IRS-Aided Non-Orthogonal ISAC Systems: Performance Analysis and Beamforming Design

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Abstract—The fundamental performance of IRS-aided communication/sensing has been extensively studied, demonstrating the benefits of IRS in improving communication rate or sensing accuracy, while that of IRS-aided integrated sensing and communication (ISAC) and the impacts of IRS on the communication-sensing tradeoff are far from being well understood. In this paper, we investigate the fundamental performance of an IRS-aided non-orthogonal ISAC (NO-ISAC) system, where a distributed IRS is deployed to assist concurrent communication and location sensing, occupying non-orthogonal time-frequency resources. We use the modified Cramer-Rao lower bound (CRLB) to characterize the joint communication-sensing performance in a unified manner, and derive its closed-form expression, revealing that IRS affects the communication-sensing tradeoff by allocating its additional spatial resources. By exploiting the modified CRLB, we propose a joint active and passive beamforming algorithm that achieves a good communication-sensing tradeoff. Numerical results demonstrate the advantage of using the unified performance metric (i.e., modified CRLB) for IRS beamforming design over using the SNR metric, and the benefits of applying more IRS elements in enlarging the communication-sensing tradeoff region. Also, we demonstrate the superiority of IRS-aided NO-ISAC systems over IRS-aided time-division ISAC systems, and show IRS-aided NO-ISAC systems can achieve comparable localization performance to IRS-aided localization systems.

Index Terms—Integrated sensing and communication (ISAC), intelligent reflecting surface (IRS), Cramer-Rao lower bound (CRLB), joint active and passive beamforming.

I. INTRODUCTION

Intelligent reflecting surface (IRS) has emerged as a revolutionary technology, which possesses advantages in extending coverage and enhancing spectrum efficiency [1]. Specifically, IRS is a digitally-controlled metasurface constructed of a large quantity of reflecting units that can proactively adjust the phase shifts and amplitudes of the incident signals to reshape the wireless propagation environment [2], [3]. Compared to traditional active relays, the IRS passively reflects the incident signal without any specialized radio-frequency processing, thereby providing a promising solution to the problems of power consumption and hardware cost [4], [5]. Moreover, IRS can be deployed in dense obstacle environments to establish virtual line-of-sight (VLoS) links to bypass signal blockage between transceivers, thus providing ubiquitous coverage efficiently and cost-effectively [6]. Owing to the above-mentioned properties, IRS has been extensively investigated in both communication and localization fields.

As for the IRS-aided communication, through joint active beamforming at the transmitter/receiver and passive beamforming at the IRS, the IRS can significantly enhance the signal-to-noise ratio (SNR) [2], [7], spectral/energy efficiency [8], [9], and data rate [10], [11], especially for users in the non-line-of-sight (NLoS) region. For example, the authors in [2] revealed that the received SNR scales with the square of the number of IRS reflecting elements in an IRS-aided communication system. The work [8] considered an energy efficiency maximization problem and demonstrated that the IRS-aided MISO communication system can achieve up to 3 times higher energy efficiency than the conventional amplify-and-forward (AF) relay system with half-duplex operations. In [10], the authors formulated a data rate maximization problem. To reduce the beamforming design complexity, an IRS element grouping strategy was adopted, where the IRS elements are divided into several groups, each of which consists of adjacent elements that are assumed to share a common reflection coefficient. Then, transmit power allocation and IRS reflection coefficients were jointly optimized. In order to fully reap the IRS beamforming gain, different IRS channel estimation methods have been adopted to acquire the channel state information (CSI). Considering the passive characteristics of the IRS, cascaded channel estimation methods based on training IRS reflection patterns, such as ON/OFF [10] and full-ON [12] patterns were proposed. Furthermore, to reduce the high channel estimation overhead caused by the vast number of IRS elements, the compressed sensing method [13] and the matrix factorization method [14] were proposed by exploiting the sparsity of mmWave channel. In addition, the deep learning method has emerged as a promising method to learn the CSI efficiently [15].

As for the IRS-aided localization, the location-related information, such as received signal strength (RSS), angle of arrival (AoA), angle of departure (AoD) as well as time
of arrival (ToA), can be extracted from the IRS VLoS link for user localization [16], [17], [18], [19], [20], [21], [22]. In [16], the authors first investigated the potential of using IRS for positioning purposes and analyzed the Cramer-Rao lower bound (CRLB) for localization with IRS. It was demonstrated that the localization performance improves quadratically with the IRS’s surface area. With the aid of IRSs, the authors in [17] proposed an RSS-based multi-user localization scheme, which achieves at least 3 times lower localization error than the traditional RSS-based localization scheme without the IRS. Later on, the authors in [22] used the IRS for the localization of a blind-zone user and proposed an angle-based positioning algorithm, which achieves centimeter-level localization accuracy.

Over the past several decades, the consistent evolution towards higher frequency bands and denser distributed massive antenna arrays of the communication/localization system results in striking similarities between them in terms of the hardware architecture, channel characteristics, as well as signal processing strategies [23], [24]. These evolutions consequently facilitate the research theme of integrated sensing and communication (ISAC), a technology that co-designs the two functionalities to leverage congested resources, offer unprecedented synergies, and acquire integration gain [25], [26]. Prompted by the ISAC technology, the works [27], [28] proposed an IRS-aided non-orthogonal ISAC (NO-ISAC) system, where uplink communication and localization for a single user are realized by using non-orthogonal/overlapped time-frequency resources at the cost of equipping partial sensing elements on the IRS, and designed sensing-assisted beamforming schemes for improving communication performance. The current literature on IRS-aided IRS mainly focuses on investigating beamforming design under different scenarios and system architectures. However, the fundamental limits of the IRS-aided NO-ISAC system are far from being well understood, especially the joint communication and sensing performance bounds and the IRS’s impacts on the fundamental communication-sensing tradeoff. For the IRS-aided NO-ISAC system, communication performance and localization performance are tightly coupled to each other due to the shared time-frequency resources. Thus, the impacts of the IRS on communication-sensing tradeoff remain unclear. Hence, it is required to devise a unified metric for characterizing the joint communication and localization performance, as well as to deeply investigate the impacts of the IRS on the joint communication-sensing performance for better guiding the design of the IRS-aided NO-ISAC system.

Motivated by the above issues, in this paper, we investigate the fundamental performance of an IRS-aided NO-ISAC system, where a distributed passive IRS is deployed to enable concurrent location sensing and downlink communication on non-orthogonal/overlapped time-frequency resources for a blind-zone user. We propose using the modified CRLB for characterizing the joint communication and location sensing performance of the IRS-aided NO-ISAC system, and derived its closed-form expression, which reveals the impacts of IRS on communication-sensing tradeoff. Based on the modified CRLB, we design a joint active and passive beamforming algorithm and investigate the tradeoff between communication performance and localization performance. Our main contributions are summarized as follows.

- We innovatively develop a modified CRLB metric for characterizing the joint communication and localization performance of the IRS-aided NO-ISAC system, and derive its closed-form expression, which allows us to deeply investigate the impacts of IRS on the tradeoff between communication and localization performances.
- By invoking the modified CRLB metric, we propose a CRLB-based beamforming algorithm that can flexibly balance communication performance and localization performance.
- Simulation results demonstrate the advantage of using the unified performance metric (i.e., modified CRLB) for IRS beamforming design over using the SNR metric for IRS beamforming design, the existence of the tradeoff region and the communication/localization saturation region, and the benefits of applying more IRS elements in enlarging the tradeoff region. Also, we demonstrate the superiority of the IRS-aided NO-ISAC system to the IRS-aided time-division ISAC (TD-ISAC) system in terms of both communication performance and location sensing performance. In addition, it is shown that the IRS-aided NO-ISAC system can achieve comparable localization performance to the IRS-aided localization system with dedicated positioning reference signals.

The remainder of this paper is organized as follows. Section II introduces the system model of the IRS-aided NO-ISAC system. Section III presents the performance characterization of the IRS-aided NO-ISAC system, while Section IV presents a CRLB-based beamforming algorithm. Numerical results are provided in Section V. Finally, Section VI concludes this paper.

Notations: The boldface upper case and boldface lower case denote matrices and vectors, respectively. The operations of transpose and Hermitian transpose are denoted by (·)T and (·)H, respectively. |·|, ∥·∥, [·], and ℘{·} respectively represent the absolute value, the Euclidean norm, the floor function, and the statistical expectation. The distribution of a complex Gaussian random variable with mean 0 and variance σ2 is denoted by CN(0, σ2). For matrices, [i,j] represents the (i, j)-th element, tr(·) is the matrix trace, diag(·) denotes the diagonal operation, θM×N and 1M×N denote the M × N all-zero matrix and all-one matrix, respectively. For vectors, [·]i denotes the i-th entry. Besides, ℘(·) is the probability of an event, and we use R(·) to denote the real part of the argument.

II. SYSTEM MODEL

We consider an IRS-aided NO-ISAC system as shown in Fig. 1, in which a distributed passive IRS is deployed to support simultaneous data transmission and localization for a blind-zone user, occupying the non-orthogonal/overlapped time-frequency resources. The distributed passive IRS is composed of 2 sub-IRSs, each of which has an L = Ly × Lz.
compared to the whole time block, we ignore it hereafter. Since such time cost is trivial addition, the first each coherence block but vary from one block to another. In positioning, simultaneously. Specifically, the demodulation for communication and AoA estimation for user received signals during the reflecting, with another sub-IRS switched off. Based on the user at time slot following the complex Gaussian distribution $\mathrm{CN}(0, \sigma^2)$. 

**A. NO-ISAC Transmission Protocol**

We consider a NO-ISAC transmission protocol as illustrated in Fig. 2, where the whole transmission duration consists of $N$ consecutive coherence blocks. Each coherence block composed of $T_c$ time slots (symbol durations) is further divided equally into two time blocks each with $T = T_c/2$ time slots. During the $i$-th time block, the $i$-th sub-IRS assists downlink data transmission and localization for the blind-zone user by reflecting, with another sub-IRS switched off. Based on the received signals during the $i$-th time block, the user conducts demodulation for communication and AoA estimation for user positioning, simultaneously. Specifically, the $T$ information-carrying symbols and the AoA pair from the $i$-th sub-IRS to the user are jointly estimated. The two sub-IRSs can act as positioning reference points since their locations are previously known. For each coherence block, the user localizes itself based on the geometric relationship between two sub-IRSs' locations and the estimated AoA pairs [29]. In this paper, both BS-IRS and IRS-user channels are modeled as quasi-static block-fading channels, which remain unchanged during each coherence block but vary from one block to another. In addition, the first $\tau_i$ time slots of the $i$-th time block are the guard time for IRS configuration. Since such time cost is trivial compared to the whole time block, we ignore it hereafter.

**B. Channel Model**

The IRS operating in the mmWave band is assumed to be deployed on high buildings with LoS paths to both the BS and the user. Hence, the channel from the IRS to the user can be expressed as [30]

$$
\mathbf{h}_{\text{IRS}} = \alpha_{\text{IRS}} \mathbf{b}_{\text{IRS}} \left( \gamma_{\text{IRS}} A_{\text{IRS}}, \varphi_{\text{IRS}} A_{\text{IRS}} \right) \mathbf{b}_H^H \left( \delta_{\text{IRS}} D_{\text{IRS}}, \varphi_{\text{IRS}} D_{\text{IRS}} \right),
$$

where $\alpha_{\text{IRS}}$ denotes the complex channel gain for the IRS-user link, $\gamma_{\text{IRS}} A_{\text{IRS}}, \varphi_{\text{IRS}} A_{\text{IRS}}$ denotes the elevation/azimuth AoA and $\delta_{\text{IRS}} D_{\text{IRS}}, \varphi_{\text{IRS}} D_{\text{IRS}}$ denotes the elevation/azimuth AoD from the IRS to the user. In addition, the array response vectors for the user and the IRS are respectively denoted by $\mathbf{b}_{\text{U}} \left( \gamma_{\text{U}} A_{\text{U}}, \varphi_{\text{U}} A_{\text{U}} \right)$ and $\mathbf{b}_l \left( \gamma_{\text{D}} D_{\text{IRS}}, \varphi_{\text{D}} D_{\text{IRS}} \right)$, whose elements are given by

$$
\mathbf{b}_{\text{U}} \left( \gamma_{\text{U}} A_{\text{U}}, \varphi_{\text{U}} A_{\text{U}} \right)_m = e^{jm_y(m)2\pi \frac{d_{\text{IRS}}}{\lambda} \sin(\gamma_{\text{U}})} \times e^{jm_z(m)2\pi \frac{d_{\text{IRS}}}{\lambda} \sin(\varphi_{\text{U}})}, m = 1, \ldots, M,
$$

$$
\mathbf{b}_l \left( \gamma_{\text{D}} D_{\text{IRS}}, \varphi_{\text{D}} D_{\text{IRS}} \right)_l = e^{jl_y(l)2\pi \frac{d_{\text{IRS}}}{\lambda} \sin(\gamma_{\text{D}})} \times e^{jl_z(l)2\pi \frac{d_{\text{IRS}}}{\lambda} \sin(\varphi_{\text{D}})}, l = 1, \ldots, L,
$$

where

$$
m_y(l) \triangleq \left\lfloor (l-1)/L_z \right\rfloor,
$$

$$
m_z(l) \triangleq m_y(l)M_z - 1,
$$

$$
l_y(l) \triangleq \left\lfloor (l-1)/L_z \right\rfloor,
$$

$$
l_z(l) \triangleq l - l_y(l)L_z - 1,
$$

$\lambda$ denotes the carrier wavelength, $d_{\text{IRS}}$ and $d_{\text{U}}$ represent the distances between two adjacent IRS elements and two adjacent user antennas, respectively.

Likewise, the channel from the BS to the IRS can be expressed as

$$
\mathbf{h}_{\text{BS}} = \alpha_{\text{BS}} \mathbf{b}_{\text{BS}} \left( \gamma_{\text{BS}} A_{\text{BS}}, \varphi_{\text{BS}} A_{\text{BS}} \right) \mathbf{b}_B^H \left( \delta_{\text{BS}} D_{\text{BS}}, \varphi_{\text{BS}} D_{\text{BS}} \right),
$$

where $\alpha_{\text{BS}}$ denotes the complex channel gain for the BS-IRS link, $\gamma_{\text{BS}} A_{\text{BS}}, \varphi_{\text{BS}} A_{\text{BS}}$ represents the elevation/azimuth AoA and $\delta_{\text{BS}} D_{\text{BS}}$ represents the AoD from the BS to the IRS. In addition, the BS array response vector is denoted by $\mathbf{b}_B \left( \delta_{\text{BS}} D_{\text{BS}}, \varphi_{\text{BS}} D_{\text{BS}} \right)$, whose elements are given by

$$
\mathbf{b}_B \left( \delta_{\text{BS}} D_{\text{BS}} \right)_n = e^{jn(n-1)2\pi \frac{d_{\text{BS}}}{\lambda} \sin(\gamma_{\text{BS}})}, n = 1, \ldots, N_z.
$$

$\lambda$ denotes the distance between two adjacent BS elements. Furthermore, we assume that $d_{\text{BS}} = d_{\text{IRS}} = \lambda/2$.

**C. Signal Model**

Without loss of generality, we focus on the $i$-th time block of the $n$-th coherence block, during which the BS sends $x^n(t)$, following the complex Gaussian distribution $\mathcal{CN}(\mu_x, \sigma_x^2)$, to the user at time slot $t \in T \triangleq \{1, \ldots, T\}$. For ease of notation, we drop the subscript $i$ and the superscript $n$ hereafter, and the received signal at the user can be expressed as

$$
y(t) = \mathbf{h}_{\text{BS}} \mathbf{H} \mathbf{B}_{\text{BS}} \mathbf{w}(t) + \mathbf{z}(t), t \in T,
$$

where the phase shift matrix of the IRS is defined as $\mathbf{\Theta} = \text{diag}(\xi)$, with the phase shift beam being $\xi = [e^{j\theta_1}, \ldots, e^{j\theta_l}, \ldots, e^{j\theta_L}]^T$. For ease of practical implementation, the phase shifts of the IRS take values...
from a finite set $F = \{0, \frac{2\pi}{2^b}, \ldots, \frac{2\pi}{2^b}(2^b - 1)\}$, where $b$ is the bit-quantization number. $w \in \mathbb{C}^{N_t \times 1}$ denotes the BS beamforming vector subject to the transmit power constraint $\|w\|^2 \leq P_t$. In addition, $z \in \mathbb{C}^{M \times 1}$ denotes the additive white Gaussian noise (AWGN), whose elements follow the complex Gaussian distribution $\mathcal{CN}(0, \sigma^2)$.

By column-wise stacking all the $y(t) \in \mathbb{C}^{M \times 1}$, $t \in T$ into an $MT \times 1$ vector, we obtain the signals received during one time block as

$$\mathbf{y} = \begin{bmatrix} y(1) \\ \vdots \\ y(T) \end{bmatrix} = \begin{bmatrix} \mathbf{h}_x(1) \\ \vdots \\ \mathbf{h}_x(T) \end{bmatrix} + \begin{bmatrix} \mathbf{z}(1) \\ \vdots \\ \mathbf{z}(T) \end{bmatrix},$$  \hspace{1cm} (11)

where

$$\mathbf{h}_x(t) = \mathbf{H}_{12U} \mathbf{\Theta} \mathbf{H}_{B2W} \mathbf{x}(t).$$ \hspace{1cm} (12)

### III. Performance Characterization of the IRS-Aided NO-ISAC System

For the considered IRS-aided NO-ISAC system, its communication and localization performances depend on the transmission of the information-carrying symbol $x(t)$ and the estimation of the AoA pair $(\gamma_{12U}^A, \varphi_{12U}^A)$, respectively. In general, the communication performance is measured by the channel capacity, while the CRLB measures the localization performance. To characterize the performances of both communication and location sensing in a unified manner, we propose to unify the communication and localization performance metrics from the CRLB perspective. First, we devise the communication CRLB metric, which is defined as the CRLB for the unbiased estimator of $x(t)$. As such, the ISAC process of simultaneous data transmission and AoA estimation is equivalent to the joint estimation of $\gamma_{12U}^A$, $\varphi_{12U}^A$, and $x(t)$, $t = 1, \ldots, T$. Motivated by this, we propose a modified CRLB (in dB) metric for characterizing the joint communication and localization performance of the NO-ISAC system, which is defined as

$$\text{CRLB}_{\text{ISAC}} \triangleq \zeta \log(\text{CRLB}_x) + (1 - \zeta) \log(\text{CRLB}_{\text{angle}}),$$ \hspace{1cm} (13)

where

$$\text{CRLB}_x \triangleq \frac{1}{T} \sum_{t=1}^{T} \text{CRLB}(x(t))/T,$$ \hspace{1cm} (14)

$$\text{CRLB}_{\text{angle}} \triangleq \left( \text{CRLB}(\gamma_{12U}^A) + \text{CRLB}(\varphi_{12U}^A) \right)/2,$$ \hspace{1cm} (15)

and $\zeta \in [0, 1]$ denotes the tradeoff factor, by adjusting which we can make a tradeoff between communication performance and localization performance. In addition, CRLB$[x(t)]$, CRLB$[\gamma_{12U}^A]$, and CRLB$[\varphi_{12U}^A]$ denote the CRLBs of $x(t)$, $\gamma_{12U}^A$, and $\varphi_{12U}^A$, respectively.

By stacking these unknown parameters (i.e., $x(t)$, $\gamma_{12U}^A$, and $\varphi_{12U}^A$) to be estimated into a $T + 2$ dimensional vector $\mathbf{\theta}$, we have

$$\mathbf{\theta} = \begin{bmatrix} x(1), \ldots, x(T), \gamma_{12U}^A, \varphi_{12U}^A \end{bmatrix}^T.$$ \hspace{1cm} (16)

Let $p(x)$ and $p(y|x)$ denote the probability density function of $x \triangleq [x(1), \ldots, x(T)]^T$ and the conditional probability density function of $y$ given $x$, respectively. Then, the joint probability density function $p(y, \mathbf{\theta})$ can be expressed as

$$p(y, \mathbf{\theta}) = p(x)p(y, \gamma_{12U}^A, \varphi_{12U}^A | x).$$ \hspace{1cm} (17)

Invoking the results in [31], we calculate the CRLB of $\theta_i$ as

$$\text{CRLB}(\theta_i) = \begin{bmatrix} J^{-1} \end{bmatrix}_{t,i},$$ \hspace{1cm} (18)

where $J \in \mathbb{R}^{(T+2) \times (T+2)}$ is the Fisher information matrix (FIM) with respect to $\mathbf{\theta}$, whose $(i, j)$-th element is given by

$$[J]_{i,j} = -\mathbb{E} \left( \frac{\partial^2 \ln p(y, \mathbf{\theta})}{\partial \theta_i \partial \theta_j} \right), i, j = 1, \ldots, T + 2.$$ \hspace{1cm} (19)

Then, we can decompose $J$ into two additive parts as

$$J = J_P + J_D,$$ \hspace{1cm} (20)

where the $(i, j)$-th elements of $J_P$ and $J_D$ are respectively defined as

$$[J_P]_{i,j} = -\mathbb{E} \left( \frac{\partial^2 \ln p(x)}{\partial \theta_i \partial \theta_j} \right),$$ \hspace{1cm} (21)

$$[J_D]_{i,j} = -\mathbb{E} \left( \frac{\partial^2 \ln p(y, \gamma_{12U}^A, \varphi_{12U}^A | x)}{\partial \theta_i \partial \theta_j} \right).$$ \hspace{1cm} (22)

By calculating $J_P$ and $J_D$, we can obtain the FIM $J$, the CRLB of $\theta$, and the modified CRLB, as summarized in the following theorem.

**Theorem 1:** The FIM with respect to $\mathbf{\theta}$ is given by (23), shown at the bottom of the next page, where

$$\mathbf{\beta}_x = \mathbf{H}_{12U} \mathbf{\Theta} \mathbf{H}_{B2W},$$ \hspace{1cm} (24)

$$\mathbf{\beta}_\gamma = \mathbf{H}_{12U} \mathbf{\gamma} \mathbf{H}_{B2W},$$ \hspace{1cm} (25)

$$\mathbf{\beta}_\varphi = \mathbf{H}_{12U} \mathbf{\varphi} \mathbf{H}_{B2W},$$ \hspace{1cm} (26)

$$[\mathbf{H}_{12U}, \gamma_{12U}^A, \mathbf{H}_{12U} \mathbf{\gamma}]_{m,i} = j \pi(m_{12U} - m_{12U}) \cos(\gamma_{12U}^A)|\mathbf{H}_{12U}|_{m,i},$$ \hspace{1cm} (27)

$$[\mathbf{H}_{12U}, \varphi_{12U}^A, \mathbf{H}_{12U} \mathbf{\varphi}]_{m,i} = j \pi(m_{12U} - m_{12U}) \cos(\varphi_{12U}^A) \times \cos(\varphi_{12U}^A)|\mathbf{H}_{12U}|_{m,i},$$ \hspace{1cm} (28)

The CRLB of $x(t)$, $\gamma_{12U}^A$, and $\varphi_{12U}^A$ are respectively given by

$$\text{CRLB}(x(t)) = \begin{bmatrix} J^{-1} \end{bmatrix}_{t,t},$$ \hspace{1cm} (29)

$$\text{CRLB}(\gamma_{12U}^A) = \begin{bmatrix} J^{-1} \end{bmatrix}_{T+1,T+1},$$ \hspace{1cm} (30)

$$\text{CRLB}(\varphi_{12U}^A) = \begin{bmatrix} J^{-1} \end{bmatrix}_{T+2,T+2}.$$ \hspace{1cm} (31)

The modified CRLB for the IRS-aided NO-ISAC system is given by

$$\text{CRLB}_{\text{ISAC}} \triangleq \zeta \log \left( \frac{1}{T} \sum_{t=1}^{T} \begin{bmatrix} J^{-1} \end{bmatrix}_{t,t} \right) + (1 - \zeta) \log \left( \frac{1}{2} \sum_{t=T+1}^{T+2} \begin{bmatrix} J^{-1} \end{bmatrix}_{t,t} \right).$$ \hspace{1cm} (32)

**Proof:** See Appendix.
Let $I(t)$ denote the mutual information between $x(t)$ and its estimator $\hat{x}(t)$. The relationship between the communication CRLB $\text{CRLB}(x(t))$ and the mutual information $I(t)$ is provided by the following corollary.

**Corollary 1:** The mutual information $I(t)$ is given by

$$I(t) = \frac{1}{2} \log \left( \frac{\sigma_x^2}{\text{CRLB}(x(t))} \right). \quad (33)$$

**Proof:** The mutual information between $x(t)$ and $\hat{x}(t)$ can be expressed as

$$I(t) = H(x(t)) - H(x(t)|\hat{x}(t)), \quad (34)$$

where $H(x(t))$ denotes the entropy of $x(t)$ and $H(x(t)|\hat{x}(t))$ denotes the conditional entropy of $x(t)$ given the estimated signal $\hat{x}(t)$. Note that, with the given estimated signal $\hat{x}(t)$, the conditional probability distribution of $x(t)$ can be expressed as

$$(x(t)|\hat{x}(t)) \sim \mathcal{CN}(\hat{x}(t), \text{CRLB}(x(t))). \quad (35)$$

According to [32], the entropy of a complex random variable is defined as the entropy of its real composite. Therefore, the mutual information $I(t)$ can be calculated as

$$I(t) = H(R\{x(t)\}) - H(R\{x(t)|\hat{x}(t)\})$$
$$= \frac{1}{2} \log \pi e \sigma_x^2 - \frac{1}{2} \log \pi e \text{CRLB}(x(t))$$
$$= \frac{1}{2} \log \left( \frac{\sigma_x^2}{\text{CRLB}(x(t))} \right). \quad (36)$$

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**IV. CRLB-BASED BEAMFORMING DESIGN**

In this section, by exploiting the modified CRLB, we propose a joint design of active beamforming at the BS and passive beamforming at the IRS, with both communication and localization performances taken into account.

We formulate the CRLB minimization problem as

**(P1):**

$$\text{min}_{\mathbf{w}, \xi} \quad \text{CRLB}_{\text{ISAC}}$$

s.t. $\|\mathbf{w}\|^2 \leq P_t,$

$$\forall l \in \mathcal{F}, \forall l = 1, \ldots, L. \quad (37b)$$

By invoking the results in Theorem 1, the objective function can be expressed as

$$\text{CRLB}_{\text{ISAC}} = \zeta \log \left( \frac{1}{T} \left| a_B^H(\delta_{B21}) \mathbf{w} \right|^2 \sum_{t=1}^{T} [J_{\xi}]^{-1}_{t,t} \right)$$
$$+ (1 - \zeta) \log \left( \frac{1}{2} \left| a_B^H(\delta_{B21}) \mathbf{w} \right|^2 \sum_{t=T+1}^{T+2} [J_{\xi}]^{-1}_{t,t} \right)$$
$$= V_\xi - 2 \log \left| a_B^H(\delta_{B21}) \mathbf{w} \right|^2, \quad (38)$$

where

$$J_\xi = \frac{\mathbf{J}}{\left| a_B^H(\delta_{B21}) \mathbf{w} \right|^2}, \quad (39)$$

$$V_\xi = \zeta \log \left( \frac{1}{T} \sum_{t=1}^{T} [J_{\xi}]^{-1}_{t,t} \right)$$
$$+ (1 - \zeta) \log \left( \frac{1}{2} \sum_{t=T+1}^{T+2} [J_{\xi}]^{-1}_{t,t} \right). \quad (40)$$

Note that, by substituting (8), (24), (25), and (26) into (39), we can extract $\|a_B^H(\delta_{B21})\mathbf{w}\|^2$ from $\mathbf{J}$, and the remainder of $\mathbf{J}$ (i.e., $J_\xi$) does not contain any terms related to $\mathbf{w}$. Therefore, $V_\xi$ in (40) is independent of $\mathbf{w}$, and we can decompose the problem (P1) into two uncoupled subproblems with respect to $\mathbf{w}$ and $\xi$, respectively.

**A. BS Active Beamforming**

The subproblem with respect to the BS beamforming vector $\mathbf{w}$ can be formulated as

**(P2):**

$$\text{max}_{\mathbf{w}} \quad \|a_B^H(\delta_{B21}) \mathbf{w}\|^2$$

s.t. $\|\mathbf{w}\|^2 \leq P_t. \quad (41b)$$

It can be easily verified that the optimal solution is

$$\mathbf{w} = \sqrt{\frac{P_t}{N_t}} a_B^H(\delta_{B21}). \quad (42)$$

**B. IRS Passive Beamforming**

The optimization subproblem with respect to the IRS phase shift beam $\xi$ can be formulated as

**(P3):**

$$\text{min}_{\xi} \quad V_\xi$$

s.t. $|\xi|_l \in \mathcal{F}, \forall l = 1, \ldots, L. \quad (43b)$$

Due to the complicated expression of the objective function as well as the non-convex phase shift constraints, the problem (P3) is difficult to solve. Responding to this, we propose an estimation of distribution algorithm (EDA) based method to optimize $\xi$, where a number of candidates for $\xi$ are created iteratively, evolving once and again until a satisfactory solution is achieved [33].

$$\mathbf{J} = \frac{2}{\sigma_x^2}$$
$$\left[ \begin{array}{c} \|\beta_x\|^2 I_{T \times T} + \mu_x R(\beta_x^H \beta_x) 1_{T \times 1} \\ \mu_x R(\beta_x^H \beta_x) 1_{T \times T} \end{array} \right]$$
$$\left[ \begin{array}{c} \|\beta_x\|^2 I_{T \times T} - \mu_x R(\beta_x^H \beta_x) 1_{T \times 1} \\ \mu_x R(\beta_x^H \beta_x) 1_{T \times T} \end{array} \right]$$
$$\left( \begin{array}{c} \mu_x^2 + \sigma_x^2 \end{array} \right) T \|\beta_x\|^2$$
$$\left( \begin{array}{c} \mu_x^2 + \sigma_x^2 \end{array} \right) T R(\beta_x^H \beta_x)$$

$$\left( \begin{array}{c} \mu_x^2 + \sigma_x^2 \end{array} \right) T \|\beta_x\|^2$$

$$\left( \begin{array}{c} \mu_x^2 + \sigma_x^2 \end{array} \right) T R(\beta_x^H \beta_x)$$

$$\right]. \quad (23)$$
First of all, we define the probability matrix corresponding to $\xi$ as

$$
P^i = \begin{bmatrix}
p_{1,1}^i & \cdots & p_{1,L}^i \\
p_{2,1}^i & \cdots & p_{2,L}^i \\
\vdots & \ddots & \vdots \\
p_{b,1}^i & \cdots & p_{b,L}^i
\end{bmatrix} \in \mathbb{C}^{2^b \times L}, \quad (44)
$$

where $i$ represents the iteration index. The $i$-th column of $P^i$ denotes the probability parameter for $[\xi]_i$, and $p_{s,l}^i$ satisfies the probability constraints $0 \leq p_{s,l}^i \leq 1$ and $\sum_{s=1}^{2^b} p_{s,l}^i = 1$. Initially, we set the probability matrix as $P^0 = \frac{1}{2^b} \times \mathbf{1}_{2^b \times L}$.

Then, in the $i$-th iteration, we randomly generate $C$ candidates $\{\xi^c\}_{c=1}^C$ according to the probability distribution, which can be expressed as

$$
\mathbb{P}\left([\xi]_l = \vartheta; P^i\right) = \begin{cases}
p_{1,1}^i, & \vartheta = \mathcal{F}(1) \\
p_{1,l}^i, & \vartheta = \mathcal{F}(s), \quad l = 1, \ldots, L, \\
p_{2^b,1}^i, & \vartheta = \mathcal{F}(2^b)
\end{cases}, \quad (45)
$$

where $\mathcal{F}(s)$ is the $s$-th entry of $\mathcal{F}$. Next, for each generated candidate $\xi^c$, we calculate the corresponding $V_{\xi}(\xi^c)$ based on (40). After that, we sort $\mathcal{V} = \{V_{\xi}(\xi^c)\}_{c=1}^C$ in ascending order. For the $C_{\text{elite}}$ smallest $V_{\xi}(\xi^c)$’s in $\mathcal{V}$, we select their corresponding phase shift beams (i.e., $\xi_{\text{elite}}^c$, $c_q = 1, \ldots, C_{\text{elite}}$) as elite candidates, based on which, we update the elements of the probability matrix in the $(i + 1)$-th iteration as

$$
p_{s,l}^{i+1} = \frac{1}{C_{\text{elite}}} \sum_{c_q = 1}^{C_{\text{elite}}} \Gamma([\xi_{\text{elite}}^c]_l, \mathcal{F}(s)),
$$

where $\Gamma([\xi_{\text{elite}}^c]_l, \mathcal{F}(s))$ denotes the judge function for phase shift judgment, which is defined as

$$
\Gamma([\xi_{\text{elite}}^c]_l, \mathcal{F}(s)) = \begin{cases}
1, & [\xi_{\text{elite}}^c]_l = \mathcal{F}(s) \\
0, & [\xi_{\text{elite}}^c]_l \neq \mathcal{F}(s)
\end{cases}. \quad (47)
$$

The above process will be repeated until the extreme difference of $\mathcal{V}$ (i.e., $|\max(\mathcal{V}) - \min(\mathcal{V})|$) is smaller than the threshold $\kappa$, which indicates that the probability matrix corresponding to $\xi$ is stable. Finally, the above-proposed modified CRLB-based EDA beamforming algorithm is summarized in Algorithm 1.

**Remark 1**: The computational complexity of Algorithm 1 is mainly composed of the number of generated candidates per iteration $C$ and the calculation of the FIM matrix $\mathbf{J}$, where the complexity of calculating $\mathbf{J}$ mainly comes from the calculation of $\beta_e$, $\beta_\gamma$, and $\beta_{e'}$, each has $O((N_t + M)L^2)$ complexity. As such, the complexity of Algorithm 1 is $O(C(N_t + M)L^2)$.

**Remark 2**: Although the IRS-aided NO-ISAC system considers the single-user case, it can be extended to the multi-user case by scheduling multiple users on different time-frequency blocks. For each user, communication and location sensing are conducted, sharing the same time and frequency resources. As such, the proposed beamforming algorithm for the single-user case can directly be applied to the multi-user case.

V. NUMERICAL RESULTS

This section presents numerical results to investigate the performance of the IRS-aided NO-ISAC system as well as to verify the effectiveness of the proposed beamforming algorithm. As shown in Fig. 3, the mobile user is on the horizontal floor with a speed of $V_U$, the BS is 10 meters (m) above the horizontal floor, and the two IRSs that are 1 m apart are 5 m above the horizontal floor. The distance between the BS and the IRS is set as $d_{B1} = 30$ m, while between the IRS and the user is set as $d_{12} = 10$ m. The path loss exponents from the BS to the IRS and from the IRS to the user are respectively set as 2.3 and 2.2, and the path loss at the reference distance of 1 m is set as 30 dB [34]. The length of each time slot is 33.33 microseconds (μs). The received SNR at the user is defined as $\text{SNR} = |\alpha_{B12}^2 \alpha_{1U}^2| / \sigma_2^2$. Unless otherwise specified, we set the system parameters as given in Table I.

A. IRS-Aided NO-ISAC System vs. IRS-Aided Localization System

In this subsection, we compare the performance of the IRS-aided NO-ISAC system with that of the IRS-aided localization system.
TABLE I
SYSTEM PARAMETERS

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $\beta$   | 45$^\circ$ | $V_U$ | 0 m/s |
| $SNR$     | 0 dB | $\mu_x$ | 0 |
| $\sigma_x^2$ | 1 | $N_t$ | 16 |
| $L$       | $16 \times 16$ | $M$ | $4 \times 4$ |
| $P_s$     | 1 | $T$ | 15 |
| $T_c$     | 30 | $N$ | 50 |
| $\zeta$   | 0.5 | $b$ | 2 |
| $C$       | 512 | $C_{\text{elite}}$ | 64 |
| $\kappa$ | $10^{-3}$ | |

Fig. 4. Performance comparison between the NO-ISAC system and the localization system.

For the former, the communication signals sent by the BS are used for both data transmission and localization, while for the latter, the positioning reference signals sent by the BS are used only for localization and are known to the user. We adopt the CRLB for angle estimation (i.e., $\text{CRLB}_{\text{angle}}$) to evaluate the localization performance, and based on Corollary 1, the communication performance of the IRS-aided NO-ISAC system is measured by the average mutual information defined as

$$I_{\text{NO-ISAC}} \triangleq \frac{1}{T} \sum_{t=1}^{T} I(t),$$

(48)

Fig. 4 compares the performance of the IRS-aided NO-ISAC system and the IRS-aided localization system, where the blue line and red line are obtained by varying the tradeoff factor $\zeta$ from 0 to 1 (denoted by the increase in marker size in Fig. 4) with the variance of the communication signal being $\sigma_x^2 = 1$ and $\sigma_x^2 = \frac{1}{4}$, respectively. The localization performance of the IRS-aided NO-ISAC system is worse than that of the IRS-aided localization system, due to the randomness of the communication signal and the requirement of allocating partial spatial resources for communication to achieve simultaneous communication and localization. Despite the loss of location sensing performance, the IRS-aided NO-ISAC system can realize concurrent communication and localization, while the IRS-aided localization system only has the location sensing function. For example, although the location sensing accuracy of the IRS-aided NO-ISAC system with $\sigma_x^2 = 1$ is 2 times lower than that of the IRS-aided localization system, the communication mutual information of 8.5 bits/symbol can be achieved in the IRS-aided NO-ISAC system. Moreover, we can observe that a high variance of the communication signal is detrimental for localization while beneficial for communication.

Fig. 5 presents the CRLB for angle estimation versus the number of time slots (per time block) $T$ with different tradeoff factors $\zeta$. It is obvious that the localization performance gap between the two systems becomes narrower as the tradeoff factor $\zeta$ decreases, indicating that allocating more spatial resources for localization could effectively improve the location sensing accuracy of the IRS-aided NO-ISAC system. Moreover, when allocating sufficient spatial resources for localization (i.e., $\zeta$ is enough small), the performance gap between the two systems gradually vanishes with the increase of the number of time slots (per time block) $T$. This is because collecting more data for localization helps suppress the adverse effect of the randomness caused by the communication signal.

Fig. 6 shows the CRLB for angle estimation versus the number of IRS elements $L$ with different variances of $x(t)$. As can be seen, the localization performance of both the IRS-aided NO-ISAC system and the IRS-aided localization system improves significantly as the number of IRS elements increases, especially in the case of a small number of IRS elements, demonstrating the advantage of using the IRS for localization. In addition, with $T = 15$, the localization...
performance of the IRS-aided NO-ISAC system remains stable as the variance of $x(t)$ increases, which indicates that the adverseness of the communication signal’s randomness on the localization performance can be eliminated by collecting enough data.

B. IRS-Aided NO-ISAC System vs. IRS-Aided TD-ISAC System

In this subsection, we compare the performance of the IRS-aided NO-ISAC system and the IRS-aided TD-ISAC system [35]. For the IRS-aided NO-ISAC system, localization and data transmission are conducted simultaneously by sending communication signals from the BS to the user during the whole time block. For the IRS-aided TD-ISAC system, as illustrated in Fig. 7, localization is conducted by sending positioning reference signals from the BS to the user in the first $T/5$ time slots of each time block, while data transmission is conducted by sending communication signals in the remaining time slots. Therefore, for communication performance comparison, the average mutual information of the IRS-aided TD-ISAC system is defined as

$$I_{TD-ISAC} = \frac{1}{T} \sum_{t=T/5}^{T} I(t). \quad (49)$$

Fig. 8(a) and Fig. 8(b) compare the performance of the IRS-aided NO-ISAC system with that of the IRS-aided TD-ISAC system versus the number of time slots (per time block) $T$. It is obvious that both the communication performance and the localization performance of the IRS-aided NO-ISAC system are superior to those of the IRS-aided TD-ISAC system, demonstrating the advantage of the IRS-aided NO-ISAC system. This superiority is due to the following two reasons. On the one hand, the IRS-aided NO-ISAC system conducts data transmission and localization simultaneously within the whole time block. On the other hand, for the IRS-aided NO-ISAC system, the adverse effect of signal randomness on localization performance can be effectively eliminated by increasing $T$. As the number of time slots (per time block) becomes larger, the advantage of the IRS-aided NO-ISAC system in terms of localization performance becomes more significant. In addition, since the number of communication symbols to be estimated per time block is proportional to $T$ and these symbols are independent of each other, the communication performance of the two systems remains stable as the number of time slots (per time block) increases.

Fig. 9(a) and Fig. 9(b) compare the performance of the IRS-aided NO-ISAC system and the IRS-aided TD-ISAC system versus the user received SNR. In the whole SNR regime, the IRS-aided NO-ISAC system outperforms the IRS-aided TD-ISAC system in terms of both communication performance and localization performance. Moreover, as the SNR increases, the communication performance gap becomes more significant, which indicates that the IRS-aided NO-ISAC system can benefit more from the increase in the user received SNR. This communication performance gap is mainly because, the IRS-aided NO-ISAC system carries out data transmission during the whole time block, while the IRS-aided TD-ISAC system carries out data transmission only within partial time slots of a time block.

Finally, in Fig. 10(a) and Fig. 10(b), we compare the performance of the IRS-aided NO-ISAC system and the IRS-aided TD-ISAC system versus the number of IRS elements. For any configuration of the number of IRS elements, the IRS-aided NO-ISAC system performs better than the IRS-aided TD-ISAC system in terms of both communication performance and localization performance. As the number of IRS elements increases, the communication performance and localization performance of both systems improve remarkably due to the increased IRS beamforming gain, and the communication advantage of the IRS-aided NO-ISAC system over the IRS-aided TD-ISAC system becomes more pronounced.
C. Tradeoff Between Communication Performance and Localization Performance

In this subsection, we first demonstrate the effectiveness of the proposed CRLB-based beamforming algorithm. Then, by adjusting the tradeoff factor $\zeta$ in the CRLB-based beamforming algorithm, we investigate the tradeoff between communication performance and localization performance. We adopt the average mutual information in (48) and the CRLB for angle estimation to measure the communication performance and localization performance of the IRS-aided NO-ISAC system, respectively.

In Fig. 11, we set $L = 20 \times 20$, and investigate the performance of the proposed modified CRLB-based EDA beamforming algorithm, which utilizes the modified CRLB and EDA as the performance metric and the optimization method, respectively. For comparison, the following four schemes are presented as benchmarks: 1) Modified CRLB-based local search (LS) beamforming algorithm that uses the modified CRLB as the performance metric and the LS method for phase shift optimization; 2) SNR-based EDA beamforming algorithm [2] that adopts the SNR as the performance metric and the EDA method for phase shift optimization; 3) Beamforming algorithm with random phase shift beam $\xi$; 4) IRS-aided TD-ISAC system, where the time ratio of communication and location sensing varies from 1/15 to 14/15, and the EDA method with the performance metric of modified CRLB is adopted for phase shift optimization. We can observe that the proposed CRLB-based EDA beamforming algorithm can achieve optimal joint communication and localization performance boundary, and can flexibly adjust the communication performance and localization performance. Specifically, first, compared to the CRLB-based LS beamforming algorithm, the proposed beamforming algorithm has a larger performance adjustable range and better joint communication and localization performance, demonstrating the effectiveness of using the EDA method for discrete phase shift optimization. Second, the proposed beamforming algorithm can flexibly make a tradeoff between communication performance and localization performance.
localization performance by adjusting the tradeoff factor $\zeta$. In contrast, the SNR-based EDA beamforming and the random beamforming algorithms not only fail to achieve optimal joint communication and localization performance boundary but also can only achieve fixed performance. These observations demonstrate the superiority of using the proposed modified CRLB for guiding the design of the IRS-aided NO-ISAC system as a unified performance metric. Third, the IRS-aided NO-ISAC system with the proposed CRLB-based EDA beamforming algorithm outperforms the IRS-aided TD-ISAC system, due to its high resource efficiency by conducting communication and sensing tasks on the same time-frequency resources.

Fig. 12 shows the tradeoff between communication performance and localization performance with different user received SNRs. We can observe that increasing the communication/localization performance would degrade the localization/communication performance, which reveals the displacement relation between communication and localization performance. By sacrificing the performance of one, the performance of another can be improved. Moreover, the communication-localization curve includes three regions, tradeoff region, communication saturation region as well as localization saturation region. In the tradeoff region, sacrificing the performance of one can effectively enhance that of another. For example, when $\text{SNR} = 0 \, \text{dB}$, by sacrificing the mutual information from 12 bits/symbol to 11 bits/symbol, the CRLB for angle estimation improves from $2 \times 10^{-9}$ to $5 \times 10^{-10}$.

In the communication/localization saturation region, despite sacrificing the performance of one a lot, little performance gain of another can be obtained. For example, when $\text{SNR} = -5 \, \text{dB}$, the communication performance only improves from 11.3 bits/symbol to 11.4 bits/symbol despite sacrificing the CRLB for angle estimation from $1 \times 10^{-8}$ to $1 \times 10^{-7}$.

Fig. 13 presents the tradeoff between communication performance and localization performance with different numbers of time slots (per time block) $T$. It can be seen that increasing the number of time slots (per time block) drastically improves the localization performance but has little effect on the communication performance. This indicates that the localization performance sacrificed for improving the communication performance could be compensated by increasing the number of time slots (per time block). For example, as the mutual information increases from 11 bits/symbol to 12 bits/symbol, the CRLB for angle estimation remains constant at $2 \times 10^{-9}$ by increasing $T$ from 5 to 15.

Fig. 14 illustrates the tradeoff between communication and localization performance with different numbers of IRS elements $L$, where we set $\beta = 0^\circ$. It can be readily seen that the adjustable range of both communication and localization performance is enlarged as the number of IRS elements increases, especially in the case of a small number of IRS elements. The reason is that more IRS elements would bring more spatial resources, which are flexibly allocated by the proposed beamforming algorithm for balancing communication and localization performance. For example, as the number of IRS elements increases from 64 to 100, the adjustable range of the mutual information is enlarged from about 4.7 bits/symbol to about 5.3 bits/symbol, while that of the CRLB for angle estimation is enlarged from 19.5 dB to 22 dB.

Finally, we investigate the impact of user mobility on the tradeoff between communication performance and localization performance in Fig. 15, where the user moves anticlockwise along the blue dotted line in Fig. 3 and we set $T = 1200$ and $L = 12 \times 12$. It is obvious that both communication and location sensing performances decrease with the user speed, especially in the high-speed scenario. This
is because higher user mobility results in channel aging, which degrades the effectiveness of the beamforming design based on the estimated CSI.

VI. CONCLUSION

In this paper, we characterized the fundamental performance of an IRS-aided NO-ISAC system by utilizing a novel unified performance metric, and designed a beamforming algorithm for improving the joint performance of communication and localization. In particular, we proposed the modified CRLB metric to characterize the joint communication and localization performance of the IRS-aided NO-ISAC system, and derived its closed-form expression. Based on the modified CRLB, we proposed a joint active and passive beamforming algorithm for balancing communication performance and localization performance. Numerical results showed that, despite the adverse effect of signal randomness on localization, with enough time slots per time block, the IRS-aided NO-ISAC system with random communication signals can achieve comparable localization performance to the IRS-aided localization system with dedicated positioning reference signals. Moreover, it was demonstrated that both the communication and localization performances of the IRS-aided NO-ISAC system are better than those of the IRS-aided TD-ISAC system. In addition, compared to using the traditional SNR metric, utilizing the proposed modified CRLB metric for beamforming not only achieves optimal joint communication and localization boundary but also flexibly makes a tradeoff between communication performance and localization performance. Investigation of the tradeoff between the two performances further verified the effectiveness of the proposed beamforming algorithm, revealed the existence of the tradeoff region and the communication/localization saturation region, and demonstrated the benefit of applying more IRS elements in enlarging the tradeoff region.

APPENDIX

To derive the FIM \( J = J_P + J_D \), we calculate \( J_P \) and \( J_D \) respectively in the following.

A. Calculate \( J_P \)

First, we calculate the \((i,j)\)-th element of \( J_P \), which is given by

\[
[J_P]_{i,j} = -E \left( \frac{\partial^2 \ln p(x)}{\partial \theta_i \partial \theta_j} \right), \quad i,j = 1, \ldots, T + 2, \tag{50}
\]

where the probability density function \( p(x) \) is expressed as

\[
p(x) = \left( \frac{\sigma_x^2}{\pi} \right)^{-T} \exp \left[ -\sum_{t=1}^{T} \frac{|x(t) - \mu_x|^2}{\sigma_x^2} \right]. \tag{51}
\]

Since the AoAs \( \gamma_{12U}^D \) and \( \phi_{12U}^D \) are constant and the transmit signals \( x(t), t = 1, \ldots, T \) are independent of each other, only the second partial derivative of \( p(x) \) with respect to \( x(t) \) may not be 0, which can be calculated according to [36] as

\[
\frac{\partial \ln p(x)}{\partial x(t)} = \frac{1}{\sigma_x^2} \frac{\partial (x(t) - \mu_x) (x(t) - \mu_x)^*}{\partial x(t)}
\]

\[
= \frac{1}{\sigma_x^2} (x(t) - \mu_x)^*, \tag{52}
\]

\[
\frac{\partial^2 \ln p(x)}{\partial x^2(t)} = \frac{1}{\sigma_x^2} \frac{\partial (x(t) - \mu_x)^*}{\partial x(t)} = 0. \tag{53}
\]

Then, we obtain \( J_P \) as

\[
J_P = 0_{(T+2) \times (T+2)}. \tag{54}
\]

B. Calculate \( J_D \)

The \((i,j)\)-th element of \( J_D \) can be expressed as (55), shown at the bottom of the page [37], where

\[
\Sigma = \sigma_x^2 I_{MT \times MT}, \tag{56}
\]

\[
[J_D(t)]_{i,j} = \frac{2}{\sigma_x^2} \Re \left\{ \mathbb{E} \left[ \begin{bmatrix} \frac{\partial h_x(t)}{\partial \theta_i} \Sigma^{-1} \frac{\partial h_x(t)}{\partial \theta_j} \\ \frac{\partial h_x(t)}{\partial \theta_i} \Sigma^{-1} \frac{\partial \Sigma}{\partial \theta_j} \end{bmatrix} \right] \right\}. \tag{57}
\]

First, we calculate the partial derivative of \( h_x(t) \) with respect to \( \theta \), \( i = 1, \ldots, T \) (i.e., \( x(1), \ldots, x(T) \)). Since the transmitted signals are independent of each other, we have

\[
\frac{\partial h_x(t_1)}{\partial x(t_2)} = \frac{\partial H_{12U} \Theta H_{12U}^H w(t_1)}{\partial x(t_2)} = \begin{cases} \beta_x, & t_1 = t_2 \\ 0_{LT \times 1}, & t_1 \neq t_2 \end{cases}, \tag{58}
\]

where \( \beta_x \triangleq H_{12U} \Theta H_{12U}^H w \).

Then, we calculate the partial derivative of \( h_x(t) \) with respect to \( \theta \), \( t \in \{T+1, T+2\} \) (i.e., \( \gamma_{12U}^A \) and \( \phi_{12U}^A \)). Noticing that \( d_{\text{IRS}} = d_{\text{user}} = \frac{\lambda}{2} \), and the URAs of both the IRS and the user lie on the \( y\) plane, we have \( \gamma_{12U}^A = -\gamma_{12U}^D \) and

\[
[J_D(t)]_{i,j} = \frac{2}{\sigma_x^2} \Re \left\{ \mathbb{E} \left[ \begin{bmatrix} \frac{\partial h_x(t)}{\partial \theta_i} \Sigma^{-1} \frac{\partial h_x(t)}{\partial \theta_j} \\ \frac{\partial h_x(t)}{\partial \theta_i} \Sigma^{-1} \frac{\partial \Sigma}{\partial \theta_j} \end{bmatrix} \right] \right\} = \sum_{t=1}^{T} [J_D(t)]_{i,j}, \quad i,j = 1, \ldots, T + 2, \tag{55}
\]
\[ \| \mathbf{v}_{12U} - \mathbf{v}_{21U} \| = \pi. \] As such, the \((m, l)\)-th element of \( \mathbf{H}_{12U} \) can be compactly written as

\[
[\mathbf{H}_{12U}]_{m,l} = \alpha_{12U} \mathbf{v}_{12U}^T \left[ (m_0(m) - l_0(l)) \mathbf{v}_{21U}^T + (m_0(m) - l_0(l)) \mathbf{v}_{21U}^T \right],
\]

where \( m = 1, \ldots, M \), \( l = 1, \ldots, L \). \( \tag{59} \)

And we can obtain the partial derivative of \( \mathbf{h}_x(t) \) with respect to \( \gamma_{12U} \) and \( \varphi_{12U} \) as

\[
\beta_{\gamma, t} \triangleq \frac{\partial \mathbf{h}_x(t)}{\partial \gamma_{12U}} = \mathbf{H}_{12U, \gamma} \mathbf{H}_{12U} \mathbf{H}_{21U} \mathbf{w}(t) \triangleq \beta_{\gamma x}(t), \tag{62} \]

\[
\beta_{\varphi, t} \triangleq \frac{\partial \mathbf{h}_x(t)}{\partial \varphi_{12U}} = \mathbf{H}_{12U, \varphi} \mathbf{H}_{12U} \mathbf{H}_{21U} \mathbf{w}(t) \triangleq \beta_{\varphi x}(t), \tag{63} \]

where the \((m, l)\)-th elements of \( \mathbf{H}_{12U, \gamma} \) and \( \mathbf{H}_{12U, \varphi} \) are respectively defined as

\[
[\mathbf{H}_{12U, \gamma}]_{m,l} \triangleq \frac{\partial [\mathbf{H}_{12U}]_{m,l}}{\partial \gamma_{12U}} = j \pi (m_2(m) - l_2(l)) \cos \left( \gamma_{12U} \right) [\mathbf{H}_{12U}]_{m,l},
\]

\[
= j \pi (m_2(m) - l_2(l)) \cos \left( \gamma_{12U} \right) [\mathbf{H}_{12U}]_{m,l}, \tag{64} \]

\[
[\mathbf{H}_{12U, \varphi}]_{m,l} \triangleq \frac{\partial [\mathbf{H}_{12U}]_{m,l}}{\partial \varphi_{12U}} = j \pi (m_2(m) - l_2(l)) \cos \left( \varphi_{12U} \right) [\mathbf{H}_{12U}]_{m,l}, \tag{65} \]

Noticing that \( \mathbf{J}_D(t) \) is a real symmetric matrix, we express it as

\[
\mathbf{J}_D(t) = \begin{bmatrix} \mathbf{J}_{D1}(t) \\ \mathbf{J}_{D2}(t) \\ \mathbf{J}_{D3}(t) \end{bmatrix}, \tag{66} \]

where \( \mathbf{J}_{D1}(t) \in \mathbb{R}^{T \times T} \), \( \mathbf{J}_{D2}(t) \in \mathbb{R}^{T \times 2} \), and \( \mathbf{J}_{D3}(t) \in \mathbb{R}^{2 \times 2} \). By substituting (62), (63), (58) into (57), we obtain

\[
[\mathbf{J}_{D1}(t)]_{i,j} = \begin{cases} \frac{2}{\sigma_x^2} \| \mathbf{v}_x \|^2, & i = j = t, \\ 0, & \text{else}, \end{cases} \tag{67} \]

\[
[\mathbf{J}_{D2}(t)]_{i,j} = \begin{cases} \frac{2 \mu_3}{\sigma_x^2} \mathcal{R} \left( \mathbf{\beta}_x^H \mathbf{\beta}_y \right), & i = j = t, \\ \frac{2 \mu_3}{\sigma_x^2} \mathcal{R} \left( \mathbf{\beta}_x^H \mathbf{\beta}_\varphi \right), & i = j = t, \end{cases} \tag{68} \]

\[
[\mathbf{J}_{D3}(t)]_{i,j} = \frac{2 (\mu_2^2 + \sigma_x^2)}{\sigma_x^2} \begin{bmatrix} \| \mathbf{v}_y \|^2 & \mathcal{R} \left( \mathbf{\beta}_y^H \mathbf{\beta}_\varphi \right) \\ \mathcal{R} \left( \mathbf{\beta}_y^H \mathbf{\beta}_\varphi \right) & \| \mathbf{\beta}_\varphi \|^2 \end{bmatrix}. \tag{69} \]

Substituting (66) to (55) yields

\[
\mathbf{J}_D = \sum_{t=1}^{T} \mathbf{J}_D(t) = \begin{bmatrix} \mathbf{J}_{D1} \\ \mathbf{J}_{D2} \\ \mathbf{J}_{D3} \end{bmatrix}. \tag{70} \]
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