The lightest neutralino in the MNSSM

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Abstract. We examine the allowed mass range of the lightest neutralino within the Minimal Non–minimal Supersymmetric Standard Model. Being absolutely stable if R-parity is conserved this lightest neutralino is a candidate for the dark matter of the universe. We establish the theoretical upper bound on the lightest neutralino mass and obtain an approximate solution for this mass.

PACS. 12.60.Jv Supersymmetric models – 14.80.Ly Supersymmetric partners of known particles – 95.35.+d Dark matter

1 Introduction

The existence of dark matter is now a well established fact. Recent astrophysical and cosmological observations indicate that the Universe contains approximately five times more exotic matter than ordinary matter. It corresponds to 22%-25% of the energy density of the Universe \cite{1}. This exotic matter exists in the form of non–baryonic, non–luminos (dark) matter. Although the microscopic composition of dark matter remains a mystery it is clear that it can not consist of any elementary particles which have been discovered so far.

The minimal supersymmetric (SUSY) standard model (MSSM) which is the best motivated extension of the SM, provides a good candidate for the cold dark matter component of the Universe. If R–parity is conserved, the lightest supersymmetric particle (LSP) in the MSSM is absolutely stable and can play the role of dark matter. In most supersymmetric scenarios the LSP is the lightest neutralino. Neutralinos naturally provide the correct relic abundance of dark matter if these particles have masses of a few hundred GeV. Furthermore in this case they behave as cold (non–relativistic) dark matter which explains well the large anisotropies in the cosmic microwave background radiation \cite{2}. In an attempt to break the \(Z_3\) symmetry, operators that are suppressed by powers of the Planck scale could be introduced, but these operators give rise to quadratically divergent tadpole contributions, which destabilise the mass hierarchy \cite{3}. The dangerous operators can be eliminated if an invariance under \(Z_2^F\) or \(Z_2^R\) symmetries is imposed \cite{4}. A linear term \(\lambda S\) in the superpotential is induced by high order operators. It is too small to affect the mass hierarchy but large enough to prevent the appearance of domain walls. The superpotential of the corresponding simplest extension of the MSSM — the Minimal Non–minimal Supersymmetric Standard Model (MNSSM) can be written as

\begin{equation}
W_{\text{MNSSM}} = \lambda S(H_d H_u) + \xi S + W_{\text{MSSM}}(\mu = 0) . \tag{1}
\end{equation}
2 Theoretical restrictions on the lightest neutralino mass

The neutralino sector in SUSY models is formed by the superpartners of neutral gauge and Higgs bosons. Since the sector responsible for electroweak symmetry breaking in the MNSSM contains an extra singlet field, the neutralino sector of this model includes one extra component besides the four MSSM ones. This is an additional Higgsino $\tilde{S}$ (singlino) which is the fermion component of the SM singlet superfield $S$. In the field basis $(\tilde{B}, \tilde{W}_3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S})$ the neutralino mass matrix reads

$$
M_{\tilde{\chi}^0} = \begin{pmatrix}
M_1 & 0 & -M_{ZSWc_\beta} & M_{ZSWs_\beta} & 0 \\
0 & M_2 & M_{ZCWc_\beta} & -M_{ZCWs_\beta} & 0 \\
-M_{ZSWc_\beta} & M_{ZCWc_\beta} & 0 & -\mu_{eff} & -\lambda v \\
M_{ZSWs_\beta} & -M_{ZCWs_\beta} & -\mu_{eff} & 0 & -\lambda \sqrt{2} \beta \\
0 & 0 & -\lambda v \sqrt{2} \beta & -\lambda v \sqrt{2} s_\beta & 0
\end{pmatrix}
$$

where $M_1$ and $M_2$ are $U(1)_Y$ and $SU(2)$ gaugino masses while $s_W = \sin \theta_W, c_W = \cos \theta_W$, $s_\beta = \sin \beta$, $c_\beta = \cos \beta$ and $\mu_{eff}$ is $\lambda s/\sqrt{2}$. Here we introduce $\tan \beta = v_2/v_1$ and $v = \sqrt{v_1^2 + v_2^2} = 246$ GeV, where $s, v_1$ and $v_2$ are the vacuum expectation values of the singlet field $S$ and of the Higgs doublets fields $H_d$ and $H_u$, respectively.

The top–left 4 × 4 block of the mass matrix (2) contains the neutralino mass matrix of the MSSM where the parameter $\mu$ is replaced by $\mu_{eff}$. From Eq. (2) one can easily see that the neutralino spectrum in the MNSSM may be parametrised in terms of

$$
\lambda, \mu_{eff}, \tan \beta, M_1, M_2.
$$

In supergravity models with uniform gaugino masses at the Grand Unification scale the renormalisation group flow yields a relationship between $M_1$ and $M_2$ at the EW scale, i.e. $M_1 \simeq 0.5 M_2$. The chargino masses in SUSY models are defined by the mass parameters $M_2$ and $\mu_{eff}$. LEP searches for SUSY particles set a lower limit on the chargino mass of about 100 GeV. This lower bound constrains the parameter space of the MNSSM restricting the absolute values of the effective $\mu$-term and $M_2$ from below, i.e. $|M_2|, |\mu_{eff}| \geq 90 - 100$ GeV.

In general the eigenvalues of the neutralino mass matrix can be complex. This prevents the establishing of any theoretical restrictions on the masses of neutralinos. In order to find appropriate bounds on the neutralino masses it is much more convenient to consider the matrix $M_{\tilde{\chi}^0} M^{-1}_{\tilde{\chi}^0}$, whose eigenvalues are positive definite and equal to the absolute values of the neutralino mass squared. In the basis $(\tilde{B}, \tilde{W}_3, -\tilde{H}_d^0 s_\beta + \tilde{H}_u^0 c_\beta, \tilde{H}_d^0 c_\beta + \tilde{H}_u^0 s_\beta, \tilde{S})$ the bottom-right 2 × 2 block of $M_{\tilde{\chi}^0} M^{-1}_{\tilde{\chi}^0}$ takes the form

$$
\begin{pmatrix}
|\mu_{eff}|^2 + \sigma^2 & \nu |\mu_{eff}| \\
\nu |\mu_{eff}| & |\mu_{eff}|^2
\end{pmatrix}
$$

where $\sigma^2 = M_Z^2 \cos 2\beta + |\nu|^2 \sin^2 2\beta$, $\nu = \lambda v/\sqrt{2}$. Since the minimal eigenvalue of any hermitian matrix is less than its smallest diagonal element at least one neutralino in the MNSSM is always light. Indeed, in the considered case the mass interval of the lightest neutralino is limited from above by the bottom–right diagonal entry of matrix (4), i.e. $|m_{\tilde{\chi}^0_1}| \leq |\nu|$. Therefore in contrast with the MSSM the lightest neutralino in the MNSSM remains light even when the SUSY breaking scale tends to infinity.

The obtained theoretical bound on the lightest neutralino mass can even be improved significantly. Since we can always choose the field basis in such a way that this 2 × 2 submatrix of $M_{\tilde{\chi}^0} M^{-1}_{\tilde{\chi}^0}$ becomes diagonal its eigenvalues also restrict the mass interval of the lightest neutralino. In particular, the absolute value of the lightest neutralino mass squared has to be always less than or equal to the minimal eigenvalue $\mu_0^2$ of the corresponding submatrix, i.e.

$$
|m_{\tilde{\chi}^0_1}|^2 \leq \mu_0^2 = \frac{1}{2} \left[ |\mu_{eff}|^2 + \sigma^2 + |\nu|^2 \right] - \sqrt{ \left[ |\mu_{eff}|^2 + \sigma^2 + |\nu|^2 \right]^2 - 4 |\nu|^2 \sigma^2 }.
$$

The value of $\mu_0$ reduces with increasing $|\mu_{eff}|$. It reaches its maximum value, i.e. $\mu_0^2 = \min \left\{ \sigma^2, |\nu|^2 \right\}$, when $\mu_{eff} \to 0$. Taking into account the restriction on the effective $\mu$–term coming from LEP searches and the theoretical upper bound on the Yukawa coupling $\lambda$ which is caused by the requirement of the validity of perturbation theory up to the Grand Unification scale ($\lambda < 0.7$) we find that $\mu_0^2 < 0.8 M_Z^2$. When $|m_{\tilde{\chi}^0_1}|$ is close to its maximum value the lightest neutralino mass is predominantly a superposition of $U(1)_Y$ gaugino and singlino.

Here it is worth to notice that at large values of the term $\mu_{eff}$ the theoretical restriction on $|m_{\tilde{\chi}^0_1}|$ (5) tends to zero independently of the value of $\lambda$. Indeed, for $|\mu_{eff}|^2 \gg M_Z^2$ we have

$$
|m_{\tilde{\chi}^0_1}|^2 \leq \frac{|\nu|^2 \sigma^2}{|\mu_{eff}|^2 + \sigma^2 + |\nu|^2}.
$$

Thus in the considered limit the lightest neutralino mass is significantly smaller than $M_Z$ even for the appreciable values of $\lambda$ at tree level.

3 Approximate solution

The masses of the lightest neutralino can be computed numerically by solving the characteristic equation $\det (M_{\tilde{\chi}^0} - \kappa I) = 0$. In the MNSSM the corresponding characteristic polynomial has degree 5 because the neutralino spectrum is described by a 5 × 5
mass matrix. After a few simple algebraic transformations we get

\[
\begin{align*}
\det(M_{\chi^0} - \kappa I) &= \left(M_1 M_2 - (M_1 + M_2)\kappa + \kappa^2 \right) \times \\
&\times \left(\kappa^3 - (\mu_{\text{eff}}^2 + \nu^2)\kappa + \nu^2\mu_{\text{eff}} \sin 2\beta \right) + \\
&+ M_Z^2 \left(\tilde{M} - \kappa \right) \left(\kappa^2 + \mu_{\text{eff}} \sin 2\beta \kappa - \nu^2 \right) = 0,
\end{align*}
\]

where \(\tilde{M} = M_1 c_{1\beta}^2 + M_2 s_{1\beta}^2\). Although one can find a numerical solution of Eq. (7) for each set of the parameters (8), it is rather interesting to explore analytically the dependence of the lightest neutralino on these parameters. In order to perform such an analysis it is worthwhile to derive at least an approximate solution of the characteristic equation (7). Such an approximate solution can be obtained in the limit when one of the eigenvalues of the mass matrix (2) goes to zero. Indeed, if \(\kappa \to 0\) we can ignore all higher order terms with respect to \(\kappa\) in the characteristic equation keeping only terms which are proportional to \(\kappa\) and \(\kappa^2\) as well as the \(\kappa\)-independent ones. In that case, Eq. (7) takes the form

\[
\alpha \kappa^2 - \beta \kappa + \gamma = 0,
\]

where

\[
\alpha = 1 + \frac{\nu^2 - M_Z^2 \mu_{\text{eff}} \sin 2\beta}{\mu_{\text{eff}}^2 + \nu^2 \left(\frac{M_1 M_2}{M_1 + M_2} + \frac{M_Z^2 \tilde{M}}{\mu_{\text{eff}}^2 + \nu^2(M_1 + M_2)}\right)}
\]

\[
\beta = \frac{M_1 M_2}{M_1 + M_2} + \left(\frac{\nu^2}{\mu_{\text{eff}}^2 + \nu^2} - \frac{M_Z^2 \tilde{M}}{\mu_{\text{eff}}^2 + \nu^2(M_1 + M_2)}\right)
\]

\[
\times \mu_{\text{eff}} \sin 2\beta - \frac{M_Z^2 \nu^2}{(M_1 + M_2)(\mu_{\text{eff}}^2 + \nu^2)},
\]

\[
\gamma = \frac{\nu^2}{\mu_{\text{eff}}^2 + \nu^2} \left(\frac{M_1 M_2}{M_1 + M_2} \mu_{\text{eff}} \sin 2\beta \right)
\]

\[
- \frac{M_Z^2}{M_1 + M_2} \left. \frac{\tilde{M}}{\mu_{\text{eff}}^2 + \nu^2}\right). \tag{10}
\]

In the MNSSM there is a good justification for applying this method. As we argued in the previous section, the mass of the lightest neutralino is limited from above and an upper bound on \(|m_{\chi^0}|\) tends to be zero with raising of \(\mu_{\text{eff}}\) or decreasing of \(\lambda\).

One can simplify the reduced form of the characteristic equation (8) even further taking into account that the second and last terms in the Eq. (9) can be neglected since they are much smaller than unity in most of the phenomenologically allowed region of the MNSSM parameter space. Then the mass of the lightest neutralino can be approximated by

\[
|m_{\chi^0}| = \text{Min} \left\{ \frac{1}{2} \left| \beta \pm \sqrt{\beta^2 - 4\gamma} \right| \right\}. \tag{12}
\]

In Figs.1–3 we plot both the numerical and the approximate solutions for the lightest neutralino mass as a function of \(\mu_{\text{eff}}, M_2\) and \(\tan \beta\). In the present study we assume that all parameters (3) appearing in the neutralino mass matrix are real. We also choose \(M_1 = 0.5 M_2\) and \(\lambda = 0.7\) which is the largest possible value of \(\lambda\) that does not spoil the validity of perturbation theory up to the GUT scale. Figs.1–3 demonstrate that the approximate solution (12) describes the numerical one with relatively high accuracy even for \(M_2 \simeq \mu_{\text{eff}} \simeq 150\) GeV, see Fig.3. One can also see that the mass of the lightest neutralino may be very small or even takes zero value for appreciable values of \(\lambda\). This happens because the determinant of the neutralino mass matrix (2) tends to zero for a certain relation between the parameters

\[
M_1 M_2 \mu_{\text{eff}} \sin 2\beta = \tilde{M} M_Z^2. \tag{13}
\]

It is worth noticing that condition (13) is fulfilled automatically when \(M_1 \sim M_2 \to 0\). Thus the absolute value of the lightest neutralino mass vanishes only once in Figs.1 and 3 and twice in Fig.2.

At large \(|\mu_{\text{eff}}|, |M_2|, |M_1| \gg M_Z\) the value of \(|m_{\chi^0}|\) decreases with raising of the absolute values of the effective \(\mu\)-term and soft gaugino masses (see Figs.1–2). From Fig.1–3 it becomes clear that the difference between the numerical and approximate solutions reduces when \(|\mu_{\text{eff}}|, |M_1|\) and \(|M_2|\) grow. If either \(|\mu_{\text{eff}}|\) or \(|M_1|\) and \(|M_2|\) are much larger than \(M_Z\), \(\beta^2 \gg \gamma\), the approximate solution for the lightest neutralino mass can be presented in a more simple form:

\[
|m_{\chi^0}| \simeq \frac{\gamma}{\beta} \approx \frac{|\mu_{\text{eff}}|^2 \sin 2\beta}{\mu_{\text{eff}}^2 + \nu^2}. \tag{14}
\]

According to Eq. (14) the mass of the lightest neutralino is inversely proportional to the term \(\mu_{\text{eff}}\). It vanishes when \(\lambda\) tends to zero. In the limit \(\lambda \to 0\) the equations for the extrema of the Higgs effective potential that determine the position of the physical vacuum
The mass of the lightest neutralino in the MNSSM as a function of $M_2$ for $\lambda = 0.7$, $M_1 = 0.5 M_2$, $\mu_{\text{eff}} = 200$ GeV. The notations are the same as in Fig. 1.

![Fig. 2](image)

The dependence of the lightest neutralino mass on $\tan \beta$ for $\lambda = 0.7$ and $M_1 = 0.5 M_2$. Solid and dashed lines correspond to $M_2 = \mu_{\text{eff}} = 250$ GeV and $M_2 = \mu_{\text{eff}} = 150$ GeV. Other notations are the same as in Fig. 1.

![Fig. 3](image)

The mass of the lightest neutralino vanishes for the lightest neutralino mass. Indeed, Eq. (14) implies that the vacuum expectation value of the singlet field rises as $M_Z/\lambda$. In other words the correct breakdown of electroweak symmetry breaking requires $\mu_{\text{eff}}$ to remain constant when $\lambda$ goes to zero. Therefore, it follows from Eq. (14) that the mass of the lightest neutralino is proportional to $\lambda^2$ at small values of $\lambda$. At this point the approximate solution (14) improves the theoretical restriction on the lightest neutralino mass derived in the previous section because for small values of $\lambda$ the upper bound (16) implies that $|m_{\chi_1^0}|$ is proportional to $\lambda$.

From Eq. (14) one can also see that the mass of the lightest neutralino decreases when $\tan \beta$ grows. The numerical results of our analysis summarised in Figs. 1–3 confirm that $|m_{\chi_1^0}|$ becomes smaller when $\tan \beta$ raises from 3 to 10. However if $\tan \beta \geq \xi = \frac{2 M_1 M_2 \mu_{\text{eff}}}{MM_Z^2}$ Eq. (14) does not provide an appropriate description for the lightest neutralino mass. Indeed, Eq. (14) implies that the mass of the lightest neutralino vanishes at large values of $\tan \beta$ while Fig. 3 demonstrates that $|m_{\chi_1^0}|$ approaches some constant non–zero value with raising of $\tan \beta$. More accurate consideration of the approximate solution (12) allows to reproduce the asymptotic behaviour of the lightest neutralino mass at $\mu_{\text{eff}}, M_2, M_1 \gg M_Z$ and large values of $\tan \beta$ ($\tan \beta \gg \xi$). It is given by

$$|m_{\chi_1^0}| \rightarrow \frac{\nu^2 M_Z^2}{\mu^2 + \nu^2} \frac{M_1 M_2}{M_1 M_2}.$$  

(15)

So once again the approximate solution (12) improves the theoretical restriction on $|m_{\chi_1^0}|$.

4 Conclusions

We have argued that the mass interval of the lightest neutralino in the MNSSM is limited from above. The upper bound on $m_{\chi_1^0}$ has been found. In the considered model $|m_{\chi_1^0}|$ does not exceed 80 – 85 GeV at tree level. The corresponding upper bound depends rather strongly on the effective $\mu$-term which is generated after the electroweak symmetry breaking. At large values of $|\mu_{\text{eff}}|$ the upper limit on $|m_{\chi_1^0}|$ goes to zero so that the mass interval of the lightest neutralino shrinks drastically. Assuming that $|m_{\chi_1^0}|$ is considerably smaller than the masses of the other neutralino states we have derived the approximate solution for the lightest neutralino mass. The obtained solution describes the numerical one with relatively high accuracy in most parts of the phenomenologically allowed parameter space. Our numerical analysis and analytic considerations show that $m_{\chi_1^0}$ decreases when $\tan \beta$ increases and the coupling $\lambda$ decreases, respectively. At small values of $\lambda$ the mass of the lightest neutralino is proportional to $\lambda^2$. The lightest neutralino mass also decreases with increasing $\mu_{\text{eff}}$, $M_1$, and $M_2$. We have shown that at large values of the effective $\mu$–term $m_{\chi_1^0}$ is inversely proportional to $\mu_{\text{eff}}$. In the allowed part of the parameter space the lightest neutralino is predominantly singlino which makes its direct observation at future colliders rather problematic.

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