Phantom cosmology and Boltzmann brains problem

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We consider the well-known Boltzmann brains problem in frames of simple phantom energy models with little rip, big rip and big freeze singularity. It is showed that these models (i) satisfy to observational data and (ii) may be free from Boltzmann brains problem. The human observers in phantom models can exist only in during for a certain period \( t < t_f \) (\( t_f \) is lifetime of universe) via Bekenstein bound. If fraction of unordered observers in this part of universe history is negligible in comparison with ordered observers than Boltzmann brains problem doesn’t appear. The bounds on model parameters derived from such requirement don’t contradict to allowable range from observational data.

I. INTRODUCTION

The discovery of accelerated expansion of the universe \([1], [2]\) led to number of new ideas/solutions in cosmology. For explanation of the cosmic acceleration the various models of so-called dark energy are proposed (for recent reviews, see \([3]-[8]\)). For dark energy the equation-of-state parameter is negative:

\[
w = p/ρ < 0, \tag{1}
\]

where \(ρ\) is the dark energy density and \(p\) is the pressure.

In principle recent observations favor to the standard cosmological model (ΛCDM-model) with a universe made up 71.3\% of vacuum energy (\(w = −1\)) and only 27.4\% of a combination of dark matter and baryonic matter \([9]\). In observational astrophysics the simple dark energy model (\(w = w_0\)) is usually considered as alternative to ΛCDM-model. In frames of this model the latest cosmological data give for \(w = −1.04^{+0.09}_{−0.10}\) \([10], [11]\).

If \(w < −1\) the violation of all four energy conditions occurs. The corresponding phantom field, which is unstable as quantum field theory \([12]\) but could be stable in classical cosmology may be naturally described by the scalar field with the negative kinetic term.

The model with constant \(w < −1\) leads to Big Rip singularity \([13]-[17]\). One note that condition \(w < −1\) is not sufficient for a singularity occurrence. Moreover, one can construct such models in which \(w\) asymptotically tends to \(−1\) and energy density increases with time or remains constant but there is no finite-time future singularity \([18]-[22]\). Of course, most evident case is when Hubble rate tends to constant (cosmological constant or asymptotically de Sitter space). Very interesting situation is related with Little Rip cosmology \([22]\) where Hubble rate tends to infinity in the infinite future (for further investigation, see \([23], [24]\)). The key point is that if \(w\) approaches \(−1\) sufficiently rapidly, then it is possible to have a model in which the time required for singularity is infinite, i.e., singularity effectively does not occur. Nevertheless, it may be shown that even in this case the disintegration of bound structures takes place in the way similar to Big Rip.

There are many advantages for this model and we shall consider one of them: such models may be free from a problem of Boltzmann brains (BB).

In \([30]\) Page has shown that if the dS universe will expand no less then \(10^{60}\) yr (with the present value of the Hubble root \(H_0 = 72 \pm 8\) km/s/Mpc) then such universe will be filled with BB: the predominant race in the universe is BB rather then ordered observers. On the other hand, the string theory prediction grants the dS universe as much time as \(t_f < \text{recurrence time} \sim e^{0.5 \times 10^{123}}\) yr \([37], [38]\) (the matter of whether it should be seconds, years or even millenniums is really unessential for such monstrous numbers). It is possible to lower this value to the \(t_f \sim e^{10^{19}}\) yr and even to the limit of \(t_f \sim e^{10^9}\) yr for models with instantons of Kachru, Pearson and Verlinde \([29]\) and with 2 Klebanov-Strassler (see \([40]\)) throats \([41]\). But, nevertheless, even with assumption that one of those models do describe our Universe, the magnitude \(t_f\) will still be way too large as compared to Page’s \(10^{60}\) yr.

To show this, following to Page, suppose that the process of observation is described by some localized positive operator \(A\), such that application of it to any state \(ψ\) leads to positive central tendency. This implies that every possible observation has some positive probability of occurrence in the given volume (e.g., as a vacuum fluctuation). Therefore, we can treat the observers as the standard quantum objects. With this in mind, Page has calculated the action for the brain of a human observer: \(S_{br} \sim 10^{16} J \times s\), and the probability \(p_{br} \sim e^{−S_{br}/ℏ} \sim e^{−10^{30}}\). Then, Page made an estimation for 4-volume for the brain (\(V_4(\text{br})\)), taken in process of making the observation: \(V_4(\text{br}) \sim e^{331} \rho_{\text{Pl}}^4\).
The crucial Page’s expression is

\[ V_4(t)p_{ord} = V_4(br)N, \]  

(2)

where \( V_4(t) \) is the total 4-volume of universe:

\[ V_4(t) = \int d^4x \sqrt{-g} \sim \int_0^t dt \rho_0^3(t), \]  

(3)

\( p_{ord} \) is the time of the \( V_4(t) \) where all ordered observations take place and \( N \) the total number of such observations.

(Following Page we can evaluate \( N \sim e^{48} \)). If \( p_{ord} > p_{br} \) then we are ordered observers rather than BBs. Using (2) and (3) we get the Page’s result: \( t < 10^{60} \) yr if we are not BB.

It were suggested some ways to avoid this conclusions (see [42], [43], [44]). In particularly, Page in [45] has suggested the solution of this problem: Our vacuum should be rather unstable and should decay within 20 Gyr (this is possible if the gravitino is superheavy). He supposed that the decay of the universe proceeds at the rate, per 4-volume, of \( A \) for the nucleation of a small bubble that then expands at practically the speed of light, destroying everything within the causal future of the bubble nucleation event. It is possible if \( A > 20 \text{Gyr}^{-4} \).

In paper we consider the BB problem in frames of phantom cosmology. In Section II the simplest phantom cosmological models with Little Rip and Big Freeze singularity are described. The next section is devoted to analysis of compatibility of these models with observational data. The optimal parameters for models are calculated from various observational data such as SNe observations, BAO and Hubble parameter data. In Section IV the mechanism permitting the dominance of BB in these cosmological models is presented. In Conclusion section some outlook is given.

II. COSMOLOGICAL MODELS WITH LITTLE RIP AND BIG FREEZE SINGULARITY

Let’s try to understand how the BB problem can be solved in frames of phantom cosmology. We shall see that although Page solution for BB problem isn’t valid in this case but another mechanism of avoiding of this problem appear.

For simplicity one consider the class of phantom energy models with equation-of-state (EoS)

\[ p = -\rho - \alpha^2 \rho_0 \left( \frac{\rho}{\rho_0} \right)^\beta. \]  

(4)

Here \( \alpha^2 \) and \( \beta \) are positive constants, \( 0 \leq \beta \leq 1 \), \( \rho_0 \) is the phantom energy density in moment of observation. Therefore for EoS parameter \( w_0 \) we have simply

\[ w_0 = -1 - \alpha^2. \]

If \( \beta = 1 \) we have simplest phantom model with constant EoS parameter.

**Phantom model from sub-quantum potential.**

The case \( \beta = 0 \) corresponds to model of sub-quantum potential. Sub-quantum potential is interesting idea which allows one to describe the accelerating of the universe without any dynamics dark energy. In this model all the speeding-up effects taking place in our universe are entirely due to the quantum effects associated with, say, background radiation. The scale factor has the form

\[ a(t) = a_0 e^{V_{SQ}t^2/4+C_0 t}, \]  

(5)

where \( V_{SQ} = \alpha^2 \rho_0 \) is the sub-quantum potential and \( C_0 \) is some constant. This model describes the acceleration of the universe faster the in dS universe but without of the Big Rip singularity.

The time-dependent EoS parameter has the form

\[ w(t) = \frac{p(t)}{\rho(t)} = -1 - \frac{V_{SQ}}{3 (V_{SQ}t^2/2 + C_0)^2}. \]  

(6)

Thus \( w < -1 \) as in phantom universe, but \( w \to -1 \) as \( t \to \infty \).

It's easy to establish the dynamics of universe filled dark energy with EoS (4) using the Friedmann equations. We will examine the future evolution of our universe from the point at which the pressure and density are dominated
by the dark energy. One can derive the following link between energy density and time for EoS written in form 
\[ p = -\rho - f(\rho), \]

\[ t = \frac{1}{\sqrt{3}} \int_{\rho_0}^{\rho} \frac{d\rho}{f(\rho)}, \quad x \equiv \sqrt{\rho}. \]  

(7)

Thus for \( 0 \leq \beta \leq 1/2 \) the singularity doesn’t effectively occur: \( \rho \to \infty \) at \( t \to \infty \). For \( \beta > 1/2 \) the big rip singularity occurs.

**Phantom model with big freeze singularity.**

One can consider dark energy model with more “rigid” EOS which is close to \( \Lambda \)CDM-cosmology. This model firstly was considered in [46], [47]. From the following EoS

\[ p(\rho) = -\beta^2 a_f \rho^{1+\epsilon/3}, \]  

(8)

where \( \beta, a_f, \) and \( \epsilon \) are positive constants. One can find the dependence of the dark energy density from the scale factor

\[ \rho = \beta^{-6/\epsilon} (a_f^\epsilon - a^\epsilon)^{-3/\epsilon}. \]  

(9)

Therefore when scale factor reaches \( a = a_f \) the dark energy density becomes infinite: the big freeze occurs.

### III. OBSERVATIONAL DATA AND MODELS WITH LITTLE RIP AND BIG FREEZE SINGULARITY

It is well-known that simplest phantom model describes observational data with sufficient accuracy. One can consider the dark energy model with EoS (8) from viewpoint of observational data for \( \beta = 0 \) (sub-quantum potential). We shall see that this two model satisfies to observational tests. Therefore in principle the dark energy model with EoS (4) for \( 0 < \beta < 1 \) are compatible with observational data. Consideration of the dark energy model with EoS (8) also shows that this model is compatible with observational data.

We compare the model predictions with data from SNe observations, the evolution of the Hubble parameter and baryon acoustic oscillation. The realistic cosmological model should be take into account that dark energy is not a single component of the universal energy. We shall see that addition of dark matter allows one to construct the cosmological models which can be matched with the modern data of observations.

**SNe observations.** We use the data for dependence of SNe Ia modulus \( \mu \) as function of redshift \( z \) from the Supernova Cosmology project [11], [48]. The theoretical relation for flat universe filled dark energy and matter (for simplicity we neglect the radiation) is

\[ \mu(z) = \mu_0 + 5 \log D_L(z). \]  

(10)

where \( D_L(z) \) is a luminosity distance, that is

\[ D_L = \frac{c}{H_0}(1 + z) \int_0^z h^{-1}(z)dz, \quad h(z) = \left[ \Omega_{m0}(1 + z)^3 + \Omega_{D0}F(z) \right]^{1/2} \]  

(11)

Here, \( \Omega_{m0} \) is the total fraction of matter density, \( \Omega_{D0} \) the fraction of dark energy energy density, and \( H_0 \) is the current Hubble parameter. The function \( F(z) = \rho_D(z)/\rho_D0 \) can be determined from the continuity equation

\[ \dot{\rho}_D - 3\frac{\dot{a}}{a} (\rho_D + p_D) = 0. \]  

(12)

For analysis of observational data one can use the \( \chi^2 \) statistics. One need to perform a uniform marginalization over free parameter \( \mu_0 \). Expanding the \( \chi^2_{SN} \) with respect to \( \mu_0 \) gives

\[ \chi^2_{SN} = A - 2\mu_0 B + \mu_0^2 C, \]  

(13)

where

\[ A = \sum_i \left( \frac{\mu_{obs}(z_i) - \mu_{th}(z_i; \mu_0 = 0)}{\sigma_i^2} \right)^2, \]
\[
\begin{array}{ccc}
    z & H_{\text{obs}}(z) & \sigma_H \\
    \text{km s}^{-1} \text{ Mpc}^{-1} & \text{km s}^{-1} \text{ Mpc}^{-1} \\
    0.090 & 69 & 12 \\
    0.170 & 83 & 8 \\
    0.270 & 77 & 14 \\
    0.400 & 95 & 17 \\
    0.480 & 97 & 62 \\
    0.880 & 90 & 40 \\
    0.900 & 117 & 23 \\
    1.300 & 168 & 17 \\
    1.430 & 177 & 18 \\
    1.530 & 140 & 14 \\
    1.750 & 202 & 40 \\
\end{array}
\]

TABLE I: Hubble parameter versus redshift data from [50].

The expression (13) has a minimum for \( \mu_0 = B/C \) at \( \bar{\chi}^2_{SN} = A - B^2/C \).

We will minimize \( \bar{\chi}^2_{SN} \) instead of \( \chi^2_{SN} \). The 68.3\% confidence level is determined by \( \Delta \chi^2 = \chi^2 - \chi^2_{\text{min}} < 2.3 \) for for two-parametric model. Similarly, the 95.4\% confidence level is determined by \( \Delta \chi^2 = \chi^2 - \chi^2_{\text{min}} < 6.17 \) [49].

**Hubble parameter.** The measurements of \( dz/dt \) from fitting stellar population models help to determine the dependence of Hubble parameter as function of redshift

\[
H(z) = -\frac{1}{1+z} \frac{dz}{dt}
\]

We use the 11 datapoints for \( H(z) \) from [50] for constraining the model parameters. These data are listed in Table I. The theoretical dependence of the Hubble parameter is

\[
H(z) = H_0 h(z).
\]

(14)

The marginalization over the parameter \( H_0 \) can be performed as in a case of analysis of SNe observations. One can minimize the quantity

\[
\bar{\chi}^2_H = A_1 - B_1^2/C_1,
\]

where

\[
A_1 = \sum_i \frac{H_{\text{obs}}(z_i)^2}{\sigma_i^2}, \quad B_1 = \sum_i \frac{H_{\text{obs}}(z_i)}{\sigma_i^2}, \quad C_1 = \sum_i \frac{1}{\sigma_i^2}.
\]

**BAO data.** For more precise determination of parameters of cosmological models we use also the BAO data. We use the measurements of the acoustic parameter \( A(z) \) from [51], where the theoretically-predicted \( A_{\text{th}}(z) \) is given by the relation

\[
A_{\text{th}}(z) = \frac{D_V(z)H_0\sqrt{\Omega_m}}{z},
\]

(15)

where \( D_V(z) \) is a distance parameter defined as

\[
D_V(z) = \left\{ (1+z)^2d_A(z)\frac{cz}{H(z)} \right\}^{1/3}.
\]

(16)
Here, \( d_A(z) \) is the angular diameter distance

\[
d_A(z) = \frac{y(z)}{H_0(1+z)}, \quad y(z) = \int_0^z \frac{dz}{h(z)}.
\]  

(17)

Using Eqs. (15)-(17) we have

\[
A_{th}(z) = \sqrt{\Omega_m^0} \left( \frac{y^2(z)}{z^2 h(z)} \right).
\]  

(18)

Using the WiggleZ \( A_{obs}(z) \) data from Table 3 of [51], we compute \( \chi_A^2 \) as

\[
\chi_A^2 = \Delta A^T (C_A)^{-1} \Delta A.
\]  

(19)

Here, \( \Delta A \) is a vector consisting of differences, \( \Delta A_i = A_{th}(z_i) - A_{obs}(z_i) \) and \( C_A^{-1} \) is the inverse of the 3 × 3 covariance matrix given in Table 3 of [51].

**Optimal parameters for little rip model.**

For sub-quantum potential

\[
F(z) = 1 + 3(1 + w_0) \ln(1 + z)
\]  

(20)

Here

\[
w_0 = -1 - \frac{V_{SQ}}{3C_0^2}.
\]

For \( w_0 = -1.024 \) the parameter \( \chi^2 = \chi_{SN}^2 + \chi_H^2 + \chi_r^2 \) reaches the minimal value 560.92 (\( \Omega_D = 0.713 \)). In the case of \( \Lambda \)CDM cosmology we have \( \chi_{min}^2 = 561.31 \) (\( \Omega_\Lambda = 0.712 \)). The value of \( w_0 \) lies in interval \(-1.125 \leq w_0 \leq -1.000 \) with 68% confidence level and in interval \(-1.185 \leq w_0 \leq -1.000 \) with 95% confidence level.

**Optimal parameters for model with big freeze singularity.**

For dimensionless Hubble parameter as function of redshift we have therefore

\[
F(z) = (1 + z)^3 \left( \frac{N_0 - 1}{N_0(1+z)^\epsilon - 1} \right)^{3/\epsilon}, \quad N_0 = (a_f/a_0)^\epsilon.
\]  

(21)

One can see that for large \( N_0 \) our model mimics \( \Lambda \)CDM cosmology with excellent precision. Therefore our model can fit the Supernova Cosmological Project data. For \( N_0 \gg 1 \) the dark energy density is nearly constant in the interval \( 0 < t < t_0 \), i.e. the model mimics a cosmological constant in the past but it leads to a finite-time future singularity.

The current EoS parameter is

\[
w_0 = - \frac{N_0}{N_0 - 1}
\]

For given value of \( w_0 \) and \( \epsilon \) one can find that such model describes the observational data with good accuracy. For example if \( \epsilon = 2 \) the parameter \( \chi^2 \) reaches the minimal value 561.09 for \( w_0 = -1.05 \) and \( \Omega_D = 0.71 \). In this case the time before final singularity is nearly 18 Gyr. For \( \epsilon = 5 \) we have the \( \chi_{min}^2 = 561.07 \) at \( w_0 = -1.08 \) and \( \Omega_D = 0.709 \). The big freeze occurs after \( \sim 7 \) Gyr. Therefore the model coincides with \( \Lambda \)CDM model in past with excellent precision but its future evolution shows radically different dynamics: universe ends quickly its existence in big freeze singularity.

**IV. RESOLUTION OF BB PROBLEM IN PHANTOM COSMOLOGY**

Let’s show that in universe with sub-quantum potential the Page’s solution of BB problem is incorrect.

To show this let suppose that the present value of \( w(0) = w_0 = -1 - \epsilon/3 \) with \( \epsilon = V_{SQ}/H_0^2 \) and the present value of Hubble roots is \( H_0 \). Then one can use (19) and (20) to present the scale factor in the form

\[
a(t) = a_0 \exp \left( \frac{H_0 t}{4} (\epsilon H_0 t + 4) \right).
\]  

(22)
Substituting (22) in (3) one get

\[ V_4(t) = \frac{i c a_0^3 e^{-3/\epsilon}}{H_0} \sqrt{\frac{\pi}{3\epsilon}} \left( \text{erf} \left[ i \left( \frac{3}{\epsilon} + \frac{\sqrt{3} H_0 t}{2} \right) \right] - \text{erf} \left[ i \sqrt{\frac{3}{\epsilon}} \right] \right). \]  

(23)

For large value of \( H_0 t = \tau \) one can estimate the expression (23) as

\[ V_4(\tau) \sim \exp \left[ \frac{3\epsilon \tau^2}{4} \right]. \]  

(24)

Now, let consider the Page’s bubble-killer. Let us take the case in which the decay of the universe proceeds by the nucleation of a small bubble that then expands at practically the speed of light, destroying everything within the causal future of the bubble nucleation event. Suppose that the bubble nucleation rate, per 4-volume, is \( A \). The probability that the spacetime would have survived to the event \( Q \) is

\[ P(Q) = e^{-AV_4(Q)}, \]

where \( V_4(Q) \) is the spacetime 4-volume to the past of the event \( Q \) in the background spacetime.

The requirement that there not be an infinite expectation value of vacuum fluctuation observations within a finite comoving 3-volume is the requirement that

\[ \int d^4x \sqrt{-g} P(Q) < \infty. \]  

(25)

In the case of the dS universe (as Page shown) the requirement (25) is valid if and only if \( \lambda > 20 \text{Gyr}^{-4} \) [43]. Now let consider the universe with sub-quantum potential and scale factor (22). One can use the the conformal time \( \eta \)

\[ \eta = \frac{cT}{a_0} \sqrt{\frac{\pi}{\epsilon}} e^{1/\epsilon} \left[ \text{erf} \left( \frac{\epsilon \tau + 2}{2\sqrt{\epsilon}} \right) - \text{erf} \left( \frac{1}{\sqrt{\epsilon}} \right) \right], \]

where \( T = 1/H_0 \). Then we have the flat FRW metric

\[ ds^2 = a^2(\eta) \left( d\eta^2 - dr^2 - r^2 d\Omega^2 \right), \]

and

\[ V_4(Q) = \frac{4\pi}{3} \int_0^\eta d\eta' a^4(\eta') (\eta - \eta')^3. \]

Thus the probability that the spacetime would have survived to the event \( Q \) is

\[ P(Q) = \exp \left[ - \frac{A}{A_m} \int_0^\tau d\xi e^{3\xi(x + 4)/4} \left( \text{erf} \left( \frac{\epsilon \tau + 2}{2\sqrt{\epsilon}} \right) - \text{erf} \left( \frac{\epsilon x + 2}{2\sqrt{\epsilon}} \right) \right) \right], \]  

(26)

where

\[ A_m = \frac{3\epsilon^{3/2} e^{-3/\epsilon}}{4\pi^{5/2}(cT)^4}. \]

Using the expression (26) we conclude that the requirement (25) is valid if and only if

\[ J \equiv \int_{1/\sqrt{\epsilon}}^{+\infty} d\xi \exp \left[ 3\xi^2 - \frac{A'}{A_m} \int_{1/\sqrt{\epsilon}}^\xi dz e^{3z^2} (\text{erf} z - \text{erf} z)^3 \right] < \infty, \]  

(27)

with \( A' = 2Ae^{-3/\epsilon}/\sqrt{\epsilon} \).

One can show that the the requirement (27) can’t be valid, since the second integral in (27) is finite as \( \xi \to \infty \). Thus \( J = \infty \) for any values of the bubble nucleation rate, per 4-volume (i.e. \( A \)). It is possible to describe a situation so: the universe with sub-quantum potential extends so fast that even the bubble growing with speed of light can’t destroy this one. Of course this conclusion is right for dark energy model with EoS (4) for arbitrary \( 0 < \beta \leq 1 \): the expansion of universe at \( \beta > 0 \) occurs more quickly than in a case of sub-quantum potential.
Thus, the Page’s mechanism does not work. Nevertheless, there is other way to decide the paradox of BB. Page’s
doomsday argument is true only in an universe which contains human-observers. But such observers can exist in
the universe filled with phantom energy only up to a certain period. The upper bound $t_{\text{max}}$ can be obtained by the
Bekenstein bound (see below). Therefore if $V_4(t_{\text{max}}) > V_4(\text{cr})$ then we have problems with such models in the light of
Page’s doomsday argument. As we shall see, this is the case only if the "fine-tuning" of $w$ take place. In the case of
general position (i.e. without "fine-tuning") cosmological models with phantom energy don’t suffer from the Page’s
doomsday argument therefore such models are more realistic then models with $\Lambda$-term.

The Bekenstein bound [52] shows that the total amount of information, which can be stored in region of radius $R$
is
$$I < I_m = 2.58 \times 10^{43} \left( \frac{M}{1 \text{ kilogram}} \right) \left( \frac{R}{1 \text{ meter}} \right) \text{ bits.} \quad (28)$$
Substituting $\rho_0 = \rho_c = 10^{-29}$ gramme/cm$^3$, $R = c/H_0$, and $H_0 \sim 70$ km/s/Mps (the current measured value of the
Hubble constant) results in
$$I_m = 0.33 \times 10^{123} \text{ bits.} \quad (29)$$
In general case, the horizon distance can be calculated as
$$R_c(t) = a(t) \int_t^{t_f} \frac{cdt'}{a(t')} \quad (30)$$
In the case of sub-quantum model we get
$$R_c(\tau) = R_{dS} e^{1/\epsilon} \sqrt{\frac{\pi}{\epsilon}} e^{(\epsilon \tau + 4)/4} \left[ 1 - \text{erf} \left( \frac{\epsilon \tau + 2}{2 \sqrt{\epsilon}} \right) \right], \quad (31)$$
where $\tau = H_0 t$, $R_{dS} = c/H_0$. At present time
$$R_c(0) = R_{dS} e^{1/\epsilon} \sqrt{\frac{\pi}{\epsilon}} \left( 1 - \text{erf} \left( \frac{1}{\sqrt{\epsilon}} \right) \right).$$
For large values of $\tau$ we get more simple expression
$$R_c(t) \sim \frac{2R_{dS}}{\epsilon \tau}, \quad (32)$$
so $R_c(t) \to 0$ as $t \to \infty$.

Now, using (32), (24) and the Page requirement
$$V_4(t) < NV_4(\text{br}) e^{S_{\text{br}}/h},$$
we get
$$\frac{3R_{dS}^2}{\epsilon R_c^2} < \frac{S_{\text{br}}}{h}, \quad (33)$$
that result in low limit on the parameter $\epsilon$:
$$\epsilon > \epsilon_{\text{min}} = \frac{3R_{dS}^2 h}{R_c^2 S_{\text{br}}}. \quad (34)$$
From the (25) one can obtain the low limit on the value of $R_c$:
$$R_c \geq \frac{h I \log 2}{2\pi Mc} \quad (35)$$
where $I$ is the amount of information encoded in a human-observer. Using (34) and (35) we get
$$\epsilon_{\text{min}} \leq 5 \times 10^{86} \frac{M}{I^2}.$$
For the $M = 100$ kg and $I \sim 10^{45}$ bits (the upper amount of information encoded in a human-observer) we get from the (36)\hfill

\[ \epsilon > \epsilon_{\text{min}} = 0.019, \]

or

\[ w_0 < -1.006. \]

From previous section one can see that this bound on $w_0$ doesn’t conflict with observational data.

**Simplest phantom model.** It is interesting to note that the same mechanism permitting to avoid dominance of BB in simplest phantom model with constant EoS parameter.

Integration of the Einstein-Friedmann equation for the flat universe filled phantom energy with $w_0 = -1 - \alpha^2$ results in

\[ a(t) = \frac{a_0}{(1 - \xi t)^{2/3\alpha^2}}, \]

\[ \rho(t) = \rho_0 \left( \frac{a(t)}{a_0} \right)^{3\alpha^2} = \frac{\rho_0}{(1 - \xi t)^2}, \]

where $\xi = \alpha^2 \sqrt{6\pi G \rho_0}$. We choose $t = 0$ as the present time, $a_0 \sim 10^{28}$ cm and $\rho_0$ to be the present values of the scale factor and the density. There, if $t = t_f = 1/\xi$, we automatically get the big rip. Using (39) one can calculate $V_4(t)$, as

\[ V_4(t) = \frac{\alpha^2 c a_0^3}{\xi(2 - \alpha^2)} \left( -\frac{1}{(1 - \xi t)^{(2 - \alpha^2)/\alpha^2}} - 1 \right). \]

Substituting (39) into the (30) gives

\[ R_c(t) = \frac{3\alpha^2 c(1 - \xi t)}{(2 + 3\alpha^2)\xi}, \]

for the case of phantom field. This, of course, means that $R_c(t) \rightarrow 0$ as $t \rightarrow t_f$. Finally, the Bekenstein Bound (28) results in

\[ I < I_m(t) \sim \frac{2.74R_c^4(t)\rho_0}{(1 - \xi t)^2} \times 10^{43}. \]

Using the superior limit of amount of information encoded in a human-observer one get (using (42) and (41))

\[ \eta \equiv 1 - \xi t > \frac{0.67\xi^2(2 + 3\alpha^2)^2}{c^2\alpha^2 \sqrt{\rho_0}}. \]

On the other hand, using (3) and the condition $V_4(t) < V_4(\text{cr})$ one get

\[ \eta < \left( \frac{\alpha^2 a_0^3 c}{\xi(2 - \alpha^2)V_4(\text{cr}) + \alpha^2 a_0^3 c} \right)^{\alpha/(2 - \alpha)}. \]

Combining (43) and (44) we have

\[ \frac{4\pi G(2 + 3\alpha^2)^2 \sqrt{\rho_0}}{c^2} < \left( \frac{a_0^3 c}{(2 - \alpha^2)V_4(\text{cr})\sqrt{6\pi G \rho_0} + a_0^3 c} \right)^{\alpha^2/(2 - \alpha^2)}. \]

Choosing $\alpha^2 \ll 1$ one get

\[ -88.48 < \log \eta < -\frac{\alpha^2}{2} \times 10^{50}, \]

so

\[ \alpha^2 < 1.77 \times 10^{-48}. \]
It is very unlikely that $\alpha^2$ is so small accurate to $10^{-48}$! Therefore it is unlikely that we have some problems with Page’s doomsday argument in phantom universe. It is interesting that if (46) is the case then $t_f > 5 \times 10^{57}$ yr.

Let consider this situation by another way. The total history of universe can be divided on two parts. The first one is the ”observable universe” which can contain human-observers and the second ”unobservable universe” where human-observers can’t exist. The $t_{\text{observ}}$ can expressed from the (43) while the total lifetime of universe $t_{\text{total}} = t_f = 1/\xi$. Therefore

$$\frac{t_{\text{observ}}}{t_{\text{total}}} < 1 - 0.93 \times 10^{-39} (2 + 3\alpha)^2.$$  

It is easy to see that for $0 < \alpha^2 < 1$ this ratio will be 1 accurate to $10^{-39}$, so the universe is ”observable” one during virtually whole it’s history. On the other hand, as we seen above, in ”observable universe” filled with phantom energy our ordered observations would be highly typical in contrast to universe filled with positive cosmological constant where our ordered observations would be highly atypical.

Therefore one can see that model with EOS (4) is free from BB for $\beta = 0$ (sub-quantum potential) and $\beta = 1$ (phantom model with constant $w$). It is obviously that the same mechanism resolves the BB problem in the case of $0 < \beta < 1$ because the cosmological dynamics for such models lies between little and big rip dynamics. Of course this conclusion is right in a case of big freeze model considered above.

V. CONCLUSION

The problem of Boltzmann brains in frames of phantom models with big freeze and little rip is considered. These models are compatible with observational tests even slightly better than ΛCDM-model and free from BB problem. The resolution of BB problem is achieved due to Bekenstein bound leading to separation of universe history on two parts in fact. The fraction of BB in ”observational” part (in which the human observers can exist) is negligible in comparison with ordered observers. The analysis of observational data shows that allowable range of $w_0$ includes such values at which the ”observable universe” consists of ordered observations mainly.
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