The Screening Mass Squared in Hot Scalar Theory
to Order $g^5$ Using Effective Field Theory

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Abstract

We study massless $g^2\Phi^4$-theory at high temperature and with zero chemical potential. Using modern effective field theory methods, we derive the screening mass squared to order $g^5$. It is demonstrated that the result is renormalization group invariant to this order in the coupling constant. Renormalization group methods are used to sum up leading logarithms of $g$.

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1 Introduction

There has been tremendous progress in perturbative calculations in thermal field theory, since the work on symmetry behaviour at finite temperature by Dolan and Jackiw more than two decades ago [1].

One major breakthrough was the discovery by Braaten and Pisarski that ordinary perturbation theory breaks down high temperature for soft external momenta $k$ ($k \sim gT$, where $g$ is the coupling constant) [2]. The usual connection between the number of loops and powers of the coupling constant is lost. The leading order

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results for physical quantities receive contribution from all orders in perturbation theory. The solution to the problem is to use an effective expansion which resums this infinite subset of diagrams and where loop corrections are truly perturbative (down by powers of $g$). This resummation program and extensions thereof, have been applied to calculate a variety of static and dynamical quantities in hot plasmas (see e.g. Ref. [3,4]). The literature is now vast, but we shall confine ourselves to briefly mention some papers relevant for the present work. The electric screening masses squared of SQED and QED have been calculated by Blaizot et al. [5] to order $e^4$ and $e^5$, respectively. The plasmon mass in $g^2\Phi^4$-theory has been calculated by Parwani to order $g^4$ in resummed perturbation theory [6]. The pressure has also been computed to order $g^4$ by Frenkel, Saa and Taylor [7], and extended to order $g^5$ by Parwani and Singh [8]. Parwani and Corianò have carried out corresponding calculations in QED to order $e^4$ [9], and the calculational frontier has again be pushed to order $e^5$ by Parwani [10]. In QCD, Arnold and Zhai have obtained the free energy to order $g^4$ [11], while Zhai and Kastening have performed a $g^5$-calculation [12].

Another major achievement is the application of effective field theory methods to the calculations of static quantities. The general idea is to take advantage of two or more well separated mass scales in the problem at hand, and construct a sequence of effective field theories which are valid at successively longer distances. The short-distance effects are then encoded in the parameters of the effective Lagrangian [13-15]. In the case of $g^2\Phi^4$-theory we have the scale $T$ which is associated with the nonzero Matsubara frequencies (their masses are $2\pi n T$, where $n$ is a positive integer) and the scale $gT$ which is associated with the zero-frequency mode (this mode acquires a thermal mass of order $gT$). Alternatively, this is the scale at which static scalar fields are screened). The strategy is to integrating out the scale $T$, or equivalently to integrating out the heavy modes, to obtain an effective Lagrangian of the light mode. The effective theory is three dimensional and the process is called dimensional reduction. In Non-Abelian gauge theories there are three momentum scales, namely $T$ which again is associated with the nonstatic modes, $gT$ which is the scale of colour electric screening and the scale $g^2T$ which is the scale of colour magnetic screening. In this case it proves useful to construct a second effective field theory by integrating out the scale $gT$ [16].

The methods of effective field theory were first applied to high temperature field theory by Ginsparg and Landsman in Refs. [17,18]. In these papers, effective three-dimensional Lagrangians were constructed by explicitly integrating out heavy modes at the one-loop level. Later, Kajantie et al. have extended this method beyond the one-loop level by matching Greens functions in the full and in the effective theory, using the effective potential [19]. This approach has mainly be used in connection with the study of phase transitions in the standard model and other theories [19,20].
There is a nice alternative to explicitly distinguishing between light and heavy modes. Instead, one writes down the most general effective three-dimensional Lagrangian which is consistent with the symmetries at high temperature. The parameters in the effective theory are then determined by the requirement that static correlators in the full theory are reproduced to some desired accuracy by the effective theory, at long distances $R \gg 1/T$. This approach has been developed by Braaten and Nieto in Ref. [21], where they applied it to $g^2\Phi^4$-theory. Combining this with renormalization group methods, they computed the screening mass squared to order $g^5\ln g$ and the pressure to order $g^6\ln g$. Later, Braaten and Nieto computed the free energy in QCD, through order $g^5$ [22], and confirmed the resummation results of Zhai and Kastening [12]. The method has also been applied by the present author to calculate the screening mass in SQED and QED to order $e^4$ and $e^5$, respectively, as well as the free energy in QED to order $e^5$ [23]. These calculations have reproduced the results first obtained by resummation [5,9,10].

In this letter we apply these ideas to scalar theory and calculate the screening mass squared to order $g^5$. Using the renormalization group techniques of Ref. [21], we also sum up leading logarithms of $T/(gT)$ from higher order of perturbation theory. In section two, we briefly discuss the effective three-dimensional theory and we determine the coefficients in the effective Lagrangian. In section three, we calculate the screening mass to order $g^5$. In section four we summarize our results and in the appendices A and B, the notation and conventions are given. We also list the sum-integrals of the full theory and the integrals of the effective theory, which are necessary for the present calculations.

## 2 The Effective Lagrangian

In this section we discuss the construction of the effective three-dimensional Lagrangian and we determine the parameters to the accuracy necessary for calculating the screening mass squared to order $g^5$. Since the results in this section appear elsewhere in the literature [18,21], the discussion will be rather brief. In particular, we refer to the paper by Braaten and Nieto [21], for a thorough discussion of this approach and calculational details.

If we denote the scalar field in the full theory by $\Phi$, the Euclidean Lagrangian for massless $g^2\Phi^4$-theory is

$$\mathcal{L}_E = \frac{1}{2}(\partial_{\mu}\Phi)^2 + \frac{g^2}{24}\Phi^4. \quad (1)$$
In the effective theory, we similarly denote the scalar field by $\phi$. The field in the three-dimensional Lagrangian can be approximately identified with the light mode in the underlying theory. At leading order in $g^2$, the relation between them is

$$\phi = \frac{1}{\sqrt{T}} \Phi.$$  \hspace{1cm} (2)

First, we must identify the symmetries of the effective Lagrangian. We have a $Z_2$-symmetry $\phi \rightarrow -\phi$, which follows from the corresponding symmetry in the full theory. There is also a three-dimensional rotational symmetry. Hence, we may write

$$L_{\text{eff}} = \frac{1}{2} (\partial_i \phi)^2 + \frac{1}{2} m^2(\Lambda) \phi^2 + \frac{\lambda(\Lambda)}{24} \phi^4 + \frac{h_1(\Lambda)}{6!} \phi^6 + h_2(\Lambda) \phi^2 \nabla^2 \phi^2 + \delta L.$$  \hspace{1cm} (3)

Here, we have indicated that the parameters in the effective Lagrangian generally depend on the ultraviolet cutoff $\Lambda$ in the effective theory\footnote{The field $\phi$ also depends on the scale $\Lambda$, but it is suppressed for notational ease.}. This is necessary in order to cancel the $\Lambda$-dependence that arises in the calculations using the three-dimensional effective Lagrangian. According to Ref. [21], $\Lambda$ can be viewed as an arbitrary factorization scale, which separates the scales $T$ and $gT$. Moreover, $\delta L$ includes all other operators which are consistent with the symmetries. The parameters $m^2(\Lambda)$ and $\lambda(\Lambda)$ are the only operators which contribute to the screening mass squared through order $g^5$. Thus for the present calculation, these are the only quantities which must be determined.

The parameters in the effective theory are determined by a matching requirement. We demand that static correlators in the full theory be the same as those in the effective theory to some desired accuracy at long distances $R \gg 1/T$. We shall carry out the matching using strict perturbation theory \[21\]. Strict perturbation corresponds to the following partition of the Lagrangian into a free piece and an interacting part:

$$L_0 = \frac{1}{2} (\partial_\mu \Phi)^2,$$  \hspace{1cm} (4)

$$L_{\text{int}} = \frac{g^2}{24} \Phi^4.$$  \hspace{1cm} (5)

The Lagrangian of the effective theory is split likewise:

$$(L_{\text{eff}})_0 = \frac{1}{2} (\partial_\mu \phi)^2,$$  \hspace{1cm} (6)

$$(L_{\text{eff}})_{\text{int}} = \frac{1}{2} m^2(\Lambda) \phi^2 + \frac{\lambda(\Lambda)}{24} \phi^4.$$  \hspace{1cm} (7)
not rearranged our Lagrangian in order to screen them. Nevertheless, this does not
prevent us from determining the parameters using this partition of the Lagrangians.
The point is that the parameters in $L_{\text{eff}}$ encode the physics on the scale $T$ and are
insensitive to the scale $gT$. So as long as we make the same incorrect assumptions in
the effective theory, the divergences cancel in the matching procedure and we can use
strict perturbation theory to compute the coefficients in $L_{\text{eff}}$. This implies that we are
free to use any suitable infrared cutoff. In the present work we use dimensional reg-
ularization. Finally, since strict perturbation theory is ordinary perturbation theory,
it is clear that the parameters in the effective Lagrangian can be written as powers
series in $g^2$.

When we match static Greens functions in the two theories, it is necessary to take
into account the different normalizations of the fields. The simple relation Eq. (2)
breaks down beyond leading order in $g^2$, and this is due to the wave function renor-
malization of $\Phi$ in the full theory. In $g^2\Phi^4$-theory, this is a two-loop effect and so is
proportional to $g^4$. It is therefore relevant for calculations first at order $g^6$, and so
we can use Eq. (2) as it stands.

The mass parameter $m^2(\Lambda)$ must be determined at next-to-leading order in $g^2$
Denoting the self-energy function of $\Phi$ by $\Sigma(k_0, k)$, we may write the static two-point
function in the full theory as

$$\Gamma^{(2)}(k_0 = 0, k) = k^2 + \Sigma(k_0 = 0, k).$$

Here, $k = |k|$. In Figs. 1 and 2 we have displayed the one and two-loop diagrams
contributing to the self-energy function. The setting sun graph is dependent on the
external momentum $(k_0, k)$, but it is consistent to evaluate this diagram at vanishing
external momentum:

$$\Sigma(k_0 = 0, k = 0) = \frac{Z_{g^2}g^2}{2} \int P \frac{1}{P^2} - \frac{g^4}{4} \int_{PQ} \frac{1}{P^2Q^4} - \frac{g^4}{6} \int_{PQ} \frac{1}{P^2Q^2(P + Q)^2}.$$

Here, $Z_{g^2}$ is the renormalization constant for the coupling $g^2$. The second term
in Eq. (3) has a linear infrared divergence for $q_0 = 0$, while the third term has a
logarithmic infrared divergence for $p_0 = q_0 = 0$. Both singularities are of course
canceled by corresponding IR-divergences in the effective theory.

Let us now turn to the self-energy function of the field $\phi$, which we denote by
$\Sigma_{\text{eff}}(k, \Lambda)$. It has a perturbative expansion which is given by the diagrams in Figs. 1
and 2, as well as the diagrams with a mass insertion. These are depicted in Fig 3.
The self-energy function of the field $\phi$ is also evaluated at $k = 0$. This implies
that $\Sigma_{\text{eff}}(0, \Lambda)$ is identically zero, since there is no mass scale in the corresponding
integrals. The two-point function in the effective theory can then be written as

$$\Gamma_{\text{eff}}^{(2)}(k, \Lambda) = k^2 + m^2(\Lambda) + \delta m^2.$$

5
Here, we have included a mass renormalization counterterm $\delta m^2$. The mass parameter $m^2(\Lambda)$ is then found by demanding

$$\Gamma^{(2)}(k_0 = 0, k) = \Gamma^{(2)\text{eff}}(k, \Lambda).$$

(11)

This implies that the mass parameter is given by

$$m^2(\Lambda) = \frac{Z g^2 g^2}{2} \sum\int P + Q \left(\frac{P^2 + Q^2}{P^2 Q^2(P + Q)^2} - \delta m^2\right).$$

(12)

The necessary sum-integrals have been listed in Appendix A. After renormalization of the coupling constant in Eq. (12), which is carried out by the substitution

$$Z g^2 = 1 + \frac{3g^2}{32\pi^2 \epsilon},$$

(13)

we are still left with a pole in $\epsilon$. This pole is canceled by $\delta m^2$ which reads

$$\delta m^2 = \frac{g^4 T^2}{24(4\pi)^2 \epsilon}.$$  

(14)

The mass parameter then becomes

$$m^2(\Lambda) = \frac{g^2 T^2}{24} \left\{ 1 + 3 \ln \frac{\mu}{4\pi T} + 4 \ln \frac{\Lambda}{4\pi T} - \gamma_E + 2 + 2 \zeta'(-1) \right\}.$$  

(15)

The mass parameter $m^2(\Lambda)$ was first obtained by Braaten and Nieto in Ref. [21]. We have used the renormalization group equation for the coupling $g^2$

$$\frac{dg^2}{d\mu} = \frac{3g^4}{16\pi^2},$$

(16)

to change the scale from $\Lambda$ to $\mu$ and so the coupling constant $g$ is then evaluated at the scale $\mu$. It is clear from Eq. (15) that $m^2(\Lambda)$ depends explicitly on the arbitrary scale $\Lambda$. The mass parameter satisfies an evolution equation, which can be derived from the requirement that the physical screening mass be independent of this arbitrary scale $\Lambda$ [21]. Alternatively, one can differentiate Eq. (15) with respect to $\Lambda$. In terms of $\lambda(\Lambda)$ the evolution equation reads

$$\Lambda \frac{dm^2}{d\Lambda} = \frac{1}{6}\left(\frac{\lambda}{4\pi}\right)^2.$$  

(17)

The physical interpretation of the mass parameter is that it is contribution from the scale $T$ to the screening mass.
Let us now move on to the quartic self-interaction. We need to know the coefficient $\lambda(\Lambda)$ in front at next-to-leading order in $g^2$. Using similar arguments as we did when we computed the mass parameter, one finds

$$
\lambda(\Lambda) = Z g^2 g^2 T - \frac{3}{2} g^4 T \int \frac{1}{p^4}.
$$

(18)

The first term on the right hand side results from matching at tree level, while the second term arises from matching at the one-loop level. The corresponding diagram is displayed in Fig. 4. After renormalization of $g^2$, we find

$$
\lambda(\Lambda) = g^2 T - \frac{3g^4 T}{(4\pi)^2} \left[ \ln \frac{\Lambda}{4\pi T} + \gamma_E \right].
$$

(19)

This coefficient was first found by Landsman in Ref. [18] by explicitly integrating out the nonzero Matsubara modes. Using the renormalization group equation for $g^2$, one can demonstrate that the quartic coupling in the effective theory is independent of the scale $\Lambda$, and so we can trade it for an arbitrary scale $\mu$.

3 The Screening Mass

In this section we calculate the screening mass squared to order $g^5$ using the effective Lagrangian. When we calculated the parameters $m^2(\Lambda)$ and $\lambda(\Lambda)$, we treated the infrared divergences incorrectly by decomposing our Lagrangians according to Eqs. (4) and (5), and Eqs. (6) and (7). In order to treat these divergences correctly, we must now incorporate the infrared cutoff provided by the mass parameter into the free part of $L_{\text{eff}}$. Thus, we split the effective Lagrangian according to

$$
(L_{\text{eff}})_0 = \frac{1}{2} (\partial_i \phi)^2 + \frac{1}{2} m^2(\Lambda) \phi^2,
$$

$$
(L_{\text{eff}})_\text{int} = \frac{\lambda(\Lambda)}{24} \phi^4.
$$

(20)

(21)

The screening mass $m_s$ is defined as the pole of the propagator at spacelike momentum [21]:

$$
m_s^2 = m^2(\Lambda) + \Sigma_{\text{eff}}(k, \Lambda), \quad k^2 = -m_s^2.
$$

(22)

The Feynman diagrams contributing to the self-energy function $\Sigma_{\text{eff}}(k, \Lambda)$ at next-to-next-to-leading order are depicted in Figs. 1–3:

$$
\Sigma_{\text{eff}}(k, \Lambda) = \frac{\lambda}{2} \int_p \frac{1}{p^2 + m^2} - \delta m^2 \frac{\lambda}{2} \int_p \frac{1}{(p^2 + m^2)^2} - \frac{\lambda^2}{4} \int_{pq} \frac{1}{(p^2 + m^2)(q^2 + m^2)} -
$$
The mass counterterm $\delta m^2$ cancels the pole in $\epsilon$ from the second two-loop diagram in Fig. 2. Similarly, the divergence of the one-loop diagram with a mass counterterm insertion cancels against a pole in $\epsilon$ arising from the third three-loop graph of Fig. 5. Moreover, this one-loop diagram also gives finite contributions to the screening mass, when $1/\epsilon$ “hits” the terms proportional to $\epsilon$. Finally, the last two-loop diagram and the last three-loop diagram depend on the external momentum $k$. The effective expansion parameter of the quartic interaction is $\lambda/m$, and in order to compute the screening mass squared consistently in powers of $\lambda/m$, we must evaluate these integrals at $k = im$. In order to see that this in fact is necessary, one can perform an expansion in the external momentum $k$, and verify that all terms are equally important for soft $k \sim gT$.

The integrals appearing in the expression for the screening mass are tabulated in Appendix B. Expanding the mass parameter in powers of $g^2$, we obtain the screening mass squared through order $g^5$:

$$m_s^2 = \frac{g^2}{24} T^2 \left\{ 1 - \frac{g}{4\pi} \sqrt{6} + \frac{g^2}{(4\pi)^2} \left[ -3 \ln \frac{\mu}{4\pi T} + 4 \ln \frac{m}{4\pi T} - 1 + 8 \ln 2 - \gamma_E + 2 + \frac{2 \zeta'(-1)}{\zeta(-1)} \right] + \frac{g^3}{(4\pi)^3} \sqrt{6} \left[ \frac{9}{2} \ln \frac{\mu}{4\pi T} - 2 \ln \frac{2m}{4\pi T} - 3 - 7 \ln 2 + \frac{7}{2} \gamma_E - \frac{\zeta'(-1)}{\zeta(-1)} \right] \right\}.$$  \hspace{1cm} (24)

Firstly, one notes that the $\Lambda$-dependence explicitly cancels in Eq. (24) to next-to-next-to-leading order in $g$. Using the running of the coupling $g^2$, it is also easy to verify that there is no dependence on the renormalization scale $\mu$ either. The result to order $g^3$ was first obtained by Dolan and Jackiw \[1\], while Braaten and Nieto computed the screening mass squared to order $g^4$. Moreover, they used the evolution equations, which the parameters in the effective theory satisfy, to sum up leading logarithms of higher order in the perturbation expansion. The result includes in particular a term
proportional to $g^5 \ln g$. This term is of course also present in Eq. (24) above. The complete result to order $g^5$ is new.

We would also like to point out that the effective field theory approach explains the very appearance of these logarithms. The mass parameter, and possibly other operators as well, depend explicitly on the factorization scale $\Lambda$ and such terms occur as logarithms of $\Lambda/T$. In the effective theory, logarithms of $m/\Lambda$ arise in perturbative calculations. In order to cancel the $\Lambda$-dependence in physical quantities, these logarithms must match, leaving logarithms of $T/m$ or $T/(gT)$. Hence, these logarithms can be attributed to the renormalization of the parameters in the effective Lagrangian \cite{21}, and in the present case it is the renormalization of the mass parameter.

The evolution equation for the mass parameter, Eq. (17), can be used to sum up the leading logarithms of $T/(gT)$. This can be done by choosing the renormalization scale $\mu = 2\pi T$ and the factorization scale $\Lambda = \frac{gT}{2\sqrt{6}}$ in the expression for the mass parameter $m^2$ \cite{21}. The screening mass squared again follows from Eqs. (22) and (23), and in terms of $m^2$ and $\lambda$ it reads

$$m^2_s = m^2\left\{1 - \frac{1}{2}\left(\frac{\lambda}{4\pi m}\right)^2\left[\frac{1}{8} - \frac{1}{2}\ln 2\right] - \left(\frac{\lambda}{4\pi m}\right)^3\left[\frac{4}{96} + \frac{11}{96}\ln 2\right]\right\}. \quad (25)$$

This is the full result to order $g^5$ and also includes leading logarithms in the form $g^{2n+3}\ln^n g$, where $n$ is a natural number. These terms are obtained by expanding the second term in Eq. (25) in powers of $g^2$.

4 Summary

In the present work we have obtained the screening mass squared to order $g^5$ using effective field theory methods. The short-distance coefficients contains the physics on the scale $T$, while the effective theory takes care of contributions to physical quantities from the scale $gT$. Thus, effective field theory methods unravel the contributions to physical quantities from the different momentum scales, and streamlines calculations, since we treat one scale in the problem at a time. This is the advantage of the effective field approach over the more conventional resummation procedure; the latter complicates calculations unnecessarily because the sum-integrals involve both scales $T$ and $m$.

We were also able to sum up leading logarithms of $T/(gT)$ arising from higher orders of perturbation theory by using the evolution equation satisfied by the mass parameter.
Our result is correct up to corrections of order $g^6$. In order to obtain the screening mass squared to this order, we must determine the mass parameter to three-loop order and also compute the four-loop diagrams in the effective theory. There are no new coefficients in the effective Lagrangian which must be determined, since the corresponding operators first contribute to $m_s^2$ at order $g^7$ or higher. The calculation should be a manageable task using the machinery developed to handle difficult multi-loop sum-integrals [11], and complicated integrals in the effective theory.

## A Sum-integrals in the Full Theory

In this appendix we give expressions for the sum-integrals used in the present work. We use the imaginary time formalism, where the four-momentum is $P = (p_0, p)$ with $P^2 = p_0^2 + p^2$. The Euclidean energy takes on discrete values, $p_0 = 2\pi nT$ for bosons. Dimensional regularization is used to regularize both infrared and ultraviolet divergences by working in $d = 4 - 2\epsilon$ dimensions, and we apply the $\overline{\text{MS}}$ renormalization scheme. We use the following shorthand notation for the sum-integrals that appear below:

$$\sum\int P f(P) \equiv \left( \frac{e^{\gamma_E} \mu^2}{4\pi} \right)^\epsilon T \sum_{p_0 = 2\pi nT} \int \frac{d^{3-2\epsilon} k}{(2\pi)^{3-2\epsilon}} f(P). \quad (A.1)$$

Then $\mu$ coincides with the renormalization scale in the $\overline{\text{MS}}$ renormalization scheme.

Arnold and Zhai have developed the machinery necessary to evaluate complicated multi-loop sum-integrals [11]. They have calculated and listed the specific sum-integrals needed in the present work, and details may be found in Ref. [11]. We list them here for the convenience of the reader

$$\sum\int_P \frac{1}{P^2} = \frac{T^2}{12} \left[ 1 + \left( 2 \ln \frac{\mu}{4\pi T} \right) + 2 + 2 \frac{\zeta'(-1)}{\zeta(-1)} \right] \epsilon + O(\epsilon^2), \quad (A.2)$$

$$\sum\int_P (P^2)^2 = \frac{1}{(4\pi)^2} \left[ \frac{1}{\epsilon} + 2 \ln \frac{\mu}{4\pi T} + 2\gamma_E + O(\epsilon) \right], \quad (A.3)$$

$$\sum\int_{PQ} P^2 Q^2 (P+Q)^2 = 0. \quad (A.4)$$

## B Integrals in the Effective Theory

In the effective three-dimensional theory we use dimensional regularization in $3 - 2\epsilon$ dimensions to regularize infrared and ultraviolet divergences. In analogy with
Appendix A, we define
\[ \int_p f(p) \equiv \left( \frac{e^{\gamma_E} \mu^2}{4\pi} \right)^\epsilon \int \frac{d^{3-2\epsilon} p}{(2\pi)^{3-2\epsilon}} f(p). \] (B.1)

Again \( \mu \) coincides with the renormalization scale in the modified minimal subtraction renormalization scheme. All the integrals necessary in the present work, except for those in Eqs. (B.4), (B.6) and (B.7) below, have been calculated by Braaten and Nieto in Ref. [21]. I have computed these remaining integrals using their methods.

\[ \int_p \frac{1}{p^2 + m^2} = -\frac{m}{4\pi} \left[ 1 + \left( 2 \ln \frac{\mu}{2m} + 2 \right) \epsilon + O(\epsilon^2) \right], \] (B.2)

\[ \int_p \frac{1}{(p^2 + m^2)^2} = -\frac{1}{8\pi m^3} \left[ 1 + \left( 2 \ln \frac{\mu}{2m} \right) \epsilon + O(\epsilon^2) \right], \] (B.3)

\[ \int_p \frac{1}{(p^2 + m^2)^3} = -\frac{1}{16\pi^2 m^3} \left[ 1 + \left( 2 \ln \frac{\mu}{2m} + 2 \right) \epsilon \right], \] (B.4)

\[ \int_{pq} \frac{1}{(p^2 + m^2)(q^2 + m^2)[(p + q + k)^2 + m^2]} \left|_{k=im} \right. = \frac{1}{(8\pi)^2} \left[ \frac{1}{\epsilon} + 6 + 4 \ln \frac{\mu}{2m} - 8 \ln 2 + O(\epsilon) \right], \] (B.5)

\[ \int_{pq} \frac{1}{(p^2 + m^2)^2(q^2 + m^2)[(p + q + k)^2 + m^2]} \left|_{k=im} \right. = \frac{1}{m^2(8\pi)^2} \left[ \ln 2 + O(\epsilon) \right], \] (B.6)

\[ \int_{pqr} \frac{1}{(p^2 + m^2)^2(q^2 + m^2)(r^2 + m^2)[(p + q + r)^2 + m^2]} = \frac{1}{m(8\pi)^3} \left[ \frac{1}{\epsilon} + 6 \ln \frac{\mu}{2m} - 4 \ln 2 + 2 + O(\epsilon) \right]. \] (B.7)

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Figure 1: One-loop correction to the two-point function.

Figure 2: Two-loop corrections to the two-point function.

Figure 3: Diagrams with a mass insertion contributing to the two-point function in the effective theory.
Figure 4: One-loop correction to the four-point function.

Figure 5: Three-loop corrections to the two-point function in the effective theory.