Piezo-magnetic energy harvesting from movement of the head

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Abstract. This paper reports the design, modeling, optimization and testing of the piezo-magnetic energy harvester that is capable of converting non-harmonic movement of the human head into electricity. The rolling magnet and doubly-clamped piezoelectric configuration of the device makes the energy harvesting from small-amplitude and low-frequency movements of the human head efficient. In addition, the device can inconspicuously be integrated with the glasses. The experimental results show that the energy harvester device could deliver the maximum instantaneous power of 0.5 µW to the impedance matched resistive load.

1. Introduction
Wearable technologies are being interwoven with the daily life. As they are becoming smarter and smaller, new efficient and cost effective powering solutions are needed. Although primary batteries have been the traditional source of power for mobile systems, energy harvesting is emerging in recent years. Energy can be harvested from almost any environment, however, since a wearable device is attached to the human body, one of the most readily available sources is body movement.

Head movement contains some kinetic energy that could be used to fully or partially power electronic head-mounted devices such as hearing aids, wearable EEG systems, handsfree Bluetooth devices and optical eyewear displays. Although human head is not as dynamic as other human parts such as arms and legs, it could be highly advantageous to power head-mounted devices for the obvious reason of proximity. Nodding or shaking of the head typically occur during conversations and it is estimated that such movements can generate a mean power of 30µW [1]. This amount of power would be sufficient to power a wearable sensor node [2].

Electromagnetic and piezoelectric transduction methods are two most competitive approaches for kinetic energy harvesting. Electromagnetic generators benefit from bulk magnets that make them suitable for low-frequency energy harvesting [3, 4]. On the other side, most piezoelectric generators have the advantages of simple mechanical structure and less peripheral components [5, 6]. The energy harvesting device presented in this paper is a combination of both transduction methods to combine their advantages. It is composed of a flexible piezoelectric composite in a doubly-clamped structure actuated by magnetic repulsive force. Piezoelectric energy harvesting with magnetic coupling has been previously used in a rotational configuration to harness human body power [7]. In contrast, the piezo-magnetic energy harvester proposed in this paper is in a rolling configuration; resulting in larger magnet velocity and stronger magnetic force. This configuration suits low-frequency and small-amplitude applications like movement of the head.
2. Energy harvesting device
The energy harvesting device is composed of a doubly-clamped piezoelectric fiber composite (PFC) beam and two permanent magnets. The PFC beam (Advanced Cerametrics, NJ, USA) comprises unidirectionally aligned piezoceramic micro-fibers with interdigitated electrodes. One of the Neodymium magnets (Amazing Magnets, CA, USA) is bonded to the PFC beam while another one is rolling along the beam length as shown in Figure 1. The rolling magnet moves back and forth while the device is tilting. The magnets are positioned such that like poles face each other and hence each time the moving magnet passes in front of the PFC magnet, the beam is deformed by the repulsive magnetic force and an electric current flows in the load circuit. The energy harvesting device is mounted on a pair of eyeglasses as shown in Fig. 2 to become wearable.

![Figure 1. Schematic representation of the energy harvesting system](image1)

![Figure 2. Piezoglasses energy harvester](image2)

3. System modeling
Fitting a beam of length $L_0$ in a gap of width $l$, where $L_0 > l$ necessitate a curvature to the beam. The equilibrium equation of an axially loaded beam is [8]:

$$y''' + n^2 y'' = 0$$

where $y$ describes the deflection of the beam, $n^2 = P/EI$, and $P$, $E$ and $I$ are the axial load, the Young’s modulus and the beam’s moment of inertia respectively. Considering the clamped-clamped boundary conditions, the equation of the buckled beam in the first mode can be written as
\[ y = \frac{h}{2} \left( 1 - \cos \frac{2\pi x}{l} \right) \]  

(2)

where \( l \) is the distance between the beam supports and \( h \) is the deflection of the center of the beam; \( y(l/2) = h \). Using the formula for calculating the arc length of the curve, the length of the deflected beam can be derived as

\[
L = \int_0^l \sqrt{1 + y'^2} \, dx = \int_0^l \sqrt{1 + \left( \frac{\pi h}{l} \sin \frac{2\pi x}{l} \right)^2} \, dx
\]

(3)

The square term is assumed to be much smaller than unity and hence Eq.(3) can be approximated by the Taylor’s series expansion.

\[
L \approx \int_0^l \left[ 1 + \frac{1}{2} \left( \frac{\pi h}{l} \sin \frac{2\pi x}{l} \right)^2 \right] \, dx = \frac{\pi^2 h^2}{4l} + l
\]

(4)

The potential energy stored in the beam due to the axial deformation can be expressed as

\[
V_a = \frac{1}{2} \frac{AE}{L_0} (L - L_0)^2 = \frac{1}{2} \frac{AE}{L_0} \left( \frac{\pi^2 h^2}{4l} + l - L_0 \right)^2
\]

(5)

where \( A \) is the cross section area of the beam. In addition to axial deformation, the bending of the beam can also be quantified as the potential energy.

\[
V_b = \frac{EI}{2} \int_0^l (y'')^2 \, dx = \frac{\pi^4 EI}{l^3} h^2
\]

(6)

The repulsive magnetic force has been measured as a function of the separation distance \( r \) between the magnets used in the prototype mechanism by using a precise scale. This force is found to be proportional to \( 1/r^3 \) and the corresponding potential energy can be calculated by

\[
V_m = -\int_r^\infty \frac{k}{r^3} \, dr = \frac{1}{2} \frac{k}{r^2}
\]

(7)

in which \( k \) is the coefficient of proportionality and \( r = \sqrt{s^2 + h^2} \).

The Lagrangian function of the entire system is defined as the subtraction of the kinetic and potential energies.

\[
L = T - V = \frac{1}{2} m \dot{h}^2 + \frac{1}{2} M \left( \dot{s}^2 + (s\dot{\theta})^2 \right) + \frac{1}{2} J \left( \frac{2\dot{\theta}}{\mu^2 d^2} \right)^2 - \left[ \frac{1}{2} AE \left( \frac{\pi^2 h^2}{4l} + l - L_0 \right)^2 + \frac{\pi^4 EI}{l^3} h^2 \right] + \frac{1}{2} \frac{k}{h^2 + s^2} + M g \sin \theta
\]

(8)

The Lagrange’s formula \( \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \) is then applied to drive the equations of motions for two generalized coordinates, \( h \) and \( s \):

\[
\begin{align*}
m\ddot{h} + c\dot{h} + \frac{2\pi^2 AEh}{4lL_0} \left( \frac{\pi^2 h^2}{4l} + l - L_0 \right) + \frac{2\pi^4 EI}{l^3} h - \frac{k h}{(h^2 + s^2)^2} = 0 \\
\left( M + \frac{4J}{\mu^2 d^2} \right) \ddot{s} - M s\dot{\theta}^2 - \frac{ks}{(h^2 + s^2)^2} + M g \sin \theta = 0
\end{align*}
\]

(9)
### Table 1. System parameters

| Parameter       | Value |
|-----------------|-------|
| Magnetic force  | $1.67 \times 10^{-6}$ Nm$^2$ |
| Fixed magnet    | $2g$  |
| Moving magnet   | $M=4g$, $d=12mm$, $J=80gmm^2$ |
| PFC             | $A=6mm^2$, $I=0.045mm^4$, $E=2.44 \times 10^{10}$ Pa, $C=10 \times 10^{-9}$ F, $d_{33}=550 \times 10^{-12}$ C/N |
| Device parameters | $l=10cm$, $R=660\Omega$ |
| Initial conditions | $s_0=5$ cm, $h_0=0.5-5$ cm, $\dot{s}_0=0$, $\dot{h}_0=0$ |

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### 4. Model validation

The theoretical model expressed in Eq. 9 is experimentally tuned and validated for $\theta=45^\circ$ and $\dot{\theta}=0$ and the results are shown in Figure 3 for different beam apex heights. All the parameters used in the theoretical model are also given in Table 1. According to Figure 3, there is a good agreement between experimental measurements and theoretical predictions. In fact, the maximum measured voltage of the device and the maximum theoretical deformation of the beam occur nearly at the same beam apex height.

### 5. System optimization

According to the derived equations of motion, Eq. (9), the magnet travel distance ($s_0$) and the piezoelectric beam apex height ($h_0$) have significant effects on the performance of the device. The more distance the magnet can travel (large $s_0$), the larger potential energy it gains and the faster it passes in front of the PFC fixed magnet, resulting in larger magnetic force and more deformation of the beam. However, this parameter is eventually limited by the size of the device. On the other hand, if the beam is too curved (large $h_0$), the separation distance is too large to produce significant magnetic force and deform the beam. Conversely, if the beam curve is too little (small $h_0$), the repulsive force component along the direction of the moving magnet is too large to let the moving magnet pass and deform the beam. Therefore, there is an optimum value for the beam apex height ($h_0$). This optimum value can be identified in Figure 3.

Having validated the theoretical model for the constant $\theta$ in Figure 3, Eq. 9 is again used to optimize the device for more realistic scenario and practical conditions in which $\dot{\theta} = \bar{\theta}f$, where $\bar{\theta}=16.5^\circ$ and $f=3.7$ Hz and are respectively average amplitude and frequency of the head movements during conversation [1]. The optimum value is then calculated to be $\tilde{h}_0=2.5$ cm.

### 6. Experimental results

A prototype is fabricated based on the derived optimum value ($\tilde{h}_0$) and is mounted on a pair of eyeglasses as shown in Figure 2. To determine the device’s approximate raw power, the PFC was terminated with the matched resistive load and its voltage was measured for a test-subject wearing the piezoelectric glasses while working with the computer as shown in Figure 4. The voltage output of the energy harvester during a ten-minute test is illustrated in Figure 5.
According to the results, the maximum measured voltage is 0.53 V that corresponds to the instantaneous power of 0.5 µW delivered to the impedance matched resistive load.

Figure 4. Experimental setup  
Figure 5. Piezo-magnetic glasses voltage output

7. Conclusions
To the best of the authors’ knowledge, the presented device is the first inertial energy harvester with the flexible piezoelectric beam that is actuated by the rolling magnet. The main advantage of the proposed energy harvester is that the velocity of the rolling magnet that excites the beam is a function of its traveled distance and the tilt angle. Therefore, the velocity and hence the kinetic energy of the rolling magnet can be quite considerable even if the host motion is very limited and slow. Consequently, the device can operate with the small-amplitude and low-frequency movements of the head for which other energy harvesting solutions are rarely efficient. Moreover, the proposed energy harvester can be unobtrusive when integrated with the glasses or any other head-mounted devices. The power output of the energy harvester device is currently much far from the expected power, however it can multiply by using heavier rolling magnets and installing more piezoelectric layers in both temples. The piezo-magnetic energy harvester can be ultimately used to power a body sensor network or any other wearable or implantable medical devices in the region of the head.

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