Bell nonlocality with no information of which particle of Alice is entangled with which particle of Bob

Ari Patrick1,* and Adán Cabello2,3,†

1Departamento de Física-Matemática, Instituto de Física, Universidade de São Paulo, São Paulo 05508-090, São Paulo, Brazil
2Departamento de Física Aplicada II, Universidad de Sevilla, E-41012 Sevilla, Spain
3Instituto Carlos I de Física Teórica y Computacional, Universidad de Sevilla, E-41012 Sevilla, Spain

We address the problem of detecting Bell nonlocality whenever Alice and Bob cannot identify which particle detected by Alice is entangled with which particle detected by Bob and the only experimental information are the intensities produced in each run of the experiment by \( N \) particles. This scenario naturally occurs in Bell experiments with parametric down conversion when the crystal is pumped by strong pulses, in Bell tests with distant sources in which particles may suffer different delays during their flight, and in Bell experiments using living cells as photo detectors. We show that, although Bell nonlocality decreases as \( N \) increases, if the parties can distinguish arbitrarily small differences of intensities and the visibility is larger than 0.98, then Bell nonlocality can still be experimentally detected with fluxes of up to \( N = 15 \) particles. We show that this prediction can be tested with current equipment, but requires the assumption of fair sampling.

I. INTRODUCTION

A. Motivation

Bell nonlocality, that is, the violation of Bell inequalities [1], is one of the most characteristic signatures of quantum theory and has a wide range of applications for communication and computation [2]. However, Bell nonlocality vanishes if:

(i) The only experimental information available to the parties (Alice and Bob) are the intensities registered by their respective detectors.

(ii) Intensities are produced by continuous fields (rather than by discrete particles).

(iii) The parties can only measure changes in intensity values of the order in which the particles have arrived to the detectors.

To explain why, let us consider the simplest Bell scenario: two parties, each of them with two measurement options, \( x \in \{1, 2\} \) for Alice and \( y \in \{1, 2\} \) for Bob, and each measurement with two possible outcomes, \( a \in \{0, 1\} \) for Alice’s measurements and \( b \in \{0, 1\} \) for Bob’s. Alice and Bob, using classical communication, can compute the marginal probability densities \( p(a_{xy}|d_{1a}|d_{1b}), p(b_{xy}|d_{1b}|d_{1a}) \), where \( I_{axy} \) and \( I_{bly} \) are the intensities registered by, respectively, Alice’s and Bob’s detectors. If conditions (i)–(iii) hold, then, in the limit \( N \gg 1 \), consistency with classical physics forces this set of marginal distributions to admit a local hidden variable model for the intensities. In this case, it is said that the intensities exhibit “macroscopic locality” [3].

Here, we investigate what happens when condition (i) holds but conditions (ii) and (iii) do not. While in a standard Bell test, every time one party chooses a measurement setting detects one particle, as illustrated in Fig. 1(a), in this work we assume that, instead of that, each party detects \( N > 1 \) particles. In addition, we assume that each party only has access to the intensities these \( N \) particles have produced but not to the order in which the particles have arrived to the detectors. See Fig. 1(b). Or, equivalently, even if a party has detected the \( N \) particles one by one, the order in which they are detected does not give information about the order in which they were emitted by the source.

We also assume that the parties can distinguish any small difference of intensity between their two detectors. The specific problem we address is what happens with the violation of a Bell inequality when, in each run of a Bell experiment (i.e., when Alice has chosen to measure \( x \) and Bob has chosen to measure \( y \)), each of Alice and Bob detects \( N > 1 \) particles (rather than a single particle); some of them in the detector corresponding to one of the two possible outcomes and the rest in the other detector.

Our motivation for investigating this problem is twofold.
From a fundamental perspective, we know that the electromagnetic field behaves as made of individual packets called photons and, therefore, intensities can be seen as produced by an accumulation of discrete particles. That is, we know that, from a fundamental perspective, condition (ii) does not hold.

In addition, from a practical perspective, there are several scenarios in which, effectively, the only experimental information available to Alice and Bob are the intensities, as in Fig. 1(b), while Alice and Bob can still distinguish small differences of intensities. The most relevant examples we have identified are the following:

(I) Bell experiments where the source uses a parametric down conversion process [4] and the nonlinear crystal is pumped with strong pulses, so each pulse produces $N$ pairs of entangled photons rather than a single pair. Here, we assume that the detectors can collect all these photons. This is possible, using, for instance, special arrays of nanowire detectors [5].

(II) Bell experiments where the source of entangled pairs is moving and far from the detectors. In addition, there may be disturbances during the propagation of the particles which make impossible to identify which particle of Alice is entangled with which particle of Bob. For example, this happens when the the source is randomly oscillating in the direction of propagation of the particles at higher speeds than the speed of propagation of the particles and/or when particles propagate a different speeds due to local disturbances. This may also affect future satellite-to-ground Bell tests in which the source is in the satellite and both Alice and Bob are in ground. In current satellite-to-ground Bell tests, Alice is in the satellite with the source [6]. This may also occur in Bell tests with hypothetical cosmic sources of entanglement.

(III) In Bell experiments that use hybrid photo detectors that incorporate living cells. For example, rod photoreceptor cells taken from the eye of a frog [7]. There, each rod has an outer segment that contains rhodosin molecules that undergoes a chemical change when exposed to light. This results in an electrical signal that is picked up by the nervous system and relayed to the brain. When submitted to a flux of photons, each photon interacts with just one rhodosin molecule [7]. If one can distinguish which electrical signal (the one corresponding to 0 or the one corresponding to 1) corresponds to a higher intensity, then we are in the case that we are considering. If this distinction would be possible in the brain, we could detect Bell nonlocality using human eyes and without needing entanglement amplification (as in [8, 9]).

In Sec. III, we discuss a fourth case designed to test our results in a more controlled environment.

The structure of the paper is as follows. In Sec. II A, we begin by assuming that both the state preparation and the detection efficiency are perfect. That is, that there is no noise and each of Alice and Bob detects $N$ particles. Then, in Sec. II B, we study the effect of noise and, in Sec. II C, we study the effect of imperfect detection. In Sec. III, we propose an experiment to test our results.

B. Bell vs contextuality experiments with intensities

Before going on, it is important to stress that the limitation that the only experimental information available is the intensities in each detector creates a fundamentally different problem in Bell experiments than in Kochen-Specker contextuality experiments [10]. In the second case, spacelike separation plays no role so in the Kochen-Specker contextuality experiment equivalent to the simplest Bell inequality experiment Bob’s measurement can be timelike separated from Alice’s. This allows us to encode each of Alice’s discrete outcomes in an extra degree of freedom (for example, path [11], time [12], or polarization [13]) before Bob’s measurement is performed. This allows us to guide the flux of discrete particles to four different detectors: one corresponding to the case the outcomes of Alice and Bob are 0, 0 (respectively), the second for the case 0, 1, the third for 1, 0, and the fourth detector for 1, 1. This trick of encoding Alice’s outcomes in an extra degree of freedom before performing Bob’s measurement allows us to recover quantum contextual correlations even using classical microwaves and classical light [14, 15]. However, this trick is not possible when there is spacelike separation between Alice’s and Bob’s measurements as is the case in Bell experiments.

II. RESULTS

A. Ideal case

We consider the simplest Bell inequality, the Clauser-Horne-Shimony-Holt [16] Bell inequality, in the version proposed by Zohren and Gill in Ref. [17], namely,

$$S \geq 1,$$

with

$$S = p(01|22) + p(10|12) + p(01|11) + p(11|21) + p(10|21) + p(00|21),$$

where $p(ab|xy)$ is the probability of obtaining outcomes $a$ and $b$ for measurements $x$ and $y$, respectively.

In the case of $N = 1$, the maximum quantum violation is

$$S = 3 - \sqrt{2} \approx 0.793,$$

and is achieved, for example, with the state

$$|\phi^+\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B),$$

where, e.g., $|0\rangle_A$ denotes that Alice’s particle is in the state represented by the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle_B$ denotes that Bob’s particle is in the state represented by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, and the following
measurement settings:

\[ M_{x=1} = \frac{1}{2} (\mathbb{I} - \sigma_x), \quad M_{x=2} = \frac{1}{2} (\mathbb{I} - \sigma_y), \quad (5a) \]
\[ M_{y=1} = \frac{1}{2} \left[ \mathbb{I} - \frac{\sigma_x + \sigma_y}{\sqrt{2}} \right], \quad M_{y=2} = \frac{1}{2} \left[ \mathbb{I} - \frac{\sigma_x - \sigma_y}{\sqrt{2}} \right], \quad (5b) \]

where \( \mathbb{I} \) denotes the identity matrix and \( \sigma_x \) the Pauli matrix in the direction \( n \). Each of these observables has two possible outcomes: 0 and 1, corresponding to the eigenvalues of the operator that represents the observable.

Now we consider the case in which, in each run of the Bell experiment, each of Alice and Bob receives a number \( N > 1 \) of particles every time they chose their measurement. \( N_0 \) of the particles end up in the detector corresponding to the outcome 0 of the measurement and \( N_1 = N - N_0 \) of the particles end up in the detector corresponding to the outcome 1. There is no information about the order in which the particles arrived. The only information are the intensities \( I_0 = kN_0 \) and \( I_1 = kN_1 \) in each detector. Using this information, each of the parties, without communicating with the other party, should provide an outcome 0 or 1. The question is which is the strategy that better preserves Bell nonlocality.

After checking all possible alternatives, we have found that an optimal strategy is the one in which each party outcomes the detector that has higher intensity. That is, if \( I_0 > I_1 \), then the party outcomes 0, while, if \( I_0 \leq I_1 \), then the party outcomes 1.

Then, for example to compute \( p_N(ab|xy) \), defined as the probability of Alice yielding the outcome \( a \) for measurement \( x \), and Bob yielding the outcome \( b \) for measurement \( y \), when each of them detect \( N \) particles, we have to sum the probabilities of all the possible ways in which \( N \) particles in Alice’s side and \( N \) particles in Bob’s side may have induced Alice to outcome \( a \) and Bob to outcome \( b \).

For example, for \( N = 2 \),

\[ p_2(01|xy) = p(00|xy)p(01|xy) + p(01|xy)p(00|xy) + p(01|xy)p(01|xy), \quad (6) \]

where \( p(00|xy)p(01|xy) \) is the probability that the first pair of particles ended in detector 0 for Alice and Bob, while the second pair ended in Alice’s detector 0 and Bob’s detector 1.

For arbitrary \( N \),

\[ p_N(00|xy) = \sum_{a_i < \frac{x}{2}} \sum_{b_j < \frac{y}{2}} \prod_{k=1}^{N} p(a_kb_k|xy), \quad (7a) \]
\[ p_N(01|xy) = \sum_{a_i < \frac{x}{2}} \sum_{b_j \geq \frac{y}{2}} \prod_{k=1}^{N} p(a_kb_k|xy), \quad (7b) \]
\[ p_N(10|xy) = \sum_{a_i \geq \frac{x}{2}} \sum_{b_j < \frac{y}{2}} \prod_{k=1}^{N} p(a_kb_k|xy), \quad (7c) \]
\[ p_N(11|xy) = \sum_{a_i \geq \frac{x}{2}} \sum_{b_j \geq \frac{y}{2}} \prod_{k=1}^{N} p(a_kb_k|xy). \quad (7d) \]

Therefore, we can define

\[ S_N = p_N(01|22) + p_N(10|12) + p_N(01|11) + p_N(11|21) + p_N(10|21) + p_N(00|21) \quad (8) \]

and compute the maximum quantum violation of the Bell inequality \( S_N \geq 1 \) as a function of the number \( N \) of particles detected by each party for states of the form (9), for different values of \( V \). For a given \( V \), the violation is higher when \( N \) is odd. For \( V = 0.95 \), the violation vanishes for \( N > 9 \), if \( N \) is odd, and for \( N > 4 \) if \( N \) is even. For \( V = 0.97 \), the violation vanishes for \( N > 17 \), if \( N \) is odd, and for \( N > 6 \) if \( N \) is even. For \( V \geq 0.99 \), there is always violation (although very small) at least up to \( N = 18 \). Obtaining the maximum quantum violation for higher values of \( N \) requires computing power that exceeds our capabilities.

Therefore, we can define

\[ S_N = p_N(01|22) + p_N(10|12) + p_N(01|11) + p_N(11|21) + p_N(10|21) + p_N(00|21) \quad (8) \]

and compute the maximum quantum violation of the Bell inequality \( S_N \geq 1 \) as a function of \( N \). The result of our calculations is presented in Fig. 2 (black points).

We have found that the maximum quantum violation is always obtained for the state given in Eq. (4) and the measurements given in Eqs. (5) and depends on whether \( N \) is even or odd. The violation of the Bell inequality is larger for odd \( N \). This is due to the fact that only when \( N \) is odd, the intensities in both detectors are always unequal, so the strategy of yielding the largest intensity as outcome partially keeps the quantum behavior. In contrast, when \( N \) is even, the intensities in both detectors are sometimes equal and, then, yielding 1 as outcome destroys any quantum correlation and degrades the violation.

### B. The effect of noise

So far, we have assumed that the state is perfect. Here, we examine the case in which the state is affected by some amount of white noise. Specifically, we assume that the prepared state is

\[ \rho = V |\phi^+\rangle \langle \phi^+ | + (1 - V) \mathbb{I}/4, \quad (9) \]

where \( V \), sometimes referred to as the visibility of the state, is not 1. For state-of-the-art photonic experiments \( V \geq 0.98 \) [18]. Here, we have computed the maximum quantum violation for \( V = 0.95, V = 0.97, \) and \( V = 0.99 \). The results are presented in Fig. 2 (red, blue, and green points, respectively).
The maximum quantum violation is always obtained for the measurements given in Eqs. (5) and depends on whether \( N \) is even or odd. Higher noise makes that the violation disappears for smaller values of \( N \). Interestingly, our results suggest that, with \( V \approx 0.99 \), it is possible to experimentally observe a statistically significant violation of the Bell inequality even with fluxes of up to 15 particles.

C. Inefficient detectors

So far, we have assumed that the detectors capture the \( N \) pairs of particles emitted by the source. However, in actual experiments detectors only capture a fraction of the particles. Therefore, an interesting question is how robust the violation of inequality \( S_N \geq 1 \) is when some of the particles are missing but the parties keep following the same strategy. That is, if one party observes \( I_0 > I_1 \) in its detectors, then the party outputs 0, and if it observes \( I_0 \leq I_1 \), then the party outputs 1.

Here, we obtain the minimum detection efficiency \( \eta_{\text{min}} \) needed to observe violation of inequality \( S_N \geq 1 \) using the strategy mentioned above, as a function of \( N \). The detection efficiency is the ratio between the number of particles detected by a detector and the number of particles emitted towards that detector. We assume that the source is heralded, \( V = 1 \), all detectors have the same detection efficiency \( \eta \), and there are no dark counts during the experiment.

For \( N = 1 \), the strategy described above is equivalent to yielding outcome 1 when no detection occurs. Then, we have the following cases:

[1.1] With probability \( \eta^2 \), both parties detect its particle. For this subensemble, \( S_1 = \frac{1 - \sqrt{2}}{2} \).

[1.2] With probability \( \eta (1 - \eta) \), Alice detects and Bob does not. Therefore, Bob observes \( I_0 = I_1 \) and always outputs 1. Therefore, for this subensemble, \( S_1 = \rho_A(0|2) + 0 + \rho_A(1|2) + 0 + 0 = \frac{1}{2} + 0 + \frac{1}{2} + 0 + 0 = \frac{3}{2} \), where \( \rho_A(0|2) \) is the probability that Alice finds the particle in detector 0 (and then outcomes 0) when she measures 2.

[1.3] With probability \( (1 - \eta)^2 \), neither Alice nor Bob detects, so each of them always outputs 1. For this subensemble, \( S_1 = 0 + 0 + 0 + 1 + 0 = 1 \).

Therefore, \( \eta_{\text{min}} \) follows from demanding that

\[
\eta^2 \left( \frac{3 - \sqrt{2}}{2} \right) + 2\eta (1 - \eta)^2 + (1 - \eta)^2 < 1,
\]

which implies

\[
\eta_{\text{min}}(N = 1) = \frac{2}{1 + \sqrt{2}} \approx 0.828.
\]

That is, there is Bell nonlocality (without making the fair sampling assumption; see below) if the detection efficiency is higher than this value. This values coincides with the one obtained for \( N = 1 \) after optimizing over all strategies [19].

Let us now suppose that \( N = 2 \). Then, we have the following cases:

[2.1] With probability \( \eta^4 \), each of Alice and Bob detects the two particles. For this subensemble, \( S_2 = \frac{21}{16} - \frac{1}{2\sqrt{2}} \), where \( S_2 \) is defined in Eq. (8).

[2.2] With probability \( 2\eta^3(1 - \eta) \), Alice detects the two particles and Bob only detects one (and thus he outputs the detector in which he found the particle). The factor 2 comes from the fact that the particle that Bob detects can be the one of the first part or the one of the second pair. To compute the value of \( S_2 \) for this subensemble, let us assume that Bob detects the first particle but not the second (the value of \( S_2 \) is the same if Bob detects the second but not the first one). Then,
same in the other case). Then,

\[
S_2 = p(00|22)p_B(1|2) + p(01|22)p_B(0|2) + p(01|22)p_B(1|2)
+ p(10|12)p_B(0|2)
+ p(00|11)p_B(1|1) + p(01|11)p_B(0|1) + p(10|11)p_B(1|1)
+ p(10|21)p_B(1|1) + p(11|21)p_B(0|1) + p(11|21)p_B(1|1)
+ p(00|21)p_B(0|1)
= [p(00|22) + p(01|22) + p(01|22) + p(10|12)
+ p(00|11) + p(01|11) + p(10|11) + p(10|21)
+ p(11|21) + p(11|21) + p(10|21) + p(00|21)] \frac{1}{2}
= 6 - \sqrt{2}.
\]

[2.4] With probability \(2\eta^2(1 - \eta)^2\), Alice detects one particle and Bob detects its entangled companion for one of the pairs but none of them detects the particle of the other pair. Then, they output what the detectors in which they found their respective particle. Therefore, the value of \(S_2\) for this subensemble is \(S_2 = 3 - \sqrt{2}\).

[2.5] With probability \(2\eta^2(1 - \eta)^2\), Alice detects one particle and Bob detects the one that it is not entangled with. Therefore, since their outputs are statistically independent, the value of \(S_2\) for this subensemble is \(S_2 = 6 \times \frac{1}{2} = \frac{3}{2}\).

[2.6] With probability \(\eta^2(1 - \eta)^2\), Alice detects the two particles and Bob none (and thus he always outputs 1). For this subensemble, \(S_2 = \frac{1}{4} + 0 + \frac{1}{2} + \frac{1}{2} + 0 + 0 = \frac{3}{2}\).

[2.7] With probability \(2\eta^2(1 - \eta)^2\), Bob detects the two particles and Alice none (and thus she always outputs 1). For this subensemble, \(S_2 = 0 + \frac{1}{4} + 0 + \frac{1}{2} + 0 + \frac{1}{2} = \frac{3}{2}\).

[2.8] With probability, \(2\eta(1 - \eta)\), Alice detects one particle and Bob none (and thus, since he observes equal intensities, he outputs 1). Therefore, the value of \(S_2\) for this subensemble is \(S_2 = \frac{1}{4} + 0 + \frac{1}{2} + \frac{1}{2} + 0 + 0 = \frac{3}{2}\).

[2.9] With probability, \(2\eta(1 - \eta)^3\), Bob detects one particle and Alice none (and thus, since she observes equal intensities, she outputs 1). Therefore, the value of \(S_2\) for this subensemble is \(S_2 = 0 + \frac{1}{2} + 0 + \frac{1}{2} + 0 + 0 = \frac{3}{2}\).

[2.10] Finally, with probability, \((1 - \eta)^4\), no one detects any particle so each of them outputs 1. The value of \(S_2\) for this subensemble is \(S_2 = 0 + 0 + 0 + 1 + 0 + 0 = 1\).

Therefore, \(\eta_{\text{min}}\) follows from demanding that,

\[
\eta^4 \left( \frac{21}{16} - \frac{1}{2\sqrt{2}} \right) + 4\eta^3(1 - \eta) \left( \frac{6 - \sqrt{2}}{4} \right) + 2\eta^2(1 - \eta)^2 \left( \frac{3 - \sqrt{2}}{2} + \frac{5}{4} \right)
+ \left[2\eta^2(1 - \eta)^2 + 4\eta(1 - \eta)^3\right] \frac{3}{2}
+(1 - \eta)^4 < 1,
\]

which implies

\[
\eta_{\text{min}}(N = 2) = 0.941. \tag{15}
\]

Similarly, for the case \(N = 3\), we have found

\[
\eta_{\text{min}}(N = 3) = 0.905. \tag{16}
\]

Calculating \(\eta_{\text{min}}(N)\) for higher values of \(N\) becomes difficult and it is not really necessary as it is clear that \(\eta_{\text{min}}(N)\) will increase with \(N\). The reason for this is that the maximum quantum violation rapidly decreases as \(N\) increases (see Fig. 2), so the fact that the parties yield a quantum-based outcome even when they do not detect all \(N\) particles is not enough to account for the possible local hidden variable models.

The problem is that, for Bell experiments with \(V \geq 0.98\), the highest experimental detection efficiencies reported are \(\eta = 0.77–0.81 [20, 21]\). Therefore, the values for the detection efficiency required to observe Bell nonlocality based on the intensities produced by \(N\) particles are too high for what is achievable with current technology: \(\eta \approx 1\) can be achieved \([22, 23]\), but at the cost of visibilities which are not enough for Bell nonlocality based on the intensities of \(N > 5\) particles.

However, even if the detection efficiency is not enough for a loophole-free Bell test, we can run an experiment adopting the fair sampling assumption. That is, selecting those runs of the experiment in which both parties detect \(N\) particles and making the assumption of fair sampling, namely, that the selected runs are a faithful subset of those that would have been obtained if detection efficiency would be perfect. This will allow us to experimentally observe Bell nonlocality using only intensities with current equipment.

III. PROPOSED EXPERIMENT

Before applying the results presented here to any of the situations (I)-(III) discussed in Sec. I A, it would be interesting to test the predictions in a more controlled experiment. With this target in mind, we propose the following modified Bell test:

(a) Suppose a heralded source that can emit an odd number \(N \leq 15\) of pairs of entangled photons, with visibility \(V > 0.98\). The pairs are emitted one by one, with temporal separation \(\tau\) between each pair. For that, we can use a source of polarization-entangled pairs of photons based on quantum dots \([24, 25]\).

(b) In the space between the source and Alice’s measurement device, we introduce beam splitters and mirrors in such a way that photons can go through paths of different lengths. See details later on. In contrast, there is only one possible path between the source and Bob’s measurement device.

(c) For each of the local measurements, we use two single-photon detectors, one for each outcome. Each of these detectors must allow us to distinguish two photons that arrive with a time difference \(\tau\). If we suitably postselect some runs we can have no information of which photon of Alice is entangled with which photon of Bob.

For example, let us assume for simplicity that \(N = 3\) and that the three photons of Alice are emitted by the source at times \(t_0, t_0 + \tau,\) and \(t_0 + 2\tau\). Suppose that each of these photons can follow a path of length \(6\tau\), or \(7\tau\), or \(8\tau\), or \(9\tau\), or \(10\tau\). Now consider those runs in which one photon is detected
at 8\(\tau\), one photon is detected at 9\(\tau\), and one photon is detected at 10\(\tau\). In these runs, Alice cannot now which is the photon of Bob each of her photons is entangled with. Still, according to the results in Fig. 2, if the visibility is high enough and adopting the assumption of fair-sampling, Alice and Bob can observe a violation of the Bell inequality \(S_N \geq 1\) for any \(N \leq 15\) (with \(N\) odd).

**IV. CONCLUSIONS**

In the “macroscopic” limit of infinite number of particles, the violation of Bell inequalities vanishes when the only experimental information are the intensities and we cannot distinguish arbitrarily small differences of intensities. However, here we have shown that, for visibilities reachable in current photonic Bell experiments, if the number of photons that reach the detectors every time a local measurement is fixed is \(N \leq 15\), then Bell nonlocality can be experimentally observed with sufficient statistical significance from the detected intensities, assuming that parties can distinguish any small differences of intensity between their detectors.

We have identified three scenarios in which this result can be useful: Bell experiments based on parametric down conversion pumped by strong pulses, Bell tests with distant moving sources of entangled pairs and/or local disturbances in the propagation of the particles, and Bell experiments using photodetectors based on living cells, including those using human eyes as detectors. In addition, we have suggested a way to experimentally test this prediction in a controlled environment.

**Note added:** After finishing this paper we have become aware of two works reaching similar conclusions [26, 27]. Secs. (IA), (IB), (II C), and (III) include material that goes beyond these works.

**ACKNOWLEDGMENTS**

We thank Bárbara Amaral, Jean-Daniel Bancal, Nicolas Brunner, Ana Predojević, Valerio Scarani, and Giuseppe Vallone for comments. AP was supported by a Grant of the Colegio de Doctorado de Física del Grupo Tordesillas and CNPq. AC was supported by Universidad de Sevilla Project Qdisc (Project No. US-15097), with FEDER funds, MINECO Project No. FIS2017-89609-P, with FEDER funds, and QuantERA grant SECRET, by MINECO (Project No. PCI2019-111885-2).

[1] J. S. Bell, On the Einstein Podolsky Rosen paradox, *Physica* **1**, 195 (1964).
[2] N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, and S. Wehner, Bell nonlocality, *Rev. Mod. Phys.* **86**, 419 (2014).
[3] M. Navascues and H. Wunderlich, A glance beyond the quantum model, *Proc. Royal Soc.* A **466**, 881 (2009).
[4] P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. Shih, New High-Intensity Source of Polarization-Entangled Photon Pairs, *Phys. Rev. Lett.* **75**, 4337 (1995).
[5] I. P. Degiovanni, S. Polyakov, A. Migdall, H. B. Coldenstrodt-Ronge, I. A. Walmsley, and F. N. C. Wong, in *Single Photon Generation and Detection, Physics and Applications*, edited by A. Migdall, S. Y. Polyakov, J. Fan, and J. C. Bienfang (Aademic Press, 2013), Chap. 7.
[6] J. Yin, Y. Cao, Y.-H. Li, J.-G. Ren, S.-K. Liao, L. Zhang, W.-Q. Cai, W.-Y. Liu, B. Li, H. Dai, M. Li, Y.-M. Huang, L. Deng, L. Li, Q. Zhang, N.-L. Liu, Y.-A. Chen, C.-Y. Lu, R. Shu, C.-Z. Peng, J.-Y. Wang, and J.-W. Pan, Satellite-to-Ground Entanglement-Based Quantum Key Distribution, *Phys. Rev. Lett.* **119**, 200501 (2017).
[7] N. Sim, M. F. Cheng, D. Bessarab, C. M. Jones, and L. A. Krivitsky, Measurement of Photon Statistics with Live Photoreceptor Cells, *Phys. Rev. Lett.* **109**, 113601 (2012).
[8] P. Sekatski, N. Brunner, C. Branciard, N. Gisin, and C. Simon, Towards Quantum Experiments with Human Eyes as Detectors Based on Chloramycin Induced Sensitive Stimulation, *Phys. Rev. Lett.* **103**, 113601 (2009).
[9] E. Pomarico, B. Sanguinetti, P. Sekatski, H. Zbinden, and N. Gisin, Experimental amplification of an entangled photon: what if the detection loophole is ignored?, *New J. Phys.* **13**, 063031 (2011).
[10] A. Cabello, Experimentally Testable State-Independent Quantum Contextuality, *Phys. Rev. Lett.* **101**, 210402 (2008).
[11] E. Amselem, M. Radmark, M. Bourennane, and A. Cabello, State-Independent Quantum Contextuality with Single Photons, *Phys. Rev. Lett.* **103**, 160405 (2009).
[12] J. Ahrens, E. Amselem, A. Cabello, and M. Bourennane, Two fundamental experimental tests of nonclassicality with qutrits, *Sci. Rep.* **3**, 2170 (2013).
[13] B. Marques, J. Ahrens, M. Nawareg, A. Cabello, and M. Bourennane, Experimental Observation of Hardy-Like Quantum Contextuality, *Phys. Rev. Lett.* **113**, 250403 (2014).
[14] D. Frustaglia, J. P. Baltanás, M. C. Velázquez-Ahumada, A. Fernández-Prieto, A. Lujambio, V. Losada, M. J. Freire, and A. Cabello, Classical Physics and the Bounds of Quantum Correlations, *Phys. Rev. Lett.* **116**, 250404 (2016).
[15] A. Zhang, H. Xu, J. Xie, H. Zhang, B. J. Smith, M. S. Kim, and L. Zhang, Experimental Test of Contextuality in Quantum and Classical Systems, *Phys. Rev. Lett.* **122**, 080401 (2019).
[16] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Proposed Experiment to Test Local Hidden-Variable Theories, *Phys. Rev. Lett.* **23**, 880 (1969).
[17] S. Zohren and R. D. Gill, Maximal Violation of the Collins-Gisin-Linden-Massar-Popescu Inequality for Infinite Dimensional States, *Phys. Rev. Lett.* **100**, 120406 (2008).
[18] H. S. Poh, S. K. Joshi, A. Ceré, A. Cabello, and C. Kurtsiefer, Approaching Tsirelson’s Bound in a Photon Pair Experiment, *Phys. Rev. Lett.* **115**, 180408 (2015).
[19] A. Garg and N. D. Mermin, Detector inefficiencies in the Einstein-Podolsky-Rosen experiment, *Phys. Rev. D* **35**, 3631 (1987).
[20] L. K. Shalm, Y. Zhang, J. C. Bienfang, C. Schlager, M. J. Stevens, M. D. Mazurek, C. Abellán, W. Amaya, M. W.
[21] W.-Z. Liu, M.-H. Li, S. Ragy, S.-R. Zhao, B. Bai, Y. Liu, P. J. Brown, J. Zhang, R. Colbeck, J. Fan, Q. Zhang, and J.-W. Pan, Device-independent randomness expansion against quantum side information, arXiv:1912.11159.

[22] B. Hensen, H. Bernien, A. E. Dréau, A. Reiserer, N. Kalb, M. S. Blok, J. Ruitenberg, R. F. L. Vermeulen, R. N. Schouten, C. Abellán, W. Amaya, V. Pruneri, M. W. Mitchell, M. Markham, D. J. Twitchen, D. Elkouss, S. Wehner, T. H. Taminiau, and R. Hanson, Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres, Nature (London) 526, 682 (2015).

[23] W. Rosenfeld, D. Burchardt, R. Garthoff, K. Redeker, N. Ortegel, M. Rau, and H. Weinfurter, Event-Ready Bell Test Using Entangled Atoms Simultaneously Closing Detection and Local-ity Loopholes, Phys. Rev. Lett. 119, 010402 (2017).

[24] H. Jayakumar, A. Predojević, T. Huber, T. Kauten, G. S. Solomon, and G. Weihs, Deterministic Photon Pairs and Coherent Optical Control of a Single Quantum Dot, Phys. Rev. Lett. 110, 135505 (2013).

[25] D. Huber, M. Reindl, Y. Huo, H. Huang, J. S. Wildmann, O. G. Schmidt, A. Rastelli, and R. Trotta, Highly indistinguishable and strongly entangled photons from symmetric GaAs quantum dots, Nat. Commun. 8, 15506 (2017).

[26] J.-D. Bancal, C. Branciard, N. Brunner, N. Gisin, S. Popescu, and C. Simon, Testing a Bell inequality in multipair scenarios, Phys. Rev. A 78, 062110 (2008).

[27] H. S. Poh, A. Cerè, J.-D. Bancal, Y. Cai, N. Sangouard, V. Scarani, and C. Kurtsiefer, Experimental many-pairs nonlocality, Phys. Rev. A 96, 022101 (2017).