\[ \mathcal{N} = 2 \] **SCFT with minimal flavor central charge**

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\textbf{ABSTRACT:} We list 4d interacting $\mathcal{N} = 2$ SCFT with minimal flavor central charge from the theory space constructed using 6d (2,0) theory. For $ADE$ and $C_N$ flavor groups, our theories saturate the bound found using bootstrap method, but other cases have higher values. We find interesting rank one SCFTs with $B_3, G_2, F_4, C_4 \times U(1), C_1 \times U(1)$ flavor symmetry. Many physical properties of these theories are also studied.
1 Introduction

Conformal central charges \((a,c)\) and flavor central charge \(k_G\) are basic invariants for a four dimensional \(\mathcal{N} = 2\) SCFT. Using conformal bootstrap method, we have following constraints on those central charges:

- The ratio \(a/c\) satisfies following constraint [1]: \(\frac{1}{2} \leq a/c \leq \frac{5}{4}\), and the upper bound is saturated by free vector multiplets and the lower bound by free hypermultiplets.

- The central charge \(c \geq \frac{11}{30}\) for interacting theory [2], and this value is saturated by simplest Argyres Douglas theory [3] (we call it \(H_0\) theory to indicate that it has trivial flavor symmetry).

- The constraint on flavor central charge is more interesting [4], and the bound is recovered in table. 1 (for interacting theory) \(^1\). There is also an interesting bound involving \(c\) and \(k_G\) [5].

| \(A_{N-1}\) | \(k \geq \frac{N}{2}\) | \(D_N\) | \(k \geq N - 2\) | \(E_6\) | \(k \geq 3\) |
| --- | --- | --- | --- | --- | --- |
| \(E_7\) | \(k \geq 4\) | \(E_8\) | \(k \geq 6\) | \(B_N\) | \(k \geq N - \frac{1}{2}\) |
| \(C_N\) | \(\frac{N}{2} + 1\) | \(G_2\) | \(k \geq \frac{5}{3}\) | \(F_4\) | \(k \geq \frac{5}{2}\) |

Table 1. The bound on flavor central charge, which comes from considering unitarity condition involving the operator \(\hat{B}_2\). There are some exceptions for lower rank classical group case: the constraint is \(A_{N-1}(N \geq 3), B_N(N \geq 4), C_N(N \geq 3), D_N(N \geq 4)\). The other bounds are: \(A_1(k_G \geq \frac{5}{2}), D_2/D_3(k_G \geq 2), B_2/C_2/B_3(k_G \geq 2)\).

\(^1\)Our convention of flavor central charge is half of that used in [4].
It is interesting to identify those theories saturating the flavor central charge bound. It is already known that the rank one AD theories of type (A1, A2, D4, E6, E7, E8) [6, 7] saturate these bounds. The purpose of this paper is to identify the remaining cases. Our strategy is to scan the theory space constructed in [8, 9], and identify the theory with minimal flavor central charge. Our findings are

- Our minimal theory saturates the flavor central charge bound for \( G = ADE \) and \( G = C_N \).
- For \( G = B_N, G_2, F_4 \), our minimal theory does not saturate the bound in table. 1.

Our bound is \( k_{B_N} \geq N - 1, k_{G_2} \geq 2, k_{F_4} \geq 3 \).

The conformal central charge \((a, c)\) and flavor central charge \(k_G\), Coulomb branch spectrum for these minimal theories are listed in table. 2. We also list whether there is an extra \(U(1)\) flavor symmetry or not.

| Flavor group \(G\) | \(k_G\) | Coulomb branch | \(a\) | \(c\) | Extra \(U(1)\) |
|-------------------|-------|----------------|-----|-----|----------------|
| \(A_{N-1}\) (N even) | \(N/2\) | \((N/2, N/2, \ldots, 2)\) | \(7N^2 - 10\) | \(1/12(N^2 - 2)\) | Yes |
| \(A_{N-1}\) (N odd) | \(N/2\) | \((N/2, N/2, \ldots, 2)\) | \(7N^2 - 10\) | \(1/12(N^2 - 2)\) | No |
| \(B_N\) (N even) | \(N - 1\) | \((N - 1, N - 3, \ldots, 3)\) | \(7N^2 - 5N - 10\) | \(1/12(2N^2 - N - 2)\) | Yes |
| \(B_N\) (N odd) | \(N - 1\) | \((N - 1, N - 3, \ldots, 2)\) | \(7N^2 - 5N - 2\) | \(1/12(2N - 1)(2N + 1)\) | No |
| \(C_N\) (N even) | \(N/2 + 1\) | \((N, N - 2, \ldots, 2, N/2)\) | \(7N^2 + 19N + 10\) | \(1/12(2N^2 + 5N + 1)\) | Yes |
| \(C_N\) (N odd) | \(N/2 + 1\) | \((N, N - 2, \ldots, 3, N/2)\) | \(7N^2 + 19N + 10\) | \(1/12(2N^2 + 5N + 1)\) | No |
| \(D_N\) (N even) | \(N - 2\) | \((N - 2, N - 4, \ldots, 2)\) | \(7N^2 - 19N + 2\) | \(1/12(2N^2 - 5N + 1)\) | Yes |
| \(D_N\) (N odd) | \(N - 2\) | \((N - 2, N - 4, \ldots, 3)\) | \(7N^2 - 19N + 2\) | \(1/12(2N^2 - 5N + 1)\) | No |
| \(E_6\) | 3 | (3) | \(41\) | \(13\) | No |
| \(E_7\) | 4 | (4) | \(39\) | \(10\) | No |
| \(E_8\) | 6 | (6) | \(35\) | \(7\) | No |
| \(G_2\) | 2 | (2) | \(4\) | \(1\) | No |
| \(F_4\) | 3 | (3) | \(41\) | \(13\) | No |

Table 2. Physical data for \(\mathcal{N} = 2\) SCFT with minimal flavor central charge among theories constructed using (2, 0) construction.

The flavor central charge can be put in the following form:

\[
k_G = h^\vee - \frac{1}{n}h, \quad G = B_N, F_2, G_2,
\]
\[
k_G = h^\vee - \frac{1}{2n}h, \quad G = C_N,
\]
\[
k_G = \frac{h}{2}, \quad G = A_{N-1},
\]
\[
k_G = \frac{h - 2}{2}, \quad G = D_N,
\]
\[
k_G = \frac{h}{6} + 1, \quad G = E_N. \quad (1.1)
\]
Here \( n \) is the lacety of the Lie algebra \( G \), see table 5 for these numbers, \( h^\vee \) the dual Coxeter number, and \( h \) the Coxeter number. For the theory without extra \( U(1) \) flavor symmetry, the corresponding 2d chiral algebra [4] is given by the Kac-Moody algebra of type \( G \) with level

\[
k_{2d} = -k_G. \tag{1.2}
\]

The corresponding level is admissible only for \( A_{\text{odd}}(N \text{ odd}) \) cases [10].

The Higgs branch of these theories can be found from the associated variety of the corresponding vertex operator algebra [11, 12]. For the theory without extra \( U(1) \) flavor symmetry and if the level is admissible, the associated variety is found in [10], and they are given by the nilpotent orbit of the corresponding Lie algebra. The associated variety of \( E_6, E_7, E_8 \) theory is also found in [13]. For other cases, we use a simpler method by computing the Higgs branch dimension from the central charge data, and guess the corresponding orbit. See table 3 for the summary. The Higgs branch is the minimal nilpotent orbit only for \( A_1, A_2, D_4, E_6, E_7, E_8 \) case.

| Flavor group | Nilpotent orbit |
|--------------|----------------|
| \( A_{N-1} (N = 2k + 1) \) | \([2, 2, \ldots, 2, 1]\) |
| \( B_N \) (N odd) | \([3, 2, \ldots, 2, 1, 1, 1, 1]\) |
| \( C_N \) (N even) | \([2, \ldots, 2, 1, 1]\) |
| \( D_N \) (N even) | \([2, \ldots, 2, 1, 1, 1]\) |
| \( E_N \) | \( A_1 \) |
| \( G_2 \) | \( G_2(a_1) \) |
| \( F_4 \) | \( A_1 \) |

Table 3. The nilpotent orbit for the Higgs branch of our minimal theory, and we list the partition for the classical group case, and the Bala-Carter label for exceptional group.

This paper is organized as follows: section II gives the detailed construction for each minimal theory, we also study many interesting properties of these theories including the Coulomb branch spectrum, 2d chiral algebra, Higgs branch, etc; section III propose a conjecture about the lower bound of flavor central charge when the Coulomb branch spectrum has a common denominator \( r \); Finally a conclusion is given in section IV.
2 Minimal theory

2.1 ADE flavor group

We can construct a large class of $\mathcal{N} = 2$ SCFT by compactifying 6d $(2,0)$ theory of type $J = ADE$ on a sphere with one irregular singularity and one regular singularity. We are interested in following irregular singularity:

$$\Phi = \frac{T}{z^{2+k/b}} + \ldots$$  \hspace{1cm} (2.1)

Here $k > -b$ and $k, b$ is copime, and $T$ is regular semi-simple. The allowed value of $b$ is

$$A_{N-1} : b|\{N, N-1\}, \quad D_N : b|\{2N-2, N\}, \quad E_6 : b|\{12, 9, 8\}, \quad E_7 : b|\{18, 14\}, \quad E_8 : b|\{30, 24, 20\}. \hspace{1cm} (2.2)$$

Here the notation means that $b$ is the divisor of the listed integers in square bracket. The SW curve is found from the spectral curve of Hitchin system: $det(x - \Phi(z)) = 0$. They can be put in following form:

$$J = A_{N-1} : x^N + \sum_{i=2}^{N} \phi_i(z)x^{N-i} = 0,$$

$$J = D_N : x^{2N} + \sum_{i=1}^{N-1} x^{2N-2i} + (\tilde{\phi}_N)^2 = 0,$$

$$J = E_6 : \phi_2(z), \phi_5(z), \phi_6(z), \phi_8(z), \phi_9(z), \phi_{12}(z),$$

$$J = E_7 : \phi_2(z), \phi_6(z), \phi_8(z), \phi_{10}(z), \phi_{12}(z), \phi_{14}(z), \phi_{18}(z),$$

$$J = E_8 : \phi_2(z), \phi_6(z), \phi_{12}(z), \phi_{14}(z), \phi_{18}(z), \phi_{20}(z), \phi_{24}(z), \phi_{30}(z). \hspace{1cm} (2.3)$$

Here $\phi_i(z)$ is a degree $i$ differential on Riemann surface. For $E_N$ case, we only list the independent differentials. The coefficients of these differentials encode the Coulomb branch spectrum of the theory.

We further assume that the regular puncture is regular semi-simple so that we have an ADE flavor symmetry. The flavor central charge of the theory is $[16–19]$:

$$k_G = h - \frac{b}{b+k}. \hspace{1cm} (2.4)$$

We find the following minimal value of flavor central charge:

- $G = A_{N-1} : b = N, b+k = 2$. The flavor central charge is $k_G = \frac{N}{2}$, and the Coulomb branch spectrum is $(\frac{N}{2}, \frac{N-2}{2}, \ldots, \frac{3}{2})$ for $N$ odd, and $(\frac{N}{2}, \frac{N-2}{2}, \ldots, 2)$ for $N$ even. If $N$ is even, there is a further $U(1)$ flavor symmetry and this theory is $SU(\frac{N}{2})$ gauge theory coupled with $N$ flavor fundamental matter.

\footnote{The flavor central charge for theory defined using only regular singularity is simple: $k_G = h^\vee$ with $h^\vee$ the dual Coxeter number [14, 15].}

\footnote{It is also possible to have non-abelian flavor symmetry from irregular singularity [16], but the flavor central charge is bigger than dual Coxeter number.}
• \( \mathbf{G} = D_N: \ b = N, b + k = 1. \ (N \geq 4) \): The flavor central charge is \( k_G = N - 2 \). The Coulomb branch spectrum is \( (N - 2, \ldots, 2) \) (N is even), or \( (N - 2, \ldots, 3) \) (N is odd). When N is even, this theory is \( Usp(N - 2) \) gauge theory coupled with \( 2N \) half fundamental flavors. When N is odd, there is an extra \( U(1) \) flavor symmetry.

• \( \mathbf{G} = E_6: \ b = 9, b + k = 1. \ k_G = 3 \). This is the rank one theory with Coulomb branch spectrum (3). The Higgs branch is the minimal nilpotent orbit of \( E_6 \).

• \( \mathbf{G} = E_7: \ b = 14, b + k = 1. \ k_G = 4 \). This is the rank one theory with Coulomb branch spectrum (4). The Higgs branch is the minimal nilpotent orbit of \( E_7 \).

• \( \mathbf{G} = E_8: \ b = 24, b + k = 1. \ k_G = 6 \). This is the rank one theory with Coulomb branch spectrum (6). The Higgs branch is the minimal nilpotent orbit of \( E_8 \).

\( E_N \) type theory is the Minahan-Nemchesky theory [7].

2.2 Non-simply laced flavor group

To get non-simply laced flavor group from \((2,0)\) type construction, we need to do outer automorphism twist. The corresponding twist is listed in table. 4:

| \( J \)   | \( A_{2N} \) | \( A_{2N-1} \) | \( D_N \) | \( E_6 \) | \( D_4 \) |
|-----------|-------------|-------------|-----------|--------|--------|
| Outer Automorphism | \( Z_2 \) | \( Z_2 \) | \( Z_2 \) | \( Z_2 \) | \( Z_3 \) |
| Invariant subalgebra \( g^\vee \) | \( B_N \) | \( C_N \) | \( B_{N-1} \) | \( F_4 \) | \( G_2 \) |

Table 4. Outer-automorphisms of simple Lie algebras and its invariant subalgebra.

• \( \mathbf{G} = B_N: \) We use \( Z_2 \) twist of \( A_{2N-1} \) theory. The action on the differential is \( \phi_k \to (-1)^k \phi_k \) [20]. This means that even degree differentials are holomorphic polynomial, and odd degree differentials have half-integral order pole. Therefore, we can only have one class of theory (unlike the untwisted theory), and the flavor central charge is given by the following formula

\[
\text{class I : } k_G = 2N - 1 - \frac{1}{2} \frac{2N}{2N + k}.
\]  

(2.5)

The minimal theory is achieved at \( k + 2N = 1 \), and \( k_G = N - 1 \). The Coulomb branch spectrum is then \( (N - 1, N - 3, \ldots, 2) \) for \( N \) odd. For \( N \) even, we have the spectrum \( (N - 1, N - 3, \ldots, 3) \), and there is an extra \( U(1) \) flavor symmetry.

• \( \mathbf{G} = C_N: \) We use \( Z_2 \) twist of \( D_{N+1} \) theory. The non-trivial action on differential is \( \tilde{\phi}_N \to -\tilde{\phi}_N \) [21]. There are two class of theories, and the flavor central charge is given by following formula:

\[
\text{class I : } k_G = \frac{1}{2}(2N + 2 - \frac{2N}{2N + k}), \text{ class II : } k_G = \frac{1}{2}(2N + 2 - \frac{2N + 2}{2N + 2k + 3}).
\]  

(2.6)

The minimal theory is achieved for class I theory with \( k + 2N = 2 \), and \( k_G = \frac{1}{2}N + 1 \). The Coulomb branch spectrum is \( (N, N - 2, \ldots, 2, \frac{N+2}{2}) \) if \( N \) is even, which is just a \( SO(N+2) \) gauge theory with \( 2N \) half fundamental hypermultiplet. The Coulomb branch spectrum is \( (N, N - 2, \ldots, 3, \frac{N+2}{2}) \) if \( N \) is odd, but there is an extra \( U(1) \) flavor symmetry for this theory.
\textbullet{} \( G = G_2 \): We use \( Z_3 \) twist of \( D_4 \) theory \([22]\). The basis of differential is \((\phi_2, \phi_4^2, \phi_4^4, \phi_6^4)\), and \( \phi_2^2, \phi_4^2 \) transform nontrivially under \( Z_3 \) group, and their order of pole has the form \( z^{n+j/3} \). We only have one class of theory with the flavor central charge

\[
k_G = 4 - \frac{1}{3} \frac{6}{k + 6}.
\]

The minimal theory is achieved at \( k + 6 = 1 \). The minimal theory has Coulomb branch spectrum 2.

\textbullet{} \( G = F_4 \): We use \( Z_2 \) twist of \( E_6 \) theory \([23]\). The \( Z_2 \) action on the differential is \( \phi_5 \rightarrow -\phi_5 \) and \( \phi_9 \rightarrow -\phi_9 \), and so they have half-integer order of pole. There are two classes of theories with following flavor central charge:

\[
class I : k_G = 9 - \frac{1}{2} \frac{12}{k + 12}, \quad class II : k_G = 9 - \frac{1}{2} \frac{8}{k + 8}.
\]

The minimal theory is achieved in class I by taking \( k + 12 = 1 \). The Coulomb branch spectrum is 3.

2.3 2d chiral algebra, central charges and Higgs branch

There is a correspondence between Schur sector of 4d \( \mathcal{N} = 2 \) SCFT and 2d vertex operator algebra \([4]\). For our theory, the corresponding 2d vertex operator algebra is just the Kac-Moody algebra of type \( G \) \([11, 18, 24]\), and the level of 2d theory is

\[
k_{2d} = -k_G.
\]

See formula \((1.1)\) for flavor central charge of 4d theory. If our theory has an extra abelian flavor symmetry, then we need to add a \( U(1) \) Kac-Moody algebra too. A level is called admissible if

\[
k = -h + \frac{p}{q}, \quad (p, q) = 1, \quad p \geq h, \quad G = ADE,
\]

\[
k = -h^\vee + \frac{p}{q}, \quad (p, q) = 1, \quad p \geq h^\vee, \quad G = BCFG,
\]

\[
k = -h^\vee + \frac{p}{nq}, \quad (p, q) = 1, \quad (p, n) = 1, \quad p \geq h, \quad G = BCFG.
\]

Here \( h \) is the Coxeter number, \( h^\vee \) is the dual Coxeter number, \( n \) is the lacety of the Lie algebra, see table. 5. The Higgs branch is the associated variety of the corresponding Kac-Moody algebra of type \( G \), and the associated variety for admissible level is found in \([10]\). In our case, only the level corresponding to \( A_N(N \ odd) \) case is admissible, and one can use the result of \([10]\) to find the corresponding Higgs branch which is listed in table. 3. The corresponding associated variety for \( E_N \) type is also considered in \([13]\).

Using above 2d/4d relation, we propose the following central charge formula \([24]\):

\[
c = \frac{1}{12} \left( \frac{-k_G \text{dim}(G)}{-k_G + h^\vee} - \frac{f}{12} \right), \quad 2a - c = \frac{1}{4} \left( \sum 2|u_i| - 1 \right).
\]

Here \( h^\vee \) is the dual Coxeter number, and \( f \) is the number of abelian flavor symmetry, and the second formula is found in \([25]\). The corresponding Lie group data is listed in table. 5.
Using above formula, we find the central charges of our theory which are listed in table 2. Once we find the central charge \((a, c)\), we use the following formula

\[
(a - c) = -\frac{\text{dim}(\text{Higgs})}{24}
\]

(2.12)

to find out the dimension of Higgs branch and then the corresponding nilpotent orbit, which is listed in table 3.

| \(A_{N-1} \) | \( N^2 - 1 \) | \( N \) | \( N \) | 1 |
| --- | --- | --- | --- | --- |
| \( B_N \) | \((2N + 1)N \) | \( 2N \) | \( 2N - 1 \) | 2 |
| \( C_N \) | \((2N + 1)N \) | \( 2N \) | \( N + 1 \) | 2 |
| \( D_N \) | \( N(2N - 1) \) | \( 2N - 2 \) | \( 2N - 2 \) | 1 |
| \( E_6 \) | 78 | 12 | 12 | 1 |
| \( E_7 \) | 133 | 18 | 18 | 1 |
| \( E_8 \) | 248 | 30 | 30 | 1 |
| \( F_4 \) | 52 | 12 | 9 | 2 |
| \( G_2 \) | 14 | 6 | 4 | 3 |

Table 5. Lie algebra data, here \( h \) is the Coexter number and \( h^\vee \) is the dual Coexter number.

### 2.4 Rank one theory

Among our minimal theory, the rank one theory are listed in the following list

\[
A_1, A_2, D_4, E_6, E_7, E_8, B_3, B_4 \times U(1), C_1 \times U(1), G_2, F_4.
\]

(2.13)

We include \( A_1 \) case here (since among our minimal theory list, it is a free theory, so we have to search again, and it is easy to find theory with minimal flavor symmetry which is called \((A_1, A_3)\) theory or \(H_1\) theory). Here \( B_3, G_2 \) theory have the same central charge \((a, c)\) as the \( D_4 \) theory, and \( B_4 \times U(1), F_4 \) theory have the same \((a, c)\) values as \( E_6 \) theory, and \( C_1 \times U(1) \) have the same \((a, c)\) value as \( A_2 \) theory.

### 3 A conjecture

The flavor central charge of our theory takes the following general form:

\[
k_G = h^\vee - \frac{1}{n} \frac{b}{k + b}.
\]

(3.1)

Here \( n \) is the lacety of Lie group \( G \). The allowed values of \( b \) are listed in table 3.

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\(^4\)We do not require \((k, b)\) coprime here and consider only the case where the irregular singularity is regular semi-simple, and these restrictions do not lose any generality in considering the lower bound of flavor central charge.
\[
\begin{array}{|c|c|c|c|}
\hline
A_{N-1} & b = [N,N-1] & D_N & b = [2N-2,N] \\
E_7 & b = [18,14] & E_8 & b = [30,24,20] \\
C_N & b = [2N,2N+2] & G_2 & b = [6] \\
B_N & b = [2N] & F_4 & b = [12,8] \\
\hline
\end{array}
\]

Table 6. The allowed value of \(b\) for flavor central charges appearing in formula 3.1.

The common denominator of Coulomb branch operators in our theory is \(r = k + b\). For \(r \neq 1\), the minimal value is achieved for \(b = h, k + b = r\). Our conjecture is that if the Coulomb branch has common denominator \(r\), the flavor central charge has the following bound:

\[ k_G \geq h \vee \frac{1}{n} \frac{h}{r}, \]  

for \(r > 1\). For \(r = 1\), we have

\[ k_G \geq k_{\text{min}}, \]  

and \(k_{\text{min}}\) is the value listed in table 2.

4 Conclusion

We performed a scan of \(\mathcal{N} = 2\) SCFT with minimal flavor central charge from theories constructed using 6d \((2,0)\) theory. Let’s make some remarks about these theories:

- A first question is the uniqueness of theory with minimal flavor central charge. We conjecture that they are unique.
- An interesting fact is that some of the theories discussed in this paper has to have extra abelian flavor symmetry. It is interesting to find out whether we can find the theory with only the simple flavor group.
- Our bound on flavor central charge is weaker than what is found in [4] for \(G = BFG\) case.
- We find rank one theory with \(B_3, G_2, F_4\) flavor group. Similar theory is also proposed in [26]. The central charges \((a,c)\) agree, but the Coulomb branch spectrum is different (their value is twice of ours). It would be interesting to find out whether these two sets of theories are the same or not. We also find new rank one theory with \(B_4 \times U(1)\) and \(C_1 \times U(1)\) flavor symmetries (their flavor symmetry might be further enhanced.).
- A quite peculiar feature for \(B_3\) and \(G_2\) theory is that they have a dimension two operator and therefore an exact marginal deformation, but there is no obvious weakly coupled gauge theory description. A relevant feature is that the exact marginal deformation appears in the differential which transforms non-trivially under the outer automorphism twist.
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