Fermi liquid theory for nonlinear transport through a multilevel Anderson impurity

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A Fermi-liquid description is presented to give a comprehensive view of the charge and heat transport through an Anderson impurity with $N$ discrete levels, at arbitrary electron filling. It is microscopically described in terms of three renormalized parameters and two nonlinear susceptibilities, which we also calculate with the numerical renormalization group up to $N = 6$. We find that in the SU($N$) symmetric case the nonlinear susceptibilities and the Fano factor for nonlinear current noise have the Kondo plateau structure near integer fillings. We also show that the nonlinear current noise for $N = 2$ exhibits a universal magnetic-field dependence in the Kondo regime.

Introduction.— Quantum impurity systems exhibit fascinating universal behavior at low temperatures. Specifically, it was shown in early years that strongly correlated low-energy states of the Kondo systems can be described as a local Fermi liquid in zero dimension. Universal Fermi-liquid behavior has also been observed in the SU($N$) symmetric case studied by Mora et al., despite that it is necessary for the SU($N$) symmetric case specifically for $N = 4, 6$ in the SU($N$) symmetric case, we find that the structures due to the Kondo and valence-fluctuation states alternatively emerge depending on electron filling. Furthermore, we show that the nonlinear current noise through a single-orbital Anderson impurity exhibits a universal magnetic-field dependence in the Kondo regime without time-reversal symmetry.

Formulation.— We consider an $N$-level Anderson impurity coupled to two leads on the left ($L$) and right ($R$):

$$\mathcal{H} = \sum_{\sigma=1}^{N} \epsilon_{d\sigma} n_{d\sigma} + \sum_{\lambda=L,R} \sum_{\sigma=1}^{N} v_{\lambda} \left( \psi_{\lambda \sigma}^\dagger d_{\sigma} + d_{\sigma}^\dagger \psi_{\lambda \sigma} \right) + \sum_{\lambda=L,R} \sum_{\sigma=1}^{N} \int_{-D}^{D} \frac{d \epsilon}{2\pi} \delta_{\sigma,\sigma'}^\epsilon c_{\epsilon,\lambda \sigma}^\dagger c_{\epsilon,\lambda \sigma'} + \frac{U}{2} \sum_{\sigma \neq \sigma'} n_{d\sigma} n_{d\sigma'}.$$  

$d_{\sigma}^\dagger$ creates an impurity electron with energy $\epsilon_{d\sigma}$, $n_{d\sigma} \equiv d_{\sigma}^\dagger d_{\sigma}$, and $U$ the Coulomb repulsion. Conduction electrons are normalized as $\{c_{\epsilon,\lambda \sigma}, \epsilon'_{\lambda',\lambda'} \} = \delta_{\lambda,\lambda'} \delta_{\sigma,\sigma'} \delta(\epsilon - \epsilon')$. The coupling between $\psi_{\lambda \sigma} \equiv \int_{-D}^{D} d \epsilon \sqrt{\rho_{\epsilon}} c_{\epsilon,\lambda \sigma}^\dagger$ and $d_{\sigma}^\dagger$ yields a resonance of the width $\Delta \equiv \Gamma_L + \Gamma_R$, with $\Gamma_{\lambda} = \pi \rho_{\epsilon} \epsilon_{\lambda}^2$, $\rho_{\epsilon} = 1/(2D)$, and $D$ the half band width.

We consider the non-equilibrium current $J$ through the quantum dot at finite bias voltage $eV \equiv \mu_L - \mu_R$:

$$J = \frac{e}{h} \sum_{\lambda} \int_{-\infty}^{\infty} d \omega \left[ J_L(\omega) - J_R(\omega) \right] T_{\sigma}(\omega, T, eV),$$

where

$$T_{\sigma}(\omega, T, eV) \equiv \frac{-4 \Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R} \text{Im} G_{\sigma}(\omega, T, eV).$$
Here, $f_{\lambda}(\omega) \equiv f(\omega - \mu_{\lambda})$ with $f(\omega) = [e^{\omega/T} + 1]^{-1}$, and $G_{\sigma}(\omega, T, eV) = [\omega - \epsilon_{d\sigma} + i\Delta - \Sigma_{\sigma}^{r}(\omega, T, eV)]^{-1}$ is the retarded Green’s function. Effects of the interaction $U$ enter through the self-energy $\Sigma_{\sigma}^{r}$, and it captures the dependences on $eV$ and $T$ through an evolution along the Keldysh time-loop contour. From the transmission probability $T_{\sigma}(\omega, T, eV)$, we can also obtain the heat current $J_{Q} = -\kappa_{QD} \delta T$, introducing a temperature difference $\delta T$ between the two leads [32]. The thermal conductivity $\kappa_{QD}$ is determined by the linear-response functions $\mathcal{L}_{QD}^{\sigma} = \int_{-\infty}^{\infty} d\omega \, \omega^{2} \langle -\frac{\partial f}{\partial \omega} \rangle T_{\sigma}(\omega, 0, 0)$ for $n = 0, 1, 2$. In addition, we also consider the noise current, defined as the symmetrized product of current operator $\delta J(t) \equiv \langle \delta \hat{J}(t) \rangle_{eV}$,

$$S_{\text{noise}}^{QD} = \int_{-\infty}^{\infty} dt \langle \delta \hat{J}(t) \delta \hat{J}(0) + \delta \hat{J}(0) \delta \hat{J}(t) \rangle_{eV},$$

where $\langle \cdots \rangle_{eV}$ denotes the Keldysh steady-state average.

**Fermi-liquid properties up to next leading order.** — We calculate these transport coefficients up to next leading order, i.e. the first two terms of the expansion with respect to $T$ and $eV$. To this end, we expand $\Sigma_{\sigma}^{r}(\omega, T, eV)$ up to terms of order $\omega^{2}$, $T^{2}$, and $(eV)^{2}$ for general $N$ extending the previous approach [22,24]. The Fermi-liquid relations which hold between the expansion coefficients due to the current conservation play an essential role in the following microscopic formulation.

The phase shift is given by $\delta_{p} \equiv \cot^{-1}(\epsilon_{d\sigma}/\Delta)$ with $\epsilon_{d\sigma} = \epsilon_{d\sigma} + \Sigma_{\sigma}^{r}(0, 0, 0)$. It plays a primary role and determines the ground-state properties: $T_{\sigma}(0, 0, 0) \propto \sin^{2}\delta_{p}$, and $\langle n_{d\sigma} \rangle = \delta_{p}/\pi$, i.e. the Friedel sum rule.

The first derivatives define the renormalization factors, $1/z_{\sigma} = 1 - \frac{\partial \Sigma_{\sigma}^{r}(0, 0, 0)}{\partial \omega}\vert_{\omega=0}$ and $\tilde{\chi}_{\sigma\sigma'} = \frac{\delta_{\sigma\sigma'} + \frac{\partial \Sigma_{\sigma}^{r}(0, 0, 0)}{\partial \omega}}{\partial \omega}\vert_{\omega=0}$ [22]. The latter one can also be written as $\tilde{\chi}_{\sigma\sigma'} = \chi_{\sigma\sigma}/\rho_{d\sigma}$ at $T = eV = 0$, in terms of the linear susceptibilities $\chi_{\sigma\sigma'} = \frac{1}{\theta T} \int_{0}^{1/T} dt \langle \delta n_{d\sigma}(\tau) \delta n_{d\sigma'}(t) \rangle$ with $\delta n_{d\sigma} = n_{d\sigma} - \langle n_{d\sigma} \rangle$, and $\rho_{d\sigma} = \sin^{2}\delta_{p}/(\pi \Delta)$. The FL relation holds between these two coefficients: $1/z_{\sigma} = \tilde{\chi}_{\sigma\sigma} [3,4].$

The second derivatives are complex: the imaginary part corresponds to the damping rate of quasiparticles of order $\omega^{2}$, $T^{2}$, and $(eV)^{2}$ [3–6,10,17]. The real part determines the energy shifts of the quasiparticle states, and satisfies the FL relation: $\frac{\partial^{2} \Sigma_{\sigma}^{r}(0, 0, 0)}{\partial \omega^{2}} = \Re \frac{\partial \Sigma_{\sigma}^{r}(0, 0, 0)}{\partial \omega}\vert_{\omega=0}$ [22,24]. The real coefficients can be expressed in terms of the nonlinear susceptibilities $[34].$

**SU($N$) symmetric case.** — We next consider the SU($N$) symmetric case in which the impurity levels are degenerate $\epsilon_{d\sigma} \equiv \epsilon_{d}$, and the linear susceptibilities $\chi_{\sigma\sigma'}$ have only two independent elements. The diagonal element determines the energy scale $T^{*} = \sqrt{4N\chi_{\sigma\sigma}}$ by which the $T$-linear specific heat is scaled as $C_{\text{imp}} = \frac{N\pi^{2}}{3} T^{*}(T/T^{*})$. It can be identified as the Kondo temperature at electron fillings where $N_{d} = \sum_{\sigma} \langle n_{d\sigma} \rangle$ approaches an integer. The off-diagonal element can be rearranged such that $R \equiv 1 - \chi_{\sigma\sigma'}/\chi_{\sigma\sigma}$ for $\sigma \neq \sigma'$, i.e. the Wilson ratio which corresponds to a dimensionless residual interaction [25].

In this work, we calculate the low-energy formulas for the transport coefficients assuming that the tunnel couplings and chemical potentials are symmetric: $\Gamma_{L} = \Gamma_{R}$ and $\mu_{L} = -\mu_{R}$. We obtain $J$ and $\kappa_{QD}$ using $\Sigma_{\sigma}^{r}(\omega, T, eV)$ expanded in such a way mentioned in the above. For

### Table I. Coefficients C’s introduced in Eqs. [3]–[7]. W’s and Θ’s represent the two- and three-body contributions, respectively.

| $C_{T}$ | $\frac{e^{2}}{4h} \left[ T_{\sigma} + \Theta_{1} + (N-1) \tilde{\Theta}_{1} \right]$ | $T_{\sigma} \equiv \left[ 1 + 2(N-1)(R-1)^{2} \right] \cos 2\delta$ |
| $C_{V}$ | $\frac{e^{2}}{4h} \left[ T_{\sigma} + \Theta_{3} + 3(N-1) \tilde{\Theta}_{1} \right]$ | $W_{\Theta} \equiv \left[ 1 + 5(N-1)(R-1)^{2} \right] \cos 2\delta$ |
| $C_{S}$ | $\frac{e^{2}}{4h} \left[ W_{\sigma} - \cos 2\delta \left( \Theta_{1} + 3(N-1) \tilde{\Theta}_{1} \right) \right]$ | $W_{\sigma} \equiv \cos 4\delta + \left[ 4 + 5 \cos 4\delta + \frac{1}{2} - \cos 4\delta \right] (N-2)(N-1)(R-1)^{2}$ |
| $C_{\sigma}^{QD}$ | $\frac{7e^{2}}{8h} \left[ W_{\sigma}^{QD} + \Theta_{1} + \frac{1}{2}(N-1) \tilde{\Theta}_{1} \right]$ | $W_{\sigma}^{QD} \equiv 10(1-\cos 2\delta) - \frac{5(N-1)(R-1)^{2}}{4} \cos 2\delta$ |
consists of two types of contributions, denoted as \( W \) and \( \Theta \). The \( W \)-part, defined in the right column of table \( \square \), represents the two-body contributions which can be described as functions of \( R \) and \( \delta \). The \( \Theta \)-part represents the three-body contributions:

\[
\Theta_1 = \frac{\sin 2\delta}{2\pi} \frac{\chi_{\sigma \sigma}}{\chi_{\sigma \sigma}^2}, \quad \Theta_{11} = \frac{\sin 2\delta}{2\pi} \frac{\chi_{\sigma' \sigma'}}{\chi_{\sigma \sigma}^2}, \quad \Theta_{111} = \frac{\sin 2\delta}{2\pi} \frac{\chi_{\sigma' \sigma'}}{\chi_{\sigma \sigma}^2}, \quad \text{for } \sigma \neq \sigma' \neq \sigma'. \tag{8}
\]

Here, \( \Theta_1 \) is a dimensionless parameter for diagonal components \( \chi_{\sigma \sigma} \), and \( \Theta_{11} \) is that for the components \( \chi_{\sigma' \sigma'} \) with \( \sigma' \neq \sigma \). The last one \( \Theta_{111} \) is defined for three different levels, which emerges for \( N \geq 3 \) but does not contribute to the coefficients \( C \)'s for the present case. Nevertheless, \( \Theta_{111} \) does contribute to the transport when the tunneling couplings and the chemical potentials may not both be symmetric.

The results listed in table \( \square \) indicate that transport properties of the SU(\( N \)) Fermi-liquid state can be described completely by five parameters, \( \delta, T^*, R, \Theta_1, \) and \( \Theta_{11} \), up to next leading order for symmetric junctions with \( \Gamma_L = \Gamma_R \) and \( \mu_L = -\mu_R \). For the first three of the five, one can choose \( (n_{d\sigma}), \chi_{\sigma \sigma}, \) and \( \chi_{\sigma' \sigma'} \) as an alternative set of independent parameters. These parameters can be determined experimentally from five different measurements, for instance, from \( T_C(0, 0, 0) \) and some of the \( C \)'s, and also from thermodynamic quantities such as the \( T \)-linear specific heat and susceptibilities.

How do these Fermi-liquid parameters evolve as the number of levels \( N \) and their position \( \epsilon_d \) vary? Since the impurity occupation \( N_d \) also varies continuously with \( \epsilon_d \), the Kondo and valence-fluctuation states alternatively emerge for \( N > 2 \); it is one of the most different points from those of the SU(2) \( \square \). In this work, we calculate the FL parameters for \( N = 4, 6 \) with the NRG using especially the interleaved algorithm for \( N = 6 \) \( \square \), choosing the Coulomb interaction to be much larger than the hybridization energy scale: \( U/(\pi \Delta) = 5.0 \).

The results are plotted in Fig. \( \square \) as functions of \( \xi_{d} \equiv \epsilon_d + (N - 1)U/2 \) for \( N = 4 \) (left panel) and \( N = 6 \) (right panel). We note that two-body correlations for \( N = 4 \), specifically \( z \) and \( R \), have already been investigated in detail \( \square \). To our knowledge, however, the nonlinear susceptibilities for \( N = 4 \) and the five FL parameters for \( N = 6 \) have not been explored so far.

Top panel of Fig. \( \square \) shows three FL parameters which are related to \( (n_{d\sigma}), \chi_{\sigma \sigma}, \) and \( \chi_{\sigma' \sigma'} \). We see that \( \sin^2 \delta \), which determines the transport of leading order, shows a flat Kondo ridge of the unitary limit \( \delta \sim \pi/2 \) near the particle-hole point \( \xi_d = 0 \) where the occupation number is almost fixed at \( N_d \approx N/2 \). It also has the plateaus at other integer filling points of \( N_d \). We can see such structures in the top panel at \( \xi_d \approx \pm U \) for \( N = 4, 6 \), and also at \( \xi_d \approx \pm 2U \) for \( N = 6 \) that will evolve to plateaus for much larger \( U \) although the slope is still not flat enough for this value of \( U \).

The renormalization factor \( z \) is suppressed in a wide region \( |\xi_{d}| \lesssim N_d^{-1}U \), seen as a broad valley in Fig. \( \square \) for both \( N = 4 \) and \( 6 \). Inside the valley region \( z \) has \( N - 1 \) minima at \( \xi_d \approx N - 2M \) for \( M = 1, 2, \ldots, N - 1 \), where the impurity filling approaches an integer \( N_d = M \). We also see that \( z \) is significantly suppressed by the interaction even in the intermediate valence states emerging in between two adjacent minima as the peak height is much smaller than the valley depth. It indicates that \( T^* \equiv z \pi \Delta/(4 \sin^2 \delta) \) becomes also small throughout \( 1 \lesssim N_d \lesssim N - 1 \). The broad valley becomes shallow as \( N \) increases, and will vanish in the large \( N \) limit \( \square \).

The Wilson ratio can be rescaled as \( \tilde{K} \equiv (N - 1)(R - 1) \) for \( N > 2 \), and is plotted in Fig. \( \square \). It is almost saturated to the strong-coupling value \( \tilde{K} = 1 \) throughout the region \( 1 \lesssim N_d \lesssim N - 1 \). Thus, the derivative of \( \tilde{K} \) becomes small \( \partial \tilde{K} / \partial N_d \approx 0 \). This also indicates the fact that not only the charge susceptibility, \( \chi_c \equiv -\partial \langle n_{d\sigma} \rangle / \partial \epsilon_d \) = \( \chi_{\sigma\sigma}(1 - \tilde{K}) \), but also its derivative \( \partial \chi_c / \partial \epsilon_d \) is suppressed in this region.

The three-body parameter \( \Theta_1 \) is plotted in the middle panel of Fig. \( \square \) together with the other two rescaled ones: \( -\Theta_{11} \equiv -(N - 1)\Theta_{11} \), and \( \Theta_{111} \equiv (N - 1)(N - 2)\Theta_{111} \).
Chiral regime near half-filling

\[ \epsilon \approx \text{constant} \]

The three-body parameters vanish at \( \xi_d = 0 \) due to the particle-hole symmetry, and are very small in the Kondo regime near half-filling \( -\frac{U}{2} \lesssim \xi_d \lesssim \frac{U}{2} \). They also have typical structures at \( \xi_d \approx \pm U \) for \( N = 4, 6 \) and also at \( \xi_d \approx \pm 2U \) for \( N = 6 \), which will evolve to the Kondo plateaus as \( U \) increases. We also find that \( \Theta_1, \Theta_2, \Theta_3 \) and \( \Theta_{11} \) take very close values over the wide range \( |\xi_d| \lesssim \frac{N_d - 1}{2} U \), which corresponds to electron-fillings of \( 1 \lesssim N_d \lesssim N - 1 \). This is caused by the suppression of change fluctuations and derivative of \( K \), mentioned above. Specifically, in the strong coupling limit \( |U| \gg \Delta \) at integer fillings \( \xi_d = \frac{N_d - 1}{2} U \) for \( M = 1, 2, \ldots, N - 1 \), low-energy states can be described by the SU(\( N \)) Kondo model. In this limit, only two of the five FL parameters, i.e., the Kondo energy scale \( T^* \) and \( \Theta_1 \), become independent as the other three are locked at \( \delta \rightarrow \pi M/N, \tilde{K} \rightarrow 1 \), and \( -\Theta_{11} \rightarrow \Theta_1 \). As the impurity level goes further away \( |\xi_d| \gtrsim \frac{N_d - 1}{2} U \), the electron filling becomes almost empty or full, and then each of the three \( \Theta \)'s approaches the non-interacting value of its own: \( \Theta_1 \rightarrow -2 \) and the other two vanish.

The coefficients \( C \)'s can also be deduced from these NRG data, and are shown in the bottom panel of Fig. 1. The difference between the \( C \)'s near half-filling \( |\xi_d| \lesssim \frac{U}{2} \) is caused by the two-body contributions \( W \)'s because the three-body ones \( \Theta \)'s almost vanish in this region. Specifically, the \( T^2 \) conductance \( C_T \) is determined mainly by \( W_T \) over a wider range \( |\xi_d| \lesssim \frac{N_d - 1}{2} U \). This is because the three-body contributions to \( C_T \) almost cancel out \( \Theta_1 + (N - 1) \Theta_{11} \approx 0 \) in this region as a result of the suppression of charge fluctuations mentioned above. Correspondingly, the \( \Theta \)'s give negative contributions on the \( T^3 \) thermal conductivity, as \( \Theta_1 + \frac{1}{2}(N - 1) \Theta_{11} \approx -\frac{i\pi}{2} \Theta_{11} \) in this region, but otherwise \( C_{\text{QD}} \) shows similar qualitative behavior as \( C_T \). The three-body correlations give positive contributions the \( (eV)^2 \) nonlinear conductance \( C_V \) as \( \Theta_1 + 3(N - 1) \Theta_{11} \approx 2 \Theta_{11} \) in the same region. Therefore, \( C_V \) is enhanced significantly at \( \xi_d \approx \pm U \) for \( N = 4 \) and at \( \xi_d \approx \pm 2U \) for \( N = 6 \), where \( -\Theta_{11} \) has a deep valley with electron fillings of \( N_d = 1 \) and \( N - 1 \).

The Fano factor, defined as a ratio of the nonlinear component of the \( S_{\text{QD}} \) current noise to that of the current \( J \) at \( T = 0 \), is given by

\[ F_K \equiv \lim_{|eV| \to 0} \frac{S_{\text{QD}}}{-2e^2|eV| \sin^2 \frac{\delta}{2}} = \frac{C_S}{C_V/3} \tag{10} \]

Here, \( C_S \) and \( C_V \) are the coefficients defined in the above. This formula covers previous result, obtained by Mora et al specifically for \( N = 2 \) at zero magnetic field [20]. Equation (10) also reproduces the other result that was derived for general \( N \) at half-filling [30], where \( \delta = \pi/2 \) and the three-body contributions vanish. Furthermore, it is also consistent with the corresponding formula for the SU(\( N \)) Kondo model by Mora et al [24].

In the left panel of Fig. 2 the Fano factor for \( N = 4 \) and 6 are plotted vs \( \xi_d \) for two different values of \( U \). By definition, \( F_K \) changes sign at zero points of \( C_S \). It also diverges at zero points of \( C_V \), where the nonlinear component of \( J \) vanishes and changes sign from the backscattering one to the forward one. Such a singularity emerges already for \( U = 0 \) at \( \xi_d = -\frac{\Delta}{2} \), and for large \( U \) it approaches the point near \( |\xi_d| \approx \frac{N_d - 1}{2} U \) which is in the valence fluctuation regime towards empty or fully occupied state. We find that \( F_K \) has a structure which evolves to the Kondo plateau for large \( U \) near integer fillings \( N_d = 1, 2, \ldots, N - 1 \) in between the two singular points. Outside this region, \( F_K \) approaches the noninteracting value \( F_K \rightarrow -1 \) for \( |\xi_d| \rightarrow \infty \). We see for both \( N = 4 \) and 6 that at the singular point \( F_K \) diverges towards different directions for large \( U \) from the ones for small \( U \). The direction is determined whether \( C_S \) is positive or negative at the zero point of \( C_V \). In contrast, for \( N = 2 \) all calculations examined so far indicate that it is negative for any \( U \gtrsim 0 \).

Magnetic-field dependence of \( C_S \) for \( N = 2 \). — We now consider effects of magnetic field \( b \) on the SU(\( N \)) and time-reversal symmetries. Specifically at half-filling for \( N = 2 \), the impurity level is given by \( \epsilon_{dt} = -\frac{U}{2} - b \) and \( \epsilon_{dt} = -\frac{U}{2} + b \), and the occupation number is fixed at \( \langle n_{d\uparrow} \rangle = \langle n_{d\downarrow} \rangle \). In this case low-energy transport up to next leading order can also be described by five parameters: the magnetization \( m_d = \langle n_{d\uparrow} \rangle - \langle n_{d\downarrow} \rangle \), two independent components of the linear susceptibilities \( \chi_{\uparrow\uparrow} = \chi_{\downarrow\downarrow} \) and \( \chi_{\uparrow\downarrow} = \chi_{\downarrow\uparrow} \), and two nonlinear susceptibilities \( \chi_{\uparrow\uparrow\uparrow\uparrow} = -\chi_{\downarrow\downarrow\downarrow\downarrow} \) and \( \chi_{\uparrow\uparrow\downarrow\uparrow} = -\chi_{\downarrow\downarrow\uparrow\downarrow} \).

While the average current \( J \) has already been studied precisely in this case [21, 24], the current noise still has not. In this work we calculate \( S_{\text{QD}} \) at finite magnetic
field $b$ up to order $(eV)^3$ at $T = 0$ [34]:

$$S_{\text{noise}} = \frac{4e^2|eV|}{h} \left[ \frac{\sin^2(\pi m_d)}{4} + C_S \left( \frac{eV}{T_K} \right)^2 + \cdots \right].$$

Here, $eV$ is scaled by the Kondo temperature defined at zero field, $T_K = T^*|_{b=0}$. Thus $C_S$ includes all effects of $b$, which enter through $m_d$, $T^* = 1/(4\chi^{[3]}\uparrow\uparrow\uparrow)$, $R = 1 - \chi^{\uparrow\downarrow}/\chi^{[3]}\uparrow\downarrow\uparrow$ and $\chi^{[3]}\downarrow\downarrow\downarrow$. In the right panel of Fig. 2 NRG results for $C_S$ are plotted as a function of $b/T_K$ for several different values of $U$. We find that $C_S$ exhibits a universal behavior for $U/(\pi \Delta) \gtrsim 2.0$. It decreases as $b$ increases for small fields, changes sign at $b \approx 0.36T_K$, takes a minimum at $b \approx 0.5T_K$, and then approaches zero at $b \gtrsim T_K$. The scaling behavior has previously been confirmed also for order $(eV)^2$ nonlinear conductance [22, 24]. Furthermore, we also find that order $T^3$ thermal conductivity exhibits the universal magnetic-field dependence [34]. These observations reflect the fact that three-body correlations $\chi^{[3]}\downarrow\downarrow\downarrow$ and $\chi^{[3]}\uparrow\uparrow\uparrow$ show the universal Kondo behavior as well as the phase shifts and linear susceptibilities.

**Conclusion.**— Low-energy asymptotic form of the transport coefficients for the SU($N$) Anderson impurity have been determined in terms of the static correlation functions as shown in table I. The NRG results of the FL parameters for $N = 4, 6$ have clarified especially the plateau structures of the nonlinear susceptibilities and transport coefficients. We have also considered the case in which magnetic field $b$ breaks the SU(2) and time-reversal symmetries, and find that the nonlinear current noise exhibits the universal $b/T_K$ dependence in a similar way as the charge and heat currents show [34]. The FL parameters can also be deduced from experiments, and then the obtained values can be used further to predict behaviors of the other unmeasured transport.

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+ \Theta^s \cos(\pi m_d) \left( \frac{a^2}{c_{\uparrow\downarrow \uparrow\downarrow \uparrow\downarrow}} \right)^2,
\text{and } C^\uparrow \uparrow \rightarrow \left( \frac{a^2}{c_{\uparrow\downarrow \uparrow\downarrow \uparrow\downarrow}} \right)^{-\frac{1}{2}} \chi^{[3]}\uparrow\uparrow\uparrow.$
I. DERIVATIONS OF THE TRANSPORT COEFFICIENTS \( C_T, C_V, C_{QD} \) AND \( C_S \)

We describe here outline of the derivation of the coefficients \( C \)'s, listed in Table I in the main text. The steady current \( J \) through the quantum dots can be calculated using the formula given in Eq. (2) with the transmission probability, defined by

\[
T_\sigma(\omega, T, eV) = \frac{-4\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R} \quad \text{Im} \quad G^\sigma_\sigma(\omega, T, eV),
\]

\[
G^\sigma_\sigma(\omega, T, eV) = \frac{1}{\omega - \epsilon_{\sigma\sigma} + i\Delta - \Sigma^\sigma_\sigma(\omega, T, eV)}. \tag{11}
\]

We also consider the thermal conductivity \( \kappa_{QD} \), which can be expressed in the form,

\[
\kappa_{QD} = \frac{1}{2\pi\hbar T} \left[ \sum_\sigma L_{n,\sigma}^{QD} - \frac{\sum_\sigma L_{0,\sigma}^{QD}}{\sum_\sigma L_{n,\sigma}^{QD}} \right]^2,
\]

\[
L_{n,\sigma}^{QD} = \int_{-\infty}^{\infty} d\omega \omega^n T_\sigma(\omega, T, 0) \left(-\frac{\partial f(\omega)}{\partial \omega}\right). \tag{12}
\]

We have deduced the coefficients \( C_T, C_V, \) and \( C_{QD} \) defined in Eqs. (5) and (7) from the low-energy expansions of the retarded self-energy \( \Sigma^\sigma_\sigma(\omega, T, eV) \) obtained up to terms of order \( \omega^2, T^2, \) and \( (eV)^2 \). We note that, in order to determine also the thermopower of quantum dots \( S_{QD} \) up to the next leading term, additional terms of order \( \omega^3 \) and \( \omega T^2 \) of the self-energy are necessary: it has been left for future studies. This is because the leading term of the thermopower at low-temperature limit already includes the derivative of the density of states \( \rho'_{\sigma\sigma} \equiv \left. \frac{\partial \rho_{\sigma\sigma}(\omega)}{\partial \omega} \right|_{\omega=0}, \)

\[
S_{QD} \equiv -\frac{1}{e|T|} \sum_\sigma L_{1,\sigma}^{QD} = -\frac{\pi^2}{3e} \sum_\sigma \rho'_{\sigma\sigma} T + O(T^3), \quad \rho'_{\sigma\sigma} = \frac{\chi_{\sigma\sigma} \sin 2\delta_{\sigma}}{\Delta}. \tag{13}
\]

The result of the low-energy expansion can be expressed in the following form in the SU(\( N \)) symmetric case, for a symmetric junction with \( \Gamma_L = \Gamma_R = \Delta/2 \) and \( \mu_L = -\mu_R = eV/2, \)

\[
\text{Im} \quad \Sigma^\sigma_\sigma(\omega, T, eV) = -\frac{\pi(N-1)}{2} \frac{\chi_{\sigma\sigma'}}{\rho_{\sigma\sigma}} \left[ \omega^2 + \frac{3}{4}(eV)^2 + (\pi T)^2 \right] + \cdots, \tag{14}
\]

\[
\epsilon_d + \text{Re} \quad \Sigma^\sigma_\sigma(\omega, T, eV) = \Delta \cos \delta + (1 - \chi_{\sigma\sigma}) \omega + \frac{1}{2} \frac{\partial \chi_{\sigma\sigma}}{\partial \epsilon_{\sigma\sigma}} \omega^2 + \frac{N - 1}{6} \frac{\chi_{\sigma\sigma'}^2}{\rho_{\sigma\sigma}} \left[ \frac{3}{4}(eV)^2 + (\pi T)^2 \right] + \cdots, \tag{15}
\]

where \( \sigma' \neq \sigma \), and \( \rho_{\sigma\sigma} = \sin^2 \delta/(\pi\Delta) \) for all \( \sigma = 1, 2, 3, \ldots, N \) in this case. These expressions are obtained by extending the higher-order Fermi-liquid relations, given previously for \( N = 2, \) to the multilevel case [23, 24], some of the proofs given in Appendix A of Ref. 23 are applicable to \( N > 2 \). Specifically, the causal vertex function \( \Gamma_{\sigma\sigma';\sigma'\sigma}(\omega, \omega'; \omega, \omega) \), defined at \( T = eV = 0 \), can be expressed in the following form up to terms of linear order in \( \omega \) and \( \omega' \), without assuming the SU(\( N \)) symmetry,

\[
\Gamma_{\sigma\sigma';\sigma'\sigma}(\omega, \omega'; \omega, \omega) \rho_{\sigma\sigma}^2 \equiv i\pi \sum_{\sigma' \neq \sigma} \chi_{\sigma\sigma'}^2 |\omega - \omega'| + \cdots, \tag{16}
\]

\[
\Gamma_{\sigma\sigma';\sigma'\sigma}(\omega, \omega'; \omega, \omega) \rho_{\sigma\sigma}^2 \rho_{\sigma'\sigma'} = -\chi_{\sigma\sigma'} + \rho_{\sigma\sigma} \frac{\partial \chi_{\sigma\sigma'}}{\partial \epsilon_{\sigma\sigma}} \omega + \rho_{\sigma'\sigma'} \frac{\partial \chi_{\sigma'\sigma'}}{\partial \epsilon_{\sigma'\sigma'}} \omega' + i\pi \chi_{\sigma\sigma'} \left| \omega - \omega' \right| \left| \omega + \omega' \right| - \left| \omega - \omega' \right| - \left| \omega + \omega' \right| + \cdots. \tag{17}
\]
where \( \sigma' \neq \sigma \). The parameter \( \tilde{\chi}_{\sigma\sigma'} = \delta_{\sigma\sigma'} + \frac{\partial \Sigma_\delta^{\text{TF}}(0,0,0)}{\partial \sigma} \) is related to the linear susceptibilities \( \chi_{\sigma\sigma'} \) through the Friedel sum rule, and thus its derivative can be expressed in terms of the non-linear susceptibilities:

\[
\chi_{\sigma\sigma'} = -\frac{\partial (n_{da})}{\partial \epsilon_{d\sigma'}} \to \rho_{da} \chi_{\sigma\sigma'}, \quad \frac{\partial \tilde{\chi}_{\sigma\sigma'}}{\partial \epsilon_{d\sigma}} = \frac{1}{\rho_{d\sigma_1}} \left( \chi_{[\sigma\sigma_1]_2}^{\sigma_3} + 2\pi \cot \delta_{\sigma_1} \chi_{\sigma_1\sigma_2} \chi_{\sigma_1\sigma_3} \right). \tag{18}
\]

The expansion coefficients for the self-energy and vertex function satisfy the Ward identities [4–6, 23]:

\[
\frac{\partial^2 \Sigma_\delta^v(\omega,0,0)}{\partial \omega^2} \bigg|_{\omega=0} = \frac{\partial^2 \Sigma_\delta^v(0,0,0)}{\partial \epsilon_{d\sigma}^2}. \tag{19}
\]

Furthermore, for the \( \omega^2 \) imaginary part: \( (\partial^2 / \partial \omega^2) \Im \Sigma_\delta^v(\omega,0,0) \bigg|_{\omega=0} = -\Im_{\omega=0} (\partial / \partial \omega) \Im \Gamma_{\sigma\sigma',\sigma}(\omega,0;\omega,\omega) \rho_{d\sigma}(0) = -\pi / \rho_{d\sigma} \sum_{\sigma' \neq \sigma} \tilde{\chi}_{\sigma\sigma'}^{\sigma'} \). In addition, order \( T^2 \) and \( (eV)^2 \) terms of the self-energy, we which denote as \( \Sigma_\delta^v(2) \), come from a single complex function \( \Psi_{\sigma}^-(\omega) \equiv \Im_{\omega=0} \sum_{\sigma'} (\partial / \partial \omega') \Gamma_{\sigma\sigma',\sigma}(\omega,\omega';\omega,\omega) \rho_{d\sigma'}(\omega') \) \[23\]:

\[
\Sigma_\delta^v(2) = \left[ (\pi T)^2 + \frac{3}{4} (eV)^2 \right] \lim_{\omega \to 0^+} \frac{\Psi_{\sigma}^-(\omega)}{6}, \quad \lim_{\omega \to 0^-} \Psi_{\sigma}^-(\omega) = \frac{1}{\rho_{d\sigma}} \sum_{\sigma' \neq \sigma} \chi_{\sigma\sigma'}^{[\sigma]} - i 3 \pi \frac{1}{\rho_{d\sigma}} \sum_{\sigma' \neq \sigma} \tilde{\chi}_{\sigma\sigma'}^{\sigma'}. \tag{20}
\]

These Fermi-liquid relations reflect the current conservation near the dot, and are essential to describe systematically the low-energy transport with a minimal set of parameters:

\[
\frac{\partial}{\partial t} (e n_{da}) + \vec{J}_{R,\sigma} - \vec{J}_{L,\sigma} = 0, \quad \vec{J}_{L,\sigma} \equiv i e v_L (\psi_{L\sigma}^\dagger d_{\sigma} - d_{\sigma}^\dagger \psi_{L\sigma}), \quad \vec{J}_{R,\sigma} \equiv -i e v_R (\psi_{R\sigma}^\dagger d_{\sigma} - d_{\sigma}^\dagger \psi_{R\sigma}). \tag{21}
\]

We have also considered the low-energy expansion of current noise \( S_{\text{noise}}^{\text{QD}} \) up to terms order \((eV)^2\) at \( T = 0 \) in order to determine the coefficient \( C_S \). Specifically, \( S_{\text{noise}}^{\text{QD}} \) is defined with respect to the symmetrized current-fluctuation operator \( \delta \hat{J}_\sigma(t) \equiv \hat{J}_\sigma(t) - \langle \hat{J}_\sigma(t) \rangle_{eV} \):

\[
S_{\text{noise}}^{\text{QD}} = \int_{-\infty}^{\infty} dt \sum_{\sigma'} i \left[ \mathcal{K}^\pm_{\sigma\sigma'}(t,0) + \mathcal{K}^\mp_{\sigma\sigma'}(t,0) \right], \quad \hat{J}_\sigma \equiv \frac{\Gamma_L \hat{J}_{R,\sigma} + \Gamma_R \hat{J}_{L,\sigma}}{\Gamma_L + \Gamma_R}. \tag{22}
\]

\[
\mathcal{K}^\pm_{\sigma\sigma'}(t,0) \equiv -i \langle \delta \hat{J}_{\sigma'}(t) \delta \hat{J}_\sigma(0) \rangle_{eV}, \quad \mathcal{K}^\mp_{\sigma\sigma'}(t,0) \equiv -i \langle \delta \hat{J}_\sigma(0) \delta \hat{J}_{\sigma'}(t) \rangle_{eV}. \tag{23}
\]

Figure 3 shows the Feynman diagrams for the current-current correlation function \( \mathcal{K}^\nu_{\nu'}_{\sigma\sigma'}(\omega,\omega';\omega) \) in the Keldysh formalism: the bubble diagram on the left and vertex corrections on the right. In order to obtain the coefficient \( C_S \), the Keldysh Green’s function \( G_{\nu\nu'}^{\sigma\sigma'}(\omega) \) has been expanded up to order \( \omega^2 \) and \( (eV)^2 \). Furthermore, the vertex function \( \Gamma_{\nu\nu';\sigma\sigma'}^{\nu_1\nu_2\nu_3\nu_4}(\omega,\omega';\omega') \) is also necessary to be expanded up to linear order in \( \omega, \omega' \) and \( eV \). We have calculated all those expansion coefficients using the approach of Yamada-Yosida, Shiba, and Yoshimori [4–6, 9] in the Keldysh formalism, and have obtained the expression presented in table III. Contributions arising from the bubble diagram \( C_S^{\text{bub}} \) and those arising from the vertex corrections \( C_S^{\text{ver}} \) can be expressed in the following form,

\[
C_S = C_S^{\text{bub}} + C_S^{\text{ver}}, \quad C_S^{\text{ver}} = \left[ \frac{7 + 5 \cos 4\delta}{2} + \frac{3}{2} (1 - \cos 4\delta) (N - 2) \right] \frac{K^2}{N - 1}. \tag{24}
\]

![FIG. 3. Feynman diagrams for the current-current correlation function \( \int_{-\infty}^{\infty} dt \mathcal{K}^{\nu\nu'}_{\sigma\sigma'}(t,0) \) at finite bias voltages \( eV \). The solid lines denote the Keldysh Green’s functions, for instance, the upper line in the diagram on the left corresponds to \( G_{\nu\nu'}^{\sigma\sigma'}(\omega) \). The shaded region in the diagram on the right represents the Keldysh vertex function \( \Gamma_{\sigma\sigma'}^{\nu_1\nu_2\nu_3\nu_4}(\omega,\omega';\omega') \). The superscripts \( \nu, \nu' \) and \( \nu_i \) (\( i = 1, 2, 3, 4 \)) specify branches of the Keldysh time-loop contour. We are using the notation in which \( \nu = - \) and \( + \) represent the forward and return paths, respectively.](image)
Here, $\bar{K} \equiv (N-1)(R-1)$ with $R \equiv 1 - \chi_{\sigma'\sigma}/\chi_{\sigma\sigma}$ for $\sigma' \neq \sigma$. Note that $C^{\text{tot}}$ includes the contributions accompanied by a factor $N - 2$ which vanishes for $N = 2$ [A. Oguri, Y. Tetatani, and S. Sakano, in preparation]. Our result of the nonlinear Fano factor, $F_K = \frac{C_v}{C_v^3}$, can be expressed explicitly in the following form,

\[
F_K = \frac{\cos 4\delta + \left[ 4 + 5 \cos 4\delta + \frac{3}{2} (1 - \cos 4\delta) (N - 2) \right] }{1 + \frac{\bar{K}^2}{N - 1}} \cos 2\delta + \Theta_1 + 3 (N - 1) \Theta_{\Pi}.
\]  

Specifically for $N = 2$, it reproduces the result of Mora et al [Eq. (11) of Phys. Rev. B 92, 075120 (2015)] which was obtained for the SU(2) symmetric case: their notation and our one correspond to each other such that $\alpha^{(1)}_\sigma/\pi = \chi_{\sigma\sigma}$, $\phi^{(1)}_{\sigma'\sigma}/\pi = -\chi_{\sigma'\sigma}$, $\alpha^{(2)}_\sigma/\pi = -\frac{1}{2} \chi_{\sigma\sigma}$, and $\phi^{(2)}_{\sigma'\sigma}/\pi = 2 \chi_{\sigma'\sigma}$ for $\sigma' \neq \sigma$. In Fig. 4 NRG results for $F_K$ for the SU(2) case are plotted with the other Fermi-liquid parameters for $N = 2$, for comparisons with those for $N = 4, 6$ shown in the main text.

Equation (25) also reproduces previous result [Sakano et al Phys. Rev. B 83 , 075440 (2011)], obtained for the particle-hole symmetric case where $\delta = \pi/2$ and the three-body contributions vanish $\Theta_1 = \Theta_{\Pi} = 0$:

\[
F_K \xrightarrow{\epsilon_d \to -(N-1)U/2} \frac{1 + \frac{9 \bar{K}^2}{N - 1}}{1 + \frac{5 \bar{K}^2}{N - 1}} \quad U \to \infty \to \frac{1 + \frac{9}{N - 1}}{1 + \frac{5}{N - 1}}.
\]  

The occupation number $\sum \sigma \langle n_{d\sigma} \rangle$ becomes integer $M = 1, 2, \ldots, N - 1$ in the strong coupling limit $U \to \infty$ at $\epsilon_d = -(M - 1/2)U$. In this case, the phase shift is given by $\delta = \pi M/N$, and the charge susceptibility takes absolute minima: $\chi_c = 0$ and $\frac{\partial \chi_c}{\partial \epsilon_d} = 0$. Thus, $\Theta_1 + (N - 1) \Theta_{\Pi} = 0$ owing to the stationary property, and it simplifies Eq. (26),

\[
F_K \to \frac{1 + \sin^2 \left( \frac{2\pi M}{N} \right)}{1 + \frac{5}{N - 1} \cos \left( \frac{2\pi M}{N} \right)} + \frac{9 - 13 \sin^2 \left( \frac{2\pi M}{N} \right)}{N - 1} + 2 \Theta_1 \cos \left( \frac{2\pi M}{N} \right).
\]  

This is consistent with the Fano factor for the SU(N) Kondo model, obtained by Mora et al [Eq. (51) of Phys. Rev. B 80, 155322 (2009), inserting some brackets for correcting minor typos].

II. NRG CALCULATIONS

NRG calculations for the SU(N) Anderson model for $N = 2, 4, 6$ have been carried out, dividing $N$ channels into $N/2$ pairs and exploiting the SU(2) spin and U(1) charge symmetries for each of the pairs, i.e. using $\prod_{i=1}^N \{ \text{SU}(2) \otimes \text{U}(1) \}_k$ symmetries. The discretization parameter $\Lambda$ and the number of retained low-lying excited states $N_{\text{trunc}}$ are chosen such that $(\Lambda, N_{\text{trunc}}) = (2, 4000)$ for $N = 2$, $(6, 10000)$ for $N = 4$, and $(20, 30000)$ for $N = 6$. We have also exploited methods of Stadler’s et al [Phys. Rev. B 93, 235101 (2017)] for $N = 6$. The truncation is performed at each step after adding states from each pair of the channels, using Oliveira’s Z-trick [Phys. Rev. B 49, 11986 (1994)] and choosing different $Z$ values for different pairs: $Z_i = 1/2 + i/N$ for the $i$-th pair ($i = 1, 2, \ldots, N/2$).
TABLE II. The coefficients $C$’s at finite magnetic fields $b$ for $N = 2$ at half filling $\epsilon_d = -U/2$.

| $C^b_T$ | $W^b_T \equiv \frac{2}{\pi} \left[ W_T^b + \Theta_1^b + \Theta_1^\Pi \right]$ | $W^b_T \equiv \left[ 1 + 2 (R - 1)^2 \right] \cos(\pi m_d)$ |
|---------|-----------------------------------------------------|-----------------------------------------------|
| $C^b_V$ | $= \frac{2}{\pi} \left[ W^b_V + \Theta_1^b + 3 \Theta_1^\Pi \right]$ | $W^b_V \equiv \left[ 1 + 5 (R - 1)^2 \right] \cos(\pi m_d)$ |
| $C^b_S$ | $= \frac{2}{\pi} \left[ W^b_S + (\Theta_1^b + 3 \Theta_1^\Pi) \cos(\pi m_d) \right]$ | $W^b_S \equiv (2 \pi m_d) + [4 + 5 \cos(2 \pi m_d)] (R - 1)^2$ |
| $C^\text{QD}_{\kappa,b}$ | $= \frac{2 \pi^2}{80} \left[ W^\text{QD}_{\kappa,b} + \Theta_1^\Pi + \frac{5}{12} \Theta_1^\Pi \right]$ | $W^\text{QD}_{\kappa,b} \equiv \left[ 1 + \frac{6}{5} (R - 1)^2 \right] \cos(\pi m_d)$ |

III. MAGNETIC-FIELD DEPENDENCE OF CURRENT NOISE FOR $N = 2$

We describe here supplemental information about the nonlinear current noise at finite magnetic field $b$, specifically for $N = 2$ at half-filling where the impurity level is given by $\epsilon_d = -U/2 - \text{sgn}(\sigma) b$: $\text{sgn}(\uparrow) = +1$ and $\text{sgn}(\downarrow) = -1$. In this case, the phase shift takes the form $\delta_\sigma = \pi \left( 1 + \text{sgn}(\sigma) m_d \right)/2$ with $m_d \equiv \langle n_{d\uparrow} \rangle - \langle n_{d\downarrow} \rangle$, and the other correlation functions have symmetry properties: $\chi_{\uparrow \uparrow} = \chi_{\downarrow \downarrow}$, $\chi_{\uparrow \downarrow} = \chi_{\downarrow \uparrow}$, $\chi_{\uparrow \uparrow}^{[3]} = -\chi_{\downarrow \downarrow}^{[3]}$, and $\chi_{\uparrow \downarrow}^{[3]} = -\chi_{\downarrow \uparrow}^{[3]}$. Thus, the transport coefficients up to the next leading order can be described by five parameters, for instance, $m_d$, $T^* = 1/(4 \chi_{\uparrow \uparrow})$, $R = 1 - \chi_{\uparrow \downarrow}/\chi_{\uparrow \uparrow}$ and the following two 3-body correlation functions,

$$\Theta_1^b \equiv -\frac{\sin(\pi m_d)}{2\pi} \frac{\chi_{\uparrow \uparrow}^{[3]}}{\chi_{\uparrow \uparrow}^2}, \quad \Theta_1^{\Pi} \equiv -\frac{\sin(\pi m_d)}{2\pi} \frac{\chi_{\uparrow \uparrow}^{[3]}}{\chi_{\uparrow \uparrow}^2}.$$

(28)

Low-energy asymptotic form of $dJ/dV$, current-current correlation function $S^\text{QD}_{\text{noise}}$, and thermal conductivity $\kappa^\text{QD}$ for this case can be written in the following forms, with the coefficients $C$’s listed in table II

$$\frac{dJ}{dV} = \frac{2 e^2}{h} \left[ C^b_T \left( \frac{\pi T}{T^*} \right)^2 - C^b_V \left( \frac{eV}{T^*} \right)^2 + \cdots \right],$$

$$S^\text{QD}_{\text{noise}} = \frac{2 e^2}{h} eV \left[ \frac{\sin^2(\pi m_d)}{4} + C^b_S \left( \frac{eV}{T^*} \right)^2 + \cdots \right],$$

$$\kappa^\text{QD} = \frac{2 \pi^2 T^*}{3h} \left[ C^{[3]}_{\text{QD}} \left( \frac{\pi T}{T^*} \right)^2 + \cdots \right].$$

(29)

(30)

(31)

In order to see the magnetic field dependences in the Kondo regime, it is preferable to rescale the next leading $(eV)^2$ and $T^2$ contributions by the Kondo temperature defined at zero field $T_K = \lim_{\sigma \rightarrow 0} T*$. This is because all effects of $b$ are absorbed into the coefficients redefined such that $C^b_T \equiv (T_K/T^*)^2 C^b_T$, $C^b_V \equiv (T_K/T^*)^2 C^b_V$, and $C^b_S \equiv (T_K/T^*)^2 C^b_S$. We have presented the NRG results for the nonlinear current noise $\overline{C^b}$ in the main text. In Fig. 5 $F^b_K \equiv C^b_K / C^b_{\nu,b}^T / 3$ and $C^\text{QD}_{\kappa,b}$ are also plotted as functions of $b/T_K$ for several different values of $U$. The nonlinear Fano factor $F^b_K$ shows the Kondo scaling behavior for strong interactions $U/(\pi \Delta) \gtrsim 2.0$ as the coefficient for the differential conductance $C^b_V$ also does [22, 23]. The universal curve of $F^b_K$ deviates significantly from the curve for $U = 0$ keeping its qualitative characteristics unchanged. We also find that the thermal conductivity $C^\text{QD}_{\kappa,b}$ exhibits the universal scaling behavior.