Skyrme pseudo-potential-based EDF parametrization for spuriousity-free MR EDF calculations

J Sadoudi1,2, M Bender1,2, K Bennaceur3,4,5, D Davesne3,4,5, R Jodon3,4,5 and T Duguet6,7

1 University Bordeaux, CENBG, UMR 5797, F-33170 Gradignan, France
2 CNRS/IN2P3, CENBG, UMR 5797, F-33170 Gradignan, France
3 Université de Lyon, F-69003 Lyon, France
4 Université Lyon 1, 43 Boulevard du 11 Novembre 1918, F-69622 Villeurbanne Cedex, France
5 CNRS-IN2P3, UMR 5822, Institut de Physique Nucléaire de Lyon, France
6 CEA-Saclay DSM/Irfu/SPhN, F-91191 Gif sur Yvette Cedex, France
7 National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA

E-mail: sadoudi@cenbg.in2p3.fr

Received 8 November 2012
Accepted for publication 28 November 2012
Published 2 May 2013
Online at stacks.iop.org/PhysScr/T154/014013

Abstract
First exploratory steps toward a pseudo-potential-based Skyrme energy density functional for spuriousity-free multi-reference calculations are presented. A qualitatively acceptable fit can be accomplished by adding simple three- and four-body contact terms to the standard central plus spin–orbit two-body terms. To achieve quantitative predictive power, higher-order terms, e.g. velocity-dependent three-body terms, will be required.

PACS number: 21.60.Jz

(Some figures may appear in colour only in the online journal)

1. Introduction

Methods based on the use of effective energy density functionals (EDFs) [1] currently provide the only set of theoretical tools that can be applied to all bound atomic nuclei but the lightest ones in a systematic manner irrespective of their mass, isospin and deformation. One underlying assumption is that of a universal effective energy functional that depends on normal and anomalous one-body density matrices and that re-sums the in-medium correlations whose details are irrelevant for low-energy nuclear structure physics. Nuclear EDF methods coexist on two distinct levels. On the first level, that is traditionally called self-consistent mean-field theory, Hartree–Fock (HF) or Hartree–Fock–Bogoliubov (HFB), a single product state provides the normal and anomalous density matrices that enter the EDF. We call this type of method a single-reference (SR) approach. Exploiting the concept of symmetry breaking as a consequence of the nuclear Jahn–Teller effect [2], collective correlations that correspond to multipole deformations or superfluidity can be easily modeled at the expense of losing good quantum numbers. On the second level, often called ‘beyond-mean-field methods’, symmetry restoration and configuration mixing can be achieved within the generator coordinate method framework. Such method is referred to as a multi-reference (MR) approach [3]. The MR techniques aim at the explicit description of correlations that are related to the small finite size of the nucleus and that neither can be easily absorbed into the EDF, nor be described by a single SR state. Typical examples are the mixing of different shapes that coexist in a nucleus or the restoration of symmetries that are broken at the SR level. Besides describing correlated ground states, the MR approach also gives access to excited states and transition moments between them taking selection rules into account.

MR techniques based on EDFs rely on the extension of the EDF to non-diagonal energy kernels. The generalized Wick theorem [4] provides the formal framework when the
EDF is defined as the expectation value of a genuine Hamilton operator. The energy functionals widely used in nuclear physics, however, are not of this form. We denote these as ‘general functional’ in what follows. The standard procedure for the MR extension of such general functionals is made by formal analogy with the Hamiltonian case, see [5, 6] and references therein. It has been pointed out a few years ago that the usual procedure to set up the non-diagonal kernels of a general EDF is ill-defined [5–9]. It gives rise to spurious contributions to the energy that can manifest themselves for example through divergences and/or finite steps when plotting the symmetry-restored energy as a function of a collective coordinate [5, 8]. It also makes MR calculations return non-zero energies when restoring negative particle numbers [8, 10]. These difficulties can be traced back to a breaking of the Pauli principle when setting up the EDF. In one way or the other, this is the case for all modern parametrizations of the nuclear EDF. It is motivated either by phenomenology or for computational reasons [1, 11].

One popular choice for a general nuclear EDF is historically based on two- and three-body contact pseudo-potentials proposed by Skyrme [12]. In the earliest adjustments of its coupling constants for nuclear EDF calculations it was used as a genuine Hamilton operator 8. This gave rise to three major problems in the resulting parametrizations: (i) the incompressibility \( K_\infty \) of homogeneous symmetric infinite nuclear matter (INM) was unavoidably much larger than the empirical one; (ii) fits that constrain other INM properties such as isoscalar effective mass \( m_0^*/m \), saturation density \( \rho_{\text{sat}} \) and symmetry energy coefficient \( a_{\text{sym}} \) to empirical values provided parametrizations that were unstable in the spin channel as indicated by the Landau parameter \( g_0 \) being smaller than \(-1\) [13] and (iii) pairing matrix elements were far too small or even repulsive. Problem (ii) could be circumvented reinterpreting the contact three-body force \( t_3 \delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_1 - \mathbf{r}_3) \) as a density-dependent two-body force \( \frac{1}{2} t_3 (1 + x_3 \hat{P}_{12}^s) \rho_0^s(\mathbf{r}_1 + \mathbf{r}_2) \delta(\mathbf{r}_1 - \mathbf{r}_3) \) with \( x_3 = 1 \), where \( \rho_0^s(\mathbf{r}) = \rho_0^1(\mathbf{r}) + \rho_0^2(\mathbf{r}) \) is the isoscalar density and \( \hat{P}_{12}^s \) the spin-exchange operator, respectively, which alters the so-called ‘time-odd’ part of the functional that governs the effective spin–spin interaction. Problem (i) could then be solved by reducing the exponent \( x_3 \) from 1 to values between about 1/6 and 1/3. And for most parametrizations of the Skyrme EDF, problem (iii) was removed by using different vertices in the particle–hole and particle–particle channels that were independently adjusted to data. In addition, some specific terms in the EDF are often suppressed or approximated to simplify the numerical treatment [1]. In the past this has led to high-precision parametrizations of the Skyrme EDF that kept a relatively simple form [1, 14] and were numerically efficient, at the expense of sacrificing the exchange symmetry of the EDF. Other variants of the nuclear EDF have been set-up directly through their energy density without any reference to an underlying effective interaction [15, 16].

The breaking of the exchange symmetry that results from this practice introduces what is known as spurious

[8] In the present context, so-called ‘density-dependent effective interactions’ do not qualify as genuine Hamilton operators or ‘pseudo-potentials’.

‘self-interaction’ in density functional theory for condensed matter [17]. An analysis of the same problem in the context of effective interactions as used in nuclear physics has been given in [18]. A similar spurious ‘self-pairing’ appears when the normal and pairing parts of the EDF are not derived from the same effective interaction as pointed out in [8]. According to [6], divergences and steps in MR EDF calculations originate from unphysical weights such self-interaction and self-pairing contributions are multiplied with. Based on this analysis, a regularization scheme that modifies those ill-defined weights while keeping the self-interaction itself has been proposed in [6] and applied with success to the case of pure particle-number projection in [8].

Not all general functionals are regularizable, though. The formalism can be applied only to functionals that correspond to polynomials in the density matrices [6, 9]. Parametrizations of a general regularizable Skyrme-type EDF of minimal form have been constructed recently [11] and are currently used to test the regularized MR EDF scheme for general configuration mixing. Constraining the EDF to polynomial form makes its fit more difficult.

In parallel, we also started the construction of functionals that are free of spuriousities from the outset by setting them up as the expectation value of a genuine Skyrme-type Hamilton operator taking all exchange and pairing terms into account such that the Pauli principle is obeyed. By construction, this removes all spurious contributions to the EDF at the price of having less independent terms in the EDF. In what follows, we denote such functionals a ‘pseudo-potential generated EDF’. This replaces the problem of how to set up a formalism for the extension of a general and flexible EDF to non-diagonal kernels for MR calculations by the problems of what is the most efficient form for a predictive pseudo potential that can be straightforwardly used in MR calculations and how to adjust it to data. This is a difficult task, as a pseudo-potential-based functional has much less independent coupling constants than a general functional of the same form. To be usable in SR and MR calculations incorporating pairing correlations, it is mandatory that the functional gives a reasonable description of the spin and pairing channels of the interaction. Otherwise the level sequence after angular-momentum projection is likely to be unrealistic. However, the difficulty to have even the correct sign of the interaction in these two channels has been among of the major motivations to abandon Skyrme-type pseudo-potential-based EDFs in favor of the general ones in the 1970s.

The aim of this contribution is to present the first explorative steps toward the construction of a predictive pseudo-potential-based EDF. We present the adjustment of a parametrization that achieves an acceptable qualitative description in all channels of the interaction by adding three- and four-body contact terms without gradients to a standard two-body Skyrme operator. It will serve to benchmark MR EDF calculations and as a reference for what can be achieved without introducing derivatives in the new terms.
2. Pseudo-potential-based EDF

The pseudo-Hamiltonian used in the present work takes the form

\[
\hat{H} = \hat{T}^{(1)} + \hat{V}^{(2Sk)} + \hat{V}^{(2C)} + \hat{V}^{(3Sk)} + \hat{V}^{(4Sk)},
\]

where \(\hat{T}^{(1)}\) is the kinetic energy operator and \(\hat{V}^{(2C)}\) the Coulomb interaction, which take their standard form. The \(\hat{V}^{(NSk)}\) are the \(N\)-body parts of the Skyrme-type pseudo-potential for which we consider here the form

\[
\hat{V}^{(2Sk)} = t_0 \left( 1 + x_0 \hat{P}_{12}^2 \right) \delta_{t,r_2} + \frac{t_1}{2} \left( 1 + x_1 \hat{P}_{12}^2 \right) \left( \hat{k}_{12}^2 \delta_{t,r_2} + \delta_{t,r_1} \hat{k}_{12}^2 \right),
\]

\[
\hat{V}^{(2C)} = t_2 \left( 1 + x_2 \hat{P}_{12}^2 \right) \hat{k}_{12} \cdot \delta_{t,r_1} \hat{k}_{12},
\]

\[
\hat{V}^{(3Sk)} = u_0 \left( \delta_{t,r_1} \delta_{t,r_2} + \delta_{t,r_2} \delta_{t,r_1} + \delta_{t,r_1} \delta_{t,r_1} \right),
\]

\[
\hat{V}^{(4Sk)} = v_0 \left( \delta_{t,r_1} \delta_{t,r_2} \delta_{t,r_3} + \delta_{t,r_2} \delta_{t,r_3} \delta_{t,r_1} + \delta_{t,r_3} \delta_{t,r_1} \delta_{t,r_2} + \cdots \right),
\]

where \(t_i, x_i, u_0\) and \(v_0\) are unknown coupling constants, \(\hat{P}_{12}^2\) denotes the spin exchange operator, \(\delta_{t,r_2}\) is the Dirac distribution, \(\hat{\sigma}\) is the vector of Pauli spin matrices and \(\hat{k}_{12}\) and \(\hat{k}_{12}'\) are the incoming and outgoing relative momenta. The three-body and four-body terms contain 3 and 12 permutations of the coordinates, respectively. The SR and MR energy kernels are calculated from

\[
\langle \Phi | \hat{H} | \Phi' \rangle = \int \! d^3r \, \mathcal{E}_H [\rho^{\phi\phi}, \kappa^{\phi\phi}, \chi^{\phi\phi}](r),
\]

where \(\rho^{\phi\phi}\) and \(\kappa^{\phi\phi}\) denote normal and anomalous transition density matrices, respectively. The resulting nuclear part of the energy can be decomposed into bilinear, trilinear and quadrilinear parts according to their content in \(\rho\) and \(\kappa\)

\[
\mathcal{E}_H = \mathcal{E}_H^{\text{exp}} + \mathcal{E}_H^{\text{sk}} + \mathcal{E}_H^{\text{pop}} + \mathcal{E}_H^{\text{space}}.
\]

The bilinear parts take the usual form [19]

\[
\mathcal{E}_H^{\text{exp}} = \sum_q \left\{ A^{\rho_1 \rho_2} \rho_1 \rho_2 + A^{\rho_1 \rho_2} \rho_1 \rho_2 + A^{\nu_1 \nu_2} \nu_1 \cdot \nu_2 \right\}
\]

\[
+ \sum_{\mu \nu} \left( A^{\nu_1 \nu_2} \left[ \nu_{\mu} \cdot s_{q_{1,1}} \right] \left[ \nu_{\mu} \cdot s_{q_{1,1}} \right] + A^{\nu_1 \nu_2} \left[ \nu_{\mu} \cdot s_{q_{1,1}} \right] \left[ \nu_{\mu} \cdot s_{q_{1,1}} \right] + A^{\nu_1 \nu_2} \left[ \nu_{\mu} \cdot s_{q_{1,1}} \right] \left[ \nu_{\mu} \cdot s_{q_{1,1}} \right] \right)
\]

3. A tentative fit

From the published work on pseudo-potential-based Skyrme EDFs from the 1970s it is clear from the outset that the simple form of equation (2) will be unlikely to reach the quality of modern parametrizations of the general Skyrme EDF. Our more modest aim is a parametrization that could be used in time-reversal-breaking SR and angular-momentum restored MR EDF calculations based on HFB-type reference states. To accomplish this, we had to add several new constraints to our fit protocol. Firstly, we had to avoid any unphysical instabilities, not just the ones related to too small values of the Landau parameters, but also finite-size instabilities of the
kind discussed in [20, 21]. Secondly, we aimed at an overall repulsive interaction in the spin channels and an attractive pairing interaction that provides gaps of realistic size. This, however, required to relax the constraints on many other properties, in particular those of nuclear matter.

To construct an acceptable starting point for a fit, we took the parameters of SIV, which has a reasonable value of \( g_0 \) and provides weakly attractive pairing, from [22]. We modified \( x_0 \) and \( t_1 \) to enhance pairing and then \( u_0, v_0 \) and, to a lesser extent, \( t_2 \) and \( x_2 \), to bring the parametrization back to a better description of symmetric INM. Simultaneously, \( x_0 \) had to be adapted to reject the onset of spin instabilities far enough above the saturation density. In a second step, the parameters were then fine-tuned to describe a set of nuclear masses and radii in addition to the nuclear matter properties with those of other parametrizations used as pseudo-potentials in the literature. They differ in higher-order terms in equation (2). SV and SHZ2 are pure two-body forces that have been used for isospin and angular-momentum projection of HF states (without pairing) [23, 24]. SIII and SIV include the three-body term, whereas SLyMR0 contains three- and four-body terms. Values obtained with a standard parametrization SLy4 of the general Skyrme EDF are given for comparison.

As expected, all the pseudo-potential-based parametrizations have difficulties to describe nuclear matter properties. In particular, their effective mass is very low except for SIII. This leads to a density of single-particle levels around the Fermi energy that is much lower than the empirical one. For SIII, the larger effective mass leads, through the interrelations between the coupling constants, to spin instability as indicated by \( g_0 < -1 \) [13]. For SIV, the reduction of the effective mass within the same functional form pushes \( g_0 \) to values around zero. However, an analysis along the lines of [21] reveals that SIV has a finite-size spin instability in the \( S = 1, T = 0 \) channel that cannot be revealed by the Landau parameters. Hence, neither SIII nor SIV can be used in their pseudo-potential form in time-reversal invariance breaking mean-field calculations. By contrast, such an analysis does not indicate unphysical instabilities for SV, SHZ2 and SLyMR0 at densities relevant for low-energy nuclear structure.

Table 1. Coupling constants of the EDF expressed in terms of the parameters of the pseudo-potential. Missing entries are zero.

| \( t_0 \) | \( t_0 x_0 \) | \( t_1 \) | \( t_1 x_1 \) | \( t_2 \) | \( t_2 x_2 \) | \( W_0 \) |
|----------|--------------|--------|-------------|--------|-------------|--------|
| \( A^{\alpha \gamma} = +\frac{1}{4} \) | \( -\frac{1}{4} \) | \( A^{\beta \gamma} = +\frac{1}{4} \) | \( +\frac{1}{4} \) | \( A^{\gamma \beta} = +\frac{1}{4} \) | \( +\frac{1}{4} \) | \( A^{\gamma \beta} = +\frac{1}{4} \) | \( -\frac{1}{4} \) |
| \( A_{\beta \gamma} = +\frac{1}{4} \) | \( -\frac{1}{4} \) | \( A_{\beta \gamma} = +\frac{1}{4} \) | \( +\frac{1}{4} \) | \( A_{\beta \gamma} = +\frac{1}{4} \) | \( +\frac{1}{4} \) | \( A_{\beta \gamma} = +\frac{1}{4} \) | \( -\frac{1}{4} \) |
| \( A_{\beta \gamma} = +\frac{1}{4} \) | \( -\frac{1}{4} \) | \( A_{\beta \gamma} = +\frac{1}{4} \) | \( +\frac{1}{4} \) | \( A_{\beta \gamma} = +\frac{1}{4} \) | \( +\frac{1}{4} \) | \( A_{\beta \gamma} = +\frac{1}{4} \) | \( -\frac{1}{4} \) |

Table 2. Coupling constants of SLyMR0.

| Parameter | \( \rho_{\text{sat}} \) (fm\(^{-3}\)) | \( E/A \) (MeV) | \( a_{\text{sym}} \) (MeV) | \( m^*_0/m \) | \( K_{\text{sat}} \) (MeV) | \( g_0 \) |
|-----------|---------------------------------|----------------|--------------------|-------------|----------------|--------|
| SIII [22]  | 0.145                          | -15.85         | 28.2               | 0.76        | 355.3          | -1.58  |
| SIV [22]   | 0.151                          | -15.96         | 31.2               | 0.47        | 324.6          | 0.06   |
| SV [22]    | 0.157                          | -16.05         | 32.8               | 0.38        | 305.7          | 0.57   |
| SHZ2 [24]  | 0.157                          | -16.27         | 42.1               | 0.38        | 309.6          | 0.27   |
| SLyMR0     | 0.152                          | -15.04         | 23.0               | 0.47        | 264.2          | 0.88   |
| SLy4 [14]  | 0.160                          | -15.97         | 32.0               | 0.69        | 229.9          | 1.38   |

Table 3. Saturation density \( \rho_{\text{sat}} \), energy per particle \( E/A \), symmetry energy \( a_{\text{sym}} \), effective mass \( m^*_0/m \), incompressibility \( K_{\text{sat}} \) and spin-Landau parameter \( g_0 \) at saturation for the parametrizations studied here.
exhibits mass residuals for isotopic and isotonic
Bender M, Heenen P-H and Reinhard P-G 2003
Reinhard P-G and Otten E-W 1984
Dobaczewski J, Stoitsov M V , Nazarewicz W and Reinhard
Bender M 2008
Ring P and Schuck P 1980
156
Lacroix D, Duguet T and Bender M 2009
Bender M, Duguet T and Lacroix D 2009
Anguiano M, Egido J L and Robledo L M 2001
Duguet T, Bender M, Bennaceur K, Lacroix D and Lesinski T
79
Duguet T and Sadoudi J 2010
86
Washiyama K, Bennaceur K, Avez B, Bender M, Heenen P-H
Fayans S A, Tolokonnikov S V , Trykov E L and Zawischa D
(erratum)
676
635
Skyrme T H R 1956
627
1
643
B¨ackman S O, Jackson A D and Speth J 1975
Chabanat E, Meyer J, Bonche P, Schaeffer R and Haensel P
Baldo M, Schuck P and Vi ˜nas X 2008
Stringari S and Brink D M 1978 Nucl. Phys. A 304 307
Perli´nska E, Rohozinski S G, Dobaczewski J and Nazarewicz
W 2004 Phys. Rev. C 69 014316

Figure 1. Neutron spectral gaps of singly magic even–even nuclei in the isotopic chains as indicated at the SR-EDF level.

Figure 2. Binding energy residuals as a function of A for singly magic nuclei for spherical SR-EDF calculations. Nuclei in isotonic and isotopic chains are connected by lines.

In figure 1, pairing properties are examined via the spectral gap $E_{\text{pair, }n} / \int d^3 r \rho_n(r)$ of neutrons as obtained from spherical HFB-type SR calculations of singly magic even–even nuclei. Empirical pairing gaps obtained from a three-point mass difference are shown for comparison. When solving HFB equations, pairing matrix elements have been multiplied with a smooth cutoff at 8.5 MeV above and below the Fermi energy. SIII, SIV, SHZ2 and SV give null pairing or at most a weak pairing in some nuclei. Only SLyMR0, for which this property was enforced during the fit, provides pairing gaps of a realistic size.

Figure 2 exhibits mass residuals for isotopic and isotonic chains of singly magic nuclei as obtained from spherical HFB-type SR calculations. The particularly large drift of the curves obtained with SLyMR0 results mainly from its very low value for the asymmetry energy coefficient $a_{\text{sym}}$, which cannot be increased without jeopardizing pairing or the time-odd terms in the EDF. We have checked that, in spite of its poor description of masses, SLyMR0 gives a reasonable description of the deformation of nuclei in the sd and pf shell regions, and of their rotational bands in cranked HFB calculations.

4. Conclusion

The simple pseudo-potential (2) allows for a description of basic nuclear properties that does not meet the standard of state-of-the-art SR-EDF calculations. However, it is sufficient for its main purpose of benchmarking multi-dimensional MR EDF calculations, which then will be free from the pathologies encountered with any modern standard parametrization. In particular, we achieved a fair description of pair correlations while avoiding (finite-size) spin-instabilities. However, the simple form used here is clearly over-constrained and higher-order terms will be needed to replicate the performance of the best available general Skyrme EDFs at the SR level. Work in that direction is underway. As a first step, the most general central three-body contact operator up to second order in gradients has been worked out recently [25]. First fits that aim at the set-up of a suitable protocol are currently underway and show promising results.

Acknowledgments

Clarifying discussions with B Avez, J Meyer and A Pastore are gratefully acknowledged. This work has been supported by the Agence Nationale de la Recherche under grant no. ANR 2010 BLANC 0407 ‘NESQ’.

References

[1] Bender M, Heenen P-H and Reinhard P-G 2003 Rev. Mod. Phys. 75 121
[2] Reinhard P-G and Otten E-W 1984 Nucl. Phys. A 420 173
[3] Bender M 2008 Eur. Phys. J. Spec. Top. 156 217
[4] Ring P and Schuck P 1980 The Nuclear Many-Body Problem (New York: Springer)
[5] Dobaczewski J, Stoitsov M V , Nazarewicz W and Reinhard P-G 2007 Phys. Rev. C 76 054315
[6] Lacroix D, Duguet T and Bender M 2009 Phys. Rev. C 79 044318
[7] Anguiano M, Egidio J L and Robledo L M 2001 Nucl. Phys. A 696 467
[8] Bender M, Duguet T and Lacroix D 2009 Phys. Rev. C 79 044319
[9] Duguet T, Bender M, Bennaceur K, Lacroix D and Lesinski T 2009 Phys. Rev. C 79 044320
[10] Duguet T and Sadoudi J 2010 J. Phys. G: Nucl. Part. Phys. 37 064009
[11] Washiyama K, Bennaceur K, Avez B, Bender M, Heenen P-H and Hellemans V 2012 Phys. Rev. C 86 054309
[12] Skyrme T H R 1956 Phil. Mag. 1 1043
[13] B¨ackman S O, Jackson A D and Speth J 1975 Phys. Lett. B 56 209
[14] Chabanat E, Meyer J, Bonche P, Schaeffer R and Haensel P 1999 Nucl. Phys. A. 627 710
Chabanat E, Meyer J, Bonche P, Schaeffer R and Haensel P 1998 Nucl. Phys. A 635 231
Chabanat E, Meyer J, Bonche P, Schaeffer R and Haensel P 1998 Nucl. Phys. A 643 641 (erratum)
[15] Fayans S A, Tolokonnikov S V , Trykov E L and Zawischa D 2000 Nucl. Phys. A 676 49
[16] Baldo M, Schuck P and Vi˜nas X 2008 Phys. Lett. B 663 390
[17] Perdew J P and Zunger A 1981 Phys. Rev. B 23 5048
[18] Stringari S and Brink D M 1978 Nucl. Phys. A 304 307
[19] Perli´nska E, Rohozinski S G, Dobaczewski J and Nazarewicz W 2004 Phys. Rev. C 69 014316
[20] Lesinski T, Bennaceur K, Duguet T and Meyer J 2006 Phys. Rev. C 74 044315
[21] Pastore A, Davesne D, Bennaceur K, Meyer J and Hellemans V 2013 Phys. Scr. T154 014014
[22] Beiner M, Flocard H, Van Giai N and Quentin P 1975 Nucl. Phys. A 238 29
[23] Satuła W, Dobaczewski J, Nazarewicz W and Rafalski M 2010 Phys. Rev. C 81 054310
[24] Satuła W, Dobaczewski J, Nazarewicz W and Werner T R 2012 Phys. Rev. C 86 054316
[25] Sadoudi J 2011 Thèse, Université Paris-Sud XI