Size of the emission source and collectivity in ultra-relativistic p-Pb collisions

Piotr Bożek\textsuperscript{a,b}, Wojciech Broniowski\textsuperscript{c,b}

\textsuperscript{a}AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, al. Mickiewicza 30, 30-059 Krakow, Poland
\textsuperscript{b}The H. Niewodniczański Institute of Nuclear Physics PAN, 31-342 Kraków, Poland
\textsuperscript{c}Institute of Physics, Jan Kochanowski University, 25-406 Kielce, Poland

Abstract

The interferometric radii in the system formed in ultra-relativistic proton-lead collisions are investigated in a framework based on event-by-event 3+1 dimensional viscous hydrodynamics. We argue that the most central p-Pb collisions undergoing collective expansion behave similarly to the peripheral nucleus-nucleus collisions. The interferometric observables can serve as signatures of the formation of an extended fireball.

Keywords: relativistic proton-nucleus collisions, relativistic hydrodynamics, interferometry, collective flow, LHC

The collective nature of the evolution of ultra-relativistic nucleus-nucleus (A-A) collisions from Relativistic Heavy-Ion Collider (RHIC) to the Large Hadron Collider (LHC) energies has been well evidenced, in particular with such phenomena as the harmonic flow or the transverse-momentum dependence of the interferometric radii. The description of the intermediate evolution stage with relativistic viscous hydrodynamics yields a successful quantitative prediction at the level of, say, 15% for the most relevant observables and for a wide range of centralities \( c \), including rather peripheral collisions up to \( c \sim 70\% \) \cite{1–7}. The application of this collective approach to the proton-nucleus, not to mention the proton-proton (p-p) collisions, is more questionable \cite{8} but very intriguing \cite{9}, as some features typical for collective phenomena have been observed in the highest-multiplicity proton-lead (p-Pb) and p-p collisions as well.

The p-Pb collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV have been investigated at the LHC \cite{13,23}, with the original motivation to study the initial state effects \cite{24} and, in particular, the saturation in QCD. Important observables in that respect are the two-particle correlations in relative pseudorapidity and azimuthal angle. The long-range correlations in rapidity are formed in the very early stage of the collision and present a dedicated probe of the initial state, on the other hand, the azimuthal correlations can be significantly modified by the final state rescattering. When a fireball of strongly interacting matter is formed, azimuthal correlations due to collective flow appear in the interaction region \cite{25,26}. For instance, in Ref. \cite{27} we have argued that the appearance of the same-side ridge structure in the correlation data in the p-Pb collisions measured at the LHC is semi-quantitatively described with event-by-event hydrodynamics, which provides a strong case for the interpretation based on collective harmonic flow. An alternative, appealing explanation of the appearance of the same-side ridge in p-Pb interactions is based on the color-glass condensate approach for the initial state \cite{28,29}. Therefore, it is important to look for independent estimates of the role of final state interactions in the dynamics.

In this Letter we argue that the behavior of the pionic interferometric radii in the most central p-Pb collisions should be used as a fingerprint of collectivity: if the experimental results follow the pattern of A-A collisions, then collective behavior is present. This concerns both the values of the radii as well as their dependence on the transverse momentum of the pair, governed by the flow \cite{30}. We evaluate the pionic interferometric radii for the most central p-Pb system with relativistic hydrodynamics and predict that in this treatment the system should closely imitate the peripheral nucleus-nucleus collision. If this is confirmed experimentally, it should serve as another strong case for the presence of collectivity in the most central p-Pb collisions. At the same time, the low-multiplicity p-Pb collisions are in our view not expected to display the above advocated behavior and should follow the p-p pattern, hinting different dynamics. Typically, the size and lifetime of the collective source formed in central p-Pb collisions is 3–4 fm \cite{31}.

The initial geometry of a small-source system formed in p-Pb collisions is dominated by fluctuations, therefore the costly machinery of event-by-event viscous hydrodynamics must necessarily be applied to properly describe the azimuthally asymmetric components of the collective flow \cite{32}. As the result, the hydrodynamic expansion in most central p-Pb collisions generates a sizable elliptic and triangular flow \cite{31}, while the two-dimensional correlation functions in relative pseudorapidity and azimuth are in

\textit{Email addresses: Piotr.Bozek@ifj.edu.pl (Piotr Bożek), Wojciech.Broniowski@ifj.edu.pl (Wojciech Broniowski)}
Figure 1: Pion interferometry radii $R_{\text{side}}$ (a), $R_{\text{out}}$ (b), and $R_{\text{long}}$ (c) for average pair momentum $k_T = 400$ MeV for different collisions systems and energies, plotted as functions of the charged particle multiplicity. The compilation of the experimental data is taken from [10, 11]. The data for the RHIC energies are from the STAR Collaboration [12, 13] and for the LHC energies from the ALICE Collaboration [14, 15]. The lines are to guide the eye. Various hydrodynamic calculations come from [16–18]. The results of this work for the p-Pb system are indicated with filled stars (standard source) and encircled stars (compact source).
semi-quantitative agreement with experimental observations [21–23], following the same mechanism as in A-A collisions [33, 34].

Interferometric correlations explored in this work are a snapshot of the emission points at the final stage [33, 36]. Fitting the Bertsch-Pratt [37, 38] formula to the same-sign two-pion correlation functions yields estimates for the femtoscopy size of the emission source. Quite remarkably, the systematics of the interferometry radii in nuclear collisions at different energies shows an approximate extended scaling with the multiplicity of the system [11, 13],

$$R \propto \left( \frac{dN}{d\eta} \right)^{1/3},$$

(1)
as shown in Fig. 1. We note that for the A-A collisions the hydrodynamic results shown in Fig. 1 are consistent with the data, properly reproducing the scaling [11] found experimentally. This stems from a generic relation between the interferometry radii and the size of fireball in A-A collisions and is found in many hydrodynamic calculations [39–41]. Admittedly, more precise probes, sensitive to the details of the collective flow profile, such as the ratio $R_{\text{side}}/R_{\text{out}}$, make the exact agreement more difficult to achieve in hydrodynamics, however, with a proper choice of the initial condition and equation of state [39–41] it can also be accomplished. The cascade model approaches also work properly for the A-A system [43, 44].

On the other hand, the interferometry radii measured in p-p collisions show a similar scaling trend but with a very different slope than the A-A case (cf. Fig. 1). This indicates that the mechanism responsible for the formation of the interferometric correlations may be distinct in elementary [45] and nuclear collisions. We note that the hydrodynamic modeling of the p-p system assuming collective expansion and rescattering in the final state is not always compatible with the data [16, 46–49].

We should admit that the analysis meets some difficulties: the extracted interferometric radii in the small p-p system depend strongly on the form of the fitted correlation function and the background subtractions, they require preservation of the conservation laws, moreover, depend on the resonance contributions or the effects of the uncertainty principle [14, 46, 47, 50]. Another important ingredient in small systems that may influence the interferometry radii and their momentum dependence is the unknown pre-thermal flow [41, 47]. All in all, the hydrodynamic predictions for the p-p pionic interferometric radii can overshoot the data by more than 50%, showing that the hydrodynamic description does not describe the p-p data in a uniform way.

The situation is hopefully different for the p-Pb system, where observed two-particle correlations [21–23] suggest the possible existence of collective flow. If this picture is true, it means that the system formed in p-Pb collisions is sufficiently large and long-lived to accommodate a hydrodynamic expansion stage. In our model, the initial condition is generated with GLISSANDO [51]. The parameters of the calculations are similar as in [31], except that they are adjusted for the collisions energy of $\sqrt{s_{\text{NN}}} = 5.02$ TeV. Thus we take the NN inelastic cross section $\sigma_{\text{NN}} = 67.7$ mb, moreover, we use a realistic (Gaussian) wounding profile [52] for the NN collisions. When the individual NN collision occurs, a source is produced, meaning deposition of energy and entropy in a location in the transverse plane and spatial rapidity. In the conventional wounded nucleon model it is assumed that the sources are located in the transverse plane in the centers of the participating nucleons. This amounts to rather large initial transverse sizes in the p-Pb system. Locating the source in the center-of-mass of the NN system [51] is also admissible, which leads to a more compact initial distribution. We use both variants, labeled standard and compact, which allows us to provide upper and lower limits for the size of the initial source and thus estimate the model uncertainty. The average rms radii for the two sets of initial conditions are 1.5 and 0.9 fm, respectively. The probabilistic nature of the Glauber model leads to initial source distributions which fluctuate event-by-event, referred to as the geometric fluctuations. No other possible sources of fluctuations in the initial phase, such as fluctuations of the color fields at smaller scales, are incorporated.

In our simulations we use the event-by-event 3 + 1-dimensional viscous hydrodynamic model as described in

---

1 We thank Larry McLerran and Adam Badak for this suggestion.
Figure 3: The model predictions for the central sections, in direction $q_{\text{side}}$, $q_{\text{out}}$ and $q_{\text{long}}$ (panels a), b) and c) respectively, of the interferometric correlation functions for the pion pairs (points) and the corresponding three-dimensional Gaussian fits (lines) for the most central ($c = 0 - 3.4\%$) p-Pb collisions.

detail in [27, 31]. The statistical emission and resonance decays at freeze-out are carried out with THERMINATOR2 [53]. The centrality 0-3.4% for the highest-multiplicity p-Pb collisions is defined in a simplified way by the condition on the number of participant nucleons, $N_{\text{part}} \geq 18$. We generate 450 distinct hydrodynamic evolutions of different initial conditions, both for the standard and compact source scenario. For each freeze-out hypersurface at $T_f = 150$ MeV we generate 1000 THERMINATOR events to increase the statistics. The expansion of the compact source is faster than for the standard source. The stronger flow in the compact source case causes an increase of the average transverse momentum by 20%, but the size of the fireball at freeze-out is similar for the two considered scenarios.

The femtoscopic correlation functions for the pion pair are obtained with the two-particle method described in detail in Ref. [54] and implemented in THERMINATOR2 [53]. Technically, the correlation functions involve pairs from the same hydro events in the numerator, and mixed pairs from different hydro events in the denominator of the correlation function

$$C(q,k) = \frac{\int d^4x_1 d^4x_2 S(x_1,p_1)S(x_2,p_2)|\Psi(k^*,r^*)|^2}{\int d^4x_1 S(x_1,p_1) \int d^4x_2 S(x_2,p_2)},$$

where $q = p_1 - p_2$ is the relative momentum of the pions, $k = (p_1 + p_2)/2$ is the average pair momentum, and the asterisk indicates the variables boosted to the pair rest frame [54]. For all the THERMINATOR events generated from the same hydrodynamic event the emission source $S(x,p)$ is the same, hence pairs from such 1000 events can be combined in the model calculation to improve statistics in the numerator. Still, by combining pairs from different freeze-out hypersurfaces, or using the event-by-event averaged initial conditions, we find that the effects of the event-by-event fluctuations of the emission source are small [17]. The interferometric radii are obtained by fitting the Gaussian shape to the correlation functions. The Coulomb effects, expected to be very small in the p-Pb system, are not taken into account in the pair wave function $\Psi$, and consistently, no Coulomb corrections are used in the fitting procedure.

The result displayed in Fig. 1 shows that our hydro predictions for the most central p-Pb system fall close to the A-A line, thus displaying the collective behavior. The distinction from the p-p trend is clearly seen, in particular for the standard-source case for $R_{\text{out}}$ and $R_{\text{long}}$, where the difference is about a factor of 2. The compact source leads to somewhat smaller interferometric radii, in particular for $R_{\text{out}}$ which is reduced by 25%. The other radii are very little affected by the initial source size.

In Fig. 2 we give the dependence of the pionic femtoscopic radii on the transverse momentum of the pair, $k_T$. Again, we show the case of the most central p-Pb collisions, as these are most likely to display collectivity. We note the expected fall-off of the radii with $k_T$ caused by the collective flow. For the compact initial distribution the
values of the radii are above 1.8 fm for $R_{\text{side}}$ and above 2 fm for $R_{\text{long}}$ and $R_{\text{root}}$. For the compact case the $R_{\text{out}}$ radius is visibly reduced, while $R_{\text{out}}$ and $R_{\text{long}}$ are only slightly smaller. Observing experimentally such large sizes (compared to the proton radius) in the most central p-Pb would demonstrate the formation of a fireball.

In femtoscopic studies the shape of the correlation function and its departure from the Gaussian form is frequently studied [44], as it carries relevant information on the dynamics of the system. We provide this information for the standard source in Fig. 3 where we plot the central sections of the correlation function. The departure from the three-dimensional Gaussian fit is clearly visible. In particular, at low momenta $q_0$ the correlation function is much sharper, rising well above the fitted Gaussian profile. Nevertheless, following the experimental works reported in Fig. 1 we use the three-dimensional Gaussian profile in extracting the interferometry radii.

To conclude, we state again the importance of the experimental femtoscopic measurements for the p-A system, which will help to determine the nature of its dynamics. The proximity to the A-A scaling line of Fig. 2 will place the system in the collective evolution mode, on the other hand, if it turns out to be close to the p-p line, elementary dynamics will be vivid. Our simple hydrodynamic calculation for the most central p-Pb system gives radii consistent with the A-A scaling. We should note, however, that in a more realistic treatment we expect some deviations due to remnants from the elementary p-p collisions, as modeled for instance in the core-corona picture. We also note that if a large size fireball is found, it could be used in quenching models to be compared with the $R_{AA}$ data.

Supported in part by Polish Ministry of Science and Higher Education, grant N N202 263438, by National Science Centre, grant DEC-2011/01/D/ST2/00772, and by PL-Grid Infrastructure.

References

[1] C. Shen, S. A. Bass, T. Hirano, P. Huovinen, Z. Qiu, et al., J.Phys. G38 (2011) 124045.
[2] D. A. Teaney, in: R. Hwa, X. N. Wang (Eds.), Quark Gluon Plasma 4, World Scientific, Singapore, 2009, p. 207.
[3] B. Schenke, S. Jeon, C. Gale, J.Phys.G G38 (2011) 124169.
[4] M. Luzum, J.Phys. G38 (2011) 124026.
[5] J.-Y. Ollitrault, F. G. Gardim, arXiv: 1210.8345 [nucl-th], (2012).
[6] U. A. Wiedemann, U. W. Heinz, Phys. Rept. 319 (1999) 145.
[7] Z.-W. Lin, C. Ko, S. Pal, Phys.Rev.Lett. 89 (2002) 152301.
[8] Q. Li, G. Graf, M. Bleicher, Phys.Rev. C85 (2012) 034908.
[9] T. Csorgo, W. Kittel, W. Metzger, T. Novak, Phys.Lett. B663 (2008) 214.
[10] K. Werner, I. Karpenko, T. Pierog, M. Bleicher, K. Mikhailov, Phys.Rev. C83 (2011) 044915.
[11] A. Kisiel, Phys.Rev. C84 (2011) 044913.
[12] Q. Li, G. Graef, M. Bleicher, arXiv: 1209.0042 [hep-ph], (2012).
[13] K. Werner, K. Mikhailov, I. Karpenko, T. Pierog, PoS WPCF2011 (2011) 019.
[14] S. Akkelin, Y. Sinyukov, Phys.Part.Nucl.Lett. 8 (2011) 959.
[15] W. Broniowski, M. Rybczynski, P. Bozek, Comput. Phys. Commun. 180 (2009) 69.
[16] M. Rybczynski, W. Broniowski, Phys. Rev. C84 (2011) 064913.
[17] M. Chojnacki, A. Kisiel, W. Florkowski, W. Broniowski, Comput. Phys. Commun. 183 (2012) 746.
[18] A. Kisiel, W. Florkowski, W. Broniowski, J. Pluta, Phys. Rev. C73 (2006) 064902.