NNI Quark-Lepton Mass Matrices in SUSY SU(5) GUT

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Abstract

We propose the Fritzsch—Branco—Silva-Marcos type fermion mass matrix, which is a typical texture in the nearest-neighbor interaction form, in SU(5) GUT. By evolution of the mass matrices with SU(5) GUT relations in the minimal SUSY standard model, we obtain predictions for the unitarity triangle of CP violation as well as the quark flavor mixing angles, which are consistent with experimental data, in the case of tan β ≃ 3.

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One of the most important unsolved problem of flavor physics is the understanding of flavor mixing and fermion masses, which are free parameters in the standard model. The observed values of those mixing and masses may provide us clues to solve this problem. Many works were made to find ansätze for quark-lepton mass matrices. The typical one is the Fritzsch ansätze [1], which is called texture zero analysis where some elements of mass matrices are required to be zero to reduce the degrees of freedom in mass matrices. As presented by Branco, Lavoura and Mota, both up- and down-quark mass matrices could always be transformed to the non-Hermitian matrices in the nearest-neighbor interaction (NNI) basis by a weak-basis transformation for the three and four generation cases [2]. Based on the NNI form, several authors have studied the quark masses and Cabibbo-Kobayashi-Maskawa (CKM) matrix [3] phenomenologically [4]–[7]. One of authors(T.I) proposed a texture, in which the up-quark mass matrix to be in the Fritzsch form and the down-quark mass matrix to be Branco–Silva-Marcos (BS) form [7]. In this letter, we call this texture F-BS one. Recently, Takasugi has shown that quark mass matrices can be transformed in general to either one of the following two form, the Fritzsch type parameterization or the BS type parameterization with retaining the NNI form for the other matrix [8]. Moreover, Takasugi and Yushimura pointed out that it is reasonable to take the BS ansätze for the down-quark mass matrix if the up-quark one is assumed to be the Fritzsch texture.

In this letter, we build a mass matrix model based on the F-BS texture in $SU(5)$ GUT. The F-BS texture may be compared with Georgi-Jarlskog texture [10], in which the zeros are forced by discrete symmetries. We do not need to force zeros by discrete symmetries because the F-BS texture is a typical case of the NNI form.

By putting $SU(5)$ GUT relations, the charged lepton mass matrix is related with the quark one. The evolution based on the SUSY renormalization group equations from the GUT scale to the $M_Z$ scale gives predictions for CKM matrix. The F-BS texture reproduces well known empirical relations,

\begin{align}
|V_{us}| &\sim \sqrt{\frac{m_d}{m_s}}, \\
|V_{cb}| &\sim \frac{m_s}{m_b}, \\
\frac{|V_{ub}|}{|V_{cb}|} &\sim \sqrt{\frac{m_u}{m_c}}.
\end{align}

Following these investigations, we take the F-BS type Yukawa matrices for quarks and leptons at the $SU(5)$ GUT scale. Yukawa matrices are written as follows:

\[
Y^U = \begin{pmatrix}
0 & a_u & 0 \\
a_u & 0 & b_u \\
0 & b_u & c_u
\end{pmatrix}, \quad Y^D = \begin{pmatrix}
0 & a_d e^{i\theta_1} & 0 \\
a_d e^{-i\theta_1} & 0 & b_d e^{i\theta_2} \\
0 & c_d & c_d
\end{pmatrix},
\]
where $Y^U$ and $Y^D$ are matrices for up-quark and down-quark, respectively. The up-quark Yukawa matrix is the Fritzsch texture, while the down-quark one is the BS texture. By assuming the 5$^*$ and 45$^*$ Higgs fields of $SU(5)$, the charged lepton Yukawa matrix $Y^E$ is given as follows:

$$
Y^E = \begin{pmatrix}
0 & a_d e^{i\theta_1} & 0 \\
0 & -3 b_d e^{i\theta_2} & c_d
\end{pmatrix}.
$$

(5)

Each entry of quark-lepton matrices is assumed to arise from the VEV of 5$^*$'s of Higgs field except for (2,3) entries in $Y^D$ and $Y^E$, which are assumed to arise from 45$^*$ of Higgs field. Therefore, the matrix $Y^U$ should be symmetric while $Y^D$ and $Y^E$ are allowed to be non-symmetric. So, parameters $a_{u(d)}$, $b_{u(d)}$, $c_{u(d)}$ are taken to be real and the phase parameters appear in $Y^D$ and $Y^E$ with $\theta_1$, $\theta_2$. Including $\tan\beta$, we have 9 parameters in the fermion mass matrix. On the other hand, there are 14 low energy observables, 9 charged fermion masses, 4 CKM mixing angles and $\tan\beta$. Thus, there are 5 predictions.

We adopt a weak hypothesis for the parameters of the matrices, the generation hierarchy,

$$a_d \ll b_d \ll c_d .$$

(6)

Then, mass eigenvalues are given in terms of those parameters as follows:

$$m_d = \frac{a_d^2}{b_d}, \quad m_s = \frac{b_d}{\sqrt{2}}, \quad m_b = \sqrt{2} c_d$$

(7)

$$m_e = \frac{1}{3} a_d^2, \quad m_\mu = 3 \frac{b_d}{\sqrt{2}}, \quad m_\tau = \sqrt{2} c_d .$$

(8)

Eliminating parameters, we obtain the $SU(5)$ GUT mass relations:

$$m_\tau = m_b ,$$

(9)

$$m_\mu = 3 m_s ,$$

(10)

$$m_e = \frac{1}{3} m_d ,$$

(11)

which is the same one in Georgi-Jarlskog texture.

In the minimal SUSY standard model, the renormalization group equations of 1-loop are [11]

$$
\frac{d}{dt} Y^U = \frac{1}{16\pi^2} \left\{ \text{tr} \left(3Y^U Y^U \dagger\right) Y^U + 3Y^U Y^U \dagger Y^U \right. \\
+ Y^D Y^D \dagger Y^U - \left( \frac{13}{9} g^2 + 3 g_2^2 + \frac{16}{3} g_3^2 \right) Y^U \right\}
$$

(12)
\[
\frac{d}{dt} Y^D = \frac{1}{16\pi^2} \left\{ \text{tr} \left( Y^E Y^{E\dagger} + 3Y^D Y^{D\dagger} \right) Y^D + 3Y^D Y^{D\dagger} Y^D \\
+ \left( \frac{7}{9}g'^2 + 3g^2 + \frac{16}{3}g_3^2 \right) Y^D \right\} 
\]

(13)

\[
\frac{d}{dt} Y^E = \frac{1}{16\pi^2} \left\{ \text{tr} \left( Y^E Y^{E\dagger} + 3Y^D Y^{D\dagger} \right) Y^E \\
+ 3Y^E Y^{E\dagger} Y^E - 3 \left( g'^2 + g^2_2 \right) Y^E \right\} 
\]

(14)

for Yukawa matrices with \( t = \ln \frac{M_2^2}{\mu^2} \) and

\[
\frac{d}{dt} \alpha_i = \frac{b_i}{2\pi} \alpha_i^2 \left( \alpha_i = \frac{g_i^2}{4\pi} , \ g_1^2 = \frac{5}{3}g'^2 , \ i = 1, 2, 3 \right), 
\]

(15)

for gauge couplings, where

\[
b_1 = \frac{3}{10}n_H + 2n_G, \\
b_2 = \frac{1}{2}n_H + 2n_G - 6, \\
b_3 = 2n_G - 9. 
\]

(16) (17) (18)

In our analysis, the GUT scale is fixed as \( M_G = 1.7 \times 10^{16}\text{GeV} \) by use of the experimental data of \( \alpha_1 \) and \( \alpha_2 \). Then we obtain \( \alpha_s(M_Z) = 0.114 \), which is almost consistent with the experimental data, \( \alpha_s(M_Z) = 0.118 \pm 0.003 \) [12]. The factor \( n_H \) and \( n_G \) are the number of Higgs doublets and fermion generations, respectively. We set \( n_H = 2 \) and \( n_G = 3 \). By numerical analysis of the renormalization group equations, the fermion mass matrices are obtained at the \( M_Z \) energy scale.

It is useful to comment on \( \tan \beta \). If \( \tan \beta \) is less than 2, the Yukawa coupling of the \( t \)-quark blows up under the GUT scale. Recent study of the proton decay suggests that \( \tan \beta \) is less than 4 [13]. Thus, \( \tan \beta \approx 3 \) is a reasonable region. Actually, our numerical results favor \( \tan \beta = 3 \). The fits with the experimental values become worse as \( \tan \beta \) increases.

Since the two phases \( \theta_1 \) and \( \theta_2 \) in \( Y^D \) and \( Y^E \) hardly affect the running of Yukawa matrices, the matrix elements \( a_{u(d)}, b_{u(d)}, c_{u(d)} \) can be adjusted by the following central values of six fermion masses at the \( M_Z \) energy scale [12][14],

\[
m_u(M_Z) = 0.0022 \pm 0.0007\text{GeV} , \ m_c(M_Z) = 0.59 \pm 0.07\text{GeV} , \\
m_t(M_Z) = 175 \pm 14\text{GeV} , \\
m_e(M_Z) = 0.486660328 \pm 0.000000143\text{MeV} , \\
m_\mu(M_Z) = 102.7288759 \pm 0.0000332\text{MeV} , \\
m_\tau(M_Z) = 1746.5^{+0.296}_{-0.266}\text{MeV} . 
\]
Then the down-quark masses are obtained as output:

\[ m_d(M_Z) = 0.0032\text{GeV}, \quad m_s(M_Z) = 0.081\text{GeV}, \quad m_b(M_Z) = 3.31\text{GeV}, \]

which are compared with the experimental values \[14\],

\[ m_d(M_Z) = 0.0038 \pm 0.0007\text{GeV}, \quad m_s(M_Z) = 0.077 \pm 0.011\text{GeV}, \quad m_b(M_Z) = 3.02 \pm 0.19\text{GeV}. \]

Thus obtained values of down-quark masses are almost consistent with the experimental data. It is remarked that due to the running of Yukawa matrices, the (2,2) entries of the quark mass matrices develop in non-negligible finite ones, which are comparable with magnitudes of (1,2) and (2,1) entries. On the other hand, the (1,1), (1,3) and (3,1) entries are almost zero at the \( M_Z \) scale. So even if Yukawa matrices of quarks are F-BS type at the GUT scale, they become to the non-NNI form at low energy scale due to the renormalization effects. On the other hand, the lepton mass matrix keeps the NNI form in the renormalization running.

If \( \theta_1 \) and \( \theta_2 \) are fixed, we can predict CKM matrix. We obtain the CKM matrix in the case of \( \theta_1 = 253^\circ \) and \( \theta_2 = 60^\circ \):

\[
|V_{CKM}| = \begin{pmatrix}
0.9757 & 0.2190 & 0.0045 \\
0.2189 & 0.9747 & 0.0458 \\
0.0082 & 0.0453 & 0.9989
\end{pmatrix}.
\]

This result is consistent with present experimental one within error bars. In particular, we get \( V_{cb} \simeq 0.045 \), which should be compared with the prediction \( V_{cb} \geq 0.05 \) in Georgi and Jarlskog texture \[15\]. The \( CP \) violating phase of the CKM matrix is also fixed. The unitarity triangle of the predicted CKM matrix is shown in Fig. 1. The vertex of this unitarity triangle is on the point \( (0.263, 0.352) \) in the \( \rho-\eta \) plane\[17\]. Thus the vertex is in the first quadrant in the \( \rho-\eta \) plane as well as the case of the ref. \[7\]. Here in order to describe the experimental allowed region, we have used the following parameters \[16\] and experimental data \[12\],

\[ B_K = 0.75 \pm 0.15, \quad f_{B_d} \sqrt{B_{B_d}} = 0.20 \pm 0.04, \]

\[
\frac{|V_{ub}|}{|V_{cb}|} = 0.08 \pm 0.02.
\]

It is noted that the predicted CKM matrix cannot reproduce the experimental data of \( |V_{ub}|/|V_{cb}| \) in the case of \( \tan \beta \geq 4 \).

In this paper, we have predicted the CKM matrix at the \( M_Z \) energy scale by assuming the F-BS texture for Yukawa matrices at the GUT scale. In the case of \( \tan \beta \simeq 3 \),
Figure 1: The unitarity triangle of CKM matrix in the case of $\tan \beta = 3$. The allowed region is given by the experimental constraints of $\epsilon_K$, $B_d - \bar{B}_d$ mixing and $|V_{ub}|/|V_{cb}|$.

the obtained CKM matrix are consistent with experiments. One may worry about our prediction $V_{cb} \simeq 0.045$ because the recent experiments favor $V_{cb} = 0.040 \pm 0.003$ [16]. There is a plausible possibility to push down this predicted value. That is to modify the $SU(5)$ GUT relations by introducing other Higgs fields. The modification may be guaranteed by the recent lattice calculations of light quark masses [18], in which the light quark masses are considerably reduced compared with the conventional ones.

We emphasize that if the $B$-factory experiments at KEK and SLAC will restrict the experimentally allowed region of the unitarity triangle in the first quadrant of the $\rho-\eta$ plane, our proposed simple model can be a candidate for the Yukawa matrices at the GUT scale.
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