Highlights of Supersymmetric Hypercharge ±1 Triplets

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Abstract

The discovery of a standard model (SM)-like Higgs boson with a relatively heavy mass $m_h$ and hints of di-photon excess has deep implication to supersymmetric standard models (SSMs). We consider the SSM extended with hypercharge ±1 triplets, and investigate two scenarios of it: (A) Triplets significantly couple to the Higgs doublets, which can substantially raise $m_h$ and simultaneously enhance the Higgs to di-photon rate via light chargino loops; (B) Oppositely, these couplings are quite weak and thus $m_h$ can not be raised. But the doubly-charged Higgs bosons, owing to the gauge group structure, naturally interprets why there is an excess rather than a deficient of Higgs to di-photon rate. Additionally, the pseudo Dirac triplet fermion is an inelastic non-thermal dark matter candidate. Light doubly-charged particles, especially the doubly-charged Higgs boson around 100 GeV in scenario B, are predicted. We give a preliminary discussion on their search at the LHC.

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The CMS and ATLAS collaborations discovered a new resonance around 126 GeV [1], the putative long-sought Higgs boson predicted by the standard model (SM). This good news consolidates supersymmetry (SUSY), which elegantly solves the gauge hierarchy problem relating with Higgs (assumed to be a fundamental spin-0 particle), as the leading candidate for new physics. Nevertheless, when we explain the Higgs boson in the popular minimal supersymmetric SM (MSSM), two problems will arise. One is that obtaining the relatively heavy mass ($m_h \simeq 126$ GeV) renders a quite serious fine-tuning [2]. The other one is that the hinted di-photon signal excess [1] can not be naturally understood. Therefore, naturally addressing these two problems simultaneously, despite the waiting-for-confirmation for the latter, gives us illuminating guide to go beyond the MSSM (BMSSM).

Actually, the relatively heavy Higgs boson mass alone may open a window for new model building. As been well known, in the MSSM the tree-level mass of the SM-like Higgs boson mass lies below the $Z$-boson mass, owing to the fact that the strength of Higgs quartic coupling is determined by the electroweak gauge couplings. New models thus should provide a significant Higgs quartic coupling either at tree- or loop-level. A well known example is the singlet extended MSSM such as the next-to MSSM (NMSSM). It possesses a large coupling between the singlet and Higgs doublets, which can give an additional Higgs quartic coupling at tree level. On top of that, it has a singlet-doublet mixing effect to raise $m_h$ [2, 3] (The related collider search please see Ref. [4]). Recently, other explorations on BMSSM along the line of raising $m_h$ include: (A) The gauge group extension, which gives non-decoupling $D$-terms [5] to lift $m_h$ (It requires the Higgs doublets to be effectively charged under the new gauge group); (B) Vector-like particles extension, which raises the Higgs boson mass at loop-level [6]; (C) $SU(2)_L$ triplet $T_0$ with hypercharge 0 extension [7, 8]. $T_0$ couples to the Higgs doublets via $\lambda_{T_0} \text{Tr} H_u T_0 H_d$ which, in raising $m_h$, is similar to the singlet-doublet coupling in the NMSSM.

However, when we further take into consideration the global fit on Higgs signal strength data [9], most of the aforementioned BMSSMs will be disfavored. In light of the best global fit, we may need new particles carrying electric charges which can directly enhance the decay width of Higgs to di-photon [9] and at the same time do not appreciably affect other partial decay widths, especially $\Gamma(h \rightarrow ZZ/WW)$. In the NMSSM, the light chargino, predicted by natural SUSY, might be a candidate of such charged particles. However, it only works when the singlet-doublet mixing is well tempered [10], which may introduce an extra fine-tuning. The triplet $T_0$ extension may improve that case, by virtue of new charged fermions from $T_0$. Indeed, they can readily enhance the di-photon rate, given a large enough $\lambda_{T_0}$ [8].

In this paper we consider the low energy SUSY incorporating $SU(2)_L$ triplets $T_{u,d}$, which
carry hypercharges $\pm 1$, respectively. Similar to Ref. [8], this BSSM is a good case following the guide mentioned at the beginning. As been noticed long ago by [11] and more recently by [12–14], if $T_{u,d}$ couple to the Higgs doublets with significant strengths, the Higgs mass can be substantially enhanced. Simultaneously, the singly-charged charginos, the mixture between these from $T_{u,d}$ and from the MSSM, are capable of enhancing the di-photon rate. Different to Ref. [8], the BMSSM considered in this paper has doubly-charged Higgs bosons. When $T_{u,d}$ weakly couple to the Higgs doublets, they can be naturally light and then be viable candidates of new charged particles (But this scenario fails in raising $m_h$). Besides, it may provide a pseudo Dirac fermion to be an inelastic non-thermal dark matter (DM) candidate.

Either in the significant or weak coupling limit, light doubly-charged particles are expected. Especially, the latter definitely predicts light doubly-charged Higgs bosons around 100 GeV. They may have promising LHC discovery prospects.

This paper is organized as follows. In Section II we study the MSSM extended with hypercharge $\pm 1$ triplets, two distinct scenarios are explored and we stress their phenomenological highlights for Higgs data and dark matter. Corresponding LHC search strategy and prospects are also briefly commented. Discussion and conclusion are casted in Section III and some necessary and complementary details are given in the Appendices.

II. HIGHLIGHTS OF SUSY WITH HYPERCHARGE $\pm 1$ TRIPLETS

Motivated by naturally explaining the recent Higgs data, including the relatively heavy SM-like Higgs boson mass and/or the hinted excess of Higgs to di-photon rate, we consider $SU(2)_L$ triplets $T_{u,d}$ extended SSMs (TSSMs). Here $T_{u,d}$ carry hypercharges $-1$ and $+1$, respectively. Note that both of them are needed for the sake of anomaly cancelation.

A. Model setup

The most relevant superpotential of the TSSM and the corresponding soft SUSY-breaking Lagrangian are given by

\[ W_H = \mu H_u \cdot H_d + \mu_T \text{Tr}(T_u T_d) + \lambda_u H_u \cdot T_u H_u + \lambda_d H_d \cdot T_d H_d, \]

\[ L_{soft} = \sum_{\Phi=H_{u,d},T_{u,d}} m_\Phi^2 |\Phi|^2 + (B_{\mu} H_u \cdot H_d + B_{\mu_T} \text{Tr}(T_u T_d) + c.c.) + (A_u H_u \cdot T_u H_u + A_d H_d \cdot T_d H_d + c.c.). \]

For simplicity, here all parameters are assumed to be real (Incorporating CP violation will bring about quite different Higgs phenomenologies [15]). $T_u$ and $T_d$, following the notation
of Ref. [11, 13], are respectively written as

\[ T_u \equiv T_u^a \sigma^a = \begin{pmatrix} T_u^+ / \sqrt{2} & -T_u^0 \\ T_u^- / \sqrt{2} & -T_u^0 \end{pmatrix}, \quad T_d \equiv T_d^a \sigma^a = \begin{pmatrix} T_d^+ / \sqrt{2} & -T_d^+ \\ T_d^0 & -T_d^+ \end{pmatrix}, \tag{3} \]

with \( \sigma^a \) the three Pauli matrices. As usual, the doublets are written as \( H_u = (H_u^+, H_u^0) \) and \( H_d = (H_d^0, H_d^-) \). For illustration, we write the superpotential terms in component form

\[ H_u \cdot T_u H_u \equiv (H_u)_\alpha \epsilon^{\alpha \beta} (T_u)_\gamma (H_u)_\gamma = \sqrt{2} H_u^+ H_u^0 T_u^- - (H_u^0)^2 T_u^0 - (H_u^+)^2 T_u^- - \]

\[ (H_u)_\alpha \epsilon^{\alpha \beta} (T_u)_\gamma (H_u)_\gamma = \sqrt{2} H_u^+ H_u^0 T_u^- - (H_u^0)^2 T_u^0 - (H_u^+)^2 T_u^-, \tag{4} \]

where the antisymmetric tensor \( \epsilon^{\alpha \beta} \) has entry \( \epsilon^{12} = -1 \).

If we want to produce the small neutrino masses, as in the type-II seesaw mechanism [16], the following lepton number violating operators are introduced:

\[ W_L = (\lambda_L)_{ij} L_i \cdot T_d L_j. \tag{5} \]

with \( i, j = 1, 2, 3 \) the family indices. Then, the tiny neutrino masses are the products of the small Yukawa couplings and triplet’s vacuum expected value (VEV): \( (m_\nu)_{ij} = (\lambda_L)_{ij} v T_d \sim 10^{-10} \) GeV. One would find that the triplets’ VEV of interest should be around the GeV scale, so the Yukawa couplings are extremely small. Then, alternatively, one may forbid such couplings and turn to the canonical seesaw mechanism. In any case, this aspect of the TSSM is not of our main concern. But we would like to stress that, the TSSM under consideration can be embedded into the supersymmetric type-II seesaw mechanism [17] without violating \( R \)–parity. In other words, the lightest sparticle LSP DM hypothesis of SUSY can be maintained here. In contrast, the models with hypercharge 0 triplets can not be embedded into the type-III seesaw mechanism and meantime respect the \( R \)–parity.

In the TSSM, \( \lambda_u \) and \( \lambda_d \) are closely related to the ensuing discussions. In one scenario studied later they will be required to be large, so we want to know the scale at which perturbitivity breaks. The relevant renormalization group equations (RGEs) are casted in Appendix. A. From it one can find, due to \( h_t \gg h_b \), the beta functions of \( \lambda_u \) and \( \lambda_d \) are asymmetric. \( \lambda_d \) is favored to be moderately larger than \( \lambda_u \). Fig. 1 shows that if they do not hit the Landau pole below the GUT-scale, the maximal \( \lambda_d \) and \( \lambda_u \) are about 0.62 and 0.45, respectively. For \( \lambda_d = 0.7 \), it hits the Landau pole at a scale \( \sim 10^{10} \) GeV. The implication to GUT is beyond the scope of this work and we comment on that in the discussion.

In the following subsections we will turn our attention to the highlights of the TSSMs, and focus on phenomenologies involving the Higgs and DM. Two different scenarios according to the magnitudes of \( \lambda_{u,d} \) are separately investigated.
FIG. 1: The plot of $\lambda_{u,d}$ versus the running scale. Two groups of boundary values at the low energy are chosen: (A) $\lambda_u = 0.3$ (thick dashed line) and $\lambda_d = 0.7$ (thick line); (B) $\lambda_u = 0.45$ (thin dashed line) and $\lambda_d = 0.62$ (thin line), showing the maximal $\lambda_{u,d}$ endured by perturbativity up to the GUT-scale.

B. Scenario A: Large $\lambda_{u,d} \sim 1$

This scenario takes the advantage in raising the SM-like Higgs boson mass and the Higgs to di-photon rate simultaneously. However, they tend to show a tension and fortunately the mixing effect can relax it.

1. Lifting the Higgs boson mass and pull-down mixing effect

The CP-even Higgs boson sector consists of four states, with two from the Higgs doublets and the rest from the Higgs triplets $T_{u,d}^0$. Mixings between Higgs doublets and triplets are induced by the triplets’ VEV, which should be no more than a few GeVs due to the bound from the $\rho$ parameter [13]. Concretely, the triplets’ VEV can be approximately determined in an analytical way:

$$v_{T_{u,d}} \equiv \langle T_{u,d}^0 \rangle \simeq \frac{v^2}{m_{T_{u,d}}^2 + \mu_T^2} M_{u,d},$$

$$M_u \equiv A_u \sin^2 \beta - \lambda_d \mu_T \cos^2 \beta - \lambda_u \mu \sin 2\beta,$$

$$M_d \equiv A_d \cos^2 \beta - \lambda_u \mu_T \sin^2 \beta - \lambda_d \mu \sin 2\beta,$$

with $\tan \beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$. These approximations are valid only for the case with heavy scalar triplets and a not anomalously large $B_{\mu T}$. It is clearly seen that $\langle T_{u,d}^0 \rangle \lesssim 1$ GeV, given a
small $\mu_T \sim 100$ GeV and large $\lambda_{u,d} \sim 1$, requires quite heavy triplets: $m_{T_{u,d}} \gtrsim \mathcal{O}(1)$ TeV. Otherwise, we have to greatly tune parameters to make $M_{u,d}$ lie around the GeV scale.

On the other hand, $m_{T_u}$ can not be too heavy when naturalness are taken into account. $H_u$ couples to $T_u$ with a large Yukawa coupling, so $m_{H_u}^2$ receives a large negative correction proportional to $m_{T_u}^2$, which, in the leading logarithmic approximation, can be estimated by means of the renomalization group equation (RGE):

$$16\pi^2 \frac{dm_{H_u}^2}{dt} = 6h^2 (m_{Q_3}^2 + m_{U_3}^2) + 12\lambda_u^2 m_{T_u}^2 + \ldots$$

(7)

This allows us to derive an upper bound ($m_{T_d}^2$ can be well beyond this bound since $T_d$ does not couple to $H_u$, but we do not consider such a hierarchy here.):

$$m_{T_u} \lesssim 1.2 \times \left( \frac{F^{1/2}}{10} \right) \left( \frac{0.5}{\lambda_u} \right) \left( \frac{30}{\log \Lambda/\text{TeV}} \right)^{1/2} \text{TeV},$$

(8)

with $F$ the degree of fine-tuning and $\Lambda$ the SUSY-breaking mediation scale. In the above estimation $\Lambda$ is set equal to the grand unification theory (GUT)-scale. But even if we take $\Lambda = 10^6$ GeV, the upper bound is merely doubled. In other words, in the large $\lambda_{u,d}$ scenario, naturalness does not permit the triplet VEVs (at least $v_{T_u}$) to be far below one GeV. This has important implications to the mixing effect.

We first investigate the TSSM specific effect on raising the SM-like Higgs boson mass $m_h$ without considering the mixing effect. It is convenient to employ the field decomposition as

$$H_u^0 = v_u + \frac{1}{\sqrt{2}} (h_1 \cos \beta + h_2 \sin \beta) + \frac{i}{\sqrt{2}} (P_1 \cos \beta + G^0 \sin \beta),$$

$$H_d^0 = v_d + \frac{1}{\sqrt{2}} (-h_1 \sin \beta + h_2 \cos \beta) + \frac{i}{\sqrt{2}} (P_1 \sin \beta - G^0 \cos \beta),$$

$$T_u^0 = v_{T_u} + \frac{1}{\sqrt{2}} (h_3 + i P_2), \quad T_d^0 = v_{T_d} + \frac{1}{\sqrt{2}} (h_4 + i P_3),$$

(9)

where $G^0$ is the Goldstone boson and $P_i$ are the CP-odd Higgs bosons. The CP-even Higgs boson mass square matrix $M_S^2$, in the basis $(h_1, h_2, h_3, h_4)$, is given by Eq. (B3). Neglecting both the doublet-doublet and doublet-triplet mixing effects for the time being, we then get $m_h^2$, which is approximated by

$$(M_S^2)_{22} = m_Z^2 \left[ \cos^2 2\beta + \frac{4}{g^2} (\lambda_d^2 \cos^4 \beta + \lambda_u^2 \sin^4 \beta) \right],$$

(10)

with $m_Z^2 = g^2 v^2$ and $g^2 = (g_1^2 + g_2^2)/2$. The new contributions originate from $\lambda_{u,d} H_{u,d} T_{u,d} H_{u,d}$, which give new quartic terms, e.g., $|F_{T_u}^a| \gtrsim \lambda_u^2 |H_u^0|^4$. Noticeably, provided that $\lambda_{u,d}$ are order one coupling constants, the tree-level Higgs boson mass will get a considerable enhancement, regardless of the value of $\tan \beta$. 

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Such an interesting feature, obviously, is attributed to the fact that the quartic terms $|H_d^4|$ and $|H_u^4|$ are generated simultaneously. To our knowledge, basically the $m_h$ dependence on the angle $\beta$ can be classified into three types: (I) The MSSM-like case where the quartic term is determined by the vector-like $D$-terms (Namely $H_{u,d}$ form a vector-like representation under the gauge group, such as $SU(2)_L \times U(1)_Y$), and takes the form of $(|H_u^0|^2 - |H_d^0|^2)^2 \propto \cos^2 2\beta$. A large $\tan \beta$ is then required to give a heavy $m_h$; (II) The NMSSM-like case where the quartic term is $|H_u^0 H_d^0|^2 \propto \sin^2 2\beta$ and thus a small $\tan \beta \sim 1$ is required to lift $m_h$. The SSMs with hypercharge-0 triplets [7, 8] also fall into this category; (III) The TSSMs considered in this work as well as the models extended by a new gauge group under which $H_u$ and $H_d$ form a chiral representation. As will be explicitly seen, the Higgs to di-photon rate excess also has a strong dependence on $\beta$. Therefore, the above classification provides a guidance to build models which can simultaneously lift $m_h$ and the Higgs to di-photon rate.

We now take the mixing effect into account, which may reduce the maximal enhancement given in Eq. (10). To see that, we consider the sub-matrix of $M_S^2$, consisting of the entries involving states 2 and 3 (namely only including the mixing with $T_0^u$). From Eq. (B3), it is not difficult to find that this $2 \times 2$ matrix shows the relation

$$\frac{1}{(M_S^2)_{22}} \frac{1}{(M_S^2)_{33}} \sim O(0.1)(M_S^2)_{23}.$$

(11)

Thereby, the mixing effect can be appreciable and thus pulls-down the lighter eigenvalue indicated by Eq. (10), with an amount estimated to be [2]

$$\Delta m_h \approx \frac{1}{2} \sqrt{(M_S^2)_{22}} \frac{(M_S^2)_{44}}{(M_S^2)_{33}(M_S^2)_{22}}$$

$$= - 2 \left( \frac{M_S^2}{m_{T_u}^2} \left( \frac{m_h}{\sqrt{(M_S^2)_{22}}} \right) \left( \frac{v^2}{m_h} \right) \right) \text{GeV.}$$

(12)

The mixing with $T_0_d$ similarly contributes to the pulling-down effect and then roughly the above estimation should be doubled.

The pulling-down mixing effect is tunable. Through a mild tuning one can obtain relatively small $M_{u,d}$, which helps to not only reduce $v_{T_{u,d}}$ but also weaken the pulling-down effect. This is consistent with the $\rho$ parameter constraint. However, sometimes the new quartic terms excessively enhance $m_h$, which will be the case when we want to further account for the di-photon excess, then we need relatively large $M_{u,d}$, or equivalently $v_{T_{u,d}}$, to strengthen the pulling-down effect. Then, from Eq. (6) it is known that negative $A_{u,d}$ are favored. For illustration, we plot these two limiting cases of mixing effect on Fig. 2.
FIG. 2: Contour plots of the amount of Higgs boson mass reduction (in GeV, dashed lines with white labels) from the doublet-triplet pulling-down mixing effect and the triplet soft mass $m_{T_u}$ (in TeV, solid lines with green labels) on the $v_{T_u} - A_u$ plane. Left panel: Large reduction by setting $\lambda_d = 0.70$, $\lambda_u = 0.45$, $v_{T_d} = -2.0$ GeV and $A_d = -300$ GeV. Right panel: Small reduction by setting $\lambda_d = 0.70$, $\lambda_u = 0.30$, $v_{T_d} = -0.5$ GeV and $A_d = 200$ GeV. Other not much relevant parameters are common to both cases: $\mu = 200$ GeV, $\mu_T = 250$ GeV, $B\mu = 400 \times 200$ GeV$^2$, $B\mu_T = 200^2$ GeV$^2$ and $\tan \beta = 1.5$ (Actually, effect of $\mu$, $\mu_T$ and $\tan \beta$ can be absorbed into $A_{u,d}$, see Eq. (6)).

2. Raising the Higgs to di-photon rate via charginos

The Higgs to di-photon rate $R_{\gamma\gamma}$ can probe charged particles beyond the SM. As usual, the rate is defined as

$$R_{\gamma\gamma} \equiv \frac{\sigma(gg \rightarrow h) \text{Br}(h \rightarrow \gamma\gamma)}{\sigma_{\text{SM}}(gg \rightarrow h) \text{Br}_{\text{SM}}(h \rightarrow \gamma\gamma)}.$$  \hspace{1cm} (13)

For a generic particle content, the decay width of Higgs to di-photon is formulated as [18]

$$\Gamma(h \rightarrow \gamma\gamma) = \frac{\alpha^2 m_h^3}{1024 \pi^3} \left| \frac{g_{hVV}}{m_V^2} Q_V^2 A_1(\tau_V) + \frac{2 g_{hff}}{m_f^2} N_{c,f} Q_f^2 A_{1/2}(\tau_f) + \frac{g_{hSS}}{m_S^2} N_{c,S} Q_S^2 A_0(\tau_S) \right|^2,$$  \hspace{1cm} (14)

with $N_c$ the color factor and $\tau_i \equiv 4m_i^2/m_h^2$. Here $V$, $f$, and $S$ denote for a vector boson, Dirac fermion and charged scalar, respectively. Their electric charges as well as couplings to the Higgs boson are labeled as $Q_V$ and $g_{hVV}$, etc. In the limits $\tau_i \gg 1$, the loop-functions
\[ A_i, \text{ take asymptotic values } A_1 \to -7, A_{1/2} \to 4/3 \text{ and } A_0 \to 1/3. \] Specified to the SM, the \( W \)-boson and top quarks dominantly contribute to Eq. (14) and their loop functions are
\[ A_1(\tau_W) \approx -8.34, \quad 3 \times (2/3)^2 A_{1/2}(\tau_t) \approx 1.84. \] (15)

To get it we have fixed \( m_h = 126 \text{ GeV}. \) In the TSSM, the singly-charged charginos and doubly-charged Higgs bosons are two promising candidates to make \( R_{\gamma\gamma} > 1. \) The doubly-charged charginos are irrelevant, which is due to the absence of their direct couplings with the SM-like Higgs boson (See the superpotential Eq. (4)). As for the doubly-charged Higgs bosons, despite of their large electric charges and moreover large couplings to the Higgs boson from D-term, are neither irrelevant. The reason is that in the large \( \lambda_{u,d} \) scenario only quite heavy \( m^2_{T_{u,d}} \) are considered. But they will play important roles in scenario B.

So, we only need to consider the contributions from the light singly-charged charginos. For a mixed Driac system with a mass matrix \( M_F, \) we have [18, 19]
\[ \sum_i 2g_{HF_i\bar{f}_i} \frac{m_{f_i}}{m_{f_i}} = \frac{1}{\sqrt{2}} \frac{\partial}{\partial v} \log \left( \det M^\dagger_F M_F \right), \] (16)
with \( f_i \) the mass eigenstates having properly heavy eigenvalues. Applying this formula to top quarks, we get
\[ \frac{2g_{ht\bar{t}}}{m_t} = \sqrt{\frac{2}{v}} \] with \( v = 174 \text{ GeV}. \) We now apply it to the singly-charged chargino system, which consists of three Dirac fermions, winos \( \tilde{W}^\pm, \) Higgsinos \( \tilde{H}^\pm_{u,d} \) as well as triplinos \( \tilde{T}^\pm_{u,d} \). Their mass terms are given by
\[ (-i\tilde{W}^-, \tilde{H}^-_d, \tilde{T}^-_u) \left( \begin{array}{ccc} M_2 & g_2 v_u & 0 \\ g_2 v_d & \mu & \sqrt{2}\lambda_d v_d \\ 0 & \sqrt{2}\lambda_u v_u & \mu_T \end{array} \right) \left( \begin{array}{c} -i\tilde{W}^+ \\ \tilde{H}^+_u \\ \tilde{T}^+_d \end{array} \right), \] (17)
where terms proportional to \( v_{T_{u,d}} \) have been safely neglected. Denoting the charginos in the mass eigenstate as \( \chi_i (i = 1, 2, 3), \) it is straightforward to calculate the effective coupling defined in Eq. (14):
\[ \frac{2g_{h\chi\chi}}{m_{\chi}} = \sum_{i=1}^{3} \frac{2g_{h\chi_i\chi_i}}{m_{\chi_i}} = \frac{2 \sin 2 \beta (2\lambda_d \lambda_u v/\mu_T + g_2^2 v/M_2)}{\sin 2 \beta (2\lambda_d \lambda_u v/\mu_T + g_2^2 v/M_2) - 2\mu/\sqrt{2}}, \] (18)
Immediately, it is seen that a small \( \tan \beta \sim 1, \) as in the MSSM, is a necessary condition to enhance the Higgs to di-photon rate by charginos [40]. Actually, from the counterpart of Eq. (18) in the MSSM, we can get Eq. (18) through the shift \( g_2^2 v/M_2 \to g_2^2 v/M_2 + 2\lambda_d \lambda_u v/\mu_T. \) Thus, in the decoupling limits \( \lambda_{u,d} \ll 1 \) or/and \( \mu_T \gg v, \) the triplinos are irrelevant to the Higgs di-photon decay and the MSSM result is recovered. Then the di-photon excess is no more than 30%, even if we work in the unrealistic limit with \( \tan \beta = 1 \) [20]. Thus, the case with non-decoupling \( \tilde{T}_{u,d} \) is of interest.
With Eq. (18), the signature strength of Higgs to di-photon is given by

\[ R_{\gamma\gamma} \approx 1 - \frac{1.84}{8.34 - 1.84} \times \frac{3}{4} \sqrt{\frac{v}{2}} \times g_{h\tilde{\chi}\tilde{\chi}} \left| \frac{2}{m_\chi} \right|^2, \]  

(19)

where we have used Eq. (15) and normalized the new physics contribution to the top quark contribution, as indicated by the second term in the above equation. Moreover, we assume that all charginos’ loop functions \( A_{1/2}(\tau_{\chi_i}) \) are equal to \( A_{1/2}(\tau_t) \). But actually the lightest chargino mass is about half of the top quark mass, and thus \( A_{1/2}(\tau_{\chi_1}) \) is slightly larger than \( A_{1/2}(\tau_t) \). As a result, Eq. (19) typically, but not appreciably, underestimates the chargino contribution.

The numerical results of \( R_{\gamma\gamma} \) and the Higgs boson mass are shown on the left panel of Fig. 3. We find that: (I) The large di-photon rate may have a tension with \( m_h = 126 \) GeV, but the pulling-down effect can relax it; (II) Only the product \( \lambda_u \lambda_d \) is relevant to determine \( R_{\gamma\gamma} \), so, in light of the discussion in Section II A, we can take \( \lambda_u = 0.4 \) and \( \lambda_d = 0.62 \) to get viable enhancements without spoiling perturbativity below the GUT-scale; (III) All charginos, including winos, should be properly light, which means that the MSSM charginos have a non-negligible contribution to \( R_{\gamma\gamma} \). This contribution helps us to obtain a relatively large \( R_{\gamma\gamma} \) with \( \lambda_u \lambda_d \) tolerated by perturbativity. To end up this subsection, we mention that the sign of \( g_{h\tilde{\chi}\tilde{\chi}} \), as in most models, is a result of parameter tuning (or artificial choice). In the following we will present a scenario to naturally understand the origin of the sign.

**C. Scenario B: The small \( \lambda_{u,d} \ll 1 \) limit**

We now switch to the scenario B characterized by very small couplings \( \lambda_{u,d} \ll 1 \). Despite of the failure of raising \( m_h \), this scenario is still of great interest, by virtue of its elegant explanation to the Higgs to di-photon excess. On top of that, this scenario can provide an inelastic dark matter (DM) candidate.

1. **Raising the di-photon rate via doubly-charged Higgs bosons**

As seen from Eq. (6), now the masses of doubly-charged Higgs bosons can be light without spoiling the \( \rho \) parameter. Then, the charginos’ contribution is reduced to the MSSM case, but instead \( H^{\pm\pm} \) is able to enhance \( R_{\gamma\gamma} \). Couplings between \( H^{\pm\pm} \) and the Higgs boson come from the \( D- \)term. The interesting point is that, by virtue of supersymmetry, the couplings are fixed by the gauge group structure of the model rather than input by hand. Explicitly,
FIG. 3: Left panel (scenario A and on the $\mu_T - \lambda_u$ plane): A comparative study on the Higgs to di-photon signature strength $R_{\gamma\gamma}$ with different wino mass $M_2$, setting $\lambda_d = 0.7$, $\tan \beta = 1.5$ and $\mu = 200$ GeV. The tree-level SM-like Higgs boson mass without considering pulling-down effect is labeled by the horizontal lines. It is allowed to be mildly heavier than 125 GeV. Two values of $M_2$ are taken: (I) $M_2 = 300$ GeV. The charigno mass lower bound $m_{\chi_1} > 94$ GeV [22] is labeled by the thick red line, and $R_{\gamma\gamma} = 1.3$ (1.5) are labeled by the thick black lines. The triangle filled by blue dashed lines denote the allowed region with $R_{\gamma\gamma} > 1.3$; (II) $M_2 = 600$ GeV. $R_{\gamma\gamma}$ and charigno mass are labeled by dashed lines. The triangle filled by red points gives $R_{\gamma\gamma} > 1.3$ and is significantly smaller than the one in case I. Right panel (scenario B): A plot of $R_{\gamma\gamma}$ varying with the doubly-charged Higgs boson mass, and two cases with 5% and 15% reduction from singly-charged Higgs boson are plotted, respectively.

they are extracted from Eq. (B1):

$$V_D := \frac{g_2^2 - g_1^2}{2} (|H_u|^2 - |H_d|^2) \left(|T_u^-|^2 - |T_d^+|^2\right) - \frac{g_1^2}{2} (|H_u|^2 - |H_d|^2) \left(|T_u^-|^2 - |T_d^+|^2\right)
= -\sqrt{2} \cos 2\beta \cos 2\theta_u \frac{m_Z^2}{v} h_2 \left(|T_u^-|^2 - |T_d^+|^2\right)
+ \sqrt{2} \cos 2\beta (\sin^2 \theta_w / \cos 2\theta_w) \frac{m_Z^2}{v} h_2 \left(|T_u^-|^2 - |T_d^+|^2\right), \quad (20)$$

where $\theta_w$ is the Weinberg angle with $\sin^2 \theta_w \approx 0.23$.

We now show that we have a way to naturally explain $R_{\gamma\gamma} > 1$, rather than the other way around. From the second line of Eq. (20) it is seen that, there is a relative sign between the two terms in the bracket. It is a consequence of $T_u^-\bar{u}$ lying at the 21-position in the $T_u$
matrix while \( T_{d}^{++} \) at the 12-position in the \( T_{d} \) matrix (or, essentially, \( T_{u} \) and \( T_{d} \) carrying opposite hypercharges.). Hence, if \( T_{d}^{++} \) is very heavy and \( T_{u}^{--} \) is sufficiently light, say due to \( m_{T_{u}}^{2} + \mu_{T}^{2} \ll m_{T_{d}}^{2} + \mu_{T}^{2} \) (Such a hierarchy can be naturally realized in the gauge mediated SUSY-breaking models by coupling \( T_{d} \) to messengers [21].), then \( T_{d}^{++} \) decouples and we have the effective coupling:

\[
\frac{g_{h|T_{u}^{--}|^{2}}}{m_{T_{u}^{--}}^{2}} = \cos 2\beta \cos 2\theta_{u} \frac{m_{Z}^{2}}{m_{T_{u}^{--}}^{2}} \frac{\sqrt{2}}{v}.
\]  (21)

The effective coupling constant given by Eq. (21) is negative, so it elegantly explains why the doubly-charged-loop constructively rather than destructively interferes with the \( W \)-loop.

The enhancement of \( R_{\gamma\gamma} \) is significant only in the large \( \tan \beta \) limit, and moreover it sensitively depends on the doubly-charged Higgs boson mass. Concretely, in the large \( \tan \beta \) limit, we have

\[
R_{\gamma\gamma} \approx \left| 1 + \frac{2.16}{8.34 - 1.84} \times \frac{m_{Z}^{2}}{m_{T_{d}^{++}}^{2}} A_{0}(\tau_{T_{u}^{--}}) + (T_{u}^{--} - \text{reduction}) \right|^2,
\]  (22)

which, actually, depends solely on the mass \( m_{T_{u}^{--}} \). The singly-charged Higgs boson with comparable mass reduces the doubly-charged Higgs boson contribution by less than 10%, which will be regarded as a constant hereafter. The right panel of Fig. 3 clearly shows that an excess \( R_{\gamma\gamma} \gtrsim 20\% \) requires a doubly-charged Higgs boson with mass below 105 GeV, even if the \( T_{u}^{--} - \text{reduction} \) is only 5%.

Comments are in orders. First, although there is a great number of works using a doubly-charged Higgs field to enhance the di-photon rate [23], here we find that only supersymmetry can naturally determine the sign of the effective coupling between the SM-like Higgs boson and the doubly-charged Higgs boson. Second, from the previous deduction it is tempting to conjecture that, this sign-determination mechanism can be generalized to any SSMs having a gauge group \( G \) and some QED-charged chiral fields, which, along with the Higgs doublets, form vector-like representations of \( G \). Third, the TSSM in scenario B has to give up the merit of lifting \( m_{h} \) [41], and we can enhance \( m_{h} \) using methods reviewed in the introduction. But the method like in the NMSSM, which requires a small \( \tan \beta \sim 1 \), may be contradict with the di-photon rate enhancement.

2. Pseudo-Dirac fermions as a non-thermal inelastic DM candidate

In scenario B, the TSSM may provide an inelastic DM candidate [25, 26] [42]. Barring large cancelation, \( \mu_{T} \) should be around \( m_{T_{d}^{++}} \simeq 100 \text{ GeV} \). Then, the neutral triplinos \( \tilde{T}_{u,d}^{0} \) can be the lightest sparticles (LSP). They are degenerate in mass and form a Dirac fermion
in limits $\lambda_{u,d} \to 0$. However, the degeneracy is lifted by the mixing with the MSSM neutralinos, which are Majorana fermions. The corresponding mass splitting is proportional to the small VEVs $v_{T_u,d}$, so it is naturally small. As a consequence, the Dirac triplino becomes pseudo Dirac.

We now study the mass splitting. In the basis $\chi^T_0 = \begin{pmatrix} -i\tilde{B}, -i\tilde{W}^3, \tilde{H}^0_d, \tilde{H}^0_u, \tilde{T}_d, \tilde{T}_u \end{pmatrix}$, the $6 \times 6$ neutralino mass matrix is

$$M_{\chi^0} \approx \begin{pmatrix} M_1 & 0 & \frac{g_1 v_d}{\sqrt{2}} & \frac{g_1 v_u}{\sqrt{2}} & \sqrt{2}g_1 v_{T_d} & -\sqrt{2}g_1 v_{T_u} \\ 0 & M_2 & \frac{g_2 v_d}{\sqrt{2}} & -\frac{g_2 v_u}{\sqrt{2}} & -\sqrt{2}g_2 v_{T_d} & \sqrt{2}g_2 v_{T_u} \\ 0 & 0 & -\mu & \frac{v_0 \lambda_d}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & \mu_T & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_T & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_T \end{pmatrix}. \tag{23}$$

The analytical analysis is difficult, but we can calculate the mass splitting in an approximate way. At tree-level, the splitting is attributed to the mixing between the triplinos and MSSM neutralinos. We consider a simplified case where mixing with wino dominates and then obtain a $3 \times 3$ neutralino mass matrix:

$$M_{\chi^0} \approx \begin{pmatrix} M_2 & -\sqrt{2}g_2 v_{T_d} & 0 \\ -\sqrt{2}g_2 v_{T_u} & 0 & \mu_T \\ 0 & \mu_T & 0 \end{pmatrix}. \tag{24}$$

We have set $v_{T_u} \ll v_{T_d}$, which is a reasonable assumption since $m_{T_u}^2 \gg m_{T_d}^2$. The mass splitting can be calculated to be

$$\delta = \frac{M_2}{M_2^2 - \mu_T^2} (\sqrt{2}g_2 v_{T_d})^2. \tag{25}$$

It is seen that, for a sub-GeV triplet VEV and moreover moderately heavy $M_2 \sim 500$ GeV, $\delta$ is at the sub-MeV scale. If the bino is considerably light, say $M_1 \sim M_2/3$ suggested by the gaugino mass unification (Actually, one can include the bino contribution by adding a similar term to Eq. (25) with the replacement $M_2 \to M_1$ and $g_2 \to g_1$), the above estimation is only about half of the actual value. When $\mu$ is rather light, it tends to reduce the splitting.

In a word, the full result accommodates a split over a rather wide range.

As been well known, DM participating in full electroweak (EW) gauge interactions has a too large annihilation cross section, say $\sigma v \sim g_2^4/16\pi\mu_T^2$, which then renders the DM relic density far smaller than $\Omega_{DM} h^2 \sim 0.1$. Although the heavy DM around one TeV has a proper annihilation rate, it is not the case considered here. We thus go beyond the thermal DM scenario. Consider a highly bino-like (So it almost does not couple to the $Z$-boson) ordinary LSP (OLSP), it acquires a proper thermal relic density and late decays into the
psuedo-Dirac fermion plus a pair of light fermions. The decay is mediated by a slepton (See a similar scenario in a different context [24],), with decay width estimated to be

$$\Gamma(\tilde{B} \rightarrow \tilde{T}_0 u, d') \sim \frac{1}{(4\pi)^3} g_1^4 V_{14}^2 \frac{M_1^5}{m_f^4} P \left( \frac{M_1^2}{\mu_T^2} \right) \sim 10^{-18} \text{GeV}. \quad (26)$$

To get the final estimation, we have set the mixing angle between bino and triplino $V_{14} \sim g_1^2 V_{14}^2 / (\mu_T m_B) \sim 10^{-4}$, slepton mass at one TeV and the phase space factor $P \sim 10^{-2}$, which in principle can be even smaller in magnitude provided sufficient degeneracy between bino and DM. Hence the decay can happen after the DM freezing-out, and the bino can transfer its number density into the DM.

A weak scale inelastic DM with sub-MeV mass splitting has interesting phenomenological applications. First, it provides a way to avoid the stringent spin-independent bound on the $Z$-mediated DM-nucleon scattering, which makes the triplino-like DM to be allowed by the DM direct detection experiments [25, 27] like XENON100 [28] (It, along with the WMAP, raises question on naturalness of the conventional neutralino LSP DM [2]). Additionally, it may be a candidate to explain the 511keV line [29] or the DAMA/LIBRA experiment result [26]. It can also be used for some other purposes [30].

But indirect detection, such as the anti-proton flux measured by PAMELA [31], places a stringent bound on the triplino DM. Ref. [32] finds that its mass below $\sim 800$ GeV may have been excluded already. That renders light triplino DM in scenario B problematic. But our main points discussed above are still meaningful, because they can be used to make the heavier inelastic triplino DM allowed by both PAMELA and XENON100.

D. The preliminary of LHC search

We have seen that, to enhance the di-photon rate, light doubly-charged particles appear in both scenarios. Concretely, in scenario A, we have a light doubly-charged triplinos $\chi^{++}$. In scenario B, besides a light $\chi^{++}$, a light doubly-charged Higgs boson $H^{++}$ around 100 GeV is predicted. In this subsection, as a preliminary discussion, we will briefly comment on their prospects for detecting but leave more quantitative studies elsewhere.

1. Doubly-charged triplino

At the LHC, $\chi^{++}$ can be pair produced via the Drell-Yan process or associated produced with a singly-charged triplino. For $\chi^{++}$ around 100 GeV, the production cross section can be as large as 10 pb (It quickly drops to a few 0.1 pb for a 200 GeV $\chi^{++}$) [33]. Moreover, the singly- and doubly-charged triplinos decay produce multi $W$-bosons, through the typical
decay topologies
\[ \chi_1^+ \to \chi_1^0 W^+, \quad \chi^{++} \to \chi_1^+ W^+ \to \chi_1^0 W^+ W^+. \] (27)

Some of the $W$-bosons may be off-shell, depending on the mass splitting among $\chi^{++}$, $\chi_1^+$ and $\chi_1^0$. If $m_{\chi^{++}} - m_{\chi_0} < m_W$, both $W$ bosons are off-shell. It is likely to be the case in scenario B where $T_{u,d}$ do not significantly couple to the Higgs doublets and thus mixing (or loop corrections) induced splitting is only at the GeV order. As a consequence, the resulting leptons are too soft to be detected and $\chi^{++}(\chi^+)$, if not long-lived, behaves as missing energy at the LHC. Such a pessimistic situation is not of interest.

We only consider cases with sufficiently large mass splittings. Scenario A is an example, where we typically have such a mass spectrum: $\chi_1^+$ has mass around 100 GeV and $m_{\chi^{++}} = \mu_T \sim 150 - 300$ GeV (see Fig. 3), while the LSP $\chi_1^0$ has an even smaller mass, e.g., 80 GeV. Now consider $\chi^{\pm\pm}$ pair-production, each $\chi^{++}$ decay produces missing energy and energetic same-sign leptons. Then the tetraleptons plus missing energy signature
\[ pp \to \chi^{++} \chi^{--} \to (\ell_1^+ \ell_2^+) (\ell_3^- \ell_4^-) E_{T}^{\text{miss}} \] (28)
has extremely suppressed backgrounds, say $\sim 10^{-3}$ fb [33], and thus has a very promising discovery prospect.

2. Doubly-charged Higgs boson

Scenario B predicts $m_{H^{++}} \simeq 100$ GeV, which is of interest. Recalling that even we work in the supersymmetric type II seesaw, extremely tiny couplings $(\lambda_L)_{ij} \sim \mathcal{O}(10^{-10})$ are required. As a result, the di-boson mode $H^{++} \to W^+ W^+$ is dominant in $H^{++}$ decays [43]. Such a light charged particle can be probed at LEP and the LHC. Before heading towards the main intent of this section, we show that $H^{++}$ promptly decays at colliders. Concretely, its width can be estimated in the massless limit:
\[ \Gamma(H^{++} \to W^+ f f') \sim \frac{g_2^6 v_T^2 m_{H^{++}}^3}{288 \pi^3 m_W^4}. \] (29)
Taking $m_{H^{++}} = 100$ GeV and $v_T = 1$ GeV, the resulted flying distance of the decaying $H^{++}$ is $\sim 10^{-7}$ mm. Thus, even taking into consideration the possible large phase space suppression, it is found that $H^{++}$ still promptly decays at colliders.

We now examine whether it has been excluded or not by LEP and/or LHC. The LEP II searches for the tetraleptons signature $\ell^+ \ell^+ \ell^- \ell^-$ [34] can be used to constrain $H^{++}$ with di-boson decay:
\[ e^+ e^- \to H^{--} H^{++} \to W^- W^- W^+ W^+. \] (30)
But the cross section of tetraleptons signature is suppressed by the fourth power of branching ratio of $W$ leptonic decay, and the LEP II bound is thus weak [34]. Similarly, at the LHC, the CMS and ATLAS same-sign lepton [35, 36] as well as multi-lepton plus missing energy searches only place weak bounds on $m_{H^{++}}$. Actually, in the light of a recent study [37], the present strongest bound is placed by the 7 TeV LHC with integrated luminosity 4.7 fb$^{-1}$ searching same-sign lepton [38], which gives a quite loose lower bound: $m_{H^{++}} > 60$ GeV. As the integrated luminosity accumulates to 20 fb$^{-1}$, the lower bound reaches 85 GeV. In summary, the current collider search results still allow a large parameter space for a light doubly-charged Higgs boson. Then, how to probe, discover and reconstruct such a particle at the future LHC is of interest, says via considering two hadronic $W-$bosons which produce signatures like

$$pp \rightarrow W^-W^-*W^+W^{++} \rightarrow (jjjj)(\ell_1^-\ell_2^-)E^\text{miss}_T. \quad (31)$$

We leave it in a specific work.

III. DISCUSSION AND CONCLUSION

Inspired by the relatively heavy Higgs boson mass and hints for the Higgs to di-photon excess, we, with guidance for naturalness, investigate low energy SUSY including light triplets with hypercharge ±1. It is found that the TSSM shows two attractive scenarios:

- One is the large $\lambda_u$ and $\lambda_d$ scenario, which is able to not only enhance $m_h$ but also di-photon rate. We point out that there is a tension between these two aspects, i.e., the latter tends to render the former excessively enhanced. Fortunately, the doublet-triplet mixing effects can substantially reduce $m_h$ to the desired value.

- Oppositely, the other one is characterized by negligibly small $\lambda_{u,d}$, which, despite of failing enhancing $m_h$, can provide an elegant explanation to the origin of the di-photon excess. On top of that, it provides an inelastic non-thermal DM candidate, i.e., the neutral pseudo Dirac triplino.

Both scenarios predict light doubly-charged objects, and thus we made a preliminary analysis of their discovery prospect at the LHC, based on the signatures from same-sign $W-$bosons.

We would like to end up this paper by commenting on the grand unification prospect. In the $SU(5)$-GUT, triplets $T_{u,d}$ come from the symmetric rank two tensors of $SU(5)$. Under the decomposition $SU(5) \rightarrow S(3)_C \times SU(2)_L \times U(1)_Y$, we have $15 = (1,3,1) \oplus (3,2,\frac{1}{6}) \oplus (6,1,-\frac{2}{3})$ and similarly for $\overline{15}$. Adding these light particles renders the gauge couplings non-perturbative below the GUT-scale. But interestingly it is consistent with scenario A, where
the Landau pole may be hit below the GUT-scale and thus non-perturbative unification is required [8]. As for scenario B, introducing magic fields can lead to the gauge coupling perturbative unification [14].

**Note added**

After the completion of this work, we noticed that [39] appeared on the arxiv. This paper, working in the supersymmetric type II seesaw model and focusing on the doubly-charged Higgs boson as the source of di-photon excess as well as its LHC implication, overlaps with a part of our discussion. Our results agree with each other.

**Appendix A: Yukawa couplings’ RGEs**

For using in the text, here we present the RGE running of the new Yukawa couplings \( \lambda_{u,d} \) as well as the running of relevant parameters, e.g., the Yukawa couplings \( h_t, h_b \) and \( h_\tau \).

\[
\begin{align*}
16\pi^2 \frac{d\lambda_u}{dt} &= \lambda_u \left[ 6h_t^\dagger h_t + 14\lambda_u^\dagger \lambda_u - 7g_2^2 - \frac{9}{5}g_1^2 \right], \\
16\pi^2 \frac{d\lambda_d}{dt} &= \lambda_d \left[ 6h_b^\dagger h_b + 2h_t^\dagger h_t + 14\lambda_d^\dagger \lambda_d - 7g_2^2 - \frac{9}{5}g_1^2 \right], \\
16\pi^2 \frac{dh_t}{dt} &= h_t \left[ 6h_t^\dagger h_t + h_b^\dagger h_b + 6\lambda_u^\dagger \lambda_u - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 \right], \\
16\pi^2 \frac{dh_b}{dt} &= h_b \left[ 6h_b^\dagger h_b + h_t^\dagger h_t + h_t^\dagger h_t + 6\lambda_d^\dagger \lambda_d - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2 \right], \\
16\pi^2 \frac{dh_\tau}{dt} &= h_\tau \left[ 3h_b^\dagger h_b + 4h_t^\dagger h_t + 6\lambda_\tau^\dagger \lambda_\tau - 3g_2^2 - \frac{9}{5}g_1^2 \right].
\end{align*}
\]

(A1)

here \( t = \ln \frac{\mu}{\mu_0} \) and \( \mu \) is the running scale. The presence of light triplets at the weak scale affects the MSSM gauge coupling runnings:

\[
\begin{align*}
16\pi^2 \frac{dg_1}{dt} &= \frac{51}{5}g_1^3, \\
16\pi^2 \frac{dg_2}{dt} &= 5g_2^3, \\
16\pi^2 \frac{dg_3}{dt} &= -3g_3^3.
\end{align*}
\]

(A2)

As expected, they will not show unification.

**Appendix B: The Higgs potential and mass matrix**

The total Higgs potential contains three parts, among which the \( F \)–term and soft term can be obtained in the text, while the \( D \)–terms with respect to the \( SU(2)_L \times U(1)_Y \) groups
are given by

\[ V_D = \frac{g_2^2}{2} \left( \text{Tr}(-T_u^t T_u + T_d^t T_d) + \frac{1}{2} \left( H_u^t H_u - H_d^t H_d \right) \right)^2 + \frac{g_2^2}{2} \sum_{a=1,2,3} \left[ \frac{1}{2} \text{Tr}(T_u^t [\sigma^a, T_u]) + \frac{1}{2} \text{Tr}(T_d^t [\sigma^a, T_d]) + \frac{1}{2} \left( H_u^t \sigma^a H_u - H_d^t \sigma^a H_d \right) \right]^2 \]

(B1)

Collecting all the terms and adopting in the fields decomposing as done in Eq. (9), we get the CP-even Higgs mass square matrix \( M_S^2 \) with entries (in the basis \( (h_1, h_2, h_3, h_4) \))

\[
(M_S^2)_{11} = M_A^2 + m_Z^2 \left( 1 + \frac{\lambda_u^2 + \lambda_d^2}{g^2} \right) \sin^2 2\beta,
\]

\[
(M_S^2)_{12} = -m_Z^2 \left[ \cos 2\beta \left( 1 + \frac{\lambda_u^2 + \lambda_d^2}{g^2} \right) - \frac{\lambda_u^2 - \lambda_d^2}{g^2} \right] \sin 2\beta,
\]

\[
(M_S^2)_{13} = m_Z^2 \left[ 2\lambda_u \mu \cos 2\beta/g - (A_u + \lambda_d \mu T) \sin 2\beta/g \right],
\]

\[
(M_S^2)_{14} = m_Z^2 \left[ 2\lambda_d \mu \cos 2\beta/g + (A_d + \lambda_u \mu T) \sin 2\beta/g \right],
\]

\[
(M_S^2)_{22} = m_Z^2 \left[ \cos^2 2\beta + \frac{4}{g^2} \left( \lambda_d^2 \cos^4 \beta + \lambda_u^2 \sin^4 \beta \right) \right],
\]

\[
(M_S^2)_{23} = -2m_Z M_u / g, \quad (M_S^2)_{24} = -2m_Z M_d / g,
\]

\[
(M_S^2)_{33} = m_Z^2 M_u, \quad (M_S^2)_{34} = B \mu_T, \quad (M_S^2)_{44} = m_Z^2 M_d,
\]

(B2)

where \( M_A^2 \equiv 2B \mu / \sin 2\beta \). In reality, in the heavy \( m_T^2 \) limit (similarly applied to \( T_d^0 \)), \( (M_S^2)_{33} \) can be simply written in a more clear way after using Eq. (6),

\[
(M_S^2)_{33} \doteq m_T^2.
\]

(B3)

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[40] This is a generic conclusion if the Dirac fermions running in the loop develop a $v$–dependence only via the mixing induced by electro-weak symmetry breaking.
[41] In principle, we can keep $\lambda_u \sim \mathcal{O}(1)$ to enhance $m_h$, but it is at the price of a big fine-tuning ($\sim 1\%$) to obtain a small $\nu_{T_d}$.
[42] Actually, Ref. [32] has also discussed such an inelastic DM, focusing on its aspect of indirect detection. We thank E. J. Chun for pointing their earlier work, and moreover the PAMELA constraints on the this DM, to us.
[43] Note that the decay channel $H^{++} \to H^+ W^+$ with a real $H^+$ is forbidden, since in the triplets $T_d$, the doubly-charged component is the lightest one. The reason can be explicitly found in Eq. (20), where $T^{++}_d$ receives a negative mass term from the coupling to the Higgs doublets while $T^{+}_d$, by contrast, gets a decrease. In other words, such a mass order is correlated with the fact that $T^{++}_d$ enhancing the di-photon rate while $T^{+}_d$ acts oppositely.