Inelastic nuclear interactions at protons multiple passage in bent crystals

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Abstract. Inelastic nuclear interactions for relativistic channeled and quasi-channeled protons in a bent crystal are theoretically studied. Multiple passage of projectiles through experimental setup was in details analyzed. Paying attention to the features observed, simulation results have been compared with known experimental data.

1. Introduction

Planar channeled and quasi-channeled projectiles travel in a crystal under small angle to the crystallographic planes interacting with the averaged field of plane’s atoms instead of separate atomic fields (see, for example, [1] and references therein). The channeling phenomenon takes place when incident angle $\theta_0$ between projectile velocity and crystallographic planes does not exceed a certain critical value $\theta_L$ known as the Lindhard angle [2]. Channeled projectiles, penetrating into the crystal at angles $\theta_0 < \theta_L$, travel between two parallel planes, which form so-called planar channel. Till the projectile transverse energy remains less than the potential barrier of a planar channel, a particle is channeled, but once the transverse energy exceeds the barrier, we deal with quasi-channeling mode of the motion (obviously, for transverse energies close to the barrier) \textsuperscript{1}. On the contrary to the channeled particles, quasi-channeled ones are not constrained within any single channel.

For a bent crystal the averaged crystal field depends on a distance to the center of a bent plane curvature (see [3, 4] for details). The beam directed along the bent planes under the channeling or quasi-channeling conditions is split onto two beams in a crystal. The first beam corresponds to channeled particles, which deflect down to bent channel at large angles from the initial direction of motion, while the second one - to quasi-channeled particles. For quasi-channeled particles, moving initially to the center of crystal curvature, the volume reflection phenomenon might be observed. Namely, on the particle trajectory we can define a "turn" point where the travel direction changes with respect to the center of curvature. Finally, the reflected particles are deflected by crystal field in the direction opposite to one of channeled particles. When quasi-channeled particles initially travel from the center of curvature, they are slightly deflected mainly due to multiple scattering on the crystal nuclei. Multiple scattering leads to the transition of projectiles from a channeled state to quasi-channeled one and vice versa.

\textsuperscript{1} At transverse energies over the barrier the particles are known as dechanneled
The first one when the projectile transits from the channeling to quasi-channeling regimes of motion is named "dechanneling". The second when the quasi-channeled projectile is captured in a channeling state - "volume capture".

The main goal of this work is to investigate the probability of inelastic nuclear interactions (INI) between channeled (as well as quasi-channeled) protons and nuclei in a bent crystal. Namely, we have analyzed the case for 120 GeV protons moving through Si crystal bent along (220) crystallographic planes at the angle $\alpha_R = 150 \, \mu\text{rad}$ with the radius $R = 1333 \, \text{cm}$ (the conditions of experiment [5]). The crystal thickness is $z_{cr} = 2 \, \text{mm}$, and the critical channeling angle for this case - $\theta_L = 19 \, \mu\text{rad}$.

Below the averaged probability of INI will be introduced, and, successfully, its dependence on crystal orientation will be investigated. Moreover, we suggest a model of experimental scheme that allows the features of multiple passage of projectiles through the crystal to be simulated. Finally, the simulation results together with the analysis of experimental data [5] will be presented.

2. Averaged probability of INI

Let designate $\sigma$ the INI cross-section. The INI probability $P$ when proton passes the way $z$ in the amorphous solid is $P = \sigma N z$ (which is valid while $P << 1$) where $N$ is the nuclear density$^2$. In aligned crystal, for both channeling and quasi-channeling conditions, the nuclear density $N$ strongly depends on the distance from crystallographic planes due to the thermal vibrations of nuclei near the equilibrium position located on a plane. In this manuscript we suppose the Gaussian distribution for the displacement of nuclei from the plane [1]. Then for aligned crystal the INI probability can be rewritten as follows:

$$P = \sigma \langle N \rangle_{tr} z,$$

where $\langle N \rangle_{tr}$ is the averaged nuclear density over the projectile trace.

Now let consider nondivergent proton beam hitting the bent crystal at the incident angle $\theta_0$ (see, in [4, 6] and references therein for details). To define the mean (averaged) nuclear density $\langle N \rangle_0$ at given crystal orientation (which is defined by the angle $\theta_0$) we have to average $\langle N \rangle_{tr}$ by all proton trajectories in a crystal [7].

Assuming that the number of protons penetrating into the crystal is large, $n_p >> 1$, we should not analyze the trajectory for each proton, but we can suggest that the averaged INI probability in a crystal volume is defined by

$$P = \sigma \langle N \rangle_0 z \quad (1)$$

for one arbitrary proton and, successfully, the full number of INI - by

$$Z = P n_p. \quad (2)$$

Hence, if the counter is mounted anywhere near the crystal for the INI registration, it records the quantity proportional to the value $Z$.

The results of simulations for the probability $P$ revealed from Eq.(1) are presented in figure 1. The trajectories were evaluated using a model presented in [4, 6]. The multiple scattering of protons on both crystal electron and nuclei was introduced based on [8, 9]. Let define $\theta_0 > 0$ for the projectiles moving from the center of curvature initially, and $\theta_0 < 0$ - in opposite case. The value $\theta_0 = 0$ corresponds to the projectile entering a crystal strongly along the tangent line to bent planes at the crystal entrance end. The INI cross-section $\sigma = 0.506 \cdot 10^{-24} \, \text{cm}^2$ was

$^2$ The nuclear density in this manuscript coincides with the concentration
taken from [10]. The designations in the figure 1 are: UAO - unaligned crystal orientation, when the deflection of protons is determined mainly by multiple scattering; VRO - volume reflection orientation, when the deflection of protons is determined mainly by the volume reflection; CO - channeling orientation, when the deflection of protons is determined mainly by the channeling.

When the parallel proton beam is directed initially along the crystallographic planes (CO) the probability of INI is reduced five times in comparison to one at unaligned crystal orientation (UAO). This result is in agreement with the experimental data [5]. At the conditions of VRO, the INI probability is about 10% higher than at UAO. The authors of [10] came to the similar conclusion. On other hand, the experimental curve in [5] for the condition of VRO appears below the curve at UAO. As further shown this effect is related to the particles multiple passage through the experimental setup. It is notable that in a whole considered interval of the angles $\theta_0$ the INI probability is less than 1%.

The shape of the curve $P(\theta_0)$ is related to the nuclear density in the area where proton moves. Channeled protons move in a central area of the channel where nuclear density is lowest. The protons traveling at the UAO conditions feel the same averaged nuclear density as in amorphous medium. On the contrary, near the turn point the reflected protons move in the area of extremly high nuclear density during long time (see, for details, in [7]). The latter proves the fact that the INI probability at VRO exceeds the ones at UAO.

3. Multiple passage of projectiles through a bent crystal

3.1. The idealized experimental setup

In experiments [5, 10] the holder with bent crystals was mounted into the pipe of storage ring resulting in the multiple passage of projectile through the setup. Obviously, the INI value should vary in comparison with a single passage case above examined. On one side, multiple passage of the projectile through the crystal should be characterized by similar features that we can observe with the crystal thickness increase, i.e. one can expect the growth of the INI number. On other side, projectiles might be large deflected from its initial direction of motion due to both channeling and volume reflection effects, and, finally, projectiles can leave the portion of a beam hitting a crystal. As a result, the deflection of particles must restrict the growth of the INI number in the multiple passage regime.

To model the situations described above, let consider a simple idealized experimental scheme (see, in figure 2). A parallel beam 1 maintains initially the number $I \gg 1$ of protons. The absorber systems 5 are placed to cut the tails of a beam. The focusing system 6 is mounted behind the absorber. Keeping the beam profile, it delivers the beam to the setup entrance. Let
Figure 2. Experimental setup scheme: 1 - the initial parallel proton beam, 2 - the bent crystal, 3 - the channeled protons, 4 - the reflected protons, 5 - the absorber, 6 - the focusing system.

mount the bent crystal 2 into the beam before the absorber 5; the number $\alpha I$ of protons hits the crystal. Hence, particles can be deflected from the initial direction due to channeling 3 or volume reflection 4 at large angles exceeding the cutting angle $\theta_b$. These particles cut by the absorber 5 will leave the beam. Other particles, namely, which either did not hit the crystal or being deflected at angles less than the cutting angle $\theta_b$, pass to the focusing system 6. The focusing system makes the beam parallel again and restores initial beam profile. After that a steered beam is turned to the crystal collimation setup.

Let introduce coefficient $\beta$ defining the part of protons hitting the crystal and being deflected by bent crystal on the angles $|\theta| < \theta_b$. Namely, a coefficient $\beta$ is the part of protons crossing the crystal and successfully passing to the focusing system. The coefficient $\beta$ can be evaluated from the angular distribution. To obtain angular distributions we used the model [4, 6] where the multiple scattering of projectiles is introduced [8, 9]. While the focusing system restores the beam profile, both coefficients $\alpha$ and $\beta$ remain unchanged from the turn to the turn; they depend on orientation as well as on position of a crystal.

3.2. The INI estimation
First of all, let define the dependence for particles number in a beam on the number of turns under the INI neglecting condition. The initial number of particles in a beam is $I_{0,1} = I_0$ at the experimental setup entrance. The number of particles hitting the crystal is $\alpha I_{0,1}$. The part of these particles that passes to the focusing system is defined by $\alpha \beta I_{0,1}$. Therefore, the full number of particles at the focusing system entrance is $I_{1,1} = (1 - \alpha)I_{0,1} + \alpha \beta I_{0,1}$. These particles are delivered to the setup entrance again. Successfully the described procedure repeats. Obviously, for the $T$-th turn the number of particles entering the setup will be $I_{0,T} = (1 - \alpha + \alpha \beta)^T - 1 I_0$.

Let now consider the problem of INI in the crystal volume. As shown in Section 2, the INI probability is quite small for one proton during single passage through the crystal. Therefore,
the proton leaving the beam due to INI does not affect the full number of particles of a beam, and Eq. (2) is valid for the INI evaluation over \( T \) turns. We can define \( Z_i = P_\alpha I_{0,i} \) as the INI number for the \( i \)-th turn, and the full INI number over \( T \) turns as follows:

\[
Z = \sum_{i=1}^{T} Z_i = P_\alpha I_0 \sum_{i=1}^{T} (1 - \alpha + \alpha \beta)^{i-1}.
\]

To make the further simplifications, one can assume that the crystal does not accept the beam completely, therefore, \( \alpha < 1 \). Additionally, the coefficient \( \beta < 1 \), if the absorber cuts protons, whereas the value \( \beta = 1 \) corresponds to the case when all protons pass to the focusing system. Therefore, at \( \beta < 1 \), we have

\[
Z = \frac{1 - (1 - \alpha + \alpha \beta)^T}{1 - \beta} P I_0. \tag{3}
\]

For the case \( \beta = 1 \) one can find that

\[
Z = T_\alpha P I_0. \tag{4}
\]

Indeed, when all particles pass to the focusing system the full number of particles does not change while condition \( Z_i \ll I_{0,i} \) is fulfilled. Hence, Eq. (4) concludes that in each turn the same number of INI, \( P_\alpha I_0 \), takes place.

On the contrary, as follows from Eq. (3), when number of turns increases the full number of nuclear reactions \( Z \) reaches the saturation characterized by the following limit

\[
Z_{\text{sat}} = \frac{P}{1 - \beta} I_0, \tag{5}
\]

which is \( \alpha \) independent. As the matter of fact, \( \alpha \) defines the time to reach the saturation.

4. INI simulation results

To eliminate the dependence on crystal position (i.e., on coefficient \( \alpha \)) we will consider further the INI number in the saturation regime. From considerations above one can conclude, the INI number depends on both incident \( \theta_0 \) and cutting \( \theta_b \) angles. It should be underlined that the coefficient \( \beta \) is strongly correlated with the angular distribution shape of protons traveled through the crystal. The detailed explanation of that fact can be found in [7]. The dependencies \( \beta(\theta_0) \) at different cutting angles \( \theta_b \) are presented in figure 3 where to higher curve by the position in a figure corresponds larger angle. All curves in figure 3 are revealed the deep in the CO area. Indeed, channeled projectiles are deflected by bent crystal at large angles \( \theta \approx -\alpha R < -\theta_b \). Hence, all channeled particles hit the absorber and leave the beam.

At small angle \( \theta_b = 1 \) \( \mu \)rad the small part of projectiles scattered by crystal passes to the focusing system and it reveals a slight variation at angular distribution (i.e. \( \theta_0 \)) change. Therefore, the corresponding curve in figure 3 is the lowest and the coefficient \( \beta \) changes slightly. On the contrary, at the largest angle \( \theta_b = 80 \) \( \mu \)rad almost all projectiles pass to the focusing system at any crystal orientation except the CO one. Therefore, one can expect \( \beta \approx 1 \) except the sharp decrease at CO.

For intermediate angles \( \theta_b = 10 \) \( \mu \)rad and \( 50 \) \( \mu \)rad the coefficient \( \beta \) in figure 3 demonstrates quite different behavior. Namely, for the case \( \theta_b = 10 \) \( \mu \)rad one can see the deep in the area \( \theta_0 \approx -\alpha R \) whereas for the case \( \theta_b = 50 \) \( \mu \)rad this deep is absent. The matter is connected with the shape of angular distribution in the transition area from VRO to UAO. Actually, when crystal is mounted at VRO, the majority of protons form the single sharp peak (see, for example, experimental data [11]). This peak is scanned at both \( \theta_b = 10 \) \( \mu \)rad and \( \theta_b = 50 \) \( \mu \)rad.
contrary, when crystal is mounted at angle $\theta_0 \approx -\alpha_R$ outgoing beam is split onto two beams. One part of projectiles moves under volume reflection condition whereas other part is deflected mainly due to multiple scattering. The observer watching the projectiles behind the absorber (at the entrance of a focusing system) scans both peaks at $\theta_b = 50 \mu\text{rad}$ simultaneously, i.e. he observes the same intensity of projectiles as in the VRO area. But under condition $\theta_b = 10 \mu\text{rad}$ the observer registers one peak only or even the deep between peaks. As the matter of fact, the number of particles passing to the focusing system decreases with respect to VRO. When the crystal orientation is changed to UAO (angle $\theta_0 < -\alpha_R$), the single peak forms once again (see, in [7] for details).

Both dependencies $\beta(\theta_0)$ (figure 3) and $P(\theta_0)$ (figure 1) define the INI number in the crystal volume (5). The number $Z_{\text{inf}}$ is presented in figure 4 for the same conditions as for coefficient $\beta$ in figure 3. The cutting angle $\theta_b$ increases from figure 4(a) to figure 4(d). For all parts of figure 4 the deep at CO appears. It is caused by deflection of channeled protons at large angles (see, in figure 3) as well as by the reduce of the INI probability (see, in figure 1). The INI number at VRO becomes larger with the increase of angle $\theta_b$. It proves the expectation: the more this angle the more protons pass many times through the crystal.

In figure 4(a) the cutting angle is very small ($\theta_b = 1 \mu\text{rad}$). Therefore, every proton passes through the crystal one time only and after that is deflected by the crystal to the absorber. In this case the ratio of INI number $Z_{\text{inf}}$ to the full number of projectiles $I_0$ is, actually, the INI probability $P$ for one proton (figure 1). The same conclusion follows directly from the formula (5) at small $\beta$.

In figure 4(b) ($\theta_b = 10 \mu\text{rad}$) one can see the deep in the area $\theta_0 \approx -\alpha_R$ whereas the similar deep is absent in figure 4(c) ($\theta_b = 50 \mu\text{rad}$) that agrees with the $\beta(\theta_0)$ analysis above presented.

At the cutting angle $\theta_b = 80 \mu\text{rad}$ (figure 4(d)) the number of protons, which suffer INI is about 40% from its initial number $I_0$ at VRO and this number approaches 100% at UAO. It should be underlined that here the saturation state has been considered. Obviously, to distinguish these quantities the huge number of turns is needed.

Let now explain the sharp oscillations of the curves $Z_{\text{inf}}/I_0$ in Figs. 4(c,d). One can notice from figure 3, the coefficient $\beta$ in the corresponding areas is $0.98 \div 0.99$. Further, from Eq.(5), the small variation in the coefficient $\beta$ leads to significant changes of the INI number $Z_{\text{inf}}$. Due to the fact that the coefficient $\beta$ is determined by the shape of corresponding angular distribution, small change of the distribution shape results in large variations of the INI number at large cutting angles. On one hand, actually, the distribution shape changes at different crystal orientation. On other hand, the Monte-Carlo method used in our simulations to obtain the angular distribution
The INI number for the saturation state $Z_{\text{inf}}$ with respect to the initial number of protons $I_0$ in dependence on the incident angle $\theta_0$ at different cutting angles $\theta_b$: (a) $\theta_b = 1 \, \mu\text{rad}$, (b) $\theta_b = 10 \, \mu\text{rad}$, (c) $\theta_b = 50 \, \mu\text{rad}$, (d) $\theta_b = 80 \, \mu\text{rad}$. The case of 120 GeV protons passing through Si crystal bend along (220) planes over the angle $\alpha_R = 150$ $\mu\text{rad}$ with the radius $R = 1333$ cm is considered.

can also contribute to small variation in the distribution shape. Both contributions form the final oscillations at UAO shown in figure 4(c) as well as at VRO shown in figure 4(d).

For the range $\theta_0 = (-75) \div (-25) \, \mu\text{rad}$ there is an interval in the VRO area in figure 4(d) where the sharp oscillations are absent. These crystal orientations are characterized by significant deflection (over angles $\theta > \theta_b$) of protons due to the volume capture effect [7, 11]. As a result, a few percents of protons leave the beam and small changes of the distribution shape does not significantly influence the INI number.

Finally, let examine next feature of the results obtained. When projectiles pass through the crystal one time, the INI probability is more at VRO than at UAO (see, figs.1 and 4(a)). When the turn’s number increases, on the contrary, this probability is more at UAO than at VRO (figure 4(b-d)). Indeed, at UAO protons are deflected on the fewer angles with respect to the case of VRO (see [7, 11]). Therefore, a proton has a chance to make more turns before it will be deflected over the angle $|\theta| > \theta_b$ at UAO than at VRO. Due to the fact that the probability of interaction growths with the increase of number of turns, at multiple passage we will have the reduced INI probability at VRO with respect to UAO. This result is in agreement with [5, 10].

5. Conclusion
In this work we have evaluated the probability of inelastic nuclear interactions when relativistic protons travel through the bent crystal. The main attention has been paid to the multiple passage of protons through the experimental setup. The idealized experiment was simulated in
order to investigate the INI at the given conditions. The scheme of experiment is "designed" to involve both a bent crystal for projectiles deflection and an absorber for deflected particles cut-off. These elements might be used to collimate a charged particle beam, and, hence, the suggested setup, idealized for simplicity, in general, works as a real collimator facility. Moreover, this theoretical experiment is free of the technical details, which make the nature of experimental data not so clear. Hence, our idealized setup can be used to better understand the real experiment features.

Presented analysis enables explaining the difference between results of simulations and experimental data published in [5]. The published data are not adapted for direct analysis. Nevertheless, the shape of our simulated curves in figure 4 is in qualitative agreement with experimental data [5]. Additionally, we would like to underline that the developed model allowed us to explain observed deeps in the INI counts when the crystal is mounted in the transition area from volume reflection to unaligned crystal orientation. Actually, it is not related to inelastic interactions themselves. It is purely the effect of the beam collimation. The mentioned deep can be either enhanced less or more or can be even absent in dependence on the cutting angle at the setup exit. The sharp oscillations at both VRO and UAO discussed above might be smoothed in experiment and unobservable due to the reasons described in [5], namely, the crystal torsion and other imperfections. Also, it should be mentioned here, in our considerations we deal with the saturation state, whereas this state could occur unreachable in experiments [5, 10] where the condition $\alpha << 1$ was realized.

The model suggested in this work based on simple mathematical considerations. It is not very exacting to the computational resources, but it gives the results which are in agreement in the well-known experimental data [5, 10]. Therefore, this model can be useful to plan the future experiments and to simulate the results.

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