Coriolis mixing of the $K = 1$ and $K = 0$ mixed symmetry states in the well deformed even–even nuclei

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Abstract The Coriolis matrix elements responsible for mixing of the $1^+ K = 1$ and $1^+ K = 0$ states are calculated in the framework of the Quasiparticle Phonon Model for several Gd and Dy isotopes. In many considered cases these matrix elements are equal to several tens of keV and are comparable with energy distances between the mixed levels. The results obtained indicates that Gd isotopes could be more suitable for finding deviations from Alaga rules in M1 transitions from $1^+$ state to the states of the ground band.

1 Introduction

The lowest lying states of the axially symmetric deformed nuclei are related to rotational degrees of freedom. The symmetries of the deformed mean field of atomic nuclei determine the possible sets of quantum numbers characterizing ground and excited states. Example of such symmetry is the invariance with respect to a rotation of $180^\circ$ around the arbitrary chosen axis perpendicular to the symmetry axis ($\mathcal{R}$ symmetry) [1]. As a consequence of this symmetry, the rotational bands with $K^\pi = 0^+$ comprise only states with even ($r = +1$) or odd ($r = -1$) angular momenta, where $r$ is $\mathcal{R}$ symmetry quantum number. The sequence of the angular momenta for $r = +1$ corresponds to the well known ground state rotational bands of axially symmetric deformed nuclei: $0^+, 2^+, 4^+, ...$. The sequence of the angular momenta for $r = -1$ should start with the $1^+$ state. However, information on the states with $I^\pi K = 1^+ K = 0$ is still practically absent. The exception is the result obtained in [2], where $K$ - mixing of the close-lying $K = 1$ and $K = 0$ states was observed. The $I^\pi K = 1^+ K = 1$ states, which are characterized by the strong M1 transition probabilities to the ground states are known under the name “mixed symmetry” states. They are well investigated and it is known that they are quite fragmented [3].

The properties of the mixed-symmetry states were analyzed in the framework of IBM-2 [4–7]. The mixing of the lowest IBM-2 $I^\pi = 1^+$ mixed-symmetry states with the other $I^\pi = 1^+$ states has been considered in [8] where the numerical value for an average mixing matrix element of $\approx 100$ keV was obtained. It is also pointed out in [8] that for the wave functions corresponding to the SU(3) limit of IBM-2 the mixing matrix element can become larger than in the vibrational limit.

Let us consider a structure of the $1^+$ mixed-symmetry states from the microscopic model point of view. The two-quasiparticle operators used to construct the excited state vectors of the even-even deformed nuclei in the intrinsic frame are divided [9] into electric type

$$A_{qq}^+ = \frac{1}{\sqrt{2}} \sum_\sigma \sigma a_{q,\sigma}^+ a_{q,\sigma}^+, \quad \tilde{A}_{qq}^+ = \frac{1}{\sqrt{2}} \sum_\sigma a_{q,\sigma}^+ a_{q,\sigma}^+,$$

(1)

and magnetic type

$$U_{qq}^+ = \frac{1}{\sqrt{2}} \sum_\sigma \sigma a_{q,\sigma}^+ a_{q,\sigma}^+, \quad \tilde{U}_{qq}^+ = \frac{1}{\sqrt{2}} \sum_\sigma a_{q,\sigma}^+ a_{q,\sigma}^+,$$

(2)

operators, which are also used for construction of the electric transition and magnetic transition operators, correspondingly. Different symmetrization of the two-quasiparticle configurations for electric and magnetic transition operators are connected with the fact that electric type transition operators are invariant with respect to the time-reverse transformation while magnetic transition operators change sign under time-reverse. The so called mixed symmetry states which are characterized by the strong M1 transitions from the ground state and weak E2 transitions to the symmetric states are
constructed using the magnetic type two-quasiparticle creation operators (2). We mention, that in the case of absence of the residual interaction two-quasiparticle states \( A_q | q_2 \{ 0 \} \) and \( U_q | q_2 \{ 0 \} \) with the same \( q_1, q_2 \) have the same excitation energies. However, they are characterized by different electromagnetic transition properties. Of course, residual forces create a difference in their excitation energies.

The experimental information on the M1 excitations with \( K = 0 \) projection was missing so far for axially deformed even–even nuclei. Only recently such information on the M1 transition properties. Of course, residual forces create a difference in their excitation energies. It was found for the two observed \( 1^+ \) states that the branching ratios deviate significantly from Alaga rule. This deviation can only be explained by mixing of the underlying basis states with projection quantum numbers \( K = 0 \) and \( K = 1 \).

It is the aim of this paper to calculate the value of the Coriolis mixing matrix element for the \( 1^+ \) states with \( K = 0 \) and \( K = 1 \) for the well deformed axially symmetric rare earth nuclei. In the case when these mixing matrix elements are not small compare to the energy distances between the mixed levels this might indicate the existance of the \( 1^+ \) states with a decay behavior deviating from the Alaga predictions. The calculations are performed below basing on the Quasiparticle-Phonon Model (QPNM) [9].

2 Brief description of the Quasiparticle-Phonon Model (QPNM)

The intrinsic Hamiltonian of the QPNM has the following structure

\[
H = H_{sp} + H_{pair} + H_{ph} + H_{pp}.
\]

The first term in (3) is the proton and neutron single particle Hamiltonian. The single particle energies are obtained from a deformed axially symmetric Woods–Saxon potential

\[
V_{sp} = V(r) + V_{ls}(r),
\]

\[
V(r) = -\frac{V_0^s}{1 + \exp(\alpha(r - R(\theta, \varphi)))},
\]

\[
V_{ls} = -\kappa p \otimes \vec{\sigma} \cdot \nabla V(r),
\]

where \((\tau = p, n)\) and

\[
R(\theta, \varphi) = R_0 \left( 1 + \beta_2 Y_{20}(\theta, \varphi) + \beta_4 Y_{40}(\theta, \varphi) \right).
\]

Here \( R_0 = r_0 A^{1/3} \) is the radius of the spherical nucleus having the same size; \( \beta_2 \) and \( \beta_4 \) are the quadrupole and hexadecapole axial deformation parameters.

The second term in (3) is a proton and neutron monopole pairing interaction. The terms \( H_{ph} \) and \( H_{pp} \) are separable interactions acting, respectively, in the particle–hole (ph) and particle–particle (pp) channels. The ph term is composed of spin-independent and spin-dependent parts

\[
H_{ph} = H_M + H_S,
\]

with

\[
H_M = -\frac{1}{2} \sum_\tau \sum_{\alpha, \beta} \sum_{\rho = \pm 1} \langle \lambda_0 | \rho \kappa_1 \rangle \kappa_1 | \lambda_0 \rangle + \rho \kappa_1 \rangle \kappa_1 | \lambda_0 \rangle \kappa_1 | \lambda_0 \rangle
\]

\[
H_S = -\frac{1}{2} \sum_\tau \sum_{\alpha, \beta} \sum_{\rho = \pm 1} \langle \lambda_0 | \rho \kappa_1 \rangle \kappa_1 | \lambda_0 \rangle + \rho \kappa_1 \rangle \kappa_1 | \lambda_0 \rangle
\]

\[
S_{\lambda_\mu_\rho}^+(\tau) S_{\lambda_\mu_\rho}^+(\tau).
\]

Above, if \( \tau = p(n) \) then \(-\tau = n(p)\). Here

\[
M_{\lambda_\mu_\rho}^+ = \sum_{q_1, q_2 \{ 2 \}} \langle q_1 | R_\lambda(r) Y_{\lambda_\mu_\rho}(\theta, \varphi) | q_2 \rangle a_{q_1,}^+ a_{q_2,}^+,
\]

\[
S_{\lambda_\mu_\rho}^+ \sum_{q_1, q_2 \{ 2 \}} \langle q_1 | R_\lambda(r) [\vec{\sigma} \otimes \vec{Y}]_{\lambda_\mu_\rho} | q_2 \rangle a_{q_1,}^+ a_{q_2,}^+.
\]

The symbols \( \kappa_0^{\lambda_\mu} \) and \( \kappa_1^{\lambda_\mu} \) denote, respectively, the isoscalar and isovector interaction constants of the spin-independent \((\lambda, \mu)\) terms. The spin-dependent analogues are \( \kappa_0^{\lambda_\mu} \) and \( \kappa_1^{\lambda_\mu} \).

The particle–particle term is spin-independent and has the form

\[
H_{pp} = -\frac{1}{2} \sum_\tau \sum_{\alpha, \beta} \sum_{\rho = \pm 1} G_{\lambda_\mu_\rho}^{\lambda_\mu_\rho}(\tau) P_{\lambda_\mu_\rho}^{\lambda_\mu_\rho}(\tau),
\]

where

\[
P_{\lambda_\mu_\rho}^{\lambda_\mu_\rho} = \sum_{q_1, q_2 \{ 2 \}} \langle q_1 | R_\lambda(r) Y_{\lambda_\mu_\rho}(\theta, \varphi) | q_2 \rangle a_{q_1,}^+ a_{q_2,}^+.
\]

The quantities \( G_{\lambda_\mu_\rho}^{\lambda_\mu_\rho} \) are the pairing strength constants. In all ph and pp terms the radial factor is \( R_\lambda(r) = dV(r)/dr \), where \( V(r) \) is the central part of the Woods–Saxon potential \((5)\).

The next step is to express the Hamiltonian in terms of the quasiparticle operators \( a_{q_1,}^+ \) and \( a_{q_2,}^+ \) and then to express the RPA phonon operators in terms of the two-quasiparticle creation and annihilation operators. Details of definition of the Hamiltonian parameters are given in [10]. The single particle spectrum used in the calculations is taken from the bottom of the potential and up to +5 MeV. The two-quasiparticle configurations with the energies up to 30 MeV are included into calculations. Because of the separable form of the interaction not all the terms of the Hamiltonian contribute to the structure of the given RPA phonon operator. The quadrupole–quadrupole interaction is the most important ingredient of the interaction part of the Hamiltonian. The spin–spin part has
the effect of pushing the spin excitations upward in energy thereby enforcing the orbital character of the low-lying states. Spin-multipole and tensor interaction have negligible effects.

The matrix elements of the Coriolis interaction
\[ H_{Coriolis} = -\frac{\hbar^2}{2\mu} (I_+ j_+ + I_- j_-), \]  
(15)
where \( I_\pm \) are components of the total angular momentum operator, acting on the Wigner functions \( D^{I}_{M}K \), and \( j_\pm \) are components of the intrinsic angular momentum operator, are calculated using the following expressions for the wave vectors of the \( 1^+ K = 1^+ 1 \) and \( 1^+ 0 \) states in the laboratory frame:
\[ |1^+MK = 1\rangle = \sqrt{\frac{3}{16\pi^2}} \left( D^{1}_{M1}Q^+(K = 1, \sigma = +1) + D^{1}_{M-1}Q^+(K = 1, \sigma = -1) \right) |0\rangle, \]  
(16)
where, due to the need to use phonon creation operators with the fixed sign of the projection of the angular momentum on the symmetry axis, operators other than (1) and (2) are used below for their construction:
\[ Q^+(K = 1, \sigma) = \frac{1}{2} \sum_{q_1q_2} \left( \psi_{q_1q_2}^{+1,K=1}(K = 1, \sigma) \cdot \chi(q_1q_2) - A_{q_1q_2}^+(K = 1, \sigma) \right), \]
where the matrix elements of the Coriolis interaction are determined by
\[ |1^+K = 1\rangle = \frac{3}{8\pi^2} \frac{1}{D^{1}_{M0}} \sum_{q_1q_2} \left( \psi_{q_1q_2}^{+1,K=1} - \psi_{q_1q_2}^{+1,K=0} U_{q_1q_2} \right) |0\rangle. \]  
(20)
The amplitudes \( \psi_{qq}^{+1,K=1} \) and \( \psi_{qq}^{+1,K=0} \) are normalized in a standard way [9].

The terms with the backward amplitudes \( \psi_{qq}^{+1,K=1} \) and \( \psi_{qq}^{+1,K=0} \) multiplied by the two-quasiparticle annihilation operators are not included in calculations because of the smallness of their effects. The final expression for the Coriolis mixing matrix element is
\[ \langle 1^+ K = 1 \vert H_{Coriolis} \vert 1^+ K = 0 \rangle \]
\[ = \frac{4\hbar^2}{3} \sum_{q_1q_2} \langle q_2, \sigma = +1 \vert j_x \vert q, \sigma = +1 \rangle v_{q_2q}^{+}(q_2, \sigma = +1) v_{q}^{+}(q, \sigma = +1) \]  
(21)
where \( \langle q_2, \sigma = +1 \vert j_x \vert q, \sigma = +1 \rangle \) is the matrix element of the single particle intrinsic angular momentum operator, \( v_{q_2q}^{+} = u_{q_2}u_{q} + v_{q_2}v_{q} \), and \( u-v \) are Bogoliubov transformation coefficients. The value of the moment of inertia is determined by the experimental energy of the first \( 2^+ \) state, according to \( \frac{h^2}{2I} = \frac{1}{2} E(2^+) \).

In calculations below it will be necessary to consider also Coriolis mixing of the \( 1^+ = 1^+ \) states having a two-phonon component as the main one. However, calculation of the corresponding matrix elements is reduced to calculation of the matrix elements between the one-phonon states.

3 Results

Calculations are carried out for \(^{156,158,160}\)Gd and \(^{160,162,164}\)Dy. The results of calculations of the Coriolis matrix elements are presented in Table 1. As it is seen from Table 1 the calculated values of the Coriolis matrix elements fluctuate significantly from one pair of states to the other pair. It is likely that the relatively stable value of the Coriolis matrix element would be expected in the case of mixing of the collective states. However, scissor \( K^\pi = 1^+ \) state is quite fragmented. The structure of the \( K^\pi = 0^+ \) states is also changes with excitation energy. The values of the calculated Coriolis matrix elements vary from 1 keV to 200 keV. In many of the considered cases the Coriolis matrix elements are comparable with an energy distances between the mixed levels. In average, the calculated matrix elements are of the same order as those obtained in [8].

It is also seen from Table 1 that the calculated matrix elements for Gd isotopes are larger in average than for Dy isotopes. It indicates that Gd isotopes could be more suitable for finding deviations from Alaga rule in M1 transitions from the \( 1^+ \) states to the states of the ground band.
Table 1 Coriolis mixing matrix elements between the magnetic type $K^\pi = 1^+$ and the nearby lying $K^\pi = 0^+$ excited states

| Nucleus | $E(K^\pi = 1^+)$ (keV) | $E(K^\pi = 0^+)$ (keV) | $|\langle 1^+, K = 1 | H_{Coriolis} | 1^+, K = 0 \rangle|_K$ (keV) |
|---------|------------------------|------------------------|---------------------------------|
| $^{156}$Gd | 2790 | 2750 | 53 |
| $^{156}$Gd | 2790 | 2800 | 68 |
| $^{156}$Gd | 2790 | 2880 | 26 |
| $^{156}$Gd | 3110 | 2750 | 209 |
| $^{156}$Gd | 3110 | 2800 | 29 |
| $^{156}$Gd | 3110 | 2880 | 82 |
| $^{158}$Gd | 3220 | 3000 | 39 |
| $^{158}$Gd | 3220 | 3130 | 11 |
| $^{158}$Gd | 3220 | 3190 | 112 |
| $^{158}$Gd | 3220 | 3230 | 5 |
| $^{158}$Gd | 3270 | 3000 | 29 |
| $^{158}$Gd | 3270 | 3130 | 18 |
| $^{158}$Gd | 3270 | 3190 | 24 |
| $^{160}$Gd | 3270 | 3230 | 25 |
| $^{160}$Gd | 3270 | 3000 | 15 |
| $^{160}$Gd | 3020 | 2760 | 52 |
| $^{160}$Gd | 3020 | 3060 | 14 |
| $^{160}$Gd | 3020 | 3060 | 78 |
| $^{160}$Dy | 2820 | 2840 | 1 |
| $^{160}$Dy | 2820 | 3020 | 1 |
| $^{160}$Dy | 2930 | 2840 | 1 |
| $^{160}$Dy | 2930 | 3020 | 60 |
| $^{162}$Dy | 3090 | 2870 | 2 |
| $^{162}$Dy | 3090 | 2930 | 35 |
| $^{162}$Dy | 3200 | 2870 | 5 |
| $^{164}$Dy | 3200 | 2930 | 1 |
| $^{164}$Dy | 3070 | 3200 | 9 |
| $^{164}$Dy | 3230 | 3200 | 3 |

Let us consider the structure of the mixed $K = 1$ and $K = 0$ states in order to find out the reasons for the large spread of the values of the Coriolis matrix elements. In the case of $^{156}$Gd the largest matrix element connects the states $| 3110 \text{keV}, K = 1 \rangle$ and $| 2750 \text{keV}, K = 0 \rangle$. The structure of the $| 3110 \text{keV}, K = 1 \rangle$ state is practically exhausted by the $[642] \otimes [401]_{nn}$ neutron two-quasiparticle component (89%). The structure of the $| 2750 \text{keV}, K = 0 \rangle$ state is determined by the $[651] \otimes [401]_{nn}$ neutron two-quasiparticle component (84%). The fact that the one-quasiparticle states $[642] 5/2^+$ and $[651] 3/2^+$ are strongly mixed by the Coriolis interaction explains the large value of the matrix element $\langle 3110 \text{keV}, K = 1 \mid H_{Coriolis} \mid 2750 \text{keV}, K = 0 \rangle = 209$ keV.

Table 2 Structure of the eigenvectors of the Hamiltonian matrix in $^{156}$Gd

| Components | Eigenvectors | | | | |
|------------|-------------|-------------|-------------|-------------|-------------|
| $K = 1$    | $| 1 \rangle$ | $| 2 \rangle$ | $| 3 \rangle$ | $| 4 \rangle$ | $| 5 \rangle$ |
| $2790 \text{keV}, K = 1$ | $-0.369$ | $-0.606$ | $0.573$ | $0.404$ | $0.072$ |
| $3110 \text{keV}, K = 1$ | $-0.386$ | $0.160$ | $-0.056$ | $-0.192$ | $0.886$ |
| $2750 \text{keV}, K = 0$ | $0.803$ | $-0.381$ | $0.171$ | $-0.150$ | $0.397$ |
| $2800 \text{keV}, K = 0$ | $0.208$ | $0.680$ | $0.634$ | $0.297$ | $0.072$ |
| $2880 \text{keV}, K = 0$ | $0.162$ | $0.019$ | $-0.488$ | $0.830$ | $0.215$ |

The large value of the matrix element (3020 keV, $K = 1$ | $H_{Coriolis} \mid 3060 \text{keV}, K = 0 \rangle = 78$ keV in $^{160}$Gd has the following explanation. The largest contribution to the structure of the $| 3020 \text{keV}, K = 1 \rangle$ state gives $[512] \otimes [521]_{nn}$ neutron two-quasiparticle component (58%). The component with the largest weight in the structure of the $| 3060 \text{keV}, K = 0 \rangle$ is the neutron two-quasiparticle component $[512] \otimes [512]_{nn}$ (63%). Strong mixing of the one-quasiparticle components $[521] 3/2^-$ and $[512] 5/2^-$ by the Coriolis interaction explains the large value of the considered matrix element.

The large values of some other Coriolis matrix elements in Gd isotopes have a similar explanation. In these cases Coriolis interaction strongly mixes, in addition to the one-quasiparticle states mentioned above, the following states: $[660] 1/2^+$ and $[651] 3/2^+$, $[523] 5/2^-$ and $[532] 3/2^-$.

Let us consider the isotopes of Dy. In the case of $^{160}$Dy there is one large matrix element: $| 2930 \text{keV}, K = 1 \rangle = 3020 \text{keV}, K = 0 \rangle = 60$ keV. The structure of the $| 2930 \text{keV}, K = 1 \rangle$ state is exhausted by the $[642] \otimes [401]_{nn}$ neutron two-quasiparticle component (98%). The structure of the $| 3020 \text{keV}, K = 0 \rangle$ state is exhausted by the $[651] \otimes [401]_{nn}$ neutron two-quasiparticle component (99%). Thus, in this case a strong mixing of the $[642] 5/2^+$ and $[651] 3/2^+$ one-quasiparticle components by Coriolis interaction explains the large value of the considered matrix element. The difference between the magnitudes of this Coriolis matrix element and the analogous matrix element considered above in $^{156}$Gd is associated with the difference in the values of the $u$–$v$ transformation coefficients in these nuclei.

However, in the case of the other Coriolis matrix elements in $^{160}$Dy situation is different. For instance, the main component of the $| 2820 \text{keV}, K = 1 \rangle = 413 \otimes [411]_{pp}$ (99%). The main component of the $| 2800 \text{keV}, K = 0 \rangle$ state is the neutron two-quasiparticle component $[521] \otimes [532]_{nn}$ (97%). Thus, the main components of the considered states are not mixed by the Coriolis interaction. The small value of the calculated matrix element $| 2820 \text{keV}, K = 1 \rangle = 2840 \text{keV}, K = 0 \rangle = 1$ keV is produced by the Coriolis mixing of the small components of the structure of the considered states.
Situation in $^{162}$Dy is somewhat different. The relatively large value of the Coriolis matrix element $\langle 3090 \text{ keV}, K = 1 | H_{\text{Coriolis}} | 2930 \text{ keV}, K = 0 \rangle = 35$ keV is produced by the contributions of several components of the considered states.

In the case of $^{164}$Dy there are no components in the structure of the considered $K = 1$ and $K = 0$ states whose Coriolis mixing is sufficiently strong. This explains small values of the Coriolis mixing matrix elements in $^{164}$Dy.

The results of diagonalization of the Hamiltonian matrix constructed using the energies of the mixing states and the values of the Coriolis matrix elements for all nuclei presented in Table 1 are shown below in Tables 2, 3, 4, 5 and 6. Eigenstates below are numbered according to increase of their excitation energy.

In the case of $^{156}$Gd the Coriolis matrix element $\langle 3110 \text{ keV}, K = 1 | H_{\text{Coriolis}} | 2750 \text{ keV}, K = 0 \rangle$, which is equal to 209 keV, is the largest one. However, the corresponding energy distance 360 keV is also large and this fact reduces the effect of this matrix element. At the same time the mixing matrix element $\langle 2790 \text{ keV}, K = 1 | H_{\text{Coriolis}} | 2750 \text{ keV}, K = 0 \rangle = 53$ keV although is four times smaller than the previous one, but is larger than the corresponding energy distance between the mixing states. In general, the structure of the eigenvectors of $^{156}$Gd shows a stronger mixing than in the rest of nuclei considered.

In the case of $^{158}$Gd the largest Coriolis matrix element corresponds to mixing of the $| 3220 \text{ keV}, K = 1 \rangle$ and $| 3190 \text{ keV}, K = 0 \rangle$ components. Their strong mixing is seen in the structure of the eigenvectors $| 2 \rangle$ and $| 6 \rangle$. All other eigenvectors have one largest component which exhausted 67% of the norm of the eigenvector $| 5 \rangle$ and more than 83% of the norm of the eigenvectors $| 1 \rangle$, $| 3 \rangle$, and $| 4 \rangle$.

The obtained structure of the eigenvectors of $^{160}$Gd is in a correspondence with the values of the Coriolis matrix elements presented in Table 1. The largest matrix elements create a mixing of the $| 2710 \text{ keV}, K = 1 \rangle$ and $| 2760 \text{ keV}, K = 0 \rangle$ components, and a mixing of the $| 3020 \text{ keV}, K = 1 \rangle$ and $| 3060 \text{ keV}, K = 0 \rangle$ components. Correspondingly, the eigenvectors $| 1 \rangle$ and $| 2 \rangle$ are a mixture of the first two components, but the structure of the eigenvectors $| 3 \rangle$ and $| 4 \rangle$ is exhausted by the other two components.

In the case of $^{162}$Dy strong mixing of the $| 2930 \text{ keV}, K = 1 \rangle$ and $| 3020 \text{ keV}, K = 0 \rangle$ components is generated by the Coriolis matrix element equal to 60 keV which is comparable with the energy distance between the mixing vectors. Just this fact is reflected in the structure of the eigenvectors $| 3 \rangle$ and $| 4 \rangle$. The structure of the eigenvectors $| 1 \rangle$ and $| 2 \rangle$ is exhausted by one component.

In the case of $^{164}$Dy the largest Coriolis matrix element corresponding to mixing of the $| 3090 \text{ keV}, K = 1 \rangle$ and $| 2930 \text{ keV}, K = 0 \rangle$ vectors is small compare to the corresponding energy distance 160 keV. However, the K-mixing can become important in this nucleus if the experimental level distance is shorter than the calculated one.

In the case of $^{164}$Dy the K-mixing is practically absent because of the smallness of the mixing matrix elements.

### Table 3 Structure of the eigenvectors of the Hamiltonian matrix in $^{158}$Gd

| Components | Eigenvectors |
|------------|--------------|
|            | $| 1 \rangle$ | $| 2 \rangle$ | $| 3 \rangle$ | $| 4 \rangle$ | $| 5 \rangle$ | $| 6 \rangle$ |
| $| 3220 \text{ keV}, K = 1 \rangle$ | -0.229 | 0.589 | 0.133 | -0.079 | -0.348 | 0.675 |
| $| 3270 \text{ keV}, K = 1 \rangle$ | -0.113 | 0.085 | -0.095 | 0.395 | 0.820 | 0.376 |
| $| 3000 \text{ keV}, K = 0 \rangle$ | 0.956 | 0.265 | 0.019 | 0.038 | 0.037 | 0.113 |
| $| 3130 \text{ keV}, K = 0 \rangle$ | 0.032 | -0.233 | 0.964 | 0.070 | 0.074 | 0.071 |
| $| 3190 \text{ keV}, K = 0 \rangle$ | 0.140 | -0.721 | -0.209 | 0.021 | -0.219 | 0.607 |
| $| 3230 \text{ keV}, K = 0 \rangle$ | 0.016 | -0.038 | 0.017 | -0.911 | 0.388 | 0.129 |
Table 6 Structure of the eigenvectors of the Hamiltonian matrix in $^{162}$Dy

| Components | Eigenvectors |
|------------|--------------|
|            | | 1 | 2 | 3 | 4 |
| 3090 keV, $K = 1$ | 0.010 | 0.205 | 0.979 | 0.002 |
| 3200 keV, $K = 1$ | 0.015 | 0.003 | -0.002 | 0.9999 |
| 2870 keV, $K = 0$ | -0.9998 | 0.008 | 0.009 | 0.015 |
| 2930 keV, $K = 0$ | -0.006 | -0.979 | 0.205 | 0.004 |

4 Conclusion

We have calculated the Coriolis matrix elements for mixing of the $1^+ K = 1$ and $1^+ K = 0$ states in several Gd and Dy isotopes. The results obtained show that the values of the matrix elements fluctuate significantly due to variation of the structure of the mixing states which is exhausted by a few configurations. In many cases the Coriolis matrix elements are equal to several tens of keV. The results obtained indicate that Gd isotopes could be more suitable for finding deviations from Alaga rule in M1 transitions from the $1^+$ states to the states of the ground band.

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