Adaptive Finite-Time Fault-Tolerant Control for Half-Vehicle Active Suspension Systems with Output Constraints and Random Actuator Failures

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Problem of adaptive finite-time fault-tolerant control (FTC) and output constraints for a class of uncertain nonlinear half-vehicle active suspension systems (ASSs) are investigated in this work. Markovian variables are used to denote terms of different random actuators failures. In adaptive backstepping design procedure, barrier Lyapunov functions (BLFs) are adopted to constrain vertical motion and pitch motion to suppress the vibrations. Unknown functions and coefficients are approximated by the neural network (NN). Assisted by the stochastic practical finite-time theory and FTC theory, the proposed controller can ensure systems achieve stability in a finite time. Meanwhile, displacement and pitch angle in systems would not violate their maximum values, which imply both ride comfort and safety have been enhanced. In addition, all the signals in the closed-loop systems can be guaranteed to be semiglobal finite-time stable in probability (SGFSP). The simulation results illustrate the validity of the established scheme.

1. Introduction

With the development of modern industrial automation, vehicles play a more and more important role in people’s production and life. Suspension as an important part, the damping effect to a great extent determines the comfort and safety of the automobile. Compared to the traditional passive and semiactive suspension systems, active suspension systems (ASSs) can provide better dynamic adjustment damping, potential road handling capacity, extreme ride comfort, and suspension deflection [1–3]. The design of complex mechanical engineering of ASSs had become a hot issue in the past two decades.

ASSs are often simplified into the full-vehicle model, half-vehicle model, and quarter-vehicle model. The actuators are parallel inserted to the components that provide external forces to increase or dissipate the energy of the ASSs and manage the tradeoffs between conflicting performance indicators. Many remarkable results were reported in [4–8] with the vertical motion of the quarter ASSs as the research topic. However, pitch motion was ignored that also directly affected the ride comfort and safety. Research on half-vehicle ASSs mainly focused on discussions of pitch motion and vertical motion [9–17]. For ASSs, some inevitable uncertainties were in the design of controllers in [9–13], such as body mass, mass moment of inertia, and modeling uncertainties. But, these control methods [9–11] did not achieve good performance in estimating real values. In [12, 13], they proposed adaptive control schemes by adding new leakage items to the update rules. In [14], the damping coefficient and spring stiffness of the tires were considered in the suspension as random uncertain parameters. The uncertain actuator was discussed and eliminated the influence by continuous-time homogeneous [15]. However, many progresses have been made for uncertain nonlinear ASSs, and a few studies were on the constraint of half-vehicle models.
The actual mechanical systems need to keep the output or states within certain ranges; otherwise, the system performance would degrade. The properties of prescribed performance control [17] and barrier Lyapunov functions (BLFs) provide effective methods to deal with constraints, and a large number of results have been obtained for output constraints of various nonlinear systems and practical systems [18–26]. The asymmetric BLFs were coped with the position constraint problem of the marine vessel [24]. Due to the limitation of physical factors in suspension fields, it is worth noting that [25, 26] reported the output and time-varying output constraint of vertical motion for the quarter ASSs. However, for the half-vehicle ASSs, considering above output constraints had been carried out.

Theoretically, the aforementioned works can only guarantee the desired system performance as time approaches infinity. However, the actual mechanical control should achieve the expected transient system performance. The design of finite-time control for nonlinear systems has attracted considerable attention. The finite-time Lyapunov stability theorem was first proposed in [27]. Based on this theory, the continuous finite-time control for nonlinear systems was proposed in [28–34], robotic manipulators in [28], switched systems in [32–34], and Markovian jump systems in [35, 36]. The concept of semiglobal practical finite-time stability (SGPFS) was proposed in different forms in [31–36]. The adaptive fuzzy finite-time control scheme of general uncertain nonlinear systems is discussed in [36]. Furthermore, Cai and Xiang expanded SGPFS to nonstrict nonlinear systems in [37]. Sui et al. expanded SGPFS for nontriangular stochastic nonlinear systems in [38]. The finite-time results of nonlinear quarter ASSs had been made in [39, 40]. The finite-time results of nonlinear half ASSs had been made in [41], but there was no constraint study on the individual states. For nonlinear strict feedback systems, both output constraint and finite-time control design had been completed in [42, 43]. However, for uncertain nonlinear half ASSs, there are few results on how to implement finite-time control associated with output constraints.

On the other hand, actuator failures are inevitable due to the influence of external environment, mechanical system failures, operation errors, and human factors in practical systems. These faults can seriously have an impact on system stability, degradation, and even catastrophic risks. Most of the above studies assumed that all actuators or sensors were in normal operation. Fault-tolerant control (FTC) strategies can compensate the faults and maintain acceptable system performance. Many methods to deal with actuator failures, such as the pseudoinverse method was in [44], model prediction method was in [45], and sliding control was in [46], by applying the adaptive backstepping techniques for linear systems in [47] and nonlinear strict feedback systems in [48–50]. Failures should be random. The actuator of states can switch between various modes in a random way. Given enough historical data, the states of the actuator can be modeled as Markovian states in [51]. In the process, the failures of different actuators also meet the requirements of different Markov processes. Each actuator is independent and can fail at any sampling time. In [52], an adaptive fault compensation for a class of nonlinear uncertain systems with random actuator faults was studied and a random function to scale actuator faults by Markov correlation variables was proposed. In [53], random faults between different actuators in the half ASSs were considered for the first time. Motivated by the above observations and existing research results, this study proposes an adaptive NN finite-time FTC scheme for uncertain nonlinear half-vehicle ASSs with output constraints. The three main advantages of the proposed scheme can be listed as follows:

1. Compared with existing adaptive FTC studies, the problem for half ASSs subject to infinite stochastic actuator failures and the states of multiaxial actuators modeled by different Markovian processes has not received enough attention. Particularly, considering finite-time control, the additional correlation terms generated by the infinitesimal generator are handled by the stochastic finite-time control theorem.

2. In comparison with existing constraints, it is asymmetry, which can restrain different outputs of place and pitch angle more reasonable and reduce the vibration in uncertain nonlinear half ASSs. Moreover, the finite-time FTC control strategy can also enable the practical control systems to realize the transient system stability.

3. In comparison with existing adaptive finite-time control, it considers a class of uncertain nonlinear half ASSs subject to stochastic actuator failures. It should dispose random terms which makes the existing stability criteria in [53–55] are invalid. Concurrently, the asymmetric output constraints for different factors have been considered. The Lyapunov proves SGPFS.

This work is organized as follows. In Section 2, the half active suspension systems and control objectives are shown. Section 3 presents the design procedures of the adaptive finite-time fault-tolerant controller designed based on stochastic Lyapunov function and zero dynamic. In Section 4, an example to show that the constructed method is effective. In Section 5, it demonstrates a conclusion about the results of this work and future work.

2. System Description and Preliminaries

2.1. Nonlinear Half-Vehicle Suspension Systems. Figure 1 shows a half-vehicle suspension model. \( M \) represents the mass of the vehicle body. \( I \) is the mass moment of inertia. \( m_f \) and \( m_r \) are the defined masses of front and rear wheels, respectively. \( D_f \) stands for the vertical displacement of the vehicle body. \( \varphi \) represents the pitch angle. \( D_f \) and \( D_r \) stand for the displacements of the front and rear vehicle body, respectively. \( D_1 \) and \( D_2 \) are the displacements of the front and rear wheels, respectively. \( D_{01} \) and \( D_{02} \) represent the road inputs of corresponding wheels. \( F_{\text{uf}}, F_{\text{ur}}, F_{\text{df}}, \) and \( F_{\text{dr}} \) are defined as the forces produced by the related stiffness. \( F_{\text{bf}}, F_{\text{br}}, F_{\text{df}}, \) and \( F_{\text{dr}} \) are defined as the forces produced by the related dampers. \( u_f \) and \( u_r \) represent the control forces of the front and rear ASSs.
For half ASSs, the state space equations of vertical motion and pitch motion are shown in the following equation:

\[
\begin{aligned}
M\ddot{x}_z + F_{fd} + F_{df} + F_{fr} + F_{dr} - u_z &= 0, \\
I\ddot{\phi} + I_2(F_{fd} + F_{df}) - I_1(F_{fr} + F_{dr}) - u_\phi &= 0, \\
m_1\ddot{r}_1 - F_{df} - F_{ft} + F_{df} + u_f &= 0, \\
m_2\ddot{r}_2 - F_{dr} - F_{fr} + F_{br} + u_r &= 0,
\end{aligned}
\]  

(1)

where \( u_z = u_r + u_f \) and \( u_\phi = l_2u_f - l_1u_r \).

The stochastic actuator failures are considered and described as follows:

\[
\begin{aligned}
u_f(t) &= g_1(r_1(t))\Psi_f(t), \\
u_r(t) &= g_2(r_2(t))\Psi_r(t),
\end{aligned}
\]

(2)

where \( \Psi_f(t) \) and \( \Psi_r(t) \) are the inputs of the front actuator and rear actuator, respectively. \( r_1(t) \) and \( r_2(t) \) are the independent irreducible right continuous homogeneous Markovian processes on the probability space \((\Omega, E, P)\), taking values in a finite set \( S = \{1, 2, \ldots, N\} \) with generator matrix \( \Pi = (Y_{pq})_{N \times N} \), where \( Y_{pq} > 0 \) is the transition rate from mode \( p \) to mode \( q \) if \( p \neq q \), and \( Y_{pp} = \sum_{q \neq p} Y_{pq} \).

In addition, \( g_1(r_1(t)) \) and \( g_2(r_2(t)) \) are the stochastic functions which represent the failure scaling factors for two different actuators and take values on the interval \([g_{1\text{min}}, 1][g_{2\text{min}}, 1]\), and \( g_{1\text{min}} > 0, g_{2\text{min}} > 0 \).

**Remark 1.** The values \( g_1(r_1(t)) \) and \( g_2(r_2(t)) \) meet as follows,

1. When \( g_1(r_1(t)) = 1 \) or \( g_2(r_2(t)) = 1 \), the front actuator or the rear actuator is healthy
2. When \( g_1(r_1(t)) \in [g_{1\text{min}}, 1] \) or \( g_2(r_2(t)) \in [g_{2\text{min}}, 1] \), there is partial failure of the corresponding actuator

Each actuator will switch randomly between above two states. The problem of FTC considered in the systems was assumed that once the actuator failed, it would keep the fault state for the rest operation. However, the failure may be intermittent, and the actuator may repetitively fail with different failure modes. Meanwhile, the modes, times, and patterns of actuator failures are essentially random. Then, actuator failures in (2) are more complicated and practical.

In order to facilitate the design and analysis of the adaptive finite-time fault-tolerant control method, the state variables need to be defined as follows:

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8
\end{bmatrix} = \begin{bmatrix}
D_x \\
\dot{D}_x \\
\Phi \\
\Phi \\
D_r \\
\dot{D}_r \\
\Phi \\
\Phi
\end{bmatrix},
\]

(3)

Then, it obtains

\[
\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= F_1 + \rho_1(g_1(r_1(t))\Psi_f(t) + g_2(r_2(t))\Psi_r(t)), \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= F_2 + \rho_2(l_2g_1(r_1(t))\Psi_f(t) - l_1g_2(r_2(t))\Psi_r(t)),
\end{aligned}
\]

(4)

\[
\begin{aligned}
\dot{x}_5 &= x_6, \\
\dot{x}_6 &= \frac{1}{m_f}(F_{fd} + F_{df} - F_{ft} + F_{br} - g_1(r_1(t))\Psi_f(t)), \\
\dot{x}_7 &= x_8, \\
\dot{x}_8 &= \frac{1}{m_r}(F_{sr} + F_{sr} - F_{fr} + F_{br} - g_2(r_2(t))\Psi_r(t)),
\end{aligned}
\]

(5)

where \( \rho_1 = 1/M, \rho_2 = 1/I, F_1 = \rho_1(-F_{fd} - F_{df} - F_{fr} - F_{dr}), \) and \( F_2 = \rho_2(-l_2(F_{fd} + F_{df}) + l_1(F_{sr} + F_{dr})) \).

The mass \( M \) and the mass moment of inertia \( I \) are uncertain due to the uncertainties of passengers and the load quality of cargoes. Therefore, \( F_1 \) and \( F_2 \) are the uncertainty and unknown functions.

### 2.2 Control Objectives

For different types of vehicles, improving ride comfort and safety is one of the most important requirements. Due to the hardware limitations, the following three requirements must be considered in the design and control process of ASSs.

First, the controllers \( u_f \) and \( u_r \) are subjected to random Markovian jumping failures in (2). The vertical displacement \( D_x \) and pitch angle motion \( \Phi \) are considered and limited in safety ranges. \( D_x \) and \( \Phi \) should be smaller which can largely enhance ride comfort.
Second, considering the driving safety, the wheels should make uninterrupted contact with the ground. It means that the dynamic tire load must be less than the static load, i.e.,

\[ |F_{df} + F_{bg}| \leq F_f, \]

\[ |F_u + F_{bg}| \leq F_r, \]  

(6)

where \( F_f + F_r = (M + m_f + m_r)g \) and \( F_f(l_1 + l_2) = m_fg(l_1 + l_2) + M_l g \).

Last, suspension constraints must be guaranteed because of the confined mechanical space, that is, suspension deflections should not exceed their maximum values.

\[ |\Delta D_f| \leq \Delta D_f^*, \]  

(7)

\[ |\Delta D_r| \leq \Delta D_r^*, \]

where \( \Delta D_f = D_c + l_2 \sin \varphi - D_1 \), and \( \Delta D_r = D_c - l_1 \sin \varphi - D_2 \).

There are some assumptions, definitions, and lemmas presented in order to facilitate the design and analysis of adaptive control scheme.

**Assumption 1.** For given constants \( k_{c_1} > 0, k_{c_2} > 0 \), and \( k_{c_3} > 0 \), and \( \bar{k}_{c_1} > 0, \bar{k}_{c_2} > 0 \), and \( \bar{k}_{c_3} > 0 \), the following results on \( y_1, y_3, \dot{y}_1, \dot{y}_3 \) hold, i.e.,

\[ -\bar{k}_{c_1} \leq y_1, \]

\[ y_1 \leq \bar{k}_{c_2}, \]  

(8)

\[ -\bar{k}_{c_3} \leq y_3, \]

\[ y_3 \leq \bar{k}_{c_3}, \]

where \( \bar{k}_{c_1} < \bar{k}_{c_1}, \bar{k}_{c_2} < \bar{k}_{c_2}, \) and \( \bar{k}_{c_3} < \bar{k}_{c_3} \), for positive parameters \( \bar{b}_{c_1}, \bar{b}_{c_2}, \bar{b}_{c_3} \), and \( \bar{b}_{c_3} \) are selected and positive parameters.

**Assumption 2.** In general, the inputs of the road and their time derivative of \( D_{01} \) and \( D_{02} \) are limited. Therefore, existing positive parameters make sure the following inequality:

\[ |D_{01}(t)| \leq \bar{D}_{01}, \]

\[ |D_{02}(t)| \leq \bar{D}_{02}, \]

\[ |\dot{D}_{01}(t)| \leq \bar{D}_{01}^*, \]

\[ |\dot{D}_{02}(t)| \leq \bar{D}_{02}^*, \]  

(9)

where the parameters \( \bar{D}_{01}, \bar{D}_{02}, \bar{D}_{01}^*, \) and \( \bar{D}_{02}^* \) are positive.

**Definition 1** (see [35]). Consider the following nonlinear systems:

\[ \dot{\zeta} = g(\zeta, u), \]

\[ g(0, 0) = 0, \]

\[ \zeta, u \in \mathbb{R}^n, \]  

(10)

where \( \zeta \) and \( u \) represent the state and input vectors, respectively. If any initial condition \( \zeta(t_0) \) satisfies \( \zeta(t_0) = \zeta_0 \), for \( t \geq t_0 + T \), the system (10) is semiglobal practical finite-time stable (SGPFS). In addition, the state vector \( \zeta(t) \) satisfies \( \|\zeta(t)\| \leq \bar{T} \), for \( t \geq t_0 + T \), where \( \bar{T} \) is a positive parameter and a deposit time \( T(\zeta_0, \zeta) \) satisfies \( 0 < T(\zeta_0, \zeta) < \infty \).

**Lemma 1** (see [36]). For any \( y_i \in R, i = 1, 2, \ldots, n, 0 < a < 1, 0 < b < 2 \), the following inequality can be constructed, namely,

\[ \left( |y_1|^2 + |y_2|^2 + \cdots + |y_n|^2 \right)^b \leq \left( |y_1|^a + |y_2|^a + \cdots + |y_n|^a \right)^b, \]

(11)

\[ \left( |y_1|^2 + |y_2|^2 + \cdots + |y_n|^2 \right)^b \leq \left( |y_1|^a + |y_2|^a + \cdots + |y_n|^a \right)^b. \]

**Lemma 2** (see [37]). Consider the Lyapunov function \( V(\zeta) \) with stochastic terms, and the following inequality holds for a nonlinear system (10), namely,

\[ \hat{L}(\zeta) \leq -\hat{C}(\zeta) + h \]

(12)

where \( L \) is the differential operator, the constants \( \ell \) and \( h \) are positive, and \( 0 < a < 1 \). \( \zeta \in \mathbb{R}^n \) and \( t \geq t_0 \). Then, stochastic trajectory of (10) is SGPFS.

**Lemma 3** (see [40]). For any \( z \) and \( \alpha \) are real variables, the following inequality can be constructed, namely,

\[ |z|^\alpha |a|^\beta \leq \frac{a}{a + b} |z|^{a+b} + \frac{b}{a + b} e^{-\frac{a+b}{a}} |a|^{a+b}, \]

(13)

where the parameters \( a, b \), and \( \mu \) are positive.

**Lemma 4** (see [23]). For all \( |y_i| \leq k_{b_1} \), the following inequality can be constructed as

\[ \log \frac{1}{k_{b_1} - y_i} \leq 2 \frac{k_{b_1}^2}{y_i^2}. \]

(14)

3. Adaptive Finite-Time Fault-Tolerant Controller Design Based on Stochastic Lyapunov Function and Zero Dynamic

For a class of uncertain nonlinear half ASSs, the universal approximation property of NN solves the uncertainty, and then, the outputs of vertical motion and pitch angle motion are constrained by using asymmetrical log-BLFs. The adaptive backstepping technique is used to address the functions with random generality variables generated by the infinitesimal generator due to random actuator failures. The Lyapunov stability is proved by the SGPFS and zero dynamic theorem.

In order to facilitate the design of controllers, there are some given coordinate transformations, i.e.,

\[ y_1 = x_1 - y_1 d, \]

\[ y_2 = x_2 - \beta_1, \]

\[ y_3 = x_3 - y_3 d, \]

\[ y_4 = x_4 - \beta_3. \]  

(15)
where $\beta_1$ and $\beta_3$ are the virtual control signals, $y_i$ for $1 \leq i \leq 4$ is the error variable, $y_{1,d}$ and $y_{3,d}$ are the desired vertical displacement and pitch angle, respectively.

3.1. Finite-Time Constraint Control Scheme Design for Vertical Motion and Pitch Angle Motion. Now, for the vertical motion of active suspension systems, more details on finite-time approach will be given in the next section.

Step 1. Select $V = V(x(t), r_1(t), r_2(t))$ as a Lyapunov function candidate in the following form:

$$V = \sum_{i=1,3} \left[ \frac{1-q(y_i)}{2} \log \frac{k_i^2}{k_i^2 - y_i^2} + \frac{q(y_i)}{2} \log \frac{k_i^2}{k_i^2 - y_i^2} \right]$$

$$+ \sum_{i=1}^{2} \left( \frac{1}{2} p_i y_{2i}^2 + \frac{1}{2} \beta_i^2 + \frac{1}{2} \theta_i^2 (r_i(t)) \right)$$

(16)

where $x(t) = (x_1(t), x_2(t), x_3(t), x_4(t))$, $y_1 = (x_1, r_1(t), r_2(t))$, $y_2 = (x_1, x_2, r_1(t), r_2(t))$, $y_3 = (x_3, r_1(t), r_2(t))$, and $y_4 = (x_3, x_4, r_1(t), r_2(t))$. The parameters $\beta_i$ and $\chi_i$ are positive, and $\theta_i$ and $\tilde{\theta}_i$ represent the errors. $\tilde{\theta}_i$ is the estimator of $\theta_i$, $\tilde{\theta}_i(r_i(t))$ is the estimator of $g_i(r_i(t))$. Then, $\tilde{\theta}_i = \theta_i - \tilde{\theta}_i$, $\tilde{\theta}_i(r_i(t)) = g_i(r_i(t)) - \hat{g}_i(r_i(t))$, $i = 1, 2$. It would make an assumption: $\forall t > 0$, $r_1(t) = p_1$, $r_2(t) = p_2$, and $p_1, p_2 \in S$.

Remark 2. For quarter ASSs, the study in [30] only showed the vehicle body's displacement constraint method by the symmetric BLFs. In contrast, we further study the pitch angle constraint of nonlinear uncertain half ASSs by asymmetric BLFs in this study.

In addition, the virtual controller signal is designed as

$$\beta_i = \frac{c_i y_i}{2} \frac{1-q(y_i)}{k_i^2 - y_i^2} + \frac{q(y_i)}{k_i^2 - y_i^2} \dot{y}_i + \dot{y}_{i,d}$$

(17)

where $i = 1, 3$. The parameter $c_i$ is a design parameter, $c_i > 0$. $k_i$ and $k_i$ are the constraint bounded of corresponding variables, respectively. Moreover,

$$q(\cdot) = \begin{cases} 1, & \cdot \geq 0, \\ 0, & \cdot < 0. \end{cases}$$

(18)

Then, according to the infinitesimal generator of $V$, it gets

$$LV = \sum_{i=1,3} \left[ (1-q(y_i)) \frac{Y_i y_i}{k_i^2 - y_i^2} + q(y_i) \frac{Y_i y_i}{k_i^2 - y_i^2} \right]$$

$$+ \sum_{i=1}^{2} \left( \frac{1}{p_i} y_{2i} + \frac{1}{\beta_i} \tilde{\theta}_i + \frac{1}{\tilde{\theta}_i} (p_i) \tilde{\theta}_i (p_i) \right)$$

(19)

+ $\sum_{i=1}^{N} Y_{P_i} V_1 + \sum_{i=1}^{N} Y_{P_i} V_2$,

where $V_1 = (x(t), p_2, q_1)$, $V_2 = (x(t), p_1, q_2)$, and $Y_{P_i} > 0$ is the transition rate, $i = 1, 2$.

Remark 3. $\sum_{i=1}^{N} Y_{P_i} V_1$ and $\sum_{i=1}^{N} Y_{P_i} V_2$ are the additional terms due to the involvement of Markovian variables $r_1(t)$ and $r_2(t)$, which do not exist in the determined situation. The extratransition rate-related terms appear in the infinitesimal generator of Lyapunov candidate function. These additions need further processing. It is a challenge that cannot be ignored in stability analysis.

From the previous definition, $i = 1, 2$, there are

$$\dot{\theta}_i(r_i(t)) = -\tilde{\theta}_i(r_i(t)),$$

$$\tilde{\theta}_i = -\tilde{\theta}_i.$$  

(20)

Combining with (19) and (20) can obtain

$$LV = \sum_{i=1,3} \frac{Y_i}{k_i^2 - y_i^2} \left( 1-q(y_i) \right) \dot{y}_i + \sum_{i=1}^{2} \frac{1}{p_i} y_{2i}$$

$$- \sum_{i=1}^{2} \left( \frac{1}{\beta_i} \tilde{\theta}_i (p_i) \tilde{\theta}_i (p_i) + \frac{1}{\tilde{\theta}_i} \tilde{\theta}_i \right)$$

(21)

$$+ \sum_{i=1}^{N} Y_{P_i} V_1 + \sum_{i=1}^{N} Y_{P_i} V_2.$$  

Furthermore, taking the derivative of $y_i$, it yields

$$\dot{y}_i = y_{i+1} + \beta_i - y_{i,d}, \quad i = 1, 3.$$  

(22)

Substituting (17) and (22) into (21), it can attain

$$\dot{y}_i = \lambda_i y_i y_{i+1} - \frac{c_i y_i}{2} \left( 1-q(y_i) \frac{1}{k_i^2 - y_i^2} + \frac{q(y_i)}{k_i^2 - y_i^2} \right)$$

$$= \lambda_i y_i y_{i+1} - \frac{c_i y_i}{2} \frac{1-q(y_i)}{k_i^2 - y_i^2} + \frac{q(y_i)}{k_i^2 - y_i^2} \frac{y_i^2}{k_i^2 - y_i^2}$$

(23)

where $\lambda_i = ((1-q(y_i)) k_i^2 - y_i^2) + (q(y_i) / (k_i^2 - y_i^2))$, $c_i$ is a positive design parameter, and $i = 1, 3$.  

Based on Lemma 3, let $z_i = (y_i^2 / 2(k_i^2 - y_i^2))$, $\alpha = 1$, $a = ((2n-1)/2n + 1)$, $n \in \mathbb{N}$, $b = 1 - a$, and $\mu = a^{(a-1)}$ obtain
\[
\left(\frac{\gamma_i^2}{2(k_i^2 - \gamma_i^2)}\right)^a \leq (1 - a)\mu + \frac{\gamma_i^2}{2(k_i^2 - \gamma_i^2)}
\]  
(24)

Therefore, it gets

\[
\frac{\gamma_i^2}{2(k_i^2 - \gamma_i^2)} \leq (1 - a)\mu - \left(\frac{\gamma_i^2}{2(k_i^2 - \gamma_i^2)}\right)^a.
\]  
(25)

Constructed in the same way, it obtains

\[
\frac{(1 - q(y_i))\gamma_i^2}{2(k_i^2 - \gamma_i^2)} \leq (1 - a)\mu - (1 - q(y_i))\left(\frac{\gamma_i^2}{2(k_i^2 - \gamma_i^2)}\right)^a.
\]  
(26)

Similarly, it yields

\[
\frac{-\gamma_i^2}{2(k_i^2 - \gamma_i^2)} \leq (1 - a)\mu - \left(\frac{\gamma_i^2}{2(k_i^2 - \gamma_i^2)}\right)^a.
\]  
(27)

According to Lemma 4, for any constant \(a\), \(0 < a < 1\), the following inequality holds:

\[
\left(\log \frac{1}{k_i^2 - \gamma_i^2}\right)^a \leq \left(\frac{\gamma_i^2}{2(k_i^2 - \gamma_i^2)}\right)^a.
\]  
(28)

Next, substituting (26)-(28) into (23), \(LV\) is expressed as

\[
LV = \sum_{i=1}^{N_1} \left[\lambda_i y_i y_i^* - c_i \left(1 - q(y_i) \log \frac{1}{k_i^2 - \gamma_i^2}\right)^a \right.
\]
\[
- c_i \left(\frac{q(y_i) \log \frac{1}{k_i^2 - \gamma_i^2}}{2} + \sum_{i=1}^N Y_{p_i} V_1 + \sum_{q=1}^N Y_{q_i} V_2 + h_1 \right.
\]
\[
+ \sum_{i=1}^2 \left(\frac{1}{\rho_1} y_2 y_2^* - \frac{1}{\lambda_i} \tilde{\eta}(p_i) \tilde{\eta}(p_i) - \frac{1}{\eta_i} \tilde{\beta}\right).
\]
(29)

where \(h_1 = \sum_{i=1,2} c_i (1 - a)\mu\).

Taking the derivative of \(y_1\) and \(y_4\), they yield

\[
\dot{y}_2 = F_1 + \rho_1 u_2 - \dot{\beta}_1,
\]
\[
\dot{y}_4 = F_2 + \rho_2 u_4 - \dot{\beta}_3,
\]
(30)

where \(\dot{\beta}_1 = (\partial \beta_1/\partial x_1) \dot{x}_1 + (\partial \beta_1/\partial y_{1,d}) \dot{y}_{1,d} + (\partial \beta_1/\partial y_{2,d}) \dot{y}_{2,d}\), \(i = 1, 3\), \(\dot{\beta}_1\) has been designed in (17). Then, \(\sum_{i=1}^{2} (1/\rho_i) y_1 y_2\) gets

\[
\frac{1}{\rho_1} y_2 \dot{y}_2 = y_2 (F_1 + u_2),
\]
\[
\frac{1}{\rho_2} y_4 \dot{y}_4 = y_4 (F_2 + u_4),
\]
(31)

where the unknown functions \(F_1\) and \(F_2\) are denoted as

\[
F_1 = \rho_1^{-1} F_1 - \rho_1^{-1} \dot{\beta}_1,
\]
\[
F_2 = \rho_2^{-1} F_2 - \rho_2^{-1} \dot{\beta}_3.
\]
(32)

Using the powerful approximating ability of NNs in [42, 43] for uncertain nonlinear systems, \(F_1\) and \(F_2\) can employ

\[
F_1 = \Theta_1 \Phi (Z_1)^T + r_1,
\]
\[
F_2 = \Theta_2 \Phi (Z_2)^T + r_2,
\]
(33)

where \(Z_1 = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, \beta_1, y_{1,d}, y_{1,d}]^T\), \(Z_2 = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, \beta_3, y_{2,d}, y_{2,d}]^T\), and \(r_i\) is the approximation error and the designed positive constant \(\bar{r}_i\) that satisfies \(|r_i| \leq \bar{r}_i\). More remarkably, \(0 < \Theta_1 \Phi (Z_1) < 1\) and \(0 < \Theta_2 \Phi (Z_2) < 1\). Furthermore, we can draw a conclusion \(0 < \Phi_1^T (Z_1) \Phi_1 (Z_1) \leq N_1\) and \(0 < \Phi_2^T (Z_2) \Phi_2 (Z_2) \leq N_2\), where \(N_1\) and \(N_2\) are the corresponding numbers of NNs nodes.

Then, applying Young’s inequality, it can get the following:

\[
y_{21} \Theta_1 \Phi (Z_1)^T \leq \frac{y^2_{21} \theta_1}{2 \delta_1^2} + \frac{\delta_1^2}{2},
\]
\[
y_{22} \Theta_2 \Phi (Z_2)^T \leq \frac{y^2_{22} \theta_2}{2 \delta_2^2} + \frac{\delta_2^2}{2}.
\]
(34)

Substituting (33)–(35) into (31), it obtains

\[
\sum_{i=1}^{2} \frac{1}{\rho_i} y_2 y_{2i} \leq y_2 \left(\frac{y_2 \theta_1}{2 \delta_1^2} + \frac{y_2}{2} + u_2\right) + y_4 \left(\frac{y_4 \theta_2}{2 \delta_2^2} + \frac{y_4}{2} + u_4\right)
\]
\[
+ \sum_{i=1}^{2} \left(\frac{1}{\rho_i} \frac{\dot{\beta}_1}{\theta_1} \frac{\dot{\beta}_3}{\theta_2}\right).
\]
(36)

Substituting (36) into (29), it can obtain

\[
LV = \sum_{i=1}^{N_1} \left[\lambda_i y_i y_i^* - c_i \left(1 - q(y_i) \log \frac{1}{k_i^2 - \gamma_i^2}\right)^a \right.
\]
\[
- c_i \left(\frac{q(y_i) \log \frac{1}{k_i^2 - \gamma_i^2}}{2} + \sum_{i=1}^N Y_{p_i} V_1 + \sum_{q=1}^N Y_{q_i} V_2 + h_1 \right.
\]
\[
+ \sum_{i=1}^{2} \left(\frac{1}{\rho_i} y_2 y_2^* - \frac{1}{\lambda_i} \tilde{\eta}(p_i) \tilde{\eta}(p_i) - \frac{1}{\eta_i} \tilde{\beta}\right) - \frac{1}{\rho_i} y_4 y_4^* - \frac{1}{\eta_i} \tilde{\beta}_3\right)
\]
\[
\leq h_2 + \sum_{i=1}^{2} \left(\frac{1}{\rho_i} \frac{\dot{\beta}_1}{\theta_1} \frac{\dot{\beta}_3}{\theta_2}\right).
\]
(37)
Design the control input \( u_z \) and \( u_p \) that are subjects to random actuator faults as
\[
\begin{align*}
  u_z &= -\lambda_1 y_1 - \frac{y_2 \hat{\theta}_1}{2-\delta_1^2} - c_1 y_2^{2n-1}, \\
  u_p &= -\lambda_3 y_3 - \frac{y_4 \hat{\theta}_2}{2-\delta_2^2} - c_4 y_4^{2n-1},
\end{align*}
\]
(38)
where the parameters \( c_2 \) and \( c_4 \) are positive.

The adaptive law is established as
\[
\begin{align*}
  \dot{\hat{\theta}}_i &= \frac{\eta_i}{2\delta_i^2} y_{2i}^2 - \kappa_i \hat{\theta}_i, \quad i = 1, 2,
\end{align*}
\]
(39)
where the designed parameter \( \kappa_i \) is positive.

Based on (1), (2), and (39), \( u_f \) and \( u_r \) are shown as
\[
\begin{align*}
  u_f &= \frac{l_1 u_z + u_q}{l_1 + l_2}, \\
  u_r &= \frac{l_2 u_z - u_q}{l_1 + l_2}.
\end{align*}
\]

The updating laws of estimated parameters are established:
\[
\begin{align*}
  \dot{\hat{\theta}}_1 (p_1) &= \text{proj}_{\hat{\theta}_1 (p_1)} \left( y_2 i y_1 \chi_1 \psi_f (t) - 2c_2 \hat{\theta}_1 (p_1) \hat{\theta}_1 (p_1) \right), \\
  \dot{\hat{\theta}}_2 (p_2) &= \text{proj}_{\hat{\theta}_2 (p_2)} \left( y_4 i y_3 \chi_2 \psi_r (t) - 2c_4 \hat{\theta}_2 (p_2) \hat{\theta}_2 (p_2) \right),
\end{align*}
\]
(40)
where \( \text{proj} \) represents the projection operator in order to avoid singular values in the denominator.

Remark 4. The adaptive estimated parameters \( \dot{\hat{\theta}}_1 (p_1) \) and \( \dot{\hat{\theta}}_2 (p_2) \) appear in the denominators in (41), which may cause the controllers to fail. Construct the projection form to avoid this situation.

Substituting (39)–(41) into (37), \( \Lambda V \) can be expressed as
\[
\begin{align*}
  \Lambda V &\leq -\sum_{i=1,3} c_i \left[\left(1 - \frac{q(y_2)}{2} \log \frac{1}{k_{ii}^2 - \eta_i^2}\right)^a + \left(\frac{q(y_2)}{2} \log \frac{1}{k_{ii}^2 - \eta_i^2}\right)^a\right] \\
  &\quad - \sum_{i=1}^2 c_2 y_{2i}^2 + \sum_{i=1}^2 c_2 \hat{\theta}_1 (p_1) \hat{\theta}_1 (p_1) + \kappa_i \hat{\theta}_i \hat{\theta}_i \\
  &\quad + \sum_{\eta_i=1}^N Y_{\eta_i} V_1 + \sum_{\eta_i=1}^N Y_{\eta_i} V_2 + h_3.
\end{align*}
\]
(42)

By Young’s inequality \( (i = 1, 2) \), they can obtain
\[
\begin{align*}
  \Lambda V &\leq -\sum_{i=1,3} c_i \left[\left(1 - \frac{q(y_2)}{2} \log \frac{1}{k_{ii}^2 - \eta_i^2}\right)^a + \left(\frac{q(y_2)}{2} \log \frac{1}{k_{ii}^2 - \eta_i^2}\right)^a\right] \\
  &\quad - \sum_{i=1}^2 c_2 y_{2i}^2 + \sum_{i=1}^2 c_2 \hat{\theta}_1 (p_1) \hat{\theta}_1 (p_1) + \kappa_i \hat{\theta}_i \hat{\theta}_i \\
  &\quad + \sum_{\eta_i=1}^N Y_{\eta_i} V_1 + \sum_{\eta_i=1}^N Y_{\eta_i} V_2 + h_3.
\end{align*}
\]
(43)
which means the conditions of the vehicle are limited. Applying the inequality (44) and (46) can be expressed as

\[
\left(\frac{\theta_i^2}{2\eta_i}\right)^a \leq (1 - a)\mu + \left(\frac{\theta_i^2}{2\eta_i}\right)^a .
\]

Furthermore, we can obtain

\[
LV \leq \sum_{i=1,3} \alpha_c \left[ \left(1 - \frac{q_i}{2} \log \frac{1}{k_{ii} - \xi_i^2}\right)^a + \left(\frac{q_i}{2} \log \frac{1}{k_{ii} - \xi_i^2}\right)^a \right] - \sum_{i=1}^2 \left[ c_{2,1} y_{2i}^a + \frac{c_i y_{2i}^a}{2\chi_i} (p_i) + \kappa_i \left(\frac{\theta_i^2}{2\eta_i}\right)^a \right] + \sum_{q_i=1}^N Y_{p_i, q_i} V_1 + \sum_{q_i=1}^N Y_{p_i, q_i} V_2 + h ,
\]

where \( h = h_3 + (1 - a)\mu \).

In addition, the number of passengers and the load conditions of the vehicle are limited. The mass \( M \) is bounded which means \( \rho_i \) is bounded, namely,

\[
LV \leq \sum_{i=1}^3 c_i \left[ \left(1 - \frac{q_i}{2} \log \frac{1}{k_{ii} - \xi_i^2}\right)^a + \left(\frac{q_i}{2} \log \frac{1}{k_{ii} - \xi_i^2}\right)^a \right] - \sum_{i=1}^2 \left[ c_{2,1} y_{2i}^a + \frac{c_i y_{2i}^a}{2\chi_i} (p_i) + \kappa_i \left(\frac{\theta_i^2}{2\eta_i}\right)^a \right] + \sum_{q_i=1}^N Y_{p_i, q_i} V_1 + \sum_{q_i=1}^N Y_{p_i, q_i} V_2 + h .
\]

From (16), (49) can be rewritten as

\[
LV \leq -\ell V^a + h + \sum_{q_i=1}^N Y_{p_i, q_i} V_1 + \sum_{q_i=1}^N Y_{p_i, q_i} V_2 ,
\]

where \( \ell = \min\left\{ -c_2 (2\rho)^a, c_4 (2\rho)^a, c_3, c_5, \kappa_1, \kappa_2, c_{g_1} + c_{g_2}\right\} \).

The following will apply the discrete expectation to address the random terms.

\[
E(LV) \leq \sum_{p_i=1}^N \sum_{p_2=1}^N E(LV_i) \pi_{p_i} \pi_{p_2} \]

\[
\leq -\ell \sum_{p_i=1}^N \sum_{p_2=1}^N E(LV_i^a) \pi_{p_i} \pi_{p_2} + h
\]

\[
+ \sum_{p_i=1}^N \sum_{p_2=1}^N E \left( \left( \sum_{q_i=1}^N Y_{p_i, q_i} V_1 + \sum_{q_i=1}^N Y_{p_i, q_i} V_2 \right) \pi_{p_i} \pi_{p_2} \right)
\]

\[
\leq -\ell \sum_{p_i=1}^N \sum_{p_2=1}^N E(LV_i^a) + h + N \left( \sum_{k=1}^N \pi_{p_k} \pi_{p_k} \right) \left\{ \sum_{k=1}^N \pi_{p_k} \pi_{p_k} \right\}
\]

\[
EV_L^a \leq -\ell E(V^a) + h .
\]
where $V_1 = V(x(t_1), r_1(t_1), r_2(t_1))$, $V_{11} = V(x(t_2), p_2, q_2)$, $V_{2} = (x(t_1), p_1, q_1)$, and $V_{11}^\gamma = V(x(t_1), r_1(t_1), r_2(t_2))$. The stability distribution law of Markovian variable $r_1(t_1)$ is $\pi_{r_1} = (\pi_{r_11}, \pi_{r_12}, \ldots, \pi_{r_1N})$, $\sum_{j=1}^N \pi_{r_1 j} = 1$, and $\pi_{r_1 j} > 0$.

The updating law (39) and (41). Meanwhile, by selecting appropriate design parameters, the tracking error of the systems can be arbitrarily small in a finite-time.

Theorem 1. The finite-time FTC control can be implemented by designing the virtual signal (17), the controller in (40), and the updating law (39) and (41). Meanwhile, by selecting appropriate design parameters, the tracking error of the systems can be arbitrarily small in a finite-time.

Proof. Through Lemma 2 in [37] and inequality (54), we can acquire that the trajectories of the all signals that satisfy $V^\gamma(y(t), \theta(t)) \leq h/(1 - \theta_0)\epsilon_{12}$, for any $t \geq T_{reach}$, in finite time, where $T_{reach}$ is defined as $T_{reach} = (1/(1 - \theta_0))V^{1-a}(y(0), \theta_1(0))) - (h/(1 - \theta_0)\epsilon_{12})^{(1-a)/a}$ with $y(0) = [y_1(0), y_2(0)]^T$. In other words, all signals of the studied closed system are SGPFs.

Furthermore, the following result can be obtained, $i = 1, 3$. $|x_i - y_i \mid \leq k_i t \leq [1 - e^{-2(h/(1 - \theta_0)\epsilon_{12})^{1/2}}] \leq k_{\text{br}}$. Thus, when $Q(1) = 0$ and $y_i < 0$, it can obtain $y_i \leq k_{\text{br}} [1 - e^{-2(h/(1 - \theta_0)\epsilon_{12})^{1/2}}] \leq k_{\text{br}}$. By using $N_{\text{br}} \leq y_i \leq N_{\text{br}}$, it yields $-k_{\text{br}} \leq y_i \leq k_{\text{br}}$. In short, $x_i$ is constrained in its limit.

3.2 Zero Dynamics and Performance Analysis. Through the analysis in Section 3.1, the signals $x_i$ and $\bar{h}_i (i = 1, 2, 3, 4, j = 1, 2)$ are bounded. Then, we are going to prove the zero dynamics that consist of the other four states $x_5, x_6, x_7, x_8$.

Therefore, substituting (38)-(41) into (5), let $y_1 = y_2 = y_3 = y_4 = 0$. It yields the following form:

$$\dot{Z} = AZ + B_1D_1 + B_0D_0 + W,$$

where

$$Z = [x_5, x_6, x_7, x_8]^T,$$

$$D_0 = [D_{\bar{h}_1}, D_{\bar{h}_2}, D_{\bar{h}_3}, D_{\bar{h}_4}]^T,$$

$$D_1 = [D_f - D_1, D_f - D_1, D_f - D_1, D_f - D_2]^T,$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -k_{f_2} & -b_{f_2} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -k_{r_2} & -b_{r_2} \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ k_{f_2} & b_{f_2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & k_{r_2} & k_{r_2} & 0 \end{bmatrix}.$$
where $V_Z = V_Z(Z, p_1, p_2)$, and $P$ is a positive matrix.

According to the infinitesimal generator in [50], (58) can be calculated at the mode of $(r_1(t), r_2(t)) = (p_1, p_2)$ as

$$LV_Z = Z^T P Z + Z^T P Z + \sum_{q_1=1}^N Y_{p_1 q_1} Z_1 + \sum_{q_1=1}^N Y_{p_2 q_1} Z_2,$$

(59)

where $Z_1 = Z_1(Z, p_1, q_1)$, and $Z_2 = Z_2(Z, p_1, q_2)$.

$$LV_Z = (AZ + B_1 D_1 + B_2 D_0 + W)^T P Z,$$

$$Z^T P (AZ + B_1 D_1 + B_2 D_0 + W) + \sum_{q_1=1}^N Y_{p_1 q_1} Z_1 + \sum_{q_1=1}^N Y_{p_2 q_1} Z_2$$

$$= Z^T (A^T P + P^T A) Z + 2Z^T P B_1 D_1 + 2Z^T P B_2 D_0 + 2Z^T P W,$$

(60)

Based on Young’s inequality, it obtains

$$2Z^T P B_0 D_0 \leq \frac{\lambda_{\text{max}}(PB_0^T B_0 P)}{\theta_0} \|Z\|^2 + \theta_0 |D|,$$

(61)

$$2Z^T P B_1 D_1 \leq \frac{\lambda_{\text{max}}(PB_1^T B_1 P)}{\theta_1} \|Z\|^2 + \theta_1 |D|,$$

$$2Z^T P W \leq \frac{\lambda_{\text{max}}(PP)}{\theta_2} \|Z\|^2 + \theta_2 |W|,$$

(62)

where the parameters $\theta_0$, $\theta_1$, and $\theta_2$ are positive.

Combining (57) and (61), (60) can be rewritten as

$$LV_Z \leq \ell_Z V_Z + h_Z + \sum_{q_1=1}^N Y_{p_1 q_1} Z_1 + \sum_{q_1=1}^N Y_{p_2 q_1} Z_2,$$

(63)

where
and we have established approach for half-vehicle ASSs. In order to facilitate the implementation of the simulation, the nonlinear stiffness and dampers are given in [6]

\[ h_Z = \theta_0 D_0^2 + \theta_1 D_1^2 + \theta_2 W^2, \]

\[ \ell_Z = \lambda_{\min} P^{(1/2)} Q P^{(1/2)} - \lambda_{\max}(P) - \lambda_{\max}(p^{(1/2)} B_0 B_0^{(1/2)}) - \lambda_{\max}(p^{(1/2)} B_1 B_1^{(1/2)}) \]

(63)

Then, same as the above discrete expectations, (62) yields

\[ EV_Z \leq e^{-\ell_0 (t-\tau)} V_{Z_0} + \frac{h_Z}{\ell_0} \left( 1 - e^{-\ell_0 (t-\tau)} \right), \]

where \( \ell_0 = \ell_Z - \ell_f \), and \( V_{Z_0}(Z(t_0), r_1(t_0), r_2(t_0)) \).

According to Lemma 2, all the signals of the zero dynamics are bounded.

Now, the control objectives will be proved. The dynamic tire loads are given as

\[ |F_{\text{sf}} + F_{\text{br}}| \leq (k_{f_2} + b_{j_2}) \left( \frac{(V_{Z_0} + h_3/\ell_Z)}{\lambda_{\min}(P)} \right)^{(1/2)} + k_{f_2} D_{11} + b_{j_2} D_{12}, \]

(65)

\[ |F_{\text{tr}} + F_{\text{br}}| \leq (k_{r_2} + b_{r_2}) \left( \frac{(V_{Z_0} + h_3/\ell_Z)}{\lambda_{\min}(P)} \right)^{(1/2)} + k_{r_2} D_{21} + b_{r_2} D_{22}. \]

Thus, selecting appropriate parameters \( \theta_1 \) and \( \theta_2 \) can ensure \( (k_{f_2} + b_{j_2}) \theta_1 + k_{f_2} D_{11} + b_{j_2} D_{12} \leq F_f, (k_{r_2} + b_{r_2}) \theta_2 + k_{r_2} D_{21} + b_{r_2} D_{22} \leq F_r \), that is, (66) holds.

Finally, we are going to learn about the suspension deflection performances. According to the analysis in Section 3.1 and 3.2, one has

\[ |\Delta D_f| \leq x_1 + l_2 \sin x_3 + x_4 \leq x_1 + l_2 x_3 + x_5 \leq k_{r_1} + l_2 k_{r_2} + \left( \frac{(V_{Z_0} + h_3/\ell_Z)}{\lambda_{\min}(P)} \right)^{(1/2)}, \]

(66)

\[ |\Delta D_r| \leq x_1 + l_2 \sin x_3 + x_7 \leq x_1 + l_2 x_3 + x_7 \leq k_{r_1} + l_2 k_{r_2} + \left( \frac{(V_{Z_0} + h_3/\ell_Z)}{\lambda_{\min}(P)} \right)^{(1/2)}. \]

Thus, let \( k_{f_2} + b_{j_2} \theta_1 + k_{f_2} D_{11} + b_{j_2} D_{12} \leq \Delta D_f \) and \( k_{r_2} + b_{r_2} \theta_2 + k_{r_2} D_{21} + b_{r_2} D_{22} \leq \Delta D_r \), and we have \( |\Delta D_f| \leq \Delta D_f \) and \( |\Delta D_r| \leq \Delta D_r \), which imply the suspension deflection performances can be achieved.

Remark 5. The works [38, 39] mainly focused on the finite-time control problem of quarter active suspension systems. Compared with the foregoing, this study not only addresses the finite-time control of half-vehicle but also further studies constraints of displacement and pitch angle of the car body. Particularly, the random faults between different actuators are studied.

### 4. Simulation Example

In this section, an example is given to demonstrate the established approach for half-vehicle ASSs. In order to facilitate the implementation of the simulation, the nonlinear stiffness and dampers are given in [6]
segmented coefficients of front damper of active suspension systems.

The road profile is an important aspect of affecting suspension performance. Thus, two different road inputs are carried out and given as follows:

**S1:** the road input of the front wheel is described as

\[
D_{01} = 0.5h \left[ 1 + \sin \left( \frac{2\pi v}{L} t \right) \right], \quad t_1 \leq t \leq t_h,
\]

\[= 0, \quad \text{otherwise}, \tag{68}\]

where \(h\) and \(L\) are the height and the length of the bump, and \(v\) is the velocity of the vehicle. The road input \(D_{02}\) for the rear wheel is implemented as \(D_{02}(t) = D_{01}(t - \tau)\). Suppose \(h = 0.05\) m, \(L = 2.5\) m, \(v = 18\) km/h, \(t_1 = 2\), \(t_h = 2.5\), and time delay \(\tau = 0.5\).

**S2:** the road input of the front wheel is presented as

\[
D_{01} = 0.5h_b \left[ 1 + \sin \left( \frac{2\pi v_b}{L} t \right) \right], \quad t_1 \leq t \leq L/v_b,
\]

\[= 0, \quad t > L/v_b, \tag{69}\]

where \(h_b = 0.08\) m, \(L = 2.5\) m, and \(v_b = 25\) km/h. The road input \(D_{02}\) is carried out the same as \(S1\).

The parameters in half-vehicle active suspension systems are briefly illustrated: \(M = 1200\) kg, \(I = 1000\) kgm², \(m_f = m_r = 100\) kg, \(k_f = k_r = 15000\) N/m, \(k_{f1} = k_{r1} = 1000\) N/m, \(k_{f2} = 200000\) N/m, \(l_1 = 1.5\) m, \(k_{r2} = 200000\) N/m, \(b_f = 1500\) Ns/m, \(b_r = 2000\) Ns/m, \(d_{f1} = d_{r1} = 1500\) Ns/m, \(d_{f2} = 1200\) Ns/m, and \(l_2 = 1.2\) m. The control parameters in this work are given as follows: \(\eta_1 = 4\), \(\eta_2 = 6\), \(\delta_1 = 3\), \(\delta_2 = 2\), \(\kappa_1 = 30\), \(\kappa_2 = 40\), \(c_1 = c_3 = 50\), \(c_2 = c_4 = 100\), \(a = 299/300\), \(k_{\alpha1} = 0.006\).
The vertical motion acceleration

\[ \begin{align*}
\text{Time (s)} & \quad 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\text{(m/s}^2) & \quad 0 & 2 & 4 & 0 & 2 & 4 & 0 & 2 & 4 & 0 & 2
\end{align*} \]

(a)

The pitch motion acceleration

\[ \begin{align*}
\text{Time (s)} & \quad 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\text{(m/s}^2) & \quad 0 & 5 & 10 & 0 & 5 & 10 & 0 & 5 & 10 & 0 & 5
\end{align*} \]

(b)

Figure 5: The accelerations of active suspension systems motion.

The updating law \( \hat{\theta}_1 \)

\[ \begin{align*}
\text{Time (s)} & \quad 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\text{0.4} & \quad 0 & 0.2 & 0.4 & 0 & 0.2 & 0.4 & 0 & 0.2 & 0.4 & 0 & 0.2
\end{align*} \]

(a)

The updating law \( \hat{\theta}_2 \)

\[ \begin{align*}
\text{Time (s)} & \quad 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\text{3} & \quad 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2
\end{align*} \]

(b)

Figure 6: Trajectories of \( \hat{\theta}_i \) (i = 1, 2).

The controller \( u_f \)

\[ \begin{align*}
\text{Force (N)} & \quad -400 & -200 & 0 & 200 & 400 \\
\text{Time (s)} & \quad 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\text{(N)} & \quad -400 & -200 & 0 & 200 & 400
\end{align*} \]

(a)

The controller \( u_r \)

\[ \begin{align*}
\text{Force (N)} & \quad -300 & -100 & 0 & 100 & 300 \\
\text{Time (s)} & \quad 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\text{(N)} & \quad -300 & -100 & 0 & 100 & 300
\end{align*} \]

(b)

Figure 7: The controllers with fault tolerant.

The pitch angle \( \phi \) is constrained under its bounded and can stabilize quickly in a finite time, as shown in Figure 4. Figure 5 shows the accelerations of vertical motion and pitch motion. It is observed that the maximums of vertical motion and pitch motion are 4 m/s\(^2\) and 5 m/s\(^2\), respectively. In addition, the stabilization of these motions is achieved in 1 sec for cases S1 and S2. The purpose of Figure 6 is to show the trajectories of \( \hat{\theta}_1 \) and \( \hat{\theta}_2 \). It indicates that for cases S1 and S2, they can converge to zero in a very short period of time. Figure 7 shows the controllers \( u_f \) and \( u_r \), and we can observe that the control forces are calculated about 300 N. It means that the designed controllers play a key role in controlling
the variation of active suspension systems. Figure 8 shows that the front wheel and rear wheel of the suspension spaces are all within the bounds. Through the above analysis, the proposed control schemes could be used to help reduce the variation of active suspension systems and achieve a good performance in a finite time.

5. Conclusions

In order to achieve good suspension performances, the finite-time control for half active suspension systems have been discussed in this study. The asymmetric BLFs have been employed to constrain vehicle body’s displacement and pitch angle in a safety range. Then, utilizing the finite-time control design theory, the adaptive controllers were proposed for vertical motion and pitch motion, respectively. Finally, the effectiveness of designed schemes is demonstrated with the help of results of simulation examples. On the one hand, in the aspect of automobile research, we would further study the battery problem combined with the distributed controller of the multiagent consistency algorithm and apply it to distributed generators in the Energy Internet such as [56, 57]. On the other hand, we apply this method in the pipeline network system such as [58] and optimal deployment of agents in the pipeline network system in [59]. The study of control problem in the pipeline network system will be an interesting research topic.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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Figure 8: The suspension spaces.
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