Generalization of the relativity theory on the arbitrary space-time geometry

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Abstract

Contemporary relativity theory is restricted in two points: (1) a use of the Riemannian space-time geometry and (2) a use of inadequate (nonrelativistic) concepts. Reasons of these restrictions are analysed in [1]. Eliminating these restrictions the relativity theory is generalized on the case of non-Riemannian (nonaxiomatizable) space-time geometry. Taking into account a progress of a geometry and introducing adequate relativistic concepts, the elementary particle dynamics is generalized on the case of arbitrary space-time geometry. A use of adequate relativistic concepts admits one to formulate the simple demonstrable dynamics of particles.

1 Introduction

Necessity of the general relativity generalization arises as a result of a geometry progress [2]. Now we know nonaxiomatizable (physical) geometries, which were unknown 20 years ago. Physical geometries are essentially the metric geometries, whose metric is free of almost all conventional restrictions. In a metric geometry there exists a problem, how one should define geometric concepts and rules of geometric objects construction. One can construct sphere and ellipsoid, which are defined in terms of metric (world function). However, one needs to impose constraints on metric (triangle axiom) even for construction of a straight line. It is unclear, how one can define the scalar product and linear dependence of vectors. The deformation principle [3] solves the problem of the geometrical concepts definition, without imposing any restrictions on the metric. The physical geometry is equipped by the deformation principle, which admits one to construct all definitions of the physical geometry as a deformation of corresponding definitions of the proper Euclidean geometry.
In the physical geometry the information on the geometry dimension and its topology appears to be redundant. It is determined by the metric (world function \([4]\)), and one may not give it independently. A physical geometry is described completely by its world function. The geometry is multivariant and nonaxiomatizable in general. The world function describes uniformly continuous and discrete geometries. As a result the dynamic equations in physical space-time geometry are finite difference (but not differential) equations. Besides, in the physical space-time geometry the particle dynamics can be described in the coordinateless form. It is conditioned by a possibility of ignoring the linear vector space, whose properties are not used in a physical geometry. It is rather uncustomary for investigators dealing with the Riemannian geometry, which is based on usage of the linear vector space properties.

There is only one uniform isotropic geometry (geometry of Minkowski) in the set of Riemannian geometries, whereas there is a lot of uniform isotropic geometries among physical geometries. In particular, let us consider the world function \(\sigma\) of the form

\[
\sigma = \sigma_M + \lambda_0 \text{sgn}(\sigma_M), \quad \lambda_0 = \frac{\hbar}{2bc}
\]

where \(\sigma_M\) is the world function of Minkowski, and \(\hbar, c, b\) are respectively quantum constant, the speed of the light and some universal constant. The space-time geometry is discrete and multivariant. Free particle motion appears stochastic (multivariant). Its statistical description is equivalent to quantum description in terms of the Schrödinger equation \([5]\).

Thus, application of the physical geometry in the microcosm admits one to give a statistical foundation of quantum mechanics and to convert the quantum principles into appearance of the correctly chosen space-time geometry. One should expect, that a consideration of a more general space-time geometry and a refusal from the Riemannian, which is conditioned by our insufficient knowledge of geometry, will lead to a progress in our understanding of gravitation and cosmology.

An arbitrary space-time geometry is described completely by the world function \(\sigma(P, P')\), given for all pairs of points \(P, P'\). Information on dimension and on topology of the geometry is redundant, as far as it may be obtained from the world function. The Riemannian geometry, which is used in the contemporary theory of gravitation, is considered usually to be the most general possible space-time geometry. However, it cannot describe a discrete geometry, or a geometry, having a restricted divisibility. The world function of the Riemannian geometry satisfies the equation

\[
\frac{\partial \sigma}{\partial x^i} g^{ik}(x) \frac{\partial \sigma}{\partial x^k} = 2\sigma, \quad \sigma(x, x') = \sigma(x', x)
\]

It means, that in the expansion

\[
\sigma(x, x') = \frac{1}{2} g_{ik}(x) \xi^i \xi^k + \frac{1}{6} g_{ikl}(x) \xi^i \xi^k \xi^l + \ldots \quad \xi^k = x^k - x'^k
\]

the metric tensor determines completely the whole world function.

Conventional gravitation equations determine only metric tensor. The world function and the space-time geometry are determined on the basis of supposition on
the Riemannian geometry. Generalization of the gravitation equations admits one to obtain the world function directly (but not only the metric tensor).

The deformation principle admits one to construct all definitions of a physical geometry as a result of deformation of definitions of the proper Euclidean geometry. One uses the fact, that the proper Euclidean geometry \( \mathcal{G}_E \) is an axiomatizable geometry and a physical geometry simultaneously. It means, that all definitions of the Euclidean geometry, obtained in the framework of Euclidean axiomatics can be presented in terms and only in terms of the world function \( \sigma_E \) of the Euclidean geometry \( \mathcal{G}_E \). Replacing \( \sigma_E \) in all definitions of the Euclidean geometry \( \mathcal{G}_E \) by a world function \( \sigma \) of some other geometry \( \mathcal{G} \), one obtains all definitions of the geometry \( \mathcal{G} \).

Definition of the scalar product \((P_0 P_1, Q_0 Q_1)\) of two vectors \(P_0 P_1\) and \(Q_0 Q_1\) and their equivalency \((P_0 P_1 \text{eqv} Q_0 Q_1)\) are the most used definitions

\[(P_0 P_1, Q_0 Q_1) = \sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_1, Q_1) \quad (1.3)\]

\[(P_0 P_1 \text{eqv} Q_0 Q_1) \quad \text{if} \quad (P_0 P_1, Q_0 Q_1) = |P_0 P_1| \cdot |Q_0 Q_1| \land |P_0 P_1| = |Q_0 Q_1| \quad (1.4)\]

They are defined in such a way in the Euclidean geometry. They are defined in the same way also in any physical geometry.

Solution of (1.4) is unique in the case of the proper Euclidean geometry, although there are only two equations, whereas the number of variables to be determined is larger, than two. For arbitrary physical geometry a solution is not unique, in general. As a result there are many vectors at the point \(P_0\), which are equivalent to vector \(Q_0 Q_1\) at the point \(Q_0\). Even geometry of Minkowski is multivariant with respect to spacelike vectors, although it is single-variant with respect to timelike vectors. Space-time geometry becomes to be multivariant with respect to timelike vectors only after proper deformation.

2 Influence of the matter distribution on space-time geometry

At the generalization of the general relativity on the case of arbitrary space-time geometry the two circumstances are important.

1. A use of the deformation principle,

2. A use of adequate relativistic concepts, in particular, a use of relativistic concept of the events nearness (See details in [6]).

Two events \(A\) and \(B\) are near, if and only if

\[\sigma(A, B) = 0 \quad (2.1)\]

In the space-time of Minkowski a variation \(\delta g_{ik}\) of the metric tensor under influence of the matter have the form

\[\delta g_{ik}(x) = -\kappa \int G_{\text{ret}}(x, x') T_{ik}(x') \sqrt{-g(x')} d^4 x', \quad \kappa = \frac{8\pi G}{c^2} \quad (2.2)\]

3
where $T_{ik}$ is the energy-momentum tensor of the matter,

$$G_{ret}(x, x') = \frac{\theta(x^0 - x'^0)}{2\pi c} \delta(2\sigma_M(x, x')),$$

and $\theta(x)$ is the Heaviside step function,

$$\theta(x) = \begin{cases} 
1 & \text{if } x > 0 \\
0 & \text{if } x < 0
\end{cases}$$

(2.3)

Appearance of world function in the $\delta$-function means, that the condition of the nearness $\sigma_M(x, x') = 0$ leads to interpretation of gravitational (and electromagnetic) interactions as a direct collision of particles. Being presented in terms of world function these formulae have the same form in any physical geometry.

$$\delta\sigma(S_1, S_2) = -G \sum_s m_s \frac{\theta((P'_lP_0Q_0)) (P'_lP_{t+1}P_0)}{(P'_lP_{t+1}P'_lP_{t+1})} \frac{(P'_lP_{t+1}P_0)}{|PQ_0|} \times ((P'_lP_{t+1}PS_1) - (P'_lP_{t+1}PS_2))^2$$

(2.4)

where $S_1, S_2$ are arbitrary points of the space-time. Summation is produced over all world lines of particles perturbing the space-time geometry. The segment $P'_lP_{t+1}$ is infinitesimal element of the world line $\mathcal{L}(s)$ of one of perturbing particles. The point $P'_l$ is near to the point $P$, which is a middle of the segment $S_1S_2$.

$$\sigma(P, P') = 0, \quad PS_1 = -PS_2$$

(2.5)

The vectors $PQ_i, i = 0, 1, 2, 3$ are basic vectors at the point $P$. Vector $PQ_0$ is timelike. If $\sigma_0$ is the unperturbed world function of space-time geometry without particles, then $\sigma = \sigma_0 + \delta\sigma$ is the world function of the space-time geometry after appearance of perturbing particles. One should use the world function $\sigma$ at calculation of scalar products in rhs of (2.4) by the formula (1.3). At first the world function $\sigma$ is unknown, and relation (2.4) is an equation for determination of $\sigma$.

Equation (2.4) is solved by the method of subsequent approximations. At the first step one calculates rhs of (2.4) by means of $\sigma_0$ and obtains $\sigma_1 = \sigma_0 + \delta\sigma_0$. At the second step one calculates rhs of (2.4) by means of $\sigma_1$ and obtains $\sigma_2 = \sigma_0 + \delta\sigma_1$ and so on.

Applying relation (2.4) to heavy pointlike particle, one obtains in the first approximation

$$\sigma_1(t_1, y_1; t_2, y_2) = \frac{1}{2} \left( 1 - \frac{4Gm}{c^2|y_2 + y_1|} \right) c^2 (t_2 - t_1)^2 - \frac{1}{2} (y_2 - y_1)^2$$

(2.6)

where $m$ is the mass of the particle.

Space-time geometry appears to be non-Riemannian already at the first approximation, although the metric tensor has the form, which it has in the conventional gravitation theory for a slight gravitational field. The next approximations do not change the situation.

Thus, the space-time geometry appears to be non-Riemannian. Furthermore, supposition on the Riemannian space-time leads to an ambiguity of the world function for large difference of times $(t_1 - t_2)$ even in the case of a gravitational field of
a heavy particle. It is conditioned by the fact, that there are many geodesics, connecting two points. It is forbidden in a physical geometry, where the world function must be single-valued.

Thus, generalization of the relativity theory on the general case of the space-time geometry is generated by our progress in geometry and by a use of adequate relativistic concepts. The deformation principle is not a hypotheses, but it is the principle, which lies in the basis of physical geometry. The uniform formalism, suitable for both continuous and discrete geometries, is characteristic for physical geometries. This formalism uses dynamic equations in the form of finite difference equations. Sometimes these equations have a form of finite relations. The uniform formalism is formulated in coordinateless form. It gets rid of necessity to consider coordinate transformations and their invariants.

3 Particle dynamics in physical space-time geometry

The contemporary elementary particle theory (EPT) is qualified usually as the elementary particle physics (EPP). However, it should be qualified more correctly as an elementary particle chemistry (EPC). The fact is that, the structure of the elementary particle theory reminds the periodical system of chemical elements. Both conceptions classify elementary particles (and chemical elements). On the basis of the classification both conceptions predict successfully new particles (and chemical elements). Both conceptions are axiomatic (but not model) constructions.

The periodical system of chemical elements has given nothing new for investigation of the atomic structure of chemical elements. One should not expect any information about elementary particle structure from contemporary EPT. For this purpose a model approach to EPT is necessary.

The simplest particle is considered usually as a point in usual 3D-space. This point is equipped by a mass and by a momentum 4-vector. One may to prescribe an electric charge and some other characteristics to the point. The aggregate of this information forms a nonrelativistic concept of a particle. This concept of a particle is based on the concept of the linear vector space, which is based in turn on the concept of axiomatizable continuous space-time geometry.

In the consecutive relativistic theory one should use another concept of a particle. The simplest particle is defined by two points \( P, P' \) in the space-time. The vector \( PP' \), formed by the two points, is a geometric momentum of the particle. Its length \( \mu = |PP'| \) is the geometric mass of the particle. The geometric mass \( \mu \) and momentum \( PP' \) are connected with conventional mass \( m \) and 4-momentum \( p \) by means of relations

\[
m = b\mu, \quad p = bcPP'
\]

where \( b \) is some universal constant, and \( c \) is the speed of the light. The electric charge appears in the 5D-geometry of Kaluza-Klein as a projection of 5-momentum
on the additional fifth dimension, which is a chosen direction. Projection on this
direction is invariant, because the direction is chosen. As a result all parameters of
a particle appear to be geometrized. A free motion of the simplest particle in the
properly chosen 5D-geometry of the space-time is equivalent to motion of a charged
particle in the given gravitational and electromagnetic fields of the Minkowskian
space-time geometry. Such a concept of a particle may be used in any space-time
geometry (nonaxiomatizable and discrete).

A particle may have a complicated structure. In this case the particle is described
by its skeleton \( \mathcal{P}_n = \{ P_0, P_1, ..., P_n \} \), consisting of \( n + 1 \) space-time points \( n = 1, 2, ... \) The question: "What does unite the skeleton points in a particle" is rele-
vant only in the space-time geometry with unlimited divisibility. In the physical geometry [2, 3, 7] the skeleton points may be connected between themselves simply as points
of a geometry with a limited divisibility.

The particle evolution is described by a chain \( C \) of connected skeletons [7, 8].

\[
C = \bigcup_s \mathcal{P}_n^{(s)}
\]  

(3.2)

Adjacent skeletons of the chain are equivalent.

\[
\mathcal{P}_n^{(s+1)} \equiv \mathcal{P}_n^{(s)} : \quad \mathbf{P}_i^{(s+1)} \mathbf{P}_k^{(s+1)} \equiv \mathbf{P}_i^{(s)} \mathbf{P}_k^{(s)} \quad i, k = 0, 1, ... n, \quad s = ... 0, 1, ... \]  

(3.3)

Points \( P_1^{(s)} \) and \( P_0^{(s+1)} \) of the chain coincide \( s = ... 0, 1 ... \) Then according to (3.3) the
leading vector \( \mathbf{P}_0^{(s)} \mathbf{P}_1^{(s)} = \mathbf{P}_0^{(s)} \mathbf{P}_0^{(s+1)} \) of skeleton \( \mathcal{P}_n^{(s)} \) is equivalent to the leading
vector \( \mathbf{P}_0^{(s+1)} \mathbf{P}_1^{(s+1)} = \mathbf{P}_0^{(s+1)} \mathbf{P}_0^{(s+2)} \) of skeleton \( \mathcal{P}_n^{(s+1)} \), i.e.

\[
\mathbf{P}_0^{(s)} \mathbf{P}_0^{(s+1)} \equiv \mathbf{P}_0^{(s+1)} \mathbf{P}_0^{(s+2)}
\]  

(3.4)

In the explicit form equations (3.2), (3.3), describing the world chain, look as
follows

\[
\begin{align*}
    (\mathbf{P}_i^{(s+1)} \mathbf{P}_k^{(s+1)}, \mathbf{P}_i^{(s)} \mathbf{P}_k^{(s)}) &= \left| \mathbf{P}_i^{(s+1)} \mathbf{P}_k^{(s+1)} \right| \cdot \left| \mathbf{P}_i^{(s)} \mathbf{P}_k^{(s)} \right|, \\
    \left| \mathbf{P}_i^{(s+1)} \mathbf{P}_k^{(s+1)} \right| &= \left| \mathbf{P}_i^{(s)} \mathbf{P}_k^{(s)} \right|, \quad P_1^{(s)} = P_0^{(s+1)}
\end{align*}
\]  

(3.5)

\[
i, k = 0, 1, ... n, \quad s = ... 0, 1, ...
\]  

(3.6)

where scalar products are defined via world functions by the relation (1.3).

Rotation of a skeleton is absent. The translational motion is carried out along the
leading vector \( \mathbf{P}_0 \mathbf{P}_1 \). Dynamics is described by means of finite difference equations.
It is reasonable, if the space-time geometry may be discrete. The leading vector
describes the evolution direction in the space-time.

The number of dynamic equations is equal to \( n(n + 1) \), whereas the number of
variables to be determined is equal to \( Nn \). Here \( N \) is the dimension of the space-
time, and \( n + 1 \) is the number of points in the particle skeleton. The difference
between the number of equations and the number of variables, which are to be
determined, may lead to different results.
1. Multivariance, i.e. ambiguity of the world chain links position, when \( n(n+1) < Nn \). It is characteristic for simple skeletons, which contain small number of points. Multivariance is responsible for quantum effects [5].

2. Zero-variance, i.e. absence of solution of equations, when \( n(n+1) > Nn \). It is characteristic for complicated skeletons, which contain many points. Zero-variance means a discrimination of particles with complicated skeletons. As a result there exist only particles, having only certain values of masses and other parameters.

Quantum indeterminacy and discrimination mechanism are two different sides of the particle dynamics. The conventional theory of elementary particles has not a discrimination mechanism, which could explain a discrete spectrum of masses.

There are two sorts of elementary particles: bosons and fermions. Boson has not its own angular momentum (spin). It is rather reasonable, because motion of elementary particles is translational. However, the fermions have a discrete spin, which looks rather unexpected at the translation motion. Spin of a fermion appears as a result of translation motion along a space-like helix with timelike axis [9, 10, 11]. The helix world line of a free particle is possible only for spacelike world line. It is conditioned by multivariance [12] of the space-time geometry with respect to spacelike vectors. This multivariance takes place even for space-time of Minkowski. This multivariance takes place for any space-time geometry. It does not vanish in the limit \( \hbar \to 0 \).

However, in the space-time geometry of Minkowski the helix world chain is impossible, because the temporal component of momentum increases infinitely. For existence of the helix world chain, the world function \( \sigma \) is to have the form

\[
\sigma = \begin{cases} 
  f(\sigma_M) & \text{if} \ |\sigma_M| < \sigma_0 \\
  \sigma_M + \lambda_0^2 \text{sgn}(\sigma_M) & \text{if} \ |\sigma_M| > \sigma_0 
\end{cases}
\]

\[
\lambda_0^2 = \frac{\hbar}{2bc}, \quad \sigma_0 = \text{const} \quad (3.7)
\]

\[
|f(\sigma_M)| < |\sigma_M| \frac{\sigma_0 + \lambda_0^2}{\sigma_0}, \quad |\sigma_M| < \sigma_0 \quad (3.8)
\]

In the conventional relativity theory the helix spacelike world lines are not considered, because one assumes, that they are forbidden by the relativity principles. Fermions are described usually by means of the Dirac equation, which needs introduction of such special quantities as \( \gamma \)-matrices. A use of \( \gamma \)-matrices generates a mismatch between the particle velocity and its mean momentum. (The quantum mechanics uses the mean momentum always [13].) This enigmatic mismatch is explained easily by means of the helix world chain. The velocity is tangent to helix, whereas the mean momentum is directed along the axis of helix.

Besides, the fermion skeleton is to contain not less, than three points. It is necessary for stabilization of the helix world line [9, 10]. Existence of the fermion is possible only at certain values of its mass, which depends on the space-time geometry (the form of function \( f \) in (3.7)) and on a choice of the skeleton points.
Thus, the spin and magnetic moment of fermions appear to be connected with spacelike world chain and with multivariance of the space-time geometry with respect to space-like vectors. At the conventional approach to geometry the spacelike world lines are considered to be incompatible with the relativity principles. Spin is associated with existence of enigmatic $\gamma$-matrices. Multivariance with respect to timelike vectors is slight (it vanishes in the limit $\hbar = 0$). Multivariance with respect to spacelike vectors is strong (it is not connected with quantum effects).

The particle motion is free in the properly chosen space-time geometry. However, the particle motion can be described in arbitrary geometry, given on the same point set, where the true geometry is given. The world function $\sigma$ of the true geometry is presented in the form

$$\sigma(P,Q) = \sigma_{K0}(P,Q) + d(P,Q)$$

(3.9)

where $d(P,Q)$ is some addition to the world function of $\sigma_{K0}(P,Q)$ of the space-time geometry of Kaluza-Klein, which is used in the given case as a basic geometry. In this geometry the particle motion ceases to be free. It turns into a motion in force fields, whose form is determined by the form of addition $d(P,Q)$.

Progress in the elementary particle dynamics is conditioned by a progress in geometry and by a use of adequate relativistic concepts. The suggested elementary particle dynamics is a model conception. It is demonstrable and simple. Multivariance of the geometry explains freely quantum effects. The zero-variance generates a discriminational mechanism, responsible for discrete characteristics of elementary particles. Mathematical technique is formulated in a coordinateless form, that gets rid of a necessity to investigate coordinate transformations and their invariants. Two-point technique of the dynamics and many-point skeletons contain a lot of information, which should be only correctly ordered. Simple principles of dynamics reduce a construction of the elementary particle theory to formal calculations of different skeletons dynamics at different space-time geometries. There is a hope, that true skeletons of elementary particles can be obtained by means of the discrimination mechanism of the true space-time geometry. At any rate, having been constructed in the framework of simple dynamic principles, this dynamics explains freely discrete spins and discrete masses of fermions and mismatch between the particle velocity and its mean momentum. These properties are described usually by introduction of $\gamma$-matrices, that is a kind of fitting.

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