On Max-SINR Receiver for HMT System over Doubly Dispersive Channel

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Abstract—In this paper, a novel receiver for Hexagonal Multicarrier Transmission (HMT) system based on the maximizing Signal-to-Interference-plus-Noise Ratio (Max-SINR) criterion is proposed. Theoretical analyses show that there is a timing offset between the prototype pulses of the proposed Max-SINR receiver and the traditional projection receiver. Meanwhile, the timing offset should be matched to the channel scattering factor of the doubly dispersive (DD) channel. The closed form timing offset expressions of the prototype pulse for Max-SINR HMT receiver over DD channel with different channel scattering functions are derived. Simulation results show that the proposed Max-SINR receiver outperforms traditional projection scheme and obtains an approximation to the theoretical upper bound SINR performance. Consistent with the SINR performance improvement, the bit error rate (BER) performance of HMT system has been further improved by using the proposed Max-SINR receiver. Meanwhile, the SINR performance of the proposed Max-SINR receiver is robust to the channel delay spread estimation errors.

Index Terms—Hexagonal Multicarrier Transmission System; Maximizing Signal-to-Interference-plus-Noise-Ratio (Max-SINR) Receiver; Doubly Dispersive (DD) channel;

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) systems with guard-time interval or cyclic prefix can prevent inter-symbol interference (ISI). OFDM has overlapping spectra and rectangular impulse responses. Consequently, each OFDM sub-channel exhibits a sinc-shape frequency response. Therefore, the time variations of the channel during one OFDM symbol duration destroy the orthogonality of different subcarriers, and result in power leakage among subcarriers, known as inter-carrier interference (ICI), which causes degradation in system performance. In order to overcome the above drawbacks of OFDM system, several pulse-shaping OFDM systems such as multiwavelets based OFDM system, OFDM based on offset quadrature amplitude modulation (OFDM/OQAM) system, et al., were proposed [1]–[8].

It is shown in [9]–[12] that signal transmission through a rectangular lattice is suboptimal for doubly dispersive (DD) channel. By using results from sphere covering theory [13], the authors have demonstrated that lattice OFDM (LOFDM) system, which is OFDM system based on hexagonal-type lattice, providing better performance against ISI/ICI [9]. However, LOFDM confines the transmission pulses to a set of orthogonal ones. In [10]–[12], the authors abandoned the orthogonality condition of the modulated pulses and proposed a multicarrier transmission scheme named as hexagonal multicarrier transmission (HMT). In HMT system, there is no cyclic prefix and data symbols of HMT signal are transmitted at the hexagonal type lattice points in TF plane. In our previous work [14]–[17], we have analyzed the system SINR performance and presented that traditional HMT receiver proposed in [10]–[12] using zero timing offset prototype pulse is a suboptimal approach in the view of SINR. The receiver prototype pulse based on Max-SINR criterion for HMT system over DD channel with exponential power delay profile and U-shape Doppler spectrum was proposed.

In this paper, we will present the receiver prototype pulses based on Max-SINR criterion for HMT system over DD channel with different channel scattering functions. Theoretical analyses show that there is a timing offset between prototype pulses of the proposed Max-SINR HMT receiver and traditional HMT receiver in the SINR point of view. The closed form timing offset expressions of prototype pulse for Max-SINR HMT receiver over the DD channel with different channel scattering functions are derived. Simulation results show that the proposed Max-SINR receiver obtains an approximation to the theoretical upper bound SINR performance and outperforms traditional receiver [10]–[12] on bit error rate (BER) performance. Meanwhile, the SINR performance of the proposed scheme is robust to the channel delay spread estimation errors.

II. HEXAGONAL MULTICARRIER TRANSMISSION SYSTEM

In HMT systems, the transmitted baseband signal can be expressed as [10]–[12]

\[
x(t) = \sum_m \sum_n c_{m,2n} g(t - mT)e^{j2\pi nFt} \\
+ \sum_m \sum_n c_{m,2n+1} g(t - mT - \frac{T}{2})e^{j2\pi nF + \frac{T}{2}t} \tag{1}
\]

where \(T\) and \(F\) are the lattice parameters, as shown in Fig.1; \(c_{m,n}\) denotes the transmitted data symbol, which is assumed to be taken from a specific signal constellation and independent and identically distributed (i.i.d.) with zero mean and average power \(\sigma_c^2\); \(m \in M\) and \(n \in N\) are the position indices in the TF plane; \(M\) and \(N\) denote the sets from which \(m, n\) can be
The ambiguity function of Gaussian pulse is defined by the spread function. In wide-sense stationary uncorrelated scattering, the elements of the Gram matrix, which leads to high implementation complexity.

As shown in [9], the orthogonalization process needs to compute all the orthogonalized pulses are constructed from a nonorthogonal set of Gaussian pulses. Without loss of generality, we use the scattering function. The prototype pulse can be modeled as a random linear operator $H$ [21],

$$
\psi_{m,n}(t) = \langle r(t), \psi_{m,n}(t) \rangle
$$

where $\psi_{m,n}(t)$ is the prototype pulse at the receiver. The energy of the received symbol $c_{m,n}^i$, after projection on the filter function $\psi_{m,n}^i(t)$ over DD channel can be expressed as

$$
E_i = \mathbb{E} \left\{ \left( \sum_{j} \sum_{m',n'} c_{m',n'} \langle H[g_{m',n'}(t)], \psi_{m,n}^j(t) \rangle + \langle w(t), \psi_{m,n}^j(t) \rangle \right)^2 \right\}
$$

Under the assumptions that WSSUS channel and source symbols $c_{m,n}^i$ are statistically independent, (6) can be rewritten as

$$
E_i = \sigma^2 \int_{\tau} \int_{\nu} S_H(\tau, \nu) \left[ \sum_{m,n} \left| A_{g,\psi}(mT + \tau, nF + \nu) \right|^2 + \sum_{m,n} \left| A_{g,\psi}(mT + T/2 + \tau, nF + F/2 + \nu) \right|^2 \right] d\tau dv + \sigma^2 \left| A_{g,\psi}(0,0) \right|^2
$$

We can see from Eq. (7) that $E_i$ is composed of the expectation symbol energy, ISI/ICI and additive noise. The SINR of the desired symbol $c_{m_0,n_0}^{i_0}$ can be expressed as

$$
R_{\text{SINR}} = \frac{\sigma^2}{E_{\text{IN}}} \int_{\tau} \int_{\nu} S_H(\tau, \nu) \left| A_{g,\psi}(\tau, \nu) \right|^2 d\tau dv
$$

where the interference-plus-noise energy $E_{\text{IN}}$ is the energy perturbation of the desired symbol $c_{m_0,n_0}^{i_0}$ from other symbols over time varying multipath fading channel $H(\tau, \nu)$. For presentation simplicity, the desired symbol $c_{m_0,n_0}^{i_0}$ is chosen as $c_1^{i_0,0}$. Hence, $E_{\text{IN}}$ can be expressed as

$$
E_{\text{IN}} = \sigma^2 \int_{\tau} \int_{\nu} S_H(\tau, \nu) \cdot \left[ \sum_{\substack{m,n,i \neq [0,0,1]}} \left| A_{g,\psi}(m + i/2T + \tau, nF + \nu) \right|^2 + \sum_{\substack{m,n,i \neq [0,0,1]}} \left| A_{g,\psi}(m + i/2T + T/2 + \tau, nF + F/2 + \nu) \right|^2 + \left| F + \nu \right|^2 \right] d\tau dv + \sigma^2 \left| A_{g,\psi}(0,0) \right|^2
$$

According to the form of channel scattering functions, we have the following two cases [10].
A. DD channel with uniform power delay profile and uniform Doppler spectrum

For the DD channel with uniform power delay profile and uniform Doppler spectrum (DD-UNI), the scattering function can be expressed as $S_H(\tau, v) = 1/(2\tau_{\text{max}} f_d)$ [23], with $\tau_{\text{max}} \geq \tau > 0, |v| < f_d$. We assume that $\psi(t) = g(t - \Delta t)e^{j2\pi\Delta f t}, |\Delta f| < f_d$, the SINR of the received signal over the DD-UNI channel can be expressed as

$$R_{\text{SIN}}^{\text{UNI}} = \frac{\sigma^2}{2\pi\tau_{\text{rms}} f_d E_{\text{SU}}^{\text{UNI}} \int_{0}^{\tau_{\text{rms}}} e^{-\sigma^2 (\tau - \Delta t)^2} d\tau} \cdot \int_{-f_d}^{f_d} e^{-\sigma^2 (\tau - \Delta t)^2} d\tau$$

The theoretical SINR upper bound of the received signal over the DD-UNI channel can be expressed as

$$R_{\text{UB}}^{\text{UNI}} = \arg \max_{\Delta t, \Delta f} R_{\text{SIN}}^{\text{UNI}}$$

The Max-SINR prototype pulse can be expressed as (see Appendix A)

$$\psi(t) = g(t - \frac{\tau_{\text{max}}}{2})$$

B. DD channel with exponential power delay profile and U-shape Doppler spectrum

For the DD channel with exponential power delay profile and U-shape Doppler spectrum (DD-EXP), the scattering function can be expressed as $S_H(\tau, v) = \frac{e^{\frac{-\tau}{\tau_{\text{rms}}}}}{\pi \tau_{\text{rms}} f_d \sqrt{1 - (v/f_d)^2}}$ [23] with $\tau > 0, |v| < f_d$, $\tau_{\text{rms}}$ denotes the channel root mean square (RMS) delay spread. We assume that $\psi(t) = g(t - \Delta t)e^{j2\pi\Delta f t}, |\Delta f| < f_d$, the theoretical SINR can be expressed as

$$R_{\text{SIN}}^{\text{EXP}} = \frac{\sigma^2}{\pi \tau_{\text{rms}} f_d E_{\text{SU}}^{\text{EXP}} \int_{0}^{\infty} e^{-\frac{\tau}{\tau_{\text{rms}}} \cdot e^{-\frac{\pi^2}{\tau_{\text{rms}}^2}} (\tau - \Delta t)^2} d\tau} \cdot \int_{-f_d}^{f_d} e^{-\sigma^2 (\tau - \Delta t)^2} d\tau$$

The theoretical SINR upper bound of the received signal over the DD-EXP channel can be expressed as

$$R_{\text{UB}}^{\text{EXP}} = \arg \max_{\Delta t, \Delta f} R_{\text{SIN}}^{\text{EXP}}$$

We can see from equation (12) and (16) that the prototype pulses of the proposed Max-SINR receiver over DD channel are functions of channel maximum delay spread and RMS delay spread, respectively. Recently, several channel estimation schemes for multicarrier modulation system with hexagonal TF lattice have been proposed in [25–27] and all this schemes are suitable for HMT system.

IV. SIMULATION AND DISCUSSION

In this section, we test the proposed Max-SINR receiver via computer simulations based on the discrete signal model. In the following simulations, the number of subcarriers for HMT system is chosen as $N=40$, and the length of prototype pulse is set to $N_g=600$. The center carrier frequency is $f_c=5$GHz and the sampling interval is set to $T_s=10^{-6}$s. The system parameters of HMT system are $F=25$kHz, $T=1 \times 10^{-3}$s and the signaling efficiency $\rho=0.8$. $\sigma$ for prototype pulse $g(t)$ is set to $\sigma T/\sqrt{3F}$. Traditional projection receiver proposed in [10–12] is named as Traditional Projection Receiver (TPR) in the following simulation results.

A. SINR Performance of the Proposed Max-SINR Receiver for HMT System

1) SINR Performance of HMT System over DD-UNI Channel: The SINR performance of different receivers with the variety of $\sigma^2/\sigma_w^2$ for HMT system over DD-UNI channel is depicted in Fig. 2. The CSF $\vartheta$ is set to 0.07 and 0.2, respectively. We can see from Fig. 2 that the SINR performance of Max-SINR receiver outperforms TPR scheme about 0.5~1.5dB at $\vartheta=0.07$ and $\sim$3dB at $\vartheta=0.2$, respectively. The SINR gap between Max-SINR receiver and theoretical SINR upper bound is smaller than 0.1dB at $\vartheta=0.07$ and 0.2, respectively. The SINR performance with the variety of channel spread factor $\vartheta$ at $\sigma^2/\sigma_w^2=20$dB over DD-UNI channel is depicted in Fig. 3. It can be seen that there is a degradation of SINR with the increasing of channel spread factor. Max-SINR receiver obtains an approximation to the theoretical upper bound SINR performance within the full range of $\vartheta$. Meanwhile, Max-SINR receiver obtains an about 3.5dB maximum SINR gain over TPR scheme at $\vartheta=0.35$.

2) In [2], the Max-SINR prototype pulse $g$ of multicarrier transmission system with rectangular TF lattice over DD channel is obtained by maximizing the generalized Rayleigh quotient $g = \arg \max \frac{\|Hg\|^2}{\|g\|^2}$. The solution is the generalized eigenvector of the matrix pair $(B, A)$ corresponding to the largest generalized eigenvalue. It is shown that there is a delay between the transmitted Gaussian prototype pulse and the received Gaussian prototype pulse. In this paper, the close form time offset expressions between the transmitted and received prototype pulse of multicarrier transmission system with hexagonal TF lattice is derived.
2) Performance of HMT System over DD-EXP Channel:
The SINR performance of different receivers with the variety of $\sigma_c^2/\sigma_w^2$ for HMT system over DD-EXP channel is depicted in Fig. 4. $\tau_{rms}f_d$ in Fig. 4 is set to 0.07 and 0.2, respectively. We can see from Fig. 4 that the SINR performance of Max-SINR receiver outperforms TPR scheme about 1~4dB at $\tau_{rms}f_d=0.07$ and 1.5~3.5dB at $\tau_{rms}f_d = 0.2$, respectively. The SINR gap between Max-SINR receiver and the theoretical SINR upper bound is smaller than 0.5dB and 0.1dB at $\tau_{rms}f_d = 0.07$ and 0.2, respectively. The SINR performance with the variety of $\tau_{rms}f_d$ at $\sigma_c^2/\sigma_w^2=20$dB over DD-EXP channel is depicted in Fig. 5. It can be seen that there is a degradation of SINR with the increasing of $\tau_{rms}f_d$. Max-SINR receiver obtains an approximation to the theoretical upper bound SINR performance within the full range of $\tau_{rms}f_d$. Max-SINR receiver achieves an about 2.5dB maximum SINR gain over TPR scheme at $\tau_{rms}f_d=0.35$.

B. BER Performance of the Proposed Max-SINR Receiver for HMT System

The BER performance of the proposed Max-SINR receiver for HMT system over the DD channel with different channel scattering functions is given in Fig. 6.

For the DD-UNI channel, channel spread factor $\tau_{max}f_d$ is set to 0.2. We can conclude from Fig. 6 that the BER performance of the proposed Max-SINR receiver for HMT system over DD-UNI channel outperforms TPR receiver about 2dB at $E_b/N_0 = 20$dB. For the DD-EXP channel, $\tau_{rms}f_d$ is set to 0.1. We can see from Fig. 6 that the proposed Max-SINR receiver over DD-EXP channel outperforms TPR receiver on the BER performance and the performance gain is about 2.5dB at $E_b/N_0 = 20$dB.

C. Robustness of the Proposed Max-SINR Receiver against Channel Delay Spread Estimation Errors

The robustness of the proposed Max-SINR receiver against channel delay spread estimation errors is depicted in Fig.
errors of $\tau_{ms}$ and $\tau_{max}$.

V. CONCLUSION

A novel receiver based on Max-SINR criterion for HMT system over DD channel with different channel scattering functions is proposed in this paper. Theoretical analyses show that there is a timing offset between the prototype pulses of the proposed Max-SINR receiver and the traditional projection receiver. The closed form timing offset expressions of prototype pulse for Max-SINR HMT receiver over DD channel with different channel scattering functions are derived. Simulation results show that the proposed Max-SINR receiver obtains an approximation to the theoretical upper bound SINR performance and outperforms traditional projection scheme on BER performance. Meanwhile, the SINR performance of the proposed prototype pulse is robust to the channel delay spread estimation errors.

APPENDIX A

PROOF OF EQUATION (12)

The SINR of the received signal over DD-UNI channel can be expressed as

$$R_{\text{SINUNI}}^{\text{UNI}} \approx \frac{\sigma^2_c}{2\sigma_w^2} \int_0^{\tau_{\text{rms}}/2} e^{-\pi \tau^2} \int_{-\tau_{\text{rms}}/2}^{\tau_{\text{rms}}/2} \sigma_\nu^2 e^{-\sigma_\nu^2 u^2} du \int_{-\tau_{\text{rms}}/2}^{\tau_{\text{rms}}/2} \sigma_\nu^2 e^{-\sigma_\nu^2 u^2} du$$

(17)

We can see from equation (17) that $R_{\text{SINUNI}}^{\text{UNI}}$ is the product of two functions $R_{\text{SINUNI}}^{\text{UNI}}(\Delta t)$ and $R_{\text{SINUNI}}^{\text{UNI}}(\Delta f)$ with respect to $\Delta t$ and $\Delta f$, respectively. Hence, the optimal timing offset $\Delta t$ and the optimal frequency offset $\Delta f$ can be obtained independently.

The optimal timing offset $\Delta t$ can be obtained by taking the partial derivative of $R_{\text{SINUNI}}^{\text{UNI}}$ with respect to $\Delta t$ and solving the partial derivative equal to zero for $\Delta t$,

$$\frac{\partial R_{\text{SINUNI}}^{\text{UNI}}}{\partial \Delta t} = 0$$

(18)

Plugging (17) in (18) and ignoring the constant items with respect to $\Delta t$, the partial derivative can be rewritten as

$$\int_0^{\tau_{\text{rms}}/2} \frac{2\pi(\tau - \Delta t)}{\sigma} e^{-\pi(\tau - \Delta t)^2} d\tau = 0$$

(19)

Under the assumption that $0 < \Delta t < \tau_{\text{max}}$, we can rewrite (19) as

$$\int_0^{\tau_{\text{max}}/2} \frac{2\pi(\tau - \Delta t)}{\sigma} e^{-\pi(\tau - \Delta t)^2} d\tau + \int_{\Delta t}^{\tau_{\text{max}}/2} \frac{2\pi(\tau - \Delta t)}{\sigma} e^{-\pi(\tau - \Delta t)^2} d\tau = 0$$

(20)

Let $\tau' = \tau - \Delta t$, we can rewrite (20) as

$$\int_0^{\tau_{\text{max}} - \Delta t} \frac{2\pi\tau'}{\sigma} e^{-\pi(\tau')^2} d\tau' = \int_0^{\Delta t} \frac{2\pi\tau'}{\sigma} e^{-\pi(\tau')^2} d\tau'$$

(21)

Hence, the solution of equation $\partial R_{\text{SINUNI}}^{\text{UNI}}/\partial \Delta t = 0$ is $\tau_{\text{max}} - \Delta t = \Delta t$, that is $\Delta t = \tau_{\text{max}}/2$, and the SINR of the received symbols obtains the maximum value while $\Delta t = \tau_{\text{max}}/2$. 

Fig. 6. The BER performance of the proposed Max-SINR receiver for HMT system over the DD channel with different channel scattering functions.

Fig. 7. The robustness of the proposed Max-SINR receiver against channel delay spread estimation errors. Estimation errors of $\tau_{\text{rms}}$ and $\tau_{\text{max}}$ for DD-EXP channel and DD-UNI channel are modeled as uniformly distributed random variables in the interval $[-\tau_{\text{rms}}/2, \tau_{\text{rms}}/2]$ and $[-\tau_{\text{max}}/2, \tau_{\text{max}}/2]$, respectively.

7. Estimation errors of $\tau_{\text{rms}}$ and $\tau_{\text{max}}$ for DD-EXP channel and DD-UNI channel are modeled as uniformly distributed random variables in the interval $[-\tau_{\text{rms}}/2, \tau_{\text{rms}}/2]$ and $[-\tau_{\text{max}}/2, \tau_{\text{max}}/2]$, respectively.

In Fig. 7, the channel spread factor $\vartheta$ of DD-UNI channel is set to 0.1. We can see from Fig. 7 that the SINR gap between the SINR upper bound and the proposed Max-SINR receiver with estimation error of $\tau_{\text{max}}$ is within 0.1dB and 0.5dB at $\sigma^2_c/\sigma^2_w=0$dB and 30dB, respectively. The SINR performance of the proposed Max-SINR receiver over DD-EXP channel at $\tau_{\text{rms}} f_d=0.1$ is given in Fig. 7. The SINR gap between the SINR upper bound and the proposed Max-SINR receiver with estimation error of $\tau_{\text{rms}}$ is within 0.1dB and 0.7dB at $\sigma^2_c/\sigma^2_w=0$dB and 30dB, respectively. SINR performance of the TPR receiver over DD-UNI and DD-EXP channel is also depicted in Fig. 7 for comparison. The proposed Max-SINR receiver outperforms TPR scheme when there exists estimation...
The optimal frequency offset $\Delta f$ can be obtained by taking the partial derivative of $R_{\text{SIN}}^{\text{UNI}}$ with respect to $\Delta f$ and solving the partial derivative equal to zero for $\Delta f$,

$$
\frac{\partial R_{\text{SIN}}^{\text{UNI}}}{\partial \Delta f} = 0
$$

(22)

Plugging (17) in (22) and let $\nu' = \nu - \Delta f$, we can rewrite (22) as

$$
\int_{0}^{f_d-\Delta f} \frac{2\pi \nu' e^{-\sigma \nu'^2} d\nu'}{\kappa(\Delta f)} = \int_{0}^{f_d+\Delta f} \frac{2\pi \nu e^{-\sigma \nu^2} d\nu}{\chi(\Delta f)}
$$

(23)

Hence, the solution of partial derivative $\frac{\partial R_{\text{SIN}}^{\text{UNI}}}{\partial \Delta f} = 0$ is $f_d - \Delta f = f_d + \Delta f$, that is $\Delta f = 0$.

**APPENDIX B**

**Proof of Equation (16)**

The SINR of the received signal can be expressed as

$$
R_{\text{SIN}}^{\text{EXP}} \approx \frac{\sigma_c^2}{\sigma_{\tau_{\text{rms},d}}} \int_{-f_a}^{f_d} e^{-\sigma \pi (v-\Delta f)^2} dv 
\cdot \left( \frac{2\pi \nu e^{-\sigma \nu^2} d\nu}{\kappa(\Delta f)} \right)
$$

(24)

where

$$
b(\Delta t) = \sqrt{\frac{\sigma}{\pi}} \int_{-\infty}^{\infty} e^{-\nu^2} d\nu = \frac{\sqrt{\pi}}{2} \text{erfc} \left( \frac{\sigma}{2\pi \tau_{\text{rms}}} - \Delta t \right)
$$

(25)

and $\text{erfc}(\cdot)$ is the complementary error function. If $x > 0$, we may obtain an approximate solution of the complementary error function $\text{erfc}(\cdot)$ by [24]

$$
\text{erfc}\left( \frac{x}{\sqrt{2}} \right) \approx \frac{2e^{-x^2}}{1.64x + \sqrt{0.76}x^2 + 1}
$$

(26)

We can see from equation (24) that $R_{\text{SIN}}^{\text{UNI}}$ is also the product of two functions with respect to $\Delta t$ and $\Delta f$, respectively. Hence, the optimal timing offset $\Delta t$ and the optimal frequency offset $\Delta f$ can be obtained independently.

The optimal timing offset $\Delta t$ can be obtained by solving the gradient $a(\Delta t)b(\Delta t)$ with respect to $\Delta t$ to zero,

$$
\frac{da(\Delta t)}{d\Delta t} b(\Delta t) + \frac{db(\Delta t)}{d\Delta t} a(\Delta t) = 0
$$

(27)

where $\frac{da(\Delta t)}{d\Delta t} = \frac{a(\Delta t)}{\tau_{\text{rms}}}$ and $\frac{db(\Delta t)}{d\Delta t} = e^{-\frac{\pi}{\tau_{\text{rms}}} - \Delta t^2}$.

Hence, equation (27) can be rewritten as

$$
b(\Delta t) = \frac{e^{-\frac{\pi}{\tau_{\text{rms}}} - \Delta t^2}}{2\tau_{\text{rms}}} \text{erfc} \left( \frac{\sqrt{\pi}}{\sigma} \left( \frac{\sigma}{2\pi \tau_{\text{rms}}} - \Delta t \right) \right)
= \frac{\sqrt{\pi}}{\tau_{\text{rms}}} \text{erfc} \left( \frac{\sqrt{\pi}}{\sigma} \left( \frac{\sigma}{2\pi \tau_{\text{rms}}} - \Delta t \right) \right)
\approx \frac{\sqrt{\pi}}{\tau_{\text{rms}}} \text{erfc} \left( \frac{\sqrt{\pi}}{\sigma} \left( \frac{\sigma}{2\pi \tau_{\text{rms}}} - \Delta t \right) \right) = \left( 1.64 \sqrt{\frac{2\pi}{\sigma}} \frac{\sigma}{2\pi \tau_{\text{rms}}} - \Delta t \right)^{-1}
$$

(28)

Equation (28) can be simplified to a quadratic equation. Under the constraint of $\Delta t > 0$, the solution of the quadratic equation can be expressed as

$$
\Delta t = \frac{\sigma}{2\pi \tau_{\text{rms}}} - \sqrt{\frac{3.28\sqrt{\pi}}{2\tau_{\text{rms}}} - \sqrt{\frac{3.28^2\sigma}{2\tau_{\text{rms}}} - 3.52}}
$$

(29)

The optimal timing offset $\Delta t$ can be obtained by solving the partial derivative $R_{\text{SIN}}^{\text{EXP}}$ with respect to $\Delta f$ to zero,

$$
\frac{\partial R_{\text{SIN}}^{\text{EXP}}}{\partial \Delta f} = 0
$$

(30)

Let $\Xi(\Delta f)$ denotes the partial derivative $\partial R_{\text{SIN}}^{\text{EXP}}/\partial \Delta f$. Plugging (24) in (30) and ignoring the constant items with respect to $\Delta f$, $\Xi(\Delta f)$ can be rewritten as

$$
\Xi(\Delta f) = \int_{-f_a}^{f_d} 2\pi \nu (v - \Delta f) e^{-\sigma \pi (v - \Delta f)^2} dv
$$

(31)

Both of the exponential function and $\sqrt{1 - (v/f_a)^2}$ are non-negative, and $2\sigma \pi (v - \Delta f)$ is a monotonically increasing function. $\Xi(\Delta f)$ can be expressed as

$$
\Xi(-\Delta f) = \int_{-f_a}^{f_d} 2\pi \nu (v + \Delta f) e^{-\sigma \pi (v + \Delta f)^2} dv
$$

(32)

Hence, $\Xi(\Delta f)$. $|\Delta f| < f_d$, is a continuous odd function. Meanwhile, $\partial^2 R_{\text{SIN}}^{\text{EXP}}/\partial \Delta f^2 = \partial \Xi(\Delta f)/\partial \Delta f$ can be expressed as

$$
\frac{\partial \Xi(\Delta f)}{\partial \Delta f} = \int_{-f_a}^{f_d} 2\pi \nu (2\pi \nu (v - \Delta f)^2 - 1) e^{-\sigma \pi (v - \Delta f)^2} dv
$$

(33)

Practical wireless channels usually satisfy that $\nu = \tau_{\text{max}} f_d < 1$ [21] and the optimal system parameter for HMT over DD channels can be chosen as $\tau_{\text{rms}} = \tau_{\text{rms},d}$ [10]. Hence, $\nu$ in (33) satisfies $\nu < 1/\Delta f^2$ and $\partial \Xi(\Delta f)/\partial \Delta f < 0$, $|\Delta f| < f_d$. We can conclude from equation (32) and (33) that $\Xi(\Delta f)$, $|\Delta f| < f_d$, is a continuous odd function and $\partial \Xi(\Delta f)/\partial \Delta f < 0$, therefore, the necessary and sufficient condition of $R_{\text{SIN}}^{\text{EXP}}$ to obtain its maximum value is $\Delta f = 0$. 

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