A New Rate Splitting Strategy for Uplink CR-NOMA Systems

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Abstract—In uplink non-orthogonal multiple access (NOMA) channels, the existing cooperative successive interference cancellation (SIC) and power control (PC) schemes lack the capability of achieving the full capacity region, which restricts the outage performance of uplink NOMA users. For the uplink cognitive radio inspired NOMA system, we propose a new rate splitting (RS) strategy to maximize the achievable rate of the secondary user without deteriorating the primary user’s outage performance. Based on the interference threshold and its own channel gain, the secondary user adaptively conducts RS, transmit power allocation and SIC, which utilizes the transmit power efficiently. The closed-form expression for the outage probability is derived for the secondary user. Numerical results show that the proposed RS scheme achieves the best outage performance for the secondary user among the existing cooperative SIC and PC schemes.

Index Terms—Rate splitting (RS), cognitive radio (CR), non-orthogonal multiple access (NOMA), outage probability.

I. INTRODUCTION

In power-domain non-orthogonal multiple access (NOMA) systems, multiple users’ signals are multiplexed on the same time/frequency resource block, which results in inter-user interference (IUI) at the receiver side. Due to the powerful capability of eliminating IUI and simple structure, successive interference cancellation (SIC) has been utilized to detect the multiplexing signals in NOMA systems [1]. Nevertheless, the decoding performance of a NOMA receiver strongly depends on how the SIC decoding order and the associated power control (PC) are optimized [2], [3].

Under the quality of service (QoS) oriented criterion, the users with more stringent QoS requirements are decoded first, while the others are decoded in the remaining stages of the SIC [2], [4]. Under the channel state information (CSI)-based criterion, the users with better CSI are firstly decoded before decoding the users with the worse CSI [1], [5]–[7]. When a fixed SIC decoding order is applied in the uplink NOMA systems, the users decoded in the primary stages of the SIC are inevitably affected by the IUI, which may result in severe outage probability error floor. Fortunately, the recently proposed QoS-guaranteed SIC (QoS-SIC) scheme in [2] and [3] avoids the outage probability error floor to some extent by adopting a hybrid decoding order, which takes into account both CSI and QoS requirements for calculating the users’ priorities in SIC. Furthermore, a new hybrid SIC (NH-SIC) scheme was proposed in [4], which introduced the PC method to increase the achievable rate.

However, the existing SIC and PC schemes cannot fully achieve the capacity region of the uplink multiple access channels (MACs) compared to the rate splitting (RS) [8], [9], hence only suboptimal outage performance can be achieved by the QoS-SIC and NH-SIC schemes. In this paper, we propose an RS strategy for the uplink cognitive radio (CR) inspired NOMA (CR-NOMA) system. Without deteriorating the outage performance of the primary user compared to that in orthogonal multiple access (OMA), the secondary user splits its signal into two streams and allocates the transmit power efficiently to each stream to maximize the achievable rate. Benefiting from advantage of the RS for achieving the full capacity region of MACs, our proposed RS scheme attains the best outage performance for the secondary user. To clarify the advantage of the proposed RS scheme, we carry out a comprehensive analysis and provide detailed comparisons over the existing SIC and PC schemes [3], [4].

II. SYSTEM MODEL AND RS STRATEGY

A. System Model

We consider an uplink CR-NOMA system consisting of a primary user $U_0$, a secondary user $U_1$, and a base-station (BS). Each node is equipped with a single antenna. The channel coefficients from $U_0$ and $U_1$ to the BS are denoted by $h_0$ and $h_1$, respectively, which are modeled as independent and identically distributed (i.i.d.) circular symmetric complex Gaussian random variables with zero mean and unit variance. We assume that the channels follow a statistical fading. In addition, we assume the same CSI acquisition procedure as that in [4] and take perfect SIC for comparing the system performance of the proposed RS scheme with the benchmarks [3], [4].

Required by the stringent QoS requirements, $U_0$ transmits at a fixed data rate. $U_1$ is allowed to share the same resource block with $U_0$ only when the $U_1$’s transmission does not deteriorate the $U_0$’s outage performance compared to the counterpart of $U_0$ in OMA. In other words, $U_0$ experiences the same outage performance as in OMA for delay-limited transmissions.
To begin with, we introduce $P_i$, $g_i$, and $\varepsilon_i$ for user $U_i$ $(i = 0, 1)$ to represent its transmit power, channel gain, and target rate related metric, respectively, where $g_i = |h_i|^2$ and $\varepsilon_i = 2^{\varepsilon_i} - 1$ with $\hat{R}_i$ denoting the target rate of $U_i$. Prior to each transmission block, the BS sends an interference threshold to $U_1$ to guarantee the QoS requirements of $U_0$. Considering that $U_0$ can achieve the same system performance as in OMA, the interference threshold is given by

$$\tau = \max \left\{ 0, \frac{P_0 g_0}{\varepsilon_0} - 1 \right\}.$$  

(1)

With respect to $\tau$, the target rate $\hat{R}_0$ is achievable for $U_0$ when $\log_2(1 + P_0 g_0) \geq \hat{R}_0$; Otherwise, $U_0$ experiences an outage due to $\log_2(1 + P_0 g_0) < \hat{R}_0$.

To enhance the transmission reliability, $U_1$ applies RS for each block of transmissions. Specifically, $U_1$ splits its signal $x_1$ into two parts $x_{1,1}$ and $x_{1,2}$ for transmissions [3], [2]. At the end of each transmission block, the received signal at the BS can be expressed as

$$y = \sqrt{P_0} h_0 x_0 + \sqrt{\alpha P_1} h_1 x_{1,1} + \sqrt{(1 - \alpha) P_1} h_1 x_{1,2} + n,$$  

(2)

where $x_0$ is the transmit signal of $U_0$, $n$ is the Gaussian background noise modeled with a normalized power, and $\alpha$ is the power allocation factor satisfying $0 \leq \alpha \leq 1$. We assume that all the signals, $\{x_0, x_{1,1}, x_{1,2}\}$, are independently coded with Gaussian code book and each signal has a unit power in expectation. The signal model in (2) is also called as rate splitting multiple access (RSMA) [8]. In this paper, we use the RS as in [2] to improve the transmission robustness of the secondary user in the considered CR-NOMA system. According to [8], the SIC decoding order $x_{1,1} \rightarrow x_0 \rightarrow x_{1,2}$ is applied at the BS receiver to achieve the benefit of RSMA. Consequently, the received SNR/SINRs for decoding $x_{1,1}$, $x_0$, and $x_{1,2}$ can be expressed as $\gamma_{11} = \frac{P_0 g_0}{\sqrt{\gamma_0}}$, and $\gamma_0 = \frac{P_0 g_0}{(1 - \alpha)P_1 g_1 + 1}$, respectively. For the primary and secondary users, the achievable rates are given by $R_0 = \log_2(1 + \gamma_0)$ and $R_1 = R_{11} + R_{12}$ with $R_{11} = \log_2(1 + \gamma_{11})$ and $R_{12} = \log_2(1 + \gamma_{12})$, respectively.

B. RS Strategy

The RS strategy is designed to obtain the maximum allowed achievable rate $\hat{R}_1$ for $U_1$, meanwhile keeping $U_0$ attaining the same outage performance as in OMA. To begin with, we assume that $x_{1,1}$ has been detected correctly in the first SIC stage using the decoding order $x_{1,1} \rightarrow x_0 \rightarrow x_{1,2}$. Then, the remaining SIC is to detect $x_0$ and $x_{1,2}$, sequentially. When $\tau = 0$, the $U_0$’s transmission is always in outage and the detection of $x_{1,2}$ will fail in the remaining SIC with decoding order $x_0 \rightarrow x_{1,2}$. Nevertheless, when $\tau > 0$ and $\gamma_{12} \leq \tau$, it can be shown that $R_0 \geq \hat{R}_0$, so that the achievable rate becomes $R_1 = R_{11} + R_{12}$ for $U_1$. To maximize the achievable rate $R_1$, the RS strategy is designed to maximize $R_{12} = \log_2(1 + \gamma_{12})$ considering that the detection of $x_{1,2}$ is interference-free in SIC.

Depending on the different values of $P_1 g_1$ and $\tau$, the RS strategy is designed as follows:

1) Case I: $\tau > 0$ and $P_1 g_1 \leq \tau$. In order to maximize $R_1$, it is needed to allocate all the transmit power $P_1$ to transmit $x_{1,2}$, such that $R_{12} = \log_2(1 + \gamma_{12})$ is maximized. Consequently, the RS strategy sets $\alpha = 0$ and $x_{1,2} = x_1$. Since $x_{1,1}$ is not transmitted, the decoding order $x_{1,1} \rightarrow x_0 \rightarrow x_{1,2}$ degrades to $x_0 \rightarrow x_{1,2}$ (or equivalently $x_0 \rightarrow x_1$). Thus, the achievable rate for $U_1$ becomes

$$R^{(I)} = \log_2 \left( 1 + P_1 g_1 \right).$$  

(3)

2) Case II: $\tau > 0$ and $P_1 g_1 > \tau$. In this case, to maximize the achievable rate $R_1$, $R_{12} = \log_2(1 + \gamma_{12})$ needs to be maximized first subject to the constraint of $\log_2(1 + \gamma_{12}) \leq \log_2(1 + \tau)$, i.e., the transmission of $x_{1,2}$ cannot decrease the outage performance of $U_0$ for the transmission of $x_0$. Obviously, the allowed maximum $R_{12} = \log_2(1 + \tau)$ is achieved by setting $\log_2(1 + \gamma_{12}) = \log_2(1 + \tau)$, which results in the optimal power allocation factor $\alpha = 1 - \frac{\tau}{P_1 g_1}$ for transmitting $x_{1,2}$ with the power $(1 - \alpha)P_1$. Meanwhile, the remaining transmit power, $\alpha P_1$, is allocated to transmit $x_{1,1}$, which yields the achievable rate $R_{11} = \log_2 \left( 1 + \frac{P_1 g_1 - \tau}{P_0 g_0 + 1 \tau} \right)$, respectively, and the achievable rate for the $U_1$’s transmission is given by

$$R^{(III)} = \log_2 \left( 1 + \frac{P_1 g_1 - \tau}{P_0 g_0 + 1 \tau} \right).$$  

(4)

To ensure that $U_0$ achieves the same outage performance as in OMA, it requires the successful detection of $x_{1,1}$ before detecting $x_0$. Nevertheless, $x_{1,1}$ cannot be detected correctly if $R^{(II)} < \hat{R}_1$. Therefore, in the proposed RS strategy, $U_1$ is allowed to transmit only if $R^{(II)} \geq \hat{R}_1$. Otherwise, $U_1$ keeps silent.

3) Case III: $\tau = 0$. In this case, the $U_0$’s transmission always encounters outage. To efficiently utilize transmit power, $U_1$ does not allocate any part of $P_1$ to transmit $x_{1,2}$ which cannot be detected correctly due to failure detection of $x_0$. To maximize the achievable rate $R_1$, the RS strategy sets $\alpha = 1$ and $x_{1,1} = x_1$. Consequently, the decoding order $x_{1,1} \rightarrow x_0 \rightarrow x_{1,2}$ degrades to $x_{1,1} \rightarrow x_0$, so that the achievable rate for $U_1$ becomes

$$R^{(III)} = \log_2 \left( 1 + \frac{P_1 g_1}{P_0 g_0 + 1} \right).$$  

(5)

Obviously, the difference between the proposed RS scheme and the QoS-SIC and NH-SIC schemes [3], [4] is that the QoS-SIC and NH-SIC schemes can attain the achievable rate in the form of $R^{(I)}$ and $R^{(II)}$, whereas only the proposed RS scheme can attain the achievable rate $R^{(III)}$ in Case II.

III. OUTAGE ANALYSIS FOR THE PROPOSED RS SCHEME

In the proposed RS scheme, the primary user experiences the same outage performance as in OMA and the outage probability experienced by the secondary user can be expressed as follows:

$$P_{\text{out}} = P_{\text{out}}^{(I)} + P_{\text{out}}^{(II)} + P_{\text{out}}^{(III)},$$  

(6)

where $P_{\text{out}}^{(I)} = \Pr\{\tau > 0, P_1 g_1 \leq \tau, R^{(I)} < \hat{R}_1\}$,

$$P_{\text{out}}^{(II)} = \Pr\{\tau > 0, P_1 g_1 > \tau, R^{(II)} < \hat{R}_1\},$$

and
$P_{\text{out}}^{(\text{III})} = \Pr\{\tau = 0, R^{(\text{III})} < \bar{R}_1\}$ denote the probabilities that $U_1$ is in outage corresponding to the RS scheme’s three operation cases, respectively.

The closed-form expression for $P_{\text{out}}^{(\text{II})}$ is provided in the following theorem.

**Theorem 1:** Corresponding to the RS scheme’s operation of Case II, the probability that $U_1$ is in outage is given by

$$P_{\text{out}}^{(\text{II})} = e^{-\eta_0} - \frac{1}{P_{1}\eta_0} \mu - e^{-\eta_0(1+\varepsilon_0)} \
\text{with } \eta_0 = \frac{\eta_1}{P_1} \text{ and}$$

$$\mu(\nu) = \begin{cases} 
\eta_0 \varepsilon_1, & \text{if } \nu = 0, \\
\frac{e^{-\eta_0(\nu+1)} - e^{-\eta_0(\nu+1)(1+\varepsilon_0)}}{\nu+1}, & \text{otherwise.}
\end{cases}
$$

**Proof:** See Appendix.

Although the operation of the proposed RS scheme in Case II is different from those of the QoS-SIC and NH-SIC schemes [6], [4], their operations in Cases I and III are the same. As a result, closed-form expressions for $P_{\text{out}}^{(\text{I})}$ and $P_{\text{out}}^{(\text{III})}$ can be retrieved from [3] and [4] as in the following theorem.

**Theorem 2:** Corresponding to the RS scheme’s operations in Cases I and III, probabilities that $U_1$ is in outage are given by

$$P_{\text{out}}^{(\text{I})} = e^{-\eta_0} - \frac{1}{P_{1}\eta_0} \mu - e^{-\eta_0(1+\varepsilon_0)} - \eta_0(1-\eta_0\mu) + 1,$$

$$P_{\text{out}}^{(\text{III})} = 1 - e^{-\eta_0} - \frac{e^{-\eta_0(1-\eta_0(P_0\eta_1+1))}}{P_0\eta_1+1},$$

respectively, where $\eta_1 = \frac{\varepsilon_1}{P_1}$.

**Corollary 1:** The outage probability experienced by the secondary user in the proposed RS scheme is given by

$$P_{\text{out}}^{(\text{II})} = 1 - e^{-\eta_0} - \frac{e^{-\eta_0(1-\eta_0(P_0\eta_1+1))}}{P_0\eta_1+1},$$

$$P_{\text{out}}^{(\text{III})} = 1 - e^{-\eta_0} - \frac{e^{-\eta_0(1-\eta_0(P_0\eta_1+1))}}{P_0\eta_1+1}.$$
sions. For comparison purposes, the CSI-based SIC (CSI-SIC) scheme is also considered [1], in which the stronger user’s signal is decoded before the weaker user’s signal.

\[ \tau = \varepsilon \text{ bits per channel use (BPCU)}, \] which results in \( \varepsilon_0 = 3 \) and \( \tau = \frac{3}{4} \). For the QoS-SIC and NH-SIC schemes, the achievable rates of \( U_1 \) are \( \bar{R}_{(1)} = \log_2 \left( 1 + \frac{100}{3} \right) \approx 3.335 \) BPCU and \( \bar{R}_{(1)}^{\text{max}} = \max\{ \log_2 \left( 1 + \frac{100}{3} \right), \log_2 \left( 1 + \frac{3}{4} \right) \} \approx 3.335 \) BPCU, respectively. For the proposed scheme, the achievable rate is \( R_{(1)} = \log_2 \left( 1 + \frac{3}{4} \right) + \log_2 \left( 1 + \frac{3}{4} \right) \approx 4.794 \) BPCU, which is much greater than those of the benchmark schemes.

**Example 2:** \[ \log_2 (1 + \tau) > \log_2 \left( 1 + \frac{P_1 g_1}{P_0 g_0 + 1} \right) \]. For this example, we set \( g_0 = g_1 = 10, P_0 = 10, P_1 = 20, \) and \( \varepsilon_0 = 3 \). The interference threshold is determined as \( \tau = \frac{97}{3} \). For the QoS-SIC and NH-SIC schemes, the achievable rate of \( U_1 \) are \( \bar{R}_{(1)} = \log_2 \left( 1 + \frac{503}{690} \right) \approx 1.575 \) BPCU and \( \bar{R}_{(1)}^{\text{max}} = \max\{ \log_2 \left( 1 + \frac{503}{690} \right), \log_2 \left( 1 + \frac{97}{3} \right) \} \approx 5.059 \) BPCU, respectively. In contrast, the proposed scheme achieves \( R_{(1)} = \log_2 \left( 1 + \frac{503}{690} \right) + \log_2 \left( 1 + \frac{97}{3} \right) \approx 6.233 \) BPCU, which is also greater than those of the benchmark schemes.

### V. Simulation Results

In this section, we present the simulation results to clarify the outage performance achieved by the proposed RS scheme and verify the accuracy of the developed analytical expressions. For comparison purposes, the CSI-based SIC (CSI-SIC) scheme is also considered [1], in which the stronger user’s signal is decoded before the weaker user’s signal.

In Fig. 1, the outage probability achieved by the proposed RS scheme is compared with the existing schemes for various transmit power settings. Particularly, we assume \( P_0 = P_1 \) in Fig. 1(a) and \( P_0 = \frac{P_1}{2} \) in Fig. 1(b), respectively. The curves in Figs. 1(a) and 1(b) verify the accuracy of the analytical result based on Theorem 1 and the approximation expression for \( P_{\text{out}}^{(II)} \). Furthermore, we can see that the proposed scheme achieves the lowest outage probability among the four schemes. Compared to the CSI-SIC and QoS-SIC schemes that cause the error floors on the outage probability, the proposed RS scheme achieves the outage probability monotonically decreasing with an increasing transmit SNR. In addition, the proposed RS scheme achieves a lower outage probability than that of the NH-SIC scheme mostly in the middle-SNR region. In the high-SNR region, the proposed scheme and the NH-SIC scheme achieve almost the same outage probability due to \( \Delta P_{\text{out}}^{(II)} \rightarrow 0 \). Similarly to the NH-SIC scheme, the proposed scheme avoids the error floor without imposing restriction on the values of \( \varepsilon_0 \) and \( \varepsilon_1 \).

In Fig. 2, we investigate the conditional outage probability of \( U_1 \) achieved by the proposed RS scheme under the Case II CSI condition. Based on the analytical results provided in Section III, we know that the outage performance differences among the existing schemes only rely on the operations for Case II. For the numerical results in Fig. 2(a), we set \( P_0 = \frac{P_1}{16} \).
provided, which clarify the superior outage performance of the proposed RS scheme over the existing SIC schemes.

APPENDIX: A PROOF OF THEOREM 1

The probability term \( P_{\text{out}}^{(II)} \) can be rewritten as

\[
P_{\text{out}}^{(II)} = \Pr \left( g_0 > \eta_0, g_1 > \frac{P_0 \varepsilon_0^{-1} g_0 - 1}{P_1} \right),
\]

\[
g_1 < \left( \frac{1 + \varepsilon_0 (1 + \varepsilon_1) - (1 + P_0 g_0)}{P_1} \right) \{ S \},
\]

where \( \eta_0 \triangleq \frac{\varepsilon_0}{P_0}, \mathbb{E}\{ \cdot \} \) stands for the expectation, and \( S \) is defined by

\[
S \triangleq \Pr \left( \frac{P_0 \varepsilon_0^{-1} g_0 - 1}{P_1} < g_1 < \left( \frac{1 + \varepsilon_0 (1 + \varepsilon_1) - (1 + P_0 g_0)}{P_1} \right) \right).
\]

For the QoS-SIC scheme, the conditional outage probability achieved by it has an error

\[
g_0 < \eta_0 \left( 1 + \varepsilon_1 \right),
\]

Consequently, the expectation for \( S \) in (15) should be taken over \( \eta_0 < g_0 < \eta_0 \left( 1 + \varepsilon_1 \right) \) and \( P_{\text{out}}^{(II)} \) can be evaluated as follows:

\[
P_{\text{out}}^{(II)} = \mathbb{E}(\eta_0 < g_0 < \eta_0 \left( 1 + \varepsilon_1 \right)) \{ S \}
\]

\[
= \int_{\eta_0}^{\eta_0 \left( 1 + \varepsilon_1 \right)} \int_{\frac{P_0 \varepsilon_0^{-1} y - 1}{P_1}}^{\frac{P_0 \varepsilon_0^{-1} y \left( 1 + \varepsilon_1 \right) - (1 + P_0 g_0)}{P_1}} e^{-x} e^{-y} dx dy.
\]

After some integration manipulations, a closed-form expression for \( P_{\text{out}}^{(II)} \) can be readily obtained, which completes the proof.

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