Calculation of the Hyperfine Magnetic Anomaly in Many-Electron Atoms

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Abstract—The precision measurements of the ratio between the hyperfine structure constants for the \( s_{1/2} \) and \( p_{1/2} \) states has allowed us to estimate the difference between the hyperfine magnetic anomalies for these levels. This work deals with the retrieval of the absolute values of the hyperfine magnetic anomalies from their difference. Taking into account the correction for the hyperfine magnetic anomaly to the \( g \) factor has made it possible to increase the determination accuracy of \( g \) factors of short-lived isotopes by more than an order of magnitude.

Keywords: hyperfine magnetic anomaly, Bohr–Weisskopf effect, Breit–Rosenthal effect

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1. INTRODUCTION

In recent years, the accuracy achieved in experiments on laser spectroscopy along with the development of atomic theory has made it possible to proceed to the verification of various nuclear models [1, 2]. The magnetic dipole hyperfine structure (HFS) constants \( A \) depend on the distributions of charge and magnetization within the atomic nucleus. In the point-like nucleus approximation, the ratio of the HFS constants for two isotopes is equal to the ratio of their \( g \) factors, \( g_L = \frac{\mu}{\mu_N I} \), where \( \mu \) and \( I \) are the magnetic moment and spin of the nucleus, respectively; \( \mu_N \) is the nuclear magneton. When considering nuclei of finite sizes, it is necessary to take into account the distribution of the magnetization within a nucleus and the behavior of the electron wave function inside the nuclear radius. The first correction is called the Bohr–Weisskopf magnetic correction [3], while the second one is termed the Breit–Rosenthal charge correction [4, 5]. These corrections disturb the proportionality between HFS constants and nuclear \( g \) factors, which is called the hyperfine magnetic anomaly (HFA):

\[
\Delta^2 \equiv \frac{A^{(1)} g_L^{(2)}}{A^{(2)} g_L^{(1)}} - 1,
\]

where \( A^{(1)} \) and \( A^{(2)} \) are the HFS constants of two isotopes, and \( g_L^{(1)} \) and \( g_L^{(2)} \) are the corresponding \( g \) factors.

When retrieving nuclear \( g \) factors from experimental data, the HFA correction is often neglected due to the lack of theoretical estimates for its value. At the same time, the correction for the HFA is usually small (smaller than 1%) [6], and this level of accuracy was reached only in recent measurements for short-lived nuclei. In addition, studies of HFS constants can serve as a useful tool for understanding the phenomenon of coexistence of different shapes in atomic nuclei [7]. Thus, the development of new methods for atomic calculations of HFS constants taking into account HFA becomes relevant and timely.

2. FINITE NUCLEAR SIZE CORRECTIONS TO HFS CONSTANTS

The observed hyperfine structure constants \( A \) can be written in the following form [8]:

\[
A = g_L \delta_A g_0 (1 - \delta)(1 - \epsilon).
\] (2)

Here, \( g_L \) is the nuclear \( g \) factor, \( g_0 \delta_A \) is the hyperfine structure constant in the case of a point-like nucleus, and \( \delta \) and \( \epsilon \) are the dimensionless corrections for the distributions of the charge (BR) and magnetization (BW) in the nucleus. Constant \( \delta_A \) does not depend on the nuclear \( g \) factor.
For hydrogen-like ions, the expression for \( A_0 \) is known in the analytical form [9, 10]:

\[
A_0 = \frac{\alpha(\alpha Z)^3 m}{\sqrt{j(j+1) m_p}} \frac{k(2\kappa n + \gamma - N)}{N^{3/2}(4\gamma^2 - 1)} mc^2. \tag{3}
\]

Here, \( \alpha \) is the fine structure constant; \( Z \) is the nuclear charge, \( m \) and \( m_p \) are the electron and proton masses, respectively; \( j \) is the total electron momentum; \( \kappa = (l-j)(2j+1) \) is the relativistic quantum number; \( N = \sqrt{n + \gamma^2 + (\alpha Z)^2} \) is the effective principal quantum number; \( n = n - \kappa / 2 \) is the radial quantum number; \( \gamma \) is the nonrelativistic principal quantum number; and \( \gamma \) is the total electron momentum. The nuclear factor is defined such that \( A_0 \) for states with \( j = 1/2 \) turns to infinity at \( \gamma = \frac{1}{2} \) or \( Z = 118 \).

For the same \( n \), the ratios \( A_{0,\text{nuc}} / A_{0,\text{p1/2}} \) and \( A_{0,\text{p1/2}} / A_{0,\text{nuc}} \) in Eqs. (2) and (5) are given by

\[
\frac{A_{0,\text{nuc}}}{A_{0,\text{p1/2}}} = 3 + 2 \left( \frac{\alpha Z}{n - 1 + \gamma / 2} \right)^2.
\]
\[
\frac{A_{0,\text{p1/2}}}{A_{0,\text{nuc}}} = \gamma / 3 \left( \frac{4\gamma^2 - 1}{2\gamma / 2} \right) \left( 1 + 3(\alpha Z)^2 \frac{5n - 4}{20n^2} \right). \tag{4}
\]

Here, we took into account that the values of \( \gamma \) are the same for the \( s_1/2 \) and \( p_{1/2} \) states and different for the \( p_{3/2} \) states. Equation (4) is consistent with the results obtained for many-electron systems in the semiclassical approximation [11].

We will consider the nucleus as a homogeneously charged and magnetized ball of the radius \( R = \left( \frac{5}{3} \langle r^2 \rangle \right)^{1/2} \), where \( \langle r^2 \rangle = \langle r \rangle^{3/2} \) is the root-mean-square radius of the nucleus. The distribution of the magnetization is determined by the spin polarization of nucleons and the orbital motion of protons.

The charge density inside the nucleus is relatively stable for different isotopes [12], whereas the magnetization distribution depends strongly on the nuclear spin and the nucleon configuration of each isotope. In order to parameterize these effects, following [13, 14], we used nuclear factor \( d_{\text{nuc}} \). Then, corrections \( \delta \) and \( \epsilon \) for selected \( Z \) and electronic state can be written as [15]

\[
\delta(R) = b_N(R / \hbar_c)^{2\gamma - 1},
\]
\[
\epsilon(R, d_{\text{nuc}}) = b_M d_{\text{nuc}}(R / \hbar_c)^{2\gamma - 1}. \tag{5}
\]

Here, \( b_N \) and \( b_M \) are the dimensionless parameters, which do not depend on the nucleus radius and the nucleus structure and which can be found numerically; and \( \hbar_c \) is the Compton electron wavelength \( \left( \hbar_c = \frac{\hbar}{mc} \right) \). The nuclear factor is defined such that \( d_{\text{nuc}} = 0 \) would correspond to a point-like magnetic dipole in the center of the nucleus, while \( d_{\text{nuc}} = 1 \) would correspond to a homogeneously magnetized ball of radius \( R \).

The parameterization of HFS constants by Eqs. (2) and (5) involves three nuclear factors \( d_{\text{nuc}}, b_N, \) and \( b_M \) and three atomic factors \( g_I, d_{\text{nuc}}, \) and \( R \) and three atomic factors \( \Delta_{\text{BR}}, \Delta_{\text{BW}}, \) and \( \Delta_{\text{FW}} \) characteristics. The atomic parameters are the same for different isotopes. Information about a nucleus can be extracted from experimentally measured HFS constants if the atomic parameters were calculated. In an isotopic series, the radius of the nucleus, \( R \), commonly varies smoothly and only by a few percent; therefore, correction \( \delta \) is practically the same for all isotopes of a selected atom.

To calculate the HFS constants, it is necessary to specify the nuclear parameters:

\[
A(g_I, d_{\text{nuc}}, R) = g_I A_0 (1 - b_N (R / \hbar_c)^{2\gamma - 1}) \times (1 - d_{\text{nuc}} b_M (R / \hbar_c)^{2\gamma - 1}). \tag{6}
\]

In order to find the atomic parameters \( A_0, b_N, \) and \( b_M \) from Eqs. (2) and (5), it is enough to vary the values of \( d_{\text{nuc}} \) and \( R \), taking \( g_I \) to be unity. By changing only the nuclear factor, we can find parameter \( b_M \). To do this, it is necessary to calculate the HFS constants in terms of two models of the nucleus: the model of a point-like magnetic dipole \( (d_{\text{nuc}} = 0) \) and that of a homogeneously magnetized ball \( (d_{\text{nuc}} = 1) \) at a fixed radius of the nucleus; then,

\[
b_M = (R / \hbar_c)^{1 - 2\gamma} \left( 1 - \frac{A(g_I, 1, R)}{A(g_I, 0, R)} \right). \tag{7}
\]

In order to find parameter \( b_N \), it is necessary to perform calculations for different nuclear radii \( R \),

\[
b_N = \frac{(A(g_I, 0, R_0) - A(g_I, 0, R_1)) \hbar_c^{2\gamma - 1}}{A(g_I, 0, R_0) \hbar_c^{2\gamma - 1} - A(g_I, 0, R_1) \hbar_c^{2\gamma - 1}}. \tag{8}
\]

After that, the third parameter can be found:

\[
A_0 = \frac{A(g_I, 0, R)}{g_I (1 - b_N (R / \hbar_c)^{2\gamma - 1})}. \tag{9}
\]

3. HYPERFINE MAGNETIC ANOMALY

By comparing the hyperfine structure constants for two isotopes with nuclear \( g \) factors \( g_I^{(1)} \) and \( g_I^{(2)} \) and close nuclear radii \( R^{(1,2)} = R \pm \varepsilon \), we can calculate the HFA value \( \Delta^2 \) using Eq. (1). In HFA \( \Delta^2 \), the contributions associated with a change in the charge distribution \( \Delta_{\text{BR}} \), and magnetization \( \Delta_{\text{BW}} \) within the nucleus can be singled out: \( \Delta^2 = \Delta_{\text{BR}}^2 + \Delta_{\text{BW}}^2 \).
Assuming the nuclear factors of the two isotopes to be \( d^{(1)}_{\text{nuc}} = d^{(2)}_{\text{nuc}} = 0 \), we obtain
\[
\frac{A(g^{(1)}_l,0,R + \gamma)}{A(g^{(2)}_l,0,R - \gamma)} = \frac{g^{(1)}_l + 2\gamma \frac{2}{A(g^{(1)}_l,0,R)} / \partial R}{A(g^{(2)}_l,0,R)}.
\] (10)

Then, the part of the HFA that is associated with a change in the charge distribution within the nucleus, \( \Delta^2_{\text{BR}}(R) \), is given by
\[
\Delta^2_{\text{BR}}(R,\gamma) = g^{(2)}_l A(g^{(2)}_l,0,R + \gamma) - 1
\]
\[
= -2(2\gamma - 1)b_n R^{2\gamma-2} / \lambda_C^{2\gamma-1}.
\] (11)

If the nuclear factors of two isotopes are equal \( d^{(1)}_{\text{nuc}} = d^{(2)}_{\text{nuc}} = d_{\text{nuc}} \) (the isotopes with the same spins and similar nuclear configurations), then a similar expression can be obtained for HFA \( \Delta^2 \):
\[
\Delta^2 = \frac{A(l,d_{\text{nuc}},R + \gamma)}{A(l,d_{\text{nuc}},R - \gamma)} - 1
\]
\[
= -2(2\gamma - 1)(b_n + d_{\text{nuc}} b_M) R^{2\gamma-2} / \lambda_C^{2\gamma-1}.
\] (12)

However, the nuclear factors of isotopes can differ quite strongly from each other, \( d^{(1)}_{\text{nuc}} \neq d^{(2)}_{\text{nuc}} \), then the difference between the radii of the isotopes can be neglected and the value of the HFA is determined by the formula
\[
\Delta^2 = (d^{(2)}_{\text{nuc}} - d^{(1)}_{\text{nuc}}) b_M \frac{R}{\lambda_C} = \epsilon_2 - \epsilon_1.
\] (13)

3.1. Nuclear Factor

In this section, we will discuss the properties of the nuclear factor \( d_{\text{nuc}} \) within the single-particle model of the nucleus. It was shown in [3, 16] that the Bohr–Weisskopf correction can be written in the following form:
\[
\epsilon = b_M(R/\lambda_C)^{2\gamma-1}(1 + \frac{2}{5} \zeta) \alpha_s + 3/5 \alpha_l \frac{R_M^2}{R^2},
\] (14)

where \( \zeta \) is the spin asymmetry parameter:
\[
\zeta = \begin{cases} 
\frac{2I - 1}{4(I + 1)} & \text{if } I = l + \frac{1}{2} \\
\frac{2I + 3}{4I} & \text{if } I = l - \frac{1}{2} 
\end{cases}
\] (15)

The coefficients \( \alpha_s \) and \( \alpha_l \) parameterize the spin and orbital contributions to the nuclear \( g \) factor: \( g_l = \alpha_s g_S + \alpha_l g_L \); then
\[
\alpha_s = \frac{g_S g_L - g_L^2}{g_l g_S - g_L}, \quad \alpha_l = 1 - \alpha_s.
\] (16)

The value \( R_M \) in (14) is the radius of the nuclear magnetization density distribution.

Thus, the following expression can be obtained for the nuclear factor \( d_{\text{nuc}} \):
\[
d_{\text{nuc}} = \left( 1 + 2 \frac{2}{5} \zeta + 3 \frac{2}{5} \alpha_l \frac{R_M^2}{R^2} \right).\] (17)

The nuclear factor depends on the configuration of nucleons and can vary significantly upon passage from one isotope to another. At the same time, \( d_{\text{nuc}} \) depends weakly on the nuclear charge \( Z \).

The simplest case, in which \( d_{\text{nuc}} = 1 \), is realized when the nuclear configuration is represented by only one valence proton in the \( s_{1/2} \) state; then \( I = 1/2, \zeta = 0, \alpha_s = 1, \) and \( R_M = R \).

For the alkali metal atoms, the contributions of electronic correlations to HFS constants can be taken into account highly accurately [17–20]; then the uncertainty in the value of the nuclear factor becomes the main reason for the error in the calculation of HFS constants. Further, we will present several estimates of the nuclear factor for the francium isotope \( ^{211}\text{Fr} \), whose nuclear configuration is represented by only one valence proton in the \( h_{9/2} \) state [1, 2].

(i) Using Eqs. (15)–(17), we can determine that \( \zeta = \frac{2}{3}, \alpha_s = -0.152, \) and \( \alpha_l = 1.152 \). Then the nuclear factor is \( d_{\text{nuc}} = 0.50 \), assuming that \( R_M = R, g_L = 1, \) and \( g_S = g_{\text{free}} = 5.586 \). For isotope \( ^{209}\text{Bi} \) with the same nuclear configuration, the nuclear factor is equal to 0.47 [10].

(ii) In [21], following the assumption by Grossman et al. [2], the parameters \( g_L = 1.16 \) and \( g_S = 0.85g_{\text{free}} \) were used, then \( d_{\text{nuc}} = 0.33 \).

(iii) The nuclear factor can be found from the ratio of the Bohr–Weisskopf corrections obtained either from the single-particle model of the nucleus or from the model of a homogeneously magnetized ball. In the latter model, the nuclear factor is 1.0. The nuclear factor obtained in this way is 0.54(21) [20].

(iv) The nuclear factor \( d_{\text{nuc}} = 0.85 \) can be obtained from the ratio of the nuclear magnetization distribution radius to the nucleus radius \( (R_M = 6.71 \text{ fm}) [17] \) and \( R = 7.28 \text{ fm} (65 \text{ fm}) [22] \) as \( d_{\text{nuc}} = (R_M/R)^2 \).

The presented examples demonstrate that the values of the nuclear factor and the Bohr–Weisskopf correction strongly depend on the used nuclear model [23, 24].

3.2. Differential Hyperfine Magnetic Anomaly

In some isotopic series, the ratios of HFS constants for low-lying atomic states \( \rho = A(s_{1/2})/A(p_{1/2}) \) were
measured with an accuracy that was sufficient to reliably retrieve the differential HFA:

\[ \Delta^2_{p_{1/2}} = \rho_{p_{1/2}}^{(0)} - 1 = \Delta^2(s_{1/2}) - \Delta^2(p_{1/2}). \]  (18)

The differential HFAs between isotopes with the same nuclear spins are very small (~10^{-3} for a neutral Fr atom [6]). Neglecting these small changes, we will monitor the redistribution of the nuclear magnetization (changes of nuclear factors) in the case of isotopes with different nuclear spins. Then the HFA is described by Eq. (13), and the ratio of the hyperfine magnetic anomalies

\[ \frac{1}{\Delta^2(s_{1/2})} = \frac{b_{s_{1/2}}}{b_{p_{1/2}}} \equiv \eta \]  (19)

is determined only by atomic parameters—by the atomic factor [25, 26].

Using the calculated value of \( \eta \) and assuming that the nuclear \( g \) factor \( g_{s_{1/2}} \) is known, we can retrieve the HFA for the \( s_{1/2} \) state and calculate the correction for the HFA to the \( g \) factor of the second isotope:

\[ \frac{1}{\Delta^2(s_{1/2})} = \frac{1}{1 - \Delta^2(p_{1/2})}, \]

\[ g_{s_{1/2}} = g_{s_{1/2}}^{(0)} \left( 1 - \frac{1}{\Delta^2(s_{1/2})} \right). \]  (20)

We can also find nuclear factor \( d_{\text{nuc}}^{(2)} \), if \( d_{\text{nuc}}^{(1)} \) is known and the differential HFA is measured:

\[ d_{\text{nuc}}^{(2)} = d_{\text{nuc}}^{(1)} + \frac{\Delta^2_{p_{1/2}}}{(1 - 1/\eta) b_{s_{1/2}}(s_{1/2}) (R/A_C)^2}. \]  (21)

4. RESULTS AND DISCUSSION

This section will be devoted to the study of the general properties of HFA. The dependence of the Bohr–Weisskopf correction on the nuclear magnetization distribution radius, \( R_M \), and the dependence of the ratio of the Bohr–Weisskopf corrections for the \( s_{1/2} \) and \( p_{1/2} \) states on the nuclear charge will be studied using hydrogen-like ions as an example.

HFS constants for hydrogen-like ions have been obtained both analytically and numerically, for which the software package [27] was used, which uses the HFD program [28] as an initial approximation. Within the one-electron approximation, an analytical expression for the atomic factor \( \eta \) for neutral atoms was obtained. The results of correlation calculations of the atomic factor for monovalent atoms known in the literature were consistent with the values obtained analytically.

4.1. Hydrogen-Like Ions

Figure 1 illustrates the dependence of the HFS constant \( A/g_{s_{1/2}} \) for the ground state of the Fr\(^{86+}\) isotope on the nuclear radius \( R \) and the nuclear magnetization density distribution radius \( R_M \). The parameters \( b_{s_{1/2}}(s_{1/2}) = 0.285(3) \) were found from (8) and (7). The parameter \( b_{p_{1/2}}(s_{1/2}) \), obtained from (9), agrees with the analytical value [10]: 398.4(6) and 397.7 THz, respectively. The approximations of the results of calculations of the HFS constants at different radii of the nucleus for \( d_{\text{nuc}} \) equal to 0 or 1, which are presented in the figure, were performed in accordance with (6) using the parameter found above. If we fix \( R = R_0 \) and vary \( R_M \) from \( R_0 \) to 0, then, in accordance with (17), the nuclear factor \( d_{\text{nuc}}(R_M) \) and the Bohr–Weisskopf correction will vary from their maximum values to zero proportionally to \( R_M^2 \).

Tables 1 and 2 present the results of calculations of the Bohr–Weisskopf (assuming that \( d_{\text{nuc}} = 1 \) and Breit–Rosenthal corrections of the \( 2s_{1/2} \) and \( 2p_{1/2} \) states of hydrogen-like ions for Z from 10 to 90. The calculated values of corrections \( \delta \) and of their
Table 1. Dependences of Bohr–Weisskopf corrections $\eta$ (5), (7) for the $2s_{1/2}$ and $2p_{1/2}$ states of hydrogen-like ions and of their ratio $\eta = \frac{\eta_1}{\eta_2}$ on nuclear charge $Z$ assuming that $d_{\text{nucl}} = 1$; radii $R$ of nuclei were taken from [29]

| $Z$ | $\eta_{(1, R)}(2s_{1/2})$, % | $\eta_{(1, R)}(2p_{1/2})$, % | $\eta$ |
|-----|----------------------|----------------------|------|
| 10  | 0.042                | 0.043                | 0.051 |
| 20  | 0.106                | 0.084                | 0.124 |
| 30  | 0.203                | 0.191                | 0.238 |
| 40  | 0.341                | 0.344                | 0.411 |
| 50  | 0.553                | 0.561                | 0.672 |
| 60  | 0.856                | 0.873                | 1.03  |
| 70  | 1.335                | 1.353                | 1.51  |
| 80  | 1.976                | 2.048                | 2.15  |
| 90  | 2.969                | 3.077                | 2.88  |

Ratios $\eta = \frac{\eta_1}{\eta_2}$ and $\eta_{BR} = \frac{\delta(\eta_1)}{\delta(\eta_2)}$ agree well with the results obtained analytically in [10]. Parameters $\eta$ and $\eta_{BR}$ rapidly decrease with increasing $Z$.

The dependences of the parameters $\eta$ for the $2s_{1/2}$, $2p_{1/2}$, and $2p_{3/2}$ states of hydrogen-like ions on the nuclear charge are presented in Table 3. The wave functions of the $p_{3/2}$ states turn to zero at $r=0$; therefore, the Bohr–Weisskopf and Breit–Rosenthal corrections for these states are zero. In this case, the HFS constants obtained from calculations for $g_f = 1$ are directly equal to $\eta_0$. For the $2s_{1/2}$ and $2p_{1/2}$ states, the parameters $\delta_{(0)}$ were obtained from (9). In this case, even at large $Z$, the parameters $\delta_{(0)}$ differ from the values obtained analytically using Eq. (3) by no more than 0.2%. The ratios $\delta_{(0),s_{1/2}}$ and $\delta_{(0),p_{1/2}}$ are described with high accuracy by Eq. (4).

4.2. Heavy Neutral Atoms

To calculate HFS constants, it is necessary to know the wave function of the outer electron at small distances, since the electron–nucleon interaction decreases rapidly with increasing distance between the electron and the nucleus [30, 31]. The Coulomb field of a nucleus at such short distances can be considered unscreened, and the radial wave functions of the $s_{1/2}$ and $p_{1/2}$ states are proportional to each other with the coefficient $Z\alpha\left(1 + \frac{Z^2\alpha^2}{4}\right)$ [32]. Using the proportionality between the radial wave functions and the estimate $\delta_{(0),s_{1/2}} = \frac{\alpha^2Z^2}{4}\left(1 + \frac{\alpha^2Z^2}{4}\right)^2$. Then the expression for $\eta$ has the form

$$\frac{1}{\eta} = \frac{3}{4}\alpha^2Z^2.$$

Taking into account the contribution of the Breit–Rosenthal corrections $k = 1 - \delta(\eta_1) + \delta(\eta_2)$ and next terms in $\alpha Z$ in expression $\delta_{(0),s_{1/2}} = \frac{\alpha^2Z^2}{4}\left(1 + \frac{\alpha^2Z^2}{4}\right)^2$, we obtain

$$\frac{1}{\eta} = \frac{3}{4}\alpha^2Z^2.$$

The dependences of parameter $k$ on $Z$ for hydrogen-like ions are presented in Table 2. The dependence of...
the atomic factor $\eta$ on the principal quantum number $n$ can be neglected.

A similar expression can be obtained for the ratio of Breit–Rosenthal corrections:

$$
\frac{1}{\eta_{\text{BR}}} = \frac{\mathcal{A}_{0,s_{1/2}} - A_{s_{1/2}}(I, 0, R)}{\mathcal{A}_{0,p_{1/2}} - A_{p_{1/2}}(I, 0, R)}.
$$

By virtue of the proportionality of the wave functions for the $s_{1/2}$ and $p_{1/2}$ states inside the nucleus, the second fractions in (22) and (25) are equal to each other, then:

$$
\frac{1}{\eta_{\text{BR}}} = k.
$$

The values of atomic factors $\eta$ obtained for neutral Au, Tl, and Fr atoms analytically from (23), (24) and from correlation calculations agree with each other (Table 4).

5. CONCLUSIONS

Using hydrogen–like ions as an example, we studied corrections to the hyperfine structure constants caused by the finiteness of the size of nuclei, namely, the Breit–Rosenthal charge correction and the Bohr–Weisskopf magnetic correction. These corrections disturb the proportionality between hyperfine structure constants and nuclear $g$ factors, which is called the hyperfine magnetic anomaly.

The distribution of the magnetization within the nucleus can vary strongly when going from one isotope to another. Therefore, magnetic corrections make main contributions to the HFA.

From the measured differential HFA values, the absolute HFA values can be retrieved by calculating atomic factor $\eta$—the ratio of the Bohr–Weisskopf corrections for the $s_{1/2}$ and $p_{1/2}$ states. In this work, the dependence of the atomic factor on the nuclear charge was studied for hydrogen–like ions and was generalized to the case of neutral atoms.

If the value of atomic factor $\eta$ was calculated and the differential anomaly was measured with sufficient accuracy, then the determination accuracy of $g$ factors of short-lived isotopes can be increased by more than an order of magnitude. Nuclear factor $d_{\text{nucl}}$ characterizes the valence configuration of nucleons. Its value can be obtained from the analysis of experimental data for complex nucleon configurations, which go outside the scope of the single-particle model of the nucleus, and it was used to verify various nuclear models.

Table 3. Dependences of parameters $\mathcal{A}$ (9) for the $2s_{1/2}$, $2p_{1/2}$, and $2p_{3/2}$ states of hydrogen-like ions in comparison with analytical values obtained using Eq. (3) [10]; last columns present numerical values of the ratios $\frac{\mathcal{A}_{0,s_{1/2}}}{\mathcal{A}_{0,p_{1/2}}}$ and $\frac{\mathcal{A}_{0,p_{1/2}}}{\mathcal{A}_{0,p_{3/2}}}$ in comparison with analytical results obtained using Eq. (4)

| $Z$ | $\mathcal{A}_{0,s_{1/2}}$, THz | $\mathcal{A}_{0,p_{1/2}}$, THz | $\mathcal{A}_{0,p_{3/2}}$, THz | $\frac{\mathcal{A}_{0,s_{1/2}}}{\mathcal{A}_{0,p_{1/2}}}$ | $\frac{\mathcal{A}_{0,p_{1/2}}}{\mathcal{A}_{0,p_{3/2}}}$ |
|-----|------------------|------------------|------------------|-----------------|-----------------|
| 10  | 0.032            | 0.032            | 0.0107           | 3.00            | 5.04            |
| 20  | 0.266            | 0.266            | 0.088            | 3.01            | 5.18            |
| 30  | 0.954            | 0.954            | 0.315            | 3.02            | 5.43            |
| 40  | 2.464            | 2.463            | 0.809            | 3.05            | 5.81            |
| 50  | 5.409            | 5.408            | 1.761            | 3.07            | 6.39            |
| 60  | 10.909           | 10.905           | 3.509            | 3.11            | 7.23            |
| 70  | 21.200           | 21.185           | 6.718            | 3.16            | 8.54            |
| 80  | 41.313           | 41.261           | 12.844           | 3.22            | 10.66           |
| 90  | 84.547           | 84.369           | 25.660           | 3.29            | 14.51           |

Table 4. Atomic factors $\eta$ for Au, Tl, and Fr atoms obtained from Eqs. (23) and (24); for hydrogen-like ions, $\eta = \frac{\epsilon(2s_{1/2})}{\epsilon(2p_{1/2})}$, while, for neutral atoms, dependences of $\eta$ on the principal quantum number can be neglected

| $\eta$   | Au  | Tl  | Fr  |
|----------|-----|-----|-----|
| (23)     | 4.01| 3.82| 3.31|
| (24)     | 3.69| 3.51| 3.07|
| Hydrogen-like ion | 4.10| 3.86| 3.26|
| Neutral atom | 3.3$^a$| 3.4(2)$^c$| 3.1(1)$^g$|
|          | 4.0(3)$^b$| 3.1$^d$| 3.36(5)$^b$|
|          |      | 2.6$^e$|      |
| Experiment |      | 2.84(78)$^f$|      |

$^a$[33]; $^b$[34]; $^c$[35]; $^d$[14]; $^e$[15]; $^f$[36]; $^g$[37]; $^h$[38].
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CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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