Investigating the properties of AGN feedback in hot atmospheres triggered by cooling-induced gravitational collapse

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ABSTRACT

Radiative cooling may plausibly cause hot gas in the centre of a massive galaxy, or galaxy cluster, to become gravitationally unstable. The subsequent collapse of this gas on a dynamical timescale can provide an abundant source of fuel for AGN heating and star formation. Thus, this mechanism provides a way to link the AGN accretion rate to the global properties of an ambient cooling flow, but without the implicit assumption that the accreted material must have flowed onto the black hole from 10s of kiloparsecs away. It is shown that a fueling mechanism of this sort naturally leads to a close balance between AGN heating and the radiative cooling rate of the hot, X-ray emitting halo. Furthermore, AGN powered by cooling-induced gravitational instability would exhibit characteristic duty cycles ($\delta$) which are redolent of recent observational findings: $\delta \propto L_X/\sigma_*$, where $L_X$ is the X-ray luminosity of the hot atmosphere, and $\sigma_*$ is the central stellar velocity dispersion of the host galaxy. Combining this result with well-known scaling relations, we deduce a duty cycle for radio AGN in elliptical galaxies that is approximately $\propto M_{BH}^{1.5}$, where $M_{BH}$ is the central black hole mass. Outburst durations and Eddington ratios are also given. Based on the results of this study, we conclude that gravitational instability could provide an important mechanism for supplying fuel to AGN in massive galaxies and clusters, and warrants further investigation.

Key words:

1 INTRODUCTION

The cooling time of X-ray emitting gas near the centres of many massive galaxies and galaxy clusters is much shorter than the Hubble time. In the absence of heat sources, significant quantities of the gas would cool and form stars. However, X-ray spectroscopy has shown that the rate at which gas cools to low temperatures is significantly lower than first expected (e.g. Peterson et al. 2001; Tamura et al. 2001; Xu et al. 2002; Sakelliou et al. 2002; Peterson et al. 2003; Kaastra et al. 2004; Peterson & Fabian 2006) suggesting that the gas is somehow being reheated.

Based on both observational and theoretical evidence, it is generally assumed that energy input by a central AGN is predominantly responsible for reheating the gas. For example, elliptical galaxies are commonly the hosts of powerful radio Active Galactic Nuclei (AGN). These sources give rise to lobes of radio emission embedded in the X-ray emitting gas which permeates massive galaxies and clusters of galaxies (e.g. Birzan et al. 2004; Best et al. 2005; Dunn et al. 2007; Rafferty et al. 2006; Best et al. 2007; Shabala et al. 2008). Moreover, recent observational studies of Brightest Cluster Galaxies (BCGs) suggest that radio AGN activity is related to the thermal state of its environment. Systems with short radiative cooling times, or a low central entropy, are more likely to exhibit active star formation, optical line-emission and jet-producing AGN (e.g. Burns 1990; Crawford et al. 1999; Cavagnolo et al. 2008; Mittal et al. 2009; Rafferty et al. 2008). This suggests that AGN activity is part of a feedback loop that can prevent the ambient hot gas from cooling, and is likely to have other important consequences for its environment.

Building on the ideas of early work (e.g. Binney & Tabor 1995; Tucker & David 1997; Ciotti & Ostriker 2001; Silk & Rees 1998), theoretical studies have drawn attention to the potential, wide-ranging

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impact of AGN feedback. For example, semi-analytic models of galaxy formation have demonstrated that, in principle, AGN heating can both reheat cooling flows and explain the exponential cutoff at the bright end of the galaxy luminosity function (e.g. Benson et al. 2003; Croton et al. 2006; Bower et al. 2006). More recently, AGN heating has been shown to be crucial in shaping the X-ray luminosity-temperature relation of massive galaxies (e.g. Puchwein et al. 2008; Bower et al. 2008; Pope 2009). Despite these findings, the fundamental details of AGN feedback remain poorly constrained. Most significantly, there is no clear consensus on how information about the thermal state of the X-ray emitting atmosphere is transmitted to the black hole at the centre of the galaxy, or cluster. Understanding this link is of great importance because it probably facilitates the observationally-inferred long-term balance between AGN heating and gas cooling, and almost certainly governs the duty cycle of AGN activity – that is, the fraction of time an AGN is active.

Generally, AGN feedback is assumed to be powered by one of the following: a) Bondi accretion of hot material in the vicinity of the black hole (e.g. Cattaneo & Tevssi& 2007; Sijacki et al. 2008; Puchwein et al. 2008; Fabian et al. 2010); b) material directly from the ambient cooling flow (e.g. Pizzolato & Soker 2003; Bower et al. 2008; Pope 2009; Pizzolato & Soker 2010). While there are plausible arguments for both, based on our current interpretation of the available observations, there are also difficulties. For example, the power output from Bondi accretion is unlikely to be sufficient to balance the gas cooling rates in massive clusters (e.g. Soker 2006). The biggest problem with models in category b) is finding a mechanism by which the central black hole receives information from the cooling flow about the thermal state of the ICM. For example, it is difficult to conceive of how material might flow all the way from the cluster cooling radius onto a central black hole, such that it prompts an AGN outburst on a useful timescale (though see Pizzolato & Soker 2003).

The model investigated here describes a mechanism that can reconcile the problem faced by models of type b). While Pizzolato & Soker (2003, 2010) focused on thermal instability of the ICM as a mechanism for delivering fuel to the AGN, we focus on the characteristics of AGN feedback that is triggered when the hot gas which resides near the centre of a galaxy becomes gravitationally unstable. That is, gas which was previously self-supporting against gravity is somehow destabilised and falls freely towards the galaxy centre, on a dynamical timescale, where it forms stars and fuels the AGN (e.g. Silk & Rees 1998; Fabian 2000; King 2008). Gravitational instabilities can be induced by merger events (e.g. Silk & Rees 1998) and the effect of radiative cooling (e.g. Birnboim & Dekel 2003). Of these two possibilities, only instabilities induced by gas cooling can lead to self-regulated AGN heating, but it is important to mention that merger-driven AGN feedback will delay the onset of gravitational instability induced by radiative cooling. The extent to which this happens is unclear, but the effect may be more significant in group and field environments (e.g. Kaviraj et al. 2010), than in clusters.

Perhaps importantly, AGN heating driven by cooling-induced gravitational instabilities has the potential to be periodic, as explained by the following argument. In massive galaxies and clusters there is a continual inflow of material from the large-scale environment, due to the effects of the ambient cooling flow. As the mass of material in the hot atmosphere near the centre of a galaxy builds up, it can become gravitationally unstable, meaning that some fraction of the gas will collapse and flow on a dynamical timescale to the centre of the gravitational potential, thereby fuelling the AGN. Once this fuel has been consumed, the AGN will be starved of new material until the nearby hot gas once again becomes gravitationally unstable. A second instability will commence after the external inflow has delivered a sufficient quantity of new material. Thus, for this mode of fuelling, the typical duration of an AGN outburst is largely governed by the local dynamical timescale, while the time between the onset of successive outbursts is controlled by the ambient cooling flow. This is notable because observations indicate that AGN are only active for a fraction of time determined by their environment and host galaxy properties (e.g. Best et al. 2003; Shabala et al. 2008). In addition, as demonstrated in Pope (2011), periodic heating can supply the energy required to balance gas cooling, with the minimum effort. Thus, periodic AGN activity appears to be an energetically favourable heating strategy. Therefore, starting from the assumption that AGN fuelling is attributable to gravitational instabilities induced by gas cooling, we derive AGN duty cycles, outburst durations, star formation rates and discuss implications for numerical simulations.

The outline of the article is as follows. In Section 2 we present the main model and place constraints on the phenomenological parameters. In Section 3 we show the expected duty cycles and outburst durations that would be associated with AGN heating that is driven by cooling-induced gravitational instability. The implications are discussed in Section 4, and the results are summarised in Section 5.

## 2 OUTLINE OF THE MODEL

In this investigation, we study the properties of AGN feedback that is driven by the gravitational collapse of hot gas in the vicinity of the supermassive black hole. Therefore, we must first describe the physical conditions of the gas in massive galaxies and clusters that can justifiably lead to gravitational instability. Following this we define the characteristic timescales of the system and the AGN power output expected from this mode of fuelling. These quantities are then used to determine the observable features predicted by this scenario.

### 2.1 The onset of gravitational instability

A self-gravitating sphere of gas will become gravitationally unstable when the sound-crossing time is greater than the gravitational free-fall time. This is the well-known Jeans criterion. Strictly speaking, the criterion only applies in the limit that the polytropic index of the gas $-\gamma_{\text{eff}}$, as defined in equation (1) – is also lower than some critical value, $\gamma_{\text{crit}} \approx 1.2$, (e.g. see Birnboim & Dekel 2003 and references therein). Since the actual adiabatic index of an ideal, non-relativistic monatomic gas is $\gamma = 5/3$, a gas of this type can only become gravitationally unstable if its behaviour is...
modified by heating, cooling and work. For example, if a parcel of gas is heated by an amount $\Delta Q$, and then does an equal amount of work on its environment (i.e. $\Delta W = \Delta Q$), the gas is defined as isothermal, since it acts to maintain a constant temperature. In this case, the polytropic index of the gas must be $\gamma_{\text{eff}} = 1$, by definition.

In the case of the hot, X-ray emitting gas that resides in massive galaxies and galaxy clusters, we envisage a central gas mass that is embedded in a pressurised ambient medium. Consequently, the scenario is somewhat reminiscent of the isothermal sphere which is prone to gravitational instability if its mass exceeds the Bonnor-Ebert limit. However, it is important to note that in this study we do not make any assumptions about the density distribution of the hot gas.

Observations clearly indicate that this gas must be subject to radiative cooling, heating from AGN and stars, and the work done by its surroundings. Therefore, it seems extremely likely that $\gamma_{\text{eff}} \neq 5/3$ for the X-ray emitting gas. However, it is unclear exactly what values the polytropic index might take because the processes acting on the gas are highly uncertain. Nevertheless, consider the behaviour of a gas mass near the centre of a massive galaxy cluster. As the gas radiates energy it loses pressure support; the weight of the overlying gas then does work and compresses it. The contraction can only persist if the compressional heating resulting from the inflow is radiated away on a timescale that is shorter than the dynamical timescale, i.e. $t_{\text{cool}} < t_{\text{dyn}}$. This is perfectly possible because a gas parcel of temperature $T$ and number density $n$, radiating via thermal bremsstrahlung, has a cooling time that is $\propto T^{3/2}/n$. As the gas cools, the temperature will fall, and the density will rise, so that the cooling time shortens. It is this condition ($t_{\text{cool}} < t_{\text{dyn}}$) that explains observations which show that the temperature of the gas remains lower near the centre of the cluster than further out. Indeed, it is precisely this argument that predicts a cooling catastrophe, and the need for AGN feedback. However, as shown below, the inflow condition also predicts the onset of a cooling-induced gravitational instability which provides a potentially informative description of AGN fuelling in hot atmospheres.

Thermodynamically, the mass of gas described above behaves like a parcel that cools down (rather than heats up) when compressed. The appropriate polytropic relation linking the temperature, $T$, and volume, $V$, of the gas parcel is $T \gamma_{\text{eff}} - 1$ = constant. Thus, the hot gas in the centres of massive galaxies and clusters acts as if $0 < \gamma_{\text{eff}} < 1$. Since this is less than all plausible values of the critical polytropic index, the hot gas is likely to be susceptible to gravitational instability.

For completeness, we also present a brief derivation to illustrate the general phenomena that influence the polytropic index of a gas. This derivation closely follows the work of Birnboim & Dekel (2003) who define the polytropic index, $\gamma_{\text{eff}}$, of a Lagrangian fluid element in terms of the logarithmic time derivative of its pressure, $P$, and density, $\rho$, such that

$$\gamma_{\text{eff}} \equiv \frac{\text{d} \ln P/\text{d}t}{\text{d} \ln \rho/\text{d}t}. \quad (1)$$

Accounting for gas cooling, the polytropic index can be expressed as

$$\gamma_{\text{eff}} \approx \gamma - \frac{n^2 \Lambda(T)}{\rho e}, \quad (2)$$

where $\gamma$ is the actual adiabatic index of the gas, $\rho$ is the rate of change of gas density and $e$ is the internal energy per unit mass. As usual, $n$ is the number density of the gas, $T$ is the gas temperature and $\Lambda(T)$ is the cooling function.

### 2.2 Characteristic timescales and flow rates

To calculate the local dynamical timescale, we refer back to the definition of the Jeans instability criterion. A self-gravitating gas mass will become gravitationally unstable when its mass exceeds some critical value that is comparable to the Jeans/Bonnor-Ebert masses. In the centre of a galaxy, the self-gravity of the gas will become important when its characteristic velocity dispersion is comparable to the central stellar velocity dispersion, $\sigma_\ast$. Thus, if a gas mass $M$ with radius $R$ located at the centre of a galaxy becomes gravitationally unstable, it will collapse on the dynamical timescale, $t_{\text{dyn}} = R/\sigma_\ast$. The average mass flow rate during this time will be

$$\dot{M} = \frac{\Delta M}{\Delta t} \approx \frac{M}{t_{\text{dyn}}} = \frac{\sigma_\ast M}{R} \quad (3)$$

We note that, for a self-gravitating cloud, the characteristic velocity dispersion of the constituent particles is related to the gravitational potential energy by the virial theorem: $\alpha GM/R = \sigma^2$, where $G$ is Newton’s gravitational constant.

In this description the numerical constant $\alpha \sim 1$ depends on the density distribution of the gas. Thus, the mass flow rate can be expressed as

$$\dot{M} = \frac{\sigma_\ast^3}{\alpha G} \sim 1000 \left(\frac{\sigma_\ast}{200 \text{ km s}^{-1}}\right)^3 \text{ M}_\odot \text{ yr}^{-1} \quad (4)$$

This is the well-known form of the dynamical mass flow rate (c.f. King 2009), modified slightly to account for arbitrary density distributions. From this it is straightforward to show that the duration of the collapse will be $t_{\text{dyn}} = \dot{M}/\dot{M}$.

Following the collapse, mass will flow towards the centre of the galaxy’s gravitational potential where a fraction is likely to be accreted by the supermassive black hole, and the remainder presumably forms stars nearby or is expelled from the galaxy as a result of feedback.

As described in the introduction, the fuelling rate provided by gravitational instability can be periodic, for the following reason. The postulated gas cloud near the galaxy centre acts as a reservoir; this reservoir is depleted when the cloud collapses, but is replenished by the inflow of new material from the larger-scale environment, which forms a new cloud. When it reaches the critical mass, the second cloud will also become gravitationally unstable and collapse, and so on.

The provenance of the material that replenishes the collapsing gas cloud is not clear, except that it must come from the ambient hot atmosphere. The simplest possibility is that the cloud is built up during two phases: 1) an initial collapse of material from slightly further out in the hot atmosphere; 2) the subsequent slow inflow of additional material due to the ambient cooling flow. If this is the case, the first phase...
of growth of the new cloud will occur on a similar dynamical timescale to the gravitational collapse of the previous cloud. However, at these early times, the new cloud is likely to be hotter and of lower mass than the previous cloud, ensuring that it can be gravitationally stable. More precisely, as long as \( \gamma_{\text{crit}} < \gamma_{\text{eff}} \approx 1.2 \), and the cloud mass is below the critical mass, the cloud will not collapse. The second phase of growth will occur on the timescale required for the cooling flow to build up the cloud mass to its gravitationally unstable limit, at which point a collapse will be initiated. Using this argument, we can estimate the average time between gravitational collapses, as shown below.

Overall, the local mass inflow rate, \( \dot{M}_{\text{ext}} \), from the cooling flow will be a slowly varying quantity governed by the difference between the time-averaged heating and cooling rates. Then, in the limit that the majority of the cloud mass is built up during this phase, the characteristic time between the triggering of successive gravitational collapses must tend to

\[
\tau = \frac{M}{\dot{M}_{\text{ext}}} \quad (5)
\]

Assuming that the AGN is only fuelled while the cloud collapses, it will be active for a fraction of time \( \delta = t_{\text{dyn}}/\tau \), known as the duty cycle. By combining equations (4) and (5), we obtain a simple functional form for the AGN duty cycle without having to explicitly calculate the AGN heating rate

\[
\delta = \frac{t_{\text{dyn}}}{\tau} = \frac{\dot{M}_{\text{ext}} R}{M \sigma_*} = \frac{\alpha G M_{\text{ext}}}{\sigma_*^2} \quad (6)
\]

Thus, in the present model, the AGN duty cycle can be straightforwardly related to the local gravitational potential, \( \sigma_* \), and the external environment through \( \dot{M}_{\text{ext}} \). This is potentially significant, because observations (e.g. Best et al. 2005; Best 2007; Shabala et al. 2008) indicate that the radio AGN duty cycle is heavily influenced by both local and environmental effects. Below, we argue that \( \dot{M}_{\text{ext}} \) is probably closely related to the mass inflow associated with the ambient cooling flow.

### 2.3 AGN heating rates

If a fraction, \( \beta \), of the inflowing mass rate, \( \dot{M} \), reaches the black hole, the accretion power output will be

\[
H = \eta \beta M c^2 \approx \eta \frac{\beta \sigma_*^3}{\alpha G} c^2 \approx 10^{47} \left( \frac{\beta/\alpha}{10^{-3}} \right) \left( \frac{\sigma_*}{200 \text{ km s}^{-1}} \right)^3 \text{ erg s}^{-1}, \quad (7)
\]

where the assumed accretion efficiency is \( \eta \approx 0.1 \). The characteristic value of \( \beta/\alpha = 10^{-3} \) is motivated below, and yields favourable comparisons with the observationally-inferred radio AGN duty cycle.

Clearly, the AGN energy injection rates associated with this fuelling mechanism can be very large. Such values are considerably larger than would be expected to arise from the accretion of nearby hot gas and, if observed in a real system, would more commonly be associated with merger-driven fuelling events. However, the reasoning above indicates that such an interpretation is not necessarily exclusive – high AGN fuelling rates can also be a consequence of gravitational instability resulting from gas cooling. In addition, it is important to remember that values estimated from equation (7) represent the instantaneous heating rate – the time-averaged values are much more modest.

By definition, the time-averaged AGN power output is written \( \dot{H} = \delta H \), which can be expanded using equations (9) and (7) to give

\[
\dot{H} = \delta H = \eta \beta \dot{M}_{\text{ext}} c^2. \quad (8)
\]

Equation (8) shows that the time-averaged AGN heating rate depends on the properties of the large scale environment, through \( \dot{M}_{\text{ext}} \). From this we also conclude that \( \dot{M}_{\text{ext}} \) will evolve until the time-averaged heating rate closely matches the ambient cooling rate of the gas. In the limit that the time-averaged AGN heating does balance gas cooling, \( \dot{H} = L_x \), equation (9) shows that

\[
\dot{M}_{\text{ext}} = \frac{L_x}{\eta \beta c} \approx 175 \left( \frac{L_x}{10^{45} \text{ erg s}^{-1}} \right) \left( \frac{\beta/10^{-3}}{10^{-3}} \right) M_\odot \text{ yr}^{-1}, \quad (9)
\]

where again we have assumed \( \eta \approx 0.1 \) and \( \beta/\alpha \sim 10^{-3} \), with \( \alpha \sim 1 \). Below, we motivate the constraints on \( \beta \) by considering the Eddington ratio of the AGN outbursts fuelled by gravitational collapse.

#### 2.4 The Eddington ratio

The Eddington ratios predicted by this model provide another method for comparison with numerical simulations and observations. Importantly, they also provide a way to check the self-consistency of the model, as outlined below.

Broadly speaking, the form of energetic output from an AGN can be predicted from its Eddington ratio, defined by

\[
R_{\text{Edd}} \equiv \frac{\dot{m}}{\dot{M}_{\text{Edd}}}, \quad (10)
\]

where \( \dot{m} \) is the black hole accretion rate, \( \dot{M}_{\text{Edd}} \) is the Eddington limited accretion rate determined by the balance between gravity and radiation pressure

\[
\dot{M}_{\text{Edd}} = \frac{L_{\text{Edd}}}{\eta c^2} = \frac{4 \pi G m_p M_{\text{BH}}}{\eta c \sigma_T}, \quad (11)
\]

where \( L_{\text{Edd}} \) is the Eddington luminosity, \( M_{\text{BH}} \) is the black hole mass, \( \sigma_T \) is the Thompson cross-section, \( m_p \) is the proton mass and the other symbols are as previously defined.

By analogy with stellar mass black holes in X-ray binaries (e.g. Kording et al. 2008), accretion rates less than \( \sim 3\% \) of the Eddington limit is radiatively inefficient so that the majority of the power output is in the form of kinetic-energy-dominated outflows of relativistic particles which are prominent radio synchrotron emitters. Outflows of this type are thought to couple strongly to the ambient gas. Conversely, accretion above this critical rate is radiatively efficient, meaning that the power output is predominantly in the form of photons, rather than kinetic outflows of particles. In this limit, radio jets may still be observed, but the efficiency of jet production is much lower than in the radiatively inefficient regime (e.g. Maccarone et al. 2003). Furthermore, in the radiatively efficient mode, it has been argued that only \( \sim 5\% \) of the accretion power is available to heat the surrounding gas (e.g. Sijacki & Springel 2006), see King (2011) for a possible explanation.

We note that the comparison with X-ray binaries is not exact, since it does not account for the existence of radio-loud quasars which produce powerful kinetic outflows at high
Eddington ratios. However, as explained below, we focus on low accretion rate objects so the distinction does not matter for the purposes of this model.

To calculate the Eddington ratios, we use the well-known relation between the black hole mass and the stellar velocity dispersion: $M_{\text{BH}} \approx 1.3 \times 10^8 (\sigma_* / 200 \text{ km s}^{-1})^4 M_\odot$, where $\sigma_*$ is the central stellar velocity dispersion (e.g. Tremaine et al. 2002). Substituting for the black hole mass where

$$R_{\text{Edd}} \approx 0.01 \left(\frac{\beta/\alpha}{10^{-3}}\right) \left(\frac{\sigma_*}{200 \text{ km s}^{-1}}\right)^{-1}, \quad (12)$$

where the assumed accretion efficiency is $\eta \approx 0.1$. Therefore, if $\beta/\alpha \lesssim 10^{-3}$, accretion following a gravitational collapse can be radiatively inefficient and produce kinetic outflows. Since $\alpha \sim 1$, this implies that $\beta \lesssim 10^{-3}$ is required for kinetic outflow production. For larger values of $\beta/\alpha$ the accretion will be radiatively efficient.

The value of $\beta$ is depends on the processes that govern how gas travels from kiloparsec scales onto the black hole. Since these processes are highly uncertain, we proceed by assigning $\beta$ a single, empirical value that encapsulates the accretion physics in the full range of AGN environments. While this is a simplification, it ensures that the results are as transparent as possible. As indicated in equation (12), we find that $\beta \sim 10^{-3}$ permits a good agreement between the model and radio observations (Best et al. 2005, Best 2007, Shabala et al. 2008). Reassuringly, equation (12) also shows that $\beta \sim 10^{-3}$ will lead to low Eddington ratios and, therefore, kinetic outflows which produce radio emission.

### 3 RESULTS

From the assumptions and derivations outlined above, we present the expected duty cycles of radio AGN fuelled by gravitationally destabilised gas, the corresponding duration of individual outbursts, and the associated heating rates resulting from shocks generated by the AGN outflow.

#### 3.1 AGN duty cycle

In this section, we give a more general derivation for the AGN duty cycle. For a constant pressure cooling flow in which external heating is not important, the classical mass flow rate, gas temperature, and X-ray luminosity are related by

$$L_X = \frac{\gamma}{\gamma - 1} M_{\text{clus}} \frac{k_B T}{\mu m_p} \quad \eta \beta c^2 \sigma_*^2$$

where $k_B$ is Boltzmann’s constant and $\mu m_p$ is the mean mass per particle.

More generally, the net inflow of mass is determined by the difference between the time-averaged cooling and heating rates. Observationally, this mass flow rate is estimated by fitting models to the X-ray spectra. Thus, we refer to the mass flow rate by its observational name, $M_{\text{spec}}$, but calculate it from the following

$$L_X - \dot{H} = \frac{\gamma}{\gamma - 1} M_{\text{spec}} \frac{k_B T}{\mu m_p} \quad \eta \beta c^2 \sigma_*^2$$

Rearranging equation (13) for the duty cycle, using equation (14), gives

$$\delta = \frac{\alpha G L_X}{\eta \beta c^2 \sigma_*^2} \left(1 - \frac{M_{\text{spec}}}{M_{\text{clus}}}\right). \quad (15)$$

Consequently, in the limit that AGN heating exactly balances gas cooling, $M_{\text{spec}} / M_{\text{clus}} \to 0$, the duty cycle simplifies to

$$\delta \to \frac{\alpha G L_X}{\eta \beta c^2 \sigma_*^2} \propto \frac{L_X}{\sigma_*^2}. \quad (16)$$

In terms of scaled quantities, the duty cycle can be expressed as

$$\delta \approx 0.1 \left(\frac{\beta/\alpha}{10^{-3}}\right)^{-1} \left(\frac{L_X}{10^{45} \text{ erg s}^{-1}}\right) \left(\frac{\sigma_*}{200 \text{ km s}^{-1}}\right)^{-3}. \quad (17)$$

Equation (17) shows that AGN fuelled by the gravitational collapse of gas in more X-ray luminous clusters should exhibit larger duty cycles. That is, those AGN should be active for a greater proportion of time, as is inferred from observations (Best et al. 2007). However, the precise scaling of the AGN duty cycle in clusters is difficult to determine because it is not clear exactly how the X-ray luminosity of the gas, and the central stellar velocity dispersion are related in clusters. Nevertheless, surveys indicate that the X-ray luminosity of a cluster scales as $L_X \propto \sigma_*^6$, where $\sigma_*$ is the velocity dispersion of the cluster potential, and that $n \sim 4 - 5$ (e.g. Mahdavi & Geller 2001). Substituting the cluster $L_X - \sigma_*$ relation into equation (17) gives $\delta \propto \sigma_*/4$. Therefore, if more massive BCGs are found in more massive clusters, the AGN duty cycle will tend to increase slowly with $\sigma_*$, which would be in qualitative agreement with observations (e.g. Best et al. 2007). Furthermore, assuming $\beta/\alpha \sim 10^{-3}$, the normalisation of the duty cycle is entirely consistent with the observational results presented by Best et al. (2007).

In contrast, the scaling relations of field elliptical galaxies yield a much simpler scaling of $\delta$ with $\sigma_*$. Under the assumption of an isothermal gravitational potential, the velocity dispersion is constant and independent of radius. Again, surveys indicate that the X-ray luminosity scales as $L_X \propto \sigma_*^6$, where $n \sim 8 - 10$ (e.g. O’Sullivan et al. 2003, Mahdavi & Geller 2001). Using this fact, the AGN duty cycle would scale as $\delta \propto \sigma_*^{n-1} \sim \sigma_*^5$. Since supermassive black hole mass scales as $M_{\text{BH}} \propto \sigma_*^5$, the duty cycle can then be said to increase as $\delta \propto M_{\text{BH}}^{5/3}$, which is consistent with observational findings (Best et al. 2005; Shabala et al. 2008).

In addition, for a massive galaxy with an X-ray luminosity of $10^{43} \text{ erg s}^{-1}$, and $\sigma_* \approx 200 \text{ km s}^{-1}$ the duty cycle would be $\delta \approx 10^{-4}$ (assuming $\beta/\alpha \sim 10^{-3}$), which is also consistent with Best et al. (2005) and Shabala et al. (2008). Consequently, both the scaling and normalisation of the duty cycle are largely consistent with observations of radio AGN in field elliptical galaxies and BCGs.

#### 3.2 Number and duration of outbursts

The AGN duty cycle is a useful quantity which provides information about how frequently an AGN is triggered in order for heating to balance the effects of radiative cooling. With additional assumptions, the duty cycle also provides some information about the duration of an individual heating event, $t_{\text{dyn}}$, and its environmental dependence. If the
time between the onset of successive AGN outbursts is \( \tau \), the number of heating events which occur during a galaxy lifetime, \( t_{\text{age}} \), must be \( N = t_{\text{age}}/\tau = \delta (t_{\text{age}}/t_{\text{dyn}}) \). Using the previous results, the expected number of AGN outbursts can be written

\[
N \approx 10 \left( \frac{\beta/\alpha}{10^{-3}} \right)^{-1} \left( \frac{t_{\text{dyn}}/t_{\text{age}}}{0.01} \right)^{-1} \left( \frac{L_X}{10^{45} \text{ erg s}^{-1}} \right) \times \left( \frac{\sigma_*}{200 \text{ km s}^{-1}} \right)^{-3}.
\]

By rearranging equation (18), the duration of an outburst can be expressed in the form \( t_{\text{dyn}} = \delta t_{\text{age}} / N \). Using equation (17) for a cluster with \( L_X \approx 10^{45} \text{ erg s}^{-1} \) and \( \sigma_* \approx 200 \text{ km s}^{-1} \), the typical outburst duration must be \( t_{\text{dyn}} \approx 0.01 t_{\text{age}} \). Taking \( t_{\text{age}} \approx 5 \text{ Gyr} \) implies \( t_{\text{dyn}} \approx 50 \text{ Myr} \), which is sufficient to explain the features AGN-blown bubbles observed in many clusters (e.g. Birzan et al. 2004, Dunn et al. 2008).

The gravitational instability model of AGN fuelling also offers an explanation for the unexpectedly large number of compact radio sources (e.g. O'Dea & Baum 1995; Shabala et al. 2008) that differs from the accretion disk variability model proposed by Czerny et al. (2000). As shown by Alexander (2000), the maximum stable length of an AGN jet propagating through an atmosphere depends on its power and the ambient density. For example, at constant jet power, a higher ambient density leads to a shorter stable jet length. Then, since the inflow of material due to gravitational instability will significantly enhance the density in the vicinity of the black hole, this enhanced density may also plausibly confine and disrupt the resultant AGN jet on kiloparsec scales. If this is correct, the jet can only propagate further outwards once the gas has been sufficiently depleted by continued accretion and star formation.

4 DISCUSSION

The model investigated above exhibits several features that are compatible with key observational characteristics of AGN feedback in both field elliptical galaxies and BCGs. Below we discuss some additional implications of the model which may be significant, but are harder to quantify.

4.1 Star formation in the host galaxy

Since only a very small fraction of the inflowing material reaches the black hole, the vast majority must either form stars, or be dragged out of the galaxy by the AGN outflow itself. As shown in Pope et al. (2010) the mass of ambient material transported by an AGN-blown bubble is approximately equal the mass of gas initially displaced by the bubble. In principle, this process can be extremely effective at removing material from the galaxy, dramatically reducing the quantity of gas available for forming stars. As a result, it is difficult to quantify star formation rates resulting from cooling-induced gravitational instability, because it is unclear how much of the collapsed gas will be retained by the galaxy, and for how long. Our current best estimate is that the mass of material available for forming stars must fall between two well-defined limits. The upper limit is the case in which all of the collapsed gas mass goes into forming stars; the lower limit is the case in which the vast majority of the collapsed gas mass is expelled by AGN feedback leaving no material available for forming stars.

Given that the collapsing gas flows inwards on a dynamical timescale, \( t_{\text{dyn}} \), with a mass flow rate \( M \), the collapsing mass can be written as \( M = \dot{M} t_{\text{dyn}} \). Then, using the fact that the instantaneous mass flow rate is \( \dot{M} = \sigma_*^2 / (\alpha G) \), the maximum mass of gas available for forming stars during each collapse, will be

\[
M = \dot{M} t_{\text{dyn}} \approx \frac{\sigma_*^3}{\alpha G} t_{\text{dyn}} \approx 10^{10} \left( \frac{\sigma_*}{200 \text{ km s}^{-1}} \right)^3 \left( \frac{t_{\text{dyn}}}{10^7 \text{ yr}} \right) M_\odot.
\]

Since a fraction \( \beta \) is assumed to be accreted by the black hole, this leaves a maximum fraction of \( 1 - \beta \) available for forming stars, in the unlikely event that no material is removed due to feedback.

However, we can estimate the mass of material removed by the AGN using simple considerations. The total energy, \( E \), injected by an AGN as a result of accreting a mass \( \beta M \), will be \( E = \eta \beta M c^2 \). If this energy does work against the pressure, \( P \), of the ambient gas, it will inflate a bubble with volume, \( V \), given approximately by \( E = 4P V \), assuming relativistic bubble contents. Using the definition of the gas pressure \( P = \rho k T / \mu m_p \), where \( \rho \) is the ambient gas density and \( T \) is temperature, we can write \( E \approx 4M_{\text{dis}} k T / \mu m_p \), where \( M_{\text{dis}} \approx \rho V \) is the mass displaced in inflating the bubble. As previously noted, Pope et al. (2010) showed that a buoyant AGN-blown bubble will lift a mass comparable to \( M_{\text{dis}}/2 \) out from the central galaxy.

The mass of gas lifted out from the centre of a galaxy by AGN feedback can be related to the mass of material that collapsed into the galaxy due to cooling-induced gravitational instability

\[
\frac{M_{\text{dis}}/2}{M} \approx \eta \beta \frac{\mu m_p c^2}{8k T} \approx 8 \left( \frac{\beta}{10^{-3}} \right) \left( \frac{T}{10^7 \text{ K}} \right)^{-1}.
\]

According to equation (20), an individual AGN outburst may remove more gas than actually flowed into the galaxy centre. While this may seem unphysical, it is not – feedback could remove all of the material that collapsed into galaxy and additional ambient gas. The fate of the gas lifted out of the galaxy will depend on the AGN power and the depth of the external gravitational potential. There are two main possibilities: 1) if the outflow injects sufficient energy, material will be permanently expelled from the galaxy; 2) if not, the outflow will temporarily lift material out of the galaxy, later allowing it to fall back inwards.

\footnote{2} It is difficult to estimate how high this material will be lifted because it depends strongly on the details of the bubble, the extra mass of material it is carrying, and the properties of the ambient atmosphere. Pope et al. (2010) demonstrated that a bubble lifting additional material will rise to a height at which the average density of the bubble, plus the lifted mass, is equal to the ambient density. At this location the buoyancy force goes to zero, and the bubble cannot rise further unless it sheds some of the material. The greater the mass carried by the bubble, the less the bubble will rise. This effect directly limits the amount of energy extracted from the bubble. Because of this, it is not possible to say that the bubble will change the gravitational potential of the lifted mass by an amount defined by: \( E = (M_{\text{dis}}/2) \Delta \Phi \).
As shown in Popel (2004), AGN fuelled at a small fraction of the ambient cooling flow rate can power outflows that are capable of ejecting material from the potential of an elliptical galaxy. However, for the deeper gravitational potentials of galaxy groups and clusters (with virial temperatures greater than 1-2 keV) there are no black holes that are massive enough to sustain outflows that can expel gas from the potential. Despite this, typical AGN outflows in galaxy groups and clusters do still affect their environment by gently redistributing the gas within the gravitational potential. Furthermore, any material that is lifted out of the centre of the host galaxy will eventually fall back inwards, thereby becoming part of a fountain-like flow.

Inspection of equation (20) leads to the expectation that AGN feedback should remove more ambient gas, per unit accreted mass, from cooler (lower mass) systems. As described above, this material can be permanently expelled from low mass elliptical galaxies. Thus, we conclude that star formation in low mass elliptical galaxies would necessarily have to occur in a limited window of opportunity: after the gravitationally unstable material has collapsed into the centre of the galaxy, and before it has been removed by AGN feedback. In other words, any reservoir of material available for forming stars will be short-lived, meaning that star formation episodes are comparatively rare in such systems.

When applied to galaxy clusters, equation (20) indicates that the mass of material lifted out of the host galaxy by AGN feedback will approximately equal the mass of material which collapses into the galaxy due to gravitational instability. As a result, AGN feedback may also be able to shut-off star formation in BCGs. However, the fountain-like flow described above would provide a reservoir of material in and around the galaxy which is available for forming stars. Consequently, it is reasonable to expect some on-going star formation in BCGs. Interestingly, this result appears to be compatible with observations: Rafferty et al. (2006) found BCG star formation rates up to \( \sim 100 \, \text{M}_\odot \, \text{yr}^{-1} \), which corresponds to \( 10^{10} \, \text{M}_\odot \) over \( 10^8 \, \text{yr} \).

Finally, considering the mass flow rates, we can show that the ratio of black hole and stellar mass growth rates must be \( \dot{M}_{\text{BH}}/\dot{M}_* \equiv \beta/(1 - \beta) \approx \beta \). This suggests that, as the age of the system becomes very large, the ratio of black hole mass to bulge mass should also tend towards \( \beta \). It is encouraging to note that a value of \( \beta \approx 10^{-3} \) agrees well with observations (e.g. Haring & Rix 2004) and is consistent with our earlier constraints obtained by comparing the theoretical AGN duty cycle with radio observations (Best et al. 2005; Shabala et al. 2008).

4.2 Possible implications for cosmological simulations

The cooling-induced gravitational instability scenario for fuelling AGN can only be captured by fluid simulations if the following are true: 1) the self-gravity of the gas is included in the calculations, and 2) the spatial resolution is fine enough to track the evolution of the gravitationally unstable region. For a collapsing mass \( M \), with a velocity dispersion \( \sigma_* \), the region of importance has a size

\[
R \sim \frac{GM}{\sigma_*^2} \approx 1 \left( \frac{M}{10^{10} \, \text{M}_\odot} \right) \left( \frac{200 \, \text{km} \, \text{s}^{-1}}{\sigma_*} \right)^{-2} \text{kpc}. \tag{21}
\]

Thus, for all plausible values of \( M \) and \( \sigma_* \), the collapsing region is likely to be smaller than the resolved spatial scales in cosmological fluid simulations. We note that the density of these small, self-gravitating regions is likely to be high (\( \sim 10^{-22} \, \text{g} \, \text{cm}^{-3} \) for \( M \sim 10^{10} \, \text{M}_\odot \) and \( \sigma_* \sim 200 \, \text{km} \, \text{s}^{-1} \)) and scales as \( \rho \sim M/R^3 \propto \sigma_*^2/M^2 \).

5 SUMMARY

The aim of this article has been to explore the properties of AGN feedback that is driven by the gravitational instability of hot gas in the locality of a supermassive black hole. While it remains unclear whether gravitational instability itself is responsible for delivering fuel to AGN, we have shown that a mechanism which behaves similarly could produce outcomes that are compatible with several key observations of radio AGN in massive galaxies and clusters. The main findings are summarised below:

(i) Gas in the centre of a galaxy which periodically becomes gravitationally unstable provides a way of linking the AGN fuelling rate to galaxy’s large scale environment without requiring gas to flow 10s of kiloparsecs before reaching the black hole.

(ii) According to this model, the AGN duty cycle scales as \( \delta \propto L_X/\sigma_*^2 \), where \( L_X \) is the X-ray luminosity of the hot gas in a cluster and \( \sigma_* \) is the stellar velocity dispersion at the centre of the galaxy which hosts the AGN.

(iii) Applying simple scaling relations to this model, we find that the duty cycle of radio AGN in massive galaxies should scale as \( \delta \propto \sigma_*^2 \propto M_{\text{BH}}^{1.5} \), in reasonable agreement with observations.

(iv) The region which collapses may be very small (< 1 kpc) and, is, therefore, difficult to capture directly in cosmological numerical simulations.

(v) The model predicts that the typical AGN outburst duration should scale as \( t_{\text{dyn}} = \delta t_{\text{age}}/N \propto L_X/(\sigma_*^2 N) \), where \( t_{\text{age}} \) is the age of the cluster and \( N \) is the number of outbursts during this time. Plausible values for \( t_{\text{age}} \) and \( N \) imply that radio AGN outbursts fuelled by gravitational collapse can extend up to \( \sim 10^7 - 10^8 \) yrs, which is consistent with observations of AGN-blown bubbles.

(vi) Using simple arguments we have shown that AGN feedback is likely to remove more gas, per unit accreted mass, from the centre of a lower mass galaxy than a BCG. By completely expelling material from lower mass elliptical galaxies, AGN feedback can dramatically reduce their star formation rates. In contrast, AGN feedback cannot completely expel gas from a galaxy cluster. As a result, there may be more material in and around BCGs which is available for forming stars. Thus, it may be more difficult to completely shut-off star formation in BCGs than in lower mass galaxies.

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REFERENCES

Alexander P., 2000, MNRAS, 319, 8
Birzan L., Rafferty D. A., McNamara B. R., Wise M. W., Nulsen P. E. J., 2004, ApJ, 607, 800
Benson A. J., Bower R. G., Frenk C. S., Lacey C. G., Baugh C. M., Cole S., 2003, ApJ, 599, 38
Best P. N., 2007, New Astronomy Review, 51, 168
Best P. N., Kauffmann G., Brinchmann J., Charlot S., Ivezić Ž., White S. D. M., 2005, MNRAS, 362, 25
Best P. N., von der Linden A., Kauffmann G., Heckman T. M., Kaiser C. R., 2007, MNRAS, 379, 894
Binney J., Tabor G., 1995, MNRAS, 276, 663
Bower R. G., Benson A. J., Malbon R., Helly J. C., Frenk C. S., Baugh C. M., Cole S., 2006, MNRAS, 370, 645
Bower R. G., McCarthy I. G., Benson A. J., 2008, MNRAS, 390, 1399
Burns J. O., 1990, ApJ, 99, 14
Cattaneo A., Teyssier R., 2007, MNRAS, 376, 1547
Cavagnolo K. W., Donahue M., Voit G. M., Sun M., 2008, ApJ, 687, 840
Ciotti L., Ostriker J. P., 2001, ApJ, 551, 131
Crawford C. S., Allen S. W., Ebeling H., Edge A. C., Fabian A. C., 1999, MNRAS, 306, 857
Croton D. J., Springel V., White S. D. M., De Lucia G., Frenk C. S., 2006, MNRAS, 376, 1547
Dunn R. J. H., Fabian A. C., Taylor G. B., 2005, MNRAS, 364, 1343
Fabian A. C., 2009, ArXiv e-prints
Fabjan D., Borgani S., Tornatore L., Saro A., Murante G., Dolag K., 2010, MNRAS, 401, 1670
Häring N., Rix H.-W., 2004, ApJ, 604, L89
Kaast M. S., Tamura T., Peterson J. R., Bleeker J. A. M., Ferrigno C., Kahn S. M., Paerels F. B. S., Piffaretti R., Branduardi-Raymont G., Böhringer H., 2004, MNRAS, 343, 415
Kaviraj S., Schawinski K., Silk J., Shabala S. S., 2010, ArXiv e-prints
King A. R., 2009, MNRAS, pp 1914–+
Körding E. G., Jester S., Fender R., 2008, MNRAS, 383, 277
Maccarone T. J., Gallo E., Fender R., 2003, MNRAS, 345, L19
Mittal R., Hudson D. S., Reiprich T. H., Clarke T., 2009, A&A, 501, 835
O’Dea C. P., Baum S. A., 1997, ApJ, 113, 148
O’Sullivan E., Ponman T. J., Collins R. S., 2003, MNRAS, 340, 1375
Peterson J. R., Fabian A. C., 2006, Physical Reports, 427, 1

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Peterson J. R., Kahn S. M., Paerels F. B. S., Kaastra J. S., Tamura T., Bleeker J. A. M., Ferrigno C., Jernigan J. G., 2003, ApJ, 590, 207
Peterson J. R., Paerels F. B. S., Kaastra J. S., Arnaud M., Reiprich T. H., Fabian A. C., Mushotzky R. F., Jernigan J. G., Sakelliou I., 2001, A&A, 365, L104
Pizzolato F., Soker N., 2005, ApJ, 632, 821
Pizzolato F., Soker N., 2010, ArXiv e-prints
Pope E. C. D., 2009, MNRAS, pp 494–+
Pope E. C. D., 2011, ArXiv e-prints
Pope E. C. D., Babul A., Pavlovski G., Bower R. G., Dotter A., 2010, MNRAS, 406, 2023
Puchwein E., Sijacki D., Springel V., 2008, ApJ, 687, L53
Rafferty D. A., McNamara B. R., Nulsen P. E. J., 2008, ApJ, 687, 899
Rafferty D. A., McNamara B. R., Nulsen P. E. J., Wise M. W., 2006, ApJ, 652, 216
Sakelliou I., Peterson J. R., Tamura T., Paerels F. B. S., Kaastra J. S., Belsole E., Böhringer H., Branduardi-Raymont G., Ferrigno C., Kennea J., Mushotzky R. F., Vestrand W. T., Worrall D. M., 2002, A&A, 391, 903
Shabala S. S., Ash S., Alexander P., Riley J. M., 2008, MNRAS, 388, 625
Short C. J., Thomas P. A., 2009, ApJ, 704, 915
Sijacki D., Pfrommer C., Springel V., Enßlin T. A., 2008, MNRAS, 387, 1403
Sijacki D., Springel V., 2006, MNRAS, 366, 397
Silk J., Rees M. J., 1998, A&A, 331, L1
Soker N., 2006, New Astronomy, 12, 38
Tamura T., Kaastra J. S., Peterson J. R., Paerels F. B. S., Mittaz J. P. D., Trudolyubov S. P., Stewart G., Fabian A. C., Mushotzky R. F., Lumb D. H., Ikebe Y., 2001, A&A, 365, L87
Tremaine S., Gebhardt K., Bender R., Bower G., Dressler A., Faber S. M., Filippenko A. V., Green R., Grillmair C., Ho L. C., Kormendy J., Lauer T. R., Magorrian J., Pinkney J., Richstone D., 2002, ApJ, 574, 740
Tucker W., David L. P., 1997, ApJ, 484, 602
Xu H., Kahn S. M., Peterson J. R., Behar E., Paerels F. B. S., Mushotzky R. F., Jernigan J. G., Brinkman A. C., Makishima K., 2002, ApJ, 579, 600