Investigating possible decay modes of $Y(4260)$ under the $D_1(2420)\bar{D} + c.c$ molecular state ansatz

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(Dated: May 7, 2014)

By assuming that $Y(4260)$ is a $D_1\bar{D}$ molecular state, we investigate some hidden-charm and charmed pair decay channels of $Y(4260)$ via intermediate $D_1\bar{D}$ meson loops with an effective Lagrangian approach. Through investigating the $\alpha$-dependence of branching ratios and ratios between different decay channels, we show that the intermediate $D_1\bar{D}$ meson loops are crucial for driving these transitions of $Y(4260)$ studied here. The coupled channel effects turn out to be more important in $Y(4260) \rightarrow D^*D^*$, which can be tested in the future experiments.

PACS numbers: 13.25.GV, 13.75.Lb, 14.40.Pq

I. INTRODUCTION

During the past years, the experimental observation of a large number of so-called $XYZ$ states has initiated tremendous efforts to unravel their nature beyond the conventional quark model (for recent reviews, see, e.g. Refs \textsuperscript{1}–\textsuperscript{5}). $Y(4260)$ was reported by the BaBar Collaboration in the $\pi^+\pi^-J/\psi$ invariant spectrum in $e^+e^- \rightarrow \gamma_{ISR}\pi^+\pi^-J/\psi$ [6], which has been confirmed both by the CLEO and Belle collaboration \textsuperscript{7, 8}. Its mass and total width are well determined as $m = 4263^{+8}_{-9}$ MeV and $\Gamma_Y = 95 \pm 14$ MeV, respectively \textsuperscript{9}. The new datum from BESIII confirms the signal in $Y(4260) \rightarrow J/\psi\phi\pi^-\pi^-$ with much higher statistics \textsuperscript{10}. The mass of $Y(4260)$ does not agree to what is predicted by the potential quark model. Further more, the most mysterious fact is that as a charmonium state with $J^{PC} = 1^{--}$, it is only “seen” as a bump in the two pion transitions to $J/\psi$, but not in any open charm decay channels like $D\bar{D}$, $D^*\bar{D} + c.c.$ and $D^*D^*$, or other measured channels. The line shapes of the cross section for $e^+e^-\rightarrow$ annihilations into $D^{(*)}$ meson pairs appear to have a dip at its peak mass 4.26 GeV instead of a bump.

Since the observation of the $Y(4260)$, many theoretical investigations have been carried out (for a review see Ref. \textsuperscript{11}). It has variously been identified as a conventional $\psi(4S)$ based on a relativistic quark model \textsuperscript{12}, a tetraquark $\bar{c}\bar{s}s$ state \textsuperscript{13}, a charmonium hybrid \textsuperscript{14–16}, hadronic molecule of $D_1\bar{D}$ \textsuperscript{17, 19, 20}, $\chi_{c1}\omega$ \textsuperscript{21}, $\chi_{c1}\rho$ \textsuperscript{21}, $J/\psi f_0$ \textsuperscript{22}, a cusp \textsuperscript{23, 27} or a non-resonance explanation \textsuperscript{28, 29} etc. The dynamical calculation of tetraquark states indicated that $Y(4260)$ can not be interpreted as P-wave $1^{--}$ state of charm-strange diquark-antidiquark, because the corresponding mass is found to be 200 MeV heavier \textsuperscript{30}. In Ref. \textsuperscript{31}, the authors also studied the possibility of $Y(4260)$ as P-wave $1^{--}$ state of charm-strange diquark-antidiquark state in the framework of QCD sum rules and arrived the same conclusion as Ref. \textsuperscript{30}. Some lattice calculations give the mass of vector hybrid within this mass region \textsuperscript{32}, which is very close to the new charmonium-like state $Y(4360)$ \textsuperscript{33}. With the $D_1\bar{D}$ molecular ansatz, a consistent description of some of the experimental observations can be obtained, such as its non observation in open charm decays, or the observation of $Z_c(3900)$ as mentioned in Ref. \textsuperscript{13}, the threshold behavior in its main decay channels are investigated in Ref. \textsuperscript{34} and the production of $X(3872)$ is studied in the radiative decays of $Y(4260)$ \textsuperscript{22}. Under such a molecular state assumption, a consistent description of many experimental observations could be obtained. However, as studied in \textsuperscript{33}, the production of an S-wave $D_1\bar{D}$ pair in $e^+e^-$ annihilation is forbidden in the limit of exact heavy quark spin symmetry, which substantially weakens the arguments for considering the $Y(4260)$ charmonium-like resonance as a $D_1\bar{D}$ molecular state.

The intermediate meson loop (IML) transition is one of the possible nonperturbative dynamical mechanisms, especially when we investigate the pertinent issues in the energy region of charmonium \textsuperscript{36–61}. During the last decade, many interesting observations were announced by Belle, BaBar, CLEO, BESIII, and so on. And in theoretical study, it is widely recognized that the IML may be closely related to a lot of nonperturbative phenomena observed in experiments \textsuperscript{14, 62}, e.g. apparent OZI-rule violations, sizeable non-$D\bar{D}$ decay branching ratios for $\psi(3770)$ \textsuperscript{14, 49}, the HSR violations in charmonium decays \textsuperscript{56, 58}, the hidden charmonium decays of the newly discovered $Z_c$ \textsuperscript{62}, etc.
In this work, we will investigate the hidden-charm decays of $Y(4260)$ and $Y(4260) \to D^{(*)} \bar{D}^{(*)}$ via $D_1 \bar{D}$ loop with an effective Lagrangian approach (ELA) under the $D_1 \bar{D}$ molecular assumption. The paper is organized as follows. In Sec. II we will introduce the ELA briefly and give some relevant formulae. In Sec. III the numerical results are presented. The summary will be given in Sec. IV.

II. THE MODEL

![Diagram](image)

FIG. 1: The hadron-level diagrams for $Y(4260) \to D^{(*)} \bar{D}^{(*)}$ with $D_1 \bar{D}$ as the intermediate states.

![Diagram](image)

FIG. 2: The hadron-level diagrams for hidden-charm decays of $Y(4260)$ with $D_1 D$ as intermediate states. $P$ denotes the pseudoscalar meson $\pi^0$ or $\eta$.

In order to calculate the leading contributions from the charmed meson loops, we need the leading order effective Lagrangian for the couplings. Based on the heavy quark symmetry and chiral symmetry [65, 66], the relevant effective Lagrangian used in this work read

$$L_{\psi D^{(*)} D^{(*)}} = g_{\psi DD} \bar{\psi} (\partial^\mu D \bar{D} - D \partial^\mu \bar{D}) - g_{\psi D^* D^*} \bar{\psi} D^* \partial^\mu D \partial^\nu \partial^\rho \partial^\sigma \bar{\psi} D \partial^\nu \partial^\rho \partial^\sigma (D \bar{D}) + D \partial_\alpha \bar{D} \partial^\alpha (D \bar{D}),$$

$$L_{h_0 D^{(*)} D^{(*)}} = g_{h_0 D^* D} \bar{h}_0 (D \bar{D} + D^* \bar{D}) + i g_{h_0 D^* D} \epsilon^{\mu \nu \rho \sigma} \partial_\mu \partial_\nu \bar{D} \partial_\rho \partial_\sigma (D \bar{D}),$$

where $D^{(*)} = (D^{(*)} D^{(*)}, D^{(*)} D^{(*)})$ and $D^{(*)} T = (D^{(*)} D^{(*)}, D^{(*)} D^{(*)})$ correspond to the charmed meson isodoublets. The following couplings are adopted in the numerical calculations,

$$g_{\psi DD} = 2g_2 \sqrt{m_\psi m_D}, \quad g_{\psi D^* D} = \frac{g_{\psi D D}}{\sqrt{m_D m_D^*}}, \quad g_{\psi D^* D^*} = \frac{g_{\psi D D}}{m_D^* m_D}. \quad (3)$$

In principle, the parameter $g_2$ should be computed with nonperturbative methods. It shows that vector meson dominance (VMD) would provide an estimate of these quantities [65]. The coupling $g_2$ can be related to the $J/\psi$ leptonic constant $f_\psi$ which is defined by the matrix element $\langle 0 | \bar{c} \gamma_\mu c | J/\psi (\mu, \epsilon) \rangle = f_\psi m_\psi \epsilon^\mu$, and $g_2 = \sqrt{m_\psi / 2 m_D f_\psi}$, where $f_\psi = 405 \pm 14$ MeV, and we have applied the relation $g_{\psi DD} = m_\psi / f_\psi$. The ratio of the coupling constants $g_{\psi DD}$ to $g_{\psi DD}$ is fixed as that in Ref. [57], i.e.,

$$\frac{g_{\psi DD}}{g_{\psi DD}} = 0.9. \quad (4)$$

In addition, the coupling constants in Eq. (2) are determined as

$$g_{h_0 D D^*} = -2g_1 \sqrt{m_{h_0} m_D m_D^*}, \quad g_{h_0 D^* D^*} = 2g_1 \sqrt{m_{h_0} m_D^*}, \quad (5)$$

with $g_1 = -\sqrt{m_{\chi_{c0}/3} / f_{\chi_{c0}}}$, where $m_{\chi_{c0}}$ and $f_{\chi_{c0}}$ are the mass and decay constant of $\chi_{c0}(1P)$, respectively [67]. We take $f_{\chi_{c0}} = 510 \pm 40$ MeV [68].
The light vector mesons nonet can be introduced by using the hidden gauge symmetry approach, and the effective Lagrangian containing these particles are as follows [69, 70],

\[
\mathcal{L}_{D^*D^*V} = ig_{D^*D^*V}\epsilon_{\alpha\beta\mu}(D_b^* \partial_\alpha D^*_a) - D_b^*(\partial^\beta)(D_a^* D^*_a) (\partial^\alpha V^\nu)_{ba} + i g_{D^*D^*V}(D_b^* \partial_\alpha D^*_a) (\partial^\beta V^\nu)_{ba} + h.c.,
\]

\[
\mathcal{L}_{DDV} = ig_{DDV}(D_b \bar{\psi}_a)(\partial^\beta V^\nu_{ba}) + i g_{DDV}(D_b \bar{\psi}_a)(\partial^\beta V^\nu_{ab}),
\]

\[
\mathcal{L}_{DD,V} = g_{DD,V}D_b^V V_{\mu a}D_a^V + g_{DD_V}(D_b^* \bar{\psi}_a)(\partial_\mu V^\nu_{ba}) + i g_{DD,V}(D_b^* \bar{\psi}_a)(\partial_\mu V^\nu_{ba}) + h.c.,
\]

\[
\mathcal{L}_{D^*D^*V} = ig_{D^*D^*V}(D_b^* \partial_\alpha D^*_a) (\partial^\beta V^\nu_{ba}) + ig_{D^*D^*V}(D_b^* \bar{\psi}_a)(\partial_\mu V^\nu_{ba}) + i g_{D^*D^*V}(D_b^* \bar{\psi}_a)(\partial_\mu V^\nu_{ab}) + h.c.,
\]

And the coupling constants read

\[
g_{DDV} = -\frac{1}{\sqrt{2}} \beta g_V,
\]

\[
g_{DD,V} = -\frac{1}{\sqrt{2}} \beta g_V = -\frac{2}{\sqrt{3}} \lambda_1 g_V \sqrt{m_D m_{D_1}},
\]

\[
g'_{DD,V} = -\frac{1}{\sqrt{2}} \beta g_V,
\]

\[
g_{D^*D^*V} = -\frac{1}{\sqrt{2}} \beta g_V,
\]

\[
g'_{D^*D^*V} = -\frac{1}{\sqrt{2}} \beta g_V = -\frac{2}{\sqrt{3}} \lambda g_V m_{D^*},
\]

where \( f_\pi = 132 \text{ MeV} \) is the pion decay constant, and the parameter \( g_V \) is given by \( g_V = m_\rho/f_\pi \) [66]. We take \( \lambda = 0.56 \text{ GeV}^{-1} \), \( g = 0.59 \) and \( \beta = 0.9 \) in our calculation [71].

The effective Lagrangian for the light pseudoscalar mesons are constructed by imposing invariance under both heavy quark spin-flavor transformation and chiral transformation [66, 72–74]. The pertinent interaction terms for this work read

\[
\mathcal{L}_{D_1D^*} = g_{D_1D^*} [3D_b^* (\partial_\mu \partial_\nu \phi)_{ab} D_a^* - D_b^* (\partial^\mu \phi)_{ab} D^*_{by}]
\]

\[
+ g_{D_1D^*} [3D_b^* (\partial_\mu \partial_\nu \phi)_{ab} D_a^* - D_b^* (\partial^\mu \phi)_{ab} D^*_{by}] + h.c.,
\]

\[
\mathcal{L}_{DD_1} = g_{DD_1} D_b (\partial_\mu \phi)_{ab} D_a^* + g_{DD_1} D_b^* (\partial_\mu \phi)_{ba} D_a^* + h.c.,
\]

\[
\mathcal{L}_{D^*D^*V} = \left( D^*(s) + D^*(0), D_v^*(s) \right) \quad \text{and} \quad \bar{D}^*(s) = \left( D^*(s) + D^*(0), D_v^*(s) \right).
\]

The 3 \times 3 Hermitian matrix for the octet of Goldstone bosons. In the chiral and heavy quark limit, the above coupling constants are

\[
g_{DD_1} = -\frac{2g}{f_\pi} \sqrt{m_D m_{D_1}},
\]

\[
g_{D^*D^*V} = \frac{\sqrt{2}}{3} \frac{g'}{\Lambda^2 f_\pi} \sqrt{m_{D^*} m_{D_{1s}}},
\]

with the chiral symmetry breaking scale \( \Lambda \approx 1 \text{ GeV} \) and the coupling \( h' = 0.65 \) [72].

By assuming \( Y(4260) \) is a \( D_1 \bar{D} \) molecular state, the effective Lagrangian is constructed as

\[
\mathcal{L}_{Y(4260)D_1D} = \frac{g}{\sqrt{2}} \left( \bar{D}_a^* \gamma^\mu D_{1a}^* \bar{D}_a^* - \bar{D}_{1a}^* \gamma^\mu D_a^* \right) + h.c.,
\]

which is an S-wave coupling. Since the mass \( Y(4260) \) is slightly below an S-wave \( D_1 \bar{D} \) threshold, the effective coupling \( g_{Y(4260)D_1D} \) is related to the probability of finding \( D_1 \bar{D} \) component in the physical wave function of the bound state,
$c^2$, and the binding energy, $\delta E = m_D + m_{D_1} - m_Y$ \cite{72, 76, 77},

$$g_{NR}^2 = 16\pi (m_D + m_{D_1})^2 c^2 \sqrt{\frac{2\delta E}{\mu}}\left[1 + O(\sqrt{2\mu r})\right],$$

(13)

where $\mu = m_D m_{D_1}/(m_D + m_{D_1})$ and $r$ is the reduced mass and the range of the forces. The coupling constants in Eq. (12) is given by the first term in the above equation. The coupling constant gets maximized for a pure bound state, which corresponds to $c^2 = 1$ by definition. In the following, we present the numerical results with $c^2 = 1$.

With the mass $m_Y = 4263^{+8}_{-9}$ MeV, and the averaged masses of the $D$ and $D_1$ mesons \cite{9}, we obtain the mass differences between the $Y(4260)$ and their corresponding thresholds,

$$m_D + m_{D_1} - m_Y = 27^{+9}_{-8} \text{ MeV},$$

(14)

and with $c^2 = 1$, we obtain

$$|y| = 14.62^{+1.11}_{-1.25} \pm 6.20 \text{ GeV}$$

(15)

where the first errors are from the uncertainties of the binding energies, and the second ones are due the the approximate nature of the approximate nature of Eq. (13).

The loop transition amplitudes for the transitions in Figs. 1 and 2 can be expressed in a general form in the effective Lagrangian approach as follows,

$$M_{fi} = \int \frac{d^4q}{(2\pi)^4} \sum_{D} \frac{T_i T_2 T_3}{a_1 a_2 a_3} F(m^2_2, q^2)$$

(16)

where $T_i$ and $a_i = q^2_i - m^2_i$ ($i = 1, 2, 3$) are the vertex functions and the denominators of the intermediate meson propagators. For example, in Fig. 2 (a), $T_i$ ($i = 1, 2, 3$) are the vertex functions for the initial $Y(4260)$, final charmonium and final light pseudoscalar mesons, respectively. $a_i$ ($i = 1, 2, 3$) are the denominators for the intermediate $\bar{D}, D^*$ and $D_1$ mesons, respectively. We introduce a dipole form factor,

$$F(m^2_2, q^2) = \left(\frac{\Lambda^2 - m^2_2}{\Lambda^2 - q^2_2}\right)^2,$$

(17)

where $\Lambda \equiv m_2 + \alpha \Lambda_{\text{QCD}}$ and the QCD energy scale $\Lambda_{\text{QCD}} = 220$ MeV. This form factor is supposed to kill the divergence, compensate the off-shell effects arising from the intermediate exchanged particle and the non-local effects of the vertex functions \cite{36, 78, 79}.

III. NUMERICAL RESULTS

![FIG. 3: (a). The $\alpha$-dependence of the branching ratios of $Y(4260) \to D\bar{D}$ (solid line) and $D^*\bar{D} + c.c.$ (dashed line). (b). The $\alpha$-dependence of the branching ratios of $Y(4260) \to D^*\bar{D}^*$.](image)
When calculating its decay widths. Its two-body decay width can be calculated as follows [80],

\[ \Gamma(Y) \text{(solid line)} \]

\[ \eta \text{(dashed line)} \]

\[ \psi \text{(dashed line)} \]

\[ \psi' \text{(solid line)} \]

\[ \psi' \text{(dashed line)} \]

\[ \eta \text{(solid line)} \]

\[ \psi \text{(solid line)} \]

\[ \psi' \text{(dashed line)} \]

Since \( Y(4260) \) has a large width 95\( \pm \) 14 MeV, so one has to take into account the mass distribution of the \( Y(4260) \) when calculating its decay widths. Its two-body decay width can then be calculated as follows [80],

\[ \Gamma(Y(4260))_{2\text{-body}} = \frac{1}{W} \int_{(m_Y-2\Gamma_Y)^2}^{(m_Y+2\Gamma_Y)^2} (2\pi)^4 \frac{d\Phi_2}{2\sqrt{s}} |M|^2 \frac{1}{\pi} \text{Im}\left( \frac{-1}{s - m_Y^2 + i m_Y \Gamma_Y} \right) \]  

\[ \int d\Phi_2 \text{ is the two-body phase space } [4] \]. \( M \) are the loop transition amplitudes for the processes in Figs. [1] and [2]. The factor \( 1/W \) with

\[ W = \frac{1}{\pi} \int_{(m_Y-2\Gamma_Y)^2}^{(m_Y+2\Gamma_Y)^2} \text{Im}\left( \frac{-1}{s - m_Y^2 + i m_Y \Gamma_Y} \right) ds \]

**FIG. 4:** (a). The \( \alpha \)-dependence of the branching ratios of \( Y(4260) \to J/\psi \eta \) (solid line) and \( J/\psi \pi^0 \) (dashed line). (b). The \( \alpha \)-dependence of the branching ratios of \( Y(4260) \to \psi' \eta \) (solid line) and \( \psi' \pi^0 \) (dashed line).

**FIG. 5:** The \( \alpha \)-dependence of the branching ratios of \( Y(4260) \to h_c \eta \) (solid line) and \( h_c \pi^0 \) (dashed line).

**TABLE I:** The predicted branching ratios of \( Y(4260) \) decays with different \( \alpha \) values. The uncertainties are dominated by the use of Eq. [13].

| Final states | \( \alpha = 0.5 \) | \( \alpha = 1.0 \) | \( \alpha = 1.5 \) | \( \alpha = 2.0 \) |
|--------------|------------------|------------------|------------------|------------------|
| \( DD \)     | \( (3.54\pm3.74) \times 10^{-6} \) | \( (4.21\pm3.78) \times 10^{-6} \) | \( (1.62\pm1.07) \times 10^{-6} \) | \( (3.94\pm3.80) \times 10^{-6} \) |
| \( D^+ D^- \) + c.c. | \( (9.86\pm5.31) \times 10^{-6} \) | \( (1.22\pm0.80) \times 10^{-6} \) | \( (4.82\pm3.18) \times 10^{-6} \) | \( (1.20\pm0.79) \times 10^{-6} \) |
| \( D^+ D^- \) | \( (2.78\pm2.91) \times 10^{-4} \) | \( (16.24\pm17.01) \times 10^{-5} \) | \( (52.21\pm54.09) \times 10^{-5} \) |
| \( J/\psi \eta \) | \( (7.43\pm3.78) \times 10^{-6} \) | \( (8.19\pm8.33) \times 10^{-6} \) | \( (2.95\pm1.07) \times 10^{-6} \) | \( (6.80\pm1.49) \times 10^{-6} \) |
| \( J/\psi \pi^0 \) | \( (3.04\pm2.94) \times 10^{-5} \) | \( (3.27\pm3.18) \times 10^{-5} \) | \( (1.19\pm0.78) \times 10^{-5} \) | \( (2.72\pm2.85) \times 10^{-5} \) |
| \( \psi' \eta \) | \( (4.34\pm4.54) \times 10^{-5} \) | \( (2.71\pm2.94) \times 10^{-5} \) | \( (6.50\pm6.31) \times 10^{-5} \) | \( (1.10\pm1.15) \times 10^{-5} \) |
| \( \psi' \pi^0 \) | \( (1.76\pm1.94) \times 10^{-6} \) | \( (9.71\pm10.17) \times 10^{-7} \) | \( (2.14\pm2.23) \times 10^{-7} \) | \( (3.43\pm3.50) \times 10^{-7} \) |
| \( h_c \eta \) | \( (3.87\pm3.33) \times 10^{-3} \) | \( (2.99\pm2.85) \times 10^{-2} \) | \( (8.20\pm8.58) \times 10^{-2} \) | \( (15.26\pm16.98) \times 10^{-2} \) |
| \( h_c \pi^0 \) | \( (1.27\pm0.84) \times 10^{-4} \) | \( (9.50\pm7.27) \times 10^{-4} \) | \( (2.54\pm1.67) \times 10^{-3} \) | \( (4.62\pm3.60) \times 10^{-3} \) |
Collaboration measured some upper limits of the ratios $Y_\text{diagrams}$ Fig. 2(a) and Fig. 2(b), the amplitudes for qualitatively estimate the contributions of the coupled-channel effects discussed in this work. Corresponding to the than $P$-wave, especially for the $S$-wave molecule with large coupling to its components, such as $Y$ containing open charmed mesons pairs and found no sign of a $40$ at $90\%$ C.L. [91], respectively. Within the parameter range considered in this work, the results displayed in Table I is greater than $0.6\%$ at $90\%$ C.L. [82]. Recently, BESIII has reported a study of a state with a mass of $4021\,\text{MeV and a width of }5$. The intermediate $D_1D$ meson loops turns out to be more important in $Y(4260) \rightarrow D^*D^*$ than that in $Y(4260) \rightarrow D\bar D$ and $D^*\bar D + c.c.$ This behavior can also be seen from Table I. As a result, a smaller value of $\alpha$ is favored in $Y(4260) \rightarrow D^*\bar D^*$. This phenomenon can be easily explained from Fig. 3(b). For the decay $Y(4260) \rightarrow D^*\bar D^*$, the off-shell effects of intermediate mesons $D_1D$ are not significant, which makes this decay favor a relatively smaller $\alpha$ value. For the decay $Y(4260) \rightarrow D\bar D$ and $D^*\bar D + c.c.$, since the exchanged mesons of the intermediate meson loops are $\rho$ and $\omega$, which makes their off-effects are relatively significant, which makes this decay favor a relatively larger $\alpha$ value.

In a fit to the total hadronic cross sections measured by BES [81], authors set an upper limit on $\Gamma_{e^+e^-}$ for $Y(4260)$ to be less than $580\,\text{eV at }90\%$ confidence level (C.L.) [82]. This implies that its branching fraction to $J/\psi\pi^+\pi^-$ is greater than $0.6\%$ at $90\%$ C.L. [82]. Recently, BESIII has reported a study of $e^+e^- \rightarrow h_c\pi^+\pi^-$, and observes a state with a mass of $4021.8 \pm 1.0 \pm 2.5\,\text{MeV and a width of }5.7 \pm 3.4 \pm 1.1\,\text{MeV in the }h_c\pi^\pm$ mass distribution, called the $Z_c(4020)$. The Belle collaboration did a comprehensive search for $Y(4260)$ decays to all possible final states containing open charmed mesons pairs and found no sign of a $Y(4260)$ signal in any of them [34, 49]. The BaBar Collaboration measured some upper limits of the ratios $B(Y(4260) \rightarrow D\bar D)/B(Y(4260) \rightarrow J/\psi\pi^+\pi^-) < 7.6$ at $95\%$ C.L. [90], $B(Y(4260) \rightarrow D^*\bar D)/B(Y(4260) \rightarrow J/\psi\pi^+\pi^-) < 34$ and $B(Y(4260) \rightarrow D^*\bar D^*)/B(Y(4260) \rightarrow J/\psi\pi^+\pi^-) < 40$ at $90\%$ C.L. [91], respectively. Within the parameter range considered in this work, the results displayed in Table I could be compatible with these available experimental limits. However, since there are still several uncertainties coming from the undetermined coupling constants, and the cutoff energy dependence of the amplitude is not quite stable, the numerical results would be lacking in high accuracy. Especially, since the kinematics, off-shell effects arised from the exchanged particles and the divergence of the loops in theses open charmed channels studied here are different, the cutoff parameter can also be different in different decay channels. We expect more precise experimental measurements on these open charmed pairs to test this point in the near future.

In Ref. [94], a nonrelativistic effective field theory (NREFT) method was introduced to study the meson loop effects in $\psi' \rightarrow J/\psi\pi^0$ transitions. And a power counting scheme was proposed to estimate the contribution of the loop effects, which is helpful to judge how important the coupled-channel effects are. This power counting scheme was analyzed in detail in Ref. [61]. Recently, the authors study that the S-wave threshold plays more important role than $P$-wave, especially for the S-wave molecule with large coupling to its components, such as $Y(4260)$ coupling to $D_1D$ in Ref. [22]. Before giving the explicit numerical results, we will follow the similar power counting scheme to qualitatively estimate the contributions of the coupled-channel effects discussed in this work. Corresponding to the diagrams Fig. 2(a) and Fig. 2(b), the amplitudes for $Y(4260) \rightarrow J/\psi\pi^0$ ($J/\psi\eta, \psi'/\pi^0, \psi'/\eta$) and $Y(4260) \rightarrow h_c\pi^0$ ($h_c\eta$)

FIG. 6: (a). The $\alpha$-dependence of the ratios of $R_1$ (solid line) and $R_2$ (dashed line) defined in Eq. (24). (b). The $\alpha$-dependence of the ratios of $r_1$ (solid line), $r_2$ (dashed line) and $r_3$ (dotted line) defined in Eq. (26).
scale as
\[ \frac{v^5}{(v^2)^3} q^3 \frac{\Delta}{v^2} \sim \frac{q^2 \Delta}{v^3}, \]  
(20) and
\[ \frac{v^5}{(v^2)^3} q^2 \frac{\Delta}{v^2} \sim \frac{q^2 \Delta}{v^3}, \]  
(21)
respectively. There are two scaling parameters \( v \) and \( q \) appeared in the above two formulæ. As illustrated in Ref. 22, \( v \) is understood as the average velocity of the intermediate charmed meson. \( q \) denotes the momentum of the outgoing pseudoscalar meson. And \( \Delta \) denotes the charmed meson mass difference, which is introduced to account for the isospin or SU(3) symmetry violation. For the \( \pi^0 \) and \( \eta \) production processes, the factors \( \Delta \) are about \( M_{D^+} + M_{D^-} - 2 M_{D^0} \) and \( M_{D^+} + M_{D^0} - 2 M_{D^-}, \) respectively. According to Eqs. (20) and (21), it can be concluded that the contributions of the coupled channel effects would be significant here since the amplitudes scale as \( O(1/v^3) \). And the branching ratio of \( Y(4260) \to h_c \pi^0 \) is expected to be larger than that of \( Y(4260) \to J/\psi \pi^0 \), because the corresponding amplitudes scale as \( O(q^2) \) and \( O(q^3) \) respectively. However, the momentum \( q \) in \( Y(4260) \to J/\psi \pi^0 \) is larger than that in \( Y(4260) \to h_c \pi^0 \), which may compensate this discrepancy to some extent.

For the open charmed decays in Fig. 4, the exchanged intermediate mesons are light vector mesons or light pseudoscalar mesons which will introduce different scale. Since we cannot separate different scales, so we just give possible numerical results in the form factor scheme.

For the hidden-charm transitions \( Y(4260) \to J/\psi \eta(\pi^0) \), we plot the \( \alpha \)-dependence of the branching ratios of \( Y(4260) \to J/\psi \eta(\pi^0) \) in Fig. 4(a) as shown by the solid and dashed lines, respectively. The \( \pi^0-\eta \) mixing has been taken into account. (Using Dashen’s theorem, one may express the mixing angle in terms of the masses of the Goldstone bosons at leading order in chiral perturbation theory and the value is about 0.01.) Some points can be learned from this figure: (1). A predominant feature is that the branching ratios are not drastically sensitive to the cutoff parameter, which indicates a reasonable cutoff of the ultraviolet contributions by the empirical form factors to some extent. (2). The leading contributions to the \( Y(4260) \to J/\psi \pi^0 \) are given by the differences between the neutral and charged charmed meson loops and also from the \( \pi^0-\eta \) mixing through the loops contributing to the eta transition. (3). At the same \( \alpha \), the branching ratios for \( Y(4260) \to J/\psi \eta \) transition are 2-3 orders of magnitude larger than that of \( Y(4260) \to J/\psi \pi^0 \). It is because that there is no cancelations between the charged and neutral meson loops.

The branching ratios of \( Y(4260) \to \psi' \eta \) (solid line) and \( Y(4260) \to \psi' \pi^0 \) (dashed line) in terms of \( \alpha \) are shown in Fig. 4(b). The behavior is similar to that of Fig. 4(a). Since the mass of \( \psi' \) is closer to the thresholds of \( DD^* \) than \( J/\psi \), it should give rise to important threshold effects in \( Y(4260) \to \psi' \eta(\pi^0) \) than in \( Y(4260) \to J/\psi \eta(\pi^0) \). At the same \( \alpha \) value, the obtained branching ratios of \( Y(4260) \to \psi' \pi^0 \) is larger than that of \( Y(4260) \to J/\psi \pi^0 \). Since the three-momentum of final \( \eta \) is only about 167 MeV in \( Y(4260) \to \psi' \eta \), which lead to a smaller branching ratios in \( Y(4260) \to J/\psi \eta \) than that in \( Y(4260) \to J/\psi \eta \) at the same \( \alpha \) value.

In Fig. 5 we plot the \( \alpha \)-dependence of the branching ratios of \( Y(4260) \to h_c \pi^0 \) (solid line) and \( Y(4260) \to h_c \eta \) (dashed line), respectively. The branching ratios for \( Y(4260) \to h_c \pi^0(\eta) \) are larger than that of \( Y(4260) \to J/\psi \pi^0(\eta) \) and \( \psi' \pi^0(\eta) \), which is consistent with the power counting analysis in Eqs. (20) and (21).

In order to study the exclusive threshold effects via the intermediate mesons loops, we define the following ratios,
\[ R_1 \equiv \frac{|M_{Y(4260) \to \psi' \eta}|^2}{|M_{Y(4260) \to J/\psi \eta}|^2}, \quad R_2 \equiv \frac{|M_{Y(4260) \to \psi' \pi^0}|^2}{|M_{Y(4260) \to J/\psi \pi^0}|^2}, \quad (22) \]
and
\[ r_1 \equiv \frac{|M_{Y(4260) \to J/\psi \pi^0}|^2}{|M_{Y(4260) \to J/\psi \pi^0}|^2}, \quad r_2 \equiv \frac{|M_{Y(4260) \to \psi' \pi^0}|^2}{|M_{Y(4260) \to \psi' \pi^0}|^2}, \quad r_3 \equiv \frac{|M_{Y(4260) \to h_c \pi^0}|^2}{|M_{Y(4260) \to h_c \pi^0}|^2}. \quad (23) \]
These ratios are plotted in Fig. 5(a) and (b), respectively. The stabilities of the ratios in terms of \( \alpha \) indicate a reasonably controlled cutoff for each channels by the form factor. Since the coupling vertices are the same for those decay channels when taking the ratio, the stability of the ratios suggests that the transitions of \( Y(4260) \to J/\psi \pi^0(\eta) \) and \( \psi' \pi^0(\eta) \) are largely driven by the open threshold effects via the intermediate \( D \bar{D} \) meson loops to some extent. The future experimental measurements of these decays can help us investigate this issue deeply.
IV. SUMMARY

In this work, we have investigated the hidden-charm decays of $Y(4260)$ and the decays $Y(4260) \to D\bar{D}$, $D\bar{D}^*$ and $D^*\bar{D}^*$ in ELA. In this calculation, $Y(4260)$ is assumed to be the $D_1\bar{D}$ molecular state. Our results show that the $\alpha$ dependence of the branching ratios are not drastically sensitive, which indicate the dominant mechanism driven by the intermediate meson loops with a fairly well control of the ultraviolet contributions.

For the hidden charmonium decays, we also carried out the power counting analysis and our results for these decays in ELA are qualitatively consistent with the power counting analysis. For the open charmed decays $Y(4260) \to D\bar{D}$, $D\bar{D}^*$ and $D^*\bar{D}^*$, the exchanged intermediate mesons are light vector mesons or light pseudoscalar mesons which will introduce different scale, so we cannot separate different scales and only give possible numerical results in the form factor scheme. For the decay $Y(4260) \to D^*\bar{D}^*$, the exchanged mesons $\pi$ is almost on-shell, so the coupled channel effects are more important than other channels studied here. We expect the experiments to search for the hidden-charm and charmed meson pairs decays of $Y(4260)$, which will help us investigate the nature and decay mechanisms of $Y(4260)$ deeply.

Acknowledgements

Authors thank Prof. Q. Zhao, Q. Wang and D.-Y. Chen for useful discussions. This work is supported in part by the National Natural Science Foundation of China (Grant Nos. 11035006, and 11275113), and in part by the China Postdoctoral Science Foundation (Grant No. 2013M530461).

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