Decay of High-Energy Astrophysical Neutrinos

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Existing limits on the non-radiative decay of one neutrino to another plus a massless particle (e.g., a singlet Majoron) are very weak. The best limits on the lifetime to mass ratio come from solar neutrino observations, and are \( \tau/m \gtrsim 10^{-4} \text{s/eV} \) for the relevant mass eigenstate(s). For lifetimes even several orders of magnitude longer, high-energy neutrinos from distant astrophysical sources would decay. This would strongly alter the flavor ratios from the expected from oscillations alone, and should be readily visible in the near future in detectors such as IceCube.

Neutrinos from astrophysical sources are expected to arise dominantly from the decays of pions and their muon daughters, which results in initial flavor ratios \( \phi_\nu: \phi_\mu: \phi_\tau \approx 1:2:0 \). The fluxes of each mass eigenstate are given by \( \phi_\nu = \sum_\alpha \phi_\nu^{\text{source}} |U_{\alpha i}|^2 \), where \( U_{\alpha i} \) are elements of the neutrino mixing matrix. For three active neutrino species (as we assume throughout) there is now strong evidence to suggest that \( \nu_\mu \) and \( \nu_\tau \) are maximally mixed and \( U_{e3} \approx 0 \). The consequent \( \nu_\mu - \nu_\tau \) symmetry means that in the mass eigenstate basis the neutrinos are produced in the ratios \( \phi_\nu: \phi_\mu: \phi_\tau = 1:1:1 \), independent of the solar mixing angle. Oscillations do not change these proportions, but only the relative phases between mass eigenstates, which will be lost. An incoherent mixture in the ratios \( 1:1:1 \) in the mass basis implies an equal mixture in any basis \( (U\hat{U}^T = I) \), and in particular the flavor basis in which the neutrinos are detected \[1\]. In this Letter we show that neutrino decay could alter the measured flavor ratios from the expected \( 1:1:1 \) in a strong and distinctive fashion.

We restrict our attention to two-body decays

\[
\nu_i \rightarrow \nu_j + X \quad \text{and} \quad \nu_i \rightarrow \tau_j + X, \quad (1)
\]

where \( \nu_i \) are neutrino mass eigenstates and \( X \) denotes a very light or massless particle, e.g. a Majoron. Vi-able Majoron models which feature large neutrino decay rates have been discussed in Ref. \[2\]. We do not consider either radiative two-body decay modes (which are constrained by photon appearance searches to have very long lifetimes \[3\]) or three-body decays of the form \( \nu \rightarrow \nu \nu \bar{\nu} \) (which are strongly constrained \[4\] by bounds on anomalous \( ZZ\nu \) couplings \[5\]). In contrast, the limits on the decay modes considered here are very weak. Beacom and Bell have shown that the strongest reliable limit is \( \tau/m \gtrsim 10^{-4} \text{s/eV} \), set by the solar neutrino data \[6\]. This limit is based primarily on the non-distortion of the Super-Kamiokande spectrum \[7\], and takes into account the potentially competing distortions caused by oscillations (see also Ref. \[8\]) as well as the appearance of active daughter neutrinos. It is very likely that the SN 1987A data place no limit at all on these neutrino decay modes, since decay of the lightest mass eigenstate is kinetically forbidden, and even a reasonable \( \bar{\nu}_1 \) flux alone can account for the data \[6\].

The strongest lifetime limit is thus too weak to eliminate the possibility of astrophysical neutrino decay by a factor of about \( 10^7 \times (L/100 \text{ Mpc}) \times (10 \text{ TeV}/E) \). A few previous papers have considered aspects of the decay of high-energy astrophysical neutrinos. It has been noted that the disappearance of all states except \( \nu_1 \) would prepare a beam that could in principle be used to measure elements of the neutrino mixing matrix, namely the ratios \( U_{e1}^2 \) : \( U_{\mu 1}^2 \) : \( U_{\tau 1}^2 \). The possibility of measuring neutrino lifetimes over long baselines was mentioned in Ref. \[11\], and some predictions for decay in four-neutrino models were given in Ref. \[12\]. We will show that the particular values and small uncertainties on the neutrino mixing parameters allow for the first time very distinctive signatures of the effects of neutrino decay on the detected flavor ratios. The expected increase in neutrino lifetime sensitivity (and corresponding anomalous neutrino couplings) by several orders of magnitude makes for a very interesting test of physics beyond the Standard Model; a discovery would mean physics much more exotic than neutrino mass and mixing alone. We will show that neutrino decay cannot be mimicked by either different neutrino flavor ratios at the source or other non-standard neutrino interactions.
A characteristic feature of decay is its strong energy dependence: $\exp\left(-L/E_{\tau}\right) = \exp\left(-Lm/ET\right)$, where $\tau$ is the rest-frame lifetime. However, we will assume that decays are always complete, i.e., that these exponential factors vanish. This is reasonable because there is a minimum $L/E$ value set by the shortest distances (typically hundreds of Mpc) and the maximum energies that will be visible in a given detector (the spectra considered are steeply falling). The assumption of complete decay means we do not have to consider the distance and intensity distributions of sources. We assume an isotropic diffuse flux of high-energy astrophysical neutrinos, and can thus neglect the angular deflection of daughter neutrinos from the trajectories of their parents [13]. It is uncertain if astrophysical sources produce the same numbers of neutrinos and antineutrinos. Though the detectors cannot distinguish neutrinos from antineutrinos, their cross sections are different, and this could cause confusion in the deduced flavor ratios. However, the antineutrino-neutrino cross section ratio is 0.7 at 10 TeV, and rapidly approaches unity at higher energies.

**Disappearance only.**— We first assume that there are no detectable decay products, that is, the neutrinos simply disappear. This limit is interesting for decay to ‘invisible’ daughters, such as a sterile neutrino, and also for decay to active daughters if the source spectrum falls sufficiently steeply with energy. In the latter case, the flux of daughters of degraded energy may make a negligible contribution to the total flux at a given energy. Since coherence will be lost we have

$$\phi_{\nu_{\alpha}}(E) = \sum_{i,\beta} \phi_{\nu_{\beta}}^{\text{source}}(E)|U_{\beta i}|^2|U_{\alpha i}|^2 \exp\left(-L/E_{\tau}(E)\right)$$

where the $\phi_{\nu_{\beta}}$ are the fluxes of $\nu_{\beta}$, $U_{\alpha i}$ are elements of the neutrino mixing matrix and $\tau_{\alpha}$ are the neutrino lifetimes in the laboratory frame. Eq. (2) corresponds to the case where decay is complete by the time the neutrinos reach Earth, so only the stable states are included in the sum.

The simplest case (and the most generic expectation) is a normal hierarchy in which both $\nu_3$ and $\nu_2$ decay, leaving only the lightest stable eigenstate $\nu_1$. In this case the flavor ratio is $U_{\alpha 1}^2 : U_{\mu 1}^2 : U_{\tau 1}^2$. Thus if $U_{e 3} = 0$

$$\phi_{\nu_{\alpha}} : \phi_{\nu_{\mu}} : \phi_{\nu_{\tau}} = \cos^2 \theta_\odot : 1/2 \sin^2 \theta_\odot : 1/2 \sin^2 \theta_\odot \simeq 6 : 1 : 1$$

where $\theta_\odot$ is the solar neutrino mixing angle, which we have set to 30°. Note that this is an extreme deviation of the flavor ratio from that in the absence of decays. It is difficult to imagine other mechanisms that would lead to such a high ratio of $\nu_\tau$ to $\nu_\mu$. Here and throughout we concentrate on the flavor ratios, since the original source fluxes are unknown. In the case of an inverted hierarchy, $\nu_3$ is the lightest and hence stable state, and so

$$\phi_{\nu_{\alpha}} : \phi_{\nu_{\mu}} : \phi_{\nu_{\tau}} = U_{\alpha 2}^2 : U_{\mu 3}^2 : U_{\tau 3}^2 = 0 : 1 : 1.$$

If $U_{e 3} = 0$ and $\theta_{\text{atm}} = 45^\circ$, each mass eigenstate has equal $\nu_\mu$ and $\nu_\tau$ components. Therefore, decay cannot break the equality between the $\phi_{\nu_{\alpha}}$ and $\phi_{\nu_{\beta}}$ fluxes and thus the $\phi_{\nu_{\alpha}} : \phi_{\nu_{\beta}}$ ratio contains all the useful information. The variation of the $\phi_{\nu_{\alpha}} : \phi_{\nu_{\beta}}$ ratio with non-zero $U_{e 3}$ (up to the maximum allowed value, $|U_{e 3}|^2 \lesssim 0.03$ [14]) is shown in Fig. 1. In the no-decay case, the variation from 1 : 1 : 1 is negligibly small. While the relative effect can be larger if neutrino decay occurs, the three cases shown are always quite distinct. In addition, the ratio of the $\nu_\mu$ and $\nu_\tau$ components can also change, e.g., Eq. (4) could be as extreme as $U_{\alpha 3}^2 : U_{\mu 1}^2 : U_{\tau 1}^2 = 3.5 : 1 : 0.3$. Hereafter, we set $U_{e 3} = 0$.

**Appearance of daughter neutrinos.**— If neutrino masses are quasi-degenerate, the daughter neutrino carries nearly the full energy of the parent. An interesting and convenient feature of this case is that we can treat the effects of the daughters without making any assumptions about the source spectra. Including daughters of full energy, we have

$$\phi_{\alpha}(E) \frac{L_{\alpha \beta}}{E_{\tau\beta}} \sum_{i,\beta} \phi_{\nu_{\beta}}^{\text{source}}(E)|U_{\beta i}|^2|U_{\alpha i}|^2$$

where $B$ is a branching fraction and stable and unstable states are denoted henceforth by $i$ and $j$ respectively.

If instead the neutrino mass spectrum is hierarchical, the daughter neutrinos will be degraded in energy with
respect to the parent, so that
\[ \phi_{\nu_\alpha}(E) \xrightarrow{\text{L} \rightarrow \text{r}_i} \sum_{ij} \phi_{\nu_\beta}^\text{source}(E)[U_{\beta i}]^2|U_{\alpha j}|^2 \]
\[ + \int_E^{\infty} dE' W_{E'E} \sum_{ij} \phi_{\nu_\beta}^\text{source}(E')[U_{\beta i}]^2|U_{\alpha j}|^2B_{j \rightarrow i}, \]
where \( E \) is the daughter and \( E' \) is the parent energy. The normalized energy spectrum of the daughter is given by
\[ W_{E'E} = \frac{1}{\Gamma(E')} \frac{d\Gamma(E',E)}{dE}. \]

If the neutrinos are Majorana particles, daughters of both helicities will be detectable (as neutrinos or antineutrinos), whereas if they are Dirac particles, daughters of one helicity will be sterile and hence undetectable. In the rest frame of the parent neutrino, the angular distributions for decays which conserve and flip helicity are proportional to \( \cos^2(\theta^*/2) \) and \( \sin^2(\theta^*/2) \) respectively, where \( \theta^* \) is the angle of the daughter neutrino with respect to the (lab frame) momentum of the parent. In the limit \( m_{\text{daughter}} \ll m_{\text{parent}} \), the corresponding energy distributions in the lab frame are \( E/E'2 \) and \( (E' - E)/E'^2 \).

In the case of Majorana neutrinos, we may drop the distinction between neutrino and antineutrino daughters and sum over helicities. Assuming the source spectrum to be a simple power law, \( E^{-\alpha} \), we find
\[ \phi_{\nu_\alpha}(E) \xrightarrow{\text{L} \rightarrow \text{r}_i} \sum_{ij} \phi_{\nu_\beta}^\text{source}(E)[U_{\beta i}]^2|U_{\alpha j}|^2 \]
\[ + \frac{1}{\alpha} \sum_{ij} \phi_{\nu_\beta}^\text{source}(E)[U_{\beta i}]^2|U_{\alpha j}|^2B_{j \rightarrow i}. \]

This is identical to the expression in Eq. (8) except for the overall factor of \( 1/\alpha \) in front of the second term. For Dirac neutrinos we detect only the daughters that conserve helicity, the effect of which is to change the numerical coefficient of the second sum in Eq. (9). Thus, although the flavor ratio will differ from the cases above, it is still independent of energy—i.e., decay does not introduce a spectral distortion of the power law. We stress that we have assumed a simple but reasonable power law spectrum \( E^{-\alpha} \); a broken power law spectrum, e.g., would lead to a more complicated energy dependence.

Uniqueness of decay signatures.— Depending on which of the mass eigenstates are unstable, the decay branching ratios, and the hierarchy of the neutrino mass eigenstates, quite different ratios result. For the normal hierarchy, some possibilities are shown in Table I.

The most natural possibility with unstable neutrinos is that the heaviest two mass eigenstates both completely decay. The resulting flavor ratio is just that of the lightest mass eigenstate, independent of energy and whether daughters are detected or not. For normal and inverted hierarchies we obtained \( 6 : 1 : 1 \) and \( 0 : 1 : 1 \) respectively. Interestingly, both cases have extreme \( \phi_{\nu_e} : \phi_{\nu_\mu} \), ratios, which provides a very useful diagnostic. Assuming no new physics besides decay, a ratio greater than 1 suggests the normal hierarchy, while a ratio smaller than 1 suggests an inverted hierarchy. In the case that decays are not complete these trends still hold, even though the limits of Eqs. (14) would not be reached. The case of incomplete decay might be identified by measuring different flux ratios in different energy ranges. It is interesting to note that complete decay cannot reproduce \( 1 : 1 : 1 \). One of the mass eigenstates does have a flavor ratio similar to \( 1 : 1 : 1 \), but it is the heavier of the two solar states and cannot be the lightest, stable state. (A possible but unnatural exception occurs if only this state decays.)

An important issue is how unique decay signatures would be. Are there other scenarios (either non-standard astrophysics or neutrino properties) that would give similar ratios? There exist astrophysical neutrino production models with different initial flavor ratios, such as \( 0 : 1 : 0 \) [15], for which the detected flavor ratios (in the absence of decay) would be about \( 0.5 : 1 : 1 \). However, since the mixing angles \( \theta_\odot \) and \( \theta_\text{atm} \) are both large, and since the neutrinos are produced and detected in flavor states, no initial flavor ratio can result in a measured \( \phi_{\nu_e} : \phi_{\nu_\mu} \) ratio anything like that of our two main cases, \( 6 : 1 : 1 \) and \( 0 : 1 : 1 \).

In terms of non-standard particle physics, decay is unique in the sense that it is “one-way”, unlike, say, oscillations or magnetic moment transitions. Since the initial flux ratio in the mass basis is \( 1 : 1 : 1 \), magnetic moment transitions between (Majorana) mass eigenstates cannot alter this ratio, due to the symmetry between \( i \rightarrow j \) and \( j \rightarrow i \) transitions. On the other hand, if neutrinos have Dirac masses, magnetic moment transitions (both diagonal and off-diagonal) turn active neutrinos into sterile states, so the same symmetry is not present. However, the process will not be complete in the same way as decay—it will average out at 1/2, so there is no way we could be left with a only single mass eigenstate.

Experimental Detectability.— Deviations of the flavor ratios from \( 1 : 1 : 1 \) due to possible decays are so extreme that they should be readily identifiable [16]. Upcoming high energy neutrino experiments, such as Ice-
Cube\cite{17}, will not have perfect abilities to separately measure the neutrino flux in each flavor. However, the quantities we need are closely related to the observables, in particular in the limit of $\nu_\mu - \nu_\tau$ symmetry ($\theta_{\text{atm}} = 45^\circ$ and $U_{e3} = 0$), in which all mass eigenstates contain equal fractions of $\nu_\mu$ and $\nu_\tau$. In that limit, the fluxes for $\nu_\mu$ and $\nu_\tau$ are always in the ratio 1 : 1, with or without decay. This is useful since the $\nu_\tau$ flux is the hardest to measure.

Detectors such as IceCube will be able to directly measure the $\nu_\mu$ flux by long-ranging muons which leave tracks through the detector. The charged-current interactions of $\nu_\mu$ produce electromagnetic showers. However, these may be hard to distinguish from hadronic showers caused by all flavors through their neutral-current interactions, or from the charged-current interactions of $\nu_\tau$ (an initial hadronic shower followed by either an electromagnetic or hadronic shower from the tau lepton decay)\cite{18}. We thus consider our only experimental information to be the number of muon tracks and the number of showers.

The relative number of shower events to track events can be related to the most interesting quantity for testing decay scenarios, i.e., the $\nu_\tau$ to $\nu_\mu$ ratio. The precision of the upcoming experiments should be good enough to test such extreme flavor ratios produced by decays. If electromagnetic and hadronic showers can be separated, then the precision will be even better.

Comparing, for example, the standard flavor ratios of 1 : 1 : 1 to the possible 6 : 1 : 1 generated by decay, the more numerous electron neutrino flux will result in a substantial increase in the number of showers compared to the number of muon events. The details of this observation depends on the range of muons generated in or around the detector and the ratio of charged to neutrino current cross sections. This measurement will be limited by the energy resolution of the detector and the ability to reduce the atmospheric neutrino background. The atmospheric background drops rapidly with energy and should be negligibly small above the PeV scale.

Discussion and Conclusions.— We have presented our results above in terms of the ratios of fluxes in each neutrino flavor. These ratios are energy-independent because we have assumed that the ratios at production are energy-independent, that all oscillations are averaged out, and that all possible decays are complete. The first two assumptions are rather generic, and the third is a reasonable simplifying assumption. In the standard scenario with only oscillations, the final flux ratios are $\phi_{\nu_\tau} : \phi_{\nu_\mu} : \phi_{\nu_\tau} = 1 : 1 : 1$. In the cases with decay, we have shown rather different possible flux ratios, for example 6 : 1 : 1 in the normal hierarchy and 0 : 1 : 1 in the inverted hierarchy. These deviations from 1 : 1 : 1 are so extreme that they should be readily measurable.

These clear and striking predictions for the effects of neutrino decay on the measured flavor ratios depend strongly on recent progress in measuring neutrino mixing parameters. In particular, it is very significant that $\theta_{23} \approx 30^\circ$\cite{19} is well below the maximal 45$^\circ$, for which Eq. (4) would instead be a much less dramatic 2 : 1 : 1. In addition, $\theta_{23} < 45^\circ$ means that $\delta m_{23}^2 > 0$ and hence that $\nu_2$ (with flavor ratios 0.7 : 1 : 1) can never be the lightest mass eigenstate. Maximal $\theta_{\text{atm}}$ and very small $U_{e3}$ also make the predictions clearer. The hierarchy of $\nu_3$ relative to the two solar states is unknown, but in either case neutrino decay will be stringently tested by upcoming measurements of astrophysical neutrinos.

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