Realizing unification in SO(10) models with one intermediate breaking scale

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We derive the threshold corrections in SO(10) grand unified models with the intermediate symmetry being flipped SU(5) × U(1) or SU(3) × SU(2) × U(1) × U(1). These models do not achieve gauge coupling unification if the matching conditions do not take threshold corrections into account. We present results showing the required size of threshold corrections for any value of the intermediate and unification scales. In particular, our results demonstrate that both of these models require large threshold corrections to allow for unification with a predicted proton lifetime above current experimental bounds.

I. INTRODUCTION

A popular type of extensions of the Standard Model (SM) are Grand Unified Theories (GUTs) [1], and especially those based on the SO(10) gauge group [2]. In order for this type of model to be allowed, it is required that the gauge couplings unify at some high energy scale, defining the scale of unification, and match to the single gauge coupling of the GUT model. This scale has to be high enough so that the predicted rate of proton decay is lower than the limits placed by the non-observation of this phenomenon.

Symmetry breaking of SO(10) down to the SM may proceed in several different chains with multiple intermediate symmetries [3]. The renormalization group (RG) running of the gauge couplings varies among these different models, which leads to some being disfavored due to either a too short prediction for the proton lifetime or a failure to unify [4]. However, this conclusion may be modified by including threshold corrections [5, 6], which are loop corrections to the matching conditions among gauge couplings imposed at the scale of symmetry breaking. This can have the effect of either saving models that predict a too short proton lifetime [7–10] or enabling unification in models that do not unify with tree-level matching conditions [11–13].

In this work, we investigate how threshold corrections may enable gauge coupling unification in non-supersymmetric models in which the SO(10) symmetry is broken via one intermediate gauge group. Specifically, we focus on two models in which the symmetry breaking proceeds through G31 ≡ SU(5) × U(1) with a flipped hypercharge embedding and G3211 ≡ SU(3) × SU(2) × U(1) × U(1). These two models were mentioned, but not investigated in Ref. [14], for which threshold corrections were computed for all other SO(10) models with zero or one intermediate symmetry. Although the models have been investigated previously in the literature [7, 19–23], we focus on the numerical computations of the size of the threshold corrections required to achieve gauge coupling unification with an experimentally allowed prediction of the proton lifetime. To this end, we compute the RG running to two-loop order and the threshold corrections to one-loop order. We further consider the effect of kinetic mixing in the G3211 model on the size of the required threshold corrections and find that the effect is small.

In Sec. II, we describe the RG running of gauge couplings and threshold corrections. Then, in Sec. III, we briefly present the model based on the G31 symmetry and describe the results for the required threshold corrections in this model. Next, in Sec. IV, we present the same for the model based on the G3211 symmetry, including the effect of kinetic mixing. Finally, in Sec. V, we summarize our findings and conclude.

II. RENORMALIZATION GROUP RUNNING, THRESHOLD EFFECTS, AND PROTON DECAY

A prerequisite of grand unification is that the gauge couplings in the low-energy model emerge from the one corresponding to the GUT symmetry, in our case SO(10). This is reconciled with their differing values at low energy by the RG running, through which the gauge couplings vary with the energy scale. To two-loop order in perturbation theory, the RG equations read

$$\frac{d\alpha_i^{-1}(\mu)}{d\ln \mu} = \frac{a_i}{2\pi} - \sum_j \frac{b_{ij}}{8\pi^2\alpha_j^{-1}(\mu)},$$  (1)

where $\alpha_i \equiv g_i^2/(4\pi)$, with $g_i$ being the gauge coupling corresponding to the $i$th gauge group in the model, and $a_i$ and $b_{ij}$ are the one- and two-loop coefficients, respectively [24, 25]. The measured values of the gauge couplings at the electroweak scale $M_Z \simeq 91.1876$ GeV are [20]

$$(\alpha_3^{-1}(M_Z), \alpha_2^{-1}(M_Z), \alpha_1^{-1}(M_Z)) \simeq (8.50, 29.6, 59.0).$$  (2)
These constitute the boundary conditions of the system of equations given by the RGEs.

At a scale $M_{m\to n}$ of symmetry breaking of a group $G_m$ to another group $G_n$, the matching conditions for the gauge couplings in the lower- and higher-energy theories including threshold corrections $\lambda^m_n$ are

$$\alpha^{-1}_n(M_{m\to n}) = \alpha^{-1}_m(M_{m\to n}) - \frac{\lambda^m_n}{12\pi}. \quad (3)$$

To one-loop order, the threshold corrections $\lambda^m_n$ are given by

$$\lambda^m_n = \sum_{i \in \text{vectors}} k_i V_i(S_i) \kappa_i + \sum_{i \in \text{scalars}} \kappa_i S_i(S_i) \ln \left( \frac{M_i}{M_{m\to n}} \right), \quad (4)$$

where $\kappa_i$, in one or two for real or complex representations, while $k_i$ and $\kappa_i$ are the multiplicities of the vector and scalar fields $V_i$ and $S_i$, respectively, and $M_i$ are the masses of $S_i$. It is assumed that all superheavy gauge boson masses lie at the symmetry breaking scale such that they do not have any mass-dependent contributions and that there are no additional fermions that contribute to the threshold corrections. For readability, we let $\eta_i = \ln(M_i/M_{m\to n})$.

The allowed scale $M_{GUT}$ is a relevant prediction, since it is related to proton decay. It gives the prediction for the proton lifetime in the most constraining channel to be

$$\tau(p \to e^+\pi^0) \simeq (7.47 \times 10^{35}) \text{ yr},$$

where $\alpha_{GUT}$ is the gauge coupling at $M_{GUT}$. This prediction is to be compared with the lower bound stemming from the non-observation of proton decay. The currently best bound comes from Super-Kamiokande [29][32] which gives $\tau(p \to e^+\pi^0) \geq 1.67 \times 10^{35}$ yr at 90% confidence level.

### III. FLIPPED SU(5)×U(1)

In the flipped SU(5)×U(1)$_X$ model, the SM hypercharge is a linear combination of the Abelian charge from within SU(5) and the external U(1)$_X$ charge. Models of this kind have been previously considered in e.g. Refs. [19][21][33][37]. The reason why the flipped hypercharge embedding is considered is that the standard embedding would only move the problem of unification from the SO(10) breaking scale to the SU(5) breaking scale.

Apart from the fermions in the 16$_F$ representation of SO(10), the model contains scalars in the 10$_H$, 45$_H$, and 126$_H$ representations of SO(10). The masses of the various scalar components are assumed to follow the survival hypothesis [35][40].

The matching conditions at $M_1$ are such that the gauge couplings corresponding to SU(2)$_L$ and SU(3)$_C$ in the SM both match to the one corresponding to SU(5). Therefore, the matching conditions at $M_1$ with threshold conditions read

$$\alpha^{-1}_{X}(M_1) = \frac{1}{24} \left[ 25\alpha^{-1}_{\text{GUT}}(M_1) - \alpha^{-1}_X(M_1) \right] + \frac{1}{12\pi}(25\lambda^{-1}_{51} - \lambda^{-1}_{S1}) = 0, \quad (5)$$

Similarly, we can write the matching condition at $M_{GUT}$ as

$$\alpha^{-1}_{E}(M_{GUT}) - \alpha^{-1}_{X}(M_{GUT}) + \frac{1}{12\pi}(\lambda^{-1}_{51} - \lambda^{-1}_{X}) = 0. \quad (6)$$

As can be seen by the dashed lines in Fig. 1, unification cannot be achieved in this model without sufficiently large threshold corrections. The reason for this is that $M_1$ is set by the intersection of $\alpha^{-1}_2$ and $\alpha^{-1}_X$, as demonstrated by Eq. [7] with threshold corrections set to zero. We then require the threshold corrections $\lambda^{-1}_{51}$ and $\lambda^{-1}_{S1}$ to be such that $M_1$ is pushed to a lower value. Since the two gauge couplings in $G_{51}$ diverge, $M_1$ should preferably be below the intersection of $\alpha^{-1}_2$ and $\alpha^{-1}_X$, or close enough

1. Different particle content is also possible. Since we are not concerned with the details of model building, we simply take the particle content required to achieve the breaking as well as provide two independent Yukawa couplings.
to it such that $\alpha_{X}^{-1}$ and $\alpha_{5}^{-1}$ can unify at $M_{\text{GUT}}$ without excessively large threshold corrections.

To illustrate the typical size of the perturbations of scalar masses $\eta_{i}$ required in order to allow for unification, we randomly sample each $\eta_{i}$ in the ranges $[-3, +3]$, $[-4, +4]$, and $[-5, +5]$ and compute the scales $M_{i}$ and $M_{\text{GUT}}$ for each sampled point. The points that corresponded to gauge coupling unification are plotted in Fig. 2 in which the ConvexHull method from the SciPy package [11] is used to illustrate the regions. Only points lying above the slanted gray (corresponding to the unphysical region $M_{\text{GUT}} < M_{i}$) and the nearly horizontal region (corresponding to a too short proton lifetime) are allowed.

As can be observed, only a few points with $\eta_{i} \in [-3, +3]$ are allowed. This illustrates that the model requires either fine-tuned or large threshold corrections in order to achieve unification. Both of these cases can be considered quite unnatural.

Since the $G_{31}$ model contains leptoquark gauge bosons, one has to take into account proton decay also in the intermediate symmetry, with a dependence on the scale $M_{i}$. Following Ref. [12], we find that values of $M_{i} \gtrsim 10^{15}$ GeV evade the bound on proton lifetime, both in the $p \rightarrow e^{+}\pi^{0}$ and $p \rightarrow n\pi^{+}$ channels. This further reduces the allowed parameter space in Fig. 2 but still allows the points with $\eta_{i} \in [-3, +3]$. However, as discussed in Ref. [13], these predictions can be largely altered by the values of the fermion mixing angles at that scale.

An example of unification is shown by the solid lines in Fig. 1. Here, we choose $\eta_{i} \in [-4, +4]$ such that $M_{i} \simeq 10^{15}$ GeV and $M_{\text{GUT}} \simeq 7 \times 10^{15}$ GeV.

### IV. $SU(3) \times SU(2) \times U(1) \times U(1)$

The other model we consider is based on the $SU(3)_{C} \times SU(2)_{L} \times U(1)_{R} \times U(1)$ hypercharge [14, 15]. We take the scalar sector to contain the $210_{G}, 10_{H}$, and $126_{H}$ representations $[16, 17]$ with the masses set by the survival hypothesis. As in Sec. III, we have the fermions in the $16_{F}$.

When matching the SM gauge couplings to those of $G_{3211}$, the SM hypercharge becomes a linear combination of the two Abelian charges of $G_{3211}$. Therefore, there is no unique matching when solving the problem from low to high energy. For this reason, we introduce an auxiliary parameter $x$ that parametrizes the relation between the two Abelian gauge couplings in $G_{3211}$ at $M_{i}$, i.e. $\alpha_{B^{-L}}^{-1}(M_{i}) = x\alpha_{R}^{-1}(M_{i})$. With this, the matching conditions at $M_{i}$ with threshold corrections read

$$
\alpha_{B^{-L}}^{-1}(M_{i}) = x \left(\frac{2}{5}x + \frac{3}{5}\right)^{-1} \left[\alpha_{1}^{-1}(M_{i}) + \frac{\lambda_{g211}^{(1)}}{12\pi}\right],
$$

$$
\alpha_{R}^{-1}(M_{i}) = \left(\frac{2}{5}x + \frac{3}{5}\right)^{-1} \left[\alpha_{1}^{-1}(M_{i}) + \frac{\lambda_{g211}^{(1)}}{12\pi}\right],
$$

$$
\alpha_{21}^{-1}(M_{i}) = \frac{\lambda_{g21}^{(1)}}{12\pi} - \frac{\lambda_{g21}^{(1)}}{12\pi},
$$

$$
\alpha_{3c}^{-1}(M_{i}) = \frac{\lambda_{g21}^{(1)}}{12\pi} - \frac{\lambda_{g21}^{(1)}}{12\pi}.
$$

At $M_{\text{GUT}}$, all gauge couplings must match to the one corresponding to SO(10). There are then three independent matching conditions given by

$$
\alpha_{B^{-L}}^{-1}(M_{\text{GUT}}) - \alpha_{R}^{-1}(M_{\text{GUT}}) + \frac{1}{12\pi}(\lambda_{B^{-L}}^{(1)} - \lambda_{R}^{(1)}) = 0,
$$

$$
\alpha_{R}^{-1}(M_{\text{GUT}}) - \alpha_{21}^{-1}(M_{\text{GUT}}) + \frac{1}{12\pi}(\lambda_{R}^{(1)} - \lambda_{21}^{(1)}) = 0,
$$

$$
\alpha_{21}^{-1}(M_{\text{GUT}}) - \alpha_{3c}^{-1}(M_{\text{GUT}}) + \frac{1}{12\pi}(\lambda_{21}^{(1)} - \lambda_{3c}^{(1)}) = 0.
$$

In order to numerically investigate this model, we first note that the threshold corrections at $M_{i}$ are negligible compared to those at $M_{\text{GUT}}$. The reason is that very few fields lie around $M_{i}$ in this model, since the $G_{3211}$ symmetry is broken down to SM by a vacuum expectation value in the $(1, 1, 1, 2)$ representation. The numerical computations are therefore performed by neglecting threshold corrections at $M_{i}$, and then checking that they indeed give rise to negligible corrections to the derived scales.

The threshold corrections then enter only through the three differences $\lambda_{B^{-L}}^{(1)} - \lambda_{R}^{(1)}$, $\lambda_{R}^{(1)} - \lambda_{21}^{(1)}$, and $\lambda_{21}^{(1)} - \lambda_{3c}^{(1)}$ in Eqs. (13)–(15). For each set of scales $M_{i}$ and $M_{\text{GUT}}$ and auxiliary parameter $x$, we can find the required values of these differences that allow the matching of the four gauge couplings in $G_{3211}$ to SO(10). Once these are found, we look for values of $\eta_{i}$ that can result in these differences. This can be done by expressing the problem

Figure 2. Possible scales $M_{i}$ and $M_{\text{GUT}}$ with successful gauge coupling unification due to threshold corrections in the $G_{31}$ model. Red regions denote solutions with $\eta_{i} \in [-3, +3]$ (dark red), $[-4, +4]$ (red), and $[-5, +5]$ (light red) and the “x” denotes the solution without threshold corrections. The horizontal and slanted gray shaded regions are ruled out by too short proton lifetime and $M_{i} > M_{\text{GUT}}$, respectively.
as a system of linear equations to be solved for the vector of $\eta_i$. The vector that has the smallest Euclidean norm is then found by the lstsq method in the NumPy package [48]. This solution is then labeled by its maximum value $\eta_{\text{max}}$. Finally, to remove the dependence on the auxiliary variable $x$, we repeat this whole procedure to find the value of $x$ for each set of scales $M_I$ and $M_{\text{GUT}}$ that minimizes the value of $\eta_{\text{max}}$.

The results of this procedure is displayed in Fig. 4 in which the smallest possible value of $\eta_{\text{max}}$ is shown for each combination of scales $M_I$ and $M_{\text{GUT}}$. The gray shaded region corresponds to the unphysical region of $M_{\text{GUT}} < M_I$ and the red line is the current limit set by the absence of proton decay. In comparing with the dashed lines in Fig. 3 we find that, with $M_I = 10^{10}$ GeV, the value of $M_{\text{GUT}}$ that requires the smallest threshold corrections corresponds to a value that lies inside the quadrilateral formed by the four gauge couplings. However, this is not allowed by the bounds on the proton lifetime. The smallest value of $\eta_{\text{max}}$ that is allowed is about $3.34$, corresponding to $M_I$ and $M_{\text{GUT}}$ being very close.

For the sake of completeness, we also investigate how kinetic mixing between the Abelian groups $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ changes the result. This introduces an off-diagonal gauge coupling, whose RG running affects that of the other gauge couplings, calculated using the PyR@TE package [49, 51]. It further affects the matching conditions Eqs. (9)–(12) at $M_I$, given e.g. in Refs. [7, 52, 53]. Finally, at $M_{\text{GUT}}$, we impose the condition that the off-diagonal coupling is zero, since the mixing is not generated by the symmetry breaking of SO(10) but rather from the RG running [48, 54].

We then perform an analysis similar to the one described above, but with the additional inclusion of kinetic mixing. This leads to slightly different values of $\eta_{\text{max}}$, from which we calculate the relative difference $\Delta \eta$. This is shown in Fig. 5 which shows that the largest relative difference found is about 20 %. However, in the region that is allowed by the proton lifetime bounds, the relative difference $\Delta \eta$ is less than about 9.7 %.
V. SUMMARY AND CONCLUSIONS

We have presented computations of the extent to which threshold corrections can allow for gauge coupling unification in two SO(10) models with SU(5) × U(1) and SU(3) × SU(2) × SU(2) × U(1) as intermediate symmetry groups. Although other possible contents are possible for these models, the results presented in this work are representative for these types of models. The RG equations have been solved numerically at two-loop level and the matching conditions among the gauge couplings at symmetry breaking scales include threshold corrections at one-loop level. Furthermore, we have investigated the effect of kinetic mixing on the results in the SU(3) × SU(2) × SU(2) × U(1) model.

In both of these models, our results have suggested that threshold corrections with \( \eta_i \in [-4, +4] \) are required to achieve unification while evading the bound from proton decay. Slightly smaller values of \( \eta_i \) are possible, but only for very small regions in parameter space, suggesting that these scenarios require fine-tuning of the scalar masses. With such large threshold corrections, the naturalness of such theories can be questioned. Furthermore, the effects of kinetic mixing have been shown to have a less than 10% effect on the required threshold corrections.

This investigation has neglected any constraints from the scalar potential. For example, there can exist correlations among the masses of the different scalar fields which restrict the allowed threshold corrections. We have also not taken into account any phenomenology relating to the value of \( M_1 \), which can place further constraints on the viability of the models. These constraints can, for example, stem from neutrino masses and leptogenesis.

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