Research Article

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Computing Topological Indices for Para-Line Graphs of Anthracene

Abstract: Atoms displayed as vertices and bonds can be shown by edges on a molecular graph. For such graphs we can find the indices showing their bioactivity as well as their physio-chemical properties such as the molar refraction, molar volume, chromatographic behavior, heat of atomization, heat of vaporization, magnetic susceptibility, and the partition coefficient. Today, industry is flourishing because of the interdisciplinary study of different disciplines. This provides a way to understand the application of different disciplines. Chemical graph theory is a mixture of chemistry and mathematics, which plays an important role in chemical graph theory. Chemistry provides a chemical compound, and graph theory transforms this chemical compound into a molecular graph which further is studied by different aspects such as topological indices. We will investigate some indices of the line graph of the subdivided graph (para-line graph) of linear-[s] Anthracene and multiple Anthracene.

Keywords: Para-line graph; nanostructures; topological indices; linear-[s] Anthracene; multiple Anthracene

1 Introduction

Chemical graph theory is a branch of mathematical chemistry that is concerned with analyses of all consequences of a connectivity in a chemical graph. Some physical properties, e.g., breaking point, can be anticipated in view of the structure of the atoms. Numerical and computational systems are viably used to show and predict the structure at an atomic level. The structures of atoms, from an numerical perspective, are graphs. Graph theory is utilized as a part of nearly every field of science, and it is likewise utilized for training, both for recreation and design.

$V(G)$ and $E(G)$ compose the vertex set and edge set of a graph respectively; $p, q \in V(G)$ are adjacent if $p$ and $q$ are end points of $u \in E(G)^*$ and $u$ is an edge whose end vertices are $p$ and $q$. The set of all neighbors of a vertex represented by $N_p$, is called the neighborhood of $p$. The count of edges that occur on a vertex is called the degree of the vertex denoted by $\xi_p$ and $S_p = \sum_{q \in N_p} \xi_p$, where $N_p = \{q \in V(G) : pq \in E(G)\}$. We can construct the line graph $L(G)$ of any graph $G$ in such a way that the edges of the original graph will become the vertices of line graph $i.e.$ two vertices $p$ and $q$ occur if and only when these vertices have a common end vertex in $G$. The para-line graph of $G$ is the line graph of the subdivision of $G$ i.e. $L(S(G))$, which will be represented as $G^*$. The subdivision graph is the graph attained from $G$ by replacing each of its edges by a path of length 2. For instance, consider $C_2H_6$ as the hydrocarbon named Ethane which is characterized as a molecular structure. The graph of $C_2H_6$ is shown in Figure 1 (a) and (b), and the line graph of subdivision $C_2H_6$ is shown in Figure 1 (c).

![Figure 1](image-url)
The topological index which is also referred to as the molecular descriptor, is a real number which describes the properties of a certain chemical compound. The study of topological indices on different chemical structures has been an area of research for all graph theorists. It is a bridge between mathematics and chemistry. The molecular descriptors which are separated into three groups depend on degree-based [16, 24], distance-based [8, 29] and spectrum-based [2, 18, 20, 21] indices. Some other topological indices have also been studied which are centred on both degrees and distances [4, 7, 11, 19].

The 1st general Zagreb index in [17] is the oldest and most used molecular descriptor and is defined as:

\[
M_a(G) = \sum_{pq \in E(G)} (\xi_p + \xi_q)^a. \tag{1}
\]

The general sum-connectivity index \( \chi_a(G) \) is defined as [30]:

\[
\chi_a(G) = \sum_{pq \in E(G)} (\xi_p + \xi_q)^a. \tag{2}
\]

The ABC is specified by Estrada in [5]. The ABC index of graph G is defined as:

\[
ABC(G) = \sum_{pq \in E(G)} \sqrt{\frac{\xi_p + \xi_q - 2}{\xi_p \xi_q}}. \tag{3}
\]

ABC\(_a\) index was presented by Ghorbani in [9] is explained as:

\[
ABC_a(G) = \sum_{pq \in E(G)} \sqrt{\frac{S_p + S_q - 2}{S_p S_q}}. \tag{4}
\]

\( R_a \) (general Randic connectivity index) of G is proposed as [1]:

\[
R_a(G) = \sum_{pq \in E(G)} (\xi_p \xi_q)^a. \tag{5}
\]

Where \( \alpha \in R \). If \( \alpha = -0.5 \), then \( R_{-0.5}(G) \) is called Randic connectivity index of G.

Vukicevic and Furtula presented the (GA) index in [28]. The GA index for graph G is defined as:

\[
GA(G) = \sum_{pq \in E(G)} \frac{2 \sqrt{\xi_p \xi_q}}{\xi_p + \xi_q}. \tag{6}
\]

GA\(_5\) was introduced by Graovac et al. in [10] is proposed as:

\[
GA_5(G) = \sum_{pq \in E(G)} \frac{2 \sqrt{S_p S_q}}{S_p + S_q}. \tag{7}
\]

The hyper-Zagreb index is suggested as:

\[
HM(G) = \sum_{pq \in E(G)} (\xi_p + \xi_q)^2. \tag{8}
\]

In 2012, Ghorbani and Azimi introduced the Zagreb indices in different form as, the 1st and 2nd multiple Zagreb index \( PM_1(G) \) and \( PM_2(G) \), 1st and 2nd Zagreb polynomial \( M_1(G, a) \) and \( M_2(G, a) \) respectively, are suggested as:

\[
PM_1(G) = \prod_{pq \in E(G)} (\xi_p + \xi_q)^a \tag{9}
\]

\[
PM_2(G) = \prod_{pq \in E(G)} (\xi_p \times \xi_q)^a \tag{10}
\]

\[
M_1(G, a) = \sum_{pq \in E(G)} \alpha (\xi_p \times \xi_q)^a \tag{11}
\]

\[
M_2(G, a) = \sum_{pq \in E(G)} \alpha (\xi_p + \xi_q)^a \tag{12}
\]

2 Applications of Topological Indices

Randic observed the correlation between the Randic index and physio-chemical properties of alkane such as boiling point, enthalpies of formation, surface area and so on. The ABC index is a very effective index in heat formation [5]. GA index is a better predictive index than the Randic Index as GA has much chosen prophetic control on the prophetic energy of the \( R_a \) [3]. The 1st and 2nd Zagreb index were very useful in the calculation of the aggregate \( \pi \)-electron energy of the molecule [12]. These molecular descriptors were suggested for the approximation of stretched carbon-skeleton [13].

3 Topological indices of para-line graphs

Para-line graphs are an attractive field of study in chemical graph theory. Ranjini et al. calculated the explicit expression for the Schultz index and Zagreb index of the para-line graphs of the wheel, helm, tadpole [25, 26]. In 2015, Su et al. [27] investigated \( \chi_a \) (general sum-connectivity index) and co-index of the above mentioned graph of wheel, tadpole. To study the para-line graphs
of ladder, tadpole and wheel, Nadeem et al. in [22] evaluated the $ABCG_4$, $GA_5$ index and calculated the generalized Randic index, general Zagreb index, $\chi_d$, $ABC$ index, $GA$ index, $ABCG_4$ index and $GA_5$ index of $TUC_4C_8[p, q]$ in [23]. Klein et al. [15] offered few applications and basic properties of the para-line graphs in chemical graph theory. Gutman also shed a light on the application of line graphs see [14]. Estrada showed the application of line graph in [6].

### 4 Results for para-line graph of linear $[s]$-Anthracene

In Figure 2, the graph of linear $[s]$-Anthracene is presented and it is denoted by $T_s$. $T_s$ has $14s$ vertices and $18s - 2$ edges.

Suppose the graph of Anthracene contains three hexagon which are connected with a square expands vertically and horizontally. Let the edge $e_{(p,q)}$ represent the count of edges connecting the vertices of degree $\xi_p$ and $\xi_q$. The graph of Anthracene holds the following edges shown in Table 2:

![Graph Image](image.png)

**Figure 2:** The molecular graph of linear $[s]$-Anthracene.

### Table 1: The distribution of edges w.r.t. degree of end vertices of every edge.

| Sr. No | $E_{(p,q)}$ | Number of Edges |
|--------|-------------|-----------------|
| 1      | $E_{(2,2)}$ | $6s + 10$       |
| 2      | $E_{(2,3)}$ | $12s - 4$       |
| 3      | $E_{(3,3)}$ | $30s - 16$      |

### Table 2: The distribution of edges w.r.t. neighbor sum of every vertex the graph of Anthracene.

| Sr. No | $S_{(p,q)}$ | Number of Edges |
|--------|-------------|-----------------|
| 1      | $S_{(4,4)}$ | 10              |
| 2      | $S_{(4,5)}$ | 4               |
| 3      | $S_{(5,5)}$ | $6s - 4$        |
| 4      | $S_{(5,8)}$ | $12s - 4$       |
| 5      | $S_{(8,8)}$ | 4               |
| 6      | $S_{(8,9)}$ | $16s - 8$       |
| 7      | $S_{(9,9)}$ | $10s - 8$       |

**Theorem 1.** Let $G'$ be the line graph of the subdivided graph (para-line graph) of $T_s$. Then

$$M_d(G') = (12s + 8)2^{a-2} + (24s - 12)3^{a+1}$$

**Proof.** In $G'$, the overall count of the vertices is $48s - 10$ which is the sum of degree two and degree three vertices $8 + 12s, 24s - 12$ respectively. As

$$M_d(G') = (12s + 8)2^{a-2} + 3^{a+1}(24s - 12).$$

**Theorem 2.** Let $G'$ be the line graph of the subdivided graph (para-line graph) of $T_s$. Then

1. $R_d(G') = (6s + 10)4^a + (12s - 4)6^a + (30s - 16)9^a$;
2. $\chi_d(G') = (6s + 10)4^a + (12s - 4)5^a + (30s - 16)6^a$;
3. $ABC(G') = \left(9\sqrt{2} + 20\right)s + 3\sqrt{2} - \frac{32}{3}$;
4. $GA(G') = \left(36 + \frac{24}{5}\sqrt{6}\right)s - 6 - 8/\sqrt{5}\sqrt{6}$.

**Proof.** The overall count of edges of $G'$ is $74s - 10$. The set of edges of $G'$ is represented as $E(G')$ which is uniformly disjointed three edge sets established based on the degrees of the adjacent(end) vertices such as $E_1(G'), E_2(G')$ and $E_3(G')$. The first set $E_1(G')$ contains the edges such as $E_{(2,2)} = 6s + 10$, the second set $E_2(G')$ holds the edges $E_{(2,3)} = 12s - 4$, and the third set $E_3(G')$ includes the edges $E_{(3,3)} = 30s - 16$, where $(2, 2), (2, 3)$ and $(3, 3)$ represent the degree of end vertices respectively. From formulas (5), (2), (3) and (6) and using Table 1, the result is proved.

**Theorem 3.** Let $G'$ be the line graph of the subdivided graph (para-line graph) of $T_s$. Then

$$ABCG_4\left(G'\right) = \left(\frac{3}{5}\sqrt{150} + \frac{12}{5}\sqrt{2} + \frac{4}{3}\sqrt{30} + \frac{40}{9}\right)\frac{3}{5}\sqrt{14}s + \frac{5}{2}\sqrt{6} + \frac{2}{5}\sqrt{35} - \frac{8}{5}\sqrt{2} - \frac{2}{3}\sqrt{30}$$

$$- \frac{1}{5}\sqrt{100} - \frac{32}{9}$$

$$GA_5(G') = \left(20 + \frac{48}{13}\sqrt{10} + \frac{192}{17}\sqrt{2}\right)s + 2 + \frac{16}{9}\sqrt{5}$$

$$- \frac{16}{13}\sqrt{10} - \frac{96}{17}\sqrt{2}$$
After some simplification, we obtain the required results.

**Proof.** In Table 3, we have

\[
ABC_4(G) = \sum_{pq \in E(G)} \sqrt{\frac{S_p + S_q - 4}{S_p S_q}}
\]

\[
ABC_4(G) = S_{(a,4)} \times \sqrt{\frac{5 + 5 - 2}{5 \times 5}} + S_{(a,5)} \times \sqrt{\frac{4 + 5 - 2}{4 \times 5}}
\]

\[
+ S_{(5,5)} \times \sqrt{\frac{5 + 5 - 2}{5 \times 5}} + S_{(5,8)} \times \sqrt{\frac{5 + 8 - 2}{5 \times 8}}
\]

\[
+ S_{(8,9)} \times \sqrt{\frac{8 + 9 - 2}{8 \times 9}} + S_{(9,9)}
\]

After some simplification, we obtain the required results.

\[
ABC_4(G') = \left( \frac{3}{5} \right) \sqrt{\frac{110 + 12}{5}} \sqrt{\frac{2}{3}} + \frac{4}{5} \sqrt{\frac{30 + 4}{9}}
\]

\[
+ \frac{1}{2} \sqrt{\frac{14}{4}} \left( s + \frac{5}{2} \sqrt{\frac{6}{5}} + \frac{2}{3} \sqrt{\frac{35}{8}} - \frac{8}{5} \sqrt{2}
\right)
\]

\[
- \frac{2}{3} \sqrt{\frac{30 + 1}{5}} \sqrt{\frac{110 - 2}{9}}
\]

Similarly, by using Table 3, we have

\[
GA_5(G) = \sum_{pq \in E(G)} \frac{2 \sqrt{S_p S_q}}{S_p + S_q}
\]

\[
GA_5(G) = S_{(a,4)} \times 2 \sqrt{\frac{4 + 5}{4 \times 5}} + S_{(a,5)} \times 2 \sqrt{\frac{4 + 5}{4 \times 5}}
\]

\[
+ S_{(5,5)} \times 2 \sqrt{\frac{5 + 5}{5 \times 5}} + S_{(5,8)} \times 2 \sqrt{\frac{5 + 8}{5 \times 8}}
\]

\[
+ S_{(8,9)} \times 2 \sqrt{\frac{8 + 8}{8 \times 8}} + S_{(8,9)} \times 2 \sqrt{\frac{9 + 9}{9 \times 9}}
\]

\[
GA_5(G') = \left( \frac{20 + \frac{48}{13} \sqrt{10 + \frac{192}{17} \sqrt{2}}}{10 - \frac{16}{9}} \right) s - 2 + \frac{16}{9} \sqrt{2}
\]

**Theorem 4.** Let \( G' \) be the line graph of the subdivided graph (para-line graph) of \( T_s \). Then

1. \( HM(G') = 1404 s - 636 \)
2. \( PM_1(G') = 4^6 s^{10} 4^{12} s^{-4} 6^{30} s^{-16} \)
3. \( PM_2(G') = 4^6 s^{10} 4^{12} s^{-4} 9^{30} s^{-16} \)

**Proof.** The edge set \( E(G') \) established on the degrees of the adjacent[end] vertices is represented as the sum of the cardinality of three sets such as \( E_1(G') \), \( E_2(G') \) and \( E_3(G') \). The cardinality of first set \( E_1(G') \), second set \( E_2(G') \) and third set \( E_3(G') \) is denoted as \( E_{(2,2)} \), \( E_{(2,3)} \) and \( E_{(3,3)} \) respectively. The \( 1^{st} \) partite edge set holds \( E_{(2,2)} = 6s + 10 \) edges, the \( 2^{nd} \) partite set holds \( E_{(2,3)} = 12s - 4 \) edges and the \( 3^{rd} \) partite set holds \( E_{(3,3)} = 30s - 16 \) edges. From Table 3, we obtain the results.

\[
HM(G') = \sum_{pq \in E(G)} (\xi_p + \xi_q)^2
\]

\[
HM(G') = \sum_{pq \in E_1(G)} (\xi_p + \xi_q)^2 + \sum_{pq \in E_2(G)} (\xi_p + \xi_q)^2
\]

\[
PM_1(G') = \prod_{pq \in E(G)} (\xi_p + \xi_q)
\]

\[
PM_1(G') = \prod_{pq \in E_1(G)} (\xi_p + \xi_q) \times \prod_{pq \in E_2(G)} (\xi_p + \xi_q)
\]

\[
PM_1(G') = 4^{L_{(2,2)}} \times 6^{L_{(2,3)}} \times 6^{L_{(3,3)}}
\]

\[
PM_2(G') = 4^{6s + 10} \times 4^{12s - 4} \times 6^{30s - 16}
\]

\[
PM_2(G') = 6^{6s + 10} \times 6^{12s - 4} \times 9^{30s - 16}
\]

**Theorem 5.** Let \( G' \) be the line graph of the subdivided graph (para-line graph) of \( T_s \). Then

1. \( M_1(G', a) = (6 s + 10) a^4 + (12 s - 4) a^5 \)
Table 3: The distribution of edges of multiple Anthracene w.r.t. degree of end vertices of every edge.

| Sr. No | \( E_{(s,r)} \) | Number of Edges |
|--------|----------------|-----------------|
| 1      | \( E_{(2,2)} \) | \( 6r + 6s + 4 \) |
| 2      | \( E_{(2,3)} \) | \( 4r + 12s - 8 \) |
| 3      | \( E_{(3,3)} \) | \( 63rs - 20r - 33s + 4 \) |

\[ + (30s - 16)a^6 \]

2. \( M_2(G', a) = (6s + 10)a^4 + (12s - 4)a^6 + (30s - 16)a^9 \)

Proof. From Table 3, we obtain the results.

\[ M_1(G', a) = \sum_{pq \in E(G)} a(\xi_s^r \xi_s) \]
\[ M_2(G', a) = \sum_{pq \in E(G)} a(\xi_s^r \xi_s) + \sum_{pq \in E(G)} a(\xi_s^r \xi_s) \]
\[ + \sum_{pq \in E(G)} a(\xi_s^r \xi_s) \]

\[ M_1(G', a) = (6s + 10)a^4 + (12s - 4)a^2 + (30s - 16)a^6 \]
\[ M_2(G', a) = \sum_{pq \in E(G)} a(\xi_s^r \xi_s) \]
\[ + \sum_{pq \in E(G)} a(\xi_s^r \xi_s) \]

\[ M_2(G', a) = (6s + 10)a^4 + (12s - 4)a^6 + (30s - 16)a^9 \]

\[ M_2(G', a) = E_{(2,2)} \times a^4 + E_{(2,3)} \times a^6 + E_{(3,3)} \times a^9 \]

5 Results for para-line graph of multiple Anthracene

In Figure 4, we presented the graph \( G' \), and it is denoted by \( T_{r,s} \), \( T_{r,s} \) has 22rs vertices and 33rs – 2r – 5s edges. Also, the para-line graph of multiple Anthracene are depicted in Figure 5.

\[ \text{Figure 4: The molecular graph of multiple Anthracene.} \]

\[ \text{Figure 5: The para-line graph of multiple Anthracene.} \]

Table 4: The distribution of edges w.r.t. neighbor sum of every vertex the graph \( G' \).

| Sr. No | \( S_{(p,q)} \) | Number of Edges |
|--------|----------------|-----------------|
| 1      | \( S_{(4,4)} \) | \( 2r + 8 \) |
| 2      | \( S_{(4,5)} \) | \( 4r \) |
| 3      | \( S_{(5,5)} \) | \( 6s - 4 \) |
| 4      | \( S_{(5,8)} \) | \( 12s - 4r - 8 \) |
| 5      | \( S_{(8,8)} \) | \( 4s \) |
| 6      | \( S_{(8,9)} \) | \( 8r + 16s - 16 \) |
| 7      | \( S_{(9,9)} \) | \( 63rs - 28r - 53s + 20 \) |

Theorem 6. Let \( G' \) be the line graph of the subdivided graph (para-line graph) of \( T_{r,s} \). Then

\[ M_a(G') = (2r + 3s)2^{a+2} + 3^{a+1}(14rs - 4r - 6s). \]

Proof. In Figure 5, we presented a graph \( G' \). In \( G' \), the overall count of the vertices is \( 63rs - 10r - 15s \) which is the sum of \( 8r + 12s \) (vertices of degree 2) and \( 42rs - 12r - 18s \) (vertices of degree 3). Hence proved.

Theorem 7. Let \( G' \) be the line graph of the subdivided graph (para-line graph) of \( T_{r,s} \). Then

1. \[ R_a(G') = (6s + 6r + 4)4^a + (4r + 12s - 8)6^a + (63rs - 20r - 33s + 4)9^a; \]
2. \[ \chi_a(G') = (6s + 6r + 4)4^a + (4r + 12s - 8)5^a \]
Proof. The total number of edges and vertices of subdivision graph $S(T_{r,s})$ are $63rs - 10r - 15s$ and $198rs - 20r - 50$ respectively. The vertices are subdivided in the following way: The count of degree 2 vertices is $8r + 12s$ and the count of degree 3 vertices is $42rs - 12r - 18s$. The overall count of edges of $G'$ are $63rs - 10r - 15s$. The set of edges $E(G')$ bifurcates into 3 partite sets established on degrees of the adjacent (end) vertices such as $E_1(G'), E_2(G')$ and $E_3(G')$. The edge partite set $E_1(G')$ contains $E_{(2,2)} = 6r + 6s + 4$ edges, where $E_{(2,2)}$ represents the edge whose both vertex has degree 2. The edge partite set $E_2(G')$ holds $E_{(2,3)} = 4r + 12s - 8$ edges, where $E_{(2,3)}$ represents the edge the vertex of which has degree 2 and second vertex has 3 and the edge partite set $E_3(G')$ contains $E_{(3,3)} = 63rs - 20r - 33s + 4$ edges, where $E_{(3,3)}$ represents the edge the vertex of which has degree 3. From formulas (5), (2), (3) and (6), the proof was established.

Theorem 8. Let $G'$ be the line graph of the subdivided graph (para-line graph) of $T_{r,s}$. Then

1. $ABC_4(G') = \left(28r + \frac{1}{2} \sqrt{14} + \frac{12}{5} \sqrt{2}\right) s + \left(\frac{3}{5} \sqrt{110} + \frac{4}{3} \sqrt{30} - \frac{211}{9}\right) s + \left(\frac{1}{2} \sqrt{6} + \frac{1}{5} \sqrt{110} + \frac{2}{5} \sqrt{35} - \frac{112}{9} + \frac{2}{3} \sqrt{30}\right) r + 2 \sqrt{6} - \frac{8}{5} \sqrt{2} - 2/5 \sqrt{110} - 4/3 \sqrt{30} + \frac{80}{9}$

2. $G_{A_5}(G') = \left(\frac{80}{13} \sqrt{10} + 99 m + \frac{288}{17} \sqrt{2} - 69\right) s + \left(-26 + \frac{16}{13} \sqrt{10} + \frac{16}{9} \sqrt{5} + \frac{96}{17} \sqrt{2}\right) r - \frac{192}{17} \sqrt{2} - \frac{32}{13} \sqrt{10} + 24$.

Proof. Let the degree sum of neighbors of end vertices $pq$ be represented as $S_{(p,q)}$. Suppose the collection of edge sets $E(G')$ is divided into 7 partite edge sets $E_1(G'), E_2(G'), E_3(G'), E_4(G'), E_5(G'), E_6(G'), E_7(G'), E_8(G'), E_9(G'), E_{10}(G')$. The edge partite set $E_4(G')$ contains $S_{(p,q)} = (4, 4) = 2r + 8$ edges, the edge partition $E_5(G')$ contains $S_{(p,q)} = (4, 5) = 4r$ edges, the edge partition $E_6(G')$ contains $S_{(p,q)} = (5, 5) = 6s - 4$ edges, the edge partition $E_7(G')$ contains $S_{(p,q)} = (5, 8) = 12s + 4r - 8$ edges, the edge partition $E_8(G')$ contains $S_{(p,q)} = (8, 8) = 4s$ edges, the edge partition $E_9(G')$ contains $S_{(p,q)} = (8, 9) = 8r + 16s - 16$ edges, and the edge partition $E_{10}(G')$ contains $S_{(p,q)} = (9, 9) = 63rs - 28r - 53s + 20$ edges. From Formulas (4) and (7), results were attained.

Theorem 9. Let $G'$ be the line graph of the subdivided graph (para-line graph) of $T_{r,s}$. Then

1. $HM(G') = 2268rs - 596r - 864s - 40$
2. $PM_1(G') = 4^{6s+6r+4}a^{4r+12s-8}d_{63rs-20r-33s+4}$
3. $PM_2(G') = 4^{6s+6r+4}a^{4r+12s-8}d_{63rs-20r-33s+4}$
4. $M_1(G', a) = (6s + 6r + 4) a^4 + (4r + 12s - 8) a^6 + (63rs - 20r - 33s + 4) a^6$
5. $M_2(G', a) = (6s + 6r + 4) a^4 + (4r + 12s - 8) a^6 + (63rs - 20r - 33s + 4) a^6$

Proof. The edge set $E(G')$ on the basis of the degree of end vertices is represented as the sum of the cardinality of three disjointed sets i.e. $E_1(G'), E_2(G')$ and $E_3(G')$. The cardinality of first set $E_1(G')$, second set $E_2(G')$ and third set $E_3(G')$ is denoted as $E_{(2,2)}, E_{(2,3)}$ and $E_{(3,3)}$ respectively. The first edge partite set holds $E_{(2,2)} = 8r + 12s$ edges, the second edge partite set holds $E_{(2,3)} = 42rs + 12r - 18s$ edges, and the third partite edge set holds $E_{(3,3)} = 63rs - 20r - 33s + 4$ edges.

Using Table 1, we obtained the following results.

$HM(G') = \sum_{pq \in E(G)} (\xi_p + \xi_q)^2$

$HM(G') = \sum_{pq \in E_1(G)} [\xi_p + \xi_q]^2 + \sum_{pq \in E_2(G)} [\xi_p + \xi_q]^2 + \sum_{pq \in E_3(G)} [\xi_p + \xi_q]^2$

$HM(G') = 16 \times E_{(2,3)} + 25 \times E_{(2,3)} + 36 \times E_{(3,3)}$

$HM(G') = 16 (6s + 6r + 4) + 25 (12s + 4r - 8)$

$HM(G') = 2268rs - 596r - 864s - 40$

$PM_1(G') = \prod_{pq \in E_1(G)} (\xi_p + \xi_q) \times \prod_{pq \in E_2(G)} (\xi_p + \xi_q)$

$PM_1(G') = \prod_{pq \in E_1(G)} (\xi_p + \xi_q) \times \prod_{pq \in E_2(G)} (\xi_p + \xi_q)$
In this paper, we have comprehended the work on these indices: \( R_a \), "\( M_a \), \( \chi_a \), \( ABC \), \( GA \), \( ABC_a \), \( GA_5 \), \( PM_1 \), \( PM_2 \), \( M_1(G, a) \), and \( M_1(G, a) \) of the line graph of the subdivided graph (para-line graph) of linear \([3]\)-Anthracene and multiple Anthracene. Randic index \( R_a \) plays a vital role in the explanation of the branching of the carbon-atom skeleton of hydrocarbons. \( PM_1(G) \) and \( PM_2(G) \) may be expressed in the QSPR study and shows a central role in the analysis of the boiling and melting point of drugs. Chemical graphs are currently being studied with the help of para-line graphs which are very important in the field of chemistry. Our upcoming work will be the emphasis on some new classes of the above mentioned graphs (para-line) of chemical structures with respect to the different topological indices.

**Ethical approval:** The conducted research is not related to either human or animal use.

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