Role of degenerate atomic levels in the entanglement and the decoherence

L. Zhou*, H. S. Song†, Y. X. Luo
Department of Physics, Dalian University of Technology, Dalian, 116023, P. R. China.

We studied the dispersive dissipation of denegerate-level atom interacting with a single linearly-polarized mode field. It is found that the degeneracy of the atomic level affects the dissipation behavior of the system as well as the subsystems. The degeneracy of the atomic level augment the periods of entanglement and increase the degree of the maxima statistical mixture states.

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I. INTRODUCTION

Since the EPR paradox was proposed, the quantum entanglement has been a interesting subject, which reveal the profound difference between quantum and classical world. Recently, entanglement as a physical resource has been used in quantum information such as quantum teleportation, superdense coding and quantum cryptography [1,2,3]. Another equally fundamental question concerns the nonexistence of coherence superposition of macroscopically distinguishable states, illustrated by the Schrödinger’s cat paradox. One of the answer to the second question stress the role of dissipation on the disappearance of coherence [4,5]. Decoherence follows from the irreversible coupling of the observed system to the outside word reservoir, and this coupling induce that decoherence of the macroscopic states would be too fast to be observed.

A atom (atoms) interaction with quantum electromagnetic field play important role in explaining these essential quantum problem, in preparation some kinds of quantum states [6,7] and in monitoring decoherence [8]. Several schemes had been proposed to generate the entangle atomic state, on the condition that the entanglement between the first atom and the cavity field can survive for long enough so that it can be transferred to a second atom via coherent interaction [6,7]. The superposition of two coherent states can be prepared and decoherence can be monitored in cavity QED. These schemes concerned with a atom or atoms interacting with field in a cavity, and the dissipation of the cavity play important role in the entanglement of subsystem and in the decoherence of the system or subsystem [9,10,17]. Therefore, in Ref. [11] the dispersive atomic evolution in a dissipative-driven cavity was studied, and the influence of dissipation on the entanglement and on the decoherence was investigated via JCM in the dispersive approximation in Ref. [12].

Although theoretical predictions based on the simple two-level model have proven to be powerful, pure two-level systems are seldom found in real experiments. In most cases, the atomic level are degenerate [13,14]. If the levels of an isolated atom are degenerate in the projection of the total electronic angular momenta on the quantization axis, we should take into account the degeneracy of atomic level [13]. In Ref.[15,16] original Jaynes-Cummings model (JCM) was generalized to the case of degenerate atomic levels. If results of degeneracy of atomic level could provide some available properties, we may turn the pure two-level atom into degenerate atom by introducing magnetic field. In
In this paper, we study dynamics of a degenerate atom interacting with the field in a dissipative cavity. In dispersive approximation, we find that the degeneracy of atomic level augment the period of disentanglement between the atom and the field and increase the degree of the maxima statistical mixture states.

II. THE JAYNES-CUMMINGS MODEL WITH DEGENERATE ATOMIC LEVEL AND THE DISPERSIVE APPROXIMATION

Let us take into account the degeneracy of atomic levels, the full set of states of the system may be written as

\[ |n, J_\alpha, m_\alpha > = |n > \cdot |J_\alpha, m_\alpha >, n = 0, 1, ..., m_\alpha = -J_\alpha, ..., J_\alpha, \alpha = b, c, \]  

where \( n \) is the number of photons in the field mode, while \( b \) and \( c \) denote the upper and lower atomic levels respectively. \( J_b \) and \( J_c \) are the values of the total electronic angular momenta of resonant levels, while \( m_b \) and \( m_c \) are their projections on the quantization axis, the Cartesian axis \( Z \), which is directed along the polarization vector of the field mode.

We assume that a degenerate-level atom interact with a single linearly-polarized mode field. Hence only that atomic transition could emit a linearly-polarized photon take part in the interaction. The Hamiltonian of the system may be written as \((\hbar = 1)\)

\[ H = \omega_0 a^+ a + \frac{1}{2} \omega (n_b - n_c) + g(a^+ S_- + a S_+), \]  

where \( \hat{a}^+ \) and \( \hat{a} \) are the operators of the creation and annihilation of photons with frequency \( \omega_0 \) in the field mode, and

\[ n_\alpha = \sum_{m_\alpha = -J_\alpha}^{J_\alpha} |J_\alpha, m_\alpha > \langle J_\alpha, m_\alpha |, \quad \alpha = b, c, \]  

are the operators of total population of resonant atomic levels \( b \) and \( c \), \( \omega \) is the frequency of the optically-allowed atomic transition \( J_b \rightarrow J_c \).

\[ S_- = \sum_m \alpha_m |J_c, m > \langle J_b, m | \]  

is the dipole moment operator of the atomic transition \( J_b \rightarrow J_c \), where

\[ \alpha_m = (-1)^{n_b - m} \begin{pmatrix} J_b & 1 & J_c \\ -m & 0 & m \end{pmatrix} \]  

is matrix elements defined through Wigner 3j-symbol, corresponding to the linearly-polarized photon.

We consider the far-off resonance limit for the atom-field interaction (dispersive interaction). The Hamiltonian take the form

\[ H_I = \frac{\delta}{2} (n_b - n_c) + g(a^+ S_- + a S_+), \]  

where the detuning \( \delta = \omega - \omega_0 \). The dipole moment operator of the atomic transition \( S_- \) and \( S_+ \) satisfy the commutation relation
\[ [S_+, S_-] = 2S_z, \quad [S_\pm, S_z] = \pm S_z, \quad (7) \]

with

\[ S_z = \frac{1}{2} \sum_m \alpha^2_m (|J_b, m\rangle \langle J_b, m| - |J_c, m\rangle \langle J_c, m|). \quad (8) \]

To solve the master equation in next section, here we take a set of unitary transformation to the Hamiltonian of Eq.(6) which is proposed in Ref.[11]. This transformation correspond to small rotation in the SU(2) group with an operator parameter

\[ H_{eff} = U_2 U_1 H U_1^+ U_2^+, \quad (9) \]

where

\[ U_1 = \exp(i \frac{\sqrt{2}g}{\delta} p S_x), \quad U_2 = \exp(i \frac{\sqrt{2}g}{\delta} q S_y), \]

and

\[ q = \frac{1}{\sqrt{2}} (a + a^+), \quad p = \frac{i}{\sqrt{2}} (a^+ - a), \]

\[ S_x = \frac{1}{2} (S_+ + S_-), \quad S_y = -\frac{i}{2} (S_+ - S_-). \]

Keeping terms up to first order in \( \sqrt{2}g/\delta \ll 1 \), we get

\[ H_{eff} = \frac{\delta}{2} (n_b - n_c) + \frac{g^2}{\delta} (R_b + R_c) + \frac{g^2}{\delta} (2a^+ a + 1) S_z, \quad (10) \]

where

\[ R_\alpha = \sum_m \alpha^2_m (|J_\alpha, m\rangle \langle J_\alpha, m|), \quad \alpha = b, c. \quad (11) \]

Note that the fore two terms of Eq. (10) commute with the effective Hamiltonian \( H_{eff} \). We can further simplify the effective Hamiltonian by the following transformation of the operator \( f \):

\[ \tilde{f} = e^{i \frac{\delta}{2} (n_b - n_c) + \frac{g}{\delta} (R_b + R_c)} \{ e^{-i \frac{\delta}{2} (n_b - n_c) + \frac{g}{\delta} (R_b + R_c)}} \cdot \]

Thus, we finally get

\[ \tilde{H}_{eff} = \Omega (2a^+ a + 1) S_z, \quad (13) \]

with \( \Omega = \frac{g^2}{\delta} \). In the next section, we will directly using the expression of Eq.(13).

**III. THE MASTER EQUATION AND ITS SOLUTION**

We assume that there is a reservoir coupled to the field in the usual way. Using the transformation of Eq. (12) to the density matrix, master equation has a standard form
\[
\frac{d\hat{\rho}}{dt} = i[\hat{\rho}, \hat{H}_{\text{eff}}] + \mathcal{D}\hat{\rho}. \tag{14}
\]

The losses in the cavity are phenomenologically represented by the superoperator \(\mathcal{D}\). At the zero temperature, we have

\[
\mathcal{D}\hat{\rho} = \kappa(2a\hat{\rho}a^+ - a^+a\hat{\rho} - \hat{\rho}a^+a), \tag{15}
\]

where \(\kappa\) is the damping constant. The density operator \(\hat{\rho}\) belongs to the set \(\mathbb{R} (\mathbb{H}_A \otimes \mathbb{H}_F)\) of the trace class operators that act in the space corresponding to the direct product of the two Hilbert space \(\mathbb{H}_A\) and \(\mathbb{H}_F\) of the atom and the field, respectively. We can represent the density operator as following:

\[
\hat{\rho} = \sum_{m,m',J_\alpha,J_\beta} \hat{\rho}_{J_\alpha J_\beta mm'}|J_\alpha, m\rangle\langle J_\beta, m'|, \text{ with } J_\alpha(J_\beta) = J_b, J_c \tag{16}
\]

where \(\omega_m = \alpha_m\Omega\). We choose \(-\) when we calculate \(\hat{\rho}_{J_b J_b mm'}\) and \(+\) corresponding to \(\hat{\rho}_{J_c J_c mm'}\). The superoperators in Eq.(18) are defined as

\[
[F, \mathcal{M}] = F, \quad [F, \mathcal{P}] = F, \quad [\mathcal{M}, \mathcal{P}] = 0. \tag{19}
\]

In the same way, we also get the Liouvillian of acting in \(\hat{\rho}_{J_b J_c mm'}\) as

\[
\mathcal{L}_{J_b J_c mm'} = -2i(\omega_m \mathcal{M} + \omega_m' \mathcal{P}) + \kappa(2F - \mathcal{M} - \mathcal{P}) \mp \kappa(\omega_m - \omega_m'). \tag{20}
\]

A similar expression of Liouvillian \(\mathcal{L}_{J_c J_b mm'}\) which act in \(\hat{\rho}_{J_c J_b mm'}\) is easy obtained except for taking conjugate of Eq.(20).

**IV. TIME EVOLUTION OF INITIAL STATE**

We assume the initial state of the system as

\[
\Psi(0) = \frac{1}{\sqrt{2}} \sum_m (\frac{1}{\sqrt{2^{J_b+1}}} |J_b, m\rangle\langle J_b, m| + \frac{1}{\sqrt{2^{J_c+1}}} |J_c, m\rangle\langle J_c, m|) \otimes |\alpha\rangle \tag{21}
\]
the atom is in the degenerate level in equal probability and it enters the cavity in a coherence superposition and finds there a coherent field state $|\alpha\rangle$, therefore initially

$$\hat{\rho}_{J_bJ_{c}m_m'}(0) = \frac{1}{2(2J_b+1)}|\alpha\rangle\langle\alpha|, \quad \hat{\rho}_{J_cJ_{c}m_{m}'}(0) = \frac{1}{2(2J_c+1)}|\alpha\rangle\langle\alpha|,$$

$$\hat{\rho}_{J_bJ_{c}m_{m}'}(0) = \hat{\rho}_{J_cJ_{c}m_{m}'}(0) = \frac{1}{2\sqrt{(2J_b+1)(2J_c+1)}}|\alpha\rangle\langle\alpha|.$$  

Solving the Eq. (17), we finally get the density matrix

$$\hat{\rho} = \frac{1}{2} \sum_{m,m'} \left\{ \frac{1}{(2J_b+1)} \exp[\Gamma(\chi_{m_{m}'}^{m}, t) + i\Theta(\chi_{m_{m}'}^{m}, t)][J_{b}, m, \alpha(t)e^{-2i\omega_{m}t}\langle J_{b}, m', \alpha(t)e^{-2i\omega_{m}t}|]ight. 
+ \frac{1}{2J_{c}+1} \exp[\Gamma(\chi_{m_{m}'}^{m}, t) - i\Theta(\chi_{m_{m}'}^{m}, t)][J_{c}, m, \alpha(t)e^{2i\omega_{m}t}\langle J_{c}, m', \alpha(t)e^{2i\omega_{m}t}|] 
+ \frac{1}{\sqrt{(2J_b+1)(2J_c+1)}}[\exp(\Gamma(\lambda_{m_{m}'}^{m}, t) + i\Theta_{m}(\lambda_{m_{m}'}^{m}, t))[J_{c}, m, \alpha(t)e^{-2i\omega_{m}t}\langle J_{c}, m', \alpha(t)e^{-2i\omega_{m}t}|] 
+ \exp[\Gamma(\lambda_{m_{m}'}^{m}, t) - i\Theta_{m}(\lambda_{m_{m}'}^{m}, t)][J_{c}, m, \alpha(t)e^{2i\omega_{m}t}\langle J_{c}, m', \alpha(t)e^{2i\omega_{m}t}|] \} \right\}$$  

(22)

where $\chi_{m_{m}'} = \omega_{m} - \omega_{m'}$, $\lambda_{m_{m}'} = \omega_{m} + \omega_{m'}$.

$$\Gamma(x, t) = -|\alpha|^2(1 - e^{-2\kappa t}) - \frac{|\alpha|^2}{\kappa^2 + x^2}[e^{-2\kappa t}(\kappa \cos 2xt - x \sin 2xt) - \kappa],$$  

(23)

and

$$\Theta(x, t) = -xt + \frac{|\alpha|^2}{\kappa^2 + x^2}[e^{-2\kappa t}(x \cos 2xt + \kappa \sin 2xt) - x],$$  

(24)

where $x$ equal to $\chi_{m_{m}'}$ and $\lambda_{m_{m}'}$, respectively. The function $\Gamma(x, t)$ in Eq. (23) embody the effect of reservoir because it vanishes for $k \to 0$.

The coherence properties of this density operator as a function of time is conveniently studied by means of the linear entropy

$$S = 1 - Tr(\rho^2).$$  

(25)

The quantity $Tr(\rho^2)$ can be taken as a measure of the degree of purity of the reduced state; for a pure state $S$ is zero but for $0 < S \leq 1$ the state corresponds to a mixture, with information effectively lost. Because the all the transformation in Eq.(9) and in Eq.(12) are unitary, hence the entropy is not affected by the transformation. Hereafter we will direct use these density operators ($\hat{\rho}_F, \hat{\rho}_A$) to gain corresponding entropy. The linear entropy of the total system is obtained from Eq.(22)

$$S = 1 - \frac{1}{4} \sum_{m,m'} \left\{ \frac{1}{(2J_b+1)^2} + \frac{1}{(2J_c+1)^2} \exp[2\Gamma(\chi_{m_{m}'}^{m}, t)] + \frac{2}{(2J_b+1)(2J_c+1)} \exp[2\Gamma(\lambda_{m_{m}'}^{m}, t)] \right\}.$$  

(26)

Note that the coherence properties of the total system is also completely governed by the presence of the reservoir, denoted by the function $\Gamma(x, t)$. This is similar to the usual dissipation of JCM [12]. However the linear entropy is the sum of "m" which is related to the value of $J_b$ and $J_c$, angular momenta of the two atomic level. This difference would result in some marvelous novel properties. In the succeeding section we will numerate some results and compare these novel properties with that of Ref. [12].

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Taking now the trace of the global density $\hat{\rho}$ on the atomic variables, we get the reduced field density

$$\hat{\rho}_F = \frac{1}{2} \sum_m \left[ \frac{1}{(2J+1)} |\alpha(t)e^{-2i\omega_m t}|^2 \right] \langle \alpha(t)e^{-2i\omega_m t}\rangle. \tag{27}$$

The linear entropy of the field is obtained by

$$S_F = 1 - \frac{1}{4} \sum_{m,m'} \left\{ \left[ \frac{1}{(2J+1)} \right] \exp(-4|\alpha(t)|^2 \sin^2 \chi_{mm'}t) + \frac{1}{(2J+1)} \exp(-4|\alpha(t)|^2 \sin^2 \lambda_{mm'}t) \right\}. \tag{28}$$

Note also that although it is the field, which is directly coupled to the reservoir, the function $\Gamma(x,t)$, characteristic function of this coupling, does not appear in the linear entropy of the field but of the atom. In order to analyze what happens to the atom, we trace out the field variables from Eq. (22) and get

$$\hat{\rho}_A = \frac{1}{2} \sum_{m,m'} \left\{ \left[ \frac{1}{(2J+1)} \right] \exp[\Gamma(\chi_{mm'}, t) + i\Theta(\chi_{mm'}, t) - |\alpha(t)|^2 (1 - e^{-2i\chi_{mm'}t})] |J_b, m\rangle \langle J_b, m'| + \frac{1}{(2J+1)} \exp[\Gamma(\lambda_{mm'}, t) - i\Theta(\lambda_{mm'}, t) - |\alpha(t)|^2 (1 - e^{-2i\lambda_{mm'}t})] |J_c, m\rangle \langle J_c, m'| \right\}. \tag{29}$$

Atomic coherence loss will be measured by its linear entropy

$$S_A = 1 - \frac{1}{4} \sum_{m,m'} \left\{ \left[ \frac{1}{(2J+1)} \right] \exp(2\Gamma(\chi_{mm'}, t) - 4|\alpha(t)|^2 \sin^2 \chi_{mm'}t) + \frac{2}{(2J+1)(2J+1)} \exp(2\Gamma(\lambda_{mm'}, t) - 4|\alpha(t)|^2 \sin^2 \lambda_{mm'}t) \right\}. \tag{30}$$

The coherence of the atom is determined by the dissipative cavity (denoted by the $\Gamma(x,t)$ function) as well as the entanglement (proportional to $|\alpha|^2$). Most important thing is that the degenerate atomic level take effect.

V. RESULTS AND DISCUSSION

The levels $b$ and $c$ in the experiments [18,19] were Rydberg states of the rubidium atom with the angular momenta $J_b = \frac{3}{2}$ and $J_c = \frac{4}{2}$ or $J_c = \frac{5}{2}$. Here we take $J_b$ and $J_c$ both are $\frac{3}{2}$, in this case,

$$\alpha_{\frac{1}{2}} = \alpha_{-\frac{1}{2}} = \frac{1}{2\sqrt{15}}, \tag{31}$$

and

$$\alpha_{\frac{3}{2}} = \alpha_{-\frac{3}{2}} = \frac{3}{2\sqrt{15}}. \tag{32}$$

According to Eq. (28), we plot the evolution of the field’s linear entropy. Note that the behavior of the coherence loss of field is not sine oscillation but we still observe that the field exhibit periodic disentanglement. As disentanglement take place, the field is in a pure state, corresponding to $S_F(t_d) = 0$. However this period $t_d$ are much longer than $t'_d = \frac{2\pi}{\Omega'}$ which are the case in the usual dissipative JCM in dispersive approximation [12]. With the parameter of our
choice, the entanglement period \( t_d = \frac{12.2}{\Omega} \). In other word, the entanglement can survive for long time. Comparing the maxima values of \( S_F \), corresponding to the maxima degree of mixture state, with that of in Ref. [12], we surprisingly find that the maxima values of \( S_F \) are greater than 0.5, the characteristic values of two statistical mixture states.

If one carefully examine the form of \( \tilde{\rho}_F \) in Eq. (27), one can see that the field are mixture of all kinds of states \( |\alpha(t)e^{\pm i\omega_m t}\rangle \). Thus the maxima degree of mixture state relate to the values of "\( m \)", and maxima values of \( S_F \) are larger than 0.5. In usual dissipative JCM, the field are mixture of the two state \( |\alpha(t)e^{\pm i\omega t}\rangle \), hence the maxima values of \( S_F \) equal to 0.5. On the other hand, different \( \omega_m \) correspond to different periods, the result of summation should take the minimum common multiple. So we can observe the longer period of entanglement. Therefore, on one hand, the degenerate atomic level increase the period of entanglement, on the other hand, it enhance the degree of maxima mixture state.

To verify the role of degeneracy of atomic level and dissipation on the coherence loss of atom and the system, we show \( S(t) \) and \( S_A(t) \) as a function of time for two values of \( \kappa \). We observe that the larger dissipation, the more rapid of the coherence loss of the atom and the system. When the atom and the field disentangle, the field is in a pure state, the atom carries alone the degree of the decoherence of the system. At the instants of disentanglement \( S(t_d) = S_A(t_d) \), while \( S_F(t_d) = 0 \). This property is the same as in the general JCM without the dissipation. We also find that the role of the degeneracy atomic level is to increase the period of entanglement and disentanglement. This is coincide to the Fig. (1). However, the increased periods of entanglement have nothing to do with the dissipation. In Fig. (3), we draw the evolution of the linear entropy of atom and the system alone with the intensity of the cavity. It is clear that the periods of the entanglement are not relate to the intensity of cavity. Note that the asymptotic value of \( S(t) \) and \( S_A(t) \) grow with the intensity and the asymptotic value break through the asymptotic limits \( \frac{1}{2} \), the characteristic of the statistical mixture. On the other hand, with the increase of the intensity of the cavity, the atom and the system lost their purity more rapidly.

**VI. CONCLUSION**

Taking into account the degeneracy of atomic level, we studied the dissipation of degenerate atom interaction with a single linearly-polarized mode field in dispersive approximation. The degeneracy of the atomic level affect the dissipation behavior of the system as well as the subsystems. We find that the degeneracy of the atomic level augment the period of entanglement between the atom and the field and increase the degree of the maxima statistical mixture states.

It is worthwhile to point out that the available of the augmented period of entanglement. The entanglement as a physical resource is available on the condition that the entanglement could keep long enough so that we can accomplish some task. For example, in Ref. [12] as we mention before, the entanglement between the first atom and the cavity field must survive long enough so as to generate the entanglement atomic state. At this point, the large period of entanglement have some advantage, although the entanglement state become complicated.
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The caption of the figures:

Fig. 1 The evolution of the field’s linear entropy where $|\alpha|^2 = 1.0$, $\kappa/\Omega = 0.01$.

Fig. 2 The linear entropy of the systems (solid line) and of the atom (dot line) as a function of $\Omega t$.

Fig. 3 Linear entropy of the systems (solid line) and of the atom (dot line) as a function of amplitude $\alpha$ where $\kappa/\Omega = 0.02$.

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