Entanglement evolution in a cascaded system with losses

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Abstract

The dynamics of a cascaded system that consists of two atom-cavity subsystems is studied by using the quantum trajectory method. Unwanted losses are included, such as photon absorption and scattering by the cavity mirrors and spontaneous emission of the atoms. Considering an initially excited two-level atom in the source subsystem, analytical solutions are obtained. The entanglement evolution is studied for the two atoms and for the two intracavity fields.

Key words: entanglement, cascaded system, unwanted losses, quantum trajectories

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1. Introduction

The concept of entanglement has been of great interest since the early days of quantum mechanics [1], and it has become of central importance in a variety of discussions on the fundamental aspects of the theory [2,3]. Unexpectedly, entanglement has also been demonstrated in classical Brownian motion [4]. Nowadays entanglement is receiving new attention in the rapidly developing fields of quantum information, quantum computation and quantum technology; for reviews, see [5,6]. In the context of entanglement preparation between atoms at separate nodes, a variety of schemes have been proposed, for example, by measuring the superpositions of light fields released from separate atomic samples or by measuring a probe light field that has interacted in a prescribed way with different samples. Due to the indistinguishability in the measurement and conditioned on the results of the measurements, the atomic system is projected onto an entangled state [7,8,9]. An unconditional preparation of entanglement has also been analyzed in the case of a cascaded system. This has been discussed for two distantly separated atoms [10,11], as well as for separate atomic ensembles [12]. More recently, the entanglement evolution for a Raman-driven cascaded system has also been analyzed [13].

In the spirit of these previous achievements, in the present contribution we will consider a cascaded open quantum system. We study the dynamics of a system that consists of two atom-cavity sub-systems $A$ and $B$. The quantum source $A$ emits a photon and the second quantum subsystem $B$ reacts on the emitted photon. Unwanted losses are included, such as photon absorption and scattering by the cavity mirrors and spontaneous emission of the atoms. Considering an initially excited two-level atom in the source subsystem, analytical solutions are obtained. Subsequently, the entanglement evolution between the two atoms, as well as between the two intracavity fields, is studied by using the concurrence.
The paper is organized as follows. In Sec. 2 the master equation describing the dynamics of the cascaded system is introduced, and the problem is solved analytically by using the quantum trajectory method. In Sec. 3 the entanglement evolution between the two atoms, or between the two intracavity fields, is analyzed. Finally, some concluding remarks are given in Sec. 4.

2. Cascaded system dynamics

In this section we analyze the dynamics of the system under study. The cascaded open quantum system consists of two atom-cavity subsystems A and B, where the source subsystem A is cascaded with the target subsystem B, cf. Fig. 1. The cavities have three perfectly reflecting mirrors and one mirror with transmission coefficient $T \ll 1$. In the two subsystems A and B we consider a two-level atomic transition of frequency $\omega_k$ (related to the atomic energy eigenstates $|1_k\rangle$ and $|0_k\rangle$) coupled to a cavity mode of frequency $\omega'_k$, where $k = a, b$ denotes the subsystem. The cavity mode is detuned by $\Delta_k$ from the two-level atomic transition frequency, $\omega_k = \omega'_k + \Delta_k$, and is damped by losses through the partially transmitting cavity mirrors. In addition to the wanted outcoupling of the field, the photon can be spontaneously emitted out the side of the cavity into modes other than the one which is preferentially coupled to the resonator. Moreover, the photon may be absorbed or scattered by the cavity mirrors. It has been shown that unwanted losses can have significant effects on the dynamical evolution of a quantum system and cannot be a priori neglected, see, e.g., Refs. [14,15].

To describe the dynamics of the system we will use a master equation formalism. This leads to the following master equation for the reduced density operator $\hat{\rho}(t)$ of the system:

$$\frac{d\hat{\rho}(t)}{dt} = \frac{i}{\hbar} \left[ \hat{H}, \hat{\rho}(t) \right] + \sum_{i=1}^5 \left[ \hat{J}_i \hat{\rho}(t) \hat{J}_i^\dagger - \frac{1}{2} \hat{J}_i^\dagger \hat{J}_i \hat{\rho}(t) - \frac{1}{2} \hat{\rho}(t) \hat{J}_i^\dagger \hat{J}_i \right].$$

(1)

The Hamiltonian is given by

$$\hat{H} = \hat{H}_A + \hat{H}_B + i\hbar \frac{\sqrt{\kappa_a \kappa_b}}{2} \left( e^{i\phi} \hat{b} \hat{a}^\dagger - e^{-i\phi} \hat{b}^\dagger \hat{a} \right),$$

(2)

where $\hat{H}_A$ and $\hat{H}_B$ describe the atom-cavity interaction in the two subsystems A and B, respectively, and, in the rotating-wave approximation, are given by

$$\hat{H}_A = \hbar g_a (\hat{a} \hat{A}_{10} + \hat{a}^\dagger \hat{A}_{01}) + \hbar \Delta_a \hat{A}_{11},$$

(3)

and

$$\hat{H}_B = \hbar g_b (\hat{b} \hat{B}_{10} + \hat{b}^\dagger \hat{B}_{01}) + \hbar \Delta_b \hat{B}_{11}.$$  

(4)

The third term in Eq. (2) describes the coupling between the two cavities [16,17]. In these expressions, $\hat{a}$ ($\hat{a}^\dagger$) is the annihilation (creation) operator for the cavity field A and similarly $\hat{b}$ ($\hat{b}^\dagger$) for the cavity field B. We have also defined $\hat{A}_{ij} = |i_a\rangle\langle j_a| (i, j = 0, 1)$, and $\hat{B}_{ij} = |i_b\rangle\langle j_b| (i, j = 0, 1)$. In addition, $g_{ab}$ is the atom-cavity coupling constant and $\kappa_k$ the cavity bandwidth. The phase $\phi$ is related to the phase change upon reflection from the source output mirror, and/or to the retardation of the source due to the spatial separation between the source and the target, cf. [18].

The jump operators $\hat{J}_i$ entering in Eq. (1) are defined by

$$\hat{J}_1 = \sqrt{\kappa_a} \hat{a} + \sqrt{\kappa_a} e^{-i\phi} \hat{b},$$

(5)

which describes photon emission by the cavities;

$$\hat{J}_2 = \sqrt{\kappa'_a} \hat{a}, \quad \hat{J}_3 = \sqrt{\kappa'_b} \hat{b},$$

(6)

are associated with photon absorption or scattering by the cavity mirrors; and

$$\hat{J}_4 = \sqrt{\Gamma_a} \hat{A}_{01}, \quad \hat{J}_5 = \sqrt{\Gamma_b} \hat{B}_{01},$$

(7)

are related to a photon spontaneously emitted by the atoms. Here $\kappa'_a$ and $\Gamma_a$ are the cavity mirrors’ absorption (or scattering) rate and the spontaneous emission rate of the two-level atom, respectively. Note that the operator $\hat{J}_i$ contains the superposition of the two fields

![Fig. 1. The cascaded open system consisting of two atom-cavity subsystems A and B.](image-url)
radiated by the two cavities due to the fact that the radiated photon cannot be associated with photon emission from either A or B separately.

In the following we will identify, for notational convenience, the state \(|a\rangle\) with the state \(|1, 0, 0, 0\rangle\), which denotes the atom A in the state \(|1_a\rangle\), the cavity A in the vacuum state, the atom B in the state \(|0_b\rangle\), and the cavity B in the vacuum state. In the state \(|b\rangle\equiv|0, 1, 0, 0\rangle\) the atom A is in the state \(|0_a\rangle\), and the cavity A is in the one-photon Fock state. Similarly, we define \(|c\rangle\equiv|0, 0, 1, 0\rangle\), \(|d\rangle\equiv|0, 0, 0, 1\rangle\), and \(|e\rangle\equiv|0, 0, 0, 0\rangle\). The state \(|a\rangle\) will be considered as the initial state of the system. It follows that the Hilbert space that describes the cascaded system under study is, in our model, spanned by the five state vectors \(|a\rangle\), \(|b\rangle\), \(|c\rangle\), \(|d\rangle\), and \(|e\rangle\).

To evaluate the time evolution of the system we use a quantum trajectory approach [18,19,20]. Let us consider the system prepared at time \(t_0 = 0\) in the state \(|a\rangle\). To determine the state vector of the system at a later time \(t\), assuming that no jump has occurred between time \(t_0\) and \(t\), we have to solve the nonunitary Schrödinger equation

\[
i \hbar \frac{d}{dt} |\tilde{\psi}_{\text{no}}(t)\rangle = \hat{H}' |\tilde{\psi}_{\text{no}}(t)\rangle, \tag{8}
\]

where \(
\hat{H}' = \hat{H} - \frac{i\hbar}{2} \sum_{k=1}^{5} J_k \hat{J}_k = \hat{H}_A + \hat{H}_B - i\hbar \left( \frac{K_a}{2} \hat{a} \hat{a}^\dagger + \frac{K_b}{2} \hat{b} \hat{b}^\dagger + \frac{\Gamma_a}{2} \hat{A}_{11} + \frac{\Gamma_b}{2} \hat{B}_{11} + \sqrt{\kappa_a \kappa_b} e^{\omega t} \hat{a} \hat{b}^\dagger \right)\)

(9)

where we have defined

\[
K_a = \kappa_a + \kappa'_a, \quad K_b = \kappa_b + \kappa'_b. \tag{10}
\]

If no jump has occurred between time \(t_0\) and \(t\), the system evolves via Eq. (8) into the unnormalized state

\[
|\tilde{\psi}_{\text{no}}(t)\rangle = \alpha(t) |a\rangle + \beta(t) |b\rangle + \gamma(t) |c\rangle + \delta(t) |d\rangle. \tag{11}
\]

In this case the conditioned density operator for the atom-cavity system is given by

\[
\hat{\rho}_{\text{no}}(t) = \frac{|\tilde{\psi}_{\text{no}}(t)\rangle\langle\tilde{\psi}_{\text{no}}(t)|}{\langle\tilde{\psi}_{\text{no}}(t)|\tilde{\psi}_{\text{no}}(t)\rangle}, \tag{12}
\]

where we indicate with “no” the fact that no jump has occurred between time \(t_0\) and \(t\).

The evolution governed by the nonunitary Schrödinger equation (8) is randomly interrupted by one of the five kinds of jumps \(\hat{J}_i\), cf. Eqs. (5)-(7). If a jump has occurred at time \(t\), \(t \in (t_0, t]\), the wave vector is found collapsed into the state \(|e\rangle\) due to the action of one of the jump operators

\[
\hat{J}_i |\tilde{\psi}_{\text{no}}(t_i)\rangle \rightarrow |e\rangle \quad (i = 1, \ldots, 5). \tag{13}
\]

The problem under study we may have only one jump. Once the system collapses into the state \(|e\rangle\), the nonunitary Schrödinger equation (8) lets it remain unchanged. In this case the conditioned density operator at time \(t\) is given by

\[
\hat{\rho}_{\text{yes}}(t) = |e\rangle\langle e|, \tag{14}
\]

where we indicate with “yes” the fact that a jump has occurred.

In the quantum trajectory method, the density operator \(\hat{\rho}(t)\) is obtained by performing an ensemble average over the different conditioned density operators at time \(t\), yielding the statistical mixture

\[
\hat{\rho}(t) = p_{\text{no}}(t) \hat{\rho}_{\text{no}}(t) + p_{\text{yes}}(t) \hat{\rho}_{\text{yes}}(t). \tag{15}
\]

Here \(p_{\text{no}}(t)\) and \(p_{\text{yes}}(t)\) are the probability that between the initial time \(t_0\) and time \(t\) no jump and one jump has occurred, respectively, where \(p_{\text{no}}(t) + p_{\text{yes}}(t) = 1\).

To evaluate \(p_{\text{no}}(t)\) we use the method of the delay function [20]. This yields the probability \(p_{\text{no}}(t)\) as the square of the norm of the unnormalized state vector:

\[
p_{\text{no}}(t) = \| \tilde{\psi}_{\text{no}}(t) \|_2^2 = \langle \tilde{\psi}_{\text{no}}(t) | \tilde{\psi}_{\text{no}}(t) \rangle = |\alpha(t)|^2 + |\beta(t)|^2 + |\gamma(t)|^2 + |\delta(t)|^2. \tag{16}
\]

From Eqs. (15) and (16) one obtains for the density operator \(\hat{\rho}(t)\) the expression

\[
\hat{\rho}(t) = |\tilde{\psi}_{\text{no}}(t)\rangle\langle \tilde{\psi}_{\text{no}}(t)| + |\epsilon(t)|^2 |e\rangle\langle e|, \tag{17}
\]

where we have defined \(|\epsilon(t)|^2 \equiv 1 - p_{\text{no}}(t)\). The physical meaning of \(|\alpha(t)|^2\), \(|\beta(t)|^2\), \(|\gamma(t)|^2\), \(|\delta(t)|^2\), and \(|\epsilon(t)|^2\) is clear. They represent the probability that at time \(t\) the system can be found either in \(|a\rangle\), \(|b\rangle\), \(|c\rangle\), \(|d\rangle\), or \(|e\rangle\).

In order to determine \(\alpha(t)\), \(\beta(t)\), \(\gamma(t)\), and \(\delta(t)\), we have to solve the nonunitary Schrödinger equation, cf. Eqs. (8) and (9). This leads us to consider the following inhomogeneous system of differential equations, similar to the one in Ref. [13].
\[
\begin{align*}
\dot{\alpha}(t) &= -i(\Delta_a - i\Gamma_a/2)\alpha(t) - ig_a\beta(t), \\
\dot{\beta}(t) &= -ig_a\alpha(t) - (K_a/2)\beta(t), \\
\dot{\gamma}(t) &= -i(\Delta_b - i\Gamma_b/2)\gamma(t) - ig_b\delta(t), \\
\dot{\delta}(t) &= -ig_b\gamma(t) - (K_b/2)\delta(t) - \sqrt{K_aK_b}\epsilon^{\phi}\beta(t).
\end{align*}
\]

The differential equations for \(\alpha(t)\) and \(\beta(t)\) can be solved independently from those for \(\gamma(t)\) and \(\delta(t)\). For the initial conditions \(\alpha(0) = 1\) and \(\beta(0) = 0\), and defining

\[
\Omega_b \equiv \sqrt{\frac{K_b^2}{4} - 4g_b^2 - iK_b\left(\Delta_b - i\Gamma_b/2\right) - \left(\Delta_b - i\Gamma_b/2\right)^2},
\]

we can write the solutions as

\[
\alpha(t) = \left[\frac{K_a/2 - i(\Delta_a - i\Gamma_a/2)}{\Omega_a}\right] \sinh\left[\frac{\Omega_a t}{2}\right] - \cosh\left[\frac{\Omega_a t}{2}\right] e^{-\left[(K_a + \Gamma_a)/4 + i\Delta_a/2\right]t},
\]

\[
\beta(t) = \frac{2ig_a}{\Omega_a} \sinh\left[\frac{\Omega_a t}{2}\right] e^{-\left[(K_a + \Gamma_a)/4 + i\Delta_a/2\right]t}.
\]

Inserting now in the inhomogeneous pair of differential equations for \(\gamma(t)\) and \(\delta(t)\) the solution obtained for \(\beta(t)\), we can determine the solutions for \(\gamma(t)\) and \(\delta(t)\). For the initial conditions \(\gamma(0) = 0\) and \(\delta(0) = 0\), we can write the solutions as

\[
\gamma(t) = g_b \left[ f_+(t)[g_+(t) + h_+(t)] - f_-(t)[g_+(t) + h_-(t)]\right],
\]

\[
\delta(t) = i\left[\frac{K_a - \Gamma_a}{4} - i\frac{\Delta_a}{2} + \frac{\Omega_a}{2}\right] f_+(t)[g_+(t) + h_+(t)] - i\left[\frac{K_b - \Gamma_b}{4} - i\frac{\Delta_b}{2} + \frac{\Omega_b}{2}\right] f_+(t)[g_+(t) + h_+(t)],
\]

where we have defined, for notational convenience,

\[
f_\pm(t) = \frac{g_b\sqrt{K_aK_b}\epsilon^{\phi}}{\Omega_a\Omega_b} e^{-\left[(K_a + \Gamma_a)/4 + i\Delta_a/2\pm\Omega_a/2\right]t},
\]

\[
g_\pm(t) = e^{-\left[(\Omega_a\pm\Omega_b)/2 - \gamma - i\Lambda\right]t} - \frac{1}{\Omega_a \pm \Omega_b} \frac{\Omega_a \pm \Omega_b}{2 - \gamma - i\Lambda},
\]

and

\[
h_\pm(t) = e^{-\left[(\Omega_a\pm\Omega_b)/2 + \gamma + i\Lambda\right]t} - \frac{1}{\Omega_a \pm \Omega_b} \frac{\Omega_a \pm \Omega_b}{2 + \gamma + i\Lambda},
\]

where \(\gamma = (K_a - K_b + \Gamma_a - \Gamma_b)/4\) and \(\Lambda = (\Delta_a - \Delta_b)/2\). In the case of equal parameters for the two subsystems \(A\) and \(B\), the solutions (21) simplify as

\[
\gamma(t) = \frac{Kg^2\epsilon^{\phi}}{\Omega^3} \left[e^{-\Omega t + \Omega t - 1} e^{-\left[(K + \Gamma)/4 - i\Delta/2 + \Omega/2\right]t} \right.
\]

\[
- \frac{Kg^2\epsilon^{\phi}}{\Omega^3} \left[e^{\Omega t - \Omega t - 1} e^{-\left[(K + \Gamma)/4 - i\Delta/2 - \Omega/2\right]t} \right],
\]

\[
\delta(t) = \frac{ik\epsilon\Delta^2}{\Omega^3} \left[\frac{K - \Gamma}{4} - \frac{\Delta}{2} + \frac{\Omega}{2} \right] \left[e^{\Omega t - \Omega t - 1} \right]
\]

\[
\left. - \frac{\Omega}{2}\right] \left[e^{-\Omega t + \Omega t - 1} e^{-\left[(K + \Gamma)/4 - i\Delta/2 + \Omega/2\right]t} \right].
\]

where we have used \(\lim_{x \to 0} \{\exp(\pm x) - 1\}/x = \pm t\), and defined \(\kappa = \kappa_a = \kappa_b\), \(K = K_a = K_b\), \(\Delta = \Delta_a = \Delta_b\), \(\Gamma = \Gamma_a = \Gamma_b\), \(g = g_a = g_b\), and \(\Omega = \Omega_a = \Omega_b\). In Fig. 2 we show the functions \(|\alpha(t)|^2\), \(|\beta(t)|^2\), \(|\gamma(t)|^2\), and \(|\delta(t)|^2\), i.e. the occupation probabilities of the state \(|a\rangle\), \(|b\rangle\), \(|c\rangle\), and \(|d\rangle\), respectively, for the case of equal parameters for the two subsystems \(A\) and \(B\), with \(g/K = 5\), \(\kappa/K = 0.9\), \(\Delta/K = 0.1\), and \(\Gamma/K = 0.2\). For these functions the phase factor \(\epsilon^{\phi}\) does not play any role.

### 3. Entanglement evolution

In the system under study, the two atoms constitute a pair of qubits. An appropriate measure of the entanglement for a two qubits system, often considered in the context of quantum information theory, is the concurrence [21]. Given the density matrix \(\rho\) for such a system, the concurrence is defined as
The concurrence $C[\rho_{\text{at}}(t)]$ between the two atoms is shown for the case of equal parameters for the two subsystems $A$ and $B$, where $g/K = 5$ and $\Delta/K = 0.1$. Moreover, $\kappa/K = 0.9$ and $\Gamma/K = 0.2$ (solid line), $\kappa/K = 0.9$ and $\Gamma/K = 0$ (dotted line), $\kappa/K = 1$ and $\Gamma/K = 0$ (dashed line).

$$C(\rho) = \max \left\{ 0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4} \right\},$$

(26)

where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ are the eigenvalues of the matrix $\tilde{\rho} = \rho(\sigma_x \otimes \sigma_y)\rho^*(\sigma_x \otimes \sigma_y)$. Here $\sigma_y$ is the Pauli spin matrix and complex conjugation is denoted by an asterisk. The concurrence varies in the range $[0, 1]$, where the values $0$ and $1$ represent separable states and maximally entangled states, respectively.

To derive an expression for the concurrence between the two atoms, let us consider the density operator that describes the system. It is obtained from the density operator $\hat{\rho}(t)$, Eq. (17), by tracing over the intracavity field states for the two subsystems, $\hat{\rho}_{\text{at}}(t) = \text{Tr}_{\text{cav}}[\hat{\rho}(t)]$, and is given by

$$\dot{\rho}_{\text{at}}(t) = |\alpha(t)|^2 |1, 0\rangle\langle 1, 0| + |\gamma(t)|^2 |0, 1\rangle\langle 0, 1| + \alpha(t)\gamma^*(t) |1, 0\rangle\langle 0, 1| + \alpha^*(t)\gamma(t) |0, 1\rangle\langle 1, 0| + \{1 - [|\alpha(t)|^2 + |\gamma(t)|^2]\} |0, 0\rangle\langle 0, 0|. $$

(27)

Considering the $4 \times 4$ density matrix $\rho_{\text{at}}(t)$, it is easy to show that the concurrence $C[\rho_{\text{at}}(t)]$ is, using Eq. (26), given by

$$C[\rho_{\text{at}}(t)] = 2|\alpha(t)||\gamma(t)|.$$  

(28)

To analyze the time dependence of the concurrence between the two atoms, let us consider the case of equal parameters for the two subsystems $A$ and $B$. Inserting the analytical solutions (20) and (25) into Eq. (28), we show in Fig. 3 the function $C[\rho_{\text{at}}(t)]$ for the parameters $g/K = 5$, $\Delta/K = 0.1$, and for different values of $\kappa/K$ and $\Gamma/K$. Since the concurrence contains only absolute values, the phase factor $e^{i\phi}$ does not play any role here.
from the density operator $\hat{\rho}(t)$, cf. Eq. (17), by tracing over the atomic states of the two subsystems, $\hat{\rho}_{\text{cav}}(t) = \text{Tr}_{\text{at}}[\hat{\rho}(t)]$. Considering now the $4 \times 4$ density matrix $\rho_{\text{cav}}(t)$ in the two intracavity-field Fock basis, the concurrence $C[\rho_{\text{cav}}(t)]$ is given by

$$C[\rho_{\text{cav}}(t)] = 2|\beta(t)||\delta(t)|.$$  \hspace{1cm} (29)

A comparison between the concurrence for the two atoms and the concurrence for the two intracavity fields is shown in Fig. 4. When the concurrence between the two atoms reaches a maximum, the concurrence between the two intracavity fields is approximately zero, and vice versa. This is related to the fact that the excitation energy is transferred between the atoms and the intracavity fields. When the atoms are unexcited, their state is separable and the intracavity fields are entangled. As a function of time, the entanglement is thus exchanged between the two atoms and the two intracavity fields.

4. Summary

The dynamics of a cascaded system that consists of two atom-cavity subsystems has been analyzed. Unwanted losses have been included, such as photon absorption and scattering by the cavity mirrors and spontaneous emission of the atoms. The evolution of the open quantum system under study has been described by means of a master equation. Considering an initially excited two-level atom in the source subsystem, analytical solutions for the dynamics of the system have been obtained. The entanglement evolution between the two atoms, constituting a two-qubit system, has been studied by using the concurrence. A similar analysis has been performed for the two intracavity fields.

The dynamical evolution of the system shows that the two initially disentangled qubits reach states of significant entanglement. It has been shown that the entanglement generated between the two atoms sensitively diminishes due to the presence of unwanted losses, which cannot be neglected in realistic quantum systems. We have also shown that the entanglement is periodically exchanged between the two atoms and the two intracavity fields.

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