How to use Model Checking for Diagnosing Fault Patterns in Petri nets
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1. INTRODUCTION

The original problem of diagnosis in discrete event systems as introduced in Sampath et al. (1995) is to determine the occurrence of a fault in a system, the fault being modelled by a single event. In this work, our objective is to consider the diagnosis of a fault event and also the diagnosis of more complex faulty behaviours (called fault patterns) that cannot be modelled by a simple event (see Jéron et al. (2006), Boussif and Ghaezl (2018)). Indeed, for some systems, a change in the order of occurrence of the events, or the joint occurrence of specific events, or the multiple occurrence of a specific type of event may reflect a faulty behaviour. This paper addresses the pattern diagnosis problem that has been introduced in Pencolé and Subias (2017) for Petri nets. The objective is to investigate several manners to solve the problem that use model-checking techniques (Clarke et al. (1999)). The advantages of modeling methods are numerous: speed, no proof building, many logics allowing to express large numbers of properties, generalization of counter-examples. Model-checking has been well studied for many years and many techniques are used to counter the main obstacle of the combinatorial explosion (e.g. abstraction, symmetries, reduction). The objective of this article is to consider a range of approaches based on model checking to solve the pattern diagnosis problem and to compare their advantages and drawbacks. The proposed approaches solve the same problem but differ in the way they represent the three elements that define a diagnosis problem: the system, the faulty behaviour and the sequence of events observed. The paper is organized as follows. The problem statement is presented in Section 2 followed by the description of the general method to solve the pattern diagnosis problem with a model-checking approach. Section 3 then presents a first method in which the faulty behaviour and the observations are directly expressed as a complex property to be verified in the system’s model. A first series of experimental results are then presented in Section 4 and compared with a method proposed in Pencolé and Subias (2017). Based on these results, a second approach is proposed in Section 5 and compared with the others. Finally, conclusions and some perspectives are given in Section 6.

2. PROBLEM STATEMENT

A model-checking problem consists in checking whether some given states (or runs) of a formal model $\mathcal{M}$ satisfy a given property $\phi$ encoded in a formula of some temporal logic (Clarke et al. (1999)). A model-checking tool checks if $\mathcal{M}$ satisfies $\phi$ (denoted $\mathcal{M} \models \phi$) i.e if $\phi$ is true in all the worlds of $\mathcal{M}$. To do so, one solution is the automata based approach of model-checking. In this case the problem is translated to a language intersection problem by converting $\mathcal{M}$ and $\neg \phi$ as Büchi automata. The Büchi automaton of $\mathcal{M}$ generates the infinitely long words of $L^\omega(\mathcal{M})$ describing the set of runs of $\mathcal{M}$ while the Büchi automaton of $\neg \phi$ generates the infinitely long
words of $L^\omega(\neg \phi)$ where $\neg \phi$ holds. The model-checker then checks for $L(M) \cap L(\neg \phi) = \emptyset$. If the intersection is empty, then $M \models \phi$, otherwise $M \not\models \phi$ and a counterexample is returned as a word of $L(M) \cap L(\neg \phi)$. Linear temporal logic (LTL) can be used to express the properties so that their statements can easily be transformed in automata. TINA is a model-checking tool that converts any LPN into a specific enriched Kripke structure $M$ (Berthomieu et al. (2004), Berthomieu et al. (2007)) and checks properties $\phi$ that are written in State/Event LTL (SE-LTL) logic. A formula $\phi$ is a SE-LTL formula if it is a universally quantified formula $\phi ::= \forall \varphi$ where $\varphi ::= cst \mid r \mid \neg \varphi \mid \varphi \lor \varphi \mid \varphi \land \varphi \mid \bigcup \varphi \mid \bigcap \varphi \mid \varphi \lor \psi \mid \varphi \land \psi$. The constant $cst$ can be $\top$ (true), $\bot$ (false), $\text{dead}$ (deadlock), $\text{div}$ (temporal divergence), $\text{sub}$ (partially known state). $r$ denotes constraints $e \Delta e$ with $\Delta \in \{\langle, <, >, \leq, \geq \}$ between arithmetical expressions $e$ involving place and transition symbols from the underlying LPN. Finally, modal operators $\bigcirc, \Box, \Diamond, \lozenge$ are respectively the operators next, always, eventually and until. SE-LTL expresses formulas about places and transitions of Petri nets, however, all along the paper, we will express SE-LTL formulas with events from $\Sigma_1 \cup \Sigma_2$. An event $e$ being the label of one or several transitions $t_1, \ldots, t_k$ of the net, by writing $e$ in any of the following formula we actually write the underlying SE-LTL formula $(t_1 \land \cdots \land t_k) \lor (t_1 \land \cdots \land t_k) \lor \cdots$ that asserts that one (and only one) transition labeled with $e$ is fired.

2.2 General principle for implementing a diagnosis function by model-checking

Algorithm 1 describes the general diagnosis algorithm implementing the diagnosis function $\Delta$ by model-checking.

$\mathcal{M}$ denotes the formal model that is investigated while $\Psi_{\Omega}$ is a SE-LTL formula corresponding to the question: are we sure that pattern $\Omega$ never occurred in a run consistent with the observation sequence $\sigma$? If the result is true, $\Delta$ can immediately returns $\Omega_{\text{Absent}}$. If not, a new question $\Psi_{\Omega}$ is asked: are we sure that $\Omega$ always occurred in all the sequences consistent with $\sigma$? If the result is true, $\Delta$ can return $\Omega_{\text{Faulty}},$ otherwise we are sure there is an ambiguity ($\Delta$ returns $\Omega_{\text{Ambiguous}}$).

Algorithm 1 Diagnosis algorithm $\Delta$ by model-checking

1. if $M \models \Psi_{\Omega}$ then
2. return $\Delta(\sigma) = \Omega_{\text{Absent}}$
3. else if $M \models \Psi_{\Omega}$ then
4. return $\Delta(\sigma) = \Omega_{\text{Faulty}}$
5. else
6. return $\Delta(\sigma) = \Omega_{\text{Ambiguous}}$
7. end if

Algorithm 1 is actually an algorithm template as we have not yet defined what is $M$, $\Psi_{\Omega}$ and $\Psi_{\Omega}$. The next sections propose different methods and compare them.

3. A PURE MODEL-CHECKING APPROACH

This section presents a method that is called pure model-checking approach (denoted PURE for short). It is called pure in the sense that the method aims at fully encoding the pattern diagnosis problem with the questions $\Psi_{\Omega}$ and $\Psi_{\Omega}$ based on the model of the system that is encoded with $M$. There are actually two subproblems to solve. The first one is: what are the runs of the system that are consistent with $\sigma$? The second one is: among these runs, what are the ones where $\Omega$ has occurred and the ones where it has not. The two formulae $\Psi_{\Omega}$ (absence of $\Omega$) and $\Psi_{\Omega}$ (occurrence of $\Omega$) are then as below:

$$
\begin{align*}
\Psi_{\Omega} & : \Phi_\sigma \Rightarrow \Phi_{\Omega} \\
\Psi_{\Omega} & : \Phi_\sigma \Rightarrow \Phi_{\Omega}
\end{align*}
$$

(1)

$\Phi_\sigma$ is the part of the formula that expresses that any run of interests in the question is consistent with the observation sequence $\sigma$ while $\Phi_{\Omega}$ (resp. $\Phi_{\Omega}$) expresses that $\Omega$ has not occurred in any run (resp. $\Omega$ has occurred in any run).

It follows that $\Psi_{\Omega}$ is true either if $\Omega$ does not occur in any run consistent with $\sigma$ or if there is no run consistent with $\sigma$. For $\Psi_{\Omega}$, in the remainder it is supposed that the observed sequence is correct and that it always exists at least one run in the model of the system that is consistent with the observation $\sigma$. Therefore, $\Psi_{\Omega}$ is true if $\Omega$ occurs in any run consistent with $\sigma$.

3.1 Definition of $\Phi_\sigma$

Let us consider now the representation of the observation sequence $\sigma$ in the SE-LTL formula ($\Phi_\sigma$). We denote $\text{NOE}$ the disjunction of all the unobservable events, $\text{NOE} = \bigvee_{1 \leq i \leq m} u_{o_i} = u_{o_1} \lor u_{o_2} \ldots \lor u_{o_m}$ with $\text{NOE} = \top$ if the net is empty. An observation sequence $\sigma = a_1 o_2$ can be described by the following formula: $\text{NOE}(\sigma) = \text{NOE}(a_1) \lor \text{NOE}(a_2)$. In other words, an observation sequence is a trace including some unobservable events ($\text{NOE}$) until ($\lor$) an observable event $a_1$ and this event $a_1$ is directly followed ($\lor$) by unobservable events ($\text{NOE}$) until ($\lor$) $a_2$.

Note that the Until operator (a $\lor b$) accepts the traces that satisfy $a$ until $b$ but also nothing before $b$. So, if no unobservable event occurs between the two observed events $a_1$ and $a_2$ in a run, this property holds in the run. In the general case an observation sequence $\sigma$ of $k = |\sigma|$ events is then recursively given by $\Phi_\sigma = \varphi_1$ with $\varphi_n$ for $n$ from $(k-1)$ to 1 such that:

$$
\begin{align*}
\varphi_k &= \text{NOE} \lor \varphi_k \\
\varphi_n &= \text{NOE} \lor (\varphi_{n+1}) \\
\end{align*}
$$

(2)

Let us consider $\sigma = a_1 o_2 a_3$, $\varphi_1$ is given by:

$$
\begin{align*}
\varphi_3 &= \text{NOE} \lor o_3 \\
\varphi_2 &= \text{NOE} \lor \text{NOE}(a_2 \lor \text{NOE}(\varphi_3)) \\
\varphi_1 &= \text{NOE} \lor \text{NOE}(a_1 \lor \text{NOE}(\varphi_2)) \\
\end{align*}
$$

then $\Phi_\sigma = \varphi_1 = \text{NOE} \lor (a_1 \lor \text{NOE}(a_2 \lor \text{NOE}(\text{NOE}(a_3))))$

3.2 Definitions of $\Phi_{\Omega}, \Phi_{\Omega}$: the single fault case

We consider first the single fault case: $\Omega$ is the occurrence of a single fault $f \in \Sigma_f$. To define $\Phi_f, \Phi_{f-1}$, we need to distinguish two cases: diagnosis problem with or without silent closure (introduced in Lamperti and Zanella (2003)). In the case that the diagnoser does not want to address
the silent closure issue (i.e. the diagnosis function does not look for what could have happened after the last observed event of σ), only unobservable events occurring before or during executions that are consistent with the observation sequence σ are considered.

**Diagnosis without silent closure**

To check that the fault has not occurred in any run (i.e. Φ_f) it is necessary to express that the fault does not occur neither at the beginning of the observation nor between observable events. Note that at this stage only the relevant runs are considered due to the double implication of Equation (1). As an example let us consider an observation sequence σ = o_1o_2, any run in which Φ_f holds must be such that it does not include f until (⊔) o_1 that is followed (○) by some events that are not f until o_2. This can be expressed by: (¬ f ⊔ (o_1 ∧ □(ϕ_2))).

From this example it appears that the formula can be expressed in a recursive way such that: Φ_f = φ_1 with φ_n for n from (k − 1) to 1.

\[
φ_k = ¬ f ⊔ o_k \quad φ_n = ¬ f ⊔ (o_n ∧ □(ϕ_{n+1})) \tag{3}
\]

Let us consider the observation sequence σ = o_1o_2o_3, φ_1 is then given recursively by:

φ_1 = ¬ f ⊔ o_3
φ_2 = ¬ f ⊔ (o_2 ∧ ○(ϕ_3))
φ_1 = ¬ f ⊔ (o_2 ∧ ○(¬ f ∧ o_3))
φ_2 = ¬ f ⊔ (o_2 ∧ (¬ f ∧ ϕ_2))
φ_1 = ¬ f ⊔ (o_2 ∧ (¬ f ⊔ (o_2 ∧ ○(¬ f ∧ o_3))))

Then Φ_f = ¬ f ⊔ (o_1 ∧ ○(¬ f ⊔ (o_2 ∧ ○(¬ f ∧ o_3))))

Now let us define Φ_f, we need to express that the fault may appear before the observed events or between the observed events. Let us consider a simple formula of two events σ = o_1o_2. Any run where Φ_f holds must include unobservable events different from f (NOE_f(f)) until f or the event o_1 directly followed by unobservable events different from f until f: NOE_f(f) ∪ (f ∨ (o_n ∧ □(NOE_f(f) ∪ f))). This can be formalized in a recursive way by: Φ_f = φ_1 with φ_n for n from (k − 1) to 1.

\[
\begin{align*}
φ_k &= NOE_f(f) ∪ f \\
φ_n &= NOE_f(f) ∪ (f ∨ (o_n ∧ □(φ_{n+1}))) \tag{4}
\end{align*}
\]

To illustrate this formula let us consider σ = o_1o_2o_3.

φ_1 = NOE_f(f) ∪ f
φ_2 = NOE_f(f) ∪ (f ∨ (o_2 ∧ ○(ϕ_3)))
φ_1 = NOE_f(f) ∪ (f ∨ (o_3 ∧ ○(NOE_f(f) ∪ f)))
φ_2 = NOE_f(f) ∪ (f ∨ (o_2 ∧ ○(ϕ_2)))
φ_1 = NOE_f(f) ∪ (f ∨ (o_2 ∧ ○(NOE_f(f) ∪ f)))
φ_2 = NOE_f(f) ∪ (f ∨ (o_3 ∧ ○(NOE_f(f) ∪ f)))

then Φ_f = φ_1 = NOE_f(f) ∪ (f ∨ (o_1 ∧ ○(NOE_f(f) ∪ f)))

**How to deal with silent closure**

By considering the silent closure, the diagnoser must now also take into account the fact that the fault can also occur after the last observed event. To check that the fault is absent all along the run, we then need to modify Φ_f. Let \( OE = \bigvee_{1≤k≤p} o_k = o_1 ∨ o_2 ... ∨ o_p \) be the disjunction of all observable events from \( Σ_o \). Let us consider an observation sequence σ = o_1o_2. The runs that we want to characterize with Φ_f do not include the fault event f until o_1, that is followed by unobservable events different from f until o_2, that is followed by unobservable events different from f until any observable event. So by generalizing, the formula Φ_f is given in a recursive way as Φ_f = φ_1 with φ_n for n from k to 1.

\[
\begin{align*}
φ_{k+1} &= ¬ f ∪ OE \\
φ_n &= ¬ f ∪ (o_n ∧ □(ϕ_{n+1})) \tag{5}
\end{align*}
\]

Suppose now that \( M \models Φ_f \Rightarrow Φ_f \) is true, the diagnosis function stops and returns \( f_{Absent} \) (see Algorithm 1). If it is not true, suppose that f occurs before the last observed event in every run such that \( Φ_f \) holds, then with the previous formula Φ_f, \( M \models Φ_f \Rightarrow Φ_f \) is true and Δ returns \( f_{Faulty} \). Finally, if there is a run, such that \( Φ_f \) holds, in which f occurs only after the last observed event, then there is also a run, such that \( Φ_f \) holds, in which f does not occur at all (it is a prefix of the previous run) so in this last case, Δ must return \( f_{Ambiguous} \). Therefore, the formula Φ_f remains unchanged when dealing with silent closure, only Φ_f is modified.

### 3.3 Extension to fault class diagnosis

We now propose to extend this pure model-checking approach to deal with a specific type of patterns called fault class. A fault class gathers a set of independent fault events denoted \( F = \{ f_1, f_2, ..., f_q \} \subseteq Σ_o \). Following Algorithm 1, the diagnosis principle is to check first that no fault of the class has occurred in a run of the system consistent with the observation sequence. If the result is true (i.e. \( M \models Φ_f \)), the diagnosis result is \( F_{Absent} \) i.e it is sure that the fault class has not occurred. Otherwise, if in any system’s run consistent with the observations σ, at least one fault of the class F has occurred, the diagnosis result is \( F_{Faulty} \). Finally, Δ returns \( F_{Ambiguous} \) if there are at least two runs consistent with σ such that one contains at least a fault of the class F and the other run does not contain any fault of the class.

**Fault class diagnosis without silent closure**

The SE-LTL formula that checks the absence of a fault class is similar to the one proposed in the case of simple fault where the simple fault event f is replaced by F that denotes the disjunction of all the faults of F: \( F = \bigvee_{f \in F} f \).

Then Φ_f = φ_1 with φ_n for n from (k − 1) to 1.

\[
\begin{align*}
φ_k &= ¬ F ∪ o_k \\
φ_n &= ¬ F ∪ (o_n ∧ □(φ_{n+1})) \tag{6}
\end{align*}
\]

Formula Φ_F is also derived from the simple case (see Equation (4)). Φ_F = φ_1 with φ_n for n from (k − 1) to 1.

\[
\begin{align*}
φ_k &= NOE_F(f) ∪ F \\
φ_n &= NOE_F(f) ∪ (F ∨ (o_n ∧ □(φ_{n+1})))
\end{align*}
\]

**Case of the silent closure**

For the absence of fault class with the silent closure, the
formula is similar to the one given for the simple fault (see Equation (5)) by replacing the event \( f \) by \( F \). To check the occurrence of the fault class, the same formula as the one used in the case without the silent closure is considered for the same reason as in the simple fault case.

4. FIRST EXPERIMENTAL RESULTS

This section describes some experimental results about the PURE approach presented here above. These experiments are performed on the example from Pencolé and Subias (2017): a product transportation system.

4.1 Product transportation system

The proposed example is a two-level system composed of two product sites (namely sites 1 and 2) at level 2 where a set of products is stored and two assembly stations at level 1 that request products from level 2. A lift is used between the two levels. Each site has a conveyor belt to move a product from the site to the lift and each station also has a conveyor belt to get the products from the lift. Figure 1 presents the system LPN model. Once a product is detected on the site 1, a Product1 signal (denoted \( Pr1 \)) is emitted. The product is then put in a box and becomes available. The box is then pushed and sent into the lift (action \( EP_{1} \) that starts with event \( P_{1} \) and ends with event \( EP_{1} \)). Site 2 behaves in a similar manner. A product from site 1 has a priority access to the lift. In the Petri net model of Figure 1, this priority relation is indicated by a dashed edge between the transitions labeled by \( P_{1} \) and \( P_{2} \). After a product has been pushed into the lift, the lift goes down (action \( Down \)) to reach level 1 (signal \( D \) detects the end of the \( Down \) action). At this level the box is directed to the station that makes the request (either request \( Req_{1} \) or request \( Req_{2} \)). If it is request \( Req_{1} \), then the box is pushed on the conveyor belt of station 1 (action \( PushReq_{1} \) that starts with event \( Req_{1} \) and ends with event \( EPReq_{1} \)). Then the conveyor belt performs a move-left action (\( LeftReq_{1} \), \( ELReq_{1} \)) to deliver the box with the product and a move-right action to go back to initial position (\( RightReq_{1} \), \( ERReq_{1} \)). The behaviour of station 2 is similar except that its conveyor belt moves right first (\( RightReq_{2} \), \( ELReq_{2} \)) and then moves left (\( LeftReq_{2} \), \( ELReq_{2} \)). Once the box has been pushed on a conveyor belt, the lift goes up (action \( Up \) ending with the emission of signal \( U \)) to level 2. In Figure 1 only underlined events are observable events.

4.2 Results

The objective in this experimental setting is to diagnose the occurrence of the event \( Req_{1} \) based on the sequence \( \sigma : Pr1 \ Pr2 \ D \ U \ D \ ERReq2 \ ELReq2 \ ELReq1 \ ERReq1 \ U \) composed of ten observable events. In the approach that we already described, the Kripke structure \( \mathcal{M} \) computed by TINA depends only on the system and has 204 states and 625 transitions. Table 1 presents the results and any line represents one step of the scenario \( \sigma \) (i.e line 1 is \( Pr1 \), line 2 is \( Pr1 \) followed by \( Pr2 \) etc). Columns 2 and 3 indicate respectively the answer of the question are we sure that the fault never occurred on a run consistent with the observation sequence \( \Psi_{-f} \) and we are sure that \( f \) always occurred in any run consistent with the observation sequence \( \Psi_{f} \). Column 4 gives the diagnosis conclusion and the last column is the computation times obtained by TINA.

Table 1. Fault event diagnosis with silent closure

| \( \Psi_{-f} \) | \( \Psi_{f} \) | Diagnosis | Time(ms) |
|---|---|---|---|
| \( Pr1 \) | T | F | Absent | 32 |
| \( Pr2 \) | T | F | Absent | 24 |
| \( D \) | F | F | Ambiguous | 22 |
| \( U \) | F | F | Ambiguous | 50 |
| \( D \) | F | F | Ambiguous | 126 |
| \( ERReq2 \) | F | F | Ambiguous | 55 |
| \( ELReq2 \) | F | F | Ambiguous | 3189 |
| \( ELReq1 \) | F | T | Faulty | 25522 |
| \( ERReq1 \) | F | T | Faulty | 240524 |
| \( U \) | x | x | x | x |

Table 2. Fault event diagnosis without silent closure

| \( \Psi_{-f} \) | \( \Psi_{f} \) | Diagnosis | Time(ms) |
|---|---|---|---|
| \( Pr1 \) | T | F | Absent | 12 |
| \( Pr2 \) | T | F | Absent | 14 |
| \( D \) | T | F | Absent | 13 |
| \( U \) | F | F | Ambiguous | 22 |
| \( D \) | F | F | Ambiguous | 153 |
| \( ERReq2 \) | F | F | Ambiguous | 197 |
| \( ELReq2 \) | F | F | Ambiguous | 1076 |
| \( ELReq1 \) | F | T | Faulty | 7121 |
| \( ERReq1 \) | F | T | Faulty | 66799 |
| \( U \) | x | x | x | x |
(the diagnosis is Absent) while the diagnosis is Ambiguous when considering the silent closure. Now regarding the computation time, results show the computation time is not satisfactory as the computation time is exponential with the size of the observation sequence and PROD clearly outperforms PURE. A detailed analysis of the set of experiments indicates the source of the complexity problem. As explained in Section 2.1, one step of the model-checking is the computation of the language $L(\neg \Phi)$, that is the conversion of the formula $\neg \Phi$ into its equivalent Büchi automaton and this conversion is unfortunately exponential to the length of formula $\Phi$ (i.e. $O(2^{|\Phi|})$) and so, by construction of the formula $\Psi_f$, $\Psi_{\neg f}$ (see Section 3), PURE is exponential in the size of $\sigma$ ($O(2^{|\sigma|})$). Looking now at the PROD method, this limitation is not present. Indeed, even if PROD follows the same algorithm (see Algorithm 1), the information stored in $M$ and in $\Psi_\Omega, \Psi_{\not\Omega}$ are fundamentally different. PROD relies on a set of specific products of LPNs to gather in one single LPN the information about how a pattern $\Omega$ matches any single run of the system and how any single run of the system is consistent with $\sigma$. The size of $M$ is in $O(2^{|\sigma|})$ in PROD while in PURE the size of $M$ does not depend on $|\sigma|$. Secondly, in PROD, the questions $\Psi_\Omega, \Psi_{\not\Omega}$ do not depend on the size of $\sigma$ as opposed to the ones from PURE. As opposed to PROD, our objective is to benefit of the expressivity of SE-LTL to write pattern $\Omega$ and not the classical transition fusion as in PROD to isolate the runs of the system consistent with the observation sequence. The result of this synchronisation is an LPN that would be converted as a Kripke structure $M$ (in $O(2^{|\sigma|})$ as in PROD) and $\Psi_\Omega, \Psi_{\not\Omega}$ would only characterize the pattern matching (independent from $\sigma$).

5. AN APPROACH OVER A MODEL SYNCHRONISED WITH THE OBSERVATIONS

This section describes the model-checking approach denoted SYN that is a compromise between PURE and PROD. The observed sequence $\sigma$ is then modeled by a LPN $O$ as a single sequence of places and transitions. A synchronised product between the LPN of the system and the LPN of the observation sequence is performed based on transition fusions (see Hack (1975)). Figure 2 presents such a synchronisation. The SYN approach is still governed by Algorithm 1 and the formula $\Psi_\Omega$ and $\Psi_{\not\Omega}$ still follow Equation (1). But now, as opposed to the PURE approach, and similarly to the PROD approach, $\Psi_\sigma$ consists in checking whether a system’s run will eventually lead to a marking where $p_{obs}$ is marked in the synchronised LPN (see Figure 2). It follows that in the SYN approach $\Phi_\sigma = \Diamond p_{obs}$ whose length does not depend on $|\sigma|$.

5.1 Diagnosis of a fault event

The objective is to check that no run consistent with the observations contains the fault event $f$ at the beginning or between two observed events. The formula $\Phi_{\neg f}$ is then given by: $\Phi_{\neg f} = \Box \neg (f \land \Diamond (\Diamond (o_k \land \bigcirc p_{obs})))$. $\Phi_{\neg f}$ asserts that fault $f$ never occurs ($\Box \neg$) if in the future ($\Diamond$) the system eventually leads ($\bigcirc$) to the firing of the last event of $\sigma$ ($o_k \land \bigcirc p_{obs}$). It is necessary to consider the sequence until the last observed event followed by $p_{obs}$ as the event $o_k$ is not a sufficient condition since the same event can be observed several times in the observation sequence, so its observation does not guarantee the end of the execution of the observation sequence. Moreover $p_{obs}$ is also not sufficient as it remains marked after the execution of the observations whereas some unobservable events can occur after the last observation. The formula $\Phi_f$ is then given by $\Phi_f = \Diamond (f \land \Diamond (o_k \land \bigcirc p_{obs}))$: it means the system will eventually ($\bigcirc$) fire $f$ followed ($\Diamond$) by a set of transitions to end as a run that is consistent with $\sigma$ ($o_k \land \bigcirc p_{obs}$). Unlike the PURE method, $\Phi_f$ has a size independent of $|\sigma|$.

5.2 Diagnosis of a fault class and multiple faults

The problem of fault class diagnosis can also be considered by replacing the event $f$ by $F$ the disjunction of all the faults of the class in the previous formulas $\Phi_{\neg f}$ and $\Phi_f$ as in the PURE method. Now we may also be interested in the diagnosis of multiple faults. Suppose for instance a fault class $F = \{f_1, f_2\}$, the multiple fault associated with $F$ is the occurrence of fault $f_1$ and $f_2$ in the same run. To diagnose such a multiple fault, the formulas $\Phi_{\neg F}, \Phi_F$ can be adapted: $\Phi_{\neg F} = \Box \neg (\varphi_2)$ and $\Phi_F = \varphi_2$ where $\varphi_2 = \Diamond (f_1 \land \Diamond (\Diamond (o_k \land \bigcirc p_{obs})))) \land (\Diamond (f_2 \land \Diamond (\Diamond (o_k \land \bigcirc p_{obs}))))$.

The problem can be generalised to $F = \{f_1, f_2, \ldots f_q\}$, thus $\Phi_{\neg F} = \Box \neg (\varphi_q)$ and $\Phi_F = \varphi_q$ where $\varphi_q$ is such that

\[
\varphi_q = (\varphi_{q-1}) \land (\Diamond (f_q \land \Diamond (\Diamond (o_k \land \bigcirc p_{obs})))) \\
\varphi_1 = (\Diamond (f_1 \land \Diamond (\Diamond (o_k \land \bigcirc p_{obs})))))
\]

(7)

5.3 Pattern diagnosis

With the SYN approach, the problem of fault event diagnosis can be extended to the fault pattern diagnosis that aims at diagnosing more complex faulty behaviours or any normal behaviour of interest (Jéron et al. (2006), Pencolé

\footnote{The next operator is used to traduce that the events $f$ and $o_k$ are not simultaneous, but it is not mandatory.}
and Subias (2017)). For instance a pattern can describe a sequence of n unobservable events: let us consider a pattern \( \Omega_1 \) corresponding to the sequence \( e_1 e_2 e_3 \). If \( \Omega_1 \) occurs in a run then the run first leads to \( e_1 \) followed in the future by \( e_2 \) that is followed in the future by \( e_3 \) followed by the last observed event followed by end of synchronisation:

\[
\Phi_{\Omega_1} = \Diamond (e_1 \land \Box \Diamond (e_2 \land \Box \Diamond (e_3 \land \Box \Diamond (\neg e_6 \land \Box \Diamond p_{obs}))))
\]

And the absence \( \Omega_1 \) in the run is characterized by the negation: \( \Phi_{\neg \Omega_1} = \Box (\neg \Phi_{\Omega_1}) \). This approach allows also to consider more complex behaviour that one might want to diagnose. Let us come back to system of Figure 1. Figure 3 describes a pattern \( \Omega_2(3,2) \) that models 3 consecutive occurrences of events of type \( Req_1 \) that are not interleaved with any occurrence of \( Req_2 \) events (the left conveyor keeps having its requests fulfilled while the right conveyor gets stuck) and this pattern should occur 2 times (the pattern has a counter). This pattern is made up of normal events but defines a behaviour that could be unexpected (both conveyors at Level 1 should get items \textit{regularly}). In this case it is necessary first to express that in the future the event \( Req_1 \) is not followed by \( Req_2 \) until \( Req_1 \) that is not followed by \( Req_2 \) until \( Req_1 \). \( \Xi = \Diamond (\neg Req_1 \land \Box (\neg Req_2 \lor \Box \Diamond Req_1) \land \Box (\neg Req_2 \lor \Box \neg Req_1)) \). Then, to check that the behaviour described above occurs twice the formula \( \Phi_{\Omega_2(3,2)} \) is given by: \( \Phi_{\Omega_2(3,2)} = \Xi \land \Box (\Xi \land \Box (\neg e_6 \land \Box \Diamond p_{obs})) \). The absence of the pattern is checked through \( \Box (\neg \Phi_{\Omega_2(3,2)}) \).

5.4 Results analysis

The SYN approach handle more complex problems than the PURE approach with satisfactory size of observations but is still not as efficient as PROD (see Table 3). Based on the pattern \( \Omega_2(n,\text{counter}) \) of Figure 3, several comparisons have been made with the PROD approach by selecting different values for \( n \) (the number of \( Req_1 \)), \( \text{counter} \) and the size of the observation sequence. As an example Table 3 shows the results of the two approaches for one occurrence of five events \( Req_1 \) without \( Req_2 \), for ten occurrences of ten consecutive events \( Req_1 \), with several sizes for the observation sequence. The SYN approach is efficient relatively to the size of the Kripke structure that only depends on the observation sequence. Nevertheless, for the same reason as for the PURE approach, SYN is limited by the length of the pattern formula. SYN must be only considered for small patterns only.

6. CONCLUSION AND FUTURE WORK

This paper addresses the diagnosis of fault patterns in the context of Petri nets. This problem is formulated as a model checking problem using the TINA tool taking benefit of the expressivity of SE-LTL. According to the knowledge encoded in the question and in the model, several approaches are presented and compared. In our perspectives, we plan to investigate alternative approaches to keep a low complexity both in the formula and in the models given to the model checker. One direction would be to consider a new way for the intersection of Petri nets (Lubat et al. (2019)). The applicability to online diagnosis is also an issue.

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