Heliosphere for a wide range of interstellar magnetic field strengths as a source of energetic neutral atoms

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ABSTRACT

Observations of the energetic neutral atoms (ENAs) of heliospheric origin by IBEX differ from expectations based on heliospheric models. It was proposed that the structure of the heliosphere may be similar to the "two-stream" model derived in 1961 by Parker for the case of strong interstellar magnetic field.

Using MHD simulations, we examine possible structure of the heliosphere for a wide range of interstellar magnetic field strengths, with different choices of interstellar medium and solar wind parameters. For the model heliospheres, we calculate the fluxes of ENAs created in the inner heliosheath, and compare with IBEX observations.

We find that the plasma flow in the model heliospheres for strong interstellar field (\(\sim 20 \, \mu G\)) has a "two-stream" structure, which remains visible down to \(\sim 5 \, \mu G\). The obtained ENA flux distribution show the features similar to the "split tail" effect observed by IBEX. In our model, the main cause of this effect is the two component (fast and slow) solar wind structure.

Subject headings: Sun: heliosphere — Sun: solar wind — ISM: magnetic fields — magnetohydrodynamics

1. Introduction

Energetic neutral atoms (ENAs) created in the distant heliosphere by energetic ion neutralization provide a means to remotely observe the distant regions of the heliosphere. Theoretical models of the large scale structure of the heliosphere are important for understanding and interpreting these observations.

The global models of the stellar wind interaction with the interstellar medium (ISM), leading to the formation of the astrospheres (the heliosphere in the case of the Sun) were first introduced in the classic work by Parker (1961). As shown by Parker, in the case of a star moving through unmagnetized interstellar plasma, the stellar wind flow, after passing the termination shock, turns ultimately in the direction opposite to the motion of the star, forming the "tail" (heliotail). This structure was indeed obtained in all models of the heliosphere based on numerical solutions of the gas dynamical or MHD equations.

Recently, another class of these models was included in the discussion. As again shown by Parker (1961), in the case of a star at rest with respect to the interstellar medium with strong magnetic field, the stellar wind may form, instead of a single astrotail, two oppositely directed streams...
or "drainage plumes", parallel and antiparallel to
the magnetic field direction. Although this model
is not strictly applicable to the case of the Sun
(which is known to move relative to the local inter-
stellar medium), a structure of this kind was tenta-
tively discussed in the context of the heliospheric
observations by IBEX and INCA (McComas et al.
2009; Krimigis et al. 2009).

In a recent study, McComas et al. (2013) fol-
lowed on the suggestion of Kivelson & Jia (2013)
and made this idea more explicit. McComas et al.
(2013) discovered that the ENA flux from the
region of the heliotail in the IBEX data shows
two regions shifted to opposite sides from the
downwind direction (anti-apex of the interstellar
medium flow), and suggested that the splitting
of the heliotail may be caused by the effect of a
strong interstellar magnetic field and subsonic in-
teraction (McComas et al. 2012). The structure
of the heliosphere would then be somewhere be-
tween the "two stream" case (corresponding to
the extremely strong magnetic field) and the more
conventional one-tail structure, corresponding to
the weak magnetic field. Kivelson & Jia (2013)
suggest also that, in the case of strong interstellar
field, ion heating by reconnection at the heliopause
may produce a new ENA source.

Among many numerical models of the helio-
sphere (astrosphere) that take the interstellar magnetic field into account, we know of only two
published models that approach close to the "two
stream" extreme field case. Florinski et al. (2004)
report a simulation for the case of interstellar
stream directed along the magnetic field, but they
found no stationary solution. For a similar case,
Pogorelov et al. (2011) found a solution with the
heliopause open in both interstellar upwind and
downwind directions. The case of very strong
magnetic field oblique to the interstellar flow was,
however, not considered. As a consequence, there
are currently no models of the heliosphere that
could be used to examine the possibility suggested
by McComas et al. (2013). Such models would
also be interesting in the case of astrospheres, for
which different combinations of the magnetic field
strength and the velocity of the star relative to
local interstellar medium may be encountered.

In this work we apply a 3D MHD code to obtain
global models of the heliosphere for a wide range
of the interstellar field strengths, from 2 µG to 20
µG. For the latter value, our models include the
analogue of the two-stream Parker model when the
star is at rest. We follow the evolution of the two-
stream structure when the magnetic field strength
decreases and its dependence on the neutral hy-
drogen background.

For these models, we calculate the directional
distribution of the ENA flux produced inside the
heliosphere, between the termination shock and
the heliopause. Our goal is to show how the global
structure of the heliosphere at different interstellar
field strengths would be reflected in the ENA ob-
servations. In particular, we find that the plasma
stream directions correspond to peaks in the ENA
flux distribution.

For moderate interstellar field strengths (2 -
5 µG) and two-component (slow + fast) solar
wind, our calculations produce the ENA flux de-
pictions reminiscent of the "split tail" effect ob-
served in the IBEX data (McComas et al. 2013;
Schwadron et al. 2014).

We use the following abbreviations: $B_{IS}$, $V_{IS}$,
$n_{IS}$, $n_H$ stand for the interstellar medium mag-
netic field strength, plasma speed, proton num-
ber density and neutral hydrogen number den-
sity, respectively. $V_{SW}$ denotes the solar wind
speed at the inner boundary and $n_{SW,1AU}$ the
solar wind electron density at 1 AU. The ENA flux
and plasma mass flux directional distributions are
presented as all-sky maps in solar ecliptic J2000
coordinates using Mollweide projection with the
ISM downwind (anti-apex) direction in the center
and the ISM upwind (apex) direction at both far
left and far right.

2. Models of the heliosphere

Our models of the heliosphere (Figs. 1-3) are
defined by time-stationary numerical solutions of
the (one-fluid) MHD equations for solar wind
plasma expanding into the magnetized interstellar
medium. The MHD code used in our calculation
was shown to be successful in treating the cases of
different interstellar magnetic field strengths and
orientations (Ratkiewicz et al. 1998).

We consider a range of values (2 to 20 µG) of
the interstellar magnetic field strength $B_{IS}$ (Table
1). Most of the remaining parameters are based on
observations. The direction of $B_{IS}$ (solar ecliptic
longitude 221°, latitude 39°) is close to the cen-
tre of the IBEX ribbon (Funsten et al. 2009). The direction and magnitude (longitude 79°, latitude −4.98°, \( V_{IS} = 23.2 \text{ km/s} \)) of the velocity of the interstellar flow are as given by McComas et al. (2012). Other interstellar and solar wind parameters \((n_{IS}, n_H, V_{SW}, n_{SW,1AU})\) are listed in Table 1. The solar wind at the inner boundary is taken either spherically symmetric or to consist of two components: the slow wind within 36° from the solar equator plane, and the fast wind elsewhere. The slow and fast components we take to have the same ram pressure (Le Chat, Issautier & Meyer-Vernet 2012). To avoid numerical reconnection, the solar magnetic field is neglected (set to zero at the inner boundary).

The neutral gas component is treated as a constant background. The value \( n_H \) of the order 0.2 cm\(^{-3}\) in the interstellar medium is presently favored (Izmodenov 2009; Bzowski et al. 2009; Zank et al. 2013). However, our MHD calculations assume \( n_H = \text{constant} \) everywhere. If this constant is equal to the interstellar value for \( n_H \), this overestimates the hydrogen density in the inner heliosphere, leading in particular to underestimation of the termination shock distance from the Sun. For this reason in most of our calculations we choose \( n_H = 0.1 \text{ cm}^{-3} \), in agreement with estimations of \( n_H \) at the termination shock (Bzowski et al. 2009) rather than 0.2 cm\(^{-3}\).

Numerical calculations are done on a spherical \((r,\theta,\phi)\) grid with logarithmic spacing for \( r \) and equal spacing for the angles. The directions of the undisturbed interstellar field \((B_{IS})\) and of the inflow velocity of the interstellar matter \((V_{IS})\) together with the position of the Sun define the \((x,y)\) plane. The angle \( \theta \) is counted from the apex direction of the interstellar matter inflow (-x axis) and the angle \( \phi \) from the y axis in the \((x,y)\) plane. The numbers of grid points \((n_r, n_\theta, n_\phi)\) equal \((348,90,180)\) for most calculations. For comparison, some calculations were done on smaller grids. The calculational domain lies between the inner boundary at \( r = 15 \text{ AU} \) (in some cases, 30 AU) and the outer boundary at \( r = 4500 \text{ AU} \).

3. Models of the energetic ion distribution and calculation of the ENA flux

We are interested in energetic neutral hydrogen atoms in the IBEX energy range \((\sim 0.7 \text{ to } \sim 4.3 \text{ keV})\) coming from the inner heliosheath (between the termination shock and the heliopause).

The main production mechanism is neutralization of the parent ions (energetic protons) by picking electrons from low energy neutral atoms entering the heliosphere from the interstellar medium. The ENAs from the IBEX ribbon (created presumably outside the heliosphere) are not considered.

Since Voyager 2 observations imply that the bulk plasma temperature downstream of the termination shock is low \((\sim 15 \text{ eV})\), we assume that most of the parent ions of the ENA derive from the pick-up protons created in the solar wind and further accelerated near the termination shock.

Our MHD simulations do not provide the pick-up ion distribution, so that it must be calculated separately.

First, inside the termination shock we replace the constant background model of the neutral hydrogen distribution used in the code by the distribution obtained by using a "hot model" approach (Thomas 1978). We take into account the ionization losses, but assume that the gravity is compensated by the radiation pressure.

The calculated neutral hydrogen distribution is used to derive the density of the pick-up protons arriving at the termination shock. An example of the result is shown in Fig. 5. The density is nonuniform, decreasing by a factor of \( \sim 2 \) towards the ISM anti-apex direction.

The distribution function of the energetic protons at the termination shock we describe by an analytical model (see the Appendix). The model satisfies the requirement (following from Voyager 2 observations) that most \((0.8)\) of the solar wind energy upstream of the shock is transferred to energetic particles instead of heating the bulk plasma. Figure 6 shows the example of the resulting spectrum on the opposite sides of the fast/slow wind boundary. The pick-up ion density in the fast wind region is lower by a factor of \( \sim 4-5 \) than in the slow wind.

The energetic proton flux in the region between the shock and the heliopause is then obtained by assuming that the particles are convected from the shock along plasma streamlines. Losses by neutralization due to charge exchange with ambient neutral hydrogen are taken into account. Also in-
cluded is the effect of adiabatic acceleration: the energy of the energetic particle (in the plasma frame) varies along a streamline.

The ENA flux $J_{ENA}$ arriving in the inner solar system we calculate by integrating the ENA production rate along the lines of sight corresponding to the grid directions:

$$J_{ENA} = \int ds J_{ion} \sigma_{cx} n_H$$

where the integral is over the distance $s$ along the line of sight (between the termination shock and the heliopause), $J_{ENA}$ and $J_{ion}$ the fluxes of the ENA and of the parent ions, respectively (both at the same energy and directed along the line of sight), $\sigma_{cx}$ the charge-exchange cross section at the ENA energy (we neglect the speed of the neutral H atoms from the background), and $n_H$ the number density of the neutral H background.

The characteristic distance for the neutralization loss for the energetic protons convected from the termination shock is given by $V/\beta_{cx}$, where $V$ is the plasma speed in the inner heliosheath and $\beta_{cx}$ the rate for charge-exchange between the energetic proton and the low energy hydrogen atom. In the IBEX energy range, $\beta_{cx}=0.7\pm1 \times 10^{-8}$ s$^{-1}$. In the region of the heliosheath not far from the termination shock, $V\approx 75$ km/s for our models with $V_{SW}=400$ km/s, and $V\approx 120$ km/s for the models with $B_{IS}=20$ $\mu$G which assume $V_{SW}=750$ km/s. The value of $V/\beta_{cx}$ is therefore 50-70 AU (400 km/s solar wind) or 80-110 AU (750 km/h solar wind). Most of the production of the ENA from the termination shock accelerated protons must therefore take place within this distance. Note, however, that the fast solar wind effects extend this range by increasing $V$.

The losses of the ENA on the way to the observation point are not included in the present calculations, but are small for the higher IBEX energies (Bzowski 2008; McComas et al. 2012).

4. Results: Structure of the model heliospheres

We concentrate on the aspects of the heliospheric structure which are most important for understanding the production of the ENA in the inner heliosheath: (1) the termination shock and the energetic ion distribution in the vicinity of the shock, (2) the structure of the plasma flow downstream from the shock (Figures 1, 2, 3), which determines the transport of the energetic ions from the termination shock region into the inner heliosheath, and (3) the shape and size of the heliopause.

In Table 1 we collected some results concerning the geometry of the heliosphere for our models: minimum ($r_{TS,min}$) and maximum ($r_{TS,max}$) distances to the termination shock, minimum distance ($r_{HP,min}$) to the heliopause, the maximum height ("height") of the heliosphere counted in the $z$ direction (perpendicular to the $(B_{IS},V_{IS})$ plane), and the width ("width") of the heliosphere along the straight line passing through the point of maximum height and perpendicular to the ISM inflow direction.

4.1. Plasma flow structure

For strong $B_{IS}$, the plasma flow between the termination shock and the heliopause has a two-stream structure. Figures 1 and 2 show the solar plasma streamlines for $B_{IS}=20$ $\mu$G starting at the termination shock in the symmetry plane of the solution (for spherically symmetric solar wind). For $V_{IS}=0$ (Figure 1), the streams are respectively parallel and antiparallel to $\vec{B}_{IS}$ as in Parker’s model (Parker 1961). When $V_{IS} \neq 0$ (Figure 2) the streams appear as bunches of almost parallel streamlines and include only a part of the flow. The streams are then deflected from the $\pm B_{IS}$ directions towards the direction of $V_{IS}$ and run approximately parallel to the "wings" of the heliopause (see next subsection).

One point to note is the effect of neutral hydrogen background on the plasma streams. For $n_H=0.1$ cm$^{-3}$ (close to the observed value), the momentum exchange between the stellar plasma and background hydrogen caused by charge-exchange interaction deflects and diffuses the streams (Fig. 2 second panel). For $n_H=0.01$ cm$^{-3}$ and smaller, the two streams are more prominent (Fig. 2 first panel) and would be recognizable even for the weaker field ($B_{IS}=5$ $\mu$G).

A quantitative representation of the two stream structure can be obtained by plotting the density of plasma streamlines. Figure 3 shows the directional distribution (projected onto the celestial sphere) of the density of solar plasma stream-
lines crossing the Sun-centered sphere of radius 300 AU for three of our models with spherically symmetric solar wind and different values of \( B_{IS} \). The streamlines start at the termination shock, with their initial points chosen to have the same directional distribution as the plasma mass flow. The streams appear as two separate density peaks near to (but shifted from) the directions parallel and antiparallel to the interstellar field. The two-stream structure is most prominent at strong \( B_{IS} \) (20 \( \mu \)G) but persists also for \( B_{IS} = 5 \mu \)G and 3 \( \mu \)G.

### 4.2. Shape of the heliopause

Asymmetric pressure of the interstellar magnetic field causes the heliosphere to expand in the \((B_{IS}, V_{IS})\) plane and contract in the perpendicular direction (Figure 4). This effect is well known from many numerical simulations.

For strong \( B_{IS} \) (20 \( \mu \)G and, if \( n_H \) is small, even for 5 \( \mu \)G) we find that the forward part of the heliopause in the \((B_{IS}, V_{IS})\) plane has the form of straight "wings" (Fig. 2). A similar shape was predicted using the Newtonian approximation (Fahr, Grzedzielski & Ratkiewicz-Landowska (1988), Czechowski & Grzedzielski (1998)). In particular, the angle \( \alpha \) between the "wings" and the x axis (the \( V_{IS} \) direction) was derived:

\[
\tan \alpha = \frac{\tan \gamma}{1 \pm V_{IS}/V_A \cos \gamma},
\]

where \( \gamma \) is the angle between the interstellar magnetic field and the x axis, and \( V_A \) is the Alfvén speed in the interstellar medium. Equation 2 is also known from the theory of Alfvén wings, which appear when a conducting body (like a satellite or a planetary magnetosphere) is moving through a magnetized plasma (Dell, Foley & Ruderman (1965); Neubauer (1980); Kivelson & Jia (2013); Saur et al. (2013)).

In Fig. 2, the directions of the Alfvén wings are shown by dashed-dotted lines. Although our results confirm the presence of straight "wings", the angle between each wing and the x axis is not in agreement with Eq. 2. Note that the interstellar plasma flow in our simulation is subsonic, so that the use of Newtonian approximation is not strictly justified.

### 4.3. Shape of the termination shock

For \( B_{IS} = 20 \mu \)G, and \( V_{IS} = 0 \) (Figure 1), we find that the shape of the termination shock is very close to a Sun-centered sphere, in agreement with Parker’s model (Parker 1961). The shock shape is weakly elongated along the magnetic field direction, but the difference between the maximum and minimum radius of the shock is only 8% of the maximum.

In the Parker model (Parker 1961), the spherical shock appeared as a consequence of assuming incompressible flow downstream. In our calculations, the plasma density downstream is not constant and the approximate symmetry of the shock must have a different explanation. A detailed study of this problem is beyond the scope of the present work.

For \( V_{IS} \neq 0 \), we find that the ratio between the Sun - termination shock distances in the ISM anti-apex (heliotail) and ISM apex (nose) directions goes down when the interstellar field strength increases. The shape of the shock is not much affected by the asymmetry of the solar wind, because of our assumption that the ram pressures of the fast and slow solar wind are equal.

### 5. Results: Angular distributions of the ENA flux

The results for directional distribution of the ENA flux calculated for the observer in the inner solar system are shown in Figures 7, 10 and 12. The projections used in our sky maps (Figs. 3, 7, 10, 12 and 14) are the same as in Fig. 6 of McComas et al. (2013) and in Fig. 10 of Schwadron et al. (2014), that is centered on the interstellar downwind direction (white dot in the middle). To help with orientation, the position of the IBEX ribbon is marked with the line of squares, and the interstellar upfield (\( B_{IS} \)) and downfield (\(-B_{IS})\) directions are marked by dots.

#### 5.1. ENA flux distributions for spherically symmetric solar wind.

Figure 4 shows our results for directional distributions of the 4.3 keV ENA flux for \( B_{IS} = 20 \mu \)G, 5 \( \mu \)G, and 3 \( \mu \)G assuming spherically symmetric solar wind. The distributions are symmetric with respect to the \((B_{IS}, V_{IS})\) plane, which is shown
as a thick black line. For the case of \( B_{IS} = 20 \) \( \mu \text{G} \) and \( B_{IS} = 5 \) \( \mu \text{G} \), the ENA flux has two peaks at directions approximately parallel and antiparallel to the interstellar field, corresponding to the two streams of the solar plasma. For \( B_{IS} = 3 \) \( \mu \text{G} \), these peaks almost completely disappear.

The reason why (for \( B_{IS} = 20 \) and \( 5\mu \text{G} \)) the two solar plasma streams are associated with high ENA flux intensity can be explained as follows (see Figs. 3 and 4). The lines-of-sight directed along the streams are close to parallel to the local plasma flow (Fig. 3), so that Eq. (30) is applicable. Since \( \beta_c L/V \gg 1 \) in the stream regions, it follows that \( J_{ENA} \approx (V/v)J_0 \). This relation agrees well with our results for the case of \( B_{IS} = 5 \) \( \mu \text{G} \) without adiabatic acceleration.

The same argument can be applied to the lines-of-sight in the heliotal. We find that the ENA flux from these directions is lower than for the streams because \( V \) and \( J_0 \) are lower for the case of the heliotal.

For \( B_{IS} = 5 \) and \( 3 \) \( \mu \text{G} \), a broad maximum of the ENA flux appears in the "nose" (ISM apex) region (Fig. 7, panel 2 and 3). This is caused by the adiabatic acceleration of the energetic protons downstream from the shock and disappears if the adiabatic acceleration is switched off (Fig. 7, panel 4 and 5).

Eq. (14) provides the estimation for the ENA flux from the "nose" direction in the absence of adiabatic acceleration. For \( B_{IS} = 5 \) \( \mu \text{G} \), the parameter \( \beta_c L/V_0 \approx 1/2 \), implying that \( J_{ENA} \approx (1/3)(V_0/v)J_0 \). This agrees well with our numerical result when the adiabatic acceleration is disregarded (\( J_0 = 110 \) cm\(^{-2}\)s\(^{-1}\)sr\(^{-1}\)keV\(^{-1}\), \( J_{ENA} = 3.8 \) cm\(^{-2}\)s\(^{-1}\)sr\(^{-1}\)keV\(^{-1}\)).

5.2. ENA flux distributions for asymmetric solar wind: The "split tail" for high energy ENA flux

The first three years of IBEX data refer to the extended solar minimum period, when the solar wind asymmetry (two-component slow + fast structure) was prominent. The results of our calculations for the case of asymmetric solar wind are collected in Figures 11, 12 and 10.

For very strong field (\( B_{IS} = 15 \) \( \mu \text{G} \), Fig. 11), the peaks from the two plasma streams are prominent.

For \( B_{IS} = 5 \) \( \mu \text{G} \) and less, the directional distribution of the ENA flux following from our models reflects primarily the two-component solar wind structure. Upstream from the shock, the slow wind is in our models restricted to the region between \( \pm 36^\circ \) solar latitude (red lines in Figs. 10 and 12). Figure 10 shows that, at low ENA energy (1.1 keV) this region corresponds to high ENA intensity belt. At high ENA energy (4.3 keV, Fig. 12) there is a similar belt, but of low ENA intensity.

This structure can be understood by observing that the ENA coming from directions within the \( \pm 36^\circ \) solar latitude derive mostly from the energetic protons carried by the shocked slow solar wind plasma. The density of \( \sim 1 \) keV energetic protons is much higher in the shocked slow wind than in the shocked fast wind (Fig. 9). This explains the high ENA intensity belt at low (\( \sim 1 \) keV) energy. On the other hand, the energetic proton flux at \( \sim 4.3 \) keV is approximately the same for the slow and fast solar wind plasma, because the high average energy of the energetic protons in the fast wind is compensated by the high density of the slow wind (Fig. 5). The reason why the 4.3 keV ENA flux is higher in the region outside the slow wind belt (Fig. 12) is that the convection time from the termination shock (and consequently the neutralization loss) for the energetic protons in the fast wind plasma is lower than in the slow wind.

For \( B_{IS} 2-5 \) \( \mu \text{G} \) (Fig. 12), we find that the low intensity belt for 4.3 keV ENA narrows down near the ISM downwind direction. This structure is very much alike to the "split tail" observed by IBEX (McComas et al. (2013), Schwadron et al. (2014)). The low ENA flux region around the ISM downwind direction is almost split into two parts (the "port" and the "starboard" lobes: McComas et al. (2013)). We find that this effect is caused by the streams of fast wind plasma entering the heliotal region. Figure 12 (second panel) shows the outline of the heliopause (thick black oval) at the distance 250 AU from the Sun, together with the outlines of the two regions in the heliotal filled by the plasma originating from the fast solar wind (thin black ovals). The flux of \( \sim 4 \) keV ENA from these fast wind plasma streams is higher than from the slow wind plasma region between them. Since the fast wind streams at large distances move to lower heliolatitudes, the two high ENA flux regions associated with them intrude into the \( \pm 36^\circ \) heliolatitude belt and partly
split the low ENA flux region.

The low 4.3 keV ENA flux region obtained from our calculations (Fig. 12) is tilted relative to ecliptic. This tilt is not dependent on $B_{IS}$ value (see Fig. 12). On the other hand, it is close to the 7.25° tilt of the solar equator (and of the slow/fast solar wind boundary: see the red lines in the figures). A similar tilt appears in the IBEX data (McComas et al., 2013, Fig. 6). It may be concluded that the main cause of the ”split tail” structure is the distribution of the slow and fast solar wind streams.

Our results concerning the ”split tail” structure remain valid when the effect of adiabatic acceleration is switched off (Fig. 12 two lower panels). This is important, because we expect that adiabatic acceleration is overestimated by our models. On the other hand, adiabatic acceleration of the energetic protons affects the production of ENA from the region near the apex of the ISM (the nose region: Fig. 12 two upper panels).

### 5.3. Comparison with IBEX

In addition to the ”split tail” structure, our results show other points of qualitative similarity with IBEX. At low energy, the ENA flux in our calculations have a peak near the tail direction (Fig. 10) which, at higher energy, evolves into a depletion (Fig. 12). A similar behaviour can be seen in the IBEX data (Schwadron et al., 2014, Fig. 10).

Compared to the ENA fluxes observed by IBEX, our calculations produce lower ENA intensity. The magnitude of the ENA flux at 4.3 keV obtained in our models is lower than the non-ribbon flux observed by IBEX by about a factor of 2 (Fig. 6 in McComas et al., 2013, Fig. 10 in Schwadron et al., 2014). For ≈1.1 keV this factor is about 4.

In Figure 14 we show our results for the slope of the ENA flux spectrum for the case of 5 $\mu$G field, asymmetric solar wind. We plot the values of $\gamma$, defined as $\gamma = - \log(J_{ENA}(E_1)/J_{ENA}(E_2))/ \log(E_1/E_2)$ for $(E_1,E_2)$=(1.1 keV,1.7 keV) and (2.7 keV, 4.3 keV). The values of the spectral index obtained in our model are lower than observed by IBEX (McComas et al., 2013, Schwadron et al., 2014), suggesting that our model underestimates the ENA flux at low energy compared to the high energy flux. On the other hand, we find that the high $\gamma$ region is similar to the ”split tail” structure in the ENA flux distribution, in agreement with IBEX observations (McComas et al., 2013, Fig. 5; Schwadron et al., 2014, Fig. 11, top panel).

### 6. Summary and conclusions

We present a set of 3D time-stationary models of the heliosphere based on numerical MHD solutions for a wide range of interstellar magnetic field strength (2-20 $\mu$G).

For the ISMF of 20 $\mu$G and the star at rest, our result is analogous to a well known Parker solution, with a spherical termination shock, and the atmosphere elongated along the $B_{IS}$ direction.

For strong interstellar field and $V_{IS} \neq 0$, we show that the plasma flow inside the heliosphere is concentrated in two streams directed close (though not exactly parallel or antiparallel) to the interstellar field direction. As a result, the forward part of the heliopause forms straight ”wings”. A similar shape was predicted using the Newtonian approximation.

When the field strength decreases to realistic values (2-5 $\mu$G), the two-stream structure becomes less prominent. This structure depends on the interstellar neutral hydrogen background: the weaker $n_H$, the more prominent the streams.

For the simulated heliospheres we calculated the distribution of the energetic protons (originating from the pick-up protons from the solar wind) and the energetic neutral atom fluxes produced by neutralization of these protons. We assumed a simple model of the energetic proton distribution at the termination shock. The results were compared with the ENA observations by IBEX. Our simulations are restricted to the ENA created inside the heliosphere, so that the ”ribbon” contribution must be excluded from the IBEX data.

We find that the structure of the ENA flux distribution at higher IBEX energy (1.7 keV and higher) in the downwind hemisphere is similar to the ”split tail” structure observed by IBEX (two low ENA flux ”lobes” shifted relative to the downwind direction). This result persists for different values (2-5 $\mu$G) of the interstellar magnetic field strength. The explanation suggested by our model is that this effect follows from the two component (fast + slow) solar wind structure, prevalent near
the low activity parts of the solar cycle. The tilt of the observed ENA flux structure is explained by the tilt of the slow/fast solar wind boundary relative to the ecliptic plane.

For spherically symmetric solar wind, as observed near solar maxima, the ENA distribution in our time-stationary model has a different form. In reality, because of long time delay (average $\sim 3$ years) before neutralization of the parent proton, we expect that the ENA distribution near solar maximum would still show some effects of the two-component solar wind.

For very strong $B_{IS}$, the ENA distributions have two prominent peaks corresponding to two plasma streams.

There are also peaks of the ENA flux that appear in the regions near the nose of the heliosphere. However, these effects disappear when adiabatic acceleration of energetic protons is neglected. Adiabatic acceleration depends on details of plasma flow and density distribution and is sensitive to numerical effects. The "split tail" structure and the two stream-related ENA peaks are not dependent on adiabatic acceleration.

Finally, we note that the recent time-dependent simulation (Zirnstein et al. 2015) found the ENA flux distribution from the heliosphere with little periodic change over the solar cycle. Their approach was different from ours. In particular, the pick-up protons were not treated separately, but assumed to form a fixed fraction of the bulk plasma.

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A. Energetic ion distribution

The method is similar to that used in Czechowski et al. (Czechowski, Hilchenbach & Hsieh 2012). The velocity distribution of energetic protons \( f(\vec{r}, v) \) is assumed to be isotropic in plasma frame. It is obtained by solving the time-stationary transport equation

\[
-\vec{V} \cdot \nabla - \frac{1}{3}(\nabla \cdot \vec{V}) \frac{\partial}{\partial \log v} f = 0 \tag{A1}
\]

where \( \vec{V} \) is the plasma velocity, \( v \) the particle velocity and \( \beta_{cx} \) is the rate for charge exchange between the energetic protons and low energy neutral hydrogen. The equation includes the effects of convection, adiabatic energy changes and neutralization loss. We assume that the proton energy is low enough to neglect spatial diffusion.

Equation (A1) together with the boundary condition at the termination shock determine the \( f(\vec{r}, v) \) along a plasma streamline \( \vec{r} = \vec{r}(s) \) parametrized by the length \( s \):

\[
f(\vec{r}(s), v(s)) = f_{\text{shock}}(\vec{r}_0, v_0) \exp \left[ - \int_0^s ds' \frac{\beta_{cx}(s')}{V(\vec{r}(s'))} \right] \tag{A2}
\]

where \( \vec{r}(s) \) and \( v(s) \) satisfy the equations \( Vd\vec{r}/ds = \vec{V}(s) \) and \( Vd\log v/\!ds = -(1/3)\nabla \cdot \vec{V} \), with the initial conditions at the termination shock \( \vec{r}(0) = \vec{r}_0 \), \( v(0) = v_0 \), respectively. If the plasma mass flow is conserved, the equation for \( v(s) \) implies \( v(s)\rho(s)^{-1/3} = \text{const.} \) Since determination of \( \nabla \cdot \vec{V} \) from numerical output is imprecise, we use the above relation to determine \( v(s) \).

We use a simple analytical model for the energetic proton distribution at the termination shock \( f_{\text{shock}}(\vec{r}, v) \) in the plasma frame

\[
f_{\text{shock}}(\vec{r}, v) = n_{\text{PU}}(\vec{r})F_{\kappa}(v, w) + n_{\text{SW}}(\vec{r})F_G(v, v_T) \tag{A3}
\]

The first term is the distribution of the accelerated pick-up protons (number density \( n_{\text{PU}} \)), and the second describes the protons from the shock-heated bulk solar wind (number density \( n_{\text{SW}} \)). \( F_{\kappa}(v, w) \) is the kappa function

\[
F_{\kappa}(v, w) = \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2) (\pi kw^2)^{3/2}} \left( 1 + \frac{v^2}{kw^2} \right)^{-(\kappa+1)} \tag{A4}
\]

We choose \( \kappa=1.65 \). The "thermal speed" parameter \( w \) we determine from the requirement that the total pick-up proton energy should be equal to 0.8 of the energy of the solar wind upstream from the shock, as observed by Voyager 2 (Richardson et al. 2008). The bulk solar wind protons are described by the Maxwellian distribution \( F_G(v, v_T) = \exp - (v/v_T)^2 / \pi^{3/2} v_T^3 \), with the thermal speed \( v_T \) equal to one half of the solar wind speed downstream. The number density \( n_{\text{SW}} \) is taken from the MHD model.

B. ENA flux estimations

Assume that the plasma flow is incompressible, so that Eq. (A1) becomes

\[
\vec{V} \cdot \nabla f = -\beta_{cx} f \tag{B1}
\]

The same equation holds for the ion flux \( J_{\text{ion}} \).

Consider first the ENA flux from the direction towards the nose of the heliosphere, along the stagnation line. Let the plasma velocity decrease linearly towards the heliopause: \( V = V_0(1 - z/L) \) where \( z \) is the distance from the shock along the stagnation line and \( L \) the distance from the shock to the heliopause.

The equation for the ion flux is

\[
\frac{dJ_{\text{ion}}}{dz} = \frac{\beta_{cx}}{V_0(1 - z/L)} J_{\text{ion}} \tag{B2}
\]
with the solution

$$J_{\text{ion}}(z) = J_0 \left(1 - \frac{z}{L}\right)^{\beta_{cx}L/V_0}$$  \hspace{1cm} (B3)

The ENA flux is then given by (see Eq. 1)

$$J_{\text{ENA}} = \frac{\beta_{cx}L/V_0}{1 + \beta_{cx}L/V_0 \frac{V_0}{v}} J_0$$  \hspace{1cm} (B4)

where $v$ is the particle speed and we have used $\beta_{cx} = \sigma_{cx} v n_H$.

Consider next the line-of-sight within one of the plasma streams assuming that the plasma speed $V$ is constant and parallel to the line-of-sight. The solution for the ion flux is then

$$J_{\text{ion}}(z) = J_0 \exp\left(-z \frac{\beta_{cx}}{V}\right)$$  \hspace{1cm} (B5)

and for the ENA flux

$$J_{\text{ENA}} = \frac{V_0}{v} J_0 \left(1 - \exp\left(- \frac{\beta_{cx}L}{V}\right)\right)$$  \hspace{1cm} (B6)

When the distance to the heliopause along the line-of-sight is large compared to $V/\beta_{cx}$, the ENA flux becomes $J_{\text{ENA}} = (V/v) J_0$. 

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Fig. 1.— Flow lines of the solar plasma for the MHD solution corresponding to the hypothetical heliosphere with the Sun at rest \((V_{IS}=0)\) relative to the interstellar medium with plasma density \(n_{IS}=0.04\ \text{cm}^{-3}\), neutral hydrogen density \(n_H=0.1\ \text{cm}^{-3}\) and very strong interstellar magnetic field \((B_{IS}=20\ \mu\text{G})\). The solar wind is spherically symmetric with \(V_{SW}=750\ \text{km/s}, n_{SW,1\text{AU}}=4.2\ \text{cm}^{-3}\). The solution is similar to the Parker model (Parker 1961). The heliopause and the termination shock are shown by thick lines.

Fig. 2.— Plasma flow lines in the \((x,y)\equiv(B_{IS},V_{IS})\) plane obtained from MHD simulations for \(B_{IS}=20\ \mu\text{G}, V_{IS}=23.2\ \text{km/s}\) and the solar wind with \(V_{SW}=750\ \text{km/s}, n_{SW,1\text{AU}}=4.2\ \text{cm}^{-3}\). The first figure corresponds to \(n_H=0.01\ \text{cm}^{-3}\) and \(n_{IS}=0.04\ \text{cm}^{-3}\), and the second to \(n_H=0.1\ \text{cm}^{-3}\) and \(n_{IS}=0.06\ \text{cm}^{-3}\). The solar wind is spherically symmetric, so that the \((x,y)\) plane is the symmetry plane. The heliopause is shown by the thick line. The dotted line shows the \(B_{IS}\) direction. The dashed lines show the Alfven wings (see Eq. 2).
Fig. 3.— Directional distribution of the solar plasma mass flux (obtained from streamline density) in units $10^4 m_p \ cm^{-2} \ s^{-1}$ at a distance 300 AU from the Sun for the cases of $B_{IS} = 20 \ \mu G$ (the first panel) 5 $\mu G$ and 3 $\mu G$. The values of $V_{IS} = 23.2 \ \text{km/s}$, $n_H = 0.1 \ \text{cm}^{-3}$, $n_{IS} = 0.06 \ \text{cm}^{-3}$ are the same for all cases. The solar wind is spherically symmetric, with $V_{SW} = 750 \ \text{km/s}$, $n_{SW,1AU} = 4.2 \ \text{cm}^{-3}$ (upper panel) and $V_{SW} = 400 \ \text{km/s}$, $n_{SW,1AU} = 5.55 \ \text{cm}^{-3}$ for the remaining cases. The projection on the celestial sphere is the same as used for the ENA distributions (Figs. 12, 10 and 7). It is centered on the anti-apex direction of the ISM flow (white circle). The interstellar field and anti-field directions are marked by black circles.

Fig. 4.— Plasma velocity distributions in the (x,y) plane (first panel) and the (x,z) plane (second panel) for $B_{IS} = 5 \ \mu G$, $n_H = 0.1 \ \text{cm}^{-3}$, $V_{IS} = 23.2 \ \text{km/s}$, $n_{IS} = 0.06 \ \text{cm}^{-3}$. The solar wind is asymmetric (fast/slow) with $V_{SW} = 750/400 \ \text{km/s}$, $n_{SW,1AU} = 1.58/5.55 \ \text{cm}^{-3}$. The outlines of the heliopause and of the termination shock are also shown. Note the flattening of the heliosphere by the asymmetric pressure of the interstellar magnetic field.
Fig. 5.— The pick-up proton density at the termination shock as a function of angle $\theta$ (counted from the inflow direction of the interstellar medium) for two choices of the angle $\phi$ corresponding to maximum (solid line) and minimum (dashed line) densities for intermediate $\theta$. The cases shown are: $B_{IS}=5 \mu G$, symmetric solar wind ($V_{SW}=400$ km/s, $n_{SW,1AU}=5.55$ cm$^{-3}$, the upper panel) and $B_{IS}=5 \mu G$, asymmetric (fast/slow) solar wind ($V_{SW}=750/400$ km/s, $n_{SW,1AU}=1.58/5.55$ cm$^{-3}$, lower panel). The low pick-up proton density region in the lower figure (dashed line) corresponds to the fast wind. The distributions have a similar form for other values of $B_{IS}$, provided that $V_{IS}$ is the same (23.2 km/s).

Fig. 6.— The proton flux as a function of energy at two selected locations on the termination shock, corresponding to slow (solid line) and fast (dashed line) solar wind, respectively. The case of $B_{IS}=5 \mu G$, asymmetric (fast/slow) solar wind ($V_{SW}=750/400$ km/s, $n_{SW,1AU}=1.58/5.55$ cm$^{-3}$) is illustrated. The flux of high energy protons up to $\sim 10$ keV is higher in the slow solar wind region because of high slow solar wind density.
Fig. 7.— ENA flux distribution (4.3 keV) in units \( \text{cm}^{-2} \text{s sr keV}^{-1} \) for spherically symmetric solar wind. The cases shown (from left to right and from top to bottom) are: \( B_{IS}=20 \ \mu \text{G}, 5 \ \mu \text{G}, 3 \ \mu \text{G}, 5 \ \mu \text{G} \) and \( 3 \ \mu \text{G} \), the two last cases corresponding to adiabatic acceleration switched off. The ENA flux for \( B_{IS}=20 \ \mu \text{G} \) and \( 5 \ \mu \text{G} \) has two peaks corresponding to two streams of the plasma flow. These streams effectively disappear for \( 3 \ \mu \text{G} \). The ENA flux from the ISM apex region is affected by adiabatic acceleration. The values of \( V_{IS}=23.2 \ \text{km/s} \), \( n_{IS}=0.06 \ \text{cm}^{-3} \), \( n_{H}=0.1 \ \text{cm}^{-3} \), \( V_{SW}=400 \ \text{km/s} \), \( n_{SW,1AU}=5.55 \ \text{cm}^{-3} \) are the same for all figures.

Fig. 8.— Streamlines ending at selected lines-of-sight within 250 AU from the Sun. Two of the lines-of-sight correspond to peaks of the ENA flux associated with two plasma streams. In addition, one line-of-sight is in the forward part of the heliosphere (the "nose" region) and one is close to the ISM anti-apex direction (heliotail). The case illustrated corresponds to the 1st panel in Fig. 7 (\( B_{IS}=20 \ \mu \text{G}, \text{symmetric solar wind} \)).
Fig. 9.— Profiles of the ENA production rate (4.3 keV) along 5 different directions corresponding to: ENA flux maximum in the south hemisphere; ENA flux maximum in the north hemisphere; vicinity of the ISM anti-apex; the midpoint between the two maxima of the flux. The case illustrated corresponds to the 4th panel in Fig. 7 ($B_{IS}=5 \mu G$, symmetric solar wind, adiabatic acceleration switched off).

Fig. 10.— ENA flux distribution (1.1 keV) in units (cm$^2$ s sr keV)$^{-1}$ for $B_{IS}=5 \mu G$ (first panel) and $B_{IS}=3 \mu G$ (second panel), for the case of asymmetric (slow+fast) solar wind, with slow wind contained within $\pm 36^\circ$ from the equator. The values of $V_{IS}=23.2$ km/s, $n_{IS}=0.06$ cm$^{-3}$, $n_{H}=0.1$ cm$^{-3}$, $V_{SW}=750/400$ km/s (fast/slow), $n_{SW,1AU}=1.58/5.55$ cm$^{-3}$ (fast/slow) are the same for both cases.

Fig. 11.— ENA flux distribution (4.3 keV) in units (cm$^2$ s sr keV)$^{-1}$ for $B_{IS}=15 \mu G$. 

Fig. 12.— ENA flux distribution (4.3 keV) in units (cm$^2$ s sr keV)$^{-1}$ for $B_{IS}$ 5 and 3 $\mu$G. The solar wind is asymmetric (fast/slow) with the slow wind contained within $\pm 36^\circ$ from the equator. The thin red lines are the projections of the solar equator and of the boundaries between the slow and fast solar wind. The second figure includes the outline of the heliopause and of the regions filled by the shocked fast solar wind plasma at the distance 250 AU from the Sun. The last two figures show the results ($B_{IS}$ = 5 and 3 $\mu$G) with adiabatic acceleration switched off. The values of $V_{IS}$=23.2 km/s, $n_{IS}$=0.06 cm$^{-3}$, $n_H$=0.1 cm$^{-3}$, $V_{SW}$=750/400 km/s (fast/slow), $n_{SW,1AU}$=1.58/5.55 cm$^{-3}$ (fast/slow) are the same for all figures.

Fig. 13.— Profiles of the ENA production rate (4.3 keV) along 4 different directions corresponding to: the port lobe; the starboard lobe; the "pinch" from the north, and the "pinch" from the south. The case illustrated corresponds to the lowest panel in Fig. 12 ($B_{IS}$=3 $\mu$G, asymmetric solar wind, adiabatic acceleration switched off).

Fig. 14.— Spectral index distributions in the 1.1-1.7 keV (first panel) and in the 2.7-4.3 keV (second panel) energy ranges for numerical solution with $B_{IS}$=5 $\mu$G, asymmetric (fast+slow) solar wind, $n_H$=0.1 cm$^{-3}$, $V_{IS}$=23.2 km/s, $n_{IS}$=0.06 cm$^{-3}$, $V_{SW}$=750/400 km/s (fast/slow), $n_{SW,1AU}$=1.58/5.55 cm$^{-3}$ (fast/slow). The projection is the same as in Figs. 5 and 6 of McComas et al. (2013).
Table 1: Parameters and results of MHD calculations

| $B_{IS}$ ($\mu$G) | $n_{IS}$ (cm$^{-3}$) | $n_{H}$ (cm$^{-3}$) | $V_{IS}$ (km/s) | $V_{SW}$ (km/s) | $n_{SW,1AU}$ (cm$^{-3}$) | $r_{TS, min}$ (AU) | $r_{TS, max}$ (AU) | $r_{HP, min}$ (AU) | height (AU) | width (AU) |
|-------------------|----------------------|----------------------|-----------------|-----------------|--------------------------|-------------------|-------------------|-------------------|-------------|-----------|
| 20                | 0.04                 | 0.04                 | 0               | 750             | 4.2                      | 36                | 40                | 95                |             |           |
| 20                | 0.06                 | 0.06                 | 0.1             | 23.2            | 750                      | 4.2               | 36                | 43                | 57          | 140        | 749       |
| 20                | 0.06                 | 0.01                 | 0.01            | 23.2            | 750                      | 4.2               | 35                | 41                | 65          | 154        | 1840      |
| 15                | 0.06                 | 0.06                 | 0.1             | 23.2            | 400                      | 5.55              | 31                | 38                | 47          | 113        | 444       |
| 15                | 0.06                 | 0.001                | 0.001           | 23.2            | 400                      | 5.55/5.55         | 31                | 39                | 45          | 130        | 483       |
| 5                 | 0.06                 | 0.06                 | 0.1             | 23.2            | 400/400                  | 5.55/5.55         | 62                | 92                | 93          | 315        | 693       |
| 5                 | 0.06                 | 0.06                 | 0.001           | 23.2            | 400                      | 5.55              | 75                | 109               | 109         | 317        | 1827      |
| 5                 | 0.06                 | 0.06                 | 0.1             | 23.2            | 400/400                  | 1.58/5.55         | 62                | 92                | 93          | 315        | 693       |
| 3                 | 0.06                 | 0.06                 | 0.1             | 23.2            | 400                      | 5.55              | 75                | 122               | 115         | 384        | 643       |
| 2                 | 0.06                 | 0.06                 | 0.1             | 23.2            | 400                      | 5.55/5.55         | 74                | 120               | 111         | 424        | 719       |
| 2                 | 0.06                 | 0.06                 | 0.1             | 23.2            | 750/400                  | 1.58/5.55         | 74                | 120               | 111         | 424        | 719       |