Nucleon properties inside compressed nuclear matter.

Jacek Rożynek
The Soltan Institute for Nuclear Studies, Hoża 69, 00-681 Warsaw, Poland

Even small departures from a nuclear equilibrium density with constant nucleon masses require an increase of its total rest energies. This process can be described as volume corrections to a nucleon rest energy, which are proportional to pressure and absent in a standard Relativistic Mean Field (RMF) with point-like nucleons. Bag model and RMF calculations show the modifications of nucleon mass, nucleon radius and a Parton Distribution Function (PDF) of Nuclear Matter (NM) above the saturation point originated from the pressure correction to the nucleon rest energy. Different scenarios of phase transition to the Quark Gluon Plasma (QGP) which include a possible constant nucleon radius or/and constant nucleon mass as a function of nuclear density are considered.

PACS numbers: 21.65.+f,24.85.+p

I. INTRODUCTION

The finite size correction to the nucleon rest energy connected with the nucleon volume $\Omega_N$ will be investigated in the compressed NM with pressure $p$. In such a compressed medium, the nucleon constituents - quarks and gluons have to do an additional work $W_N = p\Omega_N$ to keep a space $\Omega_N$ for a nucleon "bag". The nucleon mass is a result of strong interaction between, almost massless, quarks and gluons. The nuclear Drell-Yan experiments [1, 2], which measure the sea quark enhancement, we described [3] with a small 1% admixture of nuclear pions and the $M_N$ unchanged. Thus the deep inelastic phenomenology indicates that a change of the nucleon invariant mass at the saturation density in comparison to the value in vacuum is rather negligible although nuclear scalar and vector mean fields are strong [4]. Therefore, when the energy transfer from Nucleon-Nucleon (NN) interaction to the partons is absent the nucleon mass should decrease with pressure. It will involve functional corrections to a nucleon rest energy, dependent from external pressure with a physical parameter - a nucleon radius $R_0$. The Equation of State (EoS) calculated in the linear scalar-vector RMF including finite nucleon sizes [5], and presented in Fig.1 in a comparison with other models, is softer for bigger nucleon radii $R_0$ with good value [18] for a compressibility $K^{-1} \sim 235$ MeV fm$^{-3}$ for $R_0 = 0.72$ fm; see Fig.1- left panel. For a constant $R_0$, the nonlinear term [6] in a scalar potential is not needed, which shows that finite nucleon sizes can provide independently a correct nuclear stiffness of EoS like in DBHF calculations [8] shown in Fig.1. At a higher density and a stronger NN repulsion, before going to quark matter phase, we should consider the possible energy transfer to the confined space inside nucleon from NN interactions, maintaining the constant mass of the nucleon for higher densities. Other modifications connected with finite volume of nucleons, like correlations of their volumes, will be neglected. In order to discussed this issue we introduce a nucleon enthalpy $H_N$ in NM with a density $\rho$ ($\rho_0$ at equilibrium) and "external" nuclear pressure $p$

$$H_N(\rho) = M_{pr}(\rho) + p\Omega_N \text{ with } H_N(\rho_0) = M_N,$$

as a "useful" expression for the total rest energy of a immersed nucleon "bag". The nucleon mass $M_{pr}$ is possibly modified in the compressed medium. For the constant nucleon mass $M_{pr}$ in NM, the formula (1) shows that the enthalpy $H_N$ - as the total nucleon rest energy - increases. In such a case a part of nucleon-nucleon interaction energy is transferred to the extended system of strongly interacting partons. It will be discussed in detail in the next section.

Let us introduce analogously the single particle nuclear enthalpy

$$H_T^A/A = \varepsilon_A + \rho/\rho_0 = E_F$$

where $\varepsilon_A = E_A/A$ is a single nucleon energy and $E_F$ is the nucleon Fermi energy $E_F$ or chemical potential $\mu$. In order to compare energy densities let us move to the specific enthalpies which are given respectively by:

$$h_T^A(\rho) = \frac{H_T^A}{E_A} = 1 + \frac{p}{\rho \varepsilon_A(\rho)}$$

$$h_N(\rho) = \frac{H_N}{M_{pr}} = 1 + \frac{p}{\rho_{cp}(\rho)M_{pr}(\rho)}$$

*Electronic address: rozynek@fuw.edu.pl
where $\rho_{cp} = 1/\Omega_N$ is a close packing density for extended nucleons inside NM. The specific nuclear enthalpy $h_T^{\rho}(\rho_{0})$ in equilibrium density $\rho_{0}$ is smaller than the specific nucleon enthalpy $h_N(\rho_{0})$ but increases faster than the nucleon one. It is easy to show that the equality of these specific enthalpies at a certain density $\rho_{cr}$

$$h_T^{\rho}(\rho_{cr}) = h_N(\rho_{cr})$$ (4)

is equivalent to a following condition for the critical density $\rho_{cr}$

$$\rho_{cr} \varepsilon_A(\rho_{cr}) = \rho_{cp}(\rho_{cr}) M_{pr}(\rho_{cr}).$$ (5)

where the alignment of energy densities, outside and inside nucleon, takes place. Another word, energy density ($\rho \varepsilon_A$), which includes a space $\Omega_A$ between nucleons, reaches the energy density of a quark plasma ($\rho_{cp} M_{pr}$) inside nucleon therefore an ultimate de-confinement transition to the Quark-Gluon-Plasma (QGP) will take place when condition (5) or (4) is satisfied. This self-consistent condition for the will discussed in two selected regimes: a constant nucleon radius (subsection “A”) and a constant nucleon mass (subsection “B”).

II. THE NUCLEON REST ENERGY IN THE BAG MODEL IN NM

Let us discuss the relation (1) in the simple bag model where the nucleon in the lowest state of three quarks is a sphere of a volume $\Omega_N$. In a compressed medium, pressure generated by free quarks inside the bag is balanced at the bag surface not only by intrinsic confining “pressure” $B(\rho)$ but also by nuclear pressure $p$; generated e.g. by elastic collisions with other hadron bags, also derived in QMC model in a medium \[12\]. A mass $M_{pr}$ for finite $p(\rho)$ can be obtained like a general expression\[5, 9\] on a bag energy in a vacuum $E_{Bag}(R_0) \propto 1/R_0$ as a function of the radius $R_0$ with phenomenological constants - $\omega_0$, $Z_0$, and a bag “constant” $B(\rho)$. But now, in equilibrium internal parton pressure $p_B$ inside the bag is equal (cf. \[12\]), on a bag surface, nuclear pressure $p$

$$p = p_B = \frac{3\omega_0 - Z_0}{4\pi R^4} - B(\rho) \rightarrow (B(\rho) + p) R^4 = \text{const}$$

and we get the nucleon radius depending from $B + p_B$:

$$R(\rho) = \left[ \frac{3\omega_0 - Z_0}{4\pi(B(\rho) + p(\rho))} \right]^{1/4}. \quad (6)$$
FIG. 2: Left panel - pressure dependent bag “constant” $B$ for the given nucleon radius $R = 0.7 fm$ (case A - constant nucleon size) and for two different values of $R_0 = 0.7, 0.8 fm$ (case B - constant nucleon mass). Right panel - the nucleon mass $M_{pr}$ as a function of NM pressure for two constant nucleon radii $R=0.7,0.8 fm$ and the constant total rest energy $H_N$.

Thus, the pressure $p(\rho)$ between the hadrons acts on the bag surface similarly to the bag “constant” $B(\rho)$. Consequently the mass in NM

$$M_{pr}(\rho) = \frac{4}{3}\pi R^3 [4(B + p) - p] = E_{\text{Bag}}^0 \frac{R_0}{R} - p \Omega_N.$$

(7)

The scaling factor $R_0/R$ comes from a well-known model dependence ($E_{\text{bag}}^0 \propto 1/R_0$) in the spherical bag [9]. This simple radial dependence is now lost in (7) and responsible for that is the pressure dependent correction to the mass of a nucleon given by the product $p\Omega_N$. This term is identical with the work $W_N$ in (1) and disappear for the nucleon enthalpy

$$H_N(\rho) = M_{pr} + p\Omega_N = E_{\text{Bag}}^0 \frac{R_0}{R(\rho)} \propto 1/R(\rho).$$

(8)

Thus the total rest energy of the nucleon in the nuclear medium depends only from its size $R(\rho)$. This simple dependence from $R(\rho)$ reflects a scale of a confinement of partons inside compressed NM. The internal pressure $B(\rho)$ depends on the external pressure $p$ and the nucleon radius [6]

$$B = B(\rho_0)(R_0/R)^4 - p.$$

(9)

and is shown in Fig2 for the constant radius and the constant mass with a variable radius. Presented dependence of the bag “constant” $B$ from the nuclear density has a strong influence on EoS and will be discussed further in subsections A and B.

III. TOWARDS PHASE TRANSITION

Start with a discussion of a main equation [5]. The nucleon $R$ might depend on nuclear pressure which in turn depends on nuclear density. For the increasing nucleon radius $R(\rho)$, the total rest energy $H_N(\rho)$ [5] and the mass $M_{pr}$ would decrease, thus part of the nucleon rest energy would be transferred from a confined region $\Omega_N$ to an remaining internuclear space. This is not realistic possibility, otherwise the nucleon would be unstable and will start to dissolve before equilibrium. Let us focus on remaining possibilities. For decreasing $R$, the $H_N$ increases; this allows also for the constant or increasing mass $M_{pr}$ [7]. But not far from equilibrium density, where the effective NN interaction is weak, there is probably no energy transfer to the partons and the nucleon rest energy remains constant.
A. The constant nucleon radius

Therefore, begin with this “conventional” case, when nucleon interactions do not change the total rest energy $H_N(\varrho) = M_N$ of partons confined inside nucleons; thus the radius $R(\varrho)$ is constant \([3]\). It requires the work $W_N$ to keep the constant volume at the expense of the nucleon mass $M_{pr}$ \([7]\) and is obtained \([6]\) for the constant effective pressure $B_{eff} = B(\varrho) + p(\varrho) = B(\varrho_0)$. The $B(\varrho) = B(\varrho_0) - p$ gradually decreases with pressure and disappears for $p_H(\varrho) = B(\varrho_0) \approx 60$ MeV fm\(^{-3}\) \([2]\) - see Fig.3 in favor of strongly correlated colored quarks in the de-confinement phase when $\varrho \approx (0.5-0.6)$ fm\(^{-3}\). Therefore the nucleon mass is a decreasing function of pressure:

$$M_{pr}(\varrho) = M_N - p(\varrho)\Omega_N, \quad \varrho \geq \varrho_0$$  \(10\)

and is shown in Fig.2. The internal pressure $B(\varrho)$, just as the external pressure $p_H(\varrho)$ (generated by an effective meson exchange), has the same origin \([13]\) from an interaction of quarks. In fact the sum $B(\varrho) + p_H(\varrho)$ weakly depends on density in GCM \([14]\) or QMC models \([12]\) with a reasonable stiff EoS, thus the bag radius remains about constant \([6]\). It justify our “conventional” choice of the constant total rest energy $H_N$, unchanged by an increasing NN repulsion. The phase transition to the QGP will start at the critical pressure $p_{cr}^A = 60$ MeV fm\(^{-3}\), where value of $B(\varrho_{min})$ is vanish; see Fig.2. The corresponding minimal density $\varrho_{cr A}^A \geq 0.45$ fm\(^{-3}\) allowed within semi-empirical estimate \([17]\), is marked as open circle on a horizontal “A” line in a left panel of the Fig.3. The ultimate critical density $\varrho_{cr}^A$ for the phase transition with the constant radius is \([4]\):

$$\varrho_{cr}^A = \frac{(M_N - p\Omega_N)}{\varepsilon_A} \varrho_{cr} = \frac{M_N}{\varepsilon_A(\varrho_{cr})} \varrho_{cr} - p/\varepsilon_A.$$  \(11\)

The nucleon radius $R_0 = .72$ fm is assigned to a good value \([18]\) of compressibility $K^{-1} = 235$ MeV at the saturation point; compare right panel in Fig.3. The critical value, for a closed packing density $\varrho_{cp} = 0.64$ fm\(^{-3}\), can be obtained from EoS \([3]\) taking into account finite volume corrections (here for $R_0 = .72$ fm); see Fig.3\(\text{a}\). The estimation gives $\varrho_{cr}^A \sim 0.55$ fm\(^{-3}\) with the critical pressure $p_{cr}^A \sim 60$ MeV fm\(^{-3}\) and is marked on a line “A” as solid circle.

B. The constant nucleon mass

Just opposite to the case with constant radius, the constant nucleon mass $M_{pr} = M_N$, requires an increase of the total rest energy $H_N(\varrho)$ \([13]\), thus a decrease of the nucleon size $R(\varrho)$. For the constant nucleon mass, the decreasing nucleon radius satisfies a quartic equation:

$$M_N R_0/R = M_N + 4/3\pi R^3 p$$  \(12\)

The resulting radius depends from pressure and is shown in the right panel of Fig.2 for two different radii $R_0$ at equilibrium. The ultimate phase transition (energy alignment) to the QGP will take place at critical density $\varrho_{cr}^B$ and for the phase transition with the constant mass is \([6]\):

$$\varrho_{cr}^B = \frac{M_N}{\varepsilon_A} \varrho_{cr} = \frac{3M_N}{4\pi \varepsilon_B(\varrho_{cr}) R^3(\varrho_{cr})}.$$  \(13\)

The internal confining pressure $B(\varrho)$ is positive for much higher pressure in comparison with $p_{cr}^A \sim 60$ MeV fm\(^{-3}\) in case A; see Fig.2, left panel. For $R_0 = 0.72$ fm the critical pressure $p_{cr}^B \sim 90$ MeV fm\(^{-3}\), see Fig.2. However, for that pressure the nucleon is compressed to $R = 0.65$ fm which gives very high value of $\varrho_{cp} = 0.86$ fm\(^{-3}\), not compatible with the realistic EoS with constant nucleon mass (for e.g. relatively stiff EoS \([8]\) obtained from DBHF calculations do not match our critical condition \([13]\)). However it is possible to get the consistent critical values changing the initial value of $R_0 = .8$ fm with $p_{cr}^B \sim 110$ fm\(^{-3}\) and a softer EoS \([6]\) (like that, marked as “$R_0 = 0.65$” fm in Fig.4. This change in not connected directly to our dependence from compressibility presented in Fig.3, which was derived only for a constant radius $R_0$. The corresponding minimal density $\varrho_{cr 1}^B \geq 0.55$ fm\(^{-3}\) included within semi-empirical estimate is marked as open circle in a left panel of Fig.3. Now nucleons are squeezed to $R = .69$ fm with the closed packed density $\varrho_{cp} = 0.73$ fm\(^{-3}\). We get from \([13]\) the alignment density $\varrho_{cr 1}^B \sim 0.66$ fm\(^{-3}\) with the critical pressure $p_{cr}^B \sim 110$ MeV fm\(^{-3}\); marked as solid circle in Fig.3 on a line “B”.

In the widely used standard \([6]\) RMF model with point-like nucleons and constant mass, the good compressibility is fit by nonlinear changes of a scalar mean field, using two additional parameters. Our results indicate how to interpret, in this scenario, these mean field parameters proportional to the higher powers of density but here connected with an increase of the nucleon rest energy.
FIG. 3: Left panel - the EoS for NM. The area indicated by “flow constraint” taken from [17] determines the allowed course of EoS, using an analysis which extracts from the matter flow in heavy ion collisions from high pressure obtained there. Described DBHF [8] calculations with a Bonn A interaction are shown as short dashes. Right panel - Pressure dependent nucleon radius $R$ for two different values of initial radius $R_0 = 0.7, 0.8 \text{ fm}$ (case B - constant nucleon mass).

IV. THE NUCLEON PDF FOR FINITE PRESSURE

In the nuclear deep inelastic scattering on nuclei the time-distance resolution is given by variable $z$ [15]:

$$z = 1/(x M_{pr})$$

(14)

which measures the propagation time of the hit quark caring the Björken $x$ fraction of the longitudinal momentum of the nucleon of mass $M_{mid}$ in NM. Start with the scenario where the partonic mean free paths $z$ are much shorter than the average distances between nucleons. This means that partons (inside mesons) remain inside the "volume" of a given nucleon and therefore we can treat nucleons as noninteracting objects remaining on the energy shell not affected by neighboring nucleons. In the light cone formulation [15, 16], $x$ corresponds to the nucleon fraction of quark longitudinal momentum $p_q^+ = p_q^0 + p_q^3$ and is equal (in the nuclear rest frame) to the ratio $x = p_q^+/P_N^+ = \sqrt{2} p_q^+/M_{pr}$. Partons - gluons and quarks inside compressed nucleon will start to adjust their momenta to nucleon properties like a surface, volume and a mass. These squeezed extended objects exist in NM under a positive pressure and the amount of energy is required to make a room for nucleons by displacing its environment. In case B when the energy is transferred inside the nucleon to provide the constant nucleon mass the momentum sum rule integral of PDF will remain unchanged in the first approximation. However in a case A the smaller mass will reduce the sum of parton LM momenta in medium:

$$\sqrt{2} \sum_{i=1}^{N_q} p_i^+ = \sum_{i=1}^{N_q} p_i^0 = M_{pr} \quad \text{or} \quad \int_0^1 dx_M F_2^B(x_M) = 1.$$  

(15)

Now the Bjorken variable $x_M = p_q^+/P_N^+ = x M_N/M_{pr}$ which scale the deeply inelastic cross section will be smaller as smaller are the quark longitudinal momenta. If an appropriate change in the scaling will be shown experimentally in a future experiments above the equilibrium density, it will indicate the absence of the energy transfer to partons confined inside the nucleons (case A).
V. CONCLUSION

The possible energy transfer to partons inside nucleons influences strongly the stiffness of EoS and positions of phase transition to QGP. In particular, for constant nucleon radius (case A - no energy transfer) the critical density $\rho_{cr}^A \simeq 0.60 \text{ fm}^{-3}$ with the critical pressure $p_{cr}^A \sim 60 \text{ MeV fm}^{-3}$. However if the energy transfer from nucleon-nucleon interaction to the partons inside nucleons will provide the constant nucleon mass (case B) the critical density of the phase transition to QGP is shifted to $\rho_{cr}^B \simeq 0.66 \text{ fm}^{-3}$ with $p_{cr}^B \sim 110 \text{ MeV fm}^{-3}$. The energy transfer from the NN interaction to the confining partons is a not well unknown process in nuclear physics. If the process takes place it will start probably at "unbound" threshold $\rho_{unbound} \gtrsim 0.35 \text{ fm}^{-3}$ where $\varepsilon_A > M_N$ and the NN repulsion prevails; see Fig.1. Thus our estimation in the case “B” might be slightly shifted. Moreover, the energy transfer which involves the decrease of the nucleon radius can be large enough to shift the phase transition to much higher densities. Such a solution will be possible if there exist a stable bound state of 3 quarks in a dense vector meson environment. At the equilibrium density, where the attractive NN interaction prevails, the energy transfer to the partons from interaction nucleons is rather not expected, thus the nuclear compressibility is strongly reduced according to the scenario A where the the nucleon mass decreases with pressure.

This work is supported by a National Science Center of Poland, a grant DEC-2013/09/B/ST2/02897.

[1] D.M. Alde et al., Phys. Rev. Lett. 64, 2479 (1990).
[2] J. R. Smith and G. A. Miller, Phys. Rev. C 65, 015211, 055206 (2002).
[3] J. Rożyniec, Nucl. Phys. A 755, 557c (2004).
[4] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. Vol. 16 (Plenum, N. Y. 1986); R. J. Furnstahl and B. D. Serot, Phys. Rev. C 41, 262 (1990).
[5] J.Boguta, Phys. Lett. 106B, 255, (1981); N.K. Glendenning, "Compact Stars", Springer-Verlag, New York, 2000.
[6] J. Schaffner-Bielich, M. Hanauske, H. Stöcker, W. Greiner PRL, 89, 171101 (2002); P. Haensel, A.Y. Potekhin, D.G. Yakovlev, "Neutron Stars 1", 2007 Springer.
[7] C. Gross-Boelting, C. Fuchs, A. Faessler, Nuclear Physics A 648, 105 (1999); E. N. E. van Dalen, C. Fuchs, A. Faessler, Phys. Rev. Lett. 95, 022302 (2005); Fuchs J. Phys. G 35, 014049 (2008).
[8] K. Johnson, Acta Phys. Pol. B6, 865 (1975). A. Chodos et al., Phys. Rev. D 9, 3471 (1974);
[9] L. Ferroni and V. Koch, Phys. Rev. C 79, 034905 (2009).
[10] J.I. Kapusta and Ch. Gale, "Finite Temperatures Field Theory", Cambridge University Press, New York 2006.
[11] Guo Hua, J. Phys. G25, 1701 (1999).
[12] R. L. Jaffe, Los Alamos School on Nuclear Physics, CTP 1261, Los Alamos, July 1985.
[13] L. L. Frankfurt and M. I. Strikman, Phys. Rep. 160, 235 (1988).
[14] Buballa M., Nucl. Phys. A611, 393, (1996).
[15] Y. Liu, D. Gao, H. Guo, Nucl. Phys. A695, 353 (2001); R. T. Cahil, C. D. Roberts J. Praschifka, Ann. Phys. (NY), 188 (1988).
[16] E. Khan, J. Margueron, G. Colo, K. Hagino, H. Sagawa, Phys. Rev. C 82, 024322 (2010).