Lensing by galaxies in CNOC2 fields

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ABSTRACT
We have observed two blank fields of approximately 30 × 23 arcmin^2 using the William Herschel Telescope. The fields have been studied as part of the Canadian Network for Observational Cosmology Field Galaxy Redshift Survey (CNOC2), and spectroscopic redshifts are available for 1125 galaxies in the two fields. We measured the lensing signal caused by large-scale structure, and found that the result is consistent with current, more accurate measurements.

We study the galaxy–galaxy lensing signal of three overlapping samples of lenses (one with and two without redshift information), and detect a significant signal in all cases. The estimates for the velocity dispersion of an L^*_B(z = 0) = 5.6 × 10^9 h^{-2} L_B⊙ galaxy agree well for the various samples. The best-fitting singular isothermal sphere model to the ensemble-averaged tangential distortion around the galaxies with redshifts yields a velocity dispersion of \sigma^* = 130^{+15}_{-17} \text{ km s}^{-1}, or a circular velocity of V^* = 184^{+22}_{-25} \text{ km s}^{-1} for an L^*_B galaxy, in good agreement with other studies.

We use a maximum-likelihood analysis, where a parametrized mass model is compared with the data, to study the extent of galaxy dark matter haloes. Making use of all available data, we find \sigma^* = 111 \pm 12 \text{ km s}^{-1} (68.3 per cent confidence, marginalized over the truncation parameter s) for a truncated isothermal sphere model in which all galaxies have the same mass-to-light ratio. The value of the truncation parameter s is not constrained that well, and we find s^* = 260^{+124}_{-73} h^{-1} \text{ kpc} (68.3 per cent confidence, marginalized over \sigma^*), with a 99.7 per cent confidence lower limit of 80 h^{-1} \text{ kpc}. Interestingly, our results provide a 95 per cent confidence upper limit of 556 h^{-1} \text{ kpc}. The galaxy–galaxy lensing analysis allows us to estimate the average mass-to-light ratio of the field, which can be used to estimate \Omega_m. The current result, however, depends strongly on the assumed scaling relation for s.

Key words: gravitational lensing – cosmology: observations – dark matter.

1 INTRODUCTION
The (small) differential deflection of light rays by intervening structures allows us to study the projected mass distribution of the deflectors, without having to rely on assumptions concerning the state or nature of the deflecting matter. The first attempt to detect this effect, called weak gravitational lensing, was made by Tyson et al. (1984), who tried to measure the signal induced by an ensemble of galaxies. This area of astronomy blossomed with the successful measurement of the signal induced by rich clusters of galaxies at intermediate redshifts (e.g. Tyson, Wenk & Valdes 1990; Bonnet, Mellier & Fort 1994; Fahlman et al. 1994; Squires et al. 1996; Luppino & Kaiser 1997; Hoekstra et al. 1998, for an extensive review see Mellier 1999).

These studies of rich clusters were an important first step in demonstrating the feasibility of weak-lensing analyses, but nowadays more and more studies concentrate on blank fields. For example, galaxy groups have masses intermediate between clusters of galaxies and galaxies. Hoekstra, Yee & Gladders (2001b) measured the ensemble-averaged weak-lensing signal from a sample of 50 groups identified by Carlberg et al. (2001) in the Canadian Network for Observational Cosmology Field Galaxy Redshift Survey (CNOC2).

Other applications of wide field lensing are the measurement of the lensing signal caused by large-scale structure (Bacon, Refregier & Ellis 2000; Kaiser, Wilson & Luppino 2000; van Waerbeke et al. 2000, 2001, 2002; Wittman et al. 2000; Bacon et al. 2003; Hoekstra et al. 2002a; Hoekstra, Yee & Gladders 2002b; Refregier, Rhodes &
The lensing signal induced by an individual galaxy is too low to be detected, and one has to study the ensemble-averaged signal around a large number of lenses. Redshifts for the individual galaxies are useful, because they allow a proper scaling of the lensing signal around the galaxies, and they are necessary for studies of the evolution of the mass-to-light ratio of field galaxies from lensing. Hudson et al. (1998) were the first to make use of (photometric) redshifts in their galaxy–galaxy lensing analysis of the northern Hubble Deep Field (HDF). Unfortunately, the small area covered by the limited accuracy of their results.

The analysis of commissioning data of the Sloan Digital Sky Survey (SDSS) by Fischer et al. (2000) was a major step forward. Fischer et al. (2000) detected a very significant lensing signal, demonstrating the importance of the survey for the study of galaxy haloes. More recently, McKay et al. (2001) used the available redshifts for the weak-lensing analysis. The resulting number of lenses is 70 per cent completeness limit in $R$.

### 2 DATA

The Canadian Network for Observational Cosmology Field Galaxy Redshift Survey targeted four widely separated patches on the sky to study the field population of galaxies in the Universe. Redshifts of $\sim 600$ galaxies with a nominal limit of $R_s = 21.5$ were measured, resulting in a large sample of galaxies at intermediate redshifts ($z = 0.12–0.55$). A detailed description of the survey, and the corresponding data reduction is given in Yee et al. (2000). In this paper we study the dark matter properties of the galaxies targeted by CNOC2, and to this end we make extensive use of the redshifts and multicolour photometry obtained by the CNOC2 survey.

We observed the central parts of the two CNOC2 patches 1447+09 and 2148-05 (Yee et al., in preparation) using the 4.2-m William Herschel Telescope (WHT) at La Palma. The images were taken using the prime focus camera, equipped with a thinned $2048 \times 4096$ pixels$^2$ EEV10 chip, with a pixel scale of 0.237 arcsec pixel$^{-1}$. The resulting field of view of the camera is approximately $8.1 \times 16.2$ arcmin$^2$.

The patches observed in the CNOC2 survey are much larger than the field of view of the WHT prime focus camera, and we observed a mosaic of six pointings. Table 1 lists the central positions of the observed fields and the dates of the observations. The typical integration time per pointing is 1 h in $R$ (see Table 2).

### 2.1 Data reduction

The images were flat-fielded, using a master flat-field constructed from the science exposures. The images were calibrated using observations of standard stars from Landolt (1992).

The data for each pointing typically consists of three exposures of 1200 s, which were taken with small offsets. Table 2 lists the total integration time, seeing and number of detected objects for both observed fields. Note that because of the adopted weighting scheme for the weak-lensing analysis, the ‘effective’ number of galaxies is approximately 30 per cent of the total number. The data for the 1447 field have a median seeing of $\sim 0.7$ arcsec, whereas the data for the 2148 field are somewhat worse, with a seeing of 0.85 arcsec.

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**Table 1.** Positions of the centres of the fields used for the weak-lensing analysis. The final column gives the dates of the observations.

| Field | RA       | Dec      | Date          |
|-------|----------|----------|---------------|
| 1447  | 14°47′04″| 09°12′39″| 1998 May 19–21|
| 2148  | 21°48′34″| −05°56′00″| 1997 Aug 30–Sep 2|

**Table 2.** Summary of the deep WHT imaging. Column (1): identification of the pointing; (2) total integration time; (3) median seeing; (4) number of galaxies; (5) number density of galaxies; (6) 90 per cent completeness limit in $R$.

| (1)   | (2)     | (3)     | (4)    | (5)     | (6)     |
|-------|---------|---------|--------|---------|---------|
| Field | $t_{\text{exp}}$ | Seeing | No of objects | $\bar{n}$ | $m_{\text{lim}}$ |
|-------|-------------|--------|----------------|--------|-------------|
| 1447-1| 6000        | 0.75   | 5738           | 46     | 24.3        |
| 1447-2| 3600        | 0.70   | 5545           | 44     | 24.5        |
| 1447-3| 3600        | 0.65   | 6419           | 51     | 24.8        |
| 1447-4| 3600        | 0.70   | 5570           | 45     | 24.6        |
| 1447-5| 4800        | 0.70   | 6087           | 49     | 24.5        |
| 1447-6| 3600        | 0.65   | 6826           | 55     | 24.8        |
| 2148-1| 3600        | 1.0    | 4712           | 38     | 24.2        |
| 2148-2| 3600        | 0.80   | 4976           | 40     | 24.3        |
| 2148-3| 3600        | 0.75   | 5540           | 44     | 24.2        |
| 2148-4| 4800        | 0.85   | 4804           | 38     | 24.2        |
| 2148-5| 3600        | 0.90   | 4486           | 36     | 24.1        |
| 2148-6| 3600        | 0.85   | 5110           | 41     | 24.2        |
The exposures had to be remapped before the images were combined into the final image, because of focal plane field distortions. We selected and measured the positions of stars in each of the exposures and used these as input for the IRAF tasks GEOMAP and GEOTRAN. To obtain the final images that were used for the object analysis, the remapped images were simply averaged to ensure that neither cosmic ray rejection or medianing changed the shape of the point spread function (PSF) or the galaxies in a non-linear way.

Although a remapping of the images was necessary, the camera-induced distortion is small. The WHT prime focus observer’s manual lists the coefficients to estimate the telescope distortion, which is purely radial. We also calculated the distortion from a comparison of positions of bright stars that coincide with stars from the USNO catalogue. The relatively large uncertainty (≈0.3 arcsec) in the astrometry of the USNO catalogue limits the accuracy of this approach. The results, however, agree with the distortion derived from the parameters listed in the WHT observer’s manual, and we use the latter to calculate the camera distortion used in our analysis.

We find that the induced shear is small: at most 0.66 per cent in the corners of the field. The corrected weak-lensing distortion field is obtained by subtracting the camera distortion from the observed distortions (which are corrected for PSF anisotropy), as described in Hoekstra et al. (1998).

2.2 Object detection and analysis

Our analysis technique is based on that developed by Kaiser, Squires & Broadhurst (1995) and Luppino & Kaiser (1997), with a number of modifications that are described in Hoekstra et al. (1998). We analyse each pointing separately as the seeing and PSF anisotropy vary between each exposure. After the catalogues have been corrected for the various observational effects, they are combined into a master catalogue that covers the complete field that is observed.

The first step in the analysis is to detect the faint galaxy images, for which we used the hierarchical peak finding algorithm from Kaiser et al. (1995). The peak finder also provides an estimate for the Gaussian scalelength $r_g$ of the object. We select objects that were detected with a significance $\nu > 5\sigma$ over the local sky in the combined images. This galaxies are used for the weak-lensing analysis. We also run the peak finder on the images of the single exposures. We do a coincidence test on these catalogues and classify extremely small, but very significant objects as cosmic rays, which are removed from the catalogue that is used for the object analysis.

Some faint cosmic rays may hit galaxies, and consequently might not be recognized as cosmic rays. Based on the number of cosmic ray hits, and the area covered by galaxies we find that less than 0.2 per cent of the galaxies might be affected. Also, cosmic rays only introduce additional noise in the shape measurement, but do not bias the result. Consequently, we conclude that the remaining cosmic rays have a negligible effect on our results.

For the weak-lensing analysis we select objects that are detected in at least two of the shorter exposures. The objects that are detected in only one of the shorter exposures are small faint objects, which are not useful as their shape parameters are noisy. The resulting catalogues are inspected visually in order to remove spurious detections, such as spikes from saturated stars, H\textsc{ii} regions in resolved galaxies, etc.

For all detected objects we measure the apparent magnitude in an aperture with a radius of three times the Gaussian scalelength of the object, the half-light radius and the shape parameters (polarization and polarizabilities). We also estimate the error on the polarization following Hoekstra, Franx & Kuijken (2000).

2.3 PSF correction

To measure the small, lensing-induced, distortions in the images of the faint galaxies it is important to correct the shapes for observational effects, such as PSF anisotropy and seeing: PSF anisotropy can mimic a lensing signal and the correction for seeing is required to relate the measured shapes to the real lensing signal.

To do so, we follow the procedure outlined in Hoekstra et al. (1998) (which is based on Kaiser et al. 1995; Luppino & Kaiser 1997). We select a sample of moderately bright stars from our observations. These are used to characterize the PSF anisotropy and seeing. Fig. 1 shows a typical result for one of the pointings. We fit a second-order polynomial to the observed shape parameters of the stars, and this model is used to correct for the PSF anisotropy. Fig. 1(b) also shows the residual polarization of the stars after the correction. The residuals are small indicating that we can reliably correct for the PSF anisotropy.

The next step is to correct the shapes for the circularization by the PSF. The stars that were used to study the PSF anisotropy are also used to compute the ‘pre-seeing’ shear polarizability $\Gamma^P$ (Luppino & Kaiser 1997; Hoekstra et al. 1998). The measurement of $\Gamma^P$ is very noisy, and we combine the estimates of many galaxies to reduce the noise. We also found that the size of the PSF, and thus the correction, depends on the position on the chip. The variation, however, is small: approximately 10 per cent maximum. To account for this, we bin the raw polarizabilities not only in bins of $r_g$, but also as a function of position. For a given $r_g$ we fit a second-order polynomial to the median $\Gamma^P$’s as a function of position, and use the results to compute the $\Gamma^P$ for each galaxy. Fig. 2(a) shows $\Gamma^P$ as a function of $r_g$.

Objects with small values for $\Gamma^P$ require large corrections, thus increasing the noise. The weighting scheme suggested in Hoekstra et al. (2000) already gives less weight to these objects, but in addition we exclude objects with $\Gamma^P \leq 0.1$.

3 REDSHIFTS OF LENSES AND SOURCES

For a given mass distribution, the amplitude of the weak-lensing signal is proportional to the dimensionless mass surface density

$$\kappa(x) = \frac{\Sigma(x)}{\Sigma_{\text{crit}}},$$

where the critical surface density is defined as

$$\Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_{ls} D_{ls}}{D_h D_h} = \frac{c^2}{4\pi G D_h \beta}.$$  

Here $D_{ls}$, $D_l$ and $D_h$ correspond to the angular diameter distances between the observer and the source, observer and the lens, and the lens and the source, respectively. The lensing signal depends on both the redshifts of the lenses and the sources, and the dependence on the source redshift is characterized by $\beta$, which is defined as

$$\beta = \max[0, D_h/D_s].$$  

If the redshift of the lens approaches that of the source, the amplitude of the lensing signal, which is proportional to $\beta$, decreases.

Most of the lensing signal comes from galaxies that are generally too faint to be included in redshift surveys. To relate the lensing signal to a physical mass it is necessary to know the redshift distribution of the faint background galaxies. To this end, we use photometric redshift distributions from the Hubble Deep Fields North and South (Fernández-Soto, Lanzetta & Yahil 1999). Hoekstra et al. (2000) used these results for their analysis of the distant cluster MS 1054-03 ($z = 0.83$), and concluded that these redshift distributions...
Figure 1. Shape of the PSF in one of the pointings of the 1447 field; (a) the polarizations of the stars before correction; (b) residual polarization after correcting for the PSF anisotropy using a second-order polynomial fit to the data. (c) PSF anisotropy as a function of position on the chip. The left-hand panel shows the PSF anisotropy before correction. To show the higher-order components the average polarization has been subtracted. The direction of the sticks indicates the major axis of the PSF, whereas the length corresponds to the size of the anisotropy. The right-hand panel shows the residuals in the stellar polarizations after correction.

Figure 2. (a) $P'_\gamma$ as a function of $r_\delta$ for all galaxies. Owing to seeing variations the curves for the various fields blend; (b) number of galaxies with a given $r_\delta$; (c) number of galaxies with a given value of $P'_\gamma$. For the weak-lensing analysis we restrict the sample to galaxies with $P'_\gamma \geq 0.1$.

provide a good approximation of the true distribution. We use the colours of the galaxies in the HDFs to derive their $R$-band magnitude. For each lens–source pair we compute the corresponding value of $\beta$.

The lenses considered in our analysis are at much lower redshifts than MS 1054-03, and consequently the uncertainty in the value of $\beta$ is small. Based on field-to-field variation in the redshift distribution and the uncertainty caused by the finite number of galaxies in the HDFs, we estimate that the uncertainty in $\beta$ is 2 per cent.

4 LENSING BY LARGE-SCALE STRUCTURE

One of the selection criteria for the CNOC2 fields (which are described in Yee et al. 2000) is that no known nearby rich cluster should be in the field. However, the observed fields might contain distant massive clusters. In principle, such clusters can be found in a weak-lensing analysis, provided they are massive enough (e.g. Wittman et al. 2001), even if they are ‘dark’ (see Erben et al. 2000). We note that projection effects can actually introduce spurious detections (e.g. Hoekstra 2001; White, van Waerbeke & Mackey 2002).

To investigate possible structures in the CNOC2 fields, we reconstructed maps of the dimensionless surface density, using the original Kaiser & Squires (1993) algorithm. The resulting mass maps are consistent with noise maps. We find a few $3\sigma$ peaks, but no obvious counterparts are seen in the number counts of bright galaxies. The mass maps have been smoothed with a Gaussian with a FWHM of...
l arcmin, resulting in approximately 500 independent points in each map. Therefore, one expects approximately three 3σ peaks.

It is useful to estimate the mass detection limit for our data. We assume that the cluster mass profile is well described by a singular isothermal sphere (SIS):

$$\kappa = \frac{r_E}{2r}$$

(4)

where $r_E$ is the Einstein radius. The observed scatter in the shapes of the sources results in a typical uncertainty in the Einstein radius $r_E$ of $\sim 2$ arcsec. We fit SIS models out to 5 arcmin, but note that the uncertainty in the determination of the Einstein radius does not depend significantly on this particular range. Hence, we should be able to detect a cluster with $r_E > 6$ arcsec (at the 3σ level).

The velocity dispersion of the SIS model is given in km s$^{-1}$ by

$$\sigma = 186.3 \sqrt{\frac{r_E}{P}} \text{ km s}^{-1}$$

(5)

where $r_E$ is given in arcsec. Given the redshift distribution of our sources a cluster with a velocity dispersion of 630 km s$^{-1}$ at $z = 0.2$ would be detectable, whereas a cluster at $z = 0.5$ would have to have a velocity dispersion larger than 900 km s$^{-1}$. Such rich clusters would have been detected in the CNOC2 redshift survey. A detectable cluster at $z > 0.5$ would have to be even more massive. Such massive systems are very rare, and are unlikely to be in the observed fields. Based on the mass reconstructions we find no massive clusters in the CNOC2 fields.

Although we do not detect significant structures in the CNOC2 field, any large-scale structure along the line of sight will introduce an excess alignment in the shapes of the faint galaxies (compared with a random field). The measurement of this signal provides a direct measurement of the statistical properties of the large-scale mass distribution (e.g. Blandford et al. 1991; Kaiser 1992; Bernardeau, van Waerbeke & Mellier 1997; Schneider et al. 1998).

A widely used method is to look for the excess variance in an aperture: the top-hat variance. This measurement as a function of scale can be compared with predictions from cosmological models. The measurement is difficult, and residual systematics (such as an imperfect correction for PSF anisotropy) increase the observed variance.

Since the first detections (Bacon et al. 2000; Kaiser et al. 2000; Maoli et al. 2001; van Waerbeke et al. 2000; Wittman et al. 2000) tremendous progress has been made. The most recent results are based on large data sets, and yield consistent results, despite the small statistical errors (e.g. van Waerbeke et al. 2001, 2002; Bacon et al. 2003; Hoekstra et al. 2002a, b; Refregier et al. 2002). The area covered by the two CNOC2 fields is smaller than the area covered by these studies, but it is still interesting to examine our results and compare them with the predictions: although the analysis will not provide tight constraints on the cosmology, it is a useful test of whether residual systematics are present in the data.

For the analysis, we use galaxies with apparent magnitudes of 21 < $R$ < 26 as our sample of sources. The measurements of the average distortion of the two fields are presented in Table 3. To measure the excess top-hat variance as a function of aperture radius, we tile the observed fields with apertures of a given scale, compute the variance, and subtract the contribution from the intrinsic shapes of the galaxies (e.g. Bacon et al. 2000; Kaiser et al. 2000; van Waerbeke et al. 2000, 2001; Hoekstra et al. 2002a).

The results are presented in Fig. 3. The error only includes the statistical uncertainty caused by the intrinsic shapes of the sources, and does not include the contribution from cosmic variance. Also note that the points are correlated. The measured variances are small and consistent with current, more accurate measurements.

The top-hat variance is very sensitive to residual PSF anisotropy, and can be used to test the accuracy of the correction. For instance, if we do not correct the shapes of the galaxies for PSF anisotropy, we measure an excess variance $\langle \gamma^2 \rangle = (8.4 \pm 0.6) \times 10^{-3}$ at a scale of 1 arcmin – 60 times larger than the signal presented in Fig. 3. If we subtract 90 per cent (undercorrection) or 110 per cent (overcorrection) of the PSF anisotropy, the observed variance increases by a factor of ~2. Because any residual PSF anisotropy will increase the signal, this test indicates that the adopted correction yields the minimum (or close to) variance.

## 5 GALAXY–GALAXY LENSING

We use two different methods to study the lensing signal caused by the field galaxies. First, we measure the ensemble-averaged tangential distortion around the lens galaxies (galaxy–mass correlation function), which provides a robust estimate of the lensing signal. We also perform a maximum-likelihood analysis, which is described in Section 5.3.

| Field | $(g_1)$ | $(g_2)$ | $\sigma_g$ |
|-------|--------|--------|--------|
| 1447  | 0.003 34 | 0.00144 | 0.00226 |
| 2148  | 0.00323 | –0.00140 | 0.00254 |

**Figure 3.** The observed variance of $\langle \gamma^2 \rangle$ as a function of the radius of the aperture in which the shear is averaged. Note that the points are correlated. The error bars do not include the additional uncertainty caused by cosmic variance. For reference we have displayed the expected variances for three different cosmologies: SCDM ($\Omega_m = 1.0; \Omega_L = 0; \Gamma = 0.5; \sigma_8 = 0.5$), OCDM ($\Omega_m = 0.3; \Omega_L = 0; \Gamma = 0.21; \sigma_8 = 0.85$) and $\Lambda$CDM ($\Omega_m = 0.3; \Omega_L = 0.7; \Gamma = 0.21; \sigma_8 = 0.9$).
It is important to note that the measurement of the galaxy–galaxy lensing signal is much less sensitive to small residual systematics: we measure the lensing signal that is perpendicular to the line connecting many lens–source pairs. These connecting lines are randomly oriented with respect to the PSF anisotropy, and hence suppress any residual systematics. Given the results obtained for the top-hat smoothed variance (see Section 4), we expect that the effect of residual systematics is negligible.

In Table 4 we present the three subsamples of lenses that we study. The ‘faint’ and the ‘bright’ sample have magnitude limits, and we use statistical redshift distributions for the lenses to infer the average halo properties. The ‘CNOC2’ sample has the same magnitude limits as the ‘bright’ sample, but consists only of galaxies with spectroscopic redshifts. It includes approximately half of the galaxies of the ‘bright’ sample. The ‘faint’ sample is comparable to the sample of lenses studied by Brainerd et al. (1996) who used 20 < r_{lens} < 23 and 23 < r_{source} < 24. Brainerd et al. (1996) did not use fainter sources, because they did not correct for observational distortions.

5.1 Lenses selected irrespective of redshift information

The ensemble-averaged tangential distortion as a function of radius from the lens is a well established way to present the galaxy–galaxy lensing signal (e.g. Brainerd et al. 1996; Hudson et al. 1998; Fischer et al. 2000; McKay et al. 2001). The results for the ‘faint’ and the ‘bright’ sample are shown in Fig. 4. For both samples a significant lensing signal is detected. If the measured signal is caused by gravitational lensing, no signal should be observed when the sources are rotated by \( \pi/4 \). The results of this test are shown in Figs 4(b) and (d), and indeed no signal is seen.

If the lenses are distributed randomly on the sky, the tangential distortion profile can be related directly to the ensemble-averaged mass profile of the lenses. In reality the lenses cluster, and the observed signal is the convolution of the mass profile and the galaxy correlation function: the tangential shear profile is the convolution of the mass profile of the lenses. In reality the lenses cluster, and the observed signal is the convolution of the mass profile and the galaxy correlation function: the tangential shear profile measures the galaxy–mass cross-correlation function. As a result mass estimates based on the tangential shear profile are biased high, because at large distances from the lens, one measures the mass of the lens and associated galaxies. However, the lens dominates the signal on small scales.

We fit an SIS model to the tangential distortion profiles presented in Fig. 4. We do not know the redshifts of the individual lenses in these samples, and consequently we can only determine the ensemble-averaged Einstein radius \( r_E \). For the ‘faint’ sample we find a best-fitting Einstein radius \( r_E = 0.118 \pm 0.025 \) arcsec\(^2\) and for the ‘bright’ sample we obtain \( r_E = 0.176 \pm 0.045 \) arcsec\(^2\).

We examined the effect of an imperfect correction for PSF anisotropy on the determination of the Einstein radius. Even in
the extreme case that no correction is applied the derived value is changed by only 5 per cent. In the case of a 90 or 110 per cent correction of the PSF anisotropy, the signal is changed by ~1 per cent. Based on the results obtained for the top-hat smoothed variance (see Section 4), we conclude that the effect of an imperfect correction of the PSF anisotropy is much smaller than 1 per cent, and hence is negligible.

We use the effective $\beta$ for these samples (see column 6 of Table 4) to derive a mass-weighted average velocity dispersion $\langle \sigma^2 \rangle^{1/2} = 108^{+12}_{-10} \, \text{km s}^{-1}$ for the ‘faint’ sample and $\langle \sigma^2 \rangle^{1/2} = 122^{+13}_{-10} \, \text{km s}^{-1}$ for the ‘bright’ sample. The corresponding circular velocity can be calculated using $V_c = \sqrt{2}\sigma$.

The derived values of $\langle \sigma^2 \rangle^{1/2}$ depend on the selection of the sample of lens galaxies, and one cannot compare these results with findings of other studies, given the differences in sample selection. Instead we estimate the velocity dispersion (or circular velocity) of an $L_B^*$ galaxy. We assume a scaling relation between the velocity dispersion and the luminosity of the galaxy of the form

$$\sigma \propto L_B^{1/4} \quad \text{or} \quad V_c \propto L_B^{1/2}. \quad (6)$$

We also assume that the luminosity of a lens of given mass evolves with redshift $\propto (1 + z)$ (e.g. Lin et al. 1999). With these assumptions the average value of the Einstein radius (in radians) is given by

$$\langle r_E \rangle = \frac{4\pi}{c^2} \frac{\sigma^2}{\sqrt{\frac{L_B^*}{2}}} \left( \beta \frac{L_{B}^{obs}}{1 + z} \right), \quad (7)$$

where $L_{B}^{obs}$ is the observed intrinsic luminosity of the galaxy. We also introduce $\langle r_E^0 \rangle$,

$$\langle r_E^0 \rangle = \frac{4\pi}{c^2} \sigma^2 (\beta), \quad (8)$$

which is the Einstein radius of an $L_B^*$ galaxy at the average redshift of the sample of lenses.

The redshift distributions of the sources and the ‘faint’ lens sample are derived from the photometric redshift distributions of the HDF North and South (Fernández-Soto et al. 1999). For the ‘bright’ sample we use the redshift distribution from the CNOC2 survey with the proper weighting to take into account the incompleteness of the survey. To determine the rest-frame $\beta$ luminosities we use template spectra for a range in spectral types and compute the corresponding passbands as a function of redshift and galaxy colour (this procedure is similar to that described in van Dokkum & Franx 1996).

Lin et al. (1999) have studied the field galaxy luminosity function at intermediate redshift from the CNOC2 survey, and derived $M_B^*(z) = -19.18 + 5 \log h$, which corresponds to a luminosity of $7.3 \times 10^9 h^{-2} L_{B\odot}$. With our assumed luminosity evolution, this results in $L_B^*(z = 0) = 5.6 \times 10^9 h^{-2} L_{B\odot}$. Madgwick et al. (2002) derived $M_B^* = -19.55 + 5 \log h$ from the 2dF redshift survey, which probes lower redshifts. Hence, the CNOC2 value (Lin et al. 1999) is rather low, in particular with our choice of luminosity evolution. Throughout the paper we assume that the luminosity evolves $\propto (1 + z)$, but for reference we also list the results for the no-evolution scenario, in which case $L_B^*(z = 0) = 7.3 \times 10^8 h^{-2} L_{B\odot}$.

Column 8 in Table 4 lists the derived Einstein radii of an $L_B^*$ galaxy for the ‘bright’ and the ‘faint’ sample. For the ‘bright’ sample the observed $\langle r_E \rangle$ is actually very close to $\langle r_E^0 \rangle$. For the ‘faint’ sample we find that the observed $\langle r_E \rangle = 0.74 \langle r_E^0 \rangle$. Columns 9 and 10 list the velocity dispersion and circular velocity of an $L_B^*$ galaxy under the assumption that the luminosity evolves $\propto (1 + z)$. Column 11 shows the resulting velocity dispersion without luminosity evolution. Coincidentally, the values for $\sigma_*$ for the evolving and non-evolving case are very similar. Furthermore, the results for the faint and bright sample agree well with one another.

### 5.2 Lenses with redshifts from CNOC2

Redshifts for the lens galaxies are useful because they allow a proper scaling of the signals around the lens galaxies. Intrinsically faint (and therefore low-mass) galaxies, or galaxies with redshifts comparable to the source galaxies are given lower weights. We scale the observed distortion of each galaxy, and its error, such that it corresponds to that of an $L_B^*$ galaxy at the median redshift of the ‘CNOC2’ sample

$$\frac{s_{\text{scale}}}{s_{\text{obs}}} = \left[ \frac{5.6 \times 10^9 (1 + z)}{L_B^*} \right]^{1/2} \left( \frac{0.4}{\beta} \right)^2 \frac{s_{\text{obs}}}{s_{\text{E}}}, \quad (9)$$

where we assumed that the luminosity scales with the fourth power of the velocity dispersion. The value of $\beta$ is calculated for each galaxy separately, based on its redshift and the redshift distribution of the sources. The sample of lenses consists of 1125 galaxies with spectroscopic redshifts, and $17.5 < \delta \text{red} < 21.5$. We note that, because of the selection of these galaxies, the scaling in apparent magnitude is not uniform (the completeness for the fainter galaxies is lower). Fig. 5(a) shows the resulting average tangential distortion as a function of radius around an $L_B^*$ galaxy at $z = 0.56$. A significant galaxy–galaxy lensing signal is detected.

The best-fitting SIS model to the scaled tangential distortion yields an Einstein radius $r_E = 0.196 \pm 0.047 \text{arcsec}^2$. From this result we derive an average velocity dispersion of $\sigma = 130^{+15}_{-17} \, \text{km s}^{-1}$ or circular velocity of $V_c = 184^{+24}_{-22} \, \text{km s}^{-1}$ for an $L_B^*$ galaxy. If we assume no luminosity evolution, we obtain $\sigma = 129^{+15}_{-17} \, \text{km s}^{-1}$ (with $L_B^*(z = 0) = 7.3 \times 10^9 h^{-2} L_{B\odot}$).

The number of lenses in the ‘CNOC2’ sample is half that of the ‘bright’ sample, but the signal-to-noise ratio with which the lensing signal is detected is similar. Thus redshifts for the lenses are

![Figure 5](https://academic.oup.com/mnras/article-abstract/340/2/609/1002142/0.265910021424612339164718536177878?download=true)
where $s$ is a measure of the truncation radius. The total mass of this model is finite and half of the mass is contained within $r = 3/4s$. The total mass is given by

$$M_{\text{tot}} = \frac{\pi \sigma^2}{G} \times 7.3 \times 10^{12} \left(\frac{\sigma}{100 \text{ km s}^{-1}}\right)^2 \left(\frac{s}{1 \text{ Mpc}}\right).$$

The scatter in the polarizations of the galaxies is approximately constant with apparent magnitude, and it can be well approximated by a Gaussian distribution. In that case the log-likelihood follows a $\chi^2$ distribution with the number of degrees of freedom equal to the number of free model parameters (e.g. Hudson et al. 1998), and the determination of confidence intervals is straightforward. The log-likelihood is given by the sum over the two components of the polarization $e_i$ of all the source galaxies

$$\log L = -\frac{1}{2} \sum_{i,j} \left[ e_{ij} - g_{ij}(\sigma_*, s_*) P^j_f \right]^2 / \sigma_{ej},$$

where $g_{ij}$ are the model distortions, $P^j_f$ is the shear polarizability, $e_{ij}$ are the image polarizations for the $f$th galaxy and $\sigma_{ej}$ is the uncertainty in the shape measurement of the $j$th galaxy.

In our maximum-likelihood analysis we ignore the contribution from lenses outside the field of view (e.g. Hudson et al. 1998). For small fields of view this tends to lower the resulting $\sigma_*$ and $s_*$ somewhat. The area covered by our observations is much larger than the HDF North studied by Hudson et al. (1998), and we find that the effect on our estimates of $\sigma_*$ and $s_*$ is negligible.

To scale the signals of the various lenses we again use $\sigma \propto L_{B}^{1/4}$, which is based on both dynamical and observational considerations. The situation is different for the truncation parameter $s$, because there are no observational constraints. Here we will explore several cases.

If all haloes have the same value for $s$ the total mass-to-light ratio scales as $(M/L)_{\text{tot}} \propto L_{B}^{-1/2}$ (where we assume that $L \propto \sigma^4$). Thus the mass-to-light ratios of more luminous galaxies are lower. Another option is to take $(M/L)_{\text{tot}} = \text{constant}$ for all galaxies. This choice is equivalent to taking $s \propto \sigma^{2}$ (e.g. Brainerd et al. 1996; Hudson et al. 1998). The final relation we examine is $s \propto \sigma^2$, which gives $(M/L)_{\text{tot}} \propto L_{B}^{-1/2}$.

Fig. 6 shows the results for the ‘CNOC2’ sample, for which the individual redshifts of the lenses are known. It shows the likelihood contours for the parameters $\sigma_*$ and $s_*$ jointly. The result for a constant $s$ for all galaxies is presented in Fig. 6(a), and the result for $s \propto \sigma^2$ is shown in Fig. 6(b). We omit the likelihood plot $s \propto \sigma^2$, but we list the best-fitting parameters (68.3 per cent confidence limits) for all three models in Table 5(a). The results change slightly when we assume no luminosity evolution, and the values of the parameters

![Figure 6](https://academic.oup.com/mnras/article-abstract/340/2/609/1002142)

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are listed in Table 5(d). The confidence intervals on \( \sigma^* \) (\( s_* \)) listed in Tables 5(a) and (d) are obtained by marginalizing over \( s_* \) (\( \sigma^* \)). Note that these confidence intervals cannot be inferred directly from Fig. 6 (e.g. Press et al. 1992, Numerical Recipes in C, fig. 15.6.3).

We also examined the influence of an imperfect correction for PSF anisotropy. The change in the derived velocity dispersion is approximately 1 per cent if we undercorrect or overcorrect by 10 per cent. The value of \( s_* \) is changed by \( \approx 6 \) per cent in these cases. As we have demonstrated above, the actual correction for PSF anisotropy is much better than 10 per cent, and hence we find it to be negligible.

In all cases the velocity dispersion is well constrained and in good agreement with the results from the ensemble-averaged tangential distortion. The value of the truncation parameter has significant freedom, and the value of \( s_* \) depends strongly on the assumed scaling relation. We find that the value of \( s_* \) decreases when \( s \) scales with a high power of the velocity dispersion. The minimum \( \chi^2 \) for the three models are comparable, and no scaling relation for \( s \) is preferred over the other ones. Table 5(a) also lists the best estimate and the 68.3 per cent confidence limits for the total mass and mass-to-light ratio of an \( L_B^* \) galaxy. The total galaxy mass is not well constrained, mainly because of the large uncertainty in the value of the truncation parameter \( s_* \).

The mass model uses only half of the galaxies with 17.5 \( < R < \) 21.5, and therefore it ignores the contributions of the other galaxies (they are given zero mass). If the remaining galaxies were distributed randomly, their contribution to the lensing signal is that of noise. We know, however, that galaxies are clustered. Consequently, the mass model will associate the mass of the galaxies without redshifts with the galaxies in the CNOC2 sample that have measured redshifts. To examine this in more detail we study the 'bright' sample, which is selected irrespective of redshift information. Even this selection is not ideal, neighbouring galaxies that are fainter than the applied magnitude limits are still excluded. If a substantial fraction of the mass is in these galaxies both the values of \( \sigma^* \) and \( s_* \) are overestimated.

Fig. 7 shows the resulting likelihood contours. To obtain Fig. 7 we have assumed that all haloes have the same velocity dispersion, and the same value for \( s \) in arcsec. We find \( \langle \sigma^2 \rangle^{1/2} = 117^{+14}_{-13} \) km s\(^{-1}\).
and $\langle s \rangle = 88^{+43}_{-27}$ arcsec (68.3 per cent confidence). At the average redshift of the lenses, this corresponds to $\langle s \rangle = 284^{+189}_{-171} h^{-1}$ kpc. The estimate for $\langle s \rangle$ is a weighted average over the value of $s$ for all the galaxies, which are all at different redshifts. The interpretation of the result is not straightforward for the various options for the scaling relation of $s$.

To infer the best estimates for $\sigma_*$ and $s_*$ of the bright sample one has to perform a maximum-likelihood analysis in which the redshift of each individual galaxy is a free parameter, which has to be chosen such that it maximizes the likelihood. This approach is computationally not feasible, and we use another approach to obtain estimates for $\sigma_*$ and $s_*$.

We use the same method that was used for the galaxies with redshifts, but instead we create mock redshift catalogues. The redshift distribution of the galaxies in the ‘bright’ sample has been measured by the CNOC2 survey. The redshifts of the galaxies in the mock catalogues are drawn randomly for the CNOC2 survey based on the apparent $R$ magnitude of the galaxies in the ‘bright’ sample. We take the incompleteness of the survey into account when the redshifts are drawn from the survey. This mock redshift catalogue allows us to determine the best estimates for $\sigma_*$ and $s_*$, based on a maximum-likelihood analysis of the observed distortion field. We repeat this procedure 25 times, and use the average $\chi^2$ surface. The best estimates for $\sigma_*$ and $s_*$ for the three different scaling relations for $s$ are listed in Table 5(b).

Figure 7. Likelihood contours for the velocity dispersion $\sigma_*$ of an $L_*$ galaxy, and the average value of the truncation radius $\langle s \rangle$ determined from the ‘bright’ sample. We have also indicated the physical scale when $s$ is the same for all galaxies. The contours indicate 68.3, 95.4 and 99.7 per cent confidence limits on two parameters jointly. The cross indicates the best parameters. The dashed lines give models with the physical scale when $s$ is the same for all galaxies. The contours indicate 68.3, 95.4 and 99.7 per cent confidence limits on two parameters jointly.

We verified that this procedure yields an unbiased estimate, by simulating the data and applying the procedure described above. We used the derived values of $\sigma_*$ and $s_*$ (see Table 5b) as input values for the simulations. These simulations also allow us to estimate the scatter in the measurements. Compared with the situation where the redshift of each lens is known, the lack of redshifts increases the uncertainty in $s_*$ by ~50 per cent. The uncertainty in $\sigma_*$ is increased by 30 per cent. For instance, adopting $s \propto \sigma^2$, the uncertainty increases by ~45 $h^{-1}$ kpc and the uncertainty in $\sigma_*$ increases by ~3 km s$^{-1}$.

A comparison of the results presented in Table 5(b) with the results obtained for the ‘CNOC2’ sample (Table 5a) shows that the estimates for $\sigma_*$ and $s_*$ are smaller, but consistent with one another. For the case where $s \propto \sigma^2$ the ‘bright’ sample yields $\sigma_* = 111 \pm 15$ km s$^{-1}$ and $s_* = 220^{+133}_{-100}$ $h^{-1}$ kpc, compared with $\sigma_* = 126^{+15}_{-100}$ km s$^{-1}$ and $s_* = 313^{+199}_{-150}$ $h^{-1}$ kpc for the ‘CNOC2’ sample. The agreement with the results from the direct averaging method is good.

The analysis of the ‘bright’ sample has ignored any available redshift information. To make optimal use of the available information, we redo the analysis as follows. For the galaxies with redshifts we use the observed values, but for the remaining galaxies we draw redshifts and luminosities from the CNOC2 survey randomly (as was done previously for all galaxies in the ‘bright’ sample). As before, we create mock catalogues that are analysed. This procedure improves the accuracy in the estimates of $\sigma_*$ and $s_*$. The results are presented in Table 5(c). Simulations show that the lack of redshifts for all galaxies with $19.5 < R < 21$ from CNOC2 increases the uncertainty in $\sigma_*$ by 18 per cent and the uncertainty in $s_*$ by 10 per cent, compared with the situation where full redshift information is available.

5.4 Effect of galaxy groups

It is well known that galaxies cluster, and most of the galaxies reside in groups of galaxies. If the matter in galaxy groups is associated with the haloes of the group members (i.e. these haloes are indistinguishable from the haloes of isolated galaxies) the analysis presented above gives a fair estimate of the sizes of galaxy haloes. However, if a significant fraction of the dark matter in galaxy groups is distributed in a common group halo, the interpretation of the results becomes difficult.

Fischer et al. (2000) argued that galaxy groups complicate the interpretation of the galaxy–galaxy lensing signal. They measured the lensing signal out to large projected distances, and find that the distortion does not vanish at large radii, which is caused by large-scale clustering (the angular correlation function declines relatively slowly).

Compared with Fischer et al. (2000) we have the advantage that a number of galaxy groups have been identified in the CNOC2 fields (Carlberg et al. 2001). The weak-lensing signal of these groups was studied by Hoekstra et al. (2001a). They derived a mass-weighted group velocity dispersion of $\langle \sigma^2_{\text{group}} \rangle^{1/2} = 273^{+48}_{-59}$ km s$^{-1}$.

We will use the groups to examine their effect on the galaxy–galaxy lensing results for the galaxies with redshifts from the CNOC2 survey. In addition to the galaxies, we include the lensing signal from group haloes to our mass model. The groups are placed at their observed positions, and the groups are modelled as singular isothermal spheres. To study the effect of the groups on the galaxy–galaxy lensing results we increase the velocity dispersion of the ‘groups’ from 0 km s$^{-1}$ (the result without groups) up to 300 km s$^{-1}$. For each choice of the group velocity dispersion we perform a maximum-likelihood analysis and determine the best estimates for $\sigma_*$ and $s_*$. For $\sigma_{\text{group}} < 150$ km s$^{-1}$ the best estimates for $\sigma_*$ and $s_*$ vary by a few per cent. For larger group velocity dispersions the values for $\sigma_*$ and $s_*$ decrease slowly with increasing $\sigma_{\text{group}}$. For a group velocity dispersion of 300 km s$^{-1}$, the value of $\sigma_*$ has dropped by ~10 per cent, and the value of $s_*$ has decreased by ~20 per cent. We note that the minimum $\chi^2$ has increased significantly for $\sigma_{\text{group}} = 300$ km s$^{-1}$, compared with $\sigma_{\text{group}} = 0$ km s$^{-1}$.
5.5 Comparison with other lensing studies

Other studies selected different samples of lenses, used different scaling relations, or made other assumptions concerning the luminosity evolution. Fortunately, sufficient information is available to allow for a useful, if crude, comparison. We will compare the findings of some other studies with the results from our maximum-likelihood analysis.

The typical lens galaxy in the sample studied by Hudson et al. (1998) is at \( z \approx 0.6 \) and has \( M_B = -18.5 + 5 \log h \) (for \( q_0 = 0.5 \)). Hudson et al. (1998) derived a circular velocity of \( 210 \pm 40 \) km s\(^{-1}\). This corresponds to a galaxy with a luminosity of \( L_{\text{B}}(z = 0) \approx 3.2 \times 10^9 h^{-2} L_{\odot} \) for our choice of cosmology and luminosity evolution. Our weak-lensing analysis suggests a circular velocity of \( V_c = 113 \pm 12 \) km s\(^{-1}\) for such a galaxy. If we assume no luminosity evolution (as Hudson et al. 1998 did), we obtain a circular velocity of \( V_c = 130 \pm 14 \). The two results are inconsistent at the \( \sim 2 \sigma \) level. It is not clear, however, how to interpret the difference in circular velocity, as our weak-lensing analysis probes a larger physical scale than that used by Hudson et al. (1998), and the mix of galaxy types might also be different.

Fischer et al. (2000) measured the galaxy–galaxy lensing signal with much higher accuracy than Hudson et al. (1998) from SDSS data. Fischer et al. (2000) find for their sample of lenses an average mass-weighted velocity dispersion of \( \langle \sigma^2 \rangle^{1/2} \approx 145–195 \) km s\(^{-1}\) (95 per cent confidence). McKay et al. (2001) noted that the SDSS redshift survey showed that the lens redshifts used by Fischer et al. (2000) were overestimated by \( \sim 35 \) per cent. The correct range in line-of-sight velocity dispersion of the Fischer et al. (2000) analysis should be \( \langle \sigma^2 \rangle^{1/2} \approx 105–145 \) km s\(^{-1}\).

Most of the signal comes from galaxies with luminosities around \( L_{\text{B}}(z = 0) \approx 8.7 \times 10^9 h^{-2} L_{\odot} \) (Fischer, private communication). If we use an average \( B - V = 0.55 \) and the transformations from Fukugita et al. (1996), we find that the adopted B-band luminosity of our \( L^*_\text{B} \) galaxy corresponds to a \( g^\prime \)-band luminosity of \( L_{\text{g}^\prime}(z = 0) \approx 6 \times 10^9 h^{-2} L_{\odot} \). Our lensing analysis implies a velocity dispersion of \( \sigma \approx 122^{+15}_{-12} \) km s\(^{-1}\) (68.3 per cent confidence) for a galaxy with \( L_{\text{B}}(z = 0) \approx 8.7 \times 10^9 h^{-2} L_{\odot} \), which agrees well with the result of Fischer et al. (2000).

McKay et al. (2001) used a sample of galaxies with spectroscopic redshifts from the SDSS. Their sample of lenses is comparable in size to that used by Fischer et al. (2000). Using the complete sample, McKay et al. (2001) find a velocity dispersion of \( \sigma = 100–130 \) km s\(^{-1}\) (95 per cent confidence) with a best-fitting value of \( \sigma = 113 \) km s\(^{-1}\) for a galaxy with \( L_{\text{B}}(z = 0) \approx 9 \times 10^9 h^{-2} L_{\odot} \). For such a galaxy, our lensing analysis yields \( \sigma = 122^{+15}_{-12} \) km s\(^{-1}\) (68.3 per cent confidence), in excellent agreement with McKay et al. (2001).

Fischer et al. (2000) and McKay et al. (2001) also attempted to constrain the sizes of galaxy haloes. Both studies indicate that the galaxy haloes are large. Unfortunately, it is not clear what the correct value for the Fischer et al. (2000) result is. McKay et al. (2001) used the same approach as Fischer et al. (2000) and find a lower limit of \( s_{\text{min}} = 230 h^{-1} \) kpc (95 per cent confidence). McKay et al. (2001) do not use any scaling for \( s \) (they assume all haloes are the same) and their results should be compared with our results for constant \( s \) for all galaxies. We obtain an estimate of \( s = 337^{+130}_{-100} h^{-1} \) kpc, which is in good agreement with McKay et al. (2001).

Interestingly, McKay et al. (2001) find that the mass within an aperture of radius \( 260 h^{-1} \) kpc scales in proportion to the luminosity of the lens. This appears to be in contradiction with the naive expectation from the Tully–Fisher (TF) or Faber–Jackson relation, which suggests \( M \propto \sqrt{L} \). However, the latter is no longer true for truncated haloes. As discussed in Section 5.3, the relation between the mass and luminosity on large scales depends on the adopted scaling relation for \( s \) (at large radii \( M \propto \sigma^2 s \)). For instance, \( s \propto \sigma^2 \) results in \( M \propto L \) at radii (much) larger than \( s \).

Hence, the results presented by McKay et al. (2001) suggest that the truncation is smaller than \( 260 h^{-1} \) kpc. This is likely to be the case for the faint lenses, but the brighter lenses are expected to have larger haloes. However, luminous galaxies are clustered more strongly (e.g. Norberg et al. 2002) and the contribution from neighbouring galaxies could bias the lensing signal somewhat high, although the effect is expected to be small (McKay et al. 2001).

Recently, Yang et al. (2003) used numerical simulations to study the results presented by McKay et al. (2001). The galaxies in the simulations obey the TF or Faber–Jackson relation. Yang et al. (2003) find that the aperture mass within \( 260 h^{-1} \) kpc is proportional to the luminosity of the lens. The results from Yang et al. (2003), however, suggest that the linear relation found by McKay et al. (2001) is a coincidence, because they find that the halo mass scales \( \propto L_{\text{B}}^{1/2} \) (which corresponds to \( s \propto \sigma^2 \)). The mass with an aperture of radius \( 260 h^{-1} \) kpc results in a linear relation, whereas other apertures would yield different results.

Finally, we note that the results presented here are in good agreement with preliminary results from an analysis of R-band imaging data from the red-sequence cluster survey (RCS). Hoeftstra, Yee & Gladders (2002d) use lenses with \( 19.5 < R < 21 \). Their results imply \( s = 102 \pm 4 \) km s\(^{-1}\) and \( s = 198 \pm 25 h^{-1} \) kpc for an \( L_{\text{g}^\prime}(z = 0) = 5.6 \times 10^9 h^{-2} L_{\odot} \) galaxy (adopting \( s \propto \sigma^2 \)), which is in good agreement with our results.

5.6 Comparison with the TF relation

Comparison of the luminosity and the rotation velocity of gas in spiral galaxies has shown that there exists a tight relation between these two observables: the TF relation. Note, however, that weak-lensing probes the mass on a much larger scale than the rotation curve. For example, if the rotation curve is slowly declining, the lensing result will be lower (compared with the flat rotation curve). Furthermore, the sample of lenses used here is different from the sample of spiral galaxies used for the TF relation. Ideally one would like to select a sample of spiral galaxies for this comparison, but this is beyond the scope of this paper. Nevertheless, it is interesting to compare our lensing result with a local determination of the TF relation.

One of the most recent determinations of the TF relation was presented by Verheijen (2001), who studied a large sample of galaxies in the Ursa Major cluster of galaxies. The sample has the advantage that the galaxies are at a distance of 18.6 Mpc (Tully & Pierce 2000). This allowed Verheijen (2001) to study the scatter in the TF relation.

Verheijen (2001) has measured \( V_{\text{dyn}} \), the rotation velocity of the ‘flat’ part of the rotation curve, which is representative for the mass
of the galaxy. As is customary in TF studies, Verheijen (2001) used extinction-corrected luminosities. The luminosities used in our lensing analyses, however, have not been corrected for inclination and extinction. To allow a direct comparison with the weak-lensing results, we use the uncorrected magnitudes from Tully et al. (1996). The resulting TF relation is presented in Fig. 8 (open dots with error bars).

The hatched region indicates the 1σ interval around the TF relation, based on the scatter in the observations. In the comparison we use the circular velocity corresponding to the value of \( \sigma_z \) (assuming \( s \propto \sigma^3 \)) listed in Table 5(c). We assume that \( V_{\text{eff}} = \sqrt{2} \sigma_z \). The result is indicated by the large black dot. The comparison depends on the value of the Hubble parameter, because the inferred luminosities of the lenses in our analysis depend on \( h \), whereas the absolute magnitudes of the galaxies from Verheijen (2001) are based on the adopted distance to the Ursa Major cluster (and therefore do not depend on the Hubble parameter). For the comparison presented in Fig. 8 we adopted a value of \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \). Although one cannot make a direct comparison between the weak-lensing results and the TF relation (for reasons mentioned above) it is comforting that the results are rather similar.

### 6 Mass-to-Light Ratio and \( \Omega_m \)

A useful method to estimate the matter density of the Universe was proposed by Oort (1958): \( \Omega_m \) is the product of the average mass-to-light ratio of the Universe, and its luminosity density. Carlborg, Yee & Ellingson (1997) measured the mass-to-light ratios of rich clusters of galaxies, and inferred \( \Omega_m = 0.19 \pm 0.06 \). The galaxy properties of rich clusters are different from those of the field, and a large correction is needed to relate the cluster mass-to-light ratio to that of the field. Smaller corrections are required when galaxy groups are used. Hoekstra et al. (2001a) derived \( \Omega_m = 0.19 \pm 0.1 \) from their weak-lensing analysis of 50 groups. Bahcall & Comerford (2002) found \( \Omega_m = 0.17 \pm 0.05 \) from a combination of clusters and groups.

A measurement of the average mass-to-light ratio of field galaxies has the advantage that no additional corrections for the difference in stellar populations are required. An estimate for the average mass-to-light ratio of the field, i.e. the Universe as a whole, can be obtained from the results of our maximum-likelihood analysis. The inferred total mass-to-light ratio, however, depends on the assumed scaling relation for the truncation parameter \( s \) (as is apparent from column 7 in Table 5).

Lin et al. (1999) have determined the luminosity function of field galaxies from the CNOC2 survey, and we use their results to estimate the average mass-to-light ratio of the field. The results are listed in Table 6 (column 4). When \( s \propto \sigma^3 \), the average mass-to-light ratio equals that of an \( L^*_B \) galaxy. In the case where all haloes have the same value for \( s \), we have already found that the resulting mass-to-light ratio of an \( L^*_b \) galaxy is high, but in addition, fainter galaxies have even higher mass-to-light ratios. This results in a very high field mass-to-light ratio. On the other hand, if \( s \propto \sigma^4 \), faint galaxies have low mass-to-light ratios, and the field mass-to-light ratio is somewhat lower than that of an \( L^*_B \) galaxy.

To estimate the luminosity density of the Universe we use the results from Lin et al. (1999), which are also based on the CNOC2

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**Figure 8.** Plot of rotation velocity of the ‘flat’ part of the rotation curve \( V_{\text{eff}} \) versus the absolute B-band magnitude (uncorrected for inclination and extinction). The open dots with error bars indicate the measurements of Verheijen (2001), using the (uncorrected) magnitudes from Tully et al. (1996). The hatched region indicates the 1σ interval around the derived TF relation. In the comparison we use the circular velocity corresponding to the value of \( \sigma_z \) (assuming \( s \propto \sigma^3 \)) listed in Table 5(c). The result is indicated by the large black dot. The comparison depends on the value of the Hubble parameter, because the inferred luminosities of the lenses in our analysis depend on \( h \), whereas the absolute magnitudes of the galaxies from Verheijen (2001) are based on the adopted distance to the Ursa Major cluster (and therefore do not depend on the Hubble parameter). For the comparison presented in Fig. 8 we adopted a value of \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \). Although one cannot make a direct comparison between the weak-lensing results and the TF relation (for reasons mentioned above) it is comforting that the results are rather similar.

**Table 6.** Estimates for the average field mass-to-light ratio for different scaling relations of the truncation parameter \( s \). The errors correspond to 68.3 per cent confidence. (1) The scaling relation used for the truncation parameter \( s \); (2) the dependence of the total mass-to-light ratio on the luminosity; (3) total mass-to-light ratio of an \( L^*_B \) galaxy; (4) average field mass-to-light ratio at \( z = 0 \); (5) average field mass-to-light ratio at the average redshift of the sample of galaxies (\( z = 0.34 \)); (6) estimate for the matter density of the Universe \( \Omega_m \).

| (1) | (2) | (3) | (4) | (5) | (6) |
|-----|-----|-----|-----|-----|-----|
| Scaling \( s \) | Scaling \( M/L_B \) | \( M_{\text{tot}}/L_B (z = 0) \) \((hM/L_B)_{\odot}\) | \( (M/L_B)(z = 0) \) \((hM/L_B)_{\odot}\) | \( (M/L_B)(z = 0.34) \) \((hM/L_B)_{\odot}\) | \( \Omega_m \) |
| \( \propto \sigma^0 \) | \( \propto 1/\sqrt{L_B} \) | 500 \( ^{+214}_{-145} \) | 850 \( ^{+364}_{-243} \) | 634 \( ^{+272}_{-181} \) | 0.69 \( ^{+0.30}_{-0.20} \) |
| \( \propto \sigma^2 \) | Constant | 393 \( ^{+214}_{-125} \) | 393 \( ^{+214}_{-125} \) | 293 \( ^{+160}_{-93} \) | 0.32 \( ^{+0.17}_{-0.10} \) |
| \( \propto \sigma^4 \) | \( \propto \sqrt{L_B} \) | 286 \( ^{+170}_{-89} \) | 240 \( ^{+150}_{-75} \) | 168 \( ^{+112}_{-56} \) | 0.18 \( ^{+0.06}_{-0.05} \) |
Lensing by galaxies in CNOC2 fields

7 CONCLUSIONS

We observed two blank fields of approximately 30 × 23 arcmin$^2$ using the William Herschel Telescope. The fields were studied as part of the Canadian Network for Observational Cosmology Field Galaxy Redshift Survey (e.g. Yee et al. 2000; Carlberg et al. 2001), and spectroscopic redshifts are available for 1125 galaxies in the two fields. Earlier results on groups of galaxies, based on these data, were presented by Hoekstra, Franx & Kuijken (2000).

We have measured the lensing signal caused by large-scale structure. We observed that the signal is low and consistent with more accurate measurements from much larger surveys (e.g. Bacon et al. 2003; Hoekstra et al. 2002a; Refregier et al. 2002; van Waerbeke et al. 2002).

We examined the effect of an imperfect correction for PSF anisotropy, and find that the correction used here has worked well: it gives the smallest cosmic shear signal. In addition we find that the uncertainty in the correction has a negligible effect on the galaxy–galaxy lensing results.

We have studied the ensemble-averaged tangential distortion (galaxy–mass correlation function) around three subsamples of lenses. In all three cases a clear lensing signal is detected. We relate the lensing signal to an estimate of the velocity dispersion of an $L_B^*$ galaxy, and find that the results for the three samples agree well with each other. For the sample of galaxies with redshifts from the CNOC2 survey we derive $\sigma_*$ = 130$^{+22}_{-15}$ km s$^{-1}$ (or $V_+^\ast$ = 184$^{+22}_{-23}$ km s$^{-1}$). Note, however, that this measurement is slightly biased, because of the clustering of the lenses.

To study the properties of the dark matter haloes surrounding the lens galaxies, we used a maximum-likelihood analysis. This technique allowed us to derive constraints on both the velocity dispersion and the extent of the dark matter haloes (it naturally accounts for the clustering of the lenses). The value of the truncation parameter $s$ depends strongly on the assumed scaling relation, but the velocity dispersion is well constrained. Galaxy groups can complicate the interpretation of the truncation parameter $s$. We examined how galaxy groups affect our results, and we find that their effect is relatively small. Our data are not sufficient for a detailed study, but the full CNOC2 data set can be useful to this end, and studies of numerical simulations.

Under the assumption that all galaxies have the same total mass-to-light ratio, we find a value of $\sigma_*$ = 111 ± 12 km s$^{-1}$ (68 per cent confidence, marginalized over the truncation parameter $s$) for the velocity dispersion of an $L_B^*$ galaxy. For the truncation parameter we obtain $s_0 = 260^{+224}_{-51}$ h$^{-1}$ kpc (68 per cent confidence, marginalized over $\sigma_*$), with a 99.7 per cent confidence lower limit of 80 h$^{-1}$ kpc and a 95 per cent confidence upper limit of 556 h$^{-1}$ kpc. For this model we find that the average field mass-to-light ratio at $z = 0$ is $393^{+152}_{-175}$ M$_\odot$/L$_{B^*}$, comparable to what is found for rich clusters of galaxies (Carlberg et al. 1997) and galaxy groups (Hoekstra et al. 2001a).

The field mass-to-light ratio provides an estimate of $\Omega_m$, the matter density of the Universe. The results from the weak-lensing analysis provide a lower limit on $\Omega_m$, because lensing cannot detect dark matter that is distributed uniformly throughout the Universe. Unfortunately, our estimate of the field mass-to-light ratio, and consequently the derived value of $\Omega_m$, depends strongly on the assumed scaling of $s$.

The results presented here demonstrate that weak lensing is a powerful tool for studying the dark matter haloes of field galaxies, as it can probe the mass distribution out to large projected distances. In this paper we have used approximately one-quarter of the CNOC2 survey. An analysis of the full survey will improve the accuracy of the results by a factor of ~2. Furthermore, many large imaging surveys are currently underway. First results from the red-sequence cluster survey indicate that these surveys can provide accurate constraints on the dark matter haloes of field galaxies (e.g. Hoekstra, Yee & Gladders 2002d).

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REFERENCES

Bacon D., Refregier A., Ellis R.S., 2000, MNRAS, 325, 1065
Bacon D., Massey R., Refregier A., Ellis R., 2003, MNRAS, submitted (astro-ph0203134)
Bahcall N.A., Cornfield J.M., 2002, ApJ, 565, 5
Bernardeau F., van Waerbeke L., Mellier Y., 1997, A&A, 322, 1
Blandford R.D., Saust A.B., Brainerd T.G., Villumsen J.V., 1991, MNRAS, 251, 600
Bonnet H., Mellier Y., Fort B., 1994, ApJ, 427, L83
Brainerd T.G., Blandford R.D., Smail I., 1996, ApJ, 466, 623
Carlberg R.G., Yee H.C., Ellington C., 1997, ApJ, 478, 462
Carlberg R.G., Yee H.K.C., Morris S.L., Lin H., Hall P.B., Patton D.R., Sawicki M., Shepherd C.W., 2001, ApJ, 552, 427
Dell’Antonio I.P., Tyson J.A., 1996, ApJ, 473, L17
Erben T., van Waerbeke L., Mellier Y., Schneider P., Cuillandre J.-C., Castander F.J., Daniel-Fort M., 2000, A&A, 355, 23
Fahnlin G., Kaiser N., Squires G., Woods D., 1994, ApJ, 437, 56
Fernández-Soto A., Lanzetta K.M., Yahil A., 1999, ApJ, 513, 34
Fischer P. et al., 2000, AJ, 120, 1198
Fukugita M., Ichikawa T., Gunn J.E., Doi M., Shimasaku K., Schneider D.P., 1996, AJ, 111, 1748
Gorenstein M.V., Shapiro I.I., Falco E.E., 1988, ApJ, 327, 693

© 2003 RAS, MNRAS 340, 609–622
Griffiths R.E., Casertano S., Im M., Ratnatunga K.U., 1996, MNRAS, 282, 1159
Hoekstra H., 2001, A&A, 370, 743
Hoekstra H., Franz M., Kuijken K., Squires G., 1998, ApJ, 504, 636
Hoekstra H., Franz M., Kuijken K., 2000, ApJ, 532, 88
Hoekstra H. et al., 2001a, ApJ, 548, L5
Hoekstra H., Yee H.K.C., Gladders M.D., 2001b, ApJ, 558, L11
Hoekstra H., Yee H.K.C., Gladders M.D., Barrientos L.F., Hall P.B., Infante L., 2002a, ApJ, 572, 55
Hoekstra H., Yee H.K.C., Gladders M.D., 2002b, ApJ, 577, 604
Hoekstra H., van Waerbeke L., Gladders M.D., Mellier Y., Yee H.K.C., 2002c, ApJ, 577, 595
Hoekstra H., Yee H.K.C., Gladders M.D., 2002d, New Astron. Rev., 46, 767
Hudson M.J., Gwyn S.D.J., Dahle H., Kaiser N., 1998, ApJ, 503, 531
Kaiser N., 1992, ApJ, 388, 272
Kaiser N., Squires G., 1993, ApJ, 404, 441
Kaiser N., Squires G., Broadhurst T., 1995, ApJ, 449, 460
Kaiser N., Wilson G., Luppino G.A., 2000, ApJ, submitted (astro-ph/0003338)
Landolt A.U., 1992, AJ, 104, 340
Lin H., Yee H.K.C., Carlberg R.G., Morris S.L., Sawicky M., Patton D.R., Wirth G., Shepherd C.W., 1999, ApJ, 518, 533
Luppino G.A., Kaiser N., 1997, ApJ, 475, 20
McKay T.A. et al., 2001, ApJ, submitted (astro-ph/0108013)
Madgwick D.S. et al., 2002, MNRAS, 333, 133
Maoli R., van Waerbeke L., Mellier Y., Schneider P., Jain B., Bernardeau F., Erben T., Fort B., 2001, A&A, 368, 766
Mellier Y., 1999, ARA&A, 37, 127
Norberg P. et al., 2002, MNRAS, 332, 827
Oort J.H., 1958, in Stoops R., ed., La Structure et L’évolution de L’Univers, Onzième Conseil de Physique. Solvay, Bruxelles, p. 163
Press W.H., Teukolsky S.A., Vetterling W.T., Flannery B.P., 1992, Numerical Recipes in C. Cambridge Univ. Press, Cambridge
Refregier A., Rhodes J., Groth E.J., 2002, ApJ, 572, L131
Schneider P., Rix H.-W., 1997, ApJ, 474, 25
Schneider P., van Waerbeke L., Jain B., Kruse G., 1998, MNRAS, 296, 873
Smith D.R., Bernstein G.M., Fischer P., Jarvis M., 2001, ApJ, 551, 643
Squires G., Kaiser N., Babul A., Fahlman G., Woods D., Neumann D.M., Boehringer H., 1996, ApJ, 461, 572
Tully R.B., Pierce M.J., 2000, ApJ, 533, 744
Tully R.B., Verheijen M.A.W., Pierce M.J., Huang J.-S., Wainscoat R.J., 1996, AJ, 112, 2471
Tyson J.A., Valdes F., Jarvis J.F., Mills A.P., Jr, 1984, ApJ, 281, L59
Tyson J.A., Wenk R.A., Valdes F., 1990, ApJ, 349, 1
van Albada T.S., Sancisi R., 1986, R. Soc. London Phil. Trans., 320, 447
van Dokkum P.G., Franx M., 1996, MNRAS, 281, 985
van Waerbeke L. et al., 2000, A&A, 358, 30
van Waerbeke L. et al., 2001, A&A, 374, 757
van Waerbeke L., Mellier Y., Pello R., Pen U.-L., McCracken H.J., Jain B., 2002, A&A, 393, 369
Verheijen M.A.W., 2001, ApJ, 563, 694
White M., van Waerbeke L., Mackey J., 2002, ApJ, 575, 640
Wilson G., Kaiser N., Luppino G.A., 2001, ApJ, 556, 601
Wittman D.M., Tyson J.A., Kirkman D., Dell’Antonio I., Bernstein G., 2000, Nat, 405, 143
Wittman D., Tyson J.A., Margoniner V.E., Cohen J.G., Dell’Antonio I.P., 2001, ApJ, 557, 89
Yang X.-H., Mo H.J., Kauffmann G., Chu Y.-Q., 2003, MNRAS, submitted (astro-ph/0205546)
Yee H.K.C. et al., 2000, ApJS, 129, 475
Zaritsky D., White S.D.M., 1994, ApJ, 435, 599

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