Causality Aspects of the Parton Cascade Approach to Ultarelativistic Heavy Ion Reactions †

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**Introduction**

Parton cascade codes that take a space-time approach to model the microscopic processes\(^1\) of an ultrarelativistic heavy-ion interaction are – in spite of their QCD bells and whistles – by necessity based on some kind of *classical many-particle dynamics*.

**Problems:**

- space-time-based cascade models suffer from the consequences of the

  No-Interaction Theorem (NIT) \[^{[CJS63]}\]:

  The only consistent many-particle
  Hamiltonian theory that is Poincaré-covariant
  is that of a system of free particles.

- the procedure to determine the sequence of the binary parton interactions (“SBPI”) – an essential aspect of space-time-based cascade models – is by necessity an artificial and ad-hoc feature of these codes.

The only way to circumvent the NIT is to loosen its assumptions:

1. forget about Poincaré covariance \[\Rightarrow \text{VNI}\]
   
   In this approach the SBPI depends on the initially chosen frame of reference. Einstein causality remains a problem.

2. introduce a many-times formalism, e.g. by formulating the model in\(^2\) \(8N\)-dimensional phase space (\(N\) is the particle number) \[\Rightarrow \text{pcpc}\]

   The Poincaré covariance of this model seems to guarantee Einstein causality; the SBPI of the code, however, deserves closer scrutiny.

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\(^1\)Prominent examples are \textbf{VNI} and \textbf{pcpc}; cf. the OSCAR archive at http://nt3.phys.columbia.edu/people/molnard/OSCAR/.

\(^2\)We have shown previously that for \(N = 2\) all known Poincaré-covariant formalisms are equivalent to ours (cf. \[^{[PNB94]}\]).
Basic Covariant Structure of pcpc

**pcpc** is a hybrid of a *classical* dynamics approach that governs the evolution of the system between binary parton interactions, and a parton interaction model with QCD ingredients (parton distribution functions, pQCD cross sections and ‘DGLAP evolution’)\(^3\).

- The dynamical evolution of the system is parametrized by a Poincaré scalar, \(s\) (in contrast to the usual \(t\), the proper time of an external observer, as measured in some external frame)

- The phase-space variables of the \(N\) partons (quarks, gluons) are covariant 4-vectors \(x_i^\mu(s), p_i^\mu(s)\). Between binary interactions \(N\) is fixed, but can (and does) vary due to parton creation in the QCD-governed parton interactions

- The (8\(N\)-dimensional) interaction term of the Poincaré-*invariant* ‘Hamiltonian’ depends on the following Poincaré-invariant ‘4-distances’

\[
d_{ij}^2(s) := -\hat{\mathbf{x}}^2 = -(\mathbf{x}\mathbf{x}) = -\hat{\mathbf{x}}\hat{\mathbf{x}}
\]

\[
\hat{\mathbf{x}}^\mu := x^\mu - \frac{(xp)}{p^2}p^\mu
\]

\[
\begin{align*}
\mathbf{x}^\mu(s) &:= x_i^\mu(s) - x_j^\mu(s), \quad p^\mu(s) := p_i^\mu(s) + p_j^\mu(s)
\end{align*}
\]

- Binary parton interactions occur at the \(s\) determined by \(d_{ij}^2(s)\) being at a minimum\(^4\) (in the cms of partons \(i, j\), \(d_{ij}^2\) is the minimal 3-distance of approach)

- Between interactions, partons move along free trajectories:

\[
x_i^\mu(s) = \frac{p_i^\mu(s)}{m_i}(s - s_0) + x_i^\mu(s_0)
\]

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\(^3\)For details, cf. [PNB94](#).

\(^4\)The interaction term in the Hamiltonian thus can be thought of as a sum of \(\delta\)-functions.
Logical flow in pcpc

In contrast to most other cascade codes, the ‘time step’ in pcpc is not a computational artifice, but is given by the formalism itself:

- if there were only 2 partons, their interaction would occur at the value of the evolution parameter $s$ at which they would reach their minimal $d_{ij}^2(s)$ if they were indeed alone in the world,

- so a ‘time table’ is kept, containing, for every pair $i, j$ of partons, its potential minimal approach $d_{ij}^2(s)$ (i.e. their minimal approach if $i, j$ were the only partons in the whole system), and the corresponding $s$,

- then this table is searched for the smallest (‘earliest’) $s$. At this $s$ the potential interaction of the corresponding pair will actually occur (because all other potential interactions are ‘later’),

- the interaction of the pair $i, j$ will change the world lines of these two partons (and possibly create further partons). Therefore, the time table is updated, with new values of $d_{ij}^2(s)$ for all pairs involving either parton $i$ or parton $j$ (or the newly created partons); and the code loops.

Thus the code follows exactly the sequence of binary parton interactions as parametrized by the monotonically increasing Poincaré-invariant evolution parameter $s$. The logic is summarized in the following flow diagram:
Do the parton interactions preserve Einstein causality?

Prima facie this does not seem to be so: the 4-vector $x_i^\mu - x_j^\mu$ is spacelike, and so no signal can be transmitted between events $x_i$ and $x_j$. But this argument is fallacious: while it would be correct in the framework of a $6N$-dimensional phase space formalism and physical observer time, it does not follow in a many-times formalism with an $8N$-dimensional phase space.
Furthermore, it must be remembered that the representation of the (QCD) physics of the heavy-ion reaction in terms of cross sections, parton distribution functions etc. is tantamount to the description of intrinsically quantum processes in a classical terminology. But as long as we refrain from trying to look inside an individual binary parton interaction with classical concepts, Einstein causality is not infringed upon by such a model (for details on this point cf. [PNB94]).

**Is Einstein causality preserved between binary interactions?**

In terms of the cascade picture, a heavy-ion reaction can be seen as one or several disjoint graphs of connected particle world lines. In every connected subgraph, the Poincaré-covariant dynamics guarantees Einstein causality.

Separate disjoint subgraphs, however, can have no causal connection. But by construction, no signals are transmitted between them. Imagining a full quantum-mechanically description of such a system, unconnected subgraphs would correspond to subamplitudes which would simply be multiplied in obtaining the total amplitude.

**Are the initial parton positions critically important?**

There is no physically convincing argument for how to set the time components of the initial parton 4-vectors \( a_i := x_i^{\mu=0} (s = s_0) \); so these must be fixed phenomenologically with some arbitrary prescription. In ppcp they are all set to zero.

Does this imply the choice of a particular initial frame of reference, thus invalidating covariance, or spoiling the Einstein causality of the model? The answer is

**NO!**

To see this, suppose that we do not set the initial \( a_i = 0 \), but retain them as free (arbitrary) parameters. We would then find the minimum of the \( d_{ij}^2 \) to be formally dependent on the \( a_i \). But since for any two time-like 4-vectors the invariant quantity \((x \tilde{y})\) is simply \(- \vec{x} \cdot \vec{y} \big|_{\text{cms}}\), we find that the
$d_{ij}^2$ are actually independent of the (time components of the) initial parton positions. It follows that the parton interactions are implemented in a Poincaré-covariant way, even though their sequence (SBPI) does depend on how the initial parameters are chosen.

**Conclusions**

To sum up, causality is not an issue in discussing the physical validity of a cascade model that uses a space-time approach to the dynamical evolution (provided the model is Poincaré-covariant).

It is, however, important to realize that in such codes the sequence of binary parton interactions (SBPI) is to be considered an essential part of the model, and that it is necessarily phenomenological in character. In Poincaré-covariant cascade codes, such as pcpc, the SBPI (although remaining a phenomenological prescription) is independent of choice of the frame of reference in which the code is run.

The difficulties with Einstein causality incurred by non-covariance of the SBPI have been discussed many years ago [KDCDN84]. In contrast to the situations described in that paper, Einstein causality is preserved in a Poincaré-covariant model such as pcpc, both for the individual binary interactions and for the dynamic evolution of the system as a whole.

**References**

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