Reading-out the state inductively
and microwave spectroscopy of an interferometer-type charge qubit

D. Born, V.I. Shnyrkov, W. Kreck, Th. Wagner, E. Il'ichev, M. Grajcar, U. Hübner, and H.-G. Meyer

1Friedrich Schiller University, Institute of Solid State Physics, Helmholtzweg 5, D-07743 Jena, Germany
2Institute for Physical High Technology, P.O. Box 100239, D-07702 Jena, Germany

(Dated: November 9, 2018)

We implemented experimentally an interferometer-type charge qubit consisting of a single-Cooper-pair transistor closed by a superconducting loop that is in flip-chip configuration inductively coupled to a radio frequency tank circuit. The tank permits to readout the qubit state, acting as inductance measuring apparatus. By applying continuous microwave power to the quantum device, we observed inductance alterations caused by redistributions of the energy level populations. From the measured data, we extracted the energy gap between ground and upper levels as a function of the transistor quasicharge as well as the Josephson phase across both junctions.

PACS numbers: 74.50.+r, 85.25.Am, 85.25.Cp

Various quantum properties of superconducting structures with small Josephson tunnel junctions have been experimentally demonstrated by several research groups (see, e.g., Ref. 1 and references therein). The results have been widely discussed in literature because such devices might serve as prototypes for quantum bits (qubits).

In this context, the single-Cooper-pair transistor has attracted renewed attention. The device consists of two mesoscopic Josephson junctions coupled by a small island. The Hamiltonian of this system can be written as

\[ H = E_C(n - n_g)^2 - \varepsilon_J(\delta) \cos \varphi. \tag{1} \]

Here the first term on the r.h.s. is the charging energy of the island whereas the second one describes its Josephson coupling to the leads. The variable \( n = 2m \) implies the number \( m \) of excess Cooper pairs on the central electrode, the parameter \( n_g = C_g V_g / e \) is continuously controllable by the gate voltage \( V_g \) via the capacitance \( C_g \). The one-electron Coulomb energy, \( E_C = e^2 / 2C_S \), is expressed through the island’s sum capacitance \( C_S \). Furthermore, the coupling strength,

\[ \varepsilon_J(\delta) = [E_{J1}^2 + E_{J2}^2 + 2E_{J1}E_{J2}\cos\delta]^{1/2}, \tag{2} \]

is a function of the total phase across both junctions, \( \delta = \varphi_1 + \varphi_2 \) (being here a good quantum variable). \( E_{J1}, E_{J2} \) are the individual Josephson energies and \( \varphi_{1,2} \) the respective junction phases. Note that the observable \( m \) is conjugated to the island’s phase, \( \varphi = (\varphi_2 - \varphi_1)/2 \). In order to realize charge qubits, we designed the system parameters to fulfill the domination, \( E_{CP} > \varepsilon_J(\delta) \), of the Cooper pair Coulomb energy, \( E_{CP} \equiv 4E_C \), over the coupling strength \( \varepsilon_J \).

In earlier studies, microwave induced transitions between ground and upper energy states in single-charge transistors were observed by measuring the switching current or by means of the photon-assisted quasiparticle current. Later the quantum coherent oscillations in similar configurations have been detected by making use of microwave-pulsed readouts.

Recently, a new possible solution was theoretically proposed by several authors.\(^{1-4}\) The main idea is to enclose a single-Cooper-pair transistor into a superconducting inductive loop forming an interferometer. The advantage of this circuit is its low dissipation and, therefore, the remarkably weak decoherence.\(^{5-7}\) For realizing the quantum measuring process, the loop is inductively coupled to a radio frequency tank circuit.

In this paper, we introduce experimentally the conception of an interferometer-type charge qubit in conjunction with a readout procedure managed by measuring the qubit’s reverse Josephson inductance.\(^{8-11}\)

\[ L_j^{-1} = (2e/h)^2 \cdot \partial^2 H / \partial \delta^2. \tag{3} \]

We put it into practice along the lines of the conventional impedance measuring technique by means of a high-quality tank inductively coupled to the interferometer loop. Based on this method, we present results of the spectroscopic investigation of the quantum device that is manipulated by a microwave field injected via a coaxial cable (UHF line, Fig. D). These results demonstrate the utility of the used design relating to development and characterization (weak continuous quantum measuring) of future building blocks (coupled qubits) for quantum information circuits as well.

The principle of measurement is as follows: The expectation value \( \langle m | L_j^{-1}(n_g, \delta) | m \rangle \) is determined not only by the quasicharge \( n_g \) (controlled by the gate voltage) and the phase \( \delta \) (controlled by an external magnetic flux \( \Phi_z \) threading the interferometer loop) but also by the band index \( m \).\(^{12}\) Within the two-level approximation, the mean values in the upper state \( (m=1) \) and in the ground state \( (m=0) \) have the opposite sign,\(^{13}\) \( \langle 1 | L_j^{-1}(n_g, \delta) | 1 \rangle \approx -\langle 0 | L_j^{-1}(n_g, \delta) | 0 \rangle \). Consequently, a change of the band index \( m \) results in a relevant impedance change that causes a shift of the tank resonance frequency.

Quantitatively, we consider only the case when the geometrical inductance \( L_q \) of the loop is sufficiently small, ensuring in this way a unique relationship between the flux \( \Phi_z \) and the phase difference \( \delta \). This requirement can
be expressed as follows within the two-band model, \(^{11}\)

\[
L_q < 2 \left( \frac{\Phi_0}{2\pi} \right)^2 \frac{\Delta E(n_g=1, \delta = \pi)}{E_{J1}E_{J2}},
\]

(4)

where \(\Phi_0\) is the flux quantum and

\[
\Delta E = \left[ D^2(n_g) + \varepsilon_j^2(\delta) \right]^{1/2}
\]

(5)
is the local band spacing (with \(D \equiv E_{CP}(1-n_g)\)). We emphasize the fact that the coupling strength \(\varepsilon_j\) and, therefore, the gap \(\Delta E\) can be significantly reduced in the vicinity of the operating point \(n_g=1, \delta=\pi\) even for relatively large ratios \(E_{J1,2}/E_C\).

As discussed above, the observable \(L_j^{-1}\) takes different expectation values for the energy bands \(m=0\) and \(m=1\). Therefore, a redistribution of the level populations caused by microwave excitations (in competition with relaxation processes) results in a change of the quantum-statistical mean value \(\langle L_j^{-1} \rangle\). In order to prove this effect, the loop was coupled through a mutual inductance \(M = k\sqrt{L_0L_T}\) (where \(k \ll 1\) is the coupling coefficient) to the tank circuit (with known inductance \(L_T\) and quality factor \(Q\)). The tank is driven by an rf current \(I_{rf}\) at a frequency \(\omega\) close to its resonance frequency \(\omega_T\). This current generates an rf flux \(\Phi_{rf}\) threading the interferometer ring. Provided that, first, this rf flux is small, \(\Phi_{rf} \ll \Phi_0\), and, second, the condition \(L_q\langle L_j^{-1} \rangle \ll 1\) is fulfilled, variations of the interferometer inductance can be described by making use of the formula\(^{12}\)

\[
\tan \alpha \approx k^2 Q L_q\langle L_j^{-1} \rangle,
\]

(6)

where \(\alpha\) is the phase shift between drive current \(I_{rf}\) and tank voltage \(U_T\) to be measured.

The used measurement setup is shown in Fig. 1. The Nb square-shaped pancake tank coil \((L_T \approx 170 \text{ nH})\) was fabricated lithographically on an oxidized Si substrate. An external capacitance \(C_T\) is used to complete the resonance circuit. This high-quality tank \((Q \approx 700)\) is biased by the rf current \(I_{rf}\) at the resonance frequency of \(\omega_T/2\pi \approx 28 \text{ MHz}\). The rf voltage \(U_T\) across the tank circuit was measured by means of a sequence of cold and room temperature amplifiers. The angular phase shift \(\alpha\) was determined using an rf lock-in voltmeter.

The qubit under investigation is placed in the middle of the coil in flip-chip configuration. The transistor was fabricated out of Al by the conventional shadow evaporation technique. The transistor’s gate line is filtered by means of three copper powder filters (with a total length of about 35 cm) at the temperatures 2 K, 50 mK and 10 mK. The attenuation of this line is about 120 dB at 5 GHz and 300 dB at 20 GHz. Microwave irradiation for photon-assisted excitations of the quantum device is fed into the sample via an UHF line consisting of a commercial coaxial cable from room temperature to \(\sim 2 \text{ K}\) as well as via a resistive coaxial cable (known as Thermo-Coax) from \(\sim 2 \text{ K}\) to 10 mK. In order to reduce external interferences, two 20 dB commercial attenuators were installed at the temperature levels 2 K and 10 mK. The external magnetic flux \(\Phi_e\) applied to the interferometer loop is produced by a dc current \(I_{dc}\) through the tank coil. In order to feed both currents \(I_{dc}\) and \(I_{rf}\) into the tank, a simple bias tee is used (see Fig. 1).

The angular phase shift \(\alpha\) \(^{14}\) was measured as a function of the gate voltage as well as of the dc current in the tank coil that generates the external flux \(\Phi_e\). The coupling coefficient between tank and superconducting loop was obtained experimentally\(^{12}\) to be \(k = 0.026\). The mutual inductance is \(M = 0.42 \text{ nH}\), and the geometri-
In conclusion, we have successfully implemented an alternative quantum two-level system in a superconducting environment. The qubit is illuminated by a microwave with the frequency of 8.0 GHz. The periodic circular structure demonstrates the variation of the total interferometer-tank impedance due to transitions from the lower to the upper energy band. Especially, the “crater ridges” correspond to all combinations of the parameters $n_g$ and $\Phi_e$ that give the same energy gap between the respective states equal to 8.0 GHz.
FIG. 5: Top view of the \( \alpha(n_g, \Phi) \) mountain range of Fig. 4. Here the white ring approximates the ridge of the central crater leading to the fit \( E_C = 2.2 \) GHz, \( E_{J1} = 70.0 \) GHz, and \( E_{J2} = 74.4 \) GHz within the two-level model.

We thank A. Smirnov, A. Zagoskin, and A. Zaikin for fruitful discussions. We also would like to thank A.N. Omelyanchouk for comments and H. Mühlig for technical assistance. This work was partially supported by the Deutsche Forschungsgemeinschaft under contract No. KR 1172/9-2 and D-wave Sys. Inc.

* On leave from B. Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of the Ukraine, 310164 Kharkov, Ukraine.
† Electronic address: owk@rz.uni-jena.de
‡ On leave from Department of Solid State Physics, Comenius University, SK-84248 Bratislava, Slovakia.

1 A.J. Leggett, Science 296, 861, (2002).
2 D.V. Averin and K.K. Likharev, in Mesoscopic Phenomena in Solids, edited by B.L. Altshuler, P.A. Lee, and R.A. Webb (Elsevier, Amsterdam, 1991), p. 173, and M. Tinkham, in Introduction to superconductivity (McGraw-Hill, New York, 1996), 2nd ed., Chap. 7.
3 P. Joyez, P. Lafarge, A. Filipe, D. Esteve, and M. Devoret, Phys. Rev. Lett. 72, 2458 (1994).
4 T.M. Eiles and J.M. Martinis, Phys. Rev. B 50, 627 (1994).
5 D.J. Flees, S. Han, and J.E. Lukens, Phys. Rev. Lett. 78, 4817 (1997).
6 Y. Nakamura, C.D. Chen, and J.S. Tsai, Phys. Rev. Lett. 79, 2328 (1997).
7 Y. Nakamura, Yu.A. Pushkin, and J.S. Tsai, Phys. Rev. Lett. 87, 246601 (2001).
8 D. Vion, A. Aumeis, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M.H. Devoret, Science 296, 886 (2002). Especially, the authors report on band gap spectroscopy of a similar superconducting circuit (“quantrum”) that was performed by measuring the switching probabilities of interband transitions.
9 J.R. Friedman and D.V. Averin, Phys. Rev. Lett. 88, 050403 (2002).
10 A.B. Zorin, Physica C 368, 284 (2002).
11 W. Krech, M. Grajcar, D. Born, I. Zhilyaev, Th. Wagner, E. Il'ichev, and Ya. Greenberg, Phys. Lett. A 303, 352 (2002).
12 R. Rifkin and B.S. Deaver, Phys. Rev. B 13, 3894 (1976).
13 The band index can be related simply to the number \( m \) of excess pairs on the island in the pure charge states (\( \varepsilon_J \rightarrow 0; 0 < n_g < 1 \)). Within the two-band model, this procedure leads to \( |m = 0 \rangle \) (lower state) and \( |m = 1 \rangle \) (upper state), respectively (cf. Ref. 7).
14 E. Il’ichev, V. Zakosarenko, L. Fritsch, R. Stolz, H.E. Hoe- nig, H.-G. Meyer, M. Götz, A.B. Zorin, V.V. Khain, A.B. Pavolotsky, and J. Niemeyer, Rev. Sci. Instr. 72, 1882 (2001).
15 Measurements of ground band properties of a similar system were reported by M. Götz, S.A. Bogolovsky, and A.B. Zorin, in Tagungsband 8th Statusseminar BMBF Feb. 13-14, 2003, Garmisch-Partenkirchen (VDI Technologiezentrum, Düsseldorf, 2003), p. 50.