Chasing after flavor symmetries of quarks from bottom up

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Abstract

We explore a flavor structure of quarks in the standard model under the assumption that flavor symmetries exist in a theory beyond the standard model, and chase after their properties, using a bottom-up approach. We acknowledge that a flavor-symmetric part of Yukawa coupling matrix can be realized by a rank-one matrix and a democratic-type one occupies a special position, based on Dirac’s naturalness.

1 Introduction

The Yukawa sector in the standard model (SM) holds many mysteries. For instance, the origin of the fermion mass hierarchy and flavor mixing is a big riddle. There have been many intriguing attempts to explain the values of physical parameters concerning the fermion masses and flavor mixing matrices, based on the top-down approach \[1,2,3,4,5,6\], but we have not arrived at a satisfactory answer.

There are several reasons why it is difficult to understand an origin of the flavor structure. First, we have no powerful guiding principle to determine a theory beyond the SM. Although flavor symmetries are possible candidates, any evidence has not yet been discovered. If they exist at all, they might be hidden in a false bottom of the Yukawa interactions. In concrete, there are no unbroken flavor-dependent symmetries in the SM \[14,15\]. There can be several existence forms of flavor symmetries in a broken phase of an underlying theory. For instance, flavor symmetries are broken down in every interactions, they (or those sub-symmetries) survive in some interactions, or a new symmetry appears in some terms. Except for the first one, Yukawa interactions, in general, consist of flavor-symmetric and breaking parts and they are not reconstructed from experimental data alone because global U(3) symmetries emerge in the fermion kinetic terms of the

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1 The flavor structure of quarks and leptons has been studied intensively, based on various flavor symmetries \[6,7,8,9,10,11,12,13\].
SM. In other words, there is no way to determine the fermion masses and mixing angles without any excellent new concept. Furthermore, fermions in the SM do not necessarily behave as unitary bases of flavor symmetries, i.e., quarks and leptons are transformed using elements of a flavor group $G_F$ realized by non-unitary matrices if an underlying theory possesses non-canonical matter kinetic terms [16].

The world of flavor can be glimpsed from the Lagrangian in the SM by adopting Dirac’s naturalness. Here, Dirac’s naturalness means that the magnitude of dimensionless parameters on terms allowed by symmetries should be $O(1)$ in a fundamental theory and suggests that the Yukawa coupling of top quark can originate from a flavor-symmetric renormalizable interaction. In contrast, other tiny Yukawa couplings are expected to come from non-renormalizable ones suppressed by a power of a high-energy scale. Then, we obtain a conjecture that a flavor-symmetric part of up-type quark Yukawa coupling matrix can be realized by a rank-one matrix and a democratic-type one can take a peculiar position.

If flavor symmetries exist in an underlying theory and the flavor structure in the SM appears after the breakdown of $G_F$, it is desirable to study the above conjecture using suitable field variables such as unitary bases of $G_F$. Although a same conclusion ought to be obtained because of no change of physics by a choice of field variables, there is a possibility that it provides a clue to figure out the origin of flavor and it gives us a new insight of flavor physics.

In this paper, we explore the flavor structure a little further, adopting Dirac’s naturalness, and re-examine whether the above conjecture holds or not.

The outline of this paper is as follows. In the next section, we explain our setup on Yukawa interactions of quarks. In Sect. 3, we chase after properties of flavor symmetries. In the last section, we give conclusions and discussions.

## 2 Setup

We explain the setup of our analysis [16]. Our basic assumptions are as follows. (a) There are flavor symmetries beyond the SM. (b) The symmetries are broken down by the vacuum expectation values (VEVs) of flavons, on the whole. Some symmetries can survive or emerge in some terms. (c) Flavons also couple to matter fields through matter kinetic terms.

Let us start with a theory of quarks beyond the SM, described by the Lagrangian density:

$$
\mathcal{L}_{\text{quark}}^{\text{BSM}} = K_{ij}^{(q)} \overline{q}_{Li} i \not\!q_{Lj} + K_{ij}^{(u)} \overline{u}_{Ri} i \not\!u_{Rj} + K_{ij}^{(d)} \overline{d}_{Ri} i \not\!d_{Rj} - (Y_1)_{ij} \overline{q}_{Li} \not\!\phi u_{Rj} - (Y_2)_{ij} \overline{q}_{Li} \not\!\bar{\phi} d_{Rj} + \text{h.c.},
$$

where $q_{Li}$ are counterparts of left-handed quark doublets, $u_{Rj}$ and $d_{Rj}$ are those of right-handed up- and down-type quark singlets, $i, j (= 1, 2, 3)$ are family labels, summation over repeated indices is understood, $\phi$ is the Higgs doublet, $\not\!\phi = i \tau_2 \phi^*$, and h.c. stands for the Hermitian conjugation of former terms. The $K_{ij}^{(q)}$, $K_{ij}^{(u)}$, $K_{ij}^{(d)}$, $(Y_1)_{ij}$, and $(Y_2)_{ij}$ contain flavons such that $\mathcal{L}_{\text{quark}}^{\text{BSM}}$ is invariant under transformations relating to flavor.
symmetries. The $q'_L$, $u'_R$, and $d'_R$ are unitary bases of a flavor group $G_F$, and are transformed as

$$q'_L \rightarrow F_L q'_L, \quad u'_R \rightarrow F_R^{(u)} u'_R, \quad d'_R \rightarrow F_R^{(d)} d'_R, \quad \phi \rightarrow e^{i\theta} \phi,$$

(2)

where $F_L$, $F_R^{(u)}$, and $F_R^{(d)}$ are $3 \times 3$ unitary matrices which are elements of $G_F$ and family labels are omitted. From the $G_F$ invariance of $\mathcal{L}_{\text{BSM}}^{\text{quark}}$, we obtain relations:

$$F_L K^{(q)} F_L^{\dagger} = K^{(q)}, \quad F_R^{(u)} K^{(u)} F_R^{(u)\dagger} = K^{(u)}, \quad F_R^{(d)} K^{(d)} F_R^{(d)\dagger} = K^{(d)},$$

(3)

$$e^{i\theta} F_L Y_1 F_R^{(u)\dagger} = Y_1, \quad e^{-i\theta} F_L Y_2 F_R^{(d)\dagger} = Y_2.$$  

(4)

The $\mathcal{L}_{\text{BSM}}^{\text{quark}}$ describes only the part relating to quarks in new physics, and chiral anomalies are supposed to be canceled by other contributions if the $G_F$ symmetries are local.

We assume that $G_F$ changes into $H^k_F$ and $H^k_F$ after flavons acquire the VEVs at some high-energy scale $M_{\text{BSM}}$. Here, $H^k_F$ and $H^k_F$ are flavor groups of quark kinetic terms and Yukawa interactions, respectively. Then, $\mathcal{L}_{\text{BSM}}^{\text{quark}}$ turns out to be the Lagrangian density:

$$\mathcal{L}_{\text{SM}}^{\text{quark}} = k_{ij}^{(q)} \bar{q}_{L i} i \not{\!\! D} q_{L j} + k_{ij}^{(u)} \bar{u}_{R i} i \not{\!\! D} u_{R j} + k_{ij}^{(d)} \bar{d}_{R i} i \not{\!\! D} d_{R j} - (y_1)_{ij} \bar{q}_{L i} \not{\!\! D} u_{R j} - (y_2)_{ij} \bar{q}_{L i} \not{\!\! D} d_{R j} + \text{h.c.},$$

(5)

where $k_{ij}^{(q)}$, $k_{ij}^{(u)}$, and $k_{ij}^{(d)}$ are quark kinetic coefficients, and $(y_1)_{ij}$ and $(y_2)_{ij}$ are Yukawa couplings in the unitary bases of $G_F$. Note that non-canonical matter kinetic terms appear in $\mathcal{L}_{\text{SM}}^{\text{quark}}$. From Eqs. (1) and (5), the following matching conditions should be imposed on

$$k_{ij}^{(q)} = \langle K^{(q)}_{ij} \rangle, \quad k_{ij}^{(u)} = \langle K^{(u)}_{ij} \rangle, \quad k_{ij}^{(d)} = \langle K^{(d)}_{ij} \rangle, \quad (y_1)_{ij} = \langle (Y_1)_{ij} \rangle, \quad (y_2)_{ij} = \langle (Y_2)_{ij} \rangle,$$

(6)

at $M_{\text{BSM}}$. From the fact that there are no exact flavor-dependent symmetries in the SM [14, 15], the common element of $H^k_F$ and $H^k_F$ should be a flavor-independent one.

We examine a relationship between the unitary bases $(q'_L, u'_R, d'_R)$ and the SM quark fields denoted by non-prime ones $(q_L, u_R, d_R)$, and study how flavor symmetries are realized in the SM ones. The unitary bases are, in general, related to the SM ones by the change of variables as

$$q_L = N_q q'_L, \quad u_R = N_u u'_R, \quad d_R = N_d d'_R,$$

(7)

where $N_q$, $N_u$, and $N_d$ are $3 \times 3$ complex matrices which are, in general, non-unitary matrices. Under the transformation (2), the SM ones are transformed as

$$q_L \rightarrow \tilde{F}_L q_L, \quad u_R \rightarrow \tilde{F}_R^{(u)} u_R, \quad d_R \rightarrow \tilde{F}_R^{(d)} d_R, \quad \phi \rightarrow e^{i\theta} \phi,$$

(8)

where $\tilde{F}_L$, $\tilde{F}_R^{(u)}$, and $\tilde{F}_R^{(d)}$ are defined by

$$\tilde{F}_L \equiv N_q F_L N_q^{-1}, \quad \tilde{F}_R^{(u)} \equiv N_u F_R^{(u)} N_u^{-1}, \quad \tilde{F}_R^{(d)} \equiv N_d F_R^{(d)} N_d^{-1},$$

(9)

Several works on the flavor physics have been carried out based on matter kinetic terms [17, 18, 19, 20, 21, 22, 23, 24].
respectively. If \( \tilde{F}_L \), \( \tilde{F}^{(u)}_R \), and \( \tilde{F}^{(d)}_R \) belong to \( \mathbb{H}^k \), they are unbroken elements realized by unitary matrices. Otherwise, they are broken ones realized by non-unitary ones. We call fields transformed by non-unitary matrices “non-unitary bases”.

From the matching condition between the Lagrangian density (5) and that of the quark sector in the SM written by

\[
\mathcal{L}_{SM}^{\text{quark}} = \overline{q}_L i \not \! q \not \! q_L + \overline{u}_R i \not \! u \not \! u_R + \overline{d}_R i \not \! d \not \! d_R - y_{ij}^{(u)} \overline{q}_L \not \! q \not \! u_R j - y_{ij}^{(d)} \overline{q}_L \not \! q \not \! d_R j + \text{h.c.},
\]

we obtain the relations:

\[
k^{(q)}_{ij} = \left( N^+_q N_q \right)_{ij}, \quad k^{(u)}_{ij} = \left( N^+_u N_u \right)_{ij}, \quad k^{(d)}_{ij} = \left( N^+_d N_d \right)_{ij},
\]

\[
(y_1)_{ij} = \left( N^+_q y^{(u)} N_u \right)_{ij}, \quad (y_2)_{ij} = \left( N^+_d y^{(d)} N_d \right)_{ij}.
\]

Because the kinetic coefficients are hermitian and positive definite, \( k^{(q)}_{ij} \) is written by

\[
k^{(q)}_{ij} = \left( U_q^i (J_q)^2 U_q \right)_{ij},
\]

where \( U_q \) is a 3 \times 3 unitary matrix and \( J_q \) is a real 3 \times 3 diagonal matrix. Then, \( N_q, N_u, \) and \( N_d \) are parametrized by

\[
N_q = V_q J_q U_q, \quad (14)
\]

\[
N_u = (y^{(u)})^{-1} \left( N^+_q \right)^{-1} y_1 = (y^{(u)})^{-1} V_q^i \mathbf{1} U_q y_1 = V^i_{\text{diag}} (y^{(u)})^{-1} \mathbf{1} \mathbf{1} U_q y_1, \quad (15)
\]

\[
N_d = (y^{(d)})^{-1} \left( N^+_q \right)^{-1} y_2 = (y^{(d)})^{-1} V_q^i \mathbf{1} U_q y_2 = V^i_{\text{diag}} (y^{(d)})^{-1} \mathbf{1} \mathbf{1} U_q y_2, \quad (16)
\]

using \( U_q, J_q, \) a 3 \times 3 unitary matrix \( V_q, \) and the Yukawa coupling matrices \( y^{(u)}, y^{(d)}, y_1, \) and \( y_2. \) In place of \( y^{(u)} \) and \( y^{(d)}, \) the diagonalized ones \( y^{(u)}_{\text{diag}} \) and \( y^{(d)}_{\text{diag}} \) and 3 \times 3 unitary matrices \( V^{(u)}_L, V^{(d)}_L, V^{(u)}_R, \) and \( V^{(d)}_R \) are also used. The \( (y^{(u)}) \) and \( (y^{(d)}) \) are diagonalized as \( V^{(u)}_L y^{(u)} y^{(u)\dagger}_R = y^{(u)}_{\text{diag}} \) and \( V^{(d)}_L y^{(d)} y^{(d)\dagger}_R = y^{(d)}_{\text{diag}}, \) and the quark masses are obtained as

\[
V^{(u)}_L y^{(u)} y^{(u)\dagger}_R \frac{\nu}{\sqrt{2}} = y^{(u)}_{\text{diag}} \frac{\nu}{\sqrt{2}} = M^{(u)}_{\text{diag}} = \text{diag} (m_u, m_c, m_t), \quad (17)
\]

\[
V^{(d)}_L y^{(d)} y^{(d)\dagger}_R \frac{\nu}{\sqrt{2}} = y^{(d)}_{\text{diag}} \frac{\nu}{\sqrt{2}} = M^{(d)}_{\text{diag}} = \text{diag} (m_d, m_s, m_b), \quad (18)
\]

where \( \nu/\sqrt{2} \) is the VEV of the neutral component in the Higgs doublet, and \( m_u, m_c, m_t, m_d, m_s, \) and \( m_b \) are masses of up, charm, top, down, strange, and bottom quarks, respectively. Using (15) and (16), \( k^{(u)}_{ij} \) and \( k^{(d)}_{ij} \) are rewritten by

\[
k^{(u)}_{ij} = \left( y^{(u)}_1 W^{(u)}_1 \left( y^{(u)\dagger}_{\text{diag}} \right)^2 W^{(u)}_1 y_1 \right)_{ij}, \quad (19)
\]

\[
k^{(d)}_{ij} = \left( y^{(d)}_2 W^{(d)}_2 \left( y^{(d)\dagger}_{\text{diag}} \right)^2 W^{(d)}_2 y_2 \right)_{ij} = \left( y^{(d)\dagger}_2 W^{(d)}_2 V_{KM} \left( y^{(d)\dagger}_{\text{diag}} \right)^2 V_{KM}^\dagger W^{(u)}_2 y_2 \right)_{ij}, \quad (20)
\]
where $W^{(u)} \equiv V^\dagger_q V_q J_q^{-1} U_q$, $W^{(d)} \equiv V^\dagger_L V_L J_q^{-1} U_q$, and $V_{KM} \equiv V^\dagger_L V^\dagger_L$. The $V_{KM}$ is the Kobayashi–Maskawa matrix [25]. From Eq. (14) and the definition of $W^{(u)}$, we have the relations:

\[ N_q^\dagger = (W^{(u)})^{-1} V^{(u)}_L, \quad N_q = V^{(u)}_L (W^{(u)})^{-1}. \]

and, using them, we obtain the formula:

\[ k_{ij}^{(q)} = (W^{(u)})^{-1} (W^{(u)})^{\dagger} = (W^{(u)} W^{(u)})^{-1}. \]

From the definition of $V_{KM}$, we have the relation:

\[ W^{(u)} = V_{KM} W^{(d)} \quad \text{or} \quad W^{(d)} = V_{KM}^\dagger W^{(u)}. \]

Note that $W^{(u)}$ and $W^{(d)}$ are not necessarily unitary matrices. If $I_q$ is the identity matrix, $k_{ij}^{(q)}$ is the canonical one ($\delta_{ij}$) and $W^{(u)}$ and $W^{(d)}$ become unitary matrices.

### 3 Chasing after flavor symmetries

The $\mathcal{L}^{\text{quark}}_{\text{SM}}$ has been obtained from accumulated experimental data and successfully describes the physics of quarks at the weak scale. Although the quark kinetic terms of $\mathcal{L}^{\text{quark}}_{\text{SM}}$ has the global $U(3) \times U(3) \times U(3)/U(1)$ symmetry, this is an emergent symmetry and one takes care not to confuse it with flavor symmetries in an underlying theory described by $\mathcal{L}^{\text{quark}}_{\text{BSM}}$. Flavor symmetries are expected to be realized by unitary bases in $\mathcal{L}^{\text{quark}}_{\text{SM}}$, because it describes physics right after the change of flavor symmetries. As $\mathcal{L}^{\text{quark}}_{\text{SM}}$ can still retain the remnants of flavor symmetries in spite of the fact that it is equivalent to $\mathcal{L}^{\text{quark}}_{\text{SM}}$, it is favorable to examine $\mathcal{L}^{\text{quark}}_{\text{SM}}$ in the pursuit of the origin of flavor.

#### 3.1 Generic argument

We study generic properties of flavor symmetries based on $\mathcal{L}^{\text{quark}}_{\text{SM}}$. In Eqs. (19) and (20), $k_{ij}^{(u)}$ and $k_{ij}^{(d)}$ are expanded as

\[ k_{ij}^{(u)} = y_{u}^{-2} \left( y_{1}^{(u)} W^{(u)} \right)_{i1} \left( W^{(u)} y_{1} \right)_{1j} + y_{c}^{-2} \left( y_{1}^{(u)} W^{(u)} \right)_{i2} \left( W^{(u)} y_{1} \right)_{2j} + y_{t}^{-2} \left( y_{1}^{(u)} W^{(u)} \right)_{i3} \left( W^{(u)} y_{1} \right)_{3j}, \]

\[ k_{ij}^{(d)} = y_{d}^{-2} \left( y_{2}^{(d)} W^{(d)} \right)_{i1} \left( W^{(d)} y_{2} \right)_{1j} + y_{s}^{-2} \left( y_{2}^{(d)} W^{(d)} \right)_{i2} \left( W^{(d)} y_{2} \right)_{2j} + y_{b}^{-2} \left( y_{2}^{(d)} W^{(d)} \right)_{i3} \left( W^{(d)} y_{2} \right)_{3j}, \]

where $y_u$, $y_c$, $y_t$, $y_d$, $y_s$, and $y_b$ are components of $y^{(u)}_{\text{diag}}$ and $y^{(d)}_{\text{diag}}$ and are estimated at the weak scale as

\[ y_{\text{diag}}^{(u)} = (y_u, y_c, y_t) \doteq \text{diag}(1.3 \times 10^{-5}, 7.3 \times 10^{-3}, 1.0). \]
\[ y_{\text{diag}}^{(d)} = (y_d, y_s, y_b) \div \text{diag}(2.7 \times 10^{-5}, 5.5 \times 10^{-4}, 2.4 \times 10^{-2}). \]  

(27)

Physical parameters, in general, receive radiative corrections, and the above values should be evaluated by considering renormalization effects to match with their counterparts at \( M_{\text{BSM}} \).

From the requirements that the magnitude of each component in \( k_{ij}^{(u)} \) and \( k_{ij}^{(d)} \) is at most \( O(1) \) and there are no fine-tunings among terms including different couplings, we obtain the conditions:

\[
\begin{align*}
(W^{(u)} y_1)_{1j} & \leq O(y_u), \\
(W^{(u)} y_1)_{2j} & \leq O(y_c), \\
(W^{(u)} y_1)_{3j} & \leq O(y_t), \\
(W^{(d)} y_2)_{1j} & \leq O(y_d), \\
(W^{(d)} y_2)_{2j} & \leq O(y_s), \\
(W^{(d)} y_2)_{3j} & \leq O(y_b).
\end{align*}
\]

(28)

(29)

Here, we explain some existence forms of flavor symmetries. In an ordinary case, fields belong to multiplets of irreducible representations of \( G_F \) and \( \mathcal{L}^{\text{quark}}_{\text{BSM}} \) is constructed using \( G_F \)-invariant polynomials of fields. There is a case that \( G_F \) appears as an accidental one from a more fundamental theory and then fields can belong to multiplets of reducible representations effectively. The \( \mathcal{L}^{\text{quark}}_{\text{SM}} \), in general, contains \( G_F \)-invariant and non-invariant parts of irreducible multiplets. In some case, Yukawa interactions are composed of non-invariant terms alone. In other case, an accidental flavor symmetry \( G_F' \) appears partially, and \( \mathcal{L}^{\text{quark}}_{\text{SM}} \) contains invariant and non-invariant parts constructed from reducible multiplets.

As remnants of \( G_F \) in \( \mathcal{L}^{\text{quark}}_{\text{BSM}} \) or an accidentalness of \( G_F' \) in \( \mathcal{L}^{\text{quark}}_{\text{SM}} \), the kinetic coefficients \( k_{ij}^{(x)} (x = q, u, d) \) and the Yukawa couplings \( (y_1)_{ij} \) and \( (y_2)_{ij} \), in general, consist of flavor-symmetric and breaking parts and are written as

\[
\begin{align*}
k_{ij}^{(x)} &= k_{1}^{(x)} \delta_{ij} + k_{2}^{(x)} S_{ij} + \sum_{b_x} k_{3}^{(b_x)} T_{ij}^{(b_x)}, \\
(y_1)_{ij} &= \sum_{a_1} y_{a_1} S_{ij}^{(a_1)} + \sum_{b_1} \Delta y_{b_1} T_{ij}^{(b_1)}, \\
(y_2)_{ij} &= \sum_{a_2} y_{a_2} S_{ij}^{(a_2)} + \sum_{b_2} \Delta y_{b_2} T_{ij}^{(b_2)}.
\end{align*}
\]

(30)

(31)

Here, terms containing \( \delta_{ij} \) and \( S_{ij}^{(A)} (A = x, a_1, a_2) \) are flavor-symmetric parts, (strictly speaking, flavor-dependent symmetric ones except for flavor-independent ones). The \( S_{ij}^{(A)} \) are \( 3 \times 3 \) matrices (whose components take values of at most \( O(1) \)) that satisfy the following relations from the \( G_F \) or \( G_F' \) invariance:

\[
\begin{align*}
F_L S^{(q)} F_L &= S^{(q)}, \\
F_R^{(u)} S^{(u)} F_R &= S^{(u)}, \\
F_R^{(d)} S^{(d)} F_R &= S^{(d)}, \\
e^{-i\theta} F_L S^{(a_1)} F_R &= S^{(a_1)}, \\
e^{i\theta} F_L S^{(a_2)} F_R &= S^{(a_2)}.
\end{align*}
\]

(32)

(33)

Note that several \( S^{(a_1)} \)s can exist, for example, in the case that \( q' \) is a singlet but \( u' \) is a non-singlet of \( G_F \). Terms containing \( T_{ij}^{(B)} (B = b_x, b_1, b_2) \) are breaking ones. The \( T_{ij}^{(B)} \) are \( 3 \times 3 \) matrices (whose components take values of at most \( O(1) \)). For details, terms containing \( T_{ij}^{(b_x)} \) are \( H_F^x \) invariant ones and those containing \( T_{ij}^{(b_1)} \) and \( T_{ij}^{(b_2)} \) are \( H_F^y \) invariant ones. In the absence of terms containing \( T_{ij}^{(b_x)} \), there should exist those containing \( T_{ij}^{(b_1)} \).
and \(T_{ij}^{(b_2)}\) but no flavor symmetries must survive, from the fact that there are no exact flavor-dependent symmetries in the SM [14,15].

The coefficients \(k_1^{(x)}, k_2^{(x)}, k_3^{(b)}, y_{a_1}^F, \Delta y_{b_1}, y_{a_2}^F\), and \(\Delta y_{b_2}\) are dimensionless parameters, and the magnitude of their values can be a touchstone of new physics by adopting Dirac's naturalness. According to this concept, we suppose that \(k_1^{(x)} = O(1)\), \(k_2^{(x)} = (1)\), and \(|y_{a_1}^F| = O(1)\) (for some \(a_1\)) under the assumption that the relating terms originate from renormalizable interactions, and, in contrast, magnitudes of other parameters can be tiny if their interactions stem from non-renormalizable ones suppressed by a power of \(M_{BSM}\). As a comment, some \(\Delta y_{b_1}\) and \(\Delta y_{b_2}\) can be sizable if the breaking scale of flavor symmetry is near \(M_{BSM}\).

In the following, we examine whether the magnitude of each component in \(k_{ij}^{(u)}\) can be at most \(O(1)\) or not, based on \(|y_{a_1}^F| = O(1)\) (for some \(a_1\)).

By inserting the first relation of Eq. (31) into Eq. (19), we obtain the relation:

\[
k_{ij}^{(u)} = \sum_{a_1,a_1'} y_{a_1}^F y_{a_1'}^F \left( S^{(a_1')} W^{(u)} + \left(y_{a_1}^F \right)^{2 \text{diag}} W^{(u)} S^{(a_1)} \right)_{ij}
\]

\[
+ \sum_{b_1,b_1'} \Delta y_{b_1} \Delta y_{b_1'} \left( T^{(b_1')} W^{(u)} + \left(y_{a_1}^F \right)^{2 \text{diag}} W^{(u)} T^{(b_1)} \right)_{ij}
\]

\[
+ \sum_{a_1,b_1} y_{a_1}^F \Delta y_{b_1} \left( T^{(b_1)} W^{(u)} + \left(y_{a_1}^F \right)^{2 \text{diag}} W^{(u)} S^{(a_1)} \right)_{ij} + \text{h.c.},
\]

and need the conditions:

\[
y_{a_1}^F \left( W^{(u)} S^{(a_1)} \right)_{ij} \leq O(y_u), \quad y_{a_1}^F \left( W^{(u)} S^{(a_1)} \right)_{2j} \leq O(y_c), \quad y_{a_1}^F \left( W^{(u)} S^{(a_1)} \right)_{3j} \leq O(y_t),
\]

\[
\Delta y_{b_1} \left( W^{(u)} T^{(b_1)} \right)_{ij} \leq O(y_u), \quad \Delta y_{b_1} \left( W^{(u)} T^{(b_1)} \right)_{2j} \leq O(y_c), \quad \Delta y_{b_1} \left( W^{(u)} T^{(b_1)} \right)_{3j} \leq O(y_t),
\]

in order to make the magnitudes of \(k_{ij}^{(u)}\) at most \(O(1)\), unless any cancellations occur among several contributions. If the magnitude of \(\left( W^{(u)} T^{(b_1)} \right)_{ij} \) is \(O(1)\), the conditions (36) fulfill with \(\Delta y_{b_1} = O(y_u)\). In the case that the magnitude of \(\left( W^{(u)} T^{(b_1)} \right)_{ij} \) is \(O(y_u/y_c)\), that of \(\Delta y_{b_1} \) can be \(O(y_c)\). Furthermore, in the case that the magnitude of \(\left( W^{(u)} T^{(b_1)} \right)_{ij} \) and \(\left( W^{(u)} T^{(b_1)} \right)_{2j} \) are \(O(y_u/y_t)\) and \(O(y_c/y_t)\), respectively, that of \(\Delta y_{b_1} \) can be \(O(y_t)\). This suggests that a mass hierarchy of up-type quarks can be realized by the breaking part alone.

Hereafter, we consider a case with \(|y_{a_1}^F| = O(1)\) and \(\left( W^{(u)} S^{(a_1)} \right)_{3j} = O(1)\) (for some \(a_1\)) under the assumption that \(y_{a_1}^F S^{(a_1)}\) comes from a renormalizable interaction. Then, we find that \(S^{(a_1)} = \left( W^{(u)} \right)^{-1}_{ij} \left( W^{(u)} S^{(a_1)} \right)_{3j}\) up to \(O(y_c)\), in the case that the magnitude of each component of \(W^{(u)}\) is \(O(1)\), from the conditions (35). In most cases, tiny quantities of \(O(y_c)\) and \(O(y_u)\) can appear from symmetry breaking effects, and hence we suppose that \(S^{(a_1)} = \left( W^{(u)} \right)^{-1}_{ij} \left( W^{(u)} S^{(a_1)} \right)_{3j}\) holds exactly in a flavor-symmetric limit. In this
case, after a suitable unitary transformation is performed, $S^{(a_1)\dagger}S^{(a_1)}$ is diagonalized as

$$U \left( S^{(a_1)\dagger} S^{(a_1)} \right) U^\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & s \end{pmatrix},$$

where $s$ is given by

$$s = \sum_{k=1}^{3} \left| \left( W^{(u)^{-1}} \right)_{k3} \right|^2 \sum_{l=1}^{3} \left| \left( W^{(u)} S^{(a_1)} \right)_{3l} \right|^2.$$  \hspace{1cm} (38)

This implies that $S^{(a_1)}$ is a $3 \times 3$ matrix whose rank is one.

In the same way, by inserting the second relation of Eq. (31) into Eq. (20), we obtain the relation:

$$k_{ij}^{(d)} = \sum_{a_2,a_2'} y_{a_2}^F y_{a_2'}^F \left( S^{(a_2')} W^{(d)} \right)_{ij}^\dagger \left( y_{\text{diag}}^{(d)-1} \right)_{ij} W^{(d)} S^{(a_2)} + \sum_{b_2,b_2'} \Delta y_{b_2} \Delta y_{b_2'}^* W^{(d)} \left( y_{\text{diag}}^{(d)-1} \right)_{ij} W^{(d)} T^{(b_2)} + \sum_{a_2,b_2} y_{a_2}^F \Delta y_{b_2} W^{(d)} \left( y_{\text{diag}}^{(d)-1} \right)_{ij} W^{(d)} S^{(a_2)} + \text{h.c.},$$

and need the conditions:

$$y_{a_2}^F \left( W^{(d)} S^{(a_2)} \right)_{1j} \leq O(y_d), \quad y_{a_2}^F \left( W^{(d)} S^{(a_2)} \right)_{2j} \leq O(y_s), \quad y_{a_2}^F \left( W^{(d)} S^{(a_2)} \right)_{3j} \leq O(y_b),$$

$$\Delta y_{b_2} \left( W^{(d)} T^{(b_2)} \right)_{1j} \leq O(y_d), \quad \Delta y_{b_2} \left( W^{(d)} T^{(b_2)} \right)_{2j} \leq O(y_s), \quad \Delta y_{b_2} \left( W^{(d)} T^{(b_2)} \right)_{3j} \leq O(y_b).$$

in order to make the magnitudes of $k_{ij}^{(d)}$ at most $O(1)$, unless any cancellations occur among several contributions. If the magnitude of $\left( W^{(d)} T^{(b_2)} \right)_{1j}$ is $O(1)$, the conditions fulfill with $|\Delta y_{b_2}| = O(y_d)$. In the case that the magnitude of $\left( W^{(d)} T^{(b_2)} \right)_{1j}$ is $O(y_d/y_s)$, that of $|\Delta y_{b_2}|$ can be $O(y_s)$. Furthermore, in the case that the magnitude of $\left( W^{(d)} T^{(b_2)} \right)_{1j}$ and $\left( W^{(d)} T^{(b_2)} \right)_{2j}$ are $O(y_d/y_b)$ and $O(y_s/y_b)$, respectively, that of $|\Delta y_{b_2}|$ can be $O(y_b)$. This also suggests that a mass hierarchy of down-type quarks can be realized by the breaking part alone.

From (40), it is conjectured that the hierarchy of $y_{a_2}^F S^{(a_2)}$ can also stem from non-renormalizable interactions, i.e., $|y_{a_2}^F| \leq O(y_b)$, if $\left( W^{(d)} S^{(a_2)} \right)_{3j} = O(1)$. For instance, a down-type quark Yukawa coupling matrix can be obtained by the Froggatt-Nielsen mechanism of a flavor-independent charge with $F_L = e^{i\phi_1} I$ and $F_R^{(d)} = e^{i\phi_2^{(d)}} I$ from a non-renormalizable term $(Y_2)_{ij} \bar{q}_{L_i}^{(d)} \phi d_{R_j}^{(d)}$, where $(Y_2)_{ij}$ contains $(\phi/\Lambda)^n$ [6]. Here, $\phi$ is the SM-singlet scalar field with the VEV of $O(M_{\text{BSM}})$ and $\Lambda$ is a cutoff scale bigger than $M_{\text{BSM}}$. If the magnitude of
\((W^{(d)}S^{(a_2)})_{3j}\) is much bigger than that of \((W^{(d)}S^{(a_2)})_{1j}\) and \((W^{(d)}S^{(a_2)})_{2j}\) and the magnitude of each component of \(W^{(d)}\) is \(O(1)\), \(S^{(a_2)}_{1j} = \left(W^{(d)-1}\right)_{i3} \left(W^{(d)}S^{(a_2)}\right)_{3j}\) up to \(O(y_s)\). In the case that \(S^{(a_2)}_{ij} = \left(W^{(d)-1}\right)_{i3} \left(W^{(d)}S^{(a_2)}\right)_{3j}\) holds exactly, \(S^{(a_2)}_{ij}S^{(a_2)}\) is also diagonalized as the same form of \((37)\) with
\[
S = \sum_{k=1}^{3} \left| \left(W^{(d)-1}\right)_{k3} \right|^2, \tag{42}
\]
and \(S^{(a_2)}\) is also a \(3 \times 3\) matrix whose rank is one.

Under the assumption that \(|y_{a_1}^F| = O(1)\), \((W^{(u)}S^{(a_1)})_{3j} = O(1)\) and \(S^{(a_1)} = S^{(a_2)}\), the magnitude of \(|y_{a_2}^F|\) is estimated as follows. Using Eq.\((23)\), we obtain the relation:
\[
y_{a_2}^F W^{(d)} S^{(a_2)} = y_{a_2}^F V_{KM} W^{(u)} S^{(a_1)} = y_{a_2}^F \left( \begin{array}{ccc}
O(\lambda^3) & O(\lambda^3) & O(\lambda^3) \\
O(\lambda^2) & O(\lambda^2) & O(\lambda^2) \\
O(1) & O(1) & O(1)
\end{array} \right), \tag{43}
\]
where \(\lambda = \sin \theta_C \simeq 0.225\) (\(\theta_C\) is the Cabibbo angle \([26]\)), and we use the Wolfenstein parametrization \([27]\). From the conditions \((40)\) and Eq.\((43)\), we derive the inequality:
\[
|y_{a_2}^F| \leq O(y_d^F \lambda^3) = O(10^{-3}). \tag{44}
\]
In this way, we have obtained the following properties.

- The magnitude of \(y_{a_1}^F\) can be \(O(1)\) and some \(y_{a_1}^F S^{(a_1)}_{ij}\) can appear from a non-renormalizable interaction in a theory beyond the SM.
- The magnitudes of \(y_{a_2}^F\) can be \(O(y_b) = O(10^{-2})\) or less than that, and \(y_{a_2}^F S^{(a_2)}_{ij}\) can appear from non-renormalizable interactions through the Froggatt-Nielsen mechanism.
- Some \(S^{(a_1)}_{ij}\) and \(S^{(a_2)}_{ij}\) can be rank-one matrices.
- The magnitude of \(y_{a_2}^F\) can be \(O(y_d^F \lambda^3) = O(10^{-3})\) or less than that, in the case with \(|y_{a_1}^F| = O(1)\) and \(S^{(a_1)}_{ij} = S^{(a_2)}_{ij}\).

### 3.2 Peculiarity of democratic type

If \(N_q\), \(N_u\), and \(N_d\) are given, \(k_{ij}^{(q)}\), \(k_{ij}^{(u)}\), \(k_{ij}^{(d)}\), \((y_1)_{ij}\), and \((y_2)_{ij}\) are determined by Eqs.\((11)\) and \((12)\). If \(W^{(u)}\), \((y_1)_{ij}\), and \((y_2)_{ij}\) are given, \(k_{ij}^{(q)}\), \(k_{ij}^{(u)}\), and \(k_{ij}^{(d)}\) are determined by Eqs.\((22)\), \((19)\), and \((20)\). In the following, we show that the flavor-symmetric parts of \((y_1)_{ij}\) and \((y_2)_{ij}\) and parts of \(k_{ij}^{(u)}\) and \(k_{ij}^{(d)}\) constructed from them are constrained and a democratic-type matrix takes a special position, supposing that \(W^{(u)}\) is given and Dirac’s naturalness is adopted.

Let the Yukawa couplings be divided into two parts as
\[
(y_1)_{ij} = (y_1^F)_{ij} + (y_1^F)_{ij}, \quad (y_2)_{ij} = (y_2^F)_{ij} + (y_2^F)_{ij}, \tag{45}
\]
where \((y_{1}^{F})_{ij}\) and \((y_{2}^{F})_{ij}\) are flavor-symmetric parts, and \((y_{1}^{F})_{ij}\) and \((y_{2}^{F})_{ij}\) are flavor-breaking ones. Using \((y_{1}^{F})_{ij}\) and \((y_{2}^{F})_{ij}\), we define \(\tilde{k}_{ij}^{(u)}\) and \(\tilde{k}_{ij}^{(d)}\) as

\[
\tilde{k}_{ij}^{(u)} = \left( y_{1}^{F\dagger} W^{(u)\dagger} \left( y_{\text{diag}}^{(u)\!-1} \right)^2 W^{(u)} y_{1}^{F} \right)_{ij}, \\
\tilde{k}_{ij}^{(d)} = \left( y_{2}^{F\dagger} W^{(d)\dagger} \left( y_{\text{diag}}^{(d)\!-1} \right)^2 W^{(d)} y_{2}^{F} \right)_{ij}. 
\] (46) (47)

In the case that \(k_{ij}^{(q)}\) is flavor symmetric, i.e., \(F_{l} k_{ij}^{(q)} F_{l}^{\dagger} = k_{ij}^{(q)}\), or in a flavor-symmetric limit (after neglecting the breaking parts in \(k_{ij}^{(q)}\)), \(\tilde{k}_{ij}^{(u)}\) and \(\tilde{k}_{ij}^{(d)}\) also become flavor symmetric.

From the requirements that the magnitude of each component in \(k_{ij}^{(u)}\) is at most \(O(1)\), \(\tilde{k}_{ij}^{(u)}\) contains a parameter of \(O(1)\) such as \(y_{t}\) and any tiny parameters are not included, the form of \((y_{1}^{F})_{ij}\) is constrained as

\[
y_{1}^{F} = \begin{pmatrix}
l v_{1} & m v_{1} & n v_{1} \\
l v_{2} & m v_{2} & n v_{2} \\
l v_{3} & m v_{3} & n v_{3}
\end{pmatrix}, 
\] (48)

where \(l, m,\) and \(n\) are some numbers, and \(v_{1}, v_{2},\) and \(v_{3}\) are defined by

\[
v_{1} \equiv W_{12}^{(u)} W_{23}^{(u)} - W_{13}^{(u)} W_{22}^{(u)}, \quad v_{2} \equiv W_{13}^{(u)} W_{21}^{(u)} - W_{11}^{(u)} W_{23}^{(u)}, \\
v_{3} \equiv W_{11}^{(u)} W_{22}^{(u)} - W_{12}^{(u)} W_{21}^{(u)}. 
\] (49)

Eqs. (49) are derived from the orthogonality between \((W_{11}^{(u)}, W_{12}^{(u)}, W_{13}^{(u)})\) and \((v_{1}, v_{2}, v_{3})\), and \((W_{21}^{(u)}, W_{22}^{(u)}, W_{23}^{(u)})\) and \((v_{1}, v_{2}, v_{3})\). Then, \(\tilde{k}_{ij}^{(u)}\) is written by

\[
\tilde{k}_{ij}^{(u)} = \frac{1}{y_{t}^{2}} \begin{pmatrix} W_{31}^{(u)} v_{1} + W_{32}^{(u)} v_{2} + W_{33}^{(u)} v_{3} \end{pmatrix}^{2} \begin{pmatrix} l^{2} & m l & n l \\
l m & m^{2} & n m \\
l n & m n & n^{2} \end{pmatrix}. 
\] (50)

Next, we attempt to conjecture a flavor symmetry by imposing on \(F_{R}^{(u)} \tilde{k}_{ij}^{(u)} F_{R}^{(u)\dagger} = \tilde{k}_{ij}^{(u)}\).

In the case with \(l = m = n\), \(\tilde{k}_{ij}^{(u)}\) becomes a democratic-type matrix, which is proportional to the matrix:

\[
S = \begin{pmatrix} 1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \end{pmatrix}, 
\] (51)

and \((y_{1}^{F})_{ij}\) also turns out to be the democratic-type one for \(l = m = n\) and \(v_{1} = v_{2} = v_{3}\). This form has an invariance under a discrete group such as \(S_{3}\), where fields are transformed as a 3D reducible representation.\(^{3}\)

\(^{3}\) Actually, the permutations of reducible triplet

\[^{3}\] Based on an \(S_{3}\) invariant Kähler potential containing the democratic form and Yukawa couplings with the democratic form and small \(S_{3}\) breaking ones, it was pointed out that the heavy top quark mass can be attributed to a singular normalization of its kinetic term [20].
are performed by the matrices:

\[
U^\alpha = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{pmatrix}, \quad \begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1 \\
\end{pmatrix}, \quad \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0 \\
\end{pmatrix}, \quad \begin{pmatrix}
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
\end{pmatrix}, \quad \begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0 \\
\end{pmatrix}, \quad \begin{pmatrix}
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
\end{pmatrix}
\]

(52)

and \(S\) is constructed as

\[
S = \frac{1}{2} \sum_{\alpha=1}^{6} U^\alpha.
\]

(53)

The invariance is understood from the relations for any elements \(U^\beta\) (\(\beta = 1, \cdots, 6\)):

\[
\left(\sum_{\alpha=1}^{6} U^\alpha\right) U^\beta = U^\beta \left(\sum_{\alpha=1}^{6} U^\alpha\right) = \sum_{\alpha=1}^{6} U^\alpha,
\]

(54)

where \(U^\alpha U^\beta \neq U^\alpha U^\gamma\) and \(U^\beta U^\alpha \neq U^\beta U^\gamma\) for \(U^\alpha \neq U^\gamma\).

In the case with \(l = m\) and \(n \neq l\), \(\tilde{k}^{(u)}\) is proportional to the matrix:

\[
S = \begin{pmatrix}
1 & 1 & n \\
1 & 1 & n \\
n & n & n^2/l \\
\end{pmatrix},
\]

(55)

and this form has an invariance under a discrete group such as \(S_2\). For \(l \neq m\), \(m \neq n\) and \(n \neq l\), \(F^{(u)}_R\) is proportional to the identity matrix, and there is no flavor-dependent symmetry in \(\tilde{k}^{(u)}\).

In the same way, from the requirements that the magnitude of each component in \(k^{(d)}_{ij}\) is at most \(O(1)\) and any tiny parameters except for \(y_b\) are not included in \(\tilde{k}^{(d)}_{ij}\), the form of \((y^F_2)_{ij}\) is constrained as

\[
y^F_2 = \begin{pmatrix}
p w_1 & q w_1 & r w_1 \\
p w_2 & q w_2 & r w_2 \\
p w_3 & q w_3 & r w_3
\end{pmatrix},
\]

(56)

where \(p, q, r\) are some numbers, and \(w_1, w_2,\) and \(w_3\) are defined by

\[
w_1 \equiv W^{(d)}_{12} W^{(d)}_{23} - W^{(d)}_{13} W^{(d)}_{22}, \quad w_2 \equiv W^{(d)}_{13} W^{(d)}_{21} - W^{(d)}_{11} W^{(d)}_{23},
\]

\[
w_3 \equiv W^{(d)}_{11} W^{(d)}_{22} - W^{(d)}_{12} W^{(d)}_{21}.
\]

(57)

Then, \(\tilde{k}^{(d)}_{ij}\) is written by

\[
\tilde{k}^{(d)}_{ij} = \frac{1}{y^F_2} \left[ W^{(d)}_{31} w_1 + W^{(d)}_{32} w_2 + W^{(d)}_{33} w_3 \right]^2 \begin{pmatrix}
p^2 & q & r p \\
p q & q^2 & r q \\
p r & q r & r^2
\end{pmatrix}.
\]

(58)
Note that $|p|, |q|, |r| \leq O(y_b)$ in order to make the magnitude of $\tilde{k}_{ij}^{(d)}$ at most $O(1)$. We consider a flavor symmetry on down-type quarks. For $p = q = r$, $\tilde{k}_{ij}^{(d)}$ becomes the democratic one. For $v_1 = v_2 = v_3$, $w_1 = w_2 = w_3$ does not hold because of Eq. (23) and then $(y^F_{2})_{ij}$ cannot be a democratic one.

Finally, we give a comment on a case that $(y^F_{2})_{ij}$ is a democratic one $(y^F_{2})_{ij} = \tilde{y}_{2}^{F} S_{ij}$ with a complex number $\tilde{y}_{2}^{F}$. In this case, $\tilde{k}_{ij}^{(d)}$ also becomes the democratic one:

$$
\tilde{k}_{ij}^{(d)} = |\tilde{y}_{2}^{F}|^2 \left( SW^{(u)*} V_{\text{KM}} \left( y_{\text{diag}}^{(d)-1} \right)^2 V_{\text{KM}}^{*} W^{(u)} S \right)_{ij}
$$

$$
= O(\lambda^6/\lambda_3^2) |\tilde{y}_{2}^{F}|^2 \left( W_{31}^{(u)} + W_{32}^{(u)} + W_{33}^{(u)} \right)^2 S_{ij}. \quad (59)
$$

The following inequality is required

$$
|\tilde{y}_{2}^{F}| \leq O(y_d/\lambda_3) = O(10^{-3}) \quad (60)
$$

to make the magnitude of each component in $\tilde{k}_{ij}^{(d)}$ at most $O(1)$.

In this way, we find that the democratic-type one takes a special position, because it is related to a flavor symmetry such as $S_3$ and is compatible with Dirac's naturalness.

4 Conclusions and discussions

We have explored the flavor structure in the SM under the assumption that flavor symmetries exist in a theory beyond the SM, and have chased after their properties, using a bottom-up approach. We have reacknowledged that a flavor-symmetric part of Yukawa coupling matrix can be realized by a rank-one matrix and a democratic-type one occupies a special position, based on Dirac’s naturalness. Hence, it would be important to explore the origin of the democratic-type matrix. There is a possibility that it is generated by the VEVs of flavons. However, a toy model presented in [16] has a problem that it contains an unnatural fine-tuning among parameters based on a perturbative analysis. A non-perturbative effect can play a crucial role to the derivation of a specific type of terms.

There are limitations on our bottom-up approach, without any powerful principle and concept. It would be desirable to combine use of the bottom-up and top-down ones, keeping an eye on the possibility of grand unification and supersymmetry (SUSY). On a grand unification based on $SO(10)$ and $E_6$, we need an extension of Yukawa sector. Without extra matters and/or extra interactions, it is difficult to derive realistic fermion masses and flavor mixing matrices in the case that a flavor-symmetric part dominates. The reason is as follows. Both $u'_{R}$ and $d'_{R}$ belong to a common multiplet of $SO(10)$ and $E_6$, they should be transformed as a same representation of same flavor group, and their kinetic coefficients have a common one, i.e., $S^{(a_1)} = S^{(a_2)}$. Then, a common Yukawa coupling constant of $O(1)$ is not compatible with $S^{(a_1)} = S^{(a_2)}$. The SUSY can compensate for the lack of information on the flavor structure, that is, a pattern of soft SUSY breaking terms can provide useful information. It would be worth studying the flavor structure of the SM and its underlying theory by paying close attention to both matter kinetic terms and various interaction terms.
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