Anisotropic modeling of layered rocks incorporating planes of weakness and volumetric stress

Shanchao Hu¹,² | Yunliang Tan¹ | Hui Zhou³ | Wenkai Ru¹ | Jianguo Ning¹ | Jun Wang¹ | Dongmei Huang¹ | Zhen Li⁴

¹State Key Laboratory of Mining Disaster Prevention and Control Co-founded by Shandong Province and the Ministry of Science and Technology, Shandong University of Science and Technology, Qingdao, China
²State Key Laboratory of Coal Resources and Safe Mining, China University of Mining and Technology, Xuzhou, China
³State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Wuhan, China
⁴School of Civil Engineering, Henan Polytechnic University, Jiaozuo, China

Abstract

Layered rocks exhibit notable transverse isotropy and have a significant impact on the deformation and failure characteristics of underground structures. To a large extent, the mechanical properties of layered rocks are related to the structure and stress state of their bedding planes. To obtain an in-depth understanding of the deformation and yield characteristics of layered rocks, a mechanical model for layered rocks incorporating planes of weakness and volumetric stress is proposed by improving the strain-hardening/softening ubiquitous-joint model based on continuum mechanics methods. In this mechanical model, elastoplastic equations for the matrix and bedding planes of layered rocks are established. The evolution of the mechanical parameters of the matrix and bedding planes of layered rocks with the internal variable is determined. In addition, the sensitivity of the model parameters is analyzed, and a method for determining the parameters is provided to reduce the complexity in obtaining these parameters. The numerical calculation results for the laboratory tests on layered sandstone specimens under various stress levels agree relatively well with the test results, thereby validating the effectiveness of the improved model. The methods and results of this study provide a significant reference for the analysis of the deformation and failure of other layered rocks.

KEYWORDS
anisotropy, mechanical model, plastic internal variable, sensitivity analysis, volumetric stress
INTRODUCTION

Most rocks surrounding underground structures in mines consist of layered sandstone, shale, and interbedded sedimentary rocks with relatively thin beds. Many metamorphic rocks also contain a notable bedding structure. Generally, these rocks are collectively referred to as layered rocks. Currently, a large number of underground structures (eg, coal mine roadways, underground tunnels, and chemical and nuclear waste repositories) are constructed in this type of rock.1-3 In the design and stability analysis of this type of underground structure,4-6 it is necessary to consider the anisotropic deformation and failure characteristics of the layered rocks. Thus, mechanical models need to be established that can sufficiently reflect the anisotropic characteristics of layered rocks and thoroughly explain their transversely isotropic deformation and failure characteristics.7,8 These models should be based on the mechanical properties and field monitoring information of the layered rocks to improve the accuracy of the calculations of the deformation and failure of the deep surrounding layered rocks and ensure the safety and stability of deep underground structures.9,10

Critical progress has been made in research on the yield criteria and constitutive models for layered rocks. According to the differences among the assumptions and algorithms, either (a) discrete element methods or (b) equivalent continuum methods are employed to solve the computational problems of layered rocks. When used to solve problems of layered rocks, discrete element methods usually explicitly establish bedding planes or define the particle elements contained in bedding planes based on the orientation and bedding thickness. The discrete element methods offer the advantage of simulating the bedding plane-controlled failure mechanism of rocks.11,12 However, the use of noncontinuum methods to analyze large-scale engineering problems involving rocks with a relatively small bedding thickness requires large amounts of computational resources and is often infeasible.13 Layered rocks can be treated as an equivalent continuous medium (ECM); equivalent continuum methods introduce a tensor composed of the shear strength (or tensile strength) in the principal direction of the material by use of the conventional Mohr-Coulomb criterion and search for the most likely failure plane under certain stress conditions14,15. Some researchers have also proposed using the boundary element method to describe layered rocks.16 Hoek17 noted that intact rocks under in situ stress conditions and intact and fissured rocks under high stress conditions can be computed using continuum mechanics methods, which not only meets the accuracy requirements of engineering analysis but also can significantly improve the computational efficiency.

The strength of layered rocks is closely related to not only the isotropy, but also affected by the stress state.18,19 The gained experiment results indicated that the velocity of elastic wave and mechanical parameters both changed significantly with the increase in the confining pressure.20,21 Then, several constitutive models incorporating isotropy and stress state22-24 were proposed to study the deformation and failure of layered rocks under the condition of different stress state, which could provide guidance for the engineering design. It should be pointed out that several failure criteria were proposed based on the confining pressure. However, the so-called confining pressure is one kind of special stress state existing in conventional triaxial tests, and the stress state appears rarely in practical engineering. Correspondingly, the volumetric stress (namely the first invariant of stress tensor) could be calculated easily and accurately.

To achieve effective design and stability analysis of underground structures and overcome the deficiencies in current mechanical models, an ECM mechanical model for layered rocks is established considering the effect of volumetric stress. The established model is improved based on the strain-hardening/softening ubiquitous-joint model (hereinafter referred to as SU model) and considering the evolution of the mechanical parameters of layered rocks with the volumetric stress. The establishment and numerical implementation of the mechanical model are described in detail in the following sections. The improved model is also used in the simulation analysis of laboratory compression tests, and the results are compared with test results to examine the reasonableness and effectiveness of the model.

SITE DESCRIPTION AND MECHANICAL PROPERTY OF THE LAYERED ROCKS

Silty fine-grained interbedded sandstone obtained from the Ji‘ning No. 3 Coal Mine was used in the laboratory tests. The sandstone contains notable horizontal bedding and
exhibits significant transverse isotropy. The bedding planes, 0.3-2 mm in thickness, are distributed at relatively highly dispersed intervals, significantly affecting the stability and failure mode of the rocks surrounding roadways. Bed separation and roof collapse are the common failure modes in the field.

Because of the different rock properties of different regions and types, it is necessary to carry out indoor tests to find out the deformation and failure of layered rocks under different loading angle and stress state. In addition, it also needs the test data to modify the mechanical model and determine the corresponding parameters according to the evolution of mechanical parameters with loading angle and stress state. Therefore, rock samples with different bedding plane angle were prepared and conventional compression tests were carried out, and the test results have been illustrated. On the whole, there was a positive correlation between the deformation and failure of layered rocks and the loading angle and stress state.

3 | IMPROVED MECHANICAL MODEL FOR LAYERED ROCKS INCORPORATING VOLUMETRIC STRESS

Based on appropriate assumptions, a mechanical model for layered rocks considering the volumetric stress effect is established by improving the embedded SU model and using FLAC3D. The improved mechanical model for layered rocks is implemented as a plug-in DLL file into the commercial code FLAC3D by using the User Defined Constitutive Models (UDM) functionality.

3.1 | Basic assumptions

Based on the analysis of the compression test results for layered rocks, the following basic assumptions are proposed, and a mechanical model for layered rocks is established based on these assumptions:

(a) Layered rocks consist of matrix and bedding planes that extend infinitely and are parallel to one another. (b) Layered rocks are isotropic in the direction parallel to the bedding and have different physical properties in the directions parallel and perpendicular to the bedding. (c) The deformation parameters of the matrix are assumed to be constant values. The deformation parameters of the bedding plane are also set to be constant values, but not equal to those of matrix. (d) The tensile strength of the matrix is assumed to be a constant value. The tensile strength of the bedding plane is also set to be a constant value, but not equal to that of matrix.

3.2 | Criteria and model

3.2.1 | Definition of the plastic internal variable

In order to determine the evolution of mechanical parameters of marbles under different stress conditions with the plastic deformation, the plastic internal variable needs to be defined. The internal variable is used to describe the plastic deformation history of the rock material, and the interval variable could be defined in different forms, such as accumulative plastic work, current plastic strain, and accumulative plastic strain. The failure mode of rock and soil material was mainly shear yield, so the interval variable was selected as equivalent plastic shear strain. Based on the existing data, the plastic deformation of layered rocks is highly correlated with the confining pressure. Under the condition of same loading angle, the plastic strain of specimens increases as the confining pressure increases when the residual strength is reached. Therefore, it is necessary to modify the internal variable in conventional plastic mechanics to reflect the effects of the stress state. Zhou and Yang both used a representation of a plastic internal variable that considers the confining pressure, as shown in Equation (1).

$$\kappa = \int d\kappa \cdot d\kappa = \sqrt{\frac{2}{3} (d\epsilon^p : d\epsilon^p) / f (P_c / \sigma_c)}$$ (1)

where $d\epsilon^p$ is the plastic deviatoric strain tensor and is defined as $(d\epsilon^p = d\epsilon^p - tr (d\epsilon^p) I / 3)$, $d\epsilon^p$ is the plastic strain tensor, $\sqrt{\frac{2}{3} (d\epsilon^p : d\epsilon^p)}$ is the equivalent plastic shear strain increment, which is denoted by $dy^p$, $f (P_c / \sigma_c)$ is a function of the confining pressure $P_c$, and a uniaxial compressive strength $\sigma_c$ is introduced for nondimensionalization.

However, as mentioned previously, the confining pressure is merely a stress state defined to make the research and laboratory tests easier to conduct while the stress state rarely occurs in the field. In comparison, the first invariant of the stress tensor ($I_1 = \sigma_1 + \sigma_2 + \sigma_3$, namely the volumetric stress, hereafter represented by volumetric stress) can be simply and accurately calculated. Different from the function $f (P_c / \sigma_c)$ relating to the confining pressure, which is a fixed expression, the volumetric stress continuously changes and cannot be calculated using Equation (1). Hence, by improving the original variable, an internal variable that considers the volumetric stress is established as follows:

$$\kappa = \int d\kappa ,$$

$$d\kappa = \sqrt{\frac{2}{3} (d\epsilon^p : d\epsilon^p) \cdot f (I_1 / \sigma_c)} = dy^p \cdot f (I_1 / \sigma_c)$$ (2)
The internal variables defined by Equation (2) are equal under different stress state when the yield state is the same. The yield state contains two yield points of initial yield and residual yield. The initial yield is the transition point from elastic to plastic while the residual yield is the state that rock reaches its yield stress. The expression of \( f(I_1/\sigma_c) \) is determined using the following method. The elastic and plastic strain increments and the equivalent plastic shear strain increment \( d\gamma^p \) at each loading step of the yield stage are calculated based on the test data. On this basis, \( d\kappa \) containing the undetermined expression of \( f(I_1/\sigma_c) \) is obtained using Equation (2). By the cumulative summation of \( d\kappa \) from initial yield to residual yield and setting the value of \( \kappa \) to 1, an internal variable function that considers volumetric stress is obtained.

Five laboratory compression tests were conducted under confining pressures of 0, 5, 10, 20, and 30 MPa. Because the Mohr-Coulomb failure criterion is based on shear failure, only the triaxial compression test data are used to calculate the internal variable, whereas the uniaxial compression test data for nonpure shear failure are removed. Here, the calculation of the internal variable of the layered rocks matrix is described as an example. An expression of the volumetric stress containing four parameters is established as follows:

\[
f(I_1/\sigma_c) = A_m (I_1/\sigma_c)^3 + B_m (I_1/\sigma_c)^2 + C_m (I_1/\sigma_c) + D_m \quad (3)
\]

where \( A_m, B_m, C_m, \) and \( D_m \) are undetermined quantities related to the layered rocks matrix.

After substitution of Equation (3) into Equation (2) and further derivation, an expression for the internal variable of the rock matrix can be obtained by the summation of the test data points from initial yield to residual yield. The expression contains the equivalent plastic shear strain \( \gamma^p \) and the volumetric stress \( I_1 \), as shown in Equation (4).

\[
\int d\kappa = \int d\gamma^p \cdot f(I_1/\sigma_c) = \sum_{i=1}^{n} d\gamma^p_i \cdot f(I_1/\sigma_c) = 1
\]

where \( d\gamma^p_i \) and \( I_1 \) are the equivalent plastic shear strain increment and volumetric stress corresponding to each stress point, respectively.

A set of simultaneous equations consisting of four equations is generated based on the test data for the matrix of the layered rocks under four confining pressures. The corresponding undetermined coefficients are obtained by solving the set of simultaneous equations. Similarly, the coefficients related to the bedding planes of the layered rocks can also be determined: replacing \( A_m, B_m, C_m, \) and \( D_m \) in Equation (4) with the coefficients \( A_p, B_p, C_p, \) and \( D_p \), respectively; substituting the triaxial compression test data for the bedding planes into Equation (4); and solving the simultaneous equations to determine \( A_m, B_m, C_m, \) and \( D_m \).

Thus far, expressions for the internal variable of the matrix and bedding planes of the layered rocks that consider volumetric stress have been derived. In the following section, based on the test results, elastoplastic equations for the matrix and bedding planes of the layered rocks are derived, and the flow rules for layered rocks are also determined. On this basis, a mechanical model for layered rocks considering the volumetric stress effect is established.

### 3.2.2 Elastoplastic equation for the matrix of layered rock

1. **Strength softening pattern**

The compression test results show that none of the layered sandstone specimens with horizontal bedding (loading angle \( \theta = 90^\circ \), axial stress \( \sigma_1 \) perpendicular to the bedding plane) fail along the bedding planes and that the bedding has a relatively insignificant impact on the test results. Thus, the evolution of the strength parameters of the layered sandstone matrix is investigated based on the test data for the layered sandstone specimens loaded at \( \theta = 90^\circ \).

At a loading angle of \( \theta = 90^\circ \), tests are conducted under four confining pressures. By substituting the test data into Equation (4), the function for the internal variable of the matrix of the rock containing volumetric stress is obtained:

\[
\kappa = \sum_{i=1}^{n} \gamma^p_i \left[ -187.3 \left( I_{1i}/\sigma_c \right)^3 + 1033.5 \left( I_{1i}/\sigma_c \right)^2 - 1765.1 \left( I_{1i}/\sigma_c \right) + 1083.1 \right]
\]

To calculate the evolution of the strength parameters of the matrix with the internal variable, the value of the internal variable is changed from 0 to 1 at increments of 0.1:0 represents the state of initial yield while 1 represents the state of residual yield. By interpolating the deviatoric stress and internal variable under various confining pressures, the deviatoric stress functions corresponding to various confining
pressures under the same internal variable are obtained. In addition, by fitting based on the Mohr-Coulomb criterion, the strength parameters of the matrix of the rock under the same internal variable are determined. Figure 1 shows the calculation results.

As shown in Figure 1, when the matrix of the rock specimen begins to yield, the cohesion rapidly decreases. It indicates that the cracks gradually form inside the specimen and then generates propagation and coalescence rapidly. In addition, the angle of internal friction angle decreases as the internal variable increases. This result occurs because at the initial stage of the shear fracture, the cracked surface of the matrix of the rock specimen is coarse, and the action of friction is timely and sufficient; as the fracture intensifies and the cracks slip, the coarseness of the cracked surface decreases, resulting in a decrease in the coefficient of friction that is macroscopically reflected by a decrease in friction angle.

A comparison of the test results shows that the cohesion and friction angle of the matrix of layered rocks both decrease as the internal variable increases. Friction angle reaches the residual value when the internal variable is in the range of 0.8-0.9, whereas cohesion does not reach the residual value until the internal variable increases to 1.0. Evidently, the strength parameters of the layered sandstone matrix change differently as the internal variable increases. Referring to the strain hardening and softening methods used in the cohesion weakening and frictional strengthening model,\(^{30}\) the relationships between the strength parameters of the matrix and the internal variable are represented by a piecewise function (6) and (7) with intervals determined based on Figure 1.

\[
\begin{align*}
\psi &= c_0 - \kappa (c_0 - c_r) & \kappa \leq 1 \\
\psi &= c_r & \kappa > 1 \\
\varphi &= \varphi_0 - \kappa (\varphi_0 - \varphi_r) & \kappa \leq \kappa_\varphi \\
\varphi &= \varphi_r & \kappa > \kappa_\varphi
\end{align*}
\]  

(6)

(7)

where \(c_0\) and \(c_r\) are the initial and residual cohesion of matrix, respectively; \(\varphi_0\) and \(\varphi_r\) are the initial and residual internal friction angle of matrix, respectively; \(\kappa_\psi\) is the corresponding internal variable when the internal friction angle reaching its residual value.

2. Flow rule

The strain of hard brittle material (including rock) will be recovered to 0 in the elastic stage when the external stress is relieved. However, a portion of volumetric strain could not recover in the yield stage even when the external shear stress is relieved which is called dilatancy described by dilatancy angle \(\psi\). In the classic plastic theory, orthogonal flow rule is used to insure the stability of mechanical response of material.\(^{31}\) It demands that the dilatancy angle equals to the internal friction angle which could ensure the uniqueness of solution of governing equation. The orthogonal flow rule does not always hold, so the nonassociated flow rule is adopted in this article to study the dilatancy angle. Referring to some scholars’ research,\(^{28,29}\) the dilatancy angle is used to replace the internal friction angle in the Mohr-Coulomb criterion to compute the dilatancy angle of matrix and bedding plane, respectively. Then, the change rule of the dilatancy angle with plastic internal variable is analyzed and compared with the corresponding internal friction angle to clarify the rationality of nonassociated flow rule. Besides, an associated flow rule is used for tensile-plastic flow of matrix.

When elastoplastic coupling is not considered, rock deformation mainly includes elastic deformation and plastic deformation. Based on assumption (3) introduced in Section 3.1, the deformation parameters of the layered rocks are constants, that is, elastoplastic coupling is not considered. Based on the method of Vermeer,\(^{32}\) a calculation equation for the angle of dilatancy \(\psi\) expressed in the form of the plastic strain increment is derived:

\[
\psi = \arcsin \left( \frac{de^p_\psi}{-2de^p_\theta + de^p_\nu} \right)
\]

(8)

where \(de^p_\psi\) is the increment of plastic volumetric plain; \(de^p_\nu\) is the plastic first principle strain.

The calculation method for the \(\psi\) of the rock specimens under various confining pressures is the same. Here, the calculation of \(\psi\) under a confining pressure of 5 MPa loaded at \(\theta = 90^\circ\) is described as an example. First, the plastic volumetric strain \(\epsilon^p_\psi\) and the plastic first principle strain \(\epsilon^p_\nu\) are calculated and substituted into Equation (8). The relationships of \(\epsilon^p_\psi\) and the denominator \((-2\epsilon^p_\theta + \epsilon^p_\nu\)) with the equivalent plastic shear strain \(\gamma^p\) are then fitted as function (9) and (10). Figure 2 shows the fitting results.

The fitted relations are as follows:

\[
\epsilon^p_\psi = 101462(\gamma^p)^3 - 566.21(\gamma^p)^2 + 0.3825(\gamma^p) + 6e - 5
\]

(9)
By differentiating Equations (9) and (10) and substituting the results into Equation (8), an expression for $\psi$ is obtained:

$$\psi = \arcsin \frac{304386(y^p)^2 - 1132.42(y^p) + 0.3825}{-2.1544}$$ (11)

The corresponding values of $\psi$ can then be calculated by substituting the test data for various confining pressures loaded at $\theta = 90^\circ$. The values of $\psi$ corresponding to various confining pressures are successively calculated using the same method as that used for calculating the strength parameters. A negative value for $\psi$ might be obtained, and when this result occurs, $\psi$ is set to 0.29 In addition, due to the presence of an abnormal postpeak volumetric strain under a confining pressure of 20 MPa, the evolution of $\psi$ with the internal variable is of no reference value. Therefore, only the calculation results for the other three confining pressures are shown in Figure 3.

Figure 3 shows the changes in $\psi$ with the internal variable. Clearly, the $\psi$ of the layered sandstone matrix first increases and then decreases as the internal variable increases. The mechanism of these changes is described as follows. After entering the yield stage, cracks gradually form and propagate in the sandstone, which is macroscopically reflected by an increase in $\psi$. As the stress continues to increase, the slip plane is gradually smoothed, which is macroscopically reflected by a decrease in $\psi$. Because sandstone is a type of pore-cemented rock and has a relatively high porosity and a relatively loose internal structure, the dilatancy effect of this material is not significant, which is macroscopically reflected by dilatancy angle that is significantly lower than friction angle.

Based on the aforementioned analysis, the $\psi$ of the layered sandstone matrix is fitted. Considering that the values of $\psi$ are relatively similar under various confining pressures, the effects of the confining pressure are no longer considered when fitting $\psi$, and instead, only the internal variable is included. By fitting the analytical results for a confining pressure of 10 MPa, an expression depicting the changes in the $\psi$ of the matrix with the internal variable is obtained as follows:

$$\psi(\kappa) = P_1\kappa^2 + P_2\kappa + P_3 = -79.744\kappa^2 + 91.045\kappa - 8.3512$$ (12)

### 3.2.3 Elastoplastic equation for the bedding planes of the layered rock

The direct shear test and conventional triaxial compression test can both be used to determine the strength parameters of structural plane. The error between the two methods was put forward: Zhang33 noted that the strength parameters obtained by triaxial compression test were higher than that by direct shear test. The strength parameters could be determined by triaxial compression test34 while the direct shear test is also feasible.35,36 Based on the above research results and existing test data, the triaxial test data were used to determine the elastoplastic equation in the following paragraph.

Moreover, the image of the fracture of the layered sandstone (Figure 4A) shows that the fracture surface of the
layered sandstone specimen loaded at \( \theta = 30^\circ \) (the angle \( \theta \) between the bedding plane and the axial stress \( \sigma_1 \) is 30\(^\circ\)) is dark gray (the color of the sandstone bedding) and is relatively smooth, suggesting that the matrix did not reach the ultimate bearing capacity and that plastic deformation mainly occurred on the bedding planes with relatively low strength; in comparison, the specimen loaded at \( \theta = 90^\circ \) has a notable light gray fracture surface (color of the sandstone matrix, Figure 4B). Based on the compressive strengths of the sandstone specimens loaded at various \( \theta \), it is reasonable to suppose all the plastic strains are generated by the bedding planes when loading at \( \theta = 30^\circ \). Thus, the test data for the layered sandstone specimen loaded at \( \theta = 30^\circ \) are used to study the elastoplastic equation for the bedding planes.

1. Strength softening pattern

Tests were conducted under four confining pressures loaded at \( \theta = 30^\circ \). By replacing \( A_m, B_m, C_m, \) and \( D_m \) in Equation (9) with \( A_j, B_j, C_j, \) and \( D_j \), respectively, and substituting the test data into Equation (4), the function for the internal variable of the bedding planes of the layered rock containing volumetric stress is obtained:

\[
\kappa = \int d\psi \cdot [162.7 \left( I_{1i}/\sigma_c \right)^3 - 1178.3 \left( I_{1i}/\sigma_c \right)^2 + 2401.9 \left( I_{1i}/\sigma_c \right) - 922.4]
\]

(13)

The method used for calculating the internal variable of the matrix is adopted to calculate the changes in the strength parameters of the bedding with increasing internal variable. The value of the internal variable of the bedding planes is increased at increments of 0.1, and the deviatoric stress values corresponding to various confining pressures under the same internal variable are obtained. Figure 5 shows the changes in the strength parameters of the bedding planes.

As shown in Figure 5, the changes in the cohesion of the bedding planes are similar to those in the cohesion of the matrix; the cohesion of the bedding planes decreases rapidly after yielding, and the bedding planes rapidly yield and slip after the peak strength. The friction angle of the bedding planes remains constant as the internal variable increases. This result mainly occurs because the fracture surface of the sandstone specimen with tilted bedding planes is a weak sedimentary plane, and there is no significant difference between the prefracture and postfracture values of the coefficient of friction, which is macroscopically reflected by insignificant change in friction angle.

Based on the method used for the strength parameters of the matrix, a piecewise function is used to describe the relationship between the strength parameters of the bedding planes and the plastic internal variable. \( c_j \) and \( c_{jr} \) represent the initial and residual cohesion of the bedding planes, respectively. \( \phi_j \) and \( \phi_{jr} \) represent the initial and residual internal friction angle of the bedding planes, respectively. The cohesion of the bedding planes reaches the residual value when the internal variable is 1.0, whereas the friction properties (as reflected by \( \psi \)) of the bedding planes remain basically constant. Thus, the expression of the piecewise function is determined as follows:

\[
\begin{cases}
  c = c_j - \kappa (c_j - c_{jr}) & \kappa \leq 1 \\
  c = c_{jr} & \kappa > 1
\end{cases}
\]

(14)

\[
\psi = \phi_j = \phi_{jr}
\]

(15)

2. Flow rule

Figure 6 shows the values of dilatancy angle \( \psi \) of the bedding planes corresponding to various confining pressures obtained using the same method as that used for calculating the \( \psi \) of the matrix based on the test data for \( \theta = 30^\circ \).
As shown in Figure 6, the $\psi$ of the bedding planes changes to a relatively large extent only under a confining pressure of 30 MPa and shows relatively insignificant changes under other confining pressures. Based on Figure 4A, this result occurs because the layered rock slips and fractures along the weak sedimentary plane when loaded at $\theta = 30^\circ$; the fracture surface is relatively smooth, with only a few fracture points, and consequently, the change in $\psi$ is relatively insignificant after fracture. While there are relatively large differences between the $\psi$ values of the bedding planes under different confining pressures, to reduce the complexity of the model, $\psi$ is simplified to a constant value using the method of Kong, and changes in the confining pressure and internal variable are no longer considered. Based on the test results, it is reasonable to set $\psi$ to 13-20$^\circ$.

Moreover, a comparison of Figures 3 and 6 shows that the $\psi$ values of the matrix and bedding planes of the layered rocks are far lower than the corresponding $\varphi$ values, mainly because of the loose internal structure of the sandstone. Furthermore, the changes in $\psi$ differ from those in $\varphi$. Thus, it is reasonable to use a nonassociated flow rule.

### 3.4 Model deficiencies

1. The Mohr-Coulomb criterion is used to fit the deviatoric stress corresponding to the various values of the internal variable without considering the effects of the intermediate principal stress. However, the effects of the intermediate principal stress on the rock strength are relatively insignificant under intermediate to low in situ stress conditions. Therefore, the accuracy requirements of the engineering computations can be met when the intermediate principal stress is not considered.

2. The variations of the strength parameters of the layered rocks with the internal variable are represented by piecewise linear functions. Based on the fitting results, the linear functions only roughly reflect the test results but favorably reduce the complexity of the elastoplastic equations.

### 4 VALIDATION

Table 1 summarizes the model parameters for the numerical triaxial test of the improved model. Figures 7 and 8 show a comparison of the numerical calculation and laboratory test results for $\theta=90^\circ$ and $\theta=30^\circ$, respectively.

As shown in Figure 7, the numerical simulation results for the stress and strain of the layered rocks under various confining pressures loaded at $\theta=90^\circ$ agree relatively well with the test results in terms of the numerical values and trends. The simulation results reflect the mechanical characteristics, such as brittle failure and strain softening, of the rock with horizontal bedding. The volumetric strain-axial strain curves also reflect the pattern of changes in the volumetric expansion of the sandstone with increasing confining pressure relatively well. As shown in Figure 8, the numerical calculation results show a good agreement with the test results in the trend for the sandstone under various confining pressure loaded at $\theta=30^\circ$. However, there is a certain difference in the residual stage between the numerical and measured volumetric stress curves. This result occurs primarily because only the circular deformation of the middle section of each specimen was measured in the laboratory tests, whereas the numerical simulation result is the average deformation at several points along the axial direction of the surface of each specimen. In addition, neglecting the effects of the confining pressure and internal variable on the $\psi$ of the bedding planes contributes to the difference between the simulation and test results.

A comparison of Figures 7 and 8 demonstrates that the numerical simulation results reflect the transversely isotropic
characteristics of the layered rocks and describe the patterns of the changes in the strength and deformation of the layered rocks with $\theta$ relatively well. The mechanical model established based on the laboratory test results for the layered rocks can describe the transversely isotropic mechanical properties and strain softening and dilatancy properties relatively well.

5 | DISCUSSION

5.1 | Effect of volumetric stress

In order to verify the effect of stress state on the strength and deformation failure of layered rocks, numerical simulation of triaxial compression tests is carried out using the SU model built-in FLAC3D which do not take the stress state into consideration. The parameters needed in the simulation are selected in Table 1 the same as the improved model. Taking the simulation of 30 MPa confining pressure at $\theta = 90^\circ$ as example, the built-in SU model simulation with the improved model simulation and indoor test results is compared. Figure 9 shows the volumetric strain-axial strain curve obtained by SU model agree well with the test results in terms of the trend but the value varies widely. Clearly, it is necessary to take the stress state into consideration in the numerical simulation to ensure the simulation results conform to the engineering.\(^{38}\)

### 5.2 | Discussions on model parameters

#### 5.2.1 | Parameter sensitivity

Whether the material parameters used are reasonable significantly affects the numerical calculation results for rock structures.\(^{39}\) It is necessary to ensure that the value of each parameter involved in the calculation is close to the actual value.\(^{40,41}\) Currently, numerical inversion methods based on field measurements for obtaining complex intermediate to high-dimensional constitutive parameters are rapidly being developed. In particular, displacement inversion methods have been commonly used because the mechanical parameter values determined using these methods are closer to the actual values.\(^{42-44}\)

Transversely isotropic constitutive models that contain a large number of mechanical parameters are used for layered rocks. Inversion of each parameter results in a high

| Parameter type          | Parameters | Definition                                      | Value  |
|-------------------------|------------|-------------------------------------------------|--------|
| Elastic parameter       | $E_1$      | Young’s moduli parallel to isotropic plane       | 36.3 GPa |
|                         | $E_3$      | Young’s moduli perpendicular to isotropic plane | 30.1 GPa |
|                         | $\nu_1$    | Poisson’s ratio parallel to isotropic plane     | 0.20   |
|                         | $\nu_3$    | Poisson’s ratio perpendicular to isotropic plane| 0.12   |
| Strength parameter of matrix | $c_0$    | initial cohesion of matrix                      | 20.5 MPa |
|                         | $c_r$      | residual cohesion of matrix                     | 14.7 MPa |
|                         | $\phi_0$  | initial internal friction angle of matrix       | 47.6$^\circ$ |
|                         | $\phi_r$  | residual internal friction angle of matrix      | 40.3$^\circ$ |
|                         | $\sigma_t$| tensile strength of matrix                      | 7.3 MPa |
|                         | $\kappa_\phi$ | corresponding internal variable when friction angle reaching residual value | 0.9 |
| Strength parameter of bedding | $c_j$    | initial cohesion of bedding                     | 7.3 MPa |
|                         | $c_{jr}$  | residual cohesion of bedding                    | 2.9 MPa |
|                         | $\phi_j$  | initial internal friction angle of bedding      | 33$^\circ$ |
|                         | $\phi_{jr}$| residual internal friction angle of bedding     | 33$^\circ$ |
|                         | $\sigma_{tj}$ | tensile strength of bedding                    | 3.6 MPa |
| Dilatancy angle          | $\psi_j$  | dilatancy angle of bedding                      | 13$^\circ$ |
computational load and a decrease in the solution stability and causes the model to be easily trapped in local minima. In addition, the different mechanical parameters of models affect the deformation and failure of surrounding rocks to different extents. Thus, it is necessary to analyze the sensitivity of the mechanical parameters that affect the deformation and failure of surrounding rocks. Such sensitivity analysis is of great importance not only to determine the mechanical parameters that more significantly affect the deformation of the surrounding rocks but also to reduce the number of parameters that require inversion.

The improved mechanical model previously established and single-factor sensitivity analysis are used to calculate the sensitivity of the parameters of the rocks surrounding the roadway in the Ji’ning No. 3 Coal Mine. The calculation process is described as follows. (a) Based on the compression test results, the reference value and value range of each parameter are determined using the RockLab software developed based on the Hoek-Brown yield criterion. (b) A comparison of the numerical calculation and laboratory test results for $\theta = 90^\circ$.
The numerical model for roadway excavation is established. The convergence values of the side to side and top to the floor of the roadway are taken as index to determine the sensitivity of the different deformation parameters. Figure 10 shows the numerical model and the arrangement of the measurement lines (the lines connecting the measurement points are denoted k12, k13, and k23).

Figure 11 shows the relationships between certain parameters and the deformation of the surrounding rocks. Based on these relationships and the sensitivity analysis results, the model parameters are classified into three types, namely those that are significantly, moderately, and not sensitive to deformation. (a) Parameters that are significantly sensitive to deformation include $E_1$, $c_0$, $\varphi_0$, and $\varphi_f$. (b) Parameters that are moderately sensitive to deformation include $E_3$, $\sigma_r$, $\nu_1$, $\nu_3$, $c_r$, $\psi$, and $\varphi_j$. (c) Parameters that are insensitive to deformation include $E_3$.
include $c_j$, $c_{jr}$, $\phi_{jr}$, $\psi_j$, and $\sigma_{tj}$, each of which has a sensitivity level less than 0.2. Thus, test results can be directly used for these parameters. (See the list of symbols for the definitions of these parameters).

### 5.2.2 Parameter determination

Because of the structural planes in the rock mass and the scale effect of the indoor test specimen, the mechanical properties obtained by indoor test are notably different from engineering rock mass.\(^{46-48}\) Besides, as mentioned above, only a part of the mechanical parameters significantly affect the deformation and damage of the surrounding rocks. Based on the convergence value of $k_{12}$, $k_{13}$, and $k_{23}$ of the roadway in Figure 10, inversion is performed for the mechanical parameters of the rocks significantly and moderately sensitive to the deformation using intelligent back-analysis\(^ {49}\) and FLAC3D software. Longitudinal section of the excavation part of the numerical model is shown in Figure 12. On this basis, the values for the mechanical parameters of the layered surrounding rocks are determined. Table 2 summarizes the mechanical parameters for engineering calculation.

Figure 13 compares the amounts of convergence obtained by inversion and the in-site monitored values. The measured values agree relatively well with the calculation results for the displacements of the surrounding rocks, thereby validating the reliability of the inversion results and the effectiveness of the improved model.

![Figure 12](Image) Longitudinal section of the excavation part of the model

![Figure 13](Image) Comparison of the measured and calculated amounts of convergence of the roadway ($k_{23}$, $k_{12}$, and $k_{13}$ are the measuring lines of two side, roof-left side, and roof-right side of the roadway, respectively, as shown in Figure 10)

| Parameter type         | Parameters | Definition                                    | Value   |
|------------------------|------------|-----------------------------------------------|---------|
| Elastic parameter      | $E_1$      | Young’s moduli parallel to isotropic plane     | 18.5 GPa |
|                        | $E_3$      | Young’s moduli perpendicular to isotropic plane | 15 GPa  |
|                        | $\nu_1$    | Poisson’s ratio parallel to isotropic plane    | 0.2     |
|                        | $\nu_3$    | Poisson’s ratio perpendicular to isotropic plane | 0.12    |
| Strength parameter of matrix | $c_0$ | initial cohesion of matrix                    | 6.8 MPa |
|                        | $c_r$      | residual cohesion of matrix                   | 2.6 MPa |
|                        | $\phi_0$   | initial internal friction angle of matrix      | 39°     |
|                        | $\phi_r$   | residual internal friction angle of matrix     | 22°     |
|                        | $\sigma_t$ | tensile strength of matrix                    | 2.2 MPa |
|                        | $\kappa$   | corresponding internal variable when friction angle reaching residual value | 0.9    |
| Strength parameter of bedding | $c_j$ | initial cohesion of bedding                   | 5.5 MPa |
|                        | $c_{jr}$   | residual cohesion of bedding                  | 1.5 MPa |
|                        | $\phi_j$   | initial internal friction angle of bedding     | 33°     |
|                        | $\phi_{jr}$| residual internal friction angle of bedding    | 24°     |
|                        | $\sigma_{tj}$ | tensile strength of bedding                  | 1.2 MPa |
| Dilatancy angle        | $\psi_j$   | dilatancy angle of bedding                    | 15°     |
6 | CONCLUSIONS

An improved mechanical model for layered rocks that incorporating planes of weakness and volumetric stress was established by improving the SU model using continuum mechanics methods and considering the basic properties of the matrix and bedding planes of layered rocks.

The main improvements made to the SU model are summarized as follows. (a) A plastic internal variable function that considers the volumetric stress effect was defined to better meet the actual engineering conditions. This internal variable function was used as the basis for improving the mechanical model for layered rocks. (b) Based on derived expression of internal variable and indoor test data, stress-dependent parameters are proposed for both intact rock and bedding plane, respectively, in order to better reflect the complex responses of rock mass due to stress disturbance or redistribution. The strength parameters and dilatancy angle change with the stress state and internal variable in different patterns. (c) The improved model was used in the numerical simulation of laboratory tests to validate its effectiveness. (d) To reduce the complexity of the obtained parameters for the improved model, the sensitivity of the model parameters and a parameter determination method were discussed.

Several limitations are also addressed including intermediate principal stress and piecewise linear function of strength parameters. Numerical simulation of laboratory tests on a series of specimens was conducted, and the simulated values were compared with the test results. The preliminary validity of the model was confirmed through these examples.

ACKNOWLEDGMENTS

The research described in this paper was financially supported by the National Key R&D Program of China (2018YFC0604703), Natural Science Foundation of Shandong Province (ZR2017BEE013), Research Fund of the State Key Laboratory of Coal Resources and Safe Mining, CUMT (SKLCRSM19KF016), National Natural Science Foundation of China (51427803, 51574154, 51704179, 51704097, 51904166), Wuhan Yellow Crane Talents (Science) Plan of China, Major Program of Shandong Province Natural Science Foundation (ZR2018ZA0603), Key R&D Program of Shandong Province (2018GSF116003), and Scientific Research Foundation of Shandong University of Science and Technology for Recruited Talents (2017RCJJ009). We also thank American Journal Experts for the linguistic assistance during the preparation of this manuscript.

NOMENCLATURE

- $I_1$: First invariant of the stress tensor
- $\sigma_1$, $\sigma_2$, $\sigma_3$: Major, intermediate, and minor principal stresses
- $E_1$: Young's moduli within the transversely isotropic plane
- $E_3$: Young's moduli normal to the transversely isotropic plane
- $\nu_1$: Poisson's ratio characterizing lateral contraction in the plane of isotropy when tension is applied in this plane
- $\nu_3$: Poisson's ratio characterizing lateral contraction in the plane of isotropy when tension is applied in the direction normal to it
- $G_2$: Shear moduli for any plane normal to the plane of isotropy
- $\kappa$: Plastic internal variable
- $\sigma_c$: Uniaxial compressive strength
- $P_c$: Confining pressure
- $\gamma^p$: Equivalent plastic shear strain
- $\theta$: Angle between the bedding plane and the axial stress
- $c_i$, $c_r$: Initial and residual cohesion of matrix
- $\varphi_i$, $\varphi_r$: Initial and residual internal friction angle of matrix
- $c_j$, $c_{jr}$: Initial and residual cohesion of bedding
- $\varphi_j$, $\varphi_{jr}$: Initial and residual internal friction angle of bedding
- $\sigma_r$, $\sigma_{jr}$: Tensile strength of matrix and bedding
- $\psi_r$, $\psi_{jr}$: Dilatancy angle of matrix and bedding

REFERENCES

1. Guo W, Tan Y, Yu F, et al. Mechanical behavior of rock-coal rock specimens with different coal thicknesses. Geomech Eng. 2018;15(4):1017-1027.
2. Liu X, Tan Y, Ning J, Lu Y, Gu Q. Mechanical properties and damage constitutive model of coal in coal-rock combined body. Int J Rock Mech Min Sci. 2018;110:140-150.
3. Feng F, Li X, Rostami J, Peng D, Li D, Du K. Numerical investigation of hard rock strength and fracturing under polyaxial compression based on Mogi-Coulomb failure criterion. Int J Geomech. 2019;19(4):04019005.
4. Wei M, Dai F, Xu N, Liu Y, Zhao T. Fracture prediction of rocks under mode I and mode II loading using the generalized maximum tangential strain criterion. Eng Frac Mech. 2017;186:21-38.
5. Wei M, Dai F, Xu N, Liu Y, Zhao T. A novel chevron notched short rod bend method for measuring the mode I fracture toughness of rocks. Eng Frac Mech. 2018;190:1-15.
6. Yin S, Nie W, Liu Q, Hua Y. Transient CFD modelling of space-time evolution of dust pollutants and air-curtain generator position during tunneling. J Clean Prod. 2019;239:117924.
7. Meng F, Wong L, Zhou H, Yu J, Cheng G. Shear rate effects on the post-peak shear behaviour and acoustic emission characteristics of artificially split granite joints. Rock Mech Rock Eng. 2019;52(7):2155-2174.
8. Yang W, Luo G, Duan K, et al. Development of a damage rheological model and its application in the analysis of mechanical properties of jointed rock masses. Energy Sci Eng. 2019;7(3):1016-1031.

9. Zhou W, Nie W, Liu C, et al. Modelling of ventilation and dust control effects during tunnel construction. Int J Mech Sci. 2019;160:358-371.

10. Zhao T, Guo W, Tan Y, Lu C, Wang C. Case histories of rock bursts under complicated geological conditions. Bull Eng Geol Environ. 2018;77:1529-1545.

11. Donze FV, Scholtès L. Predicting The Strength of Anisotropic Shale Rock: Empirical Nonlinear Failure Criterion vs. Discrete Element Method Model. Aussois, France: Alert Doctoral School; 2017.

12. Lisjak A, Tatone BSA, Grasselli G, Vietor T. Numerical modelling of the anisotropic mechanical behavior of Opalinus Clay at the laboratory-scale using FEM/DEM. Rock Mech Rock Eng. 2014;7(1):187-206.

13. Sainsbury BL, Sainsbury DP. Practical use of the ubiquitous-joint constitutive model for the simulation of anisotropic rock masses. Rock Mech Rock Eng. 2017;50(6):1507-1528.

14. Bagheripour M, Rahgozar R, Pashnesaz H, Malekinejad M. A complement to Hoek-Brown failure criterion for strength prediction in anisotropic rock. Geomech Eng. 2011;3(1):61-81.

15. Wang J, Ning J, Qiu P, Yang S, Shang H. Microseismic monitoring and its precursory parameter of hard roof collapse in longwall faces: a case study. Geomech Eng. 2019;17(4):375-383.

16. Shen B, Siren T, Rinne M. Modelling fracture propagation in anisotropic rock mass. Rock Mech Rock Eng. 2015;48(3):1067-1081.

17. Hoek E. Underground excavations in rock. Eng Geol. 1983;19(3):244-246.

18. Ramamurthy T, Arora VK. Strength predictions for jointed rocks in confined and unconfined states. Int J Rock Mech Min Sci Geomech Abstr. 1994;31(1):9-22.

19. Nasser MH, Rao KS, Ramamurthy T. Anisotropic strength and deformational behavior of Himalayan schists. Int J Rock Mech Min Sci. 2003;40(1):3-23.

20. Fereidooni D, Khanlari GR, Heidari M, Sepahigero AA, Kolahi-azar AP. Assessment of inherent anisotropy and confining pressure influences on mechanical behavior of anisotropic foliated rocks under triaxial compression. Rock Mech Rock Eng. 2016;49(6):2155-2163.

21. Ji S, Wang Q, Marcotte D, Salisbury M, Xu Z. P wave velocities, anisotropy and hysteresis in ultrahigh-pressure metamorphic rocks as a function of confining pressure. J Geophys Res. 2007;112:B09204.

22. Prioul R, Bakulin A, Bakulin V. Nonlinear rock physics model for estimation of 3D subsurface stress in anisotropic formations: theory and laboratory verification. Geophysics. 2004;69(2):415-425.

23. Fu Y, Iwata M, Ding W, Zhang F, Yashima A. An elastoplastic model for soft sedimentary rock considering inherent anisotropy and confining-stress dependency. Soils Found. 2012;52(4):575-589.

24. Li Z, Zhou H, Jiang Y, Hu D, Zhang C. Methodology for establishing comprehensive stress paths in rocks during hollow cylinder testing. Rock Mech Rock Eng. 2018;52(4):1055-1074.

25. Hu S, Tan Y, Zhou H, et al. Impact of bedding planes on mechanical properties of sandstone. Rock Mech Rock Eng. 2017;50(8):2243-2251.

26. Zhou Y, Feng X, Xu D, Fan Q. An enhanced equivalent continuum model for layered rock mass incorporating bedding structure and stress dependence. Int J Rock Mech Min Sci. 2017;97:75-98.

27. Zhang C, Zhou H, Feng X. An index for estimating the stability of brittle surrounding rock mass: FAI and its engineering application. Rock Mech Rock Eng. 2011;44(4):401-414.

28. Zhou H, Zhang K, Feng X, Shao J, Qiu S. Elastoplastic coupling mechanical model for brittle marble. Chin J Rock Mech Eng. 2010;29(12):2398-2409 (in Chinese).

29. Yang F, Zhou H, Zhang C, Hu D, Lu J, Meng F. An elastoplastic coupling mechanical model for hard and brittle marble with consideration of the first stress invariant effect. Eur J Environ Civ Eng. 2016;1:24.

30. Hajabdolmajid V. Mobilization of strength in brittle failure of rock. Dissertation, Queen's University, Canada; 2001.

31. Drucker DC, Prager W. Soil mechanics and plastic analysis or limit design. Quarterly of Appl Math. 1995;10(2):157-165.

32. Vermeer PA. Non-associated plasticity for soils, concrete and rock. In: Herrmann HJ, Hovi JP, Luding S, editors. Physics of Dry Granular Media. Nato Asi Series (Series E: Applied Sciences) (Vol. 350); Dordrecht: Springer; 1998:163-196.

33. Zhang S, Xiao M. Error analysis of rock strength from triaxial and shear tests. J Xiangtan Min Inst. 1997;12(4):1-5 (in Chinese).

34. Ramamurthy T. Shear strength response of some geological materials in triaxial compression. Int J Rock Mech Min Sci. 2001;38(5):683-697.

35. Heng S, Guo Y, Yang C, Daemen J, Li Z. Experimental and theoretical study of the anisotropic properties of shale. Int J Rock Mech Min Sci. 2015;74:58-68.

36. Huang W, Li C, Zhang L, Yuan Q, Zheng Y, Liu Y. In situ identification of water-permeable fractured zone in overlying composite strata. Int J Rock Mech Min Sci. 2018;105:85-97.

37. Kong W, Rui Y, Dong B. Determination of dilatancy angle for geomaterials under non-associated flow rule. Rock Soil Mech. 2009;30(11):3278-3282 (in Chinese).

38. Tyrnymov A. Numerical modeling and analysis of the stress-strain state in an anisotropic rock mass by the method of graphs. J Min Sci. 2012;48(5):812-824.

39. Vatanpour N, Ghafoori M, Talouki H. Probabilistic and sensitivity analyses of effective geotechnical parameters on rock slope stability: a case study of an urban area in northeast Iran. Nat Hazards. 2014;71(3):1659-1678.

40. Wang J, Ning J, Jiang L, Jiang J, Bu T. Structural characteristics of strata overlying of a fully mechanized longwall face: a case study. J S Afr I Min Metall. 2018;118(11):1195-1204.

41. Liu Q, Nie W, Hua Y, Peng H, Liu C, Wei C. Research on tunnel ventilation systems: dust diffusion and pollution behaviour by air curtains based on CFD technology and field measurement. Build Environ. 2019;147:444-460.

42. Feng X, Zhang Z, Sheng Q. Estimating mechanical rock mass parameters relating to the Three Gorges Project permanent ship lock using an intelligent displacement back analysis method. Int J Rock Mech Min Sci. 2000;37(7):1039-1054.

43. Yazdani M, Sharifzadeh M, Kamrani K, Ghorbani M. Displacement-based numerical back analysis for estimation of rock mass parameters in Siah Bisheh powerhouse cavern using continuum and discontinuum approach. Tunn Undergr Sp Technol. 2012;28(1):41-48.

44. Guo W, Gu Q, Tan Y, Hu S. Case studies of rock bursts in tectonic areas with facies change. Energies. 2019;12(7):1330.
45. Hoek. Brief history of the Hoek-Brown criterion. Rocklab 1.0 manual. 2002.
46. Pratt H, Black A, Brown W, Brace W. The effect of specimen size on the mechanical properties of unjointed diorite. *Int J Rock Mech Min Sci*. 1972;9(4):513-516.
47. Feng F, Chen S, Li D, Hu S, Huang W, Li B. Analysis of fractures of a hard rock specimen via unloading of central hole with different sectional shapes. *Energy Sci Eng*. 2019. https://doi.org/10.1002/ese3.432
48. Huang W, Wang X, Shen Y, Feng F, Wu K, Li C. Application of concrete-filled steel tubular columns in gob-side entry retaining under thick and hard roof stratum: a case study. *Energy Sci Eng*. 2019. https://doi.org/10.1002/ese3.442
49. Feng X, Zhao H, Li S. A new displacement back analysis to identify mechanical geo-material parameters based on hybrid intelligent methodology. *Int J Numer Anal Met*. 2010;28(11):1141-1165.

How to cite this article: Hu S, Tan Y, Zhou H, et al. Anisotropic modeling of layered rocks incorporating planes of weakness and volumetric stress. *Energy Sci Eng*. 2020;8:789–803. https://doi.org/10.1002/ese3.551