta_{lb}(\mu, r)\) are illustrated in Fig. 8.

**Lemma 4:** For any \(r \in [0, 1]\), and \(\mu\) with \(\mu K_T \leq m_{max}\), we have \(\delta_{lb}(\mu, r) \leq \delta_{lb}(\mu, r)\), where \(\delta_{lb}(\mu, r)\) is given in (16) and we have defined

\[
\delta_{lb}(\mu, r) = \frac{(i + 2 - \mu K_T)K_R}{(i + 1)n_T} + \frac{(\mu K_T - 2)K_R}{(i + 2)n_T}
\]

(44)
Fig. 8. Achievable NDT $\delta_{ach}(\mu, r)$ and lower bounds $\delta_{lb}(\mu, r)$ and $\delta_{ub}(\mu, r)$: plot (a) shows Case 1 \((45a)\), plot (b) shows Case 2 \((45b)\).

for $\mu K_T \in [i, i + 1)$, with $m_{min}(r) \leq i \leq m_{max} - 1$; and

$$
\delta_{lb}^*(\mu, r) = \begin{cases} 
\frac{K_{K_T}(m^*(r) - \mu K_T)}{K_T r} + \frac{K_{r}}{m^*(r)n_T} & \text{if } K_{K_T} r > [m_{min}(r)(m_{min}(r) - 1), m_{max}^2(m_{min}(r))] \\
\frac{K_{K_T}(m_{min}(r) - \mu K_T)}{K_T r} + \delta_{lb}^*(m_{min}(r), r) & \text{if } K_{K_T} r \in [m_{max}^2(r), m_{min}(r)(m_{min}(r) + 1)]
\end{cases} \tag{45a}$$

for $\mu K_T \leq m_{min}(r)$, where $m^*(r)$ is given in \((15)\).

Proof: From Proposition \([3]\) we have the equality $\delta^*(\mu, r) = \delta_{ach}(\mu, r)$ for $\mu K_T \in \{m_{min}(r) + 1, \ldots, m_{max}\}$. Furthermore, from Lemma \([1]\) we know that the minimum NDT $\delta^*(\mu, r)$ is a convex function of $\mu$ for any $r \geq 0$. Define $g_r(\mu)$ as a subgradient of the minimum NDT $\delta^*(\mu, r)$ at $\mu \in [0, 1]$ for a fixed value of $r$. Consider any two points $\mu_1$ and $\mu_2$, where $\mu_1 K_T \in \{m_{min}(r) + 1, \ldots, m_{max}\}$ and $\mu_2$ is arbitrary. By a known convex property of convex functions (see \([16]\)), we have the inequality

$$
\delta^*(\mu_2, r) \geq g_r(\mu_1)(\mu_2 - \mu_1)K_T + \delta^*(\mu_1, r). \tag{46}
$$

Therefore, choosing $\mu_2$ so that $\mu_2 K_T = \mu_1 K_T + 1$ in \((46)\) yields

$$
g_r(\mu_1) \leq \delta^*(\mu_1 + 1/K_T, r) - \delta^*(\mu_1, r). \tag{47}
$$

For any sub-interval $\mu K_T \in [i, i + 1)$, with $m_{min}(r) \leq i \leq m_{max} - 1$, by setting $\mu_1 = (i + 1)/K_T$ in \((47)\), we have the bound $g_r((i + 1)/K_T) \leq g_{max} \triangleq \delta^*((i + 2)/K_T, r) - \delta^*((i + 1)/K_T, r)$. Combining with \((46)\) and setting $\mu_2 = \mu$, we have the inequality $\delta^*(\mu, r) \geq g_{max} \cdot (\mu K_T - i - 1) + \delta^*((i + 1)/K_T, r)$, which gives \((44)\) by Lemma 1.

For the remaining interval $\mu K_T \leq m_{min}(r)$, we distinguish the two cases illustrated in Fig. 8(a) and (b).

Case 1: $K_T r/n_T \in [m_{min}(r)(m_{min}(r) - 1), m_{min}(r)^2]$. By \([11]\) and \([15]\) in this range, we have the inequality $m^*(r) =
\[ \sqrt{K_T r/n_T} \leq \mu_{\min}(r). \] It can be directly verified that \( \delta_{lb}(\mu, r) \) in (45a) is no larger than \( \delta_{lb}(\mu, r) \) in (16a) for \( \mu K_T \leq m^*(r) \).

Instead, for \( \mu K_T / n_T \), since both \( \delta_{lb}(\mu, r) \) in (45a) and \( K_R/(\mu K_T n_T) \) are decreasing functions of \( \mu \), we have \( \delta_{lb}(\mu, r) \leq K_R/(\mu K_T n_T) \), which implies that \( \delta_{lb}(\mu, r) \leq \delta^*(\mu, r) \) in (16b), as illustrated in Fig. 8(a).

Case 2: \( K_T r/n_T \in [m_{\min}(r)^2, m_{\min}(r)(m_{\min}(r) + 1)] \). By (11) and (15) in this range, we have the inequality

\[ m^*(r) = \sqrt{K_T r/n_T} \geq m_{\min}(r). \] By setting \( \mu_1 = (m_{\min}(r) + 1)/K_T \) in (47), we have \( g_r((m_{\min}(r) + 1)/K_T) \leq g'_{\max} \frac{\delta^*}{m^*(r) + 2/K_T, r} - \delta^*((m_{\min}(r) + 1)/K_T, r). \) Combining with (46) and setting \( \mu_2 = m_{\min}(r)/K_T \), we have the inequality \( \delta^*(m_{\min}(r)/K_T, r) \geq -g'_{\max} + \delta^*((m_{\min}(r) + 1)/K_T, r) \), which gives the lower bound \( \delta_{lb}(m_{\min}(r)/K_T, r) \). It is easy to verify the inequality \( \delta_{lb}(m_{\min}(r)/K_T, r) \leq \delta_{lb}(m_{\min}(r)/K_T, r) \). Combining this with the fact that \( \delta_{lb}(\mu, r) \) in (45b) and \( \delta_{lb}(\mu, r) \) in (16a) are linear and parallel for \( \mu K_T \leq m_{\min}(r) \), we have \( \delta_{lb}(m_{\min}(r)/K_T, r) \leq \delta_{lb}(\mu, K_T) \) in this range (see Fig. 8(b)). This completes the proof.

Using the lower bound \( \delta_{lb}(\mu, r) \), we can now directly compute the gap between the achievable NDT \( \delta_{ach}(\mu, r) \) in Proposition 1 and the minimum NDT \( \delta^*(\mu, r) \). Specifically, for \( \mu K_T \in [i, i+1) \), with \( m_{\min}(r) \leq i \leq m_{\max} - 1 \), from (14) and (44), we verify that

\[ \frac{\delta_{ach}(\mu, r)}{\delta_{lb}(\mu, r)} \overset{(a)}{\leq} \frac{\delta_{ach}(\mu = i/K_T, r)}{\delta_{lb}(\mu = i/K_T, r)} = 1 + \frac{2}{i + 3i} \overset{(b)}{\leq} \frac{2}{i} \leq \frac{2}{3}, \] (48)

where inequality (a) holds because \( \delta_{ach}(\mu, r) \) and \( \delta_{lb}(\mu, r) \) are both linearly decreasing and they coincide at the endpoint \( \mu K_T = i + 1 \). For \( \mu K_T \leq m_{\min}(r) \) in Case 1, from (14) and (45a), the gap is given as

\[ \frac{\delta_{ach}(\mu, r)}{\delta_{lb}(\mu, r)} \overset{(a)}{\leq} \frac{\delta_{ach}(\mu = m_{\min}(r)/K_T, r)}{\delta_{lb}(\mu = m_{\min}(r)/K_T, r)} = 1 + \frac{2}{1/m_{\min}(r)n_T} \overset{(b)}{\leq} \frac{m_{\min}(r)}{(m_{\min}(r) + 1)^2} \overset{(c)}{\leq} \sqrt{2}, \] (49)

where inequality (a) holds because \( \delta_{ach}(\mu, r) \) and \( \delta_{lb}(\mu, r) \) decrease with the same slope and the maximum ratio is at the endpoint \( \mu K_T = m_{\min}(r) \); inequality (b) holds due to the constraints \( m^*(r) \in [\sqrt{m_{\min}(r)(m_{\min}(r) - 1), m_{\min}(r)] \); and inequality (c) holds for any \( m_{\min}(r) \geq 2 \), while for \( m_{\min}(r) = 1 \), we have \( K_T r/n_T \in [0, 1] \) and \( \mu \in [0, 1] \), it has been proved that \( \delta_{ach}(\mu, r) \) is optimal in Proposition 3. Finally, for \( \mu K_T \leq m_{\min}(r) \) in Case 2, from (14) and (45b), the gap is given as

\[ \frac{\delta_{ach}(\mu, r)}{\delta_{lb}(\mu, r)} \overset{(a)}{\leq} \frac{\delta_{ach}(\mu = m_{\min}(r)/K_T, r)}{\delta_{lb}(\mu = m_{\min}(r)/K_T, r)} = 1 + \frac{2}{m_{\min}(r) + 3m_{\min}(r)} \overset{(b)}{\leq} \frac{3}{5}, \] (50)

where inequality (a) holds as inequality (a) in (49a), completing the proof.

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