Coordination and Optimization of Supply Chain Considering the Double Fresh-keeping Effort of Supplier and 3PL

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Abstract
This paper takes a three-echelon supply chain consisting of a domestic supplier, an overseas retailer, and an overseas warehouse as the research object, and studies the decision-making and coordination problems of a single-channel three-echelon supply chain dominated by a cross-border e-commerce platform based on the dual preservation effort level of the supplier and the overseas warehouse, and also analyzes and compares the optimal decisions and profits of each member of the supply chain under the decentralized decision-making model and the centralized decision-making model. After exploring the infeasibility of wholesale price contract and wholesale price + cost-sharing contract, a wholesale price + cost sharing + revenue sharing hybrid contract is designed to optimize the coordination of the supply chain and finally examined and verified through arithmetic analysis. It is found that it is difficult to judge the overall utility of consumers under centralized decision-making, and the decisions made under decentralized decision-making are not optimal. Under the decentralized decision-making model, the wholesale price contract, and the wholesale price + cost-sharing contract can achieve supply chain coordination, but the contract will not be accepted because the former hurts the members' interests, and the latter reduces the overall profit. Therefore, a hybrid "wholesale price + cost sharing + revenue sharing" contract is designed to introduce profit redistribution and is shown to achieve perfect supply chain coordination.

Keywords
Freshness Preservation Efforts; Fresh Supply Chain; Contract; Revenue Sharing; Supply Chain Decision Making.

1. Introduction
Economic globalization has promoted the booming development of international trade, and the characteristics of e-commerce such as low cost, low risk, and high flexibility have led to the rapid development of cross-border e-commerce. In 2021, the two major platforms of China's cross-border e-commerce accounted for 26.7% and 22.4% of the market share respectively. The development of cross-border e-commerce platforms cannot be separated from the support of cross-border e-commerce logistics, and the government has introduced a series of plans to promote the construction of 100 overseas warehouses for e-commerce, attaching great importance to the development of cross-border e-commerce logistics [1].

On the other hand, with the improvement of the economic level, the living standard of the residents has been greatly improved, and consumers' demands are more diversified, paying more attention to the freshness of fresh products and the quality of goods. Cross-border fresh food e-commerce platforms as the leading and domestic suppliers, overseas warehouses consisting of a three-echelon supply chain came into being.
The increased social attention and market trends have caused many scholars at home and abroad to research the fresh food supply chain. Consumer demand has increased significantly, and product freshness and price coefficients influence consumers' willingness to purchase. Before supply chain coordination was achieved, suppliers and overseas warehouses were inclined to pay lower levels of freshness efforts to save costs, which led to a decrease in product freshness and affected platform revenues, while reducing overall supply chain performance. In this context, it is important to investigate the cooperative relationship and optimal decision of each supply chain member, and how to motivate both parties to improve the effort level to achieve supply chain coordination, to improve the performance of the cross-border fresh produce supply chain.

Research on fresh food supply chain pricing and supply chain coordination is an important part of the fresh food supply chain field.

Considering the demand for fresh produce affected by effort level, Yang Lei[2] et al. studied the optimal pricing of fresh produce supply chain under discounting strategy, and pointed out that the effort level is crucial for supply chain profit enhancement; Huai-Li Chen[3] (2013) established a multi-period ordering lot model for fresh food with shelf-life constraints to solve for the optimal ordering lot and the optimal discount price for retailers; Dye[4](2011)et al. constructed a time-varying inventory model to study the optimal freshness technology input of enterprises.

Shao-Chuan Fu[5](2020) considered the influence of risk aversion on the decision making of supply chain members and realized the effective coordination of mixed dual-channel fresh produce through "risk sharing + benefit sharing" contract; Zheng[6](2017)and others point out that freshness is an important factor influencing consumers' purchase decisions, and study supply chain coordination based on the implementation of freshness efforts by retailers and the "freshness cost sharing + revenue sharing" contract to achieve a win-win situation.; Xiao-Qiang Cai[7](2009) discussed the decisions of distributors and producers in both decentralized and centralized situations, considering that preservation effort affects both quality and quantity of fresh produce, and designs incentive schemes to coordinate the "supplier-distributor" channel to enable distributors to improve preservation effort; Xiao-Ning Cao[8](2021) considered the provision of freshness preservation efforts by suppliers and achieved the coordination of fresh dual-channel supply chain through a hybrid contract from the perspective of channel cooperation and profit maximization; Xiao Xue-Li Ma [9](2018) took the supply of seasonal agricultural products as the research object, consider the impact of preservation efforts on the quality and quantity loss of agricultural products, study the changes of system decision variables and influencing factors under the decentralized and centralized decision-making model, and design a "cost-sharing + benefit-sharing" coordination mechanism to promote supply chain coordination; Bao-Lin Zhu[10] et al.(2016) studied the coordination of three-echelon supply chains in uncertain environments and designed revenue sharing and wholesale price contracts to coordinate supply chains; Dao-Ping Wang[11] et al.(2020) constructed centralized and decentralized supply chain decision models considering the joint effects of suppliers' freshness preservation effort level and the time required to transport products on product freshness, and compared the effects of traditional cost-sharing contracts and cost-sharing contracts under Nash bargaining on supply chain coordination; Huai-Zhen Yang[12](2018) considered a three-echelon fresh agricultural products supply chain consisting of producer-distributor-retailer under the e-commerce model, solved for the optimal expected profit of the supply chain under three different decision scenarios, and demonstrated that price subsidies and revenue sharing contracts can effectively coordinate the supply chain.

The studies provide better ideas and basis for the fresh produce supply chain coordination problem, but they mainly focus on the traditional offline supply chain, and the studies considering dual freshness preservation efforts are not abundant yet. As a new form of trade,
the existing studies on cross-border e-commerce supply chains do not pay enough attention to e-commerce enterprises. Given this, this paper intends to take a three-echelon supply chain consisting of a domestic supplier, a cross-border fresh food e-commerce self-operated platform, and an overseas warehouse as the research object, and to discuss the supply chain decision and coordination problem under the dual preservation efforts of the supplier and the overseas warehouse. On this basis, a hybrid contract of "wholesale price + cost sharing + revenue sharing" is designed, and the range of contract parameters that can perfectly coordinate the supply chain is solved, to provide management insights for the operation of the fresh supply chain in the cross-border e-commerce environment.

2. Problem Description and Model Assumptions

2.1. Problem Description

The operation model of this supply chain is shown in Figure 1. Before the supply chain coordination is realized, suppliers and overseas warehouses tend to pay lower freshness effort levels to save cost, which affects the platform revenue, and reduces the overall performance of the supply chain. Therefore, we need to design a reasonable supply chain coordination mechanism to improve the supply chain's overall performance and increase each member's revenue.

In the three-echelon supply chain, the cross-border e-commerce platform is dominant in the supply chain because it has information and capital advantages. The main operation process of the supply chain is as follows.

First, cross-border e-commerce platform enterprises. The platform enterprises usually select high-quality suppliers to purchase products and then sell them through the platform. They need to determine the purchase quantity and sales price of a product based on their merchandising plans and market demand information and negotiate and sign contracts with supplier companies and overseas warehouse companies to determine the wholesale price of the product and the price of logistics services.

Second, the domestic supplier companies. The supplier enterprise determines the production service level, the quantity of the products to be produced, the wholesale price of the products, and the level of preservation efforts to be invested according to the order quantity of the products.
platform. After the signing of the contract with the cross-border e-commerce platform is completed, the supplier enterprise starts to produce the corresponding quantity of products as agreed and sends the products to the warehouse of the overseas warehouse enterprise designated by the cross-border e-commerce platform, which completes the subsequent transportation and distribution overseas.

Third, overseas warehouse enterprises. The overseas warehouse enterprise carries out the corresponding operation of the products according to the warehousing and distribution demand of the platform. It needs to determine the price of its logistics service and the level of preservation effort it needs to invest according to the logistics service contract signed with the platform, and after completing the contract, complete the warehousing and distribution of the products according to the demand and obtain the corresponding revenue.

2.2. Symbol Description

| Symbol | Description | Symbol | Description |
|--------|-------------|--------|-------------|
| $\mu$  | Preservation cost coefficient | $\gamma_s$ | The preservation effort level of Supplier |
| $\theta_c$ | Freshness preference of the consumer | $\gamma_w$ | the preservation effort level of Overseas warehouse |
| $k$ | Sensitivity coefficient for freshness effort level | $C^p$ | Production Costs of Supplier |
| $\omega$ | Product natural decay extremes | $C^i$ | Overseas warehouse logistics service cost |
| $T$ | Total time to the consumer | $p$ | Platform Sales Price |
| $TD$ | Platform Forecasted Market | $p_s$ | Wholesale prices of suppliers' products |
| $U$ | Total consumer utility | $p_w$ | Overseas warehouse logistics service price |
| $U_1$ | Consumers' product value utility | $p_{wp}$ | Wholesale prices under the wholesale price pact |
| $U_2$ | Consumer freshness utility | $P_{wp}$ | Price of logistics services under wholesale price contract |
| $\Pi_p$ | Platform profits | $\alpha$ | The percentage of supplier preservation costs are borne by the platform |
| $\Pi_s$ | Profit of suppliers | $\beta$ | The proportion of overseas warehouse preservation costs borne by the platform |
| $\Pi_w$ | Profit of overseas warehouse | $\varphi_1$ | Percentage of sales revenue shared between platform and suppliers |
| $\Pi$ | Total profit of the supply chain | $\varphi_2$ | Percentage of sales revenue shared between the platform and overseas warehouse |
3. Model Construction and Solving

3.1. Freshness Function

The freshness of fresh products will decay with time, which is by our perception. Therefore, the initial freshness of fresh products is the highest, and the initial freshness of fresh products is assumed to be 1 in this paper. To portray the change of freshness of fresh products, let the freshness function of a single fresh product at the moment \( t \) be \( \theta(t) \). The level of preservation effort refers to the level of effort of preservation measures taken by suppliers or overseas warehouses to slow down the reduction of the fresh products, which mainly depends on the capital investment and other inputs (such as preservation equipment, research, and development of preservation technology, etc.). Many scientific experiments have shown that the decay of fresh food is mainly related to the activity of bacteria, and the adoption of high-temperature sterilization, low-temperature inhibition, air insulation, and other preservation measures can inhibit the activity of bacteria to a certain extent, thus slowing down the decay of fresh food. Therefore, this paper puts forward a reasonable hypothesis: The freshness of fresh products is negatively correlated with time \( t \) and positively correlated with the level of preservation effort \( \gamma \). Based on the above analysis and concerning the freshness preservation model of Zhen-Feng Wang (2021) et al[13], this paper proposes a model of the freshness of fresh product concerning to time \( t \) and freshness preservation effort level \( \gamma \):

\[
\theta(t, \gamma) = \theta_0 + h(\gamma) - [\omega - h(\gamma)] \left( \frac{t}{T} \right)^2
\]  

(1)

where \( T \) is a known number indicating the total time required for the fresh product to reach the consumer as it can be measured analytically by previous data, \( t \in [0, T] \), \( \omega \) indicates the extreme value of the natural decay of freshness of the fresh product, and \( h(\gamma) \) indicates that the decay of freshness of the product is slowed down by the influence of the freshness preservation effort level. According to the contract theory and with reference to Xiao-Li Ji(2006)[14], Wei Wang(2021)[15] and others regarding the design of the effort level function, the effort level usually takes the form of a product and is linearly related to the objective function. Therefore, the specific form of \( h(\gamma) \) is:

\[
h(\gamma) = k\gamma
\]  

(2)

where \( k \) denotes the effort level sensitivity coefficient, \( k\gamma \in (0, \omega) \), indicating that no matter how much effort level is invested in preservation, it can only slow down the decay of product freshness, but cannot change the trend of decay. Combining equations (1) and (2), the decay model of product freshness is obtained as follows:

\[
\theta(t, \gamma) = \theta_0 + k_s\gamma_s - (\omega - k_w\gamma_w) \left( \frac{t}{T} \right)^2
\]  

(3)

The preservation effort paid by the overseas warehouse or supplier corresponds to the increase in cost, which is mainly reflected in the investment of equipment, manpower, etc. Therefore, the preservation cost paid \( C' \) is a function of the level of preservation effort and shows a positive correlation with it. Regarding the measurement of costs in supply chains, quadratic functions are prevalent and used in supply chain-related literature (Wei Wang,2021[错误!未定义书签。], Chen Fan,2020[16]). Therefore, the model in this paper on the cost of freshness is:
\[ C^r = \mu \gamma^2 \]  

(4)

where \( \mu \) is the preservation cost coefficient.

### 3.2. Demand Function

Previous studies have shown that the price factor and the product freshness factor are important factors influencing consumers' willingness to purchase [17], so consumers consider these two factors together when deciding whether to buy or not. This paper will establish a consumer utility model for consumer decision analysis. Consumer utility can better quantify consumers' decisions when they are influenced by multiple factors, and is widely used in consumer behavior and decision making, supply chain operation management and other fields[18]. According to consumer utility theory, this paper divides consumer utility \( U \) into two parts: product utility \( U_1 \) and freshness utility \( U_2 \), and \( U = U_1 + U_2 \).

We assume that consumers are heterogeneous in both product value and freshness preference, i.e., the same product brings different product utility and freshness utility to different consumers, which is consistent with our perception. Assuming that the price utility brought by the product to consumers is \( v \), it may be useful to set \( v \sim U(0,1) \), and price utility obeys a uniform distribution is a common assumption in consumer decision analysis[19]. Assume that the sales price of the unit product is \( p \), the tariff to be paid when the unit product enters the country is \( f \), the platform usually chooses to pass on the tariff to the consumer, so the sales price of the product outside the country is adjusted to \( (p + f) \), then the price utility function can be expressed as:

\[ U_1 = v - (p + f) \]  

(5)

In general, the higher the freshness of the product, the higher the utility to the consumer, which is in line with the reality. Suppose the consumer's freshness preference is \( \theta_C \), when and only when \( \theta(t, \gamma) > \theta_C \), it can bring positive freshness utility, and vice versa is negative. Without loss of generality, assuming that the coefficient of freshness utility is 1, the freshness utility function can be expressed as:

\[ U_2 = \theta(t, \gamma) - \theta_C \]  

(6)

The total utility function of the consumer can be expressed as:

\[ U = v - (p + f) + \theta(t, \gamma) - \theta_C \]  

(7)

From equation (7) we know that consumer utility is directly proportional to the value and freshness of the product and negatively proportional to the price. A consumer chooses to buy a product when and only when \( U \geq 0 \). The probability that a consumer will purchase a product at a given moment \( t \) can be expressed as:

\[ P(U \geq 0) = P(v - (p + f) + \theta(t, \gamma) - \theta_C \geq 0) = 1 - (p + f) + \theta(t, \gamma) - \theta_C \]  

(8)
Assume that the capacity of the market is 1 and substitute $\theta(t, \gamma)$ to obtain the market purchase volume $D$ at a certain moment $t$ as $D = 1 - (p + f) + \left[\theta_0 + k_s y_s - (\omega - k_w y_w) \left(\frac{t}{T}\right)^2\right] - \theta_c$. Thus the total market purchase $TD$ in the period $T$ can be obtained as:

$$TD = \left(1 + \theta_0 + k_s y_s - p - f - \theta_c + \frac{(k_w y_w - \omega)}{3}\right) T \quad (9)$$

### 3.3. Profit Function

In the three-echelon supply chain, the e-commerce platform is the dominant player, the supplier and the overseas warehouse are the followers, and their decisions depend on the decisions of the platform, and the decision sequence of each member of the supply chain is as follows:

1. The platform purchases a certain amount of products according to the demand forecast and sells them all, i.e., $Q = TD$. Assume that the cost of sales per unit of product is $C^s$. i.e., given $p_s, p_w$ and $y_s, y_w$, the platform sales price $p$ is decided to maximize profit.

2. The supplier decides the wholesale price per unit product $p_s$, the production cost per unit product $C^p$ and the level of preservation effort invested $y_s$ according to the purchasing volume of the cross-border fresh produce e-commerce platform. For the convenience of labeling, this paper uses the subscript $s$ to denote the supplier, the subscript $w$ to denote the overseas warehouse and the subscript $p$ to denote the cross-border fresh produce e-commerce platform, similarly later. The overseas warehouse decides the service price $p_w$ per unit product, the logistics service cost $C^l$ per unit product and the invested freshness preservation effort level $\gamma_w$ according to the delivery volume of the fresh e-commerce platform.

Assume that the profits of the cross-border fresh produce e-commerce platform, suppliers, overseas warehouses, and the overall profit of the supply chain are $\Pi_p, \Pi_s, \Pi_w$ and $\Pi$ respectively, and construct the corresponding profit functions. The profit function $\Pi_p = (p + f)TD - p_s TD - p_w TD - C^s TD - fTD$.

Simplification gives:

$$\Pi_p = (p - p_s - p_w - C^s)TD \quad (10)$$

The supplier's profit function $\Pi_s = p_s TD - C^p TD - C^s$. Substituting $C^p, C^s$ and simplifying, we get:

$$\Pi_s = (p_s - C^p)TD - \mu y_s^2 \quad (11)$$

The profit function for the overseas warehouse is $\Pi_w = p_w TD - C^l TD - C^w$. Substituting $C^l, C^w$ and simplifying, we get:

$$\Pi_w = (p_w - C^l)TD - \mu y_w^2 \quad (12)$$

The overall profit function of the supply chain is $\Pi = \Pi_p + \Pi_s + \Pi_w$.

$$\Pi = (p_1 - C^l - C^p - C^s)TD - \mu y_w^2 - \mu y_s^2 \quad (13)$$
3.4. Model Solution and Analysis

3.4.1. Decentralized Decision Model

In the decentralized decision model, the members of the supply chain compete to maximize their own interests. The e-commerce platform and suppliers, e-commerce platform and overseas warehouses constitute the Stackelberg game model respectively, where suppliers and overseas warehouses are the dominant players, and the platform is the follower. According to the inverse induction method, the solution steps are as follows: solve for the optimal p of the cross-border fresh produce e-commerce platform under the fixed $p_s, p_w$ and $y_s, y_w$, and then substitute into the corresponding reaction function to find the optimal $p_s, p_w$ and $y_s, y_w$.

Theorem 3-1: Under decentralized decision-making model, the optimal decision for each member of the supply chain is as follows:

(1) Cross-border fresh produce e-commerce platform

$$p^* = \frac{90\mu \left[1 + \theta_0 - f - \theta_C - \frac{\omega}{3}\right] + 18\mu(C^s+C^l + C^p) - T(C^s+C^l + C^p)(9k_s^2 + k_w^2)}{108\mu - 9k_s^2T - k_w^2T}$$

$$\Pi_p^* = \left[\frac{1 + \theta_0 + k_s y_s - f - \theta_C - p_s - p_w - C^s + \left(k_w y_w - \omega\right)}{2}\right]T$$

(2) Supplier

When $\mu > \frac{k_s^2T}{8}$, there exist unique and optimal $p_s^*, p_w^*, y_s^*, y_w^*$ such that $\Pi_p$ obtain the extreme values, where

$$p_s^* = \frac{36\mu\left[1 + \theta_0 - f - \theta_C - C^s - C^l + 2C^p - \frac{\omega}{3} - T C^p(9k_s^2 + k_w^2)\right]}{108\mu - 9k_s^2T - k_w^2T}, \quad y_s^* = \frac{9k_s T\left[1 + \theta_0 - f - \theta_C - C^s - C^l - C^p - \frac{\omega}{3}\right]}{108\mu - 9k_s^2T - k_w^2T},$$

$$\Pi_s^* = \frac{405\mu T(\frac{9}{16}k_w^2 + 1)k_w^2T(1 + \theta_0 - f - \theta_C - C^s - C^l - C^p - \frac{\omega}{3})^2}{(108\mu - 9k_s^2T - k_w^2T)^2}.$$ 

When $\mu \leq \frac{k_s^2T}{8}$, we discuss it in two cases. (1) when $\mu < \frac{(1-C^p)k_wT}{2}$, $p_s^* = 1, y_s^* = 1, \Pi_s^* = (1 - C^P)\left[\theta_0 + k_s f - \theta_C - p_w - C^s + \frac{(k_w y_w - \omega)}{3}\right]T - \mu$  (2) when $\mu \geq \frac{(1-C^p)k_wT}{2}$, $p_s^* = 1, y_s^* = 0, \Pi_s^* = (1 - C^P)\left[\theta_0 - f - \theta_C - p_w - C^s + \frac{(k_w y_w - \omega)}{3}\right]T.$

(3) overseas warehouse

when $\mu > \frac{k_w^2T}{72}$, there exists a unique $p_w^*, y_w^*$ such that $\Pi_w$ obtain the extreme values, where

$$p_w^* = \frac{36\mu\left[1 + \theta_0 - f - \theta_C - C^s - C^l + 2C^p - \frac{\omega}{3} - T C^l(9k_s^2 + k_w^2)\right]}{108\mu - 9k_s^2T - k_w^2T}, \quad y_w^* = \frac{3k_w T\left[1 + \theta_0 - f - \theta_C - C^s - C^l - C^p - \frac{\omega}{3}\right]}{108\mu - 9k_s^2T - k_w^2T},$$

$$\Pi_w^* = \frac{324\mu T(\frac{9}{16}k_w^2 + 1)T(1 + \theta_0 - f - \theta_C - C^s - C^l - C^p - \frac{\omega}{3})^2}{(108\mu - 9k_s^2T - k_w^2T)^2}.$$ 

When $\mu \leq \frac{k_w^2T}{72}$, we discuss it in two cases. (1) when $\mu < \frac{(1-C^l)k_wT}{6}$, $p_w^* = 1, y_w^* = 1, \Pi_w^* = (1 - C^l)\left[\theta_0 + k_s y_s - f - \theta_C - p_w - C^s + \frac{(k_w y_w - \omega)}{3}\right]T - \mu$  (2) when $\mu \geq \frac{(1-C^l)k_wT}{6}$, $p_w^* = 1, y_w^* = 0, \Pi_w^* = (1 - C^l)\left[\theta_0 + k_s y_s - f - \theta_C - p_w - C^s + \frac{(k_w y_w - \omega)}{3}\right]T.$

Theorem 3-1 shows that:

(1) $p_s, p_w, y_s, y_w$ are influenced by the combination of $k_s, f, \theta_C, C^s, C^l, \omega, T, \mu$ and other factors.
(2) When the increase in freshness cost per unit of preservation effort \( (\mu) \) is much less than the manufacturing cost or logistics service cost per unit of product \( (\mu < \min\left(\frac{(1-C^C)k_sT}{2}, \frac{(1-C^l)k_wT}{6}\right)) \), suppliers and overseas warehouses prefer to invest the highest level of preservation effort.

(3) As the level of preservation efforts invested by suppliers and overseas warehouses increases, the freshness of products improves, which in turn increases the sales volume of products, so the optimal product sales prices and profits of cross-border fresh produce e-commerce platforms are increased, but the profits of overseas warehouses and suppliers show a trend of first increasing and then decreasing.

(4) Higher consumer freshness preferences indicate higher consumer demand for product freshness, so cross-border fresh produce e-commerce platforms need to set lower selling prices to compensate for the lost freshness utility to consumers, and lower wholesale prices and logistics service prices for suppliers and overseas warehouses. The bias of suppliers and overseas warehouses is to invest less level of freshness effort because the freshness of the product brings limited utility to consumers.

3.4.2. Centralized Decision Model

Theorem 3-2: Under centralized decision model, the optimal decision for each member of the supply chain is as follows.

(1) When \( \mu \leq \frac{(k_w^2+9k_s^2)T}{36} \):

1) when \( k_s \geq \frac{k_w}{3} \), we divide into two cases.

\[
\begin{align*}
\text{when } & \mu > \frac{k_wT(1-C^l-C^P-C^S)}{3}, \quad p^* = 1, \gamma_s^* = 1, \gamma_w^* = 0 \quad \text{and} \quad \Pi^* = (1-C^l-C^P-C^S)\left[\left(k_s+\theta_0 - f - \theta_C - \frac{\omega}{3}\right)T\right] - \mu. \\
\text{when } & \mu \leq \frac{k_wT(1-C^l-C^P-C^S)}{3}, \quad p^*_w = 1, p^*_w = 1, \gamma_s^* = 1, \gamma_w^* = 1 \quad \text{and} \quad \Pi^* = (1-C^l-C^P-C^S)\left[\left(k_s+\theta_0 - f - \theta_C + \frac{(k_w-\omega)}{3}\right)T\right] - 2\mu.
\end{align*}
\]

2) when \( k_s < \frac{k_w}{3} \), we divide into two cases.

\[
\begin{align*}
\text{when } & \mu > k_sT(1-C^l-C^P-C^S), \quad p^*_s = 1, p^*_w = 1, \gamma_s^* = 0, \gamma_w^* = 1 \quad \text{and} \quad \Pi^* = (1-C^l-C^P-C^S)\left[\left(\theta_0 - f - \theta_C - \frac{(k_w-\omega)}{3}\right)T\right] - \mu. \\
\text{when } & \mu \leq k_sT(1-C^l-C^P-C^S), \quad p^*_s = 1, p^*_w = 1, \gamma_s^* = 1, \gamma_w^* = 1 \quad \text{and} \quad \Pi^* = (1-C^l-C^P-C^S)\left[\left(k_s+\theta_0 - f - \theta_C + \frac{(k_w-\omega)}{3}\right)T\right] - 2\mu.
\end{align*}
\]

(2) When \( \mu > \frac{(k_w^2+9k_s^2)T}{36} \), we can get the following results.

\[
\begin{align*}
p^* = & \frac{18\mu\left[1+\theta_0 - f - \theta_C + C^S + C^l + C^P - \omega\right]T(C^S+C^l+C^P)(9k_s^2+k_w^2)}{36\mu - 9k_s^2T-k_w^2T}, \quad Y_s^* = \frac{9k_sT\left[1+\theta_0 - f - \theta_C - C^S - C^l - C^P - \omega\right]}{36\mu - 9k_s^2T-k_w^2T}, \quad Y_w^* = \frac{9\mu T(1+\theta_0 - f - \theta_C - C^S - C^l - C^P - \omega)}{36\mu - 9k_s^2T-k_w^2T}, \quad \Pi^* = \frac{9\mu T(1+\theta_0 - f - \theta_C - C^S - C^l - C^P - \omega)^2}{36\mu - 9k_s^2T-k_w^2T}.
\end{align*}
\]

Theorem 3-2 shows that:

(1) \( p, Y_s, Y_w \) are influenced by the combination of \( k_s, f, \theta_C, C^S, C^l, \omega, T, \mu \) and other factors.

(2) When the increase in freshness cost \( (\mu) \) from investing a unit of freshness effort level is much smaller than the manufacturing cost or logistics service cost per unit of product \( (\mu \leq \frac{(k_w^2+9k_s^2)T}{36}) \), at least one of the supplier and the overseas warehouse will invest the highest freshness effort level. It depends on the freshness effort coefficients of suppliers and overseas warehouses, i.e., the utility of the freshness of the products that can be produced by investing a unit of freshness effort. When the freshness effort level coefficient of the supplier is much larger
than that of the overseas warehouse \((k_s \geq k_w)\), the supplier is biased to invest the highest freshness effort level, and whether the overseas warehouse invests the highest freshness effort level is still uncertain, depending on the freshness effort level cost coefficient, i.e., when the increase of the cost brought by the invested freshness effort level is small \((\mu \leq \frac{k_w T (1-C^1_C C^p - C^x)}{3})\), the overseas warehouse will put in the highest preservation effort level, otherwise it will not put in the preservation effort level. The same is true when the freshness effort level coefficient of the overseas warehouse is much larger than that of the supplier.

3.4.3. Discussion and Analysis of the Results of the Basic Model

Analyzing and comparing the optimal decision and profit of each member of the supply chain under decentralized and centralized decision-making model, we get Theorem 3-3.

Theorem 3-3: (1) When \(18 \mu - 9 k_s^2 T - k_w^2 T > 0\), \(p_{pc}^{*} > p_{pa}^{*}\). When \(18 \mu - 9 k_s^2 T - k_w^2 T < 0\), \(p_{pc}^{*} < p_{pa}^{*}\). Indicating that if the product pricing of the platform reaches the level of centralized decision-making model through supply chain coordination, overseas consumers can buy fresher fresh products at lower prices through the cross-border fresh food e-commerce platform at any moment, and the overall utility of consumers increases. However, when \(18 \mu - 9 k_s^2 T - k_w^2 T > 0\), \(p_{pc}^{*} > p_{pa}^{*}\), the pricing under the centralized decision is higher than the decentralized decision, and although consumers are able to buy fresher products, consumer utility increases, but they need to pay higher prices, and consumer utility decreases again, making it difficult to judge the overall consumer utility. Thus Lemma 3-1 is obtained.

Lemma 3-1: (1) When \(18 \mu - 9 k_s^2 T - k_w^2 T < 0\), supply chain coordination can be achieved through a suitable supply chain contract, and the overall consumer utility can be increased. When \(18 \mu - 9 k_s^2 T - k_w^2 T > 0\), the supply chain contract can enable supply chain coordination, but it may hurt consumer utility.

(2) Under centralized decision-making model, suppliers and overseas warehouses are biased to invest higher levels of preservation effort, and the overall profit of the supply chain is greater than in the case of decentralized decision-making model. This means that the decisions made under decentralized decision-making model are not optimal due to the existence of "double marginal utility", and reasonable supply chain contracts need to be designed to induce overseas warehouses and suppliers to increase the freshness effort level to achieve supply chain coordination.

Theorem 3-4: (1) When \(k_w \in (0, \frac{3\sqrt{T (28 \mu - k_s^2 T)}{T}}{28 \mu - k_s^2 T})\) or \(k_s \in (0, \frac{3\sqrt{T (252 \mu - k_w^2 T)}{3T}}{252 \mu - k_w^2 T})\), the total profit of the supply chain under decentralized decision-making model increases and then decreases as \(k_s\) or \(k_w\) increases, while the total profit of the supply chain under centralized decision-making model increases all the time.

(2) When \(\mu < \frac{9k_s^2 T + k_w^2 T}{109}\), the total supply chain profit under decentralized decision increases with the increase of \(\mu\), otherwise it decreases with the increase of \(\mu\). And the total profit of the supply chain under the centralized decision decreases with the increase of \(\mu\).

(3) In both decentralized and centralized decision-making model, the freshness effort level of suppliers and overseas warehouses is proportional to \((\theta_0 - \theta_C)\).
3.4.4. Supply Chain Coordination Contract Design

From the above analysis, the order volume of the e-commerce platform is low under decentralized decision model, mainly because the product or service cost and freshness cost of suppliers and overseas warehouses are magnified twice, which greatly increases the channel cost of products, resulting in a "double marginal effect", leading to a lower order volume of the platform.

Therefore, we first considered the wholesale price contract to reduce the double marginal effect. By solving the optimal decision, we know that under the wholesale price contract, there exists

\[ \Pi_{sp} + \Pi_{wp} = -\mu(y_{sr}^2 + y_{wr}^2) < 0, \]

i.e., one of the overseas warehouse and the supplier must have a loss, so at least one of them will not accept the wholesale price contract, and the contract cannot achieve supply chain coordination.

The main reason why the wholesale price contract cannot achieve perfect coordination in the supply chain is that the increase in costs brought about by the increase in freshness efforts of overseas warehouses and suppliers is not shared, resulting in lower profits. Therefore, we consider introducing a cost-sharing contract based on the wholesale price contract, assuming that the ratio of the platform bearing the preservation cost of suppliers and overseas warehouses is \( \alpha \) and \( \beta \). Solve that when the contract parameters satisfy \( \alpha = 1 \) and \( \beta = 1 \), the optimal decision of each member of the supply chain is the same as the centralized decision model, but the profits of suppliers and overseas warehouses are reduced compared with the decentralized decision, so the contract will not be accepted and supply chain coordination cannot be achieved.

It is easy to find that the use of wholesale price contract + cost-sharing contract makes the decision of supply chain members in the decentralized case the same as the centralized decision, but there is a certain shortage of profit distribution, so consider introducing a contract to realize the redistribution of profit. The common profit redistribution contracts are fixed-cost contract and revenue-sharing contract. Fixed-cost contracts are not widely used, so this paper will discuss the "wholesale price contract + cost-sharing contract + revenue-sharing contract" combination contract.

Assume that the model of wholesale price contract + cost-sharing contract + revenue sharing contract is the s model, in which the platform shares the sales revenue with suppliers and overseas warehouses in the ratio of \( \varphi_1, \varphi_2 \).

\[
\Pi_{ps} = [(1 - \varphi_1 - \varphi_2)p - p_{ss} - p_{ws} - C^s]TD - \alpha \mu y_{ss}^2 - \beta \mu y_{ws}^2
\]

\[\Pi_{ss} = (\varphi_1 p + p_{ss} - C^p)TD - (1 - \alpha)\mu y_{ss}^2 \]

\[\Pi_{ws} = (\varphi_2 p + p_{ws} - C^l)TD - (1 - \beta)\mu y_{ws}^2 \]

Similarly, using the inverse solution method for the solution, we get the results as follows.

\[
p_{ps}^* = \frac{(1 - \varphi_1 - \varphi_2)\left(1 + \theta_0 + k_s y_{ss}^* - f - \theta_c + \frac{(k_w y_{wc}^* - \omega)}{3}\right) + p_{ss} + p_{ws} + C^s}{2(1 - \varphi_1 - \varphi_2)}
\]

\[
y_{ss}^* = \frac{k_s T \left(1 + \theta_0 + k_s y_{ss}^* - f - \theta_c + \frac{(k_w y_{wc}^* - \omega)}{3}\right) \varphi_1 + C_p - p_{ss}}{k^2_s T \varphi_1 - 4\mu(1 - \alpha)}
\]
Let $p_{ps}^* = p_{pc}^*, \gamma_{ss}^* = \gamma_{sc}^*, \gamma_{ws} = \gamma_{wc}^*$, we get:

$$\alpha = \frac{36\mu[(1 - \varphi_1)\Delta_1 - \Delta_2 + C_p - p_{ss}^*] + T(\Delta_2 \varphi_1 + p_{ss}^* - C_p)(9k_s^2 - k_w^2)}{12\mu(\Delta_1 - 3\Delta_2)}$$

(17)

$$\beta = \frac{36\mu[(1 - \varphi_2)\Delta_1 - \Delta_2 + C_i - p_{ws}] + T(\Delta_2 \varphi_2 + p_{ws} - C_i)(9k_s^2 - k_w^2)}{12\mu(\Delta_1 - 3\Delta_2)}$$

(18)

$$\varphi_1 = \frac{\Delta_2(1 - \varphi_2) - p_{ss}^* - p_{ws}^* - C_s}{\Delta_2}$$

(19)

Combining equation (17)(18), the internal transfer price of the supply chain under this contract can be obtained as follows.

$$p_{ss} = \frac{36\mu([(1 - \varphi_1)\Delta_1 - \Delta_2]\alpha + (\Delta_1 - \Delta_2) + C_p) + T(\Delta_2 \varphi_1 - C_p)(9k_s^2 - k_w^2)}{36\mu - 9k_s^2 - k_w^2}$$

(20)

$$p_{ws} = \frac{36\mu([(1 - \varphi_2)\Delta_1 - \Delta_2]\beta + (\Delta_1 - \Delta_2) + C_i) + T(\Delta_2 \varphi_2 - C_i)(9k_s^2 - k_w^2)}{36\mu - 9k_s^2 - k_w^2}$$

(21)

To achieve supply chain coordination, the contract parameters ($\alpha, \beta, \varphi_1, \varphi_2, p_{ss}, p_{ws}$) also need to satisfy $\Pi_{ps}^* \geq \Pi_{pa}^*, \Pi_{ss}^* \geq \Pi_{sa}^*$ and $\Pi_{ws}^* \geq \Pi_{wa}^*$ to ensure that the members have sufficient incentives to accept the contract. Substituting (14)(15)(16) into the profit function in can solve for the corresponding range of contract parameters.

4. Conclusion

In this paper, a three-echelon supply chain consisting of a domestic supplier, an overseas retailer, and an overseas warehouse is studied, and the decision-making and coordination problems of a single-channel three-echelon supply chain led by a cross-border e-commerce platform are investigated based on the dual preservation efforts of the supplier and the overseas warehouse. On this basis, a hybrid contract of "wholesale price + cost sharing + revenue sharing" is designed to achieve supply chain coordination, and an analysis is conducted. It is found that the centralized decision can significantly increase the total profit of the supply chain system compared with the decentralized decision, but it is difficult to judge the overall utility of consumers because the pricing under the centralized decision is higher than that under the decentralized decision. However, because of the existence of "double marginal utility", the decisions made under the decentralized decision are not optimal. In this paper, we introduce the profit-sharing contract and design a combination of "wholesale price contract + cost-sharing contract + revenue sharing contract" and verify the existence of a feasible range of cost sharing and revenue sharing ratios to achieve perfect coordination of the supply chain through case studies. The research of this paper has certain reference value and guidance.
significance for the pricing strategy of suppliers and platforms, the dual preservation inputs of suppliers and overseas warehouses, and the coordination of supply chain contracts in the cross-border e-commerce environment.

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