Characteristic analysis of a new high-static-low-dynamic stiffness vibration isolator based on the buckling circular plate

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Abstract
The high-static-low-dynamic stiffness vibration isolator has great advantages in vibration isolation because it can decrease the natural frequency of the system while keeping the load capability, but it is usually difficult to implement because of its complex structures and installation space constraints. A high-static-low-dynamic stiffness vibration isolator composed of a buckling circular plate and a traditional linear spring is proposed in this paper. The buckling circular plate works as the negative stiffness corrector paralleled with the linear spring, which can be integrated into the sleeve. If the load is chosen properly, the static equilibrium point will be at the initial quasi-zero stiffness point. However, any changes of the load will lead the equilibrium point deviating from the initial equilibrium point. The nonlinear mathematical model of high-static-low-dynamic stiffness vibration isolator considering load imperfection is developed and its force transmissibility is analyzed with the harmonic balance method and homotopy perturbation method. The influence rule of the system parameters on it is analyzed and the corresponding results show that the force transmissibility will exhibit complicated characteristics, depending on the load imperfection, damper, and excitation force.

Keywords
High-static-low-dynamic, vibration isolator, force transmissibility, harmonic balance method, homotopy perturbation method

Introduction
The passive isolation technology is the most common approach to attenuate the harmful vibration because it is much easier to realize than active isolation and does not require the external power source. But it is also known that the traditional linear passive isolator only works when the excitation frequency is greater than \( \sqrt{2} \) times the resonance frequency of the system.¹ The frequency range of isolation would be expanded if the resonance frequency of the isolator is reduced, which can be achieved by decreasing the stiffness or increasing the mass of the system. It is usually hard to adjust the mass of the system, so the declining stiffness of the isolator and adjusting the corresponding damping ratio are general methods to reduce the vibration. However, the reduction of stiffness can usually result in the large static deformation. If the stiffness of the isolator is quite low, the static deformation would exceed the space limitation. In the past decades, high-static-low-dynamic stiffness (HSLDS) vibration isolation systems built by combining the negative stiffness structure and a linear isolator have been developed to overcome this contradiction. Ibrahim² presented a comprehensive assessment of recent developments in

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nonlinear isolators with good ultralow frequency isolation performance. Alabuzhev et al. 3 covered the fundamental theory and many prototypes of vibration isolation systems characterized with quasi-zero stiffness. Carrella et al. 4–6 used two inclined springs to produce negative stiffness which counteracts the positive stiffness provided by the vertical spring and studied both the force transmissibility and displacement transmissibility of the isolator. Besides, there are many other means of negative stiffness mechanism including cam-roller-spring mechanism, 7 bi-stable structures, 8 magnetic springs, 9 and scissor-like structures. 10,11 The fact about the benefits of non-linearity has given rise to a growing interest in the study of nonlinear isolators. 12,13 Moreover, some mathematic method has been used to solve the nonlinear dynamic equations such as the homotopy perturbation method (HPM) proposed by He and co-workers 14–19 which can be a useful and effective way. The buckling circular cylinder adopting a variational formulation is also discussed by He. 20

From the reviews above, if the negative stiffness mechanism is chosen carefully, the combined nonlinear isolator can exhibit quasi-zero stiffness at the static equilibrium point with the rated load. 21,22 However, it is very difficult to keep working position at the desired static equilibrium point in the reality due to the underloaded or overloaded condition. When the load changes, the static equilibrium point will deviate from the setpoint. Huang and Liu et al. 23–25 studied on the effect of the system imperfections on the dynamic response of a HSLDS vibration isolator using the compressed Euler beams as the negative stiffness mechanism.

The objective of this paper is to design a new HSLDS vibration isolator consisting of the buckling circular plate and the linear spring and analyze its force transmissibility under the overloaded and underloaded condition. The outline of the rest of this paper is organized as follows. In the next section, a simple structure of HSLDS vibration isolator using buckling mode of circular plate as the negative stiffness mechanism is presented. The HSLDS vibration isolator with the overloaded or underloaded condition is described in the “The HSLDS vibration isolator working under overloaded or underloaded condition” section. In the “Force transmissibility characteristic of the HSLDS isolator” section, the nonlinear mathematical model of HSLDS vibration isolator with the overloaded imperfection is developed and the effect of system parameters on its frequency response characteristic is analyzed with the harmonic balance method (HBM) and the HPM. Finally, some conclusions are drawn in the final section.

Structure of vibration isolator with HSLDS

It is required to develop simple HSLDS vibration isolators with minimum number of elements, preferably with only one that provides the best simplicity, and thus high reliability. The structure of HSLDS vibration isolator is shown in Figure 1, which consists of a buckling circular plate and a linear spring. The buckling circular plate acts as the negative stiffness corrector while the linear spring works as the positive stiffness element. The HSLD stiffness is obtained by connecting the circular plate and the linear spring in parallel while the weight of the isolated object is sustained by the linear spring. The structure of designed vibration isolator is simple and reliable. It has been verified that the restoring force is the cubic of the displacement and the stiffness is the quadratic of the displacement.

Figure 1. The structure of HSLDS vibration isolator.
The HSLDS vibration isolator working under overloaded or underloaded condition

If HSLDS isolator works under the rated load, the negative stiffness of buckling circular plate cancels out positive stiffness spring. It exhibits the quasi-zero stiffness characteristic as shown in Figure 2. As mentioned above, it is difficult to keep the load balanced at the initial static equilibrium point. The vibration isolator with overloaded or underloaded condition is discussed in this paper, only in terms of with load imperfection. It means that negative stiffness still counteracts positive stiffness, but the true static equilibrium position deviates from the ideal static equilibrium position due to overload or underload, as shown in Figures 3 and 4.

Force transmissibility characteristic of the HSLDS isolator

Nonlinear mathematical model of HSLDS vibration isolator

The HSLDS vibration isolator can be simplified as one-degree-freedom vibration isolation system with the nonlinear elastic restoring force as shown in Figure 5, which is different from the linear isolator system. From the
“Structure of vibration isolator with HSLDS” section, the elastic force of the HSLDS vibration isolator can be written as follows

\[
f_k = k(k_1' y + k_3'y^3) \quad (1)
\]

where \( y \) is the displacement of the isolated object, \( k \) is the stiffness of the linear spring; \( k_1' \) is the dimensionless coefficient indicating the extent of negative stiffness corrector which cancels out part of the positive stiffness, and \( k_3' \) is the dimensionless coefficient which is related to the deformation structure of buckling circulate plate. When \( k_1' = 0 \), the vibration isolator is a quasi-zero stiffness vibration isolator. When \( k_1' = 1 \), the isolator turns out to be a linear isolator. The non-dimensional stiffness of the HSLDS vibration isolator is presented in Figure 6.

When the system suffers from a harmonic excitation force \( F = F_0 \cos \omega t \), the dynamic equation of the system can be written as the Duffing equation

\[
m\ddot{y} + c\dot{y} + k(k_1' y + k_3'y^3) = F_0 \cos \omega t \quad (2)
\]

where \( m \) and \( c \) denote the load mass and damper, respectively. To simplify the mathematical formulation, the following dimensionless parameters are introduced

\[
\tau = \omega_n t, \quad \omega_n^2 = \frac{k}{m}, \quad \Omega = \frac{\omega}{\omega_n}, \quad \frac{dy}{dt} = \omega_n \frac{dy}{d\tau}, \quad \frac{d^2y}{dt^2} = \omega_n^2 \frac{d^2y}{d\tau^2} \quad (3)
\]
Substituting equation (3) into equation (2), the dimensionless equation can be obtained
\[ \ddot{u} + 2\dot{\zeta} \dot{u} + k_1' u + k_3' u^3 = f_0 \cos(\Omega \tau) \]  
(4)

where \( u = \frac{\xi}{r}, \zeta = \frac{c_{\infty}}{2k}, f_0 = \frac{F_0}{kr}, \) and \( r \) represents the radius of the circular plate. If the vibration isolator with rated load \( m = m_0 \), it will balance at the initial static equilibrium point \( u_0 = 0 \). Here we define \( \dot{u}_0 = u_0/r \) as the non-dimensional offset. According to Figures 3 and 4, if the isolator with overloaded mass \( m = m' \), it will balance at \( u = u_0 > 0 \) (it is assumed to be \( u = -u_0 < 0 \), when it is underloaded). At the static equilibrium point, the system satisfies
\[ \ddot{u} + 2\dot{\zeta} \dot{u} + k_1'(u \pm \dot{u}_0) + k_3'(u \pm \dot{u}_0)^3 = f_0 \cos(\Omega \tau) \]  
(5)

This equation can also be converted into another form
\[ \ddot{u} + 2\dot{\zeta} \dot{u} + a_1u \pm a_2u^2 + a_3u^3 = f_0 \cos(\Omega \tau) \]  
(6)

where \( a_1 = k_1' + 3k_3' \dot{u}_0^2, a_2 = 3k_3' \dot{u}_0, \) and \( a_3 = k_3' \). Here the notation “+” is for overload and the notation “−” is for underload. Setting \( \ddot{x} = u \pm a_2/3a_3, \) equation (6) can be rewritten as follows
\[ \frac{d^2\ddot{x}}{dt^2} + 2\dot{\zeta} \frac{d\ddot{x}}{dt} + b_1\ddot{x} + b_3\dddot{x} = f_0 \cos(\Omega \tau) \pm b_0 \]  
(7)

where \( b_0 = a_1a_2/3a_3^2 - 2a_3^2/27a_3^2, b_1 = a_1 - a_2^2/3a_3, \) and \( b_3 = a_3. \) It is supposed that the solution to equation (7) can be given in the form
\[ \ddot{x}(t) = A_0 + A_1 \cos(\Omega \tau + \theta) \]  
(8)

where \( A_0, A_1 \) is the response amplitude of constant item and harmonic item, respectively, and \( \theta \) is the phase angle. Substituting equation (8) into equation (7), equation (8) can be obtained with HBM as follows
\[
\begin{align*}
    b_1A_0 + b_3A_1^2 &+ \frac{3}{2}b_3A_0A_1^2 = \pm b_0 \\
    -\Omega^2A_1 + b_1A_1 + 3b_3A_0^2A_1 + \frac{3}{4}b_3A_1^2 &+ f_0 \cos \theta \\
    -2\dot{\zeta}\Omega A_1 &+ f_0 \sin \theta
\end{align*}
\]  
(9)

Figure 6. Non-dimensional stiffness of the HLSDS vibration isolator for different \( k_1'. \)
By eliminating θ and \( A_1 \) and considering the overload case \( u = u_0 > 0 \), equation (10) can be obtained as follows
\[
25b_3^4A_0^6 + (35b_1b_3^2 - 20\Omega^2b_3^2)A_0^5 + 15b_0b_3^2A_0^4 + (11b_1^2b_3 + 4\Omega^4b_3^2 + 4\Omega^4b_1 + 16\zeta^2\Omega^2b_1 - 16b_0b_3^2)A_0^3 \\
+ (2b_0b_1b_3 + 16\Omega^2b_0b_3)A_0^4 + (b_1^2 - 4\Omega^2\Omega^2 + 4\Omega^4b_1 + 16\zeta^2\Omega^2b_1 - 9b_0^2b_3^2 + 6b_3b_0A_0^2)A_0^3 \\
+ (b_1^2 - 16\zeta^2\Omega^2b_0A_0^2 + (4\Omega^2 - b_0b_3)A_0 - b_0^2) = 0
\] (10)

Thus, we can get the amplitude–frequency curve between \( A_0 \) and \( \Omega \) with the known values of \( b_0 \), \( b_1 \), \( b_3 \), \( f_0 \), and \( \zeta \). By eliminating \( \theta \) and \( A_0 \), the implicit function of \( A_1(\Omega) \) can also be given in equation (11)
\[
\left( - \Omega^2A_1 + b_1A_1 + 3b_3A_1 \sqrt{b_0 \over 2b_3} + \sqrt{b_0^2 \over 4b_3^2} + (2b_1 + 3b_3A_1^2)^2 \over 216b_3^2 \right) + \left( b_0 \over 2b_3 - \sqrt{b_0^2 \over 4b_3^2} + (2b_1 + 3b_3A_1^2)^2 \over 216b_3^2 \right)^2 + (2\zeta\Omega A_1)^2 - f_0^2 = 0
\] (11)

The force transmissibility is defined as the ratio of the amplitude of the dynamic force transmitted to the rigid foundation, to the amplitude of the excitation force. It is aimed at evaluating the force isolation performance of a vibration isolator and its definition is given
\[
T = \left| {f_T \over f_0} \right|
\] (12)

where \( T \) denotes the force transmissibility, \( f_T \) is the dynamic force transmitted to the rigid foundation, and \( f_0 \) is the excitation force. The amplitude of \( f_T \) can be presented as follows
\[
|f_T| = \sqrt{f_{Te}^2 + f_{Td}^2}
\] (13)

where \( f_{Te} \) denotes the elastic restoring force and \( f_{Td} \) denotes the damping force.

**Effects of system parameters on the force transmissibility of isolator working under rated load condition**

If the vibration isolator works under rated load, it will balance at the initial static equilibrium point \((\ddot{u}_0 = 0)\) and exhibit the quasi-zero stiffness characteristic. The force transmitted to the rigid foundation is given as
\[
f_T = 2\zeta \dot{u} + k'_3u^3
\] (14)

Thus, the amplitude of the transmitted force can be obtained by substituting equation (8) into equation (14)
\[
|f_T| = \sqrt{(2\zeta A_1\Omega)^2 + \left( {3 \over 4}k'_3A_1^3 \right)^2}
\] (15)

The amplitude of the excitation force can be obtained from equation (11), so the force transmissibility of the HLSDS vibration isolator can be written in the logarithmic form
\[
T = 20\log \left( \sqrt{(2\zeta A_1\Omega)^2 + \left( {3 \over 4}k'_3A_1^3 \right)^2} / f_0 \right)
\] (16)

The effect of the stiffness ratio \( k'_3 \) on force transmissibility is shown in Figure 7. When the stiffness ratio \( k'_3 \) rises the resonant frequency increases. But the force transmissibility remains unchanged in the high frequencies regardless of different stiffness ratios and the isolation frequency band is reduced.
The effect of the damping ratio $\zeta$ on force transmissibility is illustrated in Figure 8. The resonant frequency and the peak value of the transmissibility decline as the damper ratio increases, but the transmissibility increases in the high frequency region. It indicates that the vibration isolation effect deteriorates when the damping ratio is raised. So, the damper degrades the efficiency of vibration isolation in the high frequency domain as in the linear system.

The effect of the excitation force ratio $f_0$ on force transmissibility is shown in Figure 9. It can be clearly observed that the resonant frequency goes up and the isolation frequency band is reduced as the excitation force ascends. Moreover, increasing the excitation force ratio has little effect on the isolation performance in the high frequency region.
Effects of system parameters on the force transmissibility of isolator working under overloaded condition

If the vibration isolator works under overload or underload, it will balance at the new static equilibrium point \((\tilde{u}_0 \neq 0)\), the solution to equation (7) is supposed to be given as follows

\[
u(s) = A_0 + A_1 \cos(\Omega s + \phi)
\]  

(17)

where \(A_0 = A_0 - \frac{x_2}{3}x_3\). Thus, the elastic restoring force of isolator can be given as follows

\[
f_{Te} = a_1 u + a_2 u^2 + a_3 u^3
\]  

(18)

Substituting equation (17) into equation (18), one obtains

\[
f_{Te} = f_{T0} + f_{T1} \cos(\Omega s + \phi)
\]  

(19)

\[
f_{T0} = x_1 A_0' + 2x_2 A_0' A_1 + 3x_2 A_1^2 / 2 + 3x_3 A_0' A_1^3 / 2
\]  

(20)

\[
f_{T1} = x_1 A_1 + 2x_2 A_0' A_1 + 3x_3 A_1^3 / 4 + 3x_3 A_0' A_1^2 A_1
\]  

(21)

Considering \(A_0' = A_0 - \frac{x_2}{3}x_3\), \(x_1 = 3k_3' \tilde{u}_0^2\), \(x_2 = 3k_3' \tilde{u}_0\), and \(x_3 = k_3'\), equation (21) can be simplified as follows

\[
f_{T1} = 3k_3' A_1^3 / 4 + 3x A_0^2 A_1
\]  

(22)

Then, the force transmissibility of isolator can be given as follows

\[
T = \sqrt{\left(f_{T1}^2 + (2\zeta \Omega A_1)^2\right)} / f_0
\]  

(23)

Substituting equation (22) into equation (23), the force transmissibility of isolator can be rewritten in the logarithmic form

\[
T = 20 \log\left(\sqrt{(3k_3' A_1^3 / 4 + 3k_3' A_0^2 A_1)^2 + (2\zeta \Omega A_1)^2} / f_0\right)
\]  

(24)
Based on above equations, it can be inferred that the force transmissibility of the isolator working under overloaded condition is related to stiffness ratio $k_3^3$, damping ratio $\zeta$, excitation force $f_0$, and equilibrium position $\bar{u}_0$.

The effect of the excitation force $f_0$ and static equilibrium position $\bar{u}_0$ on force transmissibility is shown in Figure 10. It can be seen that the force transmissibility goes up as the excitation force $f_0$ increases, when the isolator works at the quasi-zero stiffness point ($\bar{u}_0 = 0$). However, if the static equilibrium position deviates from quasi-zero stiffness point ($\bar{u}_0 \neq 0$), the isolation effect could become better as the excitation force $f_0$ is increasing. It can also be found that the resonance frequency increases as the offset $\bar{u}_0$ increases and the frequency band of isolation gets narrow. When the isolator balances at the new static equilibrium point $\bar{u}_0 = 0.05$ or 0.1, it exhibits softening-to-hardening stiffness characteristics as the excitation force $f_0$ increases. So the excitation force $f_0$ and static equilibrium position $\bar{u}_0$ has a lot to do with the vibration isolation performance of the HLDS vibration isolator. Besides, the effect of the static equilibrium position $\bar{u}_0$ on the isolation is greater than the excitation force $f_0$.

The effect of the excitation force $f_0$ and the damping ratio $\zeta$ on force transmissibility is depicted in Figure 11. It can be found that the peak value of transmissibility will reduce as the damping ratio increases. When the excitation force amplitude and the damping ratio are relatively small, the isolator exhibits the linear isolator characteristic. If the excitation force amplitude rises gradually, the force transmissibility curve will bend. At this time, the system shows the stiffness characteristic of softening-to-hardening. It can be inferred that damping coefficient $\zeta$ can make the resonance peak value smaller, while, the excitation force $f_0$ can make the isolator exhibit more complex.

The effect of the stiffness ratio $k_3^3$ and static equilibrium position $\bar{u}_0$ on force transmissibility is shown in Figure 12. It can be seen that if the system balances at the initial quasi-zero stiffness point ($\bar{u}_0 = 0$), the stiffness ratio $k_3^3$ only affects the resonance frequency of the isolation system and has little effect on the vibration isolation of the system in high frequency region. It is obvious that the peak value of force transmissibility and resonance frequency of the isolator system increase as stiffness ratio $k_3^3$ is increasing. Increasing equilibrium position $\bar{u}_0$ not only raises the resonance frequency of the system, but also increases the force transmissibility. Moreover, the transmissibility curve will bend, and the stiffness characteristic of the system will soften when the static equilibrium position $\bar{u}_0$ changes. This may lead to the instability of vibration isolation system.

The effect of the stiffness ratio $k_3^3$ and the damping ratio $\zeta$ on force transmissibility is presented in Figure 13. The whole curve of force transmissibility shifts slightly to right as the stiffness ratio $k_3^3$ increases. The peak value of force transmissibility reduces as damping ratio $\zeta$ is increasing. The vibration isolation system shows the characteristics
of softening with the tiny damping ratio. Increasing the stiffness ratio $k_3'$ and reducing damping ratio $\zeta$ result in the peak value of force transmissibility rising and affect the vibration isolation effect in low frequency region.

**Effects of system parameters on the force transmissibility of isolator working under overloaded condition using HPM**

The vibration isolator working under overload or underload condition has been discussed in the “Nonlinear mathematical model of HSLDS vibration isolator” section, and equation (6) can be rewritten as follows

$$\ddot{u} + 2\zeta \dot{u} + x_1 u \pm x_2 u^2 + x_3 u^3 - f_0 \cos(\Omega t) = 0$$

(25)
The dependent equation is solved with HPM in this section. For this purpose, the linear terms, nonlinear terms, and the analytic function are separated in the following form

\[ L(u) = \ddot{u} + 2\zeta \dot{u} + \omega^2 u, \quad N(u) = \omega_1 u^2 + \omega_2 u^3, \quad f(\tau) = f_0 \cos(\Omega \tau) \]  

The above formulas can be rewritten

\[ L(u) + N(u) - f(\tau) = 0 \]  

Then, the following homotopy equation is constructed

\[ H(x, q) = L(x) - L(u_0) + q[N(x) + L(u_0) - f(\tau)] = 0 \]  

whose solution can be expressed as \( x(\tau, q) \), and \( q \) is the embedded parameter \( q \in [0, 1] \). When \( q = 0 \), we can get

\[ H(x, 0) = L(x) - L(u_0) = 0 \]  

When \( q = 1 \), we can also get

\[ H(x, 1) = L(x) + N(x) - f(\tau) = 0 \]  

So when \( q \) varies from 0 to 1, the solution \( x(\tau, q) \) changes from \( u_0(\tau) \) to \( u(\tau) \). Considering \( q \) as a small parameter, the solution can be expressed in power series as follows

\[ x = x_0 + qx_1 + q^2 x_2 + \cdots \]  

When \( q = 1 \), the solution is

\[ u = \lim_{q \to 1} x = x_0 + x_1 + x_2 + \cdots \]
Substituting equation (32) into equation (28), we obtain
\[ L(x_0 + qx_1 + q^2 x_2 + \cdots) - L(u_0) + q[N(x_0 + qx_1 + q^2 x_2 + \cdots) + L(u_0) - f(\tau)] = 0 \] (33)

In order to make the coefficient of the \( q^0 \) and \( q^1 \) the same, we can obtain
\[ q^0 : \begin{cases} L(x_0) - L(u_0) = 0 \\ x_0(0) = \bar{u}_0, \dot{x}_0(0) = 0 \end{cases} \] (34)
\[ q^1 : \begin{cases} \ddot{x}_1 + 2\zeta \dot{x}_1 + x_1 + \ddot{\bar{u}}_0 + 2\zeta \dot{\bar{u}}_0 + x_1 \mu_0 + x_2 x_0^2 + x_3 x_0^3 - f_0 \cos(\Omega \tau) = 0 \\ x_1(0) = 0, \dot{x}_1(0) = 0 \end{cases} \] (35)

From the above equation (34), the initial solution to equation (28) is supposed as follows
\[ x_0(\tau) = u_0(\tau) = \bar{u}_0 \cos(\Omega \tau) \] (36)

Substituting equation (36) into equation (35), we can obtain
\[ \ddot{x}_1 + 2\zeta \dot{x}_1 + x_1 + \Omega^2 \bar{u}_0 \cos(\Omega \tau) - 2\Omega \zeta \bar{u}_0 \sin(\Omega \tau) + \cdots \\
\qquad x_1 \bar{u}_0 \cos(\Omega \tau) + x_2 \bar{u}_0^3 \cos(\Omega \tau) + x_3 \bar{u}_0^3 \cos(\Omega \tau) - f_0 \cos(\Omega \tau) = 0 \] (37)

It can further be rewritten as follows
\[ \ddot{x}_1 + 2\zeta \dot{x}_1 + x_1 + \Omega^2 \bar{u}_0 \cos(\Omega \tau) - 2\Omega \zeta \bar{u}_0 \sin(\Omega \tau) + \cdots \\
\qquad x_1 \bar{u}_0 \cos(\Omega \tau) + \frac{x_2 \bar{u}_0^3 \cos(\Omega \tau) + 1}{2} + \frac{x_3 \bar{u}_0^3 \cos(\Omega \tau) + 3 \cos(3\Omega \tau)}{4} - f_0 \cos(\Omega \tau) = 0 \] (38)

By a simple calculation, we have
\[ \ddot{x}_1 + 2\zeta \dot{x}_1 + x_1 + \left[(x_1 + \Omega^2 \bar{u}_0) + \frac{3}{4} x_3 \bar{u}_0^3 - f_0 \right] \cos(\Omega \tau) - 2\Omega \zeta \bar{u}_0 \sin(\Omega \tau) + \cdots \\
\qquad \frac{x_2 \bar{u}_0^3 \cos(2\Omega \tau)}{2} + \frac{x_3 \bar{u}_0^3 \cos(3\Omega \tau)}{4} + \frac{x_3 \bar{u}_0^3}{2} = 0 \] (39)

\[ x_1(0) = 0, \dot{x}_1(0) = 0 \]

It is easy to solve \( x_1(\tau) \) from equation (39). So we can get the approximate solution \( u(\tau) \) to equation (25). Thus, the response amplitude \( U \) can be obtained by Fourier transform and the elastic restoring force of isolator can be given as follows
\[ f_{Te} = x_1 U + x_2 U^2 + x_3 U^3 \] (40)

The force transmissibility of isolator can also be calculated as follows
\[ T = 20 \log \left( \sqrt{f_{Te}^2 + (2\zeta \Omega U)^2} / f_0 \right) \] (41)

Here, the same parameters in the “Effects of system parameters on the force transmissibility of isolator working under overloaded condition” section are taken, and the effect of the static equilibrium position \( \bar{u}_0 \) on force
transmissibility is calculated with HBM and HPM to make a comparison, as shown in Figure 14. It can be seen that the HPM results are in good agreement with HBM results.

**Conclusions**

The force transmissibility of the HSLDS vibration isolator working under overload conditions using a buckling circular plate as the negative stiffness is discussed in this paper. The HPM results are in good agreement with HBM. They both indicate that the peak value of force transmissibility can be reduced when the excitation force amplitude decreases or the damping ratio rises under rated load conditions. Besides, when excitation force amplitude increases or the damping ratio descends under overloaded conditions, the system will exhibit softening-to-hardening characteristic, resulting in poor performance of vibration isolation in low frequency region. But, the peak value of the force transmissibility will go down slightly as the excitation force is raised under overloaded conditions. The static equilibrium position has similar effects on the vibration isolation system as the stiffness ratio. They can both increase the resonance frequency and force transmissibility, but the stiffness ratio mainly affects the low frequency region. In summary, the force transmissibility characteristic is dominated by the position of the static equilibrium point (i.e. the load condition of the system), the excitation force amplitude, damping ratio, and stiffness ratio. The HLSDS vibration isolator works at the ideal static equilibrium point with rated load performing better in vibration isolation. When the HLSDS isolator works under the overloaded condition and excitation force is small, it can still achieve excellent vibration isolation in the low frequency domain, which outperforms the linear vibration isolator.

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