Deuteron: properties and analytical forms of wave function in coordinate space

V. I. Zhaba

Uzhgorod National University, Department of Theoretical Physics,
54, Voloshyna St., Uzhgorod, UA-88000, Ukraine

(Received June 26, 2017)

Key words: Deuteron; wave function; approximation; analytic form; polarization.

PACS: 13.40.Gp, 13.88.+e, 21.45.Bc, 03.65.Nk

Abstract

Static parameters of the deuteron, obtained by the wave functions for various potential models, have been chronologically systematized. The presence or absence of knots near the origin of coordinates for the radial wave function of the deuteron have been shown. Analytical forms for the deuteron wave function in coordinate space have been reviewed. Both analytical forms and parameterizations of the deuteron wave function, which are necessary for further calculations of the characteristics of the processes involving the deuteron, have been provided. In addition, the asymptotic behaviors of deuteron wave function near the origin of coordinates and for large values of distance have been analyzed in the paper. Minimization of the number of numerically calculated coefficients for new analytical forms as a product of exponential function $r^n$ by the sum of the exponential terms $A_i \exp(-a_i r^3)$ have been done. The optimum is $N=7-10$.

1. Introduction

Deuteron is the most elementary nucleus. He consists of the two strongly interacting elementary particles: a proton and a neutron. The simplicity and evidentness of the deuteron’s structure makes it a convenient laboratory for studying and modeling nucleon-nucleon forces. Now, deuteron has been well investigated both experimentally and theoretically.

The experimentally determined values of static properties of the deuteron are in very much good agreement with the experimental data. Owever despite that, there still are some theoretical inconsistencies and problems. For example, in latest papers one (for OBE [1], Bonn [2] potentials) or both (for Soft core Reid68 [3], Moscow [4], renormalized OPE and TPE chiral [5] potentials) components of the radial wave function in coordinate space have knots near the origin of the coordinates. The existence of knots in the wave functions of the basic and sole state of the deuteron is the evidence of inconsistencies and inaccuracies in implementation of numerical algorithms in solving similar problems. Or it is connected with features of potential models for the description of a deuteron. The way the choice of numerical algorithms influences the solution is shown in Refs. [6, 7, 8]. The knots of the wave function in coordinate representation are analyzed in more detail in the following sections of the article.

Besides, it should be noted that the deuteron wave function in momentum space in the scientific literature is presented ambiguously. In particular, in the S- component [9, 10, 11, 12] (or in S- and D- components [13, 14]), there is an excess knot in the middle of interval for values of momentum.

It should also be noted that such potentials of the nucleon-nucleon interaction as Bonn [2], Moscow [4], Nijmegen group potentials (NijmI, NijmII, Nijm93 [15, 16]), Argonne v18 [17], Paris [18], NLO, NNLO and N3LO [19], Idaho N3LO [20] or Oxford potential [21] have quite a complicated structure and cumbersome representation. Example, the original potential Reid68 was parameterized on the basis of the phase analysis by Nijmegen group and was called as updated regularized version - Reid93. The parametrization was done for 50 parameters of the potential, where value $\chi^2/N_{data}=1.03$ [15, 16].

Besides, the deuteron wave function (DWF) in coordinate space can be presented as a table: through respective arrays of values of radial wave functions. It is sometimes quite difficult and inconvenient to operate with such arrays of numbers during numerical calculations. And the program code for numerical calculations is bulky, overloaded and unreadable. Therefore, it is feasible to obtain simpler and comfortabler analytical forms of DWF representation. It is further possible on the basis to calculate the form factors and tensor polarization, characterizing the deuteron structure.

DWFs in a convenient form are necessary for use in calculations of polarization characteristics of the deuteron, as well as to evaluate the theoretical values of spin observables in $dp$ scattering [21].

In addition to introduction, the first section and conclusions, the article is composed of six more sections. The second section deals with the deuteron wave function: main peculiarities and scientific interest in its studying. The
third section describes the basic properties of the deuteron. The numerical values of theoretical calculation results and experimental data are presented in convenient tables. The fourth and the fifth sections provide a description of basic analytical forms of DWF in the coordinate representation. The sixth section describes the ”improved” analytical forms of DWF. The seventh section suggests new analytical forms of DWF used in modern scientific literature. Coefficients for new analytical forms in the form \( r^{n_i} A_i \exp(-a_i \cdot r^3) \) have been calculated.

The main objectives of the research in this paper are to systematize the analytical forms of DWF in the coordinate representation, calculate and analyze the coefficients for new analytical forms.

2. Deuteron wave function

Wave function describes quant-mechanical system and is the basic characteristic of microobjects. Knowledge of deuteron wave function allows receiving the maximal information on system and theoretically to calculate the characteristics measured on experiment. Deuteron wave function find as the decision of system of coupled Schrodinger equations.

Deuteron wave functions write down as the sum of wave functions for \( ^3S_1 \)- and \( ^3D_1 \)- state \[22\]

\[
\Psi_d = \psi_S + \psi_D = \frac{u(r)}{r} Y_{101}^1 + \frac{w(r)}{r} Y_{121}^1,
\]

where \( u(r) \) and \( w(r) \) are radial deuteron wave functions for states with the orbital moments \( l=0 \) and \( 2 \); \( Y_{JLS}^M(\theta, \phi) \) are spherical harmonics, that are determined by orbital moment \( L \), spin \( S \), the full moment \( J = L + S \) and his projection \( M \) to an axis \( z \). For deuteron: \( S=1; J = M = S=1 \).

The condition of normalization for DWF \( \Psi_d \) can be written down as

\[
p_S + p_D = \int_0^\infty \left( u^2(r) + w^2(r) \right) dr = 1,
\]

where \( p_S \) and \( p_D \) are probabilities to find out deuteron in \( S \)- and \( D \)- state accordingly.

Taking into account spherical harmonics, it is possible to write down system of the coupled differential equations of the second order for deuteron

\[
\begin{align*}
\frac{d^2 u}{dr^2} + \left( -k^2 - U_1 \right) u = \sqrt{8} U_T w, \\
\frac{d^2 w}{dr^2} + \left( -k^2 - \frac{6}{r^2} - U_2 \right) w = \sqrt{8} U_T u.
\end{align*}
\]

Here \( U_1, U_2 \) are normalized potentials of channels \( l = 0; 2 \); \( U_3 \) are tensor component NN- interaction; \( U_i(r) = \frac{2\mu}{k_i^2} V_i(r); k_i^2 = \frac{2\mu}{\hbar^2} E \) is wave number.

About the beginning of coordinates wave function D- state \( w(r) \) has small value, because the repellent centrifugal barrier \( \hbar^2 (l+1) \) will prevail on small distances. Outside of radius for action of forces the behaviour for \( w(r) \) also is determined by this barrier which sets asymptotic as \[23\]:

\[
w(r) \sim C \exp(-\gamma r) \left[ 1 + \frac{3}{\gamma r} + \frac{3}{(\gamma r)^2} \right].
\]

In paper \[24\] it was specified that one can divide the main models into four categories: 1) the models based on quantum chromo dynamics; 2) the effective field theory is another outstanding approach to NN problem; 3) the boson exchange models; 4) the almost pure phenomenological NN potentials. Last decades the second and fourth groups of potentials are more often and are more intensively used for the description of properties for deuteron and character of his interaction with easy nucleus.

On Fig. 1 is shown interest of researchers to deuteron and to its properties according to the quoted literature in this article. Obvious not fading interest. It is connected first of all to studying those processes and interactions where the direct participant is deuteron. And knowledge its DWF is necessary for a substantiation and an explanation of corresponding models. Thus it is necessary to interpret the received experimental data, in particular tensor
polarization.

Fig. 1. Interest of researchers to deuteron

3. Deuteron properties

Based on the known DWFs one can calculate the deuteron properties:

deuteron radius $r_m$

$$r_d = \frac{1}{2} \left\{ \int_0^\infty r^2 \left[ u^2(r) + w^2(r) \right] dr \right\}^{1/2};$$

the quadrupole moment $Q_d$

$$Q_d = \frac{1}{20} \int_0^\infty r^2 w(r) \left[ \sqrt{8} u(r) - w(r) \right] dr;$$

the magnetic moment $\mu_d$

$$\mu_d = \mu_s - \frac{3}{2} \left( \mu_s - \frac{1}{2} \right) P_D;$$

the D- state probability $P_D$

$$P_D = \int_0^\infty w^2(r) dr;$$

the “D/S- state ratio” $\eta$

$$\eta = A_D/A_S;$$

the triplet effective range $\rho$.

In a formula for $\mu_d$ size $\mu_s = \mu_n + \mu_p$ is the sum of the magnetic moments of a neutron and proton. Value of the calculated magnetic moment of a deuteron is given in nuclear magnetons $\mu_N$.

Values of these static properties for deuteron that were designed for different potential models or wave functions of a various origin are resulted in Table 1. Knots for radial DWFs $u(r)$ and $w(r)$ are designated as $r_u$ and $r_w$.

Table 1. Deuteron properties
| Years | Potential or DWF | $r_u$ (fm) | $r_w$ (fm) | $E_d$ (MeV) | $r_m$ (fm) | $Q_d$ (fm$^2$) | $P_D$ (%) | $\eta$ | $A_S$ (fm$^{-1/2}$) | Ref. |
|-------|------------------|-----------|-----------|-------------|----------|------------|----------|------|----------------|-----|
| 1940  | Neutral theory (zero cut-off) | - | - | 0.270 | 6.8 | [25] |
| 1940  | Neutral theory (straight cut-off) | - | - | 0.261 | 6.63 | [25] |
| 1941  | Results of Rarita-Schwinger | - | 2.17 | 1.97 | 0.325 | 7.60 | [26] |
| 1954  | Results of Brueckner-Watson ($V_T$=-500 MeV) | - | - | 1.86 | 0.277 | 5.10 | [27] |
| 1955  | Trial functions | 2.227 | 0.28 | 7.11 | [28] |
| 1955  | Gartenhaus DWF | - | - | 0.29-0.308 | 6.8-7.0 | [29] |
| 1956  | Pion-theoretical wave function | 0.4 | 0.4 | 0.28 | 5-8 | 0.0245 | [30] |
| 1956  | Variational wave function | - | - | 17 | [31] |
| 1958  | Hulthen type DWF | - | - | - | - | - | [32] |
| 1959  | GT-Potential | 0.4 | 0.4 | 2.288 | 0.263 | 6.3 | [33] |
| 1960  | Hamada | - | - | 1.7 | 0.273 | 6.7 | 0.0258 | [34] |
| 1960  | Pion-theoretical DWF | - | 0.15 | 0.26 | 7 | [35] |
| 1961  | Hamada | - | - | 9.9 | 0.029 | [36] |
| 1962  | Hamada-Johnston | - | - | 2.226 | 0.285 | 6.97 | 0.02656 | [37] |
| 1963  | Martin’s method | - | - | 0.137 | 4 | [38] |
| 1964  | Hulthen wave function | 0.15 | - | - | - | - | [39] |
| 1964  | separable potential | - | - | 2.225 | 3.2 | [40] |
| 1966  | Hamada-Johnston (analytic) | - | - | 0.282 | 7 | 0.0269 | [41] |
| 1966  | Hamada-Johnston (Hulthen) | - | - | - | - | - | [41] |
| 1966  | Hamada-Johnston-Partovi | 0.5 | 0.5 | - | - | - | [41] |
| 1966  | Soft core | - | - | 2.227 | - | - | [42] |
| 1968  | Relative harmonic oscillator basis | - | - | 2.1 | 0.325 | 3.6 | [43] |
| 1968  | Effective nucleon-nucleon potential (A, B, F variants) | - | - | 1.99; 2.20; 2.13 | 0.272; 0.266; 0.227 | 1.94; 1.97; 2.59 | [44] |
| 1968  | Soft core Reid68 | 0.01 | 0.01 | 2.2246 | 0.27964 | 6.4696 | 0.02622 | 0.87758 | [3] |
| 1968  | Hard core Reid68 | 0.38 | 0.38 | 2.2246 | 0.277 | 6.497 | 0.0259 | 0.88034 | [3] |
| 1969  | Non-static OBEP (set 1) | - | - | 2.2 | 0.26 | 6.3 | [45] |
| Year  | Event Description                      | L | T | Potential Value | Error Value | Comment        |
|-------|----------------------------------------|---|---|-----------------|-------------|----------------|
| 1969  | ... (set 2)                             | 2.3| 0.25| 5.4            | 0.2642      |                |
| 1970  | Modified HJ v1                          | 0.4| 0.4| 2.226          | 0.2845      | 6.953          |
| 1970  | Modified HJ v3                          | 0.4| 0.4| 2.2256         | 0.2867      | 6.964          |
| 1970  | Modified HJ v9                          | 0.4| 0.4| 2.2680         | 0.2869      | 7.050          |
| 1971  | Velocity dependent potentials from the various models: distributed mass scalar | - | - | 2.224 | 0.275 | 4.6 |
| 1971  | L$^2$ force                             | - | - | 2.224 | 0.262 | 4.0 |
| 1971  | Contact term                            | - | - | 2.224 | 0.258 | 4.9 |
| 1971  | Phenomenological charge dependent       | - | - | 2.224 | 0.240 | 4.1 |
| 1972  | OBEP                                    | 2.2 | 0.26 | 6.3  |
| 1973  | Local nucleon-nucleon potential A       | 2.224 | 0.262 | 4.43 |
| 1973  | ... B                                   | 2.224 | 0.262 | 5.25 |
| 1973  | ... C                                   | 2.224 | 0.279 | 5.45 |
| 1973  | UT101                                   | 0.6; 0.8 | 0.6; 0.8 | 0.279 |
| 1973  | UT102                                   | 0.7 | 0.7 | 0.279 |
| 1973  | UT103                                   | 0.6; 0.9 | 0.6; 0.9 | 0.279 |
| 1974  | Boundary condition model                | 2.2262 | 0.2774 | 5.20 | 0.02617 | 0.8858 |
| 1974  | Reid hard core                          | 2.2247 | 0.2769 | 6.49 | 0.02584 | 0.8774 |
| 1974  | Yale                                    | 2.1939 | 0.2757 | 6.95 | 0.02505 | 0.8804 |
| 1974  | Hamada-Johnston                         | 2.2710 | 0.2837 | 7.02 | 0.02686 | 0.8921 |
| 1974  | Bryan-Scott potential                   | 2.1841 | 0.2589 | 5.44 | 0.02375 | 0.8687 |
| 1974  | Ueda-Green I potential                  | 1.9556 | 0.2811 | 5.47 | 0.02291 | 0.8455 |
| 1974  | Ueda-Green II potential                 | 2.2052 | 0.2797 | 6.01 | 0.02567 | 0.8881 |
| 1974  | Ueda-Green III potential                | 2.5315 | 0.2605 | 4.93 | 0.02817 | 0.9349 |
| 1974  | Separable potential                     | 2.223 | 0.288 | 7 | 0.0437 |
| 1975  | Approximation for Yale potential        | 2.1888 | 0.276 | 6.95 |
| 1975  | RSC                                     | - | - | 0.280 | 6.47 |
| 1975  | RHC                                     | 0.5 | 0.5 | 0.277 | 6.50 |
| 1975  | HJ potential                            | 0.5 | 0.5 | 0.284 | 6.95 |
| 1975  | RHC+Baker transf. of u(r)               | - | - | 0.276 | 6.50 |
| 1975  | RSC+u-w twist                           | - | 1.2 | 0.268 | 4.35 |
| Year | Model                                                                 | Parameters | p̄ (fm) | τ (fm) | R (fm) | S (fm) |
|------|----------------------------------------------------------------------|------------|--------|--------|--------|--------|
| 1975 | RSC+UT101                                                             | 0.8        | 0.8    | 0.279  | 6.47   |        |
| 1975 | OBEP HM                                                               | 2.224      | 1.86   | 0.284  | 5.75   |        |
| 1975 | OBEP SCH                                                              | 2.910      | 1.79   | 0.249  | 4.85   |        |
| 1975 | OBEP GTG                                                              | 2.985      | 1.76   | 0.252  | 4.88   |        |
| 1975 | OBEP UNG                                                              | 2.511      | 1.81   | 0.266  | 4.40   |        |
| 1975 | Refitted OBEP SCH'                                                    | 2.224      | 1.85   | 0.284  | 5.82   |        |
| 1975 | Refitted OBEP GTG'                                                    | 2.223      | 1.85   | 0.296  | 6.10   |        |
| 1975 | Refitted OBEP GTG"                                                   | 2.227      | 1.82   | 0.285  | 5.67   |        |
| 1975 | Meson exchange model F_0F_1'                                           | -          | -      | 2.227  | 6.17   |        |
| 1975 | One-boson-exchange potential                                          | 0.48       | 0.48   | 2.224644 | 5.92    | 0.0251 |
| 1975 | OBEH(R)                                                               | 0.4        | -      | 2.231  | 0.2747 | 6.23   |
| 1975 | OBEH(NR)                                                              | 2.232      | -      | 0.2721 | 5.57   |        |
| 1975 | OBEG(R)                                                               | -          | -      | 2.227  | 0.2740 | 6.14   |
| 1975 | OBEG(NR)                                                              | 2.205      | -      | 0.2720 | 5.58   |        |
| 1975 | OBEV(R)                                                               | -          | -      | 2.205  | 0.2745 | 5.63   |
| 1975 | OBEV(NR)                                                              | 2.244      | -      | 0.2698 | 5.23   |        |
| 1975 | Super-soft-core potential                                            |            | -      | 2.2245 | 0.282  | 5.92   |
| 1976 | OBEP Holinde-Machleidt model                                          | -          | -      | 2.224  | 1.86   | 0.284  |
| 1976 | OBEP Holinde-Machleidt model                                          | 2.2246     | 1.79   | 0.2864 | 4.32   |        |
| 1976 | Exact, Kim-Vasavada’s, Brysk-Michalik’s DWF                           | -          | -      | 0.288  | 4      |        |
| 1977 | Analytic wave function                                               | -          | -      | -      | 0.288  | 4      |
| 1977 | RSC potential with pion Compton wavelength                            | 0.3        | 0.3    | 0.2732 | 4.5    | 6.5    |
| 1978 | Analytic wave function                                               | 0.25-0.5   | 0.3-0.5| 0.2775 | 5.39   | 0.0255 | 0.8015 |
| 1978 | KLS                                                                    | -          | -      | 2.16   | 0.093  | 0.32   |
| 1978 | Graz I                                                                 | -          | -      | 2.225  | 0.288  | 2.63   |
| 1978 | Mongan II                                                             | -          | 1.2    | 2.223  | 0.275  | 1.12   |
| 1978 | Low-energy nucleon-nucleon potential from Regge-pole theory           | -          | -      | 0.2775 | 5.39   | 0.0255 | 0.8015 |
| 1979 | Interactions in the core region                                       | 0.5        | 0.5    | 0.279  | 5.45   |        |
| 1979 | Super soft-core potential                                            | -          | -      | 0.279  | 5.45   |        |
| 1979 | OBE (λ=0)                                                             | 0.2        | -      | 4.74   |        |
| 1979 | OBE (λ=0.4)                                                           | 0.2        | -      | 4.78   |        |
| 1979 | OBE (λ=1.0)                                                           | 0.25       | 0.5    | 3.60   |        |
| 1979 | OBEP model                                                            | 2.22464    | 0.284  | 6.36   | 0.0261 | 0.797  |
| Year | Potential/Model                  | S | D0 | E0 | W| References |
|------|----------------------------------|---|----|----|--|-------------|
| 1980 | Paris potential                  | - | -  | 2.2249 | 0.279 | 5.77 | 0.02608 | [18] |
| 1980 | Four-component relativistic models | 0.3 | 0.2-0.6 | | | | | [72] |
| 1980 | S potential                      | | | 0.286 | 6.7 | 0.026 | [73] |
| 1980 | SF potential                     | | | 0.285 | 4.0 | 0.027 | [73] |
| 1980 | QT interactions                  | | | 0.352 | 4.1 | 0.038 | [73] |
| 1981 | YY7                              | | | 1.722 | 0.283 | 7.0 | 0.029 | [74] |
| 1981 | YY4                              | | | 1.723 | 0.283 | 4.0 | 0.029 | [74] |
| 1981 | T4D-2                            | | | 1.744 | 0.282 | 4.0 | -0.004 | [74] |
| 1981 | T4D-1                            | | | 1.201 | 0.282 | 4.0 | -0.004 | [74] |
| 1981 | Urbana potential                 | 2.225 | | 0.273 | 5.2 | 0.025 | [75] |
| 1984 | PEST potential                   | 2.2249 | | 0.279 | 5.77 | 0.0261 | [76] |
| 1984 | FSP                              | 0.5  | 0.5  | 2.2246 | 1.9549 | 0.2727 | 6.315 | 0.02544 | 0.8766 | [77] |
| 1984 | Mehdi-Gupta parametrization      | | | 0.1978-0.2745 | 2-6 | | | [78] |
| 1984 | Mehdi-Gupta parametrization      | | | 0.2252-0.2813 | 2-6 | | | [78] |
| 1984 | Argonne v14                      | - | - | 2.2250 | 0.286 | 6.08 | 0.0266 | 0.845 | [79] |
| 1984 | Argonne v28                      | - | - | 2.2250 | 0.286 | 6.13 | 0.0265 | 0.846 | [79] |
| 1984 | Realistic superdeep local NN-potential (Moscow) | 0.55 | 0.55 | 2.2246 | 1.9611 | 0.2860 | 6.78 | 0.0269 | 0.8814 | [80] |
| 1985 | BEST potential                   | 2.225 | | 0.2855 | 4.58 | 0.0267 | 0.8950 | [81] |
| 1986 | Quark compound bag model (b=1.2 fm) | | | | 5.33 | 0.02609 | 0.8945 | [82] |
| 1986 | ... (b=1.4 fm)                   | | | | 4.66 | 0.02609 | 0.8757 | [82] |
| 1986 | ... (b=1.6 fm)                   | | | | 4.26 | 0.02609 | 0.8884 | [82] |
| 1986 | Positive short range tensor model potential | - | 0.8 | 2.22464 | 1.9726 | 0.2860 | | 0.02639 | 0.8847 | [83] |
| 1987 | NN potentials with six-quark core radius b=1fm | 2.22462 | 1.96 | 0.276 | 5.7 | 0.0258 | [84] |
| 1987 | ... b=1.2fm                      | 2.22462 | 1.99 | 0.286 | 5.3 | 0.0263 | [84] |
| 1987 | Certov- Mathelitsch-Moravcsik DWF up | 0.1 | | 1.959-1.975 | 0.280 | 4;6;8 | 0.0261 | 0.88688 | [85] |
| 1987 | Microscopic meson-quark cluster model (set A) | - | - | 0.266 | 5.23 | | | [86] |
| 1987 | ... (set B)                      | - | - | 0.268 | 5.33 | | | [86] |
| 1987 | OBEP full model                  | - | 0.3 | 2.2246 | 2.0016 | 0.2807 | 4.249 | 0.02668 | 0.9046 | [87] |
| 1987 | OBEPQ                            | - | 0.04;0.5 | 2.2246 | 1.9684 | 0.274 | 4.38 | 0.0262 | 0.8862 | [87] |
| 1987 | OPE                              | 0.25 | 0.25 | | | 6 | 0.0262 | [88] |
| Year | Type / Model | Method | \( \lambda \) (fm\(^{-3}\)) | \( s \) | \( s' \) | \( F \) | \( S \) | \( T \) | \( U \) | \( V \) |
|------|-------------|--------|----------------|--------|--------|--------|--------|--------|--------|--------|
| 1988 | Nonlocal potential \((\lambda=5\text{fm}^{-3})\) | - - | 2.22448 | 1.96880 | 0.23953 | 4.9989 | 0.02198 | 0.8861 | [89] |
| 1988 | Nonlocal potential \((\lambda=375\text{fm}^{-3})\) | 0.5 0.8 | 2.22466 | 1.98547 | 0.30670 | 8.8181 | 0.02570 | 0.8856 | [89] |
| 1988 | Phenomenological realistic DWF | | 1.953 | 0.286 | 0.0268 | 0.8800 | [90] |
| 1989 | OBEPA | - 0.05; 0.4 | 2.22452 | 1.9693 | 0.274 | 4.38 | 0.0263 | 0.8867 | [91] |
| 1989 | OBEPB | - 0.02 | 2.22461 | 1.9688 | 0.278 | 4.99 | 0.0264 | 0.8860 | [91] |
| 1989 | OBEPC | - 0.01 | 2.22459 | 1.9674 | 0.281 | 5.61 | 0.0266 | 0.8850 | [91] |
| 1989 | Quark compound bag model QCB82 | 0.4 0.4 | 2.224574 | 0.2777 | 5.34 | 0.02593 | 0.8891 | [92] |
| 1989 | ... QCB86 | 0.6 0.6 | 2.224574 | 0.2786 | 5.47 | 0.02597 | 0.8894 | [92] |
| 1990 | Quark cluster model (set A and B) | - - | | | | | | | |
| 1990 | Quark compound bag model \((b=1.2\text{fm})\) | 2.2249 | 1.9725 | 0.279 | 5.30 | 0.0261 | 0.8874 | [94] |
| 1990 | Quark compound bag model \((b=1.35\text{fm})\) | 2.2249 | 1.9751 | 0.278 | 4.66 | 0.0261 | 0.8889 | [94] |
| 1991 | Padua potential | - - | 2.2249 | 1.9725 | 0.279 | 5.3 | 0.0261 | 0.8874 | [95] |
| 1992 | Full folded-diagram potential | 2.2244 | 0.2796 | 5.22 | 0.0264 | 0.8886 | [96] |
| 1992 | Moscow NN model | 0.65 - | 2.2245 | 1.9592 | 0.2859 | 6.75 | 0.0269 | [97] |
| 1993 | Nonlocal potential | 0.5 0.5 | 2.2242 | 1.953 | 0.2862 | 6.544 | 0.0287 | 0.8898 | [98] |
| 1993 | Coupled-coupled folded-diagram potential | 2.2245 | 0.285 | 5.58 | 0.0267 | 0.8927 | [99] |
| 1994 | OPE \((R=0.8906313)\) | - - | | 1.9366 | 0.2751 | 5.862 | 0.02653 | 0.86952 | [100] |
| 1994 | Inversion potential | - - | 2.224579 | 1.9702 | 0.2816 | 5.91 | 0.0264 | 0.8860 | [101] |
| 1994 | Nijm-3 | - - | 2.224576 | 1.9672 | 0.2705 | 5.53 | 0.0252 | 0.8848 | [101] |
| 1994 | Quantum inversion by Newton-Fulton \((original)\) | - 1.3 | 2.232139 | 1.85 | 0.2852 | 5.58 | 0.0267 | 0.8927 | [99] |
| 1994 | Newton-Fulton \((wrong)\) | - 1.8 | 2.232139 | 1.935 | 0.0925 | 1.00 | 0.01807 | 0.8753 | [101] |
| 1994 | Newton-Fulton \((correct)\) | - - | 2.232139 | 1.947 | 0.2310 | 6.77 | 0.01808 | 0.8753 | [101] |
| 1994 | Quark cluster model | - - | 2.2246 | 1.9657 | 4.91 | 0.0261 | 0.8765 | [102] |
| 1994 | Nijm I | 2.224575 | 0.2719 | 5.664 | 0.0253 | 0.8841 | [15] |
| 1994 | Nijm II | 2.224575 | 0.2707 | 5.635 | 0.0252 | 0.8845 | [15] |
| 1994 | Reid 93 | 2.224575 | 0.2703 | 5.699 | 0.0251 | 0.8853 | [15] |
| 1994 | Nijm 93 | 2.224575 | 0.2706 | 5.755 | 0.0252 | 0.8842 | [15] |
| 1995 | Complex Kohn variational | - - | 2.2208 | | | | | | |
| 1995 | OBEPR, OBEPR(A), OBEPR(B) | - 0.2 | | | | | | | |
| 1995 | Argonne v18 | - - | 2.22457 | 1.967 | 0.270 | 5.76 | 0.0250 | 0.8850 | [17] |
| 1995 | NijmI, NijmII, Reid93 | - - | | | | | | | |
| Year   | Model                                      | Parameters          | Description                                      | Values   | Notes |
|--------|--------------------------------------------|---------------------|--------------------------------------------------|----------|-------|
| 1996   | SDA                                        | 2.2246, 1.965       |                                                  | 0.275    |       |
| 1996   | SDB                                        | 2.2246, 1.9649      |                                                  | 0.2750   |       |
| 1996   | SDC                                        | 2.2246, 1.964       |                                                  | 0.2749   |       |
| 1996   | SDD                                        | 2.2246, 1.9657      |                                                  | 0.2750   |       |
| 1996   | Reid, Paris, Urbana, Argonne v18          | -                   |                                                  |          |       |
| 1996   | Resonating-group method (RGM-F)            | 2.274, 1.933        |                                                  | 0.2752   | 5.391 |
| 1996   | FSS                                        | -                   |                                                  |          |       |
| 1996   | RGM-H                                      | 2.224, 1.986        |                                                  | 0.2750   | 4.998 |
| 1996   | Effective chiral Lagrangian model fitted   | 2.15, 0.246         |                                                  |          |       |
| 1996   | One solitary boson exchange potential (OS- | 2.224, 1.966        |                                                  | 0.2845   | 5.879 |
| 1998   | Moscow A                                   | 0.5, 0.5            |                                                  |          |       |
| 1998   | Moscow B                                   | 0.5, 0.5            |                                                  |          |       |
| 1998   | Moscow C                                   | 0.5, 0.5            |                                                  |          |       |
| 1999   | OPE                                        | 0.8, 0.8            |                                                  |          |       |
| 2000   | NLO                                        | -                   |                                                  |          |       |
| 2000   | NNLO                                       | 1.1, -              |                                                  |          |       |
| 2000   | NNLO-Δ                                     | 2.18, 0.237         |                                                  |          |       |
| 2000   | Local NN Potential                         | 0.5, 0.271          |                                                  |          |       |
| 2000   | Argonne V18                                | -                   |                                                  |          |       |
| 2001   | Bonn C                                     | -                   |                                                  |          |       |
| 2001   | FSS2 (Isospin basis)                       | -                   |                                                  |          |       |
| 2001   | FSS2 (Particle basis, Coulomb off)         | 2.2261, 1.9599      |                                                  |          |       |
| 2001   | FSS2 (Particle basis, Coulomb on)          | 2.2309, 1.9582      |                                                  |          |       |
| 2001   | CD-Bonn                                    | -                   |                                                  |          |       |
| Year  | Description                                | Parameters | Details                           |
|-------|--------------------------------------------|------------|-----------------------------------|
| 2001  | Separable potentials with the Laguerre form factors | 0.2        | 0.281, 5.729, 0.0252, 0.8845       |
| 2001  | Idaho-A                                    | -          | 2.224575, 1.9756, 0.281, 4.41, 0.0256, 0.8846 |
| 2001  | Idaho-B                                    | -          | 2.224575, 1.9758, 0.284, 4.94, 0.0255, 0.8846 |
| 2003  | Nij1 transformed                           | 0.6, 0.4   |                                   |
| 2003  | Nij2 transformed                           | -          |                                   |
| 2003  | DBS model NN                               | 0.6, 0.5   | 2.2245 , 2.004, 0.286, 5.42, 0.0259, 0.8846 |
| 2003  | DBS model NN + 6q                          | 2.22454, 1.972, 0.275, 5.22, 0.0264, 0.8864 |
| 2003  | Idaho N3LO (500)                           | 2.224575, 1.978, 0.285, 4.51, 0.0256, 0.8843 |
| 2004  | Exponential potential                      | 0.5        | 2.2246, 1.960, 0.283, 6.22, 0.0265, 0.8846 |
| 2004  | Modified Moscow                            | 2.22453, 1.956, 0.286, 6.776, 0.0269, 0.8864 |
| 2003  | N3LO transformed                           | 0.5        | 2.224575, 1.9877, 4.271, 0.0252, 0.8845 |
| 2004  | N3LO transformed                           | -          | 2.224575, 1.9997, 5.620, 0.0256, 0.8846 |
| 2004  | DBS model NN + 6q                          | 2.22454, 1.9680, 0.275, 5.22, 0.0264, 0.8864 |
| 2005  | OPE-η (LO)                                 | -          | 1.9423, 0.1321, 6, 0, 0.8752       |
| 2005  | OPE-pert (NLO)                             | -          | 1.6429, 0.4555, 0, 0.51, 0.7373    |
| 2005  | Nonrelativistic DWF                        | -          | 2.2245, 2.108, 0.285, 6.2, 0.0263, 0.8861 |
| 2005  | NLO                                        | -          | 2.171-2.186, 1.973-1.974, 0.273-0.275, 3.46-4.29, 0.0256-0.0257, 0.868-0.873 |
| 2005  | NNLO                                       | -          | 2.189-2.202, 1.970-1.972, 0.271-0.275, 3.53-4.93, 0.0255-0.0256, 0.874-0.879 |
| 2005  | N3LO                                       | 0.5        | 2.216-2.223, 1.973-1.985, 0.264-0.268, 2.73-3.63, 0.0254-0.0255, 0.882-0.883 |
| 2006  | Moscow                                    | 0.5        | 2.2246, 1.9639, 0.2674, 0.02714, 0.8892 |
| 2007  | Moscow                                    | 0.5        |                                   |
| 2007  | Renormalized OPE and TPE chiral potentials | up 0.5, up 0.5 | 0.02633, 0.02564                  |
| 2007  | MT wave function                          | -          | 2.224966, 1.972, 0.2731, 6.2, 0.0253, 0.8629 |
| 2007  | JISP16                                     | -          | 2.224576, 1.9647, 0.2915, 4.136, 0.0252, 0.8629 |
| 2008  | LO χET, NNLO χET                           | up 0.5, up 0.6 | 1.90-2.06, 0.276-0.359, 6.98-10.08, 0.0251-0.0302, 0.845-0.925 |
| 2008  | OPE                                        | 0.45, 0.5  | 2.224575, 1.9351, 0.2762, 7.88, 0.02634, 0.8681 |
| 2008  | HB-TPE set IV                             | 2.224575, 1.967, 0.276, 8, Input, 0.884, 0.884 |
| 2008  | RB-TPE set IV                             | 0.1-0.6, 0.55 | 2.224575, 1.8526, 0.3087, 22.99, 0.03198, 0.8226 |
| 2008  | RB-TPE set η                              | 0.5-0.8    | 2.224575, 1.96776, 0.2749, 5.59, 0.02566, 0.88426 |
| 2009  | NNLO                                       | 0.5        |                                   |
| 2009  | LO                                         | 0.2, 0.5   | 1.9351, 0.2762, 7.31, 0.02633, 0.8681 |
|       |                                            |            |                                   |
| Year | Method                  | Parameters         | r_m (fm) | Q_d (fm^2) | η          |
|------|-------------------------|--------------------|----------|------------|------------|
| 2009 | NLO-∆                   | 0.1-0.6, 0.1-0.8   | 1.963    | 0.274      | 5.9        | 0.884      |
| 2009 | N2LO-∆                  | 0.1-0.7, 0.1-0.6   | 1.980    | 0.279      | 5.9        | 0.892      |
| 2010 | Oxford potential        |                    | 2.2246   | 1.9767     | 0.2871     | 5.604      | 0.0262     | 0.8918     |
| 2010 | GWU PWA                 |                    | 2.224575 | 1.9557     | 0.2852     | 0.0256     | 0.8764     |
| 2010 | Nijm PWA93              |                    | 2.224575 | 1.9673     | 0.2884     | 0.0256     | 0.8845     |
| 2011 | Yakawa Potential        |                    | 2.228    |            |            |            |
| 2012 | Hulthen wave function   |                    | 1.9645   | 0.2679     | 5.62       | 0.02493    | 0.8829     |
| 2014 | Coarse-grained NN       |                    | 1.9689   | 0.2658     | 5.30       | 0.02473    | 0.8854     |
| 2014 | Statistical error analysis for potentials | | 1.9744   | 0.2645     | 5.30       | 0.02448    | 0.8885     |
| 2014 | Standard Wood-Saxon potential | | 1.9532   | 0.2769     | 6.659      |            |            |
| 2014 | Generalized Wood-Saxon potential | | 1.7269   | 0.2818     | 5.056      |            |            |
| 2014 | Modified Wood-Saxon potential | | 1.9532   | 0.2836     | 4.86       |            |            |
| 2015 | Idaho N3LO (500)        |                    | 2.2246   | 1.975      | 0.275      | 4.51       | 0.0256     | 0.8843     |
| 2015 | Juelich N3LO (550/600)  |                    | 2.2196   | 1.977      | 0.266      | 3.28       | 0.0254     | 0.8820     |
| 2015 | Improved N3LO (R=0.8fm) | 0.5, 0.8           | 2.2246   | 1.970      | 0.268      | 3.78       | 0.0255     | 0.8843     |
| 2015 | ... (R=0.9mm)           | 0.5                | 2.2246   | 1.972      | 0.271      | 4.19       | 0.0255     | 0.8845     |
| 2015 | ... (R=1.0fm)           |                    | 2.2246   | 1.975      | 0.275      | 4.77       | 0.0256     | 0.8845     |
| 2015 | ... (R=1.1fm)           |                    | 2.2246   | 1.979      | 0.279      | 5.21       | 0.0256     | 0.8846     |
| 2015 | ... (R=1.2fm)           |                    | 2.2246   | 1.982      | 0.283      | 5.58       | 0.0256     | 0.8846     |
| 2015 | FSS2                    |                    | 2.2206   | 1.961      | 0.270      | 5.52       | 0.0252     |            |
| 2015 | Nonlocal potentials with chiral TPE including Δ resonances, Model a | | 2.224575 | 1.948      | 0.257      | 4.94       | 0.0245     | 0.8777     |
| 2015 | ... Model b             |                    | 2.224574 | 1.975      | 0.268      | 5.29       | 0.0248     | 0.8904     |
| 2015 | ... Model c             |                    | 2.224575 | 1.989      | 0.269      | 5.55       | 0.0246     | 0.8964     |

Experimental values [10, 140] of static properties for deuteron it is specified in Table 2.

**Table 2. Experimental properties for deuteron**

| Properties          | Values                  | Ref. |
|---------------------|-------------------------|------|
| Spin                | 1                       |      |
| Mean life           | Stable                  |      |
| Mass (u)            | 2.01410219(11)          | 140  |
| Mass (MeV)          | 1875.61282(16)          | 140  |
| Magnetic moment (µ_N)| 0.8574382308(72)        | 140  |
| E_d (MeV)           | 2.22456612(48)          | 10   |
| r_m (fm)            | 1.975(3)                | 10   |
| Q_d (fm^2)          | 0.2859(3)               | 10   |
| η                   | 0.0256(4)               | 10   |

According to the General mathematical theorem on the number of knots of eigenfunctions of boundary value
problems \[111\] the function describing the ground state of the particle becomes zero only at the ends of the interval, and inside it she knots will have.

In paper \[118\] S.B.Dubovichenko considering the possibility of the existence of knots VFD. If we consider the deuteron as a six-quark system, in accordance with a generalized Levinson theorem \[142,143\] triplet S phase scattering starts with 360° and singlet with 180° up 220°. In the D wave, there is a single bound state is enabled, which, together with the S wave determines the ground state of the deuteron.

Therefore, the availability of knots due to the numerical calculations or used potential model.

4. **Analytical forms of DWF in the years 1939-1969**

When describing DWF in the coordinate representation using terms such as “analytical shape (form)”, its “approximation” or “parameterization”. Familiar in the first place, the term “analytical form” is used as the obtained solution of a system of coupled equations. Later in the works is a expression used to refer to records HFD resulting approximation.

Analytical forms of deuteron wave function are provided with use according to the designations specified in the quoted literature.

The work written by Flugge \[144\] in 1939 was one of the first works on research of a deuteron and its quadruple moment. For calculations such deuteron functions for S- and D- states were used

\[
\psi_S = \frac{(ab)^{3/2}}{\sqrt{8\pi}} e^{-(ab)^2} e^{-\frac{1}{2}(ab^2 r^2)}
\]

\[
\psi_D = \frac{(ab)^{7/2}}{\sqrt{2176\pi}} r^2 e^{-(ab)^2} e^{-\frac{1}{2}(ab^2 r^2)}
\]

where \(a=1.3\) and \(\alpha=1.34\).

H.A. Bethe \[25\] was one of the first considered the deuteron is a mixture of \(3S_1\) and \(3D_1\) state. Then the complete wave function is

\[
\psi = \frac{1}{r} [\chi(r) F_{10M} + \phi(r) F_{12M}]
\]

where \(F_{JLM}\) are the angular functions; \(\chi\) and \(\phi\) are the radial wave functions of the S and D component.

The radial deuteron wave functions satisfy the two coupled differential equations

\[
\frac{d^2 \chi}{dr^2} = A \chi - B \sqrt{2} \phi,
\]

\[
\frac{d^2 \phi}{dr^2} = \left( A + B + \frac{1}{2} \right) \phi - B \sqrt{2} \chi,
\]

where

\[
A = ae^{-r/3r} + \varepsilon^2,
\]

\[
B = ae^{-r} \left( \frac{1}{r_0^2} + \frac{1}{r_0^2} + \frac{1}{3r_0} \right).
\]

The potential was be cut off at small distances, therefore we consider the two alternatives:

1) Zero cut-off \(r < r_0\)

\[
A = B = 0;
\]

2) Straight cut-off \(r < r_0\)

\[
A = A_0 = ae^{-r_0/3r_0} + \varepsilon^2;
\]

\[
B = B_0 = ae^{-r_0} \left( \frac{1}{r_0^2} + \frac{1}{r_0^2} + \frac{1}{3r_0} \right).
\]

The outside solution is pairs

\[
\chi_1 = e^{-\varepsilon z} + \frac{a}{\varepsilon} e^{-r} G(r),
\]

\[
\phi_1 = -\frac{2a}{\varepsilon} \left\{ \left[ \frac{2\varepsilon^3 + 3\varepsilon - 3}{r^2} + \frac{(2\varepsilon - 1)e^2}{r} \right] e^{-(1+\varepsilon)r} + \frac{3 - 4\varepsilon^2}{2\varepsilon} F(r) \right\};
\]
\[ \chi_2 = -\frac{\sqrt{2}a}{r_0} \left\{ \left[ \frac{1}{r} + \frac{1+\varepsilon}{r^2} + \frac{2\varepsilon-1}{r} \right] e^{-(1+\varepsilon)r} + \frac{3-4\varepsilon^2}{12r^2} G(r) \right\}, \]
\[ \varphi_2 = \left( \frac{1}{r} + \frac{\varepsilon}{r} + \frac{e^2}{3} \right) e^{-sr} + a \left\{ H(r) e^{-(1+\varepsilon)r} + \left[ \frac{3}{8r^2} - \frac{5}{16\varepsilon r} + \frac{1}{12r} \right] F(r) \right\}; \]

where

\[ G(r) = -e^{-sr} Ei (-r) + e^{sr} Ei (-r(1+2\varepsilon)), \]
\[ F(r) = -e^{-sr} Ei (-r) \left( \frac{1}{r} + \frac{\varepsilon}{r} + \frac{e^2}{3} \right) + e^{sr} Ei (-r(1+2\varepsilon)) \left( \frac{1}{r^2} - \frac{\varepsilon}{r} + \frac{e^2}{3} \right), \]
\[ H(r) = \frac{1}{6r^3} + \left( -\frac{3}{16\varepsilon r} + \frac{3}{16\varepsilon r^2} + \frac{3}{8r^2} - \frac{1}{4r} + \frac{1}{6} + \frac{1}{12r} \right) \frac{1}{r^2} + \frac{(2e^2+3)(2e-1)}{48e^2r}. \]

The inside solution is next pairs

\[ \chi_3 = z - 0.009z^5 - ... + \log z(0.02z^5 + ...) + b(0.1667z^3 - ...) + b \log z(...) + ...; \]
\[ \varphi_3 = 0.01298z^5 + 0.01202z^7 + ... - \log z(0.2828z^5 + ...) + b(0.0113z^5 + ...) + ...; \]

\[ -\chi_4 = 0.07071z^5 + 0.002405z^7 + ... + b(0.004z^7 + ...), \]
\[ \varphi_4 = z^3 + 0.00714z^5 + ... + b(0.0714z^5 + ...) + ...; \]

where

\[ z = \sqrt{B_0 r}; \]
\[ b = \frac{A_0}{B_0} = \frac{\frac{1}{3}a + r_0e^2r_0}{a \left( \frac{1}{3} + \frac{1}{r_0} + \frac{1}{r_0^2} \right)}; \]

\[ r_0 = 0.4 \text{fm}. \]

W. Rarita and J. Schwinger obtain the following differential equations for the \(^3\text{S}_1\) and \(^3\text{D}_1\) radial deuteron wave functions [20]

\[ \frac{d^2u}{dr^2} + \frac{M}{r^2} \left[ E + J \right] u = -2^{3/2} \gamma \frac{M}{r^2} Jw, \]
\[ \frac{d^2w}{dr^2} - \frac{6w}{r^2} + \frac{M}{r^2} \left[ E + (1 - 2\gamma)J \right] u = -2^{3/2} \gamma \frac{M}{r^2} Jw. \]

Outside the range of interaction these coupled equations are readily integrable. The result of such decisions

\[ u(r > r_0) = Ae^{-\alpha(r-r_0)}, \]
\[ w(r > r_0) = Be^{-\alpha(r-r_0)} \left( 1 + \frac{3}{ar} + \frac{3}{(av)^2} \right), \]

where \( \alpha = \sqrt{M|E_0|/h^2}, |E_0| = -E = 2.17\text{MeV}. \)

At distances less than \( r_0 \) the differential equations for the ground state wave function will be written in the following form

\[ \left( \frac{d^2}{dr^2} + \kappa^2 \right) u(r) = -\lambda^2 w(r), \]
\[ \left( \frac{d^2}{dr^2} - \frac{6}{r^2} + \kappa^2 \right) w(r) = -\lambda^2 u(r). \]

Here introduced the next notation

\[ \kappa^2 = \frac{M(V_0 - |E_0|)}{h^2}, \]
\[ \kappa'^2 = \frac{M((1 - 2\gamma)V_0 - |E_0|)}{h^2}, \]
\[ \lambda^2 = \frac{2^{3/2} \gamma M^2 V_0}{h^2}. \]

The procedure adopted was the expansion for deuteron wave functions \( u(r) \) and \( w(r) \) in infinite power series
The constants $A_n$, $B_n$, $C_n$, $D_n$ satisfy the recursion formulas

\[
\begin{align*}
(n + 1)(n + 2)A_{n+1} + (2n + 3)C_n + (\kappa r_0)^2 A_{n-1} &= -(\lambda r_0)^2 B_{n-3}, \\
(n + 1)(n + 2)C_{n+1} + (\kappa r_0)^2 C_{n-2} &= -(\lambda r_0)^2 D_{n-3}, \\
n(n + 5)B_n + (2n + 5)D_n + (\kappa' r_0)^2 B_{n-2} &= -(\lambda r_0)^2 A_n, \\
n(n + 5)D_n + (\kappa' r_0)^2 D_{n-2} &= -(\lambda r_0)^2 C_{n-1}.
\end{align*}
\]

The criterion for continuity of the logarithmic derivatives of function $u(r)$ and $w(r)$ gives two simple equations

\[
\begin{align*}
\frac{r_0}{r} \frac{du}{dr} \bigg|_{r=r_0} &= -\alpha r_0, \\
\frac{r_0}{r} \frac{dw}{dr} \bigg|_{r=r_0} &= -2 + \frac{(\alpha r_0)^2(1+\alpha r_0)}{(\alpha r_0)^2 + 3\alpha r_0 + 3),
\end{align*}
\]

which suffice to amply determine $B_0/A_0$ and $V_0$ for a given choice of parameters $r_0$ and $\gamma$.

The constants $A$ and $B$ may be derived from the known normalization condition:

\[
\int_0^\infty (u^2 + w^2)dr = \int_0^\infty (u^2 + w^2)dr + \frac{A^2}{2\alpha} + \frac{B^2}{2\alpha} \left(1 + \frac{6(1+\alpha r_0)^2}{(\alpha r_0)^2}\right) = 1.
\]

The final set of constants was calculated as $V_0/|E_0| = 6.4; \gamma = 0.775; r_0 = 2.8\times10^{-13}$ cm.

Inside the range interaction a general expansion for DWF is \[145\]

\[
\begin{align*}
u(r) &= \sum_i A_i(\kappa_i r)^{1/2} J_{1/2}(\kappa_i r) = \sum_i u_i(r), \\
w(r) &= \sum_j B_j(\lambda_j r)^{1/2} J_{1/2}(\lambda_j r) = \sum_j w_j(r),
\end{align*}
\]

where $u_i$ and $w_j$ are the modes in terms of Bessel functions of order one-half or five-halves. The wave-lengths ($\kappa_i$ and $\lambda_j$) of these modes are determined by the continuity of the logarithmic derivative.

Also different set of modes for the radial functions were taken as an exponential times a power series for the interparticle distance:

\[
\begin{align*}
u(r) &= \sum_i A_i r^\frac{3}{2} \exp(-\lambda r) = \sum_i u_i(r), \\
w(r) &= \sum_j B_j r^{j+2} \exp(-\mu r) = \sum j w_j(r).
\end{align*}
\]

The parameters $\lambda$ and $\mu$ for radial DWF are practically fixed by minimizing the energy.

In Ref. \[146\] it is investigated the radial dependence of the tensor force in the Deuteron. The find the solutions of coupled Schrodinger equations for DWF with methods are similar to the ones used by Rarita and Schwinger. Such ranges and them regions are considered.

A. Range of tensor force equal to range of ordinary force: $\varepsilon = 1$.

For region I $r_0 \geq r \geq 0$ were received solutions

\[
\begin{align*}
u &= \sum_n (A_n + C_n \ln x)x^n, \\
w &= \sum_n (B_n + D_n \ln x)x^n.
\end{align*}
\]

For region II $\infty \geq r \geq r_0$ solutions is

\[
\begin{align*}
u &= A \exp\{-\alpha(r - r_0)\} + C \exp\{\alpha(r - r_0)\}, \\
u &= B \exp\{-\alpha(r - r_0)\} \left[1 + \frac{3}{\alpha r} + \frac{3}{(\alpha r)^2}\right] + D \exp\{\alpha(r - r_0)\} \left[1 - \frac{3}{\alpha r} + \frac{3}{(\alpha r)^2}\right].
\end{align*}
\]
B. Range of tensor force less than range of ordinary force: $\varepsilon_I$. For region I: $\varepsilon_I r_0 \geq r \geq 0$ solutions

$$u = \sum_n (A_n + C_n \ln y) y^n,$$
$$w = \sum_n (B_n + D_n \ln y) y^n.$$

For region II $r_0 \geq r \geq \varepsilon_I r_0$ solutions

$$u = A' \sin(\kappa r) + C' \cos(\kappa r),$$
$$w = B' \left[ \sin(\kappa r) + \frac{3}{\kappa^2} \cos(\kappa r) - \frac{3}{\kappa^2} \sin(\kappa r) \right] +$$
$$+ D' \left[ \cos(\kappa r) - \frac{3}{\kappa^2} \sin(\kappa r) - \frac{3}{\kappa^2} \cos(\kappa r) \right].$$

For region III $\infty \geq r \geq r_0$ solutions is the same as [1].

C. Range of tensor force greater than range of ordinary force: $\varepsilon_J I$.

For region I $r_0 \geq r \geq 0$ solutions is the same as [3].

For region II $\varepsilon_I r_0 \geq r \geq r_0$ solutions

$$u = \sum_n (A'_n + C'_n \ln y) y^n,$$
$$w = \sum_n (B'_n + D'_n \ln y) y^n.$$

For region III $\infty \geq r \geq \varepsilon_I r_0$ solutions

$$u = A \exp \{ -\alpha (r - \varepsilon_I r_0) \} + C \exp \{ \alpha (r - \varepsilon_I r_0) \},$$
$$u = B \exp \{ -\alpha (r - \varepsilon_I r_0) \} \left[ 1 + \frac{3}{\alpha r_0} + \frac{3}{(\alpha r_0)^2} \right] + D \exp \{ \alpha (r - \varepsilon_I r_0) \} \left[ 1 - \frac{3}{\alpha r_0} + \frac{3}{(\alpha r_0)^2} \right].$$

Pairs of the equations for these areas are specified in work [146]. The series coefficients satisfy the recurrence formulas:

$$(n + 1)(n + 2)A_{n+2} + (2n + 3)C_{n+2} + aA_n + cB_n = 0,$$
$$(n + 1)(n + 2)C_{n+2} + aC_n + cD_n = 0,$$
$$(n - 1)(n + 4)B_{n+2} + (2n + 3)D_{n+2} + bB_n + cA_n = 0,$$
$$(n - 1)(n + 4)D_{n+2} + bD_n + cC_n = 0.$$

Here it is used following abbreviations

$$a = (\kappa r_0)^2; a' = (\alpha r_0)^2; b = (\kappa' r_0)^2; b' = (\alpha' r_0)^2; c = (\lambda r_0)^2;$$
$$\alpha = \frac{\sqrt{M E_0}}{\hbar}; \alpha' = \frac{\sqrt{M (E_0 + 2\gamma V_0)}}{\hbar};$$
$$\kappa = \frac{\sqrt{M V_0 - E_0}}{\hbar}; \kappa' = \frac{\sqrt{M (1 - 2\gamma)V_0 - E_0}}{\hbar};$$
$$\lambda = \frac{\sqrt{2^3/2 \gamma M V_0}}{\hbar}; x = \frac{r}{r_0}; y = \frac{r}{\varepsilon_I r_0}.$$

At the outside of potentials NN interaction $u(x)$ and $w(x)$ have following form [27]

$$u(r) = N \exp \left( -r/\xi \right),$$
$$w(r) = N' \exp \left( -r/\xi \right) [3(\xi/r)^2 + 3(\xi/r) + 1],$$

where constant $\xi$ is determined from the binding energy of deuteron. The coupled equations [2] have two independent solutions, which satisfy the boundary its conditions and are denoted by $\psi_1 = (u_1, w_1), \psi_2 = (u_2, w_2)$. Any solution of (2) is given by

$$\psi_1 + \alpha \psi_2 = (u_1 + \alpha u_2, w_1 + \alpha w_2).$$

For core radius $r_0$
\[ u_1(r_0) + \alpha u_2(r_0) = 0, \]
\[ w_1(r_0) + \alpha w_2(r_0) = 0, \]
therefore, \( r_0 \) is the zero point of determinant \[ \begin{vmatrix}
  u_1(x) & u_2(x) \\
  w_1(x) & w_2(x)
\end{vmatrix} \]
and \( \alpha \) is given by
\[ \alpha = -\frac{u_1(r_0)}{u_2(r_0)}, \]
Static parameters determined \( \alpha \) by
\[ \alpha = \frac{-(bX - B) \pm \sqrt{D}}{aX - A}, \]
where \( A, B, C, a, b, c \) are some integrals quadratic of wave functions
\[ a = \int (u_2^2 + w_2^2) \, dr; \quad b = \int (u_1 u_2 + w_1 w_2) \, dr; \quad c = \int (u_1^2 + w_1^2) \, dr; \]
\[ D = X^2(b^2 - ac) - X(2bB - Ac - aC) + (B^2 - AC). \]

The assumed potentials confine the physical value \( X \) to some limited region. For example, numerical results are

given below with \( V_C = -500 \text{MeV}; \ V_T = -500 \) or \( 300 \text{MeV}. \)

In the method for the solution of the deuteron problem and its application to a regular potential were applied
such sets trial functions \(^{28}\)
\[
\begin{cases}
  u = a \tau e^{-\mu r}, \\
  w = b \tau e^{-\mu r},
\end{cases}
\]
or
\[
\begin{cases}
  u = a \tau^2 e^{-\mu r}, \\
  w = b \tau^3 e^{-\mu r},
\end{cases}
\]
where \( w_0 = \frac{1}{\sqrt{2}} \psi_{10}; \ \psi_{3i} \) are Laguerre functions.

A nucleon-nucleon potential which is a well-defined static limit of a phenomenological covariant interaction is
suggested in paper \(^{31}\). For this model have used a variational wave function with the correct behavior at the origin
and at infinity:
\[
u(r) = e^{-r} - e^{-\alpha r},
\]
\[
w(r) = N \left[ \left( \frac{3}{r^2} + \frac{3}{r} + 1 \right) e^{-r} - \left( \frac{3}{r^2} + \frac{3\alpha}{r} + \frac{3\alpha^2 - 1}{2} + \frac{\alpha r(\alpha^2 - 1)}{2} \right) e^{-\alpha r} \right],
\]
where \( \alpha = 5 \) and \( N = 0.1 \) are the approximate values of the variational parameters.

For normalization \( \int_0^\infty (u^2(r) + w^2(r)) \, dr = 1 \) of pion-theoretical deuteron function its record will be as analytical
expression \(^{30}\)
\[
\begin{align*}
u(r) &= 1.039 \exp^{-0.32r} - 1.392^{-2.36r}; \\
w(r) &= 0.02624 \left\{ 1 + \frac{3}{0.328r} + \frac{3}{(0.328r)^2} \right\} \exp^{-0.328r} - \frac{1.298}{r^2} \exp^{-0.962r},
\end{align*}
\]
In \(^{32}\) are desirable to approximate the Gartenhaus wave function from the cut-off meson theory \(^{29}\) by an
analytic expression. They can be usefully in the various integrals for calculates phenomena involving the deuteron.
Three such approximations of varying degrees of accuracy are specified further.

Approximation 1. The best Hulthen type wave function defined by the such form
\[
u(r) = C \left( e^{-\alpha r} - e^{-\beta r} \right).\]
Its parameters $C$ and $\alpha$ are agree with the asymptotic behavior of the Gartenhaus $S$- function, and $\beta$ find from the normalization of the two functions according to formulas

$$\int_0^\infty u^2 dr = 4.025;$$

$$\int_0^\infty w^2 dr = 0.29.$$  

The received values of these parameters: $C=1.85$ or $1.91$; $\alpha=0.232$; $\beta=1.202$.

Approximation 2 and 3. Next even better approximation only as sum of exponential functions has the forms

$$u(r) = \begin{cases} C (1 - e^{-1.59r}) (e^{-0.232r} - e^{-1.59r}), \\ C (1 - e^{-2.5r}) (1 - e^{-1.59r}) (e^{-0.232r} - e^{-1.90r}). \end{cases}$$

A good approximation to the D function using only exponential functions is the following:

$$w(r) = \begin{cases} 0.658r^3, \ 0 \leq r \leq 0.63 \\ 2.34r^3e^{-2r}, \ 0.63 \leq r \leq 2.1 \\ 0.147e^{-0.256r} + 0.810e^{-0.577r}, \ 2.1 \leq r \leq +\infty \end{cases}$$

which agrees with the Gartenhaus function for D- state everywhere within 4 percent.

For relativistic DWF (In particular for S- state) the authors [147] find as

$$\psi(r) = \sum_0^\infty A_q(r)G_{0q}^2\left(\frac{\pi}{2}\right),$$

where $G_{0q}^2$ are Gegenbauer polynomial at argument $\pi/2$. The radial DWFs in coordinate and momentum space are Bessel-Fourier transforms to each other:

$$A_q(r) = \frac{(-1)^q}{(2\pi)^2} \int_0^\infty A_q(p) \frac{J_{2q+1}(pr)}{pr} p^3 dp.$$ 

In work [148] are investigated the elastic scattering of high energy neutron by deuteron, using DWFs calculated making use of the meson theoretical potential:

1) The DWF with the hard core:

$$u(r) = \begin{cases} N \{\exp(-\alpha(r - r_C) - \exp(-\beta(r - r_C))}, \ r \leq r_C; \\ 0, \ r \leq r_C. \end{cases}$$

Here $N^2 = \frac{\alpha\beta(\alpha+\beta)}{2\pi(\alpha-\beta)^2}$.

2) The DWF without the hard core:

$$u(r) = N \{\exp(-\alpha r) - \exp(-\beta r)\}.$$ 

For the deuteron state in work [35] was considered the pion-theoretical wave function given in [30]. Thus

$$\psi = \frac{1}{\sqrt{4\pi}} \left[ \frac{u(r)}{r} - \frac{1}{\sqrt{8}} S_{12} \frac{w(r)}{r} \right] \frac{1}{\sqrt{2}} (\xi_1\eta_2 - \xi_2\eta_1) \chi_m^l.$$ 

The plane wave approximation is the conventional form for purpose:

$$\psi_f = \frac{1}{\sqrt{2}} \{\xi_1\eta_2 \exp[ikr] - \xi_2\eta_1 \exp[-ikr]\} \chi_m^l.$$
Here $k$ is the relative propagation vector of the nucleons; $\xi_i, \eta_i$ are the isotopic spin wave functions in a proton and a neutron states; $\chi_m$ is the triplet spin function.

For simplicity of calculation for photodisintegration of the deuteron in the high energy range, are used the following analytical form which approximates the deuteron wave function very well in the outer region \[35\]:

$$u(r) = A_S \left[ e^{-\alpha r} - e^{-\beta r} \right],$$
$$w(r) = D_1 e^{-\alpha_1 r} + D_2 e^{-\alpha_2 r} + D_3 e^{-\alpha_3 r}.$$  

The parameters are chosen as:

$$A_S = 1.039; \quad D_1 = 0.111; \quad \alpha_1 = 0.4;$$
$$\alpha = 0.328; \quad D_2 = 0.656; \quad \alpha_2 = 1;$$
$$\beta = 1.972; \quad D_3 = -0.767; \quad \alpha_3 = 2.$$  

The wave function and them parameters reproduce result of calculations for the deuteron parameters: $P_D = 7\%, Q = 2.6 \times 10^{27}$.

In paper \[149\] the deuteron wave-functions used are of the Hulthen-Sugawara type \[150\].

$$\psi_D(r) = \frac{N}{\sqrt{4\pi}} \left\{ \frac{u_g(r)}{r} + \frac{S_{12} w_g(r)}{\sqrt{8}} \right\} \chi_m,$$

where

$$u_g(r) = \cos \varepsilon_g \left[ 1 - e^{-\beta(x=x_C)} \right] e^{x};$$
$$w_g(r) = \sin \varepsilon_g \left[ 1 - e^{-\gamma(x-x_C)} \right]^{1/2} e^{x} \left[ 1 + \frac{3(1-e^{-\gamma x})}{x} + \frac{3(1-e^{-\gamma x})^2}{x^2} \right];$$

$N^2=7.6579 \times 10^{-12}$ cm$^{-1}$; $x = \alpha r$; $x_C = \alpha r_C$; $\alpha=0.2316$ fm$^{-1}$; $r_C$ are hard-core radius. Two values were select for D- probabilities as

$\beta = 7.961; \quad \gamma = 3.798; \sin\varepsilon_g=0.0266$ for 4% D- state;
$\beta = 7.451; \quad \gamma = 4.799; \sin\varepsilon_g=0.02486$ for 6% D- state.

The numerical deuteron wave function using the Yale nucleon-nucleon potential has been approximated by analytic expressions \[39\] that contained only exponential functions. A first approximation consisted with Hulthen function for S- wave of the form

$$u_1(r) = A e^{-\alpha r} - B e^{-\beta r}.$$  

Value of parameters $A$ and $\alpha$ are determined by the asymptotic behaviour of the radial wave function, $B$ by the boundary conditions at the hard core and $\beta$ by the required normalization from the S- state. The result for these parameters is

$$A = 1.04965; \quad \alpha = 0.331; \quad B = 2.57955; \quad \beta = 2.900.$$  

An improved approximation to $u(x)$ is obtained with the function

$$u_2(r) = \left( 1 + 1.039 e^{-5r} - 8 e^{-10.58r} \right) \left( 1.0459 e^{-0.331 r} - 2.5702 e^{-2.9r} \right).$$  

Fit the Yale D- state data were received a suitable approximation with a function of the form

$$w(r) = \left\{ \begin{array}{ll}
A e^{-\alpha r} - B e^{-\beta r}, & 0.35 \leq r \leq 3.416; \\
C e^{-\gamma r} + D e^{-\delta r}, & 3.416 \leq r.
\end{array} \right.$$  

The values of the constants are

$$A = 0.46354; \quad \alpha = 0.6636; \quad B = 0.24479; \quad \beta = 5.4183;$$
$$C = 0.13436; \quad \gamma = 0.417; \quad D = 0.85599; \quad \delta = 1.1703.$$  

For the Schrodinger equations for the deuteron radial wave functions are look for a solution of this equation having the following form \[35\]:
\[
\begin{pmatrix}
u(r) \\
w(r)\
\end{pmatrix} = a \begin{pmatrix} f_1(r) \\
g_1(r) \\
\end{pmatrix} \exp[-\kappa r] + b \begin{pmatrix} f_2(r) \\
g_2(r)\
\end{pmatrix} \exp[-\kappa r].
\]

For the deuteron wave function in both S and D states is constructed following Martin’s method. He allows to written down the analytical solutions as

\[
u(r) = A e^{-\kappa r} \left[ 1 + \frac{2}{1} e^{-\alpha r} \rho^+(\alpha) d\alpha + H \frac{2}{1} e^{-\alpha r} \rho^-(\alpha) d\alpha \right],
\]

\[
w(r) = A e^{-\kappa r} \left[ H + \frac{2}{1} e^{-\alpha r} \sigma^+(\alpha) d\alpha + H \frac{2}{0} e^{-\alpha r} \sigma^-(\alpha) d\alpha \right],
\]

where \( A = a + b; H = \frac{a-b}{2}; \rho^\pm = \frac{1}{2} (\rho_1 \pm \rho_2); \sigma^\pm = \frac{1}{2} (\sigma_1 \pm \sigma_2) \). In a Martin’s method it was considered that

\[
f_\lambda = 1 + \int_0^\infty \rho_\lambda(\alpha) e^{-\alpha r} d\alpha;
\]

\[
g_\lambda = \eta_\lambda + \int_0^\infty \sigma_\lambda(\alpha) e^{-\alpha r} d\alpha;
\]

are solutions of modified equations Schrödinger

\[
f_\lambda'' - 2\kappa f_\lambda' - U_C f_\lambda = U_T g_\lambda,
\]

\[
g_\lambda'' - 2\kappa g_\lambda' - (6/r^2 + U_m) g_\lambda = U_T f_\lambda.
\]

Are considered the “inner” part of the interaction in the wave functions themselves by adding two terms for the two-pion exchange and the repulsive nucleon core. For couple of functions \( u(r) \) and \( w(r) \) by solutions will be the following form as (it dearly fixes the normalization of the functions):

\[
u(r) = e^{-\kappa r} \left[ 1 + \frac{2}{1} e^{-\alpha r} \rho^+(\alpha) d\alpha + H \frac{2}{1} e^{-\alpha r} \rho^-(\alpha) d\alpha + \gamma_1 e^{-\xi_1 r} + \gamma_2 e^{-\xi_2 r} \right],
\]

\[
w(r) = e^{-\kappa r} \left[ H + \frac{2}{1} e^{-\alpha r} \sigma^+(\alpha) d\alpha + H \frac{2}{0} e^{-\alpha r} \sigma^-(\alpha) d\alpha + \gamma_3 e^{-\xi_1 r} + \gamma_4 e^{-\xi_2 r} \right],
\]

where \( H, \gamma_i, \xi_i \) are parameters to be fixed. This representation for DWFs with tensor forces.

In paper [151] was assumed that the true wave function is a sum of the “outer” part found from the well-known OPE potential, and an “inner” part. The “outer” part more slowly than “inner” part vanishes exponentially with an exponent between one and two pion masses. Białkowski [38] have proposed the wave function of the form

\[
\begin{cases}
u(r) = u_{\text{outer}} + u_{\text{inner}}, \\
w(r) = w_{\text{outer}} + w_{\text{inner}},
\end{cases}
\]

\[
u_{\text{outer}} = A e^{-\kappa r} \left[ 1 + \int \frac{\rho^+(\alpha) e^{-\alpha r} d\alpha}{\alpha(\alpha + 2 \kappa)} + H \int \frac{\rho^-(\alpha) e^{-\alpha r} d\alpha}{\alpha(\alpha + 2 \kappa)} \right],
\]

\[
w_{\text{outer}} = A e^{-\kappa r} \left[ H + \int \frac{\sigma^+(\alpha) e^{-\alpha r} d\alpha}{\alpha(\alpha + 2 \kappa)} + H \int \frac{\sigma^-(\alpha) e^{-\alpha r} d\alpha}{\alpha(\alpha + 2 \kappa)} \right],
\]

\[
\begin{cases}
u_{\text{inner}} = A e^{-\kappa r} \left[ \gamma_1 e^{-\xi_1 r} + \gamma_2 e^{-\xi_2 r} \right], \\
w_{\text{inner}} = A e^{-\kappa r} \left[ \gamma_3 e^{-\xi_1 r} + \gamma_4 e^{-\xi_2 r} \right].
\end{cases}
\]

Except these forms, are also such forms for “inner” part DWF [151] as

\[
\begin{cases}
u_{\text{inner}} = A e^{-\kappa r} \left[ \gamma_1 e^{-\xi_1 r} + \gamma_2 e^{-\xi_2 r} \right], \\
w_{\text{inner}} = A e^{-\kappa r} \left[ \gamma_3 + \gamma_4 \right] e^{-\xi_2 r};
\end{cases}
\]
In the work [41] authors have approximated the coordinate space wave functions by a sum of exponentials or Hankel functions. The deuteron S state can then be viewed as an extension of the known Hulthen wave function. The wave functions in coordinate space have the form

\[
\begin{aligned}
\psi_{\text{inner}} &= Ae^{-kr} \left[ \gamma_1 e^{-\xi_1 r} - \gamma_2 e^{-\xi_2 r} \right], \\
\psi_{\text{outer}} &= Ae^{-kr} \left[ \gamma_3 e^{-\xi_3 r} - \gamma_4 e^{-\xi_4 r} \right],
\end{aligned}
\]

where \( h_2 \) is the spherical Hankel function \( xh_2(ix) = e^{-x} \left[ 1 + 3/x + 3/x^2 \right] \); \( \alpha = \sqrt{M\varepsilon} \) is given by the deuteron binding energy \( \varepsilon \). Fitted pole positions and residues are denoted by \( \varepsilon_j, C_j \). Coefficient \( N \) is normalization for wave function in terms of the deuteron effective range \( \rho \)

\[
N^2 = \frac{2\alpha}{1 - \alpha \rho(-\varepsilon, -\varepsilon)}.
\]

The calculated values of parameters were provided as \( \alpha = 0.2338 \text{fm}^{-1} \); \( N = 0.8896 \text{fm}^{-1/2} \); \( \rho = 0.0269 \).

The deuteron wave function may be expanded [43] in the complete set of relative oscillator functions \( \phi_{nl} \) \( (s=1; j=1; l=0 \text{ or } 2) \)

\[
\psi = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \alpha_{nl} \phi_{nl},
\]

where [152]:

\[
\phi_{nl}(r, b) = \sqrt{\frac{2\Gamma(n + l + \frac{3}{2})}{b^n n!}} \left(\frac{r^l}{b^l \Gamma(l + \frac{3}{2})}\right)^{1/2} F\left(-n \mid l + \frac{3}{2} \left| \frac{r^2}{b^2} \right.\right).
\]

5. **Analytical forms of DWF in the years 1970-1999**

Yamaguchi’s separable tensor potential generates a deuteron wave function in momentum space. Fourier transformation produces wave function in coordinate space [153]

\[
\begin{aligned}
\psi(r) &= e^{-\alpha r} - e^{-\beta r}, \\
\psi(r) &= \frac{\alpha^2(\beta^2 - \alpha^2)t}{(\gamma^2 - \alpha^2)^2} r^2.
\end{aligned}
\]

where the asymptotic ratio of D to S wave

\[
\eta = \lim_{r \to \infty} \left[ \frac{\psi(r)}{\psi(r)} \right] = \alpha^2(\beta^2 - \alpha^2)t
\]

Function \( w(r) \) is proportional to \( r^2 \):

\[
\lim_{r \to 0} w(r) = \frac{\eta(\gamma^2 - \alpha^2)^2}{8\alpha^2} r^2.
\]

Using function \( u(r) \) and \( w(r) \) it is possible to find the central potential \( V_C(r) \) and the tensor potential \( V_T(r) \). For this reason Burnap et all. solve the coupled equations for radial DWF. In the result is written down the local potentials corresponding to Yamaguchi’s form factors as
\[ V_C = \frac{-\hbar^2 (\beta^2 - \alpha^2)}{M} \left[ -\frac{w(t + 1)}{2u} e^{-\gamma r} + \frac{1 - w/\sqrt{2}}{u} e^{-\beta r} \right] \left( u - \frac{w}{\sqrt{2}} - \frac{w^2}{u} \right)^{-1}, \]

\[ V_T = \frac{-\sqrt{8\hbar^2}}{M} \left[ \frac{(\gamma^2 - \alpha^2)^2}{2\alpha^2} \eta(\gamma r + 1) e^{-\gamma r} - \frac{w(\beta^2 - \alpha^2)}{u} e^{-\beta r} \right] \left( u - \frac{w}{\sqrt{2}} - \frac{w^2}{u} \right)^{-1}. \]

Parameters \( \beta, \gamma, t \) are definite in [153], thus \( \alpha = 0.2316 \text{fm}^{-1} \).

Humberston and Wallace offered some series of analytic approximations [46] to the deuteron wave function for the modified Hamada-Johnston potential. The solution for coupled equations for the radial components DWF must satisfy the boundary conditions:

\[ u(x_0) = 0, \quad u(x) \approx e^{-\kappa r}, \]

\[ w(x_0) = 0, \quad w(x) \approx e^{-\kappa r} \left( 1 + \frac{3}{\kappa r} + \frac{3}{(\kappa r)^2} \right), \]

where \( x_0 = 0.343 \text{fm} \) is the hard-core radius.

Equations for the radial components of the S- and D-state wave functions was then transformed to

\[ \begin{align*}
\left\{ \frac{d^2}{dy^2} + \frac{2}{y} \frac{d}{dy} - \frac{s^2}{y^2} - A(y) \right\} \tilde{u}(y) - B(y)\tilde{w}(y) &= 0, \\
\left\{ \frac{d^2}{dy^2} + \frac{2}{y} \frac{d}{dy} - \frac{6}{y^2} - C(y) \right\} \tilde{w}(y) - B(y)\tilde{u}(y) &= 0,
\end{align*} \]

where \( y = 1/r; \tilde{u}(y) = u(r); \tilde{w}(y) = w(r); \)

\[ A(y) = U_C(r)/y^4; \quad B(y) = 2\sqrt{2}U_T(y)/y^4; \quad C(y) = [U_C(r) - 2U_T(r) - 3U_{LS}(r) - 3U_{LL}(r)]/y^4. \]

Here \( U_j(r) \) is components a nucleon-nucleon potential.

Forms of analytic approximations to the solution of coupled equations were obtained for the modified and unmodified Hamada-Johnston potentials. It was applied the Rayleigh-Ritz variational method to the deuteron binding energy. The trial function for the deuteron as

\[ \begin{align*}
&u(r) = e^{-\alpha r} \left( 1 - e^{-\delta(r - r_0)} \right) \sum_{i=1}^{L} c_i e^{-(i-1)\alpha r} = \sum_{i=1}^{L} c_i \phi_i, \\
w(r) = e^{-\alpha r} \left( 1 - e^{-\rho(r - r_0)} \right) \left( 1 + \frac{3}{\alpha r} + \frac{3}{(\alpha r)^2} \right) \sum_{i=1}^{N} d_i e^{-(i-1)\alpha r} = \sum_{i=1}^{N} c_{M+i} \phi_{M+i}.
\end{align*} \]

Here \( \hbar^2 \alpha^2 / M = -E_\alpha \) and \( \delta, \rho, c_i \ (i=1, \ldots, L), d_i \ (i=1, \ldots, N) \) are variational parameters.

DWFs for Reid soft-core potential are selected according to [50]:

(a) particular Haftel-Tabakin cases [154]:

\[ u(r) = C_0 e^{-\alpha_0 r}(1 - \beta_0 r), \quad w(r) = C_2 e^{-\alpha_2 r}(1 - \beta_2 r), \]

(b) "fixed-range" cases:

\[ g(r) = \alpha(1 - p)p^a \left[ 1 - bp^c + (b - 1)p^d \right]; 0 \leq r \leq e, \]

where \( p = 1 - r/e \). Appropriate parameters and properties of the unitary transformations are presented as UT8, 13, 18, 22, 23 for case (a) and UT101, 102, 103 for case (b).

The resulting form of the separable potentials [53] is

\[ v = \sum_n |v_n \langle \lambda_n (v_n) ; \]

\[ \langle l, p | v_n \rangle = \sum_m b_{n,l;m} u_{t;m}(p); \]

21
where DWF in momentum space
\[ u(p) = \frac{1}{(p^2 + \alpha_m^2)^{\frac{3}{2}}}; \]
\[ w(p) = \frac{1}{(p^2 + \alpha_m^2)^{\frac{3}{2}}}. \]

The Fourier transforms of DWF in momentum space are
\[ u(r) = -\frac{1}{8} \sqrt{\frac{\pi}{2}} \frac{1}{\alpha_m} \exp(-\alpha_m r), \]
\[ w(r) = -\frac{1}{8} \sqrt{\frac{\pi}{2}} r \exp(-\alpha_m r). \]

To determine the unitary pole approximations for a concrete potential model, in [54] are done calculations the two-nucleon bound state wave functions in momentum space. The partial wave Schrodinger equation appropriate to S- and D- state it is written down as
\[ \lambda \left( \frac{d^2}{dr^2} - \frac{6}{r^2} \delta_{L2} - k_d^2 \right) u_l(r) = \sum_{L=0,2} V_{lL}(r) u_L(r), \]
where
\[ k_d^2 = E_d/\lambda; E_d \] are deuteron binding energy. To solves the coupled equations components of the radial deuteron wave function \( u_l(r) \), use expressions
\[ u_l(r) = 0 \text{ for } r < r_c, \]
\[ u_l(r) = -\sqrt{\frac{\pi}{2}} \lambda \sum_{j=1}^{N} \alpha^j \phi^j_l(r) \text{ for } r \geq r_c, \]
where \( r_c \) are hard-core radius; \( \alpha^j \) are the expansion coefficients. The effect of the hard core be incorporated by the modification
\[ \phi^j_0(r) = \exp(-k_d r) - \eta^j_0 \exp(-a_j r), \]
\[ \phi^j_1(r) = 2k_d^2 A_{5/2}(k_d r) - \eta^j_2 \left[ 2a_j^2 A_{5/2}(a_j r) - (k_d^2 - a_j^2)a_j r A_{3/2}(a_j r) \right]. \]

where
\[ A_{5/2}(\mu r) = \left( 1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2} \right) e^{-\mu r}, \]
\[ A_{3/2}(\mu r) = \left( 1 + \frac{1}{\mu r} \right) e^{-\mu r}. \]

Here \( a_j \) \((j=1,N)\) are predetermined ranges chosen between 0.7 and 20.0 fm. This approximation was applied for the group potentials of different types: hard core (Reid hard core [3], Hamada- Johnston [37], Yale [155]), soft core (Reid soft core and Alternate Reid soft core [3]), super soft core (Tourreil-Sprung A, B and C [49]) and velocity dependent (Bryan-Scott, Bryan-Gersten, Stagat [47], Riewe, and Green, Ueda-Green II).

In work [55] is submitted Baker transformation as
\[ \tilde{u}(r) = \sqrt{\frac{dR}{dr}} u(R(r)), \]
where
\[ R = r + a + 2\beta \ln \left[ \frac{1 + \sqrt{1 + \rho \exp(-r/\beta)}}{1 + \sqrt{1 + S}} \right], \]
\( a \) are hard-core radius; \( S \) is determined by the asymptotic
\[ \lim_{r \to \infty} [R(r) - r] = 0. \]

Besides in work [55] are specified exotic shapes by DWF UT101 [50] two DWFs obtained from RSC wave functions by a unitary transformation designed for lower the D-state probability
$$\ddot{u}(r) = C(r)u(r) + S(r)w(r),$$
$$\ddot{w}(r) = -S(r)u(r) + C(r)w(r),$$

where

$$S(r) = A_t r \frac{\tanh(r/\gamma) \exp(-(r - \rho)/\tau)}{1 + \exp(-(r - \rho)/\tau)},$$

$$C(r) = \sqrt{1 - S^2(r)}.$$

Here parameters chosen are $A_t = 0.4472$; $\gamma = 0.02\text{fm}$; $\tau = 0.02\text{fm}$; $\rho = 0.8$ or $1.9\text{fm}$.

Accordant [65] the Hulthen wave function for S state DWF

$$u(r) = N \left(e^{-\gamma r} - e^{-\beta r}\right), \beta >> \gamma,$$

where $\gamma = \sqrt{M\varepsilon} = 0.2316\text{fm}^{-1}$; $\beta$ be determined from the triplet effective range parameter with the value $r_0 = 1.75\text{fm}$ as approximately

$$\beta = \frac{3 - \gamma r_0 + \sqrt{\gamma^2 r_0^2 - 10\gamma r_0 + 9}}{2r_0} = 5.98\gamma.$$

The normalization constant $N$ in terms of the effective range as

$$N^2 = \frac{2\gamma}{1 - \gamma r_0} = 0.783.$$

Wave function for D-state choose explicitly as

$$w(r) = \eta N \left(1 - e^{-\tau r}\right)^5 e^{-\gamma r} \left(1 + \frac{3}{\gamma r} + \frac{3}{\gamma^2 r^2}\right),$$

Multiplication is considered the asymptotic by an interpolating factor.

Formulas for calculation of values of the D-state percentage and for the quadrupole moment will be respectively

$$P_D = \eta^2 N^2 \sum_{n=1}^{4} \frac{a_n}{(n-1)!} \sum_{q=0}^{10} \left(\frac{10}{q}\right) (-1)^{n-q} (2\gamma + q\tau)^{n-1} \ln(2\gamma + q\tau) + \eta^2 N^2 \sum_{q=0}^{10} \left(\frac{10}{q}\right) \frac{(-1)^q}{2\gamma + q\tau};$$

$$Q = \frac{\eta N^2}{\sqrt{15}} \sum_{n=0}^{5} b_n \sum_{q=0}^{5} \left(\frac{5}{q}\right) (-1)^{qn} \left[\frac{1}{(q\tau + 2\gamma)^n + 1} - \frac{1}{(q\tau + \gamma + \beta)^n + 1}\right] -$$

$$- \frac{\eta^2 N^2}{20} \left\{ \sum_{n=0}^{2} c_n \sum_{q=0}^{10} \left(\frac{10}{q}\right) (-1)^{qn} \ln(2\gamma + q\tau) + \sum_{n=0}^{2} c_n \sum_{q=0}^{10} \left(\frac{10}{q}\right) \frac{(-1)^{n-q}}{(n-1)! (2\gamma + q\tau)^{n-1}} \right\},$$

where

$$a_n = \left(\frac{1}{\gamma}, \frac{6}{\gamma^2}, \frac{15}{\gamma^3}, \frac{18}{\gamma^4}, \frac{9}{\gamma^5}\right);\ b_n = \left(\frac{3}{\gamma^2}, \frac{3}{\gamma^3}, 1\right);\ c_n = \left(\frac{15}{\gamma^3}, \frac{6}{\gamma^4}, 1\right);\ d_n = \left(\frac{18}{\gamma^4}, \frac{9}{\gamma^5}\right).$$

The calculated values of parameters: $\tau = 1.09\text{fm}^{-1}$; $\eta = 0.025$ for $P = 7\%$ and $\tau = 0.83\text{fm}^{-1}$; $\eta = 0.029$ for $P = 4\%$.

In [66] DWF modelled on that of the Reid soft-core potential (RSCP) outside $1.5\lambda\pi$;

$$\psi_L(r) = \begin{cases} \sum_{i=1}^{8} a_{Li} r^{i-1}, \ r < 1.5\lambda\pi; \\
\psi_{L(RSCP)}(r), \ r \geq 1.5\lambda\pi, \end{cases}$$
where $\lambda_\pi$ is the pion Compton wavelength. In radial wave functions five of the coefficients $a_{Li}$ are determined by: 1) continuity of DWF together with its first and second derivatives of the RSCP at $1.5\lambda_\pi$; 2) $u(0)=0; w(0)=0$; 3) adjusting a D-state percentage (4.5-6.5%) and the overall normalization as 1.

In Refs. [67] and [70] it is considered electron-deuteron tensor polarization and the short range behavior of the deuteron wave function. Interactions for twelve classes varying in the core region obtained using form factor for the unitary transformation

$$g(r) = \begin{cases} C(R-r)^\alpha(1-\beta r), & r \leq R; \\ 0, & r > R. \end{cases}$$

where $R=0.7$ fm; $\alpha=2.1$. The constant $C$ is determined by the normalization condition. At a choice $\alpha \geq 2$ from that the transformed DWF will be continuous and continuous it first and second derivatives at $R$. Calculations are compared for super soft-core (SSC) potential [49]. The tensor polarization for the recoil deuterons in $ed$ scattering are calculated as

$$P_e = \frac{2G_0G_2 + G_2^2/\sqrt{2}}{G_0^2 + G_2^2}.$$

Its values in the range 0.625-0.668.

Lomon-Feshbach, Holinde-Machleidt and four-component relativistic models were used for research elastic electron-deuteron scattering at high energy [72].

In coordinate space expansion in Hulthen functions of different range is presented as

$$\frac{u(r)}{r} = \sqrt{\frac{\pi}{2}} \sum_i c_i \frac{\exp(-\beta_i r)}{r}.$$

If we calculate the $n$th moment of the coefficients as

$$M_n = \sum_i c_i \beta_i^n,$$

then the reduced wave function $u(r)$ will go like $r^n$ at the origin.

The normalized solutions of the Schrödinger equation select in [156] as

$$u(r) = N \left[ u_1(r) + \eta u_2(r) \right],$$
$$w(r) = N \left[ w_1(r) + \eta w_2(r) \right].$$

The experimental values of deuteron observables severely restrict values of $\eta$. For placing upper and lower bounds for $\eta$ it is used Schwarz’s inequality

$$U_2 W_2 \geq X^2 + \sqrt{\frac{1}{2} X W_2 + \frac{1}{8} W_2^2}.$$

The condition for the existence of a solution is

$$\Delta(R, \eta) = Y^2 - 4X^2 - \sqrt{2}XY \geq 0,$$

where

$$X = X(R, \eta) = \sqrt{50Q} - \int_R^\infty r^2(uw - \sqrt{\frac{T}{8}}w^2)dr = V_2 - \sqrt{\frac{T}{8}}W_2;$$

$$Y = Y(R, \eta) = 4 \left< r^2 \right> - \int_R^\infty r^2(u^2 + w^2)dr = U_2 + W_2;$$

$$U_n = \int_0^R r^n u^2 dr; \quad V_n = \int_0^R r^n uwdr; \quad W_n = \int_0^R r^n w^2 dr.$$
Value $p_D$ it is determined with a condition as

$$p_D = \int_0^\infty w^2 dr = W_0 + Z;$$

$$p_D > Z + \frac{X^2(1 + sgnX)}{2U_4};$$

where $Z = Z(R, \eta) = \int R w^2 dr$.

In paper [74] were presented DWFs from Yamaguchi type form factors with 4% or 7% deuteron D-stare probability (designations YY4 and YY7). Also are obtain a new set T4D-1 (T4D-2) which has the rank-1 (rank-2) separable potential with the first (second) form factor of T4D.

It should be noted that the most popular, the quoted and used parametrization of DWF are the analytical forms offered by the Paris group. Known numerical values of radial DWF in coordinate representation for the Paris potential can be approximated by means of convenient decompositions [157] in a such form:

$$u(r) = \sum_{j=1}^N C_j \exp \left( -m_j r \right),$$

$$w(r) = \sum_{j=1}^N D_j \exp \left( -m_j r \right) \left[ 1 + \frac{3}{m_j r} + \frac{3}{(m_j r)^2} \right],$$

where $N=13; m_j = \beta + (j-1)m_0; \beta = \sqrt{ME_d}; m_0=0.9\text{fm}^{-1}$. $M$ is nucleon mass, $E_d$ is binding energy of deuteron. The boundary conditions as $r \to 0$

$$u(r) \to r, \quad w(r) \to r^3.$$

The asymptotics behavior of the deuteron wave functions for large values of $r \to \infty$ are

$$u(r) = A_S \exp(-\beta r),$$

$$w(r) = A_D \exp(-\beta r) \left[ 1 + \frac{3}{\beta r} + \frac{3}{(\beta r)^2} \right],$$

The last coefficients of an analytical form were determined by formulas

$$C_n = - \sum_{j=1}^{n-1} C_j;$$

$$D_{n-2} = \frac{m_{n-2}^2}{(m_n^2 - m_{n-2}^2)(m_{n-1}^2 - m_{n-2}^2)} \left[ -m_{n-1}^2 m_n^2 \sum_{j=1}^{n-3} D_j m_j \right]$$

$$\left( m_{n-1}^2 + m_n^2 \right) \sum_{j=1}^{n-3} D_j - \sum_{j=1}^{n-3} D_j m_j^2 \right]$$

and taking into account conditions

$$\sum_{j=1}^N C_j = 0; \quad \sum_{j=1}^N D_j = \sum_{j=1}^N D_j m_j^2 = \sum_{j=1}^N \frac{D_j}{m_j^2} = 0.$$

The accuracy of parametrization is characterized by the values

$$I_S = \left( \int_0^\infty [u(r) - u_{aprox}(r)]^2 dr \right)^{1/2},$$

$$I_D = \left( \int_0^\infty [w(r) - w_{aprox}(r)]^2 dr \right)^{1/2}.$$

Model radial DWF [85] according to parametrization [5] [157] are constructed to facilitate the exploration of dependencies on the percentage D state and on the small-, medium-, and large-distance parts of DWF. Parametrization
them is such are the oscillator wave functions for the ground state and the level with two excitation quanta. The ratio between the extent to which \( \alpha \)

\[ u(r) = \eta A_S \frac{e^{-\gamma r}}{r}; \]

\[ u(r) = \eta A_S \frac{1 + \frac{3}{7r} + \frac{3}{(7r)^2}}{r} \]

will be of value in determining the \( I(J;L) \), and hence the calculated polarizability. Further is investigated the extent to which \( \alpha \) and \( r \) are in fact determined by \( A_S \) and \( \eta \).

DWF [159] must belong to the area of the Hilbert space orthogonal to the trivial solution. therefore the orthogonalization is straightforward for the Paris wave function \( u(r) \) and \( w(r) \)

\[ \tilde{u}(r) = \frac{u(r) - C \Phi_0(r)}{\sqrt{1 - C^2}}; \]

\[ \tilde{w}(r) = \frac{w(r)}{\sqrt{1 - C^2}}. \]

Here \( b \) is the oscillator width parameters; constant \( C \) equal to the product \( \langle u | \Phi_0 \rangle \); \( \Phi_0(r) \) is the eigenfunction of the norm kernel calculated in oscillator basis

\[ \Phi_0(r) = \left[ \frac{2}{\pi} \frac{3}{b^2} \right]^{1/4} r \exp \left\{ -\frac{3r^2}{4b^2} \right\}. \]

The modified DWF takes the form

\[ \tilde{u}(r) = A u(R) \frac{1}{2} \sin \alpha \Phi_0(r) - \frac{1}{2} \cos \alpha \Phi_2(r); r \leq R; \]

\[ \tilde{u}(r) = A u(r); r \geq R; \]

\[ \tilde{w}(r) = A w(r), \]

where \( R \) is certain radius, when for \( r \leq R \) the wave function is determined by six quarks dynamics; \( \Phi_0(r) \) and \( \Phi_2(r) \) are the oscillator wave functions for the ground state and the level with two excitation quanta. The ratio between them is such

\[ \Phi_2(r) = \Phi_0(r) \sqrt{\frac{3}{2} \left( 1 - \frac{r^2}{b^2} \right)}. \]

In paper [160] a method has been obtained which determines as whether or not the long-range part for potential model of a two-body is consistent with measured deuteron properties and independent of the short-range behaviour. For the determination outer part of the deuteron wave function was to construct two independent solutions of the coupled Schrodinger equations \( \left( \begin{array}{c} u_1 \\ w_1 \end{array} \right) \) and \( \left( \begin{array}{c} u_2 \\ w_2 \end{array} \right) \) in the region \( r \geq R \). Further are used the asymptotic boundary conditions as

\[ \left( \begin{array}{c} u_1 \\ w_1 \end{array} \right) \to \left( \begin{array}{c} e^{-x} \\ \eta_0 xk_2(x) \end{array} \right); \left( \begin{array}{c} u_2 \\ w_2 \end{array} \right) \to \left( \begin{array}{c} 0 \\ xk_2(x) \end{array} \right), \]

where \( x = \alpha r, xk_2(x) = e^{-x} (1 + 3/x + 3/x^2) \). The first solution corresponds to \( \eta = \eta_0 \) and for other solution \( \eta \) take a linear combination:

\[ \left( \begin{array}{c} u \\ w \end{array} \right) \eta = \left( \begin{array}{c} u_1 \\ w_1 \end{array} \right) + (\eta - \eta_0) \left( \begin{array}{c} u_2 \\ w_2 \end{array} \right). \]
In [161] is specified fit the electromagnetic form factors of the deuteron on the basis of nonrelativistic wave functions

\[ u(r) = N \left[ e^{-\alpha r} - \sum_i c_i^S e^{-\beta_i^S r} \right]; \]
\[ w(r) = \rho N \left[ \alpha r h_2(i\alpha r) - \sum_i \left( \frac{\beta_i^D}{\alpha r} \right)^2 c_i^D \beta_i^D r h_2(i\beta_i^D r) \right], \]

where \( xh_2(ix) = [1 + 3/x + 3/x^2] \exp(-x) \). Asymptotics at \( r \to 0 \) for the S and D state will be as

\[ u(r) = r \text{ or } \sum_i c_i^S = 1; \]
\[ u(r) = r^3 \text{ or } \left\{ \begin{array}{l}
\sum_i c_i^S = \alpha; \\
\sum_i (\beta_i^S)^2 c_i^S = \alpha^2;
\end{array} \right. \]
\[ w(r) = r^3 \text{ or } \left\{ \begin{array}{l}
\sum_i (\beta_i^D)^2 c_i^D = \alpha^2; \\
\sum_i (\beta_i^D)^4 c_i^D = \alpha^4.
\end{array} \right. \]

For separable potentials with and without tensor force are presented calculation of deuteron form factors [78], which are expressed through radial DWF in configuration space. The expressions Mehdi-Gupta parametrization for the radial DWF are:

\[ u(r) = A \left( e^{-\alpha r} - e^{-\beta r} \right) + B r e^{-\beta r}, \]
\[ w(r) = C \left[ \frac{\alpha^2}{\beta} \left( e^{-\alpha r} - e^{-\gamma r} \right) - \frac{\gamma (\gamma^2 - \alpha^2)}{6} r e^{-\gamma r} \right. \]
\[ + \left. \left( \frac{1}{\beta^2} + \frac{\alpha}{\gamma} \right) e^{-\alpha r} - \left( \frac{1}{\beta^2} + \frac{\gamma}{\alpha} + \frac{\gamma^2 - \alpha^2}{2 \alpha^2} \right) e^{-\gamma r} \right], \]

where \( C = \frac{3\sqrt{3\pi N t}}{(\gamma^2 - \alpha^2)^2} \). Coefficients \( A \) and \( B \) for shape-1:

\[ A = \sqrt{2\pi N} \frac{\beta}{\beta^2 - \alpha^2}; \quad B = 0; \]

and for shape-2:

\[ A = \sqrt{2\pi N} \frac{\beta}{(\beta^2 - \alpha^2)^2}; \quad B = -\frac{\pi N}{\sqrt{2\beta(\beta^2 - \alpha^2)}}. \]

The two-body parameters represented as ratios \( \beta/\alpha \) and \( \gamma/\alpha \). The D- state probability \( P_D \) is given by

\[ P_D = \frac{N^2 \pi^2 \ell^2 (5\alpha + \gamma)}{8\gamma(1 + \gamma)^5}. \]

The following parameterization of DWF for realistic superdeep local NN- potential (Moscow) was written down as gaussian expansions [80]

\[ u(r) = r \sum_{i=1}^{N_S} a_i \exp(-\alpha_i r^2), \]
\[ w(r) = r^3 \sum_{i=1}^{N_D} b_i \exp(-\beta_i r^2), \]

where

\[ \alpha_i = \frac{\alpha_0}{41.47} g_i^{7/2} \left[ \frac{\pi(2i-1)}{4N_S} \right], \]
\[ \beta_i = \frac{\beta_0}{41.47} g_i^{7/2} \left[ \frac{\pi(2i-1)}{4N_D} \right], \]

\( \alpha_0 = 31, 9; \beta_0 = 164; N_S = N_D = 30. \)
In [82] are considered quark compound bag (QCB) and six quark bag models and are inquire into the values of $P_{QCB}$ and $P_{6q}$ predicted by the QCB model. For illustration the method first consider a "toy model" of the S-wave deuteron without the NN interaction

$$u(r) = N \begin{cases} -\gamma_1 sh(\kappa r) + \gamma_2 \sin(\beta r), r \leq b, \\ \exp(-\kappa r), r \geq b. \end{cases}$$

Calculated value were $P_{QCB}=0.9\%$; $P_{6q}=17\%$. The general expression for the deuteron wave function in the QCB model it will be written down as

$$u_l(r) = N \begin{cases} b_1 u_1^{[1]}(r) + b_2 u_2^{[2]}(r), r \leq b; \\ u_1^{ext}(r), r > b, \end{cases}$$

where $N$ is the normalization factor, $u_1^{ext}$ are DWF derived from the external potential, and $u_1^{[1]}$, $u_2^{[2]}$ are the two linear independent solutions of Schrodinger equation in the inner region. The constants $b_1$ and $b_2$ are defined from the attaching condition of the internal and external wave functions at $r = b$. Thus are established the upper limit on $1.2 \text{fm} \leq b \leq 1.6 \text{fm}$ ($P_{QCB} \leq 1\%$).

In [90] are consider a more general case for [150] and [65] by including additional terms as such follows

$$u(r) = A_S(1 - e^{-\tau r}) e^{-\alpha r} \sum_{i=0}^n C_i \exp(-\alpha_i r),$$

$$u(r) = A_S(1 - e^{-\sigma r})^5 x A_S(1 - e^{-\sigma r})^5 e^{-\alpha r} \sum_{i=0}^m D_i \exp(-\alpha_i r),$$

where $\alpha=0.2315370 \text{ fm}^{-1}$; $\tau=5\alpha$; $\sigma=1.99 \text{ fm}^{-1}$; $\eta=0.025$; $k_2(\alpha r)$ - terms of the spherical Bessel function:

$$k_2(\alpha r) = \left(1 + \frac{3}{\alpha r} + \frac{3}{(\alpha r)^2}\right) e^{-\alpha r}.$$

Also in [90] are calculate the simplest phenomenological realistic deuteron wave function given by [41] and [82]

$$u(r) = A_S(1 - e^{-\tau r}) e^{-\alpha r},$$

$$u(r) = \eta A_S(1 - e^{-\sigma r})^5 e^{-\alpha r} \left(1 + \frac{3}{\alpha r} + \frac{3}{(\alpha r)^2}\right).$$

Values are obtained for the parameters $\eta$, $\tau$, $\sigma$, when the indicated values of $A_S$ and $r_d$ are used as input.

In [91] present a quark compound bag (QCB) parameterization in r-space. Details of this parameterization are given in ref. [82]. In terms of S- and D- waves (respectively $l=0;2$) one has

$$u_l(r) = N \begin{cases} b_1 u_1^{[1]}(r) + b_2 u_2^{[2]}(r), r \leq b; \\ b_1 u_1^{ext}(r), r > b, \end{cases}$$

where $N$ is the normalization factor; $u_1^{ext}(r)$ are the DWF derived from the assumed external potential; $u_1^{ext}(r)$ may be parameterized in terms of Yukawa functions

$$u_0^{ext}(r) = \sum_{j=1}^m C_j \exp(-m_j r);$$

$$u_2^{ext}(r) = \sum_{j=1}^m D_j \exp(-m_j r) \left(1 + \frac{3}{m_j r} + \frac{3}{(m_j r)^2}\right).$$

In [91] are present the QCB model parameters for $b=1.2$ and $1.35 \text{ fm}$ that were selected as representative solutions. In Ref. [162] are used in calculations the DWF in the Hulthen form

$$\phi_d(r) = \sqrt{\frac{\alpha \beta (\alpha + \beta)}{2\pi(\alpha - \beta)^2}} \frac{e^{-\alpha r} - e^{-\beta r}}{r},$$

where $\alpha=0.23 \text{ fm}^{-1}$; $\beta=1.61 \text{ fm}^{-1}$. It is necessary for receiving the deuteron formation rate

$$A = \frac{3\pi^3}{r_0 \nu \tau} \int_0^\infty r |\phi_d(r)|^2 \exp \left(-\frac{r^2}{4r_0^2}\right) \text{erfi}(ar) dr,$$
\[ a = \frac{v^\tau}{2r_0\sqrt{r_0^2 + v^2r^2}}; \quad erf(i) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt. \]

It is also possible to remember also the parameterization of function received for Moscow NN model \[97\] \((N=24)\)

\[ u(r) = r \sum_{i=1}^{N} a_i \exp\left(-\alpha_i r^2\right), \]
\[ w(r) = r^3 \sum_{i=1}^{N} b_i \exp\left(-\beta_i r^2\right). \]

6. “Improved” analytical forms of DWF

In some papers the above table of values and coefficients for the parameterization \[5\] \[157\] and calculated for him DWF. It is about papers \[85\], \[87\], \[91\], where there are, although not insignificant, the obvious knots to DWF near the origin! In addition, there is an obvious failure to comply with mandatory conditions for the summation of the coefficients

\[ \sum_{j=1}^{N} C_j = 0; \quad \sum_{j=1}^{N} D_j = 0. \]

In the comparative Table 3 shows the results of the summation of the coefficients of these works.

For my numerical calculations the resulting coefficients are shown Table 4. The following is the corresponding Fig. 2 where there are knots for WDF.

**Table 3. The results of the summation of the coefficients**

| Model      | \(\Sigma C_i\)   | \(\Sigma D_i\)   |
|------------|-------------------|-------------------|
| \(\psi_{\text{MM}}\) [85] | -0.00002  | -0.000055  |
| \(\psi_{\text{SL}}\) [85] | 0.00008  | -0.000325  |
| \(\psi_{\text{BL}}\) [85] | -0.00003  | -0.000565  |
| \(\psi_{\text{BS}}\) [85] | 0.00018  | 0.000335  |
| OBEPA [91] | -5.326E-06 | -6.499E-08 |
| OBEPB [91] | -2.471E-06 | 1.799E-08  |
| OBEPB [91] | 5.171E-06  | 4.710E-07  |

**Table 4. Coefficients \(D_i\) for OBEPC and \(\psi_{\text{LS}}^{\text{BB}}\)**

| Model      | \(\psi_{\text{BS}}^{\text{BB}}\) |
|------------|---------------------------------|
| OBEPC      | -0.6333000560135323454  | 14.8969971094926  |
|            | 1.5303945785856441734   | -11.2303580952213 |
|            | -0.9017034635680319384  | 2.835625985728753  |
Fig. 2. “Improved” DWF for OBEPC and ψ_{LS}^{6B}.

So, more accurately calculate the coefficients for the relevant parameterizations of DWF.

7. New analytical forms of DWF

In 2000-x years are new analytical forms of deuteron wave function. Except the mentioned parametrization, in literature there is one more analytical form [163] for DWFs:

\[
\begin{align*}
    u(r) &= \sum_{i=1}^{N} A_i \exp(-a_i r^2), \\
    w(r) &= r^2 \sum_{i=1}^{N} B_i \exp(-b_i r^2).
\end{align*}
\]

This parametrization was used [118] for Nijmegen potentials groups (NijmI, NijmII, Nijm93 and Reid93). Thus value \(N=13\).

For explanation D-state of deuteron and correct asymptotic behavior are received nonrelativistic deuteron wave function [121]:

\[
\begin{align*}
    u(r) &= \frac{N}{\sqrt{4\pi}} \sum_{k=1}^{n_u} C_k \exp(-\alpha_k r), \\
    u(r) &= \frac{N}{\sqrt{4\pi}} \rho \sum_{k=1}^{n_u} D_k \exp(-\beta_k r) \left(1 + \frac{3}{\beta_k r} + \frac{3}{(\beta_k r)^2}\right), \\
    N &= \sqrt{\sum_{k,j=1}^{n_u} C_k C_j \frac{1}{\alpha_k + \alpha_j} + \rho^2 \sum_{k,j=1}^{n_u} D_k D_j \frac{1}{\beta_k + \beta_j}}.
\end{align*}
\]

where \(\alpha_i, \beta_i, C_i, D_i, N, \rho\) are the real model parameters; \(n_u = n_w = 3\). The form of asymptotics in the limit \(r \to 0\) was assumed as: \(u(r) \to r^2; w(r) \to r^3\). The set of parameters has to meet conditions

\[
\begin{align*}
    \sum_k C_k = 0; \sum_k C_k \alpha_k = 0; \sum_k D_k = 0; \sum_k D_k \frac{1}{\beta_k^2} = 0.
\end{align*}
\]

In the limit \(r \to \infty\) the deuteron wave functions must have such known asymptotic form

\[
\begin{align*}
    u(r) &\to e^{-\alpha r}, \\
    w(r) &\to e^{-\alpha r} \left(1 + \frac{3}{\alpha r} + \frac{3}{(\alpha r)^2}\right),
\end{align*}
\]

where \(\alpha = \sqrt{M \varepsilon}/\hbar = 0.2316\text{fm}^{-1}; \varepsilon=2.2245\text{MeV}\) is the deuteron binding energy. Then after the application of the condition of equations (10) to the deuteron wave functions in forms (9) leads to the relations for model parameters \(\alpha_1 = \beta_1 = \alpha\) and 

The charge and quadrupole deuteron form factors and the structure function are defined by values of parameters of model. By using this wave function has calculated the differential cross section of the elastic deuteron-nucleus...
\[ u(r \to 0) = r^{10/4} \frac{\Gamma(\frac{1}{4})}{231 \sqrt{\pi}} \sum_j \frac{C_j}{m_j^{3/2}}, \]
\[ u(r \to \infty) = r^{11/4} \frac{231^{3/4} \Gamma(3/4)}{231 \sqrt{\pi}} e^{-\alpha r}, \]

where \( \Gamma(x) \), \( K_v(x) \) are Euler and McDonald functions; \( \alpha = 15/4 \).

If in addition to the conditions \( \sum_{j=1}^{N} C_j = 0 \) for the S-wave function the condition is imposed \( \sum_{j=1}^{N} C_j m_j^2 = 0 \), then in the vicinity of zero the wave function has the following form
\[ u_0(r) = r + ar^3; u_0^\prime(0) = 0. \]

In Ref. [167] research is conducted for pion electromagnetic structure without asymptotic decomposition. It was used the following wave function of Coulomb interaction at small distances and linear confinement
\[ u(r) = N_T \exp \left( -ar^{3/2} - \beta r \right), \] where \( \alpha = \frac{2}{3} \sqrt{2aM} \); \( \beta = bM \); \( a \) and \( b \) are parameters of linear and Coulomb parts of potential respectively.

The paper [168] contains description of spin-dependent observables in elastic proton-deuteron scattering on the basis of a generalized diffraction model. This would have parameterization for DWFs in coordinate space. To parameterize the DWFs under consideration are employed used the sum of Gaussian functions with account the behavior of the wave functions at \( r=0 \)
\[ u(r) = r \sum_{j=1}^{m} C_{0j} \exp(-A_{0j} r^2), \] \[ w(r) = r^3 \sum_{j=1}^{m} C_{2j} \exp(-A_{2j} r^2), \]

where \( m=5 \). The functions fitted on the basis of numerical values for the CD-Bonn and dressed dibaryon model (DBM) functions in the intervals 0-20 fm with a step of 0.1 fm.

In [130] are specified results of calculations for the deuteron quadrupole momentum \( Q \) by using experimental phase shifts for partial-wave analysis of GWU (George Washington University) [169] and Nijmegen [170]. Also the deuteron parameters (deuteron quadrupole moment \( Q \), the deuteron asymptotic D/S and the deuteron asymptotic normalization constant \( A_N \)) and correlation between them for the group potentials is studied. This dependence is represent in the form \( Q/\eta = a + b A_N^2 \), where \( a = 3.92464 \text{fm}^2 \); \( b = 8.71829 \text{fm}^3 \).

Influence of the D- state component of DWF [65] on the application of the Trojan horse method it was shown in [132].

Parametrization formulas in a form according to [157] are applied approximations of DWF for potential charge-dependent Bonn (CD-Bonn) [2], model FSS2 with the Coulomb exchange kernel [9], and calculated in three different schemes (isospin basis and particle basis with Coulomb off or Coulomb on) and fss2 baryon-baryon interaction [12] at \( N=11 \), and also for MT model [124] when \( N_S=16; N_D=12 \).

Parametrization Dubovichenko [118] is improved in works [171, 172, 173, 174]. Minimization of values \( \chi^2 \) is carried out \( 10^{-4} \). Using deuteron wave functions in coordinate and space representations, are designed a component of a tensor of sensitivity polarization of deuterons \( T_{20} \) [175] polarization transmission \( K_0 \), tensor analyzing power \( A_y \) and tensor-tensor transmission of polarization \( K_y \) [176]. The obtained outcomes are compared to the published experimental and theoretical outcomes.

For deuteron wave function in configuration representation for potential Argonne v18 are designed numerical coefficients of analytical forms [177]
\[
\begin{cases}
  u(r) = \sum_{i=1}^{20} A_i \exp(-a_i r^3), \\
  w(r) = r^2 \sum_{i=1}^{20} B_i \exp(-b_i r^3).
\end{cases}
\]
The coefficients of the four approximating dependencies for the numerical values of DWFs for four realistic phenomenological potentials Nijmegen group have been numerically calculated. The analytical forms are chosen as the product of the power function \( r^n \) for the sum of exponential terms \[178\]:

\[
\begin{align*}
    u(r) &= r \sum_{i=1}^{N} A_i \exp(-a_i r^2), \\
    w(r) &= r \sum_{i=1}^{N} B_i \exp(-b_i r^2), \\
    u(r) &= r^2 \sum_{i=1}^{N} A_i \exp(-a_i r^3), \\
    w(r) &= r^2 \sum_{i=1}^{N} B_i \exp(-b_i r^3).
\end{align*}
\]

The behavior of the value \( \chi^2 \) depending on the number of expansion terms \( N_i \) has been studied. With the account of the minimum values of \( \chi^2 \) for these forms we have built DWFs in the coordinate space, which do not contain superfluous knots. The calculated parameters of the deuteron are in good agreement with theoretical and experimental results. For DWFs in coordinate and momentum space it is calculated such polarization characteristics: the tensor polarization \[179\] (values \( t_{20}(p) \), \( t_{21}(p) \), \( t_{22}(p) \)) in the range of 0-7 pulse \( \text{fm}^{-1} \). The value of \( t_{20}(p) \) for potentials Nijmegen group in good agreement with literature results for other potential nucleon-nucleon of models and with experimental data's. The results of the deuteron tensor polarization \( t_{ij}(p) \) give some information about the electromagnetic structure of the deuteron. And when known tensor analyzing power it is possible to calculate the differential cross section of double scattering.

To solve the system of associated Schrödinger equations that describe the radial DWF \( u \) and \( w \)

\[
\begin{align*}
    u'' - \alpha^2 u &= f(r), \\
    w'' - (\alpha^2 + \frac{6}{r^2}) w &= g(r)
\end{align*}
\]

parameterizations were proposed back in 1955 \[28\]:

\[
\begin{align*}
    f(r) &= \sum_{n=0}^{\infty} c_n \psi_{1n}(r), \\
    g(r) &= \sum_{n=0}^{\infty} d_n \psi_{1n}(r).
\end{align*}
\]

They can be generalized for the DWF approximation as such analytical forms through Laguerre functions \[28\]:

\[
\begin{align*}
    u(r) &= \sum_{n=0}^{11} A_n \psi_{3n}(r), \\
    w(r) &= \sum_{n=0}^{11} B_n \psi_{3n}(r),
\end{align*}
\]

where \( \psi_{3n}(r) \) - Laguerre functions \((n=0,1,2,3,\ldots)\):

\[
\psi_{3n}(r) = \frac{2\alpha\sqrt{2\alpha}}{n!\sqrt{(n+1)(n+2)}} \exp(\alpha r) \frac{d^n}{dr^n} \left(r^{n+2} \exp(-2\alpha r)\right),
\]

\[
\psi_{30} = \sqrt{\alpha} \exp(-\alpha r) (2\alpha r),
\]

\[
\psi_{31} = 2\sqrt{\frac{\alpha}{3}} \exp(-\alpha r) \left(3\alpha r - 2\alpha^2 r^2\right),
\]

\[
\psi_{32} = 2\sqrt{\frac{2\alpha}{3}} \exp(-\alpha r) \left(3\alpha r - 4\alpha^2 r^2 + \alpha^3 r^3\right),
\]
\[ \psi_{33} = 2\sqrt{10\alpha} \exp(-\alpha r) \left( \alpha r - 2\alpha^2 r^2 + \alpha^3 r^3 - \frac{2}{15} \alpha^4 r^4 \right), \]

\[ \psi_{34} = \sqrt{\frac{5\alpha}{3}} \exp(-\alpha r) \left( 6\alpha r - 16\alpha^2 r^2 + 12\alpha^3 r^3 - \frac{16}{5} \alpha^4 r^4 + \frac{4}{15} \alpha^5 r^5 \right), \]

\[ \psi_{35} = 2\sqrt{\frac{7\alpha}{3}} \exp(-\alpha r) \left( 3\alpha r - 10\alpha^2 r^2 + 10\alpha^3 r^3 - 4\alpha^4 r^4 + \frac{2}{3} \alpha^5 r^5 - \frac{4}{105} \alpha^6 r^6 \right). \]

The coefficients of analytical forms through Laguerre functions for the deuteron wave function in coordinate space for NijmI, NijmII, Nijm93, Reid93 and Argonne v18 potentials have been numerically calculated in [180]. Near the beginning of coordinates there are some small oscillations for DWFs, but despite of it designed static parameters well coincide with original values.

Parameterizations [28] and [163] can be generalized for the DWF approximation as such analytical forms:

\[
\begin{align*}
\begin{cases}
    u(r) = r^A \sum_{i=1}^{N} A_i \exp(-a_i r^3), \\
    w(r) = r^B \sum_{i=1}^{N} B_i \exp(-b_i r^3).
\end{cases}
\] \tag{11}
\]

Given \(N=11\), search for an index of function of a degree \(r^n\) has been carried out, appearing as a factor before the sums of exponential terms of the analytical form [11]. Best values appeared to be \(n=1.47\) and \(n=1.01\) for \(u(r)\) and \(w(r)\) accordingly. Hence, the factors before the sums in (11) can be chosen as \(r^{3/2}\) and \(r^1\) [181]:

\[
\begin{align*}
\begin{cases}
    u(r) = r^{3/2} \sum_{i=1}^{N} A_i \exp(-a_i r^3), \\
    w(r) = r \sum_{i=1}^{N} B_i \exp(-b_i r^3).
\end{cases}
\] \tag{12}
\]

Despite cumbersome and time-consuming calculations and minimizations of \(\chi^2\) (to the value smaller than 10\(^{-7}\)), it was necessary to approximate numerical values of DWF, the arrays of numbers of which made up 8394 values in an interval \(r=0-25\) fm for potentials Nijm, NijmII, Nijm93 and Reid93 [14], and 15002 values in an interval \(r=0-15\) fm for potential Argonne v18 [17].

The accuracy of parametrization (12) is characterized by:

\[ \chi^2 = \frac{1}{n-p} \sum_{i=1}^{N} \left( y_i - f(x_i; a_1, a_2, ..., a_p) \right)^2, \]

where \(n\) - the number of points of the array \(y_i\) of the numerical values of DWF in the coordinate space; \(f\) - approximating function of \(u\) (or \(w\)) according to the formulas (12); \(a_1, a_2, ..., a_p\) - parameters; \(p\) - the number of parameters (coefficients in the sums of formulas (12)). Hence, \(\chi^2\) is determined not only by the shape of the approximating function \(f\), but also by the number of the selected parameters.

The approximation can be made on the whole interval, or divided into a few distinct sites: around the origin in the maximum and descending function. But this complicates further generalization for the form of the wave function.

Coefficients and DWFs (12) for NijmI, NijmII, Nijm93, Reid93 and Argonne v18 potentials it is resulted in works [182] [183]. A detailed comparison of the obtained values of \(t_{20}(p)\) (the scattering angle \(\theta=70^0\)) for these potentials with the up-to-date experimental data of JLAB t20 [184] [185] and BLAST [179] [180] collaborations. There is a good agreement is for the momentas \(p=1-4\) fm\(^{-1}\).

If we consider normalization \(\int (u^2 + w^2)dr = 1\) for DWFs (11), we can write this condition using the corresponding coefficients as

\[
\sum_{i=1}^{N} \left( \frac{2^{2/3} \Gamma \left[ \frac{4}{3} \right]}{12a_i^{4/3}} A_i^2 + \frac{B_i^2}{6b_i} \right) = 1.
\]
In this paper it has been used parameterization (12) and it is made minimization of quantity of the designed coefficients. Dependence $\chi^2$ from the number of expansion terms $N$ is resulted in Tables 5 and 6 separately for functions $u(r)$ and $w(r)$. At increase for value $N$ reduction of size $\chi^2$ for $u(r)$ (potential Reid93) is precisely shown in Fig. 3. The coefficients of new analytical forms for DWF in coordinate space for NijmI, NijmII, Nijm93, Reid93 and Argonne v18 potentials have been numerically calculated (Tables 7-11). The obtained wave functions (Fig. 4 and 5) do not contain any superfluous knots.

Based on the known DWFs (12) and them coefficients (Tables 7-11) one can calculate the deuteron properties (Table 12): deuteron radius $r_m$, the quadrupole moment $Q_d$, the $D$-state probability $P_D$ and the magnetic moment $\mu_d$. They are in good agreement with the theoretical (Table 1) and experimental (Table 2) data.
Fig. 4. Deuteron wave function \( u(r) \)

Fig. 5. Deuteron wave function \( w(r) \)

Table 5. Values \( \chi^2 \) for \( u(r) \)

| \( N \) | NijmI | NijmII | Nijm93 | Reid93 | Av18 |
|-------|-------|--------|--------|--------|------|
| 3     | 2.61E-04 | 3.78E-04 | 2.64E-04 | 2.85E-04 | 2.24E-05 |
| 4     | 1.13E-04 | 2.13E-04 | 6.66E-05 | 1.16E-04 | 3.26E-06 |
| 5     | 1.76E-05 | 1.92E-04 | 3.92E-05 | 9.47E-05 | 6.45E-07 |
| 6     | 1.18E-05 | 1.92E-04 | 3.93E-05 | 9.49E-05 | 4.27E-07 |
| 7     | 1.08E-05 | 1.93E-04 | 3.61E-05 | 9.51E-05 | 4.24E-07 |
| 8     | 1.27E-06 | 1.91E-04 | 3.62E-05 | 9.31E-05 | 4.25E-07 |
| 9     | 1.30E-06 | 1.94E-04 | 3.63E-05 | 9.33E-05 | 4.10E-07 |
| 10    | 8.03E-06 | 1.92E-04 | 3.64E-05 | 9.36E-05 | 4.03E-07 |
| 11    | 7.01E-06 | 1.93E-04 | 3.64E-05 | 9.38E-05 | 4.04E-07 |

Table 6. Values \( \chi^2 \) for \( w(r) \)

| \( N \) | NijmI | NijmII | Nijm93 | Reid93 | Av18 |
|-------|-------|--------|--------|--------|------|
| 3     | 2.56E-05 | 2.72E-05 | 3.10E-05 | 2.82E-05 | 4.46E-06 |
| 4     | 2.81E-06 | 2.73E-06 | 3.45E-06 | 3.20E-06 | 3.58E-06 |
| 5     | 8.46E-07 | 4.94E-07 | 7.50E-07 | 1.01E-06 | 3.58E-06 |
| 6     | 6.52E-07 | 2.77E-07 | 4.49E-07 | 7.96E-07 | 3.58E-06 |
| 7     | 6.53E-07 | 2.68E-07 | 4.29E-07 | 7.97E-07 | 4.30E-07 |
| 8     | 6.52E-07 | 2.59E-07 | 4.20E-07 | 7.99E-07 | 4.31E-07 |
| 9     | 6.56E-07 | 2.58E-07 | 4.21E-07 | 8.00E-07 | 4.32E-07 |
| 10    | 6.58E-07 | 2.60E-07 | 4.19E-07 | 7.75E-07 | 4.32E-07 |
| 11    | 6.84E-07 | 2.61E-07 | 4.23E-07 | 7.86E-07 | 4.31E-07 |

Table 7. Coefficients \( A_i, a_i, B_i, b_i \) (NijmI)
| i | $A_i$ | $a_i$ | $B_i$ | $b_i$ |
|---|---|---|---|---|
| 1 | 0.00065826309 | 0.00023679107 | -0.15255613149 | 3.97342104241 |
| 2 | 0.04005454537 | 0.01185654423 | 0.00081779640 | 0.00060963929 |
| 3 | 0.01595367314 | 0.00350795838 | 0.04767548848 | 0.22931527246 |
| 4 | 0.08430196890 | 0.03911783132 | 0.05400988310 | 0.06026407442 |
| 5 | 0.15463267892 | 0.12780158776 | 0.00599092790 | 0.00361063044 |
| 6 | 0.00466297503 | 0.00098105003 | 0.03019488106 | 0.22931528067 |
| 7 | 0.02431014420 | 4.49214201019 | 0.02219993378 | 0.01578590745 |

Table 8. Coefficients $A_i$, $a_i$, $B_i$, $b_i$ (NijmII)

| i | $A_i$ | $a_i$ | $B_i$ | $b_i$ |
|---|---|---|---|---|
| 1 | 0.00085796423 | 0.00026671465 | -0.16642454661 | 4.70854300660 |
| 2 | 0.05148122494 | 0.01601293924 | 0.00093664870 | 0.00066353652 |
| 3 | 0.00615093450 | 0.00118853942 | 0.01905670791 | 0.00400272658 |
| 4 | 0.11879048059 | 0.05800341799 | 0.00671655150 | 0.25657778517 |
| 5 | 0.09493719253 | 0.25110911883 | 0.05706931441 | 0.06798490110 |
| 6 | 0.09222467415 | 0.25162731987 | 0.01956153732 | 0.25657784999 |
| 7 | 0.08013045048 | 0.25146410450 | 0.02445879052 | 0.01769592277 |
| 8 | 0.02054925493 | 0.00451468381 | 0.01976865474 | 0.25657778490 |
| 9 | 0.00466297503 | 0.00098105003 | 0.03019488106 | 0.22931528067 |

Table 9. Coefficients $A_i$, $a_i$, $B_i$, $b_i$ (Nijm93)

| i | $A_i$ | $a_i$ | $B_i$ | $b_i$ |
|---|---|---|---|---|
| 1 | 0.00098878586 | 0.00028491272 | -0.16660171842 | 5.01389130303 |
| 2 | 0.13571076271 | 0.07486587795 | 0.00042682696 | 0.00042106442 |
| 3 | 0.12400573946 | 0.32247840206 | 0.08172183510 | 0.26642919122 |
| 4 | 0.14274952979 | 0.32247840078 | -0.13944102365 | 0.19870727465 |
| 5 | 0.06212705650 | 0.02002538042 | 0.09969921461 | 0.11841769371 |
| 6 | 0.00719613810 | 0.00132675713 | 0.05422237632 | 0.26560362260 |
| 7 | 0.02453978059 | 0.00531708637 | 0.03879995997 | 0.26603771790 |
| 8 | 0.00329896732 | 0.00224533303 | 0.00905020138 | 0.03188653927 |
| 9 | 0.01264941157 | 0.00905020138 | 0.03368130195 | 0.03188653927 |

Table 10. Coefficients $A_i$, $a_i$, $B_i$, $b_i$ (Reid93)

| i | $A_i$ | $a_i$ | $B_i$ | $b_i$ |
|---|---|---|---|---|
| 10 | 0.03368130195 | 0.03188653927 | 0.03368130195 | 0.03188653927 |

Table 11. Coefficients $A_i$, $a_i$, $B_i$, $b_i$ (Av18)

Table 12. Deuteron properties

8. Conclusions

Static properties of the deuteron ($E_d$, $r_m$, $Q_d$, $P_D$, $\eta$, $A_S$), obtained by DWFs for potential models, have been chronologically systematized. The presence or absence of knots near the origin of coordinates for the radial DWF have been shown. The forms, methods of obtaining and asymptotic behaviors of analytic forms for DWFs in the coordinate space have been analyzed.

Parameterization in the form of (12) has been used and the number of expansion coefficients has been minimized. Dependence of $\chi^2$ on the number of expansion terms $N$ parameterization (12) is shown separately for the functions $u(r)$.
| \( i \) | \( A_i \) | \( a_i \) | \( B_i \) | \( b_i \) |
|---|---|---|---|---|
| 1 | 0.00085859852 | 0.00026661728 | -0.15112264940 | 5.27032534000 |
| 2 | 0.02106750054 | 0.00457697258 | 0.00032452355 | 0.0036454826 |
| 3 | 0.00620290953 | 0.00119165642 | -0.11845656993 | 0.1569022137 |
| 4 | 0.11117085753 | 0.25958967300 | 0.05880984638 | 0.21866245485 |
| 5 | 0.12121298131 | 0.06065662499 | 0.05571966351 | 0.21983692771 |
| 6 | 0.09712838391 | 0.25946221474 | 0.02824394399 | 0.02520241662 |
| 7 | 0.05353538391 | 0.01656444102 | 0.04817589390 | 0.2194235873 |
| 8 | 0.05686488199 | 0.25952108904 | 0.0025375036 | 0.00184332786 |
| 9 | 0.09143245521 | 0.09410956021 | 0.09410956021 | 0.09410956021 |
| 10 | 0.0099601651 | 0.00723824613 | 0.00723824613 | 0.00723824613 |

| \( i \) | \( A_i \) | \( a_i \) | \( B_i \) | \( b_i \) |
|---|---|---|---|---|
| 1 | -2.31737065809 | 0.35274596179 | -0.16442140495 | 4.27551981801 |
| 2 | 0.02659110800 | 0.00732058085 | 0.02869362316 | 0.0294360502 |
| 3 | -0.28877337807 | 5.38000986025 | 0.00074392415 | 0.00074392415 |
| 4 | 0.99922786191 | 0.36067988379 | 0.05707754763 | 0.09089766773 |
| 5 | 0.11094070754 | 0.06921749879 | 0.01167238661 | 0.00953758791 |
| 6 | 0.0107521733 | 0.00225041184 | 0.00370587438 | 0.0028124630 |
| 7 | 0.00274077964 | 0.000549265934 | 0.08509685731 | 0.30274669392 |
| 8 | 0.65409540449 | 0.26723780768 | 0.41270669658 | 0.41270669658 |
| 9 | 0.99185931871 | 0.41270669658 | 0.41270669658 | 0.41270669658 |
| 10 | 0.05693893648 | 0.02247865903 | 0.02247865903 | 0.02247865903 |

| Potential | \( r_m \) (fm) | \( Q_d \) (fm\(^2\)) | \( P_D \) (%) | \( \mu_d \) |
|---|---|---|---|---|
| NijmI | 1.96616 | 0.271372 | 5.65618 | 0.847577 |
| NijmII | 1.96711 | 0.270014 | 5.62972 | 0.84727 |
| Nijm93 | 1.96543 | 0.270362 | 5.74951 | 0.847045 |
| Reid93 | 1.96819 | 0.270162 | 5.69023 | 0.847383 |
| Argonne v18 | 1.95471 | 0.268201 | 5.75946 | 0.846988 |
and \( w(r) \). The optimum is \( N = 7-10 \). The resulting wave functions do not contain any extra knots. Calculations have been done for realistic phenomenological potentials NijmI, NijmII, Nijm93, Reid93 and Argonne v18. What is more, analytical forms of DWF by such authors as ertov, Mathelitsch, Moravcsik and Machleidt have been “improved”.

The resulting DWFs for the group of potential models can be applied to calculate polarization characteristics of the deuteron (tensor polarization \( t_{20} \), sensitivity tensor component to polarization of deuterons \( T_{20} \), polarization transmission \( K_0 \) and tensor analyzing power \( A_{yy} \), etc. \([176]\)). The results will allow studying the deuteron electromagnetic structure, its form-factors and differential cross section of double scattering in more detail in future.

References

[1] W.W. Buck, F. Gross, \textit{Phys. Rev. D} \textbf{20}, 2361 (1979).
[2] R. Machleidt, \textit{Phys. Rev. C} \textbf{63}, 024001 (2001).
[3] Jr.R.V. Reid, \textit{Ann. Phys. (NY)} \textbf{50}, 411 (1968).
[4] V.I. Kukulin, V.N. Pomerantsev, A. Faessler et al., \textit{Phys. Rev. C} \textbf{57}, 535 (1998).
[5] E.R. Arriola, M.P. Valderrama, \textit{Eur. Phys. J. A} \textbf{31}, 549 (2007).
[6] I. Haysak and V. Zhaba, \textit{Visnyk Lviv Univ. Ser. Phys.} \textbf{44}, 8 (2009).
[7] I.I. Haysak and V.I. Zhaba, \textit{Uzhhorod Univ. Scien. Herald. Ser. Phys.} \textbf{36}, 100 (2014).
[8] V.S. Bokhinyuk, V.I. Zhaba, O.M. Parlag, \textit{Uzhhorod Univ. Scien. Herald. Ser. Phys.} \textbf{31}, 111 (2012).
[9] Y. Fujiwara, T. Fujita, M. Kohno et al., \textit{Phys. Rev. C} \textbf{65}, 014002 (2001).
[10] M. Garcon, J.W. van Orden, \textit{Advanc. Nucl. Phys.} \textbf{26}, 293 (2001).
[11] S. Veerasamy and W.N. Polyzou, \textit{Phys. Rev. C} \textbf{84}, 034003 (2011).
[12] K. Fukukawa, M. Baldo, G.F. Burgio et al., \textit{Phys. Rev. C} \textbf{92}, 065802 (2015).
[13] B. Loiseau, L. Mathelitsch, W. Plessas, \textit{Nuovo Cimento A} \textbf{97}, 77 (1987).
[14] F. Gross, A. Stadler, \textit{Phys. Rev C} \textbf{82}, 034004 (2010).
[15] V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen, J.J. de Swart, \textit{Phys. Rev. C} \textbf{49}, 2950 (1994).
[16] J.J. de Swart, R.A.M.M. Klomp, M.C.M. Rentmeester, Th.A. Rijken, \textit{Few-Body Syst. Suppl.} \textbf{8}, 438 (1995).
[17] R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, \textit{Phys. Rev. C} \textbf{51}, 38 (1995).
[18] M. Lacombe, B. Loiseau, J.M. Richard et al., \textit{Phys. Rev. C} \textbf{21}, 861 (1980).
[19] E. Epelbaum, W. Glockle, U.-G. Meiβner, \textit{Nucl. Phys. A} \textbf{747}, 362 (2005).
[20] C. Downum, J.R. Stone, T. Barnes et al., \textit{AIP Conf. Proc.} \textbf{1257}, 538 (2010).
[21] V.P. Ladygin and N.B. Ladygina, \textit{J. Phys. G: Nucl. Part. Phys.} \textbf{23}, 847 (1997).
[22] J.M. Blatt, V.F. Weisskopf, \textit{Theoretical nuclear physics} (Wiley, New York, 1958).
[23] G.E. Brown, A.D. Jackson, \textit{The nucleon-nucleon interaction} (North-Holland, Amsterdam, 1976).
[24] M. Naghdhi, \textit{Phys. Part. Nucl. Lett.} \textbf{11}, 410 (2014).
[25] H.A. Bethe, \textit{Phys. Rev.} \textbf{57}, 390 (1940).
[26] W. Rarita, J. Schwinger, \textit{Phys. Rev.} \textbf{59}, 436 (1941).
[27] M. Matsumoto, W. Watari, \textit{Prog. Theor. Phys.} \textbf{12}, 503 (1954).
[28] F. Cap, W. Gröbner, *Nuovo Cimento* **1**, 1211 (1955).
[29] S. Gartenhaus, *Phys. Rev.* **100**, 900 (1955).
[30] J. Iwadare, S. Otsuki, R. Tamagaki, W. Watari, *Prog. Theor. Phys.* **16**, 455 (1956).
[31] H.P. Noyes, S.P. Pandya, *Phys. Rev.* **102**, 269 (1956).
[32] M.J. Moravcsik, *Nucl. Phys.* **7**, 113 (1958).
[33] K.V. Laurikainen, O. Varho, *Nucl. Phys.* **12**, 606 (1959).
[34] T. Hamada, *Prog. Theor. Phys.* **24**, 126 (1960).
[35] M. Matsumoto, *Prog. Theor. Phys.* **23**, 597 (1960).
[36] T. Hamada, *Prog. Theor. Phys.* **25**, 247 (1961).
[37] T. Hamada, I.D. Johnston, *Nucl. Phys.* **34**, 382 (1962).
[38] G. Bialkowski, *Nuovo Cimento* **29**, 201 (1963).
[39] H. Kottler, K.L. Kowalski, *Nucl. Phys.* **53**, 334 (1964).
[40] F. Tabakin, *Ann. Phys. (NY)* **30**, 51 (1964).
[41] I.J. McGee, *Phys. Rev.* **151**, 772 (1966).
[42] H. Eikemeier, H.H. Hackenbroich, *Z. Physik* **195**, 412 (1966).
[43] J.P. Elliott, A.D. Jackson, *Nucl. Phys. A* **121**, 279 (1968).
[44] C.W. Nestor Jr, K.T.R. Davies, S.J. Krieger, M. Baranger, *Nucl. Phys. A* **113**, 14 (1968).
[45] K. Erkelenz, K. Holinde, K. Bleuler, *Nucl. Phys. A* **139**, 308 (1969).
[46] J.W. Humberston, J.B.G. Wallace, *Nucl. Phys. A* **141**, 362 (1970).
[47] R.W. Stagat, F. Rieme, A.E.S. Green, *Phys. Rev. C* **3**, 552 (1971).
[48] K. Holinde, K. Erkelenz, R. Alzetta, *Nucl. Phys. A* **194**, 161 (1972).
[49] R. de Tourreil, D.W.L. Sprung, *Nucl. Phys. A* **201**, 193 (1973).
[50] J.P. Vary, *Phys. Rev. C* **7**, 521 (1973).
[51] H. Arenhovel, H.G. Miller, *Z. Physik* **266**, 13 (1974).
[52] W. Fabian, H. Arenhovel, H.G. Miller, *Z. Physik* **271**, 93 (1974).
[53] S.C. Pieper, *Phys. Rev. C* **9**, 883 (1974).
[54] I.R. Afnan, J.M. Read, *Phys. Rev. C* **12**, 293 (1975).
[55] F. Coester, A. Ostebeet, *Phys. Rev. C* **11**, 1836 (1975).
[56] K. Holinde, R. Machleidt, *Nucl. Phys. A* **247**, 495 (1975).
[57] A.D. Jackson, D.O. Riska, B. Verwest, *Nucl. Phys. A* **249**, 397 (1975).
[58] M.M. Nagels, T.A. Rijken, J.J. de Swart, *Phys. Rev. D* **12**, 744 (1975).
[59] T. Obinata, M. Wada, *Prog. Theor. Phys.* **53**, 732 (1975).
[60] R. de Tourreil, B. Rouben, D.W.L. Sprung, *Nucl. Phys. A* **242**, 445 (1975).
[61] K. Holinde, R. Machleidt, *Nucl. Phys. A* **256**, 479 (1976).
[62] K. Holinde, R. Machleidt, *Nucl. Phys. A* **256**, 497 (1976).
[63] H.J. Weber, *Nucl. Phys. A* **264**, 365 (1976).
[64] J. Weiss, Czech. *J. Phys. B* **26**, 603 (1976).
[65] R.J. Adler, T.K. Das, A.F. Filho, *Phys. Rev. C* **16**, 1231 (1977).
[66] N.J. McGurk, H. Fiedeldey, *Nucl. Phys. A* **281**, 310 (1977).
[67] L.J. Allen, H. Fiedeldey, *Few Body Syst. Nucl.* **82**, 57 (1978).
[68] L. Mathelitsch, H. F. K. Zingl, *Nuovo Cimento A* **44**, 81 (1978).
[69] M.M. Nagels, T.A. Rijken, J.J. de Swart, *Phys. Rev. D* **17**, 768 (1978).
[70] L.J. Allen, H. Fiedeldey, *Phys. Rev. C* **19**, 641 (1979).
[71] M.M. Nagels, T.A. Rijken, J.J. de Swart, *Phys. Rev. D* **20**, 1633 (1979).
[72] R.G. Arnold, C.E. Carlson, F. Gross, *Phys. Rev. C* **21**, 1426 (1980).
[73] G.H. Lamot, N. Giraud, C. Fayard, *Nuovo Cimento A* **57**, 445 (1980).
[74] Y. Koike, Y. Taniguchi, M. Sawada, J. Sanada, *Prog. Theor. Phys.* **66**, 1899 (1981).
[75] I.E. Lagaris, V.R. Pandharipande, *Nucl. Phys. A* **359**, 331 (1981).
[76] J. Haidenbauer, W. Plessas, *Phys. Rev. C* **30**, 1822 (1984).
[77] V.I. Kukulin, V.N. Pomerantsev, V.M. Krasnopol’sky, P.B. Sazonov, *Phys. Lett. B* **135**, 20 (1984).
[78] S.S. Mehdi, V.K. Gupta, *Pramana* **22**, 497 (1984).
[79] R.B. Wiringa, R.A. Smith, T.L. Ainsworth, *Phys. Rev. C* **29**, 1207 (1984).
[80] V.M. Krasnopol’sky, V.I. Kukulin, V.N. Pomerantsev, P.B. Sazonov, *Phys. Lett. B* **165**, 7 (1985).
[81] J. Haidenbauer, Y. Koike, W. Plessas, *Phys. Rev. C* **33**, 439 (1986).
[82] Yu.S. Kalashnikova, I.M. Narodetskii, A.I. Veselov, *Z. Phys. A* **323**, 205 (1986).
[83] M.W. Kermode, S.G. Cooper, S. Klarsfeld, *Phys. Lett. B* **174**, 357 (1986).
[84] M. Beyer, H.J. Weber, *Phys. Rev. C* **35**, 14 (1987).
[85] A. Certov, L. Mathelitsch, M. J. Moravesik, *Phys. Rev. C* **36**, 2040 (1987).
[86] H. Ito, A. Faessler, *Nucl. Phys. A* **470**, 626 (1987).
[87] R. Machleidt, K. Holinde, Ch. Elster, *Phys. Rep.* **149**, 1 (1987).
[88] S. Righi, M. Rosa-Clot, *Z. Phys. A* **326**, 163 (1987).
[89] M.M. Mustafa, E.S. Zahran, *Phys. Rev. C* **38**, 2416 (1988).
[90] J.A. Oteo, *Can. J. Phys.* **66**, 478 (1988).
[91] R. Machleidt, *Adv. Nucl. Phys.* **19**, 189 (1989).
[92] H. Dijk, B.L.G. Bakker, *Nucl. Phys. A* **494**, 438 (1989).
[93] A. Buchmann, Y. Yamauchi, A. Faessler, *Prog. Part. Nucl. Phys.* **24**, 333 (1990).
[94] I.L. Grach, Yu.S. Kalashnikova, I.M. Narodetskii, J. Phys. G 16, 63 (1990).
[95] T.A. Minelli, A. Pascolini, C. Villi, Nuovo Cimento A 104, 1589 (1991).
[96] J. Haidenbauer, K. Holinde, M. B. Johnson, Phys. Rev. C 45, 2055 (1992).
[97] V.I. Kukulin, V.N. Pomepntsev, Prog. Theor. Phys. 88, 159 (1992).
[98] M.M. Mustafa, Phys. Rev. C 47, 473 (1993).
[99] J. Haidenbauer, K. Holinde, M.B. Johnson, Phys. Rev. C 48, 2190 (1993).
[100] D.W.L. Sprung, W. van Dijk, E. Wang et al., Phys. Rev. C 49, 2942 (1994).
[101] H. Kohlhoff, H.V. von Geramb, Quant. Invers. Theor. Applic. 427, 314 (1994).
[102] A. Valcarce, A. Buchmann, F. Fernandez, A. Faessler, Phys. Rev. C 50, 2246 (1994).
[103] C.F. de Araujo Jr., S.K. Adhikari, L. Tomio, J. Comput. Phys. 118, 200 (1995).
[104] M.I. Levchuk, Few-Body Syst. 19, 77 (1995).
[105] P. Doleschall, Nucl. Phys. A 602, 60 (1996).
[106] J.L. Forest, V.R. Pandharipande, S.C. Pieper et al., Phys. Rev. C 54, 646 (1996).
[107] Y. Fujiwara, C. Nakamoto, Y. Suzuki, Phys. Rev. C 54, 2180 (1996).
[108] C. Ordóñez, L. Ray, U. van Kolck, Phys. Rev. C 53, 2086 (1996).
[109] L. Jade, Phys. Rev. C 58, 96 (1998).
[110] K.A. Gridnev, V.B. Soubbotin, V.B. Stepukov et al., Eur. Phys. J. A 6, 21 (1999).
[111] E. Epelbaum, W. Glockle, Ulf-G. Meiβner, Nucl. Phys. A 671, 295 (2000).
[112] S.B. Dubovichenko, I. I. Strakovsky, Phys. Atom. Nucl. 63, 582 (2000).
[113] S.A. Zaitsev, E.I. Kramar, J. Phys. G 27, 2037 (2001).
[114] D.R. Entem, R. Machleidt, Proceedings of the 7th International Spring Seminar on Nuclear Physics, Maiori, Italy, 2001, p. 113.
[115] A. Amghar, B. Desplanques, Nucl. Phys. A 714, 502 (2003).
[116] M.M. Kaskulov, P. Grabmayr, Intern. Jour. Mod. Phys. E 12, 449 (2003).
[117] D.R. Entem, R. Machleidt, Phys. Rev. C 68, 041001 (2003).
[118] S.B. Dubovichenko, Properties of light atomic nucleus in potential cluster model (Daneker, Almaty, 2004).
[119] A.M. Shirokov, A.I. Mazur, S.A. Zaytsev et al., Phys. Rev. C 70, 044005 (2004).
[120] M.P. Valderrama, E.R. Arriola, Phys. Rev C 72, 054002 (2005).
[121] Yu.A. Berezhnoy, V.Yu. Korda, A.G. Gakh, Intern. Jour. Mod. Phys. E 14, 1073 (2005).
[122] V.A. Knyr, V.G. Neudatchin, N.A. Khokhlov, Phys. Atom. Nucl. 69, 2034 (2006).
[123] N.A. Khokhlov, V.A. Knyr, V.G. Neudatchin, Phys. Rev. C 75, 064001 (2007).
[124] A.F. Krutov, V.E. Troitsky, Phys. Rev. C 76, 017001 (2007).
[125] A.I. Mazur, A.M. Shirokov, J.P. Vary et al., Bull. Russ. Academ. Scien.: Physics 71, 754 (2007).
[126] R. Higa, M.P. Valderrama, E.R. Arriola, Phys. Rev. C 77, 034003 (2008).
[127] C.-J. Yang, Ch. Elster, D.R. Phillips, *PoS CD09:064* (2009).

[128] M.P. Valderrama, A. Nogga, E.R. Arriola, D.R. Phillips, *Eur. Phys. J. A* 36, 315 (2008).

[129] M.P. Valderrama, E.R. Arriola, *Phys. Rev. C* 79, 044001 (2009).

[130] V.A. Babenko, N.M. Petrov, *Phys. Atom. Nucl.* 74, 352 (2011).

[131] M.R. Shojaei, A.A. Rajabi, T. Karimi, *Appl. Phys. Research* 3, 122 (2011).

[132] L. Lamia, M. La Cognata, C. Spitaleri et al., *Phys. Rev. C* 85, 025805 (2012).

[133] R.N. Perez, J.E. Amaro, E.R. Arriola, *Phys. Rev. C* 88, 024002 (2013).

[134] R.M.Id Betan, *Phys. Lett. B* 730, 18 (2014).

[135] R.N. Perez, J.E. Amaro, E.R. Arriola, *Phys. Rev. C* 89, 024004 (2014).

[136] R.N. Perez, J.E. Amaro, E.R. Arriola, *Phys. Rev. C* 89, 064006 (2014).

[137] B. Rezaei, A. Dashtimoghadam, *Jour. Theor. Appl. Phys.* 8, 203 (2014).

[138] E. Epelbaum, H. Krebs, U.-G. Meißner, *Eur. Phys. J. A* 51, 53 (2015).

[139] M. Piarulli, L. Girlanda, R. Schiavilla et al., *Phys. Rev. C* 91, 024003, (2015).

[140] N. Takigawa, K. Washiyama. *Fundamentals of Nuclear Physics* (Springer Japan, Tokyo, 2017).

[141] R. Courant, D. Hilbert, *Methods of Mathematical Physics* (Interscience, New York, 1953).

[142] V.G. Neudatchin, Y.F. Smirnov, *Modern problems of optics and atomic physics* (Kiev. State. Univ., Kiev, 1974).

[143] V.I. Kukulin, V.G. Neudatchin, Y.F. Smirnov, *PEPAN* 10, 1236 (1979).

[144] S. Flügge, *Z. Phys.* 113, 587 (1939).

[145] W. Rarita, *Phys. Rev.* 74, 1799 (1948).

[146] W.G. Guindon, *Phys. Rev.* 74, 145 (1948).

[147] M. Gourdin, J.T.T. Van, *Nuovo Cimento* 14, 1051 (1959).

[148] Y. Sakamoto, T. Sasakawa, *Prog. Theor. Phys.* 21, 879 (1959).

[149] A. Donnachie, *Nucl. Phys.* 32, 637 (1962).

[150] L. Hulthen, M. Sugawara, *In Handbook der Physik* (Springer-Verlag, Berlin, 1957).

[151] G. Bialkowski, *Nuovo Cimento* 32, 1809 (1964).

[152] J.P. Elliott, A.D. Jackson, H.A. Mavromatis et al., *Nucl. Phys. A* 121, 241 (1968).

[153] C. Burnap, J.S. Levinger, B. Siebert, *Phys. Lett. B* 33, 337 (1970).

[154] M.I. Haftel, F. Tabakin, *Phys. Rev. C* 3, 921 (1971).

[155] K.E. Lassila, M.H. Hull, H.M. Ruppel et al., *Phys. Rev. 26*, 881 (1962).

[156] S. Klarsfeld, J. Martorell, D.W.L. Sprung, *Nucl. Phys. A* 352, 113 (1981).

[157] M. Lacombe, B. Loiseau, J.M. Richard et al., *Phys. Lett. B* 101, 139 (1981).

[158] M.H. Lopes, J.A. Tostevin, R.C. Johnson, *Phys. Rev. C* 28, 1179 (1983).

[159] A. Deloff, *Z. Phys.* 316, 49 (1984).
[160] S. Klarsfeld, J. Martorell, D.W.L. Sprung, J. Phys. G 10, 165 (1984).
[161] P. Locher, A. Svarc, Z. Phys. A 316, 55 (1984).
[162] S. Mrowczynski, Phys. Lett. B 277, 43 (1992).
[163] S.B. Dubovichenko, Phys. Atom. Nucl. 63, 734 (2000).
[164] A.F. Krutov, V.E. Troitsky, N.A. Tsirova, Theor. Phys. 5, 17 (2004).
[165] A.F. Krutov, V.E. Troitsky, N.A. Tsirova, Vestnik SamGU 3, 100 (2006).
[166] A. Krutov, V. Troitsky, N. Tserova, PoS LC054 (2008).
[167] E.S. Gamzova, A.F. Krutov, V.E. Troitsky, N.A. Tsirova, Theor. Phys. 10, 32 (2009).
[168] M.N. Platonova, V.I. Kukulin, Phys. Atom. Nucl. 73, 86 (2010).
[169] R. A. Arndt, I. I. Strakovsky, R. L. Workman, Phys. Rev. C 62, 034005 (2000).
[170] V.G.J. Stoks, R.A.M. Klomp, M.C.M. Rentmeester, J.J. de Swart, Phys. Rev. C 48, 792 (1993).
[171] V.I. Zhaba, Ukr. J. Phys. 61, 949 (2016).
[172] V.I. Zhaba, Probl. Atom. Sci Tech. 3, 154 (2016).
[173] V.I. Zhaba, Kharkov. Univ. Bull., Phys. Ser. 23, 36 (2015).
[174] V.I. Zhaba, Visnyk Lviv Univ., Ser. Phys. 51, 77 (2016).
[175] V.A. Karmanov, Yad. Fiz. 34, 1020 (1981).
[176] V.P. Ladygin, N.B. Ladygina, Yad. Fiz. 65, 188 (2002).
[177] V.I. Zhaba, Prykarpat. visnyk NTSh, Number Ser. 1, 139 (2016).
[178] V.I. Zhaba, J. Phys. Stud. 20, 3101 (2016).
[179] M. Garson, J. Arvieux, D.H. Beck et al., Phys. Rev. C 49, 2516 (1994).
[180] V.I. Zhaba, Electr. Journ. Theor. Phys. 13, 161 (2016).
[181] V.I. Zhaba, Cherkasy Univ. Bull., Phys. and Mathem. Scienc. 349, 50 (2015).
[182] V.I. Zhaba, Nucl. Phys. Atom. Energy 17, 22 (2016).
[183] V.I. Zhaba, Mod. Phys. Lett. A 31, 1650139 (2016).
[184] D. Abbott et al., Phys. Rev. Lett. 84, 5053 (2000).
[185] D. Abbott et al., Eur. Phys. J. A 7, 421 (2000).
[186] C. Zhang, M. Kohl, T. Akdogan et al., Phys. Rev. Lett. 107, 252501 (2011).