We study static neutron stars in the context of a class of non-minimally coupled inflationary potentials, the universal attractors. Universal attractors are known to generate a viable inflationary era, and they fall into the same category of inflationary phenomenology as the \( R^2 \) model and other well-known cosmological attractors. We present the essential features of universal attractors in both the Einstein and Jordan frame, and we extract the Tolman-Oppenheimer-Volkoff equations in the Einstein frame using the usual notation of theoretical astrophysics. We use a python 3 based double shooting numerical code for our numerical analysis and we construct the \( M - R \) graphs for the universal attractor potential, using piecewise polytropic equation of state the small density part of which is the WFF1 or the APR or the SLy equation of state. As we show, all the studied cases predict larger maximum masses for the neutron stars, and all the results are compatible with the GW170817 constraints imposed on the radii of the neutron stars.

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Introduction

The last three decades have brought cosmology and astrophysics to the mainstream of physics, since the observation of dark energy [1] and the direct detection of gravitational waves [2] [3] have altered the way of thinking on how the Universe works in small and large scales. Neutron stars (NS) [4–8] currently are in the interest of many scientific areas, like nuclear theory [9–18], high energy physics [19–23], modified gravity [24–32] and astrophysics [33–40].

There is strong evidence coming from the observations on dark energy that modified gravity in its various forms [41–48] actually plays a fundamental role on large scales. Also at the astrophysical level, it is possible to generate large or extremely large neutron star masses and solve several fundamental equation of state (EoS) related problems of neutron stars [27, 28]. Hence it is probable that general relativity (GR) by itself may not suffice to describe NSs, hence some extension of GR might be compelling. In this work we shall consider NSs in hydrodynamic equilibrium in the context of non-minimally coupled scalar-tensor theories. This subject is very well studied in the theoretical astrophysics literature, see Refs. [49–62] for an important stream of articles on this subject. We shall consider some not so well known in the theoretical astrophysics literature non-minimal coupled theories, those of cosmological attractors [63–100]. Specifically, in this work we shall consider the class known as universal attractors [70], and we shall investigate the implications of such non-minimally coupled scalar field theories on static NSs in the Einstein frame. These models are known to provide a uniform inflationary phenomenology and belong to the larger class of cosmological attractors, which provide a viable inflationary phenomenology compatible with the latest Planck data [101].

We shall solve numerically the Tolman-Oppenheimer-Volkoff (TOV) equations, using an “LSODA” integrator python 3 based code, which is a modification of [102], and with regard to the EoS for the nuclear matter, we shall assume that the EoS is a piecewise polytropic EoS [103] [104], with the low density part being the WFF1 [105], the SLy [106], of the APR EoS [107], and from it we shall calculate the Jordan frame mass numerically.

This paper is organized as follows: In section I we review the essential features of the universal attractors in the context of cosmology, and we shall demonstrate how these models provide a viable inflationary era. In section II we discuss these models in the context of theoretical astrophysics notation and physical units, and we solve numerically the TOV equations for the three distinct EoSs. In the same section we qualitatively discuss the phenomenological features of the NSs for the universal attractor potentials. Finally the conclusions follow in the end of the paper.

I. ESSENTIAL FEATURES OF UNIVERSAL ATTRACTOR THEORIES

Universal attractors belong to a large class of cosmological attractors studied in Refs. [63–100]. All these cosmological attractor models originate from various forms of Jordan frame non-minimally coupled scalar theories, but the Einstein frame counterpart yield a quite similar inflationary phenomenology, for generic non-minimal coupling. In this section we shall briefly review how the universal attractors inflationary theory is obtained. The notation we shall use is frequently used in cosmological contexts, so we use natural units for this section. In the next section where
we study the Einstein frame NS phenomenology, we switch to Geometrized units. The conventions and formalism of conformal transformations in cosmological contexts, we refer the reader to Refs. [109, 110].

We start off with the Jordan frame action of a non-minimally coupled scalar field \( \phi \),

\[
S_J = \int d^4x \left[ f(\phi) R - \frac{\omega(\phi)}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) \right] + S_m(g_{\mu \nu}, \psi_m),
\]

with \( \psi_m \) denoting the Jordan frame perfect matter fluids, with \( P \) and energy density \( \epsilon \). The universal attractors in the Jordan frame have the following non-minimal coupling,

\[
f(\phi) = \frac{M_p^2}{2} \left( 1 - \xi \phi^2 \right),
\]

where \( \xi \) is a positive constant coupling, and the reduced Planck mass is defined as follows,

\[
M_p = \frac{1}{\sqrt{8\pi G}},
\]

where \( G \) is the gravitational constant of Newtonian gravity. Moreover, the universal attractors in the Jordan frame have the following scalar potential,

\[
U(\phi) = U_0 f^2(\phi) \left( \frac{\phi}{M_p} \right)^{2n},
\]

with \( n \) being some positive number. Now if the following conformal transformation is performed in the Jordan frame action with metric \( g_{\mu \nu} \),

\[
\tilde{g}_{\mu \nu} = \Omega^2 g_{\mu \nu},
\]

we obtain the Einstein frame action with metric \( \tilde{g}_{\mu \nu} \), where the “tilde” will denote the Einstein frame quantities. If we use [109 [110],

\[
\Omega^2 = \frac{2}{M_p^2} f(\phi),
\]

we may obtain a minimal coupled scalar theory in the Einstein frame,

\[
S_E = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{M_p^2}{2} \tilde{R} - \frac{\zeta(\phi)}{2} \tilde{g}^{\mu \nu} \tilde{\partial}_\mu \phi \tilde{\partial}_\nu \phi - V(\phi) \right] + S_m(\Omega^{-2} \tilde{g}_{\mu \nu}, \psi_m),
\]

where,

\[
\zeta(\phi) = \frac{M_p^2}{2} \left( 3 \left( \frac{df}{d\phi} \right)^2 + 2\omega(\phi) \right),
\]

and the potential \( V(\phi) \) is written in terms of the Jordan frame potential as,

\[
V(\phi) = \frac{U(\phi)}{\Omega^2},
\]

thus in view of Eqs. (10) and (9), the Einstein frame potential for the universal attractors reads,

\[
V(\phi) = \frac{U_0 M_p^4}{4} \left( \frac{\phi}{M_p} \right)^{2n}.
\]

The Einstein frame scalar field \( \phi \) can be made canonical by using the following transformation,

\[
\left( \frac{d\phi}{d\phi} \right) = \sqrt{\zeta(\phi)},
\]
hence the Einstein frame action becomes,

$$S_E = \int d^4 x \sqrt{-\tilde{g}} \left[ \frac{M_p^2}{2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \tilde{\partial}_\mu \varphi \tilde{\partial}^\nu \varphi - V(\varphi) \right] + S_m(\Omega^2 \tilde{g}_{\mu\nu}, \psi_m). \tag{12}$$

The Einstein frame matter fluids are coupled to the conformal factor so these are not perfect, because the energy momentum tensor satisfies,

$$\tilde{\partial}^\mu \tilde{T}_{\mu\nu} = -\frac{d}{d\varphi} \ln \Omega \tilde{T} \tilde{\partial}_\nu \phi. \tag{13}$$

Hereafter we shall assume that,

$$\Omega(\varphi) \ll \frac{3 M_p^2}{2} \Omega'(\varphi), \tag{14}$$

which by using the analytic form of $\Omega$ for the universal attractors, the above condition can be written as follows,

$$1 - \frac{\xi \varphi^2}{M_p^2} \ll \frac{6 \xi^2 \varphi^2}{M_p^2}. \tag{15}$$

By substituting the analytic form of $f(\varphi)$ from Eq. (2) into Eq. (8), we have,

$$\frac{d\varphi}{d\phi} = \sqrt{1 + \frac{\xi \varphi^2}{M_p^2} + \frac{6 \xi^2 \varphi^2}{M_p^2}} \left(1 + \frac{\xi \varphi^2}{M_p^2}\right), \tag{16}$$

so in view of the assumption (14), we easily obtain from Eq. (16) the following,

$$\varphi = -\sqrt{\frac{3}{2} M_p \ln \left(1 - \frac{\xi \varphi^2}{M_p^2}\right)}, \tag{17}$$

or equivalently,

$$\frac{\varphi^2}{M_p^2} = \frac{1 - e^{-\sqrt{\frac{3}{2} \pi \varphi^2}}}{\xi}. \tag{18}$$

Hence, by substituting Eq. (18) in Eq. (10) we finally obtain the Einstein frame scalar potential in terms of the canonical scalar field $\varphi$,

$$V(\varphi) = \frac{U_0 M_p^4}{4 \xi^n} \left(1 - e^{-\sqrt{\frac{3}{2} \pi \varphi^2}}\right)^n. \tag{19}$$

Let us set for convenience $V_0 = \frac{U_0 M_p^4}{4 \xi^n}$, and by taking into the Planck constraints [101] on the amplitude $\Delta_s^2$ of single canonical scalar field fluctuations,

$$\Delta_s^2 = 2.2 \times 10^{-9}, \tag{20}$$

where $\Delta_s^2$ is,

$$\Delta_s^2 = \frac{1}{24 \pi^2} \frac{V(\varphi_f)}{M_p^4} \frac{1}{\epsilon(\varphi_f)}, \tag{21}$$

the parameter $V_0$ is constrained to be,

$$V_0 \sim 9.6 \times 10^{-11} M_p^4. \tag{22}$$

Let us elaborate further on the constraint we just quoted, namely the parameter $V_0$. This parameter is constrained by the Planck data in a model independent way, using the BK15 constraint on $r$ (which is $r = 16 \epsilon$), see for example Eq. (32) of the Planck 2018 constraints on inflation, page 14 [101]. In that equation the tensor-to-scalar ratio is used,
while we replaced \( r = 16\epsilon \) in Eq. (21). Also note that in our notation the amplitude of the scalar fluctuations is \( \Delta^2_s \) while in the Planck data this is denoted as \( A_s \). Therefore the constraint of Eq. (32) of Ref. [101] is equivalent to our constraint (upper bound) Eq. (20), which if we substitute the maxim allowed values of the slow-roll index \( \epsilon(\varphi_f) \) (or equivalently the maximum allowed value of the tensor-to-scalar ratio), we obtain the constraint (22) of our paper. This is obtained in a general and model-independent way and does not rely on the specifics of the model used, it is solely based on the Planck constraints on canonical scalar field inflation.

Clearly the constraint on \( V_0 \) is an upper bound and we thus focused on this upper bound case in our paper. Definitely the parameter \( V_0 \) can take smaller values, but we used the upper bounds values for the slow-roll parameters \( \epsilon \) and \( r \), thus we focused our analysis on the maximum value for the scale of inflation \( V_0 \). In principle one could use lower values for the scale of inflation, until for example \( V_0^{4/3} \sim 10^{12} \text{GeV} \), which is the low-scale inflation constraint (we used the slow-roll relation \( \frac{3M^2}{2\epsilon_s} \sim V \)), but this would not change drastically the parameter \( V_0 \), plus one should explain how the low-scale inflation scenario occurs. Hence in our approach we used the most plausible values for the scale of inflation, inherently connected to the ordinary scale of inflation, not the low-scale of inflation.

Note that \( \varphi_f \) and \( \epsilon \) in Eq. (21) are the value of the canonical scalar field in the Einstein frame at the end of inflation and the first slow-roll index. The canonical scalar theory in the Einstein frame with the potential (19), which has a scale of inflation, inherently connected to the ordinary scale of inflation, not the low-scale of inflation.

\[
S_E = \int d^4x\sqrt{-\tilde{g}} \left[ \frac{M^2}{2} \tilde{R} - \frac{1}{2}(\tilde{g}^{\mu\nu}\tilde{\partial}_\mu\varphi\tilde{\partial}_\nu\varphi) - V_0 \left( 1 - e^{-\sqrt{\frac{2}{3}}\frac{\pi\epsilon}{\sqrt{\varphi}}} \right)^N \right],
\]

yields a viable inflationary phenomenology and has an attractor behavior resulting to the following spectral index of primordial scalar curvature perturbations \( n_s \) and tensor-to-scalar ratio, at leading order in the large \( e \)-folding number \( N \),

\[
n_s = 1 - \frac{2}{N}, \quad r = \frac{12}{N^2}.
\]

The above observational indices for inflation are identical to the ones corresponding to the \( R^2 \) model and other inflationary phenomenological models. A useful expression for the action (23) is the following,

\[
S_E = \int d^4x\sqrt{-\tilde{g}} \left[ \frac{1}{16\pi G} \tilde{R} - \frac{1}{2}(\tilde{g}^{\mu\nu}\tilde{\partial}_\mu\varphi\tilde{\partial}_\nu\varphi) - \frac{16\pi GV(\varphi)}{16\pi G} \right],
\]

and recall that \( M^2 = \frac{1}{8\pi G} \). The above form of the action is convenient for the universal attractor theory in the Einstein frame in the context of theoretical astrophysics notation. Also let us comment that the action (23) is identical to the one corresponding to the \( R^2 \) model for \( n = 2 \), and this is the case we shall also study. However, the two theories yield only identical inflation but the two theories are not the same because the conformal factor \( \Omega \) and the resulting coupling to the matter fluids are not the same, so these two theories look like the same but are not the same. We evince this feature in the next section.

\section{Neutron Stars in the Einstein Frame with Universal Attractors}

Let us now study the universal attractor potentials in astrophysical contexts. We shall use Geometrized units \( G = c = 1 \), and also we adopt the notation and conventions of Ref. [49]. The Jordan frame action of a non-minimally coupled scalar field in the presence of perfect matter fluids \( \psi_m \) is,

\[
S = \int d^4x\sqrt{-g} \left[ f(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi) \right] + S_m(\psi_m, g_{\mu\nu}).
\]

By conformally transforming the above action using,

\[
\tilde{g}_{\mu\nu} = A^{-2}g_{\mu\nu}, \quad A(\phi) = f^{-1/2}(\phi),
\]

we obtain the Einstein frame action which is,

\[
S = \int d^4x\sqrt{-\tilde{g}} \left[ \frac{\tilde{R}}{16\pi} - \frac{1}{2}\tilde{g}_{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{V(\varphi)}{16\pi} \right] + S_m(\psi_m, A^2(\phi)g_{\mu\nu}),
\]
where $\phi$ is the Einstein frame canonical scalar field, which is related to the scalar field $\phi$ as follows,

$$\frac{d\phi}{d\phi} = \frac{1}{\sqrt{4\pi}} \sqrt{\frac{3}{4} \frac{1}{f^2} \left( \frac{df}{d\phi} \right)^2 + \frac{1}{4f}},$$

(29)

while the potential $V(\phi)$ is,

$$V(\phi) = \frac{U(\phi)}{f^2}.$$  

(30)

Now the universal attractors case corresponds to the choices,

$$f(\phi) = 1 - \xi \phi^2, \quad U(\phi) = U_0 f^2(\phi) \phi^{2n},$$

(31)

and for these choices, Eq. (29) becomes,

$$\frac{d\phi}{d\phi} = \frac{1}{\sqrt{16\pi}} \sqrt{1 - \xi \phi^2 + 12\xi^2 \phi^2}. $$

(32)

Now, for the case and notation at hand, the assumption of Eq. (14) reads,

$$1 - \xi \phi^2 \ll 12\xi^2 \phi^2,$$

(33)

and in view of this assumption, we can integrate Eq. (29) to obtain,

$$\phi^2 = \frac{1}{\xi} \left( 1 - e^{-4\sqrt{\pi}} \right),$$

(34)

and now due to Eq. (27) the conformal factor $A(\phi)$ reads,

$$A(\phi) = e^{2\sqrt{\pi} \phi}.$$  

(35)

In addition, a useful quantity which shall be used in the following is the function $\alpha(\phi)$ defined as follows,

$$\alpha(\phi) = \frac{dA(\phi)}{d\phi},$$

(36)

hence in the case of universal attractors,

$$a(\phi) = \alpha = 2 \frac{\sqrt{\pi}}{3}. $$

(37)

It is worth noting that when $f(\phi) \rightarrow \infty$ then $A(\phi) \rightarrow 0$, thus at this limit the Einstein and Jordan frame are not equivalent, see for example [111], however we do not have to worry for this limit, since during inflation, the values of the scalar field are of the order of the Planck mass, while even at astrophysical contexts, such as interior and exterior of scalar-tensor neutron stars, the values of the scalar field are significantly smaller than the Planck mass. Also, using (34) and (37) the potential in the Einstein frame takes the final form,

$$V = V_0 \left( 1 - e^{-2\alpha \phi} \right)^n,$$

(38)

where $V_0 = \frac{U_0}{\xi^2}$. From this point we shall assume that $n = 2$, thus let us use the constraints on $V_0$ from the previous section to determine the values of $V_0$ always working in Geometrized units with $G = 1$. By comparing the actions (25) and (38), we have $V_0 = 16\pi V_0$, so $V_0 \approx 7.62094 \times 10^{-12}$. Hence, since $V_0 = \frac{U_0}{\xi^2}$, we can choose $U_0 = 1$ and $\xi = 36.2239 \times 10^4$, and we can observe that for this choice we shall also explicitly check whether the constraint (33) holds true, in the Jordan frame. We shall consider static NSs in the Einstein frame. Thus, the spherically symmetric static metric which describes the NS used in this paper is,

$$ds^2 = -e^{\nu(r)} dt^2 + \frac{dr^2}{1 - \frac{2m(r)}{r}} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2),$$

(39)
FIG. 1: $M - R$ graphs for the universal attractor model compared to the GR case, for the WFF1 EoS (upper), the APR EoS (middle) and the SLy EoS (bottom). The GW170817 indicate that for a NS of mass $M \sim 1.6 M_{\odot}$, the predicted radii for the universal attractor models must be larger than $R = 10.68^{+15}_{-0.04}$ km. Also the GW170817 event indicates that the radii corresponding to the maximum mass must be larger than $R = 9.6^{+14}_{-0.03}$ km. All the GW170817 constraints are satisfied as it can be seen in the plots.

where the function $m(r)$ stands for the gravitational mass of the NS with circumferential radius $r$. It is worth discussing here the issue of exterior and interior spacetime for the static neutron star. In standard GR contexts where the scalar field is absent, the exterior spacetime of the neutron star is Schwarzschild, however in the presence of the
scalar field the spacetime is uniformly given by the metric \([39]\), see for example \([19]\). It is the aim of any study in scalar-tensor astrophysics to find, numerically, the metric functions \(\nu(r)\) and \(\frac{1}{1-2m(r)}\). Obviously no matching conditions are required at the surface of the star, because the TOV equations will yield a continuous solution for the metric functions, starting from the interior of the star, extending to the surface of the star and these solutions will describe the star asymptotically, thus at \(r \to \infty\). At exactly this point, the numerical infinity, the exterior spacetime will be a Schwarzschild spacetime, and this is the only condition imposed on the metric functions \(\nu(r)\) and \(\frac{1}{1-2m(r)}\).

These have to be matched with the corresponding Schwarzschild ones. The solutions for the metric functions \(\nu(r)\) and \(\frac{1}{1-2m(r)}\) will be obtained numerically by solving the TOV equations continuously from the interior until the numerical infinity. The difference between the interior of the star and the exterior is that the exterior does not have contribution from the matter of the star, thus the pressure and the energy density outside the star are zero. However, the potential is not switched off, thus it affects the metric function solutions even outside the star. This is the major difference from the matter of the star, thus the pressure and the energy density outside the star are zero. However, the potential is still present thus it affects the star beyond its surface. This is exactly why no matching outside the star is needed at the surface. The numerical solutions of the TOV equations will yield continuous uniform solutions for the quantities. Also the interior and the interior spacetime are uniformly described by the metric \((39)\), see for example \([49]\). It is the aim of any study in scalar-tensor astrophysics to find, numerically, the metric functions \(\nu(r)\) and \(\frac{1}{1-2m(r)}\).

The initial conditions for the TOV equations are,

\[
P(0) = P_c, \quad m(0) = 0, \quad \nu(0) = -\nu_c, \quad \varphi(0) = \varphi_c, \quad \omega(0) = 0.
\]

(45)

The condition \(m(0) = 0\) means that the gravitational mass for zero radius is zero. This however does not make the metric function \(\nu(r)\) to be non-zero at zero radius. The exact value \(\nu(0) = -\nu_c\) will be obtained by the double shooting method. With regard to the EoS, we shall use a piecewise polytropic EoS \([103, 104]\) (see also \([102]\), with the low density parameters corresponding to the SLy, WFF1 or the APR EoSs. For the piecewise polytropic EoS, the relation between the energy density and pressure is,

\[
\epsilon(\rho) = (1 + \alpha_i)\rho + \frac{K_i}{\Gamma_i - 1}\rho^{\Gamma_i}, \quad \alpha_i = \frac{\epsilon(\rho_{i-1})}{\rho_{i-1}} - 1 - \frac{K_i}{\Gamma_i - 1}\rho_{i-1}^{\Gamma_i - 1},
\]

(46)

where the \(i\) refers to the three different pieces of the polytropic equation of state. Let us elaborate on this further, the energy density and the pressure in each of the three piecewise density intervals \(\rho_{i-1} \leq \rho \leq \rho_i\) satisfy the polytropic relation,

\[
P = K_i\rho^{\Gamma_i},
\]

(47)
Hereafter we shall identify with the Jordan and Einstein frame radii of the NS are related as follows,

\[ M = K_{i+1} \rho_{i}^{\Gamma_{i+1}} = K_{i} \rho_{i}^{\Gamma_{i}}, \]

and from the above relations, the parameters \( K_2 \) and \( K_3 \) are obtained, given \( K_1, \Gamma_1, \Gamma_2, \Gamma_3 \), or equivalently, given the initial pressure \( p_i \) and for given parameters \( \Gamma_2 \), and \( \Gamma_3 \), which are not chosen arbitrarily. Upon integrating the first thermodynamic law for barotropic fluids,

\[ \frac{d\epsilon}{\rho} = -Pd\frac{1}{\rho}, \]

in conjunction with the continuity requirement at each piece of the polytropic, yields Eq. (46). Now let us discuss the gravitational mass issue for the NS. The gravitational mass of the NS which we shall consider is the ADM mass in the Jordan frame. Thus when we extract the numerical results, we need to transform the obtained Einstein frame ADM mass to the Jordan frame. We define the auxiliary functions \( \mathcal{K}_E \) and \( \mathcal{K}_J \) in Geometrized units,

\[ \mathcal{K}_E = 1 - \frac{2m}{r_E}, \quad \mathcal{K}_J = 1 - \frac{2m_J}{r_J}, \]

which are basically the metric functions in the Einstein and Jordan frames, with \( m \) and \( m_J \) being the gravitational mass confined in a radius \( r \). The metric functions along with the metric radius parameter in the two frames are related as follows,

\[ \mathcal{K}_J = A^{-2} \mathcal{K}_E, \quad r_J = A r_E, \]

and in addition, the Jordan and Einstein frame ADM masses are,

\[ M_J = \lim_{r \to \infty} \frac{r_J}{2} (1 - \mathcal{K}_J), \quad M_E = \lim_{r \to \infty} \frac{r_E}{2} (1 - \mathcal{K}_E). \]

Taking the asymptotic limit of Eq. (51), we obtain,

\[ \mathcal{K}_J (r_E) = \left( 1 + \alpha(r_E) \frac{d\phi}{dr} r_E \right)^2 \mathcal{K}_E (r_E), \]

where \( r_E \) denotes the radius in the Einstein frame asymptotically (not at the numerical infinity though, slightly smaller) and in addition \( \frac{d\phi}{dr} = \frac{d\phi}{dr} \bigg|_{r=r_E} \). Upon combining Eqs. (50)-(53) we acquire

\[ M_J = A(\phi(R_E)) \left( M_E - \frac{r_E^2}{2} \alpha(\phi(r_E)) \frac{d\phi}{dr} \left( 2 + \alpha(\phi(r_E)) r_E \frac{d\phi}{dr} \right) \left( 1 - \frac{2M_E}{r_E} \right) \right). \]

Finally, the Jordan and Einstein frame radii of the NS are related as follows,

\[ R = A(\phi(R_s)) R_s. \]

Hereafter we shall identify with \( M \) the Jordan frame mass of Eq. (54), that is \( M = M_J \) measured in solar masses, and the radius in the Jordan frame \( R \) shall be expressed in kilometers. At this point we shall proceed to the presentation of our numerical analysis of the TOV equations. We shall solve the TOV equations numerically using a python 3 based numerical code which is a variant form of the pyTOV-STT code using the “LSODA” integrator, in order to extract the Jordan frame mass and the circumferential radius of the NS. The method includes a double shooting method in order to find the optimal values for the the metric function \( \nu \), and for the scalar field \( \varphi \), at the center of the NS, which make the values of the scalar field and of the metric function to vanish at the numerical infinity, with the latter being chosen in kilometers to be \( r \sim 67.94378528694695 \) km.

In Fig. 1 we present the \( M - R \) graphs of the universal attractor models when compared to the corresponding GR curves, for the WFF1 EoS (upper), for the APR EoS (middle) and for the SLy EoS (bottom plot). In order to produce Fig. 1, we numerically solved the TOV equations and we extracted the masses and radii corresponding to 160 central densities, namely \( M(\rho_c) \) and \( R(\rho_c) \) and then we generated the \( M - R \) graph using the resulting values for the mass and radii for each of these 160 central densities. Also in Fig. 2, we compare the universal attractors \( M - R \) curves, for the three distinct EoSs. A first interesting result is that the WFF1 EoS for the universal attractor model is compatible with the GW170817 constraint derived in Ref. [20], which indicates that \( M \sim 1.6M_\odot \) NSs must have radii in the
FIG. 2: Comparison of the $M - R$ graphs for the universal attractor model for the WFF1, APR and SLy EoSs. The GW170817 indicate that for a NS of mass $M \sim 1.6 M_\odot$, the predicted radii for the universal attractor models must be larger than $R = 10.68^{+15}_{-0.04}$km. Also the GW170817 event indicates that the radii corresponding to the maximum mass must be larger than $R = 9.6^{0.14}_{-0.03}$km. All the GW170817 constraints are satisfied as it can be seen in the plots.

TABLE I: Masses with $\frac{M}{M_\odot} \simeq 1$ and the Corresponding Radii of Static NS for the Universal Attractors and for GR

| Model          | APR EoS | SLy EoS | WFF1 EoS |
|----------------|---------|---------|----------|
| GR $M_{APR} = 1.0261 M_\odot$ | $M_{SLy} = 1.033 M_\odot$ | $M_{WFF1} = 1.025 M_\odot$ |
| Universal Attractors $M_{APR} = 1.092 M_\odot$ | $M_{SLy} = 1.079 M_\odot$ | $M_{WFF1} = 1.075 M_\odot$ |
| Universal Attractors Radii $R_{UNI} = 11.463$km | $R_{UNI} = 11.827$km | $R_{UNI} = 10.590$km |

range $R = 10.68^{+15}_{-0.04}$km. This is in contrast to the GR case, where the WFF1 EoS is excluded by the GW170817 data. Secondly, in all the studied cases, the maximum mass NS configurations for the three distinct EoSs, satisfy the second constraint of GW170817 derived in Ref. [36], which indicates that the maximum mass configurations must have radii larger than $R = 9.6^{0.14}_{-0.03}$km. Also in Table I we present the radii of the static NS for all the EoS in both GR and the universal attractors models, for which $\frac{M}{M_\odot} \simeq 1$. Note that the limit $R \to 0$ is never reached by the neutron stars, because these are GR objects bound from gravity solely even in the context of scalar-tensor gravity. In contrast, strange stars could reach the limit of very small radii. Furthermore, in Table II we gather the data for the maximum masses and the corresponding radii, for the GR NS and the universal attractors NS, and for all the EoSs studied in this paper. Finally, we need to explicitly check whether the approximation of Eq. (33), and in Fig. 3 we present the fraction of $\frac{\xi^2}{1 - \xi^2}$ versus the central densities, for the three distinct EoSs, and for the values of the scalar field at the surface of the star. As it can be seen, the constraint of Eq. (33) is safely satisfied for the values of the parameter $\xi$ we used in this article. Also for brevity we did not include the case for the values of the scalar field at the center of the star, in which case the approximation of Eq. (33) is satisfied.

TABLE II: Maximum Masses and the Corresponding Radii of Static NS for the Universal Attractors and for GR

| Model          | APR EoS | SLy EoS | WFF1 EoS |
|----------------|---------|---------|----------|
| GR $M_{max} = 2.18739372 M_\odot$ | $M_{max} = 2.04785291 M_\odot$ | $M_{max} = 2.12603999 M_\odot$ |
| Universal Attractors $M_{max} = 2.41712697 M_\odot$ | $M_{max} = 2.27234095 M_\odot$ | $M_{max} = 2.32003695 M_\odot$ |
| Universal Attractors Radii $R = 10.54678577$km | $R = 10.56752764$km | $R = 9.9118728$km |
Concluding Remarks

In this paper we studied the phenomenology of NSs for universal attractor non-minimally coupled scalar theories of inflation. The universal attractors are known in cosmological contexts since these provide a viable inflationary era and also belong to a larger class of cosmological attractors that are similar to the $R^2$ inflation. We investigated how the universal attractors can be obtained in the strong coupling limit, that is for large $\xi$, so the large coupling limit constraint must be satisfied by the resulting values of the scalar field in the Einstein frame. After demonstrating the essential features of the universal attractor theories, we used the theoretical astrophysics context and we found all the quantities that are involved in the Einstein frame TOV equations. We solved numerically the TOV equations using a double shooting method of a python 3 code, and we constructed the $M-R$ graphs for all the different EoS we studied. The resulting numerical values for the masses and radii of the NSs were the Jordan frame ones, calculated from the resulting Einstein frame quantities delivered by the numerical code. The results of our analysis are interesting since we demonstrated that the WFF1 EoS which was excluded by the GW170817 data in the context of GR, it is not anymore excluded for the universal attractors model, since for a NS of mass $M \sim 1.6 M_\odot$, the predicted radii for the universal attractor models are larger than $R = 10.68^{+15}_{-0.04}$ km. Also all the three distinct EoS for the universal attractor models, predict higher maximum radii compared to the GR ones, and moreover all the radii respect the constraint of the GW170817 event which indicates that the radii corresponding to the maximum masse must be larger than $R = 9.69^{+14}_{-0.04}$ km. A crucial issue we did not address is related to the question whether the attractor property satisfied by the inflationary theories, is also satisfied by the NSs. Work is in progress in this line of research.
Finally, let us comment that the inflationary era obviously has no effect on the neutron stars for the universal attractor potential. Obviously the two eras are not connected, since the scalar field value during inflation is much larger compared to the values of the scalar field in and outside of the neutron star. The only constraint coming from the inflationary era is on the parameter $V_0$, the constant coupling of the scalar potential, and this is required satisfy Eq. (22).

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[103] J. S. Read, B. D. Lackey, B. J. Owen and J. L. Friedman, Phys. Rev. D 79 (2009), 124032
[104] J. S. Read, C. Markakis, M. Shibata, K. Uryu, J. D. E. Creighton and J. L. Friedman, Phys. Rev. D 79 (2009), 124033
[105] R. B. Wiringa, V. Fiks and A. Fabrocini, Phys. Rev. C 38 (1988), 1010-1037 doi:10.1103/PhysRevC.38.1010
[106] F. Douchin and P. Haensel, Astron. Astrophys. 380 (2001), 151 doi:10.1051/0004-6361:20011402 [arXiv:astro-ph/0111092 [astro-ph]].
[107] A. Akmal, V. R. Pandharipande and D. G. Ravenhall, Phys. Rev. C 58 (1998), 1804-1828 doi:10.1103/PhysRevC.58.1804 [arXiv:nucl-th/9804027 [nucl-th]].
[108] R. Arnowitt, S. Deser and C. W. Misner, Phys. Rev. 118 (1960), 1100-1104 doi:10.1103/PhysRev.118.1100
[109] D. I. Kaiser, Phys. Rev. D 52 (1995), 4295-4306 doi:10.1103/PhysRevD.52.4295 [arXiv:astro-ph/9408044 [astro-ph]].
[110] Valerio Faraoni, Cosmology in Scalar-Tensor Gravity, Springer 2004
[111] K. Bhattacharya, B. R. Majhi and D. Singleton, JHEP 07 (2020), 018 doi:10.1007/JHEP07(2020)018 [arXiv:2002.04743 [hep-th]].
[112] S. D. Odintsov and V. K. Oikonomou, [arXiv:2103.07725 [gr-qc]].