ON THE ORIGIN OF THE SCATTER BROADENING OF FAST RADIO BURST PULSES AND ASTROPHYSICAL IMPLICATIONS

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ABSTRACT

Fast radio bursts (FRBs) have been identified as extragalactic sources that can probe turbulence in the intergalactic medium (IGM) and their host galaxies. To account for the observed millisecond pulses caused by scatter broadening, we examine a variety of possible electron density fluctuation models in both the IGM and the host galaxy medium. We find that a short-wave-dominated power-law spectrum of density, which may arise in highly supersonic turbulence with pronounced local dense structures of shock-compressed gas in the host interstellar medium (ISM), can produce the required density enhancements at sufficiently small scales to interpret the scattering timescale of FRBs. This implies that an FRB residing in a galaxy with efficient star formation in action tends to have a broadened pulse. The scaling of the scattering time with the dispersion measure (DM) in the host galaxy varies in different turbulence and scattering regimes. The host galaxy can be the major origin of scatter broadening, but contributes to a small fraction of the total DM. We also find that the sheet-like structure of the density in the host ISM associated with folded magnetic fields in a viscosity-dominated regime of magnetohydrodynamic (MHD) turbulence cannot give rise to strong scattering. Furthermore, valuable insights into the IGM turbulence concerning the detailed spatial structure of density and magnetic field can be gained from the observed scattering timescale of FRBs. Our results favor the suppression of micro-plasma instabilities and the validity of the collisional-MHD description of turbulence properties in the collisionless IGM.

Key words: intergalactic medium – radio continuum: general – turbulence

1. INTRODUCTION

A population of bright millisecond radio transients known as fast radio bursts (FRBs) has been discovered and has attracted increasing attention in recent years (e.g., Lorimer et al. 2007; Thornton et al. 2013; Masui et al. 2015; Keane et al. 2016; Petroff et al. 2016; Spitler et al. 2016). The large dispersion measure (DM) values and high Galactic latitudes of these events provide strong observational evidence of their extragalactic origin (e.g., Katz 2016b).

As one of the important observational parameters of FRBs, the pulse broadening timescale (i.e., pulse width with the intrinsic timescale subtracted) is a result of the multi-path scattering during the propagation of radio waves through a turbulent medium. The Galactic contribution in pulse broadening can be easily eliminated since the Galactic pulsars at high latitudes visually possess orders of magnitude smaller broadening timescales than FRBs (Bhat et al. 2004; Krishnakumar et al. 2015; Cordes et al. 2016; Katz 2016b).\(^4\) Non-Galactic contributions may arise from the intergalactic medium (IGM) and the host galaxy medium. The empirical relation between the scattering measure and DM in the IGM estimated by Macquart & Koay (2013) demonstrates that the scattering per unit DM in the IGM is orders of magnitude smaller than that in the Galactic interstellar medium (ISM). The possibility of prominent intergalactic scattering was disputed by Luan & Goldreich (2014) because of the incompatibility between the excessive heating of Kolmogorov turbulence with a small outer scale and inefficient cooling of the IGM. The IGM was also disfavored as the location of scattering by Katz (2016a, 2016b) based on the non-monotonic dependence of pulse widths on the intergalactic dispersion.\(^5\) Apart from the above arguments, according to the catalog of known FRBs provided by Petroff et al. (2016), some FRBs have a scattering time longer than 1 ms (at 1 GHz), while the others have unresolved scattering tails, for which the upper limit is set by the time resolution, but the actual scattering can be much weaker (\(<1\) ms). Intuitively, the observational facts that some FRBs have greater DMs but narrower pulses and that both resolved and unresolved pulses exist imply that the scatter broadening is not a common feature originating from the IGM that every FRB pulse traverses through, but more likely attributed to the diverse environments local to FRBs. That is, the host galaxy is the most promising candidate for interpreting the strong scattering events (see Yao et al. 2016 for a different point of view). However, the host contribution depends on the progenitor location and line-of-sight (LOS) inclination. It is expected to be negligibly small for sightlines passing through a host galaxy’s outskirts, similar to the case of our Galaxy at high Galactic latitudes. For this reason, it was suggested that the pulse broadening is produced by the highly turbulent and dense medium in the immediate vicinity of the FRB (Katz 2016b). But since the scattering material is in strong association with the burst, the resulting pulse width is likely entirely intrinsic, and the scenario is restricted to specific FRB progenitor models involving young stellar populations (Masui et al. 2015; Spitler et al. 2016).

\(^4\) For the low-latitude FRBs, i.e., FRB 010621 (Keane et al. 2012), FRB 150418 (Keane et al. 2016), and FRB 121102 (Spitler et al. 2014), only upper limits on the broadening time are available. Even for these FRBs, the NE2001 model of Galactic scattering (Cordes & Lazio 2002) predicts that the Galactic contribution to the scattering timescale is below the threshold of detectability.

\(^5\) One caveat of this argument is that the empirical scattering measure-DM relationship in the Milky Way has a large dispersion (Johnston et al. 1998; Bhat et al. 2004). However, after correcting for such a scatter, Caleb et al. (2016) still could not interpret the FRB scattering data.
A proper interpretation of the temporal broadening of FRBs entails comprehensive modeling of the electron density fluctuations and related turbulence properties in both the ISM and IGM. A Kolmogorov spectrum of both velocity and magnetic fluctuations was predicted by the Goldreich & Sridhar (1995) theory for Alfvénic turbulence and later confirmed by magnetohydrodynamic (MHD) simulations (Cho & Vishniac 2000; Maron & Goldreich 2001; Cho & Lazarian 2003; Beresnyak & Lazarian 2009). The observationally measured electron density power spectrum in the diffuse ionized ISM is also consistent with a Kolmogorov-like power law over a wide range of scales spanning over 10 decades, known as “the big power law in the sky” (Armstrong et al. 1995; Chepurnov & Lazarian 2010). In earlier studies on the scatter broadening of FRB pulses (e.g., Macquart & Koay 2013; Luan & Goldreich 2014; Cordes et al. 2016), the Kolmogorov model of turbulence has been commonly adopted. However, the spectral form of density fluctuations can be affected by the magnetization and compressibility of the local turbulent medium. The density fluctuations do not track the Kolmogorov velocity spectrum, but exhibit a steeper spectrum in a strongly magnetized subsonic turbulence, and a shallower one in supersonic turbulence (Beresnyak et al. 2005; Kowal et al. 2007). As a general result of both compressible MHD simulations and hydrodynamic simulations, supersonic turbulence effectively generates a complex system of shocks that correspond to regions of converging flows and concentration of mass (Padoan et al. 2001, 2004; Kim & Ryu 2005; Kritsuk et al. 2006). Kim & Ryu (2005) explicitly showed that the density power spectrum becomes shallower with increasing sonic Mach number $M_s$. Notice that $M_s$ varies in different ISM phases. The warm ionized medium (WIM) has $M_s$ of the order of unity (Haffner et al. 1999; Kim & Ryu 2005; Hill et al. 2008) and hence Kolmogorov density distribution (Armstrong et al. 1995; Chepurnov & Lazarian 2010), while in other colder and denser phases in the inner Galaxy with a higher compressibility (i.e., larger $M_s$, Larson 1981; Heiles & Troland 2003), a shallower density spectrum is naturally expected (Kim & Ryu 2005; Burkhardt et al. 2015). Significant deviation from the Kolmogorov law and flattening of the density spectrum are indicated from, e.g., spectroscopic observations (Lazarian 2006, 2009; Hennebelle & Faiglerone 2012), scattering measurements of the Galactic pulsars with high DMs (Löhmer et al. 2001, 2004; Bhat et al. 2004; Lewandowski et al. 2013, 2015; Krishnakumar et al. 2015; Xu & Zhang 2016b), and rotation measure fluctuations at low Galactic latitudes (Haverkorn et al. 2004, 2008; Xu & Zhang 2016a). Accompanying the shallowness of the spectral slope of density fluctuations, substantial discontinuous structures in density emerge at small scales due to supersonic compressions. The corollary is to significantly strengthen the scatter-broadening effect. Besides the spectral slope, the distinct properties of turbulence in different ISM phases are also manifested in the volume filling factor of density structures. The volume filling factor of cold and dense phases, such as the cold neutral medium and molecular clouds, is smaller than that of the WIM by order(s) of magnitude (Tielens 2005; Haverkorn & Spangler 2013). The small-scale overdense structures embedded in these phases produced by the supersonic turbulence are supposed to have a further smaller filling factor.

In view of the theoretical arguments and observational facts, we consider the spectrum of density fluctuations with a much shallower slope than the Kolmogorov one as a physically motivated possibility of inducing enhanced scattering. Moreover, we also take into account the microscale density fluctuations associated with the microphysical properties of turbulence, which include the density perturbations caused by the mirror instability in the collisionless regime of MHD turbulence (Hall 1980), and the sheet-like configuration of density generated by the magnetic folds in the viscosity-damped regime of MHD turbulence (Goldreich & Sridhar 2006; Lazarian 2007). We will examine whether they can serve as an alternative source of strong scattering.

In this work, to identify the separate roles of the IGM and host galaxy in temporal smearing and probe the environmental conditions of FRBs, we examine the scattering effect of different models of electron density fluctuations pertaining to distinct turbulence regimes, including a detailed analysis on both the Kolmogorov and shallower density power spectra, and an exploratory investigation on other not well-determined but potentially important models of density structures. On the other hand, with the radio signals traveling across cosmological distances, the investigation of the scatter broadening of FRBs offers a promising avenue for probing the IGM turbulence, which remains a highly controversial and elusive subject concerning whether a collisional-MHD treatment is still valid for the dynamics of the weakly collisional IGM (Santos-Lima et al. 2014) or the large-scale dynamics is dramatically affected by the microscale instabilities (Schekochihin et al. 2008).

This paper is organized into four sections. In Section 2, we focus on the power-law model of electron density fluctuations and the effect of shallowness of the spectral slope on temporal broadening. In Section 3, we generalize the analysis and evaluate the scattering strength of other alternative models of electron density fluctuations on the basis of the observed scattering timescale. Implications and conclusions are presented in Section 4.

2. ELECTRON DENSITY FLUCTUATIONS ARISING FROM A TURBULENT CASCADE

2.1. Temporal Broadening

A power-law spectrum of the plasma density irregularities is commonly applied in studies on radio wave propagation (Lee & Jokipii 1976; Rickett 1977, 1990), which is also reinforced by growing observational evidence of interstellar density fluctuations (Armstrong et al. 1995; Chepurnov & Lazarian 2010). We assume that the scattering effect is introduced by electron density fluctuations that arise from a turbulent cascade and the relevant spectrum takes the form (Rickett 1977; Coles et al. 1987)

$$P(k) = C_0^2 k^{-\beta} e^{-\left(k \theta_0 \right)^2} , \quad k > L^{-1} ,$$  

(1)

which is cast as a power-law spectrum in the inertial range of turbulence,

$$P(k) = C_0^2 k^{-\beta} , \quad L^{-1} < k < \theta_0^{-1} ,$$  

(2)

where $L$ and $\theta_0$ are the outer and inner scales, corresponding to the injection and dissipation scales of turbulent energy. The spectral index $\beta$ is suggested to be within the range $2 < \beta < 4$ on observational grounds (e.g., Lee & Jokipii 1975; Rickett 1977; Romani et al. 1986). Intuitive insight to the properties of turbulence can be gained from the value of $\beta$. At the critical index $\beta = 3$, density fluctuations, which scale as
$\delta n_e \propto k^{3-\beta/2}$, are scale-independent. That is, the density fluctuations with the same amplitude exist at all scales. Notice that $\delta n_e$ represents the rms amplitude of density fluctuations. Following the power-law statistics studied in, e.g., Lazarian & Pogosyan (2000, 2004, 2006) and Esquivel & Lazarian (2005), we consider the density spectrum in both the long-wave-dominated regime with $\beta > 3$ and the short-wave-dominated regime with $\beta < 3$. The density field in the former case is dominated by large-scale fluctuations, but in the latter case is localized in small-scale structures.

Both long- and short-wave-dominated density spectra are a confirmed reality in compressible MHD turbulence (Beresnyak et al. 2005; Kowal et al. 2007). In the WIM phase of Galactic ISM, which corresponds to the transonic turbulence, the power-law spectrum of electron density fluctuations has been convincingly demonstrated to have a unique slope consistent with the Kolmogorov spectrum ($\beta = 11/3$) on scales spanning from $10^6$ to $10^{17}$ m (Armstrong et al. 1995; Chepurnov & Lazarian 2010). On the other hand, in colder and denser phases of the ISM in the Galactic plane, the turbulence becomes highly supersonic and shocks are inevitable, which produce large density contrasts and a short-wave-dominated density spectrum (Kim & Ryu 2005). The density spectra with $\beta < 3$ have been extracted from ample observations by using different tracers and techniques (e.g., Stutzki et al. 1998; Deshpande et al. 2000; Swift 2006; Xu & Zhang 2016a; also see Table 5 in the review by Lazarian 2009 and Figure 10 in the review by Hennebelle & Falgarone 2012). In addition, the Kolmogorov density spectrum also fails to reconcile with the observationally measured scaling relation between scatter-broadening time and frequency for highly dispersed pulsars (e.g., Löhmer et al. 2001, 2004; Bhat et al. 2004; Lewandowski et al. 2013). In view of the diversity of ISM phases and properties of the associated turbulence, it is necessary to perform a general analysis incorporating both long- and short-wave-dominated spectra of density fluctuations.

The normalization of the power spectrum depends on the steepness of the slope. From the density variance

$$\langle (\delta n_e)^2 \rangle = \int P(k) d^3k$$

and assuming $L \gg l_0$, we find

$$C_N^2 \sim \frac{\beta - 3}{2(2\pi)^{4/3}} (\delta n_e)^2 L^{3-\beta}, \quad \beta > 3,$$

$$C_N^2 \sim \frac{3 - \beta}{2(2\pi)^{4/3}} (\delta n_e)^2 l_0^{3-\beta}, \quad \beta < 3.$$  (4a)  (4b)

It shows that the turbulent power characterized by density perturbation $\delta n_e$ concentrates at $L$ for $\beta > 3$ and $l_0$ for $\beta < 3$. Thus $\delta n_e$ is the density perturbation at the correlation scale of turbulence, which is $L$ for a long-wave-dominated spectrum and $l_0$ for a short-wave-dominated spectrum.

As the radio waves propagate through a turbulent plasma, multi-path scattering causes temporal broadening of a transient pulse (e.g., Rickett 1990; Cordes & Rickett 1998). On a straight-line path of length $D$ through the scattering medium, the integrated phase structure function is defined as the mean square phase difference between a pair of LOSs with a separation $r$ on the plane transverse to the propagation direction, $D_0 = \langle (\Delta \Phi)^2 \rangle$. Given the spectral form of Equation (1) with $2 < \beta < 4$, and under the condition $r \ll L \ll D$, $D_0$ has expressions (Coles et al. 1987; Rickett 1990)

$$D_0 \sim \pi r_c^2 \lambda^2 S M_0^{\beta - 2} r^2, \quad r < l_0,$$  (5a)

$$D_0 \sim \pi r_c^2 \lambda^2 S M_0^{\beta - 2} r^2, \quad r > l_0,$$  (5b)

where $r_c$ is the classical electron radius and $\lambda$ is the wavelength. The scattering measure $S M$ is the integral of $C_N^2$ along the LOS path through the scattering region, and characterizes the scattering strength. Here we consider a statistically uniform turbulent medium, with the turbulence properties independent of the path length. Thus the SM is simplified as

$$S M \sim C_N^2 D.$$  (6)

By applying $C_N^2$ expressed in Equation (4) in the SM, the structure function $D_0$ is applicable for both a long-wave-dominated spectrum of turbulence on scales below the correlation scale ($L$) and a short-wave-dominated spectrum on scales above the correlation scale ($l_0$). In the WIM phase of Galactic ISM, which corresponds to the transonic turbulence, the power-law spectrum of electron density

$$D_0 \sim \pi r_c^2 \lambda^2 S M_0^{\beta - 2} r^2, \quad r < l_0.$$  (7a)

$$D_0 \sim \pi r_c^2 \lambda^2 S M_0^{\beta - 2} r^2, \quad r > l_0.$$  (7b)

In a particular case when $r_{\text{diff}}$ coincides with $l_0$, equaling $r_{\text{diff}}$ from the above equation and $l_0$ yields

$$r_{\text{diff}} \sim (\pi r_c^2 \lambda^2 S M_0^{\beta - 2})^{-1}, \quad r_{\text{diff}} < l_0.$$  (8)

It means that the physical parameters involved in the scattering process should satisfy the condition

$$r_{\text{diff}} \sim (\pi r_c^2 \lambda^2 S M_0^{\beta - 2})^{-1}, \quad \text{for } r_{\text{diff}} < l_0.$$  (9)

for $r_{\text{diff}}$ to be smaller than $l_0$, and

$$r_{\text{diff}} \sim (\pi r_c^2 \lambda^2 S M_0^{\beta - 2})^{-1}, \quad \text{for } r_{\text{diff}} > l_0.$$  (10)

In the presence of the inner scale of the density power spectrum, $D_0$ exhibits a break in the slope at $r = l_0$ and steepens at smaller scales. The quadratic scaling of $D_0$ with $r$ at $r < l_0$ comes from the Gaussian distribution of density fluctuations $\exp(-k^2 r_c^2)$ below the inner scale (Equation (1)).

For the multi-path propagation in the strong scattering regime, $r_{\text{diff}}$ characterizes the coherent scale of the random phase fluctuations and the density perturbation on $r_{\text{diff}}$ dominates the scattering strength, with the angular and 6

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6. Besides introducing the power-law spectrum in Fourier space, the structure function can be also derived by employing the real-space statistics. For instance, the rotation measure structure function calculated by using the correlation function within the inertial range of turbulence in Lazarian & Pogosyan (2016) has the scaling consistent with that shown in Equation 5(b) (see Equations (148) and (149) in Lazarian & Pogosyan 2016) in the case of a thick Faraday screen.
temporal broadening given by (Rickett 1990; Narayan 1992)
\[ \theta_{\text{sc}} = \frac{\lambda}{2\pi r_{\text{diff}}}, \]  
and
\[ \tau_{\text{sc}} = \frac{D}{c} \theta_{\text{sc}}^2 = \frac{D_{\text{eff}}^2 \lambda_0^2}{4 \pi^2 c (1 + z_q)^2} r_{\text{diff}}^{-2}. \]  
The above formulae pertain to the Galactic scattering medium, but should be modified when the scattering plasma is located at a cosmological distance. In the observer’s frame, the wavelength is \( \lambda_0 = \lambda(1 + z_q) \), where \( z_q \) is the redshift of the scattering material. By also taking into account the LOS weighting, which depends on the location of the scattering material along the LOS (Gwinn et al. 1993; Macquart & Koay 2013), the temporal broadening becomes
\[ W = \tau_{\text{sc, obs}} = (1 + z_q) \frac{D_{\text{eff}}^2 \lambda_0^2}{4 \pi^2 c} r_{\text{diff}}^{-2}. \]  

Here \( D \) in Equation (13) is replaced by the effective scattering distance \( D_{\text{eff}} = D_{\text{eff}} D_{\text{qp}}/D_{\text{p}} \), with \( D_{\text{p}}, D_{\text{qp}}, \) and \( D_{\text{d}} \) as the angular diameter distances from the observer to the source, from the source to the scattering medium, and from the observer to the scattering medium. Accordingly, SM is also replaced with the weighted SM as adopted in Cordes & Lazio (2002),
\[ \text{SM} \sim C_D D_{\text{eff}}. \]  

\( D_{\text{eff}} \) is comparable to \( D_{\text{d}} \) in the case of Galactic scattering, and comparable to \( D_{\text{qp}} \) when the scattering medium is close to the source. In both cases, \( D_{\text{eff}} \) serves as a good approximation of the path length through the scattering region, and thus Equation (15) is appropriate for estimating the actual SM. But we caution that for a thin scattering screen located somewhere between the source and the observer, its thickness, i.e., the path length that should be used for calculating SM, is in fact far smaller than the value of \( D_{\text{eff}} \).

In combination with Equations 4(a), 7, and 15, the approximate expression of \( W \) in the case of \( \beta > 3 \) can be obtained from Equation (14),
\[ W \sim \frac{D_{\text{eff}}^2 \lambda_0^4 \beta^4}{4 \pi^2 c} (1 + z_q)^2 \left( \frac{\delta n_e}{n_e} \right)^2 \left( \frac{L}{L_0} \right)^{\beta - 4} L^{-1}, \quad r_{\text{diff}} < l_0, \]  
and
\[ W \sim \frac{D_{\text{eff}}^2 \lambda_0^4 \beta^4}{4 \pi^2 c} (1 + z_q)^2 \left( \frac{\delta n_e}{n_e} \right)^2 L^{-1/2}, \quad r_{\text{diff}} > l_0. \]  
The observationally measured wavelength dependence of the pulse width can make a distinction between the scenarios with \( r_{\text{diff}} \) below or exceeding \( l_0 \), which, however, is limited by the insufficient accuracy of the current data (Thornton et al. 2013; Luan & Goldreich 2014). Nevertheless, it is evident that in both situations \( W \) decreases with increasing \( L \). A given pulse width imposes a constraint on the outer scale of turbulence. In particular, when \( r_{\text{diff}} < l_0 \), \( W \) also decreases with increasing \( l_0 \). Moreover, in terms of the dispersion measure \( DM = n_e D_{\text{eff}} \) of the scattering medium, where \( n_e \) is the electron density averaged along the LOS passing through the scattering region, \( W \) in Equation (16) is rewritten as
\[ W \sim \frac{r_e^2 \lambda_0^4}{4 \pi c} \left( 1 + z_q \right)^3 \left( \frac{\delta n_e}{n_e} \right)^2 \left( \frac{L}{L_0} \right)^{\beta - 4} L^{-1} \text{DM}^2, \quad r_{\text{diff}} < l_0, \]  
and
\[ W \sim \frac{r_e^2 \lambda_0^4}{4 \pi^2 c} \left( 1 + z_q \right)^2 \left( \frac{\delta n_e}{n_e} \right)^2 \left( \frac{L}{L_0} \right)^{\beta - 4} \times \left( \frac{L}{L_0} \right)^{\beta - 4} \text{DM}^{-2}, \quad r_{\text{diff}} > l_0. \]  

In the case of \( \beta < 3 \), from Equations 4(b), 7, and 15, \( W \) can be estimated as
\[ W \sim \frac{D_{\text{eff}}^2 r_e^2 \lambda_0^4}{4 \pi c} \left( 1 + z_q \right)^3 \left( \frac{\delta n_e}{n_e} \right)^2 L^{-1} \text{DM}^2, \quad r_{\text{diff}} < l_0, \]  
and
\[ W \sim \frac{D_{\text{eff}}^2 r_e^2 \lambda_0^4}{4 \pi^2 c} \left( 1 + z_q \right)^2 \left( \frac{\delta n_e}{n_e} \right)^2 \left( \frac{L}{L_0} \right)^{\beta - 4} \times \left( \frac{L}{L_0} \right)^{\beta - 4} \text{DM}^{-2}, \quad r_{\text{diff}} > l_0. \]  

Instead of \( L \), \( W \) in this case only places a constraint on \( l_0 \). When \( r_{\text{diff}} < l_0 \), an excess of temporal broadening requires \( l_0 \) to be comparable to \( r_{\text{diff}} \), so \( l_0 \) should be sufficiently small, while when \( r_{\text{diff}} > l_0 \), a larger \( l_0 \) is more favorable. The relation between \( W \) and DM can be also established:
\[ W \sim \frac{r_e^2 \lambda_0^4}{4 \pi c} \left( 1 + z_q \right)^3 \left( \frac{\delta n_e}{n_e} \right)^2 l_0^{-1} \text{DM}^2, \quad r_{\text{diff}} < l_0, \]  
and
\[ W \sim \frac{r_e^2 \lambda_0^4}{4 \pi^2 c} \left( 1 + z_q \right)^2 \left( \frac{\delta n_e}{n_e} \right)^2 \left( \frac{l_0}{L_0} \right)^{\beta - 4} \times \left( \frac{l_0}{L_0} \right)^{\beta - 4} \text{DM}^{-2}, \quad r_{\text{diff}} > l_0. \]  

By comparing Equations 17(a), 19(a), and 19(b), one can see that the dependence of \( W \) on DM is determined by both the relation between \( r_{\text{diff}} \) and \( l_0 \), and the spectral properties of density fluctuations. In general, \( W \) increases more drastically with DM at a smaller \( \beta \) in the case of \( r_{\text{diff}} > l_0 \), and has its mildest dependence on DM as \( W \propto \text{DM}^2 \) in the case of \( r_{\text{diff}} < l_0 \), irrespective of the value of \( \beta \). Also, the density perturbation \( \delta n_e \) at \( L \) for a long-wave-dominated spectrum of density fluctuations is close to \( n_e \) averaged over a large scale, while \( \delta n_e \) at \( l_0 \) for a short-wave-dominated spectrum can considerably exceed the background \( n_e \) due to turbulent compression in shock-dominated flows. More exactly, following the power-law behavior, the ratio of the density perturbation at \( l_0 \) to that at \( L \) when \( \beta < 3 \) is
\[ \frac{\delta n_e(l_0)}{\delta n_e(L)} = \frac{\delta n_e}{\delta n_e(L)} = \left( \frac{L}{l_0} \right)^{1/2}. \]  

Therefore, with a higher density perturbation and a smaller scale \( l_0 \) instead of \( L \) involved, a short-wave-dominated spectrum of density fluctuations provides much stronger scattering than a long-wave-dominated one when the DMs are the same.

2.2. Applications in the IGM and the Host Galaxy ISM

To elucidate the millisecond scattering tail observed for some FRBs (Lorimer et al. 2007; Thornton et al. 2013), we next consider the IGM and the FRB host galaxy as two possible sources responsible for the scattering timescale.
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(1) Scattering in the IGM: Growing observational evidence supports the presence of the IGM turbulence (e.g., Rauch et al. 2001; Zheng et al. 2004; Meiksin 2009; Lu et al. 2010) and the Kolmogorov-type turbulence in clusters of galaxies (Muragia et al. 2004; Schuecker et al. 2004; Vogt & Enßlin 2005). Supercomputer simulations show that the turbulent motions inside clusters of galaxies are subsonic, and are transonic or mildly supersonic in filaments (Ryu et al. 2008), which agrees with the observational detection of subsonic turbulence in, e.g., the Coma cluster (Schuecker et al. 2004) and the core of the Perseus cluster (Churazov et al. 2004). Down to small scales, theoretical studies suggest the existence of Alfvénic turbulence with a spectrum dictated by the Kolmogorov scaling (Schekochihin & Cowley 2006), which is supported by the observed spectrum of magnetic energy in the core region of the Hydra cluster (Vogt & Enßlin 2005). Based on these signatures obtained so far, the IGM turbulence is unlikely to be highly supersonic and thus unlikely to possess a short-wave-dominated density spectrum, especially on scales small enough to be important for diffractive scattering. Therefore, to numerically evaluate the temporal broadening for propagation of radio waves through the diffuse IGM, we consider a long-wave-dominated spectrum ($\beta > 3$) of turbulent density and adopt the generally accepted Kolmogorov turbulence model with $\beta = 11/3$. Meanwhile, the choice of parameters should also be made to fulfill the conditions indicated by Equations (9) and (10) in cases of $r_{\text{diff}} < l_0$ and $r_{\text{diff}} > l_0$, respectively. Inserting Equations 4(a), 15, and $\beta = 11/3$ into Equations (9) and (10) yields

$$L \left( \frac{l_0}{L} \right)^{\frac{5}{4}} > (\pi r e D e x^{2} (\delta n e)^{2})^{-1}, \quad r_{\text{diff}} < l_0, \quad (21a)$$

$$L \left( \frac{l_0}{L} \right)^{\frac{5}{4}} < (\pi r e D e x^{2} (\delta n e)^{2})^{-1}, \quad r_{\text{diff}} > l_0. \quad (21b)$$

We now rewrite $W$ from Equation (16) in terms of typical parameters for the IGM,

$$W \sim \frac{0.065}{(1 + z_q)^3} \left( \frac{D_{\text{eff}}}{1 \text{ Gpc}} \right)^2 \left( \frac{\lambda_0}{1 \text{ m}} \right)^4 \times \left( \frac{\delta n e}{10^{-7} \text{ cm}^{-3}} \right)^2 \left( \frac{l_0}{L} \right)^{-\frac{1}{4}} \left( \frac{L}{10^{-2} \text{ pc}} \right)^{-1} \text{ ms}, \quad r_{\text{diff}} < l_0, \quad (22a)$$

$$W \sim \frac{4.9}{(1 + z_q)^{3.4}} \left( \frac{D_{\text{eff}}}{1 \text{ Gpc}} \right)^{2.2} \left( \frac{\lambda_0}{1 \text{ m}} \right)^{4.4} \times \left( \frac{\delta n e}{10^{-7} \text{ cm}^{-3}} \right)^{2.4} \left( \frac{L}{10^{-2} \text{ pc}} \right)^{-0.8} \text{ ms}, \quad r_{\text{diff}} > l_0. \quad (22b)$$

The value of $W$ at $r_{\text{diff}} < l_0$ depends on the disparity between $L$ and $l_0$, according to Equation 21(a), which satisfies

$$l_0 > 2.4 \times 10^{-6} (1 + z_q)^{1.2} \left( \frac{D_{\text{eff}}}{1 \text{ Gpc}} \right)^{-0.6} \left( \frac{\lambda_0}{1 \text{ m}} \right)^{-1.2} \times \left( \frac{\delta n e}{10^{-7} \text{ cm}^{-3}} \right)^{-1.2} \left( \frac{L}{10^{-2} \text{ pc}} \right)^{-0.6}. \quad (23)$$

With the lower limit of $l_0/L$ in the above expression adopted, we get the same result in both cases that for a low-redshift source the outer scale $L$ on the order of $10^{-2} \text{ pc}$ can lead to the pulse duration of $\sim 5 \text{ ms}$ at 0.3 GHz frequency ($\lambda = 1 \text{ m}$). The derived outer scale of turbulence seems unreasonably small compared with the expected injection scale ($>100 \text{ kpc}$) of turbulence induced by cluster mergers (Subramanian et al. 2006) or cosmological shocks (Ryu et al. 2008, 2010). Also, as pointed out by Luan & Goldreich (2014), a serious difficulty is that such a small outer scale is accompanied by a turbulent heating rate at

$$\tau_{\text{heat}}^{-1} \sim \frac{c_s}{L} = 0.005 \left( \frac{L}{10^{-2} \text{ pc}} \right)^{-1} \left( \frac{T}{10^5 \text{ K}} \right)^{\frac{1}{2}} \text{ yr}^{-1}, \quad (24)$$

where $c_s$ is the sound speed. The typical IGM temperature $T$ ranges from $10^5$ to $10^7 \text{ K}$ (Bykov et al. 2008; Ryu et al. 2008). The heating rate is so high that it is incompatible with the cooling rate, which is comparable to the inverse Hubble time. With regards to a Kolmogorov cascade with the turbulent energy injected at a scale considerably larger than $\sim 10^{-2} \text{ pc}$, the resulting electron density fluctuations in the IGM make a negligible contribution to the observed temporal scattering.

Due to the high heating rate at small scales in the IGM, any small-scale density enhancement would be rapidly erased by the thermal streaming motions in the IGM (Cordes et al. 2016). For this reason, the scenario in which the scattering medium is concentrated and localized in a thin layer in the IGM may not reflect the reality. Based on this questionable assumption, one tends to overestimate the contribution to the pulse broadening from the IGM.

(2) Scattering in the host galaxy ISM: In the multiphase ISM of the Galaxy, the distribution of the electron density fluctuations throughout the diffuse WIM is described by a Kolmogorov spectrum (Armstrong et al. 1995; Chepurnov & Lazarian 2010), but exhibits a much shallower spectrum in the supersonic turbulence prevalent in inner regions of the Galaxy (e.g., Lazarian 2009; Hennebelle & Falgarone 2012). By assuming that the host galaxy of an FRB is similar to the Galaxy and the general properties of turbulence are applicable, we next attribute the strong scattering to the propagation of radio waves through the ISM of the host galaxy and analyze the scattering effects from the Kolmogorov and short-wave-dominated density spectra, respectively.

We again start with the Kolmogorov power law of turbulence. The resulting $W$ from Equation (16) is

$$W \sim \frac{0.065}{(1 + z_q)^3} \left( \frac{D_{\text{eff}}}{1 \text{ kpc}} \right)^2 \left( \frac{\lambda_0}{1 \text{ m}} \right)^4 \times \left( \frac{\delta n e}{10^{-7} \text{ cm}^{-3}} \right)^2 \left( \frac{l_0}{L} \right)^{-\frac{1}{4}} \left( \frac{L}{10^{-2} \text{ pc}} \right)^{-1} \text{ ms}, \quad r_{\text{diff}} < l_0, \quad (25a)$$

$$W \sim \frac{1.9}{(1 + z_q)^{3.4}} \left( \frac{D_{\text{eff}}}{1 \text{ kpc}} \right)^{2.2} \left( \frac{\lambda_0}{1 \text{ m}} \right)^{4.4} \times \left( \frac{\delta n e}{10^{-7} \text{ cm}^{-3}} \right)^{2.4} \left( \frac{L}{10^{-3} \text{ pc}} \right)^{-0.8} \text{ ms}, \quad r_{\text{diff}} > l_0. \quad (25b)$$

Here the normalization of $D_{\text{eff}}$ is assigned a typical galaxy size and $\delta n e$ the electron density in diffuse ISM, i.e., $\delta n e \sim n e$. At
\( r_{\text{diff}} < l_0 \), using Equation 21(a), we have

\[
l_0 > \frac{3.8 \times 10^{-8}(1 + z_q)^{1.2}}{L} \left( \frac{D_{\text{eff}}}{1 \text{ kpc}} \right)^{0.6} \left( \frac{\lambda_0}{1 \text{ m}} \right)^{1.2} \frac{\delta n_e}{\left( 10^{-2} \text{ cm}^{-3} \right)}^{1.2} \left( \frac{L}{10^{-3} \text{ pc}} \right)^{-0.6} .
\]

(26)

Substituting the lower limit of the ratio \( l_0/L \) into Equation 25(a) yields the consistent result on the value of \( W \) as in the case of \( r_{\text{diff}} > l_0 \). We see that \( L \) inferred from the millisecond pulse broadening is far smaller than the injection scale of the turbulence throughout the Galactic WIM, which is suggested to be on the order of \( \sim 100 \text{ pc} \) by measuring the spectra of interstellar density fluctuations (Armstrong et al. 1995; Chepurnov & Lazarian 2010). It is also below the smaller outer scale of a few parsecs of the turbulence found in the Galactic spiral arms (Minter & Spangler 1996; Haverkorn et al. 2004). This heightens the challenge to interpreting the driving mechanism of the Kolmogorov turbulence as well as the cooling efficiency in the host galaxy.

A plausible solution is that a short-wave-dominated spectrum of electron density fluctuations, which is extracted from the observations of the inner Galaxy, also applies in the ISM of the host galaxy. As the density power spectrum becomes flat in supersonic turbulence, if the turbulent ISM of the host galaxy through which the LOS traverses contains highly supersonic turbulent motions and as a result is characterized by numerous small-scale clumpy density structures, we expect that the spectrum of electron density fluctuations deviates from the Kolmogorov power law and has \( \beta < 3 \).

In the above calculations, we assume that the volume filling factor \( f \) of the scattering material is comparable to unity, which is valid for a long-wave-dominated density spectrum characterized by large-scale density fluctuations. For small-scale clumpy density structures described by a short-wave-dominated density spectrum, however, it is necessary to consider that only a fraction of the volume is filled by the overdense regions and replace \( \delta n_e \) with \( \sqrt{f} \delta n_e \). In the case of the Galactic ISM, the WIM phase where the Kolmogorov density spectrum is present has \( f \sim 25\% \). In contrast, the filling factors of the cold neutral medium and molecular clouds are as low as 1\% and 0.05\% (Tielens 2005; Haverkorn & Spangler 2013). In these colder and denser phases, which only fill a small fraction of the volume, the short-wave-dominated density spectrum gives rise to small-scale density structures with the spatial profile of the density field characterized by peaks of mass as a result of strong shocks (see Figure 2 in Kim & Ryu 2005). Therefore, the small-scale density structures created within these phases have an even smaller value of \( f \). Accordingly, we include the effect of a small filling factor in the case of a short-wave-dominated density spectrum, so as to reach a more realistic evaluation of the scattering produced by the supersonic turbulence in the host ISM.

In the case of \( r_{\text{diff}} < l_0 \), by inserting Equations 4(b) and (15) into Equation (9), we find

\[
l_0 > \frac{\pi r_{\text{diff}}^{2} D_{\text{eff}} \lambda_{b}^{2} (\delta n_{e})^{2}}{\lambda_{0}^{2}}^{-1}
\]

\[
= 1.3 \times 10^{8}(1 + z_{q})^{2} \left( \frac{D_{\text{eff}}}{1 \text{ kpc}} \right)^{-1} \left( \frac{\lambda_{0}}{1 \text{ m}} \right)^{-2} \frac{\delta n_{e}}{\left( 10^{-1} \text{ cm}^{-3} \right)^{2}} \text{ cm.}
\]

(27)

Given the parameters adopted in the above expression, the minimum \( l_0 \) is comparable to the inner scale of the density spectrum in the Galactic ISM inferred from observations (Spangler & Gwinn 1990; Armstrong et al. 1995; Bhat et al. 2004). By using a larger value of \( l_0 \) and substituting the normalization parameters into Equation 18(a) for a short-wave-dominated spectrum of density fluctuations, we get

\[
W \sim \frac{6.5}{(1 + z_{q})^{3}} \left( \frac{D_{\text{eff}}}{1 \text{ kpc}} \right)^{2} \left( \frac{\lambda_{0}}{1 \text{ m}} \right)^{4} \left( \frac{f}{10^{-6}} \right) \left( \frac{\delta n_{e}}{10^{-1} \text{ cm}^{-3}} \right)^{2} \left( \frac{l_0}{10^{10} \text{ pc}} \right)^{-1} \text{ ms.}
\]

(28)

As \( r_{\text{diff}} \) is below the inner scale of density power spectrum, the scaling presented in the above equation is independent of the spectral slope \( \beta \) of density fluctuations. It shows that clumps of electron density \( 0.1 \text{ cm}^{-3} \) and size \( 10^{-10} \text{ cm} \) (~1 mm) which occupy a small fraction of the volume of the host galaxy, would be adequate to produce the observed scattering delay.

Individual clumps of excess electrons have been included for modeling the Galactic distribution of electrons and scattering properties of Galactic ISM (Pynzar & Shishov 1999; Cordes & Lazio 2003; Cordes et al. 2016). The clumpy component of the ionized plasma introduced in these studies are associated with discrete H II regions or supernova remnants with a characteristic scale of \( \sim 1 \text{ pc} \) (Haverkorn et al. 2004). However, based on Equation (28) we note that the density fluctuations appearing on parsec scales, unless the local density is extraordinarily high, are unable to cause the intense scattering related with some FRBs. In contrast, we consider much smaller-scale density structures corresponding to a short-wave-dominated density spectrum with a sufficiently small inner scale. If the host galaxy medium is dominated by supersonic turbulence, in accordance with the concentrated density distribution induced by shock compression, the spectral form is dominated by the formation of small-scale density fluctuations and exhibits a rather shallow slope.

Compared with the above situation with \( r_{\text{diff}} < l_0 \) (Equation (28)), the density spectrum in the case of \( r_{\text{diff}} > l_0 \) can lead to a significantly larger degree of scattering due to the stronger dependence of \( W \) on the physical parameters involved (see Equations 18(b) and 19(b)). When the \( \beta \) value can be determined, the scaling relations presented in Equation 18(b) (or Equation 19(b)) can be used to constrain the turbulence properties. This small-scale properties of turbulent density can account for more pronounced scattering observed for some FRBs, and can also provide a plausible scattering source for the Galactic pulsars with high DMs (Xu & Zhang 2016b). It implies that, with similar properties to that of the Galaxy, the...
host galaxy is adequate to provide the observed scattering strength for an FRB.

(3) Locations of scattering and dispersion: The above results inform us that a long-wave-dominated power-law spectrum, e.g., the Kolmogorov spectrum, of electron density fluctuations with a reasonably large outer scale of turbulence in both the diffuse IGM and the host galaxy medium are incapable of producing the millisecond scattering tail. A short-wave-dominated electron density spectrum with $\beta < 3$ from the ISM of the host galaxy can easily render the host galaxy a strong scatterer. The excess fluctuation power at small scales characterized by a short-wave-dominated density spectrum gives rise to enhanced diffractive scattering and thus strong temporal broadening of a transient pulse.

A short-wave-dominated spectrum of density fluctuations in Galactic ISM can also produce the desired amount of scattering for FRBs. However, most of the known FRBs were discovered at high Galactic latitudes in directions through the WIM Galactic ISM can also produce the desired amount of scattering characterized by a short-wave-dominated density spectrum with a reasonably large outer scale of turbulence in both the diffuse IGM and the host galaxy medium are incapable of producing the millisecond scattering tail. A short-wave-dominated electron density spectrum with $\beta < 3$ from the ISM of the host galaxy can easily render the host galaxy a strong scatterer. The excess fluctuation power at small scales characterized by a short-wave-dominated density spectrum gives rise to enhanced diffractive scattering and thus strong temporal broadening of a transient pulse.

A short-wave-dominated spectrum of density fluctuations in Galactic ISM can also produce the desired amount of scattering for FRBs. However, most of the known FRBs were discovered at high Galactic latitudes in directions through the WIM component of the ISM, where the turbulence is transonic at high Galactic latitudes in directions through the WIM Galactic ISM can also produce the desired amount of scattering characterized by a short-wave-dominated density spectrum with a reasonably large outer scale of turbulence in both the diffuse IGM and the host galaxy medium are incapable of producing the millisecond scattering tail. A short-wave-dominated electron density spectrum with $\beta < 3$ from the ISM of the host galaxy can easily render the host galaxy a strong scatterer. The excess fluctuation power at small scales characterized by a short-wave-dominated density spectrum gives rise to enhanced diffractive scattering and thus strong temporal broadening of a transient pulse.

It is commonly accepted that the diffuse IGM makes an unimportant contribution to scattering. Instead, intervening galactic halos along the LOS are appealed to for explaining the observed scattering (Yao et al. 2016). Indeed, if the intervening ISM happens to be in a state of supersonic turbulence, and located close to us with a small reduction factor, which depends on redshift, the intervening galaxy would dominate the scattering. However, we regard this scenario as implausible because for a source at a cosmological distance, the probability for the LOS to intersect with an intervening galaxy is very low, e.g., $\leq 5\%$ within $z_g \sim 1.5$ (Macquart & Koay 2013; Roeder & Verheuil 1969), and the probability for the intervening ISM to be supersonically turbulent is further lower. This is in contradiction with the fact that around half of the known FRBs have detectable scattering tails (Petroff et al. 2016).

After identifying the host galaxy medium as the most promising candidate for dominating the observed scattering, we see from Equation (28) that

$$W \sim \frac{6.5}{(1 + z_g)^3} \left( \frac{\lambda_0}{1 \text{ m}} \right)^4 \left( \frac{f}{10^{-6}} \right) \left( \frac{\delta n_e}{n_e} \right)^2 \left( \frac{\rho}{10^{-10} \text{ pc}} \right)^{-1} \left( \frac{\text{DM}}{100 \text{ pc cm}^{-3}} \right)^2 \text{ ms.} \quad (29)$$

The dependence of $W$ on DM is affected by the turbulence properties in the surrounding ISM of the source. Under the condition of a short-wave-dominated spectrum of density fluctuations, strong scattering does not entail large DM in the host medium. As the Galactic contribution to the total DM is minor compared with its extragalactic component (Cordes et al. 2016), the IGM is most likely the dominant location for the observed DMs of FRBs.

The FRB data exhibit considerable scatter around any modeled (Caleb et al. 2016) or fitted (Yao et al. 2016) scattering time–DM relation. After considering an order-of-magnitude scatter similar to the case of Galactic pulsars (Bhat et al. 2004), one still cannot reach a satisfactory fit of the intergalactic scattering model to the FRB data (Caleb et al. 2016). As suggested in Caleb et al. (2016), the LOS-dependent inhomogeneity in the Galactic ISM (Johnston et al. 1998; Cordes & Lazio 2002) may not apply to the IGM, which further poses difficulty for the IGM scattering scenario. Besides, by plotting the scattering time versus DM for high-Galactic-latitude FRBs, Katz (2016a) claimed that no correlation between the two variables can be seen. More plausibly, scattering and dispersion are separately dominated by the host galaxy and the IGM. As shown above, the scattering time is largely affected by the turbulence properties (e.g., $\beta > 3$ or $\beta < 3$) and scattering regimes ($\tau_{\text{diff}} < 10$ or $\tau_{\text{diff}} > 100$). Therefore, the variation of the scattering time for FRBs can be attributable to the diverse interstellar environments of their host galaxies. From the observational point of view, it is also necessary to point out that the estimated scattering time is subject to effects such as the signal-to-noise ratio and limited temporal resolution due to dispersion smearing, leading to non-negligible uncertainties in the observationally measured scattering time–DM relation.

3. ALTERNATIVE MODELS OF ELECTRON DENSITY FLUCTUATIONS

Besides the turbulent cascade, different magnetic field structures associated with other processes, such as plasma instabilities and fluctuation dynamo, can also induce electron density fluctuations. In the strong scattering regime, the rms phase perturbation is

$$\tau_{\text{sc}} = \frac{\lambda}{4\pi c} \frac{\tau_{\text{diff}}^2}{\delta n_e (\lambda)} \quad (30)$$

from which Equations (16) and (18) can be recovered (see the Appendix). In a simple case when the fluctuating density $\delta n_e$ has a characteristic scale $d$, the above expression leads to

$$\tau_{\text{sc}} = \frac{\lambda}{4\pi c} \frac{(\delta n_e (d))^2}{d} \quad (31)$$

which in the observer’s frame is

$$W = \frac{\Delta\nu_{\text{eff}}^2 \lambda_{\text{eff}}^4}{4\pi c (1 + z_g)^3} \frac{(\delta n_e (d))^2}{d} \quad (32)$$

In the strong scattering regime, the rms phase perturbation is greater than 1 rad, i.e., $\sqrt{\Delta\nu_{\text{sc}}^2} > 1$ (e.g., Rickett 1990; Cordes & Lazio 1991; Luan & Goldreich 2014). Accordingly, we have a lower limit of $d$ at a given density perturbation

$$d > (1 + z_g)^2 \frac{\tau_{\text{diff}}^2 \lambda_{\text{diff}}^4 \delta n_e (d)}{\tau_{\text{diff}}^2 \lambda_{\text{diff}}^4 \delta n_e (d)} \quad (33)$$

or a lower limit of $\delta n_e (d)$ when $d$ is determined,

$$\frac{(\delta n_e (d))^2}{(1 + z_g)^2} \approx \frac{\tau_{\text{diff}}^2 \lambda_{\text{diff}}^4 \delta n_e (d)}{\tau_{\text{diff}}^2 \lambda_{\text{diff}}^4 \delta n_e (d)} \quad (34)$$

The derivations of the above equations are presented in the Appendix. In the following analysis, we will apply these
relations and the observational constraint on the scattering timescale to investigate the scattering effect of other possibilities of density fluctuations.

3.1. Electron Density Fluctuations Arising from the Mirror Instability in the IGM

For intergalactic plasmas, the ion collision frequency $\nu_{\text{ii}}$ is much lower than the cyclotron frequency $\Omega_i$, and accordingly, the mean free path of ions (Braginskii 1965)

$$\lambda_{\text{mfp}} = \frac{\nu_{\text{hi},i}}{\nu_{\text{ii}}} = \frac{3\sqrt{2}}{4\pi}\frac{(k_B T)^2}{\ln \Lambda e^4 n_i}$$

$$= 2.15 \times 10^{21} \left(\frac{\ln \Lambda}{10}\right)^{-1} \left(\frac{T}{10^{5} \text{ K}}\right)^{\frac{1}{2}} \left(\frac{n_i}{10^{-7} \text{ cm}^{-3}}\right)^{-1} \text{ cm}$$

(35)

is significantly larger than the ion gyroradius

$$l_i = \frac{\nu_{\text{hi},i}}{\Omega_i} = \frac{\nu_{\text{hi},i} m_i e}{e B}$$

$$= 4.2 \times 10^9 \left(\frac{T}{10^{5} \text{ K}}\right)^{\frac{1}{2}} \left(\frac{B}{0.1 \mu \text{G}}\right)^{-1} \text{ cm},$$

(36)

where $\nu_{\text{hi},i} = (2k_B T/m_i)$ is the ion thermal speed, and $k_B$, $\ln \Lambda$, $n_i$, and $e$ are the Boltzmann constant, Coulomb logarithm, ion number density, and speed of light, respectively. The magnetic field strength $B$ is taken as the inferred value from the Faraday rotation measurements of polarized extragalactic sources (Ryu et al. 1998; Xu et al. 2006). We also treat the IGM as a fully ionized hydrogen plasma, so ions have the same charge $e$ and mass $m_i = m_H$ as protons.

The weakly collisional and magnetized IGM is subject to firehose and mirror instabilities driven by pressure anisotropies with respect to the local magnetic field direction (Fabian 1994; Carilli & Taylor 2002; Schekochihin et al. 2005; Rincon et al. 2015). The instability growth rate increases with wave numbers, resulting in fluctuating magnetic fields peaking at a plasma microscale comparable to the ion gyro-scale $l_i$ (Schekochihin & Cowley 2006). The compressive mirror instability induces variations in density, which are anti-correlated with the magnetic field variations. The fluctuations in density and magnetic field are related as (Hall 1980)

$$\frac{\delta n_e}{n_e} \sim \frac{\delta B}{B},$$

(37)

where $\delta n_e$, $\delta B$ and $n_e$, $B$ are the fluctuating and uniform components of electron density and magnetic field strength, respectively. If the density perturbation $\delta n_e(d)$ at $d = l_i \sim 4.2 \times 10^9$ cm (Equation (36)) is sufficient to account for strong scattering, Equation (34) sets the lower limit of density perturbation at $d$,

$$\delta n_e(d) > 5.5 \times 10^{-9}(1 + z_q) \left(\frac{D_{\text{eff}}}{1 \text{ Gpc}}\right)^{-\frac{1}{2}} \left(\frac{\lambda_0}{1 \text{ m}}\right)^{-1} \times \left(\frac{T}{10^{5} \text{ K}}\right)^{-\frac{1}{2}} \left(\frac{B}{0.1 \mu \text{G}}\right)^{\frac{1}{2}} \text{ cm}^{-3}.$$ 

(38)

Inserting the above expression and Equation (36) into Equation (32) results in

$$W > \frac{1.5 \times 10^3}{1+z_q} \left(\frac{D_{\text{eff}}}{1 \text{ Gpc}}\right) \left(\frac{\lambda_0}{1 \text{ m}}\right)^{2} \left(\frac{T}{10^{5} \text{ K}}\right)^{-1} \left(\frac{B}{0.1 \mu \text{G}}\right)^{2} \text{ ms}.$$ 

(39)

The predicted timescale is obviously inconsistent with the observed FRB pulses with millisecond or shorter durations. To accommodate the observations, the saturated amplitude of the density fluctuations and the associated magnetic fluctuations generated by plasma instabilities should remain at a marginal level, so that the strong scattering cannot be realized. We can see from Equation (38) that by adopting an average electron density as $n_e = 10^{-7} \text{ cm}^{-3}$, a conservative estimate of the magnetic field and density perturbations near $l_i$ is

$$\frac{\delta B}{B} \sim \frac{\delta n_e}{n_e} < \frac{5.5 \times 10^{-9} \text{ cm}^{-3}}{10^{-7} \text{ cm}^{-3}} = 0.055.$$ 

(40)

This result suggests that although the micro-plasma instabilities have a fast growth rate in comparison with the large-scale turbulent motions, they are mostly suppressed over the fluid timescale. As demonstrated by earlier works, the enhanced particle scattering originating from the plasma instabilities can effectively relax the pressure anisotropy and increase the collision rate. As a result, both the turbulent cascade over small scales and efficient magnetic field amplification can be facilitated (Lazarian & Beresnyak 2006; Santos-Lima et al. 2014). This naturally explains the magnetization and turbulent motions in the IGM inferred from the observations (e.g., Ryu et al. 2008). By taking into account the relaxation effect of pressure anisotropy, the collisionless MHD simulations carried out by Santos-Lima et al. (2014) exhibit the statistical properties of turbulence similar to that of collisional-MHD turbulence, which justifies a collisional-MHD description of collisionless plasmas at the intracluster medium (and IGM) conditions. The observed pulse widths of transient radio sources at cosmological distances, like the FRBs, offer a strong argument supporting the above picture of the IGM turbulence, whereas the model of nonlinear evolution of the plasma instabilities with a secular growth of small-scale magnetic field fluctuations to large amplitudes, $\delta B/B \sim 1$, is disfavored (Schekochihin et al. 2008; Rincon et al. 2015).

3.2. Electron Density Fluctuations Arising from a Folded Structure of Magnetic Fields in the IGM

Corresponding to the large mean free path of ions in the IGM, the viscosity parallel to magnetic field lines is

$$\nu_{\parallel} = \lambda_{\text{mfp}} \nu_{\text{hi},i} = \frac{3(k_B T)^2}{2\pi \ln \Lambda e^4 n_i}$$

$$= 8.7 \times 10^{27} \left(\frac{\ln \Lambda}{10}\right)^{-1} \left(\frac{T}{10^{5} \text{ K}}\right)^{\frac{1}{2}} \left(\frac{n_i}{10^{-7} \text{ cm}^{-3}}\right)^{-1} \text{ cm}^2 \text{s}^{-1}.$$ 

(41)
It damps the turbulent cascade at a large viscous scale, which can be obtained by equaling the turbulent cascading rate
\[ \tau_{\text{cas}}^{-1} = \frac{v_f}{l} = k^2 \nu_f \]
with the viscous damping rate \( k^2 \nu_f \). Here we use the Kolmogorov scaling, where \( v_f \) is the turbulent velocity at scale \( l \), \( V_L \) is the turbulent velocity at the injection scale \( L \), and \( k = 1/l \) is the wavenumber. The viscous scale calculated by using the parallel viscosity is
\[ l_0 = L \frac{L}{V_L} \frac{V_L^2}{\nu_f^2} = 3.78 \times 10^{21} \left( \frac{L}{100 \text{kpc}} \right)^{\frac{2}{3}} \left( \frac{V_L}{100 \text{ km s}^{-1}} \right)^{-\frac{2}{3}} \left( \frac{\nu_f}{(10^{-7} \text{ cm}^{-3})} \right)^{\frac{1}{2}} \text{ cm}. \]

The viscous-scale eddies are responsible for the random stretching of magnetic field lines that drives an exponential growth of the initially weak magnetic energy at a rate equal to the viscous-eddy turnover rate. As mentioned in Section 3.1, the particle scattering in the presence of the plasma instabilities makes the effective parallel viscosity sufficiently small, and thus the corresponding dynamo growth rate becomes fast (Schekochihin & Cowley 2006; Santos-Lima et al. 2014), so that the kinematic dynamo process can be efficient enough to generate strong magnetic fields within the cluster lifetime.

In addition, the ordinary Spitzer resistivity in the IGM is negligibly small (Spitzer 1956),
\[ \eta = \frac{e^2 \sqrt{m_e m_i} \ln \Lambda}{4 (k_B T)^{\frac{3}{2}}} = 3.05 \times 10^5 \left( \frac{\ln \Lambda}{10} \right) \left( \frac{T}{10^5 \text{ K}} \right)^{\frac{3}{2}} \text{ cm}^2 \text{s}^{-1}. \]

Thus the magnetic Prandtl number \( P_m = v_f/\eta \sim 10^{22} \) (Equations (41) and (44)) in the IGM is high, and magnetic fluctuations can be developed in the viscosity-damped regime of MHD turbulence (Cho et al. 2002, 2003; Lazarian et al. 2004). During the dynamo growth of magnetic energy, the stretched magnetic fields form a folded structure in the sub-viscous range, with the field variation across the field lines at the viscous scale \( l_0 \) and the field direction reversal at the resistive scale (Schekochihin et al. 2004; Goldreich & Sridhar 2006; Lazarian 2007; Brautknecht 2015). The folded magnetic fields compress gas into dense sheet-like structures. Such dense sheets have been invoked to explain the formation of the small ionized and neutral structures (SINS) in the partially ionized ISM (Dieter et al. 1976; Heiles 1997; Stanimirović et al. 2004) by Lazarian (2007), and is also proposed as the source of extreme diffractive scattering in the Galactic center by Goldreich & Sridhar (2006).

However, as regards the fully ionized IGM environment, the persistence of the folded structure of magnetic fields is speculative. First, the folded structure, especially its curved part, is unstable to the plasma instabilities and the resulting thickness of the fold can be much larger than the resistive scale (Schekochihin et al. 2005). Moreover, not only can the parallel viscosity be effectively reduced, but the viscosity perpendicular to magnetic field lines also substantially decreases with increasing field strength (Simon 1955),
\[ \nu_{f, \perp} = \frac{3 k_B T v_i}{10 \pi^2 m_i} \left( \frac{\ln \Lambda}{10} \right) \left( \frac{T}{10^5 \text{ K}} \right)^{\frac{1}{2}} \times \left( \frac{n_i}{10^{-7} \text{ cm}^{-3}} \right) \left( \frac{B}{0.1 \mu \text{G}} \right)^{-2} \text{ cm}^2 \text{s}^{-1}. \]

It implies that the turbulent motions perpendicular to magnetic field lines are undamped at the viscous scale \( l_0 \) (Equation (43)) derived from the parallel viscosity and can initiate a cascade of Alfvénic turbulence at smaller scales down to the cutoff scale determined by the much smaller perpendicular viscosity, which tends to violate the preservation of the folded structure of magnetic fields on scales below \( l_0 \).

In the following analysis, we nevertheless presume that the magnetic fields appear in folds with undetermined thickness at scales below \( l_0 \), and the local magnetic perturbation is determined by the equilibrium between the turbulent energy at \( l_0 \) and the magnetic-fluctuation energy,
\[ \delta B = \sqrt{4 \pi \rho_0 v_0} = \sqrt{4 \pi \rho_0 V_L \left( \frac{l_0}{L} \right)^{\frac{3}{2}}}. \]

where \( \rho_0 = m_H n_i \) is the average mass density of ions. In pressure equilibrium, the density perturbation across the sheet of folded fields is approximately given by the ratio between the local magnetic and gas pressure (Lazarian 2007),
\[ \frac{\delta n_e}{n_e} \sim \frac{P_B}{P}. \]

with the magnetic pressure \( P_B = (\delta B)^2/8\pi \), and the thermal pressure
\[ P_g = P_i + P_e = n_i k_B T + n_e k_B T = 2 n_i k_B T, \]

where the number density of ions \( n_i \) and electrons \( n_e \) are equal. Therefore, we can get (Equations (43), (46)–(48))
\[ \frac{\delta n_e}{n_e} = \frac{(\delta B)^2}{16 \pi n_i k_B T} = \frac{m_i V_L^2}{4 k_B T L^2} \]
\[ = 0.16 \left( \frac{L}{100 \text{kpc}} \right)^{\frac{1}{2}} \left( \frac{V_L}{100 \text{ km s}^{-1}} \right)^{\frac{3}{2}} \times \left( \frac{\ln \Lambda}{10} \right)^{\frac{1}{2}} \left( \frac{T}{10^5 \text{ K}} \right)^{\frac{1}{2}} \left( \frac{n_i}{10^{-7} \text{ cm}^{-3}} \right)^{-\frac{1}{2}}. \]

By taking \( \delta n_e (d)/n_e \sim 0.16 \) from above expression and \( n_e = 10^{-7} \text{ cm}^{-3} \), the condition of strong scattering requires (Equation (33))
\[ d > 5.0 \times 10^8 (1 + z_E)^2 \left( \frac{D_{\text{dc}}}{1 \text{ Gpc}} \right)^{-1} \left( \frac{\lambda_0}{1 \text{ m}} \right)^2 \text{ cm}, \]

with the lower limit smaller than \( l_1 \) (Equation (36)). It implies that the density perturbation we adopt for the folded structure at any sub-viscous scale can contribute to strong scattering. We have demonstrated in Section 2.2 that the intergalactic scattering is likely weak. Therefore, in accordance with the observationally determined scattering timescale \( W \), the
viscosity-dominated regime, a distinctive spectral slope of terms of one-dimensional magnetic energy spectrum in the vicinity of the density gradient, which can be calculated from Equations (41), (43), and (44),

\[
\begin{align*}
\frac{d}{W} & > \frac{D_{\text{eff}}^2 \chi^2}{4 \pi c (1 + z_\Lambda)^3} \frac{(\delta n_e (d))^2}{W} \\
& = \frac{5.2 \times 10^{13}}{(1 + z_{\Lambda})^3} \frac{D_{\text{eff}}^2 \chi^2}{1 \text{ Gpc}} \left( \frac{\lambda_0}{1 \text{ m}} \right)^4 \left( \frac{W}{1 \text{ ms}} \right)^{-1} \text{ cm. (51)}
\end{align*}
\]

As expected, it is larger than the resistive scale, which can be calculated from Equations (41), (43), and (44),

\[
\begin{align*}
l_R & = l_0 P_m^{-2} = 2.2 \times 10^{10} \left( \frac{L}{100 \text{ kpc}} \right)^{\frac{7}{10}} \left( \frac{V_i}{100 \text{ km s}^{-1}} \right)^{-\frac{2}{3}} \\
& \times \left( \frac{\ln \Lambda}{10} \right)^{\frac{7}{10}} \left( \frac{T}{8000 \text{ K}} \right)^{-\frac{7}{10}} \left( \frac{n_i}{0.1 \text{ cm}^{-3}} \right)^{-\frac{7}{10}} \text{ cm. (52)}
\end{align*}
\]

For the small-scale folded magnetic fields generated by fluctuation dynamo, besides the geometrical structure that is related with the scattering effects on radiation propagation, in terms of one-dimensional magnetic energy spectrum in the viscosity-dominated regime, a distinctive spectral slope of \(k^{-1}\) has been analytically derived by Lazarian et al. (2004) and numerically confirmed by Cho et al. (2002). The detection of such a spectral index and comparison between the measured spectral cutoff scale and the lower limit of sheet thickness in Equation (51) can verify the existence of the folded magnetic fields and provide more definite information on the properties of the viscosity-damped regime of turbulence.

3.3. Electron Density Fluctuations Arising from a Folded Structure of Magnetic Fields in the Host Galaxy Medium

The folded structure of magnetic fields in the sub-viscous range of turbulence can also be present in the ISM of the host galaxy. We next follow the similar calculations as shown above, but use the environment parameters for the Galactic WIM (McKee & Ostriker 1977), which account for most of the ionized gas within the Galactic ISM (Haffner et al. 2009) and are taken as an example of the fully ionized phase of the host galaxy medium.

Given the parallel viscosity (Equation (41))

\[
\nu_i = 1.6 \times 10^9 \left( \frac{\ln \Lambda}{10} \right)^{-1} \left( \frac{T}{8000 \text{ K}} \right)^{\frac{7}{10}} \left( \frac{n_i}{0.1 \text{ cm}^{-3}} \right)^{-\frac{7}{10}} \text{ cm}^2 \text{s}^{-1},
\]

and the Spitzer resistivity (Equation (44))

\[
\eta = 1.3 \times 10^7 \left( \frac{\ln \Lambda}{10} \right) \left( \frac{T}{8000 \text{ K}} \right)^{-\frac{7}{10}} \text{ cm}^2 \text{s}^{-1},
\]

the WIM phase has a large \(P_m\),

\[
\begin{align*}
P_m & = \frac{\nu_i}{\eta} = 1.2 \times 10^{11} \left( \frac{\ln \Lambda}{10} \right)^{-2} \\
& \times \left( \frac{T}{8000 \text{ K}} \right)^{\frac{2}{3}} \left( \frac{n_i}{0.1 \text{ cm}^{-3}} \right)^{-1}.
\end{align*}
\]

The resulting resistive scale

\[
l_R = l_0 P_m^{-\frac{2}{3}} = 7.2 \times 10^8 \left( \frac{L}{30 \text{ pc}} \right)^{\frac{7}{10}} \left( \frac{V_i}{10 \text{ km s}^{-1}} \right)^{-\frac{2}{3}} \\
\times \left( \frac{\ln \Lambda}{10} \right)^{\frac{7}{10}} \left( \frac{T}{8000 \text{ K}} \right)^{-\frac{7}{10}} \left( \frac{n_i}{0.1 \text{ cm}^{-3}} \right)^{-\frac{7}{10}} \text{ cm. (56)}
\]

is smaller than the ion mean free path (Equation (35))

\[
\lambda_{\text{mfp}} = 1.4 \times 10^{13} \left( \frac{\ln \Lambda}{10} \right)^{-1} \left( \frac{T}{8000 \text{ K}} \right)^{2} \left( \frac{n_i}{0.1 \text{ cm}^{-3}} \right)^{-1} \text{ cm},
\]

and thus falls in the collisionless regime. Similar to the IGM plasma, the folded structure of magnetic fields can be significantly affected by the plasma instabilities and turbulent cascade at small scales. Nevertheless, to seek the possibility of enhanced scattering introduced by different structures of magnetic fields arising in the host galaxy medium, we suppose that the folded fields survive at scales below the viscous scale \(l_0\) (Equation (43)),

\[
l_0 = \frac{l_0 P_m^{-\frac{2}{3}}}{4 \pi k T \Gamma_{\text{L}}} = 7.8 \times 10^{14} \left( \frac{L}{30 \text{ pc}} \right)^{\frac{7}{10}} \left( \frac{V_i}{10 \text{ km s}^{-1}} \right)^{-\frac{2}{3}} \\
\times \left( \frac{\ln \Lambda}{10} \right)^{\frac{7}{10}} \left( \frac{T}{8000 \text{ K}} \right)^{-\frac{7}{10}} \left( \frac{n_i}{0.1 \text{ cm}^{-3}} \right)^{-\frac{7}{10}} \text{ cm. (58)}
\]

In the case of the WIM, the turbulent cascade along the long-wave-dominated Kolmogorov spectrum over an extended inertial range leads to small turbulent fluctuations at \(l_0\). So the corresponding density perturbation given by Equation (49) is relatively small,

\[
\frac{\delta n_e}{n_e} = \frac{m_i V_e^2 l_0^2}{4 \pi k T \Gamma_{\text{L}}} = 1.6 \times 10^{-4} \left( \frac{L}{30 \text{ pc}} \right)^{\frac{7}{10}} \left( \frac{V_i}{10 \text{ km s}^{-1}} \right)^{-\frac{2}{3}} \\
\times \left( \frac{\ln \Lambda}{10} \right)^{\frac{7}{10}} \left( \frac{T}{8000 \text{ K}} \right)^{-\frac{7}{10}} \left( \frac{n_i}{0.1 \text{ cm}^{-3}} \right)^{-\frac{7}{10}}. \quad (59)
\]

It follows that to fulfill the strong scattering condition, the characteristic scale of the density fluctuations should be sufficiently large (Equation (33)),

\[
d > 5.3 \times 10^{8} (1 + z_\Lambda)^{\frac{2}{3}} \left( \frac{D_{\text{eff}}}{1 \text{ kpc}} \right)^{-1} \left( \frac{\lambda_0}{1 \text{ m}} \right)^{-1} \text{ cm},
\]

where \(\delta n_e (d)/n_e \sim 1.6 \times 10^{-4}\) and \(n_e = 0.1 \text{ cm}^{-3}\) are used. But in the meantime, as the density perturbation is rather weak, only with a small value of \(d\) can the millisecond pulse duration be reached (Equation (32))

\[
d \leq 4.9 \times 10^{7} \left( \frac{D_{\text{eff}}}{1 \text{ kpc}} \right)^{\frac{2}{3}} \left( \frac{\lambda_0}{1 \text{ m}} \right)^{4} \left( \frac{W}{1 \text{ ms}} \right)^{-1} \text{ cm}. \quad (61)
\]

The thickness of the sheet-like structure in the density field is expected to be larger than \(l_R\) (Equation (56)) due to the effect of plasma instabilities (Schekochihin et al. 2005), and thus larger
than the value indicated from the above equation, leading to insignificant pulse broadening.

This result shows that the density fluctuations induced by the folded structure of magnetic fields in the WIM-like environment are inadequate to render the host galaxy a strong scatterer. It has been suggested earlier by Goldreich & Sridhar (2006) that the large density contrast associated with the folded fields suffices for interpreting the extreme scattering of radio waves taking place in the Galactic center. Besides different environment parameters employed, as the major difference between our analysis and their work, we use the local magnetic field fluctuations with the magnetic energy equal to the turbulent energy at the viscous scale in deriving the density perturbation, rather than the magnetic field coherent on the scale of the largest turbulent eddy taken in Goldreich & Sridhar (2006), which has a much stronger strength than the perturbed field on the scale of the smallest eddy. The scenario described in Goldreich & Sridhar (2006) can be realized when the forcing scale of turbulence is comparable to the viscous scale and the inertial range of turbulence is absent. Otherwise the folded fields only emerge in the sub-viscous region with larger-scale magnetic perturbations irrelevant in determining the local density structure.

It is also necessary to point out that we use the isotropic Kolmogorov scaling for analytical simplicity in Sections 3.2 and 3.3. But in fact, as the magnetic field becomes dynamically important, anisotropic MHD turbulence develops with the turbulent eddies more elongated along the local magnetic field direction toward smaller scales. Then the Goldreich & Sridhar (1995) scaling applies as a more appropriate description of the relation between the parallel and perpendicular scales with respect to the local magnetic field. If one takes into account the effect of turbulence anisotropy in the above calculations, the viscous damping rate $k^2\nu_\parallel$ is replaced by $k^2_{\perp}\nu_\parallel$, and the latter is relatively small. Here $k_\parallel$ and $k_\perp$ are the parallel and perpendicular components of wavevector $k$. Accordingly, the viscous scale is shifted downward and the corresponding density fluctuations are further reduced (Equation (49)), leading to a less important contribution of the plasma sheets in scatter broadening.

4. DISCUSSION AND CONCLUSIONS

We analyzed various models of electron density fluctuations and examined their effects on broadening FRB pulse widths. Different from earlier studies (e.g., Macquart & Koay 2013; Luan & Goldreich 2014) where the Kolmogorov turbulence is conventionally adopted for describing the spatial power spectrum of density fluctuations, our study is devoted to a general form of the density spectrum, as well as other density structures induced by physical processes including plasma instabilities and fluctuation dynamo in both the IGM and ISM of the host galaxy.

Macquart & Koay (2013) evaluated the strength of scattering in the IGM by assuming a Kolmogorov spectrum and a sufficiently low outer scale of turbulence. Our calculation under similar turbulence conditions yields detectable intergalactic scattering. We disfavor this picture because as pointed out by Luan & Goldreich (2014), an outer scale smaller than \( \sim 10^{24} \) cm entails too large turbulent heating rate to be compatible with the cooling rate in the realistic IGM. Yao et al. (2016) suggested the importance of the IGM in both dispersion and scattering of FRBs and empirically determined a flat DM-dependence $\propto DM^{1.3}$ of the scattering timescale, which to our knowledge is inconsistent with the predictions of existing scattering theories. Furthermore, when confronted to the observational data of known FRBs, non-monotonic dependence of pulse widths on DMs is obviously seen (Katz 2016a, 2016b), e.g., FRB 1107039 has larger DM but shorter scattering timescale in comparison with FRB 110220 (Thornton et al. 2013). The considerable scatter around any single W-DM relation can be hardly interpreted as sightline-to-sightline scatter since the probability of encountering an intervening galaxy along the LOS is quite low (Macquart & Koay 2013). An alternative scenario that the host galaxy dominates both dispersion and scattering was raised in Cordes et al. (2016; see also Xu & Han 2015). Their analysis was restricted to the Kolmogorov turbulence model and based on a specific relation between the broadening time and DM, $W \propto DM^2$, which corresponds to Equation 17(a) at $\beta = 11/3$ and $r_{\text{shift}} < r_0$ in this work. Our general discussion on the spectral properties of density fluctuations overcomes this limitation and enables us to gain new physical insight. We find that a short-wave-dominated spectrum of turbulent density in the host galaxy medium provides a plausible explanation of the pulse broadening of FRBs. A single relation between the scattering and dispersion in the host galaxies for all FRBs is inappropriate because of the widely diverse turbulence properties in different host galaxies. The strong scattering effect can naturally arise as a consequence of a short-wave-dominated density spectrum and in the meantime the host-galaxy component of the total DM is small, supportive of the dominant intergalactic contribution to dispersion and cosmological distances of FRBs.

A short-wave-dominated spectrum of density fluctuations is commonly observed in the inner Galaxy where the turbulent flows are highly supersonic and shock-dominated (Lazarian 2009; Hennebelle & Falgarone 2012; Falceta-Gonçalves et al. 2014). The turbulent energy is predominantly injected by stellar sources such as stellar winds and protostellar outflows, indicative of active star formation (Haverkorn et al. 2008). If an FRB resides in the center region of a galaxy with intense ongoing star formation where the power spectrum of density field becomes flat, evident temporal broadening independent of the inclination angle of the host galaxy is expected. We caution that the situation regarding FRBs with discernible scattering tails is complicated by the fact that the observed pulse width can contain both the host galaxy component and the intrinsic one. Therefore, extra care is needed when the pulse width is used as a discriminator between different progenitor models (Keane et al. 2016).

Among the diverse FRB progenitor models, some are indicative of rich and turbulent ISM environment with intense star formation. The discovery of repeating bursts from FRB 121102 (Spitler et al. 2016) supports an origin of young neutron stars, from which giant radio pulses may be sporadically produced (Connor et al. 2016; Cordes & Wasserman 2016; Lyubarsky & Ostrovskaya 2016). These young neutron stars are likely to be found in star-forming regions where the requirement to produce strong scatter broadening can be easily met. The magnetar giant flare model (Popov & Postnov 2013; Kulkarni et al. 2014; Katz 2016a) also relates FRBs with young neutron stars, which mark the star-forming regions of galaxies. For other repeating FRB models (e.g., Dai et al. 2016; Gu et al. 2016), scattering effect can also manifest to the observer if their preferential environment is characterized by a high star...
formation rate. As for the non-repeating FRBs with distinct cosmological origins, the blazar model (Falcke & Rezzolla 2014; Zhang 2014) invokes delayed collapse of a supra-massive neutron star to a black hole after it loses centrifugal support, with a timescale ranging from minutes (Zhang 2014) to thousands of years (Falcke & Rezzolla 2014) after the birth of the neutron star. Plausibly, if the supra-massive neutron star comes from collapse of a massive star, this model is also related with star formation activity and satisfies the external condition for pulse broadening. Another categories of FRB progenitor systems invoke catastrophic events involving compact star mergers such as double neutron stars, neutron star–black hole, and double black hole mergers (e.g., Piro 2012; Totani 2013; Liu et al. 2016; Wang et al. 2016; Zhang 2016a, 2016b). The star formation process is usually not relevant in such events. So, unless the merger delay timescale is shorter than Myr, as expected in some prompt merger scenarios, the scattering mechanism introduced in this work does not apply to these FRBs.

In contrast to the consideration of extensively distributed scattering medium in this work, the ad hoc thin screen scattering model applies when the scattering matter is concentrated in a local region. By assuming a uniform distribution of the density irregularities along the LOS through the scattering region, a thick scattering screen behaves similar to a thin screen, except that the depth passing through the extended scattering medium should be replaced by a much smaller screen thickness in the latter case. According to Equation (81), extraordinary high density contrast is required to compensate for the dramatic decrease of $D$ and account for strong scattering. This localized density excess is too large to be produced and confined in diffuse IGM or ISM (Katz 2014), but could possibly be associated with the FRB source and located in its immediate vicinity (Masui et al. 2015). For this reason, we relate the thin screen scattering scenario to the intrinsic pulse width and exclude it from our analysis on the scattering effect arising in more diffuse media.

The microscale instabilities are an important physical ingredient in many fundamental processes such as heat conduction (Chandran & Cowley 1998), dynamo growth of magnetic fields (Schekochihin & Cowley 2006), and acceleration of cosmic rays (Lazarian & Beresnyak 2006) in the IGM. The evolution of instabilities are directly related to the magnetic field geometry and intensity at scales smaller than the particle mean free path. Multiple observational techniques have been utilized to measure the extragalactic large-scale (>1 kpc) magnetic fields (Kronberg 1994; Carilli & Taylor 2002; Govoni & Feretti 2004; Xu et al. 2006), but detailed information on small-scale magnetic field structures is still inaccessible due to the limited spatial resolution. As exemplified in this work, the pulse durations of FRBs pose an upper bound on the amplitude of density and magnetic fluctuations, and a lower bound on their characteristic scale, which can be potentially exploited as an observational approach of studying the properties of collisionless regime of the IGM turbulence.

The sheet-like structures of density in the viscosity-damped regime of MHD turbulence are unlikely to dominate the strong scattering of radio waves as suggested in earlier studies (e.g., Goldreich & Sridhar 2006). In the presence MHD turbulence cascade, not only can the rigidity of the folded magnetic field structure easily break down, but also the local magnetic variation on the viscous scale fails to produce sufficient density fluctuations.

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APPENDIX

GENERALIZED FORMALISM OF TEMPORAL BROADENING

The derivation of the diffractive scattering formalism presented in Section 2 for a power-law spectrum of electron density fluctuations can be further generalized. We first write the phase structure function as

$$D_\Phi \sim \pi r_e^2 \lambda^2 D (\delta n_e(r))^2 r. \quad (62)$$

At the diffractive scale $r_{\text{diff}}$, $D_\Phi = 1$ is satisfied and there is

$$\pi r_e^2 \lambda^2 D (\delta n_e(r_{\text{diff}}))^2 r_{\text{diff}} = 1. \quad (63)$$

Substituting $r_{\text{diff}}$ from the above equation in Equation (13) leads to a general form of the scattering timescale,

$$\tau_{\text{sc}} = \frac{D \Delta^2}{4\pi c} r_{\text{diff}}^2 = \frac{D^2 r_e^2 \lambda^4 (\delta n_e(r_{\text{diff}}))^2}{4\pi c r_{\text{diff}}}. \quad (64)$$

We first use Equation (64) to reproduce the expressions of $\tau_{\text{sc}}$ corresponding to a spatial power spectrum of density fluctuations derived in Section 2. In the case of $r_{\text{diff}} < l_0$, the scattering effect is dominated by the inner scale $l_0$ of the density spectrum. From Equations 11(a) and 62, the diffractive scale is

$$r_{\text{diff}} = \frac{l_0}{D_\Phi(l_0)} = [\pi r_e^2 \lambda^2 D (\delta n_e(l_0))^2]^{-1/2} l_0^{1/2}, \quad (65)$$

where the electron density perturbation at $l_0$ depends on the spectral shape,

$$\langle \delta n_e(l_0)^2 \rangle = \langle \delta n_e^2 \rangle \left( \frac{l_0}{L} \right)^{\beta - 3} , \quad \beta > 3, \quad (66a)$$

$$\langle \delta n_e(l_0)^2 \rangle = \langle \delta n_e^2 \rangle , \quad \beta < 3. \quad (66b)$$

Thus $r_{\text{diff}}$ has the form

$$r_{\text{diff}} = (\pi r_e^2 \lambda^2 D (\delta n_e)^2) L^{\beta - 4} l_0^{-1}, \quad \beta > 3, \quad (67a)$$

$$r_{\text{diff}} = (\pi r_e^2 \lambda^2 D (\delta n_e)^2) L^{3 - \beta} l_0^{-1}, \quad \beta < 3. \quad (67b)$$

It recovers Equation 7(a) in combination with Equations 4 and 6. Using Equation (63) together with Equation (65), we find

$$\frac{\langle \delta n_e(r_{\text{diff}})^2 \rangle}{r_{\text{diff}}} = \frac{\langle \delta n_e^2 \rangle}{l_0} \left( \frac{l_0}{L} \right)^{\beta - 4} \frac{1}{r_{\text{diff}}} = \frac{\langle \delta n_e(l_0)^2 \rangle}{l_0}. \quad (68)$$

Inserting this into Equation (64) and considering Equation 66(a) yields

$$\tau_{\text{sc}} = \frac{D^2 r_e^2 \lambda^4}{4\pi c} \langle \delta n_e^2 \rangle \left( \frac{l_0}{L} \right)^{\beta - 4} L^{-1}, \quad \beta > 3, \quad (69a)$$

$$\tau_{\text{sc}} = \frac{D^2 r_e^2 \lambda^4}{4\pi c} \langle \delta n_e^2 \rangle l_0^{-1}, \quad \beta < 3, \quad (69b)$$

which after we incorporate the $(1 + z_0)$ factor and replace $D$ with $D_{\Phi}$ have the same expressions as $W$ in Equations 16(a) and 18(a).
When $r_{\text{diff}}$ resides within the inertial range, $r_{\text{diff}} > l_0$, the density perturbation at $r_{\text{diff}}$ can be given according to the power-law scaling of the spectrum,

$$ (\delta n_e(r_{\text{diff}}))^2 = (\delta n_e)^2 \left( \frac{r_{\text{diff}}}{L} \right)^{\beta-3}, \quad \beta > 3, \quad (70a) $$

$$ (\delta n_e(r_{\text{diff}}))^2 = (\delta n_e)^2 \left( \frac{r_{\text{diff}}}{l_0} \right)^{\beta-3}, \quad \beta < 3. \quad (70b) $$

It can be equivalently written as

$$ (\delta n_e(r_{\text{diff}}))^2 = \frac{SM}{D} r_{\text{diff}}^{\beta-3}. \quad (71) $$

Substituting this into Equation (63) gives

$$ r_{\text{diff}} = (\pi r_c^2 \lambda^3 SM)^{-1}, \quad (72) $$

which recovers Equation 7(b). From both Equations (71) and (72), we can now get

$$ (\delta n_e(r_{\text{diff}}))^2 = \frac{SM}{D} \frac{1}{r_{\text{diff}}^\beta} = (\pi r_c^2 \lambda^3)^{-\beta} D^{-1} SM^{-\beta}. \quad (73) $$

Thus $\tau_{sc}$ from Equation (64) in this case becomes

$$ \tau_{sc} = \frac{D \pi r_c^2 \lambda^3}{4 \pi^2 c^3} \frac{1}{r_{\text{diff}}^\beta} SM^{-\beta}. \quad (74) $$

We can further derive (Equations (4) and (6))

$$ \tau_{sc} = \frac{D \pi r_c^2 \lambda^3}{4 \pi^2 c^3} (\delta n_e)^2 \frac{L}{r_{\text{diff}}} \frac{1}{r_{\text{diff}}^\beta}, \quad \beta > 3, \quad (75a) $$

$$ \tau_{sc} = \frac{D \pi r_c^2 \lambda^3}{4 \pi^2 c^3} (\delta n_e)^2 \frac{l_0}{r_{\text{diff}}^\beta}, \quad \beta < 3. \quad (75b) $$

After adding the $(1 + z_q)$ factor to the above expressions and using $D_{\text{eff}}$ instead of $D$, we obtain the same results in the observer’s frame as in Equations 16(b) and 18(b).

When the density irregularities are characterized by a density perturbation $\delta n_e(d)$ and a length scale $d$, similar to the case of a density power spectrum with $r_{\text{diff}} < l_0$, the phase structure function can be simplified (Scheuer 1968),

$$ D_b \sim \pi r_c^2 \lambda^3 D (\delta n_e(d))^2 d. \quad (76) $$

Strong scattering occurs when $d$ exceeds $r_{\text{diff}}$, which is

$$ r_{\text{diff}} = \sqrt{\frac{1}{D_b}} = \left[ \pi r_c^2 \lambda^3 D (\delta n_e(d))^2 \right]^{-1/2}. \quad (77) $$

The condition $d > r_{\text{diff}}$ (i.e., $\sqrt{D_b} > 1$) sets a minimum $d$ when $\delta n_e$ is provided,

$$ d > \left[ \pi r_c^2 \lambda^3 D (\delta n_e(d))^2 \right]^{-1}, \quad (78) $$

or a minimum density perturbation at a given $d$,

$$ (\delta n_e(d))^2 > \left[ \pi r_c^2 \lambda^3 D d \right]^{-1}. \quad (79) $$

From the relation Equation (63) and the expression of $r_{\text{diff}}$ in Equation (77), we get

$$ \frac{(\delta n_e(r_{\text{diff}}))^2}{r_{\text{diff}}} = \frac{(\pi r_c^2 \lambda^3)^{-\beta} r_{\text{diff}}^{-\beta}}{d} = \frac{1}{d} \frac{(\delta n_e(d))^2}{d}. \quad (80) $$

So the general form of $\tau_{sc}$ in Equation (64) in this situation becomes

$$ \tau_{sc} = \frac{D^2 r_c^4 \lambda^4 (\delta n_e(d))^2}{4 \pi c^3 d}. \quad (81) $$

At the observer’s wavelength $\lambda_0$, Equations (78) and (79) become

$$ d > (1 + z_q)^2 \left[ \pi r_c^2 \lambda_0 D_{\text{eff}} (\delta n_e(d))^2 \right]^{-1}, \quad (82) $$

$$ (\delta n_e(d))^2 > (1 + z_q)^2 \left[ \pi r_c^2 \lambda_0 D_{\text{eff}} \right]^{-1}, \quad (83) $$

and the pulse scatter-broadening measurement from Equation (81) in the frame of the observer is

$$ W = \frac{D_{\text{eff}}^2 r_c^4 \lambda_0^4 (\delta n_e(d))^2}{4 \pi c (1 + z_q)^3 d}. \quad (84) $$

Equations (82)–(84) impose observational constraints on the density perturbation and its characteristic scale that the density fluctuation model under consideration must satisfy.

REFERENCES

Armstrong, J. W., Rickett, B. J., & Spangler, S. R. 1995, ApJ, 443, 209
Berjesnyak, A., & Lazarian, A. 2009, ApJ, 702, 1190
Berjesnyak, A., Lazarian, A., & Cho, J. 2005, ApJL, 624, L93
Bhat, N. D. R., Cordes, J. M., Camilo, F., Nice, D. J., & Lorimer, D. R. 2004, ApJ, 605, 759
Brigninski, S. I. 1965, RpPP, 1, 205
Brathwaite, J. 2015, MNRAS, 450, 3201
Burkhart, B., Collins, D. C., & Lazarian, A. 2015, ApJ, 808, 48
Bykov, A. M., Paerels, F. B. S., & Petrovian, V. 2008, SSRv, 134, 141
Caleb, M., Flynn, C., Bailes, M., et al. 2016, MNRAS, 458, 708
Carilli, C. L., & Taylor, G. B. 2002, ARA&A, 40, 319
Chandran, B. D. G., & Cowley, S. C. 1998, PIPhS, 80, 3077
Chepurnov, A., & Lazarian, A. 2010, ApJ, 710, 853
Cho, J., & Lazarian, A. 2003, MNRAS, 345, 325
Cho, J., Lazarian, A., & Vishniac, E. T. 2002, ApJL, 566, L49
Cho, J., Lazarian, A., & Vishniac, E. T. 2003, ApJ, 595, 812
Cho, J., & Vishniac, E. T. 2000, ApJ, 539, 273
Chzewkowski, E., Forman, W., Jones, C., Sunyaev, R., & Böhringer, H. 2004, MNRAS, 347, 29
Coles, W. A., Rickett, B. J., Codona, J. L., & Freihlich, R. G. 1987, ApJ, 315, 666
Connor, L., Sievers, J., & Pen, U.-L. 2016, MNRAS, 458, L19
Cordes, J. M., & Lazio, T. J. W. 2002, arXiv:astro-ph/0207156
Cordes, J. M., & Lazio, T. J. W. 2003, arXiv:astro-ph/0301598
Cordes, J. M., & Rickett, B. J. 1998, ApJ, 507, 846
Cordes, J. M., & Wasserman, I. 2016, MNRAS, 457, 232
Cordes, J. M., Wharton, R. S., Spitzer, L. G., Chatterjee, S., & Wasserman, I. 2016, arXiv:1605.05890
Dai, Z. G., Wang, J. S., Wu, X. F., & Huang, Y. F. 2016, arXiv:1603.08207
Deshpande, A. A., Dwarkanath, K. S., & Goss, W. M. 2000, ApJ, 543, 227
Dieter, N. H., Welch, W. J., & Romney, J. D. 1976, ApJL, 206, L113
Esquivel, A., & Lazarian, A. 2005, ApJ, 631, 320
Fabian, A. C. 1994, ARA&A, 32, 277
Falcetta-González, D., Kowal, G., Falgarone, E., & Chian, A. C.-L. 2014, NPGeo, 21, 587
Falcke, H., & Rezzolla, L. 2014, A&A, 562, A137
Goldreich, P., & Sridhar, S. 1995, ApJ, 438, 763
Goldreich, P., & Sridhar, S. 2006, ApJL, 640, L159
Govoni, F., & Feretti, L. 2004, IMPED, 13, 1549
Gu, W.-M., Dong, Y.-Z., Liu, T., Ma, R., & Wang, J. 2016, ApJL, 823, L28
Gwinn, C. R., Bartel, N., & Cordes, J. M. 1993, ApJ, 410, 673
Haffner, L. M., Reynolds, R. J., & Tuffe, S. L. 1999, ApJ, 523, 223
