Spherical collapse in a dark energy model with reconstructed effective equation of state

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Abstract

The present work deals with the study of spherical collapse of matter density contrast in a late-time cosmological model with a reconstructed effective equation of state. The linear and nonlinear evolution of matter density contrast are studied. The variation of the critical density at collapse of the spherical overdense region along redshift are also investigate. Further the number count of collapsed object or the dark matter halos, equivalent to the number count of galaxy clusters, are also studied. Two different halo mass function formulations, namely the Press-Schechter mass function and the Sheth-Tormen mass function, are adopted to study the cluster number count along the redshift. Similar analysis is also carried out for \( w \)CDM dark energy model to have a direct comparison with the reconstructed effective equation of state model. These two models are highly degenerate at the background and linear level of matter perturbation. But the nonlinear evolution of matter overdensity breaks the degeneracy. The reconstructed effective equation of state model shows a substantial suppression in the cluster number count compared to \( w \)CDM for redshift \( z > 0.5 \), and for \( z < 0.5 \), the number count is slightly higher than that of \( w \)CDM.

Keywords: Cosmology, dark energy, equation of state, spherical collapse, cluster number count.

1 Introduction

The discovery of cosmic acceleration [1, 2, 3] and its further confirmations from latest cosmological observations [4, 5, 6, 7] have opened a new arena of research in cosmology. To explain the phenomenon of cosmic acceleration with the regime of General Relativity, it is indispensable to assume the existence of an exotic component in the energy budget of the universe. This exotic component is dubbed as dark energy. The alleged acceleration is generated due to the

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effective negative pressure of dark energy. With the unprecedented technological advancement in cosmological observations, different cosmological parameters are constrained to a very high level of accuracy. But it hardly provides any information about the physical entity of dark energy. The dark energy properties have their signature on the dynamics of the universe at background and perturbative level. The present work is devoted to study the nonlinear evolution matter density perturbation for a reconstructed dynamical dark energy model. The reconstructed model is based on a parametrization of the total or effective equation of state of the energy sector [8]. For a comparison, the same analysis has also been carried out for $w_{CDM}$ dark energy model in the present context.

In the analysis, a semi-analytic approach, namely the spherical collapse model [9, 10, 11], is adopted to study the evolution of matter overdensity. The basic idea is to study the evolution of spherical homogeneous overdensity using the fully nonlinear equation derived from Newtonian hydrodynamics. The overdensity region is assumed to be spherically symmetric and to have a uniform density which is higher than the background density of dark matter. It is considered as a closed sub-universe expanding with Hubble flow. But the expansion slows down and after reaching a maximum radius, it starts compression and eventually collapses due to gravitational attraction. A virilization of gravitational potential and thermal energy due to random motion needs to be introduced in this context to explain the finite size of the collapsed object. The effect of dark energy on the clustering of dark matter can be probed in the spherical collapse model if matter overdensity. Several studies in this direction are there in literature [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. Another numerically sophisticated approach to study the nonlinear evolution of cosmological perturbation and formation of large scale structure in the universe is the N-body simulation [29, 30, 31, 32].

As already mentioned that the evolution of matter overdensity is effected by the background cosmology, it is obvious that the nature of dark energy would have its signature on the collapse of matter overdensities and the formation of large scale structure of the universe. In the present work, the spherical collapse of matter overdensity is studied for a reconstructed dynamical dark energy scenario. Further, the distribution of the galaxy cluster number along the redshift is studied for the to different dark energy models which are degenerate at background level. The linear and nonlinear evolution of matter density contrast are studied. Critical density contrast at collapse is defined as the value of linear density contrast when the nonlinear value diverges. Critical density contrast is important to study the spherical collapse and formation of large scale structure. The objects, formed due to the collapse of dark matter overdensity are called the dark matter halos. The galaxy clusters are embedded in the dark matter halos due to gravitation. The distribution of galaxy clusters follows the same distribution as the dark matter halos. Thus the observed distribution of galaxy cluster number count is the probe of the distribution of dark matter halos in the universe. For a comparison, the same analysis has also been carried out for another dark energy model, namely the $w_{CDM}$. The reconstructed $w_{eff}$ model was found to be highly consistent with $w_{CDM}$ at background level [8]. The prime endeavour of the present work
is to investigate whether degenerate dark energy models can produce distinguishable effect on the collapse of matter overdensity and formation of large scale structure.

The paper is presented as follows. In section 2, the reconstructed dynamical dark energy model and the observational constraints are discussed. The spherical collapse scenario and the critical density at collapse and the virial overdensity are studied in section 3. In section 4 the formulations of halo mass function and the galaxy cluster number count of are discussed. Finally, in section 5, it has been concluded with overall summarization of the results.

2 Reconstructed effective equation of state

The constructed model of latetime cosmology, adopted in the present work to study the clustering of dark matter, is based on the parametrization of effect equation of state of the energy budget in the universe [8]. In a spatially flat FLRW universe, the Friedmann equations are written as,

\[ 3H^2 = 8\pi G \rho_{tot}, \]  
\[ 2\dot{H} + 3H^2 = -8\pi G p_{tot}, \]  

where \( \rho_{tot} \) and \( p_{tot} \) are the total energy density and the total pressurelike contribution of the components in the energy budget. The effective equation of state is defined as,

\[ w_{eff} = \frac{p_{tot}}{\rho_{tot}}. \]  

The phenomenological parametrization was introduced as,

\[ w_{eff}(z) = -\frac{1}{1 + \alpha(1 + z)^n}, \]  

where \( z \) is the redshift, and \((\alpha, n)\) are the model parameters. The constraints on the model was obtained as [8], \( \alpha = 0.444 \pm 0.042 \), \( n = 2.907 \pm 0.136 \) at \( 1\sigma \), combining the data from observational measurements of Hubble parameter, the distance modulus measurements of type Ia supernovae, baryon acoustic oscillation and CMB distance prior measurements. The reconstructed \( w_{eff} \) model is found to be highly degenerate with the \( w_{CDM} \) dark energy model. In case of \( w_{CDM} \), the dark energy equation of state parameter is kept as a constant free parameter \( w \), instead of fixing at \( w = -1 \) as in \( \Lambda CDM \). The reconstructed \( w_{eff} \) model mimics the \( \Lambda CDM \) for \( n = 3 \), well within the \( 1\sigma \) confidence region. The constraints on \( w_{CDM} \) parameters, obtained in [8] using the same sets of data, are \( w = -0.981 \pm 0.031 \), \( \Omega_{m0} = 0.296 \pm 0.011 \) at \( 1\sigma \). For the \( w_{eff} \) model, the values of matter density parameter obtained is \( \Omega_{m0} = 0.296 \pm 0.011 \) at \( 1\sigma \), exactly same to that of \( w_{CDM} \). These two models are also found to be highly degenerate on kinematical parameter space. The observational constraints on these two models are utilized for the analysis in the present work.
The matter density contrast is defined as $\delta = \Delta \rho_m / \rho_m$ where $\Delta \rho_m$ is the deviation from homogeneous matter density $\rho_m$. The overdense region initially grows in size due to Hubble expansion. But it gathers mass due to gravitational attraction. Due to the increasing gravitational force, at certain time, it ceases expansion and starts collapsing. Gravitational collapse of the overdense regions is the fundamental process to form the large scale structure in the universe. To understand the dynamics of the structure formation, it is essential to study the non-linear evolution of the matter overdensities. A spherical collapse model [9, 10, 11] is the simplest approach to probe the evolution of the matter density contrast at the nonlinear regime. It is a semi-analytic approach that assumes the overdense regions are spherically symmetric. The nonlinear differential equation, that governs the time evolution of the matter density contrast is,

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G \rho_m \delta (1 + \delta) - \frac{4}{3} \frac{\dot{\delta}^2}{1 + \delta} = 0. \tag{5}$$

The linear version of equation (5) is given as,

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G \rho_m \delta = 0. \tag{6}$$
Figure 2: The linear (left panels) and nonlinear (right panels) evolution of matter density contrast $\delta(a)$ for the reconstructed $w_{\text{eff}}$ model for different values of the models parameters $\alpha$ and $n$. The initial conditions for the numerical solutions of the evolution equations are fixed at $a_i = 0.001$ as $\delta(a_i) = 0.0042$ and $\delta'(a_i) = 0.0$.

Using scale factor ‘a’ as the argument of differentiation in equation (5) yield,

$$\delta'' + \left( \frac{h'}{h} + \frac{3}{a} \right) \delta' - \frac{3\Omega_{m0}}{2a^3h^2} \delta(1 + \delta) - \frac{4}{3} \frac{\delta'^2}{(1 + \delta)} = 0. \quad (7)$$

Similarly the liner equation (eq. 6) is given as,

$$\delta'' + \left( \frac{h'}{h} + \frac{3}{a} \right) \delta' - \frac{3\Omega_{m0}}{2a^3h^2} \delta = 0. \quad (8)$$

Equation (7) and (8) are studied numerically for the reconstructed $w_{\text{eff}}$ model and $w_{\text{CDM}}$ model. The initial conditions for the numerical solutions of equation (7) and (8) are fixed at $a = 0.001$.
when the universe was mainly matter dominated. Figure 1 shows the linear and nonlinear evolutions of $\delta(a)$ for these two models. In figure 1 the boundary values are fixed as $\delta(a_i) = 0.0042$ and $\delta'(a_i) = 0.0$. The plots clearly show that the linear evolution of $\delta(a)$ is almost indistinguishable for these two models. But the nonlinear evolution is not degenerate. The $w_{eff}$ model allows the collapse of the overdense region slight earlier than the $w_{CDM}$ scenario. In figure 2, the effects of variation in the values of the model parameters $\alpha$ and $n$ are studied. The linear evolution is very less effected by the change in parameter values. On the other hand, the nonlinear evolution is found to be suppressed with the increase in the values of $\alpha$ and $n$.

Another quantity of interest to study the clustering of dark matter in a spherically collapsing scenario is the critical density contrast at collapse ($\delta_c$). It is defined as the value of the linear density contrast at the redshift where the non-linear density contrast diverges. Changing the initial condition in the differential equation (equation (7)), we can figure out the critical density contrast $\delta_c$ as a function of redshift at the collapse ($z_c$). The curves of $\delta_c(z_c)$ for the $w_{eff}$ and $w_{CDM}$ model are shown in figure 3. The critical density at collapse $\delta_c$ as a function of redshift is also important to study the distribution of galaxy cluster number count or the number dark matter halos.

4 Halo mass function and cluster number count

In this section, the distribution of the number density of collapsed object of a given mass range is studied. The collapsed objects are called the dark matter halos. The baryonic matter follows
the distribution of dark matter. Thus the distribution of dark matter halos can be tracked by observing the distribution of galaxy clusters. In semi-analytic approach two different mathematical formulation halos mass is used to evaluate the distribution of number of collapsed objects or the galaxy clusters along the redshift. The first one is the Press-Schechter formalism [33] and the other one, which is a generalization of the first one, is called the Sheth-Tormen formalism [34]. The mathematical formulations of halo mass function are based on the assumption of a Gaussian distribution of the matter density field. The comoving number density of collapsed object (galaxy clusters) at a certain redshift $z$ having mass range $M$ to $M + dM$ can be expressed as,

$$\frac{dn(M, z)}{dM} = -\frac{\rho_{m0}}{M} \frac{d\ln \sigma(M, z)}{dM} f(\sigma(M, z)), \quad (9)$$

where $f(\sigma)$ is called the mass function. The mathematical formulation of the mass function, proposed by Press and Schechter [33], is given as,

$$f_{PS}(\sigma) = \sqrt{\frac{2}{\pi}} \frac{\delta_c(z)}{\sigma(M, z)} \exp \left[-\frac{\delta_c^2(z)}{2\sigma^2(M, z)}\right]. \quad (10)$$

The $\sigma(M, z)$ is the corresponding rms density fluctuation in a sphere of radius $r$ enclosing a mass $M$. This can be expressed in terms of the linearized growth factor $g(z) = \delta(z)/\delta(0)$, and the rms of density fluctuation at a fixed length $r_8 = 8h^{-1}\text{Mpc}$ as,

$$\sigma(z, M) = \sigma(0, M_8) \left(\frac{M}{M_8}\right)^{-\gamma/3} g(z), \quad (11)$$

where $M_8 = 6 \times 10^{14}\Omega_{m0}h^{-1}M_\odot$, the mass within a sphere of radius $r_8$ and the $M_\odot$ is the solar mass. The $\gamma$ is defined as

$$\gamma = (0.3\Omega_{m0}h + 0.2) \left[2.92 + \frac{1}{3} \log \left(\frac{M}{M_8}\right)\right]. \quad (12)$$

Finally the effective number of collapsed objects between a mass range $M_i < M < M_s$ per redshift and square degree yield as,

$$\mathcal{N}(z) = \int_{1\text{deg}^2} d\Omega \left(\frac{c}{H(z)} \left[\frac{z}{H(x)}\right]^2\right) \int_{M_i}^{M_s} \frac{dn}{dM} dM. \quad (13)$$

The number count of collapsed object along the redshift is studied for the $w_{eff}$ model and $w$CDM model using equation (13). The values of Hubble constant $H_0$ and the $\sigma_8$ are fixed at the Planck-$\Lambda$CDM measurements as $H_0 = 67.66 \pm 0.42 \text{ km s}^{-1}\text{Mpc}^{-1}$, $\sigma_8 = 0.8102 \pm 0.0060$ (CMB power spectra+CMB lensing+BAO) [7]. As the models are consistent with $\Lambda$CDM at 1$\sigma$ level, the $\Lambda$CDM estimated values of these parameters can be safely used in the present context. The
number count distribution, obtained for the Press-Schechter mass function formula (equation 10) are shown in figure 4. The difference of number count distribution for these two models $\Delta N(z) = (N_{wCDM}(z) - N_{w_{eff}}(z))$, scaled by $N_{w_{eff}}(z)$ are also shown in figure 4. The results shows a suppression of cluster number count for the $w_{eff}$ model at redshift $z > 0.5$. At very low redshift, the number count for $w_{eff}$ model is found to be slightly higher than that of $w_{CDM}$ model.

Though the Press-Schechter formalism is successful to depict a general nature of the distribution of galaxy cluster number count, it suffers from the prediction of higher abundance of galaxy cluster at low redshift and lower abundance of clusters at high redshift compared to the result obtained in simulation of dark matter halo formation [35]. To alleviate this issue, a modified mass function formula is proposed by Sheth and Tormen [34], which is given as,

$$ f_{ST}(\sigma) = A \sqrt{\frac{2}{\pi}} \left[ 1 + \left( \frac{\sigma^2(M,z)}{a\delta_c^2(z)} \right)^p \right] \frac{\delta_c(z)}{\sigma(M,z)} \exp \left[ -\frac{a\delta_c^2(z)}{2\sigma^2(M,z)} \right]. $$

(14)

The Sheth-Tormen mass function formula, given in equation (14), introduce three new parameters $(a, p, A)$ and for the values $(1, 0, \frac{1}{2})$ the Sheth-Torman mass function actually become the Press-Schechter mass function. In the present work, while studying the distribution of cluster number count using Sheth-Tormen mass function formula, the values of the parameter $(a, p, A)$ are fixed at
Figure 5: The left panel shows the cluster number count plots as a function of redshift obtained using the Sheth-Tormen mass function formula. The plots for the reconstructed $w_{\text{eff}}$ model and $w$CDM model are shown. The right panel shows the difference of cluster number count $\Delta N = N_{\text{wCDM}} - N_{\text{weff}}$, normalized by $N_{\text{weff}}$.

$(0.707, 0.3, 0.322)$ as obtained from the simulation of dark matter halo formation [35]. The number count distribution, obtained for the Sheth-Tormen mass function formula (equation 14) are shown in figure 5. The deviation of cluster number count of these two models in this case follows the same pattern as it was in the Press-Schechter formulation. The Sheth-Tormen formalism predicts slightly higher cluster number at high redshift compared to the Press-Schechter formalism.

5 Conclusion

In the present work, two degenerate dark energy models, namely the reconstructed $w_{\text{eff}}$ model and the $w$CDM model, have been utilized to study the nonlinear evolution of matter overdensity. The semi-analytic prescription of spherical collapse of matter overdensity is adopted for the present purpose. It is observed that the linear evolution of matter density contrast is almost degenerate for these two models but the nonlinear evolution breaks the degeneracy (figure 1). The effect of variation of parameter values is found to be more prominent in the nonlinear evolution compared to the linear evolution of matter density contrast (figure 2). The critical density at collapse as function of redshift has a similar pattern in these two models, and for the $w_{\text{eff}}$ model it attains a slightly higher value at very low redshift (figure 3).

The number count of dark matter halos or the galaxy clusters has also been also been stud-
ied for these two models at late-time cosmology. It is observed that the cluster number count is substantially suppressed in case of the $w_{\text{eff}}$ model at higher redshift ($z > 0.5$) compared to that of $w_{\text{CDM}}$. At very low redshift ($z < 0.5$) the cluster number is slightly higher for the $w_{\text{eff}}$ model. This pattern of difference in the cluster number count between these two models is similar in both, the Press-Schechter and Sheth-Tormen formalism. The Sheth-Tormen formalism has a higher cluster number at high redshift compared to the Press-Schechter formalism.

The present study shows that the evolution of nonlinear perturbation is highly sensitive to the slight change in dark energy model. It is efficient to break the degeneracy in the models. Spherical collapse model of matter overdensity is an useful method to study the effect dark energy properties on large scale structure of the universe. The results of such studies would be useful for selection of dark energy models based on future observations like South Pole Telescope, eROSITA etc.

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