Use the Keys Pre-Distribution KDP-scheme for Mandatory Access Control Implementation

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Abstract

The possibility of use the keys preliminary distribution KDP-scheme for mandatory access control realization in the distributed systems with user’s hierarchy is considered. The modified keys preliminary distribution algorithm is suggested. It is developed a method for creation of subsets family for solution this task.

Keywords: keys pre-distribution scheme, KDP-scheme, security model, mandatory access control.

1 Introduction

Mandatory differentiation of access is more rigorous in comparison with a discretionary analog. The centralized security subsystem is necessary for its realization. At system there has to be a uniform center of a decision making comparing mandates of access. This problem is easily solved in local systems. There are some difficulties for the distributed systems. Now this problem is solved on the basis of open keys certificates. Such decision cannot be considered satisfactory. Certificates use sluggish asymmetric cryptographic algorithms. The system based on certificates uses the center of confirmation.

Qualitatively other algorithm can be constructed on the basis of keys preliminary distribution schemes. In this case the role of the central server comes down only to key materials distribution. Network’s subscribers calculate keys of information exchange self-contained. The main problem consists that widely known keys preliminary distribution schemes [1, 2] provide information exchange for each user with everyone. Modification of such schemes is necessary for accounting of security policy of system. Modifications of the Blom’s keys preliminary distribution scheme, considering the forbidden channels it is suggested in work [3]. The organization of simplex channels for the same scheme is realized in article [4]. The solution similar task on the
basis of the KDP scheme is proposed in articles [5, 6]. These works are focused on realization of discretionary security policy. Mandatory security policy demands accounting of hierarchy, both subjects, and objects. The decision on the basis of hash functions is suggested in article [7]. However this approach does not allow realizing exchange between users taking into account hierarchy.

The purpose of this article is development of the keys preliminary distribution scheme allowing realizing mandatory security policy in the distributed computing systems.

2 Keys preliminary distribution scheme

Mandatory access control uses a security tags set which form an algebraic lattice. Security tags are both at users, and at informational objects. At request for access there is a comparison of security tags. The decision is made on the basis of some logical condition.

Let’s designate the set of users in the distributed system $U$. For users of system there is an order relation. We will be limited to the order relation described by the graph in the form of a tree. Dominance of the user $u_i$ over the user $u_j$ we will designate $u_i > u_j$. Also the situation when users are incomparable with each other is possible. Let’s use then designation $u_i <> u_j$. We will be limited to a case of mandatory access control in which only informational streams from below up are resolved. This case corresponds to mandatory security policy on ensuring confidentiality of information. In this case for two users $u_i > u_j$ is resolved only informational stream from $u_j$ to $u_i$. For two incomparable users informational streams in both parties are forbidden.

Let’s set the task to formation the key scheme allowing communicating according to mandatory access control. For this purpose we will construct the keys preliminary distribution scheme calculating pair keys only for the allowed channels.

For the solution this task we modify the KDP scheme of keys preliminary distribution. For system without access control in the KDP scheme key materials is formed in based of set $K = \{k_1, ..., k_n\}$. Key materials beforehand are sent to all users via secure channels. For development of pair keys the system subsets $S = \{S_1, ..., S_m\}$ of set $\{1, ..., n\}$ is used. $m$ – number of users in system. The set $S$ is open. For information exchange with the user $u_j$ the user $u_i$ takes subsets $S_i \cap S_j$. Further he calculates the elements entering in the product of sets.
\( S_{ij} = S_i \cap S_j \). The pair key is calculated with use the key materials \( K \), and subsets \( S_{ij} \):

\[
\kappa_{ij} = \oplus k_l \quad (l \in S_{ij}).
\]

The same operations are carried out by the user \( u_j \) when obtaining the message from \( u_i \).

The scheme described above allows carrying out exchange of messages for each user with everyone in both directions. We modify the scheme, having entered into it asymmetry of keys \( k_{ij} \neq k_{ji} \). For this purpose also we use the key materials \( K \) and a set \( S \). For calculation the key of encrypting for the channel from \( u_j \) to \( u_i \) we use the difference of two sets:

\[
\Delta S_{ij} = \begin{cases} 
    S_i \setminus S_j, & \text{if } S_i \cap S_j = \emptyset \\
    S_i \cap S_j, & \text{otherwise}
\end{cases}
\]

\[
k_{ij} = \bigoplus_{l \in S_{ij}} k_l.
\]

Such approach leads to automatic implementation the requirement of keys asymmetry. For reading messages the user \( u_i \) \((i = 1, ..., m)\) will use keys \( k_{ij} \) \((j = 1, ..., m)\), and for sending messages – keys \( k_{ji} \) \((j = 1, ..., m)\). The suggested scheme is based on the symmetric encrypting that accelerates processes of encrypting and decrypting.

We realize the ban on channels of information exchange. For this purpose we will demand that the corresponding pair keys were zero \( k_{ji} = 0 \), that is \( S_{ij} = \emptyset \). From here we receive requirements to a set of subsets \( S \). The most widespread approach to creation the set of \( S \) is uses of the Sperner’s families [1]. The Sperner’s family [2] is called the family of subsets \( D = \{D_1, ..., D_n\} \) such that, if \( D_i \cap D_j \subseteq D_t \), that either \( t = i \), or \( t = j \). In the unmodified KDP scheme on the basis of elements \( D_i \) the Shperner’s family are formed \( S_{ij} \). We use similar approach for the solution the problem. Let’s create the Shperner’s family with the quantity of elements equal to number of users \( D = \{D_1, ..., D_m\} \). We will form a set \( S \), moving on a tree of users hierarchy leaves to a root. Let’s allocate "leave’s" users \( u_1, ..., u_l \), where \( l \) – quantity of leave’s tops on the tree. Let’s equate, the elements of a set \( S \) corresponding to them, to Sperner’s family elements \( S_i = D_i \) \((i = 1, ..., l)\). Let’s rise from leaves to a tree root. If the top of \( u_i \) has the closest descendants \( u_{i1}, ..., u_{ik} \), then to this user there corresponds the set:

\[
S_i = S_{i1} \cup S_{i2} \cup ...S_{ik} \cup D_i.
\]

This algorithm of formation the set \( S \) leads to realization of the required condition the mandatory access control: if \( u_i > u_j \), then \( S_i \supset S_j \), and \( S_i \setminus S_j \neq \emptyset \), but \( S_j \setminus S_i = \emptyset \). Thus, users
can create a pair key only for the allowed communication channels. Also the requirement for incomparable users is fulfilled: if \( u_i \not< u_j \), then \( S_j \setminus S_i = \emptyset \) and \( S_i \setminus S_j = \emptyset \).

### 3 Example of keys preliminary distribution scheme

Let’s consider implementation the suggested scheme on a simple example. In system seven users are authorized. The hierarchy of users is shown in the figure 1.

![Figure 1: Hierarchy of users.](image)

Let’s define the key materials set with 15 one byte elements:

\[
\begin{align*}
    k_1 &= 00100100, & k_2 &= 10101010, & k_3 &= 01010101, & k_4 &= 11011011, \\
    k_5 &= 11101110, & k_6 &= 00010001, & k_7 &= 10010010, & k_8 &= 10110110, \\
    k_9 &= 00110000, & k_{10} &= 11101110, & k_{11} &= 10110001, & k_{12} &= 11101110, \\
    k_{13} &= 00101101, & k_{14} &= 11010010, & k_{15} &= 01111111.
\end{align*}
\]

We will set the Sperner’s family as follows:

\[
\begin{align*}
    D_1 &= \{1, 2\}, & D_2 &= \{3, 4\}, & D_3 &= \{5\}, & D_4 &= \{6, 7, 8\}, \\
    D_5 &= 9, 10, & D_6 &= \{11, 12, 13\}, & D_7 &= \{14, 15\}.
\end{align*}
\]

For sheet tops the sets \( S \) are defined as

\[
\begin{align*}
    S_4 &= D_4 = \{6, 7, 8\}, & S_5 &= D_5 = \{9, 10\}, & S_6 &= D_6 = \{11, 12, 13\}, & S_7 &= D_7 = \{14, 15\}.
\end{align*}
\]
For other users

\[ S_2 = S_4 \cup S_5 \cup D_2 = \{3, 4, 6, 7, 8, 9, 10\}, \quad S_3 = S_6 \cup S_7 \cup D_3 = \{5, 11, 12, 13, 14, 15\}, \]
\[ S_1 = S_2 \cup S_3 \cup D_1 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}. \]

Process of formation sets \( S \) is presented in the figure 2.

![Diagram depicting the formation sets](image)

Figure 2: Formation sets \( S \).

For calculation of pair keys we will define set differences

\[ \Delta S_{12} = S_1 \setminus S_2 = \{1, 2, 5, 11, 12, 13, 14, 15\}, \]
\[ \Delta S_{13} = S_1 \setminus S_3 = \{1, 2, 3, 4, 6, 7, 8, 9, 10\}, \]
\[ \Delta S_{14} = S_1 \setminus S_4 = \{1, 2, 3, 4, 9, 10, 11, 12, 13, 14, 15\}, \]
\[ \Delta S_{15} = S_1 \setminus S_5 = \{1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 15\}, \]
\[ \Delta S_{16} = S_1 \setminus S_6 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15\}, \]
\[ \Delta S_{17} = S_1 \setminus S_7 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}, \]
\[ \Delta S_{21} = S_2 \setminus S_1 = \emptyset, \quad \Delta S_{23} = S_2 \setminus S_3 = \emptyset, \]
\[ \Delta S_{24} = S_2 \setminus S_4 = \{3, 4, 9, 10\}, \]
\[ \Delta S_{25} = S_2 \setminus S_5 = \{3, 4, 6, 7, 8\}, \]
\[ \Delta S_{26} = S_2 \setminus S_6 = \emptyset, \quad \Delta S_{27} = S_2 \setminus S_7 = \emptyset, \]
\[ \Delta S_{31} = S_3 \setminus S_1 = \emptyset, \quad \Delta S_{32} = S_3 \setminus S_2 = \emptyset, \quad \Delta S_{34} = S_3 \setminus S_4 = \emptyset, \quad \Delta S_{35} = S_3 \setminus S_5 = \emptyset, \]
\[ \Delta S_{36} = S_3 \setminus S_6 = \{5, 14, 15\}, \]
\[ \Delta S_{37} = S_3 \setminus S_7 = \{5, 11, 12, 13\}, \]
\[ \Delta S_{41} = S_4 \setminus S_1 = \emptyset, \quad \Delta S_{42} = S_4 \setminus S_2 = \emptyset, \quad \Delta S_{43} = S_4 \setminus S_3 = \emptyset, \quad \Delta S_{45} = S_4 \setminus S_5 = \emptyset, \]
\[ \Delta S_{46} = S_4 \setminus S_6 = \emptyset, \quad \Delta S_{47} = S_4 \setminus S_7 = \emptyset, \]
\[ \Delta S_{51} = S_5 \setminus S_1 = \emptyset, \quad \Delta S_{52} = S_5 \setminus S_2 = \emptyset, \quad \Delta S_{53} = S_5 \setminus S_3 = \emptyset, \quad \Delta S_{54} = S_5 \setminus S_4 = \emptyset, \]
\[ \Delta S_{56} = S_5 \setminus S_6 = \emptyset, \quad \Delta S_{57} = S_5 \setminus S_7 = \emptyset, \]
\[ \Delta S_{61} = S_6 \setminus S_1 = \emptyset, \quad \Delta S_{62} = S_6 \setminus S_2 = \emptyset, \quad \Delta S_{63} = S_6 \setminus S_3 = \emptyset, \quad \Delta S_{64} = S_6 \setminus S_4 = \emptyset, \]
\[ \Delta S_{65} = S_6 \setminus S_5 = \emptyset, \quad \Delta S_{67} = S_6 \setminus S_7 = \emptyset, \]
\[ \Delta S_{71} = S_7 \setminus S_1 = \emptyset, \quad \Delta S_{72} = S_7 \setminus S_2 = \emptyset, \quad \Delta S_{73} = S_7 \setminus S_3 = \emptyset, \quad \Delta S_{74} = S_7 \setminus S_4 = \emptyset, \]
\[ \Delta S_{75} = S_7 \setminus S_5 = \emptyset, \quad \Delta S_{76} = S_7 \setminus S_6 = \emptyset. \]

Pair keys will be defined by equalities:

\[ k_{12} = k_1 \oplus k_2 \oplus k_5 \oplus k_{11} \oplus k_{12} \oplus k_{14} \oplus k_{15} = 10111110, \]
\[ k_{13} = k_1 \oplus k_2 \oplus k_3 \oplus k_4 \oplus k_6 \oplus k_7 \oplus k_8 \oplus k_9 \oplus k_{10} = 11000011, \]
\[ k_{14} = k_1 \oplus k_2 \oplus k_3 \oplus k_4 \oplus k_9 \oplus k_{10} \oplus k_{11} \oplus k_{12} \oplus k_{13} \oplus k_{14} \oplus k_{15} = 00101000, \]
\[ k_{15} = k_1 \oplus k_2 \oplus k_3 \oplus k_4 \oplus k_5 \oplus k_6 \oplus k_7 \oplus k_8 \oplus k_{11} \oplus k_{12} \oplus k_{13} \oplus k_{14} \oplus k_{15} = 00000101, \]
\[ k_{16} = k_1 \oplus k_2 \oplus k_3 \oplus k_4 \oplus k_5 \oplus k_6 \oplus k_7 \oplus k_8 \oplus k_9 \oplus k_{10} \oplus k_{14} \oplus k_{15} = 10000000, \]
\[ k_{17} = k_1 \oplus k_2 \oplus k_3 \oplus k_4 \oplus k_5 \oplus k_6 \oplus k_7 \oplus k_8 \oplus k_9 \oplus k_{10} \oplus k_{11} \oplus k_{12} \oplus k_{13} = 01011110, \]
\[ k_{21} = 0, \quad k_{23} = 0, \]
\[ k_{24} = k_3 \oplus k_4 \oplus k_9 \oplus k_{10} = 01111000, \]
\[ k_{25} = k_3 \oplus k_4 \oplus k_6 \oplus k_7 \oplus k_8 = 10111011, \]
\[ k_{26} = 0, \quad k_{27} = 0, \]
\[ k_{31} = 0, \quad k_{32} = 0, \quad k_{34} = 0, \quad k_{35} = 0, \]
\[ k_{36} = k_5 \oplus k_{14} \oplus k_{15} = 01100011, \]
\[ k_{37} = k_5 \oplus k_{11} \oplus k_{12} \oplus k_{13} = 10011111, \]
\[ k_{41} = 0, \quad k_{42} = 0, \quad k_{43} = 0, \quad k_{45} = 0, \quad k_{46} = 0, \quad k_{47} = 0, \]
\[ k_{51} = 0, \quad k_{52} = 0, \quad k_{53} = 0, \quad k_{54} = 0, \quad k_{56} = 0, \quad k_{57} = 0, \]
\[ k_{61} = 0, \quad k_{62} = 0, \quad k_{63} = 0, \quad k_{64} = 0, \quad k_{65} = 0, \quad k_{67} = 0, \]
\[ k_{71} = 0, \quad k_{72} = 0, \quad k_{73} = 0, \quad k_{74} = 0, \quad k_{75} = 0, \quad k_{76} = 0. \]

The constructed keys preliminary distribution scheme satisfies the hierarchy of subjects shown in the figure 1. Only information channels are resolved "from below-up". Incomparable users also cannot communicate.
The suggested scheme can be used also in systems with the hierarchy of users other than a tree. Let’s review an example in the figure 3.

Figure 3: Hierarchy of users.

By the same principle elements of the set $S$ are calculated (the Figure 4.)

Figure 4: Hierarchy of users.

The keys contradicting mandatory security policy are equal to zero.
Conclusion

The scheme suggested in this article allows realizing keys preliminary distribution of a symmetric enciphering for the distributed systems with user’s hierarchy. As well as in case of the KDP scheme the Sperner’s families are used. However KDP scheme allows forming bidirectional channels of information exchange whereas the suggested scheme is focused on simplex channels. The suggested modification of the keys preliminary distribution scheme does not increase the size of key materials. It is an indispensable condition for using this approach to creation the protected systems.

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