Astrophysical $S$ factor and rate of $^7\text{Be}(p, \gamma)^8\text{B}$ direct capture reaction in a potential model

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Abstract

The astrophysical $^7\text{Be}(p, \gamma)^8\text{B}$ direct capture process is studied in the framework of a two-body single-channel model with potentials of the Gaussian form. A modified potential is constructed to reproduce the new experimental value of the $S$-wave scattering length and the known astrophysical $S$ factor at the Gamow energy, extracted from the solar neutrino flux. The resulting potential is consistent with the theory developed by Baye [Phys. Rev. C 62 (2000) 065803] according to which the $S$-wave scattering length and the astrophysical $S$ factor at zero energy divided by the square of ANC are related. The obtained results for the astrophysical $S$ factor at intermediate energies are in good agreement with the two data sets of Hammache et al. [Phys. Rev. Lett. 86, 3985 (2001); *ibid.* 80, 928 (1998)]. Linear extrapolation to zero energy yields $S_{17}(0) \approx (20.5 \pm 0.5)$ eV b, consistent with the Solar Fusion II estimate. The calculated reaction rates are substantially lower than the results of the NACRE II collaboration.

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I. INTRODUCTION

The astrophysical capture process $^7\text{Be}(p, \gamma)^8\text{B}$ is the most important nuclear reaction of the pp-chain in the Solar Fusion Model and in stellar nucleosynthesis [1–4]. A realistic estimate of the reaction rate of this process is crucial for the solution of the solar neutrino problem. The core temperature of the Sun can be determined through the measurements of the $^8\text{B}$ neutrino flux with a precision of about 9% [5]. The rate of the $^7\text{Be}(p, \gamma)^8\text{B}$ reaction is used for modeling this solar neutrino flux.

Many original research papers have been published in addition to reviews [2, 3]. Direct measurements face difficulties due to large Coulomb forces at low energies [6–12]. Coulomb dissociation of $^8\text{B}$ in the field of a heavy target have been experimentally studied in Refs. [13–17]. However, none of these experimental studies could reach the energies below the solar Gamow window at 0.019 MeV. As a result, the extrapolated value $S_{17}(0)$ of the astrophysical $S$ factor in the "Solar Fusion II" (SF II) workshop

$$S_{17}(0) = (20.8 \pm 0.7_{\text{exp}} \pm 1.4_{\text{theor}}) \text{ eV b}$$

has a large uncertainty [3].

From the theory point of view, potential models [18–21], R-matrix parametrization [22], microscopic models [23–25], three-body model [26], \textit{ab-initio} calculations [27, 28], Skyrme-Hartree-Fock theory [29] and halo effective-field theory [30] have been developed. Results of most theoretical studies for $S_{17}(0)$ belong to the aforementioned uncertainty range of the SF II estimate. In Ref. [31] the reaction $^{10}\text{B}(^7\text{Be},^8\text{B})^9\text{Be}$ was used for extracting the asymptotic normalization coefficient (ANC) $C$ for the the virtual transition $p+^7\text{Be} \rightarrow ^8\text{B}$. Similarly, in Ref. [32] the ANC was extracted from the data of the $^7\text{Be}(d,n)^8\text{B}$ transfer reaction. In Ref. [33] the ANC was derived from the experimental cross section of the $^{13}\text{C}(^7\text{Li},^8\text{Li})^{12}\text{C}$ charge-conjugate reaction. As first established in Ref. [34], the astrophysical $S$ factor at low energies is mainly determined by the ANC. The idea is widely used for estimating the astrophysical $S$ factor of capture reactions [31, 32, 35].

Besides the value of the astrophysical $S$ factor at zero energy, the most important property is the energy dependence of the astrophysical $S$ factor at low energies below the Gamow window. In Ref. [36] the coefficients of the Taylor expansion in terms of energy around zero energy have been found for a given potential. Due to the fact that the largest contribution
to the astrophysical $S$ factor of the process at low energies comes from the initial $S$-wave $p + ^{7}$Be scattering state, in Ref. [37] the dependence of $S(0)/C^2$ and $S'(0)/S(0)$ on the $S$-wave $p + ^{7}$Be scattering length have been studied in details and important formulas have been derived.

On the other hand, recently in Ref. [38] the most precise experimental values $a_{o1} = 17.34^{+1.11}_{-1.33}$ fm and $a_{o2} = -3.18^{+0.55}_{-0.50}$ fm for the $s$-wave scattering lengths have been obtained in the spin=1 and spin=2 channels, respectively. Additionally, a new data for the astrophysical $S$ factor at the Gamow energy has been extracted from the solar neutrino flux [39] to be

$$S_{17}(19^{+6}_{-5} \text{ keV}) = (19.0 \pm 1.8) \text{ eV b.}$$ (2)

The aim of the present study is to estimate $S_{17}$ and corresponding reaction rates in the potential model which reproduces new values of the $S$-wave scattering length and of $S_{17}$ at the Gamow energy. This work is based on a single-channel potential model [21]. First we examine and optimize the $S$-wave potential parameters by fitting to the new value of $a_{o1}$, then we fit the bound $^3P_2$ state potential parameters based on the new values of $S_{17}$ at the Gamow energy found in Ref. [39], as described above. Then consistency of the resulting potential with the theory of Ref. [37] is examined.

In Section 2 the theoretical model is described. Section 3 contains the numerical results. Conclusions are drawn in the last section.

II. THEORETICAL MODEL

A. Wave functions

In the single-channel potential model [40-42], the initial and final state wave functions are defined as

$$\Psi_{iS}^{J} = \frac{u^{(lSJ)}_{E}(r)}{r} \{Y_{l}^{(r)}(\hat{r}) \otimes \chi_{S}(\xi)\}_{JM}$$ (3)

and

$$\Psi_{fS'}^{J_f} = \frac{u^{(l^{'}S'^{'}J_f)}_{E}(r)}{r} \{Y_{l^{'}f}^{(r)}(\hat{r}) \otimes \chi_{S'^{'}}(\xi)\}_{J_fM_f},$$ (4)

respectively. The initial $p-^{7}$Be scattering states in the $^3S_1$, $^3P_0$, $^3P_1$, $^3P_2$, $^3D_1$, $^3D_2$, $^3D_3$, $^3F_3$ partial waves are described by the radial wave functions which are solutions of the two-body
Schrödinger equation

\[
-\frac{\hbar^2}{2\mu} \left( \frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} \right) + V^{lSJ}(r) \right] u_E^{(lSJ)}(r) = Eu_E^{(lSJ)}(r), \tag{5}
\]

where \( \mu \) is the reduced mass of \( p \) and \( ^7\text{Be}(3/2^-) \), \( 1/\mu = 1/m_1 + 1/m_2 \), and \( V^{lSJ}(r) \) is a two-body potential in the partial wave with the orbital angular momentum \( l \), spin \( S \) and total angular momentum \( J \). The wave function \( u^{(l,S'J_f)}(r) \) of the final \( ^3P_2 \) ground state is calculated as a solution of the bound-state Schrödinger equation. The Schrödinger equation is solved using the Numerov algorithm. The cross section, the astrophysical \( S \) factor and the reaction rates are estimated using the accurate wave functions of the initial and final states. The initial scattering wave function is found subject to the asymptotic condition

\[
u_E^{(lSJ)}(r) \rightarrow \cos \delta_{lSJ}(E) F_l(\eta, kr) + \sin \delta_{lSJ}(E) G_l(\eta, kr), \tag{6}
\]

where \( k \) is the wave number of the relative motion, \( \eta \) is the Zommerfeld parameter, \( F_l \) and \( G_l \) are regular and irregular Coulomb functions, respectively, and \( \delta_{lSJ}(E) \) is the phase shift in the \( (l, S, J) \)th partial wave.

The \( p-^7\text{Be} \) two-body potential has the Gaussian form [21]:

\[
V^{lSJ}(r) = V_0 \exp(-\alpha_0 r^2) + V_c(r), \tag{7}
\]

where the Coulomb part is taken in a point-like potential form [21].

B. Cross sections of the radiative-capture process

The cross sections for radiative-capture process can be expressed as [2, 21]

\[
\sigma(E) = \sum_{J_f, \lambda} \sigma_{J_f, \lambda}(\Omega), \tag{8}
\]

where \( \Omega = E \) or \( M \) (electric or magnetic transition), \( \lambda \) is a multiplicity of the transition, \( J_f \) is the total angular momentum of the final state. For a particular final state with total angular momentum \( J_f \) and multiplicity \( \lambda \) we have [2]

\[
\sigma_{J_f, \lambda}(\Omega) = \sum_J \frac{(2J_f + 1) 32\pi^2 (\lambda + 1)}{|S_1||S_2|} \frac{k^{2\lambda+1} C^2(S)}{\hbar \lambda ([\lambda]!!)^2},
\]

\[
\times \sum_i \frac{1}{k_i^2 \epsilon_i} |\langle \Psi_{J_f, \lambda}^{i}\|M^\Omega \|\Psi_{lSJ_f}^{i} \rangle|^2, \tag{9}
\]

4
where \( l \) and \( l_f \) are the orbital momenta of the initial and final states, respectively; \( k_i \) and \( v_i \) are the wave number and speed of the \( p-^7\text{Be} \) relative motion in the entrance channel, respectively; \( S_1 \) and \( S_2 \) are spins of the clusters \( p \) and \( ^7\text{Be} \), \( k_\gamma = E_\gamma / \hbar c \) is the wave number of the photon corresponding to energy \( E_\gamma = E_{\text{th}} + E \), where \( E_{\text{th}} \) is the threshold energy for the breakup reaction \( \gamma + ^8\text{B} \rightarrow ^7\text{Be} + p \). Constant \( C^2(S) \) is a spectroscopic factor [2].

Within the potential approach where the bound and scattering properties (energies, phase shifts and scattering length) are reproduced, a value of the spectroscopic factor must be taken equal to 1 [35]. We also use short-hand notations \([S] = 2S + 1\) and \([\lambda]!! = (2\lambda + 1)!!\).

The reduced matrix elements are evaluated between the initial \( \Psi_{iS}^J \) and final \( \Psi_{fS'}^J \) state wave functions. The electric transition operator in the long-wavelength approximation reads as

\[
M^E_{\lambda\mu} = e \sum_{j=1}^{A} Z_j l_j^\lambda Y_{\lambda\mu}(\vec{r}_j'), \tag{10}
\]

where \( \vec{r}_j' = \vec{r}_j - \vec{R}_{\text{cm}} \) is the position of the \( j \)th particle in the center of mass system. Its reduced matrix elements can be evaluated as [2]

\[
\langle \Psi_{fS'}^J \| M^E_{\lambda\mu} \| \Psi_{iS}^J \rangle = e \left[ Z_1 \left( \frac{A_2}{A} \right)^\lambda + Z_2 \left( \frac{-A_1}{A} \right)^\lambda \right] \delta_{SS'} \times (-1)^{J+l+S} \left( \frac{[\lambda][l][J]}{4\pi} \right)^{1/2} C^{J,0}_{\lambda M} \left\{ J \ l \ S \ \begin{vmatrix} l_f \ l_f \ S_f \end{vmatrix} \right\} \int_0^\infty u^{(S_f)}_E(r) r^\lambda u_f^{(l_f S_f)}(r) dr,
\]

where \( A_1, A_2 \) are the mass numbers of the clusters in the entrance channel, \( A = A_1 + A_2 \).

The magnetic transition operator reads as [2]

\[
M^M_{\mu} = \sqrt{\frac{3}{4\pi}} \sum_{j=1}^{A} \left[ \mu_N Z_j \hat{l}_{j\mu} + 2\mu_j \hat{S}_{j\mu} \right] \tag{12}
\]

\[
= \sqrt{\frac{3}{4\pi}} \left[ \mu_N \left( \frac{A_2 Z_1}{AA_1} + \frac{A_1 Z_2}{AA_2} \right) \hat{l}_{r\mu} + 2(\mu_1 \hat{S}_{1\mu} + \mu_2 \hat{S}_{2\mu}) \right],
\]

where \( \mu_N \) is the nuclear magneton, \( \mu_j \) is the magnetic moment and \( \hat{l}_{j\mu} \) (\( \mu = -1, 0, +1 \)) is the projection of the orbital angular momentum of \( j \)th particle. The projection of the orbital angular momentum of the relative motion is denoted as \( \hat{l}_{r\mu} \). The magnetic M1 transition operator consists of the orbital and spin parts:

\[
M^M_{1\mu} = \sqrt{\frac{3}{4\pi}} \left[ M_1(l) + M_1(S) \right]. \tag{13}
\]
The orbital part of the reduced matrix elements of the magnetic M1 transition operator reads as

\[
\langle \Psi_{J_f}^l S_f || M_1(l) || \Psi_{J_i}^s S_i \rangle = \mu_N \left( \frac{A_2 Z_1 + A_1 Z_2}{AA_1} \right) \sqrt{l(l+1)[J][l]}
\]

\[
\times (-1)^{\kappa_1} \left\{ \begin{array}{c} l \\ J_f \\ 1 \\ l \end{array} \right\} \delta_{l_f,l} \delta_{S,S'} I_{i_f},
\]

where the exponential part of the phase factor \( \kappa_1 = S + 1 + J + l \). The spin part of the magnetic M1 transition operator for the first particle (proton)

\[
\langle \Psi_{J_f}^l S_f || M_1^M(S_1) || \Psi_{J_i}^s S_i \rangle = 2 \mu_p (-1)^{\kappa_2} \sqrt{S_1(S_1+1)[S][S][S'][J]}
\]

\[
\times \left\{ \begin{array}{c} S_1 \\ S_2 \\ S \\ 1 \\ S' \end{array} \right\} \left\{ \begin{array}{c} S \\ l \\ J \\ 1 \\ S' \end{array} \right\} \delta_{l_f,l} I_{i_f},
\]

with the exponential part of the phase factor \( \kappa_2 = S_1 + S_2 + 2S + l + J_f \). In the above formula and everywhere we set \( S_1 = S_p = 1/2, S_2 = S(7\text{Be}) = 3/2 \) and \( S' = S = 1 \) due to the use of the single-channel approximation. The spin part of the reduced matrix elements of the M1 transition operator for the second particle (\( ^7\text{Be} \)) reads as

\[
\langle \Psi_{J_f}^l S_f || M_1^M(S_2) || \Psi_{J_i}^s S_i \rangle = 2 \mu_{^7\text{Be}} (-1)^{\kappa_3} \sqrt{S_2(S_2+1)[S][S][S'][J]}
\]

\[
\times \left\{ \begin{array}{c} S_2 \\ S_1 \\ S \\ 1 \\ S' \end{array} \right\} \left\{ \begin{array}{c} S \\ l \\ J \\ 1 \\ S' \end{array} \right\} \delta_{l_f,l} I_{i_f},
\]

where \( \kappa_3 = S_1 + S_2 + S + S' + l + J_f \) and the overlap integral is given as

\[
I_{i_f} = \sqrt{\frac{3}{4\pi}} \int_0^{\infty} u_E(lSJ)(r)u(lSJ')(r)dr.
\]

In the above equations the magnetic momenta are taken as \( \mu_p = 2.792847 \mu_N \) and \( \mu_{^7\text{Be}} = -1.398 \mu_N \) for the first and second particles, respectively.

Finally, the astrophysical \( S \) factor of the process is expressed in terms of the cross section with the help of the equation [43]

\[
S(E) = E \sigma(E) \exp(2\pi\eta).
\]
III. NUMERICAL RESULTS

A. Details of the calculations and interaction potentials

The Schrödinger equation in the entrance and exit channels is solved with the two-body $p-^7\text{Be}$ central potentials of the Gaussian form as defined in Eq. (7) with the corresponding point-like Coulomb part. For consistency we use the same model parameters as in the aforementioned paper: $\hbar^2/2\text{[a.m.u]}=20.7343$ MeV fm$^2$, $m_p = A_1$ a.m.u. $=1.0072764669$ a.m.u., $m_{^7\text{Be}} = A_2$ a.m.u. $= 7.014735$ a.m.u. The scattering wave function $u_E(r)$ of the relative motion is obtained by solving the Schrödinger equation using the Numerov method with an appropriate potential subject to the boundary condition specified in Eq. (6).

The depth $V_0$ and width $\alpha_0$ of the $p-^7\text{Be}$ potentials $V_D$ and $V_M$ in different partial waves.

| $^{2S+1}L_J$ | $V_0$, (MeV) | $\alpha_0$, (fm$^{-2}$) | $E_{FS}^B$, (MeV) |
|-------------|--------------|-------------|-----------------|
| $^3S_1$     | -343.0       | 1.0         | -110.13         |
| $^3S_1(V_M)$ | -100.0       | 0.876       | -2.42           |
| $^3P_0$     | -580.0       | 1.0         | -102.25         |
| $^3P_1$     | -709.85      | 0.83        | -205.38         |
| $^3P_2$     | -330.414634  | 0.375       | -96.59          |
| $^3P_2(V_M)$ | -300.5003    | 0.34        | -87.86          |
| $^3D_1$     | -343.0       | 1.0         | -               |
| $^3D_2$     | -116.04      | 0.095       | -20.45          |
| $^3D_2(V_M)$ | -193.0       | 0.15        | -37.92          |
| $^3D_3$     | -343.0       | 1.0         | -               |
| $^3F_3$     | -104.555     | 0.055       | -15.99          |

The depth $V_0$ and width $\alpha_0$ of the $p-^7\text{Be}$ potentials are given in Table II. We use two parameter sets for the original potential $V_D$ from Ref. [21] and the modified potential $V_M$, respectively. These potentials differ from each other only in the $^3S_1$, $^3P_2$ and $^3D_2$ partial waves. The last column of the table contains energies of the forbidden states in the $^3S_1$, $^3P_1$, $^3P_2$, $^3P_3$, $^3D_2$ partial waves. The parameters of the modified $V_M$ potential are fitted to reproduce the scattering length $a_{01}$ in the $^3S_1$ partial wave, binding energy of the $^8\text{B}(2^+, 1)$...
ground state and the experimental astrophysical $S$ factor at the Gamow energy in the $^3P_2$ partial wave, and the experimental astrophysical $S$ factor around the $^3D_2$ resonance.

First we examine how the scattering length $a_{01}$ is described with the original $V_D$ potential in the $^3S_1$ wave. As discussed in the Introduction, the most realistic experimental data $a_{01}^{exp} = 17.34^{+1.11}_{-1.33}$ fm [38] for the spin=1 channel should be reproduced by the $p-^7$ Be potential. However, the original $V_D$ potential yields an estimate of $a_{01}^{th}=-0.26$ fm, which does not reproduce even the sign of the data. In Table I we present the fitted parameters of the new modified potential $V_M$ in the $^3S_1$ partial wave which yields an estimate of $a_{01}^{th}=17.34$ fm for the scattering length. The parameters of the modified potential $V_M$ in the $^3P_2$ bound channel are adjusted according to two conditions. The first condition for the potential is the binding energy $E_b=0.1375$ MeV of the $^8B(2^+, 1)$ ground state. The second condition comes from Eq.(2). It represents the experimental value of the astrophysical $S$ factor at the Gamow solar energy. The last condition could as well be replaced by the relation

$$S_s(0)/C^2 \approx 35.6(1 - 0.0014a_{01}) \text{ eV b fm} \approx 34.74 \text{ eV b fm}, \quad (19)$$

from Ref. [37]. This relationship connects the scattering length with the ANC and the astrophysical $S$ factor at zero energy due to the transition from the initial $S$ scattering wave. In other words, the above two conditions from Eq. (2) and Eq. (19) should be equivalent. Since Eq. (19) needs extrapolated value of the astrophysical $S$ factor at $E = 0$, its uncertainty is quite large. This is why we use Eq. (2) to define the potential parameters in the $^3P_2$ partial wave. Then consistency of the new potential with the relation in Eq. (19) will be examined. The original $V_D$ and the modified $V_M$ potentials yield values $C^2 = 0.496 \text{ fm}^{-1}$ and $C^2 = 0.538 \text{ fm}^{-1}$, respectively, for the ANC of the bound $^3P_2$ state. Finally, the parameters of the modified potential in the partial $^3D_2$ wave are chosen to reproduce the astrophysical $S$ factor in the second resonance region around $E = 3$ MeV.

In Fig. 1 we show the description of the phase shift in the $^3S_1$, $^3P_2$ and $^3D_2$ partial waves. As can be seen from the figure, the potentials $V_D$ and $V_M$ yield similar phase-shift description in the partial waves $^3P_2$ and $^3D_2$, but display significantly different description in the partial $^3S_1$ wave channel.
FIG. 1. Phase shifts in the $^3S_1$ (a), $^3P_2$ (b) and $^3D_2$ (c) partial waves of the $p-^7\text{Be}$ scattering state with potentials $V_D$ and $V_M$.

B. The astrophysical $S$ factor and the reaction rates of the $^7\text{Be}(p, \gamma)^8\text{B}$ capture process

The astrophysical $S$ factor and reaction rates of the $^7\text{Be}(p, \gamma)^8\text{B}$ direct radiative-capture process presented below are calculated with the potentials $V_D$ and $V_M$. The partial astrophysical $S$ factors estimated with the above potential models and their combination for the initial $^3S_1$ channel are presented in Fig. 2 (panel a). As can be seen, the potential model $V_M$ yields results quite different from the $V_D$ model ones for both absolute values and energy dependence of the $S$ factor. This is, firstly, due to the fact that these models yield different values for the scattering length $a_{01}$ and, secondly, due to the relation between the astro-
physical $S$ factor and the scattering length $a_{01}$ given in Eq. (19). On the other hand, the value of $S_s(0.6\text{keV})/C^2 = 35.18 \text{ eV b fm}$, calculated for the $^3S_1(V_M) \rightarrow ^3P_2(V_D)$ transition with a combined potential model is larger than the value of 34.74 eV b fm from Eq. (19). The corresponding estimate for the $^3S_1(V_M) \rightarrow ^3P_2(V_M)$ transition at the energy $E = 0.6$ keV is about 34.57 eV b fm, which is more consistent with the underlying theory [37].

In panel (b) of Fig. 2 we show the partial astrophysical $S$ factors estimated for the initial $^3D_2$ resonance channel. Here the parameters of the model $V_M$ have been adjusted to reproduce the experimental astrophysical $S$ factor around the resonance energy. Below we see that this is possible.

![Graph](image)

**FIG. 2.** Astrophysical $S$ factors for the $^7\text{Be}(p, \gamma)^8\text{B}$ synthesis reaction due to the E1 transitions $^3S_1 \rightarrow ^3P_2$ (a) and $^3D_2 \rightarrow ^3P_2$ (b) estimated within the potential models $V_D$ and $V_M$, and their combination.

Figure 3 compares the partial astrophysical $S$ factors for different initial scattering channels obtained within the potential model $V_M$. One can see that the most important contri-
bution at low energies comes from the initial $^3S_1$ channel due to the electric E1 transition. The E1 transitions from the initial $^3D_1$, $^3D_2$ and $^3D_3$ scattering channels altogether yield a contribution that is less than the contribution from the main $^3S_1$ channel by an order of magnitude at low energies. However, they become comparable at energies beyond the resonance region. The partial M1 transition from the initial $^3P_1$ scattering wave and E1 transition from the $^3D_2$ wave are responsible for the first and second resonances at energies 0.633 MeV and 2.988 MeV, respectively.

FIG. 3. The partial E1, E2 and M1 components of the astrophysical $S$ factor for the $^7\text{Be}(p, \gamma)^8\text{B}$ capture process within the $V_M$ potential model.

FIG. 4. Astrophysical $S$ factor for the $^7\text{Be}(p, \gamma)^8\text{B}$ synthesis reaction within the potential models $V_D$ and $V_M$ in comparison with available experimental data.
In Fig. 4 we present the total astrophysical $S$ factor of the $^7\text{Be}(p, \gamma)^8\text{B}$ process obtained within the potential models $V_D$ and $V_M$. As can be seen from the figure, the results for the potential model $V_M$ are mostly consistent with the two data sets of Hammache et al. [10, 11]. Other measurements [12, 14, 17, 39] show higher values in the vicinity of the resonance.

A behavior of the astrophysical $S$ factor near zero energy is more complex. Our estimates within the $V_M$ potential model are $S_{17}(1\text{ keV}) = 19.64\text{ eV b}$ and $S_{17}(0.6\text{ keV}) = 20.07\text{ eV b}$. A linear extrapolation to the zero energy yields

$$S_{17}(0) \approx (20.5 \pm 0.5)\text{ eV b},$$

which is consistent with the SF II estimate [3] quoted in Eq. (1).

Finally, estimated reaction rates within the models $V_D$ and $V_M$ are presented in Table II and Fig. 5. In the second and third columns of the table, "the most effective" energy $E_0$ and the width of the Gamow window $\Delta E_0$ are given [2]. One can note that our theoretical results are substantially lower than the estimates of the NACRE II collaboration [44] and Du et al. [45].

![Graph](image.png)

**FIG. 5.** Reaction rates of the direct $p + ^7\text{Be} \rightarrow ^8\text{B} + \gamma$ capture process within the $V_D$ and $V_M$ potential models normalized to the experimental data by the NACRE collaboration [2]

**IV. CONCLUSIONS**

The astrophysical $^7\text{Be}(p, \gamma)^8\text{B}$ direct capture process has been studied within the two-body potential model using the single-channel approximation. The modified potential is
TABLE II. Theoretical estimations of the direct $^7\text{Be}(p,\gamma)^8\text{B}$ capture reaction rate in the temperature interval $10^6 \, \text{K} \leq T \leq 10^{10} \, \text{K}$ ($0.001 \leq T_0 \leq 10$).

| $T_0$ | $E_0$ (MeV) | $\Delta E_0$ (MeV) | $N_A(\sigma v)$ (cm$^2$mol$^{-1}$s$^{-1}$) | $T_0$ | $E_0$ (MeV) | $\Delta E_0$ (MeV) | $N_A(\sigma v)$ (cm$^2$mol$^{-1}$s$^{-1}$) |
|-------|--------------|---------------------|---------------------------------|-------|--------------|---------------------|---------------------------------|
|       | $V_D$        | $V_M$               |                                 |       | $V_D$        | $V_M$               |                                 |
| 0.001 | 0.003        | 0.001               | $4.99 \times 10^{-38}$         | 0.120 | 0.072       | 0.063               | $8.15 \times 10^{-4}$          |
| 0.002 | 0.005        | 0.002               | $5.19 \times 10^{-39}$         | 0.130 | 0.076       | 0.067               | $1.34 \times 10^{-3}$          |
| 0.003 | 0.006        | 0.003               | $1.21 \times 10^{-24}$         | 0.140 | 0.079       | 0.072               | $2.09 \times 10^{-3}$          |
| 0.004 | 0.007        | 0.004               | $7.77 \times 10^{-22}$         | 0.150 | 0.083       | 0.076               | $3.12 \times 10^{-3}$          |
| 0.005 | 0.009        | 0.005               | $6.07 \times 10^{-20}$         | 0.160 | 0.087       | 0.080               | $4.51 \times 10^{-3}$          |
| 0.006 | 0.010        | 0.005               | $1.87 \times 10^{-18}$         | 0.180 | 0.094       | 0.088               | $8.63 \times 10^{-3}$          |
| 0.007 | 0.011        | 0.006               | $2.87 \times 10^{-17}$         | 0.200 | 0.101       | 0.096               | $1.51 \times 10^{-2}$          |
| 0.008 | 0.012        | 0.007               | $2.72 \times 10^{-16}$         | 0.250 | 0.117       | 0.116               | $4.58 \times 10^{-2}$          |
| 0.009 | 0.013        | 0.007               | $1.82 \times 10^{-15}$         | 0.300 | 0.132       | 0.135               | $1.06 \times 10^{-1}$          |
| 0.010 | 0.014        | 0.008               | $9.35 \times 10^{-15}$         | 0.350 | 0.146       | 0.153               | $2.07 \times 10^{-1}$          |
| 0.011 | 0.015        | 0.009               | $3.90 \times 10^{-14}$         | 0.400 | 0.160       | 0.172               | $3.60 \times 10^{-1}$          |
| 0.012 | 0.015        | 0.009               | $1.38 \times 10^{-13}$         | 0.500 | 0.186       | 0.207               | $8.53 \times 10^{-1}$          |
| 0.013 | 0.016        | 0.010               | $4.27 \times 10^{-13}$         | 0.600 | 0.210       | 0.240               | $1.66 \times 10^{0}$           |
| 0.014 | 0.017        | 0.010               | $1.18 \times 10^{-12}$         | 0.700 | 0.232       | 0.273               | $2.85 \times 10^{0}$           |
| 0.015 | 0.018        | 0.011               | $2.97 \times 10^{-11}$         | 0.800 | 0.254       | 0.306               | $4.50 \times 10^{0}$           |
| 0.016 | 0.019        | 0.012               | $6.92 \times 10^{-12}$         | 0.900 | 0.275       | 0.337               | $6.69 \times 10^{0}$           |
| 0.018 | 0.020        | 0.013               | $3.07 \times 10^{-11}$         | 1.000 | 0.295       | 0.368               | $9.46 \times 10^{0}$           |
| 0.020 | 0.022        | 0.014               | $1.11 \times 10^{-10}$         | 1.500 | 0.386       | 0.516               | $3.18 \times 10^{1}$           |
| 0.025 | 0.025        | 0.017               | $1.44 \times 10^{-9}$          | 2.000 | 0.468       | 0.656               | $6.45 \times 10^{1}$           |
| 0.030 | 0.028        | 0.020               | $1.01 \times 10^{-8}$          | 2.500 | 0.543       | 0.790               | $1.03 \times 10^{2}$           |
| 0.040 | 0.034        | 0.025               | $1.72 \times 10^{-7}$          | 3.000 | 0.613       | 0.919               | $1.45 \times 10^{2}$           |
| 0.050 | 0.040        | 0.030               | $1.27 \times 10^{-6}$          | 4.000 | 0.743       | 1.168               | $2.38 \times 10^{2}$           |
| 0.060 | 0.045        | 0.035               | $5.82 \times 10^{-6}$          | 5.000 | 0.862       | 1.407               | $3.41 \times 10^{2}$           |
| 0.070 | 0.050        | 0.040               | $1.95 \times 10^{-5}$          | 6.000 | 0.973       | 1.638               | $4.52 \times 10^{2}$           |
| 0.080 | 0.055        | 0.045               | $5.27 \times 10^{-5}$          | 7.000 | 1.078       | 1.863               | $5.70 \times 10^{2}$           |
| 0.090 | 0.059        | 0.049               | $1.22 \times 10^{-4}$          | 8.000 | 1.179       | 2.082               | $6.93 \times 10^{2}$           |
| 0.100 | 0.063        | 0.054               | $2.50 \times 10^{-4}$          | 9.000 | 1.275       | 2.297               | $8.18 \times 10^{2}$           |
| 0.110 | 0.068        | 0.058               | $4.69 \times 10^{-4}$          | 10.00 | 1.368       | 2.507               | $9.45 \times 10^{2}$           |

constructed to reproduce the new experimental value of the $S$-wave scattering length and the known astrophysical $S$ factor at the Gamow energy, extracted from the solar neutrino flux. The modified potential is consistent with the theory of Baye [36] which connects the $S$-wave scattering length with the astrophysical $S$ factor at zero energy divided by the square of ANC.

The results obtained for the astrophysical $S$ factor within the modified potential approach are in accordance with the data of Hammache et al. in contrast to those obtained using
the original potential by Dubovichenko et al. [21]. The value of the astrophysical $S$ factor extrapolated to zero energy is found to be $S_{17}(0) \approx (20.5 \pm 0.5)$ eV b which is consistent with the SF II estimates [3]. However, the calculated reaction rates are lower than the results of the NACRE II collaboration [44].

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