The Near-infrared Tip of the Red Giant Branch. I. A Calibration in the Isolated Dwarf
Galaxy IC 1613

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Received 2017 December 15; revised 2018 February 28; accepted 2018 March 2; published 2018 April 27

Abstract

Based on observations from the FourStar near-infrared camera on the 6.5 m Baade-Magellan telescope at Las Campanas, Chile, we present calibrations of the 1HK luminosities of stars defining the tip of the red giant branch (TRGB) in the halo of the Local Group dwarf galaxy IC 1613. We employ metallicity-independent (rectified) T-band magnitudes—constructed using J-, H-, and K-band magnitudes and both (J – H) and (J – K) colors to flatten the upward-sloping red giant branch tips as otherwise seen in their apparent color–magnitude diagrams. We describe and quantify the advantages of working at these particular near-infrared wavelengths, which are applicable to both the Hubble Space Telescope (HST) and the James Webb Space Telescope (JWST). We also note that these same wavelengths can be accessed from the ground for an eventual tie-in to Gaia for absolute astrometry and parallaxes to calibrate the intrinsic luminosity of the TRGB. Adopting the color terms derived from the IC 1613 data, as well as the zero points from a companion study of the Large Magellanic Cloud, whose distance is anchored by geometric distances of detached eclipsing binaries, we find a true distance modulus of 24.32 ± 0.02 (statistical) ±0.05 mag (systematic) for IC 1613, which compares favorably with the recently published multi-wavelength, multi-method consensus modulus of 24.30 ± 0.05 mag by Hatt et al.

Key words: distance scale – galaxies: distances and redshifts – galaxies: stellar content – infrared: stars – Magellanic Clouds – stars: Population II

Supporting material: machine-readable table

1. Introduction

In recent years, the tip of the red giant branch (TRGB) has emerged as a superb method for measuring the distances to nearby galaxies (e.g., Lee et al. 1993; Sakai et al. 2004; Rizzi et al. 2007; Mager et al. 2008; Jang & Lee 2017). This feature, i.e., the sudden truncation of the red giant branch (RGB) in luminosity, is explained by stellar astrophysics: theoretically, it is predicted that the upward evolution of the luminosities of low-mass stars (≤1.6 M☉; Girardi et al. 2000) on the RGB will have a sharp termination as the temperature of the helium core reaches ~10^8 K, resulting in a lifting of the degeneracy of the core, and allowing stable helium burning to occur as the star rapidly decreases in luminosity, settling onto the horizontal branch (e.g., Iben & Renzini 1984; Salaris & Cassisi 1997). TRGB distances have been measured in practice by applying edge-detection techniques to the color–magnitude diagrams (CMDs) of galaxies to mark the transition point between the foreground and the RGB, one method being the Sobel filter (for a review see Rizzi et al. 2007); or, alternatively fitting a model to the observed luminosity function (e.g., Méndez et al. 2002; Conn et al. 2011).

It is established that the bolometric luminosity at the so-called “He flash” is weakly dependent on metallicity at or below solar values (see Figure 5.19 of Salaris & Cassisi 2005), but there are wavelength-dependent observational effects based on the metal content of stellar atmospheres. In the optical (e.g., B, V), the effects of line blanketing suppress the emergent blackbody radiation, which translates into a downward-sloping TRGB as redder, higher metallicity stars appear progressively fainter relative to their metal-poor counterparts. As this radiation inevitably re-emerges as continuum flux in the near- and mid-infrared, the bolometric corrections in the near-infrared (NIR) reverse their sign so that TRGB stars become brighter (both in zero point and as a function of color). Theoretical studies predict that bolometric corrections in the I-band (~8000 Å) will largely compensate for the effects of metallicity (Salaris & Cassisi 1998), such that the I-band luminosity is largely independent of color and metallicity. The positions of the TRGB in these three wavelength regimes (visual, red, and near-infrared) are schematically illustrated in Figure 1.

The “flattness” of the I-band TRGB has been borne out empirically (starting with Lee et al. 1993), making it the standard passband for TRGB studies. Despite the success and simplicity of the I-band TRGB as a distance indicator, there are two major advantages to extending the TRGB technique to NIR passbands: first, the effects of reddening are much diminished over optical wavelengths (e.g., A_V = 0.18 × A_I versus A_J = 0.48 × A_I; Rieke & Lebofsky 1985), and second, as the bolometric corrections in the NIR reverse their sign so that TRGB stars become brighter, they are more easily observable...
and the existence of numerous independent distance measurements in the literature, all make IC 1613 an ideal target for such a calibration. Moreover, in a companion paper (Hoyt et al., submitted) we provide an absolute calibration of the NIR-TRGB using the Large Magellanic Cloud (LMC), which we apply to this work to obtain a NIR-TRGB distance to IC 1613.

It is worth noting that over time, the calibration of the TRGB zero point, in the I-band, say, has been remarkably stable at the ±0.05 mag level (e.g., Da Costa & Armandroff 1990; Lee et al. 1993; Rizzi et al. 2007; Jang & Lee 2017) despite cautionary estimates of the systematic uncertainty to the contrary. After decades of gradual progress in measuring the structure and distance scale of the TRGB, optical, and NIR, the accuracy and precision of the method are now improving at a considerable pace. High signal-to-noise observations of local galaxies—e.g., this study of IC 1613 and the companion study of the LMC by Hoyt et al.—are allowing for unprecedented resolution of TRGB structure to robustly extend the method into the NIR. Beyond individual studies of galaxies, in the near future, Gaia will also provide accurate and precise trigonometric parallaxes for thousands of RGB stars in the Milky Way that will independently set the TRGB distance scale in multi-wavelength regimes. These substantially more accurate distance scales will coincide with the launch of the James Webb Space Telescope (JWST), allowing the application of the NIR-TRGB to distances that are over three times greater than currently possible with the I-band TRGB, thereby allowing a significant increase in the number of accurate distances for the calibration of Type Ia supernovae that are used in direct measures of $H_0$.

2. Observations and the Multi-wavelength Sample

The near-infrared $JHK$ imaging data sets used here are described in detail in Scowcroft et al. (2013). Here, we note that the FourStar imager has a field of view of 10′8 × 10′8 and a resolution of 0.′159 pixel$^{-1}$, and that the observations were centered on the main body of IC 1613 so as to optimize detection of Population I (disk) Cepheids. This field of view is still large enough to cover a significant portion of IC 1613’s outer disk, as is demonstrated in Figure 2.

Figure 3 shows a plot of signal-to-noise ratio (S/N) in each of the three ($JHK$) bands as a function of magnitude. Of particular interest in the context of this paper is the S/N obtained at the level of the TRGB. These values are identified in the plot, and range from 70:1 (∓0.014 mag) in the $K$-band, to 120:1 (∓0.009 mag) in the $H$-band 130:1 (∓0.008 mag) in the $J$-band.

Figure 4 shows the $J$ versus ($J − K$) color–magnitude diagram (CMD) for all 23,525 sources having $JHK$ photometry down to a $J$-band magnitude of about 22 mag. As is immediately apparent at these wavelengths, hot and blue stars, which usually prominently define the high-mass main sequence (MS) in optical CMDs, are still present, but their dominance in optical CMDs is much suppressed in the NIR. The blue limit of the MS is broadly outlined by the upward-sloping line to the left of Figure 4 at a color of ($J − K$) ∼ 0.0 mag. These stars evolve up in luminosity and redward in color, eventually slowing in their evolution at the red supergiant (RSG) sequence, also vertically marked in Figure 4 at ($J − K$) ∼ 0.8 mag. These stars make up the most prominent features in the NIR CMD, attributable to Population I stars.

The remainder of the CMD in Figure 4 is dominated by Population II stars: the RGB, the asymptotic giant branch

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10 An additional parameter worthy of comment is the mass range of stars contributing to the TRGB. (Cassisi & Salaris 2013, their Figure 7.4) show that for ages between 8 and 14 Gyr there is virtually no sensitivity of the J-band tip magnitude to age. Even more recently, Lee et al. (2017) have shown that the tip magnitude in the J-band is also unchanged at the 0.005 mag level, for an even larger age range of 5−3 Gyr. Nevertheless, we would still argue that it is dangerous to attempt to measure the TRGB in disk fields made up composite stellar populations overlapping the tip and coexisting with interstellar dust intrinsic to those local lines of sight.
The RGB is the dominant feature: its stars rise up from the significantly fainter MS turnoff of this old population (well below the plotted data) and gradually move to the red by a few tenths of a magnitude in $(J-K)$ color and by nearly 2 mag in luminosity. The dramatic drop in star counts in the CMD above the termination of the RGB (at about $J = 19$ mag) marks the beginning of the more rapidly evolving AGB. This phase continues upward in luminosity for about a magnitude, before it seamlessly merges with the extended AGB, which quickly fans out to both higher luminosities ($J \sim 18$ mag) and redder colors, $(J-K) > 2$ mag. For the remainder of this paper, our attention will be drawn to the central portion of this CMD, where we will focus on the apparent discontinuity in the RGB luminosity function which

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Figure 2. Spatial distribution of stars in IC 1613 contained in the merged JHK catalog. The field of view is $10^\prime \times 10^\prime$. The inscribed circle [4°0 in radius, centered at R.A.(2000) = 16°2090 and decl.(2000) = +2°1159] shows the inner disk region that has been excluded from the TRGB analysis due to the higher source density and the risk of crowding. At right is the detected source density, which is dominated by red giant branch stars, as seen in the color–magnitude diagram presented in Figure 4.

Figure 3. The run of S/N as a function of magnitude for our near-infrared data ($J$—black, $H$—red, $K$—blue). Vertical lines mark the approximate magnitude at which the TRGB is measured, giving values for the $S/N = 130, 110$ and 70 for $J, H$, and $K$, respectively.

Figure 4. Full, near-infrared $J$ vs. $(J-K)$ color–magnitude diagram for the $10^\prime/8 \times 10^\prime/8$ IC 1613 field of view containing 23,525 stars. Major features are labeled, including the asymptotic giant branch (AGB), red supergiants (RSG), extended-AGB stars (ext-AGB), and red giant branch (RGB). The shallow, upward-sloping line in the middle of the diagram marks the termination of the RGB luminosity function, with AGB stars above the discontinuity and first-ascent RGB stars below. The downward-slanting broken line shows the trace of a $K$-band limiting magnitude of 21.
corresponds to the termination of the upward luminosity evolution of the first-ascent RGB population, i.e., the TRGB.

To generally avoid contamination from evolved intermediate-aged stars in the main disk of IC 1613, we selected only those stars outside of the main body of the galaxy, as shown in Figure 2. The boundary for the outer-disk sample was chosen such that the majority of the stars seen in the CMD are Population II. Out of the 23,525 stars in our catalog, approximately half (12,358 stars) are in our halo sample. We further filtered sources for a S/N of $\geq$10, resulting in 6070 high S/N stars. The CMD of this high S/N, predominantly lower-density population of stars is given in Figure 5. Table 1 contains the positions, $JHK$ magnitudes, and errors for a sample of 10 stars from the full catalog, the remainder of which can be found in the machine-readable table.

3. Identifying the Tip of the RGB

As described earlier and displayed in Figure 1, it is predicted that the stars defining the end of the (hydrogen shell-burning) evolution of first-ascent RGB stars will individually trace differently sloping lines of peak luminosity. These lines are a function of color (any color). Longward (or shortward) of the $I$-band in wavelength, the slope of the termination points of the RGB must be positive (or negative) as a function of color. The longward prediction is affirmed in and illustrated by Figures 4 and 5.

In Figure 6, we see a zoomed-in view of the IC 1613 outer-disk CMD in $H$ versus $(J - K)$, focusing on the upper two magnitudes of the RGB. The upward-sloping TRGB is marked in two ways: first, there is a solid straight line slanting across the CMD having the slope of the TRGB trace (see a description of the methodology below) and second, stars that fall within $\pm 0.02$ mag of the fiducial TRGB line given above are shown as circled dots. These stars define the TRGB in the color range $0.85 < (J - K) < 1.05$ mag (i.e., metallicity range) found in IC 1613 old Population II stars. Circled stars outside this color range are also shown so as to better delineate a transformation (discussed later in the text) of this line into the $J$ and $K$ versus $(J - K)$ CMDs.

The fiducial lines in Figure 6 were produced in the following self-consistent way: we visually made a first approximation of the slope and zero point of the TRGB locus in the $H$ versus $(J - K)$ CMD. Using that slope the data were then transformed into the $T[H,(J-K)]$ plane as first introduced and implemented in Madore et al. (2009). The transformed $T$-magnitude CMD is given in Figure 7. We then constructed the $T$-band luminosity function by marginalizing over color in the $T$-band CMD. The marginalized $T$-band luminosity function was constructed having bins of width equal to 0.01 mag. The GLOESS-smoothed edge detector was iteratively applied to the luminosity function where the slope of the TRGB locus was perturbed around the initial approximation until a minimum width in the tip detection output was achieved.

This solution is shown graphically in Figure 8, where the tip is detected at 20.01 mag. The filter response at that magnitude has a width of $\sigma = 0.065$ mag. With 125 stars contributing to the response at that magnitude, the formal error on the TRGB magnitude is $\pm 0.006$ mag. We conservatively adopt $\pm 0.01$ mag as the statistical uncertainty in measuring the peak. The long solid lines in Figures 6 and 7 correspond to the adopted $H$-band TRGB solution discussed above.

In panels (a) and (b) of Figure 9, we show the CMDs for the $J$-band and for the longer-wavelength $K$-band, respectively, holding the $(J - K)$ colors in common across the CMDs. The solid lines in these two cases correspond to the self-consistent mapping of the $H$-band solution, as well as the stars marked in the $H$ versus $(J - K)$ CMD mapped into these $J$ and $K$ CMD planes. This makes mathematical and physical sense; however, exceptions (generated by noise) will arise for individual stars, when photometric measurement uncertainties scatter stars differentially around the same line seen from

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11 Madore et al. (2009) found a downward trend of $\Delta I/\Delta(V-I) = +0.20 \pm 0.05$ mag/mag in NGC 4258; Rizzi et al. (2007) earlier found essentially the same value $0.22 \pm 0.02$ using a larger sample of galaxies, resulting in a higher-quoted precision in the slope.

12 GLOESS stands for Gaussian-windowed, Locally weighted Scatterplot Smoothing, a modern variant on a method first developed by Cleveland (1979). We first used Gaussian windowing for Cepheid light-curve smoothing in Persson et al. (2004), subsequently in Scowcroft et al. (2011, 2016), and most recently in Monson et al. (2017) for RR Lyrae light-curve smoothing, where additional details and descriptions are given. Most recently, Hatt et al. (2017) have discussed the application of GLOESS to luminosity-function smoothing and TRGB detection.

13 Translations, rotations, scalings, reflections, shear, dilations, and compressions all fall under the umbrella of affine transformations. Affine transformations all have one thing in common: before and after an affine transformation collinear points remain collinear, and parallel lines remain parallel. The lines and points in Figures 6 and 9, for example, undergo an affine rotation and so preserve their relative ordering when they are seen in Figures 7 and 9.
bandpass to bandpass. The corresponding $T$-magnitude plots for these CMDs are given in panels (c) and (d) of Figure 9.

As stated in Section 2, the result above is based on the outer-disk sample of stars. The question naturally arises as to whether there is any detectable difference in the detected tip magnitude between the global sample and the more restricted outer-disk sample. The simple answer is no; we detect no statistically significant difference between the inner and outer regions of IC 1613. Nevertheless, as this paper was being finalized, we endeavored to obtain new FourStar imaging further out into the actual halo of IC 1613. Those observations were made, but they were taken under poorer seeing conditions and the S/N at the tip was significantly degraded with respect to the first data set being discussed here. However, the new data were sufficient to show that at the $\pm0.02$ mag level, there is still no gradient in the apparent magnitude of the TRGB in going from the central regions out into the extended halo of this galaxy. Independent confirmation of this lack of any radial gradient in the TRGB is also found in the recent literature. Sibbons et al. (2015), for example, obtained high-quality $JHK$ stellar photometry over a 0.8 square degree region centered on IC 1613. They studied both the azimuthal and radial dependence of the TRGB magnitude and concluded that “there is no overall trend” in either case (specifically, their Figure 12 shows no gradient out to 4 kpc). However, we remain focused on the outer-disk component of the data, which is more than substantial enough in star count for a high-fidelity measurement of the TRGB and its structure, in order to ensure a clean and convincing calibration result.

### Table 1
Near-infrared JHK Photometry of Stars in IC 1613

| ID     | $\alpha$ | $\delta$ | $J$  | $\sigma$ | $H$  | $\sigma$ | $K$  | $\sigma$ |
|--------|----------|----------|------|----------|------|----------|------|----------|
| 01603  | 01 04 59.93 | +02 01 17.86 | 18.77 | 0.03     | 18.21 | 0.03     | 17.69 | 0.03     |
| 29036  | 01 04 51.30 | +02 01 18.04 | 21.72 | 0.11     | 20.96 | 0.17     | 20.46 | 0.20     |
| 23646  | 01 04 38.88 | +02 01 18.18 | 21.61 | 0.08     | 21.03 | 0.20     | 21.22 | 0.31     |
| 15705  | 01 04 42.84 | +02 01 18.40 | 19.90 | 0.02     | 19.37 | 0.03     | 19.24 | 0.05     |
| 24995  | 01 04 43.08 | +02 01 18.50 | 21.96 | 0.11     | 21.23 | 0.14     | 20.69 | 0.28     |
| 07510  | 01 04 51.93 | +02 01 18.54 | 19.43 | 0.01     | 18.70 | 0.02     | 18.57 | 0.03     |
| 24468  | 01 04 58.15 | +02 01 18.83 | 21.15 | 0.06     | 20.36 | 0.09     | 20.31 | 0.12     |
| 17323  | 01 04 40.09 | +02 01 18.97 | 20.30 | 0.03     | 19.70 | 0.04     | 19.64 | 0.07     |
| 02009  | 01 05 5.71  | +02 01 19.22 | 20.71 | 0.06     | 19.94 | 0.08     | 19.07 | 0.06     |
| 25222  | 01 04 57.83 | +02 01 19.44 | 21.06 | 0.05     | 20.50 | 0.08     | 20.48 | 0.14     |

**Note.** Sample photometry for 10 stars of the full catalog, the remainder of which can be accessed in the machine-readable table.

(This table is available in its entirety in machine-readable form.)

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**Figure 6.** $H$ vs. $(J-K)$ color–magnitude diagram for Population II stars in the halo of IC 1613. The solid line shows the adopted slope and position of the TRGB sequence as a function of the metallicity-sensitivity of the $H$-band magnitude to the $(J-K)$ color. Circled dots are $\pm0.02$ mag above and below the line defining the TRGB. They are also individually cross-identified in each of the two panels in Figure 9. In moving from $H$ to $J$ and $K$, their independent photometric errors inflate the $H$-band defined TRGB stars away from the fiducial line. The two flanking lines covering the color range of the TRGB are arbitrarily displaced in magnitude by $\pm0.10$ mag so as to visually highlight the scatter in the color range of the TRGB.

**Figure 7.** Rectified $H$ vs. $(J-K)$ color–magnitude diagram for stars in the halo of IC 1613. All of the data and the TRGB fiducial line have had an affine transformation applied to them such that the TRGB run with color is neutralized. It is in this projection that the marginalized luminosity function shown in Figure 10 was made. Three short, horizontal lines mark the color width over which the TRGB was measured. The magnitude separation of the outer two lines is reset to $\pm0.05$ mag. Stars within $\pm0.02$ mag of the central line are shown as circled dots highlighting the tip and tracing its formal extrapolation across the CMD. These same defining stars are re-identified in each of the other CMDs.
Figure 8. Near-infrared luminosity function and TRGB detection for Population II stars in IC 1613. Straight lines marking the AGB, RGB and tip discontinuity are shown for illustrative purposes only; they are not used anywhere in the analysis.

4. Relative Zero-point Calibrations

Undertaking tip detection independently for each of the slope-corrected $JHK$ luminosity functions give the following apparent magnitude relations for the TRGB in IC 1613:

\begin{align}
J & = 19.21 - 0.85[(J - K) - 1.00], \\
H & = 18.40 - 1.62[(J - K) - 1.00], \\
K & = 18.21 - 1.85[(J - K) - 1.00].
\end{align}

Errors on the slopes are $\pm 0.12, 0.22$ and $0.27$ in $J$, $H$ and $K$, respectively, and the equivalent calibrations in terms of $(J - H)$ colors are

\begin{align}
J & = 19.24 - 1.11[(J - H) - 0.80], \\
H & = 18.44 - 2.11[(J - H) - 0.80], \\
K & = 18.26 - 2.41[(J - H) - 0.80].
\end{align}

Errors on these slopes are $\pm 0.15, 0.26,$ and $0.36$ in $J$, $H$, and $K$, respectively. Random errors on the zero points are $\pm 0.03$ mag for all three bands. We discuss the color–color relation in the following section as well as in the Appendix and only report the result for the IC 1613 TRGB here:

\begin{equation}
(J - H) = 0.77(J - K) + 0.04.
\end{equation}

In a companion paper on the NIR-TRGB calibration in the LMC (Hoyt et al. 2018) we derive an absolute calibration of the NIR-TRGB method using stars in the bar that are co-extensive with the detached eclipsing binaries geometric distance moduli that give a mean of $18.49 \pm 0.02$ (statistical) $\pm 0.05$ (systematic) mag (Pietrzyński et al. 2013). In Hoyt et al., we give the following absolute calibrations:

\begin{align}
M_J & = -5.14 - 0.85[(J - K)_0 - 1.00], \\
M_H & = -5.94 - 1.62[(J - K)_0 - 1.00], \\
M_K & = -6.14 - 1.85[(J - K)_0 - 1.00],
\end{align}

\begin{align}
M_J & = -5.13 - 1.11[(J - H)_0 - 0.80], \\
M_H & = -5.93 - 2.11[(J - H)_0 - 0.80], \\
M_K & = -6.13 - 2.41[(J - H)_0 - 0.80].
\end{align}

Applying these LMC calibrations to the IC 1613 data, and adding in quadrature the LMC statistical and systematic error estimates, gives an average distance modulus of $24.32 \pm 0.02$ (statistical) $\pm 0.05$ mag (systematic), which is in excellent agreement with the distance modulus of 24.30 mag and its uncertainty ($\pm 0.05$ mag) adopted by Hatt et al. (2017). Furthermore, our derived value of 24.32 mag is, to within the errors, the same as the multi-wavelength Cepheid distance modulus of 24.29 $\pm 0.03$ mag, most recently determined by Scowcroft et al. (2013), which, in turn is also equal to the median Cepheid distance modulus of 24.31 mag, based on 51 published Cepheid distances found in NED-D. Finally, our value agrees with the median distance modulus of 24.30 mag determined from 14 RR Lyrae distance moduli listed in NED.

As discussed in B. F. Madore & W. L. Freedman (2018, in preparation) the three ($JHK$) solutions for the relations of TRGB magnitude with color are tightly coupled mathematically. For instance, once the color/color relation and one of the color/tip-magnitude relations are adopted then all of the other color/magnitude combinations are mathematically determined. In the Appendix, we discuss this particular path to a calibration as an external consistency check on the more traditional approach given above.

4.1. Reddening Considerations

Using multi-wavelength observations of Cepheids, Scowcroft et al. (2013) determined a total line of sight reddening $E(B - V) = 0.05$ mag into the main body of IC 1613. Adopting a ratio of total-selective absorption of $R_V = 3.1$ and conversion factors of 0.282, 0.190 and 0.114 for mapping $E(JHK)$ values into $JHK$ extinctions (Cardelli et al. 1989) the main-body extinctions would be $A_J = 0.044$ $A_H = 0.033$ and $A_K = 0.018$ mag. However, our stars are in the outer disk of IC 1613, which is almost certainly devoid of any appreciable amounts of dust. NED quotes a Galactic foreground reddening of $E(JHK)$ values into $JHK$ reddening. In his own words he states “The data require $E(B - V) \leq 0.03$, and it could be zero. In any case, it is almost certainly smaller than $E(B - V) = 0.07 \ldots$” (emphasis ours). Taking each of these points into consideration we adopt a foreground correction of $E(B - V) = 0.03$ mag and propagate it into the derivation of the three true distance moduli, averaged, and given above.

5. Comparisons with Theory

Based on fitting isochrone models from Bressan et al. (2012), Wu et al. (2014) present a calibration of the TRGB luminosity (in the F110W and F160W Hubble Space Telescope (HST) filter magnitudes which closely match $J$ and $H$ ground-based magnitudes) as a function of color. In the color range $(J - H) = 0.70$ to 0.95 mag they find $M_J = -4.81$ mag and $M_H = -5.61$ mag at our fiducial color of $(J - H)_0 = 0.80$ mag. These are to be compared with our values of $M_J = -5.10 \pm 0.03$ mag and $M_H = -5.90 \pm 0.03$ mag with the differences in both bands being $+0.29$ mag, in the sense that we are brighter than the Wu et al. (2014) values. Our respective slopes agree at the 12% level over the quoted color range, with the Wu et al. (2014) slopes being steeper than ours.
Using different published theoretical models (primarily the BASTI set from Pietrinferni et al. 2004; Cordier et al. 2007), Bellazzini (2008) gives color-dependent calibrations of the TRGB using the $(J-K)$ color. Here, we give his calibrations (which, it should be noted are offset from the original models by +0.24 mag, so as to fit his globular cluster distances, recent on $(J-K)=1.00$ mag: $M_J = -5.37 - 1.08 [(J-K)_0 - 1.00]$, $M_H = -6.20 - 1.64 [(J-K)_0 - 1.00]$, and $M_K = -6.37 - 2.08 [(J-K)_0 - 1.00]$. Across all bands, the Bellazzini (2008) calibration is found to be −0.22 mag brighter than our adopted calibration via IC 1613. In addition, his slopes are all consistently higher (by about 10%) than the slopes derived in our study. This current offset between zero points is within the uncertainties allowed by the already comparably large zero-point offsets applied by Bellazzini (2008) to get correspondence between theory and the observations available to them at their time of writing.

These two independent comparisons of our calibrations with theory find no consensus, even as to the sign of the difference. Put into stark contrast, the two theoretically based TRGB calibrations in the NIR differ from each other by over 0.5 mag. We take this as a further indication of the real need for establishing a calibration empirically.

There are further implications regarding the color–color relation between Bellazzini (2008) and our result (given in Section 4 and further discussed in the Appendix). Pairwise differencing the Bellazzini (2008) $J$ and $H$ versus $(J-K)$ equations gives rise to his version of the color–color $(J-H)$ versus $(J-K)$ relation, viz., $(J-H)_0 = 0.56 (J-K)_0 + 0.27$. This slope (0.56) is shallower and the intercept (0.27 mag) is redder in $(J-H)$ than ours (0.77 and 0.06 mag, respectively). This comparison is more clearly seen in Figure 10, where we have plotted the Bellazzini (2008) calibration as a slanting broken line, and our empirical calibration as the solid line. This
theory-based calibration is at variance with the Frogel et al. (1978) individual data points, and divergent both with the fit to the Milky Way data (the solid line) and with the IC 1613 stellar data being presented here. These current results and those forthcoming by Gaia will offer increasingly accurate constraints on the theoretical calibration of the TRGB.

Finally, we note that as this paper was coming to completion, Serenelli et al. (2017) published an extensive compilation and update of theoretical models of the TRGB covering a wide range of stellar masses and metallicities, defining the bolometric magnitude at the tip as a function of effective temperature. Applying a variety of modern bolometric corrections then allowed them to map these models into commonly adopted photometric bandpasses, specifically $VJ$ in the optical and, most importantly for our purposes, $JK$ in the near-infrared. Figure 11 shows an adaptation of their Figure 11 overplotted with our $J$ and $K$ versus $(J-K)$ calibrations. The correspondence is gratifying and clearly favors the slope of the MARCS and PHOENIX bolometric corrections, as discussed in their paper, over the other two contenders within the narrow range of colors ($0.85 < (J-K) < 1.05$) available to us for the IC 1613 calibration. The difference between the Serenelli et al. (2017) zero point and ours (pivoting at $(J-K) = 1.00$) is about 0.1 mag, with the theoretical value being fainter. This difference is well within the 0.25–0.30 mag full range of TRGB tip-magnitude systematic uncertainty quoted by Serenelli et al. (2017) in their Section 5.1. This difference might now be inverted to put interesting constraints on the stellar interior input physics.

5.1. Multi-wavelength Trends and Correlations

In Figure 12, we show the multi-wavelength ($JHK$) relation of the absolute magnitude zero point of the TRGB (left panel) and its slope (right panel). The solid lines joining the data points suggest that both functions are monotonically increasing, but slowing in their mutual growth as a function of wavelength passing from the near to the mid-infrared. Figure 13 shows that the slopes and magnitudes are tightly coupled. The origin and explanation of this multi-wavelength property of the TRGB is the subject of discussion in an upcoming paper (B. F. Madore & W. L. Freedman 2018, in preparation).

6. Conclusions

Using $JHK$ near-infrared photometry of red giant stars in the outer disk of the Local Group dwarf galaxy, IC 1613, we have quantified the relationship of the absolute magnitudes of the tip of the TRGB stars as a function of bandpass and color, and presented a preliminary calibration of the increasing (mean) absolute magnitude of the TRGB as a function of increasing wavelength. At a fiducial color of $(J-H) = 0.80$ mag the near-infrared absolute magnitudes for the TRGB are 1.1, 1.9 and 2.1 mag brighter at $J, H$ and $K$, respectively than the $I$-band absolute magnitude of the tip. This means that, at the same signal-to-noise as in the $I$-band, the near-infrared TRGB can be used to probe cosmological distances that are up to 3/6/7 times larger, corresponding to regions that are 27/216/343 times more voluminous, respectively, than are currently accessible to $I$-band TRGB applications configuration.

We thank the Carnegie Institution for Science and the University of Chicago for their continuing generous support of our long-term research into the expansion rate of the Universe. The near-infrared observations discussed here were taken with the FourStar camera on the Baade 6.5 m telescope at Las Campanas, Chile. This research has made use of the NASA/IPAC

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14 These estimates, of course, also depend on the relative sensitivities of the detectors for a given telescope. For HST if, using the on-line Exposure Time Calculators on the HST website, the comparison is made between $I$-band observations using ACS/F814W and $J$-band observations using WFC3-IR/F110W then, the same signal-to-noise can be reached for a distance modulus 1.9 mag larger (i.e., a factor of 2.4 times more distant) for the same integration time, using the WFC3 configuration rather than using ACS.
Extragalactic Database (NED) which is operated by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. Support for this work was provided by NASA through grant number HST-GO-13691.003-A from the Space Telescope Science Institute, which is operated by AURA, Inc., under NASA contract NAS 5-26555. MGL was supported by a grant from the National Research Foundation (NRF) of Korea, funded by the Korean Government (MSIP) (NRF-2017R1A2B400463). Finally, we thank the anonymous referee for their detailed, helpful and constructive comments.

Appendix

Color–Color Relations for RGB Stars in the Near-infrared

Central to the methodology undertaken here, in deriving a self-consistent calibration of the multi-wavelength slopes and zero points of the TRGB in the near-infrared, is the adoption of an independent calibration of the (distance-independent) color–color relation between the \((J - H)\) and \((J - K)\) intrinsic colors of K and M-type giant stars defining the TRGB. One such calibration is implicit in the tabulation of adopted mean colors for Milky Way giants (and dwarfs) given in Table 12 of Frogel et al. (1978). We have fit for the slope and zero point of the \((J - H) - (J - K)\) color–color relation using the six fiducial data points spanning K0 to K5, which are representative of the giants defining the TRGB. We find that

\[
(J - H)_0 = 0.77(J - K)_0 + 0.06 \text{ (Milky Way)}
\]  

with uncertainties on the slope and zero point being \(\pm 0.012\) and \(\pm 0.008\) mag, respectively.

This solution and the full complement of giant-star colors from Frogel et al. (1978) are shown in Figure 10 overlaid on our sample of halo (large dots) and main body (small points) stars in IC 1613. Although there is significant scatter in the IC 1613 data, the overall trend is extremely clear and the fit to the Milky Way giants is seen to be an excellent (and totally independent) representation of the extragalactic sample. However, it must be said that the Galactic sample was not chosen to be representative of the the highest-luminosity RGB stars (i.e., tip stars) alone, and so it may be biased if surface-gravity-dependent color terms are being averaged over in this field sample.

To evaluate this possibility or other indications of non-universality of the TRGB calibration, we give the near-infrared color–color relations for TRGB stars in the LMC and in IC 1613 as discussed in the main text and in Hoyt et al. (2018).

We find that

\[
(J - H)_0 = 0.77(J - K)_0 + 0.03 \text{ (LMC)},
\]

\[
(J - H)_0 = 0.77(J - K)_0 + 0.04 \text{ (IC 1613)},
\]

Figure 12. Zero point and slope of the TRGB with \((J - H)\) color as a function of bandpass. Slopes are plotted in the right panel. Zero points, measured at \((J - H) = 0.80\) mag, are plotted in the left panel. The general trend is for the characteristic absolute magnitude of the trace of the TRGB with color to become brighter with increasing wavelength and for the slope to increase as well. Both quantities appear to be starting to level off around \(2 \mu m\).

Figure 13. Absolute magnitude of the TRGB as a function of TRGB slope for \(JH\) and \(K\) bandpasses.
where the slopes were fixed. The agreement at the ±0.02 mag level is encouraging and will be the subject of further scrutiny in future papers.

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