Neutron stars and the transition to superconducting quark matter

M. Baldo\textsuperscript{a}, M. Buballa\textsuperscript{b,c,}\textsuperscript{*}, G.F. Burgio\textsuperscript{a}, F. Neumann\textsuperscript{b}, M. Oertel\textsuperscript{d}, and H.-J. Schulze\textsuperscript{a}

\textsuperscript{a}INFN, Sezione di Catania, 57 Corso Italia, 95129 Catania, Italy
\textsuperscript{b}Institut für Kernphysik, TU Darmstadt, Schlossgartenstr. 9, 64289 Darmstadt, Germany
\textsuperscript{c}Gesellschaft für Schwerionenforschung, Planckstr. 1, 64291 Darmstadt, Germany
\textsuperscript{d}IPN-Lyon, 43 Bd du 11 Novembre 1918, 69622 Villeurbanne Cédex, France

Abstract

We explore the relevance of color superconductivity inside a possible quark matter core for the bulk properties of neutron stars. For the quark phase we use a Nambu–Jona-Lasinio (NJL) type model, extended to include diquark condensates. For the hadronic phase, a microscopic many-body model is adopted, with and without strangeness content. In our calculations, a sharp boundary is assumed between the hadronic and the quark phases. For NJL model parameters fitted to vacuum properties we find that no star with a pure quark core does exist. Nevertheless the presence of color superconducting phases can lower the neutron star maximum mass substantially. In some cases, the transition to quark matter occurs only if color superconductivity is present. Once the quark phase is introduced, the value of the maximum mass stays in any case below the value of two solar masses.

Key words: dense matter, equation of state, hadron-quark phase transition, neutron stars, color superconductivity
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1 Introduction

According to QCD, the fundamental theory of strong interaction, nuclear matter is viewed as the confined phase of quark-gluon matter, where chiral symmetry is broken and quarks

\textsuperscript{*} Corresponding author
\textit{Email address: michael.buballa@physik.tu-darmstadt.de} (M. Buballa).
are bound inside nucleons. At large enough density, nuclear matter is expected to undergo a phase transition to the deconfined phase, where quarks and gluons are free to move in the medium and chiral symmetry is restored. Unfortunately, this transition is not well understood, since QCD lattice calculations cannot yet be performed at large density, i.e., at large chemical potential. Experimentally, in ultra-relativistic heavy ion collisions the transition to the deconfined phase is expected to occur at high temperature but essentially at zero baryon density. On the other hand, neutron stars are believed to contain very high baryon density in their interior, where the transition to the deconfined phase could occur. It has been argued by several authors that the bulk properties of neutron stars can be strongly affected by the presence of a core where a quark phase or a mixed hadron-quark phase is present. The fact that the quark phase - if present - is likely to be a color superconductor has recently attracted much attention (For reviews see [1,2] and references therein). In general, the condensation energy associated with the superconductivity is only a small fraction of the total energy, and the bulk properties of the equation of state (EOS) are hardly affected. This is because only a small fraction of fermions around the Fermi surface participates effectively to pairing, and the condensation energy is not proportional to the pairing gap $\Delta$, but to $\Delta^2/E_F$, where $E_F$ is the Fermi energy. These considerations equally apply to color superconductivity in quark matter. However, as recently argued [3], the structure of neutron stars appears to be sensitive to the properties of the possible quark phase and even a relatively small change in the quark equation of state could affect the predictions of neutron star properties.

In this paper we study the consequence of introducing explicitly color superconducting phases in the deconfined quark matter for the structure and bulk properties of neutron stars. To this purpose, we describe the hadronic phase by the EOS derived from a microscopic many-body theory [4]. For the quark matter we adopt a three-flavor Nambu-Jona Lasinio (NJL) type model, which has been recently extended to include diquark condensates [5,6,7]. In contrast to bag models which are often employed to describe quark phases, the NJL model dynamically generates a density-dependent effective bag constant and density dependent effective quark masses which can be considerably larger than the bare masses. It has been shown earlier that this has important consequences for the (non-) existence of absolutely stable strange quark matter [8] and the color-flavor (un-) locking phase transition in quark matter [5]. In the context of neutron star interiors the NJL model has been employed in Refs. [9] and [10]. It was found by both groups that quarks exist at most in a small mixed phase regime but not in a pure quark core. This could be traced back to the relatively large values for effective bag constant and the effective strange quark mass which results from the calculation. On the other hand these calculations did not include diquark condensates. As recently discussed within a bag model [3] the latter effectively lower the bag constant and could have sizable consequences for the hadron-quark phase transition and for the properties of compact stars. It is therefore interesting to study these effects for an NJL model equation of state.

Depending on the flavors which participate in a diquark condensate one can distinguish between several color superconducting quark phases, most important the two-flavor superconducting (2SC) phase [11,12] where only up and down quarks are paired and the
color-flavor locked (CFL) phase [13] which contains $ud$, $us$, and $ds$ pairs. In principle this can give rise to a large number of globally electric and color neutral mixed quark phases which are, however, unlikely to be stable if Coulomb and surface effects are included [7]. Therefore in this letter we discuss only homogeneous neutral quark phases which have been constructed first in Ref. [6]. We have checked, however, that the use of the mixed quark phase equation of state of Ref. [7] yields practically the same results.

Similarly, we assume a sharp (first order) transition from the hadronic to the quark phase. This is motivated by the results of Ref. [14] where it was found that a quark-hadron mixed phase is unlikely to be stable for reasonable values of the surface tension. We will briefly comment on the possible consequences of a mixed phase in Sec. 4.

## 2 Hadronic and quark EOS

We start with the description of the hadronic phase. The EOS is based on the Brueckner–Bethe–Goldstone (BBG) many-body theory, which is a linked cluster expansion of the energy per nucleon of nuclear matter (see Ref. [15], chapter 1 and references therein). It has been shown that the non-relativistic BBG expansion is well convergent [16,17], and the Brueckner-Hartree-Fock (BHF) level of approximation is accurate in the density range relevant for neutron stars. The basic ingredient in this many-body approach is the Brueckner reaction matrix $G$, which is the solution of the Bethe–Goldstone equation

\[ G[n; \omega] = v + \sum_{k_a k_b} \frac{|k_a k_b)}{\omega - e(k_a) - e(k_b)} Q G[n; \omega], \tag{1} \]

where $v$ is the bare nucleon-nucleon (NN) interaction, $n$ is the nucleon number density, and $\omega$ the starting energy. The single-particle energy $e(k)$ (assuming $\hbar=1$ here and throughout the paper),

\[ e(k) = \frac{k^2}{2m} + U(k), \tag{2} \]

and the Pauli operator $Q$ determine the propagation of intermediate baryon pairs. The Brueckner–Hartree–Fock (BHF) approximation for the single-particle potential $U(k)$ using the continuous choice prescription is

\[ U(k) = \text{Re} \sum_{k' \leq k_F} \langle kk' | G[n; e(k) + e(k')] | kk' \rangle_a, \tag{3} \]

where the subscript “$a$” indicates antisymmetrization of the matrix element. Due to the occurrence of $U(k)$ in Eq. (2), they constitute a coupled system that has to be solved in
a self-consistent manner for several Fermi momenta of the particles involved. In the BHF approximation the energy per nucleon is

\[
\frac{E}{A} = \frac{3}{5} \frac{k_F^2}{2m} + \frac{1}{2n} \Re \sum_{k,k' \leq k_F} \langle kk' \mid G[n; e(k) + e(k')] \mid kk'\rangle_a. \tag{4}
\]

In the calculations reported here we have used the Paris potential [18] as the two-nucleon interaction and the Urbana model as three-body force [19,20]. The corresponding nuclear matter EOS fulfills several requirements, namely (i) it reproduces the correct nuclear matter saturation point \(\rho_0\) [4], (ii) the incompressibility is compatible with the values extracted from phenomenology, (iii) the symmetry energy is compatible with nuclear phenomenology, (iv) the causality condition is always fulfilled.

Recently, we have included the hyperon degrees of freedom within the same approximation to calculate the nuclear EOS needed to describe the NS interior [21]. We have included the \(\Sigma^-\) and \(\Lambda\) hyperons. To this purpose, one needs also nucleon-hyperon and hyperon-hyperon interactions [21,22]. However, because of a lack of experimental data, the hyperon-hyperon interaction has been neglected in the first approximation in this work, whereas for the nucleon-hyperon interaction the Nijmegen soft-core model [23] has been adopted.

In neutron stars one has to consider matter in beta equilibrium, where electrons and eventually muons coexist with baryons, while neutrinos are considered to escape from the star. The EOS for the beta equilibrated matter can be obtained once the hadron matter is known, together with the chemical potentials of different species as a function of total baryon density. Since the procedure is standard, we do not give further details of the calculations.

For the quark phase we consider a leptonic contribution from electrons and muons and a quark contribution. The former is treated in free gas approximation. For the latter we employ the model defined by the Lagrangian

\[
\mathcal{L}_{\text{eff}} = \bar{\psi}(i\not\!\!\!\!\!D - \hat{m})\psi + \mathcal{L}_{qq} + \mathcal{L}_{qq}, \tag{5}
\]

where \(\psi\) denotes a quark field with three flavors and three colors. The mass matrix \(\hat{m}\) has the form \(\hat{m} = \text{diag}(m_u, m_d, m_s)\) in flavor space. We consider an NJL-type interaction with a quark-quark part

\[
\mathcal{L}_{qq} = H \sum_{A=2,5,7} \sum_{A'=2,5,7} (\bar{\psi} i\gamma_5 \tau_A \lambda_{A'} C \bar{\psi}^T)(\psi^T C \bar{\psi}^T \tau_A \lambda_{A'} \psi) \tag{6}
\]

and a quark-antiquark part

\[
\mathcal{L}_{qq} = G \sum_{a=0}^8 \left[ (\bar{\psi} \tau_a \psi)^2 + (\bar{\psi} i\gamma_5 \tau_a \psi)^2 \right].
\]
\[- K \left[ \det_f \left( \bar{\psi} (1 + \gamma_5) \psi \right) + \det_f \left( \bar{\psi} (1 - \gamma_5) \psi \right) \right]. \tag{7} \]

Here \( \tau_i \) and \( \lambda_j \) are \( SU(3) \)- (Gell-Mann-)matrices in flavor and color space, respectively. \( \tau_0 = \sqrt{\frac{2}{3}} \) \( 1_f \) is proportional to the unit matrix in flavor space \(^{*}\). In order to determine the energy density and the pressure of the system we calculate the mean-field (grand canonical) thermodynamic potential \( \Omega(T = 0, \{ \mu_i \}) \) in the presence of three quark-antiquark condensates \( \langle \bar{u}u \rangle, \langle \bar{d}d \rangle \), and \( \langle \bar{s}s \rangle \), and three possible diquark condensates corresponding to \( ud, us \), and \( ds \) pairing in the scalar color- antitriplet channel. The different chemical potentials \( \{ \mu_i \} \) are associated with the conserved charges of the system and are adjusted such that we deal with color and electrically neutral quark matter in beta equilibrium. More details about the model can be found in Refs. [5,6,7], where it has been employed to study color superconducting quark matter.

Since the model is not renormalizable we have to specify a regularization procedure. For simplicity we will use a three-dimensional momentum cutoff \( \Lambda \). The values of \( \Lambda = 602.3 \) MeV, the two coupling constants \( G = 1.835/\Lambda^2 \) and \( K = 12.36/\Lambda^5 \) as well as the current quark masses \( m_u = m_d = 5.5 \) MeV, \( m_s = 140.7 \) MeV are taken from Ref. [24], where they have been adjusted to reproduce masses and decay constants of the pseudoscalar meson nonet. For the quark-quark coupling constant we take \( H = G \), as motivated in Ref. [7]. This is likely to be an upper limit of the realistic values for this coupling constant.

We should mention that we neglect possible contributions of the “pseudo”-Goldstone bosons of broken chiral symmetry to \( \Omega \) in the CFL phase. Since in neutral CFL matter the electric charge chemical potential vanishes \([6,7]\) no charged bosons are excited. The contributions of a possible \( K_0 \)-condensate \([25]\) are expected to be small (see, e.g., the discussion in Ref. [3]) and will therefore be neglected.

\section{Hadron-quark phase transition}

In Fig. 1 the pressure of neutral hadron and quark matter in beta equilibrium is displayed as a function of the baryon chemical potential. For the hadronic case, we consider the microscopic EOS including only nucleons and leptons as well as also including hyperons (dashed and dotted lines, respectively). The presence of strange matter considerably softens \([21]\) the EOS, which results in a steeper increase of pressure as a function of baryon chemical potential.

The pressure of quark matter with and without color superconductivity (CS) is indicated by the bold lines. In both EOS strangeness is included. (For CS in neutral NJL quark matter without strangeness see \([27]\)). The EOS without CS (bold dashed line) is identical to the one employed in Refs. \([8,9]\). Here the strangeness content is zero below \( \mu_B = \)

\(^{*}\) In this form the interaction is understood to be used at mean-field level in Hartree approximation.
Fig. 1. Pressure of neutral hadron and quark matter in beta equilibrium as a function of baryon chemical potential: BHF calculation without hyperons (dashed) and with hyperons (dotted), relativistic mean field EOS with $K = 240$ MeV [26] (dashed-dotted), and NJL quark matter (bold lines) without (dashed) and with CS (solid). The open square indicates the transition point from the 2SC phase to the CFL phase.

1295 MeV and rises smoothly above this point. This is quite different for the case with color superconductivity (solid line). Strictly speaking, this line corresponds to two phases, separated by a first order phase transition at $\mu_0 \simeq 1240$ MeV: the 2SC phase for $\mu_B < \mu_0$ and the CFL phase for $\mu_B > \mu_0$. At $\mu_B = \mu_0$ (open square in the figure) the number density of strange quarks increases discontinuously from almost zero in the 2SC phase to 1/3 of the total quark number density in the CFL phase. Thereby the total baryon density jumps from less than four to more than five times nuclear matter density. This corresponds to a sudden increase of the slope of the curve, which is clearly visible in the figure.

Starting from the surface of a neutron star, the baryon density and the baryon chemical potential increase as one moves inside, until a possible transition to quark matter occurs. In the considered plot of pressure vs. baryon chemical potential, the transition is marked by the crossing of the hadron EOS with the quark matter EOS. If only nucleons are included in the hadronic sector, this crossing indeed occurs. However, the presence of CS substantially lowers the density at which the transition happens (see Table 1). Obviously, this is closely related to the kink at the 2SC-CFL transition point: In the 2SC phase the introduction of color superconductivity leads only to a moderate enhancement $\delta p$ of the pressure which can be attributed to some additional binding caused by the formation of Cooper pairs. However, in the CFL phase $\delta p$ grows faster and thereby reduces the critical chemical potential for the hadron-quark phase transition by about 165 MeV. As discussed above, this increased slope corresponds to a higher density which is mainly due to the sudden appearance of a large amount of strange quarks in the CFL phase.
The microscopic hadronic EOS which includes hyperons does not present any crossing with the quark EOS up to large density, even if color superconducting phases are included. In that case there would be no transition to a quark phase at densities relevant for neutron stars. However, it has already been found in Ref. [21] that this EOS gives a maximum neutron star mass of about 1.25 solar masses (see next section), which is below the observational limit of 1.44 solar masses [28]. Therefore, this EOS cannot describe the whole interior of neutron stars, and either this particular EOS or the NJL description of the quark phase must be ruled out. But even if we keep the considered quark EOS, the appearance of hyperons cannot be excluded, since the microscopic EOS depends on the adopted hyperon-hyperon interaction, at least at high density. In fact, while the hyperon-nucleon interaction is constrained, to some extent, by hyperon-nucleus phenomenology, very little is known about the hyperon-hyperon interaction potential. It is therefore possible that the hyperon-hyperon interaction becomes strongly repulsive at higher density, and the corresponding EOS is actually stiffer than the adopted one. To illustrate the relevance of this uncertainty, we have considered a phenomenological hadronic EOS based on relativistic mean field scheme of Ref. [26] with $K = 240$ MeV (hereafter called ‘G240’). Without hyperons this EOS turns out to be very similar to our microscopic EOS. Once hyperons are introduced, the microscopic (dotted) and mean field (dashed-dotted) EOS are quite different above the hyperon threshold, which indicates that the hyperon interactions are quite different in the two cases. With the mean field EOS, a phase transition is now possible, but only if CS is included in the quark EOS. Due to this uncertainty on the hyperon interactions, it is not possible to firmly establish if and in which place the quark phase can appear once hyperons are considered. As we will see, this uncertainty will not affect our main conclusions.

In Fig. 2 the energy density as a function of pressure is displayed for the various EOS discussed above. We have also indicated the points where the hadron-quark phase transitions take place, as obtained from Fig. 1. The open circles and the full squares mark the

| transition | BHF(N,l) → w/o CS | BHF(N,l) → CFL | G240 → CFL |
|------------|------------------|----------------|------------|
| $\mu_B$ (MeV) | 1478 | 1312 | 1397 |
| $\rho_B^{(h)}/\rho_0$ | 4.6 | 3.7 | 6.5 |
| $\rho_B^{(q)}/\rho_0$ | 7.9 | 6.4 | 7.7 |
| $\Delta$ (MeV) | — | 115 | 120 |
| $M_u$ (MeV) | 10 | 28 | 22 |
| $M_s$ (MeV) | 283 | 261 | 236 |
| $B_{eff}$ (MeV/fm$^3$) | 176 | 208 | 234 |

Table 1
Various quantities at the phase transition points from hadronic matter to NJL quark matter (see Figs. 1 and 2): chemical potential, baryon density in the hadronic phase ($\rho_B^{(h)}$) and in the quark phase ($\rho_B^{(q)}$) in units of normal nuclear matter density $\rho_0 = 0.17$ fm$^{-3}$, average diquark gap $\Delta = \sqrt{(\Delta_{ud}^2 + \Delta_{us}^2 + \Delta_{ds}^2)/3}$, effective quark masses, and effective bag constant.
phase transition from the microscopic EOS (with nucleons only) to quark matter with and without CS, respectively. The asterisks correspond to the transition from the mean field EOS (with hyperons) to the quark EOS with CS.

As discussed above, the quark EOS with CS (solid line) contains a first order phase transition from the 2SC phase to the CFL phase. At the transition point \( p \approx 80 \text{ MeV/fm}^3 \), this leads to a strong discontinuity in the energy density. As a consequence, the effect of including color superconducting phases in the NJL model is qualitatively different from the behavior in a bag model, like in Ref. [3]. In a bag model, the formation of Cooper pairs more or less acts like a negative contribution to the bag constant, reducing the energy density and enhancing the pressure for a given chemical potential. In our case the situation is more complicated. If we compare the solid line with the bold dashed line we see that color superconductivity enhances the energy density in a certain regime of pressure. The reason is again the fact that the CFL phase always contains a large amount of (relatively heavy!) strange quarks which strongly contribute to the energy density, whereas the NJL quark matter without CS contains no or very few strange quarks up to much larger values of \( p \).

For our later discussion it is useful to introduce effective bag model parameters to characterize the NJL EOS. To that end one inserts the effective quark masses and diquark gaps of the NJL model into the bag model expression for the pressure and defines an effective bag constant \( B_{\text{eff}} \) by equating the result with the pressure obtained in the NJL model. Note, however, that in contrast to a bag model \( B_{\text{eff}} \), the gaps, and effective quark masses are density dependent quantities. In Table 1 they are given for the chemical potentials corresponding to the phase transition points from the hadronic phase. Compared with most bag models, both, the strange quark mass and the bag constant, are relatively large.
As we will see below, this has important consequences for the structure of compact stars.

4 Neutron star structure

For a given EOS the mass of a static compact star as a function of its radius can be obtained by solving the Tolman-Oppenheimer-Volkoff equation. The resulting curves for the EOS constructed above are displayed in Fig. 3. The corresponding maximum masses and maximum central energy densities are listed in Table 2. We are particularly interested in the possible existence of hybrid stars with a pure quark core. According to the results of Sec. 3 there are three cases where such a hybrid star could be expected, namely for the microscopic EOS without hyperons combined with a quark EOS with or without CS (solid and bold dashed line, respectively) and for the relativistic mean field EOS combined with the quark EOS with CS (bold dashed-dotted line). As a consequence of the discontinuous energy density at the transition point (see Fig. 2) the phase transitions manifest themselves by cusps in the mass-radius relation. It turns out, however, that in all three cases this cusp is strong enough to render the star unstable. Hence, if our EOS are correct, no star with a pure quark core can exist.

This result seems to be rather insensitive to the choice of the hadronic EOS and must mainly be attributed to the quark EOS derived within the NJL model. Recently, Alford and Reddy have found stable hybrid stars with pure quark cores described within a bag model EOS [3]. The hadronic EOS employed in that analysis are comparable to our
Table 2

| EOS                  | $M_G^{max}/M_\odot$ | $\varepsilon_c^{max}$ (MeV/fm$^3$) |
|----------------------|----------------------|-------------------------------------|
| BHF(N,l)             | 2.07                 | 1409                                |
| BHF(N,l) + NJL without CS | 1.97                 | 884                                 |
| BHF(N,l) + NJL with CS | 1.77                 | 672                                 |
| BHF(N,H,l)           | 1.25                 | 1442                                |
| G240                 | 1.55                 | 1434                                |
| G240 + NJL with CS   | 1.55                 | 1279                                |

Maximum gravitational mass $M_G^{max}$ and corresponding central energy density $\varepsilon_c^{max}$ for the various EOS.

microscopic EOS without hyperons. For a bag constant $B = 137$ MeV/fm$^3$ and a strange quark mass $M_s = 200$ MeV the authors found that a star with a pure quark core is unstable without CS, but stable if a (CFL) diquark gap of 100 MeV is chosen. If we compare the above numbers with the effective quantities listed in Table 1, we see that the NJL model is characterized by relatively large values of the strange quark mass and the effective bag constant. This leads to considerably larger energy densities in the quark phase which are finally responsible for the instability.

Of course, the existence of a mixed phase, instead of a sharp transition, would smooth out the cusps in Fig. 3 and a quark component could be present in the core of the neutron star. However, the value of the maximum mass is not expected to be modified by a substantial amount.

5 Conclusions

We have combined various hadronic EOS with an NJL model EOS with and without color superconductivity to construct the hadron-quark phase transition and to investigate the structure of compact stars. Our results do not allow for a pure quark phase in the interior of a neutron star. In some cases we find no phase transition at all while in other cases the star becomes unstable as soon as the phase transition occurs. Both phenomena are mainly due to the relatively large values of the effective strange quark mass and the effective bag constant in the NJL model. Considering non-superconducting quark matter we thus confirm the results of Ref. [9]. Including color superconductivity does not change these findings.

Of course, the properties of the NJL EOS depend on model parameters. In this letter we have adopted the parameters of Ref. [24] which have been adjusted to vacuum properties. Although this is common practice in this field, it is very well possible that these parameters are not appropriate to describe quark matter at densities relevant for compact stars. Thus our arguments could also be turned around: If there were strong hints for the existence
of pure quark cores in compact stars, this would indicate a considerable modification of the effective NJL-type quark interactions in dense matter †.

If there are no stable quark cores in compact stars, color superconductivity is unlikely to exist in any natural surrounding. Nevertheless, the possibility of CS could have an effect on the maximum mass of neutron stars, as is evident from the difference between the solid line and the bold dashed line in Fig. 3. While excluding CS the maximum mass can be close to a value of two solar masses, including CS the maximum mass is reduced to 1.77 solar masses in our calculations. (If we had chosen a smaller value for the quark-quark coupling constant \( H \) in Eq. (6) the maximum mass would of course come out somewhere in between these two values.) Introducing hyperons softens the EOS further, and the corresponding maximum mass is further lowered. If the softening is large, as, e.g., in the case of the EOS of Ref. [21], the neutron star becomes unstable already before the transition density to a possible quark matter core is reached. In this case the maximum mass falls below the observational limit.

In conclusion, the inclusion of CS in the quark phase keeps the neutron star maximum mass well below two solar masses, independently of the details of the hadronic EOS. The observation of a neutron star with a mass well above two solar masses would seriously question our present view on the EOS of quark matter and on the structure of the quark matter phase.

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† After submission of our manuscript, two groups have reported the construction of a stable hybrid star with a pure quark matter core described within two-flavor NJL-type models including color superconductivity, but without strange quarks. This is in strong contradiction to our results where even a phase transition to quark matter seems to be impossible without strange quarks. In Ref. [29] this was achieved within a standard NJL model with relatively light up and down quarks, corresponding to a relatively small effective bag constant. The authors of Ref. [30] find stable quark cores if the integrals are regularized by Gaussian form factors, but not for a sharp cut-off. In both papers the hadronic phase is described within relativistic mean field models. These findings call for a systematic survey of the parameter dependence of the results and of the influence of possible extensions and modifications of the model.

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