Damping Identification and Prediction for Laminated Composite Plates

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Abstract. Laminate composite structures are widely used in different sectors of industry due to their remarkable stiffness and damping performance. The present study focuses on the damping identification and prediction for laminated composite plates. The damping model based on the strain energy method was used for damping coefficients identification and for modal loss factor prediction. The study concerns with the damping coefficients identification for a single layer of an unidirectionally laminated plate. The reliability of the proposed identification method is validated by numerical modal loss factor prediction for a plate made of the layers in question but having different geometric characteristics. Further, the predictions of the modal loss factors were also validated by a vibration test. All the vibration tests necessary for the current study were performed in a non contact manner using laser vibrometer. The results delivered from the identification method were proven to be credible for modal loss factor prediction.

1. Introduction
Laminate composite structures are widely used in the form of plates, beams, or cylindrical shells. Besides their exceptional stiffness properties, the composites structures exhibit highly preferable damping characteristics. The damping identification as well as damping prediction have been intensively studied considering laminated composite plates and beams [1, 2, 3, 4, 5]. As these two procedures are mutually dependent on each other, the identification procedure seems to be more crucial in this relation. It is so, because the identification procedure delivers the necessary information on the damping coefficients which are further used to predict the modal damping of structures composed of a material in question. Among the methods for damping coefficients identification, the inverse technique based on dynamic response has been appreciated the most [6, 7, 8, 9]. The inverse technique combines the experimental and numerical analyses. In the numerical analysis, a proper selection of the applied damping model is the core issue, since it is used for the both, identification and prediction of damping.

The study concerns with the damping coefficients identification of a single layer of an unidirectionally laminated plate. In the present work the damping model based on the strain energy method [2, 10, 11] was used for damping coefficients identification and for modal loss factor prediction. The damping model was
combined with the finite element solutions. The identification was performed using an inverse technique which was based on the design of experiments and the response surface methodology [12]. The reliability of the identification method was validated by numerical modal loss factors predictions for a plate made of the same material but having different geometric characteristics. Further the predictions of the modal loss factors were also validated by vibration test. All the vibration tests necessary for the current study were performed in a non-contact manner using a laser vibrometer.

2. Materials and samples
Damping properties of two laminated plates were investigated in the present study. One plate with a rectangular shape was used for the damping coefficients identification purpose. The second plate, square in shape, was used for predictions of modal loss factors based on the identified damping coefficients. Both plates were built up of the same carbon/epoxy prepreg material by means of the vacuum bagging. The plates had length \( a \), width \( b \), ply thickness \( t \), and total height \( h \) (Table 1). The plates were composed of 16 layers of plies having material properties as listed in Table 2. A global Cartesian co-ordinate system \((x, y, z)\) was set-up for the plates, with \( z \) axis downward and normal to the reference surface (Figure 1a). For each layer a lamina co-ordinate system \((1, 2, 3)\) was defined with the direction 1 along the lamina fibres, direction 2 transverse to the fibres direction, and 3 - through the thickness direction. A lamination angle \( (\Phi) \) of a single layer was defined between the \( x \)-axis and the \( l \)-axis.

3. Inverse technique
The inverse technique based on experiment design and the response surface methodology is used in the present study for damping coefficients identification. The whole procedure is given schematically on Figure 1b and can be summarised as follows. In the first step a plan of experiments (DoE) is developed depending on the number of identification parameters \( l \) and number of experiments \( p \). Then, the finite element analyses are performed in the reference points of the experiment design and plate’s eigenfrequencies and corresponding eigenvectors are calculated numerically. Based on these data the modal loss factors are then calculated for each reference point. The results of these calculations are then taken to determine simple functions using the response surface methodology. In parallel, a vibration test is carried out with the purpose to extract frequency response function (FRF) which is used for experimental modal loss factor assessment. An identification of damping coefficients is performed in

| Sample | \( a \) [mm] | \( b \) [mm] | \( h \) [mm] | \( t \) [mm] | Layers | \( \Phi \) [deg] |
|--------|---------|---------|--------|--------|--------|----------|
| Plate 1 | 290.0   | 200.0   | 2.6    | 0.1625 | 16     | 0        |
| Plate 2 | 300.0   | 300.0   | 2.6    | 0.1625 | 16     | 0        |

**Table 1.** Geometric characteristics of the laminated plates used in the investigation.

| Material         | \( E_1 \) [GPa] | \( E_2 \) [GPa] | \( G_{12} \) [GPa] | \( v_{12} \) | \( \rho \) [kg/m\(^3\)] |
|------------------|-----------------|-----------------|-------------------|-------------|-----------------|
| Carbon/Epoxy     | 116.80          | 8.85            | 4.10              | 0.36        | 1546.60         |

**Table 2.** Properties of the constituent plies of the two laminated plates used in the investigation.
3.1. Response surface methodology

Response surface methodology (RSM) is a mathematical procedure for empirical model building [13]. By carefully generated DoE, the aim of RSM is to optimize an output variable (a response) which is affected by several independent input variables (identification parameters). The DoE is a set of runs (experiments), in which changes are made in the input variables in order to observe the changes in the response. The input variables are denoted as \( x_1, x_2, \ldots, x_l \) and the response is denoted as \( y \). The relationship between the response and the input variables can be represented as:

\[
y = f(x_1, x_2, \ldots, x_l)
\]  

The RSM starts with finding an approximation to the true functional relationship between \( y \) and the input variables. For the most cases, the approximations are taken in the form of first or second-order polynomials. The domain of these polynomials is enclosed in the region of the input variables.

3.2. Identification parameters

The parameters to be identified were specific damping capacity coefficients (SDC coefficients) of constituent layers of the laminated rectangular composite plate (Plate 1). The unidirectional layer of the plate was considered homogeneous and transversely isotropic with respect to the fibre direction. In the current investigation a thin plate is considered with a linear elastic material and the plane stress state. For such a case, a vector of three SDC coefficients corresponding to the in plane lamina directions is required to be identified:

\[
x = [x_1, x_2, \ldots, x_l] = [\psi_{11}, \psi_{12}, \psi_{22}]
\]  

3.3. Design of experiments

The main part of the RSM is the design of experiments (DoE). The objective of the DoE is the selection of the sample points (reference points) in which the response should be calculated. A location of a single reference point is defined by the value and number of the identification parameters. The initial informations for developing the design are therefore a number of identification parameters \( l \) and the number of experiments \( p \). The reference points shall be distributed as regularly as possible in the narrow domain of the identification parameters.
3.4. Objective function
The objective function is expressed as an error estimator describing the discrepancy between the experimental modal loss factors and the corresponding modal loss factors calculated from RMS approximation polynomials:

$$\Theta(x) = \sum_{n=1}^{I} w_n \left[ \frac{(\eta_{n}^{\text{EXP}})^2 - (\eta_{n}^{EM}(x))^2}{(\eta_{n}^{\text{EXP}})^2} \right]^2$$ (3)

where \(\eta_{n}^{\text{EXP}}\) are the experimentally determined modal loss factors; \(\eta_{n}^{EM}\) are the modal loss factors calculated from the approximation polynomials; \(x\) is the vector of unknown parameters; \(I\) is the total number of the modal loss factors considered in the analysis, \(w_n\) is the integer having value 1 or 0 and was used for the selection of the modal loss factors which entered the objective function.

3.5. Optimisation method
In order to find the minimum of the objective function, the following optimisation problem was solved

$$\min \Theta(x) \quad \text{Subject to constraints:} \quad x^L_i \leq x_i \leq x^U_i, \ i = 1, 2, \ldots, I$$ (4)

where \(x = [x_1, x_2, \ldots, x_l]\); \(x^L_i\) and \(x^U_i\) are lower and upper bounds of the identification parameters, respectively. The Hook-Jeeves Direct Search method was used to find the optimal unknown vector which minimise the objective function.

4. Numerical Model
The numerical models of the laminated plates were developed using the Finite Element Method (FEM). The finite element code SIMULIA/ABAQUS was used to model the laminated plates with the use of SR4 layered shell elements. The elements’ displacement field was described according to the First Order Shear Deformation Plate Theory. The linear elastic material model was applied for the plates. For the need of the current study the eigenvalue problem of undamped free vibrations was represented as:

$$(K - \omega^2_n M) \phi_n = 0$$ (5)

where \(K\) and \(M\) are the stiffness and mass matrices of the plate, respectively; \(\phi_n\) are the eigenvectors (mode shapes) of the corresponding eigenvalues \(\omega_n = 2\pi f_n\), where \(f_n\) are eigenfrequencies. The Linear perturbation Frequency analysis was solved with the Lanczos mode-extraction method. The eigenvalues and the corresponding eigenvectors of the FFFF plate (all edges free) were then extracted. The graphical representation of the FE model is given on Figure 2.

5. Damping model
An applied damping model for the laminated plates was based on the modal strain energy method [3, 10]. According to the method, the specific damping capacity (SDC) \(\Psi_n\) of a plate is defined as the ratio of the total dissipated energy \(\Delta U_n\) to the maximum strain energy \(U_n\) stored in the plate during a stress cycle at \(n^{th}\) mode of vibration:

$$\Psi_n = \frac{\Delta U_n}{U_n} = 2\pi \eta_n$$ (6)
Figure 2. Finite Element Models of laminated plates.

where $\eta_n$ is the modal loss factor. The maximum strain energy is calculated using the mode shapes of the laminated plate obtained from the solution of the undamped vibration as stated in [14].

In regard to the strain energy and the finite element model, the selected elements of the strain and stress tensors are related to each other as follows:

$$U_{ij,k}^e = \frac{1}{2} \int_{V_k^e} \varepsilon_{ij,k}^e \cdot \sigma_{ij,k}^e \cdot dV_k^e$$  \hspace{1cm} (7)

where $U_{ij,k}^e$ is strain energy, $\varepsilon_{ij,k}^e$ are the strain components, $\sigma_{ij,k}^e$ are the stress components, $V_k^e$ is the volume of the layer $k$ of the finite element $e$, $ij = 11, 12, 22$ are directions in the lamina coordinate system. For the laminated plate composed of the same layers, the total strain energy of the plate is given as:

$$U_n = (U_{11})_n + (U_{12})_n + (U_{22})_n$$  \hspace{1cm} (8)

where the each $U_{ij}$ component is the sum of the strain energies stored in all layers of the plate’s elements on a given $ij$ direction. Likewise, the dissipated energy is given as:

$$\Delta U_n = \psi_{11}(U_{11})_n + \psi_{12}(U_{12})_n + \psi_{22}(U_{22})_n$$  \hspace{1cm} (9)

where the $\psi_{ij}$ terms are the SDC coefficients responsible for energy dissipation on a given $ij$ direction.

Having calculated the strain energy $U_n$ and the dissipated energy $\Delta U_n$ the modal loss factor can be calculated according to the Eq. 6.

6. Experimental set-up

In the current research a non-contact method for vibration sensing is applied using the scanning laser vibrometer POLYTEC PSV-400-B. The PSV-400-B system is built of a scanning head, a controller, a junction box, a PC station, and a power amplifier (Figure 3). The PSV-400-B system has the ability
of scanning procedure (multipoint measurements). This gives the possibility for storing an average Frequency Response Function (FRF) and provides a high quality of operational mode shapes of a specimen. The scanning procedure is as follows. A specimen is driven to vibrate by means of a loudspeaker which is provided with an input signal generated by the internal generator. As a result, the specimen starts vibrating within the frequency range of the input signal. The vibrometer controller automatically moves the measurement laser beam (using scanner mirrors) to the each point of the defined scanning grid and senses its back-scattered light. The photo-detector (highly sensitive digital decoder VD-07) measures the time depended vibration velocity and validates measurements with the signal-to-noise ratio. After the measurements have been performed at each point, the averaged time response is transformed to the frequency domain using Fast Fourier Transform. As a result, the FRF of the specimen is obtained. The estimation of the modal loss factors based on the method proposed in [15], and it makes use of the Real part of the mobility FRF:

$$\eta_{n}^{EXP} = \frac{1 - \left(\frac{f_n^b}{f_n^a}\right)^2}{1 + \left(\frac{f_n^b}{f_n^a}\right)^2}$$  \hspace{1cm} (10)

where $\eta_{n}^{EXP}$ is the experimentally measured modal loss factor, $n$ stands for a mode number; $f_n^a$ and $f_n^b$ are frequencies estimated from the Real part of the FRF (Figure 3b). A more comprehensive description of the experimental set-up can be found in [16].

7. Identification and prediction procedure

7.1. Identification of SDC coefficients

A unidirectional laminated composite plate (Plate 1) was used for the identification procedure. The identification procedure was built up with Isight software and is given schematically on Figure 4. The finite element analysis was performed at first on the numerical model of the laminated plate. The selected eigenfrequencies and eigenvectors were then extracted - 8 flexural modes in the range of $0 \div 600$ Hz (Table 3). The strain and stress components corresponding to the extracted eigenvectors were stored in the Excel 1 file. To do so, each shell element was decomposed to separate layers and the strain $\varepsilon_{ij}$ and stress $\sigma_{ij}$ components of the layers were extracted. In addition the volumes of the layers of each
elements were also extracted from the finite element model. The obtained volumes were multiplied by the strain and stress components giving the strain energy of the single layer of an element. Summing up all the layers gave the strain energy stored in the single element. The strain energy of the laminated plate for a particular mode shape was obtained by summing up all the energies of the individual elements. Then, the initial values of the SDC coefficients of the layers were used to calculate the dissipated energy components. Finally, the SDC of the whole plate \( \Psi_n \) could be calculated according to Eq. 6 as well as the modal loss factor \( \eta_{nFEM} \) according to Eq. 6. The procedure was repeated for all extracted mode shapes.

The DoE was performed next. Three SDC coefficients were chosen as the identification parameters. The Latin Hypercube Technique was used for developing of the design of experiments. The design domain for the SDC coefficients was defined as: \( \psi_{11} = 0.0038 \pm 20\% \), \( \psi_{12} = 0.040 \pm 20\% \), \( \psi_{22} = 0.040 \pm 20\% \). What follows, the number of the input variables was set to \( l = 3 \) and the number of the reference points was set to \( p = 100 \). An example of the generated DoE is given on Figure 4b. Calculations of the SDC \( \Psi_n \) of the plate for each point of the DoE were performed using the data of the mode shapes stored in Excel 1 file. After having calculated the \( \Psi_n \), the modal loss factors \( \eta_{nFEM} \) were calculated using Eq. 6. Then, the first order approximation polynomials were developed by applying response surface methodology on the calculated \( \eta_{nFEM} \). These polynomials were then exported to the Excel 2 file and served for a rapid calculation of \( \eta_{nFEM}^FEM(x) \) in terms of SDC coefficients \( \psi_{11} \), \( \psi_{12} \), and \( \psi_{22} \). These \( \eta_{nFEM}^FEM(x) \) values were then used to build the objective function given by Eq. 3.

Next, the vibration tests were performed. The excitation range was set up to \( 0 \div 600 \) Hz, and the frequency response functions (FRFs) were stored. The Real parts of the FRFs were then used to assess the modal loss factors. From the obtained FRFs, 8 resonances were captured. However, the mode shape type \((1,2)\) was of a poor quality and was not used for the estimation of the modal loss factor. On that reason 7 resonances were used for the modal loss factors \( \eta_{nEXP} \) calculations. The calculations were performed according to the Eq. 10 and the results were stored in Excel 2 file.

In the last step, the identification problem was defined. The objective function was defined as given in Eq. 3. The identification parameters were subjected to the following constraints: \( 0.0036 \leq \psi_{11} \leq 0.0054 \), \( 0.048 \leq \psi_{12} \leq 0.072 \), \( 0.040 \leq \psi_{22} \leq 0.060 \). In order to eliminate the mode shape \((1,2)\) from the objective function, the integer \( w_n \) was set to 0 as follows: \( w_3 = 0 \). The optimisation was performed with the Hooke-Jeeves Direct Search method with the process termination after 40 iteration and linear scaling of the identification parameters.
### Table 3. Experimental results.

| Resonance (n) | Mode shape (u,v)* | $f_n^{\text{EXP}}$ [Hz] | $\eta_n^{\text{EXP}} \cdot 10^{-3}$ |
|---------------|-------------------|--------------------------|-----------------------------------|
| 1             | 1,1               | 75.2                     | 10.36                             |
| 2             | 0,2               | 157.6                    | 8.99                              |
| 3             | 1,2               | 225.1                    | -                                 |
| 4             | 2,0               | 278.6                    | 0.74                              |
| 5             | 2,1               | 317.7                    | 2.74                              |
| 6             | 0,3               | 445.5                    | 6.97                              |
| 7             | 2,2               | 465.1                    | 6.75                              |
| 8             | 1,3               | 504.9                    | 7.66                              |

* u - nodal line perpendicular to the x-axis; v - nodal line perpendicular to the y-axis

7.2. Loss factor prediction

Having obtained the vector of the desired damping coefficients given as $x = [\psi_{11}, \psi_{12}, \psi_{22}] = [0.42; 6.23; 4.78] \cdot 10^{-2}$, the modal loss factors of the square laminated composite plate (Plate 2) were predicted numerically. To do so, the proposed damping model was applied and the procedure given in Section 7.1 was repeated omitting the steps required for identification purpose. In order to validate the predicted modal loss factor values, the vibration test was performed on the Plate 2 in a manner as described in Section 6. The experimental mode shapes used for damping prediction, the corresponding FEM eigenvectors, and modal loss factors are given in Table 5.

8. Results

The inverse method was applied for the identification of the SDC coefficients for the rectangular laminated plate - Plate 1. The experimentally measured modal loss factors, corresponding resonances and the mode shape types are listed in Table 3. The results of the identification procedure are given in Table 4. The reliability of the applied identification method was verified by the numerical predictions of the modal loss factors for the square laminated plate - Plate 2. For this purpose the identified SDC coefficients were used. In order to validate the numerical predictions, the experimental modal loss factors of the Plate 2 were also extracted from the vibration test. As given in Table 5 a good convergence was found between the numerical predictions and the experimental results. The predicted values of the modal loss factors differs less than 10% in most cases. As a matter of fact, the discrepancy of the results can be further reduced by application of the vacuum chamber for vibration testing. Besides, the frequency resolution of the extracted FRFs shall be set up to be lower what might have an influence on the results convergence.
Table 5. Mode shapes and their corresponding modal loss factors - experimentally measured (EXP) and numerically predicted (FEM ABAQUS).

| (u, v)* | EXP | FEM ABAQUS | \( f^u, \text{Hz} \) | \( f^b, \text{Hz} \) | \( \eta^{EXP} \) | \( (\eta^{FEM})^{**} \) | Difference % |
|---------|-----|------------|----------------|----------------|---------------|----------------|--------------|
| (1, 1)  |     |            | 45.45          | 45.00          | 9.95          | 9.82           | 1.27         |
| (0, 2)  |     |            | 69.75          | 69.25          | 7.19          | 7.66           | 6.42         |
| (1, 2)  |     |            | 120.75         | 119.75         | 8.32          | 9.05           | 8.87         |
| (0, 3)  |     |            | 194.00         | 192.50         | 7.76          | 7.64           | 1.63         |
| (2, 1)  |     |            | 263.30         | 262.75         | 2.09          | 1.88           | 10.11        |
| (2, 2)  |     |            | 327.00         | 325.75         | 4.90          | 4.31           | 11.96        |
| (0, 4)  |     |            | 387.75         | 385.00         | 7.12          | 7.60           | 6.79         |
| (2, 3)  |     |            | 444.75         | 441.75         | 6.77          | 6.20           | 8.32         |

*| | *u - nodal line perpendicular to the x - axis; v - nodal line perpendicular to the y - axis
| | **Predicted with ABAQUS using the identified SDC coefficients vector \( x = [0.42; 6.23; 4.78] \cdot 10^{-2} \)

9. Conclusions
Two laminated plates were investigated in the present work. One plate, rectangular in shape, was used to utilise the inverse method for the identification of the SDC coefficients. The second plate made of the same material but rectangular in shape, was used for modal loss factor prediction based on the identified SDC coefficients. By doing so, the reliability of the proposed inverse method was verified and was proven to be credible. Moreover, the numerical predictions were also validated by experimental vibration test and a good results convergence was found. However, further research shall be focused on the SDC coefficients identification for plates being in vacuum conditions.
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