Time-scale dependence of correlations among foreign currencies

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Summary. For the purpose of elucidating the correlation among currencies, we analyze daily and high-resolution data of foreign exchange rates. There is strong correlation for pairs of currencies of geographically near countries. We show that there is a time delay of order less than a minute between two currency markets having a strong cross-correlation. The cross-correlation between exchange rates is lower in shorter time scale in any case. As a corollary we notice a kind of contradiction that the direct Yen-Dollar rate significantly differs from the indirect Yen-Dollar rate through Euro in short time scales. This result shows the existence of arbitrage opportunity among currency exchange markets.

Key words. Foreign exchange, cross-correlation, interaction.

1 Introduction

It is widely known that foreign exchange rates sometimes show very violent fluctuations characterized by power law distributions [T. Mizuno][K. Matia]. A very important report appeared recently that arbitrage chances exist by considering three currency markets simultaneously [Y. Aiba]. This result is due to the fact that the distribution of return of rotation transaction of three currencies show a similar fat-tail. As statistical laws among foreign exchange rates in the short time scale are not known sufficiently, we investigate correlations among foreign exchange rates in this paper.
Table 1. The set of Daily data for 25 kinds of foreign exchange we examined.

| Base Currency | Target Currencies |
|---------------|-------------------|
| U.S. Dollar   | Australian Dollar, Brazilian Real, British Pound, Canadian Dollar, Chilean Peso, Colombian Peso, Czech Koruna, Euro, Hungarian Forint, Indonesian Rupiah, Japanese Yen, Mexican Peso, New Zealand Dollar, Norwegian Kroner, Peruvian New Sole, Philippine Peso, Polish Zloty, Russian Ruble, Singapre Dollar, Slovakian Koruna, South African Rand, South Korean Won, Swedish Krona, Swiss Franc, Thai Baht |

Correlations among financial stocks in markets have been actively discussed. The strength of synchronization among stocks was analyzed by using the ultrametric spaces and they discussed about the portfolio of stock [R. N. Mantegna, H. E. Stanley]. The interaction of companies was investigated by analyzing return of many stocks and a directed network of influence among companies was defined [L. Kullmann]. It will be noticed in the present paper that the correlation among foreign exchange rates resembles the stock case.

In the following section we discuss the cross-correlation between exchange rates with no time difference to show the relation between synchronization of exchange rate. Then, we show the maximum correlation between exchange rates is observed with nonzero time shift, namely, the direction of influence is discussed.

2 The correlation among foreign exchange rates

We first analyze a set of daily data provided by Exchange Rate Service [ERS] for 25 exchange rates for about 3 years from January ‘99 to August ‘01 as listed in Table 1. We first estimate cross-correlation functions for these exchange rates measured by USD (United States Dollar). The largest correlation value (≈0.95) is observed for a pair of CHF(Swiss Franc)/USD and EUR(Euro)/USD. As demonstrated in Fig.1 it is evident that these exchange rates are remarkably synchronized. There are cases with negative correlation values as found for the pair of MXP(Mexican-Peso)/USD and CHF(Swiss Franc)/USD whose correlation value is −0.23. It is known that correlations between geographically closer currencies tend to have larger correlation and there exist key currencies in each area, for example, Euro for West Europe, Yen for Asia, Hungarian Forint for East Europe and Australian Dollar for Oceania [H. Takayasu and M. Takayasu].

Although we can find such large correlations among currencies in daily data, we can expect low correlations in short time scale as it is common that dealers in major banks tend to work with a single foreign exchange market. In order to clarify this tendency we examine tick-by-tick data provided by Reuters for about 4 months from March ‘02 to July ‘02. In Fig.2 we plot the correlation value as a function of coarse-grained time-scale for a pair of CHF/USD and EUR/USD. From this figure
we notice that the correlation vanishes if we observe the high-resolution data with the precision of seconds and the correlation value is about 0.5 in the time scale of 5 minutes (300 sec.). From these results it is understood that these two currency markets, CHF/USD and EUR/USD, are working independently in very short time-scale.
In order to clarify the nature of short time interaction among currencies we calculate the cross-correlation with a time shift, namely, we observe the correlation of these two markets with a time difference by the following equation,

$$C(dt) = \frac{\langle dp_A(t) \cdot dp_B(t + dt) \rangle - \langle dp_A(t) \rangle \langle dp_B(t + dt) \rangle}{\sigma_A \cdot \sigma_B},$$

where, $dp_A(t)$ is the rate change in the market $A$ at time $t$, $dp_B(t + dt)$ is the rate change in the market $B$ at time $t + dt$, $\sigma$ is the standard deviation of rate changes in each market. In Fig.3, we show the correlation value between CHF/USD at time $t + dt$ and EUR/USD at time $t$ as a function of time difference $dt$. Here, we show two plots for different coarse-graining time-scales, 60 sec. and 120 sec. In both cases it is found that the largest value of correlation is observed around $dt = 10$ seconds, which implies that in an average sense the EUR/USD market is going about 10 second ahead and the CHF/USD market is following it.

4 Currency correlation in short time scale

Here, we discuss about value of currency correlation in each foreign exchange market in a short time scale. From Fig.2, it is noticed that the correlation between foreign exchanges is very low in a short time scale. Namely, each exchange rate is changing rather independently. In order to clarify this property, we analyze the exchange rate of Yen-Dollar by analyzing a set of tick-by-tick data provided by CQG for about 2 years from February ’99 to March ’02.

We introduce two definitions of Yen-Dollar rate: One definition of JPY/USD is the usual transaction rate and the other JPY/USD is defined through Euro as
Fig. 4 The cross-correlation of JPY (Japanese Yen)/USD (U.S. Dollar) and \( \{ \text{EUR (Euro)/USD} \} \times \{ \text{JPY/EUR} \} \).

Fig. 5 Triangular arbitrage opportunity. A full line is JPY/USD and a dashed line is \( \{ \text{EUR (Euro)/USD} \} \times \{ \text{JPY/EUR} \} \). Triangular arbitrage occurred at the time of.

\[ \{ \text{EUR/USD} \} \times \{ \text{JPY/EUR} \} \]. Here, all exchange rates are given by the middle rate (= (Bid rate + Ask rate) / 2). In Fig. 4 we plot the cross-correlation value of these exchange rates in different time scales. The correlation value is not unity in the time scale less than about 1 hour. This result clearly shows that the value of a currency differs in different markets in the time scale less than an hour. This is a kind of self-contradiction of markets causing the occurrence of triangular arbitrage opportunity as shown in Fig. 5 [Y. Aiba].

5 Discussion

We have clarified the detail properties of correlation among foreign exchanges; the short time correlation is generally very small even between the pair of currencies showing large correlation in daily data. Within the time scale of a minute we can observe the direction of influence from one currency market to others. In very short time scale we can find contradiction of exchange rates between JPY/USD and
\( \{\text{EUR/USD}\} \times \{\text{JPY/EUR}\} \). For example, if you observe carefully the two rates, JPY/USD and \( \{\text{EUR/USD}\} \times \{\text{JPY/EUR}\} \), then you can buy Yen cheaper in one market and can sell it with higher rate in the other market even taking into account the effect of the spread (=Ask rate – Bid rate) that is about 0.05%. Although, no time lag is considered regarding the actual transactions, it is now clear how triangular arbitrage opportunity appears.

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