A Constituent Quark-Meson Model for Heavy Meson Decays

Aldo Deandrea

Centre de Physique Théorique, CNRS Luminy, Case 907, F-13288 Marseille Cedex 9, France

Abstract

I describe a model for heavy meson decays based on an effective quark-meson lagrangian. I consider the heavy mesons $S$ with spin and parity $J^P = (1^+, 0^+), \ H$ with $J^P = (1^-, 0^-)$ and $T^\mu$ with $J^P = (2^+, 1^+)$, i.e. $S$ and $P$ wave heavy-light mesons. The model is constrained by the known symmetries of QCD in the $m_Q \rightarrow \infty$ for the heavy quarks and chiral symmetry in the light quark sector. Using a very limited number of free parameters it is possible to compute several phenomenological quantities, e.g. the leptonic $B$ and $B^{**}$ decay constants; the three universal Isgur-Wise form factors: $\xi, \tau_{3/2}, \tau_{1/2}$, describing the semi-leptonic decays $B \rightarrow D^{(*)}\ell\nu, B \rightarrow D^{**}\ell\nu$; the strong and radiative $D^*$ decays; the weak semi-leptonic decays of $B$ and $D$ into light mesons: $\pi, \rho, A_1$. An overall agreement with data, when available, is achieved.

To appear in the Proceedings of the
XXIXth International Conference on High Energy Physics
Vancouver, B.C., Canada, 23–29 July 1998

CPT-98/P.3695
September 1998
A CONSTITUENT QUARK-MESON MODEL FOR HEAVY MESON DECAYS

ALDO DEANDREA

Centre de Physique Théorique, CNRS Luminy, Case 907, F-13288 Marseille Cedex 9, France
E-mail: deandrea@cpt.univ-mrs.fr

I describe a model for heavy meson decays based on an effective quark-meson lagrangian. I consider the heavy mesons $S$ with spin and parity $J^P = (1^+, 0^+),$ $H$ with $J^P = (1^-, 0^-)$ and $T^\mu$ with $J^P = (2^+, 1^+),$ i.e. $S$ and $P$ wave heavy-light mesons. The model is constrained by the known symmetries of QCD in the $m_Q \to \infty$ for the heavy quarks and chiral symmetry in the light quark sector. Using a very limited number of free parameters it is possible to compute several phenomenological quantities, e.g. the leptonic $B$ and $B^{**}$ decay constants; the three universal Isgur-Wise form factors: $\xi, \tau_{3/2}, \tau_{1/2},$ describing the semi-leptonic decays $B \to D^{(*)}\ell\nu; B \to D^{**}\ell\nu;$ the strong and radiative $D^*$ decays; the weak semi-leptonic decays of $B$ and $D$ into light mesons: $\pi, \rho, A_1.$ An overall agreement with data, when available, is achieved.

1 The Model

The model described in the present paper is based on an effective quark-meson lagrangian. I consider the heavy mesons as they can be described by a very limited number of free parameters. The model is suitable for the description of higher spin heavy mesons as they can be included in the formalism in a very easy way (see also). On the contrary the inclusion of higher order corrections, albeit possible, requires the determination of new free parameters, which proliferate as new orders are added to the expansion. In this sense the model allows a simple and intuitive approach to heavy-meson processes if it is kept at lowest order, while it loses part of its predictive power if corrections have to be included.

1.1 Heavy meson field

In order to implement the heavy quark symmetries in the spectrum of physical states the wave function of a heavy meson has to be independent of the heavy quark flavor and parity. It can be characterized by the total angular momentum $s_\ell$ of the light degrees of freedom. To each value of $s_\ell$ corresponds a degenerate doublet of states with angular momentum $J = s_\ell \pm 1/2.$ The mesons $P$ and $P^*$ form the spin-symmetry doublet corresponding to $s_\ell = 1/2$ (for charm for instance, they correspond to $D$ and $D^*$).

The negative parity spin doublet $(P, P^*)$ can be represented by a $4 \times 4$ Dirac matrix $H,$ with one spinor index for the heavy quark and the other for the light degrees of freedom.

An explicit matrix representation is:

$$H = \frac{1 + \gamma}{2} \left[ P^\mu_{\gamma} \gamma^\mu - P \gamma_5 \right]$$

$$\bar{H} = \gamma_0 H^\dagger \gamma_0 \ .$$

Here $v$ is the heavy meson velocity, $\epsilon_{\mu} P_{\alpha \mu} = 0$ and $M_H = M_P = M_{P^*}.$ Moreover $\gamma H = -H \gamma = \bar{H} \gamma = -\gamma \bar{H} = H$ and $P^\mu$ and $P$ are annihilation operators normalized as follows:

$$\langle 0 | P | Q \bar{q}(0^-) \rangle = \sqrt{M_H}$$

$$\langle 0 | P^\mu | Q \bar{q}(1^-) \rangle = \epsilon^\mu \sqrt{M_H} \ .$$

The formalism for higher spin states was introduced by Falk and Luke. I shall consider only the $S$ and $P$-waves of the system $Q\bar{q}.$ The heavy quark effective theory predicts two distinct multiplets, one containing a $0^+$ and a $1^+$ degenerate state, and the other one a $1^-$ and a $2^+$ state. In matrix notation, analogous to the one used for the negative parity states, they are described by

$$S = \frac{1 + \gamma}{2} \left[ P^\mu_{\gamma} \gamma^\mu \gamma_5 - P_0 \right]$$

$$\bar{S} = \gamma_0 H^\dagger \gamma_0 \ .$$

1
introduced using the chiral lagrangian: 

\[ \chi = P^\mu\nu\gamma_\mu - \sqrt{\frac{3}{2}} P^\nu_\mu \gamma_5 \left( g^{\mu\nu} - \frac{1}{3} \gamma^\mu (\gamma^\nu - v^\nu) \right) \]

(6)

These two multiplets have \( s_\ell = 1/2 \) and \( s_i = 3/2 \) respectively, where \( s_\ell \) is conserved together with the spin \( s_Q \) in the infinite quark mass limit because \( \tilde{f} = s_\ell + s_Q \).

1.2 Meson-Quark Interaction

The light degrees of freedom, i.e. the light quark fields \( \bar{Q} \) and the pseudo-scalar \( SU(3) \) octet of mesons \( \pi \) are introduced using the chiral lagrangian:

\[ \mathcal{L}_{\ell\ell} = \bar{\chi} i D^\mu \gamma_\mu + g_A A^\mu \gamma_5 \chi - m \bar{\chi} \chi \\
+ \frac{f^2}{8} \partial_\mu \Sigma \partial^\mu \Sigma. \]

(7)

Here \( D_\mu = \partial_\mu - i \not{v} \), \( \xi = \exp(i\pi/f_\pi) \), \( \Sigma = \xi^2 \), \( f_\pi = 130 \) MeV and

\[ \gamma^\mu = \frac{1}{2}(\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger) \]

\[ A^\mu = \frac{i}{2}(\xi^\dagger \gamma^\mu \xi - \xi \gamma^\mu \xi^\dagger). \]

(8)

The term with \( g_A \) is the coupling of pions to light quarks; it will not be used in the sequel. It is a free parameter, but in NJL model \( g_A = 1 \).

One can introduce a quark-meson effective lagrangian involving heavy and light quarks and heavy mesons. At lowest order one has:

\[ \mathcal{L}_{hd} = \bar{Q} i \not{v} \cdot \partial Q_v - \left( \bar{\chi}(H + S + i T_\mu \frac{D^\mu}{\Lambda_\chi}) Q_v + h.c. \right) \]

\[ + \frac{1}{2G_3} \text{Tr}[(\bar{H} + \bar{S})(H - S)] + \frac{1}{2G_4} \text{Tr}[\bar{T}_\mu T^\mu] \]

(9)

where the meson fields \( H, S, T \) have been defined is section 1.1. \( Q_v \) is the effective heavy quark field, \( G_3, G_4 \) are coupling constants and \( \Lambda_\chi \) (= 1 GeV) has been introduced for dimensional reasons. Lagrangian \( \mathcal{H} \) has heavy spin and flavor symmetry. This lagrangian comprises three terms containing respectively \( H, S \) and \( T \). Note that the fields \( H \) and \( S \) have the same coupling constant. In doing this one assumes that this effective quark-meson lagrangian can be justified as a remnant of a four quark interaction of the NJL type by partial bosonization.

1.3 Cut-Off Prescription

The cut-off prescription is the way in which part of the dynamical information regarding QCD is introduced in the model, this is why it is crucial and is part of the definition of the model. As the heavy mesons are described consistently with HQET, the heavy quark propagator in the loop contains the residual momentum \( k \) which arises from the interaction with the light degrees of freedom. It is natural to assume an ultraviolet cut-off on the loop momentum of the order of \( \Lambda \approx 1 \) GeV, even if the heavy quark mass is larger than the cut-off.

In the infrared the model is not confining and its range of validity can not be extended below energies of the order of \( \Lambda_{QCD} \). In practice one introduces an infrared cut-off \( \mu \), to take this into account.

The cut-off prescription is implemented via a proper time regularization (a different choice is followed in \( \mathcal{H} \)). After continuation to the Euclidean space it reads, for the light quark propagator:

\[ \int d^4k_E \frac{1}{k^2_E + m^2} \rightarrow \int d^4k_E \int_{\mu^2}^{1/\Lambda^2} ds e^{-s(k^2_E + m^2)} \]

(10)

where \( \mu \) and \( \Lambda \) are infrared and ultraviolet cut-offs.

Reasonable values are \( \Lambda \approx 1 \) GeV, \( m \approx \mu \approx 10^2 \) MeV. The cut-off prescription is similar to the one in \( \mathcal{H} \) with \( \Lambda = 1.25 \) GeV; the numerical results are not strongly dependent on the value of \( \Lambda \). The constituent mass \( m \) in the NJL models represents the order parameter discriminating between the phases of broken and unbroken chiral symmetry and can be fixed by solving a gap equation, which gives \( m \) as a function of the scale mass \( \mu \) for given values of the other parameters. In the second paper of Ebert et al. \( \mathcal{H} \) the values \( m = 300 \) MeV and \( \mu = 300 \) MeV are used and we shall assume the same values. As shown there, for smaller values of \( \mu, m \) is constant (=300 MeV) while for much larger values of \( \mu \), it decreases and in particular it vanishes for \( \mu = 550 \) MeV.

2 Analytical and Numerical Results

2.1 Decay constants

The lepton decay constants \( F \) and \( F^+ \) are defined as follows:

\[ \langle 0 | \bar{q} \gamma^\mu \gamma_5 Q | H(0^-, v) \rangle = i \sqrt{M_H} v^\mu F \]

(11)

\[ \langle 0 | \bar{q} \gamma^\mu Q | S(0^+, v) \rangle = i \sqrt{M_S} v^\mu F^+. \]

(12)

and they can be computed by a loop calculation, where the heavy meson interacts with the heavy and light quarks (via the interaction introduced in \( \mathcal{H} \)) and then those interact with the current. The result is

\[ F = \frac{\sqrt{Z_H}}{G_3} \]

(13)

\[ F^+ = \frac{\sqrt{Z_S}}{G_3} \]

(14)

where \( Z_H \) and \( Z_S \) are the field renormalization constants. Detailed results can be found in Deandrea et al. \( \mathcal{H} \). Values
Table 1: Renormalization constants and couplings. $\Delta_H$ in GeV; $G_3$, $G_4$ in GeV$^{-2}$, $Z_f$ in GeV$^{-1}$.

| $\Delta_H$ | $1/G_3$ | $Z_H$ | $Z_S$ | $Z_T$ | $1/G_4$ |
|------------|---------|-------|-------|-------|---------|
| 0.3        | 0.16    | 4.17  | 1.84  | 2.95  | 0.15    |
| 0.4        | 0.22    | 2.36  | 1.14  | 1.07  | 0.26    |
| 0.5        | 0.345   | 1.14  | 0.63  | 0.27  | 0.66    |

Table 2: $\hat{F}$ and $\hat{F}^+$ for various values of $\Delta_H$. $\Delta_H$ in GeV, leptonic constants in GeV$^{3/2}$.

| $\Delta_H$ | $\hat{F}$ | $\hat{F}^+$ |
|------------|-----------|-------------|
| 0.3        | 0.33      | 0.22        |
| 0.4        | 0.34      | 0.24        |
| 0.5        | 0.37      | 0.27        |

for the renormalization constants and couplings can be read in Table 1 for three values of the parameter $\Delta_H$.

The numerical results for the decay constants can be found in Table 2. Neglecting logarithmic corrections, $\hat{F}$ and $\hat{F}^+$ are related, in the infinite heavy quark mass limit, to the leptonic decay constant $f_B$ and $f^+$. For example, for $\Delta_H = 400$ MeV, one obtains from Table 2:

$$f_B \simeq 150 \text{ MeV} \quad (15)$$

$$f^+ \simeq 100 \text{ MeV} \quad (16)$$

2.2 Semi-leptonic Decays and Form Factors

As an example of the quantities that can be analytically calculated in the model, one can examine the Isgur-Wise function $\xi$:

$$\langle D(v') | \bar{c} \gamma_\mu (1 - \gamma_5) b | B(v) \rangle = \sqrt{M_B M_D} \times C_{cb} \xi(\omega) (v_\mu + v'_\mu)$$  
(17)

$$\langle D^+(v', \epsilon) | \bar{c} \gamma_\mu (1 - \gamma_5) b | B(v) \rangle = \sqrt{M_B M_D} C_{cb} \xi(\omega) \times [(\epsilon_{\mu\nu\alpha}\epsilon^{*\nu\alpha \nu'} - (1 + \omega) \epsilon^{*\nu\alpha} \epsilon'_{\nu\alpha})$$  
(18)

where $\omega = v \cdot v'$ and $C_{cb}$ is a coefficient containing logarithmic corrections depending on $\alpha_s$; within our approximation it can be put equal to 1: $C_{cb} = 1$. We also note that, in the leading order we are considering here $\xi(1) = 1$.

One finds

$$\xi(\omega) = Z_H \left[ \frac{2}{1 + \omega} I_3(\Delta_H) + \left( m + \frac{2\Delta_H}{1 + \omega} \right) I_5(\Delta_H, \Delta_H, \omega) \right]$$  
(19)

The $\xi$ function is plotted in Fig. 1.

The integrals $I_3$, $I_5$ can be found in the Appendix.

One can compute in a similar way the form factors describing the semi-leptonic decays of a meson belonging to the fundamental negative parity multiplet $H$ into the positive parity mesons in the $S$ and $T$ multiplets. Examples of these decays are

$$B \to D^{**} \ell \nu \quad (20)$$

where $D^{**}$ can be either a $S$ state (i.e. a $0^+$ or $1^+$ charmed meson having $s_\ell = 1/2$) or a $T$ state (i.e. a $2^+$ or $1^+$ charmed meson having $s_\ell = 3/2$).

The decays in (20) are described by two form factors $\tau_{1/2}, \tau_{3/2}$ which can be computed by a loop calculation similar to the one used to obtain $\xi(\omega)$. The result is

$$\tau_{1/2}(\omega) = \frac{\sqrt{Z_H Z_S}}{2(1 - \omega)} \left[ I_3(\Delta_S) - I_3(\Delta_H) + (\Delta_H - \Delta_S + m(1 - \omega)) I_5(\Delta_H, \Delta_S, \omega) \right]$$  
(21)

and

$$\tau_{3/2}(\omega) = -\frac{\sqrt{Z_H Z_T}}{\sqrt{3}} \times \left[ \frac{1}{2} \left( -1 - \omega + \omega^2 + \omega^3 \right) \left( -3 S(\Delta_H, \Delta_T, \omega) - (1 - 2\omega) S(\Delta_T, \Delta_H, \omega) + (1 - \omega^2) T(\Delta_H, \Delta_T, \omega) - 2(1 - 2\omega) U(\Delta_H, \Delta_T, \omega) \right) \right]$$  
(22)

where the integrals $S, T, U$ are defined in the Appendix.

The numerical results are reported in Table 3. For a comparison with other calculations of these form factors see [8].
An important test of our approach is represented by the Bjorken sum rule, which states

$$\rho^2_{\text{BW}} = \frac{1}{4} + \sum_k \left[ |\tau_{1/2}(k)|^2 + 2 |\tau_{3/2}(k)|^2 \right]. \quad (23)$$

Numerically we find that the first excited resonances, i.e. the $S$ and $T$ states ($k = 0$) practically saturate the sum rule for all the three values of $\Delta_H$.

### 2.3 Strong decays

The model can be used to calculate strong coupling constants, such as those concerning the decays:

$$H \to H\pi \quad (24)$$

$$S \to H\pi \quad (25)$$

The constant $g_{D^* D\pi}$ is related to the strong coupling constant of the effective meson field theory $g$ appearing in the heavy meson effective lagrangian

$$\mathcal{L} = ig \text{Tr}(\overline{H}H^\mu \gamma_5 A_\mu) + [ih \text{Tr}(\overline{H}S^\mu \gamma_5 A_\mu) + \text{h.c.}]. \quad (26)$$

by the relation

$$g_{D^* D\pi} = \frac{2m_D}{f_\pi} g \quad (27)$$

valid in the $m_Q \to \infty$ limit.

Numerically one gets

$$g = 0.456 \pm 0.040 \quad (28)$$

where the central value corresponds to $\Delta_H = 0.4$ GeV and the lower (resp. higher) value corresponds to $\Delta_H = 0.3$ GeV (resp. $\Delta_H = 0.5$ GeV). In an analogous way one obtains

$$h = -0.85 \pm 0.02 \quad (29)$$

The details of the calculation can be found in Deandrea et al. Once the coupling constants are calculated, it is possible to make predictions for branching ratios in strong heavy mesons decays. The results are given in Table 3 and are in good agreement with experimental data.

### 3 Conclusions

Starting from an effective lagrangian at the level of mesons and constituent quarks, one can calculate meson transition amplitudes by evaluating loops of heavy and light quarks. In this way it is possible to compute the Isgur-Wise function, the form factors $\tau_{1/2}$ and $\tau_{3/2}$, the leptonic decay constant $F$ and $F^*$, and many other quantities, such as radiative and strong couplings and decays which are only briefly mentioned in this short note (see for details and ). The agreement with data, when available, is very good in most cases. The model is able to describe a number of essential features of heavy meson physics in a simple and compact way.

### Acknowledgments

I acknowledge the support of a “Marie Curie” TMR research fellowship of the European Commission under contract ERBFMBICT960965. The subject described in this paper is based on work in collaboration with R. Gatto, G. Nardulli, A. Polosa and in the early stage of the work with N. Di Bartolomeo. I wish to thank them all. Centre de Physique Théorique is Unité Propre de Recherche 7061.

### Appendix

The integrals used in the paper are listed in this appendix. A more exhaustive list of integrals with proper time regularization useful for calculations in the model described here can be found in the appendix of .

$$I_1 = \frac{iN_c}{16\pi^2} \int_{m^2}^{\text{reg}} \frac{d^4 k}{(k^2 - m^2)} = \frac{N_c m^2}{16\pi^2} \Gamma(-1, m^2/\Lambda^2, m^2/\mu^2)$$

$$I_3(\Delta) = \frac{iN_c}{16\pi^2} \int_{m^2}^{\text{reg}} \frac{d^4 k}{(k^2 - m^2)(v \cdot k + \Delta + i\epsilon)}$$

$$= \frac{N_c}{16\pi^{3/2}} \int_{1/\Lambda^2}^{1/\mu^2} \frac{ds}{s^{3/2}} e^{-s(m^2 - \Delta^2)} \left(1 + \text{erf}(\Delta \sqrt{s})\right)$$
Finally one has:

\[ \sigma(x, \Delta_1, \Delta_2, \omega) = \frac{\Delta_1 (1 - x) + \Delta_2 x}{\sqrt{1 + 2 (\omega - 1) x + 2 (1 - \omega) x^2}} \]

is the error function and erf is the error function. In order to keep \( I_5 \) and \( I_6 \) in a short form one can introduce the function:

\[
I_5 (\Delta_1, \Delta_2, \omega) = \int_0^1 dx \frac{1}{1 + 2x^2(1 - \omega) + 2x(\omega - 1)} \times \\
\left[ \frac{6}{16\pi s^2} \int_1^{1/\mu^2} ds e^{-s(m^2 - \sigma^2)} s^{-1/2} (1 + \text{erf}(\sigma s)) + \frac{6}{16\pi^2} \int_1^{1/\mu^2} ds e^{-s(m^2 - 2\sigma^2)} s^{-1} \right]
\]

\[
I_6 (\Delta_1, \Delta_2, \omega) = I_1 \int_0^1 dx \frac{1}{1 + 2x^2(1 - \omega) + 2x(\omega - 1)} \times \\
\left[ \frac{N_c}{16\pi s} \int_0^{1/\mu^2} ds e^{-s(m^2 - \sigma^2)} \left\{ \sigma [1 + \text{erf}(\sigma s)] \right\} \times \\
\left[ 1 + 2s(m^2 - \sigma^2) \right] + 2 \sqrt{\frac{s}{\pi}} e^{-s \sigma^2} \left[ \frac{3}{2s} + (m^2 - \sigma^2) \right] \right]
\]

\[
S(\Delta_1, \Delta_2, \omega) = \Delta_1 I_3(\Delta_2) + \omega (I_1 + \Delta_2 I_3(\Delta_2)) \\
+ \Delta_1^2 I_5(\Delta_1, \Delta_2, \omega) \\
T(\Delta_1, \Delta_2, \omega) = m^2 I_5(\Delta_1, \Delta_2, \omega) + I_6(\Delta_1, \Delta_2, \omega) \\
U(\Delta_1, \Delta_2, \omega) = I_1 + \Delta_2 I_3(\Delta_2) + \Delta_1 I_3(\Delta_1) \\
+ \Delta_2 \Delta_1 I_5(\Delta_1, \Delta_2, \omega)
\]

References

1. For further references see the review papers: H. Georgi, contribution to the Proceedings of TASI 91, R.K. Ellis ed., World Scientific, Singapore,1991; B. Grinstein, contribution to High Energy Phenomenology, R. Huerta and M.A. Peres eds., World Scientific, Singapore, 1991; N. Isgur and M. Wise, contribution to Heavy Flavours, A. Buras and M. Lindner eds., World Scientific, Singapore,1992; M. Neubert, Phys. Rep. 245 259 (1994).
2. R. Casalbuoni et al., Phys. Rep. 281 145 (1997).