Integrated approach of superiority and inferiority ranking method with interval-valued intuitionistic fuzzy cognitive map (IVIFCM) for analyzing the online resource tools used in teaching and learning during Covid-19 in India

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Abstract

Fuzzy Cognitive Maps (FCM) are used to model the relationships between criteria and to handle high level of uncertainty. Multi criteria decision making (MCDM) is fascinating method to choose the optimal alternative from multiple alternatives according to some criteria. To deal with uncertainty in high complex non-linear problems FCMs are used whereas in problems involving decision making, to estimate the strength of the relationship is difficult. Inter-Valued Intuitionistic Fuzzy Cognitive Map is used to find the precise values in multi criteria decision making. In the recent researches to prioritize the products various ranking methods are applied. SIR-Superiority and Inferiority ranking method is a novel one with two types of flows superiority and inferiority through which the set of alternatives are ranked partially or completely. In this paper, the new integrated approach of Interval-Valued intuitionistic Fuzzy Cognitive Map (IVIFCM) with SIR is introduced and the effectiveness of the approach along with practicality is verified through the illustrative example of analyzing the best online resource tools used in higher education during the pandemic season in most of the countries with various criteria.

Keywords

Multi-Criteria Decision Making, Fuzzy Cognitive Map, Interval-Valued intuitionistic Fuzzy Cognitive Map, Intuitionistic Fuzzy, Superiority and Inferiority Ranking method.

AMS Subject Classification

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1. Introduction

The sudden major and abrupt change the world underwent recently with the outbreak of Covid 19 is unforgettable. Evidently Covid 19 has great impact with schools, colleges
and universities. In the recent past years India is able to use technology to find apprehension combing excellence and equity [2]. As a result, technology has supported to sustain in all the possible ways and means during pandemic not sparing education too. Online teaching and learning replaced traditional classes as immediate remedy throughout the world. Higher Education has regularized the classes through the institutional LMS and online resource tools for scheduling virtual classes and for conducting examinations. Private schools, colleges and universities have adopted it quickly. Though technology makes things easier and accessible it is limiting [6]. It is essential to analyze the resource tool for effective teaching and learning. To choose the best tool, multi criteria decision making is involved. It involves choosing optimal alternative from multiple alternatives according to some criteria.

It is familiar that Fuzzy Cognitive Maps are fuzzy signed weighted digraphs in which nodes and edges represent concepts and causal relationships between concepts respectively. In decision making incorporating a high level of uncertainty is done with the help of with the help of FCM which appeals for supporting decisions. It is difficult to take decision under dynamic and uncertain environment. Since finding the precise values of concepts and causal relationship are considered difficult, the generalizations of FCM like Intuitionistic Fuzzy Set theory with membership degree, non-membership degree and hesitation degree helpful in decision making [4]. In solving MCDM problems IVIFSs were used to find the preferences of the criteria partially. IVIFSs are found suitable to describe the decision problems where alternatives and criteria values could not be expressed as numerical values or fuzzy sets [3]. To solve MCDM problems different methods like TOPSIS, VIKOR, CODAS, ELECTRE, PROMETHEE are used, etc. Sometimes MCDM problems cannot be solved by standard methods if they have imprecise representation of criteria, contradictory decisions of decision makers and if they have interactions. So in order to model the interactions among criteria IVIFCM is chosen in which criteria is represented as IVIFSs because of its additional degree of hesitation [3]. Finally in the selection process to ensure the consistency and integrity of the information, choosing the best alternatives, different ranking methods are applied.

In this integrated approach, to analyze and overcome the uncertainty in choosing the online tool for teaching and learning in education, Superiority- Inferiority Ranking is combined in order to make use of the new information like S-matrix and L-matrix with S-flow and I-flow reflecting the attitude of DMs towards criteria derived from the decision matrix along with interval valued-intuitionistic fuzzy cognitive map method. Finally the preferable alternative is chosen from the mapping of the criterion descending or ascending order.

### 2. Preliminaries

#### 2.1 Intuitionistic Fuzzy Sets [1]

Intuitionistic fuzzy sets are sets whose elements have membership and non-membership degree, were introduced by as Krassimir Atanasov in 1983 an extension of Lofti Zadeh’s Fuzzy set. Intuitionistic fuzzy set is defined as follows:

Let A be a intuitionistic fuzzy finite set in the universe set X. If x is an element of a set X then

\[ A = \{(x, \mu_A(x), \gamma_A(x)) \mid x \in X\} \]

Where \( \mu_A(x) \) is the membership degree and \( \gamma_A(x) \) is the non-membership degree of element x which belongs to the intuitionistic fuzzy finite set A, where \( 0 \leq \mu_A(x) \leq 1, 0 \leq \gamma_A(x) \leq 1 \) and \( 0 \leq \mu_A(x) + \gamma_A(x) \leq 1 \). Hesitation degree is defined as

\[ \pi_A(x) = 1 - \mu_A(x) - \gamma_A(x) \]

#### 2.2 Interval - Valued Intuitionistic Fuzzy Sets

(Atanassov, K., & Gargov, G. (1989)[7])

To express a stronger uncertainty that prevails in the decision making Atanassov and Gargov introduced interval-valued intuitionistic fuzzy sets in 1989 which is the extension of IFSs and IVFSs.

Let the Int \([0,1]\) contains the set of all closed subintervals \(M[0,1] \). An interval-valued intuitionistic fuzzy set A in universe X can be defined as consider A on X be an Interval valued intuitionistic fuzzy set which is defined as

\[ A = \{(x, [\mu_A(x), \mu^I_A(x)], [\gamma_A(x), \gamma^I_A(x)]) \mid x \in X\} \]

in which \([\mu_A(x), \mu^I_A(x)]\) represent the interval membership function and \([\gamma_A(x), \gamma^I_A(x)]\) represent the interval non-membership function.

Addition and Multiplication operators of interval valued intuitionistic fuzzy set:

\[ A \oplus B = \{<x, [\mu^I_A(x) + \mu^I_B(x) - \mu_A(x) \cdot \mu_B(x), \mu_A(x) + \mu^I_A(x) \cdot \mu^I_B(x)], \gamma_A(x) \cdot \gamma^I_B(x) \mid x \in X\} \]

\[ A \otimes B = \{<x, [\mu^I_A(x) + \mu^I_B(x) - \mu_A(x) \cdot \mu_B(x), \gamma_A(x) \cdot \gamma^I_B(x) \mid x \in X\} \]

#### 2.3 Interval - Valued Intuitionistic Fuzzy Cognitive Map [4]

Kosko and Dickerson defined \( C,E > \) as a pair for FCM. Here the set of concepts is represented by C and adjacency matrix is represented by E. \( e_{ij} \in [-1,1] \) is the crisp weight contained in adjacency matrix. \( c_j \in C \) is the concept represented in the form of fuzzy set. The connection between the concepts are represented depend either in the form of positive or negative. To figure dynamic procedure FCM is
According to the IVIFS, every concept \( c_j(t) \in [0, 1] \) explains the position of FCM in the iteration \( t \). The position for successive iteration \( t+1 \) is evaluated as:

\[
c_i(t+1) = f\left(c_i(t) + \sum_{j=1}^{N} c_j(t) \times e_{ji}\right)
\]

According to the IVIFS, every concept \( c_j \in C \) is replaced. After that, in the iteration \( t \) the position of it concept is calculated as

\[
c_i(t) = \left\{ [\mu^L_{ci}(x), \mu^U_{ci}(x)], [\nu^L_{ci}(x), \nu^U_{ci}(x)] \right\}(t)
\]

Consider the weight \( e_{ji} \in E \) is also denoted by IVIFS.

\[
e_{ji}(t) = \left\{ [\mu^L_{ej}(x), \mu^U_{ej}(x)], [\nu^L_{ej}(x), \nu^U_{ej}(x)] \right\}(t)
\]

Applying the addition and multiplication operators, the succeeding concepts position are determined by \( c_i(t+1) \)

\[
c_i(t+1) = f\left(\{[\mu^L_{ci}(x), \mu^U_{ci}(x)], [\nu^L_{ci}(x), \nu^U_{ci}(x)]\}(t) + \sum_{j=1}^{N-1} \{[\mu^L_{ej}(x), \mu^U_{ej}(x)], [\nu^L_{ej}(x), \nu^U_{ej}(x)]\}(t) \right) \bigotimes \{[\mu^L_{ej}(x), \mu^U_{ej}(x)], [\nu^L_{ej}(x), \nu^U_{ej}(x)]\}(t)
\]

(2.1)

### 2.4 Normalized Euclidean Distance: [5]

If \( \alpha_1 = ([a_1, b_1], [c_1, d_1]) \) and \( \alpha_2 = ([a_2, b_2], [c_2, d_2]) \) are two interval-valued intuitionistic fuzzy numbers then the Normalized distance between them is given by

\[
D(\alpha_1, \alpha_2) = \sqrt{\left((a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2 + (d_1 - d_2)^2\right)}
\]

### 2.5 Intersection Principles [5]

If two alternatives \( x_k \) and \( x_k \) are considered their partial ranking structure \( R^+ = \{P, I, R\} \) as it satisfies

1. If \((x_kP_{>x_k} \text{ and } x_kP_{<x_k}) \text{ or } (x_kI_{>x_k} \text{ and } x_kI_{<x_k}) \text{ or } (x_kI_{>x_k} \text{ and } x_kP_{<x_k}) \) \( x_k \) is preferred \( x_k \).
2. If \((x_kI_{>x_k} \text{ and } x_kI_{<x_k}) \), \( x_k \) is indifferent \( x_k \).
3. If \((x_kP_{>x_k} \text{ and } x_kP_{<x_k}) \) or \((x_kP_{>x_k} \text{ and } x_kP_{<x_k}) \) is incomparable \( x_k \) or \( x_k \).

### 3. Algorithm for Interval-Valued Intuitionistic Fuzzy Cognitive Map - SIR ranking method

This new algorithm is framed by combining Interval-Valued Intuitionistic Fuzzy Cognitive map with Superiority and Inferiority Ranking method [3,5]

**Step 1:** With the help of experts to obtain decision matrix of IVIF through linguistic variable to evaluate the alternatives with various criteria

**Step 2:** From the experts to obtain FCM matrix with values through linguistic variable.

**Step 3:** From the FCM graph and IVIFCM matrix calculate final decision matrix

**Step 4:** To calculate IVIFS PIS and NIS (Positive and Negative Ideal Solution) [5] using

\[
a^+ = \left\{ [\mu^L_{a^+}(x), \mu^U_{a^+}(x)], [\nu^L_{a^+}(x), \nu^U_{a^+}(x)] \right\}
\]

(3.1)

\[
a^- = \left\{ [\mu^L_{a^-}(x), \mu^U_{a^-}(x)], [\nu^L_{a^-}(x), \nu^U_{a^-}(x)] \right\}
\]

**Step 5:** Calculate performance function in order to confirm the IVIF S-matrix and I-matrix

**Performance function**

\[
f(x_i) = \frac{D_j(a_{ij}, a^-)}{D_j(a_{ij}, a^+)} + D_j(a_{ij}, a^+) \quad (3.1)
\]

where \( 0 \leq f(x_i) \leq 1 \) and if \( f(x_i) \rightarrow 1 \) then \( a_{ij} \rightarrow a^+ = ([1, 1], [0, 0]) \)

(i) To get the preference intensity:

\[
A_k(x_i, x_j) = \theta_k(f_k(x_i) - f_k(x_j)) = \theta_k(d)
\]

The function \( \theta_k(d) \) is an increasing function the real number to \([0, 1]\). Generally it is one of the generalized six threshold function, here two threshold function are considered namely

\[
\begin{align*}
0.02 & \text{ if } d > 0 \\
0.01 & \text{ if } d \leq 0
\end{align*}
\]

(ii) Obtain the S-matrix and I-matrix in the following way:

\[
S_k(x_i) = \sum_{j=1}^{m} P_k(x_j, x_i) = \sum_{j=1}^{m} \theta_k(f_k(x_j) - f_k(x_i))
\]

\[
I_k(x_i) = \sum_{j=1}^{m} P_k(x_j, x_i) = \sum_{j=1}^{m} \theta_k(f_k(x_j) - f_k(x_i))
\]

**Step 6:** Calculate S-flow and I-flow from S-matrix and I-matrix using the following formula

**S-Flow:**

\[
\Theta^+ = \sum_{j=1}^{m} a_j S_j(x_i)
\]

\[
= \left( \begin{array}{c}
1 - \prod_{j=1}^{m} (1 - a_j)^{S_j(x_i)} \\
1 - \prod_{j=1}^{m} (1 - b_j)^{S_j(x_i)}
\end{array} \right) \cdot \left( \begin{array}{c}
\prod_{j=1}^{m} (c_j)^{S_j(x_i)} \\
\prod_{j=1}^{m} (d_j)^{S_j(x_i)}
\end{array} \right)
\]

(3.2)

**I-Flow:**

\[
\Theta^- = \sum_{j=1}^{m} a_j I_j(x_i)
\]

\[
= \left( \begin{array}{c}
1 - \prod_{j=1}^{m} (1 - a_j)^{I_j(x_i)} \\
1 - \prod_{j=1}^{m} (1 - b_j)^{I_j(x_i)}
\end{array} \right) \cdot \left( \begin{array}{c}
\prod_{j=1}^{m} (c_j)^{I_j(x_i)} \\
\prod_{j=1}^{m} (d_j)^{I_j(x_i)}
\end{array} \right)
\]

(3.3)

**Step 7:** Ranking the alternatives

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(i) To obtain the superiority ranking and inferiority ranking of the alternatives

(ii) When the ranking is done according to the intersection principle, to obtain a partial ranking structure \( R^* = \{P, I, R\} \).

**Step 8**: To get the desirable alternative, map the complete ranking

### 4. Problem Description

During Covid pandemic season the education sector faces lot of challenges along with other various sectors. This panic situation is overcome to certain percentage through online resource tools. But it requires decision making to choose the alternative with respect to multiple criteria. To analyze the practicality of the new integrated approach of IVIFCM with SIR method, introduced in this paper the above mentioned issue of choosing best online tools in higher education is considered. The alternatives are X1 -Microsoft Teams, X2-Zoom, X3-Google Meet and X4-Institutional LMS are analyzed with respect to the following criteria

C1- Exclusive Features or Conferencing Features,
C2-Simple and Streamlined,
C3-Web Features,
C4-Pricing& User Friendly
C5-Security

The values are obtained as linguistic variables for alternatives with respect to criteria as given in Table 1. As a next step, the interval-valued intuitionistic fuzzy numbers are obtained by changing the linguistic variables

#### Table 1. Linguistic Values for IVIF Decision matrix

| Linguistic term | IVIFS values |
|----------------|-------------|
| Excellent      | [0.8,0.9][0.0,0.1] |
| Good           | [0.6,0.7][0.2,0.3] |
| Fair           | [0.4,0.5][0.3,0.4] |
| Poor           | [0.2,0.3][0.5,0.6] |
| V. Poor        | [0.1,0.2][0.7,0.8] |

#### Table 2. Linguistic Values for FCM graph

| Linguistic term | IVIFS values |
|----------------|-------------|
| Very Good      | [0.7,0.8][0.1,0.2] |
| Good           | [0.6,0.7][0.2,0.3] |
| Fair           | [0.5,0.6][0.3,0.4] |
| Poor           | [0.3,0.4][0.4,0.5] |
| V. Poor        | [0.2,0.3][0.5,0.6] |

#### Table 3. IVIF decision matrix in the form of Linguistic Variables

|   | C1   | C2   | C3   | C4   | C5   |
|---|------|------|------|------|------|
| X1 | 3    | 4    | 4    | 4    | 4    |
| X2 | 5    | 5    | 4    | 5    | 4    |
| X3 | 5    | 5    | 5    | 4    | 4    |
| X4 | 3    | 4    | 4    | 4    | 5    |

#### Table 4. Converted IVIF decision matrix

|   | C1   | C2   | C3   | C4   | C5   |
|---|------|------|------|------|------|
| X1 | [0.4,0.5] | [0.4,0.5] | [0.6,0.7] | [0.6,0.7] | [0.6,0.7] |
| X2 | [0.6,0.7] | [0.8,0.9] | [0.8,0.9] | [0.6,0.7] | [0.8,0.9] |
| X3 | [0.8,0.9] | [0.8,0.9] | [0.8,0.9] | [0.6,0.7] | [0.6,0.7] |
| X4 | [0.4,0.5] | [0.6,0.7] | [0.6,0.7] | [0.6,0.7] | [0.8,0.9] |

#### Step 1 : Obtain IVIF decision matrix from the expert using a linguistic variable to evaluate the alternative depending on the criteria

#### Step 2 : Using linguistic variable Experts give opinion about the strength of influence among the criteria

#### Figure 1.

#### Step 3 : The final value is obtained for the alternative depending on criteria using the decision matrix (initial value) and the influence values (Influence of one criteria over the other) using the equation (2.1)

#### Step 4 : Positive Ideal Solution and Negative Ideal Solution are given in Table 6.

#### Step 5 : The performance function \( f_k(x_i) \) is calculated using equations (3.1)

\[
f_k(x_i) = \begin{bmatrix}
0 & 0 & 0.0125 & 0.0202 & 0.4574 \\
0.9274 & 0.9274 & 1 & 1 & 1 \\
1 & 0.8941 & 0.7344 & 0.5973 & 0.7906 \\
0.4360 & 0.4916 & 0.3639 & 0.3275 & 0.8667
\end{bmatrix}
\]

#### Step 6 : S-flow and I-flow is calculated using the equations (3.2) and (3.3)
Table 5. Final value of the decision matrix

| X1    | X2    | X3    | X4    |
|-------|-------|-------|-------|
|       |       |       |       |
|       |       |       |       |
| [0.82,0.95] | [0.8,0.91] | [0.86,0.95] | [0.91,0.97] |
| [0.013,0.049] | [0.00166,0.05848] | [0.0099,0.0166] | [0.0037,0.0255] |
|       |       |       |       |
|       |       |       |       |
| [0.96,0.98] | [0.96,0.99] | [0.96,0.95] | [0.97,0.99] |
| [0.00144,0.01359] | [0.000931] | [0.000586] | [0.00026,0.00203] |
|       |       |       |       |
|       |       |       |       |
| [0.96,0.99] | [0.00140412] | [0.00140412] | [0.005,0.0078] |
| [0.000062] | [0.088,0.96] | [0.088,0.96] | [0.03172,0.02156] |
|       |       |       |       |
|       |       |       |       |
| [0.88,0.96] | [0.00570,0.03152] | [0.0858,0.03477] | [0.00766,0.0312] |
| [0.0130,0.049] | [0.00161,0.05848] | [0.00990,0.0312] | [0.005,0.0255] |

Table 6. Positive Ideal Solution

| C1    | C2    | C3    | C4    | C5    |
|-------|-------|-------|-------|-------|
| [0.96,0.99] | [0.96,0.99] | [0.96,0.90] | [0.97,0.99] | [0.94,0.99] |
| [0.00062] | [0.00104] | [0.000586] | [0.00026,0.00203] | [0.006812] |

Table 7. Negative Ideal Solution

| C1    | C2    | C3    | C4    | C5    |
|-------|-------|-------|-------|-------|
| [0.82,0.93] | [0.8,0.91] | [0.86,0.95] | [0.91,0.97] | [0.84,0.93] |
| [0.0130,0.049] | [0.0161,0.05848] | [0.00990,0.0312] | [0.005,0.0255] | [0.0135,0.01568] |

Table 8. S-Flow and I-Flow of the alternatives

| Alternatives | S-Flow  | I-Flow  |
|--------------|---------|---------|
| X1           | S ⊤> x1 | S ⊤< x1 |
| [0.18,0.2323] | -0.5194 | (0.1651,0.7852) |
| (0.6927,0.7584) | (0.7116,0.7726) | -0.1476 |
| X2           | S ⊤> x1 | S ⊤< x1 |
| [0.1455,0.1905] | -0.5972 | (0.1632,0.7874) |
| (0.737,0.7973) | (0.7096,0.7712) | -0.1446 |
| X3           | S ⊤> x1 | S ⊤< x1 |
| [0.1401,0.1834] | -0.6147 | (0.1608,0.7903) |
| (0.7493,0.8035) | (0.7136,0.7744) | -0.1497 |
| X4           | S ⊤> x1 | S ⊤< x1 |
| [0.1711,0.2235] | -0.5334 | (0.1478,0.8076) |
| (0.6984,0.7619) | (0.7415,0.7978) | -0.185 |

Step 7 : The descending order of S-flow and ascending order of I-flow is given as

\( \mathbb{R}^S_\succ : S(\theta^>(x_1)) > S(\theta^>(x_4)) > S(\theta^>(x_2)) > S(\theta^>(x_3)) \)

\( \mathbb{R}^S_\prec : S(\theta^<(x_4)) < S(\theta^<(x_3)) < S(\theta^<(x_1)) < S(\theta^<(x_2)) \)

Step 8 : Mapping is got with complete ranking using (definition 2.5) [5,8]

6. Conclusion

In the proposed tool IVIFCM-SIR by integrating the IVIFCM and SIR ranking method for MCDM situations, it is found that the tool is more accurate. It reveals the intensity of...
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criterion influence in preferring the alternative and the attitude of the decision makers towards criterion while uncertainty prevails in higher degree among criteria and its values. The proposed method gives superiority and inferiority flows and thus ranking of the alternatives is done more accurately. The effectiveness of the tool is proved through the illustration of online resource tools for teaching and learning used during the pandemic season as one of the quick remedy in all the fields of planning, administration and very specially in the education sector with easily accessible and availability as four major platforms Microsoft teams (X1), Zoom (X2), Google Meet (X3) and Institutional Learning Management System (X4) were few considered among the many available.

Here the most desirable and preferable alternative is X1 (Microsoft Teams) and X4 (Institutional LMS) is the second preferable, with the slight difference between them as -0.5194/-0.1476 (S-flow and I-flow of X1) and -0.5334/-0.185 (S-flow and I-flow of X4). Hence this integrated IVIFCM-SIR ranking tool gives an optimal alternative which is very helpful for the decision makers. The tool can be extended to Neutrosophic set etc for future works.

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