The approach to the analysis of the dynamic of non-equilibrium open systems and irreversibility

Vyacheslav Somsikov

Laboratory of Physics of the geogeliocosmic relation, Institute of Ionosphere, Almaty, 480020, Kazakhstan

Abstract

The approach to the analysis of the dynamic of non-equilibrium open systems within the framework of the laws of classical mechanics on the example of a hard-disk system is offered. This approach was based on Hamilton and Liouville generalized equations which was deduced for the subsystems of the nonequilibrium system. With the help of generalized Liouville equation it was obtained that two types of dynamics are possible: reversible and irreversible. The connection between the dynamical parameter -generalized field of forces, and entropy is established. The estimation of characteristic time of establishment of equilibrium in the thermodynamic limit is realized. It is shown how from the condition of irreversibility of a hard-disk system, the condition of irreversibility for the rarefied system of potentially interacting particles follows. The explanation of the mechanism of irreversibility is submitted.

Introduction. In connection with a modern physics, the Newton’s equation is a background of dynamical picture of the world. It is caused by the fact that all four known fundamental interactions of elementary particles are potential. Potentiality of forces causes reversibility of the Newton equation in time. Hence, dynamics of all natural systems consisting of elementary particles should be reversible. But irreversibility is a basis of evolution. In fundamental physics irreversibility determines the contents of the second law of thermodynamics. According to this law there is function $S$ named entropy, which can grow for isolated systems only, achieving a maximum in an equilibrium state. Thus, the fundamental physics includes areas contradicting each other: reversible classical mechanics and irreversible thermodynamics.

So, the rigorous substantiation of the second law of thermodynamics within the framework of the classical mechanics laws is one of the primary tasks of modern physics. This problem keenly reveals difficulties and limits of capability resources of classical methods of physics. Despite of numerous attempts, this problem does not have satisfactory solution up to date. Thus, the question how one can describe
irreversible processes on the basis of the reversible equations of the classical mechanics, are discussed and studied [1, 5, 8, 13, 14]. The solution of this question is necessary for development of the theory of irreversible processes on dynamic basis. In order to find an answer for this question, we studied the evolution of the non-equilibrium colliding hard-disk system [10-12].

Here we are going to show, how one can study of the mechanism of establishment of the equilibrium state within the framework of the classical mechanics laws if one replaces the canonical Hamilton and Liouville equations with the generalized equations, applied for nonholonomic and open systems.

The main idea of our approach to analyzes of non-equilibrium system is splitting of conservative nonequilibrium system into interacting subsystems, research of dynamics of one of them with help of the generalized Liouville equation, is not brought to the assumption of potentiality of their forces of interaction.

The work is constructed as follows. Using D’Alambert and the motion equations for disks, we obtain generalized Lagrange, Hamilton and Liouville equations for disk subsystem, which is selected from the whole disk system. Then the analysis of two types of evolution of nonequilibrium system - reversible and irreversible, was made. The existence of these types of evolution follows from the condition of compatibility of the generalized Liouville equation for subsystem and the canonical Liouville equation, applicable for whole system. Estimation of characteristic parameter of establishment of the equilibrium state was realized. Then, it is shown how the condition of irreversibility for the potentially interacting and rarefied disks system follows from the condition of irreversibility for a hard-disk system. The formula that determines the connection between increment entropy for full non-equilibrium system and the generalized field of forces of subsystems was obtained. In summary, the analysis of the results is submitted.

Mathematical apparatus. We used an approach of pair interaction of disks. Their motion equations are deduced with help of the matrix of pair collisions. As complex plane this matrix is given [10]:

\[ S_{kj} = \begin{pmatrix} a & -ib \\ -ib & a \end{pmatrix} \]

where \( a = d_{kj} \exp(i\vartheta_{kj}) \); \( b = \beta \exp(i\vartheta_{kj}) \); \( d_{kj} = \cos\vartheta_{kj} \); \( \beta = \sin\vartheta_{kj} \); \( i \) - an imaginary unit; \( k \)- and \( j \)- numbers of colliding disks; \( d_{kj} \)- the impact parameter (IP), determined by distance between centers of colliding disks in the Cartesian plane system of coordinates with axes of \( x \) and \( y \), in which the \( k \)-disk swoops on the lying the \( j \)- disk along the \( x \) - axis. The scattering angle \( \vartheta_{kj} \) varies from 0 to \( \pi \). In consequence of collision the transformation of disks velocities can be presented in such form: \( V_{kj}^+ = S_{kj} V_{kj}^- \) (a), where \( V_{kj}^- \) and \( V_{kj}^+ \) - are bivectors of velocities of \( k \) and \( j \) - disks before \((-\)) , and after \((+) \) collisions, correspondingly; \( V_{kj} = \{V_k, V_j\} \), \( V_j = V_{jx} + iV_{jy} \) - are complex velocities of the incident disk and the disk - target with corresponding components to the \( x \) - and \( y \) - axes. The collisions are considered to be central, and friction is neglected. Masses and diameters of disks “d” are accepted to be equal to 1. Boundary conditions are given as either periodical or in form of hard walls. From (a) we can obtain equations for the change of velocities of
colliding disks [10-12]:

\[
\begin{pmatrix}
\delta V_k \\
\delta V_j
\end{pmatrix} = \varphi_{kj} \begin{pmatrix}
\Delta_{kj}^- \\
-\Delta_{kj}^+
\end{pmatrix}.
\] (1)

Here, \( \Delta_{kj} = V_k - V_j \) - are relative velocities, \( \delta V_k = V_k^+ - V_k^- \), and \( \delta V_j = V_j^+ - V_j^- \) - are changes of disks velocities in consequence of collisions, \( \varphi_{kj} = i\beta e^{i\theta_{kj}} \).

That is, Eq. (1) can be presented in the differential form of Somsikov, V.M. [2001]:

\[
\dot{V}_k = \Phi_{kj} \delta(\psi_{kj}(t)) \Delta_{kj}
\] (2)

where \( \psi_{kj} = [1 - |l_{kj}|/|\Delta_{kj}|] \); \( \delta(\psi_{kj}) \)-delta function; \( l_{kj}(t) = z_{kj}^0 + \int_0^t \Delta_{kj} dt \) - are distances between centers of colliding disks; \( z_{kj}^0 = z_k^0 - z_j^0 \), \( z_k^0 \) and \( z_j^0 \) - are initial values of disks coordinates; \( \Phi_{kj} = i(l_{kj}(\Delta_{kj}))/(|l_{kj}|/|\Delta_{kj}|) \).

The Eq. (2) determines transformation of velocity of \( k \)-disk when it collides with \( j \)-disk. A right side of Eq.(2) represents force depending from \( \Delta_{kj} \). Therefore the equation (2) is not a Newtonian one [4].

Let’s take a disk system, which consist of \( N \) disks, when \( N \to \infty \) and \( L^2 \to \infty \), where \( N/L^2 = \text{finit} \); and \( L^2 \) - is area occupied by disks. Then divide this system on \( R \) subsystems, so that in each subsystem will be \( T >> 1 \) disks. Therefore, \( N = RT \). Let energy of the system is equal \( E = \text{const} \). It is equal to the sum of internal energies of all subsystems and interaction energies between subsystems. Let us select one of them, which we call \( p \)-subsystem. Let us examine \( \delta W_a^p \) - the virtual work of active forces made above \( p \)-subsystem. In the general case, this work can be presented us follows: \( \delta W_a^p = \sum_{k=1}^{T} \sum_{s=1}^{N-T} F_{ks}^p \delta r_{k} = \sum_{k=1}^{T} F_{ks}^p \delta r_{k} \), where \( k = 1, 2, 3...T \) -disks number of the \( p \) -subsystem, \( s = 1, 2, 3...N-T \) -external disk number to the \( p \) -subsystem of the disk, which interaction with \( k \)-disk of \( p \)-subsystem, \( F_{ks}^p \) - the interaction force of \( k \)- and \( s \) -disks, \( \delta r_{k} \) - virtual displacement of the \( k \)-disk, \( F_{ks}^p = \sum_{s=1}^{N-T} F_{ks}^p \).

Here we use that the virtual work of the interaction force of internal disks of a \( p \)-subsystem is equal to zero.

In case of the pair interaction approaching, the virtual work of external forces gets the form: \( \delta W_a^p = \sum_{k=1}^{T} F_{ks}^p \delta r_{k} = \sum_{k=1}^{T} \dot{V}_k \delta r_{k} \), because in this case \( F_{ks}^p = F_{ks}^p \). The inertial force can be presented so: \( \delta W_{in}^p = \sum_{k=1}^{T} \dot{V}_k \delta r_{k} \). The sum of the active and the inertial forces is called the effective force. The principle of D’Alambert asserts that the work of effective forces is always equal to zero [2], i.e.

\[
\delta W_{q}^p = \delta W_{in}^p - \delta W_{a}^p.
\] (3)

The feature above virtual work means, that in generally it is not reduced to a complete differential.

From the motion equations for a hard disks it follows [11]: \( \sum_{p=1}^{R} (\sum_{k=1}^{T} F_{ks}^p) \delta r_{k} = 0 \). Therefore the total of the active, and the total inertial works for all subsystems at any moment of time is equal to zero, i.e. \( \sum_{p=1}^{R} \delta W_{in}^p = \sum_{p=1}^{R} \delta W_{a}^p = 0 \). This equaling can take place in two cases: when the sum of nonzero members is equal to zero, and when each member of the sum is equal to zero. It is obviously that the second case,
appropriate to an equilibrium state, takes place when \( T \to \infty \). For this case, with the help of the motion equations for a hard disks, it is possible to record:

\[
\sum_{k=1}^{T} \dot{V}_k = \sum_{k=1}^{T} \Phi_{ks} \delta(\psi_{ks}(t)) \Delta_{ks}(t) = \sum_{k=1}^{T} F_{k} = 0, \tag{4}
\]

Equality to zero of the right-hand side of the equation (4) means, that the selected \( p \)-subsystem is in a stationary state.

To obtain the general Lagrange equation for \( p \)-subsystem, let’s transform D’Alambert equation (3) by multiplied it by \( dt \), and integrated it over an interval from \( t = t_1 \) to \( t = t_2 \). In the general case we have:

\[
\int_{t_1}^{t_2} \delta \dot{W}_q^p dt = \int_{t_1}^{t_2} \sum_{k=1}^{T} \left[ \frac{d}{dt} V_k - \sum_{j \neq k}^{T} F_{kj}^p \right] \delta r_k dt =
\]

\[
\delta \int_{t_1}^{t_2} \sum_{k=1}^{T} V_k^2 dt - \int_{t_1}^{t_2} \sum_{k=1}^{T} \left( F_{kj}^p + \sum_{j \neq k} F_{jk}^p \right) \delta r_k dt - \int_{t_1}^{t_2} \sum_{k=1}^{T} V_k \delta r_k |_{t_1}^{t_2} \tag{b}
\]

(b) In equation (b) the member \( \sum_{j \neq k} F_{kj}^p \) determines the force of interaction a internal disks of the \( p \)-subsystem. Then \( k \) and \( j \)-colliding disks from the \( p \)-subsystem. The member \( F_{k}^p \) is the force on the \( p \)-subsystem. If it is demanded that on the ends an interval \([t_1, t_2]\) the virtual displacements are zero, the last member in (b) will be equal to zero.

Let’s neglect nonequilibrium inside a subsystem (this is a typical assumption for local equilibrium). Then for internal forces of interaction of subsystem disks, we can set in the conformity such a function dependent on coordinates, \( U(r_1, r_2, ... r_T) \), for which the following condition is satisfied:

\[
\int_{t_1}^{t_2} \sum_{k=1}^{T} \sum_{j \neq k} F_{kj}^p \delta r_k dt = -\int_{t_1}^{t_2} U(r_1, r_2, ... r_T) dt.
\]

Here \( r_1, r_2, ... r_T \) - coordinates of disks of the \( p \)-subsystem disks. In general case it is impossible, to present acting on \( p \)-subsystem active forces, as a gradient of a force function [2]. In this case the equation (b) can be written as:

\[
\int_{t_1}^{t_2} \delta \dot{W}_q^p dt = \int_{t_1}^{t_2} \sum_{k=1}^{T} \left( \frac{d}{dt} V_k + \frac{\partial L_p}{\partial r_k} - \frac{\partial L_p}{\partial \dot{r}_k} \right) \delta r_k dt = 0 \tag{5}
\]

In eqn. (5) we denote \( L_p = \sum_{k=1}^{T} \frac{V_k^2}{2} + U(r_1, r_2, ... r_T) \). Therefore, if the interaction of disks will be potential, the \( L_p \) will include also internal potential energy of the \( p \)-subsystem - \( U(r_1, r_2, ... r_T) \). Because for any variations integral in equation (5) will be equal to zero, the next expression is carried out:

\[
\sum_{k=1}^{T} \left( \frac{d}{dt} V_k + \frac{\partial L_p}{\partial r_k} - \frac{\partial L_p}{\partial \dot{r}_k} \right) = \sum_{k=1}^{T} F_{k}^p = F_p \tag{6}
\]

We denote \( \sum_{k=1}^{T} F_{k}^p = F_p \).

The equation (6) is a generalized equation of Lagrange for a \( p \)-subsystem. So, \( F_p \), is the polygenic force acting on the \( p \)-subsystem which dependent from its dynamic. When the \( F_p = 0 \), the equation (6) transforms to a canonical equation of Lagrange for an equilibrium, conservative system.

Let us derive Hamilton’s equations for \( p \)-subsystem. The differential for \( L_p \) can be written as:

\[
dL_p = \sum_{k=1}^{T} \left( \frac{\partial L_p}{\partial r_k} dr_k + \frac{\partial L_p}{\partial V_k} dV_k \right) + \frac{\partial L_p}{\partial \dot{r}_k} dt, \text{ where } \frac{\partial L_p}{\partial \dot{r}_k} = p_k \text{ - is disks momentum.}
\]

With the help of Lagrange transformation, it is possible to get:

\[
d[\sum_{k=1}^{T} p_k V_k - L_p] = \sum_{k=1}^{T} \left( -\frac{\partial L_p}{\partial r_k} dr_k + V_k dp_k \right) - \frac{\partial L_p}{\partial \dot{r}_k} dt.
\]





Because \( \frac{\partial H_p}{\partial t} = -\frac{\partial L_p}{\partial t} \), where \( H_p = \sum_{k=1}^{T} p_k V_k - L_p \), we will have from (6):

\[
\frac{\partial H_p}{\partial r_k} = -\dot{p}_k + F^p_k \tag{7}
\]

\[
\frac{\partial H_p}{\partial p_k} = V_k \tag{8}
\]

These are the general Hamilton equations for the selected \( p \)-subsystem. The external forces, which acted on \( p \)-subsystem, presented in a right-hand side an equation (7).

Using equations (7,8), we can find the Liouville equation for \( p \)-subsystem. For this purpose, let’s to take a generalized current vector - \( J_p = (\dot{r}_k, \dot{p}_k) \) of the \( p \)-subsystem in a phase space [13]. From equations (7,8), we find:

\[
\text{div} J_p = \sum_{k=1}^{T} \left( \frac{\partial}{\partial r_k} V_k + \frac{\partial}{\partial p_k} \dot{p}_k \right) = \sum_{k=1}^{T} \frac{\partial}{\partial p_k} F^p_k \tag{9}
\]

The differential form of the particle number conservation law in the subsystem is a continuity equation:

\[
\frac{\partial f_p}{\partial t} + \text{div} (J_p f_p) = 0,
\]

where \( f_p = f_p(r_k, p_k, t) \) - the normalized distribution function of disks in the \( p \)-subsystem. With the help of the continuity equation and equation (9) for a divergence of a generalized current vector in a phase space, we can get:

\[
\frac{df_p}{dt} = -f_p \sum_{k=1}^{T} \frac{\partial}{\partial p_k} F^p_k. \tag{10}
\]

Equation (10) is a Liouville equation for \( p \)-subsystem. It has a formal solution:

\[
f_p = \text{const} \cdot \exp \left[ -\int_0^t \left( \sum_{k=1}^{T} \frac{\partial}{\partial p_k} F^p_k \right) dt \right].
\]

The equation (10) is obtained from the common reasons. It is suitable for any interaction forces of subsystems. Thus, the equation (10) is applicable to analyze any nonequilibrium open systems. In particularly, it can be used for explanation of irreversibility. The right side of (10), \( \sum_{k=1}^{T} \frac{\partial}{\partial p_k} F^p_k \), is the integral of collisions. This integral can be obtained from the motion equations of the systems element. For example, for a hard disks system it can be found with the help of the Eq. (2).

The stationary nonequilibrium state of the system can be supported by external constant stream of energy. Because \( \frac{df_p}{dt} = \frac{\partial f_p}{\partial t} + \sum_{k=1}^{T} \left( V_k \frac{\partial f_p}{\partial r_k} + \dot{p}_k \frac{\partial f_p}{\partial p_k} \right) \), then from (10) for stationary case we have:

\[
\sum_{k=1}^{T} \left( V_k \frac{\partial f_p}{\partial r_k} + \dot{p}_k \frac{\partial f_p}{\partial p_k} \right) = -f_p \sum_{k=1}^{T} \frac{\partial}{\partial p_k} F^p_k, \tag{11}
\]

The equation (11) is similarly to the equation, which described fluctuations of the distribution function in nonequilibrium gas [6]:

\[
V_k \frac{\partial f_p}{\partial r_k} = St f_p, \tag{12}
\]

Here, \( St f_p \)- is integral of collisions.
Let’s consider the important interrelation between descriptions of dynamics of separate subsystems and dynamics of system as a whole. As the expression, \( \sum_{p=1}^{R} \sum_{k=1}^{T} F_k^p = 0 \), is carried out, the next equation for the full system Lagrangian, \( L_R \), will have a place:

\[
\frac{d}{dt} \frac{\partial L_R}{\partial \dot{V}_k} - \frac{\partial L_R}{\partial V_k} = 0
\]

(13)

and the appropriate Liouville equation: \( \frac{\partial f}{\partial t} + V_k \frac{\partial f}{\partial \dot{V}_k} + p_k \frac{\partial f}{\partial \omega_k} = 0 \) The function, \( f_R \), corresponds to the full system. The full system is conservative. Therefore, we have: \( \sum_{p=1}^{R} \text{div} J_p = 0 \). This expression is equivalent to the next equality: \( \frac{d}{dt} (\sum_{p=1}^{R} \ln f_p) = \frac{d}{dt} (\ln \prod_{p=1}^{R} f_p) = (\prod_{p=1}^{R} f_p) \frac{d}{dt} (\prod_{p=1}^{R} f_p) = 0 \). So, \( \prod_{p=1}^{R} f_p = \text{const} \). In an equilibrium state we have \( \prod_{p=1}^{R} f_p = f_R \). Because the equality \( \sum_{p=1}^{R} F_p = 0 \) is fulfilled during all time, we have that equality, \( \prod_{p=1}^{R} f_p = f_R \), is a motion integral. It is in agreement with Liouville theorem about conservation of phase space [4]. So, only in two cases the Liouville equation for the whole system is in agreement with the general Liouville equation for selected subsystems: if the condition \( \int_{0}^{t} \left( \sum_{k=1}^{T} \frac{\partial F_k^P}{\partial \dot{V}_k} \right) dt \to \text{const} \) is satisfied when \( t \to \infty \), or when, \( \left( \sum_{k=1}^{T} \frac{\partial F_k^p}{\partial \omega_k} \right) \), is a periodic function of time. The first case corresponds to the irreversible dynamics, and the second case corresponds to reversible dynamics. So, non-potentiality of interaction forces of subsystems causes in an opportunity of existence of irreversible dynamics. Let’s consider, how these two types of dynamics can be realized.

**Irreversible dynamics.** The condition (c) occurs if the next condition is satisfied: \( \frac{\partial}{\partial t} F_p \leq 0 \), i.e., \( F_p(0, \omega_0) \geq F_p(t, \omega) = S^t F_p(0, \omega_0) \). These conditions determine irreversibility. Here \( S^t \) - a operator of evolution or a phase stream; \( \omega \) - a point of phase space. Let’s show that performance of these conditions follows from the law of momentum conservation of colliding disks. Firstly, we consider a simple case. Let us have \( P \) pairs of colliding disks. At that, \( P \) of them are resting disks, and \( P \) disks swoop on the latter with equal velocities directed along axis \( x \). Let distances between colliding disks are equal, therefore all collisions occur through equal interval of time, \( \tau \). In this case the total relative velocity of disks along \( x \) axis before collision would be equal to \( P \Delta k(t) \). After collisions the inequality \( P \Delta k(t) \geq \sum_{k=1}^{T} \Delta k(t + \tau) \) will take place, where \( \Delta k(t) \) are projections of relative velocity of disks on \( x \) axis. Equality would take place only if all collisions were frontal. So, as it follows from equation (2) the force may only decrease because \( F_p \) is proportional to \( \sum_{k=1}^{T} \Delta k \). Decreasing of \( F_p \) would take place in general case because inequality \( F_p \neq 0 \) takes place when the subsystem has average velocity which distinct from the average velocities of external disks. As a result, the system would come to stationary state and the condition for it, \( \sum_{k=1}^{T} \Delta k \), would be satisfied. It is corresponds to the stationary state when \( F_p = 0 \). From here, the conclusion that the decrease of force, \( F_p \), comes to homogeneous distribution of system energy, is follows.

Let’s consider the stability of the equilibrium state on the example of hard-disks system, and estimate characteristic time of decreasing force, \( F_p \). For this purpose we analyze solution’s stability Eq.2 in the equilibrium point.

The evolution of the \( p \)-subsystem is determined by the vector-column, \( \vec{V}_p^P \), which components is
speed of disks of the \( p \)-subsystem: \( \vec{V}_k^p = \{V_k^p\}, k = 1, 2, 3...T \). Some of the evolution’s properties of this subsystem will be determined by the studying of the sum of it components. Let us designate this sum as \( \Upsilon_p \). Carrying out the summation in Eq.(2) on all disks of the \( p \)-subsystem, we shall obtain:

\[
\dot{\Upsilon}_p = \sum_{k=1}^{T} \Phi_{ks} \Delta_{ks} = F_P
\]  

(14)

The equation (14) describes the change of a total momentum, acting onto the \( p \)-subsystem as a result of collisions. The relaxation of a total momentum to zero is equivalent to relaxation to zero of the force, \( F_P \). Now let us show, if the mixing property for a disks system is carried out, the homogeneous distribution of impact parameters of disks have place as well. In accordance to definition of the mixing condition, we have [7]: \( \mu(\delta)/\mu(d) = \delta/d \) where, \( \mu(d) \), is a measure corresponding to the total value of impact parameter - "d"; \( \delta \) - is an arbitrary interval of the impact parameter and, \( \mu(\delta) \), is a corresponding measure. The fulfillment of the mixing condition means the proportionality between the number of collisions of disks, falling at the interval, "\( \delta \)", and the length of this interval. It allows to say that distribution of the impact parameters is homogeneous. As it is well known [7, 13], for mixed systems the condition of depletion of correlations have place. For the equation (14) this condition can be written down so:

\[
< \Phi_{ks} \Delta_{ks} > = < \Phi_{ks} > < \Delta_{ks} > \text{ i.e. average from two multiplied functions is equal to multiplication of these functions average. The } \Phi_{ks} \text{ is dependent from impact parameters, and } \Delta_{ks} \text{ is dependent from relative velocities of colliding disks. Therefore this condition is similar to a condition of independence of coordinates and momenta that is widely used in the statistical physics Landau [1976]. Thus, it is possible to execute summation in the multiplier, } \Phi_{ks} \text{, on impact parameters, independent from summation of expression}

\Delta_{ks} \text{ on relative speeds of colliding disks. Then under the condition of the homogeneous distribution of impact parameters and when } T \gg \infty \text{, we can transit from summation to integration. So, we will have}\n
\[
\phi = 1/T \lim_{T \to \infty} \sum_{k=1}^{T} \varphi_{ks} = \frac{1}{T} \int_{-1}^{1} \varphi_{ks} d(\cos \vartheta) = -\frac{2}{3}, \text{ where } G = 2 \text{ is the normalization factor.}
\]

Taking it into account, we will have from eq. (14):

\[
\dot{\Upsilon}_p = -\frac{2}{3} \sum_{k=1}^{T} \Delta_{ks}.
\]

(15)

The negative factor in the right side equation (15) means, that the force, \( F_P \), will decrease. The stability of a stationary point \( p \)-subsystem can be established with the help of the motion equation (2). Let the point, \( Z_0 \), be a stationary point, in which the, \( F_P \), effecting on \( p \)-subsystem, is equal to zero. From the Lyapunov’s theorem about stability follows that the point, \( Z_0 \), is asymptotically stable if any deviation from it will be attenuated. Let us expand the left and right sides of the equation (16) into series by small parameter, \( \nu \), of perturbation of velocities of disks of the \( p \)-subsystem, near point, \( Z_0 \), and keep terms to first order. The expansion of the left side of the equation (16) gives: \( \dot{\nu} = \sum_{k=1}^{T} \dot{\varepsilon}_k \), where the summation is carried out on components of the variation, \( \nu \). In the expansion of the right side, the only remaining part is, \( -\frac{2}{3} \sum_{k=1}^{T} \varepsilon_k = -2/3\nu \). A contribution into the expansion will be given by collisions of disks of
the \( p \)-subsystem, with disks of its complement. So, we have: \( \dot{v} = -\frac{2}{3}v \). This equation means, that any system deviation from an equilibrium state will decreases. Hence, the stationary point at performance of a mixing condition is steady. Stability is provided by occurrence of returning force, \( F_p \), at a deviation of a subsystem from an equilibrium point. We shall note, that the equation (16) also follows from the theory of fluctuations (see Eq. [12]) where it is proved from other facts [3].

Reversibility dynamics. According to (c) and to the general concept of reversible dynamics the following inequalities should take place [7]: \( F_p(t + n\tau_0, \omega) = F_p(t, \omega) \), \( \Delta\Gamma_0 = S(t + n\tau_0)\Delta\Gamma_0(t) \), where \( \Delta\Gamma_0 \) is an element of volume of the phase space occupied with the subsystem at the moment, \( t \); \( \tau_0 \) is period of system’s return in the initial point; \( n = 1, 2, 3... \); \( \omega \in \Delta\Gamma_0 \). This condition takes place when a vector of the force, \( F_p \), rotates without change of its module. This is an example, demonstrating existence of such points, there is a system consisting of disks located on plane with a hard walls. If at the initial moment of time this system was at phase point, in which velocities of all disks are perpendicular to one of the walls, and all impacts of disks are frontal; so the condition of the reversibility would take place. I.e. in this case the system comes back periodically to the initial point. Therefore, such points are accepted to call as periodic or cyclic. Probability of system’s return is determined by probability to be system in these points at the moment of preparation and by stability these points. Really, if at the initial moment of time the system is not in one of these points, the reversible dynamic is impossible and the system will never come to these points. Otherwise, it might leave this point, but that contradicts the definition of reversibility.

It is possible to assume that for system of disks at \( N \to \infty \), probability to appear in cyclic point is very small. Then, the probability of reversible dynamic is very small also, though it is vary from zero. Cyclic points are determined by symmetry of boundary conditions and groups of systems’ symmetry, which can be found with help of matrix’s collision. I.e. probability of reversibility of the system depends on geometrical characteristics of its elements and boundary conditions. Research of these points, for example, their measures and stability, has basic interest for process of evolution [14]. So, we can say that reversibility is possible only at the event when at the moment of preparation of nonequilibrium system appears in one of cyclic points. Such interpretation of reversibility coincides with point of view of Einstein, according to which, probabilistic description of the system is dictated by probability to take one of the points in phase space in moment of the system preparation. But the dynamics of the system should be determined.

I.e. the statistical description of systems is connected not with probabilistic the nature of processes, but with opportunity of use of such description for the analysis of dynamics of many-body systems.

Irreversibility of rarefied systems of potentially interacting disks. Let’s consider, how , the irreversibility for rarefied systems of potentially interacting disks is follow from the property of irreversibility for hard disks. Let us take into account, that pair collision approach is convenient for rarefied system. Dynamics of potentially interacting disks is described by the Newton’s equation. In this equation the
force, \( F_{ks} \), between \( k \) and \( s \) disks is expressed by meaning of scalar function, \( U \): \( F_{ks} = -\frac{\partial U}{\partial r_{ks}} \), where \( U \) is potential energy, \( r_{ks} \) is distance between disks. It is possible to take a characteristic radius of interaction disks, \( R_{int} \) for rarefied system, which is much less than length of free path, \( l_c \), i.e. \( l_c \gg R_{int} \). Disks can be considered to be free when their distance up to the nearest neighbor disks is more than \( R_{int} \). Let’s consider scattering of two disks.

In coordinate system of the centre of weights, character of scattering is determined by the formula of [4]:

\[
\varphi_0 = \int_{r_{min}}^{R_{int}} \frac{\rho dr}{r^2 \sqrt{1 - \frac{\rho^2}{m^2 (\Delta_0_{ks})^2}}}.
\]

Here, \( \varphi_0 \) is a scattering angle; \( r \) is a distance between scattering disks; \( \rho \) is an impact parameter; \( r_{min} \) is a square root of the formula under radical; \( m \) is mass of disks; \( \Delta_0_{ks} \) a velocity of swoop disk, which is equal to relative velocity of interaction disks in laboratory co-ordinate system. In coordinate system of the centre of weights \( m = 1/2 \) (the weight of a disk has been accepted to equal unit).

It is easy to show that velocity of disks, after they abandon the area of interaction, is possible to be determined under formulas for hard-disks, if to make replacement in them at formula of velocity (see [4]):

\[
\theta_{ks} = |\pi - 2\varphi_0|/2, \cos \theta_{ks} = d_{ks}.
\]

Here, \( \theta_{ks}, d_{ks} \), are an angles of scattering of the disk and an impact parameter accordingly. Thus, trajectories of disks under condition of \( r \geq R_{int} \) are possible to be determined with the help of formulas (2) without integration of the Newton equation of inner area of interaction.

For rarefied system of disks the conditions, \( l_c/V \gg t_{int} \), take place. Here, \( t_{int} = R_{int}/V \) is characteristic time during which the disks are in the interaction region, \( V \) is characteristic velocity of disks. In this case we can make transition, \( R_{int} \to 0 \). Such transition corresponds to approach of hard collisions. I.e. dynamics of the system is described by the equation (2) rigorously. It follows from here that dynamics of rarefied system can be studied both with the help of the equation of Newton and the equation (2). At \( R_{int} \to 0 \) or \( t_{int} \to 0 \) the calculations results in both cases completely coincide. Hence, dynamics of rarefied system of potentially interacting particles may be described with help of motion equation (2). So, the rarefied system of potentially interacting particles, as well as system of hard colliding particles also can possess irreversible dynamics.

**Interrelation of the generalized field of force with entropy.** If we know the generalized field of forces, it is possible to obtain deviation of entropy for given nonequilibrium condition of system from volume entropy of the equilibrium state. Really, let the nonequilibrium system come to equilibrium. Then work of the generalized field of forces would go on increase of internal energy of the system Landau, L.D. [1976]. In this case entropy deviation, \( \Delta S \), can be determined with the help of following expression Somsikov, V.M. [2003]:

\[
\Delta S = \sum_{p=1}^{R} \left\{ \frac{T}{E_p} \sum_{k=1}^{T} F_k^p dr_k \right\}
\]

(16)

where \( E_p \) is kinetic energy of subsystem (all subsystems have \( T \) number of the disks). I.e. with help of
equations (17) the dynamic parameter of system - generalized field of forces is connected with thermodynamic parameter - entropy. And as the system aspires to achieve a condition with the minimal internal work appropriate to the minimal value of a generalized field of forces, the equation (17) corresponds to the principle of the minimal entropy production. It is not difficult to obtain (for example when \( R = 2 \)) that the entropy increment for all system may be only positive (though in some subsystems it can be negative). This conclusion corresponds to existing results connected with entropy change in the system at occurrence in them of some structures, for example, turbulent structure. So, Klimontovich, Yu.L. [1] with the help of the S-theorems, offered by him, proved decreasing of the entropy when formations of structures have a place. The basic difference of the formula (17) from the formula for entropy offered by Yu.L. Klimontovich, is that the deviation entropy, \( \Delta S \), is determined by relative to the nonequilibrium condition, which accepted as "back-ground"; but in the formula (17) \( \Delta S \) is determined by deviation of entropy an nonequilibrium state from an equilibrium state.

**Conclusion.** So, we shown an opportunity of explanation of irreversible dynamics on the basis of formalism of classical mechanics, if this formalism expands by inclusion to it of the generalized Hamilton and Liouville equations. Formally, irreversibility follows from the right side of the generalized Liouville equation. The nature of this equation is connected with appearing in the system of the generalized field of forces, when it deviates from equilibrium, and dependence its forces from particles velocities. This dependence is caused by that that moving particles create this field of forces. So one can submit the following explanation of irreversibility. Let the nonequilibrium system, which velocity of center of weights, \( V_0 \), is equal to zero. Because of nonequilibrium, velocity of the center of weights some subsystems will differ from zero. I.e. these subsystems will move relatively each other. Then the corresponding part of kinetic energy of the subsystems which connected with this relative subsystem moving will be redistributed between them proportionally to their relative velocity. This energy will go on increasing of system entropy. As a result the relative velocity of subsystems will go to zero and the system will come to an equilibrium state with maximal entropy. This is a physical essence of irreversibility.

As in equilibrium state the generalized field of force is equal to zero, so in this case, the generalized Hamilton and Liouville equations transform into the canonical equations. And therefore in equilibrium state Poincaré’s recurrence theorem (see. [13]) is applicable. Dynamic of such systems is completely reversible. But reversibility is possible not far from equilibrium state. So, it is because the factors, which determining irreversibility, has the second order of value in relation to linear disturbances.

Thus, the analysis of evolution of open nonequilibrium systems becomes possible within the framework of the laws of classical mechanics only on the basis of the generalized Hamilton and Liouville equations.
References

[1] Klimontovich, Yu.L.: *Statistical theory of the open system* Moscow, 1995.

[2] Lanczos, C.: *The variation principles of mechanics*, Second edition University of Toronto Press, 1962.

[3] Landau, L.D.: *Statistical physics* Part 1. Nauka, Moscow, 1976.

[4] Landau, L.D. & Lifshits, Ye.M.: *Mechanics*, Nauka, Moscow, 1973.

[5] Lebowitz, J.L.: Boltzmann’s entropy and time’s arrow *Physics Today* (1993), September, 32-38.

[6] Lifshits, Ye.M. & Pitaevsky A.P.: *Phys. kinetics*, Nauka, Moscow, 1979.

[7] Loskutov, F.U. & Mihailov, A.S.: *Introduction to synergetic*, Moscow, 1999.

[8] Petrosky, T. & Prigogine, I.: The Extension of classical Dynamics for unstable Hamiltonian systems, *Computers Math. Applic.*, V. 34. No. 2-4. (1997), 1-44.

[9] Sinai, Ya.G.: Dynamical system with elastic reflection ergodynamic properties of scattering billiards, *Uspekhi Mat. Nauk*, (1970), V. 25, 141-192.

[10] Somsikov, V.M.: Non-recurrence problem in evolution of a hard-disk system, *Intern. Jour. Bifurc. And Chaos*, 11, No 11, (2001), 2863-2866.

[11] Somsikov, V.M.: Some approach to the Analysis of the Open Nonequilibrium systems, *AIP* 20, (2002), 149-156.

[12] Somsikov, V.M.: The mechanism of irreversibility in a hard-disks system, *Problems of the evolution of the open systems* (Almaty), V.1, (2003) 49-60.

[13] Zaslavsky, G.M.: *The stochastic of dynamics system* (Nauka, Moscow), 1984.

[14] Zaslavsky, G.M. Chaotic dynamic and the origin of Statistical laws, *Physics Today* August, Part 1, (1999), 39-45.