Shot Noise for Entangled Electrons with Berry Phase

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We study shot noise for entangled electrons in a 4-lead beam-splitter with one incoming lead driven by adiabatically rotating magnetic fields. We propose a setup of an adiabatically rotating magnetic field therefor, which is appropriate for an electron beam to transport through. Using the scattering matrix approach, we find that shot noise for the singlet and that for the entangled triplet oscillates between bunching and antibunching due to the influence of the Berry phase. It provides us a new approach for testing the Berry phase in electron transport on the basis of entanglement.

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Since the quantum interference and quantum entanglement in electron transport become very important, the study of biparticle system has absorbed much attention recently. The extension of the scattering matrix approach \textsuperscript{1} to deal with a biparticle system in a beam-splitter \textsuperscript{2,3} is instructive \textsuperscript{4}. Within the extension, entangled states are identified via shot noise measurement. Some application of the extended approach involves dealing with entangled states affected by an interaction, such as the Rashba interaction \textsuperscript{5}. Because the interaction affects entangled states, it will finally affect the transport current and shot noise which explicitly involve the effects of interaction. This provide us an approach to probe the properties of the interaction. As to the Berry phase \textsuperscript{6}, it is studied very well theoretically and experimentally \textsuperscript{7} in a single-particle system. However, the interest in the Berry phase of a biparticle state has just been aroused since the work of Sjöqvist \textsuperscript{8}. Two aspects of the Berry phase for a biparticle system are very important. One is how the Berry phase changes due to the interparticle interaction \textsuperscript{8,9,10}, the other is how it affects the biparticle state \textsuperscript{11}. All those advancements lead us to a way to observe the Berry phase in electron transport with an entangled state driven by adiabatically rotating magnetic fields.

In this Letter, we study the transport properties of entangled electrons in a beam-splitter configuration \textsuperscript{2,3} with an incoming lead driven by adiabatically rotating magnetic fields. We propose a setup which is appropriate for an electron beam to transport through therefor. The Berry phase affects the spin states of a single electron significantly. The application of this setup can make entangled state into a superposition of the singlet and triplets. We calculate shot noise for one outgoing lead by means of scattering matrix approach \textsuperscript{1}. We find the Berry phase affects the entangled state and shot noise for the mutated state oscillates as a function of the Berry phase, which helps us to observe the Berry phase in electron transport on the basis of entanglement.

The conventional adiabatically rotating magnetic field is considered around a fixed point, but it is not suitable for an electron to transport. We propose a setup which is suitable for an electron beam to transport through. The setup consists of a cylinder, two magnets, two magnetic stripes and two metal stripes. The cylinder is displaced between the two magnets, around whose surface the two magnetic stripes and the two metal stripes are wound helically, as shown in Fig. 1 schematically. Our purpose is to make electrons travelling along the axis of the cylinder feel a right-handed rotating magnetic field. Thus we need to realize the two components of the rotating magnetic field as follows. The constant axial component $\vec{B}_z$ is produced by the two magnets. Each magnet is made with a hole coaxial to the cylinder, so that electrons can enter or leave the cylinder through the hole. In our setup, the radial component $\vec{B}_\perp$ distributes uniformly but rotates helically along the axis of the cylinder, which is produced in the following way. Starting from two opposite positions at one end of the cylinder, we wind two

![Figure 1: Schematic view of a design of an adiabatically rotating magnetic field. (a): the design of an adiabatically rotating magnetic field, which is appropriate for an electron beam to transport through it. It consists of two magnets to produce a constant magnetic field component along the $z$ axis, two magnetic stripes to produce a helically radial magnetic field component and two metal stripes to produce a radial electric field. (b): the configuration of the radial magnetic field component and the radial electric field.](Image)
magnetic stripes around its surface respectively with a right-hand uniform helix such that they undergo a twist of $2\pi$ when arriving at the other end. We keep the inward poles of the two magnetic stripes opposite. These two magnetized stripes clearly produce the wanted component $\vec{B}_\perp$. However, when an electron moves along the axis of the cylinder, it will suffer a Lorentz force arising from this component. The Lorentz force will make the moving electron deviate away from the axis of the cylinder. In order to prevent such derivation, an electronic force to balance the Lorentz force is necessary. This is realized by two parallel metal stripes wounded alternately with the two magnetic stripes and connected to the cathode and the anode of a battery respectively. The radial magnetic field component and the electrical field along the axis of the cylinder are illustrated in Fig. 1(b). Thus the magnetic field in the cylinder takes the form

$$\vec{B}(z) = B\vec{u}(z) = B\left(\sin \vartheta \cos \left(\frac{2\pi}{L} z + \varphi\right), \sin \vartheta \sin \left(\frac{2\pi}{L} z + \varphi\right), \cos \vartheta\right),$$

(1)

where $B = \sqrt{B_1^2 + B_2^2}$, $\vartheta = \tan^{-1}(B_1/B_2)$, $L$ is the length of the cylinder, and $\varphi$ refers to the azimuthal angle of the radial magnetic field component $\vec{B}_\perp$ at $z = 0$. Both $B$ and $\vartheta$ can be adjusted experimentally. Thus the magnetic field is parameterized.

The magnetic field brings about a Zeeman energy to electrons moving in the cylinder. Since the Lorentz force on the moving electron is expected to be balanced by an extra-electric force as aforementioned, the movement of the electron is governed by the following Hamiltonian

$$H(z) = \frac{\mu}{2} \vec{B}(z) \cdot \vec{\sigma},$$

(2)

where $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ denotes the Pauli operator and $\mu = g\mu_B$ the coupling constant in which $g$ is the Landé factor and $\mu_B = \frac{e\hbar}{2m}$ the Bohr magneton. Then the local energy eigenstates of the Hamiltonian (2) is given by

$$|\uparrow_n(z)\rangle = \cos \frac{\vartheta}{2} |\uparrow\rangle + e^{i\left(\frac{2\pi}{L} z + \varphi\right)} \sin \frac{\vartheta}{2} |\downarrow\rangle,$$

$$|\downarrow_n(z)\rangle = -\sin \frac{\vartheta}{2} |\uparrow\rangle + e^{i\left(\frac{2\pi}{L} z + \varphi\right)} \cos \frac{\vartheta}{2} |\downarrow\rangle.$$  

(3)

(4)

where $|\uparrow\rangle$ and $|\downarrow\rangle$ refers to the two eigenstates of $\sigma_z$ operator. The eigenenergies of $|\uparrow_n(z)\rangle$ and $|\downarrow_n(z)\rangle$ are $\mu B/2$ and $-\mu B/2$ respectively.

If an electron is travelling along the axis of the cylinder at speed $v$, it will feel that the rotating frequency of the magnetic field $\omega = 2\pi v/L$. We can control electrons’ speed $v$, let $\omega = \frac{v}{L} \ll \frac{\mu B}{2\hbar}$, so that the adiabatic condition is satisfied. As the magnetic field at $z = L$ returns to its original direction at $z = 0$, i.e. $\vec{B}(L) = -\vec{B}(0)$, the evolution of the electron state is cyclic. According to the adiabatic theorem in quantum mechanics, the local energy eigenstates (3) and (4) acquire extra phase factors in addition to the dynamical phase factors:

$$|\uparrow_n(L)\rangle = e^{i\delta} e^{i\gamma_B} |\uparrow_n(0)\rangle,$$

$$|\downarrow_n(L)\rangle = e^{-i\delta} e^{-i\gamma_B} |\downarrow_n(0)\rangle,$$

(5)

(6)

where $\delta = \int_0^L \frac{\mu B}{2\hbar} B(z) dz$ is the dynamical phase and $\gamma_B = \pi \cos \vartheta$ is the Berry phase.

Now we consider that electrons polarizing along the $z$ direction are injected into the field. The incoming states may have $|\chi^{\uparrow}\rangle_{\text{in}} = |\uparrow\rangle$ and $|\chi^{\downarrow}\rangle_{\text{in}} = |\downarrow\rangle$. Below we study their evolution in the adiabatically rotating magnetic fields. At $z = 0$, we expand the incoming states into the local energy eigenstates:

$$|\chi^{\uparrow}\rangle_{\text{in}} = \cos \frac{\vartheta}{2} |\uparrow_n(0)\rangle - \sin \frac{\vartheta}{2} |\downarrow_n(0)\rangle,$$

(7)

$$|\chi^{\downarrow}\rangle_{\text{in}} = e^{-i\varphi} \left( \sin \frac{\vartheta}{2} |\uparrow_n(0)\rangle + \cos \frac{\vartheta}{2} |\downarrow_n(0)\rangle \right).$$

(8)

Then the corresponding outgoing states for the electrons reaching $z = L$ can be obtained by means of an unitary evolution:

$$|\chi^{\uparrow}\rangle_{\text{out1}} = \left( \sin \frac{\vartheta}{2} e^{-i\delta} e^{-i\gamma_B} + \cos \frac{\vartheta}{2} \right) |\uparrow\rangle + \sin \frac{\vartheta}{2} \cos \frac{\vartheta}{2} e^{i\varphi} \left( e^{i\delta} e^{i\gamma_B} - e^{-i\delta} e^{-i\gamma_B} \right) |\downarrow\rangle,$$

$$|\chi^{\downarrow}\rangle_{\text{out1}} = \sin \frac{\vartheta}{2} \cos \frac{\vartheta}{2} e^{-i\varphi} \left( e^{i\delta} e^{i\gamma_B} - e^{-i\delta} e^{-i\gamma_B} \right) |\uparrow\rangle + \left( \sin^2 \frac{\vartheta}{2} e^{i\delta} e^{i\gamma_B} + \cos^2 \frac{\vartheta}{2} e^{-i\delta} e^{-i\gamma_B} \right) |\downarrow\rangle.$$ 

(9)

(10)

To guarantee the adiabatic condition being satisfied, the change of the dynamical phase $\delta$ should be much larger than the Berry phase shift $\gamma_B$ induced by the adiabatic excursion of the system’s parameters. Thus the sepa-
rivation of $\gamma_B$ from the total phase requires very precise control of the system in experiment\textsuperscript{12}. As is known the spin-echo technique \textsuperscript{11, 13} is applicable for cancelling out the dynamical phase shift. We apply a second setup similar to what we described in Fig. \textsuperscript{1} but turning each magnet to the opposite direction and rotating the cylinder by $\pi$. This provides a rotating magnetic field with the opposite direction in comparison to the first one, i.e., $\varphi \rightarrow \pi - \varphi$ and $\varphi \rightarrow \pi + \varphi$ in Eq. \textsuperscript{11}. Let the electron just passing through the first setup inject into the second setup. Clearly, the outgoing states \textsuperscript{11} and \textsuperscript{11} are the incoming states with respect to the second setup. The outgoing states after passing through the second cylinder are obtained as follows

\begin{align}
|\chi^\uparrow\rangle_{\text{out}2} &= \left( \sin \frac{\varphi}{2} e^{-2i\gamma_B} + \cos \frac{\varphi}{2} e^{2i\gamma_B} \right) |\uparrow\rangle + \sin \frac{\varphi}{2} e^{-i\varphi} \left( e^{2i\gamma_B} - e^{-2i\gamma_B} \right) |\downarrow\rangle, \\
|\chi^\downarrow\rangle_{\text{out}2} &= \sin \frac{\varphi}{2} e^{i\varphi} \left( e^{2i\gamma_B} - e^{-2i\gamma_B} \right) |\uparrow\rangle + \left( \sin \frac{\varphi}{2} e^{2i\gamma_B} + \cos \frac{\varphi}{2} e^{-2i\gamma_B} \right) |\downarrow\rangle,
\end{align}

in which the dynamical phases are cancelled completely. Eqs. \textsuperscript{11} and \textsuperscript{12} exhibit that the rotating magnetic fields alter the polarization direction of electron spins. It therefore suggests a route for spin control \textsuperscript{14} merely by rotating the cylinder by $\frac{\pi}{2}$.

\begin{figure}[h]
\centering
\includegraphics[width=\linewidth]{fig2.png}
\caption{The sketch picture of a 4-lead beam-splitter, in which two adiabatically rotating magnetic fields are displaced in lead 1.}
\end{figure}

Now we are in the position to study the transport properties of electrons in an entangled system with one subsystem evolving in adiabatically rotating magnetic fields. We suggest a scheme based on the one presented in Ref. \textsuperscript{1, 5}, but displacing two afore-mentioned adiabatically rotating magnetic fields in one incoming lead to construct `spin echo' \textsuperscript{11, 13}; Fig. \textsuperscript{4}. Entangled electron pairs are generated by the entangler. Of each pair, one electron enters path 1 via lead 1 while the other enters path 2 via lead 2. We set the entangler to keep injected electrons polarizing along the $z$ direction. Thus we have incoming states $|\pm\rangle_{\text{in}} = \frac{1}{\sqrt{2}} (|\uparrow\rangle \otimes |\downarrow\rangle \pm |\downarrow\rangle \otimes |\uparrow\rangle)$. Because lead 1 is driven by the adiabatically rotating magnetic fields, the incoming states will be affected and change. However, there is no coupling between paths 1 and 2. Hence the Hilbert space of the electron entangled system is the direct product of two subspaces. It means that the two subsystems evolve independently. The states of the subsystem 1 via path 1 undergoes an adiabatic evolution given by Eqs. \textsuperscript{11} and \textsuperscript{12}, while those of the subsystem 2 via path 2 keeps their initial spin states. After passing through the fields’ region, we obtain the following outgoing states

\begin{align}
|\pm\rangle_{\text{out}} &= \frac{1}{\sqrt{2}} \left[ \cos \frac{\varphi}{2} |\uparrow\rangle + \sin \frac{\varphi}{2} e^{-i\varphi} \left( e^{2i\gamma_B} - e^{-2i\gamma_B} \right) |\downarrow\rangle \right], \\
\frac{1}{\sqrt{2}} &\left[ \cos \frac{\varphi}{2} e^{i\varphi} \left( e^{2i\gamma_B} - e^{-2i\gamma_B} \right) |\downarrow\rangle + \cos \frac{\varphi}{2} e^{-i\varphi} \left( e^{2i\gamma_B} - e^{-2i\gamma_B} \right) |\uparrow\rangle \right],
\end{align}

which is significantly affected by the Berry phase.

The outgoing electrons arrive at the beam-splitter and scatter. We calculate shot noise for the scattering current in one outgoing lead. Shot noise is regarded as a nonequilibrium current fluctuation arising from the discrete nature of the charge flow (at zero temperature) \textsuperscript{5, 13}. The current fluctuation around its mean value in lead $\mu$ at a time $t$ is given by

$$\delta I_\mu(t) = \hat{I}_\mu(t) - \langle \hat{I}_\mu(t) \rangle.$$ 

Usually, shot noise is defined as the Fourier transform of the symmetrized current-current autocorrelation function between leads $\mu$ and $\nu$:

$$S_{\mu\nu}(\omega) = \frac{1}{2} \int \langle \delta I_\mu(t) \delta I_\nu(0) + \delta I_\nu(t) \delta I_\mu(0) \rangle e^{i\omega t} dt.$$  

The current in lead $\mu$ in the scattering approach is

$$\hat{I}_\mu(t) = \frac{e}{\hbar} \sum_{\alpha\beta} \int d\epsilon d\epsilon' e^{i(\epsilon - \epsilon')t/\hbar} a_{\alpha}^\dagger(\epsilon) A_{\alpha\beta}(\mu; \epsilon, \epsilon') a_{\beta}(\epsilon'),$$

with the brevity notations:

$$A_{\alpha\beta}(\mu; \epsilon, \epsilon') = \delta_{\mu\alpha} \delta_{\beta\beta} 1 - s_{\mu\alpha}(\epsilon) s_{\beta\beta}(\epsilon'),$$

$$a_{\alpha}^\dagger = (a_{\alpha\uparrow}^\dagger, a_{\alpha\downarrow}^\dagger),$$

where $a_{\alpha\sigma}(\epsilon)[a_{\alpha\sigma}(\epsilon)]$ denotes the creation (annihilation) fermionic operator for an electron with spin $\sigma$ and energy.
\( \epsilon \) in lead \( \alpha \). The setup shown in Fig. 2 involves four leads, and the single-particle scattering matrix elements to describe it are \( s_{31} = s_{42} = r \), and \( s_{41} = s_{32} = t \), where \( r \) and \( t \) denote the reflection and transmission amplitudes at the beam splitter, respectively. We assume that there is no backscattering, \( s_{12} = s_{34} = s_{\text{ons}} = 0 \). The unitarity of the scattering matrix implies \( |r|^2 + |t|^2 = 1 \), and \( \text{Re}|r^*t| = 0 \). At zero temperature, we calculate the shot noise for the outgoing state \(|\alpha\rangle\) in the lead 3, and find its zero-frequency component

\[
S^\pm_{33} = \frac{2e^2}{\hbar \nu} (1 - T) f^\pm
\]

where \( T = |t|^2 \) stands for the beam-splitter transmission, \( \nu \) for the density of states in lead 3 which can be determined by the mean value of the transport current \(|\langle I_3 \rangle| = \frac{e}{h} \nu \) and

\[
f^+ = 1 - \delta_{\epsilon_1\epsilon_2} \left( 1 - \frac{2\gamma_B^2}{\pi^2} \sin^2(2\gamma_B) \right),
\]

\[
f^- = 1 - \delta_{\epsilon_1\epsilon_2} \cos(4\gamma_B)
\]

respectively for the states \(|+\rangle_{\text{out}}\) and \(|-\rangle_{\text{out}}\) with \( \epsilon_1 \) and \( \epsilon_2 \) the discrete energies of the paired electrons.

![Image of Fig. 3: Fano factor \( f \) as a function of the Berry phase \( \gamma_B \). The solid line denotes the Fano factor for the state \(|+\rangle_{\text{out}}\) and the dashed line for the state \(|-\rangle_{\text{out}}\).](image)

In conclusion, the Berry phase affects an electron’s states evolving in adiabatically rotating magnetic fields. Applying this means, we change the entangled state into a linear superposition of the singlet and triplets. The changed states are affected by the Berry phase significantly, and the Fano factors for them in a beam-splitter are single valued functions of the Berry phase, which provides a new approach to detect the Berry phase in spin transport on the basis of entanglement.

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