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Collective amplitude mode fluctuations in a flat band superconductor formed at a semimetal surface

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We study the fluctuations of the amplitude (i.e., the Higgs-Anderson) mode in a superconducting system of coupled Dirac particles proposed as a model for possible surface or interface superconductivity in rhombohedral graphite. This system also serves as a generic model of a topological semimetal with an interaction driven transition on its surface. We show that the absence of Fermi energy and vanishing of the excitation gap of the collective amplitude mode in the model leads to a large fluctuation contribution to thermodynamic quantities such as the heat capacity. As a consequence, the mean-field theory becomes inaccurate indicating that the interactions lead to a strongly correlated state. We also present a microscopic derivation of the Ginzburg-Landau theory corresponding to this model.

A topological flat band spectrum can emerge as a surface or interface state of topological semimetals.1,2 Such a spectrum has been proposed to occur in various semimetals, such as rhombohedrally stacked graphite3–5, at dislocation interfaces in bernal stacked graphite6, edges of zig-zag graphene7, three-dimensional honeycomb structures8, inversion-symmetric crystals9, antiperovskite Cu3PdN10, a polymorph of Ca3P211 and three-dimensional graphene networks12. A flat band can also take place at the interfaces between crystalline topological and non-topological insulators13. Such a state has a singular density of states which can, in the presence of an attractive pairing between the particles, lead to superconductivity with an unusually high critical temperature14,15. This mechanism has been suggested as an explanation to the unusually high critical temperature $T_c$ in some IV-VI semiconductor heterostructures16–18 and in various graphite based materials19–21. In the Hubbard model, related work has also been done to study superconductivity in the presence of a flat band22 and with Fermi energy close to van Hove singularities23.

We study here the effect of fluctuations on the superconducting properties of flat band superconductors in topological semimetals. We use the particular model proposed in Refs. 4 and 5 and also numerically analyzed in Ref. 24 for rhombohedrally stacked graphite. However, we argue that the form of the dispersion (1) and therefore most of the qualitative conclusions of this paper apply to all flat band surface states of topological semimetals. The model consists of $N$ individual graphene layers with s-wave pairing between the Dirac electrons coupled to form a stack of rhombohedral graphite. The flat band is formed on the surfaces of the stack where the superconductivity also appears (see Fig. 1). A peculiarity of this model is the closing of the gap in the fermionic excitation spectrum at the limit of a large number of layers even with an isotropic s-wave mean-field order parameter as can be seen from the surface state spectrum (see Fig. 1).5

$$E_p^2 = (1 - p^2/p_{FB}^2) \gamma^2 (\Delta_0^2 + \xi_p^2),$$

where $\xi_p = \gamma_1 |p/p_{FB}|^N$, $\gamma_1$ is the interlayer coupling constant, $p_{FB} = \gamma_1/v_F$ is the width of the flat band and $\Delta_0$ is the mean-field order parameter. (We use everywhere $\hbar = k_B = 1$.) This expression is valid for $|p/p_{FB}| < 1 - 1/N$.25 The gap closes at the flat band edge along the line $|p| = p_{FB}$ in the bulk limit $N \to \infty$. This is due to the fact that there the surface states penetrate into the bulk and surface superconductivity cannot create a gap for the bulk excitations. This hence applies to all topological semimetals with a flat band surface state. Note that this spectrum is qualitatively different than, for example, in d-wave superconductors, where the gap in the fermionic spectrum vanishes in a set of points in the momentum space.

Even without making detailed calculations, simple arguments as for why fluctuations are expected to matter in this model can be made. Typically the magnitude of fluctuations is characterized by the Ginzburg number $G_i \sim 1 - T^*/T_c$, which is related to the temperature $T^*$ above which the fluctuations dominate the thermodynamical properties. In a 2D BCS superconductor, the Ginzburg number is given by $G_i \sim T_c/E_F$, where $E_F$ is the Fermi energy. For the flat band, $E_F = 0$, leaving no other relevant energy scale available to be compared with $T_c$ which suggests5 that $G_i \sim 1$. However, this argument is based on the analysis close to $T_c$ and it does not tell how low temperatures are needed to get rid of the effect of fluctuations. Below we show that due to the line node at $|p| = p_{FB}$ in the superconducting state, the fluctuation contribution is relevant down to $T = 0$.

The aim of this paper is to show that in this model, the amplitude fluctuations provide a large contribution to thermodynamic observables (namely, the heat capacity). We show this both within the Ginzburg-Landau theory (close to $T_c$) and within a more general calculation based on functional integral formalism for the partition function.
A microscopic calculation from the general action for our model, given in the next section, yields the following values for the Ginzburg-Landau coefficients:

\[
\alpha_1 = \frac{p_{FB}^2}{48\pi T_c^2 T_c} \frac{T - T_c}{T_c}, \quad \alpha_2 = \frac{1}{32\pi T_c^2}, \quad \beta = \frac{p_{FB}^2}{16\pi^2 T_c^2}, \quad \gamma = \frac{2\gamma_e^2}{8\pi T_c N'}. \tag{4}
\]

From these coefficients one can extract a coherence length \( \xi^2 \sim \alpha_2/|\alpha_1| = 3T_c/(2p_{FB}^2(T_c - T)) \), which is very short \((\xi \sim v_F/\gamma_1 \ll v_F/\Delta)\) for \( T \) sufficiently below \( T_c \) indicating the strong coupling nature of the superconductivity.

To consider fluctuations for \( T < T_c \), we expand around the mean-field value of \( \Delta_0^2 = -(\alpha_1 - \gamma)/2\beta \) as \( \Delta(x) = (\Delta_0 + \delta(x))e^{i\phi(x)} \). The fluctuation part of the action can then be written in terms of four fluctuation modes:

\[
S_1 = \int d^2x \frac{\alpha_2}{4\beta}|\alpha_1 - \gamma|(|\nabla \phi^+|^2), \quad S_2 = \int d^2x \left[ \frac{\alpha_2}{2} (|\nabla \delta|^2 + |\alpha_1 - \gamma| \delta^2) \right],
\]

\[
S_3 = \int d^2x \gamma|\alpha_1 - \gamma| \phi_2^2 + \frac{\alpha_2 |\alpha_1 - \gamma| (|\nabla \phi^-|^2)}, \quad S_4 = \int d^2x \left[ \frac{\alpha_2}{2} (|\nabla \delta|^2 + |\alpha_1 - 2\gamma| \delta^2) \right] \tag{5}
\]

so that \( S_{GL} = S_{MF} + S_1 + S_2 + S_3 + S_4 \), where the mean-field part is given by \( S_{MF} = -(\alpha_1 - \gamma)^2/(2\beta) \). The four fluctuation modes can be identified as the total phase (Nambu-Goldstone) mode (S1), the total amplitude (Higgs-Anderson) mode (S2), the relative phase (Leggett) mode (S3), and the relative amplitude mode (S4). In this new diagonal basis, the four independent fields are defined as \( \phi_\pm = \phi_1 \pm \phi_N \) and \( \delta_\pm = \delta_1 \pm \delta_N \). In the limit \( N \to \infty \), the two phase and the two amplitude modes become identical and the action separates into two identical standard fluctuating Ginzburg-Landau actions with coefficients given by (4). The standard Ginzburg-Landau theory calculations can then be applied\(^{27}\) and we find that the total amplitude mode contribution to the heat capacity below \( T_c \) becomes\(^{28}\)

\[
\delta C_{amplitude} = \frac{\delta}{3\pi} \left( -\frac{T}{T_c - T} \right)^2 A. \tag{6}
\]

Here \( A \) is the surface area of the sample. In the lowest order in \( T_c - T \), the phase mode does not contribute to the heat capacity. Setting \( \delta C_{amplitude}(T^*) \) equal to the heat capacity jump at the transition,

\[
\Delta C = Ap_{FB}^2 \frac{5}{6\pi} \sim \frac{\alpha_1^2}{\beta} \left( \frac{T_c}{T_c - T} \right)^2 A, \tag{7}
\]

and solving for temperature, \( T^* \), yields the Ginzburg number

\[
G_i \equiv \frac{T_c - T^*}{T_c} = \frac{2}{5}. \tag{8}
\]

This is of the order of unity as expected from purely dimensional arguments. While at the reduced temperature...
indicated by the Ginzburg number, the Ginzburg-Landau approach is not, strictly speaking, valid, this nevertheless gives us an estimate of the size of the fluctuations.

To understand the large fluctuations and compare them to conventional superconductors, we note that in both cases the fluctuation contribution and the heat capacity jump are proportional to the same Ginzburg-Landau coefficients, but the microscopic values of the coefficients differ greatly. In a conventional two-dimensional BCS superconductor $\alpha_1 = N(0)(T - T_c)/T_c^2$, $\alpha_2 = 7\zeta(3)N(0)v_F^2/(32\pi^2 T_c^3)$ and $\beta = 7\zeta(3)N(0)/(16\pi^2 T_c^3)$, where $\zeta(x)$ is the Riemann zeta function, $N(0)$ is the density of states at the Fermi level and $v_F$ is the Fermi velocity. There is one additional parameter compared to the case of the flat band model. This leads to different scales for the heat capacity jump and the fluctuation heat capacity in two-dimensional BCS superconductors

$$\delta C_{\text{amplitude}} \sim \frac{T_c}{E_F |T - T_c|} \Delta C$$

resulting in a much smaller Ginzburg number $G_i \sim T_c/E_F$ compared to the flat band superconductors.

At low temperatures, the Ginzburg-Landau approach is invalid and we need a microscopic theory to account for the fluctuations. For the surface state, we can use the Bogoliubov-de Gennes equation derived in Ref. 4 to find that the action, after integrating out the Fermionic degrees of freedom, has the form

$$Z = \int \mathcal{D}\xi \mathcal{D}\Delta \exp\left\{-\frac{V}{gT} \sum_p (|\Delta_{i,p}|^2 + |\Delta_{N,p}|^2) + \frac{\tau}{T} \sum_p \left[ \Delta_{i,p} \cdot \mathcal{F}_p \right] \right\},$$

where $\mathcal{F}_p = \left( \begin{array}{cc} -W_q + \frac{\delta}{gT} & D_q \\ -\chi_q & -W_q + \frac{\delta}{gT} \end{array} \right)$ and

$$\delta \Delta_q = \frac{1}{2} \sum_p \sum_{\xi} \bar{\delta} \Delta_q^\dagger \left( \begin{array}{cccc} -W_q + \frac{\delta}{gT} & D_q \\ -\chi_q & -W_q + \frac{\delta}{gT} \end{array} \right) \bar{\delta} \Delta_q$$

and the mean-field part by

$$S_{MF} = -4 \sum_p \ln \left[ \cosh \left( \frac{E_p}{2T} \right) \right] + \frac{2V}{gT} |\Delta_0|^2. \quad (13)$$

In the latter we have performed the summation over the Matsubara frequencies and $E_p$ are the positive quasiparticle energies given by Eq. (1). We have here written the action in a convenient matrix form with the vector fluctuation fields given by $\delta \Delta_q = (\delta \Delta_{1,q}, \delta \Delta_{1,-q}, \delta \Delta_{N,q}, \delta \Delta_{N,-q})^T$. The matrix elements in the fluctuation action are given in terms of three po-

larization operators. Expressions for them are

$$W_q = \sum_p \left( \frac{\bar{\delta} \Delta_q}{E_p + \Delta_0^2 + \xi_p^2 + \xi_{p-q}^2} \right) \left( \frac{\bar{\delta} \Delta_q}{E_p - \Delta_0^2 + \xi_p^2 + \xi_{p-q}^2} \right),$$

$$D_q = \sum_p \left( \frac{\bar{\delta} \Delta_q}{E_p + \Delta_0^2 + \xi_p^2 + \xi_{p-q}^2} \right) \left( \frac{\bar{\delta} \Delta_q}{E_p - \Delta_0^2 + \xi_p^2 + \xi_{p-q}^2} \right),$$

$$\chi_q = \sum_p \left( \frac{\bar{\delta} \Delta_q}{E_p + \Delta_0^2 + \xi_p^2 + \xi_{p-q}^2} \right) \left( \frac{\bar{\delta} \Delta_q}{E_p - \Delta_0^2 + \xi_p^2 + \xi_{p-q}^2} \right). \quad (14)$$

As for the Ginzburg-Landau action, the four eigenmodes
of the fluctuation action correspond to relative and total phase fluctuations and relative and total amplitude fluctuations. Also, again, in the limit of a large number of layers, only the total amplitude and total phase modes survive and the third polarization operator, $X_q$, vanishes.

From the Ginzburg-Landau calculation, we know that the dominating contribution to the thermodynamics in the large $N$ limit comes from the amplitude fluctuation part. Taking into account only this mode, the fluctuation contribution to the free energy, $\delta F = -T \ln \delta Z \approx \delta F_{\text{amplitude}}$, becomes

$$\delta F_{\text{amplitude}} = 2T \sum_q \ln \left( 1 - \frac{W_q - D_q}{V/gT} \right).$$

(15)

We calculate $\delta F_{\text{amplitude}}$ as well as the heat capacity, $\delta C_{\text{amplitude}} = -T \partial^2 \partial_T \delta F_{\text{amplitude}}$, numerically. The result for the free energy is shown in Fig. 2. For zero temperature, the fluctuation contribution goes to zero, but grows larger than the mean-field contribution as the temperature increases. Above the critical temperature, $\delta F_{\text{amplitude}}$ saturates into a finite value.

![Fig. 2.](image)

**FIG. 2.** (Color online) Free energy contribution from the amplitude fluctuations and from the mean-field calculation.

For the heat capacity, the numerical result is shown in Fig. 3. At low temperatures we find that the fluctuation contribution to the heat capacity becomes equal to the mean field heat capacity $C_{MF}$, both given by

$$C(T \ll T_c) = \frac{A \rho^2_B \pi}{6 \Delta_0} \frac{T}{\Delta_0}. \quad (16)$$

For $C_{MF}$ Eq. (16) straightforwardly follows from Eq. (13). On the other hand, the fact that $\delta C_{\text{amplitude}}$ is also given by Eq. (16) can be analytically understood by assuming that it comes from a free boson contribution corresponding to the amplitude mode dispersion, which can be determined from the corresponding eigenvalue of the matrix in the fluctuation action (12) (the method is described for example in\textsuperscript{29}). Namely, this way we find that for $T = 0$ and $N \rightarrow \infty$ the amplitude mode dispersion is approximatively given by

$$E_{p,\text{ampl}}(T = 0) = \Delta_0 p^2 / p_B^2. \quad (17)$$

and the corresponding heat capacity is indeed given by Eq. (16). For higher temperatures, the fluctuation contribution completely dominates over the mean field contribution. At $T_c$, the heat capacity diverges as $\sim 1/|T - T_c|$ as predicted by the Ginzburg-Landau theory. The temperature at which the fluctuation heat capacity equals the mean-field heat capacity jump at the transition $\Delta C$ is also shown (red curve). Inset shows the low-temperature behaviour of the three curves.

![Fig. 3.](image)

**FIG. 3.** (Color online) Specific heat contribution from the amplitude fluctuations (green curve) and from the mean-field calculation (blue curve). For comparison the fluctuation contribution for a conventional BCS superconductor with $T_c = 10^{-4}E_F$ and equal heat capacity jump at the transition $\Delta C$ is also shown (red curve). Inset shows the low-temperature behaviour of the three curves.
closes and the contribution from the fermionic mode and the bosonic amplitude mode can be of the same order of magnitude at low temperatures. Therefore, finite $N$ corrections to our result would likely diminish the fluctuation contribution, improving the validity range of the mean-field theory.

Since vanishing of the superconducting excitation gap along a 1D line is a general property of all topological semimetals with flat band surface state, we expect that our prediction of large fluctuation contribution and massless collective excitation spectrum applies to all materials within this class. However, this is not a general property to all superconducting states in systems exhibiting a flat normal state spectrum, as some of them may well exhibit a gapful superconducting spectrum.

Flat bands can also emerge at the surface in superconductors with a nodal order parameter structure possibly resulting in correlated states$^{30-34}$. Similarly as in the present situation, the fluctuation contribution to the heat capacity may also dominate the mean field contribution in these correlated states, so that the mean field theory becomes inaccurate and the interactions lead to a strongly correlated state. In the bulk of these superconductors, the presence of the nodal points leads to a decrease of the amplitude mode masses$^{35,36}$, but contrary to the superconducting topological semimetal surface states, this mass does not vanish.

We consider here only the effect of the amplitude mode to thermodynamics since we assume that it dominates over the phase mode contribution. At reasonably high temperatures it follows from general Ginzburg-Landau theory that the amplitude mode contribution dominates because of its singularity at $T = T_c$. At lower temperatures, the phase mode contribution to thermodynamic properties in a charged system is also influenced by the coupling to the electromagnetic field$^{35,36}$. In the calculation of this coupling it should be taken into account that although the order parameter appears only close to the surface the supercurrent flows also in the bulk$^{37}$.

Lastly, we want to emphasize that we do not consider the coupling of the fluctuations to external fields. The large contribution from the amplitude fluctuations to the heat capacity might or might not appear in other observables depending on whether they couple to this degree of freedom or not. For example, it is known that the electromagnetic field does not couple to the amplitude mode at $T = 0$ directly$^{38}$ and thus such a large fluctuation effect is not necessarily present in the charge current.

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capacity depends also on the choice of the momentum cut-off\textsuperscript{25}.

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