Quantum states and local projective measurement in a relativistic field

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Abstract. In an explicitly covariant system with point-like detectors coupled to a relativistic quantum field, the Gaussian states of the combined system and the reduced states of the detectors derived from them are consistent, but not have to be covariant, in different frames of different observers. Even in the presence of spatially local projective measurements, which result instantaneous wave functional collapses on different time-slices in different reference frames, the consistency still holds.

1. Introduction
Recently the new field on the relativistic quantum information (RQI) is emerging between relativity and quantum information science (QIS) [1]. People are applying the knowledge newly developed in the QIS to relativistic systems, to broaden the range of applications of the QIS on the one hand, and to deepen understanding on both relativity and quantum physics on the other. The central issue of the RQI is the consistency of relativistic locality and quantum nonlocality. The latter in quantum mechanics (QM) manifests when combining quantum entanglement of two or more parties with quantum measurement on one of these parties. This is why most of the efforts in RQI have been put on quantum entanglement in different reference frames: How to transform the measures or degrees of entanglement covariantly [1, 2]? How to entangle and what to be entangled for quantum fields living in a non-trivial spacetime such as the Rindler space or the black hole geometry [3, 4]? What are the entanglement dynamics of quantum fields in dynamical background spacetime such as the expanding universe [5, 6, 7], and the entanglement dynamics of atoms or detectors as an open quantum system [8, 9, 10, 11, 12, 13]? In the extreme cases such as black holes or even worm holes, can the RQI offer new insights into them [14, 15, 16, 17]?

As much attention has been drawn to quantum entanglement, quantum measurement in relativistic systems is also of great interest. Aharonov and Albert have shown that indirect measurement on quantum objects localized in space and time is consistent with relativistic QM [18] and quantum states make sense only within a given frame. In Ref. [19] Aharonov and Albert further concluded that there is no covariant description of measurement in terms of time evolution of quantum states. Even in the simplest case they illustrated, with no wave function collapse between initial and final time-slices, they found that quantum states have to be parametrized by many local times if the system consists of many local quantum objects. Diósi pointed out that in this case it would be simpler to describe such processes using Heisenberg operators, which are naturally covariant in a relativistic model [20].
One kind of models quite popular for studying entanglement dynamics in RQI is the detector theory \([21, 22]\), where the detectors (point-like quantum objects with internal degrees of freedom) are always coupled with a relativistic quantum field locally at their positions. One may wonder whether instantaneous projective measurement is still consistent in the presence of relativistic quantum fields \([23]\). This question concerns those direct projective measurements in the interaction region rather than in the asymptotic region, while such kind of measuring process cannot be described by a Hamiltonian. To our knowledge, a general argument for the consistency in such situations is still lacking. Thus we are looking at the consistency in a specific detector-field model below by analyzing the evolution of the Gaussian states with or without projective measurement in a direct extension of standard QM.

A brief introduction to our toy model and the mode function expansion we are using are given in Sections 2 and 3 respectively. Then we discuss the time-slicing independence of the reduced state of a point-like object without being measured in Section 4. The consistency of quantum states of the combined system and the reduced states of the detectors when we extend the instantaneous projective measurement in QM to relativistic quantum field theory are illustrated in Sections 5 and 6. Finally in Section 7 we summarize our findings and apply them to the case of quantum teleportation using two entangled detectors with one at rest, the other uniformly accelerated in Minkowski space.

2. RSG detector theory in Minkowski space
To make our discussions more concrete, let us consider a model with one or more Raine-Sciama-Grove (RSG) detectors \([24]\) coupled to a massless scalar field in \((1+1)\)D Minkowski space, described by the action

\[
S = -\frac{1}{2} \int d^2 x \sqrt{-g} \partial_\alpha \Phi \partial^\alpha \Phi + \sum_d \left\{ \frac{1}{2} \int d\tau_d \left[ (\partial_d Q_d)^2 - \omega_d^2 Q_d^2 \right] + \lambda \int d\tau_d \partial_d Q_d \int d^2 x \Phi(x^0, x^1) \delta^2(x^{\alpha} - z^d_\alpha(\tau_d)) \right\},
\]

where \(\alpha = 0, 1\), \(g\) is the determinant of the metric tensor \(g_{\alpha\beta}\) of the background spacetime, \(\Phi\) is a massless scalar field, \(Q_d\) is the harmonic oscillator (HO) inside the detector \(d\) with \(d = A\) for one-detector case, \(d = A, B\) for two-detector case, also \(\tau_d\) and \(z^d_\alpha(\tau_d)\) are the proper time and the prescribed trajectory of the detector \(d\), respectively, and \(\partial_d \equiv d/d\tau_d\). The combined system in (1) is a linear system. It is so simple that in some cases analytic results have been obtained in the whole parameter range without any approximation, but still it is complicated enough to give nontrivial results and insights.

The conjugate momenta of \(\Phi_{,x^1}\) and \(Q_d\) can be obtained from the action (1), then one can write down the Hamiltonian defined on the whole time-slice \(x^1 \in \mathbb{R}\) and evolving in time \(x^0\). Quantizing the Hamiltonian gives the Schrödinger equation, whose solutions are the wave functionals of the detectors and the field \(\psi(Q_d, \Phi_{,x^1}; x^0)\). However, different observers may have different reference frames, where time-slices are differently foliated. Since the field is defined on the whole time-slices, the Hamiltonian as well as the quantum state or the wave functional of the field in the Schrödinger equation are also defined on the whole time-slice. So the quantum states and their histories for the same system could look very different in different frames. Moreover, if the postulate of projective measurement in QM can be extended to relativistic quantum field theory, then the wave functional of a field on the whole time-slice would be collapsed by a measurement localized in space if the relevant degrees of freedom are entangled. This implies that the wave functionals in different frames could collapse on different time-slices passing through the same measurement event, if the event is local in spacetime.
Figure 1. The alternative coordinate \((\eta, \xi)\) given by (2) with \(A = 1/2\) in \(t-x\) diagram of \((1+1)D\) Minkowski space. The solid and dashed curves are constant \(\eta\) and \(\xi\) hypersurfaces, respectively.

It has been shown by Aharonov and Albert \[18\] that quantum states in relativistic quantum mechanics make sense only in a given frame. Quantum states defined on two different time slices but intersecting at some spacetime points could be very different even for the sector of the dynamical variables defined right on the intersecting region \[19\]. Thus one can compare two wave functionals of a field defined in two different frames only when the time-slices they are living on are totally the same. For example, in an alternative coordinate in \((1+1)\) dimensional Minkowski space,

\[
\eta = t - A \sin t \cos x, \quad \xi = x - A \sin x \cos t,
\]

with constant \(A < 1\) (illustrated in Figure 1), \(\eta\)-slices and conventional \(t\)-slices will overlap at \(t = \eta = n\pi\) with integer \(n\). Off those moments, \(\eta\)-slices are the wavy ones in Figure 1, where \(t\)-slices would be the horizontal straight lines. So if the initial state is assigned at \(t = 0\) time-slice, both the observer in conventional \(t-x\) frame and the observer in the \(\eta-\xi\) frame will agree with the same initial state. Then at \(t = \eta = \pi, 2\pi, 3\pi, \ldots\), two observers can make a clear comparison on the quantum states defined on that time-slice.

3. Mode functions

From the Hamiltonian the Heisenberg equations of motion for operators read

\[
\begin{align*}
\partial_\alpha \hat{Q}_d (\tau_d) + \omega_d^2 \hat{Q}_d (\tau_d) &= -\lambda \partial_\alpha \hat{\Phi}(z_d^0 (\tau_d)), \\
\sqrt{-g} \Box \hat{\Phi}(x^\alpha) &= \lambda \sum_d \int d\tau_d \partial_\alpha \hat{Q}_d \delta^2 (x^\alpha - z_d^0 (\tau_d))
\end{align*}
\]

where \(\Box \equiv \sqrt{-g}^{-1} \partial_\alpha \sqrt{-g} g^{\alpha\beta} \partial_\beta\). By virtue of the linearity of the system, operators at \(x^0\) are linear combinations of the operators defined at the initial moment \[25\]:

\[
\begin{align*}
\hat{Q}_d (\tau_d (x^0)) &= \sum_{d'} \left[ \phi_{d'}^0 (\tau_d) \hat{Q}_{d'}^0 (\tau_d) + f_{d'}^0 (\tau_d) \hat{P}_{d'}^0 (\tau_d) \right] + \int dy \left[ \phi_{y}^0 (\tau_d) \hat{\Phi}_{y}^0 (\tau_d) + f_{y}^0 (\tau_d) \hat{\Pi}_{y}^0 (\tau_d) \right], \\
\hat{\Phi}_{x_1} (x^0) &= \sum_{d'} \left[ \phi_{x_1}^0 (x^0) \hat{Q}_{d'}^0 (x^0) + f_{x_1}^0 (x^0) \hat{P}_{d'}^0 (x^0) \right] + \int dy \left[ \phi_{x_1}^0 (x^0) \hat{\Phi}_{y}^0 (x^0) + f_{x_1}^0 (x^0) \hat{\Pi}_{y}^0 (x^0) \right],
\end{align*}
\]
from which $\hat{P}_d(x^{0})$ and $\vec{F}_x(x^{0})$ can be derived. Here $\hat{\Omega}_{\mu}^{[n]} \equiv \hat{\Omega}_{\mu}(t_n)$ and all the “mode functions” $\phi_{\mu}^{0}(x^{0})$ and $f_{\nu}^{\mu}(x^{0})$ are real functions of time, which can be related to those in $k$-space in Ref. [25] (with different initial conditions.) Inserting the above expansions into (4), one obtains the field equations for mode functions similar to (4), which gives $\phi_{x_1}^{\mu}(x^{0}) = \phi_{x_1}^{\mu(0)}(x^{0}) + \phi_{x_1}^{\mu(1)}(x^{0})$ where, for proper initial conditions, the homogeneous solutions are $\phi_{x_1}^{\mu(0)} = 0$ and

$$\phi_{y_1}^{\mu(0)}(T) = \int \frac{dk}{2\pi} e^{\text{i}k(x^1-y^1)} \cos \omega_k T = \frac{1}{2} \left[ \delta(x^1-y^1+T) + \delta(x^1-y^1-T) \right], \quad (7)$$

with $\omega_k = |k|$. Then one would see that the homogeneous solutions $\phi_{x_1}^{\mu(0)}$ can be interpreted as vacuum fluctuations of the field, and $\phi_{x_1}^{\mu(1)}(x^{0})$ behave like retarded relativistic fields sourced by the point-like detectors. Inserting the solutions of $\phi_{x_1}^{\mu}$ into (3), one finds that $\phi_{x_1}^{\mu}$ behaves like those HO damped by the back reaction to the field, and $\phi_{x_1}^{\mu}$ behave like damped HO driven by vacuum fluctuations of the field $\phi_{x_1}^{\mu}(z_d^{\mu}(\tau_d^{\mu}))$ at the positions of the detectors $z_d^{\mu}(\tau_d^{\mu})$, both HO are living in the point-like detectors and not extended in space.

Comparing the expansions (5) and (6) of two equivalent continuous evolutions, one from $t_0$ to $t_1$ then from $t_1$ to $t_2$, the other from $t_0$ all the way to $t_2$, one can see that the mode functions have the following identities,

$$\phi_{\mu}^{[20]}(x^{0}) = \sum_{d'} \left[ \phi_{\nu}^{\mu(10)}(d') + f_{\nu}^{(10)}(d') \right] + \int dx' \left[ \phi_{\nu}^{\mu(10)}(d') + f_{\nu}^{\mu(10)}(d') \right], \quad (8)$$

$$f_{\nu}^{[20]}(x^{0}) = \sum_{d'} \left[ \phi_{\nu}^{\mu(10)}(d') + f_{\nu}^{(10)}(d') \right] + \int dx' \left[ \phi_{\nu}^{\mu(10)}(d') + f_{\nu}^{\mu(10)}(d') \right], \quad (9)$$

where $F_{[mm]} = F(t_m - t_n)$, $\pi_{d}^{\mu}(\tau_d^{\mu}(t)) \equiv \partial_d \phi_{d}^{\mu}(\tau_d^{\mu}(t)) + \lambda \phi_{\pi_d^{\mu}(t)}(t)$, $\pi_{x_1}^{\mu}(t) \equiv \partial_t \phi_{x_1}^{\mu}(t)$, $p_{d}^{\mu}(\tau_d^{\mu}(t)) \equiv \partial_d \phi_{d}^{\mu}(\tau_d^{\mu}(t)) + \lambda \phi_{\pi_d^{\mu}(t)}(t)$ and $p_{x_1}^{\mu}(t) \equiv \partial_t \phi_{x_1}^{\mu}(t)$ with $\mu, \nu = \{d\} \cup \{x_1\}$. Similar identities for $\pi_{d}^{\nu}$ and $p_{d}^{\nu}$ can be derived straightforwardly from (8) and (9). Such identities can be interpreted as the Huygens’ principle of the mode functions, and can be verified by inserting particular solutions of the mode functions into the identities.
4. Time-slicing scheme and quantum states

Suppose at \( t_0 = \eta_0 = 0 \) (when \( \tau_A = 0 \) for all detectors) the combined system of the detectors and the field is initially in a Gaussian state, which could be pure or mixed. Then the quantum state will always be Gaussian because of the linearity of the interaction. It will be the most convenient to work with the quantum state in the \((K, \Delta)\)-representation of the Wigner functional [26], which is the double Fourier transform of the conventional Wigner functional,

\[
\rho[K, \Delta] = \int \mathcal{D}\Sigma e^{iK \Sigma} \bar{\rho} \left[ \Sigma - (\Delta/2), \Sigma + (\Delta/2); x^0 \right]
\]

\[
= \exp \left( -\frac{1}{2\hbar} \left[ K^\mu \mathcal{Q}_{\mu\nu} K^\nu - 2\Delta^\mu \mathcal{R}_{\mu\nu} K^\nu + \Delta^\mu \mathcal{P}_{\mu\nu} \Delta^\nu \right] \right),
\]

(10)

where \( \bar{\rho}(Q_d, \Phi_{x^1}),(Q'_d, \Phi'_{x^1}); x^0 \) is the density matrix in the conventional representation, \( \mu, \nu = \{d\} \cup \{x^1\} \), and the time-dependent factors \( \mathcal{Q}_{\mu\nu}, \mathcal{P}_{\mu\nu}, \) and \( \mathcal{R}_{\mu\nu} \) are actually the symmetric two-point correlators \( \langle A, B \rangle \equiv \langle AB + BA \rangle/2 \), for they are obtained by \( \langle \Phi_\mu, \Phi_\nu \rangle = \hbar \delta_{\mu\nu}/(2\pi\hbar) \rho[K, \Delta]|_{\Delta = 0} = \hbar \mathcal{Q}_{\mu\nu}, \langle \Pi_\mu, \Pi_\nu \rangle = \hbar \delta_{\mu\nu}/12 \rho[K, \Delta]|_{\Delta = 0} = \hbar \mathcal{P}_{\mu\nu}, \) and \( \langle \Pi_\mu, \Phi_\nu \rangle = \hbar \mathcal{R}_{\mu\nu} \rho[K, \Delta]|_{\Delta = 0} \). With projective measurement, the form of the time-dependent factors \( \mathcal{Q}_{\mu\nu}, \mathcal{P}_{\mu\nu}, \) and \( \mathcal{R}_{\mu\nu} \) in terms of two-point correlators will keep unchanged from initial to final time-slices.

The reduced state of a detector (say, the detector \( A \)) at some moment \( x^0 \) is obtained by tracing out all other degrees of freedom in the density matrix,

\[
\rho_R(Q_A, Q'_A; x^0) = \text{Tr}_\phi \bar{\rho}(Q_A, \Phi_{x^1}),(Q'_A, \Phi'_{x^1}); x^0) \equiv \int \mathcal{D}\Phi_{x^1} \bar{\rho}(Q_A, \Phi_{x^1}),(Q'_A, \Phi_{x^1}); x^0),
\]

or in \((K, \Delta)\)-representation,

\[
\rho_R[K^A, \Delta^A; x^0] = \rho[(K^A, K^{x^1}), (\Delta^A, \Delta^{x^1}); x^0]|_{K^{x^1} = \Delta^{x^1} = 0}
\]

\[
= \exp \left( -\frac{1}{2\hbar} \left[ K^A \mathcal{Q}_{AA}(x^0) K^{A} - 2\Delta^A \mathcal{R}_{AA}(x^0) K^{A} + \Delta^A \mathcal{P}_{AA}(x^0) \Delta^A \right] \right).
\]

(12)

The reduced state \( \rho_R \) of the detector can be completely determined by the two-point correlators of the detector \( A \), namely, \( \mathcal{Q}_{AA}(x^0) = \langle \hat{Q}_A^2(\tau_A(x^0)) \rangle, \mathcal{P}_{AA}(x^0) = \langle \hat{P}_A^2(\tau_A(x^0)) \rangle, \) and \( \mathcal{R}_{AA}(x^0) = \langle \hat{Q}_A(\tau_A(x^0)), \hat{P}_A(\tau_A(x^0)) \rangle \), which are actually parametrized by the proper time \( \tau_A \) of the detector \( A \). So when the fields and possibly other detectors defined on different time-slices but passing through the worldline of the detector \( A \) at the same spacetime point associated with its proper time \( \tau_A \) are traced out, the same reduced state of \( A \) will be obtained. In other words, the reduced state of a detector is independent of the time-slicing outside the detector.

Looking more closely, one can see that the two-point correlators of the detector \( A \) are combinations of the mode functions and the initial data, the latter is in the form of correlators evaluated at the initial moment \( t = t_0 \). E.g.,

\[
\langle \hat{Q}_A^2(\tau_A) \rangle = \phi_A^A(\tau_A) \phi_A^A(\tau_A) \langle \hat{\psi}_A^0(\tau_A)^2 \rangle_0 + \int dxdy \phi_A^x(\tau_A) \phi_A^y(\tau_A) \langle \hat{\phi}_x^0(\tau_A), \hat{\phi}_y^0(\tau_A) \rangle_0 + \ldots.
\]

(13)

Here \( \langle \ldots \rangle_0 \) denotes the expectation values taken from the quantum state right after \( t = t_n \). One can see that all the relevant degrees of freedom are those in the past lightcone of the spacetime point \( z^0(\tau_A) \) where the reduced state of the detector is defined (cf. Figure 2). So we can say that the reduced state is causal and independent of different degrees of freedom living on different time-slices in different frames because those degrees of freedom are outside the past lightcone and so irrelevant.
The reduced states of two or more detectors and the degree of entanglement derived from them are also parametrized by \( \tau_A \) only and independent of time-slicing outside the detectors, provided that there is no projective measurement on any detector or field during the evolution. As we have observed in [10], the evolution of the two-point correlators of the detectors in one coordinate (e.g. \( \langle Q_A(\tau_A(t)), Q_B(\tau_B(t)) \rangle \) in \( t \)) could be quite different from the ones in a different coordinate (e.g. \( \langle Q_A(\eta), Q_B(\eta) \rangle \) in \( \eta \)), since \( \tau_A \) and \( \tau_B \) evolve in different ways in different frames.

5. Projective measurement on detectors

Suppose the detector \( A \) is located at \( x = 0 \) in Figure 1 and started to evolve at its proper time \( \tau_A = t = 0 \). If a local measurement is done on the detector at some moment \( 0 < t_1 \), then which time-slice, \( t_1 \)- or \( \eta_1 \)-slice \( (\eta_1 \equiv \eta(t_1)) \), will the wave functional of the combined system collapse on? If both collapses occur for different observers, will the two post-measurement states (PMS) be “identical” [23]?

Suppose a Gaussian measurement is done on the detector \( A \) at \( t = t_1 \in (0, \pi) \) when the quantum state on the \( t_1 \)-slice collapses to \( \rho = \rho_A \otimes \rho_A \) for the observer in the \( t-x \) frame, where we assume

\[
\rho_A = \exp \left( -\frac{1}{\hbar} \left[ g_A (\Delta_A)^2 + \frac{1}{g_A} (K_A)^2 \right] \right)
\]

(14)
is a squeezed coherent state for the detector \( A \) with constant \( g_A \). From (14) one obtains

\[
\rho_A = \exp \left( -\frac{1}{\hbar} \left[ K^{\mu} \tilde{Q}_{\mu \nu} K^\nu - 2\Delta^{\mu} \tilde{R}_{\mu \nu} K^\nu + \Delta^\mu \tilde{P}_{\mu \nu} \Delta^\nu \right] \right)
\]

(15)
with \( \mu, \nu = \{ x \} \cup \{ d \} - \{ A \}, \) \( \hbar Q_{\mu \nu} = \langle \hat{\Phi}^{[1]}_{\mu}, \hat{\Phi}^{[1]}_{\nu} \rangle_1, \) \( \hbar R_{\mu \nu} = \langle \hat{\Phi}^{[1]}_{\mu}, \hat{\Pi}^{[1]}_{\nu} \rangle_1, \) and \( \hbar P_{\mu \nu} = \langle \hat{\Pi}^{[1]}_{\mu}, \hat{\Pi}^{[1]}_{\nu} \rangle_1, \) where

\[
\langle \hat{\Theta}^{[1]}_{\mu}, \hat{\Theta}^{[1]}_{\nu} \rangle_1 = \langle \hat{\Theta}^{[1]}_{\mu}, \hat{\Theta}^{[1]}_{\nu} \rangle_0 + J^{[1,0]}_A (\hat{\Theta}^{[1]}_{\mu}, \hat{\Theta}^{[1]}_{\nu})
\]

(16)
with \( \Theta = \Phi \) or \( \Pi, \) \( O_{\mu \nu}^{[n]} = O_{\mu \nu}(t_1 - t_2), \) and \( I^{[n]}_A \) and \( J^{[n]}_d \) being combinations of two-point correlators [23]. For the observer in the \( \eta-\xi \) frame, similar projection occurs but we assume the wave functional collapses on the \( \eta_1 \)-slice instead. Then, started at \( t_1 \) and \( \eta_1 \), both PMS evolve to \( t_2 = \pi = \eta_2, \) when \( t \) and \( \eta \)-slices overlap so two observers can make a comparison between these two quantum states.

In the conventional \((t, x)\) coordinates of Minkowski space, the two-point correlators at \( t_2 \) determining the wave functional can be expressed as combinations of the mode-functions evolving from \( t_1 \) to \( t_2 \), together with the initial data on the \( t_1 \)-slice in the form of the correlators of the field at spatial points on the slice, e.g.,

\[
\hbar \tilde{Q}_{xy}(t_2) = \langle \hat{Q}^{[2]}_x, \hat{Q}^{[2]}_y \rangle_2 = \text{Tr} \hat{Q}^{[2]}_x \hat{Q}^{[2]}_y \hat{\rho} \sim \int dx' dy' \phi^{[2]}_x(x', \eta', \rho) \phi^{[2]}_y(x', \eta', \rho) \langle \hat{\Phi}^{[1]}_x, \hat{\Phi}^{[1]}_y \rangle_1 + \cdots
\]

(17)
where \( x' \) and \( y' \) are points on the \( t_1 \)-slice. It turns out that the dependence on the data on the \( t_1 \)-slice in (17) is removable by expressing the \( \langle \rangle_1 \) correlators as (16) with (5) and (6) inserted, then using (8), (9) and similar identities for \( \pi^{[\nu]}_\mu \) and \( p^{[\nu]}_\mu \) to replace the \( \int dx' \) and \( \int dy' \) terms. The two-point correlators, and therefore the \( (K, \Delta) \) representation of the quantum state on the \( t_2 \)-slice, end up with functionals of \( F^{[2] }_A, F^{[2] }_A, F^{[2] }_A, F^{[1] }_A, F^{[1] }_A, F^{[2] }_A \) with \( F = \phi, f, \pi, p, \) and the initial data in the two-point correlators evaluated at \( t_0 \).

The quantum state at \( t_2 \) in the \( \eta-\xi \) frame has exactly the same functional form, except that the mode functions here are those in the \( \eta-\xi \) frame. Now \( F^{[2] }_A, F^{[2] }_A, \) and \( F^{[2] }_A \) are HO in
the point-like detector, $F_x^{[21]}$ and $F_x^{[20]}$ are retarded fields sourced from the detector, while $F_x^{[20]}$ are superpositions of vacuum fluctuations and retarded fields. All of them are explicitly covariant and independent of the data on the $t_1$-slice outside the detector. Thus the quantum states collapsed in two different frames are identical up to a coordinate transformation when compared on the same time-slice at $t_2 = \eta_2 = \pi$.

The result in the case with two successive measurements on a single detector at $t = t_1$ and $t_2$, $t_1 < t_2 < t_3 = \pi$ is similar. The two-point correlators at $t_3$ can be written in functionals of the covariant functions and the initial data at $t_0$. They are independent of the data on $t_1$, $\eta_1$, $t_2$, or $\eta_2$-slice outside the detector, so the quantum state at $t_3$ is still independent of the time-slices on which the wave functional collapsed.

What happens if two measurement events are done separately on two detectors? Consider the case with two spatially separated detectors, $Q_A$ is put at $x = -\pi$ and $Q_B$ at $x = 0$, so both are at rest in the $t$- and $\eta$-$\xi$ frames. Suppose a local measurement is done on $Q_A$ at $t_1$ and $\eta_1(t_1, -\pi) = t_1 + A \sin t_1$, $0 < A < 1$, another local measurement on $Q_B$ at $t_2 > t_1$ but $\eta_2(t_2, 0) = t_2 - A \sin t_2 < \eta_1$: the two measurement events are spacelike separated so that the time order of these two events can be altered in coordinate transformation. In this case, the two-point functions at $t_3 = \eta_3 = \pi$ can still be written in functionals independent of the data on the $t_1$, $\eta_1$, $t_2$, or $\eta_2$-slice outside the detectors, e.g. in the $t$-$x$ frame,

\[
\langle \hat{\Phi}_x, \hat{\Phi}_y \rangle_3 = \frac{\hbar}{2 g_B^2} \phi_x^{[32]} \phi_y^{[32]} \frac{f_B^{[32]} f_B^{[32]}}{f_B^{[32]}} + \frac{\hbar}{2 g_A^2} \phi_x^{[31]} \phi_y^{[31]} \frac{f_A^{[31]} f_A^{[31]}}{f_A^{[31]}} + \frac{\hbar}{2 g_A^2} f_A^{[31]} f_A^{[31]} + \frac{1}{J_B^{[1,0]}} T_x^{[1,0]}(T_y^{[0,0]} + T_y^{[1,1]} J_B^{[2,1]}) (18)
\]

where

\[
T_x^{[0]} = \phi_x^{[0]}(\phi_x^{[30]} - \phi_x^{[31]} \phi_A^{[10]} - f_A^{[31]} \phi_A^{[10]} - f_B^{[32]} f_B^{[32]} - f_A^{[31]} f_A^{[31]} - f_B^{[32]} f_B^{[32]} - f_B^{[32]} f_B^{[32]}).
\]

By expressing all correlators $\langle \rangle_1$ in $J_B^{[2,1]}$ and $J_B^{[1,1]}$ in terms of $\langle \rangle_0$ by (16), both $J_B^{[2,1]}$ and $J_B^{[1,1]}$ will become functionals of $T_x^{[0]}$, too. So the final expression of the wave functional at $t_3$ depends only on the covariant mode functions in (18) and (19) and vacuum fluctuations from the $t_0$-slice. Further, using computer algebra it is straightforward to verify that when the retarded mutual influences $F_B^{[21]}$ vanish, $I_A^{[1,2]}(T_x^{[2]} + T_x^{[2]})/J_A^{[1,2]} + I_B^{[2,0]}(T_x^{[0]} + T_x^{[0]})/J_B^{[2,0]}$ obtained in the $\eta$-$\xi$ frame has the same functional form as $I_A^{[1,0]}(T_x^{[0]} + T_x^{[0]})/J_A^{[1,0]} + I_B^{[2,1]}(T_x^{[1]} + T_x^{[1]})/J_B^{[2,1]}$ in the $t$-$x$ frame. Thus the time order of the spacelike separated measurement events does not matter.

If the two measurement events are timelike separated, then different observers in different frames will recognize the same time order of the two events. In this case the retarded mutual influences will come into play so the resulted PMS will be different from those in the reversed time order, while the consistency of quantum states at the final time-slice is obvious after knowing the results in the previous cases.

6. Reduced state of a detector with its entangled partner being measured

Let us consider a similar setup as the previous case but now only a local measurement is performed on the detector $A$ at some $t_1 \in (0, \pi)$. Then at $t_1$ or $\eta_1$ the quantum state collapses to $\rho_A \otimes \rho_A$ on $t$- or $\eta$-slice, depending on the observer’s frame. The reduced PMS of the detector $B$ still has the same form as (12) with $A$ there replaced by $B$.

Before the detector $B$ enters the future lightcone of the measurement event on $A$, namely, when $t = t_M < t_1 + \pi$ in $t$-$x$
frame, the two-point correlators of the detector $B$ is either in the original, uncollapsed form, e.g. $\langle \hat{Q}_B^2(t_M) \rangle_0$, or in the collapsed form evolved from the PMS with the correlators evaluated at $t_1$ or $\eta_1$,

$$
\langle \hat{Q}_B^2(t_M) \rangle = \left[ \sum_d \left( \phi^{[M1]}_B \hat{Q}_d^{[1]} f^{[M1]}_B + f^{[M1]}_B \hat{P}_d^{[1]} \right) + \int dx \left( \phi^{x[M1]}_B \phi^{[1]}_x + f^{x[M1]}_B \pi^{[1]}_x \right) \right]^2_1 \equiv \frac{\hbar^2}{2 g_A} \left( \phi^{A[M1]}_B \right)^2 + \frac{\hbar^2}{2 g_A} \left( f^{A[M1]}_B \right)^2 + \langle \left( \hat{\Upsilon}_B^{[M0]} \right)^2 \rangle_0 + \frac{1}{f^{[1,0]}_A} \left( \hat{T}_B^{[M0]}, \hat{\Upsilon}_B^{[M0]} \right),
$$

(20)

depending on the observer’s frame. Here we have used the Huygens’ principles (8) and (9), and defined

$$
\hat{T}_B^{[M0]} \equiv \phi^{[0]}_A \left[ \phi^{[10]}_B - \phi^{A[M1]}_B \phi^{[10]}_A - f^{A[M1]}_B \pi^{[10]}_A \right] + \hat{\Pi}_A^{[0]} \left[ f^{[M0]}_B - \phi^{A[M1]}_B f^{[10]}_A + f^{A[M1]}_B \phi^{[10]}_A \right] .
$$

(21)

Note that before the detector $B$ enters the lightcone, $\phi^{A[M1]}_B = f^{A[M1]}_B = 0$, such that $\hat{\Upsilon}_B^{[M0]} = \phi^{[0]}_B$; $\hat{T}_B^{[M0]} = \hat{P}_B^{[M0]}$; and $\hat{T}_B^{[M0]} \equiv Q_B^{[M0]}$.

No matter in which frame the system is observed, the correlators in the reduced state of the detector $B$ must have become the collapsed ones like (20) at the moment the detector $B$ is entering the future lightcone of the measurement event on the detector $A$, namely, when $t_M = t_1 + \pi$ in the $t$-$x$ frame. After that moment the retarded mutual influences will reach $B$ so those $\hat{P}_B^{A[M1]}$ become nonzero and get involved in the correlators of $B$, though some information of measurement has entered the correlators of $B$ via $I_A$ and $J_A$ much earlier.

7. Summary and Application

In summary, we found that the Gaussian states of the detector-field system in different frames if started with the same initial state defined on the same fiducial time-slice will evolve to the same state on the same final time-slice (up to a spatial coordinate transformation). Neither the choice of coordinate between the initial and final time-slices nor the presence of local projective measurements during the evolution will violate this consistency. We believe such a consistency could be more generic in relativistic quantum field theory. Furthermore, the reduced states of the detectors are parametrized by the proper times of the detectors and independent of the time-slicing outside the detectors. If one detector of an entangled pair is measured, then the reduced state of the other detector will have different histories for different observers before the moment it enters the future lightcone of the measurement event. After that moment the reduced state of the other detector becomes identical.

An application of the above analysis is quantum teleportation in a relativistic system. Following the setup in Ref. [27] by Shiozawa for quantum teleportation, suppose Alice is at rest in Minkowski space and Rob is uniformly accelerated, each holds one Unruh-DeWitt (UD) detector of an entangled pair ($A$ and $B$), and Alice holds the third UD detector ($C$) whose quantum state is unknown. Suppose Alice is teleporting the quantum state of $C$ to Rob. At the moment $t_1$ before Alice goes beyond the event horizon of Rob (Figure 3 (left)) Alice performs a joint measurement on $A$ and $C$. Then the wave functional of the combined system (including the quantum fields that these UD detectors are coupled with) will collapse at that moment either on the Minkowski time-slice (gray horizontal line in Figure 3 (left)), or on the quasi-Rindler time-slice (the gray long-dashed line in the same plot: it almost overlaps the Rindler time-slice in the R-wedge but the part in the L-wedge has been bent to the region with positive $t$ to make the whole time-slice located after the initial time-slice), or whatever time-slice may be passing through the same measurement event in some observer’s frame. All the PMS of the combined
Figure 3. Setup for quantum teleportation from Alice (dotted dark line) to Rob (dashed dark curve). The straight line $t = x$ is the event horizon of Rob, and the shaded region represents the retarded field from the detector $A$ after the joint measurement on $A$ and $C$ by Alice, or the future lightcone of the joint measurement event. The joint measurement is done before the moment that Alice goes beyond the event horizon of Rob (left), or after that moment (right).

system in different frames will evolve to the same state when compared on the same time-slice after the measurement, according to the analysis in Section 5.

The reduced state of the detector $B$ consists of the two-point correlators of $Q_B$ and $P_B$. It has a sudden change from the uncollapsed to the collapsed one at $\tau_B = \tau_c$ in Figure 3 (left), as observed in the conventional Minkowski frame, or at $\tau'_c$, as observed in the quasi-Rindler frame. Such a sudden change occurs at different spacetime points for different observers. In other words, when observed at some moment $\tau_B = \tau_M$ before Rob enters the future lightcone of the joint measurement event, those correlators of $B$ may either in the uncollapsed form, or in a collapsed form as shown in (20), depending on the observer. In the latter case, the correlators of $B$ consist of the mode functions $F_{B}^{[M0]}$, $F_{B}^{[M0]}$, and $F_{B}^{[M0]}$ ($F = \phi, f, \pi, p$), the information about the outcome of the joint measurement on $A$ (in $I_A$ and $J_A$), as well as the two-point correlators evaluated on the initial time-slice at $t_0$ only. Either collapsed or uncollapsed all the correlators of $B$ are independent of the data on the time-slice that the wave functional collapsed on except those right at the position of the detector $A$.

If Rob never performs any further measurement on $B$ before entering the future lightcone of the joint measurement event done by Alice, certainly he will have no idea that $B$ is in the uncollapsed or collapsed state. But at the moment Rob is entering the future lightcone of the joint measurement event, all these reduced state of $B$ must have become identical, namely, the collapsed one, with the same combination of the mode functions. So quantum teleportation after Rob receives the classical information sent by Alice will give a definite result consistent in all frames.

Suppose Rob performs a measurement on $B$ before entering the future lightcone of the joint measurement event to see which state $B$ is in. In some observer’s frames $B$’s reduced state at the moment of Rob’s measurement is in the collapsed state, then the outcome will have some dependence of the outcome of $A$ since $A$ and $B$ were entangled initially. In other observer’s frames, $B$ is still in the uncollapsed state right before the measurement, then this measurement will result a wave functional collapse of the combined system, later in these frames the outcome
of Alice’s joint measurement will have the same correlation with the outcome of $B$. Both kinds of histories interpreted by different observers will be consistent \textit{a posteriori}, but both could not help Rob to conclude $B$ was in the uncollapsed or collapsed state right before the measurement. Rob cannot obtain the full knowledge about the quantum state of $B$ by only one single measurement. If Rob and Alice share an ensemble of many copies of the entangled pairs, in the reference frames that Rob performs the measurement first, using the outcomes Rob can recognize the reduced state of $B$ as uncollapsed by quantum state tomography. In the reference frames that Rob performs the measurement after Alice, the exact collapsed state of one copy of the entangled pair is different from another after the measurements by Alice and not predictable either by Alice or by Rob, while the distribution of the collapsed state is determined by the uncollapsed state of $A$ and $B$ right before Alice’s measurement. So the reduced state of detector $B$ recognized by Rob from his measurement on this ensemble of the collapsed states will have no difference from the uncollapsed one. Rob still cannot tell whether the quantum state of each copy of $B$ before his measurement is collapsed or not.

If the joint measurement on $A$ and $C$ is performed after Alice went beyond the event horizon of Rob (Figure 3 (right)), the mutual influences $F_{BA}$ will never reach Rob, though it appears that some information of measurement could enter the collapsed reduced state of $B$ via $I_A$ and $J_A$ in (20). Similar to the previous case with Rob still outside the future lightcone of the joint measurement, the functional form of the correlators in the reduced state of $B$ will be suddenly changed at the moment of the projective measurement, and different observers will recognize different spacetime points where this change occurs. But again Rob will never know whether the state of $B$ is in the uncollapsed state or collapsed. When $\tau_B$ goes to infinity or after Rob makes a measurement on $B$, these reduced states in different frames will become identical.

Above discussion indicates that in ultraweak coupling limit with mutual influences neglected, the approximated reduced state of $B$ after Rob enters the future lightcone of the measurement event will be almost independent of the time-slice on which the wave functional collapsed. Indeed, even the wave functional is collapsed by hand on the future lightcone, numerical calculation shows that the approximated reduced state of $B$ after Rob enters the lightcone is almost identical to the one collapsed on the $t$-slice, within the error of the ultraweak coupling approximation. This will largely simplify the numerical work for quantum teleportation between Alice and Rob.

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