The $\Lambda$CDM growth rate of structure revisited

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We re-examine the growth index of the concordance $\Lambda$ cosmology in the light of the latest 6dF and WiggleZ data. In particular, we investigate five different models for the growth index $\gamma$, by comparing their cosmological evolution using observational data of the growth rate of structure formation at different redshifts. Performing a joint likelihood analysis of the recent supernovae type Ia data, the Cosmic Microwave Background shift parameter, Baryonic Acoustic Oscillations and the growth rate data, we determine the free parameters of the $\gamma(z)$ parametrizations and we statistically quantify their ability to represent the observations. We find that the addition of the 6dF and WiggleZ growth data in the likelihood analysis improves significantly the statistical results. As an example, considering a constant growth index we find $\Omega_m0 = 0.273 \pm 0.011$ and $\gamma = 0.586^{+0.079}_{-0.074}$.

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1. INTRODUCTION

The high-quality cosmological observational data (e.g. supernovae type Ia, CMB, galaxy clustering, etc), accumulated during the last two decades, have enabled cosmologists to gain substantial confidence that modern cosmology is capable of quantitatively reproducing the details of many observed cosmic phenomena, including the late time accelerating stage of the Universe. A variety of studies have converged to a cosmic expansion history involving a spatially flat geometry and a cosmic dark sector formed by cold dark matter and some sort of dark energy, endowed with large negative pressure, in order to explain the observed accelerating expansion of the Universe [1–9] (and references therein).

In spite of that, the absence of a fundamental physical theory, regarding the mechanism inducing the cosmic acceleration, has given rise to a plethora of alternative cosmological scenarios. Most are based either on the existence of new fields in nature (dark energy) or in some modification of Einstein’s general relativity, with the present accelerating stage appearing as a sort of geometric effect. In order to test the latter possibilities, it has been proposed that measuring the so called growth index, $\gamma$, could provide an efficient way to discriminate between modified gravity models and dark energy (hereafter DE) models which adhere to general relativity.

The accurate determination of the growth index is considered one of the most fundamental tasks on the interface between Astronomy and Cosmology. Its importance stems from the fact that there is only a weak dependence of $\gamma$ on the equation of state parameter $w(z)$, as has been found in Linder & Cahn [10], which implies that one can separate the background expansion history, $H(z)$, constrained by a large body of cosmological data (SNIa, BAO, CMB), from the fluctuation growth history, given by $\gamma$. Assuming a homogeneous dark energy, it was theoretically shown that for DE models within general relativity the growth index $\gamma$ is well approximated by $\gamma \approx \frac{3w-1}{6w-1}$ (see [11,12,13]), which boils down to $\approx 6/11$ for the $\Lambda$CDM cosmology $w(z) = -1$. Notice, that in the case of the braneworld model of Dvali, Gabadadze & Porrati [14] we have $\gamma \approx 11/16$ (see also [10,13,17]), while for the $f(R)$ gravity models we have $\gamma \approx 0.41 - 0.21z$ for $\Omega_m0 = 0.27$ [18,20].

On the other hand, the existence of inhomogeneous DE. In section 4, a joint statistical analysis based on the WiggleZ growth rate data. The structure of the paper is as follows. Initially in section 2, we briefly discuss the background cosmological equations. The basic theoretical elements of the growth index are presented in section 3, where we extend the original Polarski & Gannouji method [33] for a general family of $\gamma(z)$ parametrizations. Notice that the current theoretical approach does not treat the possibility of having inhomogeneous DE. In section 4, a joint statistical analysis based on the Union 2 set of type Ia su-
pernovae (SNIa: [34]), the shift parameter of the Cosmic Microwave Background (CMB: [9]), the observed Baryonic Acoustic Oscillations (BAOs: [35]) and the observed linear growth rate of clustering, measured mainly from the 2dF, VVDS, SDSS, 6dF and WiggleZ redshifts catalogs, is used to constraint the growth index model free parameters. Finally, we draw our main conclusions in section 5.

2. THE BACKGROUND EVOLUTION

In this section, it will be assumed that the universe is a self-gravitating fluid described by general relativity, and endowed with a spatially flat homogeneous and isotropic geometry. In addition, we also consider that it is filled by non-relativistic matter plus a DE component (or some effective mechanism that simulates it), and whose equation of state (hereafter EoS), \( p_{DE} = w(a)\rho_{DE} \), is driving the present accelerating stage. Following standard lines, the Hubble flow reads:

\[
\frac{H^2(a)}{H_0^2} = E^2(a) = \Omega_{m0} a^{-3} + \Omega_{DE} a^{-3} \int_0^1 \frac{da}{a} \frac{\ln[1+w(a)]}{1+\ln[1+w(a)]},
\]

(2.1)

where \( a(z) = 1/(1+z) \) is the scale factor of the universe, \( E(a) \) is the normalized Hubble flow, \( \Omega_{m0} \) is the dimensionless matter density at the present epoch, \( \Omega_{DE0} = 1 - \Omega_{m0} \) denotes the DE density parameter and \( w(a) \) its EoS parameter. On the other hand, we can express the EoS parameter in terms of \( E(a) = H(a)/H_0 \) using the Friedmann equations as

\[
w(a) = -1 - \frac{2a \frac{d\ln E}{da}}{1 - \Omega_{m}(a)}.
\]

(2.2)

where

\[
\Omega_{m}(a) = \frac{\Omega_{m0} a^{-3}}{E^2(a)}.
\]

(2.3)

Differentiating the latter and utilizing Eq. (2.2) we find that

\[
\frac{d\Omega_{m}}{da} = \frac{3}{a} w(a) \Omega_{m}(a) [1 - \Omega_{m}(a)].
\]

(2.4)

Since the exact nature of the DE has yet to be found, the above DE EoS parameter encodes our ignorance regarding the physical mechanism powering the late time cosmic acceleration.

The methodology described above can also be applied to the framework of modified gravity (see [37, 38]). In this case, instead of using the exact Hubble flow through a modification of the Friedmann equation one may consider an equivalent Hubble flow somewhat mimicking Eq. (2.1). The key point here is that the accelerating expansion can be attributed to a kind of "geometrical" DE contribution. Now, since the matter density (baryonic+dark) cannot accelerate the cosmic expansion, we perform the following parametrization [37, 38]:

\[
E^2(a) = \frac{H^2(a)}{H_0^2} = \Omega_{m0} a^{-3} + \Delta H^2.
\]

(2.5)

Naturally, any modification to the Friedmann equation of general relativity may be included in the last term of the above expression. After some algebra one may also derive, using Eqs. (2.2) and (2.5), an effective ("geometrical") dark energy EoS parameter, given by:

\[
w(a) = -1 - \frac{1}{3} \frac{d\ln \Delta H^2}{d\ln a}.
\]

(2.6)

Notice that we will use the above quantities in the next section.

3. THE EVOLUTION OF THE LINEAR GROWTH FACTOR

Here, we briefly discuss the basic equation which governs the behavior of the matter perturbations on sub-horizon scales and within the framework of any DE model, including those of modified gravity ("geometrical dark energy"). At the sub-horizon scales the DE component is expected to be smooth and thus it is fair to consider perturbations only on the matter component of the cosmic fluid [39]. In the framework of the homogeneous DE, the evolution equation of the matter fluctuations, for cosmological models where the DE fluid has a vanishing anisotropic stress and the matter fluid is not coupled to other matter species (see [40, 41, 42, 43, 44, 45, 46, 47, 48]), is given by:

\[
\ddot{\delta}_m + 2\dot{H}\delta_m = 4\pi G_{\text{eff}} \rho_m \delta_m
\]

(3.1)

where \( \rho_m \) is the matter density and \( G_{\text{eff}}(t) = G_N Q(t) \), with \( G_N \) denoting Newton’s gravitational constant.

For those cosmological models which adhere to general relativity, \( Q(t) = 1 \), \( G_{\text{eff}} = G_N \), the above equation reduces to the usual time evolution equation for the mass density contrast [40], while in the case of modified gravity models (see [40, 41, 42, 43, 44, 45]), we have \( G_{\text{eff}} \neq G_N \) (or \( Q(t) \neq 1 \)). In this context, \( \delta_m(t) \propto D(t) \), where \( D(t) \) is the linear growing mode (usually scaled to unity at the present time).

Solving Eq. (3.1) for the concordance ΛCDM cosmology\(^1\), we derive the well known perturbation growth factor (see [40]):

\[
D(z) = \frac{5\Omega_{m0} E(z)}{2} \int_{z}^{\infty} \frac{(1 + u)du}{E^3(u)}.
\]

(3.2)

In this work we use the above equation normalized to unity at the present time. Obviously, for \( E(z) \approx
\( \Omega_m^{1/2} (1 + z)^{3/2} \) it gives the standard result \( D(z) \approx a = (1 + z)^{-1} \), which corresponds to the matter dominated epoch, as expected.

Now, for any type of DE, an efficient parametrization of the matter perturbations is based on the growth rate of clustering [46]

\[
 f(a) = \frac{d \ln \delta_m}{d \ln a} \approx \Omega_m^\gamma(a) \tag{3.3}
\]

where \( \gamma \) is the so called growth index (see Refs. [10–13, 37, 40]) which plays a key role in cosmological studies as we described in the introduction, especially in the light of recent large redshift surveys (like the 6dF [47] and the WiggleZ [48, 49]; and references therein).

**A. The generalized growth index parametrization**

Inserting the first equality of Eq. (3.3) into Eq. (3.1) and using simultaneously Eq. (2.2) and \( \frac{d}{dt} = H \frac{d}{d \ln a} \), we derive after some algebra, that

\[
 a^2 \frac{df}{da} + f^2 + X(a)f = \frac{3}{2} \Omega_m(a)Q(a) \tag{3.4}
\]

where

\[
 X(a) = \frac{1}{2} - \frac{3}{2} w(a) \left[ 1 - \Omega_m(a) \right]. \tag{3.5}
\]

Now, we consider that the growth index varies with cosmic time. Transforming equation (3.4) from a to redshift \( \frac{d}{da} = - (1 + z)^{-1} \frac{d}{dz} \) and utilizing Eqs. (3.3) 2.4, we simply derive the evolution equation of the growth index \( \gamma = \gamma(z) \) (see also [33]). Indeed this is given by:

\[
 -(1 + z) \gamma' \ln (\Omega_m) + \Omega_m^\gamma + 3w(1 - \Omega_m)(\gamma - \frac{1}{2}) + \frac{1}{2} = \frac{3}{2} Q_0 \Omega_m^{1-\gamma}, \tag{3.6}
\]

where prime denotes derivative with respect to redshift. At the present epoch the above equation takes the form:

\[
 -\gamma'(0) \ln (\Omega_m) + \Omega_m^{\gamma(0)} + 3w(1 - \Omega_m)(\gamma(0) - \frac{1}{2}) + \frac{1}{2} = \frac{3}{2} Q_0 \Omega_m^{1-\gamma(0)}, \tag{3.7}
\]

where \( Q_0 = Q(z = 0) \) and \( w_0 = w(z = 0) \).

Over, the last few years there have been many theoretical speculations regarding the functional form of the growth index and indeed various candidates have been proposed in the literature. Here we phenomenologically parametrize \( \gamma(z) \) by the following general relation

\[
 \gamma(z) = \gamma_0 + \gamma_1 y(z). \tag{3.8}
\]

The latter equation can be seen as a first order Taylor expansion around some cosmological quantity such as \( a(z), \ z \) and \( \Omega_m(z) \). Interestingly, for those \( y(z) \) functions which satisfy \( y(0) = 0 \) [or \( \gamma(0) = \gamma_0 \)] one can write the parameter \( \gamma_1 \) in terms of \( \gamma_0 \). In this case \( \gamma(0) = \gamma_1 y(0) \), using Eq. (3.7) we obtain

\[
 \gamma_1 = \frac{\Omega_m^{\gamma(0)} + 3w_0(\gamma(0) - \frac{1}{2})(1 - \Omega_m) + \frac{3}{2} Q_0 \Omega_m^{1-\gamma(0)} + \frac{1}{2}}{\gamma(0) \ln \Omega_m} \tag{3.9}
\]

Note that for the rest of the paper we concentrate on the usual \( \Lambda \)CDM cosmology and thus we set \( Q(z) = 1 \).

Let us now briefly present various forms of \( \gamma(z) \), \( \forall z \):

- **Constant growth index** (hereafter \( \Gamma_0 \) model): Here we set \( \gamma_0 \) strictly equal to zero, thus \( \gamma = \gamma_0 \).
- **Expansion around \( z = 0 \)** (see [33]; hereafter \( \Gamma_1 \) model): In this case we have \( y(z) = z \). Note however, that this parametrization is valid at relatively low redshifts \( 0 \leq z \leq 0.5 \). In the statistical analysis presented below we utilize a constant growth index, namely \( \gamma = \gamma_0 + 0.5 \gamma_1 \) for \( z > 0.5 \).
- **Interpolated parametrization** (hereafter \( \Gamma_2 \) model): Since \( \Gamma_1 \) model is valid at low redshifts we propose to use a new formula \( y(z) = ze^{-z} \) that connects smoothly low and high-redshifts ranges. The above formula can be viewed as a combination of \( \Gamma_1 \) model with that of Dossett et al. [23]. For \( z \gg 1 \) we have \( \gamma_\infty \approx \gamma_0 \).
- **Expansion around \( a = 1 \)** ([51, 32]; hereafter \( \Gamma_3 \) model): Here the function \( y \) becomes \( y(z) = \frac{1 - a(z)}{a(z)} \). Obiously, at large redshifts \( z \gg 1 \) we get \( \gamma_\infty \approx \gamma_0 + \gamma_1 \).
- **Expansion around \( \Omega_m = 1 \)** ([52]; hereafter \( \Gamma_4 \) model): In this parametrization we have \( y(z) = 1 - \Omega_m(z) \) implying that \( y(0) = 1 - \Omega_m(0) \neq 0 \). As we have already mentioned above, the latter condition means that we can not write \( \gamma_1 \) in terms of \( \gamma_0 \). However, considering a constant equation of state parameter \( w(z) = w_0 = const. \) one can write \( \gamma_1 \) only in terms of \( \gamma(0) \),

\[
 \gamma_0 = \frac{3(1 - w_0)}{5 - 6w_0}, \quad \gamma_1 = \frac{3}{125} \frac{(1 - w_0)(1 - 3w_0/2)}{(1 - 6w_0/5)^3}. \tag{3.10}
\]

Since at large redshifts \( \Omega_m \approx 1 \) we can write \( \gamma_\infty \approx \gamma_0 \).

To conclude, for the \( \Gamma_1, \Gamma_2 \) and \( \Gamma_3 \) parametrizations one can show that \( y(0) = 0 \) and \( y'(0) = 1 \), respectively. Evidently, based on the above discussion it becomes clear that Eq. (3.9) is satisfied for any type of DE model. Therefore, for the case of the \( \Lambda \)CDM cosmology with \( \gamma_0 \approx 6/11 \) and \( \Omega_m = 0.274 \), Eq. (3.3) provides \( \gamma_1 \approx -0.0477 \), while for the case of the \( \Gamma_4 \) model we obtain \( \gamma_1 \approx 0.01127 \) (see Eq. 3.10).

2 Concerning the \( \gamma_1 \) parameter, Gong et al. [51] found a rather different value \( \gamma_1 = \frac{3}{125} \frac{(1 - w_0)(1 - 3w_0/2)}{(1 - 6w_0/5)^3} \).
4. OBSERVATIONAL CONSTRAINTS

In the following we briefly present some details of the statistical method and on the observational sample that we adopt in order to constrain the free parameters of the growth index, presented in the previous section.

A. The Growth data

The growth data that we will use in this work based on the 2dF, VVDS, SDSS, 6dF and WiggleZ galaxy surveys, for which the observed growth rate of structure, \( f_{\text{obs}}(z) \), is provided as a function of redshift. In Table 1 we quote the precise numerical values of the data points with the corresponding errors. This is an expanded version of the data appearing in Fig.5.

| Index | \( z \) | \( f_{\text{obs}} \) | Refs. | Symbols |
|-------|-------|----------|-----|--------|
| 1     | 0.15  | 0.49 ± 0.14 | [21, 52, 53] | solid circles |
| 2     | 0.35  | 0.70 ± 0.18 | [54] | solid circles |
| 3     | 0.55  | 0.75 ± 0.18 | [55] | solid circles |
| 4     | 0.77  | 0.91 ± 0.36 | [21] | solid circles |
| 5     | 1.40  | 0.90 ± 0.24 | [56] | solid circles |
| 6     | 2.42  | 0.74 ± 0.24 | [23, 57] | solid circles |
| 7     | 3.00  | 1.46 ± 0.29 | [58] | solid circles |
| 8     | 0.067 | 0.58 ± 0.11 | [47] | open triangles |
| 9     | 0.22  | 0.60 ± 0.10 | [48] | open circles |
| 10    | 0.41  | 0.70 ± 0.07 | [48] | open circles |
| 11    | 0.60  | 0.73 ± 0.07 | [48] | open circles |
| 12    | 0.78  | 0.70 ± 0.08 | [48] | open circles |

B. The overall Likelihood analysis

In order to constrain the cosmological parameters of the concordance \( \Lambda \)CDM model one needs to perform a joint likelihood analysis, involving the cosmic expansion data such as SNIa, BAO and CMB shift parameter together with the growth data. Up to now, due to the large errors of the growth data with respect to the cosmic expansion data, various authors preferred to constrain first
\( \Omega_{m0} \) using SNIa/BAO/CMB and then to fit \( \Omega^{\gamma}_{m}(z) \) to the growth data \( f_{\text{obs}}(z) \) alone. Of course, in the light of the 6dF and WiggleZ growth data it would be worthwhile to simultaneously constrain the \((\Omega_{m0}, \gamma)\) pair. In particular, we use the Union 2 set of 557 SNIa of Amanullah et al. \(^{3}\), the shift parameter of the CMB \(^{9}\) and the observed BAOs (see \[^{35}\]). The overall likelihood function\(^4\) is given by the product of the individual likelihoods according to:

\[
\mathcal{L}_{\text{tot}}(\mathbf{p}) = \mathcal{L}_{E}(\Omega_{m0}) \times \mathcal{L}_{f}(\mathbf{p}) \quad (4.1)
\]

where

\[
\mathcal{L}_{E}(\Omega_{m0}) = \mathcal{L}_{SNIa} \times \mathcal{L}_{CMB} \times \mathcal{L}_{BAO}. \quad (4.2)
\]

Since likelihoods are defined as \( \mathcal{L}_{j} \propto \exp(-\chi_{j}^{2}/2) \), it translates into an addition for the joint \( \chi_{\text{tot}}^{2} \) function:

\[
\chi_{\text{tot}}^{2}(\mathbf{p}) = \chi_{E}(\Omega_{m0}) + \chi_{f}^{2}(\mathbf{p}) \quad (4.3)
\]

with

\[
\chi_{E}(\Omega_{m0}) = \chi_{SNIa}^{2} + \chi_{CMB}^{2} + \chi_{BAO}^{2}. \quad (4.4)
\]

Note that the \( \chi_{f}^{2} \) is given by

\[
\chi_{f}^{2}(\mathbf{p}) = \sum_{i=1}^{N_f} \left[ \frac{f_{\text{obs}}(z_i) - \Omega_{m}(z_i)\gamma(z_i; \mathbf{p})}{\sigma_i} \right]^{2} \quad (4.5)
\]

where \( \sigma_i \) is the observed growth rate uncertainty. Evidently, the essential free parameters that enter in Eq. (4.3) are: \( \mathbf{p} = (\Omega_{m0}, \gamma_0, \gamma_1) \).

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\(^{3}\) The expansion data and the corresponding covariances can be found in [http://supernova.lbl.gov/Union/](http://supernova.lbl.gov/Union/) and in the paper of Zhang et al. \[^{61}\].

\(^{4}\) Likelihoods are normalized to their maximum values. In the present analysis we always report 1\(\sigma\) uncertainties on the fitted parameters. The total number of expansion data points used here is \( N_{E} = 559 \), while the associated degrees of freedom is: \( \text{dof} = N_{E} + N_{f} - n_{\text{df}} - 1 \), where \( N_{f} \) is the number of growth entries used in the statistical analysis and \( n_{\text{df}} \) is the model-dependent number of fitted parameters. Note that the uncertainty of the fitted parameters will be estimated, in the case of more than one such parameters, by marginalizing one with respect to the others.

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**FIG. 2:** The variance \( \Delta \chi^{2} = \chi_{\text{tot}}^{2} - \chi_{\text{min}}^{2} \) around the best fit \( \gamma \) value when we marginalize over \( \Omega_{m0} = 0.273 \). The solid and the dashed line correspond to the total sample (see Table I) and to the 6dF/WiggleZ data respectively. Note that the cross corresponds to \( (\gamma, \Delta \chi_{\text{tot}}^{2}) = (6/11, 1) \).

1. **Constant growth index**

First of all we utilize the \( \Gamma_{6} \) parametrization \((\gamma = \gamma_{0}, \gamma_{1} = 0\); see section 3A). Therefore, the corresponding statistical vector \( \mathbf{p} \) contains only two free parameters namely, \( \mathbf{p} = (\Omega_{m0}, \gamma_{0}, 0) \). We sample \( \Omega_{m0} \in [0.1, 0.6] \) and \( \gamma \in [0.1, 1.3] \) in steps of 0.001. In figure 1 we present the \( 1\sigma, 2\sigma \) and \( 3\sigma \) confidence levels in the \((\Omega_{m0}, \gamma)\) plane for the old growth+SNIa/CMB/BAO (left panel) and the total growth+/SNIa/CMB/BAO data (right panel) respectively. Using the old growth data (see entries 1-7 in Table I) it is evident that although the \( \Omega_{m0} \) parameter is tightly constrained \((0.274 \pm 0.012) \), the \( \gamma \) parameter remains weakly constrained, \( \gamma = 0.607^{+0.197}_{-0.179} \). As can be seen in the right panel of figure 1, the strong \( 2\sigma \)-degeneracy is broken when we include the 6dF and the WiggleZ growth data in the joint likelihood analysis. Indeed using the total growth data-set (see solid line in the left panel of figure 1) we find that the overall likelihood function peaks at \( \Omega_{m0} = 0.273 \pm 0.011 \) and \( \gamma = 0.586^{+0.074}_{-0.071} \) \((\chi_{\text{min}}^{2}/\text{dof} \simeq 549.3/568) \), while using only the 6dF/WiggleZ data (dashed line; \( N_{f} = 5 \), entries 8-12) we obtain \( \gamma = 0.589^{+0.087}_{-0.082} \) \((\chi_{\text{min}}^{2}/\text{dof} \simeq 545.3/567) \). Furthermore, it becomes evident that using the overall growth data-set together with the expansion cosmological data we decrease the \( 2\sigma \) surface area (see the left panel of figure 1) by a factor of \( \sim 2.5 \). Hereafter we call this quantity ”reduction factor” and is indicated by \( S \) (closely related to the “figure-of-merit” definition), defined as the ratio of the surface area of the \( 2\sigma \) contour using the old growth data to that of the total growth data-set. It is also interesting to mention that the best fit value of \( \Omega_{m0} \) is in excellent agreement with that provided by WMAP7 \((\Omega_{m0} = 0.273; \text{Komatsu et al. \[^{9}\]}) \). In figure 2 we plot the variation of \( \Delta \chi^{2} = \chi_{\text{tot}}^{2} - \chi_{\text{min}}^{2} \) around the best \( \gamma \) fit value when we marginalize over \( \Omega_{m0} = 0.273 \).

The above \( \gamma \) best fit results are in agreement (within \( 1\sigma \)) with the theoretically predicted value of \( \gamma \simeq 6/11 \) (see straight line in figure 1). Also our growth index results are in agreement with previous studies. For example, Di Porto & Amendola \[^{22}\] obtained \( \gamma = 0.60^{+0.40}_{-0.30} \), Gong \[^{13}\] measured \( \gamma = 0.64^{+0.17}_{-0.15} \) while Nessier & Perivolaropoulos \[^{12}\] found \( \gamma = 0.67^{+0.20}_{-0.17} \). Comparing the error bars among the various best fit values it is interesting to mention that including in the likelihood analysis the 6dF and the WiggleZ data we manage to reduce the error budget by \( \sim 50\% \). Finally it is interesting to mention that our \( \gamma \) parameter is in excellent agreement with those found based on the \( f\sigma_{8} \) estimator. Indeed
and Hudson & Turnbull [29] found γ and grand distribution levels) in the (γ, γ) plane of the (upper panel) and Γ (bottom panel) parametrizations (see section 3A). In the left panels we present the contours that correspond to the old growth rate data (see Table I, entries 1-7) while the right panels show the likelihood contours for the overall sample including the 6dF/WiggleZ data. We also include the theoretical ΛCDM (Ω_m0 = 0.273; crosses) pair (γ, γ) = (6/11, -0.0477).

Samushia et al. [49] found γ = 0.584 ± 0.112, Rapetti et al. [62] obtained γ = 0.576 ± 0.058, Basilakos & Pouri [28] and Hudson & Turnbull [29] found γ = 0.602 ± 0.055 and γ = 0.619 ± 0.054 respectively.

2. Time varying growth index

Now we concentrate on the γ(z) parametrizations, presented in section 3A. Now the statistical vector becomes: p = (Ω_m0, γ_0, γ_1). Notice that we sample γ_0 ∈ [0.1, 1.3] and γ_1 ∈ [-2.2, 2.2] in steps of 0.001. Since the expansion data put strong constraints on the value of Ω_m0, we find that for all Γ_1-4 models, the joint likelihood function peaks at Ω_m0 = 0.273 ± 0.011.

In figures 3 and 4 we present (after we marginalize over Ω_m0 = 0.273) the results of our analysis for the Γ_1, Γ_2, Γ_3 and Γ_4 models in the (γ_0, γ_1) plane.

In the left panels we show the contours using the old growth rate data (see Table I, entries 1-7: [13, 15, 60]) while in the right panel one can see results for the total sample including that of the 6dF/WiggleZ. The theoretical (γ_0, γ_1) values in the ΛCDM model indicated by the crosses (see section 3A).

From the left panels of figures 3 and 4, it becomes clear that using the old growth rate data-set (entries 1-7) we are unable to place constraints on the (γ_0, γ_1) parameters. On the other hand, utilizing the overall growth rate sample, we find:

![FIG. 3: Likelihood contours (for \( \Delta \chi^2 = -2 \ln L/\ln L_{\text{max}} \) equal to 2.30, 6.18 and 11.83, corresponding to 1σ, 2σ and 3σ confidence levels) in the (γ_0, γ_1) plane of the (upper panel) and Γ (bottom panel) parametrizations (see section 3A). In the left panels we present the contours that correspond to the old growth rate data (see Table I, entries 1-7) while the right panels show the likelihood contours for the overall sample including the 6dF/WiggleZ data. We also include the theoretical ΛCDM (Ω_m0 = 0.273; crosses) pair (γ_0, γ_1) = (6/11, -0.0477).]

![FIG. 4: The Likelihood contours for Γ_3 (upper panel) and Γ_4 (bottom panel). For more definitions see caption of figure 3. Here the crosses correspond to the theoretical (γ_0, γ_1) pair provided in section 3A [Γ_3: (6/11, -0.0477) and Γ_4: (6/11, 0.0113)].]

| Model | γ_0       | γ_1       | Δν |
|-------|-----------|-----------|----|
| Γ_0   | 0.586^{+0.034}_{-0.038} | 0.074     | 0  |
| Γ_1   | 0.49^{+0.12}_{-0.12} | 0.305^{+0.345}_{-0.318} | 4.6 |
| Γ_2   | 0.456^{+0.12}_{-0.11} | 0.587^{+0.502}_{-0.464} | 3.9 |
| Γ_3   | 0.461^{+0.12}_{-0.11} | 0.513^{+0.448}_{-0.414} | 3.4 |
| Γ_4   | 0.879^{+0.11}_{-0.11} | 0.551^{+0.590}_{-0.448} | 2.8 |

(a) Γ_1 parametrization: In this case the likelihood function peaks at \( γ_0 = 0.49^{+0.12}_{-0.12} \) and \( γ_1 = 0.305^{+0.345}_{-0.318} \) with \( \chi^2_{\text{min}}/\text{dof} \sim 549/567 \). Interestingly, the addition of five more points (6dF and WiggleZ growth data) in the statistical analysis provides a significant improvement in the derived (γ_0, γ_1) constraints. In particular, using the overall data-set we decrease the 2σ surface area (see left upper panel of fig.3) by a factor of S ∼ 4.6. Actually, one would expect such an improvement because the 6dF and the WiggleZ surveys measure f(z) to within 9 – 17% in every redshift bin, in contrast to the old growth rate data [13, 15, 60] in which the corresponding accuracy lies in the interval 20 – 40%.

(b) Γ_2 and Γ_3: Obviously, these parametrizations provide similar contours and thus they are almost equivalent as far as their statistics are concerned. In particular, the best fit values are: (i) for Γ_2 we have \( γ_0 = 0.456^{+0.12}_{-0.12} \), \( γ_1 = 0.587^{+0.502}_{-0.464} \) and (ii) for Γ_3 model we obtain \( γ_0 = 0.461^{+0.12}_{-0.11} \), \( γ_1 = 0.513^{+0.448}_{-0.414} \). In both cases the reduced \( \chi^2_{\text{min}}/\text{dof} \) is \( \sim 548.4/567 \). Notice that the reduction
factor” here is $S \sim 3.9$ and $3.4$ respectively.

(c) $\Gamma_4$ parametrization: In this case although the $\gamma_0$ is strongly degenerate with $\gamma_1$, the likelihood function peaks at $\gamma_0 = 0.875^{+0.12}_{-0.11}$ and $\gamma_1 = -0.551^{+0.50}_{-0.46}$ with $\chi^2_{\text{min}}/\text{dof} \simeq 548.4/567$. Also we find that $S \sim 2.8$.

We would like to stress that the predicted $(\gamma_0, \gamma_1)$ solutions of the $\Gamma_{1-4}$ parametrizations remain close to the $1\sigma$ borders (see crosses in figs. 3,4). In the top panel of figure 5, we present the evolution of the growth rate of structure, using the $\Gamma_0$ parametrization together with the growth data scaled to $(\Omega_m, \gamma) = (0.273, 0.586)$. In the bottom panel, we present the relative difference between the $\Gamma_0$ parametrization and all the rest $\Gamma_{1-4}$ models, ie, $\Delta_f(z) = [\Omega_m^{\gamma}(z) - \Omega_m^{0.586}(z)]/[\Omega_m^{0.586}(z)]$. The relative growth rate difference of the various fitted $\gamma(z)$ models with respect to that of $\Gamma_0$ (with $\gamma = 0.586$), $\Delta_f(z)$ indicates that the $\Gamma_{0-4}$ models have a very similar redshift dependence for $z \geq 0.4$ (with $|\Delta_f(z)| \leq 0.05$), while all the models show large such deviations for $z < 0.4$, reaching $|\Delta_f| \simeq 0.2$ at the lowest redshifts.

Finally, in Table II, one may see a more compact presentation of our results for the total sample, including the ”reduction factor” due to the presence of the 6dF/WiggleZ data.

![Graph](image)

**FIG. 5:** *Top panel:* Comparison of the observed and theoretical evolution of the growth rate of clustering $f(z) = \Omega_m^{0.586}(z)$ [see solid line: $\Gamma_0$ parametrization, $\Omega_m = 0.273$]. The different growth datasets are represented by different symbols (see Table I for definitions). *Bottom panel:* The relative difference, $\Delta_f(z)$, between the $\Gamma_0$ and the rest of the $\Gamma_{1-4}$ parametrizations. In particular the different lines correspond to the following pairs: $\Gamma_1 - \Gamma_0$ (short dashed line), $\Gamma_2 - \Gamma_0$ (long dashed line), $\Gamma_3 - \Gamma_0$ (dotted line) and $\Gamma_4 - \Gamma_0$ (dot-dashed line).

5. CONCLUSIONS

In this article we provide a general growth index evolution model $\gamma(z)$, based on phenomenology, which is valid for all possible non-interacting dark energy models, including those of modified gravity. Armed with our general $\gamma$ evolution model it is straightforward to apply the Polarski & Gannouji approach to various $\gamma(z)$ models. In the context of the concordance $\Lambda$ cosmology, we investigate the ability of five growth index parametrizations (including a constant one) to represent a variety of observational growth rate of structure data, based mainly on 2dF, SDSS, VVDS, 6dF and WiggleZ measurements. To this end we perform a joint likelihood analysis of the recent expansion data (SNIa, CMB shift parameter and BAOs) together with the growth rate of structure data, in order to determine the free parameters of the $\gamma(z)$ parametrizations and to statistically quantify their ability to represent the observations.

The comparison shows that all $\gamma$ parametrizations fit at an acceptable level the current growth data, as indicated by the their reduced $\chi^2$ values. Considering a constant growth index we can place tight constraints, up to $\sim 15\%$ accuracy, on the $\gamma$ parameter. Indeed, for the total growth rate data-set (see Table I) we find that $\gamma = 0.586^{+0.079}_{-0.074}$, while using only the 6dF and the WiggleZ growth data we obtain $\gamma = 0.583^{+0.087}_{-0.082}$, which is in agreement with the theoretically predicted value of $\gamma \simeq 6/11$. Under the assumption that the growth index varies with time we find that the $(\gamma_0, \gamma_1)$ parameter solution space of all growth index parametrizations, accommodate the theoretical $(\gamma_0, \gamma_1)$ values at $2\sigma$ level. We also observe that the inclusion of the new 6dF and WiggleZ data reduce significantly the $(\gamma_0, \gamma_1)$ parameter solution space. Despite the latter improvement we find that the majority of the $\gamma(z)$ parametrizations still suffer from the $\gamma_0 - \gamma_1$ degeneracy, implying that more and accurate data are essential.
