A Note on the $4D$ Antisymmetric Tensor Field Model in Curved Space-Time

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Abstract. Using the BRS techniques, we prove the existence of a local and nonlinear symmetry of the gauge fixed action of the antisymmetric tensor field model in curved background.

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1 Introduction

Topological field theories \[1\] are mathematically as well as physically interesting \[2\]. In four dimensions we have two interesting topological field theories, The topological Yang–
Mills model \[2\] and the antisymmetric tensor field model \[3\]. The algebraic renormalization \[2\] of the topological Yang–
Mills field theory was carried out in \[4\] and then extended to curved space–time in \[5\]. On the other hand, the algebraic renormalization of the
antisymmetric tensor field model (in the flat space–time limit) was first done in \[6\] and then generalized to a curved space–time admitting a covariantly constant vector field in \[7\]. In this work we will go a step further and try to generalize the analysis of \[7\] to an
arbitrary, curved, Riemannian manifold.

The paper is organized as follows, in section 2 we give the gauge fixed action and display
the BRS transformations of all the fields appearing in the model. Next, in section 3 we
derive the on–shell local supersymmetry–like transformations, i.e. the anticommutator of
the BRS operator and of the local susy–like operator leads to Lie derivative . In section 4
we extend the on–shell analysis (of section 3) to the off–shell level. Here we will see that
the local susy–like Ward operator, when it acts on the total action describing the model,
gives rise to a hard breaking which is quadratic in the quantum fields. In order to eliminate
this quadratic breaking we introduce auxiliary fields. The result of this construction
is that the anticommutator of the linearized Slavnov operator and the Ward operator
of the new symmetry does not close on diffeomorphisms for certain fields. Furthermore, the
most important fact is that the total action is invariant under this new, local, nonlinear (and not susy–like) symmetry.

On the other hand, one could use this symmetry (first one has to show its validity at all
order of perturbation theory) to prove the finiteness of the 4D antisymmetric tensor field
model in a general class of curved manifolds: generalizing the results of \[7\].

2 The model

First let us consider the following classical action in curved space–time

\[
\Sigma_{\text{inv}} = - \frac{1}{4} \int_{\mathcal{M}} d^4x \varepsilon_{\mu \nu \rho \sigma} F^{a}_{\mu \nu} B_{\rho \sigma}^{a},
\]

where \(\mathcal{M}\) is a curved manifold described by the Euclidean metric \(g_{\mu \nu}\). The field strength
is described by

\[
F_{\mu \nu}^{a} = \partial_{\mu} A_{\nu}^{a} - \partial_{\nu} A_{\mu}^{a} + f^{abc} A_{\mu}^{b} A_{\nu}^{c},
\]

where \(A_{\mu}^{a}\) is the gauge field which belongs to the adjoint representation of a compact Lie
group whose structure constants are denoted by \(f^{abc}\). The antisymmetric tensor field \(B_{\mu \nu}^{a}\)
is also Lie algebra valued and \(\varepsilon^{\mu \nu \rho \sigma}\) is the Levi–Civita tensor density\[6\] of weight +1. On

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\[More details about the subject of algebraic renormalization can be found in the last reference of \[8\].

\[In this paper we denote the inverse of the metric by \(g^{\mu \nu}\) and its determinant by \(g\). Under diffeo-

morphisms, \(\sqrt{g}\) behaves like a scalar density of weight +1, whereas the volume element density \(d^4 x\) has

weight -1. The Levi–Civita antisymmetric tensor density \(\varepsilon^{\mu \nu \rho \sigma}\) has weight +1 and \(\varepsilon_{\mu \nu \rho \sigma}\) has weight \(-1\).
the other hand, the action (1) possesses two kinds of invariance, given by
\[
\delta(1) A^a_{\mu} = -(D_{\mu} \theta)^a = -\left( \partial_{\mu} \theta^a + f^{abc} A^b_{\mu} \theta^c \right),
\]
\[
\delta(1) B^a_{\mu\nu} = f^{abc} g^b B^c_{\mu\nu},
\]
and
\[
\delta(2) A^a_{\mu} = 0,
\]
\[
\delta(2) B^a_{\mu\nu} = -(D_{\mu} \varphi_{\nu} - D_{\nu} \varphi_{\mu})^a,
\]
\[\theta^a\] and \[\varphi^a\] are local parameters. Now, by choosing a Landau gauge the gauge-fixing part of the action is given by \[7\]
\[
\Sigma_{gf} = -s \int_M d^4x \sqrt{g} \left[ g^{\alpha\beta} \partial_{\alpha} \tilde{c}^a A^a_{\nu} + g^{\rho\sigma} g^{\sigma\beta} \partial_{\rho} \tilde{c}^a B^a_{\mu\nu} + g^{\mu\nu} \partial_{\mu} \phi^a \xi^a_{\nu} - g^{\mu\nu} \partial_{\nu} \phi^a \xi^a_{\mu} - \tilde{\phi}^a \lambda^a \right] - \frac{1}{2} \int_M d^4x \varepsilon^{\mu\nu\rho\sigma} f^{abc} \partial_{\mu} \tilde{c}^a \partial_{\rho} \tilde{c}^b \phi^c,
\]
As already computed in \[7\], the extended BRS transformations of all fields introduced so far read
\[
s A^a_{\mu} = -(D_{\mu} c)^a = -\left( \partial_{\mu} c^a + f^{abc} A^b_{\mu} c^c \right),
\]
\[
s B^a_{\mu\nu} = -(D_{\mu} \xi^a_{\nu} - D_{\nu} \xi^a_{\mu}) + f^{abc} B^b_{\mu} c^c + \varepsilon^{\mu\nu\rho\sigma} f^{abc} \sqrt{g} g^\rho g^\sigma (\partial_{\rho} \tilde{c}^a) \phi^c,
\]
\[
s \xi^a_{\mu} = (D_{\mu} \phi)^a + f^{abc} b^b \xi^c_{\mu},
\]
\[
s \phi^a = f^{abc} b^b \phi^c,
\]
\[
s c^a = \frac{1}{2} f^{abc} b^b c^c,
\]
\[
s \tilde{c}^a = b^a, \quad s b^a = 0,
\]
\[
s \xi^a_{\mu} = \tilde{h}^a_{\mu}, \quad s \tilde{h}^a_{\mu} = 0,
\]
\[
s \tilde{\phi}^a = \omega^a, \quad s \omega^a = 0,
\]
\[
s e^a = \lambda^a, \quad s \lambda^a = 0,
\]
\[
s g_{\mu\nu} = \tilde{g}_{\mu\nu}, \quad s \tilde{g}_{\mu\nu} = 0.
\]
The vector \[\xi^a_{\mu}\] is the ghost field for the symmetry (3) whereas \[c^a\] is the ghost field for the gauge symmetry (1). Each of the couples of fields \[(\tilde{c}^a, b^a), (\tilde{h}^a, \mu^a), (\tilde{\phi}^a, \omega^a)\] and \[(e^a, \lambda^a)\] contains an antighost and the corresponding Lagrange multiplier fields.
We could extend the BRS transformations (see last line of (6)) by letting \(s\) acting on the metric \(g_{\mu\nu}\) because, at the level of the gauge fixed action \(\Sigma_{inv} + \Sigma_{gf}\), the metric appears only in a BRS exact expression \[7\], a fact which guarantees its non-physical character.
It turned out \[7\] that the BRS operator, constructed above, is nilpotent on-shell. More precisely,
\[
s^2 B^a_{\mu\nu} = -\varepsilon^{\mu\nu\rho\sigma} f^{abc} \delta \left( \Sigma_{inv} + \Sigma_{gf} \right) \phi^c\] and \(s^2 = 0\) for the other fields.

Furthermore, we have the following useful identity
\[
\varepsilon^{\mu\nu\rho\sigma} = \frac{1}{g} g^{\mu\alpha} g^{\nu\beta} g^{\rho\gamma} g^{\sigma\lambda} \varepsilon_{\alpha\beta\gamma\lambda}.
\]
3 The on–shell analysis

In the flat space–time limit the authors of [4] constructed, besides the BRS transformations, a further symmetry of the gauge fixed action, the so called vector supersymmetry–like transformations. Their analysis was generalized [7] to the case of a curved space–time admitting a covariantly constant vector field. In both cases the supersymmetry–like transformations were rigid transformations. In this paper we make a further step and try to construct a local supersymmetry–like transformations (at least on–shell), so let us begin by proposing the following transformations

\[
\begin{align*}
\delta_{(\eta)} A^a_\mu &= -\varepsilon_{\mu\nu\rho\sigma} \eta^\nu \sigma^a \partial_\alpha \xi^a_{\beta} , \\
\delta_{(\eta)} B^a_{\mu\nu} &= -\varepsilon_{\mu\nu\rho\sigma} \eta^\rho \sigma^a \partial_\alpha \overline{c}^a , \\
\delta_{(\eta)} c^a &= -\eta^\mu A^a_\mu , \\
\delta_{(\eta)} \overline{c}^a &= 0 , \\
\delta_{(\eta)} h^a_\mu &= \mathcal{L}_\eta c^a , \\
\delta_{(\eta)} \xi^a_\mu &= \eta^\nu B^a_{\mu
u} , \\
\delta_{(\eta)} \overline{\xi}^a_\mu &= 0 , \\
\delta_{(\eta)} h^a_\mu &= \mathcal{L}_\eta \overline{c}^a , \\
\delta_{(\eta)} \phi^a &= \eta^\mu c^a , \\
\delta_{(\eta)} \overline{\phi}^a &= 0 , \\
\delta_{(\eta)} \omega^a &= \mathcal{L}_\eta \overline{\phi}^a , \\
\delta_{(\eta)} e^a &= 0 , \\
\delta_{(\eta)} \lambda^a &= \mathcal{L}_\eta e^a , \\
\delta_{(\eta)} g_{\mu\nu} &= 0 , \\
\delta_{(\eta)} \hat{g}_{\mu\nu} &= \mathcal{L}_\eta g_{\mu\nu} ,
\end{align*}
\]

where \( \mathcal{L}_\eta \) is the Lie derivative and \( \eta^\mu \) is the vector parameter of the transformations with ghost number +2. It turns out that the on–shell algebra takes the following form

\[
\{ s, \delta_{(\eta)} \} = \mathcal{L}_\eta + \text{equ. of motion}
\]

for all fields, except for the antisymmetric tensor field \( B^a_{\mu\nu} \) where we get

\[
\{ s, \delta_{(\eta)} \} \ B^a_{\mu\nu} = \mathcal{L}_\eta B^a_{\mu\nu} + \varepsilon_{\mu\nu\rho\sigma} \eta^\rho \sigma^a \partial_\alpha \phi^b \phi^c .
\]
Now to repair this shortcoming we add to the gauge fixed action the following expression

$$\Sigma_{K,M} = \int d^4x (K^a_{\mu}\Xi^{a\mu} - M^a_{\mu}s\Xi^{a\mu})$$

(11)

with

$$\Xi^{a\mu} = - f^{abc}\sqrt{g}g^{\mu\alpha}\partial_{\alpha}\bar{\phi}^b \phi^c.$$  

(12)

The two auxiliary fields $K^a_{\mu}$ and $M^a_{\mu}$ transform under the BRS and $\delta_{(\eta)}$ operators according to

$$sM^a_{\mu} = K^a_{\mu}, \quad sK^a_{\mu} = 0, \quad \delta_{(\eta)}M^a_{\mu} = 0, \quad \delta_{(\eta)}K^a_{\mu} = \mathcal{L}_\eta M^a_{\mu}.$$  

(13)

In an easy way we can prove that

$$\{s, \delta_{(\eta)}\} = \mathcal{L}_\eta,$$  

(14)

for the two auxiliary fields. The advantage of introducing the auxiliary fields $K^a_{\mu}$ and $M^a_{\mu}$ is that (10) will get the same form as (9). Indeed,

$$\mathcal{B}_{\mu\nu} = \mathcal{L}_\tau \mathcal{B}_{\mu\nu} + \varepsilon_{\mu\nu\rho\sigma}\eta^{\rho}\left(\frac{\delta A_{\sigma}}{\delta A^a_{\sigma}} - \frac{\delta \Sigma}{\delta K^a_{\sigma}}\right)$$

(15)

So, in this way we have constructed an on–shell local supersymmetry–like transformations which anticommute with the BRS operator and lead to Lie derivative (9). Next, we want to know if the transformations, generated by the operator $\delta_{(\eta)}$, give rise to a symmetry of the gauge fixed action. This question is investigated in the next section where we also display the off–shell algebra.

4 The off–shell analysis

In order to generalize the above results to the off–shell level we first introduce external sources which couple to the non–linear BRS transformations (3)

$$\Sigma_{ext} = \int_{\mathcal{M}} d^4x \left[ \Omega^{a\mu}(sA^a_{\mu}) + \gamma^{a\mu\nu}(sB^a_{\mu\nu}) + L^a(s\phi^a) + D^a(s\xi^a) \right] + \int d^4x \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}f^{abc}\gamma^{a\mu\nu}\gamma^{b\rho\sigma}\phi^c,$$

(16)

with dimensions, weights and ghost numbers as given in table 3.
To eliminate the nonlinear expression in (19) we first add to the action the BRS exact
the context of the renormalization procedure.

The nonlinear breaking (21) is quadratic in the quantum fields, then it is not harmless in

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Table 3: Dimensions, ghost numbers and weights of the external sources.

From the transformations  and the second line of (13) (and due to presence of the external sources) we get the Ward operator \( \mathcal{W}_S^{(\eta)} \) such that

\[
\mathcal{W}_S^{(\eta)} = \int d^4x \left[ -\varepsilon_{\mu\nu\rho\sigma} \Theta^{(\eta)}(\gamma^{\alpha\mu}\nabla^\alpha + \sqrt{g}g^{\alpha\beta} \partial^\alpha \xi^\beta) \frac{\delta}{\delta A^\alpha_{\mu}} - \eta^{\nu} A^\nu_{\rho} \frac{\delta}{\delta b^{\rho}} - \varepsilon_{\mu\nu\rho\sigma} \eta^{\nu}(\Omega^{\alpha\sigma} + \sqrt{g}g^{\alpha\sigma} \partial^\alpha \xi^\sigma) \frac{\delta}{\delta B_{\mu}^{\alpha\nu}} + \eta^{\nu} B_{\mu}^{\alpha\nu} \frac{\delta}{\delta \phi^\nu} - \eta^{\nu} \xi^\alpha \frac{\delta}{\delta \phi^\alpha} + \mathcal{L}_{\eta} \xi^\alpha \delta \xi^\alpha_{\mu} + \mathcal{L}_{\eta} \phi^\nu \delta \phi^\nu_{\mu} - \eta^{\nu} D_{\mu}^{\alpha} \frac{\delta}{\delta \phi^\alpha} - \eta^{\nu} L^{\alpha}_{\mu} \frac{\delta}{\delta \phi^\alpha} \right].
\]

After tedious calculations, the corresponding Ward identity takes the form

\[
\mathcal{W}_S^{(\eta)}(\Sigma_{inv} + \Sigma_{gf} + \Sigma_{K,M} + \Sigma_{ext}) = \Delta_{(\eta)}^{cl},
\]

where the classical breaking \( \Delta_{(\eta)}^{cl} \) split in linear and quadratic parts

\[
\Delta_{(\eta)}^{cl} = \Delta_{(\eta)}^{L} + \Delta_{(\eta)}^{Q}.
\]

with,

\[
\Delta_{(\eta)}^{L} = \int d^4x \left[ -\gamma^{\mu\nu} \mathcal{L}_{\eta} B_{\mu}^{\alpha} - \Omega^{\alpha\mu} \mathcal{L}_{\eta} A_{\mu}^{\alpha} + L^{\alpha}_{\mu} \mathcal{L}_{\eta} \phi^\alpha_{\mu} - D_{\mu}^{\alpha} \mathcal{L}_{\eta} \xi^\alpha_{\mu} - \varepsilon_{\mu\nu\rho\sigma} \Omega^{\mu\nu} \eta^{\rho} s(\sqrt{g}g^{\rho\sigma} \partial^\rho \xi^\sigma) - \varepsilon_{\mu\nu\rho\sigma} \gamma^{\mu\nu} \eta^{\rho} s(\sqrt{g}g^{\rho\sigma} \partial^\rho \xi^\sigma) \right]
\]

and

\[
\Delta_{(\eta)}^{Q} = \int d^4x \left[ -\varepsilon_{\mu\nu\rho\sigma} s(\sqrt{g}g^{\rho\sigma} \partial^\rho \xi^\sigma) s(\sqrt{g}g^{\rho\sigma} \partial^\rho \xi^\sigma) - s(\sqrt{g}g^{\rho\sigma} \partial^\rho \phi^\sigma) s(\sqrt{g}g^{\rho\sigma} \partial^\rho \xi^\sigma) \right].
\]

The nonlinear breaking (21) is quadratic in the quantum fields, then it is not harmless in the context of the renormalization procedure.

To eliminate the nonlinear expression in (19) we first add to the action the BRS exact integral

\[
\Sigma_1 = \int d^4x \left[ -L_\mu \bar{\gamma}^\mu + R_\mu s \bar{\gamma}^\mu + W^\mu \Psi_\mu - Z^\mu s \Psi_\mu + \Gamma^\mu \Theta^{(\eta\mu)} - J^\mu s \Theta^{(\eta\mu)} + P^a \Lambda^a + Q^a s \Lambda^a \right]
\]
such that
\[
\begin{align*}
\Upsilon^\mu &= \varepsilon^{\mu\rho\sigma} \partial_\nu \tilde{c}^\alpha \partial_\rho \tilde{c}_\sigma, \\
\Psi_\mu &= \tilde{\phi}^\alpha \partial_\mu \tilde{e}^\alpha, \\
\Theta^{a\mu} &= \sqrt{g} g^{\mu\rho} \partial_\nu \tilde{\phi}^\alpha, \\
\Lambda^a &= f^{abc} \sqrt{g} g^{\mu\rho} M^b_\mu \partial_\rho \tilde{\phi}^c. 
\end{align*}
\] (23)

All new auxiliary fields introduced in (22) transform in a BRS doublets
\[
\begin{align*}
s R_\mu &= L_\mu, & s L_\mu &= 0, \\
s Z^\mu &= W^\mu, & s W^\mu &= 0, \\
s J^a_\mu &= I^a_\mu, & s I^a_\mu &= 0, \\
s Q^a &= P^a, & s P^a &= 0. 
\end{align*}
\] (24)

Furthermore, under the \(\delta_{(\eta)}\) operation, they transform as
\[
\begin{align*}
\delta_{(\eta)} R_\mu &= g_\mu \eta^\rho, & \delta_{(\eta)} L_\mu &= \mathcal{L}_\eta R_\mu - \dot{\gamma}^\rho_\mu \eta^\rho, \\
\delta_{(\eta)} Z^\mu &= -\sqrt{g} \eta^\rho, & \delta_{(\eta)} W^\mu &= \mathcal{L}_\eta Z^\mu + \eta^\rho s \sqrt{g}, \\
\delta_{(\eta)} J^a_\mu &= \eta^a B^a_\mu, & \delta_{(\eta)} I^a_\mu &= \mathcal{L}_\eta J^a_\mu - s(\eta^a B^a_\mu), \\
\delta_{(\eta)} Q^a &= \eta^a \xi^a. & \delta_{(\eta)} P^a &= \mathcal{L}_\eta Q^a - s(\eta^a \xi^a). 
\end{align*}
\] (25)

|   | \(L_\mu\) | \(R_\mu\) | \(W^\mu\) | \(Z_\mu\) | \(I^a_\mu\) | \(J^a_\mu\) | \(P^a\) | \(Q^a\) |
|---|--------|--------|--------|--------|--------|--------|--------|--------|
| dim | -1     | -1     | -1     | -1     | 1      | 1      | 0      | 0      |
| \(\phi\pi\) | 2      | 1      | 2      | 1      | 2      | 1      | 3      | 2      |
| weight | 0      | 0      | 1      | 1      | 0      | 0      | 0      | 0      |

Table 1: Dimensions, ghost numbers and weights of auxiliary fields.

For all auxiliary fields, the anticommutator of the BRS operator \(s\) and \(\delta_{(\eta)}\) closes on the Lie derivative plus equations of motion.

On the other hand, the Ward identity (18) gets promoted to
\[
\mathcal{W}^S_{(\eta)}(\Sigma) = \mathcal{W}^S_{(\eta)}(\Sigma) + \mathcal{V}^S_{(\eta)}(\Sigma) = \Delta^L_{(\eta)}, \quad (26)
\]
where \(\mathcal{V}^S_{(\eta)}(\Sigma)\) is nonlinear, and its expression is
\[
\begin{align*}
\mathcal{V}^S_{(\eta)}(\Sigma) &= \int d^4 x \left(g_\mu \eta^\rho \frac{\delta \Sigma}{\delta R_\mu} + (\mathcal{L}_\eta R_\mu - \dot{\gamma}_\mu \eta^\rho) \frac{\delta \Sigma}{\delta L_\mu} - \sqrt{g} \eta^\rho \frac{\delta \Sigma}{\delta Z^\rho} + \eta^\rho B^a_\mu \frac{\delta \Sigma}{\delta J^a_\mu} + + (\mathcal{L}_\eta Z^\mu + \eta^\rho s \sqrt{g}) \frac{\delta \Sigma}{\delta W^\mu} + (\mathcal{L}_\eta J^a_\mu - \eta^\nu \dot{\gamma}_a^\mu \eta^\nu) \frac{\delta \Sigma}{\delta I^a_\mu} + \eta^\nu \xi^a \frac{\delta \Sigma}{\delta Q^a} + \right. \\
&+ \left. (\mathcal{L}_\eta Q^a - \eta^\rho \frac{\delta \Sigma}{\delta P^a} \right) \frac{\delta \Sigma}{\delta P^a} \right) 
\] (27)

The complete gauge fixed action is now given by
\[
\Sigma = \Sigma_{\text{inv}} + \Sigma_{gf} + \Sigma_{K,M} + \Sigma_{\text{ext}} + \Sigma_1. \quad (28)
\]

6
It obeys the Slavnov identity

\[ S(\Sigma) = 0, \]

where

\[ S(\Sigma) = \int d^4x \left( \frac{\delta \Sigma}{\delta \gamma} \frac{\delta}{\delta B_{\mu\nu}} + \frac{\delta \Sigma}{\delta \Omega} \frac{\delta A_{\mu\rho}}{\delta \rho_{\mu\rho}} + \frac{\delta \Sigma}{\delta \xi} \frac{\delta D_{\mu}}{\delta \rho_{\mu\rho}} \right. \]

\[ + b^a \frac{\delta \Sigma}{\delta \bar{c}^a} + h^a_{\mu} \frac{\delta \Sigma}{\delta \xi^{\mu}} + \omega^a \frac{\delta \Sigma}{\delta \phi^a} + \lambda^a \frac{\delta \Sigma}{\delta \phi^a} + g_{\mu} \frac{\delta \Sigma}{\delta g_{\mu\nu}} + K^{\mu} \frac{\delta \Sigma}{\delta M_{\mu}} + L_{\mu} \frac{\delta \Sigma}{\delta R_{\mu}} \]

\[ \left. + W^a \frac{\delta \Sigma}{\delta Z^a} + \left( \frac{\delta \Sigma}{\delta J_{\mu}} + P^a \frac{\delta \Sigma}{\delta Q^a} \right) \right). \]  

(30)

It is straightforward to verify that the corresponding linearized Slavnov operator is given by

\[ S_{\Sigma} = \int d^4x \left( \frac{\delta \Sigma}{\delta \gamma} \frac{\delta}{\delta B_{\mu\nu}} + \frac{\delta \Sigma}{\delta \Omega} \frac{\delta A_{\mu\rho}}{\delta \rho_{\mu\rho}} + \frac{\delta \Sigma}{\delta \xi} \frac{\delta D_{\mu}}{\delta \rho_{\mu\rho}} \right. \]

\[ + b^a \frac{\delta \Sigma}{\delta \bar{c}^a} + h^a_{\mu} \frac{\delta \Sigma}{\delta \xi^{\mu}} + \omega^a \frac{\delta \Sigma}{\delta \phi^a} + \lambda^a \frac{\delta \Sigma}{\delta \phi^a} + g_{\mu} \frac{\delta \Sigma}{\delta g_{\mu\nu}} + K^{\mu} \frac{\delta \Sigma}{\delta M_{\mu}} + L_{\mu} \frac{\delta \Sigma}{\delta R_{\mu}} \]

\[ \left. + W^a \frac{\delta \Sigma}{\delta Z^a} + \left( \frac{\delta \Sigma}{\delta J_{\mu}} + P^a \frac{\delta \Sigma}{\delta Q^a} \right) \right). \]  

(31)

At the functional level, the invariance of the classical action (28) under diffeomorphisms can be expressed by an unbroken Ward identity

\[ W^D(\varepsilon) \Sigma = 0, \]

\[ W^D(\varepsilon) = \int d^4x \sum_f (L_f) \frac{\delta}{\delta f}, \]  

(32)

where \( W^D(\varepsilon) \) denotes the corresponding Ward operator,

for all fields \( f \). The vector parameter of the diffeomorphism transformations is denoted by \( \varepsilon^\mu \), and it carries ghost number +1.

Next, we display the complete nonlinear algebra of the Slavnov operator and the Ward operator \( W^D(\varepsilon) \). To this end, let \( \Gamma \) be an arbitrary functional depending on the fields of the model, then

\[ S_{\Gamma} S(\Gamma) = 0, \]

\[ S_{\Gamma} W^D(\varepsilon) \Gamma + W^D(\varepsilon) S(\Gamma) = 0, \]

\[ \{ W^D(\varepsilon), W^D(\varepsilon') \} \Gamma = -W^D(\{\varepsilon,\varepsilon'\}) \Gamma. \]  

(34)

Now if the functional \( \Gamma \) is a solution of the Slavnov identity and of the Ward identity of diffeomorphisms, then the off–shell algebra (34) reduces to the linear algebra

\[ S_{\Sigma} S_{\Sigma} = 0, \]

\[ \{ S_{\Sigma}, W^D(\varepsilon) \} = 0, \]

\[ \{ W^D(\varepsilon), W^D(\varepsilon') \} = -W^D(\{\varepsilon,\varepsilon'\}). \]  

(35)
with the Lie brackets
\[
\{\varepsilon, \varepsilon'\}^\mu = \mathcal{L}_\varepsilon \varepsilon'^\mu
\]  

(36)

Contrary to other topological field theories [5] [9], the anticommutator of the linearized Slavnov operator (31) with the linearized Ward operator does not give rise to the diffeomorphisms Ward operator. Indeed, for certain fields we get \(\{\mathcal{S}_\Sigma, \mathcal{W}_S^{(\eta)}\} = \mathcal{W}_D^{(\varepsilon)}\) and for other fields the anticommutator does not close on \(\mathcal{W}_D^{(\varepsilon)}\). As a simple example, one can let the above anticommutator acting on the two auxiliary fields \(M_\mu^a\) and \(K_\mu^a\).

The main result of this paper is that we could, after introducing auxiliary fields, construct a local and nonlinear symmetry of the 4D antisymmetric tensor field model in a curved manifold. The next natural step to do is to show the renormalizability of the new symmetry. It turns out that if this symmetry is valid at the quantum level then it would be very useful in showing the finiteness of the model in a big class of curved manifolds. This would be the generalisation of the results of [4].

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\[\text{First one has to linearize } \mathcal{W}_S^{(\eta)}\text{ and then let the anticommutator } \{\mathcal{S}_\Sigma, \mathcal{W}_S^{(\eta)}\}\text{ acting on the different fields.}\]
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