Plato’s Fire and the Neutrino Mass Matrix

Ernest Ma

Physics Department, University of California, Riverside, California 92521

Abstract

With the accumulation of many years of solar and atmospheric neutrino oscillation data, the approximate form of the $3 \times 3$ neutrino mixing matrix is now known. The theoretical challenge is to understand where this mixing matrix comes from. Recently, a remarkable fact was discovered that for a specific pattern of the neutrino mass matrix at a high scale, any flavor-changing radiative correction will automatically lead to the desired mixing matrix. It was also discovered that the required specific pattern at the high scale can be maintained by the non-Abelian discrete symmetry $A_4$ which is also the symmetry group of the regular tetrahedron, one of five perfect geometric solids known to Plato who associated it with the element “fire”. I discuss this recent development and add to it a new and very simple mechanism for the implementation of the flavor-changing radiative correction.

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1 Introduction

With the recent addition of the SNO (Sudbury Neutrino Observatory) neutral-current data [1], the overall picture of solar neutrino oscillations [2] is becoming quite clear. Together with the well-established atmospheric neutrino data [3], the $3 \times 3$ neutrino mixing matrix is now determined to a very good first approximation by

$$

\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta / \sqrt{2} & \cos \theta / \sqrt{2} & -1 / \sqrt{2} \\
\sin \theta / \sqrt{2} & \cos \theta / \sqrt{2} & 1 / \sqrt{2}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},

(1)

$$

where $\nu_{1,2,3}$ are assumed to be Majorana neutrino mass eigenstates. In the above, $\sin^2 2\theta_{\text{atm}} = 1$ is already assumed and $\theta$ is the solar mixing angle which is now known to be large but not maximal [4], i.e. $\tan^2 \theta \simeq 0.4$. The $U_{e3}$ entry has been assumed zero but it is only required experimentally to be small [4], i.e. $|U_{e3}| < 0.16$.

Denoting the masses of $\nu_{1,2,3}$ as $m_{1,2,3}$, the solar neutrino data [1, 2] require that $m_2^2 > m_1^2$ with $\theta < \pi/4$, and in the case of the favored large-mixing-angle solution [4],

$$

\Delta m_{\text{sol}}^2 = m_2^2 - m_1^2 \simeq 5 \times 10^{-5} \text{ eV}^2.

(2)

$$

The atmospheric neutrino data [3] require

$$

|m_3^2 - m_{1,2}^2| \simeq 2.5 \times 10^{-3} \text{ eV}^2,

(3)

$$

without deciding whether $m_3^2 > m_{1,2}^2$ or $m_3^2 < m_{1,2}^2$.

The big question now is what the neutrino mass matrix itself should look like, in order that Eq. (1) be obtained. Since neutrino oscillations measure only the differences of the squares of the mass eigenvalues, it is obvious that the answer will not be unique [4]. On the other hand, a pattern supported by an underlying symmetry would be better motivated than an ad hoc hypothesis. In this Brief Review, a recent interesting development in this direction is presented, with details for a more general readership as well as something entirely new.
In Sec. 2, it is shown how Eq. (1) may be automatically obtained from any flavor-changing radiative correction of a particular neutrino mass matrix at a high scale. This of course requires physics beyond the Standard Model. In Sec. 3, the symmetry which allows us to have that special neutrino mass matrix at the high scale is identified as Plato’s “fire”, i.e. the non-Abelian discrete symmetry $A_4$. In Sec. 4, it is shown how the irreducible representations of $A_4$ are just right to allow for three arbitrary charged-lepton masses while maintaining three degenerate neutrino masses at the high scale. In Sec. 5, a new and very simple mechanism for flavor-changing radiative corrections is proposed, which gives realistic values for the neutrino mass differences of Eqs. (2) and (3). In Sec. 6, there are some concluding remarks.

2 Getting the Right Neutrino Mixing Matrix With Almost Nothing

Given the particle content of the Standard Model, lepton masses come from

$$L_{int} = f_{ij} (\nu, e)_{iL} e_{jR} \left( \phi^+ \right) + \lambda_{ij} (\nu_i \phi^0 - e_i \phi^+)(\nu_j \phi^0 - e_j \phi^+) + H.c.,$$

where the second term is the effective dimensional-five operator [7] for Majorana neutrino masses. Note that $\lambda_{ij}$ has the dimension of inverse mass, hence any such neutrino mass must be proportional to the square of $v = \langle \phi^0 \rangle$ divided by a large mass, i.e. “seesaw” in character whatever its origin [8]. At some high scale, let

$$f_{ij} = U_L^T \begin{pmatrix} f_e & 0 & 0 \\ 0 & f_\mu & 0 \\ 0 & 0 & f_\tau \end{pmatrix} U_R,$$

then in the $\nu_{e,\mu,\tau}$ basis, $\lambda_{ij}$ becomes $U_L^T \lambda_{ij} U_L$ which is of course arbitrary without further assumptions. Suppose however that for some reason,

$$U_L^T \lambda_{ij} U_L \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$
then a remarkable thing happens when one-loop flavor-changing radiative corrections are added, i.e. Eq. (1) will be automatically obtained, as shown below. Note that the form of Eq. (6) is crucial for this to hold. If it were simply proportional to the identity matrix, Eq. (1) would not be the result.

From the high scale to the electroweak scale, one-loop radiative corrections will change Eq. (6) to

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{pmatrix} + R \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{pmatrix} R^T, \tag{7}
\]

where the radiative correction matrix is assumed to be of the most general form, i.e.

\[
R = \begin{pmatrix}
r_{ee} & r_{e\mu} & r_{e\tau} \\
r_{e\mu}^* & r_{\mu\mu} & r_{\mu\tau} \\
r_{e\tau}^* & r_{\mu\tau}^* & r_{\tau\tau}
\end{pmatrix}. \tag{8}
\]

Thus the observed neutrino mass matrix is given by

\[
\mathcal{M}_\nu = m_0 \begin{pmatrix}
1 + 2r_{ee} & r_{e\tau} + r_{e\mu}^* & r_{e\mu} + r_{e\tau}^* \\
r_{e\mu} + r_{e\tau} & 2r_{\mu\tau} & 1 + r_{\mu\mu} + r_{\tau\tau} \\
r_{e\tau} + r_{e\mu} & 1 + r_{\mu\mu} + r_{\tau\tau} & 2r_{\mu\tau}^*
\end{pmatrix}. \tag{9}
\]

Then using the redefinitions:

\[
d_0 \equiv r_{\mu\mu} + r_{\tau\tau} - r_{\mu\tau} - r_{\mu\tau}^*, \tag{10}
\]

\[
d \equiv 2r_{\mu\tau}, \tag{11}
\]

\[
d' \equiv r_{ee} - \frac{1}{2}r_{\mu\mu} - \frac{1}{2}r_{\tau\tau} - \frac{1}{2}r_{\mu\tau} - \frac{1}{2}r_{\mu\tau}^*, \tag{12}
\]

\[
d'' \equiv r_{e\mu}^* + r_{e\tau}, \tag{13}
\]

the neutrino mass matrix becomes

\[
\mathcal{M}_\nu = m_0 \begin{pmatrix}
1 + d_0 + \delta + \delta^* + 2\delta' & \delta'' & \delta''^* \\
\delta'' & \delta & 1 + \delta_0 + (\delta + \delta^*)/2 \\
\delta''^* & 1 + \delta_0 + (\delta + \delta^*)/2 & \delta^*
\end{pmatrix}. \tag{14}
\]

Without any loss of generality, \(\delta\) may be chosen real by absorbing its phase into \(\nu_\mu\) and \(\nu_\tau\) and \(d_0\) set equal to zero by redefining \(m_0\) and the other \(\delta'\)s.
In the Standard Model, there are no flavor-changing leptonic interactions, thus $\delta = \delta'' = 0$ and Eq. (14) does not lead to neutrino oscillations at all. However, if there is some new physics which allows all the $\delta$’s to be nonzero, then Eq. (14) is exactly diagonalizable if $\delta''$ is real, and the result is Eq. (1) independent of the actual values of $m_0$, $\delta$, $\delta'$, or $\delta''$, subject only to the constraint

$$\tan \theta = \frac{\sqrt{2} \delta''}{\sqrt{\delta'^2 + 2\delta''^2} - \delta'}, \quad \text{with } \delta' < 0.$$  

(15)

The mass eigenvalues do depend on $m_0$ and the $\delta$’s and they are given by

$$m_1 = m_0(1 + 2\delta + \delta' - \sqrt{\delta'^2 + 2\delta''^2}),$$  

(16)

$$m_2 = m_0(1 + 2\delta + \delta' + \sqrt{\delta'^2 + 2\delta''^2}),$$  

(17)

$$m_3 = -m_0.$$  

(18)

Thus the relevant $\Delta m^2$ parameters for atmospheric and solar neutrino oscillations are respectively

$$(\Delta m^2)_{\text{atm}} = (\Delta m^2)_{32} = (\Delta m^2)_{31} \simeq 4\delta m_0^2, \quad (\Delta m^2)_{\text{sol}} = (\Delta m^2)_{12} \simeq 4\sqrt{\delta'^2 + 2\delta''^2}m_0^2.$$  

(19)

To obtain $U_{e3} \neq 0$, let $Im\delta'' \neq 0$ but small, then

$$U_{e3} \simeq \frac{iIm\delta''}{\sqrt{2}\delta},$$  

(20)

and the above expressions are corrected by the replacement of $\delta'$ with $\delta' + (Im\delta'')^2/2\delta$, and $\delta''$ by $Re\delta''$. This results in the following interesting relationship:

$$\left[\frac{(\Delta m^2)_{12}}{(\Delta m^2)_{32}}\right]^2 \simeq \left[\frac{\delta'}{\delta} + |U_{e3}|^2\right]^2 + 2\left[\frac{Re\delta''}{\delta}\right]^2.$$  

(21)

It has thus been shown that with Eq. (6) at the high scale, a desirable neutrino mixing matrix is automatically generated by arbitrary radiative corrections, including the possibility of CP violation which is predicted here to be maximal because $U_{e3}$ is purely imaginary.
3 Plato’s Fire

The patterns of Eqs. (5) and (6) should be simultaneously maintained by some symmetry, but it looks impossible. However, there is in fact a solution, and it is based on the non-Abelian discrete symmetry $A_4$. What is $A_4$ and why is it special?

Around the year 390 BCE, the Greek mathematician Theaetetus proved that there are five and only five perfect geometric solids. The Greeks already knew that there are four basic elements: fire, air, water, and earth. Plato could not resist matching them to the five perfect geometric solids and for that to work, he invented the fifth element, i.e. quintessence, which is supposed to hold the cosmos together. His assignments are shown in Table 1.

Table 1: Properties of Perfect Geometric Solids

| solid     | faces | vertices | Plato | Group |
|-----------|-------|----------|-------|-------|
| tetrahedron | 4     | 4        | fire  | $A_4$ |
| octahedron | 8     | 6        | air   | $S_4$ |
| icosahedron | 20    | 12       | water | $A_5$ |
| hexahedron | 6     | 8        | earth | $S_4$ |
| dodecahedron | 12    | 20       | ?     | $A_5$ |

The group theory of these solids was established in the early 19th century. Since a cube (hexahedron) can be imbedded perfectly inside an octahedron and the latter inside the former, they have the same symmetry group. The same holds for the icosahedron and dodecahedron. The tetrahedron (Plato’s “fire”) is special because it is self-dual. It has the symmetry group $A_4$, i.e. the finite group of the even permutation of 4 objects. The reason that it is special for the neutrino mass matrix is because it has 3 inequivalent one-dimensional irreducible representations and 1 three-dimensional irreducible representation exactly. Its character table is given below.
Table 2: Character Table of $A_4$

| class | n | h | $\chi_1$ | $\chi_2$ | $\chi_3$ | $\chi_4$ |
|-------|---|---|---------|---------|---------|---------|
| $C_1$ | 1 | 1 | 1       | 1       | 1       | 3       |
| $C_2$ | 4 | 3 | $\omega$ | $\omega^2$ | 0       |         |
| $C_3$ | 4 | 3 | $\omega^2$ | $\omega$ | 0       |         |
| $C_4$ | 3 | 2 | 1       | 1       | 1       | $-1$    |

In the above, $n$ is the number of elements, $h$ is the order of each element, and

$$\omega = e^{2\pi i/3}$$

is the cube root of unity. The group multiplication rule is

$$\begin{align*}
\mathbf{3} \times \mathbf{3} &= \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}.
\end{align*}$$

4 Details of the $A_4$ Model

Using $A_4$, we now have the following natural assignments of quarks and leptons:

$$(u_i, d_i)_L, \ (\nu_i, e_i)_L \sim \mathbf{3}.$$  \hspace{1cm} (24)$$  

$$u_{1R}, \ d_{1R}, \ e_{1R} \sim \mathbf{1}.$$  \hspace{1cm} (25)$$  

$$u_{2R}, \ d_{2R}, \ e_{2R} \sim \mathbf{1}'.$$  \hspace{1cm} (26)$$  

$$u_{3R}, \ d_{3R}, \ e_{3R} \sim \mathbf{1}''.$$  \hspace{1cm} (27)$$

Heavy fermion singlets are then added [10]:

$$U_{iL(R)}, \ D_{iL(R)}, \ E_{iL(R)}, \ N_{iR} \sim \mathbf{3}.$$  \hspace{1cm} (28)$$

Together with the usual Higgs doublet and new heavy singlets:

$$(\phi^+, \phi^0) \sim \mathbf{1}, \ \chi_i^0 \sim \mathbf{3}.$$  \hspace{1cm} (29)$$

7
With this structure, charged leptons acquire an effective Yukawa coupling matrix $\bar{e}_iL e^j R \phi^0$ which has 3 arbitrary eigenvalues (because of the 3 independent couplings to the 3 inequivalent one-dimensional representations) and for the case of equal vacuum expectation values of $\chi_i$, i.e.

$$\langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle = u, \quad (30)$$

the unitary transformation $U_L$ which diagonalizes $f_{ij}$ of Eq. (5) is given by

$$U_L = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \quad (31)$$

The corresponding $\lambda_{ij}$ of Eq. (6) is fixed by $A_4$ to be proportional to the identity matrix; thus the effective neutrino mass operator, i.e. $\nu_i \nu^j \phi^0 \phi^0$, is proportional to

$$U_L^T U_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (32)$$

exactly as desired [9, 10].

To derive Eq. (32), the validity of Eq. (30) has to be proved. This is naturally accomplished in the context of supersymmetry. Let $\hat{\chi}_i$ be superfields, then its superpotential is given by

$$\hat{W} = \frac{1}{2} M_\chi (\hat{\chi}_1 \hat{\chi}_1 + \hat{\chi}_2 \hat{\chi}_2 + \hat{\chi}_3 \hat{\chi}_3) + h_\chi \hat{\chi}_1 \hat{\chi}_2 \hat{\chi}_3. \quad (33)$$

Note that the $h_\chi$ term is invariant under $A_4$, a property not found in $SU(2)$ or $SU(3)$. The resulting scalar potential is

$$V = |M_\chi \chi_1 + h_\chi \chi_2 \chi_3|^2 + |M_\chi \chi_2 + h_\chi \chi_3 \chi_1|^2 + |M_\chi \chi_3 + h_\chi \chi_1 \chi_2|^2. \quad (34)$$

Thus a supersymmetric vacuum ($V = 0$) exists for

$$\langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle = u = -M_\chi / h_\chi, \quad (35)$$

proving Eq. (30), with the important additional result that the spontaneous breaking of $A_4$ at the high scale $u$ does not break the supersymmetry.
5 New Flavor-Changing Radiative Mechanism

The original $A_4$ model \[9\] was conceived to be a symmetry at the electroweak scale, in which case the splitting of the neutrino mass degeneracy is put in by hand and any mixing matrix is possible. Subsequently, it was proposed \[10\] as a symmetry at a high scale, in which case the mixing matrix is determined completely by flavor-changing radiative corrections and the only possible result happens to be Eq. (1) if CP is conserved. This is a remarkable convergence in that Eq. (1) is also the phenomenologically preferred neutrino mixing matrix based on the most recent data from neutrino oscillations.

We should now consider the new physics responsible for the $\delta$'s of Eq. (14). Previously \[10\], arbitrary soft supersymmetry breaking in the scalar sector was invoked. It is certainly a phenomenologically viable scenario, but lacks theoretical motivation and is somewhat complicated. Here I propose a new and much simpler mechanism, using a triplet of charged scalars under $A_4$, i.e. $\eta^+_i \sim 3$. Their relevant contributions to the Lagrangian of this model is then

$$L = f \epsilon_{ijk} (\nu_i \nu_j - e_i \nu_j) \eta_k^+ + m_i^2 \eta^+_i \eta^-_j.$$  \hspace{1cm} (36)

Whereas the first term is invariant under $A_4$ as it should be, the second term is a soft term which is allowed to break $A_4$, from which the flavor-changing radiative corrections will be calculated.

The neutrino wave-function renormalizations are depicted in Fig. 1, where the indices $i, j, k$ are all different. Let

$$\begin{pmatrix} \eta_e \\ \eta_\mu \\ \eta_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix},$$  \hspace{1cm} (37)

where $\eta_{1,2,3}$ are mass eigenstates with masses $m_{1,2,3}$. Using Eqs. (11) to (13), the $\delta$'s are then all easily calculated. They are all finite and independent of an arbitrary overall mass scale.
Figure 1: Neutrino wave-function renormalizations.

as shown below.

\[
\delta = -\frac{f^2}{4\pi^2} \sum_{i=1}^{3} U_{\mu i}^* U_{\tau i} \ln m_i^2, \\
\delta' = -\frac{f^2}{8\pi^2} \sum_{i=1}^{3} \left( \frac{1}{2} |U_{\mu i} - U_{\tau i}|^2 - |U_{ei}|^2 \right) \ln m_i^2, \\
\delta'' = -\frac{f^2}{8\pi^2} \sum_{i=1}^{3} (U_{\mu i}^* U_{ei} + U_{\tau i} U_{ei}^*) \ln m_i^2.
\]

As an example, consider

\[
U = \begin{pmatrix}
\frac{1}{\sqrt{2}} & -1/\sqrt{2} & 0 \\
\frac{c}{\sqrt{2}} & \frac{c}{\sqrt{2}} & s \\
-s/\sqrt{2} & -s/\sqrt{2} & c
\end{pmatrix},
\]

then

\[
\delta = \frac{f^2 sc}{8\pi^2} \left( \ln \frac{m_1^2}{m_3^2} + \ln \frac{m_2^2}{m_3^2} \right), \\
\delta' = \frac{f^2 (1 - 2sc)}{32\pi^2} \left( \ln \frac{m_1^2}{m_3^2} + \ln \frac{m_2^2}{m_3^2} \right), \\
\delta'' = \frac{f^2 (s - c)}{16\pi^2} \ln \frac{m_1^2}{m_2^2}.
\]

To obtain Eqs. (2), (3), and \(\tan^2 \theta \simeq 0.4\), using Eqs. (15) and (19), a possible solution is

\[
s = 0.638, \quad c = 0.770, \quad \frac{m_3^2}{m_1^2} = 1.728, \quad \frac{m_3^2}{m_2^2} = 1.573, \quad f^2 m_0^2 = 0.1 \text{ eV}^2,
\]

where \(m_0\) is the approximate common mass of all 3 neutrinos, as measured in neutrinoless double beta decay \[11\].
To determine the absolute values of $m_{1,2,3}$, the most stringent condition comes from the flavor-changing radiative decay $\mu \rightarrow e\gamma$. Its amplitude is given here by

$$\mathcal{A} = \frac{ef^2 c m_{\mu}}{192\pi^2} \left( \frac{1}{m_1^2} - \frac{1}{m_2^2} \right) e^{\frac{\lambda}{4}} \bar{\nu}_e \sigma_{\lambda\nu} (1 + \gamma_5) \mu. \quad (46)$$

The resulting branching fraction is then

$$B(\mu \rightarrow e\gamma) = 4.24 \times 10^{-10} f^4 \left( \frac{1 \text{ TeV}}{m_1} \right)^4. \quad (47)$$

Using the present experimental upper bound of $1.2 \times 10^{-11}$, the constraint

$$m_1/f > 2.44 \text{ TeV} \quad (48)$$

is obtained.

## 6 Concluding Remarks

In conclusion, recent experimental progress on neutrino oscillations points to a neutrino mixing matrix which can be derived in a remarkable way through radiative corrections of an underlying $A_4$ symmetry at some high scale. This scheme \[10\] automatically leads to $\sin^2 2\theta_{atm} = 1$ and a large (but not maximal) solar mixing angle. A new and very simple radiative mechanism using a triplet of heavy charged scalars (at the TeV scale) is proposed which leads to realistic values of the neutrino-oscillation parameters. To the extent that the Yukawa coupling $f$ should not be too big, the value of $m_0$ measured in neutrinoless double beta decay should not be much less than its current upper bound of about 0.4 eV.

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