Modeling bias in supermassive black hole spin measurements

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X-ray reflection spectroscopy (or iron line method) is a powerful tool to probe the strong gravity region of black holes, and currently is the only technique for measuring the spin of supermassive black holes. While all the available relativistic reflection models assume thin accretion disks, we know that several sources accrete near or above the Eddington limit and therefore must have thick accretion disks. In this Letter, we estimate the systematic error on the spin measurement when a source with a thick accretion disk is fitted with a thin disk model. Our results clearly show that the spin can be significantly overestimated. Current spin measurements of sources with high mass accretion rate are therefore not reliable.

Most galaxies are thought to harbor a supermassive black hole at their center [1]. Reliable mass and spin measurements are crucial to study the physics and the astrophysics of black holes. The mass is relatively easy to infer, by studying the motion of stars or gas orbiting the black hole [2, 3]. The spin measurement is more challenging because the spin has no gravitational effects in Newtonian gravity and its measurement thus requires the study of relativistic effects near the compact object. X-ray reflection spectroscopy (or the iron line method) is a powerful technique to measure black hole spins [4–7] and even to test general relativity in the strong field regime [8–11]. Moreover, it is currently the only method to measure the spin of supermassive black holes, while spin measurements for stellar-mass black holes are also possible with the continuum-fitting method [12, 13] and gravitational waves [14].

Current relativistic reflection models assume that the accretion disk is geometrically thin and that the inner edge is at the innermost stable circular orbit (ISCO) or, if the disk is truncated, at a larger radius. Numerical and observational studies show that accretion disks are geometrically thin with the inner edge at the ISCO when the source is in the thermal state accreting between a few percent to up to 30% of its Eddington limit [15, 16]. However, especially in the case of supermassive black holes, we know that a source can easily exceed this 30% bound. The supermassive black holes in narrow-line Seyfert 1 galaxies, which are the most common target for X-ray reflection spectroscopy spin measurements, are thought to accrete near or even above the Eddington limit [17–20]. In such a case, the gas pressure should make the accretion disk geometrically thick and the inner edge of the disk is likely inside the ISCO radius [21]. The question then is whether we can fit the spectra of these sources with a thin disk reflection model for measuring the black hole spin, and how large is the resulting systematic error on the spin measurement.

While geometrically thin and optically thick accretion disks are well described by the Novikov-Thorne model [15], there is currently no good and simple model for the description of thick disks. The only available semi-analytic model to describe the accretion disk of sources accreting near or above the Eddington limit is the so-called Polish donut model [21]. In such a model, the disk is still non-self-gravitating, but the gas pressure is taken into account. Unlike the Novikov-Thorne model in which the disk structure is completely determined by the conservation of rest-mass, energy, and angular momentum of the accreting gas, the Polish donut model is less constrained. It requires the relation between the fluid angular velocity and the fluid angular momentum per unit energy at the inner edge of the disk, \( \Omega = \Omega(l) \), as well as the fluid angular momentum per unit energy at the inner edge of the disk, \( l_{\text{in}} \).

The simplest Polish donut model is the configuration with \( l = \text{constant} \), which turns out to be marginally stable with respect to axisymmetric perturbations [22]. Accretion disks in the Kerr spacetime only exist for \( l_{\text{ms}} < l < l_{\text{mb}} \), where \( l_{\text{ms}} \) and \( l_{\text{mb}} \) are, respectively, the angular momentum per unit energy of the marginally stable equatorial circular orbit (or ISCO) and of the marginally bound equatorial circular orbit [21, 23, 24]. Indeed, for \( l < l_{\text{ms}} \) the fluid angular momentum is too low and there is no stable accretion disk (the fluid directly plunges onto the black hole), while, for \( l > l_{\text{mb}} \), the fluid angular momentum is too high and there is no accretion (there is just a fluid rotating around the black hole).

In the presence of an accretion disk, when \( l_{\text{ms}} < l < l_{\text{mb}} \), the inner edge of the disk, \( R_{\text{in}} \), is between the marginally bound orbit and the ISCO, i.e. \( R_{\text{mb}} < R_{\text{in}} < R_{\text{ISCO}} \) [21, 23, 24]; that is, for a given black hole spin, the inner edge of a Polish donut disk is always smaller than the inner edge of a Novikov-Thorne disk. Since the exact location of the inner edge of the disk is crucial in reflection spin measurements, we can already suspect that systematic uncertainties related to the use of an incorrect disk model to fit the data of a source can be important. It is indeed quite surprising that this problem has not been investigated before. In Ref. [25], the authors study the reflection spectrum of a thick disk, but not in the case of high mass accretion rate and their disk is actually truncated before the ISCO radius. In Ref. [26], the authors calculate the reflection spectrum of a thin disk
with finite thickness.

In order to estimate the impact of the disk structure on spin measurements with the iron line method, we use the code presented in Ref. [27] to calculate the theoretical reflection spectrum of a Polish donut disk. We use the XILVER model [28, 29] to calculate the non-relativistic reflection spectrum at the emission point in the rest-frame of the gas in the disk, and we use our ray-tracing code to convolve this non-relativistic spectrum into a relativistic spectrum of a thick disk.

At this point, we need to choose the properties of our source and of our instruments to simulate the observation of a black hole with a thick accretion disk. As X-ray mission, we consider NICER [30], which has a good energy resolution near the iron line and is thus suitable for X-ray reflection spectroscopy measurements. We assume that the source has a very simple spectrum, just a power-law component and a reflection component. In XSPEC language [31]:

\[
\text{TBABS}\times(\text{POWERLW} + \text{REFLECTION})
\]

where TBABS takes the Galactic absorption into account [32], POWERLW is the power-law component generated by inverse Compton scattering of thermal photons from the disk off free electrons in some hot gas close to the black hole [33], and REFLECTION is our theoretical reflection model for the Polish donut disk. Considering a photon flux of \(1.4 \times 10^{-10} \text{ erg cm}^{-2} \text{ s}^{-1}\) and an exposure time of about 420 ks, we get 50 million counts in the energy range 0.2-12 keV. The simulated data are then fitted with the XSPEC model

\[
\text{TBABS}\times\text{RELXILL}
\]

where RELXILL is the most popular relativistic reflection model and assumes that the accretion disk is thin [34, 35]. Since the reflection fraction parameter, \(R_f\), is assumed to be free in RELXILL, the model describes both the power-law component from the corona and the reflection spectrum from the disk. Note that we are assuming an overoptimistic observation with 50 million counts (NICER cannot observe a source for a long time, but we can imagine to combine many observations together), but this will be used later to point out that even high quality data can be fitted well with an incorrect model.

The best-fit tables of our simulations are reported in the Supplemental Material (SM). We consider two possible disk inclination angles with respect to the line of sight of the observer: \(i = 20^\circ\) and \(i = 35^\circ\). These values are relatively low, but the inner part of the disk gets obscured for higher inclination angles in the Polish donut model. This is true even in the case of real observations of supermassive black holes: relativistic reflection features are observed only for sources with low disk inclination angles, because dusty tori present in active galactic nuclei otherwise prevent direct observation of the reflection spectrum from the inner part of the accretion disk. In our simulations, we also consider two different disk configurations, with an inner edge at two different distances from the central black hole. As discussed in [27], the location of the inner edge of the disk can be conveniently parametrized by the outer edge of the disk, \(R_{\text{out}}\), and here we choose \(R_{\text{out}} = 20 M\) and \(R_{\text{out}} = 40 M\). As \(R_{\text{out}}\) increases, the inner edge of the disk \(R_{\text{in}}\) approaches \(R_{\text{mb}}\), but we cannot increase too much the value of \(R_{\text{out}}\) because, otherwise, the inner part of the disk is only visible for observers with very low viewing angles. When \(R_{\text{out}} = 20 M\), the maximum height of the Polish donut disk is \(h_{\text{max}} \approx 10 M\). In the case \(R_{\text{out}} = 40 M\), we have \(h_{\text{max}} \approx 25 M\).

It is remarkable that we always find very good fits; that is, even if we fit the reflection spectrum of a thick disk with a model that assumes a thin disk, the fit is good, in the sense that the reduced \(\chi^2\) is close to 1 and the ratio plot between the simulated data and the best-fit model does not show unresolved features. Such a result is even more remarkable considering that we are analyzing 50 million count observations, which are very optimistic for supermassive black holes. This means that the fit itself cannot tell us that the model employed to fit the data is incorrect. However, some model parameters are clearly estimated incorrectly, in the sense that the statistical uncertainties from the fits is much smaller than the systematic uncertainties due to the incorrect model used to fit the data.

Since our main focus is on the systematic effects on the spin measurement, we show the discrepancy between the input spin parameters used in the simulations and the output spin parameters found from the best-fit with RELXILL in Fig. 1. To illustrate the possible effect of a high mass accretion rate on current spin measurements, we also show two spin measurements of supermassive black holes that are thought to accrete near or above their Eddington limit. In the case of 1H0707–495, the spin measurement is \(a_\ast > 0.98\) in Refs. [7, 36], and in Fig. 1 we show this measurement with the horizontal gray region. For Ton S180, the spin measurement is \(a_\ast = 0.92^{+0.02}_{-0.09}\) in Ref. [7], corresponding to the yellow region in Fig. 1. If we consider the simulations for \(i = 35^\circ\) and \(i = 40 M\), we see that the spin of 1H0707–495 may be as low as \(a_\ast \approx 0.85\), while in the case of Ton S180 the actual value of the spin parameter may be as low as \(a_\ast \approx 0.4\). Fig. 1 does not show the fit errors on the output spin parameters because the uncertainties are too small at this scale (see SM for more details).

The emissivity profile in our simulations is described by a power-law with input emissivity index \(q = 9\). We chose quite a high value in order to limit the effects related to the size of the disk. Regardless, high emissivity indices are often preferred in the fits of reflection dominated spectra. Besides, we have also run a few simulations with \(q = 6\), finding the same qualitative results, so our conclusions do not change for emissivity indices somewhat lower than 9. For even lower emissivity indices, like \(q = 3\), the effects of the size of the Polish donut disk become important and the fits are not good. We have also run a few simulations with the response files of XMM-

\[
\chi^2 = \frac{1}{2} \sum \left(\frac{\text{obs} - \text{calc}}{\text{error}}\right)^2
\]

where \(\text{obs}\) and \(\text{calc}\) are the observed and calculated fluxes, respectively, and \(\text{error}\) are the uncertainties of the observations. The reduced \(\chi^2\) is calculated by dividing \(\chi^2\) by the number of degrees of freedom, which is the number of observations minus the number of parameters in the model. The best-fit model is the one that minimizes \(\chi^2\).
Newton to see possible effects related to the choice of the X-ray mission, but we obtain similar results, confirming that our conclusions are robust.

Concerning the interpretation of our results, the key point is the dependence on the location of the inner edge of the accretion disk, which plays a crucial role in spin measurements using the iron line method. Fig. 2 shows the input spin parameter in our simulations and the corresponding ISCO radius, inner edge of the Polish donut disk, and ISCO radius corresponding to the output spin parameter obtained from the fits. As we can see, the inner edges of the Polish donut disks turn out to be close to the inner edges of the thin disks of the best-fit models, with only a moderate dependence on the viewing angle of the observer. Note, however, that the effect of disk thickness on the recovered spin does become more important as the disk becomes thicker and the spin measurement cannot be solely explained by the location of the disk’s inner edge.

The Polish donut model is surely a toy-model, but our analysis clearly shows that even high quality data of the reflection spectrum of a thick disk can be fitted well with a thin disk model, but with the result that the spin estimate is incorrect by a significant margin. Note that reliable spin measurements of supermassive black holes are important, for instance, for studying the growth and merger history of galaxies: very fast-rotating black holes are only possible if the black hole accretes for a long time from its accretion disk (low galaxy merger rate), while frequent black hole mergers lead to black holes with a moderate value of the spin parameter (high galaxy merger rate) [37].
Conclusions — All the current relativistic reflection models assume a geometrically thin accretion disk, but several sources accrete at a rate that makes their disks geometrically thick. This issue is particularly relevant for supermassive black holes, because X-ray reflection spectroscopy is currently the only technique that can measure the spin of these objects. Spin measurements obtained in this way are inevitably affected by systematic uncertainties, but surprisingly the problem has not garnered enough attention, probably because of the lack of alternative relativistic models. In this Letter, we have shown that employing a thin disk reflection model to fit the data of a source accreting with a thick disk can lead to significantly overestimating the black hole spin. We thus claim that current reflection spin measurements of sources with high mass accretion rate are not reliable.

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[1] J. Kormendy and D. Richstone, Ann. Rev. Astron. Astrophys. 33, 581 (1995).
[2] A. M. Ghez et al., Astrophys. J. 689, 1044 (2008) [arXiv:0808.2870 [astro-ph]].
[3] J. Casares and P. G. Jonker, Space Sci. Rev. 183, 223 (2014) [arXiv:1311.5118 [astro-ph.HE]].
[4] L. W. Brenneman and C. S. Reynolds, Astrophys. J. 652, 1028 (2006) [astro-ph/0608502].
[5] L. Brenneman, Measuring Supermassive Black Hole Spins in Active Galactic Nuclei (Springer, 2013), doi:10.1007/978-1-4614-7771-6 arXiv:1309.6334 [astro-ph.HE].
[6] C. S. Reynolds, Space Sci. Rev. 183, 277 (2014) [arXiv:1302.3260 [astro-ph.HE]].
[7] D. J. Walton, E. Nardini, A. C. Fabian, L. C. Gallo and R. C. Reis, Mon. Not. Roy. Astron. Soc. 428, 2901 (2013) [arXiv:1210.4503 [astro-ph.HE]].
[8] C. Bambi, A. Cardenas-Avendano, T. Dauser, J. A. Garcia and S. Nampalliwar, Astrophys. J. 842, 76 (2017) [arXiv:1607.00596 [gr-qc]].
[9] Z. Cao, S. Nampalliwar, C. Bambi, T. Dauser and J. A. Garcia, Phys. Rev. Lett. 120, 051101 (2018) [arXiv:1709.00219 [gr-qc]].
[10] A. Tripathi, S. Nampalliwar, A. B. Abdikamalov, D. Ayzenberg, C. Bambi, T. Dauser, J. A. Garcia and A. Marinucci, Astrophys. J. 875, 56 (2019) [arXiv:1811.08148 [gr-qc]].
[11] Y. Zhang, A. B. Abdikamalov, D. Ayzenberg, C. Bambi and S. Nampalliwar, Astrophys. J. 884, 147 (2019) [arXiv:1907.03084 [gr-qc]].
[12] S. N. Zhang, W. Cui and W. Chen, Astrophys. J. 482, L155 (1997) [astro-ph/9704072].
[13] J. E. McClintock et al., Class. Quant. Grav. 28, 114009 (2011) [arXiv:1103.0748 [astro-ph.HE]].
[14] B. P. Abbott et al. [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. 116, 061102 (2016) [arXiv:1602.03837 [gr-qc]].
[15] R. F. Penna, J. C. McKinney, R. Narayan, A. Tchekhovskoy, R. Shafee and J. E. McClintock, Mon. Not. Roy. Astron. Soc. 408, 752 (2010) [arXiv:1003.0966 [astro-ph.HE]].
[16] J. F. Steiner, J. E. McClintock, R. A. Remillard, L. Gou, S. Yamada and R. Narayan, Astrophys. J. 718, L117 (2010) [arXiv:1006.5729 [astro-ph.HE]].
[17] T. A. Boroson and R. F. Green, Astrophys. J. Suppl. 80, 109 (1992).
[18] D. Grupe, S. Komossa, K. M. Leighly and K. L. Page, Astrophys. J. Suppl. 187, 64 (2010) [arXiv:1001.3140 [astro-ph.CO]].
[19] M. Gliozzi, I. E. Papadakis, D. Grupe, W. P. Brinkmann, C. Raeth and L. Kedziora-Chudczer, Astrophys. J. 717, 1243 (2010) [arXiv:1005.4933 [astro-ph.HE]].
[20] E. Kara, J. A. Garcia, A. Lohfink, A. C. Fabian, C. S. Reynolds, F. Tombesi and D. R. Wilkins, Mon. Not. Roy. Astron. Soc. 468, 3489 (2017) [arXiv:1703.09815 [astro-ph.HE]].
[21] M. Abramowicz, M. Jaroszynski and M. Sikora, Astron. Astrophys. 63, 221 (1978).
[22] J. A. Font and F. Daigne, Mon. Not. Roy. Astron. Soc. 334, 383 (2002) [astro-ph/0203403].
[23] J. M. Bardeen, W. H. Press and S. A. Teukolsky, Astrophys. J. 178, 347 (1972).
[24] Z. Li and C. Bambi, JCAP 1303, 031 (2013) [arXiv:1212.5848 [gr-qc]].
[25] S. M. Wu and T. G. Wang, Mon. Not. Roy. Astron. Soc. 378, 841 (2007) [arXiv:0705.1796 [astro-ph]].
[26] C. Taylor and C. S. Reynolds, Astrophys. J. 855, 120 (2018) [arXiv:1712.05418 [astro-ph.HE]].
[27] S. Riaz, D. Ayzenberg, C. Bambi and S. Nampalliwar, Mon. Not. Roy. Astron. Soc. (in press) [arXiv:1908.04969 [astro-ph.HE]].
[28] J. Garcia and T. Kallman, Astrophys. J. 718, 695 (2010) [arXiv:1006.0485 [astro-ph.HE]].
[29] J. Garcia, T. Dauser, C. S. Reynolds, T. R. Kallman, J. E. McClintock, J. Wilms and W. Eikmann, Astrophys. J. 768, 146 (2013) [arXiv:1303.2112 [astro-ph.HE]].
[30] K. C. Gentadre et al., Proc. SPIE Int. Soc. Opt. Eng. 9905, 99051H (2016).
[31] K. A. Arnaud, Astronomical Data Analysis Software and Systems V, 101, 17 (1996).
[32] J. Wilms, A. Allen and R. McCray, Astrophys. J. 542, 914 (2000) [astro-ph/0008425].
[33] C. Bambi, Annalen Phys. 530, 1700430 (2018) [arXiv:1711.10256 [gr-qc]].
[34] T. Dauser, J. Garcia, J. Wilms, M. Bock, L. W. Brenneman, M. Falanga, K. Fukumura and C. S. Reynolds, Mon. Not. Roy. Astron. Soc. 430, 1694 (2013) [arXiv:1301.4922 [astro-ph.HE]].
[35] J. Garcia et al., Astrophys. J. 782, 76 (2014) [arXiv:1312.3231 [astro-ph.HE]].
[36] A. Zoghbi, A. Fabian, P. Uttley, G. Miniutti, L. Gallo, C. Reynolds, J. Miller and G. Ponti, Mon. Not. Roy.
SUPPLEMENTAL MATERIAL

The available relativistic reflection models employ the Novikov-Thorne accretion disk model, which completely ignores the gas pressure and thus describes geometrically thin and optically thick disks [15, 16]. Moreover, the disk is approximated as infinitesimally thin, so the particles of the gas follow nearly geodesic circular orbits in the equatorial plane. However, in reality, as the mass accretion rate increases, the gas pressure becomes more and more important, and the thickness of the disk increases. The Polish donut model takes the gas pressure into account and can describe geometrically and optically thick accretion disks, which form when the mass accretion rate is near or above the Eddington limit of the source [21, 24]. Fig. 3 shows the shape of the Polish donut disks employed in the simulations for \( a_*=0.95 \), but different spins have quite similar disks.

For the study presented in the Letter, we simulate four sets of observations.

1. Simulations A: the observer viewing angle is \( i=20^{\circ} \) and the outer edge of the Polish donut disk is \( R_{\text{out}}=20 \, M \).
2. Simulations B: the observer viewing angle is \( i=20^{\circ} \) and the outer edge of the Polish donut disk is \( R_{\text{out}}=40 \, M \).
3. Simulations C: the observer viewing angle is \( i=35^{\circ} \) and the outer edge of the Polish donut disk is \( R_{\text{out}}=20 \, M \).
4. Simulations D: the observer viewing angle is \( i=35^{\circ} \) and the outer edge of the Polish donut disk is \( R_{\text{out}}=40 \, M \).

For every set, we consider seven input spin parameters: \( a_*=0.4, 0.6, 0.8, 0.85, 0.9, 0.95, 0.98 \). In all simulations, the intensity profile of the reflection spectrum is described by a power-law with emissivity index \( q=9 \), the ionization parameter is set to \( \log \xi=3.1 \) (\( \xi \) in units erg cm s\(^{-1} \)), iron abundance is assumed to have the Solar value (\( A_{Fe}=1 \)), the coronal spectrum is described by a power-law component with photon index \( \Gamma=2 \) and cutoff energy \( E_{\text{cut}}=300 \, \text{keV} \).

The simulated data are fitted with the XSPEC model TBABS×RELXILL, where the column density in TBABS and the cutoff energy in RELXILL are frozen to their input value. The former can be inferred from other observations, while the latter cannot be constrained and does not play a significant role for data up to 12 keV, as it is in the case of NICER. The reflection fraction, \( R_f \), is left free in the fit, so RELXILL is used to describe both the power-law component from the corona and the reflection spectrum from the disk.

The best-fit values of our fits are shown in Tab. I (simulations A1-A7), Tab. II (simulations B1-B7), Tab. III (simulations C1-C7), and Tab. IV (simulations D1-D7). The ratios between simulated data and best-fit models are shown in Fig. 4, and we see that there are no unresolved features, namely the theoretical model can provide a good fit. Fig. 5 shows the capability of our model to recover the input inclination angle and Fig. 6 shows the capability of recovering the emissivity index of the intensity profile of the reflection component. The input value of most model

![Image](image.png)

FIG. 3. Shape of the Polish donut disks for \( R_{\text{out}}=20 \, M \) (red solid curve) and \( 40 \, M \) (green dashed curve) when the spin parameter is \( a_*=0.95 \).
TABLE I. Best-fit values for simulations A1-A7. In all these simulations, the disk inclination angle $i = 20^\circ$ and the outer radius of the Polish donut disk is $R_{\text{out}} = 20 M$. The reported uncertainties correspond to 90% confidence level for one relevant parameter. * indicates that the parameter is frozen in the fit.

| Simulation A1 | Simulation A2 | Simulation A3 | Simulation A4 |
|---------------|---------------|---------------|---------------|
| $N_{\text{H}}/10^{20}$ cm$^{-2}$ | $6.74$ | $6.74^*$ | $6.74$ | $6.74^*$ | $6.74$ | $6.74^*$ |

| RELXILL | | | | |
| $q$ | $9.00^{+0.20}_{-0.20}$ | $9.00^{+0.20}_{-0.20}$ | $9.00^{+0.20}_{-0.20}$ | $9.00^{+0.20}_{-0.20}$ |
| $i$ [deg] | $20^{+2.1}_{-2.1}$ | $20^{+2.1}_{-2.1}$ | $20^{+2.1}_{-2.1}$ | $20^{+2.1}_{-2.1}$ |
| $a_*$ | $0.40^{+0.007}_{-0.007}$ | $0.40^{+0.007}_{-0.007}$ | $0.40^{+0.007}_{-0.007}$ | $0.40^{+0.007}_{-0.007}$ |
| log $\xi$ | $3.1^{+0.009}_{-0.009}$ | $3.1^{+0.009}_{-0.009}$ | $3.1^{+0.009}_{-0.009}$ | $3.1^{+0.009}_{-0.009}$ |
| $A_{\text{Fe}}$ | $1.00^{+0.10}_{-0.10}$ | $1.00^{+0.10}_{-0.10}$ | $1.00^{+0.10}_{-0.10}$ | $1.00^{+0.10}_{-0.10}$ |
| $E_{\text{cut}}$ [keV] | $300$ | $300^*$ | $300$ | $300^*$ |
| $R_{t}$ | $-1.0^{+0.003}_{-0.003}$ | $-1.0^{+0.003}_{-0.003}$ | $-1.0^{+0.003}_{-0.003}$ | $-1.0^{+0.003}_{-0.003}$ |
| $\chi^2/\nu$ | $1244.57/1171$ | $1196.93/1171$ | $1219.05/1171$ | $1140.97/1171$ |

| Simulation A5 | Simulation A6 | Simulation A7 |
|---------------|---------------|---------------|
| $N_{\text{H}}/10^{20}$ cm$^{-2}$ | $6.74$ | $6.74^*$ | $6.74$ | $6.74^*$ | $6.74$ | $6.74^*$ |

| RELXILL | | | | |
| $q$ | $9.00^{+0.20}_{-0.20}$ | $9.00^{+0.20}_{-0.20}$ | $9.00^{+0.20}_{-0.20}$ | $9.00^{+0.20}_{-0.20}$ |
| $i$ [deg] | $20^{+2.1}_{-2.1}$ | $20^{+2.1}_{-2.1}$ | $20^{+2.1}_{-2.1}$ | $20^{+2.1}_{-2.1}$ |
| $a_*$ | $0.90^{+0.004}_{-0.004}$ | $0.90^{+0.004}_{-0.004}$ | $0.90^{+0.004}_{-0.004}$ | $0.90^{+0.004}_{-0.004}$ |
| log $\xi$ | $3.1^{+0.009}_{-0.009}$ | $3.1^{+0.009}_{-0.009}$ | $3.1^{+0.009}_{-0.009}$ | $3.1^{+0.009}_{-0.009}$ |
| $A_{\text{Fe}}$ | $1.00^{+0.013}_{-0.013}$ | $1.00^{+0.013}_{-0.013}$ | $1.00^{+0.013}_{-0.013}$ | $1.00^{+0.013}_{-0.013}$ |
| $E_{\text{cut}}$ [keV] | $300$ | $300^*$ | $300$ | $300^*$ |
| $R_{t}$ | $-1.0^{+0.003}_{-0.003}$ | $-1.0^{+0.003}_{-0.003}$ | $-1.0^{+0.003}_{-0.003}$ | $-1.0^{+0.003}_{-0.003}$ |
| $\chi^2/\nu$ | $1176.69/1171$ | $1188.97/1171$ | $1179.94/1171$ | $1179.94/1171$ |

Parameters are recovered pretty well, but there are a few exceptions. The emissivity index $q$ is always underestimated. The spin parameter, which is the topic of our Letter, can be significantly overestimated. It is remarkable that the fits are all very good, with the reduced $\chi^2$ close to 1. This means that the quality of the fit cannot tell us that our disk model is incorrect. We fit the data of a thick disk with a thin disk model and we get a good fit but an incorrect estimate of the spin parameter. This is the take away message of our work.

Lastly, we have run Markov chain Monte Carlo (MCMC) simulations for two different spin parameters: $a_*=0.6$ (simulations A2, B2, C2, and D2) and $a_*=0.95$ (simulations A6, B6, C6, and D6). The resulting corner plots are reported, respectively, in Fig. 7 and Fig. 8, where we can see that there is a correlation between the estimates of the inclination angle and of the emissivity index as well as between the estimates of the inclination angle and the spin parameter, while there is no correlation between the estimate of the spin parameter and the emissivity index.
The radius of the Polish donut disk is \( R \) parameter.

| TABLE II. Best-fit values for simulations B1-B7. In all these simulations, the disk inclination angle \( i = 20^\circ \) and the outer radius of the Polish donut disk is \( R_{\text{out}} = 40 \, M \). The reported uncertainties correspond to 90\% confidence level for one relevant parameter. * indicates that the parameter is frozen in the fit. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | Simulation B1    | Simulation B2    | Simulation B3    | Simulation B4    |
|                 | Input           | Fit             | Input           | Fit             | Input           | Fit             |
| \( N_H / 10^{20} \, \text{cm}^{-2} \) | 6.74            | 6.74*           | 6.74            | 6.74*           | 6.74            | 6.74*           |
| \( \chi^2/\nu \) | 1213.41/1171    | 1231.81/1171    | 1197.38/1171    | 1090.89/1171    |
|                 | =1.03622        | =1.05193        | =1.02253        | =0.931588       |

| Table entries |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| RELXILL         |                 |                 |                 |                 |
| \( q \)         | 9               | 6.16^{+0.19}_{-0.21} | 9               | 6.5^{+0.2}_{-0.2} | 9               | 6.58^{+0.21}_{-0.15} |
| \( i \) [deg]   | 20              | 21.2^{+0.8}_{-0.9}    | 20              | 20^{+1}_{-1}    | 20              | 19.4^{+2.5}_{-1.0} |
| \( a_r \)       | 0.40            | 0.823^{+0.008}_{-0.007} | 0.60            | 0.888^{+0.006}_{-0.005} | 0.80           | 0.944^{+0.005}_{-0.004} |
| \( \log \xi \)  | 3.1             | 3.082^{+0.008}_{-0.007} | 3.1             | 3.073^{+0.008}_{-0.008} | 3.1             | 3.084^{+0.008}_{-0.008} |
| \( A_Fe \)      | 1               | 0.972^{+0.013}_{-0.010} | 1               | 0.963^{+0.012}_{-0.010} | 1               | 0.976^{+0.012}_{-0.012} |
| \( \Gamma \)    | 2               | 1.998^{+0.002}_{-0.002} | 2               | 2.000^{+0.002}_{-0.002} | 2               | 1.998^{+0.003}_{-0.002} |
| \( E_{\text{cut}} \) [keV] | 300            | 300*             | 300*            | 300*            | 300*            | 300*            |
|                 | -1.0            | 0.692^{+0.027}_{-0.020} | -1.0            | 0.79^{+0.03}_{-0.03} | -1.0            | 0.86^{+0.04}_{-0.04} |
|                 | 1275.99/1171    | 1205.22/1171    | 1189.23/1171    |
|                 | =1.08966        | =1.02922        | =1.01556        |
| Simulation C1 | Simulation C2 | Simulation C3 | Simulation C4 |
|--------------|--------------|--------------|--------------|
| \( N_H/10^{20} \text{ cm}^{-2} \) | 6.74 | 6.74* | 6.74 | 6.74* |
| RELXILL | | | | |
| \( q \) | 9 | 8.0^{+0.3}_{-0.3} | 9 | 7.4^{+0.3}_{-0.3} |
| \( i \) [deg] | 35 | 36.9^{+1.3}_{-1.3} | 35 | 33.8^{+1.8}_{-1.9} |
| \( a_* \) | 0.40 | 0.792^{+0.021}_{-0.023} | 0.60 | 0.849^{+0.019}_{-0.022} |
| \( \log \xi \) | 3.1 | 3.081^{+0.010}_{-0.011} | 3.1 | 3.075^{+0.013}_{-0.012} |
| \( A_{Fe} \) | 1 | 0.974^{+0.018}_{-0.016} | 1 | 0.968^{+0.023}_{-0.016} |
| \( \Gamma \) | 2 | 1.996^{+0.002}_{-0.002} | 2 | 1.998^{+0.002}_{-0.002} |
| \( E_{\text{cut}} \) [keV] | 300 | 300* | 300 | 300* |
| \( R_t \) | -1.0 | 0.436^{+0.13}_{-0.013} | -1.0 | 0.456^{+0.020}_{-0.018} |
| \( \chi^2/\nu \) | 1185.89/1171 | 1127.37/1171 | 1129.73/1171 | 1128.35/1171 |

| Simulation C5 | Simulation C6 | Simulation C7 |
|--------------|--------------|--------------|
| \( N_H/10^{20} \text{ cm}^{-2} \) | 6.74 | 6.74* | 6.74 | 6.74* |
| RELXILL | | | | |
| \( q \) | 9 | 5.64^{+0.23}_{-0.18} | 9 | 6.2^{+0.4}_{-0.2} |
| \( i \) [deg] | 35 | 21^{+4}_{-3} | 35 | 25^{+4}_{-3} |
| \( a_* \) | 0.90 | 0.956^{+0.007}_{-0.005} | 0.95 | 0.972^{+0.007}_{-0.005} |
| \( \log \xi \) | 3.1 | 3.079^{+0.012}_{-0.015} | 3.1 | 3.069^{+0.011}_{-0.012} |
| \( A_{Fe} \) | 1 | 0.993^{+0.063}_{-0.016} | 1 | 0.979^{+0.017}_{-0.017} |
| \( \Gamma \) | 2 | 2.000^{+0.003}_{-0.003} | 2 | 2.003^{+0.003}_{-0.003} |
| \( E_{\text{cut}} \) [keV] | 300 | 300* | 300 | 300* |
| \( R_t \) | -1.0 | 0.56^{+0.03}_{-0.03} | -1.0 | 0.58^{+0.05}_{-0.03} |
| \( \chi^2/\nu \) | 1216.36/1171 | 1178.76/1171 | 1111.73/1171 |

**TABLE III.** Best-fit values for simulations C1-C7. In all these simulations, the disk inclination angle \( i = 35^\circ \) and the outer radius of the Polish donut disk is \( R_{\text{out}} = 20 \ M \). The reported uncertainties correspond to 90\% confidence level for one relevant parameter. * indicates that the parameter is frozen in the fit.
TABLE IV. Best-fit values for simulations D1-D7. In all these simulations, the disk inclination angle $i = 35^\circ$ and the outer radius of the Polish donut disk is $R_{\text{out}} = 40 \, M$. The reported uncertainties correspond to 90% confidence level for one relevant parameter. $^\ast$ indicates that the parameter is frozen in the fit.

| Simulation D1 | Simulation D2 | Simulation D3 | Simulation D4 |
|---------------|---------------|---------------|---------------|
| $N_{\text{H}}/10^{20} \, \text{cm}^{-2}$ | 6.74 | 6.74$^\ast$ | 6.74 | 6.74$^\ast$ | 6.74 | 6.74$^\ast$ |
| RELXILL      |               |               |               |               |               |               |
| $q$           | 9             | 7.6$^{+0.3}_{-0.3}$ | 9             | 7.3$^{+0.4}_{-0.4}$ | 9             | 6.6$^{+0.5}_{-0.4}$ | 9             | 7.2$^{+0.3}_{-0.5}$ |
| $i \, [\text{deg}]$ | 35            | 37.4$^{+1.7}_{-1.7}$ | 35            | 36.7$^{+2.5}_{-2.9}$ | 35            | 35$^{+4}_{-4}$ | 35            | 39$^{+4}_{-5}$ |
| $a_\ast$      | 0.40          | 0.835$^{+0.20}_{-0.023}$ | 0.60          | 0.900$^{+0.016}_{-0.022}$ | 0.80          | 0.963$^{+0.009}_{-0.009}$ | 0.85          | 0.982$^{+0.005}_{-0.007}$ |
| log $\xi$     | 3.1           | 3.092$^{+0.001}_{-0.011}$ | 3.1           | 3.080$^{+0.012}_{-0.002}$ | 3.1           | 3.081$^{+0.012}_{-0.012}$ | 3.1           | 3.085$^{+0.008}_{-0.008}$ |
| $A_{Fe}$      | 1             | 0.991$^{+0.054}_{-0.017}$ | 1             | 1.001$^{+0.071}_{-0.019}$ | 1             | 0.991$^{+0.043}_{-0.017}$ | 1             | 0.995$^{+0.027}_{-0.017}$ |
| $\Gamma$      | 2             | 1.995$^{+0.002}_{-0.003}$ | 2             | 1.996$^{+0.002}_{-0.003}$ | 2             | 1.998$^{+0.003}_{-0.003}$ | 2             | 1.997$^{+0.002}_{-0.003}$ |
| $E_{\text{cut}} \, [\text{keV}]$ | 300           | 300$^\ast$ | 300           | 300$^\ast$ | 300           | 300$^\ast$ | 300           | 300$^\ast$ |
| $R_t$         | $-1.0$        | 0.459$^{+0.022}_{-0.016}$ | $-1.0$        | 0.496$^{+0.025}_{-0.023}$ | $-1.0$        | 0.56$^{+0.03}_{-0.03}$ | $-1.0$        | 0.62$^{+0.03}_{-0.02}$ |
| $\chi^2/\nu$  | 1199.27/1171  | 1204.24/1171 | 1156.41/1171 | 1221.69/1171 |
|               | =1.02414      | =1.02839     | =0.987538     | =1.04329     |

| Simulation D5 | Simulation D6 | Simulation D7 |
|---------------|---------------|---------------|
| $N_{\text{H}}/10^{20} \, \text{cm}^{-2}$ | 6.74 | 6.74$^\ast$ | 6.74 | 6.74$^\ast$ | 6.74 | 6.74$^\ast$ |
| RELXILL      |               |               |               |               |               |               |
| $q$           | 9             | 7.5$^{+0.5}_{-0.5}$ | 9             | 9.7$^{+0.5}_{-0.5}$ | 9             | 9.6$^{+1.3}_{-1.3}$ |
| $i \, [\text{deg}]$ | 35            | 42$^{+4}_{-4}$ | 35            | 53.0$^{+1.4}_{-2.1}$ | 35            | 52.5$^{+1.8}_{-6.4}$ |
| $a_\ast$      | 0.90          | 0.992$^{+0.003}_{-0.005}$ | 0.95          | 0.9980$^{+0.003}_{-0.005}$ | 0.98          | 0.9980$^{+0.003}_{-0.005}$ |
| log $\xi$     | 3.1           | 3.076$^{+0.012}_{-0.013}$ | 3.1           | 3.091$^{+0.013}_{-0.012}$ | 3.1           | 3.072$^{+0.014}_{-0.014}$ |
| $A_{Fe}$      | 1             | 0.980$^{+0.031}_{-0.020}$ | 1             | 0.971$^{+0.020}_{-0.024}$ | 1             | 0.980$^{+0.031}_{-0.026}$ |
| $\Gamma$      | 2             | 2.000$^{+0.005}_{-0.003}$ | 2             | 1.995$^{+0.003}_{-0.004}$ | 2             | 2.001$^{+0.007}_{-0.004}$ |
| $E_{\text{cut}} \, [\text{keV}]$ | 300           | 300$^\ast$ | 300           | 300$^\ast$ | 300           | 300$^\ast$ |
| $R_t$         | $-1.0$        | 0.66$^{+0.04}_{-0.07}$ | $-1.0$        | 0.767$^{+0.042}_{-0.022}$ | $-1.0$        | 0.70$^{+0.04}_{-0.05}$ |
| $\chi^2/\nu$  | 1169.03/1171  | 1145.59/1171 | 1075.16/1171 | 1075.16/1171 |
|               | =0.998319     | =0.978301     | =0.918156     | =0.918156     |
FIG. 4. Data to best-fit model ratios for simulations A1-A7 (top left panel), B1-B7 (top right panel), C1-C7 (bottom left panel), and D1-D7 (bottom right panel).
FIG. 5. Input spin parameter of the simulations vs best-fit inclination angle obtained from RELXILL. The error bars show the fit uncertainties. The horizontal black dashed and blue dotted lines mark the input inclination angles of the simulations, respectively $i = 35^\circ$ and $i = 20^\circ$.

FIG. 6. Input spin parameter of the simulations vs best-fit emissivity index obtained from RELXILL. The error bars show the fit uncertainties. The horizontal blue dotted line marks the input emissivity index of the simulations.
FIG. 7. Corner plots for the emissivity index $q$, the spin parameter $a_*$, and the inclination angle after the MCMC run when the input spin parameter is $a_* = 0.60$. The input inclination angle is $i = 20^\circ$ in the top panels and $i = 35^\circ$ in the bottom panels. The outer radius of the Polish donut disk is 20 gravitational radii in the left panels and 40 gravitational radii in the right panels.
FIG. 8. Corner plots for the emissivity index $q$, the spin parameter $a_*$, and the inclination angle after the MCMC run when the input spin parameter is $a_* = 0.95$. The input inclination angle is $i = 20^\circ$ in the top panels and $i = 35^\circ$ in the bottom panels. The outer radius of the Polish donut disk is 20 gravitational radii in the left panels and 40 gravitational radii in the right panels.