Consensus of networked double integrator systems under sensor bias

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Summary

A novel distributed control law for consensus of networked double integrator systems with biased measurements is developed in this article. The agents measure relative positions over a time-varying, undirected graph with an unknown and constant sensor bias corrupting the measurements. An adaptive control law is derived using Lyapunov methods to estimate the individual sensor biases accurately. The proposed algorithm ensures that position consensus is achieved exponentially in addition to bias estimation. The results leverage recent advances in collective initial excitation-based results in adaptive estimation. Conditions connecting bipartite graphs and collective initial excitation are also developed. The algorithms are illustrated via simulation studies on a network of double integrators with local communication and biased measurements.

KEYWORDS

adaptive control, multi-agent systems, nonlinear control

1 | INTRODUCTION

Consensus of networked double integrators has been studied extensively in control literature and several globally convergent controllers have been proposed.1-5 This degree of interest is because the double integrator is one of the most fundamental block in any control system. Applications of double integrators include feedback linearizable nonlinear mechanical and aerospace systems such as free-rigid body motion, manipulator motion and spacecraft rotation. The consensus algorithms thus obtained can be further extended to complex nonlinear systems. Multi-agent systems with double integrator dynamics have been extensively studied in literature, see for example Reference 6 and the references therein. A comprehensive survey of the several consensus results in literature can be found in Reference 7. The preceding references, however, assume perfect measurements or extraneous disturbances only.

The motivation for the problem of consensus under sensor bias originates from mechanical systems that have only relative position and absolute velocity measurements available for feedback. However, relative position sensors suffer from errors such as bias in measurements. Unknown biases can appear during the functioning of various sensors such as rate gyros, accelerometers, magnetometers, altimeters, range sensors, etc. These biases can be an outcome of inaccurate sensor calibration, environmental conditions, etc. The presence of bias deteriorates the performance of control laws on the network, and may result in stability issues.8-10 Specifically, bias in relative position feedback could drive the agents to infinity, if not compensated. It is, therefore, of interest to estimate the biases and possibly nullify their effect on the network. In the context of the continuous system, bias uncertainties in measurements are in general, sparsely studied. In the context of a single rigid-body system, “gyro bias” is the most commonly addressed bias uncertainty and has been studied in detail in several references.11-13 However, the literature on adaptive estimation and compensation of ‘position’
sensor bias is somewhat limited and the only relevant contributions known to the authors are References 14 and 15. There have of course been parallel approaches using nonsmooth control laws, where disturbance rejection is possible for both single and networked second-order systems subject to knowledge of bounds on the bias uncertainty which is then modeled as a bounded disturbance. Such nonsmooth laws for disturbance rejection in double integrator systems have been explored in References 16-19. A distributed adaptive dynamic surface controller for a leader-follower multi-agent network of high-order nonlinear system is developed by the authors in Reference 20. Considering a constant communication graph and in the presence of unknown nonlinearities, unknown time-varying actuator defects, it is demonstrated that the tracking errors converge to a small neighborhood of the origin. The References 21 and 22 effectively address designing neural and barrier Lyapunov function controllers, respectively, for nonlinear time-delay systems, in the presence of actuator faults and various other constraints. However, References 20-22 use Lyapunov–Krasovskii functionals to compensate the unknown faults and time-delays and not explicitly take into account bias estimation. An approach to estimating measurement inconsistencies using an output regulation-based technique is presented in Reference 23. Accurate estimation in Reference 23, however, requires a unique constant, graph structure.

For uncertain networked double integrators (sensor bias being one such uncertainty), conventional adaptive control laws 24-28 require the regressor function to be persistently exciting (PE) or collectively persistently exciting (C-PE) for parameter convergence. Recently, several methods have been proposed to get rid of the PE condition for parameter convergence. Reference 29 proposes an adaptive algorithm that uses both instantaneous state data and past measurements for the adaptation process. This scheme ensures parameter estimation errors converge to zero exponentially, subject to the satisfaction of a finite-time excitation condition. In the same spirit, Reference 30 proposes a PI-like (Proportional-Integral controllers) parameter update law that guarantees parameter convergence with a relaxation of the PE condition, namely initial excitation (IE) on the regressor. Reference 31 is an extension of Reference 30, where the authors develop a distributed composite adaptive synchronization algorithm for multiple uncertain Euler–Lagrange (EL) systems to ensure parameter convergence using the collective-IE (C-IE) condition. The method proposed in Reference 30 obviates the need for data storage and memory allocation required in concurrent learning-based adaptive control methods. 32

There have been several strides in consensus under bounded disturbance and zero-mean noise. In Reference 33, a leader-follower consensus control for a network of double integrators is proposed for follower measurements corrupted by (zero-mean) noise. This control law ensures that consensus tracking is achieved in the mean square sense for both fixed and switching communication networks. However, no bias errors are accounted for in this work. Reference 34 shows a Kalman filter inspired technique for consensus which is input to state stable. As would be expected, the accuracy of the cooperation objective is directly related to the power level of the communication noise. Reference 35 analyzes the asymptotic properties of linear consensus algorithms in the presence of bounded measurement errors. Here, consensus is not guaranteed with respect to all possible noise realizations. In Reference 36, a novel self-triggering co-ordination scheme for finite time consensus is proposed in the presence of unknown but bounded noise affecting the communication channels. Bias errors with unknown bounds are not the subject of study in any of the above articles.

We now summarize some of the results that lead up to the current work on consensus under measurement bias with unknown bounds. Reference 37 proposes an adaptive control law in the presence of unknown constant bias for a double integrator network. This controller ensures bounded closed-loop signals in the presence of sensor bias which would not be the case in the absence of adaptation. However, convergence only to a neighborhood of consensus can be shown. Reference 37 is based on the results in Reference 14 which addresses the problem of accommodating unknown sensor bias in a direct MRAC setting for a single agent. In Reference 38, the authors present a consensus algorithm for synchronization of double integrators over directed graphs in the presence of constant bias with unknown bounds. Here, the authors assume the existence of a bias error on each communication channel. Similar to Reference 37, here too convergence “near” a common equilibrium point is guaranteed. Reference 39 shows an extension of References 37 and 38 to develop a distributed consensus tracking algorithm for spacecraft in formation modeled as an EL network with similar bounded performance results. For a fixed communication graph, exact constant estimation of a constant bias and consensus in single integrator agents are demonstrated in Reference 40. An undirected, connected, and non-bipartite graph network is shown to be necessary and sufficient for estimation of the full bias vector.

In comparison with existing literature, the novel contributions of this article are as below.

1. An adaptive control law for exact bias estimation and consensus is developed over References 37-39. One sensor attached to each node is considered and all relative measurements from a node are assumed to be affected by the same bias in contrast to Reference 38.
2. As opposed to Reference 40 that considers only the bias-estimation problem on single-integrators, we look at an adaptive consensus control problem in parallel to bias estimation.

3. The aforementioned article further requires a constant and hence connected, non-bipartite graph (for all time). We consider a time-varying communication topology in the derivations. The analysis is based on the joint-connectivity property and a joint non-bipartite property is shown to be necessary for adaptive bias consensus as proposed here.

4. A collective initial excitation-based adaptive controller is employed for the first time in bias estimation problems over networks. This removes the requirement for the graph to be non-bipartite for all time.

This paper is organized as follows. Section 2 introduces mathematical notation, necessary lemmas and in brief, graph theory. In Section 3, we formulate the consensus problem over a network of double integrator systems. We develop an adaptive control law for achieving consensus and bias estimation in Section 4. A discussion on the choice of control gains and the collective initial excitation condition on the regressor matrix are presented in Section 5. Section 6 presents numerical simulations validating our algorithm. Conclusions are presented in Section 7.

2 | PRELIMINARIES

In this section, we present several mathematical notations, lemmas, assumptions, and a concise introduction of graph theory that forms the basis of the problem formulation.

2.1 | Notation

\( \mathbb{R}^+ \) denotes nonnegative reals. Kronecker product is denoted by \( \otimes \). The Euclidean norm of a vector \( x \) is denoted by \( \| x \| \) and the corresponding induced matrix Euclidean norm by \( \| A \| \) for a matrix \( A \). A diagonal matrix with elements \( d_1, d_2, \ldots, d_n \) on the diagonal is represented by \( \text{diag}(d_1, \ldots, d_n) \). The \( n \times n \) identity matrix and zero matrix are denoted by \( I_n \) and \( 0_n \), respectively; a \( n \)-dimensional vector of ones is denoted \( 1_n \). For a matrix \( A \), the maximum and minimum eigenvalues are respectively denoted by \( \lambda_{\text{max}}(A) \) and \( \lambda_{\text{min}}(A) \). For a symmetric matrix \( \Gamma \), the notation \( \Gamma > 0 \) (\( \Gamma < 0 \)) is used to denote a positive-definite (negative-definite) matrix. For a matrix signal, \( A(\cdot) : \mathbb{R}^+ \to \mathbb{R}^{p \times p} \), we define \( \| A \|_\infty \triangleq \sup_{t \geq 0} \| A(t) \| \), the signal infinity norm. Time and initial condition arguments for all state variables (variables with dynamics) are uniformly omitted for notational simplicity. Similarly, function arguments for the control variables are suppressed, and they are made clear through explicit expressions proposed later in the manuscript.

2.2 | Graph theory

Consider a network of \( n \) agents interacting with each other over a time-varying graph.

We define the interaction graph as a function of time \( (t \geq 0) \) through the tuple, \( G(t) \triangleq (\mathcal{V}, \mathcal{E}(t)) \), where \( \mathcal{V} \triangleq \{1, \ldots, n\} \) is a node set and \( \mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V} \) is an edge set signifying interaction between nodes\(^6\) at time instant \( t \).”

If an edge \( (i, j) \in \mathcal{E}(t) \), then node \( i \) is called a neighbor of node \( j \) with \( j \) being the head node and \( i \) being the tail node indicating information flow from \( i \) to \( j \). The set of neighbors of a node \( i \) at time \( t \), is denoted by \( \mathcal{N}_i(t) \). In an undirected graph, \( (j, i) \in \mathcal{E}(t) \) if and only if \( (i, j) \in \mathcal{E}(t) \). An undirected graph \( G(t) \) is instantaneously connected if there is an undirected path between every pair of distinct nodes. The adjacency matrix, \( \mathbb{R}^{n \times n} \ni \mathcal{A}(t) = [a_{ij}(t)] \), is defined such that \( a_{ij}(t) > 0 \) if \( (j, i) \in \mathcal{E}(t) \) and \( a_{ij}(t) = 0 \) if \( (j, i) \notin \mathcal{E}(t) \). We assume no self edges are present and hence, \( a_{ii}(t) = 0 \) for all \( t \). For an undirected graph, \( A \) is symmetric. The degree matrix of the graph \( G \) is, \( D(t) \triangleq \text{diag} \left( \sum_{j=1}^{n} a_{ij}(t), \ldots, \sum_{j=1}^{n} a_{nj}(t) \right) \in \mathbb{R}^{n \times n} \) and the Laplacian matrix, \( \mathcal{L}(t) \triangleq [l_{ij}(t)] \in \mathbb{R}^{n \times n} \) is computed as:

\[
\mathcal{L}(t) = D(t) - \mathcal{A}(t)
\]

\[
l_{ij}(t) = \sum_{j=1, j \neq i}^{n} a_{ij}(t), \quad l_{ij}(t) = -a_{ij}(t), \ i \neq j.
\]
As evident from above, $\mathcal{L}(t)$ is symmetric for undirected graphs. Further, $\mathcal{L}(t)$ has both row and column sums zero indicating that $0$ is an eigenvalue with a corresponding eigenvector being $\mathbf{1}_n$ (vector of ones), that is, $\mathcal{L}(t)\mathbf{1}_n = \mathbf{1}_n^T \mathcal{L}(t) = 0$. Another symmetric matrix of interest is the signless Laplacian defined as $Q(t) \triangleq D(t) + A(t) = [Q_{ij}(t)] \in \mathbb{R}^{n \times n}$ where,

$$Q_{ii}(t) = \sum_{j=1,j\neq i}^{n} a_{ij}(t), \quad Q_{ij}(t) = a_{ij}(t), i \neq j.$$ 

For an undirected graph $\mathcal{G}(t)$, both $\mathcal{L}(t)$ and $Q(t)$ are positive semi-definite matrices. A union graph denoted $\bigcup_{\tau \in [t,t+T]} \mathcal{G}(\tau)$ is the graph formed by Adjacency matrix elements, $\bar{a}_{ij}(t) \triangleq \int_{t}^{t+T} a_{ij}(\tau) \, d\tau$. The union graph, as defined, is the graph obtained by collecting all the edges in the sub-graphs appearing over a time-interval $[t, t+T]$.

**Definition 1.** (Reference 41) For a time-varying graph $\mathcal{G}(t)$ with adjacency matrix elements $a_{ij}(t)$, the weighted incidence matrix $\mathcal{H} : \mathbb{R}^+ \to \mathbb{R}^{n \times \frac{n(n+1)}{2}}$ is defined as,

$$\mathcal{H}(t) \triangleq \hat{h}_{ij}(t) \triangleq \begin{cases} \sqrt{a_{ij}(t)}, & \text{if } e_j = (i,j) \\ -\sqrt{a_{ij}(t)}, & \text{if } e_j = (j,i) \\ 0, & \text{otherwise.} \end{cases}$$

**Remark 1.** In the aforementioned definition, for undirected graphs, it is standard practice to choose an arbitrary orientation (information flow direction). This has no effect on the graph Laplacian and can always be computed as, $\mathcal{L}(t) = \mathcal{H}(t)\mathcal{H}^T(t)$.

**Definition 2.** (Reference 42) At any given time ‘$t$’, an undirected graph $\mathcal{G}(t)$ is called bipartite if there exists a disjoint partition of the node set denoted as $\mathcal{V} = \mathcal{V}_+(t) \cup \mathcal{V}_-(t)$ such that all edges in $\mathcal{G}(t)$ are between the node sets, and there are no edges within the node set. Mathematically, for all $(i,j) \in \mathcal{E}(t)$, $i \in \mathcal{V}_k(t) \Rightarrow j \in \mathcal{V} \setminus \mathcal{V}_k(t)$ for $k \in \{+, -\}$. A graph is called jointly (non-)bipartite over $[t, t+T]$ if the corresponding union graph $\bigcup_{\tau \in [t,t+T]} \mathcal{G}(\tau)$ is (non-)bipartite.

**Remark 2.** The above definition implies that the graph need not necessarily be (non-)bipartite for all time instants between $[t, t+T]$, but the graph obtained by collecting all the edges in the sub-graphs appearing over the time interval is (non-)bipartite.

**Definition 3.** (Reference 41) The time-varying graph $\mathcal{G}(t)$ is termed jointly ($\delta, T$)-connected if there are two real numbers $\delta > 0$ and $T > 0$ such that the edges $(j, i)$ satisfying,

$$\int_{t}^{t+T} a_{ij}(s) \, ds \geq \delta, \quad i, j \in \mathcal{V},$$

form a connected graph over $\mathcal{V}$ for all $t \geq 0$.

**Definition 4.** (Persistence of excitation) A locally integrable function $\phi : \mathbb{R}^+ \to \mathbb{R}^{nxm}$ is said to be persistently exciting if there exist positive constants $\mu_1, \mu_2,$ and $T$ such that,

$$\mu_1 I_n \leq \int_{t}^{t+T} \phi(\tau)\phi^T(\tau) \, d\tau \leq \mu_2 I_n, \quad \forall t \geq 0.$$ 

**Definition 5.** (Reference 30) (Initial excitation) A locally integrable function $\phi : \mathbb{R}^+ \to \mathbb{R}^{pxq}$ is said to be initially exciting if there exist constants $T, \eta > 0$ such that,

$$\int_{t_0}^{t_0+T} \phi^T(\tau)\phi(\tau) \, d\tau \geq \eta I_q, \quad \text{some } t_0 \geq 0.$$ 

The extension of persistence and initial excitation conditions to multi-agent systems are termed collective persistence of excitation (C-PE) and collective initial excitation (C-IE), respectively. These are defined below.

**Definition 6.** A set of bounded, locally integrable signals $\phi_i : \mathbb{R}^+ \to \mathbb{R}^{pxi}, \forall i = \{1, \ldots, n\}$, are C-PE, if there exist constants $T > 0$ and $\gamma > 0$ such that,
\[
\int_{t_0}^{t+T} \sum_{i=1}^{n} \phi_i(t) \phi_i(t) \, dt \geq \gamma I_n, \quad \forall t \geq t_0 \geq 0.
\]

**Definition 7.** A set of bounded, locally integrable signals \( \phi_i : \mathbb{R}^+ \to \mathbb{R}^{k_{\times p}}, \forall i \in \{1, \ldots, n\} \), are C-IE, if there exist constants \( \bar{T} > 0 \) and \( \gamma > 0 \) such that,
\[
\int_{t_0}^{t_0 + \bar{T}} \sum_{i=1}^{n} \phi_i(t) \phi_i(t) \, dt \geq \gamma I_n, \quad \text{some } t_0 \geq 0.
\]

The following assumption is intrinsic to the subsequent results.

**Assumption 1.** The network graph, \( \mathcal{G}(t) \), is undirected and jointly \((\delta, T)\)-connected for some \( \delta, T > 0 \). We assume the same graph \( \mathcal{G}(t) \) for both relative measurements as well as information exchange. Further, we assume existence of an \( a_M > 0 \) such that \( a_{ij}(t) \leq a_M \) for all \( i, j \in \{1, 2, \ldots, n\} \) and for all \( t \geq 0 \).

### 2.3 Fundamental results

We state a few results from graph theory and consensus analysis to be used subsequently.

**Proposition 1.** (Reference 42) A graph \( \mathcal{G} \) is bipartite if and only if \( \mathcal{G} \) has no cycle of odd length.

**Proposition 2.** (Reference 43) The smallest eigenvalue of the signless Laplacian matrix\( Q = D + A \) of an undirected and connected graph is equal to zero if and only if the graph is bipartite. In case the graph is bipartite, zero is a simple eigenvalue.

Theorem 3.4 from Reference 44 has been rephrased as follows.

**Proposition 3.** Consider the time-varying dynamics,
\[
\dot{x} = -\sigma N(t)N^\top(t)x, \quad x(0) = x_0,
\]
with \( N : \mathbb{R}^+ \to \mathbb{R}^{k_{\times p}} \) being a piecewise continuous matrix function. If \( N(\cdot) \) is persistently exciting (Definition 4), then the above dynamics admits a Lyapunov function
\[
V(t,x) = \frac{1}{2} x^\top [\pi I_k + S(t)] x,
\]
where,
\[
S(t) = 2\delta_T I_k - \frac{2}{T} \int_{t_0}^{t_0 + T} \int_{t}^{t_0 + T} N(\tau)N^\top(\tau) \, d\tau \, d\tau,
\]
and the positive constants \( \pi, \delta_T \) are defined as,
\[
\delta_T \triangleq T |N(\cdot)N^\top(\cdot)|_{\infty},
\]
\[
\pi \triangleq 1 + \frac{2\sigma^2 \delta_T^3}{\mu_1}.
\]

Further, the states of the dynamical system (1) are uniformly exponentially stable at the origin.

The following result was established as part of the proof in Reference 41 (section II.1A).

**Proposition 4.** The following hold for an undirected graph \( \mathcal{G}(t) \) over \( n \) nodes, with Laplacian \( \mathcal{L}(t) \) and weighted incidence matrix \( \mathcal{H}(t) \) (Definition 1).

- \( \left( \mathcal{L}(t) + \frac{1_{n}^\top}{n} \right) = \left( \mathcal{H}(t) + \frac{1_{n} h^\top(t)}{\sqrt{n}} \right) \left( \mathcal{H}(t) + \frac{1_{n} h^\top(t)}{\sqrt{n}} \right)^\top \), where \( h(t) \) is unit vector in the kernel of \( \mathcal{H}(t) \).
- The graph \( \mathcal{G}(t) \) is jointly \((\delta, T)\)-connected (Definition 3) for some \( \delta, T > 0 \) if and only if \( \left( \mathcal{H}(t) + \frac{1_{n} h^\top(t)}{\sqrt{n}} \right) \) is persistently exciting (Definition 4).
3 | BIAS ESTIMATED CONSENSUS

The objective of this article is to develop a distributed consensus algorithm for a network of double integrator systems in the presence of a constant unknown bias corrupting the relative measurements of position while ensuring estimation of all biases by each agent. The interaction between the agents is modeled by an undirected and jointly (\(\delta, T\))-connected graph, \(\mathcal{G}(t)\), with an associated Laplacian \(\mathcal{L}(t)\). The input-output model for each agent representing a node in \(\mathcal{G}(t)\) is expressed as the following double integrator equation,

\[
\begin{align*}
\ddot{q}_i &= u_i, \\
q_i(0) &= q_{i0}; \quad \dot{q}_i(0) = \dot{q}_{i0}; \quad i = 1, 2, \ldots, n \\
y_i &= [z_{ij}, \dot{q}_i]^\top, \quad i = 1, 2, \ldots, n, \; j \in \mathcal{N}_i(t),
\end{align*}
\]

where state \(q_i \in \mathbb{R}^m\) is the vector of generalized coordinates (called “positions” in general with their derivatives being “velocities”), \(u_i(\cdot) \in \mathbb{R}^m\) is a distributed, time-varying feedback, \(y_i = [z_{ij}, \dot{q}_i] \in \mathbb{R}^{2m}\) is the information state available to each agent with \(\mathbb{R}^m \ni z_{ij} \triangleq (q_i - q_j + b_i)\) and \(b_i \in \mathbb{R}^m\) is the constant, unknown sensor bias. The estimate of sensor bias \((\hat{b}_i)\) for each agent \(k\) will be denoted \(\hat{b}_k\) in the sequel. Each agent \(k\) has an estimate of all sensor biases \((\hat{b}^k = [\hat{b}^k_1, \hat{b}^k_2, \ldots, \hat{b}^k_n]^\top)\) and a dynamic update law is designed for the same.

The control objective in this article is to design a distributed feedback \(u_i\) so that the closed-loop solutions of (2) satisfy,

\[
\begin{align*}
\lim_{t \to \infty} (q_i - q_j) &= 0, \quad \forall i, j \in \{1, 2, \ldots, n\} \\
\lim_{t \to \infty} \dot{q}_i &= 0 \quad \forall i \in \{1, 2, \ldots, n\} \\
\lim_{t \to \infty} (b_i - \hat{b}_i) &= 0 \quad \forall i, k \in \{1, 2, \ldots, n\}
\end{align*}
\]

(Bias estimated consensus)

The following assumption delineates the information available for the design of the feedback law \(u_i\).

**Assumption 2.** Agents can measure their own velocities \(\dot{q}_i\); neighbors measure relative velocities \(\dot{q}_i - \dot{q}_j\) and relative position corrupted by a constant unknown bias \((z_{ij} = q_i - q_j + b_i)\). Further, neighbors exchange their measurement of relative positions \((z_{ji} = q_j - q_i + b_i)\) and their estimate of biases \((\hat{b}^j)\) with each other.

4 | CONTROL LAW DESIGN

We prescribe a controller \(u_i\) for (2) to satisfy our control objective (Bias estimated consensus) where \(b_i, \; i = 1, 2, \ldots, n\) are assumed to be unknown, constant measurement biases.

In this article, we consider the following distributed control algorithm,

\[
u_i = k(t)\left(-\dot{\hat{q}}_i - \frac{1}{2} \sum_{j \in \mathcal{N}_i} a_{ij}(t)[z_{ij} + z_{ji}]\right) + w_i,
\]

where, \(k : \mathbb{R}^+ \to \mathbb{R}^+\) is a positive valued uniformly bounded function (there exists \(k_M > 0\) such that \(k(t) \leq k_M\) for all \(t \geq 0\)) to be prescribed later, \(q = [q_1, \ldots, q_n]^\top\) and \(w_i(\cdot) \in \mathbb{R}^m\) is an auxiliary control term. The implementation of the term \(z_{ij}\) in the control law requires that all neighbors’ relative position measurements (corrupted by bias) be communicated to each agent. This is guaranteed by Assumption 2. The individual control expressions above can now be collected to specify the feedback for the entire double integrator network as follows:

\[
u = -k(t)\dot{q} + \frac{k(t)}{2} [\mathcal{L}(t) \otimes I_m] b - k(t) [D(t) \otimes I_m] b + w,
\]

where \(u, \; b, \; \text{and} \; w\) are the column stacked vectors of \([u_1, \ldots, u_n]^\top, [b_1, \ldots, b_n]^\top\) and \([w_1, \ldots, w_n]^\top\) respectively. Let \(\overline{\mathcal{L}}(t) \triangleq \mathcal{L}(t) \otimes I_m\) and \(\overline{D}(t) \triangleq D(t) \otimes I_m\), then (3) can be simplified as,
\[ u = -k(t)\overline{D}(t)b - k(t)\dot{q} + \frac{k(t)}{2}\overline{L}(t)b + w. \]  

(4)

We also obtain the network dynamics from (2) as,

\[ \ddot{q} = u. \]  

(5)

Substituting the control law (4) in (5), we obtain the following closed-loop network dynamics,

\[ \ddot{q} + k(t)\dot{q} + k(t)\left(\overline{D}(t) - \frac{1}{2}\overline{L}(t)\right)b = w. \]  

(6)

It is worth noting that \( \overline{D}(t) - \overline{L}(t)/2 = Q(t)/2 \otimes I_m \equiv \overline{Q}(t)/2 \). (6) can be written in a standard regressor-parameter form as,

\[
\begin{bmatrix}
\ddot{q}, & k(t)\dot{q}, & \frac{1}{2}k(t)\overline{Q}(t)
\end{bmatrix}
\begin{bmatrix}
1 & 1 & b^T
\end{bmatrix} = w,
\]

with \( Y : \mathbb{R}^m \times \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^{mn \times (mn+2)} \) denoting the regressor and \( \theta \in \mathbb{R}^{mn+2} \) being the constant, unknown parameter.

We can also write, corresponding to each agent:

\[
Y_i = \begin{bmatrix}
\ddot{q}_i, & k(t)\dot{q}_i, & \frac{1}{2}k(t)[Q_i(t)I_m, Q_{i2}(t)I_m, \ldots, Q_{in}(t)I_m]
\end{bmatrix}.
\]  

(7)

\( Y_i \) and \( w_i \) are available for each agent, that is, \( Y_i \in \mathbb{R}^{mn \times (mn+2)} \) corresponds to each \( m \) rows of \( Y \), \( w_i \in \mathbb{R}^m \) corresponds to each \( m \) rows of \( w \). Additionally, we have corresponding to each agent, \( Y_i \theta = w_i \). Each agent has an adaptive estimate of the unknown parameter vector \( \theta \) for all \( i = 1, \ldots, n \) which is an over-parametrization of \( b \) denoted \( \hat{\theta} \in \mathbb{R}^{mn+2} \). \( \hat{\theta} = \begin{bmatrix} 1 & 1 & b^T \end{bmatrix} \) where \( b^i = [\hat{b}^i_1, \hat{b}^i_2, \ldots, \hat{b}^i_n]^T \). We define the agent parameter error as, \( \check{\theta}^i \equiv [0 \ 0 \ b^i]^T \), where \( \check{b}^i = [b_1 - \hat{b}^i_1, \ldots, b_n - \hat{b}^i_n]^T = [\hat{b}^i_1, \ldots, \hat{b}^i_n]^T \). We now define,

\[ s_i = \ddot{q}_i + \lambda \left( q_i + \frac{\check{b}^i}{2} \right), \quad \lambda > 0, \ i = 1, 2, \ldots, n, \]  

(8)

where \( \check{b}^i = b_i - \hat{b}^i \). Further we assign, \( s \equiv [s_1, \ldots, s_n]^T \), \( \check{b}^{\text{new}} \equiv [\check{b}^1, \ldots, \check{b}^n]^T \) and obtain \( s = \ddot{q} + \lambda (q + \check{b}^{\text{new}}/2) \).

Note - \( \check{b}^i \) has been used in (8), and not \( \check{b}^i \) as \( b^i = b_i - \check{b}^i \) will require bias estimate information of \( b_i \) computed by agent \( k \not\in \mathcal{N}_i \), which may not be available with agent \( i \).

Taking the derivative of \( s \) and substituting from (6),

\[
\begin{aligned}
\dot{s} &= \ddot{q} + \lambda \left( q + \frac{\check{b}^{\text{new}}}{2} \right) = -k(t)\dot{q} - \frac{1}{2}k(t)\overline{Q}(t)b + w + \lambda \left( \ddot{q} + \frac{\check{b}^{\text{new}}}{2} \right) \\
&= \begin{bmatrix}
0_{mn \times 1} & 0_{mn \times 1} & -\frac{1}{2}k(t)\overline{Q}(t)
\end{bmatrix}
\begin{bmatrix} 1 & 1 & b^T \end{bmatrix} - k(t)\dot{q} + w + \lambda \left( \ddot{q} + \frac{\check{b}^{\text{new}}}{2} \right).
\end{aligned}
\]  

(9)

The corresponding dynamics for each agent \( s_i \in \mathbb{R}^m \), is given by

\[
\dot{s}_i = Z_i \theta - k(t)\dot{q}_i + w_i + \lambda \left( \ddot{q}_i + \frac{\check{b}^i}{2} \right),
\]

where \( Z_i \in \mathbb{R}^{mn \times (mn+2)} \) is defined similar to \( Y_i \). We now define the second part of the control, \( w \) at each agent node as,

\[
w_i = k(t)\dot{q}_i - Z_i \hat{\theta}^i - \lambda \left( \ddot{q}_i + \frac{\check{b}^i}{2} \right) - \sigma \sum_{j \in \mathcal{N}_i} a_{ij}(t)(s_i - s_j),
\]  

(10)
for some $\sigma > 0$, which can be written in an implementable form as,

$$w_i = k(t)\ddot{q}_i - \lambda \dot{q}_i - \frac{\lambda \dot{b}_i}{2} + k(t)\sum_{j \in N_i} a_{ij}(t)\ddot{b}_j - \sigma \sum_{j \in N_i} a_{ij}(t)(s_i - s_j) - \frac{k(t)}{2}\sum_{j \in N_i} a_{ij}(t)(\dot{b}_i - \dot{b}_j).$$  \hspace{1cm} (11)

The above yields for the entire network the following,

$$w = k(t)\dot{q} - \lambda \left(\dot{q} + \frac{\dot{b}_{\text{new}}}{2}\right) - \sigma \mathcal{L}(t)s - Z_{\text{new}}\dot{\theta},$$  \hspace{1cm} (12)

where $\dot{\theta} \triangleq [\dot{\theta}^1, ..., \dot{\theta}^n]^T$ and $Z_{\text{new}}(\dot{q}, t) \in \mathbb{R}^{mn \times (mn+2)}$ is defined as

$$Z_{\text{new}}(\dot{q}, t) \triangleq \text{diag}(Z_1(\dot{q}_1, t), Z_2(\dot{q}_2, t), ..., Z_n(\dot{q}_n, t)).$$

Since the bias, $b$, is constant we have, $\dot{b} = \dot{b}_i$ and hence is implementable in (11). Further, though $s_i$ is not implementable (due to the $(\dot{q}_i + \dot{b}_i/2)$ term in $s_i$), $(s_i - s_j) = \dot{q}_i - \dot{q}_j + \frac{\lambda}{2} \{z_j - z_i - (\dot{b}_j - \dot{b}_i)\}$ is, and that is what appears in the control law (11). Further, the implementation of the term $(\dot{b}_i - \dot{b}_j)$ requires neighbors to exchange their bias estimates (Assumption 2).

The control law $u_i$ after substituting for $w_i$ from (11) is given by:

$$u_i = \frac{k(t)}{2}\sum_{j \in N_i} a_{ij}(t)\left(\ddot{b}_j - \dot{b}_i\right) - \lambda \dot{q}_i - \frac{\lambda \dot{b}_i}{2} - \sigma \sum_{j \in N_i} a_{ij}(t)(s_i - s_j) - k(t)\sum_{j \in N_i} a_{ij}(t)\left(\dot{b}_j\right).$$  \hspace{1cm} (13)

Substituting (10) in (9),

$$\dot{s} = Z_{\text{new}}\dot{\theta} - \sigma \mathcal{L}(t)s,$$  \hspace{1cm} (14)

where $\dot{\theta} \triangleq [\dot{\theta}^1, ..., \dot{\theta}^n]^T$, $\dot{\theta}^i = \theta - \dot{\theta}_i$.

### 4.1 Bias estimation

The matrix $Y_i(\dot{q}_i, \ddot{q}_i, t)$ in (7) is dependent on the acceleration term $\ddot{q}_i$, and so cannot be used in our adaptation law for bias estimation. In order to facilitate relaxation of the persistence of excitation condition, a filter is designed for each agent as proposed in Reference 31,

$$Y_{F_i} = -\beta Y_{F_i} + Y_i(\dot{q}_i, \ddot{q}_i, t), \hspace{1cm} Y_{F_i}(0) = 0$$

$$w_{F_i} = -\beta w_{F_i} + w_i, \hspace{1cm} w_{F_i}(0) = 0,$$  \hspace{1cm} (15)

where $\beta > 0$ is the scalar filter gain, $Y_{F_i} \in \mathbb{R}^{m \times (mn+2)}$ and $w_{F_i} \in \mathbb{R}^m$. Solving the above equations explicitly we obtain,

$$Y_{F_i}(t) = e^{-\beta t}\int_0^t e^{\beta r}Y_i(\dot{q}_i, \ddot{q}_i, r) \, dr$$

$$w_{F_i}(t) = e^{-\beta t}\int_0^t e^{\beta r}w_i \, dr.$$  \hspace{1cm} (16) (17)

Utilizing the relation $Y_i(\dot{q}_i, \ddot{q}_i, t)\theta = w_i$ we get $Y_{F_i}\theta = w_{F_i}$ from (16) and (17). $Y_{F_i}$ in (15) cannot be solved explicitly as $Y_i(\dot{q}_i, \ddot{q}_i, t)$ is not measured. Therefore, we split $Y_i(\dot{q}_i, \ddot{q}_i, t)$ into measured and non-measured parts as,

$$Y_i(\dot{q}_i, \ddot{q}_i, t) = Y_1(\dot{q}_i) + Y_2(\dot{q}_i, t),$$
where
\[
Y_1(\dot{q}_i) = \begin{bmatrix} \dot{q}_i, & 0_{m \times 1}, & 0_{m \times mn} \end{bmatrix}
\]
\[
Y_2(\dot{q}_i, t) = \begin{bmatrix} 0_{m \times 1}, & k(t)q_i, & \frac{1}{2}k(t)[Q_1(t)I_m, Q_2(t)I_m, \ldots, Q_n(t)I_m] \end{bmatrix}
\]

This implies that \( Y_{F_i} = Y_{F_1} + Y_{F_2} \), where,
\[
Y_{F_1} = -\beta Y_{F_1} + Y_1, \quad Y_{F_1}(0) = 0
\]
\[
Y_{F_2} = -\beta Y_{F_2} + Y_2, \quad Y_{F_2}(0) = 0.
\]

Since \( Y_2(\dot{q}_i, t) \) is known, \( Y_{F_i} \) can be solved online using (18) by employing a numerical integration scheme. We solve \( Y_{F_i} \) analytically as follows,
\[
Y_{F_i}(t) = e^{-\beta t} \int_0^t e^{\beta r} Y_1(\dot{q}(r))dr = \begin{bmatrix} e^{-\beta t} \int_0^t e^{\beta r} \dot{q}(r)dr & 0 & 0 \end{bmatrix}.
\]

The elements of \( Y_{F_i} \) can be evaluated using integration by parts as follows,
\[
Y_{F_i}(t) = \begin{bmatrix} e^{-\beta t}[e^{\beta t}\dot{q}_i(t) - \dot{q}_i(0)] - e^{-\beta t} \int_0^t \beta e^{\beta r} \ddot{q}_i(r)dr & 0 & 0 \\
\dot{q}_i(t) - e^{-\beta t} \dot{q}_i(0) - h_i(t) & 0 & 0 \\
0 & 0 & 0 
\end{bmatrix}.
\]

∀ \( i = 1, \ldots, n \) and
\[
\dot{h}_i = \beta \dot{q}_i - \beta h_i, \quad h_i(0) = 0.
\]

\( Y_{F_i}, w_{F_i} \) are filtered regressor and filtered control for each agent \( i \), respectively. \( Y_{F_i} \) can now be used in our adaptation law. Additionally, for bias estimation, we will make use of the double filtered regressor and control law introduced in References 30 and 31.

\[
Y_{IF_i} = Y_{F_i}^T Y_{F_i}, \quad Y_{IF_i}(0) = 0,
\]
\[
w_{IF_i} = Y_{F_i}^T w_{F_i}, \quad w_{IF_i}(0) = 0.
\]

where \( Y_{IF_i} \in \mathbb{R}^{(mn+2) \times (mn+2)}, w_{IF_i} \in \mathbb{R}^{mn+2} \).

**Fact 1.** (Reference 30) Integrating (19) and (20) and using the relation \( Y_{F_i}^T \theta = w_{F_i} \), it can be verified that,
\[
Y_{IF_i} \theta = w_{IF_i}, \quad \forall t \geq 0.
\]

**Fact 2.** (Reference 30) The solution \( Y_{IF_i}(t) \) of (19) is a nonnegative and nondecreasing function of time.

The adaptive control law for bias estimation is now chosen as,
\[
\dot{\hat{\theta}}^i = \mu_F Y_{F_i}^T (w_{F_i} - Y_{F_i}^T \hat{\theta}^i) + \mu_{IF}(w_{IF_i} - Y_{IF_i}^T \hat{\theta}^i) + \sum_{j \in N_i} a_{ij}(\hat{\theta}^j - \hat{\theta}^i), \forall i = 1, \ldots, n,
\]

which using Fact 1 can be written as,
\[
\dot{\hat{\theta}} = -\mu_F \phi_F \theta - \mu_{IF} \phi_{IF} \hat{\theta} - L \otimes I_{(mn+2)} \hat{\theta},
\]
\[
\ddot{\hat{b}} = \begin{bmatrix} \ddot{\hat{\theta}}^3 & \ddot{\hat{\theta}}^4 & \cdots & \ddot{\hat{\theta}}_{(mn+2)} \end{bmatrix}^T
\]

for constant \( \mu_F, \mu_{IF} > 0 \) and arbitrary initial conditions. \( \ddot{\hat{\theta}}^{(k)} \) denotes the \( k \)-th element of \( \dot{\hat{\theta}}^i \) and so on. \( \phi_F, \phi_{IF} \in \mathbb{R}^{(mn+2) \times (mn+2)} \) are block diagonal matrices and are defined as,
\[
\phi_F \triangleq \text{diag} \left( Y_{F_1}^T Y_{F_1}, \ldots, Y_{F_n}^T Y_{F_n} \right)
\]
\[
\phi_{IF} \triangleq \text{diag} \left( \int_0^t Y_{F_1}^T Y_{F_1} dt, \ldots, \int_0^t Y_{F_n}^T Y_{F_n} dt \right).
\]

Figure 1 provides the block diagrams for the distributed control law and the parameter estimation law in order to make the controller design transparent. In Figure 2, a block schematic for the entire bias estimated control scheme is provided. The proposed control technique for bias estimated consensus for networked double integrator systems is then shown via a flow diagram in Figure 3.

Further, we consider the following assumption and a corresponding proposition.

**Assumption 3.** The set of filtered regressors \( Y_{F_i} \) are C-IE as per Definition 7.

**Remark 3.** It is worth noting that the solution \( Y_{F_i} \) in the assumption above depends on the initial conditions of the closed-loop state \( \dot{q}_i(0) \). The collective initial excitation condition is therefore not necessarily uniform with respect to initial data.

**Proposition 5.** (Reference 31) Provided Assumption 3 holds, the matrix \( M(t) = \mathcal{L} \otimes I_{(mn+2)} + \phi_{IF} \) appearing in (23) is uniformly strictly positive definite over the time window \([T, \infty)\) that is,

\[
\xi^T M(t) \xi > 0, \quad \forall t \geq T,
\]

\( \forall \xi \in \mathbb{R}^{mn+2} \).

We are now ready to state the primary result of this article.
**Theorem 1.** Consider the multi-agent network with the agent dynamics given by (2) interacting over an undirected graph $\mathcal{G}(t)$. If Assumptions 1–3 hold, then the control law given by

$$u = Z_{new} \tilde{\theta} - \sigma \overline{L}(t) s - \lambda \dot{q} - \lambda \frac{\dot{b}_{new}}{2},$$

with bias adaptation law (22), guarantees that $\lim_{t \to \infty} (q_i - q_j) = 0$ (for all $i, j \in \{1, 2, \ldots, n\}$), $\lim_{t \to \infty} \dot{q} = 0$ and $\lim_{t \to \infty} (b_i - b_k) = 0$ (for all $i, k \in \{1, 2, \ldots, n\}$) exponentially (beyond the collective initial excitation window, i.e., $t > T$) for sufficiently large $\mu_F > 0$, while ensuring that the trajectories of the closed-loop system given by (9), (12), and (23) are uniformly bounded.

**Remark 4.** While the result stated in Theorem 1 pertains to the consensus problem, the same idea extends to the trajectory tracking problems for a known bounded, smooth trajectory $r(t)$ known to the agents. The error variable $e \triangleq (q - r)$ is used in the results and control design instead of $q$.

**Proof.** The closed-loop system using (14), (23), and (21) can be written in the following matrix structure,

$$\frac{d}{dt} \begin{pmatrix} s \\ \tilde{\theta} \end{pmatrix} = \begin{pmatrix} -\sigma I_{mn} & Z_{new}(\dot{q}, t) \\ 0 & -\mu_F \phi_F - \mu_{IF} \phi_{IF} - \mathcal{L} \otimes I_{(mn+2)} \end{pmatrix} \times \begin{pmatrix} \overline{L}(t) s \\ \tilde{\theta} \end{pmatrix}$$

We now define a new consensus error variable $e \triangleq (I_{mn} - \frac{1}{n} 1_n^\top \otimes I_m)s = \left(s - \sum_{i=1}^n s_i/n\right)$. The dynamics in the new error state variables are,

$$\dot{e} = -\sigma \left(\overline{L}(t) + \frac{1_n 1_n^\top}{n} \otimes I_m\right) e + \left(I_{mn} - \frac{1_n 1_n^\top}{n} \otimes I_m\right) Z_{new}(\dot{q}, t) \tilde{\theta}$$

$$\dot{\tilde{\theta}} = -\left(\mu_F \phi_F + \mu_{IF} \phi_{IF} + \mathcal{L} \otimes I_{(mn+2)}\right) \tilde{\theta}.$$  

(24)
Arriving at the first equation is a straightforward application of the definition of $c$, computing its derivative according to the preceding dynamics and noting the fact that $\sigma \overline{L}(t)s = \sigma \overline{L}(t) \left( e + \frac{1}{n} I_n s \right) + \left( \frac{1}{n} I_n \otimes I_m \right) \left( I_{mn} - \frac{1}{n} I_n \otimes I_m \right) = 0$.

Similar transformation equations appear in consensus analysis in Reference 41 (section IIA).

By applying Theorem 4 we have,

$$\left( \overline{L}(t) + \frac{1}{n} I_n \otimes I_m \right) = \left( N(t) \otimes I_m \right) \left( N(t) \otimes I_m \right)^T,$$

where $N(t) \triangleq \left( H(t) + \frac{1}{n} I_n \right)$. It is also evident from Theorem 4 that $N(t)$ is persistently exciting since $G(t)$ is jointly $(\delta, T)$-connected. For the error dynamics (24) we choose the following candidate Lyapunov function motivated by Theorem 3,

$$V(t, e, \bar{\theta}) = \frac{1}{2} e^T \left[ \pi I_{mn} + S(t) \right] e + \frac{1}{2} \bar{\theta}^T \bar{\theta},$$

where $S(t)$ and constants $\pi, \delta_T$ are as defined in Theorem 3 with $N(t)$ defined above. It is immediately evident from the definition of $S(t)$ and $\delta_T$ that,

$$0 \leq S(t) \leq 2\delta_T I_{mn},$$

and the derivative of $S(t)$ can be computed as,

$$\dot{S}(t) = 2N(t)N^T(t) - \frac{2}{T} \int_t^{t+T} N(\tau)N^T(\tau) \, d\tau.$$ 

It is therefore obvious that $V(t, e, \bar{\theta}) \geq 0.5(\pi\|e\|^2 + \|\bar{\theta}\|^2)$ and therefore is a positive definite function. The directional derivative of $V(t, e, \bar{\theta})$ along the dynamics (24) can now be computed as,

$$\dot{V}(t, e, \bar{\theta}) = -\left( \pi \sigma - 1 \right) e^T N(t)N^T(t)e - \sigma e^T S(t)N(t)N^T(t)e - \frac{1}{T} e^T \int_t^{t+T} N(\tau)N^T(\tau) \, d\tau e$$

$$+ e^T \left[ \pi I_{mn} + S(t) \right] I_{mn} Z_{new}(\bar{\theta}, t) \bar{\theta} - \bar{\theta}^T \left( \mu_F \phi_F + \mu_{HF} \phi_{HF} + L \otimes I_{(mn+2)} \right) \bar{\theta},$$

where $I_{mn} \triangleq \left( I_{mn} - \frac{1}{n} I_n \otimes I_m \right)$ is used as a placeholder. We now compute an upper bound for $\dot{V}(t, e, \bar{\theta})$ as below. Keeping in mind that $\phi_F \geq 0, \phi_{HF} \geq 0$ and Proposition 5 we obtain,

$$\dot{V}(t, e, \bar{\theta}) \leq -\left( \pi \sigma - 1 \right) \|N(t)e\|^2 - \frac{\mu_1}{T} \|e\|^2 + \frac{\sigma \gamma}{2} \|N(t)e\|^2$$

$$+ \frac{\sigma}{2\gamma} \|S(t)N(t)e\|^2 \|e\|^2 - \mu_{HF} \lambda_{min}(M(t)) \|\bar{\theta}\|^2$$

$$+ \|\pi I_{mn} + S(t)\| \|I_{mn}\| \|Z_{new}(\bar{\theta}, t)\| \|e\| \|\bar{\theta}\|$$

$$\leq -\left( \pi \sigma - 1 - \frac{\sigma \gamma}{2} \right) \|N(t)e\|^2 - \left( \frac{\mu_1}{T} - \frac{\sigma}{2\gamma} \|N\|_\infty^2 \|S\|_\infty^2 \right) \|e\|^2$$

$$+ z_M(\pi + \|S\|_\infty) \|I_{mn}\| \|e\| \|\bar{\theta}\| - \mu_{HF} \lambda_{min}(M(t)) \|\bar{\theta}\|^2,$$

where the first inequality is based on norm upper bounding, utilizing the persistence condition on $N(t)$ (Definition 4) and applying the Young’s inequality to bound mixed terms in $e$ using a constant $\gamma > 0$. The second inequality is arrived at by utilizing the $\| \cdot \|_\infty$ norm of matrix functions and the fact that $\|Z_{new}(\bar{\theta}, t)\| \leq z_M$ for some $z_M > 0$. Such a $z_M$ exists due to the bound on $k(t)$ and the upper bound on edge weights $\delta_j(t)$ (Assumption 1). We now note that $\|S\|_\infty \leq 2\delta_T$ and make the following choice of constants in the above inequality,

$$\gamma = \frac{4\sigma T^2 \delta^2}{\mu_1} \|N\|_\infty^2; \quad \pi = \frac{1}{\sigma} + \frac{2\sigma T \delta^2}{\mu_1} \|N\|_\infty^2.$$
which leads to,

\[
\dot{V}(t, c, \tilde{\theta}) \leq -\frac{\mu_1}{2T} \|\epsilon\|^2 + \gamma \|I_{mn}\| \|\epsilon\| \|\tilde{\theta}\| - \mu_{IF} \lambda_{\min}(M(t)) \|\tilde{\theta}\|^2
\]

\[
\leq -\left(\frac{\mu_1}{2T} - \frac{\beta^0}{2\gamma^0}\right) \|\epsilon\|^2 - \left(\mu_{IF} \lambda_{\min}(M(t)) - \frac{\beta^0 \gamma^0}{2}\right) \|\tilde{\theta}\|^2
\]

where the final inequality is an application of the Young’s inequality with some \(\gamma^0 > 0\) and \(\beta^0 \triangleq \gamma \|I_{mn}\|\). We note that \(M(t) \geq M(t_0 + \bar{T}) > 0, \forall t \geq t_0 + \bar{T}\), which implies \(\exists c > 0\) such that \(\lambda_{\min}(M(t)) \geq c > 0\) using the argument as in Fact 2. Therefore, the following choice of constants guarantees exponential convergence of the \((c, \tilde{\theta})\) dynamics to the origin.

\[
\gamma^0 > \frac{\beta^0 T}{\mu_1}, \quad \mu_{IF} > \frac{\beta^0 T}{2 \mu_1 \lambda_{\min}(M(t))}
\]

We can now argue from the convergence of \(c\) that \(\tilde{L}(t)s \to 0\) exponentially. Now employing the definition of \(s_i\) in (8) and accounting for the fact that \(s_i - s_j \to 0\) for all \(i, j \in \{1, 2, \ldots, n\}\) and \(\tilde{b} \to 0\) exponentially, we are left with,

\[
\frac{d}{dt}(q_i - q_j) = -\lambda(q_i - q_j) + \gamma(t),
\]

where \(\gamma : \mathbb{R}^+ \to \mathbb{R}^m\) denotes an exponentially decaying function. This immediately shows that \(\lim_{t \to \infty}(q_i - q_j) = 0\) and the convergence is exponential. This implies that \((\tilde{L}(t)q, \tilde{L}(t)\dot{q}) \to (0, 0)\) exponentially, from the above equation. We use these facts to carry out an asymptotic analysis of the closed-loop by substituting \(w\) from (12) in (6). Since, \(\bar{b}, \tilde{b} \to 0\), we have the dynamics, in the limit, as \(\dot{q} = -\lambda\tilde{q}\), which immediately proves that \(\lim_{t \to \infty}\tilde{q} = 0\) exponentially using similar arguments as before.

For proof of boundedness, we note that in the \(\tilde{\theta}\) dynamics of (24), \(\mu_F, \mu_{IF} > 0\) and \(\phi_F, \phi_{IF}, L \otimes I_{(mn+2)}\) are symmetric positive semidefinite matrices at each \(t \geq 0\). Therefore we immediately have \(\|\tilde{\theta}\| \leq \|\tilde{\theta}(0)\|\). It is already known that the unforced \((\tilde{\theta} = 0)\) dynamics of \(c\) is exponentially stable\(^{41,44}\) and from the fact that the forcing term is bounded, we can conclude boundedness of \(\|c\|\) irrespective of the collective initial excitation on \(Y Fi\). Therefore \(\|\tilde{L}(t)s\| = \|\tilde{L}(t)c\|\) is also uniformly bounded. Therefore, employing the definition of \(s_i\) in (8) we obtain,

\[
s_i - s_j = \dot{q}_i - \dot{q}_j + \lambda(q_i - q_j) = \psi(t),
\]

where \(\psi : \mathbb{R}^+ \to \mathbb{R}^{mn}\) is a uniformly bounded function (\(\|\psi(t)\| \leq \psi_M\)). We have used the boundedness of \(\tilde{b}\) to arrive at the above. Solving the above equation allows us to conclude that \(\|(q_i - q_j)\|\) and \(\|(\dot{q}_i - \dot{q}_j)\|\) are uniformly bounded.

**Remark 5.** We note that the first equation in the system (24) is identical to the consensus dynamics studied in Reference 44 and similar in structure to Reference 41 (section IIA) if the forcing term due to \(\tilde{\theta}\) vanishes. The \(\tilde{\theta}\) term is a result of the unknown sensor bias being studied in this article and evolves according to network properties embedded in \(\phi_F, \phi_{IF}\), and \(Z_{new}(\tilde{q}, t)\).

## 5 Choosing Gains and C-IE Condition on Regressors

A central condition for exponential convergence of the bias estimation error to zero is Assumption 3. We have,

\[
Y_F(t) = e^{-\beta t} \int_0^t e^{\beta \tau} Y_F(\tau, \tilde{q}, \tilde{q}, t) \, d\tau
\]

where from (7), \(Y(\tilde{q}_i, \tilde{q}_j, t) = \left[q_i, k(t)q_i, \frac{1}{2}k(t)Q(t)I_m, Q_2(t)I_m, \ldots, Q_m(t)I_m\right]\).

For Assumption 3 to be satisfied, it is required that the integral of \(\sum_{i=1}^{mn} Y_F^TY_F\) over the initial finite time window spans the \(mn + 2\) dimensional space. We will now prove that, if the set of regressors \(Y_i\)'s are C-IE then the set of filtered regressors, \(Y_F\)'s, are also C-IE which further implies that \(Y_{F_i}\)'s are C-IE.

**Proposition 6.** The sufficient condition for the set of \(Y_i\)'s \((i \in \{1, 2, \ldots, n\})\) to be C-IE is that the set of \(Y_i\)'s are C-IE.
Proof. We proceed along the line of proof given in Reference 45 (proposition 4.1). Consider an arbitrary unit vector $v \in \mathbb{R}^{(mn+2)}$ and define the following variables:

$$K_i \triangleq Y_i v
$$
$$K_{Fi} \triangleq Y_{Fi} v.$$

Let us assume that the set of regressors $Y_i$’s are C-IE. The above proposition can now be proved by contradiction. Suppose that $Y_i$’s are not C-IE. Then, $\exists v \in \mathbb{R}^{(mn+2)}$ such that

$$\int_{t_0}^{t_0+T} \sum_{i=1}^{n} \left(K_i^T(r) K_{Fi}(r)\right) dr = 0,$$

which implies that, $K_{Fi}(t) = 0, \forall i, \forall t \in [t_0, t_0 + T]$. Therefore, $\dot{K}_i(t) = 0 \forall i$ and $t \in (t_0, t_0 + T)$. By definition, we have,

$$\dot{K}_{Fi} = -\beta K_{Fi} + K_i,$$

which indicates that $K_i(t)$ is zero for all $i, \forall t \in (t_0, t_0 + T)$. This contradicts the fact that $Y_i$’s are C-IE. Hence, the set of $Y_i$’s being C-IE implies that the set of $Y_i$’s are C-IE.

\[\square\]

We now derive a necessary condition to be able to conclude collective Initial Excitation (C-IE) on the set of regressors, $Y_i(\hat{q}, \hat{q}, t)$. Since, $Y = [Y_1, Y_2, \ldots, Y_n]^T$ we can write,

$$\sum_{i=1}^{n} Y_i^T Y_i(\hat{q}, \hat{q}, t) = Y^T Y(\hat{q}, \hat{q}, t) = \begin{pmatrix} A(\hat{q}, \hat{q}, t) & B(\hat{q}, \hat{q}, t) \\ B^T(\hat{q}, \hat{q}, t) & C(\hat{q}, t) \end{pmatrix},$$

where, $A(\hat{q}, \hat{q}, t) = \begin{pmatrix} \hat{q}^T \dot{q} & \hat{q}^T k(t) \hat{q} \\ k(t) \hat{q}^T \hat{q} & k^2(t) ||\hat{q}||^2 \end{pmatrix}$,

$$B(\hat{q}, \hat{q}, t) = \frac{k(t)^2}{2} \hat{q}^T \overline{Q}(t),$$

$$C(t) = \frac{k^2(t)}{4} \overline{Q}(t) \overline{Q}(t).$$

All arguments in the preceding equation have been deliberately removed for the sake of brevity and clarity. The functions $A(\hat{q}, \hat{q}, t)$ and $C(t)$ as defined above, map into positive semi-definite matrices by definition. We now state the result pertinent to this section.

**Lemma 1.** If $Y_i(\hat{q}, \hat{q}, t)$’s are collectively initially exciting as per Definition 7, then the matrix functions, $\int_{t_0}^{T} A(\hat{q}, \hat{q}, t) dt$ and $\int_{t_0}^{T} C(t) dt$ are positive definite. Further, if the graph, $G(t)$ is jointly $(\delta, T)$-connected for some $\delta > 0$, then it is also jointly non-bipartite over $[0, \max\{T, \overline{T}\}]$.

**Proof.** Let us assume for contradiction that $0 \in \text{spec} \left\{ \int_{0}^{T} C(t) dt \right\}$. Let the eigenvalues be ordered as $0 \leq \beta_2 \leq \cdots \leq \beta_{mn}$.

We already know that $\int_{0}^{T} \sum_{i=1}^{n} Y_i^T Y_i(\hat{q}, \hat{q}, t) dt \geq 0$ which indicates that all eigenvalues are non-negative. Let us assume these are ordered as $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_{(mn+2)}$. Since $\int_{0}^{T} C(t) dt$ is a principal submatrix of $\int_{0}^{T} \sum_{i=1}^{n} Y_i^T Y_i(\hat{q}, \hat{q}, t) dt$, we can use the Cauchy’s interlacing and inclusion theorem (Reference 46, theorem 8.4.5) to conclude that, $\lambda_1 \leq 0 \leq \lambda_3$. Since $\int_{0}^{T} \sum_{i=1}^{n} Y_i^T Y_i(\hat{q}, \hat{q}, t) dt \geq 0$, the only possibility is $\lambda_1 = 0$. This immediately implies that the set of $Y_i(\hat{q}, \hat{q}, t)$’s are not collectively initially exciting, thus contradicting our premise. Similar arguments can be used to claim that $\int_{0}^{T} A(\hat{q}, \hat{q}, t) dt > 0$.

From $\int_{0}^{T} C(t) dt > 0$ along with the facts that $k(t) > 0$, we can immediately conclude $\int_{0}^{T} \overline{Q}(t) dt \geq \lambda_c I_{nn}$ for some $\lambda_c > 0$. This immediately implies that $\int_{0}^{\max\{T, \overline{T}\}} \overline{Q}(t) dt > 0$. The union graph of $G(t)$, denoted $\cup_{t \in [0, \max\{T, \overline{T}\}]} G(t)$, is connected by the joint $(\delta, T)$-connectedness assumption. The positive definiteness of the signless Laplacian for the union
graph \( \left( \int_{0}^{\max(T, \bar{T})} Q(t) \, dt \right) \) and the joint connectivity of the union graph allows us to invoke Theorem 2 and conclude that the union graph \( \bigcup_{t \in [0, \max(T, \bar{T})]} G(t) \) is non-bipartite, that is, \( G(t) \) is jointly non-bipartite over \([0, \max(T, \bar{T})]\). □

Remark 6. We note here that Reference 40 utilizes the non-bipartite graph structure to propose exponential bias estimators. The graph is, however, assumed to be constant. The necessary condition above allows the use of time-varying graphs that are only jointly non-bipartite (as opposed to at each time instant) and persists only for a finite time \([0, \max(T, \bar{T})]\).

Based on Lemma 1, assuming that we have a jointly non-bipartite union graph over \([0, \max(T, \bar{T})]\), the primary purpose of \( k(t) \) is to ensure that \( \int_{0}^{T} A(\dot{q}, \ddot{q}, t) \, dt \) becomes positive definite. While there is no direct way to prescribe such a function for all possible initial conditions and system parameters, we introduce multiple frequency components through the time dependence in \( k(t) \) to make the columns of \( A(\dot{q}, \ddot{q}, t) \) linearly independent over sub-intervals.

6 SIMULATION RESULTS

We now present simulation studies to verify Theorem 1 for a network of double integrators interacting via an undirected graph \( G(t) \) and bias corrupted measurements. We consider the translational dynamics of \( n \) identical quadrotors given by

\[
\ddot{q}_i = \begin{bmatrix} 0 \\ 0 \\ -9.8 \end{bmatrix} + \mathbf{R}_i \dot{e}_3 \frac{\tau_i}{M}, \quad \forall i = 1, 2, \ldots, n,
\]

where the position vector is denoted by \( q_i = (x_i, y_i, z_i)^T \in \mathbb{R}^3 \), \( e_3 = (0, 0, 1)^T \), \( M \in \mathbb{R}^+ \) is the mass of the quadrotor. \( \mathbf{R}_i \in SO(3) \) is the 3 × 3 orthogonal rotation matrix from the quadrotor body frame to the inertial frame. The feedback \( \tau_i(\cdot) \in \mathbb{R} \) is the sum of thrust forces from the individual motors in each quadrotor. Typical tracking control of the quadrotor consists of an inner loop attitude control\(^{17} \) which modulates \( \mathbf{R}_i \) while the outer loop translation control is designed assuming full linear actuation in (26). Therefore, (26) can be treated as a double integrator model for our purposes by assuming a new control \( u_i = \mathbf{R}_i \dot{e}_3 \frac{\tau_i}{M} \). Corresponding to a control \( u_i \) the \( \mathbf{R}_i \) and \( \tau_i \) are calculated at each instant of time. As a result, each agent’s \( \mathbf{R}_i \) matrix is a time varying quantity and is assumed to be efficiently tracked by an “inner-loop” control. For these simulations we have considered \( n = 5 \).

Two adjacency matrices are used, one corresponding to a non-bipartite, connected graph \( (\mathcal{A}_b) \) and another corresponding to a bipartite, connected graph \( (\mathcal{A}_c) \).

\[
\mathcal{A}_b = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{A}_c = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}.
\]

The graphs corresponding to \( \mathcal{A}_b \) and \( \mathcal{A}_c \) are shown in Figures 4 and 5, respectively.

![Connected non-bipartite graph](image-url)
The actual adjacency matrix is obtained by cycling periodically (period $T = 8$ s) between subgraphs of $A_b$ for the first $t = 8$ s and then between subgraphs of $A_c$ for the rest of the time. This allows for a jointly non-bipartite and connected graph in the initial phase of the simulations and a jointly bipartite, connected graph beyond.

The initial position, velocity, and the bias in relative measurement of the position for the $i$th quadrotor are given by

$$\begin{bmatrix}
i\pi/7; i\pi/3; i\pi/3 \\
0.1i - 0.7; -0.1i + 0.6; 0.1i + 0.7 \\
0.1i + 0.1; 0.1i + 0.1; 0.1i + 0.1 \\
\end{bmatrix},$$

respectively, for $i = 1, 2, ..., 5$. $\hat{\theta}$ is initialized to the zero vector for $i = 1, 2, ..., 5$. The gain constants $\sigma, \mu_F, \mu_{IF}, \lambda$, and $\beta$ are chosen to be 0.2, 0.020, 15, 0.5, and 0.5, respectively, and $k(t) = 1 + 0.5\cos^2(t) + 0.5\sin^2(2t)$. The gain chosen helps introduce multiple frequency components through the time dependence in $k(t)$. The chosen $k(t)$ ensures that $\int_0^T A(\ddot{q}, \ddot{q}, t) \, dt$ becomes positive definite, which guarantees collective initial excitation on the regressor $Y_i$'s. The design parameter $\sigma$ is a scalar gain used to alter the consensus rate of the $s_i$ variables defined in (8), while $\lambda$ controls the rate of position consensus, once $s_i$'s are synchronized. The stability of filters (18) is maintained by the scalar gain $\beta, \mu_F$, and $\mu_{IF}$ are two additional scalar gains that can be used to regulate the rate of parameter convergence. The design parameters $\sigma, \lambda, \beta, \mu_F$ are free to be chosen as long as they are strictly positive. The control gain $\mu_{IF}$ is selected to be sufficiently large since they must meet the inequality (25).

Figure 6 is the phase-plane evolution of the three positions. As is evident, starting from different initial conditions, they converge to consensus. Similarly, all the three velocities in Figure 7 and the bias estimation errors given by $\tilde{b} = [\tilde{b}_1^1, \tilde{b}_2^1, \tilde{b}_3^1, \tilde{b}_4^1, \tilde{b}_5^1]^T$ in Figure 8 converge to zero during the simulation horizon. The final plot, Figure 9, is for the verification of Lemma 1, wherein we claim that the collective initial excitation of the regressors necessitates a jointly non-bipartite graph. Figure 9 plots the determinant of $\int_t^{t+T} Q(\tau) \, d\tau$ for all $t$, keeping $T = 4$ s as the cycling period. We see that the $\int_t^{t+T} Q(\tau) \, d\tau$ is positive definite over an initial period of time and beyond this is singular. This verifies, by Proposition 2 that $G(t)$ determined by $A(t)$ is jointly non-Bipartite over a finite initial window.
Remark 7. The practical implementation of this approach necessitates measurements of the relative positions of network agents in a three-dimensional space. To the best of our knowledge, there are not any commercialized sensors available right now for determining the relative position of dynamic agents. There are ultra-wide-band (UWB) modules that can only be used to measure the ranges between two agents. Research is being done to create algorithms that estimate the relative position of the agents by utilizing a network of multiple UWB modules and combining their data with other sensory data using inertial sensors, or an inertial measurement unit, but they are still far from being reliable and stable solutions.48
7 | CONCLUSION

In this article we propose a novel distributed adaptive controller to estimate bias in relative position measurements along with guaranteed exact consensus in a network of double-integrator systems. It is shown that joint \((\delta, T)\)-connectivity and joint non-Bipartite properties of the graph are necessary for bias estimation and consensus. In future work, we seek to explore more general measurement errors and nonlinear agent dynamics. Time-varying bias and directed time varying communication graphs on consensus under erroneous relative measurements are features we also anticipate adding as part of our future work. We must be aware that when there are time-varying biases, the unknown parameter is not a constant but rather a time-varying signal. While dealing with time-varying biases among a group of dynamic agents, it will also be crucial to take into account the effect of zero-mean noises with time-varying deviations on the relative position measurements and the effectiveness of the proposed solution. Consensus analysis will be challenging for the case of directed communication graphs due to the asymmetry of the Laplacian matrix.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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