Non-Markovianity through quantum coherence in an all-optical setup

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We investigate through both theory and experiment the application of quantum coherence of a single qubit as a measure of non-Markovianity in an open quantum system. As a first result, it is shown that distance-based quantum coherence minimized over local unitary transformations is equivalent to geometric measures of quantum correlations. This result is obtained for arbitrary distance (or pseudo-distance) measures in a multipartite scenario. Then, by taking into account the contractivity of trace-preserving quantum operations, we employ quantum coherence as a quantifier of non-Markovianity for quantum operations that preserve incoherence in a quantum system. This quantifier can be implemented through a single qubit, spending less resources than the usual multi-qubit characterizations of non-Markovianity through correlation measures. As an example, we consider one qubit under non-Markovian amplitude damping, for which we analytically evaluate the optimization over initial states required by the non-Markovian measure. Then, we experimentally realize this system through an optical setup with an intense laser beam simulating a single-photon polarization, with the system-environment interaction encoded in the propagation path. The experiment illustrates the quantification of non-Markovianity through the revivals of the single-qubit coherence, which are suitably controllable in terms of the non-Markovian strength.

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I. INTRODUCTION

Quantum coherence is a fundamental feature of quantum mechanics and is an important physical resource in quantum information. It is closely related to fascinating quantum phenomena in nature, arising from the superposition principle that single quantum systems and generalizing to multipartite quantum correlations in many-body systems. Indeed, the coherent superpositions of states in Hilbert space, in combination with the quantization of observables, signals the departure of quantum mechanics from the classical realm.

Quantum optical methods provide an important set of tools for the manipulation of coherence, and indeed, at its basis lies the formulation of the quantum theory of coherence. Optical setups are largely used to investigate quantum information tasks once one can encode qubits in the degrees of freedom of light such as propagation path, polarization, and transverse modes. For instance, entanglement, quantum computation, quantum gates, quantum cryptography, and teleportation have been investigated by encoding qubits in the degrees of freedom of light. In parallel, recent works have been exploring linear optical setups with an intense laser beam for investigating quantum features. This interesting approach has been used to investigate the emergence of topological phases in the evolution of a pair of entangled qubits, environment induced entanglement, Bell’s inequality, Mermin’s inequality for entangled tripartite systems, cryptography, teleportation, and conditional operations that emulate quantum gates. In this scenario, linear optical circuits associated to intense laser beams can be used to simulate single-photon experiments. Such approach is a suitable tool to test the quantum features that we deal with in this work.

Inspired by the recent developments about the quantitative characterization of coherence, our purpose here is to investigate through both theory and experiment the application of quantum coherence of a single qubit as a measure of non-Markovianity in an open quantum system. Classical and quantum correlations between two or more subsystems have been previously theoretically and experimentally applied to characterize a non-Markovian evolution. These approaches are based on the definition of a Markovian evolution in terms of completely positive (CP)-divisibility of a dynamical map, which provides a suitable measure of non-Markovianity as defined in Ref. [30]. While such correlations characterize the quantum features of a system with at least two parties, quantum coherence is already defined for a single system, which provides the simplest scenario to access quantum superpositions. Then, as a first step, we show that quantum coherence in a multiparticle system minimized over local unitary transformations is closely related to general geometric quantum correlation measures for arbitrary distance (or pseudo-distance) measures adopted. Therefore, it is somewhat intuitive that quantum coherence may be a useful tool to quantify non-Markovianity. Indeed, as a further result, we show that this is indeed the case for incoherent dynamical maps, i.e. dynamical maps that preserve incoherence in a quantum system. By taking this result into account, we experimentally realize a single-qubit amplitude damping channel through an optical setup with an intense laser beam simulating a single-photon polarization, with the system-environment interaction encoded in the propagation path. The experiment provides a convenient framework to illustrate the quantification of non-Markovianity through quantum coherence.
Markovianity through the revivals of the single-qubit coherence, which are controllable in terms of the non-Markovian strength.

The paper is organized as follows. In Sec. II we review the definition of quantum coherence measures. In Sec. III we analyze the connections of quantum coherence with quantum correlations measures in a generalized distance-based framework. In Sec. IV we turn our attention to the quantification of non-Markovianity by quantum coherence, with an analytical example for the amplitude damping channel. In Sec. V we present the experimental results for the realization of the single-qubit evolution under non-Markovian amplitude damping. In Sec. VI we summarize our results and present our conclusions.

II. QUANTUM COHERENCE MEASURES

Let us begin our discussion describing the incoherent states and the incoherent operations. Coherence is naturally a basis-dependent concept [1]. For this reason, we need first to define an orthonormal local reference basis \( |r⟩ = |r1⟩ ⊗ \cdots ⊗ |r_N⟩ \) for an \( N \)-partite system represented in a \( d \)-dimensional Hilbert space \( \mathcal{H} \). The density matrices acting on \( \mathcal{H} \) that are diagonal in this specific basis form the set of incoherent density operators \( I \) acting on \( \mathcal{H} \). Therefore, all density operators of the form

\[
\delta = \sum_{r=1}^{d} p_r |r⟩⟨r|,
\]

with the set \( \{p_r\} \) denoting a probability distribution, are incoherent (\( δ ∈ I \)) [3]. For Markovian quantum open systems, the quantum operations are described by completely positive and trace-preserving (CPTP) maps in terms of a set of Krauss operators \( \{K_n\} \) satisfying \( \sum_n K_n^† K_n = I \) [31]. By definition, incoherent operations are a subset of quantum operations with the restriction \( K_n T K_n^† \subseteq \mathcal{I} \) for all \( n \). Thus, the incoherent completely positive and trace-preserving (ICPTP) operations, which act as \( \Phi_{\text{ICPTP}}(δ) = \sum_n K_n^† δ K_n \), transform incoherent states into incoherent states, i.e., for any \( δ ∈ \mathcal{I} \), \( \Phi_{\text{ICPTP}}(δ) ∈ \mathcal{I} \) [1][3].

The \( d \)-dimensional maximally coherent state is defined by [3]

\[
|Ψ_d⟩ = \frac{1}{\sqrt{d}} \sum_{r=1}^{d} |r⟩,
\]

for which any \( ρ \) acting on the same Hilbert space can be generated with certainty by merely incoherent operations \( \Phi_{\text{ICPTP}} \) on it. It is important to note that Eq. (2) is independent of a specific measure for coherence and serves as a unit for defining coherence measures.

Baumgratz et al. [3] have defined a set of conditions that should be satisfied by any proper quantum coherence measure \( C(ρ) \), as follows:

(C1) Nonnegativity: \( C(ρ) ≥ 0 \) for all states \( ρ \), with \( C(δ) = 0 \) iff \( δ ∈ \mathcal{I} \).

(C2a) Monotonicity: \( C(ρ) \) does not increase under ICPTP maps, i.e., \( C(ρ) ≥ C(Φ_{\text{ICPTP}}(ρ)) \).

(C2b) Strong Monotonicity: Monotonicity under selective measures on average, i.e., \( C(ρ) ≥ \sum p_n C(ρ_n) \), where the post-measurement states are given by \( ρ_n = K_n^† ρ K_n/n \) with probabilities \( p_n = \text{tr}[K_n^† ρ K_n] \) for any set of incoherent Kraus operators satisfying \( \sum_n K_n^† K_n = I \) and \( K_n T K_n^† ⊂ \mathcal{I} \) for all \( n \). This condition quantifies the intuition that coherence should not increase under incoherent measurements even if one has access to the individual measurements outcomes.

(C3) Convexity: Nonincreasing under mixing of quantum states, i.e., \( \sum_n p_n C(ρ_n) ≥ C(\sum_n p_n ρ_n) \) for any set of states \( \{ρ_n\} \) and any \( p_n ≥ 0 \) with \( \sum_n p_n = 1 \).

A quantity \( C(ρ) \) which fulfills conditions (C1)-(C3) is called a coherence monotone. Some examples are the relative entropy of coherence and \( l_1 \) norm of coherence [3], the distillable coherence and coherence cost [32][33], geometric coherence [34], coherence monotones from entanglement [34] and the robustness of coherence [35]. A general distance-based coherence quantifier is defined as [3]

\[
C(ρ) = D(ρ, δ_{\text{min}}),
\]

where \( D(ρ, δ_{\text{min}}) \) is the distance (or pseudo-distance) between \( ρ \) and the closest incoherent state \( δ_{\text{min}} \). A bona fide distance is positive, unitarily invariant, and contractible under CPTP maps. An example of a distance-based coherence is the trace norm of coherence, defined by [3]

\[
C(ρ) = \| ρ − δ_{\text{min}} \|_1,
\]

where \( \| A \|_1 = \text{tr} \sqrt{A^† A} \) denotes the trace norm of the matrix \( A \). This measure satisfies conditions (C1)-(C3) for one-qubit [36][37]. Remarkably, in this case \( δ_{\text{min}} = ρ_{\text{diag}} \), where \( ρ_{\text{diag}} \) corresponds to the diagonal part of \( ρ \) in a given local basis, and the trace norm of coherence has the same expression as the \( l_1 \) norm of coherence, reading [38][39]:

\[
C(ρ) = 2|ρ_{12}|,
\]

with \( ρ_{12} = ⟨0|ρ|1⟩ \) denoting the off-diagonal element of the one-qubit density matrix \( ρ \). The equality between the trace norm of coherence and the \( l_1 \) norm of coherence is also maintained for two-qubit Bell diagonal states, where the closest incoherent states \( δ_{\text{min}} \) to a state \( ρ \) is always its diagonal part \( ρ_{\text{diag}} \). However, it was shown by Bromley et al. that this equivalence cannot be extended to general two-qubit states [38].

III. CONNECTIONS WITH QUANTUM CORRELATIONS MEASURES

Since coherence is a basis-dependent concept, even local unitary operation can change the quantum coherence in a given state. A basis-free measure of quantum coherence can be defined by the minimization over all local unitary transformations [40]:

\[
C_{\text{free}}(ρ) = C(U_{\text{min}} ρ U_{\text{min}}^†),
\]

(C2a) Monotonicity: \( C(ρ) \) does not increase under ICPTP maps, i.e., \( C(ρ) ≥ C(Φ_{\text{ICPTP}}(ρ)) \).

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where \( U_{\text{min}} \) represents a local unitary operation that minimizes \( C(\rho) \). By optimizing coherence as in Eq. (6), we then show in Theorem 1 below that \( C_{\text{free}}(\rho) \), defined for arbitrary multipartite systems and distance measures, is equivalent to the geometric quantum correlation measure, which reads

\[
Q(\rho) = D(\rho, \chi_{\text{min}}). \tag{7}
\]

Notice that \( Q(\rho) \) corresponds to the distance between \( \rho \) and its closest classical state \( \chi_{\text{min}} \). The set \( C \) of classical states is formed by any state that can be written as

\[
\chi = \sum_{i=1}^{d} p_i |s_i\rangle \langle s_i|,
\]

where \( \{p_i\} \) is a probability distribution and \( \{|s_i\} = \{|s_1\} \otimes \cdots \otimes |s_N\rangle \) is an orthonormal local basis for an \( N \)-partite system represented in a \( d \)-dimensional Hilbert space \( \mathcal{H} \). Here \( \{|s\rangle\} \) is not taken as a fixed orthonormal local reference basis \( \{|r\rangle\} \) such as in Eq. (1), revealing that incoherent states form a significantly smaller subset of the set of classical states, \( I \subset C \).

**Theorem 1.** Let \( \rho \) be a density operator acting on a multipartite Hilbert space \( \mathcal{H} \). Then, the basis-free quantum coherence measure \( C_{\text{free}}(\rho) \) is equal to the geometric quantum correlation measure \( Q(\rho) \).

**Proof.** Let us consider the general distance-based quantum coherence measure given by Eq. (3), with \( \rho \) acting on a multipartite Hilbert space. As \( D \) is invariant under unitary operations, i.e., \( D(U\rho U^\dagger, U\sigma U^\dagger) = D(\rho, \sigma) \), then:

\[
C_{\text{free}}(\rho) \equiv \min_{U, \delta} D(U\rho U^\dagger, \delta)
\]

\[
= \min_{U, \delta} D(U^\dagger U\rho U^\dagger U, U^\dagger \delta U)
\]

\[
= \min_{U, \delta} D(\rho, U^\dagger \delta U). \tag{9}
\]

Since \( \delta = \sum_r p_r |r\rangle \langle r| \), it follows that:

\[
U^\dagger \delta U = \sum_r p_r |r\rangle \langle r| U = \sum_r p_r |s_r\rangle \langle s_r|,
\]

where we take \( |s\rangle \equiv U^\dagger |r\rangle \). Thus, we can write

\[
\min_{U, \delta} D(\rho, U^\dagger \delta U) = \min_{|s_i\rangle, |s_i\rangle} D(\rho, \sum_s p_s |s\rangle \langle s|).
\]

From Eqs. (11), (9) and (8), we conclude that

\[
C_{\text{free}}(\rho) = D(\rho, \chi_{\text{min}}), \tag{12}
\]

where \( \chi_{\text{min}} \) is a classical state that minimizes \( D(\rho, \chi) \). Consequently, \( C_{\text{free}}(\rho) = Q(\rho) \). \( \square \)

Thus, for multipartite systems, we conclude that \( C_{\text{free}}(\rho) \) is equivalent to a geometric quantum correlation measure for all bona fide distance (or pseudo-distance)-based measure of coherence. Theorem 1 includes a previous result obtained for the relative entropy of discord, where the basis-free quantum coherence is calculated by the relative entropy of coherence [40]. For unipartite systems, it is always possible to choose an orthonormal basis such that \( \rho \) is diagonal and, as \( C_{\text{free}}(\rho) \) is unitarily invariant, we conclude that \( C_{\text{free}}(\rho) = 0 \). On the other hand, non-vanishing results can be obtained for other orthonormal basis, e.g., the computational basis. This will be our strategy when defining a coherence-based measure of non-Markovianity.

**IV. MEASURING NON-MARKOVIANITY THROUGH QUANTUM COHERENCE**

We now discuss a method of quantifying non-Markovianity through quantum coherence. Markovian evolution washes out correlations in a quantum system, making quantum correlation measures monotonic under local CPTP maps [27, 30, 42]. Following the same line of thought, based on the monotonically decreasing behavior of quantum coherence measures under ICPTP maps, given by the condition (2a), we have that for Markovian dynamics, it follows that \( dC(\rho(t))/dt \leq 0 \), where \( C(\rho) \) is a proper quantum coherence measure. Thus, any violation of this monotonicity \( dC(\rho(t))/dt > 0 \) at any time \( t \) will provide an indication of non-Markovianity. From this non-monotonicity of quantum coherence measures, we can define a quantifier of non-Markovianity as [44]:

\[
N_C(\Phi) = \max_{\rho(0)} \int_{d\rho(0)>0} \frac{d}{dt} C(\rho(t))dt, \tag{13}
\]

where the maximization is taken over all initial states \( \rho(0) \). Hence, \( N_C(\Phi) \) quantifies the degree of non-Markovianity for dynamical maps that preserve incoherence and lead to the interpretation of the reservoir memory effect like a backflow of the maximum amount of quantum coherence on the initial state, after the state has been subject to a noisy channel for a certain time.

In order to quantitatively analyze the quantifier of non-Markovianity \( N_C(\Phi) \), we will consider the coherence of a single qubit and take it in the computational basis \( \{|0\rangle, |1\rangle\} \). As an illustration, we will investigate the non-Markovian amplitude damping (AD) channel described by the following incoherent Kraus operators [14, 45]:

\[
K_0 = |0\rangle\langle 0| + \sqrt{p(t)}|1\rangle\langle 1|, \quad K_1 = \sqrt{1-p(t)}|0\rangle\langle 1|, \tag{14}
\]

where

\[
p(t) = e^{-\Gamma t} \left\{ \cos \left( \frac{\sqrt{2\gamma} t}{2} \right) + \frac{1}{\sqrt{\gamma}} \sin \left( \frac{\sqrt{2\gamma} t}{2} \right) \right\}^2.
\]

Here \( \alpha = 2\gamma/\Gamma \), being \( \gamma \) the system-reservoir coupling constant and \( \Gamma \) the decay rate of the qubit. The parameter \( \Gamma \), defining the spectral width of the coupling, is related to the

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1 This Kraus operators fullfil the condition \( K_i K_i^\dagger \in I \). The proof is straightforward if we consider the incoherent state \( \delta = \sum_i \delta_i |i\rangle \langle i| \) we obtain \( K_i \rho K_i^\dagger = \delta_i |0\rangle\langle 0| + \delta_i |1\rangle\langle 1| \in I \) and \( K_i \delta K_i^\dagger = \delta_i (1-p) |0\rangle\langle 0| \in I \).
reservoir correlation time through \( \tau_s \approx \Gamma^{-1} \). On the other hand, \( \gamma \) depends on the qubit relaxation time by \( \tau_s \approx \gamma^{-1} \). In the weak coupling regime, i.e. for \( 0 \leq \alpha \leq 1 \), \( p(t) \) is monotonically decreasing. In this regime, the time of relaxation \( \tau_s \) is greater than the reservoir correlation time \( \tau_r \). The behavior of \( p(t) \) is essentially a Markovian exponential decay controlled by \( \gamma \). Whereas in the strong coupling regime \( \alpha > 1 \), the reservoir correlation time is greater than the relaxation time and non-Markovian effects become relevant.

Let us now calculate the degree of non-Markovianity \( N_C(\Phi) \) for one-qubit system under non-Markovian AD noise. Firstly, from Eqs. (5) and (14), we get the quantum coherence dynamics:

\[
C(t) = C(0) \sqrt{p(t)},
\]

where \( C(0) = 2\rho_{12}(0) \). For \( 0 \leq \alpha \leq 1 \), \( C(t) \) is monotonically decreasing and, consequently, \( N_C(\alpha) = 0 \). On the other hand, for \( \alpha > 1 \), \( C(t) \) shows an oscillatory decay with the maximums and minimums occurring at \( t_m^\text{max} = 2\gamma \Gamma^{-1} (\alpha - 1)^{-1/2} m \) and \( t_m^\text{min} = t_{m+1}^\text{max} - 2\Gamma^{-1} (\alpha - 1)^{-1/2} \arctan \sqrt{\alpha - 1} (m = 0, 1, 2, 3...) \), respectively. In this case, Eq. (15) can be written as

\[
N_C(\alpha) = \max_{\rho(0)} \sum_{m=1}^{\infty} \left| C(t_m^\text{max}) - C(t_m^\text{min}) \right| (\alpha > 1),
\]

being \( \{(t_m^\text{min}, t_m^\text{max}) \} (m \neq 0) \) the set of all intervals of time such that \( dC(\rho)/dt > 0 \). Then, using Eq. (15), we obtain:

\[
N_C(\alpha) = \max_{\rho(0)} C(0) \sum_{m=1}^{\infty} e^{-x^2/2} \sqrt{\alpha - 1} (\alpha > 1).
\]

The maximization over \( \rho(0) \) is satisfied when \( C(0) = 1 \), i.e., when \( \rho_{12}(0) = 1/2 \). In addition, \( \sum_{m=1}^{\infty} e^{-x^2/2} \sqrt{\alpha - 1} = (e^{x^2/2} - 1)^{-1} \). Thus, we find a compact analytical expression for the degree of non-Markovianity in terms of the parameter \( \alpha \):

\[
N_C(\alpha) = \begin{cases} (e^{x^2/2} - 1)^{-1} & \alpha > 1, \\ 0 & 0 \leq \alpha \leq 1. \end{cases}
\]

The behavior of \( N_C(\alpha) \) is illustrated in Fig. [1]. In particular, \( N_C(\alpha) \approx \sqrt{\alpha}/\pi \) in the strong non-Markovian regime (\( \alpha \gg 1 \)).

The AD-channel effect on the one qubit system state can be regarded in the following map

\[
|0\rangle_S |0\rangle_R \rightarrow |0\rangle_S |0\rangle_R, \quad \text{(19)}
\]

\[
|1\rangle_S |0\rangle_R \rightarrow \sqrt{p(t)} |1\rangle_S |0\rangle_R + \sqrt{1-p(t)} |0\rangle_S |1\rangle_R, \quad \text{(20)}
\]

where the label \( S \) and \( R \) stands for system and reservoir states, respectively. The ground state of the qubit (\( |0\rangle_S \)) remains unchanged along the evolution, as shown in Eq. (19). On the other hand, the excited state of the qubit (\( |1\rangle_S \)) can decay with probability \( [1 - p(t)] \), as it can be seen by Eq. (20).

Note that for \( t \rightarrow \infty \), \( [1 - p(t)] \rightarrow 1 \), which means the excited qubit will decay after long time interaction with the channel. As discussed before, depending on the system-reservoir coupling constant \( \alpha \) we can achieve Markovian or Non-Markovian behavior. For a maximally coherent superposition state \( |\psi_+\rangle = \frac{1}{\sqrt{2}} (|0\rangle_S + |1\rangle_S) \) we have

\[
|\psi_+\rangle_S |0\rangle_R \rightarrow \frac{1}{\sqrt{2}} [ |0\rangle_S |0\rangle_R + \sqrt{p(t)} |1\rangle_S |0\rangle_R \\
+ \sqrt{1-p(t)} |0\rangle_S |1\rangle_R ].
\]

For \( t = 0 \), no interaction between the system and the reservoir happens, which means \( p(t = 0) = 1 \). Then, the state of the system is the initial superposition, as it can be seen in the right side of Eq. (21). After a long time interaction, \( p(t \rightarrow \infty) = 0 \), the superposition is lost and the system state decays to its ground state, for which coherence vanishes. The Non-Markovian behavior can be witnessed by observation of oscillations on the coherence values along the evolution.

V. THE EXPERIMENTAL REALIZATION

In order to experimentally investigate Non-Markovian signatures by means quantum coherence in the AD channel, we used propagation direction (path) and the polarization degrees of freedom of light. The system states are encoded in the polarization degree of freedom and the reservoir is encoded in the path degree of freedom. Regarding the state of polarization of a single-photon as two level system, we can associate the horizontal and vertical polarization state with the ground state \( (|H\rangle \equiv |0\rangle_S) \) and excited state \( (|V\rangle \equiv |1\rangle_S) \), respectively. Regarding the path, we can associate orthogonal directions, \( k_0 \) and \( k_1 \), corresponding to the reservoir ground state \( |0\rangle_R \) and excited state \( |1\rangle_R \), respectively. By using the polarization labels for the system states, the maps given by Eq. (19) and (20)
can be regarded as follows

$$|H\rangle_S |0\rangle_R \rightarrow |H\rangle_S |0\rangle_R,$$ (22)

$$|V\rangle_S |0\rangle_R \rightarrow \sqrt{p(t)} |V\rangle_S |0\rangle_R + \sqrt{1-p(t)} |H\rangle_S |1\rangle_R.$$ (23)

Then, for a maximally coherent superposition state ($|\psi_+\rangle$) the map presented in Eq. (21) can be written as

$$|\psi_+\rangle_S |0\rangle_R \rightarrow \frac{1}{\sqrt{2}} ( |H\rangle_S |0\rangle_R + \sqrt{p(t)} |V\rangle_S |0\rangle_R$$

$$+ \sqrt{1-p(t)} |H\rangle_S |1\rangle_R ).$$ (24)

We performed the experiment with an intense laser beam once it simulates the single-photon experiment and its results present the essence of the phenomenon we are studying. The use of intense laser beam for simulate quantum tasks has been used frequently in the literature [15]-[26]. In this approach, a laser beam polarized at +45° can be regarded as the analog of the maximally coherent state, given by Eq. (2) for $d = 2$. Then, we represent the H and V polarization respectively as $|H\rangle$ and $|V\rangle$, in order to directly associate the polarization quantum state.

The linear optical circuit used in the experiment is presented in Fig 2. A DPSS laser (532nm, 1.5mW power, horizontally polarized) passes through a half wave plate (HWP1@22.5°) with its fast axis making an angle of 22.5° by respect the horizontal in order to prepare a +45° polarized beam, i.e., the analogue to the state

$$|\psi_+\rangle_S \equiv |+\rangle_S = \frac{1}{\sqrt{2}} ( |H\rangle + |V\rangle ).$$ (25)

The resulting path is associated with the reservoir ground state. Then, after the HWP1, we have the input state $|+\rangle_S |0\rangle_R$. This step is highlighted in Fig 2 in the block preparation. It is worth to mention that we can prepare different input states just by rotating HWP1.

The AD-channel starts with a polarized beam splitter (PBS1) that transmits H-polarization component ($|H\rangle$) and reflects the V-polarization one ($|V\rangle$). In the reflection arm a HWP2@45° turns $|V\rangle \rightarrow |H\rangle$ and this beam is transmitted in the PBS2 remaining in the path $|0\rangle_R$. The component $|H\rangle$ transmitted in PBS1 passes through the HWP3@θ. By setting $\theta = +45°$, we transform $|H\rangle \rightarrow |V\rangle$ and the beam is reflected in PBS2 going also to the path $|0\rangle_R$. A piezoelectric ceramic (PZT) placed in one mirror controls the difference of phase $\Delta \phi$ between the two arms. This device allows us to perform a coherent superposition of $|H\rangle$ and $|V\rangle$ components in the output path $|0\rangle_R$. We can adjust $\Delta \phi = 0$ and recover the +45° polarized light, i.e., $|+\rangle_S |0\rangle_R$ state. This case simulates the instant $t = 0$, when the system has not yet interacted with the reservoir and the coherence is kept maximal.

The HWP3@θ emulates the parameter $p(t)$. Let us consider the HWP3 fast axis with an arbitrary angle $\theta$ by respect the horizontal. After HWP3@θ we have the following transformation

$$|H\rangle \rightarrow \cos(2\theta) |H\rangle + \sin(2\theta) |V\rangle.$$ (26)

Then, by adjusting $\Delta \phi = 0$, the polarization of the beam in the path $|0\rangle_R$ is

$$|\phi_0(0)\rangle_S |0\rangle_R = \frac{1}{\sqrt{2}} [ |H\rangle + \sin(2\theta) |V\rangle ] |0\rangle_R.$$ (27)

On the other hand, for the path $|1\rangle_R$ we have only the horizontal component of the polarization produced by HWP3@θ

$$|\phi_1(0)\rangle_S |1\rangle_R = \frac{1}{\sqrt{2}} [ \cos(2\theta) |H\rangle ] |1\rangle_R,$$ (28)

which corresponding to the damping produced by the channel with the reservoir stated receiving a quantum of energy. The complete map of the AD-channel circuit can be written as

$$|\psi_+\rangle_S |0\rangle_R \rightarrow |\phi_0(0)\rangle_S |0\rangle_R + |\phi_1(0)\rangle_S |1\rangle_R$$

$$\rightarrow \frac{1}{\sqrt{2}} [ |H\rangle_S |0\rangle_R + \sin(2\theta) |V\rangle_S |0\rangle_R$$

$$+ \cos(2\theta) |H\rangle_S |1\rangle_R ].$$ (29)

By comparing Eq. (29) and Eq. (24) we identify

$$\cos(2\theta) = \sqrt{1-p(t)}.$$(30)

Then, for each $\gamma t$ we can associate a corresponding angle $\theta$ of the HWP3@θ. The state associated with the path $|0\rangle_R$ of the AD-channel output is directed in the tomography block that performs a polarization state tomography [47]. PBS3 measure the polarization in $|H\rangle, |V\rangle$ basis, HWP4@22.5° associated with PBS3 proceed measurements in the diagonal basis ({$|\pm\rangle$}), and the sequence of a quarter wave plate QWP@0°, HWP4@22.5°, and PBS3 measure in the right-handed and left-handed circular polarization basis ({$|R\rangle, |L\rangle$}). The intensity of each component is projected on a screen and recorded in a single image by a charged-coupled-device (CCD) camera. The normalized intensity $I_{\beta}/I_{\alpha}$ ($\beta = A, B$) plays the role of the probabilities in the density matrix $\rho_{ij}$ reconstruction. Depending on the measurement basis $A = H, +, +$, and $R$, while $B = V, -$, and $L$, respectively. As an initial result, Fig 3 shows the false color images for $\theta = +45°$, which corresponds to the initial state in AD-channel ($t = 0$). The tomographic measured basis are indicated to the left of each image. The respective normalized intensities are showed to the right of each the image.
By means of the normalized intensities we can obtain the corresponding density matrix \[ \rho = \begin{pmatrix} 0.5021 & 0.4904 - 0.0136i \\ 0.4904 + 0.0136i & 0.4978 \end{pmatrix}, \] (31)
which is in agreement with the theoretical prediction. By using Eq. (5) the experimental coherence is \( C(0) = 0.98 \pm 0.03 \), very close to the expected unit value. The error comes from the limited visibility of the interferometer and the intensity sensitivity of the CCD camera.

Let us now discuss the quantifier of non-Markovianity \( N_C(\alpha) \), evaluated from the experimental data. The values of \( \alpha \) considered in the experiment are pointed out in the theoretical plot exhibited in Fig. 1. For the Markovian evolution (\( \alpha = 0.4 \)) no revivals are observed, and \( N_C(\alpha) = 0 \), as expected by Eq. (18). For the weak non-Markovian behavior (\( \alpha = 20 \)) we can clearly identify five local maximum revivals of coherence in Fig. 3. For this case we experimentally found \( N_C(\alpha) = 0.86 \), which is close to the value \( N_C(\alpha) = 0.92 \) obtained from the theoretical expression given by Eq. (18) when 5 revivals are considered. On the other hand, by integrating from 0 to \( t \rightarrow \infty \) (infinite revivals) we obtain \( N_C(\alpha) = 0.95 \). This difference between the total integration and the finite revivals become more accentuated for larger \( \alpha \). For the case when \( \alpha = 200 \), we experimentally observed 3 revivals for the time range of the experiment, which produces \( N_C(\alpha) = 1.90 \) in agreement with the theoretically calculated value \( N_C(\alpha) = 1.95 \) for 3 revivals. However, in this case, the comparison between calculation using experimental values in Eq. (16) and integration of Eq. (18) is not appropriate, since only 3 revivals have been considered in the experiment. By integrating Eq. (18) we find \( N_C(\alpha) = 4.01 \) [see also Fig. 1]. Hence, for a strong non-Markovian regime, the computation of \( N_C \) requires that the observation time is sufficiently long in order for the experimental values obtained from Eq. (16) for finite revivals are able to reproduce the theoretical result predicted by Eq. (18) for infinite revivals.
VI. CONCLUSIONS

We have investigated through both theory and experiment distance-based quantum coherence quantifiers and non-Markovianity characterization. In the context of basis-free measures of coherence, we have shown the equivalence between the minimum amount of coherence obtained by the minimization over local unitary transformations and multipartite geometric measures of quantum correlation. These correspondences hold for arbitrary distance (or pseudo-distance) measures, enhancing the relations between different types of quantum correlations.

In addition, we have shown that, under the allowed incoherent operation criterion, the monotonicity of the valid coherence measure may be affected by a partial backflow of the previously lost information of the system to the environment. This is similar to the cases of non-Markovian effects on the distinguishability between two different states and other quantum information measures [28–30, 42, 46]. Our numerical and analytical results for AD non-Markovian noise have indeed indicated that coherence by trace-norm captures the non-Markovianity features of the one-qubit system. An advantage of this method is that it involves only a single qubit, implying a simpler process of optimization and empirical realization. By taking this result into account, we experimentally realized a single-qubit AD channel through an all-optical setup where we used the polarization degree of freedom to encode the qubit and the propagation path to encode the system-environment interaction. The experiment has been performed in an intense laser beam regime, which is an appropriated approach for study quantum features as used in several works in the literature [15–20]. The non-Markovian quantifier \( N_C(\alpha) \) has been experimentally obtained and the results are in agreement with theory. A generalized investigation can be implemented for two-qubit systems by encoding, e.g., the second qubit in the transverse degree of freedom of light. Such a simple and reliable system could be useful to enlighten the relationship between coherence and quantum correlation measures in a two-qubit scenario, where coherence minimized under local unitaries may be nonvanishing. This is left for future research.

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