Numerical investigation on vapor transfer in unsaturated soil during freezing

Jidong Teng 1, Sheng Zhang 1, Wuming Leng 1) and Daichao Sheng 1)

1) Assistant Professor, Department of Geotechnical Engineering, Central South University, Changsha 410075, China.
2) Associate Professor, Department of Geotechnical Engineering, Central South University, Changsha 410075, China.
3) Professor, Department of Geotechnical Engineering, Central South University, Changsha 410075, China.

ABSTRACT

This paper presents a new approach for modelling moisture migration in unsaturated freezing soil, in which the evaporation and condensation processes of water vapor are taken into account. Comparing predicted date with the measured result of one-dimensional soil freezing test shows proposed model is capable of effectively simulating the freezing process of unsaturated soil. Parametric analysis is carried out to clarify the role of vapor in moisture migration during freezing, which shows that vapor largely contributes to ice formation, occupying around 10%–60% of total water fluxes in an unsaturated and closed system. The result also shows that, total ice content and vapor flux percentage increases and decreases with the increase of initial water content, respectively. The higher the temperature gradient, the greater the vapor flux percentage, while the total ice content is insensitive to the temperature gradient. Peak values exist for the variations of total ice content and vapor flux against saturated hydraulic conductivity.

Keywords: unsaturated soil, freezing, vapor flow, numerical model

1 INTRODUCTION

The soil freezing process is a coupling movement of heat and pore water transfer. In the freezing process of a fully saturated soil with sufficient water supply, only the water increase in the frozen zone and consequently the total frost heave are of concern, and the problem can be solved by ignoring the effect of water content variation and skeletal deformation in unfrozen zone (Zhou et al., 2014). When analyzing the freezing behavior of an unsaturated soil, however, the water migration is complicated due to temporal and spatial variations of water content and temperature in both frozen and unfrozen zone. As a result, aiming to simply and quickly solve the simultaneous equations of modelling heat-mass transfer in unsaturated freezing soil, most researcher preferred to neglect the process of vapor flow (Harlan, 1973; Hu et al., 1992; Zhou et al., 2014; etc.). As a matter of fact, water vapor flow can represent a major part of overall water flow especially in the case that temperature gradient is remarkable. The recent experimental evidences also proved that vapor flow in freezing soil could lead to large amount of ice formation, even dominate the moisture migration (Eigenbrod and Kennespohl, 1996; Guthrie et al., 2006; Wang et al., 2012). Additionally, water vapor flow significantly affects the movement of heat, since it drives a substantial amount of energy as the latent heat of vaporization (Sakai et al., 2006). Therefore, it is important to take vapor flow into account when modelling the process of soil freezing, and also, it is essential to clarify the role of vapor transfer in unsaturated freezing soil.

In theory of water-heat coupling movement in unsaturated soil, Philip and de Vries (1957), as the pioneer, developed mathematical model to describe coupled movement of liquid water and water vapor under nonisothermal conditions. Milly (1984) adopted a matric head based formulation to modify the volumetric water content based Philip and de Vries (1957) model. Nassar and Horton (1989) further extended by including osmotic effects on liquid and water vapor movement. Sakai et al. (2009) presented a coupled model for water and vapor movement with condensation and evaporation under nonisothermal and low water content. Although numerical models of the simultaneous movement of liquid water, vapor, and heat in unsaturated soil have been proposed in the past, few simulations have been performed with considering soil freezing, that is to say, existing theories mainly demonstrate the limited situation of soil temperature over than 0, the coupling theory of heat and mass movement in unsaturated soil under both negative and positive temperature is not fully understood.

In this paper, a numerical model is presented to simulate the one-dimensional moisture-mass transfer in unsaturated freezing soil, which considers the processes
of evaporation and condensation of water vapor, to evaluate its performance, one-dimensional soil freezing tests would be presented to compare with the predicted result. The role or contribution in freezing of unsaturated soil is therefore elucidated on basis of a parametric study.

2 THEORY

Some essential assumptions are stated as follows:
(a) The freezing front always locates at the depth where soil temperature is 0.
(b) The vapor evaporates and condenses in unfrozen part that is up to the freezing front, in the frozen zone only liquid water flows feeding the ice formation.
(c) The hysteresis in unsaturated soil is neglected.

2.1 Liquid water and vapor flow

For the unfrozen and frozen soil as a whole, the water flow is continuous. An improved Darcy’s law as given by Philip and de Vries (1957) is adopted to describe the liquid water flux \( q_L \), which considers both the flow driven by temperature gradient and pressure head gradient. Fick’s law for vapor flux \( q_v \), can also be separated into two (isothermal and isothermal) components. Thus the liquid water and vapor flux in unsaturated soil can be given as follows,

\[
\begin{align*}
q_L &= q_{L\theta} + q_{LT} = -K_{L\theta} \frac{\partial h}{\partial z} - K_{LT} \frac{\partial T}{\partial z} \\
q_v &= q_{v\theta} + q_{vT} = -K_{v\theta} \frac{\partial h}{\partial z} + K_{vT} \frac{\partial T}{\partial z}
\end{align*}
\]

(1a)

(1b)

where \( q_{L\theta} \) and \( q_{LT} \) are isothermal and thermal liquid water fluxes (m s\(^{-1}\)), respectively; \( q_{v\theta} \) and \( q_{vT} \) are isothermal and thermal vapor fluxes (m s\(^{-1}\)), respectively. \( h \) is the pressure head (m), \( T \) is the temperature (K), and \( z \) is the spatial coordinate positive upward (m). \( K_{L\theta} \) (m s\(^{-1}\)) and \( K_{LT} \) (m\(^2\) K\(^{-1}\) s\(^{-1}\)) are the isothermal and thermal hydraulic conductivities for liquid water fluxes due to gradients in \( h \) and \( T \), respectively; \( K_{v\theta} \) (m s\(^{-1}\)) and \( K_{vT} \) (m\(^2\) K\(^{-1}\) s\(^{-1}\)) are the isothermal and thermal vapor hydraulic conductivities, respectively (Teng et al., 2013).

Supposing that the vapor evaporates in unfrozen soil, condenses into liquid water, and then flows and get frozen in frozen part, the volumetric water content here can be divided into three components accounting for liquid water, ice lensing and vapor content. Liquid water flow mainly results in the quantitative change of liquid water content and ice content, while vapor flow leads to the variation of vapor content. By using mass conservation law, the following expressions are given as:

\[
\frac{\partial \theta_L}{\partial t} + \frac{\rho_v}{\rho_w} \frac{\partial \theta_v}{\partial t} = -\frac{\partial q_L}{\partial z} - E
\]

(2a)

where \( \theta_L \), \( \theta_v \), and \( \theta_r \) are the liquid water content, pore ice content, and water vapor content (in form of an equivalent water content, m\(^3\) m\(^{-3}\)), respectively, and \( E \) represents the evaporation or condensation rate (s\(^{-1}\)), \( \rho_v \) and \( \rho_w \) are the densities of ice and liquid water, respectively, \( t \) is time (s).

Substituting Eqs. (1a) and (1b) into Eqs. (2a) and (2b) respectively and making combination, the governing equation for one-dimensional flow of liquid water and vapor in an unsaturated frozen soil can be obtained

\[
\frac{\partial \theta}{\partial t} = \frac{\partial \theta_L}{\partial t} + \frac{\partial \theta_v}{\partial t} + \frac{\rho_v}{\rho_w} \frac{\partial \theta_r}{\partial t} = -\frac{\partial q_L}{\partial z} - \frac{\partial q_v}{\partial z} \]

\[
= \frac{\partial}{\partial z} \left[ K_{L\theta} \frac{\partial h}{\partial z} + K_{LT} \frac{\partial T}{\partial z} + K_{v\theta} \frac{\partial h}{\partial z} + K_{vT} \frac{\partial T}{\partial z} \right]
\]

(3)

where \( \theta \) is the total volumetric water content (m\(^3\) m\(^{-3}\)).

2.2 Soil hydraulic properties

Neglecting the hysteresis between drying and wetting curves, the van Genuchten model (van Genuchten, 1980) is adopted to simulate the Soil Water Retention Curve (SWRC), as follows:

\[
\Theta = \left[1 + \left(\frac{\alpha h}{\theta_r}\right)^m\right]^{-m}
\]

(4)

where, \( \Theta \) is the effective saturation (unitless), \( \Theta=(\theta_1-\theta_r)/(\theta_s-\theta_r) \), \( \theta_s \) and \( \theta_r \) are the saturated and residual water content (m\(^3\) m\(^{-3}\)), respectively; \( \alpha \) (m\(^{-1}\)), \( n \) (unitless), and \( m \) (=1\(-1/n\)) are empirical shape parameters. Without considering the ice lens in frozen soil for obstructing the flow of remained liquid water, the isothermal unsaturated hydraulic conductivity of liquid water in both frozen and unfrozen soil is modeled following Mualem (1976):

\[
K_{L\theta} = K_s \Theta^\prime \left[1 - (1 - \Theta^\prime)^{m_1}\right]^2
\]

(5)

where \( K_s \) is the saturated hydraulic conductivity (m s\(^{-1}\)), \( l \) is fitting parameter, which was suggested as 0.5 by Mualem (1976).

The definitions of thermal hydraulic conductivity of liquid water \( K_{LT} \), and hydraulic conductivities of vapor due to temperature \( K_{vT} \), and water head \( K_{v\theta} \) refers to Saito et al. (2006):

\[
K_{LT} = K_{L\theta} (hG_{vT} \frac{1}{\gamma_0} \frac{dy}{dt})
\]

(6)

\[
K_{v\theta} = \frac{D}{\rho_v} \left(\frac{M_g}{RT} H_t\right)
\]

(7)

\[
K_{vT} = \frac{D}{\rho_v} \eta H_t \left(\frac{dp_w}{dt}\right)
\]

(8)

respectively, in Eq. (6), \( G_{vT} \) is the gain factor, which
amends the temperature dependence of the SWRC. \( \gamma \) is the surface tension of soil water (J m\(^{-2} \)), and \( \gamma_0 \) is the surface tension at 25 \( ^\circ \)C (\( = 71.89 \) g s\(^{-2} \)). In Eqs. (7) and (8), \( M \) is the molecular weight of water (\( = 0.018 \) kg mol\(^{-1} \)), \( g \) is the gravitational acceleration (m/s\(^2 \)), \( R \) is the universal gas constant (\( = 8.341 \) J mol\(^{-1} \) K\(^{-1} \)), \( H_r \) is the relative humidity (unitless), \( \eta \) is an enhancement factor (unitless), \( \rho_{sv} \) is the saturated vapor density (kg m\(^{-3} \)).

\( D \) is the vapor diffusivity in soil (m\(^2\) s\(^{-1} \)), it can be derived from the diffusivity of water vapor in air, as follows,

\[
D = \frac{\tau_0 D_s}{\theta_i^2 \theta_s} \quad \text{(9)}
\]

where \( \tau \) is the tortuosity factor, \( \theta_s \) is the air-filled porosity (m\(^3\) m\(^{-3} \)), \( D_s \) is the diffusivity of water vapor in air (m\(^2\) s\(^{-1} \)). The definitions of the related parameters are presented in Table 1.

| Parameter | Equation |
|-----------|----------|
| Surface tension of liquid water \( \gamma \) (J m\(^{-2} \)) | \( \gamma = 75.6 \times 10^{-3} \text{erg cm}^{-1} \) |
| Saturated vapor density \( \rho_{sv} \) (kg m\(^{-3} \)) | \( \rho_{sv} = \exp(31.37-6014.79T^{-0.92}\times 10^{-4}T) \times 10^{17} \text{g cm}^{-3} \) |
| Enhancement factor \( \eta \) (unitless) | \( \eta = 9.5 + 30 \theta_i - 8.5(\exp(1+2.6\theta_i)^{-1}+\theta_i^{-1}) \) |
| Relative humidity \( H_r \) (unitless) | \( H_r = \exp(hMg/RT) \) |
| Vapor diffusivity in air \( D_i \) (m\(^2\) s\(^{-1} \)) | \( D_i = 1.2 \times 10^{-5} \text{cm}^2 \text{s}^{-1} \) |
| Volumetric heat capacity of soil \( C_v \) (MJ m\(^{-3} \) K\(^{-1} \)) | \( C_v = C_i + C_{iv} + C_{f^1} + C_{f^2} \) |
| Latent heat of water vaporization \( L_w \) (Kg \( \text{kg}^{-1} \)) | \( L_w = 2.501 \times 10^5 \text{J} \text{kg}^{-1} \) |

Note: \( f_c \) is the mass fraction of clay in the soil (dimensionless), \( \theta_s \) is volumetric fraction of the solid phase (m\(^3\) m\(^{-3} \)).

### 2.3 Heat transport and soil thermal properties

Considering vaporization, condensation, and freezing in unsaturated soil, the governing equation of heat conservation can be given as (de Vries, 1958; Nassar and Horton, 1992):

\[
C_p \frac{\partial T}{\partial t} + L_a \rho_v \frac{\partial (\theta_0)}{\partial t} - L_v \frac{\partial \theta}{\partial t} + \frac{\partial}{\partial z} \left[ \lambda(0) \frac{\partial T}{\partial z} \right] - C_v \frac{\partial q_v}{\partial z} - L_i \rho_v \frac{\partial \theta}{\partial z} - C_{iv} \frac{\partial q_{iv}}{\partial z} = 0 \quad \text{(10)}
\]

where, \( C_p \) is the density of the v umetric heat capacities of unsaturated soil (MJ m\(^{-3} \) K\(^{-1} \)), which equals to the solid \( C_s \) (\( = 1.92 \) MJ m\(^{-3} \) K\(^{-1} \)), liquid \( C_l \) (\( = 4.18 \) MJ m\(^{-3} \) K\(^{-1} \)), air \( C_i \) (\( = 0.63 \) KJ m\(^{-3} \) K\(^{-1} \)) and ice \( C_{iv} \) (\( = 2.10 \) MJ m\(^{-3} \) K\(^{-1} \)) phases multiplied by their respective volumetric fractions (de Vries, 1958). \( L_w \) is the latent heat of water vaporization (J kg\(^{-1} \)), the expressions of \( C_p \) and \( L_w \) are presented in Table 1. \( L_i \) is the latent heat of water freezing (J kg\(^{-1} \)). \( \lambda(0) \) in Eq. (10) is the soil thermal conductivity (W m\(^{-1} \) K\(^{-1} \)), which can be defined by the following equation,

\[
\lambda(0) = b_1 + b_2 \theta_i + b_3 \theta_i^{0.5} \quad \text{(11)}
\]

where \( b_1 \) (\( = 0.228 \) W m\(^{-1} \) K\(^{-1} \)), \( b_2 \) (\( = -2.406 \) W m\(^{-1} \) K\(^{-1} \)), and \( b_3 \) (\( = 4.909 \) W m\(^{-1} \) K\(^{-1} \)) are empirical regression parameters. In Eq. (10), the three terms on the left-hand side represent energy changes in soil, the latent heat of the vapor phase, the heat generated during water freezing, the negative sign indicates the exothermic process for water freezing. The four terms on the right-hand side of Eq. (10) are soil heat flow by conduction, the convection of sensible heat with flowing water, the transfer of latent heat by vapor diffusion, and the transfer of sensible heat by vapor diffusion, respectively (Sakai et al., 2009).

It should be noted that the four variables of \( \theta_L, \theta_i, \theta_s \), and \( T \) exist in the two simultaneous equations of Eqs. (3) and (10) that governing the moisture and heat transport in unsaturated frozen soil, the other two equations is necessary to solve the simultaneous equations. The one is presented as follows,

\[
\theta_L + \theta_i + \theta_s = \theta_i \quad \text{(12)}
\]

To derive the second equation, the generalized Clapeyron equation is introduced to set up the relationship between pressure of unfrozen liquid water and that of pore ice when these two phase are in equilibrium in frozen soil, which is expressed as (Sheng, 2014),

\[
du_w = du_i + \frac{L_i}{gT_0} dT \quad \text{(13)}
\]

where \( u_w \) and \( u_i \) are the unfrozen water pressure (in unit of a water head, m) and pore ice pressure (m), \( T_0 \) is the freezing point of water (K), and noted that \( T \) in this equation is in unit of \( ^\circ \)C. Neglecting the overburden loading, \( u_i \) remains practically unchanged, that is to say, the first term on right hand of Eq. (13) changes into 0. Moreover, it has been validated that the pore water pressure in frozen soil equals to that in unsaturated soil with the same liquid water content (Harlan, 1973), thus \( u_w \) has the equivalent meaning with \( h \) in Eq. (4). Integrating both sides of Eq. (13) and then substituting into Eq. (4), one can get the function accounting for one-to-one correspondence between unfrozen liquid water content \( \theta_L \) and soil temperature \( T \) in frozen soil, as follows,

\[
\theta_L = \left( \theta_i - \theta_s \right) \left[ 1 + \left( \frac{LT}{gT_0} \right)^{\alpha} \right]^{m} + \theta_i \quad \text{(14)}
\]
the Eq. (14) above is also named as Soil Freezing Characteristic Curve (SFCC), and it is the additional information for solving the simultaneous equations.

3 EXPERIMENTAL VALIDATION

The equations have been numerically solved based on the finite difference method, to evaluate the performance, the proposed model is used to analyze one example on basis of published experimental data.

Wang et al. (2012) performed a one-dimensional freezing experiment, during which a partially saturated soil column was frozen from the top to the bottom without an external water supply. The soil material was loess of Xi’an area in China, the liquid limit and plastic limit are 30.2% and 17.8%, respectively. The soil column was 30 cm in height and 7.5 cm in diameter, two kinds of initial water content were controlled in the measurement, 13.6% and 20.8% in gravity water content, for these two cases separately included two cases of controlling soil density $\rho_d$, 1.40 g/cm$^3$ and 1.28 g/cm$^3$, thus totally four conditions were carried out in this test. For each cases, soil sample was subjected to a temperature gradient of top -10 ºC and bottom 19 ºC in 7 days. During the test, the side wall of the sample was adiabatic, the water content and temperature distributions were measured at the end of test.

Fig.1. The computed and measured profiles of volumetric water content (a) and soil temperature (b), Case 1: the initial gravity water content is 13.6%, the freezing period is 7 days.

Fig.2. The computed and measured profiles of volumetric water content (a) and soil temperature (b), Case 2: the initial gravity water content is 20.8%, the freezing period is 7 days.

Fig.1 illustrates the simulated profiles of water content (a) and soil temperature (b) with comparing with the measured result. It can be seen that the volumetric water content decreases in the unfrozen zone and increases in the frozen zone, it is because that no water was supplied during soil freezing, the liquid and vapor water was “sucked” from unsaturated zone. It can be observed that both the simulated result of water content and temperature agrees reasonable well with the experimental result. The predicted water content at the freezing front is somehow lower while at the frozen zone is slightly higher than the measured result, it is due to the indeterminacy of hydraulic conductivity in frozen part.

Fig.2 simulated the test result of case 2 with a higher initial water content, it shows that the increase in water content frozen region of small density loess is smaller than with the big density loess, and the freeze frontal water content increase in big density loess is relatively smaller than in small density loess. Comparing Fig.1 (a) and Fig.2 (a), it is found that the greater initial water content, the greater increase in the water content of freezing front. Same with case 1, an acceptable simulated result can be generated from the proposed model except for the water content at freezing front. In a word, the proposed model is capable of effectively simulating the freezing process of unsaturated soil.

4 PARAMETRIC ANALYSIS

In this section, we aim to evaluate the effects of
relevant factors on soil freezing in an unsaturated soil, including initial water content, temperature gradient and saturated hydraulic conductivity. The control condition is designed as that a homogeneous soil column in height of 1 m is subjected to a temperature gradient of 20 °C in 30 days, the top and bottom temperature are -10 °C and 10 °C, respectively, the soil properties in the controlled case are given in Table 2, the initial profiles of water content and temperature are constantly 15% and 5 °C, respectively. During soil freezing, no water flows at the lower boundary. Two indexes, total ice content $\Theta_i$ and percentage of vapor content $\theta_v/\theta_i$ in unfrozen part, are chosen as assessment criteria. Here the $\Theta_i$ is defined as

$$\Theta_i = \int_0^{d_i} \theta_i$$

where $d_i$ is the frost depth (cm), thus $\Theta_i$ represents the amount of ice in the magnitude of height, while $\theta_v/\theta_i$ denotes the percentage of vapor flow in total water flow of the unfrozen part.

Table 2. The soil hydraulic parameters in control case, the parameters originate from van Genuchten (1980) model.

| Parameter  | $K_s$ (cm/d) | $\alpha$ (cm$^{-1}$) | $\theta_i$ (m$^3$/m$^3$) | $\theta_v$ (m$^3$/m$^3$) | $n$ (m) |
|------------|--------------|---------------------|--------------------------|--------------------------|--------|
| Value      | 20.0         | 0.00546             | 0.065                    | 0.49                     | 2.32   |

![Fig.3](image)

Fig.3. The effect of initial water content on freezing behavior of unsaturated soil, initial water contents are 10%, 12%, 15%, 18% and 20%.

Fig.3 illustrates the total ice content and percentage of vapor content against the initial volumetric water content, the initial water content was assigned as five values from 10% to 20%. It can be observed that the total ice content $\Theta_i$ gradually increase with the increase of initial water content; it is easily imagine that much more water can migrate from the unfrozen part if the initial water content is higher. A maximum value of $\Theta_i$, however, seems to appear at around 12 cm, which indicates the ice formation would terminate even if giving a high water content. It is deduced that the “sucking force” at freezing front affects the amount of water flowed into frozen part, which limits the increase of ice formation. Additionally, the percentage of vapor content $\theta_v/\theta_i$ decreases with the increase of initial water content, the range is about from 60% to 15%, it implies that the vapor flux cannot be neglected in an unsaturated soil with closed system, especially when the water content is low.

![Fig.4](image)

Fig.4 displays the variations of $\Theta_i$ and $\theta_v/\theta_i$ against temperature. When adding a couple of equivalent temperature of positive and negative values on the bottom and top boundary of soils, respectively, the higher the temperature, the more the vapor flux generated in the soil. It can be demonstrated like that higher temperature increases the vaporization rate in soil body, and its higher gradient supplies greater driving force for vapor flow. It should be noted that the temperature gradient produces tiny influence on the total ice content, which smoothly increase in the range of 9.0–9.5 cm. The reason may be that the water content profile in unfrozen zone reaches to an equilibrium state, at the status, little water can be driven by the temperature.

![Fig.5](image)

Fig.5. The effect of saturated hydraulic conductivity $K_s$ on freezing behavior of unsaturated soil, the selected values of $K_s$ are 0.1, 1, 20, 100 and 500 cm/d.
Fig. 5 shows the developments of $\Theta_t$ and $\theta_s/\theta$ with saturated hydraulic conductivity $K_s$. Focusing on the total water content $\Theta_t$, it can be seen that $\Theta_t$ firstly decreases and then increases with the increase of saturated hydraulic conductivity, an inflection point of minimum value appears at around $K_s=20\text{cm/d}$. As to the variation of $\theta_s/\theta$, it initially increases with the increase $K_s$, a reduction occurs after a peak value of about $K_s=100\text{cm/d}$. It should be noted that there is an optimal value of $K_s$ for the amount of ice formation and vapor flux, their relationship cannot be linear.

5 CONCLUSIONS

This paper presents a coupled model of heat, liquid water, and vapor movement in unsaturated freezing soil, the model of simultaneous equations consists two equations governing mass and heat transfer, respectively, one equation for mass conservation, and one equation derived from Clapeyron equation. The advantage of this model is simulating the freezing process of unsaturated soil.

Parametric analysis is carried out to clarify the role of vapor in moisture migration during freezing, the result shows that vapor largely contributes to ice formation, occupying about 20%–60% of total water fluxes in an unsaturated and closed system under regular temperature gradient, it indicates the vapor flow in freezing soil cannot be neglected. The result also shows that, total ice content and vapor flux increases and decreases with the increase of initial water content, respectively. The higher the temperature gradient, the greater the vapor flux percentage, while the total ice content is insensitive to the temperature gradient. Peak values exist both for the variations of total ice content and that of vapor flux against saturated hydraulic conductivity.

ACKNOWLEDGEMENTS

This research was supported by National Basic Research Program of China (2014CB047001).

REFERENCES

1) de Vries, D.A. (1958): Simultaneous transfer of heat and moisture in porous media. Trans. Am. Geophys. Union, 39, 909-916.
2) Eigenbrod, K. D. and Kenneppohl G. J. A. (1996): Moisture Accumulation and Pore Water Pressures at Base of Pavements. Transportation Research Record, 1546, 151-161.
3) Guthrie, W., Hermansson, A. and Wolffinden, K. (2006): Saturation of Granular Base Material Due to Water Vapor Flow during Freezing: Laboratory Experimentation and Numerical Modeling. Cold Regions Engineering, 1-12. doi: 10.1061/40836(210)66.
4) Harlan, R.L. (1973): Analysis of coupled heat-fluid transport in partially frozen soil. Water Resources Research, 9 (5), 1314-1323.
5) Hu, H., Yang, S. and Lei, Z. (1992): A numerical simulation for heat and moisture transfer during soil freezing. Journal of Hydraulic Engineering, 7, 1-8. (in Chinese).
6) Milly, P.C.D. (1984): A simulation analysis of thermal effects on evaporation. Water Resource Research, 20, 1087-1098.
7) Mualem, Y. (1976): A new model for predicting the hydraulic conductivity of unsaturated porous media. Water Resources Research, 12, 513-522.
8) Nassar, I.N. and Horton, R. (1989): Water transport in unsaturated nonisothermal salty soil: II. Theoretical development. Soil Science Society of America Journal, 53, 1330-1337.
9) Nassar, I.N. and Horton, R. (1992): Simultaneous transfer of heat, water, and solute in porous media: I. Theoretical development. Soil Science Society of America Journal, 56, 1350-1356.
10) Philip, J.R. and de Vries, D.A. (1957): Moisture movement in porous materials under temperature gradient. Trans. Am. Geophys. Union, 38, 222-232.
11) Sakai, M., Toride, N., Šimůnek, J. (2009): Water and vapor movement with condensation and evaporation in a sandy column. Soil Science Society of America Journal, 73(3), 707-717.
12) Saito, H., Šimůnek J. and Mohanty, B.P. (2006): Numerical analysis of coupled water, vapor, and heat transport in the vadose zone. Vadose Zone Journal, 5, 784-800.
13) Sheng, D., Zhang, S., Niu, F. and Cheng, G. (2014): A potential new frost heave mechanism in high-speed railway embankments. Géotechnique, 64(2), 144-154.
14) Teng, J.D., Yasufuku, N., Liu, Q. and Liu, S.Y. (2013): Analytical solution for soil water redistribution during evaporation process. Water Science and Technology, 68 (12), 2545-2551.
15) van Genuchten, M.Th. (1980): A closed-form equation for predicting hydraulic conductivity of unsaturated soils. Soil Science Society of America Journal, 44, 892-898.
16) Wang, T., Wang, J. and Zhang, L. (2012): Experimental research on moisture migration in freezing unsaturated loess. Journal of Xi’an University of Architecture & Technology (Natural Science Edition), 44(1), 7-13 (In Chinese).
17) Zhou, J., Wei, C., Li, D. and Wei, H. (2014): A moving-pump model for water migration in unsaturated freezing soil. Cold Regions Science and Technology, 104-105, 14-22.