MLEM Image Reconstruction Algorithm for Transmission Tomographic Gamma Scanning in Drummed Nuclear Wastes

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Abstract: In this paper, 7 different samples and two radioactive isotopes (⁶⁰Co and ¹³⁷Cs) are used to fill a standard industrial waste bucket for simulating the real situation of drummed nuclear wastes. A transmission tomographic gamma scanning (TGS) measurement is carried out based on the TGS system independently developed by the project group for transmission image reconstruction. The data is processed by the MLEM iterative algorithm, it’s able to reconstruct the actual distribution of inhomogeneous media within the drum and to correct attenuation coefficients values of media under particular emission energies. The results show: the MLEM iterative algorithm can reconstruct clear TGS transmission images with accurate resolution of different densities which accord with the actual distributions of inhomogeneous media within the drum; the quality of reconstructed images tend to improve with the increase of transmission energy; the corrected attenuation coefficients values of 72 voxels within the drum under emission energy: 661.661keV, 1173.238keV and 1332.513keV are consistent with the reference values, which proves the validity of this method.

Keywords: Tomographic Gamma Scanning, Transmission Measurement, MLEM Iterative Algorithm, Image Reconstruction

1. Introduction

As one of the most advanced non-destructive assay methods with gamma ray, tomographic gamma scanning (TGS) technology is able to give accurate quantitative measurements and qualitative analysis of nuclear wastes and determine the distribution of selected radioisotopes in sealed containers [1,2]. In TGS, the transmission measurements and reconstruction are the basis for the subsequent emission reconstruction, so in this paper, major techniques of transmission TGS including: scanning mode, voxels division and reconstruction method are discussed. And with ideal flexibility and fewer limits of projection data [3], iterative algorithms have become the mainstream in transmission TGS reconstruction, there are a large number of researches on iterative image reconstruction algorithms over the recent twenty years [4-7]. MLEM, as a mature iterative algorithm, is used for the non-linear solution of TGS transmission measurement equation in this study, and is able to provide accurate reconstructed images as corrections for the following emission measurement.

2. Image reconstruction techniques

2.1 TGS transmission measurement equation

According to Beer-Lambert theorem, the TGS transmission measurement equation at measuring position i (or through transmission path i) is mathematically stated as:

\[ N_i = N_0 \exp(-\sum_{j=1}^{n} T_{ij} \mu_j) \]  

\[ N_i, N_0 \] are the detected full-energy peak counting rates before and after placing the waste drum in the TGS device; \( T_{ij} \) is an \( i \times j \) matrix, each element represents the linear absorption thickness (or the track length) gamma rays have produced while passing through voxel \( j \) with detector at measuring position \( i \); \( \mu_j \) is the linear attenuation coefficient of voxel \( j \).

By introducing a parameter \( V_i \); the negative logarithm of transmissivity, the TGS transmission measurement equation can be stated as equation 3 [8]:

\[ V_i = -\ln\left(\frac{N_i}{N_0}\right) \]  

\[ V_i = \sum_{j=1}^{n} T_{ij} \mu_j \]
2.2 MLEM iterative algorithm

The substance of TGS transmission image reconstruction is solving the equation mentioned above to obtain the linear attenuation coefficient $\mu_j$. In this work, MLEM (Maximum Likelihood Expectation Maximization) [9-11], as a statistical iterative algorithm with preferable anti-noise capability, is used for the solution of the transmission equation 3. The mathematical iterative formula of MLEM is as follows [12]:

$$X_j^{(k+1)} = \frac{X_j^{(k)}}{\Sigma i=1 \alpha_{ij} \Sigma i=1 \frac{\alpha_{ij} b_{ij}}{(a_i, X_j^{(k)})}} \quad \text{(4)}$$

$X_j^{(k)}$ is the linear attenuation coefficient of the $j$th voxel after $k$th iteration; $\alpha_{ij}$ is an element from track matrix which shows the track length gamma rays produced within the $j$th voxel during the $i$th measurement; $b_i$ represents the negative logarithm of transmissivity obtained by the $i$th measurement.

3. TGS transmission experiment

3.1 TGS experimental set-up

As in Fig.1, the TGS system used in this work mainly consists of four parts: transmission source, platform, a HPGe detector and data-acquisition system. The scanning mode is multi-layered: by elevating the objective platform, the scanning of drum vertically divided into several layers. In each layer, the horizontal measurement is a 2D process achieved by combining translation movement (translating the source-detector pair) with rotation movement (rotating the drum platform). Add up all layers’ reconstruction results, the attenuation coefficients map of the whole drum is obtained. Since the scanning mode and reconstruction method for each layer are nearly the same, in this paper we only carry out TGS experiment and build reconstruction images on one layer of the drum.

To simulate the real situation of a nuclear waste drum as in industrial production, 7 kinds of samples including polyethylene (S1, density: 1.037 g.cm$^{-3}$), concrete(S2, density:2.017 g.cm$^{-3}$), glass(S3, density:1.438 g.cm$^{-3}$), fiber(S4, density:0.206 g.cm$^{-3}$), plastic(S5, density:1.411 g.cm$^{-3}$), aluminum(S6, density:2.762 g.cm$^{-3}$) and water(S7, S8, density: 1.000 g.cm$^{-3}$) are used as the unevenly distributed packing mediums. The specific distribution of the simulated mediums within drum is presented in fig.2.

In order to obtain the linear attenuation coefficient of every small region in the nuclear waste drum, we divide the drum into several voxels by the way of polar-coordinates. Assuming that each voxel has homogeneous mediums, the round scanned layer will be divided into 4 circles of same width where the inner two of them will be split every 30° and the outer two is 15°, as in fig.3, the total number of voxels is 72.

137Cs (emitted γ energy:662 keV; source activity: 1.829×105 Bq) and 60Co (emitted γ energy:1173 keV and 1332 keV; source activity: 3.271×105 Bq) are chosen to be the emission sources within the drum. And the transmission source for the transmission measurement is $^{152}$Eu (source activity: 2.892×108 Bq), which is able to emit over 30 kinds of characteristic energies ranging from low to high. In this study, we mainly reconstruct images under 6 characteristic energies which has relatively larger branching ratios: 122 keV, 344 keV, 779 keV, 964 keV, 1112 keV and 1408 keV.
3.3 Transmission scanning measurement
At each elevation, the scanning process includes translation motion and rotation motion, as presented in fig.4: the initial translation position (position 1) of the source-detector pair is 35 centimeters (d1) from the centre point, and start from there the pair will move away from the centre point at a distance of 70 centimeters each time. There are 4 translations (d1, d2, d3 and d4 in fig.4) and the final translation position is 245 centimeters (d4) from the centre point. After every translation of the source-detector pair, the platform along with drum will rotate clockwise and be detected and recorded every 15° (θ), after a full turn there will be 24 rotation positions at each translation position, so 96 measurement positions in total.

3.4 Reference value measurement of linear attenuation coefficients
To accurately estimate the reconstruction quality, the reference values of linear attenuation coefficients are required, they are obtained by carrying out direct attenuation measurements on 7 kinds of filling materials used in this experiment: as shows in Fig.5, put a sample between the transmission source collimating orifice and the detector collimating orifice at a specific distance to make sure the conical space formed by two holes will be completely blocked, so the width of gamma ray beam can be less than the width of the sample. In that way, the linear attenuation coefficients of gamma rays passing through 7 kinds of samples under three characteristic energies (662 keV, 1173 keV and 1332 keV) can be obtained.

4. Reconstruction results
4.1 TGS transmission image reconstruction
The detected data obtained from transmission scanning are projected into the iteration of MLEM method as in Equation 3, and after certain iterations, the final constructed linear attenuation coefficients of 72 voxels under 6 characteristic energies (122 keV, 344 keV, 779 keV, 964 keV, 1112 keV and 1408 keV) can be reconstructed, based on that, the transmission reconstruction images under 6 characteristic energies are obtained as in Fig.6.
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Referring to the results above, conclusions are made that images reconstructed by MLEM method are consistent with the real distribution of samples within drum (showed in fig.2), the specific location of all kinds of samples and linear attenuation coefficients of all voxels in this layer are clear and relatively accurate. And the reconstruction quality is proportional to the value of transmitted gamma ray energy, reconstructed image under 1408 keV agrees best with the actual situation in both image effects and data accuracy. Sample with a very low linear attenuation coefficient (like S4) will make the display of itself in the reconstructed image not obvious.

4.2 Reference values verification

According to the relation of attenuation coefficients and energies, linear attenuation coefficients under 3 emission energies (662 KeV，1173 KeV，1332 KeV) are reconstructed based on the reconstructed values of 6 characteristic energies. And reconstructed values are used to verify reference values, the results are displayed in the following Table 1:

| Samples     | 662 keV A/cm⁻¹ | 662 keV error (%) | 1173 keV A/cm⁻¹ | 1173 keV error (%) | 1332 keV A/cm⁻¹ | 1332 keV error (%) |
|-------------|----------------|------------------|----------------|------------------|----------------|------------------|
| polyethylene (S1) | 0.0782 | 0.0858 | -8.79 | 0.0621 | 0.0661 | -6.11 | 0.0591 | 0.0632 | -6.49 |
| concrete (S2) | 0.1975 | 0.1674 | 1.79 | 0.1484 | 0.1272 | 1.66 | 0.1372 | 0.1212 | 3.33 |
| glass (S3) | 0.1244 | 0.1264 | -1.57 | 0.0919 | 0.0979 | -6.13 | 0.0830 | 0.0909 | -8.70 |
| fiber (S4) | 0.0118 | 0.0119 | -1.48 | 0.0070 | 0.0094 | -25.11 | 0.0062 | 0.0091 | -32.21 |
| plastic (S5) | 0.1408 | 0.1179 | 19.35 | 0.0918 | 0.0898 | 2.20 | 0.0846 | 0.0834 | 1.31 |
| aluminum (S6) | 0.1716 | 0.1949 | -11.97 | 0.1418 | 0.1596 | -11.17 | 0.1363 | 0.1551 | 12.16 |
| water (S7) | 0.1107 | 0.0854 | -29.62 | 0.0860 | 0.0654 | -31.50 | 0.0801 | 0.0613 | -30.60 |

Tab.3 shows that linear attenuation coefficients of 7 samples reconstructed by MLEM iteration algorithm match well with reference values, numerical error are in a range of 1.31%~32.21%; samples with small attenuation coefficients tend to have larger reconstruction error; the reconstruction effects are relatively better when samples are in bigger physical size. The main error sources include: ①the detected layer in the drum hasn’t been completely filled with samples, resulting in errors in distribution of linear attenuation coefficients and artifacts in the reconstructed images. ②The calculation error caused during iterative steps of MLEM method. ③The counting error in the process of transmission scanning measurement.

5. Conclusion

In this paper, MLEM statistical iteration algorithm is applied for TGS transmission reconstruction. The reconstructed images under 6 characteristic energies show good efficiency and quality, we can clearly distinguish the location of each sample material and linear attenuation coefficient of each voxel which are consistent with the true distribution of the drum. By using reconstructed linear attenuation coefficients maps under 6 characteristic energies for correction, linear attenuation coefficients of all samples under the 3 gamma ray energies emitted from the drum (662

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KeV, 1173 KeV, 1332 KeV) are in good agreement with the reference values. The feasibility of this method in TGS transmission image reconstruction of drummed nuclear waste has been illustrated.

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