Upper bound on the momentum scale in noncommutative phase space of canonical type

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Abstract - We find a stringent upper bound on the momentum scale in noncommutative phase space of canonical type on the basis of studies of the perihelion shift of the Mercury planet by taking into account features of the description of the motion of a macroscopic body in the space with noncommutativity of coordinates and noncommutativity of momenta. Using results for precession of the perihelion of the Mercury planet from ranging to the MESSENGER spacecraft we obtain an upper bound for the parameter of momentum noncommutativity $10^{-83}$ kg m$^2$/s$^2$ which is many orders less than known in the literature.

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Introduction. - Recently the idea to describe the features of space structure at the Planck scale considering modifications of commutation relations for coordinates and momenta has attracted much attention. In the noncommutative phase space of canonical type relations for operators of coordinates and operators of momenta are as follows:

$[X_i, X_j] = i\hbar \theta_{ij}$, $[X_i, P_j] = i\hbar (\delta_{ij} + \sigma_{ij})$, $[P_i, P_j] = i\hbar \eta_{ij}$, \hspace{1cm} (1) (2) (3)

with $\theta_{ij}$, $\eta_{ij}$, $\sigma_{ij}$ being elements of constant matrices (see, for instance, [1,2]). In the classical limit from (1)–(3) one obtains the corresponding Poisson brackets,

$\{X_i, X_j\} = \theta_{ij}$, \hspace{1cm} (4) $\{X_i, P_j\} = \delta_{ij} + \sigma_{ij}$, \hspace{1cm} (5) $\{P_i, P_j\} = \eta_{ij}$. \hspace{1cm} (6)

The relations (1)–(3) lead to the minimal areas $\hbar |\theta_{ij}|$, $\hbar |\eta_{ij}|$ in the configurational and momentum spaces, respectively. The detailed studies of the quantum of an area in noncommutative space are presented in [3].

Various quantum and classical problems were studied in noncommutative space. Among them, for example, are free particles [4–7], hydrogen atom [4,8–19], classical systems with various potentials [20–27] and many others. The studies are important for finding the influence of space quantization on the properties of physical systems and for estimating the values of parameters of noncommutativity.

Quite stringent upper bounds on the parameters of coordinate noncommutativity were obtained on the basis of studies of the perihelion shift of the Mercury planet in noncommutative phase space [21,22], in noncommutative phase space [23]. These results correspond to the minimal area which is close to the squared Planck length. Extremely strong upper bounds on the minimal length (many orders less than the Planck length) were obtained on the basis of studies of the perihelion shift of the Mercury planet in the frame of deformed commutation relations $[X_i, P_j] = i\hbar((1 + \beta P^2)\delta_{ij} + \beta' P_i P_j)$ (here $\beta$, $\beta'$ are constants) [28,29], in the curved Snyder space [30]. The authors of paper [28] obtained the upper bound for the minimal length which is 33 orders less than the Planck length.

We would like to note here that the phenomenology of the models considered in [28–30] and the phenomenology of the model described by (1)–(3) are different. The model given by (1)–(3) is the minimal area model, in the frame of the model one can always have a null distance in any spatial direction. Models studied in [28–30] are characterized by commutation relations for coordinates and momenta which in contrast to noncommutative phase space of canonical type (1)–(3) are functions of coordinates and momenta. Because of the momentum dependence of commutator $[X_i, P_j]$ the model studied in [28,29] is the minimal length model. Similarly, commutation relations
which describe curved Snyder space lead to the minimal length [31].

In papers [25,32,33] it was concluded that the extremely small upper bounds presented in [21–23,28,30] can be re-examined to more relevant ones, taking into consideration features of the description of the motion of a macroscopic body in quantum space, namely taking into account that the motion of the center of mass of a macroscopic body is described by effective parameters which are less than parameters corresponding to the elementary particles. In this paper we show that taking into consideration the features of the description of the motion of a macroscopic body in noncommutative phase space a quite strong upper bound on the momentum scale in the space can be obtained on the basis of studies of the influence of noncommutativity on the perihelion shift of the Mercury planet.

Studies of a particle with mass \( m \) in the gravitational field \( -k/X \) (\( k \) is a constant, \( X = \sqrt{\sum \lambda(X)} \) in noncommutative phase space of canonical type (4)–(6)) were done in [23]. Examining the planar motion of the particle and considering \( \sigma_{ij} = \sum_k \theta_{ik} \eta_{jk} / 4, \theta_1 = \theta_2 = \eta_1 = \eta_2 = 0, \theta = \theta_3, \eta = \eta_3 \) with \( \delta_{ij} = \epsilon_{ijk} \theta_{jk} / 2, \eta_i = \epsilon_{ijk} \eta_{jk} / 2, \) up to the first order in the parameters of noncommutativity the following expression for the perihelion shift of its orbit was obtained:

\[
\Delta \phi_{nc} = 2\pi \left( \frac{m^2 k}{a^3(1-e^2)^{3/2}} \theta + 2 \frac{a^3(1-e^2)^{3/2}}{m^2 k} \eta \right),
\]

where \( a, e \) are the semi-major axis and eccentricity. The result was applied to the case of the Mercury planet, substituting its mass, parameters of the orbit into expression (7). On the basis of the analysis of the values of multipliers \( \sqrt{mk} / \sqrt{a^3(1-e^2)^{3/2}} \), and \( 2 \sqrt{a^3(1-e^2)^{3/2}/e^2 \sqrt{mk}} \), with \( k = GM, X \) (\( G \) is the gravitational constant, \( M \) is the mass of the Sun) the contribution of the second term in (7) was ignored because of its smallness. Comparing the result (7) with the observed perihelion shift for the Mercury planet the upper bound \( \sqrt{\Delta \phi_{nc}} \leq 6.3 \cdot 10^{-34} \) m which is close to the Planck length was obtained [23].

In the present paper we study the influence of noncommutativity on the perihelion shift of the Mercury planet, taking into consideration features of the description of the motion of a macroscopic body in noncommutative phase space. Namely we take into account that the motion of a composite system in space can be noticed that the motion of a many-particle system was studied in four-dimensional (3D configurational and 3D momentum space) noncommutative phase space constructed with the help of the generalization of the parameters of noncommutativity to tensors [39].

Let us discuss the features of the description of a composite system in six-dimensional (3D configurational and 3D momentum space) noncommutative phase space of canonical type characterized by relations (4)–(6). Noncommutative algebra for coordinates and momenta of different particles can be written in the following form:

\[
\begin{align*}
\{X_i^{(n)}, X_j^{(m)}\} &= \delta_{nm} \sigma_{ij}^{(n)} , \\
\{X_i^{(n)}, P_j^{(m)}\} &= \delta_{nm} \eta_{ij}^{(n)} , \\
\{P_i^{(n)}, P_j^{(m)}\} &= \delta_{nm} \eta_{ij}^{(n)},
\end{align*}
\]

where \( n, m \) label the particles. Parameters \( \sigma_{ij}^{(n)} \), \( \eta_{ij}^{(n)} \) are considered to be different for different particles. Coordinates and momenta satisfying (8), (10) can be represented by coordinates and momenta \( x_i^{(n)}, p_i^{(n)} \) which satisfy the ordinary relations \( \{x_i^{(n)}, x_j^{(m)}\} = \{p_i^{(n)}, p_j^{(m)}\} = 0 \), \( \{x_i^{(n)}, p_j^{(m)}\} = \delta_{nm} \delta_{ij} \) as

\[
\begin{align*}
X_i^{(n)} &= x_i^{(n)} - \frac{1}{2} \sum_j \sigma_{ij}^{(n)} p_j^{(n)} , \\
P_i^{(n)} &= p_i^{(n)} + \frac{1}{2} \sum_j \eta_{ij}^{(n)} x_j^{(n)}. 
\end{align*}
\]

Calculating the Poisson brackets \( \{X_i^{(n)}, P_j^{(m)}\} \), one obtains

\[
\sigma_{ij}^{(n)} = \sum_k \frac{\theta_{ik}^{(n)} \eta_{jk}^{(n)}}{4}.
\]

(see [1,2,23]).
For coordinates and momenta of the center of mass of a composite system (a macroscopic body) made of \( N \) particles of masses \( m_n \), taking into account (8)–(10), one has

\[
\{ X^c_i, X^c_j \} = \theta_{ij}^c, \quad (14)
\]

\[
\{ X^c_i, P^c_j \} = \delta_{ij} + \sum_n \mu_n \sigma_{ij}^{(n)}, \quad (15)
\]

\[
\{ P^c_i, P^c_j \} = \eta_{ij}^c, \quad (16)
\]

where \( X^c_i = \sum_n \mu_n X_i^{(n)} \), \( P^c_i = \sum_n p_i^{(n)} \), \( \mu_n = m_n/M \), \( M = \sum_n m_n \), parameters \( \theta_{ij}^c \) and \( \eta_{ij}^c \) are defined as

\[
\theta_{ij}^c = \sum_n \mu_n^2 \sigma_{ij}^{(n)}, \quad (17)
\]

\[
\eta_{ij}^c = \sum_n \eta_{ij}^{(n)}. \quad (18)
\]

It is worth emphasizing that there is a relation of the parameters of noncommutativity \( \theta_{ij}^c \) with respect to the parameters \( \theta_{ij}^{(n)} \) and an increase of the parameters \( \eta_{ij}^c \) with respect to \( \eta_{ij}^{(n)} \). The center of mass of a nonelementary object sees smaller effective noncommutativity of the coordinates and larger effective noncommutativity of the momenta with respect to the elementary particles forming it. Note that in the case of a system of \( N \) particles with the same mass \( m_n = m \) and parameters \( \theta_{ij}^{(n)} = \theta_{ij} \), \( \eta_{ij}^{(n)} = \eta_{ij} \), from (17), (18) we find \( \theta_{ij}^c = \theta_{ij}/N, \eta_{ij}^c = N \eta_{ij} \).

It is worth mentioning that relations for coordinates and momenta of the center of mass of a composite system are not the same as relations of noncommutative algebra for particles forming the system (8)–(10). Poisson brackets for coordinates of the center of mass and Poisson brackets for momenta of the center of mass are equal to the effective parameters of noncommutativity (17), (18) and in relation (15) one has \( \sum_n \mu_n \sigma_{ij}^{(n)} = \sum_n \mu_n \sum_k \theta_{ik} \eta_{kj}^{(n)}/4 \neq \sum_k \theta_{ik} \eta_{kj}^{(n)}/4 \).

In addition, it is worth mentioning that in noncommutative phase space of canonical type the motion of the center of mass is not independent of the relative motion. For coordinates and momenta of the center of mass of coordinates and momenta of the relative motion defined in the traditional way one has

\[
\{ X^c_i, \Delta X_j^{(n)} \} = \mu_n \theta_{ij}^{(n)} - \sum_m \mu_m \theta_{im}^{(m)}, \quad (19)
\]

\[
\{ P^c_i, \Delta P_j^{(n)} \} = \eta_{ij}^{(n)} - \mu_n \sum_m \eta_{jm}^{(m)}, \quad (20)
\]

\[
\{ \Delta X_i^{(n)}, P_j^{(n)} \} = \sigma_{ij}^{(n)} - \sum_m \mu_m \sigma_{im}^{(m)}, \quad (21)
\]

\[
\{ X^c_i, \Delta P_j^{(n)} \} = \mu_n (\sigma_{ij}^{(n)} - \sum_m \mu_m \sigma_{ij}^{(m)}), \quad (22)
\]

where \( \Delta X_i^{(n)} = X_i^{(n)} - X^c_i \), \( \Delta P_i^{(n)} = P_i^{(n)} - \mu_n P^c_i \).

Let us consider the following conditions on the parameters \( \theta_{ij}^{(n)} \), \( \eta_{ij}^{(n)} \):

\[
\theta_{ij}^{(n)} m_n = \gamma_{ij}, \quad (23)
\]

\[
\eta_{ij}^{(n)} m_n = \alpha_{ij}, \quad (24)
\]

here \( \gamma_{ij}, \alpha_{ij} \) are constants which do not depend on mass. From (13), (23), (24) we have that parameters \( \sigma_{ij}^{(n)} \) are the same for different particles,

\[
\sigma_{ij}^{(n)} = \sum_k \gamma_{ik} \alpha_{jk} / 4 = \sigma_{ij}. \quad (25)
\]

We would like to stress that if conditions (23)–(25) are satisfied, namely if the parameters of coordinate noncommutativity are inversely proportional to mass, parameters of momentum noncommutativity are proportional to mass and, therefore, parameters \( \sigma_{ij}^{(n)} \) are the same for particles with different masses, the relations (19)–(22) have the form

\[
\{ X^c_i, \Delta X_j^{(n)} \} = \{ P^c_i, \Delta P_j^{(n)} \} = 0, \quad (26)
\]

\[
\{ \Delta X_i^{(n)}, P_j^{(n)} \} = (X^c_i, \Delta P_j^{(n)}) = 0. \quad (27)
\]

So, in this case the motion of the center of mass of a body can be considered independently of the relative motion. In addition, taking into account (13), (17), (18), (23)–(25), one has

\[
\sigma_{ij} = \sum_k \theta_{ik} \eta_{jk}^{(n)} / 4 = \sum_k \theta_{ik} \eta_{jk}^{(n)} / 4. \quad (28)
\]

So, the relations for coordinates of the center of mass reproduce relations of noncommutative algebra for coordinates and momenta of particles (8)–(10) with effective parameters of noncommutativity

\[
\theta_{ij}^c = \gamma_{ij} M, \quad \eta_{ij}^c = \mu_n. \quad (29)
\]

Note that the effective parameters of noncommutativity which correspond to a composite system do not depend on its composition and are determined by its mass \( M \) (29), (30), similarly as parameters of noncommutativity corresponding to the individual particles are determined by their masses (23), (24).

In addition we would like to mention that on the conditions (23)–(25) the weak equivalence principle is recovered in noncommutative phase space. Note that the expression for the perihelion shift (7) depends on mass [23]. It is a consequence of violation of the equivalence principle in noncommutative phase space of canonical type. The effect of noncommutativity on the implementation of the equivalence principle was studied in [25,40–46]. In [44] it was concluded that the equivalence principle holds in the sense that an accelerated frame of reference is locally
equivalent to a gravitational field, unless noncommutative parameters are anisotropic, $\eta_{xy} \neq \eta_{yz}$. In [25,26,43] it was shown that the weak equivalence principle can be recovered in noncommutative space of canonical type, in four-dimensional noncommutative phase space, in rotationally invariant noncommutative phase space, considering parameters of noncommutativity to be dependent on mass. This conclusion can be generalized to the case of algebra (4)–(6). If relations (23), (24) hold, the trajectory of a particle (a body) in gravitational field $V(X)$ does not depend on its mass and composition. For a particle with mass $m$, considering the Hamiltonian

$$H = \frac{p^2}{2m} + mV(X),$$

and taking into account relations (4)–(6) one finds

$$\dot{X}_i = \sum_j (\delta_{ij} + \sigma_{ij}) \frac{P_j}{m} + \sum_j m\theta_{ij} \frac{\partial V}{\partial X_j},$$

$$\dot{P}_i = -m \sum_j (\delta_{ij} + \sigma_{ij}) \frac{\partial V}{\partial X_j} + \sum_{j} \eta_{ij} \frac{P_j}{m}. \tag{32}$$

Note that if relations (23)–(25) hold one can rewrite (32), (33) as

$$\dot{X}_i = \sum_j (\delta_{ij} + \sigma_{ij}) P'_j + \sum_j \eta_{ij} \frac{\partial V}{\partial X_j}, \tag{34}$$

$$\dot{P}'_i = -\sum_j (\delta_{ij} + \sigma_{ij}) \frac{\partial V}{\partial X_j} + \sum_j \alpha_{ij} P'_j. \tag{35}$$

Equations (34), (35) do not depend on mass, therefore, $X_i(t)$ and $P'_i(t)$ ($P'_i = P_i/m$) do not depend on mass, too. So, the weak equivalence principle which states that the motion of a particle in a gravitational field is independent of its mass and composition is preserved.

In the case of a macroscopic body, if relations (23)–(25) are satisfied, the motion of the center of mass of a body in noncommutative phase space can be studied independently of the relative motion, the coordinates and the momenta of the center of mass satisfy noncommutative algebra with the effective parameters of noncommutativity (29), (30) which do not depend on the composition of the body. Therefore, for a macroscopic body in a gravitational field the equations of motion have the form (32), (33) with parameters (29), (30) and can be rewritten as (34), (35). So, the motion of a body in a gravitational field in noncommutative phase space does not depend on its mass and composition and the weak equivalence principle is satisfied.

Note also that if conditions (23), (24) hold, namely if $\theta m = \gamma$, $\eta/m = \alpha$ ($\gamma$, $\alpha$ are constants which do not depend on mass) the expression for the perihelion shift (7) can be rewritten as

$$\Delta \phi_{nc} = 2\pi \left( \sqrt{\frac{k}{a(1-e^2)}} \gamma + \frac{2}{e^2} \sqrt{a^3(1-e^2)^3 \frac{\alpha}{k}} \right) \tag{36}$$

The perihelion shift does not depend on mass.

Besides it is worth noting that due to relations (23)–(25) the motion of a free particle in noncommutative phase space does not depend on its mass and a system of free particles with the same initial velocities does not fly away. Namely, if relations (23)–(25) hold for a free particle one can write eqs. (34), (35) with $V = 0$. The solutions of these equations $X_i(t)$, $P'_i(t)$ do not depend on mass. From eqs. (34), (35) with $V = 0$, taking into account (23)–(25), one finds

$$X_i(t) = A_{i1} \cos \left( \frac{\sqrt{\eta_{12}^2 + \eta_{23}^2 + \eta_{31}^2} t}{m} \right) + A_{i2} \sin \left( \frac{\sqrt{\eta_{12}^2 + \eta_{23}^2 + \eta_{31}^2} t}{m} \right) + A_{i3} =$$

$$A_{i1} \cos (\tilde{\alpha} t) + A_{i2} \sin (\tilde{\alpha} t) + A_{i3}, \tag{37}$$

where $A_{ij}$ are elements of the matrix

$$\tilde{\alpha} = (1 + \hat{\sigma}) \begin{pmatrix} C_{2\alpha_{31}} \tilde{\alpha} - C_{1\alpha_{12}} \alpha_{23} & C_{2\alpha_{31}} \tilde{\alpha} + C_{2\alpha_{12}} \alpha_{23} & C_{3\alpha_{23}} \\ \alpha_{23} + \alpha_{31} & \alpha_{23} + \alpha_{31} & \alpha_{12} \\ \alpha_{23} + \alpha_{31} & \alpha_{23} + \alpha_{31} & \alpha_{12} \end{pmatrix}.$$

Here constants $C_i$ are determined by the initial velocities $v_0$,

$$(1 + \hat{\sigma}) \hat{B} \hat{C} = \dot{v}_0, \tag{40}$$

where

$$\hat{B} = \begin{pmatrix} -\alpha_{12}\alpha_{23} & \alpha_{23} \tilde{\alpha} & \alpha_{23} \\ \alpha_{23} + \alpha_{31} & -\alpha_{23} \tilde{\alpha} & \alpha_{31} \\ 1 & 0 & 1 \end{pmatrix}, \tag{41}$$

$$\hat{C} = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix}, \quad \dot{v}_0 = \begin{pmatrix} v_{01} \\ v_{02} \\ v_{03} \end{pmatrix}. \tag{42}$$

Matrix $\hat{\sigma}$ has elements $\sigma_{ij}$ (13). If conditions (23)–(25) hold, taking into account (37), one has that the velocity of the center of mass of a system of free particles with the same initial velocities is equal to the velocities of particles forming it, $X_i(t) = \sum_n \mu_n X_i(t) = X_i(t)$ and the relative velocities are equal to zero, $\Delta X_i(t) = \dot{X}_i(t) = 0$. We would like to stress that if relations (23)–(25) are not preserved, the trajectory of a free particle in noncommutative phase space depends on its mass and free particles with the same initial velocities do not move together.
So, if conditions (23)–(25) are satisfied, the motion of the center of mass of a composite system is independent of the relative motion; coordinates and momenta of the center of mass satisfy noncommutative algebra with parameters of noncommutativity which are independent of its composition; the trajectory of a free particle does not depend on its mass; a system of free particles with the same initial velocities does not fly away; the weak equivalence principle is recovered in noncommutative phase space (4)–(6).

At the end of this section we would like to note that the idea to relate parameters of deformed algebra with mass opens the possibility to obtain important results in deformed space with minimal length [47], in noncommutative space of canonical type [26], in twist-deformed space [45], in a space with Lie-algebraic noncommutativity [46].

Estimation of upper bounds on the parameters of noncommutativity on the basis of studies of the precession of Mercury's perihelion. – Let us study the influence of noncommutativity of the coordinates and noncommutativity of the momenta of the perihelion shift of the Mercury planet taking into account features of the description of the motion of a macroscopic body in noncommutative phase space.

As was shown in the previous section, the motion of a macroscopic body in noncommutative phase space is described by effective parameters of noncommutativity (17), (18). So, parameters \( \theta, \eta \) in (7) which are determined as \( \theta = \theta_1 = \theta_2, \eta = \eta_1 = \eta_2 \) (see [23]) should be replaced by \( \theta^c = \sum_n \hbar^2 \theta (n), \eta^c = \sum_n \eta (n) \), where \( \theta (n), \eta (n) \) are parameters of noncommutativity corresponding to particles with masses \( m_n \) which form the planet \( (\theta (n) = \theta_1 \equiv \theta_2 \equiv \theta_n^c, \eta (n) = \eta_1 \equiv \eta_2 \equiv \eta_n^c) \). So, we have

\[
\begin{align*}
\Delta \phi_{\text{nc}} &= \Delta \phi + \Delta \phi_1, \\
\Delta \phi &= 2\pi \sqrt{\frac{G m_S^3 m_S}{a^3 (1 - e^2)^3}} \theta^c, \\
\Delta \phi_1 &= \frac{4\pi}{c^2} \sqrt{\frac{a^3 (1 - e^2)^3}{G m_M^3 m_S}} \eta^c.
\end{align*}
\]

Writing (43) we assume that the influence of the relative motion on the motion of the center of mass of the Mercury planet can be neglected and take into account that \( k = G m_S, \ G \) is the gravitational constant, \( m_S \) is the mass of the Sun.

Observed perihelion precession rate which cannot be explained by the Newtonian gravitational effects of other planets and asteroids, solar oblateness is

\[
\Delta \phi_{\text{obs}} = 42.9779 \pm 0.0009 \text{ arc-seconds per century} = 2\pi (7.98695 \pm 0.00017) \cdot 10^{-8} \text{ radians/revolution},
\]

(see, for instance, [34]). This advance is explained by relativistic effects (Lense-Thirring, gravitoelectric effect) [34]. From general relativity predictions the perihelion precession rate is \( \Delta \phi_{\text{GR}} = 2\pi (7.98744 \cdot 10^{-8}) \text{ radians/revolution} \) (see, for instance, [34]). Comparing the perihelion shift caused by noncommutativity with

\[
\Delta \phi_{\text{obs}} - \Delta \phi_{\text{GR}} = 2\pi (-0.00049 \pm 0.00017) \cdot 10^{-8} \text{ radians/revolution},
\]

and assuming that \( |\Delta \phi_{\eta} - \Delta \phi_{\text{GR}}| \leq 2\pi \cdot 10^{-11} \text{ radians/revolution} \) at 3\( \sigma \), one can write

\[
|\Delta \phi_{\eta}| \leq 2\pi \cdot 10^{-11} \text{ radians/revolution}.
\]

A similar assumption was considered in [28] for the estimation of the minimal length in the deformed space, and in [21–23] for the estimation of the parameters of noncommutativity in noncommutative space of canonical type.

Since either of two contributions \( \Delta \phi_\theta, \Delta \phi_\eta \) to \( \Delta \phi_{\text{nc}} \) could be equal to zero, to estimate the orders of the parameters of noncommutativity we consider the following inequalities:

\[
\begin{align*}
|\Delta \phi_\theta| &\leq 2\pi \cdot 10^{-11} \text{ radians/revolution}, \\
|\Delta \phi_\eta| &\leq 2\pi \cdot 10^{-11} \text{ radians/revolution}.
\end{align*}
\]

So, using (44), (45), one obtains

\[
\begin{align*}
|h|\theta^c| &\leq 3.6 \cdot 10^{-63} \text{ m}^2 = 1.4 \cdot 10^7 l_p^2, \\
|\hbar|\eta^c| &\leq 6.5 \cdot 10^{-30} \text{ kg}^2 \text{ m}^2 / \text{ s}^2 = 2.3 \cdot 10^{25} \text{ eV} / \text{ c}^2 = 1.5 \cdot 10^{-31} \text{ (E}_p / \text{ c})^2,
\end{align*}
\]

where \( l_p \) is the Planck length and \( E_p \) is the Planck energy. Let us find the effective parameters of noncommutativity \( \theta^c, \eta^c \) corresponding to the Mercury planet. Taking into account (18) for the effective parameter of momentum noncommutativity one has

\[
\eta^c = N_{\text{nuc}} \eta^{(\text{nuc})} + N_e \eta^{(e)},
\]

where \( N_{\text{nuc}} \) is the number of nucleons and \( N_e \) is the number of electrons in the planet, \( \eta^{(\text{nuc})}, \eta^{(e)} \) are parameters of noncommutativity corresponding to nucleons and electrons, respectively. The main contribution to the mass of the planet comes from nucleons. Therefore, their number can be calculated as \( N_{\text{nuc}} \approx m_M / m_{\text{nuc}}, \ m_{\text{nuc}} \) is the mass of a nucleon. Taking into account that the number of electrons in the planet is equal to the number of protons \( N_p \) and \( N_p \approx N_{\text{nuc}} / 2 \), one has \( N_e \approx N_{\text{nuc}} / 2 \). The nucleons are made of three quarks, so for the parameter of noncommutativity corresponding to nucleons we can write \( \eta^{(\text{nuc})} = 3 \eta^{(q)} \) (\( \eta^{(q)} \) is the parameter of momentum noncommutativity corresponding to a quark).

As a result, assuming that parameters of noncommutativity corresponding to the elementary particles (electrons and quarks) are of the same order, on the basis of (53) one obtains

\[
\eta^c \approx 3 N_{\text{nuc}} \eta^{(q)} + \frac{N_{\text{nuc}} \eta^{(e)}}{2} \approx \frac{m_M}{m_{\text{nuc}}} \eta^{(\text{nuc})}.
\]
So, on the basis of (52), for the parameter of momentum noncommutativity, corresponding to nucleons, we find
\[
h |\theta^{(\text{nuc})}| \leq 3.3 \cdot 10^{-80} \text{kg}^2 \text{m}^2 / \text{s}^2 = 1.2 \cdot 10^{-25} (\text{eV}/c)^2 = 7.8 \cdot 10^{-82} (E_P/c)^2.
\] (55)
Analogically, taking into account the expression for the effective parameter of coordinate noncommutativity (17) for the Mercury planet one has
\[
\theta^c = N_e \theta^{(\text{nuc})} \frac{m_n^2}{m_M^2} + N_c \theta^{(\text{nuc})} \frac{m_e^2}{m_M^2} \approx \frac{\theta^{(\text{nuc})} m_{\text{nuc}}}{m_M},
\] (56)
where \(\theta^{(\text{nuc})}, \theta^c\) are parameters of noncommutativity corresponding to nucleons and electrons, \(m_e\) is the mass of an electron. The details of the calculation of the effective parameter of coordinate noncommutativity for the Mercury planet can be found in [25]. So, from (51) we find
\[
h |\theta^{(\text{nuc})}| \leq 7.2 \cdot 10^{-13} \text{m}^2 = 2.8 \cdot 10^{57} \text{I}_2.
\] (57)
The upper bound for the parameter of coordinate noncommutativity (57) is in agreement with the result obtained on the basis of studies of neutrons in a gravitational well in noncommutative space [48]. Note that the upper bound for the parameter of momentum noncommutativity (55) is quite strong. The result (55) is 13 orders less than that obtained on the basis of studies of neutrons in a gravitational quantum well in noncommutative phase space [49].

We would like to mention that the same results for the upper bounds for the parameters of noncommutativity (55), (57) can be obtained considering relations (23), (24). If conditions (23), (24) hold taking into account (29), (30) we obtain \(\eta^{(\text{nuc})} / m_{\text{nuc}} = \eta^e / m_M, \theta^{(\text{nuc})} m_{\text{nuc}} = \theta^c m_M\) that correspond to (54), (56).

On the basis of inequalities (51), (52), considering relations (23), (24), (29), (30), one can estimate the upper bounds on the parameters of noncommutativity, corresponding to different particles. For electrons we have \(\eta^e = \eta^e m_e / m_M, \theta^e = \theta^c m_M / m_e\) and
\[
|\eta^{(\text{nuc})}| \leq 1.3 \cdot 10^{-79} \text{m}^2 = 5 \cdot 10^{40} \text{I}_2,
\] (58)
\[
|\theta^{(\text{nuc})}| \leq 1.8 \cdot 10^{-33} \text{kg}^2 \text{m}^2 / \text{s}^2 = 6.3 \cdot 10^{-29} (\text{eV}/c)^2 = 4.2 \cdot 10^{-83} (E_P/c)^2.
\] (59)
The upper bound (58) is not strong. Because of the reduction of the effective parameter of coordinate noncommutativity with respect to parameters of noncommutativity corresponding to elementary particles, to find a strong upper bound for the parameters of coordinate noncommutativity on the basis of studies of the motion of a macroscopic body the experimental results with very high accuracy are needed. The result (59) is 17 orders less than the upper bound obtained examining the effect of noncommutativity on the hyperfine structure of the hydrogen atom in noncommutative phase space [14]. From (59) we obtain that the upper bound on the momentum scale is
\[
\sqrt{h |\eta^{(\text{nuc})}|} \leq 4.2 \cdot 10^{-42} \text{kg} \cdot \text{m/s} = 7.9 \cdot 10^{-15} \text{eV}/c = 6.5 \cdot 10^{-43} E_P/c.
\] (60)
To analyze the obtained result it is worth comparing it with the known values. From the Heisenberg uncertainty relation one has \(\Delta P \geq h / \Delta X\). For the distance corresponding to the diameter of the observable universe \(8.8 \cdot 10^{26} \text{m} [50]\) from this relation we find \(\Delta P \geq 6 \cdot 10^{-62} \text{kg} \cdot \text{m/s}\). The obtained upper bound (60) is far from this value, one has \(\sqrt{h |\eta^{(\text{nuc})}| / \Delta P} = 7 \cdot 10^{19}\).

Since constants \(\alpha, \gamma\) are the same for particles with different masses and composition and play the role of universal parameters, let us estimate the upper bounds for them. For constants \(\alpha = \alpha_{12}, \gamma = \gamma_{12}\), taking into account (29), (30) on the basis of (51), (52) we obtain
\[
|\gamma| \leq 1.1 \cdot 10^{-5} \text{s} = 2.1 \cdot 10^{18} T_P,
\] (61)
\[
|\alpha| \leq 1.9 \cdot 10^{-19} \text{s}^{-1} = 10^{-62} T_P^{-1},
\] (62)
where \(T_P\) is the Planck time.

**Conclusion.** – A space with noncommutativity of coordinates and noncommutativity of momenta of canonical type (4)–(6) has been considered. The features of the description of the motion of a composite system in the space have been discussed. We have shown that if parameters of noncommutative algebra are related with mass, namely if conditions (23)–(25) are satisfied, the motion of the center of mass of a composite system is independent of the relative motion, the system of free particles with the same initial velocities does not fly away, the weak equivalence principle is recovered in the noncommutative phase space (4)–(6).

The perihelion shift of the Mercury planet has been studied, taking into consideration the features of the description of the motion of a macroscopic body in noncommutative phase space. On the basis of the results of these studies and the results for the precession of Mercury’s perihelion from ranging to the MESSENGER spacecraft we have estimated the upper bound on the parameters of noncommutativity corresponding to nucleons. We have concluded that taking into account expressions for the effective parameters of noncommutativity (54), (56), corresponding to the Mercury planet, an extremely strong upper bound for the parameter of coordinate noncommutativity presented in [23] can be reexamined to more relevant result (57) and a quite stringent upper bound on the parameter of momentum noncommutativity can be found. Namely we have found an upper bound \(3.3 \cdot 10^{-80} \text{kg}^2 \text{m}^2 / \text{s}^2\) for the parameter of momentum noncommutativity corresponding to nucleons. This result is 13 orders less than that obtained studying neutrons in gravitational field in noncommutative phase space [49]. Also, using results for the effective parameters of noncommutativity (51), (52) and considering relations (23), (24), (29), (30) we have found upper bounds for the parameters of noncommutativity corresponding to electrons (58), (59) and estimate the values of parameters \(\alpha, \gamma\) (61), (62). The upper bound for the parameter of momentum noncommutativity corresponding to electrons...
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Upper bound on the momentum scale

1.8 · 10^{-83} kg^2 m^2 s^{-2} is 17 orders less than that obtained on the basis of studies of the hydrogen atom in noncommutative phase space \[14\].

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