Periodic interference structures in the time-like proton form factor

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Abstract

An intriguing and elusive feature of the timelike hadron form factor is the possible presence of an imaginary part associated to rescattering processes. We find evidence of that in the recent and precise data on the proton timelike form factor measured by the BABAR collaboration. By plotting these data as a function of the 3-momentum of the relative motion of the final proton and antiproton, a systematic sinusoidal modulation is highlighted in the near-threshold region. Our analysis attributes this pattern to rescattering processes at a relative distance of 0.7-1.5 fm between the centers of the forming hadrons. This distance implies a large fraction of inelastic processes in pp interactions, and a large imaginary part in the related $e^+e^- \to pp$ reaction because of unitarity.

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Electromagnetic hadron form factors (FFs) are fundamental quantities which describe the internal structure of the hadron (for a recent review see Ref. [1]). FFs enter explicitly in the coupling of a virtual photon with the hadron electromagnetic current, and can be directly compared to hadron models which describe dynamical properties of hadrons. They are experimentally accessible through the knowledge of the differential cross section and the polarization observables. Efforts are presently directed, on one side, to increase the precision and, on the other side, to extend the kinematic range of the measurements.

Traditionally most data on FFs come from electron-proton elastic scattering. It is assumed that the interaction occurs through the exchange of a single virtual photon which carries a four momentum transfer squared $q^2$. In this kinematical region (space-like form factors, SLFFs) the virtual photon-proton coupling and the electron-proton scattering cross section are described via two real FFs, electric, $G_E$, and magnetic, $G_M$.

FFs have been also studied in the time-like region of momentum transfer squared (TLFFs). They are measured through the reactions

$$e^+ + e^- \to \bar{p} + p,$$  \hspace{1cm} (1)

$$\bar{p} + p \to e^+ + e^-,$$  \hspace{1cm} (2)

where a hadron pair is formed by or annihilated into a virtual photon. In the following we will refer to the former reaction, when not otherwise specified.

Assuming single photon exchange, the unpolarized cross section contains the squared moduli of two TLFFs (electric and magnetic FF), which are complex functions of $q^2$. The imaginary part is expected to be large and in-formation on the relative phase between $G_E$ and $G_M$ can be extracted from single spin polarization experiments [2], which are presently out of reach. In this letter evidence

for periodic structures in TLFF data is reported and related to their complex nature.

Many models for the hadron coupling to the virtual photon have been developed and applied to the calculation of SLFFs. Some may be applied or analytically continued to the TL region (see [1] for a detailed review). This is the case for approaches based on vector meson dominance [3,4] and dispersion relations [5,6] where the complex nature of TLFFs arises naturally. Constituent quark models in light front dynamics may be applied [4], as well as approaches based on AdS/QCD correspondence [8]. More recently, a phenomenological picture has been proposed for an interpretation of FFs in both the SL and TL regions [9].

The angular distribution allows in principle for the individual determination of the electric and magnetic TLFF, but until now the luminosity has not been sufficient for a precise measurement of the angular distribution. The results of the various experiments are therefore compared on the basis of a generalized FF [10], which is proportional to the unpolarized cross section $\sigma$:

$$|F_p|^2 = \frac{3\beta q^2 \sigma}{2\pi \alpha^2 \left(2 + \frac{1}{\tau}\right)},$$  \hspace{1cm} (3)

(where $\alpha = e^2/(4\pi), \beta = \sqrt{1 - 1/\tau}, \tau = \frac{q^2}{4M^2}), M$ is the proton mass.

Even in these simplified terms, it has long been difficult to analyse with precision the behaviour of the data over a broad kinematic range because of the uncertainties and of the matching of data from different experiments which covered limited $q^2$ regions. The recent data by the BABAR collaboration [11,12] cover with reasonable continuity a region ranging from slightly over the $\bar{p}p$ threshold to $q^2 \approx 36$ GeV$^2$. In particular about 30 data points have been extracted in the region $q^2 < 10$ GeV$^2$. 


with a relative error lower than 10%. These features allow for a refined analysis of the systematic behavior of the timelike form factor, where large-scale and small-scale (in $q^2$ sense) properties of the data distribution may be scrutinized.

From now on, we use the expression "near-threshold region" to indicate a $q^2$-range extending from the threshold of the $pp$ channel up to $q^2 \approx 9 \text{ GeV}^2$ (with the convention $c = \hbar = 1$). In this kinematic region, two different scales participate: on one side, the total energy of the colliding $e^+e^-$ pair is $> 2M \approx 1.9 \text{ GeV}$, while the kinetic energy of the created $\bar{p}p$ pair is relatively small. Therefore one may expect to observe complex effects where a highly relativistic formation picture expressed in terms of quarks and gluons coexists with non relativistic interactions of two slow hadrons leaving the formation zone.

Proton-antiproton interactions in the near-threshold region have been studied in experiments at LEAR (see references therein for previous data) and more recently at AD [15]. These measurements could not separate spin channels and, as in the case of the single effective scattering region [13, 14] and references therein for previous data) and Watson’s final state theorem [16] applied to reaction (1). Near the threshold this is rigorously stated by the total annihilation cross section $\bar{p}p \rightarrow X \neq \bar{p}p$ (or $n + n$) is pretty large. Unitarity says that a large imaginary part is present in the amplitude for each of the $\bar{p}p$ state presents a "hole" of size 1 fm. Within 400 GeV/c of the threshold, this property is demonstrated by several counter-intuitive phenomena, in particular the equality of $\bar{p}p$, $\bar{p}D$, and $\bar{p}^4\text{He}$ annihilation cross sections at small energies (see [17]), and the suppressed effect of the electric charge in $\bar{p}$-nucleus annihilations with the paradoxical effect of $\bar{n}$ cross sections on heavy nuclei exceeding $\bar{p}$ ones by a factor of 2, in the nonrelativistic limit, $p\bar{p}$ coincides with the momentum of the relative motion of the pair in its centre of mass frame. The usefulness of this variable presumes that the process may be divided into two stages: formation and rescattering, where the latter involves energies on a smaller scale than the former. This means that the amplitude for the process is the sum of a leading term due to a "bare formation" process taking place on a time scale $1/\sqrt{q^2}$, and a relatively small perturbation associated to rescattering and other hadronization processes taking place on a larger time scale.

A consequence of this assumption is that the measured form factors can be fitted by a function of the form:

$$F(p) = F_0(p) + F_{\text{osc}}(p),$$

where:

- $F_0(p)$ is the translation in terms of the variable $p$ of a well known fit that has been extracted from data in the full range $4M^2 < q^2 < 36 \text{ GeV}^2$ (see equations and references later) ignoring small-scale oscillations.

- $F_{\text{osc}}(p)$ reproduces GeV-scale or sub-GeV-scale irregularities in the lower part of the $p$ range. We name it "oscillation fit" for later clarified reasons.

In order to look in the data for signals of final state effects at small kinetic energies, it is more convenient to introduce variables directly related to the relative motion of the hadron pair. In the following we will use the 3-momentum $p$ of one of the two hadrons in the frame where the other one is at rest:

$$p \equiv \sqrt{E^2 - M^2}, \quad E \equiv q^2/(2M) - M.$$

Apart from a factor of 2, in the nonrelativistic limit, $p$ coincides with the momentum of the relative motion of the pair in its centre of mass frame. The usefulness of this variable presumes that the process may be divided into two stages: formation and rescattering, where the latter involves energies on a smaller scale than the former. This means that the amplitude for the process is the sum of a leading term due to a "bare formation" process taking place on a time scale $1/\sqrt{q^2}$, and a relatively small perturbation associated to rescattering and other hadronization processes taking place on a larger time scale.

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most precise data in the near-threshold region, and on the other side they cover with continuity a very large kinematic range. Both properties are necessary for our analysis. The fitting procedure is the one of Minuit package of root.cern.ch \[22\], based on a \(\chi^2\)-minimization procedure.

To satisfy Eq. \[7\] a good choice of the regular background term \(F_0(p)\) is needed.

The generalized FF has been consistently extracted at \(e^+e^-\) colliders and antiproton facilities. It shows a decreasing behavior as function of \(q^2\), which on the world data up to 2006 is reproduced in the experimental papers according to the function:

\[
|F_{\text{scaling}}(q^2)| = \frac{A}{(q^2)^2 \log^2(q^2/\Lambda^2)},
\]

where \(q^2\) is expressed in GeV\(^2\). A good fit of the data prior to BABAR last results was obtained with \(A = 40\) GeV\(^{-4}\) and \(\Lambda = 45\) GeV\(^2\). The functional form of eq. \[8\] is driven by an extension of the dipole model in TL region, and follows the scaling laws of QCD.

Based on Ref. \[31\], in order to avoid ghost poles in \(\alpha_s\), the following modification was suggested:

\[
|F_{\text{scaling}+\text{corr}}(q^2)| = \frac{A}{(q^2)^2 \left[ \log^2(q^2/\Lambda^2) + \pi^2 \right]},
\]

In this case the best fit parameters are \(A = 72\) GeV\(^{-4}\) and \(\Lambda = 0.52\) GeV\(^2\).

The TLFF data from the BABAR collaboration \[11\] \[12\] were obtained from the reaction

\[e^+e^- \rightarrow \bar{p}p + \gamma,\]

where the photon is preferentially emitted in the entrance channel. These data extend from the threshold to \(q^2 \approx 36\) GeV\(^2\) and are consistent with a steeper \(q^2\)-dependence \[32\]:

\[
|F_{\text{BABAR}}(q^2)| = \frac{A}{(1 + q^2/m_a^2)[1 - q^2/0.71]^2},
\]

\[A = 7.7\ \text{GeV}^{-4}, \ m_a^2 = 14.8\ \text{GeV}^2\]

The world data are shown in Fig. \[1\] as a function of the transferred momentum \(q^2\), and compared with the results from Eq. \[8\] (blue,dash-dotted line), Eq. \[9\] (red,dashed line), and Eq. \[11\] (black,solid line). The near-threshold region is highlighted in the insert of Fig. \[1\].

In the following, the reference background term is \(F_0(p)\), expressed as a function of \(p\):

\[F_0(p) \equiv F_{\text{BABAR}}[q^2(p)].\]

As evident from Fig. \[1\] this choice is the best one not to produce an artificial \(F_{\text{osc}}(p)\) at large momenta, that would not satisfy Eq. \[7\].

In Fig. \[2a\] the BABAR data are plotted as a function of \(p\). After fitting the BABAR data with the smooth function of Eq. \[11\], the result of the fit is subtracted from the data. This difference \(D\) (i.e., data minus \(F_0(p)\)) is shown in Fig. \[2b\] and exhibits a damped oscillatory behavior, with regularly spaced maxima and minima. Assuming that the first maximum is at \(p = 0\), the distance between this, the 2nd and the 3rd maximum is \(\Delta p \approx 1.1\) GeV. The first minimum is precisely half way between \(p = 0\) and the 2nd maximum, the second minimum is slightly closer to the 3rd maximum. After the 3rd maximum the oscillations of the data are within the error bars.

This behavior is fitted with the 4-parameter function

\[F_{\text{osc}}(p) \equiv A \exp(-Bp) \cos(Cp + D),
\]

\[A = -0.05, \ B = 0.73\ \text{GeV}^{-1}, \ C = 5.53\ \text{GeV}^{-1}, \ D = -9.44.\]

The relative size of the oscillating term over the regular one is \(\sim 10\%\). The parameter \(B\) indicates that the damping range of the oscillations of Fig. \[2b\] is \(1/B \approx 1.4\) GeV. \(F_0(p)\) decreases by a factor \(1/e\) within about 1.5 GeV. Therefore, the relative magnitude of the oscillations and of the regular background does not change much at increasing \(p\), although increasing relative errors.
make the oscillations indetectable for $p > 3$ GeV ($q^2 > 10$ GeV$^2$).

At asymptotically large $q^2$ values, the Phragmén-Lindelöff theorem requires that the imaginary part of TLFFs vanishes, which implies that rescattering disappears. So, although we expect a large-momentum suppression of the relative weight of the rescattering terms, this is not seen in the range $q^2 < 10$ GeV$^2$ where error bars allow us to distinguish systematic from statistical oscillations in the data.

The periodicity and the simple shape of the oscillations seem to exclude a random arrangement of maxima and minima of heterogeneous origin. Rather, they indicate a unique interference mechanism behind all the visible modulation.

For example, during the intermediate stages of $\bar{p}p$ formation it has been suggested that charge and color are distributed in a highly inhomogeneous but organized way, with relevant space correlations. Within such a structure it is natural that rescattered waves follow well separated paths with possible interference effects.

Since we do not know the precise rescattering mechanism we cannot identify the sources of rescattered waves, but we may gain some clue on their space distribution. Let $\vec{r}$ be the space variable that is conjugated to $\vec{p}$ via three-dimensional Fourier transform. We may identify $r$ as the distance between the centers of the two forming or formed hadrons, in a frame where one is at rest. Let $M_0(r)$ and $M(r)$ be the Fourier transforms of the regular background fit and of the complete fit:

$$F_0(p) = \int d^3\vec{r} \exp(i\vec{p} \cdot \vec{r}) M_0(r)$$ (14)

$$F(p) = F_0(p) + F_{\text{osc}}(p) = \int d^3\vec{r} \exp(i\vec{p} \cdot \vec{r}) M(r).$$ (15)

$M_0(r)$ is shown in Fig. 3 left panel. The most relevant feature is that $M_0(r)$ decreases by 7 orders of magnitude for $r$ ranging from 0 and to 2 fm. The decrease is regular and almost constant on a semilog scale, and $M_0(r)$ is steep near the origin too. From a mathematical point of view this follows from the fact that at the threshold of the $\bar{p}p$ channel the function $F_0(p) = F_{\text{BABAR}}(q^2)$ is a regular, continuous, and rapidly decreasing function of $q^2$. This can be interpreted by the fact that both $F_0(p)$ and its transform $M_0(r)$ are expression of those asymptotic quark-counting rules that permit exclusive $\bar{p}p$ production at the condition that the final quarks and antiquarks are formed within a small region. Near threshold, the size of this region is $\lesssim 0.1$ fm, much smaller than the standard hadron size.

In the right panel of Fig. 3 $M(r)$ is superimposed to $M_0(r)$. We notice that these two functions do not differ for $r < 0.7$ fm, and that the physical reason of the data oscillation must be searched for in processes taking place in the $r$-range 0.7-1.5 fm.

This range is important because it includes the distances corresponding to the largest annihilation probability in the phenomenological $\bar{p}p$ interactions in the near-threshold region (see the parameters of the potentials used to fit $\bar{p}p$ annihilation data, and the discussion on the relevance of the surface parameters in these works). At a distance of 1 fm a relevant part of rescattering must involve physical or almost physical hadrons that annihilate into groups of 2-10 mesons. As discussed
above, this means a large contribution to the imaginary part of the amplitude for $\gamma^* \rightarrow p\bar{p}$ from the Cutkoswky cuts applied to all the 2-step processes like $\gamma^* \rightarrow n\pi \rightarrow p\bar{p}$, where $n\pi$ is a state composed by $n$ on-shell pions (or other mesons).

Summarizing the main conclusions of this work, a systematic modulation pattern in the timelike proton form factor measured by the BABAR collaboration in the near-threshold region has been highlighted. This modulation presents periodical features with respect to the momentum $p$ associated with the relative motion of the final hadrons. It suggest an interference effect involving rescattering processes at moderate kinetic energies of the outgoing hadrons. These processes take place when the centres of mass of the produced hadrons are separated by more than 0.7 fm, most likely around 1 fm. For this reason at least a relevant part of rescattering must consist of interactions between phenomenological or almost phenomenological protons and antiprotons. Since these interactions are characterized by a huge amount of inelasticity, unitarity arguments imply the presence of a large imaginary part in the form factor.

The relative errors of the data increase with $q^2$, making us able to detect the modulation for $q^2 < 10 \text{ GeV}^2$, but its relative magnitude of about 10 % is constant in this range, suggesting the interesting possibility that this modulation could be observed at larger $q^2$ in future data with relative error < 10 %.

Precise measurements in the near threshold region are ongoing at BESIII (BEPCII), on the proton as well as on the neutron, bringing a new piece of information. The measurement of TL FFs in a large $q^2$ range will be possible at PANDA (FAIR).

[1] S. Pacetti, R. Baldini Ferroli, and E. Tomasi-Gustafsson, Phys.Rept. 550-551, 1 (2015).
[2] A. Dubnickova, S. Dubnicka, and M. Rekalo, Nuovo Cim. A109, 241 (1996).
[3] R. Bijker and F. Iachello, Phys.Rev. C69, 068201 (2004), nucl-th/0405028.
[4] C. Adamuscin, S. Dubnicka, A. Dubnickova, and P. Weisenpacher, Prog.Part.Nucl.Phys. 55, 228 (2005), hep-ph/0510316.
[5] M. Belushkin, H.-W. Hammer, and U.-G. Meissner, Phys.Rev. C75, 035202 (2007), hep-ph/0608337.
[6] E. L. Lomon and S. Pacetti, Phys.Rev. D85, 113004 (2012), 1201.6126.
[7] J. de Melo, T. Frederik, E. Pace, and G. Salme, Phys.Lett. B581, 75 (2004), hep-ph/0311369.
[8] S. J. Brodsky and G. F. de Teramond, Phys.Rev. D77, 056007 (2008), 0707.3559.
[9] E. Kuraev, E. Tomasi-Gustafsson, and A. Dbezysi, Phys.Lett. B712, 240 (2012), 1106.1670.
[10] G. P. Lepage and S. J. Brodsky, Phys.Rev.Lett. 43, 545 (1979).
[11] J. Lees et al. (BaBar Collaboration), Phys.Rev. D87, 092005 (2013), 1302.0055.
[12] J. Lees et al. (BaBar Collaboration) (2013), 1308.1795.
[13] A. Zenoni, A. Bianconi, G. Bonomi, M. Corradini, A. Donzella, et al., Phys.Lett. B461, 413 (1999).
[14] A. Zenoni, A. Bianconi, F. Bocci, G. Bonomi, M. Corradini, et al., Phys.Lett. B461, 405 (1999).
[15] A. Bianconi, M. Corradini, M. Hori, M. Leali, E. Lodi Rizzini, et al., Phys.Lett. B704, 461 (2011).
[16] K. M. Watson, Phys.Rev. 88, 1163 (1952).
[17] A. Bianconi, G. Bonomi, M. Bussa, E. Lodi Rizzini, L. Venturelli, et al., Phys.Lett. B483, 353 (2000), nucl-th/0002015.
[18] C. Batty, E. Friedman, and A. Gal, Nucl.Phys. A689, 721 (2001), nucl-th/0010006.
[19] E. Friedman, Nucl.Phys. A925, 141 (2014), 1402.3968.
[20] V. Matveev, R. Muradyan, and A. Tavkhelidze, Teor.Mat.Fiz. 15, 332 (1973).
[21] S. J. Brodsky and G. R. Farrar, Phys.Rev.Lett. 31, 1153 (1973).
[22] R. Brun and F. Rademakers, Nucl.Instrum.Meth. A389, 81 (1997).
[23] M. Ablikim et al. (BES Collaboration), Phys.Lett. B630, 14 (2005), hep-ex/0506059.
[24] M. Ambrogiani et al. (E835 Collaboration), Phys.Rev. D60, 032002 (1999).
[25] M. Andreotti, S. Bagnasco, W. Baldini, D. Bettoni, G. Borreani, et al., Phys.Lett. B559, 20 (2003).
[26] A. Antonelli, R. Baldini, M. Bertani, M. Biagini, V. Bidoli, et al., Phys.Lett. B334, 431 (1994).
[27] D. Bisello, S. Limentani, M. Nigro, L. Pescara, M. Posocco, et al., Nucl.Phys. B224, 379 (1983).
[28] D. Bisello et al. (DM2 Collaboration), Z.Phys. C48, 23 (1990).
[29] B. Delcourt, I. Derado, J. Bertrand, D. Bisello, J. Bizot, et al., Phys.Lett. B86, 395 (1979).
[30] T. Pedlar et al. (CLEO Collaboration), Phys.Rev.Lett. 95, 261803 (2005), hep-ex/0510005.
[31] D. Shirkov and I. Solovtsov, Phys.Rev.Lett. 79, 1209 (1997), hep-ph/9704333.
[32] E. Tomasi-Gustafsson and M. Rekalo, Phys.Lett. B504, 291 (2001).
[33] E. Titchmarsh, The Theory of Functions, Oxford science publications (Oxford University Press, 1939), ISBN 9780198533498.