Solitary electromagnetic waves propagation in the asymmetric oppositely-directed coupler

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We consider the electromagnetic waves propagating in the system of coupled waveguides. One of the system components is a standard waveguide fabricated from nonlinear medium having positive refraction and another component is a waveguide produced from an artificial material having negative refraction. The metamaterial constituting the second waveguide has linear characteristics and a wave propagating in the waveguide of this type propagates in the direction opposite to direction of energy flux. It is found that the coupled nonlinear solitary waves propagating both in the same direction are exist in this oppositely-directed coupler due to linear coupling between nonlinear positive refractive waveguide and linear negative refractive waveguide. The corresponding analytical solution is found and it is used for numerical simulation to illustrate that the results of the solitary wave collisions are sensible to the relative velocity of the colliding solitary waves.

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I. INTRODUCTION

The waveguide structure fabricated from the two closely placed waveguides is of common use in the fiber and integrated optics. Coupling between the waveguides is due to tunnel penetration of light from one waveguide into another waveguide [1, 2]. This coupler preserves direction of light propagation, and for this reason it is named a directed coupler. It was found [3, 4] that the steady state pair of electromagnetic pulses can exist in the extended directed coupler or twin-core fibers [5]. Sometimes it termed soliton. There is a large body of publications devoted to investigation of the soliton generation and its propagation, see for example [6, 7, 8, 9].

Recent progress in nanofabrication has lead to the design of new materials with highly unusual optical properties [11, 12, 13, 14, 15, 16, 17, 18], of which negative refraction is an example. Negative refraction occurs in media in which the wave vector of the electromagnetic wave is antiparallel to the Poynting vector [21, 22, 23]. In the particular case when the real parts of the dielectric permittivity and magnetic permeability in the medium simultaneously take on negative values in some frequency range, the property of negative refraction will appear. The existence of such media was demonstrated experimentally first in the microwave and then in the near-infrared ranges [12, 13, 14, 15, 16, 17]. Negative refraction can be realized for the media with spatial dispersion [20] which is large enough and in the photonic crystals [21, 23, 24]. Recently bulk metamaterials that show negative refraction for all incident angles in the visible region were presented [27, 28].

The unusual properties of negative refractive (NR) index materials reveal themselves most prominently when the refractive index of the same medium can be positive in one spectral region and negative in another [22, 29, 30, 31]. New wave propagation phenomena can also be expected when a wave passes through, or is localized near, an interface between such a material and a conventional dielectric, i.e., positive refractive (PR) index material [32, 33, 34, 35]. We can refer to these cases as negative-positive-refraction ones. The intriguing example of the negative-positive refraction medium is the coupler, where one of the waveguides is fabricated from a material with a negative refractive index. This device acts as a (distributed) mirror. The radiation entering one waveguide leaves the device through the other waveguide at the same end but in the opposite direction. For this reason, this device can be called a oppositely-directed coupler (ODC). The ODC is known to support the propagation of linear waves with a gap in their spectrum (forbidden zone). It has been shown recently [36] that due to this gap the nonlinear oppositely directed coupler is bistable. Bistability results from the multi-valued dependence of the transmission coefficient on the input-wave power. It is noteworthy that this effect has no analogy in conventional directed couplers consisting of uniform waveguides.
without a mirror-based feedback mechanism. It was found that if the waveguide nonlinear optical properties are characterized by the third order susceptibility, a coupled pair of the steady state pulses (each one is localized in its own waveguide) can exist in ODC.

The fabrication of the transparent nonlinear NR waveguide is a complicated and still unresolved problem. The linear NR waveguide could be more suitable for realization. However there is question is it possible a quasisolitonic regime for the waves in such an antisymmetric coupler consisting of a nonlinear conventional dielectric waveguide coupled with a linear NR waveguide. In this paper we consider the extended asymmetric oppositely-directed coupler (AODC). We found that the pulses are very robust against perturbations. It allows a definite conclusion that the steady state pulses are very close to the lossesless materials. Slowly varying envelope of the electric field

\[ E \]

These values are assumed to be real, that corresponds to the lossless materials. Slowly varying envelope of the electric field \( E_\ell(z,t) \) in \( j \)-th channel is written as \( E_\ell(z,t) = A_0q_\ell(z,t), v_{gj} \) is the group-velocity of the wave in \( j \)-th channel, \( (j = 1, 2) \). Here we assume that the group-velocity dispersion can be neglected. In the following we also assume that the synchronism condition is satisfied: \( \Delta \beta = 0 \). It is convenient to use new normalized variable

\[ Q_1 = \sqrt{K_{21}}q_1 e^{-i \Delta \beta z}, \quad Q_2 = \sqrt{K_{12}}q_2 e^{i \Delta \beta z}, \]

\[ \zeta = z/L_c, \quad \tau = t^{-1}(t - z/V_0), \quad L_c = (K_{12}K_{21})^{-1/2}, \]

\[ t_0 = L_c(v_{g1} + v_{g2})/2v_{g1}v_{g2}, \quad V_0^{-1} = (v_{g1} - v_{g2})/2v_{g1}v_{g2}. \]

In the normalized variables the ultimate system of equations \( 1 \) for the AODC reads as

\[ i \left( \frac{\partial}{\partial \zeta} + \frac{\partial}{\partial \tau} \right) Q_1 + Q_2 + r|Q_1|^2Q_1 = 0, \]

\[ i \left( \frac{\partial}{\partial \zeta} - \frac{\partial}{\partial \tau} \right) Q_2 - Q_1 = 0. \]

The parameter of nonlinearity is

\[ r = \frac{2\pi \omega_0}{cK_{21}\sqrt{K_{12}K_{21}}} \sqrt{\frac{\mu_1(\omega_0)}{\varepsilon_1(\omega_0)}} A_0 \chi_{eff}^{(3)}. \]

Let us consider the linear wave limit of the AODS equations. From \( 2 \) one can get

\[ i \left( \frac{\partial}{\partial \zeta} + \frac{\partial}{\partial \tau} \right) Q_1 + Q_2 = 0, \]

\[ i \left( \frac{\partial}{\partial \zeta} - \frac{\partial}{\partial \tau} \right) Q_2 - Q_1 = 0. \]

If we take the Fourier transformation

\[ Q_{1,2} = \int_{-\infty}^{+\infty} \tilde{Q}_{1,2} e^{-iv\tau + i\kappa x} \frac{dk d\omega}{4\pi^2}, \]
the equations (3) result in the following linear system of equation

\[(\nu - \kappa)\tilde{Q}_1 + \tilde{Q}_2 = 0,\]
\[(\nu + \kappa)\tilde{Q}_2 + \tilde{Q}_1 = 0.\]

This system of equations has the nonzero solution only if the corresponding determinant

\[
\det \begin{pmatrix} \nu - \kappa & 1 \\ 1 & \nu + \kappa \end{pmatrix}
\]

is equal to zero. That leads to the dispersion relation

\[
\nu(\kappa) = \pm \sqrt{1 + \kappa^2}.
\]

(4)

Thus the spectrum of the linear waves has the gap \(\Delta \nu_g = 2\). This gap is characteristic feature for a distributed mirror [39]. Hence, the AODC in linear wave limit acts as a mirror.

In following consideration it is suitable to take the real variables form of equations (2). By using the real variables \(Q_{1,2} = a_{1,2} e^{i\Phi_{1,2}}\) one obtains

\[
\left( \frac{\partial}{\partial \zeta} + \frac{\partial}{\partial \tau} \right) a_1 = a_2 \sin \Phi,
\]
\[
\left( \frac{\partial}{\partial \zeta} - \frac{\partial}{\partial \tau} \right) a_2 = a_1 \sin \Phi,
\]
\[
\left( \frac{\partial}{\partial \zeta} + \frac{\partial}{\partial \tau} \right) \Phi_1 = \frac{a_2}{a_1} \cos \Phi + ra_1^2,
\]
\[
\left( \frac{\partial}{\partial \zeta} - \frac{\partial}{\partial \tau} \right) \Phi_2 = -\frac{a_1}{a_2} \cos \Phi,
\]

where \(\Phi = \Phi_1 - \Phi_2\). From the amplitude equation it follows that

\[
\frac{\partial}{\partial \zeta} (a_2^2 - a_1^2) = \frac{\partial}{\partial \tau} (a_1^2 + a_2^2).
\]

Hence,

\[
\frac{\partial}{\partial \zeta} \int_{-\infty}^{+\infty} (a_2^2 - a_1^2) \, d\tau = (a_1^2 + a_2^2) \bigg|_{-\infty}^{+\infty}.
\]

(6)

In the case of solitary waves, for which the electromagnetic fields vanish at infinity are considered, the right part of (6) is equal to zero. It leads to integral of motion

\[
\int_{-\infty}^{+\infty} (a_2^2 - a_1^2) \, d\tau = \text{const.}
\]

(7)

It is the modified Manley-Rowe relation. As usually, in the case of the quadratic nonlinear PR media Manley-Rowe relation looks like

\[
\int_{-\infty}^{+\infty} (a_2^2 + a_1^2) \, d\tau = \text{const.}
\]

The difference of the expression (7) from conventional Manley-Rowe relation is explained by the fact that the energy flux for the waves in one waveguide opposite in direction to energy flux of other waveguide, while their wave vectors are approximately the same. This flux pattern is an inherent feature of the negative-positive refraction media.
III. ANALYTICAL SOLUTION — THE STEADY-STATE PULSE

To consider the solitary steady state waves in AODC by similar way as in [38] we have start from the equations (5). Suppose that solutions of these equations are depend only on single variable

\[ \eta = \frac{\zeta + \beta \tau}{\sqrt{1 - \beta^2}} \]

with free parameter \( \beta \). Suppose \( u_1 = \sqrt{1 + \beta a_1} \) and \( u_2 = \sqrt{1 - \beta a_2} \). The system of the equations (5) takes the following form

\[ \frac{\partial}{\partial \eta} u_1 = u_2 \sin \Phi, \quad \frac{\partial}{\partial \eta} u_2 = u_1 \sin \Phi, \] (8)

\[ \frac{\partial}{\partial \eta} \phi_1 = \frac{u_2}{u_1} \cos \Phi + \Theta u_1^2, \] (9)

\[ \frac{\partial}{\partial \eta} \phi_2 = -\frac{u_1}{u_2} \cos \Phi, \] (10)

where

\[ \Theta = \frac{r}{1 + \beta} \sqrt{1 - \beta^2}. \]

We can also write an equation for the phase difference

\[ \frac{\partial}{\partial \eta} \Phi = \left( \frac{u_1}{u_2} + \frac{u_2}{u_1} \right) \cos \Phi + \Theta u_1^2. \] (11)

The phase equations can be used to get one equation for the phase difference. Finally, the total system of equations reads

\[ \frac{\partial}{\partial \eta} u_1 = u_2 \sin \Phi, \] (12)

\[ \frac{\partial}{\partial \eta} u_2 = u_1 \sin \Phi, \] (13)

\[ \frac{\partial}{\partial \eta} \Phi = \left( \frac{u_1}{u_2} + \frac{u_2}{u_1} \right) \cos \Phi + \Theta u_1^2. \] (14)

We are looking for a solution in a form of the solitary wave, it corresponds with the following boundary condition

\[ a_{1,2} \rightarrow 0 \quad \text{at} \quad \eta \rightarrow \pm \infty. \]

From the equation (12) and (13) it follows \( u_1^2 = u_1^2 \), or \( u_1 = \epsilon u_2 \), where \( \epsilon = \pm 1 \). Hence, the system of equations (12)-(14) is reduced to following pare of equations

\[ \frac{\partial}{\partial \eta} u_1 = \epsilon u_1 \sin \Phi, \] (15)

\[ \frac{\partial}{\partial \eta} \Phi = 2 \epsilon \cos \Phi + \Theta u_1^2. \] (16)

Multiplying the last equation by \( u_1^2 \sin \Phi \) and taking into account the equation (15), we get the second integral of motion

\[ u_1^2 \left( \cos \Phi + \frac{\epsilon \Theta}{4} u_1^2 \right) = C_2. \] (17)

Due to the boundary condition, the value of this integral is equal to zero. As we are looking for a non zero solution of (15) and (16), one can write

\[ \cos \Phi + \frac{\epsilon \Theta}{4} u_1^2 = 0. \] (18)
Substitution of (18) into (15) leads to
\[(du_1/d\eta)^2 = u_1^2 \left(1 - (\Theta/4)^2 u_1^4\right).\]
Choosing the variable \(u_1 = w^{-1/2}\) reduces this expression into equation
\[(dw/d\eta)^2 = 4 \left(w^2 - (\Theta/4)^2\right),\]
which has the following solution
\[w(\eta) = (\Theta/4) \cosh 2(\eta - \eta_2).\]
Thus, the solution of (15) and (16) is
\[u_2^2(\eta) = u_2^2(\eta) = \frac{4/\Theta}{\cosh 2(\eta - \eta_2)}.\]
By using the phase’s equations and (19) one can write
\[\frac{\partial}{\partial \eta} \phi_1 = \frac{3}{4} \Theta u_1^2(\eta) = \frac{3}{\cosh 2(\eta - \eta_2)},\]
\[\frac{\partial}{\partial \eta} \phi_2 = \frac{1}{4} \Theta u_1^2(\eta) = \frac{1}{\cosh 2(\eta - \eta_2)}\]
that yields
\[\phi_1(\eta) = \phi_1(-\infty) + 3 \arctan (\exp 2(\eta - \eta_2)),\]
\[\phi_2(\eta) = \phi_2(-\infty) + \arctan (\exp 2(\eta - \eta_2)).\]
The phases \(\phi_{1,2}(-\infty)\) should be chosen in such a way that (18) is satisfied. As the solitary wave’s amplitude tends to zero \(u_{1,2}(\infty) \to 0\), the relation (18) at infinity reduces to
\[\cos \Phi(-\infty) = \cos(\phi_1(-\infty) - \phi_2(-\infty)) = 0.\]
In the numerical simulation we choose \(\phi_1(-\infty) = 0\) and \(\phi_2(-\infty) = -\pi/2\).
The amplitudes \(a_{1,2}\) are defined by the following expressions
\[a_1^2(\eta) = \frac{4}{\Theta(1 + \beta) \cosh 2(\eta - \eta_2)},\]
\[a_2^2(\eta) = \frac{4}{\Theta(1 - \beta) \cosh 2(\eta - \eta_2)}.\]
The expressions (22) and (23) describe the steady state solitary wave propagating in AODC under consideration. This wave looks like a gap soliton propagating in the nonlinear Bragg grating, however there are no any periodic structures. Gap in the linear wave spectrum (13) is due to the flux pattern in this negative-positive refraction media.
Some remarks are worthy to be made about the free parameter \(\beta\). The negative value of the parameter \(\beta\) corresponds to the solitary wave propagating in the direction of the axis \(\zeta\). The solitary wave characterized by positive value of the parameter \(\beta\) propagates in the opposite direction. The large amplitudes (more powerful solitary waves) correspond to large positive values of the parameter \(\beta\). For the negative values of the parameter \(\beta\) the quasisolitons with smaller amplitudes have smaller values of the parameter \(\beta\) however the absolute value of the velocity determined by the parameter \(\beta\) is larger for the less powerful solitary waves. We will refer to the Fig.1 later in discussion the influence of the linear losses at the quasisoliton.

IV. NUMERICAL SIMULATION

It is common knowledge that the completely integrable evolution nonlinear equations have a special solutions describing elastic interaction of solitary waves [40]. These waves are named solitons. It is our opinion that the system of equations (2) does not belong to the class of completely integrable equations. Hence the solution of these equations does not
FIG. 1: (Color online) The plot for the solitary wave amplitude $a_{1,2}$ dependence on parameter $\beta$. Red solid curve corresponds to the amplitude of the pulse in the nonlinear PR channel, dashed green curve is for the NR channel.

FIG. 2: Crossing collision between two solitary waves with $\beta = -0.9$ and $\beta = -0.5$. Left panel is for the solitary wave in the PR waveguide, right panel is for the solitary wave in the NR waveguide.

represent true soliton. We will denote them as quasisolitons. To investigate interaction between the steady state solitary waves (22) and (23) the numerical simulation was pursued.

Numerical solution of the system of equations (2) is performed using the scheme of the finite differences. The conservation of the first integral of motion (7) is used to control the computational error. In the consequent an integration step over the evolution variable $\tau$ is set to $h\tau = 0.0005$. The nonlinearity is set to $r = 1$. (The smaller nonlinearities correspond to the quasisolitons with the same profiles but the smaller amplitudes (defined by the parameter $\Theta$ which is linearly proportional to the nonlinearity coefficient.)

It was found that the results of the collision depend on the relative velocity of the pulses. Fig 2 illustrates collision between quasisolitons characterized by $\beta = -0.9$ and $\beta = -0.5$. The relative interaction velocity is 0.4. The quasisolitons drop some radiation after interaction and weakly radiating wave could be noticed in the left panel of the picture as a result of collision. Velocities and the amplitudes of the resulting quasisolitons are slightly different from the parameters of the quasisolitons before the collision due to loss of some part of energy.

Decrease the difference between velocities of colliding pulses results in long-range interaction between pulses. The interaction distance depends on the relation of the velocities. In the Fig 3 is shown interaction of two quasisolitons characterized by $\beta = -0.7$ and $\beta = -0.5$. The relative interaction velocity is 0.2. The quasisolitons exchange energy at the distance and form a transitory bound state.

It was mentioned before that the quasisolitons with the same absolute values of velocity have different amplitudes. To investigate the robustness of the quasisoliton depending on its energy, at the Fig 4 we present results of modeling
the collision between two steady-state solitary waves with the same absolute values of the velocity \( \beta = 0.7 \) and \( \beta = -0.7 \). A more energetic quasisoliton with \( \beta = 0.7 \) remains unchanged after collision and the less energetic quasisoliton with \( \beta = -0.7 \) loose some radiation and changes its trajectory.

Thus, the collision of two steady state pulses with different velocities has shown significant robustness. Small amplitude radiation appearing after collision attests that the quasisoliton eventually be disappeared.

For the current state of fabrication technology the losses in the real NR materials are considerable. High value of losses renders steady state pulse propagation impossible. Nevertheless, the role of small losses would be considered. To check the influence of linear losses in the NR channel on the solitary wave formation and propagation, the model equations (2) were modified by including the additional term in the equation for a NR channel

\[
i \left( \frac{\partial}{\partial \zeta} + \frac{\partial}{\partial \tau} \right) Q_1 + Q_2 + r|Q_1|^2 Q_1 = 0
\]

(24)

\[
i \left( \frac{\partial}{\partial \zeta} - \frac{\partial}{\partial \tau} - \gamma \right) Q_2 - Q_1 = 0
\]

(25)

The Fig. 5 illustrates the evolution of same pulses as in the Fig. 4 placed at the boundaries however the linear losses in the NR channel are taken into consideration \( \gamma = 0.05 \). Comparing these two pictures one may conclude that even small losses affect considerably the propagation properties of the solitary waves. Propagating in the dissipative
medium these pulses loose energy and the pulse with positive $\beta$ slows down to sustain the steady state regime. The pulse with negative $\beta$ slightly accelerates to support the quasi-steady regime of propagation (notice a Fig. 1).

V. CONCLUSION

We considered the nonlinear solitary waves propagating in the nonlinear opposite-directed coupler. One of its components is a nonlinear waveguide made from the material with positive index of refraction. Another channel is fabricated from linear dielectric material with negative refraction index. For the system of coupled waves the analytical solution for the electromagnetic waves in the form of steady state solitary waves is found. It is interesting to notice that the wave in the nonlinear channel affects the wave propagation in the neighboring linear channel. This influence results in coupling and a steady propagation of the solitary waves in both waveguides, the wave in the nonlinear waveguide draws the wave in the linear NR waveguide.

It was shown that the result of interaction of the quasisolitons depends on their velocities. Provided that relative velocity of the two colliding quasisolitons is large, the pulses collide almost like the solitons. A weak radiation or a weak pulse emerges as a result of collision. At small relative velocities of the quasisolitons the interaction between two colliding solitary waves could result in a strong energy exchange between colliding solitary waves and formation of the temporarily coupled state of the interacting quasisolitons. This phenomena is known for the case of nonlinear directed coupling. In the case under consideration the formation of the coupled state demands for an additional investigation. It could be possible that a long-living coupled state of two solitary waves is exists there.

The influence of the linear losses in the NR waveguide on the existence of the solitary waves was studied. The wave loses its energy as it propagates through the dissipative medium. In order to compensate the losses the entering pulse in a form of the solitary wave characterized by positive value of parameter $\beta$ (its propagation direction is opposite to the coordinate axis $\zeta$) permanently transforms into another solitary wave, less energetic, having smaller value of the parameter $\beta$. It results in slowing down the wave in the lossy medium. After that the wave stops in the waveguide and changes the direction of its propagation. Oppositely the entering pulse in a form of the solitary wave characterized by negative parameter $\beta$ (its propagation direction is coincides to the coordinate axis $\zeta$) tends to increase its velocity (i.e. to decrease parameter $\beta$ which is negative) to sustain the steady-state regime of propagation in presence of the losses.

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FIG. 5: Two solitary waves with $\beta = 0.7$ and $\beta = -0.7$ in a system with the lossy NR channel $\gamma = 0.05$. Left panel is for the solitary wave in the PR waveguide, right panel is for the solitary wave in the NR waveguide.
[1] A. Yariv, and P. Yeh, *Optical waves in crystals* (John Wiley and Sons, New York, Chichester, Brisbane, Toronto, Singapore, 1984).

[2] A. Yariv, IEEE J.Quant.Electron, 9, 919 (1973).

[3] S. R. Friberg, Y. Silberberg, M. K. Oliver, M. J. Andrejco, M. A. S aifi, and P. W. Smith, Appl. Phys. Lett. 51, 1135 (1987)

[4] D. R. Heatley, E. M. Wright, G. I. Stegeman, Appl. Phys. Lett. 53, 172 (1988).

[5] A. Hasegawa, *Optical Solitons in Fibers* (Springer-Verlag, Berlin, 1990).

[6] S. R. Friberg, Y. Silberberg, M. J. Andrejco, M. A. S aifi, and P. W. Smith, Appl. Phys. Lett. 51, 1135 (1987).

[7] D. R. Heatley, E. M. Wright, G. I. Stegeman, Appl. Phys. Lett. 53, 172 (1988).

[8] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[9] S. R. Friberg, Y. Silberberg, M. J. Andrejco, M. A. S aifi, and P. W. Smith, Appl. Phys. Lett. 51, 1135 (1987).

[10] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[11] S. R. Friberg, Y. Silberberg, M. J. Andrejco, M. A. S aifi, and P. W. Smith, Appl. Phys. Lett. 51, 1135 (1987).

[12] D. R. Heatley, E. M. Wright, G. I. Stegeman, Appl. Phys. Lett. 53, 172 (1988).

[13] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[14] S. R. Friberg, Y. Silberberg, M. J. Andrejco, M. A. S aifi, and P. W. Smith, Appl. Phys. Lett. 51, 1135 (1987).

[15] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[16] S. R. Friberg, Y. Silberberg, M. J. Andrejco, M. A. S aifi, and P. W. Smith, Appl. Phys. Lett. 51, 1135 (1987).

[17] D. R. Heatley, E. M. Wright, G. I. Stegeman, Appl. Phys. Lett. 53, 172 (1988).

[18] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[19] S. R. Friberg, Y. Silberberg, M. J. Andrejco, M. A. S aifi, and P. W. Smith, Appl. Phys. Lett. 51, 1135 (1987).

[20] D. R. Heatley, E. M. Wright, G. I. Stegeman, Appl. Phys. Lett. 53, 172 (1988).

[21] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[22] S. R. Friberg, Y. Silberberg, M. J. Andrejco, M. A. S aifi, and P. W. Smith, Appl. Phys. Lett. 51, 1135 (1987).

[23] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[24] S. R. Friberg, Y. Silberberg, M. J. Andrejco, M. A. S aifi, and P. W. Smith, Appl. Phys. Lett. 51, 1135 (1987).

[25] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[26] S. R. Friberg, Y. Silberberg, M. J. Andrejco, M. A. S aifi, and P. W. Smith, Appl. Phys. Lett. 51, 1135 (1987).

[27] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[28] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[29] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[30] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[31] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[32] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[33] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[34] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[35] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[36] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[37] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[38] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[39] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).

[40] A. Yariv, IEEE J. Quant. Electron, 9, 919 (1973).