Numerical simulations of dynamical gluinos in $SU(3)$ Yang-Mills theory: first results.

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In a numerical Monte Carlo simulation of $SU(3)$ Yang-Mills theory with dynamical gluinos we have investigated the behaviour of the expectation value of the scalar and pseudoscalar gluino condensates in order to determine the phase structure. Preliminary results are presented as a function of the hopping parameter.

1. INTRODUCTION

In the last years there has been a great progress in the understanding of the non-perturbative properties of supersymmetric gauge theories. Because of their highly symmetric nature, supersymmetric quantum field theories are best suited for analytical studies, which sometimes lead to exact solutions \[1\]. The basic assumption about the non-perturbative dynamics of supersymmetric Yang-Mills (SYM) theory is that there is confinement and spontaneous chiral symmetry breaking \[3\].

1.1. Supersymmetric Yang-Mills theory

Since local gauge symmetries play a very important role in nature, there is a particular interest in supersymmetric gauge theories. The simplest examples are SYM theories, which are supersymmetric extensions of pure gauge theories. We shall pay our attention to the SYM action with $N = 1$, where $N$ is the number of pairs of supersymmetry generators $Q_{i\dot{\alpha}}, \overline{Q}^{i\dot{\alpha}}$ ($i = 1, 2, \ldots, N$). This theory is a Yang-Mills theory with a Majorana fermion in the adjoint representation.

The action for such a $N = 1$ SYM theory with a $SU(N_c)$ gauge group is given by

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) + \frac{1}{2} \overline{\Psi}(x) \gamma_\mu D_\mu \Psi(x), \quad (1)$$

where $\Psi(x)$ is the spinor field, and $F^a_{\mu\nu}(x)$ the field strength tensor, $a \in \{1, \ldots, N_c^2 - 1\}$.

Introducing a non-zero gluino mass $m_\tilde{g}$ breaks supersymmetry “softly”. Such a mass term is

$$m_\tilde{g}(\lambda^a \lambda_\alpha + \overline{\lambda}^{\dot{\alpha}} \lambda^{\dot{\alpha}}) = m_\tilde{g} \overline{\Psi} \Psi. \quad (2)$$

Here in the first form the Majorana-Weyl components $\lambda, \overline{\lambda}$ are used, in the second form the Dirac-Majorana field $\Psi$. The Yang-Mills theory of a Majorana fermion in the adjoint representation is similar to QCD: besides the special Majorana-feature the only difference is that the fermion is in the adjoint representation and not in the fundamental one. As there is only a single Majorana adjoint “flavour”, the global chiral symmetry of $N = 1$ SYM is $U(1)_{\lambda}$.

The $U(1)_{\lambda}$-symmetry is anomalous: for the corresponding axial current $J_\mu^5 = \overline{\Psi} \gamma_\mu \gamma_5 \Psi$, with a gauge group $SU(N_c)$, we have

$$\partial^\mu J_\mu^5 = \frac{N_c g^2}{32 \pi^2} \epsilon^{\mu\nu\rho\tau} F^a_{\mu\nu} F^a_{\rho\tau}. \quad (3)$$

However the anomaly leaves a $Z_{2N_c}$ unbroken.
this can be seen by noting that the transformations
\[ \Psi \mapsto e^{-i\varphi} \Psi, \quad \overline{\Psi} \mapsto \overline{\Psi} e^{-i\varphi} \gamma_5 \] (4)
are equivalent to
\[ m_{\tilde{g}} \mapsto m_{\tilde{g}} e^{-2i\varphi} \gamma_5 \] (5)
and
\[ \Theta_{\text{SYM}} \mapsto \Theta_{\text{SYM}} - 2N_c \phi , \] (6)
where \( \Theta_{\text{SYM}} \) is the \( \theta \)-parameter of the gauge dynamics. Since \( \Theta_{\text{SYM}} \) is periodic with period \( 2\pi \), for \( m_{\tilde{g}} = 0 \) the \( U(1)_{\lambda} \) symmetry is unbroken if
\[ \varphi = \varphi_k = \frac{k\pi}{N_c}, \quad (k = 0, 1, \ldots, 2N_c - 1) . \] (7)

The discrete global chiral symmetry \( Z_{2N_c} \) is expected to be spontaneously broken to \( Z_2 \) by the non-zero gluino condensate \( \langle \overline{\Psi}(x) \Psi(x) \rangle \neq 0 \). The consequence of this spontaneous chiral symmetry breaking is the existence of a first order phase transition at zero gluino mass \( m_{\tilde{g}} = 0 \). In the case of \( N_c = 2 \), there exist two degenerate ground states with opposite signs of the gluino condensate. An interesting point is the dependence of the phase structure on the gauge group: instanton calculations \( \text{[3]} \) at \( \Theta_{\text{SYM}} = 0 \) give \( N_c \) degenerate vacua \( (k = 0, \ldots, N_c - 1) \) with
\[ \langle \overline{\lambda} \lambda \rangle = e^{\frac{\alpha_{\text{SYM}}^3}{2N_c}} e^{\frac{2\pi i k}{N_c}} . \] (8)

The coexistence of \( N_c \) vacua implies a first order phase transition at \( m_{\tilde{g}} = 0 \). Recently Kovner and Shifman have suggested the existence of an additional massless phase with no chiral symmetry breaking \( \text{[3]} \). In the case of \( SU(3) \), there are at least three degenerate vacua and for \( m_{\tilde{g}} < 0 \) we expect that \( \Theta_{\text{SYM}} = \pi \).

2. LATTICE FORMULATION

No lattice gauge theory exists with an exact supersymmetry. This is because lacking lattice generators of the Poincaré group, it is impossible to fulfill the (continuum) algebra of SUSY transformations. Another problem is represented by the balancing between bosonic and fermionic modes required by SUSY: the naive lattice fermion formulation produces too many fermions.

Curci and Veneziano \( \text{[3]} \) have proposed a simple solution: instead of trying to have an exact version of SUSY on the lattice, the requirement is that, like chiral symmetry, it should only be recovered in the continuum limit, tuning the bare parameters (gauge coupling \( g \), gluino mass \( m_{\tilde{g}} \)) to the supersymmetric point.

2.1. Actions

The Curci-Veneziano action of \( N = 1 \) SYM is based on Wilson fermions. The effective action obtained after integrating the gluino field is given by
\[ S_{CV} = \beta \sum_{pl} (1 - \frac{1}{2} \text{Tr} U_{pl}) - \frac{1}{2} \log \det Q[U] \] (9)
The fermion matrix for the gluino \( Q \) is
\[ Q_{yv,xu} = \delta_{yx} \delta_{vu} - K \sum_{\mu = \pm} \delta_{y,x+\mu} (1 + \gamma_{\mu}) V_{vu,x\mu} \] (10)
with the gauge link in the adjoint representation \( V_{vu,x\mu}[U] = \text{Tr}(U_{x \mu}^\dagger \tau_v U_{x \mu} \tau_u) \).

2.2. Monte Carlo simulation

The renormalized gluino mass is obtained from the hopping parameter \( K \) as
\[ m_{R\tilde{g}} = \frac{Z_m(a\mu)}{Z_{a\mu}} \left[ \frac{1}{K} - \frac{1}{K_0} \right] = Z_m(a\mu) m_{\tilde{g}} \] (11)
Here \( K_0 = K_0(\beta) \) gives the \( \beta \)-dependent position of the phase transition and \( \mu \) is the renormalization scale. The renormalized gluino condensate is obtained by additive and multiplicative renormalizations:
\[ \langle \overline{\Psi}(x) \Psi(x) \rangle_{R(\mu)} = Z(a\mu) \left[ \langle \overline{\Psi}(x) \Psi(x) \rangle - b_0(a\mu) \right] . \]

A first order phase transition should show up as a jump in the expectation value of the gluino condensate at \( K = K_0 \). By tuning the hopping parameter \( K \) to \( K_0 \) for a fixed gauge coupling \( \beta \) one expects to see a two peak structure in the distribution of the gluino condensate. By increasing the volume the tunneling between the two ground states becomes less and less probable and at some point practically impossible. It is possible to see
this phase diagram in our simulations by measuring the chiral and pseudo chiral gluino condensate: the order parameter of the supersymmetry phase transition at zero gluino mass is the value of the gluino condensate

$$\rho \equiv \frac{1}{\Omega} \sum_x (\overline{\Psi}(x)\Psi(x)) .$$

(12)

Additionally, for $K \geq K_0 (m_{\tilde{g}} \leq 0)$ a spontaneous CP-violation, indicated by a nonvanishing pseudo condensate $<\overline{\Psi}(x)\gamma_5\Psi(x)> \neq 0$, is expected.

We determine the value of $\rho$ on a gauge configuration by stochastic estimators

$$\frac{1}{N_\eta} \sum_{i=1}^{N_\eta} \sum_{x,y} (\eta_{y,i}, Q_{y,x}^{-1} \eta_{x,i}) .$$

(13)

Outside the phase transition region the observed distribution of $\rho$ can be fitted well by a single Gaussian but in the transition region a good fit can only be obtained with two Gaussians. For $SU(2)$ results are shown in fig. 1.

The hopping parameter $K_0$, corresponding to zero gluino mass, is indicated by a first order phase transition which is due to the spontaneous discrete chiral symmetry breaking $Z_6 \rightarrow Z_2$.

Figure 1. The preliminary results for the gluino condensate.

We have investigated the dependence of the distribution of the gluino condensate and the pseudo condensate as a function of the hopping parameter, starting from a lattice volume $L^3 \cdot T = 4^3 \cdot 8$. This lattice is, however, still not very large in physical units. Therefore the expected two-peak structure is not yet very well developed, nevertheless we have high statistics. For $K = 0.195$ fig. 2 shows the distribution of the gluino condensate. The distribution indicates that we are near the phase transition. Outside this region, we can fit the distribution with a single Gaussian. Presently, we are calculating on a bigger lattice volume $(L^3 \cdot T = 6^3 \cdot 12)$ in order to separate the two-peak structure. On the other hand, our present results on the smaller lattice do not show any signal for a pseudo condensate.

Figure 2. The probability distribution of the gluino condensate for $K = 0.195$ at $\beta = 5.6$ on a $4^3 \cdot 8$ lattice.

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