Abstract — This article aims at studying sounding rocket (SR) class, or those vehicles launched from the earth and carrying out various experiments at high altitudes. After the end of their mission, rockets should deliver a payload to a given point or to give range.

In general, design tasks are divided into direct and inverse ones. This paper focuses on direct designing tasks. In this respect, the set values are maximum range, payload mass and restrictions on the sounding rocket construction. As a result, a SR general design technique is proposed. This technique includes selection of the SR initial parameters, the number of stages, the relative weights of the fuel components, the specific thrust of the engines for each stage and the initial transverse load on the SR. After selecting the fuel composition, engine design features and rocket as an object, the starting mass can be represented as a complex mathematical function. Moreover, predetermined maximum range, rocket payload, selected fuel components, structural design, materials, design parameters allow for a definite SR weight determination. Finally, overall and power characteristics of the SR are proposed within this technique. Such characteristics are accepted as design parameters.

Index Terms — Rocket, design, structure, parameter, sounding rocket, solid rocket motor (SRM), liquid rocket motor (LRM).

I. INTRODUCTION

Nowadays guided rockets have different classifications. In addition, their tasks and design procedures are varied significantly. Moreover, modern space technology is characterized by an increasing complexity of the tasks to be solved by modern space vehicles. The efficiency in solution of such problems significantly depends upon technical characteristics of the on-board engineering systems ensuring the functioning of the spacecraft. Due to the relatively small weight of modern scientific instruments, even "seemingly small increase of payload weight can significantly extend the program of research and experiments implemented by that spacecraft" [1].

The tasks and design procedures volumes may vary significantly. In this research studying sounding rocket (SR) class. That vehicle launched from the Earth and used for solving various experiments at the high altitude (Fig. 1). After a mission solution, rockets should deliver a payload to a given point or to a given range. In general, design tasks are divided into direct and inverse ones. In this publication namely the direct task was solved. As set values are accepted maximum range \( L_{\text{max}} \), payload mass \( m_{\text{PL}} \) and restrictions on the sounding rocket construction.

In practice, in the process of SR creating, instead of the costs, the SR starting mass is considered as a value proportional to the costs, under certain assumptions.

Fig. 1. The Maxus 5 sounding rocket on the launch pad at Esrange on 31 March 2003 [2].

The technique for solving the direct design problem provides for the following:

- initial selection of design parameters and fuel;
- SR weight analysis;
- SR ballistic design.

In the design process used tables and graphs obtained on the basis of experience and research design typical ballistic guided rockets developed previously [3]-[8]. It should be noted that the design process is iterative, i.e., the results repeatedly reviewed and refined. The proposed further design method may be used in carrying out experimental studies preceding the initial stages of the SR design.

Computer simulations processes enable rapid implementation of all design stages for varying conditions interactively.

In Fig. 2 shows a general design diagram of SR ballistic type. The content of the algorithms used by the modules shown in the diagram are described in subsequent publications [8]-[10].
II. THE INITIAL STAGE OF SELECTING SOUNDING ROCKETS DESIGN PARAMETERS

A. Design Parameters

For the adopted mission program for the active part of trajectory, the SR speed at the end of this part and the flight range depend on the following parameters:

1) the number of stages \( n \);
2) the relative weights of fuel stages \( \mu_{K_i} \);
3) the specific engine thrust for each stage \( P_{\text{spec,}i} \);
4) thrust-to-weight ratio steps, characterized by coefficients \( \lambda_{ii} \);
5) the initial load on the SR transverse midsection \( P_{M1} \).

In other words, the SR range can be represented as a next function:

\[
L = f_1(\mu_{K1}, P_{\text{spec,}ii}, \lambda_{ii}, P_{M1}), i = 1, 2, \ldots, n. \tag{1}
\]

In this case, the following two factors must be taken into account:

1. The specific thrust of the engine \( P_{\text{spec,}ii} \) depending on fuel composition, the engine design and magnitude of the pressure in the combustion chamber \( p_a \) and in the outlet section of the nozzle \( p_a' \).

The relative weight of fuel in the stages \( \mu_{K1} \) be expressed by the duty ratio of the first stage fuel \( \mu_{K1} \) namely [11]:

\[
\begin{align*}
\mu_{K2} &= \chi_2 \mu_{K1} \\
\mu_{K3} &= \chi_3 \mu_{K2} = \chi_2 \chi_3 \mu_{K1} \\
& \quad \vdots \\
\mu_{Kn} &= \chi_{n-1} \mu_{Kn-1} = \chi_1 \chi_2 \cdots \chi_{n-1} \mu_{K1}
\end{align*}
\]

where \( \chi_w = \frac{\mu_{K(i+1)}}{\mu_{K1}} \) - the weight ratio of adjacent stages.

Then we can represent the function (1) in a new form:

\[
L = f_1(\mu_{K1}, \chi_{1-1}, \lambda_{ii}, p_{Kii}, p_{ai}, P_{M1}), i = 1, 2, \ldots, n. \tag{2}
\]

The same relationship can be viewed as an inverse function of the form:

\[
\mu_{K1} = f_1(L, \chi_{1-1}, \lambda_{ii}, p_{Kii}, p_{ai}, P_{M1}), i = 1, 2, \ldots, n. \tag{3}
\]

III. Fuels Selection and Their Characteristics Determination

Among the few energy sources that are used to create jet propulsion, chemical molecular fuel still has the widest application.

In the SR design process necessary to select the optimum composition of existing components of fuels on the following criteria [12]:

1) standard specific thrust \( P_{\text{spec,}ii} \);
2) thermodynamic parameters of the combustion products (gas constant \( R \), temperature \( T \), the adiabatic index \( k \) or isentropic index \( n_{is} \));
3) average fuel density \( \rho_f' \);
4) the burning rate (for SRE) \( U \), or the residence time of the combustion products in the chamber (for LRE) \( T \);
5) the characteristics of mechanical strength of solid fuels with regard to their rheology;
6) cost characteristics of fuel components.

C. Fuels Selection and Their characteristics Determination

The standard specific thrust is a characteristic energy of
fuel capacity. Theoretical specific thrust is standard at equilibrium flow of combustion products and at high pressures in the combustion chamber \(p_{ch} = 40 \text{ bar}\) and at the nozzle exit \(p_a = 1.0 \text{ bar}\). This operation of the nozzle is assumed in design mode.

The specific thrust values for various fuel components are obtained as a result of thermodynamic calculations, then that results are tabulated and entropy diagrams are calculated too. To account for losses in the combustion chambers, a standard specific thrust \(P_{Spec,st}\) to be reduced by 4.5%. Thus, the standard thrust shown as:

\[
P_{Spec,st}^{br} = (0.95 \div 0.96)P_{Spec,st}.
\]

(7)

For solid fuels containing aluminum, it is necessary to take into account additional losses in specific thrust caused by the two-phase gas flow, i.e., in presence of gaseous combustion products of particulate matter \(Al_2O_3\).

The standard specific thrust in this case is calculated by the following empirical dependencies [8]:

\[
P_{Spec,st}^{br} = P_{Spec,st}[(1 - 17a + 0.009a^2) \times 10^{-2}],
\]

(8)

where \(a\) – the aluminum content is in weight percent.

In case of the absence of tables and entropy diagrams, the engine specific thrust at the calculated mode of the nozzle operation can be calculated by the following empirical dependencies [8]:

\[P_{Spec,st}^{calc} = P_{Spec,st} + 19.4 + 0.76p_{ch} - 70p_a + 25p_a^2.
\]

(9)

\[P_{Spec,st}^{calc} = P_{Spec,st} + 21 + 0.76p_{ch} - 70p_a + 25p_a^2.
\]

(9)

b) for solid propellants:

In formulas (9) and (10), the specific thrust is measured in seconds, and the pressure in the combustion chamber and at the nozzle exit are in bars.

The specific thrust in a vacuum is given by [8]:

\[
P_{Spec,vac} = P_{Spec}^{calc} + \frac{RT}{\delta p_{Spec} p_{ch}} \left( \frac{p_a}{p_{ch}} \right)^{k-1}\left(1 - \frac{p_a}{p_{ch}}\right).
\]

(11)

where \(R\) – the gas constant; \(T\) – combustion temperature; \(k\) – adiabatic exponent.

The calculated dependence for determining specific thrust at any altitude has the next form:

\[
P_{Spec,vac} = P_{Spec}^{calc} + \frac{RT}{\delta p_{Spec} p_{ch}} \left( \frac{p_a}{p_{ch}} \right)^{k-1}\left(1 - \frac{p_a}{p_{ch}}\right).
\]

(12)

where \(p_a\) – the air pressure at an altitude above the surface of the Earth.

Specific thrust on the ground is calculated at the atmospheric pressure of \(p_W = 1.01\). To calculate the function \((\frac{p_a}{p_{ch}})^{k-1}\) designer may use the table at Ref. [13].

The combustion temperature of the fuel can be calculated by the formula [13]:

\[T = T_{std} + 1.12(p_{ch} - 40),
\]

(13)

where \(T_{std}\) – standard combustion temperature determined by the results of thermodynamic calculations; \(p_{ch}\) – pressure in the combustion chamber (bar).

Formula (13) is valid under the temperature condition: 3000 °K < \(T_{std}\) < 3500 °K.

The gas constant \(R\) and the adiabatic exponent \(k\) are weakly dependent on the pressure in the combustion chamber. Therefore, only their standard values \(R_{st}\) and \(k_{st}\) are used for calculations.

The density of the liquid fuel is calculated by the formula [10]:

\[\rho_f = \frac{(1+k)\rho_{ox}\rho_f}{\rho_{ox}+k\rho_f},
\]

(14)

where \(\rho_{ox}\) – oxidant density, \(\frac{kg}{m^3}\); \(\rho_f\) – fuel density, \(\frac{kg}{m^3}\); \(K\) - ratio of the second consumption of oxidizer and fuel agents.

The solid propellant density is determined from the next expression:

\[\rho_j = \sum_j \rho_j g_j p_j,
\]

(15)

where \(\rho_j\) – is the density of the \(j\) - source component; \(g_j\) – weight fraction of component \((j=1,2,\ldots,n)\).

The density of modern solid propellants covers a relatively narrow range: from 1680 to 1800 \(\frac{kg}{m^3}\).

The most important characteristic of solid propellants is their burning rate. The burning rate is determined by the nature of the fuel, the ratio of its components, and essentially depends on external factors: the pressure in the combustion chamber \(p_{ch}\) and the initial temperature of solid propellant grain \(T_{PG}\);

The dependence of burning rate on pressure in the chamber is expressed by the empirical expressions [9]:

\[u = a p_{ch}^b,
\]

(16)

\[u = b + a p_{ch}^b.
\]

(17)

The dependence of the burning rate on the initial temperature of the solid propellant grain is characterized by the temperature coefficient of the burning rate:

\[a_T = \frac{\Delta u}{\Delta T_{PG}},
\]

(18)

where \(\Delta u\) – change in combustion rate; \(\Delta T_{PG}\) – interval changes the initial solid propellant grain temperature.

For modern fuels \(a_T = 0.02 \div 0.03\) [10].

For solid rocket propellants, in addition to the chemical composition, it is necessary to select the solid propellant grain shape. When choosing a solid propellant grain shape, consider:
1. The combustion surface remains constant during engine operation.
2. Large second gas flow.
3. Large fill factor of the combustion chamber with propellant.
4. Simplicity of manufacturing technology and installation of the solid propellant grain.

In practice, frequently used solid propellant grain telescopic form, solid propellant grain with longitudinal slits and with star-grain channel (transitioning) (Fig. 3, [10]).

The burning of such solid propellant grains occurs on the internal surfaces, while the external, and sometimes end surfaces, are covered with an inert reservation. The solid propellant grain may be bonded to the inner surface of the combustion chamber.

The main geometric characteristics of the solid propellant grain:
1) the outer diameter of the propellant grain \( d_g \);
2) relative solid propellant grain length:
\[
\bar{L}_g = \frac{l_g}{d_g}
\]  
where \( l_g \) — length of the charge;
3) the relative diameter of the inner channel (for solid propellant grain with slots and star-grain channels):
\[
\bar{d}_{sg} = \frac{d_{sg}}{d_g}
\]  
where \( d_{sg} \) — the inner channel diameter;
4) the relative length of the slits to the charge with longitudinal slits (Fig. 4):
\[
\bar{h}_{sg} = \frac{h}{d_g} = 0.3\bar{L}_{sg} - 0.3,
\]  
During ballistic design of SR with SRMs, the difference between the outer and inner diameters of the cylindrical part of the combustion chamber and the outer diameter of the charge can be neglected. Therefore, for the \( n \)-th stage, we can take
\[
d_{gi} = d_i.
\] (22)

The relative diameter of the inner channel (if the absence of constraints on the strength of the solid propellant) is chosen from the next condition:
\[
\frac{A_{free}}{A_{crit}} = \frac{d_g^2}{d_{crit}^2} \leq 1.5,
\] (23)
where \( A_{free} \) — area of the fuel-free cross section of the combustion chamber before starting the engine; \( A_{crit} \) and \( d_{crit} \) — area and diameter of the nozzle critical section.

For a solid propellant grain bonded to a combustion chamber, provided that there are limitations on its strength, the internal diameter of the channel \( d_g \) is selected from the ratio:
\[
d_K = (0.3 + 0.5)d_g,
\] (24)

III. RESULTS AND DISCUSSION

As a first approximation, it is possible to use the graphs that describe the dependence of the SR range on the ratio of the starting mass to the payload mass and the number of stages, calculating the SR starting mass for a given mission range for a different number of stages (Fig. 5-7). In this case, the most profitable number of stages can be selected.
4. In the range of distances, single-stage and two-stage SRs with LREs have practically the same starting mass ($m_0 / m_{PL} \approx 6 + 7$); in the same mission ranges, the same starting masses ($m_0 / m_{PL} \approx 7$) have two- and three-stage SRs with solid propellant rocket engines.

5. Two-stage SRs with LREs can provide flight for almost any range, including intercontinental ones, at $m_0 / m_{PL} \approx 50$ SR are able to bring a payload into a basic circular orbit.

6. Three-stage SRs with LREs, starting from ranges $L=2,000\div3,000$ km, have a slightly lower starting mass than two-stage ones. However, the gain in the starting mass does not exceed $10\%$.

7. Increasing the SR with SRM number of stages over 3 does not significantly increase in flight distance or decrease the starting weight.

8. Increasing the SR number of stages with SRM in excess of 3 results in no significant increase in flight distance or the reduction of the starting weight.

9. SR reliability with the number of stages is markedly reduced.

10. Given these factors, the most advantageous number of stages $n$ for SR is usually one less than the number of stages at which it has a starting mass.

11. After selecting the number of stages, from the graph it is easy to find the first approximation of the SR starting mass $m_0$.

### IV. CONCLUSION

Analysis charts (Fig. 5-7) are leads to the following conclusions:

1. For a single-stage SR, a significant increase in flight speed $V_{max}$ and range $L$ occurs only until $m_0 / m_{PL} \approx 50$; with a relative weight of the fuel $\mu_k = 0.90 \div 0.92$ single-stage SR with LREs can reach mission ranges $5,000\div6,000$ km.

2. For single-stage SR with SRMs, the possible mission ranges are much lower, since the specific thrust of modern solid propellant rocket engines is $10\%\text{--}15\%$ less than the specific thrust of the LRE and SR with SRMs having lower fuel filling values 11.

3. It can be considered that ballistic possibilities for single stage SR and for 2-stages SR with solid rocket propellants with practically identical.

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