Lifting M-theory to Two-Time Physics

Itzhak Bars, Cemsinan Deliduman, Djordje Minic

Department of Physics and Astronomy
University of Southern California
Los Angeles, CA 90089-0484, USA

Abstract

M-theory has different global supersymmetry structures in its various dual incarnations, as characterized by the M-algebra in 11D, the type IIA, type-IIB, heterotic, type-I extended supersymmetries in 10D, and non-Abelian supersymmetries in the AdS$_n \times S^m$ backgrounds. We show that all of these supersymmetries are unified within the supersymmetry OSp(1/64), thus hinting that the overall global spacetime symmetry of M-theory is OSp(1/64). We suggest that the larger symmetries contained within OSp(1/64) which go beyond the familiar symmetries, are non-linearly realized hidden symmetries of M-theory. These can be made manifest by lifting 11D M-theory to the formalism of two-time physics in 13D by adding gauge degrees of freedom. We illustrate this idea by constructing a toy M-model on the worldline in 13D with manifest OSp(1/64) global supersymmetry, and a number of new local symmetries that remove ghosts. Some of the local symmetries are bosonic cousins of kappa supersymmetries. The model contains 0-superbrane and p-forms (for p=3,6) as degrees of freedom. The gauge symmetries can be fixed in various ways to come down to a one time physics model in 11D, 10D, AdS$_n \times S^m$, etc., such that the linearly realized part of OSp(1/64) is the global symmetry of the various dual sectors of M-theory.

1 This research was partially supported by the US. Department of Energy under grant number DE-FG03-84ER40168.
1 Perspective on hidden symmetries

M-theory is defined in 11 dimensions, with one time and ten space coordinates. It has an extended global supersymmetry characterized by 32 supercharges $Q_\alpha$ and 528 abelian bosonic charges that include the momentum $P_\mu$, the two-brane charge $Z_{\mu\nu}$, and the five-brane charge $Z_{\mu_1\cdots\mu_5}$ [1]–[7].

$$\{Q_\alpha, Q_\beta\} = \gamma^{\mu}_{\alpha\beta} P_\mu + \gamma^{\mu\nu}_{\alpha\beta} Z_{\mu\nu} + \gamma^{\mu_1\cdots\mu_5}_{\alpha\beta} Z_{\mu_1\cdots\mu_5}. \tag{1}$$

The charges $P_\mu$, $Z_{\mu\nu}$, $Z_{\mu_1\cdots\mu_5}$ commute among themselves and with $Q_\alpha$. In addition, the 11-dimensional SO(10,1) Lorentz generator $J_{\mu\nu}$ has non-trivial commutation rules with $Q_\alpha$, $P_\mu$, $Z_{\mu\nu}$, $Z_{\mu_1\cdots\mu_5}$ that correspond to the classification of these charges as spinor, 1-form, 2-form, 5-form respectively in 11-dimensions. We refer to the algebra satisfied by $J_{\mu\nu}$, $Q_\alpha$, $P_\mu$, $Z_{\mu\nu}$, $Z_{\mu_1\cdots\mu_5}$ as the M-algebra.

It is well known that physical input in four dimensions, such as the absence of massless interacting particles with spin higher than two, constrains the maximum number of supersymmetries to 32 [8]. This refers to the maximum number of $Q_\alpha$ that commute with the momentum operators $P_\mu$, as is the case in the M-algebra. However, there is no physical restriction on the number of non-linearly realized supersymmetries that do not commute with $P_\mu$. In particular, it is possible to have supersymmetries that do not commute with the Hamiltonian or $P_\mu$, while being supersymmetries of the action. For example, this is the case with the well known special superconformal symmetry generated by the fermions $S_\alpha$ in any superconformal theory in any dimension. In addition, it has been discovered recently that very simple familiar systems have previously unnoticed hidden symmetries of the type SO($d$, 2), that are not symmetries of the Hamiltonian [9] [10] but are symmetries of the action. Such symmetries are made manifest by lifting the system to the formalism of two-time physics by the addition of gauge degrees of freedom together with new gauge symmetries [3] [11].

In this paper we provide arguments that the supersymmetries of the different dual versions of M-theory are all unified within OSp(1/64), with 64 supercharges. We will show that the M-algebra is a subalgebra of OSp(1/64) without any contractions. The extra 32 supercharges do not commute with $P_\mu$ within OSp(1/64). We suggest they are symmetries of the “action” of M-theory. The symmetry structures that emerge in this way suggest that M-theory could admit a two-time physics formulation with a total of 13 dimensions.

There has been a number of hints that M-theory may contain various two-time structures [11], [12]–[20]. In this paper we will show a new embedding of the symmetries of M-theory in a higher structure suggested by the formalism of two-time physics. We show that when the M-algebra is extended by adding the 11D conformal generator $K^\mu$, the closure requires the full OSp(1/64). Duality and 11D covariance suggest that $K^\mu$ is a hidden symmetry in M-theory. The point is that the 10D covariant type-IIB superalgebra, as well as heterotic and type I superalgebras can be obtained from the same OSp(1/64), and the 10D type-IIA superalgebra
is just the dimensional reduction of the M-theory algebra. While $K^\mu$ is non-linearly realized and remains “hidden” in the 11D version of M-theory, some of its components are linearly realized in a dual version of M-theory, so $K^\mu$ is actually present non-perturbatively. A further clue is the presence of conformal symmetry in some corners of M-theory as noted through the CFT-AdS correspondance [21]-[24].

These points will be illustrated in a specific toy M-model [25] with a world line action that includes 13D p-form degrees of freedom $a^{\hat{M}_1\cdots\hat{M}_p} (\tau)$ for $p = 3, 6$ in addition to the zero brane degrees of freedom $X^{\hat{M}} (\tau), P^{\hat{M}} (\tau), \Theta^\alpha (\tau)$ that normally exist in a worldline formalism [10].

The model introduces new concepts of local symmetries, including one that is a bosonic cousin of kappa supersymmetry. Various gauge choices can be found to yield the M-algebra in 11D, the type IIA, type-IIB, heterotic, type-I extended supersymmetries in 10D, and non-Abelian superalgebras in the AdS$_n \times S^m$ backgrounds. Thus, the symmetries of different corners of the moduli space of M-theory emerge as different gauge choices in this model [25].

2 Subgroup chains

We first show that the M-algebra is contained in OSp(1/64). The supergroup OSp(1/64) has 64 fermionic generators $Q_\hat{\alpha}$ and 2080 bosonic generators $S_\hat{\alpha}\hat{\beta}$ that form a 64×64 symmetric matrix. The Lie superalgebra is

$$\{Q_\hat{\alpha}, Q_\hat{\beta}\} = S_{\hat{\alpha}\hat{\beta}}, \quad [S_{\hat{\alpha}\hat{\beta}}, Q_\hat{\gamma}] = C_{\hat{\alpha}\hat{\beta}\hat{\gamma}} Q_\hat{\gamma} + C_{\hat{\beta}\hat{\gamma}\hat{\alpha}} Q_\hat{\alpha}$$

$$[S_{\hat{\alpha}\hat{\beta}}, S_{\hat{\gamma}\hat{\delta}}] = C_{\hat{\alpha}\hat{\gamma}S_{\hat{\beta}\hat{\delta}}} + C_{\hat{\beta}\hat{\gamma}S_{\hat{\alpha}\hat{\delta}}} + C_{\hat{\beta}\hat{\delta}S_{\hat{\alpha}\hat{\gamma}}} + C_{\hat{\alpha}\hat{\delta}S_{\hat{\beta}\hat{\gamma}}}$$

The $S_{\hat{\alpha}\hat{\beta}}$ form the Lie algebra of Sp(64) and the constant antisymmetric matrix $C_{\hat{\alpha}\hat{\beta}}$ is the metric of Sp(64). A matrix representation of OSp(1/64) is given by 65×65 supermatrices. We are interested in re-expressing these generators in various bases, that are related to the basis above by unitary transformations, such that various spacetime interpretations can be given to those bases. For this purpose several branchings of Sp(64) will be used:

$$A : \quad Sp (64) \supset SU^* (32) \otimes SO (1, 1)$$

$$\supset Sp^* (32) \otimes SO (1, 1) \supset SO (10, 2) \otimes SO (1, 1) \supset \cdots$$

$$B : \quad Sp (64) \supset SO^* (32) \otimes SO (2, 1)$$

$$\supset SU^* (16) \otimes U (1) \otimes SO (2, 1)$$

$$\supset SO (9, 1) \otimes SO (2, 1) \otimes U (1) \supset \cdots$$

$$C : \quad Sp (64) \supset SO (11, 2) \supset \cdots$$

The (*) indicates an appropriate analytic continuation that contains the non-compact groups listed. There is another chain of interest that involves a supergroup that will come up in our
discussion (we are not listing a complete set of branchings).

\[ OSp(1/64) \supset OSp(1/32) \otimes Sp(32) \supset \cdots \]  

(5)

We list the first step of the decomposition of the 64 \oplus 2080 representations for the A,B,C branches. For the A-branch we have the \( SU^* (32) \otimes SO (1, 1) \) representations

\[
\begin{align*}
64 &= 32^{1/2} \oplus \overline{32}^{-1/2}, \\
2080 &= \left( 1^0 \oplus 1023^0 \right)_{32,\overline{32}} \oplus 528^\dagger_{(32,\overline{32})} \oplus 528^-_{(\overline{32},32)}
\end{align*}
\]  

(6)

where the superscripts correspond to the \( SO(1, 1) \) charge, and subscripts indicate the products of the supercharges that produce those representations (\( s \) and \( a \) stand for symmetric and antisymmetric product respectively). For the B-branch we have the \( SO^* (32) \otimes SO (2, 1) \) representations

\[
\begin{align*}
64 &= (32, 2), \\
2080 &= \left( 1_{(32,\overline{32})}, 3_{(2,2)} \right) \oplus \left( 527_{(32,\overline{32})}, 3_{(2,2)} \right) \oplus \left( 496_{(32,\overline{32})}, 1_{(2,2)} \right)
\end{align*}
\]  

(7)

For the C-branch we have the \( SO (11, 2) \) representations

\[
\begin{align*}
64 &= 64, \\
2080 &= 78 (\tilde{J}_2) \oplus 286 (\tilde{J}_3) \oplus 1716 (\tilde{J}_6)
\end{align*}
\]  

(10)

where the \( \tilde{J}_{\tilde{M}\tilde{N}} \), \( \tilde{J}_{\tilde{M}_1\tilde{M}_2\tilde{M}_3} \), \( \tilde{J}_{\tilde{M}_1\ldots\tilde{M}_6} \) are p-forms in 13D. The \( \tilde{J}_p \) can be represented by p-products of 13D 64\times64 gamma matrices. For \( p = 2, 3, 6 \) these are 64\times64 symmetric matrices that represent the 2080 components of \( S_{\tilde{\alpha}\tilde{\beta}} \) in a 13D spinor basis. The A,B,C branches in this paper are related to the A,B,C branches discussed in S-theory \[13\]. The C branch provides a \( SO(11, 2) \) covariant 13D interpretation for M-theory as we will see with an explicit toy M-model.

\section{11D and 12D interpretations}

If \( SO(11, 2) \) of the C-branch is interpreted as the conformal group in 11-dimensions then one may re-classify all the generators as representations of the Lorentz subgroups \( SO(10, 1) \times SO (1, 1) \) in 11-dimensions and 2-dimensions contained in 13D. To do so, re-label the 13 dimensions with two sets, one in 10+1 and the other in 1+1 dimensions, \( \tilde{M} = \mu \oplus m \) where \( \mu = 0, 1, \ldots, 10 \) and \( m = (+', -') \), with the metric in the extra two dimensions taken in a lightcone type basis \( \eta^{+',-'} = -1 \). This conformal basis emerges naturally as one of the gauge choices in two-time physics \[2\] \[10\] \[25\] as will be discussed later in this paper. The 64-spinor may be re-labelled as \( Q_{\tilde{\alpha}} \sim Q_{\alpha}^{1/2} \oplus S_{\alpha}^{-1/2} \) with \( \alpha \) denoting the 32-spinor in 11D and \( \pm \frac{1}{2} \) denoting the two chiral spinors in 2D. Then the generators \( J_{\tilde{M}\tilde{N}} \) of the conformal group are identified as \( J^{\mu\nu}-SO(10, 1) \) Lorentz transformations, \( J^{+\mu} \equiv P^\mu \) -translations, \( J^{-\mu} \equiv K^\mu \) -special conformal transformations, and
\[ J^{+\prime -\prime} \equiv D \text{-dilatations.} \] All the generators have definite dimensions under the commutation relations with \( D \) which generates the SO(1,1) subgroup.

\[
D = 1: \quad J^{+\mu} \equiv P^\mu, \quad J^{+\mu\nu} \equiv Z^{\mu\nu}, \quad J^{+\mu_1\ldots\mu_5} \equiv Z^{\mu_1\ldots\mu_5} \tag{11}
\]

\[
D = \frac{1}{2}: \quad Q_{\alpha}^\frac{1}{2} \equiv Q_\alpha \tag{12}
\]

\[
D = 0: \quad \begin{cases} J^{\nu}, \quad J^{+\prime -\prime} \equiv D, \quad J^{+\prime -\prime}, \quad J^{+\mu} \equiv J^\mu, \quad J^{\mu_1\mu_2\mu_3}, \\ J^{+\mu_1\ldots\mu_4} \equiv J^{\mu_1\ldots\mu_4}, \quad J^{\mu_1\ldots\mu_6} \equiv \varepsilon^{\mu_1\ldots\mu_6\mu_7\ldots\mu_5} J_{\mu_1\ldots\mu_5} \end{cases} \tag{13}
\]

\[
D = -\frac{1}{2}: \quad Q_{\alpha}^{-\frac{1}{2}} \equiv S_\alpha \tag{14}
\]

\[
D = -1: \quad J^{-\mu} \equiv K^\mu, \quad J^{-\mu\nu} \equiv \tilde{Z}^{\mu\nu}, \quad J^{-\mu_1\ldots\mu_5} \equiv \tilde{Z}^{\mu_1\ldots\mu_5} \tag{15}
\]

The dimensions \( D = \pm 1, \pm \frac{1}{2}, 0 \) provide a 5-gradation of the superalgebra such that under the commutation rules the dimensions add \([X_{D_1}, X_{D_2}] \sim X_{D_1+D_2}\). If \( D_1 + D_2 \) is not one of the dimensions listed, the result of the commutator is zero. Then we see that the \( D \geq 0 \) operators \( J^{\mu\nu}, Q_{\alpha}^1/2, P^\mu, Z^{\mu\nu}, Z^{\mu_1\ldots\mu_5} \), form the M-algebra: the anti-commutator \( \{Q_\alpha, Q_\beta\} \) contains only the 528 generators \( P^\mu, Z^{\mu\nu}, Z^{\mu_1\ldots\mu_5} \) (D=1) which commute among themselves and with \( Q_{\alpha}^1/2 \) (D=1/2), while \( J^{\mu\nu} \) (D=0) generates the Lorentz transformations SO(10,1). Hence, the 11D M-algebra is contained in OSp(1/64) as a subalgebra without considering any contractions!

OSp(1/64) is the smallest simple supergroup that includes the M-algebra as a sub-algebra without resorting to contractions. As argued above, if the special conformal generator \( K^\mu \) is also included along with the M-algebra, then all other generators of OSp(1/64) must also be included for consistency with the 5-grading and Jacobi identities. Thus, conformal symmetry in 11D together with the M-algebra demand OSp(1/64).

The 11D interpretation fits into the higher algebraic structures contained in the A-branch (4). The spinor decomposes as 64=32^{1/2} \oplus 32^{-1/2} under SU^*(32) \otimes SO(1,1). Each 32-spinor corresponds to the fundamental representation of Sp^*(32) \subset SU^*(32) and furthermore they are classified as real Weyl spinors of SO(10,2) of same chirality. This classification defines the A-envelop of the 5-graded 11D superconformal algebra described above such that 11D is embedded in 12D. One may then combine the 11D operators \( J_\mu \oplus J_{\mu\nu} \) into a 12D generator \( J_{MN} \), and the 5-form \( J_{\mu_1\ldots\mu_5} \) may be written as a 6-form \( J_{M_1\ldots M_6} \) that is self-dual in 12D. The \( J_{MN} \) form the SO(10,2) subgroup listed in the A-branch, and together with the \( J_{M_1\ldots M_6} \) they make up the 528-adjoint of Sp(32). Similarly, the remaining operators make up complete 12D representations, such as \( P_1 \oplus Z_2 = Z_{MN} \) and \( Z_5 = Z_{M_1\ldots M_6}^+ \) and \( J_3 \oplus J_4 = J_{M_1\ldots M_4} \), etc. Finally all of these are put together as SU^*(32) \otimes SO(1,1) representations as in the first step of the A-branch in (6-7). This shows that the operators \( J_{MN}, Z_{MN}, Z_{M_1\ldots M_6}^+, Q_\alpha \) which form a 12D envelop for the M-algebra, as used in several applications of S-theory, also fit in the two-time formalism given in this paper.
4 Type-IIA, IIB, heterotic, type-I interpretations

The 10D type-IIA version follows from re-classifying the 11D basis of $[\mathbb{I}^{11} \mathbb{I}^{12}]$ under SO(9, 1) of 10D. The 32-spinor supercharge $Q^{1/2}_\alpha$ in 11D becomes the two opposite chirality supercharges $16+\overline{16}$ of type IIA. Similarly the 32-spinor of special superconformal generator $S^{-1/2}_\alpha$ becomes the two opposite chirality special superconformal supercharges $16+\overline{16}$ of type IIA. The 11D M-algebra is rewritten trivially in 10D in the type IIA basis.

In the C/B-branch, the type-IIB version follows from re-classifying the 64-spinor of 13D as the spinor $\times$ spinor of 10D$\oplus$3D, namely $Q_\alpha = Q_{\alpha a} \oplus S_{\alpha a}$ where $\alpha, \dot{\alpha}$ denote the real spinors $16, \overline{16}$ of SO(9, 1) and $a$ denotes the doublet spinor of SO(2, 1). Both sets of spinors $Q_{\alpha a}, S_{\alpha a}$ are real, and there is no relation between them via hermitian conjugation. The $Q_{\alpha a}$ play the role of the two 10D supersymmetry generators of type IIB, while the two $S_{\alpha a}$ play the role of the two 10D special superconformal generators of type IIB. Similarly the vector of SO(11, 2) is decomposed by using $\tilde{M} = \tilde{\mu} + \tilde{m}$ with $\tilde{\mu} = 0, 1, \cdots, 9$ and $\tilde{m} = +', -', 2'$. The $\tilde{m} = \pm'$ components are the same as the $\pm'$ components used to reduce 13D to 11D, and the 2' component is a re-naming of the 11th dimension in SO(10, 1). The anti-commutators in the superalgebra take the sketchy form

$$\{Q_{\alpha a}, Q_{\beta b}\} = P^{\mu}_{\tilde{\mu}} \oplus Z^{\tilde{m}}_{\mu_1 \cdots \mu_3} \oplus Z_{\tilde{\mu}_1 \mu_2 \mu_3}$$

$$\{S_{\alpha a}, S_{\beta b}\} = K^{\tilde{m}}_{\mu} \oplus Z^{\tilde{m}}_{\mu_1 \cdots \mu_5} \oplus Z_{\tilde{\mu}_1 \mu_2 \mu_3}$$

$$\{Q_{\alpha a}, S_{\beta b}\} = D_{\tilde{m} \tilde{n}} \oplus J_{\mu_1 \tilde{n}} \oplus X_{\mu_1 \cdots \mu_4} \oplus \chi \oplus J_{\mu_1 \tilde{m}} \oplus X_{\mu_1 \cdots \mu_4}$$

The map between the 13D notation and the 10D+3D notation follows from $\tilde{M} = \tilde{\mu} \oplus \tilde{m}$

$$Q_\alpha \sim Q_{\alpha a} \oplus S_{\alpha a}$$

$$J_{\tilde{M}_1 \tilde{M}_2} \sim J_{\mu_1 \mu_2} \oplus (P^{\mu}_{\tilde{\mu}} + K^n_{\mu}) \oplus D_{\tilde{m}_1 \tilde{m}_2}$$

$$J_{\tilde{M}_1 \tilde{M}_2 \tilde{M}_3} \sim \left(Z_{\mu_1 \mu_2 \mu_3} - \tilde{Z}_{\mu_1 \mu_2 \mu_3}\right) \oplus J_{\mu_1 \mu_2} \oplus (P^{\mu}_{\tilde{\mu}} - K^n_{\mu}) \varepsilon_{\mu_1 \mu_2 \mu_3} \varepsilon_{\tilde{m}_1 \tilde{m}_2 \mu_3}$$

$$J_{\tilde{M}_1 \cdots \tilde{M}_6} \sim \varepsilon_{\mu_1 \cdots \mu_6 \mu_3 \cdots \mu_4} X^{\mu_1 \cdots \mu_4} \oplus \left[Z_{\mu_1 \cdots \mu_5} \oplus Z^{\tilde{m}}_{\tilde{\mu}_1 \cdots \tilde{\mu}_5}\right]$$

$$\varepsilon_{\mu_1 \cdots \mu_4} \varepsilon_{\tilde{m}_1 \tilde{m}_2 \mu_3} \oplus \left[Z_{\mu_1 \cdots \mu_5} + \tilde{Z}_{\tilde{\mu}_1 \cdots \tilde{\mu}_5}\right]$$

where $\oplus$ is direct sum, but $\pm$ imply ordinary addition or subtraction, $\varepsilon_{\tilde{m}_1 \tilde{m}_2 \mu_3}$ is the SO(2, 1) invariant Levi-Civita tensor, and $Z^{\tilde{m}}_{\mu_1 \cdots \mu_5}, \tilde{Z}^{\tilde{m}}_{\tilde{\mu}_1 \cdots \tilde{\mu}_5}$ are self-dual and anti self-dual respectively in 10D.

The SO(9, 1) generators are $J_{\mu_1 \mu_2}$ and the SO(2, 1) generators are $D_{\tilde{m} \tilde{n}}$. The operators labelled with $\tilde{m}$ are triplets of SO(2, 1) while the others are singlets. The singlet operator $\chi$ is written in terms of the 13D operators as $\chi = J^{\pm', -'2'}$. Its commutation rules with the 64 spinors $Q_\alpha$ is $[\chi, Q_\alpha] \sim (\Gamma^{\pm', -'2'}Q)_\alpha$. Since one may write $\Gamma^{+', -'2'} = \Gamma^{01 \cdots 9}$ for 64$\times$64 gamma matrices $[\mathbb{I}^{13}]$, the operator $\chi$ acts like the chirality operator on the 10D spinors $Q_{\alpha a}, S_{\alpha a}$. Therefore $\chi$
provides a 5-grading for the OSp(1/64) operators based on their 10D chirality

\[ \chi = 1 : \ P_1^1 \oplus Z_5^1 \oplus Z_3 \]  
\[ \chi = \frac{1}{2} : \ Q_{aa} \]  
\[ \chi = 0 : \ \chi \oplus D^2 \oplus J_2 \oplus J_2^1 \oplus X_4 \oplus X_4^1 \]  
\[ \chi = -\frac{1}{2} : \ S_{aa} \]  
\[ \chi = -1 : \ K_1^1 \oplus \tilde{Z}_5^1 \oplus \tilde{Z}_3 \]  

The bosonic generators \( P_1^1, Z_5^1 \) etc. are labelled by numbers in the subscripts and superscripts \( Z_p^q \) that correspond to \( p \)-forms in 10D and \( q \)-forms in 3D respectively. So the commutation rules for OSp(1/64) may be written in a graded chirality basis in the form \( [X_{\chi_1}, X_{\chi_2}] = X_{\chi_1+\chi_2} \). If the chirality \( \chi_1 + \chi_2 \) does not exist the result of the commutator is zero. The \( \chi \geq 0 \) operators \( J_2, D^2, \chi, Q_{aa}, P_1^1, Z_5^1, Z_3 \) define the B-algebra. The chirality grading shows that \( P_1^1, Z_5^1, Z_3 \) commute with each other as well as with \( Q_{aa} \), while the SO(9,1) \( \otimes \) SO(2,1) \( \otimes \) U(1) subgroup, consisting of \( J_2, D^2, \chi \), map the operators \( Q_{aa}, P_1^1, Z_5^1, Z_3 \) into themselves.

We have established that the B-algebra, that is essential for understanding M-theory in a IIB basis, is included in OSp(1/64) as a subalgebra without any contractions. Furthermore, since it fits into the C-branch it is consistent with the SO(11,2) symmetry of two-time physics and the idea that the 10D+3D basis is arrived at as a gauge choice in two-time physics. Indeed this is true in the toy M-model discussed later.

These C-branch 13D=10D+3D results intersect the B-branch 10D+3D basis as can be seen by the following larger classification under the B-branch (B) that include the envelops SU\(^*\) (16) \( \otimes \) U (1) and SO\(^*\) (32) \( \otimes \) U (1)

\[ (496, 1) = [\chi \oplus (J_2 \oplus X_4)](16 \times 16)_{(2 \times 2)_a} \oplus (Z_3)(16 \times 16)_{a(2 \times 2)} \oplus (\tilde{Z}_3)(16 \times 16)_{a(2 \times 2)} \]  
\[ (1, 3) = (D^2)(16 \times 16)_{s(2 \times 2)} \]  
\[ (527, 3) = (P_1^1 \oplus Z_5^1 \oplus Z_3^1)(16 \times 16)_{s(2 \times 2)} \oplus (K_1^1 \oplus \tilde{Z}_5^1 \oplus \tilde{Z}_3^1)(16 \times 16)_{s(2 \times 2)} \oplus (J_2^1 \oplus X_4^1)(16 \times 16)_{s(2 \times 2)} \]

The products of SO(9,1) \( \otimes \) SO(2,1) spinor representations \( Q = (16, 2) \) and \( S = (\overline{16}, 2) \) that produce the various 10D and 3D forms are indicted. The SU\(^*\) (16) \( \otimes \) U (1) subgroup of SO\(^*\) (32) is generated by \( (J_2 \oplus X_4) \oplus \chi \). The forms in independent parentheses (\( \cdots \)) correspond to irreducible representations under SU\(^*\) (16) \( \otimes \) SO(2,1) \( \otimes \) U (1). Their collection in each line correspond to SO\(^*\) (32) \( \otimes \) SO(2,1) representations as given in (B). The 496 is the adjoint representation of SO\(^*\) (32) and the 527 is the symmetric traceless tensor of SO\(^*\) (32). These 2080 generators form the algebra of Sp(64).

It was shown in [20] that the B-algebra may be written in an SL(2, Z) (U-duality) basis instead of the spacetime SO(2,1) =SL(2, R) basis given above. These two bases are related.
to each other by a deformation that involves the IIB string coupling constant $z = a + ie^{-\phi}$ (axion and dilaton moduli), and its SL(2, Z) properties $z' = (az + b)/(cz + d)$. The deformed B-algebra may be used to perform certain non-perturbative computations at any value of the string coupling constant (see [20]). The entire superalgebra OSp(1/64) may be rewritten in the SL(2, Z) basis by following the prescription in [20]. Therefore the observations in S-theory in an IIB basis may now be interpreted as observations in the two-time physics version of M-theory taken in a particular gauge.

The 10D A and B bases are obviously related to each other since they both occur in the C-branch. The map between these two corresponds to a rearrangement of the 64 fermions that are in the spinor representation of SO(11, 2). This map is clearly related to T-duality as discussed in [13], and in the present context of two-time physics it is interpreted as just a gauge transformation from one fixed gauge to another fixed gauge.

The heterotic and type-I superalgebras in 10D are then obtained as in [13] from the IIB basis since the 4 and the 8 spinors are rewritten in 32+32 form. This is possible because one can work in a OSp(1/64) representation space labelled by the commuting operators $P_{\mu}^{m} \oplus Z_{\mu_{1} \cdots \mu_{5}}^{m} \oplus Z_{\bar{\mu}_{1} \bar{\mu}_{2} \bar{\mu}_{3}}^{m}$. The heterotic or type-I sectors may be viewed as BPS-like sectors in which some of these charges are related to each other as discussed in the second paper in [13].

5 \ AdS$_{n}$ \otimes \ S$^{m}$ bases

In the C-branch, starting with SO(11, 2) one can come down to the basis labelled by the subgroups SO(3, 2) \otimes SO(8) or to SO(6, 2) \otimes SO(5), which are the isometries of the spaces AdS$_{4}$\times S$^{7}$ and AdS$_{7}$\times S$^{4}$ respectively. The reduction is obtained by rewriting the 13D label $\tilde{M} = \tilde{\mu} \oplus \tilde{m}$, where $\tilde{\mu}$, $\tilde{m}$ are labels for the vectors of SO(3, 2),SO(8) or SO(5),SO(6, 2). Also, the 64-spinor is rewritten in 32+32 form $Q_{\tilde{\alpha}} = \psi_{\tilde{\alpha} a}^{+} \oplus \psi_{\tilde{\alpha} a}^{-}$. Each 32=4×8 since $\tilde{\alpha}$ denotes the 4-spinor for Sp(4) \sim SO(3, 2) or SO(5), and $\alpha$, $\tilde{\alpha}$ denote the two spinors 8$_{\pm}$ for SO(8) or SO(6, 2). For SO(3, 2) \otimes SO(8) both $\psi_{\tilde{\alpha} a}^{+}$ and $\psi_{\tilde{\alpha} a}^{-}$ are real since the corresponding spinors are real. Hence, they each have 32 real and independent components. For SO(5) \otimes SO(6, 2) they are in a complex basis since the 4 and the 8$_{\pm}$ are pseudo-real. However, the pseudo-reality condition still gives 32 real and independent components in each of the $\psi_{\tilde{\alpha} a}^{+}$, $\psi_{\tilde{\alpha} a}^{-}$. The anti-commutators in the superalgebra take the sketchy form

$$\{\psi_{\tilde{\alpha} a}^{+}, \psi_{\tilde{\beta} b}^{+}\} = J_{\tilde{\mu} \rho}^{+} \oplus J_{\tilde{\mu} \rho}^{\tilde{m} \tilde{n}} \oplus X_{\tilde{\mu}}^{\tilde{m} \tilde{n}} \oplus X_{\tilde{\mu} \rho}^{\tilde{m} \tilde{n} \cdots \tilde{m}_{4}} \tag{32}$$
$$\{\psi_{\tilde{\alpha} a}^{-}, \psi_{\tilde{\beta} b}^{-}\} = J_{\tilde{\mu} \rho}^{-} \oplus J_{\tilde{\mu} \rho}^{\tilde{m} \tilde{n}} \oplus X_{\tilde{\mu}}^{- \tilde{m} \tilde{n}} \oplus X_{\tilde{\mu} \rho}^{- \tilde{m} \tilde{n} \cdots \tilde{m}_{4}} \tag{33}$$
$$\{\psi_{\tilde{\alpha} a}^{+}, \psi_{\tilde{\beta} b}^{-}\} = Y_{\tilde{\mu} \rho}^{\tilde{m}} \oplus Y_{\tilde{\mu} \rho}^{\tilde{m}_{1} \tilde{m}_{2} \tilde{m}_{3}} \oplus Y_{\tilde{\mu} \rho}^{\tilde{m} \tilde{m}_{1} \tilde{m}_{2} \tilde{m}_{3}} \oplus Y_{\tilde{\mu} \rho}^{\tilde{m} \tilde{m}_{1} \tilde{m}_{2} \tilde{m}_{3}} \tag{34}$$
The map to the 13D operators \( \hat{J}_{2,3,6} \) can be easily established through \( \hat{M} = \hat{\mu} \oplus \hat{m} \). The \( X^{\pm \hat{m}_1 \cdots \hat{m}_4} \) are self or anti-self dual in the 8-dimensions labelled by \( m \). The operators \( \psi_{\hat{a}a}^+ J_2^+, J_2^\pm, \pm X_1^\pm, X_2^{\pm 4} \) form \( \text{OSp}(1/32)_\pm \) sub-supergroups, but \( \text{OSp}(1/32)_+ \) does not commute with \( \text{OSp}(1/32)_- \) since \( \left\{ \psi_{\hat{a}a}^+, \psi_{\hat{b}b}^- \right\} \) is not zero. However, \( \text{OSp}(1/64) \supset \text{OSp}(1/32)_+ \otimes \text{Sp}(32)_- \). The generators \( J_2^+ \oplus J^{2 \pm} \) form \( \text{SO}(3,2)_+ \otimes \text{SO}(8)_- \) or \( \text{SO}(5,2)_+ \) subgroups embedded in each of the \( \text{Sp}(32)_\pm \). How is \( \text{OSp}(1/64) \) superalgebra related to the familiar \( \text{AdS} \times S^7 \) supersymmetries \( \text{OSp}(8/4) \) or \( \text{OSp}(6,2/4) \)? These are not sub-supergroups of \( \text{OSp}(1/64) \). From our analysis it can be seen that \( \text{OSp}(8/4) \) or \( \text{OSp}(6,2/4) \), which includes \( \psi_{\hat{a}a}^+ J_2^+, J_2^\pm \), gets enlarged by the addition of the non-Abelian operators \( X_1^{\pm 2}, X_2^{\pm 4} \) into \( \text{OSp}(1/32)_+ \) (it is not possible to set these operators to zero naively since they are non-Abelian and they cannot be simultaneously diagonalized in a quantum theory). In turn, \( \text{OSp}(1/32)_+ \) is the sector of \( \text{OSp}(1/64) \) that is a singlet under \( \text{Sp}(32)_- \).

We speculate that if the CFT-AdS conjecture \([21]\) corresponds to a corner of \( M \)-theory then the enlargement of the superalgebra probably does occur on the CFT side from the point of view of the \( N=8 \) Super Yang-Mills theory in 3D. The symmetry of this theory in perturbation theory to all orders is \( \text{OSp}(8/4) \). As shown in \([10]\), by taking the \( \text{SO}(3,2) \) indices \( \hat{\mu} = \mu, \pm' \) and \( \psi_{\hat{a}a}^+ \sim Q_{\hat{a}a}^{1/2} \otimes S_{\hat{a}a}^{-1/2} \), the conformal supersymmetry of the \( \text{AdS}_4 \times S^7 \) background can be rewritten in the compact form of Eq.\((32)\), excluding the \( X_1^{\pm 2} \) and \( X_2^{\pm 4} \) generators. How can one see the enlargement to \( \text{OSp}(1/32)_+ \)? One begins with non-perturbative field configurations that turn on the central extensions \( \left\{ Q_{\hat{a}a}^{1/2}, Q_{\hat{b}b}^{1/2} \right\} \sim X^{\pm \hat{m}\hat{n}}_{\hat{\mu}=\hat{\mu}'} \); then the conformal symmetry \( K_{\mu} \) requires all \( X^{\pm \hat{m}\hat{n}}_{\hat{\mu}=-\hat{\mu}'} \), \( X^{\pm \hat{m}\hat{n}}_{\hat{\mu}=\hat{\mu}'} \), and their non-Abelian nature generates \( X^{\pm \hat{m}_1 \cdots \hat{m}_4} \), thus completing the \( \text{OSp}(1/32)_+ \) superalgebra of Eq.\((32)\). This argument shows that the inclusion of non-perturbative physics in the \( \text{AdS}_4 \times S^7 \) background could be described by the \( \text{Sp}(32)_-\)singlet sector of \( \text{OSp}(1/64) \). The singlet sector can arise as a result of a contraction that would be related to the limits \([21]\) one must take to establish the AdS-CFT correspondence.

Similarly, in the \( C \)-branch, starting with \( \text{SO}(11,2) \) we can come down to a 12D\( ' \) basis by separating the 13th spacelike dimension \( \hat{M} = M \oplus \hat{1}' \). This gives \( 64=32_L + 32_R \) where \( 32_{L,R} \) are the two \( \text{SO}'(10,2) \) spinors \( \psi^L, \psi^R \) of opposite chirality (contrast 12D\( ' \) to the 12D of the \( A \)-branch which gave same chirality). Next we separate 12D\( ' = 6D + 6D \) so that \( \text{SO}'(10,2) \rightarrow \text{SO}(6,2) \otimes \text{SO}(6) \) which is the isometry group of the \( \text{AdS}_5 \times S^5 \) space. Consider the \( 32_L \) real components \( \psi_{\alpha a}^L \) written in the \( (4,4) \) complex spinor basis with \( \alpha \) and \( a \) denoting the complex 4 spinors of \( \text{SU}(2,2) = \text{SO}(4,2) \) and \( \text{SU}(4) = \text{SO}(6) \) respectively. Since the basis is complex we need to consider the hermitian conjugates \( \left( \psi_{\alpha a}^{L,R} \right)^\dagger = \bar{\psi}_{\hat{a}a}^{L,R} \) where \( \hat{\alpha} \) and \( \hat{a} \) denote the complex 4 spinors. Then the 12D\( ' \) basis of \( \text{OSp}(1/64) \) is reduced to the 6D+6D basis and it takes the following sketchy form

\[
\left\{ \psi_{\alpha a}^L, \bar{\psi}_{\beta b}^L \right\} \sim J_2^L \oplus J_2^\pm \oplus J_L \oplus X_2^{L2}, \quad (35)
\]

\[
\left\{ \psi_{\alpha a}^L, \psi_{\beta b}^L \right\} \sim Z_1^{L1} \oplus Z_3^{L3}, \quad \text{and h.c.} \quad (36)
\]
where the generators \( J^a_p, X^a_p, Y^a_p, Z^a_p, W^a_p \) are labelled with numbers in subscripts or superscripts that are \( \text{SO}(4, 2) \text{ p-forms or } \text{SO}(6) \text{ q-forms, and } \text{h.c.} \) stand for hermitian conjugate relations. The map to the 13D operators \( \tilde{J}_{2,3,6} \) can be easily established through the reduction \( M = \tilde{\mu} \oplus \tilde{m} \oplus \Gamma \) where \( \tilde{\mu}, \tilde{m} \) are labels for the vectors of \( \text{SO}(4, 2), \text{SO}(6) \).

\( J^2_L \oplus J^2_R \) generate \( \text{SU}(2) \otimes \text{SU}(4) \) which is the isometry of \( \text{AdS}_5 \times S^5 \), i.e. \( \text{SO}(4, 2) \otimes \text{SO}(6) \subset \text{SO'}(10, 2) \). The supersymmetry algebra in this background is \( \text{SU}(2, 2/4) \). What is the relation of \( \text{OSp}(1/64) \) and \( \text{SU}(2, 2/4) \)? As before, this is not a sub-supergroup. Again, upon the inclusion of the charges \( J_L \oplus X^{L2} \oplus Z^{L1} \oplus Z^{L3} \) it can be seen that \( \text{SU}(2, 2/4) \) is enlarged into \( \text{OSp}(1/32)_L \) as in the first two lines of the equations above (see also [27] [28]). Then we see that \( \text{SU}(1/64) \supset \text{OSp}(1/32)_L \otimes \text{Sp}(32)_R \) and the sector that is singlet under \( \text{Sp}(32)_R \) is described by \( \text{OSp}(1/32)_L \).

If one starts with the superconformal \( \text{N}=4 \) super Yang-Mills theory in 4D, the perturbative symmetry is \( \text{SU}(2, 2/4) \). By including central extensions that correspond to non-perturbative backgrounds such as monopoles and dyons one turns on the central charge \( Z^{Lm} \) which is part of \( Z^{LI} \). Conformal symmetry requires the full \( Z^{LI} \) and its hermitian conjugate \( \left(Z^{LI}\right)^\dagger \). Their commutators generate all the other remaining charges to complete the \( \text{OSp}(1/32)_L \) superalgebra. Thus the inclusion of non-perturbative physics in the \( \text{AdS}_5 \times S^5 \) background could be described by the \( \text{Sp}(32)_R \)-singlet sector of \( \text{OSp}(1/64) \). The singlet sector can arise as a result of a contraction that may be related to the limits [21] one must take to establish the AdS-CFT correspondence.

### 6 Toy M-Model, cousins of kappa symmetry

Our ideas can be dynamically illustrated with a toy M-model on the worldline with a new set of gauge symmetries. The main point is that in this model the various dual bases described above emerge naturally by making appropriate gauge choices within the same theory. Conversely, one can transform one basis to another dual basis by making gauge transformations, thus imitating duality in M-theory. Here we only outline the general structure of such a model, leaving the details to a future paper [25]. Similar structures could be constructed for any group \( G \) and a choice of subgroup \( H \) as outlined at the end of [14].

Consider the supergroup \( G = \text{OSp}(1/64) \) with the subgroup \( H = \text{SO}(11, 2) \). Let \( X^M_i = (X^M, P^M) \) represent position / momentum \( \text{SO}(11, 2) \) vectors. The 0-brane vectors \( X^M_i (\tau) \) form an \( \text{Sp}(2) \) doublet. \( \text{Sp}(2) \) is gauged by including the gauge potentials \( A^{ij} (\tau) \). This
gauge symmetry introduces first class constraints whose solution requires two-timelike dimensions as explained in \[3\]. In addition, consider the 65×65 matrix which is a group element \(g(\tau) \in \text{OSp}(1/64)\). It is a singlet under \(\text{Sp}(2)\). The subgroup \(H = \text{SO}(11, 2)\) acting simultaneously on the left side of \(g\) and on the vectors \(X^\hat{M}_i\) is gauged. The gauge potential is \(\Omega^{\hat{M}\hat{N}}(\tau)\).

With this information we can write covariant derivatives as in \[10\]

\[
D_\tau X^\hat{M}_i = \partial_\tau X^\hat{M}_i - \varepsilon_{ik} A^{kj} X^\hat{M}_i - \Omega^{\hat{M}\hat{N}} X_{i\hat{N}}, \tag{41}
\]

\[
D_\tau g = \partial_\tau g - \frac{1}{4} \Omega^{\hat{M}\hat{N}} (\Gamma^{\hat{M}\hat{N}} g). \tag{42}
\]

Consider the part of the Cartan connection \((D_\tau g)g^{-1}\) restricted to the subgroup \(H\)

\[
(D_\tau g g^{-1})^H = \frac{1}{32} \text{Str} \left( \Gamma^{\hat{M}\hat{N}} D_\tau g g^{-1} \right) = \frac{1}{32} \text{Str} \left( \Gamma^{\hat{M}\hat{N}} \partial_\tau g g^{-1} \right) - \Omega^{\hat{M}\hat{N}} \tag{43}
\]

A Lagrangian that is invariant under the gauge symmetry \(\text{Sp}(2) \times \text{SO}(11, 2)\) is given by

\[
\mathcal{L} = \frac{i}{2} \varepsilon^{ij} D_\tau X^\hat{M}_i X^\hat{N}_j \eta_{\hat{M}\hat{N}} + \frac{i}{2} \left[ (D_\tau g g^{-1})^H \right]^2 + \frac{i}{2} \left( \varepsilon^{ij} X^\hat{M}_i X^\hat{N}_j \right)^2. \tag{44}
\]

This action is invariant under the global symmetry \(\text{OSp}(1/64)\) that acts linearly on the right side of \(g(\tau)\). On the left side of \(g(\tau)\) the evident symmetry is smaller \(H = \text{SO}(11, 2)\) because of the presence of \(\Omega\), but as we will see there is a much bigger local symmetry that is realized non-linearly. Thus the model is realized in the C-branch \(\text{OSp}(1/64)\). As we saw above, the C-branch contains all the interesting spacetime interpretations at its intersections with the A and B branches and with the \(\text{OSp}(1/32)\) branch.

The group element \(g(\tau)\) can be written in the form \(g = ht\) where \(h \in H\) and \(t \in G/H\). We can use the \(H = \text{SO}(11, 2)\) gauge symmetry to choose a unitary gauge by eating away \(h(\tau)\). Using the equations of motion (or doing the path integral) one may solve for \(\Omega^{\hat{M}\hat{N}} = \bar{\Omega}^{\hat{M}\hat{N}} + L^{\hat{M}\hat{N}}\), with \(\bar{\Omega}^{\hat{M}\hat{N}} = \frac{1}{32} \text{Str} (\Gamma^{\hat{M}\hat{N}} \partial_\tau tt^{-1})\) and substitute back into the action to find the version of the action given in \[10\]

\[
\mathcal{L} = \partial_\tau X_1 \cdot X_2 - A^{ij} X^\hat{M}_i X^\hat{N}_j - \frac{1}{32} \text{Str} \left( \Gamma^{\hat{M}\hat{N}} \partial_\tau tt^{-1} \right) L^{\hat{M}\hat{N}}. \tag{45}
\]

where \(L^{\hat{M}\hat{N}} = \varepsilon^{ij} X^\hat{M}_i X^\hat{N}_j = X^{\hat{M}} p^{\hat{N}} - X^{\hat{N}} p^{\hat{M}}\) is the orbital angular momentum in 13D. A total derivative has been dropped in the first term of \(\mathcal{L}\). With this we see that the canonical formalism gives \(X^\hat{M}_1 = X^{\hat{M}}\) as the position and \(X^\hat{M}_2 = P^{\hat{M}}\) as the momentum of the 13D zero-brane.

We parametrize \(t(a(\tau), \Theta(\tau))\) it terms of 64 fermionic \(\Theta_{\hat{a}}(\tau)\) and 286+1716 bosonic \(a^{\hat{M}_1\hat{M}_2\hat{M}_3}(\tau), a_{\hat{M}_1\ldots\hat{M}_6}(\tau)\) degrees of freedom that correspond to 13D p-forms with \(p=3,6\). As described in \[10\], the global symmetry \(g \subset \text{OSp}(1/64)\) acts on the right side of \(t(\tau)\) and it must be compensated by a field dependent, gauge restoring transformation \(h^{-1}(a, \Theta; g)\) on the left side

\[
t(a, \Theta) \rightarrow t(a', \Theta') = h^{-1}t(a, \Theta) g. \tag{46}
\]
$h(\tau)$ acts as a field dependent $\text{SO}(11, 2)$ gauge transformation on $\tilde{\Omega}^{\tilde{M}\tilde{N}}(a, \Theta)$ and $X^\tilde{M}_i$. The action remains invariant because it was built as a gauge invariant under $\text{SO}(11, 2)$ and globally invariant under $\text{OSp}(1/64)$.

The action in this gauge involves the following fields on the worldline $X^\tilde{M}_i(\tau)$, $A^{ij}(\tau)$, $a_3(\tau)$, $a_6(\tau)$, $\Theta(\tau)$. The $X^\tilde{M}_i(\tau), \Theta^\tilde{a}(\tau)$ may be interpreted as the 13D superspace of two-time physics as in [10], however the bigger dimensions in the present case, as opposed to the 3,4,6D cases of [10], require the extra bosonic degrees of freedom $a_3, a_6$. From the superalgebra point of view these are closely associated with the $p$-brane charges $\tilde{J}_3, \tilde{J}_6$ of the C-branch, hence the $a_p$ may be thought of as the $\tau$-component of $(p + 1)$-form gauge potentials $A_{p+1}$ for $p = 3, 6$.

There is also additional local symmetry beyond $\text{SO}(11, 2)$ that originates with transformations on the left side of $t(a, \Theta)$ with fermionic as well as bosonic parameters $b_3(\tau), b_6(\tau), \kappa^\tilde{a}(\tau)$ that form a generalization of kappa supersymmetry. The transformation is

$$t(a, \Theta) \rightarrow t(a^\prime, \Theta^\prime) = h^{-1}t(b, \kappa)\ t(a, \Theta),$$

where again $h(b, \kappa; a, \Theta)$ is the induced local Lorentz transformation. For infinitesimal $b, \kappa$ we have $t = 1_{65} + b_3\Gamma^3 + b_6\Gamma^6 + \kappa^\tilde{a}F^\tilde{a}$ where $\Gamma^{3,6}$ and $F^\tilde{a}$ are $65 \times 65$ matrices that provide representations of the generators of $G/H$. Then we find that the Lagrangian is invariant under these additional gauge symmetries provided the parameters $\kappa, b_3, b_6$ are constrained by the following relations (where $\sim X_i \cdot X_j$ means proportional up to field dependent functions with appropriate indices)

$$L_{\tilde{M}\tilde{N}}(\Gamma^{\tilde{M}\tilde{N}} \kappa)_{\tilde{a}} \sim L_{[\tilde{M}_1 \cdots \tilde{M}_5]_\tilde{N}} \sim L_{[\tilde{M}_1 \tilde{M}_2 \cdots \tilde{M}_6]_\tilde{N}} \sim X_i \cdot X_j.$$  \hfill (48)

The left side looks like Lorentz transformations on $\kappa, b_3, b_6$ with a Lorentz parameter $L_{\tilde{M}\tilde{N}}$. Then the part proportional to $X_i \cdot X_j$ is cancelled by choosing $\delta A^{ij}$ so that $\mathcal{L}$ is invariant (see [10] for an explicit example). These gauge symmetries are more than sufficient to remove the ghosts in $\Theta, a_3, a_6$ associated with the extra timelike dimension in 13D. We will call these gauge symmetries “extended kappa symmetries”.

We may now make various gauge choices for $\text{Sp}(2), \text{SO}(11, 2)$, and extended kappa symmetries, that are manifestly covariant under various spacetime interpretations (one-time) as discussed in the purely algebraic discussion in the first part of this paper. To do this we reorganize the degrees of freedom $X^\tilde{M}_i(\tau), a_3(\tau), a_6(\tau), \Theta^\tilde{a}(\tau)$ according to the representations of the various subgroups of $\text{SO}(11, 2)$ as we did for the generators of $\text{OSp}(1/64)$. Some of the pieces of $X_i, a_3, a_6, \Theta$ are set to zero or constants by gauge fixing. Then part of $\text{OSp}(1/64)$ is realized linearly on the remaining degrees of freedom and the remainder is realized non-linearly. For such examples in simpler cases without supersymmetry see [4] and with supersymmetry see [10]. This discussion makes it evident that we can choose gauges that would realize the 11D M-algebra, the 10D type IIA, type IIB, heterotic, type-I supersymmetries, or the superalgebras describing the $\text{AdS}_n \times S^m$ backgrounds. These gauge choices provide different looking toy
models that have the supersymmetries of the various corners of the moduli space of M-theory \cite{25}. Since they are all derived from the same unified 13D theory by gauge choices, they can be transformed into each other by gauge transformations that correspond to dualities in M-theory.

The toy M-model given here has a much richer set of possible gauge choices along the lines of \cite{9} that tie together systems such as free particles, Hydrogen atom, harmonic oscillators, curved spaces, and even non-relativistic systems with more general potentials, etc. This suggests that M-theory may have similar properties. It appears that M-theory could be lifted to two-time physics with a global symmetry OSp(1/64). We are hopeful that M-theory will be better understood by studying it covariantly in 13D in the formalism of two-time physics in the presence of various gauge degrees of freedom and new gauge symmetries of the type described here.

References

[1] P. K. Townsend, hep-th/9507048.

[2] I. Bars, Phys. Rev. D54 (1996) 5203 or hep-th/9604139; hep-th/9604200.

[3] P. K. Townsend, hep-th/9612121, hep-th/9712004, hep-th/9708034; D. Sorokin and P.K. Townsend, hep-th/9709007.

[4] J. W. van Holten and A. van Proeyen, J. Phys. A15 (1982) 3763.

[5] R. D’Auria and P. Fré, Nucl. Phys. B201 (1981) 101.

[6] P. Hořava, Phys. Rev. D59 (1999) 046004, hep-th/9712130.

[7] M. Günaydin, D. Minic, Nucl. Phys. B523 (1998) 145, hep-th/9802047; M. Günaydin, Nucl. Phys. B528 (1998) 432, hep-th/9803138.

[8] W. Nahm, Nucl. Phys. B135 (1978) 149.

[9] I. Bars, C. Deliduman and O. Andreev, Phys. Rev. D58 (1998) 066004, or hep-th/9803188; I. Bars, Phys. Rev. D58 (1998) 066006, or hep-th/9804028; I. Bars and C. Deliduman, Phys. Rev. D58 (1998) 106004, or hep-th/9806085; I. Bars, hep-th/9810023, to appear in Phys. Rev. D.; for a review see, I. Bars, hep-th/9809034.

[10] I. Bars, C. Deliduman and D. Minic, hep-th/9812161.

[11] M. Duff and M. P. Blencowe, Nucl. Phys. B310 (1988) 387.

[12] C. Vafa, Nuc. Phys. B469 (1996) 403.
[13] I. Bars, Phys. Rev. D55 (1997) 2373 or hep-th/9607112; hep-th/9608061. Phys. Lett. B403 (1997) 257 or hep-th/9704054.

[14] D. Kutasov and E. Martinec, Nucl. Phys. B477 (1996) 652; Nucl. Phys. B477 (1996) 675.

[15] H. Nishino and E. Sezgin, Phys. Lett. B388 (1996) 569.

[16] I. Bars and C. Kounnas, Phys. Lett. B402 (1997) 25; Phys. Rev. D56 (1997)3664.

[17] E. Sezgin, Phys. Lett. B403 (1997) 265.

[18] I. Bars and C. Deliduman, Phys. Rev. D56 (1997) 6579 or hep-th/9707213; Phys. Lett. B417 (1998)24 or hep-th/9710066.

[19] E. Sezgin and Rudychev, hep-th/9711128; Phys. Lett. B424 (1998) 60.

[20] H. Nishino, Phys. Lett. B426 (1998) 64 or hep-th/9710141; Nucl. Phys. B542 (1999) 217 or hep-th/9807193; hep-th/9901104.

[21] J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231 or hep-th/9711200.

[22] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B428 (1998) 105 or hep-th/9802109.

[23] E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253 or hep-th/9802150.

[24] the relevant supergroups were studied, for example, in I. Bars and M. Günaydin, Comm. Math. Phys. 91 (83) 31 ; M. Günaydin and N. Marcus, Class. and Quant. Grav. 2 (1985) L11; L19; M. Günaydin and N. Warner, Nucl. Phys. B272 (1986) 99; M. Günaydin, P. van Nieuwenhuizen and N. Warner, Nucl. Phys. B255 (1985) 63; M. Pernici, K. Pilch, P. van Nieuwenhuizen, Nucl. Phys. B259 (1985) 460.

[25] I. Bars, C. Deliduman and D. Minic, A toy M-model, in preparation.

[26] I. Bars, Phys. Rev. D56 (1997) 7954 or hep-th/9706185.

[27] B. Craps, J. Gomis, D. Mateos and A. van Proeyen, hep-th/9901060.

[28] S. Ferrara and M. Porrati, hep-th/9903241.