Thermally activated switching rate of a nanomagnet in the presence of spin torque

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The current dependence of the spin torque switching rate in a thermally activated region of an in-plane magnetized system was studied. The logarithm of the switching rate depended nonlinearly on current in the high-current region, \( I \lesssim I < I^* \), where \( I_c \) and \( I^* \) are critical currents distinguishing the stability of magnetization. We also found that the attempt frequency had a minimum around \( I_c \), and that the attempt frequency at \( I_c \) was three orders of magnitude smaller than that at zero current, contrary to the assumption in previous analyses of experiments that it remains constant.

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I. INTRODUCTION

The escape problem of a Brownian particle from a meta-stable state is ubiquitous in many fields of science, such as chemical reaction of molecules\textsuperscript{1–7}. Spin-torque-driven magnetization switching\textsuperscript{8–29} in nanostructured ferromagnets in the thermally activated region also belongs to this problem, which has been extensively studied because of its potential application to spintronics devices such as magnetic random access memory (MRAM) and magnetic sensor. The observation of the magnetization switching provides us important information, such as the retention time of the MRAM. More than a decade has passed since the first experimental and theoretical works on spin torque switching in the thermally activated region\textsuperscript{10,11}.

The spin torque switching can be regarded as the Brownian motion in the presence of a non-conservative force, contrary to the switching by magnetic field, which is a conservative force defined as the gradient of a potential. The lack of a general method to formulate the switching rate in the presence of the non-conservative force is an unresolved problem in statistical physics\textsuperscript{17,18}. Therefore, many assumptions have been used in the previous theories of the spin torque switching\textsuperscript{14–16}. However, recent works\textsuperscript{19–24} have revealed the limits of the applicability of previous theories. For example, the switching rate has been assumed to obey the Arrhenius law, \( \nu = f e^{-\Delta} \), with linear scaling of the switching barrier, \( \Delta = \Delta_0 (1 - I/I_c) \), where \( f \), \( \Delta_0 \), \( I \), and \( I_c \) are the attempt frequency, the thermal stability, the current, and the critical current of the precession around the easy axis, respectively\textsuperscript{14–16}. However, the linear scaling is valid only in the low-current region\textsuperscript{24}, while a relatively large current has been applied in experiments\textsuperscript{20–22} to observe the switching quickly. The use of the linear scaling leads to an error of the estimation of the thermal stability\textsuperscript{22}. Another issue is that the transition state theory previously adopted\textsuperscript{16} cannot estimate the switching rate under a low damping limit\textsuperscript{25}, while the Gilbert damping constant of materials typically used in spintronics application are very low\textsuperscript{25}, i.e., \( \alpha = 10^{-3} - 10^{-2} \). These facts prompted us to revisit the theory of spin torque switching in a thermally activated region.

In this paper, we study the spin-torque-switching rate of an in-plane magnetized system using the mean first passage time approach\textsuperscript{23,25,27}. The introduction of the effective energy density enables us to calculate the switching rate even in the presence of the non-conservative force. The switching rate showed a nonlinear dependence on the current in the high-current region \( (I_c \lesssim I < I^*) \) on a logarithmic scale, where \( I^* \simeq 2.7 \alpha \) is the spin torque switching current at zero temperature. The attempt frequency was strongly suppressed around \( I_c \) contrary to the assumption in previous experimental analysis that it remains constant\textsuperscript{10–12}. For example, the attempt frequency at \( I_c \) was three orders of magnitude smaller than that at the zero current. The theoretical approach presented in this paper is useful for the escape problem of a Brownian particle under a non-conservative force when the magnitude of the non-conservative force is much smaller than that of the conservative force.

The paper is organized as follows. In Sec. \textsuperscript{III} the Fokker-Planck equation for the magnetization dynamics in the energy space is introduced based on the small damping assumption. In Sec. \textsuperscript{IIII} the current dependence of the switching rate, as well as that of the attempt frequency, is calculated by using the mean first passage time approach. Section \textsuperscript{IX} is devoted to the conclusion.

II. FOKKER-PLANCK EQUATION IN ENERGY SPACE

Figure \textsuperscript{II} (a) schematically shows an in-plane magnetized system, where the \( x \) and \( z \) axes are normal to the film plane and parallel to the in-plane easy axis, respectively. The unit vectors pointing in the magnetization directions of the free and the pinned layers are denoted as \( \mathbf{m} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \) and \( \mathbf{n}_p = e_z \), respectively. Here, we assume that the magnetization dynamics is well described by the macrospin model. The macrospin assumption is, at least for the grand state, guaranteed by the spin torque diode experiment\textsuperscript{28} in which the os-
oscillation frequency of the free layer magnetization agrees with the ferromagnetic resonance frequency derived by the macrospin model. The positive constant is defined as the electron flow from the free layer to the pinned layer. The energy density of the in-plane magnetized system is

\[
E = -\frac{MH_K}{2}(\mathbf{m} \cdot \mathbf{e}_z)^2 + \frac{4\pi M^2}{2}(\mathbf{m} \cdot \mathbf{e}_z)^2,
\]

where \(M, H_K,\) and \(-4\pi M\) are the magnetization, the uniaxial anisotropy field along the \(z\)-axis, and the demagnetization field along the \(x\)-axis, respectively. The magnetic field is defined as \(H = -\partial E/(\partial M\mathbf{m})\). Figure 1(b) schematically shows the constant energy curves of \(E\) in \((\theta, \phi)\) space. The in-plane magnetized system has two low-energy regions around the energy minima at \(\mathbf{m} = \pm \mathbf{e}_z\) corresponding to \(E = -MH_K/2 \equiv E_K\). These two low-energy regions are separated by the saddle point \(\mathbf{m} = \pm \mathbf{e}_y\) at which the energy density \(E_s = 0\). We named the low-energy region, \(E_K \leq E \leq E_s\), around \(\mathbf{m} = +\mathbf{e}_z(-\mathbf{e}_z)\) as region 1 (2). The area outside regions 1 and 2 corresponds to the high-energy region. The magnetization dynamics is described by the Landau-Lifshitz-Gilbert equation with the random torque,

\[
\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H} - \gamma H_s \mathbf{m} \times (\mathbf{n}_p \times \mathbf{m}) - \gamma \mathbf{m} \times \mathbf{h} + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt},
\]

where \(\gamma\) and \(\alpha\) are the gyromagnetic ratio and the Gilbert damping constant, respectively. The spin torque strength, \(H_s = \hbar I_s/(2eMV)\), consists of the current \(I_s\), the spin polarization \(\eta_s\), and the volume of the free layer. The components of the random field, \(h_k (k = x, y, z)\), satisfy the fluctuation-dissipation theorem \(\langle h_i(t)h_j(t') \rangle = (2D/\gamma^2)\delta_{ij}\delta(t-t')\), where the diffusion coefficient \(D = \alpha\gamma k_B T/(MV)\) consists of the Boltzmann constant \(k_B\), and the temperature \(T\).

During the switching between the regions 1 and 2, the magnetization precesses on the constant energy curve around the easy axis. The precession period is determined by the anisotropy fields, \(H_K\) and \(4\pi M\), and is typically on the order of 1 ms \(^2\). On the other hand, the switching time is determined by the damping, the spin torque, and the random field, and is on the order of 1 µs-1 ms, depending on the current magnitude \(I_s\). Such long time scale of the switching is due to the fact that the correlation function of the random torque, which induces the switching, is proportional to the small parameter \(\alpha^2\). Therefore, the precession period on the constant energy curve is much shorter than the switching time. Also, because of the large demagnetization field due to the thin film geometry, as soon as the energy exceeds the saddle points energy, the magnetization moves from region 1 (2) to 2 (1) by the precession around the demagnetization field, and relaxes to region 2 (1). Therefore, the dominant contribution to the switching rate is the time climbing the potential well of region 1 or 2. Thus, we average the magnetization dynamics on the constant energy curve in regions 1 and 2, and neglect the high-energy region. The averaged dynamics is described by the Fokker-Planck equation in the energy space, which can be derived from Eq. (2) and is given by

\[
\frac{\partial P}{\partial t} + \frac{\partial J}{\partial E} = 0,
\]

where \(P = P(E, t|E', t')\) is the transition probability function of the magnetization direction from the state \((E', t')\) to \((E, t)\). The probability current is

\[
J = -\frac{\alpha M \mathcal{M}_\alpha}{\gamma \tau} \frac{dE}{E} P - D \left(\frac{M}{\gamma}\right)^2 \mathcal{M}_\alpha \frac{\partial P}{\partial E}. \quad (4)
\]

We use the approximation \(1 + \alpha^2 \approx 1\) because the present theory is based on the small damping assumption. The effective energy density for the region \(i\) is defined as

\[
\varepsilon_i(E) = \int_{E_s}^{E} dE' \left[1 - \frac{\mathcal{M}_e(E')}{\alpha \mathcal{M}_e(E')}\right], \quad (5)
\]

where the lower boundary of the integral, \(E_s\), is chosen to make the effective energy density continuous at the boundary of the regions 1 and 2. The precession period on the constant energy curve, \(\tau = \frac{4}{\gamma \sqrt{H_K(4\pi M - 2E/M)}}\), is given by

\[
\tau = \frac{4}{\gamma \sqrt{H_K(4\pi M - 2E/M)}} K \left\{ \frac{4\pi M(H_K + 2E/M)}{H_K(4\pi M - 2E/M)} \right\} \left[ \frac{4\pi M(H_K + 2E/M)}{H_K(4\pi M - 2E/M)} \right], \quad (6)
\]

\[
\mathcal{M}_\alpha = 4\gamma \frac{4\pi M(4\pi M - 2E/M)}{H_K} K \left[ \frac{4\pi M(H_K + 2E/M)}{H_K(4\pi M - 2E/M)} \right] + H_K E \left[ \frac{4\pi M(H_K + 2E/M)}{H_K(4\pi M - 2E/M)} \right], \quad (7)
\]
The physical meanings of $I_{c1}$ and $I_{c2}$ are that, for region $1$, the state $m = e_z$ is destabilized by the current $I > I_c$ while the magnetization switches without the thermal fluctuation for $I > I_c^*$. Therefore, in terms of the current, the thermally activated region is defined by $I < I_c^*$. It should also be noted that the steady state solution of Eq. [3] in the region $i$ is the Boltzmann distribution with the effective energy density, i.e., $P/E \propto e^{-\mathcal{E}_i(E)/V/(k_B T)}$.

Figure 2 shows the typical dependences of $\mathcal{E}_i$ on $E$ for $I \leq I_c$ and $I_c < I \leq I_c^*$, where the values of the parameters are $M = 1000$ emu/c.c., $H_K = 200$ Oe, $V = \pi \times 80 \times 35 \times 2.5$ nm$^3$, $\eta = 0.8$, $\gamma = 1.764 \times 10^7$ rad/(Oe-s), and $\alpha = 0.01$, respectively. We denote the energy density corresponding to the local minimum of the effective energy density as $E^*$, which for region 1 is located at

$$E^*(\text{region 1}) = \begin{cases} E_K & (I \leq I_c) \\ \text{solution of } d\mathcal{E}_1/dE = 0 & (I_c < I < I_c^*) \end{cases}. \quad (11)$$

The minimum of $\mathcal{E}_2$ always locates at $E^* = E_K$.

### III. MEAN FIRST PASSAGE TIME APPROACH TO SWITCHING RATE

The mean first passage time, which characterizes how long the magnetization stays in the energy range $E^* \leq E \leq E_s$ of the region $i$, is defined as

$$T_i(E) = \int_0^E dt \int_{E^*}^{E_s} dE_i P(E_i, t|E, 0). \quad (12)$$

The equation to determine the mean first passage time is obtained from the adjoint of Eq. [11], and is given by

$$\frac{\alpha M.\mathcal{M}_i d\mathcal{E}_i dT_i}{\gamma E dE} - D \left( \frac{M}{\gamma} \right) \frac{1}{E} \frac{d}{dE} \mathcal{M}_i \frac{dT_i}{dE} = 1. \quad (13)$$

We use the reflecting and the absorbing boundary conditions at $E = E^*$ and $E = E_s$, respectively: that is, $dT_i(E^*)/dE = 0$ and $T_i(E_s) = 0$. Then, the mean first passage time is given by

$$T_i(E) = \frac{\gamma V}{\alpha M k_B T} \int_{E_s}^{E} dE_i \int_{E}^{E_s} dE_2 \frac{\tau(E_2)}{\mathcal{M}_i(E_1)} \times \exp \left( \frac{[\mathcal{E}_i(E_1) - \mathcal{E}_i(E_2)]V}{k_B T} \right). \quad (14)$$

The switching rate from region $i$ to region $j$ is

$$\nu_{ij} = \frac{d\mathcal{E}_j(E)}{dE} \frac{1}{d\mathcal{E}_i(E_j)/dE + d\mathcal{E}_i(E_j)/dE} T_i(E^*). \quad (15)$$

Here, we assume that once the magnetization reaches the saddle point, the probability of it moving to the regions $i$ or $j$ is proportional to the gradient of the effective energy, i.e., the deterministic force acting on a Brownian particle. For a conservative system, Eq. [15] is $1/(2T_i)$. The switching probability $P$ and the switching current distribution $dP/dI$ measured in the experiments can be calculated from Eq. [15]. For example, for $I > 0$, the switching probability from $m = e_z$ to $m = -e_z$ is $P \approx 1 - e^{-\int_{0}^{t_s} \nu_{21}(t) dt}$, which should be noted that $\lim_{t \to t_s} \nu_{21}(t) = 0$ because region 1 is no longer stable due to the spin torque; thus, the magnetization immediately switches to region 2. For the same reason, $\lim_{t \to t_s} \nu_{21}(t_{21}) = 0$.

Equations [14] and [15] indicate that the switching rate cannot be expressed as the Arrhenius law, in general. However, it is convenient to introduce the switching
barrier and the attempt frequency as

$$\Delta_i = \frac{[\varepsilon_i(E_0) - \varepsilon_i(E_1)]V}{k_BT},$$

$$f_{ij} = \nu_{ij} e^{\Delta_i}.$$  \hspace{1cm} (17)

The current dependence of the switching barrier was extensively studied in Ref.\(^{24}\). In the low-current region, \(I < I_c\), corresponding to the high barrier limit, \(\Delta_i \gg \frac{k_BT}{E_1}E_1\), the exponential terms in Eq. (14) are dominated by \(E_1 = E_a\) and \(E_2 = E_K\), respectively. Using the Taylor expansion of \(\varepsilon_i\), \(\Delta_i\) can be approximated as

$$\Delta_i(E_K) \approx \frac{\gamma k_BT \tau(E_K) e^{\Delta_i}}{\alpha MV \cdot \mathcal{M}_a(E_a) [d\varepsilon_i(E_K)/dE][d\varepsilon_i(E_a)/dE]}.$$  \hspace{1cm} (18)

where \(\mathcal{M}_a(E_a) = 4\gamma/\sqrt{H_K + 4\pi M}\) and \(\tau(E_K) = 2\pi/|\gamma/\sqrt{H_K + 4\pi M}|\). Then, the switching rate obeys the Arrhenius law as follows:

$$\nu_{ij} = \frac{\alpha MV \cdot \mathcal{M}_a(E_a)}{2\gamma k_BT \tau(E_K)} \left(1 \mp \frac{I}{I_c}\right) \left[1 - \left(\frac{I}{I_c}\right)^2\right] \times \exp \left[-\Delta_0 \left(1 \mp \frac{I}{I_c}\right)\right],$$  \hspace{1cm} (19)

where \(\Delta_0 = M H_K V/(2k_B T)\) is the thermal stability. The term \((1 \mp I/I_c)\) of Eq. (19) arises from \(d\varepsilon_i(E_K)/dE\) in Eq. (15), while the term \(1 - (I/I_c)^2\) of Eq. (19) arises from \(d\varepsilon_i(E_a)/dE\) in Eqs. (13) and (18), respectively. The current \(I_c\) is defined as \(I_c = \int_{E_K}^{E_2} (dE/|E_K|) \alpha \cdot \mathcal{M}_a(E_a)/\mathcal{M}_a(E_a)\), which satisfies \(I_c < I < I_c^*\). Although the linear scaling of the switching barrier appears in this low-current region, the scaling current is not the switching current, as argued in Refs.\(^{14-16}\). This means that the previous analyses of the experiments underestimate the real value of the switching current.\(^{10,11,14,15,16}\) Equation (19) can be directly reproduced by applying Brown’s approach\(^{20}\) to Eq. (3), as shown in Appendix.

Equation (17) becomes zero in the zero-dissipation limit \((\alpha \to 0)\) because the correlation function of the thermal field, which induces the switching, is proportional to \(\alpha\) according to the fluctuation-dissipation theorem. However, the switching rate based on the transition state theory\(^{16}\) given by \(\nu_{ij} = \exp[-\Delta_0 (1 \mp I/I_c)]/\tau(E_K)\) remains finite in the zero-dissipation limit. This problem has already been pointed out in the case of the magnetic field switching.\(^{25}\) The terms except for \(1/\tau(E_K)\) in Eq. (19) can be regarded as correction terms to the transition state theory used in Ref.\(^{16}\).

Figure 3 (a) shows the dependence of the switching rate \(\nu_{ij}\) on the current numerically obtained from Eq. (14), where \(\nu_{ij}\) are normalized by the values at \(I = 0\). The values of the parameters are those used in Fig. 2 with \(T = 300\) K. The analytical solution, Eq. (19), for \(I < I_c\) is shown by dots, and shows good agreement with the numerical result. The switching rate in the high-current region, \(I > I_c\), is shown in Fig. 3 (b). One of the main results in this paper is the nonlinear dependence of \(\nu_{ij}\) in the relatively high-current region on the logarithmic scale, while the linear dependence has been widely used in previous works\(^{14,15}\) by assuming the linear scaling of the switching barrier and the constant attempt frequency.

The current dependence of the attempt frequency, \(f_{ij}\), is shown in Fig. 4. The attempt frequency, \(f_{ij}\), decreases with increasing current for \(I \leq I_c\), while it increases for \(I_c < I < I_c^*\). The discontinuity of the slope of \(f_{ij}\) around \(I_c\) arises for the following reason. According to Eqs. (15) and (18), the attempt frequency for \(I < I_c\) is approximately proportional to the gradient of \(\varepsilon_i\) at its minimum, \(d\varepsilon_i(E_0)/dE\), which decreases with increasing current. Here, \(d\varepsilon_i(E_0)/dE\) arises from the Taylor expansion of \(\varepsilon_i\) in Eq. (14). On the other hand, \(d\varepsilon_i(E_a)/dE\) for \(I_c < I < I_c^*\) is zero at the minimum of \(\varepsilon_i\) as shown by Eq. (11). Then, \(f_{ij}\) is approximately proportional
to the curvature of $\delta_1$ at its minimum, $d^2\delta_1(E^*)/dE^2$, which increases with increasing the current. Thus, the attempt frequency shows a minimum around $I_c$. The attempt frequency in Fig. 4 strongly depends on the current. For example, the attempt frequency at $I_c$ is three orders of magnitude smaller than that at zero current. Contrary to this result, the attempt frequency has been generally assumed to be constant in previous experimental analyses.  

IV. CONCLUSION

In summary, the spin-torque-switching rate of an in-plane magnetized system was studied. The current dependence of the switching rate was obtained numerically, and an analytical formula in the low-current region was derived. The logarithm of the switching rate depends nonlinearly on the current in the high-current region. The switching barrier linearly depends on the current in the low-current region, which guarantees the validity of the previous theories, although the scaling current $I_c$ is not identical to the switching current. The attempt frequency has a minimum around the critical current $I_c$, and exhibits a strong current dependence while it has been assumed to be constant in previous experimental analyses.

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Appendix A: Brown’s Approach to Eq. (19)

The switching rate in the high barrier limit, Eq. (19), can be obtained by using the Brown’s approach. To this end, it is convenient to use $W = M\mathcal{P}/(\gamma \tau)$, instead of $\mathcal{P}$. In terms of $W$, Eqs. (3) and (4) can be expressed as

$$\frac{\gamma \tau}{M} \frac{\partial W}{\partial t} + \frac{\partial J}{\partial E} = 0,$$

(A1)

$$J = -\frac{\alpha k_B T}{V} \mathcal{M} e^{-\delta V/(k_B T)} \frac{\partial}{\partial E} e^{\delta V/(k_B T)} W.$$

(A2)

To guarantee $\Delta_i \gg 1$, we assume that $I < I_c$, i.e., $E^* = E_K$. Then, $\Delta_i$ is given by $\Delta = \Delta_0 (1 \mp I/I_c)$. In the high barrier limit, the probability functions near the minima of $\delta$ are approximately the Boltzmann distribution functions, while a tiny constant flow of the probability current crosses over the saddle point. The probability functions of the region 1 and 2 around $E_K$ are expressed as

$$W_i(E) = W_i(E_K) \exp \left\{-\frac{[\delta_i(E) - \delta_i(E_K)]V}{k_B T}\right\},$$

(A3)

where $i = 1, 2$, $W_i(E_K) = W_i(E_o) \exp\{[\delta_i(E_o) - \delta_i(E_K)]V/(k_B T)\}$, and $W_i(E_o)$ is the probability function at the saddle point satisfying $W_i(E_o) = W_2(E_o)$. Since the probability functions show sharp peaks around $E_K$, and rapidly decreases by approaching to $E_o$, the integrals of the probability functions,

$$n_i = \frac{\gamma}{M} \int_{E_K}^{E_o} dE W_i(E) \tau(E),$$

(A4)

are independent of the upper boundaries, $E_o$, which are arbitrary points located in the region 1 and 2 close to $E_o$. Equation (A4) can be expressed as

$$n_i = W_i(E_K) \exp[\delta_i(E_K)V/(k_B T)] I_i,$$

where $I_i$ are given by

$$I_i = \frac{\gamma}{M} \int_{E_K}^{E_o} dE \exp \left[-\frac{\delta_i(E)V}{k_B T}\right] \tau(E).$$

(A5)

The exponential term in Eq. (A5) rapidly decreases from $E = E_K$ to $E = E_o$. Thus, by using the Taylor expansion of $\delta_i$ around $E = E_K$ and using the fact that $\lambda E = -d\delta_i/dE \neq 0$ for $I < I_c$, $I_i$ can be approximated to

$$I_i \simeq \frac{\gamma k_B T \tau(E_K)}{MV (d^2\delta_i(E_K)/dE^2)} e^{-\delta_i(E_K)V/(k_B T)}.$$

(A6)

The double sign means the upper for region 1 and the lower for region 2.
Next, we consider the flow of the probability current from region 1 to region 2 crossing the saddle point. Equation (A2) can be rewritten as

\[ \frac{JV}{\alpha k_BT} e^{\psi(E) V/(k_BT)} = -\frac{\partial}{\partial E} e^{\psi(E) V/(k_BT)} W. \] (A7)

By assuming the divergenceless current\(^{22}\), the integral of Eq. (A7) over \([E_1, E_s]\) is given by

\[ \frac{JV}{\alpha k_BT} \int_{E_1}^{E_s} dE e^{\psi_1(E) V/(k_BT)} = W_1(E_1) e^{\psi_1(E_1) V/(k_BT)} - W_1(E_s) e^{\psi_1(E_s) V/(k_BT)}. \] (A8)

We also integrate Eq. (A7) over \([E_2, E_s]\) by changing the sign of the probability current \(J\). Then, we obtain the following equation;

\[ \frac{JV}{\alpha k_BT} I_\alpha = W_1(E_1) e^{\psi_1(E_1) V/(k_BT)} - W_2(E_2) e^{\psi_2(E_2) V/(k_BT)}; \] (A9)

where the right hand side is identical to \((n_1/I_1) - (n_2/I_2)\).

On the other hand, \(I_\alpha\) is given by

\[ I_\alpha = \int_{E_1}^{E_s} dE e^{\psi_1(E) V/(k_BT)}/\mathcal{M}_\alpha + \int_{E_2}^{E_s} dE e^{\psi_2(E) V/(k_BT)}/\mathcal{M}_\alpha \]

\[ \approx \frac{k_BT}{\mathcal{M}_\alpha(E_s)V} \left[ \frac{e^{\psi_1(E_s) V/(k_BT)}}{d\delta_1(E_s)/dE} + \frac{e^{\psi_2(E_s) V/(k_BT)}}{d\delta_2(E_s)/dE} \right]. \] (A10)

The probability current satisfies \(dn_1/dt = -dn_2/dt = -J\). Thus, we obtain the following rate equation between the region 1 and 2;

\[ \frac{dn_1}{dt} = -\frac{dn_2}{dt} = -n_1\nu_{12} + n_2\nu_{21}. \] (A11)

where the switching rate from the region \(i\) to the region \(j\) is \(\nu_{ij} = \alpha k_BT/(I_i I_\alpha V)\). By using Eqs. (9), (10), (A6), and (A10), the explicit form of the switching rate is given by

\[ \nu_{ij} = \alpha M V \mathcal{M}_\alpha(E_\nu) \left( 1 \pm \frac{I}{I_c} \right) \left[ 1 - \left( \frac{I}{I_c} \right)^2 \right] e^{-\Delta t}, \] (A12)

where the double sign means the upper for \((i, j) = (1, 2)\) and the lower for \((2, 1)\). Equation (A12) is identical to Eq. (19). The solutions of Eq. (A11) for constant current in time with the initial condition \(n_1(t = 0) = 1\) and \(n_2(t = 0) = 0\) are given by

\[ n_1 = \frac{\nu_{21}}{\nu_{12} + \nu_{21}} + \frac{\nu_{12}}{\nu_{12} + \nu_{21}} e^{-(\nu_{12} + \nu_{21})t}, \] (A13)

\[ n_2 = \frac{\nu_{12}}{\nu_{12} + \nu_{21}} - \frac{\nu_{21}}{\nu_{12} + \nu_{21}} e^{-(\nu_{12} + \nu_{21})t}. \] (A14)

For the positive current, \(\nu_{12} \gg \nu_{21}\), and therefore \(n_1 \approx e^{-\nu_{21}t}\) and \(n_2 = 1 - e^{-\nu_{12}t}\), where \(n_2\) corresponds to the switching probability measured in the experiments.

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By using the boundary condition \( \partial (P/\tau)/\partial E = 0 \) and the continuity of \( J \) at \( E = E_s \), the probability in the region \( i \) close to the saddle point is given by \( \frac{(d\xi_j/dE)\Gamma}{[(d\xi_i/dE) + (d\xi_j/dE)]} \), which determines the rate moving to the region \( j \).

For MRAM application, the thermal stability \( \Delta_0 \) larger than 40 is required. In such sample, the switching barrier at \( I = I_c \) is on the order of unity.

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