MIXING OF NEUTRAL B MESONS AND FACTORIZATION

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Abstract

A brief review of checking the factorization hypothesis for matrix element of $B^0 - \bar{B}^0$ mixing within operator product expansion and QCD sum rules is given. Both perturbative and power corrections are considered.

Talk given at III German-Russian Workshop on Theoretical Progress in Heavy Quark Physics, Dubna, 20-22 May 1996
1. Introduction

Determination of the exact pattern of CP violation is one of the most important problems of modern particle physics. The system of neutral B mesons can provide some useful experimental information on the subject. To decipher this information and convert it into some knowledge of theoretical parameters of standard model or some extended model one needs quite accurate theoretical calculation of corresponding observables within the adopted theory. At present an essential obstacle in getting precise theoretical estimates for characteristics of CP violation related processes is a necessity of computing hadronic matrix elements that is a completely nonperturbative problem. The most popular approximation for estimating such elements is the factorization, or vacuum saturation, hypothesis the justification of which is quite unclear. In the present note we very briefly review some recent results of checking the validity of the factorization hypothesis with analytical methods. We consider three point correlator for computing power corrections violating the factorization approximation and two point correlator for computing perturbative ones.

2. Power corrections: three point correlator

In order to develop a machinery of operator product expansion and QCD sum rules\(^1,2,3\) we use here a three point correlator of the form\(^4,5\)

\[
\Pi_{\mu\nu}(p, p') = i^2 \int dx dy e^{ipx - ip'\gamma_5 y} \langle 0 | T J_\mu(x) O(0)_{\Delta B=2} J_\nu(y) | 0 \rangle
= p_\mu p'_\nu \Pi_1(p^2, p'^2, q^2) + \ldots = p_\mu q_\nu \Pi_2(p^2, p'^2, q^2) + \ldots
\]  

(1)

where \( q = p' - p, q^2 = 0, \) \( J_\mu = \bar{d}\gamma_\mu \gamma_5 b \) is an interpolating current for B meson,

\[
\langle 0 | J_\mu | B^0(p) \rangle = i f_B p_\mu.
\]

The operator \( O_{\Delta B=2} \) is chosen with a standard normalization

\[
O_{\Delta B=2} = \bar{d}\gamma_\mu (1 + \gamma_5) d b \gamma_\mu (1 + \gamma_5) d
\]

and the parameter \( B_B \) is defined by the relation

\[
\langle B^0 | O_{\Delta B=2} | B^0 \rangle = \frac{8}{3} f_B^2 m_B^2 B_B.
\]

Within factorization approximation \( B_B = 1. \) For higher reliability we take for our analysis two invariant functions \( \Pi_{1,2}(p^2, p'^2, q^2) \) that appear in the expression for the three point correlator Eq. (1). The dispersion relation dictates the following representation for these functions after using saturation with the low lying resonance state

\[
\Pi_i(p^2, p'^2)|_{q^2=0} = \int ds ds' \frac{\rho(s, s')}{(s - p^2)(s' - p'^2)} = \frac{8/3 f_B^2 m_B^2 B_B}{(p^2 - m_B^2)(p'^2 - m_B^2)} + \ldots
\]
For both theoretical and physical parts of sum rules the factorization corresponds to
the following representation of the amplitudes
\[ \Pi_{\mu\nu}^{\text{fact}}(p, p') = \frac{8}{3} T_{\mu\beta}(p) T_{\nu\beta}(p'), \]
where
\[ T_{\mu\beta}(p) = i \int dx e^{ipx} \langle 0 | T J_\mu(x) \bar{b}(0) \gamma_\beta (1 + \gamma_5) d(0) | 0 \rangle, \]
and gives the value \( B_B = 1 \) for the parameter that describes the ratio of exact matrix
element to the factorized one.

Leading contributions of power corrections that are not caught within the factorization
approximation are given by some specific diagrams with external vacuum fields. The first
one is given by the gluon condensate\(^5\)
\[ \Delta \Pi^G_1 = -\frac{1}{48 \pi^2} \langle GG \rangle (pp') (5 r(p^2) r(p'^2) + e(p^2) e(p'^2)), \]
\[ \Delta \Pi^G_2 = \Delta \Pi^G_1 - \frac{1}{48 \pi^2} \langle GG \rangle p^2 (r(p^2) r(p'^2) + e(p^2) e(p'^2)) + \frac{1}{24 \pi^2} \langle GG \rangle e(p^2) g(p'^2) \]
where
\[ r(p^2) = \int_0^1 dx \frac{x}{-p^2 x + m^2}, \quad e(p^2) = \int_0^1 dx \frac{x - 2x^2}{-p^2 x + m^2}, \]
\[ g(p^2) = -\int_0^1 dx (1 - 2x) \ln(1 - xp^2 / m^2), \]
m is the \( b \) quark mass, \( \langle GG \rangle = \langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle \).

Next contributions are due to condensates of operators with dimension five in mass
units. They are
\[ \Delta \Pi^q_1 = 0, \quad \Delta \Pi^q_2 = -\frac{m \langle g \bar{d} \gamma_\mu G^{\mu\nu} d \rangle}{6 \pi^2 (p^2 - m^2)} \int_0^1 dx \frac{x(x + 1/2)}{-p^2 x + m^2}. \]
Finally, four quark operators contribute the amount that is not taken into account by the
factorization approximation for vacuum expectation values
\[ \Delta \Pi_1^{4q}(p, p') = \Delta \Pi_2^{4q}(p, p') = -\frac{2(\langle F G (1 + \gamma_5) d \rangle)^{\text{non-fact}}}{(p^2 - m^2)(p'^2 - m^2)}. \]

As in most applications of operator product expansion we do not include operators
with dimension higher than six in our analysis. Numerical results are obtained after using
Borel transformation in both \( p^2 \) and \( p'^2 \) independently and putting \( M^2 = M'^2 \) afterwards.
For both invariant functions there is a fairly wide window of stability with respect to
change of the Borel parameter. The numerical results are rather stable and reveal only
small violation of factorization. Namely, if \( B_B = 1 + \Delta B_B \) then for both invariant functions and for wide range of parameters \( (m_0^2, \langle \bar{q}q \rangle, \ldots) \) we get \[^5\]

\[-\Delta B_B = 0 \div 0.1.\]

This estimate is very conservative. The absolute value of deflection from factorization approximation is about \(-0.05\) for the parameter \( B_B \). Main uncertainty is due to poor knowledge of numerical value of \( f_B \) on which there is a strong dependence (to the fourth power).

**3. Perturbative corrections: two point correlator**

Perturbative corrections of order \( \alpha_s \) that violate the factorization approximation are connected with genuine three loop massive diagrams. Their computation with layout for sum rules technique based on three point correlator with independent Borel transformation with regards to kinematical variables \( p^2 \) and \( p'^2 \) can not be done at present because of technical complexity. Therefore we turn to two point correlator\[^6,7\] and introduce a quantity\[^8\]

\[
T(x) = \langle 0 | T O_{\Delta B=2}(x) O_{\Delta B=2}(0) | 0 \rangle.
\]

The leading term of \( \alpha_s \) expansion for the above correlator has an expression in the configuration space that reads

\[
T_0(x) = 2N_c^2 \left( 1 + \frac{1}{N_c} \right) 16S'(x, m)S(-x, 0)S'(x, m)S(-x, 0)
\]

\[
= 2 \left( 1 + \frac{1}{N_c} \right) \text{tr}[S(x, m)S(-x, 0)] \text{tr}[S(x, m)S(-x, 0)] = 2 \left( 1 + \frac{1}{N_c} \right) \Pi_5(x) \Pi_5(x) \tag{2}
\]

where \( S(x, m) \) is the free fermion propagator and \( N_c \) stands for the number of quark colors. The prime means taking only the part of the propagator that is proportional to a \( \gamma \) matrix. The function \( \Pi_5(x) = \langle 0 | T j_5(x)j_5(0) | 0 \rangle \) is the two point correlator associated to the current \( j_5 = \bar{b}i\gamma_5d \). Thus one observes a complete factorization in this order.

Eq. (2) can be rewritten in the form

\[
T_0(x) = 2 \left( 1 + \frac{1}{N_c} \right) \Pi_{\mu\nu}(x) \Pi^{\mu\nu}(x), \quad \tag{3}
\]

where \( \Pi^{\mu\nu}(x) = \langle 0 | T j_L^\mu(x)j_L^\nu(0) | 0 \rangle \) and \( j_L^\mu = \bar{b}_L\gamma^\mu d_L \). The Lorentz decomposition in \( x \)-space reads

\[
\Pi^{\mu\nu}(x) = (-\partial^\nu \partial^\nu + g^{\mu\nu} \partial^2) \Pi_T(x^2) - \partial^\mu \partial^\nu \Pi_L(x^2)
\]

that again demonstrates an explicit factorization in the configuration space to leading order in \( \alpha_s \).
The dispersion representation in $x$-space for any two point correlator $\Pi_j(x)$ ($j = T, L, 5$) has the form

$$i\Pi_j(x^2) = \int_{s_j}^{\infty} r_j(s) D(x, s) ds$$

(4)

where $D(x, s)$ is a free boson propagator with the “mass” $\sqrt{s}$. The spectral functions $r_j$ read to leading order in $\alpha_s$

$$r_L^{(0)}(s) = \frac{N_c}{16\pi^2} z(1 - z)^2, \quad r_T^{(0)}(s) = \frac{N_c}{48\pi^2} (1 - z)^2 (2 + z), \quad r_5^{(0)}(s) = \frac{m^2 N_c}{8\pi^2} \frac{(1 - z)^2}{z}$$

where $z = m^2/s$, $m$ is the $b$ quark mass.

The spectral function $\rho(s)$ of the full correlator $T(x)$ is defined in the same way as in Eq. (4). To first order in $\alpha_s$ it can be expressed in terms of the spectral functions $r_j(s)$ associated to the two-line correlators in the form

$$\rho(s) = \int r_1(s_1) r_2(s_2) \Phi(s; s_1, s_2) ds_1 ds_2$$

where

$$\Phi(s; s_1, s_2) = \frac{1}{16\pi^2 s} \sqrt{(s - s_1 - s_2)^2 - 4s_1 s_2}$$

is the two-body phase space factor.

To leading order in $1/N_c$, one can write the correlator $T(x)$ as a product of two two-line correlators. It is worthwhile to notice that this decomposition is gauge invariant and finite, i.e. it does not require any renormalization.

The full spectral density $\rho(s)$ has been computed numerically to the first order in $\alpha_s$ with a heavy use of known results for two loop massive diagrams obtained earlier (e.g.9) and the program of symbolic computation REDUCE. We analyze our results concentrating on presentation of the entire spectral density $\rho(s)$ as a sum of factorizable and nonfactorizable pieces

$$\rho(s) = \rho_0(s) \left(1 + \Delta\rho_f(s) + \Delta\rho_{nf}(s)\right).$$

Nonfactorizable part of the spectral density is given by one gluon exchange diagrams of a two-line correlator.

Within sum rules approach one works with moments

$$M_i(s_{th}) = \int_{4m_b^2}^{s_{th}} \rho(s) s^{-i} ds$$

that are decomposed as

$$M_i(s_{th}) = M_i^0(s_{th}) \left(1 + \Delta M_i^f(s_{th}) + \Delta M_i^{nf}(s_{th})\right)$$
according to the decomposition of the spectral density.

A set of input parameters for numerical estimates is \( \Lambda_{\overline{\text{MS}}}^{(5)} = 175 \text{ MeV}, m_b = 4.6 \text{ GeV} \). The moments of the factorizable spectral density are almost independent of the power \( i \) of the weight function \( s^{-i} \). The nonfactorizable correction does not exceed a 15% level with respect to the full factorized spectral density. Corrections of order \( \alpha_s \) (factorizable+nonfactorizable) are large for both the spectral density itself and its moments. Depending on the energy \( s \) they can reach a magnitude of 100% with respect to the leading term. The nonfactorizable corrections measured in terms of the fully factorized (lowest order + \( \alpha_s \) terms) spectral density are moderate. All the factorizable corrections to the correlator, however, can be absorbed into the calculation of the decay constant \( f_B \) from two point correlator with two quark lines, in such a way that the relevant corrections to the \( B_B \) parameter are only due to nonfactorizable ones. Results for spectral density itself and for its moments for different values of integration regions are collected in Tables 1,2 taken from ref.\(^8\).

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\[
\begin{array}{|c|c|c|c|}
\hline
s/m_b^2 & \Delta \rho_f & \Delta \rho_{nf} & \Delta \rho_{nf}/(1 + \Delta \rho_f) \\
\hline
5.5 & 1.03 & 0.02 & 0.01 \\
6.0 & 0.95 & 0.21 & 0.11 \\
6.5 & 0.89 & 0.29 & 0.15 \\
7.0 & 0.84 & 0.32 & 0.17 \\
\hline
\end{array}
\]

Table 1: Normalized spectral densities

\[
\begin{array}{|c|c|c|c|c|}
\hline
i & s_{th}/m_b^2 & \Delta M_f & \Delta M_{nf} & \Delta M_{nf}/(1 + \Delta M_f) \\
\hline
0 & 5.5 & 1.07 & -0.16 & -0.08 \\
 & 6.0 & 0.99 & 0.11 & 0.06 \\
 & 6.5 & 0.93 & 0.23 & 0.12 \\
 & 7.0 & 0.88 & 0.29 & 0.15 \\
5 & 5.5 & 1.08 & -0.21 & -0.10 \\
 & 6.0 & 1.00 & 0.08 & 0.04 \\
 & 6.5 & 0.94 & 0.20 & 0.11 \\
 & 7.0 & 0.89 & 0.27 & 0.14 \\
10 & 5.5 & 1.09 & -0.27 & -0.13 \\
 & 6.0 & 1.01 & 0.02 & 0.01 \\
 & 6.5 & 0.96 & 0.16 & 0.08 \\
 & 7.0 & 0.91 & 0.23 & 0.12 \\
\hline
\end{array}
\]

Table 2: Normalized moments of spectral densities