Sequential Monte Carlo for on-line estimation of the heat loss coefficient

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ABSTRACT
The calibration of building energy models based on in-situ sensor information is generally performed after the measurement period, using all data in a single batch. Alternatively, on-line parameter estimation proposes updating a model every time a new data point is available: this allows observing a direct relation between external events and the identifiability of parameters. The present study uses the Sequential Monte Carlo method to train a RC model, and thus estimate a Heat Loss Coefficient, and other parameters, sequentially. Results show the direct impact of solicitations (solar irradiance and indoor heat input) on this estimation, in real time. The method is validated by comparing its results with the Metropolis-Hastings algorithm for off-line estimation.

KEYWORDS
Bayesian inference; on-line; SMC

INTRODUCTION
The calibration of simplified building energy models using in-situ measurements is now a widespread research topic (Rouchier, 2018). It is commonly performed for two general types of applications: the characterisation of intrinsic building performance (Heo et al. 2012; Bauwens and Roels, 2014), and the identification of a model for predictive purposes, for instance in the aim of model predictive control. State-space models, which include the simplified resistor-capacitor (RC) model structures, are a popular choice for both applications. When written as a set of Stochastic Differential Equations, they allow accounting for modelling approximations (Madsen and Holst, 1995) and offer a more reliable parameter estimation than deterministic models (Rouchier et al. 2018).

Parameter estimation is typically performed off-line: measurements of indoor and outdoor conditions are first carried in a test building, and data is processed after the experiment in a single batch. An interesting challenge is to carry parameter estimation on-line, during the observation period: starting from an initial guess for parameter values, these estimates are updated sequentially, every time a new observation becomes available. The motivation is twofold: first, it would allow using the measurement period for computations, thus reducing the total time of the procedure (Raillon and Ghiaus, 2017). Second and foremost, it would allow directly observing which phenomena “bring information” to the parameters, by correlating the reduction in their estimation uncertainty with observed events.

Bayesian inference offers the possibility of on-line estimation with Sequential Monte-Carlo (SMC) methods (Doucet et al. 2000). Originally developed for the sequential estimation of states (Handschin 1970), SMC was later adapted to state and parameter estimation (Kantas et al. 2015). Building physics applications are scarce and very recent (Raillon and Ghiaus, 2017), but may become more common due to the motivations listed above. The present paper
applies SMC for the on-line estimation of the heat loss coefficient (HLC) of a test building. Starting from a highly uncertain prior knowledge of HLC, the target is to dynamically observe what leads its estimation to narrow down to a more precise value. The identifiability of HLC regarding available data is then discussed.

CASE STUDY

Test case
This study uses measurements that were carried in the Round Robin Test Box (RRTB), within the framework of the IEA EBC Annex 58 (Jimenez et al. 2016). This experimental test cell has a cubic form with dimensions of one cubic meter, identical wall components on all sides and one window with dimensions 60x60 cm$^2$. It was installed outdoors, in the LECE laboratory at Plataforma Solar de Almeria, in the South East of Spain. Experiments were carried during a 43 days period in the winter of 2013-2014. More information on the conditions of the test is available in the Annex 58 report (Jimenez et al. 2016).

Figure 1. Measured indoor and outdoor temperature, heating power and solar irradiance

A period of six days was chosen for the present investigation, shown on Fig. 1. The indoor temperature is left free floating during the first three days, although it is impacted by daily peaks of solar irradiance. Then, a ROLBS heating signal was imposed inside the test box during the remaining three days. The motivation behind the choice for this sequence of measurements is that these boundary conditions are not very informative at first, then become more informative: we expect to witness their effects on the evolution of the estimation of the heat loss coefficient.

Model
The test box is represented by a 2R2C model described by:

\[
\begin{bmatrix}
\dot{T}_e(t) \\
\dot{T}_i(t)
\end{bmatrix} = \begin{bmatrix}
\frac{1}{R_1 C_1} & \frac{1}{R_2 C_1} \\
\frac{1}{R_1 C_2} & \frac{1}{R_2 C_2}
\end{bmatrix} \begin{bmatrix}
T_e(t) \\
T_i(t)
\end{bmatrix} + \begin{bmatrix}
0 & \frac{k_1}{C_1} \\
0 & \frac{k_2}{C_2}
\end{bmatrix} \begin{bmatrix}
q(t) \\
I_{\text{sol}}(t)
\end{bmatrix} + w(t)
\]

\[
y(t) = [1 0] \begin{bmatrix}
T_e(t) \\
T_i(t)
\end{bmatrix} + v(t)
\]

where $T_i$, $T_a$ and $T_e$ are the indoor, ambient (outdoor) and envelope temperatures. The model has two states $T_e$ (unobserved) and $T_i$ (observed); $q$ (W) is the indoor heating power; $I_{\text{sol}}$ (W/m$^2$) is the solar irradiance on a southern vertical plane; $R_1$ and $R_2$ (m$^2$.K/W) are two thermal resistances, $C_1$ and $C_2$ (J/K) are thermal capacities, and $k_1$ and $k_2$ (m$^2$) are two solar
aperture coefficients, one for each state of the model. \( w(t) \) denotes a Wiener process that represents modelling errors with an incremental covariance \( Q_c \) (Madsen and Holst, 1995), and \( v(t) \) is the measurement error of the indoor temperature, normally distributed white noise with zero mean and variance \( R_c \). Both are considered unknown and will be estimated along with the other static parameters of the model, which are all denoted by a single vector \( \theta \).

The stochastic model described by Eq. 1 and 2 must be discretized to specify its evolution between discrete time coordinates.

\[ x_t = F x_{t-1} + G u_t + w_t \]  
\[ y_t = H x_t + v_t \]  

(3) (4)

where \( x_t \) denotes the vector of states at the time coordinate \( t \), and \( y_t \) denotes the observations. The reader is referred to (Madsen and Holst, 1995) and (Rouchier 2018) for more details regarding the discretization steps.

Given a state transition probability \( p(x_t | \theta, x_{t-1}, u_t) \) (Eq. 3) and an observation probability \( p(y_t | x_t) \) (Eq. 4), filtering produces \( p(x_t | y_{1:T}, \theta) \), the probability distribution function of each state given measurements and parameter values, and the marginal likelihood function \( L_y(\theta) = p(y_{1:T} | \theta) \). The Kalman filter algorithm is not described here for the sake of concision, but has been described by many authors including (Madsen and Holst, 1995; Rouchier 2018).

**BAYESIAN PARAMETER ESTIMATION**

The target of the on-line parameter estimation exercise is to assess the value of all static parameters of the model, at each time coordinate of the measurement period: the expected output is a sequence of posterior distributions \[ \{p(\theta | y_{1:T}, t, \theta), t \in 1 \ldots T\} \], where \( T \) is the number of data points in the measurement period. This sequential estimation is performed by the SMC algorithm. For the sake of validation, the estimation has also been carried in a “traditional” off-line fashion with the Metropolis-Hastings algorithm. Both methods are described below.

**Off-line estimation: Marginal Metropolis Hastings**

The Marginal Metropolis Hastings (MMH) algorithm is part of the family of Markov Chain Monte Carlo (MCMC) methods. Given one batch of data \( y_{1:T} \), a parameter prior \( p(\theta) \) and a model specification (Eq. 3 and 4), the MMH algorithm returns a finite sequence of samples \( \{\theta_n, n \in 1 \ldots N\} \) approximating the posterior distribution \( p(\theta | y_{1:T}) \). Examples of applications to the calibration of building energy models include (Heo et al. 2012; Rouchier et al. 2018).

The algorithm employs a Kalman filter to compute the states \( p(x_{1:T} | \theta, y_{1:T}) \) and likelihood \( L_y(\theta) \) associated to each proposal for \( \theta \). If the state-space model (Eq. 3) is non-linear, this filter can be replaced by a particle filter: this approach is known as Particle Markov Chain Monte Carlo (PMCMC).

**On-line estimation: Sequential Monte Carlo**

The SMC algorithm for parameter estimation is an adaptation of particle filtering for state variables. The foundation of this method is the Importance Sampling paradigm as described by (Cappé et al. 2007): simulating samples under an instrumental distribution and then approximating the target distributions by weighting these samples using appropriately defined importance weights. The reader is referred to (Cappé et al. 2007) and (Kantas et al. 2015) for a deeper explanation of SMC and its application to parameter estimation. The method used
here is inspired from the Iterated Batch Importance Sampling algorithm (Chopin 2002). It is described on Fig. 2.

Figure 2. Principle of the Sequential Monte Carlo algorithm

The algorithm starts with the generation of a population of $N_\theta$ particles drawn from a prior distribution $p(\theta)$. Each parameter is assigned an initial state $x_0$ and weight. At each time step $t$, a Kalman filter computes the states $x_t^{(j)}$ and likelihood $L_t^{(j)}$ associated to each particle $\theta_t^{(j)}$. By this operation, the population of particles is updated so that at each time $t$ they are a properly weighted sample from $p(\theta \cup y_{1:t})$. After several time steps, there is a risk that only a few of the initial particles are significantly more likely than the others and concentrate most of the total weight: a resampling step is then performed to generate a new population of particles from the most influential ones, and a MCMC rejuvenation step then restores the diversity of particles (Murray 2013).

Resampling does not occur every time a new observation becomes available, but only when required: this is measured by the effective number of particles that significantly contribute to the total weight of all particles (Murray 2013). This operation decreases the number of unique particles, hence the subsequent MCMC rejuvenation step that restores diversity. The choice of the proposal distribution for the MCMC rejuvenation step was proposed by (Chopin 2002) and ensures a reasonable acceptance ratio while leaving $p(\theta \cup y_{1:t})$ invariant. The rejuvenation step makes the algorithm quite computationally expensive, since the total likelihood of all particles must be recalculated every time resampling occurs. This problem is mitigated by the fact that particles can be resampled independently, making this effort parallelizable.

RESULTS

The MMH and SMC algorithms were used to estimate the parameter vector $\theta$, either off-line or on-line, using the batch of measurements shown on Fig. 1 from the RRTB test cell represented by a 2R2C model (Eq. 1). First, the sufficiency of the 2R2C model to recreate the indoor temperature time series in checked. This is ensured by the autocorrelation function of the residuals after parameter estimation shown on Fig. 3.
As pointed out by (Madsen and Holst, 1995), the low value of this function ensures that residuals are uncorrelated and thus are close to white noise, in accordance with the hypothesis is the model. This observation allows us to analyze the parameter estimation results. All estimation results from both MMH and SMC are assembled into Fig. 4.

First, Fig. 4(a) shows the progression of the estimation of the parameter $R_1$ by SMC during the six days of measurements. The blue area denotes the mean and the 95% confidence region of $R_1$ at each time coordinate. The indoor heating power and solar irradiance are superimposed over it and allows the interpretation of the estimation. During the first three days of measurements, the value of $R_1$ can be seen to narrow down to a more confident estimation during the day, when the solar irradiance is positive. An important uncertainty however remains on this parameter. The heating is then turned on inside the test box, which causes the estimation to narrow down more abruptly. These results are consistent with the general knowledge that some indoor solicitations are necessary to make a building energy model identifiable. The originality of the present study is that we can observe the information brought to the parameter estimates in real time.
The performance of SMC is then compared to the MMH algorithm, which estimates parameters from a single batch of data at once. Fig. 4(a), 4(b) and 4(c) respectively show estimates of the heat loss coefficient $HLC = \frac{1}{R_1 + R_2}$, the total heat capacity $C = C_1 + C_2$ and the total solar aperture $k = k_1 + k_2$. Each plot shows the prior distribution of these properties, and their estimation after 2 days, 4 days and 6 days of measurements. The MMH algorithm had to be run separately with each measurement length, while the SMC algorithm only had to run once to return all results. Starting from vague Gaussian prior distributions for each parameter, both methods resulted in mostly matching results.

CONCLUSION
The present study uses the Sequential Monte Carlo method to train a RC model, and thus estimate a Heat Loss Coefficient, and other parameters, sequentially. Results show the direct impact of solicitations (solar irradiance and indoor heat input) on this estimation, in real time. The method is validated by comparing its results with the Metropolis-Hastings algorithm for off-line estimation.

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