Abstract—Digital Twin allows creating virtual representations of complex physical systems. However, making the Digital Twin behavior matching with the real system is challenging due to the number of unknown parameters. Its search can be done using optimization-based techniques, producing a family of models based on different system datasets. So, a discrimination criterion is required to determine the best Digital Twin model. This paper presents an information theory-based discrimination criterion to determine the best Digital Twin model resulting from a behavioral matching process. The Information Gain of a model is employed as a discrimination criterion. Box-Jenkins models are used to define the family of models for each behavioral matching result. The proposed method is compared with other information-based metrics and the \( \nu \)-gap metric. As a study case, the discrimination method is applied to the Digital Twin for a real-time vision feedback infrared temperature uniformity control system. Obtained results show that information-based methodologies are useful for selecting an accurate Digital Twin model representing the system among a family of plants.

Index Terms—Digital Twin, Behavioral Matching, Model Discrimination, Information Gain, \( \nu \)-gap metric.

I. INTRODUCTION

Technologies like Artificial Intelligence (AI), Data Analytics, or Edge computing can increase the smartness in manufacturing and automatic control. Among them, Digital Twin (DT) that can be defined as a precise, virtual copies of complex systems. These models mirror almost every facet of a system [1] with applications in Unmanned Autonomous systems [2], power distribution, and smart grid [3]–[5], or smart transportation [6]. It is supported by a multidomain simulation models built based on the knowledge about each subsystem that conforms the real system as its constitutive physic laws. Considering that the Digital Twin simulation model should replicate the system actual behavior, a systematic approach to determine the unknown parameters in the DT is required. An optimization procedure can be used to find these parameters based on the system data called behavioral matching to match with the real system inputs and outputs response. Although, the optimization can return different parameters for different operating conditions of the system, producing a family of models. Some works perform the behavioral matching using metaheuristic algorithms to find the optimal parameters for power systems, ultra-precision machinery, optimal trajectory searching, or task scheduling [7]–[10]. However, there is no criteria to choose the best possible model between a set family of models. This paper introduces an information-based model discrimination method for Digital Twin behavioral matching results. It takes the behavioral matching results obtained for different operating points of the system and calculates a set of transfer function models of the Digital Twin of different complexity, which are evaluated using the Information Gain criterion proposed on [11], the normalized Akaike Information Criterion (nAIC) [12], Bayesian Information Criterion (BIC) [13], and the Minimum Description Length (MDL) to determine the model architecture with the best trade-off between complexity and overfitting. So, the \( \nu \)-gap metric [14] is also employed to select the best set of parameters based on the determined models of the system. The methodology is applied to assess the Digital Twin behavioral matching results performed for a temperature uniformity control system. The main contribution of this paper is employing information-based metrics to determine the best model during the behavioral matching process in Digital Twin applications, resulting from the presence of parametric uncertainty at different operation points of the real system.

II. INFORMATION BASED METRICS FOR BEHAVIORAL MATCHING MODELS DISCRIMINATION

A. Information Gain

Information Gain [11] is based on the Kolmogorov complexity \( K \). It is defined by (1) for a finite sequence of letters \( x \), drawn using a finite alphabet \( A \), being \( A \) the output alphabet of a computer \( F \), with \( p \) as a finite sequence of letters drawn using the input alphabet \( B \) for \( F \), with \( l(p) \) as the length of \( p \). So that the Kolmogorov Complexity is the length of the shortest program required to compute \( x \). Thus, \( K_f(x) \) is a suitable measure of the smallest amount of information to obtain \( x \). Considering that \( K(x) \) is hard to compute, it can be related with the Shannon information in a random variable \( 2 \), with \( H(x) \) as the entropy of \( x \) and \( H(x|y) \) the conditional entropy of \( x \) with respect to another random variable \( y \).

\[
K_f(x) = \left\{ \begin{array}{ll}
\min_{p} & l(p), s.t. F(p) = x \\
\infty & \text{if no such } p \text{ exist}
\end{array} \right.
\]

\[
J(y : x) = H(x) - H(x|y)
\]
So that, if \( x \) and \( y \) are random sequences from an alphabet \( A \), the algorithmic information of the sequence \( y \) regarding sequence \( x \) is given by in terms of the Kolmogorov complexity by (3), where \( I(y : x) \) is a measure of how much \( x \) relays on \( y \) for its calculation.

\[
I(y : x) = K(x) - K(x|y)
\]

This idea can be applied for model assessment, considering that the system observations can be divided in two datasets, one explained by the model \( x \) and another one that helps to explain the first dataset \( y \). Thus, the quality of the model can be judged using a program to compute \( x \) from \( y \) and measure its length bounded by \( K(x|y) \). As these value is lower, it indicates that the model represents better the system. Assume \( x \) is given by (6), with \( U \) and proposed models for the system. For any model, the length \( x \) of the system observations can be divided in two datasets, \( \hat{X} \) and \( \hat{Y} \), where \( \hat{X} = (x_1, x_2, ..., x_{\frac{N}{2}}) \) and \( \hat{Y} = (y_1, y_2, ..., y_{\frac{N}{2}}) \) for some \( m, n \geq 0, N > 0 \). Each pair of observations \( u_i, y_i \) can be coded by a small integer \( r \) representing a numeration system \((r = 2 \text{ and } r = 10 \text{ for binary and decimal})\). Besides, a model \( F \) for the system \( (u, y) \) can be defined as a computer program \( p \) that calculates the system output \( y \) based on its input \( u \). So that, \( F \) can be defined as \( (4) \) where \( C_i \) is a subset \( C_i = (A_i, B_i) \) with \( A_i = u_i \) and \( B_i = y_i \).

\[
F(p, i, C_i) = y_i^r, \quad i = 0, 1, ..., N
\]

From 4, the shortest model \( F \) of \( S \) is the one that uses the information \( I((1, C_1), (2, C_2), ..., (N, C_N), y) \) more efficiently. However, calculating \( I \) from (3) is not possible due to the unknown of \( K_F(y) \) and \( K_F(y|C) \), so only known models can be compared. For any system \( S \), a trivial model \( t \) can be defined from the beginning, by reading the output \( y \) from a look-up table. So, for any model \( p \) of \( S \), the Information Gain \( I() \) is defined by (5)

\[
I(p) = l(t) - l(p)
\]

where \( l(t) \) and \( l(p) \) corresponds to the lengths of the trivial and proposed models for the system. For any model, the length is given by (6), with \( l_{program}() \) as the length of the computer program that describes the model, and \( l_{table}() \), as the length of the lookup table.

\[
l() = l_{program}() + l_{table}()
\]

In the case of \( t \), the look-up table corresponds to the system outputs observations. For the model \( p \), the look-up table records the difference between the system output \( y \) and the estimated output \( \hat{y} \) given by the model \( p \), quantifying the error or missing behavior captured by the model.

To calculate the length of the look-up tables \( L_{table}() \) for \( t \) and \( p \), these should be codified, assuming that each element in the table corresponds to a rational number that will be scaled and represented using a numeration system \( r \). So, the code-length function \( l() \) for each \( n \) element in the table is defined by (7), were \([ \cdot ]\) represents the floor operation.

\[
L(n) = [\log_r|n|] + 1
\]

In this paper, a decimal numeration \((r = 10)\) system is used for look-up table codification, treating each table element \( n \) as a high order integer, removing decimal period and adding the corresponding sign to \( n \). For example, if \( n = 10.34 \), it is codified as " + 1034" returning a length of 5, or if \( n = -0.45 \) its codification is " - 45" returning a length of 3 always removing the leading zeros. Thus, the look-up table length is given by (8).

\[
L_{table}(n) = \sum_{i=1}^{n} L(i).
\]

Likewise, to calculate the program length \( l_{program}() \), a similar codification rule is applied, based on the number of code lines and commands required by a programming language to implement the model of the system. According to 5, some rules can be set to quantify the program length. Initially, an extended alphabet of 26 characters plus ten digits (0-9) and special symbols (\#,%,+,,,..) are considered. Each character or digit in the code increases the program length by 1. However, the variable names and reserved words of the programming language, only increase the program length by 1. In [11], the models were implemented using ALGOL68, but in this paper, the models are be implemented in Matlab. Notice that as the Information Gain of the system \( I(p) \) increases, the model \( p \) offers a better explanation of the system behavior. Dividing \( I(p) \) by \( l(t) \) return the explanation degree of the model \( p \), bounded between 0-1, where a value of 1 indicates the best level of explanation the system behavior by the model.

B. Normalized Akaike Information Criterion

According to [12], the Akaike Information Criterion (AIC) returns a measurement of the model quality like testing the model in the presence of different datasets. This criterion compares the family of models information entropy via the Kullback-Leibler divergence. Thus, the most accurate model among a family of models is the one with the smallest AIC value. This criterion penalize the complexity of the system, increasing for systems with bigger structures and number of parameters. There are different AIC criterion forms. In this paper, the normalized AIC is calculated, which is given by (9), where \( N \) is the number of samples, \( \epsilon(t) \) is a vector of the prediction errors, \( \theta_n \) is the vector of estimated parameters, \( n_y \) the number of model outputs and \( n_p \) the number of estimated parameters. In addition, the Bayesian Information Criterion (BIC) [13] can be calculated from AIC, which is given by (11).

\[
nAIC = N\log(det(\frac{1}{N}\sum_{t=1}^{N}\epsilon(t, \hat{\theta} ) (\epsilon(t, \hat{\theta} )^T ) ) + 2n_p \frac{N}{N} \]

\[
BIC = N\log(det(\frac{1}{N}\sum_{t=1}^{N}\epsilon(t, \hat{\theta} ) (\epsilon(t, \hat{\theta} )^T ) ) + N \times (n_y \times \log(2\pi) + 1) + n_p \times \log(N) \]
C. Minimum Description Length

The Minimum Description Length (MDL) is a information theory based index for evaluating model complexity, penalizing the number of parameter required to represent the system behavior [15]. MDL can be calculated using (11), where $V_{ml}$ is the loss function of the model for the estimated model parameters $\theta$, $d$ is the number of parameters in the system, and $N$ the length of the output observations vector.

$$MDL = V_{ml}(\hat{\theta}(z), z)(1 + \frac{d}{N})\ln(N)$$  \hspace{1cm} (11)

D. $\nu$ gap metric

The Vinnicombe $\nu$ gap metric [14] is a measurement of distance between two LTI dynamic systems $P_1$ and $P_2$, with right coprime factorization $P_1 = N_1M_1^{-1}$ and $P_2 = N_2M_2^{-1}$ given by (12). It can be used as a stability indicator for robust control design. The $\nu$ gap metric is always bounded between 0 and 1. As the value is close to zero the $P_1$ and $P_2$ are more similar with a stability margin degradation less than the $\nu$ gap metric value. In this paper, the $\nu$ gap metric is employed to measure the similarity between the family of models resulting from the behavioral matching, for this reason, the $\nu$ gap metric is calculated between each model from the behavioral matching, creating a triangular $\nu$ gap matrix, which, the smaller column cumulative summation will indicate the best model.

$$\delta_{\nu}(P_1, P_2) = \max_{\nu} \|(I+P_2P_1^{*})^{-\frac{1}{2}}(P_1-P_2)(I+P_1P_1^{*})^{-\frac{1}{2}}\|_{\infty}$$  \hspace{1cm} (12)

III. DEVELOPMENT FRAMEWORK FOR DIGITAL TWIN APPLICATIONS

A framework to developing Digital Twin models is shown in Fig.1. It is composed of five steps corresponding to the target system definition, system documentation, Multidomain simulation, DT assembly and behavioral matching, and the DT evaluation and deployment. In the first step, the current status of the physical system to be replicated via Digital Twin is recognized between two possible scenarios. The first one is Conceptual design, where a physical prototype is not available, and DT is employed for the initial designing task. In the second one, the physical system is operating, and DT is a supporting tool to improve system operation. In the second step, all the system available information is collected to create an accurate representation, including the control algorithms, sensors and actuators datasheets, troubleshooting and system domain knowledge. In the third step, a set of simulation models is built to represent the real system behavior, defining the system simulation domains according to the system physical, constitutive laws and the appropriated computational tools for multiphysics simulation. With the virtual models, the behavioral matching is performed. It consist of determining the unknown parameters of the system using real data collected for the system for different operating point through optimization techniques. Thus the simulation behavior of the Digital Twin can match the real asset. Finally, the Digital Twin is ready for real-life validation and deployment, running in parallel with the real system and being feed with live data streams to perform further analysis like prognosis or fault detection.

IV. STUDY CASE: REAL-TIME VISION FEEDBACK INFRARED TEMPERATURE UNIFORMITY CONTROL

The real-time vision feedback infrared temperature uniformity control presented in Fig.2 is employed in this paper as a study case for developing its Digital Twin based on the proposed framework in Section III. As shown in Fig.2, the system is composed of a Peltier cell (M1) that works as a heating element, a thermal infrared camera (M2) as a temperature feedback sensor. The LattePanda board (M3) running Windows 10 64-bits executing Matlab in hardware in the loop configuration. The power applied to the Peltier cell is managed with a power driver (M4) with Pulse Width Modulation (PWM). The platform is equipped with a battery (M5). This system employs a PID controller with antwindup for temperature uniformity control. Table.I presents a summary of the system components properties. More details about the system implementation and real tests can be found in [16], [17]. The multiphysics simulation model is presented in Fig.3. It is divided in four simulation domains. The first domain is the Electrical, composed by the power driver, the Battery and semiconductor on the Peltier module. The second one corresponds to the Thermal domain defined by the peltier heat transfer and the thermal properties of the heat sink. The third domain corresponds to the fluids, given by the airflow pumped into the heat sink to keep its temperature constant. Finally, the fourth domain corresponds to the Digital Domain, composed by the PID control algorithm and communication interfaces. In this paper, the Electric, Thermal, and Digital domains are replicated in the Digital Twin using Matlab Simulink and Simscape as multidomain simulation packages.
The Peltier module is modeled using (13)-(15), where $\alpha$ is the Seebeck coefficient, $R$ is the electrical resistance, $K$ is thermal conductance, $T_A, T_B$ are hot/cold side temperatures, $Q_A, Q_B$ are the hot/cold side thermal flows, and $I, V$ the applied voltage and current. Likewise, the dynamic change of the heat flow $Q$ in the hot side of the Peltier is given by (16), where $C$ is the specific heat of the Peltier device and $m$ is the specific mass of the module.

$$Q_A = \alpha T_A I - \frac{1}{2} I^2 R + K(T_A - T_B)$$

$$Q_B = \alpha T_B I - \frac{1}{2} I^2 R + K(T_B - T_A)$$

$$V = \alpha(T_B - T_A) + IR$$

$$Q = Cm \frac{dT}{dt}$$

The behavioral matching is required to determine the values of $\alpha, R, K, C$. Based on the Peltier datasheet, some reported experimental measurements [18], [19], and previous experience manipulating the system; the initial guess for the behavioral matching process are defined in Table II. A set of real tests is performed to get real data from the system, consisting of applying different step reference signals, as shown in Fig.4 to the system to evaluate its behavior for four different setpoints $30^\circ C, 50^\circ C, 70^\circ C$ and $90^\circ C$. The control signal $u$, the system output temperature $y$ and the reference signal $r$ are registered for each setpoint to determine $\alpha, R, K, C$. The nonlinear recursive least squares algorithm and the Matlab design optimization toolbox [20] is employed at each case to find the values of $\alpha, R, K, C$ matching the output and control action curves of the physical system with the Digital Twin. The sum of squared error is employed as a cost function for the parameter fitting problem defined by (17), where $e(k)$ are the system residuals and $N$ the number of data samples. It is important to notice that $R = 3.3\Omega$, which was physically measured. The obtained parameters $\alpha, K, C$ for each setpoint are presented in Table III. It can be observed that the Peltier thermal parameters vary among the setpoints, indicating parametric uncertainty on the system showing a significant difference with the parameters reported in Table II. As example, Fig.5 shows the Digital Twin response for $50^\circ C$ setpoint with the parameters set obtained from behavioral matching registered in Table III, confirming the presence of uncertainty also in the Digital Twin. For this reason, applying model discrimination techniques is required to determine the most accurate set of parameters for the Digital Twin.

$$F(x) = \sum_{k=0}^{N} e(k) \times e(k)$$

### A. Digital Twin Model discrimination

The information-based metrics presented in section II are employed to perform the model discrimination assessment for the Digital Twin, which requires a model of the Digital Twin to determine the nominal set of parameters of the system. Considering that the behavioral matching uses the temperature $y$ and control $u$ action of the system to determine the parameters for a specific reference signal $r$, a single-input multiple-output (SIMO) system for the Digital Twin is

### TABLE I: Thermal system documentation

| Component                        | Features                                                        |
|----------------------------------|-----------------------------------------------------------------|
| FLIR lepton Thread               | Wavelength: 8 to 14 µm Resolution: 80x60 pixels Accuracy: ± 0.5°C |
| Infrared thermal Camera          |                                                                  |
| TEC1-12706 Peltier Module         | $Q_{\text{max}} = 50W$ $\Delta T_{\text{max}} = 75^\circ C$ $I_{\text{max}} = 6.4A$ $V_{\text{max}} = 16.4V$ |
| MC33926 DC Power Driver          | Input: 0-5 V Output: 0-12V Peak Current: 5A                    |
| Lattepanda board                 | 5 inch Windows 10 64 bits PCIntel Atom µp 4GB of RAM Built-in Arduino Leonardo board |

### Fig. 3: Assembled DT multidomain simulation

### Fig. 4: Peltier system responses for different steps

### TABLE II: Peltier Thermal parameters

| Parameter | Datasheet | Measurement [19] | Experience |
|-----------|-----------|------------------|------------|
| $\alpha$  | 53 mv     | 40 mv            | 75 mv      |
| $R$       | 1.8 Ω     | 6 Ω              | 3.3 Ω      |
| $K$       | 0.5555 K/W| 0.3333 K/W       | 0.3808 K/W |
| $C$       | 15 J/K    | 15 J/K           | 31.4173 J/K|

### TABLE III: Behavioral matching results for different setpoints

| Setpoint | $30^\circ C$ | $50^\circ C$ | $70^\circ C$ | $90^\circ C$ |
|----------|--------------|--------------|--------------|--------------|
| $\alpha$ | 96.3mv       | 82.5mv       | 21.1mv       | 29.5mv       |
| $R$      | 3.3Ω         | 3.3Ω         | 3.3Ω         | 3.3Ω         |
| $K$      | 0.3K/w       | 0.35K/w      | 0.286K/w     | 0.38K/w      |
| $C$      | 34.9J/K      | 31.93J/K     | 11.1J/K      | 13.7J/K      |
parameters for a setpoint of 50°C.

![Figure 5: Digital Twin uncertainty for a setpoint of 50°C](image)

**V. CONCLUSIONS AND FUTURE WORKS**

In this paper, a model discrimination methodology was presented for Digital Twin assessment based on information criteria. It is employed to determine the most suitable parameters set after behavioral matching of a Digital Twin in the presence of parametric uncertainty. A SIMO transfer function model is employed to represent the overall behavior of the Digital Twin, choosing the most suitable model based on multiple information indices to define a model with the best trade-off between complexity and overfitting. Thus, the \( \nu \)-gap metric is be applied to determine the best set of parameters based on the optimal models of the Digital Twin. The model is proposed in Fig.6. It is composed by two transfer functions one between \( y(k)/r(k) \) and other for \( u(k)/r(k) \). The goal of this SIMO model is to consider \( y \) and \( u \) in the model assessment with the same reference signal. The order of the SIMO model should be of the lowest order possible to satisfy the Occam’s razor condition, it means reducing the model complexity to avoid overfitting. For this reason, four Box-Jenkins models (BJ) given by (18) are identified for \( B(z), C(z), F(z), D(z) \) for each set of parameters in Table III, conforming a 2x1 transfer function matrix. As example, Table IV shows the polynomial coefficients for the BJ models obtained for \( y(k)/r(k) \) and \( u(k)/r(k) \) using the second set of parameters for a setpoint of 50°C.

\[
y(z) = \frac{B(z)}{F(z)} u(z) + \frac{C(z)}{D(z)} e(z) \tag{18}\]

Now, the model discrimination criteria are calculated for the identified SIMO system for each set of parameters presented in Table III. In the case of Information Gain, each BJ model is evaluated as a difference equation employing only the transfer function part of (18). From (5), the Information Gain is given by the difference between the trivial model and the BJ model \( l(BJ) \). Likewise, the length of each program is calculated using the coding rules proposed in section 2, which is implemented in Matlab with a length of 15, being the same for all the trivial models. Regarding the look-up table for the trivial model \( t \), it is coded using the rules in section 2, and its length depends on each real setpoint response.

The implementation of BJ models is also performed in Matlab with a length of \( l(BJ) = 176 \). Considering that the same code works for any of the proposed BJ models, the code length \( l(BJ) \) keeps constant at each calculation. Regarding the look-up table, it is calculated as \( y - \hat{y} \), where \( y \) is the physical system response, and \( \hat{y} \) is the response obtained from each BJ model evaluated. Again, its length depends on \( y - \hat{y} \) and is calculated using the rules in section 2. Finally, the total Information Gain of the SIMO model is calculated as the sum of the individual Information Gains from \( y(k)/r(k) \) and \( u(k)/r(k) \). In this case, the most suitable model is the one with the higher Information Gain, it means, the one that provides more information about the system. The trivial and BJ models codes can be found in https://github.com/tartanus/Information-Gain-Criterion.

Considering that only one criterion may not be enough to choose the most suitable model for the system, the nAIC, BIC, and MDL Information Gain criteria are calculated for the SIMO system, using the expressions (9)-(12). Table V shows the calculation of the information criteria for each MISO BJ model regarding its corresponding dataset. As can be observed, the Information Gain shows that for setpoints 50°C and 90°C, a second-order BJ model is enough to represent the system dynamics, while for setpoints 30°C and 70°C, models of third and fourth-order are more representative for that specific datasets. It is important to say that the Information Gain method is sensitive to the decimal precision of the measurements and the look-up table. On the other hand, using the nAIC, BIC, and MDL criteria, the second model BJ order is the best model to represent the system dynamics. So, we can say that based on the multiple assessment metrics employed, a second-order BJ model represents the Digital Twin dynamic with the best trade-off between complexity and overfitting.

Using the best SIMO model, the next step consists of determining the nominal set of parameters of the Digital Twin, that works for multiple operating points. In that sense, the \( \nu \)-gap metric is calculated for the second-order BJ models obtained for each operating point. Thus, the set of parameters with the less cumulative \( \nu \)-gap metric determines the nominal set of parameters, considering that \( \nu \)-gap metric measures the distance between the models based on the \( H_\infty \) norm presented in (12). The obtained result of the \( \nu \)-gap metric for the second-order BJ models are 2.93, 2.74, 2.22, and 2.28 for the 30°C, 50°C, 70°C, and 90°C setpoints respectively. It can be observed that the smallest value of \( \nu \)-gap metric is given for the third set of parameters corresponding to a setpoint of 70°C. So that, we can say that these values of \( \alpha, R, K, C \) correspond to the nominal operation parameters for the Digital Twin.
assessments performed for the Digital Twin for a real-time vision infrared temperature uniformity control system shows that the estimated parameters are closer to the values reported by the manufacturer. As future works, the introduction of statistical methods like maximum likelihood, fishe information, and stochastic assessment techniques is proposed to improve the results of this method and make it more general for its application to complex systems.

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