Resonance-Parton Duality in \((e, e'\pi)\) off Nucleons

Murat Kaskulov and Ulrich Mosel
Institut fuer Theoretische Physik, Universitaet Giessen, D-35392 Giessen, Germany

November 14, 2011

Abstract

Regge-pole based descriptions of pion-electroproduction on nucleons have given a very good description of the longitudinal components of the cross sections. However, these very same models grossly underestimate the transverse components. A related problem appears in QCD-based scaling arguments that predict the predominance of longitudinal over transverse electroproduction of pions by terms \(\propto Q^2\). However, data from JLAB, Cornell and DESY, covering a wide kinematical range \(1 < Q^2 < 11 \text{ GeV}^2\) and \(2 \text{ GeV} < W < 4 \text{ GeV}\), do not show this expected behavior. We address here this issue of the transverse response in pion-electroproduction by considering the contributions of high-lying \((W > 2 \text{ GeV})\) nucleon resonances to pion production. The coupling strengths and form factors are obtained through resonance-parton duality. We show that in a wide range of electron energies and four-momentum transfers such a model describes all the available data very well.

1 Introduction

Electroproduction of mesons in the deep inelastic scattering (DIS), is a modern tool which permits to study the structure of the nucleon on the partonic level. Exclusive channels in DIS are of particular importance. For instance, arguments based on asymptotic QCD predict that the transverse component of the cross section for the exclusive reaction \(p(e, e'\pi^+)n\) falls off with \(1/Q^8\) while the longitudinal component falls with \(1/Q^6\) \cite{1, 2}. Exclusive pion production can be used to check this prediction and to determine in which kinematical regime these simple scaling laws become effective. In the following we discuss these reactions; we draw here on our publications \cite{3, 4, 5} where further details can be found.

At Jefferson Laboratory (JLAB) the exclusive reaction \(p(e, e'\pi^+)n\) has been investigated for a range of photon virtualities up to \(Q^2 \simeq 5 \text{ GeV}^2\) at an invariant mass of the \(\pi^+n\) system around the onset of deep–inelastic regime, \(W \simeq 2 \text{ GeV}\) \cite{6, 7, 8}. A separation of the cross section into the transverse \(\sigma_T\) and longitudinal \(\sigma_L\) components has been performed. The data show that \(\sigma_T\) is large at JLAB energies \cite{7}. At \(Q^2 = 3.91 \text{ GeV}^2\) \(\sigma_T\) is by about a factor of two larger than \(\sigma_L\), contrary to the QCD-based expectations, and at \(Q^2 = 2.15 \text{ GeV}^2\) it has same size as \(\sigma_L\). Previous measurements at values of \(Q^2 = 1.6 (2.45) \text{ GeV}^2\) \cite{6} show a similar problem in the understanding of \(\sigma_T\). Even at smaller JLAB \cite{8} and much higher Cornell \cite{9} and DESY \cite{10} values of \(Q^2\) there is a disagreement between the scaling expectations and experimental data.

The longitudinal cross section \(\sigma_L\) is well understood in terms of the pion quasi–elastic knockout mechanism \cite{11} because of the pion pole at low \(-t\) . This makes it possible to study the charge form factor of the pion at momentum transfer much bigger than in the scattering of pions from atomic electrons \cite{12}. However, the model of Ref. \cite{13}, which is based on such a picture and which is generally
considered to be a guideline for the experimental analysis and extraction of the pion form factor, underestimates largely the transverse response \(\sigma_T\) at high values of \(Q^2\) \[7\].

## 2 Transverse strength in electroproduction

The model of Ref. [13] can be represented by the three amplitudes shown in Fig. 1. The leftmost

\[
\begin{array}{ccc}
\text{e} & \gamma & \pi \\
\pi & N & e \\
N & e & \gamma
\end{array}
\]

Figure 1: The diagrams describing the hadronic part of the \(\pi^+\)– electroproduction amplitude at high energies. The leftmost diagram shows the \(t\)-channel contribution, the middle one the contribution of vector meson exchange and the rightmost one gives the nucleon Born term.

Reggeized \(t\)-channel term dominates the longitudinal strength which thus always dominates at forward momenta, with relatively small contributions from vector meson exchange (middle graph). The right graph, involving a nucleon-pole \(s\)-channel diagram, is necessary for gauge invariance; its main, but small, contribution is to the transverse cross section. At the kinematics of the relevant electroproduction experiments the energies are high enough to excite also nucleon resonances; the invariant masses of the \((\gamma^* p)\) system are between approximately 2 and 4 GeV. We identify these high-lying resonances with partonic excitations leading to DIS \([14]\) and invoke the correspondence principle in going from an inclusive final DIS state to the exclusive pion production \([15]\). During this transition the transverse strength of DIS remains intact. We thus expect that the inclusion of these resonance excitations into the \(s\)-channel diagram enhances the transverse scattering while leaving the longitudinal strength originating in the \(t\)-channel diagram intact. The resonance excitations are also \(s\)-channel contributions which – because of their special coupling – are gauge invariant by themselves \([16]\). Adding such a resonance (DIS) contributions to the Born term in the model of \([13]\) constitutes the central point of our model which also contains an improved treatment of gauge invariance accounting for the difference in the electromagnetic form factors for pions and proton \([3, 4, 5]\).

In the experiments quoted the invariant masses \(W\) of the \((\gamma^*, N)\) system are all \(W > 2\) GeV, i.e. they all lie above the region of well-established, separated nucleon resonances. Thus, the Born term, that has become a sum over individual resonances, can be replaced by an integral over resonances with average coupling constants and form factors

\[
B = \sum_i r(M_i)c(M_i) \frac{F(Q^2, M_i^2)}{s - M_i^2 + i0^+} \Rightarrow \int_{M_i^2}^\infty dM_i^2 \rho(M_i^2) r(M_i^2)c(M_i^2) \frac{F(Q^2, M_i^2)}{s - M_i^2 + i0^+},
\]

where \(r(M_i)\) and \(c(M_i)\) are the electromagnetic and strong couplings, respectively, and \(F(Q^2, M_i^2)\) is the electromagnetic form factor. Here \(\rho(M_i^2)\) is the density of resonances with mass \(M_i\). So far unknown are here the couplings \(r\) and \(c\) and the form factors \(F\) in Eq. (1).

Our aim is to maintain the transverse character of DIS, which follows from a parton picture, when going to the exclusive limit of a resonance decay. We thus have to establish a connection between these two pictures. Bloom and Gilman \([17, 18]\) (BG) have shown that the total DIS strength follows closely
the average behavior of nucleon resonances (for a more recent review see also \[19\]). We, therefore, now use this BG duality in its local form

\[
F_p^2(x_B, Q^2) = \sum_i (M_i^2 - M_p^2 + Q^2)W(Q^2, M_i)\delta(s - M_i^2),
\]

where \(x_B\) stands for the Bjorken scaling variable and the deep inelastic structure function \(F_p^2(x_B, Q^2)\) is expressed as a sum of resonances. \(W(Q^2, M_i)\) defines the \(i\)th resonance contribution to the \(\gamma^*p\) forward scattering amplitude; it is essentially the electromagnetic coupling constant \(r(M_i)\) times the resonance form factor \(F(Q^2, M_i)\). Eq. (2) links the partonic content of nucleon resonances with their hadronic structure. Since the density of resonances in Eq. (1) is a steeply increasing function of invariant mass, it follows from Eq. (2) that the electromagnetic coupling to nucleon resonances must be decreasing with mass since \(F_2\) is finite. This leads to a natural cut-off for the number of resonances and makes the integral in Eq. (1) finite.

In a further step the combined nucleon-resonance contribution to the s-channel is absorbed into an effective Born-term with a nucleon pole only, but a modified form factor \(F_s\) which now contains all the effects of the resonances

\[
B = \frac{F_s(Q^2, s)}{s - M_p^2 + i0^+}.
\]

Parameterizing the form factor in a dipole form leads to the conclusion that the cut-off must increase with the mass of the resonance, a result that is well known from early studies of BG duality \[20\]. We, therefore, expect that the effective form factor \(F_s\) is considerably harder than that of the nucleon alone. This is indeed born out by the calculations as can be seen in Fig. 2.

In Fig. 3 we compare the result of our calculations with the \(p(\gamma^*, \pi^+)n\) data from JLAB \[21\] for unseparated cross sections \(d\sigma_U/dt\), at values of \(W \simeq 2.2 \div 2.4\) GeV and for different values of \((Q^2, \varepsilon)\) bins. The square symbols connected by solid lines describe the model results. The discontinuities in the curves result from the different values of \((Q^2, W, \varepsilon)\) for the various \(-t\) bins. The data are very well reproduced by the present model in the measured \(Q^2\) range from \(Q^2 \simeq 1\) GeV\(^2\) up to 5 GeV\(^2\). In Fig. 3 we also show the contributions of the longitudinal \(\varepsilon d\sigma_L\) (dash-dotted curves) and transverse \(d\sigma_T\) (dashed curves) cross sections to the total unseparated cross sections (solid curves) for the lowest and highest average values of \(Q^2 = 1.1\) GeV\(^2\) and \(Q^2 = 4.7\) GeV\(^2\). The cross sections at high values of \(Q^2\)
Figure 3: The differential cross sections \(d\sigma_U/dt = d\sigma_T/dt + \varepsilon d\sigma_L/dt\) in exclusive reaction \(p(\gamma^*, \pi^+)n\) in the kinematics of the \(\pi\)-CT experiment at JLAB \[21\]. The square symbols connected by solid lines describe the model results. The discontinuities in the curves result from the different values of \((Q^2, W, \varepsilon)\) for the various \(-t\) bins. The dash-dotted and dashed curves describe the contributions of the longitudinal \(\varepsilon d\sigma_L\) and transverse \(d\sigma_T\) cross sections, respectively, to the total unseparated cross sections (solid curves) for the lowest and highest average values of \(Q^2 = 1.1\) GeV\(^2\) and \(Q^2 = 4.7\) GeV\(^2\).

are flat and totally transverse. At forward angles a strong peaking of the cross section at \(Q^2 = 1.1\) GeV\(^2\) comes from the large longitudinal component in this case. The off-forward region is transverse. This behavior agrees with the results from \[3\]. As we shall see, the same behavior is observed in the DIS regime at HERMES \[10\] where the value of \(W\) is higher. At HERMES, because of the Regge shrinkage of the \(\pi\)-reggeon exchange and smaller transverse component, the forward peak just has a steeper \(-t\)-dependence \[4\].

In Fig. 4 we show that, indeed, this very same picture also works remarkably well at the much higher momentum transfers \(Q^2\) and invariant masses \(W\) reached in the HERMES experiment \[10\]. Even in the kinematical windows 4 < \(Q^2\) < 11 GeV\(^2\) the difference between the dash-dash-dotted curve (no resonance contributions) and the dashed curve (resonances included) shows the dramatic impact of the resonance contributions which are essential in describing the cross section at larger \(-t\). Just contrary to the situation in the JLAB experiment the longitudinal cross section at HERMES determines the total differential cross section at small \(-t\).

As the HERMES kinematics are quite close to those expected for JLAB at 12 GeV we predict that again high-lying resonances determine the transverse cross sections at larger \(-t\). In Fig. 23 in Ref. \[5\] detailed predictions for the \(L/T\) separated cross sections are given both for \(\pi^+\) and \(\pi^-\) production. In particular we predict that \(\pi^-\) production is largely longitudinal.

Fig. 5 shows the results of the calculations in comparison with the data of Ref. \[6, 7\] for all four separated cross sections. The solid lines that represent the results of our calculations describe all cross sections, the longitudinal, the transverse and the interference ones, very well. The transverse strength is nearly entirely built up by the resonance contributions that are considerably bigger and fall off much more weakly with \(Q^2\) than the ones obtained from the nucleon Born term alone (see Fig. 2). The resonances also contribute about 30% to the longitudinal cross section at forward angles (small \(-t\)
Figure 4: $-t + t_{\min}$ dependence of the differential cross section $d\sigma_U/dt = d\sigma_T/dt + \epsilon d\sigma_L/dt$ in exclusive reaction $p(\gamma^*, \pi^+)n$ at HERMES. The experimental data are from Ref. [10]. The calculations are performed for the average values of $(Q^2, x_B)$ in a given $Q^2$ and Bjorken $x_B$ bin. The solid curves are the full model results. The dash-dotted curves correspond to the longitudinal $\epsilon d\sigma_L/dt$ and the dashed curves to the transverse $d\sigma_T/dt$ components of the cross section. The dash-dash-dotted curves describe the results without the resonance/partonic effects. From [5].

where the major contribution comes from the $t$-channel graph with the nucleon Born term alone. For the interference cross sections $\sigma_{TT}$ and $\sigma_{LT}$ the sign even changes when the resonances are taken into account.

This model has recently been extended to the photo- and electroproduction of $\pi^0$ [22]; again a very good description of all available data is reached without any new parameters. The transition from photoproduction ($Q^2 = 0$) to electroproduction shows that the resonance contributions assume a larger and larger role with increasing $Q^2$. While at the photon point they just fill in the diffractive dip in the otherwise Regge-dominated cross section at higher $Q^2$ they become more and more dominant.

3 Summary

The large transverse strength observed in various experiments on exclusive pion production has been a long-standing puzzle. We have resolved this open problem by adding to the usual $t$-channel + Born term description the contribution of high-lying nucleon resonances as an effective description of DIS excitations. By using quark-hadron duality we have been able to link the properties of these resonances to the partonic degrees of freedom. We thus treat this contribution to the exclusive production as the limiting case of inclusive DIS processes [3, 4] that are predominantly transverse. Such a model, that makes use only of quite general average properties of nucleon resonances, is able to describe all the available exclusive electroproduction data for pions. In particular the transverse response of nucleons is determined by their resonance/parton excitations. A discussion of the relevance of the present results for color transparency measurements in exclusive electroproduction of pions off nuclear can be found in [23, 24].
Figure 5: $-t$ dependence of $L/T$ partial transverse $d\sigma_T/dt$, longitudinal $d\sigma_L/dt$ and interference $d\sigma_{TT}/dt$ and $d\sigma_{LT}/dt$ differential cross sections in exclusive reaction $p(\gamma^*, \pi^+)n$. The experimental data are from the $F_\pi$-2 [6] and $\pi$-CT [7] experiments at JLAB. The numbers displayed in the plots are the average $(Q^2, W)$ values. The dashed curves correspond to the exchange of the $\pi$-Regge trajectory alone. The dash-dotted curves are obtained with the on-mass-shell form factors in the nucleon-pole contribution and exchange of the $\rho(770)/a_2(1320)$-trajectory. The solid curves describe the model results with the resonance contributions. The data points in each $(Q^2, W)$ bin correspond to slightly different values of $Q^2$ and $W$ for the various $-t$ bins. The calculations are performed for values of $Q^2$ and $W$ corresponding to the first $-t$ bin. The histograms for $d\sigma_T/dt$ are the results from [3]. From [5].
Acknowledgments

This work has been supported by DFG through the TR16 and by BMBF.

References

[1] S. J. Brodsky, G. R. Farrar, Phys. Rev. D11, 1309 (1975).
[2] J. C. Collins, L. Frankfurt and M. Strikman, Phys. Rev. D 56, 2982 (1997).
[3] M. M. Kaskulov, K. Gallmeister, U. Mosel, Phys. Rev. D78, 114022 (2008).
[4] M. M. Kaskulov, U. Mosel, Phys. Rev. C80, 028202 (2009).
[5] M. M. Kaskulov, U. Mosel, Phys. Rev. C81, 045202 (2010).
[6] T. Horn et al., Phys. Rev. Lett. 97, 192001 (2006).
[7] T. Horn et al., Phys. Rev. C 78, 058201 (2008).
[8] V. Tadevosyan et al., Phys. Rev. C 75, 055205 (2007).
[9] C. J. Bebek et al., Phys. Rev. D 17, 1693 (1978).
[10] A. Airapetian et al. [ HERMES Collaboration ], Phys. Lett. B659, 486-492 (2008).
[11] V. G. Neudatchin et al., Nucl. Phys. A 739, 124 (2004).
[12] J. D. Sullivan, Phys. Lett. B 33, 179 (1970).
[13] M. Vanderhaeghen, M. Guidal and J. M. Laget, Phys. Rev. C 57, 1454 (1998); M. Guidal, J. M. Laget and M. Vanderhaeghen, Nucl. Phys. A 627, 645 (1997).
[14] G. Domokos, S. Kovesi-Domokos and E. Schonberg, Phys. Rev. D 3, 1184 (1971).
[15] J. D. Bjorken, J. B. Kogut, Phys. Rev. D8, 1341 (1973).
[16] G. Penner, U. Mosel, Phys. Rev. C66, 055212 (2002).
[17] E. D. Bloom and F. J. Gilman, Phys. Rev. Lett. 25, 1140 (1970).
[18] E. D. Bloom and F. Gilman, Phys. Rev. D 4, 2901 (1971).
[19] W. Melnitchouk, R. Ent, C. Keppel, Phys. Rept. 406, 127-301 (2005).
[20] M. Elitzur, Phys. Rev. D 3, 2166 (1971).
[21] X. Qian et al., Phys. Rev. C81, 055209 (2010).
[22] M. M. Kaskulov, [arXiv:1105.1993 [nucl-th]].
[23] M. M. Kaskulov, K. Gallmeister, U. Mosel, Phys. Rev. C79, 015207 (2009).
[24] M. M. Kaskulov, U. Mosel, [arXiv:1103.1602 [nucl-th]].