An integrated approach based on two-objective optimization and 2SFCA model for health-care facility location-allocation problems

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Abstract. Public health-care facilities are essential to all communities, and their location has long been an important issue in urban planning. With the steady growth of Shenzhen’s population, the government needs to decide where health-care facilities should be located to improve the equity of accessibility, and further more to raise the residential needs and satisfaction for the entire population. As a large immigrate city, Shenzhen is a complex socio-ecological system which make it hard to find the best solution that meets all of the objectives. In this article, we establish a multi-objective model to examine the problem of where health-care facilities should be located to improve accessibility for the entire population and decrease the cost of building new facilities. In particular, the model accounts for the different requirements of varying age groups. We used data from Shenzhen and two heuristic algorithms to create the model. The strategy proposed here can be adopted in urban planning and help policymakers reach better decisions.

1. Introduction
Community-based health services are recognized as the most important basic public service facilities and the most effective interventions for a large part of NCDs (chronic and non-communicable diseases) [1]. In cities with high population density such as Shenzhen, health-care resources are unevenly distributed and in short supply, which may negatively affect public health, especially during an epidemic outbreak; the rational allocation of community health-care facilities is thus an urgent problem to be solved in these contexts. Most existing allocation models are static and do not consider facility accessibility or urban residents behavioral characteristics [2]. Moreover, these models only recommend potential locations for building facilities, ignoring potential priorities; this results in low practicability. So in this article, we establish a multi-objective model to examine the problem of where health-care facilities should be located to improve accessibility for the entire population and decrease the cost of building new facilities.

This paper takes Shenzhen, one of China’s rapidly developed cities, as the study area. Cities like Shenzhen with high population density and limited health-care resources require not only accessibility in health-care facilities but also the cost of building new health-care facility, moreover, people with different acceptable travel distance is also an important factor which should be considered in the location-allocation model. Thus, we select four objectives in Shenzhen’s community-based health facility location-allocation problem: (1) minimize inequity of accessibility; (2) maximize accessibility for the whole city; (3) minimize the total costs of locating a new community-based health facility, the
total costs including the fixed cost of building a new facility and the travelling cost from each allocated demand point to the facility; (4) maximize the total numbers of persons who can receive services within the specified maximum distances and the capacity constraints of the facility.

From the above objectives, we can find that the objective of accessibility and equity can be increased by adding more health-care facilities. In this paper, we first use the 2SFCA (two-step floating catchment area) model to calculate the accessibility of each demand point, and then we propose a two-objective location-allocation model that both service and cost aspects are considered, the two-objective problem is considered as a NP-hard problem, we use two algorithms based on greedy algorithm to solve it[4], finally, we combine the results of accessibility and two-objective optimization to find the location and urgency of new health-care facilities.

This paper is organized as follows. Section 2 introduces the background of Shenzhen including its population and the data used in this paper. Section 3 describes the 2SFCA model for calculating accessibility of each demand point and the two-objective location-allocation model for community-based health facility in Shenzhen. Section 4 describes the solving algorithm in detail and discuss the computational results. Section 5 concludes this paper.

2. Background of Shenzhen and data source

Shenzhen is a special economic region of the People’s Republic of China, located in the southern part of the country. According to Shenzhen statistics bureau's annual statistical yearbook of 2018, the total population of Shenzhen is about 12.5283 million, and the data is still growing, such a large population brings great challenges to the public service facilities in the city. Figure 1 shows the population density of Shenzhen. Shenzhen’s topography is mountainous, thus the population distribution is heterogeneous, so the health-care facilities are required to provide health-care services to the large and heterogeneously distribution residents.

The data of community-based health-care facilities were obtained from the website of Shenzhen municipal health commission (http://wjw.sz.gov.cn/), we use Baidu API to find the latitude and longitude of each facility, and the spatial distribution of Shenzhen’s community-based health-care facilities are shown in Figure 1.

![Figure 1. the spatial distribution of Shenzhen’s community-based health-care facilities and the population density](image)

3. The problem statement and methodology

3.1. Framework of location-allocation problem

The framework of location-allocation problem in this paper is shown in Figure 2. First, we use the 2SFCA model to calculate accessibility for each demand point. We then propose a multi-objective location-allocation model that considers both service and cost aspects. As mentioned above, the multi-objective problem is considered an NP-hard problem, and we use two algorithms based on the greedy
algorithm to solve it. Finally, we combine the results on accessibility and the multi-objective optimization to identify the locations and levels of urgency for new health-care facilities.

The input data was composed of the land use of Shenzhen, the set of demand point, the existing location of community-based health-care facilities and the set of potential facility sites. Shenzhen’s topography is mountainous, thus there are some regions can not be used as health-care facilities. We divided Shenzhen into 1km*1km grids, and get rid of grids that cannot build facilities, finally, we got 1217 grids that can be the set of potential facility sites, which is shown in Figure 3.

### Data
- The land use
- The existing community-based health-care facilities
- The population
- The set of potential facility sites

### Step-1 Calculating the accessibility of each demand point $I$
- Data cleaning and data standardization
- Two-step floating catchment area model
- Accessibility of each demand point $I$

### Step-2 determining the optimal locations of community-based health-care facilities
- Two-objective optimization model
- Service objective
- Cost objective
- Does the capacity of facility $J$ is full?
  - Remove $j$ from $J$
- Allocate demand to facility $J$
- Calculating the two objectives to find the optimal set of facility

Figure 2. framework of location-allocation problem

Figure 3. the location of potential facility sites

### 3.2. Calculating accessibility

The calculation of the spatial distribution of public resources mainly depends on its spatial accessibility. Spatial accessibility reflects the interactivity of land use and transportation and is used to measure the convenience of people to use various public resources. Access to health-care is a complex concept and is interpreted differently by policy makers, researchers, and the general public. Researches describing access between clients and the system, identify five key dimensions of access: availability, accessibility, accommodation, affordability, and acceptability[5]. The accessibility
describes the geographical location of health services in relation to the location of clients by considering the geographical factors (such as transportation, travel time, distance and cost).

In this paper, spatial accessibility was measured using a two-steps floating catchment areas (2SFCA) method. The 2SFCA model is a special case of gravity models. It is a comprehensive and flexible method to measure the accessibility through the influence of comprehensive facility supply, population demand and the distance between supply and demand points [6].

The 2SFCA model can be divided into the following two steps:

1. Calculate the supply/demand ratio for each facility

\[
R_j = \frac{S_j}{\sum_{i\in[d_{ij}<D_j]} G(d_{ij},D_j) P_i} 
\]

where \( R_j \) is the supply/demand ratio for facility \( j \), \( S_j \) is the capacity of facility \( j \), \( P_i \) is the population of demand point \( i \), \( d_{ij} \) is the walking distance between facility \( j \) and demand point \( i \), \( D_j \) is the maximum travel distance.

\[
G(d_{ij},D_j) = \begin{cases} 
    e^{-\frac{1}{2} \frac{d_{ij}^2}{D_j^2}}, & d_{ij} \leq D_j \\
    0, & d_{ij} > D_j 
\end{cases} 
\]

\( G(d_{ij},D_j) \) is a Gaussian function, it is a continuous function, which has the advantages of small attenuation rate in the near and far region and large attenuation rate in the middle region.

2. Calculate the accessibility of each demand point

\[
A_i = \sum_{j \in [d_{ij} \leq D_j]} G(d_{ij},D_j) \cdot R_j 
\]

3.3. Two-objective optimization model

Location problems consist of clients and a set of potential sites where facilities can be located. Location problems are classified into four categories based on objective function criteria: facility location problems, \( p \)-median problems, \( p \)-center problems, covering problems. The objective of facility location problems is to find a place to locate a facility in order to minimize the total setup cost and travel cost; the objective of \( p \)-median problem is to determine the locations of \( p \) facilities and their assigned demand point in order to minimize the total cost; \( p \)-center problems’ objective is to minimize the maximum distance between facility and demand point; covering problems are to find the minimum number of facilities to cover all demand points[7]. Previous studies have showed that \( p \)-median model and maximal covering model with single objective are used by the most. In this paper, we undertake to use multi-objective location-allocation with cost and service objectives.

The two-objective optimization model is defined below.

\[
\begin{align*}
\min & \sum_j f_j L_j + \sum_{ij} d_{ij} C_j \\
\max & \sum_i \left( \theta^{f}_{ij} \varphi^f_{ij} + \theta^{n}_{ij} \varphi^n_{ij} \right)
\end{align*} 
\]

Subject to:

\[
\begin{align*}
\varphi^f_{ij} & \leq L_j, \forall i \in I, \forall j \in J \\
\varphi^n_{ij} & \leq L_j, \forall i \in I, \forall j \in J \\
\sum_j \varphi^f_{ij} & = 1, \forall i \in I, \forall j \in J \\
\sum_j \varphi^n_{ij} & = 1, \forall i \in I, \forall j \in J \\
\sum_j \left( P_j^f \varphi^f_{ij} + P_j^n \varphi^n_{ij} \right) & \leq C_j
\end{align*} 
\]

The cost objective function minimizes the total costs of location-allocation problem. The first term describes the fixed cost for new health-care facilities, the second term describes the traveling costs
from demand point $i$ to health-care facility $j$. The service objective maximizes the total number persons who can receive services from the health-care facilities within the specified maximum distances. It’s worth noting that we divide the demand into flexible demand and non-flexible demand according to their age. According to our survey, the elderly and children generally choose community-based health-care facilities within 1km scale, while the scale for other people are generally 2km scale.

The first two constraints ensure that there is an opening facility in location $j$ if any flexible or non-flexible demand is allocated to that location. The third and fourth constraints ensure that each demand point is assigned to some facilities, maybe one or more. The last constraint ensures that the service capacity of each facility is sufficient to handle the allocated demand.

4. Computational results

4.1. Results of accessibility

Through the 2SFCA model described in section 3.2, we calculated the accessibility of community-based health-care facility in Shenzhen. If the result is equal or bigger than 1, it means that the health-care services meets the demand of residents in that region. The Figure 4 shows that the accessibility of residential areas is low and the inequity is high, so the government need to build more health-care facility to meet the demands of local residents.

![Figure 4 the results of accessibility](image)

It can be seen from the ratio of supply and demand that most of the community-based health-care service capacity in Shenzhen is far beyond the range it can afford, and only a few facilities can meet the demand. For this situation, the government can not only build new health-care facilities, but also upgrade the facility’s service capacity by increasing the number of doctors and expanding the scale.

4.2. Location-allocation results

4.2.1. Initialization.

Based on the spatial data of Shenzhen, the number of demand points is 2199, and the number of potential facility sites is 1217. According to Shenzhen community health service establishment plan (2016-2020), variables in the model are set as follow: $D^f = 2km$, $D^n = 1km$, $C_j = 20000$, $f_j = 150000$.

According to the existing community-based health-care facility location and its service capacity, we can calculate the current needs of each demand point which is one of the input data of the model, the result is shown in Figure 5.
4.2.2. Solving algorithms
The hill climbing heuristic is a path-based local search method and is strongly dependent upon the starting positions for the search[8]. For the purposes of this paper, the hill climbing heuristic was applied due to its inherent simplicity and effectiveness. Moreover, it is frequently preferred in comparison with more complex search algorithm such as simulated annealing algorithm to avoid falling into local optimization.

Hill Climbing uses a kind of gradient to guide the direction of search. Each iteration consists in choosing randomly a solution in the neighborhood of the current solution and retains this new solution only if it improves the fitness function.

Simulated annealing consists in given an initial solution iteratively improves it by selecting another neighbor solution changing the services of some facilities and comparing them according a fitness function[9]. Then it usually moves towards the best solution and repeats the process until convergence or for a given amount of time. The algorithm has the particularity of allowing some “bad moves” that consist in going (sometimes) towards the worst solution to avoid getting stacked on local optimums or flat regions.

Table 1 shows the result of hill climbing and simulated annealing, we can see that the result of simulated annealing is greater than hill climbing, so we use the result of simulated annealing as the optimal result.

| Algorithm            | Cost       | Service  | Time     |
|----------------------|------------|----------|----------|
| Hill climbing        | 49839907   | 2634723  | 446.64 s |
| Simulated annealing  | 49045089   | 2638489  | 486.84 s |
The spatial distribution of optimal community health-care facilities is represented in Figure 6. Compared to the population density in Figure 1 and the accessibility in Figure 4, we find that most of the population suffers from poor accessibility in the existing configuration, but after optimization, more people experience better accessibility.

![Figure 6. location-allocation results](image)

The new community-based health-care facilities tend to be located in more heavily populated areas. The figure shows that the government should build more health-care facility in the area with the larger number of people. Locating the new health-care facilities in these places simultaneously increase the equity and improves the total accessibility.

5. Conclusion
Location-allocation problems have played a major role in urban studies and government decision. This paper considers how to locate and allocate community-based health-care facilities over a geographically heterogeneous region. This paper uses an integrated approach based on two-objective optimization and 2SFCA model to find the optimal tradeoffs between the objectives in locating health-care facilities. After reviewing the strengths and weaknesses of the existing health-care facility allocation literature, we developed a model for the optimal allocation of community health-care facilities during a pandemic. In the model, we consider two objectives: (1) minimizing the total costs of a new community-based health facility; (2) maximizing the total service to residents. In particular, the model accounts for the different requirements of varying age groups. In this article, we used Shenzhen as the study area to validate the model, and the results indicate that the strategy proposed here can be adopted in urban planning and contribute to health policy.

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