Ratio of Isoscalar to Isovector Core Polarization for Magnetic Moments

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In calculations of isoscalar magnetic moments of odd-odd N=Z nuclei it was found that for medium to heavy mass nuclei large-scale shell model calculations yielded results which were very close to those obtained with the much simpler single-j shell model. To understand this we compare isoscalar and isovector core-polarization configuration-mixing contributions to the magnetic moments of mirror pairs in first order perturbation theory, using a spin dependent delta interaction. We fit the strength of the delta interaction by looking at isovector and isoscalar mirror pairs. We then use the same interaction to calculate corrections due to first order core polarization of the magnetic moments of odd-odd nuclei.

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Previously S. Yeager et al.[1] compared the results of both large-scale and single-j shell model calculations of isoscalar magnetic g factors of odd-odd N=Z nuclei with the experimental data. There it was noted that for large A the g factors, both experimental and calculated, approached the value of 0.5. Detailed comparisons with experiment were made in [1]. In the present work the emphasis will be on a theory vs. theory discussion of these results, based on the observation there that as A increases the results of the large-scale shell-model calculations for these odd-odd N=Z nuclei, were becoming very close to the single-j shell-model results. This is shown in Table 1. We see that for light nuclei, e.g. 6Li, there are significant differences between the results of the two models. However, the differences become much smaller as A increases.

Overall, the average absolute deviation between the two columns for nuclei heavier than 6Li is about 0.02 (i.e. about 4%).

A related problem is that of the magnetic moments of mirror pairs. We can form isoscalar and isovector combinations defined thus:

\[ \mu(IS) = (\mu(\text{oddproton}) + \mu(\text{oddneutron}))/2 \]  

\[ \mu(IV) = (\mu(\text{oddproton}) - \mu(\text{oddneutron}))/2 \]

In this work we will focus on the mirror pairs 57N, 57Cu and the odd-odd nucleus 58Cu. Our plan of attack is to use a spin dependent surface delta interaction \(-G(1+xP_{\sigma})\delta(r_1-r_2)\) to calculate deviations from the Schmidt limits for the above 3 nuclei. Only one parameter G enters into the calculation for \(\mu(IV)\) and hence we will use this moment to fit G. We will then examine the values of the spin exchange parameter needed to explain the isoscalar magnetic moments i.e. \(\mu(IS)\) and \(\mu(58Cu)\). Indeed, the main thrust of this work will be to show the importance of the spin exchange in explaining why the isoscalar moments are so close to the single-j(Schmidt) results.

For the mirror pairs we have the following information for the J=3/2− ground states, where we give the values in units of nuclear magnetons:

\(\mu^{57}\text{Ni})=-0.7975\) from T. Ohtsubo et al.[6].
\(\mu^{57}\text{Cu})=2.582(7)\) from T.E. Cocolios et al. [7].

The latter is quite different from a previous value of 2.0 in Ref [8].

From the above we find, using (1)

\(\mu(IS) =0.892 \mu(IV)=1.690\). The Schmidt values are 0.940 and 2.853 respectively. The deviations of the magnetic moments (expt.-theor.) are respectively -0.0480 and -1.163.

The deviations of the g factors are (expt.-theor.) are respectively -0.0320 and -0.775. An good estimate of the deviation from the Schmidt value for 58Cu (with J=1) is the above isoscalar value -0.03184. Unfortunately the error bars for the measurement on this nucleus are too large.

The ratio of deviations (IS/IV) is 0.04256 i.e. the isoscalar deviation is much smaller than the isovector one.

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Table I: Comparison of large-scale shell model and single-j shell model results for isoscalar g factors of N=Z odd-odd nuclei.

| Nucleus | J | Large scale | Single-j | single-j values |
|---------|---|-------------|----------|----------------|
| ²H      | ¹⁺ | 0.88        | 0.88     | s₁/₂           |
| ⁶Li     | ¹⁺ | 0.87        | 0.63     | p₃/₂           |
| ¹⁰B     | ³⁺ | 0.61        | 0.63     | p₃/₂           |
| ¹⁴N     | ¹⁺ | 0.32        | 0.37     | p₁/₂           |
| ¹⁸F     | ³⁺ | 0.62        | 0.58     | d₅/₂           |
| ⁵⁺      | ⁵⁺ | 0.58        | 0.58     | d₅/₂           |
| ²²Na    | ³⁺ | 0.59        | 0.58     | d₅/₂           |
| ²⁶Al    | ¹⁺ | 0.52        | 0.58     | d₅/₂           |
| ²⁶K     | ⁴⁺ | 0.57        | 0.58     | d₅/₂           |
| ³⁸V     | ³⁺ | 0.41        | 0.42     | d₅/₂           |
| ⁶⁰Cu    | ¹⁺ | 0.63        | 0.63     | p₃/₂           |
To understand why this is so we introduce configuration mixing via perturbation (PT) theory following Arima and his group \[2,3\]. We thus calculate the core-polarization configuration-mixing corrections to these magnetic moments that are calculated in the single-j shell model. The PT expression for these corrections simplifies if we use particle-hole, rather than particle-particle, interaction matrix elements (see e.g., Eq. 5 in Mavromatis et al. \[4\]). This is because for each isospin there is only one particle-hole matrix element with total angular momentum one, while there are several such particle-particle matrix elements. The expression is as follows.

\[
\Delta \mu(T) = 2(j/((j+1)2j+1)))^{1/2} V(ph, T) \ast j' || \mu(T)|| j > \frac{1}{(1+x)^{P\sigma}}\]

(1+ x \ P\sigma) where \(P\sigma\) is the spin exchange operator \((1+ \sigma_1 \sigma_2)/2\)

The expression for the particle-hole matrix elements with this delta interaction is \[6\]
\[
V(ph,0)= G/2 R(1-2x) M
\]
\[
V(ph,1)= G/2 RM
\]
where \(M= (2j_v+1) ((2j_v+1)(2j_v+1))^{1/2} || (j_v j_v 1/2 -1/2 0) (j_v' j_v' 1/2 -1/2 0) (1/2 -1/2 0) (1/2 1/2 -1) (1/2 1/2 -1) R = 4/π \int R_1(r) R_2(r) R_3(r) R_4(r) r^2 dr
\]
We then find \(V(ph,0)/V(ph,1) = (1-2x)\).

A popular choice for \(x\) is 1/3. With this choice (1-2x) is equal to 1/3, i.e., the first bracket in Eq. (2) makes the ratio IS/IV = 1/3 for the core polarization corrections. But there is also the second bracket in Eqn. (2), which we now evaluate.

Since \(L = J - 9\),
\[
\bar{\mu}_L = g_L L + g_S S = g_L L + (g_S - g_L) S
\]
\[
\bar{\mu}\text{ can thus connect to the same L value, i.e., the spin-orbit partners J=L+1/2 and J=L-1/2.}
\]

Then
\[
< j' || \bar{\mu} || j > = < j' || g_L L || j > + (g_S - g_L) < j' || S || j > \]

The first term vanishes since \(j \neq j'\) so
\[
< j' || \bar{\mu} || j > \sim (g_S - g_L).
\]

All in all we find \(\Delta \mu(IS)/\Delta \mu(IV) = (1-2x)^* (g_S - g_L)/(g_S - g_L) IS/(g_S - g_L) IV\)

For the second bracket in Eq. (2), we need to evaluate the ratio of \((g_S - g_L) IS / (g_S - g_L) IV\).

The relevant values are \[1\]

Isoscalar: \(g_L = 0.5, \ g_S = 0.88, \ (g_S - g_L) = 0.38\)

Isovector: \(g_L = 0.5, \ g_S = 4.71, \ (g_S - g_L) = 4.21\).

This leads to a ratio of \((g_S - g_L) IS / (g_S - g_L) IV = 0.38/4.21 = 0.09\). From eq.(2) we find

\[
\Delta \mu(1) = \frac{8800 MeV}{1000 MeV}. \text{To get the g factor we divide this by 1.5(that would be } \Delta \mu\text{ for the J=1+ state of }^{58}\text{Cu).}
\]

If we take \(\Delta E = \epsilon_{f_7/2} - \epsilon_{f_5/2} = -5.0 \text{ MeV and if we adjust } G\bar{R} \text{ so that } \mu(IS) - \mu(IS)_{Schmidt} = -1.123 \text{ we obtain } G\bar{R}=0.648.
\]

Then we ask what value in the spin exchange operator will yield the isoscalar difference \(\mu(IS) - \mu(IS)_{Schmidt}=-0.0478\). We thus find that this value of x is 0.264.

This is to be compared with x=1/3 for which the coupling of T=0 in the particle-particle channel is twice that for T=1 (i.e. \((1+x)/(1-x)=2\)).

Alternately we can use renormalized values of \(g_S\) and \(g_L\) due to second order perturbation theory and corrections and meson exchange currents. A simple choice is to only change the isovector couplings--make \(g_S(IV)=0.7\) and \(g_L(IV)\) free and change \(g_L(IV)\) from 0.5 to 0.6. These changes will make the isovector contribution smaller. We now will have

\[
\bar{g}(S(IV)-g_L(IV))=2.697 \text{ (as compared with the free value of 4.21).}
\]

The new value of x is a somewhat larger 0.349.
The main point of this paper is to show how the spin exchange term in the nucleon-nucleon interaction affects isoscalar and isovector moments. The $(1-2x)$ factor in the $T=0$ particle-hole channel suppresses the isoscalar magnetic moment deviations from the Schmidt value by about a factor of two or three relative to the isovector ones.

We briefly add that in the single $j$ shell of both neutrons and protons the isoscalar $g$ factor is the same as the Schmidt. - not so in isovector case. This is relevant to the case of $^{45}$V $J=3^+$ and the $J=7/2^-$ mirror pairs $^{45}$Ti and $^{45}$V. The isoscalar $g$ factor is

$$g(IS) = \frac{(g_{j\pi} + g_{j\nu})}{2}$$

It does not depend on the details of the function.

The isovector term is

$$\frac{[g(45V) - g(45Ti)]}{2}$$

is given by

$$\frac{(g_{j\pi} - g_{j\nu})}{2} \sum |D(j_p,j_n)|^2 \left( J_p(J_p+1) - J_n(J_n+1) \right)$$

where $|D|^2$ is the probability that in a state of total angular momentum $J$ the protons couple to $J_p$ and the neutrons to $J_n$. Only if the even number of nucleons would couple to zero would we get the Schmidt value in the isovector case. Unfortunately the magnetic moment of $^{45}$V has not yet been measured.

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