Output Series-Parallel Connection of Passivity-Based Controlled DC-DC Converters: Theoretical Generalization of Stability

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Abstract—The series-parallel method of dc-dc converters is utilized in various domains of electrical engineering for improved power conversion. Previous studies have proposed and classified the control schemes for the series-parallel converters. However, they have several restrictions and a lack of diversity. The purpose of this paper is to propose passivity-based control (PBC) for the series-parallel connection of dc-dc converters. First, PBC for the dc-dc converters is generally explained. Then, it is theoretically shown that the output series-parallel converters regulated by PBC are asymptotically stable. The series-parallel converters are numerically simulated to confirm asymptotic stability. In the simulation, PBC maintained the stability of the series-parallel converters with diverse circuit topologies, parameters, and steady-states, due to its robust feature. The result of this paper will contribute to allowing various dc-dc converters to have a wide variety of output connections.

Index Terms—Series-parallel converters, dc-dc converters, passivity-based control, asymptotic stability, hybrid power systems.

I. INTRODUCTION

The series-parallel connection of multiple dc-dc converters is a technique utilized in various fields of electrical engineering [1]–[9]. It offers several advantages over a single, high-power, centralized converter such as lower component stresses, increased reliability, ease of maintenance, and improved thermal management [1]–[4]. In previous research, this technique has achieved higher efficiency in photovoltaic (PV) systems [4]–[6], the load sharing design of high-power converters [1]–[3], forming a hybrid power system by combining the distributed power sources [7]–[9], etc. The series-parallel technique of dc-dc converters is vital for improved power conversion and the combined use of power sources.

References [1], [2] have classified the load sharing methods for the paralleled converters. Similarly, series-parallel converters were investigated in [3]. However, these considerations were explicitly aimed to obtain equal load sharing among specific converters. Thus, they are incapable of being applied to the systems which require the connection of various converters or arbitrary load sharing, such as PV systems or hybrid power systems. These systems typically have diverse operating modes depending on the power flow of each converter, hence their control strategies must be designed to cover the wide range of operation. For example, bifurcations and instabilities of a PV-battery hybrid power system are analyzed in [7]. It is necessary to construct a control scheme that widely covers the circuit topologies and the operating range of the dc-dc converters for the diverse series-parallel connection.

There are two categories for the control of dc-dc converters; linear and nonlinear. Previous researches in this area, including the above-mentioned references, have primarily been based on linear control. However, the switched power converters are inherently nonlinear systems [10], [11]. Besides, series-parallel converters have mutual interactions, in contrast to a single dc-dc converter. Obviously, linear controllers are unlikely to give robust solutions and optimum performance for the series-parallel converters [12]. In this study, we adopt passivity-based control (PBC) [13]–[17], which is a class of a nonlinear control scheme, for the regulation of the series-parallel converters. It aims to achieve the asymptotic stability of passive systems by adding some damping to the system’s storage function. Since PBC is based on the physical properties of the system, it gives simple and robust control rules [13]–[15].

A system composed of passive subsystems is a passive system [18]. Series-parallel converters consist of dc-dc converters which are inherently passive systems. Hence, it is expected from the perspective of energy that the series-parallel converters are asymptotically stable by applying PBC to each converter. In the previous research, the regulation of Ćuk converters connected in parallel by PBC is discussed [19]. Recently, PBC of paralleled boost and buck converters was experimentally verified in [20]. The application of PBC to...
the hybrid power systems is considered in [21], [22]. Yet, these considerations are limited to specific circuit configurations. Generalized results for the series-parallel converters and their PBC have not been presented.

From this viewpoint, we theoretically discuss the asymptotic stability of the output series-parallel dc-dc converters regulated by PBC. The output series-parallel connection of dc-dc converters is shown in Fig. 1. This circuit configuration describes systems such as PV systems and hybrid power systems that combine several power sources by the connection of the converters. Throughout the paper, the output series-parallel converters are shown to be asymptotically stable with the implementation of PBC to each converter. The stability is confirmed from the perspectives of energy and Lyapunov stability theory. The results suggest that PBC can allow various types of dc-dc converters connected in series-parallel at the output. This feature is verified based on the numerical simulation for the series-parallel converters.

The paper is organized as follows. In section II, we first introduce the general system model of the switched dc-dc converters. Then, the PBC of the dc-dc converters is explained. Some examples are given to show that the model and the PBC can be applied to various types of dc-dc converters.

In section III, a variable representing the mutual interaction is defined to generalize the output series-parallel connection. The application of PBC to each converter is shown to stabilize the series-parallel converters theoretically. The results of the numerical simulation are shown in section IV to confirm the theoretical arguments. Finally, section V is the conclusion.

II. PASSIVITY-BASED CONTROL OF DC-DC CONVERTERS

In this section, a general model for the dc-dc converters is first introduced. Then, PBC for the dc-dc converters is explained in detail. The control rules for boost, buck, and buck-boost converters are derived as examples. The circuit elements mentioned in this section are assumed ideal.

A. DC-DC Converter Model

As a general model for the dc-dc converters,

$$ A\dot{x} = \{J(s) - R\} x + g(s) u, \quad (1) $$

is assumed. The dot (') is a notation for a time differentiation. The symbols in Eq. (1) are denoted in Table I where $n, m, l \in \mathbb{N}$ are natural numbers. This model is essentially a port-controlled Hamiltonian system [23] with minor modifications. Moreover, Eq. (1) is an averaged model [14]–[16], [24] based on the assumption that the switches have sufficiently fast switching frequency. Thus, the model is approximated to be a smooth system controlled by the duty ratios $s \in [0, 1]^l$. We can confirm that Eq. (1) describes a variety of dc-dc converters from [16].

The steady-state equation i.e. the null dynamics of Eq. (1) is obtained as

$$ \{J(s) - R\} x + g(s) u = 0, \quad (2) $$

by setting the differential terms to zero. The desired state $x = x_d$ and the corresponding duty ratios $s = s_d$ are determined as the constants to satisfy Eq. (2). Hereafter, the subscript ‘d’ indicates the desired value. The control aims to achieve the asymptotic stability at $x = x_d$ by modifying the duty ratios $s$ at the transient state.

The error $\tilde{x} = x - x_d$ satisfies

$$ A\dot{\tilde{x}} = \{J(s) - R\} \tilde{x} + \tilde{g}(s) \tilde{u}, \quad (3) $$

on Eq. (1) and (2). The input structure $\tilde{g}$ and input $\tilde{u}$ are defined as

$$ \tilde{g}(s) = [J(s) - R \ g(s)], \ \tilde{u} = \begin{bmatrix} x_d \\ u \end{bmatrix}. \quad (4) $$

Then, the system is interpreted as a port network shown in Fig. 2. The system description of Eq. (3) simplifies the control task to achieve the asymptotic stability at $\tilde{x} = 0$. Thus, we consider PBC in the following discussions by Eq. (3).

B. Passivity-Based Control

The storage function of Eq. (3) is defined as

$$ \mathcal{H} = \frac{1}{2} x^T A\dot{x}. \quad (5) $$

Eq. (3) has its minimum at $\tilde{x} = 0$. The time derivative of the storage function Eq. (5) is

$$ \dot{\mathcal{H}} = \tilde{x}^T A\dot{\tilde{x}} = -\tilde{x}^T R \tilde{x} + \tilde{x}^T \tilde{g}(s) \tilde{u}. \quad (6) $$

The first term on the right hand side $-\tilde{x}^T R \tilde{x}$ is the dissipation of the system, which is negative at $\tilde{x} \neq 0$. On the other hand, the second term $\tilde{x}^T \tilde{g}(s) \tilde{u}$ corresponds to the power supplied from the externals.

When the externally supplied power satisfies

$$ \tilde{x}^T \tilde{g}(s) \tilde{u} < 0, \ \tilde{x} \neq 0, \quad (7) $$

the hybrid power systems is considered in [21], [22]. Yet, these considerations are limited to specific circuit configurations. Generalized results for the series-parallel converters and their PBC have not been presented.

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When the externally supplied power satisfies

$$ \tilde{x}^T \tilde{g}(s) \tilde{u} < 0, \ \tilde{x} \neq 0, \quad (7) $$
the remaining energy dissipation guarantees
\[ \mathcal{H} = -\dot{x}^T R \dot{x} + \dot{x}^T g(s) \dot{u} < 0, \quad \dot{x} \neq 0. \] (8)

The storage function \( \mathcal{H} \) plays the role of a Lyapunov function of Eq. (3) and ensures the asymptotic stability at \( \dot{x} = 0 \). Therefore, PBC for the dc-dc converters is given as a control rule for the duty ratios \( s \) given to satisfy the condition Eq. (7).

C. Examples of PBC for DC-DC Converters

1) Boost Converter: Fig. 3(a) is the circuit schematic of the boost converter. The boost converter is modeled as
\[ \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} i \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & -(1-\mu) \\ (1-\mu) & -1/R \end{bmatrix} \begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} E, \] (9)

which is in the form of Eq. (1). The steady-state equations are obtained as
\[ \begin{aligned}
\mu &= 1 - \frac{E}{v}, \\
Ei &= \frac{v^2}{R}.
\end{aligned} \] (10)

The desired state \( [i \ v]^T = [i_d \ v_d]^T \) and the desired duty ratio \( \mu_d \in [0, 1] \) are determined as constants to satisfy Eq. (10).

Putting \( \dot{x} = [i \ i_d \ v - v_d]^T \) gives the storage function of
\[ \mathcal{H} = \frac{1}{2} \dot{x}^T A \dot{x} = \frac{1}{2} \begin{bmatrix} i - i_d \\ v - v_d \end{bmatrix}^T \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} i - i_d \\ v - v_d \end{bmatrix} + \frac{1}{2} C(v - v_d)^2. \] (11)

In order to form Eq. (11) as a Lyapunov function,
\[ \dot{x}^T g(\mu) \dot{u} = \begin{bmatrix} i - i_d \\ v - v_d \end{bmatrix}^T \begin{bmatrix} 0 & -(1-\mu) \\ (1-\mu) & -1/R \end{bmatrix} \begin{bmatrix} i_d \\ v_d \end{bmatrix} + \mu \begin{bmatrix} i_d \\ v_d \end{bmatrix} E, \] (12)

which is satisfied by
\[ \mu = \mu_d - k(i - i_d), \quad k > 0. \] (17)

Therefore, Eq. (17) is the PBC for the buck converter.

3) Buck-boost Converter: PBC for the buck-boost converter also follows the same derivation process. The buck-boost converter shown in Fig. 3(c) is modeled as
\[ \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} i \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & -(1-\mu) \\ (1-\mu) & -1/R \end{bmatrix} \begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} \mu \\ 0 \end{bmatrix} E, \] (18)

from the Kirchhoff’s rules. According to Eq. (7), the condition for the asymptotic stability is
\[ \dot{x}^T g(\mu) \dot{u} = \begin{bmatrix} i - i_d \\ v - v_d \end{bmatrix}^T \begin{bmatrix} 0 & -(1-\mu) \\ (1-\mu) & -1/R \end{bmatrix} \begin{bmatrix} i_d \\ v_d \end{bmatrix} E \] (19)

which is satisfied by
\[ \mu = \mu_d - k(i - i_d) + k(i - i_d) E < 0, \] (20)

which is the PBC for the buck-boost converter.

III. OUTPUT SERIES-PARALLEL CONNECTION OF DC-DC CONVERTERS

In this section, we theoretically confirm that the output series-parallelled converters regulated by PBC are asymptotically stable. First, the dc-dc converter model is introduced with additional external input variables representing the mutual interaction between the converters. Then, certain circuit restrictions are given to the variables in order for the
model to describe a series or parallel connection. Through the process, the output series-parallel converters are shown to be reclassified into the general dc-dc converter models of Eq. (3). Thus, the asymptotic stability of the series-parallel converters is discussed based on Eq. (7), which is the condition also obtained from the general model. Finally, it is shown that the dc-dc converters regulated by PBC can be connected in series-parallel at the output while maintaining their asymptotic stability.

A. DC-DC Converter with Output Interaction

Fig. 4 shows the dc-dc converter model with an output interaction. The interaction appears as the current \( j \) flowing into the converter through the output load voltage \( v \). The dc-dc converter with output interaction is modeled as

\[
A \dot{x} = \{J(s) - R(s)\} \dot{x} + \tilde{g}(s) \dot{u} + b,
\]

(21)

where \( b \in \mathbb{R}^n \) implies the interaction from the output. Here, \( b \) can be interpreted as an additional external input for the model. The storage function of Eq. (21) is held as

\[
\mathcal{H} = \frac{1}{2} \dot{x}^T A \dot{x}.
\]

However, its time derivative is altered to be

\[
\dot{\mathcal{H}} = \dot{x}^T A \dot{x} = -\dot{x}^T R(s) \dot{x} + \dot{x}^T \tilde{g}(s) \dot{u} + \dot{x}^T b,
\]

(23)

where the term \( \dot{x}^T b \) is the energy supplied from the output interaction. Since the interaction is given as the current \( j \) flowing into the voltage \( v \), we have

\[
\dot{x}^T b = j v.
\]

Thus, the derivative of the storage function becomes

\[
\dot{\mathcal{H}} = -\dot{x}^T R(s) \dot{x} + \dot{x}^T \tilde{g}(s) \dot{u} + j v.
\]

B. Output Series-Parallel Connection

Fig. 5(a) shows a pair of dc-dc converters with output interactions. The subscripts ‘1’ and ‘2’ correspond to the converters #1 and #2, respectively. The output port of each converter has the output voltage \( v \), the output load \( R \), and the current \( j \). We can describe either series or parallel connection of the converters by giving some restrictions to these variables.

The differential equations describing the pair of dc-dc converters shown in Fig. 5(a) are

\[
\begin{aligned}
A_1 \dot{x}_1 &= \{J_1(s) - R_1\} \dot{x}_1 + \tilde{g}_1(s) \dot{u}_1 + b_1, \\
A_2 \dot{x}_2 &= \{J_2(s) - R_2\} \dot{x}_2 + \tilde{g}_2(s) \dot{u}_2 + b_2.
\end{aligned}
\]

(26)

Note that these equations are independent due to the fact that no conditions are fixed to the variables yet. Eq. (26) is rewritten by using

\[
\begin{aligned}
\dot{x}_{12} &= \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}, \quad \dot{u}_{12} = \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, \\
J_{12} &= \begin{bmatrix} J_1 \\ 0 \\ 0 \\ J_2 \end{bmatrix}, \quad R_{12} = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}, \\
\tilde{g}_{12} &= \begin{bmatrix} \tilde{g}_1 \\ 0 \\ 0 \\ \tilde{g}_2 \end{bmatrix}, \quad b_{12} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad s_{12} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}.
\end{aligned}
\]

(28)

Eq. (27) is obviously written in the form of Eq. (21), which was defined to generally describe the dc-dc converters with interaction. Then, the storage function of Eq. (27) becomes

\[
\dot{\mathcal{H}}_{12} = \frac{1}{2} \dot{x}_{12}^T A_{12} \dot{x}_{12}.
\]

(29)

The time derivative of the storage function is obtained as

\[
\dot{\mathcal{H}}_{12} = \dot{x}_{12}^T A_{12} \dot{x}_{12} = -\dot{x}_{12}^T R_{12} \dot{x}_{12} + \dot{x}_{12}^T \tilde{g}_{12}(s) \dot{u}_{12} + \dot{x}_{12}^T b_{12},
\]

(30)

where

\[
\dot{x}_{12}^T b_{12} = \dot{x}_1^T b_1 + \dot{x}_2^T b_2 = j_1 \dot{v}_1 + j_2 \dot{v}_2,
\]

(31)

from Eq. (28).

Fig. 5(b) is the schematic of the output ports connected in series. In this configuration, both loads have identical current.
flow. Therefore, the converters interact through the voltage \( w' \), depicted in Fig. 5(b). The variable \( w' \) represents the voltage imbalance among the output ports caused by the connection. The pair of output ports in Fig. 5(a) would imply the series connection when the conditions
\[
\begin{align*}
\frac{\dot{v}_1}{R_1} &= \frac{\dot{v}_2}{R_2}, \\
\frac{1}{R_1} &= \frac{J_1}{j_1}, \quad \frac{1}{R_2} = \frac{J_2}{j_2} = -\frac{w'}{R_2},
\end{align*}
\]  
are satisfied. The first equation is the current equality of the loads. The second equation is the condition for the currents \( j_1 \) and \( j_2 \). Here, the contribution of the voltage \( w' \) is transformed equivalently as the currents \( j_1 \) and \( j_2 \) flowing into the outputs.

On the other hand, Fig. 5(c) shows the parallel connection of the output ports. In the case, the output loads must have the same voltage. Therefore, the current \( j' \) flowing between the load would imply the interaction caused by the connection. The conditions
\[
\begin{align*}
\dot{v}_1 = \dot{v}_2, \\
\dot{v}_1 = j', \quad \dot{v}_2 = -j',
\end{align*}
\]  
must be satisfied for the pair of output ports in Fig. 5(a) to represent the parallel connection.

By substituting the conditions for the series connection Eq. (32) to Eq. (31), we obtain
\[
\dot{x}_{12}^T b_{12} = j_1 \dot{v}_1 + j_2 \dot{v}_2 = w' \frac{\dot{v}_1}{R_1} - w' \frac{\dot{v}_2}{R_2} = 0.
\]  
Similarly, substituting the conditions for the parallel connection Eq. (33) to Eq. (31) results in
\[
\dot{x}_{12}^T b_{12} = j_1 \dot{v}_1 + j_2 \dot{v}_2 = j' \dot{v}_1 - j' \dot{v}_2 = 0.
\]  
These equations suggest that the interactions of power caused by either series or parallel connection have sign-reversal relationship between the two converters. Furthermore, above results immediately lead to \( b_{12} = 0 \) since \( \dot{x}_{12} \) is the state variable. Thus, a pair of dc-dc converters connected in series or parallel at the output is written as
\[
A_{12} \dot{x}_{12} = \{ J_{12}(s_{12}) - R_{12} \} \dot{x}_{12} + g_{12}(s_{12}) \dot{u}_{12},
\]  
from Eq. (27). Obviously, this equation is in the form of Eq. (4), which represents the general model of the dc-dc converters. Eq. (36) and \( b_{12} = 0 \) are natural in the sense that no additional interactions are occurring to the series or parallel connected converters. Consequently, the output series or parallel connection of a pair dc-dc converters is shown to come back to the general form of the dc-dc converters Eq. (4).

By simply repeating the above process, we are able to confirm that an arbitrary output series-parallel connection of dc-dc converters is also represented in the general form of Eq. (4). We have theoretically shown that Eq. (4) generally represents the output series-parallel dc-dc converters.

C. Stabilization by PBC
The time derivative of the storage function \( H_{12} \) is
\[
\dot{H}_{12} = -\dot{x}_{12}^T R_{12}(s_{12}) \dot{x}_{12} + \dot{x}_{12}^T g_{12}(s_{12}) \dot{u}_{12},
\]  
where
\[
\begin{align*}
\dot{x}_{12}^T R_{12}(s_{12}) \dot{x}_{12} &= x_{12}^T R_{12} x_{12} < 0, \quad \dot{x}_{12} \neq 0, \\
\dot{x}_{12}^T g_{12}(s_{12}) \dot{u}_{12} &= \dot{x}_{12}^T g_{12} u_{12} = \dot{x}_{12}^T g_{12} u_{12} = \dot{x}_{12}^T g_{12} u_{12}.
\end{align*}
\]  

The condition for the storage function \( H_{12} \) to be the Lyapunov function is
\[
\dot{x}_{12}^T g_{12}(s_{12}) \dot{u}_{12} < 0, \quad \dot{x}_{12} \neq 0,
\]  
which is satisfied when
\[
\begin{align*}
\dot{x}_{12}^T g_{12}(s_{12}) \dot{u}_{12} &= \dot{x}_{12}^T g_{12} u_{12} < 0, \quad \dot{x}_{12} \neq 0, \\
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\end{align*}
\]  

Note that Eq. (39) corresponds to the condition Eq. (7), which was originally aimed to guarantee in order to achieve the asymptotic stability of the system. Eq. (40) shows that the PBC for each individual converter assures Eq. (39). This implies that the dc-dc converters regulated by PBC maintain their asymptotic stability whether they are connected in series or parallel at the output. In other words, series or parallel connected converters can be asymptotically stabilized by the application of PBC to each converter. This is also natural in the sense that the series-parallel connection is sharing the dissipation at the output.

Repeating the above process, again, leads to the conclusion that an arbitrary series-parallel connection of dc-dc converters regulated by PBC is asymptotically stable. PBC of each converter guarantees the monotonic decrease of the storage function for the whole series-parallelled converters. Note that the restrictions made in the above arguments were only for the circuit constraints due to the output connection. It implies the robust feature of PBC that it does not restrict the diverse circuit topologies, parameters, and steady-states of the series-parallelled converters. This theoretical result is confirmed in the next section by the numerical simulation.

IV. NUMERICAL SIMULATION

In this section, the PBC of the output series-parallelled converters is examined based on numerical simulation using MATLAB/Simulink.

A. Setups for Simulation

The simulated circuit is shown in Fig. 5. It is composed of boost, buck, and buck-boost converters. The subscripts ‘1’, ‘2’, and ‘3’ correspond to the boost, buck, and buck-boost converter, respectively. The circuit parameters for the simulated circuit are shown in Table II. Here, the parameters are chosen to be diverse. The parameter \( k \) is the feedback control gain set for the PBC of each converter. The load resistance is \( R = 12 \Omega \). The switches and the diodes are considered as ideal.

In the simulation, the PBC rules explained in section II are applied to each converter. The switches of boost, buck, and buck-boost converters are governed by Eq. (13), Eq. (17), and Eq. (20), respectively. The calculated duty ratios are A/D converted to the switching signal by pulse width modulation (PWM) at 1 MHz.
Fig. 6. Boost, buck, and buck-boost converters connected in series-parallel.

| #1 Boost          | #2 Buck          | #3 Buck-boost     |
|------------------|-----------------|------------------|
| Parameter Values | Parameter Values | Parameter Values |
| $L_1$ 470 µH     | $L_2$ 500 µH    | $L_3$ 330 µH     |
| $C_1$ 10 µF      | $C_2$ 33 µF     | $C_3$ 20 µF      |
| $E_1$ 18 V       | $E_2$ 40 V      | $E_3$ 24 V       |
| $k_1$ 0.02       | $k_2$ 0.3       | $k_3$ 0.02       |

TABLE III
INITIAL STATES OF DC-DC CONVERTERS

| #1 Boost          | #2 Buck          | #3 Buck-boost     |
|------------------|-----------------|------------------|
| Variable State   | Variable State  | Variable State   |
| $i_1$ 1.4 A      | $i_2$ 1.3 A     | $i_3$ 2.8 A      |
| $v_1$ 10 V       | $v_2$ 16 V      | $v_3$ 12 V       |

TABLE IV
DESIERED STATES OF DC-DC CONVERTERS

| #1 Boost          | #2 Buck          | #3 Buck-boost     |
|------------------|-----------------|------------------|
| Variable State   | Variable State  | Variable State   |
| $i_{1d}$ 1.950 A | $i_{2d}$ 2.025 A| $i_{3d}$ 3.375 A |
| $v_{1d}$ 36 V    | $v_{2d}$ 20 V   | $v_{3d}$ 16 V    |
| $\mu_{1d}$ 0.5   | $\mu_{2d}$ 0.5  | $\mu_{3d}$ 0.4   |

The initial state of the circuit at $t = 0$ is given as Table III. The desired state for the control is shown in Table IV. This desired state is determined for the converters to have various voltage, current, and power levels at the steady-state.

B. Results

The simulated waveforms of the series-parallel converters are shown in Fig. 7. It can be seen from Fig. 7 that the waveforms converge to the desired state shown in Table IV. Thus, the asymptotic stability of the series-parallel boost, buck, and buck-boost converters was numerically confirmed.

Fig. 7. Numerical waveforms of the series-parallel converters.

It was also confirmed that the diversity in each converter did not violate the asymptotic stability, due to the robust feature of PBC. The simulation showed that the PBC allowed diverse voltage, current, and power levels, among various circuit parameters in the series-parallel connection.

V. CONCLUSION

In this paper, we examined the asymptotic stability of the output series-parallel dc-dc converters regulated by PBC. We introduced the dc-dc converter model with the additional variable representing the output interaction to describe the series-parallel connection of the converters. Then, the output series-parallel converters were shown to be reclassified into the general dc-dc converter model. This feature enabled us to discuss the asymptotic stability of the series-parallel converters based on the condition obtained from the general converter model. Consequently, the converters regulated by PBC were theoretically shown to maintain their stability at the output series-parallel connection. In other words, the output series-parallel converters were shown to be asymptotically stable by the PBC of each converter. This theoretical result was verified in the numerical simulation of series-parallel boost, buck, and buck-boost converters. PBC allowed diverse circuit topologies, parameters, and steady-states of the output series-parallel dc-dc converters.

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