Pulse compression favorable frequency modulated thermal wave imaging for non-destructive testing and evaluation: an analytical study

Anju Rani and Ravibabu Mulaveesala
Department of Electrical Engineering, Indian Institute of Technology Ropar, Indian Institute of Technology Ropar, Bara Phool, Birla Seed Farms, Rupnagar, Punjab, 140001, India
E-mail: ravibabucareiitd@yahoo.co.in

Keywords: non-destructive testing, frequency modulated thermal wave imaging, pulse compression, correlation coefficient

Abstract
InfraRed Thermography (IRT) is one of the non-destructive testing and evaluation (NDT&E) approach widely used for testing and evaluation of wide variety of materials such as metals, semiconductors and composites. Among the widely used Thermal NDT&E (TNDT&E) approaches for better depth resolution and sensitivity for detection of defects located at different depths inside the test specimen recently proposed correlation based approach gained importance due to its enhanced defect detection capabilities. The present paper introduces a novel one-dimensional analytical solution for the frequency modulated excitation scheme under adiabatic boundary conditions for detection of flat bottom holes as defects in a mild steel sample. The performance of the Pulse Thermography (PT), Lock-in Thermography (LT) and Frequency Modulated Thermal Wave Imaging (FMTWI) methods are highlighted their defect detection capabilities have been compared by adopting the recently introduced correlation based post-processing approach. Finally, the proposed analytical method has been validated with the results obtained from the commercially available finite element based software.

1. Introduction

Active infrared thermography (IRT) is an efficient non-destructive testing and evaluation (NDT&E) technique widely used for the inspection of large areas in a short span of time. Active thermography allows heating the sample surface with an external stimulus in order to generate thermal waves inside the test sample. The thermal waves being dispersive in nature diffuse through the sample thickness and interact with the defects present inside the sample due to change in thermal diffusivity, thus generating the change in surface temperature [1, 2]. The surface temperature exhibits time delays depending upon the frequency spectrum (bandwidth) of the input excitation and defect depths present inside the test sample. Popular active IRT techniques such as pulse thermography (PT), pulse phase thermography (PPT) and lock-in thermography (LT) has been used widely from the last three decades for the detection of defect depths, thickness, thermal diffusivity and effusivity measurements [3–8]. PT allows heating of the sample surface with a high peak power heat source for a short duration to record temperature evolution. But the non-uniform surface heating, requirement of high peak power and non-uniform emissivity limits its applicability. PPT is analogous to PT except being a phase analysis approach it is less sensitive to the non-uniform heating and non-uniform emissivity but still require a very high peak power heat sources. LT on the other hand uses a periodically modulated heat sources to heat the sample surface. But due to mono-frequency excitation, the LT approach requires repetitive experimentation for analyzing each defect depth thus making it a time consuming method. Therefore in order to eradicate the limitations associated with the discussed methods, frequency modulated thermal wave imaging (FMTWI) was introduced [9, 10]. FMTWI is a non-periodic thermal wave imaging technique which sweeps the entire range of frequencies in a single experimentation cycle using moderate peak power heat sources.
The present paper proposes a novel analytical solution for defect depth estimation by solving the heat transfer equation under adiabatic boundary conditions with no heat source and sink inside the test specimen for a frequency modulated input excitation in a mild steel sample. The surface temperature calculated for different depth locations are then post-processed using correlation based pulse compression favorable data processing approach for better depth resolvability [10, 11]. Further, the temporal data obtained from the analytical solution for frequency modulated thermal wave imaging; lock-in thermography and pulse thermography are compared to know defect detection capability of the proposed methods. Finally, the proposed solution was validated with the results obtained from the commercially available FEM software for the same mild steel sample.

The work presented in this paper is organized as follows: section 2 introduces the proposed one-dimensional analytical solution for defect detection for the FMTWI input heat flux. Section 3 discusses the post-processing techniques used for the depth prediction followed by simulation studies explained in section 4. The comparative analysis of the proposed FMTWI technique with respect to pulsed thermography and lock-in thermography techniques was carried out in section 5. Finally, the present work was concluded in section 6.

2. Theory

In FMTWI, a frequency modulated heat flux is used to excite the sample surface based on governing heat diffusion equation given by [6]:

\[
\frac{\partial T(x, t)}{\partial t} - \alpha \frac{\partial^2 T(x, t)}{\partial x^2} = 0
\]  

(1)

where \( T(x, t) \) - temperature response obtained using equation (1) at spatial location \( x \) at time \( t \) under given boundary conditions and \( \alpha \) - thermal diffusivity.

The set of boundary condition and initial conditions corresponding to insulated surfaces at \( x = L \) is given in the following equations [10]:

\[-K \left. \frac{\partial T(x, t)}{\partial x} \right|_{x=0} = Q(t), \quad 0 \leq x \leq L, \quad t > 0 \quad (2.a)\]

\[-K \left. \frac{\partial T(x, t)}{\partial x} \right|_{x=L} = 0, \quad 0 \leq x \leq L, \quad t > 0 \quad (2.b)\]

\[T(x, t)|_{t=0} = T_0, \quad t = 0 \quad (2.c)\]

where \( K \) is thermal conductivity, \( L \) is total thickness of the sample, \( T_0 \) is the initial sample temperature and \( Q(t) \) defines the frequency modulated input heat flux given by:

\[Q(x, t)|_{x=0} = Q_0 \left[ 1 + \sin \left( \frac{2\pi}{f_i T} + \frac{B t^2}{2T} \right) \right] \]

(3)

where \( Q_0 \) - amplitude of the modulating signal, \( f_i \) - initial frequency, \( B \) - bandwidth, \( T \) - total duration of the frequency modulated input heat flux.

The inhomogeneous boundary conditions (BC) are reduced to homogeneous BC in the form of:

\[T(x, t) = T_1(x, t) + T_2(x, t) \]

(4)

such that \( T_1(x, t) \) and \( T_2(x, t) \) represents the homogenous and non-homogeneous part of the heat diffusion equation which satisfies the boundary conditions (2.a)–(2.c) given in the form of:

\[-\left. \frac{\partial T_1(x, t)}{\partial x} \right|_{x=0} = 0 \quad (5.a)\]

\[\left. \frac{\partial T_1(x, t)}{\partial x} \right|_{x=L} = 0 \quad (5.b)\]

\[-\left. \frac{\partial T_1(x, t)}{\partial x} \right|_{x=L} = 0 \quad (5.c)\]

\[T_1(x, t)|_{t=0} = T_0 \quad (5.d)\]

and

\[-\left. \frac{\partial T_2(x, t)}{\partial x} \right|_{x=0} = 0 \quad (6.a)\]

\[\left. \frac{\partial T_2(x, t)}{\partial x} \right|_{x=L} = 0 \quad (6.b)\]

\[\left. \frac{\partial T_2(x, t)}{\partial x} \right|_{x=L} = U(x, t) \]

(6.c)
\[ \frac{\partial T_i(x, t)}{\partial x} \bigg|_{x=0} = 0 \quad (6.c) \]
\[ T_i(x, t) \big|_{x=L} = 0 \quad (6.d) \]

where \( U(x, t) = \frac{Q(t)}{KL} \)

The homogeneous auxiliary equation (5.a) can be solved using Fourier series to obtain general solution in the form of:

\[ T_i(x, t) = \sum_{n=0}^{\infty} \left[ A_n \sin \left( \frac{n\pi x}{L} \right) + B_n \cos \left( \frac{n\pi x}{L} \right) \right] e^{-\frac{\alpha n^2 \pi^2 t}{L^2}} \quad (7.a) \]

where \( A_n \) and \( B_n \) are the integration constants obtained for the \( n \)th eigen value \( \gamma \).

The corresponding eigen value is obtained by applying the given boundary and initial conditions (5.b)–(5.d), we obtain \( \gamma = n\pi/L \) for \( n = 0, 1, 2, \ldots, \infty \) and:

\[ T_i(x, t) = \sum_{n=0}^{\infty} B_n \cos \left( \frac{n\pi x}{L} \right) e^{-\frac{\alpha n^2 \pi^2 t}{L^2}} \quad (7.b) \]

Therefore,

\[ T_i(x, t) = \left[ B_o + \sum_{n=1}^{\infty} B_n \cos \left( \frac{n\pi x}{L} \right) \right] e^{-\frac{\alpha n^2 \pi^2 t}{L^2}} \quad (7.c) \]

where,

\[ B_o = \frac{1}{L} \int_0^L T_0 dx, \]

and

\[ B_n = \frac{2}{L} \int_0^L T_0 \cos \left( \frac{n\pi x}{L} \right) dx \quad (7.d) \]

Similarly, the non-homogeneous equation (6.a) can be solved using Duhamel’s theorem:

\[ T_2(x, t) = \sum_{n=0}^{\infty} \cos \left( \frac{n\pi x}{L} \right) \left[ \int_0^t C_n(\tau) e^{-\frac{\alpha n^2 \pi^2 (t-\tau)}{L^2}} d\tau \right] \quad (8.a) \]

where \( C_n \) is the integration constant for \( n \)th eigen value.

\[ T_2(x, t) = \int_0^t C_0(\tau) + \sum_{n=1}^{\infty} \cos \left( \frac{n\pi x}{L} \right) \left[ \int_0^t C_n(\tau) e^{-\frac{\alpha n^2 \pi^2 (t-\tau)}{L^2}} d\tau \right] \quad (8.b) \]

where

\[ C_0(\tau) = \frac{1}{L} \int_{x=0}^L U(x, \tau) dx \]

and

\[ C_n(\tau) = \frac{2}{L} \int_{x=0}^L U(x, \tau) \cos \left( \frac{n\pi x}{L} \right) dx \quad (8.c) \]

Considering, the frequency modulated thermal wave input heat flux \( Q(t) \) given by equation (3) is applied on the surface of the sample \( x = 0 \) with insulation on the other end \( x = L \) discussed in equations (2.a)–(2.c), the thermal response at the surface \( T(0, t) \) can be calculated in the form of:

\[ T(0, t) = T_0 + \frac{Q_o}{2KL} \left( \frac{4}{\lambda c} (1 - e^{-\alpha \lambda t}) + 2kt + (-1)^{1/4}e^{-\alpha \lambda t} \right) \]

\[ \left\{ e^{\frac{i\pi \xi}{\pi}} (\text{Erf} [\xi_1] - \text{Erf} [\xi_2]) + e^{\frac{i\pi \eta}{\eta}} (\text{Erf} [\eta_1] - \text{Erf} [\eta_2]) \right\} \]

\[ + \sqrt{2} \left\{ \cos \theta [\text{FresnelS} \lambda_1] + \text{FresnelC} \lambda_1 \right\} \]

\[ + \sqrt{2} \left\{ \sin \theta [\text{FresnelC} \lambda_1] - \text{FresnelS} \lambda_1 \right\} \quad (9) \]

Following constants were calculated while incorporating the solution:

\[ \lambda_1 = \lambda \sqrt{\frac{2T}{B}} \quad (10) \]
Figure 1. (a) Schematic diagram of mild steel sample with six flat bottom holes as defects of 10 mm diameter each (all dimensions in mm), and (b) illustrates the meshing used in FEM software for model the mild steel sample.

\[ \chi_2 = (Bt + Tf_j) \sqrt{\frac{2}{BT}} \]  \hspace{2cm} (11)

\[ \delta_1 = (\imath \alpha \lambda^2 - 2\pi f_j) \sqrt{\frac{T}{4B\pi}} \]  \hspace{2cm} (12)

\[ \delta_2 = (\imath \alpha T \lambda^2 - 2\pi (Bt + Tf_j)) \frac{1}{\sqrt{4TB\pi}} \]  \hspace{2cm} (13)

\[ \gamma_1 = (\imath \alpha \lambda^2 + 2\pi f_j) \sqrt{\frac{T}{4B\pi}} \]  \hspace{2cm} (14)

\[ \gamma_2 = (\imath \alpha T \lambda^2 + 2\pi (Bt + Tf_j)) \frac{1}{\sqrt{4TB\pi}} \]  \hspace{2cm} (15)

\[ \eta = \frac{\pi Tf_j^2}{B} \]  \hspace{2cm} (16)
The defects lying beneath the surface of the sample alters the heat transfer which in turn affects the surface temperature. The temperature responses obtained from equation (9) over the sample surface observes a mean rise in temperature magnitude and is a function of defect depth and the applied input heat flux. The presented analytical solution was carried out for other popular thermographic techniques such as pulsed thermography and lock-in thermography and later compared after applying post-processing techniques for defect depth.

\[
\vartheta = \frac{\alpha}{\sqrt{\beta}}
\]

(17)

and

\[
\varsigma = f_i^2 - \frac{\alpha^2 \lambda^4}{4\pi^2}
\]

(18)

**Figure 2.** Thermal response obtained for a frequency modulated excitation over the test sample having flat bottom hole defects using analytical solution: (a) incident heat flux used for frequency modulated thermography, (b) Temperature response obtained for incident frequency modulated heat flux, (c) Mean removed temperature response, and (d) Normalized cross-correlation coefficients.
estimation. The post-processing techniques were applied to obtain zero mean thermal responses for the recorded signals by removing the mean rise using a first order polynomial fitting curve.

3. Correlation based pulse compression techniques

The pulse compression (PC) technique is one of the popular techniques used in radar applications for better sensitivity and enhanced range resolution even in the presence of random noise thus improving the signal to noise ratio. In the present consist to infrared thermography the correlation based pulse compression technique is executed by cross correlating (CC) the mean removed thermal responses obtained over the defect regions with

![Diagram](image-url)
that of the mean removed thermal response of the reference or sound region given as below \cite{12, 13}:

\[
CC(\tau) = \int_{-\infty}^{+\infty} T_d(t) \cdot T_i(\tau + t) dt
\]

(19)

where \(T_d(t)\) and \(T_i(t)\) are the thermal responses obtained over the defective regions and reference thermal response respectively.

The CC technique concentrates the entire supplied thermal energy into a \textit{sinc} shaped compressed pulse and its peak value and time shift depends on the finite attenuation and delay depending on the depth of the defect inside the material. The obtained correlation coefficient of compressed pulses contributes towards the contrast in the correlation image at any time instant, thus distinguishing the defect locations from that of the sound regions \cite{14, 15}.
4. Numerical modeling

The finite element modeling (FEM) and analysis was carried out using commercially available FEM software to compute the thermal response to the frequency modulated excitation for a mild-steel sample containing six flat bottom hole defects at different depths. The layout of the mild steel sample having dimensions $125 \times 105 \times 9.74$ (all dimensions in mm) is shown in figure 1(a). The implicit backward differentiation formula (BDF) under fine meshing was used with 1000 time steps for duration of 100 s and the thermal response was recorded for the FMTWI technique as depicted in figure 1(b).

A similar analysis was carried out for lock-in and pulse thermographic techniques and compared with the proposed FMTWI technique for defect estimation. The mild steel sample was configured by heating the sample surface with 1 kW heat flux for duration of 100 s. The variation in the surface temperature is observed during the active heating at the front surface of the sample at a frame rate of 20 Hz during 100 s of the experimentation.

5. Results and discussions

In order to validate the detectability of the proposed analytical method, a finite element modeling and simulation study has been carried out over a mild steel sample of thickness 9.74 mm. The sample contains six flat bottom hole defects of 10 mm diameter located at various depths as shown in figure 1. The front surface of mild steel sample is illuminated to different excitation schemes (FMTWI, LT and PT) under adiabatic boundary conditions. Figure 2(a) depicts frequency modulated heat flux incident over mild steel sample with a 2 kW peak power heat source with a frequency sweep of 0.01 to 0.1 Hz for 100 s. The temperature response has been recorded over the sample surface using equation (9) for all the defect depths as shown in figure 2(b) for 100 s duration. Figure 2(b) shows that shallow defects have high temperature in comparison to the deeper defects.

The mean rise in transient temperature response is removed using first order polynomial fitting from the obtained data. This process is repeated for all the defects located at various depths inside the sample to obtain mean removed temperature response as depicted in figure 2(c) which is further post-processed using correlation based data processing approach. Figure 2(d) represents depth scanning performance using correlation coefficients. Correlation based pulse compression approach is performed by cross-correlating the zero mean temperature response obtained for defected regions with zero mean temperature response for a chosen non-defective sound region given by equation (19).

The same process is repeated for lock-in thermography, where a sinusoidal heat flux with 2 kW peak power with modulating frequency of 0.055 Hz for duration of 100 s is illuminated over mild steel sample shown in figure 3(a). The temperature response is obtained at the sample surface under adiabatic boundary conditions using equation (9) to obtain figure 3(b). The mean removed temperature response is as shown in figure 3(c) is used for time domain correlation based data processing approach. The correlation profile obtained for the LT approach is shown in figure 3(d). The correlation approach in LT fails to provide advantage of signal compression therefore it does not help in any improvement in the depth resolution.

Lastly, pulse thermography technique was implemented for pulse shaped incident heat flux of 5 kW peak power for duration of 20 s shown in figure 4(a). The thermal response for defect depths at sample surface is depicted in figure 4(b). Further, correlation coefficient values obtained from cross-correlating thermal response of defect depths with sound response on mean removed thermal response is shown in figures 4(c) and (d) respectively. The compressed pulse obtained in PT shows distribution of almost equal energy in main lobe and side lobe therefore diminishes the advantage of pulse compression approach. Finally, it can be concluded that FMTWI technique along with correlation based pulse compression approach shows better performance for

| Table 1. Error Analysis for analytical and simulated models For FMTWI, LT and PT. |
|---------------------------------------------------------------|
| Defect Depth (mm)    | Normalized Correlation Coefficient Error |
|----------------------|------------------------------------------|
|                       | FMTWI | LT    | PT    |
| 1.20                 | 0     | 0     | 0     |
| 1.37                 | 0.0002 | 0.0006 | 0.001 |
| 1.97                 | 0.0016 | 0.0019 | 0.0019 |
| 2.13                 | 0.0018 | 0.0021 | 0.0028 |
| 2.32                 | 0.0019 | 0.0022 | 0.0032 |
| 3.42                 | 0.0021 | 0.0033 | 0.0047 |
Figure 5. Comparison of obtained temperature responses for analytical and simulated results for FMTWI for defects located at different depths: (a) 1.2 mm, (b) 1.37 mm, (c) 1.97 mm, (d) 2.13 mm, (e) 2.32 mm, and (f) 3.42 mm.
Figure 6. Comparison of obtained correlation responses for analytical and simulated results for FMTWI for defects located at different depths: (a) 1.2 mm, (b) 1.37 mm, (c) 1.97 mm, (d) 2.13 mm, (e) 2.32 mm, and (f) 3.42 mm.
detecting defects in comparison to pulse based and mono-frequency lock-in thermograhic technique as shown in table 1.

The performance of proposed defect detection approach is analyzed by comparing the analytical model as shown in figures 6 and 7 with simulated FEM model for the same mild steel sample shown in figure 1. All the simulation were carried out under heat transfer module where input heat flux is illuminated over the sample surface while the opposite end is kept insulated keeping the sample at room temperature.

The time dependent study is carried under implicit backward differentiation formula (BDF) solver with strict time stepping to record temperature response over the sample surface. The temperature responses for different defect depths (a)–(f) are compared in the analytical and simulated model. Figure 5 shows high similarity between the analytical and simulated models for six defect depths (a)–(f) ranging from 1.2 mm–3.42 mm respectively.

6. Conclusion

The defect detection capability of proposed analytical model under adiabatic boundary conditions has been presented for frequency modulated thermal wave imaging technique (FMTWI) using correlation based pulse compression data processing approach for mild steel sample. Further, analytical model has been extended to other thermographic techniques; pulse thermography (PT) and lock-in thermography (LT) to evaluate defect detection capability using correlation based approach. The thermal response obtained over the sample from analytical model is compared with finite element simulated model (FEM). Results shown high similarity in the thermal response obtained for different defect depths between the analytical and simulated model. Also,
FMTWI technique along with correlation based post processing approach exhibits enhanced sensitivity and resolution for defect detection by concentrating maximum supplied energy in its main lobe. However, correlation approach is not up to the mark to provide similar depth scanning capabilities for LT and PT as these techniques fails to concentrate the energy as in case of FMTWI.

Data availability statement

No new data were created or analysed in this study.

ORCID iDs

Ravibabu Mulaveesala https://orcid.org/0000-0001-7351-0982

References

[1] Maldague X P V 2001 Theory and Practice of Infrared Technology for Nondestructive Testing’ (New York: Wiley)
[2] Vavilov V P 2017 Thermal nondestructive testing of materials and products: a review Russ. J. Nondestr. Test. 53 707–30
[3] Busse G 1979 Optoacoustic phase angle measurement for probing a metal Appl. Phys. Lett. 35 759–60
[4] Busse G, Wu D and Karpen W 1992 Thermal wave imaging with phase sensitive modulated thermography J. Appl. Phys. 71 3962–5
[5] Maldague X and Sergio M 1996 Pulse phase infrared thermography J. Appl. Phys. 79 2694–8
[6] Mulaveesala R and Tuli S 2006 Theory of frequency modulated thermal wave imaging for nondestructive subsurface defect detection Appl. Phys. Lett. 89 191913
[7] Sakagami T and Kubo S 2002 Applications of pulse heating thermography and lock-in thermography to quantitative nondestructive evaluations Infrared Phys. Technol. 43 211–8
[8] Wu D and Busse G 1998 Lock-in thermography for nondestructive evaluation of materials Revue générale de Thermique 37 693–703
[9] Ghali V S and Mulaveesala R 2010 Frequency modulated thermal wave imaging techniques for non-destructive testing Insight: Non-Destructive Testing & Condition Monitoring 52 475–80
[10] Rani A and Mulaveesala R 2020 Depth resolved pulse compression favourable frequency modulated thermal wave imaging for quantitative characterization of glass fibre reinforced polymer Infrared Phys. Technol. 110 103441
[11] Almond D P and Lau S K 1994 Defect sizing by transient thermography I: an analytical treatment J. Phys. D: Appl. Phys. 27 1063–9
[12] Arora V, Siddiqui J A, Mulaveesala R and Muniyappa A 2014 Hilbert transform-based pulse compression approach to infrared thermal wave imaging for sub-surface defect detection in steel material Insight: Non-Destructive Testing and Condition Monitoring 56 350–2
[13] Laureti S, Starra S, Malekohammadi H, Burrascano P, Hutchins D, Senni L, Silipigni G, Maldague X and Ricci M 2018 The use of pulse-compression thermography for detecting defects in paintings NDT & E International 98 147–54
[14] Rani A and Mulaveesala R 2020 Investigations on pulse compression favourable thermal imaging approaches for characterization of glass fibre reinforce polymers Electron. Lett. 56 995–8
[15] Mulaveesala R, Arora V, Dua G, Rani A, Kher V, Sharma A and Kaur K 2020 Pulse compression favorable thermal wave imaging methods for testing and evaluation of carbon fibre reinforced polymer Proc. SPIE 11409 114090S