Two Component Theory of Neutrino Flavor Mixing

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Abstract

Neutrino flavor mixing is discussed in terms of two-component coupled left-handed flavor fields. This is to take into account the fact that the weak interaction couples only to left-handed fields. The flavor fields are written through a rotation matrix, as a linear combination of left-handed free fields. In order to obtain properly normalized wave functions directly from those free fields, states of mixed helicity have to be considered. Neutrino flavor oscillation amplitudes are also derived.

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I. INTRODUCTION

In Ref. [1] we have discussed neutrino flavor mixing by assuming that neutrinos are described by Dirac fields. However to take into account the fact that neutrinos are created with negative chiralities, we were forced to consider only left-handed wave functions as observable wave functions.

In this paper we want to consider a Lagrangian which is only a function of left-handed, but not right-handed fields.

More specifically, neutrino flavor mixing is discussed in terms of the two-component fields $\psi_{eL}$ and $\psi_{\mu L}$, defined by

$$\psi_{eL} = \frac{1}{2}(1 - \gamma_5)\psi_e, \quad \psi_{\mu L} = \frac{1}{2}(1 - \gamma_5)\psi_\mu,$$

with $\psi_e$ and $\psi_\mu$ Dirac fields. These fields satisfy the following Lagrangian [2]

$$\mathcal{L} = \mathcal{L}_e + \mathcal{L}_\mu + \mathcal{L}_{\text{int}},$$

$$\mathcal{L}_e = i\bar{\psi}_{eL}\gamma \cdot \partial \psi_{eL} - \frac{m_e}{2}(\bar{\psi}_{eL}\psi_{eL} + \bar{\psi}_{eL}\psi_{eL}),$$

$$\mathcal{L}_\mu = i\bar{\psi}_{\mu L}\gamma \cdot \partial \psi_{\mu L} - \frac{m_\mu}{2}(\bar{\psi}_{\mu L}\psi_{\mu L} + \bar{\psi}_{\mu L}\psi_{\mu L}),$$

$$\mathcal{L}_{\text{int}} = -\frac{\delta}{2}[\bar{\psi}_{eL}\psi_{\mu L} + \bar{\psi}_{\mu L}\psi_{eL} + \bar{\psi}_{eL}\psi_{\mu L} + \bar{\psi}_{\mu L}\psi_{eL}],$$

where $m_e$ and $m_\mu$ are the electron and muon neutrino masses respectively, $\delta$ is the coupling constant, and $\psi^c_{eL}$ and $\psi^c_{\mu L}$ are the so called charge-conjugate fields defined in the chiral representation by

$$\psi^c_{eL} = -i\gamma^2(\psi^\dagger_{eL})^T, \quad \psi^c_{\mu L} = -i\gamma^2(\psi^\dagger_{\mu L})^T.$$

We assume that neutrinos are produced through some weak interaction process and then they mix in flavor accordingly to the Lagrangian given by Eq. (2).

The Lagrangian given above allows for exact diagonalization. The rotation matrix between the coupled fields $\psi_{eL}$ and $\psi_{\mu L}$ and the free uncoupled fields $\psi_{1L}$ and $\psi_{2L}$, which diagonalizes Eq. (2), is the same as the one obtained in Ref. [1].
The neutrino left-handed fields $\psi_{eL}$ and $\psi_{\mu L}$ are written as linear combinations of the left-handed free fields $\psi_{1L}$ and $\psi_{2L}$ through this rotation matrix. The two-component free fields $\psi_{1,2L}$ are described by the Lagrangian of the Majorana type 

$$\mathcal{L} = i\bar{\psi}_L \gamma \cdot \partial \psi_L - \frac{m}{2}(\bar{\psi}_L \psi^c_L + \bar{\psi}^c_L \psi_L).$$

(4)

Therefore there is no distinction in this theory between particles and anti-particles.

Neutrino flavor wave functions are derived as matrix elements of the fields $\psi_{1L}$ and $\psi_{2L}$ between the vacuum state and one-particle states. In order to obtain properly normalized wave functions directly from those fields, a suitable combination of one particle states has to be considered, as described in the following section.

Neutrino flavor oscillation probability amplitudes are also derived. The standard neutrino oscillation probabilities, derived in the literature from a quantum mechanical treatment, are recovered in the field theory treatment only in the relativistic limit. The same limitation occurs also for Dirac fields.

The paper is organized as follows. In Sec. 2, the lagrangian density, given by Eq. (4) is considered. In Sec. 3, the Lagrangian (2) is diagonalized and neutrino flavor oscillations are discussed. The last section closes with some concluding remarks.

II. LEFT-HANDED (MAJORANA) FERMIONS

As stated in the Introduction, we will review and study here the following Lagrangian

$$\mathcal{L} = i\bar{\psi}_L \gamma \cdot \partial \psi_L - \frac{m}{2}(\bar{\psi}_L \psi^c_L + \bar{\psi}^c_L \psi_L),$$

(5)

which differs by a total derivative from the Majorana lagrangian density (see for example [4–8]).

In the chiral representation the left-handed fields $\psi_L$ and $\psi^c_L$ are

$$\psi_L = \frac{1}{2}(1 - \gamma_5)\psi = \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \quad \psi^c_L = -i\gamma^2 \psi^{cT} = \begin{pmatrix} 0 \\ i\sigma^2 \phi^{cT} \end{pmatrix},$$

(6)
where \( \psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix} \), \( \phi \) and \( \chi \) are two-component fields, in particular

\[
\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \phi^\dagger = \begin{pmatrix} \phi_1^* \\ \phi_2^* \end{pmatrix}.
\]  

(7)

In terms of the fields \( \phi \) and \( \phi^\dagger \), the Lagrangian given by Eq. (5) becomes

\[
\mathcal{L} = i\phi^\dagger \sigma \cdot \partial \phi + \frac{im}{2} (\phi^T \sigma^2 \phi - \phi^\dagger \sigma^2 \phi^\dagger T),
\]

(8)

with \( \sigma = (1, -\vec{\sigma}) \).

Because the matrix \( \sigma^2 \) is anti-symmetric the mass terms \( \phi_\alpha \sigma_{\alpha \beta}^2 \phi_\beta \) and \( \phi_\alpha^\dagger \sigma_{\alpha \beta}^2 \phi_\beta^\dagger \) are identically zero. Therefore the Lagrangian given by Eq. (8) makes sense only if the fields \( \phi \) and \( \phi^\dagger \) are considered as Grassmann fields, i.e. their components satisfy the conditions

\[
\phi_1 \phi_2 + \phi_2 \phi_1 = 0, \quad \phi_1^* \phi_2^* + \phi_2^* \phi_1^* = 0.
\]

(9)

The equation of motion for the two-component field \( \phi \) is

\[
i\sigma \cdot \partial \phi - im\sigma^2 \phi^\dagger T = 0.
\]

(10)

We first solve Eq. (10) for the momentum \( p \) along the z-axis and then we generalize the result to three dimensions. We consider the ansatz

\[
\phi_p(z, t) = a_p u_p e^{-iEt} e^{ipz} + a_p^\dagger v_p e^{iEt} e^{-ipz},
\]

(11)

where \( a_p \) and \( a_p^\dagger \) are Grassmann numbers and \( u \) and \( v \) are two component c-number spinors

\[
u_p = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad v_p = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.
\]

(12)

By substituting Eq. (11) into Eq. (10) we obtain two set of solutions, one is

\[
u_1 = \frac{m}{E + p} v_2^*, \quad v_1 = \frac{-m}{E + p} u_2^*,
\]

(13)

and the other one is
\[ u_2 = \frac{-m}{E - p} v_1^*, \quad v_2 = \frac{m}{E - p} u_1^*. \tag{14} \]

The solutions are equivalent if

\[ E = \pm \sqrt{p^2 + m^2}. \tag{15} \]

The solution to Eq. (10) for a given momentum \( p \) and positive energy \( E = \sqrt{p^2 + m^2} \) can be written as

\[
\phi_p(z, t) = a(p, 1) v_2^* \frac{m}{E + p} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-iEt} e^{ipz} + a^\dagger(p, 1) v_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{iEt} e^{-ipz} \\
+ a(p, 2) u_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-iEt} e^{ipz} - a^\dagger(p, 2) \frac{m}{E + p} u_2^* \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{iEt} e^{-ipz}. \tag{16} \]

The solution corresponding to \( E = -\sqrt{p^2 + m^2} \) is equivalent to the one above with the substitution \( p \to -p' \), therefore in this theory we have only one type of particle.

It is easy to see that the components of \( \phi_p(z, t) \) satisfies the condition, given by Eq. (9), if the operators \( a(p, 1) \), \( a(p, 2) \) satisfy the Grassman algebra

\[ \{a(p, i), a^\dagger(p, j)\} = \delta_{ij}, \tag{17} \]

and

\[ |u_2| = |v_2|. \tag{18} \]

The phase between \( u_2, v_2 \) can be chosen in such a way that \( |u_2| = |v_2| = \lambda_p \), where \( \lambda_p \) is a normalization constant.

The general solution is given therefore by

\[
\phi(z, t) = \frac{1}{\sqrt{L}} \sum_p \lambda_p \left[ \left( \frac{m}{E + p} a(p, 1) \chi^{(1)} + a(p, 2) \chi^{(2)} \right) e^{-iEt} e^{ipz} \\
+ \left( a^\dagger(p, 1) \chi^{(2)} - \frac{m}{E + p} a^\dagger(p, 2) \chi^{(1)} \right) e^{iEt} e^{-ipz} \right], \tag{19} \]

with

\[
\chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{20} \]
The result given by Eq. (19) can be generalized in three spatial dimensions as follows. Given the spin base \( \chi^{(1)}, \chi^{(2)} \), such that

\[
\sigma \cdot p \chi^{(1)} = p \chi^{(1)}, \quad \sigma \cdot p \chi^{(2)} = -p \chi^{(2)},
\]

(21)

with \( p = |p| \), for example

\[
\begin{align*}
\chi^{(1)} &= \left( e^{-\frac{i}{2} \cos(\theta/2)} \right), \\
\chi^{(2)} &= \left( -e^{-\frac{i}{2} \sin(\theta/2)} \right),
\end{align*}
\]

(22)

we can write the two spinors \( u_p \) and \( v_p \) defined in Eq. (12) as a linear combination of the spin base defined in Eq. (21). The ansatz, given in Eq. (11), can be therefore generalized as

\[
\phi_p(x, t) = a_p(u_1 \chi^{(1)} + u_2 \chi^{(2)}) e^{-iEt + ip \cdot x} + a^\dagger_p(v_1 \chi^{(1)} + v_2 \chi^{(2)}) e^{iEt - ip \cdot x}.
\]

(23)

By inserting Eq. (23) into the equation of motion (10) and by noticing that

\[
i\sigma^2 \chi^{*(1)} = -\chi^{(2)}, \quad i\sigma^2 \chi^{*(2)} = \chi^{(1)},
\]

(24)

we get the same solutions as given in Eqs. (13), (14), (15), (17) and (18).

Therefore the general solution can be written as

\[
\phi(x, t) = \frac{1}{\sqrt{V}} \sum_p \lambda_p \left[ \left( \frac{m}{E + p} a(p, 1) \chi^{(1)} + a(p, 2) \chi^{(2)} \right) e^{-ip \cdot x} \right. \\
\left. + \left( a^\dagger(p, 1) \chi^{(2)} - \frac{m}{E + p} a^\dagger(p, 2) \chi^{(1)} \right) e^{ip \cdot x} \right],
\]

(25)

In terms of its components, the field \( \phi(x, t) \) can be written as

\[
\begin{align*}
\phi^{(1)}(x, t) &= \frac{1}{\sqrt{V}} \sum_p \lambda_p \frac{m}{E + p} \left[ a(p, 1) e^{-ip \cdot x} - a^\dagger(p, 2) e^{ip \cdot x} \right], \\
\phi^{(2)}(x, t) &= \frac{1}{\sqrt{V}} \sum_p \lambda_p \left[ a(p, 2) e^{-ip \cdot x} + a^\dagger(p, 1) e^{ip \cdot x} \right].
\end{align*}
\]

(26)

The constant \( \lambda_p \) is chosen in such a way that the equal time anti-commutation relations

\[
\{ \phi^{(i)}(x, t), \phi^{(j)}(x', t) \} = \delta_{ij} \delta^{(3)}(x - x'), \quad \{ \phi^{(i)}(x, t), \phi^{(j)}(x', t) \} = 0, \quad \{ \phi^{(i)}(x, t), \phi^{(j)}(x', t) \} = 0,
\]

(27)
hold, i.e.

\[ \lambda_p = \sqrt{\frac{E+p}{2E}}. \]  \hspace{1cm} (28)

The field \( \phi(x, t) \) describes a particle which can be in two different states of helicity, corresponding to the states \( \chi^{(1)} \) and \( \chi^{(2)} \). However the physical interpretation of the field \( \phi(x, t) \) poses some problems. For example, the one-particle state \( |p, i \rangle = a^\dagger p_i | 0 \rangle \) gives wave functions which are not properly normalized

\[ <0|\phi(x, t)|p, 1 > = \frac{1}{\sqrt{V}} \lambda_p \frac{m}{E+p} \chi^{(1)} e^{-ip \cdot x}, \]  \hspace{1cm} (29)

\[ <0|\phi(x, t)|p, 2 > = \frac{1}{\sqrt{V}} \lambda_p \chi^{(2)} e^{-ip \cdot x}. \]

The only way to obtain properly normalized wave functions is to consider states such as

\[ |\phi >_{pL} = |p, 1 > + |p, 2 >. \]  \hspace{1cm} (30)

The wave function associated with \( |\phi >_{pL} \) is

\[ \phi(x, t)_L = \frac{1}{\sqrt{V}} \lambda_p \left[ \frac{m}{E+p} \chi^{(1)} + \chi^{(2)} \right] e^{-ip \cdot x}. \]  \hspace{1cm} (31)

This wave function describes a left-handed particle which is in a state of mixed helicity with the positive helicity suppressed by the factor \( \frac{m}{E+p} \).

### III. FLAVOR MIXING

The Lagrangian given by Eq. (2) can be written in terms of the fields \( \phi_e \) and \( \phi_\mu \) where

\[ \psi_{eL} = \begin{pmatrix} \phi_e \\ 0 \end{pmatrix}, \quad \psi_{\mu L} = \begin{pmatrix} \phi_\mu \\ 0 \end{pmatrix} \]  \hspace{1cm} (32)

as

\[ L = i \bar{\psi}_e \sigma \cdot \partial \psi_e + \frac{im_e}{2} \left( \phi_e^T \sigma^2 \phi_e - \phi_e^T \sigma^2 \phi_e^\dagger \right) \]

\[ + i \bar{\psi}_\mu \sigma \cdot \partial \psi_\mu + \frac{im_\mu}{2} \left( \phi_\mu^T \sigma^2 \phi_\mu - \phi_\mu^T \sigma^2 \phi_\mu^\dagger \right) \]

\[ + i \frac{\delta}{2} \left[ \phi_e^T \sigma^2 \phi_\mu - \phi_e^T \sigma^2 \phi_\mu^\dagger + \phi_\mu^T \sigma^2 \phi_e - \phi_\mu^T \sigma^2 \phi_e^\dagger \right]. \]  \hspace{1cm} (33)
It is possible to see that the rotation matrix $U$, defined in $[1]$ as

$$U = \begin{pmatrix} \frac{1}{\sqrt{1+M_1^2}} & \frac{M_1}{\sqrt{1+M_1^2}} \\ \frac{M_1}{\sqrt{1+M_1^2}} & -\frac{1}{\sqrt{1+M_1^2}} \end{pmatrix},$$

(34)

with $M_1$ given by

$$M_1 = \frac{m_\mu - m_e + R}{2\delta}, \quad R = \sqrt{(m_\mu - m_e)^2 + 4\delta^2}$$

(35)

when applied to the fields $\phi_1$ and $\phi_2$

$$\phi_\nu = \begin{pmatrix} \phi_e \\ \phi_\mu \end{pmatrix} = U \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix},$$

(36)

uncouples the Lagrangian given by Eq. (33). The uncoupled Lagrangian is given by

$$\mathcal{L}_D = [i\phi_1 \sigma \cdot \partial \phi_1 + \frac{im}{2}(\phi_1^T \sigma^2 \phi_1 - \phi_1^T \sigma^2 \phi_1^T)]$$

$$+ [i\phi_2 \sigma \cdot \partial \phi_2 + \frac{im}{2}(\phi_2 \sigma^2 \phi_2 - \phi_2 \sigma^2 \phi_2^T)]$$

(37)

with

$$m_{1,2} = \frac{1}{2}[(m_e + m_\mu) \pm R].$$

(38)

The fields $\phi_1$ and $\phi_2$ which have been discussed in Sec. 2, are free fields of masses $m_1$, $m_2$

respectively

$$\phi_1(x, t) = \frac{1}{\sqrt{V}} \sum_\mathbf{p} \lambda_{1p} \left[ \left( \frac{m}{E_1 + p} a_1(\mathbf{p}, 1) \chi^{(1)} + a_1(\mathbf{p}, 2) \chi^{(2)} \right) e^{-iE_1 t} e^{i\mathbf{p} \cdot \mathbf{x}} \right]$$

$$+ \left( a_1^T(\mathbf{p}, 1) \chi^{(2)} - \frac{m}{E_1 + p} a_1^T(\mathbf{p}, 2) \chi^{(1)} \right) e^{iE_1 t} e^{-i\mathbf{p} \cdot \mathbf{x}},$$

(39)

$$\phi_2(x, t) = \frac{1}{\sqrt{V}} \sum_\mathbf{p} \lambda_{2p} \left[ \left( \frac{m}{E_2 + p} a_2(\mathbf{p}, 1) \chi^{(1)} + a_2(\mathbf{p}, 2) \chi^{(2)} \right) e^{-iE_2 t} e^{i\mathbf{p} \cdot \mathbf{x}} \right]$$

$$+ \left( a_2^T(\mathbf{p}, 1) \chi^{(2)} - \frac{m}{E_2 + p} a_2^T(\mathbf{p}, 2) \chi^{(1)} \right) e^{iE_2 t} e^{-i\mathbf{p} \cdot \mathbf{x}},$$

(40)

where

$$E_1 = \sqrt{p^2 + m_1^2}, \quad E_2 = \sqrt{p^2 + m_2^2},$$

(41)

$$\lambda_{1p} = \sqrt{\frac{E_1 + p}{2E_1}}, \quad \lambda_{2p} = \sqrt{\frac{E_2 + p}{2E_2}},$$

(42)
The electron and neutrino field operators \( \phi_e \) and \( \phi_\mu \) are related to the diagonal (uncoupled) field operators \( \phi_1 \) and \( \phi_2 \) through the rotation matrix \( U \) defined in Eq. (34).

For a given momentum \( \mathbf{p} \) and spin \( i = 1, 2 \), there are two possible one-particle states, one associated with the field \( \phi_1 \) and the other one with the field \( \phi_2 \), i.e.

\[
a_1^\dag(\mathbf{p}, i)|0> = |1(\mathbf{p}, i)>, \quad a_2^\dag(\mathbf{p}, i)|0> = |2(\mathbf{p}, i)>. 
\]

The wave function associated with the state

\[
|\phi_1 >_{pL} = |1(\mathbf{p}, 1) > + |1(\mathbf{p}, 2) >, 
\]

is

\[
\phi_{L\nu}(\mathbf{x}, t) = \begin{pmatrix} \phi_{Le}(\mathbf{x}, t) \\ \phi_{L\mu}(\mathbf{x}, t) \end{pmatrix} = \begin{pmatrix} <0|\phi_e(\mathbf{x}, t)|\phi_1 >_{pL} \\ <0|\phi_\mu(\mathbf{x}, t)|\phi_1 >_{pL} \end{pmatrix} = \begin{pmatrix} \frac{1}{(1+M_1^2)^{1/2}} \\ \frac{M_1}{(1+M_1^2)^{1/2}} \end{pmatrix} \frac{1}{\sqrt{4\pi}} \lambda_{1p} [\frac{m}{E_1+p} \chi^{(1)} + \chi^{(2)}] e^{\mathbf{p} \cdot \mathbf{x}} e^{-iE_1 t}. 
\]

This represents a plane wave of mixed helicity. In any location inside the volume \( V \) there is a probability equal to \( \frac{1}{1+M_1^2} \) of finding the neutrino in the electron flavor and probability equal to \( \frac{M_1^2}{1+M_1^2} \) of finding it in the muon flavor.

Similar consideration can be applied for the other state of given momentum \( |\phi_2 >_{pL} = |2(\mathbf{p}, 1) > + |2(\mathbf{p}, 2) >. \)

To be able to describe neutrino flavor oscillations, we need to consider a linear combinations of the states \( |\phi_1 >_{pL} \) and \( |\phi_2 >_{pL} \), such as

\[
|\phi >_{L} = A|\phi_1 >_{pL} + B|\phi_2 >_{pL}, 
\]

with

\[
|A|^2 + |B|^2 = 1.
\]

The matrix element
\begin{equation}
< 0|\phi_e(x, t)|\phi >_L = \phi_{eL}(x, t) = \frac{1}{\sqrt{V}} \frac{1}{\sqrt{1 + M_1^2}} \left[ A\lambda_{1p}\left[ \frac{m}{E_1 + p}\chi^{(1)} + \chi^{(2)} \right] e^{-iE_1 t} + M_1 B\lambda_{2p}\left[ \frac{m}{E_2 + p}\chi^{(1)} + \chi^{(2)} \right] e^{-iE_2 t} \right] e^{ip \cdot x},
\end{equation}

(48)
gives the probability amplitude of finding a neutrino of momentum \( p \) at the space-time point \((x, t)\) with the electron flavor. In the same way, the matrix element

\begin{equation}
< 0|\phi_\mu(x, t)|\phi >_L = \phi_{\mu L}(x, t) = \frac{1}{\sqrt{V}} \frac{1}{\sqrt{1 + M_1^2}} \left[ M_1 A\lambda_{1p}\left[ \frac{m}{E_1 + p}\chi^{(1)} + \chi^{(2)} \right] e^{-iE_1 t} - B\lambda_{2p}\left[ \frac{m}{E_2 + p}\chi^{(1)} + \chi^{(2)} \right] e^{-iE_2 t} \right] e^{ip \cdot x},
\end{equation}

(49)
is the probability amplitude for the muon flavor.

The coefficients \( A \) and \( B \) in Eqs. (48) and (49) are determined through the initial boundary conditions.

However, it is possible only in the relativistic limit, when the term \( \frac{m}{E_1 + p} \approx 0 \) in Eqs. (48), (49), to have only one given flavor. This limitation occurs also for Dirac fields. The procedure illustrated in [1] to obtain neutrino flavor oscillation amplitudes is valid in the relativistic limit, because the condition given by Eq. (47) holds only in that limit. Other authors by using different approaches have found the same limitation. [9,10]

In the relativistic limit, Eqs. (48)(49) can be approximated as

\begin{equation}
\phi_{eL}(x, t) \simeq \frac{1}{\sqrt{V}} \frac{e^{ip \cdot x}}{\sqrt{1 + M_1^2}} (Ae^{-iE_1 t} + M_1 Be^{-iE_2 t})\chi^{(2)},
\end{equation}

(50)

\begin{equation}
\phi_{\mu L}(x, t) \simeq \frac{1}{\sqrt{V}} \frac{e^{ip \cdot x}}{\sqrt{1 + M_1^2}} (AM_1 e^{-iE_1 t} - Be^{-iE_2 t})\chi^{(2)}.
\end{equation}

(51)
The coefficients \( A \) and \( B \) are determined from the initial boundary condition. Suppose for example that at \( t = 0 \)

\begin{equation}
\phi_{\mu L}(x, t = 0) = 0,
\end{equation}

(52)
so we have only the electron flavor present. The other one is obtained by the normalization condition
By imposing the boundary conditions given by Eq. (52) and Eq. (53) we obtain the following flavor wave functions

$$
\phi_{eL}(x, t) = \frac{e^{ip \cdot x}}{\sqrt{V}} \frac{1}{1 + M_1^2} \left[ e^{-iE_1 t} + M_1^2 e^{-iE_2 t} \right] \chi^{(2)},
$$

(54)

$$
\phi_{\mu L}(x, t) = \frac{e^{ip \cdot x}}{\sqrt{V}} \frac{M_1}{1 + M_1^2} \left[ e^{-iE_1 t} - e^{-iE_2 t} \right] \chi^{(2)}.
$$

(55)

These amplitudes squared give the standard neutrino oscillation probabilities as described in [1].

IV. CONCLUSIONS

We have discussed a model to describe neutrino flavor mixing which takes into account the fact that neutrinos are created through weak interaction, which couples only to left-handed fields. The flavor wave functions are in a superposition of states of mixed helicities. The standard neutrino oscillation probabilities are obtained in the relativistic limit.
REFERENCES

[1] Elisabetta Sassaroli, “Flavor Oscillations in Field Theory”, MIT-CTP-2571, hep-ph/9609476.

[2] S. M. Bilenky and B. Pontecorvo, Phys. Rep. 41, 225 (1978).

[3] K. M. Case, Phys. Rev. 107, 307 (1957).

[4] F. Boehm and P. Vogel, Physics of Massive Neutrinos (Cambridge University Press, Cambridge, England (1987)).

[5] B. Kayser, F. Gibrat-Debu, and F. Perrier, The Physics of Massive neutrinos (World Scientific, Singapore, 1989)

[6] J. N. Bahcall, Neutrino Physics and Astrophysics (Cambridge University Press, 1989).

[7] R. N. Mohapatra, and P. B. Pal, Massive Neutrinos in Physics and Astrophysics (World Scientific, Singapore, 1991).

[8] C. W. Kim and A. Pevsner, Neutrinos in Physics and Astrophysics (Harwood Academic Publishers, 1993).

[9] C. Giunti, C. K. Kim, and U. W. Lee, Phys. Rev D 45, 2414 (1992).

[10] M. Blasone and G. Vitiello, Ann. Phys. 244, 283 (1995); E. Alfinito, M. Blasone, A. Iorio, and G. Vitiello, Phys. Lett. B 362.