Different Type of Nasties Using in Linear Programming Problem

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Abstract. An innovative procedure for identifying a nasty numbers with the help of twin nasties using Big-M method in linear programing problem is existing in this paper.  Linear programming is a way for defining an optimum schedule of interdependent events in observation of the existing resource. A definition of twin nasties and the interesting properties of nasty number is summarized. A new method of relating Big-M method with twin nasties using maximization procedure to get optimal solution which is a nasty number is shown in a clear way.

1. Introduction
Linear programming is a technique for determining an optimum schedule of interdependent events in sight of the accessible resources. In many application want to find a maximum or minimum value. Constraints are expressed as inequalities. The solution set of the system of inequalities made up of the constraints all the feasible solutions of a linear programming problem. The function that want to maximize or minimize is called the objective function. Mathematical programming is used to discover the optimal solution to a problem that has in need of a decision or set of decisions around how best to use a set of restricted resources to succeed a state aim of objectives. Big-M method is an iterative technique for solving a linear programming problem in finite number of steps.

An outer layer view about, nasty number, twin nasties, independent nasty, double nasties and its interesting properties are being discovered along with examples. Nasty numbers is a fascinating number. And is also applied in Fibonacci number, Lucas number and etc. A different manner of concerning Big-M method with twin nasties using maximization procedure to get optimal solution which is also a nasty number is displayed in this article. Some more interesting patterns of numbers namely nasty, jarasandha and dhuruva, these numbers have been presented in [1-2] and also explained how to discover the nasty numbers and their characterizations. In [3], different Pythagorean triangles related with nasty numbers are obtained. In linear programming, there are some interesting algorithms to solve the problems for various methods and many hidden connection presented in [4-20]. In 2014 Karpagam A and Sumathi P proposed an innovative method for correlating fuzzy in linear

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programming problems are obtained in [9-10]. Recently in [19-20], proposed a new approach of solve nasty in linear programming problems are obtained.

2. Nasty Number
The sum of the factors of nasty numbers in one pair is equal to the difference between the factors in another pair of nasty numbers and the nasty number is equal to the product of each pair.

Thus a non-negative integer \( F^1 \) is a nasty number.

If \( F^1 = f_1 \times f_2 = f_3 \times f_4 \) and
\[
 f_1 + f_2 = f_3 - f_4.
\]
Where \( f_1, f_2, f_3, f_4 \) are non-negative integers.

2.1. Examples
(1) \( F^1 \) with four different factors is 6 and it is nasty because,
\[
6 = 3 \times 2 = 6 \times 1
\]
and
\[
3 + 2 = 6 - 1
\]
(2) If \( F^1 \) is a nasty number, then clearly \( f_q^2 \) \( F^1 \) is also a nasty number for every non-zero integer values of \( f_q \).

So \( 6 f_q^2 \) is nasty for \( f_q^2 = 1, 2, 3, \ldots \)

When \( f_q^2 = 2 \) and \( 6 f_q^2 = 24 \) which is nasty since the factors of 24 are given by
\[
1, 2, 3, 4, 6, 8, 12, 24.
\]
\[
24 = 12 \times 2 = 6 \times 4
\]
and
\[
12 - 2 = 6 + 4
\]

3. Twin Nasties
Some consecutive multiples of 6 are nasties like 24, 30; 54, 60; 210, 216; 330, 336; 480, 486; 540, 546; 720 and 726. They are called twin nasties. And the number of twin nasties are infinite.

4. Independent Nasty Number
A nasty number \( F^1 \) is call an independent nasty number, it cannot be obtained from another nasty number \( p \) by multiplying it with \( f_q^2 \) where \( f_q \) is any non-zero integer.

4.1. Example
6 is an independent nasty number while 24 is not. From the list of nasty numbers the following are independent nasty numbers.
6, 30, 60, 84, 180, 210, 330, 504, 546, 630, 924 and 990.

5. Double Nasties
Some positive integers are double nasties. That is, there are some positive integers which can be the areas of two primitive Pythagorean triangles.

5.1. Example
210 is the area of two primitive. Pythagorean triangles (20, 21, 29) and (35, 12, 37).

Again 840 is the area of 3 primitive Pythagorean triangles (40, 42, 58), (70, 24, 74) and (112, 15, 113).

6. Properties of Nasty Numbers
- A number \( F^1 \) is nasty if and only if it is of the form \( f_q^2 (gh) (g^2 - h^2) \), where \( (g, h) = 1, g > h \)
  and \( g \) and \( h \) are natural numbers of different parity and \( f_q \) is some positive integer.
- Every nasty number is divisible by 6.
- If \( 6p \) is a nasty number and \( p \) is a prime then \( p = 5 \).
- A nasty number is never a square.
- No nasty number except 6 is perfect.
- No nasty number can have 2 or 8 in its unit’s place.
The product of three consecutive integers is nasty.
The product of three consecutive Fibonacci numbers is nasty.
The product of four consecutive Lucas number is nasty.
There are infinitely many nasty numbers which are products of two consecutive integers.
Every nasty number is divisible by 6.

7. Formulation of Linear Programming Problem for Finding Nasty Number using Twin Nasties

7.1. Algorithm

Step 1: Take the twin nasties and keep it as objective function for linear programming problem.

Step 2: Objective function should be a maximization type.

Step 3: Formulate the objective function as using the twin nasties by decreasing order.

Step 4: Construct $b_i$ with factors of twin nasties by satisfies the properties of nasty number.

Step 5: If the value of $f_1, f_2, f_3, f_4$ repeats in another nasty, then omit it while formulating the constraints.

Step 6: Frame the constraints which should be a divisor of the corresponding $b_i$.

Step 7: Solve the linear programming problem by using Big-M method.

Step 8: The obtained optimal solution is a nasty number.

7.2. General Form of Linear Programming Problem for the Proposed Algorithm

Consider the twin nasties with their factors

Maximization $z = k_1 x_1 + k_2 x_2$

$k_1, k_2$ is the coefficient of $x_1$ and $x_2$ in such a way that their considered twin nasties is in decreasing order.

Subject to the constraints

$$a_{11} x_1 + a_{12} x_2 \leq b_1$$
$$a_{21} x_1 + a_{22} x_2 \leq b_2$$
$$a_{31} x_1 + a_{32} x_2 \leq b_3$$
$$a_{41} x_1 + a_{42} x_2 \leq b_4$$
$$a_{51} x_1 + a_{52} x_2 \leq b_5$$
$$a_{61} x_1 + a_{62} x_2 \leq b_6$$
$$a_{71} x_1 + a_{72} x_2 \leq b_7$$
$$a_{81} x_1 + a_{82} x_2 \leq b_8$$

$x_1, x_2 \geq 0$

Here $a_{ij} > 0$, where $i = 1$ to $m$ and $j = 1$ to $n$.

Using Big-M method

Max $z = F^t$. 
Which is again a nasty number.

7.3. Examples

Example: 1

Consider the twin nasties with their factors
54 and 60 is a twin nasties.
1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 are the factors of 60.
f₁ = 5, f₂ = 12, f₃ = 20, f₄ = 3.
1, 2, 3, 6, 9, 18, 27, 54 are the factors of 54.
f₁ = 6, f₂ = 9, f₃ = 18, f₄ = 3.

Max z = 60 x₁ + 54 x₂.
Subject to the constraints

\[
x₁ + 3x₂ \leq 3
\]
\[
2x₁ + 3x₂ \leq 6
\]
\[
3x₁ + 3x₂ \leq 9
\]
\[
2x₁ + 9x₂ \leq 18
\]
\[
x₁ + 5x₂ \leq 5
\]
\[
3x₁ + 4x₂ \leq 12
\]
\[
2x₁ + 10x₂ \leq 20
\]
\[
x₁, x₂ \geq 0.
\]

Using Big-M method
Max z = 180.
180 is a nasty number.

Example: 2

Consider the twin nasties with their factors
24 and 30 is a twin nasties.
1, 2, 3, 5, 6, 10, 15, 30 are the factors of 30.
f₁ = 2, f₂ = 15, f₃ = 10, f₄ = 3.
1, 2, 3, 4, 6, 8, 12, 24 are the factors of 24.
f₁ = 2, f₂ = 12, f₃ = 4, f₄ = 6.

Max z = 30 x₁ + 24 x₂.
Subject to the constraints

\[
x₁ + 2x₂ \leq 2
\]
\[
2x₁ + 2x₂ \leq 4
\]
\[
2x₁ + 3x₂ \leq 6
\]
\[
3x₁ + 4x₂ \leq 12
\]
\[
x₁ + 3x₂ \leq 3
\]
\[
2x₁ + 5x₂ \leq 10
\]
\[
3x₁ + 5x₂ \leq 15
\]
\[
x₁, x₂ \geq 0.
\]

Using Big-M method
Max z = 60.
60 is a nasty number.

8. Appendices
A different tactic of getting nasty number as optimal solution with the help of twin nasties using Big-M method in linear programming problem had been presented in this article and the suggested tactic had been analysed for several twin nasties in linear programming problems.

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