Impact-parameter analysis of the new TOTEM pp data at 13 TeV: confirmation of the black disk limit excess.

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We revisit a discussion on the impact-parameter dependence of proton-proton elastic scattering amplitude. This analysis allows to reveal the asymptotic properties of hadron interactions. New data clearly indicate that the impact-parameter elastic scattering amplitude exceeds the black disk limit at 13 TeV c.m.s. energy of the LHC reaching a value of $\text{Im} H(s,0) = 0.568 \pm 0.001$. The inelastic overlap function reaches its maximum value at $b \simeq 0.4$ fm with a clear dip at $b = 0$. New analysis is consistent with smooth energy evolution of the elastic scattering amplitude and confirms the earlier conclusion on the black disk limit excess observed at 7 TeV.

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I. INTRODUCTION

Due to a rapid growth of experimentally available proton-proton collision energy we are provided with a unique opportunity to test the general asymptotic properties of hadronic collisions. In particular, asymptotic at $s \to \infty$ ratio of elastic to total cross-section $\sigma_{el}/\sigma_{tot}$ can range from 1/2, the so called black disk limit, corresponding to maximal elastic unitarity contribution from inelastic channel, to 1, corresponding to maximal partial amplitudes allowed by unitarity. This ratio is directly related to the value of elastic amplitude at zero impact parameter.

In 1980 U. Amaldi and K. R. Schubert [1] suggested an approach to reconstruct hadronic elastic amplitude dependence on impact parameter from differential cross-section data. It was first applied to available data at $\sqrt{s} = 546$ GeV by T. Fearnley in 1985 [2]. Only in 2014 the modified method was applied to the latest elastic pp scattering data at $\sqrt{s} = 7$ TeV [3]. The analysis yielded an intriguing result, showing, for the first time, that black disk limit is violated. Such feature of the elastic pp amplitude also casts doubt on eikonal approximation related to this limit. While at 7 TeV the effect was small, it was expected to increase with collision energy. This observation inspired a number of publications, analyzing phenomenological and theoretical consequences of elastic amplitude behavior, such as the recent talk by V.A. Petrov and A.P. Samokhin [4] and a few other examples [5, 6]. Finally, in 2018 TOTEM collaboration has presented pp differential cross-section data at 13 TeV [7] that allows us to confirm previous observation. In this paper we present the results of elastic amplitude impact parameter dependence reconstruction from new TOTEM data using an updated analysis.

II. ANALYSIS

The general approach follows that of the previous publication [3]. Here we provide a short summary and a description of updated uncertainty calculation method. The starting point is the impact-parameter representation of hadronic amplitude $H(s,b)$ defined by the transformation of the elastic scattering amplitude $A(s,t)$

$$H(s,b) = \frac{1}{8\pi s} \int_0^\infty dq q J_0 \left(\frac{qb}{k_{im}}\right) A(s,t),$$

$$A(s,t) = \frac{8\pi s}{\int_0^\infty dq b J_0 \left(\frac{qb}{k_{im}}\right) H(b,s),}$$

where $k_{im} = 0.1973269718$ GeV fm, $J_0$ is a Bessel function and $q^2 \equiv -t$. Normalization of $A(s,t)$ is chosen to be

$$\sigma_{tot} = \frac{k_{mb}}{s} \text{Im} A(s,0), \quad \frac{d\sigma}{dt} = \frac{k_{mb}}{16\pi s^2} |A(s,t)|^2$$

where $k_{mb} = 0.389379338$ mb GeV$^2$ and $\sigma_{tot}$ is the value of total pp cross-section in millibarns.

The elastic amplitude dependence on momentum transfer $t$ was first parameterized, for the experimentally available region $8 \times 10^{-4}$ GeV$^2 \leq |t| \leq 3.83$ GeV$^2$, using the original form introduced in previous analysis [3]. It is referred to as a standard parameterization

$$A(t) = s \left\{ i\alpha \left[ A_1 e^{\alpha b t/2} + (1-A_1) e^{\alpha b t/2} \right] - i A_2 e^{b t/2} - A_2 \rho (1-t/\tau)^{-4} \right\},$$

where

$$\alpha = (1-i\rho) \left( \sigma_{tot}/k_{mb} + A_2 \right)$$

$$A_1 = 0.0035 \pm 0.0013, A_2 = 0.005 \pm 0.0015, \rho = 0.7 \pm 0.3, \tau = 0.0022 \pm 0.0019.$$
and $\rho$ parameter, $\rho \equiv \text{Re} A(s,0)/\text{Im} A(s,0)$, is taken from the experiment. Additionally, the exponential parameterization was used in a general form

$$A(t) = s \times (\text{Re} A + i \text{Im} A),$$

where real and imaginary parts are

$$\text{Re} A = A_1 e^{b_1 t} + A_2 e^{b_2 t},$$

$$\text{Im} A = A_3 e^{b_1 t} + A_4 e^{b_2 t} + A_5 (1 - t/\tau)^{-4}.$$ (8)

Two variants are considered, referred to as exponential $(3+1)$ and exponential $(3+2)$ parameterizations, that correspond to cases where either $A_5$ or $A_2$ is fixed at zero. The relations between unknown constants $A_i$ were determined by requiring parameterizations to automatically satisfy equations $\text{Im} A(s,0) = s\sigma_{\text{tot}}/k_{\text{mb}}$ and $\text{Re} A(s,0)/\text{Im} A(s,0) = \rho$ for arbitrary parameter values

$$A_3 = \sigma_{\text{tot}}/k_{\text{mb}} - A_1 - A_2 - A_p,$$

$$A_4 = \rho \sigma_{\text{tot}}/k_{\text{mb}} - A_5.$$ (9)

Parameters were fitted to $d\sigma/dt$ data using expression \([3]\) with $\sigma_{\text{tot}}$ and $\rho$ considered free parameters, limited according to their experimental values and uncertainties.

**a. Data set and low-$t$ region description.** Two TOTEM data sets were combined to have the widest possible $t$ range covered. The low-$t$ data set \([3]\) is fully compatible with the most recent results for larger momentum transfer \([7]\). To improve the quality of the parameterization at low values of $t$, a Coulomb term in the simplest form \([9]\) was added to all three parameterization variants

$$A_C(t) = \frac{s}{t} \left\{ \frac{8\pi \alpha_{\text{EM}}}{(1 + t/t_0)^3} \exp \left\{ -i \left[ \alpha_{\text{EM}} \left( \gamma + \ln \left| \frac{B}{2} \right| \right) \right] \right\},$$

where $\alpha_{\text{EM}} \approx 0.007297$ is the fine-structure constant, $t_0 = 0.71 \text{ GeV}^2$, $\gamma = 0.577$ and $B = 20.4 \text{ GeV}^{-2}$ is the first cone slope of differential cross-section determined by exponential fit (the value coincides with the one obtained by TOTEM \([8]\)). We emphasize that this parameterization was chosen for its simplicity, rather than physical essence, in order to achieve better data description at low $t$ and, consequently, improve the overall fit quality. An actual analysis of the Coulomb-nuclear interference region is performed by TOTEM Collaboration \([8]\). The normalization-related systematic uncertainty was excluded from both data sets when performing the fits. Best-fit parameter values and corresponding $\chi^2$ values are given in the Table \(I\) and the fits with residuals are shown in Fig. \(I\).

**b. Imaginary part of the impact-parameter amplitude\([4]\).** Imaginary part of the elastic amplitude in the

$$d\sigma/dt$$

where $\text{Re}$ and $\text{Im}$ parts are

$$\text{Re} A = A_1 e^{b_1 t} + A_2 e^{b_2 t},$$

$$\text{Im} A = A_3 e^{b_1 t} + A_4 e^{b_2 t} + A_5 (1 - t/\tau)^{-4}.$$ (8)

While data were fitted in the full experimental region of momentum transfer, for calculating the hadronic amplitude we need to exclude the region where Coulomb contribution to cross-section is non-negligible. We define a point $t_1$ that is determined by condition $d\sigma_{\text{exp}}/dt = -1 < 0.01$, where $d\sigma_{\text{exp}}/dt$ is the experimental cross-section and $d\sigma/dt$ is the model hadronic cross-section, determined from parameterizations \([4]\) and \([6]\). Since this point differs between parameterizations, we chose the maximum value out of three which is $t_1 = 1.56 \times 10^{-2} \text{ GeV}^2$ coming from exponential $(3+2)$

\[1\] T. Fearnley \([2]\) used the profile function $\Gamma(s,b) = -2iH(s,b)$.
The parameterizations (4) and (6). The constant $A$ is set using optical theorem. Its (assuming the quoted uncertainty to be 1)

evaluations from the experimental points were propagated. At each value of impact parameter $b$, the amplitude imaginary part $\text{Im} H(s, b)$ values were fitted with the Gaussian function and the corresponding variance was used as an uncertainty estimate.

The uncertainties of the real part of impact amplitude were estimated from the standard error propagation technique

$$ \Delta \text{Re} H(s, b) = \left[ \frac{\partial \text{Re} H(s, b)}{\partial p_i} C_{ij} \frac{\partial \text{Re} H(s, b)}{\partial p_j} \right] + \left( \frac{\partial \text{Re} H(s, b)}{\partial \sigma_{\text{tot}}} \right)^2 + \left( \frac{\partial \text{Re} H(s, b)}{\partial \rho} \right)^2 \Delta \sigma_{\text{tot}}^2 \ ,$$

where $p_i$ are the fit parameters and $C_{ij}$ is the parameter covariance matrix given by minimization routine, while $\Delta \sigma_{\text{tot}}$ and $\Delta \rho$ are experimental uncertainties of total cross-section and $\rho$ parameter.

d. Fit quality. Fit quality is assessed by distribution of normalized residuals defined as

$$ r(t_i) = \frac{\frac{\text{d} \sigma}{\text{d} t} \bigg|_{t_i} - \kappa_{\text{mb}} / (16 \pi^2) \ | \text{A}_{\text{param}}(s, t_i) |^2}{\delta_{\frac{\text{d} \sigma}{\text{d} t}} \bigg|_{t_i}} ,$$

where $\text{A}_{\text{param}}$ is the particular parameterization and $\delta_{\frac{\text{d} \sigma}{\text{d} t}}$ is the experimental uncertainty in the corresponding point. Fits are presented visually together with the residuals distribution in Fig. 1.

III. RESULTS

We have considered three parameterizations, (4) and (6), for pp elastic scattering amplitude at $\sqrt{s} = 13$ TeV, in order to assess the relative importance of elastic amplitude real part $\text{Re} A(s, t)$ for $H(s, b)$ reconstruction and the overall sensitivity of the process to the particular functional form of the real part. Reconstructed functions $H(s, b)$ and $G_{\text{inel}}(s, b)$ are presented in Fig. 2a and 10. Figure 2b shows a scaled-up version of $G_{\text{inel}}(s, b)$ plot near the maximum.

The behavior of imaginary part of the amplitude as a function of $t$ is practically identical in three parameterizations used, since it is mostly determined by the differential cross-section data. Only considerable differences can be observed near the dip region. Real part, however, is less constrained, which allows us to assess its effect on the final quantities. A derived plot of $\rho(t)$ function is presented in Fig. 3 to illustrate the variances in

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2 The inelastic overlap function alone is not enough to draw conclusions about asymptotic regime as shown in previous analyses [1, 2, 3].
amplitude real part behavior. Figure 2A shows that in impact-parameter representation real part of the amplitude is different between the parameterizations, but the effect on the imaginary part behavior is negligible.

The main conclusion from comparison of the different amplitude parameterizations is the following. Influence of the amplitude real part, given by \( \rho(t) \), on the values of \( H(s, b) \) is small despite the fact that \( \rho(t) \) behavior differs significantly between the parameterizations considered. This is not surprising since \( \rho(t) \) is still small in the first cone region (\( |t| \lesssim 0.4 \text{ GeV}^2 \)), which is a main contribution in \( H(s, b) \) integral. Thus, we can conclude that the reliability of the \( H(s, b) \) extraction method is quite high. From the Figures 2A and 2B we can clearly see two effects of the black disk limit excess:

1. Imaginary part of the amplitude at \( b = 0 \) exceeds 1/2, \( \text{Im} H(s, b = 0) \rightarrow 1/2 \), and at 13 TeV the effect is much more visible than at 7 TeV [3]. The values, reached with the three parameterizations considered, are \( \text{Im} H(s, b) = 0.568 \pm 0.001 \) (exponential (3+1)), \( 0.568 \pm 0.002 \) (exponential (3+2)) and \( 0.572 \pm 0.001 \) (standard).

2. For the first time it can be reliably concluded that \( G_{\text{inel}}(s, b) \) has maximum at \( b = b_0 > 0 \). Due to the fact that the \( [\text{Re} H(s, b_0)]^2 \ll G_{\text{inel}}(s, b_0) \), the value \( b_0 \) almost coincides with the point \( b_1 \) where \( \text{Im} H(s, b_1) = 1/2 \).

We would like to note that the first results on \( H(s, b) \) at \( \sqrt{s} = 13 \text{ TeV} \) confirming the black disk limit excess were presented at the 4th Elba Workshop on Forward Physics @ LHC Energy in the talks of E. Martynov and A.D. Martin [11].

Recently some similar estimates were published [12], although without error analysis. It is claimed that effect is too small and the value they obtained, \( \text{Im} H(s, 0) = 0.513 \), is consistent with the statement that the amplitude does not exceed black disk limit [4]. Taking into account our results above we can not agree with this statement.

### IV. CONCLUSION

The black disk limit excess leads to unitarity saturation characterized by reflective scattering mode dominance [13]. Its main feature is a negativity of the elastic scattering matrix element \( S(s, b) \) (where \( b \) is an impact parameter of the colliding hadrons; note that angular momentum \( l = b \sqrt{s/2} \) leading to the asymptotic dominance of the reflective elastic scattering and peripheral form of the inelastic overlap as a function of the impact parameter. The corresponding elastic scattering decoupling from the multiparticle production occurs initially at small values of the impact parameter \( b \) expanding to larger values with increase of energy. Such behavior corresponds to increasing self-dumping of inelastic contributions to unitarity equation [14].

The \( b \)-dependence of the scattering amplitude as well as the inelastic overlap function should be considered as a collision geometry. It should be emphasized that the collision geometry describes the hadron interaction region but not the matter distribution inside of the individual colliding hadrons.

For qualitative discussion it is convenient to assume smallness of the real part of the elastic scattering amplitude in the impact parameter representation \( H(s, b) \) and substitute \( H \rightarrow iH \). This assumption is related to unitarity saturation, meaning that, at \( s \rightarrow \infty \) with \( b \), \( \text{Im} H(s, b) \rightarrow 1 \) and \( \text{Re} H(s, b) \rightarrow 0 \). However, alternatives exist where \( \text{Im} H(s, b) \rightarrow H_0 > 1/2 \) but \( \text{Re} H(s, b) \rightarrow 0 \) at \( s \rightarrow \infty \). It is claimed [3] that accounting the real part of the elastic scattering amplitude \( H \) should lead to the central dependence of \( G_{\text{inel}} \) on \( b \).
However, both previous and present analyses demonstrate that peripheral behavior of $G_{\text{inel}}$ is still observed even when the real part of the impact-parameter amplitude is non-negligible. It is also important to note that peripheral mode is achieved without $G_{\text{inel}}$ reaching unitarity limit of 1/4. This is especially visible with the parameterizations introduced in present analysis.

The present analysis confirms results of the previous one performed for lower energy of 7 TeV. We demonstrate that the elastic scattering amplitude exceeds the black disk limit at $\sqrt{s} = 7$ and 13 TeV, and that inelastic overlap function is peripheral at both energies. Results are also consistent with smooth energy dependence of the elastic scattering amplitude at LHC energies and one can conclude that reflective scattering mode dominates in the central region, that widens with energy ($b \lesssim 0.4$ fm at 13 TeV; $b \lesssim 0.3$ fm at 7 TeV).

We would like to note that the important consequence of obtained results on $H(s, b)$ and $G_{\text{inel}}(s, b)$, is that the pp and $\bar{p}p$ scattering models, based on eikonal approach with the black disk limit, must be substantially modified to be in agreement with experimental data.

FIG. 3: Ratios $\rho(t) \equiv \frac{\text{Re}A}{\text{Im}A}$ in three parameterizations for pp elastic scattering amplitude (color online).

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