Atmospheric Neutrino Oscillations in Three-Flavor Neutrinos. II

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We analyze the atmospheric neutrino experiments of SuperKamiokande (535 days) using the three-flavor neutrino framework with the mass hierarchy \( m_1 \approx m_2 \ll m_3 \). We study the event ratios of sub-GeV, multi-GeV and upward through-going muons zenith angle distributions. Taking into account atmospheric and terrestrial experiments, we obtain the allowed region of mass and mixing parameters \( \Delta m_{21}^2, \sin^2 2\theta_{12}, \Delta m_{23}^2, \theta_{13}, \) and \( \theta_{23} \). The mass parameter \( \Delta m_{23}^2 \) is restricted to \( 0.01 \text{ eV}^2 \) – \( 0.0002 \text{ eV}^2 \). For mixing parameters, there is no difference between the large \( \theta_{12} \) angle solution and the small one, and \( 0^\circ < \theta_{13} < 23^\circ \) and \( 29^\circ < \theta_{23} < 61^\circ \) for \( \Delta m_{23}^2 = 0.002 \text{ eV}^2 \), \( 4^\circ < \theta_{13} < 22^\circ \) and \( 38^\circ < \theta_{23} < 54^\circ \) for \( \Delta m_{23}^2 = 0.0002 \text{ eV}^2 \).

§1. Introduction

SuperKamiokande experiments 1) have observed a definite atmospheric neutrino anomaly and confirmed the hypothesis of neutrino oscillation. In their two-flavor mixing analyses of the sub-GeV and multi-GeV zenith angle distribution, it has been found that the \( \nu_\mu \leftrightarrow \nu_\tau \) oscillation is preferred over the \( \nu_\mu \leftrightarrow \nu_e \) oscillation, and the range of mass parameter \( \Delta m^2 \) is from \( 3 \times 10^{-4} \text{ eV}^2 \) to \( 0.8 \times 10^{-2} \text{ eV}^2 \). 1)

Observing neutrino oscillations is one of the most important experiments suggesting new physics beyond the Standard Model. To construct a model beyond the Standard Model, it is necessary to confirm the mixing and mass parameters of three neutrinos in the low energy region, although there is no restriction on number of neutrinos in the very high energy region. 2) The three-flavor neutrino scenario 3) is found to be consistent with the large \( \nu_\mu \leftrightarrow \nu_\tau \) oscillation and the solar neutrino anomaly 4) and terrestrial neutrino experiments. 5)-7) K2K, MINOS and CERN-Gran Sasso long-baseline experiments 8) are in preparation to confirm the \( \nu_\mu \leftrightarrow \nu_e \) and \( \nu_\mu \leftrightarrow \nu_\tau \) oscillation precisely. Then it becomes important to examine the mass and mixing parameters in the three-flavor neutrinos framework 3), 9) precisely.

In this paper, we analyze the SuperKamiokande atmospheric neutrino experiment (535 days) of sub-GeV, multi-GeV and upward through-going muons zenith angle distributions in the three-flavor neutrino framework with a hierarchy of neutrino masses \( m_1 \approx m_2 \ll m_3 \). In a previous work, 10) we analyzed the zenith angle distributions in the SuperKamiokande atmospheric neutrino experiment (325.8 days). 11) In that work, we analyzed the double ratios of the sub-GeV and multi-GeV zenith angle distributions neglecting the matter effects of the Earth and the difference between incident neutrinos and detected charged leptons.

In the three-flavor neutrino framework with the hierarchy \( m_1 \approx m_2 \ll m_3 \), there
are the 5 parameters $\theta_{12}$, $\theta_{13}$, $\theta_{23}$, $\Delta m_{12}^2 = m_1^2 - m_2^2$, and $\Delta m_{23}^2 = m_3^2 - m_2^2$ concerned with neutrino oscillation. For solar neutrino deficit, the MSW solution (12, 13) taking into account the matter effects predicts the large angle solution $\sin^2 2\theta_{12} = 0.6 - 0.9$, $\Delta m_{12}^2 = 4 \times 10^{-6} - 7 \times 10^{-5}$ eV$^2$ and small angle solution $\sin^2 2\theta_{12} = 0.003 - 0.01$, $\Delta m_{12}^2 = 3 \times 10^{-6} - 1.2 \times 10^{-5}$ eV$^2$ for $\theta_{13} = 0^\circ - 20^\circ$, and these large and small angle solutions are merged for $\theta_{13} = 25^\circ - 50^\circ$. The vacuum solution for solar neutrinos is obtained as $\Delta m_{12}^2 \sim 10^{-10}$ eV$^2$. In terrestrial neutrino experiments, Fogli et al. (15) have thoroughly analyzed and obtained the allowed regions in the $\tan^2 \theta_{13}$-$\tan^2 \theta_{23}$ plane for various values of the mass parameter $\Delta m_{23}^2$. We also analyzed the terrestrial neutrino experiments (10) and obtained results similar to those of Fogli et al. The allowed region in the $\tan^2 \theta_{13}$-$\tan^2 \theta_{23}$ plane was more restricted for larger values of $\Delta m_{23}^2$, and the allowed region spread out for smaller values. In this paper, we combine the results obtained from the atmospheric neutrino experiments of SuperKamiokande and the terrestrial experiments to obtain the allowed regions for mixing and mass parameters in three-flavor neutrinos.

§2. Neutrino oscillation in three-flavor neutrinos

The unitary matrix $U$ representing the neutrino mixing is defined as

$$v_{\alpha} = \sum_{\beta=1}^{3} U_{\alpha \beta} v_{\beta}, \quad \alpha, \beta = e, \mu, \tau,$$

where the states $v_{\alpha}$ and $v_{\beta}$ are the flavor and mass eigenstate of neutrinos, respectively. This mixing matrix corresponds to the CKM matrix $U_{\text{CKM}}^\dagger$ for the quark sector. We parameterize the unitary matrix neglecting the $CP$ violation phases as

$$U = \exp(i\theta_{23}\lambda_7) \exp(i\theta_{13}\lambda_5) \exp(i\theta_{12}\lambda_2)$$

$$= \begin{pmatrix}
    c_{\nu_{e}}^\nu_{\mu} & c_{\nu_{e}}^\nu_{\tau} & s_{\nu_{e}}^\nu_{\mu} \\
    -s_{\nu_{e}}^\nu_{\mu} c_{\nu_{e}} & c_{\nu_{e}}^\nu_{\tau} & s_{\nu_{e}}^\nu_{\tau} \\
    s_{\nu_{e}}^\nu_{\mu} c_{\nu_{e}} & -s_{\nu_{e}}^\nu_{\tau} & s_{\nu_{e}}^\nu_{\tau}
\end{pmatrix},$$

$$c_{ij}^\nu = \cos \theta_{ij}^\nu, \quad s_{ij}^\nu = \sin \theta_{ij}^\nu,$$

where the $\lambda_i$ are the Gell-Mann matrices.

The probabilities for transitions the $v_{\alpha} \rightarrow v_{\beta}$ are written as

$$P(v_{\alpha} \rightarrow v_{\beta}) = |\langle v_{\beta}(t)|v_{\alpha}(0)\rangle|^2 = \delta_{\alpha \beta} + p_{12}^{12} + p_{12}^{31} S_{12} + p_{23}^{23} S_{23} + p_{31}^{31} S_{31},$$

$$p_{12}^{12} = -2\delta_{\alpha \beta}(1 - 2U_{\alpha \beta}^2) + 2(U_{\alpha \beta}^2 U_{\gamma \beta}^2 + U_{\alpha \gamma}^2 U_{\beta \gamma}^2 - U_{\alpha \gamma}^2 U_{\beta \beta}^2),$$

$$p_{23}^{23} = -2\delta_{\alpha \beta}(1 - 2U_{\alpha \beta}^2) + 2(-U_{\alpha \beta}^2 U_{\gamma \beta}^2 + U_{\alpha \gamma}^2 U_{\beta \beta}^2 - U_{\alpha \gamma}^2 U_{\beta \gamma}^2),$$

$$p_{31}^{31} = -2\delta_{\alpha \beta}(1 - 2U_{\alpha \beta}^2) + 2(U_{\alpha \beta}^2 U_{\gamma \beta}^2 - U_{\alpha \gamma}^2 U_{\beta \gamma}^2 - U_{\alpha \gamma}^2 U_{\beta \beta}^2).$$
where $S_{ij}$ is the term representing the neutrino oscillation:

$$S_{ij} = \sin^2 1.27 \frac{\Delta m_{ij}^2}{E} L.$$

Here $\Delta m_{ij}^2 = |m_i^2 - m_j^2|$, and $E$ and $L$ are measured in units of eV$^2$, GeV and km, respectively.

The values of the neutrino masses are not known precisely, but two mass parameters are necessary to account for the solar neutrino experiments and the atmospheric neutrino experiments. In the former, mass parameter $\Delta m^2$ is obtained as $10^{-4}$ to $10^{-5}$ eV$^2$ (4), (13), (16) or $\sim 10^{-10}$ eV$^2$ (14) and in the latter, $\Delta m^2$ is obtained as $10^{-1}$ to $10^{-3}$ eV$^2$ (11), (16).

Then it seems that the most reasonable mass hierarchy is that in which the lower two masses of the three neutrinos are very close and the third is rather far from them. Thus we assume that the three neutrino masses have a mass hierarchy obeying

$$m_1 \approx m_2 \ll m_3. \tag{5}$$

With the hierarchy Eq. (5), $\Delta m_{12}^2 \ll \Delta m_{23}^2 \simeq \Delta m_{13}^2$, Eq. (3) for the transition probabilities $P(\nu_\alpha \to \nu_\beta)$ can be rewritten as

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 2(1 - 2U_{i_3}^2 - U_{i_1}^2 - U_{i_2}^2 + U_{i_1}^2 + U_{i_2}^2)S_{12} - 4U_{i_1}^2(1 - U_{i_3}^2)S_{23},$$

$$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha) = 2(U_{i_1}^2U_{i_3}^2 + U_{i_2}^2U_{i_3}^2 - U_{i_3}^2U_{i_1}^2)S_{12} + 4U_{i_3}^2U_{i_1}^2S_{23}. \tag{6}$$

In the atmospheric neutrino experiments, especially in the sub-GeV ones, the matter of the Earth has an important effects. Matter effects are induced by the quantity $A = 2\sqrt{2}E G_F N_e$. In the Earth, $N_e = 2.4$ mol/cm$^3$ for the mantle, and $N_e = 5.5$ mol/cm$^3$ for core. Then we approximate the density of the Earth to be uniform as $N_e = 3$ mol/cm$^3$. In this case $A \sin 2\theta_{13}/2m_3^2 \ll 1$, and we can approximate the mixing matrix $U$ as

$$U^M = \exp(i\lambda_7 \theta_{23}) \exp(i\lambda_5 \theta_{13}) \exp(i\lambda_2 \theta_{12}^M), \tag{7}$$

where

$$\sin 2\theta_{12}^M = \frac{\Delta m_{12}^2}{\Delta m_{12}^{M2}} \sin 2\theta_{12},$$

$$\Delta m_{12}^{M2} = m_2^{M2} - m_1^{M2}, \quad \Sigma = m_1^2 + m_2^2,$$

$$m_{1,2}^{M2} = \frac{1}{2} \left\{ (\Sigma + A \cos^2 \theta_{13}) \mp \sqrt{(A \cos^2 \theta_{13} - \Delta m_{12}^2 \cos 2\theta_{12})^2 + (\Delta m_{12}^2 \sin 2\theta_{12})^2} \right\},$$

$$m_3^{M2} = m_3^2 + A \sin^2 \theta_{13}.$$
compared with the angle in observing all of the Earth, and $A$ is small compared with $m^2$. In this approximation, we can take the expressions for the transition probability with matter effects as

$$
P^M(\nu_\alpha \rightarrow \nu_\beta) = 1 - 2(1 - 2U^2_{\alpha\beta} - U^2_{\alpha3} - U^2_{\beta3} + U^2_{\alpha3} + U^2_{\beta3})S^M_{12} - 4U^2_{\alpha3}U^2_{\beta3}S^M_{23},
$$

$$
P^M(\nu_\beta \rightarrow \nu_\alpha) = 2(U^2_{\beta2}U^2_{\alpha1} + U^2_{\beta3}U^2_{\alpha3} - U^2_{\alpha3}U^2_{\beta3})S^M_{12} + 4U^2_{\alpha3}U^2_{\beta3}S^M_{23},
$$

where

$$
S^M_{ij} = \sin^2 \frac{\Delta m_{ij}^2}{2E} - L.
$$

§3. Numerical analyses of neutrino oscillations

3.1. Atmospheric neutrinos

Evidence for an anomaly in atmospheric neutrino experiments was pointed out in reports of the Kamiokande collaboration\textsuperscript{17,18} and IMB collaboration\textsuperscript{19} using water-Cherenkov experiments. More recently, reports of SuperKamiokande collaboration\textsuperscript{3.1} have provided more precise results on the anomaly in atmospheric neutrinos. In our previous papers,\textsuperscript{10} we have analyzed the double ratios $R(\mu/e) \equiv R_{\text{Exp}}(\mu/e)/R_{\text{MC}}(\mu/e)$ of atmospheric neutrinos. In this work, we analyze the event ratio $N_{\text{Exp}}(l_\alpha)/N_{\text{MC}}(l_\alpha)$, where $l_\alpha$ represents $\mu$ and $e$.

The zenith angle distributions of sub-GeV, multi-GeV events and upward through-going $\mu$ fluxes are taken from the SuperKamiokande 535 day experiments.\textsuperscript{1} The data are tabulated in Table I. These values are taken from the experimental event data and Monte-Carlo simulations which are given graphically in Ref. 1). The $\mu$-like events include the fully contained and partially contained events. Errors represent statistical ones only. In the above data, sub-GeV experiments detect the visible-energy less than 1.33 GeV.

The zenith angle $\theta$ dependent events $dN_{\text{Exp}}(l_\alpha)/d\cos\theta$ and $dN_{\text{MC}}(l_\alpha)/d\cos\theta$ are defined as

$$
\frac{dN_{\text{Exp}}(l_\alpha)}{d\cos\theta} = \sum_\beta \int \epsilon_\alpha(E_\alpha)\sigma_\alpha(E_\nu, E_\alpha, \psi)F_\beta(E_\nu, \theta - \psi)P^M(\nu_\beta \rightarrow \nu_\alpha)dE_\alpha dE_\nu d\cos\psi,
$$

$$
\frac{dN_{\text{MC}}(l_\alpha)}{d\cos\theta} = \int \epsilon_\alpha(E_\alpha)\sigma_\alpha(E_\nu, E_\alpha, \psi)F_\alpha(E_\nu, \theta - \psi)dE_\alpha dE_\nu d\cos\psi,
$$

where the summation $\sum_\beta$ is taken over $\nu_\mu$ and $\nu_e$. Processes of $\bar{\nu}_\mu$ and $\bar{\nu}_e$ are contained in these expressions. $\epsilon_\alpha(E_\alpha)$ is the detection efficiency of the detector for $\alpha$-type charged lepton with energy $E_\alpha$ and $\sigma_\alpha(E_\nu, E_\alpha, \psi)$ is the differential cross section of scattering $l_\alpha$ with energy $E_\nu$ of an incident $\nu_\alpha$ with energy $E_\alpha$, where the angle $\psi$ is the scattering angle relative to the direction of the incident $\nu_\alpha$. $F_\beta(E_\nu, \theta)$ is the incident $\nu_\beta$ flux with energy $E_\nu$ at the zenith angle $\theta$. $P^M(\nu_\alpha \rightarrow \nu_\beta)$ is the transition probability with the matter effects expressed in Eq. (8) and it depends on the energy $E_\nu$ and the distance $L$. This distance depends on the zenith angle $\theta$ as...
Table I. e-like, µ-like atmospheric neutrino data and upward through-going muons fluxes of SuperKamiokande experiments (535 day data). These values are the ratios of experimental data and Monte-Carlo simulations which are obtained from the graphs in Ref. 1). µ-like events include fully contained and partially contained events, and errors represent statistical ones only.

| Sub-GeV data |  |  |  |
|--------------|---|---|---|
| cos θ range  | e-like event ratio N_{Exp}/N_{MC} | µ-like event ratio N_{Exp}/N_{MC} |  |
| -1.0 - -0.6  | 1.37 ± 0.09 | 0.56 ± 0.04 |  |
| -0.6 - -0.2  | 1.12 ± 0.08 | 0.71 ± 0.05 |  |
| -0.2 - 0.2   | 1.18 ± 0.08 | 0.74 ± 0.05 |  |
| 0.2 - 0.6    | 1.05 ± 0.08 | 0.86 ± 0.06 |  |
| 0.6 - 1.0    | 1.15 ± 0.08 | 0.82 ± 0.05 |  |

| Multi-GeV data |  |  |  |
|----------------|---|---|---|
| cos θ range    | e-like event ratio N_{Exp}/N_{MC} | µ-like event ratio N_{Exp}/N_{MC} |  |
| -1.0 - -0.6    | 1.35 ± 0.21 | 0.56 ± 0.07 |  |
| -0.6 - -0.2    | 1.10 ± 0.16 | 0.57 ± 0.07 |  |
| -0.2 - 0.2     | 1.13 ± 0.15 | 0.79 ± 0.07 |  |
| 0.2 - 0.6      | 1.42 ± 0.18 | 1.02 ± 0.09 |  |
| 0.6 - 1.0      | 1.18 ± 0.20 | 1.03 ± 0.10 |  |

| Upward through-going µ data |  |  |  |
|-----------------------------|---|---|---|
| cos θ range                 | µ flux ratio N_{Exp}/N_{MC} |  |  |
| -1.0 - -0.9                 | 0.77 ± 0.14 |  |  |
| -0.9 - -0.8                 | 0.79 ± 0.12 |  |  |
| -0.8 - -0.7                 | 0.58 ± 0.11 |  |  |
| -0.7 - -0.6                 | 0.96 ± 0.13 |  |  |
| -0.6 - -0.5                 | 0.73 ± 0.10 |  |  |
| -0.5 - -0.4                 | 0.80 ± 0.10 |  |  |
| -0.4 - -0.3                 | 0.74 ± 0.10 |  |  |
| -0.3 - -0.2                 | 0.92 ± 0.10 |  |  |
| -0.2 - -0.1                 | 1.02 ± 0.10 |  |  |
| -0.1 - 0.0                  | 1.09 ± 0.10 |  |  |

$L = \sqrt{(r+h)^2 - r^2 \sin^2 \theta - r \cos \theta}$, where $r$ is the radius of the Earth, and $h$ is the altitude of the production point of atmospheric neutrinos.

Information concerning $F_\beta(E_\mu, \theta)$ for the multi-GeV case is given in Refs. 20), 21) and 23). Information concerning $F_\beta(E_\mu, \theta)$, including the geomagnetic effects for the sub-GeV case, is taken from Ref. 22). Other information concerning $\epsilon_\alpha(E_\alpha)$ and $\sigma_\alpha(E_\mu, E_\alpha, \psi)$ is given in Ref. 24). The upward through-going muon fluxes were simulated in Ref. 24. Explicit calculation of Eq. (10) is explained in Appendix A.

Since $P^M(\nu_\alpha \rightarrow \nu_\beta)$ is a function of $\Delta m^2_{12}, \Delta m^2_{23}, \theta_{12}, \theta_{13}$ and $\theta_{23}$, the ratio $(dN_{Exp}(E_\alpha)/d \cos \theta)/(dN_{MC}(E_\alpha)/d \cos \theta)$ of the zenith angle distributions is a function of $\Delta m^2_{12}, \Delta m^2_{23}, \theta_{12}, \theta_{13}, \theta_{23}$ and $\theta$. We analyze the atmospheric neutrino data fixing the parameters $\Delta m^2_{12}$ and $\sin^2 2\theta_{12}$ (which have been determined from the solar neutrino experiments$^{4,13,10}$) as $\Delta m^2_{12} = 3 \times 10^{-5}$ eV$^2$ and $\sin^2 2\theta_{12} = 0.7$, which correspond to the large angle solution, and $\Delta m^2_{12} = 10^{-5}$ eV$^2$ and $\sin^2 2\theta_{12} = 0.005$, which corresponds to the small angle solution. We treat the ratios of the zenith angle
Fig. 1. The plots of allowed regions in the $\tan^2 \theta_{13}$-$\tan^2 \theta_{23}$ plane determined by the zenith angle distributions of SuperKamiokande 535 day data. These figures correspond to the large $\theta_{12}$ angle solution, $\Delta m_{12}^2 = 3 \times 10^{-5}$ eV$^2$ and $\sin^2 2\theta = 0.7$. In these figures, broken thick, broken thin and dotted curves denote the regions allowed in 99%, 95% and 90% C.L., respectively. Figures (a)–(d) show the plots of sub-GeV experiments.

We treat $\chi^2$ defined as

$$\chi^2 = \sum_{i, \alpha} \left\{ \frac{(N_{\text{Exp}}(l_\alpha)/N_{\text{MC}}(l_\alpha))_{i}^{\text{cal}} - (N_{\text{Exp}}(l_\alpha)/N_{\text{MC}}(l_\alpha))_{i}^{\text{data}}}{(\sigma_{st})_{i}^2 + (\sigma_{sy})_{i}^2} \right\}^2. \quad (11)$$

For the sub-GeV and multi-GeV experiments, the summation over $i$ is over 1 to 5 bin of zenith angle, and for the upward through-going $\mu$, over 1 to 10 bin. The summation over $l_\alpha$ is over $\mu$ and $e$ for the sub-GeV and multi-GeV experiments. We estimate the value of $\chi^2$ on the various values of $\Delta m_{23}^2$, $\theta_{13}$ and $\theta_{23}$. $\sigma_{st}$ represents the statistical
error and $\sigma_{\text{sys}}$ represents systematic error. For the values of $\sigma_{\text{sys}}$, we assume the values corresponding to 10% of the magnitudes of $N_{\text{MC}}$. These values are deduced from the graphs given by the report of the SuperKamiokande collaboration.\(^1\) In Fig. 1, we give the contour plots of $\chi^2$ in the $\tan^2 \theta_{13}$-$\tan^2 \theta_{23}$ plane for various $\Delta m_{23}^2$. We give the plots for the sub-GeV experiment in Figs. 1(a)–(d) and the plots for the multi-GeV in Figs. 1(e)–(h) for the large $\theta_{12}$ angle solution, $\Delta m_{12}^2 = 3 \times 10^{-5}$ eV$^2$ and $\sin^2 2\theta = 0.7$. In these figures, broken thick, broken thin and dotted curves denote the regions allowed in 99%, 95% and 90% C.L., respectively. The combinations of the sub-GeV and multi-GeV experiments are shown in Figs. 1(i)–(l). Figure 2 represents the case of the small $\theta_{12}$ angle solution, $\Delta m_{12}^2 = 10^{-5}$ eV$^2$ and $\sin^2 2\theta = 0.005$.

From these plots, we can obtain the following results for the mixing parameters:

1. $\Delta m_{23}^2$ is allowed from $10^{-2}$ eV$^2$ to $2 \times 10^{-4}$ eV$^2$. The minimum value of $\chi^2$ is obtained as 14.5 at $\Delta m_{23}^2 = 7 \times 10^{-4}$ eV$^2$, $\tan^2 \theta_{13} = 6 \times 10^{-2}$ and $\tan^2 \theta_{23} = 2$.

The minimum value of $\chi^2$ in the restriction $\theta_{13} = 0$, which corresponds to $\nu_{\mu} - \nu_{\tau}$...
mixing, is 17 at $\Delta m^2_{23} = 2 \times 10^{-3}$ eV$^2$.

(2) $\nu_e$-$\nu_\tau$ mixing is small, and $\nu_\mu$-$\nu_\tau$ mixing is large; $\tan^2 \theta_{13} < 10^{-1}$ and $\tan^2 \theta_{23} \sim 1$.

(3) There is no significant difference between the large $\theta_{12}$ solution and the small one.

The result that $\Delta m^2_{23}$ is allowed from $10^{-2}$ eV$^2$ to $2 \times 10^{-4}$ eV$^2$ and the minimum value of $\chi^2$ with the restriction that $\theta_{13} = 0$ is obtained at $\Delta m^2_{23} = 2 \times 10^{-3}$ eV$^2$ is the same as the result obtained by the SuperKamiokande collaboration.\(^1\)

Plots of the upward through-going $\mu$ are shown in Figs. 3(a)–(d) and the combinations of the sub-GeV, multi-GeV and upward through-going $\mu$ flux in Figs. 3(e)–(h) for the large $\theta_{12}$ angle solution. The plots for the small $\theta_{12}$ angle solution are similar to Fig. 3. From these results, we can say that

(1) the mixing parameters allowed in the sub-GeV and multi-GeV zenith angle distributions can explain the upward through-going $\mu$ flux experimental data, and

Fig. 1. The plots of allowed regions in the $\tan^2 \theta_{13}$-$\tan^2 \theta_{23}$ plane. Figures (i)–(l) show the plots of the combinations of the sub-GeV and multi-GeV experiments.
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\[ \Delta m_{23}^2 = 0.1 \text{ eV}^2 \]

99%CL

\[ \tan^2 \theta_{13} \]

\[ 10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 10^0 \quad 10^1 \quad 10^2 \quad 10^3 \]

\[ \tan^2 \theta_{23} \]

\[ 10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 10^0 \quad 10^1 \quad 10^2 \quad 10^3 \]

Fig. 2. The plots of allowed regions on \( \tan^2 \theta_{13} \)-\( \tan^2 \theta_{23} \) plane determined by the zenith angle distributions of the combination of sub-GeV and multi-GeV \( \mu \) SuperKamiokande 535 day data. These figures correspond to the small \( \theta_{12} \) angle solution, \( \Delta m_{12}^2 = 10^{-5} \text{ eV}^2 \) and \( \sin^2 2 \theta = 0.005 \). In these figures, broken thick, broken thin and dotted curves denote the regions allowed in 99%, 95% and 90% C.L., respectively.

(2) for large values of \( \Delta m_{23}^2 \sim 10^{-1} - 10^{-2} \text{ eV}^2 \), the values near 45° of \( \theta_{23} \) are excluded, because the deficit from 1 of the event ratio \( \frac{N_{\text{Exp}}(\mu)}{N_{\text{MC}}(\mu)} \) is not so large (about 0.2) for the upward through-going \( \mu \) flux compared with multi-GeV experiments.

3.2. Terrestrial neutrinos

Through many terrestrial experiments, the strong restrictions have been imposed on neutrino mixing parameters and masses. In terrestrial experiments, there are two types of experiments, short-baseline and long-baseline experiments.\(^8\) In this paper, we analyze the short-baseline experiments. Accelerator experiments searching for the appearance of \( \nu_\tau \) in \( \nu_\mu \) were performed by E531, CHORUS and NOMAD,\(^5\) with...
The plots of allowed regions in the $\tan^2 \theta_{13}$-$\tan^2 \theta_{23}$ plane determined by the zenith angle distributions of the upward through-going $\mu$ SuperKamiokande data for large $\theta_{12}$ angle solution. There is no difference between the large $\theta_{12}$ angle solution and small one. In these figures, the broken thick, broken thin and dotted curves denote the regions allowed in 99%, 95% and 90% C.L., respectively. Figures (a)–(d) show the plot of the upward through-going $\mu$ experiments.

The results

\begin{align}
P(\nu_\mu \to \nu_e) &< 2 \times 10^{-3} \quad (90\% \text{ C.L.}), \\
L/E &\sim 0.02. 
\end{align}

(12)

Accelerator experiments searching for the $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations were performed by E776, KARMEN and LSND,\textsuperscript{6} with the results

\begin{align}
P(\nu_\mu \to \nu_e) &< 1.5 \times 10^{-3} \quad (90\% \text{ C.L.}), \quad \text{E776}, \\
L &\sim 1 \text{ km}, \quad E \sim 1 \text{ GeV}, \\
P(\bar{\nu}_\mu \to \bar{\nu}_e) &< 3.1 \times 10^{-3} \quad (90\% \text{ C.L.}), \quad \text{KARMEN}, \\
L &\sim 17.5 \text{ m}, \quad E < 40 \text{ MeV}, 
\end{align}

(13a)

(13b)
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\[ \Delta m_{23}^2 = 0.1 \text{ eV}^2 \]

99%CL (29DF)

\[ \frac{10^{-3}}{10^{-1}} \]

\[ \tan^2 \theta_{13} \]

\[ \frac{10^{-3}}{10^{-1}} \]

\[ \tan^2 \theta_{23} \]

\[ \Delta m_{23}^2 = 0.01 \text{ eV}^2 \]

99%CL

95%CL (29DF)

90%CL

\[ \frac{10^{-3}}{10^{-1}} \]

\[ \tan^2 \theta_{13} \]

\[ \frac{10^{-3}}{10^{-1}} \]

\[ \tan^2 \theta_{23} \]

\[ \Delta m_{23}^2 = 0.001 \text{ eV}^2 \]

99%CL

95%CL (29DF)

90%CL

\[ \frac{10^{-3}}{10^{-1}} \]

\[ \tan^2 \theta_{13} \]

\[ \frac{10^{-3}}{10^{-1}} \]

\[ \tan^2 \theta_{23} \]

\[ \Delta m_{23}^2 = 0.0001 \text{ eV}^2 \]

99%CL

95%CL (29DF)

90%CL

\[ \frac{10^{-3}}{10^{-1}} \]

\[ \tan^2 \theta_{13} \]

\[ \frac{10^{-3}}{10^{-1}} \]

\[ \tan^2 \theta_{23} \]

Fig. 3. The plots of allowed regions in the \( \tan^2 \theta_{13} - \tan^2 \theta_{23} \) plane for the combinations of sub-GeV, multi-GeV and upward through-going \( \mu \) experiments.

\[ P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = 3.4^{+2.0}_{-1.8} \pm 0.7 \times 10^{-3}, \quad \text{LSND}, \]

\[ \quad L = 30 \text{ m}, \quad E \sim 36-60 \text{ MeV}. \]

(13c)

Accelerator experiments searching for the disappearance of \( \nu_\mu \) were carried out by CDHSW, and nuclear power reactor experiments searching for the disappearance of \( \bar{\nu}_e \) were carried out by BUGEY and CHOOZ: \(^7\)

\( \nu_\mu \)-disappearance experiment, CDHSW,

\[ 0.26 \text{ eV}^2 < \Delta m^2 < 90 \text{ eV}^2 \]

are excluded for maximal mixing,

\[ \quad \Delta m^2 = 130, 885 \text{ m}, \quad E \sim 3 \text{ GeV}, \]

(14a)

\[ 1 - P(\bar{\nu}_e \rightarrow \bar{\nu}_e) < 10^{-2} \quad (90\% \text{ C.L.}), \quad \text{BUGEY}, \]

\[ \quad L = 15, 40, 95 \text{ m}, \quad E \sim 1-6 \text{ MeV}. \]

(14b)

\[ P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 0.98 \pm 0.04 \pm 0.09, \quad \text{CHOOZ}, \]

\[ \quad L = 1 \text{ km}, \quad E \sim 3 \text{ MeV}. \]

(14c)
We give the contour plots of allowed regions in the $\tan^2 \theta_{13}$-$\tan^2 \theta_{23}$ plane determined by the probability $P$ expressed in Eq. (6) satisfying experimental data in Eqs. (12), (13) (except LSND data (13c)) and (14) in Fig. 4. The allowed regions are the left- and right-hand sides surrounded by curves. The curves represent the boundary of 90 % C.L. of $P$. $\Delta m_{12}^2$ and $\sin^2 2\theta_{12}$ are fixed as $10^{-5}$ eV$^2$ and 0.8, respectively. $\Delta m_{23}^2$ is fixed to 0.1 eV$^2$ (a), 0.01 eV$^2$ (b), 0.001 eV$^2$ (c) and 0.0001 eV$^2$ (d). Dotted lines show the allowed regions determined by LSND data.

Although we fix the parameters $\Delta m_{12}^2$ and $\theta_{12}$ as $\Delta m_{12}^2 = 10^{-5}$ eV$^2$ and $\sin^2 2\theta_{12} = 0.8$, and used various values of the parameter $\Delta m_{23}^2$ between 0.1 eV$^2$ and 0.0001 eV$^2$. Although we fix the parameters $\Delta m_{12}^2$ and $\theta_{12}$ as $\Delta m_{12}^2 = 10^{-5}$ eV$^2$ and $\sin^2 2\theta_{12} = 0.01$, the results are not changed, because the probability $P$ of a terrestrial neutrino is insensitive to the $\nu_e$-$\nu_\mu$ mixing parameters $\Delta m_{12}^2$ and $\sin^2 2\theta_{12}$. The dotted lines show the allowed regions restricted by LSND data Eq. (13c). The allowed regions determined by LSND data are very restricted, then hereafter we do not consider it.
3.3. **Allowed regions of mixing parameters**

Here we discuss the allowed regions for the mixing parameters $\Delta m^2_{12}$, $\sin^2 2\theta_{12}$, $\Delta m^2_{23}$, $\theta_{13}$ and $\theta_{23}$ satisfying the atmospheric neutrino and terrestrial neutrino experiments. $\Delta m^2_{12}$ and $\sin^2 2\theta_{12}$ have been determined in analyses considering the MSW effects of the solar neutrino experiments: \(^4\), \(^13\), \(^10\)

\[
(\Delta m^2_{12}, \sin^2 2\theta_{12}) = \begin{cases} 
(4 \times 10^{-6} - 7 \times 10^{-5} \text{ eV}^2, 0.6 - 0.9), & \text{large angle solution} \\
(3 \times 10^{-6} - 1.2 \times 10^{-5} \text{ eV}^2, 0.003 - 0.01), & \text{small angle solution}
\end{cases}
\]

for $\theta_{13} = 0^\circ - 20^\circ$.

Observing the allowed regions obtained in terrestrial neutrino experiments (Fig. 4) and the allowed regions obtained in atmospheric neutrino experiments including sub-GeV and multi-GeV data (Figs. 1 and 2), we obtain the allowed regions consistent with these experiments. The allowed regions of $\theta_{13}$ and $\theta_{23}$ for the large $\theta_{12}$ angle solution are as follows:

- For $\Delta m^2_{23} = 0.1 \text{ eV}^2$, no allowed region
- For $\Delta m^2_{23} = 0.01 \text{ eV}^2$, $(\theta_{13} < 13^\circ, 28^\circ < \theta_{23} < 37^\circ, 53^\circ < \theta_{23} < 62^\circ)$
- For $\Delta m^2_{23} = 0.005 \text{ eV}^2$, $(\theta_{13} < 9^\circ, 28^\circ < \theta_{23} < 62^\circ)$
- For $\Delta m^2_{23} = 0.002 \text{ eV}^2$, $(\theta_{13} < 23^\circ, 29^\circ < \theta_{23} < 61^\circ)$
- For $\Delta m^2_{23} = 0.001 \text{ eV}^2$, $(\theta_{13} < 20^\circ, 29^\circ < \theta_{23} < 61^\circ)$
- For $\Delta m^2_{23} = 0.0005 \text{ eV}^2$, $(\theta_{13} < 24^\circ, 35^\circ < \theta_{23} < 55^\circ)$
- For $\Delta m^2_{23} = 0.0002 \text{ eV}^2$, $(4^\circ < \theta_{13} < 22^\circ, 38^\circ < \theta_{23} < 54^\circ)$
- For $\Delta m^2_{23} = 0.0001 \text{ eV}^2$, no allowed region

For the small $\theta_{12}$ angle solution, a similar solution is obtained. We cannot see a significant difference between the large $\theta_{12}$ angle solution and small one from these results.

If we adopt the 90% C.L. of $\chi^2$ for the zenith angle dependence of atmospheric neutrinos, the range of the mass parameter $\Delta m^2_{23}$ is restricted as

\[
\Delta m^2_{23} = 0.01 \text{ eV}^2 \sim 0.0002 \text{ eV}^2.
\]

The value of $\Delta m^2_{23}$ at the minimum of $\chi^2$ under the constraint of $\theta_{13} = 0$ is obtained near $\tan^2 \theta_{23} = 1$ as

\[
\Delta m^2_{23} = 2 \times 10^{-3} \text{ eV}^2 \text{ with } \chi^2 = 17.
\]

Under the no constraint of $\theta_{13}$, the value of $\Delta m^2_{23}$ at the minimum of $\chi^2$ is obtained at $\tan^2 \theta_{13} = 6 \times 10^{-2}$ and $\tan^2 \theta_{23} = 2$ as

\[
\Delta m^2_{23} = 7 \times 10^{-4} \text{ eV}^2 \text{ with } \chi^2 = 14.5.
\]

These results (17a) and (17b) are the same as those for SuperKamiokande (Ref. 1) and similar to those for three-flavor neutrino analysis (the last paper in Ref. 9)).
The zenith angle distribution of the upward through-going $\mu$ flux can be explained by the mixing parameter ranges (16) restricted by the zenith angle distributions of sub-GeV and multi-GeV experiments and terrestrial experiments, except for the values near $45^\circ$ of $\theta_{23}$ for large values of $\Delta m_{23}^2 (\sim 10^{-1} - 10^{-2} \text{eV}^2)$. This exception is caused from the small discrepancy from 1 (about 0.2) of the ratio $N_{\text{Exp}}(\mu)/N_{\text{MC}}(\mu)$ in upward through-going $\mu$ flux compared with multi-GeV events.

We now comment on the present results and ones obtained in our previous analysis. In a previous study, we analyzed the double ratios of the zenith angle distributions of atmospheric neutrinos neglecting the matter effects of the Earth and the smearing effects, and obtained the results that the mass of $\Delta m_{23}^2$ is rather large and that the large $\theta_{12}$ angle solution is favored over the small one. Analyzing the event ratio in the present study, the upper limit of the mass parameter $\Delta m_{23}^2$ is limited to a small value. The difference between the large $\theta_{12}$ angle solution and the small one disappear by considering the matter effects of the Earth.

§4. Conclusion

We analyzed the atmospheric neutrino experimental data of SuperKamiokande in the three-flavor neutrino framework with the mass hierarchy $m_1 \approx m_2 < m_3$ and obtained the allowed regions of the parameters $\Delta m_{12}^2$, $\sin^2 2\theta_{12}$, $\Delta m_{23}^2$, $\theta_{13}$ and $\theta_{23}$. We studied the event ratios of the sub-GeV, multi-GeV and upward through-going muons zenith angle distributions. From these atmospheric experiments and terrestrial ones, the allowed regions of the mass parameters $\Delta m_{23}^2$ are restricted in the range $0.01 \text{eV}^2 - 0.0002 \text{eV}^2$, and the value of $\Delta m_{23}^2$ at the minimum $\chi^2$ under the restriction $\theta_{13} = 0$ is $2 \times 10^{-3} \text{eV}^2$. For mixing parameters, there is no difference between the large $\theta_{12}$ angle solution and the small one. The obtained results are $\theta_{13} < 13^\circ$, $28^\circ < \theta_{23} < 37^\circ$ and $53^\circ < \theta_{23} < 62^\circ$ for $\Delta m_{23}^2 = 0.01 \text{eV}^2$, $\theta_{13} < 23^\circ$ and $29^\circ < \theta_{23} < 61^\circ$ for $\Delta m_{23}^2 = 0.002 \text{eV}^2$, $\theta_{13} < 24^\circ$ and $35^\circ < \theta_{23} < 55^\circ$ for $\Delta m_{23}^2 = 0.0005 \text{eV}^2$, $4^\circ < \theta_{13} < 22^\circ$ and $38^\circ < \theta_{23} < 54^\circ$ for $\Delta m_{23}^2 = 0.0002 \text{eV}^2$. In our three neutrino analyses for the SuperKamiokande atmospheric neutrino experiments, large $\nu_\mu-\nu_\tau$ mixing is favored, although small mixing $\nu_e-\nu_\mu$ mixing is allowed at the same time.

The present analysis has used the approximation that the density of the Earth is constant. Presently, we are analyzing a case in which the density of the Earth is realistic. Furthermore, the result obtained in this work is considered to be significant in the analysis of long-baseline experiments.

Appendix A

Explicit Calculation of Eqs. (10a) and (10b)

In the multi-GeV case, the information concerning the Super-Kamiokande detection efficiency $\epsilon_\alpha(E_{\alpha})$ is not known to us, but the detected charged lepton event number $f_\alpha(E_{\alpha})$ is given in the paper of the SuperKamiokande collaboration. The
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\[ f_\alpha(E_\alpha) = \int \epsilon_\alpha(E_\alpha) \sigma_\alpha(E_\nu, E_\alpha, \psi) F_\alpha(E_\nu, \theta - \psi) dE_\nu d\cos \psi d\cos \theta. \]  \hspace{1cm} (A.1)

We define the effective zenith angle dependent neutrino flux \( f_\alpha(E_\alpha, \theta) \) as

\[ f_\alpha(E_\alpha, \theta) = f_\alpha(E_\alpha) \frac{F_\alpha(E_\alpha, \theta)}{\int \_1 \_1 d\cos \theta F_\alpha(E_\alpha, \theta)}, \]  \hspace{1cm} (A.2)

where we use the zenith angle distribution given in Ref. 21 for \( F_\alpha(E_\alpha, \theta) \). Furthermore, we approximate the \( \psi \) dependence of the differential cross section \( \sigma_\alpha \) by a smearing Gaussian function \( n(\psi) \) as

\[ n(\psi) = \frac{2}{\sqrt{\pi} \tan(\psi_0/2)} \frac{1}{(1 + \cos \psi) \sin \psi} \exp \left\{ -\frac{\tan^2(\psi/2)}{\tan^2(\psi_0/2)} \right\}, \]  \hspace{1cm} (A.3)

where the resolution \( \psi_0 \) is taken as 17°. Then, we estimate \( dN_\text{Exp}/d\cos \theta \) by the integral

\[ dN_\text{Exp}/d\cos \theta = \int n(\psi) f_\beta(E_\beta, \theta - \psi) P_M(\nu_\beta \rightarrow \nu_\alpha) dE_\beta d\cos \psi. \]  \hspace{1cm} (A.4)

In the sub-GeV case, \( \sigma_\alpha(E_\nu) = \int \sigma_\alpha(E_\nu, E_\alpha, \psi) d\cos \psi dE_\alpha \) is calculated in Ref. 24). \( \epsilon_\alpha(E_\alpha) \) is nearly 1 in the sub-GeV region. Thus we estimate the detected lepton event number with zenith angle dependence \( f_\alpha(E_\nu, \theta) \), using the approximation \( \theta = \psi \) in \( F_\alpha(E_\nu, \theta - \psi) \) as

\[ f_\alpha(E_\nu, \theta) = \epsilon_\alpha(E_\nu) \int \sigma_\alpha(E_\nu, E_\alpha, \psi) F_\alpha(E_\nu, \theta) dE_\alpha d\cos \psi = \epsilon_\alpha(E_\nu) \sigma_\alpha(E_\nu) F_\alpha(E_\nu, \theta), \]  \hspace{1cm} (A.5)

where information concerning \( F_\alpha(E_\nu, \theta) \) is taken from Ref. 22). Then, introducing the smearing Gaussian function \( n(\psi) \) as the multi-GeV case, we express \( dN_\text{Exp}/d\cos \theta \) as

\[ dN_\text{Exp}/d\cos \theta = \int n(\psi) f_\beta(E_\nu, \theta - \psi) P_M(\nu_\beta \rightarrow \nu_\alpha) dE_\nu d\cos \psi, \]  \hspace{1cm} (A.6)

where the resolution \( \psi_0 \) is taken as 60° for the sub-GeV case.

For upward through-going muons fluxes, the event number \( f_\alpha(E_\alpha) \) of through-going muons is estimated in Ref. 24). Thus, using this information and the same treatment as in the multi-GeV case, we calculate the zenith angle \( \theta \) dependent events of upward through-going muons.

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