The Tajmar effect from quantised inertia

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Abstract – The Tajmar anomaly is an unexplained acceleration observed by gyroscopes close to, but isolated from, rotating rings cooled to 5 K. The observed ratio between the gyroscope and ring accelerations was $3 \pm 1.2 \times 10^{-8}$ for clockwise rotations and about half this size for anticlockwise ones. Here, this anomaly is predicted using a new model that assumes that the inertial mass of the gyroscope is caused by Unruh radiation that appears as the ring and the fixed stars accelerate relative to it, and that this radiation is subject to a Hubble-scale Casimir effect. The model predicts that the sudden acceleration of the ring causes a slight increase in the inertial mass of the gyroscope, and, to conserve momentum the gyroscope must move with the ring with an acceleration ratio of $2.67 \pm 0.24 \times 10^{-8}$ for clockwise rotations and $1.34 \pm 0.12 \times 10^{-8}$ for anticlockwise ones, in agreement with the observations. The model predicts that in the southern hemisphere the anomaly should be larger for anticlockwise rotations instead, and that with a significant reduction of the mass of the disc, the decay of the effect with vertical distance should become measurable.

Introduction. – It has been found experimentally by [1–3] that when rings of niobium, aluminium, stainless steel and other materials are cooled to 5 K and spun, then accelerometers and laser gyroscopes, not in frictional contact, show a small unexplained acceleration in the same direction as the ring, with a size $3 \pm 1.2 \times 10^{-8}$ times the acceleration of the ring for clockwise rotations, and about half that value for anticlockwise ones. This is called the Tajmar effect and is similar to the Lense-Thirring effect (frame-dragging) predicted by General Relativity, but is 20 orders of magnitude larger and shows the added parity violation. The effect has not yet been reproduced in another laboratory.

[4] proposed an explanation for the anomaly that relies on a Higgs mechanism that causes the graviton to gain mass. This theory was called the gravitometric London effect (MiHsC), but has been discredited because the inception of the Tajmar effect (at about 25 K) does not coincide with the superconducting transition temperature, only with very low temperatures.

The model suggested here as an explanation for the effect was proposed by [5]. It assumes that the inertial mass of an object is caused by Unruh radiation which is generated by the object’s acceleration relative to other matter, and that this radiation is subject to a Hubble-scale Casimir effect (in which longer Unruh waves are increasingly disallowed). The model could be called modified inertia due to a Hubble-scale Casimir effect (MiHsC) or perhaps quantised inertia, and was tested by [5] on the Pioneer anomaly (observed by [6]). In [5] the inertial mass ($m_I$) was defined as

$$m_I = m_g \left(1 - \frac{\beta \pi^2 c^2}{|a| \Theta} \right),$$

where $m_g$ is the gravitational mass, $\beta = 0.2$ (empirically derived by Wien as part of Wien’s law), $c$ is the speed of light, $\Theta$ is the Hubble diameter ($2.7 \times 10^{26}$ m, from [7]) and the magnitude of the acceleration ($a$) in this case was the acceleration of the Pioneer craft relative to their main attractor: the Sun. This model predicted a small loss of inertial mass for the Pioneer spacecraft that increased their Sunward acceleration by an amount close to the observed Pioneer anomaly (beyond 10 AU from the Sun).

Reference [8] applied MiHsC to the unexplained velocity jumps observed in Earth flybys of interplanetary probes (the flyby anomalies observed by [9]) and found that these anomalies could be reproduced quite well if the acceleration in eq. (1) was taken to be that of the spacecraft relative to all the particles of matter in the spinning Earth. Using MiHsC and the conservation of momentum the predicted anomalous jump for spacecraft passing by the spinning Earth was

$$dv' = \frac{\beta \pi^2 c^2}{\Theta} \left( \frac{v_2}{|a_2|} - \frac{v_1}{|a_1|} \right),$$

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where \( a_1 \) and \( a_2 \) were the average accelerations of all the matter in the spinning Earth seen from the point of view of the incoming \( (a_1) \) and outgoing \( (a_2) \) craft. This formula predicted a slightly lower inertia for spacecraft close to the Earth’s spin axis, meaning that, by conservation of momentum, their speed increases by an amount similar to the observed flyby anomalies (except for the EPOXI flyby, which was much further from the Earth than the others and is discussed later).

The Tajmar effect is similar to the flyby anomalies, although instead of being an anomalous acceleration of a spacecraft close to a spinning planet, it is an anomalous acceleration of a laser gyroscope close to a cold spinning ring. Therefore, in this paper MiHsC is applied to the Tajmar effect, but also, following Mach’s principle, in this paper the effect of the relative accelerations of the fixed stars on inertia are also considered. A previous paper by the author [10] applied the same theory, conserving momentum relative to the fixed stars, and explained Tajmar’s setup A and B numerically (though the reference frame used was incorrect) and setup B’s rotation direction. However, the rotation direction in setup B may have had a clockwise bias [11]. In this paper the analysis is the same except that momentum is conserved relative to the moving ring and the model explains very well the results from setup A [3] (and setup B if the bias is corrected) including the observed parity violation in setup A. As shown here, MiHsC predicts that performing the experiment in the southern hemisphere, the gyroscope should still follow the ring’s rotation, but the parity violation should be greater for anticlockwise ring rotation instead of for clockwise. This is an important correction to the prediction made by [10] where the gyroscope’s rotation was predicted to be clockwise in the north, and anticlockwise in the south.

**Method and results.** – The assumed experimental setup is that of [3] and their setup configuration A, and is shown in fig. 1, with a rotating super-cooled ring of radius \( r \). Three laser gyroscopes of mass \( m \) are located above the ring. The fixed stars are also shown schematically. They have a huge combined mass, but are very far away.

For the laser gyroscope situated just above the ring we can assume a conservation of momentum parallel to the ring’s edge (subscript “\( r \)”) \( m_r v_{gr1} = m_r v_{gr2} \).

This is similar to eq. (2), which was derived from MiHsC for the flyby anomalies. For this new case, the initial and final accelerations \( (a_{g1} \) and \( a_{g2} \) of the gyroscope with respect to all the surrounding masses now need to be defined. First we assume that because of cooling the temperature-dependent acceleration of nearby atoms is small. We can say that the acceleration relative to the atoms in the Earth is zero since the experiment is solidly fixed to the Earth. So, before the ring accelerates the gyroscope sees only an acceleration of the fixed stars since it is on the spinning Earth. These are far away, but their combined mass is huge. The rotational acceleration with respect to the fixed stars \( (a_s) \) of an object fixed to the Earth at the latitude of Seibersdorf in Austria where the experiment was performed (at 48°N) is the same as the Coriolis acceleration: \( f \) where \( f \sim 0.0001 \text{s}^{-1} \) in mid-latitudes, and \( v \), the spin velocity of the Earth at this latitude is 311 m/s, so \( a_s = 0.0311 \text{m/s}^2 \). To this should be added the acceleration due to the Earth’s orbit around the Sun (so we have: \( a_s = (0.0311 + 0.006) \text{m/s}^2 \)). The acceleration due to the Sun’s orbit around the galaxy is far smaller and can be neglected. So in the above formula \( a_{g1} = 0.0371 \text{m/s}^2 \).

The sudden acceleration of the Tajmar ring causes an acceleration of \( a_R = 33 \text{rad/s}^2 = 2.5 \text{m/s}^2 \) (since the radial position of the gyroscope was 0.075 m). Therefore \( a_{g2} = f n (a_s, a_R) \). However, to find the average acceleration we have to consider the relative importance of the fixed stars and the ring for determining the inertia of the gyroscope. As in [10] we will assume here that the importance of an object for the inertia of another one is equivalent to its gravitational importance, which is proportional to its mass over the distance squared. The details of this assumption do not effect the final result as we will see. Therefore \( a_{g2} \) is

\[
a_{g2} = \frac{m_R}{r_R} a_s + \frac{m_R}{r_R} a_{gr} \ ,
\]

\[
\text{Fig. 1: Schematic showing the experiment of [3], their setup A. The ring, the lower, middle and upper laser gyroscopes, some dimensions and the fixed stars are shown.}
\]
where \( m_s \) is the mass of all the fixed stars, and \( r_s \) is their mean distance away and \( m_R \) is the mass of the ring and \( r_R \) is its distance away. Using eq. (6) in eq. (5) gives

\[
v_{gr2} - v_{gr1} = \frac{\beta \pi^2 c^2}{\theta} \left( \frac{v_{gr2}}{\frac{M}{a_{g2} + \frac{118a_{g2}}{118}} - \frac{v_{gr1}}{a_{gs}}} \right)
\]

These values were approximated as a total stellar mass of \( m_s \sim 2.4 \times 10^{32} \text{ kg} \) from [12], at a distance of \( r_s \sim 2.7 \times 10^{26} \text{ m} \) \( (r_s = 2c/H) \), derived from the Hubble constant, \( H \), from [7], and a ring mass of \( m_R \sim 0.336 \text{ kg} \) (stainless steel has a density of about 8000 kg/m\(^3\)), and the ring had a circumference of \( 2 \pi \times 0.075 \text{ m} \) with a width of 0.006 m and a ring distance of \( r_R \sim 0.0533 \text{ m} \) (the estimated vertical distance from the center of the lower gyroscope to the ring). Using these values we have

\[
v_{gr2} - v_{gr1} \sim \frac{\beta \pi^2 c^2}{\theta} \left( \frac{v_{gr2}}{0.33a_{g2} + \frac{118a_{g2}}{118}} - \frac{v_{gr1}}{a_{gs}} \right).
\]

We can therefore neglect \( a_{g2} \) in the denominator of the first term in brackets (this simplification similarly applies to the aluminum and niobium rings) to give

\[
v_{gr2} - v_{gr1} \sim \frac{\beta \pi^2 c^2}{\theta} \left( \frac{v_{gr2}}{a_{g2}} - \frac{v_{gr1}}{a_{gs}} \right).
\]

Now, since the ring is spinning fast at \( t = 2 \) the first term in the brackets is small (since \( a_{g2} = v_{gr2}' \)) so we can say

\[
dv' \sim \frac{\beta \pi^2 c^2}{\theta} \left( - \frac{v_{gr1}}{a_{gs}} \right).
\]

Differentiating with respect to time to find the resulting anomalous acceleration and neglecting changes in \( a_{gs} \) (for now),

\[
da' \sim -\frac{\beta \pi^2 c^2}{\theta} a_{gr} \frac{-6.6 \times 10^{-10}}{0.0371} a_{gr}.
\]

Changing \( a_{gr} \) to \( a_{rg} \) (which involves a sign change) we get

\[
da' \sim \frac{\beta \pi^2 c^2 a_{rg}}{\theta} \frac{6.6 \times 10^{-10}}{0.0371} a_{rg} \sim 1.78 \pm 0.16 \times 10^{-8} a_{rg}.
\]

The \( a_{rg} \) is the rotational acceleration of the ring with respect to the gyroscope so eq. (12) implies that the anomalous rotational acceleration of the gyroscope will be in the same direction as the ring, as observed by [3] for setup A and the predicted acceleration ratio is \( 1.78 \pm 0.16 \times 10^{-8} \) in agreement with the observed value which was \( 3 \pm 1.2 \times 10^{-8} \). The predicted error of 0.16 was derived by assuming a 9\% error in the Hubble constant (and therefore the estimated Hubble diameter) following [7]. Of the two ratios compared here, one is a observed velocity ratio and one is a predicted acceleration ratio. These are the same because the acceleration of the gyro (\( da' \)) is not a spin-up of the gyro’s field coil itself but an acceleration of the gyro along the ring’s edge, in the same radius circle as the ring. Therefore in the formula \( a = v^2/r \) \( r \) is the same in both cases, so the \( v \) and along-ring acceleration ratios are the same.

At the top speed of the ring the anomalous gyroscope output (its spin) was equal to one third of the Earth’s rotation, according to [3]. The \( a_{gs} \) in eqs. (10)–(12) then also depends on the gyroscope’s rotation. If the ring (and then the gyro) rotate clockwise (anticlockwise) then this is counter to the Earth’s rotation and so the original acceleration of the gyro with respect to the fixed stars: \( a_{gs} \) decreases by one third, therefore 1/\( a_{gs} \) increases by 1.5 times, and so the predicted anomalous signal from eq. (12) for clockwise rotations increases from 1.78 \( \pm 0.16 \times 10^{-8} \) to 2.67 \( \pm 0.24 \times 10^{-8} \) (the observation was 3 \( \pm 1.2 \times 10^{-8} \)). If the ring and gyroscope rotate anticlockwise then this adds to the Earth’s own spin, it increases \( a_{gs} \) by one third, so 1/\( a_{gs} \) decreases to 75\% of its original value and the predicted signal decreases to 1.34 \( \pm 0.12 \times 10^{-8} \) and this agrees with the observed anticlockwise value which was about half of the clockwise value: 1.5 \( \pm 1.2 \times 10^{-8} \), according to [3] (their fig. 2).

**Discussion.** – In the model used here (MIHsC) the inertial mass of the gyroscope is assumed to be determined by Unruh radiation that appears as it accelerates with respect to every other mass in the universe. The Unruh radiation is also subject to a Hubble-scale Casimir effect. Before its surroundings are cooled the gyroscope sees large accelerations due to the vibration of nearby atoms, it is surrounded by Unruh radiation of short wavelengths, MIHsC has only a small effect, and the inertial mass of the gyroscope is close to its gravitational mass. The nearby atomic accelerations reduce when the surroundings (the cryostat) are cooled, so that the inertia of the gyroscope is more sensitive to the accelerations of the fixed stars (it is on the rotating Earth). This is a small acceleration, so the Unruh waves it sees are long and a greater proportion are disallowed by MIHsC, so the gyroscope’s inertial mass (from eq. (1)) falls very slightly below its gravitational mass. In this case it loses \( 2 \times 10^{-8} \) kg for every kilogram of mass. However, when the nearby ring accelerates, the gyroscope suddenly sees the higher accelerations of the ring, the Unruh waves shorten, fewer are disallowed, and its inertial mass increases again following eq. (1). The important point is that to conserve momentum with respect to the ring, the heavier gyroscope must accelerate in the same direction as the ring. A further complication is that when the gyroscopes do start to spin with the ring, it changes its original acceleration with respect to the fixed stars, producing the parity violation.

MIHsC (eq. (1)) violates the equivalence principle, but not in a way that could be detected by the usual torsion balance experiment. These experiments measure
the differential attraction of two balls on a cross bar suspended on wire, towards distant masses by detecting tiny twists in the wire (e.g., [13]). With MiHsC these two balls would have equal accelerations with respect to the distant masses (being rigidly connected) so their inertial masses would be modified equally by MiHsC, and there will be no twist in the wire, and no apparent violation of equivalence.

Equation (7) predicts a decay in the MiHsC effect with distance. For this experiment, setup A, at 1 m, 5 m, 20 m and 56 m away the effect is predicted by eq. (7) to be 0.03%, 0.8%, 11.5% and 50% smaller. The decay at 20 m may be detectable, but cryostats this long are difficult to find. The effect should decay more quickly outside the cryostat, tending to a decay proportional to one over distance squared due to nearby thermal accelerations, but this needs to be studied in more detail. (See [10] for a more detailed discussion of the decay.) Calculations using eq. (7) show that a change in the ring velocity does not change the measured gyro/ring ratio at the lower gyroscope (0.0553 m away) much, but it does change the decay of the effect with distance. For example, a reduction in ring acceleration by a factor of 10000 (a reduction of velocity by a factor of 100) will increase the decay of the MiHsC effect with distance, so that the upper gyroscope 0.2283 m away would see a decrease of 13.6%, but the gyroscope may not be able to measure the lower velocities. A better way to achieve the same result would be to reduce the ring’s mass. If this was reduced by a factor of 10000 the gyro/ring acceleration ratio would remain the same at the lower gyroscope 0.0553 m away, but the gyroscope 0.2283 m away should now see a 13.6% drop in the effect. Tajmar’s group could try this test with their existing equipment.

As discussed in the introduction, [8] applied MiHsC quite successfully to predicting the flyby anomalies. Subsequent data obtained from J. Campbell of NASA (at a flyby workshop organised by the International Space Science Institute, ISSI, in Bern, Switzerland) shows that the flyby anomaly for the EPOXI spacecraft was zero, whereas MiHsC predicts a large anomaly. A difference with the EPOXI flyby was that it has a periapse radius of 49835 km [14] from the Earth whereas the other flybys were closer, typically 7000 km away. As stated by [10], in MiHsC the effect of one body on the inertia of another decays as its mass over its distance squared and therefore the sensitivity of the EPOXI craft’s inertia to the spin of the Earth should be lower, and its sensitivity to other (more constant) relative accelerations from other Solar System bodies should be more important. The flyby anomalies can be predicted more successfully, if the accelerations within the Sun are assumed to be 1 m/s^2. It is not known whether this value is realistic.

A previous paper by the author applied the same theory [10], but it is now thought that the reference frame used was wrong. The new prediction is that performing the experiment in the southern hemisphere the gyroscope should still follow the ring’s rotational direction, but the anomalous signal should be greater for anticlockwise ring rotations instead of for clockwise ones in the north.

Conclusions. – The anomalous accelerations observed by laser gyroscopes close to rotating super-cooled rings (see [3], for setup A) can be predicted by a theory (MiHsC) that assumes that the inertial mass of the gyroscope is caused by Unruh radiation that appears because of its mutual acceleration with the fixed stars and the spinning ring, and that this radiation is subject to a Hubble-scale Casimir effect.

The parity violation observed by [3] for setup A can also be explained by MiHsC as a secondary effect brought on by the movement of the gyro relative to the fixed stars. This spin with respect to the stars, changes the gyro’s inertia and its anomalous acceleration depending on the spin direction, in good agreement with the observations.

It is proposed that the validity of MiHsC in this case could be tested by reducing the mass of the ring in [3] setup A by a factor of 10000 or more, and looking for a measurable decay of the effect with vertical distance.

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