CONTROVERSIES ON AND A REASONING FOR EXISTENCE OF THE LIGHT $\sigma$-PARTICLE

Shin Ishida

Atomic Energy Research Institute, College of Science and Technology
Nihon University, Kanda-Surugadai, Chiyoda-ku, Tokyo 101-0062, Japan

ABSTRACT

The light $\sigma$-particle is, regardless of the strong criticism, reviving recently due to the works done from various sides. I review essential points of the controversies (especially related to our works) and of their answer: Conventionally a large concentration of the iso-scalar $S$-wave $2\pi$ events below 1 GeV (being, correctly, due to the $\sigma$-production), which is observed in most of production processes, is interpreted as a mere background from the viewpoint of, so called, universality argument. However, I show, by resorting to a simple field theoretical model, that the argument is not correct and the production process has “its own value” independent of the scattering process. Thus it is suggested that the present index “$f_0(400-1200)$ or $\sigma$” in PDG’98 is to be changed as “$\sigma(400-800)$” in the PDG 2000.

1 Introduction

1.1 Recent short history of the $\sigma$-particle and the related works

Recently the many works\footnote{In this talk I refer only to the recent works after 1980. As for the old references on the $\sigma$-particle see the works referred in S.Ishida et al.’96 in References.} suggesting existence of the light $\sigma$-particle both theoretically and phenomenologically, had been published and the $\sigma$-particle was revived in
the newest lists of PDG’96 and ’98 after missing more than two decades, although still with an obscure index \( f_0(400-1200) \) or \( \sigma \). Among them our group\(^2\) of collaboration had also given rather strong evidences for its existence through a series of papers,\(^1\) while received a serious criticism. In this talk I shall summarize the essential points of controversies and explain our answers to the criticism clearly, leading to the suggestion as is given in the abstract:

From early 1980’s the importance of the \( \sigma \) in relation to the dynamical chiral symmetry breaking had been stressed by the works.\(^2\) Possible evidences suggesting for existence of the \( \sigma \) in production processes had been given in the works.\(^3\) The reanalyses of the \( \pi \pi \) scattering phase shifts, leading to the rather strong evidences for the \( \sigma \) were done by the works.\(^4\)

The results of our series of works were reported in the several occasions,\(^5\) of which criticisms are found in the references.\(^6\)\

Some useful arguments and discussions, which make the crucial points clear, were given in the references.\(^7\)

1.2 Outline of controversies

First possible evidence for direct \( \sigma \)-production, which was obtained in a proton proton central collision process at 450GeV/c

\[
pp \rightarrow p_f X^0 p_s, \quad X^0 \rightarrow n\gamma, \quad X^0 = (\pi\pi)
\]

was reported\(^8\) by the GAMS KEK-subgroup at Manchester, Hadron’95. The obtained \( (\pi^0\pi^0) \) mass spectra are given in Fig. 1. They fitted the spectra by the Variant Mass and Width (VMW) method, representing the invariant production amplitude as a coherent sum of Breit-Wigner amplitudes of resonances, \( X^0 = f_0(975), f_2(1275) \) and \( f_c(500) \). There the huge concentration of \( S \)-wave events below 1 GeV, to which similar spectra had also been found in other experiments\(^9\) and taken as the mere background before, was interpreted as being due to \( f_c(500) = \sigma(500) \)-particle production.

However, this interpretation of \( \sigma \) production was severely criticized\(^8\) in the summary talk of the Hadron’95 from the so-called “Universality Argument.” It says “claims of a narrow \( \sigma(500) \) in the GAMS results cannot be correct as”

1 No \( \sigma \) is seen in the \( \pi\pi \) scattering.
2 Unitarity demands the production amplitude \( F \) to be consistent with the scattering amplitude \( T \).

\(^2\) The members are S.Ishida, T.Komada and H.Takahashi(Nihon University); M.Y.Ishida(Tokyo Institute of Technology); K.Takamatsu(Miyazaki University); and T.Ishida and T.Tsuru(KEK).
Due to this serious criticism the GAMS group himself had taken a very cautious attitude on the $\sigma$-particle to state formally that “In summary the analysis of $\pi^0\pi^0$ system \ldots confirms a large concentration of $S$-wave events below 1 GeV, which interferes with $f_0(980)$ destructively \ldots. This would be compatible with a broad $S$-wave state \ldots but its coherence with the known $\pi\pi$-scattering phase shifts is still the object of controversy that bears basic non-perturbative QCD concepts.”

On the other hand our group had reanalyzed, in replying to the criticism the $\pi\pi$ phase shifts by using the Interfering (Breit-Wigner) Amplitude (IA) method which satisfies the elastic unitarity automatically and shown that the $\sigma$-particle actually exists. Furthermore, our group had also investigated, in replying to the criticism the relation between the scattering and the production amplitudes and shown our $F$ in the VMW method and $T$ in the IA method satisfy consistently the unitarity of $S$-matrix. Meanwhile, there have been opened some useful arguments to make clear the critical points. Through the above processes I believe that now the answers to all the criticisms have been given.

\section{$\pi\pi$-scattering amplitude and reanalyses of phase shifts}

We made recently a reanalysis of the old CERM-Munich '73 and '74 data of $\pi\pi$ phase shifts and found a strong evidence for existence of $\sigma$-particle. There we applied the IA-method, which satisfies the elastic unitarity automatically and is parametrized only in terms of physically meaningful quantities as masses and widths of resonances. In a simple case of one $(\pi\pi)$-channel and two resonant ($\sigma$ and $f_0$) particles the partial $S$-wave $S$-matrix in the IA method is given as follows:

\begin{equation}
S = S^{\text{Res}}S^{\text{BG}}, \quad S^{\text{Res}} = S^\sigma S^{f_0}, \quad S^{(i)} = e^{2i\delta^{(i)}}, \quad \delta = \delta^\sigma + \delta^{f_0} + \delta^{BG}, \quad (2)
\end{equation}
where $S_{\text{Res}}(S_{\text{BG}})$ corresponds to $S$-matrix in the case of pure resonant (background) scattering and the $\delta^{(i)}$ represent the phase shifts due to the respective pure scattering cases. The unitarity of total $S$-matrix $S$ is reduced to the unitarity of each “component $S$-matrix” $S^{(i)}$;

$$SS^\dagger = S^\dagger S = 1 \leftrightarrow S^{(i)}S^{(i)\dagger} = S^{(i)^\dagger}S^{(i)} = 1.$$  \hspace{1cm} (3)

The scattering amplitude $a(S \equiv 1 + 2ia)$ due to resonances $a_{\text{Res}}$ is given as

$$a_{\text{Res}} = a_{\text{BW}}^\sigma + a_{\text{BW}}^f + 2ia_{\text{BW}}^\sigma a_{\text{BW}}^f, \hspace{1cm} (4)$$

where $a_{\text{BW}}^{\sigma(f)}$ represents the Breit-Wigner amplitude of the $\sigma(f)$ resonance ($a_{\text{BW}}^\sigma \equiv \rho g_\sigma^2/(m_\sigma^2 - s - i\rho g_\sigma^2)$ etc., $\rho = \sqrt{1 - 4m_\pi^2/s}/(16\pi)$). The last term in the r.h.s. of Eq.(3) represents an “interference” between the $\sigma$ and $f$ (B.W.) amplitudes. The physical reason for obtaining the different result even with using the same experimental data is our introduction of “negative background phase” $\delta_{\text{BG}}$ of hard core type

$$\delta_{\text{BG}} = -|p_1|r_c,$$ \hspace{1cm} (5)

where $|p_1| = \sqrt{s/4 - m_\pi^2}$ is the pion momentum in the 2$\pi$ CM system and the $r_c$ a parameter. The physical origin of the $\delta_{\text{BG}}$ is able to be reduced \cite{11} to the compensating repulsive interaction guaranteed by the chiral symmetry, \cite{12} and it is describable quantitatively in the framework of linear $\sigma$ model including the $\rho$-meson contribution. \cite{12} The results of our re-analyses are given in Fig. 2(a), while in Table 1 I compare the essential points and the results of our analysis with those of the conventional one. \cite{13} In our analysis the introduction of repulsive $\delta_{\text{BG}}$ with

| Table 1: Comparison between the fit with $r_c \neq 0$ and with $r_c = 0$ in our PSA. The latter corresponds to the conventional analyses thus far made. |
|---------------------------------------------------------------|
| $r_c \neq 0$ ($\chi^2/N_f = 23.6/30$) | $r_c = 0$ ($\chi^2/N_f = 163.4/31$) |
|---------------------------------|---------------------------------|
| $\delta^{\text{tot}} = \delta_{f_0(980)} + [\delta_{(600)} + \delta_{\text{BG}}]^{\text{pos.}}_{\sigma(600)}$ | $\delta^{\text{tot}} = \delta_{f_0(980)} + [\delta_{\text{BG}}^{\text{pos.}}]_{\text{"$\sigma$" (equivalent to $\epsilon(900)$)}}$ |
| $m_{\sigma}$ | 585 ± 20 (535 ~ 675) | 920 |
| $\Gamma_{\sigma}$ | 385 ± 70 | 660 |
| $\sqrt{s_{\text{pole}}/\text{MeV}}$ | (602 ± 26) − i(196 ± 27) | 970-1320 |
| $r_c$ | 3.03 ± 0.35 GeV$^{-1}$ (0.60±0.07 fm) | (−) |
Figure 2: $I=0$ $\pi\pi$ scattering phase shift. (a) Best fit to the standard $\delta^0_0$. The dotted line labeled “$r_c=0$” represents the conventional fit without the repulsive background. (b) $\chi^2$, $M_\sigma$ and $g_\sigma$ versus $r_c$. (c) Fits to the upper and lower $\delta^0_0$.

$r_c \sim 3\text{GeV}^{-1}$ (0.60fm, about the structural size of pion) plays a crucial role for the existence of $\sigma(600)$. The sum of the large attractive $\delta_\sigma(600)$, contribution due to $\sigma(600)$, and the large repulsive $\delta_{BG}$ gives a small positive phase shift, which was treated, in the conventional analysis, as a background (or broad $\epsilon(900)$) contribution $[\delta_{BG}^{\text{pos}}]$. Note that the fit with $r_c=0$ in our analyses corresponds to the conventional analyses without the repulsive $\delta_{BG}$ thus far made. In this case the mass and width of “$\sigma$” becomes large, and the “$\sigma$”-Breit-Wigner formula can be regarded as an effective range formula describing a positive background phase. The corresponding pole position is close to that of $\epsilon(900)$ in Ref. [13]. In this case the value of $\chi^2$ is $\chi^2 = 163.4$, worse by 140 than that in our best fit.

3 Production amplitude and its relation to scattering amplitude

3.1 General problem

We found also some evidences[1, 2] for existence of the $\sigma$-particle as an intermediate state of the $\pi\pi$ system in the production processes[3] by analyzing the data.

\[\text{Recently we have made a preliminary analysis of the } m_{\sigma^{0,\pi^0}} \text{ spectra in the process } p\bar{p} \rightarrow 3\pi^0 \text{ observed in the crystal barrel experiment, and found that they are reasonably well understood as due to production of the } \sigma \text{ with } m_\sigma \approx 700 \text{ MeV and } \Gamma_\sigma \approx 600 \text{ MeV in addition to the resonances.}\]
obtained through the $pp$ central collision experiment by GAMS\textsuperscript{1,2} and the data in the $J/\psi \to \omega \pi \pi$ decay reported by DM2 collaboration\textsuperscript{3). In the analyses we applied the Variant Mass and Width (VMW)-method\textsuperscript{4} where the production amplitude is represented by a sum of the $\sigma$ and $f_0$ Breit-Wigner amplitudes with relative phase factors

$$\frac{r_\sigma e^{i\theta_\sigma}}{m_\sigma^2 - s - i\sqrt{s}\Gamma_\sigma(s)} + \frac{r_{f_0} e^{i\theta_{f_0}}}{m_{f_0}^2 - s - i\sqrt{s}\Gamma_{f_0}(s)},$$

(6)

The general problem to be examined is whether our applied methods of analyses are consistent with the unitarity of $S$ matrix: The scattering amplitude $\mathcal{T}$ must satisfy the elastic unitarity and the production amplitude $\mathcal{F}$ must have, in case that the initial state has no strong phase, the same phase as $\mathcal{T}$: $\mathcal{T} \propto e^{i\delta} \to \mathcal{F} \propto e^{i\delta}$ (FSI; Final-State-Interaction theorem). Conventionally, the more restrictive relation between $\mathcal{F}$ and $\mathcal{T}$ is required on the basis of the “universality,”\textsuperscript{4,5}

$$\mathcal{F} = \alpha(s) \mathcal{T}$$

(7)

with a slowly varying real function $\alpha(s)$ of $s$. I have already shown that our $\mathcal{T}$ in the IA method satisfies the elastic unitarity automatically. The remaining problem is whether our $\mathcal{F}$ in the VMW method is consistent with the FSI theorem or not.

3.2 Basic consideration

Here I shall describe our general line of thought on the strong interaction of hadrons, our relevant problem. It is a residual interaction of QCD among color-singlet bare-hadrons, which are the stable bound states of quark and anti-quark systems. First let us consider an old example of the strong interaction among pions and nucleons. Before knowing the quark physics, the $\rho$ and the $\Delta$ were resonances of $2\pi$ system and $\pi N$ system, respectively, produced through the strong interaction among the basic pion and nucleon fields. However, presently after knowing the quark physics, the $\rho$ and the $\Delta$ should also be treated as basic fields equally as the $\pi$ and the $N$: The stable bare particle $\bar{\rho}$ ($\bar{\Delta}$) as the bound state of $q\bar{q}$ ($qqq$) system becomes the unstable physical particle $\rho_{\text{phys}}$ ($\Delta_{\text{phys}}$) after switching on the strong interaction among bare-particles $\bar{\pi}, \bar{\rho}, \bar{N}$ and $\bar{\Delta}$. In this example an $S$-matrix $S$ consistent with the unitarity is obtained, in the framework of (local) field theory, following considered there.

\textsuperscript{4} It was named\textsuperscript{3} historically after the following reason. The mass and width of “a” resonant particle, which is misinterpreted as one resonance instead of actual two overlapping resonances, are observed variably depending upon the respective processes.
In our relevant problem of scalar mesons, we should take as basic fields $\bar{\sigma}$ and $\bar{f}$ as well as $\bar{\pi}$. Here we take a viewpoint that the $\sigma$ and the $f$ are some intrinsic quark-dynamics states (possibly to be relativistic $S$-wave $q\bar{q}$ states). In this case we set up (as a simple example) the strong interaction Hamiltonian

$$H_{\text{int}}^{\text{scatt}} = \sum_{\alpha=\sigma,f} \bar{g}_\alpha \bar{\alpha} \pi \pi + \bar{g}_{\pi \pi} (\pi \pi)^2, \quad H_{\text{int}}^{\text{prod}} = \sum_{\alpha=\sigma,f} \bar{\xi}_\alpha \bar{\alpha} P^P + \bar{\xi}_{\pi \pi} P^P, \quad (9)$$

where $\bar{g}$ and $\bar{\xi}$ are real coupling constants, “$P$” denoting a relevant production channel. Due to the (former) interaction ($\text{P}$) the stable bare states $\bar{\pi}, \bar{\sigma}$ and $\bar{f}$ change into the physical states denoted as $\pi = (\bar{\pi})$, and $\sigma$ and $f$ with finite widths. Then we can derive the scattering and production amplitudes following the standard procedure of quantum field theory.

The general structure of $T$ and $F$ is shown schematically in Fig. 3, where shaded ellipses represent the final state interaction of the $2\pi$ system. It is worthwhile to note that correctly both the mechanisms in Fig. 3 should be taken into account. As a result the $T$ and $F$ are, in principle, mutually independent quantities, reflecting the coupling constants $\bar{g}_\alpha$ and $\bar{\xi}_\alpha$ being so.

In the conventional treatment, where only the former mechanism is taken into account, the function $\alpha(s)$ in Eq. (7) becomes

$$\alpha(s) = \frac{\bar{\xi}_{\pi \pi}}{\bar{g}_{\pi \pi}} = \text{const.} \quad (10)$$

---

5 We suppose that a theory of strong interaction among local hadron fields is valid as a low energy effective theory of QCD.
Figure 4: Scattering and production mechanism in a simple field-theoretical model of resonance dominative case. The production amplitude is obtained, following the mechanism shown in the figure, by replacing the first $\pi\pi$-coupling constant $\tilde{g}$ in $T$ with the production coupling $\tilde{\xi}$. The $F$ amplitude obtained in this way automatically satisfies the FSI theorem.

This leads to essentially the same $2\pi$-mass spectra in any production process as in the scattering process, which is evidently inconsistent with experimental facts. Accordingly in the conventional analysis the $\alpha(s)$ is assumed to have the form (which is generally not varying slowly)

$$\alpha(s) = \sum_n \alpha_n s^n / (s - s_0^T), \quad (11)$$

introducing the physically meaningless parameters $\alpha_n$, and fixing the value of $s_0^T$, the zero-point of $T$, from the scattering experiments. This procedure implies that production experiments generally lose their values in seeking for resonant particles. In the correct treatment considering both the former and the latter mechanisms, the direct peak of the $\pi\pi$ mass spectra due to the $\alpha$-particle production is to be observed in the production process, if its production coupling constant $\tilde{\xi}_\alpha$ is dominant, in conformity with our intuition. Thus the production experiments have generally their own values independent from the scattering experiments.

3.3 Justification of IA method and VMW method

In the previous work\footnote{In our model the parameters $\alpha_n$ and $s_0^T$ are determined from physical quantities $\tilde{g}, \tilde{\xi}$ and $\tilde{m}$.} resorting to the above model we have derived our methods of analyses, the IA method for $T$ and the VMW method for $F$, and shown their consistency with the FSI theorem. The obtained formulas of the amplitudes (derived as solutions of Schwinger-Dyson equations shown in Fig. 4) were\footnote{In our model the parameters $\alpha_n$ and $s_0^T$ are determined from physical quantities $\tilde{g}, \tilde{\xi}$ and $\tilde{m}$.}

$$T = \mathcal{K}/(1 - i\rho\mathcal{K}), \quad \mathcal{K} = \tilde{g}_\sigma^2 / (\tilde{m}_\sigma^2 - s) + \tilde{g}_j^2 / (\tilde{m}_j^2 - s),$$

$$F = \mathcal{P}/(1 - i\rho\mathcal{K}), \quad \mathcal{P} = \mathcal{K}(\tilde{g}_\sigma^2 \rightarrow \tilde{\xi}_\sigma \tilde{g}_\sigma \text{ etc.}) \quad (12)$$
in the “bare-state representation.” These formulas of $\mathcal{T}$ and $\mathcal{F}$ are rewritten into the forms of Eq. (11) and Eq. (12), respectively, in the “physical state representation.”

The consistency of the amplitudes $\mathcal{F}$ and $\mathcal{T}$ are easily seen from Eq. (12) since $\mathcal{K}$ and $\mathcal{P}$ are real and their phases come only from their common denominator $(1 - i\rho\mathbf{K})$.

4 Summary and concluding remarks

I have explained that our methods of analyses, the Interfering Amplitude method for treating the $\pi\pi$ scattering process and the VMW method for the $\pi\pi$ production process (which were effective in leading to evidences for the $\sigma$-existence) are consistent with the unitarity of $S$-matrix. Thus the conventional treatments along the line of universality argument are proved to be not correct. Accordingly I have stressed that production experiments of resonant particles have generally their own value independent of scattering experiments.

It is considered that confirmation of the $\sigma$-particle with low mass and vacuum quantum number, which possibly appears in various processes, and its right treatment is crucially important for hadron physics.

Finally, on the basis of the present talk, I propose that the present index “$f_0(400-1200)$ or $\sigma$” in PDG’98 is to be corrected as $\sigma(400-800)$ in the PDG future edition.

The present speaker acknowledges deeply to the committee of this workshop for giving him this nice opportunity of presentation. I should like also to express our sincere gratitude, representing all members of our collaboration, to professor Montanet for his fair and warm interest in our works.

References

1. T. Ishida, in proceedings of Hadron’95 (Manchester).
   S. Ishida et al., Prog. Theor. Phys. 95, 745(1996); 98, 1005(1997).
S. Ishida, M. Y. Ishida, T. Ishida, K. Takamatsu and T. Tsuru, 1 plenary and 4 parallel session talks in proceedings of Hadron’97(BNL).

7 The $r_\alpha$ and $\theta_\alpha$ in Eq. (1) are expressed in terms of $\bar{g}_\alpha$, $\bar{\xi}_\alpha$, and $\lambda_\alpha (= M_\alpha - i\rho g_\alpha^2)$ and shown to be almost $s$-independent except for the threshold region. However, we must note on the following: In the VMW-method essentially the three new parameters, $r_\sigma$, $r_f$ and $\lambda_f$, are independent of the scattering process, characterize the relevant production processes. Presently they are represented by the two production coupling constants, $\bar{\xi}_\sigma$ and $\bar{\xi}_f$. Thus, among the three parameters there exists one constraint due to the FSI-theorem.

8 Here we treat a simple case of the resonance-dominative case, including only the virtual two-$\pi$ meson effects. In Eq. (12) we also made simplification by identifying the “$K$-matrix states” with the bare states. As for details see Ref. [8].
2. R. Delbourgo and M. D. Scadron, Phys. Rev. Lett. 48, 379 (1982).
   T. Hatsuda and T. Kunihiro, Prog. Theor. Phys. 74, 765 (1985).

3. N. N. Achasov and G. N. Shestakov, Phys. Rev. D49, 5779 (1994).
   R. Kaminski, L. Lesniak and J. P. Maillet, Phys. Rev. D50, 3145 (1994).
   N. A. Tornqvist and M. Roos, Phys. Rev. Lett. 76, 1575 (1996).
   M. Harada, F. Sannino and J. Schechter, Phys. Rev. D54, 1991 (1996).
   S. Ishida et al. ('96), in Ref. 1).

4. M. R. Pennington, summary talk in proceedings of Hadron'95 (Manchester).

5. E. Klempt, summary talk in proceedings of Hadron'97 (BNL).

6. M. R. Pennington, hep-ph/9710456. M. Y. Ishida et al., hep-ph/9802272. M. D. Scadron, hep-ph/9710317.

7. T. Akeson et al., Nucl. Phys. B264, 154 (1986).
   A. Breakstone et al., Z. Phys. C31, 185 (1986).
   T. A. Armstrong et al., Z. Phys. C51, 351 (1991).

8. D. Alde et al., Phys. Lett. B397, 350 (1997).

9. M. Y. Ishida, S. Ishida and T. Ishida, Prog. Theor. Phys. 99, 1031 (1998).

10. M. Y. Ishida in Ref 1); Nucl. Phys. A629, 148c (1998); Prog. Theor. Phys. 96, 853 (1996).

11. S. Weinberg, Phys. Rev. Lett. 17, 616 (1966). M. D. Scadron in Ref 8).

12. M. Y. Ishida, see the talk in this workshop.

13. K. L. Au, D. Morgan and M. R. Pennington, Phys. Rev. D35, 1633 (1987).

14. J. E. Augustin et al., Nucl. Phys. B320, 1 (1989).

15. S. Ishida et al., Prog. Theor. Phys. 88, 89 (1992).

16. I. J. Aitchson, Nucl. Phys. A189, 417 (1972). S. U. Chung et al., Ann. Physik. 4, 404 (1995).