Light and Strange Quark Masses with Dynamical Wilson Fermions.

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We determine the masses of the light and the strange quarks in the $\overline{MS}$-scheme using our high-statistics lattice simulation of QCD with dynamical Wilson fermions. For each of our three sea quarks we have analyzed our data at five different values of the valence quark mass, enabling us to parameterize our fit results in the $(m_{\text{sea}}, m_{\text{valence}})$ plane. For the light quark mass we find $m_{\text{light}}^{\overline{MS}}(2\,\text{GeV}) = 2.7(2)\,\text{MeV}$, which is lower than in quenched simulations. Applying a new method, which we propose to extract the strange quark mass in a sea of two dynamical light quarks, we obtain $m_{\text{strange}}^{\overline{MS}}(2\,\text{GeV}) = 140(20)\,\text{MeV}$.

1. Introduction

More than 25 years after the introduction of quarks as the constituents of hadrons the absolute values of the light quark masses, as given in the current Particle Data Book \cite{PDG}, are still among the most poorly known fundamental parameters of the Standard Model. Chiral perturbation theory (\chi PT) has been very successful in fixing the ratios of light quark masses \cite{Bijnens:2003as} but the overall scale is beyond its possibilities. QCD sum-rules \cite{Shifman:1978bx} have produced a wealth of predictions but in a large range of values and systematic errors are hard to pin down. Given the shortcomings and problems of these two methods, the lattice, as a first-principles non-perturbative method, would appear to be the most promising candidate for a reliable extraction of the absolute quark mass values. In this talk I present results of the SESAM collaboration for the masses of the light and strange quarks obtained from the currently largest and statistically most precise lattice QCD simulation with 2 flavours of dynamical Wilson fermions \cite{Hoeber:2017wcy}.

2. Method and Status

Calculating quark masses on the lattice consists of three steps: firstly, a hadron mass is matched to its experimental value to fix the bare lattice quark mass under consideration. Typically, the pseudoscalar meson mass, having the smallest statistical lattice-error of all hadrons, is used. This step also invokes \chi PT, often only to lowest order, to relate the pseudoscalar mass to the bare quark mass. Secondly, the lattice scale is set in a similar manner by matching some experimentally well known quantity such as the $\rho$ mass or the $1S - 2S$ energy level splitting in the $\Upsilon$ spectrum. Finally, perturbation theory is used to convert the bare quark mass to that of the $\overline{MS}$ scheme.

Quite a large number of simulations have been carried out in this manner in the quenched approximation of QCD. Internal fermion loops are totally neglected in these simulations for want of sufficient compute power. Recent compilations and interpretations of these results are given in \cite{Beneke:2010gr, Grayers:2015}. The authors of \cite{Beneke:2010gr, Grayers:2015} have each attempted to produce quark mass values in the limit of vanishing lattice spacing. Although the analysis differ greatly in approach and method the overall result is very much the same: the isospin symmetric light quark and the strange quark masses are both predicted by quenched lattice QCD to be smaller than the sumrule results \cite{Bijnens:2003as}. For full QCD, very few results have been obtained so far but these would seem to indicate even smaller quark masses; however, no reliable continuum extrapolation can be performed so far.
3. SESAM analysis

SESAM has recently completed the generation of gauge configurations with two dynamical Wilson fermions at three different values of the dynamical mass and for a lattice extent of $16^3 \times 32$. The bare coupling of $\beta = 5.6$ corresponds to a cutoff $a^{-1} \simeq 2.33(6)\text{GeV}$ (matching the $\rho$). 200 lattices are analyzed per sea-quark mass, giving $m_\pi/m_\rho = 0.841(6), 0.755(8),$ and $0.69(2)$. Correlators are evaluated for 5 values of the valence quark mass at each sea quark mass value.

3.1. Light Quark Mass

To extract the light quark mass from the pseudoscalar (PS) and vector (V) meson mass ratio $m_\pi/m_\rho$ we attempt to fit the masses of mesons with valence quark masses equal to sea quark masses (characterized by the argument $ss$) according to first order $\chi$PT:

$$m^2_{PS}(ss) = c m_{sea}^4, \quad m_V(ss) = m_{crit} + b m^2_{PS}.$$(1)

Figure 1 shows that this ansatz works very well (although a quadratic term in the vector data cannot be excluded). The condition

$$\frac{m_{PS}(m^4_q)}{m_V(m^4_q)} = \left(\frac{m_\pi}{m_\rho}\right)_{\exp} = 0.1785$$

yields

$$m_{\overline{MS}}^0(2\text{GeV}) = \frac{1}{2}(m_u + m_d) = 2.7(2)\text{MeV}. \quad(3)$$

The lattice result has been converted to the $\overline{MS}$ scheme using boosted 1-loop perturbation theory and the scale was set by the $\rho$-mass.

3.2. Strange Quark Mass

So far, we have used our data with equal sea and valence quark masses to extract the light quark mass in a sea of light quarks. To simulate the strange quark we need to introduce valence quarks that are unequal to the dynamical quarks. The meson masses with strange valence quarks can then be extrapolated to the sea of light quarks. To this end, encouraged by eq. 1, we

2If, instead, we match the $1S - 2S$ splitting in the $\Upsilon$, the scale turns out to be $\simeq 2.48(14)\text{GeV}$.
make the all-linear ansatz:

\[
m^2_{PS}(sv) = c'm^\text{sea}_q + d'm^\text{val}_q,
\]

\[
m^2_{PS}(vv) = c''m^\text{sea}_q + \frac{1}{2}d''(m^\text{val}_{1q} + m^\text{val}_{2q}).
\]

The arguments \(sv\) and \(vv\) denote the valence quark content so that, for example, \(vv\) means neither of the valence quark masses is equal to the sea quark mass.

Equations 4 and 5 seem to show that four additional parameters are needed to parametrize the pseudoscalar meson mass in the entire \((m^\text{sea}, m^\text{valence})\) plane. However, the consistency of equations 4 and 5 with eq. 1 can be used to reduce the number of unknown parameters to 3 and similar arguments hold for the vector particle. Thus, we attempt fits to the combined parametrization

\[
\begin{pmatrix}
  m^2_{PS,ss} \\
  m^2_{PS,sv} \\
  m^2_{PS,vv}(vv)
\end{pmatrix}
= \begin{pmatrix}
  c & 0 \\
  c - d' & d' \\
  c - d'' & d''
\end{pmatrix}
\begin{pmatrix}
  m^\text{sea} \\
  m^\text{val}.
\end{pmatrix}
\]

and similarly for the vector meson. Figure 2 displays our data and the combined fits according to this ansatz. Again, we find the parametrization to work very well but cannot exclude small contributions from higher order \(\chi\)PT.

The strange quark is now determined from the (redundant) system of conditions

\[
\begin{align*}
  \left( \frac{m_K}{m_\rho} \right)_{\text{exp}} &= 1.16, \quad \left( \frac{m_{K^*}}{m_\rho} \right)_{\text{exp}} = 0.642, \\
  \left( \frac{m_\Phi}{m_\rho} \right)_{\text{exp}} &= 1.326,
\end{align*}
\]

which has to be evaluated at \(m^\text{sea}_q = m^l_q\). All three conditions lead to consistent results for \(m^s\); we average and include the spread into the statistical error. After conversion into the \(\overline{MS}\) scheme we obtain

\[
m^s_{\overline{MS}}(2\text{ GeV}) = 140(20)\text{ MeV}.
\]

4. Discussion

Our dynamical results can be compared to the analogous quenched values at similar lattice spacing (corresponding to \(\beta_{\text{quenched}} = 6.0\)):

\[
\begin{align*}
  m^\text{light}_{\overline{MS}}(2\text{ GeV}) &= 5.5(5)\text{ MeV} \\
  m^\text{strange}_{\overline{MS}}(2\text{ GeV}) &= 166(15)\text{ MeV}
\end{align*}
\]

quenched (8)

Figure 2. Simultaneous fit of all pseudoscalar and vector data to eq. 6. Symbols: \(\star = m_{ss} - -; \; \diamond = m_{sv} - -; \; \square = m_{vv} - -\). All plots in lattice units.
Errors due to the finite lattice spacing and the finite volume are not included in \(\ref{4}, \ref{5}\) and \(\ref{8}\).

Compared to the quenched result (at the same cutoff), which is in good agreement with previous lattice calculations \(\ref{5}\), we observe a much smaller dynamical light quark mass, whereas the strange masses are compatible within errors. It is important to keep in mind that the dynamical light and strange quark masses have been obtained with two slightly different methods: the light quark is obtained in two-flavour QCD whereas evaluating the strange quark mass necessitates using valence quarks unequal to the sea quarks. This may explain why the ratio of light to strange quark masses, so tightly constrained by \(\chi PT\), is not reproduced by the two dynamical values of eqs. \(\ref{3}\) and \(\ref{7}\).

To illustrate this matter further, it is instructive to examine the values of the light quark masses at fixed sea quark masses. The surprising result is that extrapolating the valence quark to 0 is not sufficient to make the pseudoscalar masses \(m_{PS}^{ss}(sv)\) and \(m_{PS}^{sv}(vv)\) vanish at fixed sea quark masses. Instead, we need to tune the valence quark masses to negative values. The reason for this can be traced back to the definition of the quark masses, which, for Wilson-like theories, where chiral symmetry is broken, are defined relative to the critical value of the hopping parameter \(\kappa_c\). In the dynamical theory, the critical point is taken at vanishing sea quark, \(\kappa_c = \kappa_c(m_{sea} = 0)\), \(m_q = \frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)\), where the mass of the pseudoscalar truly vanishes. If instead, we define a variable, sea quark mass dependent critical kappa at each sea quark value we find much larger quark masses (5.7(4), 5.6(3), 5.4(3) MeV) which are similar to the quenched quark mass. The situation is displayed schematically in figure 3. \(\Delta_1\) is the value of the light quark mass we obtain in the full theory from \(m_{PS,ss}^2\) whereas values such as \(\Delta_2\) are obtained from fixed sea-quark measurements (partially quenched). We see from figure 3 that the light quark masses from finite sea-quark mass values do not extrapolate to the value \(\Delta_1\). Thus, to extract the light quark mass, it is necessary to work with \(m_{PS,sv}^2\); the data for \(m_{PS,sv}^2\) and \(m_{PS,vv}^2\) can be used as a consistency check but are not sufficient to obtain \(m_{light}\).

To consolidate our results we are presently calculating the quark masses using an alternative method in which the bare quark mass is defined by means of the PCAC relation,

\[
m_q = \frac{\partial_1 \langle A_4(x)P(0) \rangle}{2 \langle P(x)P(0) \rangle},
\]

(9)

where \(A\) is the axial current and \(P\) the pseudoscalar density. This method may be less sensitive to renormalisation effects as the previous one, thus giving an important cross-check.

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