QUARKONIUM POLARIZATION
IN THE NRQCD FACTORIZATION FRAMEWORK∗

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The NRQCD factorization approach for calculating inclusive production of heavy quarkonium gives unambiguous predictions for the polarization of quarkonium states. The factorization formula for polarized states can be obtained by using the threshold expansion method to calculate the short-distance coefficients and then using symmetries of NRQD to reduce the NRQCD matrix elements. A particularly dramatic prediction of the NRQCD factorization framework is that prompt psi’s and psi-primes’s produced at the Tevatron should be predominantly transversely polarized at large transverse momentum.

1. Introduction

The NRQCD factorization approach is a systematic framework for analyzing the inclusive cross sections and annihilation decay rates of heavy quarkonium. The cross section (or decay rate) is factored into short-distance coefficients that are computable using perturbation theory and long-distance NRQCD matrix elements. The matrix elements scale as a definite power of \( v \), the typical relative velocity of the heavy quark in quarkonium. Thus the cross section can be organized into a double expansion in powers of \( v \) and powers of \( \alpha_s(m_c) \), where \( m_c \) is the heavy quark mass.

The NRQCD factorization framework makes definite, and in some cases rather dramatic, predictions for the dependence of the cross section on the polarization of the quarkonium. Below, I outline the NRQCD factorization approach as it applies to polarized quarkonium states and then summarize the applications that have been carried out thus far.

2. NRQCD Factorization

The general expression for the cross section for the inclusive production of a quarkonium state \( H \) with four-momentum \( P \) is

\[
\sum_X d\sigma(12 \to H(P) + X) = \frac{1}{4E_1E_2v_{12}} \frac{d^3P}{(2\pi)^32Ep}
\]

∗Invited talk presented at the Quarkonium Physics Workshop, University of Illinois at Chicago, June 1996.
where \( T_{12 \rightarrow H(P)+X} \) is the T-matrix element and the sum on the right side includes integration over the phase space of the additional particles. This cross section involves both “short distances” of order \( 1/m_c \) or smaller and “long distances” of order \( 1/(m_c v) \) or larger. The production of the \( c\bar{c} \) pair involves short distances, because the parton processes that produce a \( c\bar{c} \) pair always involve particles that are off their mass shells by amounts of order \( m_c \) and can therefore propagate only over short distances. The binding of the \( c \) and \( \bar{c} \) into the state \( H \) involves long distances, because gluons whose wavelengths are comparable to or larger than the size of the bound state, which is of order \( 1/(m_c v) \), play a large role in the binding.

If the cross section in (1) is sufficiently inclusive, short-distance and long-distance effects can be factored. (If there are hadrons in the initial state, it is also necessary to restrict the four-momentum of the quarkonium to be significantly different from that of the initial hadrons in order to avoid contributions from diffractive scattering.) Like many other factorization “theorems” of perturbative QCD, the factorization of quarkonium cross sections has not been proven with complete rigor, but it is a plausible generalization of the factorization theorems for the Drell-Yan production of muon pairs and for heavy quark production. The factorization relies on cancellations between soft partons that are emitted by the \( c\bar{c} \) pair and soft partons that are exchanged between the \( c\bar{c} \) pair and other jet-like collections of collinear partons. After taking into account these cancellations, the cross section can be factored into short-distance and long-distance parts. The short-distance parts involve the production of a \( c\bar{c} \) pair with small relative momentum plus hard partons. The \( c\bar{c} \) pair that emerges from the short-distance part is essentially point like on the scale of the quarkonium wavefunction. The long-distance parts involve the formation of \( H \) plus soft partons from the pointlike \( c\bar{c} \) pair.

The standard factorization methods of perturbative QCD produce an expression for the cross section that involves an integral over the relative momentum of the \( c \) and \( \bar{c} \) that form the quarkonium. Long-distance and short-distance effects can be further untangled by expanding the short-distance factors in powers of the relative momentum \( q \) and absorbing the integration over \( q \) into the long-distance parts. The resulting long-distance factors can be expressed as matrix elements in an effective field theory called nonrelativistic QCD (NRQCD). The T-matrix elements in (1) that survive after soft-parton cancellations can be expressed in the form

\[
T_{12 \rightarrow H(P)+X_H+X_S} \approx \sum_n \hat{T}_{12 \rightarrow c\bar{c}(P,n)+X_H} \langle H + X_S | \psi^\dagger K_n \chi (x = 0) | 0 \rangle ,
\]

where the sum includes all possible color and angular-momentum states of the \( c\bar{c} \) pair. The factor \( \hat{T}_{12 \rightarrow c\bar{c}(P,n)+X_H} \) can be interpreted as a T-matrix element for producing a \( c\bar{c} \) pair in the state \( n \) plus the hard partons \( X_H \). The second factor on the right is a matrix element in NRQCD between the vacuum state and a state that in the asymptotic future consists of the quarkonium \( H \) at rest plus the soft partons \( X_S \). The local operator \( \psi^\dagger K_n \chi \) creates a pointlike \( c\bar{c} \) pair in the state \( n \).
After inserting the expression (2) for the T-matrix elements into (1), we can rearrange the cross section into a form in which the short-distance and long-distance contributions are factored:

\[ \sum_X (2\pi)^4 \delta^4(k_1 + k_2 - P - k_X) \left| T_{12 \to H(P) + X} \right|^2 \]
\[ \approx \sum_{mn} \left( \sum_{X_H} \left( \hat{T}_{12 \to c\bar{c}(P,m) + X_H} \right)^* \hat{T}_{12 \to c\bar{c}(P,n) + X_H} \right) \times \left( \sum_{X_S} \langle 0 | \chi^\dagger K^\dagger m \psi | H + X_S \rangle \langle H + X_S | \psi^\dagger K_n \chi | 0 \rangle \right). \] (3)

The short-distance factor is

\[ C_{mn}(k_1, k_2, P) = \sum_{X_H} \left( \hat{T}_{12 \to c\bar{c}(P,m) + X_H} \right)^* \hat{T}_{12 \to c\bar{c}(P,n) + X_H}. \] (4)

The long-distance factor is

\[ \langle O_{mn}^H \rangle = \langle 0 | \chi^\dagger K^\dagger m \psi | P_H \psi^\dagger K_n \chi | 0 \rangle, \] (5)

where \( P_H \) projects onto states that in the asymptotic future contain the quarkonium state \( H \) plus soft partons:

\[ P_H = \sum_{X_S} | H + X_S \rangle \langle H + X_S |. \] (6)

Inserting (3) into (1), we obtain the NRQCD factorization formula for the inclusive cross section:

\[ \sum_X d\sigma(12 \to H(P) + X) = \frac{1}{4E_1E_2\sin^2 \theta_{12}} \frac{d^3P}{(2\pi)^3 2E_P} \sum_{mn} C_{mn}(k_1, k_2, P) \langle O_{mn}^H \rangle. \] (7)

We can write a similar factorization formula for the inclusive decay rate of the quarkonium \( H \) via the annihilation of the \( c\bar{c} \) pair. The general formula for the decay rate is

\[ \sum_X d\Gamma(H \to X) = \frac{1}{2M_H} \sum_X (2\pi)^4 \delta^4(P - k_X) \left| T_{H(P) \to X} \right|^2, \] (8)

where \( P = (M_H, 0) \). This can be expressed in the factored form

\[ \sum_X d\Gamma(H \to X) = \frac{1}{2M_H} \sum_{mn} C_{mn} \langle H | O_{mn} | H \rangle, \] (9)

where the \( C_{mn} \)’s are short-distance coefficients. The long-distance factors are expectation values in the quarkonium state of local four-fermion operators of the form \( O_{mn} = \psi^\dagger K^\dagger m \chi^\dagger K_n \psi \).
3. Short-distance Coefficients

Since the coefficients $C_{mn}$ in (7) and (9) involve only short distances of order $1/m_c$ or larger, they can be expressed as perturbation series in $\alpha_s(m_c)$. These coefficients are known at tree level for many processes, and in a few cases they are known at the one-loop level. Most of these coefficients have been obtained by calculating the perturbative cross section for producing a $c\bar{c}$ pair in a state with a prescribed nonrelativistic wavefunction. The inclusive cross section is sensitive only to the behavior of the wavefunction near the origin. The analogs of the NRQCD matrix elements (5) for the $c\bar{c}$ state can also be calculated using perturbation theory in terms of the prescribed wavefunction. Knowing the cross section and the matrix elements, we can read off the short-distance coefficients. Unfortunately, this method is not sufficiently general to determine all the short-distance coefficients, especially for polarized quarkonium states.

The threshold expansion method, developed recently by Braaten and Chen, is a general prescription for calculating the short-distance coefficients. It is based directly on the NRQCD factorization approach, and can be readily applied to polarized quarkonium states.

3.1. Threshold expansion method

The threshold expansion method relies on the fact that the short-distance coefficients in (3) are insensitive to the long-distance effects that bind the $c\bar{c}$ pairs into the quarkonium state $H$. Thus the factorization formula will hold with the same short-distance coefficients if we replace $H$ by asymptotic perturbative states $c\bar{c} = c\bar{c}(q, \xi, \eta)$ that consist of a $c$ and a $\bar{c}$ with relative momentum $q$ and spin/color state that is represented by the Pauli spinors $\xi$ and $\eta$. To completely determine the short-distance coefficients, we need to use different states $c\bar{c}$ and $c\bar{c}'$ in the $T$-matrix element and in its complex conjugate. The resulting matching prescription is

$$\sum_X (2\pi)^4 \delta^4(k_1 + k_2 - P - k_X) \langle T_{12 \to c\bar{c}'(P)+X} T_{12 \to c\bar{c}(P)+X}\rangle_{pQCD} \approx \sum_{mn} C_{mn}(k_1, k_2, P) \langle 0|\chi^\dagger K_m \psi P_{c\bar{c}', c\bar{c}} \psi^\dagger K_n \chi|0\rangle_{pNRQCD},$$

where the projection operator in the matrix element is

$$P_{c\bar{c}', c\bar{c}} = \sum_{XS} |c\bar{c}' + X_S\rangle \langle c\bar{c} + X_S|.$$

The left side of (10) is to be calculated using perturbative QCD, and then expanded in powers of the relative momenta $q$ and $q'$. The matrix elements on the right side are to be calculated using perturbative NRQCD, and then expanded in powers of $q$ and $q'$. The coefficients $C_{mn}$ are then determined by matching these expansions order by order in $\alpha_s$.

3.2. Example
We illustrate the threshold expansion method by carrying out one of the simplest matching calculations. It gives the short-distance coefficient corresponding to the parton process $q\bar{q}\rightarrow c\bar{c}$. The T-matrix element for this process is

$$T_{12\rightarrow c\bar{c}} = g^2 \frac{1}{F^2} \bar{v}(k_2)\gamma_\mu T^a u(k_1) \bar{u}(p)\gamma^\mu T^a v(p).$$  \hspace{1cm} (12)$$

Making a nonrelativistic expansion of the spinors of the $c$ and $\bar{c}$, this reduces to

$$T_{12\rightarrow c\bar{c}} = \frac{g^2}{2m_c} \bar{v}(k_2)\gamma_\mu T^a u(k_1) \ L^I_i \bar{\epsilon}^I \gamma^\mu T^a \eta,$$  \hspace{1cm} (13)$$

where $L^I_i$ are elements of the boost matrix that transforms from the rest frame of the $c\bar{c}$ pair to the frame in which it has total four-momentum $P$. Multiplying by the complex conjugate of $T_{12\rightarrow c\bar{c}}$ and averaging over initial spins and colors, we obtain

$$\langle T_{12\rightarrow c\bar{c}}^* \rangle = \frac{4\pi^2 \alpha_s^2}{9} \left[ \delta^{ji} - \hat{\hat{z}}^j \hat{\hat{z}}^i \right] \eta^I \gamma^\mu T^a \bar{\epsilon}^I \gamma^\mu T^a \eta,$$  \hspace{1cm} (14)$$

where $\hat{\hat{z}}$ is a unit vector in the direction of the momenta of the colliding $c$ and $\bar{c}$. The spinor factor can be expressed in terms of an NRQCD matrix element:

$$\langle \chi^I \sigma^j T^a \psi | c\bar{c}^I \rangle \langle c\bar{c}^I | \psi \rangle T^a \chi \rangle_{p\text{NRQCD}} = 4m_c^2 \eta^I \gamma^\mu T^a \bar{\epsilon}^I \gamma^\mu T^a \eta.$$  \hspace{1cm} (15)$$

Using the matching prescription (10), the short-distance coefficient of the matrix element $\langle \chi^I \sigma^j T^a \psi | c\bar{c}^I \rangle \langle c\bar{c}^I | \psi \rangle T^a \chi \rangle$ is

$$C_{ij} = (2\pi)^4 \delta^{ij} \left( k_1 + k_2 - P \right) \frac{\pi^2 \alpha_s^2}{9m_c^2} \left[ \delta^{ji} - \hat{\hat{z}}^j \hat{\hat{z}}^i \right].$$  \hspace{1cm} (16)$$

Inserting the short-distance coefficient (16) into the factorization formula (7) and integrating over the phase space of the quarkonium, we get an expression for the inclusive cross section:

$$\sum_X \sigma(q\bar{q} \rightarrow H + X) = \delta(s - 4m_c^2) \frac{\pi^2 \alpha_s^2}{9m_c^2} \left[ \delta^{ji} - \hat{\hat{z}}^j \hat{\hat{z}}^i \right] \langle \chi^I \sigma^j T^a \psi | \mathcal{P}_H \psi \rangle \langle \chi^I \sigma^j T^a \chi \rangle.$$  \hspace{1cm} (17)$$

This is a term in the factorization formula for the inclusive cross section of any quarkonium state $H$, whether polarized or unpolarized. There are additional terms of order $\alpha_s^3$ from the process $gg \rightarrow c\bar{c}$. All other parton processes give terms with short-distance coefficients of order $\alpha_s^4$ or higher.

4. NRQCD Matrix Elements

The long-distance factors in the NRQCD factorization formulas are expressed as matrix elements of local four-fermion operators in NRQCD. Since long-distance effects in QCD are inherently nonperturbative, the NRQCD matrix elements can...
only be calculated using nonperturbative methods like lattice gauge theory. There are effective lattice prescriptions for calculating the matrix elements $\langle H | O_{mn} | H \rangle$ that appear in quarkonium decay rates. Unfortunately, these methods cannot be used to calculate directly the production matrix elements $\langle O_{mn}^H \rangle$. The problem lies in implementing on the lattice the projection defined by (5). In the absence of nonperturbative calculations, the only alternative is to treat the NRQCD matrix elements as phenomenological parameters to be determined by experiment.

### 4.1. Model-independent framework

The factorization formula (5) provides a model-independent framework for analyzing quarkonium production. In any reasonable model for the production of quarkonium through short-distance parton processes, the inclusive cross section can be expressed in the factored form (5). The model can therefore be reduced to a set of assumptions about the NRQCD matrix elements. Until recent years, most calculations of quarkonium production were carried out using either the color-singlet model or the color-evaporation model. In the color-singlet model, the quarkonium is assumed to be simply a color-singlet $c\bar{c}$ pair in an appropriate angular-momentum state. Only one NRQCD matrix element is assumed to be important, and it can be expressed in terms of the $c\bar{c}$ wavefunction, or one of its derivatives, evaluated at the origin. In the color-evaporation model, the color and angular-momentum quantum numbers of the quarkonium are simply ignored. The NRQCD matrix elements are assumed to be calculable in perturbation theory up to an overall normalization constant that depends on the state $H$. This model implies that the matrix elements scale like $v^{3+D}$, where $v$ is a small parameter and $D$ is the number of covariant derivatives $D$ in the operator $O_{mn}^H$.

NRQCD predicts a much more intricate hierarchy among the matrix elements. The matrix element $\langle O_{mn}^H \rangle$ defined in (6) scales like $v^{3+D_{+}E_{+}2M}$, where $D$ is the number of covariant derivatives $D$ that appear in the operator and $E$ and $M$ are the number of chromoelectric and chromomagnetic transitions that are required for $c\bar{c}$ pairs in the states created by $\psi^\dagger K_m \chi$ and $\psi^\dagger K_n \chi$ to reach the dominant Fock state of $H$. These velocity-scaling rules determine the approximate magnitudes of NRQCD matrix elements. By keeping only those matrix elements that scale with the fewest powers of $v$, we can reduce their number sufficiently that a phenomenological approach becomes tractable. One should keep in mind, however, that the importance of a term in the cross section is determined not only by the magnitude of the matrix element but also by the magnitude of its short-distance coefficient.

### 4.2. Reducing the matrix elements

The matrix elements can be further simplified by using symmetries of NRQCD. To illustrate the simplifications, we will use matrix elements of the $J/\psi(\lambda)$, where the polarization state is specified by the helicity $\lambda$.

#### 4.2.1. Rotational symmetry
Rotational symmetry is an exact symmetry of NRQCD. It implies, for example, that the matrix element for $H = \psi(\lambda)$ in (17) must be a linear combination of the tensors $\delta_{ij}$, $U_{\lambda j}U_{i\lambda}^\dagger$, and $U_{\lambda j}U_{i\lambda}^\dagger$, where $U_{i\lambda}$ is the unitary matrix that transforms vectors from the spherical basis to the Cartesian basis. If that matrix element is summed over the polarizations of the $\psi$, the only possible tensor is $\delta_{ij}$. The matrix element must therefore satisfy

$$\sum_\lambda \langle \chi^I \sigma^j T^a \psi \mathcal{P}_{\psi(\lambda)} \psi^I \sigma^i T^a \chi \rangle = \frac{3}{3} \langle \chi^I \sigma^j T^a \psi \mathcal{P}_{\psi} \psi^I \sigma^i T^a \chi \rangle. \quad (18)$$

4.2.2. Heavy-quark spin symmetry

Heavy-quark spin symmetry is an approximate symmetry of NRQCD that holds up to corrections of order $v^2$. The symmetry follows from the fact that the spin of the heavy quark is conserved at leading order in $v^2$ in NRQCD. It implies, for example, that the matrix element for $H = \psi(\lambda)$ in (17) must be proportional to $U_{\lambda j}U_{i\lambda}^\dagger$:

$$\langle \chi^I \sigma^j T^a \psi \mathcal{P}_{\psi(\lambda)} \psi^I \sigma^i T^a \chi \rangle \approx \frac{U_{\lambda j}U_{i\lambda}^\dagger}{3} \langle \chi^I \sigma^j T^a \psi \mathcal{P}_{\psi} \psi^I \sigma^i T^a \chi \rangle. \quad (19)$$

Summing over helicities, we recover (18).

4.2.3. Vacuum-saturation approximation

The vacuum-saturation approximation can only be applied to specific color-singlet matrix elements. If the matrix element is expressed in the form (5), the operators $\psi^I K_m \chi$ and $\psi^I K_n \chi$ must create pointlike $c\bar{c}$ pairs in the dominant Fock state of the quarkonium. In the vacuum-saturation approximation, the projection operator $P_H$ defined in (6) is replaced by $|H\rangle\langle H|$, which corresponds to keeping only the vacuum term in the sum over soft states $X_S$. This is a controlled approximation in NRQCD, holding up to corrections that are of order $v^4$. The vacuum saturation approximation can be illustrated by the following matrix element for $J/\psi$ production:

$$\langle 0 | \chi^I \sigma^j \psi \mathcal{P}_{\psi(\lambda)} \psi^I \sigma^i \chi | 0 \rangle \approx \langle 0 | \chi^I \sigma^j \psi \psi^I \sigma^i \chi | 0 \rangle \langle \psi(\lambda) | \psi^I \sigma^i \chi | 0 \rangle. \quad (20)$$

The vacuum-saturation approximation can also be used for decay matrix elements:

$$\langle \psi | \chi^I \sigma^j \psi \cdot \psi^I \sigma^i \chi | \psi \rangle \approx \sum_\lambda \langle \psi(\lambda) | \psi^I \sigma^i \chi | 0 \rangle^2. \quad (21)$$

The vacuum-to-quarkonium matrix element $\langle \psi | \psi^I \sigma \chi | 0 \rangle$ that appears on the right sides of (20) and (21) is proportional to the wavefunction at the origin. This matrix element can be easily calculated using lattice simulations of NRQCD. Thus the vacuum-saturation approximation provides a way to calculate certain production matrix elements on the lattice.

5. Polarization Predictions
The NRQCD factorization formulas apply equally well to any quarkonium state $H$, including a polarized state. Since the short-distance coefficients are independent of $H$, the dependence on $H$ enters only through the NRQCD matrix elements. In particular, the dependence on the polarization comes only from the matrix elements. In many cases, it is completely determined by the symmetries of NRQCD.

The color evaporation model predicts that quarkonium states are always produced unpolarized. Both the color-singlet model and the NRQCD factorization approach give nontrivial predictions for polarized quarkonium. There have been many calculations of polarization effects in the color-singlet model. In most cases, the resulting predictions are not particularly dramatic. Below, we discuss several examples for which the calculations have been extended to include color-octet mechanisms predicted by the NRQCD factorization formalism. In one case, we find a very dramatic prediction.

5.1. Spin alignment at the Tevatron

The NRQCD factorization framework has led to a dramatic change in our understanding of the production of charmonium at large transverse momentum in $p\bar{p}$ collisions. As pointed out by Braaten and Yuan in 1993, the cross section for $p\bar{p} \rightarrow \psi + X$ at sufficiently large transverse momentum $p_T$ is dominated by gluon fragmentation. It can be factored into the cross section for producing a gluon with large transverse momentum and a fragmentation function:

$$d\sigma(p\bar{p} \rightarrow \psi(P) + X) = \int_0^1 dz \, d\hat{\sigma}(p\bar{p} \rightarrow g(P/z) + X) \, D_{g\rightarrow\psi}(z).$$

The fragmentation function $D_{g\rightarrow\psi}(z)$ gives the probability that the jet initiated by the gluon includes a $\psi$ carrying a fraction $z$ of the gluon momentum. Using the NRQCD factorization approach, the fragmentation function can be expressed in the form

$$D_{g\rightarrow\psi}(z) = \sum_{mn} d_{mn}(z) \langle O^H_{mn} \rangle.$$ 

The matrix element that is leading order in $v$ is $|\langle \psi|\psi^{\dagger}\sigma^{\chi}|0\rangle|^2$, which scales like $v^3$. It has a short-distance coefficient of order $\alpha_s^3$. Using this term in the fragmentation function (23), the cross section predicted by (22) is about a factor of 30 below recent data on prompt $\psi$ production at the Tevatron from the CDF detector.

In 1995, Braaten and Fleming suggested that the gluon fragmentation function for the $\psi$ might actually be dominated by a term that represents a color-octet production mechanism. The matrix element is $\langle \chi^{\dagger}\sigma^{k}T^{a}\psi \, P_{\psi} \, \psi^{\dagger}\sigma^{k}T^{a}\chi \rangle$, which is of order $v^7$ and measures the probability of producing a $\psi$ from a pointlike $c\bar{c}$ pair in a color-singlet $^3S_1$ state. The reason this matrix element might be important is that its short-distance coefficient is of order $\alpha_s$. The enhancement from the two fewer powers of $\alpha_s$ can overcome the suppression by $v^4$. The leading-order expression for
this term in the fragmentation function is

\[ D_{g\to\psi}(z) = \frac{\pi\alpha_s}{96m_c^2} \delta(1-z) \langle \chi^\dagger \sigma^k T^a \psi \ P_\psi \ \psi^\dagger \sigma^k T^a \chi \rangle. \]  

(24)

The \( p_T \)-dependence predicted by this mechanism is in agreement with the CDF data. The normalization depends on the unknown matrix element in (24). The value of the matrix element required to fit the CDF data is consistent with suppression by a factor of \( v^4 \) relative to the corresponding color-singlet matrix element \(|\langle \psi | \psi^\dagger \sigma \chi |0 \rangle|^2\).

Cho and Wise\(^9\) pointed out in 1995 that this production mechanism has dramatic implications for the polarization of the \( \psi \). At leading order in \( \alpha_s \), the \( \psi \)'s produced by gluon fragmentation will be 100% transversely polarized. The radiative corrections were examined by Beneke and Rothstein\(^10\), who concluded that the spin alignment at large \( p_T \) will remain greater than 90%. The dominant corrections to the spin alignment at the values of \( p_T \) measured at the Tevatron come from nonfragmentation contributions to the cross section\(^11\). At large \( p_T \), they fall like \( 1/p_T^2 \) relative to gluon fragmentation. Unfortunately, the calculations required to obtain the prediction for the spin alignment as a function of \( p_T \) have not yet been carried out.

The predictions for the spin alignment of the \( \psi' \) are identical to those for the \( \psi \). The spin alignment for the \( \psi' \) is easier to measure, because, in the case of the \( \psi \), one has to take into account the effects of the radiative decays \( \chi_{cJ} \to \psi \gamma \). It should be possible to make at least a crude measurement of the spin alignment of the \( \psi' \) from existing CDF data.

5.2. Spin alignment in \( Z^0 \) decay

The color-singlet model predicts that the cross sections for producing charmonium in \( Z^0 \) decay are too small to be observed at LEP. As pointed out by Cheung, Keung, and Yuan and by Cho\(^12\), the dominant contribution comes instead from a color-octet production mechanism involving the same matrix element that appears in the fragmentation function (24). Using the value of this matrix element obtained by fitting the CDF data, they found that the rate is almost an order of magnitude larger than predicted by the color-singlet model and thus large enough to be observed. The spin alignment of the \( \psi \) in \( Z^0 \) decay was also calculated by Cheung, Keung, and Yuan. Unfortunately, the alignment is predicted to be small and is completely unobservable in the present data sample from LEP.

5.3. Spin alignment in \( B \) decay

In the production of \( \psi \) from \( B \) decay, color-singlet contributions are suppressed by a near cancellation between Wilson coefficients in the effective weak hamiltonian. Color-octet production mechanisms are therefore important, in spite of the \( v^4 \) suppression of the matrix element\(^13\). The spin alignment of the \( \psi \)'s that are produced in the decay \( B \to \psi + X \) were recently calculated by Fleming et al.\(^14\). It depends sensitively on the values of 3 independent color-octet matrix elements, including
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the one that appears in [24]. A measurement of the spin alignment would therefore place strong constraints on these matrix elements.

6. Conclusions

The factorization formulas (1) and (2) provide a model-independent framework for analyzing heavy quarkonium production and annihilation rates. All the short-distance factors can be calculated systematically using the threshold expansion method. The long-distance factors are defined in terms of NRQCD matrix elements. The decay matrix elements can be computed using lattice simulations of NRQCD, but most of the production matrix elements must be treated as phenomenological parameters. The relative magnitudes of the matrix elements are predicted by the velocity-scaling rules of NRQCD. These magnitudes have a pattern that is very different from that assumed in the color-singlet model or in the color-evaporation model.

The NRQCD factorization approach gives unambiguous predictions for the polarization of heavy quarkonium states. These predictions are inescapable consequences of this framework. The polarization predictions from NRQCD factorization can be dramatic. An example is the spin alignment of prompt $\psi$ and $\psi'$ at the Tevatron, which is predicted to be greater than 90\% at the largest values of $p_T$.

An experimental measurement of this spin alignment would be a crucial test of the NRQCD factorization framework.

Acknowledgements

This work was supported in part by the United States Department of Energy, Division of High Energy Physics, under grant DE-FG02-91-ER40690.

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