Turbulence budget in transitional plane Couette flow with turbulent stripe

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Abstract. On the basis of direct numerical simulations, we report the transport process of Reynolds stresses with respect to the turbulent-stripe structure (TSS) in a plane Couette flow at a transitional Reynolds number. The transitional structure of TSS is partially turbulent with coexisting laminar regions, oriented obliquely to the streamwise direction. To explain the inclination of the turbulent bands against the mean flow direction, we investigated the budgets for the turbulent energy, considering the non-zero Reynolds shear stresses, including $v'w'$ and $uw'$ (here, $u'$, $v'$, and $w'$ denote the velocity fluctuations in the streamwise, wall-normal, and spanwise directions, respectively). We found that, although they hardly contributed to the turbulent production, their spanwise gradient was a key factor in sustaining a secondary flow in the spanwise direction.

1. Introduction

Intermittently-turbulent flows exhibiting a periodic alternation of turbulent and laminar bands have recently attracted interest in exploring the reverse transition from turbulence to laminar. In the last decade, a transitional structure similar to the equilibrium turbulent puff of a pipe flow in the transitional regime was found to be oblique to the main flow in transitional channel flows; we denote it as ‘turbulent-stripe structure (TSS)’.

Tsukahara et al. (2005) performed a direct numerical simulation (DNS) with a moderate aspect-ratio domain of $L_x \times L_y \times L_z = 51.2\delta \times 2\delta \times 22.5\delta$—here, $x$, $y$, and $z$ denote the streamwise, wall-normal, and spanwise directions, respectively, and $\delta$ denotes the channel half width—and demonstrated that there existed a single oblique band of the TSS in the plane Poiseuille flow. On the other hand, a TSS in plane Couette flow (PCF) was probably first found experimentally by Prigent et al. (2002). Further observations of the TSS have since been reported. Barkley & Tuckerman (2005) carried out simulations in a tilted computational domain that was inclined with respect to the mean flow direction in the PCF. They revealed that, once the Reynolds number $Re_w$ (based on $\delta$ and half the wall velocity difference) decreased from 500 to 350, a TSS appeared spontaneously and maintained itself. Moreover, they considered the force balance responsible for maintaining the pattern and the Reynolds-number dependency of the wavelength and angle of the pattern (Barkley & Tuckerman, 2005, 2007; Tuckerman & Barkley, 2011). Adding to such numerical studies within frameworks of a tilted geometry with a minimal domain and of a semi-realistic model (Manneville & Lagha, 2007), DNS studies on the formation of TSSs in the PCF were performed using very huge domains by a few research
groups (Duguet et al., 2010; Tsukahara et al., 2009, 2010). The transition accompanied by the TSS, although well-described experimentally and numerically, is still far from being completely elucidated in terms of the sustaining mechanism of the inclination and the localized turbulence.

In the present study, we performed DNS with a moderate aspect-ratio domain size of the order of the distance between stripes, which allows us to capture one single oblique band of the TSS in the PCF. We investigated the transportation of kinetic energy with respect to the TSS, based on the Reynolds equation (i.e., the conditionally ensemble-averaged Navier-Stokes equation), to confirm the presence of an association between secondary flows and the Reynolds stresses including \( \nu'w' \) and \( w'u' \)—these two averaged values are essentially zero in a fully-turbulent regime, but not zero in flows with the TSS.

2. Numerical procedures of DNS and conditional ensemble average

The objective flow field is an incompressible plane Couette flow, as given in Fig. 1. Throughout this paper, \( u, v \) and \( w \) denote the velocities in \( x, y, \) and \( z \) directions, respectively. The configuration for DNS is a fully-developed (statistically steady) PCF, which is driven by the relative movement of both walls. The upper plate moves to the right and the lower plate to the left in the figure. The fundamental equations are the continuity equation and the Navier-Stokes equations:

\[
\frac{\partial u_i}{\partial x_i} = 0, 
\]

\[
\frac{\partial u_i^*}{\partial \tau^*} + u_j^* \frac{\partial u_i^*}{\partial x_j^*} = -\frac{\partial p^*}{\partial x_i^*} + \frac{1}{Re_w} \frac{\partial}{\partial x_j^*} \left( \frac{\partial u_i^*}{\partial x_j^*} \right),
\]

where all quantities with the superscript * are normalized by \( U_w \) and/or \( \delta \). The Reynolds number as a control parameter for the PCF is defined as \( Re_w = U_w \delta / \nu \), where \( \nu \) is the kinematic viscosity of fluid. We focus on the flow for \( Re_w = 350 \), at which a TSS can be expected to occur spontaneously. We acquired the present flow by decreasing the Reynolds number from a higher level of \( Re_w = 700 \) (without TSS) to 350. The size of the domain can capture a single localized turbulent region \( (L_x \times L_y \times L_z = 136\delta \times 2\delta \times 68\delta) \), The number of grid points are \( N_x \times N_y \times N_z = 1024 \times 64 \times 512 \). The periodic boundary condition is imposed in the horizontal (\( x \) and \( z \)) directions, and the non-slip condition is applied to the wall surfaces.
In order to study various (local) turbulence statistics with respect to the TSS, we adopt the moderately large size of computational domain, whose lengths are comparable to the spatial distances of the TSS in each horizontal direction (Tsukahara et al., 2009). Therefore, a single band of TSS can be captured in the present domain. Due to the periodic boundary condition, the band becomes stable once it is formed throughout the domain. As can be seen from a visualized flow field displayed in Fig. 2, the TSS exists in the form of the single band crossing the domain. Let us define \( z' \)-axis as the coordinate parallel to the diagonal line, i.e., statistically homogenous direction of the flow field. A spatial-averaged value in the \( z' \) direction and a fluctuation from the mean value are defined as:

\[
\tilde{f}(x, y) = \frac{1}{T} \int_{0}^{T} \int_{0}^{L_{z'}} f(x, y, z', t)dz'dt,
\]

\[
 f'(x, y, z) = f(x, y, z) - \tilde{f}(x, y)
\]

This conditional ensemble averaging (\( z' \)-averaging) can remove large-scale fluctuations due to the TSS (and the secondary flow) from the fluctuating component.

3. Flow field and region definition with for TSS

The mean velocity and pressure distributions in the \( x-y \) plane are shown in Fig. 3. Note that Fig. 3(a) presents the difference between \( \bar{u}^{+} \) and the conventional ensemble averaging value (viz., \( \bar{u} = (L_{x}L_{z}T)^{-1} \iint \int udz dt \)), exhibiting large-scale motions. As seen in the figure, large-scale regions of positive and negative fluctuations appear occupying the whole width in the wall-normal direction, each extending over a long distance (half length of the computational domain) in the streamwise direction. It can be confirmed that the large-scale motion occurs
also in the spanwise and wall-normal velocity components, as given in Figs. 3(b) and 3(c). This spontaneously-generated secondary flow is a typical feature of flows accompanied by TSS (Barkley & Tuckerman, 2007; Tsukahara et al., 2006).

To discuss the characteristics of the local turbulent and quasi-laminar regions by using statistical techniques, a certain criterion should be determined to distinguish between them. In this study, we chose four streamwise locations: the maximum of the turbulent kinetic energy, at the channel center \((y = 0)\), as given by the line (I) on Figs. 3 and 4; the minimum of \(\tilde{k}^+\), (III); the positive peak of \(\tilde{u}^+ - \overline{u}^+\), (II); and the negative peak of \(\tilde{u}^+ - \overline{u}^+\), (IV). The large-scale positive and negative fluctuations of the streamwise component, i.e., forward and backward motions relative to the the mean flow, collide with each other around the line (I), which causes highly disordered motions (turbulence) and the secondary flow in the spanwise direction. As can be clearly seen from Fig. 4(b), the region of (II)-(III)-(IV) may be defined as the quasi-laminar region and the other of (IV)-(I)-(II) is as the turbulent region. Moreover, we labelled the four regions demarcated by the lines, as the following, in upper channel: the region from line (IV) to (I), turbulent region 1 (TR1); (I)–(II), turbulent region 2 (TR2); (II)–(III), quasi-laminar region 1 (LR1); and (III)–(IV), quasi-laminar region 2 (LR2).

Figure 4(c) shows the distribution of an inertial term of \(\tilde{u}^+ \partial_x \tilde{u}^+\). When we focus on the upper half of channel, its positive region appears in LR1, namely, in the flow incoming to the quasi-laminar region. On the other hand, a negative one exists in TR1. This implies that the mean flow past the turbulent region is accelerated in the upstream of quasi-laminar region, and the flow decelerated in the upstream of turbulent region. Therefore, the regions labelled as TR1 and LR1 in the channel upper side are regarded as the rear interfaces of the turbulent region and of the quasi-laminar region, respectively.

4. Reynolds stresses and their budgets

In this section, we focus on the relation between the Reynolds stress and the mean flow field and investigate the budget of the transport equation for the Reynolds stress. Figure 5 shows the wall-normal distribution of a Reynolds shear stress, \(-\overline{u'v'}\), averaged in the conventional manner, namely, ensemble averaging in time and in the horizontal directions. Here, the stress and the wall-normal height are normalized using the friction velocity \(u_r\). Also shown are the profiles

\[\tilde{k}^+ = \frac{1}{2} \left( \overline{u'^+u'^+} + \overline{v'^+v'^+} + \overline{w'^+w'^+} \right), \]
of \( v'^+ w'^+ \) and \( w'^+ u'^+ \) for \( \text{Re}_w = 350 \). It is found that the magnitudes of \( v'^+ w'^+ \) and \( w'^+ u'^+ \) are as large as 3\% and 30\% of \( -u'^+ v'^+ \), respectively. These terms are negligible in the case of \( \text{Re}_w = 700 \) (or should be theoretically zero in a fully-developed turbulent Couette flow). Once a single turbulent band appears in the transitional Reynolds-number regime, the symmetry of mean flow is broken about the spanwise direction in conjunction with the secondary flow. Then the non-zero mean spanwise velocity (\( \overline{w}^+ \neq 0 \)) gives rise to the additional mean shear stresses, especially, \( u'^+ w'^+ \). The peak wall-normal height of \( u'^+ w'^+ \) is at \( y^+ = 11 \), as given in Fig. 5, and equivalent to the peak location of turbulent energy (figure not shown here).

Figure 6 shows wall-normal variations of \( u'^+ u'^+ \) at different streamwise positions in each region. It is observed that the profiles of \( u'^+ u'^+ \) are basically opposite signs for turbulent and quasi-laminar regions. This may represent consistency with that the spanwise secondary flow changes direction for those regions, as shown in Fig. 3(c). In TR1, the magnitude of \( u'^+ u'^+ \) is as large as that of \( -u'^+ v'^+ \) given in Fig. 5, which implies for \( u'^+ u'^+ \) to play an important role in sustaining TSS and the isolated turbulent region. Therefore, we investigate the energy transport equations for Reynolds stresses including \( u'^+ u'^+ \), as shown in the next section.

### 4.1. Budgets of Reynolds stress and Turbulent energy

Each budget term of the transport equation for a Reynolds stress \( u_i'^+ u_j'^+ \), in the fully-developed channel flow field, can be expressed as,

\[
\frac{\text{Du}_i'^+ u_j'^+}{\text{Dt}^+} = P_{ij} + T_{ij} + \Pi_{ij} + D_{ij} - \varepsilon_{ij} = 0, \tag{6}
\]

Production: \( P_{ij} = -\left( u_i'^+ u_k'^+ \frac{\partial u_j'^+}{\partial x_k'^+} + u_j'^+ u_k'^+ \frac{\partial u_i'^+}{\partial x_k'^+} \right) \tag{7} \]

Turbulent diffusion: \( T_{ij} = -\frac{\partial}{\partial x_k'^+} \left( u_i'^+ u_j'^+ u_k'^+ \right) \tag{8} \]

Vel. pressure-grad. corr. (VPG): \( \Pi_{ij} = u_j'^+ \frac{\partial p'^+}{\partial x_j'^+} + u_i'^+ \frac{\partial p'^+}{\partial x_i'^+} \tag{9} \]

Molecular diffusion: \( D_{ij} = \frac{\partial^2}{\partial x_k'^2} \left( u_i'^+ u_j'^+ \right) \tag{10} \]

Dissipation: \( \varepsilon_{ij} = 2 \left( \frac{\partial u_i'^+}{\partial x_k'^+} \frac{\partial u_j'^+}{\partial x_k'^+} \right) \tag{11} \]

### 4.2. Production & Dissipation

In the fully-developed turbulent PCF, the total shear stress \( \tau_{\text{total}} \) and the production of the streamwise turbulent intensity \( P_{11} \) can be expressed as

\[
\tau_{\text{total}}^+ = 1 = -u'^+ v'^+ + \frac{\partial \overline{\tau}^+}{\partial y^+}, \tag{12}
\]

and

\[
P_{11} = -2u'^+ v'^+ \frac{\partial \overline{\tau}^+}{\partial y^+}, \quad \left( \frac{\partial \overline{\tau}^+}{\partial x^+} = \frac{\partial \overline{\tau}^+}{\partial z^+} = 0 \right) \tag{13},
\]
The production term of turbulent kinetic energy was also examined in detail. As shown in Fig. 8, \( u'^+ w'^+ \partial_z \tilde{w}^+ \) occupied about 97% of the turbulent energy, whereas \( w'^+ u'^+ \partial_z \tilde{w}^+ \) and \( v'^+ v'^+ \partial_y \tilde{w}^+ \) were found to be very small. This argues that the Reynolds shear stresses of \( w'^+ u'^+ \) and \( v'^+ w'^+ \) would not contribute to the turbulence production.

From these equations, the maximum of \( P_{11} \) and its peak location are theoretically derived as,

\[
P_{11,\text{max}} = \frac{1}{2} \quad \text{at} \quad y = \left\{ y \mid \frac{\partial \tilde{t}^+}{\partial y^+} = \frac{1}{2} \right\}.
\]

As for the flow accompanied by TSS, the production in TR1 is found to be locally larger than 0.5 for \( y^+ > 7 \), as shown in Fig. 7(a). The production terms for \( v'^+ v'^+ \) and \( w'^+ w'^+ \) are very small but non-zero. The VPG term dominantly works as the energy gain term for these two components \( (v'^+ v'^+ \) and \( w'^+ w'^+) \) as compared to the production term, and is almost balances with the dissipation terms in each domain. This trend is the same as that in the fully turbulent flow.

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Figure 7. Budget of Reynolds normal stress in respective regions: ——, TR1; - - - - , TR2; ○, LR1; and □, LR2. (a) \( u'^+ u'^+ \) (b) \( v'^+ v'^+ \) (c) \( w'^+ w'^+ \). Production (black), VPG (red), and dissipation (blue).

Figure 8. Production terms of turbulent kinetic energy \( \tilde{k}^+ \). For black lines, ——, \( 2 u'^+ u'^+ \partial_x \tilde{u}^+ \); - - - - , \( 2 u'^+ v'^+ \partial_y \tilde{u}^+ \); - - - - - , \( 2 u'^+ w'^+ \partial_z \tilde{u}^+ \). For red, ——, \( 2 u'^+ v'^+ \partial_x \tilde{v}^+ \); - - - - , \( 2 v'^+ v'^+ \partial_y \tilde{v}^+ \); - - - - - , \( 2 v'^+ w'^+ \partial_z \tilde{v}^+ \). For blue, ——, \( 2 w'^+ u'^+ \partial_x \tilde{w}^+ \); - - - - , \( 2 w'^+ v'^+ \partial_y \tilde{w}^+ \); - - - - - , \( 2 w'^+ w'^+ \partial_z \tilde{w}^+ \).

Figure 9. Schematic of the turbulent-energy transportation process between turbulent and quasi-laminar regions for \( Re_\theta = 350 \).
4.3. VPG

The VPG can be decomposed into the terms of pressure strain and pressure diffusion.

\[
\Pi_{ij} = \left( p^+ \frac{\partial u_i^+}{\partial x_j^+} + p^+ \frac{\partial u_j^+}{\partial x_i^+} \right) - \left( \frac{\partial u_i^+ p^+}{\partial x_j^+} + \frac{\partial u_j^+ p^+}{\partial x_i^+} \right)
\]

The pressure-strain term has two important roles. One of roles is well known as the redistribution term, which represents the energy transfer to other diagonal components. Another role is to discreate Reynolds shear stress in non-diagonal component. Generally, it works to make turbulence isotropic. Figure 7 reveals that the VPG terms become large in the turbulent region (peculiarly in TR2) and small in the quasi-laminar region (peculiarly in LR1).

Figure 9 shows schematically the transport process of turbulent energy from the production to the dissipation. Note that the sum of the turbulent kinetic energy produced from the mean-flow energy in the whole flow field is 100%. About 73% of the total energy is generated in the turbulent region (TR1 and TR2), and the other 27% in the quasi-laminar region. In the turbulent region, 6% energy is redistributed from \( w^+ w^+ \) to \( v^+ v^+ \), and another 16% to \( w^+ w^+ \). Their counterparts in the quasi-laminar region are 2% and 5%, respectively. The redistributions from \( w^+ w^+ \) were in the ratio 7:3 for \( w^+ w^+ \) and \( v^+ v^+ \) in both regions, revealing a difference from the ratio 6:4 for high Reynolds numbers, i.e., \( Re_w = 700 \). It is interesting to note that the energy transport between the turbulent and quasi-laminar regions is negligibly-small, and hence the produced turbulent energy is dissipated in each region where it occurs. As for TSS in the Poiseuille flow, Kaneko et al. (2010) reported that the energy transferred by the advection term to the quasi-laminar region is 3% of the total produced turbulent energy and that 2% is changed adversely to the mean-flow energy. The present result might uncover different features in TSS between the Couette flow and the Poiseuille flow.

5. Reynolds equation

The Reynolds equation for \( \tilde{u}_i^+ \) is defined as

\[
\tilde{u}_i^+ \frac{\partial \tilde{u}_i^+}{\partial x_i^+} + \tilde{v}^+ \frac{\partial \tilde{u}_i^+}{\partial y^+} + \tilde{w}^+ \frac{\partial \tilde{u}_i^+}{\partial z^+} = \frac{1}{Re_w} \nabla^2 \tilde{u}_i^+ - \frac{\partial \tilde{u}_i^+ u_j^+}{\partial x_j^+} - \frac{\partial \tilde{u}_j^+ u_i^+}{\partial y^+} - \frac{\partial \tilde{u}_j^+ w_i^+}{\partial z^+}.
\]

Here, the time-derivative term is neglected. The advective term in the left-hand side of Eq. (16) represents the momentum transfer by the mean flow (including the secondary flow), with which the sum of the pressure, viscous, and Reynolds-stress terms are balanced. We investigated those terms in each region, in order to discuss the contribution of the Reynolds-stress tensor \( u_i^+ u_j^+ \) with the mean flow field \( \tilde{u}_i^+ \). Moreover, we aimed to identify aspects of the dominant force balances, which plays a key role in sustaining the turbulent-laminar pattern.

Figure 10 shows the wall-normal distributions of various forces for the streamwise momentum component. The pressure force is found to be quite small, while the advective, viscous and Reynolds-stress forces are balanced in the whole area. The distributions of other forces show point-symmetric profile with respect to the channel center. In the turbulent region, the viscous and Reynolds-stress forces are almost comparable and balances, as shown in Fig. 10(a). The advective force of the fluid moving is in the same direction with the viscous force. In the quasi-laminar region, the advective forces is a dominating term and balances with the the viscous force:
Figure 10. Balances of forces in the streamwise direction. Curves show advective force(——), Reynolds stress force(- - - -), viscous force(○ ) and pressure gradient(●).

Figure 11. Advective, Viscous and Reynolds stress terms in the streamwise direction in turbulent region. Curves(a) show $\partial_x^2 u'^2$ (——), $\partial_y^2 u'^2$ ( - - - - ), $\partial_z^2 u'^2$ (● ). Curves(b) show $\bar{u}'\partial_x \bar{u}'$ (——), $\bar{v}'\partial_y \bar{u}'$ ( - - - - ), $\bar{w}'\partial_z \bar{u}'$ (● ). Curves(c) show $\partial_x u'^2$ (——), $\partial_y u'^2$ ( - - - - ), $\partial_z u'^2 w'^2$ (● ).

see Fig. 10(b). These results show a qualitatively good agreement with those obtained using a minimal-PCF simulation by Barkley & Tuckerman (2007).

Figure 11 shows the decomposition of the viscous terms (a), the advective terms (b), and Reynolds stress terms (c), in the turbulent region. It is observed that the advective term of $\bar{u}'\partial_x \bar{u}'$ is dominant, while $\nabla^2 \bar{u}'$ and the Reynolds-stress term are dominated by $\partial_y^2 \bar{u}'$ and $\partial_y u'^2 v'^2$, respectively.

With respect to the spanwise momentum transfer, we found that the distribution of the Reynolds-stress force term for $\bar{w}'$ (see Eq. (16)) revealed a close similarity to that of $\bar{w}'$, as shown in Fig. 12(b). Based on this, it can be suggested that the term of the spanwise gradient of Reynolds stresses play a role in sustaining the spanwise secondary flow in TSS.

6. Conclusion

We studied the turbulent stripe in a plane Couette flow using DNS and obtained the following results. Figure 13 illustrates the schematic of the role of the Reynolds stress in TSS. Through the pressure-strain terms, the $u'u'$ energy is redistributed to the other components of $v'v'$ and $w'w'$ (as presented by the green arrows in the figure). Those averaged redistribution were in the ratio 7:3 for $\overline{w'^2 w'^2}$ and $\overline{v'^2 v'^2}$, The magnitude of the Reynolds stresses of $\overline{v'^2 w'^2}$ and $\overline{w'^2 u'^2}$ are as large as 3% and 30% of $-\overline{u'^2 v'^2}$, respectively, but their contributions to the turbulent production are almost negligible. The Reynolds stress $\overline{v'^2 w'^2}$ and $\overline{w'^2 u'^2}$ are generated as shown.
Figure 12. Quasi-mean spanwise velocity $\tilde{w}^+ [-0.08, 0.08]$ (a) and Reynolds-stress force term $\partial \tilde{w}^+ u_j^+ / \partial x_j^+ [-0.4, 0.4]$ (b) in an $(x-y)$ frame. Color ranges from blue to red.

Figure 13. Schematic of the role of the Reynolds stress.

by the blue arrows. We found a close similarity between distributions of the quasi-mean spanwise velocity and of the Reynolds-stress force terms, $\partial \tilde{w}^+ u_j^+ / \partial x_j^+$. Based on this, we conjectured that the gradient of the Reynolds stress containing the spanwise velocity fluctuation ($w'$) would contribute the secondary flow in the spanwise direction (as indicated by the orange arrows).

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