A toy model for weak interaction based on condensed gauge bosons

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(Dated: January 2, 2018)

We construct a toy model for weak interaction based on the assumption that gauge bosons form condensates. We then discuss the model predictions calculated from the effective Feynman rules which are derived through computing the effective action.

PACS numbers: 11.15.-q, 12.15.-y, 12.60.-i

I. INTRODUCTION

It is known that physics relating to the electroweak interactions is well described by the standard model (SM) [1, 2] in which Higgs particle plays an important role.

In this paper, from mathematical interest, we construct a toy model based on the assumption of condensed gauge bosons. The motivation to consider the gauge boson condensation is on the fact that such hypothetical condensate can give the effective masses to the gauge bosons. Here we first construct a toy model then discuss the properties of the obtained model. We believe that this type of speculations is useful for considering possible model constructions and their restrictions by experiments.

II. TOY MODEL

The Lagrangian for the SU(2) gauge field coupled to fermions is given by

\[ L = \bar{\psi} (i \gamma \partial - m) \psi - \frac{1}{4} (\partial_\mu A^a_v - \partial_\nu A^a_\mu)^2 - \frac{1}{4} g^2 (f_{abc} A^a_v A^b_\mu A^c_\nu), \]

where \( \psi \) is the fermion doublet field, \( \psi = (\nu_e, e)^T \), \( m \) is the diagonal mass matrix, \( m = \text{diag}(m_{\nu_e}, m_e) \), \( A^a_\mu \) is the SU(2) gauge field, \( g \) is the coupling constant, \( \tau^a = \sigma^a / 2 \) with the Pauli matrices \( \sigma^a \) and \( f_{abc} = i \epsilon^{abc} \) is the structure constant for SU(2) gauge sector being the antisymmetric tensor.

By using the following notations,

\[ W^\pm_\mu = \frac{1}{\sqrt{2}} (A^1_\mu \pm i A^2_\mu), \quad Z_\mu = A^3_\mu, \]

the Lagrangian can be rewritten as

\[ \mathcal{L} = \bar{\psi} (i \gamma \partial - m) \psi + \mathcal{L}_K + \mathcal{L}_1 + \mathcal{L}_3 + \mathcal{L}_4, \]

with the kinematic term

\[ \mathcal{L}_K = - i (\partial_\mu W^+_\nu) (\partial^\nu W^-_\mu) + (\partial_\mu W^+_\nu) (\partial^\nu W^-_\mu) - \frac{1}{4} (\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2, \]

the interaction with fermions,

\[ \mathcal{L}_1 = \frac{g}{\sqrt{2}} \bar{\psi} e^\gamma \nu W^+_\mu + \frac{g}{\sqrt{2}} \bar{\psi} e^\gamma \nu W^-_\mu + \frac{g}{2} \bar{\psi} e^\nu \nu Z_\mu - \frac{g}{2} \bar{\psi} e^\gamma \nu Z_\mu, \]

and the interacting terms for gauge bosons,

\[ \mathcal{L}_3 = -g [(\partial_\mu W^+_\nu)(W^{-\mu} Z^\nu - Z^{\mu} W^-) + (\partial_\mu W^-_\nu)(Z^{\mu} W^- - W^{\mu} Z^\nu) + (\partial_\mu Z_\nu)(W^{\mu} W^- - W^{-\mu} W^+)], \]

\[ \mathcal{L}_4 = \frac{1}{2} g^2 (W^+_\mu W^-_\mu)^2 - \frac{1}{2} g^2 (W^+_\mu W^+_{\mu})(W^-_\nu W^-_{\nu}) + g^2 (W^+_\nu W^-_\nu) Z_\mu Z^\nu - g^2 (W^+_\mu Z^\mu)(W^-_\nu Z^\nu). \]

What is actually observed in experiments are the n-point functions which should be calculated from the effective Feynman rules derived from the effective action, \( \Gamma = \int d^4 x (\mathcal{L}) + \cdots \), e.g.,

\[ i D^{-1}_Z(x, y) = \frac{\delta^2 \Gamma}{\delta Z_\mu(x) \delta Z_\nu(y)}, \]

then one has to evaluate the effective action for the sake of deriving the model predictions [3]. In evaluating the effective action, we assume that the gauge boson fields have non-zero expectation values,

\[ \langle W^+_\mu W^-_\mu \rangle = g_{\mu \nu} \phi_W, \quad \langle Z_\mu Z^\nu \rangle = g_{\mu \nu} \phi_Z, \]

where we call \( \phi_W \) and \( \phi_Z \) as the gauge boson condensates. The motivation of speculating this hypothesis comes from the Bose Einstein condensate at extremely low temperature in condensed matter physics [4]. Due to the non-zero values of the condensates, there appear the mass terms in the effective propagators for the gauge bosons, which will be discussed in the next section.

III. EFFECTIVE MASSES FOR GAUGE BOSONS

Performing the functional derivative of the effective action, one obtains the following effective propagators in the Feynman gauge,

\[ D_W(p) = \frac{-ig^{\mu \nu}}{p^2 - M_W^2}, \quad D_Z(p) = \frac{-ig^{\mu \nu}}{p^2 - M_Z^2}. \]
where
\[ M_W^2 = 3g^2(\phi_W + \phi_Z), \quad M_Z^2 = 6g^2\phi_W. \] (11)

From the experiments we have \( M_W = 80.2\text{GeV}, M_Z = 91.2\text{GeV}, \) then we see \( g^2\phi_W = (37.2\text{GeV})^2, \) \( g^2\phi_Z = (27.5\text{GeV})^2. \) Thus the expectation values of \( \phi_W \) and \( \phi_Z \) are different; here we think the difference stems from the different electric charge between \( W \) and \( Z \) bosons.

IV. FERMION INTERACTIONS

It is known that the left and right handed fermions interact differently with the gauge bosons. This means that the effective couplings, \( g_W^L \) and \( g_W^R \) for left and right handed interactions calculated from
\[ \frac{\delta^3 \Gamma}{\partial \phi_L(\mu) \partial \phi_L(\nu) \partial W_\mu} \rightarrow \gamma^\mu g_W^{L(\mu)}, \] (12)
lead different values, namely, \( g_W^L \neq g_W^R \). Based on the experimental results we usually set \( g_W^L = 0 \), which indicates that the \( W \) bosons do not effectively interact with right handed fermions via weak force.

Similarly, the effective couplings for \( Z \) boson and \( \nu_e, e \) can be denoted by \( g_Z^{\nu_e}, g_Z^e \), \( g_Z^{\nu_e}, g_Z^e \), and \( g_Z^{\nu_e} \), \( g_Z^e \), where we also set \( g_Z^{\nu_e} = 0 \) for the empirical reason. Theoretically, it might be possible to evaluate the forms of the effective couplings through carefully calculating the effective action. While it is more practical to fix the values of effective couplings by the experimental observations.

Here, it may be interesting to make the comparison with the forms appearing in the SM. The counterparts become,
\[ g_W^L \rightarrow \frac{g_{\text{SM}}}{\sqrt{2}}, \quad g_W^R \rightarrow 0, \quad g_Z^{e, \nu_e} \rightarrow \frac{g_{\text{SM}}}{2\cos \theta_W}, \quad g_Z^{\nu_e} \rightarrow 0, \]
\[ g_Z^e \rightarrow g_{\text{SM}} \frac{1}{\cos \theta_W} \left( \frac{1}{2} + \sin^2 \theta_W \right), \quad g_Z^{\nu_e} \rightarrow g_{\text{SM}} \frac{1}{\cos \theta_W} \sin^2 \theta_W, \]
where \( g_{\text{SM}} \) represents the coupling strength in the SM and \( \theta_W \) does the Weinberg angle.

V. POSSIBLE ENERGY LEVELS FOR CONDENSATES

Finally, we shall present the further possibility of extending the current toy model.

We think that the objects formed by the condensates have finite size in the coordinate space, namely \( \phi(t, r) \neq 0 \) for finite \( r \), but \( \phi(t, r) \rightarrow 0 \) for \( r \rightarrow \infty \). As seen in solving the Schrödinger equation in quantum mechanics, the above restriction leads discreet energy levels of the condensates. Therefore, we expect that the condensates can have various energy levels,
\[ \phi_W^{(n)}, \quad \phi_Z^{(n)}. \] (13)
with energies which would give different masses to gauge bosons where the straightforward generalization of Eq. (11) become
\[ M_W^{(n,m)} = 3g^2(\phi_W^{(n)} + \phi_Z^{(m)}), \quad M_Z^{(n)} = 6g^2\phi_W^{(n)}. \] (14)
Thus, we believe that the extended version of our model has the possibility of describing the properties of the gauge bosons with heavier masses.

The energy levels of the condensates can be, in principle, evaluated by solving the Schrödinger equation if one can calculate the potential derived from the non-Abelian gauge interaction.

VI. CONCLUDING REMARKS

We have considered the toy model by assuming the condensed gauge bosons. We find that the gauge bosons can have effective masses if the condensation occurs. We believe that the speculation presented here is also useful in considering models for QCD.

ACKNOWLEDGMENTS

The author is supported by Ministry of Science and Technology (Taiwan, ROC), through Grant No. MOST 103-2811-M-002-087.

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