Analysis and optimization of electromagnetic scattering characteristics of two-dimensional metal airfoil covered with metamaterial

Ji Liu¹, Zhaowen Yan¹, Jinzu Ji²,³, Qi Chang² and Hang Song²

¹School of Electronics and Information Engineering, Beihang University, Beijing 100191, China
²School of Aeronautic Science and Engineering, Beihang University, Beijing 100191, China
³E-mail: jijinzu@buaa.edu.cn

Abstract. Airfoil is one of the strong scattering sources of aircraft, so it is of great significance to study the stealth characteristics of airfoil for the reduction of the radar cross section of the whole aircraft. As is known, the airfoil of aircraft is the main component that generates lift. Shaping the airfoil will reduce the aerodynamic performance of the aircraft. However, the emergence of metamaterials allows aircraft designers to achieve a good balance between stealth and aerodynamics. Considering the high cost of stealth test, numerical simulation is proposed to guide the design of metamaterial stealth. Then, the bistatic radar cross section (RCS) of airfoil covered with metamaterial is researched in this paper. We employ the 2-D auxiliary differential equation finite-difference time-domain (ADE-FDTD) to simulate the airfoil’s stealth characteristics. The two-dimensional situation can be regarded as a special approximate when electromagnetic waves are perpendicularly incident on the three-dimensional infinitely long airfoil. The tuning parameters including coating thickness, Drude frequency and collision frequency are considered on the airfoil’s bistatic RCS. The calculation program verified its correctness by simulating the perfect lens characteristics of electromagnetic waves propagating in the metamaterial plate. The numerical experiments show that the meta-material Drude frequency and collision frequency mechanism are more complicated than the coating thickness. The dual-station RCS increases at some angles and decreases at other angles than metal airfoils. But by choosing appropriate metamaterial parameters, airfoils with excellent stealth effect can be obtained.

1. Introduction

Stealth technology is significant to protect aircraft from radar detection. Reducing the radar cross section (RCS) of the airfoils are indispensable for the stealth of the whole aircraft, as the airfoils are the strong scattering sources of the aircraft. However, the airfoils are the main component that generates lift, the aerodynamic configure is essentially immutable. Shape stealth is to design the target's contour and reflects the radar wave back away from the enemy radar receiving antenna, so that it returns a weak signal to the radar receiving antenna [1-3]. In order to achieve the balance between aerodynamic and stealth, we need to consider some technologies that do not change the aerodynamic shape of the aircraft. A mature technique for attenuation of electromagnetic waves is the use of absorbing materials. The absorbing material attenuates and reduces the reflection of electromagnetic
waves by increasing the impedance matching ratio with the free space. Absorbing material coatings generally use magnetic materials with large magnetic loss and dielectric loss as the absorbing carrier [4-5], including ferrite, magnetic metal and alloy powder.

However, the stealth shape of the aircraft affects its aerodynamic characteristics and it is difficult to achieve a balance between aerodynamic and stealth by relying solely on shape stealth. Fortunately, metamaterials seem to have great potential and promise in stealth technology. In References [6-7], a tunable terahertz metamaterial absorber with two bands is proposed, which can achieve the characteristics of both broadband and single peak absorption. Mohammad Alibakhshikenari proposed a novel on-chip antenna using standard CMoS-technology based on metasurface implemented on two-layers polyimide substrates with a thickness of 500 μm, which exhibit a very large bandwidth of 0.350–0.385 THz with an average radiation gain and efficiency of 8.15dBi and 65.71%, respectively [8].

An effective method is presented for suppressing mutual coupling between adjacent radiating elements which is based on metasurface isolation for multiple-input multiple-output (MIMO) and synthetic aperture radar (SAR) systems. It is achieved by choking surface current waves induced over the patch antenna by inserting a cross-shaped metasurface structure between the radiating elements [9].

A novel configuration for a 2×2 microstrip patch antenna based on the metamaterial concept exhibiting a super-wide impedance bandwidth, extending from 20 GHz to 120 GHz for S11<-15 dB [10]. In Reference [11], metamaterial electromagnetic bandgap decoupling structures between the radiating elements are demonstrated to suppress mutual coupling between adjacent radiating elements in array antennas. Reference [12] shows a novel meta-surface wall reducing mutual coupling between adjacent microstrip patch antennas implemented on the same substrate and operating in the THz-band of 139–141 GHz, which is achieved by inserting a meta-surface wall between the radiating elements.

Reference [13] proposed a novel planar microstrip array antenna based on a simplified composite right/left-handed metamaterial transmission line (SCRLH-TL) for application in circularly polarized synthetic aperture radar (CP-SAR) systems operated in UHF, L, S, and C-Bands. Reference [14] presented a backfire-to-endfire leaky-wave antenna based on metamaterial transmission-lines with ability to scan from -25° to +45°. Reference [15] researched the radar cross section (RCS) reduction of scattering of perfectly electric conductive (PEC) cylinder coated with plasma and metamaterial by finite-difference time-domain (FDTD) method.

FDTD method is a time-domain algorithm [16-18], which shows unique advantages involving the electromagnetic scattering calculation of dispersion medium or arbitrary shape and complex structure [19]. Therefore, the ADE-FDTD method is used to analyze the effects of different metamaterial parameters on the bistatic RCS of the airfoil [20]. The two-dimensional situation can be regarded as a special approximate when electromagnetic waves are perpendicularly incident on the three-dimensional infinitely long airfoil. Airfoils in the real world are all three-dimensional, but when the ratio of the length of the airfoils to the wavelength large enough, it can be considered as approximately satisfying the infinite length condition.

2. Formulation

2.1. ADE FDTD method
Metamaterials made from assemblies of multiple elements usually arranged in repeating patterns can affect waves of electromagnetic radiation or sound in an effective manner. Those that exhibit a negative index of refraction for particular wavelengths are called negative-index metamaterials. They are dual-dispersion media, their dielectric coefficient and magnetic constitutive parameters are related to frequency. The dielectric constant and permeability are negative at the same time, so we need to consider the expressions of the electrical coefficient term and the magnetic constitutive parameter term of Maxwell equation. Drude dispersive medium model are utilized to research metamaterial in this paper. According to the constitutive relation
\[ \mathbf{D}(\omega) = \varepsilon(\omega) \mathbf{E}(\omega) = \varepsilon_0 \left[ \varepsilon_{\infty} + \chi(\omega) \right] \mathbf{E}(\omega) \]
\[ = \varepsilon_0 \varepsilon_{\infty} \mathbf{E}(\omega) + \varepsilon_0 \chi(\omega) \mathbf{E}(\omega) \]
\[ \mathbf{B}(\omega) = \mu(\omega) \mathbf{H}(\omega) = \mu_0 \left[ \mu_{\infty} + \chi_m(\omega) \right] \mathbf{H}(\omega) \]
\[ = \mu_0 \mu_{\infty} \mathbf{H}(\omega) + \mu_0 \chi_m(\omega) \mathbf{H}(\omega) \]  

Where \( \mathbf{D} \) is the electric flux, \( \mathbf{B} \) is the magnetic flux, \( \mathbf{E} \) and \( \mathbf{H} \) are electric field and magnetic field intensity, \( \omega \) is the frequency of electromagnetic waves, \( \varepsilon_0 \) and \( \mu_0 \) are permittivity and permeability in vacuum, respectively, \( \varepsilon_{\infty} \) and \( \mu_{\infty} \) are relative permittivity and relative permeability of infinite frequency, respectively, \( \chi_e \) and \( \chi_m \) are the electric and magnetic polarizability in Drude media. Maxwell's equations can be rewritten as

\[ \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \sigma \mathbf{E} = \varepsilon_0 \varepsilon_{\infty} \frac{\partial \mathbf{E}}{\partial t} + \sigma \mathbf{E} + \mathbf{J}_p \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} - \sigma_{\infty} \mathbf{H} = -\mu_0 \mu_{\infty} \frac{\partial \mathbf{H}}{\partial t} - \omega \mathbf{H} - \mathbf{J}_m \]

Where the frequency domain of polarized current and polarized magnetic current is

\[ \mathbf{J}_p(\omega) = j \omega \sigma_0 \chi_e(\omega) \mathbf{E}(\omega) \]
\[ \mathbf{J}_m(\omega) = j \omega \mu_0 \chi_m(\omega) \mathbf{H}(\omega) \]  

Where \( \mathbf{J}_p, \mathbf{J}_m \) are polarization current and polarization magnetic current in Drude media. The curl equations can be discretized as

\[ \left[ \nabla \times \mathbf{H} \right]_{n-1}^{n+1} = \varepsilon_0 \varepsilon_{\infty} \frac{\mathbf{E}_{n}^{n+1} - \mathbf{E}_{n}^{n-1}}{2 \Delta t} + \sigma \frac{\mathbf{E}_{n}^{n+1} + \mathbf{E}_{n}^{n-1}}{2} + \frac{\mathbf{J}_p^{n+1} - \mathbf{J}_p^{n}}{2 \Delta t} \]
\[ \left[ \nabla \times \mathbf{E} \right]_{n} = -\mu_0 \mu_{\infty} \frac{\mathbf{H}_{n}^{n+1} - \mathbf{H}_{n}^{n-1}}{2 \Delta t} - \sigma_{\infty} \frac{\mathbf{H}_{n}^{n+1} + \mathbf{H}_{n}^{n-1}}{2} - \frac{\mathbf{J}_m^{n+1} - \mathbf{J}_m^{n-1}}{2 \Delta t} \]  

The operator transition relation \( j \omega \rightarrow \partial / \partial t \) is employed to transform the frequency domain to the time domain. It can be written as

\[ \frac{\mathbf{J}_p^{n+1} - \mathbf{J}_p^{n}}{\Delta t} = \frac{\mathbf{J}_p^{n+1}}{2 \Delta t} + \mathbf{J}_p^{n-1} + \frac{\mathbf{J}_p^{n+1} - \mathbf{J}_p^{n}}{2} = -\varepsilon_0 \varepsilon_{\infty} \frac{\mathbf{E}_{n}^{n+1} + \mathbf{E}_{n}^{n-1}}{2} \]
\[ \frac{\mathbf{J}_m^{n+1} - \mathbf{J}_m^{n-1}}{\Delta t} = \frac{\mathbf{J}_m^{n+1}}{2 \Delta t} + \mathbf{J}_m^{n-1} + \frac{\mathbf{J}_m^{n+1} - \mathbf{J}_m^{n-1}}{2} = -\mu_0 \mu_{\infty} \frac{\mathbf{H}_{n}^{n+1} + \mathbf{H}_{n}^{n-1}}{2} \]

Where \( \omega_{pe} \) and \( \omega_{pm} \) are Drude frequencies, \( \nu_{ce} \) and \( \nu_{cm} \) are collision frequencies. It can be seen that electric and magnetic parameters takes the same dispersive model in this paper. The polarization current term can be rewrite as [21]

\[ \mathbf{J}_p^{n+1} = k_p \mathbf{J}_p^{n} + \beta_p \left( \mathbf{E}_{n}^{n+1} + \mathbf{E}_{n}^{n-1} \right) \]
\[ \mathbf{J}_m^{n+1} = k_m \mathbf{J}_m^{n-1} + \beta_m \left( \mathbf{H}_{n}^{n+1} + \mathbf{H}_{n}^{n-1} \right) \]  

Where

\[ k_p = \frac{2 - \nu_{ce} \Delta t}{2 + \nu_{ce} \Delta t}, \beta_p = \frac{\varepsilon_0 \varepsilon_{\infty} \Delta t}{2 + \nu_{ce} \Delta t}, k_m = \frac{2 - \nu_{cm} \Delta t}{2 + \nu_{cm} \Delta t}, \beta_m = \frac{\mu_0 \mu_{\infty} \Delta t}{2 + \nu_{cm} \Delta t} \]

Furthermore, the iterative formulas of electric and magnetic fields can be obtained
The ability of the target to reflect electromagnetic waves can be represented by the radar cross section (RCS), which is an important parameter used to measure the stealth ability of the target. The radar cross section of a two-dimensional target is also called the scattering width and can be defined as follows

\[
E^{z+1} = \frac{2\varepsilon_e \varepsilon_m + \beta_z \Delta t - \sigma z \Delta t}{2\varepsilon_e \varepsilon_m - \beta_z \Delta t + \sigma z \Delta t} \cdot E^z + \frac{2\Delta t}{2\varepsilon_e \varepsilon_m - \beta_z \Delta t + \sigma z \Delta t} \left\{ \left[ \nabla \times H \right]^{z+1} - \frac{1}{2} (1 + k_z) J^z \right\}
\]

\[
H^{z+1} = \frac{2\mu_e \mu_m - \beta_z \Delta t - \sigma z \Delta t}{2\mu_e \mu_m - \beta_z \Delta t + \sigma z \Delta t} \cdot H^z - \frac{2\Delta t}{2\mu_e \mu_m - \beta_z \Delta t + \sigma z \Delta t} \left\{ \left[ \nabla \times E \right]^{z+1} + \frac{1}{2} (1 + k_z) J^z \right\}
\]

The ability of the target to reflect electromagnetic waves can be represented by the radar cross section (RCS), which is an important parameter used to measure the stealth ability of the target. The radar cross section of a two-dimensional target is also called the scattering width and can be defined as follows

\[
\sqrt{\sigma} = \lim_{R \to \infty} \sqrt{2 \pi R} \frac{\hat{e}_s \cdot E_s}{\hat{e}_i \cdot E_i}
\]  

(9)

Where \( \hat{e}_s \) is polarization direction of receiving antenna, \( \hat{e}_i \) is transmitting antenna polarization direction, \( E_s \) and \( E_i \) are the scattering electric field and the incident electric field, respectively. The bistatic angle is defined as the angle between \( \hat{e}_s \) and \( \hat{e}_i \).

2.2. Perfect lens feature verification

In metamaterial, due to its negative refractive index, convergence effect will occur when incident wave passes through metamaterial plate. As long as the plate is thick enough, convergence point will be formed in left-handed material and focusing phenomenon will occur.

The planar lensing effect of metamaterials is simulated by the mentioned algorithm, where the parameters of Drude media \( \nu_e = 0 \), \( \omega_p = \sqrt{\varepsilon_0} \) here is aim to let the refractive index of the metamaterial is \(-1\). The incident waveform is the sine wave.

The thickness of the metamaterial plate is four wavelengths, and the distance between the current source and the center of the metamaterial plate is four wavelengths. The computational model structure is shown in Figure 1 and the simulation results are shown in Figure 2.

![Figure 1. Schematic diagram of metamaterial plate model.](image)
Figure 2. Electromagnetic waves evolve over time in a metamaterial plate.

In Figure 2(a), the electromagnetic wave has not yet propagated to the metamaterial plate. We can clearly see that the electromagnetic wave propagates in the surrounding vacuum centering on the current source. In Figure 2(b), the wave front has reached the bottom of the metamaterial and refraction occurs at the boundary between the metamaterial and the vacuum. Although the electric field is continuous, part of the electromagnetic wave propagation direction is opposite to that in Figure 2(a). In Figure 2(c) and Figure 2(d), electromagnetic waves converge inside the metamaterial plate. The phase velocity of electromagnetic waves propagating in the metamaterial is opposite to that in vacuum. After passing through the metamaterial flat plate, the electromagnetic waves also converge on the other side similar to the convex lens, which verifies the possibility of using the metamaterial for the superlens.

3. Stealth performance of airfoil covered with metamaterial
The airfoil coated with metamaterial is exhibited in Figure 3. The airfoil’s chord length is 4 m. The incident excitation is transverse magnetic (TM) wave. The calculation region is divided into 200×200 grids and the mesh size are 1/20 of the wavelength. The time stability factor is 0.5 and ten-cell perfectly matched layer is utilized to truncate the computational boundary. The distance between the connection boundary and truncation boundary is thirty cells length.
Figure 3. The airfoil coated with metamaterial model: (a) Geometric structure diagram; (b) Schematic diagram of section division.

3.1. The influence of different coating thickness of metamaterial
Firstly, the influence of different thickness metamaterials on the scattering characteristics of airfoil is considered under the same Drude frequency and collision frequency. The metamaterial Drude frequency is 18GHz and collision frequency is 10GHz.

Figure 4. RCS of metamaterial coated metallic airfoil with different thickness.

It can be seen from Figure 4 that the greater the coating thickness is, the greater attenuation of the target RCS will be and the better the stealth effect will be. In addition, it can be seen that the attenuation amplitude of the target RCS will decrease when the target coating exceeds a certain thickness. The results show that the mean RCS decreases by 6.73 dB after the metal airfoil coated with 0.0025 m metamaterial in the 60° to 180° angular domain, and the maximum reduction in the entire angular domain is 11.49 dB. After coated with 0.05 m metamaterials in the 60° to 180° angular domain, the mean RCS decreased by 9.52 dB, and the maximum reduction in the entire angular domain was 20.01 dB. The mean RCS decreased by 10.82 dB after the metal airfoil coated with 0.075m metamaterial in the 60° to 180° angular domain, and the maximum reduction in the entire angular domain was 31.57 dB. After coating 0.1m metamaterials in the 60° to 180° angular domain, the mean RCS decreased by 10.55 dB, and the maximum reduction in the entire angular domain was 42.56 dB. The mean reduction effect of RCS with coating thickness of 0.075m was 60.72% higher than that with coating thickness of 0.025 m.
When no metamaterial is applied, the RCS first decreases with the increase of the bistatic angle, and then maintains a more stable level. When the coating thickness is 0.025m and 0.05m, the RCS decreases with the bistatic angle and then increases. When the coating thickness is increased to 0.075m and 0.1m, the overall change trend is the same as before, but the number of maximum peaks increases significantly.

3.2. The influence of different Drude frequencies of metamaterial
The effects of different Drude frequencies on the scattering properties of metamaterial airfoil with the same coating thickness and collision frequency are considered. The collision frequency of the metamaterial is 10 GHz and the coating thickness is 0.05 m.

![Figure 5. RCS of metamaterial coated metallic airfoil at different Drude frequencies.](image)

In Figure 5, it's not really difficult to discover that different Drude frequencies have different attenuation of electromagnetic waves. Increasing Drude frequency in a certain range can increase the reduction of target RCS. The results show that the average RCS decreases by 2.21 dB and the maximum reduction is 4.21 dB in the whole angular domain after the metal airfoil coated with the metamaterial with Drude frequency of 5GHz in the 60° to 180° angular domain. Coating the metal airfoil with a Drude frequency of 10 GHz metamaterial in the 60° to 180° angle domain, the mean RCS decreased by 5.33 dB, and the maximum reduction was 7.77 dB in the whole angle domain. The average RCS decreased by 9.52 dB coated metamaterials with a Drude frequency of 18.8 GHz in the 60° to 180° angular domain, and the maximum reduction was 20.01 dB in the entire angular domain. The effect of Drude frequency on RCS is not obvious when the bistatic angle is within the range of 0°-60°. Within the 120°-170° range, the RCS decreases significantly as the Drude frequency increases.

3.3. The impact of different metamaterial collision frequencies
The effects of different collision frequencies on the scattering characteristics of metamaterial airfoil were considered under the same coating thickness and Drude frequency. The Drude frequency of metamaterials was 18.8 GHz and the coating thickness was 0.05 m.

In Figure 6, the attenuation of electromagnetic wave is different with different collision frequencies. The attenuation of airfoil RCS can be increased by increasing the collision frequency appropriately. The calculation result shows that the average RCS of the metal airfoil with a collision frequency of 5 GHz metamaterial is reduced by 6.73 dB in 60° to 180° angle domain, and the maximum reduction value is 22.58 dB in the entire angle domain. At 60° to 180° angle range, the average RCS of the metal
airfoil coated with a collision frequency of 10 GHz is reduced by 9.52 dB, and the maximum reduction value in the entire angular domain is 20.01 dB. When the collision frequency increases to 15 GHz, the average RCS is reduced by 11.90 dB in the angular domain of 60° to 180°, and the maximum reduction value is 30.63 dB in the entire angular domain. The average RCS is reduced by 13.73 dB with a collision frequency of 20 GHz in the 60° to 180° angular domain, and the maximum reduction value is 25.28 dB in the entire angular domain. When the collision frequency increases to 30 GHz, the RCS average value is reduced by 17.49 dB in the 60° to 180° angular domain, and the maximum reduction is 41.90 dB in the entire angular domain.

![Figure 6](image.png)

**Figure 6.** RCS of metamaterial coated metallic airfoil at different collision frequencies.

When the collision frequency is 5 GHz and 10 GHz, the overall variation trend of RCS decreases first and then increases with the increase of the bistatic angle. When the collision frequency increases to 15 GHz, 20 GHz and 30 GHz, the number of crest values increases significantly. When the bistatic angle is 60°-160°, the RCS decreases with the increase of collision frequency.

4. Conclusions

The ADE-FDTD method is utilized to investigate a two-dimensional metal airfoil covered with metamaterial. The accuracy of ADE-FDTD algorithm proposed in this paper are validated via numerical simulation of convergence of a metamaterial plate. Numerical experiments show that the stealth characteristics of metamaterial airfoils are closely related to coating thickness, Drude frequency and collision frequency. Then, the impacts of different metamaterial parameters on the airfoil stealth performance are researched. Compared to the metal airfoil with no coating, it is obvious that metamaterial coating has better stealth effect. And the RCS attenuation will be greater when the metamaterial thickness increases. Specifically, in the wide angular domain, the airfoil coated with metamaterial can reduce RCS of the side and backward directions, but increase the forward RCS. Increasing the Drude frequency can significantly reduce the bistatic RCS of the metamaterial airfoil, especially in the bistatic angle range of 120°-170°. But the effect of Drude frequency on RCS is not obvious when the bistatic angle is within the range of 0°-60°. When the collision frequency is 5 GHz and 10 GHz, the overall variation trend of RCS decreases first and then increases with the increase of the bistatic angle. When the collision frequency increases to 15 GHz, 20 GHz and 30 GHz, the number of crest values increases significantly. When the bistatic angle is 60°-160°, the RCS decreases with the increase of collision frequency. By tuning the metamaterial coating, Drude frequency and collision frequency, we can obtain the desired stealth effect of the airfoil. It can be concluded that
metamaterials can greatly change the scattering characteristics of the airfoil, which can provide a new idea for aircraft stealth technology.

References

[1] Mahmoud S F and Habib M K 1998 Design of a two layer microwave absorber[J]. Journal of Electromagnetic Waves and Applications 12(8) 1005-1014
[2] Swarner W G and Peters Jr L 1963 Radar cross sections of dielectric or plasma coated conducting spheres and circular cylinders[J]. IEEE Transactions on Antennas and Propagation 11(5) 558-569
[3] Lock K W, Mark R and Igor A 1996 Conceptual study of stealth plasma antenna[C]. Proceed of the 1996 IEEE International Conference on Plasma Science 261
[4] Matsumoto M and Miyata 1997 Thin electromagnetic wave absorber for quasi-microwave band containing aligned thin magnetic metal particles[J]. IEEE Transactions on Magnetics 33(6) 4459-4464
[5] Lan C H, Hu X W and Jiang Z H 2008 Interaction of electromagnetic waves with two-dimensional metal covered with radar absorbing material and plasma Plasma Sci. Technol 10(6) 717-723
[6] Zhang Y, Cen C, Liang C, Yi Z, Chen X, Li M and Zhang G 2019 Dual-band switchable terahertz metamaterial absorber based on metal nanostructure Results in Physics 14 102422
[7] Landy N I, Sajuyigbe S, Mock J J, Smith D R and Padilla W J 2008 Perfect metamaterial absorber Physical Review Letters 100(20) 207402
[8] Alibakhshikenari M, Virdee B S, Chan H S, et al. 2020 High-Gain Metasurface in polyimide on-chip Antenna Based on cRLH-TL for Sub-terahertz integrated circuits[J]. Scientific Reports 10(1) 1-9
[9] Alibakhshikenari M, Virdee B S, Chan H S, et al. 2019 Surface wave reduction in antenna arrays using metasurface inclusion for MIMO and SAR systems[J]. Radio Science 54(11) 1067-1075
[10] Alibakhshikenari Mohammad, et al. 2019 Super-wide impedance bandwidth planar antenna for microwave and millimeter-wave applications[J]. Sensors 19(10) 2306
[11] M Alibakhshikenari, M Khalily, Virdee B S, Chan H S, R A Abd-Alhameed and E Limiti 2019 Mutual-Coupling Isolation Using Embedded Metamaterial EM Bandgap Decoupling Slab for Densely Packed Array Antennas[J]. IEEE Access 7 pp 5182-51840
[12] Alibakhshikenari Mohammad, et al. 2018 Meta-surface Wall Suppression of Mutual Coupling between Microstrip Patch Antenna Arrays for THz-band Applications Progress In Electromagnetics Research Letters 75 105-111
[13] Alibakhshikenari Mohammad, Virdee Bal Singh and Ernesto Limiti 2018 Wideband planar array antenna based on SCRLH-TL for airborne synthetic aperture radar application Journal of Electromagnetic Waves and Applications 32(12) 1586-1599
[14] Alibakhshikenari Mohammad, et al. 2018 A novel monofilar-ArchiMedean metamaterial inspired leaky-wave antenna for scanning application for passive radar systems Microwave and Optical Technology Letters 60(8) 2055-2060
[15] Ji Jinzu, et al. 2018 Scattering reduction of perfectly electric conductive cylinder by coating plasma and metamaterial[J]. Optik 161 98-105
[16] Liu S B, Liu S Q and Yuan N C 2006 FDTD simulation of bistatic scattering by conductive cylinder covered with inhomogeneous time-varying plasma[J]. Plasma Science Technology 8(2) 190-194
[17] Liu S, Zhong S, et al. 2008 Finite-difference time-domain algorithm for dispersive media based on Runge-Kutta exponential time differencing method International Journal of Infrared and Millimeter Waves 29(3) 323-328
[18] J P Berenger 1994 A perfectly matched layer for the absorption of electromagnetic waves Journal of Computational Physics 114(2) 185-200
[19] K S Yee 1966 Numerical solution of initial boundary value problems involving Maxwell’s equations in isotropic media *IEEE Transactions on Antennas and Propagation* 14(3) 302-307

[20] Y Takayama and W Klaus 2002 Reinterpretation of the auxiliary differential equation method for FDTD *IEEE Microwave and Wireless Components Letters* 12(3) 102-104

[21] Ji Jinzu, et al. 2018 Research on scattering characteristics of metamaterials based on ADE-FDTD[J]. *Optik* 164 402-406