Entanglement between external degrees of freedom of atoms via Bragg deflection

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Abstract

We suggest that atoms undergoing Bragg deflection from a cavity field introduce entanglement between their external degrees of freedom. The atoms interact with an electromagnetic cavity field which is far detuned from atomic transition frequency and is in superposition state. We provide a set of experimental parameters in order to perform the suggested experiment within the framework of the presently available technology.

03.67.-a, 42.50.-p, 03.65.-w
I. INTRODUCTION

In 1935, Einstein, Podolsky and Rosen questioned the legitimacy of quantum mechanics by using entangled states [1]. Their work started enormous philosophical discussions which led to improve the understanding of the subject. The quantum entangled states have proved to be a foundation stone in devising techniques to perform quantum computation [2], quantum teleportation [3] and quantum cryptography [4]. It has made it vital to understand the roots of entanglement in quantum systems in its details.

The generation of entanglement has been performed successfully between two electromagnetic cavities [5], multimodes of a single electromagnetic cavity [6], internal states of atoms [7], ions [8] and Bose-Einstein condensates [9]. Moreover entanglement between angular momenta of a single atom [10] and between atoms in dark states [11] has been suggested. In this paper we present technique to develop entanglement between external degrees of freedom of atoms which are defined by means of their momentum states. Our suggested scheme relies on Bragg deflection of atoms from a cavity field. As a manifestation of quantum duality, a matter wave passing through an optical crystal at a certain angle displays Bragg deflection. We discuss that two noninteracting matter waves incident on a cavity field in superposition, generate entangled states comprising EPR-Bell basis by controlling atom-field interaction times. We extend our suggested scheme to generate GHZ [12,13] entangled states. Later, we discuss that presently available experiments on atomic Bragg scattering from electromagnetic fields [14,15] may be considered to realize our theoretical work in experiments.

We consider two supercooled atoms propagating with centre of mass momentum $P_1$ and $P_2$, and interacting simultaneously with a quantized standing wave cavity field. The field inside the cavity is in a superposition state $1/\sqrt{2}(|0\rangle + |n_0\rangle)$, where $|0\rangle$ is vacuum state and $|n_0\rangle$ is any other Fock state. We take the frequency of the electromagnetic field far detuned from the transition frequency of the two level atoms. Atom-field large detuning ensures that atoms do not exit the cavity field in excited state and there is no spontaneous emission.
which contributes photon in arbitrary direction. Thus interaction of atoms does not alter the field state and both the atoms experience the field in the same state during the time of interaction, that is either $|0\rangle$ or $|n_0\rangle$.

The effective Rabi frequency of each atom interacting with the field in state $|n\rangle$, where $n$ is either 0 or $n_0$, becomes $|g|^2 n / 2\Delta$. Here, $g$ expresses coupling constant, and $\Delta = \nu - w$ indicates detuning between the field frequency, $\nu$, and the atomic transition frequency, $w$.

In order to study atomic deflection from the cavity field we consider that the incident atoms propagate making an angle $\theta$ with the normal to the cavity field. We apply Fresnel approximation to the atomic motion, and, therefore we consider atomic momentum component along the cavity field very small compared to the component along the normal to the cavity field. Hence, we treat the atomic motion along the normal to the cavity field classically.

We may express the evolution of the atoms interacting with the cavity field by the effective Hamiltonian,

$$
\hat{H}_{\text{eff}} = \frac{\hat{P}_x^2}{2M} + \frac{\hat{P}_z^2}{2M} - \frac{\hbar |g|^2}{2\Delta} \sum_{j=1,2} \hat{n} \sigma_+^{(j)} \sigma_-^{(j)} (\cos 2kx + 1),
$$

which is obtained in presence of dipole approximation, rotating wave approximation and secular approximation. Here, $\hat{P}_{x,j}$, is the momentum operator describing momentum component along the cavity field of each $j$th atom for $j = 1, 2$, and $M$ indicates their respective mass. Moreover, $\sigma_+^{(j)}$ and $\sigma_-^{(j)}$ are the corresponding atomic raising and lowering operators and $\hat{n}$ describes field number operator. Above Hamiltonian is separable for atoms 1 and 2.

This suggests that we may write the wave function of the system of two atoms, $A_1$ and $A_2$ and field, $F$ as

$$
|\Psi(A_1, A_2, F)\rangle = \frac{1}{\sqrt{2}} \sum_{l=-\infty}^{\infty} \sum_{l'=-\infty}^{\infty} \left[ C_{0,P_l}(t)C_{0,P_{l'}}(t)|P_l^{(1)}, P_{l'}^{(2)}, 0\rangle + C_{n_0,P_l}(t)C_{n_0,P_{l'}}(t)|P_l^{(1)}, P_{l'}^{(2)}, n_0\rangle \right],
$$

where, $C_{n,P_l}(t)$ is the probability amplitude of atom 1 (2) exiting with momentum $P_l$ ($P_{l'}$) in presence of field with $n$ photons. By comparing atomic scattering with optical Bragg scattering [16], we [17, 21] can develop a condition on initial momentum of the incident
atom, viz. \( P_{l_0} = \frac{\hbar}{2} \, \Delta \), for which atomic Bragg scattering may occur. Here, \( l_0 = \pm 2, \pm 4, \pm 6 \) etc., which correspond to first, second, third order of Bragg scattering, respectively. By changing the atomic momentum component, \( P_{l_0} \), parallel to the cavity field, we can change the order of Bragg scattering. During interaction with the field for each complete Rabi cycle, momentum transferred to the atom by the field is either zero or \( 2 \Delta \) \[19\]. Thus momentum of the exiting atom is given as \( P_l = P_{l_0} + l \, \Delta \), where, \( l \) is an even integer.

Hence, by substituting the effective Hamiltonian, defined in Eq. (1), and the wave function given in Eq. (2) of our system in time dependent Schrödinger equation, we get separate sets of infinite coupled rate equations for probability amplitudes \( C_{n,P_l}^{(j)} \), for each \( j \)th atom. We may express the set of coupled rate equations as

\[
\frac{i}{\hbar} \frac{\partial C_{n,P_l}^{(j)}(t)}{\partial t} = w_{\text{rec}}(l + l_0)C_{n,P_l}^{(j)}(t) - \frac{\chi n}{2} \left( C_{n,P_l+2\Delta}^{(j)}(t) + C_{n,P_l-2\Delta}^{(j)}(t) \right). \tag{3}
\]

Here, \( w_{\text{rec}} = \frac{\hbar \Delta^2}{2M} \) is recoil frequency of the atom and \( \chi = \frac{|g|^2 n}{2 \Delta} \) is effective Rabi frequency. In Bragg deflection, recoil frequency of the deflected atom, is much larger than effective Rabi frequency \[17,22\], that is \( w_{\text{rec}} \gg \chi n \). Also, conservation of energy provides us \( l = 0 \) and \( l = -l_0 \), which leads to two possible directions of propagation for the deflected atom, one with momentum \( P_{l_0}^{(j)} \) and the other with momentum \( P_{-l_0}^{(j)} \), respectively. We solve the set of \( l_0/2 \) coupled equations, from \( l = 0 \) to \( l = -l_0 \), adiabatically and obtain \[23\] two coupled equations as

\[
\frac{i}{\hbar} \frac{\partial C_{n,P_{l_0}+l}^{(j)}(t)}{\partial t} = A_n C_{n,P_{l_0}+l}^{(j)}(t) - \frac{1}{2} B_n C_{n,P_{l_0}+l}^{(j)}(t), \tag{4}
\]
\[
\frac{i}{\hbar} \frac{\partial C_{n,P_{l_0}-l}^{(j)}(t)}{\partial t} = A_n C_{n,P_{l_0}-l}^{(j)}(t) - \frac{1}{2} B_n C_{n,P_{l_0}-l}^{(j)}(t), \tag{5}
\]

where,

\[
A_n = \begin{cases} 
-\frac{\chi n/2}{w_{\text{rec}}(l_0-2)!} & \text{for } l_0 \neq 2, \\
0 & \text{for } l_0 = 2,
\end{cases} \tag{6}
\]

and

\[
|B_n| = \begin{cases} 
\frac{\left(\chi n\right)^{l_0}}{(2w_{\text{rec}})^{\frac{l_0}{2}-1}[l_0-2](l_0-4)\cdots4.2]^2} & \text{for } l_0 \neq 2, \\
\chi n & \text{for } l_0 = 2.
\end{cases} \tag{7}
\]
From Eqs. (4) and (5), we obtain the probability amplitudes of atoms exiting with momentum $P_{+l_0}$ and that of exiting with momentum $P_{-l_0}$ as

$$C^{(j)}_{n,P_{+l_0}}(t) = e^{-iA_nt/2} \left[ C^{(j)}_{n,P_{+l_0}}(0) \cos \left( \frac{1}{2} B_n t \right) + C^{(j)}_{n,P_{-l_0}}(0) \sin \left( \frac{1}{2} B_n t \right) \right],$$

$$C^{(j)}_{n,P_{-l_0}}(t) = e^{-iA_nt/2} \left[ C^{(j)}_{n,P_{-l_0}}(0) \cos \left( \frac{1}{2} B_n t \right) + C^{(j)}_{n,P_{+l_0}}(0) \sin \left( \frac{1}{2} B_n t \right) \right].$$

Hence, as a result of Bragg deflection the probability of finding the exiting atom in either of the two propagation directions flips, as a function of interaction time $t$, with frequency $|B_n|/2 \ [24]$.

As defined in Eq. (2), we may express the combined state of the two deflected atoms and field, at any interaction time, $t$, as

$$|\Psi(A_1, A_2, F)\rangle = \frac{1}{\sqrt{2}} \sum_{l=+l_0,-l_0} \sum_{l'=+l_0,-l_0} \left[ C^{(1)}_{n,P_1}(t) C^{(2)}_{n,P_1'}(t) |P_1^{(1)}\rangle P_1^{(2)}\rangle, + C^{(1)}_{n,P_1}(t) C^{(2)}_{n,P_1'}(t) |P_1^{(1)}\rangle P_1^{(2)}\rangle, + C^{(1)}_{n,P_1}(t) C^{(2)}_{n,P_1'}(t) |P_1^{(1)}\rangle P_1^{(2)}\rangle, + C^{(1)}_{n,P_1}(t) C^{(2)}_{n,P_1'}(t) |P_1^{(1)}\rangle P_1^{(2)}\rangle, + C^{(1)}_{n,P_1}(t) C^{(2)}_{n,P_1'}(t) |P_1^{(1)}\rangle P_1^{(2)}\rangle, + C^{(1)}_{n,P_1}(t) C^{(2)}_{n,P_1'}(t) |P_1^{(1)}\rangle P_1^{(2)}\rangle, + C^{(1)}_{n,P_1}(t) C^{(2)}_{n,P_1'}(t) |P_1^{(1)}\rangle P_1^{(2)}\rangle, + C^{(1)}_{n,P_1}(t) C^{(2)}_{n,P_1'}(t) |P_1^{(1)}\rangle P_1^{(2)}\rangle, + C^{(1)}_{n,P_1}(t) C^{(2)}_{n,P_1'}(t) |P_1^{(1)}\rangle P_1^{(2)}\rangle, + C^{(1)}_{n,P_1}(t) C^{(2)}_{n,P_1'}(t) |P_1^{(1)}\rangle P_1^{(2)}\rangle, + C^{(1)}_{n,P_1}(t) C^{(2)}_{n,P_1'}(t) |P_1^{(1)}\rangle P_1^{(2)}\rangle, + C^{(1)}_{n,P_1}(t) C^{(2)}_{n,P_1'}(t) |P_1^{(1)}\rangle P_1^{(2)}\rangle, + C^{(1)}_{n,P_1}(t) C^{(2)}_{n,P_1'}(t) |P_1^{(1)}\rangle P_1^{(2)}\rangle, + C^{(1...
\[ |\Psi(A_1,A_2,F)\rangle = \frac{1}{\sqrt{2}} \left[ |P_{+l_0}^{(1)},P_{-l_0}^{(2)},0\rangle + e^{-i\varphi}|P_{-l_0}^{(1)},P_{+l_0}^{(2)},n_0\rangle \right], \tag{13} \]

which is a three party entangled state, that is, between two atomic external degrees of freedom and cavity field. We may get the same entangled state if one atom interacts for a time \( t = s\pi/B_n \) with the cavity field, whereas the other atom with a time difference of \( 2r\pi/B_n \), where \( r \) is an even integer. This makes \( \varphi = (s + r)\pi A_{n_0}/B_n \). We may extract the entanglement of the external degrees of freedom of the atoms by making measurement over the cavity field state, which yields,

\[ |\Psi(A_1,A_2)\rangle = \frac{1}{\sqrt{2}} \left[ |P_{+l_0}^{(1)},P_{-l_0}^{(2)}\rangle + e^{-i\varphi}|P_{-l_0}^{(1)},P_{+l_0}^{(2)}\rangle \right]. \tag{14} \]

In case, we keep the interaction time of one atom as \( s\pi/B_n \), while let the other atom interact for an interaction time different by \( 2r'\pi/B_n \), where \( r' \) is an odd integer, we get the entangled state, as

\[ |\Psi(A_1,A_2)\rangle = \frac{1}{\sqrt{2}} \left[ |P_{+l_0}^{(1)},P_{-l_0}^{(2)}\rangle - e^{-i\varphi'}|P_{-l_0}^{(1)},P_{+l_0}^{(2)}\rangle \right], \tag{15} \]

where, \( \varphi' = (s + r')\pi A_{n_0}/B_n \).

We may generate the other two entangled states of the Bell basis by preparing the two atoms in the same initial momentum states \(|P_{+l_0}\rangle\) or \(|P_{-l_0}\rangle\), and let them interact with the cavity field. In case the field is in vacuum state, \(|0\rangle\), the incident atoms pass undeflected, whereas, in presence of field state, \(|n_0\rangle\), the probability amplitudes of the incident atoms oscillate as a function of interaction time. Now following our above discussion, the interaction of one atom for a time \( s\pi/B_n \) and the other for a time difference of \( 2r\pi/B_n \), and, later, a measurement over the cavity field state, leads us to the entanglement of the external degrees of freedom of atoms as

\[ |\Psi(A_1,A_2)\rangle = \frac{1}{\sqrt{2}} \left[ |P_{+l_0}^{(1)},P_{+l_0}^{(2)}\rangle + e^{-i\varphi}|P_{-l_0}^{(1)},P_{-l_0}^{(2)}\rangle \right]. \tag{16} \]

For the same initial conditions of the system but interaction times different by an amount \( 2r'\pi/B_n \) of the two atoms with the cavity field, we get the entangled state as
\[ |\Psi (A_1, A_2)\rangle = \frac{1}{\sqrt{2}} \left[ |P^{(1)}_{+l_0}, P^{(2)}_{+l_0}\rangle - e^{-i\varphi'} |P^{(1)}_{-l_0}, P^{(2)}_{-l_0}\rangle \right]. \]

(17)

Hence, our scheme leads us to generate complete set of Bell basis.

Equations (16) and (17) provide a direct extension of our work to develop GHZ entangled state between external degrees of freedom of atoms. We consider that initially we prepare more than two atoms in the same momentum state \(|P_{+l_0}\rangle\) or \(|P_{-l_0}\rangle\), and, let them interact simultaneously with the cavity field. Bragg deflection of the incident atoms ensures the generation of GHZ state, \(\frac{1}{\sqrt{2}} \left( |P^{(1)}_{+l_0}, P^{(2)}_{+l_0}, \ldots, P^{(k)}_{+l_0}\rangle \pm e^{-i\varphi} |P^{(1)}_{-l_0}, P^{(2)}_{-l_0}, \ldots, P^{(k)}_{-l_0}\rangle \right)\). Here an interaction time of \(s\pi/B_{n_0}\), leads to positive sign and \(\varphi = ks\pi A_{n_0}/2B_{n_0}\), however, any atom interacting for an interaction time difference of \(2r'\pi/B_{n_0}\) will lead to negative sign and \(\varphi = [(k-1)s + 2r']\pi A_{n_0}/2B_{n_0}\), where \(k\) indicates the number of interacting atoms.

We may realize the suggested scheme in laboratory by using the experimental set up of Ref. [15]. We propagate rubidium atoms of mass \(M = 1.42 \times 10^{-25}\) Kg, through an optical quantum field of wavelength \(\lambda = 0.8\) \(\mu\)m. Therefore, the atoms experience a recoil frequency, \(w_{rec} = 2\pi \times 3.8\) kHz, while passing through the field, in presence of a detuning by an amount \(\Delta = 2\pi \times 80\) MHz. We find that \(g = 2\pi \times 112\) kHz, such that, \(\chi \approx 0.02w_{rec}\), which ensures Bragg deflection of incident atoms. We make the times of interaction of the two atoms with the cavity field different by controlling their initial momentum components along the normal to the cavity field, as required to generate entangled states expressed in Eqs. (13) and (17). We may apply our suggested scheme in order to engineer external degrees of freedom entanglement between different isotopes of same material, between atoms of different materials and between an atom and an ion.
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