Microscopic Calculations of $^8$He+p Elastic Scattering Cross Sections

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Abstract. A microscopic approach to calculate the optical potential (OP) with the real part obtained by a folding procedure and with the imaginary part inherent in the high-energy approximation is applied to study the $^8$He+p elastic scattering at energies of tens of MeV/nucleon (MeV/N). The OP's and the cross sections are calculated using different models for the neutron and proton densities of $^8$He. The role of the spin-orbit potential is studied. Comparison of the calculations with the available experimental data on the elastic scattering differential cross sections at beam energies of 15.7, 26, 32, 66 and 73 MeV/N is performed. The problem of the ambiguities of the depths of each component of the optical potential is considered by means of the imposed physical criterion related to the known behavior of the volume integrals as functions of the incident energy. It is shown also that the role of the surface absorption is rather important, in particular for the lowest incident energies (e.g., 15.7 and 26 MeV/N). The present approach, which uses only parameters that renormalize the depths of the OP, can be applied along with other methods using microscopically calculated optical potentials.

1. Introduction

The experiments with intensive secondary radioactive nuclear beams have made it possible to investigate the structure of light nuclei near the neutron and proton drip lines as well as the mechanism of scattering of the weakly bound nuclei. A widely used way to study the structure of exotic nuclei is to analyze their elastic scattering on protons or nuclear targets at different energies. Here we would like to mention, for example, the experiments on scattering of helium isotopes on protons at incident energies $E_{\text{inc}}$ less than 100 MeV/N, in particular, for $^8$He at energy 15.7 [1], 26 [2], 32 [3, 4], 66 [3, 4] and 73 MeV/N [3, 4, 5].

The experimental data on differential and total reaction cross sections of processes with light exotic nuclei have been analyzed using a variety of phenomenological and microscopic methods. Among the latter methods we note, e.g. the microscopic analysis based on the coordinate-space $g$-matrix folding method [6, 7, 8, 9, 10, 11, 12, 13], as well as works where the real part of OP (ReOP) is microscopically calculated using the folding approach (e.g. [14, 15, 16, 17, 18, 19, 20, 21, 22, 23]). Usually the imaginary part of the OP’s (ImOP) and the spin-orbit (SO) terms have been determined phenomenologically. Thus, the OP’s have a number of fitting parameters.
The main aim of this work is to calculate differential cross sections of elastic $^8$He+p scattering at energies less than 100 MeV/N studying the possibility to describe the existing experimental data by calculating microscopically not only the real (in a double-folding procedure) but also the imaginary optical potential (instead of using phenomenological one) within the high-energy approximation (HEA) and using a minimal number of fitting parameters. In this respect, we want to establish the limits of the applicability of the HEA OP for calculations of differential elastic cross sections of the $^8$He+p scattering for different regions of angles and incident energies. The standard folding OP includes an exchange term and, correspondingly, the non-linear effects of renormalization of the real and imaginary parts of microscopic optical potentials, orbit forces and regularization of the NN forces used in folding calculations. Also, we consider the contribution of various physical quantities and features, such as microscopically obtained spin-orbit forces and regularization of the NN forces used in folding calculations. Also, we consider effects of renormalization of the real and imaginary parts of microscopic optical potentials, the differences or similarities of various models for the $^8$He structure which are used in the description of the experimental data on the cross section of the $^8$He+p elastic scattering. We study the density dependence of the effective NN interaction, as well as the sensitivity of the results to the predictions of different theoretical models for the densities of $^8$He. In addition, the nuclear surface effects are also studied by introducing an additional surface term in OP. This is related to investigations of the lowest energy limit of the applicability of the HEA OP in $^8$He+p elastic scattering. Also we pay attention to the energy dependence of the renormalization parameters, as well as to the respective volume integrals. We note the necessity to analyze the differential cross sections estimating simultaneously the values of the total reaction cross section. This would give an additional test of the various ingredients of the approach.

2. Theoretical scheme

2.1. The optical potential

The general expression for the optical potential used to calculate the $^8$He+p elastic scattering differential cross sections has the form:

$$U_{\text{opt}}(r) = N_R V^F(r) + i N_I W^H(r) + 2 \lambda^2 \left\{ N_R^S O V^F_0 \frac{1}{r} \frac{df_r(r)}{dr} + i N_I^S O W^H_0 \frac{1}{r} \frac{df_l(r)}{dr} \right\} (1s),$$

(1)

where $V^F$ (calculated within the folding approach) and $W^H$ (calculated within the HEA) are the real and imaginary contributions to the OP, respectively, and $V^F_0$ and $W^H_0$ are the depths of the SO optical potential obtained simultaneously from the approximation of the volume real and imaginary OP’s by Woods-Saxon (WS) form. In Eq. (1) $N_R$, $N_I$, $N_R^S O$, and $N_I^S O$ are additionally introduced strength parameters that renormalize the depths of the different components of the OP.

The real part of the nucleon-nucleus OP is assumed to be a result of a single folding of the effective NN potential and the nuclear densities. It involves the direct and exchange parts (for more details, see, e.g. [19, 20, 21] and also [17]):

$$V^F(r) = V^D(r) + V^{EX}(r).$$

(2)

In Eq. (2) the direct part ($V^D$) is composed of the isoscalar (IS) and isovector (IV) contributions, correspondingly:

$$V^D_{IS}(r) = \int \rho_2(r_2) g(E) F'(\rho_2) v^{D}_{00}(s) dr_2,$$

(3)

$$V^D_{IV}(r) = \int \delta \rho_2(r_2) g(E) F'(\rho_2) v^{D}_{00}(s) dr_2.$$

(4)

In Eqs. (3) and (4) the energy dependence of the effective NN interaction is taken in the usually used form:

$$g(E) = 1 - 0.003E.$$
Also, for the NN potentials \( v^{D}_{10} \) and \( v^{D}_{31} \) we use the expression from [21] for the CDM3Y6-type of the effective interaction based on the solution of the equation for the \( g \)-matrix, in which the Paris NN potential has been used. The density dependence of the effective interaction is taken in the following form:

\[
F(\rho) = C \left[ 1 + \alpha e^{-\beta \rho(r)} - \gamma \rho(r) \right],
\]

where \( C = 0.2658 \), \( \alpha = 3.8033 \), \( \beta = 1.4099 \text{ fm}^3 \), and \( \gamma = 4.0 \text{ fm}^3 \).

The isoscalar part of the exchange contribution to the ReOP has the form (see, e.g. [17]):

\[
V_{E}^{\text{EX}}(r) = g(E) \int \rho_2(r_2, r_2 - s) F\left[ \rho_2\left(\frac{r_2 - s}{2}\right)\right] \times v_{00}^{\text{EX}}(s) j_0[k(r)s] d\mathbf{r}_2.
\]

In Ref. [17] the so-called complex HEA optical potential has been applied to explain the available data on the \(^6\text{He}+p\) elastic differential cross sections and energies less than 100 MeV/N. The HEA OP was derived in [24] on the basis of the eikonal phase inherent in the optical limit of the Glauber theory. It can be obtained as a folding of form factors of the nuclear density and the NN amplitude \( f_{NN}(q) \) [24, 25]:

\[
U_{opt}^H = V^H + iW^H = -\frac{\hbar v}{(2\pi)^2} (\bar{\sigma}_{NN} + i\bar{\sigma}_{NN}) \times \int_0^\infty dq q^2 j_0(qr) \rho_2(q)f_{NN}(q).
\]

In (8) \( \bar{\sigma}_{NN} \) and \( \bar{\sigma}_{NN} \) are, respectively, the NN total scattering cross section and the ratio of the real to imaginary parts of the forward NN scattering amplitude both averaged over the isospin of the nucleus.

### 2.2. The density distributions of \(^8\text{He}\)

In the calculations of the OP’s we use the following point-nucleon density distributions of \(^8\text{He}\):

1. The Tanihata densities deduced in [26] by means of comparison of the measured total reaction cross section of \(^6,8\text{He}^{+12}\text{C}\) at 800 MeV with the respective expression from [27] derived in the framework of the optical limit of the Glauber theory;
2. The LSSM densities calculated in a complex \(4\hbar\omega\) shell model space [22, 23] using the WS basis of single-particle wave functions with realistic exponential asymptotic behavior;
3. The densities obtained in [28, 29] with accounting for the NN central-type short-range Jastrow correlations.

### 3. Results and discussion

In this Section we present the results of the calculations of the microscopic OP’s and the respective \(^8\text{He}+p\) elastic scattering differential cross sections at energies \( E_{inc} < 100 \text{ MeV/N} \). In principle, the OP’s do not contain free parameters, but they depend on the density distribution of the target nucleus. This allows one to test advanced theoretical methods that give predictions for the density distribution. In Fig. 1 in logarithmic and linear scales are shown the proton \( \rho_p(r) \), neutron \( \rho_n(r) \) and matter \( \rho(r) \) densities of \(^8\text{He}\) obtained in different models. Also, for comparison, the known COSMA densities [30, 31] are presented. We note that among them only the LSSM densities have a realistic exponential asymptotics, whereas the others have a Gaussian one. The results for the JCM densities are given for the value of the correlation parameter \( \beta = 2.5 \text{ fm}^{-1} \) in the Jastrow correlation factor \( 1 - \rho \beta r^2 \), where \( r \) is the distance between neutrons. It was shown in Refs. [28, 29] that the inclusion of this factor causes a slight increase of the density in the central part of the nucleus.

The discussion that follows is based on the fitting procedure, where the additionally introduced strength parameters \( N_R^-, N_I^-, N_R^{SO}, N_I^{SO} \) are varied step by step. So, we start
Figure 1. Total ((a) and (a’)), point-proton (b) and point-neutron (c) densities of $^8$He from the model of Tanihata [26], COSMA [30, 31], LSSM [22, 23] and JCM calculations [28, 29].

from the case $N_R=N_I=1$, $N_R^{SO}=N_I^{SO}=0$, then fit successively both coefficients $N_R$ and $N_I$, and after that the values of $N_R^{SO}$ and $N_I^{SO}$. First, we give in Fig. 2 the results of our methodical calculations of the cross sections for different energies (15.7, 26, 32, 66 and 73 MeV/N) using the densities of $^8$He from LSSM, Tanihata and JCM approaches in the case when $N_R=N_I=1$ and $N_R^{SO}=N_I^{SO}=0$ (i.e. without spin-orbit interaction). It can be seen that the behavior of the cross sections for a given energy and interval of angles is weakly sensitive to the choice of the model for the density of $^8$He. In spite of this uncertainty we choose for the further applications the LSSM density since it has a realistic exponential behavior in the peripheral region of the nucleus.

Later, as a next step, we allow the ”depth” of each of the parts of the OP (1) in our semi-microscopic models to vary in order to find the optimal values of the parameters $N_R$, $N_I$, $N_R^{SO}$ and $N_I^{SO}$ by a fitting procedure to the available experimental data for the cross sections. In Fig. 3 we present the results of our calculations of $^8$He+p elastic scattering cross sections for various energies and the LSSM density with the fitted values of the parameters $N_R$, $N_I$, $N_R^{SO}$ and $N_I^{SO}$. The values of these renormalization parameters are given in Table 1 together with the predicted total reaction cross sections. The results obtained using the values of the parameters from the first line of this Table for each energy are given by solid line in Fig. 3, while those from the second line for each energy are given by dashed line.

The difficulties in the description of the cross sections at low energies can be overcome by adding the contribution of a surface part of OP to the volume form. For this reason we consider
Figure 2. The $^8$He$+p$ elastic scattering cross sections at different energies calculated using $U_{\text{opt}}$ [Eq. (1)] for values of the parameters $N_R=N_I=1$ and $N_{SO_R}=N_{SO_I}=0$. The used densities of $^8$He are LSSM, Tanihata, and JCM ($\beta=2.5$ fm$^{-1}$). Experimental data are taken for 15.7 [1], 26 [2], 32 [3, 4], 66 [3, 4] and 73 MeV/N [3, 4, 5].

also the contribution of the surface potential:

$$U'_{\text{opt}}(r) = U_{\text{opt}}(r) - i4\alpha N_S \frac{dV^F(r)}{dr},$$  \hspace{1cm} (9)

where the first term in the right-hand side is the expression for the OP given by Eq. (1) (in which the ImOP is taken in the form of $V^F(r)$) and the second term is responsible for the surface effects. We would like to note that, in particular, for the lowest incident energy, the combination of the microscopically folded real and imaginary parts in the form of $V^F$ is more appropriate. In Eq. (9) $\alpha$ is the diffuseness parameter of $V^F(r)$ fitted by the WS form.

We present in Fig. 4 our results for the cross section in the case of $E=15.7$ MeV/N obtained using the LSSM density of $^8$He. The calculations are performed by fitting the strength parameters $N_R$, $N_I$, $N_{SO_R}$, $N_{SO_I}$ entering Eqs. (1) and (9) and the depth parameter $N_S$ of the surface term of the OP [Eq. (9)]. It is seen from Fig. 4 that the inclusion of the surface
contribution to the imaginary part of the OP improves the agreement with the experimental data, especially for small angles and in the region of the cross section minimum. Obviously, for more successful description of the cross sections at low energies (15.7 and 26 MeV/N) our method has to be modified and improved by an inclusion of virtual excitations of inelastic and decay channels of the reactions.

4. Conclusions
The results of the present work can be summarized as follows:

i) The optical potentials and cross sections of \(^8\text{He}+\text{p}\) at \(E=15.7, 26, 32, 66\) and 73 MeV/N elastic scattering were calculated and comparison with the available experimental data was performed.

(a) The ReOP \((V^F)\) was calculated microscopically using the folding procedure and M3Y effective interaction based on the Paris NN potential.

(b) The ImOP \((W^H)\) was calculated within the HEA.

(c) Different model densities of protons and neutrons in \(^8\text{He}\) were used in the calculations: Tanihata, COSMA, LSSM and JCM.

**Figure 3.** The \(^8\text{He}+\text{p}\) elastic scattering cross sections at different energies calculated using \(U_{opt}\) [Eq. (1)] for various values of the renormalization parameters \(N_R\), \(N_I\), \(N_R^{SO}\) and \(N_I^{SO}\) (presented in Table 1) giving the best agreement with the data. The used density of \(^8\text{He}\) is LSSM. Experimental data are taken for 15.7 [1], 26 [2], 32 [3, 4], 66 [3, 4] and 73 MeV/N [3, 4, 5].

**Figure 4.** The \(^8\text{He}+\text{p}\) elastic scattering cross section at energy \(E=15.7\) MeV/N using LSSM density of \(^8\text{He}\). The fitted values of \(N_R\), \(N_I\), and \(N_S\) are given. Experimental data are taken from [1].
Table 1. The renormalization parameters $N_R$, $N_I$, $N_{SO}^R$, and $N_{SO}^I$ obtained by fitting the experimental data in Fig. 3 in the case of LSSM density. The energies are in MeV/N and the total reaction cross sections $\sigma_R$ are in mb.

| $E$ | $N_R$ | $N_I$ | $N_{SO}^R$ | $N_{SO}^I$ | $\sigma_R$ |
|-----|-------|-------|------------|------------|------------|
| 15.7 | 1.0   | 0.236 | 0          | 0          | 603.6      |
| 15.7 | 0.9   | 0.1   | 0.107      | 0.040      | 693        |
| 26   | 0.422 | 0.104 | 0.090      | 0.010      | 275.11     |
| 26   | 0.439 | 0.144 | 0.087      | 0.023      | 377.22     |
| 32   | 0.438 | 0.036 | 0.096      | 0          | 71.9       |
| 32   | 1.0   | 0.374 | 0          | 0          | 419.5      |
| 66   | 0.876 | 0.071 | 0          | 0          | 55.7       |
| 66   | 0.854 | 0.086 | 0          | 0          | 65.9       |
| 73   | 0.875 | 0.02  | 0          | 0          | 1.48       |
| 73   | 0.869 | 0.01  | 0.010      | 0.002      | 1.22       |

(d) The SO contribution to the OP was included in the calculations.
(e) The cross sections were calculated by numerical integration of the Schrodinger equation by means of the DWUCK4 code using all interactions obtained (Coulomb plus nuclear optical potential).
ii) The results show that the LSSM densities of $^8\text{He}$ which have more diffuse tails at larger $r$ than the densities based on Gaussians lead to a better agreement with the data for the $^8\text{He}+p$ elastic scattering at different energies.
iii) It was shown that, generally, at energies $E > 25$ MeV/N a good agreement with the experimental data for the differential cross sections can be achieved using OP with calculated both $V^F$ and $W^H$ varying mainly the volume part of the OP and neglecting the SO contribution.
iv) The explanation of the $^8\text{He}+p$ cross sections at lower energies ($E < 25$ MeV/N) needs accounting for the effects of the nuclear surface. In this case the use of ImOP of the HEA type is limited. A more successful explanation of the cross section at low energies could be given by inclusion of polarization contributions due to virtual excitations of inelastic and decay channels of the reactions.
v) It was shown that the effects of the Jastrow central short-range NN correlations on the OP’s and on the shape of differential cross sections are weak.
vi) The problem of the ambiguity of the values of the parameters $N_R$, $N_I$, $N_{SO}^R$, and $N_{SO}^I$ when the fitting procedure is applied to a limited number of experimental data is considered. Physical criteria imposed in our work [18] on the choice of the values of the parameters $N$ were the known behavior of the volume integrals as functions of the incident energy in the interval $0 < E_{inc} < 100$ MeV/N, as well as the values of the total cross section of scattering and reaction.

Concluding, we would like to note that the microscopic approach presented in our work can be used along with other more sophisticated methods mentioned above.

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