Generalized couplings between an Abelian $p$-form and a $(3,1)$ mixed symmetry tensor field

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Abstract

The consistent interactions between a single, free, massless tensor gauge field with the mixed symmetry $(3,1)$ and an Abelian $p$-form are investigated in the framework of the BRST formalism combined with cohomological techniques. Under the assumptions on smoothness, locality, Lorentz covariance, and Poincaré invariance of the deformations, supplemented by the requirement that the interacting Lagrangian is at most second-order derivative, it is proved that for every value $p \geq 1$ of the form degree there are consistent couplings between the Abelian form and the massless $(3,1)$ gauge field.

1 Introduction

Tensor fields in “exotic” representations of the Lorentz group, characterized by a mixed Young symmetry type $[1, 2, 3, 4, 5, 6, 7]$, held the attention lately on some important issues, like the dual formulation of field theories of spin two or higher $[8, 9, 10, 11, 12, 13, 14]$, the impossibility of consistent cross-interactions in the dual formulation of linearized gravity $[15]$, or a Lagrangian first-order approach $[16, 17]$ to some classes of massless or partially massive mixed symmetry type tensor gauge fields, suggestively resembling to the tetrad formalism of General Relativity. An important matter related to mixed symmetry type tensor fields is the study of their consistent interactions, among themselves as well as with higher-spin gauge theories $[18, 19, 20, 21, 22, 23, 24, 25, 26]$. The most efficient approach to this problem is the cohomological one, based on the deformation of the solution to the master equation $[27]$. The purpose of this paper is to investigate the consistent interactions between a single free massless tensor gauge field $t_{\alpha\mu\nu}$ with the mixed symmetry of a two-column Young diagram of
the type \((3, 1)\) and one Abelian form \(A_{\mu_1...\mu_p}\). It is worth mentioning the duality of the free massless tensor gauge field \(t_{\lambda \mu \nu |\alpha}\) to the Pauli-Fierz theory in \(D = 6\) dimensions and, in this respect, the recent developments concerning the dual formulations of linearized gravity from the perspective of \(M\)-theory \[25, 29, 30\].

Our analysis relies on the deformation of the solution to the master equation by means of cohomological techniques with the help of the local BRST cohomology, whose component in the \((3, 1)\) sector has been reported in detail in \[31\]. Under the hypotheses of smoothness in the coupling constant, locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the preservation of the number of derivatives on each field, we prove that for every Abelian \(p\)-form there exists a deformation of the solution to the master equation, which provides nontrivial cross-couplings. This case corresponds to a \((p + 4)\)-dimensional space-time and is described by a deformed solution that stops at order two in the coupling constant. The interacting Lagrangian action contains only mixing-component terms of order one and two in the coupling constant. At the level of the gauge transformations, only those of the Abelian form are modified at order one in the coupling constant with a term linear in the antisymmetrized first-order derivatives of some gauge parameters from the \((3, 1)\) sector such that the gauge algebra and the reducibility structure of the coupled model are not modified during the deformation procedure, being the same like in the case of the starting free action. It is interesting to note that if we require the PT invariance of the deformed theory, then no interactions occur. Although it is not possible to construct interactions that deform the gauge algebra, our results (see \[32\]) are interesting since these seem to be the first cases where mixed symmetry type tensor fields allow nontrivial cross-couplings.

2 The free theory and the BRST symmetry

We start from a “free” Lagrangian action written as a sum between the Lagrangian action \(S_0 \left[ t_{\mu \nu \lambda |\rho} \right]\) of the tensor field with the mixed symmetry \((3, 1)\) and the Lagrangian action \(S^A \left[ A_{\mu_1...\mu_p} \right]\) of an abelian \(p\)-form,

\[
S_0 \left[ t_{\mu \nu \lambda |\rho}, A_{\mu_1...\mu_p} \right] = S_0 \left[ t_{\mu \nu \lambda |\rho} \right] + S^A \left[ A_{\mu_1...\mu_p} \right],
\]

where

\[
S^A \left[ A_{\mu_1...\mu_p} \right] = -\frac{1}{2(p + 1)!} \int d^D x F_{\mu_1...\mu_{p+1}} F^{\mu_1...\mu_{p+1}}
\]

and

\[
S_0 \left[ t_{\lambda \mu \nu |\alpha} \right] = \int d^D x \left( \frac{1}{2} \left( \partial^\rho t_{\lambda \mu \nu |\alpha} \partial_\rho t_{\lambda \mu \nu |\alpha} - \partial_\alpha t_{\lambda \mu \nu |\alpha} \partial^\beta t_{\lambda \mu \nu |\beta} \right) \right. \\
- \frac{3}{2} \left( \partial_\alpha t_{\lambda \mu \nu |\alpha} \partial^\rho t_{\rho \mu \nu |\alpha} + \partial^\rho t_{\lambda \mu \nu |\alpha} \partial_\rho t_{\lambda \mu \nu |\alpha} \right) \\
\left. + 3 \left( \partial_\alpha t_{\lambda \mu \nu |\alpha} \left( \partial_\lambda t_{\mu \nu} + \partial_\rho t^{\rho \mu} \partial^\lambda t_{\lambda \mu} \right) \right) \right).
\]
The dimension of the space-time satisfies the inequality
\[ D \geq \max (5, p + 1). \] (4)

The field strength of the abelian form from the formula (2) is defined in the standard manner,
\[ F_{\mu_1...\mu_{p+1}} = \partial_{[\mu_1} A_{\mu_2...\mu_{p+1}]}, \quad D > p. \] (5)

Everywhere in this paper we employ the flat Minkowski metric of ‘mostly plus’ signature \( \sigma_{\mu\nu} = \sigma_{\mu\nu} = (-, +, +, +, \ldots) \). We remember \( A_{\mu_1...\mu_p} \) is an antisymmetric tensor and the mixed symmetry (3, 1) of the tensor field \( t_{\lambda\mu\nu|\alpha} \) means it is antisymmetric in the first three indices and satisfies the identity
\[ t_{[\lambda\mu\nu|\alpha]} \equiv 0. \] (6)

The functional \( S_0 [t_{\mu\nu\lambda|\rho}] \) is invariant to the known gauge transformations (see [31]) of the tensor field \( t_{\lambda\mu\nu|\alpha} \),
\[ \delta_{\epsilon,\chi} t_{\lambda\mu\nu|\alpha} = 3 \partial_{\alpha} \epsilon_{\lambda\mu\nu} + \partial_{[\lambda} \epsilon_{\mu\nu|\alpha} + \partial_{[\lambda} \chi_{\mu\nu]|\alpha], \] (7)
where \( \epsilon_{\lambda\mu\nu} \) is an arbitrary, antisymmetric tensor field and \( \chi_{\lambda\mu\nu|\alpha} \) has the mixed symmetry (2, 1) (it is antisymmetric in the first two indices and satisfy the identity \( \chi_{\lambda\mu\nu|\alpha} \equiv 0 \)). The gauge symmetries (7) are off-shell second stage reducible, because the right side of the formula (7) vanishes if we use the replacements
\[ \epsilon_{\mu\nu|\alpha} \rightarrow \epsilon^{(\omega,\psi)}_{\mu\nu|\alpha} = -\frac{1}{2} \partial_{[\mu} \omega_{\nu]\alpha]}, \] (8)
\[ \chi_{\mu\nu|\alpha} \rightarrow \chi^{(\omega,\psi)}_{\mu\nu|\alpha} = \partial_{[\mu} \psi_{\nu]\alpha} + 2 \partial_{\alpha} \omega_{\mu\nu} - \partial_{[\mu} \omega_{\nu]|\alpha], \] (9)
where \( \omega_{\mu\nu} \) is an arbitrary, antisymmetric tensor field, \( \Psi_{\mu\nu} \) an arbitrary, symmetric tensor field, while (8)- (9) vanish by the replacements
\[ \omega_{\mu\nu} \rightarrow \omega^{(\theta)}_{\mu\nu} = \partial_{[\mu} \theta_{\nu]\alpha}], \] (10)
\[ \psi^{(\theta)}_{\mu\nu} = -3 \partial_{[\mu} \theta_{\nu]}]. \] (11)

The most general object invariant to the gauge transformations (7) is the curvature tensor, defined by
\[ K^{\lambda\mu\nu\xi|\alpha\beta} = \partial_{\alpha} \partial_{[\lambda} K^{\mu\nu\xi]|\beta] - \partial_{\beta} \partial_{[\lambda} K^{\mu\nu\xi]|\alpha\beta] - \partial_{[\lambda} K^{\mu\nu\xi]|\alpha\beta]} \equiv 0, \] (12)
which has the mixed symmetry (4, 2) and satisfies Bianchi type identities: algebraic, \( K^{[\lambda\mu\nu\xi|\alpha\beta]} = 0 \), and differential, \( \partial^{[\kappa\lambda} K^{\mu\nu\xi]|\alpha\beta]} = 0 \). We can express the field equations of \( t_{\lambda\mu\nu|\alpha} \) with the help of the curvature tensor,
\[ \frac{\delta S_0}{\delta t_{\lambda\mu\nu|\alpha}} \equiv -T^{\lambda\mu\nu|\alpha} \approx 0, \] (13)
\[ T^{\lambda\mu\nu|\alpha} = K^{\lambda\mu\nu|\alpha} - \frac{1}{2} \sigma^{\alpha[\lambda} K^{\mu\nu]} \]. (14)
An important property of field equations (13) is they can be written as

\[ T^{\lambda\mu\nu|\alpha} = \partial_\xi \partial_\beta \Phi^{\lambda\mu\nu|\alpha\beta} , \]  

(15)

where \( \Phi^{\lambda\mu\nu|\alpha\beta} \) is an antisymmetric tensor, separately, in the first two indices, respectively in the last two,

\[ \Phi^{\lambda\mu\nu|\alpha\beta} = -\sigma^{\alpha|\lambda \sigma \mu|\beta} t^{\nu \xi} - \sigma^{\alpha|\nu \sigma \xi|\beta} t^{\lambda \mu} + \sigma^{\alpha|\lambda \sigma |\beta} t^{\mu \xi} + \sigma^{\alpha|\mu \sigma |\beta} t^{\lambda \xi} + \sigma^{\alpha|\lambda \mu \nu ||\beta} - \sigma^{\beta|\lambda \mu \nu ||\alpha} . \]  

(16)

The functional \( S^A_0 [A_{\mu_1...\mu_p}] \) has the gauge symmetries

\[ \delta_{(1)} \rho A_{\mu_1...\mu_p} = \partial_{[\mu_1} (^{(1)} A_{\mu_2...\mu_p]} , \]  

(17)

where the gauge parameter \( (^{(1)} \rho_{\mu_2...\mu_p] \) is an arbitrary, antisymmetric tensor field. The gauge transformations (17) are off-shell reducible, vanishing if we use the replacements

\[ {^{(1)} \rho}_{\mu_1...\mu_{p-1}} = \partial_{[\mu_1} {^{(2)} \rho}_{\mu_2...\mu_{p-1}] } \Rightarrow \delta_{^{(1)} \rho} {^{(2)} A}_{\mu_1...\mu_p} = 0 , \]  

(18)

where \( {^{(2)} \rho}_{\mu_2...\mu_{p-1]} \) is an arbitrary, antisymmetric tensor field. The relation (18) represents the first order reducibility, the Lagrangian action \( S^A_0 [A_{\mu_1...\mu_p}] \) describing a \( p-1 \) reducible gauge theory. The reducibility relation in the \( k \) order is

\[ {^{(k)} \rho}_{\mu_1...\mu_{p-k}} = \partial_{[\mu_1} {^{(k+1)} \rho}_{\mu_2...\mu_{p-k}] } \Rightarrow \delta_{^{(k)} \rho} {^{(k+1)} A}_{\mu_1...\mu_p} = 0 , \]  

(19)

(For \( k = p, \) \( {^{(k)} \rho} \) has no indices.) It follows the theory described by \( S^A_0 [A_{\mu_1...\mu_p}] \) has the Cauchy order \( p+1 \), so the “free” theory described by the action +1 has the Cauchy order \( p+1 \) if \( p > 3 \), or \( 4 \) if \( p \leq 3 \). This affirmation will be important later, when we will need the cohomology \( H_1 (\delta|d|) \). The field strength \( F_{\mu_1...\mu_{p+1}} \) defined in (5) represents for the abelian p-form the analogous of the curvature tensor for the tensor field \( t_{\lambda\mu\nu|\alpha} \), being the most general object invariant to the gauge transformations (17). It can be used to construct the action (2) and the field equations for the abelian p-form,

\[ \frac{\delta S^A_0}{\delta A_{\mu_1...\mu_p}} = \frac{1}{p!} \partial^\lambda F_{\lambda\mu_1...\mu_p} . \]  

(20)

The BRST complex corresponding to the tensor field \( t_{\lambda\mu\nu|\alpha} \) contains the fermionic ghosts \( \{ \eta_{\lambda\mu\nu}, G_{\mu\nu|\alpha} \} \) associated to the gauge parameters \( \{ \epsilon_{\lambda\mu\nu}, \chi_{\mu\nu|\alpha} \} \) from (7), the bosonic ghosts \( \{ C_{\mu\nu}, C_{\nu\alpha} \} \) due to the reducibility parameters in order one \( \{ \omega_{\mu\nu}, \psi_{\nu\alpha} \} \) from (8)-(9), the fermionic ghosts \( C_{\nu} \) corresponding to the reducibility parameters in order two \( \theta_{\nu} \) from (10)-(11), the antifields \( t^{*\lambda\mu\nu|\alpha} \)
of the tensor field $t_{\lambda \mu \nu | \alpha}$ and the antifields $\{ \eta^{\ast \lambda \mu \nu | \alpha}, \{ C^{\ast \mu \nu | \alpha}, C^{\ast \nu | \alpha} \}$ associated to the ghosts. The ghosts have the same properties as the associated reducibility parameters,

$$
\begin{align*}
\eta_{\lambda \mu \nu} &= -\eta_{\mu \lambda \nu} = -\eta_{\nu \mu \lambda}, \quad G_{\mu \nu | \alpha} = -G_{\nu \mu | \alpha}, \quad G_{[\mu \nu | \alpha]} \equiv 0, \\
C_{\mu \nu} &= -C_{\nu \mu}, \quad C_{\nu \alpha} = C_{\alpha \nu},
\end{align*}
$$

(21)

and, also, the antifields have the properties of the fields and the ghosts associated to,

$$
\begin{align*}
t^{\ast \lambda \mu \nu | \alpha} &= -t^{\ast \mu \lambda \nu | \alpha} = -t^{\ast \lambda \nu \mu | \alpha} = -t^{\ast \nu \mu \lambda | \alpha}, \\
C^{\ast \mu \nu} &= -C^{\ast \nu \mu}, \quad C^{\ast \nu | \alpha} = C^{\ast \alpha | \nu}.
\end{align*}
$$

(22)

The BRST generators of the complex corresponding to the abelian $p$-form are the $p$-form $A_{\mu_1 \ldots \mu_p}$ and its antifield $A_{\ast \mu_1 \ldots \mu_p}$, the ghosts $(^{(k)}\xi_{\mu_1 \ldots \mu_p} - k)_{k=1 \over 2}$ related to the gauge parameters and the reducibility functions from (17)-(19), the antifields $(^{(k)}\xi_{\ast \mu_1 \ldots \mu_p - k})_{k=1 \over 2}$ associated to the ghosts (all these generators are antisymmetric tensors). The antifields have the properties

$$
\begin{align*}
\varepsilon (A_{\ast \mu_1 \ldots \mu_p}) &= 1, \quad \varepsilon (^{(k)}\xi_{\ast \mu_1 \ldots \mu_p - k}) = (k + 1) \mod 2, \\
\text{agh} (A_{\ast \mu_1 \ldots \mu_p}) &= 1, \quad \text{pgh} (A_{\ast \mu_1 \ldots \mu_p}) = 0, \\
\text{agh} (^{(k)}\xi_{\ast \mu_1 \ldots \mu_p - k}) &= k, \quad \text{pgh} (^{(k)}\xi_{\ast \mu_1 \ldots \mu_p - k}) = 0.
\end{align*}
$$

(26)

(27)

(28)

(We remember: $\varepsilon =$ Grassmann parity, agh = antighost number, pgh = pureghost number.) The ghosts have the properties

$$
\begin{align*}
\varepsilon (^{(k)}\xi_{\mu_1 \ldots \mu_p - k}) &= k \mod 2, \quad \text{pgh} (^{(k)}\xi_{\mu_1 \ldots \mu_p - k}) = k, \quad \text{agh} (^{(k)}\xi_{\mu_1 \ldots \mu_p - k}) = 0.
\end{align*}
$$

(29)

We know the BRST differential decompose as $s = \delta + \gamma$. The action of the Koszul-Tate differential $\delta$ on the BRST complex of the abelian form is given by the formulas

$$
\begin{align*}
\delta (A_{\mu_1 \ldots \mu_p}, ^{(k)}\xi_{\mu_1 \ldots \mu_p - k}) &= 0, \\
\delta A_{\ast \mu_1 \ldots \mu_p} &= -\frac{1}{p!} \partial^\lambda F_{\lambda \nu_1 \ldots \nu_p},
\end{align*}
$$

(30)

(31)
\[
\begin{align*}
\delta^{(1)} \xi_{\mu_1 \ldots \mu_{p-1}} &= -p \partial^\mu A^*_{\mu_1 \ldots \mu_{p-1}}, \\
\delta^{(k+1)} \xi_{\mu_1 \ldots \mu_{p-k-1}} &= (-)^{k+1} (p-k) \partial^\mu (k)^* \\
\gamma A_{\mu_1 \ldots \mu_p} &= \partial_{[\mu_1} \xi_{\mu_2 \ldots \mu_p]}, \\
\gamma \xi_{\mu_1 \ldots \mu_{p-k}} &= \partial_{[\mu_1} \xi_{\mu_2 \ldots \mu_{p-k]}}, \\
\gamma \xi &= 0.
\end{align*}
\]

For the BRST complex of the tensor field \( t_{\lambda \mu \nu \rho} \), one has the degrees

\[
\begin{align*}
pgh(t_{\lambda \mu \nu | \rho}) &= 0, \\
pgh(G_{\mu \nu | \alpha}) &= 1 = pgh \left( G^*_{\mu \nu | \alpha} \right), \\
pgh(C_{\mu \nu}) &= 2 = pgh \left( C_{\nu \alpha} \right), \\
pgh(G^*_{\mu \nu | \alpha}) &= 0, \\
pgh(C^*_{\mu \nu}) &= pgh \left( \bar{C}^+_{\mu \nu} \right) = pgh \left( C^*_{\mu \nu} \right) = 0, \\
agh(t_{\lambda \mu \nu | \alpha}) &= \gamma t_{\lambda \mu \nu | \alpha} = \gamma \left( t_{\lambda \mu \nu | \alpha} \right) = \gamma \left( t^{* \lambda \mu \nu | \alpha} \right) = 1, \\
agh(C_{\mu \nu}) &= \gamma C_{\mu \nu} = \gamma \left( C_{\nu \alpha} \right) = \gamma \left( C^*_{\mu \nu} \right) = 0, \\
agh(G_{\mu \nu | \alpha}) &= \gamma G_{\mu \nu | \alpha} = \gamma \left( G_{\mu \nu | \alpha} \right) = \gamma \left( G^*_{\mu \nu | \alpha} \right) = 0, \\
agh(C^*_{\mu \nu}) &= \gamma C^*_{\mu \nu} = \gamma \left( C^*_{\mu \nu} \right) = \gamma \left( C^*_{\mu \nu} \right) = 0.
\end{align*}
\]

and the actions of \( \delta \) and \( \gamma \)

\[
\begin{align*}
\gamma t_{\lambda \mu \nu | \alpha} &= 3 \partial_{\alpha} \eta_{\lambda \mu \nu} + \partial_{[\lambda} \eta_{\mu \nu | \alpha} + \partial_{[\lambda} G_{\mu \nu | \alpha}, \\
&= 4 \partial_{\alpha} \eta_{\lambda \mu \nu} + \partial_{[\lambda} \eta_{\mu \nu | \alpha} + \partial_{[\lambda} G_{\mu \nu | \alpha}, \\
\gamma \eta_{\lambda \mu \nu} &= -\frac{1}{2} \partial_{[\lambda} C_{\mu \nu]}, \\
\gamma G_{\mu \nu | \alpha} &= 2 \partial_{\alpha} C_{\mu \nu} - \partial_{[\mu} C_{\nu | \alpha} + \partial_{[\mu} C_{\nu | \alpha}, \\
&= 2 \partial_{\alpha} C_{\mu \nu} - 3 \partial_{[\mu} C_{\nu | \alpha} + \partial_{[\mu} C_{\nu | \alpha}, \\
\gamma C_{\mu \nu} &= \partial_{[\mu} C_{\nu | \alpha}, \\
\gamma C_{\mu \nu} &= \gamma C_{\mu \nu} = -3 \partial_{(\nu} C_{\alpha)} = 0, \\
\gamma C_{\mu \nu} &= \gamma C_{\mu \nu} = \gamma C_{\mu \nu} = \gamma C_{\mu \nu} = \gamma C_{\mu \nu} = 0, \\
\gamma t^{* \lambda \mu \nu | \alpha} &= \gamma t_{\lambda \mu \nu | \alpha} = \gamma G^{* \mu \nu | \alpha} = \gamma C^{* \mu \nu} = \gamma C^{* \nu \alpha} = \gamma C^{* \mu \nu} = 0, \\
\delta t_{\lambda \mu \nu | \alpha} &= \delta \eta_{\lambda \mu \nu} = \delta G_{\mu \nu | \alpha} = \delta C_{\mu \nu} = \delta C_{\nu \alpha} = \delta C_{\nu \alpha} = 0.
\end{align*}
\]
\[
\delta t^* \lambda^{\mu \nu|\alpha} = T^{\lambda^{\mu \nu|\alpha}}, \quad \delta \eta^* \lambda^{\mu \nu} = -4 \partial_{\lambda} t^* \lambda^{\mu \nu|\alpha}, \quad (52)
\]
\[
\delta G^* \mu^{\nu|\alpha} = -\partial_{\alpha} \left( 3 t^* \lambda^{\mu \nu|\alpha} - t^* \mu^{\nu \alpha|\lambda} \right), \quad (53)
\]
\[
\delta C^* \mu^{\nu} = 3 \partial_{\lambda} \left( G^* \mu^{\nu|\lambda} - \frac{1}{2} \eta^* \lambda^{\mu \nu} \right), \quad \delta C^* \nu^{\alpha} = \partial_{\mu} G^* (\nu^{\alpha}), \quad (54)
\]
\[
\delta C^* = 6 \partial_{\mu} \left( C^* \mu^{\nu} - \frac{1}{3} C^* \mu^{\nu} \right). \quad (55)
\]

We know the BRST differential has a canonical action, \( s \cdot = (\cdot, S) \), generated by the solution of the master equation \((S, S) = 0\),
\[
S = S^t + S^A, \quad (56)
\]
where
\[
S^t = S_0 \left[ t_{\lambda^{\mu \nu|\alpha}} \right] + \int d^D x \left( 3 \partial_{\alpha} \eta_{\lambda^{\mu \nu}} + \partial_{[\lambda} \eta_{\mu^{\nu|\alpha]} + \partial_{\lambda} \G^{\mu^{\nu||\alpha}} \right) - \frac{1}{2} \eta^* \lambda^{\mu \nu} \partial_{\lambda} C^{\mu^{\nu}}, \quad (57)
\]
and
\[
S^A = S_0^A \left[ A_{\mu_1 \ldots \mu_p} \right] + \int d^D x \left( A^{* \mu_1 \ldots \mu_p} \partial_{\mu_1} (1) \right) \xi_{\mu_2 \ldots \mu_p}, \quad (58)
\]

3 \quad \( H (\gamma) \) and \( H (\delta |d) \)

The cohomological method for the computation of the interactions is known (see \([27, 31]\)) and it is based on the deformation of the solution \((56)\) of the master equation. The deformation has to satisfy also the master equation, so the components of the deformation \( \bar{S} = S + gS_1 + g^2 S_2 + \ldots \) have to satisfy a chain of equations
\[
(S_1, S) = 0, \quad (59)
\]
\[
\frac{1}{2} (S_1, S_1) + (S_2, S) = 0, \quad (60)
\]
\[
(S_1, S_2) + (S, S_3) = 0, \quad (61)
\]
\[
\ldots \quad (62)
\]

obtained by the projection of the equation \((\bar{S}, \bar{S}) = 0\) on the different orders of the coupling constant \( g \). The nonintegrated density of the first order deformation in the coupling constant, \( S_1 = \int d^D x a \), satisfies the local equation
\[
\delta a = \partial_{\mu} j^\mu \quad (63)
\]
and it has three independent components,

\[ a = a^{t-t} + a^{p-p} + a^{t-p}. \] (64)

\[ a^{t-t}, \ a^{p-p} \text{ and } a^{t-p} \] are independent solutions of the equation (63). \( a^{t-t} \) is formed only by objects from the sector of the tensor field \( t_{\lambda \mu \nu |\alpha} \), \( a^{p-p} \) has components only from the sector of the abelian p-form and \( a^{t-p} \) should generate the interactions between the tensor field \( t_{\lambda \mu \nu |\alpha} \) and the p-form, so every term that belongs to \( a^{t-p} \) is necessary a product between elements from the BRST complex of the tensor field with mixed symmetry \((3, 1)\) and elements from the BRST complex of the abelian form. It was proved in \cite{31} that \( a^{t-t} = 0 \), while for \( a^{p-p} \) we will use in a space-time with the dimension \( D = 2p+1 \) the solution

\[ a^{p-p}(D=2p+1) = \epsilon^\mu_1...\mu_{2p+1} A_{\mu_1...\mu_p} F_{\mu_{p+1}...\mu_{2p+1}} \] (65)

and in a space-time with the dimension \( D = 3p + 2 \) the solution

\[ a^{p-p}(D=3p+2) = \epsilon^\mu_1...\mu_{3p+2} A_{\mu_1...\mu_p} F_{\mu_{p+1}...\mu_{2p+1}} F_{\mu_{2p+2}...\mu_{3p+2}} \] (66)

(see \cite{38}).

The component that could generate the cross-interactions, \( a^{t-p} \), is determined by the equation

\[ sa^{t-p} = \partial_\mu j^\mu, \] (67)

where \( j^\mu \) is a local current. We solve the equation (67) decomposing \( a^{t-p} \) according to the antighost number (we suppose this decomposition contains a finite number of terms, see \cite{33, 34, 36}),

\[ a^{t-p} = \sum_{k=0}^{I} a^{t-p}_k, \text{ agh} \left( a^{t-p}_k \right) = k, \text{ gh} \left( a^{t-p}_k \right) = 0, \text{ \varepsilon} \left( a^{t-p}_k \right) = 0. \] (68)

If we take account of \( s = \delta + \gamma \), the equation (67) is equivalent to a chain of equations

\[ \gamma a^{t-p}_I = 0, \] (69)

\[ \delta a^{t-p}_k + \gamma a^{t-p}_{k-1} = \partial_\mu \left( \frac{(k-1)!}{w} \right)^I, I \geq k \geq 1. \] (70)

where \( \left( \frac{(k-1)!}{w} \right)^I \) are local currents with \( \text{agh} \left( \frac{(k-1)!}{w} \right)^I \) = \( k \), while we have noted by \( I \) the greatest antighost number from the decomposition \( (68) \) (see \cite{27, 33, 34} for further details). We say the chain (69)-(70) is consistent if all its equations have solutions (the equations are solved from the greatest antighost number of the chain, to the zero order). Also, it is possible that two chains of the type (69)-(70) (one with the greatest antighost number \( I \) and the other with the greatest antighost number \( J \), each of this two chains separately inconsistent and the consistency stopping at same antighost number for both chains) together to be consistent and to form a solution of the equation (67).
The solution of the equation (69) pertains to $H(\gamma)$. The generators of this cohomology associated to the tensor field $t_{\lambda\mu\nu|\alpha}$ were calculated in [31] and these are all the antifields from the BRST complex of $t_{\lambda\mu\nu|\alpha}$ (noted with $\Pi^*\Delta$), their derivatives, the curvature tensor $K_{\lambda\mu\nu\xi|\alpha\beta}$ and its derivatives, the ghosts $F_{\nu\lambda\mu\rho}=\partial_{\nu}[t_{\lambda\mu\rho}]$ and $C_{\mu}(F_{\mu\nu\lambda\rho}\in H^1(\gamma), C_\mu\in H^3(\gamma))$, while from the BRST complex of the abelian form we have as generators of $H(\gamma)$ all the antifields and their derivatives, the field strength $F_{\mu1...\mu_{p+1}}$ and the ghost $\xi (\xi \in H^p(\gamma))$. Therefore, up to $\gamma$-exact terms, the general solution for (69) has the form

$$a_{I}^{t-p} = \alpha_{I}^{t-p} ([\pi^*\Theta], [K_{\lambda\mu\nu\xi|\alpha\beta}], [F_{\mu1...\mu_{p+1}}]) e^I (\xi, C_{\nu}, F_{\lambda\mu\nu\alpha})$$

(71)

where we have noted by $\pi^*\Theta = (\Pi^*\Delta, A^*_{\mu1...\mu_p}, (\xi^*_{\mu1...\mu_{p-k}}))$ all the antifields.

$\alpha_{I}^{t-p}$ are called “invariant polynomials” (the invariant polynomials are objects with zero pureghost number and $\gamma$-closed) and they introduce into the interactions study the local cohomology of the Koszul-Tate differential, because for the chain (69)-(70) to have solutions it is necessary the invariant polynomials to be $\delta$-closed modulo the exterior space-time differential $d$, $\delta\alpha_{I}^{t-p} = \partial_{\mu}\beta_{I}^{\mu}$.

(72)

The objects $\beta_{I}^{\mu}$ from the previous formula may be chosen invariant polynomials (see [33, 34, 31]), while about the trivial solutions $\alpha_{I}^{t-p} = \delta\alpha_{I+1} + \partial_{\mu}\lambda_{I}^{\mu}$ first it can be proved the objects $\alpha_{I+1}$ and $\lambda_{I}^{\mu}$ are invariant polynomials and, second, we can remove from the first order deformation the terms formed with trivial invariant polynomials. Hence, we are interested, in fact, about the local cohomology of the Koszul-Tate differential in the space of the invariant polynomials, $H^{inv}_{I}(\delta|d)$. If we denote the Cauchy order of the theory (1) by $\text{Ord}$, the theorems regarding the local cohomologies prove that

$$H_{I}(\delta|d) = 0, \text{ if } I > \text{Ord},$$

(73)

therefore

$$H^{inv}_{I}(\delta|d) = 0, \text{ if } I > \text{Ord},$$

(74)

so the first order deformation will contain only terms with the antighost number less or equal than the Cauchy order. In our theory (1), if the degree of the abelian form is $p > 3$ the Cauchy order is $\text{Ord}= p + 1$, else the Cauchy is $\text{Ord}= 4$. Consequently, for the greatest antighost number in the first order deformation we will have three main possibilities, analysed separately in the sequel. In all the cases we will analyse it is maintained the requirement that the possible interacting Lagrangian has the maximum derivative order two.
I \geq 5

This case appears if \( p \geq 4 \), all the cohomological groups \( H^\text{inv}_I(\delta|d) \) being non-trivial for \( I \leq p + 1 \). If \( I \geq 5 \), the last term from the decomposition (68) will be linear in the antifield \( \xi \) from the sector of an abelian form with \( p \geq 4 \),

\[
a^I_{\mu_1...\mu_{p-I+1}} = (I-1)^*_{\mu_1...\mu_{p-I+1}} e^I_{\mu_1...\mu_{p-I+1}} \left( \xi \right)_{\mu_1...\mu_{p-I+1}} (p)_{\xi,C_{\nu},F_{\lambda\mu\nu}},
\]

(75)

\( e^I_{\mu_1...\mu_{p-I+1}} \) are nontrivial objects from \( H(\gamma) \), with the pureghost number \( I \leq p + 1 \), constructed from the ghosts \( \xi, C_{\nu}, F_{\lambda\mu\nu} \) and, possibly, the metric and the Levi-Civita symbols, thus to have \( p-I+1 \) Lorentz indices. We are interested in the computation of the cross-interactions, so \( a^I_{\mu_1...\mu_{p-I+1}} \) has to contain at least one ghost from the sector of the tensor field with the mixed symmetry (3, 1). The restriction imposed to the derivative order constrains \( a^I_{\mu_1...\mu_{p-I+1}} \) to depend of one ghost \( F_{\lambda\mu\nu}, \) at most. If we want the object \( e^I_{\mu_1...\mu_{p-I+1}} \) to depend of the ghost \( \xi \) from the sector of the abelian form, then the only possibility is \( I = p + 1 \) and

\[
a^I_{\mu_1...\mu_{p-I+1}} = f^{\lambda\mu\nu}(p+1)^*(p)_{\xi,C_{\nu}},
\]

(76)

where \( f^{\lambda\mu\nu} \) is constant (because \( \text{gh} \left( (p)_{\xi,C_{\mu}} \right) = p + 3 \), the product \( (p)_{\xi,C_{\mu}} \) can not appear in \( a^I_{\mu_1...\mu_{p-I+1}} \)). It is not possible to construct the object \( a^I_{\mu_1...\mu_{p-I+1}} \) with the form (76), because of the space-time dimension, \( D \geq 5 \).

We remain with two possibilities: either \( e^I_{\mu_1...\mu_{p-I+1}} \), it is formed only from the ghosts \( C_{\mu} \), or from the ghosts \( C_{\mu} \) and one ghost \( F_{\lambda\mu\nu} \). We will analyse each of these possibilities. In the first case, the general form of \( a^I_{\mu_1...\mu_{p-I+1}} \) is

\[
a^I_{\mu_1...\mu_{p-I+1}} = f^{\mu_1...\mu_{p-I+1}|\nu_1...\nu_N}(I-1)^*_{\mu_1...\mu_{p-I+1}} C_{\nu_1...\nu_N},
\]

(77)

where \( f^{\mu_1...\mu_{p-I+1}|\nu_1...\nu_N} \) is a constant tensor, antisymmetric separately in the groups of indices \( \mu_1...\mu_{p-I+1} \), respectively \( \nu_1...\nu_N \). Furthermore, the following conditions are satisfied,

\[
I = 3N, 5 \leq I \leq p + 1 \Rightarrow N \geq 2.
\]

(78)

We deduce from (77) the equation \( \delta a^I_{\mu_1...\mu_{p-I+1}} + \gamma a^I_{\mu_1...\mu_{p-I+1}} = \partial_{\mu_1}^{\mu} \) has the solution

\[
a^I_{\mu_1...\mu_{p-I+1}} = \frac{N(p-I+2)}{6} f^{\mu_1...\mu_{p-I+1}|\nu_1...\nu_N} \times
\]

\[
(I-2)^{\lambda}_{\mu_1...\mu_{p-I+1}} C_{\nu_1...\nu_N},
\]

(79)
while the equation \( \delta a_{I-2}^{t-p} + \gamma a_{I-2}^{t-p} = \partial_{\mu} j_{I-2}^{\mu} \) has the solution

\[
a_{I-2}^{t-p} = \frac{N (p - I + 2) (p - I + 3) f_{\mu_1...\mu_p-1}||\nu_1...\nu_N} \xi_{\mu_1...\mu_p-1} \times \left( \frac{N - 1}{6} C_{\nu_1...\nu_{N-2}} C'_{\nu_{N-1}} - C_{\nu_1...\nu_{N-1}} G'_{\nu_{N-1}} \right).
\]

The action of \( \delta \) on \( a_{I-2}^{t-p} \) from (80) is

\[
\delta a_{I-2}^{t-p} = \partial_{\mu} p_{I-3} + \gamma \left\{ \frac{N (p - I + 2) (p - I + 3) (p - I + 4) f_{\mu_1...\mu_p-1}||\nu_1...\nu_N} \xi_{\mu_1...\mu_p-1} \times \left( \frac{N - 2}{18} C_{\nu_1...\nu_{N-3}} C'_{\nu_{N-2}} C'_{\nu_{N-1}} \right) \right. \left. \times \left( \nu_1...\nu_{N-1} G_{\nu_{N-1}} \right) + \frac{1}{3} C_{\nu_1...\nu_{N-1}} G'_{\nu_{N-1}} \right\} + \frac{N (p - I + 2) (p - I + 3) (p - I + 4) f_{\mu_1...\mu_p-1}||\nu_1...\nu_N} \xi_{\mu_1...\mu_p-1} \times \left( \frac{N - 1}{6} C_{\nu_1...\nu_{N-2}} C'_{\nu_{N-1}} - C_{\nu_1...\nu_{N-1}} G'_{\nu_{N-1}} \right) \right. \left. \times \left( \nu_1...\nu_{N-1} G_{\nu_{N-1}} \right) \right\}.
\]

(Remark about the convention we use for the indices: if \( N = 2 \) the term containing the product \( C_{\nu_1...\nu_{N-3}} \) does not appear anymore, while the term with the product \( C_{\nu_1...\nu_{N-2}} \) will not contain anymore this product.) The last term from the expression (81) of \( \delta a_{I-2}^{t-p} \) pertains to \( H(\gamma) \) and the equation \( \delta a_{I-2}^{t-p} + \gamma a_{I-2}^{t-p} = \partial_{\mu} j_{I-2}^{\mu} \) has no solution.

The second possibility we analyse for \( 5 \leq I \leq p + 1 \) is that \( a_{I}^{t-p} \) has the form

\[
a_{I}^{t-p} = f_{\mu_1...\mu_p-1}||\lambda_{\nu_1...\nu_M} \xi_{\mu_1...\mu_p-1} \times \left( \frac{N - 1}{6} C_{\nu_1...\nu_{M-2}} C'_{\nu_{M-1}} - C_{\nu_1...\nu_{M-1}} G'_{\nu_{M-1}} \right) \right\}.
\]

where \( f_{\mu_1...\mu_p-1}||\lambda_{\nu_1...\nu_M} \) is a constant tensor, antisymmetric separately in the three groups of indices \( (\mu_1...\mu_p-1, \lambda_{\nu_1...\nu_M}, \nu_1...\nu_M) \)

\[
I = 3M + 1, 5 \leq I \leq p + 1 \Rightarrow M \geq 2.
\]

From (82), it follows the equation \( \delta a_{I-1}^{t-p} + \gamma a_{I-1}^{t-p} = \partial_{\mu} j_{I-1}^{\mu} \) has the solution

\[
a_{I-1}^{t-p} = \frac{(-1)^M}{3} \partial_{\lambda} t_{\mu p}||\alpha C_{\nu_1...\nu_M} \times \left( \frac{M - 1}{6} F_{\lambda_{\nu_1...\nu_M}} \right),
\]

(84)
The action of $\delta$ on $a^{t-p}_{I-1}$ is

$$\delta a^{t-p}_{I-1} = \partial_\mu \rho^{\mu}_I - 2 + \gamma \{(p - I + 2)(p - I + 3) \times \}

\begin{aligned}
&f^{\mu_1 \ldots \mu_p-1}||f_{\mu_1 \ldots \mu_p-1}|| (I-3)^{\alpha \beta} \\
&\times \\
\left[ \frac{(-1)^M}{18} \partial(\lambda t_{\mu \nu})|_\alpha C_{\nu_1 \ldots \nu_{M-1}} C'_{\beta_{\nu_{M}}} + \frac{M(M-1)}{6} f_{\mu \nu} C_{\nu_1 \ldots \nu_{M-2}} C'_{\beta_{\nu_{M-1}}} C'_{\alpha_{\nu_{M}}} + \right. \\
&\left. \frac{M}{12} f_{\mu \nu} C_{\nu_1 \ldots \nu_{M-1}} G'_{\beta \alpha} \right]
\end{aligned}

\begin{aligned}
&+ \left. \frac{(-1)^{M+1}}{6} (p - I + 2)(p - I + 3) f^{\mu_1 \ldots \mu_p-1}||f_{\mu_1 \ldots \mu_p-1}|| \times \right. \\
&\left. (I-3)^{\alpha \beta} \\
&\times \\
\left[ \frac{(-1)^{M+1}}{6} (p - I + 2)(p - I + 3) f^{\mu_1 \ldots \mu_p-1}||f_{\mu_1 \ldots \mu_p-1}|| \times \right. \\
&\left. \frac{M}{12} f_{\mu \nu} C_{\nu_1 \ldots \nu_{M-1}} G'_{\beta \alpha} \right]
\end{aligned}

\text{and the equation } \delta a^{t-p}_{I-1} + \gamma a^{t-p}_{I-2} = \partial_\mu \rho^{\mu}_I \text{ has no solution.}

5 \quad 0 < I \leq 4

For $I \leq 4$, the cohomological group $H_I(\delta|d)$ may have, depending on the degree of the abelian form, two types of generators: the antifields with antighost number $I$ from the sector of the tensor field with the mixed symmetry $(3,1)$ $t_{\lambda_{\mu \nu}}|_\alpha$ and the antifields with antighost number $I$, if these exist, from the sector of the abelian form. The equation (69) has two independent solutions, one linear in the antifields from the sector of the tensor field $t_{\lambda_{\mu \nu}}|_\alpha$ and the other linear in the antifields from the sector of the abelian form. We will analyse separately the chains of equations of the type (69)-(70), starting from each of these two independent solutions. The consistency for each of these chains goes separately until the antighost number one, and if both chains are inconsistent in order one, it is possible that together to be consistent also in the zero order.

5.1 Invariant polynomials generated by the antifields from the sector $(3,1)$

The necessary condition to generate the cross-interactions is that $a^{t-p}_I$ to mix the objects from the sector $(3,1)$ and the objects from the sector of the abelian form, so we need at least the form $(a^{t-p}_I)_{I \leq 4} = (\text{antifields} (3,1)) \times (\text{form ghosts})$.

The only ghost corresponding to the abelian form nontrivial in $H(\gamma)$ is $\xi$ and it must to have the pureghost number less or equal than four. It follows that in this case we can take into account only the abelian forms with the degrees $p \leq 4$. In the sequel, the analysis will depend on the degree $p$ of the abelian form.
If \( p = 4 \) and \( I = 4 \) the general form of the last component from (68) is
\[
a^{4-p}_4 = f^\mu C^*_\mu \xi \quad (86)
\]
and, because \( f^\mu \) is constant, it can not be constructed.

5.1.1 3-forms, \( p = 3 \)

If \( p = 3 \) and \( I = 4 \) the last component from (68) can be constructed in a space-time with the dimension \( D = 5 \),
\[
a^{3-p(D=5)}_3 = \epsilon^{\mu_1 \ldots \mu_5} C^*_\mu_1 \partial_{[\mu_2} \eta_{\mu_3 \mu_4 \mu_5]} \xi \quad (87)
\]
The equation \( \delta a^{3-p(D=5)}_3 + \gamma a^{3-p(D=5)}_3 = \partial^\mu j_\mu \) has the solution
\[
a^{3-p(D=5)}_3 = -2\epsilon^{\mu_1 \ldots \mu_5} C^*_\mu_1 \left( \partial_{[\mu_2} t_{\mu_3 \mu_4 \mu_5]} \xi - 3\partial_{[\mu_2} \eta_{\mu_3 \mu_4 \mu_5]} \xi \right), \quad (88)
\]
but the consistency stops here (the equation \( \delta a^{3-p(D=5)}_3 + \gamma a^{2-p(D=5)}_2 = \partial^\mu j_\mu \) has no solution \( a^{2-p(D=5)}_2 \), because in
\[
\delta a^{3-p(D=5)}_3 = \partial^\lambda \rho \lambda + \gamma \left( \epsilon^{\mu_1 \ldots \mu_5} G^*_\rho \lambda |_{\mu_1} \left( -4\partial_{[\mu_2} t_{\mu_3 \mu_4 \mu_5]} \xi + 6\partial_{[\mu_2} \eta_{\mu_3 \mu_4 \mu_5]} \xi \right) - 2\epsilon^{\mu_1 \ldots \mu_5} G^*_\rho \lambda |_{\mu_1} \partial_{[\mu_2} t_{\mu_3 \mu_4 \mu_5]} \xi \right), \quad (89)
\]
the last term is nontrivial in \( H^3 (\gamma) \), so \( \delta a^{3-p(D=5)}_3 \neq \partial^\mu j_\mu + (\gamma - \text{exact}) \).

If \( p = 3 \) and \( I = 3 \) we can construct the last component from (68) in a space-time with the dimension \( D \geq 5 \),
\[
a^{3-p}_3 = C^*_\mu \xi \quad (90)
\]
The equation \( \delta a^{3-p}_3 + \gamma a^{2-p}_2 = \partial^\mu j_\mu \) has the solution
\[
a^{3-p}_3 = -2G^*_\rho \lambda |_{\mu} \xi, \quad (91)
\]
while for \( \delta a^{3-p}_2 + \gamma a^{2-p}_1 = \partial^\mu j_\mu \) we find the solution
\[
a^{3-p}_1 = 3T^\rho \lambda |_{\mu} \xi, \quad (92)
\]
From the last formula it follows
\[
\delta a^{3-p}_1 = 3T^\rho \lambda |_{\mu} \xi, \quad (93)
\]
where $T^{\rho \lambda \mu | \alpha}$ are the functions appearing in the field equations of $t_{\lambda \mu \nu | \rho}$ (see (14)). These functions may be expressed using $F^{\lambda \mu \nu \xi | \alpha \beta}$ (see (14)), following the contraction $T^{\rho \lambda \mu | \mu} = (\frac{D}{2} - 2) F^{\rho \lambda \alpha \beta | \alpha \beta}$, which we introduce in (93) and we obtain

$$
\delta a_{1}^{t-p} = \partial^{\lambda} \rho_{\lambda} - 3 \left( \frac{D}{2} - 2 \right) \partial^{[\rho \mu \lambda \alpha \beta]} | \beta \cdot \gamma A_{\rho \lambda}. \quad (94)
$$

The last term from (94) is not $\gamma$-closed, so $a_{1}^{t-p}$ from (92) is not consistent (however, we will see later the chain of the type (69)-(70) starting from the component (93) will be consistent together with a chain depending on the antifields from the sector of the 3-form).

For $p = 3$ and $I \leq 2$, the last component from (98) can not be constructed.

### 5.1.2 2-forms, $p = 2$

If $p = 2$, $I = 4$ the general form of the last component from (98) is

$$
a_{4}^{t-p} = f^{\mu_{1} \cdots \mu_{6}} C^{*}_{\mu_{1} \mu_{2}} \partial_{[\mu_{3} \eta_{4} \mu_{5} \mu_{6}]} \partial_{[\mu_{7} \mu_{2} \eta_{7} \mu_{5} \mu_{6}]} \xi, \quad (95)
$$

and can not be constructed, because of the restriction on the derivative order.

If $p = 2$, $I = 3$ we can construct the last component from (98) in a space-time with the dimension $D = 6$,

$$
a_{3}^{t-p(D=6)} = e^{\mu_{1} \cdots \mu_{6}} C^{*}_{\mu_{1} \mu_{2}} \partial_{[\mu_{3} \eta_{4} \mu_{5} \mu_{6}]} \xi. \quad (96)
$$

The equation $\delta a_{3}^{t-p(D=6)} + \gamma a_{2}^{t-p(D=6)} = \partial^{(2)} \mu^{j} \gamma \mu$ has the solution

$$
a_{3}^{t-p(D=6)} = 3 e^{\mu_{1} \cdots \mu_{6}} \left( G^{*}_{\mu_{1} \mu_{2}} \lambda - \frac{1}{2} \eta^{*}_{\mu_{1} \mu_{2}} \lambda \right) \left( -\frac{1}{3} \partial_{[\mu_{3} t_{4} \mu_{5} \mu_{6}]} [\lambda] \xi + \partial_{[\mu_{3} \eta_{4} \mu_{5} \mu_{6}]} [\xi] \right) \lambda, \quad (97)
$$

but the consistency ends here, because in

$$
\delta a_{2}^{t-p(D=6)} = \partial^{\lambda} \rho_{\lambda} + \gamma \left( g_{\mu_{1} \cdots \mu_{6}} t^{*}_{\mu_{1} \mu_{2} [p]} \left( -\frac{1}{3} \partial_{[\mu_{3} t_{4} \mu_{5} \mu_{6}]} [\lambda] \xi + \frac{1}{2} \partial_{[\mu_{3} \eta_{4} \mu_{5} \mu_{6}]} (A^{\rho}) \right) \right) + \frac{1}{6} \left( e^{\mu_{1} \cdots \mu_{6}} t^{*}_{\mu_{1} \mu_{2} [p]} \partial_{[\mu_{3} t_{4} \mu_{5} \mu_{6}]} [p] \xi \right), \quad (98)
$$

the last term is nontrivial in $H^{3} (\gamma)$ and the equation $\delta a_{2}^{t-p(D=6)} + \gamma a_{1}^{t-p(D=6)} = \partial^{(1)} \mu^{j} \gamma \mu$ has no $a_{1}^{t-p(D=6)}$ solution.

If $p = 2$, $I = 2$, the last component from (98) is (in a space time with the dimension $D \geq 5$)

$$
a_{2}^{t-p} = \eta^{* \mu \nu \lambda} \partial_{[\mu A_{\nu \lambda}] \xi} \quad (99)
$$
The previous object is not consistent, because in
\[\delta a^t_p = \partial_{\lambda} \rho^\lambda + \gamma \left( t^{*\mu\nu\lambda}\rho \partial_{[\mu} A_{\nu\lambda]} \right)^{(1)} + t^{*\mu\nu\lambda}\rho \partial_{[\mu} A_{\nu\lambda]} \xi, \quad (100)\]
the last term is not \(\gamma\)-exact.

5.1.3 Vector fields, \(p = 1\)
This case was analysed in [32].

5.2 Invariant polynomials generated by the antifields from the sector of an abelian form
The condition that the solution of the equation \(\gamma a_I = 0\) to mix the objects from the BRST complex (3, 1) and the objects from the BRST complex of an abelian form forces the invariant polynomials linear in the antifields from the sector of an abelian form, \(a_I\) to depend on the ghosts from the sector (3, 1). It is simple to prove \(a_I\) do not depend in this case on the ghosts of the abelian form, namely we don’t have solutions of the type \(a_I = \text{form antifields} \times \text{form ghosts} \times \text{ghosts}\) (3, 1).

Such a solution would have the general form
\[a_I = f^{\mu_1\mu_2\mu_3\mu_4} \xi \partial_{[\mu_1} \eta_{\mu_2\mu_3\mu_4]}, \quad I = p + 1, \quad (101)\]
that can not constructed concretely, because of the condition on the space-time dimension \(D \geq 5\). Therefore, we will search for the component with the greatest antighost number from (68) the solution of the type \(a_I = \text{form antifields} \times \text{form ghosts} \times \text{ghosts}\) (3, 1). Our analysis will not depend so much on the degree of the abelian form, as in the case when the invariant polynomials where generated by the antifields from the sector (3, 1).

5.2.1 \(I = 4\)
If \(I = 4, p \geq 3\) we can construct the last component from (68) in a space time with the dimension \(D = p + 2 \geq 5\),
\[a_4^{t-p} = \epsilon^{\mu_1...\mu_{p+2}} \xi^{(3)*}_{\mu_1...\mu_{p+3}} C_{\mu_{p+2}} \partial_{[\mu_{p+1} \eta_{\mu_p\mu_{p+1}\mu_{p+2}}]}, \quad (102)\]
The equation \(\delta a_4^{t-p} + \gamma a_3^{t-p} = \partial_{\mu} j^{(2)*}\) has the solution
\[a_3^{t-p} = -(p - 2) \epsilon^{\mu_1...\mu_{p+2}} \xi_{\lambda_{\mu_1...\mu_{p+3}}} \left( \frac{1}{6} C^\lambda_{\mu_{p+2}} \partial_{[\mu_{p+1} \eta_{\mu_p\mu_{p+1}\mu_{p+2}}]} + \frac{1}{3} C_{\mu_{p+2}} \partial_{[\mu_{p+1} t_{\mu_p\mu_{p+1}\mu_{p+2}}]} \right), \quad (103)\]
from which it follows

$$\delta a_3^{l-p} = \partial^l j_\lambda - \gamma \left[ (p - 2)(p - 1) \epsilon^{\mu_1 \ldots \mu_{p+2}} \xi_{\rho \lambda \mu_1 \ldots \mu_{p-3}} \times \right.$$  

$$\left. \left( \frac{1}{12} G'_{\rho \lambda | \mu_{p-3}} \partial_{[\mu_{p-1} \eta_{\mu_{p+1} \mu_{p+2}]} + \frac{1}{18} c^\lambda_{\mu_{p-2}} \partial_{[\mu_{p-1} t_{\mu_{p+1} \mu_{p+2}}]^{\rho | \lambda, \mu}] \right) \right) - (p - 2)(p - 1) \epsilon^{\mu_1 \ldots \mu_{p+2}} \xi_{\rho \lambda \mu_1 \ldots \mu_{p-3}} C_{\mu_{p-2}} \partial_{[\mu_{p-1} t_{\mu_{p+1} \mu_{p+2}}]^{\rho | \lambda, \mu}] \right) \right) - (p - 2)(p - 1) \epsilon^{\mu_1 \ldots \mu_{p+2}} \xi_{\rho \lambda \mu_1 \ldots \mu_{p-3}} C_{\mu_{p-2}} \partial_{[\mu_{p-1} t_{\mu_{p+1} \mu_{p+2}}]^{\rho | \lambda, \mu}] \right) \right) \right) \right) .$$

The last term from the previous expression is nontrivial in $H^3(\gamma)$, so $\delta a_3 \neq \partial^l \rho \lambda + (\gamma - \text{exact})$ and the consistency of the object $a_3^{l-p}$ from (102) ends here.

### 5.2.2 $I = 3$

If $I = 3$, the general form of the last component from (68) is

$$a_3^{l-p} = f^{\mu_1 \ldots \mu_{p-1}} (2)^* \xi_{\mu_1 \ldots \mu_{p-2}} C_{\mu_{p-1}},$$

and it can be constructed concretely only for $p = 3$,

$$a_3^{l-p} = (2)^* \xi \ C_{\mu}.$$  

(105)

(106)

It follows from (106) the solution of the equation $\delta a_3^{l-p} + \gamma a_2^{l-p} = \partial_{\mu} j_2^{\mu}$,

$$a_2^{l-p} = (2)^* \xi \ C_{\mu},$$

(107)

Next, the equation $\delta a_2^{l-p} + \gamma a_1^{l-p} = \partial_{\mu} j_1^{\mu}$ has the solution

$$a_1^{l-p} = -\frac{1}{2} A^{* \lambda \nu \mu} \eta_{\lambda \nu \mu},$$

(108)

and the action of $\delta$ on $a_1^{l-p}$ from (105) is

$$\delta a_1^{l-p} = \partial_{\mu} \rho^{\mu} + \gamma \left( \frac{1}{6 \cdot 3!} A^{* \lambda \nu \mu} \partial_{[\rho t_{\lambda \nu \mu]}^{\mu}] |^p \right) - \frac{1}{6 \cdot 3!} \gamma A^{* \lambda \nu \mu} \partial_{[\rho t_{\lambda \nu \mu]}^{\mu}] |^p \right)$$

(109)

and the consistency stops here, because the last term from (109) is not $\gamma$-exact (Remark: still, we will use this chain, starting from (106), together with the chain starting from (67) to obtain a solution for the equation (67)).

### 5.2.3 $I = 2$

If $I = 3$, the general form of the last component from (68) is

$$a_2^{l-p} = f^{\mu_1 \ldots \mu_{p+2}} (1)^* \xi_{\mu_1 \ldots \mu_{p-1}} \partial_{[\mu_{p+1} \eta_{\mu_{p+2} \mu_{p+3}]} \partial_{[\mu_{p+4} \eta_{\mu_{p+5} \mu_{p+6} \mu_{p+7}]}},$$

(110)

and can not be constructed because of the condition on the derivative order (if it was consistent, the component $a_2^{l-p}$ from the previous formula would produce in a possible $a_0^{l-p}$ terms with three derivatives).
5.2.4 $I = 1$

If $I = 1$, the decomposition (58) has two terms,

$$a^{t-p} = a_0^{t-p} + a_1^{t-p}. \quad (111)$$

The last term from (111) has the properties agh $(a_1^{t-p}) = 1$, pgh $(a_1^{t-p}) = 1$ and satisfies the equation $\gamma a_1^{t-p} = 0$, with the solution

$$a_1^{t-p} = \epsilon^{\mu_1 \ldots \mu_{p+4}} A^*_p \mu \partial_{[\mu_{p+1} \eta_{p+2} \mu_{p+3} \mu_{p+4}]} \lambda, \quad (112)$$

for every value of the abelian form degree ($p \geq 1$), in a space time with the dimension $D = p + 4 \geq 5$. The equation $\delta a_1^{t-p} + \gamma a_0^{t-p} = \partial_{(0)}^\mu j$ has the solution

$$a_0^{t-p} = \frac{1}{3p!} \epsilon^{\mu_1 \ldots \mu_{p+4}} F_{\lambda \mu_1 \ldots \mu_p} \partial_{[\mu_{p+1} \eta_{p+2} \mu_{p+3} \mu_{p+4}]} \lambda. \quad (113)$$

In this moment we have discovered, starting from (111), the first order deformation of the solution of the master equation for the theory (1),

$$S_1 = \int d^D x \epsilon^{\mu_1 \ldots \mu_{p+4}} \left( A^*_p \mu \partial_{[\mu_{p+1} \eta_{p+2} \mu_{p+3} \mu_{p+4}]} \right) + \frac{1}{3p!} F_{\lambda \mu_1 \ldots \mu_p} \partial_{[\mu_{p+1} \eta_{p+2} \mu_{p+3} \mu_{p+4}]} \lambda. \quad (114)$$

[Remark: if $I = 1$, $p \geq 3$ we could construct, apparently, in a space-time with the dimension $D = p + 2 \geq 5$

$$a_1^{t-p} = \epsilon^{\mu_1 \ldots \mu_{p+2}} A^*_p \mu \partial_{[\mu_{p+1} \eta_{p+2} \mu_{p+3} \mu_{p+4}]} \sigma^{\lambda p}, \quad (115)$$

but

$$\epsilon^{\mu_1 \ldots \mu_{p+2}} A^*_p \mu \partial_{[\mu_{p+1} \eta_{p+2} \mu_{p+3} \mu_{p+4}]} \sigma^{\lambda p} = - \frac{3}{p} \epsilon^{\mu_1 \ldots \mu_{p+2}} A^*_p \mu \partial_{[\mu_{p+1} \eta_{p+2} \mu_{p+3} \mu_{p+4}]} \sigma^{\lambda p} = 0. \quad (116)$$

]

6 $I = 0$

In this case the first order deformation contains only a component with antighost number zero, $a^{t-p} = a_0^{t-p}$, and we have to solve the equation

$$\gamma a_0^{t-p} = \partial_{\mu} m_\mu^{t-p}, \quad (117)$$

where $a_0^{t-p}$ depends only on the tensor field (3, 1) and the abelian form (because $pgh a_0^{t-p} = 0$), and $m_\mu^{t-p} \neq 0$ (the case $m_\mu^{t-p} = 0$ is easily eliminated, see [31, 32]). To find the solutions for (117), we shall adopt the method used in [31]
slightly modified for cross-interactions. We introduce two counting operators, one for the mixed symmetry tensor field and its derivatives,

\[ N^{(t)} = t_{\lambda \mu |p} \frac{\partial}{\partial t_{\lambda \mu |p}} + \sum_{k>0} \partial_{\mu_1} \cdots \partial_{\mu_k} t_{\lambda \mu |p} \frac{\partial}{\partial (\partial_{\mu_1} \cdots \partial_{\mu_k} t_{\lambda \mu |p})}, \]  

(118)

and the other for the abelian form and its derivatives,

\[ N^{(A)} = A_{\mu_1 \cdots \mu_p} \frac{\partial}{\partial A_{\mu_1 \cdots \mu_p}} + \sum_{k>0} \partial_{\nu_1} \cdots \partial_{\nu_k} A_{\mu_1 \cdots \mu_p} \frac{\partial}{\partial (\partial_{\nu_1} \cdots \partial_{\nu_k} A_{\mu_1 \cdots \mu_p})}. \]  

(119)

The solution for (117) is written as a sum of eigen solutions for the counting operators (118)-(119),

\[ a_{t-A} = \sum_{k,l} a_{kl} \]  

(120)

and it can be proved that every component \( a_{tkl} \) from \( a_{t-A} \) is separately a solution for (117). Therefore, we search a solution \( a_{t-A} \) for the equation (117) that is in the same time an eigen solution for \( N^{(t)} \) and for \( N^{(A)} \). We denote the functional derivatives of \( a_{t-A} \) by

\[ D_{\mu \nu \lambda |p} = \delta a_{t-A} / \delta t_{\mu \nu \lambda |p}, \quad D_{\mu_1 \cdots \mu_p} = \delta a_{t-A} / \delta A_{\mu_1 \cdots \mu_p}. \]  

(121)

and using the integration by parts the actions of the operators \( N^{(t)} \) and \( \gamma \) on \( a_{t-A} \) are

\[ N^{(t)} a_{t-A} = D_{\mu \nu \lambda |p} t_{\mu \nu \lambda |p} + \partial_{\mu} n^{\mu}, \]  

(122)

where we are not interested by the concrete form of the divergences. It follows from (122) that (117) has solutions only if the functional derivatives satisfy the conditions

\[ \partial_{\mu} D_{\mu \nu \lambda |p} = 0, \quad \partial_{\nu} D_{\mu \nu \lambda |p} = 0, \]  

(123)

\[ \partial_{\mu} D_{\mu_2 \cdots \mu_p} = 0. \]  

(124)

(123) implies further using the generalized cohomology of the space-time exterior differential that \( D_{\mu \nu |p} \) must have the form

\[ D_{\mu \nu |p} = \partial_{\alpha} \partial_{\beta} \Phi_{\mu \nu |p}^{\alpha \beta}, \]  

(125)

where the tensor field \( \Phi_{\mu \nu |p}^{\alpha \beta} \) has the same mixed symmetry as the curvature tensor. We can reconstruct the form of the solution for the equation (117) using (121), (125) and the fact that \( a_{t-A} \) is eigen solution for \( N^{(t)} \), \( N^{(t)} a_{t-A} = k a_{t-A} \). Hence, up to a negligible divergence, we obtain a necessary condition for \( a_{t-A} \) to be a solution to (117),

\[ a_{t-A} = 1 / 8k \Phi_{\mu \nu |p}^{\alpha \beta} K_{\mu \nu |p}^{\alpha \beta}. \]  

(126)
Furthermore we remark that at the pure ghost number zero $\gamma$ splits in $\gamma = \gamma^{(t)} + \gamma^{(A)}$ ($\gamma^{(t)}$ acts only on the mixed symmetry tensor field and its derivatives and $\gamma^{(A)}$ only on the abelian form and its derivatives) and in the equation (117) every component of $\gamma$ must give separately a total derivative. This remark helps us to obtain, using almost the same computation technique as in [31], that the tensor field $\Phi_{\mu \nu \lambda \alpha | \beta \rho}$ in (126) is linear in the fields (it cannot depend on the derivatives of the fields due to the restraint imposed on the derivative order).

Because we study the cross-interactions, we consider that $\Phi_{\mu \nu \lambda \alpha | \beta \rho}$ depends only on the abelian form and the general form of the solution for the equation (117) is

$$a_0^{-A} = f_{\mu \nu \lambda \alpha | \beta \rho}^{(t)} \mu_1 \ldots \mu_p K_{\mu \nu \lambda \alpha | \beta \rho},$$

where $f_{\mu \nu \lambda \alpha | \beta \rho}^{(t)} \mu_1 \ldots \mu_p$ is a constant tensor. There is only one solution, for an abelian two-form,

$$a_0^{-A} = \sigma_{\mu \nu} \sigma_{\rho \beta} A_{\mu \nu} K^{\mu \nu \lambda | \alpha \beta},$$

but it proves to be $s$-exact modulo $d$,

$$a_0^{-A} = s \left( \frac{2}{4 - D} A_{\mu \nu} t^{\mu \nu} + \frac{4}{3 (4 - D)} \sigma_{\mu \nu} G^{\nu \lambda | \mu} + \frac{2}{3 (4 - D)} \xi^{(2)} C_{\mu \nu} \right) + \partial_{\mu} \left( \frac{4}{4 - D} \xi^{(1)} \mu \nu - \frac{4}{3 (4 - D)} \sigma_{\mu \nu} G^{\nu \lambda | \mu} \right).$$

7 Solutions for the first order deformation

There are four independent solutions for the first order deformation of the solution of the master equation. The first two have the nonintegrated densities given by the formulas (65)-(66) and depend only on the abelian form (see [37, 38]),

$$S^{(1)}_1 = \int d^{2p+1} x \epsilon_{\mu_1 \ldots \mu_{2p+1}} A_{\mu_1 \ldots \mu_p} F_{\mu_{p+1} \ldots \mu_{2p+1}},$$

$$S^{(2)}_1 = \int d^{2p+2} x \epsilon_{\mu_1 \ldots \mu_{2p+2}} A_{\mu_1 \ldots \mu_p} F_{\mu_{p+1} \ldots \mu_{2p+1}} F_{\mu_{2p+2} \ldots \mu_{3p+2}}.$$
and the two chains are consistent in the zero order together for
\[ k = 6 \cdot 3! (4 - D) , \]  
with the next solution of the equation \( \delta a_1^{i-p} + \gamma a_0^{i-p} = \partial_{\mu} j_{i}^{\mu} , \]
\[ a_0^{i-p} = (4 - D) A^{\rho \lambda \mu} \partial_{[\theta} t_{\rho \lambda \mu]} |^\theta . \]  
The formulas (132)-(136) give us as the components of a solution for the first order deformation in the case of the cross-interactions for an abelian 3-form and the tensor field \( t_{\lambda \mu | \rho} \),
\[ S^{(3)}_1 = \int d^D x \left( C^{\mu} \right)_\mu (3) + k \xi + k \xi C_\mu - 2 G^{\star \lambda \mu} C_\mu (2) + \right. \]  
\[ 3 t^{\star \rho \lambda \mu} (1) \left( \xi |_{\mu \rho \lambda} - \frac{k}{2} A^{\rho \lambda \mu} \eta_{\lambda \mu} + (4 - D) A^{\rho \lambda \mu} \partial_{[\theta} t_{\rho \lambda \mu]} |^\theta . \]  
The fourth independent solution for the first order deformation has the components written in the formulas (112)-(113),
\[ S^{(4)}_1 = \int d^{p+4} x \left( A^{\mu} \right)_{\mu \rho p+1} \partial_{[\mu p+1} \eta_{\rho p+2 \mu p+3} t_{\mu p+4]} + \frac{1}{3!} F^{\lambda \mu \rho \sigma} \partial_{[\mu p+1} t_{\rho p+2 \mu p+3} t_{\mu p+4]} |^\lambda . \]  
We remark, first, the solutions \( S^{(1)}_1 \) and \( S^{(2)}_1 \) can not appear together, because of the space-time dimensions incompatibility (2p + 1 \( \neq \) 3p + 2, \( \forall p \geq 1 \)). Second, \( S^{(1)}_1 \) and \( S^{(3)}_1 \) can appear together only if 2p + 1 = p + 4 (p = 3, D = 7), while \( S^{(2)}_1 \) and \( S^{(4)}_1 \) can appear together only if 3p + 2 = p + 4 (p = 1, D = 5; this case was analysed in [32]). To conclude, for an abelian 3-form the general solution for the first order deformation is
\[ S_1 = \delta_{D,7} \left( c_1 S^{(1)}_1 + c_4 S^{(4)}_1 \right) + \delta_{D,11} c_2 S^{(2)}_1 + c_3 S^{(3)}_1 , \]  
where \( c_1, c_2, c_3, c_4 \) are constants, while for an abelian form with \( p \notin \{1, 3\} \) the general solution is only \( S^{(4)}_1 \).

8 Higher order deformations

Second order deformation \( S_2 \) is the solution of the equation (60). The first order deformation for the case of the cross-interactions between an abelian \( p \)-form and the mixed symmetry (3, 1) tensor field has the general form (135) for every value \( p \geq 1 \) of the form degree. For the particular case of a three-form (\( p = 3 \)) the first order deformation is given by (139), but in our estimations it is consistent
(i.e. the equation (60) has a solution) only if the coefficients \(c_1, c_2, c_3\) vanish. So, we will take into account only the solution \(S_1^{(4)}\) given by (138) for the first order deformation and the first term in (138) is

\[
\frac{1}{2} (S_1, S_1) = \frac{1}{2} \int d^D x \frac{1}{3p} \epsilon_{\nu_1 \ldots \nu_{p+1} \ldots \mu_{p+4}} \epsilon_{\nu_{p+1} \ldots \nu_{p+3} \ldots \nu_{p+4}} \times \\
\partial_\lambda \partial_{\mu_{p+1}} \partial_{\eta_{p+2}} \partial_{\mu_{p+3}} \partial_{\mu_{p+4}} \partial_{\nu_{p+1}} \partial_{\nu_{p+2}} \partial_{\nu_{p+3}} \partial_{\nu_{p+4}} [\lambda],
\]

(140)

from which using the identity

\[
\epsilon_{\nu_1 \ldots \nu_{p+1} \ldots \mu_{p+4}} \epsilon_{\mu_{p+1} \ldots \mu_{p+3} \ldots \mu_{p+4}} = (-1)^{p+1} \frac{1}{p!} \delta_{\mu_{p+1} \ldots \mu_{p+4}} \delta_{\nu_{p+1} \ldots \nu_{p+4}}
\]

(141)

it follows

\[
\frac{1}{2} (S_1, S_1) = \frac{1}{2} \int d^D x s \left( (-1)^{p+1} \frac{32}{3} \partial^{[\lambda} t^{\mu \nu \rho]} \partial_{[\lambda} \partial_{\alpha} t^{\mu \nu \rho]} \right).
\]

(142)

Therefore we have for the second order deformation the solution

\[
S_2 = \int d^D x \left( (-1)^{p-1} \frac{16}{3} g \partial^{[\lambda} t^{\mu \nu \rho]} \partial_{[\lambda} \partial_{\alpha} t^{\mu \nu \rho]} \right).
\]

(143)

Because \((S_1, S_2) = 0\) we can choose the deformations of the order higher than two to vanish,

\[
S_k = 0, \quad k \geq 3.
\]

(144)

9 Conclusions

The deformation of the solution of the master equation for the theory described by the action (11) is consistent in a space-time with the dimension \(D = p+4\), if the development according to the antighost number of the first order deformation has only two terms (111), that with the antighost number one being (112). The components of the deformation \(\bar{S} = S + gS_1 + g^2 S_2\) are written \(S\) in (56), \(S_1\) in (114) and \(S_2\) in (143). The terms with the antighost number one from \(\bar{S}\) represents the deformed gauge transformations. Thus, we observe that the gauge transformations of the tensor field \(t_{\lambda \mu \nu |\alpha}\) are unchanged, while the gauge transformations of the abelian form are

\[
\delta_{\mu_1}^{(1)} \partial_\rho \epsilon = \partial_{\mu_1}^{(1)} \rho \mu_2 \ldots \mu_p + g e_{\mu_1 \ldots \mu_p} \lambda \mu \nu \rho A^*_\lambda \mu_1 \ldots \mu_p \partial_\lambda \partial_\mu \partial_\nu \partial_\rho,
\]

(145)

where \(\epsilon_{\mu \nu \rho}\) are the gauge parameters from (7). The terms with the antighost number zero from \(\bar{S}\) represent the Lagrangian action of the interacting theory, which can be written as

\[
S_L \left[ t_{\mu \nu |\lambda |\rho}, A_{\mu_1 \ldots \mu_p} \right] = S_0 \left[ t_{\mu \nu |\lambda |\rho} \right] - \frac{1}{2} \frac{1}{(p+1)!} \int d^{p+4} x \bar{F}_{\mu_1 \ldots \mu_{p+1}} \bar{F}^{\mu_1 \ldots \mu_{p+1}},
\]

(146)
where $\bar{F}_{\mu_1...\mu_{p+1}}$ are the most general objects depending on the abelian form, invariant under the gauge transformations (7) and (145),

$$\bar{F}_{\mu_1...\mu_{p+1}} = F_{\mu_1...\mu_{p+1}} - \frac{g}{3} \epsilon^{[\lambda}_{\mu_1} F^{\mu_\nu\rho]} |\epsilon^{\mu_2...\mu_{p+1}]_\lambda\mu_{\nu}\rho}, \quad (147)$$

($F_{\mu_1...\mu_{p+1}}$ is defined in (5).) The terms which contain products of the abelian form with the mixed symmetry $(3, 1)$ tensor field does not represent interaction vertices, being known in the literature as “mixing terms”.

References

1. T. Curtright, Generalized gauge fields, Phys. Lett. B165 (1985) 304
2. T. Curtright, P. G. O. Freund, Massive dual fields, Nucl. Phys. B172 (1980) 413
3. C. S. Aulakh, I. G. Koh, S. Ouvry, Higher spin fields with mixed symmetry, Phys. Lett. B173 (1986) 284
4. J. M. Labastida, T. R. Morris, Massless mixed symmetry bosonic free fields, Phys. Lett. B180 (1986) 101
5. J. M. Labastida, Massless particles in arbitrary representations of the Lorentz group, Nucl. Phys. B322 (1989) 185
6. C. Burdik, A. Pashnev, M. Tsulaia, On the mixed symmetry irreducible representations of the Poincaré group in the BRST approach, Mod. Phys. Lett. A16 (2001) 731 [hep-th/0101201]
7. Yu. M. Zinoviev, On massive mixed symmetry tensor fields in Minkowski space and (A)dS [hep-th/0211233]
8. C. M. Hull, Duality in gravity and higher spin gauge fields, JHEP 0109 (2001) 027 [hep-th/0107149]
9. X. Bekaert, N. Boulanger, Tensor gauge fields in arbitrary representations of $GL(D, \mathbb{R})$: duality & Poincaré lemma, Commun. Math. Phys. 245 (2004) 27 [hep-th/0208058]
10. X. Bekaert, N. Boulanger, Massless spin-two field $S$-duality, Class. Quantum Grav. 20 (2003) S417 [hep-th/0212131]
11. X. Bekaert, N. Boulanger, On geometric equations and duality for free higher spins, Phys. Lett. B561 (2003) 183 [hep-th/0301243]
12. H. Casini, R. Montemayor, L. F. Urrutia, Duality for symmetric second rank tensors. II. The linearized gravitational field, Phys. Rev. D68 (2003) 065011 [hep-th/0304228]
[13] N. Boulanger, S. Chockaert, M. Henneaux, A note on spin-\(s\) duality, JHEP 0306 (2003) 060 [hep-th/0306023]

[14] P. de Medeiros, C. Hull, Exotic tensor gauge theory and duality, Commun. Math. Phys. 235 (2003) 255 [hep-th/0208155]

[15] X. Bekaert, N. Boulanger, M. Henneaux, Consistent deformations of dual formulations of linearized gravity: A no-go result, Phys. Rev. D67 (2003) 044010 [hep-th/0210278]

[16] Yu. M. Zinoviev, First order formalism for mixed symmetry tensor fields [hep-th/0304067]

[17] Yu. M. Zinoviev, First order formalism for massive mixed symmetry tensor fields in Minkowski and (A)dS spaces [hep-th/0306292]

[18] A. K. Bengtsson, I. Bengtsson, L. Brink, Cubic interaction terms for arbitrarily extended supermultiplets, Nucl. Phys. B227 (1983) 41

[19] M. A. Vasiliev, Cubic interactions of bosonic higher spin gauge fields in AdS(5), Nucl. Phys. B616 (2001) 106 [hep-th/0106200]; Erratum-ibid. B652 (2003) 407

[20] E. Sezgin, P. Sundell, 7–D bosonic higher spin theory: symmetry algebra and linearized constraints, Nucl. Phys. B634 (2002) 120 [hep-th/0112100]

[21] D. Francia, A. Sagnotti, Free geometric equations for higher spins, Phys. Lett. B543 (2002) 303 [hep-th/0207002]

[22] X. Bekaert, N. Boulanger, S. Chockaert, No Self-Interaction for Two-Column Massless Fields, J. Math. Phys. 46 (2005) 012303 [hep-th/0407102]

[23] N. Boulanger, S. Chockaert, Consistent deformations of [p,p]-type gauge field theories, JHEP 0403 (2004) 031 [hep-th/0402180]

[24] C. C. Ciobăcă, E. M. Cioroianu, S. O. Salău, Cohomological BRST aspects of the massless tensor field (k,k), Int. J. Mod. Phys. A19 (2004) 4579 [hep-th/0403017]

[25] N. Boulanger, S. Leclercq, S. Chockaert, Parity violating vertices for spin-3 gauge fields, Phys.Rev. D73 (2006) 065019 [hep-th/0509118]

[26] X. Bekaert, N. Boulanger, S. Chockaert, Spin three gauge theory revisited, JHEP 0601 (2006) 052 [hep-th/0508048]

[27] G. Barnich, M. Henneaux, Consistent couplings between fields with a gauge freedom and deformations of the master equation, Phys. Lett. B311 (1993) 123 [hep-th/9304057]

[28] C. M. Hull, Strongly coupled gravity and duality, Nucl. Phys. B583 (2000) 237 [hep-th/0004195]
[29] C. M. Hull, Symmetries and compactifications of (4, 0) conformal gravity, JHEP 0012 (2000) 007 [hep-th/0011215]

[30] H. Casini, R. Montemayor, L. F. Urrutia, Dual theories for mixed symmetry fields. Spin two case: (1, 1) versus (2, 1) Young symmetry type fields, Phys. Lett. B507 (2001) 336 [hep-th/0102104]

[31] C. Bizdadea, C. C. Ciobîrcă, E. M. Cioroianu, I. Negru, S. O. Saliu, S. C. Săraru, Interactions of a single massless tensor field with the mixed symmetry (3,1). No-go results, J. High Energy Phys. JHEP 0310 (2003) 019

[32] C. Bizdadea, C. C. Ciobîrcă, I. Negru, S. O. Saliu, Couplings between a single massless tensor field with the mixed symmetry (3,1) and one vector field, Phys. Rev. D74 (2006) 045031

[33] G. Barnich, F. Brandt, M. Henneaux, Local BRST cohomology in the antifield formalism. I. General theorems, Commun. Math. Phys. 174 (1995) 57 [hep-th/9405109]

[34] G. Barnich, F. Brandt, M. Henneaux, Local BRST cohomology in gauge theories, Phys. Rept. 338 (2000) 439 [hep-th/0002245]

[35] M. Henneaux, Space-time locality of the BRST formalism, Commun. Math. Phys. 140 (1991) 1

[36] G. Barnich, F. Brandt, M. Henneaux, Local BRST cohomology in the antifield formalism. II. Application to Yang-Mills theory, Commun. Math. Phys. 174 (1995) 93 [hep-th/9405194]

[37] M. Henneaux, B. Knaepen, C. Schomblond, Characteristic cohomology of p-form gauge theories, Commun.Math.Phys. 186 (1997) 137 [hep-th/9606181]

[38] M. Henneaux, B. Knaepen, All consistent interactions for exterior form gauge fields, Phys.Rev. D56 (1997) R6076 [hep-th/9706119]