Development of axial asymmetry in the neutron-rich isotope $^{110}$Mo

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Abstract. The neutron-rich nucleus $^{110}$Mo has been investigated by means of $\gamma$-ray spectroscopy following the $\beta$-decay of $^{110}$Nb, produced using in-flight fission of a $^{238}$U beam at 345 MeV/nucleon at the RIBF facility. In addition to the ground-band members reported previously, spectroscopic information on the low-lying levels of the quasi-$\gamma$ band built on the second $2^+$ state at 494 keV has been obtained for the first time. The experimental finding of the second $2^+$ state being lower than the yrast $4^+$ level suggests that axially-asymmetric $\gamma$ softness is substantially enhanced in this nucleus. The experimental results are compared with model calculations based on the general Bohr Hamiltonian method. The systematics of the low-lying levels in even-even $A \approx 110$ nuclei is discussed in comparison with that in the neutron-rich $A \approx 190$ region, by introducing the quantity $E_S/E(2^+_1) - E(4^+_1)$, as a global signature of the structural evolution involving axial asymmetry.

1. Introduction

One of the long-standing unsolved issues in nuclear structure studies is the rare occurrence of well-deformed oblate (disc-like) shapes in nuclear ground states, in contrast to the large abundance of prolate (cigar-like) deformations [1]. A simple explanation for this empirical fact can be suggested in Ref. [2] based on the properties of quantized orbits in deformed potentials. According to Ref. [3], the cubic term of deformation in the liquid drop formula is also effective in lowering the energy of the prolate shape relative to the oblate one. In actual nuclei, however, it is expected that a subtle interplay between the single-particle and collective degrees of freedom plays a significant role in the shape polarization. Under a certain condition that multiple energy minima coexist at prolate and oblate deformation in the potential energy surface [4], the two different shapes can compete, and presumably interact, leading to the nuclear shape being soft with respect to the $\gamma$ degree of freedom, where $\gamma$ represents a deviation from axial symmetry of the ellipsoidal shape; $\gamma = 0^\circ$ and $60^\circ$ for axially symmetric, prolate and oblate shapes, respectively, and $30^\circ$ for a maximally asymmetric nucleus that has three different radii in Cartesian coordinates. The absence of well-deformed oblate ground states in nature may be associated partly with such a transitional character of axially-asymmetric $\gamma$-soft nuclei. Hence, it is certainly necessary to explore deformed nuclei at the critical point of the prolate-to-oblate transition, if a proper understanding of the mechanisms underlying the spontaneous symmetry breaking (Jahn-Teller effect) in nuclear-shape deformation is to be reached.

In the present work, we have investigated neutron-rich $Z \approx 40$, $A \approx 110$ nuclei, in which the phase transitions from prolate, via $\gamma$-soft, to oblate shapes are predicted to occur with increasing number of neutrons [5]. A similar type of the shape evolution is suggested for Hf.
Figure 1. Partial level scheme of $^{110}$Mo established in the present work (left). The widths of arrows represent relative intensities of $\gamma$ rays. $\gamma$-ray energy spectrum measured in coincidence with $\beta$ rays detected within 250 ms after implantation of $^{110}$Nb (right). Contaminants from $^{110}$Tc and $^{109}$Mo are indicated with filled and open triangles, respectively.

W, and Os isotopes with $A \approx 190$ [6]. These are the regions where the Fermi surfaces for protons and neutrons concurrently lie at the upper halves of the respective major shells, and the rotation alignment of both types of nucleons in the high-$j$ orbits is expected to enhance the oblate stability [7, 8]. The recent observation of a possible oblate-shape isomer in $^{109}$Nb [9] motivates one to probe the $N = 68$ isotope $^{110}$Mg, with the particular aim of studying its $\gamma$-soft nature as a fingerprint for the prolate-oblate shape transition.

2. Experimental procedures
Experiments were carried out at the RIBF facility [10] in collaboration with RIKEN Nishina Center and CNS, University of Tokyo. Neutron-rich $A \approx 110$ nuclei were produced via in-flight fission of $^{238}$U$^{86+}$ projectiles at 345 MeV/nucleon, incident on a beryllium target with a thickness of 3 mm. The average beam intensity was approximately 0.3 pnA during the experiment. The nuclei of interest were separated and transported through the BigRIPS spectrometer [11, 12], operated with a 6-mm-thick wedge-shaped aluminum degrader at the first dispersive focal plane for purification of the secondary beams. An additional degrader placed at the second dispersive focus served as a charge stripper to remove fragments that were not fully stripped. The identification of nuclei by their atomic number and the mass-to-charge ratio was achieved on the basis of the $\Delta E$-TOF-$B\rho$ method, where $\Delta E$, TOF, and $B\rho$ denote energy loss, time of flight, and magnetic rigidity, respectively.

A total of $5.2 \times 10^4$ $^{110}$Nb ions were implanted into an active stopper consisting of nine double-sided silicon-strip detectors (DSSSD) stacked compactly. Each DSSSD has a thickness of 1 mm with a 50 mm $\times$ 50 mm active area segmented into sixteen strips on both sides in the vertical and horizontal dimensions. The DSSSDs also served as detectors for electrons following $\beta$-decay and internal-conversion processes. The implantation of an identified particle was associated with the subsequent electron events that were detected in the same DSSSD pixel. Gamma rays were detected by four Compton-suppressed Clover-type Ge detectors arranged around the DSSSD telescope in a close geometry. Further details of a particle-identification spectrum and data-analysis techniques are given in Ref. [9].

3. Results
The left panel in Fig. 1 exhibits the level scheme of $^{110}$Mo, established by means of $\beta$-delayed $\gamma$-ray spectroscopy following the decay of $^{110}$Nb. Prior to the present work, the ground-state
band in $^{110}$Mo has been known up to the $10^+$ state by measuring the prompt $\gamma$ rays from the spontaneous fission of $^{248}$Cm [13]. In addition to the $214$, $386$, and $532$-keV $\gamma$ rays that belong to the ground-state band, seven new transitions have been unambiguously observed in a singles $\gamma$-ray spectrum measured in coincidence with $\beta$ rays subsequent to implantation of $^{110}$Nb, as shown in the right panel in Fig. 1.

The second $2^+$ level ($2^+_2$) is proposed at 494 keV, decaying by the 281- and 494-keV transitions which directly feed the yrast $2^+$ and $0^+$ states, respectively. The assignment of the $2^+_2$ state at 494 keV is justified, since only the energy sum for the 281–214-keV cascade agrees with the energy of the 494-keV $\gamma$ ray within experimental errors among the newly observed $\gamma$ rays. This observation is consistent with the decay pattern from the $2^+_2$ state to the lower-lying $2^+_1$ and $0^+_1$ levels, as confirmed for even $^{42}$Mo and $^{44}$Ru isotopes in this neutron-rich region [14].

The 207-keV transition is assigned as feeding the $2^+_2$ state on account of the consistency in energy with the fission at 701 keV, which is most likely the $3^+$ member of the quasi-$\gamma$ band decaying also via the parallel deexcitation pathway that consists of the 214- and 487-keV transitions. The $\gamma$ rays at 421 and 463 keV are proposed to be the $4^+_1 \rightarrow 2^+_2$ and $5^+_1 \rightarrow 3^+_2$ transitions, respectively, based on the systematics of the quasi-$\gamma$-band levels for lighter Mo isotopes [14]. The assignment of $5^+$ for the 1164-keV state is positively supported by the observation of a possible deexcitation to the $4^+_1$ state via the 564-keV $\gamma$ ray. Furthermore, the measured intensity of the 532-keV $\gamma$ ray depopulating the yrast $6^+$ state at 1132 keV implies a sizable value of spin ($\approx 5\hbar$) for the $\beta$-decaying state in $^{110}$Nb, consistent with the argument above on the population of the $4^+_2$ and $5^+_1$ levels at 916 and 1164 keV, respectively, in the $\beta$ decay of $^{110}$Nb.

4. Discussion

Figure 2(a) exhibits the systematics of the low-lying levels in even $^{42}$Mo isotopes with neutron numbers ranging from 62 to 68, including the new result obtained for $^{110}$Mo$_{68}$ in the present work. It can be seen that the excitation energies of the $2^+_1$ and $4^+_1$ states reach a minimum at $N = 64$; an examination of the $E(4^+_1)/E(2^+_1)$ ratio suggests that the maximum quadrupole deformation of the ground state occurs for $^{106}$Mo$_{64}$ [13]. Meanwhile, the $2^+_2$ level falls down in energy as the neutron number increases toward $N = 68$. Indeed, the proposed $2^+_2$ state at 494
Figure 3. Experimental and calculated level energies (in keV) for the ground-state (left) and quasi-γ bands (middle) in $^{110}$Mo. The model calculations are based on the general Bohr Hamiltonian approach with the CHFB+IB method using the SIII (A) and SLy4 (B) versions of the Skyrme interaction, and the CHFB+LQRPA method using the P+Q force (C). Right panels show potential energy surface calculations for $^{110}$Mo with the SIII (A) and SLy4 (B) versions of the Skyrme interaction. The energy minima are indicated with filled circles.

keV in $^{110}$Mo is lowest in energy of the $2^+_2$ levels identified to date for the even-even nuclei in this neutron-rich region. The $2^+_2$ states in deformed nuclei are associated with the bandhead of the quasi-γ band. Therefore, the presence of the rather low-lying $2^+_2$ state indicates that the degree of axial asymmetry increases, irrespective of its nature being either dynamic or static.

The experimental excitation energies of the $2^+_2$, $4^+_1$, and $2^+_2$ states in even Mo isotopes are reproduced well in a microscopic theory based on the general Bohr Hamiltonian approach [16], in which the potential energy and inertial functions (mass parameters) are calculated using the constrained Hartree-Fock-Bogoliubov (CHFB) method with the Skyrme effective interaction. For $^{110}$Mo, the $2^+_2$ state is predicted to lie at about 500 keV (see Fig. 6 in Ref. [16]), being in good agreement with the energy assigned in the present work. The level energies of the ground-state and quasi-γ bands calculated using the SIII and SLy4 versions of the Skyrme interaction are shown in Figs. 3(A) and 3(B), respectively.

We also performed a similar calculation with the pairing-plus-quadrupole (P+Q) model including a quadrupole pairing [17]. The parameters in the P+Q Hamiltonian are fixed so as to reproduce the Skyrme-HFB calculation with the SLy4 interaction. In this model, the inertial functions are calculated with the local quasiparticle random-phase approximation (LQRPA), in which the contributions from the time-odd mean fields are taken into account. The result of this calculation is shown in Fig. 3(C).

Figure 3 provides the comparison of the experimental levels with the predictions of the two theoretical frameworks. The agreement between the observed and calculated level energies is very satisfactory for both the ground-state and quasi-γ bands. In particular, it is noteworthy that the observation for the $2^+_2$ state being lower than the $4^+_1$ level is reproduced in all of the calculations. The quantity $R_b = B(E2; 2^+_2 \rightarrow 2^+_1)/B(E2; 2^+_2 \rightarrow 0^+_1)$ can be extracted from the measured intensity ratio for a pair of γ rays deexciting the $2^+_2$ state on the assumption that the $2^+_2 \rightarrow 2^+_1$ transition has a pure $E2$ multipolarity. In the current work, an experimental value of...
$R_b = 10 \pm 3$ has been obtained; this is in reasonable agreement with calculated values, $R_b = 33.9$ for the CHFB+IB with SLy4 and $R_b = 9.4$ for the CHFB+LQRP A with P+Q.

In the right panels in Fig. 3, the potential energy surface calculations for $^{110}$Mo exhibit local minima at the prolate ($\gamma \approx 0^\circ$) and oblate ($\gamma \approx 60^\circ$) sides using the SIII and SLy4 versions of the Skyrme interaction, respectively. However, the exact location of the energy minimum is not important in characterizing the collective level properties of the heavier Mo isotopes, because an overall profile in the potential energy surface spreads over the $\gamma$ degree of freedom. Consequently, the level structure of $^{110}$Mo is ascribed to its $\gamma$-soft nature rather than the rigid deformation of any kind.

The systematic behavior of the low-lying level energies in the Mo isotopes compares well with that in neutron-rich $^{74}$W nuclei [15], as is evident from Figs. 2(a) and 2(b); the $2^+_2$ states are lower than the $4^+_1$ levels at $N = 68$ and 116 in the Mo and W isotopes, respectively. Such a crossing of the $4^+_1$ and $2^+_2$ states has also been observed for neutron-rich $A \approx 110$ $^{44}$Ru nuclei and $A \approx 190$ $^{76}$Os isotopic chains, in both of which this energy trend is followed by an increase in the $2^+_2$ energy at neutron numbers higher than 68 and 116 [18, 19].

The comparison of the structural evolution in the $A \approx 110$ and 190 nuclei can be highlighted by introducing the quantity $E_S/E(2^+_1)$, where $E_S$ denotes the energy difference between the $2^+_2$ and $4^+_1$ states. The results are plotted in Figs. 2(c) and 2(d) against the neutron number for selected isotopes in each region. For $^{48}$Pd and $^{78}$Pt isotopes, which are representative of $\gamma$-soft nuclei [20, 21], the deduced values of $E_S/E(2^+_1)$ are within a narrow range below zero. In the $\gamma$-independent limit of the Wilets-Jean model [22], the quantity $E_S$ is zero because the $2^+_2$ and $4^+_1$ states are completely degenerate. On the other hand, the $2^+_2$ state goes under the $4^+_1$ level in the nucleus with a rigid-triaxial shape for $\gamma \approx 25^\circ$ in the Davydov and Filippov model [23]; at the extreme of triaxiality ($\gamma = 30^\circ$), $E_S/E(2^+_1) = -0.67$. Therefore, nuclei with negative values of $E_S/E(2^+_1)$ between these two extremes are most likely characterized by $\gamma$-soft potentials with shallow minima at the average $\gamma$ value close to 30$^\circ$. This argument on the deformed potential with some $\gamma$ dependence is consistent with the observation of the $3^+_1$ state being depressed relative to the $4^+_2$ level for the Pd and Pt isotopes in the regions of interest [14]; these quasi-$\gamma$-band levels would be degenerate in the extreme $\gamma$-independent limit [22].

In Fig. 2(c), a steeper fall of the $E_S/E(2^+_1)$ values is found for the $^{42}$Mo isotopes compared to the $^{44}$Ru isotopes as the neutron number increases. The lighter isotopes $^{104}$Mo$_{62}$ and $^{106}$Mo$_{64}$ are known to exhibit rather $\gamma$-vibrational behavior, as demonstrated by the observation of the properties expected for rotational bands built on one- and two-$\gamma$-phonon states [24, 25]. Consequently, the observed rapid decrease in $E_S/E(2^+_1)$ in the Mo isotopic chain reflects the structural change from a nearly axial rotor with the small-amplitude $\gamma$ vibrations to a large-amplitude $\gamma$-soft dynamics. Similar trends can be seen for the $^{74}$W and $^{76}$Os isotopes up to $N = 116$, but the observed variation is more dramatic than that in the $A \approx 110$ nuclei with the same $N$ dispersions, as shown in Fig. 2(d). One intriguing feature of the empirical $E_S/E(2^+_1)$ plots in Figs. 2(c) and 2(d) is the near coincidence at around $-0.5$, observed for both the $N = 68$ and 116 isotones. This may suggest that the empirical value of $E_S/E(2^+_2) \approx -0.5$ is characteristic of the critical-point nuclei in terms of maximum $\gamma$ softness between prolate and oblate shapes. However, to elucidate the relation between the systematic behavior of $E_S/E(2^+_1)$ and the underlying cause for the structural evolutions, further theoretical investigations are necessary.

5. Conclusions

The level structure of $^{110}$Mo has been investigated following the $\beta$-decay of $^{110}$Nb, populated via in-flight fission of a $^{238}$U beam. In addition to the known levels of the ground-state band, several new levels, including a candidate for the $2^+_2$ quasi-$\gamma$-band state, have been identified. This is the most neutron-rich Mo isotope for which spectroscopic information on the low-lying
level structure has been obtained. The observed level energies and the $B(E2)$ ratio for a pair of $\gamma$ rays from the $2^+_1\rightarrow 2^+_2$ state are reproduced well by the model calculations based on the general Bohr Hamiltonian approach. The potential energy surface calculations indicate that the low-lying level structure of $^{110}$Mo is of a $\gamma$-soft nature. The energy systematics of the low-lying levels in $A \approx 110$ Mo isotopes compares well with that in $A \approx 190$ W isotopes. The enhancement of axially-asymmetric $\gamma$ softness in $^{110}$Mo is illuminated with the empirical plot of the quantity $E_S/E(2^+_1) = (E(2^+_2) - E(4^+_1))/E(2^+_1)$. In future experiments, it will be of particular interest to investigate the level properties of heavier Mo isotopes, in which more stable oblate shapes are predicted.

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