Transient Model of Brushless Doubly Fed Reluctance Machine using 
ABC Reference Frame

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Abstract—This paper presents a complete derivation of brushless doubly fed reluctance machine (BDFRM) model in ABC reference frame using winding function theory. Extensive numerical simulations are conducted, using MATLAB/SIMULINK software, to investigate the BDFRM model capabilities under different operating modes.

Keywords—Brushless Doubly Fed Reluctance Machine (BDFRM), Modeling, Winding Function Theory, ABC reference frame, fail-safe mode of operation.

I. INTRODUCTION

BDFRM is one of the new trends in partial variable speed wind turbine generators offering more advantages due to absences of brushes which improves it reliability making it more attractive specially for offshore wind applications. In addition to its “fail-safe” mode of operation which enables its robust operation in spite of the failure on its inverter or secondary stator. BDFRM has also reluctance rotor design which makes its rotor easier in manufacturing with less mechanical stresses in addition to absence of copper losses on rotor. On the other hands; the main drawbacks of BDFRM are the larger machine size due to its low torque to volume ratio, complex rotor design and complex controllability [1]

BDFRM has two standard 3-phase windings on its stator. One connected directly to the grid and called the primary or power winding and the other is indirectly connected to the grid through back to back converter and is called the secondary or control winding. The rotor is salient pole reluctance rotor with no windings and its shaft is coupled to the wind turbine through a step-up gearbox. Fig.1 show the detailed construction of BDFRM.

Number of poles of primary winding, secondary winding, and salient rotor must not be the same in order to produce useful torque. Also, primary winding is supplied by grid frequency while secondary winding is supplied by slip frequency. Construction and operation details will be handled in details through this paper.

Due to existence of two frequencies, the most commonly modeling technique used for this machine is park’s d-q model using two arbitrary rotating reference frames \( \omega \) and \( \omega_r - \omega \). The first one is for primary winding quantities and the other one is for secondary winding quantities. Where, \( \omega \) is any arbitrary rotating speed, while \( \omega_r \) is rotor speed [2-9]. This modeling technique is suitable for
machine control purpose as it eliminates the dependence of inductances on rotor position and simplifies modeling calculations. However, in some cases the need for more detailed modeling techniques is required to model some special studies (e.g. study of winding inter-turn short circuit effects on machine performance). This paper will present a guide on how to build a detailed model for BDFRM in ABC reference frame utilizing winding function theory for inductance calculations.

II. BDFRM TRANSIENT MODEL USING ABC REFERENCE FRAME

The model is built considering that both primary and secondary windings are consisting of multiple inductive circuits coupled together, and current in each circuit is considered as an independent variable. The analysis is based on the following assumptions [10]:

- The iron of the machine will be assumed to have infinite permeability ($\mu_{\text{iron}} = \infty$).
- The stator windings of the machine will be assumed to be adequately modeled as spatially sinusoidal distributed windings.
- $p_r = p + q$, where $p_r$ is number of rotor poles, $p, q$ are number of pole pairs of primary and secondary windings respectively.
- The air gap of the machine will be modelled using a sinusoidal air gap function as below [10]:
  \[ g^{-1}(\theta, \theta_{\text{rm}}) = m + n \cos(p_r(\theta - \theta_{\text{rm}})) \]  
  Where, $\theta$ is the mechanical angle around the machine with respect to phase A, $\theta_{\text{rm}}$ is the rotor mechanical angle with respect to phase A, $m, n$ are constants so, $m > n > 0$ and:
  \[ g_{\min} = \frac{1}{m-n}, \quad g_{\max} = \frac{1}{m+n} \]  
  or \[ m = \frac{g_{\max} - g_{\min}}{2g_{\max}g_{\min}}, \quad n = \frac{g_{\max} - g_{\min}}{2g_{\max}g_{\min}} \]  
  Where, $g_{\min}$ and $g_{\max}$ are minimum and max air gap length between rotor and stator respectively.

2.1. BDFRM voltage equations

The stator comprises two 3 phase windings as shown in Fig.1 which can be represented by:

\[ [v] = [R][i] + \frac{d}{dt} [\lambda] \]  
And,

\[ [v] = [v_A \quad v_B \quad v_C \quad v_a \quad v_b \quad v_c]^T \]  
\[ [i] = [i_A \quad i_B \quad i_C \quad i_a \quad i_b \quad i_c]^T \]  
\[ [\lambda] = [\lambda_A \quad \lambda_B \quad \lambda_C \quad \lambda_a \quad \lambda_b \quad \lambda_c]^T \]  
\[ [R] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_B & 0 & 0 & 0 & 0 \\ 0 & 0 & r_C & 0 & 0 & 0 \\ 0 & 0 & 0 & r_a & 0 & 0 \\ 0 & 0 & 0 & 0 & r_B & 0 \\ 0 & 0 & 0 & 0 & 0 & r_c \end{bmatrix} \]  

Where, $v_A, v_B, v_C, v_a, v_b, v_c$ are primary and secondary winding voltages respectively, $i_A, i_B, i_C, i_a, i_b, i_c$ are primary and secondary winding currents respectively, $\lambda_A, \lambda_B, \lambda_C, \lambda_a, \lambda_b, \lambda_c$ are primary and secondary winding flux linkages respectively and can be calculated as follow:

\[ [\lambda] = \begin{bmatrix} [L_p] \\ [L_{ps}] \\ [L_s] \end{bmatrix} [i] \]  
And,

\[ [L_p] = \begin{bmatrix} L_{AA} + L_{IA} & L_{AB} & L_{AC} \\ L_{BA} & L_{BB} + L_{IB} & L_{BC} \\ L_{CA} & L_{CB} & L_{CC} + L_{Ic} \end{bmatrix} \]  
\[ [L_s] = \begin{bmatrix} L_{aa} + L_{ia} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} + L_{ib} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} + L_{ic} \end{bmatrix} \]
\[ L_{PS} = \begin{bmatrix} L_{AA} & L_{Ab} & L_{Ac} \\ L_{Ba} & L_{BB} & L_{BC} \\ L_{Ca} & L_{Cb} & L_{CC} \end{bmatrix} \]  
(13)

\[ L_{SP} = \begin{bmatrix} L_{aA} & L_{bA} & L_{cA} \\ L_{aB} & L_{bB} & L_{cB} \\ L_{aC} & L_{bC} & L_{cC} \end{bmatrix} \]  
(14)

Where \([L_P]\) is primary winding inductance matrix, \([L_S]\) is secondary winding inductance matrix, and \([L_{PS}]\) is mutual inductance matrix between primary and secondary winding. \(L_{ii}\) are winding self-inductances, \(L_{ij}\) are winding leakage inductances, \(L_{ij}\) are mutual inductances between phases, \(i, j = [A, B, C, a, b, c]\), and \(i \neq j\).

### 2.2. Inductance Calculations using Winding Function Theory

According to winding function theory; inductance of winding “i” and winding “j” can be calculated as [10]:

\[ L_{ij} = \mu_r r l \int_0^{2\pi} g^{-1}(\theta, \theta_{rm})N_i(\theta, \theta_{rm})N_j(\theta, \theta_{rm})\,d\theta \]  
(15)

Where \(\mu_r\) is the permeability of free space, \(r\) is the effective radius of the machine, \(l\) is the length of the machine, \(N_i(\theta, \theta_{rm})\) is the winding function of winding \(i\), \(N_j(\theta, \theta_{rm})\) is the winding function of winding \(j\).

For sinusoidal distributed stator windings; both winding function of primary and secondary windings can be written as:

\[ N_{A}(\theta, \theta_{rm}) = n_p \cos(p\theta) \]  
(16)

\[ N_{B}(\theta, \theta_{rm}) = n_p \cos\left(p\theta - \frac{2\pi}{3}\right) \]  
(17)

\[ N_{C}(\theta, \theta_{rm}) = n_p \cos(p\theta + \frac{2\pi}{3}) \]  
(18)

\[ N_{a}(\theta, \theta_{rm}) = n_s \cos(q\theta) \]  
(19)

\[ N_{b}(\theta, \theta_{rm}) = n_s \cos\left(q\theta - \frac{2\pi}{3}\right) \]  
(20)

\[ N_{c}(\theta, \theta_{rm}) = n_s \cos\left(q\theta + \frac{2\pi}{3}\right) \]  
(21)

Where, \(n_p, n_s\) are half of effective number of turns of primary and secondary windings respectively and equal to \(k_w \times n, k_w\) is winding factor and \(n\) is number of actual number of turns.

Using (1), (15), and (16 to 21); Inductances formulas for BDFRM can be obtained as below:

\[ L_{AA} = L_{BB} = L_{CC} = L_{mp} \]  
(22)

\[ L_{aa} = L_{bb} = L_{cc} = L_{ms} \]  
(23)

\[ L_{AB} = L_{AC} = L_{BC} = L_{BA} = L_{CA} = L_{CB} = -\frac{1}{2}L_{mp} \]  
(24)

\[ L_{ab} = L_{ac} = L_{bc} = L_{ba} = L_{ca} = L_{cb} = -\frac{1}{2}L_{ms} \]  
(25)

\[ L_{Aa} = L_{aA} = L_{psmax} \cos(p_r \theta_{rm}) \]  
(26)

\[ L_{Bc} = L_{CB} = L_{psmax} \cos(p_r \theta_{rm}) \]  
(27)

\[ L_{Ca} = L_{aC} = L_{psmax} \cos(p_r \theta_{rm} + \frac{2\pi}{3}) \]  
(30)

\[ L_{Cc} = L_{cC} = L_{psmax} \cos(p_r \theta_{rm} + \frac{2\pi}{3}) \]  
(31)

\[ L_{Ab} = L_{bA} = L_{psmax} \cos(p_r \theta_{rm} - \frac{2\pi}{3}) \]  
(33)

\[ L_{Ba} = L_{aB} = L_{psmax} \cos(p_r \theta_{rm} - \frac{2\pi}{3}) \]  
(34)
Where \( L_{mp} = \mu_0 r l_2^2 m \pi \), \( L_{ms} = \mu_0 r l_2^2 m \pi \), and \( L_{psmax} = 0.5 \pi r l_p n_2 n_1 \). From (13), (14), (26)-34 it found \( [L_{PS}] \) and \( [L_{SP}] \) are identical. From (22) to (25); both \([L_p]\) and \([L_s]\) contains constants. From (4) and (11):

\[
[v] = [R][i] + \begin{bmatrix} [L_p] & [L_{PS}] \\ [L_{SP}] & [L_s]\end{bmatrix} \begin{bmatrix} \frac{d[i]}{dt} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ [L_{SP}] \end{bmatrix} \begin{bmatrix} [L_{PS}] \end{bmatrix} i
\]

(35)

### 2.3. Electromagnetic Torque Equation

From energy conversion principles; the electromechanical torque can be calculated from [12]:

\[
T_{em} = p_r [i_p]^T \frac{d[L_{PS}]}{d\theta_r} [i_s] \tag{36}
\]

Assuming idle sinusoidal primary and secondary currents, the general form for primary and secondary currents are:

\[
i_p = I_p \sin(\omega_p t - \alpha)
\]

(37)

\[
i_s = I_s \sin(\omega_s t - \beta)
\]

(38)

Where \( I_p \) and \( I_s \) are primary and secondary current magnitudes, \( \alpha \) and \( \beta \) are primary and secondary currents phase angles. From (26 to 34); the general form of primary to secondary mutual inductance is

\[
L_{ps} = L_{psmax} \cos(p_r \theta_{rm} - \delta)
\]

(39)

Where \( \delta \) is constant and equals to 0 or \( \pm \frac{2\pi}{3} \).

Substitute from (37), (38), and (39) in to (36); it can be proved that useful torque will be produced from BDFRM only if:

\[
\omega_r = \omega_p + \omega_s
\]

(40)

Where \( \omega_r \) is rotor electrical angular speed, \( \omega_p \) and \( \omega_s \) are primary and secondary angular frequencies respectively.

As primary winding is directly connected to the grid, \( \omega_p \) is constants and depends only grid frequency. Thus \( \omega_s \) must be controlled with the help of rotor speed feedback according to equation (40).

### 2.4. Electromechanical Equation

The rotor mechanical equation can be written as:

\[
T_{em} = J \frac{d\omega_{rm}}{dt} + F \omega_{rm} + T_L
\]

(41)

Where \( J \) is rotor moment of inertia, \( \omega_{rm} \) is rotor mechanical angular speed, \( F \) is friction coefficient, and \( T_L \) is the load torque.

### III. SIMULATION RESULTS AND DISCUSSIONS

Simulation study using MATLAB/SIMULINK is performed for 1.5KW BDFRM with parameter shown in table 1 as follow:

| Table 1. BDFRG Prototype Parameters and Ratings |
|-----------------------------------------------|
| **Parameter**                  | **Value**            | **Unit**     |
| Rotor inertia \((J)\)           | 0.1 kg \cdot m^2     |             |
| Friction coefficient \((F)\)    | Zero (ignored)       |             |
| Primary resistance \((R_p)\)    | 11.1 \(\Omega\)      | Primary power \((P_p)\) 1.5 kW |
| Secondary resistance \((R_s)\)  | 13.5 \(\Omega\)      | Stator currents \((I_{sp})\) 2.5 A rms |
| Primary winding self-inductance \((L_{mp})\) | 0.2706 H | Stator voltage \((v_{ps})\) 400 V rms |
| Primary winding leakage inductance \((L_{ip})\) | 0.0041 H | Stator frequency \((f_p)\) 50 Hz |
| Secondary self-inductance \((L_{ms})\) | 0.3762 H | Winding connections \(Y/Y\) |
| Secondary winding leakage inductance \((L_{is})\) | 0.0057 H | Stator poles \((p/q)\) 6/2 |

The simulation is done for no-load and full load of 19Nm under different mode of operations to check model capabilities.
3.1. Starting of BDFRM
To avoid inverter overloading during starting of the machine [2,3,8,9]; BDFRM is started as an induction machine by shorting the secondary windings. Fig.2 shows starting period of the BDFRM, the machine started from standstill and reached to the steady state speed of 750 RPM on 0.75sec. BDFRM behalf as a four poles induction machine with average starting torque of 8 Nm and max torque of 18.5 Nm.

![Fig.2. BDFRM rotor speed and torque during starting at no-load](image)

Fig.2. BDFRM rotor speed and torque during starting at no-load

Fig.3 and 4 shows primary and secondary currents during starting of the machine. The primary current magnitude was 8.5A at starting and dropped to 4.5A at steady state while secondary current starts at 6A and dropped to zero at steady state.

![Fig.3. BDFRM Primary winding starting current](image)

Fig.3. BDFRM Primary winding starting current

![Fig.4. BDFRM Secondary winding starting current](image)

Fig.4. BDFRM Secondary winding starting current

3.2. BDFRM “fail-safe” mode of operation (BDFRM as an Induction Machine)
As mentioned before; one of the BDFRM advantages is its “fail-safe” mode of operation. Where BDFRM act as an induction machine when its secondary windings are shorted, thus it can work during inverter failures.

Fig.5 to 7 shows BDFRM performance while shorting its secondary winding. Both generating and motoring modes of operation are simulated. It noticed that rotor speed depends on mechanical load applied.

![Fig.5. BDFRM rotor speed and torque during “fail-safe” mode of operation (Induction machine mode)](image)
3.3. BDFRM Synchronous, Super-Synchronous, and Sub-Synchronous mode of operation

In [2-10], different types of control are presented for BDFRM including scalar or V/F, Vector control (VC), Field oriented control (FOC), and Direct torque control (DTC). To illustrate capabilities of ABC model presented to simulate different modes of operation; DTC control is simulated for synchronous, super-synchronous, and sub-synchronous modes of operations under constant load of 9 Nm. Maximum torque per inverter ampere (MTPIA) strategy is utilized. Fig. 8 to 10 show speed, torque, primary and secondary currents for the machine. The machine is operated as a generator (Load torque = -9 Nm) in the three modes of operation; Synchronous mode at synchronous speed of 750 RPM during the period from 2 sec to 3.5 sec, Super-Synchronous mode at speed of 850 RPM during the period from 4 sec. to 6 sec, then Sub-Synchronous mode at speed of 650 RPM during the period of 7 sec to 8.5 Sec.

IV. CONCLUSION

This paper explains detailed derivation of BDFRM model in ABC frame based on winding function theory. Simulation study using MATLAB/SIMULINK program is conducted using DTC control to proof capability of the model to present different modes of operation under different loading levels. The model requires higher processing time than traditional d-q model but it has the capability to simulate more details than d-q model as each winding has its own differential equation. It is also worth to mention that effect of stator slots, rotor saliency, and winding distribution can also be simulated if inductance equations are modified to consider these effects instead of assuming sinusoidal distribution of both primary and secondary windings.
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