Analysis of a two grade system when Interdecision times have exponential geometric distribution

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Abstract: Consider any single graded marketing organization where depletion of manpower occurs since decisions, exit of personnel etc.. There is an assumption that the depletion due to voluntary exit is correlated. By assuming that the inter-involuntary exit times, inter-breaking decision times forms different modified renewal processes, estimated mean and estimated variance of time to recruitment are determined. The stochastic model assuming that intercontact times between successive contacts as correlated random variables are proposed. Shock models with intercontact time have been obtained by assuming the threshold distribution as exponential. In this paper, it is assumed that threshold follows exponential geometric distribution.

Keywords: Inter-voluntary exit times, Inter-involuntary exit times, Inter-breaking decision times, Threshold, Shock model

1. Introduction
It is very common in business world to face the availability and shortage of Manpower periods. These two periods are liable to be dependent on each other due to various reasons such as larger number of employees leaving the organization, unavailability of suitable persons in the job market, stringent recruitment policies and such similar reasons. There are many reasons for an employee to leave the organization. It may be for higher salary or to join family or for higher education etc. As loss and shortcomings are inevitable and fund management are to be done during busy period and in the recruitment period, one may have to speed up recruitments using different strategies in order to start business early. The duration of busy period and the duration to recruit employees are random and they occur alternately in a business organization. When a busy period is long, one may like to speed up recruitment so as to start the next busy period early. For a single graded system, many authors worked out estimated mean and estimated variance of the time to recruitment by a univariate CUM policy of recruitment for many distributions of the threshold for the depletion of manpower in a system for independent as well as correlated interdecision times. This paper study the work of Uma[16] using a bivariate CUM policy of recruitment when the interdecision times form an exponential geometric process. A stochastic model to estimate the expected mean to recruitment for a manpower model and expected variance of recruitment derived under the assumption that the inter-arrival times between contacts may be treated as correlated random variables and the threshold distribution follows exponential-
geometric distribution is discussed. For a detailed study of exponential-geometric distribution, one can refer to Adamidis and Loukas. Shock model with correlated intercontact times has been studied by Sathiyamoorthi. In developing this model the results of Gurland has been used.

2. Notations:

\[ X_i \]  
We assume that \( X_1, X_2, \ldots, X_n \) are continuous i.i.d random variables, with probability distribution function \( g(.) \) and c.d.f \( G(.) \).

\[ Y_i \]  
Random variable threshold following exponential-geometric distribution has parameter \( g_1, g_2, \ldots, g_{18} \leq 3, g_{2869}, g_{18} \leq 3, g_{2870}, g_{18} \leq 3, g_{2871}, g_{18} \leq 3, g_{2872} \), with probability distribution function \( h(.) \) and c.d.f \( H(.) \).

\[ U_i \]  
a continuous random variable denoting the inter-arrival times between successive contacts with probability distribution function \( r(.) \) and c.d.f \( R(.) \).

\[ g_k(.) \]  
Denotes the probability distribution function of the random variable \( \sum_{j=0}^{k} X_j \).

\[ \zeta \]  
is correlation coefficient between \( x_i \) and \( x_j \), \( i \neq j \).

\[ V_k(t) \]  
Probability of exactly \( k \) contacts in \([0,t]\).

\[ Z_k \]  
random variable denote the time with probability distribution function \( l(.) \) and c.d.f \( L(.) \).

\[ l^*(s) \]  
denotes Laplace transform of \( l(t) \).

\[ f^*(s) \]  
denotes Laplace transform of \( f(t) \).

3. Result

The threshold variable \( Y \) has exponential-geometric distribution with parameters \( a_1, a_2, a_3, a_4 \), so that,

\[
P(Z_p^A < c)P(Z_p^B < c) = (Z_p^A - e^{-a_1c})(Z_p^A - a_2e^{-a_1c})^{-1}(Z_p^B - e^{-a_2c})(Z_p^B - a_4e^{-a_2c})^{-1}
\]

If we assume it follow exponential distribution with parameter \( a_3, a_4 \) then

\[
P[\sum_{j=1}^{k} X_j < Y] = [k_1(1 - a_3) \sum_{j=1}^{k} \frac{a_3^{j-1}}{k_1 + ja_3}] [k_2(1 - a_4) \sum_{j=1}^{k} \frac{a_4^{j-1}}{k_2 + ja_4}]
\]

and \( S(t) = \sum_{k=0}^{\infty} (R_K(t) - R_{k+1}(t)) [k_1(1 - a_3) \sum_{j=1}^{k} \frac{a_3^{j-1}}{k_1 + ja_3}] [k_2(1 - a_4) \sum_{j=1}^{k} \frac{a_4^{j-1}}{k_2 + ja_4}] \)

\[
L(t) = 1 - S(t) \text{ is called the prevalence function}
\]

\[
L(t) = 1 - \sum_{k=0}^{\infty} (R_K(t) - R_{k+1}(t)) [k_1(1 - a_3) \sum_{j=1}^{k} \frac{a_3^{j-1}}{k_1 + ja_3}] [k_2(1 - a_4) \sum_{j=1}^{k} \frac{a_4^{j-1}}{k_2 + ja_4}] \quad \ldots \quad (1)
\]

If the intercontact timings are independent, it is easy to obtain the joint distribution of their sum, but if they are correlated the determination of the distribution of \( U_1 + U_2 + \cdots + U_n \) is very complex in any general case but Gurland has obtained of \( Z_k = \sum_{j=1}^{k} u_j \)

When \( U_i \)'s are a sequence of constantly correlated random variables, each having exponential distribution with p.d.f

\[
R(u) = e^{-u^p}, \quad p > 0, \quad 0 < u < \infty
\]
such that the correlation coefficient between any \( x_i \) and \( x_j \), \( i \neq j \) is \( \xi \).

\[
R_p (u) = P \left[ Z_p \leq u \right] = (1 - \xi) \sum_{j=0}^{\infty} \frac{(\tau \varphi)^j}{(1 - \tau \varphi)^{j+1}} \frac{\varphi}{(1+\tau \varphi)^{j+1}(p+j-1)!} \quad \cdots (2), \quad \varphi = \frac{1 - \tau}{\tau}
\]

\[
\chi (k,u) = \int_0^u e^{-\varphi} \varphi^{p-1} d\varphi
\]

Laplace transform of \( Z_k \) is given by,

\[
\mathcal{L} \{Z_k\} = \sum_{g=18}^{30} \left( \sum_{g=18}^{73} \ldots \sum_{g=30}^{3} \sum_{g=18}^{73} \right) \prod_{g=18}^{30} \frac{1 - \xi_g}{\xi_g} = (1 - \xi) \prod_{g=18}^{30} \frac{1 - \xi_g}{\xi_g} + \ldots (3)
\]

Laplace transform of \( l(t) \) is given by,

\[
\mathcal{L} \{l(t)\} = \sum_{g=18}^{30} \left( \sum_{g=18}^{71} \ldots \sum_{g=30}^{3} \sum_{g=18}^{71} \right) \prod_{g=18}^{30} \frac{1 - \xi_g}{\xi_g} + \ldots (3)
\]

Where,

\[
R_p (s) = \frac{1}{(1+ws)^p} \prod_{g=18}^{30} \frac{1 - \xi_g}{\xi_g} \frac{1}{(1+\tau \varphi)^{1+(p+1)\varphi}} \quad \cdots (3)
\]

Taking first order differentiation,

\[
\frac{\partial R_p (s)}{\partial s} = \frac{-p}{\varphi} \quad \text{and} \quad \frac{\partial R_{p+1} (s)}{\partial s} = \frac{p+1}{\varphi}
\]

And second order differentiation,

\[
\frac{\partial^2 R_p (s)}{\partial s^2} = \frac{p(1-\tau^2) + p^2(1+\tau^2)}{\varphi^2} \quad \text{and} \quad \frac{\partial^2 R_{p+1} (s)}{\partial s^2} = \frac{p^2(1+3\tau^2-2\tau) + p(1+\tau)^2 + 2}{\varphi^2}
\]

Hence \( E(T) \) or \( ETR = - \frac{\partial^2 \Psi (s)}{\partial s^2} \big|_{s=0} \)

\[
ETR = \left( \frac{1}{\varphi^2} \right) \left[ k_1 (1 - a_3) \sum_{g=18}^{30} \frac{a_{g,j-1}^{k_1 + j/a_1}}{k_2 (1 - a_4) \sum_{g=18}^{30} \frac{a_{g,j-1}^{k_2 + j/a_2}}{k_2 + j/a_2}} \right] \quad \text{(on simplification)}
\]
And  $E(T^2) = \frac{\partial^2 E(x)}{\partial x^2}|_{x=0}$

$= 2\sum_{p=0}^{\infty} \left[ \frac{p^2 \tau (r-1) + p \tau (r+1) + 1}{\varphi^2} \right] [\sum_{j=1}^{k} \alpha_{j-1} \sum_{j=1}^{k} \frac{\alpha_{j-1}}{k_{2}+j_{2}}]^{2}$

Hence $VTR = E(T^2) - [E(T)]^2$

Therefore $VTR = \sum_{K=0}^{\infty} \left[ \frac{2 p^2 \tau (r-1) + 2 p \tau (r+1) + 1}{\varphi^2} \right] [\sum_{j=1}^{k} \alpha_{j-1} \sum_{j=1}^{k} \frac{\alpha_{j-1}}{k_{2}+j_{2}}]^{2}$

If $\tau = 0$,  

$VTR = \frac{1}{\varphi^2} [\sum_{j=1}^{k} \alpha_{j-1} \sum_{j=1}^{k} \frac{\alpha_{j-1}}{k_{2}+j_{2}}]^{2}$  

…………….(4)

#### Numerical Data

| $\varphi$ | $ETR$   | $VTR$  |
|---|---|---|
| 0.00.1 | 0.19335 | 20.19 |
| 0.00.2 | 0.9668 | 5.0475 |
| 0.00.3 | 0.6445 | 2.2433 |
| 0.00.4 | 0.4834 | 1.2619 |
| 0.00.5 | 0.3867 | 0.8076 |
| 0.00.6 | 0.3225 | 0.5608 |
| 0.00.7 | 0.2762 | 0.4120 |
| 0.00.8 | 0.2417 | 0.3154 |
| 0.00.9 | 0.2148 | 0.2493 |
Table 2.

| $k_{\perp}$ | ETR   | VTR   |
|------------|-------|-------|
| 00.1       | 00.0967 | 00.002 |
| 00.2       | 00.1934 | 00.0081 |
| 00.3       | 00.29   | 00.0182 |
| 00.4       | 00.3867 | 00.0323 |
| 00.5       | 00.4834 | 00.0505 |
| 00.6       | 00.5801 | 00.0727 |
| 00.7       | 00.6767 | 00.0989 |
| 00.8       | 00.7734 | 00.1292 |
| 00.9       | 00.8701 | 00.1635 |

Figure 1.
4. Conclusion.

From the table 1, we observe that for fixed $k_1, k_2, a_1, a_2, a_3, a_4$, and $\tau$, when $\varphi$ increases which means that the average inter-arrival time become smaller, so the mean time decreases and also variance time decreases.

From the table 2, we observe that for fixed and $k_2, a_1, a_2, a_3, a_4$, and $\varphi$, when $k_1$ increases then it is seen that the mean time and variance time increases.

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