Stability and Mass Parabola in Integrated Nuclear Model

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Abstract Following our previous introduction of nuclear quark-like model, in this paper, a more precise formula is presented for nuclear binding energy in the context of modified Integrated Nuclear Model (INM). INM is based upon quark-like model with three basic assumptions from which the nuclear binding energy and magic numbers are easily obtained. INM is modified here to give the nuclear mass parabola and most stable nuclei in each nuclei group. Our findings are compared with Liquid Drop Model (LDM), unmodified Integrated Nuclear Model (INM) and experimental data for most stable nuclei.

Keywords Stability, Mass Parabola, Integrated Nuclear Model, Binding Energy

1. Introduction

Nuclear physicists have been struggling hard for so many years to present a simple and complete nuclear model from which the characteristics of nuclei can be explained and comprehended. The existing successful nuclear models can only explain certain characteristics of the nuclei and make no comments about other nuclear properties. For example Liquid Drop Model (LDM) was presented by Von Weizsacker [1] and then was extended by Bohr and Wheeler [2]. This model gives the nuclear binding energy formula in terms of A and Z [3] as follow:

\[ B(A,Z) = a_v A - a_s A^{2/3} - a_v Z (Z - 1) A^{-1/3} - a_n (N - \frac{Z A}{2} - 1 \pm \eta) + \delta \]

In formula (1) \( \delta \) is a dimensionless constant defined to have a range from 90 to 100 [10]. The coefficient \( a \) is a dimensionless constant defined to have a range from 90 to 100 [10].

Liquid Drop Model (LDM) has been successful in the calculation of binding energy, mass parabola and most stable isotopes. However this model fails to predict other properties of nuclei such as, the magic numbers and nuclear magnetic moments. On the other hand Nuclear Shell Model [4] which is based upon Schrodinger equation solution with selected potential such as rounded edge potential well, predict the magic numbers and nuclear magnetic moments by using spin-orbit couplings in a relatively complicated manners[5,6].

We have presented the Integrated Nuclear Model (INM) based upon nuclear quark-like model from which all magic numbers are easily obtained and a new magic number is predicted [7,8]. This model is also used to find the magnetic dipole moment of deuteron with greater precision [9]. Using INM, the nuclear binding energy formula was also obtained from quark-like model of nuclei [10] and is given as follow:

\[ B(A,Z) = \left[ A - \left( \frac{(N^2 - Z^2) + \delta(N - Z)}{3Z} + 3 \right) \right] \times \frac{M_N c^2}{a} \text{ A > 5} \] (2)

In formula (2) \( \delta \) is a dimensionless constant defined to have a range from 90 to 100 [10]. The coefficient \( a \) is a dimensionless constant defined to have a range from 90 to 100 [10].

Formula (2) provides the nuclear binding energy for most of the stable nuclei in terms of only one coefficient namely \( a \) which is simpler than LDM with several coefficients. However this formula needs to be modified to give us the mass parabola in the same way as LDM.

In this paper the modified nuclear binding energy formula is presented in order to find the nuclear mass parabola and the stability of isobaric groups of nuclides. The determination of the mass parabola itself is an indication of the validity of the INM which is based upon the quark structure of the nuclei instead of nucleon structure.

2. Modified Nuclear Binding Energy Formula in INM

In the binding energy formula (2) presented in INM a coefficient \( a \) is introduced which varies between 90 to 100 for all stable nuclides with A > 5. The coefficient \( a \) may be called “nuclear stability coefficient”.

A careful investigation of the stability coefficient for many stable nuclides indicates the fact that \( a \) depends upon atomic number (Z) and mass number (A). In fact for isobar nuclides, the stability coefficient \( a \) is proportional to atomic number (Z) whereas for isotopic nuclides the stability coefficient \( a \) is inversely proportional to the mass number (A). Therefore
Further analysis of most of the stable nuclides allows proper modification of formula (2) as follow:

\[
B(A,Z) = \left[ A - \left( \frac{(N^2 - Z^2) + \delta(N - Z)}{3(Z - k)} \right) + 3 \right] \times \frac{A^{n+\delta}M_N}{126(Z - k)} \quad A > 5
\]  

(3)

In which the coefficient \( k, s \) and \( n \) are defined as

\[ s = \begin{cases} 
0.0003, & N,Z \text{ even} \\
-0.0003, & N,Z \text{ odd} 
\end{cases} \]

\[ k = \begin{cases} 
2, & Z \leq 118 \\
0, & Z > 118 
\end{cases} \]

\[ n = 0.87 \text{ to } 0.88 \]

As can be seen, only one free coefficient namely, \( n \) exist that has a limited range of fine tuning and all other coefficients are known constants.

Now with this modified formula (3), one can find the mass parabolas and stability line.

First we write the nuclear mass as follow:

\[
M(A,Z) = AM_N + (M_H - M_N)Z - \left[ A - \left( \frac{(N^2 - Z^2) + \delta(N - Z)}{3(Z - k)} \right) + 3 \right] \times \frac{A^{n+\delta}M_N}{126(Z - k)}
\]

(4)

Differentiating \( M(A,Z) \) with respect to \( Z \) and equating it to zero give us \( Z_A \) at which \( M(A,Z) \) is minimum. After carrying out the standard calculation, we end up with an equation of power of three in \( Z \) such as:

\[
aZ^3 + bZ - c = 0
\]

(5)

Equation (5) is solved with Maple program and we get one real solution:

\[
Z_A = k + \frac{1}{6} \sqrt[3]{\frac{108c + 12\sqrt{3}}{a^2} \left( \frac{4b^3 + 27c^2a}{a} \right) a^2} \]

(6)

Fig.(1) shows the mass parabolas for odd \( A \) (\( A = 135 \)) and for even \( A \) (\( A = 102 \)) in which \( Z_A \) is shown by down ward arrows. For odd \( A \) with \( s = 0 \) only one parabola is obtained; for even \( A \) isobars we get two parabolas.

As shown in fig.(1) for \( A = 135 \) only one stable isobar exist at \( Z = 56 \) which is very close to our finding at \( Z_A = 55.8 \) but for even \( A \) nuclides depending upon the distance between two obtained parabolas, we get several stable even-even isobars. Here for \( A = 102 \) there are two stable isobars at \( Z = 44 \) and \( Z = 46 \) and for \( Z = 44 \) we get the most stable one, since this is closer to the calculated value namely, \( Z_A = 44.3 \). For LDM for \( A = 135 \), \( Z_A = 55.7 \) and for \( A = 102 \, Z_A = 44.7 \) [11]. It is seen that our calculated values of \( Z_A \) is closer to the experimental data as compared to the LDM.
Finally in table (1) a comparison is made between our findings of nuclear binding energy and the results of other models and experimental data.

3. Conclusion

Determination of the stability mass parabola and the nuclear stability line in modified INM and comparison of the most stable nuclei obtained from this model, with the experimental data and LDM is an indication of the validity of the modified INM which is based upon quark-like model of nuclei. Also comparison of nuclear binding energy obtained from modified INM and experimental data, results in a closer match than LDM for most of the light, medium and heavy nuclei. Other characteristics of nuclides are being tested by INM in our research group and the results are promising.
| Z | Nucleus | A | B(EXP) MeV | B(LDM) MeV | B(INM) MeV | B(Modified INM) MeV |
|---|---------|---|------------|------------|------------|---------------------|
| 1 | H       | 1 | -26.461    |            |            |                      |
| 2 | He      | 4 | 28.296     | 21.9452    |            | 28.27305            |
| 3 | Li      | 6 | 31.994     | 27.64      |            | 31.7322             |
| 3 | Li      | 7 | 39.244     | 38.3835    |            | 38.9815             |
| 4 | Be      | 9 | 58.165     | 56.6316    |            | 59.3114             |
| 5 | B       | 10| 64.751     | 63.0939    | 30.10864   | 65.2428             |
| 5 | B       | 11| 76.205     | 75.0627    | 33.58272   | 77.4225             |
| 6 | C       | 12| 92.162     | 87.749     | 54.71667   | 91.8896             |
| 6 | C       | 13| 97.108     | 93.629     | 65.03467   | 96.9228             |
| 7 | N       | 14| 104.659    | 99.6605    | 75.73481   | 104.2934            |
| 7 | N       | 15| 115.492    | 112.2803   | 92.19658   | 113.8012            |
| 8 | O       | 16| 127.619    | 123.7138   | 96.69506   | 127.5033            |
| 8 | O       | 17| 131.763    | 130.9744   | 104.82993  | 130.1135            |
| 8 | O       | 18| 139.807    | 141.24997  | 127.94649  | 138.5735            |
| 9 | F       | 19| 147.801    | 149.6775   | 139.15385  | 145.9720            |
| 10 | Ne      | 20| 160.645    | 160.15493  | 147.91676  | 158.8530            |
| 10 | Ne      | 21| 167.406    | 168.363    | 160.75488  | 167.4783            |
| 10 | Ne      | 22| 177.77     | 179.44476  | 167.29278  | 177.7011            |
| 11 | Na      | 23| 186.564    | 188.0092   | 178.76377  | 186.7211            |
| 12 | Mg      | 24| 198.257    | 196.68558  | 186.66229  | 197.1147            |
| 12 | Mg      | 25| 205.588    | 205.5993   | 198.70651  | 206.2926            |
| 12 | Mg      | 26| 216.681    | 217.2668   | 206.02692  | 214.5963            |
| 13 | Al      | 27| 224.952    | 224.1192   | 217.4098   | 222.5858            |
| 14 | Si      | 28| 236.537    | 233.089    | 225.38779  | 231.1055            |
| 14 | Si      | 29| 245.011    | 242.5576   | 236.6431   | 246.7711            |
| 14 | Si      | 30| 255.62     | 254.6751   | 244.7457   | 255.7162            |
| 15 | P       | 31| 262.917    | 260.9052   | 255.17021  | 264.1575            |
| 16 | S       | 32| 271.781    | 269.23215  | 261.40635  | 272.0444            |
| 16 | S       | 33| 280.442    | 279.1541   | 271.82458  | 280.8788            |
| 16 | S       | 34| 291.839    | 291.6321   | 280.5625   | 288.9729            |
| 17 | Cl      | 35| 298.21     | 297.29795  | 292.09649  | 297.3346            |
| 17 | Cl      | 36| 317.101    | 317.675    | 299.71709  | 313.3686            |
| 18 | Ar      | 36| 306.717    | 305.02832  | 309.3663   | 304.5650            |
| 18 | Ar      | 40| 343.811    | 346.7388   | 343.29829  | 343.6705            |
| 19 | K       | 39| 333.724    | 333.24958  |            | 335.6762            |
| 19 | K       | 41| 351.619    | 354.4661   | 334.60819  | 352.6519            |
| 20 | Ca      | 40| 342.052    | 340.41858  | 350.20724  | 342.5981            |
| 20 | Ca      | 44| 380.96     | 383.66084  | 379.8567   | 382.8547            |
| 21 | Sc      | 45| 387.848    | 390.6604   | 389.4375   | 391.2262            |
| 22 | Ti      | 46| 398.193    | 399.52474  | 398.22944  | 398.8393            |
| 22 | Ti      | 47| 407.073    | 408.53494  | 409.04261  | 407.5979            |
| 22 | Ti      | 48| 418.7      | 419.9271   | 426.51645  | 419.2554            |
| 22 | Ti      | 49| 426.842    | 427.26614  | 437.60839  | 427.4059            |
| 22 | Ti      | 50| 437.781    | 437.02429  | 437.60839  | 436.8867            |
| 23 | V       | 50| 434.794    | 434.6673   | 437.60839  | 436.8867            |
| Z   | Nucleus | A  | B(\text{EXP}) MeV | B(\text{LDM}) MeV | B(\text{INM}) MeV | B(\text{Modified INM}) MeV |
|-----|---------|----|-------------------|-------------------|-------------------|-----------------------------|
| 23  | V       | 51 | 445.416           | 455.536           | 463.39155        | 481.1957                    |
| 24  | Cr      | 52 | 456.349           | 468.80385         | 490.5518         | 524.93075                   |
| 25  | Mn      | 55 | 517.313           | 502.61932         | 516.2997         | 529.02083                   |
| 26  | Fe      | 58 | 506.459           | 526.846           | 595.7451         | 550.87284                   |
| 27  | Co      | 63 | 551.385           | 569.4077          | 558.6975         | 572.5846                    |
| 28  | Ni      | 66 | 569.212 559.098   | 562.37022         | 576.4769         | 594.627                     |
| 29  | Cu      | 68 | 595.387           | 603.8851          | 609.72888        | 622.65537                   |
| 30  | Zn      | 71 | 601.996           | 620.99784         | 631.17806        | 645.29982                   |
| 31  | Ga      | 74 | 618.951           | 612.65537         | 647.61204        | 650.43133                   |
| 32  | Ge      | 77 | 645.665           | 655.4886          | 655.8966         | 679.32321                   |
| 33  | As      | 80 | 679.99 696.866    | 682.06496         | 696.6787         | 679.6684                    |
| 34  | Se      | 83 | 686.321           | 697.35903         | 693.54105        | 695.4010                    |
| 35  | Br      | 86 | 704.37            | 689.2439          | 701.12258        | 687.4491                    |
| 36  | Kr      | 89 | 714.274           | 705.81675         | 712.55477        | 703.7878                    |
| 37  | Rb      | 92 | 721.737           | 715.7677          | 721.61748        | 714.8280                    |
| 38  | Sr      | 95 | 732.258           | 722.76424         | 730.68841        | 723.0077                    |
| 39  | Sr      | 98 | 749.235           | 731.78274         | 748.85597        | 730.9761                    |
| 40  | Y       | 101| 739.283           | 746.12903         | 742.09493        | 747.9164                    |
| 41  | Zr      | 104| 757.856           | 739.5233          | 760.81913        | 739.0302                    |
| 42  | Zr      | 107| 757.356           | 755.05569         | 745.0301         | 756.5981                    |
| 43  | Zr      | 110| 768.469           | 748.7453          | 754.44401        | 748.3195                    |
| 44  | Zr      | 113| 775.538           | 756.09715         | 772.1865         | 768.2167                    |
| 45  | Zr      | 116| 783.893           | 765.42723         | 774.67303        | 776.0138                    |
| 46  | Zr      | 119| 791.087           | 772.8487          | 784.95789        | 783.4294                    |
| 47  | Zr      | 122| 799.222           | 781.019           | 794.88848        | 791.9112                    |
| 48  | Zr      | 125| 814.677           | 788.70082         | 796.30213        | 800.2144                    |
| 49  | Zr      | 128| 805.765           | 798.32021         | 815.99203        | 814.4118                    |
| 50  | Zr      | 131| 814.04797         | 806.51142         | 805.9039         |                             |

Table (contd.)
| Z  | Nucleus | A | B(ESP) MeV | B(LDM) MeV | B(INM) MeV | B(Modified INM) MeV |
|----|---------|---|------------|------------|------------|---------------------|
| 42 | Mo      | 92 | 796.508    | 793.09932  | 795.94785  | 795.4930           |
| 42 | Mo      | 95 | 821.625    | 820.5965   | 817.88079  | 821.6087           |
| 42 | Mo      | 96 | 830.779    | 830.48545  | 827.97895  | 829.9008           |
| 42 | Mo      | 98 | 846.243    | 846.8345   | 848.34648  | 846.2422           |
| 44 | Ru      | 99 | 852.255    | 851.80259  | 857.39063  | 852.8166           |
| 44 | Ru      | 100| 861.928    | 861.94355  | 858.94508  | 861.2846           |
| 44 | Ru      | 101| 868.73     | 869.4767   | 869.40782  | 869.5955           |
| 44 | Ru      | 102| 874.844    | 878.87415  | 870.67943  | 875.7178           |
| 44 | Ru      | 104| 893.083    | 894.40381  | 891.54317  | 893.5445           |
| 45 | Rh      | 103| 884.163    | 885.0934   | 880.17114  | 885.1863           |
| 46 | Pd      | 104| 892.82     | 892.71256  | 889.22905  | 892.3174           |
| 46 | Pd      | 105| 899.914    | 900.53543  | 899.97871  | 900.7118           |
| 46 | Pd      | 106| 909.474    | 910.189    | 910.81159  | 908.9562           |
| 46 | Pd      | 108| 925.239    | 926.29915  | 922.81499  | 924.9800           |
| 46 | Pd      | 110| 940.207    | 941.11778  | 944.64934  | 940.3585           |
| 47 | Ag      | 107| 915.263    | 915.80492  | 919.77497  | 916.1749           |
| 47 | Ag      | 109| 931.727    | 932.86781  | 932.11781  | 932.5995           |
| 48 | Cd      | 110| 940.646    | 940.7986   | 940.98553  | 939.9041           |
| 48 | Cd      | 111| 947.622    | 948.22807  | 952.19342  | 948.0861           |
| 48 | Cd      | 112| 957.016    | 957.45559  | 953.35906  | 956.1212           |
| 48 | Cd      | 113| 963.556    | 964.25463  | 964.54063  | 964.0060           |
| 48 | Cd      | 114| 972.599    | 972.84985  | 975.82258  | 971.7369           |
| 49 | In      | 113| 963.094    | 963.41231  | 962.12798  | 963.5100           |
| 49 | In      | 115| 979.404    | 979.7077   | 984.90679  | 979.5262           |
| 50 | Sn      | 116| 988.684    | 987.89353  | 983.20639  | 986.9973           |
| 50 | Sn      | 118| 1004.955   | 1003.8319  | 1006.25447 | 1005.2137          |
| 50 | Sn      | 120| 1020.546   | 1018.5973  | 1018.68817 | 1020.0715          |
| 51 | Sb      | 121| 1026.325   | 1025.6927  | 1027.54763 | 1027.0924          |
| 51 | Sb      | 123| 1042.097   | 1040.1801  | 1040.04681 | 1043.4808          |
| 52 | Te      | 126| 1066.369   | 1063.5988  | 1061.3598  | 1066.7824          |
| 52 | Te      | 128| 1081.439   | 1076.7653  | 1085.47929 | 1082.4686          |
| 52 | Te      | 130| 1095.941   | 1088.937   | 1097.80741 | 1097.6144          |
| 53 | I       | 127| 1072.577   | 1070.8771  | 1069.98274 | 1073.9530          |
| 54 | Xe      | 129| 1087.651   | 1085.9613  | 1090.45272 | 1089.1120          |
| 54 | Xe      | 131| 1103.512   | 1100.1915  | 1103.42321 | 1105.2131          |
| 54 | Xe      | 132| 1112.448   | 1107.9009  | 1115.85749 | 1113.0778          |
| 55 | Cs      | 133| 1118.528   | 1115.3434  | 1124.19407 | 1117.6565          |
| 56 | Ba      | 137| 1149.681   | 1144.0675  | 1145.90412 | 1148.4568          |
| 56 | Ba      | 138| 1158.293   | 1151.5421  | 1158.66304 | 1158.9851          |

Table (contd.)
| Z | Nucleus | A | B(EXP) MeV | B(LDM) MeV | B(INM) MeV | B(Modified INM) MeV |
|---|---------|---|------------|------------|------------|---------------------|
| 57 | La | 138 | 1155.774 | 1151.2653 | 1153.8554 | 1155.7274 |
| 57 | La | 139 | 1164.551 | 1159.105 | 1166.7345 | 1136.5646 |
| 58 | Ce | 140 | 1172.692 | 1167.8856 | 1174.3925 | 1170.7473 |
| 58 | Ce | 142 | 1185.29 | 1181.675 | 1187.9478 | 1186.3789 |
| 59 | Pr | 141 | 1177.919 | 1174.1085 | 1181.6624 | 1177.7014 |
| 60 | Nd | 142 | 1185.142 | 1181.5574 | 1188.5679 | 1184.4542 |
| 60 | Nd | 143 | 1191.266 | 1188.3505 | 1189.3183 | 1192.5988 |
| 60 | Nd | 144 | 1199.083 | 1196.7746 | 1202.6147 | 1200.6467 |
| 60 | Nd | 146 | 1212.403 | 1211.0565 | 1216.5176 | 1213.3904 |
| 62 | Sm | 152 | 1253.104 | 1253.5745 | 1258.4994 | 1252.2646 |
| 62 | Sm | 154 | 1266.94 | 1266.5934 | 1272.4654 | 1267.1604 |
| 63 | Eu | 151 | 1244.141 | 1245.4718 | 1250.9251 | 1243.8002 |
| 63 | Eu | 153 | 1258.998 | 1260.0164 | 1252.0625 | 1259.4052 |
| 64 | Gd | 156 | 1281.598 | 1281.9731 | 1286.0473 | 1281.7931 |
| 64 | Gd | 158 | 1295.896 | 1295.468 | 1300.3523 | 1296.8405 |
| 64 | Gd | 160 | 1309.29 | 1308.1556 | 1314.5448 | 1308.1365 |
| 65 | Tb | 159 | 1302.027 | 1301.9809 | 1306.8200 | 1300.7987 |
| 66 | Dy | 162 | 1324.106 | 1323.6067 | 1327.5617 | 1322.9576 |
| 66 | Dy | 163 | 1330.377 | 1329.4064 | 1332.8106 | 1330.3749 |
| 67 | Ho | 164 | 1338.035 | 1336.7617 | 1342.0879 | 1337.6889 |
| 68 | Er | 165 | 1344.256 | 1343.3234 | 1348.2736 | 1345.0398 |
| 68 | Er | 166 | 1351.572 | 1351.0262 | 1354.1255 | 1352.1740 |
| 68 | Er | 167 | 1358.008 | 1357.0604 | 1354.4578 | 1359.0660 |
| 68 | Er | 168 | 1365.779 | 1364.6314 | 1368.9572 | 1367.0472 |
| 69 | Tm | 169 | 1371.352 | 1370.6194 | 1374.6657 | 1370.7159 |
| 70 | Yb | 172 | 1392.764 | 1391.7813 | 1395.1819 | 1392.6360 |
| 70 | Yb | 173 | 1399.131 | 1397.6674 | 1395.4145 | 1399.9972 |
| 70 | Yb | 174 | 1406.595 | 1405.069 | 1410.2421 | 1407.2110 |
| 71 | Lu | 175 | 1412.106 | 1411.0893 | 1415.6698 | 1410.8362 |
| 71 | Lu | 176 | 1418.394 | 1416.9087 | 1415.8533 | 1418.0879 |
| 72 | Hf | 177 | 1425.185 | 1424.3349 | 1421.1023 | 1425.2612 |
| 72 | Hf | 178 | 1432.811 | 1431.9414 | 1436.1315 | 1431.4239 |
| 72 | Hf | 180 | 1446.297 | 1444.9382 | 1451.4315 | 1446.5547 |
| 73 | Ta | 181 | 1452.24 | 1450.9735 | 1456.5802 | 1453.2859 |
| 74 | W | 182 | 1459.335 | 1458.1089 | 1461.4052 | 1459.8504 |
| 74 | W | 183 | 1465.525 | 1464.0784 | 1461.6230 | 1464.6875 |
| 74 | W | 184 | 1472.937 | 1471.526 | 1476.9029 | 1471.2362 |
| 75 | Re | 185 | 1478.341 | 1477.0087 | 1481.6746 | 1477.8280 |
| 75 | Re | 187 | 1491.877 | 1490.2904 | 1497.3695 | 1492.8904 |

Table (contd.)
| Z  | Nucleus | A  | B(EXP) MeV | B(LDM) MeV | B(INM) MeV | B(Modified INM) MeV |
|----|---------|----|------------|------------|------------|---------------------|
| 76 | Os      | 189| 1505.007   | 1503.2513  | 1502.04693  | 1503.9947           |
| 76 | Os      | 190| 1512.799   | 1510.5523  | 1517.73611  | 1510.5094           |
| 76 | Os      | 192| 1526.116   | 1523.036   | 1533.53906  | 1525.0828           |
| 77 | Ir      | 191| 1518.088   | 1516.0377  | 1522.14121  | 1517.1743           |
| 77 | Ir      | 193| 1532.058   | 1529.0561  | 1538.08144  | 1531.9626           |
| 78 | Pt      | 194| 1539.577   | 1536.145   | 1542.34012  | 1538.6738           |
| 78 | Pt      | 195| 1545.682   | 1541.8699  | 1542.38144  | 1546.0526           |
| 78 | Pt      | 196| 1553.604   | 1549.0352  | 1558.40616  | 1553.3518           |
| 79 | Au      | 197| 1559.386   | 1554.5095  | 1562.51729  | 1560.1086           |
| 80 | Hg      | 198| 1566.489   | 1561.0472  | 1566.36429  | 1566.7061           |
| 80 | Hg      | 199| 1573.153   | 1566.9765  | 1566.48393  | 1574.1315           |
| 80 | Hg      | 200| 1581.181   | 1574.3316  | 1582.67354  | 1581.4803           |
| 80 | Hg      | 201| 1587.411   | 1579.9487  | 1582.63325  | 1588.7515           |
| 80 | Hg      | 202| 1595.165   | 1586.9882  | 1598.96888  | 1595.9441           |
| 81 | Tl      | 203| 1600.87    | 1592.4387  | 1602.80964  | 1602.7903           |
| 81 | Tl      | 205| 1615.072   | 1604.989   | 1619.26432  | 1617.1500           |
| 82 | Pb      | 206| 1622.325   | 1611.9829  | 1622.92633  | 1624.0399           |
| 82 | Pb      | 207| 1629.063   | 1617.5005  | 1622.8084   | 1631.2074           |
| 82 | Pb      | 208| 1636.431   | 1624.4233  | 1639.51406  | 1638.2975           |
| 83 | Bi      | 209| 1640.23    | 1629.8385  | 1643.0243   | 1640.8417           |
| 90 | Th      | 232| 1766.687   | 1769.1834  | 1764.19153  | 1767.0470           |
| 92 | U       | 235| 1783.864   | 1785.8988  | 1786.7108   | 1782.1712           |
| 92 | U       | 238| 1801.69    | 1804.1704  | 1803.59006  | 1802.6499           |

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