UNIQUENESS OF RS2 TYPE THICK BRANES SUPPORTED BY A SCALAR FIELD

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We study thick brane world models as $\mathbb{Z}_2$-symmetric domain walls supported by a scalar field with an arbitrary potential $V(\phi)$ in 5D general relativity. Under the global regularity requirement, such configurations (i) have always an AdS asymptotic far from the brane, (ii) are only possible if $V(\phi)$ has an alternating sign and (iii) $V(\phi)$ should satisfy a certain fine-tuning type equality. Thus a thick brane with any admissible $V(\phi)$ is a regularized version of the RS2 brane immersed in the AdS\textsubscript{5} bulk. The thin brane limit is realized in a universal manner by including an arbitrary thick brane model in a one-parameter family, where the parameter $a$ is associated with brane thickness; the asymptotic value of $V(\phi)$ (related to $\Lambda_5$, the effective cosmological constant) remains $a$-independent. The problem of ordinary matter confinement on the brane is discussed for a test scalar field. Its stress-energy tensor is found to diverge at the AdS horizon for both thin and thick branes, making a serious problem for this class of brane world models.

1. Introduction

According to the widely discussed brane world concept, the standard-model particles are confined on a hypersurface, called a brane, which is embedded in a higher-dimensional space called the bulk. This interesting and intriguing possibility from the viewpoint of the physical world outlook traces back to the 80s \cite{1}, and the present-day interest in it is largely related to the developments in superstring/M-theories \cite{2}. Various aspects of brane-world particle physics, gravity and cosmology are discussed in the recent review articles (\cite{3}, see also references therein).

Most of the studies are restricted to infinitely thin branes with delta-like localization of matter. This kind of models can, however, be only treated as an approximation since any fundamental underlying theory, be it quantum gravity or string theory, must contain a fundamental length beyond which a classical space-time description is impossible. It is therefore necessary to justify the infinitely thin brane approximation as a well-defined limit of a smooth structure, a thick brane, obtainable as a solution to coupled gravitational and matter field equations.

In Ref. \cite{4} we, like many others, tried to describe a thick brane in the framework of 5D general relativity as a domain wall separating two different states of a scalar field. Scalar field structures with arbitrary potentials were studied analytically, assuming $\mathbb{Z}_2$ symmetry with respect to the middle plane of the wall. The consideration was restricted to Poincaré branes, i.e., flat domain walls. A reason was that most of the existing problems are clearly seen even in these comparatively simple systems; moreover, in the majority of physical situations, the inner curvature of the brane itself is much smaller than the curvature related to brane formation, therefore the main qualitative features of Poincaré branes should survive in curved branes.

This work confirmed and generalized the well-known results obtained in a number of specific models (see, among others, \cite{6–10}): in the framework of 5D general relativity, a globally regular thick brane always has an anti-de Sitter (AdS) asymptotic and is only possible if $V(\phi)$ has an alternating sign. In other words, it can be stated that any regular thick Poincaré brane supported by a scalar field is a smooth version of the well-known Randall-Sundrum second model (RS2) \cite{11}.

The problem of a well-defined thin brane limit was also discussed in \cite{4} and was solved partly, using as an example exactly solvable models with scalar field potentials consisting of two constant steps. The RS2 limit, with the corresponding (fine-tuned) relation between the brane tension $\sigma$ and the bulk cosmological constant $\Lambda_5$, was found to be independent of the two shape factors of the potential (the steps’ relative height and thickness) which were kept constant in the transition to zero thickness. Examples of similar transitions for other specific potentials have also been found previously \cite{4,8}.

In this paper we re-consider this problem and indicate a simple algorithm which, given any regular thick...
brane model, includes it into a one-parameter family of similar models with a parameter \( a \) related to brane thickness. The limit \( a \to 0 \) corresponds to the RS2 model. We thus prove the existence of a correct thin brane limit in a universal way for thick brane models with potentials of arbitrary shape.

In the concluding section 5 we also briefly discuss the problem of ordinary matter confinement using as an example a test scalar field. It turns out that its stress-energy tensor diverges on the AdS horizon, this feature being common for a thin RS2 brane and all its thick counterparts.

2. Field equations and boundary conditions

We consider general relativity in a 5D space-time, where we distinguish the usual four coordinates \( x^\mu, \mu = 0,1,2,3 \) and the fifth coordinate \( x^4 = z \), to be used for describing the direction across the brane. The action is taken in the form \( S = \int \sqrt{g} L \, dx^4 \), where \( g = |\det(g_{AB})| \), \( A, B = 0,1,2,3,4 \) and \( g_{AB} \) is the 5D metric tensor; the Lagrangian density has the form

\[
L = \frac{R}{2\kappa^2} + L_\phi, \quad L_\phi = \frac{1}{2} \partial^A \phi \partial_A \phi - V(\phi),
\]

(\( R \) is the 5D Ricci scalar and \( \kappa \) the 5D gravitational constant) and leads to the Einstein-equations

\[
R^B_A - \frac{1}{2} \kappa^2 R^B - \kappa^2 T^B_A - \kappa^2 [\partial_A \phi \partial^B \phi - \delta^B_A L_\phi], \quad \partial_A (\sqrt{g} g^{AB} \partial_B \phi) = -\sqrt{g} \, dV/d\phi
\]

where Eq. (3) is a consequence of (2) due to the Bianchi identities.

We seek regular solutions describing a static domain wall (thick Minkowski brane), possessing \( \mathbb{Z}_2 \) symmetry with respect to the hypersurface \( z = 0 \).

Consider Eqs. (2), (3) for \( \phi = \phi(z) \) and the metric

\[
ds_5^2 = e^{2F(z)} \eta_{\mu\nu} dx^\mu dx^\nu - e^{8F(z)} dz^2,
\]

where \( \eta_{\mu\nu} \) is the 4D Minkowski metric and we have chosen the harmonic coordinate \( z \), such that \( \sqrt{g} g^{zz} = -1 \). As a result, the 5D Ricci tensor acquires an especially simple form:

\[
R^0_0 = R^1_1 = R^2_2 = R^3_3 = -e^{-8F} F'', \quad R^z_z = -4e^{-8F} (F'' - 3F'^2)
\]

The Kretschmann scalar \( K = R_{ABCD} R^{ABCD} \) is

\[
K = 4\left[ e^{-5F}(e^{-3F} F')^2 \right]^2 + 6\left( e^{-4F} F' \right)^4.
\]

For the metric (2), \( K \) is a sum of squares of all nonzero components \( R_{ABCD} \) of the Riemann tensor, therefore its finite value is necessary and sufficient for finiteness of all algebraic curvature invariants.

The 5D Einstein equations (2) in our case reduce to

\[
F'' = -\frac{2\kappa^2}{3} e^{8F} V, \quad 3(-F'' + 4F'^2) = \kappa^2 \phi'^2, \quad F'^2 = \frac{\kappa^2}{6} \left( \frac{1}{2} \phi'^2 - e^{8F} V \right),
\]

where (1) is a first integral of the other two equations. The scalar field equation (3) reads

\[
\phi'' = e^{8F} dV/d\phi
\]

and is also a consequence of (2) and (8). Eqs. (7) and (8) may be taken as a complete set of equations for \( F(z) \) and \( \phi(z) \); it is third-order and requires three boundary conditions.

Now, the \( \mathbb{Z}_2 \) symmetry assumption dictates the boundary condition \( F'(0) = 0 \). Then, assigning \( F'(0) = 0 \) by a proper choice of the time scale, we arrive at an unambiguous value of \( \phi'^2: \phi'^2(0) = 2V(0) \). So \( F(z) \) is an even function while \( \phi(z) \) is an odd one. A complete set of boundary conditions at \( z = 0 \), compatible with \( \mathbb{Z}_2 \) symmetry, is

\[
F(0) = F'(0) = 0, \quad \phi(0) = 0.
\]

Thus for any fixed function \( V(\phi) \), there is no free parameter in the equations and boundary conditions, i.e., the solution is already uniquely determined. The requirement of regularity at large \( z \) should thus lead to an additional constraint on the function \( V(\phi) \), leading to a “fine tuning” between the brane and bulk parameters.

3. Regularity conditions

Let us briefly reproduce the reasoning of Ref. [1] on regularity of the solutions at the asymptotic \( z \to \infty \).

Regularity of the metric [see (4)] implies \( |b'(z)| < \infty \) where \( b(z) \equiv e^{-4F} \). Regularity of the geometry leads, via the Einstein equations, to finiteness of certain scalar field characteristics. On the whole, we should require

\[
|b'(z)| < \infty, \quad |V(\phi)| < \infty, \quad \phi(z)|\phi'(z)| < \infty.
\]

Eq. (8) leads to \( b''(z) > 0 \). Since \( b'(0) = 0 \), this means that \( b(u) \) increases at \( z > 0 \) and inevitably grows to infinity at large \( z \) at least linearly for any nontrivial solution. The growth is precisely asymptotically linear due to (5), \( b' \to \text{const} > 0 \), and hence \( F' \approx -1/(4z) \) at large \( z \).

Returning to (2) and integrating, we obtain

\[
F'(z) = \nabla(z) := -\frac{2}{3} \kappa^2 \int_0^z e^{8F} V \, dz.
\]

For regular solutions we necessarily have

\[
\nabla(\infty) = 0.
\]
This is the above-mentioned fine-tuning condition in terms of the potential $V$. The integral $\mathbf{V}(z)$ is the invariant full potential energy per unit 3-volume in the layer from zero to $z$. Since $e^{4F} = 1/b^2 > 0$, a nontrivial potential $V(\phi)$ must change its sign at least once to yield $\mathbf{V}(\infty) = 0$.

It is easy to show that in regular solutions $\phi' = o(1/z)$. Therefore, $e^{4F} \phi' \to 0$ at large $z$, and Eq. (14) shows that $V$ tends to a finite negative value. If

$$b(z) = e^{-4F} \approx k z, \quad k = \text{const} > 0, \quad z \to \infty, \quad (15)$$

then

$$\kappa^2 V \big|_{z=\infty} = \Lambda_5 = -\frac{3k^2}{8}, \quad (16)$$

where $\Lambda_5$ is the effective cosmological constant.

Thus $V(\phi)$ changes its sign at least once, tends to a negative value as $z \to \infty$, and the integral $\mathbf{V}(\infty)$ is zero. If $\phi$ tends to a finite value $\phi_\infty$, $V(\phi)$ has a minimum at $\phi = \phi_\infty$.

This behaviour is not unique: despite $\phi' = o(1/z)$, one cannot exclude that $\phi \to \infty$, though slower than $\ln z$ (it can be, for instance, $\phi' \sim 1/\ln \ln z$ and $\phi \sim \ln \ln z$). Then $V(\phi)$ tends to the value (16) as $\phi \to \infty$.

In any case, due to oddness of $\phi(z)$, the values $\phi(+\infty) = -\phi(-\infty) \neq 0$ make the domain wall topologically stable. The postulated $Z_2$ symmetry implies that $V(\phi)$ is an even function. Its asymptotic value, $V(z = \pm \infty) = \Lambda_5/k^2 < 0$ plays the role of a cosmological constant at the bulk asymptotic, and the metric is asymptotically anti-de Sitter (AdS). The values $z = \pm \infty$ then correspond to an anti-de Sitter horizon.

4. Thin brane limit

Consider the thin brane limit of regular solutions with finite $\phi_\infty$, leaving aside the above case of slowly growing $\phi(z)$. We thus have $V(\phi)$ with a minimum at $\phi = \phi_\infty$, and there holds the “fine tuning” condition $\mathbf{V}(\infty) = 0$.

The action $S$ with the Lagrangian $L = L_G + L_\phi$ may be split into the bulk and brane parts,

$$S = S_{\text{bulk}} + S_{\text{brane}}, \quad (17)$$

$$S_{\text{bulk}} = -\int R - \frac{2\Lambda_5}{2k^2} - \sqrt{g} d^5 x, \quad \Lambda_5/k^2 = V(\phi_\infty), \quad (18)$$

$$S_{\text{brane}} = \int \left( \frac{1}{2} \partial_\phi \phi' \partial^\phi \phi + V(\phi_\infty) - V(\phi) \right) \sqrt{g} d^5 x. \quad (19)$$

The brane action may be presented in the form

$$S_{\text{brane}} = -\int \sigma d^4 x,$$

$$\sigma = \int_{-\infty}^{\infty} \left[ -\frac{1}{2} \partial_\phi \phi' \partial^\phi \phi + V(\phi) - V(\phi_\infty) \right] e^{4F} dz, \tag{20}$$

where the quantity $\sigma$ can be regarded as the brane tension. It is equal to the total scalar field energy per unit 3-volume on the brane, in which the potential energy is counted from the vacuum level $V_\infty$.

Randall and Sundrum’s second model (RS2) of a thin brane [11] is based on the splitting (17): assuming a delta-like matter distribution characterized by the tension $\sigma$ and using the Israel matching condition for the 5D metric, they found the fine-tuning condition

$$6\Lambda_5 = -\kappa^4 \sigma^2. \tag{21}$$

In our approach, a transition to a thin brane can be carried out along a family of solutions with different potentials but a fixed value of $\Lambda_5$, which determines a length scale $1/\sqrt{\Lambda_5}$ in the bulk, independent of the brane thickness. This corresponds to a brane as a domain wall between two vacua with equal and fixed energy densities.

Then, if the thin brane concept is correct, we should expect that, independently of the specific form of the potential,

$$\lim_{a \to \infty} \frac{\Lambda_5}{k^4 a^2} = \frac{1}{6}, \quad (22)$$

the parameter $a$ characterizing the brane thickness.

To consider the problem, let us introduce more convenient variables

$$f(z) := \frac{2\kappa}{\sqrt{3}} \phi(z), \quad (v(f) := \frac{8}{3} \kappa^2 V(\phi)). \quad (23)$$

Eqs. (19) are then rewritten as

$$V = b b'' - b'^2 = b^2 f'^2 - b'^2, \quad (24)$$

$$f'^2 = b''/b. \quad (25)$$

The boundary conditions at $z = 0$ are

$$b(0) = 1, \quad b'(0) = 0, \quad f(0) = 0. \quad (26)$$

Let there be any function $b(z)$ describing a thick brane in an AdS background. This means that $b(z)$ satisfies Eqs. (24) and (25) and the following boundary conditions at $z = 0$ (location of the brane) and $z \to \infty$ (AdS horizon):

$$b(0) = 1, \quad b'(0) = 0, \quad b(z) = k z + \text{const} \quad (z \to \infty) \quad (27)$$

so that the potential $v \to -k^2 = \text{const} \ (k > 0)$ as $z \to \infty$. The tension $\sigma$ is expressed as

$$\sigma = \frac{3}{8\kappa^2} \int_{-\infty}^{0} dz \left( \frac{b''}{b} + \frac{k}{b} \right). \quad (28)$$

A well-defined thin brane limit means that this thick-brane solution should be included in a family of solutions with a parameter $a$ (to be interpreted as a thickness parameter), say, $B(a, z)$ such that $B(1, z) = b(z)$, which should satisfy the following requirements:

1) At each fixed $a > 0$, the function $B(a, z)$ should satisfy Eqs. (27) with the same constant $k$, i.e., it should be a thin-brane solution with the same cosmological constant $\Lambda_5 = -\frac{3}{8} k^2$. (The potential $v$ should evidently
be $a$-dependent since its limiting form should be $v = -k^2$ at all $z \neq 0$.)

2) At each fixed $z \neq 0$, the function $B(a, z) \to 1 + k|z|$ as $a \to 0$, i.e., the metric should tend to the AdS metric with the properly chosen time scale at $z = 0$.

Then the limiting solution corresponds to the RS2 thin brane. In particular, the relation between the brane tension $\sigma$ and $\Lambda_5$ should be as in the RS2 model, i.e.,

$$\sigma = \frac{3k}{(2\kappa^2)}. \quad (29)$$

The following function $B(a, z)$ evidently satisfies the above requirements 1) and 2):

$$B(a, z) = 1 - a + ab \left(\frac{z}{a}\right), \quad (30)$$

where $b()$ means the functional dependence specified in the original function $b(z)$. Substituting $B(a, z)$ instead of $b(z)$ into (28) and considering the limit $a \to 0$, we see that the second term of the integrand yields precisely half the desired value (24), namely, $3k/(4\kappa^2)$. The other half is given by the first term, since, in the limit $a \to 0$, we have $B'' \to 2k\delta(z)$.

It is of interest to note that only half of the RS2 brane tension is related to an energy concentrated on the brane and described by the first term in (28), while the other half is distributed in the bulk proportionally to $(1 + k|z|)^{-2}$.

The limiting form of the potential is

$$v(z) \bigg|_{a=0} = 2k\delta(z) - k^2. \quad (31)$$

5. Concluding remarks. The confinement problem

We have continued a general study of regular domain walls (thick branes) supported by a minimally coupled scalar field with an arbitrary potential in 5D general relativity. It has been previously shown [1] that the only kind of asymptotic for such walls is AdS, that the potentials $V(\phi)$ able to create such configurations have an alternating sign and satisfy the fine tuning condition [12]. We have now confirmed in a general form that the zero thickness limit of such branes is well defined and conforms to the RS2 brane world model.

The family of solutions (30), derived from any appropriate solution (which is entirely characterized by the function $b(z)$), is probably the simplest possible family that realizes the thin brane limit, but it is evidently not unique.

Explicit examples of a transition to thin branes with some special potentials $V(\phi)$ were studied previously (see [8, 9] and references therein) with the same result.

It should be stressed, however, that, to be considered as models of our Universe, brane world models like those discussed here must satisfy two major requirements: (i) ordinary matter should be confined to the brane to account for the fact that extra dimensions are not observed, and (ii) Newton’s law of gravity should be reproduced on the brane in a non-relativistic limit. These issues, which have already been treated in a number of papers (among others, [3, 12, 14, 15]) turn out to be quite nontrivial, and we hope to discuss them in future publications. Let us briefly illustrate the confinement problem, using as an example a test scalar field $\chi$ in the background of a thick brane with the metric $\text{d} s^2 = A(\nu)^2 \text{d} \nu^2 + \lambda^2 \text{d} \chi^2$.

Consider a scalar field $\chi$ with the Lagrangian

$$L_\chi = \frac{1}{2} \partial_\chi \chi \partial^4 \chi - \frac{1}{2} m_0^2 \phi^* \chi + \frac{1}{2} \lambda \phi^2 \chi \chi, \quad (32)$$

where $\phi^*$ is the complex conjugate field, and the last term describes a possible interaction between $\chi$ and the wall scalar field $\phi$; $\lambda$ is a coupling constant. The influence of $\chi$ on the wall structure is neglected. The field $\chi(x^4)$, which may be interpreted as the wave function of a $\chi$-particle, satisfies the Klein-Gordon-Fock equation

$$\frac{1}{\sqrt{g}} \partial_\mu \left(\sqrt{g} g^{\mu\nu} \partial_\nu \chi\right) = (\lambda \phi^2 - m_0^2) \chi, \quad (33)$$

which is linear and homogeneous with respect to $\chi$. Its coefficients depend on $\nu$ only, while the coordinates $x^\mu$ are cyclic variables. The canonically conjugate momenta $p_\mu = (E, p)$ are integrals of motion, and we can present $\chi(x^4)$ in the form

$$\chi(x^4) = X(\nu) \exp(-ip_\mu x^\mu), \quad \mu = 0, 1, 2, 3. \quad (34)$$

The function $X(\nu)$ determines the $\chi$ field distribution across the brane and satisfies the linear homogeneous equation

$$X'' + \sqrt{g}(p^\mu p_\mu + \lambda \phi^2 - m_0^2)X = 0. \quad (35)$$

We can consider a $\chi$-particle to be localized on the brane if the stress-energy tensor (SET) of the $\chi$ field, $T^\mu_\nu[\chi]$, is finite in the whole 5-space and decays sufficiently rapidly at large $z$. If $T^\mu_\nu[\chi]$ somewhere tends to infinity, this evidently violates the test field assumption.

As an evident necessary condition of localization, one can require converging $\chi$ field energy per unit 3-volume of the brane, i.e.,

$$E_{\text{tot}}[\chi] = \int_{-\infty}^{\infty} T^t_\nu \sqrt{g} d\nu dz = \int_0^\infty e^{8F} \left[ e^{-2F(E^2 + p^2)} X^2 + (m_0^2 - \lambda \phi^2) X^2 + e^{-8F} X^2 \right] dz < \infty. \quad (36)$$

The inequality (36) evidently implies a finite norm of the $\chi$ field defined as

$$\|\chi\|^2 = \int_{-\infty}^{\infty} \sqrt{g} \chi'^2 dz = \int_{-\infty}^{\infty} e^{-8F} X^2 dz. \quad (37)$$

Eq. (36) is rewritten as

$$\chi'' + \left[ e^{6F}(E^2 - p^2) + e^{8F}(\lambda \phi^2 - m_0^2) \right] \chi = 0. \quad (38)$$
The term $e^{8F}(\lambda \phi^2 - m_0^2)$ describes interaction of a $\chi$-particle with the brane. If $\lambda = 0$, it is purely gravitational, while $\lambda \neq 0$ describes an additional, non-gravitational interaction between $\phi$ and $\chi$.

Recall now that the space-time regularity requirement leads to $V(\infty) = 0$, and $e^{4F} \sim 1/z$. The term $\sim e^{8F}$ in Eq. (35) vanishes faster than the one with $e^{6F}$, and the equation determining the behavior of $\chi$ at large $z$ is $X'' + e^{6F}(E^2 - \nu^2)X = 0$, or, due to (35),

$$X'' + \frac{\partial}{z^{3/2}}X = 0,$$

$$\vartheta^2 = (E^2 - \nu^2) \left[ 2\sqrt{\frac{4}{3}\kappa|V(\phi_\infty)|} \right]^{-3/2}. \quad (39)$$

Its asymptotic solution is

$$X = Cz^{3/8} \sin(4\vartheta z^{1/4} + \varphi_0), \quad z \to \infty, \quad (40)$$

where $C$ and $\varphi_0$ are integration constants. We see that the wave function (40) not only does not vanish as $z \to \infty$, but oscillates with an increasing amplitude. As a result, the SET components $T^\mu_\nu[\chi]$ are infinite at $z = \infty$, i.e., at the AdS horizon. Moreover, the integral (36) diverges since, due to the term proportional to $X'^2 \sim z^{-1/4}$, it behaves as $\int z^{-3/4} dz$. Meanwhile, the normalization integral (37) converges since the integrand behaves as $z^{-5/4}$. The latter result is sometimes treated as a sufficient condition for localization, but, in our view, it is not the case since the very existence of the brane configuration is put to doubt if the test field SET is somewhere infinite.

The origin of the above divergence is the gravitational field which repels matter from the brane and pulls it to the AdS horizon. The universal AdS asymptotic leads to the solution (40) whose form is insensitive to the test field mass and to its possible interaction with the brane-supporting field $\phi$.

For this reason we think that this class of models is unsatisfactory from a physical viewpoint. One of the ways in a search for brane world models free from this problem is to consider higher-dimensional bulks, and we hope to discuss them in our future publications.

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