SUPERNova Ia CONSTRAINTS ON A TIME-VARIABLE COSMOLOGICAL “CONSTANT”

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ABSTRACT

The energy density of a scalar field $\phi$ with potential $V(\phi) \propto \phi^{-\alpha}$, $\alpha > 0$, behaves like a time-variable cosmological constant that could contribute significantly to the present energy density. Predictions of this spatially flat model are compared to recent Type Ia supernova apparent magnitude versus redshift data. A large region of model parameter space is consistent with current observations. (These constraints are based on the exact scalar field model equations of motion, not on the widely used time-independent equation of state fluid approximation equations of motion.) We examine the consequences of also incorporating constraints from recent measurements of the Hubble parameter and the age of the universe in the constant and time-variable cosmological constant models. We also study the effect of using a noninformative prior for the density parameter.

Subject headings: cosmology: observations — large-scale structure of universe — supernovae: general

1. INTRODUCTION

Current observations are more consistent with low-density cosmogonies dominated by cold dark matter (CDM). For recent discussions see, e.g., Ratra et al. (1999a), Kravtsov & Klypin (1999), Doroshkevich et al. (1999), Colley et al. (2000), Freudling et al. (1999), Sahni & Starobinsky (1999), Bahcall et al. (1999), and Donahue & Voit (1999). The simplest low-density CDM models have either flat spatial hypersurfaces and a constant or time-variable cosmological “constant” $\Lambda$ (see, e.g., Peebles 1984; Peebles & Ratra 1988, hereafter PR; Efstathiou, Sutherland, & Maddox 1990; Stompor, Górski, & Banday 1995; Caldwell, Dave, & Steinhardt 1998) or open spatial hypersurfaces and no $\Lambda$ (see, e.g., Gott 1982, 1997; Ratra & Peebles 1994, 1995; Górski et al. 1998).

While these models are consistent with most recent observations, there are notable exceptions. For instance, recent applications of the apparent magnitude versus redshift test based on Type Ia supernovae (SNe Ia) favor a nonzero $\Lambda$ (see, e.g., Riess et al. 1998, hereafter R98; Perlmutter et al. 1999a, hereafter PP99), although not with great statistical significance (Drell, Loredo, & Wasserman 2000).

On the other hand, the open model is favored by the following:

1. Measurements of the Hubble parameter $H_0 (= 100 h$ km s$^{-1}$ Mpc$^{-1}$) that indicate $h = 0.65 \pm 0.1$ at 2 $\sigma$ (see, e.g., Suntzeff et al. 1999; Biggs et al. 1999; Madore et al. 1999), and measurements of the age of the universe that indicate $t_0 = 12 \pm 2.5$ Gyr at 2 $\sigma$ (see, e.g., Reid 1997; Gratton et al. 1997; Chaboyer et al. 1998). This is because the resulting central $H_0 t_0$ value is consistent with a low-density open model with nonrelativistic-matter density parameter $\Omega_m \approx 0.35$ and requires a rather large $\Omega_\Lambda \approx 0.6$ in the flat constant-$\Lambda$ case.

2. Analyses of the rate of gravitational lensing of quasars and radio sources by foreground galaxies that require a large $\Omega_\Lambda \geq 0.38$ at 2 $\sigma$ in the flat constant-$\Lambda$ model (see, e.g., Falco, Kochanek, & Munoz 1998).

3. Analyses of the number of large arcs formed by strong gravitational lensing by clusters (Bartelmann et al. 1998; Meneghetti et al. 2000).

4. The need for mild antibiasing to accommodate the excessive intermediate- and small-scale power predicted in the COBE DMR-normalized flat constant-$\Lambda$ CDM model with a scale-invariant spectrum (see, e.g., Cole et al. 1997).

While the time-variable $\Lambda$ model has not yet been studied to the same extent as the open and flat-$\Lambda$ cases, it is likely that it can be reconciled with most of these observations (see, e.g., PR; Ratra & Quillen 1992, hereafter RQ; Frieman & Waga 1998; Perlmutter, Turner, & White 1999b; Wang et al. 2000; Efstathiou 1999).

We emphasize that most of these observational indications are tentative and certainly not definitive. This is particularly true for constraints derived from a $\chi^2$ comparison between model predictions and cosmic microwave background (CMB) anisotropy measurements (see, e.g., Ganga, Ratra, & Sugiyama 1996; Lineweaver & Barbosa 1998; Baker et al. 1999; Rocha 1999); see discussions in Bond, Jaffe, & Knox (1998) and Ratra et al. (1999b). More reliable constraints follow from model-based maximum likelihood analyses of the CMB anisotropy data (see, e.g., Górski et al. 1995; Yamamoto & Bunn 1996; Ganga et al. 1998; Ratra et al. 1999b; Rocha et al. 1999), which make use of all the information in the CMB anisotropy data and are based on fewer approximations. However, this technique has not yet been used to analyze enough data sets to provide robust statistical constraints. Kamionkowski & Kosowsky (1999) review what might be expected from the CMB anisotropy in the near future.

In this paper we examine constraints on a constant and time-variable $\Lambda$ that follow from recent Type Ia supernova apparent magnitude versus redshift data and recent measurements of $H_0$ and $t_0$. We focus here on the favored scalar field ($\phi$) model for a time-variable $\Lambda$ (PR; Ratra & Peebles 1988, hereafter RP), in which the scalar field potential $V(\phi) \propto \phi^{-\alpha}$, $\alpha > 0$, at low redshift.$^1$ This scalar field could

$^1$ Other potentials have been considered, e.g., an exponential potential (see, e.g., Luccinchi & Matarrese 1985; RP; Ratra 1992; Wetterich 1995; Ferreira & Joyce 1998), or one that gives rise to an ultralight pseudo-Nambu-Goldstone boson (see, e.g., Frieman et al. 1995; Frieman & Waga 1998), but such models are either inconsistent with observations or do not share the more promising features of the inverse power law potential model.
Fig. 1.—PDF confidence contours for the spatially flat time-variable $\Lambda$ scalar field model, with potential $V(\phi) \propto \phi^{-\alpha}$, derived using the three SN Ia data sets. The $\alpha = 0$ axis corresponds to the spatially flat time-independent $\Lambda$ case. Confidence contours in panels a–c run from $-2$ to $+3$ starting from the lower left-hand corner of each panel. Panel a shows those derived from all the R98 SNe, while panel b is for R98 SNe excluding the $z = 0.97$ one, and panel c is for the P99 fit C data set. Panel d compares the $\pm 2$ $\sigma$ limits from the three data sets: all R98 SNe (long-dashed lines); R98 SNe excluding the $z = 0.97$ one (short-dashed lines); and the P99 fit C SNe (dotted lines).

have played the role of the inflation at much higher redshift, with the potential $V(\phi)$ dropping to a nonzero value at the end of inflation and then decaying more slowly with increasing $\phi$ (PR; RP). See Peebles & Vilenkin (1999), Perrotta & Baccigalupi (1999), and Giovannini (1999) for a specific model and observational consequences of this scenario. A potential $\propto \phi^{-\alpha}$ could arise in a number of high-energy particle physics models; see, e.g., Binétruy (1999), Kim (1999), Choi (1999), Banks, Dine, & Nelson (1999), Brax & Martin (1999), Masiero, Pietroni, & Rosati (2000), and Bento & Bertolami (1999) for specific examples. It is conceivable that such a setting might provide an explanation for the needed form of the potential, as well as for the needed very weak coupling of $\phi$ to other fields (RP; Carroll 1998; Kolda & Lyth 1999; but see Periwal 1999 and Garriga, Livio, & Vilenkin 2000 for other possible explanations for the needed present value of $\Lambda$).

A scalar field is mathematically equivalent to a fluid with a time-dependent speed of sound (Ratra 1991). This equivalence may be used to show that a scalar field with potential $V(\phi) \propto \phi^{-\alpha}$, $\alpha > 0$, acts like a fluid with negative pressure and that the $\phi$ energy density behaves like a cosmological constant that decreases with time. This energy density could come to dominate at low redshift and thus help reconcile low dynamical estimates of the mean mass density with a spatially flat cosmological model. Alternative mechanisms that also rely on negative pressure to achieve this result have been discussed (see, e.g., Özer & Taha 1986; Freese et al. 1987; Turner & White 1997; Chiba, Sugiyama, & Nakamura 1997; Özer 1999; Waga & Miceli 1999; Overduin 1999; Bucher & Spergel 1999). However, these mechanisms either are inconsistent or do not share a very appealing feature of the scalar field models. For some of the scalar field potentials mentioned above, the solution of the equations of motion is an attractive time-dependent fixed point (RP; PR; Wetterich 1995; Ferreira & Joyce 1998; Copeland, Liddle, & Wands 1998; Zlatev, Wang, & Steinhardt 1999; Liddle & Scherrer 1999; Santiago & Silbergleit 1998;
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Uzan (1999). This means that for a wide range of initial conditions the scalar field $\phi$ evolves in a manner that ensures that the cosmological constant contributes a reasonable energy density at low redshift. Of course, this does not resolve the (quantum-mechanical) cosmological constant problem.

There have been many studies of the constraints placed on a time-variable $\Lambda$ from Type Ia supernova apparent magnitude versus redshift data (see, e.g., Turner & White 1997; Frieman & Waga 1998; Garnavich et al. 1998; Hu et al. 1999; Cooray 1999; P99; Perlmutter et al. 1999b; Wang et al. 2000; Efstathiou 1999). However, except for the analysis of Frieman & Waga (1998), who use the earlier Perlmutter et al. (1997) data, these analyses have mostly made use of the time-independent equation of state fluid approximation to the scalar field model for a time-variable $\Lambda$. (Perlmutter et al. 1999b and Efstathiou 1999 do go beyond the time-independent equation of state approximation by also approximating the time dependence of the equation of state; however, they do not analyze the exact scalar field model.)

A major purpose of this paper is to use the newer supernova data of R98 and P99 to derive constraints on a time-variable $\Lambda$ without making use of the time-independent equation of state fluid approximation to the scalar field model.

In the analyses here we use the most recent R98 and P99 data to place constraints on a constant and time-variable $\Lambda$. We note, however, that this is a young field and insufficient understanding of a number of astrophysical processes and effects (the mechanism responsible for the supernova, evolution, environmental effects, intergalactic dust, etc.) could render this a premature undertaking (see, e.g., et al. Hožich 1998; Aguirre 1999; Drell et al. 2000; Umeda et al. 1999; Riess et al. 1999; Wang 2000). Furthermore, other cosmological explanations (time-variable gravitational or fine-structure “constants”) could be consistent with the data (see, e.g., Amendola, Corasaniti, & Occhionero 1999; Barrow & Magueijo 2000; García-Berro et al. 1999).

In addition to analyzing just the supernova data, we also incorporate constraints from recent measurements of $H_0$ and $t_0$ and derive a combined likelihood function which we use to constrain both a constant and a time-variable $\Lambda$. We also examine the effect on the model-parameter constraints caused by varying the prior for $\Omega_0$.

We emphasize that the tests examined in this paper are not sensitive to the spectrum of inhomogeneities in the models considered. This spectrum (possibly generated by
2. COMPUTATION

We examine three sets of supernova apparent magnitudes. We use the MLCS magnitudes for the R98 data, both including and excluding the unclassified SN 1997ck at $z = 0.97$ (with 50 and 49 SNe Ia, respectively). The third set are the corrected/effective stretch factor magnitudes for the 54 fit C SNe Ia of P99. In all three cases we assume that the measured magnitudes are independent. We also assume that SNe Ia at high and low $z$ are not significantly different (Drell et al. 2000, Riess et al. 1999, and Wang 2000 consider the possibility and consequences of evolution), and that intergalactic dust has a negligible effect (Aguirre 1999 considers the effects of dust).

Our analysis of the data of R98 is similar to that described in their § 4, with a few minor modifications. For the time-independent $\Lambda$ model we compute the likelihood function $L(\Omega_0, \Omega_\Lambda, H_0)$ for a range of $\Omega_0$ spanning the interval 0–2.5 in steps of 0.1, for a range of $\Omega_\Lambda$ spanning the interval $-1$ to 3 in steps of 0.1, and for a range of $H_0$ spanning the interval 50–80 km s$^{-1}$ Mpc$^{-1}$ in steps of 0.5 km s$^{-1}$ Mpc$^{-1}$.

Our analysis of the P99 data is similar to theirs, with some modifications. Specifically, we work with their corrected/effective magnitudes and so need only compute a three-dimensional likelihood function $L(\Omega_0, \Omega_\Lambda, M_B)$. Here $M_B$ is their “Hubble constant–free” $B$-band absolute magnitude (see P99) related to $H_0$ by $M_B = -19.46 - 5\log_{10} H_0 + 25$ (determined by us from results in P99). For the time-independent $\Lambda$ model we compute this likelihood function for a range of $\Omega_0$ spanning the interval 0–3 in
steps of 0.1, for a range of $\Omega_\Lambda$ spanning the interval $-1.5$ to
3 in steps of 0.1, and for a range of $\mathcal{M}_\Lambda$ spanning the interval
$-3.95$ to $-2.95$ in steps of 0.05 (which corresponds to about the same interval in $H_0$ used in our analyses of the
R98 data). Note that in our P99 time-independent $\Lambda$ model
plots we do not show the likelihood for the whole $\Omega_0$–$\Omega_\Lambda$
region over which we have computed it.

The spatially flat time-variable $\Lambda$ model we consider is the
scalar field model with potential $V(\phi) \propto \phi^{-\alpha}$, $\alpha > 0$.
When $\alpha = 0$ the model tends to the flat constant-$\Lambda$ case and when $\alpha \to \infty$ it approaches the Einstein–de Sitter
model. It is discussed in detail in PR, RP, and RQ, and we derive the predicted distance moduli for the SNe using expressions
given in these papers.

As shown in these papers, the scalar field behaves like a
fluid with a constant (but different) equation of state in each
epoch of the model. For instance, in the CDM- and baryon-
dominated epoch of the model, it obeys the equation of
state $p_\phi = \omega_\phi \rho_\phi$ (relating the pressure and energy density of
the scalar field), where

$$w_\phi = -\frac{2}{\alpha + 2};$$  \hspace{1cm} (1)

see, e.g., equation (2) of RQ, or see Zlatev et al. (1999) for a
more recent derivation. We shall also have need for an
average equation-of-state parameter, used by Perlmuter et
al. (1999b) and Wang et al. (2000),

$$w_{\text{eff}} = \frac{\int_0^{\Omega_0} da \Omega_\phi(a)w_\phi(a)}{\int_0^{\Omega_0} da \Omega_\phi(a)},$$  \hspace{1cm} (2)

where $\Omega_\phi$ is the scalar field density parameter and $a$ is the
scale factor (with $a_0$ being the present value).

In this model, for the R98 data we compute the likelihood
function $L(\Omega_0, \alpha, H_0)$, and for the P99 data the likelihood
function $L(\Omega_0, \mathcal{M}_\Lambda)$. In both cases we evaluate the
likelihood function for a range of $\Omega_0$ spanning the interval 0.05–
0.95 in steps of 0.025, for a range of $\alpha$ spanning the interval
0–8 in steps of 0.5, and for the same range of $H_0$ or $\mathcal{M}_\Lambda$ as in
the time-independent $\Lambda$ cases discussed above.

We marginalize these three-dimensional likelihood functions
by integrating over $H_0$ for the R98 data) or $\mathcal{M}_\Lambda$ (for
the P99 data) and derive two-dimensional likelihood functions,
$L(\Omega_0, \mathcal{M}_\Lambda)$ for the constant-$\Lambda$ model and $L(\Omega_0, \alpha)$
for the spatially flat time-variable $\Lambda$ case. These
two-dimensional likelihood functions are used to derive highest
posterior density limits (see Ganga et al. 1997 and references
therein) in the $(\Omega_0, \mathcal{M}_\Lambda)$ or $(\Omega_0, \alpha)$ planes. In what follows we
consider 1, 2, and 3 $\sigma$ confidence limits which include
68.27%, 95.45%, and 99.73% of the area under the likelihood
function.

When marginalizing over a parameter or deriving a limit
from the likelihood functions, we consider a number of different
prions. We first consider a uniform prior in the parameter
integrated over, set to zero outside the range
considered for the parameter.

We also incorporate constraints from measurements that
indicate $H_0 = 65 \pm 7$ km s$^{-1}$ Mpc$^{-1}$ at 1 $\sigma$ (see, e.g., Biggs
et al. 1999; Madore et al. 1999), by using the prior

$$p(H_0) = \frac{1}{\sqrt{2\pi}(7 \text{ km s}^{-1} \text{ Mpc}^{-1})} \times \exp \left[ -\frac{(H_0 - 65 \text{ km s}^{-1} \text{ Mpc}^{-1})^2}{2(7 \text{ km s}^{-1} \text{ Mpc}^{-1})^2} \right],$$  \hspace{1cm} (3)

and from measurements that indicate $t_0 = 12 \pm 1.3$ Gyr at
1 $\sigma$ (i.e., 0.5 Gyr added to the globular cluster age estimate of
Chaboyer et al. 1998),

$$p(t_0) = \frac{1}{\sqrt{2\pi}(1.3 \text{ Gyr})} \times \exp \left[ -\frac{(t_0 - 12 \text{ Gyr})^2}{2(1.3 \text{ Gyr})^2} \right],$$  \hspace{1cm} (4)

with $\alpha$ replacing $\Omega_\Lambda$ in this expression in the spatially flat
time-variable $\Lambda$ case. Note that Figure 2 of Chaboyer et al.
(1998) may be used to establish that a Gaussian prior of the
form of equation (4) is a good approximation to the shifted
Monte Carlo globular cluster age distribution. See Ganga
et al. (1997) for a discussion of Gaussian priors in a related
context. This method of incorporating constraints from
measurements of $H_0$ and $t_0$ is not identical to the classical
$H_0$, $t_0$ cosmological test (see, e.g., § 13 of Peebles 1993).

Finally, since $\Omega_0$ is a positive quantity, we also consider
the noninformative prior (Berger 1985, p. 82),

$$p(\Omega_0) = \frac{1}{\Omega_0}.$$  \hspace{1cm} (5)

Since $\Omega_0$ is bounded from below by the observed lower limit
on the baryon density parameter ($\sim 0.01$–0.05, depending
on the data used; see, e.g., Olive, Steigman, & Walker 2000),
this prior does not result in an infinity.

### 3. RESULTS AND DISCUSSION

Figure 1 shows the posterior probability density distribution
function (PDF) confidence contours for the time-variable $\Lambda$
scalar field model, derived using the three data
sets discussed above. Figure 1d shows that constraints from the
different three data sets are quite consistent. At 2 $\sigma$ a large
region of the parameter space of these spatially flat
models (with a constant or time-variable $\Lambda$) is consistent
with the SN Ia data. These data favor a smaller $\Omega_0$, as $\alpha$
is increased from zero.

A fluid with a time-independent equation of state $p = w\rho$, $w < 0$, has often been used to approximate the scalar field
with potential $V(\phi) \propto \phi^{-2}$ in the time-variable $\Lambda$ model.
The solid lines in Figure 2a show the confidence contours for
such a fluid model, derived using the P99 fit C data.

These confidence contours are consistent with those shown
in Figure 1 of Perlmuter et al. (1999b) (but see discussion
below), Figure 10 of Wang et al. (2000), and Figure 4 of
Efstathiou (1999). The dashed lines in Figure 2a show the
exact scalar field model contours of Figure 1c, transformed
using the relation between $w_\phi$ and $\alpha$ in the CDM- and
baryon-dominated epoch (eq. [1]), which comes to an end
just before the present. The two sets of contours agree near
$w \sim -1$, as they must, since at $w = -1$ this is the flat
constant-$\Lambda$ model and $\Lambda$ does behave exactly like a fluid

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3 A more complete analysis would need to account for the uncertainty in this (0.5 Gyr) numerical value. This would likely weaken the effect of this prior.

4 We thank R. Gott for emphasizing this prior; a more complete discussion of it may be found in J. R. Gott et al. (2000, in preparation).

I. Wasserman has noted that such a prior is probably more appropriate for a parameter that sets the scale for the problem, such as $H_0$ here (see § B.2 of Drell et al. 2000; § VII of Jaynes 1968 gives a more general discussion). It is, however, still of interest to determine how the conclusions depend on the choice of prior.
with a time-independent equation of state. However, the
two sets of confidence contours differ significantly at larger
$w$. Since the scalar field $w_\phi$ eventually switches over to $-1$
in the scalar field–dominated epoch; RP), it is unclear what
significance should be ascribed to this difference. We stress,
however, that since the time-independent equation of state
fluid model is an approximation to the time-variable $\Lambda$
scalar field model, constraints on model-parameter values
that are based on the fluid model approximation are only
approximate (and possibly indicative). In passing, we note
that Perlmutter et al. (1999b) use $w_{\text{eff}}$ (eq. [2]) and not $w$
to parameterize the fluid model constraints. Figure 2b shows
contours of constant $w_{\text{eff}}$ as a function of $\alpha$ and $\Omega_0$
in the time-variable $\Lambda$ scalar field model. In a large part of model
parameter space $w_{\text{eff}}$ is a sensitive function of both $\alpha$ and $\Omega_0$
and hence is not the best parameter to use to describe the
time-variable $\Lambda$ scalar field model.

Figure 3 shows the effects of incorporating constraints
based on $H_0$ and $t_0$ measurements (eqs. [3] and [4]). Panel
a shows that adding the $H_0$ constraint does not significantly
alter the contours derived from the SN Ia data alone. This is
expected, since the value of $H_0$ used here is very close to the
value that is indicated by the SN Ia data (see R98).
However, incorporating the $t_0$ constraint, $t_0 = 12 \pm 1.3$
Gyr at 1 $\sigma$, does significantly shift the contours (panel b).
This is because the SN Ia data alone favor a higher $t_0$,
$14.2 \pm 1.7$ Gyr (R98) or $14.5 \pm 1.0$ (0.63/$h$) Gyr (P99).

Figure 4 shows constraints on the time-variable $\Lambda$ model,
from the three different SN Ia data sets used in conjunction
with the $H_0$ and $t_0$ measurements (eqs. [3] and [4]). The
main effect of incorporating the $H_0$ and $t_0$ constraints is to
increase the favored values of $\Omega_0$; a weaker effect is the
disfavoring of larger values of $\alpha$. Even this extended set of
data does not tightly constrain model-parameter values.
Figure 5 shows the corresponding constraints on the constant-Λ model (from the SN Ia, $H_0$, and $t_0$ measurements). Again, the major effect of including the $H_0$ and $t_0$ data is to increase the favored values of $\Omega_\Lambda$. A weaker effect is that it reduces the odds against reasonable open models (Görski et al. 1998), but not by a large factor.

Figures 6 and 7 show the effects of using the noninformative prior, $1/\Omega_\Lambda$, of equation (5). The major effect is a decrease in the favored values of $\Omega_\Lambda$. In the time-variable Λ case there is also a slight increase in the favored values of $\alpha$ (see Fig. 6), while in the constant-Λ case there is a mild reduction in the odds against reasonable open models (see Fig. 7). If the PDF was narrower (i.e., if the error bars on the data were smaller), changing from the flat to the noninformative prior would not result in as large a change in the confidence contours.

4. CONCLUSION

Recent SN Ia data do favor models with a constant or time-variable Λ over an open model without a Λ. However, this is not at a very high level of statistical significance. Also, the incomplete understanding of a number of astrophysical effects and processes (evolution, intergalactic dust, etc.) means that these results are preliminary and not yet definitive.

The constraints on the time-variable Λ model derived here are based on the exact scalar field model equations of motion, not on the widely used time-independent equation of state fluid approximation equations of motion.

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Fig. 6.—PDF confidence contours (1, 2, and 3 $\sigma$) derived from the P99 fit C SNe, for the spatially flat time-variable $\Lambda$ scalar field model. Panel $a(b)$ ignores (accounts for) the $H_0$ and $t_0$ constraints (eqs. [3] and [4]). The solid (dotted) lines use the noninformative $1/\Omega_0$ (flat) prior. The dotted lines in panel $a(b)$ are the same as the solid lines in Fig. 1c (4c).

Fig. 7.—PDF confidence contours (1, 2, and 3 $\sigma$) derived from the R98 SNe excluding the $z = 0.97$ one, for the time-independent $\Lambda$ model. Panel $a(b)$ ignores (accounts for) the $H_0$ and $t_0$ constraints (eqs. [3] and [4]). The solid (dotted) lines use the noninformative $1/\Omega_0$ (flat) prior. The dotted lines in panel $a(b)$ are the same as the dotted (solid) lines in Fig. 5b. The dot-dashed lines are described in the caption of Fig. 5. In the noninformative prior cases the likelihood function is computed down to $\Omega_0 = 0.01$. 
