Non-fragile infinite time control for networked Lipschitz nonlinear system

Weiguo Ma¹,a and Xia Xu¹,b*
¹College of Electrical Engineering, Nantong University, Nantong, China
²Corresponding author¹email: wgma@ntu.edu.cn
Abstract. The non-fragile infinite time control is investigated for networked Lipschitz nonlinear system with quantization and incomplete sensor data in this paper. The state of the system is quantized before transmitted to the controller over the network. The Markov Chain is adopted to describe the transmission process. The non-fragile infinite time controller which makes the system stable and performance index minimum is designed in form of linear matrix inequality. A numerical example is given to demonstrate the effectiveness of the proposed method.

1. Introduction
Lipschitz system is a kind of common nonlinear systems, which can describe many physical processes [1]. When the components of the system such as sensor, controller and actuator are distributed in a broad space, the information can be transmitted through the network. Networked control system has the advantages of low cost, good scalability, easy installation and maintenance, which has become the trend of control system [2, 3]. However, the introduction of network also results in some issues. For instance, the sensor samples the state of the controlled system. The measurement data need to be quantized before transmitted to the controller over the network. In the process of transmission, the data packet may be lost due to network congestion or interference.

When the system model is established, the factors which have little effect on the system are usually ignored such as small parameter change of system components, friction force and external interference. Therefore, there will be errors between the obtained and the actual model of the system.

The infinite time control is a widely used control method. For a given controlled system, the weighted matrix of the performance index is taken as constant, and the upper limit of the integral is infinite. The infinite time linear quadratic optimal control problem is addressed for systems with stochastic disturbance and input constraint [4]. The infinite horizon H2/H∞ controller for discrete-time time-varying linear systems subject to Markov jump parameters and state-multiplicative noise is designed [5]. The infinite time optimal tracking control for a class of discrete-time nonlinear systems is studied using the greedy heuristic dynamic programming iteration algorithm [6]. The parallel and non parallel distributed infinite horizon model predictive control is considered for fuzzy discrete systems [7].

Due to the influence of some factors such as analog to digital conversion accuracy and word length, there exist uncertainties in the implementation of the controller. Therefore, the design of non-fragile controller is of great significance. For linear system with actuator saturation and disturbances, Liu and Yang design the non-fragile dynamic output feedback controller [8]. The stabilization of uncertain networked control systems with additive time-varying delays is addressed by using non-fragile...
sampled-data control in [9]. Bu et al consider the non-fragile distributed fault estimation for time-varying systems through sensor networks over a finite horizon [10]. The finite time dissipative analysis and non-fragile control are investigated for a class of uncertain discrete time switched linear systems [11].

In this paper, the non-fragile infinite time control for Lipschitz nonlinear system with quantization and incomplete measurement data is considered. The designed controller makes the closed loop system stable. The performance index is minimized.

2. Problem formulation

The diagram of the non-fragile infinite time control for Lipschitz nonlinear system considered in this paper is shown in Figure 1.

![Diagram of the non-fragile infinite time control system](image)

The controlled system is described by

\[
\begin{aligned}
    x(k+1) &= (G + \Delta G)x(k) + (H + \Delta H)u(k) + (L + \Delta L)f(k, x(k)) \\
    y(k) &= (C + \Delta C)x(k)
\end{aligned}
\]  

where \( x(k) \in \mathbb{R}^n \) is the state of the system, \( u(k) \in \mathbb{R}^p \) is the input of the system, \( y(k) \in \mathbb{R}^m \) is the output of the system, \( f(k, x(k)) \) is the Lipschitz nonlinear function, i.e. \( \|f(k, x_i(k)) - f(k, x_j(k))\| \leq F\|x_i(k) - x_j(k)\| \). \( G, H, L, C \) and \( F \) are known constant matrices with appropriate dimensions, \( \Delta G, \Delta H, \Delta L, \Delta C \) are uncertainties of the system satisfying

\[
[\Delta G, \Delta H, \Delta L, \Delta C] = D F(k) [E_G, E_H, E_L, E_C], \quad F_i^T(k) F_i(k) \leq I
\]

where \( D, E_G, E_H, E_L, E_C \) are known constant matrices with appropriate dimensions.

The sensor samples the state of the system and converts it into digital signal. In this paper, a logarithmic quantizer is adopted. The output of the quantizer is

\[
x_q(k) = [I + F_i(k)] x(k)
\]

where \( F_i^T(k) F_i(k) \leq \delta^2 I, \quad \delta = (1 - \rho)/(1 + \rho), \quad \rho \) is the quantization density.

Due to the influence of network congestion or external interference, the measurement data may be lost in the process of transmission to the controller over the network. The process of transmission is described by a Markov Chain with two states. The state set is \( \Phi = \{0, 1\} \). The transition probability matrix \( \Pi = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix} \). \( \alpha(k) = 0 \) implies that the data packet is dropped, the previous data is used as the input of the controller. \( \alpha(k) = 1 \) means that the data packet is transmitted successfully. The input of the controller is

\[
x_q(k) = [1 - \alpha(k)] x_q(k - 1) + \alpha(k) x_q(k), \quad \alpha(k) = i = \{0, 1\}
\]

The system performance index is given as

\[
J = \sum_{k=0}^{\infty} \left[ x^T(k) Q x(k) + u^T(k) R u(k) \right]
\]

where \( Q \) and \( R \) are known symmetric positive definite matrices.
For the controlled system and performance index, the following non-fragile controller is designed to minimize the performance index.

\[ u(k) = (K_{c(k)} + \Delta K)x_c(k) \]  

where \( \Delta K = DF_1(k)E_k \), \( F_1^T(k)F_1(k) \leq I \), \( D \), \( E_k \) are known constant matrices with appropriate dimensions.

Defining \( x_a(k) = \begin{bmatrix} x(k) \\ x(k-1) \end{bmatrix} \), the closed loop system is formulated as

\[
\begin{cases}
x_a(k+1) = G_{cl}x_a(k) + L_{cl}f(k, x(k)) \\
y(k) = C_{cl}x_a(k)
\end{cases}
\]

where \( G_{cl} = \begin{bmatrix} G + \Delta G & (H+\Delta H)(K_1 + \Delta K) \\ 0 & I \end{bmatrix} \), \( G_{cl1} = \begin{bmatrix} G_{cl11} \\ 0 \end{bmatrix} \), \( G_{cl11} = G + \Delta G + (H+\Delta H)(K_1 + \Delta K)(I + F_2(k)) \), \( L_{cl0} = L_{cl1} = \begin{bmatrix} L + \Delta L \\ 0 \end{bmatrix} \), \( C_{cl0} = C_{cl1} = [C + \Delta C, 0] \).

3. Main results

The non-fragile infinite time optimal controller for Lipschitz nonlinear systems is designed in this section.

Theorem 1 For the Lipschitz nonlinear system (1) and the performance index (5), if there exist symmetric positive definite matrices \( X_{cl1}, X_{cl2}, X_{cl3}, X_{cl4}, W_0, W_1 \), matrices \( M_0, M_1 \), scalars \( \varepsilon_1 > 0 \), \( \varepsilon_2 > 0 \), \( \varepsilon_3 > 0 \), satisfying the following linear matrix inequalities

\[
\Omega = \begin{bmatrix}
\Omega_{11} & * & * & * & * & * & * \\
0 & \Omega_{22} & * & * & * & * & * \\
\Omega_{31} & \Omega_{32} & \Omega_{33} & * & * & * & * \\
\Omega_{41} & \Omega_{42} & 0 & \Omega_{44} & * & * & * \\
\Omega_{51} & 0 & 0 & 0 & \Omega_{55} & * & * \\
\Omega_{61} & \Omega_{62} & 0 & 0 & 0 & \Omega_{66} & * \\
\Omega_{71} & 0 & 0 & 0 & 0 & 0 & \Omega_{77} \\
\Omega_{81} & 0 & \Omega_{83} & \Omega_{84} & 0 & \Omega_{86} & 0 & \Omega_{88} \\
\end{bmatrix} < 0
\]

\[
A = \begin{bmatrix}
A_{11} & * & * & * & * & * & * \\
0 & A_{22} & * & * & * & * & * \\
A_{31} & A_{32} & A_{33} & * & * & * & * \\
A_{41} & A_{42} & 0 & A_{44} & * & * & * \\
A_{51} & 0 & 0 & 0 & A_{55} & * & * & * \\
0 & 0 & A_{63} & A_{64} & A_{65} & A_{66} & * & * \\
A_{71} & A_{72} & 0 & 0 & 0 & A_{76} & A_{77} & * \\
A_{81} & 0 & 0 & 0 & 0 & 0 & 0 & A_{88} \\
A_{91} & 0 & A_{93} & A_{94} & 0 & 0 & 0 & 0 & A_{99} \\
\end{bmatrix} < 0
\]
where \( \Omega_1 = \text{diag}\{X_{\Omega_1},-X_{\Omega_2}\} \), \( \Omega_2 = -\varepsilon I \), \( \Omega_3 = \begin{bmatrix} G_{\Omega_1} & H_{\Omega_1}^T \\ 0 & X_{\Omega_2} \\ \end{bmatrix} \), \( \Omega_4 = \begin{bmatrix} \varepsilon L \\ \end{bmatrix} \), \( \Omega_5 = \text{diag}\{-\pi_{\Omega_1}^* X_{\Omega_1},-\pi_{\Omega_2}^* X_{\Omega_2}\} \),

\[
\Omega_6 = \begin{bmatrix} G_{\Omega_1} & H_{\Omega_1}^T \\ 0 & X_{\Omega_2} \\ \end{bmatrix}, \quad \Omega_7 = \begin{bmatrix} \varepsilon L \\ \end{bmatrix}, \quad \Omega_8 = \text{diag}\{-\pi_{\Omega_1}^* X_{\Omega_1},-\pi_{\Omega_2}^* X_{\Omega_2}\}.
\]

Taking the difference of \( \mathcal{V} \), one can obtain

\[
\mathcal{V}_t = \begin{bmatrix} E_kX_{\Omega_1} & E_{\Omega_1}X_{\Omega_2} \\ 0 & 0 \\ \end{bmatrix} \frac{\partial \mathcal{V}}{\partial X_{\Omega_1}} + \begin{bmatrix} 0 \\ \end{bmatrix} \frac{\partial \mathcal{V}}{\partial X_{\Omega_2}} + \begin{bmatrix} -\varepsilon \mathcal{L} \\ -\varepsilon \mathcal{L} \\ \end{bmatrix} - \begin{bmatrix} X_{\Omega_1} \\ X_{\Omega_2} \\ \end{bmatrix} \frac{\partial \mathcal{V}}{\partial \Omega_1} - \begin{bmatrix} 0 \\ \end{bmatrix} \frac{\partial \mathcal{V}}{\partial \Omega_2} - \begin{bmatrix} X_{\Omega_1} \\ X_{\Omega_2} \\ \end{bmatrix} \frac{\partial \mathcal{V}}{\partial \Omega_3} - \begin{bmatrix} 0 \\ \end{bmatrix} \frac{\partial \mathcal{V}}{\partial \Omega_4}.
\]

As \( \mathcal{F}^{T}(k)f(k) \leq x^T(k)F \mathcal{F}x(k) \), there exist \( \varepsilon > 0 \) such that \( \varepsilon x^T(k)F \mathcal{F}x(k) - \varepsilon f^{T}(k)f(k) \geq 0 \).
where \( \Xi^3 = \left[ \begin{array}{c}
\Xi_{11}^3 + \Xi_{21}^3 + U_1^* \\

\Xi_{21}^3 - \Xi_{22}^3 \end{array} \right] \), \( U_1 = \begin{bmatrix} \varepsilon F^T F & 0 \\
0 & 0 \end{bmatrix} \), \( \Xi^2 = \left[ \begin{array}{c}
\Xi_{11}^2 \\

\Xi_{21}^2 \end{array} \right] \), \( \Xi_{11}^1 = Q + \alpha(k)(I + \\
F_2(k))(K + \Delta K)^T R \alpha(k)(K + \Delta K)(I + F_2(k)) \), \( \Xi_{21}^2 = (1 - \alpha(k))(K + \Delta K)^T R \alpha(k)(K + \Delta K)(I + F_2(k)) \), \\
\Xi_{22}^2 = (1 - \alpha(k)) (K + \Delta K)^T R (K + \Delta K) \).

Let \( X_{01} = P_{01}^1 \), \( X_{02} = P_{02}^1 \), \( X_{11} = P_{11}^1 \), \( X_{12} = P_{12}^1 \), \( X_{01} = P_{01}^1 \), \( W_0 = \varepsilon_{41} P_{01}^1 P_{01}^1 \), \( W_1 = \varepsilon_{41} P_{11}^1 P_{11}^1 \), \\
\( M_0 = K_n P_{02}^1 \), \( M_1 = K_n P_{11}^1 \), \( \epsilon_i = \epsilon_i^1 \). Multiplying \( \text{diag} \left\{ P_{011}, P_{021}, \epsilon_1, I, I, I, I, P_{011}, \epsilon_1, I, I, I, P_{011}, I, I \right\} \) left and right on both sides of (8) and (9) respectively, one has \( \Xi^3 < 0 \). Then, \( \Delta V[x_a(k)] < 0 \). The closed loop system is stable.

As \( \sum_{k=0}^{\infty} x^T(k)Qx(k) + u^T(k)Ru(k) + \Delta V[x_a(k)] \) < \( 0 \), \\
(8) \sum_{k=0}^{\infty} x^T(k)Qx(k) + u^T(k)Ru(k) - x^T_a(0)P_{\sigma(0)}x_a(k) < 0 \).

Since \( E \{ x(0) \} = 0 \), \( E \{ x(0) \} = 1 \), the performance index \( J < x^T_a (0) P_{\sigma(0)} x_a(k) < \text{Trace}(X_{\sigma(0)11}^{-1}) \).

This completes the proof.

Theorem 2 For the Lipschitz nonlinear system (1) and the performance index (5), if the following optimization problem has solution \( \bar{N}_2, K'_0 = \bar{M}_0 \bar{X}_{022}^{-1}, K'_1 = \bar{M}_1 \bar{X}_{111}^{-1} \) are the non-fragile infinite time optimal controllers of the system (1).

\[
\begin{align*}
\min_{N_2} \text{Trace}(N_2) \\
\text{s.t.} & \ a \mathcal{Q} < 0 \\
& \ A < 0 \\
& \begin{bmatrix} N_1 & * \\
 I & X_{011} \end{bmatrix} > 0 \\
& \begin{bmatrix} N_1 & * \\
 I & X_{111} \end{bmatrix} > 0
\end{align*}
\]

where \( N_2 = (X_{01}, X_{02}, X_{111}, X_{122}, W_0, W_1, M_0, M_1, \epsilon_1, \epsilon_2, \epsilon_3, N_0) \).

Proof. If \( \bar{N}_2 \) is the solution of the optimization problem, the conditions (a) and (b) hold. \( K'_0 \) and \( K'_1 \) are the non-fragile infinite time controllers. By Schur complement lemma, (c) and (d) is equivalent to \( N_1 - X_{\sigma(0)11}^{-1} > 0 \). Namely, \( \text{Trace}(X_{\sigma(0)11}^{-1}) < \text{Trace}(N_2) \), so the performance index is minimized. This completes the proof.

4. Numerical example

Consider the nonlinear system (1) with the following parameters:

\[
\begin{align*}
G &= \begin{bmatrix} 0.06 & 0.09 \\
0.17 & 0.24 \end{bmatrix}, \\
H &= \begin{bmatrix} 0.08 \\
0.2 \end{bmatrix}, \\
L &= \begin{bmatrix} 0.02 \\
0.03 \end{bmatrix}, \\
C &= [0.1, 0.1], \\
D &= 0.02, \\
E_G &= \begin{bmatrix} 0.07 & 0.04 \\
0.02 & 0.05 \end{bmatrix}, \\
E_H &= \begin{bmatrix} 0.03 \\
0.06 \end{bmatrix}, \\
E_L &= \begin{bmatrix} 0.04 \\
0.02 \end{bmatrix}, \\
E_C &= \begin{bmatrix} 0.05, 0.02 \end{bmatrix}, \\
f(k) &= 0.02 \times \sin(k), \\
F &= \begin{bmatrix} 1 & 0 \\
0 & 1 \end{bmatrix}, \\
Q &= \begin{bmatrix} 0.2 & 0 \\
0 & 0.1 \end{bmatrix}, \\
R &= 0.2, \\
\rho &= 0.8, \\
E_k &= \begin{bmatrix} 0.01, 0.02 \end{bmatrix}, \\
P &= \begin{bmatrix} 0.36 & 0.64 \\
0.15 & 0.85 \end{bmatrix}.
\end{align*}
\]
The transmission process of the data is shown in Figure 2. Solving the linear matrix inequalities (8) and (9) yields the non-fragile infinite time controller $K_0 = \begin{bmatrix} 0.0145 \\ 0.0120 \end{bmatrix}$, $K_1 = \begin{bmatrix} 0.0980 \\ 0.2241 \end{bmatrix}$.

Assume that the system state $x(0) = [0.5, 0.2]$. The trajectory of the system state controlled by the designed controller is shown in Figure 3. It can be concluded that the Lipschitz nonlinear system is stable under the action of the designed controller.

5. Conclusion
The non-fragile infinite time controller for Lipschitz nonlinear system is designed considering the effect of data quantization and packet drop out. The controller gain matrices can be obtained by solving a set of linear matrix inequalities. A numerical example is presented to demonstrate the effectiveness of the proposed method.

Acknowledgments
This work was supported in part by Natural Science Foundation of Nantong under grant JC2018146.

References
[1] Zhu, F. L. and Han, Z. Z. (2002) A note on observers for Lipschitz nonlinear systems. IEEE T. Automat. Contr., 47: 1751-1754.
[2] Park, P., Ergen, S. C., Fischione, C., Lu, C. Y. and Johansson, K. H. (2018) Wireless network design for control systems: a survey. IEEE Commun. Surv. Tut., 20: 978-1013.
[3] Li, T. X., Zhang, W. A. and Yu, L. (2019) Improved switched system approach to networked control systems with time-varying delays. IEEE T. Contr. Syst. T., 27: 2711-2717.
[4] Chmielewski, D. J. and Manousiouthakis, V. (2005) On constrained infinite-time linear quadratic optimal control with stochastic disturbances. J. Process Contr., 15: 383-391.
[5] Ma, H. J., Zhang, W. H. and Hou, T. (2012) Infinite horizon $H_2/H_\infty$ control for discrete-time time-varying Markov jump systems with multiplicative noise. Automatica, 48: 1447-1454.
[6] Zhang, H. G., Wei, Q. L. and Luo, Y. H. (2008) A novel infinite-time optimal tracking control scheme for a class of discrete-time nonlinear systems via the greedy HDP iteration algorithm. IEEE T. Syst. Man Cy. B, 38: 937-942.
[7] Xia, Y. Q., Yang, H. J., Shi, P. and Fu, M. Y. (2010) Constrained infinite-horizon model predictive control for fuzzy-discrete-time systems. IEEE T. Fuzzy Syst., 18: 429-436.
[8] Liu, D. and Yang, G. H. (2018) Event-triggered non-fragile control for linear systems with actuator saturation and disturbances. Inform. Sciences, 429: 1-11.
[9] Muthukumar, P., Arunagirinathan, S. and Lakshmanan, S. (2019) Nonfragile sampled-data control for uncertain networked control systems with additive time-varying delays. IEEE T. Cybernetics, 49: 1512-1523.
[10] Bu, X. Y., Dong, H. L., Wang, Z. D. and Liu, H. J. (2019) Non-fragile distributed fault estimation for a class of nonlinear time-varying systems over sensor networks: the finite-horizon case. IEEE T. Signal Inf. Pr., 5: 61-69.

[11] Xia, J. W., Gao, H., Liu, M. X., Zhuang, G. M. and Zhang, B. Y. (2018) Non-fragile finite-time extended dissipative control for a class of uncertain discrete time switched linear systems. J. Franklin I., 355: 3031-3049.