Lepton Mass Hierarchy and Neutrino Mixing

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Abstract

We speculate that the mass spectrum of three neutrinos might have a normal hierarchy as that of three charged leptons or that of three up-type (or down-type) quarks. In this spirit, we propose a novel parametrization of the \(3 \times 3\) lepton flavor mixing matrix. Its mixing angles \(\theta_l\) and \(\theta_\nu\) can be related to the mass ratios \(m_e/m_\mu\) and \(m_1/m_2\) in a specific texture of lepton mass matrices with vanishing (1,1) elements: \(\tan \theta_l = \sqrt{m_e/m_\mu}\) and \(\tan \theta_\nu = \sqrt{m_1/m_2}\). The latter relation, together with solar and atmospheric neutrino oscillation data, predicts \(0.0030 \text{ eV} < \sim m_1 < \sim 0.0073 \text{ eV}\), \(0.009 \text{ eV} < \sim m_2 < \sim 0.012 \text{ eV}\) and \(0.042 \text{ eV} < \sim m_3 < \sim 0.058 \text{ eV}\). The smallest neutrino mixing angle is found to be \(\theta_{13} \approx \theta_l/\sqrt{2} \approx 3^\circ\), which is experimentally accessible in the near future.

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The observed pattern of quark flavor mixing [1] remains one of the major puzzles in particle physics. On account of the hierarchical structures of up- and down-quark Yukawa couplings in the standard model, a natural and useful representation of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix reads as follows [2]:

$$V = \begin{pmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\varphi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \begin{pmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} s_u s_d c + c_u c_d e^{-i\varphi} & s_u c_d c - c_u s_d e^{-i\varphi} & s_u s \\ c_u s_d c - s_u c_d e^{-i\varphi} & c_u c_d c + s_u s_d e^{-i\varphi} & c_u s \\ -s_d s & -c_d s & c \end{pmatrix},$$  \hspace{1cm} (1)

where $s_u \equiv \sin \vartheta_u$, $s_d \equiv \sin \vartheta_d$, $c \equiv \cos \vartheta$, etc. This parametrization arises automatically from the quark mass matrices, if the hierarchy of quark masses and their chiral evolution are taken into consideration. It is not only convenient for the description of heavy flavor physics (e.g., $\tan \vartheta_u = |V_{ub}/V_{cd}|$, $\tan \vartheta_d = |V_{td}/V_{us}|$ and $\sin \vartheta = \sqrt{|V_{ub}|^2 + |V_{cd}|^2}$ hold exactly), but also suggestive for the understanding of flavor dynamics. For instance, a variety of models of quark mass matrices predict [3]

$$\tan \vartheta_u = \sqrt{\frac{m_u}{m_c}},$$

$$\tan \vartheta_d = \sqrt{\frac{m_d}{m_s}},$$  \hspace{1cm} (2)

which are very close to reality and might be exact. The merit of Eq. (2) is that it provides a phenomenological bridge between the smallness of quark mixing angles and the hierarchy of quark masses.

A question is whether the phenomenon of lepton flavor mixing, which has recently been verified by the solar [4], atmospheric [5], reactor [6] and accelerator [7] neutrino oscillation experiments, can be described in a way analogous to Eq. (1). The answer is certainly affirmative. If neutrinos are Majorana particles, the $3 \times 3$ lepton flavor mixing matrix can be expressed as $V = UP$, where

$$U = \begin{pmatrix} c_l & s_l & 0 \\ -s_l & c_l & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\varphi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \begin{pmatrix} c_\nu & -s_\nu & 0 \\ s_\nu & c_\nu & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} s_l c_\nu c + c_l c_\nu e^{-i\varphi} & s_l c_\nu c - c_l s_\nu e^{-i\varphi} & s_l s_\nu \\ c_l s_\nu c - s_l c_\nu e^{-i\varphi} & c_l c_\nu c + s_l s_\nu e^{-i\varphi} & c_l s_\nu \\ -s_\nu s & -c_\nu s & c \end{pmatrix},$$  \hspace{1cm} (3)

with $c_l \equiv \cos \vartheta_l$, $c_\nu \equiv \cos \vartheta_\nu$, $s \equiv \sin \theta$, and so on; and $P = \text{Diag} \{ e^{i\varphi}, e^{i\varphi}, 1 \}$ denotes the Majorana phase matrix. In the approximation that solar and atmospheric neutrino oscillations are nearly decoupled, the three mixing angles of $U$ can simply be related to those of solar [4], atmospheric [5] and CHOOZ [6] neutrino oscillations:

$$\theta_{12} \approx \vartheta_\nu, \quad \theta_{23} \approx \theta, \quad \theta_{13} \approx \vartheta_{12} \sin \theta.$$  \hspace{1cm} (4)
This novel parametrization is therefore a convenient option to describe the present neutrino oscillation data. It is more convenient than the so-called “standard” parametrization [1], when they are applied to deriving the one-loop renormalization-group equations of relevant neutrino mixing and CP-violating parameters [8].

The main purpose of this paper is to explore a possible relationship between the neutrino mixing pattern and the lepton mass hierarchy. At first, we speculate that the neutrino mass spectrum might have a normal hierarchy. Such a hierarchy is certainly weaker than the mass hierarchy of three charged leptons. Then we conjecture that the neutrino mixing angles $\theta_1$ and $\theta_\nu$ could be related to the lepton mass ratios $m_e/m_\mu$ and $m_1/m_2$ in a way similar to Eq. (2). This hypothesis allows us to determine the magnitudes of three neutrino masses from current experimental data. It actually implies an interesting texture of lepton mass matrices with vanishing (1,1) elements, which can be reconstructed in terms of the lepton mixing parameters. We find a simple but instructive possibility to decompose $U$, such that the resultant charged-lepton and neutrino mass matrices take a parallel form. The implications of this lepton mass texture will also be discussed.

2 First of all, let us emphasize that three mixing angles of $U$ have direct meanings. The angle $\theta$ describes the flavor mixing between the second and third lepton families. In comparison, $\theta_l$ primarily describes the $e$-$\mu$ mixing in the charged-lepton sector, and $\theta_\nu$ is essentially relevant to the $\nu_e$-$\nu_\mu$ mixing in the neutrino sector. In analogy to the quark sector, it makes sense to conjecture

$$\tan \theta_l = \frac{m_e}{m_\mu},$$
$$\tan \theta_\nu = \sqrt[2]{\frac{m_1}{m_2}},$$

at least as a leading-order approximation for the lepton mass models. One can see that Eq. (5) is just in analogy with the empirically successful Eq. (2) for quark mixing. The possibility to obtain Eq. (5) from a specific texture of lepton mass matrices will be discussed later.

The consequences of Eq. (5) are quite non-trivial. Because of $m_e/m_\mu \approx 0.00484$ [1], we immediately obtain $\theta_l \approx 4^\circ$ from Eq. (5). This result indicates that $\theta_{13} \approx \theta_l/\sqrt{2} \approx 3^\circ$ for $\theta \approx 45^\circ$, as one can see from Eq. (4). Such a small $\theta_{13}$ is certainly consistent with current experimental data, $0^\circ \leq \theta_{13} \leq 10^\circ$ at the 99% confidence level [9]. Indeed, $\theta_{13} \approx 3^\circ$ is equivalent to $\sin^2 2\theta_{13} \approx 0.01$, which is accessible in a few planned reactor neutrino oscillation experiments (e.g., the Daya-Bay experiment [10]). The very interesting point is that the smallness of $\theta_l$ (or $\theta_{13}$) is attributed to the strong mass hierarchy of $e$ and $\mu$ in our conjecture. It is therefore natural and suggestive for model building.

On the other hand, the observed value $\theta_\nu \approx 33^\circ$ [9] together with Eq. (5) implies that the mass hierarchy of $\nu_1$ and $\nu_2$ (the mass eigenstates of $\nu_e$ and $\nu_\mu$) must be rather weak. In particular, Eq. (5) is qualitatively consistent with the experimentally-established facts $m_1 < m_2$ and $\theta_\nu < 45^\circ$ (or equivalently $\theta_{12} < 45^\circ$). Note that solar and atmospheric neutrino oscillations are associated with the neutrino mass-squared differences $\Delta m^2_{21} \equiv m_2^2 - m_1^2$ and $\Delta m^2_{32} \equiv m_3^2 - m_2^2$, respectively. Note also that $\sin^2 \theta_\nu = m_1/(m_1 + m_2)$ and $\cos^2 \theta_\nu = m_2/(m_1 + m_2)$ hold as a consequence of Eq. (5). Then we arrive at
If neutrinos are Majorana particles, their (effective) mass matrix

\[
m_1^2 = \frac{\sin^4 \theta_\nu}{\cos 2 \theta_\nu} \Delta m_{21}^2 ,
\]

\[
m_2^2 = \frac{\cos^4 \theta_\nu}{\cos 2 \theta_\nu} \Delta m_{21}^2 ,
\]

\[
m_3^2 = \frac{\cos^4 \theta_\nu}{\cos 2 \theta_\nu} \Delta m_{21}^2 + \Delta m_{32}^2 .
\]  

(6)

Current experimental data yield $\Delta m_{21}^2 \approx 8 \times 10^{-5}$ eV$^2$ and $|\Delta m_{32}^2| \approx 2.5 \times 10^{-3}$ eV$^2$ [9]. Because of $\cos^4 \theta_\nu / \cos 2 \theta_\nu \approx 1.2$ for $\theta_\nu \approx 33^\circ$, the positiveness of $m_3^2$ forbids the possibility of $\Delta m_{32}^2 < 0$. Then we obtain $m_1 \approx 0.0041$ eV, $m_2 \approx 0.0097$ eV and $m_3 \approx 0.051$ eV from Eq. (6). Two independent neutrino mass ratios are $m_1/m_2 \approx 0.42$ and $m_2/m_3 \approx 0.19$. It becomes clear that three neutrino masses have a normal but relatively weak hierarchy, namely, $m_1 : m_2 : m_3 \approx 1 : 2.4 : 12.5$ as a typical result.

The numerical dependence of two neutrino masses $m_1$ and $m_2$ on the mixing angle $\theta_\nu$ is explicitly illustrated in Fig. 1, where $\Delta m_{21}^2 = (7.2 \cdots 8.9) \times 10^{-5}$ eV$^2$ and $30^\circ \leq \theta_\nu \leq 38^\circ$ (at the 99% confidence level [9]) have typically been input. One can see that $m_1$ may change from 0.0030 eV to 0.0073 eV when $\theta_\nu$ increases from 30° to 38°. In comparison, the allowed range of $m_2$ is somehow narrower: 0.009 eV $\lesssim m_2 \lesssim 0.012$ eV. Note that $m_3$ is essentially insensitive to $\theta_\nu$, because $\Delta m_{32}^2 \ll \Delta m_{21}^2$ and $m_3 \approx \sqrt{\Delta m_{32}^2}$ is a very good approximation. Given $\Delta m_{32}^2 = (1.7 \cdots 3.3) \times 10^{-3}$ eV$^2$ at the 99% confidence level [9], we immediately obtain $0.042$ eV $\lesssim m_3 \lesssim 0.058$ eV from Eq. (6).

3 | If neutrinos are Majorana particles, their (effective) mass matrix $M_\nu$ must be symmetric. Without loss of generality, one may redefine the right-handed charged-lepton fields to make the charged-lepton mass matrix $M_l$ to be symmetric too. In this basis, $M_l$ and $M_\nu$ can be diagonalized by the following transformations:

\[
O_l^\dagger M_l O_l^* = \begin{pmatrix}
\lambda_e & 0 & 0 \\
0 & \lambda_\mu & 0 \\
0 & 0 & \lambda_\tau
\end{pmatrix},
\]

\[
O_\nu^\dagger M_\nu O_\nu = \begin{pmatrix}
\lambda_1 & 0 & 0 \\
0 & \lambda_2 & 0 \\
0 & 0 & \lambda_3
\end{pmatrix},
\]  

(7)

where $O_l$ and $O_\nu$ are unitary matrices, $\lambda_\alpha$ (for $\alpha = e, \mu, \tau$) and $\lambda_i$ (for $i = 1, 2, 3$) stand respectively for the mass eigenvalues of charged leptons and neutrinos. Note that $|\lambda_\alpha| = m_\alpha$ and $|\lambda_i| = m_i$ hold. Alternatively, one may absorb the negative signs of $\lambda_\alpha$ or $\lambda_i$ into $O_l$ or $O_\nu$. Note also that the signs of $\lambda_\alpha$ has no physical significance, but those of $\lambda_i$ can affect the effective mass term of the neutrinoless double-beta decay

\[
\langle m \rangle_{ee} = \left| \sum_{i=1}^{3} (m_i V_{ei}^2) \right| .
\]  

(8)

Because the neutrino mass spectrum has a normal hierarchy in our scenario, the magnitude of $\langle m \rangle_{ee}$ is strongly suppressed. It is in general expected that $\langle m \rangle_{ee} \lesssim \mathcal{O}(10^{-3})$ eV holds,
as a direct consequence of $m_3 \sim 0.05$ eV and $\theta_{13} \leq 10^\circ$ (or $|V_{e3}| \leq 0.17$). This expectation is actually independent of the details of a specific neutrino mass model, only if it predicts a normal neutrino mass hierarchy.

It has been mentioned above that the lepton flavor mixing matrix $V$ is in general written as $V = UP$. For simplicity, we take $U = O_1^l O_\nu$ and attribute the probably negative signs of $\lambda_i$ into $P$. To reproduce the parametrization of $U$ in Eq. (3), we may simply express $O_1$ and $O_\nu$ as

$$O_1 = \begin{pmatrix} e^{i\phi_x} & 0 & 0 \\ 0 & c_x & s_x e^{i\phi_x} \\ 0 & -s_x e^{-i\phi_x} & c_x \end{pmatrix} \begin{pmatrix} c_l & -s_l & 0 \\ s_l & c_l & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$O_\nu = \begin{pmatrix} e^{i\phi_y} & 0 & 0 \\ 0 & c_y & s_y e^{i\phi_y} \\ 0 & -s_y e^{-i\phi_y} & c_y \end{pmatrix} \begin{pmatrix} c_\nu & -s_\nu & 0 \\ s_\nu & c_\nu & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{9}$$

where $c_x \equiv \cos \theta_x$, $s_y \equiv \sin \theta_y$, and so on. We find that Eq. (3) can be reproduced from $U = O_1^l O_\nu$ in four special cases:

- Case A: $\phi_x = 0$ and $\phi_y = 0$. We obtain $\theta = \theta_y - \theta_x$ and $\phi = \phi_x - \phi_y$.
- Case B: $\phi_x = \pi$ and $\phi_y = 0$. We obtain $\theta = \theta_x + \theta_y$ and $\phi = \phi_x - \phi_y$.
- Case C: $\phi_x = 0$ and $\phi_y = \pi$. We obtain $\theta = -\left(\theta_x + \theta_y\right)$ and $\phi = \phi_x - \phi_y$.
- Case D: $\phi_x = \pi$ and $\phi_y = \pi$. We obtain $\theta = \theta_x - \theta_y$ and $\phi = \phi_x - \phi_y$.

If $\theta_x$ and $\theta_y$ are related to the ratios of charged-lepton and neutrino masses, then it is likely to determine the mixing angle $\theta$.

With the help of Eqs. (7) and (9), we obtain the charged-lepton and neutrino mass matrices as follows:

$$M_l = \begin{pmatrix} (\lambda_e c_l^2 + \lambda_\mu s_l^2) e^{2i\phi_x} & (\lambda_e - \lambda_\mu) c_x c_l s_l e^{i\phi_x} & (\lambda_\mu - \lambda_e) s_x c_l s_l e^{i\phi_x} \\ (\lambda_e - \lambda_\mu) c_x c_l s_l e^{i\phi_x} & (\lambda_e s_x^2 + \lambda_\mu c_l^2) c_x^2 + s_x^2 & (\lambda_\mu - \lambda_e s_x^2 - \lambda_\mu c_l^2) c_x s_x \\ (\lambda_\mu - \lambda_e) s_x c_l s_l e^{i\phi_x} & (\lambda_\mu - \lambda_e s_x^2 - \lambda_\mu c_l^2) c_x s_x & (\lambda_\mu s_x^2 + \lambda_\mu c_l^2) s_x^2 + c_x^2 \end{pmatrix},$$

$$M_\nu = \begin{pmatrix} (\lambda_1 c_\nu^2 + \lambda_2 s_\nu^2) e^{2i\phi_y} & (\lambda_1 - \lambda_2) c_\nu c_\nu s_\nu e^{i\phi_y} & (\lambda_2 - \lambda_1) s_\nu c_\nu s_\nu e^{i\phi_y} \\ (\lambda_1 - \lambda_2) c_\nu c_\nu s_\nu e^{i\phi_y} & (\lambda_1 s_\nu^2 + \lambda_2 c_\nu^2) c_\nu^2 + s_\nu^2 & (\lambda_2 - \lambda_1 s_\nu^2 - \lambda_2 c_\nu^2) c_\nu s_\nu \\ (\lambda_2 - \lambda_1) s_\nu c_\nu s_\nu e^{i\phi_y} & (\lambda_2 - \lambda_1 s_\nu^2 - \lambda_2 c_\nu^2) c_\nu s_\nu & (\lambda_1 s_\nu^2 + \lambda_2 c_\nu^2) s_\nu^2 + c_\nu^2 \end{pmatrix}. \tag{10}$$

Setting $(M_l)_{11} = (M_\nu)_{11} = 0$ and taking the opposite signs for $\lambda_e$ (or $\lambda_1$) and $\lambda_\mu$ (or $\lambda_2$), we immediately arrive at

$$\tan \theta_1 = \sqrt{\frac{\lambda_e}{\lambda_\mu}} = \sqrt{\frac{m_e}{m_\mu}},$$

$$\tan \theta_\nu = -\sqrt{\frac{\lambda_1}{\lambda_2}} = \sqrt{\frac{m_1}{m_2}}. \tag{11}$$
We see that the conjecture made in Eq. (5) can be regarded as a consequence of the condition \((M_l)_{11} = (M_\nu)_{11} = 0\) which is imposed on the textures of \(M_l\) and \(M_\nu\) in Eq. (11). The parallel form and texture zeros of \(M_l\) and \(M_\nu\) imply that they might originate from the same flavor dynamics in an underlying theory.

4] What about the unknown angles \(\theta_x\) and \(\theta_y\)? Without loss of generality, we require both of them to lie in the first quadrant. One might conjecture that \(\theta_x\) and \(\theta_y\) have the following relations with the mass ratios \(m_\mu/m_\tau\) and \(m_2/m_3\):

\[
\sin \theta_x = \sqrt{\frac{m_\mu}{m_\tau}},
\]

\[
\sin \theta_y = \sqrt{\frac{m_2}{m_3}}.
\]  

Then we arrive at \(\theta_x \approx 14.1^\circ\) and \(\theta_y \approx 26.1^\circ\) by inputting the experimental value \(m_\mu/m_\tau \approx 0.0594\) [1] and the afore-obtained result \(m_2/m_3 \approx 0.19\). It turns out that only case B mentioned above (i.e., \(\varphi_x = \pi\) and \(\varphi_y = 0\)) is favored to achieve a large and positive value for the atmospheric neutrino mixing angle \(\theta\): \(\theta = \theta_x + \theta_y \approx 40.2^\circ\). This result, equivalent to \(\sin^2 2\theta_{23} \approx 0.97\), is certainly consistent with current experimental data. A much larger value of \(\theta\) can be achieved, if we take a larger value for the mass ratio \(m_2/m_3\) in its allowed range.

In Fig. 2, we illustrate the numerical region of \(\theta = \theta_x + \theta_y\) by taking account of Eqs. (6) and (12) together with current experimental data on \(\Delta m_{21}^2\), \(\Delta m_{32}^2\) and \(\theta_\nu\) [9]. One can see that the lower and upper bounds of \(\theta\) are about 37.5\(^\circ\) and 46\(^\circ\), respectively. In particular, the maximal atmospheric neutrino mixing (i.e., \(\theta \approx 45^\circ\)) can be achieved when \(\theta_\nu > 36^\circ\) is input. The simple correlation between \(\theta_\nu\) and \(\theta\) shown in Fig. 2 provides a straightforward way to experimentally test the validity of this phenomenological scenario in the near future.

It seems more difficult to fix the unknown phase parameters \(\phi_x\) and \(\phi_y\) in a plausible way, although their difference \(\phi = \phi_x - \phi_y\) is a physical parameter for CP violation. If CP were a good symmetry in the lepton sector (i.e., \(\phi = 0\) held and there were no non-trivial Majorana phases either), one could simply set \(\phi_x = \phi_y = 0\). But it is more likely that there exists large leptonic CP violation, which might even lead to some observable effects in the long-baseline neutrino oscillations [11]. In a few phenomenological models or Ansätze of lepton mass matrices (see, e.g., Refs. [12] and [13]), \(\phi \approx 90^\circ\) or the so-called “maximal CP violation” has been discussed. Such discussions might make sense, as current experimental data on quark flavor mixing and CP violation do favor \(\phi \approx 90^\circ\) [14]. It is therefore interesting to speculate whether \(\phi = \varphi = 90^\circ\) can be realized in a specific model with lepton-quark symmetry.

Given Eq. (12), the textures of \(M_l\) and \(M_\nu\) in Eq. (10) can then be illustrated as

\[
M_l \approx 0.94m_\tau \begin{pmatrix}
0 & 0.0043e^{i\phi_x} & 0.0011e^{i\phi_y} \\
0.0043e^{i\phi_x} & 0.0041 & -0.27 \\
0.0011e^{i\phi_x} & -0.27 & 1
\end{pmatrix},
\]

\[
M_\nu \approx 0.78m_3 \begin{pmatrix}
0 & 0.14e^{i\phi_y} & -0.094e^{i\phi_y} \\
0.14e^{i\phi_y} & 0.13 & 0.56 \\
-0.094e^{i\phi_y} & 0.56 & 1
\end{pmatrix},
\]  

(13)
where we have taken \( \lambda_e = m_e \) (or \( \lambda_1 = m_1 \)) and \( \lambda_\mu = -m_\mu \) (or \( \lambda_2 = -m_2 \)) by convention and used \( m_1/m_2 \approx 0.42 \) and \( m_2/m_3 \approx 0.19 \) as typical inputs. A normal hierarchy does manifest itself in \( M_l \) and \( M_\nu \). As expected, the structural hierarchy of \( M_\nu \) is relatively weak. The parallel form and hierarchical textures of \( M_l \) and \( M_\nu \), together with their texture zeros, should be helpful for further model building.

Finally let us point out that Eqs. (5) and (12) or their analogues can also be reproduced, at least in the leading-order approximation, from the Fritzsch-type texture of lepton mass matrices with \( [15] \) or without \( [16] \) the seesaw mechanism.

Starting from the speculation that the mass spectrum of three neutrinos might have a normal hierarchy in analogy with that of three charged leptons or three up-type (or down-type) quarks, we have explored a possible relationship between the neutrino mixing pattern and the lepton mass matrices. A novel parametrization, which has proved to be very useful in the description of quark flavor mixing, is now used to describe the phenomenon of lepton flavor mixing. Its mixing angles \( \theta_l \) and \( \theta_\nu \) are conjectured to relate to the mass ratios \( m_e/m_\mu \) and \( m_1/m_2 \) in a straightforward way: \( \tan \theta_l = \sqrt{m_e/m_\mu} \) and \( \tan \theta_\nu = \sqrt{m_1/m_2} \).

We find that this hypothesis allows us to determine the magnitudes of three neutrino masses and the smallest (CHOOZ) neutrino mixing angle \( \theta_{13} \) from current experimental data. The typical numerical results are \( 0.0030 \text{ eV} \lesssim m_1 \lesssim 0.0073 \text{ eV}, \ 0.009 \text{ eV} \lesssim m_2 \lesssim 0.012 \text{ eV} \) and \( 0.042 \text{ eV} \lesssim m_3 \lesssim 0.058 \text{ eV} \), together with \( \theta_{13} \approx \theta_l/\sqrt{2} \approx 3^\circ \). The latter is experimentally accessible in the near future. We have also pointed out a simple but instructive possibility to decompose the lepton mixing matrix, such that the charged-lepton and neutrino mass matrices can be reconstructed in a parallel form. It turns out that the texture zeros of their (1,1) elements just lead to the afore-conjectured expressions of \( \theta_l \) and \( \theta_\nu \).

We remark that the structural parallelism, normal hierarchy and texture zeros could be the useful starting points of view to build phenomenologically-favored models, in order to deeply understand the generation of charged-lepton and neutrino masses and the pattern of their Yukawa-coupling matrices. A similar emphasis has actually been made for the quark sector \([17]\). Because the parametrization in Eq. (1) for quark mixing and that in Eq. (3) for lepton mixing are very analogous to each other, one may even speculate whether such a novel representation of flavor mixing can provide a transparent description of the possible lepton-quark unification in a more fundamental flavor theory.

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FIG. 1. Changes of two neutrino masses $m_1$ and $m_2$ with the mixing angle $\theta_{\nu}$ as predicted by Eq. (6), where $\Delta m_{21}^2 = (7.2 \cdots 8.9) \times 10^{-5}$ eV$^2$ and $30^\circ \leq \theta_{\nu} \leq 38^\circ$ [9] have been input.
FIG. 2. The range of $\theta = \theta_x + \theta_y$ predicted by Eq. (12) and constrained by current data on $\Delta m^2_{21}$, $\Delta m^2_{32}$ and $\theta_\nu$ in our scenario. The inputs are $\Delta m^2_{21} = (7.2 \cdots 8.9) \times 10^{-5}$ eV$^2$, $\Delta m^2_{32} = (1.7 \cdots 3.3) \times 10^{-3}$ eV$^2$ and $30^\circ \leq \theta_\nu \leq 38^\circ$ [9].