TREATMENT OF BARYONIC RESONANCES IN THE RQMD APPROACH INCLUDING SCALAR–VECTOR MEAN FIELDS

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Abstract

In the relativistic Quantum Molecular Dynamics (RQMD) approach baryons are described within the framework of covariant hamilton constraint dynamics. The inclusion of a relativistic mean field results in a quasiparticle picture for the baryons. This requires to distinguish between canonical and kinetic variables of the particles. As resonances we include the $\Delta(1232)$ and the $N^*(1440)$ resonance. The resonance masses are distributed according Breit-Wigner functions. However, the scalar self energy leads to a shift in the masses and introduces an additional medium dependence. Consequences of this description on resonance and pion dynamics are discussed.

1 Introduction

In relativistic heavy ion collisions, e.g. in the SIS energy range excited nucleon states play a decisive role in the reaction dynamics. Most important is the $\Delta(1232)$ resonance which can reach abundances comparable to nuclear matter saturation density \cite{1}, however also the $N^*(1440)$ plays a non-negligible role. The decay and rescattering of these resonances are the predominant sources of meson production. In order to use such mesons as a source of information of the hot and compressed phase in heavy ion reactions the understanding of their origin, i.e. of the properties of excited nucleon states appear to be indispensable. Microscopic studies of the excitation and deexcitation of nucleons can be performed within the framework of transport models like Quantum Molecular Dynamics (QMD) \cite{2} and the BUU-type models.

In the covariant extension of QMD, i.e., the Relativistic QMD (RQMD) \cite{3,4} up to now only static Skyrme forces were used. These generalized Skyrme forces are treated as scalar potentials in the framework of Constrained Hamil-
ton Dynamics. But the full Lorentz-structure of the nucleon-nucleon interaction contains large scalar and vector components. Effects of these relativistic forces were already studied in the framework of covariant generalizations of models of the BUU-type [5, 6, 7], mostly in the framework of the Walecka model [8] and its non-linear extensions, but not yet in RQMD. In Ref. [11] the formulation of a scalar-vector RQMD has been given. Here we study in particular the influence of the relativistic mean field on the resonance dynamics.

2 RQMD with scalar-vector mean fields

The relativistic self-energy \( \Sigma = \Sigma_s - \gamma_\mu \Sigma^\mu \) contains scalar and vector components. Thus, one has to distinguish between canonical momenta \( p_i \) and bare masses \( M_i \) on the one hand and kinetic momenta \( p_i^\ast \) and effective (Dirac) masses \( m_i^\ast \) on the other hand. The latter correspond to the quasiparticles dressed by the surrounding medium which obey the in-medium Dirac equation

\[
(\gamma_\mu p_i^{\ast \mu} - m_i^\ast) u_i^\ast = 0
\]

where \( u_i^\ast \) is an in-medium spinor or a Rarita spinor in the case of a \( \Delta \).

In the formalism of Constrained Hamiltonian Dynamics [9, 10], the 8N dimensional phase space of N interacting relativistic particles is reduced to 6N dimensions by 2N constraints which fix the individual energies and by the mass-shell constraints parametrize the world lines by fixing the relative time coordinates.

With respect to Eq. (1) the N on-mass-shell conditions for the four-momenta are given in terms of the quasiparticles

\[
K_i = (p_i^{\mu} - \Sigma_i^{\mu})(p_i^{\mu} - \Sigma_i^{\mu}) - (M_i - \Sigma_{is})^2 = p_i^{\mu} p_i^{\ast \mu} - m_i^{\ast 2} = 0 ,
\]

which contain scalar and vector self-energies. Hence, the constraints, Eq.(2), take the full Lorentz structure of the relativistic mean field into account. In the relativistic Hartree approximation the scalar and vector self-energies are proportional to the scalar density \( \rho_s \) and the baryon current \( B^\mu \), respectively

\[
\Sigma_s = \Gamma_s \rho_s , \quad \Sigma^\mu = \Gamma_v B^\mu .
\]

The proportionality factors \( \Gamma_{s,v} \) in Eq. (3) may either be constants as in the Walecka model [8] or, if one includes higher order medium effects, density dependent functions as discussed, e.g, in Ref. [11]. Here we apply the non-linear Walecka model (NL2) which can be expressed in the form of Eq. (3) with

\[
\Gamma_s(\rho_s) = \frac{g_s^2}{m_s^2 - B \Phi(\rho_s) - C \Phi^2(\rho_s)}
\]
where $\Phi$ is the scalar $\sigma$–meson field [11]. The scalar density and the baryonic current

$$\rho_s(q_i,p_i) = \sum_{j \neq i} \rho_{ij} \ , \quad B_\mu(q_i,p_i) = \sum_{j \neq i} \rho_{ij} u_{j\mu} \quad (5)$$

are determined in terms of the scalar two-body densities

$$\rho_{ij} = \frac{1}{(4\pi L)^{3/2}} \exp \left( \frac{q_{Tij}^2}{4L} \right) \ , \quad (6)$$

with $u_j^\mu = p_j^\mu/m_j^*$ being the 4-velocity of particle $j$ and $q_{Tij}$ the invariant center-of-mass distance of the particles $i$ and $j$ as defined in Ref. [3].

The total Hamiltonian is given by the $N$ on-shell constraints, Eq. (2), and by $N-1$ time fixation constraints $\phi_i$ chosen as in [3]. These time constraints ensure that interacting particles have equal times in their center-of-mass frame. A final constraint fixes the global time evolution parameter. The Hamiltonian

$$H = \sum_{i=1}^{N} \lambda_i K_i + \sum_{i=N+1}^{2N-1} \lambda_i \phi_i \quad (7)$$

generates the equations of motions for the canonical conjugated coordinates and momenta

$$dq_i^\mu/d\tau = [H, q_i^\mu] \ , \quad dp_i^\mu/d\tau = [H, p_i^\mu] \quad (8)$$

where $[A,B]$ means the Poisson bracket of phase space functions $A$ and $B$. Notice that in the present formalism canonical and kinetic momenta are no independent quantities. To integrate the set of above equations (8), one has to determine the unknown Lagrange multipliers $\lambda_i(\tau)$. This can be done using the fact that the complete set of $2N$ constraints must be fulfilled during the whole time evolution, i.e. $dK_i/d\tau = d\phi_i/d\tau = 0$. If the Dirac’s first class condition is fulfilled, i.e. $[K_i, K_j] = 0$, the Hamiltonian is reduced to

$$H = \sum_{i=1}^{N} \lambda_i K_i \ , \quad \lambda_i = \Delta_{iN}^{-1} \quad (9)$$

where $\Delta_{ji} = [K_j, \chi_i]$ is a submatrix of the complete constraint matrix $C_{ji}$ [3, 4].

In this approach the dynamics are to high extent determined by the Lagrange multipliers $\lambda_i(\tau)$. Thus in Fig.1 we show the time evolution of $\lambda$ for a representative nucleon in a central Ca+Ca collision at 1.85 A.GeV for both, a pure mean field calculation and including binary collisions. It is clearly seen that $\lambda$ is a smooth function in time if its evolution is governed only by the mean
field dynamics. However, in the high density phase \((10 \text{fm/c} \leq t \leq 20 \text{fm/c})\) strong fluctuations occur which are due to binary collisions. These fluctuations represent rapid changes in the momenta and—in the case of a resonance creation or decay—the masses of the particles.

![Graph](image)

**Fig. 1.** Time evolution of a lagrange multiplier in a central Ca+Ca reaction at 1.85 A.GeV. A full calculation including binary collisions (solid) is compared to a pure mean field calculation (dashed).

The inclusion of a relativistic mean field which introduces an additional momentum dependence into the model is essential in order to retain the correct dynamics at incident energies above about 1 A.GeV. In **Fig. 2** we compare the present approach to the standard RQMD [3, 4] which uses static Skyrme forces. From **Fig. 2** which shows the transverse flow per particle for the reaction Ar+KCL at 1.8 A.GeV under minimum bias condition it is clearly seen that the Skyrme forces result in far too less repulsive mean fields whereas NL2 is in good agreement with the data.

### 3 Baryonic resonances

The mean field propagation described in the previous section is the same for nucleons and baryonic resonances. In the latter case the bare nucleon mass in Eq. (3) has to be replaced by the corresponding resonance mass \(M_R\), i.e. \(m^*_{R} = M_R - \Sigma_{sR}\). Thus we apply the same mean field to nucleons and resonances, i.e., the coupling strength of the respective mesons is assumed to be identical.
Since we restrict to non-strange mesons this appears to be reasonable. In the present approach the coupling functions $\Gamma_{s,v}$ have to be interpreted as effective quantities which parametrize the mean field rather than elementary vertex functions which further justifies this assumption.

![Graph showing transverse flow per particle for the reaction Ar+KCl at 1.8 A.GeV.](image)

Fig. 2. Transverse flow per particle for the reaction Ar+KCl at 1.8 A.GeV. Static Skyrme forces (soft/hard) are compared to the non-linear Walecka model NL2 and to data from Ref. [12].

For the inelastic nucleon-nucleon channels we include the $\Delta(1232)$ as well as the $N^*(1440)$ resonance with the cross sections of Ref. [13]. The lifetimes of the resonances are determined through their energy and momentum dependent decay widths

$$\Gamma(|p|) = \frac{a_1|p|^3}{(1 + a_2|p|^2)(a_3 + |p|^2)}\Gamma_0$$

which originates from the $p$-wave representation of the resonances. In Eq. (10) $p$ is the momentum of the created pion (in GeV/c) in the resonance rest frame. According to Ref. [13] the values $a_1=22.83$ (28.8), $a_2=39.7$ and $a_3=0.04$ (0.09) are used for the $\Delta$ ($N^*$) and the bare decay widths are taken as $\Gamma_0^{\Delta}=120$ MeV and $\Gamma_0^{N^*}=200$ MeV.

Since only the quasiparticles, i.e. kinetic momenta and effective mass lie on the mass-shell all collisions are performed in the kinetic center-of-mass frame with $\sqrt{s} = \sqrt{P_{CM}^2 + m_N^2} + \sqrt{P_{CM}^2 + m_R^2}$. In the case of an inelastic collision, e.g. $NN \rightarrow NR$ the final momentum can be evaluated

$$|p_{CM}^*| = \frac{1}{2\sqrt{s}} \sqrt{\lambda(s^*, m_N^*, m_R^*)}$$

(11)
with \( \lambda = [s^* - (m^*_N + m^*_R)^2][s^* - (m^*_N - m^*_R)^2] \). The probability distribution for the effective resonance mass is given by a Breit-Wigner distribution

\[
A(m^*_R) = \frac{1}{\pi} \frac{\Gamma/2}{(m^*_R - m^*_R)^2 + (\Gamma/2)^2}
\]

(12)

with \( m^*_R = M^*_R - \Sigma sR \). The maximal possible bare mass of the created resonance is then restricted by

\[
M^*_R^{\text{max}} = \sqrt{s^* - m^*_N + \Sigma sR}.
\]

(13)

Fig. 3. Shift of the maximal resonance mass by the scalar self-energy in nuclear matter (solid, left scale). In addition the effective nucleon mass \( m^* \) is shown (dashed, right scale). The non-linear Walecka model NL2 is applied.

Fig.3 shows the medium dependence of \( M^*_R^{\text{max}} \) created in a \( NN \rightarrow NR \) collision. Here we have choosen the momenta of the incident nuclei \( (p^*_1 = -p^*_2) \) to be 0.6 GeV/c in the nuclear matter rest frame. It is seen that \( M^*_R^{\text{max}} \) is considerably enhanced by the presence of the medium, i.e. by the attractive scalar self-energies which is correlated to the reduction of the effective mass \( m^* \) also shown in Fig.3. Thus the probability for the excitation of higher resonances is generally enhanced in the presence of relativistic mean fields. This effect is even more pronounced when models are applied which result in larger fields than NL2 as, e.g., the original Walecka model [8].
The reabsorption processes ($\pi N \rightarrow \Delta, N^*$) are treated as described in Ref. [15] again adopting the cross sections from Ref. [13].

Finally in Fig. 4 we show the pion $p_t$-spectrum obtained in a 1.85 A.GeV Ni+Ni collision. The contributions from pions originating from $N^*$-resonances are shown separately and it is seen that these pions contribute over all by about 10% to the total yield. However, $N^*$ pions are in particular relevant for the high energetic part of the spectrum. Here the contributions from $N^*$ are more than 20%. Since high energetic pions are supposed to be most likely produced in the early phase of the reaction [14] they really probe the high density phase and thus can give signals on the medium dependence of the resonances.

![Graph](image)

**Fig. 4.** Transverse $\pi^0$ spectrum in a Ni+Ni collision at 1.85 A.GeV under minimal bias condition. The contribution from pions originating from $N^*$ are shown separately.

To summarize we have applied relativistic mean fields with scalar and vector components in the formalism of Hamilton Contrain Dynamics. This results in a quasiparticle picture for baryons and resonances. The attractive scalar fields lead thereby to a shift of the resonance mass distributions towards higher values and thus reduces the thresholds for the excitation of high lying resonances. Concerning heavy ion reactions in the SIS domain we found that the $N^*$ gives important contributions to the pion spectra. In particular high energetic pi-
ons will be well adopted to study the influence of medium effects on baryonic resonances.

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