Few-nucleon bound states using the unsymmetrized HH expansion

M Gattobigio$^1$, A Kievsky$^2$ and M Viviani$^2$

$^1$ INLN, Université de Nice-Sophia Antipolis, CNRS, 1361 route des Lucioles, 06560 Valbonne, France
$^2$ Istituto Nazionale di Fisica Nucleare, Sezione di Pisa, Largo B. Pontecorvo 3, I-56127, Pisa, Italy
E-mail: Mario.Gattobigio@inln.cnrs.fr

Abstract. The hyperspherical harmonic basis is used to describe bound states in an $A$-body system up to six bodies. In this basis the kinetic energy matrix is diagonal whereas the potential matrix is presented in a compact form, well suited for numerical implementation. The basis is neither symmetrized nor antisymmetrized, as required in the case of identical particles. However, after the diagonalization of the Hamiltonian matrix, the eigenvectors reflect the symmetries present in it, and the identification of the physical states is possible, as it will be shown in specific cases. As an example we solve the case of $A = 3, 4, 5, 6$ particles interacting through a short-range central interaction with and without the inclusion of the Coulomb potential.

1. Introduction

The ab initio description of light nuclear systems, starting from the nucleon-nucleon (NN) interaction, requires well established methods to solve the Schrödinger equation. Among them, the Green’s function Monte Carlo (GFMC) method has been extensively used to describe light nuclei up to $A = 10$ and the no-core shell model (NCSM) up to $A = 12$ [1, 2]. In the $A \leq 4$ systems, well established methods for treating both bound and scattering states exist as the Faddeev equations ($A = 3$) and the Faddeev-Yakubovsky equations ($A = 4$) in configuration or momentum space, and the Hyperspherical Harmonic (HH) expansion. All these methods have proven to be of great accuracy and they have been tested using different benchmarks [3, 4, 5].

The HH method provides a systematic way of constructing a complete basis for the expansion of the $A$-particle wave function. In the specific case of applications to nuclear physics, the wave function has to be antisymmetric and, therefore, the HH basis has been managed to produce basis states having well defined properties under particle permutations. However, it is possible to use the HH basis without a previous symmetrization procedure [6]. It has been observed that the eigenvectors of the Hamiltonian matrix reflects the symmetries present in it, even if it has been constructed using the non-symmetrized basis. The only requirement was to include all the HH basis elements having the same grand angular quantum number $K$. It is a property of the HH basis that basis elements having well defined behavior under particle permutation can be constructed as a linear combination of HH elements having the same value of $K$. Accordingly, the diagonalization procedure generates eigenvectors with a well defined permutation symmetry that can be organized in accordance with the irreducible representations of the permutation group of $A$ objects, $S_A$. Identifying those eigenvectors with the desired symmetry, the corresponding
energies can be considered variational estimates. In particular, in Ref. [6], it was possible to identify a subset of eigenvectors and eigenvalues corresponding exactly to those that would be obtained performing the preliminary symmetrization of the states. It should be noticed that the simplicity of using the HH basis without a preliminary antisymmetrization step has to be counterbalanced with the large dimension of the matrices to be diagonalized. However, at present, different techniques are available to treat (at least partially) this problem.

As mentioned, one of the main problems in using the HH basis is its large degeneracy, resulting in very large matrices. On the other hand, the potential energy matrix, expressed as a sum of pairwise interactions, cannot connect arbitrary basis elements differing in some specific quantum numbers. This means that in some representation each pairwise-interaction term is represented by a sparse matrix. The matrix representation of the potential $V(1, 2)$, constructed in terms of basis elements in which the quantum numbers of particles (1, 2) are well defined, is sparse in $A \geq 3$ systems. Its matrix elements connecting basis elements with different quantum numbers labelling states which do not involve particles (1, 2) are zero. A problem arises when the matrix elements of the generic term $V(i, j)$, defining the interaction between particles $(i, j)$, has to be calculated using basis elements in which the quantum numbers of particles $(i, j)$ are not well defined. One operative way to solve this problem consists in rotating the basis to a system of coordinates in which particles $(i, j)$ have well defined quantum numbers. This makes the matrix $V(i, j)$ sparse. Conversely, the rotation matrix is not sparse but it can be expressed as a product of sparse matrices, each one representing a rotation which involves a permutation of particles of successive numbering. After these manipulations the potential energy matrix results in a sum of products of sparse matrices suitable for numerical implementations.

2. The unsymmetrized HH basis
Following Refs. [6, 7], we present a brief overview of the properties of the HH basis and its implementation without generating basis elements with well defined permutation symmetry. We start with the following definition of the Jacobi coordinates for an equal-mass $A$-body system with Cartesian coordinates $r_1 \ldots r_A$

$$x_{N-j+1} = \sqrt{\frac{2j}{j+1}} (r_{j+1} - X_j), \quad j = 1, \ldots, N, \quad (1)$$

where $N = A - 1$ and $X_j = \sum_{i=1}^{j} r_i / j$. For a given set of Jacobi coordinates $x_1, \ldots, x_N$, we can introduce the hyperradius $\rho$

$$\rho = \left( \sum_{i=1}^{N} x_i^2 \right)^{1/2} = \left( \frac{2}{A} \sum_{i=1}^{A} (r_i - X)^2 \right)^{1/2} = \left( \frac{2}{A} \sum_{j>i}^{A} (r_j - r_i)^2 \right)^{1/2}, \quad (2)$$

with $X = \sum_{i=1}^{A} r_i / A$ the center of mass, and the hyperangular coordinates $\Omega_N$

$$\Omega_N = (\hat{x}_1, \ldots, \hat{x}_N, \phi_2, \ldots, \phi_N), \quad (3)$$
with the hyperangles \( \phi_i \) defined via

\[
\begin{align*}
x_N &= \rho \cos \phi_N \\
x_{N-1} &= \rho \sin \phi_N \cos \phi_{N-1} \\
&\vdots \\
x_i &= \rho \sin \phi_N \cdots \sin \phi_{i+1} \cos \phi_i \\
&\vdots \\
x_2 &= \rho \sin \phi_N \cdots \sin \phi_3 \cos \phi_2 \\
x_1 &= \rho \sin \phi_N \cdots \sin \phi_3 \sin \phi_2.
\end{align*}
\]

The explicit expression for the HH functions, having well defined values of \( LM \), is

\[
\mathcal{Y}^{LM}_{[K]}(\Omega_N) = \left[ \prod_{j=2}^{N} \mathcal{P}_{K_j}^{\alpha_l, \alpha_{K_j-1}}(\phi_j) \right] \left[ Y_{l_1}(\hat{\mathbf{x}}_1) \otimes Y_{l_2}(\hat{\mathbf{x}}_2) | L_2 \ldots \otimes Y_{L_{N-1}}(\hat{\mathbf{x}}_{N-1}) | L_{N-1} \otimes Y_N(\hat{\mathbf{x}}_N) \right]_{LM},
\]

with the indicated coupling scheme. The hyperspherical polynomial is

\[
\mathcal{P}_{K_j}^{\alpha_l, \alpha_{K_j-1}}(\phi_j) = N_n^{\alpha_l, \alpha_{K_j}}(\cos \phi_j)^{l_j}(\sin \phi_j)^{K_j-1}P_n^{\alpha_{K_j-1}, \alpha_l}(\cos 2\phi_j).
\]

The set of quantum numbers \([K]\) includes the \( n_2 \ldots n_N \) indices of the Jacobi polynomials, the \( l_1 \ldots l_N \) angular momenta of the particles and the intermediate couplings \( L_2 \ldots L_{N-1} \). The \( K_j \) quantum numbers are defined as

\[
K_j = \sum_{i=1}^{j} (l_i + 2n_i), \quad n_1 = 0, \quad K \equiv K_N,
\]

\( K \equiv K_N \) is known as the grand angular momentum, and \( N_n^{\alpha_l, \alpha_{K_j}} \) is a normalization factor. For the definition of the \( \alpha_{K_j} \), where \( a \) can be either an angular momentum \( l_j \) or a quantum number \( K_j \), one needs to introduce the hyperspherical-binary-tree structure [8]. For example the tree of figure 1 corresponds to the choice of hyperangles given by Eq. (4), in which the coefficients specialize to \( \alpha_{K_j} = K_j + 3j/2 - 1 \) and \( \alpha_{l_j} = l_j + 1/2 \).

Hyperspherical functions constructed on top of different hyperspherical-coordinate definitions can be related using the \( T \)-coefficients [9, 10]. Schematically, these coefficients relate the following tree structures

\[
\begin{array}{c}
\begin{tikzpicture}
  \node (l_i) at (0,0) {$l_i$};
  \node (K_{i-1}) at (1.5,1.5) {$K_{i-1}$};
  \node (K_i) at (1.5,3) {$K_i$};
  \node (K_{i+1}) at (3,4.5) {$K_{i+1}$};
  \node (phi_{i-1}) at (1.5,2) {$\phi_{i-1}$};
  \node (phi_i) at (1.5,1) {$\phi_i$};
  \draw (l_i) edge (K_{i-1});
  \draw (K_{i-1}) edge (K_i);\end{tikzpicture}
\end{array} = \sum_{n_{i-1}=0}^{N_i} T_{n_{i-1} n_i K_i}^{\alpha_{K_{i-1}} \alpha_{n_{i-1}} \alpha_i},
\]

where \( K_i = K_{i-1} + l_i + 2n_i = K_{i+1} + l_i + 2\hat{n}_i \). The explicit definition of the coefficients is given in Ref. [6]. Let us call \( \mathcal{Y}^{LM}_{[K]}(\Omega_N) \) the HH basis element constructed in terms of a set of Jacobi
coordinates in which the \( i \)-th and \( i + 1 \)-th Jacobi vectors result from the transposition between particles \( j, j + 1 \)

\[
x_i' = -\frac{1}{j} x_i + \frac{\sqrt{(j + 1)^2 - 2(j + 1)}}{j} x_{i+1}
\]

\[
x_{i+1}' = \frac{\sqrt{(j + 1)^2 - 2(j + 1)}}{j} x_i + \frac{1}{j} x_{i+1},
\]

with all the other vectors equal to the original ones (transposed basis). The coefficients

\[
A_{LM}^{i,[K][K']} = \int d\Omega_N |Y_{LM}^{[K]}(\Omega_N)|^* Y_{LM}^{[K']}(\Omega_N),
\]

are the matrix elements of a matrix \( A_{LM}^{i} \) that allows to express the transposed HH basis elements in terms of the reference basis. The coefficients \( A_{LM}^{i,[K][K']} \) form a very sparse matrix and they can be calculated analytically using the \( T \)-coupling coefficients and the Raynal-Revai matrix elements. A generic rotation between the reference HH basis and a basis in which the last Jacobi vector is defined as \( x_N' = r_j - r_i \) can be constructed as successive products of the \( A_{LM}^{i,[K][K']} \) coefficients. Defining \( Y_{LM}^{[K]}(\Omega_N) \) the HH basis element constructed in terms of a set of Jacobi coordinates in which the \( N \)-th Jacobi vector is defined \( x_N' = r_j - r_i \), this coefficient can be given in the following form

\[
B_{ij,[K][K']}^{LM} = \int d\Omega |Y_{LM}^{[K]}(\Omega_N)|^* Y_{LM}^{[K]}(\Omega_N) = [A_{LM}^{1} \cdots A_{LM}^{n}]_{[K][K']}.
\]

The particular values of the indices \( i_1, \ldots, i_n \), labelling the matrices \( A_{LM}^{i_1}, \ldots, A_{LM}^{i_n} \), depend on the pair \((i, j)\). The matrix

\[
B_{ij}^{LM} = A_{i_1}^{LM} \cdots A_{i_n}^{LM},
\]
is written as a product of the sparse matrices $A^{LM}_{ij}$.

We consider now the potential energy of an $A$-body system constructed in terms of two-body interactions

$$V = \sum_{i<j} V(i,j). \quad (13)$$

In terms of the HH basis it results

$$\sum_{ij} V_{ij}(\rho) = \sum_{ij} [B^{LM}_{ij}] V_{12}(\rho) B^{LM}_{ij}, \quad (14)$$

where the matrix elements of the matrix $V_{12}(\rho)$ are defined as

$$V^{(1,2)}_{[K],[K']}(\rho) = \langle \gamma^{LM}_{[K]}(\Omega_N)|V(1,2)|\gamma^{LM}_{[K']}(\Omega_N) \rangle = \delta_{l_1,l'_1} \cdots \delta_{l_N,l'_N} \delta_{L_1,K_2} \cdots \delta_{L_N,K'_N} \times \int d\phi_N (\cos \phi_N \sin \phi_N)^2 \mathcal{P}^\alpha_{K_N} \mathcal{P}^\beta_{K_{N-1}} V(\rho \cos \phi_N) \mathcal{P}^\alpha_{K_N} \mathcal{P}^\beta_{K_{N-1}}(\phi_N). \quad (15)$$

Each term of the sum in Eq.(14) results in a product of sparse matrices, a property which allows an efficient implementation of matrix-vector product.

3. Results
In this section we present results for $A = 3 - 6$ systems obtained by a direct diagonalization of the Hamiltonian of the system. The corresponding Hamiltonian matrix is obtained using the following orthonormal basis

$$\langle \rho \Omega | m [K] \rangle = \left( \beta^{(\alpha+1)/2} \sqrt{\frac{m!}{(\alpha+m)!}} L^{(\alpha)}_m(\beta \rho) e^{-\beta \rho/2} \right) \gamma^{LM}_{[K]}(\Omega_N), \quad (16)$$

where $L^{(\alpha)}_m(\beta \rho)$ is a Laguerre polynomial with $\alpha = 3N - 1$ and $\beta$ a variational non-linear parameter. The matrix elements of the Hamiltonian are obtained after integrations in the $\rho, \Omega$ spaces. They depend on the indices $m, m'$ and $[K], [K']$ as follows

$$\langle m' [K'] | H | m [K] \rangle = -\frac{\hbar^2 \beta^2}{m} \left( T^{(1)}_{m'm} - K(K+3N-2)T^{(2)}_{m'm} \right) \delta_{[K']|K} + \sum_{ij} \sum_{[K''][K''']} B^{ij,LM}_{[K][K'']} B^{ij,LM}_{[K'''][K']} V^{m,m'}_{[K][K'']} \quad (17)$$

The matrices $T^{(1)}$ and $T^{(2)}$ have an analytical form and are given in Ref. [6]. The matrix elements $V^{m,m'}_{[K][K']}$ are obtained after integrating the matrix $V_{12}(\rho)$ in $\rho$-space (we will call the corresponding matrix $V_{12}$). Introducing the diagonal matrix $D$ such that $\langle [K'] | D | [K] \rangle = \delta_{[K],[K']} K(K+3N-2)$, and the identity matrix $I$ in $K$-space, we can rewrite the Hamiltonian schematically as

$$H = -\frac{\hbar^2 \beta^2}{m} (^{(1)} T \otimes I + ^{(2)} T \otimes D) + \sum_{ij} [B^{LM}_{ij}] V_{12} B^{LM}_{ij}, \quad (18)$$

in which the tensor product character of the kinetic energy is explicitly given.
As a central potential we have chosen the Volkov potential

$$V(r) = V_R e^{-r^2/R_i^2} + V_A e^{-r^2/R_A^2},$$

with $V_R = 144.86$ MeV, $R_1 = 0.82$ fm, $V_A = -83.34$ MeV, and $R_2 = 1.6$ fm. The nucleons are considered to have the same mass chosen to be equal to the reference mass $m$ and corresponding to $\hbar^2/m = 41.47$ MeV fm$^2$. With this parametrization of the potential, the two-nucleon system has a binding energy $E_{2N} = 0.54592$ MeV and a scattering length $a_{2N} = 10.082$ fm. This potential has been used several times in the literature making its use very useful to compare different methods [11, 12, 13, 14]. The use of central potentials in general produces too much binding, in particular the $A = 5$ system results bounded. Conversely, the use of the $s$-wave version of the potential produces a spectrum much closer to the experimental situation. This is a direct consequence of the weakness of the nuclear interaction in $p$-waves. Accordingly, we analyze both versions of the potential, the central Volkov potential and the $s$-wave projected potential. The results are obtained after a direct diagonalization of the Hamiltonian matrix of Eq.(17) including $m_{\text{max}} + 1$ Laguerre polynomials with a fix value of $\beta$, and all HH states corresponding to maximum value of the grand angular momentum $K_{\text{max}}$. The scale parameter $\beta$ can be used as a non-linear parameter to study the convergence in the index $m = 0, 1, \ldots, m_{\text{max}}$, with $m_{\text{max}}$ the maximum value considered. In the present analysis the convergence will be studied with respect to the index $K_{\text{max}}$, therefore, the number of Laguerre polynomials at each step, $m_{\text{max}} + 1$, will be sufficiently large to guarantee independence from $\beta$ of the physical eigenvalues and eigenvectors. We found that $m_{\text{max}} + 1 \approx 20$ Laguerre polynomials (with proper values of $\beta$) were sufficient for an accuracy of 0.1% in the calculated eigenvalues.

In table 1 we present the pattern of convergence for selected bound states in $A = 3, 4, 5, 6$ systems using the Volkov potential acting in all partial waves or acting only in $s$-wave. For $A = 3, 4$ the ground state of the system corresponds to the states $(L, S, T)J^\pi = (0, 1/2, 1/2)1/2^+ (A = 3)$ and $(L, S, T)J^\pi = (0, 0, 0)0^+ (A = 4)$. In the case of systems with $A > 4$ the spatially-symmetric state cannot be antisymmetrized using the corresponding spin-isospin functions. In the $A = 5$ system the deepest two levels having positive parity and $L = 0$ correspond to a completely symmetric state (the irreducible representation of $S_5$ [5]). The third level belongs to the irreducible representation of $S_5$ [4 1]; it can be antisymmetrized using the $A = 5$ spin-isospin functions with $S = 1/2, T = 1/2$, therefore it represents the lowest level of the $(L, S, T)J^\pi = (0, 1/2, 1/2)1/2^+$ state of five nucleons. The convergence of this state in terms of $K_{\text{max}}$ is given in the fourth column of the table 1. It can be observed that the [4 1] state is not bounded, in agreement with the fact that the $A = 5$ nucleus does not exist. Its energy results to be above the threshold of 30.42 MeV describing an $^4$He nucleus plus a fifth nucleon far away (here the Coulomb interaction has not been included). In the case of the $A = 6$ system we concentrate the analysis in the $(L, S, T)J^\pi = (0, 0, 1)0^+$ and $(0, 1, 0)1^+$ states which, using a central potential, and disregarding the Coulomb interaction, are degenerate. Including the Coulomb interaction between two protons, the first state has the quantum numbers of $^6$He. A direct diagonalization of the six body Hamiltonian using the non-symmetrized HH basis, with the Volkov potential, produces a spectrum in which the first two levels belongs to the [6] irreducible representation of $S_6$. The third level belongs to the [5 1] representation whereas the fourth level belongs to the [4 2] representation, and it is the first one that can be symmetrized using the $A = 6$ spin-isospin functions with $S = 0, T = 1$ or $S = 1, T = 0$. The convergence pattern of this level in terms of $K_{\text{max}}$ is shown in the last column of table 1. Similar to the $A = 5$ case, the Volkov potential acting in all waves predicts large binding energies. Using the $s$-wave potential a much more reasonable value of $\approx 34$ MeV is obtained for this level.

For $A = 6$, when the Coulomb interaction between two protons is considered, the degeneracy of the [4 2] level (of dimension 9) is broken and four different states appear. It is possible to identify the physical state looking at the symmetry of the four neutrons. One of the states
Table 1. Ground state of the $A = 3, 4$ systems (in MeV) and selected states of the $A = 5, 6$ systems (see text) using the all-waves (upper part) and $s$-wave (lower part) Volkov potential as a function of the maximum grand angular quantum number $K_{\text{max}}$.

| $K_{\text{max}}$ | $A = 3$ | $A = 4$ | $K_{\text{max}}$ | $A = 5$ | $K_{\text{max}}$ | $A = 6$ |
|------------------|---------|---------|------------------|---------|------------------|---------|
| 0                | 7.7075  | 28.580  | 4                | 22.043  | 2                | 61.142  |
| 10               | 8.4157  | 30.278  | 8                | 25.568  | 6                | 63.377  |
| 20               | 8.4623  | 30.416  | 12               | 27.043  | 10               | 65.354  |
| 30               | 8.4647  | 30.418  | 16               | 27.862  | 14               | 66.201  |
| 40               | 8.4649  | 30.418  | 20               | 28.371  | 18               | 66.437  |
|                  |         |         | 24               | 28.719  | 22               | 66.491  |
| SVM [12]         | 8.46    | 30.42   | 43.00            | 66.25   |                  |         |
| Ref. [11]        | 8.462   | 30.406  | 42.383           |         |                  |         |

| $K_{\text{max}}$ | $A = 3$ | $A = 4$ | $K_{\text{max}}$ | $A = 5$ | $K_{\text{max}}$ | $A = 6$ |
|------------------|---------|---------|------------------|---------|------------------|---------|
| 0                | 7.7075  | 28.580  | 3                | 24.317  | 2                | 24.793  |
| 10               | 8.3812  | 30.116  | 7                | 26.971  | 6                | 30.723  |
| 20               | 8.4283  | 30.250  | 11               | 27.971  | 10               | 32.244  |
| 30               | 8.4307  | 30.252  | 15               | 28.521  | 14               | 33.075  |
| 40               | 8.4309  | 30.252  | 19               | 28.823  | 18               | 33.561  |
|                  |         |         | 23               | 29.005  | 22               | 33.814  |

belongs to the $[4]$ representation, two belong to the $[3 1]$ representation and the last one belongs to the $[2^2]$ representation of $S_4$. This last state is the only one that can be antisymmetrized using the spin functions of four neutrons having $S_n = 0$. Moreover, the proton state is spatially symmetric and therefore can be antisymmetrized with the spin function $S_p = 0$ making a total $S = 0$ state. This can be seen as

\[
[4^2] \rightarrow [1^2] \otimes [3 1] + [2] \otimes [4] + [2] \otimes [3 1] + [2] \otimes [2^2], \tag{20}
\]

where only $[2] \otimes [2^2]$ is physical and describes the ground state of $^6\text{He}$. When the Coulomb interaction is extended to three particles, the symmetry breaking is $S_6 \rightarrow S_3 \otimes S_3$, and the split reads

\[
[4^2] \rightarrow [2 1] \otimes [3] + [2 1] \otimes [2 1] + [3] \otimes [3] + [3] \otimes [2 1], \tag{21}
\]

with the only physical state, describing $^6\text{Li}$, being $[2 1] \otimes [2 1]$. In table 2 we show the results for the $L = 0$ state.

4. Conclusions
We have shown the power and the flexibility of the unsymmetrized HH approach. In particular we were able to include basis states up to $K = 22$ for a six-body system (corresponding to a basis set of 38 798 760 elements using 11 Laguerre polynomials). For these large matrices it is possible to restrict the search of the eigenvectors to those having a particular symmetry using a symmetry-adapted Lanczos method [15]. Essentially, starting with a vector having the desired symmetry, after each iteration of the matrix-vector product, the new vector is projected onto the sub-space of the selected symmetry. Following Ref. [15], an intermediate purification step is also implemented. This method has the characteristic of finding eigenvalues corresponding to eigenvectors of one particular symmetry simplifying the search procedure and the identification of the eigenvectors. The extension of this method to treat realistic NN potentials is in progress.
Table 2. Binding energies calculated with Volkov’s potential in s-wave, for $A = 6$ particles and $L = 0$, with and without Coulomb interaction, using 11 Laguerre’s polynomials.

| $K_{\text{max}}$ | $N_{\text{HH}}$ | $E_6^{[\text{He}]_{[2]} \otimes [2]}$ (MeV) | $E_6^{[\text{Li}]_{[2]} \otimes [2]}$ (MeV) |
|------------------|------------------|--------------------------------------|---------------------------------------|
| 2                | 15               | 24.064                               | 22.974                                |
| 4                | 120              | 28.016                               | 26.988                                |
| 6                | 680              | 29.935                               | 28.947                                |
| 8                | 3045             | 30.851                               | 29.889                                |
| 10               | 11427            | 31.446                               | 30.496                                |
| 12               | 37310            | 31.908                               | 30.964                                |
| 14               | 108810           | 32.275                               | 31.334                                |
| 16               | 288990           | 32.558                               | 31.620                                |
| 18               | 709410           | 32.762                               | 31.827                                |
| 20               | 1628328          | 32.912                               | 31.980                                |
| 22               | 3527160          | 33.016                               | 32.087                                |

References

[1] Pieper S C, Varga K and Wiringa R B 2002 Phys. Rev. C 66 044310
[2] Navrátil P, Gueorguiev V G, Vary J P, Ormand W E and Nogga A 2007 Phys. Rev. Lett. 99 042501
[3] Kievsky A et al. 1998 Phys. Rev. C 58 3085
[4] Lazauskas R et al. 2005 Phys. Rev. C 71 034004
[5] Kamada H et al. 2001 Phys. Rev. C 64 044001
[6] Gattobigio M, Kievsky A and Viviani M 2011 Phys. Rev. C 83 024001
[7] Gattobigio M, Kievsky A, Viviani M and Barletta P 2009 Phys. Rev. A 79 032513
[8] Vilenkin N Ya, Kuznetsov G I and Smorodinskii Ya A 1966 Sov. J. Nucl. Phys. 2 645
[9] Kil’dyushov M S 1972 Yad. Fiz. 15 197 [1972 Sov. J. Nucl. Phys. 15 113]
[10] Kil’dyushov M S 1972 Yad. Fiz. 16 217 [1973 Sov. J. Nucl. Phys. 16 117]
[11] Barnea N 1999 Phys. Rev. A 59 1135
[12] Varga K and Suzuki Y 1995 Phys. Rev. C 52 2885
[13] Viviani M, Kievsky A and Rosati S 2005 Phys. Rev. C 71 024006
[14] Timofeyuk N K 2002 Phys. Rev. C 65 064306
[15] Wang X-G and Carrington T Jr 2001 J. Chem. Phys. 114 1473