Mirror Diffusion of Cosmic Rays in Highly Compressible Turbulence Near Supernova Remnants

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Abstract

Recent gamma-ray observations have revealed inhomogeneous diffusion of cosmic rays (CRs) in the interstellar medium (ISM). This is expected, as the diffusion of CRs depends on the properties of turbulence, which can vary widely in the multiphase ISM. We focus on the mirror diffusion arising in highly compressible turbulence in molecular clouds (MCs) around supernova remnants (SNRs), where the magnetic mirroring effect results in significant suppression of diffusion of CRs near CR sources. Significant energy loss via proton–proton interactions due to slow diffusion flattens the low-energy CR spectrum, while the high-energy CR spectrum is steepened due to the strong dependence of mirror diffusion on CR energy. The resulting broken power-law spectrum of CRs matches well the gamma-ray spectrum observed from SNR/MC systems, e.g., IC443 and W44.

Unified Astronomy Thesaurus concepts: Galactic cosmic rays (567); Supernova remnants (1667); Molecular clouds (1072); Magnetohydrodynamics (1964)

1. Introduction

The diffusion of cosmic rays (CRs) is a fundamental physical ingredient that is involved in diverse astrophysical processes, including solar modulation (Potgieter 2013), magnetospheric shielding of extrasolar Earth-like planets (Grießmeier et al. 2015), diffusive shock acceleration (Axford et al. 1977), ionization in molecular clouds (MCs) and star formation (Schlickeiser et al. 2016; Padovani et al. 2018), and driving galactic winds (Ipaich 1975).

Gamma-ray observations provide a powerful tool to probe the diffusion of CRs in the vicinity of CR sources (Di Sciascio 2019; Semenov et al. 2021). Recent gamma-ray observations suggest significant suppression of diffusion of CRs with respect to the average Galactic value around, e.g., pulsar wind nebulae (Abeysekara et al. 2017; Evoli et al. 2018) and supernova remnants (SNRs) (Torres et al. 2010; Li & Chen 2012; Funk 2017), which is also indicated by the enhanced ionization rate in shocked clumps and molecular clouds (MCs) around SNRs (Indriolo et al. 2010; Ceccarelli et al. 2011). In addition, steep gamma-ray spectra are found in SNRs interacting with MCs (Li & Chen 2012; Ackermann et al. 2013; Cardillo et al. 2014), as well as in the Gould Belt clouds (Blasi et al. 2012; Neronov et al. 2012).

Theoretically, with the development of fundamental theories of magnetohydrodynamic (MHD) turbulence in the past two decades (Goldreich & Sridhar 1995; Lazarian & Vishniac 1999), significant progress has been made in our understanding on diffusion of CRs. Naturally, the diffusion of CRs depends on the properties of MHD turbulence with which they interact. Compressible MHD turbulence can be decomposed into Alfvén, slow, and fast modes (Lithwick & Goldreich 2001; Cho & Lazarian 2002, 2003). The Alfvénic component with a Kolmogorov energy spectrum turns out to be inefficient in CR pitch-angle scattering due to its anisotropy (Chandran 2000; Yan & Lazarian 2002; Xu & Lazarian 2020). It also leads to $D(E) \propto E^{-3/2}$, where $D$ is the diffusion coefficient in the direction parallel to the magnetic field and $E$ is the CR energy (Beresnyak et al. 2011; Lazarian & Xu 2021), instead of the $D(E) \propto E^{1/3}$ that is expected for isotropic Kolmogorov turbulence. By contrast, fast modes, despite their relatively small energy fraction compared with Alfvén modes (Cho & Lazarian 2002), play a dominant role in scattering CRs especially in cold interstellar phases (Yan & Lazarian 2004; Xu & Lazarian 2018, 2020), as well as in galaxy clusters (Brunetti & Lazarian 2007, 2011), and thus they are important for determining the diffusion of CRs. In the multiphase interstellar medium (ISM), with the varying compressibility and magnetization of the medium and driving condition of turbulence (Gaensler et al. 2011; Lazarian et al. 2018; Xu & Zhang 2020; Ha et al. 2021), both the properties of turbulence and the turbulence-dependent diffusion of CRs are expected to be spatially inhomogeneous.

More recently, Lazarian & Xu (2021) (henceforth LX21) investigated the mirroring effect in compressible MHD turbulence on CR diffusion. In the presence of magnetic mirrors generated by compressible slow and fast modes, $D$ corresponding to the scattering by fast modes becomes smaller because the range of pitch angles where scattering dominates over mirroring is smaller than $[0°, 90°]$ (Cesarsky & Kulsrud 1973; Xu & Lazarian 2020). Moreover, LX21 identified a new diffusion mechanism of the CRs for which mirroring dominates over scattering. They found that, unlike static magnetic mirrors that trap CRs (Cesarsky & Kulsrud 1973), due to the superdiffusion of magnetic fields induced by Alfvénic turbulent motions (Lazarian & Vishniac 1999; Xu & Yan 2013; Lazarian & Yan 2014), CRs that interact with magnetic mirrors propagate diffusively along the magnetic field, which is termed the “mirror diffusion.” As the “mean free path” is determined by the size of compressible magnetic fluctuations, the mirror diffusion enabled by fast modes usually has a smaller $D$ than that resulting from the scattering by fast modes (LX21).

To illustrate the application of the mirror diffusion in the ISM, in this paper we focus on the diffusion of CRs in an MC-like environment with highly compressible turbulence (e.g., Bally et al. 1987; Xu & Zhang 2016, 2017) and thus non-
negligible fast modes. As the mirror diffusion in general has a smaller $D$ compared with other diffusion mechanisms in compressible MHD turbulence, we consider it as the dominant diffusion mechanism that governs the diffusion behavior of CRs near their acceleration sites. The MCs in the vicinity of SNRs serve as the target dense material for the CR protons that escape from the SNRs where they are accelerated, resulting in gamma-ray emission via neutral pion decay. By formulating the distribution function of CRs that undergo both mirror diffusion and its energy loss due to pion production taken into account. In Section 4, we apply the mirror diffusion to SNR/MC systems and compare the analytically derived gamma-ray spectra with the observed ones. The discussion and summary of our main results can be found in Sections 5 and 6, respectively.

2. Mirror Diffusion of CRs

In compressible MHD turbulence, the diffusion of CRs is determined by their interactions with both Alfvénic and compressible components of MHD turbulence. The energy fraction of the compressible component depends on the driving mechanism of turbulence, as well as the sonic Mach number $M_s = V_L/c_s$ and the Alfvén Mach number $M_A = V_L/V_A$ of the medium (Cho & Lazarian 2002; Federrath et al. 2011; Lim et al. 2020), where $V_L$ is the turbulent velocity at the injection scale $L$ of turbulence, $c_s$ is the sound speed, and $V_A = B/\sqrt{4\pi \rho}$ is the Alfvén speed with the magnetic field strength $B$ and gas density $\rho$. In highly compressible interstellar media with a large $M_A$, such as MCs (e.g., Bally et al. 1987; Padoan et al. 1999; Mac Low & Klessen 2004), a non-negligible fraction of compressible fast and slow modes is expected (Vazquez-Semadeni et al. 2000; Xu & Hu 2021).

Magnetic compressions induced by fast and slow modes act as magnetic mirrors. CRs with the gyroradius $r_g = v_\perp/\Omega$ smaller than the variation scale of the magnetic field and the pitch-angle cosine

$$\mu < \sqrt{b_k / B_0 + b_k}$$

are subject to the mirror force, and they can be reflected by magnetic mirrors when they move along converging field lines. Here, $v_\perp$ is the CR speed perpendicular to the magnetic field, $\Omega = qB_0/\gamma mc$ is the CR gyrofrequency with $q$, $m$, and $\gamma$ as the electric charge, mass, and Lorentz factor of the CR particle, $B_0$ is the mean magnetic field strength, $c$ is the light speed, and $b_k$ is the magnetic perturbation at the wavenumber $k$ of compressible modes.

In the meantime, the shear Alfvénic turbulent motions mix magnetic field lines and cause their superdiffusion in the direction perpendicular to the mean magnetic field (Lazarian & Vishniac 1999; Lazarian & Yan 2014). The anisotropic scaling of strong MHD turbulence is (Goldreich & Sridhar 1995; Lazarian & Vishniac 1999)

$$l_\parallel = \frac{V_A}{L_d} \frac{1}{l_D^2}, \quad l_\perp = \frac{1}{l_D}$$

where $l_\parallel$ and $l_\perp$ are the parallel and perpendicular sizes of a turbulent eddy measured relative to the local mean magnetic field and the corresponding diffusion mechanism of turbulence, as well as the sonic Mach number $M_s < 1$, there are (Lazarian 2006; Xu et al. 2019)

$$V_d = V_l M_A, \quad L_d = l_\perp = L_{Ma}^2, \quad (3)$$

and for super-Alfvénic turbulence with $M_A > 1$, there are

$$V_d = V_A, \quad L_d = l_\parallel = L_{Ma}^{-3}, \quad (4)$$

This means that, over a parallel distance $s$ along the turbulent magnetic field, the wandering field lines have spread by a distance comparable to the transverse scale corresponding to $s$, which is

$$\delta_s^2 = L^{-1} M_A^2 s^3, \quad \delta_\perp < l_\parallel$$

for sub-Alfvénic turbulence, and

$$\delta_s^2 = L^{-1} M_A^3 s^3, \quad \delta_\perp < l_\parallel$$

for super-Alfvénic turbulence (see Lazarian & Yan 2014 for the magnetic field diffusion in other turbulence regimes). With the increase of $s$, the field lines are distorted by larger and larger eddies. As a result, $\delta_s^2$ has a stronger dependence on $s$ than the linear dependence, and the magnetic fields exhibit super-diffusion in perpendicular direction.

Due to the superdiffusion of field lines on all length scales within the inertial range of strong MHD turbulence, infinitely many magnetic field lines can be stochastically advected to each point in space (Eyink et al. 2011). In the presence of both Alfvénic and compressible components of MHD turbulence, when a CR particle is reflected by a magnetic mirror, the stochasticity of field lines makes it unable to trace back the same field line, and thus it cannot be trapped and bounce back and forth between two magnetic mirrors. This is different from the case with the magnetic mirrors arising from compressional MHD waves that was usually considered in the literature (e.g., Cesarsky & Kulsrud 1973). The resulting stochastic encounters with different magnetic mirrors lead to diffusive behavior of CRs in the direction parallel to the magnetic field, which is termed “mirror diffusion” ($LX21$).

At each $\mu$, the mirroring effect is dominated by the magnetic fluctuation at (Cesarsky & Kulsrud 1973; LX21)

$$k^{-1} = L \left( \frac{\delta B_k}{B_0} \right)^{-4} \mu^8. \quad (7)$$

Here, we consider that fast modes dominate the mirroring effect, which is true when fast and slow modes have comparably large energy fractions (Xu & Lazarian 2020), and adopt the scaling of fast modes (Cho & Lazarian 2002)

$$b_{fk} = \delta B_f (kL)^{-4}, \quad (8)$$

where $b_{fk}$ and $\delta B_f$ are the magnetic fluctuations of fast modes at $1/k$ and $L$, respectively. Given the $\mu$-dependent step size of
mirror diffusion in Equation (7), we have the corresponding \( \mu \)-dependent parallel diffusion coefficient (LX21)

\[
D_{||,f}(\mu) = v \mu k^{-1} = vL \left( \frac{\delta B_f}{B_0} \right)^4 \mu^9,
\]

\[
\mu_{\text{min},f} < \mu < \mu_c,
\]

\[
v \mu r_g, \quad \mu < \mu_{\text{min},f}.
\]

Here,

\[
\mu_{\text{min},f} \approx \frac{b_f(r_g)}{B_0}
\]

corresponds to the minimum scale (= \( r_g \)) of magnetic fluctuations for the mirror bouncing (Cesarsky & Kulsrud 1973), where \( b_f(r_g) \) is the magnetic fluctuation at \( r_g \). The upper limit of \( \mu \) is given by the cutoff value \( \mu_c \), which is determined at the balance between mirroring and scattering (Xu & Lazarian 2020)

\[
\mu_c \approx 1.2 \left[ \frac{14 \delta B_f^2}{\pi B_0^2} \frac{v}{L \Omega} \right]^{\frac{1}{2}},
\]

where \( v \) is the CR speed. The factor 1.2 is introduced here for the analytical approximation to better agree with the numerical evaluation of \( \mu_c \), and we note that \( \mu_c \) cannot exceed the maximum \( \mu = \sqrt{\delta B_f/(B_0 + \delta B_f)} \) for mirroring. Only at \( \mu < \mu_c \) is the parallel diffusion of CRs dominated by mirroring. At \( \mu > \mu_c \), the scattering by fast modes is more important in determining the parallel diffusion (Xu & Lazarian 2020). Finally, we have the pitch-angle integrated parallel diffusion coefficient of CRs under the mirroring effect as (LX21)

\[
D_{||,f} = \int_0^{\mu_c} D_{||,f}(\mu) d\mu \approx \frac{1}{10} vL \left( \frac{\delta B_f}{B_0} \right)^4 \mu_c^{10},
\]

under the assumption of an isotropic pitch angle distribution. Its dependence on CR energy \( E \) is determined by the \( E \) dependence of \( \mu_c \).

In Figure 1, we present the numerically calculated \( D_{||} \) for CR protons under the mirroring effect in compressible MHD turbulence with (a) \( \delta B_f = 0.5B_0 \) and (b) \( \delta B_f = B_0 \). Dots and solid lines represent numerical calculation (see Appendix) and analytical estimate (Equations (11) and (12)).

![Figure 1](image_url)

**Figure 1.** \( D_{||} \) of CR protons under the mirroring effect in compressible MHD turbulence with (a) \( \delta B_f = 0.5B_0 \) and (b) \( \delta B_f = B_0 \). Dots and solid lines represent numerical calculation (see Appendix) and analytical estimate (Equations (11) and (12)).

The mirror diffusion in high energy is larger than in low energy.

3. Distribution Function and Energy Spectrum of CRs

The distribution function \( f(r, E) \), i.e., the number of CR protons of energy \( E \) at a position \( r \) per unit volume and unit energy interval in a stationary state, can be found by solving (Syrovatskii 1959; Ginzburg & Syrovatskii 1964)

\[
D \Delta f - \frac{\partial}{\partial E} (Sf) + Q = 0.
\]

We consider that \( D \) has a power-law dependence on \( E \) with the power-law index \( \alpha \),

\[
D = D_\perp E^\alpha,
\]

and neglect its spatial dependence by assuming that the properties of MHD turbulence in the local environment, i.e., an
The normalized energy distribution of CRs at a closer distance with their energy loss is negligible. By inserting Equation (16) into Equation (21), we approximate the CR energy spectrum as
\[
f(r, E) \approx \frac{Q_r}{S_r E} \frac{1}{(4\pi\lambda)^{\frac{1}{2}}} E^{-\gamma_0 - \frac{1}{\alpha}} \exp \left[ -\frac{(r - r_0)^2}{4\lambda} \right],
\]
with the integrated energy distribution modified due to diffusion. In the above expression, we assume a spherical symmetry for simplicity with \(d = \sqrt{r^2 - r_0^2}\). The corresponding energy spectrum of CRs is
\[
\frac{dN}{dE} = \iint dr f(r, E).
\]
As an illustration, in Figure 2 we present the numerical solution to Equation (14) by assuming a spherical symmetry. We use \(f = 0\) at \(r = X\) as the boundary condition, where \(r = d\) is the radial distance from the source at the center of the sphere, and \(X\) is the radius of the sphere. In fact, \(f\) already approaches 0 before reaching \(X\), due to the energy loss (see below).
Table 1

| Distance (kpc) | \(B_0 (\mu G)\) | \(L (\text{pc})\) | \(n_H (\text{cm}^{-3})\) | \(\gamma_0\) | \(\alpha\) | Range of \(r (\text{pc})\) | \(Q_\gamma (\text{erg}^{-1/\gamma_0} \text{s}^{-1})\) |
|---------------|-----------------|-----------------|-----------------|-------------|---------|-----------------|------------------|
| IC 443        | 1.5             | 70              | 30              | 20          | 2.2     | 10/11           | 2–10             | 2.5 \times 10^{35}|
| W44           | 2.9             | 70              | 30              | 100         | 2.4     | 10/11           | 1.6–2            | 3.0 \times 10^{36}|

Figure 2(a) shows the evolution of \(f(r, E)\) (normalized by its value \(f(0, E_{\text{inj}})\) at the source position \((r=0)\) and the minimum CR energy \(E_{\text{min}}\) with \(r\) (normalized by \(X\)) and \(E\) (normalized by \(E_{\text{inj}}\)). We note that the parameters adopted here do not correspond to a specific astrophysical object, so we only use their normalized values for the purpose of illustration. At a distance closer to the source (the upper line in Figure 2(b)), the energy loss is insignificant for most energies under consideration, and the energy distribution is consistent with Equation (26). It is steeper than the injected distribution because of diffusion. At a distance farther from the source (the lower line in Figure 2(b)), as lower-energy CRs diffuse more slowly, they suffer more energy loss via hadronic interactions with the surrounding interstellar matter. As a result, the injected energy distribution for low-energy CRs cannot be seen. The energy scaling is regulated only by energy loss and is close to Equation (23), as we analytically expect. Higher-energy CRs still follow the energy distribution as in Equation (26), with the transition energy \(E_d\) given by Equation (25). Naturally, \(E_d\) increases with \(d\), but decreases with increasing \(D_r\). With highly suppressed diffusion, the energy-loss effect can be easily seen within the energy range of interest in the vicinity of the CR source.

We find that the energy-dependent diffusion and energy loss together lead to a smoothly broken power law for the CR energy distribution. The slope of \(dN/dE\) of higher-energy CRs depends on both the injected energy distribution and \(E\)-dependence of diffusion coefficient.

4. Application to IC 443 and W44

To exemplify the mirror diffusion of CRs in cold interstellar phases with highly compressible turbulence, we consider the scenario of an SNR interacting with an MC. The accelerated CR protons at the SN shock propagate in the surrounding MC and undergo the mirror diffusion when interacting with the compressible turbulence. To study the gamma-ray emission from the dense MC illuminated by the CR protons, we adopt the above theoretical model for describing the energy distribution of CRs involving both energy-dependent diffusion and energy loss due to pion production. We take the parallel diffusion coefficient for mirror diffusion of relativistic CRs as

\[
D = 10^{25} \left( \frac{E}{\text{GeV}} \right)^{10/11} \text{cm}^2 \text{s}^{-1},
\]

based on the calculation in Section 2 and the results in Figure 1. For simplicity, we consider super-Alfvénic turbulence and thus do not distinguish between parallel and perpendicular diffusion in the global frame of reference (Yan & Lazarian 2008). We use the above \(D\) as the isotropic diffusion coefficient, but bear in mind this would not apply to sub-Alfvénic turbulence. For the energy loss due to the inelastic interactions of CR protons with the gas, we adopt the model and the value of \(\sigma_{p, \text{inel}}\) in Kelner et al. (2006).

We consider IC 443 and W44, for which the hadronic process mainly contributes to the gamma-ray emission, as confirmed based on the characteristic spectral feature expected from neutral pion decay (Giuliani et al. 2011; Ackermann et al. 2013). The parameters of these two SNR/MC systems are listed in Table 1. The distance, \(B_0\), and \(n_H\) are taken from Giuliani et al. (2011) and Ackermann et al. (2013). \(B_0\) of tens of \(\mu G\) and \(L\) of tens of pc are typical values for MCs (e.g., Crutcher et al. 2010; Qian et al. 2018; Ha et al. 2021). \(\gamma_0\) should be in the range \(2.1–2.4\) for shock acceleration (Strong et al. 2007; Castellina & Donato 2013). We adopt a larger \(\gamma_0\) for W44, due to its particularly steep high-energy gamma-ray spectrum compared with other middle-aged SNRs (Cardillo et al. 2014). Using a spherical symmetry, the CR proton spectrum is

\[
\frac{dN}{dE} = \int 4\pi r^2 f(r, E) dr, \tag{29}
\]

with the range of \(r\) for integration provided in Table 1. A broad range of \(r\) for IC 443, with a broad range of transition energies corresponding to different distances from the source (see Section 3), gives rise to an extended bump of the energy spectrum toward low energies. For W44 with a higher energy loss rate due to the higher gas density and a narrower spectral bump, the range of \(r\) is accordingly smaller. The resulting CR proton spectra are presented in Figure 3(a). Their corresponding gamma-ray spectra in Figure 3(b) are calculated using the model by Kelner et al. (2006). As a comparison, the observational data taken from Ackermann et al. (2013) are also displayed in Figure 3(b). The value of \(Q_\gamma\) is determined to match the measured gamma-ray flux. We note that our approximate expression of \(D\) (Equation (28)) for relativistic CRs does not apply to CRs with \(E \lesssim 1\) GeV.

We see that, with a suppressed \(D\) of mirror diffusion in the highly compressible turbulence in an MC, the energy loss effect is significant and thus modifies the low-energy gamma-ray spectrum. Moreover, the steep energy scaling of \(D\) of mirror diffusion explains the steep high-energy gamma-ray spectrum.

5. Discussion

The diffusion of CRs in MHD turbulence has the diffusion coefficient \(D\) dependent on the properties of MHD turbulence, which include—but are not limited to—the slope of magnetic power spectrum. Other important properties, e.g., the scale-dependent anisotropy of Alfvén and slow modes (Goldreich & Sridhar 1995; Lazarian & Vishniac 1999; Chandran 2000; Yan & Lazarian 2002; Xu & Lazarian 2020), and the nonlinear decorrelation of turbulent magnetic fields (Lynn et al. 2012; Xu & Lazarian 2018; Demidem et al. 2020), can significantly affect the diffusion associated with the pitch angle scattering of CRs. The mirror diffusion of CRs arises from both the mirroring effect of compressible modes and superdiffusion of magnetic fields induced by Alfvén modes, and it has
dependence on the energy fractions of different modes (LX21). Given the specific turbulence properties in the local environment, the corresponding $D$ and its $E$-dependence apply.

As theoretically expected and observationally confirmed, the interstellar turbulence has varying properties in different gas phases and regions (e.g., Armstrong et al. 1995; Padoan et al. 1999; Elmegreen & Scalo 2004; Chepurnov & Lazarian 2010; Chepurnov et al. 2010; Xu & Zhang 2020; Hu et al. 2021a). By combining the measurements of $M_s$ and $M_A$ (Gaensler et al. 2011; Tofflemire et al. 2011; Lazarian et al. 2018; Xu & Hu 2021; Hu et al. 2021b) that characterize the basic properties of turbulence and the theoretical model on the turbulence-dependent diffusion of CRs, one can obtain a more realistic description of CR diffusion in the multiphase ISM and a better interpretation of CR-related observations, e.g., gamma-ray spectra.

Here, we only consider the case with the mirror diffusion dominated by fast modes by assuming their energy fraction is not small. When the energy fraction of fast modes is so small that slow modes dominate the mirror diffusion, a different $D$ with a different $E$ dependence is expected (LX21). As the energy fractions of slow and fast modes depend on the local $M_s$ and $M_A$ values (Choi & Lazarian 2002), their measurements would be necessary for a self-consistent study of CR diffusion in SNR/MC systems and around other CR sources. This would be also helpful for distinguishing between the models using diffusion and acceleration of CRs (e.g., Fang et al. 2013) to explain the gamma-ray spectra.

In partially ionized MCs, the ion-neutral collisional damping of fast modes can affect the scattering of CRs (Xu et al. 2015, 2016; Xu & Lazarian 2018). The damping effect on mirror diffusion of CRs will be addressed in our future studies. In addition, if the measured $M_A$ is small, the anisotropic diffusion in sub-Alfvenic turbulence (Xu & Yan 2013) should be taken into account when modeling the gamma-ray emission.

The effect of energy loss on modifying the CR energy spectrum in dense MCs was earlier discussed in, e.g., Gabici et al. (2007). The main difference is that we use a physically motivated $D$ and specify the shape of CR energy spectrum in both loss- and diffusion-dominated energy ranges. This is also the main difference in comparison with other studies on CR diffusion in SNR/MC systems, e.g., Ohira et al. (2011); Li & Chen (2012).

6. Summary

The turbulence-dependent diffusion of CRs is spatially inhomogeneous in the multiphase ISM. In the MCs with highly compressible turbulence in the vicinity of SNRs, the mirror diffusion of CRs resulting from the mirroring effect of compressible magnetic fluctuations and superdiffusion of turbulent magnetic fields is expected. As the mirror diffusion is more inefficient than other diffusion processes associated with the pitch angle scattering of CRs, it is likely to be the dominant diffusion process near CR sources, accounting for the gamma-ray emission from the MCs surrounding SNRs.

Based on the analysis of the distribution function of CRs that undergo both mirror diffusion and energy loss through hadronic interactions with the ambient material, we found a significant energy loss effect on low-energy CRs because of their highly suppressed diffusion in the MCs. As a result, the CR energy spectrum has a smoothly broken power-law shape, with the low-energy part flattened due to the energy loss and the high-energy part steepened due to the diffusion. This explains the shape of the gamma-ray spectra of IC 443 and W44. The diffusion coefficient $D$ of mirror diffusion has a distinctive steep energy scaling in comparison with other diffusion mechanisms, which naturally explains the steep high-energy gamma-ray spectra of SNR/MC systems.

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Software: MATLAB (MATLAB 2018).

Appendix

We provide here the equations used for the numerical calculation of $D_i$ in Section 2. They are all available in LX21, but still presented here for completeness. The mirroring rate of
fast modes is (Cesarsky & Kulsrud 1973; Xu & Lazarian 2020)
\[
\Gamma_{b,k} = \frac{1}{\mu} \frac{d\mu}{dt} = \frac{v}{2B_0} \frac{1 - \mu^2}{\mu} b_{b,k} \mu
\]
\[
= \frac{v}{2L} \frac{\delta B_j}{B_0} \frac{\sqrt{\frac{r_g}{L}} \frac{1 - \mu^2}{\mu}}
\]
(1)
at \mu > \mu_{\text{min},t}, and
\[
\Gamma_{b,k} = \frac{v}{2B_0} \frac{1 - \mu^2}{\mu} b_{b,k} \mu
\]
\[
= \frac{v}{2r_g} \frac{\delta B_j}{B_0} \frac{\sqrt{\frac{r_g}{L}} \frac{1 - \mu^2}{\mu}}
\]
(2)
at \mu < \mu_{\text{min},t}.

Slow modes are passively mixed by Alfvén modes (Lithwick & Goldreich 2001). By using the same anisotropic scaling of Alfvén modes (Cho et al. 2002),
\[
b_{b,k} = \frac{\delta B_j}{B_0} \frac{r_g}{L} = \frac{\delta B_j}{B_0}, \quad (3)
\]
where \(b_{b,k}\) is the magnetic fluctuation of slow modes at \(k\), and \(k_{||}\) and \(k_{\perp}\) are the perpendicular and parallel components of \(k\) with respect to the local magnetic field (Lazarian & Vishniac 1999), the mirroring rate of slow modes is (Xu & Lazarian 2020)
\[
\Gamma_{b,s} = \frac{v}{2B_0} \frac{1 - \mu^2}{\mu} b_{b,k} \mu
\]
\[
= \frac{v}{2r_g} \frac{\delta B_j}{B_0} \frac{\sqrt{\frac{r_g}{L}} \frac{1 - \mu^2}{\mu}}
\]
(4)
at \mu > \mu_{\text{min},s}, where
\[
\frac{\delta B_j}{B_0} = \sqrt{\frac{\delta B_j}{B_0}} = \sqrt{\frac{r_g}{L}}
\]
(5)
\[
\mu_{\text{min},s} = \frac{\sqrt{b_{b,k} \mu \mu}}{\frac{r_g}{L}} = \sqrt{\frac{r_g}{L}}
\]
(6)
and \(b_{b,k}(r_g)\) is the magnetic fluctuation at \(k_{||} = 1/r_g\). The mirroring rate at \(\mu < \mu_{\text{min},s}\) is
\[
\Gamma_{b,s} = \frac{v}{2B_0} \frac{1 - \mu^2}{\mu} b_{b,k} \mu
\]
\[
= \frac{v}{2r_g} \frac{\delta B_j}{B_0} \frac{\sqrt{\frac{r_g}{L}} \frac{1 - \mu^2}{\mu}}
\]
(7)
For the mirror diffusion induced by slow modes, the \(\mu\)-dependent parallel diffusion coefficient is
\[
D_{||,s}(\mu) = v_{\mu} k_{||}^{-1} \frac{v_{\mu} L^{\frac{1}{2}}}{\mu^{5}} \mu_{\text{min},s} < \mu < \mu_c.
\]
(8a)
\[
D_{\perp,s}(\mu) = v_{\mu} \mu_{\text{min},s}, \quad \mu < \mu_{\text{min},s}.
\]
(8b)
Under the assumption of an isotropic pitch angle distribution, the corresponding pitch-angle integrated diffusion coefficient is
\[
D_{||,s} \approx \int_0^{\mu_c} D_{||,s}(\mu) d\mu = \frac{1}{6} v_{\mu} L^{\frac{1}{2}} \mu_{\text{min},s}^5.
\]
(9)
For the pitch-angle scattering of CRs, we consider the gyroresonant scattering with the resonance function in the quasilinear approximation,
\[
R = \pi \delta (\omega_k - v || k || + \Omega),
\]
(10)
where \(v_{\mu}\) is the CR speed parallel to the magnetic field and \(\omega_k\) is the wave frequency. The pitch-angle diffusion coefficient for gyroresonant scattering by fast modes is (Voelk 1975)
\[
D_{\mu,j,f} = C_{\mu} \int d^3k \frac{k_j^2}{k^2} [J_j'(\chi)]^2 I_j(k) R(k),
\]
with
\[
C_{\mu} = (1 - \mu^2) \frac{\Omega^2}{B_0^2}, \quad x = \frac{k_j v_{\perp}}{\omega_k}, \quad \frac{k_j r_g}{\Omega}.
\]
(12)
The energy spectrum is (Cho & Lazarian 2002)
\[
I_j(k) = C_f k^{-\frac{d}{2}}, \quad C_f = \frac{1}{16\pi} \delta B_j^2 L^{-}\frac{3}{2}.
\]
(13)
The pitch-angle diffusion coefficients for gyroresonant scattering by Alfvén and slow modes are (Voelk 1975)
\[
D_{\mu,j,A} = C_{\mu} \int d^3k x\frac{x^2}{k^2} [J_j'(\chi)]^2 I_A(k) R(k),
\]
and
\[
D_{\mu,j,s} = C_{\mu} \int d^3k \frac{k_j^2}{k^2} [J_j'(\chi)]^2 I_s(k) R(k),
\]
(15)
where the magnetic energy spectra are (Cho et al. 2002)
\[
I_A(k) = C_A k_{\perp}^{\frac{d}{2}} \exp \left( -L \frac{k_j}{k_{||}} \frac{r_g}{L} \right),
\]
\[
C_A = \frac{1}{6\pi} \delta B_j^2 L^{-}\frac{3}{2}
\]
(16)
for Alfvén modes, and
\[
I_s(k) = C_s k_{\perp}^{\frac{d}{2}} \exp \left( -L \frac{k_j}{k_{||}} \frac{r_g}{L} \right),
\]
\[
C_s = \frac{1}{6\pi} \delta B_j^2 L^{-}\frac{3}{2}
\]
(17)
for slow modes.

In compressible MHD turbulence, we consider the scattering rate with contributions from all three modes,
\[
\Gamma_{\text{tot}} = \frac{1}{\mu_{\text{c},s}} \frac{1}{\mu_{\text{c},t}} \frac{1}{\mu_{\text{c},f}}.
\]
(18)
The value of \(\mu_c\) at the balance between mirroring and scattering is determined by equalizing \(\Gamma_{\text{tot}}\) with max[\(\Gamma_{b,f}, \Gamma_{b,s}\)]. Then, under the assumption of an isotropic pitch angle distribution, \(D_{\parallel}\) for mirror diffusion is calculated as
\[
D_{\parallel} = \int_0^{\mu_c} D_{\parallel,\mu} d\mu.
\]
(19)
where \(D_{\parallel,\mu}(\mu)\) is the \(\mu\)-dependent parallel diffusion coefficient for mirroring by the compressible modes with a larger mirroring rate.

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