Neutralino Exchange Corrections to the Higgs Boson
Mixings with Explicit CP Violation

Tarek Ibrahim\textsuperscript{a} and Pran Nath\textsuperscript{b}

\textsuperscript{a}. Department of Physics, Faculty of Science, University of Alexandria, Alexandria, Egypt
\textsuperscript{b}. Department of Physics, Northeastern University, Boston, MA 02115-5000, USA

Abstract

A calculus for the derivatives of the eigen values of the neutralino mass matrix with respect to the CP violating background fields is developed and used to compute the mixings among the CP even and the CP odd Higgs sectors arising from the inclusion of the neutralino sector consisting of the neutralino, the Z boson, and the neutral Higgs bosons ($\chi_i^0 - Z - h^0 - H^0$) exchange in the loop contribution to the effective potential including the effects of large CP violating phases. Along with the stop, sbottom, stau and chargino-W-charged Higgs ($\chi^+ - W - H^+$) contributions computed previously the present analysis completes the one loop corrections to the Higgs boson mass matrix in the presence of large phases. CP violation in the neutral Higgs sector is discussed in the above framework with specific focus on the mixings of the CP even and the CP odd sectors arising from the neutralino sector. It is shown that numerically the effects of the neutralino exchange contribution on the mixings of the CP even and the CP odd sectors are comparable to the effects of the stop and of the chargino exchange contributions and thus the neutralino exchange contribution must be included for a realistic analysis of mixings in the CP even and the CP odd sectors. Phenomenological implications of these results are discussed.
1 Introduction

CP violation in supersymmetric theories via soft supersymmetry breaking parameters has received a considerable degree of attention since the beginning of the formulation of supersymmetric models[1]. Recently, there has been enhanced interest in the investigation of their effects due to the realization that supersymmetric theories may allow for large CP violating phases[2] consistent with the electric dipole moment of the electron and of the neutron[3]. Such a situation can arise because of several possibilities, such as the SUSY spectrum being heavy[2], due to internal cancellations[4] and due to the possibility that the CP phases may reside in the third generation and consequently their effects on the first two generation EMDs are suppressed[5]. Of course it is possible that a more unified framework may determine the combination of phases that enter the EDMs to be small[6]. However, we shall investigate here the possibility that the phases are large and the EDM constraints are satisfied by one of the methods discussed above so that the sparticle spectrum is consistent with the naturalness constraints (see, e.g., Ref.[7]). In this case their effects on low energy physics can be quite significant and a number of low energy phenomena have been discussed including the effect of CP phases. These include the effect of CP phases on g-2[8], on dark matter[9], on the trileptonic signal[10], on baroyogenesis[11], and on other low energy phenomena[12]. Another area where the effect of CP phases has been discussed is the Higgs sector[13, 14, 15, 16, 17, 18]. It is well known that loop corrections to the Higgs masses and mixings are important[19]. In fact in the absence of the loop corrections the lightest Higgs boson mass must lie below $M_Z$ which is already experimentally excluded and it is the presence of the loop corrections that raises its value above $M_Z$. An interesting phenomenon arises if the loop corrections have CP violating phases. In this case it has been pointed out that a significant mixing can occur between the CP even and the CP odd neutral Higgs sectors of the theory[13]. In Refs.[13, 14, 15, 16, 17, 18] the effect of CP phases via the stop and sbottom exchanges was carried out. Further, in the work of Ref.[16] it was pointed out that the effect of chargino loop corrections can be quite significant and in fact the CP effects from the chargino exchange may even dominate the CP effects from the stop-sbottom exchange for the case of large $\tan \beta$.

In this paper we give an analysis of the one loop correction to the Higgs boson mass including the neutralino-Z boson-neutral Higgs exchange including the CP violating phases. The inclusion of the CP dependent neutralino exchange correc-
tions are more intricate relative to the stop-sbottom exchanges and the chargino exchanges. This is due to the fact that the stop-sbottom exchange and the chargino exchange involve diagonalization of only $2 \times 2$ squark and chargino mass matrices and thus the evaluation of their contribution can be carried out analytically in a straightforward fashion. For the case of the neutralino exchange the neutralino mass matrix is a $4 \times 4$ object and its diagonalization analytically is more intricate and a straightforward technique for the analysis is wieldy. In this paper we develop a calculus for the derivatives of the eigen values of the neutralino mass matrix to obtain an explicit analytic expression for the neutralino exchange contribution. The outline of the rest of the paper is as follows. In Sec.2 we give the Higgs potential and discuss the minimization conditions in the presence of the CP violating phases. In Sec.3 we discuss the calculus for the computation of derivatives of the eigen values of the neutralino mass matrix. In Sec.4 we use the technique of Sec.3 and compute the one loop contributions to the Higgs boson mass matrix from the neutralino-Z-neutral Higgs boson exchange. Discussion of the numerical results is given in Sec.5. Conclusions are given in Sec.6. Some further details of the analysis are given in Appendices A and B.

2 CP Phases and Minimization of Higgs Potential

We begin by defining the soft SUSY breaking parameters for the mSUGRA case. Here the low energy physics for the CP conserving case is parametrized by $m_0$, $m_{\tilde{g}}$, $A_0$, and $\tan \beta$ where $m_0$ is the universal scalar mass, $m_{\tilde{g}}$ is the universal gaugino mass, $A_0$ is the universal trilinear coupling, and $\tan \beta = \frac{v_2}{v_1}$ is the ratio of the Higgs VEVs, where the VEV of $H_2$ gives mass to the up quarks and the VEV of $H_1$ gives mass to the down quarks and the leptons. In the presence of CP violation mSUGRA allows for only two CP violating phases which can be taken to be $\theta_{\mu_0}$, and $\alpha_{A_0}$ where $\theta_{\mu_0}$ is the phase of the Higgs mixing parameter $\mu_0$ and $\alpha_{A_0}$ is the phase of $A_0$. The analysis of this paper, however, will be more general, valid for the MSSM parameter space. The Higgs sector in MSSM at the one loop level is described by the scalar potential $V(H_1, H_2) = V_0 + \Delta V$ where

$V_0 = m_0^2 |H_1|^2 + m_2^2 |H_2|^2 + (m_3^2 H_1 H_2 + H.C.)$

$+ \frac{(g_2^2 + g_1^2)}{8} |H_1|^4 + \frac{(g_2^2 + g_3^2)}{8} |H_2|^4 - \frac{g_2^2}{2} |H_1 H_2|^2 + \frac{(g_2^2 - g_1^2)}{4} |H_1|^2 |H_2|^2$
\[
\Delta V = \frac{1}{64\pi^2} \sum_i c_i (2J_i + 1)(-1)^{2J_i}(M_i^4(H_1,H_2)(\log\frac{M_i^2(H_1,H_2)}{Q^2} - \frac{3}{2}))
\] (1)

Here \( m_1^2 = m_{H_1}^2 + |\mu|^2, m_2^2 = m_{H_2}^2 + |\mu|^2, m_3^2 = |\mu B| \) and \( m_{H_{1,2}} \) and \( B \) are the soft SUSY breaking parameters, \( \Delta V \) is the one loop correction to the effective potential\[21, 22\] and includes contributions from all the fields that enter MSSM consisting of the standard model fields and their superpartners, i.e., the sfermions, the gauginos and higgsinos\[22\]. The sum over \( i \) in Eq.(1) runs over particles with spin \( J_i \) and \( c_i (2J_i + 1) \) counts the degrees of the \( i \)th particle, and \( Q \) is the renormalization group running scale. It is well known that the one loop corrections to the effective potential can make significant contributions to the Higgs vacuum expectation values in the minimization of the effective potential\[22\].

In general the effective potential depends on the CP violating phases and its minimization will lead to induced CP violating effects on the Higgs vacuum expectation values\[13\]. It is found convenient to parameterize the Higgs VEVs in the presence of CP violating effects in the following form

\[
(H_1) = \left( \frac{H_1^0}{H_1^-} \right) = \frac{1}{\sqrt{2}} \left( \frac{v_1 + \phi_1 + i\psi_1}{H_1^-} \right), \quad (H_2) = \left( \frac{H_2^+}{H_2^-} \right) = \frac{e^{i\theta_H}}{\sqrt{2}} \left( \frac{v_2 + \phi_2 + i\psi_2}{H_2^-} \right)
\] (2)

where \( \theta_H \) is in general non-vanishing as a consequence of the minimization conditions. Thus the minimization of the potential with respect to the fields \( \phi_1, \psi_1, \phi_2, \psi_2 \) gives

\[
\frac{1}{v_2}(\partial\Delta V/\partial\psi_1)_0 = m_3^2 \sin \theta_H
\]

\[-\frac{1}{v_1}(\partial\Delta V/\partial\phi_1)_0 = m_1^2 + \frac{g_2^2 + g_1^2}{8}(v_1^2 - v_2^2) + m_3^2 \tan \beta \cos \theta_H
\] (3)

and

\[
\frac{1}{v_1}(\partial\Delta V/\partial\psi_2)_0 = m_3^2 \sin \theta_H
\]

\[-\frac{1}{v_2}(\partial\Delta V/\partial\phi_2)_0 = m_2^2 - \frac{g_2^2 + g_1^2}{8}(v_1^2 - v_2^2) + m_3^2 \cot \beta \cos \theta_H
\] (4)

In the above the subscript 0 stands for the fact that we are evaluating the relevant quantities at the point \( \phi_1 = \phi_2 = \psi_1 = \psi_2 = 0 \). We note in passing that in Eqs.(3) and (4) only one of the two equations that involve the variation with respect to \( \psi_1 \) and \( \psi_2 \) is independent\[15\].
3 Calculus for Derivatives of Eigen Values of Neutralino Mass Matrix

As mentioned in Sec.1, in previous analyzes computations of the CP dependent loop corrections from the stop sbottom and from the chargino- W- charged Higgs sectors have been carried out. In these analyzes one was able to analytically obtain the eigen values by diagonalizing the $2 \times 2$ squark matrices and the $2 \times 2$ charginio mass matrix and then differentiate them analytically to obtain the loop correction to the Higgs mass matrix. As also pointed out in Sec.1 for the neutralino exchange case the situation is more difficult since the neutralino mass matrix is a $4 \times 4$ matrix and the analytic solutions for the eigen values of the neutralino (mass) matrix are not easily obtained. Here we expand on a technique introduced in Ref.[22] to derive a calculus for the derivatives of the eigen values for the neutralino mass matrix. This technique is valid for an arbitrary high order eigen value equation. We shall show that quite remarkably even though one cannot analytically solve for the eigen values one can analytically solve for the derivatives of the eigen values with respect to the background fields in terms of the eigen values and the parameters that appear in the eigenvalue equation. To illustrate the procedure we consider an nth order eigen value equation

$$F(\lambda) = Det(M^\dagger M - \lambda I) = \lambda^n + c^{(n-1)} \lambda^{n-1} + c^{(n-2)} \lambda^{n-2} + .. + c^{(1)} \lambda + c^{(0)} = 0 \quad (5)$$

Here the co-efficients are explicit functions of the background fields

$$\Phi_\alpha = \{\phi_1, \phi_2, \psi_1, \psi_2\} \quad (6)$$

while the eigen values are implicit functions of the background fields through the satisfaction of the eigen value equation. Eq.(5) has n eigen values which we denote by $\lambda_i \ (i = 1, 2, .., n)$. From Eq.(5) it follows that

$$\frac{\partial \lambda_i}{\partial \Phi_\alpha} = -\left(\frac{D_\alpha F}{D_\lambda F}\right)_{\lambda = \lambda_i} \quad (7)$$

and

$$\frac{\partial^2 \lambda_i}{\partial \Phi_\alpha \partial \Phi_\beta} = \left[-\frac{D_\alpha F D_\beta F D^2_\lambda F}{(D_\lambda F)^3} + \frac{D_\alpha F D_\beta D_\lambda F + D_\beta F D_\alpha D_\lambda F}{(D_\lambda F)^2} - \frac{D_\alpha D_\beta F}{D_\lambda F}\right]_{\lambda = \lambda_i} \quad (8)$$

where $D_\lambda$ differentiates the $\lambda$ dependence in $F$

$$D_\lambda F(\lambda) = \frac{dF}{d\lambda} \quad (9)$$
and $D_{\alpha}$ differentiates only the co-efficients in Eq.(5), i.e.,

$$D_{\alpha}F = c_{\alpha}^{(n-1)}\lambda^{(n-1)} + c_{\alpha}^{(n-2)}\lambda^{(n-2)} + \ldots + c_{\alpha}^{(1)}\lambda + c_{\alpha}^{(0)}$$

(10)

$D_{\alpha}D_{\beta}F$ are similarly defined where $c_{\alpha}^{(k)}$ etc are replaced with $c_{\alpha\beta}^{(k)}$ where

$$c_{\alpha}^{(k)} = \frac{\partial c_{\alpha}^{(k)}}{\partial \Phi_{\alpha}}, \quad c_{\alpha\beta}^{(k)} = \frac{\partial^2 c_{\alpha}^{(k)}}{\partial \Phi_{\alpha} \partial \Phi_{\beta}}$$

(11)

and the derivatives $D_{\alpha}D_{\lambda}$ are defined in an obvious way. We note in passing that $D_{\alpha}$ and $D_{\lambda}$ commute

$$[D_{\alpha}, D_{\lambda}] = 0$$

(12)

Eqs.(7) and (8) are the central equations of our analysis. It is easy to check that for the $2 \times 2$ matrix case, e.g., for the stop and the chargino exchanges, they give exactly the results gotten by explicit differentiation of the eigen values. However, now these equations provide us with a technique of analyzing cases where the analytic solutions to the eigen values are not available.

4 Neutralino, Z and neutral Higgs loop contributions

As mentioned above the CP dependent contributions to the Higgs boson masses from stop and sbottom exchanges have been discussed at length in the literature[13, 14, 15, 16, 17, 18]. More recently the CP dependent chargino-W-charged Higgs contributions were also discussed[19]. In this work we use the technique discussed in Sec.3 to compute the contribution from the neutralino -Z - neutral Higgs exchange. The loop correction in this sub sector is given by

$$\Delta V(\chi_i^0, Z, h^0, H^0) = \frac{1}{64\pi^2} \left( \sum_{i=1}^{4} (-2)M_{\chi_i^0}^4 (\log \frac{M_{\chi_i^0}^2}{Q^2} - \frac{3}{2}) + 3M_Z^4 (\log \frac{M_Z^2}{Q^2} - \frac{3}{2}) + M_{h^0}^4 (\log \frac{M_{h^0}^2}{Q^2} - \frac{3}{2}) + M_{H^0}^4 (\log \frac{M_{H^0}^2}{Q^2} - \frac{3}{2}) \right)$$

(13)

The neutralino mass matrix is given by

$$M_{\chi^0} = \begin{pmatrix}
\tilde{m}_1 & 0 & -\frac{g_1}{\sqrt{2}}H_1^0 & \frac{g_1}{\sqrt{2}}H_2^0 \\
0 & \tilde{m}_2 & \frac{g_2}{\sqrt{2}}H_1^0 & -\frac{g_2}{\sqrt{2}}H_2^0 \\
-\frac{g_1}{\sqrt{2}}H_1^0 & \frac{g_2}{\sqrt{2}}H_1^0 & 0 & -\mu \\
\frac{g_1}{\sqrt{2}}H_2^0 & \frac{g_2}{\sqrt{2}}H_2^0 & -\mu & 0
\end{pmatrix}$$

(14)
where \( \mu = |\mu|e^{i\theta_\mu}, \tilde{m}_1 = |\tilde{m}_1|e^{i\xi_1} \) and \( \tilde{m}_2 = |\tilde{m}_2|e^{i\xi_2} \). We note that in the supersymmetric limit \( M_{\chi^0_0} = (0, 0, M_Z, M_Z) \) and \( (M_{h^0}, M_{H^0}) = (M_Z, 0) \) and consequently in this limit the loop corrections from this sub sector vanish. We return now to the full analysis and follow the method described in Ref. [16] to minimize the potential and compute the loop corrections. First we give the determination of \( \theta_H \) from the minimization constraints including the stop, the sbottom, the stau, the chargino and neutralino contributions. One finds that \( \theta_H \) is given by the equation

\[
m_3^2 \sin \theta_H = \frac{1}{2} \beta_{h_t} |\mu| |A_t| \sin \gamma_t f_1(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + \frac{1}{2} \beta_{h_b} |\mu| |A_b| \sin \gamma_b f_1(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) + \frac{1}{2} \beta_{h_\tau} |\mu| |A_\tau| \sin \gamma_\tau f_1(m_{\tilde{\tau}_1}^2, m_{\tilde{\tau}_2}^2) \]

\[
+ \frac{1}{2} \beta_{h_\mu} |\mu| |A_\mu| \sin \gamma_\mu f_1(m_{\tilde{\mu}_1}^2, m_{\tilde{\mu}_2}^2) - \frac{g_2^2}{16\pi^2} |\mu| |\tilde{m}_1| \sin \gamma_2 - \frac{g_2^2}{16\pi^2} |\mu| |\tilde{m}_1| \sin \gamma_1 \]

\[
+ M_{\chi_0^0}^2 (g_2^2 |\tilde{m}_1|^2 + |\mu|^2) |\tilde{m}_2| |\mu| \sin \gamma_2 + \frac{g_1^2}{16\pi^2} |\mu| |\tilde{m}_1| |\mu| \sin \gamma_1 \]

\[
+ (-g_2^2 |\tilde{m}_1|^2 |\mu|^2 |\tilde{m}_2| |\mu| \sin \gamma_2 - \frac{g_1^2}{16\pi^2} |\mu|^2 |\tilde{m}_1|^2 |\mu| \sin \gamma_1) \]  \tag{15}
\]

where

\[
D_j \equiv (D_j F)_{\lambda=\Lambda_j} = 4M_{\chi^0_0}^6 + 3aM_{\chi^0_0}^4 + 2bM_{\chi^0_0}^2 + c \]

\[
\beta_{h_t} = \frac{3h_t^2}{16\pi^2}, \quad \beta_{h_b} = \frac{3h_b^2}{16\pi^2}, \quad \beta_{h_\tau} = \frac{3h_\tau^2}{16\pi^2} \]

\[
\gamma_t = \alpha_{A_t} + \theta_\mu, \quad \gamma_b = \alpha_{A_b} + \theta_\mu, \quad \gamma_\tau = \alpha_{A_\tau} + \theta_\mu, \quad \gamma_1 = \xi_1 + \theta_\mu, \quad \gamma_2 = \xi_2 + \theta_\mu \]  \tag{16}
\]

and where \(a,b,c\) are defined in Appendix A and \(f_1(u, v)\) is given by

\[
f_1(u, v) = -2 + \log\frac{uv}{Q^4} + \frac{v + u}{v - u} \log\frac{v}{u} \]  \tag{17}
\]

To construct the mass squared matrix of the Higgs scalars we need to compute the quantity

\[
M_{\alpha\beta}^2 = (\frac{\partial^2 V}{\partial \Phi_\alpha \partial \Phi_\beta})_0 = M^{2(0)}_{\alpha\beta} + \Delta M^2_{\alpha\beta} \]  \tag{18}
\]

where \(M^{2(0)}_{\alpha\beta}\) is the contribution from \(V_0\) and \(\Delta M^2_{\alpha\beta}\) is the contribution from \(\Delta V\) where \(\Phi_\alpha (\alpha = 1 - 4)\) are defined by Eq.(6) and as already mentioned earlier the subscript 0 means that we set \(\phi_1 = \phi_2 = \psi_1 = \psi_2 = 0\) after evaluating the mass matrix. The loop contribution \(\Delta M^2_{\alpha\beta}\) arising from the neutralino- Z-neutral Higgs sector is given by
\[ \Delta M^2_{\alpha\beta} = \frac{1}{32\pi^2} \text{Str} \left( \frac{\partial^2 M^2}{\partial \Phi_\alpha \partial \Phi_\beta} \log \frac{M^2}{Q^2} + \frac{M^2}{Q^2} \right) \]

Computation of the 4 × 4 Higgs mass matrix in the basis of Eq.(6) gives

\[
\begin{pmatrix}
M_Z^2 c_\beta^2 + M_A^2 s_\beta^2 + \Delta_{11} & -(M_Z^2 + M_A^2) s_\beta c_\beta + \Delta_{12} & \Delta_{13} s_\beta & \Delta_{13} c_\beta \\
-(M_Z^2 + M_A^2) s_\beta c_\beta + \Delta_{12} & M_Z^2 s_\beta^2 + M_A^2 c_\beta^2 + \Delta_{22} & \Delta_{23} s_\beta & \Delta_{23} c_\beta \\
\Delta_{13} s_\beta & \Delta_{23} s_\beta & (M_Z^2 + M_A^2) s_\beta^2 & (M_Z^2 + M_A^2) s_\beta c_\beta \\
\Delta_{13} c_\beta & \Delta_{23} c_\beta & (M_Z^2 + M_A^2) s_\beta c_\beta & (M_Z^2 + M_A^2) c_\beta^2
\end{pmatrix}
\]

where \( c_\beta(s_\beta) = \cos(\beta \sin(\beta)) \) and \( m_A^2 \) is given by

\[
m_A^2 = (\sin(\beta \cos(\beta))^{-1}(-m_3^2 \cos\theta + \frac{1}{2} \beta_{a_1} |A_t||\mu| \cos\gamma t f_1(m_{t_1}^2, m_{t_2}^2) + \frac{1}{2} \beta_{a_2} |A_b||\mu| \cos\gamma f_1(m_{b_1}^2, m_{b_2}^2) + \frac{g_2^2}{16\pi^2} |\tilde{m}_2||\mu| \cos\gamma f_1(m_{\tilde{c}_1}^2, m_{\tilde{c}_2}^2) - \frac{1}{16\pi^2} \sum_{j=1}^{4} \frac{M_{\chi_j}^2}{D_j} (\log(\frac{M_{\chi_j}^2}{Q^2}) - 1) |M_{\chi_j}^2| (-g_2^2 |\mu||\tilde{m}_2| \cos\gamma_2 - g_1^2 |\mu||\tilde{m}_1| \cos\gamma_1)
\]

\[
+ M_{\chi_j}^2 (g_2^2 |\tilde{m}_1|^2 + |\mu|^2 |\tilde{m}_2| \cos\gamma_2 + g_1^2 (|\tilde{m}_2|^2 + |\mu|^2) |\mu||\tilde{m}_1| \cos\gamma_1)
\]

The first term in the second brace on the right hand side of Eq.(21) is the tree term, while the second, the third, the fourth and the fifth terms come from the stop, sbottom, stau and chargino exchange contributions. The remaining contributions in Eq.(21) arise from the neutralino sector. The \( \Delta \)'s appearing in Eq.(20) can be decomposed as follows

\[
\Delta_{\alpha\beta} = \Delta_{\alpha\beta} + \Delta_{\alpha\beta} + \Delta_{\alpha\beta} + \Delta_{\alpha\beta} + \Delta_{\alpha\beta}
\]

where \( \Delta_{\alpha\beta} \) is the contribution from the stop (and top) exchange in the loops, \( \Delta_{\alpha\beta} \) is the contribution from the sbottom (and bottom) exchange in the loops, \( \Delta_{\alpha\beta} \) is the contribution from the stau (and tau) exchange, \( \Delta_{\alpha\beta} \) is the contribution from the chargino (and W and charged Higgs) exchange in the loops, and \( \Delta_{\alpha\beta} \) is the contribution arising from the neutralino (and Z and neutral Higgs exchange) in the loops. The computations of \( \Delta_{\alpha\beta} \), \( \Delta_{\alpha\beta} \), \( \Delta_{\alpha\beta} \), and \( \Delta_{\alpha\beta} \) have been given before and would not be reproduced here. We compute here only the \( \Delta_{\alpha\beta} \) arising from the \( (\chi_i^0 - Z - h^0 - H^0) \) exchange. The \( \Delta_{\alpha\beta} \) are listed below.
\[ \Delta_{11\chi^0} = -\frac{1}{16\pi^2} \sum_{j=1}^{4} M_{\chi_j}^2 (\ln\left(\frac{M_{\chi_j}^2}{Q^2}\right) - 1) \]

\[
\left\{ -\left( a_1 M_{\chi_j}^6 + b_1 M_{\chi_j}^4 + c_1 M_{\chi_j}^2 + d_1 \right)^2 \frac{D_j^3}{(12 M_{\chi_j}^4 + 6a M_{\chi_j}^2 + 2b)} \\
+ \frac{2(a_1 M_{\chi_j}^6 + b_1 M_{\chi_j}^4 + c_1 M_{\chi_j}^2 + d_1)(3a_1 M_{\chi_j} + 2b_1 M_{\chi_j} + c_1)}{D_j^3} \right\} \\
 \left( -\frac{1}{16\pi^2} \sum_{j=1}^{4} \frac{(a_1 M_{\chi_j}^6 + b_1 M_{\chi_j}^4 + c_1 M_{\chi_j}^2 + d_1)^2}{D_j^2} \ln\left(\frac{M_{\chi_j}^2}{Q^2}\right) + \frac{3}{128\pi^2} (g_1^2 + g_2^2) v_1^2 \ln\left(\frac{M_{\chi_j}^2}{Q^2}\right) \\
- \frac{1}{32\pi^2} \left( \frac{1}{16} \frac{A_0^2}{(M_{H^0}^2 - M_{H^0}^2)^2} f_2(M_{H^0}^2, M_{H^0}^2) - \frac{1}{16} (g_1^2 + g_2^2) v_1 \ln\left(\frac{M_{H^0}^2}{M_{H^0}^2}\right) \right) \right)(23) \\
\]

where \( D_j \) is defined in Eq.(16) and \( f_2 \) is defined by

\[ f_2(u, v) = -2 + \frac{v + u}{v - u} \ln\frac{v}{u} \]  

\[ \Delta_{22\chi^0} = -\frac{1}{16\pi^2} \sum_{j=1}^{4} M_{\chi_j}^2 (\ln\left(\frac{M_{\chi_j}^2}{Q^2}\right) - 1) \]

\[
\left\{ -\left( a_2 M_{\chi_j}^6 + b_2 M_{\chi_j}^4 + c_2 M_{\chi_j}^2 + d_2 \right)^2 \frac{D_j^3}{(12 M_{\chi_j}^4 + 6a M_{\chi_j}^2 + 2b)} \\
+ \frac{2(a_2 M_{\chi_j}^6 + b_2 M_{\chi_j}^4 + c_2 M_{\chi_j}^2 + d_2)(3a_2 M_{\chi_j} + 2b_2 M_{\chi_j} + c_2)}{D_j^3} \right\} \\
 \left( -\frac{1}{16\pi^2} \sum_{j=1}^{4} \frac{(a_2 M_{\chi_j}^6 + b_2 M_{\chi_j}^4 + c_2 M_{\chi_j}^2 + d_2)^2}{D_j^2} \ln\left(\frac{M_{\chi_j}^2}{Q^2}\right) + \frac{3}{128\pi^2} (g_1^2 + g_2^2) v_2^2 \ln\left(\frac{M_{\chi_j}^2}{Q^2}\right) \\
- \frac{1}{32\pi^2} \left( \frac{1}{16} \frac{B_0^2}{(M_{H^0}^2 - M_{H^0}^2)^2} f_2(M_{H^0}^2, M_{H^0}^2) - \frac{1}{16} (g_1^2 + g_2^2) v_2 \ln\left(\frac{M_{H^0}^2}{M_{H^0}^2}\right) \right) \right)(25) \\
\]

\[ \Delta_{12\chi^0} = -\frac{1}{16\pi^2} \sum_{j=1}^{4} M_{\chi_j}^2 (\ln\left(\frac{M_{\chi_j}^2}{Q^2}\right) - 1) \]

\[
\left\{ -\left( a_1 M_{\chi_j}^6 + b_1 M_{\chi_j}^4 + c_1 M_{\chi_j}^2 + d_1 \right)(a_2 M_{\chi_j}^6 + b_2 M_{\chi_j}^4 + c_2 M_{\chi_j}^2 + d_2)(12 M_{\chi_j}^4 + 6a M_{\chi_j}^2 + 2b) \right\} \\
 \left( -\frac{1}{16\pi^2} \sum_{j=1}^{4} \frac{M_{\chi_j}^2}{D_j^3} \ln\left(\frac{M_{\chi_j}^2}{Q^2}\right) - 1 \right) \]
\[
\Delta_{13\chi^0} = -\frac{1}{16\pi^2} \sum_{j=1}^{4} \frac{M_{\chi_j}^2}{D_j^2} (\ln\left(\frac{M_{\chi_j}^2}{Q^2}\right) - 1) \frac{1}{\sin \beta}
\]

\[
\Delta_{23\chi^0} = -\frac{1}{16\pi^2} \sum_{j=1}^{4} \frac{M_{\chi_j}^2}{D_j^2} (\ln\left(\frac{M_{\chi_j}^2}{Q^2}\right) - 1) \frac{1}{\cos \beta}
\]

\[
\Delta_{33\chi^0} = -\frac{1}{16\pi^2} \sum_{j=1}^{4} \frac{M_{\chi_j}^2}{D_j^2} (\ln\left(\frac{M_{\chi_j}^2}{Q^2}\right) - 1)
\]
The parameters $a, b, c$ and the derivatives $a_i, b_i, c_i, d_i$ (i=1,2, etc.) that appear in Eqs.(23-29) are defined in Appendices A and B. Eqs.(23-29) constitute the main new theoretical results of this paper. These results along with the computations of $\Delta \alpha \beta, \Delta \alpha \beta \tau, \Delta \alpha \beta \chi^+$ give a complete determination of the CP dependent one loop contributions to the Higgs boson masses and mixings. As has been noted before it is preferable to work with a $3 \times 3$ matrix rather than the $4 \times 4$ matrix of Eq.(20). The desired $3 \times 3$ matrix can be gotten from Eq.(20) by going to the basis

$$\psi_{1D} = \sin \beta \psi_1 + \cos \beta \psi_2, \quad \psi_{2D} = -\cos \beta \psi_1 + \sin \beta \psi_2$$

(30)

In this basis the field $\psi_{2D}$ is the zero mass Goldstone boson and decouples while the remaining (mass)$^2$ matrix in the basis $\phi_1, \phi_2, \psi_{1D}$ is given by

$$M^2_{Higgs} = \begin{pmatrix}
M^2_Z c_\beta^2 + M^2_A s_\beta^2 + \Delta_{11} & -(M^2_Z + M^2_A) s_\beta c_\beta + \Delta_{12} & \Delta_{13} \\
-(M^2_Z + M^2_A) s_\beta c_\beta + \Delta_{12} & M^2_Z s_\beta^2 + M^2_A c_\beta^2 + \Delta_{22} & \Delta_{23} \\
\Delta_{13} & \Delta_{23} & (M^2_A + \Delta_{33})
\end{pmatrix}$$

(31)

We label the eigen values for this case $m^2_{H_1}, m^2_{H_2}, m^2_{H_3}$ corresponding to the eigen states $H_1, H_2, H_3$. These eigen states are in general admixtures of the CP even and the CP odd states due to the mixing generated by $\Delta_{13}$ and $\Delta_{23}$. Thus the CP even-odd mixings arise from $\Delta_{13}$ and $\Delta_{23}$ and these are nonvanishing only in the presence of CP violation and vanish when the phases go to zero and one recovers the usual result of two distinct (one CP even and the other CP odd) Higgs sectors. We note in passing that $\Delta_{33}$ also vanishes in the limit when the CP phases go to zero. This was also the behavior that was observed when the contributions from the stop, sbottom, stau and chargino exchanges were considered. Since the main point of this work is to study the phenomenon of CP even-odd mixing the main focus of our analysis is the computation of $\Delta t \gamma$ and specifically of $\Delta_{13}$ and $\Delta_{23}$ which are the basic sources of mixings between the CP even and the CP odd sectors. We order the eigen values of Eq.(31) in such a way that in the limit of no CP violation one has $(m_{H_1}, m_{H_2}, m_{H_3}) \rightarrow (m_{H}, m_{h}, m_{A})$ and $(H_1, H_2, H_3) \rightarrow$
\( (H, h, A) \) where (h, H) are (light, heavy) CP even Higgs and A is the CP odd Higgs in the absence of CP violation.

5 Discussion of the Neutralino Exchange Contribution to CP even CP odd Higgs Mixing

The analytical results given above are quite general as they apply to the MSSM parameter space. However, the MSSM parameter space is quite large. Thus for a numerical study of the CP effects including those from the neutralino sector we will work with a constrained set of parameters consisting of the parameter space \( m_0, m_\frac{1}{2}, m_A, |A_0|, \tan \beta, \theta_\mu, \alpha_{A_0}, \xi_1, \xi_2 \) and \( \xi_3 \). Starting with these all other low energy parameters are obtained by a renormalization group evolution by running the parameters from the GUT scale down to the electro-weak scale. Of course one is free to utilize the formulae derived above for the more general MSSM parameter space. As discussed in Sec.1 one can satisfy the EDM constraints in the presence of large phases. This can come about in a variety of ways. As pointed out in Sec.1 one possibility is that the internal cancellations can occur which allow for large phases consistent with the EDM constraints. The other possibility is that that CP phases appear only in the third generation which suppresses their contributions to the EDMs of the quarks and the leptons in the first two generations to achieve consistency with the experimental constraints. There also exist scenarios which are linear combinations of these two. For the purpose of this analysis we do not revisit the problem of the satisfaction of the EDM constraints. Rather we shall assume that regions of the parameter space exist where such constraints are satisfied and examine the effect of the phases on the Higgs masses and mixing. Specifically we are interested in the effects of the neutralino exchange contributions on \( \Delta_{13} \) and \( \Delta_{23} \), and thus, on the mixings of the CP even and the CP sectors.

It was pointed out in Sec.4 that the neutralino, the Z and the neutral Higgs exchanges together form a sub sector so that in the supersymmetric limit one finds that the one loop correction to the effective potential from this sub sector vanishes. This phenomenon is similar to what was also seen in the exchange of the chargino, the W and the charged Higgs where the contribution from that sector to the effective potential vanishes in the supersymmetric limit. It was also seen in the analysis of the chargino-W-charged Higgs exchange that the CP even-odd mixing arising from this sector was roughly Q independent because of the sum of the three
separate contributions within this sector. A very similar situation is also realized in the neutralino sector. Here again because of the contributions from the neutralino, the Z and the neutral Higgs exchanges their sum contribution to the CP even-odd mixing is roughly scale independent. However, unlike the chargino-W- charged Higgs exchange where one could demonstrate the above phenomenon analytically, here one has to demonstrate it numerically due to the more analytically complex nature of the results. This is exhibited in Fig.1 where a plot the percentage of the CP even component $\phi_1$ and the CP odd component $\psi_{1D}$ of $H_1$ as a function of Q is given. The analysis shows an approximate independence in Q of the CP even-odd mixing. We turn now to a discussion of other aspects of the analysis below.

In Fig.2 we plot the quantity $\Delta_{13}$ as a function of the CP phase of the U(1) gaugino mass $\xi_1$. The plots exhibited in Fig.2 contain the stop, the sbottom, the stau the chargino and the neutralino exchange contributions. Among the above exchanges the neutralino exchange contribution is the only one that depends on $\xi_1$, and thus the variation of $\Delta_{13}$ with $\xi_1$ arises only from this exchange. From Fig.2 the size of the neutralino exchange contribution can be seen to be fairly substantial. Specifically, the analysis of Fig.2 shows that the neutralino exchange contribution is comparable to the effects from the stop and chargino exchanges. A plot of $\Delta_{23}$ vs $\xi_1$ is given Fig.3. As in Fig.2 one finds that $\Delta_{23}$ is quite sensitive to the CP violating phase $\xi_1$. As in Fig.2 here again the neutralino exchange contribution is comparable to the stop and the chargino exchange contribution. An analysis of the percentage of the CP even component $\phi_1$ of $H_1$ (upper curves) and of the percentage of the CP odd component $\psi_{1D}$ of $H_1$ (lower curves) arising from the exchange of the stop, the sbottom, the stau, the chargino and the neutralino sector contributions as a function of $\xi_1$ is given in Figs.4. As expected from the analysis of Fig.2 and Fig.3 one finds that there is a significant mixing between the CP even and the CP odd components of $H_1$. Further, as also expected from the analysis of Figs 2 and 3, the CP even and CP odd components of $H_1$ show a reasonably strong dependence on $\xi_1$.

An analysis of the CP even and CP odd mixing in $H_1$ as a function of the SU(2) gaugino phase is given in Fig.5. Unlike Figs.2-4, where the entire $\xi_1$ dependence arose from the neutralino exchange contribution here the $\xi_2$ dependence of the CP even and CP odd components of $H_1$ arises from two sources, i.e., from the chargino and the neutralino exchange contributions. Because of this the dependence of the CP even and CP odd components on $\xi_2$ is much stronger than on $\xi_1$ as may be
seen by comparing the plots of Figs. 2-4 with the plots of Fig.5. In Fig.6 a plot of the percentage of the CP even component \( \phi_1 \) of \( H_1 \) (upper sets) and the CP odd component \( \psi_{1D} \) of \( H_1 \) (lower sets) arising from the exchange of the stop, the sbottom, the stau, the chargino and the neutralino sector contributions is given as a function of \( \theta_\mu \). In this case we find that the dependence of the CP even and the CP odd components on \( \theta_\mu \) is also very strong. Indeed in this case the mixings between the CP even and the CP odd states can be maximal depending on the value of \( \theta_\mu \). The strong dependence on \( \theta_\mu \) can be understood as due to the fact that all contributions, i.e., the stop, the sbottom, the stau, the chargino, and the neutralino contributions, depend on \( \theta_\mu \). This in contrast to the dependence on \( \xi_1 \) which arises only from the neutralino exchange.

Finally, in Fig.7 we give an analysis of the percentage of the CP even component \( \phi_1 \) of \( H_1 \) (upper sets) and the CP odd component \( \psi_{1D} \) of \( H_1 \) (lower sets) arising from the exchange of the stop, the sbottom, the stau, the chargino and the neutralino sector contributions as a function of \( \tan \beta \). We find that the CP even and the CP odd mixings show a strong dependence on \( \tan \beta \). A similar strong dependence on \( \tan \beta \) was seen also in previous analyses[16]. We note that the inclusion of the neutralino contribution further sharpens the \( \tan \beta \) dependence and one finds that the CP even (odd) component can vary from 100% (0%) to less than 60% (more than 40%) as \( \tan \beta \) is varied. This sharper behavior of the amplitudes with \( \tan \beta \) arises from the additional contributions from the neutralino, the neutral Higgs and the Z boson exchanges. An analysis similar to the above can be carried out for the case of the \( H_2 \) and \( H_3 \) fields. In the analysis of chargino exchange contributions it was found that the CP odd component of \( H_2 \) is rather small while the analysis of \( H_3 \) parallels the analysis of \( H_1 \) with the only difference that the roles of the CP even and the CP odd components is reversed. Much the same situations occurs in this case and thus we omit the detailed discussion of these states.

6 Conclusions

In this paper we have developed a calculus for the derivatives of the eigen values of the neutralino mass matrix with respect to the background fields which are in general dependent on CP violating phases. The calculus allows one to deduce the derivatives of the eigen values of the neutralino mass matrix analytically even though the eigen values themselves cannot be gotten analytically in a
compact form. We use this calculus to obtain analytical results for the neutralino-Z-neutral Higgs exchange contribution to the masses and mixings in the CP even-CP odd neutral Higgs sector. The above computation along with the stop-top, the sbottom-bottom, the tau-stau and the chargino-W- charged Higgs exchange contribution computed previously provide us with a complete one loop contribution to the Higgs mass matrix with the inclusion of CP phases. This full one loop result was then used to discuss the phenomenon of CP violation in the neutral Higgs sector. The numerical analysis shows that the mixings between the CP even and the CP odd sectors are significantly affected by the neutralino exchange contribution. The mixing of the CP even and the CP odd Higgs sector have many important consequences\cite{13,14,15}. Thus one consequence is that CP even-odd mixing affects the couplings of the Higgs bosons with quarks and leptons and this effect can be discerned in Higgs searches in collider experiments. Another important implication is that the CP even-odd mixing will affect the relic density analysis and thus modify the parameter space allowed by the relic density constraints. Further, since the couplings of the quark and leptons with the Higgs are affected due to the CP even-odd mixing there will also be an effect of these mixings on detection rates in the direct searches for dark matter. It would be interesting to carry out an analysis of these phenomena.

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7 Appendix A: Neutralino eigen values and derivatives

The characteristic equation for the square of the neutralino mass is

\[ F(\lambda) = \text{Det}(M_{\chi^0}^\dagger M_{\chi^0} - \lambda I) = 0 \]

where \( \lambda \) represents the square of the neutralino mass eigen values. It can be expanded as

\[ F(\lambda) = \lambda^4 + a\lambda^3 + b\lambda^2 + c\lambda + d = 0 \]  \hspace{1cm} (32)
In the above a, b, c and d are computed using Eq.(14). The computation of the co-efficients is done to leading and to next to the leading order in an expansion in $M_Z^2/M_S^2$ where $M_S$ stands for the soft SUSY parameters. Thus, e.g., $a$ is expanded to $O(M_S^2)$ and $O(M_Z^2)$ orders (it is actually exact when expanded to this order), $b$ is expanded to $O(M_S^4)$ and $O(M_S^2M_Z^2)$ orders etc. The analysis for $a$, $b$ and $c$ ($d$ does not enter in Eqs.(23-29) and is not exhibited) gives

$$a = -[|\tilde{m}_1|^2 + |\tilde{m}_2|^2 + 2|\mu|^2 + 2M_Z^2]$$

$$b = |\tilde{m}_1|^2|\tilde{m}_2|^2 + |\mu|^4 + 2|\mu|^2(|\tilde{m}_1|^2 + |\tilde{m}_2|^2) + M_Z^2(|m_1|^2 + |m_2|^2 + 2|\mu|^2 + (|\tilde{m}_1|^2 - |\tilde{m}_2|^2)\cos 2\theta_W - 4\cos \beta \sin \beta C_W^2 |\tilde{m}_2||\mu| \cos \gamma_2 - 4\cos \beta \sin \beta S_W^2 |\tilde{m}_1||\mu| \cos \gamma_1)$$

where $C_W^2 = \frac{g_2^2}{g_1^2 + g_2^2}$ and $S_W^2 = \frac{g_1^2}{g_1^2 + g_2^2}$

$$c = -2|\mu|^2|\tilde{m}_1|^2|\tilde{m}_2|^2 - |\mu|^4(|\tilde{m}_1|^2 + |\tilde{m}_2|^2) + 4M_Z^2 \sin \beta \cos \beta |\mu|(|\tilde{m}_1|^2 + |\mu|^2)S_W^2 |\tilde{m}_1| \cos \gamma_1 + (|\tilde{m}_1|^2 + |\mu|^2)C_W^2 |\tilde{m}_2| \cos \gamma_2$$

The derivatives $\partial \lambda_i / \partial \Phi_\alpha$ can be gotten explicitly as follows:

$$\frac{\partial \lambda_i}{\partial \Phi_\alpha} = -\frac{a_\alpha \lambda^3 + b_\alpha \lambda^2 + c_\alpha \lambda + d_\alpha}{4\lambda^3 + 3a\lambda^2 + 2b\lambda + c}|_{\lambda = \lambda_i}$$

The second derivatives are given by

$$\frac{\partial^2 \lambda_i}{\partial \Phi_\alpha \partial \Phi_\beta} = \left[ -\frac{(a_\alpha \lambda^3 + b_\alpha \lambda^2 + c_\alpha \lambda + d_\alpha)(a_\beta \lambda^3 + b_\beta \lambda^2 + c_\beta \lambda + d_\beta)(12\lambda^2 + 6a\lambda + 2b)}{(4\lambda^3 + 3a\lambda^2 + 2b\lambda + c)^3} + \frac{(a_\alpha \lambda^3 + b_\alpha \lambda^2 + c_\alpha \lambda + d_\alpha)^2}{(4\lambda^3 + 3a\lambda^2 + 2b\lambda + c)^2} \right] \frac{(a_\beta \lambda^3 + b_\beta \lambda^2 + c_\beta \lambda + d_\beta)}{(3a_\beta \lambda^2 + 2b_\beta \lambda + c_\beta)} + \frac{(a_\beta \lambda^3 + b_\beta \lambda^2 + c_\beta \lambda + d_\beta)}{(4\lambda^3 + 3a\lambda^2 + 2b\lambda + c)^2} \left[ \frac{(a_\alpha \lambda^3 + b_\alpha \lambda^2 + c_\alpha \lambda + d_\alpha)^2}{(4\lambda^3 + 3a\lambda^2 + 2b\lambda + c)^2} \right] \times \left[ (3a_\alpha \lambda^2 + 2b_\alpha \lambda + c_\alpha) - \frac{(a_\alpha \lambda^3 + b_\alpha \lambda^2 + c_\alpha \lambda + d_\alpha)}{(4\lambda^3 + 3a\lambda^2 + 2b\lambda + c)} \right] |_{\lambda = \lambda_i}$$

where

$$a_\alpha = \frac{\partial a}{\partial \Phi_\alpha}, \quad a_{\alpha\beta} = \frac{\partial^2 a}{\partial \Phi_\alpha \partial \Phi_\beta}$$

(38)
8 Appendix B: List of parameters

The explicit evaluation of the co-efficients $a_1, b_1, c_1, d_1$ is given below

\begin{align*}
    a_1 &= - (g_1^2 + g_2^2) v_1 \\
    b_1 &= -g_2^2 |\mu| |\tilde{m}_2| v_2 \cos \gamma_2 - g_1^2 |\mu| |\tilde{m}_1| v_2 \cos \gamma_1 \\
        &\quad + v_1 |\tilde{m}_1|^2 g_2^2 + |\tilde{m}_2|^2 g_1^2 + (g_1^2 + g_2^2) |\mu|^2 \\
    c_1 &= g_2^2 (|\tilde{m}_1|^2 + |\mu|^2) |\mu| |\tilde{m}_2| v_2 \cos \gamma_2 \\
        &\quad + g_1^2 (|\tilde{m}_2|^2 + |\mu|^2) |\mu| |\tilde{m}_1| v_2 \cos \gamma_1 \\
        &\quad - g_2^2 |\mu|^2 |\tilde{m}_1|^2 v_1 - g_1^2 |\mu|^2 |\tilde{m}_2|^2 v_1 \\
    d_1 &= -g_2^2 |\mu|^3 |\tilde{m}_1|^2 |\tilde{m}_2| v_2 \cos \gamma_2 \\
        &\quad -g_1^2 |\mu|^3 |\tilde{m}_2|^2 |\tilde{m}_1| v_2 \cos \gamma_1
\end{align*}

(39)

The co-efficients $a_2, b_2, c_2, d_2$ can be gotten from $a_1, b_1, c_1, d_1$ with the following interchanges

\begin{align*}
    a_2 &= a_1 (v_1 \leftrightarrow v_2), \\
    b_2 &= b_1 (v_1 \leftrightarrow v_2), \\
    c_2 &= c_1 (v_1 \leftrightarrow v_2), \\
    d_2 &= d_1 (v_1 \leftrightarrow v_2)
\end{align*}

(42)

The co-efficients $a_3, b_3, c_3, d_3$ are given as follows

\begin{align*}
    a_3 &= 0 \\
    b_3 &= -g_2^2 |\tilde{m}_2| |\mu| v_2 \sin \gamma_2 - g_1^2 |\tilde{m}_1| |\mu| v_2 \sin \gamma_1 \quad (43) \\
    c_3 &= g_2^2 (|\tilde{m}_1|^2 + |\mu|^2) |\tilde{m}_2| |\mu| v_2 \sin \gamma_2 + g_1^2 (|\tilde{m}_2|^2 + |\mu|^2) |\tilde{m}_1| |\mu| v_2 \sin \gamma_1 \\
        &\quad + g_2^2 |\tilde{m}_1|^2 |\mu|^3 v_2 \sin \gamma_2 - g_1^2 |\tilde{m}_2|^2 |\mu|^3 |\tilde{m}_1| v_2 \sin \gamma_1 \quad (44) \\
    d_3 &= -g_2^2 |\tilde{m}_1|^2 |\mu|^3 |\tilde{m}_2| v_2 \sin \gamma_2 - g_1^2 |\tilde{m}_2|^2 |\mu|^3 |\tilde{m}_1| v_2 \sin \gamma_1 \quad (45)
\end{align*}
The co-efficients $a'_3, b'_3, c'_3, d'_3$ can be gotten from $a_3, b_3, c_3, d_3$ with the following interchanges

$$a'_3 = a_3(v_1 \leftrightarrow v_2), \quad b'_3 = b_3(v_1 \leftrightarrow v_2), \quad c'_3 = c_3(v_1 \leftrightarrow v_2), \quad d'_3 = d_3(v_1 \leftrightarrow v_2)(46)$$

$A_0$ and $B_0$ are given by

$$A_0 = 2(g_1^2 + g_2^2)v_1(M_Z^2 - M_{A_0}^2) \cos 2\beta + (g_1^2 + g_2^2)v_2(M_Z^2 + M_{A_0}^2) \sin 2\beta \quad (47)$$

$$B_0 = -2(g_1^2 + g_2^2)v_2(M_Z^2 - M_{A_0}^2) \cos 2\beta + (g_1^2 + g_2^2)v_1(M_Z^2 + M_{A_0}^2) \sin 2\beta \quad (48)$$
Figure Captions

Fig.1: Plot of the CP even component $\phi_1$ of $H_1$ (upper curves) and the CP odd component $\psi_{1D}$ of $H_1$ (lower curves) including the stop, sbottom, stau, chargino and neutralino sector contributions as a function of the scale $Q$. The common parameters are $m_A = 300$, $\tan \beta = 15$, $m_0 = 100$, $m_{\tilde{t}} = 500$, $\xi_1 = .4$, $\xi_2 = .5$, $\alpha_0 = .3$, $|A_0| = 1$. The curves with circles are for $\theta_\mu = 0.1$ and with squares for $\theta_\mu = 0.2$ where all masses are in GeV and all angles are in radians.

Fig.2: Plot of $\Delta_{13}$ including the stop, sbottom, stau, chargino and neutralino sector contributions vs the U(1) gaugino phase $\xi_1$. The common input for all the curves are $m_0 = 100$, $m_{\tilde{t}} = 500$, $M_A = 300$, $|A_0| = 1$, $\alpha_0 = 0.3$, $\xi_2 = 0.5$ and $Q = 320$. The five curves correspond to the pairs of $\tan \beta$ and $\theta_\mu$ values as follows. The curve with $\Delta_{13} = 301$ at $\xi_1 = 0$ corresponds to $\tan \beta = 5$, $\theta_\mu = .4$. Similarly the curves with values of $\Delta_{13} = 406$ at $\xi_1 = 0$ correspond to $\tan \beta = 6$, $\theta_\mu = .6$, $\Delta_{13} = 416$ at $\xi_1 = 0$ correspond to $\tan \beta = 8$, $\theta_\mu = .8$, and $\Delta_{13} = 579$ at $\xi_1 = 0$ correspond to $\tan \beta = 15$, $\theta_\mu = .3$ where all masses are in GeV and all angles are in radians.

Fig.3: Plot of $\Delta_{23}$ including the stop, sbottom, stau, chargino and neutralino sector contributions vs the U(1) gaugino phase $\xi_1$ for the same input parameters as in Fig.2. The curves with the same symbols as in Fig.2 have the same common inputs.

Fig.4: Plot of the CP even component $\phi_1$ of $H_1$ (upper curves) and the CP odd component $\psi_{1D}$ of $H_1$ (lower curves) including the stop, sbottom, stau, chargino and neutralino sector contributions as a function of the U(1) gaugino phase $\xi_1$ for the same inputs as in Fig.2. The curves with the same symbols as in Fig.2 have the same common inputs.

Fig.5: Plot of the CP even component $\phi_1$ of $H_1$ (upper curves) and the CP odd component $\psi_{1D}$ of $H_1$ (lower curves) including the stop, sbottom, stau, chargino and neutralino sector contributions as a function of the $\xi_2$. The common parameters are: $m_A = 300$, $Q = 320$, $m_0 = 100$, $m_{\tilde{t}} = 500$, $\alpha_0 = .3$, $|A_0| = 1$, $\theta_\mu = .4$. For the curves with diamonds $\tan \beta = 15$, $\xi_1 = 1.5$, for squares $\tan \beta = 8$, $\xi_1 = 1.5$, for triangles $\tan \beta = 8$, $\xi_1 = 0.5$, and for circles $\tan \beta = 10$, $\xi_1 = 1.5$ where all masses are in GeV and all angles are in radians.
Fig.6: Plot of the CP even component $\phi_1$ of $H_1$ (upper curves) and the CP odd component $\psi_{1D}$ of $H_1$ (lower curves) including the stop, sbottom, stau, chargino and neutralino sector contributions as a function of $\theta_\mu$. The common parameters are $m_A = 300$, $Q = 320$, $m_0 = 100$, $m_{\tilde{\tau}} = 500$, $\xi_2 = .5$, $\alpha_0 = .3$, $|A_0| = 1$. For curves with diamonds $\tan \beta = 15$, $\xi_1 = 1.5$, for squares $\tan \beta = 8$, $\xi_1 = 1.5$, and for triangles $\tan \beta = 8$, $\xi_1 = 0.5$ where all masses are in GeV and all angles are in radians.

Fig.7: Plot of the CP even component $\phi_1$ of $H_1$ (upper curves) and the CP odd component $\psi_{1D}$ of $H_1$ (lower curves) including the stop, sbottom, stau, chargino and neutralino sector contributions as a function of $\tan \beta$. The common input parameters for the curves are $m_A = 300$, $Q = 320$, $m_0 = 100$, $m_{\tilde{\tau}} = 500$, $\xi_1 = .5$, $\xi_2 = .5$, $\alpha_0 = .3$, and $|A_0| = 1$. For the curves with diamonds, $\theta_\mu = .4$, for squares $\theta_\mu = .6$, and for triangles $\theta_\mu = .8$ where all masses are in GeV and all angles are in radians.
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$% \text{ of } \phi_1 \text{ and } \psi_{1D} \text{ in } H_1$
% of $\phi_1$ and $\psi_{1D}$ in $H_1$ versus $\xi_1$ (radians)
% of φ₁ and ψ₁D in H₁ vs ξ₂ (radians)
\% of $\phi_1$ and $\psi_{1D}$ in $H_1$ vs $\tan \beta$