Saturated control without velocity measurements for planar robots with flexible joints

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Abstract

In this work, we propose a passivity-based controller that addresses the problem of set point regulation for planar robots with two links and flexible joints. Moreover, the controller is saturated and does not require velocity measurements. Additionally, we present experiments that corroborate the theoretical results of this note.

Keywords.- Port-Hamiltonian systems, passivity-based control, saturation, asymptotic stabilization.

1 Introduction

Energy-based models, e.g., the Euler-Lagrange (EL) and port-Hamiltonian (pH) frameworks, have been extensively used to represent mechanical systems, see for example [1,2,3,4,5,6]. One of the main advantages of these modeling approaches is that they provide a systematic procedure to obtain mathematical models that capture the nonlinear phenomena and preserve conservation laws present in physical systems.

A well-known property of mechanical systems is that they are passive, loosely speaking this means that these systems are not able to generate energy by themselves. Particularly, in the EL and pH representations, this passivity property can be verified by considering as storage function the total energy of the system. Accordingly, a natural way to control passive systems is to design controllers that give a desired shape to the energy of the closed-loop system. This process is known as energy-shaping and it is the main idea of several passivity-based control (PBC) approaches.

In this work, we focus on the pH representation of planar robots with flexible joints, where the objective is to address the problem of set-point regulation for this class of systems. Additionally, we are interested in controllers that can overcome two common issues that arise during practical implementation, namely, the lack of sensors to measure the velocities and limitations in the actuators, particularly, the necessity of saturated signals to ensure the safety of the equipment. Following these ideas, the main contribution of this work is the design of a controller that solves the set-point regulation problem for planar robots with flexible joints. The aforementioned controller has the following appealing properties:

- The closed-loop system preserves the pH structure and, consequently, the passivity property.
- The control design does not require the solution of partial differential equations (PDEs).
- The control signals are constrained to a desired interval. Thus, for implementation purposes, it is not necessary to include additional saturation blocks to prevent damage to the motors.
- The control design only requires position measurements. Therefore, the controller can be implemented without the necessity of filters or observers that estimate the velocities.

The outline of this paper is as follows. First, we provide the model of the system and the problem formulation in Section 2. Section 3 is devoted to the control design, where we present two saturated controllers and the experimental results derived from their implementation. Finally, we give closure to this note with some concluding remarks and future work in Section 4.

Notation: We denote the \( n \times n \) identity matrix as \( I_n \), and the \( n \times s \) matrix of zeros as \( 0_{n \times s} \). Consider the vector \( x \in \mathbb{R}^n \), the square matrix \( A \in \mathbb{R}^{n \times n} \), the function \( f : \mathbb{R}^n \to \mathbb{R} \), and the mapping \( F : \mathbb{R}^n \to \mathbb{R}^m \). Then: we denote the \( i \)-th element of \( A \) as \( A_{ii} \), the \( i \)-th element of \( x \) as \( x_i \), the symmetric part of \( A \) is given by \( \text{sym}(A) := \frac{1}{2}(A + A^\top) \). When \( A = A^\top \), \( A \) is said to be positive definite, \( A > 0 \), or positive semi-definite, \( A \geq 0 \), if and only if \( x^\top Ax > 0 \), \( x^\top Ax \geq 0 \) for all \( x \neq 0_n \), respectively. If \( A > 0 \), we denote the Euclidean weighted-norm as \( \|x\|_A^2 := x^\top Ax \). We define the differential operator \( \nabla_x f := \left( \frac{\partial f}{\partial x} \right) \) and \( \nabla^2_x f := \frac{\partial^2 f}{\partial x^2} \). For \( F \), we define the \( ij \)-th element of its \( n \times m \) Jacobian matrix as \( (\nabla_x F)_{ij} := \frac{\partial F_i}{\partial x_j} \). When clear from the context the subindex in \( \nabla \) is omitted. For any \( F \) and the distinguished element \( x_\ast \in \mathbb{R}^n \), we define the constant matrix \( F_\ast := F(x_\ast) \). All mappings are supposed smooth enough.
2 Model and problem setting

The system to be controlled consists of two links, each one attached to a motor shaft through a spring. As is stated above, we adopt a pH model to characterize the behavior of the system. Therefore, we consider as state vector the above, we adopt a pH model to characterize the behavior of the system. Therefore, we consider as state vector the positions \( q \in \mathbb{R}^2 \) and the momenta \( p \in \mathbb{R}^2 \) related to the links and motors, respectively.

where the vectors \( q \in \mathbb{R}^2 \), \( q_m \in \mathbb{R}^2 \) denote the angular position of the links and the motors, respectively; while, \( p \in \mathbb{R}^2 \) represent the momenta of the links, and the momenta of the motors are given by \( p_m \in \mathbb{R}^2 \). Hence, the system dynamics is expressed as

\[
\begin{bmatrix}
\dot{q} \\
\dot{p}
\end{bmatrix} =
\begin{bmatrix}
0_{4 \times 4} & I_4 \\
-I_4 & -R_2
\end{bmatrix}
\begin{bmatrix}
\nabla_q H(q, p) \\
\nabla_p H(q, p)
\end{bmatrix} +
\begin{bmatrix}
0_4 \\
B
\end{bmatrix} u
\]

\[ H(q, p) = \frac{1}{2} \| p \|_2 M^{-1}(q) p + \frac{1}{2} \| q_m - q \|_2 \]

with

\[
M(q_{s_2}) = \begin{bmatrix}
M_1(q_{s_2}) & 0_{2 \times 2} \\
0_{2 \times 2} & M_m
\end{bmatrix}
\]

\[
M_1(q_{s_2}) = \begin{bmatrix}
a_1 + a_2 + 2b \cos(q_{s_2}) & a_2 + b \cos(q_{s_2}) \\
a_2 + b \cos(q_{s_2}) & a_2
\end{bmatrix}
\]

\[
R_2 = \begin{bmatrix}
D_t & 0_{2 \times 2} \\
0_{2 \times 2} & D_m
\end{bmatrix}
\]

\[
M_m = \text{diag}(I_{m_1}, I_{m_2})
\]

\[
D_t = \text{diag}(D_{t_1}, D_{t_2})
\]

\[
D_m = \text{diag}(D_{m_1}, D_{m_2})
\]

\[
K_s = \text{diag}(k_{s_1}, k_{s_2})
\]

\[
B = \begin{bmatrix}
0_2 \\
I_2
\end{bmatrix}
\]

where \( a_1, a_2 \) and \( b \) are constants related to the moment of inertia (Mol) of the links; and \( u \in \mathbb{R}^2 \) is the input vector which corresponds to the torques of the motors. All the parameters are positive and their physical meaning is explained in Table 5 in the Appendix of this note.

Problem setting

the objective of this work is to stabilize system \( \mathcal{P} \) to a constant point that belongs to the set

\[ \mathcal{E} := \{ q \in \mathbb{R}^4 \mid q_l = q_m, p = 0_4 \} \]

Furthermore, we consider that only measurements of the positions \( q \) are available; and the elements of the controller are constrained to given intervals, that is, \( u(t) \in \mathcal{U} \) for all \( t \geq 0 \), where \( \mathcal{U} := [-u_{\text{max}}, u_{\text{max}}] \times [-u_{\text{max}}, u_{\text{max}}] \).

3 Control design

In this section, we present three controllers that stabilize the planar robot to a reference. The first controller is based on the PI-PBCs reported in [8], which is used as a starting point towards the development of the saturated controllers. The second controller satisfies the requirements established in Section 2 nonetheless, the experiments exhibit a steady state error. Finally, the third controller is an extension of the previous one, where an integral-like action is added to eliminate the steady state error, this is corroborated in the experimental results.

3.1 Preliminary PI controller

In [8], a constructive procedure to stabilize pH systems is proposed, an advantage of this approach over other PBC techniques is that the control law is obtained without the necessity of solving PDEs. Furthermore, the controllers derived from this approach can be interpreted as PI regulators, where the feedback signal is the passive output of the system. In Proposition 1 we provide a modified PI that stabilizes system \( \mathcal{P} \) to a desired reference, and where the passive output is given by the velocities of the motors. Then, based on this PI controller, in subsequent sections we develop a control law that satisfies the requirements imposed in Section 2.

Proposition 1 Consider system \( \mathcal{P} \) in closed-loop with the controller\footnote{For an alternative pH representation of this system, we refer the reader to [7].}

\[ u = -K_{P_m} q_m - K_I (q_m - q_s) - K_{P_l} \dot{q}_l \]

where \( q_s \in \mathbb{R}^2 \) is the desired position of the links, and the matrices \( K_{P_m}, K_{P_l}, K_I \in \mathbb{R}^{2 \times 2} \) verify

\[ K_{P_m} > 0, \quad K_I > 0, \quad D_m + K_{P_m} - \frac{1}{2} K_{P_l}^{-1} K_{P_l} > 0. \]

Then, the following statements hold true.

(i) The closed-loop system admits a pH representation, that is

\[ \begin{bmatrix}
\dot{q} \\
\dot{p}
\end{bmatrix} =
\begin{bmatrix}
0_{4 \times 4} & I_4 \\
-I_4 & J_{P_l} - R_{P_l}
\end{bmatrix}
\begin{bmatrix}
\nabla_q H_{P_l}(q, p) \\
\nabla_p H_{P_l}(q, p)
\end{bmatrix} \]

where

\[
R_{P_l} :=
\begin{bmatrix}
D_l & \frac{1}{2} K_{P_l}^{-1} \\
\frac{1}{2} K_{P_l} & D_m + K_{P_m}
\end{bmatrix}
\]

\[
J_{P_l} := \begin{bmatrix}
0_{2 \times 2} & K_{P_l}^{-1} \\
-K_{P_l} & 0_{2 \times 2}
\end{bmatrix}
\]

\[ H_{P_l}(q, p) := H(q, p) + \frac{1}{2} \| q_m - q_s \|_2 K_s. \]

\footnote{We recall that \( \nabla_p H(q, p) = M^{-1}(q)p \).}
(ii) The point
\[ x_* := (q_*, q_m, p_1, p_m) = (q_*, q_*, 0_2, 0_2) \]
is an asymptotically stable equilibrium of the closed-loop system with Lyapunov function \( H_{P_1}(q, p) \).

Proof: Note that, based on a Schur complement analysis, (9) implies that \( H_{P_1} > 0 \). Moreover,
\[ J_{P_1} - R_{P_1} = \begin{bmatrix} -D_1 & 0_2 \\ -K_{P_1} & -D_m - K_{P_m} \end{bmatrix}. \]

Hence, replacing (9) in (2) we get (6) implies that
\[ (6) \quad \text{implies that} \]

Hence, replacing (6) in (2) we get (3).

Due to space constraints, in the sequel, during the development of the proofs, the arguments of the functions are omitted when they are clear.

3.2 Saturated control without velocity measurements

Although the controller defined in (5) stabilizes the system at the desired point, it clearly requires information of the velocities. Moreover, it is not possible to ensure that the control signals remain in the range of operation of the motors. Accordingly, to overcome the aforementioned issues, we propose two modifications to the control law:

- To replace the integral term \( K_l(q_m - q_*) \) with a saturated function.
- To inject damping without the necessity of measuring the velocities.

A method to inject damping without velocity measurements, for mechanical systems, is proposed in [10]. The main idea of this methodology is to propose a virtual state that is linearly related to the positions, then, this new state is used to inject damping into the closed-loop system. Proposition 2 establishes one of the main contributions of this note, where a combination of the damping injection approach reported in [10] with the PI of Proposition 1 is proposed.

Proposition 2 Let the controller state vectors \( x_{c_1}, x_{c_m} \in \mathbb{R}^2 \). Define the function
\[ \Phi_l(z_l) := \sum_{i=1}^2 \frac{\alpha_l}{\beta_l} \ln(\cosh(\beta_l z_l)) \]
where \( \alpha_l, \alpha_m, \beta_l, \beta_m \in \mathbb{R} > 0 \). Consider the dynamics
\[ \begin{align*}
\dot{x}_{c_1} &= -R_{c_1} \nabla x_{c_1} \Phi_l(z_l(q_1, x_{c_1})) \\
\dot{x}_{c_m} &= -R_{c_m} \left[ \nabla x_{c_m} \Phi_m(z_m(q_m, x_{c_m})) + K_c x_{c_m} \right] 
\end{align*} \]
where \( R_{c_1}, R_{c_m}, K_c \in \mathbb{R}^{2 \times 2} \) are positive definite constant matrices verifying
\[ R_{c_1} - \frac{1}{4} (D_1^{-1} + D_m^{-1}) > 0. \]

Consider the control law
\[ u = -\nabla z_l \Phi_l(z_l(q_1, x_{c_1})) - \nabla z_m \Phi_m(z_m(q_m, x_{c_m})). \]

Then:

(i) The elements of the input vector \( u \) are saturated.

(ii) Consider system (2) in closed-loop with (17). Hence, the dynamics of the augmented state space \( \zeta := [q_1^T, p_1^T, x_{c_1}^T, x_{c_m}^T]^T \) admit a pH representation.
(iii) The point \( \zeta_* = (x_*, 0_2, 0_2) \) is an asymptotically stable equilibrium of the closed loop system with Lyapunov function

\[
H_\zeta(\zeta) = H(q, p) + \Phi_l(z_l(q_l, x_m)) + \Phi_m(z_m(q_m, x_{cm}) + \frac{1}{2}\|x_{cm}\|^2_{K_c}.
\]

Proof: To proof (i), note that, since vector take the form

\[
\begin{bmatrix}
\alpha_1 \tanh(\beta_1 z_l) \\
\alpha_2 \tanh(\beta_2 z_l)
\end{bmatrix}
\]

Then, the control law \( u \) reduces to

\[
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix} = \begin{bmatrix}
\alpha_1 \tanh(\beta_1 z_l) + \alpha_m \tanh(\beta_m z_m) \\
\alpha_2 \tanh(\beta_2 z_l) + \alpha_m \tanh(\beta_m z_m)
\end{bmatrix}.
\]

To proof (ii), note that, since

\[
\begin{align*}
\nabla_{\Bar{z}} \Phi_l &= \begin{bmatrix}
\alpha_1 \tanh(\beta_1 z_l) \\
\alpha_2 \tanh(\beta_2 z_l)
\end{bmatrix} \\
\nabla_{\Bar{z}} \Phi_m &= \begin{bmatrix}
\alpha_1 \tanh(\beta_1 z_l) \\
\alpha_2 \tanh(\beta_2 z_l)
\end{bmatrix}
\end{align*}
\]

Thus, \(- (\alpha_1 + \alpha_m) \leq u_i \leq (\alpha_1 + \alpha_m).\)

To proof (iii) note that, from \( (20) \) and \( (28) \), we have

\[
H_\zeta = \left( \nabla H_\zeta \right)^\top F_\zeta \nabla H_\zeta \leq 0,
\]

which implies that \( H_\zeta(\zeta) \) is non-increasing. Moreover,

\[
\begin{align*}
z_l &= 0_2 \quad \Rightarrow \quad (\nabla_{q_l} \Phi_l) = (\nabla_{z_l} \Phi_l) = 0_2, \\
z_m &= 0_2 \quad \Rightarrow \quad (\nabla_{q_m} \Phi_m) = (\nabla_{z_m} \Phi_m) = 0_2.
\end{align*}
\]

Therefore,

\[
(\nabla H_\zeta)^\top = \begin{bmatrix}
0_2 \\
0_2
\end{bmatrix},
\]

Furthermore, since

\[
\begin{align*}
\nabla^2 \Phi_l &= \nabla^2_{z_l} (\nabla_{q_l} \Phi_l) = \nabla^2_{z_l} (\nabla_{z_l} \Phi_l) = \nabla^2_{z_l} \Phi_l \\
\nabla^2 \Phi_m &= \nabla^2_{z_m} (\nabla_{q_m} \Phi_m) = \nabla^2_{z_m} (\nabla_{z_m} \Phi_m) = \nabla^2_{z_m} \Phi_m
\end{align*}
\]

some straightforward computations show that

\[
\begin{align*}
(\nabla H_\zeta)^\top = \begin{bmatrix}
K_S + A & 0_{4 \times 4} \\
0_{4 \times 4} & M^* \quad A
\end{bmatrix}
\quad > 0,
\end{align*}
\]

where

\[
A := \text{diag}\{\beta_1 \alpha_1, \beta_2 \alpha_2, \beta_m \alpha_m, \beta_m \alpha_m, \beta_m \alpha_m, \beta_m \alpha_m\},
\]

\[
K_S := \begin{bmatrix}
K_S & -K_S \\
-K_S & K_S
\end{bmatrix},
\]

Accordingly, from \( (31) \) and \( (32) \), \( \arg\min\{H_\zeta(\zeta)\} = \zeta_.\)

Hence, the asymptotic stability property is proved by invoking Lyapunov theory. Moreover, to proof the asymptotic stability property, note that

\[
\begin{align*}
\dot{H}_\zeta = 0 \iff \left\{ \begin{array}{l}
\nabla_p H_\zeta = 0 \quad \Rightarrow \quad p = 0_4 \\
\nabla_{\Bar{z}} H_\zeta = 0_2 \quad \Rightarrow \quad \Bar{z}_l = 0_2 \\
\nabla_{x_{cm}} H_\zeta = 0_2 \quad \Rightarrow \quad K_c x_{cm} = -\nabla_{q_m} \Phi_m
\end{array} \right.
\end{align*}
\]

Further, from \( (10) \), it follows that

\[
\text{sym}(F_\zeta) = \begin{bmatrix}
0_{4 \times 4} & 0_{4 \times 4} & 0_{4 \times 2} \\
0_{4 \times 4} & 0_{4 \times 4} & 0_{4 \times 2} \\
0_{4 \times 2} & 0_{4 \times 2} & 0_{2 \times 2} \\
0_{4 \times 2} & 0_{4 \times 2} & -R_{cm}
\end{bmatrix} \leq 0.
\]
and we study three cases for different values of the parameters $\alpha_{l_i}$, $\alpha_{m_i}$ which are shown in Table 1.

| Case | values for $\alpha_{l_i}$ | values for $\alpha_{m_i}$ |
|------|--------------------------|--------------------------|
| C1   | 0.8                      | 0.4                      |
| C2   | 0.2                      | 1                        |
| C3   | 0                        | 1.2                      |

The three cases under study illustrate the effect of the term $\nabla_z \Phi(z_l(q_l, x_{c_l}))$ in the behavior of the closed-loop system. This term is interpreted as damping in the links of the planar robot, therefore, it is expected that as the values $\alpha_{l_i}$ increase, the oscillations in the response decrease. Accordingly, the experiments for the three cases are carried out under the same initial conditions, $\zeta(0) = 0_{12}$, and the same reference, $q_\ast = (-1, 1)$. Figures 1, 2 depict the results of case C1, Figures 3, 4 show the results of case C2, and Figures 5, 6 correspond to the results of case C3. From the aforementioned plots, we conclude the following:

- The existence of steady state error can be observed for all the cases. This situation is probably a consequence of phenomena that are not taken into account in the model, e.g., nonlinear friction terms. Moreover, from the experiments, we notice the actuators are not able to provoke any displacement when $u_i \in [-0.12, 0.12]$. Therefore, the positions remain constant, even when the control signals are different from zero. This is particularly notorious in Figure 2.

- There exists a trade off between the damping injected to the links and the magnitude of the steady state error. Furthermore, a similar relationship takes place between the magnitude of $\alpha_{l_i}$ and the oscillations, where, a greater magnitude of these values yields into an important attenuation in the oscillations. Both relations can be noticed in Figures 1, 3 and 5.

From a theoretical point of view, the control law (17) solves the problem stated in Section 2, that is, the controller de-
3.3 Eliminating the steady state error

Customarily, the steady state error is eliminated by the negative feedback of an integral term of the error between some measurements and the reference. In our case, such an error is the difference between the positions of the motors and the reference, that is, \((q_m - q_*)\). Moreover, the derivative of this error is given by \(\dot{q}_m\), which is the passive output of the system. Hence, following the approach adopted in this note, the term that eliminates the steady state error is given by a double integrator of the passive output of the system. Hence, following the approach adopted in this note, the term that eliminates the steady state error is given by a double integrator of the passive output of the system. Hence, following the approach adopted in this note, the term that eliminates the steady state error is given by a double integrator of the passive output of the system. Hence, following the approach adopted in this note, the term that eliminates the steady state error is given by a double integrator of the passive output of the system. Hence, following the approach adopted in this note, the term that eliminates the steady state error is given by a double integrator of the passive output of the system.

Proposition 3 Consider the vector state \(\sigma \in \mathbb{R}_2\) whose dynamics are given by

\[
\dot{\sigma} = \nabla^2 \Phi_\sigma(\sigma)(q_m - q_*) - K_\sigma \sigma, \tag{35}
\]

where the matrix \(K_\sigma \in \mathbb{R}^{2 \times 2}\) and the function \(\Phi_\sigma : \mathbb{R}^2 \to \mathbb{R}\) are defined as

\[
\Phi_\sigma(\sigma) := \sum_{i=1}^{2} \frac{\alpha_{\sigma_i}}{\beta_{\sigma_i}} \ln(\cosh(\beta_{\sigma_i} \sigma_i)), \tag{36}
\]

\[
K_\sigma := \text{diag} \{k_{\sigma_1}, k_{\sigma_2}\}, \tag{37}
\]

with \(\alpha_{\sigma_i}, \beta_{\sigma_i}, k_{\sigma_i} \in \mathbb{R}_{>0}\). Consider \(F_\xi, \) given in \((34)\) and assume that \(R_{c_1}\) verifies \((10)\). Consider the control law

\[
u = -\nabla z_i^T \Phi_l(q_l, x_c) - \nabla z_m^T \Phi_m(q_m, x_c) - \nabla \Phi_\sigma(\sigma), \tag{38}
\]

with \(z_l, z_m, \Phi_l, \Phi_m\) defined as in \((11)\). Fix a reference \(q_*\) and define the matrices

\[
A_\sigma := \text{diag}(\beta_{\sigma_1} \alpha_{\sigma_1}, \beta_{\sigma_2} \alpha_{\sigma_2});
\]

\[
A_{\xi_1} := \begin{bmatrix} 0 & -A_\sigma \\ A_\sigma & 0 \end{bmatrix}, \quad A_{\xi_2} := \begin{bmatrix} 0_2 & A_\sigma \\ 0_2 & 0_8 \end{bmatrix}, \tag{39}
\]

\[
A := \begin{bmatrix} F_\xi(\nabla^2 H_\xi)_* \quad A_{\xi_1} \\ A_{\xi_2}^T & -K_\sigma \end{bmatrix},
\]

where \((\nabla^2 H_\xi)_*\) is given in \((32)\). Then:

(i) The elements of the input vector \(u\) are saturated.

(ii) Consider system \((2)\) in closed-loop with \((38)\). If the matrix \(A\) is Hurwitz, then \(\xi^* = [q_1^T, q_2^T, 0_{10}]^T\) is a (locally) asymptotically stable equilibrium point of the closed-loop system.

Proof: To proof (i), note that

\[
\nabla \Phi_\sigma = \begin{bmatrix} \alpha_{\sigma_1} \tanh(\beta_{\sigma_1} \sigma_1) \\ \alpha_{\sigma_2} \tanh(\beta_{\sigma_2} \sigma_2) \end{bmatrix}. \tag{40}
\]

Furthermore, from the proof of item (i) in Proposition \((2)\), we get

\[
u_i = -\alpha_i \tanh(\beta_{\sigma_i} z_i) - \alpha_m \tanh(\beta_{\sigma_m} z_m) - \alpha_\sigma \tanh(\beta_{\sigma_\sigma} \sigma_i)
\]

Moreover,

\[-(\alpha_i + \alpha_m + \alpha_\sigma) \leq \nu_i \leq \alpha_i + \alpha_m + \alpha_\sigma.\]

To proof (ii), define the new state space \(\xi := [\xi^T, \sigma^T]^T\), and the error \(\bar{\xi} := \xi - \xi^*\). Then, some lengthy but straightforward computations show that the linearization of closed-loop system, around \(\xi^*\), is given by

\[
\dot{\bar{\xi}} = A \bar{\xi},
\]

The proof is completed by applying Lyapunov’s Indirect Method, see Chapter 4 of [9]. □

While the main theoretical contributions of this document are the results of Proposition \((2)\) with the implementation of the control law \((38)\) the closed-loop system exhibits a better performance in terms of steady state error and oscillations. Below, we report the experimental results of this implementation.

Experimental results

The control law \((38)\) is implemented in the robot arm. Towards this end, we select the control matrices as

\[
R_c = \text{diag}\{25, 40\}, \quad K_c = 0.1 I_2, \quad R_{cm} = 0.25 I_2, \quad K_m = I_2, \tag{41}
\]

The rest of the control parameters is given in Table \((2)\).
Moreover, we fix the reference $\xi = (-1, 1)$, and we carry out the experiments under initial condition $\xi(0) = 0$. Note that with this selection of the parameters of the controller, we ensure that the matrix $A$, defined in (39), is Hurwitz and consequently the point $\zeta$ is an asymptotically stable equilibrium of the closed-loop system. Figure 7 depicts the positions of the links and motors, where the error of the final position with respect to the desired reference is almost zero. On the other hand, Figure 8 shows the control signals, where the value of both signals is close to zero. From the plots we conclude the implementation of controller (17) has in overall a better performance than controller (17). Moreover, in this case the trade off between the attenuation of the oscillations and the steady state error and the attenuation of the oscillations in its behavior. Furthermore, after the addition of this extra term, the property of saturation in the control law is preserved.

As future work, we aim to extend the proposed methodology to pH systems, in different domains, that can be stabilized with PI-PBC, e.g., electrical circuits, electromechanical systems or fluid systems.

Table 3 contains the information about the physical parameters of the robot arm 2 DOF serial flexible joint by Quanser. These parameters were taken from the datasheet of the robot and [11].

| Parameter | Physical meaning | Value | Units |
|-----------|------------------|-------|-------|
| $a_1$     |                  | 0.148 | $kg \cdot m^2$ |
| $a_2$     |                  | 0.073 | $kg \cdot m^2$ |
| $b$       |                  | 0.086 | $kg \cdot m^2$ |
| $I_{m_1}$ | Mol of motor 1   | 0.217 | $kg \cdot m^2$ |
| $I_{m_2}$ | Mol of motor 2   | 0.007 | $kg \cdot m^2$ |
| $D_{l_1}$ | Damping on link 1| 0.038 | $N \cdot m \cdot s/rad$ |
| $D_{l_2}$ | Damping on link 2| 0.03  | $N \cdot m \cdot s/rad$ |
| $D_{m_1}$ | Damping on motor 1| 8.435| $N \cdot m \cdot s/rad$ |
| $D_{m_2}$ | Damping on motor 2| 0.136| $N \cdot m \cdot s/rad$ |
| $k_{s1}$  | Spring constant 1| 9     | $[N \cdot m/rad]$ |
| $k_{s2}$  | Spring constant 2| 4     | $[N \cdot m/rad]$ |

The experiments reported in Section 8 were carried out using the Matlab/Simulink interface. Where,

$$u_{\text{max1}} = u_{\text{max2}} = 1.2[A].$$

Note that, the saturation value is given in terms of the currents supplied to the motors, nonetheless, these cur-
rents satisfy a linear relationship with the torques of the motors. The details of these linear relations are provided by Quanser.

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