Vector boson polarizations in the decay of the
Standard Model Higgs

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ABSTRACT: The kinematic distributions of the lepton pairs produced in the decay of
the Standard Model Higgs to ZZ and WW are related to the polarization fractions of
the virtual vector bosons. The full amplitude can be decomposed analytically into a sum
of polarized terms. Several observables, in particular the invariant mass of two charged
leptons, one from each of the bosons, and the lepton angular distribution in the vector
boson center of mass are shown to be sensitive to the boson polarizations.
1 Introduction

Vector Boson (VB) polarizations have attracted a great deal of attention in recent times. On the one hand, for single boson inclusive production, they can be unambiguously predicted in the Standard Model. On the other hand, Vector Boson Scattering of longitudinally polarized is a crucial probe of the ElectroWeak Symmetry Breaking mechanism.

Since experiments can only observe the boson decay products within a limited subset of the full phase space, extracting vector polarizations is not straightforward.

VB polarizations at the LHC have been studied in a number of papers [1–6]. Both CMS and ATLAS have measured the W polarization fractions in the W+ jets [7, 8] channel and in t\bar{t} events [9, 10]. Z polarization fractions at the LHC have been measured in [11, 12]. The first polarization measurement at 13 TeV has been performed by ATLAS in WZ production [13].

In [14–16] a simple and natural way to define cross sections corresponding to vector bosons of definite polarization has been proposed. This allows to use polarized templates in fitting the data. Recently MadGraph5_aMC@NLO has introduced the possibility of generating polarized amplitudes [17].

In this paper, I discuss VB polarizations in the decay of the Standard Model Higgs to ZZ and WW, where only one of the VB can be on mass shell. The process is so simple that the decomposition of the full amplitude can be performed analytically, yielding a compact and transparent expression. The polarization fractions in Higgs decay are completely determined as in the case of single boson inclusive production. Their measurement would provide a test of the SM. This is a new take on a process which has been studied [18–23] since long before the discovery of the Higgs [24, 25]. Precise predictions, including QCD and ElectroWeak NLO corrections, have been been provided in refs. [26–28]. An analysis including dimension six EFT operators can be found in [29].
The Higgs decay into four fermions has been studied experimentally, albeit with limited statistics, in order to determine the spin and parity of the Higgs, to set limits on its coupling strength and anomalous couplings to vector bosons [30–34].

In sect. 2 I recall the main ingredients needed in the analysis. In sect. 3–4 I discuss a number of observables whose distributions depend on the VB polarizations. Finally, in sect. 5 I discuss how the distributions are modified in the presence of leptonic cuts in a simple LHC-like framework.

2 Vector boson polarizations and angular distribution of its decay products

Let us consider an amplitude in which a weak vector boson decays to a final state fermion pair. In the Unitary Gauge, it can be expressed as

$$\mathcal{M} = \mathcal{M}_\mu k^\mu - M^2 + i\Gamma M \left( -g^{\mu\nu} + \frac{k^\mu k^\nu}{M^2} \right) J^\mu_h(f, f'),$$

(2.1)

where

$$J^h_\mu(f, f') = \left[ -i g \bar{\psi}^h \gamma_\mu \left( c_L P_L + c_R P_R \right) \psi^h \right].$$

(2.2)

$M$ and $\Gamma$ are the vector boson mass and width, respectively, while $c_R$ and $c_L$ are the right and left handed couplings of the fermions to the $W^\pm (Z)$, as shown in tab. 1, and $h$ denotes the chirality of the fermion.

The polarization tensor, even when $k^2 \neq M^2$, can be expressed in terms of four polarization vectors [35]:

$$-g^{\mu\nu} + \frac{k^\mu k^\nu}{M^2} = \sum_{\lambda=1}^{4} \varepsilon^{\mu}_\lambda(k)\varepsilon^{\nu*}_\lambda(k).$$

(2.3)

|     | $c_L$          | $c_R$          | $g_{HVW}$        |
|-----|----------------|----------------|------------------|
| $W$ | $1/(s\sqrt{2})$ | 0              | $M_W/s$          |
| $Z$ | $(I_{W,f} - s^2 Q_f)/(s c)$ | $-s Q_f/c$ | $M_Z/(s c)$    |

Table 1. Weak couplings. $c = \cos \theta_W = M_W/M_Z$ , $s = \sin \theta_W$

In a frame in which the off shell vector boson propagates along the $(\theta_V, \phi_V)$ axis, with three momentum $\kappa$, energy $E$ and invariant mass $\sqrt{Q^2} = \sqrt{E^2 - \kappa^2}$, the polarization
vectors read:
\[ \varepsilon_L^\mu = \frac{1}{\sqrt{2}} \left(0, \cos \theta_V \cos \phi_V + i \sin \theta_V \sin \phi_V, \cos \theta_V \sin \phi_V - i \cos \theta_V, -\sin \theta_V\right) \quad \text{(left)}, \]
\[ \varepsilon_R^\mu = \frac{1}{\sqrt{2}} \left(0, -\cos \theta_V \cos \phi_V + i \sin \theta_V \sin \phi_V, -\cos \theta_V \sin \phi_V + i \cos \theta_V, \sin \theta_V\right) \quad \text{(right)}, \]
\[ \varepsilon_A^\mu = \sqrt{\frac{Q^2 - M^2}{Q^2 M^2}} \left(E, \kappa \sin \theta_V \cos \phi_V, \kappa \sin \theta_V \sin \phi_V, \kappa \cos \theta_V\right) \quad \text{(auxiliary)}.
\]

On shell, the auxiliary polarization is zero and the longitudinal polarization reduces to the usual expression.

Consider the decays \( H \to Z^*Z^* \to \mu^+\mu^-e^+e^- \) and \( H \to W^+W^+ \to \bar{\nu}_\mu\mu^-e^+\nu_e \) in the center of mass of the Higgs. At lowest order the decay is described by a single, double resonant, diagram. The corresponding amplitude is
\[ \mathcal{M}_{h_1h_2} = -ig_{\mu
u}h d^{h_1}(f_1,f_1') \sum \frac{\epsilon_\lambda^\mu(k_1)\epsilon_\lambda^\nu(k_1)}{k_1^2 - M^2 + i\Gamma M} \sum \frac{\epsilon_\lambda^\nu(k_2)\epsilon_\lambda^\nu(k_2)}{k_2^2 - M^2 + i\Gamma M} d^{h_2}(f_2,f_2') \]
\[ = -ig_{\mu
u}h d^{h_1}(f_1,f_1') \left( \epsilon_0^\rho(k_1)\epsilon_0^\rho(k_2) \epsilon_0(k_1) \cdot \epsilon_0(k_2) + \epsilon_1^\rho(k_1)\epsilon_1^\rho(k_2) + \epsilon_R^\rho(k_1)\epsilon_R^\rho(k_2) \right) d^{h_2}(f_2,f_2') \frac{1}{(k_1^2 - M^2 + i\Gamma M)(k_2^2 - M^2 + i\Gamma M)} \quad \text{(2.5)}
\]

Notice that, since \( M_H < 2 M_V \), the double pole approximation of refs.\[36–40\] is not applicable.

Defining the decay amplitudes of the Vector Bosons as
\[ \mathcal{M}_{\lambda,h}^D(i) = J_{\mu}^{h_i}(f_i,f_i') \varepsilon_\lambda^\mu \quad \text{(2.6)} \]
and using \( \varepsilon_R^* = -\varepsilon_L^R \) one obtains
\[ \mathcal{M}_{h_1h_2} = -ig_{\mu
u}h \frac{f_0 \mathcal{M}_{0,h_1}^D(1)\mathcal{M}_{0,h_2}^D(2) \epsilon_0(k_1) \cdot \epsilon_0(k_2) - f_L \mathcal{M}_{L,h_1}^D(1)\mathcal{M}_{L,h_2}^D(2) - f_R \mathcal{M}_{R,h_1}^D(1)\mathcal{M}_{R,h_2}^D(2)}{(k_1^2 - M^2 + i\Gamma M)(k_2^2 - M^2 + i\Gamma M)} \quad \text{(2.7)}
\]
where we have introduced factors \( f_0, f_L, f_R \) to keep track from which vector polarization each term in the final result originates. \( f_0, f_L, f_R \) are equal to one in the Standard Model, and one can envisage to measure them experimentally as a test of the SM.

The decay amplitudes of the Vector Bosons depend on their polarization and the fermion chirality, which we denote as \(+/–\). In the rest frame of the \( ff' \) pair, they are:
\[ \mathcal{M}_{0,–}^D = ig_c^L 2E \sin \theta, \quad \mathcal{M}_{0,–}^D = ig_c^R 2E \sin \theta, \quad \text{(2.8)} \]
\[ \mathcal{M}_{L,–}^D = ig_c^L \sqrt{2}E \left(1 - \cos \theta\right) e^{-i\phi}, \quad \mathcal{M}_{L,–}^D = -ig_c^R \sqrt{2}E \left(1 + \cos \theta\right) e^{-i\phi}, \quad \text{(2.9)} \]
\[ \mathcal{M}_{R,–}^D = ig_c^L \sqrt{2}E \left(1 + \cos \theta\right) e^{i\phi}, \quad \mathcal{M}_{R,–}^D = -ig_c^R \sqrt{2}E \left(1 - \cos \theta\right) e^{i\phi}, \quad \text{(2.10)} \]
where \((\theta, \phi)\) are polar and azimuthal angles of the positively charged lepton (antineutrino in the \( W^- \) case), relative to the boson direction in the laboratory frame. Notice that, if the boson propagates in the negative \( z \) direction \( \phi \to -\phi \). If \( Q_\lambda^2 \) \((\lambda = 1,2)\) are the
invariant masses squared of the two fermion pairs, \( E \) in eqs.(2.8–2.10) is equal \( \sqrt{Q_1^2/2} \). For massless leptons, the decay amplitude for the auxiliary polarization is zero because \( c_A^\mu \) is proportional to the four–momentum of the virtual boson. Eqs.(2.8–2.10) show that each polarization is uniquely associated with a specific angular distribution of the charged lepton, even when the \( V \) boson is off mass shell and the notion of a vector boson with definite polarization is ill–defined.

The squared amplitude, summed over fermion polarizations, becomes:

\[
\mathcal{M}^2 = \frac{4 g_{HV}^2 Q_1^2 Q_2^2}{(Q_1^2 - M^2)^2 + \Gamma^2 M^2} \left[ + f_L^2 \left( c_R^4 (1 + \cos \theta_1)^2 (1 + \cos \theta_2)^2 + c_L^2 c_R^2 (1 + \cos \theta_1)^2 (1 - \cos \theta_2)^2 \
+ c_L^2 c_R^2 (1 - \cos \theta_1)^2 (1 + \cos \theta_2)^2 + c_L^4 (1 - \cos \theta_1)^2 (1 - \cos \theta_2)^2 \right) \right]
+ f_R^2 \left( c_L^4 (1 + \cos \theta_1)^2 (1 + \cos \theta_2)^2 + c_L^2 c_R^2 (1 + \cos \theta_1)^2 (1 - \cos \theta_2)^2 \
+ c_L^2 c_R^2 (1 - \cos \theta_1)^2 (1 + \cos \theta_2)^2 + c_R^4 (1 - \cos \theta_1)^2 (1 - \cos \theta_2)^2 \right) \right]
+ 4 K^2 f_0 f_L (c_L^2 + c_R^2)^2 \sin^2 \theta_1 \sin^2 \theta_2 
- 4 K f_0 \left( f_L \left( c_R^4 (1 + \cos \theta_1)^2 (1 + \cos \theta_2) + c_L^2 (1 - \cos \theta_1)^2 (1 - \cos \theta_2) \right) \
- c_L^2 c_R^2 ((1 + \cos \theta_1) (1 - \cos \theta_2) + (1 - \cos \theta_1) (1 + \cos \theta_2)) \right) \sin \theta_1 \sin \theta_2 \sin \phi 
+ 2 f_L f_R (c_L^2 + c_R^2)^2 \sin^2 \theta_1 \sin^2 \theta_2 \cos(2 \phi) \right],
\]

where \( \phi = \phi_1 - \phi_2 \).

\( K \) denotes the product of longitudinal polarization vectors:

\[
K = \varepsilon_0(k_1) \cdot \varepsilon_0(k_2) = \frac{M_H^2 - Q_1^2 - Q_2^2}{\sqrt{4 Q_1^2 Q_2^2}},
\]

since, in the Higgs rest frame we have

\[
E_1 = \frac{m_H^2 + Q_1^2 - Q_2^2}{2M_H}, \quad E_2 = m_H - E_1, \quad \kappa_{1,2} = \frac{\sqrt{(m_H^2 + Q_1^2 - Q_2^2)^2 - 4m_H^2 Q_1^2}}{2M_H}.
\]

\( K \) is larger when the invariant masses of the virtual vector bosons are small and, therefore, longitudinal polarizations yield a larger fraction of soft fermion pairs.

The interference terms in eq.(2.11) cancel when the squared amplitude is integrated over the full range of the angle \( \phi \), or, equivalently, when the charged lepton can be observed for any value of \( \phi \).
Notice that the azimuthal modulation depends quite strongly on the polar angles of the two decays. The amplitude of the oscillation is maximal when both $\theta_1$ and $\theta_2$ are equal to $\pi/2$ and becomes zero when either angle is zero or $\pi$.

Figure 1. Distribution of the invariant mass of the $\ell^-\ell^+$ pairs. The curves on the right are normalized to unit integral.

Figure 2. Angular distribution of the positively charged lepton in the $Z$ CoM.

3 The $H \rightarrow ZZ \rightarrow 4 \ell$ channel

Using $M_H = 125.25$ GeV, $M_Z = 91.19$ GeV, $\Gamma_Z = 2.50$ GeV, $\sin(\theta_W)^2 = 0.23$, $\alpha = \frac{1}{127}$ the differential decay width with respect to $\phi$ in $H \rightarrow ZZ$ is

$$\frac{d\Gamma}{d\phi} = (4.216 f_0^2 + 1.376 (f_L^2 + f_R^2) - 7.8 \times 10^{-2} f_0 (f_L + f_R) \cos \phi$$

$$+ 0.688 f_L f_R \cos(2 \phi)) \times 10^{-7} \frac{\text{MeV}}{\text{degree}},$$

(3.1)

Taking $f_0 = f_L = f_R = 1$ it becomes

$$\frac{d\Gamma}{d\phi} = (6.968 - 0.156 \cos \phi + 0.688 \cos(2 \phi)) \times 10^{-7} \frac{\text{MeV}}{\text{degree}},$$

(3.2)
in good agreement with refs. [26, 28]. The coefficient of the \( \cos(2\phi) \) term is about 10% of the constant one and about four times larger than the coefficient of the \( \cos\phi \) term. The presence of a large contribution proportional to \( \cos(2\phi) \) was pointed out in [26], while the smaller term proportional to \( \cos\phi \) went unnoticed. The longitudinal longitudinal component accounts for about 60% of the partial width while each of the left left and right right components contribute 20%.

**Figure 3.** Invariant mass distribution of the \( e^+\mu^+ \) pairs. No lepton cut is applied. The curves on the right are normalized to unit integral.

**Figure 4.** Azimuthal separation, in the Higgs center of mass system, between \( e^+ \) and \( \mu^+ \) for ZZ events in the absence of cuts.

One could wonder whether the distributions discussed in this note have any chance of being measured. We recall that CMS, with 35 \( fb^{-1} \) at 13 TeV, collected about 50 four lepton events on the Higgs peak [34, 41]. For comparison, Run 2 has provided about 140 \( fb^{-1} \) to each large experiment; Run 3 is expected to accumulate about 200 \( fb^{-1} \) at 14 TeV and finally HL-LHC will deliver 3000 \( fb^{-1} \) [42, 43]. Therefore we can expect of the order of 500 events by the end of Run 3 and thousands of additional events from HL-LHC.

Fig. 1 shows the distribution of the invariant mass of the same flavour, opposite charged leptons. The curves in the right hand side plot are normalized. In addition to the expected peak at the Z mass, the curves display a wide increase at small invariant masses. The
secondary peak is wider and extends to smaller value for the longitudinally polarized virtual Z’s than for the transversely polarized ones.

Fig. 2 presents the distribution of the angle between the positively charged lepton and the direction of flight of the Z boson, see eq.(2.11). The longitudinally polarized part is distributed as \( \sin^2 \theta \). The right and left terms depend only mildly on the angle, showing a weak preference for the forward(backward) direction in the right(left) polarized case. These distributions coincide with the familiar ones for on shell Z decay even though in \( H \to ZZ \) each Z is on mass shell only about 50% of the times.

In fig. 3 we study the invariant mass of the two positively charged leptons. This quantity has the interesting property of depending on all five independent variables which describe the decay of the Higgs boson to four fermions. On the right hand side we show the same curves normalized to unit integral.

The \( e^+\mu^+ \) invariant mass shows some dependence on the underlying vector boson polarizations: the longitudinal longitudinal result is more peaked that the LL and RR ones. It is harder than the LL distribution. The RR curve is the widest one, with a tail at larger invariant masses. The contribution of the interference terms in eq.(2.11) is not zero. However, it is about two orders of magnitude smaller than those in fig. 3 and, therefore, not plotted.

Fig. 4 shows the azimuthal separation of the two positively charged leptons in the Higgs center of mass system, with the decay axis in the \( z \) direction. The result agrees with eq.(3.2). The RL interference term provides the bulk of the azimuthal dependence. The term proportional to \( \cos \phi \) is due to the interference between the longitudinal component and the R and L ones. NLO Electroweak corrections for the \( \Delta \phi \) differential distribution have been computed in refs. [27, 28]. They are about -1% for \( \Delta \phi = \pi \) and +4% for \( \Delta \phi = 0 \).

4 The \( H \to WW \to e\mu\nu\nu \) channel

Using \( M_W = 80.38 \) GeV, \( \Gamma_W = 2.10 \) GeV, the differential decay width with respect to \( \phi \) in \( H \to WW \) is

\[
\frac{d\Gamma}{d\phi} = (1.762 f_0^2 + 0.576 (f_L^2 + f_R^2)) + (1.275 f_0 (f_L + f_R)) \cos \phi + 0.651 f_L f_R \cos(2 \phi)) \times 10^{-5} \text{ MeV degree}.
\]  

(4.1)

Taking \( f_0 = f_L = f_R = 1 \) it becomes

\[
\frac{d\Gamma}{d\phi} = (2.913 + 2.550 \cos \phi + 0.651 \cos(2 \phi)) \times 10^{-5} \text{ MeV degree}.
\]  

(4.2)

Eq.(4.2) shows that, in the \( W \) case, the coefficient of the \( \cos \phi \) term is comparable in magnitude with the constant term and about four times larger than the coefficient of the \( \cos(2 \phi) \) one, in general agreement with ref. [28] which, however, shows a different though related variable, the difference in azimuth in the laboratory transverse plane, which is easier to measure. Notice that for the negatively charged W, the \( \ell^- \) has opposite three momentum in the W rest frame compared to the antiparticle, whose distribution is described in eq.(2.11).
This implies that for the negatively charged lepton $\phi \rightarrow \pi + \phi$ and $\cos \theta \rightarrow -\cos \theta$. The longitudinal longitudinal component accounts for about 60% of the partial width while the two left left and right right components contribute 20%.

Fig. 5 presents the distribution of the angle between the positively charged lepton and the direction of flight of the $W$ boson in the reference frame of the latter. The longitudinal polarized part is distributed as $\sin^2 \theta$, while the right and left terms are proportional to $(1 \pm \cos \theta)^2$ respectively, as in on shell $W$ decays.

![Figure 5](image)

**Figure 5.** Angular distribution of the positively charged lepton in the $W$ CoM.

![Figure 6](image)

**Figure 6.** Invariant mass distribution of the $e^-\mu^+$ pair. No lepton cut is applied. The curves on the right are normalized to unit integral.

In fig. 6 we study the invariant mass of the two charged leptons. On the right hand side we show the same curves normalized to unit integral. In the $W$ case this variable has the additional advantage of not requiring the identification of the rest frame of the $W$ pair which is notoriously extremely difficult to determine because of the two neutrinos in the final state.

The $\ell^-\ell^+$ invariant mass again shows some dependence on the underlying vector boson polarizations: the RR and LL curves are identical, as expected, and softer than the longitudinal longitudinal result. The contribution of the interference terms in eq.(2.11) are
Figure 7. Azimuthal separation, in the laboratory frame, between $e^+$ and $\mu^-$ for $WW$ events in the absence of cuts.

not zero, however, they are about two orders of magnitude smaller than those in fig. 6.

Fig. 7 shows the azimuthal separation of the two charged leptons in the laboratory frame. Eq.(4.2) shows that in the Higgs center of mass system all diagonal terms are independent of the angular separation in the plane orthogonal to the decay axis. In the lab, however, all polarization combinations depend non trivially on the difference in azimuth between the leptons. The full distribution favors small separations. At large values of $\Delta \phi$, there is a partial cancellation between the longitudinal longitudinal and transverse components, and the longitudinal transverse interferences. Large interferences in $\Delta \phi$ have also been reported in polarized $W^+W^-$ production at the LHC [6]. NLO Electroweak corrections for the $\Delta \phi$ differential distribution have been computed in ref. [28]. They are about 2.5% for $\Delta \phi = \pi$ and 3.5% for $\Delta \phi = 0$.

5 A preliminary assessment of the effect of cuts in the LHC environment

In this section I investigate whether the differences of the kinematical distributions in the decay of polarized vector bosons survive in the LHC environment, where acceptance cuts and additional requirements, to improve the separation of signal from background, are necessary. Starting from a sample of $gg \rightarrow H$ events at leading order, the Higgs boson has been subsequently decayed to four leptons according to eq.(2.11), with a uniform angular distribution of the decay axis. In this simplified setup, all affects due to the transverse momentum of the Higgs boson are neglected.

The set of cuts for the $ZZ$ case have been extracted from ref. [32] by CMS.

- $p_T\ell > 7$ GeV, $|\eta| < 2.5$ (acceptance)
- $12$ GeV $< m_{\ell^+\ell^-} < 120$ GeV, $m_{4\ell} > 70$ GeV
- $\Delta R_{\ell\ell} > 0.02$, $m_{\ell^+\ell^-} > 4$ GeV (veto on soft, collinear pairs)
- $N_\ell(pT > 20$ GeV $) > 0$, $N_\ell(pT > 10$ GeV $) > 1$ (high $pT$ leptons)
Fig. 8 shows the mass distribution of the $e^+\mu^+$ pairs for each of the six combinations of $Z$ polarizations, for the sum of the RR, LL and longitudinal longitudinal contributions and for the sum of all contributions. The shape of the RR, LL and longitudinal longitudinal distributions are very similar to the ones in the inclusive case while the normalization decreases by about a factor of three. The LR interference term is small but not negligible. Its contribution is positive at the small and large end of the mass range, while it is negative in the peak region $m_{e^+\mu^+} \approx 30$ GeV.

The transverse momentum distribution of the $e^+$ for $ZZ$ events is shown in fig. 9 for the RR, LL and longitudinal longitudinal cases. The interference contributions are negligible. The three distribution show small differences. As expected, they exhibit two broad peaks at about half the value of the preferred lepton pair masses in fig. 1.

Fig. 10 presents the distribution of the angle between the positively charged lepton and the direction of flight of the $Z$ boson for the RR, LL and longitudinal longitudinal contributions. The interference terms are negligible. The shape of the distributions are very similar to those in fig. 2.
Figure 10. Angular distribution of the positively charged lepton in the $Z$ CoM for the RR, LL and longitudinal contributions. The interference terms are negligible.

Fig. 11 presents the azimuthal separation between $e^+$ and $\mu^+$ in the plane transverse to the Higgs decay axis. The full result is shown alongside the contribution of each polarization combination. Most of the curves are almost flat with a modest decrease at $\Delta \phi = 0, \pi$. The exception is the LR term which behaves basically as $\cos(2\phi)$. The ratio of the different contributions are very similar to those in fig. 4.

Figure 11. Azimuthal separation, in the Higgs center of mass system, between $e^+$ and $\mu^+$ for $ZZ$ events in the presence of cuts.

The set of cuts for the $WW$ case have been taken from ref. [44] by ATLAS.

- $p_T \ell > 15$ GeV, $|\eta_\ell| < 2.5$ (acceptance)
- $10$ GeV < $m_{\ell^+\ell^-}$ < $55$ GeV, $N_\ell(pT > 22\, \text{GeV}) > 0$
- $p_T \ell\ell > 30$ GeV, $pT_{\text{miss}} > 20$ GeV
- $\Delta\phi_{\ell\ell} < 1.8, \Delta\phi_{(\ell\ell)pT_{\text{miss}}} > \pi/2$

Fig. 12 shows the mass distribution of the $e^+\mu^-$ pairs for each of the six combinations of $W$ polarizations, for the sum of the RR, LL and longitudinal contributions.
and for the sum of all contributions. Contrary to the inclusive case, the interference terms, particularly those involving one longitudinal and one transverse \( W \), are large, contributing a sizable fraction of the cross section.

The transverse momentum distribution of the \( e^+ \) for WW events is shown in fig. 13 for each of the six combinations of W polarizations, for the sum of the RR, LL and longitudinal longitudinal contributions and the sum of all contributions. There is a clear discontinuity at \( p_{T_e} = 22 \) GeV related to the requirement of at least one charged lepton with such transverse momentum. The interference terms involving one longitudinal and one transverse \( W \) are large. The LR contribution is very small.

Fig. 14 shows the azimuthal separation of the two charged leptons in the laboratory frame for each of the six combinations of W polarizations, for the sum of the RR, LL and longitudinal longitudinal contributions and for the sum of all contributions. The distribution exhibits a sharp drop at \( \Delta \phi_{\ell\ell} = 1.8 \), about 103 degrees, due to the veto on back to back leptons. This eliminates the large \( \Delta \phi \) region where the longitudinal transverse interferences are negative.
Figure 14. Azimuthal separation, in the laboratory frame, between $e^+$ and $\mu^-$ for $WW$ events in the presence of cuts.

6 Conclusions

In this note I have shown that the amplitude for the Higgs decay to four fermions can be analytically reformulated in terms of the polarizations of the intermediate vector bosons. The vector polarizations can be reconstructed analyzing the kinematic distributions of the final state leptons, providing a new test of the Standard Model.

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References

[1] Z. Bern et al., Left-Handed $W$ Bosons at the LHC, Phys. Rev. D84 (2011) 034008, [arXiv:1103.5445].

[2] W. J. Stirling and E. Vryonidou, Electroweak gauge boson polarisation at the LHC, JHEP 07 (2012) 124, [arXiv:1204.6427].

[3] A. Belyaev and D. Ross, What Does the CMS Measurement of $W$-polarization Tell Us about the Underlying Theory of the Coupling of $W$-Bosons to Matter?, JHEP 08 (2013) 120, [arXiv:1303.3297].

[4] J. Baglio and N. Le Duc, Fiducial polarization observables in hadronic $WZ$ production: A next-to-leading order QCD+EW study, JHEP 04 (2019) 065, [arXiv:1810.11034].

[5] J. Baglio and L. D. Ninh, Polarization observables in $WZ$ production at the 13 TeV LHC: Inclusive case, Commun. Phys. 30 (2020), no. 1 35–47, [arXiv:1910.13746].

[6] A. Denner and G. Pelliccioli, Polarized electroweak bosons in $W^+ W^-$ production at the LHC including NLO QCD effects, [arXiv:2006.14867].
[7] CMS Collaboration, S. Chatrchyan et al., Measurement of the Polarization of W Bosons with Large Transverse Momenta in W+Jets Events at the LHC, Phys. Rev. Lett. 107 (2011) 021802, [arXiv:1104.3829].

[8] ATLAS Collaboration, G. Aad et al., Measurement of the polarisation of W bosons produced with large transverse momentum in pp collisions at $\sqrt{s} = 7$ TeV with the ATLAS experiment, Eur. Phys. J. C72 (2012) 2001, [arXiv:1203.2165].

[9] ATLAS Collaboration, M. Aaboud et al., Measurement of the W boson polarisation in t$\bar{t}$ events from pp collisions at $\sqrt{s} = 8$ TeV in the lepton+jets channel with ATLAS, arXiv:1612.02577.

[10] CMS Collaboration, V. Khachatryan et al., Measurement of the W boson helicity fractions in the decays of top quark pairs to lepton + jets final states produced in pp collisions at $\sqrt{s} = 8$TeV, Phys. Lett. B762 (2016) 512–534, [arXiv:1605.09047].

[11] ATLAS Collaboration, G. Aad et al., Measurement of the angular coefficients in $Z$-boson events using electron and muon pairs from data taken at $\sqrt{s} = 8$ TeV with the ATLAS detector, JHEP 08 (2016) 159, [arXiv:1606.00689].

[12] CMS Collaboration, V. Khachatryan et al., Angular coefficients of $Z$ bosons produced in pp collisions at $\sqrt{s} = 8$ TeV and decaying to $\mu^+\mu^-$ as a function of transverse momentum and rapidity, Phys. Lett. B750 (2015) 154–175, [arXiv:1504.03512].

[13] ATLAS Collaboration, M. Aaboud et al., Measurement of $W^\pm Z$ production cross sections and gauge boson polarisation in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector, Eur. Phys. J. C 79 (2019), no. 6 535, [arXiv:1902.05759].

[14] A. Ballestrero, E. Maina, and G. Pelliccioli, W boson polarization in vector boson scattering at the LHC, JHEP 03 (2018) 170, [arXiv:1710.09339].

[15] A. Ballestrero, E. Maina, and G. Pelliccioli, Polarized vector boson scattering in the fully leptonic $WZ$ and $ZZ$ channels at the LHC, JHEP 09 (2019) 087, [arXiv:1907.04722].

[16] A. Ballestrero, E. Maina, and G. Pelliccioli, Different polarization definitions in same-sign $WW$ scattering at the LHC, arXiv:2007.07133.

[17] D. Buarque Franzosi, O. Mattelaer, R. Ruiz, and S. Shil, Automated predictions from polarized matrix elements, JHEP 04 (2020) 082, [arXiv:1912.01725].

[18] A. Soni and R. Xu, Probing CP violation via Higgs decays to four leptons, Phys. Rev. D 48 (1993) 5259–5263, [hep-ph/9301225].

[19] D. Chang, W.-Y. Keung, and I. Phillips, CP odd correlation in the decay of neutral Higgs boson into $Z\ Z$, $W^+\ W^-$, or $t\ anti-t$, Phys. Rev. D 48 (1993) 3225–3234, [hep-ph/9303226].

[20] A. Skjold and P. Osland, Angular and energy correlations in Higgs decay, Phys. Lett. B 311 (1993) 261–265, [hep-ph/9303294].

[21] T. Arens and L. Sehgal, Energy spectra and energy correlations in the decay $H \rightarrow ZZ \rightarrow \mu^+\mu^-\mu^+\mu^-$, Z. Phys. C 66 (1995) 89–94, [hep-ph/9409396].

[22] C. Buszello, I. Fleck, P. Marquard, and J. van der Bij, Prospective analysis of spin- and CP-sensitive variables in $H \rightarrow ZZ \rightarrow l(1) + l(1) - l(2) + l(2)$ at the LHC, Eur. Phys. J. C 32 (2004) 209–219, [hep-ph/0212396].

[23] S. Choi, D. Miller, M. Muhlleitner, and P. Zerwas, Identifying the Higgs spin and parity in decays to $Z$ pairs, Phys. Lett. B 553 (2003) 61–71, [hep-ph/0210077].
[24] CMS Collaboration, S. Chatrchyan et al., *Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC*, Phys. Lett. **B716** (2012) 30–61, [arXiv:1207.7235].

[25] ATLAS Collaboration, G. Aad et al., *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, Phys. Lett. **B716** (2012) 1–29, [arXiv:1207.7214].

[26] A. Bredenstein, A. Denner, S. Dittmaier, and M. Weber, *Precise predictions for the Higgs-boson decay $H \rightarrow WW/ZZ \rightarrow 4$ leptons*, Phys. Rev. **D74** (2006) 013004, [hep-ph/0604011].

[27] S. Boselli, C. M. Carloni Calame, G. Montagna, O. Nicrosini, and F. Piccinini, *Higgs boson decay into four leptons at NLOPS electroweak accuracy*, JHEP **06** (2015) 023, [arXiv:1503.07394].

[28] L. Altenkamp, M. Boggia, and S. Dittmaier, *Precision calculations for $h \rightarrow WW/ZZ \rightarrow 4$ fermions in a Singlet Extension of the Standard Model with Prophecy4f*, JHEP **04** (2018) 062, [arXiv:1801.07291].

[29] S. Boselli, C. M. Carloni Calame, G. Montagna, O. Nicrosini, F. Piccinini, and A. Shivaji, *Higgs decay into four charged leptons in the presence of dimension-six operators*, JHEP **01** (2018) 096, [arXiv:1703.06667].

[30] ATLAS Collaboration, G. Aad et al., *Determination of spin and parity of the Higgs boson in the WW$^* \rightarrow e\nu\mu\nu$ decay channel with the ATLAS detector*, Eur. Phys. J. **C75** (2015), no. 5 231, [arXiv:1503.03643].

[31] ATLAS Collaboration, G. Aad et al., *Study of the spin and parity of the Higgs boson in diboson decays with the ATLAS detector*, Eur. Phys. J. **C75** (2015), no. 10 476, [arXiv:1506.05669]. [Erratum: Eur.Phys.J.C 76, 152 (2016)].

[32] CMS Collaboration, A. M. Sirunyan et al., *Measurements of properties of the Higgs boson decaying into the four-lepton final state in pp collisions at $\sqrt{s} = 13$ TeV*, JHEP **11** (2017) 047, [arXiv:1706.09936].

[33] CMS Collaboration, A. M. Sirunyan et al., *Measurements of properties of the Higgs boson decaying to a $W$ boson pair in pp collisions at $\sqrt{s} = 13$ TeV*, Phys. Lett. **B791** (2019) 96, [arXiv:1806.05246].

[34] CMS Collaboration, A. M. Sirunyan et al., *Measurements of the Higgs boson width and anomalous $HVV$ couplings from on-shell and off-shell production in the four-lepton final state*, Phys. Rev. **D99** (2019), no. 11 112003, [arXiv:1901.00174].

[35] A. Kadeer, J. G. Körner, and U. Moosbrugger, *Helicity analysis of semileptonic hyperon decays including lepton mass effects*, Eur. Phys. J. **C59** (2009) 27–47, [hep-ph/0511019].

[36] A. Aeppli, F. Cuypers, and G. J. van Oldenborgh, *$O(\Gamma)$ corrections to $W$ pair production in $e^+e^-$ and $\gamma\gamma$ collisions*, Phys. Lett. **B314** (1993) 413–420, [hep-ph/9303236].

[37] A. Aeppli, G. J. van Oldenborgh, and D. Wyler, *Unstable particles in one loop calculations*, Nucl. Phys. **B428** (1994) 126–146, [hep-ph/9312212].

[38] A. Denner, S. Dittmaier, M. Roth, and D. Wackeroth, *Electroweak radiative corrections to $e^+e^- \rightarrow WW \rightarrow 4$ fermions in double pole approximation: The RACOONWW approach*, Nucl. Phys. **B587** (2000) 67–117, [hep-ph/0006307].

[39] M. Billoni, S. Dittmaier, B. Jäger, and C. Speckner, *Next-to-leading order electroweak
corrections to $pp \to W^+W^- \to 4$ leptons at the LHC in double-pole approximation, JHEP 12 (2013) 043, [arXiv:1310.1564].

[40] B. Biedermann, M. Billoni, A. Denner, S. Dittmaier, L. Hofer, B. Jäger, and L. Salfelder, Next-to-leading-order electroweak corrections to $pp \to W^+W^- \to 4$ leptons at the LHC, JHEP 06 (2016) 065, [arXiv:1605.03419].

[41] ATLAS, CMS Collaboration, A. Sopczak, Precision measurements in Higgs sector at ATLAS and CMS, PoS FFK2019 (2020) 006, [arXiv:2001.05927].

[42] CMS Collaboration, C. Collaboration, Vector Boson Scattering prospective studies in the ZZ fully leptonic decay channel for the High-Luminosity and High-Energy LHC upgrades.

[43] HL-LHC, HE-LHC Working Group Collaboration, P. Azzi et al., Standard Model Physics at the HL-LHC and HE-LHC, arXiv:1902.04070.

[44] ATLAS Collaboration, M. Aaboud et al., Measurements of gluon-gluon fusion and vector-boson fusion Higgs boson production cross-sections in the $H \to WW^* \to e\nu\mu\nu$ decay channel in $pp$ collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector, Phys. Lett. B789 (2019) 508–529, [arXiv:1808.09054].