A TEST OF CP SYMMETRY IN POSITRONIUM

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The aim of this CP symmetry test in positronium is to measure the CP violation amplitude parameter \( C_{CP} \). This is derived from the measurement of the asymmetry in an angular distribution of the photons from the decay of the ortho-positronium in a magnetic field. The Standard Model prediction for \( C_{CP} \) is a value of the order of \( 10^{-9} \). Thus the observation of a larger \( C_{CP} \) value would be a signal of physics beyond the Standard Model. A previous measurement has found \( C_{CP} \) consistent with zero, with an uncertainty of \( \sim 10^{-2} \). We have investigated the possibility of using the existing ETHZ-INRM-IN2P3 BGO crystal detector, set-up for positronium physics studies, to improve the sensitivity on the \( C_{CP} \) measurement. Preliminary calculations indicate that, using such an apparatus, with some modification, in a magnetic field of 4 kGauss, \( C_{CP} \) could be measured with an uncertainty in the range between \( \sim 10^{-4} \) and \( \sim 10^{-3} \), depending mainly on the uncertainty in the asymmetry measurement and the angular resolution of the photon detectors. If \( C_{CP} \) is less than \( \sim 10^{-4} \), the experimental technique outlined here appears to be inadequate to observe a CP violating effect and new techniques or different observables must be exploited for better sensitivity.

Keywords: Positronium; CP symmetry test; BGO crystal detector.

1. Introduction

In the context of the Standard Model of particle physics, violation of the discrete symmetry CP (C=charge conjugation, P=parity operation) and time reversal T arise in the quark sector only through the Cabibbo-Kobayashi-Maskawa (CKM) matrix. A number of measurements, mainly in the K and B meson sectors, allow to constrain the mixing angles and the phase of this matrix and to check the consistency of the model (see e.g. Ref. 2 for a review). This model, however, does not explain the observed pattern of mixing and CP violation in the quark sector. The situation in the lepton sector is different. Within the Standard Model, the lepton sector shows invariance under the discrete symmetry transformations CP and T. In the Standard Model, CP violating effects in a lepton system can arise only through higher order corrections from the CP violation in the quark sector,

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and they are expected to be extremely small. However, the recent observation of neutrino oscillations implies the existence of a lepton mixing matrix the so-called Pontecorvo-Maki-Nagakawa-Sakata (PMNS) matrix, which is analogous to the CKM matrix for quarks. If it contains non-zero CP-violating phases, then it can induce CP-violating effects in the neutrino and the charged lepton sectors. If the PMNS matrix is the only source of CP violation in the lepton sector, then neutrino oscillations are likely to be the only manifestation of CP violation in the lepton sector, as the CP-violating effects induced on charged leptons by the PMNS matrix are expected to be unobservably small (see e.g. Ref. [4] for a recent review). Thus, experimental evidence for genuine CP violation and/or T violation in a charged lepton system would be signature of new interactions among leptons, with an associated leptonic mixing matrix containing CP violating phases. A compilation of present experimental results of CP and T symmetry tests for leptons can be found in Ref. [5].

It was realized since several years that the positronium, an electron-positron bound state, can serve as a testing ground for different discrete symmetries as C, P, T, CP and CPT in the charged lepton sector (see Ref. [6] for a theoretical review). Previous and new experimental results on C and CPT symmetry tests in positronium have been reported [7].

In this paper we concentrate on CP symmetry tests in the positronium system. A previous experiment is described in Ref. [8]. The experiment aims to measure the angular correlation between the positronium spin and the momenta of the photons from the positronium decay. As proposed in Ref. [9], if CP violating interactions are acting in the positronium system, then (at least) one CP violating term must contribute to the positronium lagrangian. This term should be proportional to:

\[
(S \cdot \hat{k}_1)(S \cdot \hat{k}_1 \times \hat{k}_2)
\]  \(\text{(1)}\)

Here \(S\) is the positronium spin, while \(\hat{k}_1\) and \(\hat{k}_2\) are the unit vectors in the directions of the highest and second highest energy photons from the three-photon decay of positronium. Indeed, the quantity in eq. (1) is non-zero only for the spin 1 state of the positronium, the ortho-positronium (o-Ps), which decays into three photons. The \(\hat{k}_1\) and \(\hat{k}_2\) unit vectors identify a plane (see Fig. 1). The first product in eq. (1) is the cosine of the angle between the o-Ps spin and the highest energy photon, which we define as \(\cos \theta_1\). The second product in eq. (1) is the cosine of the angle between the o-Ps spin and the normal to the \(\hat{k}_1-\hat{k}_2\) plane (see Fig. 1), which we define as \(\cos \theta_2\). Then the quantity in eq. (1) can be rewritten as \(\cos \theta \equiv \cos \theta_1 \cos \theta_2\). If there is a CP violating term in the lagrangian, then the measured number of events as a function of \(\cos \theta\) should be described by

\[
N(\cos \theta) = N_0(1 + C_{CP} \cos \theta)
\]  \(\text{(2)}\)

with the CP violation amplitude parameter, \(C_{CP}\), different from zero. Within the Standard Model, \(C_{CP}\) in the positronium system is expected to be very small, of the order of \(10^{-9}\) (see Ref. [10]). The measured distribution \(N(\cos \theta)\) should
show an asymmetry in $\cos \theta$ so that $N(\cos \theta_+) - N(\cos \theta_-) = 2N_0 C_{CP} \cos \theta$ for $\cos \theta_+ = -\cos \theta_- = \cos \theta$. The quantity $C_{CP}$ can be determined by measuring the rate of events $N_+$ for a given $\cos \theta_+ = \cos \theta$ and $N_-$ for $\cos \theta_- = -\cos \theta$. In practice, $N_+$ is the number of events in which $\hat{k}_2$ forms an angle $\theta_{12}$, smaller than 180 degrees, with $\hat{k}_1$ and the B field forms an angle $\theta_2$ smaller than 90 degrees with the normal to the o-Ps decay plane (see Fig. 2). In the $N_-$ events, $\hat{k}_2$ forms an angle $2\pi - \theta_{12}$ with $\hat{k}_1$ and the B field forms an angle $2\pi - \theta_2$ with the normal to the o-Ps decay plane. For these events the normal to the o-Ps decay is reversed with respect to the $N_+$ events, by flipping the direction of $\hat{k}_2$ specularly with respect to $\hat{k}_1$ (see Fig. 2). Then the asymmetry

$$A = \frac{(N_+ - N_-)}{(N_+ + N_-)} = C_{CP} \cos \theta$$

(3)

allows to derive the experimental value of $C_{CP}$.

2. Experimental method

The measurement of the asymmetry $A$ implies that $\cos \theta$ in eq. 3 is a well defined quantity in the experiment. In turn, this implies that the spin $S$ direction is defined. This direction can be selected using an external magnetic field.

Let us recall the Ps spin states:

- the singlet, with $S=0$, $m=0$, for the parapositronium (p-Ps);
- the triplet, with $S=1$, $m=1, 0, -1$, for the orthopositronium (o-Ps).

An external magnetic field $\vec{B}$ can be used to align the o-Ps spin parallel ($m=1$), perpendicular ($m=0$) or antiparallel ($m=-1$) to the field direction. However, the magnetic field does not only align the spin, but also perturbs and mixes the $m=0$ states. Thus, two new states are possible for the Ps system: the perturbed singlet and the perturbed triplet $m=0$ states. Their lifetimes depend on the $\vec{B}$ field intensity. The perturbed singlet state has a lifetime shorter than 1 ns (as the unperturbed one), which is not relevant in the measurement described here, because too short compared to the typical detector time resolution of 1 ns. The perturbed triplet lifetime as a function of the magnetic field intensity $B$ is shown in Fig. 3 (calculated using the lifetime formula in Ref. 10). One sees that for values of $B$ of few kGauss the lifetime of the triplet $m=0$ state can be substantially reduced with respect to the unperturbed lifetime. Thanks to this effect, applying an external magnetic field, it is possible to separate the $m=0$ from the $m=\pm1$ states, by the different lifetimes of the perturbed $m=0$ and unperturbed $m=\pm1$ states. The values of the B field can be optimized for maximum separation: it is found that for $B=4$ kGauss, corresponding to a perturbed lifetime of 30 ns, the separation is optimal.

Fig. 4 shows the positronium decay time spectrum without and with an external magnetic field of 4 kGauss, corresponding to a perturbed lifetime of the triplet $m=0$ state of 30 ns (see Fig. 3). For this calculation, the population of each of the four
states, one singlet \( (m=0) \) and three triplet \( (m=1, 0, -1) \) states, is taken to be the same. The \( m=0 \) states are mixed by the magnetic field and their lifetime modified, depending on the magnetic field intensity. The unperturbed singlet lifetime is 0.125 ns, while the perturbed singlet lifetime for \( B=4 \) kGauss is 0.522 ns. This value is used for the lifetime of the singlet \( m=0 \) state, in the presence of the magnetic field. The unperturbed triplet lifetime used in the calculation is 132 ns, as measured in SiO\(_2\) aerogel target of Ref. 11. The perturbed triplet \( m=0 \) lifetime for \( B=4 \) kGauss is 30 ns. Note that the triplet \( m=\pm1 \) states are unperturbed, thus they have a lifetime of 132 ns, without or with the external magnetic field.

The measurement of the asymmetry \( A \) is actually done in the following way. The direction and intensity of the \( B \) field are fixed. The \( \hat{k}_1 \) and \( \hat{k}_2 \) detectors are also fixed. In this way \( \cos \theta \) is defined. For each event, the Ps decay time and the energies of the three photons from the o-Ps decay are measured. The off-line analysis requires that the highest energy photon be in the \( \hat{k}_1 \) detector within an energy range \( \Delta E_1 = E_{1\max} - E_{1\min} \). The second highest energy photon must be recorded in the \( \hat{k}_2 \) detector within an energy range \( \Delta E_2 = E_{2\max} - E_{2\min} \). Then the \( N_+ \) and \( N_- \) events are counted to determine the asymmetry in eq. 3.

The measurement of the asymmetry \( A \) is performed for both the perturbed \( m=0 \) states, selecting events with Ps decay time between 10 and 60 ns (“perturbed time window” in Fig. 4), and for the unperturbed \( m=\pm1 \) states, selecting events with Ps decay time between 60 and 270 ns (“unperturbed time window” in Fig. 4). Let us define \( A_u \) and \( A_p \) the asymmetry measurements for the perturbed and unperturbed states, respectively. In this way two independent asymmetry measurements are available. Furthermore it is easy to demonstrate that if \( A_u = C_{CP} \cos \theta \) then \( A_p = -C_{CP} \cos \theta \). Thus, the quantity

\[
A = (A_u - A_p)/2
\]

provides a measurement of \( C_{CP} \) which is free of decay-time-independent systematics. Decay-time-dependent systematics, on the contrary, are not canceled out in eq. 4 and they must be considered and included in the total uncertainty on \( A \), as explained in the next section.

The measured asymmetry is related to the \( C_{CP} \) parameter by the simple formula

\[
A = C_{CP} Q .
\]

The quantity \( Q \) is ideally the \( \cos \theta \) value in eq. 4. However, due to the experimental uncertainty on the \( \cos \theta \) value, to the backgrounds affecting the asymmetry measurement, and to other uncertainties, the measured asymmetry value is actually reduced with respect to an ideal case with no uncertainties. Comparing eq. 5 with eq. 4 the quantity \( Q \) can be written as

\[
Q = f \cos \theta ,
\]

with \( f < 1 \) in a real experiment. The quantity \( Q \) is the product of several factors, accounting for effects and uncertainties deteriorating the asymmetry measurement.
Fig. 1. Definition of the geometry and of the quantities used for the experimental tests of CP symmetry in the positronium system. The vectors $\hat{k}_1$ and $\hat{k}_2$, the directions of the first and second highest energy photons, respectively, identify the decay plane of the o-Ps. The vector S indicates the o-Ps spin.

Fig. 2. Definition of $N_+$ and $N_-$ events: $N_+$ is the number of events where the normal to the o-Ps decay plane forming an angle smaller than 90 degrees with the B field vector (defined as normal UP in the picture). Flipping $\hat{k}_2$, with respect to $\hat{k}_1$, as shown in the picture, inverts the normal to the o-Ps decay plane (defined as normal DOWN in the picture).

with respect to the case of an ideal detectors. It can be determined, for a given detector geometry, by a Monte Carlo simulation and by measurements. The uncertainty on $C_{CP}$ is determined by the uncertainties on the measured $A$ value and on the evaluated Q value. The uncertainty on $C_{CP}$ determines the level of sensitivity of the experiment to a hypothetical CP violating interaction.

3. A new CP symmetry test experiment

In the previous experiment to test CP invariance the $C_{CP}$ value is measured to be

$$C_{CP} = -0.0056 \pm 0.0154.$$  (7)
The total uncertainty includes both statistical and systematic uncertainties. This result is obtained by the asymmetry measurement and the evaluated value of $Q$ (see eq. 5). The evaluated $Q$ value (called analyzing power $S_{an}$ in eq. 5) is $Q = 0.072 \pm 0.15$. The value of $\cos \theta$, for $\theta_1 = 55^\circ$ and $\theta_2 = 45^\circ$, is $\cos \theta = 0.41$, resulting into $f = Q/\cos \theta = 0.17$, about a factor of 6 smaller than for a perfect detector ($f = 1$). It implies that in a new experiment, at the best $Q$ can be increased by a factor of $\sim 6$, as compared to the previous experiment. The quantity $Q$ is evaluated as the product of several factors: the dominant factors are related to the (decay-time dependent) background and to the uncertainties on $\theta_1$ and $\theta_2$ values, determined by the photon detector angular resolution.

The final asymmetry measurement gives:

$$A_{final} = -0.0004 \pm 0.0010(stat.) \pm 0.0004(syst.1) \pm 0.0001(syst.2) = -0.0004 \pm 0.0011 \, .$$

(8)

The statistical uncertainty on $A_{final}$, with few $10^6$ events collected, amounts to less than $10^{-3}$. The systematic uncertainty on $A_{final}$ is determined by two effects, which are both decay-time-dependent, i.e. affecting differently the measurements made in the unperturbed and perturbed time windows (recall definition of $A$ by eq. 4).

The first systematic uncertainty on $A_{final}$ in eq. 8 is due to the subtraction of background induced mainly by events with two back-to-back annihilation photons (from e.g. collisional quenching of unperturbed triplet Ps or magnetic quenching...
Fig. 4. The positronium decay time spectrum without and with an external magnetic field of 4 kGauss, corresponding to a perturbed lifetime of the triplet $m = 0$ state of 30 ns (see Fig. 1). For this calculation, the population of each of the four states, one singlet ($m=0$) and three triplet ($m=1,0,-1$) states, is taken to be the same. The $m=0$ states are mixed by the magnetic field and their lifetime modified, depending on the magnetic field intensity. The unperturbed singlet lifetime is $\tau_s = 0.125$ ns, while the perturbed singlet lifetime for $B=4$ kGauss is 0.522 ns. This value is used for the lifetime of the singlet $m=0$ state, in the presence of the magnetic field. The unperturbed triplet lifetime is $\tau_{o-Ps} = 132$ ns, while the perturbed triplet $m=0$ lifetime for $B=4$ kGauss is 30 ns. Note that the triplet $m=\pm 1$ states are unperturbed thus they have a lifetime of 132 ns, without or with the magnetic field.

of perturbed triplet Ps), mimicking a $3\gamma$ event. This is the case if one of the 511 keV photon from the two-photon decay makes Compton scattering and deposits in the $k_2$ detector an energy in the required $\Delta E_2$ range. The second systematic uncertainty is introduced by the correction for diffusion of the Ps atoms during their lifetime in the positronium formation region.

An additional systematic uncertainty on the $A_{\text{final}}$ value (not included in eq. 8) may be introduced by the effect of the shadowing of the crystals by the coils of the permanent magnet (see Fig. 1 in Ref. 8), used to create the magnetic field over the volume of the o-Ps forming region. To obtain a magnetic field of 4 to 5 kGauss, as needed, there are two options: either the use of a small permanent magnet, generating a field just around the Ps forming region, or a large magnet, capable to contain the photon detector. The first option has the advantage that the magnetic field affects practically only the Ps forming region. The disadvantage is the effect of the shadowing of the crystals by the magnet coils. The second option (large magnet) is envisaged for a new CP symmetry test experiment outlined here. The advantage of this option is that the detector is inside the magnetic field, so
Fig. 5. Schematic view of the BGO crystal barrel calorimeter used to detected the photons from the o-Ps decay: (a) detector front view and definition of the $k_1$ and $k_2$ vectors, for this arrangement: the outer crystal ring is used for the photon measurements, the inner crystal rings (shaded region) are used as veto; (b) detector side view, showing the crystal outer ring for photon detection and the inner rings (shaded region) for veto: only the photons detected by the outer crystals in the window between the veto crystals are considered for the $C_{CP}$ measurement. This arrangement allows to improve the crystal angular resolution for photon detection, as compared to the case where no veto is used.

there is no shadowing of the crystals by the magnet coils. In addition the field uniformity is expected to be better than $\pm 1\%$. The disadvantage of this option is that conventional photomultipliers cannot be used inside the strong magnetic field. It is instead possible to use e.g. large area avalanche photodiodes (see e.g. Ref. [12] and references therein).

The new detector set-up is sketched in Fig. 5. The photon detector is made of BGO crystals. They have an hexagonal cross section with an inner diameter of 5.5 cm and a length of 20 cm. The crystals are the same as those used in the ETHZ-INRM-IN2P3 experiment of Ref. [11] and expected to be used in the experiments proposed in Ref. [13] and Ref. [14]. About hundred of these crystals are available, from a previous experiment (see Ref. [15]). The positronium formation and tagging methods can be similar to those described in Ref. [11]. The crystals surround the positronium formation and decay region. In the set-up shown in Fig. 5 24 crystals are used for a barrel detector. The advantage of having a complete crystal barrel surrounding the Ps decay region is that several asymmetry measurements can be done for different photon angles, without any intervention on the detector between the measurements. The different asymmetry measurements, when combined together, are expected to result in a reduction of systematic uncertainties, due to geometrical effects, inducing fake asymmetries.
Fig. 6. Schematic side view of the crystal barrel placed in a solenoidal magnet, generating a magnetic field in the direction shown in the picture. The direction of the B field with respect to \( \hat{k}_1 \) and to the the normal to the \( \hat{k}_1 \)-\( \hat{k}_2 \) plane is chosen to be 45 degrees, to maximize the sensitivity of the \( C_P \) measurement.

The crystals are placed \( \sim 20 \) cm from the o-Ps decay region. Thus the solid angle covered by one crystal is much larger than the \( \sim 2\% \) of \( 4\pi \) used for the measurement in (indeed the photon detectors where mounted in lead shields to reduce the photon detection area). Such a poor angular resolution for the present photon detectors would result into an unacceptably large uncertainty on the photon angle measurements. Thus, for a comparable or better sensitivity, than in the previous experiment, we need at least a comparable photon angular resolution. A way to improve this resolution is by reducing the photon measuring area. This could be realized by reducing the crystal size, cutting the crystals into smaller ones. Although technically feasible, this is not envisageable in the short term, because the crystals are being used also for other experiments, where the bigger size is preferred. An alternative way to reduce the photon measuring area is by shielding part of the (rectangular) face of the crystal exposed to the photons, so that only a small window is actually used to detect the photons for the \( C_P \) measurement. An efficient shield can be realized by additional BGO crystals, in a set-up as the one shown in Fig. 5. Two inner rings, each of 18 BGO crystals (shaded area), can be used as a veto. A small distance \( d \) is left among the two inner rings (Fig. 5(b)). In this way, if \( l \) is the shorter side of the crystal rectangular face exposed to photons, a

\[ \text{The overall size of the detector is determined by the number of available crystals and the space available for the detector inside the magnet.} \]
photon measuring window of area $l \times d$ is defined on this face of the outer crystal. If a signal is detected in the veto crystals, then the event is rejected. The event is accepted by the off-line analysis if three photons, $\hat{k}_1$, $\hat{k}_2$ and $\hat{k}_3$ are recorded in the photon measuring window of the respective outer ring crystals, with energies $E_1$, $E_2$ and $E_3$ in the required energy ranges. An angular resolution comparable to the previous experiment is obtained if the distance between the inner veto rings is about 1 cm. Then the photon measuring window on the outer crystals has an area of $l \times d = 5.5 \times 1$ cm$^2$, resulting into $(5.5/20^2) \sim 1.4\%$ of $4\pi$.

The detector is placed inside a solenoidal magnet (see Fig[4]), generating a uniform and constant magnetic field of 4 kGauss, along the direction of the magnet longitudinal axis. The crystal calorimeter must be held in place by a rigid mechanical structure. The B field direction must not be perpendicular to $\hat{k}_1$ and not lying in the $\hat{k}_1$-$\hat{k}_2$ plane, for a non-zero value of the CP violating term (eq[1]). The $\cos\theta$ value is maximum (best sensitivity to $C_{CP}$) if the B vector is lying in the plane identified by $\hat{k}_1$ and the normal to the $\hat{k}_1$-$\hat{k}_2$ plane, and forming an angle of 45° with $\hat{k}_1$. In this case $\cos\theta = 0.5$.

4. Estimate of the detector sensitivity

To understand how to optimize the sensitivity of the detector we must consider the uncertainties affecting the $C_{CP}$ measurement. They are the uncertainties on the $A$ asymmetry measurement and on $Q$. In the following, we quantify the uncertainty on $C_{CP}$ as a function of the quantities $A$ and $Q$ and their uncertainties. Let us define $\Delta C_{CP}$, $\Delta A$ and $\Delta Q$ the uncertainties on $C_{CP}$, $A$ and $Q$, respectively. Then, the relative error on $C_{CP}$ is given by:

$$\frac{\Delta C_{CP}}{C_{CP}} = \left| \frac{\Delta A}{A} \right|^2 + \left| \frac{\Delta Q}{Q} \right|^2 .$$

which can be written as:

$$\left| \frac{\Delta C_{CP}}{C_{CP}} \right|^2 = \frac{1}{Q^2} \left[ \left| \frac{\Delta A}{C_{CP}} \right|^2 + \left| \Delta Q \right|^2 \right] .$$

The uncertainty on $C_{CP}$ is then:

$$\Delta C_{CP} = \frac{1}{Q} \left[ |\Delta A|^2 + C_{CP}^2 |\Delta Q|^2 \right]^{1/2}$$

which for $C_{CP} << 1$ can be approximated by

$$\Delta C_{CP} \approx \frac{|\Delta A|}{Q} ,$$

where $\Delta A$ is the total uncertainty, combining both the statistical and systematic uncertainties.

The statistical uncertainty on $A$ is determined by the event statistics:

$$\Delta A_{stat} \sim \sqrt{2/(N_+ + N_-)} .$$
For $N_+ + N_-$ of the order of $10^{10}$, then the statistical uncertainty is $\Delta A_{\text{stat}} \sim 10^{-5}$. A systematic uncertainty is resulting from the subtraction of the two-photon annihilation background which, as explained in the previous Section, affects differently the asymmetry measurements in the perturbed and unperturbed time windows. To see how a decay-time-dependent background affects the asymmetry measurement, let us write the asymmetry expression including a decay-time-dependent background component. This is given by

$$A_{\text{meas}} = \frac{N_+ - N_- + B}{N_+ + N_- + B} \approx A + \frac{B}{N_+ + N_-}. \quad (10)$$

In this equation, $A$ is defined as in eq. 3 and $B$ is the background counts (the approximate expression is valid for $B/(N_+ + N_-) \ll 1$). Combining the asymmetry measurements in the perturbed and unperturbed time windows we obtain:

$$\frac{A_{u_{\text{meas}}} - A_{p_{\text{meas}}}}{2} = A + \frac{B_u}{(N_+ + N_-)_u} - B_p \frac{1}{(N_+ + N_-)_p}. \quad (11)$$

The background contributions from $B_u$ and $B_p$ can be evaluated by a Monte Carlo simulation and subtracted to the measured asymmetry. The uncertainty on the background evaluation ($syst.1$ in eq. 8) contributes to the total asymmetry uncertainty.

We find that, for the detector set-up considered here, requiring three “good” photons (detected in the windows of the $\hat{k}_1$, $\hat{k}_2$ and $\hat{k}_3$ outer crystals), with $450 < E_1 < 550$ keV, with $300 < E_2 < 400$ keV, and with $150 < E_3 < 250$ keV, the probability of observing a background event, in the perturbed time window, where higher background is expected from the two-photon annihilation, is $B_p/(N_+ + N_-)_p < 10^{-6}$. In the unperturbed window we set this background contribution conservatively to zero so that the uncertainty in the difference in eq. 11 is maximized. Thus, we assign a background subtraction systematic uncertainty of $10^{-6}$, while it was measured to be $4 \times 10^{-4}$ in the previous experiment. The requirement of measuring a third photon in the specific energy range and angle (while only two photons were measured in 8) results into a reduction of the expected background from two-photon annihilation.

The additional systematic uncertainty ($syst.2$ in eq. 8), introduced by the correction for diffusion of the Ps atoms during their lifetime in the positronium formation region, appears to be the most difficult to evaluate and a detailed study is needed to understand how to control it at a level lower than the measured $10^{-4}$.

For what concerns the $Q$ value, entering in eq. 9, we have seen that its maximum expected value for an ideal detector, is a factor of about 6 greater than the measured value of 0.072 in the previous detector. A realistic value of $Q$ for the present detector is obtained by considering $\cos \theta = 0.5$ (was 0.42 in the previous experiment) and $f \approx 0.3$, improved by a a factor of about two with respect to the $f$ value in the previous measurement ($f = 0.17$). This improvement is due to the background reduction in this set-up, already considered in the discussion of the asymmetry systematic uncertainty from two-photon background subtraction.
In summary, with \( \Delta A_{\text{stat}} \sim 10^{-5} \), \( \Delta A_{\text{syst1}} \approx 10^{-6} \) and in the optimistic scenario where \( \Delta A_{\text{syst2}} \) can be reduced at the \( 10^{-5} \) level (in comparison to \( 10^{-4} \) in the previous experiment), using \( Q = 0.3 \times 0.5 = 0.15 \), we get a total error \( \Delta C_{CP} \sim 10^{-4} \) (in this case statistical and systematic uncertainty are of the same order). In a more pessimistic scenario, where \( \Delta A_{\text{syst2}} \) is not reduced significantly with respect to \( 10^{-4} \), then \( \Delta A_{\text{syst2}} \) dominates the total uncertainty, resulting into \( \Delta C_{CP} \sim 7 \times 10^{-4} \).

5. Conclusions

The measurement of CP violating effects in the lepton sector would be signature of physics beyond the Standard Model. The positronium offers the possibility to test the CP symmetry in a charged lepton system. The Standard Model predicts for the CP violation amplitude parameter in positronium, \( C_{CP} \), a value of the order of \( 10^{-9} \). The present measurement of \( C_{CP} \) is consistent with zero at the 1% level of precision. We have outlined an experimental set-up to possibly improve on this precision. The experiment is based on the use of the ETHZ-INRM-IN2P3 BGO crystal calorimeter in a magnetic field of 4 kGauss. We have made a preliminary evaluation of the sensitivity to \( C_{CP} \) expected for such an experiment. We find that the sensitivity range \( \Delta C_{CP} \sim 10^{-4} - 10^{-3} \) can be accessible to this experiment. The upper end of this range is determined by the systematic uncertainty on the asymmetry measurement, dominated by the effect of the Ps atom diffusion during its lifetime in the Ps formation region. In general, a reduction of \( \Delta C_{CP} \) can be achieved by reducing the total uncertainty on the asymmetry measurement and increasing the resolving power \( Q \). The latter could be obtained by using a photon detector with a finer angular resolution than for the detector presented here. However, if \( C_{CP} \) is non zero but less than \( \sim 10^{-4} \), the experimental technique outlined here appears to be inadequate to observe a CP violating effect and new techniques or different observables must be exploited for better sensitivity.

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