Analysis and design of halo orbits in cis lunar space

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Abstract. This paper examines the formation of halo-orbits by a spacecraft around L₁ and L₂ libration points of the Earth-Moon system. Perturbations from the gravitation of the Sun and the non-centrality of the gravitational fields of the Moon and the Earth are taken into account (LP165p model). The proposed perturbed model is an intermediate stage between the dynamic model of the three bodies of the Earth-Moon system and the high-precision non-periodic ephemeris model. The paper proposes two strategies for maintaining halo orbits. The first is to use a small impulse performed on each half of the orbit. The second is to use continuation methods to form a periodic orbit that does not require additional corrections. The essence of the second strategy is to use the perturbation periodicity to generate a special multi-circular halo orbit resonating with the perturbation period. The results of these two approaches are presented for various perturbed models.

1. Introduction

Lunar reconnaissance is the foundation of deep space exploration, it requires the search for suitable orbits for the operation of the spacecraft. There are many different types of orbits around the Moon. Several special orbits around the Moon have been comprehensively discussed in [1]. In [2], a project called "Lunar Orbital Platform-Gateway" was proposed, thus, halo orbits in the cis lunar space have become an essential research topic [2].

For real missions, the orbital analysis in the unperturbed model is far from sufficient, since many main perturbation factors must be taken into account [3-6]. For halo orbits, it is necessary to take into account the existence of the main multiple disturbing factors for keeping the orbit. In the Earth-Moon three-body system, the Sun's gravity is a sufficient disturbance. And the uneven gravitational disturbance of the Moon is also an important disturbing force, since the halo orbits in this system are relatively close to the Moon. Since the Moon's gravitational field is very complex, this is far from enough to consider the Moon's contraction only.

Thus, the purpose of this work is to analyze and design halo orbits around the libration points L₁ and L₂ of the Earth-Moon system, taking into account the perturbations from the Sun's gravity, the non-centrality of the gravitational fields of the Moon and the Earth.

2. Unperturbed motion

The unperturbed dimensionless equation of satellite motion in the CR3BP model (Figure 1) has the form:

\[ \ddot{x} - 2\dot{y} = U_x, \quad \ddot{y} + 2\dot{x} = U_y, \quad \ddot{z} = U_z \]  \hspace{1cm} (1)
Here the pseudopotential $U$ is defined as

$$U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2},$$

where $U_x$, $U_y$, and $U_z$ are the partial derivatives of $U$ with respect to the position variables, $r_1$ is the distance from the satellite to the Earth, $r_2$ is the distance from the satellite to the Moon. Mass parameter $\mu$, distance, velocity and time for the rotating Earth-Moon system are described in [7].

There are interesting periodic orbital solutions in (1), which Farkukhar first called the halo orbit in [8]. There is a special periodic orbit in the plane of the Moon’s motion - the Lyapunov orbit [9]. But here, for the purposes of mission and orbit analysis, only general 3D halo orbits are discussed.

$X$ is a state vector: $X = \{x, y, z, \dot{x}, \dot{y}, \dot{z}\}^T$, therefore, equation (1) is rewritten as: $\dot{X} = f(X)$. First order variational equations are obtained, which lead to the following vector differential equation:

$$\Delta\dot{X} = A(t)\Delta X$$

where $A(t) = \begin{bmatrix} 0_{6 \times 1} & I_{6 \times 6} \\ U_x & \Omega \end{bmatrix}$, $U_x = \begin{bmatrix} U_{xx} & U_{xy} & U_{xz} \\ U_{yx} & U_{yy} & U_{yz} \\ U_{zx} & U_{zy} & U_{zz} \end{bmatrix}$, $U_x = \frac{\partial U_x}{\partial x}$, $\Omega = \begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

According to this differential equation, the periodic halo orbits are symmetric about the x-z plane [10], so the initial state vector takes the form: $X_0 = \{x_0, 0, z_0, 0, \dot{x}_0, 0\}^T$, which is perpendicular to the x-z plane. So the initial condition is fixed with a first order extension:

$$\phi(X_0 + \Delta X, T_{t/2} + \Delta t) = \phi(X_0, T_{t/2}) + \frac{\partial \phi(X_0, T_{t/2})}{\partial X} \Delta X + \frac{\partial \phi(X_0, T_{t/2})}{\partial t} \Delta t,$$

where $\Delta X = \{\Delta x, 0, \Delta z, 0, \Delta \dot{x}, 0\}^T$, $\phi(X_0 + \Delta X, T_{t/2} + \Delta t) = \{x', 0, z', 0, \dot{x}', 0\}^T$ , * represents what is initially unknown. Here we define $\frac{\partial \phi}{\partial X} = \Phi(t, t_0)$ as the state transition matrix, then:

$$\Phi(t, t_0) = A(t)\Phi(t, t_0), \Phi(t_0, t_0) = I,$$
In (5), we consider \( t_0 = 0, t = T_{1/2} \), which means that the time when the state vector returns to the x-z plane for the first time \( \partial \phi / \partial t \) is defined as the half-cycle of the orbit, and has been replaced by \( f(\phi) \).

Considering only the second, fourth and sixth lines in (5), the system becomes:

\[
\begin{bmatrix}
\Phi_{21} & \Phi_{23} & \Phi_{25} & f_3 \\
\Phi_{41} & \Phi_{43} & \Phi_{45} & f_4 \\
\Phi_{61} & \Phi_{63} & \Phi_{65} & f_6 \\
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta z \\
\Delta \gamma \\
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
0 \\
\end{bmatrix}
\]

In (6) there are three equations with four unknown variables. According to the study of halo orbits in [9-11], we can conclude that the motion in the z direction is a simple harmonic, and the motion in the x-y plane is related. This is expected since the family must be parameterized with an amplitude outside the \( Az \) plane. Thus, assuming \( \Delta z = 0 \) in (6), we continue iteratively from (1), correcting the initial conditions \( X_n \) to \( \bar{u} = 0, \bar{w} = 0 \). The new initial condition will be \( X_{\text{new}} = X_{\text{old}} + \Delta X \).

In the Figure 2 we can see that orbits are parameterized by \( Az \), the change in \( Az \) is not monotonic. For the orbits of the halo around the points \( L_1 \) and \( L_2 \) of the Earth-Moon system orbital period range is 8-12 days, maximum \( |Az| \) is 112,000 km, and the maximum distance from the orbits of the halo to the Moon is 74,000 km.

Equation (4) is integrated over one orbital period \( T \) to obtain the monodromy matrix \( \Phi(T,0) \) and then analyze the stability of the orbit by calculating the eigenvalues of this monodromy matrix. These eigenvalues, denoted from \( \lambda_1 \) to \( \lambda_6 \), serve as characteristics of the orbit. Recall from Lyapunov’s theorem that the eigenvalues occur in reverse pairs if they are real, or in conjugate pairs if they are complex [5, 9, 12]:

\[
\lambda_1, \lambda_2, \lambda_3 = 1, \lambda_4 = 1/ \lambda_1, \lambda_5 = 1/ \lambda_2, \lambda_6 = 1,
\]

Figure 2. Orbital periods for \( |Az| \) and Max\[|x|\].
Thus, each pair of eigenvalues is combined into three stability indices.

\[ \nu_i = \frac{1}{2} (\lambda_i + \frac{1}{\lambda_i}), i = 1, 2, 3. \] (8)

It is worth noting that the family of orbits contains two branches, symmetric to the x-y plane, which are called the north and south orbits. The southern family is symmetrical to the northern family about the x-y plane [9, 13], so we can analyze only one of them. At present, one of the goals of the Moon exploration is the region near the South Pole of the Moon due to the potential existence of frozen volatiles [14]. Therefore, in this article, the southern orbit family is selected for analysis, but the conclusions are also valid for the corresponding northern orbit:

3. Perturbed motion

As shown in Figure 3, in the rotating Earth-Moon system the X-axis connects the Earth and the Moon, the Z-axis is directed along the Moon's orbital angular momentum around the Earth, and the Y-axis completes the right-handed system. The x-y plane is the Moon's motion plane, and the x'-y' plane is the Moon's equatorial plane. But the movement of the moon's axis of rotation is extremely complex, and the angle between these two planes changes only within a small angle. In addition, there is no single and precise calculation formula for this task. Thus, in this article, the problem is simplified to assume that the lunar equatorial plane and the lunar plane of motion are the same.

![Figure 3. Lunar fixed coordinate system in the rotating Earth-Moon system.](image)

Then there are the following coordinate transformations:

\[
\begin{bmatrix}
  x' \\
  y' \\
  z'
\end{bmatrix} =
\begin{bmatrix}
  -1 & 0 & 0 \\
  0 & -1 & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}. \] (9)

Here \((x, y, z)^T\) is the Earth-Moon rotating system, \((x', y', z')^T\) is the Earth-Moon rotating system centered on the Moon, \((x', y', z')^T\) is the fixed frame of the Moon's body, \((l, 0, 0)^T\) is the position of the Moon in the \((x, y, z)^T\) system.

When considering the LNGP model, we use \(U^*\) instead of \(U\) in (1):

\[ U^* = \frac{1}{2} (x^2 + y^2) + \frac{1 - \mu}{r_1} + \frac{\mu}{r_2} + U^0. \] (10)

Here \(U^0\) is the dimensionless perturbed potential of the LNGP model, it is transformed from the dimensional potential \(u^0\):

\[ u^0 = \frac{f m}{r} \sum_{n=2}^{N_{max}} \sum_{k=0}^{n} \left( \frac{r_0}{r} \right)^n P_n (\sin \phi)(C_{nk} \cos k \lambda + S_{nk} \sin k \lambda), \] (11)

where \(r_0\) is the mean equatorial radius of the Moon, \(P_n\) are the conjugate Legendre functions of \(n\) degree and \(m\) order, \(C_{nk}\) and \(S_{nk}\) are dimensionless coefficients of decomposition of lunar gravity, \(r, \phi, \lambda\) the spherical coordinates of the spacecraft in the fixed coordinate system of the Moon, \(f m\) is the lunar gravitational constant. \(N_{max} = 165\) for LP165p model.
4. Solar Gravity Perturbation Model (SGP)

The SGP model assumes that the Earth and Moon move in circular orbits around their common barycenter B1, while the Sun and B1 move in circular orbits relative to the Earth-Moon-Sun barycenter B2, as shown in Figure 4.

**Figure 4.** The position of the Sun in the Earth-Moon rotating system.

Here we also assume that these two planes of motion coincide. This is not a coherent model: the perturbing acceleration from the Sun does not affect the motion of the Earth and the Moon [5].

Now let us replace $U$ in (1) with the potential expression $U^*$:

$$U^* = \frac{1}{2} (x^2 + y^2) + \frac{1}{r_1} - \frac{\mu}{r_2} + \frac{m}{r} (x, y, z),$$

(12)

where $m$ is the dimensionless mass of the Sun, $a$ is the dimensionless distance between the Earth-Moon barycenter and the Sun, $x$, $y$, $z$, are the components of the Sun position, expressed in terms of the Earth-Moon rotating system. The $x$, $y$, $z$, components are defined as follows:

$$\begin{align*}
x, y, z &= a \begin{bmatrix} \cos(\theta_s) \\ \sin(\theta_s) \\ 0 \end{bmatrix}, \quad \theta_s = \omega_s t + \theta_{\phi_s},
\end{align*}$$

(13)

where $\omega_s = -0.9253001$ is the dimensionless angular velocity of the Sun in the rotating Earth-Moon system, and the angle $\theta_s$ of the Sun is measured from the x-axis to the vector of the Sun's position. The system depends on time and does not admit an integral of motion, and in accordance with the relative position of the celestial bodies in this disturbed system, there are two periods: the synodic period of the Sun and the Moon (29.5 days) and the sidereal period of the Earth and the Moon (27.2 days).

From Table 1 it can be seen that the deviations of the libration points in the cislunar space are very small due to the LNGP (about 10 m). Whereas the deflection due to SGP is related to the position of the Sun and is very large (about 300 km).

**Table 1.** Coordinates of libration points $L_1$ and $L_2$ in different models.

| Model                  | $L_1$                                      | $L_2$                                      |
|------------------------|--------------------------------------------|--------------------------------------------|
| Unperturbed model      | $X=0.836918007$, $Y=0$, $Z=0$             | $X=1.155679913$, $Y=0$, $Z=0$             |
| LNGP model (165×165)   | $X=0.836917976$, $Y=5.613090221e^{-11}$, $Z=2.397169737e^{-10}$ | $X=1.155679946$, $Y=9.272120099e^{-11}$, $Z=2.243495069e^{-10}$ |
| SGP model $\theta_{\phi_s}=0$ | $X=0.836085318$, $Y=0$, $Z=0$             | $X=1.153946922$, $Y=0$, $Z=0$             |
| SGP model $\theta_{\phi_s}=45^\circ$ | $X=0.836718509$, $Y=-0.001699103$, $Z=0$ | $X=1.155178482$, $Y=-0.004391103$, $Z=0$ |
Figure 5 shows the influence of SGP on L₁ and L₂ points during one synodic period. Within 29.5 days, the instantaneous equilibrium point rotates twice around its original position (red dot). In addition, the deviation of the position of point L₂ is more dependent on solar gravity than on point L₁.

![Figure 5](image-url)

**Figure 5.** Positions of L₁ and L₂ points during one synodic period.

From Table 2 we know that in the cislunar space the influence of gravity on the L₂ point is $10^5$ times greater than that of the LNGP. The acceleration of the disturbance from the pressure of solar radiation has the smallest value. Since the impact of LNGP is small compared to other perturbation factors, it can be ignored when analyzing short-period orbits. But for a long-term mission, this is a relatively important interfering force in the following analysis.

**Table 2.** Basic disturbance at L₂ point in cislunar space.

| Perturbation | Acceleration $a$ [m/s²] |
|--------------|------------------------|
| Earth        | $7.359 \times 10^{-5}$ |
| Moon         | $3.509 \times 10^{-5}$ |
| Sun          | $3.721 \times 10^{-5}$ |
| LNGP         | $6.721 \times 10^{-10}$ |
| Sun radiation| $4.612 \times 10^{-11}$ |

The halo orbit from Figure 2 (period = 11.684 days) is calculated in the x-y plane and shown in Figure 6. When only the SGP is considered and no additional corrections are taken into account, the deviation of the satellite is $1.4669 \times 10^4$ km from the home position after one circle. It cannot form a periodic orbit. When only LNGP is considered, the satellite deviates from its original position by 0.1895 km after one circle and deviates 12.0249 km after two circles. This means that the satellite can go around the halo region in a short time, but after 3 circles there is a clear deflection.

![Figure 6](image-url)

**Figure 6.** Impact of SGP and LNGP on halo orbits.
The above analysis shows the effects of SGP and LNGP in cislunar space and describes the disturbed motion of a spacecraft in halo orbits. Thus, a dynamic system considering LNGP and SGP can be considered as a more accurate intermediate dynamic three-body system. In the next part, two orbit-keeping strategies are described.

5. Retention of halo-orbits for SGP model
In small impulse method, a small impulse is performed in each half-cycle of the orbit, so that in the next half of the part, the spacecraft returns to the x-z plane at a speed perpendicular to the x-z plane, which forms a quasi-halo orbit. Although this method does not always maintain the original halo orbit, this method can effectively maintain the orbit in a specific area [4]. To a large extent, this method has been successfully applied in real China missions by the two Chinese lunar satellites CHANG'E 2 and Queqiao.

Let the initial vector of the spacecraft coordinates at step i be \( \mathbf{X}_i = [x_i, y_i, z_i] \), \( \mathbf{X}_{+i} \) represents the corrected vector of the satellite coordinates, and \( \mathbf{X}_{+i} = \mathbf{X}_i + \Delta \mathbf{V} \), where \( \Delta \mathbf{V} \) is the impulse. Then replace \( \Delta \mathbf{V} = [0, 0, 0, \Delta x_i, \Delta y_i, \Delta z_i] \) with \( \Delta \mathbf{X} \) in (3) so that equation (6) changes to the following:

\[
\begin{bmatrix}
\Phi_{24} & \Phi_{25} & \Phi_{26} \\
\Phi_{44} & \Phi_{45} & \Phi_{46} \\
\Phi_{64} & \Phi_{65} & \Phi_{66}
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
\bar{a} \\
\bar{w}
\end{bmatrix}
\tag{14}
\]

Thus, using equations (1), (2), (6), (11), (12) and (13) for iterative calculation, long-term station maintenance can be achieved in the SGP model. Meanwhile, equation (4) is used to calculate the monodromy matrix obtained at each i stage to analyze the stability of the orbits. In fig. 10 the shown quasi-halo orbits were formed by stable halo orbits at L1 point using this method with 50 corrections (25 revolutions).

It can be seen from Figure 7 that this station hold strategy can effectively keep the satellite in periodic orbits for a long time in the SGP model. The modified quasi-halo orbits have changed insignificantly compared to the orbits in the unperturbed model.

**Figure 7.** Quasi-halo orbit at L1 point.
Figure 8 shows the period, $dv_x$, $dv_y$ and $v_i$ for $L_1$ and $L_2$ points.

![Graphs showing period, $dv_x$, $dv_y$, and $v_i$ for $L_1$ and $L_2$ libration points.](image)

**Figure 8.** Characteristics of a quasi-halo orbit at $L_1$ and $L_2$ libration points.

Let us now describe the continuation method. There are certain periodic orbits in the SGP model that do not require additional correction, the period of which resonates with a synodic period called a multicircular halo orbit (MC-Halo). For the orbit to be periodic in this perturbed model, it is sufficient that the orbit intersects the $x$-$z$ plane perpendicularly when the Sun is in the apse (The position of the Sun corresponding to an odd number $Q$ is opposite to its initial position, and the Sun's position corresponding to an even number $Q$ is its initial position.) [15].

Synodic resonance orbits are characterized by an integer ratio between synodic period and orbital period. This relationship is represented as where the halo revolves around rotating $N$ circles, while the Sun revolves around $Q$ circles.

Figure 9 shows the curve of the period of the family of halo-orbits $L_1$ and $L_2$ of the Earth-Moon. Vertical dotted lines correspond to correct ($Q$:N). There are infinite sets ($Q$:N) since they are discretely distributed. It should be noted that $N$ grows faster than $Q$, which will cause numerical difficulties [15].

![Graph showing orbital period and corresponding resonance ratios.](image)

**Figure 9.** Orbital period and corresponding resonance ratios.

The following three steps are used to obtain MC-Halos. At the first step, the initial parameters of the orbit from fig. 9 are selected according to the planned value ($Q$:N) using a dichotomy. At the second step, make $\Delta t = 0$ in (6), and the initial orbit parameters with exact period can be obtained using the iterative process mentioned above. Finally, the Lobatto IIIa algorithm is used to generate MC-Halo. The collocation polynomial gives a $C^1$-continuous solution that is uniform in the integration domain with high precision. Mesh selection and error control are based on the remainder of the solution [16]. The collocation method uses a grid of points to subdivide the integrating range into sub-intervals. A solver can determine a numerical solution by solving a system of global algebraic equations derived from boundary and configuration conditions imposed on all subintervals. The solver then estimates the numerical solution error at each subinterval. If the solution does not meet the tolerance criteria, the
The solver adjusts the mesh and repeats the process. It is necessary to indicate the points of the initial value, as well as the initial approximation of the solution at each point [16, 17].

The halo orbit obtained in the second step is integrated for N periods, and all state vectors are used as the initial mesh for the Lobatto IIIa algorithm. Then, using a continuous process, ε is determined to transform (11) into form (14), where ε = 0 represents the system without solar gravity disturbance and ε = 1 represents the SGP system. Gradually increasing ε from 0 to 1, the unperturbed model is gradually transformed into a perturbed model, thereby obtaining MC-Halo in the SGP model

\[
U^* = \frac{1}{2} \left( x^2 + y^2 \right) + \frac{1 - \mu}{r_1} + \frac{\mu}{r_2} + \frac{\epsilon m}{a^3} (x, x + y, y + z, z)
\]  

Figures 10 and 11 show the selected halo orbits in the unperturbed model and MC-Halo in the SGP model at L₁ and L₂ points, respectively, in which the MC-Halo period at point L₁ is equal to 2 synodic periods, while L₂ has 1 synodic period. MC-Halo has a clear deformation, and the orbit is symmetric about the x-z plane.

Table 3 shows the stability indices of the two sets of coupled orbits. The stability index for MC-Halo with synodic resonance Q2N5 associated with unstable modes is less \(1.2460 \times 10^9\) than the initial halo orbits integrated over N periods \(1.5744 \times 10^9\), i.e. 1 period MC-Halo. A solution built in the SGP model from this periodic, linearly stable initial assumption is unstable.
Table 3. Stability indices for selected MC-halo.

| Type of orbit          | MC-Halo Q2N5 | MC-Halo Q1N3 |
|------------------------|--------------|--------------|
| Halo orbit, 1 period   | 39.6890      | 1            |
|                        | 1            | 0.0334       |
|                        | -0.9377      | -0.6736      |
| Halo orbit, N period   | 1.5744 × 10^9| 1            |
|                        | 1            | -0.1002      |
|                        | 0.2027       | 0.7983       |
| MC-Halo, 1 period      | 1.2460 × 10^9| -34.1692     |
|                        | 1.0098       | 0.2902       |
|                        | 0.0218       | 0.9721       |

Thus, both of these methods are effective for keeping the station in the cislunar space. In particular, the small impulse method can maintain the stability of the halo orbits in the SGP model, but requires additional fuel consumption. Continuation methods can be used to form a periodic orbit without additional correction, but this can lead to greater orbital deformation and affect the implementation of mission objectives. Therefore, it is necessary to choose the station storage method according to the actual mission requirements.

6. Conclusion

In this paper, a perturbed dynamic model that takes into account the non-uniform gravitational disturbance of the Moon and the solar gravitational disturbance is proposed as an intermediate model between the CR3BP model and a real ephemeris model of high accuracy. Based on this model, the deviation of the spatial position of the L₁ and L₂ points from the disturbing force is calculated.

Two efficient station-keeping strategies were proposed. The first strategy is to use a small impulse to correct the orbits of each half-period forming the quasi-halo orbits that can maintain the stability of the halo orbit by using additional fuel, while the orbital structure and period do not transform significantly. The second is to use continuous methods and the Lobatto IIIa algorithm to generate a periodic multicircular halo orbit that resonates with a synodic period. This method does not require additional fuel to run the station. However, the linear stability of the orbit is poor, and if the satellite is closer to the Moon, the orbital structure will change greatly due to the lunar gravity.

The perturbed dynamic model presented in this article can be used as a simplified model of a high quality real ephemeris model, which will provide a more accurate expression for preliminary orbit construction.

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