Renormalizability of a quark-gluon model with soft BRST breaking in the infrared region

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Abstract

We prove the renormalizability of a quark-gluon model with a soft breaking of the BRST symmetry, which accounts for the modification of the large distance behavior of the quark and gluon correlation functions. The proof is valid to all orders of perturbation theory, by making use of softly broken Ward identities.

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1 Introduction

The Gribov-Zwanziger framework \[1, 2, 3\] consists in restricting the domain of integration in the Feynman path integral within the Gribov horizon. It has motivated extended studies of nonperturbative tools for investigating the infrared behavior of the gluon and ghost correlation functions, see \[4\] for the use of modified Schwinger-Dyson equations for QCD, \[5, 6, 7, 8\] for recent analytical results and \[9, 10, 11\] for numerical data obtained through lattice simulations.

Zwanziger has been able to show that the restriction of the path integral within the Gribov horizon for the gluon can be achieved by adding to the Fadeev–Popov action in the Landau gauge a local action, depending on new fields, with well-defined interactions with the gluons and their Fadeev-Popov ghosts in the Landau gauge \[2, 3\]. However, this local action violates the BRST symmetry by a soft term, so that BRST symmetry is only enforced in the scaling limit. The yet unorthodox point of view that the BRST symmetry can be broken in the IR region of QCD was heuristically anticipated by Fujikawa \[12\]. It is not in contradiction with any given physical principle, since it is by no means necessary that the QCD microscopic theory possesses a unitary sector for its partons, namely the quarks and gluons, to warrantee unitarity properties among the sector of bound states that constitute its spectrum. The only requirement is that the modified theory remains renormalizable and that the BRST symmetry is recovered in the ultraviolet region, in order to suitably describe the asymptotic properties in terms of almost deconfined partons, as predicted by asymptotic freedom and short distance expansion. From a physical point of view, it is gratifying for the quark-gluon model introduced in \[13\] that the modification of the usual Feynman propagators into a Gribov-type propagators eliminates from the beginning all partons from the spectrum, since their modified propagators have no poles on the real axis, a property that anticipates quite well the confinement.

What justified our previous work \[13\] is that the genuine geometrical approach of Zwanziger leaves aside the quarks, which do not participate to the Gribov phenomenon, while the idea of a parton model suggests that quarks and gluons should be treated on the same footing, and quark propagators should have an analogous behavior as the gluon one in the infra-red domain. An idea for getting such modified quark propagators was thus needed, which goes beyond the Gribov question. A generalization of the work of Zwanziger was also needed \[6, 7, 8\] in order to achieve a different behavior for the shape of the genuine Gribov-gluon propagator \(D(q^2)\) that vanishes at \(q^2 = 0\), for accommodating recent lattice simulations that seem to indicate that the gluon propagator goes to a non-vanishing constant at very small \(q^2\) \[9, 10, 11\].

To provide such an improved, local and renormalizable, quantum field theory that gives the wanted modifications in the infra-red region, both for quarks and gluons, we proposed in \[13\] the following picture. Given a theory of partons (eg quark and gluons in four dimensions, or a scalar fields in dimensions such that the ultraviolet divergences are renormalizable or super-renormalizable), it can always be coupled to a topological field theory made of new fields arranged as BRST trivial doublets, in such a way that the partons are already confined at the tree level by their mixing with the unphysical fields of the topological
field theory. We found that such a mixing can be generally allowed by a soft breaking of the BRST symmetry. Here, confinement is meant in a very simple way. The propagators of all fields have only poles at complex positions. This implies that the theory has no vacuum for the partons and all observables are made of composite operators, defined by solving the cohomology of the BRST operator. These composite operators can be renormalized in the standard way, with expectation values related to the parameters of the soft breaking mechanism. This idea was inspired by the algebraic characterization of the local terms that Zwanziger introduced to complete the Faddeev–Popov action for the gluons. Part of the Zwanziger action can be recognized as a topological action, which involves bosons and fermions that transform under the BRST symmetry as a system of two BRST trivial doublets, and it is BRST-exact. The remaining part of Zwanziger’s action breaks the BRST symmetry in a soft way, and yields a Gribov propagator for the transverse gluon.

The addition of the new fields arranged as BRST-exact doublets eventually allows for the introduction of massive parameters that can be used to modify the infrared behavior of the theory, without changing the set of observables. The necessity of breaking the BRST symmetry can be easily understood within this framework. If the added action were BRST-exact, nothing would be changed for the predictions of the original parton theory. Indeed, the observables, defined as the cohomology of the BRST operator, are the same and the effect of integrating over the new fields would only consist in multiplying the partition function by one. So, if the new field dependence is through BRST-invariant terms (and thus BRST-exact terms because they transform as BRST-exact doublets), there is no way to improve the infra-red behavior of the amplitudes. One can notice that the possibility of adding such a BRST-exact terms can be related to the formal invariance of the path integral under arbitrary changes of variables for the parton fields. By doing such changes of variables and playing with Lagrange multipliers and determinants formula, one can indeed recover the class of BRST-exact action that are bilinear in the new fields that have the opposite statistics to the partons. To obtain the wanted modification of the infra-red behavior of parton correlators, one must go further, and consider the possibility of an explicit soft breaking of the BRST symmetry. This, of course, can modify the parton propagators in the low energy domain, without changing their ultraviolet behavior, provided the theory is renormalizable. This will also modify the numerical values of the observables, by giving them a dependence from the parameters of the soft breaking.

In the case of the improved quark-gluon theory, the properties of the breaking and the respective consequences for the theory can be summarized as follows:

- The breaking term is soft, meaning that its dimension is smaller than the space-time dimension. As a consequence, the breaking can be neglected in the ultraviolet region, where one recovers the notion of exact BRST symmetry.

- The BRST operator preserves nilpotency. Moreover, the set of physical operators of the theory, identified with the cohomology classes of the BRST operator, is left unmodified. This occurs

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1 More precisely, the propagators display violation of reflection positivity, a feature which invalidates the interpretation of partons as excitations of the physical spectrum of the theory.

2 In string theory, this idea has been already used by adding so-called “topological packages” to any given 2d-string action, which allows one to show in this way possible relationships between the different string models [14, 15, 16].
because the new additional fields are introduced as BRST doublets. As such, they do not alter the cohomology of the BRST operator.

- The soft breaking of the BRST symmetry is introduced in a way compatible with the renormalizability. More precisely, it is associated to a quadratic term in the fields obtained by demanding that the additional fields couple linearly to the original fields of the theory, so that the resulting propagators turn out to be modified in the infrared region. As a consequence, a given correlation function can display a different behavior, when going from the deep ultraviolet to the infrared region in momentum space.

- The soft BRST symmetry breaking is meant to be an explicit breaking, i.e. it is not a spontaneous symmetry breaking, which would give rise to Goldstone massless fields, and thus to a quite different framework, as it will be explained elsewhere.

- All massive parameters should be related to the unique scale of the theory, namely $\Lambda_{\text{QCD}}$, by requiring that the massive soft parameters satisfy suitably gap type equations which allow to determine them in a self-consistent way. This is the case, for example, of the massive Gribov parameter $\gamma$ which is fixed by a gap equation \[2, 3, 5, 6, 7\], see Sect.4.

Following the procedure described above, in \[13\] a model accounting for a soft BRST symmetry breaking giving rise to a modification of the long distance behavior of the quark propagator was established. In fact, the model predicts

$$
\langle \psi(k)\bar{\psi}(-k) \rangle = \frac{i\gamma_\mu k_\mu + A(k)}{k^2 + \mathcal{M}^2(k)} ,
$$

instead of the standard Dirac propagator for the quarks $\frac{i\gamma_\mu k_\mu}{k^2}$. In this equation

$$
A(k) = \frac{2M_1^2 M_2}{k^2 + m^2} ,
$$

is a function depending on the soft breaking mass parameters $M_1$, $M_2$ and $m$. From expression (2) one sees that the function $A(k)$ vanishes in the deep ultraviolet region where the usual perturbative behavior for the propagator is recovered. It is worth mentioning that expression (2) provides a good fit for the dynamical mass generation for quarks in the infrared region in the Landau gauge, as reported by lattice numerical simulations of the quark propagator \[17, 18\]. We recall that the function (2) is analogous to the one appearing in the gluon propagator within the Gribov-Zwanziger framework \[2, 3\]. In fact, as discussed in \[13\], the Gribov-Zwanziger action can be recovered through the introduction of a soft breaking, related to the appearance of the Gribov parameter $\gamma$. This parameter is needed to implement the restriction to the Gribov horizon, which turns out to be at the origin of the soft breaking of the BRST symmetry \[6, 7\]. In particular, for the tree level gluon propagator, one finds

$$
\langle A_\mu^a(k)A_\nu^b(-k) \rangle = \delta^{ab} \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{1}{k^2 + \mathcal{M}^2(k)} ,
$$

with

$$
\mathcal{M}^2(k) = \frac{\gamma^4}{k^2 + \mu^2} ,
$$

(4)
where the second mass parameter $\mu$ accounts for the nontrivial dynamics of the auxiliary fields needed to localize Zwanziger’s horizon function, see [6, 7].

It is of course striking how similar are the modified gluon and quark propagators, in the sense that the introduction of the massive parameters by the above mechanism has eliminated the infra-red problem for both type of fields. Within the context of plain perturbation theory, these parameters can be finetuned, and even be used as plain infra-red regulators that are controlled by softly broken Ward identities. However, in a confining theory like QCD, they must be related to physical observables, and non-perturbatively computed as a function of the basic parameter of the theory, $\Lambda_{\text{QCD}}$. The clear advantage of our approach is that the modified theory is no more submitted to the Gribov ambiguity and the parton (quark and gluons) propagators have no poles at real values. The model excludes their appearance in the spectrum from the beginning, and its perturbative expansion can be possibly compared to the prediction of non-perturbative approach such as the lattice formulation in the Landau gauge. The method solves in a quite simple conceptual way the necessity of introducing massive parameters, not only for the Gribov ambiguities but also for defining the scale of the chiral symmetry breaking.

The goal of this paper is to prove an elementary, but necessary property of this model: it has to be multiplicatively renormalizable, to ensure that the breaking of the BRST symmetry remains soft, at any given finite order of perturbation theory, and that no further parameters than those which modify the infra-red behavior of propagators can appear. We will do this by employing the algebraic renormalization [19]. Notice that such a proof can be generalized to other cases, where one may wish to introduce an infra-red cut-off for other theories with nontrivial gauge invariance and control its effect by softly broken Ward identities, within the context of locality. This could be of relevance for the study of non-exactly solvable super-renormalizable theories, like 3-dimensional gauge theories, where one must control the way the infra-red regularization is compatible with the BRST symmetry, or even for gravity, when it is computed at a finite order of perturbation theory.

The paper is organized as follows. In Section 2 we provide a short overview of the model and of its soft BRST symmetry breaking. In Section 3 the algebraic proof of the renormalizability is given. In Section 4 we consider the inclusion of the Gribov-Zwanziger action and we discuss the renormalizability of the resulting model. Finally, in Section 5 we collect our conclusions.

2 The quark model and its BRST soft breaking term

We start with the Yang-Mills action quantized in the Landau gauge,

$$S_{\text{inv}} = \int d^4x \left( \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\psi}^i (\gamma_\mu)_{\alpha\beta} D_{\mu}^{ij} \psi^j + ib^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_{\mu}^{ab} c^b \right), \quad (5)$$

where

$$D_{\mu}^{ij} = \partial_\mu \delta^{ij} - ig(T^a)^{ij} A_\mu^a, \quad (6)$$
is the covariant derivative in the fundamental representation of the $SU(N)$ gauge group, with generators $(T^a)^{ij}$, and

$$D^a_{\mu} = \partial^a_{\mu} = -g f^{abc} A^c_{\mu},$$

(7) is the covariant derivative in the adjoint representation. The first set of small Latin indices $\{i,j,\ldots\} \in \{1,\ldots,N\}$ will be used to denote the fundamental representation, while the second set $\{a,b,\ldots,h\} \in \{1,\ldots,N^2 - 1\}$ will be employed for the adjoint representation. The set of Greek indices $\{\alpha,\beta,\gamma,\delta\}$ stand for spinor indices. The remaining Greek indices will denote space-time indices.

The action (5) is invariant under the BRST transformations, namely

$$sA^a_{\mu} = -D^a_{\mu} c^b,$$
$$s\psi^i_{\alpha} = -ig c^a (T^a)^{ij} \psi^j_{\alpha},$$
$$s\bar{\psi}^i_{\alpha} = -ig \bar{\psi}^i_{\alpha} c^a (T^a)^{ji},$$
$$s c^a = \frac{1}{2} g f^{abc} c^b c^c,$$
$$s \bar{c}^a = ib^a,$$
$$s b^a = 0.$$  

(8)

Following [13], a model for the dynamical quark mass generation can be constructed by introducing two BRST doublets of spinor fields $(\xi^i, \theta^i)$ and $(\eta^i, \lambda^i)$, transforming as

$$s\xi^i_{\alpha} = \theta^i_{\alpha}, \quad s\theta^i_{\alpha} = 0,$$
$$s\eta^i_{\alpha} = \lambda^i_{\alpha}, \quad s\lambda^i_{\alpha} = 0.$$  

(9)

The propagation of these fields is described by the following BRST-exact action

$$S_{\xi\lambda} = s \int d^4x \left( -\eta^i_{\alpha} \partial^2 \xi^i_{\alpha} + \bar{\xi}^i_{\alpha} \partial^2 \eta^i_{\alpha} + m^2 (\eta^i_{\alpha} \xi^i_{\alpha} - \bar{\xi}^i_{\alpha} \eta^i_{\alpha}) \right)$$

$$= \int d^4x \left( -\bar{\lambda}^i_{\alpha} \partial^2 \lambda^i_{\alpha} + \xi^i_{\alpha} \partial^2 \bar{\lambda}^i_{\alpha} - \bar{\eta}^i_{\alpha} \partial^2 \theta^i_{\alpha} + \bar{\theta}^i_{\alpha} \partial^2 \eta^i_{\alpha} + m^2 (\bar{\lambda}^i_{\alpha} \xi^i_{\alpha} + \xi^i_{\alpha} \bar{\lambda}^i_{\alpha} + \bar{\eta}^i_{\alpha} \theta^i_{\alpha} - \bar{\theta}^i_{\alpha} \eta^i_{\alpha}) \right),$$  

(10)

where $m$ is a mass parameter. Further, we introduce the coupling of the spinors $(\xi^i, \theta^i)$ and $(\eta^i, \lambda^i)$ with the matter field $\psi^i_{\alpha}$,

$$S_M = \int d^4x \left( M_1 (\bar{\psi}^i_{\alpha} \psi^i_{\alpha} + \bar{\psi}^i_{\alpha} \xi^i_{\alpha}) - M_2 (\bar{\lambda}^i_{\alpha} \psi^i_{\alpha} + \bar{\psi}^i_{\alpha} \bar{\lambda}^i_{\alpha}) \right).$$  

(11)

Evidently, the action (10) is BRST-invariant. This is not the case of $S_M$, which will give rise to a soft breaking of the BRST symmetry, parameterized by the two soft mass parameters $M_1, M_2$. In fact,

$$sS_M = \Delta,$$  

(12)
where

\[
\Delta = \int d^4x \left( M_1^2 \left( \bar{\theta}^\alpha i \psi_\alpha - \bar{\psi}_\alpha i \theta^\alpha \right) + igM_1^2 \epsilon^a \left( \bar{\xi}_\alpha^i (T^a)^{ij} \psi_\alpha^j - \bar{\psi}_\alpha^i (T^a)^{ij} \xi_\alpha^j \right) \\
+ igM_2^2 \epsilon^a \left( \bar{\lambda}_\alpha^i (T^a)^{ij} \psi_\alpha^j - \bar{\psi}_\alpha^i (T^a)^{ij} \lambda_\alpha^j \right) \right). \tag{13}
\]

Notice that \( \Delta \) is a soft breaking, \textit{i.e.} it is of dimension less than four.

Performing the integration over the auxiliary spinor fields, yields a nonlocal action for \( \psi \) and \( \bar{\psi} \),

\[
S_\psi = \int d^4x \left( \bar{\psi}_\alpha^i \left( \gamma^\mu \right)_{\alpha\beta} D^\mu \psi_\beta^j - 2M_1^2 M_2 \bar{\psi}_\alpha^i \left( \frac{1}{\partial^2 - m^2} \right) \psi_\alpha^i \right). \tag{14}
\]

It is important to emphasize that the introduction of the auxiliary spinor fields can be seen as a tool for localizing the nonlocal term \( \Delta \) that appears in the fermionic sector, in the same way as the nonlocal Gribov-Zwanziger horizon function is cast in local form through the introduction of a suitable set of auxiliary BRST doublet fields \[2, 3, 5, 7\].

### 2.1 Introducing sources for controlling the soft symmetry breaking

In order to prove that the model described by the action

\[
S = S_{\text{inv}} + S_{\xi\lambda} + S_M , \tag{15}
\]

is multiplicatively renormalizable, we follow the procedure outlined by Zwanziger in the study of the Gribov horizon in the Landau gauge \[2, 3\]. It amounts to embedding the action \( S \) in a larger model, \( S \rightarrow S_0 \), displaying exact BRST invariance. This is achieved by treating the breaking term \( \Delta \) as a composite operator, which is introduced into the theory through a suitable set of external sources. The original action \( S \) is thus recovered from the extended action \( S_0 \) by demanding that the sources acquire a particular value, which we shall refer to as the physical value. The renormalizability of \( S \) follows thus by proving the renormalizability of the extended action \( S_0 \).

In order to introduce the extended invariant action \( S_0 \), we make use of the following set of sources \((J,H), (\bar{J},\bar{H}), (K,G), (\bar{K},\bar{G})\) and \((N,P)\), assembled in BRST doublets \[3\], \textit{i.e.}

\[
\begin{align*}
   sJ_{ij}^{\alpha\beta} &= H_{ij}^{\alpha\beta} , \quad sH_{ij}^{\alpha\beta} = 0, \\
   s\bar{J}^{ij}_{\alpha\beta} &= \bar{H}^{ij}_{\alpha\beta} , \quad s\bar{H}^{ij}_{\alpha\beta} = 0, \\
   sK_{ij}^{\alpha\beta} &= G_{ij}^{\alpha\beta} , \quad sG_{ij}^{\alpha\beta} = 0, \\
   s\bar{K}^{ij}_{\alpha\beta} &= \bar{G}^{ij}_{\alpha\beta} , \quad s\bar{G}^{ij}_{\alpha\beta} = 0, \\
   sN &= P , \quad sP = 0. \tag{16}
\end{align*}
\]

\(^3\)For future purpose, we recall that all quantum numbers of fields and sources are displayed in Tables \[1\] and \[2\] including the charge \( Q_{4, \mathcal{N}} \) that will be defined through expression \( \mathcal{Q}_{23} \).
The invariant action that accounts for the extra spinor fields and for the breaking term is thus defined as

$$ S_{JK} = s \int d^4 x \left( -\bar{\eta}_0^i \partial^2 \xi^i_\alpha + \bar{\xi}_\alpha \partial^2 \eta^i_\alpha + P(\bar{\eta}_0^i \xi^i_\alpha - \bar{\xi}_\alpha \eta^i_\alpha) + \sigma NP \right. $$

$$ \left. + J_{ij}^{ij} \bar{\xi}_\alpha \psi^i_\beta + J_{ij}^{ij} \bar{\psi}^i_\beta \alpha + K_{ij}^{ij} \bar{\xi}_\alpha \psi^i_\beta + K_{ij}^{ij} \bar{\psi}^i_\beta \alpha \right) , \tag{17} $$

which, explicitly, reads

$$ S_{JK} = \int d^4 x \left( -\bar{\lambda}_0^i \partial^2 \xi^i_\alpha - \bar{\xi}_\alpha \partial^2 \lambda^i_\alpha - \bar{\eta}_0^i \partial^2 \theta^i_\alpha + P(\bar{\lambda}_0^i \xi^i_\alpha + \bar{\xi}_\alpha \lambda^i_\alpha + \bar{\eta}_0^i \theta^i_\alpha - \theta^i_\alpha \eta^i_\alpha) + \sigma P^2 \right. $$

$$ \left. + H_{ij}^{ij} \bar{\xi}_\alpha \psi^i_\beta + \bar{H}_{ij}^{ij} \bar{\psi}^i_\beta \alpha - J_{ij}^{ij} \left( \bar{\theta}_0^i \psi^i_\beta - \bar{\xi}_\alpha i g c^a (T^a)^{jk} \psi^k_\beta \right) - J_{ij}^{ij} \left( i g \bar{\psi}^k_\beta c^a (T^a)^{kj} \xi^i_\alpha - \psi^i_\beta \theta^i_\alpha \right) \right) $$

$$ + G_{ij}^{ij} \bar{\lambda}_\alpha \psi^i_\beta + \bar{G}_{ij}^{ij} \bar{\psi}^i_\beta \lambda^i_\alpha + K_{ij}^{ij} \bar{\lambda}_\alpha i g c^a (T^a)^{jk} \psi^k_\beta - K_{ij}^{ij} i g \bar{\psi}^k_\beta c^a (T^a)^{kj} \lambda^i_\alpha \right) , \tag{18} $$

whose BRST invariance is manifest. The parameter $\sigma$, in expression (15), is a dimensionless parameter needed for renormalization purposes. For the so called physical values of the external sources, we have

$$ J_{ij}^{ij} \bigg|_{\text{phys}} = J_{ij}^{ij} \bigg|_{\text{phys}} = 0 , $$

$$ K_{ij}^{ij} \bigg|_{\text{phys}} = K_{ij}^{ij} \bigg|_{\text{phys}} = 0 , $$

$$ H_{ij}^{ij} \bigg|_{\text{phys}} = H_{ij}^{ij} \bigg|_{\text{phys}} = M^2 \delta_{ij} \delta_{\alpha\beta} , $$

$$ G_{ij}^{ij} \bigg|_{\text{phys}} = G_{ij}^{ij} \bigg|_{\text{phys}} = -M_2 \delta_{ij} \delta_{\alpha\beta} , $$

$$ P_{\text{phys}} = m^2 , $$

$$ N_{\text{phys}} = 0 . \tag{19} $$

The BRST-invariant extended action $S_0$ is thus defined as

$$ S_0 = S_{\text{inv}} + S_{\xi\lambda} + S_{JK} + S_{\text{ext}} . \tag{20} $$

It is easily checked that the starting action $S$, eq.(15), is recovered from the extended action $S_0$ when taking the physical values of the sources, eq.(19), namely

$$ S_0 \bigg|_{\text{phys}} = S + \int d^4 x \sigma m^4 . \tag{21} $$

It is worth noticing that the action $S_0$ possesses an additional $U(4N)$ global symmetry, provided by

$$ Q^{ij}_{\alpha\beta} S_0 = 0 , \tag{22} $$

where

$$ Q^{ij}_{\alpha\beta} = \int d^4 x \left( \xi^i_\alpha \delta \delta \xi^j_\beta - \bar{\xi}_\beta \delta \delta \xi^j_\beta + \lambda^i_\beta \delta \delta \lambda^j_\alpha - \bar{\lambda}_\alpha \delta \delta \lambda^j_\alpha + \theta^i_\alpha \delta \delta \theta^j_\alpha - \bar{\theta}_0^i \delta \delta \theta^j_\alpha + \bar{\eta}_0^i \delta \delta \eta^i_\alpha + \bar{\eta}_0^j \delta \delta \eta^j_\alpha + J^{jk}_{\beta\alpha} \delta \delta J^{ij}_{\alpha\beta} \right. $$

$$ \left. - \bar{J}^{ik}_{\delta\alpha} \delta \delta J^{jk}_{\delta\alpha} + K^{ik}_{\beta\sigma} \delta \delta K^{jk}_{\beta\sigma} - \bar{K}^{ik}_{\delta\beta} \delta \delta K^{jk}_{\delta\beta} + G^{ik}_{\alpha\sigma} \delta \delta G^{jk}_{\beta\sigma} - \bar{G}^{ik}_{\delta\alpha} \delta \delta G^{jk}_{\delta\alpha} + H^{ik}_{\beta\sigma} \delta \delta H^{jk}_{\beta\sigma} - \bar{H}^{ik}_{\delta\beta} \delta \delta H^{jk}_{\delta\beta} \right) . \tag{23} $$

8
The trace of the operator $Q^{ij}_{\alpha\beta}$, i.e. $Q^{\alpha\alpha}_{ii} = Q_{4N}$, defines a new conserved charge, and thus an additional quantum number for the fields and sources, allowing for the introduction of a multi-index $I = (i, \alpha)$, with $I \in \{1, \ldots, 4N\}$, for the fields $\phi = (\xi, \eta, \lambda, \theta)$ and sources $\Gamma = (K, J, H, G)$. Accordingly, we shall set
\[
\phi^i_\alpha = \phi^I, \quad \Gamma^{ij}_{\alpha\beta} = \Gamma^{ij}_{I\beta}.
\] (24)

From now on, we shall make use of the multi-index notation.

## 3 Algebraic proof of the renormalizability

Let us face now the issue of the renormalizability of the extended action $S_0$, a task which we shall undertake by making use of the algebraic renormalization [19]. Let us start by establishing the set of Ward identities fulfilled by the action $S_0$. To that purpose, we add external sources, $L^a, \Omega^a_\mu, Y^i_\alpha$ and $\bar{Y}^i_\alpha$, coupled to the non-linear BRST variations of the fields, namely
\[
S_{\text{ext}} = s \int d^4x \left( \Omega^a_\mu A^a_\mu + L^a c^a + \bar{Y}^i_\alpha \psi^i_\alpha + \bar{\psi}^j Y^j_i \right)
\]
\[
= \int d^4x \left( -\Omega^a_\mu D_\mu c^a + \frac{1}{2} gf^{abc} L^a c^b c^c - ig \bar{Y}^i_\alpha \psi^i_\alpha (T^a)^{ij} \psi^j_\beta - ig \bar{\psi}^j Y^j_i (T^a)^{ji} \bar{\psi}^i_\alpha \right),
\] (25)
with
\[
s\Omega^a_\mu = sL^a = sY = s\bar{Y} = 0.
\] (26)

The term (25) allows one to convert the BRST symmetry into the corresponding Slavnov-Taylor identity. Thus, to prove the renormalizability of the model we shall consider the more general action
\[
\Sigma = S_0 + S_{\text{ext}}.
\] (27)
3.1 Ward Identities

The complete action \[ (27) \] fulfills a rich set of Ward Identities, namely:

- The Slavnov-Taylor identity
  \[ S(\Sigma) = 0, \]  
  where
  \[
  S(\Sigma) = \int d^4x \left( \frac{\delta \Sigma}{\delta Y^a_\alpha} \frac{\delta \Sigma}{\delta \psi^a_\alpha} + \frac{\delta \Sigma}{\delta Y^a_\alpha} \frac{\delta \Sigma}{\delta \bar{\psi}^a_\alpha} + \frac{\delta \Sigma}{\delta c^a} \frac{\delta \Sigma}{\delta L^a} + \frac{i b^a}{\delta c^a} + \theta^I \frac{\delta \Sigma}{\delta \xi^I} + \bar{\lambda}^I \frac{\delta \Sigma}{\delta \bar{\eta}^I} \right) + \bar{\theta}^I \frac{\delta \Sigma}{\delta \bar{\xi}^I} - \lambda^I \frac{\delta \Sigma}{\delta \bar{\eta}^I}
  \]\n
- The gauge condition:
  \[ \frac{\delta \Sigma}{\delta b^a} = i \partial_\mu A^a_\mu, \]  

- The antighost equation:
  \[ \frac{\delta \Sigma}{\delta \bar{c}^a} + \partial_\mu \frac{\delta \Sigma}{\delta \bar{\Omega}_\mu^a} = 0, \]  

- The ghost equation:
  \[ G^a_\alpha \Sigma = \Delta^a_{cl}, \]  

  with
  \[
  G^a_\alpha = \int d^4x \left( \frac{\delta}{\delta c^a} - i f^{abc} \frac{\delta}{\delta c^b} \frac{\delta}{\delta \bar{c}^c} - ig (T^a)^{jk} \left( J^{l}_{\alpha} \frac{\delta}{\delta H^l_{\alpha}} + K^l_{\alpha} \frac{\delta}{\delta G^l_{\alpha}} \right) - ig (T^a)^{kj} \left( J^{l}_{\alpha} \frac{\delta}{\delta H^l_{\alpha}} + K^l_{\alpha} \frac{\delta}{\delta G^l_{\alpha}} \right) \right),
  \]\n
  and
  \[ \Delta^a_{cl} = \int d^4x \left( g f^{abc} \bar{\Omega}^b_\mu A^c_\mu + g f^{abc} L^b \bar{c}^c + ig Y^i_\alpha \left( T^a \right)^{ij} \bar{\psi}^j_\alpha - ig \bar{\psi}^i_\alpha \left( T^a \right)^{ij} Y^j_\alpha \right).
  \]

- The classical rigid invariance
  \[ R^{IJ}_\Sigma = 0, \]  

  where
  \[
  R^{IJ} = \int d^4x \left( \frac{\lambda^I}{\delta \xi^I} + \frac{\lambda^J}{\delta \xi^J} + \frac{\tilde{H}^{lk}_{\alpha}}{\delta G^{lk}_{\alpha}} - \frac{H^{lk}_{\alpha}}{\delta G^{lk}_{\alpha}} + \frac{J^{lk}_{\alpha}}{\delta K^{lk}_{\alpha}} \right).
  \]

- The equations of motion of the doublet fields:
  \[
  \frac{\delta \Sigma}{\delta \bar{\eta}^I} = -\partial^2 \theta^I + P\theta^I,
  \]
  \[
  \frac{\delta \Sigma}{\delta \bar{\xi}^I} = -\partial^2 \bar{\eta}^I - P\bar{\eta}^I - J^I_{\alpha} \bar{\psi}^j_\alpha,
  \]
  \[
  \frac{\delta \Sigma}{\delta \lambda^I} + K^I_{\alpha} \frac{\delta \Sigma}{\delta Y^j_{\alpha}} = -\partial^2 \lambda^I + P\lambda^I + H^{lj}_{\alpha} \bar{\psi}^j_\alpha,
  \]
  \[
  \frac{\delta \Sigma}{\delta \bar{\xi}^I} + J^I_{\alpha} \frac{\delta \Sigma}{\delta Y^j_{\alpha}} = -\partial^2 \lambda^I + P\lambda^I + H^{lj}_{\alpha} \bar{\psi}^j_\alpha.
  \]
The Ward identity for the source $N$:
\[ \frac{\delta \Sigma}{\delta N} = 0 . \] (38)

3.2 The invariant counterterm

In order to characterize the most general invariant counterterm which can be freely added to all orders in perturbation theory [19], we perturb the classical action $\Sigma$ by adding an integrated local polynomial $\Sigma^{\text{count}}$ of dimension bounded by four, and with vanishing ghost number as well as $Q_{4N}$ charge. We demand thus the perturbed action, $(\Sigma + \epsilon \Sigma^{\text{count}})$, where $\epsilon$ is an expansion parameter, fulfills, to the first order in $\epsilon$, the same Ward identities fulfilled by the classical action $\Sigma$, i.e. eqs. (28)-(38). This requirement gives rise to the following constraints for the counterterm $\Sigma^{\text{count}}$:

\[ B_{\Sigma} \Sigma^{\text{count}} = 0 , \] (39)

\[ \frac{\delta}{\delta \theta^0} \Sigma^{\text{count}} = 0 , \] (40)

\[ \left( \frac{\delta}{\delta c} + \partial_\mu \frac{\delta}{\delta \Omega_{\mu}^a} \right) \Sigma^{\text{count}} = 0 , \] (41)

\[ G_{a}^{\text{count}} = 0 , \] (42)

\[ R_{IJ}^{\text{count}} = 0 , \] (43)

\[ \frac{\delta \Sigma^{\text{count}}}{\delta N} = 0 , \] (44)

\[ \frac{\delta \Sigma^{\text{count}}}{\delta \eta^I} = 0 , \] (45)

\[ \frac{\delta \Sigma^{\text{count}}}{\delta \theta^I} = 0 , \] (46)

\[ \frac{\delta \Sigma^{\text{count}}}{\delta \chi^I} + K_{IJ}^{\text{count}} \frac{\delta \Sigma^{\text{count}}}{\delta Y_{\alpha}^I} = 0 , \] (47)

\[ \frac{\delta \Sigma^{\text{count}}}{\delta \xi^I} + J_{IJ}^{\text{count}} \frac{\delta \Sigma^{\text{count}}}{\delta Y_{\alpha}^I} = 0 . \] (48)

where the operator $B_{\Sigma}$ in eq. (39) stands for the nilpotent linearized Slavnov-Taylor operator,

\[
B_{\Sigma} = \int d^4 x \left( \frac{\delta \Sigma}{\delta \Omega_{\mu}^a} \frac{\delta}{\delta A_{\mu}^a} + \frac{\delta \Sigma}{\delta A_{\mu}^a} \frac{\delta}{\delta \Omega_{\mu}^a} - \frac{\delta \Sigma}{\delta \psi_{\alpha}^i} \frac{\delta}{\delta Y_{\alpha}^i} - \frac{\delta \Sigma}{\delta Y_{\alpha}^i} \frac{\delta}{\delta \psi_{\alpha}^i} + \frac{\delta \Sigma}{\delta \psi_{\alpha}^i} \frac{\delta}{\delta Y_{\alpha}^i} \\ + \frac{\delta \Sigma}{\delta c} \frac{\delta}{\delta L} + \frac{\delta \Sigma}{\delta L} \frac{\delta}{\delta c} - i b^a \frac{\delta}{\delta c^a} + \bar{\theta}^I \frac{\delta}{\delta \xi^I} + \bar{\lambda}^I \frac{\delta}{\delta \eta^I} + \lambda^I \frac{\delta}{\delta \eta^I} + \frac{\delta}{\delta \chi^I} + \bar{H}_{IJ}^{\text{count}} \frac{\delta}{\delta K_{IJ}^{\text{count}}} + \bar{G}_{IJ}^{\text{count}} \frac{\delta}{\delta K_{IJ}^{\text{count}}} + P \frac{\delta}{\delta N} \right).
\] (49)

The first constraint, eq. (39), identifies the invariant counterterm as the solution of the cohomology of the
operator $\mathcal{B}_\Sigma$ in the space of the integrated local field polynomials of dimension four. From the general results on the cohomology of Yang-Mills theories \cite{19}, it follows that $\Sigma^{\text{count}}$ can be written as

$$
\Sigma^{\text{count}} = \frac{a_0}{4} \int d^4x F_{\mu \nu}^a F_{\mu \nu}^a + \mathcal{B}_\Sigma \Delta^{(-1)},
$$

(50)

where $\Delta^{(-1)}$ is a local integrated polynomial in all fields and sources, with dimension four, ghost number minus one and vanishing $Q_{4N}$ charge, namely

$$
\Delta^{(-1)} = \int d^4x \left( a_1 A_\mu^a \Theta^a_\mu + a_2 \partial_\mu c^a A_\mu^a + a_3 c^a L^a + a_4 c^a \theta^a + a_5 \frac{g}{2} f^{abc} c^b c^c + a_6 \bar{\lambda}^I K^{Ij}_\alpha \psi^j_\alpha \\
+ a_7 \bar{\psi}^j_\alpha K^{Ij}_\alpha \lambda^I + a_8 \bar{\eta}^I G^{Ij}_\alpha \psi^j_\alpha + a_9 \bar{\psi}^j_\alpha G^{Ij}_\alpha \eta^I + a_{10} \bar{\theta}^I K^{Ij}_\alpha \theta^j_\alpha + a_{11} \bar{\eta}^j_\alpha K^{Ij}_\alpha \theta^I + a_{12} \bar{\xi}^I G^{Ij}_\alpha \eta^j_\alpha \\
+ a_{13} \bar{Y}^j_\alpha G^{Ij}_\alpha \xi^I + a_{14} \bar{\xi}^I J^{Ij}_\alpha \psi^j_\alpha + a_{15} \bar{\psi}^j_\alpha J^{Ij}_\alpha \xi^I + a_{16} \bar{\psi}^i_\alpha \psi^i_\alpha + a_{17} \bar{\eta}^i_\alpha \psi^i_\alpha + a_{18} \bar{H}^{Ij}_\alpha J^{Ij}_\alpha + a_{19} \bar{J}^{Ij}_\alpha H^{Ij}_\alpha \\
+ a_{20} c^a c^a K^{Ij}_\alpha G^{Ij}_\alpha + a_{21} \bar{c}^a \bar{c}^a N + a_{22} A_\mu^a A_\mu^a N + a_{23} A_\mu^a A_\mu^a G^{Ij}_\alpha K^{Ij}_\alpha + a_{24} G^{Ij}_\alpha \partial^2 K^{Ij}_\alpha + a_{25} G^{Ij}_\alpha G^{Ij}_\alpha N \\
+ a_{27} \bar{\theta}^I \partial^2 \xi^I + a_{28} \bar{\xi}^I \partial^2 \eta^I + a_{29} A_\mu^a A_\mu^a \bar{\xi}^I \eta^I + a_{30} A_\mu^a A_\mu^a \bar{\xi}^I \eta^I + a_{31} c^a \bar{c}^a \bar{\xi}^I \xi^I \\
+ a_{32} c^a \bar{c}^a \bar{\xi}^I \eta^I + a_{33} \bar{P} \bar{\xi}^I \xi^I + a_{34} \bar{P} \bar{\xi}^I \eta^I + a_{35} c^a \bar{c}^a K^{Ij}_\alpha G^{Ij}_\alpha + a_{36} G^{Ij}_\alpha \partial^2 K^{Ij}_\alpha + a_{37} P N \\
+ a_{38} \bar{\xi}^I \lambda^I N + a_{39} \bar{\lambda}^I \xi^I N + a_{40} \bar{\theta}^I \eta^I N + a_{41} \bar{\eta}^I \theta^I N + a_{42} A_\mu^a A_\mu^a G^{Ij}_\alpha K^{Ij}_\alpha \right).
$$

(51)

The coefficients \{a_0, a_1, a_2, \ldots, a_{42}\} in expressions (50) and (51) stand for arbitrary constants parameters.

After a straightforward analysis using the conditions (39)-(48) it turns out that the only non-vanishing coefficients are: \(a_0, a_1, a_2, a_6, a_7, a_{14}, a_{15}, a_{16}, a_{17}, a_{37}\), with the following relations between them

$$
\begin{align*}
 a_2 &= a_1, \\
 a_{16} &= a_{15} = a_7, \\
 -a_{17} &= a_{14} = a_6.
\end{align*}
$$

(52)

and

$$
\begin{align*}
 a_{16} &= a_{15} = a_7, \\
 -a_{17} &= a_{14} = a_6.
\end{align*}
$$

(53)

Then, after redefining

$$
\begin{align*}
 a_6 + a_7 &\quad \mapsto \quad a_2, \\
 a_{37} &\quad \mapsto \quad \sigma a_3,
\end{align*}
$$

(54)

for the form of the final allowed counterterm one finds

$$
\begin{align*}
\Sigma^{\text{count}} &= \int d^4x \left( \left( \frac{a_0 + 4a_1}{4} \right) F_{\mu \nu}^a F_{\mu \nu}^a - a_1 \partial_\mu c^a \Theta^a_\mu - a_1 \partial_\mu c^a \partial_\mu c^a - a_1 i g \bar{\psi}^j_\alpha (\gamma_\mu)_{\alpha \beta} T^{\alpha \beta} \psi^j_\beta A^a_\mu \\
&\quad - a_2 \psi^j_\alpha D^j_\mu (\gamma_\mu)_{\alpha \beta} \psi^j_\beta + a_3 \sigma P^2 \right),
\end{align*}
$$

(55)

which corresponds to the usual Yang-Mills counterterm in the Landau gauge with the addition of an energy vacuum term, $\sigma P^2$, related to the mass $m$. 

12
3.3 Stability

It remains now to discuss the stability of the model, *i.e.* to check that the counterterm $\Sigma^{\text{count}}$ can be reabsorbed in the classical action $\Sigma$ by means of a multiplicative redefinition of the coupling constant $g$, of the parameter $\sigma$, and of the fields and sources \[19\], namely

\[ \Sigma(g, \sigma, \phi, \Phi) + \epsilon \Sigma^{\text{count}} = \Sigma(g_0, \sigma_0, \phi_0, \Phi_0) + O(\epsilon^2) , \]  

(56)

where $\phi$ stands for all fields and $\Phi$ for the sources,

\[ \phi \in \{ A, \psi, c, \tilde{c}, b, \xi, \tilde{\xi}, \lambda, \tilde{\lambda}, \theta, \tilde{\theta}, \eta, \tilde{\eta}\} , \]
\[ \Phi \in \{ \Omega, L, Y, \tilde{Y}, J, \tilde{J}, K, \tilde{K}, G, \tilde{G}, H, \tilde{H}, N, P\} . \]  

(57)

Thus, by defining

\[ \phi_0 = Z_{\phi}^{1/2} \phi , \]
\[ \Phi_0 = Z_{\Phi} \Phi , \]
\[ g_0 = Z_g g , \]
\[ \sigma_0 = Z_\sigma \sigma , \]

(58)

we obtain

\[ Z_A^{1/2} = 1 + \frac{\epsilon}{2} (a_0 + 2a_1) , \]
\[ Z_g = 1 - \epsilon a_0 , \]
\[ Z_\psi^{1/2} = Z_{\tilde{\psi}}^{1/2} = 1 + \frac{\epsilon}{2} a_2 , \]
\[ Z_\sigma = 1 + \epsilon a_3 . \]  

(59)

Expressions \[60\] constitute the independent renormalization factors. All the remaining factors can be expressed in terms of the renormalization factors appearing in eq.\[59\]. In fact, for the Lagrange multiplier and Faddeev-Popov ghost fields we have

\[ Z_b = Z_A^{-1/2} , \]
\[ Z_c^{1/2} = Z_{\tilde{c}}^{1/2} = Z_g^{-1/2} Z_A^{-1/4} , \]  

(60)

while the renormalization of the external BRST sources are found

\[ Z_\Omega = Z_g^{-1/2} Z_A^{-1/4} , \]
\[ Z_L = Z_A^{1/2} , \]
\[ Z_Y = Z_Y = Z_g^{-1/2} Z_A^{1/4} Z_\psi^{-1/2} . \]  

(61)

As expected from \[55\], the renormalization properties of the usual Yang-Mills sector are preserved. For the doublet fields we obtain

\[ Z_\xi^{1/2} = Z_{\tilde{\xi}}^{1/2} = Z_\lambda^{1/2} = Z_{\tilde{\lambda}}^{1/2} = 1 , \]
\[ Z_\theta^{1/2} = Z_{\tilde{\theta}}^{1/2} = Z_\eta^{-1/2} = Z_{\tilde{\eta}}^{-1/2} = Z_g^{1/2} Z_A^{-1/4} . \]  

(62)
Finally, for the remaining sources we have
\[
Z_H = Z_H = Z_G = Z_G = Z^{-1/2}_\psi,
\]
\[
Z_J = Z_J = Z_K = Z_K = Z^{-1/2}_gZ_\tau^{1/4}Z^{-1/2}_\psi,
\]
\[
Z_P = 1 .
\]

This ends the proof of the multiplicative renormalizability of the model proposed in this article. For completeness, let us give the expression of the bare action written in terms of the renormalized fields and parameters:
\[
\Sigma = \int d^4x \left( \frac{1}{2} Z_A (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) \partial_\mu A^a_\nu + Z_g Z_A^{3/2} g f^{amn} \partial_\mu A^n_\mu A^m_\nu + \frac{1}{4} Z_g Z_A^2 g^2 f^{abc} f^{amn} A^n_\mu A^n_\nu A^m_\mu A^m_\nu 
+ Z^\psi (\gamma_\mu)_{\alpha\beta} \partial_\mu \psi^\beta - i Z_g Z^\psi Z^\mu A^\mu A^\nu g^{-1/2} (\gamma_\mu)_{\alpha\beta} A^\mu_\mu (T^a)^ij \psi^j_\beta + i b^\alpha \partial_\mu A^a_\mu + i g \bar{\psi}_a \psi^a (T^a)^ij \psi^j_\alpha
- i g^\rho (T^a)^ij Y_\rho^i - Z^{-1/2}_g Z^\mu (\partial_\mu c^a + \Omega^a_\mu) \partial_\mu c^a + (\partial_\mu c^a + \Omega^a_\mu) g f^{abc} A^c_\mu A^b_\nu + 1 2 g L^a f^{abc} c^b c^c
- \bar{\lambda}^i \partial_\mu \lambda^i - \bar{\xi}^i \partial_\mu \xi^i - \bar{\eta}^i \partial_\mu \eta^i + m \bar{\lambda}^i \xi^i + m \bar{\xi}^i \lambda^i + m \bar{\eta}^i \eta^i - m \partial_\mu \lambda^i
+ M^2_1 \bar{\psi}_a \psi^a - M^2_2 \bar{\psi}_a \psi^a - M^2_3 \bar{\psi}_a \psi^a + Z \sigma \sigma^2 \right).
\]

From expression (64), one can obtain the renormalized version of (14), namely
\[
S^\text{ren}_\psi = \int d^4x \left( Z^\psi (\gamma_\mu)_{\alpha\beta} \partial_\mu \psi^\beta - i Z_g Z^\psi Z^\mu A^\mu A^\nu g^{-1/2} (\gamma_\mu)_{\alpha\beta} A^\mu_\mu (T^a)^ij \psi^j_\beta - 2 Z_g M^2_1 M^2_2 \bar{\psi}_a \left( \frac{1}{g^2 - m^2} \right) \psi^a_\alpha \right).
\]

4 Inclusion of the Gribov-Zwanziger term

4.1 A brief overview of the Gribov-Zwanziger action and of its soft BRST breaking term

The Gribov-Zwanziger framework can enable one to take into account the existence of the Gribov copies, which affect the Landau gauge. This is done by restricting the domain of integration in the Feynman path integral to the so-called Gribov region Ω, defined as the set of fields fulfilling the Landau gauge condition and for which the Faddeev-Popov operator, \( \mathcal{M}^{ab} = -\partial_\mu D^{ab}_\mu (A) \), is strictly positive
\[
\Omega = \{ A^a_\mu \; , \; \partial_\mu A^a_\mu = 0 \; , \; \mathcal{M}^{ab} > 0 \},
\]

As shown in [2] [3], the implementation of the restriction to the region Ω is done by adding to the starting action a nonlocal term, known as the horizon function, namely
\[
S_{\text{GZ}} = -g^2 \gamma^4 \int d^4x f^{abc} A^a_\mu [ (\partial \cdot D)^{-1} ]^{ad} f^{dec} A^c_\mu .
\]

\[4\text{See } [20] \text{ for an introduction to the subject of the Gribov copies.} \]
The parameter $\gamma$ has the dimension of a mass and is known as the Gribov parameter. It is not a free parameter, being determined in a self consistent way through the gap equation

$$\frac{\delta \Gamma}{\delta \gamma^2} = 0 ,$$

where $\Gamma$ stands for the effective action evaluated in the presence of the horizon function $\delta \Gamma$. Despite of its nonlocal character, the term $S_{GZ}$ can be cast in local form by introducing a suitable set of auxiliary fields $(\varphi_{\mu}^{ab}, \bar{\varphi}_{\mu}^{ab}, \omega_{\mu}^{ab}, \bar{\omega}_{\mu}^{ab})$,

$$e^{-S_{GZ}} = \int D\varphi D\bar{\varphi} D\omega D\bar{\omega} e^{-S_{\text{Local}}^{GZ}} ,$$

$$S_{\text{GZ}}^{\text{Local}} = \int d^4x \left( -\varphi_{\mu}^{ac} \partial_{\nu} D_{\nu}^{ab} \varphi_{\mu}^{bc} + \bar{\omega}_{\mu}^{ac} \partial_{\nu} D_{\nu}^{ab} \omega_{\mu}^{bc} + (\partial_{\nu} \bar{\omega}_{\mu}^{ac}) g f^{abcd} \varphi_{\mu}^{de} D_{\nu}^{ef} e^x \right) + g \gamma^2 \int d^4x \left( f^{abc} (\varphi_{\mu}^{ab} - \bar{\varphi}_{\mu}^{ab}) A_{\mu}^c - \frac{4}{g} (N^2 - 1) \gamma^2 \right) .$$

Here, $(\varphi_{\mu}^{ab}, \bar{\varphi}_{\mu}^{ab})$ form a pair of complex commuting fields, while $(\omega_{\mu}^{ab}, \bar{\omega}_{\mu}^{ab})$ form a pair of complex anti-commuting fields. These fields are assembled in BRST doublets

$$s\varphi_{\mu}^{ab} = \omega_{\mu}^{ab} , \quad s\omega_{\mu}^{ab} = 0 ,$$

$$s\bar{\varphi}_{\mu}^{ab} = \bar{\omega}_{\mu}^{ab} , \quad s\bar{\omega}_{\mu}^{ab} = 0 ,$$

and, as pointed out in [5, 6, 7, 13], the local action (70) gives rise to a soft breaking of the BRST symmetry, due to the presence of the Gribov parameter $\gamma$. In fact, it turns out that expression (70) can be written as

$$S_{\text{GZ}}^{\text{Local}} = S_{\varphi\omega} + g \gamma^2 \int d^4x \left( f^{abc} (\varphi_{\mu}^{ab} - \bar{\varphi}_{\mu}^{ab}) A_{\mu}^c - \frac{4}{g} (N^2 - 1) \gamma^2 \right) ,$$

with

$$S_{\varphi\omega} = -s \int d^4x \bar{\omega}_{\mu}^{ac} \partial_{\nu} D_{\nu}^{ab} \varphi_{\mu}^{bc} = \int d^4x \left( -\varphi_{\mu}^{ac} \partial_{\nu} D_{\nu}^{ab} \varphi_{\mu}^{bc} + \bar{\omega}_{\mu}^{ac} \partial_{\nu} D_{\nu}^{ab} \omega_{\mu}^{bc} + (\partial_{\nu} \bar{\omega}_{\mu}^{ac}) g f^{abcd} \varphi_{\mu}^{de} D_{\nu}^{ef} e^x \right) ,$$

so that

$$sS_{\text{GZ}}^{\text{Local}} = \gamma^2 \Delta \gamma ,$$

$$\Delta \gamma = \int d^4x \left( g f^{abc} \omega_{\mu}^{ab} A_{\mu}^c - g f^{abc} (\varphi_{\mu}^{ab} - \bar{\varphi}_{\mu}^{ab}) D_{\mu}^{cd} e^x \right) .$$

In order to keep control of the soft BRST breaking term, we proceed as before and introduce a set of external sources $(U_{\mu\nu}^{ab}, \bar{U}_{\mu\nu}^{ab}, V_{\mu\nu}^{ab}, \bar{V}_{\mu\nu}^{ab})$ transforming as

$$sV_{\mu\nu}^{ab} = U_{\mu\nu}^{ab} , \quad sU_{\mu\nu}^{ab} = 0 ,$$

$$s\bar{U}_{\mu\nu}^{ab} = \bar{V}_{\mu\nu}^{ab} , \quad s\bar{V}_{\mu\nu}^{ab} = 0 ,$$

(75)
and whose physical values are defined by
\[ V^{ab}_{\mu \nu} \mid_{\text{phys}} = -\bar{V}^{ab}_{\mu \nu} \mid_{\text{phys}} = -\gamma^2 g^{ab} \delta_{\mu \nu}, \]
\[ U^{ab}_{\mu \nu} \mid_{\text{phys}} = \bar{U}^{ab}_{\mu \nu} \mid_{\text{phys}} = 0. \] (76)

Thus, we can replace the breaking term in eq.(72) by the following BRST-invariant source term
\[ S_{\text{source}} = s \int d^4 x \left( \bar{U}^{ab}_{\mu \nu} D^{ac}_{\mu} \varphi^{cb} + V^{ab}_{\mu \nu} D^{ac}_{\mu} \bar{\omega}^{cb} - \bar{U}^{ab}_{\mu \nu} V^{ab}_{\mu \nu} \right) \]
\[ = \int d^4 x \left( \bar{V}^{ab}_{\mu \nu} D^{ac}_{\mu} \varphi^{cb} - \bar{U}^{ab}_{\mu \nu} \right) D^{ac}_{\mu} \varphi^{cb} + gf^{acd}(D_{\mu}^{de} \varphi^{eb}) \bar{\varphi}^{cb} \]
\[ + U^{ab}_{\mu \nu} D^{ac}_{\mu} \bar{\omega}^{cb} + V^{ab}_{\mu \nu} [D^{ac}_{\mu} \varphi^{cb} + gf^{acd}(D_{\mu}^{de} \varphi^{eb})] \bar{\omega}^{cb} - (\bar{U}^{ab}_{\mu \nu} U^{ab}_{\mu \nu} - \bar{V}^{ab}_{\mu \nu} V^{ab}_{\mu \nu}) \] . (77)

Notice that the source term \( S_{\text{source}} \) gives back the original BRST soft breaking term when the sources attain their physical values (76). In fact, after a little algebra, one finds
\[ S_{\text{source}} \mid_{\text{phys}} = g\gamma^2 \int d^4 x \left( f^{abc}(\varphi^{ab}_{\mu} - \bar{\varphi}^{ab}_{\mu}) A^c_{\mu} - \frac{4}{g}(N^2 - 1)\gamma^2 \right) . \] (78)

We are now ready to discuss the inclusion of the Gribov-Zwanziger term (70) into our starting action \( \Sigma \), eq.(27). To that purpose we consider the more general action
\[ \Sigma_{\text{tot}} = \Sigma + S_{\varphi \omega} + S_{\mu} + S_{\text{source}} , \] (79)

where
\[ S_{\mu} = \mu^2 s \int d^4 x \bar{\omega}^{ab}_{\mu} \varphi^{ab}_{\mu} = \mu^2 \int d^4 x \left( \bar{\varphi}^{ab}_{\mu} \varphi^{ab}_{\mu} - \bar{\varphi}^{ab}_{\mu} \omega^{ab}_{\mu} \right) . \] (80)

As discussed in \([6, 7]\), the term \( S_{\mu} \) takes into account the nontrivial dynamics of the auxiliary localizing fields \( (\bar{\varphi}^{ab}_{\mu} \varphi^{ab}_{\mu}) \). The introduction of the BRST-invariant term \( S_{\mu} \) follows from the observation that the dimension two condensate \( \langle \bar{\varphi}^{ab}_{\mu} \varphi^{ab}_{\mu} - \bar{\omega}^{ab}_{\mu} \omega^{ab}_{\mu} \rangle \) has a nonzero value for non-vanishing Gribov parameter \( \gamma \), namely
\[ \langle \bar{\varphi}^{ab}_{\mu} \varphi^{ab}_{\mu} - \bar{\omega}^{ab}_{\mu} \omega^{ab}_{\mu} \rangle = \frac{3(N^2 - 1)}{64\pi} 2^{1/2} g N^{1/2} \gamma^2 . \] (81)

The existence of this condensate is taken into account through the mass parameter \( \mu \) which, in a way similar to the Gribov parameter \( \gamma \), is determined by a variational principle, see \([6, 7]\).

### 4.2 Renormalizability of the quark-gluon model in the presence of the Gribov-Zwanziger term

In order to discuss the renormalizability of expression (79), one uses the Ward identities that have already been established in \([3, 21, 5, 6, 7]\). Moreover, it exhibits the following symmetry
\[ \Theta_{\mu \nu}^{ab} \Sigma_{\text{tot}} = 0 , \] (82)
where the operator $\Theta_{\mu\nu}^{ab}$ is given by

$$
\Theta_{\mu\nu}^{ab} = \int d^4 x \left( \phi_{ac}^{\mu} \frac{\delta}{\delta \phi_{bc}^{\nu}} - \phi_{bc}^{\mu} \frac{\delta}{\delta \phi_{ac}^{\nu}} + \omega_{ac}^{\mu} \frac{\delta}{\delta \omega_{bc}^{\nu}} - \omega_{bc}^{\mu} \frac{\delta}{\delta \omega_{ac}^{\nu}} + V_{ac}^{\mu\sigma} \frac{\delta}{\delta V_{bc}^{\nu\sigma}} - \bar{V}_{ac}^{\mu\sigma} \frac{\delta}{\delta \bar{V}_{bc}^{\nu\sigma}} + U_{ac}^{\mu\sigma} \frac{\delta}{\delta U_{bc}^{\nu\sigma}} - \bar{U}_{ac}^{\mu\sigma} \frac{\delta}{\delta \bar{U}_{bc}^{\nu\sigma}} \right).
$$

(83)

The Ward identity (82) expresses the invariance of the action (79) under a global $U(4(N^2-1))$ transformation. This symmetry works exactly as the global $U(4N)$ symmetry associated with the spinor sector of the theory through the operator (23). These global symmetries ensure in fact that no mixing terms between the two set of BRST doublet fields, i.e. $(\bar{\phi}_{ab}^{\mu},\phi_{ab}^{\mu},\bar{\omega}_{ab}^{\mu},\omega_{ab}^{\mu})$ and $(\xi_{i\alpha},\theta_{i\alpha},\eta_{i\alpha},\lambda_{i\alpha})$, arise in the allowed counterterm. All Ward identities of the Gribov-Zwanziger action remain valid in the present case. This is also the case of the identities (28)-(38). Of course, the Slavnov-Taylor identity (28) is supplemented by suitable extra terms accounting for the BRST new doublets of the gluon sector, (71) and (75),

$$
S(\Sigma) \rightarrow S(\Sigma_{\text{tot}}) + \int d^4 x \left( \omega_{ab}^{\mu} \frac{\delta \Sigma_{\text{tot}}}{\delta \phi_{ab}^{\mu}} + \bar{\phi}_{ab}^{\mu} \frac{\delta \Sigma_{\text{tot}}}{\delta \bar{\omega}_{ab}^{\mu}} + U_{ab}^{\mu\nu} \frac{\delta \Sigma_{\text{tot}}}{\delta V_{ab}^{\mu\nu}} + \bar{V}_{ab}^{\mu\nu} \frac{\delta \Sigma_{\text{tot}}}{\delta \bar{V}_{ab}^{\mu\nu}} \right).
$$

(84)

Also, the ghost equation (32) needs a little modification which, due to the presence of the Gribov-Zwanziger term, generalizes to

$$
\mathcal{G}^a \rightarrow \mathcal{G}^a + gf^{abc} \int d^4 x \left( \phi_{bd}^{\mu} \frac{\delta}{\delta \omega_{cd}^{\mu}} + \bar{\omega}_{bd}^{\mu} \frac{\delta}{\delta \bar{\phi}_{cd}^{\mu}} + V_{bd}^{\mu\nu} \frac{\delta}{\delta U_{cd}^{\mu\nu}} + \bar{U}_{bd}^{\mu\nu} \frac{\delta}{\delta \bar{U}_{cd}^{\mu\nu}} \right),
$$

(85)

while the classical breaking term (34) remains unmodified. The previous algebraic analysis can be now repeated for the more general action (79). The final output is that the action (79) remains renormalizable to all orders.

5 Conclusion

In this work we have considered a model that accounts for a modification of the infrared behavior of quark and gluon propagators in Yang-Mills theories. This is achieved through the introduction of suitable mass parameters which give rise to a soft breaking of the BRST symmetry, as outlined in [13].

Being soft, the breaking term can be neglected in the ultraviolet region, where the standard massless quark propagator is recovered as well as the notion of exact BRST invariance. Moreover, in the infrared region the quark propagator turns out to be deeply modified, as shown by expression (2). The physical reasoning behind the introduction of the soft BRST breaking and of the ensuing modification of the propagator relies on quark confinement and on the breaking of the chiral symmetry, both occurring in the non-perturbative infrared region. It is worth remarking that the quark propagator (2) is in fact in qualitative agreement with the fitting formulas employed in the numerical studies of the quark two-point function through lattice simulations in the Landau gauge [17, 18].
The main result of the present article is the analysis of the renormalizability of the model, which we have shown to hold at any given finite orders of perturbation theory, by making use of the algebraic renormalization \cite{19}. The inclusion of the Gribov-Zwanziger term which enables us to implement the restriction to the Gribov region $\Omega$ has also been taken into account. Despite the presence of the soft BRST breaking term, the renormalizability of the model is guaranteed by the large set of Ward identities, eqs.\eqref{eq:28}-\eqref{eq:58}, which can be established.

We expect that the mechanism of introducing non-perturbative infrared effects through the soft breaking of the BRST symmetry \cite{13} applies as well to other kinds of models, including supersymmetric and topological field theories.

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