Particle Entanglement in Rotating Gases

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In this paper, we investigate the particle entanglement in 2D weakly-interacting rotating Bose and Fermi gases. We find that both particle localization and vortex localization can be indicated by particle entanglement. We also use particle entanglement to show the occurrence of edge reconstruction of rotating fermions. The different properties of condensate phase and vortex liquid phase of bosons can be reflected by particle entanglement and in vortex liquid phase we construct the same trial wave function with that in [Phys. Rev. Lett. 87, 120405 (2001)] from the viewpoint of entanglement to relate the ground state with quantum Hall state. Finally, the relation between particle entanglement and interaction strength is studied.

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I. INTRODUCTION

Quantum entanglement, which is considered to be the most non-classical phenomenon in the quantum world, has being attracting much attention in last decades. On the one hand, it has been identified as a key resource in many aspects of quantum information theory, such as quantum teleportation, quantum key distribution and quantum computation [1]. On the other hand, the concept of entanglement is proven to be useful in condensed matter systems [2], such as in spin chain, Bose-Einstein condensate and electronic quantum Hall effect.

As a typical condensed matter system, the two-dimensional (2D) rotating atom gas is a very interesting field because it opens a door to many important quantum phenomena [3,4]. Many excellent experimental and theoretical papers focus on the formation and melt of vortex lattice [5], many-body energy spectrum [6,7], its analogy to quantum Hall effect of electrons in a magnetic field [8–13], particle localization and vortex localization [14–17], the comparison between the results of exact quantum numerical solution and mean-field theory [18] and so on. However, the entanglement in this system has not been investigated as extensively as in other condensed matter systems, for example in spin chain systems.

In one of our previous work, we use some quantum information concepts, such as fidelity susceptibility and particle entanglement to study the properties of 2D weakly-interacting rotating Bose-Einstein condensate trapped in a harmonic potential [19]. We find that the single-particle entanglement, defined as the von Neumann entropy of the single-particle reduced density operator, can indicate some angular momentum of real ground states and reflect the properties of condensed phase (single-vortex state) and vortex liquid phase. This indicates that in 2D rotating atom (both boson and fermion) gas, some other important properties of this rotating system may be shown by entanglement.

In this paper, we mainly investigate the particle entanglement of 2D weakly-interacting rotating atom gas trapped in a harmonic potential in disk geometry. In order to compare the results of bosons and fermions, we choose repulsive Coulomb interaction. For a fixed particle number \( N \), we calculate the single-particle entanglement \( S_1 \) of the ground states in a series of subspaces of fixed angular momentum \( L \). We find that both particle localization and vortex localization can be reflected by the oscillation of \( S_1 \) with the increase of \( L \). This oscillation is universal for bosons and fermions. Then we study the single-particle entanglement of the ground states in subspaces whose angular momentum are special functions of \( N \). For the subspace \( L = N \), through the comparison between the results of bosons and fermions, we find that the edge reconstruction of rotating fermions can be indicated by the single-particle entanglement. For the subspace \( L = (N - \bar{N})(N + N - k)/k \) with \( k \geq 1 \) an integer and \( \bar{N} \) the smallest non-negative integer making \( N - \bar{N} \) to be divisible by \( k \), we relate its subspace ground state with quantum Hall state from the viewpoint of entanglement and construct the same trial wave function as that in Ref.[8]. Finally, we study rotating bosons trapped in a quadratic plus quartic trap interacting with contact potential, both attractive and repulsive. We find both single-particle and two-particle entanglement of the ground state in subspace \( L = N \) and \( L = N(N - 1) \) increase with the interaction strength for fixed \( N \). However, when the interaction is attractive, the single-particle entanglement of the ground state in subspace \( L = N \) does not decay with \( N \), verifying that attractive bosons do not condensate [20].

II. MODEL

We focus our attention on the 2D rotating spinless Bose and polarized Fermi atom gases. In rotating reference,
the Hamiltonian of $N$ atoms is as follows:

$$
\mathcal{H} = \sum_{i=1}^{N} \left\{ -\frac{\hbar^2}{2m} \nabla_i^2 + V(r_i) - L_z i \Omega \right\} + \sum_{i<j=1}^{N} U(r_i, r_j),
$$

where $V(r)$ is the trap potential, $-L_z \Omega$ is the rotating energy with rotation frequency $\Omega$ and $U$ is the interaction energy. $U > (\langle 0 \rangle)$ represents repulsive (attractive) interaction. Typically, $V(r) = \frac{1}{2} m \omega^2 r^2$ is a harmonic potential. $U$ can be either a short-range contact interaction $\delta(r_i - r_j)$ or a long-range Coulomb interaction $\frac{1}{|r_i - r_j|}$.

In the weakly-interacting limit, we can use single-particle lowest Landau level (LLL) wave function $\varphi_i(z) = \frac{1}{\sqrt{4\pi}} l^{-1/2} e^{-|z|^2/2}$, where $z = (x + iy) \sqrt{m \omega / \hbar}$ is the dimensionless position of a particle, to make second-quantization of Eq. (1). The obtained Hamiltonian in the subspace of fixed angular momentum $L_z = L \hbar$ takes the form:

$$
\mathcal{H}_L = (L + N) \hbar \omega - L \hbar \Omega + U_0 \sum_{i,j,k,l=0}^{L} U_{i,j,k}(a_i^\dagger a_j^\dagger a_k a_l),
$$

where $|U_0|$ represents the strength of interaction and

$$
U_{i,j,k,l} = \int dz_1 dz_2 \varphi_i^*(z_1) \varphi_j^*(z_2) U(z_1, z_2) \varphi_k(z_2) \varphi_l(z_1).
$$

For contact interaction, the expression is simple:

$$
U_{i,j,k,l} = \frac{1}{2\pi^2} \frac{\langle \delta_{j,k} \rangle_{\hat{\rho}_{i}}}{\langle \hat{\Phi}_{L}^{i} \rangle_{\hat{\rho}_{i}}} \delta_{i+j,k+l}. \quad \text{But for Coulomb interaction, the expression is a little complicated [21]:}
$$

$$
U_{i,j,k,l} = \delta_{i+j,k+l} \sum_{\ell=0}^{L} \frac{\Gamma(i + j + 3/2) \Gamma(i + t + 1/2)}{\Gamma(i + s + t + 3/2) \Gamma(i + s + t + 1/2)} \langle A_{\ell}^{i,j} B_{\ell}^{i,j} - A_{\ell}^{i,j} B_{\ell-1}^{i,j} - A_{\ell-1}^{i,j} B_{\ell}^{i,j} - A_{\ell-1}^{i,j} B_{\ell-1}^{i,j} \rangle,
$$

where

$$
A_{\ell}^{i,j} = \sum_{i=0}^{r} \frac{r!}{\ell! (r-i)! i!} \Gamma(i + 1/2) \Gamma(i + t + 1/2),
$$

$$
B_{\ell}^{i,j} = \sum_{i=0}^{r} \frac{r!}{\ell! (r-i)! i!} \Gamma(i + 1/2) \Gamma(i + t + 1/2) \times (2i + t + 1/2).
$$

$\Gamma(x) = \int_{0}^{\infty} t^{x-1} \exp(-t) dt$ is the usual Gamma function.

After diagonalizing Eq. (2) numerically by Lanczos algorithm, we obtain its subspace ground state $|\Phi_L\rangle$ in angular momentum Fock representation. Here we consider entanglement between particles [22] (In Ref. [23], the entanglement between angular momentum orbits is investigated to obtain the topological entropy in rotating Bose-Einstein condensate). We define the single-particle reduced density operator of $|\Phi_L\rangle$ as $(\rho_{i})_{ij} = \frac{\langle a_i^\dagger a_j \rangle_{L}}{N} \delta_{i,j} \langle a_i^\dagger a_i \rangle_{L}$ with $(\langle \rangle)_{L} = \langle \Phi_L | \cdot | \Phi_L \rangle$. The single-particle entanglement of $|\Phi_L\rangle$ is just the von-Neumann entropy of $\rho_1$:

$$
S_1 = -\text{Tr}(\rho_1 \ln \rho_1) = \ln N - \frac{1}{N} \sum_{i=0}^{L} \langle a_i^\dagger a_i \rangle_{L} \ln \langle a_i^\dagger a_i \rangle_{L}.
$$

For bosons, it’s possible that $\langle a_i^\dagger a_j \rangle_{L} = N$ for one $i$ and $\langle a_i^\dagger a_j \rangle_{L} = 0$ for $j \neq i$. Therefore $S_1 \geq 0$. For fermions, because $\langle a_i^\dagger a_j \rangle_{L} \leq 1$, we can find that $S_1 \geq \ln N$. Moreover, considering the dimension of single-particle Hilbert space is $L$, we have $S_1 \leq \ln L$ for both bosons and fermions.

Similarly, we can define the two-particle reduced density operator of $|\Phi_L\rangle$ as $(\rho_{ij})_{ijkl} = \frac{1}{N(N-1)} \langle a_i^\dagger a_j a_k a_l \rangle_{L} \delta_{i+j,k+l}$. And its two-particle entanglement as $S_2 = -\text{Tr}(\rho_{2} \ln \rho_{2})$.

III. SINGLE-PARTICLE ENTANGLEMENT

In this section we consider single-particle entanglement of both bosons and fermions interacting with each other by repulsive Coulomb potential.

First we fix the particle number $N$ to calculate the single-particle entanglement in a series of subspace ground states $|\Phi_L\rangle$. It’s known that at extreme angular momentum $L$, a few particles ($N$ is small) in a harmonic trap localize to Wigner molecules, independent of their bosonic or fermionic statistics. While at moderate angular momentum, if the particle number $N$ is much larger than the number of vortices, the localization happens in vortices. The localization of particles (vortices) can be studied by particle (hole) pair correlation function and can be reflected by the regular oscillation of quantum many-body energy spectrum with $L$. As we show below, the single-particle entanglement of $|\Phi_L\rangle$ also oscillates with $L$ and can reflect particle and vortex localization very well. In Fig.1 we fix particle number $N$ to calculate single-particle entanglement $S_1$ of the ground state of bosons in every angular momentum subspace $L \geq 2$. With the increase of $L$, $S_1$ not only has the tendency to become larger but also shows oscillation. To see the oscillation clearer, we also plot $\ln L - S_1$, the difference between $S_1$ and its upper bound, in Fig.1 For $N = 6$, a series of local maxima of $\ln L - S_1$ appear: $L=6,10,12,15,18,20,25,30,36,40,42,45,48,50,55,60,65,70,75$ etc. The angular momentum of the real ground state is $L_g=6,10,12,15,20,24,29,30,36,40,42,45,48,50,55,60,65,70,75$ etc. Therefore similar to the case of bosons with contact interaction, the local maxima of $\ln L - S_1$ of the subspace ground states can indicate some angular momentum of real ground state [19]. One can note $L$ can always be written as $5k$ or $6k$ with $k$ a positive integer. This reflects the spatial symmetry of the particles in the subspace ground state, namely particle localization. The series $L = 5k$ is associated with a (1,5) pentagon ring
structure with one particle at the center, while the series $\mathcal{L} = 6k$ is associated with a (0,6) hexagon ring structure. When $L$ is large enough ($L \geq 50$), the series $\mathcal{L} = 5k$ dominate. When $L$ is relatively small, the series $\mathcal{L} = 5k$ and $\mathcal{L} = 6k$ compete with each other and the spatial symmetry of particles can be observed by full $N$-point correlation function of the subspace ground state \cite{17}. For relative large $N$ such as $N = 12$ and $N = 20$, the oscillations of $S_1$ and $\ln L - S_1$ are different from those for $N = 6$. For $N = 12$, $S_1$ and $\ln L - S_1$ oscillate with a period of $P_L = 2$ between $L = 20$ and $L = 24$ corresponding to the two-vortex state and they oscillate with a period of $P_L = 3$ between $L = 24$ and $L = 36$ corresponding to the three-vortex state. For $N = 20$, $S_1$ and $\ln L - S_1$ oscillate with a period of $P_L = 2$ between $L = 32$ and $L = 42$ corresponding to the two-vortex state and they oscillate with a period of $P_L = 3$ between $L = 42$ and $L = 54$ corresponding to the three-vortex state. This phenomenon reflects vortex localization.

Due to Pauli exclusion principle, the smallest angular momentum of $N$ fermions is $M = N(N - 1)/2$, whose subspace ground state single-particle entanglement is $\ln N$. Therefore when considering fermions, sometimes it is convenient to use $\Delta L = L - M$ and $\Delta S_1 = S_1 - \ln N$. In Fig\[2\] we fix particle number $N$ to calculate single-particle entanglement $\Delta S_1$ of the ground state of fermions in every angular momentum subspace $\Delta L \geq 0$. The oscillation of $\Delta S_1$ is very clear so we do not need to plot the $\ln L - S_1$ as what we did for bosons. On the one hand, there is similarity between the oscillations of subspace ground state single-particle entanglement of bosons and fermions. For $N = 6$, the positions $\Delta L$ of local minima of fermionic $\Delta S_1$ are nearly the same with those $L$ for bosons, reflecting the particle localization in the subspace ground state. For $N = 12$ and $N = 20$, $\Delta S_1$ oscillates with a period of $P_L = 2, 3$ and even 4 successively, reflecting the vortex localization. On the other hand, there also exists difference between boson and fermion case. For bosons, the average angular momentum per particle $\ell_{(2)}$ at which $S_1$ begins to oscillate with a period of $P_L = 2(3)$ is nearly a constant. When $N = 12$, we have $\ell_2 = 20/12 \approx 1.67$ and $\ell_3 = 24/12 = 2.0$. When $N = 20$, we have $\ell_2 = 32/20 = 1.6$ and $\ell_3 = 42/20 = 2.1$. This is consistent with the conclusion of mean-field theory that two vortices form at $\ell_2 \approx 1.7$ and three vortices form at $\ell_3 \approx 2.1$ \cite{3}. But for fermions, $\ell_{(2)}$ is not a constant. When $N = 12$, we have $\ell_2 = 14/12 \approx 1.17$ and $\ell_3 = 24/12 = 2.0$. When $N = 20$, we have $\ell_2 = 20/20 = 1.0$ and $\ell_3 = 33/20 = 1.65$. Moreover, from Fig\[4\] we can see that the change of the distribution of eigenvalues of the single-particle reduced density operator of bosons with the increase of $L$ is different from that of fermions. For bosons, the first two (three) eigenvalues of the single-particle reduced density operator in two- (three-) vortex state of bosons are always relatively small with the increase of $L$. But for fermions, the smallest two (three) values in the trough of eigenvalues of the single-particle reduced density operator in two- (three-) vortex state of fermions move towards the center with the increase of $L$.

Next we study the single-particle entanglement of bosons of the ground state in some special subspaces, whose angular momentum are functions of particle number $N$. The first subspace is $L = N$. In Fig\[4\] we plot $S_1$ of the ground state in the subspace $L = N$ for bosons. One can see that $S_1$ decays with $N$ monotonically, just like in the case of contact interaction \cite{10}, demonstrating a property of condensate phase. The second subspace is $L = (N - \bar{N})(N + \bar{N} - k)/k$ with $k \geq 1$ an integer, where $\bar{N}$ is the smallest non-negative integer making $N - \bar{N}$ to

\[\begin{align*}
\text{FIG. 1: (Color Online) The subspace ground state single-particle entanglement $S_1$ and the difference between $S_1$ and its upper bound $\ln L$ for $N=6,12,20$ bosons interacting through repulsive Coulomb potential. The oscillations of $S_1$ and $\ln L - S_1$ can reflect either particle localization (for $N = 6$) or vortex localization (for $N = 12$ and $N = 20$).}
\end{align*}\]
be divisible by $k$. For example, when $k = 1$, $\bar{N} = 0$ and when $k = 2$, $\bar{N} = 0$ for even $N$ and $\bar{N} = 1$ for odd $N$. In Fig. 2 we plot $S_1$ of the subspace ground states corresponding to $k = 1, 2, 3, 4$. We can see for all of them $S_1 \approx \ln(2N/k - 1)$, demonstrating a strongly-correlated property of vortex liquid phase. This phenomenon implies from an entanglement view that the ground states in these subspaces $L = (N - \bar{N})(N + \bar{N} - k)/k$ may have a close relation with quantum Hall states. Recalling that for a quantum Hall state in spherical geometry, the single-particle entanglement is exactly $S_1 = \ln(N/\nu - \sigma + 1)$, where $\nu$ is the filling factor and $\sigma$ is called shift, we can extract some information of those subspace ground states from their single-particle entanglement, although we are dealing with disk geometry. Comparing with our results, we can find that the ground state of subspace $L = (N - \bar{N})(N + \bar{N} - k)/k$ is a quantum Hall state with filling factor $\nu = k/2 = \lim_{N \to \infty} \frac{N(N-1)}{2L}$ and when $N$ is divisible by $k$ the shift of its counterpart in spherical geometry is $\sigma = 2$. We can express the single-particle entanglement in another way $S_1 \approx \ln[2(N/k - 1) + 1]$. It’s known that for a Laughlin state of $N_0$ particles in spherical geometry with filling factor $\nu = 1/m$, $S^L_1 = \ln[m(N_0 - 1) + 1]$. Therefore in the viewpoint of entanglement, when $N/k$ is an integer, our subspace ground state behaves approximately like a Laughlin state of $N/k$ particles with $m = 2$, namely in our subspace ground state one particle is only entangled with $N/k - 1$ particles in the form of Laughlin state. Enlightened by this fact, we can express the subspace ground state $\Psi^k$ simply as a product of $k$ Laughlin states, each of which consists of $N/k$ particles and has a filling factor $1/2$. Then considering the symmetry of

FIG. 2: The subspace ground state single-particle entanglement $\Delta S_1 = S_1 - \ln N$ for $N=6,12,20$ fermions interacting through repulsive Coulomb potential. The oscillation of $\Delta S_1$ can reflect either particle localization (for $N = 6$) or vortex localization (for $N = 12$ and $N = 20$).

FIG. 3: (Color Online) The eigenvalues of the single-particle reduced density operator of bosons (left column) and fermions (right column). For bosons, the first two (three) eigenvalues of the single-particle reduced density operator in two- (three-) vortex state of bosons are always relatively small with the increase of $L$. But for fermions, the smallest two (three) values in the trough of eigenvalues of the single-particle reduced density operator in two- (three-) vortex state of fermions move towards the center with the increase of $L$. For example, for 20 fermions with $\Delta L = 26$ (39), the 7th and 8th (7th, 8th and 9th) eigenvalues are in the bottom of the trough.
in spherical geometry has a shift \( \sigma \) particles. One can verify that the total angular momentum is \( \nu \) for bosons and \( \nu \) for fermions. When \( N/k \) is not an integer, namely \( N \neq k \), we can slightly change the form of \( \Psi^k \):

\[
\Psi^k = \mathcal{S} \left[ \prod_{i<j \in A_1} (z_i - z_j)^2 \right. \\
\left. \prod_{l<m \in A_N} (z_l - z_m)^2 \right]
\]

where the set \( A_i \) has \( N/k \) particles and the symbol \( \mathcal{S} \) indicates symmetrization over all partitions of \( N \) particles into sets \( A_i \). Now we obtain the same trial quantum Hall state as in Ref.\[8\] but we achieve this in an entanglement manner. It has been checked that \( \Psi^k \) has large overlap with the ground state obtained by exact diagonalization \[8\]. For bosons, we obtain: (the exponential factor is omitted and we suppose \( N \) is divisible by \( k \))

\[
\Psi^k = \mathcal{S} \left[ \prod_{i<j \in A_1} (z_i - z_j)^2 \right. \\
\left. \prod_{l<m \in A_N} (z_l - z_m)^2 \right]
\]

where the set \( A_i \) has \( N/k \) particles and the set \( B_i \) has \( (N - N/k) \) particles. One can verify that the total angular momentum of this state is just \( L = \tilde{N}(\frac{N-N/k}{k} + 1) + (k - \tilde{N}) \frac{N-N/k}{k} - 1 = (N - \tilde{N})(N + \tilde{N} - k)/k \).

At last we study the single-particle entanglement of fermions of the ground state in some special subspaces as a comparison with bosons. The first subspace is \( \Delta L = N \). In Fig.4 we plot \( \Delta S_1 \) of the ground state in the subspace \( \Delta L = N \) for fermions. When \( N \) is small, \( \Delta S_1 \) decays with \( N \) monotonically similar to bosons. But when \( N > 17 \), \( \Delta S_1 \) suddenly jumps to a higher position and then begins to decay again accompanied by oscillation. This is because the distribution of the eigenvalues of the single-particle reduce density operator changes qualitatively from \( N = 17 \) to \( N = 18 \) (Fig.5). When \( N \leq 17 \), the subspace ground state is a central single-vortex state, while when \( N > 17 \), the subspace ground state is a candidate of an off-center double-vortex state (see \( N = 18, 20, 22, 40 \) in Fig.5). Therefore entanglement can indicate the edge reconstruction of rotating fermions \[24\]. A similar phenomenon of electrons in quantum dot that reconstruction of the maximum density droplet begins from the edge rather than the dot center if the electron number exceeds \( N \approx 15 \) is reported in Ref.\[24\]. The second subspace is \( \Delta L = (N - \tilde{N})(N + \tilde{N} - k)/k \). Through an analysis similar to that for bosons, we can obtain that \( S_1 \approx \ln[(1 + 2/k)N - 2] \) and find that the ground state in this subspace is a quantum Hall state with filling factor \( \nu = k/(k+2) \) and when \( N/k \) is an integer its counterpart in spherical geometry has a shift \( \sigma = 3 \) (see Fig.6).

**IV. BOSONS IN QUADRATIC PLUS QUARTIC TRAP**

So far we only consider the case in which the trap potential is a harmonic potential \( V(r) = \frac{1}{2} m \omega^2 r^2 \). This leads to the irrelevance of the subspace ground state with the interaction strength \( U_0 \), because in Eq.\[2\] the subspace ground state is uniquely determined by \( \sum_{i,j,k=0} U_{i,j,k} a_i^\dagger a_j^\dagger a_k a_i \). Consequently, the subspace ground state entanglement is also irrelevant of \( U_0 \). In this section, we consider a quadratic plus quartic trap, namely

\[
V(r) = \frac{1}{2} m \omega^2 r^2 \left[ 1 + \lambda \left( \frac{r}{a_0} \right)^2 \right].
\]
where \( a_0 = \sqrt{\frac{\hbar}{2m}} \) is the oscillator characteristic length.

The rotating Bose and Fermi gas confined in such an anharmonic potential is investigated in Refs.\[20, 25\]. In the weakly-interacting limit, the subspace Hamiltonian can be written as

\[
H_L = \sum_{i=0}^{L} \epsilon_i a_i^\dagger a_i - L\hbar \Omega + U_0 \sum_{i,j,k,l=0}^{L} U_{i,j,k,l} a_i^\dagger a_j^\dagger a_k a_l, \tag{3}
\]

where

\[
U_{i,j,k,l} = \int dz_1 dz_2 \varphi_i^\ast(z_1) \varphi_j^\ast(z_2) \varphi_k(z_2) \varphi_l(z_1).
\]

The single-particle LLL wave function \( \varphi_i \) and its eigenenergy \( \epsilon_i \) are solved by Numerov method. The ground state of Eq.(3) is dependent on \( U_0 \), making it possible for us to investigate the relation between entanglement and interaction. For simplicity we suppose that the bosons interact with each other through contact potential (either attractive or repulsive), so that \( U_{i,j,k,l} = \int dz_1 dz_2 \varphi_i^\ast(z_1) \varphi_j^\ast(z_2) \varphi_k(z_2) \varphi_l(z_1) \). In order to keep the LLL approximation valid, throughout the calculation we restrict the range of \( U_0 \) in the interval \([-0.05\hbar \omega, 0.05\hbar \omega]\).

We consider both the single-particle and two-particle entanglement of the ground state in the subspace \( L = N \) and \( L = N(N - 1) = 20 \).

In this paper, we consider the particle entanglement in rotating Bose and Fermi gases. When the particle number \( N \) is fixed, for both bosons and fermions,

FIG. 6: (Color Online) The eigenvalues of the single-particle reduced density operator of the ground state in the subspace \( \Delta L = N \) for fermions with repulsive Coulomb interaction. It can be seen that at \( N = 18 \), the distribution of eigenvalues changes qualitatively.

FIG. 7: (Color Online) Entanglement of fermions with repulsive Coulomb interaction. (a) Black cubic: The entanglement of the ground state in the subspace \( \Delta L = N(N - 1) \), namely \( k = 1 \). Red circle: The entanglement of the ground state in the subspace \( \Delta L = (N - 1)^2/2 \) for odd \( N \) and \( \Delta L = N(N - 2)/2 \) for even \( N \), namely \( k = 2 \). (b) Black cubic: The entanglement of the ground state in the subspace \( \Delta L = (N - \bar{N})(N + \bar{N} - 3)/3 \), namely \( k = 3 \). Red circle: The entanglement of the ground state in the subspace \( \Delta L = (N - \bar{N})(N + \bar{N} - 4)/4 \), namely \( k = 4 \).

FIG. 8: (Color Online) Entanglement of fermions with contact interaction, both attractive and repulsive (\( \lambda = 0.005 \) in our calculation and \( U_0 \) in unit \( \hbar \omega \)). (a) The single-particle and two-particle entanglement of the ground state in the subspace \( L = N = 5 \). (b) The single-particle and two-particle entanglement of the ground state in the subspace \( N = 5, L = N(N - 1) = 20 \).

V. SUMMARY

In this paper, we consider the particle entanglement in rotating Bose and Fermi gases. When the particle number \( N \) is fixed, for both bosons and fermions,
through investigating the single-particle entanglement of the ground state in various subspaces with fixed angular momentum \( L \), we find that the phenomena of particle localization and vortex localization can be indicated by the single-particle entanglement. Moreover, we study the single-particle entanglement of the ground state in some special subspaces. For the subspace \( L = N \), through the comparison between the results of bosons and fermions, we find that the edge reconstruction of rotating fermions can be indicated by the single-particle entanglement. For the subspace \( L = (N - \bar{N})(N + \bar{N} - k)/k \), we relate the subspace ground state with quantum Hall state from the viewpoint of entanglement and construct the same trial wave function as that in Ref. [8]. For bosons, different properties of condensate phase and vortex liquid phase is reflected by entanglement. At last, we study the relation between entanglement and interaction by introducing a quartic trap.

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