Reply to a comment of I. Schmelzer

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March 16, 2011

Abstract

This is a reply to the comment of Schmelzer [1] on my earlier paper [2]. In contrast to Schmelzer’s claims, it is shown that passive locality does not correspond to the completeness condition of EPR. Nevertheless, it is argued that Schmelzer has found several errors in my article which, fortunately, can be corrected. In spite of Schmelzer’s arguments, the Fritsche-Haugk theory can be used to define a local hidden variable theory for the singlet state.

Completeness of hidden variable theories that violate passive locality

In his comment [1], Schmelzer argues that the passive locality condition

\[ P_{\phi_{jA} \otimes \phi_{jB}} (\sigma_{jA} \cap \sigma_{jB} | F_j S) = P_{\phi_{jA} \otimes \phi_{jB}} (\sigma_{jA} | F_j S) P_{\phi_{jA} \otimes \phi_{jB}} (\sigma_{jB} | F_j S) \]  

(1)

would be equal to the “completeness” condition of EPR [3]. Furthermore, Schmelzer writes that his Eq. (9): \[ P(A, B | \vec{a}, \vec{b}, \lambda) = P(A | \vec{a}, \vec{b}, \lambda) P(B | \vec{a}, \vec{b}, \lambda) \] would correspond to Eq. (1). We will show in this note that both claims are wrong.

Schmelzer assumes that there would be some intrinsic particle property \( \lambda \) that simultaneously determines the results at the separated detectors \( A \) and \( B \) for all conceivable pairs of axes \( \vec{a}, \vec{b} \). In that case, one could describe the experiment with a single probability space and write \( P(A, B | \vec{a}, \vec{b}, \lambda) \) for the probabilities of the outcomes at \( A \) and \( B \). However, Faris has proven in [4] that Bell’s inequalities hold trivially in this model.

As shown on p. 234-235 of [2], we can integrate \( P(A = \uparrow \cap A = \uparrow | \vec{a}, \vec{b}, \lambda) \), \( P(A = \uparrow \cap A = \downarrow | \vec{b}, \vec{c}, \lambda) \), \( P(A = \uparrow \cap A = \downarrow | \vec{c}, \vec{d}, \lambda) \), and get the probabilities of events. Those events are mutually exclusive, since the outcomes at \( A \) for the same axis are opposite in all pairs of those events. Therefore, we can write:

\[ P(A = \uparrow \cap A = \uparrow | \vec{a}, \vec{b}) + P(A = \uparrow \cap A = \downarrow | \vec{b}, \vec{c}) + P(A = \uparrow \cap A = \downarrow | \vec{c}, \vec{d}) \leq 1. \]  

(2)

The event \( A = \downarrow \) with axis \( \vec{b} \) is equivalent to \( B = \uparrow \) with the same axis \( \vec{b} \), or \( \{ A = \downarrow, \vec{b} \} \equiv \{ B = \uparrow, \vec{b} \} \).

Similarly, we have \( \{ A = \downarrow, \vec{c} \} \equiv \{ B = \uparrow, \vec{c} \} \) and \( \{ A = \downarrow, \vec{a} \} \equiv \{ B = \uparrow, \vec{a} \} \). In computing probabilities, one can substitute equivalent events. Therefore, we finally arrive at

\[ P(A = \uparrow \cap B = \uparrow | \vec{a}, \vec{b}) + P(A = \uparrow \cap B = \uparrow | \vec{b}, \vec{c}) + P(A = \uparrow \cap B = \uparrow | \vec{c}, \vec{d}) \leq 1, \]  

(3)

which is one version of Bell’s inequality. Eq. (2), from which Bell’s inequality was derived, does not depend on any locality condition. Hence, with a single probability measure \( P \), Bell’s inequalities can never be violated and we have to find an entirely different mathematical description.

For example, the particles may be created with properties that are changed later by the measurement instruments when the particles are flying through the devices. We do not require for each device

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to change the properties of the particles that are flying through the spatially separated station. Hence, this kind of local contextuality does not require instantaneous signaling. In such models, the outcomes at the detectors are generated by axis dependent random variables $\phi_{j\vec{A}\vec{a}}$ and $\phi_{j\vec{B}\vec{b}}$. As shown in [2], we then get one probability measure $P_{\phi_{j\vec{A}\vec{a}}\otimes\phi_{j\vec{B}\vec{b}}}$ for each combination of axes. On this family of probability spaces, there exists an analogue of Eq. (3). This model can also reproduce the Kochen-Specker theorem, see [5].

Since the spin operators for different axes do not commute, a measurement of the outcome generated by $\phi_{j\vec{A}\vec{a}}$ for axis $\vec{a}$ will disturb the outcome generated by $\phi_{j\vec{B}\vec{b}}$ for axis $\vec{b} \neq \vec{a}$. Hence, in case of a measurement for axes $\vec{a}$ at $A$ and $\vec{b}$ at $B$, the outcomes of $\phi_{j\vec{A}\vec{a}}$ are, due to the measurement at $A$ with $\vec{a}$, no more equivalent to the outcomes at $B$ with axis $\vec{b}$. This is the reason why one cannot do the substitutions in $P_{\phi_{j\vec{A}\vec{a}}\otimes\phi_{j\vec{B}\vec{b}}}$ that would lead to something like Eq. (2).

In [1], Schmelzer argues that the passive locality condition of Eq. (1) could be somehow related to EPR’s completeness condition: “Every element of the physical reality must have a counterpart in the physical theory”. According to the proofs in [2, 4], passive locality implies for an actively local theory that the outcomes at $A$ and $B$ for axes $\vec{a}$ and $\vec{b}$ would have to be predetermined by a third, combined event $x_{j\vec{a}\vec{b}} \in F_{\vec{J}\vec{S}}$. EPR write at ¶2 of p. 780 in [3] that they consider the correlated outcomes of the entangled quantum state to be the elements of reality. However, the absence of a common cause that determines the values of two sets of physical quantities, e.g. momentum and position in a particle pair, can neither prevent the simultaneous existence, nor the simultaneous description of those physical quantities by a theory. Therefore, a stochastic theory that violates passive locality can, in principle, provide all elements of the physical reality.

On the other hand, the outcomes of a theory that violates passive locality seem not to fit into the following definition of EPR: “If, without disturbing the system in any way, we can predict with certainty the value of a physical quantity, then there exists an element of reality corresponding to that physical quantity”. With the failure of passive locality, the measured outcomes must be the result of a spontaneous stochastic decision that happens after the particles have arrived at the measurement devices. For Stern-Gerlach magnets, the probabilities of the results of this decision are given by Eqs. (A2a) and (A2b) of [6]. According to those formulas, a Stern-Gerlach magnet locally changes the spin state of an incoming particle if the magnet is set to polar angles $\theta \neq \tilde{\theta}$, where $\tilde{\theta} = 180^\circ$ with respect to the $\hat{z}$ axis in which the spin of the incoming particles was prepared. The final values of the individual outcomes therefore depend on the axis of the magnet.

Note that the rotational invariance of the singlet state does not imply that no such disturbance happens in course of the measurement process. In case the experimenters at $A$ and $B$ choose their polar angles $\theta_a, \theta_b$ such that $\theta_a = \theta_b \neq \tilde{\theta}$, they only modify their local outcomes in the same way. This preserves the exact correlations between the outcomes at the two stations and even the probabilities of the events may remain the same. Nevertheless, the individual outcomes of the singlet state that are prepared by the source with respect to the $\hat{z}$ axis are transformed by Eqs. (A2a) and (A2b) of [6] with an axis dependent transition probability.

Due to Eqs. (A2a) and (A2b) of [6] and the failure of passive locality, the values of the measured spin components must be the result of a stochastic decision that depends on the local setting of the Stern-Gerlach magnet. However, the experimenters at one station, e.g. at $A$, can not know the setting $\vec{b}$ of the remote station $B$. Hence, the physicists at $A$ can, strictly speaking, not predict the results of $B$, since the values of $B$’s outcomes are spontaneously generated during the measurement at $B$ and depend on $B$’s setting $\vec{b}$.

That their values must be spontaneously generated by correlated stochastic processes is, however, no argument against the simultaneous existence of the physical quantities that correspond to the eigenvalues of several non-commuting operators. For example, one could use axis dependent families of stochastic processes $\phi_{j\vec{A}\vec{a}}$ and $\phi_{j\vec{B}\vec{b}}$ and Eqs. (A2a) and (A2b) of [6] to generate unobservable spin outcomes at times $t$ for all axes during the flight of the two particles to the measurement stations. This model would be complete, since every element of the physical reality would have a counterpart in the theory. However, one would have to make sure that in case of a measurement process, the
random outcomes of the stochastic processes change their values in the following way: At first, the unmeasured outcomes for the same axes at the two stations must not be equivalent to each other anymore. Secondly, the measured outcomes at the two stations must, in case the separated devices at A and B have the same settings, be exactly correlated. Furthermore, the measured outcomes at the two stations must not be predetermined by a common cause. For all this, however, no interaction between the separated random variables at A and B is necessary since correlations between random variables, even if the correlations are exact, do not imply causation.

One reason why one could need such a theory might be the following: Neutrons are particles that are bounded by quantum chromodynamics to a confinement radius of approx. 0.7fm. Confinement is a feature of quantum chromodynamics that does not depend on observation or any details of a measurement process. Nevertheless, one can do interferometry experiments with neutrons. Moreover, the interferometers used for that sort of experiments are macroscopic devices with dimensions of several centimeters. Due to QCD’s confinement, it would seem unplausible to believe that a neutron does not have a position in case one measures its momentum. On the other hand, a theory, where outcomes are predetermined would have to violate active locality and therefore never could be written manifestly covariant. It is the opinion of the author that stochastic hidden variable theories are the only way to explain how a neutron that is bounded to a size of 0.7fm makes its way through a macroscopic interferometer.

**Reply to Schmelzer’s arguments on section 4.2**

In [1], Schmelzer argues that after splitting the superposition up as in Eq. (72) of [2], there is no explicit way which forbids the calculation of observables in the separated systems. Hence, it would be impossible to distinguish whether the probability amplitude in Eq. (70) from [2] or the amplitude in Eq. (75) from [2] is valid. Fortunately, this problem is not present in states like

$$\Psi(\vec{r},t) = \psi_1(\vec{r},t)\sqrt{\rho_{10}}e^{i\varphi_{10}}|+\rangle + \psi_2(\vec{r},t)\sqrt{\rho_{20}}e^{i\varphi_{20}}|−\rangle,$$

(4)

with amplitudes $\rho_+ = |\psi_1(\vec{r},t)|^2\rho_{10}$ and $\rho_− = |\psi_2(\vec{r},t)|^2\rho_{20}$. Similar considerations hold for the probability amplitudes $\rho_{++}, \rho_{−−}, \rho_{+-},$ and $\rho_{−+}$ that are given in [4] for the state of the EPR experiment

$$\Psi(\vec{r}_1,\vec{r}_2,t) = \frac{1}{\sqrt{2}} (1 \cdot \psi(\vec{r}_1,t)|+\rangle \otimes (−\sin (\vartheta/2)) \cdot \psi(\vec{r}_2,t)|+\rangle$$

$$+ 1 \cdot \psi(\vec{r}_1,t)|+\rangle \otimes \cos (\vartheta/2) \cdot \psi(\vec{r}_2,t)|−\rangle$$

$$− 1 \cdot \psi(\vec{r}_1,t)|−\rangle \otimes \cos (\vartheta/2) \cdot \psi(\vec{r}_2,t)|+\rangle$$

$$− 1 \cdot \psi(\vec{r}_1,t)|−\rangle \otimes \sin (\vartheta/2) \cdot \psi(\vec{r}_2,t)|−\rangle).$$

(5)

The form of $\rho_{++}, \rho_{−−}, \rho_{+-},$ and $\rho_{−+}$ allows to divide the two particle Pauli equation that is associated with Eq. [5] into several equations of motion in completely the same way as it is done in Eq. (152) of [2]. Since angle dependent constant $\vartheta$ is associated with the $\vec{r}_2$ particle system, a change of $\vartheta$ does not modify the equations of motion for the $\vec{r}_1$ particles. However, the method does only work with states that are similar to Eq. [5] and not with general entangled states. Therefore section 4.6 must be retracted. The second problem of sect. 4.2 is that the proposed exchange procedure between the trajectories would result in identical average velocities $\vec{u}$ and $\vec{v}$. Then, one would be restricted to states whose superposed terms only differ by some constant. The correction of this can be found in this reply.

Schmelzer claims in his comment [1], that constants of the form $\sqrt{\rho_{10}}e^{i\varphi_{10}}$ in Eqs. (4)–(5) would become irrelevant factors during the derivations in [2], and therefore, my model would be unable to derive the states of Eqs. (4)–(5). This is, however, not true. In the Fritsche-Haugk theory, the velocities $\vec{u}, \vec{v}$ are only ensemble averages of the particle trajectories $\vec{x}_j(t)$, which are the hidden variables. Two ensembles of particles may have the same average velocity, but still can contain different numbers of trajectories. Therefore, those numbers must be specified separately. Consider a superposition like Eq. (4), with $N = 2$ superposed states. As noted in [2], constants like $\sqrt{\rho_{10}}$ do not appear in $\vec{u}, \vec{v}$. Hence,
the assumption that \( \sqrt{\rho_{00}} \) is determined via
\[
\rho_{00} = \frac{1}{\int \psi_1(\vec{r},t) \ast \psi_1(\vec{r},t) \, d^3r} \sum_{i} S_i \tag{6}
\]
with \( S_i \) as the number of trajectories \( \vec{x}_j(t) \) in the ensemble described by \( \psi_1(\vec{r},t) \), seems to be the only consistent interpretation. We will see later in this note that factors like \( \sqrt{\rho_{00}} \) can result directly from the trajectory exchange mechanism that is present in the theory.

In the Fritsche-Haugk theory, the phases of the wave function are given by
\[
\varphi = \frac{m_0}{\hbar} \int_{\vec{r}_a}^{\vec{r}_b} \vec{v} \, d\vec{r} - \frac{m_0}{\hbar} \int_{t_0}^{t} i \beta(t') \, dt'. \tag{7}
\]
For particles with spin in a Stern-Gerlach magnet, an argument was described in [2] that determines \( \sqrt{\rho_{00}} e^{i\varphi_{00}} \) of Eq. (4) from the rotational symmetry of the experiment. As this argument is time independent, the factors \( \varphi_{00} \) are determined solely by \( \frac{m_0}{\hbar} \int_{\vec{r}_a}^{\vec{r}_b} \vec{v} \, d\vec{r} \), which is connected via \( \vec{r}_a \) and \( \vec{r}_b \) to the geometry of the experimental situation. Fritsche and Haugk derive the Schrödinger equation from Langevin equations where only particle velocities appear. This is carried over to the equations for the ensemble average velocities \( \bar{u}, \bar{v}, \bar{r} \). The constant phase factors are therefore determined in the same way as in classical mechanics. In order to describe the position of a classical particle correctly, one has to integrate its velocity and then fix the integration constants with additional boundary terms that arise from the experimental situation.

**Corrections of problems in paragraphs 9-16 of section 6.2**

Unfortunately, the details in 9-16 of sect. 6.2 only describe a rather complicated deterministic model for exact spin anti-correlations with respect to the \( \vec{z} \) axis that are given by the state
\[
\Psi(\vec{r}_1, \vec{r}_2t) = \frac{1}{\sqrt{2}} (\psi(\vec{r}_1, t)|+\rangle \otimes \psi(\vec{r}_2, t)|-\rangle - \psi(\vec{r}_1, t)|-\rangle \otimes \psi(\vec{r}_2, t)|+\rangle). \tag{8}
\]

The model did not include the measurement process that generates outcomes with axis dependent probabilities. As shown in [2, 18], an actively local theory can already violate Bell’s inequalities if passive locality does not hold for a single one of three axes. Hence, the correlations of Eq. (8) with respect to one distinguished axis \( \vec{z} \) can be described without the violation of passive locality. In fact, Schmelzer is correct when he writes that the passive locality condition holds in the deterministic model in 9-16 of sect. 6.2.

In the following, we will define a stochastic hidden variable model for the state of Eq. (5) that violates Bell’s inequalities. After converting Eqs. (8) and (9) into equations of motion in the same way as Eq. (152) of [2], we observe that Eq. (6) describes more sub-ensembles than Eq. (8). Furthermore, by Eq. (6) the numbers of the particles in those ensembles are different between Eq. (5) and Eq. (8). Hence, during the transition from Eq. (8) to Eq. (9), there must exist some procedure that decides for each \( \vec{r}_j(t) \) and \( \vec{r}_2(t) \) in the \( \vec{r}_1 \) and \( \vec{r}_2 \) ensembles of Eq. (8), whether one, or both, or even none of the trajectories should change the spin system. As only one trajectory from each pair might change its spin state, we get two more particle ensembles for Eq. (9) than are described by Eq. (8).

Due to the form of Eqs. (8) and Eq. (9), the trajectory exchange mechanism that transforms Eq. (9) into Eq. (8) must leave \( \vec{u} \) and \( \vec{v} \) of the already existing ensembles invariant. Moreover, the newly created ensembles in Eq. (9) must have the same average velocity than the systems from Eq. (8). On the other hand, the process must change the numbers of trajectories in the spin ensembles such that the probabilities of finding a particle pair are given by the amplitudes \( \rho_{++}, \rho_{--}, \rho_{+-} \) and \( \rho_{+-} \) of Eq (5). Due to the form of those amplitudes, the correlations between the outcomes that
the trajectory exchange process generates must depend on the settings of the separated measurement devices. Nevertheless, the relations

$$\vec{u}^N = (\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_N), \quad \vec{v}^N = (\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_N)$$

hold in the Fritsche-Haugk theory in general. Furthermore, the constants in Eq. (5) that depend on the angle of the station where the $\vec{r}_2$ particles arrive do only modify the numbers of trajectories in the ensembles of the $\vec{r}_2$ particle system. Thus, there is nothing that forbids to the define active locality as an additional property of the trajectory exchange mechanism that constructs Eq. (5).

Note that a similar stochastic process is present in the Fritsche-Haugk theory for single particle states. In order to derive the Schroedinger equation, one has to interchange trajectories between certain $A$ and $B$ ensembles until their average velocities are equal. In [7], the behavior of this stochastic process was only defined in terms of random variables, probability amplitudes, ensemble averages and correlations of outcomes. In case of Eq. (5), one additionally has the result that this mechanism cannot be controlled by a common cause, since if it were, it could not violate passive locality. This is the difference between Eq. (5) and a classical mixture of product states. Although the probability amplitudes of the latter may look similar than those of Eq. (5), a classical mixture has a different density operator and can only produce correlations between two outcomes that can be explained by a common predetermination.

References

[1] I. Schmelzer, Comment on a paper by B. Schulz on Bell’s inequalities; arXiv:0910.4740
[2] B. Schulz, Ann. Phys. (Berlin) 18, 231 (2009)
[3] A. Einstein, B. Podolski, N. Rosen, Phys. Rev. 47, 777 (1935)
[4] W. G. Faris, Probability in Quantum Mechanics, appendix to: D.Wick, The Infamous Boundary: Seven Decades of Controversy in Quantum Physics (Birkhauser, Boston, 1995)
[5] B.R. La Cour, Phys. Rev. A 79, 012102 (2009)
[6] D. Greenberger, M. Horne, A. Shimony, A. Zeilinger, Am. J, Phys. 58, 1131, Appendix B (1990)
[7] L. Fritsche, M. Haugk, Ann. Phys. (Berlin) 12, 371 (2003)
[8] E. Nelson, Annals of the New York Academy of Sciences, 480, 533 (1986)