Abstract  Logic’s historically central mission has been to provide formally precise descriptions of logical consequence. This was done with two broad expectations in mind. One was that a pre-theoretically recognizable concept of consequence would be present in the ensuing formalization. The other was that the formalization would be mathematically mature. The first expectation calls for conceptual adequacy. The other calls for technical virtuosity. The record of the past century and a third discloses a tension between the two. Accordingly, logicians have sought a reasoned, if delicate, rapprochement, one in which each expectation would be given its due, but well-short of free sway. Recent developments have imperiled this perestroika. One is logic’s massive and often rivalrous pluralism, and the cheapening relativism to which it beckons. This is exacerbated by the long-acknowledged part that the formal representations of logic distort the logical particles of natural language. The present paper discusses what might be done about this.

Keywords  Analysis · Conceptual adequacy · Consequence · Explication · Generic entailment · Logical particle · Mathematical virtuosity · Semantic penumbra · Similarity · Synthesis

1 Entailment

Philosophers are much taken with the validity and invalidity of arguments, with what follows from what, and whether this entails that.¹ Entailment is a concept more invoked than studied,² although most philosophers in the broadly analytical tradition will have had some minimal theoretical acquaintance with it, whether as students of the mandatory graduate course in symbolic logic, or as teachers of an introductory class (or module). By quite a large margin, philosophers who have had at least this level of exposure to it will be favourably drawn to two commonplace claims about the formal logic of deduction. One is that it provides an accurate definition of entailment. The other is that its logical particles—for example, its sentential connectives—distort the meanings of their natural language counterparts. Of course, a good number of philosophers who subscribe to this pair of claims have more than a minimal acquaintance with logic. There are plenty of them for whom logic is an active research programme. I will have something to say about this group in Sect. 3. For the present, it suffices to emphasize the readiness of philosophers at large (if I may call them that) to accept the two propositions presently in view.

In the matter of entailment, the dominant view is that necessary and sufficient for \( U \)’s entailment of \( w \) is the logical impossibility jointly of \( U \)’s truth and \( w \)’s falsity. Not all the philosophers I intend by the designation “philosophers at large” are happy with this definition. But most are; and until further notice I shall simply assume that they are right. With

¹ Coinage of “entails” for the converse of “follows from” is G.E. Moore’s. See Moore (1922).
² Although not completely neglected. See for example Woods (1965) and Bennett (1969).
a nod to its lineage in logics of strict implication, let’s call this the *strict condition* on entailment.

The prevailing view about connectives is that the schematization manuals that map natural language constructions (from English, say) to a logic’s formal structures assign the connectives of English to symbols that misrepresent what they mean in English.

This is embarrassing. At least, it is embarrassing on its face. A formal system is designed in such a way that its logical particles play a semantically indispensable role in specifying the entailment relation. But now we must ask, “How is this possible?” How is it possible for a logic to get entailment right if the very same system gets entailment’s logical particles wrong? This is an issue important enough to have a name. I propose that we call it the Right-Wrong Problem.⁴

There is, of course, an intuitive answer (of sorts) to the Right-Wrong Problem. It is that while logic gets the logical particles wrong, there is nevertheless enough similarity between how they actually are and how they are represented by logicians to defeat the supposition that Right-Wrong is a genuine problem. This is not a meritless idea. But before we take to it with any theoretical seriousness, we should try to specify the relation of purported similarity and demonstrate its key properties. To this end, I shall call upon the concept of *semantic penumbra*, to be introduced just below. Later I will test the usefulness of what I will call *generic relations* in logic.

### 2 Semantic Penumbra

To see how the Right-Wrong Problem arises, let’s begin with classical first order logic (CFL). For my purposes here we might just as well have chosen a modal logic such as S5. I will have occasion to mention modal approaches a bit later. But since everyone is familiar with CFL, we’ll start with it. Virtually everyone also agrees that CFL’s principal target is to provide clear and rigorous definitions of a family of interlocking notions centering around the relation of logical entailment, to demonstrate the key properties of these notions and, to the degree possible, to facilitate their recognition.⁴ In plainer words, logic’s principal job is to say what these concepts *mean*. A significant factor in performing these tasks is that CFL’s target properties are defined for syntactic constructions from the system’s formal language. Key to it all is the semantic interpretations the system attaches to its logical particles: “∼”, “∧”, “∨”, “⊃”, “≡”, “∃”, for each of which in turn there is a purported counterpart in English: “not”, “and”, “or”, “if … then”, “if and only if” and “some”. If getting entailment (etc.) right is CFL’s *principal target*, then dealing with negation, conjunction, disjunction, conditionality, bi-conditionality and quantification is its *subsidiary target*.⁵

No one should seriously suppose that the connectives and their respective counterparts mean the same. There is no sense of “if … then” in English for which Φ’s falsity provides that if Φ then ψ for arbitrary ψ, and for which ψ’s truth does the same. That is, there is no sense of “if … then” in English for which Φ ⊃ ψ is a way of saying that if Φ then ψ.

Even CFL’s negation doesn’t quite fit negation in English. Russell noted long ago that there is a use of “not” in English in which ¬Not-Φ can be true even though Φ is not false. (Φ might be “The present king of France is bald.”).⁶

We have it, then, that CFL gets the English particles wrong—not dead wrong perhaps, but wrong enough to notice. Yet there is a substantial consensus that it gets entailment right.⁷ But why would this be so? How could a logic that gets “if … then” wrong turn out to get “is a consequence of” right? Whereupon the Right-Wrong Problem.

The answer to which I will now give some provisional attention is that the CFL connectives are in the *semantic penumbra* of their counterpart expressions in English. Consider first the case in which the semantic gap between a CFL connective and its counterpart in English is starkest: the connectives “⊃” and “if … then”. Notwithstanding their difference in meaning, these expressions exhibit a certain similarity⁸:

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⁴ There is an analogous problem about how scientific models represent their target systems by way of idealizations that fail on the ground. For more of this, see Woods and Rosales (2010).

⁵ This is not to say that the sentential operators aren’t interesting. Jennings (1994) is a very good book on disjunction and Horn (1989) is the same for negation, as are Bennett (2003); Nute and Cross (2002) and Lycan (2001) for conditionality. The point is that the job of a theory of deduction is to get deduction right. It is not the job of a theory of deduction to pronounce upon negation, disjunction and conditionality, save in ways that may facilitate its progress with deduction (which in its semantic sense is what entailment is).

⁶ Russell (1905). There is also the large class of cases in which the “not” in “Not-Φ” serves as the denial or commitment-retraction of Φ, short of affirming the truth of its negation.

⁷ In CFL, Φ entails ψ just in case there is no valuation making Φ true on any interpretation that doesn’t also make ψ true on that interpretation. More loosely, there is no possible way of making Φ true without making ψ true. This, of course, is the strict condition.

⁸ Giere (1988) has proposed a contextually sensitive relation of similarity between a model and its target system. Da Costa and French (2003) characterize this relation as a partial isomorphism, analyzed as a kind of partial structure.
The truth of \( \neg(\Phi \land \neg \psi) \) is necessary and sufficient for the truth of \( \Phi \supset \psi \), and is necessary, though not sufficient, for the truth of \( \text{if } \Phi \text{ then } \psi \).

Accordingly, we may say that, while “\( \supset \)” and “\( \text{if ... then} \)” don’t mean the same, “\( \supset \)” is in the semantic penumbra of “\( \text{if ... then} \)”.

Consider next the less strained case of the negation connectives “\( \neg \)” and “\( \text{not} \)”:

- The falsity of \( \Phi \) is necessary and sufficient for the truth of \( \neg \Phi \), but is sufficient (and not necessary) for the truth of \( \neg \Phi \).

Accordingly, while “\( \neg \)” and “\( \text{not} \)” don’t mean the same, “\( \neg \)” is in the semantic penumbra of “\( \text{not} \)”.

Generalizing: A CFL particle \( t \) is in the semantic penumbra of its English counterpart \( t^* \) if and only if a condition necessary and sufficient for the truth of a \( t \)-statement is either necessary only or sufficient only for the truth of the corresponding \( t^* \)-statement.

In my dictionary, one of the listings for “penumbra” is any area of partial shade. How things are in its lighter parts are some indication of how they are in the more shaded parts. Something of that meaning is preserved here, with formal terms operating in the lighter areas of the partial semantic shade of their English counterparts.

Semantic penumbra carry an interesting suggestion about getting things right. We’ve already noticed that getting the particles right is not a condition on CFL’s getting entailment right. If we wanted to put this more accurately, we could say that what our examples show is that if you want to get entailment right, it is not necessary to get your logical particles as right. It suffices that you get them right enough. Semantic penumbra offer a test of this enoughness. You’ve handled your logical particles well enough if they are in the semantic penumbra of their counterparts in English. Let us call this the semantic penumbra thesis. The semantic penumbra thesis is, with an appropriate tentativeness, the preferred solution of the Right-Wrong Problem.

Over the years some philosophers have been much exercised by CFL’s failure to get the connectives right. This, they say, is a representation problem. So it is. There is a good deal of inclination to regard this misrepresentation as a bad problem. If the semantic penumbra thesis is true, these critics are right in the first instance but wrong in the second. CFL does misrepresent the English connectives, but the damage is merely local – that is, it compromises the system’s subsidiary ambitions without laying a glove on its principal ambition.

Suppose this is so. Suppose that the semantic penumbra thesis offers a satisfactory resolution of the Right-Wrong Problem. Wouldn’t this mean that these critics should consider revising their position on the misrepresentations occasioned by deductive logic’s logical particles? Shouldn’t they now concede that while those misrepresentations do indeed exist they are no longer grounds for complaint? If this were right, it would be methodologically important. If Right-Wrong is left unanswered, then the uncritical acceptance by philosophers at large of logic’s handling of entailment would be suspect. It would counsel some reconsideration. On thinking it over, we might elect to accept the definition but refuse the formalization in which it is couched. Or we might decide to keep the formal link and forgo the definition. Both are consequential options, each requiring the rejection of uncritical acceptance. The upshot of the first option is the abandonment of formal logic as the elucidater of one of philosophy’s key analytical tools. The upshot of the second option is that we admit to a lesser grasp of entailment than we have routinely supposed ourselves to have.

On the other hand, all the messiness is avoided if Right-Wrong is a real problem and semantic penumbrancy is a good resolution of it.

2.1 An Objection

An interesting feature of the definition of semantic penumbra is that it allows for some of CFL’s formal connective to be in the semantic penumbra of English connectives that are not, under the standard schematization mappings, their counterparts in English. Thus it is necessary and sufficient for the truth of \( \neg(\Phi \land \psi) \) that \( \Phi \) and \( \psi \) both be true; yet this is also a sufficient condition of \( \neg(\Phi \lor \psi) \) in English. Similarly, for the truth of \( \neg(\Phi \land \psi) \) in CFL it is necessary and sufficient that either \( \Phi \) is true or \( \psi \) is true; yet this is also a necessary condition of the truth of \( \neg(\Phi \land \psi) \).

Footnote 10 continued

of logicians, arrives at the strange conclusion that (among Englishmen) we may conclude from a man’s red hair that he is doctor, or from his being a doctor that (whatever appearances may say to the contrary) his hair is red” (MacColl, 1908a, p. 152). For further dis-satisfactions with how CFL represents the logical particles of English, see Adams (1988), Stove (1986), McGee (1985) and Jacquette (1999). While there certainly are difficulties arising from CFL’s treatment of the particles, these particular criticisms are I believe largely misplaced. See here Woods (2004), chapter 3.
In English. So “∧” is in the semantic penumbra of “or”, and “∨” is in the semantic penumbra of “and”. But since “∧” is also in the semantic penumbra of “and” and of “or”, we would seem to have it that sometimes CFL connectives are in the semantic penumbra of different and inequivalent English connectives. Wouldn’t this extend to the idea of semantic penumbra too much latitude for its own good? I think not, since by an obvious extension of our definition the “and” and “or” of English are also in one another’s semantic penumbra. Rightly so, “and” and “or” are each others duals. So there is bound to be some degree of penumbrial overlap.

3 Making-Clear and Clear-Making

I have already remarked that philosophers for whom formal logic is a research programme tend to be favourably inclined towards the two theses which (I say) generate the Right-Wrong question for logic. But there are important exceptions. While these propositions may enjoy majority support of this group, there are members of it who reject their presuppositions. One is that since there is a right way for entailment to be, and a right way also for negation and the other sentential operations to be, there is a right way for their resembling counterparts to be, namely, relevantly similar to how entailment, negation and the rest actually are. Doubts about this are consequential. For if, in particular, there is no way for entailment to be, what would there be for a formalized representation of entailment to be similar to? How could anyone harbouring these doubts coherently attribute a Right-Wrong Problem to logic? I want in this present section to develop this point in a bit more detail, but not before mentioning that if these doubts could be made to stand, they would present philosophers at large with a second problem. If there is no fact of the matter about what entails what, then what is to be made of entailment’s utter predilection for discerning, in arguments for and against philosophical claims, the presence or absence of entailment?

For most of its history since 1879, logicians have had a widely shared understanding of their duty to their principal targets – entailment and the others. Entailment would be subject to two theoretical constraints. On the one hand, the concept of entailment would have to be mathematically well-defined. Its unpacking would have to be clear, precise, systematic, rigorous and general. Its definition would also have to facilitate recognizability, if not outright guarantee it. A treatment of entailment that met these conditions would be technically adequate. It would exhibit the right kind of mathematical virtuosity.

Logicians of the period also assigned themselves a second duty. Logic’s mathematically adequate provisions for entailment would also have to serve as a conceptual elucidation of it, one which gave an improved understanding of it. When a logic met this further condition, its treatment of entailment would be conceptually faithful.

Applying formal methods to a concept constitutes a formalization of it. A concept’s meaning in pre-formalized linguistic practice is its intuitive meaning or, equivalently, reflects the intuitive concept. Consider now the question, “What is achieved by the formalization of a concept?” Historical practice suggests four different answers to this concept-engagement question.

**Analysis** The formalization of concept K explicitizes the meaning it has in pre-formalized linguistic practice; that is, it articulates K’s intuitive meaning.

**Explication** The formalization of a concept K preserves its intuitive meaning but does so in ways that gives to K a more aggressive clarity than it had pre-theoretically.

**Rational reconstruction** The formalization of a concept K involves an additional attribution to K of features not present in pre-formalized linguistic practice, but in a way that retains enough of the intuitive concept to make it intelligible to say that the rational construction at hand is a formalization of it.

**Stipulation** The formalization constitutes a nominal definition of a concept lacking a prior presence in pre-formalized linguistic practice, of a concept, that is, that lacks an intuitive predecessor.

In the first instance, analysans and analysandum are taken as synonymous. In the next two cases, the equivalences are more in the range of partial synonymy. In case four, there is no trace of it.

The distinction between analysis and stipulation is roughly Kant’s contrast between analysis and synthesis. Analysis, says Kant, is the business of making concepts clear, and synthesis the business of making clear concepts, that is, the business of making them up. Analysis is the purview of philosophy and synthesis the province of mathematics. Explication and rational reconstruction are hybrids, with explication more analysis-like and rational reconstruction tilting rather more towards stipulation.

There are, of course, grey areas at each of these borders, but here are some quick examples. Some probability theorists think that to the extent that the intuitive meaning of probability resides in how prior probabilities are compounded, that aspect of its meaning is captured analytically by the axioms of the probability calculus. Some mathematicians take the view that the axioms of number theory offer an explication of the intuitive concept of number. The
The same distinction is also present, as Quine famously quipped, one person’s explication is another’s stipulation.

The war-cry of formal philosophy in this period was, although not in these exact words, “Mathematical virtuosity is the preferred route to conceptual adequacy.” A paradigm of this sentiment was the doctrine of contextual eliminability underlying Russell’s theory of descriptions, itself - according to Ramsey – a paradigm of philosophy. The motto animated the practice of philosophy in the tradition of Russell, Carnap and Quine, to name an additional three. It is an intriguing idea – akin to having your cake and eating it too. Most deductive logicians in the mainstream three. It is an intriguing idea – akin to having your cake and eating it too. Most deductive logicians in the mainstream

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it is well-stocked with (ever enlarging) constituencies of theorists whose syntheticism is not moderate, but extreme. The presence of this radical syntheticism – the doctrine that logic is what you make it, that and nothing more – is discernible in what is now a quite standard reaction to logic’s sprawling and apparently rivalrous pluralism. It is a nihilistic relativism according to which, as mentioned just above, there is no fact of the matter about entailment except as provided for in some or other logical system, hence no fact about their respective target concepts that makes any one system a more accurate logic than any other, or its entailment relation more similar than others to how entailment really is. There is a shorter way of saying the same thing. Among practicing logicians there is a large and growing consensus that what truly matters in modern logic is mathematical virtuosity, not conceptual fidelity; that the goal of conceptual fidelity is sentimental naïveté.

4 Instrumental Nihilism

It would be wrong to think of logic’s relativistic nihilism as an intrinsically corrosive abandonment of intellectual seriousness in logic.\(^{15}\) An early adumbration of this nihilism is Carnap’s Principle of Tolerance (1934/1937), according to which logic is both stipulationist and unattended by antecedent facts of the matter. For Carnap, logic is indeed what you make it, and the goodness or badness of its theorems owes nothing to their fidelity or lack of it to pre-existing logical facts.\(^{16}\) But Carnap was far from permissive about logic. A logic’s merits would be a function of what it was wanted for in the first place. For example, if your goal is the reduction of arithmetic to logic, there is reason to like the logic of Principia Mathematica. Accordingly, it is not quite true to deny the existence of facts of the matter about logic. There are instrumental facts and there are the facts achieved by stipulation. It is an instrumental fact that PM is good for logicism. It is a stipulated fact that the axioms of infinity and reducibility are true there.

4.1 Experimental Nihilism

This is a further variation of nihilism in logic, one that sets it apart from nihilism of the Carnapian instrumentalist sort.

For lack of a name, I’ll simply call it experimental nihilism, typified in nonlogical circles by circumstances attending Riemann’s construction of non-Euclidean geometry, in which, among other things, the parallel postulate fails. No one, least of all its inventor, took Riemannian geometry as a description of real space. Its virtues were technical, notably its proof of consistency relative to the consistency of Euclidean geometry. It was a significant piece of mathematical virtuosity but a dead loss on the score of conceptual adequacy. Riemann’s was not a semantic analysis of the concept space. Given the way that space really was, Riemann’s was the wrong way to conceptualize it, until, unexpectedly, the arrival of general relativity theory, in which it emerges that Riemann’s space is the empirically well confirmed description of big, or global, space.

There is no reason to think that Riemann himself anticipated this Einsteinean application of his abstract theory. Even so, non-Euclidean geometry was ready to hand—on the shelf, so to speak—and was available for Einstein’s later appropriation. But this couldn’t have happened in the first place had it not been for the impressive technical virtuosity of Riemann’s achievement. People would not have paid attention to it otherwise.\(^{17}\)

Imagine a situation like Riemann’s, but with a variation. A logician is building a logic and, like Riemann, his interests are mathematically virtuosic. He wants his logic to be sound and complete, or he wants its axioms to be categorical, and so on. In the course of realizing these objectives he has no expectation of conceptual adequacy, and no interest in it. But like Riemann, he publishes his logic on the understanding that in the fullness of time it may turn out to have an application. In that case, we could say that our logician’s mathematical virtuosity was attended by an experimental intent. “Here is another logic”, we can imagine him saying, “Who knows what its uses might turn out to have.” Note, however, that when the paper is submitted to a journal the editors will accept it or reject it for its virtuosity or lack of it as a piece of mathematical contrivance. Applications, if such there be, are entirely the contingent byproduct of a favourable judgment of technical finesse.

How widespread is this experimentalism in logic? The short answer is: “Just read at random any published piece

\(^{15}\) Of course, there is in principle what we might call the option of mindless nihilism, according to which since there are no facts of the matter about what entailment actually is, all logics of entailment are false (for there is nothing that could make them true). There are lots of nihilist logicians, but none of the mindless breed.

\(^{16}\) Quine says that theories are conceptualizations “of our own making” (1981, p. 2). Edington says that they “are put-up jobs” (quoted in Quine 1971).

\(^{17}\) I don’t want to leave the impression that Riemann was simply playing about until one day, hey presto! the general theory of relativity popped into view. Riemann’s geometry was a formidable piece of mathematics. It generalized Gauss’s work on surfaces to n dimensions. When n = 3, Riemann’s geometry is the result. Riemann didn’t think the 3D case described physical space or—since scaling up is a dubious assumption in physics—that it ever would prove to do so. Still, he did think it a physically possible hypothesis, hence a description of how the world might conceivably be.
in any of the mainstream logic journals, in the past forty years.”

Nihilism in logic raises an interesting question. Although conceptual fidelity might not long survive as a condition of adequacy for logic, aren’t some logics as a matter of fact more conceptually faithful than others? By this I mean, don’t the meanings assigned to the symbol for entailment (“$\vdash$”), by some logics come closer than others to the meaning assigned by English to the word for entailment? It may or may not be the case that what English takes entailment to be is the way that entailment actually is. But suppose we were interested in having a deeper appreciation of what philosophers understand themselves to be invoking when they attribute the relation of entailment or when they regret its absence. Wouldn’t a conceptually faithful logic be the thing to aim for? And wouldn’t this reactivate the Right-Wrong question? Wouldn’t there now be room to ask how it is that logic might do pretty well at capturing the meaning of “entails” in English, while not doing as well with the meaning in English of “not”, “if … then”, and so on? Wouldn’t this revive the penumbral answer to that question? Couldn’t we say that the connective of logic enable an at least moderately synthetical representation of entailment in English by being in the semantic penumbra of their counterparts in English?

5 Losing Negation

Whatever its attractions, the semantic penumbra thesis is not without its problems. Here is one of them. Negation, we said, begins at home. When “$\sim$” is classical negation, it is in the semantic penumbra of “not”. It is necessary and sufficient for the truth of “$\sim \Phi$” that $\Phi$ be false, and it is at least sufficient for the truth of “$\neg \Phi$” in English that $\Phi$ be false. But consider now negation of a rather different cast. One of the mainstream approaches to a relevant logic of entailment is first order entailment (FDE). One of the main approaches to FDE’s semantics is the so-called Routley-star semantics. Here, briefly, are some of the technicalities. In Routley’s possible worlds semantics for FDE, every world $w$ has a unique buddy $w^*$, called its star world. An interpretation of FDE is a structure $\langle W, *, v \rangle$ in which $W$ is a non-empty set of elements which we might informally think of as worlds, * is a function from worlds to worlds satisfying the condition that $w^{**} = w$, and $v$ is a truth value assignment. Negation is now defined as: $\nu_\pi (\sim \Phi) = T$ if and only if $\nu_\pi (\Phi) = F$. That is, it is necessary and sufficient for the truth in this world of “$\sim \sim$” that $\Phi$ be false in its star world. Call this the star condition.

Up to now, I have sided with the mainstream in thinking that entailment in English is correctly defined by the strict condition; that is, that for the entailment of $\psi$ by $\Phi$ it is necessary and sufficient that it be logically impossible that $\Phi$ and yet $\sim \psi$. As befits its status as a logic relevance, FDE accepts the strict condition as necessary, but not sufficient, for entailment, the other condition being that $\psi$ be relevantly connected to $\Phi$. For present purposes we needn’t settle this question. Suffice it to say that virtually everyone involved in this matter thinks that FDE gets entailment at least half-right.

One of FDE’s most interesting features is that its notion of negation answers to nothing that qualifies as negation in any sense of “not” in English. This is acknowledged by its inventors and supporters. FDE’s “negation” is negation in no discernible sense. This makes it easy to see that the star condition excludes star negation from the semantic penumbra of negation in English. FDE gets negation dead wrong. Yet it gets entailment at least half right. So it can’t be a blanket condition on getting entailment even somewhat right that we also get the connectives at least somewhat right.

18 A sociological observation: The Journal of Symbolic Logic used to make room in its reviews section for philosophical contributions to the subject. In time, this mandate was transferred to its sister journal, the Bulletin of Symbolic Logic, which was soon dominated by mathematical work. A further adjustment, intended to accommodate philosophical interests, was the Review of Symbolic Logic, which too, since its recent inception has been dominated by mathematical pieces. Meanwhile, even the Journal of Philosophical Logic which was the Association’s predecessor organ of the Review was rife with a mathematical emphasis. In a way, this is as it should be. It reflects the fact that in the past generation or so, logic largely ceased being a humanities discipline and has passed its remit to mathematics and computer science.

19 Consider here Gödel’s platonic approach to sets. There is, says Gödel, a way in which sets actually are, hence a way of conceiving them as they actually are. However, owing among other things to the limiting contingencies of human thinking, there are different ways of conceiving of sets, none of which captures sets as they actually are.

20 Routley and Routley (1972). “First order” denotes a system in which entailment statements don’t themselves embed entailment statements.

21 See, in addition to Routley (1972), Priest (2001/2008). Restall (1999) is an exception. Restall has learned to love star-negation. So has Nick Griffin (2011).

22 Thus calling to mind Quine’s words about a different though related thing. Speaking of the dialethic approach to entailment, Quine writes: “To turn to a popular extravaganza, what if someone were to reject the law of non-contradiction and so accept an occasional sentence and its negation both as true? … My view … is that neither party knows what he is talking about. They think that they are talking about negation, $\sim \sim$, ‘not’; but surely the notation ceased to be recognizable as negation when they took to regarding some conjunctions of the form $p \& \sim p$ as true, and stopped regarding such sentences as implying all others” (Quine, 1982, p. 81).
5.1 Re-Thinking Right-Wrong

FDE imposes the strict condition on entailment. That is, FDE imposes a condition formulable in those words. But how can the following three things be true: that “~” means nothing recognizable as negation, that “~” occurs essentially in the wording of the strict condition, and that the ensuing meaning of “entails” is close enough to its meaning in English to warrant our saying that this very wording gives the strict condition for that very sense of entailment—that is, its sense in English?

It’s not a sellable proposition. If the definiens of a definiendum invokes no recognizable concept, the definiendum itself imbibes that limitation. FDE’s star-negation is one in which negation is unrecognizable. Equally, then, FDE’s entailment is one in which entailment is unrecognizable. FDE’s “[~]” isn’t in the semantic penumbral of “entails” in English. So there is no Right-Wrong Problem for FDE. FDE is wrong through and through. FDE has a Wrong–Wrong Problem.

Star worlds have a further feature which suits them to the semantics of paraconsistency. Star worlds are susceptible of non-detonating inconsistency. One might see in the semantics of paraconsistency. Star worlds are susceptible to negation-inconsistency, but not absolute inconsistency. The inconsistency of a star world doesn’t make everything true there.

Footnote 24 continued

It might be objected that, owing to their susceptibility to inconsistency, we shouldn’t think of star worlds as possible worlds. We should instead use a more neutral name such as “set-up”. I don’t mind. Nothing I have to say against star negation is predicated on the idea that, except as a façon de parler, some possible worlds are actually inconsistent. Let us note that in the mainstream semantics for intensional logics, there is not one word of instruction about what possible worlds are, beyond their purely formal characterization as possible but not actual falsity there, there is in English no sense of the expression “it is not the case that…” in which if Φ is possibly but not actually false then it is not the case that Φ. So (I said) there is in this sense of “~” nothing recognizable as negation.

If it is right to ascribe a Wrong–Wrong Problem to FDE, unsettling consequences arise. For now there is reason to doubt even the modified Right-Wrong Problem for CFL. The modified Right-Wrong thesis is that it gets entailment in English more right than the English connectives. We had no difficulty in seeing that the unrecognizability of negation in FDE’s “~” made for the unrecognizability of entailment in FDE’s “[~]”. The same applies here, only less dramatically. That the English operations are only somewhat recognizable in CFL’s connectives makes entailment equally recognizable in CFL’s “[~]”, namely somewhat only. CFL’s “[~]” does no better as an analysis of entailment in English than it does an analysis of the operations to which the English connectives correspond. It is a truth of English that Φ entails ψ if and only if it is necessarily the case that if Φ then ψ. In CFL, this is likened to “Necessarily, Φ ⊃ ψ.”. But since there is no sense of “if … then” in English for which “Φ ⊃ ψ” provides that if Φ then ψ, then “Necessarily, Φ ⊃ ψ” doesn’t provide that necessarily if Φ then ψ. That is, “Necessarily, Φ ⊃ ψ” doesn’t capture the English meaning of “entails”.

The gap between English and CFL is no trifling matter. It presents us with nothing less than a Somewhat Wrong-Somewhat Wrong Problem for CFL. If CFL gets the connectives of English somewhat wrong, it also gets entailment in English at least that wrong.

How big a nuisance is the Somewhat Wrong-Somewhat Wrong Problem? By the lights of strict conceptual conservatism, it reflects the failure to analyze entailment as it actually is. But by a moderate syntheticism backed by the requisite semantic penumbra, not only does it do as well with entailment as a formal system is able to do, but its provisions achieve a philosophically tenable balance between technical and conceptual adequacy.

5.1.1 Reclaiming Star Negation?

It might be objected that I’ve been too hard on star negation. Consider “~” under the star interpretation. I say that if the truth of ~Φ in a given world is guaranteed by Φ’s possible but not actual falsity there, there is in English no sense of the expression “it is not the case that…” in which if Φ is possibly but not actually false then it is not the case that Φ. So (I said) there is in this sense of “~” nothing recognizable as negation.

Somewhat more technically, star worlds are susceptible to negation-inconsistency, but not absolute inconsistency. The inconsistency of a star world doesn’t make everything true there.

Footnote 24 continued

It might be objected that, owing to their susceptibility to inconsistency, we shouldn’t think of star worlds as possible worlds. We should instead use a more neutral name such as “set-up”. I don’t mind. Nothing I have to say against star negation is predicated on the idea that, except as a façon de parler, some possible worlds are actually inconsistent. Let us note that in the mainstream semantics for intensional logics, there is not one word of instruction about what possible worlds are, beyond their purely formal characterization as possible but not actual falsity there, there is in English no sense of the expression “it is not the case that…” in which if Φ is possibly but not actually false then it is not the case that Φ. So (I said) there is in this sense of “~” nothing recognizable as negation.
Against this, as a cursory examination of FDE will show, star negation appears to behave remarkably like real (as anyhow recognizable) negation in certain contexts. Double negation holds for star negation, as do modus tolens and the De Morgan equivalences. Isn’t star negation negation-like in these contexts? Doesn’t this make negation recognizable there?

This gives us two arguments consider:

| Argument 1 | Argument 2 |
|------------|------------|
| 1. Star negation is negation-like | 1. Negation is not recognizably present in star negation |
| 2. So negation is recognizably present in star negation | 2. Star negation is negation-like |
| 3. So non-negation can sometimes behave in negation-like ways | |

Argument 1, we might be inclined to think, gives more weight to an inferential semantics for negation, whereas argument 2 leans towards a truth conditional approach. This is a large question for which there is no space here.

Even so, it seems to me that the distinction I purport between behaving like negation and being negation is not subject to mere dismissal. After all, the star function * also behaves like negation. It obeys double star: \( w^{**} = w \). Double star is like double negation, but the star function is clearly not negation. Nothing recognizable as negation is present there.

6 Modal Ambiguities

In what we’ve been saying so far, CFL has been a stand-in for one’s favourite formal logic of entailment. Some may think, as Lewis himself did (and MacColl before him), that modal logic is a better instrument for that purpose, made so by the fact that, in addition to entailment itself, it makes the possibility involved in its definition principal rather than subsidiary target. Both logics call upon possibility in their definitions of entailment. But wouldn’t a logic that tells us how possibility actually goes give us a richer grasp of entailment? Yes it would. At least, it would to the extent to which possibility is recognizable in its possibility symbol “\( \Diamond \)”.

Consider the Lewis systems for propositional logic: S1, S2, S3, S4, S5, as well as the later S6, S7 and S8. Although the S-systems have a common wording for entailment – \( \Phi \) entails \( \psi \) if and only if \( \neg \Diamond (\Phi \land \neg \psi) \) they are different and not always compatible logics. Figure 1 charts these differences and incompatibilities.

Figure 1 discloses but a small part of the sprawling and often rivalrous pluralism mentioned earlier. There are more theories of necessity and possibility than you can shake a stick at, and more still of entailment. The logics of Fig. 1 are slim pickings, each arising in the period between 1912 and 1932. By 1969, the number of normal propositional logics of the modalities was easily fifty, and since then the wheels of the journals have ground ceaselessly.\(^{26}\) Modal pluralism disturbs the repose of modest syntheticism.

Moderate syntheticism says that as regards a system’s target properties technical adequacy—mathematical virtuosity—must not be allowed to erase conceptual adequacy. This we have lately taken to mean that if \( C \) is the target in question then there is a sense of “\( C \)” in English which the logic in question honours. But it is clear on inspection that the logics of Fig. 1 provide for eight different concepts of possibility (and necessity)—never mind the over fifty of the Hughes and Creswell list. Since in all these S-systems \( \Phi \) entails \( \psi \) iff \( \neg \Diamond (\Phi \land \neg \psi) \), the eight-wise ambiguity of “\( \Diamond \)” generates eight different semantic treatments of entailment.

No matter the exact number, it is obvious that

a. the number of senses in English of possibility and entailment is radically smaller than the senses in the adumbrated sprawling modal pluralism of present day logic;

and

b. it is only with respect to this smaller subset that the project of moderate syntheticism has any chance of success.

How is membership in this comparatively small subset determined? Here, too, it would be natural to think of the penumbral option. But this is seriously mistaken. Attributes of semantic penumbra are fixed point attributions. That is, they take as given a contextually indicated

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\(^{25}\) Which I borrow from Dagfinn Follesdal’s 1962 PhD thesis, reissued as Follesdal (2008).

\(^{26}\) Hughes and Cresswell (1968/1996) has a chart listing this multiplicity.
meaning in English against which to test the formal terms in question. Again, take classical "~" as an example. We say that "~" is the semantic penumbrum of "not" in English if the truth of \( \sim \Phi \) is sufficient, but not necessary, for the truth in English of \( \not \Phi \). In so saying, we are holding "not" to some fixed, and typically undeclared, meaning. Suppose that there were a hundred different and not always compatible logics of "~". How many of these "~"s would be in the semantic penumbrum of not's in English? The answer would depend on how many negational meanings of "not" English actually has.\(^{27}\) The doctrine of semantic penumbra doesn’t answer that question.

This is problematic. Consider the Lewis system S2. S2 is a non-normal logic in S2. Unlike S7, say it is not a theorem of S2 that every statement is possibly possible. But S2 is consistent with the possible possibility of every sentence (including \( \not \Phi \land \sim \Phi \)). It is not remotely credible that there is in English a sense of possibility according to which every statement of English is possibly possible. There is no sense of “possible” in English consistent with the possible possibility of every English sentence. But the “\( \diamond \)" of S2 has a meaning such that for any \( \Phi \), it is consistent to say that \( \diamond \not \Phi \). So we have it trivially that “\( \diamond \)\(^{S2}\)" lacks a semantic penumbrum in English.

As we see, determining that a formal term lacks a corresponding meaning in English is sometimes easy to do. But this is far from so in the general case. To see why, it is instructive to compare S2 with S4 and S5. Consider those sentences \( \Phi \) entailed in those respective systems by the arbitrary set of sentences. These respectively are the valid sentences of S4 and S5. For each such we have it that \( \sim \diamond \sim \Phi \). There are modal logicians for whom the meaning of “\( \diamond \)" in S5 is such that “is an S5-valid sentence” means “is true solely in virtue of logical form”; and the meaning of “\( \sim \)" in S4 is such that “is an S4-valid sentence” means “is provable solely in virtue of logical form.”\(^{28}\) This, if true, reflects different things that “\( \diamond \)" means in S4 and S5. Similarly, if true, it reflects the different things that “entails” means in S4 and S5. It is natural to want to know whether S4 and S5 are synthetically adequate logics with regard to these different meanings. If so, then, on the suggestion presently in view, in each system “\( \diamond \)" is in the semantic penumbrum of one of a pair of senses of possibility in English, and “\( \models \)" is in the semantic penumbrum of one of a pair of senses of entailment in English. To determine whether these conditions are met it is necessary to find among the meanings of “possible” in English precisely that pair whose elements serve respectively as the penumbras of “\( \diamond \)\(^{S4}\)" and “\( \diamond \)\(^{S5}\)".

Suppose that I am wrong about this. Suppose that there are at least as many senses of “possible” in English as may be required to give S4 and S5 (and other normal systems) a decent shot at attaining synthetic adequacy. Then it might also be true that the number of such senses of possibility in English simply out-run the number of senses of “entails” in English. (The more common synonym is “implies”. Let us adopt that usage now.) Thus we might have it that the “\( \sim \)”, “\( \sim \)”, and “\( \diamond \)" of these logics are in the semantic penumbrum of their English counterparts for one or other of these senses and yet that, owing to the paucity of senses of “implies” in English, there would be at least one case in which the implication defined by “\( \land \)”, “\( \sim \)”, and “\( \diamond \)" would not have a sense in English that would qualify “implies" as the semantic penumbrum of “\( \models \)". If so, the definien of implication would be penumbral but not the definiendum, occasioning therewith the loss of synthetic adequacy for “\( \models \)".\(^{29}\)

Let us say it again: Semantic penumbrum attributions are fixed point attributions. If a meaning of an English term is readily enough to hand to discern either a necessary or sufficient condition for an application of it, then any corresponding formal particle whose full definition is known can be put to the test of semantic penumbra. But where putative English meanings aren’t at fixed points, two difficulties arise. One is the problem of bringing sought-for meanings to heel in a principled way. The other, relatedly, is the problem of managing a disciplined engagement of the semantic penumbrum test.

There are philosophers galore—not least of whom is Quine\(^{30}\)—who disdain all talk of meaning in philosophy.

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\(^{27}\) “Negational” as opposed to “emphatic”. The person who says, “I will never, never, never, never set foot in Dry Gulch again” is not announcing an intention to return. The innermost occurrence of “never” is negational. The other three are for emphasis.

\(^{28}\) See Burgess (2009), p. 65.

\(^{29}\) Burgess identifies truths by logical form alone as alethic modalities, and provabilities by logical form alone as apodictic modalities, concerning which he displays a caution appropriate to our present reflections: “… nothing said so far constitutes even an informal ‘proof’ that no formula not a theorem of S4 is correct for apodictic modality, or that no formula not a theorem of S5 is correct for alethic modality. And indeed there is no generally accepted informal argument for the first claim, though a convincing one can be given for the second claim” (Burgess 2009, p. 65; emphasis added to contrast argument with proof).

\(^{30}\) Quine (1951) and (1960), chapter 2.
The main complaint is that we lack the means to put them to systematic and load-bearing use. A particular version of this complaint is that lacking clear identity conditions, meanings are hostile to individuation. There may be something to say for these reservations, but I note with optimism the determination of some philosophers to subject concepts to the discipline of a proper theoretical articulation.31

7 Generic Entailment

“No, of course meanings are fraught. But the fact remains, doesn’t it, that S5’s concept is like the one in English, and is like enough to make it possible for people interested in the entailments of English to learn something about them by attending to what entailments are like in S5?” It is an interesting suggestion, well worth some sympathetic attention. If it were true, the following might also be true. It might be true that there is a concept of entailment which is instantiated by pairs of formal sentences in the ways chronicled by S5, and which is also instantiated in English by, among others, those informal entailments to which genus-species and determinate-determinable relations give rise. These entailments are as “The table is square” entails “The table is rectangular” and “The shirt is red” entails “The shirt is coloured”). Thus it would be the same concept differently instantiated—with different extensions—in S5 and English.

Promising as it might be, the present idea is of little value without some effort to specify the genus-species relation for concepts. Let us see. Take the strict condition on any reading of it in which “~” is recognizably as negation, “∧” as conjunction and “◊” as possibility, thus satisfying the semantic penumbrum condition in each instance. Let F be the family of distinct sets entailments generated by fulfillment of the strict condition on each such reading of it. Supposing now that there is a generic relation of entailment, we might also suppose that each member of F is a different species of that generic relation. Moreover, since on each reading entailment is defined via negation, conjunction and possibility, these readings too define species of the generic operations of negation, conjunction and possibility. Here, too, particle similarity would be specieshood of a common genus.

One of the attractions of the genus-species approach to the logic of entailment is that it permits recognition of a good part of logic’s pluralism without the necessity of a nihilistic relativism. Here is why. There are different and incompatible ways of being a species of mammal—think of whales and ourselves for example. Although being mammalian is what all mammals of all species are, there is no way of being a mammal except by being of a species of mammal. There are massive differences between species of a common genus, as between—to vary the example—the ostrich and the hummingbird. There is no serious scientific theory about mammalianism in general. Any prospects for one would be parasitic on what we already know of the various species of it. No theory of the whale’s mammalian nature owes anything to a prior theory of mammality, as such or to what if anything is disclosed about mammalianism in everyday pre-theoretical English. It is not at all difficult to see why. Genera are abstractions from multiplicities. Species are prior. Genera are after the fact.

It is a common mistake not to notice that whales are mammals. The mistake arises from taking the most familiar examples of it—Marilyn Monroe, Bossy the cow, and Lassie’s mom—as paradigmatic. Whales are too unlike to pass easy mammalian muster. Of course this is wrong. Winnie the whale is not one jot less mammalian than Bossy the cow. In essence, it is the mistake of elevating Bossy to the status of genus and of excluding Winnie for not being a cow.

Suppose that we say that the entailments of CFL, S4 and S5 (and the other members of F) are of this genus species. If this were so, there would be something in virtue of which this is so, namely, that they are all relations that preserve truth. Here, too, there would be no theory of generic entailment that wasn’t deeply parasitic on the logics of F. Here, too, the logics of F would be prior. The theory of generic entailment will be an afterthought. Accordingly, no specific theory of entailment would owe anything to the theory of generic entailment. There is no serious theory of truth-preservation as such. Theories of truth-preservation are theories of particular ways in which to be truth-preserving.

It is interesting to reflect on the place of “entails” in English in this scheme. I daresay some would see it as the genus. Others would see it as one of the species (albeit one without much of a developed theory). There is useful instruction in both these alternatives. If entailment in English were the genus, this would explain why there isn’t a serious theory of it, and it would free the members of F.

31 For state of the art information about the concept-project, see Margolis and Laurence (1999). We might note in passing that the chief backer of the “no identity, no entity” thesis is Quine, who has a word to say about what he takes a thing’s identity conditions actually to be. That is, for the concept of identity condition Quine himself furnishes no identity conditions. Even so, it is clear from Quine’s writings that the main targets of his criticisms are intensional entities—meanings, propositions, propositional attitudes, properties, concepts and modalities. On a plausible reading, we might take a thing’s identity conditions, the conditions that individuate it, to be its theory. A theory of something provides a principled basis for sorting what’s true of it from what’s false of it, which surely is one good way of saying what it is to be a thing of that kind. Quine also requires theories to be formulable in suitably interpreted first order languages. A first order language is one that excludes expressions for intensional entities. So an intensional entity lacks identity conditions just because it cannot be described in a language that excludes intensional entities!
from a fidelity to it beyond truth-preservation. If, on the other hand, the English variety were a species rather than the genus, then, aside from logical truth-preservation, no member of F would owe it any more conceptual fidelity than any other. But what is this, if not the retirement of the conceptual adequacy requirement and, with it, the demand that for membership in F negation, conjunction and possibility be recognizable in “∼”, “∧”, and “◊”. All that would now be needed is that truth-preservation be recognizably present in them all. That would allow into the fold the non-normal logics as well—e.g., S1, S2, and S3. But if the genus-species approach lets these logics in, it is no longer necessary that their connectives be in the semantic penumbra of “not”, “and” and “possible” in English. Whereupon conceptual adequacy as a constraint on logics of entailment collapses.

8 Re-Thinking Analysis

In the old way of thinking, conceptual analysis is archeology. Intuitive concepts are excavated from their sites in English usage and brought up into the light of day, where they are dusted off and cleaned up. Let C be such a concept. Then a fluent speaker of English will know the meaning of “C” without in the general case knowing how to state it. There is nothing particularly odd in this limitation. It is but a variation of the anti-KK hypothesis, according to which it is possible to know that Φ without knowing that you do. (But be warned: Lots of philosophers think that KK-hypothesis is true.) In their native origination, we could say (varying the metaphor) that intuitive concepts hide their semantic lights under a bushel. They linger in the semantic shade. They exhibit a degree of semantic shyness. The driving idea behind conceptual analysis is that a philosophical understanding of C-ness can be got by processes that bring the intuitive concept of C out of the shade and into the bright light of semantic articulation. Here the philosopher’s goal is to provide a rendering that removes or at least mitigates this semantic shyness.

There are problems with this. Let C be a philosopher’s analysandum and CC its analysans. The analysis will be rendered by a statement crudely in the form “x is C if and only if x is CC”. The analysis will fail not only if the biconditional fails but also if there isn’t a sufficient understanding of C to make it possible to see that the biconditional is true. Near and far, philosophers have been troubled by this. One finds stirrings of concern in the _Meno_’s ancient problem of knowledge and more recently in Langford’s paradox of analysis.32 The heart of it all is that C wouldn’t be a candidate for philosophical analysis if it weren’t semantically unforthcoming in a way that impedes its intelligibility. How then can it be determined that CC removes or at least mitigates the impediment of it such as to make it CC’s repair of it indiscernible? Suppose, on the other hand, that C presents no problem on the score of intelligibility. In that case the biconditional, if true, is trivial.

Let us see in a bit more detail how this bears on the paradox of analysis. If the deployment of an intuitive, unanalyzed concept is epistemically subpar—to the point of philosophical untenability—then the biconditional that links an analysandum to its analysans is itself either epistemically subpar or trivial, hence in each case philosophically untenable. To see why, consider again the biconditional that gives the analysis, “x is C iff x is CC”. Consider also the persons for whom the biconditional offers the intended clarification. Then either “C” is understood only implicitly—one understands it under a semantic shadow—or one has an explicit command of it. If the former is true, the epistemic subparness attaching to one’s grasp of “C” spreads (in a variation of Gresham’s law) to the entire biconditional.33 If the latter is true, then no improvement in one’s understanding of “C” is provided by one’s understanding of “CC”. Either way, the biconditional is philosophically untenable.

The desire for conceptual analysis is embedded in a critical pair of epistemological assumptions. One is that the employment of concepts with which one has only an implicit acquaintance (or mainly so) is epistemically suspect. (For how can it be said that you know what “C” means if you are unable to state the meaning of “C”?) The other, relatedly, is that made-up concepts are epistemically impotent. (For how can it be said that you know what “C” gives you a knowledge of what it is to be a C?)

Make no mistake. Sometimes one’s understanding of “C” is improved by one’s ability to state its meaning. Sometimes it is nice to have such articulations at hand. Similarly, there are certainly ways of assigning a meaning to “C” that don’t in the least advance one’s interest in knowing what it is to be a C. Suppose we decided to generalize such cases and to condemn them all as such. Then, in addition to the paradox of analysis, we would have landed ourselves in a massive epistemological scepticism. There would be two things wrong with this. One is that it would cost us our knowledge of most of mathematics and science. Even more generally, it would cost us the assumption that knowledge is valuable. These are big and complex issues, well beyond what there is space for here. But let it be noted that perhaps the single most dramatic discouragement of this epistemological orientation is the massive success of post-paradox set theory. Who but a

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32 Langford (1942).

33 According to Gresham’s Law, bad money drives out good.
philosophical kill-joy would deny us a knowledge of sets on grounds that some of its most central provisions are creations of our own devising and that its concept of set is at variance with how the term “sets” appears to function in pre-theoretical English? This, if true, is a fact of fundamental epistemological importance. For if synthetic theories can afford us a knowledge of things, then making things up cannot as such be epistemologically disqualifying. 34

For good or ill, highly synthetic theories are linked to quite different epistemological presumptions. One is that, in the absence of suitably direct observational linkages, the optimum way of getting a theoretically satisfying command of what it is to be a C is to stipulate meanings for “C” that enable the production of the requisite theorems in the requisite numbers. This suggests a rather striking alternative to the notion of conceptual clarification forwarded by proponents of conceptual analysis. Seen that way, clarifying a concept is lifting it from the semantic shadows of intuitive usage into the light of full disclosure, a light that enables is fuller meaning to be discerned. (The light of reason, on some tellings). On the other hand, seen in the way of synthesis, clarifying a concept is not bringing hidden meanings to the surface, but rather giving it meanings of which there was no antecedent trace—darkly or otherwise. Accordingly, the meanings by which conceptual elucidation would be achieved would be value-added contributions of the clarifier’s creation—in Edington’s words, a put-up job.

There is in these speculations promise of comfort for the genus-species approach to the logic of entailment. If we take as generic the relation of unqualified truth-preservation, we have the latitude to think that conceptual clarification will come not from bringing antecedent generic meanings into the light, but rather from supplementing the generic with value-added meanings wrought by the logician for the purpose of proving theorems about entailment in the requisite numbers and with the desired rigour. Since there is no fixed single way of adding such value, the theorist is free to ambiguate the generic in as many ways as facilitate the production of theorems, while honouring the over-arching requirement of truth-preservation. Of course, it will happen as a matter of out-and-out contingency that some of the species generated thus will treat entailment in a more familiar light (e.g. S5) than others (e.g. S2). It may be that the whole enterprise of theory-building is linked in a causal way to our fondness for conceptual familiarity when we can find it. If so, it is more a matter for psychology than

logic. Being prompted by conceptual familiarity is neither a condition on theoretical adequacy nor—beyond truth-preservation—anything close to an invariant feature of it.

Suppose, then that all this is true. Suppose the genus-species approach to entailment has legs. Then, correspondingly, something else lacks legs. It is the assumption that semantic penumbras are load-bearing features of the logic of entailment. 35

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34 Related is the all but universal proclivity of the mature natural sciences to subject their principal targets to conceptual change. Thus one gets to know something of what it is to be a C by, among other things, changing the meaning of “C”. See here Thagard (1992).

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