SU(3) in \(D\) decays: From 30% symmetry breaking to \(10^{-4}\) precision

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Flavor SU(3) symmetry, including 30% first order SU(3) breaking, has been shown to describe adequately a vast amount of data for charmed meson decays to two pseudoscalar mesons and to a vector and a pseudoscalar meson. We review a recent dramatic progress achieved by applying a high order perturbation expansion in flavor SU(3) breaking and treating carefully isospin breaking. We identify a class of U-spin related \(D^0\) decays to pairs involving charged pseudoscalar or vector mesons, for which high-precision nonlinear amplitude relations are predicted. Symmetry breaking terms affecting these relations are fourth order U-spin breaking, and terms which are first order in isospin breaking and second order in U-spin breaking. The predicted relations are shown to hold within experimental errors at a precision varying between \(10^{-3}\) and \(10^{-4}\), in agreement with estimates of high order terms. Improved branching ratio measurements for \(D^0 \rightarrow K^+\rho^-\), \(K^{*+}\pi^-\) and for decay modes involving three other pairs of charged pseudoscalar and vector mesons could further sharpen two of these precision tests. We also obtain amplitude relations for \(D^0\) decays to pairs of neutral pseudoscalar mesons, and relations for rate asymmetries between decays involving \(K^0_S\) and \(K^0_L\), which hold up to second order U-spin breaking at a level of several percent.

I Introduction

A useful tool for studying hadronic decay amplitudes of charmed mesons is approximate flavor SU(3) symmetry. First order symmetry breaking corrections in amplitudes, due to the light quark mass term in the Standard Model Lagrangian, are expected to be of order \((m_s - m_{u,d})/\Lambda_{\text{QCD}} \sim f_K/f_\pi - 1 \sim 0.2 - 0.3\).

In the seventies, shortly after the discovery of charm, SU(3) group theory has been used to obtain amplitude relations for charmed meson decays into pairs involving two pseudoscalar mesons or a pseudoscalar and a vector meson [1]. This study was extended in the early nineties to include numerous first order SU(3) breaking terms one of which was assumed to dominate over the others [2, 3]. A diagrammatic approach [4], equivalent to SU(3) group theory, has been developed and applied to fit data for branching ratios as they have been
accumulated [5]. Assumptions made in these studies about SU(3) breaking were often model-dependent. Other studies of these decay processes went much beyond SU(3) by assuming factorization of hadronic amplitudes [6]. A recent SU(3) fit to current data of charmed meson decays to two pseudoscalar mesons worked reasonably well when including two of the numerous first order SU(3) breaking terms (of order 30%) available in a group theoretical approach [7].

While flavor SU(3), including 30% first order symmetry breaking, has been shown to describe adequately a vast amount of data of hadronic decays of charmed mesons, this has not provided a precision test. For this matter one would hope to develop a perturbative expansion in reasonably small SU(3) breaking parameters, in which only high order symmetry breaking terms survive certain relations among amplitudes. A small step in this direction was made in Ref. [8], searching for linear relations among amplitudes for two-body and quasi two-body charmed meson decays in which first order SU(3) breaking terms cancel. Testing these relations, expected to hold within several percent, requires in most cases measuring relative strong phases between amplitudes which is a highly challenging task.

The purpose of this paper is to review, expand and improve new results obtained in recent work published in two short reports [9, 10] applying a high order perturbation expansion in flavor SU(3) breaking and treating carefully isospin breaking. We will identify a class of $D^0$ decays, for which high-precision nonlinear relations among magnitudes of amplitudes hold. The lowest order symmetry breaking terms affecting these relations will be shown to be fourth order SU(3) breaking terms, and terms which are first order in isospin breaking and second order in SU(3) breaking [11].

One major motivation for this work is searching for signals of new physics. Very precise relations as discussed here, which would fail at some high order flavor symmetry breaking, could provide such signatures. For great convenience we will use U-spin, an SU(2) subgroup of flavor SU(3), rather than applying the full SU(3) group structure. Our U-spin expansion is, of course, consistent with SU(3) expansions in Refs. [3, 7, 8].

In Section II we identify sets of four two-particle final states in $D^0$ decays, each consisting of pairs involving charged pseudoscalar and vector mesons. In a given set two of these states and a linear combination of the other two form a U-spin triplet, playing an important role in $D^0$–$\bar{D}^0$ mixing. Our discussion in Sections III - VII studies in detail one of these sets denoted by $D^0 \rightarrow P^+P^-$ consisting of $D^0 \rightarrow \pi^+K^-$, $K^+\pi^-$, $K^+K^-$, $\pi^+\pi^-$. We derive U-spin symmetry relations for these processes in Section III and study first order and arbitrary order U-spin breaking corrections in Sections IV and V, respectively. In Section VI we obtain a high-precision relation, obeyed up to fourth order U-spin breaking, for ratios of amplitudes of the above four processes. Section VII investigates first order isospin breaking terms occurring in this relation, showing that they are also suppressed by factors associated with second order U-spin breaking. Sections VIII, IX and X discuss experimental tests in $D^0 \rightarrow P^+P^-$, $D^0 \rightarrow V^+P^-$ and $D^0 \rightarrow P^+V^-$, respectively, where $V^\pm$ denote charged vector mesons. In Section XI we study $D^0$ decays into pairs of neutral pseudoscalar mesons, deriving amplitude relations and $K_S^0 - K_L^0$ rate asymmetry relations which hold up to second order U-spin breaking. A short conclusion is given in Section XII.
II  \(D^0\) decays to two-body \(U = 1\) states

An SU(2) subgroup of flavor SU(3) that is useful for studying charmed mesons is U-spin [12]. The quark pair \((d, s)\) behaves like a doublet under this group while the \(u\) quark and the heavier \(c, b\) and \(t\) quarks are singlets. U-spin symmetry leads to interesting relations among amplitudes of hadronic \(D\) decays [13, 14]. It also implies the vanishing of \(D^0 - \bar{D}^0\) mixing up to second order U-spin breaking [15], which underlies the vanishing of \(D^0 - \bar{D}^0\) mixing within full flavor SU(3) [16]. This behavior of \(D^0 - \bar{D}^0\) mixing under U-spin has been shown to follow from a cancellation up to second order U-spin breaking of mixing contributions from intermediate U-spin triplet states. The high order U-spin breaking perturbation expansion that will be studied in this paper works well, as we will show, for these two-body or quasi two-body \(D^0\) decays to \(U = 1\) states.

One class of \(U = 1\) two-body states involves pairs of opposite charge pseudoscalar or vector mesons. Our study will focus on these final states for which a high-order U-spin breaking expansion is applicable. Another class of processes involves decay into pairs of neutral mesons. In this case two-body final states do not have well-defined values of U-spin. Instead, linear superpositions of final states have \(U = 1\). Consequently in these decays a U-spin expansion works well for certain linear combinations of decay amplitudes.

We start by classifying single meson states of positive or negative charge as doublets of U-spin. Since a pair of \(d\) and \(s\) quark and their antiquarks form two U-spin doublets, \((d, s)\) and \((\bar{s}, -\bar{d})\), one has two doublets of pseudoscalar mesons

\[
P^+ = \begin{pmatrix} K^+ \\ -\pi^+ \end{pmatrix} \equiv \begin{pmatrix} u\bar{s} \\ -ud \end{pmatrix}, \quad P^- = \begin{pmatrix} \pi^- \\ K^- \end{pmatrix} \equiv \begin{pmatrix} d\bar{u} \\ s\bar{u} \end{pmatrix},
\]

and two doublets of vector mesons

\[
V^+ = \begin{pmatrix} K^{*+} \\ -\rho^+ \end{pmatrix}, \quad V^- = \begin{pmatrix} \rho^- \\ K^{*-} \end{pmatrix}.
\]

One can then form two-particle states of charge zero in four different forms, \(P^+ P^-, V^+ P^-, P^+ V^-\) and \(V^+ V^-\).

In the next several sections we will study the four processes \(D^0 \to P^+ P^-\), \(P^+ P^- = \pi^+ K^-\), \(K^+ \pi^-\), \(\pi^+ \pi^-\), and \(K^+ K^-\), for which most precise data exist. This discussion is also applicable to the other three sets of processes, \(D^0 \to V^+ P^-\), \(P^+ V^-\) and \(D^0 \to V^+ V^-\). For \(D^0 \to V^+ V^-\) one may, in principle, treat separately S, P and D-wave amplitudes, or amplitudes for longitudinal polarizations, and for transverse polarizations which are mutually parallel and perpendicular to each other. We will not discuss further these latter challenging decay modes for which no branching ratios have been measured [17].

III  U-spin symmetry limit

As a starting point we derive amplitude relations in the U-spin symmetry limit. The four possible two-particle states \(|P^+ P^-\rangle\) can be written in the form of three U-spin triplet states
and one singlet state:

\[ |K^+\pi^-\rangle = |1, 1\rangle, \quad |\pi^+K^-\rangle = |0, -1\rangle, \quad \frac{1}{\sqrt{2}}|K^+K^+ - \pi^+\pi^-\rangle = |1, 0\rangle, \quad (3) \]

\[ \frac{1}{\sqrt{2}}|K^+K^- + \pi^+\pi^-\rangle = |0, 0\rangle. \quad (4) \]

The charm-changing weak Hamiltonian has a simple transformation property under U-spin. Its three pieces responsible for Cabibbo-favored (CF), singly Cabibbo-suppressed (SCS) and doubly Cabibbo-suppressed (DCS) decays transform, when normalized suitably, like three components of a U-spin triplet operator denoted \((U = 1, U_3 = -1, 0, +1)\):

\[ H_{CF}^W = \frac{G_F}{\sqrt{2}} \cos^2 \theta_C (\bar{s}c)(\bar{u}d) = -\cos^2 \theta_C(1, -1), \]

\[ H_{SCS}^W = \frac{G_F}{\sqrt{2}} \cos \theta_C \sin \theta_C [(\bar{s}c)(\bar{u}s) - (\bar{d}c)(\bar{u}d)] = \sqrt{2} \cos \theta_C \sin \theta_C(1, 0), \]

\[ H_{DCS}^W = -\frac{G_F}{\sqrt{2}} \sin^2 \theta_C (\bar{d}c)(\bar{u}s) = -\sin^2 \theta_C(1, +1). \quad (5) \]

We have suppressed the chiral structure of V-A operators, using \(V_{ud} = V_{cs} = \cos \theta_C, V_{us} = -V_{cd} = \sin \theta_C\) for Cabibbo-Kobayashi-Maskawa (CKM) matrix elements.

Charmed meson decays are dominated by real CKM matrix elements associated with the first two quark families. For the most part we will be using a parametrization of the CKM matrix up to terms which are fourth order in \(\lambda \equiv V_{us}\) \(^\dagger\). In Section VI we will discuss the effect of higher order terms in \(\lambda\) on four ratios of amplitudes studied in this section.

Virtual \(b\) quarks in penguin amplitudes may lead to tiny direct CP asymmetries in SCS decays of order \(\left(\alpha_s(m_b^2)/\pi\right)(|V_{cb}V_{ub}|/|V_{cs}V_{us}|) \sim 10^{-4}\) \(^\dagger\), depending on the final state. No CP asymmetries of this order are expected in CF and DCS decays. An order of magnitude larger asymmetries may occur in SCS decays which are subject to potential penguin amplitude enhancement \(^\dagger\). No CP asymmetries at these small levels have been measured so far. We will seek amplitude relations which hold at this high precision but not at higher accuracy. Errors in amplitudes are half the errors measured in decay rates. Thus we will neglect in our discussion direct CP asymmetries, assuming that branching ratios for \(D^0 \to f\) are given by averages measured for decay processes and their CP conjugates,

\[ \mathcal{B}(D^0 \to f) = \mathcal{B}(D^0 \to f)_{CPav} \equiv \frac{1}{2}[\mathcal{B}(D^0 \to f) + \mathcal{B}(\bar{D}^0 \to \bar{f})]. \quad (6) \]

The \(D^0\) is a U-spin singlet. Denoting hadronic matrix elements in the U-spin symmetry limit by a superscript \((0)\) and defining a reduced matrix element, \(A \equiv \langle 1, U_3| (1, U_3)|0, 0\rangle\), one obtains from Es. (3) and (5)

\[ \langle \pi^+K^-|H_{CF}^W|D^0(0)\rangle \cos^2 \theta_C = A, \quad (7) \]
\[
\frac{\langle K^+K^- - \pi^+\pi^- | H^{\text{SCS}}_{W}|D^0 \rangle^{(0)}}{\cos \theta_C \sin \theta_C} = 2A ,
\]
(8)
\[
\frac{\langle K^+\pi^- | H^{\text{DCS}}_{W}|D^0 \rangle^{(0)}}{-\sin^2 \theta_C} = A .
\]
(9)

Eq. (4) leads to
\[
\langle K^+K^- + \pi^+\pi^- | H^{\text{SCS}}_{W}|D^0 \rangle^{(0)} \propto \langle 0,0 | (1,0) | 0,0 \rangle = 0 .
\]
(10)

Using (8) this implies
\[
\frac{\langle K^+K^- | H^{\text{SCS}}_{W}|D^0 \rangle^{(0)}}{\cos \theta_C \sin \theta_C} = \frac{\langle \pi^+\pi^- | H^{\text{SCS}}_{W}|D^0 \rangle^{(0)}}{-\cos \theta_C \sin \theta_C} = A .
\]
(11)

Thus the four amplitudes in (7), (9) and (11) denoted by the decay final state, \(A(f) \equiv \langle f | H_{W}|D^0 \rangle\) have simple ratios in the U-spin symmetry limit [1].

\[
A^0(\pi^+K^-) : A^0(K^+K^-) : A^0(\pi^+\pi^-) : A^0(K^+\pi^-) = 1 : \tan \theta_C : -\tan \theta_C : -\tan^2 \theta_C .
\]
(12)

We note that a derivation of the two ratios, \(A^0(\pi^+K^-)/A^0(\pi^+\pi^-) = -\tan^2 \theta_C\) and \(A^0(\pi^+\pi^-)/A^0(K^+K^-) = -1\), uses only symmetry under \(d \leftrightarrow s\) reflection [14] implying,

\[
\langle \pi^+K^-|(\bar{s}c)(\bar{u}d)|D^0 \rangle = \langle K^+\pi^-|(\bar{d}c)(\bar{u}s)|D^0 \rangle ,
\]
\[
\langle K^+K^-|(\bar{s}c)(\bar{u}s) - (\bar{d}c)(\bar{u}d)|D^0 \rangle = -\langle \pi^+\pi^-|(\bar{s}c)(\bar{u}s) - (\bar{d}c)(\bar{u}d)|D^0 \rangle ,
\]
(13)
and does not require full SU(2) U-spin symmetry. This full symmetry is required for the relation between these two pairs of processes.

**IV First order U-spin breaking**

First order U-spin breaking corrections to the amplitudes (7), (9) and (11) are obtained by multiplying the weak Hamiltonian or the final states by an \(s-d\) spurion mass operator, \(M_{\text{Ubrk}} \propto (\bar{s}s) - (\bar{d}d)\) [2]:

\[
\langle f | H^{\text{eff}}_{\text{W}}|D^0 \rangle^{(1)} = \langle f | H^{\text{eff}}_{\text{W}} M_{\text{Ubrk}}|D^0 \rangle + \langle M_{\text{Ubrk}} f | H^{\text{eff}}_{\text{W}}|D^0 \rangle .
\]
(14)

While for CF and DCS decays one has simply

\[
H^{\text{CF,DCS}}_{\text{W}} M_{\text{Ubrk}} = H^{\text{CF,DCS}}_{\text{W}} M_{\text{Ubrk}} ,
\]
(15)

the effective Hamiltonian for SCS decays obtains at first order an additional nonperturbative \(s+d\) penguin term \(P_{s+d}\) due to an \(s-d\) mass difference [20]:

\[
H^{\text{SCS}}_{\text{W}} M_{\text{Ubrk}} = H^{\text{SCS}}_{\text{W}} M_{\text{Ubrk}} + P_{s+d} .
\]
(16)
The $U = 0$ penguin amplitude in SCS decays interferes with opposite signs with the $U = 1$ tree amplitudes in $D^0 \to K^+K^-$ and in $D^0 \to \pi^+\pi^-$. This may potentially increase the first amplitude and decrease the second. This effect of the penguin amplitude has been pointed out very early in Refs. [21, 22, 23], and has been studied recently in Refs. [20, 24, 25] with its implication on CP asymmetries in these processes.

Eqs. (15) and (16) imply different first order U-spin breaking corrections in $D^0 \to \pi^+K^-$ and $D^0 \to K^+\pi^-$, on the one hand, and in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$, on the other. We will show now that the corrections within each of these two pairs of processes have equal magnitudes and opposite signs when normalized by the corresponding U-spin symmetric amplitudes. We will first follow a full SU(2) U-spin argument presented in [9], and then derive this result in a simpler manner using $d \leftrightarrow s$ reflection.

Consider first Eq. (14) for decays to the two U-spin triplet states, $|f_1\rangle = |\pi^+K^-\rangle = -|1, -1\rangle$ and $|f_2\rangle = |K^+\pi^-\rangle = |1, 1\rangle$, to which (15) applies. $H^{\text{CF}}_{\text{eff}}$ and $H^{\text{DCS}}_{\text{eff}}$ transform like $(1, -1)$ and $(1, +1)$ while the $s-d$ spurion mass operator $M_{\text{Ubrk}}$ behaves like $(1, 0)$. Since the $D^0$ is a U-spin singlet only the triplet operators in the products $H^{\text{DCS,CF}}_{\text{eff}}M_{\text{Ubrk}} \propto (1, \pm 1)(1, 0)$ contribute to the triplet final states $\pm|1, \pm 1\rangle$, and only the triplet states in $M_{\text{Ubrk}}|1, \pm 1\rangle \propto (1, 0)|1, \pm 1\rangle$ obtain contributions from the triplet Hamiltonian operator. Thus the two terms in (14) involve coupling of the product $(1, \pm 1) \otimes (1, 0)$ into $(1, \pm 1)$, where $\pm$ signs correspond to $K^+\pi^-$ and $\pi^+K^-$ final states. Using a property of Clebsch-Gordan coefficients,

$$(1, 1; n, 0|1, 1) = (-1)^n(1, -1; n, 0|1, -1),$$

we therefore find

$$\frac{\langle \pi^+K^-|H^{\text{CF}}_{\text{eff}}|D^0\rangle^{(1)}}{\cos^2\theta_C} = \frac{\langle K^+\pi^-|H^{\text{DCS}}_{\text{eff}}|D^0\rangle^{(1)}}{\sin^2\theta_C}.$$  

That is, first order U-spin breaking terms contribute equally but with opposite signs to $D^0 \to \pi^+K^-$ and $D^0 \to K^+\pi^-$, when normalized by corresponding U-spin symmetric amplitudes given in Eqs. (17) and (19):

$$\frac{\langle \pi^+K^-|H^{\text{CF}}_{\text{eff}}|D^0\rangle^{(1)}}{\langle \pi^+K^-|H^{\text{CF}}_{\text{eff}}|D^0\rangle^{(0)}} = -\frac{\langle K^+\pi^-|H^{\text{DCS}}_{\text{eff}}|D^0\rangle^{(1)}}{\langle K^+\pi^-|H^{\text{DCS}}_{\text{eff}}|D^0\rangle^{(0)}} \equiv -\epsilon_1.$$  

We denote by $\epsilon_1$ the U-spin breaking term in $D^0 \to K^+\pi^-$ normalized by its U-spin invariant amplitude.

A simpler and rather immediate derivation of (19) may be obtained by applying $d \leftrightarrow s$ reflection to Eq. (14) for $f = \pi^+K^-$. Noting that the $s-d$ spurion mass operator $M_{\text{Ubrk}}$ changes sign under this reflection, one has

$$\langle \pi^+K^-|\langle \bar{s}c\rangle(\bar{u}d)M_{\text{Ubrk}}|D^0\rangle + \langle M_{\text{Ubrk}}(\pi^+K^-)|\langle \bar{s}c\rangle(\bar{u}d)|D^0\rangle = -\langle K^+\pi^-|\langle \bar{d}c\rangle(\bar{u}s)M_{\text{Ubrk}}|D^0\rangle - \langle M_{\text{Ubrk}}(K^+\pi^-)|\langle \bar{d}c\rangle(\bar{u}s)|D^0\rangle.$$  

This leads directly to (18) and (19).

This short argument applies also to first order U-spin breaking in SCS decays, $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$, since the penguin operator $P_{s+d}$ in (16) also changes sign under
d \leftrightarrow s \text{ reflection. Therefore,}
\begin{align}
\frac{\langle \pi^+ \pi^- | H_{\text{eff}}^{\text{SCS}} | D^0 \rangle^{(1)}}{\langle \pi^+ \pi^- | H_{W}^{\text{SCS}} | D^0 \rangle^{(0)}} &= -\frac{\langle K^+ K^- | H_{\text{eff}}^{\text{SCS}} | D^0 \rangle^{(1)}}{\langle K^+ K^- | H_{W}^{\text{SCS}} | D^0 \rangle^{(0)}} = -\epsilon_2 ,
\end{align}
where \( \epsilon_2 \) denotes the U-spin breaking term in \( D^0 \rightarrow K^+ K^- \) normalized by its U-spin invariant amplitude. Furthermore, the change in sign between first order terms in \( D^0 \rightarrow K^+ K^- \) and \( D \rightarrow \pi^+ \pi^- \) applies separately to contributions of the two operators on the right-hand side of (16) representing tree and penguin amplitudes. This leads one to expect that the U-spin breaking parameter \( \epsilon_2 \), involving both tree and penguin amplitudes, is larger than \( \epsilon_1 \) which involves only tree amplitudes [20].

Combining the results (19) and (21) with the zeroth order Eqs. (7), (9) and (11) one obtains the following first order expressions for decay amplitudes:
\begin{align}
A(D^0 \rightarrow \pi^+ K^-) &= \cos^2 \theta_C A(1 - \epsilon_1) , \\
A(D^0 \rightarrow K^+ \pi^-) &= -\sin^2 \theta_C A(1 + \epsilon_1) , \\
A(D^0 \rightarrow \pi^+ \pi^-) &= -\cos \theta_C \sin \theta_C A(1 - \epsilon_2) , \\
A(D^0 \rightarrow K^+ K^-) &= \cos \theta_C \sin \theta_C A(1 + \epsilon_2) .
\end{align}

V  U-spin breaking of arbitrary order

U-spin breaking of order \( n \) in decay amplitudes \( \langle f | H_{\text{eff}} | D^0 \rangle \) is obtained by introducing in the Hamiltonian or in the final state a total of \( n \) powers of the \( s - d \) spurion mass operator, applying (16) to SCS decays. Generalizing the argument for a relative negative sign in first order breaking, based on a change of sign of \( M_{\text{brk}} \) and \( P_s + P_d \) under \( d \leftrightarrow s \) reflection, we conclude that a negative relative sign applies to odd \( n \) and a positive sign to even \( n \):
\begin{align}
\frac{\langle \pi^+ K^- | H_{\text{eff}}^{\text{CF}} | D^0 \rangle^{(n)}}{\cos^2 \theta_C} &= (-1)^n \frac{\langle K^+ \pi^- | H_{\text{eff}}^{\text{SCS}} | D^0 \rangle^{(n)}}{-\sin^2 \theta_C} , \\
\frac{\langle K^+ K^- | H_{W}^{\text{SCS}} | D^0 \rangle^{(n)}}{\cos \theta_C \sin \theta_C} &= (-1)^n \frac{\langle \pi^+ \pi^- | H_{W}^{\text{SCS}} | D^0 \rangle^{(n)}}{-\cos \theta_C \sin \theta_C} .
\end{align}

Thus we may sum up:
\begin{align}
A(D^0 \rightarrow \pi^+ K^-) &= \cos^2 \theta_C A[1 - \epsilon_1 + a_1 \epsilon_1^2 - a_1' \epsilon_1^3 + ...] , \\
A(D^0 \rightarrow K^+ \pi^-) &= -\sin^2 \theta_C A[1 + \epsilon_1 + a_1 \epsilon_1^2 + a_1' \epsilon_1^3 + ...] , \\
A(D^0 \rightarrow \pi^+ \pi^-) &= -\cos \theta_C \sin \theta_C A[1 - \epsilon_2 + a_2 \epsilon_2^2 - a_2' \epsilon_2^3 + ...] , \\
A(D^0 \rightarrow K^+ K^-) &= \cos \theta_C \sin \theta_C A[1 + \epsilon_2 + a_2 \epsilon_2^2 + a_2' \epsilon_2^3 + ...] .
\end{align}

While the complex U-spin breaking parameters \( \epsilon_{1,2} \) and the nonperturbative coefficients \( a_{1,2}, a_{1,2}' ... \) are not calculable from first principles, one expects the first two parameters to be around \( 0.2 - 0.3 \) (\( \epsilon_2 \) being larger in magnitude than \( \epsilon_1 \) for the above-mentioned argument) and the coefficients to be of order one, \( |a_{1,2}| \sim |a_{1,2}'| \sim 1 \).
Expanding magnitudes of amplitudes up to and including third order we find

\[
|1 \pm \epsilon + a\epsilon^2 \pm a'\epsilon^3| = 1 \pm \text{Re} \epsilon + \frac{1}{2}(\text{Im} \epsilon)^2 + \text{Re}(a\epsilon^2) \pm \text{Re}(a'\epsilon^3) + \frac{1}{2}\text{Re} \epsilon (\text{Im} \epsilon)^2 \pm \text{Im} \epsilon \text{Im}(a\epsilon^2) .
\] (26)

In the next section we will use this expansion for studying ratios of magnitudes of decay amplitudes, identifying relations among ratios in which U-spin breaking terms up to third order cancel, thus being sensitive to tiny fourth order U-spin breaking. We will then argue that there is no need to go beyond third order in (26) for showing that fourth order terms do not cancel in these relations.

VI High-precision relation among ratios of amplitudes

We define four ratios of amplitudes:

\[
R_1 \equiv \frac{|A(D^0 \to K^+\pi^-)|}{|A(D^0 \to \pi^+K^-)| \tan^2 \theta_C} , \\
R_2 \equiv \frac{|A(D^0 \to K^+K^-)|}{|A(D^0 \to \pi^+\pi^-)|} , \\
R_3 \equiv \frac{|A(D^0 \to K^+K^-)| + |A(D^0 \to \pi^+\pi^-)|}{|A(D^0 \to \pi^+K^-)| \tan \theta_C + |A(D^0 \to K^+\pi^-)| \tan^{-1} \theta_C} , \\
R_4 \equiv \sqrt{\frac{|A(D^0 \to K^+K^-)||A(D^0 \to \pi^+\pi^-)|}{|A(D^0 \to \pi^+K^-)||A(D^0 \to K^+\pi^-)|}} .
\] (27)

These four ratios are not mutually independent. They obey a trivial identity,

\[
R_4 = R_3 \sqrt{\frac{1 - [(R_2 - 1)/(R_2 + 1)]^2}{1 - [(R_1 - 1)/(R_1 + 1)]^2}} .
\] (28)

We will prove a nontrivial relation involving $R_3$ and $R_4$ and another nonlinear function of $R_1$ and $R_2$, that holds up to maximal fourth order U-spin breaking.

Using (26) we start by expanding the two ratios $R_1$ and $R_2$ up to third order U-spin breaking:

\[
R_1 = 1 + 2[\text{Re} \epsilon_1 + (\text{Re} \epsilon_1)^2] + \mathcal{O}(\epsilon_1^3) , \\
R_2 = 1 + 2[\text{Re} \epsilon_2 + (\text{Re} \epsilon_2)^2] + \mathcal{O}(\epsilon_2^3) .
\] (29)

These two ratios involve first order corrections given by $2\text{Re} \epsilon_1$ and $2\text{Re} \epsilon_2$. Second order corrections in these ratios are given by squares of these same real parts with no dependence on the unknown parameters $a_1$ and $a_2$. Thus measurements of $R_1$ and $R_2$ provide a way for calculating $\text{Re} \epsilon_1$ and $\text{Re} \epsilon_2$ up to third order corrections. Solutions for $\text{Re} \epsilon_1$ and $\text{Re} \epsilon_2$ using
Eqs. (29) should include the U-spin symmetry limit, requiring $\text{Re} \epsilon_1 = 0$ and $\text{Re} \epsilon_2 = 0$ for $R_1 = 1$ and $R_2 = 1$, respectively, rather than $\text{Re} \epsilon_1 = -1$ and $\text{Re} \epsilon_2 = -1$. This implies

$$
\text{Re} \epsilon_1 = \frac{1}{2} \left( \sqrt{2R_1 - 1} - 1 \right) + \mathcal{O}(\epsilon_1^3), \quad \text{Re} \epsilon_2 = \frac{1}{2} \left( \sqrt{2R_2 - 1} - 1 \right) + \mathcal{O}(\epsilon_2^3). \quad (30)
$$

As we will see immediately, these two first order U-spin breaking parameters do not determine only $R_1$ and $R_2$ but also the difference $R_3 - R_4$.

The two ratios $R_3$ and $R_4$, in which first and third order terms cancel \cite{26, 27}, may be expanded up to fourth order:

$$
R_3 = 1 + \frac{1}{2}[(\text{Im} \epsilon_2)^2 - (\text{Im} \epsilon_1)^2] + \text{Re} [a_2 \epsilon_2^2 - a_1 \epsilon_1^2] + \mathcal{O}(\epsilon_1^4, \epsilon_2^4),
$$

$$
R_4 = 1 - \frac{1}{2} \text{Re} (\epsilon_2^2 - \epsilon_1^2) + \text{Re} (a_2 \epsilon_2^2 - a_1 \epsilon_1^2) + \mathcal{O}(\epsilon_1^4, \epsilon_2^4)
$$

$$
= 1 + \frac{1}{2}[(\text{Im} \epsilon_2)^2 - (\text{Im} \epsilon_1)^2] + \text{Re} (a_2 \epsilon_2^2 - a_1 \epsilon_1^2) - \frac{1}{2} [(\text{Re} \epsilon_2)^2 - (\text{Re} \epsilon_1)^2]
$$

$$
+ \mathcal{O}(\epsilon_1^4, \epsilon_2^4). \quad (31)
$$

These two ratio are noticed to differ by second order U-spin breaking terms depending solely on $\text{Re} \epsilon_1$ and $\text{Re} \epsilon_2$:

$$
R_3 - R_4 = \frac{1}{2}[(\text{Re} \epsilon_2)^2 - (\text{Re} \epsilon_1)^2] + \mathcal{O}(\epsilon_1^4, \epsilon_2^4). \quad (32)
$$

Using

$$
(\text{Re} \epsilon_i)^2 = \frac{1}{4} \left( \sqrt{2R_i - 1} - 1 \right)^2 + 2 \text{Re} \epsilon_i \mathcal{O}(\epsilon_i^3) = \frac{1}{4} \left( \sqrt{2R_i - 1} - 1 \right)^2 + \mathcal{O}(\epsilon_i^4), \quad i = 1, 2, \quad (33)
$$

one obtains the the following nonlinear relation among the four ratios of amplitudes, which holds up to tiny fourth order U-spin breaking terms:

$$
\Delta R \equiv R_3 - R_4 + \frac{1}{8} \left[ \left( \sqrt{2R_1 - 1} - 1 \right)^2 - \left( \sqrt{2R_2 - 1} - 1 \right)^2 \right] = \mathcal{O}(\epsilon_1^4, \epsilon_2^4). \quad (34)
$$

This relation may also be obtained by expanding the identity (28) to first order in $[(R_{1,2} - 1)/(R_{1,2} + 1)]^2$ and applying (29) and (30). Fourth order terms in $(\text{Re} \epsilon_i)^2$ are proportional to $\text{Re} \epsilon_i$. It can be easily shown that this is not the case for terms of this order occurring in $R_3$, $R_4$ and in their difference. Therefore one concludes that fourth order terms do not cancel in $\Delta R$ and in any higher order expansion of the square root in (28). Thus, in hindsight, there is no way of obtaining a U-spin breaking relation of higher order than (34) and one does not need to go beyond third order in (26).

In our derivation of (34) we have used a parametrization of the CKM matrix up to terms which are fourth order in $\lambda \equiv V_{us} = 0.2253 \pm 0.008$ \cite{17}. We now study the effects of higher
order terms in $\lambda$ on $R_i$ and on this relation. Including $\lambda^4$ and $\lambda^5$ terms in the CKM matrix one has [28]

\begin{align*}
V_{ud} &= 1 - \frac{1}{2} \lambda^2 - \frac{1}{8} \lambda^4 , \\
V_{us} &= \lambda , \\
V_{cd} &= -\lambda + \frac{1}{2} A^2 \lambda^5 [1 - 2(\rho + i\eta)] , \\
V_{cs} &= 1 - \frac{1}{2} \lambda^2 - \frac{1}{8} \lambda^4 - \frac{1}{2} A^2 \lambda^4 ,
\end{align*}

(35)

where $A = 0.82 \pm 0.02, \rho = 0.12 \pm 0.02, \eta = 0.36 \pm 0.02$ [17].

We define a small parameter $\xi$ determining the effect of $\lambda^4$ and $\lambda^5$ terms on ratios of amplitudes:

$$1 + \xi \equiv \frac{1 - \frac{1}{2} A^2 \lambda^4 |1 - 2(\rho + i\eta)|}{1 - \frac{1}{2} A^2 \lambda^4} .$$

(36)

Using the above values of $\lambda, A, \rho$ and $\eta$ we find $|\xi| \lesssim 10^{-4}$. While $R_4$ is unaffected by $\xi$, $R_3$ and the two quadratic terms in (34) obtain corrections suppressed by $\xi$ and by first order U-spin breaking. These corrections, respectively $-\frac{1}{2} \xi (\text{Re} \, \epsilon_1 + \text{Re} \, \epsilon_2), +\frac{1}{2} \xi \text{Re} \, \epsilon_1$ and $+\frac{1}{2} \xi \text{Re} \, \epsilon_2$, cancel each other. The remaining corrections in (34), suppressed by $\xi$ and by second order U-spin breaking, are much below the level of $10^{-4}$ and may be safely neglected relative to tiny fourth order U-spin breaking terms of order $10^{-3}$ or $10^{-4}$.

Taking $\epsilon_i \sim 0.2$ as a typical value for first order U-spin breaking, the nonlinear relation $\Delta R = 0$ is expected to hold at a very high precision of order $\epsilon_i^4 \sim 10^{-3}$. We will confirm this prediction in Section VIII. At this high precision one cannot ignore first order isospin breaking terms, which one would generally assume to be around $(m_d - m_u) \Lambda_{\text{QCD}} \sim 10^{-2}$. We will study these terms in the next section showing that, in fact, they are suppressed by both isospin breaking parameters and by second order U-spin breaking.

VII First order isospin breaking

First order isospin breaking is introduced by multiplying the weak Hamiltonian by a $d - u$ spurion mass operator,

$$M_{\text{brk}} \propto (\bar{d} d - \bar{u} u) = \frac{1}{2} (\bar{d} d + \bar{s} s) - \bar{u} u + \frac{1}{2} (\bar{d} d - \bar{s} s) ,$$

(37)

transforming like a combination of a U-spin singlet and triplet. Isospin breaking contributions of the U-spin singlet operator in the four amplitudes (25) are identical when normalized by suitable CKM factors, and may be absorbed into the U-spin symmetric amplitude $A$. This is true also for $U = 0$ isospin breaking electromagnetic interactions because the $d$ and $s$ quarks have identical charges.

Contributions of the triplet operator in (37) follow the signs of first order U-spin breaking corrections. They are represented by two distinct first order isospin breaking parameters,
\( \delta_1 \) - for U-spin triplet states \( \pi^+K^- \) and \( K^+\pi^- \), and \( \delta_2 \) - for \( K^+K^- \) and \( \pi^+\pi^- \), the two component of a U-spin singlet state. Thus

\[
\begin{align*}
A(D^0 \to \pi^+K^-) &= \cos^2 \theta_C A(1 - \epsilon_1 + a_1 \epsilon_1^2 - a'_1 \epsilon_1^3 - \delta_1 + \ldots) , \\
A(D^0 \to K^+\pi^-) &= -\sin^2 \theta_C A(1 + \epsilon_1 + a_1 \epsilon_1^2 + a'_1 \epsilon_1^3 + \delta_1 + \ldots) , \\
A(D^0 \to \pi^+\pi^-) &= -\cos \theta_C \sin \theta_C A(1 - \epsilon_2 + a_2 \epsilon_2^2 + a'_2 \epsilon_2^3 - \delta_2 + \ldots) , \\
A(D^0 \to K^+K^-) &= \cos \theta_C \sin \theta_C A(1 + \epsilon_2 + a_2 \epsilon_2^2 + a'_2 \epsilon_2^3 + \delta_2 + \ldots) .
\end{align*}
\]

(38)

Instead of (26) we now expand:

\[
|1 \pm \epsilon + ae^2 + a' \epsilon^3 \pm \delta| = 1 \pm \text{Re} \epsilon + \frac{1}{2} \text{Im} \epsilon^2 + \text{Re}(ae^2) \pm \text{Re}(a' \epsilon^3) + \frac{1}{2} \text{Re} \epsilon \text{Im} \epsilon^2 \pm \text{Re} \delta + \text{Im} \delta \text{Im} \epsilon .
\]

(39)

This characteristic amplitude expansion includes two new terms, ±Re \( \delta \) and +Im \( \delta \) Im \( \epsilon \), the latter involving suppression by both isospin and U-spin breaking parameters. We will show that terms of this order do not affect the nonlinear relation (34).

The expansion of the four ratios of amplitudes now includes terms which are first order in isospin breaking and other terms suppressed by both isospin and U-spin breaking:

\[
\begin{align*}
R_1 &= 1 + 2[\text{Re} \epsilon_1 + (\text{Re} \epsilon_1)^2] + 2\text{Re} \delta_1 + 4\text{Re} \delta_1 \text{Re} \epsilon_1 + O(\epsilon_1^3) + O(\delta_1 \epsilon_1^2) , \\
R_2 &= 1 + 2[\text{Re} \epsilon_2 + (\text{Re} \epsilon_2)^2] + 2\text{Re} \delta_2 + 4\text{Re} \delta_2 \text{Re} \epsilon_2 + O(\epsilon_2^3) + O(\delta_2 \epsilon_2^2) ,
\end{align*}
\]

(40)

\[
\begin{align*}
R_3 &= 1 + \frac{1}{2}[(\text{Im} \epsilon_2)^2 - (\text{Im} \epsilon_1)^2] + \text{Re}(a_2 \epsilon_2^2 - a_1 \epsilon_1^2) + (\text{Im} \delta_2 \text{Im} \epsilon_2 - \text{Im} \delta_1 \text{Im} \epsilon_1) \\
&+ O(\epsilon_1^3) + O(\delta_1, \epsilon_1, \delta_2, \epsilon_2) , \\
R_4 &= 1 + \frac{1}{2}[(\text{Im} \epsilon_2)^2 - (\text{Im} \epsilon_1)^2] + \text{Re}(a_2 \epsilon_2^2 - a_1 \epsilon_1^2) + (\text{Im} \delta_2 \text{Im} \epsilon_2 - \text{Im} \delta_1 \text{Im} \epsilon_1) \\
&- \frac{1}{2}[(\text{Re} \epsilon_2)^2 - (\text{Re} \epsilon_1)^2] - (\text{Re} \delta_2 \text{Re} \epsilon_2 - \text{Re} \delta_1 \text{Re} \epsilon_1) + O(\epsilon_1^4) + O(\delta_1, \epsilon_1, \delta_2, \epsilon_2) .
\end{align*}
\]

(41)

Eqs. (40) imply for \( i = 1, 2 \)

\[
\text{Re} \epsilon_i = \frac{1}{2} \left( \sqrt{2R_i - 1} - 1 \right) - \text{Re} \delta_i - 2\text{Re} \delta_i \text{Re} \epsilon_i + O(\delta_i \epsilon_i) + O(\epsilon_i^3) ,
\]

(42)

or

\[
(\text{Re} \epsilon_i)^2 = \frac{1}{4} \left( \sqrt{2R_i - 1} - 1 \right)^2 - 2\text{Re} \delta_i \text{Re} \epsilon_i + O(\delta_i \epsilon_i^2) + O(\epsilon_i^4) .
\]

(43)

Eqs. (41) lead to

\[
R_3 - R_4 = \frac{1}{2} [(\text{Re} \epsilon_2)^2 - (\text{Re} \epsilon_1)^2] + (\text{Re} \delta_2 \text{Re} \epsilon_2 - \text{Re} \delta_1 \text{Re} \epsilon_1) + O(\epsilon_1^4) + O(\delta_1, \epsilon_1, \delta_2, \epsilon_2) .
\]

(44)
Consequently isospin breaking terms of the form $\text{Re} \delta_2 \text{Re} \epsilon_2 - \text{Re} \delta_1 \text{Re} \epsilon_1$ cancel in $\Delta R$:

$$\Delta R \equiv R_3 - R_4 + \frac{1}{8} \left[ (\sqrt{2R_1} - 1 - 1)^2 - (\sqrt{2R_2} - 1 - 1)^2 \right] = \mathcal{O}(\epsilon_1^4, \epsilon_2^4) + \mathcal{O}(\delta_1 \epsilon_1^2, \delta_2 \epsilon_2^2).$$  \tag{45}$$

It is remarkable that isospin breaking modifies the nonlinear relation (34) by terms which are suppressed by both first order isospin breaking and second order U-spin breaking factors. Taking $\delta_i \sim 10^{-2}$, $\epsilon_i \sim 0.2 - 0.3$, these terms are expected to be at most $10^{-3}$, similar in magnitude to fourth order U-spin breaking terms affecting this relation.

**VIII Experimental tests in $D^0 \to P^+ P^-$**

We will now apply current experimental data to study the hierarchy among U-spin breaking terms of increasing order. Our final goal is testing the predicted amplitude relation (45). Hadronic decay amplitudes ($A$) are obtained from measured branching ratios ($B$) by eliminating phase space factors depending on final particles center-of-mass 3-momenta ($p$), and on the $D$ meson mass and its lifetime ($M_D$ and $\tau_D$),

$$|A| = M_D \sqrt{\frac{8\pi B}{\tau_D p}}.$$  \tag{46}

In our calculation of amplitudes we will disregard a common factor $M_D \sqrt{8\pi/\tau_D}$ which cancels in the four ratios $R_i$. Values for measured branching ratios \[17\], center-of-mass momenta, and amplitudes defined in this manner are quoted in Tables I.

Note that all four amplitudes include a factor $B_{\pi K}^{1/2}$ corresponding to the branching fraction of the Cabibbo-favored decay $D^0 \to \pi^+ K^-$. We have included no error in the amplitude for this process as the other three branching ratios (including errors) have been measured relative to this process \[17\]. The relative errors in the amplitudes of these three processes are all below the level of one percent. We will assume no correlation between these errors, which have been measured in three independent analyses for different final states. The high precision achieved recently by the CDF, LHCb and Belle collaborations in measuring the DCS amplitude is remarkable \[29\], as it required time-dependent separation between this highly suppressed decay and $D^0 - D^0$ mixing followed by the CF decay.

Using values of amplitudes given in Tables I and $\tan \theta_C = 0.23125 \pm 0.00082$ \[17\] we calculate the four ratios $R_i$ defined in Eq. (27),

$$R_1 = 1.115 \pm 0.012,$$
$$R_2 = 1.811 \pm 0.020,$$
$$R_3 = 1.052 \pm 0.008,$$
$$R_4 = 1.008 \pm 0.007.$$  \tag{47}

Errors in the ratios have been obtained by adding in quadrature errors in the relevant amplitudes.
Table I: Branching fractions and amplitudes for $D^0 \to P^+ P^-$ decays [17]

| Decay mode | Branching fraction | $p$ (GeV/c) | $|A| = \sqrt{B/p}$ (GeV/c)$^{-1/2}$ |
|-------------|--------------------|-------------|----------------------------------|
| $D^0 \to \pi^+ K^-$ | $\mathcal{B}_{\pi K} = (3.88 \pm 0.05) \times 10^{-2}$ | 0.861 | $1.078 \mathcal{B}_{\pi K}^{1/2}$ |
| $D^0 \to K^+ \pi^-$ | $(3.56 \pm 0.06) \times 10^{-3} \mathcal{B}_{\pi K}$ | 0.861 | $(0.06430 \pm 0.00054) \mathcal{B}_{\pi K}^{1/2}$ |
| $D^0 \to \pi^+ \pi^-$ | $(3.59 \pm 0.06) \times 10^{-2} \mathcal{B}_{\pi K}$ | 0.922 | $(0.1973 \pm 0.0016) \mathcal{B}_{\pi K}^{1/2}$ |
| $D^0 \to K^+ K^-$ | $(10.10 \pm 0.16) \times 10^{-2} \mathcal{B}_{\pi K}$ | 0.791 | $(0.3573 \pm 0.0028) \mathcal{B}_{\pi K}^{1/2}$ |

We have seen that the first two ratios involve first order U-spin breaking terms. These terms, depending on two distinct U-spin breaking parameters $\epsilon_1$ and $\epsilon_2$, are considerably larger in $R_2$ than in $R_1$. This has been anticipated in the discussion below Eq. (21). Using (42), in which we neglect first order isospin breaking and third order U-spin breaking, we calculate reasonably small U-spin breaking parameters,

$$\text{Re} \, \epsilon_1 = 0.054 \pm 0.005,$$

$$\text{Re} \, \epsilon_2 = 0.310 \pm 0.006.$$  \hspace{1cm} (48)

The other two ratios, $R_3$ and $R_4$, given in (41) in terms of $\epsilon_{1,2}$ and coefficients $a_{1,2}$ of order one, deviate from one by second order U-spin breaking terms (we neglect terms suppressed by both isospin and U-spin breaking and fourth order terms in U-spin breaking),

$$R_3 - 1 = \frac{1}{2} [(\text{Im} \, \epsilon_2)^2 - (\text{Im} \, \epsilon_1)^2] + \text{Re} \,(a_2 \epsilon_2^2 - a_1 \epsilon_1^2) = 0.052 \pm 0.08,$$

$$R_4 - 1 = \text{Re} \, [(a_2 - \frac{1}{2}) \epsilon_2^2 - (a_1 - \frac{1}{2}) \epsilon_1^2] = 0.008 \pm 0.007.$$  \hspace{1cm} (49)

The hierarchy between the first order parameters in (48) and the second order terms calculated in (49) confirms and justifies the perturbative approach we have applied in this study to U-spin breaking. Without having a method for calculating the nonperturbative coefficients $a_i$, the almost exact cancellation of second order terms in $R_4$ seems to be accidental. In view of the small value of $\text{Re} \, \epsilon_1$ and the much larger value of $\text{Re} \, \epsilon_2$ this approximate cancellation seems to imply $a_2 \simeq 1/2$.

Having shown that second order U-spin breaking terms are a few percent, one expects fourth order terms to be of order $10^{-3}$, comparable in magnitude or larger than terms which are first order in isospin breaking and second order in U-spin breaking. Let us now check this prediction in the relation (45) which contains on the right-hand side terms of these two kinds. Using the definitions of $R_i$ in (27), and adding in quadrature errors in amplitudes [rather than errors in $R_i$ given in (47)], we obtain

$$\Delta R = (-3.2 \pm 0.4) \times 10^{-3}.$$  \hspace{1cm} (50)

This confirms our prediction.
IX  Experimental tests in $D^0 \to V^+ P^-$

As mentioned, the discussion in Sections III - VII applies also to three other classes of processes involving one or two charged vector mesons, $D^0 \to V^+ P^-, D^0 \to P^+ V^-$ and $D^0 \to V^+ V^-$. In particular, a nonlinear vector relation similar to (45) holds in each one of these classes with a precision depending on the size of U-spin breaking. In this section we summarize concisely the situation relevant to this question in $D^0 \to V^+ P^-$, consisting of the four processes, $D^0 \to \rho^+ \ K^-, D^0 \to K^{*+} \pi^-, D^0 \to \rho^+ \pi^-$ and $D^0 \to K^{*+} K^-$. We denote first order U-spin breaking and isospin breaking parameters in these processes by $\epsilon'_{1,2}$ and $\delta'_{1,2}$, respectively, in analogy to $\epsilon_{1,2}$ and $\delta_{1,2}$ in $D^0 \to P^+ P^-$. 

Defining four ratios of amplitudes $R'_i$ in analogy with (27),

\[
\begin{align*}
R'_1 & \equiv \frac{|A(D^0 \to K^{*+} \pi^-)|}{|A(D^0 \to \rho^+ K^-)| \tan^2 \theta_C}, \\
R'_2 & \equiv \frac{|A(D^0 \to K^{*+} K^-)|}{|A(D^0 \to \rho^+ \pi^-)|}, \\
R'_3 & \equiv \frac{|A(D^0 \to K^{*+} K^-)| + |A(D^0 \to \rho^+ \pi^-)|}{|A(D^0 \to \rho^+ K^-)| \tan \theta_C + |A(D^0 \to K^{*+} \pi^-)| \tan^{-1} \theta_C}, \\
R'_4 & \equiv \sqrt{\frac{|A(D^0 \to K^{*+} K^-)||A(D^0 \to \rho^+ \pi^-)|}{|A(D^0 \to \rho^+ K^-)||A(D^0 \to K^{*+} \pi^-)|}},
\end{align*}
\]

one obtains a sum rule analogous to (45):

\[
\Delta R' \equiv R'_3 - R'_4 + \frac{1}{8} \left[ (\sqrt{2R'_1 - 1} - 1)^2 - (\sqrt{2R'_2 - 1} - 1)^2 \right] = O(\epsilon_1^{14}, \epsilon_2^{14}) + O(\delta_1^{12}, \delta_2^{12}) .
\]

Magnitudes of amplitudes for the P-wave decays $D^0 \to V^+ P^-$ are obtained from corresponding branching ratios using

\[
|A| = M_D \sqrt{\frac{8\pi B}{\tau_D p^3}} .
\]

We will disregard again the factor $M_D \sqrt{8\pi/\tau_D}$ since we are only concerned with ratios of amplitudes. Values for measured branching ratios $\mathbb{I}$, center-of-mass momenta, and amplitudes defined in this manner are quoted in Tables II. Relative errors in CF and SCS amplitudes are reasonably small, between two and three percent. In contrast, the relative error in the DCS amplitude $|A(D^0 \to K^{*+} \pi^-)|$, obtained by the CLEO and BaBar collaborations through Dalitz plot analyses of $D^0 \to K_S \pi^+ \pi^-$ $\mathbb{II}$, is quite large, $+26\%$. This large asymmetric error limits considerably the precision of $R'_1, R'_3$ and $R'_4$. Adding in quadrature errors in relevant amplitudes, measured independently for different three-body final states, we calculate

\[
R'_1 = 0.971^{+0.257}_{-0.148} ,
\]
Table II: Branching fractions and amplitudes for \( D^0 \to V^+ P^- \) decays \[17\]

| Decay mode | Branching fraction | \( p \) (GeV/c) | \( |A| = \sqrt{\mathcal{B}/p^2} \) (GeV/c)\(^{-3/2} \) |
|------------|-------------------|-----------------|---------------------------------|
| \( D^0 \to \rho^+ K^- \) | 0.108 ± 0.007 | 0.675 | 0.593 ± 0.019 |
| \( D^0 \to K^{*+} \pi^- \) | \((3.42^{+1.80}_{-1.02})\times10^{-4}\) | 0.711 | \(0.0308^{+0.0081}_{-0.0046}\) |
| \( D^0 \to \rho^+ \pi^- \) | \((9.8 \pm 0.4)\times10^{-3}\) | 0.764 | 0.148 ± 0.003 |
| \( D^0 \to K^{*+} K^- \) | \((4.38 \pm 0.21)\times10^{-3}\) | 0.610 | 0.1389 ± 0.0033 |

\[
\begin{align*}
R_2' &= 0.939 \pm 0.029 , \\
R_3' &= 1.061^{+0.082}_{-0.140} , \\
R_4' &= 1.061^{+0.083}_{-0.142} .
\end{align*}
\] (54)

Expansions similar to \[40\] and \[41\] apply to these four ratios in terms of \( \epsilon_{1,2}' \) and \( \delta_{1,2}' \).

The leading U-spin breaking corrections in \( R_2' \) and \( R_3' \) are first order, while in \( R_3' \) and \( R_4' \) they are second order. The measured values of the first two ratios imply

\[
\begin{align*}
\text{Re } \epsilon_1' &= \frac{1}{2} \left( \sqrt{2R_1'} - 1 - 1 \right) = -0.015^{+0.118}_{-0.083} , \\
\text{Re } \epsilon_2' &= \frac{1}{2} \left( \sqrt{2R_2'} - 1 - 1 \right) = -0.032 \pm 0.016 .
\end{align*}
\] (55)

We note that the numerical value of \( \text{Re } \epsilon_2' \) is smaller by about an order of magnitude than the value of \( \text{Re } \epsilon_2 \) calculated in \[48\]. That is, first order U-spin breaking in SCS \( D^0 \to V^+ P^- \) decays is about an order of magnitude smaller than in corresponding \( D^0 \to P^+ P^- \) decays. This seems to imply that an enhancement of the U-spin breaking penguin amplitude suggested to occur in \( D^0 \to P^+ P^- \) \[19\] is not at work in \( D^0 \to V^+ P^- \). The other U-spin breaking parameter, \( \text{Re } \epsilon_1' \), does not involve a penguin amplitude. In spite of the current large error in this parameter the first of Eqs. (55) favors strongly \( |\text{Re } \epsilon_1'| \leq 0.1 \), suggesting that a value close to that measured for \( |\text{Re } \epsilon_1| \) is not unlikely.

These values of \( \text{Re } \epsilon_{1,2}' \) imply that typical second order U-spin breaking terms in \( R_{3,4}' \) are around one percent. Confirming this prediction and obtaining a more precise value for \( \text{Re } \epsilon_1' \) requires a substantial improvement in the measurement of \( \mathcal{B}(D^0 \to K^{*+} \pi^-) \). In the meantime we use the values measured for \( |\text{Re } \epsilon_{1,2}'| \) to argue that fourth order U-spin breaking terms should be around \( 10^{-4} \). This is also expected to be the magnitude of terms in \( \Delta R' \) suppressed by both isospin breaking and by second order U-spin breaking. Thus, due to smaller U-spin breaking parameters in \( D^0 \to V^+ P^- \) relative to \( D^0 \to P^+ P^- \) one predicts \( \Delta R' \) to be about an order of magnitude smaller than \( \Delta R \). Using the amplitudes in Table \[17\] we calculate

\[
\Delta R' = (0.2^{+3.2}_{-0.5}) \times 10^{-4} ,
\] (56)

where the error is dominated by the error in \( |A(D^0 \to K^{*+} \pi^-)| \). This confirms our prediction. A more precise test could be achieved by improving the measurement of the branching ratio for this process.
fraction of ratios (3.18 ± 0.29) × 10\(^{-4}\) and (2.97 ± 0.19) × 10\(^{-4}\), respectively. The \(D^0 \rightarrow K^+\pi^0\) events involve interference of DCS decays with \(D^0\overline{D}^0\) mixing followed by CF decays. Evidence for \(D^0\overline{D}^0\) mixing at 3.2 standard deviation was presented by Babar [33], measuring the fraction of \(K^+\rho^\circ\) in these events to be (39.8 ± 6.5)%). (No interference would have implied \(\mathcal{B}(D^0 \rightarrow K^+\rho^-) \sim 1.2 \times 10^{-4}\).) More work is needed for resolving the effect of \(D^0\overline{D}^0\) mixing on these events, and for obtaining a solid measurement of \(\mathcal{B}(D^0 \rightarrow K^+\rho^-)\).

We define ratios of amplitudes \(R''_i\) in analogy with (27),

\[
\begin{align*}
R''_1 & \equiv \frac{|A(D^0 \rightarrow K^+\rho^-)|}{|A(D^0 \rightarrow \pi^+K^*)|}\frac{1}{\tan^2 \theta_C}, \\
R''_2 & \equiv \frac{|A(D^0 \rightarrow K^+K^-)|}{|A(D^0 \rightarrow \pi^+\rho^-)|}, \\
R''_3 & \equiv \frac{|A(D^0 \rightarrow K^+K^-)| + |A(D^0 \rightarrow \pi^+\rho^-)|}{|A(D^0 \rightarrow \pi^+K^*)|\tan \theta_C + |A(D^0 \rightarrow K^+\rho^-)|\tan^{-1} \theta_C}, \\
R''_4 & \equiv \sqrt{\frac{|A(D^0 \rightarrow K^+K^-)||A(D^0 \rightarrow \pi^+\rho^-)|}{|A(D^0 \rightarrow \pi^+K^*)||A(D^0 \rightarrow K^+\rho^-)|}}.
\end{align*}
\]

We denote first order U-spin breaking and isospin breaking parameters in these amplitudes by \(\epsilon''_{1,2}\) and \(\delta''_{1,2}\), respectively.

In the absence of a solid measurement of \(|A(D^0 \rightarrow K^+\rho^-)|\) one can only calculate \(R''_2\). Neglecting isospin breaking and third order U-spin breaking one obtains

\[
R''_2 = 1 + 2 \left[ \text{Re} \epsilon''_2 + (\text{Re} \epsilon''_2)^2 \right] = 0.786 \pm 0.036,
\]

which implies

\[
\text{Re} \epsilon''_2 = -0.122 \pm 0.024.
\]
That is, the magnitude of the U-spin breaking parameter in SCS $D^0 \to P^+ V^-$ decays is intermediary between corresponding parameters in $D^0 \to P^+ P^-$ (Re $\epsilon_2 = 0.310 \pm 0.006$) and $D^0 \to V^+ P^-$ (Re $\epsilon'_2 = -0.032 \pm 0.016$). Namely, no significant U-spin breaking penguin enhancement applies to $D^0 \to P^+ V^-$. One expects a similar or smaller magnitude for Re $\epsilon''_2$.

The two ratios $R''_3$ and $R''_4$ deviate from one by second order U-spin breaking terms [see Eqs. (31)] which are expected to be at most a few percent. Using $R''_3 = 1 \pm 0.05$, where we include a conservative uncertainty of 5% due to second order U-spin breaking corrections, one obtains the following prediction for $B(D^0 \to K^+ \rho^-)$ [34],

$$|A(D^0 \to K^+ \rho^-)| = 0.0237 \pm 0.0025 \implies B(D^0 \to K^+ \rho^-) = (1.73 \pm 0.36) \times 10^{-4} \ . \ (60)$$

For comparison, assuming $R''_4 = 1 \pm 0.05$ implies a very similar prediction,

$$|A(D^0 \to K^+ \rho^-)| = 0.0235 \pm 0.0028 \implies B(D^0 \to K^+ \rho^-) = (1.70 \pm 0.40) \times 10^{-4} \ . \ (61)$$

This value of $|A(D^0 \to K^+ \rho^-)|$ would imply Re $\epsilon''_2 = 0.08 \pm 0.06$, comparable in magnitude to Re $\epsilon'_2$ and in agreement with expectation.

Taking for $|A(D^0 \to K^+ \rho^-)|$ the value in (61), using the three measured amplitudes quoted in Table III, and assuming no error correlation between the four amplitudes, one obtains

$$\Delta R'' \equiv R''_3 - R''_4 + \frac{1}{8} \left[ \left( \sqrt{2R''_1 - 1} - 1 \right)^2 - \left( \sqrt{2R''_2 - 1} - 1 \right)^2 \right] = (0.8^{+0.2}_{-0.6}) \times 10^{-4} \ . \ (62)$$

This value, which is similar to (50), is in agreement with our prediction that, in view of the above values of Re $\epsilon''_{1,2}$, fourth order U-spin breaking terms and isospin breaking terms suppressed by second order U-spin breaking should be of order $10^{-4}$. The positive error in $\Delta R''$ is dominated by the uncertainty assumed in the unmeasured DCS amplitude. One still awaits a solid measurement of $B(D^0 \to K^+ \rho^-)$ which would test the prediction (61). The larger negative error in $\Delta R''$, originating in errors on the three measured amplitudes in Table III, may be reduced by improving the corresponding branching ratio measurements.

**XI \quad D^0 \text{ decays to pairs of neutral pseudoscalars**

In the U-spin symmetry limit one obtains simple amplitude relations for $D^0$ decays to pairs of light neutral pseudoscalar mesons [35]. We will go through the symmetry argument first, extending it to include U-spin breaking at arbitrary order in CF and DCS decays. We will then demonstrate a few amplitude relations which hold up to second order U-spin breaking. Other relations of this kind have been studied in Ref. [8] in the framework of flavor SU(3).

In the symmetry limit one neglects $\eta - \eta'$ mixing which is due to first order U-spin breaking represented by the spurion mass operator $M_{Ubrk}$. Thus, we will write amplitudes for $\eta = \eta_8$ in our discussion of the symmetry limit, while $\eta_8$ will be used explicitly when introducing U-spin breaking. When discussing amplitudes, rates and asymmetries for $\eta_8$ we will assume (as has been assumed in Ref. [8]) knowledge of amplitudes including a relative phase for $\eta$.
and $\eta'$ and favored values of the mixing angle. Taking $\eta = \eta_8 \equiv (2s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{6}$, the following superpositions of single neutral particle states belong to a U-spin triplet,

$$|K^0\rangle \equiv |d\bar{s}\rangle = |1,1\rangle , \quad |\bar{K}^0\rangle \equiv |s\bar{d}\rangle = -|1,-1\rangle , \quad \frac{1}{2}(\sqrt{3}|\eta\rangle - |\pi^0\rangle) \equiv |s\bar{s} - d\bar{d}\rangle/\sqrt{2} = |1,0\rangle ,$$

while the orthogonal U-spin singlet is

$$\frac{1}{2}(|\eta\rangle + \sqrt{3}|\pi^0\rangle) \equiv |s\bar{s} + d\bar{d} - 2u\bar{u}\rangle/\sqrt{6} = |0,0\rangle .$$

Two-particle states in an S-wave are obtained by symmetrizing products of single particle states.

Symmetrized products of two $U = 1$ single-particle states consist of $U = 0$ and $U = 2$ (with $U_3 = \pm 1,0$) states, while the product of two $U = 0$ states is pure $U = 0$. $D^0$ decay matrix elements of the $U = 1$ weak Hamiltonian vanish for each one of these five states. This implies the following five U-spin symmetry relations: (For short notation we denote amplitudes by their final states.)

$$\sqrt{3}A^{(0)}(K^0\eta) - A^{(0)}(K^0\pi^0) = 0 ,$$
$$\sqrt{3}A^{(0)}(\bar{K}^0\eta) - A^{(0)}(\bar{K}^0\pi^0) = 0 ,$$
$$A^{(0)}(K^0\bar{K}^0) = 0 ,$$
$$A^{(0)}(\eta\eta) + A^{(0)}(\pi^0\pi^0) = 0 ,$$
$$\sqrt{3}A^{(0)}(\eta\pi^0) + \sqrt{2}A^{(0)}(\pi^0\pi^0) = 0 .$$

Hadronic matrix elements for symmetrized products of $U = 1, U_3 = \pm 1,0$ and $U = 0$ states are then given by a single $U = 1$ amplitude $A$:

$$\sqrt{3}A^{(0)}(K^0\eta) = A^{(0)}(K^0\pi^0) = -\sin^2 \theta_C A ,$$
$$\sqrt{3}A^{(0)}(\bar{K}^0\eta) = A^{(0)}(\bar{K}^0\pi^0) = \cos^2 \theta_C A ,$$
$$\sqrt{3}A^{(0)}(\eta\pi^0) = \sqrt{2}A^{(0)}(\eta\eta) = -\sqrt{2}A^{(0)}(\pi^0\pi^0) = \sqrt{2}\cos \theta_C \sin \theta_C A .$$

Note that since we symmetrized final states also for identical particles, corresponding amplitudes have been divided by $\sqrt{2}$ in order that their squares give decay rates.

The above amplitude expressions are analogous to the U-spin symmetry expressions for decays into pairs of charged pseudoscalar mesons. Some of these relations follow merely from $d \leftrightarrow s$ symmetry. Since the $U = 1$ and $U = 0$ superpositions of $\pi^0$ and $\eta$ states in \((63)\) and \((64)\) are respectively antisymmetric and symmetric with respect to $d \leftrightarrow s$ reflection one has

$$\langle \bar{K}^0(\sqrt{3}\eta - \pi^0)|(\bar{s}\bar{c})(\bar{u}\bar{d})|D^0\rangle = -\langle K^0(\sqrt{3}\eta - \pi^0)|(\bar{d}\bar{c})(\bar{u}s)|D^0\rangle ,$$
$$\langle \bar{K}^0(\eta + \sqrt{3}\pi^0)|(\bar{s}\bar{c})(\bar{u}\bar{d})|D^0\rangle = \langle K^0(\eta + \sqrt{3}\pi^0)|(\bar{d}\bar{c})(\bar{u}s)|D^0\rangle ,$$

\[(69)\]
implying
\[
\frac{\sqrt{3}A^{(0)}(\bar{K}^0\eta) - A^{(0)}(\bar{K}^0\pi^0)}{\cos^2\theta_C} = \frac{\sqrt{3}A^{(0)}(K^0\eta) - A^{(0)}(K^0\pi^0)}{\sin^2\theta_C} = 0 ,
\]
\[
\frac{A^{(0)}(\bar{K}^0\eta) + \sqrt{3}A^{(0)}(\bar{K}^0\pi^0)}{\cos^2\theta_C} = \frac{A^{(0)}(K^0\eta) + \sqrt{3}A^{(0)}(K^0\pi^0)}{\sin^2\theta_C} = \frac{4}{\sqrt{3}} A .
\] (70)

The right-hand sides follow using (66) and (67) based on the full SU(2) structure of U-spin.

U-spin breaking of order \(n\) in CF and DCS amplitudes is introduced by multiplying transition operators or final states by a total of \(n\) powers of the \(s-d\) spurion mass operator \(M_{\text{Urbrk}}\) which changes sign under \(d \leftrightarrow s\). Consequently one has
\[
\frac{\sqrt{3}A^{(n)}(\bar{K}^0\eta_8) - A^{(n)}(\bar{K}^0\pi^0)}{\cos^2\theta_C} = (-1)^n \frac{\sqrt{3}A^{(n)}(K^0\eta_8) - A^{(n)}(K^0\pi^0)}{\sin^2\theta_C} ,
\]
\[
\frac{A^{(n)}(\bar{K}^0\eta_8) + \sqrt{3}A^{(n)}(\bar{K}^0\pi^0)}{\cos^2\theta_C} = (-1)^n \frac{A^{(n)}(K^0\eta_8) + \sqrt{3}A^{(n)}(K^0\pi^0)}{-\sin^2\theta_C} .
\] (71)

Denoting first order U-spin breaking parameters in these two pairs of processes by \(\epsilon_0\) and \(\epsilon'_0\), one may expand the above four linear combinations of amplitudes to arbitrary order,
\[
\sqrt{3}A(\bar{K}^0\eta_8) - A(\bar{K}^0\pi^0) = \cos^2\theta_C A[\epsilon_0 - a_0\epsilon^2_0 + ...] ,
\]
\[
\sqrt{3}A(K^0\eta_8) - A(K^0\pi^0) = -\sin^2\theta_C A[\epsilon_0 + a_0\epsilon^2_0 + ...] ,
\] (72)
\[
\frac{\sqrt{3}}{4}[A(\bar{K}^0\eta_8) + \sqrt{3}A(\bar{K}^0\pi^0)] = \cos^2\theta_C A[1 - \epsilon'_0 + a'_0\epsilon^2_0 + ...] ,
\]
\[
\frac{\sqrt{3}}{4}[A(K^0\eta_8) + \sqrt{3}A(K^0\pi^0)] = -\sin^2\theta_C A[1 + \epsilon'_0 + a'_0\epsilon^2_0 + ...] .
\] (73)

where \(|a_0| \sim |a'_0| \sim 1\).

Eqs. (72) imply a linear amplitude relation in which first order U-spin breaking terms cancel,
\[
[\sqrt{3}A(\bar{K}^0\eta_8) - A(\bar{K}^0\pi^0)]\tan^2\theta_C + \sqrt{3}A(K^0\eta_8) - A(K^0\pi^0) = 0 .
\] (74)

This relation has also been obtained using a general first order SU(3) breaking expansion [8], in which a dozen SU(3) breaking parameters contributing to these processes cancel in this relation. We note that, while it follows from Eqs. (73) that the linear relation
\[
[A(\bar{K}^0\eta_8) + \sqrt{3}A(\bar{K}^0\pi^0)]\tan^2\theta_C - [A(K^0\eta_8) + \sqrt{3}A(K^0\pi^0)] = \frac{4}{\sqrt{3}} \sin^2\theta_C A
\] (75)

is also free of first order U-spin breaking, the U-spin invariant amplitude \(A\) on the right-hand side is not necessarily invariant under the full flavor SU(3) group when including other states. This is just like the amplitude \(A\) defined above (7) for decays to charged particles.

The four amplitudes for two body decays involving \(K^0\) or \(\bar{K}^0\) and \(\pi^0\) or \(\eta_8\) have the following first order expansions:
\[
A(\bar{K}^0\pi^0) = \cos^2\theta_C A\left[1 - \epsilon'_0 - \frac{1}{4}\epsilon_0 + \mathcal{O}(\epsilon_0^2, \epsilon'_0^2)\right] ,
\]
\[
A(K^0\eta_8) = -\sin^2\theta_C A\left[1 + \epsilon'_0 + \frac{1}{4}\epsilon_0 + \mathcal{O}(\epsilon_0^2, \epsilon'_0^2)\right] ,
\]
\[ \sqrt{3} A(K^0 \eta_8) = \cos^2 \theta_C A \left[ 1 - \epsilon_0' + \frac{3}{4} \epsilon_0 + O(\epsilon_0'^2, \epsilon_0^2) \right] , \]

\[ A(K^0 \pi^0) = -\sin^2 \theta_C A \left[ 1 + \epsilon_0' - \frac{1}{4} \epsilon_0 + O(\epsilon_0'^2, \epsilon_0^2) \right] , \]

\[ \sqrt{3} A(K^0 \eta_8) = -\sin^2 \theta_C A \left[ 1 + \epsilon_0' + \frac{3}{4} \epsilon_0 + O(\epsilon_0'^2, \epsilon_0^2) \right] . \] (76)

This implies that the two ratios of amplitudes in (66) and (67), which are equal in the U-spin symmetry limit, are also equal when including first order U-spin breaking:

\[ \frac{\sqrt{3}|A(K^0 \eta_8)|}{|A(K^0 \pi^0)|} = \frac{\sqrt{3}|A(K^0 \eta_8)|}{|A(K^0 \pi^0)|} = 1 + \text{Re} \epsilon_0 + O(\epsilon_0, \epsilon_0^2) , \] (77)

\[ \frac{|A(K^0 \eta_8)|}{|A(K^0 \pi^0)| \tan^2 \theta_C} = \frac{|A(K^0 \eta_8)|}{|A(K^0 \eta_8)| \tan^2 \theta_C} = 1 + 2\text{Re} \epsilon_0' + O(\epsilon_0'^2, \epsilon_0^2) . \] (78)

Branching ratio measurements of the above two DCS decay modes are not feasible because a final state neutral kaon is identified in a $K_S$ or a $K_L$ state. This involves an interference between CF and DCS decays. A method for measuring this interference has been proposed in Ref. [36]. Defining a rate asymmetry between decays involving $K_S^0$ and $K_L^0$,

\[ R(D^0, M^0) \equiv \frac{\Gamma(D^0 \to K_S^0 M^0) - \Gamma(D^0 \to K_L^0 M^0)}{\Gamma(D^0 \to K_S^0 M^0) + \Gamma(D^0 \to K_L^0 M^0)} , \quad (M^0 = \pi, \eta, \eta') \] (79)

one obtains, to leading order in the ratio of DCS and CF amplitudes,

\[ R(D^0, M^0) = -\frac{2\text{Re}[A(K^0 M^0)A^*(K^0 M^0)]}{|A(K^0 M^0)|^2} . \] (80)

Eqs. (70) predict equal asymmetries for $M^0 = \pi$ and $M^0 = \eta_8$ up to second order U-spin breaking. The two asymmetries are given by

\[ R(D^0, \eta_8) = R(D^0, \pi^0) = 2\tan^2 \theta_C \left[ 1 + 2\text{Re} \epsilon_0' + O(\epsilon_0'^2, \epsilon_0^2) \right] . \] (81)

Comparing Eqs. (78) and (81) we find

\[ \frac{|A(K^0 M^0)|}{|A(K^0 M^0)|} = \frac{1}{2} R(D^0, M^0) \left[ 1 + O(\epsilon_0'^2, \epsilon_0^2) \right] , \quad (M^0 = \pi^0, \eta_8) . \] (82)

That is, although the branching fractions for DCS decays $D^0 \to K^0 \pi^0$ and $D^0 \to K^0 \eta_8$ cannot be measured directly, they may be obtained up to second order U-spin breaking corrections from corresponding CF branching fractions and $K_S^0 - K_L^0$ asymmetries:

\[ B(D^0 \to K^0 M^0) = \frac{1}{4}[R(D^0, M^0)|^2 B(D^0 \to \bar{K}^0 M^0) \left[ 1 + O(\epsilon_0'^2, \epsilon_0^2) \right] , \quad (M^0 = \pi, \eta_8) . \] (83)

Table [IV] summarizes current relevant information on branching ratios and amplitudes for $D^0$ decays into pairs of neutral pseudoscalars. We do not include the $\eta \eta$ mode (and decays involving the $\eta'$), as $D^0 \to \eta_8 \eta_8$ would include $D^0 \to \eta' \eta'$ which has zero phase space.
Table IV: Branching fractions and amplitudes for $D^0$ decays to pairs of neutral pseudoscalar mesons $[17]$

| Decay mode | Branching fraction | $p$ (GeV/c) | $|A| = \sqrt{B/p}$ (GeV/c)^{-1/2} |
|------------|-------------------|--------------|----------------------------------|
| $D^0 \to K^0_S \pi^0$ | $(1.19 \pm 0.04) \times 10^{-2}$ | 0.860 | -- |
| $D^0 \to K^0_L \pi^0$ | $(1.00 \pm 0.07) \times 10^{-2}$ | 0.860 | -- |
| $D^0 \to \bar{K}^0 \pi^0$ | $(2.29 \pm 0.07) \times 10^{-2}$ | 0.860 | $0.163 \pm 0.002$ |
| $D^0 \to \pi^0 \pi^0$ | $(8.20 \pm 0.35) \times 10^{-4}$ | 0.923 | $0.0298 \pm 0.0006$ |
| $D^0 \to \eta \pi^0$ | $(6.8 \pm 0.7) \times 10^{-4}$ | 0.846 | $0.0284 \pm 0.0014$ |

$a$ Branching ratio calculated as twice the average of $B(D^0 \to K^0_S \pi^0)$ and $B(D^0 \to K^0_L \pi^0)$.

An estimate of U-spin breaking is given by a ratio of SCS and CF decay amplitudes which equals one in the symmetry limit [see (67) (68)],

$$\frac{|A(\pi^0 \pi^0)|}{|A(K^0 \pi^0)| \tan \theta_C} - 1 = -0.21 \pm 0.02 \ , \quad (84)$$

Another quantity measuring U-spin breaking is

$$\frac{\sqrt{3}|A(\bar{K}^0 \eta)|}{|A(K^0 \pi^0)|} - 1 = 0.14 \pm 0.05 \ . \quad (85)$$

In order to determine $Re \epsilon_0$ from (77) one would have to know also $|A(\bar{K}^0 \eta')|$ and the relative strong phase between this amplitude and $A(\bar{K}^0 \eta)$, using a favored value for the $\eta - \eta'$ mixing angle $[37]$. The other U-spin breaking parameter, $Re \epsilon'_0$, is obtained from (81) using a $K^0_S - K^0_L$ asymmetry measurement by the CLEO collaboration $[38]$, $R(D^0, \pi^0) = 0.108 \pm 0.035$,

$$Re \epsilon'_0 = \frac{R(D^0, \pi^0) - 2 \tan^2 \theta_C}{4 \tan^2 \theta_C} = 0.00 \pm 0.16 \ . \quad (86)$$

Arguments favoring small U-spin breaking in the asymmetries $R(D^0, M^0)$ have been presented in Ref. $[39]$ adopting a diagrammatic flavor SU(3) approach.

The numerical values of $Re \epsilon'_0$ in (86) and of the above other two measured U-spin breaking quantities imply that second order U-spin breaking terms in Eqs. (77) – (83) are at most of order several percent. Neglecting these contributions involves an approximation that is quantitatively similar to the one used to obtain (80), where terms which are second order in the ratio of DCS and CF amplitudes have been neglected.

**XII Conclusion**

We described a new approach to hadronic $D^0$ decay amplitudes applying a perturbative expansion in U-spin breaking parameters and treating isospin breaking carefully. We have
identified a class of two-body and quasi two-body decays involving charged pseudoscalars ($P$) and vector mesons ($V$), for which in each case adequate hierarchies have been shown to occur between U-spin breaking terms of increasing order.

Nonlinear amplitude relations were predicted in each one of three cases, $D^0 \rightarrow P^+P^-$, $D^0 \rightarrow V^+P^-$ and $D^0 \rightarrow P^+V^-$, which hold up to fourth order U-spin breaking and isospin breaking terms suppressed also by second order U-spin breaking. The three predicted relations have been shown to hold experimentally at a very high precision varying between $10^{-3}$ and $10^{-4}$, in agreement with our estimates of high order terms. More precise tests require a first robust measurement of $B(D^0 \rightarrow K^+\rho^-)$ and improving branching ratio measurements for $D^0$ decays to $K^{++}\pi^-, \pi^+K^{*-}, \pi^+\rho^-$ and $K^+K^{*-}$. So far no unexpected flavor symmetry breaking down to this very low level has been found, which would indicate physics beyond the standard model. This provides useful constraints on new $|\Delta C| = 1$ operators, potentially originating in new physics at energies much above a TeV [40].

Finally, we also studied decays to two neutral pseudoscalar mesons, deriving much less precise amplitude relations and relations for rate asymmetries between decays involving $K^0_S$ and $K^0_L$, which hold up to second order U-spin breaking terms at a level of several percent.

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