Cyclic Arbitrage in Decentralized Exchange Markets

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In May 2020, Uniswap V2 was officially launched on Ethereum. Uniswap V2 allows users to create trading pools between any pair of cryptocurrencies, without the need for ETH as an intermediary currency. Uniswap V2 introduces new arbitrage opportunities: Traders are now able to trade cryptocurrencies cyclically: A trader can exchange currency \( A \) for \( B \), then \( B \) for \( C \), and finally \( C \) for \( A \) again through different trading pools. Almost surely, the three floating exchange rates are not perfectly in sync, which opens up arbitrage possibilities for cyclic trading.

In this paper, we study cyclic arbitrages in Decentralized Exchanges (DEXes) of cryptocurrencies with transaction-level data on Uniswap V2, observing 285,127 cyclic arbitrages over eight months. We conduct a theoretical analysis and an empirical evaluation to understand cyclic arbitrages systematically. We study the market size (the revenue and the cost) of cyclic arbitrages, how cyclic arbitrages change market prices, how cyclic arbitrageurs influence other participants, and the implementations of cyclic arbitrages.

Beyond the understanding of cyclic arbitrages, this paper suggests that with the smart contract technology and the replicated state machine setting of Ethereum, arbitrage strategies are easier implemented in DEXes than in Centralized Exchanges (CEXes). We find that deploying private smart contracts to implement cyclic arbitrages is more resilient to front-running attacks than applying cyclic arbitrages through public (open-source) smart contracts.

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1 INTRODUCTION

We are contemporary witnesses to what may become a major disruption in markets and trading. It started when some Reddit users speculated that the short-selling of GameStop shares had reached unnaturally high values. In late January 2021, various popular stock brokers halted the possibility of buying (but not selling) GameStop shares. United States national securities regulators even considered removing GameStop stocks from the market. A heated discussion of conspiracy theories ensued, and a lot of people started asking whether the whole financial system is rigged.

Even though the New York Stock Exchange is considered a free market where the price of stocks is self-regulated by the trades, centralized trading platforms (we call these CEX as in Centralized Exchange in this paper) apparently can manipulate trading activities in the market. In such an environment, investors cannot protect their rights because their assets are under the control of CEX operators.

To eliminate the dominance of CEXes and enable free transactions between traders, blockchain-based Decentralized Exchanges (DEXes) have been introduced to cryptocurrency markets. A DEX ensures decentralized and non-custodial trading. Traders sell and buy assets on a DEX by interacting with DEX smart contracts through on-chain transactions, while no centralized authority is involved throughout the whole process. Considering the recent developments in CEXes, DEXes may be the future.

Uniswap V2 is the most prominent example of a DEX of cryptocurrencies. After eight months of its official launch on Ethereum [1], as of January 2021, Uniswap V2 has amassed total market liquidity of over 3 billion USD and attracted a trading volume of over 800 million USD per day. Uniswap V2 is an Automated Market Maker (AMM) DEX, which uses algorithmic mechanisms to perform market making, i.e., enabling the purchase and sale of cryptocurrencies (tokens) and setting prices for each trade automatically. In AMM DEXes, the exchange rate between cryptocurrencies of each trade is determined by the predefined algorithms and the market liquidity reserves. All information is public to everyone due to the transparent nature of blockchains. Therefore, arbitrageurs can prejudge the price of each upcoming transaction in advance and exploit the price difference to their advantage across different markets [10, 32, 33, 38].

In this paper, we study a new arbitrage behavior, i.e., cyclic arbitrage, observed in Uniswap V2. While Uniswap V1 only allowed trading from/to ETH, V2 allows users to create trading pools between any pair of tokens, without the need for ETH as an intermediary currency [2]. Each pool then allows for automatic trading between a pair of tokens. Assume we have three currencies A, B, C, and three liquidity pools between any two tokens: A ⇔ B, B ⇔ C and C ⇔ A. Each of these pools has an exchange rate, which changes over time, depending on the recent trades in that pool. Given these three pools, arbitrageurs are now able to trade currencies cyclically: A trader can exchange currency A for B in A ⇔ B, then B for C in B ⇔ C, and finally C for A in C ⇔ A again, to benefit themselves from the deviations of exchange rates (the particular case with three

Fig. 1. An example cyclic arbitrage. Transaction 0xcf7094... in Uniswap V2, executed on 30th October 2020.
tokens is called triangular arbitrage [15, 27]). The combination of these three exchanges (swaps) of tokens is a cyclic arbitrage (transaction). Figure 1 shows an example of a cyclic arbitrage executed on October 30th, 2020. The arbitrageur traded 285.71 USDC in Uniswap V2 through four different liquidity pools across USD Coin, Tether USD, Seal Finance, Keep3rV1, and finally received 303.68 USDC. The revenue of this cyclic arbitrage is 17.97 USDC (18 USD).

Previous studies have examined the idea of cyclic (triangular) arbitrage in CEX foreign exchange markets [3–5, 14] and CEX cryptocurrency markets [8, 16, 28, 31]. However, they have not studied real market behaviors. Uniswap V2 is the first real possibility to quantitatively study cyclic trading. How many trades are made by cyclic arbitrageurs? How much revenue they take from the markets? And how seriously these cyclic arbitrages influence the market price? To our knowledge, this is the first paper to study the traders’ behavior from transaction-level data to understand cyclic arbitrages and the first paper to study cyclic arbitrages in DEXes. We conduct a mixed-method study of a theoretical analysis and an empirical evaluation to understand cyclic arbitrages systematically. We analyze when cyclic arbitrage is profitable in AMM DEXes, how cyclic arbitrages change market prices, and how cyclic arbitrageurs influence other participants. We also evaluate the market size (the revenue and the cost) of cyclic arbitrages and cyclic arbitrages’ implementations. We believe that our findings could be partially transferred to cyclic trading in CEXes.

We collected transaction data of Uniswap V2 from May 4, 2020 to January 23, 2021. We show larger deviations in exchange rates between cryptocurrencies in Uniswap V2 across different trading pools, resulting in many potential arbitrage opportunities for cyclic transactions. If the product of the exchange rates (arbitrage index) along the trading cycle is larger than the products of transaction fees, then arbitrageurs can receive positive revenue by cyclic transactions. We classify 21,830,282 transactions in Uniswap V2 across 29,235 trading pools and observe 285,217 cyclic transactions over eight months. From May 2020 to January 2021, the total cyclic arbitrage revenue exceeds 31,727.11 ETH, while the daily amount of arbitrage revenue is often more than 200 ETH.

The cost of implementing arbitrage strategies is another fundamental factor for understanding cyclic arbitrages. Like other on-chain transactions in Ethereum, miners ask for a fee to execute the cyclic transactions. On top of that, arbitrageurs also have to pay a transaction fee to the liquidity providers who reserve cryptocurrencies in the trading pools. Moreover, there is a risk that the cyclic transactions are not executed at all, in particular, if the cyclic trade was not profitable anymore at the time of execution. This may happen if the exchange rates changed between submitting a transaction and executing the transaction, for instance, if another arbitrageur’s (cyclic) trade was executed earlier. In this case, miners will still ask for a fee for the failed executions. We measure the revenue, the cost for successful cyclic transactions, the net profit, and the cost for all cyclic transactions (including successful and failed). More than 93% of successful transactions have a positive profit. The total cost of executing (successful and failed) cyclic transactions is 9,520.23 ETH, only accounting for 30% of the revenue of cyclic transactions. We also find a significant variance in success rates of two different implementations of cyclic transactions, i.e., deploying private smart contracts AND directly calling the public (open-source) Uniswap V2 smart contracts, yielding at 62% and 28.4%, respectively. Because it is hard for front-running attackers to recognize the trading behavior from private smart contracts, cyclic transactions under this implementation are more resilient to front-running attacks, while others that directly call the public Uniswap V2 smart contract from EOAs can be easily front-run.

Frequent performance of cyclic arbitrages also influences the market prices and other market participants. Cyclic arbitrages balance the exchange rates across the market, improving the overall market efficiency, which is good for traders (who trade in a fair market). As a consequence, cyclic arbitrages are also good for the liquidity providers (since these arbitrages also increase the trading
volume, and as such their income from fees). Since cyclic arbitrageurs can deploy atomic transactions to guarantee their income, it is a win for everybody involved in DEXes.

Cyclic arbitrages are not limited to Uniswap V2. Traders can also perform cyclic arbitrages across different DEXes. We compare the block-wise market price of cryptocurrencies in Uniswap V2 and Sushiswap, i.e., an AMM DEX forked from Uniswap V2. We find that price deviations of 15 popular token pairs between these two DEXes are significant in 12% of blocks, even though Uniswap V2 and Sushiswap use the same market mechanism and the same cryptocurrencies are traded in them. This indicates many cyclic arbitrage opportunities across different DEXes.

This paper makes the following contributions by studying cyclic arbitrages in DEXes. First, we provide insights into the trading behavior of cyclic arbitrageurs in an emerging DEX market, by studying a new transaction-level data source of cryptocurrency markets. We extend the research scope from potential arbitrage opportunities to real arbitrage implementations. Second, we propose the cyclic arbitrage model in AMM DEXes and analyze how traders are motivated by unbalanced market prices, and how they influence the market prices. Third, we provide the measurement of state-of-the-art value extraction of (cyclic) arbitrages in DEXes. Fourth, we analyze the resistance of different implementations of trading strategies to front-running attack, which inspires future research on preventing front-running. Finally, we suggest that DEXes can contribute novel understandings of trading behavior in financial markets because traders can freely exchange their assets without being dependent on a third party, and blockchain technologies enable traders to explore more trading strategies.

2 BACKGROUND

2.1 Ethereum

Ethereum is a public blockchain platform, which supports Turing complete smart contract functions. Compared to earlier blockchain systems, such as Bitcoin, Ethereum is not only implementing a decentralized system that allows peer-to-peer transactions but also realizing “the world computer”. Smart contracts are “software” running on this “computer”. Ethereum deploys a decentralized virtual machine, i.e., Ethereum Virtual Machine (EVM), to execute smart contract code.

There are two kinds of accounts in Ethereum, namely externally owned accounts (EOAs), i.e., those controlled by human users with the corresponding private key, and smart contract accounts (CAs), controlled by executable code [36]. Each account has a fixed address, which serves as the identity of the account in the Ethereum system. The system keeps account states for each account. The state of an EOA is the current balance of ETH, while the state of a CA is the value of its variables. States of accounts are stored in an independent key-value database, i.e., stateDB.

An Ethereum transaction is broadcast by an EOA user to the Ethereum network. EOAs can send three types of transactions: simple transaction, of which recipient is another address (account) to transfer the native currency, ETH; smart contract creation transaction, of which recipient is the null address to create a new smart contract; smart contract execution transaction, of which recipient is a smart contract address to execute a specific function of that contract. When a transaction is recorded in blocks by miners, the operation corresponding to the message takes effect. The miner who creates the block modifies the state of corresponding accounts based on the messages. Each step of the miner’s operation consumes a certain amount of gas, and the amount of gas consumed in each block is capped. Users need to specify a gas price for the operation execution when sending transactions. The fee paid by the initiator of transactions to miners is determined by the amount of gas consumed and the gas price (gas fee = gas price × gas consumption).
After the transaction is executed, a transaction receipt will be generated and included in Ethereum blocks. The receipt includes the information on whether the transaction has been executed successfully, the gas fee, the identity of the transaction and the block, and other information of events generated during the transaction execution.

2.2 Smart Contract and ERC-20 Token Standard

A smart contract is a set of programs written in high-level languages, e.g., Solidity. Creators compile them into executable byte-code and send them within a transaction to the null address, declaring the creation of a smart contract, while the source code will not be published. The address of the contract will be determined by the sender’s address and the transaction nonce. After the contract creation, all addresses can send transactions to the new smart contract to invoke functions.

Based on the support of the smart contract, users can create cryptocurrency other than ETH on Ethereum. These smart contracts have to follow some standards, and the most widely used one is the ERC20 standard, which requires an approve function and a transferFrom function. When an address (account) \( addr_a \) calls the function \( \text{approve}(addr_b, v) \), then the address \( addr_b \) can transfer at most \( v \) tokens in total from \( addr_a \) to other accounts. After this approval, \( addr_b \) can transfer \( v' \) tokens from \( addr_a \) to another account \( addr_c \) by calling the function \( \text{transferFrom}(addr_a, addr_c, v') \), where \( \sum v' \leq v \).

2.3 AMM DEXes

DEXes are smart contracts on Ethereum. Users send messages to DEXes addresses to invoke functions for performing market operations. AMM DEXes support three kinds of operations regarding the token pair, i.e., creating the trading (liquidity) pool between the pair, adding/removing liquidity, and exchanging tokens. We take Uniswap V2 as an example to present these operations.

2.3.1 Creating Liquidity Pools. In AMM DEXes, exchanges between two tokens are conducted through a liquidity pool, i.e., a smart contract that keeps the pair of tokens. There are two participants involved in the market: the liquidity provider and the trader. Providers reserve their tokens in the liquidity pool, while traders exchange their tokens with the liquidity pool. Because providers contribute to the market liquidity, they benefit from the transaction fees incurred with transactions in DEXes.

Assume we have two tokens \( A \) and \( B \), and we want to create a liquidity pool between \( A \) and \( B \) on Uniswap V2. We first send a smart contract execution transaction with the ERC20 smart contract address of \( A \) and \( B \) to the Uniswap V2 to claim the creation of the liquidity pool \( A \leftrightarrow B \). The smart contract will then check whether the pool between \( A \) and \( B \) exists according to the addresses of two tokens. If not, Uniswap V2 smart contract will create a new liquidity pool \( A \leftrightarrow B \), i.e., a new smart contract reserving these two tokens.

2.3.2 Adding/Removing Liquidity. After creating the liquidity pool between \( A \) and \( B \), liquidity providers can provide a pair of tokens to the liquidity pool. Liquidity providers need to approve the Uniswap V2 address to transfer their \( A \) token and \( B \) token from their address to the liquidity pool address. When Uniswap V2 contract receives a call of adding liquidity from liquidity providers, it will invoke the transferFrom function in ERC20 contracts to transfer tokens from the provider’s address to the liquidity pool address.

If there are no tokens reserved in the liquidity pool, providers can supply any amount of \( A \) and \( B \) to the liquidity pool, and the pool will return \( lp \) tokens as proof of the deposit. If the amounts of \( A \) and \( B \) provided by the providers are \( a \) and \( b \), respectively, then the provider will get \( \sqrt{a \times b} \) \( lp \) tokens. Meanwhile, the total supply of \( lp \) token of \( A \leftrightarrow B \) pool is \( l = \sqrt{a \times b} \).
If there is already \(a\) of token \(A\) and \(b\) of token \(B\) in the liquidity pool, a provider can reserve \(\delta_a\) of its asset \(A\) and \(\delta_b\) of its asset \(B\) in the liquidity pool simultaneously, where \(\frac{\delta_a}{\delta_b} = \frac{a}{b}\). Then he will earn \(\delta_l = l \cdot \frac{\delta_a}{\delta_b} \) \(lp\) tokens for the \(A \leftrightarrow B\) and the total supply of \(lp\) token becomes \(l + \delta_l\).

Providers can also remove their tokens from the liquidity pool. The amount of tokens providers can redeem is related to the amount of \(lp\) token they own. Assume a provider has \(\delta_l \) \(lp\) token of the liquidity pool \(A \leftrightarrow B\) and the total supply of \(lp\) token is \(l\). The provider can withdraw \(\delta_a\) of \(A\) and \(\delta_b\) of \(B\) from the \(A \leftrightarrow B\) pool with \(\delta_l' \leq \delta_l \) \(lp\) token, where \(\frac{\delta_a}{a} = \frac{\delta_b}{b} = \frac{\delta_l'}{l}\). The \(\delta_l'\) of \(lp\) token will be burned (destroyed) after he redeems the money and the total supply of \(lp\) token becomes \(l - \delta_l'\).

2.3.3 Exchanging Assets. In AMM DEXes, tokens are not exchanged between two traders but between the trader and the liquidity pools. The execution of exchanging assets is realized in two steps. First, the trader sends his tokens to the liquidity pool. Second, the liquidity pool computes the exchange rate and returns the targeted token to the trader.

Assume a trader wants to exchange \(\delta_a\) of \(A\) for \(B\) token and the liquidity of \(A\) and \(B\) are \(a\) and \(b\). He first needs to let the Uniswap V2 address get the approval for transferring his \(A\) token to other accounts. After receiving the exchange order from the trader, the Uniswap V2 contract then transfers \(\delta_a\) of \(A\) to the liquidity pool address and returns \(\delta_b\) of \(B\) back to the trader.

The following equation always holds during the exchange: 
\[
a \cdot b = (a + \delta_a \cdot r_1) \cdot (b - \frac{\delta_b}{r_2}),
\]

where \(r_1\) and \(r_2\) denote the transaction fee ratio in asset \(A\) and \(B\) respectively. In Uniswap V2, \(r_1 = 0.997\) and \(r_2 = 1\), which indicates that the transaction fee is equal to 3% of \(\delta_a\). The remaining liquidity in the pool equals to 
\[
(a + \delta_a, b - \frac{r_1 \cdot \delta_b \cdot \frac{\delta_a}{a + r_1 \cdot \delta_a}}{a + r_1 \cdot \delta_a})
\]

and the amount of \(lp\) token does not change.

2.4 Atomic Transactions

In Ethereum, traders are allowed to issue atomic transactions. They can deploy smart contracts that contain several exchanges through different trading pools within one transaction. These exchanges will be executed atomically, i.e., no other transaction can happen between them. Because Ethereum is a replicated state machine, the system state is changed only if the whole transaction executes successfully. If the transaction fails because a particular condition cannot be met (e.g., insufficient gas fees and prerequisites on the market state), the whole transaction will not be executed, and the state of all accounts will revert to the original state before the transaction. An arbitrageur can submit a transaction by deploying smart contracts with a precondition that the transaction will be executed only if there is a positive income, which ensures no negative trading revenue.

2.5 Arbitrage in Cryptocurrency Markets

Research on trading and arbitrage in cryptocurrencies is still in its beginning. The majority of papers either focuses on theoretical analysis of the behavior of miners and traders in blockchain systems [7, 12, 13, 20–22, 24, 26, 29, 34, 35], or consider how cryptocurrencies influence financial markets as a potential payment and transaction mechanisms [6, 9, 11, 17, 19, 30].

Some studies [8, 16, 25, 28, 31] have noted changes in the prices of cryptocurrencies in CEXes of cryptocurrencies. Makarov and Schoar [25] have used transaction-level data to understand trading strategies in CEXes of cryptocurrencies. They analyzed price deviations and potential arbitrage opportunities of Bitcoin, Ethereum, and Ripple for 34 CEXes across 19 countries. They observed significant market segmentation among different countries and suggested that capital controls are the main reasons for market segmentation. Nan and Kaizoji [28] focused on the potential triangular arbitrage with Bitcoin, Euro and U.S. dollar and modelled with a bivariate GARCH model. Nevertheless, their analyses are limited to potential arbitrage opportunities, while they haven’t found any real arbitrage behaviors across different markets. Meanwhile, the arbitrage strategy...
they studied has many constraints to be implemented in real markets, such as cross-border capital controls and instantaneous transfer of bitcoins between different CEXes.

Recently, DEXes have attracted attention worldwide as an emerging alternative of CEXes for exchanging cryptocurrencies. Daian et al. [10] have analyzed the fundamental weakness of DEXes: slow (on-chain) trading. Since transactions are broadcast in the Ethereum network, adversaries can observe profitable transactions before they are executed and place their own orders with higher fees to front-run the target victim. Front-running attackers bring threats to the market and system stability. Arbitrageurs optimize network latency aggressively and conduct priority gas auctions to front-run profitable trades [23], which results in excessive transaction fees affecting normal users in blockchain ecosystems. Moreover, because of the high miner-extractable value, fee-based forking attacks and time-bandit attacks are created and bring systemic consensus-layer vulnerabilities. Zhou et al. [38] and Qin et al. [32] studied sandwiching attacks, i.e., combinations of front- and back-running, in DEXes. When observing a victim transaction, attackers place one order just before it (front-run) and place an order just after it (back-run) to benefit themselves through the variance of the exchange rates. Qin et al. [33] considered how attackers manipulate the price oracle of DEXes. These attackers take advantage of loopholes in DEXes market mechanism to reap benefits maliciously and profit themselves by harming the target victim. Qin et al. [32] measured arbitrages in DEXes. However, their measurements are incomplete and limited to very few cryptocurrencies, and they have not provided any analysis on arbitrage strategies.

Compared to previous studies, our work fills the following three gaps. First, we provide a systemic study on cyclic arbitrages by modeling AMM DEXes theoretically and analyzing transaction-level data of DEXes empirically. We not only understand the mechanism of cyclic arbitrages in DEXes and the impact of cyclic arbitrages on the market, but also quantify the arbitrage value for cyclic transactions. Meanwhile, we consider how cyclic arbitrageurs balance the revenue and the cost of cyclic transactions in real DEXes. Second, we introduce a more generalized arbitrage strategy in DEXes. Other than front-running attacks and sandwich attacks, which require arbitrageurs to monitor all non-executed transactions broadcast in the Ethereum network, cyclic arbitrages only need information published on Ethereum blocks. Traders can easily play cyclic arbitrages in DEXes, which benefits both the market efficiency and stability. Third, we consider how different implementations of arbitrage strategies influence the success rate of arbitrage transactions. Because there are many arbitrageurs front-running profitable transactions in DEXes, not every cyclic arbitrage can be executed successfully. We find that deploying private smart contracts is more immune to front-running attacks than calling open-source smart contract functions of DEXes.

3 DATA DESCRIPTION

The main data for this project are transactions in the largest DEX (Uniswap V2). Every successful transaction in Uniswap V2 is recorded on Ethereum blocks. To collect market data of Uniswap V2, we download Ethereum blocks and analyze from block 10000835 (where Uniswap V2 has been deployed, 4th May 2020) to block 11709847 (23rd January 2021). Every trading pair in Uniswap V2 is recorded in the UniswapV2Factory contract. We query this contract to get 29,235 available trading pairs until 23rd January 2021 and contract addresses of these trading pools. We collect all 21,830,282 transactions interacting with liquidity pools of Uniswap V2 and the transaction receipts.

For each successful exchange between two tokens, a Swap event will be generated by the trading pair contract and be included in the transaction receipt. A Swap event includes six parameters: sender, which is the initiator of the exchange; to, which is the receiver of the output tokens;
amount0In, amount1In, amount0Out, amount1Out, which denote the amount of token0 spent by the initiator, the amount of token1 spent by the initiator, the amount of token0 received by the receiver, and the amount of token1 received by the receiver, respectively. If a trade exchanges $x$ of token0 for $y$ token1, then \((\text{amount0In}, \text{amount1In}, \text{amount0Out}, \text{amount1Out}) = (x, 0, 0, y)\). With the information included in the \textit{Swap} event, we can identify the direction and the amount of the exchange. If several \textit{Swap} events are involved in the same transaction receipt, the trading path of these \textit{Swap} events can form a cycle, and the amount of the output token in the previous \textit{Swap} is exactly equal to the amount of the input token of the next \textit{Swap}, then we identify a successful cyclic transaction$^2$.

To compute the successful rate of cyclic arbitrage, we also need the information of failed cyclic transactions. A \textit{Swap} event is recorded in the transaction receipt only if the exchange has been executed successfully. We cannot identify whether a failed transaction is for the purpose of cyclic arbitrage if the smart contract is private. Therefore, we count all failed transactions initiated by arbitrageurs who have performed cyclic transactions. To perform a cyclic arbitrage, arbitrageurs will call smart contract functions with an encoded signature, which is recorded in the transaction information. We collect signatures of all failed transactions interacting with cyclic arbitrage smart contracts and compare them with signatures in successful arbitrages. If a failed transaction’s signature matches a successful one, we consider the failed transaction as a potential failed cyclic transaction. For those failed transactions interacting with public (open-source) smart contracts (such as Uniswap V2 Router contract), we can directly identify whether the failed transactions are for the purpose of cyclic arbitrage.

To better understand the relationship between prices in different DEXes, we collect 1,129,715 transactions interacting with Sushiswap, another AMM DEX, which is forked from Uniswap V2 since 28th August 2020. We query the Sushiswap contract to get available trading pairs. We compute the exchange rate between 668 token pairs (until 23rd January 2021) and compare the price deviation of the same trading pairs which exist in both Uniswap V2 and Sushiswap.

4 CYCLIC ARBITRAGES

4.1 Non-equilibrium of Exchange Rate

In Uniswap V2, users can create trading pools for any pair of tokens, while the exchange rate between these two tokens only depends on the liquidity reserved in the smart contract and amount of trading tokens. Assume we have three tokens $A, B, C$ and three liquidity pools between them. If we want to exchange $A$ for $C$ in Uniswap V2 markets, we can either trade in $A \rightleftharpoons C$ directly, or trade token $A$ for $B$ in $A \rightleftharpoons B$, then $B$ for $C$ in $B \rightleftharpoons C$ to complete the transaction. The exchange rates between $A$ and $C$ through these two trading paths are not the same.

We denote $a_1$ and $b_1$ as liquidity of $A$ and $B$ in $A \rightleftharpoons B$, $b_2$ and $c_2$ as liquidity of $B$ and $C$ in $B \rightleftharpoons C$, and $c_3$ and $a_3$ as liquidity of $C$ and $A$ in $A \rightleftharpoons C$. If we directly exchange $a_3$ of token $A$ through the $A \rightleftharpoons C$ pool, then we get $\delta_c$ of token $C$, where $\delta_c = \frac{n \cdot r_2 \cdot c_1 \cdot \delta_a}{a_2 \cdot a_1 + r_1 \cdot c_2 \cdot \delta_a}$.

If we take token $B$ as an intermediate, then the output amount of $C$ is $\delta'_c = \frac{r_1^2 \cdot r_2 \cdot b_1 \cdot c_2 \cdot \delta_a}{a_1 \cdot b_2 + r_1 \cdot r_2 \cdot b_1}$. An example has been shown in Figure 2. We can trade 0.005 ETH for 0.0001466 WBTC through the $ETH \rightleftharpoons WBTC$ pool, or trade 0.005 ETH for 0.1694 LINK in the $ETH \rightleftharpoons LINK$ pool first and then obtain 0.0001496 WBTC through the $LINK \rightleftharpoons WBTC$ pool. The direct exchange rate between ETH and WBTC is 34.1:1, while if we use LINK as an intermediate, the exchange rate is 33.4:1, demonstrating a price deviation between ETH and WBTC in Uniswap V2.

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$^2$Cyclic transaction data (transaction receipts) is available at https://bit.ly/3aSXvtK

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To better understand how frequently price deviations appear in Uniswap V2 and how seriously unbalanced the market is, we form a block-wise arbitrage index. We take ten popular tokens (Appendix B Table 1) in Uniswap V2 over different time periods and compute their arbitrage index in each block. In a given block, we first compute the reserve ratio (exchange rate) between the token (token $A$) and ETH (token $C$) reserved in the exchange pool. Then, we take another token (token $B$) as an intermediate and compute the exchange rate between $A$ and ETH through two exchange pools ($A \leftrightarrow B$ and $B \leftrightarrow ETH$). We take the high price divided by the low price as the arbitrage index of token $A$ with ETH at the block. For example, in Figure 2, the arbitrage index between WBTC and ETH through LINK is $\frac{976825}{28730} \times \frac{2789}{947934} \approx 1.069$. Finally, we take the highest arbitrage index among these ten tokens as the arbitrage index of the block (arbitrage index of each token pair is shown in Appendix B).

Figure 3 shows the block-wise arbitrage index in Uniswap V2. Since block 10098323 (19th May), the arbitrage index appears because the pool between TRB and DAI has been created. There are 40,465 blocks (3.4%) with an arbitrage index larger than 2, where the highest arbitrage index is 960, appearing at block 10667283 (15th August 2020) between ETH and XAMP when the trading pair just been created in Uniswap V2. When a trading pair is created on Uniswap V2, the creator will reserve some tokens in the liquidity pool as initial liquidity, which may deviate far from other prices.

The average arbitrage index is 1.328, while 71.5% of blocks exist arbitrage opportunity with index larger than 1.1. Even though the arbitrage index with these ten tokens between ETH can already indicate significant arbitrage opportunities in Uniswap V2 over time, the arbitrage index here is not maximal. Traders may get a better exchange rate through another intermediate token. Moreover, there might be a better arbitrage index between other token pairs in Uniswap V2 at each block.
4.2 Existence of Arbitrage Opportunities

The unbalanced exchange rates in the market enable arbitrageurs to engage in cyclic transactions to benefit themselves. Cyclic transactions exploit divergent exchange rates between markets. Arbitrageurs can submit a series of exchanges that start and end with the same currency, i.e., $A \rightarrow B \rightarrow C \rightarrow D \ldots \rightarrow A$.

Whether a cyclic transaction is profitable depends on the exchange rates between tokens, which are predefined by the market mechanism. Assume an arbitrageur trades $\delta_a$ of token $A$ across the market of three tokens $A$, $B$, and $C$. He has two possible trading strategies, namely exchanging $\delta_a$ through $A \rightarrow B \rightarrow C \rightarrow A$ path or $A \rightarrow C \rightarrow B \rightarrow A$ path. We denote $U_{ABCA}$ and $U_{ACBA}$ as the utility of these two strategies, respectively, where,

$$U_{ABCA} = \delta_a' - \delta_a = \left( \frac{r_1' \cdot r_2'}{b_2 \cdot c_1 + r_1 \cdot r_2 \cdot b_1 \cdot c_2 + r_1' \cdot r_2' \cdot b_1 \cdot c_2} - 1 \right) \cdot \delta_a$$

$$U_{ACBA} = \delta_a' - \delta_a = \left( \frac{r_1' \cdot r_2'}{c_2 \cdot b_1 + r_1 \cdot r_2 \cdot c_1 \cdot b_2 + r_1' \cdot r_2' \cdot c_1 \cdot b_2} - 1 \right) \cdot \delta_a$$

The arbitrageur can benefit from the cyclic transaction if either $U_{ABCA} > 0$ or $U_{ACBA} > 0$, which only depends on the amount of tokens stored in the liquidity pools and the amount of tokens that traders would like to exchange. When an Ethereum block is published, the information of liquidity pools’ balance is available to all traders. Therefore, traders can quickly determine whether there is a revenue for cyclic transactions by monitoring public information.

With Equation 1, arbitrageurs can only know whether a cyclic transaction has positive income. Nonetheless, it is not clear where exists arbitrage opportunities among 29,235 trading pools. Based on the amount of tokens reserved in the liquidity pools, traders can also determine whether an arbitrage opportunity exist in a trading cycle.

**Lemma 4.1.** For a given cycle $A \rightarrow B^1 \rightarrow \ldots \rightarrow B^n \rightarrow A$ with $n + 1$ tokens, there exists an arbitrage opportunity for the cyclic transaction if the arbitrage index (product of exchange rates)

$$\frac{b_1 \cdot b_2 \cdot \ldots \cdot b_n \cdot a_{n+1}}{a_1 \cdot b_1 \cdot \ldots \cdot b_n \cdot a_{n+1}} > \frac{1}{\prod_{i=1}^{n+1} r_i^2}.$$  

Meanwhile, the arbitrage cannot benefit from the reversed direction $A \rightarrow B^n \rightarrow \ldots \rightarrow B^1 \rightarrow A$ for cyclic transactions.

4.3 Cyclic Arbitrages in Uniswap V2

Until 23rd January 2021, we find 3,285 EOAs calling 685 private CAs to perform 275,127 cyclic transactions and 136 EOAs calling the Uniswap V2 smart contract directly to perform 10,000 cyclic transactions. These cyclic transactions start with 555 unique tokens, while more than 93% of them start with Wrapped ETH (Figure 4), which is an ERC20 token equivalent to ETH. Other popular starting tokens are main stable coins (the value of a stable coin is pegged to 1 USD) or tokens with a high price. When computing the revenue of cyclic transactions in Uniswap V2, we only consider those that start with ETH because the majority of the proceeds of cyclic transactions are in ETH. Moreover, the exchange rate between ETH and other tokens fluctuates dramatically, which is difficult to unify. Arbitrageurs generate 31,727.11 ETH (42 million USD) revenue by 265,560 cyclic transactions in Uniswap V2.

Figure 5 shows the daily number of cyclic transactions in Uniswap V2. Figure 6 shows the cyclic arbitrage market size, denominated in ETH. The black line shows the daily revenue of cyclic transactions, and the dotted orange line shows the cumulative revenue of cyclic transactions.
Arbitrageurs started to perform cyclic arbitrage from 20th May, two weeks after the start of Uniswap V2 (4th May 2020). From 20th May to 7th August, the market has been growing in both the transaction number, reaching one thousand, and the daily revenue, reaching 100 ETH. Later, the market enters a relatively active period until 27th December, where arbitrageurs often perform over 1,500 cyclic transactions for a daily revenue of 100-1,000 ETH. At most, there are 3,872 arbitrages in the market on one day (4th October), gaining 1,195 ETH in profit (2nd September). Recently, the market matures into a more steady and consistent period. The daily number of cyclic transactions keeps at 800, and arbitrageurs take 50-100 ETH revenue from the market every day.

Even though more and more arbitrageurs joined the market and arbitrageurs have gained more revenue every day, the average revenue of cyclic transactions does not vary too far. Figure 7 presents the relationship between daily number of cyclic transactions and the average revenue of them. At the early stage of the market (from 20th May to 7th August), very few arbitrageurs are active in the market. Sometimes they can find an arbitrage opportunity with high revenue and drive the daily average revenue per transaction to 2.2 ETH (28th June, with 143 cyclic transactions). Later on, more and more arbitrage opportunities with small revenue have also been seized. The average daily revenue of cyclic transactions (0.124 ETH) can better represent how much arbitrageurs can earn from each transaction.

Fig. 4. The starting token as cyclic arbitrages.

Fig. 5. Daily number of cyclic arbitrages.

Fig. 6. Revenue of cyclic transactions in Uniswap V2 (in ETH). The black line shows a daily market size. The dotted orange line shows the cumulative market size.

Fig. 7. Average revenue of cyclic transactions per day in Uniswap V2 (in ETH). The scatter plot coloring indicates the number of daily cyclic arbitrage transactions.
4.4 Single Arbitrage Performance

Figure 8 shows the statistics of cycle length of cyclic transactions. Some cyclic transactions involve multiple cycles within one transaction. For example, transaction 0x329da4... contains 15 exchanges through 5 cycles of length 3 starting from ETH. For those transactions, we consider them as five cycles of length 3. In total, 287,097 cycles appear in 285,217 cyclic transactions.

Moreover, we find some cyclic transactions with negative income: 2,848 transactions perform on cycles with length 2. For example, transaction 0x19f2ec... contains two cycles of length 2. According to the Lemma 4.1, these transactions are not profitable because the arbitrage index is always 1. Most of these traders do not aim for positive income from cyclic transactions, while they were attending some other decentralized finance projects, such as trading volume mining [18], to get profits.

Lemma 4.1 suggests that the longer the cycle, the more restricted requirement is imposed on the arbitrage index for profitable cyclic transactions. Other than those cycles of length two, more cyclic arbitrages are performed within shorter cycles. In particular, 87.77% of cycles are length 3. Figure 9 shows the average revenue of cycles with different lengths larger than 2 and demonstrates the trend that arbitrageurs can get higher revenue in shorter cycles.

Why arbitrageurs prefer cycles with short lengths? There are three main reasons.

First, as we showed in Lemma 4.1, the required arbitrage index increases exponentially with the length of cycles, for example, in Uniswap V2, if we exchange token $A$ through a cycle of length 3, then the arbitrage index should be higher than $1.003^3$ for a profitable cyclic transaction. Nonetheless, if the length of the cycle is 6, then the arbitrage index should be higher than $1.003^6$, which is much harder than finding arbitrage opportunities in cycles with fewer tokens.

Second, arbitrageurs can easily benefit from trading in markets with three tokens. As shown in Figure 9, there are already many arbitrage opportunities in cycles of three tokens. Arbitrageurs are more likely to search for arbitrage opportunities in the market with fewer tokens first, which can already satisfy their trading demand. They may move to cycles with more tokens if they cannot find appropriate arbitrage opportunities in cycles with fewer tokens.

Third, long cycles are more susceptible to market changes. There is a latency between arbitrageurs submitting transactions and the execution of the transactions. During this period, other trades may change the exchange rate between tokens. Because long cyclic transactions are involved with more tokens and more trading markets, the exchange rate is more likely to deviate from the expected rate because of the influence of other trade. The main goal of cyclic arbitrage is to take the risk-free revenue from the arbitrage strategy. Therefore, to reduce the risk of market changes, arbitrageurs prefer trading through shorter cycles.
5 COST OF CYCLIC ARBITRAGES

Although arbitrageurs submit their transaction requests as soon as they observe arbitrage opportunities, there is still a risk that the arbitrageurs will not benefit from the transactions. Miners collect transactions that have not been executed from the Ethereum network and order them in the newly generated block according to miners own wishes (normally according to the gas price). Traders have to pay gas fees to miners for executing their transactions. Some transactions that interact with the same liquidity pools may be executed prior to the cyclic transactions. When the cyclic transaction executes, the exchange rate might deviate from the rate at which the trader submitted the transactions, which results in a negative revenue, or the revenue is smaller than the gas fee.

Ethereum allows arbitrageurs to hedge such risks by deploying smart contracts. Because Ethereum is a replicated state machine, the system state is changed if the whole transaction (including several exchanges) executes successfully. Arbitrageurs can let the transaction fail if it results in a negative revenue. While this seems risk-free for cyclic arbitrageurs, there are costs as well. In particular, no matter the transaction has been successfully executed or not, the Ethereum miners ask for a fee from arbitrageurs to execute the smart contract. If the cyclic trade was not profitable anymore at the time of execution, the miners will still ask for a fee.

What are the costs of the arbitrageurs? In this section, we study the cost for implementing cyclic arbitrages in real markets.

5.1 Successful Transactions

Figure 10 shows the distribution of the gas fee of successful cyclic transactions. Notice that a cyclic transaction might involve several cycles. However, it is impossible to distinguish the gas fee for each cycle separately. Therefore, we only consider the net profit of the entire transaction. More than half of transactions spend less than 0.02 ETH as the gas fee to miners, and only 11.8% of transactions spend more than 0.05 ETH to execute the transaction. The gas fee (7,869.14 ETH) only accounts for 24.9% of the total revenue (31,727.11 ETH) of cyclic transactions. Figure 11 compares arbitrage income and net profit for cyclic arbitrages. Only 6.7% of cyclical arbitrages result in negative profits. Gas fees drive revenue per cyclic arbitrage from 0.1-0.3 ETH gross revenue to 0-0.1 ETH net profit, where the majority of cyclic arbitrages (68.2%) falling in this range.

The cost of a successful cyclic transaction (gas fee) is determined by the gas price and the amount of gas used by miners to run the contract (gas fee = gas price $\times$ gas consumption). To minimize the gas fee, arbitrageurs can either decrease the gas price (may result in lower priority of their transactions and longer waiting time for miners to execute), or optimize the executing efficiency of their transaction. Figure 12 shows the optimization of transaction execution over time. The average gas consumption per transaction has been decreasing from 400,000 to 250,000 between May and
January. Meanwhile, the gas price is extremely high in August and September (Figure 13), resulting in a sharp downtrend of the mean gas used (yellow and red plots are lower than blue plots at similar time points). Because if arbitrageurs want a positive profit when the gas price is very high, they have to optimize their gas used per transaction during the period.

### 5.2 Failed Transactions

What are the costs for failed attempts? Notice that arbitrageurs perform cyclic transactions with two different implementations: deploying a private smart contract to call exchange functions of Uniswap V2 (275,127 transactions, 685 CAs), and directly calling Uniswap V2 from EOAs (10,000 transactions, 136 EOAs). We analyze the success rate of cyclic transactions under these two different implementations and find a significant variance.

Figure 14 shows the distribution of the success rate of 685 CAs. As we have declared in Section 3, because all these smart contracts are private, we can not distinguish whether a failed transaction aims to cyclic arbitrage. Therefore, we count all failed transactions involving the smart contract as failed cyclic transactions, making the success rate we count here is lower than the real number. Arbitrageurs have performed 442,800 transactions, and 275,127 successful transactions have been observed. The overall success rate is 62%. We find most arbitrageurs (368 out of 685) have a success rate higher than 90%.

Some arbitrageurs have a low success rate because of the defective design of their smart contracts. In particular, some of them have already been updated to a more advanced version. Nevertheless, smart contracts cannot be directly modified after being deployed on blockchains. Arbitrageurs
can only create a new CA to perform arbitrages. Due to the anonymity of blockchain systems, we cannot recognize which contracts belong to the same arbitrageurs. But we observe some examples: 0xb8db34... (success rate 27%) stopped performing cyclic arbitrages since the middle of January and 0xd0ca12... (success rate 94%) started cyclic arbitrages one day after the previous account stopped trading. The same groups of EOAs have interacted with these two smart contracts, indicating that these two CAs may belong to the same arbitrageur with high probability, and the same for other low success rate CAs.

Figure 15 shows the distribution of the net profit of 685 CAs. Because we only consider revenue acquired by cyclic transactions starting with ETH but the cost for all transactions, the real net profit is higher than what we show here. The total net profit of cyclic arbitrageurs is 22,206.88 ETH (70% of the revenue), while 60.3% of arbitrageurs have a positive income. Although these numbers do not look perfect, several reasons suggest that the risk of cyclic arbitrages is negligible. First, for those experienced arbitrageurs who have performed more than 100 cyclic arbitrages, 103 out of 109 have a positive net profit, suggesting that cyclic arbitrage is profitable over the long term. Second, some arbitrageurs may give up the smart contract if it does not perform well and get a positive income from the newly deployed smart contract. Their old smart contracts stay in a negative profit state and will never be changed. Third, we overestimate the number of failed transactions. Some arbitrageurs spend more than 100 ETH on failed transactions. We infer that these costs are caused by failed executions of other market behavior. Each failed cyclic transaction only costs the gas fee, less than 0.2 ETH. If the cyclic arbitrage strategy is not profitable and loses the money up to such an amount (100 ETH), then traders will give it up much earlier for sure.

For those arbitrageurs who use public functions in Uniswap V2 to perform cyclic transactions, they have a much lower success rate than private smart contract arbitrageurs. Since Uniswap V2 is open-source, we can identify if the failed transactions are for the purpose of cyclic arbitrage. Arbitrageurs made 35,263 arbitrage attempts interacting with the Uniswap V2 smart contract, yielding a success rate of 28.4% (10,000 out of 35,263). More importantly, arbitrageurs have costs higher than their revenue. The total gas fee for these transactions is 338.78 ETH, while the revenue of success arbitrages is only 208.69 ETH, resulting in a significant loss of money.

6 IMPACT ON MARKET PRICES AND PARTICIPANTS

In this section, we consider how cyclic arbitrages change the market and influence on other market participants (normal traders and liquidity providers). Cyclic arbitrages balance the exchange rates across different pools. This is good for traders (who trade in a fair market), and as a consequence, also for the liquidity pool providers (since the increased trust in the system will also increase the number of trades, and as such, their income from transaction fees).

6.1 Market Price Balance and Normal Trades

First, we consider how cyclic arbitrages change the market price of tokens. Cyclic arbitrages can effectively balance the price between tokens through different trading paths and provide fair markets for normal traders.

**Lemma 6.1.** The market price after cyclic transactions is more balanced than the price before the cyclic transactions.

Figure 16 shows the change of the arbitrage index within the cycles before and after the transaction. Apart from what we have shown in the figure, more than one-third of cyclic transactions happen when the arbitrage index is larger than 1.1 (119,621 out of 285,217). Our result shows that the market price is always balanced after cyclic transactions, where the arbitrage index of these cycles always becomes smaller than the previous index. Normal traders can exchange their assets
in a more fair market. Even though arbitrageurs may not balance the market prices perfectly and remain price deviations in the market, most of them (147,011 out of 285,217) do not leave any cyclic arbitrage opportunity. The arbitrage index is smaller than $1.003^3$ after these cyclic arbitrages, which is the minimum requirement on the arbitrage index for a positive revenue of cyclic transactions of length 3.

### 6.2 Influence on Liquidity Providers

Other than traders, liquidity providers also perform an important role in DEXes. Liquidity providers reserve their tokens in liquidity pools, enabling traders to exchange their assets with the pools. Liquidity providers contribute to the one-sided trading mechanism of DEXes. Traders are trading with the liquidity pools without the participation of another trader.

Moreover, the contributions of liquidity providers also influence price stability. Assume there are two markets between $A$ and $B$. In the first market, the liquidity pool reserves $a = 10000$ and $b = 1000$, while in the second market, the liquidity pool reserves $a' = 100$ and $b' = 10$. The exchange rates between $A$ and $B$ are $10 : 1$ in both markets. After an exchange of $10$ $A$ with $B$, the exchange rate in the first market moves to $\frac{10010}{999} = 10.02$, while the exchange rate in the second market moves to $\frac{110}{9} = 12.1$. The exchange rate in the first market is more stable than the second because of the contributions of liquidity providers.

To reward liquidity providers and attract more liquidity providers to reserve their money in liquidity pools, traders pay a transaction fee to liquidity providers for each transaction. In Uniswap V2, 0.3% of trading volume will remain in the liquidity pool as the transaction fee.

When playing the arbitrage strategy, cyclic arbitrageurs do not care about the exact exchange rate between two tokens in a liquidity pool but whether the market price is balanced across different pools. Cyclic arbitrageurs may move the exchange rate between tokens away from what other traders expect. Other traders have to perform more trades in the market to drive the exchange rate between tokens to the price they have in mind. Thus, cyclic transactions can increase the number of trading in DEXes, resulting in more transaction fees to liquidity providers.

**Lemma 6.2.** If the market is efficient, liquidity providers gain more transaction fees if cyclic transactions appear during the convergence of the market prices.

We analyze the relationship between the daily change of $lp$ tokens (represents the total contribution of liquidity providers) and the daily number of cyclic transactions in six liquidity pools where cyclic arbitrages always happen (Appendix D), i.e. $ETH ⇛ YAM$, $ETH ⇛ BADGER$, $ETH ⇛ REVV$, $ETH ⇛ RWS$, $ETH ⇛ HEGIC$, $ETH ⇛ XFI$. We find that the correlation coefficients are almost zero for the first four pools and are around 0.3 for the last two pools.
In real markets, the behavior of liquidity providers is influenced by many other factors, such as the price of tokens in other exchanges, the trading volume through the liquidity pool, and other liquidity providers’ market performance. Moreover, the majority of liquidity providers lack information about cyclic transactions and how these transactions benefit themselves. Even though liquidity providers’ behaviors seem highly likely irrelevant to the involvement of cyclic arbitrageurs, Lemma 6.2 suggests that cyclic arbitrages do not hurt liquidity providers’ benefits. The participant of cyclic arbitrageurs does not influence liquidity providers’ engagement in DEXes.

7 ARBITRAGE ACROSS DIFFERENT DEXES

Price deviations not only appear within the same DEX, but also spread across different DEXes. Even for the same trading pair of tokens, exchange rates in different DEXes are also not the same, enabling traders to perform more cyclic arbitrages across liquidity pools of multiple DEXes, even arbitrages with cycles of length 2.

We collect the block-wise exchange rate (the ratio of token volume reserved in the liquidity pool) of fifteen popular trading pairs in both Uniswap V2 and Sushiswap and compute the block-wise arbitrage index between two exchanges. The arbitrage index of a trading pair is calculated by dividing the high exchange rate between the two markets by the low exchange rate.

Figure 17 shows the arbitrage index of these 15 token pairs between the trading pool in Uniswap V2 and Sushiswap over time. From Lemma 4.1, we know that cyclic arbitrages of length 2 are profitable if the arbitrage index is larger than 1.006. One-third of these 15 token pairs yield an average arbitrage index larger than the threshold, and 12% of block-wise arbitrage index among all these token pairs is larger than the arbitrage threshold, indicating great opportunities for cyclic arbitrages across different DEXes. Here we only collect the 15 largest liquidity pools in Uniswap V2 and Sushiswap, where the trades frequently happen across two DEXes. The market price is expected to be balanced in these pools. We can infer that for other pools with less liquidity, where not many trades perform across different DEXes, the arbitrage index may be higher than what we show here.
8 DISCUSSION

8.1 Arbitrage Strategies in DEXes and CEXes

Cyclic arbitrage opportunities appear in CEXes of cryptocurrencies when there are price deviations across different markets. For example, when the price of bitcoin in a US market (such as Coinbase) is above the price in a Japanese market (such as bitFlyer), an arbitrageur can buy bitcoins in Japan, sell them in the US, and then exchange US dollars for Japanese YEN [25] (trading through a cycle of Japanese YEN, bitcoin, and US dollars). However, there are many constraints to arbitrage in CEXes, such as transaction costs between different CEXes, cross-border capital controls, the latency of transactions, and limited trading options. Even though arbitrage opportunities can be easily found and persist for several hours, it is still not feasible for traders to implement arbitrage strategies across CEXes. This section mainly discusses how arbitrage strategies can be implemented in DEXes overcoming constraints in CEXes and suggests further studies to consider DEXes to analyze trading behavior in financial markets.

When traders want to sell their cryptocurrencies in CEXes, they have to first transfer their assets to the platform. This may take multiple minutes to hours. After that, they can trade with their assets on CEXes. Traders also need to pay the gas fee to the miners to realize the transfer of cryptocurrencies between the two CEXes. Nowadays, there are dozens of CEXes all over the world, and arbitrage opportunities may appear between any two of them. Traders can either move their assets frequently across different CEXes for arbitrage. In this case, they have to wait a long time for cryptocurrency transactions and pay a lot of gas fees. Alternatively, they may stick to a few CEXes and only perform arbitrages within them. In this case, they may lose many arbitrage opportunities. Moreover, after depositing cryptocurrencies in CEXes, traders already lose control of their assets. Recently, one of the leading CEXes has halted cryptocurrency withdrawals from their platforms [37], indicating the risk that the CEXes operator might misappropriate traders’ assets.

In DEXes, traders always (at any state of the transaction) keep their assets within their own accounts (wallets). DEXes smart contracts only swap assets between trading account (the protocol of ERC20 tokens allows DEXes smart contract accounts to remove assets from approval accounts) but not reserve traders’ assets in their own accounts. Therefore, traders can easily trade across different DEXes, even within a single transaction. They can efficiently utilize price deviations in different markets.

Another advantage of implementing arbitrage strategies in DEXes is that DEXes are not under cross-border capital control. In many CEXes, traders exchange cryptocurrencies with fiat currencies. For example, if arbitrageurs want to perform arbitrages within bitFlyer and Coinbase, they have to exchange the fiat currency of Japan and the US to ensure that their profits are not stuck in a country. In such a case, capital controls play an important role in implementing arbitrage strategies in CEXes. Capital controls do not influence trades in DEXes because all trading assets in DEXes are cryptocurrencies, which is stored in Ethereum or other blockchain systems. There is no need for traders to exchange fiat currencies immediately after cyclic arbitrage because they already earn profits in the same cryptocurrency that they own.

Finally, DEXes enable more trading options based on blockchain technologies. As we have shown in Section 2.4, traders can perform atomic transactions to ensure the consistency of several exchanges, which is the basis of the implementation of cyclic arbitrages. Meanwhile, smart contract technology ensures positive revenue of the transaction, which limits the risk of losing money. Moreover, other decentralized finance applications support more arbitrage possibilities. For example, flash loan applications [10], allow traders to borrow any amount of money (less than the amount reserved in the flash loan smart contracts) to apply arbitrage strategies if they can return the money back within the same transaction. Combined with cyclic arbitrages, traders do not need to keep...
any assets in their accounts to apply arbitrage transactions. They can easily use a huge amount of money to benefit themselves from price deviations, which greatly encourages more traders to participate in DEXes. Meanwhile, if their arbitrage transactions fail or do not have enough money to pay back to flash loan contracts, the system state will be reverted to the original state and ensures the safety of the money reserved in flash loan smart contracts.

These advantages of DEXes over CEXes suggest that arbitrage is happening more often in DEXes. As a result, exchange rates in DEXes will have less arbitrage potential. This again means that regular DEX customers have to worry less that the exchange rate is far from the truth.

8.2 Resilience to Front-running

Despite clear benefits in implementing arbitrage strategies in DEXes, the fundamental weakness of DEXes still keeps many traders resistant to the new markets: on-chain trades are somewhat slow. The average time of generating an Ethereum block is 15 seconds. Attackers may front-run transactions: They may observe an arbitrage transaction before the generation of the block and place their own transaction with higher fees.

In Section 5.2, we present the significant success rate of two different implementations of cyclic transactions. In general, two reasons can lead to the fail of cyclic arbitrages. The first one is a (random) change of the market state. Other transactions that are performed in advance of the cyclic transactions may change the exchange rate of the trading pair, resulting in a negative revenue. Secondly, there are front-running attacks [10] from other arbitrageurs. Especially for those transactions interacting with public smart contracts, attackers can easily understand the arbitrage strategy of the transaction and actively create their own transactions in front of the original one to grab the benefits. In such a case, the original cyclic transaction will also fail because the front-running attackers have already balanced the market prices.

The probability that a cyclic transaction failed because of the price fluctuations should be similar with two different implementations. Therefore, the resistance to front-running determines the success rate of the two different implementations. Assume that the price fluctuations influence $x\%$ of transactions, then $1 - \frac{\text{success rate}}{1-x\%}$ of cyclic transactions have been front-run by attackers. Given the huge variance between the success rate of two implementations of cyclic transactions (62% vs. 28.4%), our empirical analysis suggests that deploying private smart contracts can be resilient to at least 30% more of front-running attacks than directly calling public smart contracts.

For front-running attackers, if the smart contract is not open-source, they have to replay the transaction locally to identify whether the transaction is profitable. If the smart contracts use authentication schemes such that only their own EOAs can call smart contract functions, then it is more challenging for attackers to copy the arbitrage strategy in a short time.

9 CONCLUSION

This paper provides a comprehensive study on cyclic transactions in AMM DEXes. We take Uniswap V2 as an example, analyzing traders' behavior in the emerging cryptocurrencies exchanges and observing how they benefit themselves from the recurring deviations in exchange rates across different liquidity pools.

We show that price deviations appears in the market since two weeks after the beginning of Uniswap V2 and provide a certain market condition on price deviations (arbitrage index) when arbitrageurs can earn positive revenue with cyclic transactions. By deploying atomic and reversible cyclic transactions, arbitrageurs can easily take the risk-free/low risky revenue, yielding a total revenue of 42 million USD within eight months. We further analyze the implementations of cyclic arbitrages. We consider the gas fees of cyclic transactions and find that only 6.7% of cyclic
transactions result in a negative profit. Moreover, we notice that some cyclic transactions may fail during the execution because they cannot bring a positive revenue. We study the success rate of two different implementations: deploying private smart contracts, using open-source smart contracts. A significant variance between their success rates has been observed. Even though failed transactions bring additional costs to cyclic arbitrageurs, cyclic arbitrages are still profitable over the long term. Cyclic arbitrages significantly balance the market price. Meanwhile, cyclic arbitrages do not influence liquidity providers’ behavior and even provide additional transactions to these market contributors. We also extend the scope of cyclic transactions in DEXes. We show that price deviations also spread to other DEXes, such as Sushiswap. Cyclic arbitrageurs can find more arbitrage possibilities across different DEXes.

This paper not only studies cyclic arbitrages in DEXes, but also provide insights into two other areas. First, for arbitrages in cryptocurrency markets, this paper suggests that implementations of arbitrages are more feasible in DEXes than CEXes, which will benefit further study in trader behavior in cryptocurrency markets. Second, for preventing front-running, this paper suggests private smart contract implementations can efficiently defend the on-chain attack, inspiring further studies in this direction.
REFERENCES

[1] Hayden Adams. 2020. Uniswap V2 Mainnet Launch! Technical Report. Uniswap.

[2] Hayden Adams. 2020. Uniswap V2 Overview. Technical Report. Uniswap.

[3] Yukihiro Aiba, Naomichi Hatano, Hideki Takayasu, Kouhei Marumo, and Tokiko Shimizu. 2002. Triangular arbitrage as an interaction among foreign exchange rates. *Physica A: Statistical Mechanics and its Applications* 310, 3-4 (2002), 467–479.

[4] Yukihiro Aiba, Naomichi Hatano, Hideki Takayasu, Kouhei Marumo, and Tokiko Shimizu. 2003. Triangular arbitrage and negative auto-correlation of foreign exchange rates. *Physica A: Statistical Mechanics and its Applications* 324, 1-2 (2003), 253–257.

[5] Yukihiro Aiba, Naomichi Hatano, Hideki Takayasu, Kouhei Marumo, and Tokiko Shimizu. 2004. Triangular arbitrage in the foreign exchange market. In *The Application of Econophysics*. Springer, 18–23.

[6] Susan Athey, Ivo Parashkevov, Vishnu Sarukkai, and Jing Xia. 2016. Bitcoin pricing, adoption, and usage: Theory and evidence. (2016).

[7] Zeta Avarikioti, Lioba Heimbach, Yuyi Wang, and Roger Wattenhofer. 2020. Ride the lightning: The game theory of payment channels. In *International Conference on Financial Cryptography and Data Security*. Springer, 264–283.

[8] Sanghyun Bai and Fred Robinson. 2019. Automated Triangular Arbitrage: A Trading Algorithm for Foreign Exchange on a Cryptocurrency Market.

[9] Rainer Böhme, Nicolas Christin, Benjamin Edelman, and Tyler Moore. 2015. Bitcoin: Economics, technology, and governance. *Journal of economic Perspectives* 29, 2 (2015), 213–38.

[10] Philip Daian, Steven Goldfeder, Tyler Kell, Yunqi Li, Xueyuan Zhao, Iddo Bentov, Lorenz Breidenbach, and Ari Juels. 2020. Flash boys 2.0: Frontrunning in decentralized exchanges, miner extractable value, and consensus instability. In *2020 IEEE Symposium on Security and Privacy (SP)*. IEEE, 910–927.

[11] David Easley, Maureen O’Hara, and Soumya Basu. 2019. From mining to markets: The evolution of bitcoin transaction fees. *Journal of Financial Economics* 134, 1 (2019), 91–109.

[12] Ittay Eyal. 2015. The miner’s dilemma. In *2015 IEEE Symposium on Security and Privacy*. IEEE, 89–103.

[13] Ittay Eyal and Emin Gün Sirer. 2014. Majority is not enough: Bitcoin mining is vulnerable. In *International conference on financial cryptography and data security*. Springer, 436–454.

[14] Daniel J Fenn, Sam D Howison, Mark McDonald, Stacy Williams, and Neil F Johnson. 2009. The mirage of triangular arbitrage in the spot foreign exchange market. *International Journal of Theoretical and Applied Finance* 12, 08 (2009), 1105–1123.

[15] Henry N Goldstein. 1964. The implications of triangular arbitrage for forward exchange policy. *The Journal of Finance* 19, 3 (1964), 544–551.

[16] Paz Grimberg, Tobias Lautinger, and Damon McCoy. 2020. Empirical analysis of indirect internal conversions in cryptocurrency exchanges. arXiv preprint arXiv:2002.12274 (2020).

[17] Campbell R Harvey. 2016. Cryptofinance. Available at SSRN 2438299 (2016).

[18] Alpha Homora. 2020. *Trading Volume Mining*. Technical Report. Alpha Homora.

[19] Gur Huberman, Jacob Leshno, and Ciamac C Moallemi. 2017. Monopoly without a monopolist: An economic analysis of the bitcoin payment system. *Bank of Finland Research Discussion Paper* 27 (2017).

[20] Aggelos Kiayias, Elias Koutsoupias, Maria Kyropoulou, and Yiannis Tselekovounis. 2016. Blockchain mining games. In *Proceedings of the 2016 ACM Conference on Security and Privacy (SP)*. 365–382.

[21] Elias Koutsoupias, Philip Lazos, Foluso Ogunlana, and Paolo Serafino. 2019. Blockchain mining games with pay forward. In *The World Wide Web Conference*. 917–927.

[22] Yujin Kwon, Dohyun Kim, Yunnok Son, Eugene Vasserman, and Yongdae Kim. 2017. Be selfish and avoid dilemmas: Fork after withholding (faw) attacks on bitcoin. In *Proceedings of the 2017 ACM SIGSAC Conference on Computer and Communications Security*. 195–209.

[23] Michael Lewis. 2014. *Flash boys: a Wall Street revolt*. WW Norton & Company.

[24] Hanqing Liu, Na Ruan, Rongtian Du, and Weijia Jia. 2018. On the strategy and behavior of bitcoin mining with n-attackers. In *Proceedings of the 2018 on Asia Conference on Computer and Communications Security*. 357–368.

[25] Igor Makarov and Antoinette Schoar. 2020. Trading and arbitrage in cryptocurrency markets. *Journal of Financial Economics* 135, 2 (2020), 293–319.

[26] Francisco J Marmolejo-Cossío, Eric Brigham, Benjamin Sela, and Jonathan Katz. 2019. Competing (semi-) selfish miners in Bitcoin. In *Proceedings of the 1st ACM Conference on Advances in Financial Technologies*. 89–109.

[27] Frank McCormick. 1979. Covered interest arbitrage: unexploited profits? Comment. *Journal of Political Economy* 87, 2 (1979), 411–417.

[28] Zheng Nan and Taisei Kaizoji. 2019. Bitcoin-based triangular arbitrage with the Euro/US dollar as a foreign futures hedge: modeling with a bivariate GARCH model. *Quantitative Finance and Economics* 3, 2 (2019), 347–365.
[29] Kartik Nayak, Srijan Kumar, Andrew Miller, and Elaine Shi. 2016. Stubborn mining: Generalizing selfish mining and combining with an eclipse attack. In *2016 IEEE European Symposium on Security and Privacy (EuroS&P)*. IEEE, 305–320.

[30] Emiliano Pagnotta and Andrea Buraschi. 2018. An equilibrium valuation of bitcoin and decentralized network assets. *Available at SSRN 3142022* (2018).

[31] Lukáš Pichl, Zheng Nan, and Taisei Kaizoji. 2020. Time series analysis of ether cryptocurrency prices: Efficiency, predictability, and arbitrage on exchange rates. In *Advanced studies of financial technologies and cryptocurrency markets*. Springer, 183–196.

[32] Kaihua Qin, Liyi Zhou, and Arthur Gervais. 2021. Quantifying Blockchain Extractable Value: How dark is the forest? *arXiv preprint arXiv:2101.05511* (2021).

[33] Kaihua Qin, Liyi Zhou, Benjamin Livshits, and Arthur Gervais. 2020. Attacking the DeFi Ecosystem with Flash Loans for Fun and Profit. *arXiv preprint arXiv:2003.03810* (2020).

[34] Geoffrey Ramseyer, Ashish Goel, and David Mazieres. [n.d.]. Scaling On-Chain Asset Exchanges via Arrow-Debreu Exchange Markets. ([n. d.]).

[35] Ayelet Sapirshtein, Yonatan Sompolinsky, and Aviv Zohar. 2016. Optimal selfish mining strategies in bitcoin. In *International Conference on Financial Cryptography and Data Security*. Springer, 515–532.

[36] Gavin Wood et al. 2014. Ethereum: A secure decentralised generalised transaction ledger. *Ethereum project yellow paper* 151, 2014 (2014), 1–32.

[37] Hu Yue and Denise Jia. 2017. China’s OKEx halts cryptocurrency withdrawals after founder arrested. *Academia. edu-Share research* (2017).

[38] Liyi Zhou, Kaihua Qin, Christof Ferreira Torres, Duc V Le, and Arthur Gervais. 2020. High-Frequency Trading on Decentralized On-Chain Exchanges. *arXiv preprint arXiv:2009.14021* (2020).
A APPENDIX: PROOFS OF THE LEMMAS

Lemma A.1 (Lemma 4.1). For a given cycle $A \rightarrow B^1 \rightarrow \ldots \rightarrow B^n \rightarrow A$ with $n + 1$ tokens, there exists arbitrage opportunity for the cyclic transaction if the arbitrage index $\frac{a_{n+1}b_1^1b_2^2\ldots b_n^n}{a_1b_1^1b_2^2\ldots b_{n+1}^{n+1}} > \frac{r_1}{r_1^{n+1}r_2}$. Meanwhile, the arbitrage cannot benefit from the reversed direction $A \rightarrow B^n \rightarrow \ldots \rightarrow B^1 \rightarrow A$ for cyclic transactions.

We first give a general expression of cyclic transactions with more than three tokens.

Lemma A.2. For a cyclic transaction through a path with $n + 1$ edges $A \rightarrow B^1 \rightarrow B^2 \rightarrow \ldots \rightarrow B^n \rightarrow A$, the first deviation of the utility function at $\delta_a = 0$ is $\frac{\partial U}{\partial \delta_a} \bigg|_{\delta_a=0} = \frac{r_1^{n+1}r_2^{n+1}a_{n+1}b_1^1b_2^2\ldots b_n^n}{a_1b_1^1b_2^2\ldots b_{n+1}^{n+1}} - 1$.

Proof. We can observe that the transaction through $A \equiv B$ and $B \equiv C$ is equivalent to an exchange of $\delta_a$ through liquidity pool $A'/C'$, where $a' = \frac{a_1b_1}{b_1+r_1r_2b_1}$, and $c' = \frac{b_1r_1r_2b_1}{b_1+r_1r_2b_1}$.

We prove this lemma by induction. We first show it is correct when $n = 2$.

We take the utility function of a cyclic transaction of three tokens as shown in Equation 1 and compute the first deviation of the function,

$$\frac{\partial U_{AB^1B^1A}}{\partial a} \bigg|_{a=0} = \frac{r_1^3r_2^3a_1b_1^1b_2^2}{a_1b_1^1b_2^2} - 1 \quad (2)$$

For $n > 2$, the inductive hypothesis is that the equation is true for $n$:

$$\frac{\partial U}{\partial \delta_a} \bigg|_{\delta_a=0} = \frac{r_1^{n+1}r_2^{n+1}a_{n+1}b_1^1b_2^2\ldots b_n^n}{a_1b_1^1b_2^2\ldots b_{n+1}^{n+1}} - 1.$$

The inductive step is to prove the equation for $n + 1$:

If we exchange $\delta_a$ through $A/B^1$ pool to get $\delta_b^1$ and obtain $\delta_b^2$ from $B^1/B^2$ by trading $\delta_b^1$ right after, these two atomic transactions can be equivalent to a single transaction with $\delta_c$ in a virtual pool $A_c/B_c$, where $a_c = \frac{a_1b_1}{b_1+r_1r_2b_1}$, and $b_c = \frac{r_1r_2b_1b_2}{b_1+r_1r_2b_1}$.

We assume that the statement is correct for any $n$-path cyclic transaction. Consider a cyclic transaction through a $n + 1$-path $A \rightarrow B^1 \rightarrow B^2 \rightarrow \ldots \rightarrow B^n \rightarrow A$, which is equivalent to a $n$-path cyclic transaction through $A_0 \rightarrow B_0 \rightarrow \ldots \rightarrow B^n \rightarrow X$, where the first deviation of the utility function at $\delta_a = 0$ is,

$$\frac{\partial U}{\partial \delta_a} \bigg|_{\delta_a=0} = \frac{r_1^n r_2^n a_{n+1} b_1^1 \ldots b_n^n}{a_1 b_1^1 b_2^2 \ldots b_{n+1}^{n+1}} - 1 \quad (3)$$

By the principle of mathematical induction: the first deviation of the utility function at $\delta_a = 0$ is $\frac{r_1^{n+1}r_2^{n+1}a_{n+1}b_1^1b_2^2\ldots b_n^n}{a_1b_1^1b_2^2\ldots b_{n+1}^{n+1}} - 1$, for any $n \geq 2$.

Proof of Lemma 4.1. Assume that we would like to exchange $\delta_a$ of token $A$ through the cycle. When $\delta_a = 0$, both the value of $U_{AB^1B^1A}$ and the value of $U_{AB^nB^1A}$ are 0.

Then we consider the first and the second derivative of the utility function at $\delta_a = 0$. 

, Vol. 1, No. 1, Article . Publication date: May 2021.
\[
\frac{\partial U_{AB^nB^nA}}{\partial \delta_a} \bigg|_{\delta_a=0} = r_1^{n+1} \cdot r_2^{n+1} \cdot a_{n+1} \cdot b_1^i \cdot b_2^i \cdot \ldots \cdot b_n^i - 1
\]  

If arbitrageurs can make profit in this trading direction, then,

\[
0 < \frac{\partial U_{AB^nB^nA}}{\partial \delta_a} \bigg|_{\delta_a=0} < 1
\]  

which implies that \( \frac{r_1^{n+1} \cdot r_2^{n+1} \cdot a_{n+1} \cdot b_1^i \cdot b_2^i \cdot \ldots \cdot b_n^i}{a_1 \cdot b_1^i \cdot b_2^i \cdot \ldots \cdot b_n^i} < 1 \), then \( \frac{\partial U_{AB^nB^nA}}{\partial \delta_a} \bigg|_{\delta_a=0} < 0 \).

By computing the second derivative of \( U_{AB^nB^nA} \), we know that \( \frac{\partial^2}{\partial \delta_a^2} U_{AB^nB^nA} \) is negative for all \( \delta_a \in R^+ \). \( U_{AB^nB^nA} \) is a monotone decreasing function in its domain, and the maximum value of \( U_{AB^nB^nA} \) is 0 at \( \delta_a = 0 \). Therefore, there is no opportunity for arbitrage through the reversed direction \( A \rightarrow B^n \rightarrow \ldots \rightarrow B^1 \rightarrow A \) for cyclic transactions.

Arbitrageurs cannot benefit from trading \( \delta_a \) through \( A \rightarrow B^n \rightarrow \ldots \rightarrow B^1 \rightarrow A \), sequentially. Therefore, arbitrageurs have at most one direction to benefit themselves with cyclic transactions.

\( \square \)

**Lemma A.3** (Lemma 6.1). The market price after cyclic transactions is more balanced than the price before the cyclic transactions. In other words, there are fewer arbitrage opportunities after the cyclic transactions.

**Proof.** Consider a cyclic transaction through trading path \( \delta_a \) through \( A \rightarrow B^n \rightarrow \ldots \rightarrow B^1 \rightarrow A \).

A cyclic transaction is profitable through the path if the product of exchange rates before the transaction satisfies

\[
\frac{a_{n+1} \cdot b_1^i \cdot b_2^i \cdot \ldots \cdot b_n^i}{a_i \cdot b_1^i \cdot b_2^i \cdot \ldots \cdot b_n^i} > \frac{1}{r_1^{n+1} r_2^{n+1}}.
\]

After the transaction, the product have to satisfy

\[
\frac{(a_{n+1} - \delta_a) \cdot (b_1^i - \delta b_1^i) \cdot (b_2^i - \delta b_2^i) \cdot \ldots \cdot (b_n^i - \delta b_n^i)}{(a_i + \delta a) \cdot (b_1^i + \delta b_1^i) \cdot (b_2^i + \delta b_2^i) \cdot \ldots \cdot (b_n^i + \delta b_n^i)} > \frac{r_1^{n+1} \cdot r_2^{n+1}}{r_1^{n+1} r_2^{n+1}}.
\]

Otherwise, there is no income for arbitrageurs from this transaction. Thus, this cyclic transaction will not be executed.

Before the cyclic transaction, for two tokens in the market, such as \( A \) and \( B_i \), there are two market exchange rates between them, namely \( \frac{a_i}{b_1^i} \cdot \frac{b_1^i}{b_2^i} \cdot \ldots \cdot \frac{b_i^{i-1}}{b_i^i} \) and \( \frac{a_{n+1}}{b_{n+1}^i} \cdot \frac{b_{n+1}^i}{b_{n+2}^i} \cdot \ldots \cdot \frac{b_n^i}{b_{n+1}^i} \). The arbitrage index of these two prices is exactly \( \frac{a_{n+1} \cdot b_1^i \cdot b_2^i \cdot \ldots \cdot b_n^i}{a_i \cdot b_1^i \cdot b_2^i \cdot \ldots \cdot b_n^i} \), which is larger than \( \frac{r_1^{n+1} \cdot r_2^{n+1}}{r_1^{n+1} r_2^{n+1}} \).

Similarly, after the cyclic transaction, the arbitrage index of two market exchange prices between \( A \) and \( B_i \) is less than \( \frac{1}{r_1^{n+1} r_2^{n+1}} \). Therefore, the market price is more balanced after cyclic transactions.

\( \square \)

**Lemma A.4** (Lemma 6.2). If the market is efficient, liquidity providers gain more transaction fees if cyclic transactions appear during the convergence of the market prices.

**Proof.** Consider the exchange rate between tokens \( A \) and \( B \) is \( \frac{r}{r'} = r \) at time point \( t \), and rational traders would like to move this rate to \( r' \) by a transaction \( T \)

Assume there is cyclic transaction \( T^c \) happens at \( t \) and moves the exchange rate to \( r_c \). We analyze how this cyclic transaction changes the profit of liquidity providers.
If \( r_c = r' \), which means that \( T_c \) already moves the market price as other traders want. There is no need for an additional transaction \( T \). Cyclic transaction \( T_c \) does not change the profit of liquidity providers while it just substitutes of the original trade \( T \).

Otherwise, \( r_c \neq r' \), which means there will be another transaction \( T' \) to move the market price to \( r' \). Then we compare the profit of liquidity providers with \( T \), and with the combination of \( T_c \) and \( T' \).

We assume that \( T = (+\delta_a, -\delta_b) \), where traders would like to exchange \( \delta_a \) of A with \( \delta_b \) of B from the market. There are three scenarios according to the cyclic transaction \( T_c \) and \( T' \).

- \( T_c = (+\delta'_a, -\delta'_b) \) and \( T' = (-\delta'_a, +\delta'_b) \): We can deduce that \( \delta'_a > \delta_a \). It turns out that \( T_c \) leaves \( \delta'_a \cdot (1 - r_1) \) of token A as the transaction fee in the liquidity pool, which is more than what \( T \) can contribute to the liquidity pool. Therefore, liquidity providers earn more transaction fees because of the cyclic transaction.
- \( T_c = (-\delta'_a, +\delta'_b) \) and \( T' = (+\delta'_a, -\delta'_b) \): We can deduce that \( \delta'_a > \delta_a \). It turns out that \( T' \) leaves \( \delta'_a \cdot (1 - r_1) \) of token A as the transaction fee in the liquidity pool, which is more than what \( T \) can contribute to the liquidity pool. Therefore, liquidity providers earn more transaction fees because of the cyclic transaction.
- \( T_c = (+\delta'_a, -\delta'_b) \) and \( T' = (+\delta'_a, -\delta'_b) \): The cyclic transaction \( T_c \) leaves \( \delta'_a \cdot (1 - r_1) \) of token A as transaction fee in the liquidity pool. Because the product of tokens’ volume is a constant in Uniswap V2, after the cyclic transaction \( T_c \), the product of tokens’ volume is increased by \( \delta'_a \cdot (1 - r_1) \cdot (b - \delta'_b) \). Meanwhile, the exchange rates between A and B after \( T \) and \( T' \) are the same. Therefore, \( T_c \) and \( T' \) leave more transaction fees in the liquidity pool then \( T \).

\[ \square \]

B APPENDIX: BLOCK-WISE ARBITRAGE INDEX

| token A | token B | token A | token B | token A | token B | token A | token B |
|---------|---------|---------|---------|---------|---------|---------|---------|
| YAM     | yCRV    | XFI     | XSP     | FRM     | FRMX    | REVV    | LCX     |
| XRT     | RWS     | $BASED  | sUSD    | HEGIC   | zHEGIC  | XAMP    | TOB     |
| BADGER  | WBTC    | TRB     | DAI     |         |         |         |         |

Table 1. Tokens for computing the block-wise arbitrage index in Uniswap V2.

Fig. 18. Block-wise arbitrage index of YAM in Uniswap V2.
Fig. 19. Block-wise arbitrage index of XRT in Uniswap V2.

Fig. 20. Block-wise arbitrage index of BADGER in Uniswap V2.

Fig. 21. Block-wise arbitrage index of XFI in Uniswap V2.

Fig. 22. Block-wise arbitrage index of $BASED in Uniswap V2.
APPENDIX: ARBITRAGEUR PERFORMANCE

Figure 28 shows the daily arbitrage revenue of the top 10 arbitrageurs we observed with the highest total revenue. These arbitrageurs are (private) smart contracts that implement arbitrage strategies.
Fig. 27. Block-wise arbitrage index of XAMP in Uniswap V2.

Fig. 28. Revenue for top 10 cyclic arbitrageurs over time.

Fig. 29. The number of cyclic transactions performed on top cycles over time.

Only creators or other authorized licensed EOAs have access to call them (an arbitrageur may use multiple EOAs to initiate arbitrage transactions). The figure suggests a dual oligopoly market from May to August where two arbitrageurs (0x860bd2... and 0x693c18...) dominate the revenue of cyclic transactions. Since September, more competitors have appeared in the market. Even though 0x860bd2... still leads in the market for most of the time, other arbitrageurs can share the revenue of the market and sometimes outperform 0x860bd2....

D APPENDIX: POPULAR TRADING CYCLES

Figure 29 demonstrates that the most popular trading cycle over time. Due to the relatively short lifespan of cryptocurrencies, most of them are only active during a certain period of time, around
one month. After the active period, the exchange rates between these tokens become stable, and the market size of these tokens shrinks, resulting in less cyclic arbitrage opportunities.