Axial Couplings on the World-Line

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October 16, 1995

Abstract

We construct a world-line representation for the fermionic one-loop effective action with axial and also vector, scalar, and pseudo-scalar couplings. We use this expression to compute a few selected scattering amplitudes. These allow us to verify that our method yields the same results as standard field theory. In particular, we are able to reproduce the chiral anomaly. Our starting point is the second order formulation for the Dirac fermion. We translate the second order expressions into a world-line action.

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Recently it has been argued \cite{1, 2, 3, 4, 5, 6} that conventional quantum field theory, at least in some problems, can be profitably substituted by a first quantized formalism dealing with relativistic particles on a world-line. This is a very old idea \cite{7, 8, 9, 10}, but for a long time it was only applied to the description of one-loop determinants and propagators, whereas in ref. \cite{3, 4} a unified description for whole classes of higher loop Feynman graphs was proposed. The Bern-Kosower formalism \cite{11, 12, 13}, which triggered much of the recent research in this subject, was derived from string theory. It beautifully simplifies tree and one-loop calculations in field theory but, unfortunately, becomes very difficult to develop for higher loops. This is one of the reasons why Strassler’s observation \cite{1} that their one-loop rules can already be derived in the first quantized relativistic particle approach, \textit{i.e.}, in a one-dimensional QFT on the world-line, is extremely useful. Indeed, already in the calculation of one-loop effective actions this leads to impressive simplifications \cite{2, 14} compared to a conventional heat kernel approach. Higher loop-calculations \cite{3, 4}, even though still at a preliminary stage, look very promising.

In order to perform the analog of conventional QFT calculations it is necessary to know the proper world-line Lagrangian. Whereas for scalar $\phi^4$ or $\phi^3$ potentials and vector gauge coupling to scalar and Dirac particles this is well known, only very recently we constructed a world-line Lagrangian \cite{15} for the Yukawa couplings of scalars and pseudo-scalars to Dirac particles.

In this letter we present a general formalism for the simultaneous coupling of abelian vector, axial vector, scalar, and pseudo-scalar background fields to Dirac particles in loops in the world-line formalism. Our starting point is the second order description for the fermionic one-loop effective action. The expression for the effective action in the second order formulation allows us to justify the expressions for the effective action in the world-line formalism which we propose later on. Whereas in the examples of ref. \cite{15} only even numbers of outer pseudo-scalars were allowed\footnote{We thank D. Geiser-Gagné for pointing this out to us.} here we do not have that restriction. Among the examples we present here, we also compute the chiral anomaly.

Let us begin with the second order description for the one-loop effective action of a fermion in an arbitrary background. The main difference between the usual (first order) description of a Dirac particle and the second order description is in the form of the propagator. The first order propagator for a Dirac particle in Euclidean space is $i/(p^2 + m)$. In the second order description, one studies basically the square of the Dirac equation. Therefore, the propagator is now of Klein-Gordon type $1/(p^2 + m^2)$. The space-time prop-
agator generated in the world-line formalism (and string theory) is usually of the second type, independent of the nature of the propagating field. For this reason, the second order formalism \cite{16,13,17} is an important link in establishing the connection between expressions in the world-line formalism and ordinary Feynman diagram results \cite{13,15}.

In Euclidean space, the Lagrangian for a Dirac particle is given by

\[ L = \bar{\psi} O \psi, \]  

where

\[ O = (\partial_\mu + igV_\mu + ig_5 \gamma_5 A_\mu)\gamma^\mu - im - i\lambda\phi + \gamma_5 \lambda' \phi', \]  

with the convention that \( \{\gamma_\mu, \gamma_\nu\} = -2\delta_{\mu\nu} \) and \( \gamma^\dagger_\mu = -\gamma_\mu \).

The corresponding one-loop effective action is

\[ \Gamma = \text{Tr} \log O = \frac{1}{2} \{ \text{Tr} (\log O + \log O') + \text{Tr} (\log O - \log O') \} . \]  

To arrive at a second order formulation, one can pick an arbitrary operator \( O' \) with the restriction that the free part in \( OO' \) is quadratic. However, a good choice of \( O' \) is important for an efficient perturbation theory derived from (3). In particular, it is convenient if we can choose \( O' \) such that the second term vanishes. It is also convenient if it can be arranged such that in \( OO' \) no covariant derivative acts to the right, except for those in the kinetic term.

For a general \( O \) as given in (2), there is no choice for the operator \( O' \) which satisfies both criteria. For the usual choice \( O' = \gamma_5 O \gamma_5 \), the second term vanishes since \( \text{det} O = \text{det} O' \). However, in the presence of axial and pseudo-scalar couplings, the second criterion mentioned above cannot be satisfied at the same time. This makes the translation of the expressions in the second order formalism into expressions in the world-line formalism difficult. Instead we choose

\[ O' \equiv O^\dagger = (\partial_\mu + igV_\mu - ig_5 \gamma_5 A_\mu)\gamma^\mu + im + i\lambda\phi + \gamma_5 \lambda' \phi'. \]  

As we will see later on, this choice makes it straightforward to translate (3) into a world-line expression.

Rewriting the first term in (3) as \( \log OO^\dagger \), we see that the operator which generates the real part of the effective action in the second order formalism is

\[ OO^\dagger = -D_\mu D^\mu + g\sigma^{\mu\nu} V_\mu V_\nu + g_5 \gamma_5 \sigma^{\mu\nu} A_\mu A_\nu + i\lambda\gamma^\mu \partial_\mu \phi - \gamma_5 \lambda' \gamma^\mu \partial_\mu \phi' \\
+ 2(im + i\lambda\phi - \gamma_5 \lambda' \phi')ig_5 \gamma_5 A_\mu \gamma^\mu + m^2 + 2m\lambda\phi + \lambda^2 \phi^2 + \lambda'^2 \phi'^2, \]  

\[ (5) \]
where $D_\mu = \partial_\mu + igV_\mu + ig_5\gamma_5A_\mu$. We use $V_{\mu\nu}$ and $A_{\mu\nu}$ to denote the field-strength tensors.

The term which is a bit more difficult to handle is the second term in (3), corresponding to the imaginary part of the effective action $\Gamma$. The imaginary part of $\Gamma$ is generated by processes involving at least one axial vector or one pseudo-scalar. This follows immediately from the fact that for $O' = \gamma_5O\gamma_5$ the second term vanishes while the first term gives the same result as for $O' = O$. If we view the effective action primarily as the generating functional for one-particle irreducible Green’s functions the next step is clear: Instead of looking at the original term we look at the term after one functional differentiation. Using

$$\frac{\delta}{\delta U} \text{Tr}(\log O - \log O') = \text{Tr} \left( \frac{\delta O}{\delta U} \frac{1}{O} - \frac{\delta O'}{\delta U} \frac{1}{O'} \right)$$

(6)

and the cyclicity of the trace we can rewrite the derivative of the second term of (3) as

$$\text{Tr} \left\{ \frac{\delta O}{\delta U} O' - O \frac{\delta O'}{\delta U} \right\} \frac{1}{O'O'}$$

(7)

where $U$ is either $A_\mu$ or $\phi'$. This way, the expression we have to study is of the form of some operator times a second order propagator. From equations (3, 4) we can construct Feynman rules for their perturbative evaluation (tables 1, 2).

The perturbative evaluation of (3) using the rules from table 1 is the standard procedure. To evaluate the extra terms generated by (4), one has to take one vertex from table 2 (corresponding to the term in braces) and all others from table 1. All propagators are of Klein-Gordon type. Using this set of rules it is straightforward to compute the one-particle irreducible one-loop Green’s functions.
\[ V_\mu g(p_\mu + 2q_\mu - i\sigma_{\mu\nu}p_\nu) \]

\[ A_\mu g_5\gamma_5(p_\mu + 2q_\mu - 2m\gamma_\mu) \]

\[ \phi - \lambda(p_\nu\gamma^\nu + 2m) \]

\[ \phi' - i\lambda'\gamma_5p_\nu\gamma^\nu \]

\[ V_\mu  
\]

\[ A_\mu  
\]

\[ 2g^2\delta_{\mu\nu} \]

\[ -2g^2\delta_{\mu\nu} \]

\[ 2\lambda g_5\gamma_5\gamma_\mu \]

\[ 2i\lambda'g_5\gamma_5\gamma_\mu \]

\[ -2\lambda^2 \]

\[ -2\lambda^2 \]

\[ V_\mu \]

\[ A_\nu \]

\[ -2g g_5\gamma_5\delta_{\mu\nu} \]

Table 1: Feynman rules for the modified second order formalism \((5)\). We use \( p (= i\partial) \) to denote the momentum of the incoming boson, and \( q \) for the momentum of the incoming fermion. A global factor of 1/2 and a negative sign for the fermion loop are required.

\[ A_\mu - g_5\gamma_5(i\sigma_{\mu\nu}(p_\nu + 2q_\nu) - p_\mu) \]

\[ \phi' - i\lambda'\gamma_5(\gamma_\nu(p_\nu + 2q_\nu) - 2m) \]

\[ 2ig^2\sigma_{\mu\nu} \]

\[ 2i\lambda'g_5\gamma_5\gamma_\nu \]

\[ V_\nu \]

\[ \phi' \]

\[ 2igg_5\gamma_5\sigma_{\mu\nu} \]

\[ -2\lambda'\gamma_5 \]

Table 2: Feynman rules for the extra terms (braces in eq. \((7)\)). Besides the ordinary second order calculation, one also has to compute diagrams where one vertex is taken from this table and all others are ordinary vertices from table \(1\).

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Now let us look at some examples. For the calculations we present here, we use dimensional regularisation. We verified, however, that the same results are obtained using Pauli-Villars regularisation. Since in this paper we only deal with one-loop processes, we use a naive scheme for the treatment of $\gamma_5$ in dimensional regularisation [18].

To warm up, let us compute the axial-axial two-point function. In principle, we have to compute four terms: The regular term (eq. (5)) and the extra term (eq. (7)) for diagrams (a) and (b) of figure 1. As it turns out, only the regular terms contribute while the extra terms vanish. The result of this calculation is for arbitrary, even dimension

$$\langle A_\mu A_\nu \rangle_{(a)} = \frac{n_F}{2} g_5^2 (4\pi)^{-d/2} \frac{(m^2)^{d/2-1}}{2} \Gamma(1 - d/2) \delta_{\mu\nu},$$

$$\langle A_\mu A_\nu \rangle_{(b)} = \frac{n_F}{2} g_5^2 (4\pi)^{-d/2} \int_0^1 dx \left\{ -\Gamma(1 - d/2) 2 \delta_{\mu\nu} (x(1-x)p^2 + m^2)^{d/2-1} $$

$$+ \Gamma(2 - d/2)(-\delta_{\mu\nu}(p^2 + 4m^2) + 4x(1-x)p_\mu p_\nu) \times $$

$$\times (x(1-x)p^2 + m^2)^{d/2-2} \right\},$$

where $n_F = \text{tr}(1)$ is the number of fermionic degrees of freedom. This is related to the standard first order result

$$\langle A_\mu A_\nu \rangle = -2n_F g_5^2 (4\pi)^{-d/2} \int_0^1 dx \Gamma(2 - d/2)(x(1-x)p^2 + m^2)^{d/2-2} \times$$

$$\times \left\{ g_{\mu\nu}(x(1-x)p^2 + m^2) - x(1-x)p_\mu p_\nu \right\}$$

by integration by parts and by $\Gamma$-function identities.

A more interesting example is given by the axial-vector and vector pseudoscalar two-point functions in two dimensions. With those two processes we can study the axial current Ward identity and the chiral anomaly in two dimensions. It turns out that for this case, the contribution from the regular second order expression vanishes. Instead, the process is described using the extra Feynman rules (table 2). Here, we chose the vertex to be a special vertex for the coupling to the axial current. The other vertex for the vector current is an ordinary vertex. Besides this, the calculation uses standard Feynman techniques.

$$\langle A_\mu V_\nu \rangle_{(a)} = -2igg_5 (4\pi)^{-d/2} (m^2)^{d/2-1} \Gamma(1 - d/2) \epsilon_{\mu\nu}$$

$$\langle A_\mu V_\nu \rangle_{(b)} = igg_5 (4\pi)^{-d/2} \int_0^1 dx \left\{ \Gamma(1 - d/2) 2 \epsilon_{\mu\nu} (x(1-x)p^2 + m^2)^{d/2-1} $$

$$+ \Gamma(2 - d/2)(\epsilon_{\mu\nu} p^2 - 4x(1-x)\epsilon_{\mu\rho} p_\rho p_\nu) \times $$

$$\times (x(1-x)p^2 + m^2)^{d/2-2} \right\}.$$

and

$$\langle PV_\nu \rangle = -2mg^2 \lambda \epsilon_{\mu\nu} p_\mu \Gamma(2 - d/2) \int_0^1 (x(1-x)p^2 + m^2)^{d/2-2}.$$
Again, these expressions are equivalent to the results of an ordinary, first order calculation. If we check the axial current Ward identity

$$ \frac{i}{g_5} \langle A_\mu V_\nu \rangle = \frac{2m}{\Lambda'} \langle PV_\nu \rangle $$

(12)

we find, as expected, on the right hand side an extra, anomalous term

$$ 4g(4\pi)^{-d/2} \int_0^1 dx \Gamma(2-d/2) \epsilon_{\mu\nu} p_\mu (x(1-x)p^2 + m^2)^{d/2-1}. $$

(13)

A first order field-theory calculation verifies that this is indeed the axial anomaly in two dimensions.

Our goal now is to translate these results into a world-line formulation. Since we study the couplings to an internal fermion we start from the usual description of a spinning particle by a supersymmetric world-line Lagrangian [19, 20, 21] using a curved superspace description. The particle is described by a superfield $X^\mu(\tau, \theta) = x^\mu(\tau) + \theta \sqrt{e} \psi^\mu(\tau)$, where $x$ is a normal commuting number, and $\theta$ is a Grassmann variable. To keep the manifest reparametrisation invariance of the super world-line, we introduce the super-einbein $\Lambda = e + \theta \sqrt{e} \chi$. Furthermore, to couple the spinning particle to scalar and pseudo-scalar fields, we need two auxiliary fields $\bar{X}, X' = \sqrt{e} \psi_{5,6} + \theta x_{5,6}$ [15]. Since we deal with a curved superspace on the world-line, we also have to distinguish the two derivatives $D_\theta = \Lambda^{-1/2} \left( \frac{\partial}{\partial \theta} - \theta \frac{\partial}{\partial \tau} \right)$ and $D_\tau = \Lambda^{-1} \frac{\partial}{\partial \tau}$. Translating the second order action (5) into world-line form we suggest the world-line action

$$ S = \int d\tau d\theta \Lambda^{1/2} \left( \frac{1}{2} D_\tau X \cdot D_\theta X + \frac{1}{2} \Lambda^{-1} \bar{X} \cdot D_\theta \bar{X} + \frac{1}{2} \Lambda^{-1} X' \cdot D_\theta X + g_5 \bar{X} X' D_\theta X_\mu A_\mu + \Lambda^{-1/2} i \lambda X' \Phi' + i g D_\theta X_\mu V_\mu + \Lambda^{-1/2} i \lambda X \Phi \right). $$

(14)

Before we can do any calculations using this action, we have to fix the reparametrisation invariance by fixing $\Lambda$. For calculations on the circle, a valid and convenient gauge is $e = 2, \chi = 0$. In this gauge, the world-line action in component notation is

$$ S = \int d\tau \left( \frac{x^2}{4} + \frac{x_5^2}{4} + \frac{x_6^2}{4} + \frac{1}{2} \dot{\psi} \dot{\psi} + \frac{1}{2} \dot{\psi}_5 \dot{\psi}_5 + \frac{1}{2} \dot{\psi}_6 \dot{\psi}_6 
\right.

+ 2i \lambda (x_5 \Phi - 2 \psi_5 \psi \cdot \partial \Phi) + 2i \lambda' (x_6 \Phi' - 2 \psi_6 \psi \cdot \partial \Phi')

$$

$$ + g_5^2 \left[ \dot{\psi}_5 \dot{\psi}_6 (\dot{x}_\mu A_\mu + 2 \psi_\mu \psi_\nu \partial_\nu A_\mu) + (\dot{\psi}_5 x_6 - \psi_6 x_5) \psi_\mu A_\mu \right]

- ig (\dot{x}_\mu V_\mu + 2 \psi_\mu \psi_\nu \partial_\nu V_\mu \right). $$

(15)
A mass term for the fermion is generated by shifting $\phi \rightarrow \phi + m/\lambda$ \[15\].

Having this world-line action is not all. For example, one can check immediately that it is impossible to generate diagrams with an odd number of pseudo-scalars. This corresponds to the situation in the second order formalism, so we also need to describe the terms corresponding to (6). They can all be generated from

$$
\Gamma'_U = \frac{\delta}{\delta U} i 3m \Gamma = -\frac{n_F}{2} \int_0^\infty \frac{dT}{T} \int \mathcal{D}x \mathcal{D}\bar{x} \mathcal{D}x' (-1)^F \Omega_U e^{-S} \quad (16)
$$

where we have $U = A_\mu$ or $\phi'$. We can translate the terms in table 2 in a suggestive manner. The inserted operators are then

$$
\begin{align*}
\Omega_A &= -g_5 \int_0^T d\tau \left( \psi_\mu \psi_\nu \tilde{x}_\mu - \frac{i}{2} \tilde{p}_\mu \right) \psi_5 \psi_6 \\
&= -g_5 \int_0^T d\tau d\theta \left\{ \left( \nabla_\tau X \cdot D_\theta X + \Lambda^{-1} \tilde{X} \cdot D_\theta \tilde{X} + \Lambda^{-1} X' \cdot D_\theta X' \right) D_\theta X_\mu \\
& \quad - \Lambda^{-1} \tilde{p}_\mu \right\} \tilde{X} X' \\
\Omega_{\phi'} &= -i \lambda' \int_0^T d\tau \left( \frac{1}{2} \psi_\nu \tilde{x}_\nu + \frac{1}{2} \psi_5 x_5 \right) \psi_6 \\
&= -i \lambda' \int_0^T d\tau d\theta \left( \nabla_\tau X \cdot D_\theta X + \Lambda^{-1} \tilde{X} \cdot D_\theta \tilde{X} + \Lambda^{-1} X' \cdot D_\theta \tilde{X}' \right) X' \\
& \quad \left(17\right)
\end{align*}
$$

The quartic vertices in table 2 are generated by $\delta$-functions which appear in some of the world-line Green’s functions. The explicit $p_\mu$ in the terms is really a derivative acting on $U$.

The operator $(-1)^F$ anti-commutes with all fermionic fields and this way implements the usual $\gamma_5$ in the Dirac algebra. The presence of $(-1)^F$ changes the boundary conditions for world-line fermions from anti-periodic to periodic and is known to play a role in the computation of anomalies [22, 23]. This and the introduction of the auxiliary fields $X'$ and $\tilde{X}$ are the only new ingredients in the action.

All these terms are of a surprisingly simple form, which is almost a product of the kinetic part of the Lagrangian in (14) with the interaction part of the field under consideration, all projected to the lower component of the superfield. This is not manifestly supersymmetric.

Before we can perform any calculations in this world-line formalism, we have to expand the action in components and perform the integral over the auxiliary fields. This step is necessary to resolve some ambiguities in the evaluation of products of Green’s functions on the world-line (Similar ambiguities appeared in the world-line calculations of [24]). These ambiguities arise since $\hat{G}_B$ and $\langle x_5 x_5 \rangle$ contain a $\delta$-function while $\hat{G}_B$ and $G_F$ contain a
step-function. After the removal of the auxiliary fields from the world-line action, we can always use $G_F^2 = 1$ in all calculations, even for identical arguments (relevant if multiplied by a $\delta$-function). An example of this ambiguity occurs in the calculation of the axial current two-point function. We see that both the $\psi_5\psi_6\dot{x}_\mu A_\mu$ term and the $(\psi_5x_6 - \psi_6x_5)\psi_\mu A_\mu$ term in (15) generate $G_F^2(u)\delta(u)$. However, to reproduce the second order calculation, only the first term is allowed to contribute while the contribution from the second term has to vanish.

Integrating out the auxiliary fields removes the second term in the axial coupling in (15) and introduces quartic couplings instead. The resulting world-line action bears a much closer relation to the second order expression for the effective action (5). Some of the subtleties mentioned here will be further discussed in a forthcoming publication [25].

Now let us look at our examples again. The evaluation of the axial-vector axial-vector two-point function is fairly simple. The calculation follows the usual world-line rules [1, 2, 15]. For the axial vector two-point function we find

$$\langle A_\mu A_\nu \rangle = \frac{n_F}{2} g_5^2 \int_0^\infty \frac{dT}{T} (4\pi T)^{-d/2} T^2 \int_0^1 du \left\{ -\frac{\delta_{\mu\nu}}{T} (p^2 - \frac{1}{T} \dot{G}_B + 4m^2) \right. \\
-\left. p_\mu p_\nu ((\dot{G}_B)^2 - 1) \right\} \exp(-T(m^2 + p^2G_B))$$

(18)

where $G_B$ is the bosonic Green’s function on the circle. Replacing $G_B = u(1 - u)$, $\dot{G}_B = 1 - 2u$, and $\ddot{G}_B = 2\delta(u) - 2$ it is easy to check that this is indeed the result from eq. (8). In the Green’s function, the $\delta$-function is understood to be on the circle. As in the second order calculation, the extra terms do not play a rôle in this process.

More interesting is the computation of the vector-axial vector and pseudo-scalar vector two-point functions. We can see immediately that the direct term generated by (18) vanishes. So we have to look at the terms generated by (15). Here, the $(-1)^F$ appears, which corresponds to the presence of a $\gamma_5$ in the field-theory calculation. This means that the boundary conditions for the fermions get changed and both fermions and bosons have periodic boundary conditions. In this case there are zero modes for the fermions which we have to take into account. Furthermore, the fermionic Green’s function is changed and we have

$$G_F^{\text{(periodic)}} = \dot{G}_B,$$

(19)

i.e., the world-line supersymmetry is no longer broken by the boundary
conditions. We get now

\[ \langle A_\mu V_\nu \rangle = -ig g_5 \int_0^\infty \frac{dT}{T} (4\pi T)^{-d/2} e^{-T m^2} \times \]
\[ \times T^2 \int_0^1 du \left[ \langle \psi_\mu \psi_\nu \rangle'' \langle \dot{x}_\rho, 1 \dot{x}_\nu, 2 e^{-ip x_1} e^{ip x_2} \rangle 
+ \langle \psi_\nu \psi_\rho \rangle'' \langle e^{-ip x_1} e^{ip x_2} \rangle 
+ 2ip_\rho \langle \psi_\mu, 1 \psi_\nu, 2 \psi_\delta, 2 \rangle'' \langle e^{-ip x_1} e^{ip x_2} \rangle \right]. \quad (20) \]

Here the extra subscripts 1, 2 denote the vertex insertion which the fields belong to. Evaluating this term we notice that we have to include one zero mode for each fermion present. For \( \psi_5 \) and \( \psi_6 \) this is already done. For the \( \psi_\mu \), this is indicated by the modified contraction \( \langle \rangle'' \). In \( n \) dimensions this automatically produces a factor \( \prod_{i=1}^n \psi_\mu_i \) which is nothing but the \( n \)-dimensional \( \epsilon \)-tensor. The result is then

\[ \langle A_\mu V_\nu \rangle = -ig g_5 \int_0^\infty \frac{dT}{T} (4\pi T)^{-d/2} e^{-T m^2} \times \]
\[ \times \int_0^1 du \left[ \epsilon_{\mu \nu} \mathcal{G}_B - \epsilon_{\mu \alpha} P_\alpha P_\nu \mathcal{G}_B^2 + \epsilon_{\nu \beta} P_\mu P_\beta \right] e^{-T G_B p^2}, \quad (21) \]

where we used the identity \( \epsilon_{\mu \nu} p^2 = \epsilon_{\mu \alpha} P_\alpha P_\nu - \epsilon_{\nu \beta} P_\mu P_\beta \). We use this identity again and substitute the world-line Green’s functions to recover the second order result \( (10) \). For the pseudo-scalar vector coupling we find

\[ \langle P V_\nu \rangle = 2\lambda' g m p_\rho \int_0^\infty \frac{dT}{T} (4\pi T)^{-d/2} e^{-T m^2} T^2 \int_0^1 du \langle \psi_\nu \psi_\rho \rangle'' e^{-T G_B p^2} \]
\[ = -2\lambda' g m \epsilon_{\rho \nu} p_\rho \Gamma(2 - d/2) \int_0^1 du (m^2 + u(1 - u)p^2)^{d/2-2}. \quad (22) \]

Both expressions agree with the second order results obtained above. This implies that also in the world-line formalism we were able to compute both the axial current Ward identity and the axial anomaly. It is possible, even though slightly more cumbersome, to do the same calculation using a Pauli-Villars regulator instead of dimensional regularisation. The final result is not affected by this choice.

In four dimensions, we can do the same for the triangle graphs with one or three axial currents \( [25] \). Again, the \( \epsilon \)-tensors are produced by the zero-modes of the fermion fields.

We have seen how useful the second order formalism is as a tool for constructing of world-line actions for a spinning particle in a loop. By constructing an appropriate second order representation for a theory with Dirac fermions we were able to find a world-line representation which reproduces exactly the second order expressions we started with. This indicates strongly
that the world-line formalism suggested here indeed reproduces the usual field theory results. An important new ingredient are auxiliary fields \( X' \) and \( \bar{X} \), and the inclusion of \((-1)^F\) in the representation of \( \gamma_5 \). This allowed us to treat processes with an odd number of vertices with \( \gamma_5 \)-couplings, especially those connected to the chiral anomaly. A computerized higher order effective action calculation to verify the agreement between the standard approach and the world-line formalism is in preparation [25]. Furthermore it is interesting to see if this kind of world-line formulation of axial couplings allows for the generalization to multi-loop processes and to processes with open fermion lines.

In a recent preprint [26], which we received while finishing this letter, D’Hoker and Gagné also introduce the operator \((-1)^F\) for the calculation of the imaginary part of the effective action as a mean to generate the \( \epsilon \)-tensor from the fermionic zero modes. They do not include axial vector couplings, though. Furthermore, they derive an elegant integral representation for the imaginary part of the effective action. This representation has the advantage of providing a closed form for the imaginary part which is still missing in our approach. The resulting perturbation theory introduces an additional Feynman parameter-like integration. It seems to us that the construction of the world-line representation as we present it here, including the superfield formulation, can also be done starting from their integral representation.

An alternative construction of the Yukawa coupling of a spinning particle and a bosonic representation of \( \gamma_5 \) are given in [27].

Acknowledgments: We acknowledge Jan Willem van Holten for useful discussions, and Jan Eeg for drawing our attention to the relevance of effective actions involving axial currents and for early collaboration. One of us (C. S.) would like to thank the Aspen Center for Physics for hospitality.

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