Can contribution from magnetic-penguin operator with real photon to $B_s \to \ell^+\ell^-\gamma$ in the standard model be neglected?

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Abstract

Using the $B_s$ meson wave function extracted from non-leptonic $B_s$ decays, we reevaluate the rare decays $B_s \to \ell^+\ell^-\gamma$, ($\ell = e, \mu$) in the standard model, including two kinds of contributions from magnetic-penguin operator with virtual and real photon. We find that the contributions from magnetic-penguin operator $b \to s\gamma$ with real photon to the exclusive decays, which is regarded as to be negligible in previous literatures, are large, and the branchings of $B_s \to \ell^+\ell^-\gamma$ are nearly enhanced by a factor 2. With the predicted branching ratios at order of $10^{-8}$, it is expected that the radiative dileptonic decays will be detected in the LHC-b and B factories in near future.

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I. INTRODUCTION

The standard model (SM) of electroweak interaction has been remarkably successful in describing physics below the Fermi scale and is in good agreement with the most experiment data. Thanks to the efforts of the B factories and LHC, the exploration of quark-flavor mixing is now entering a new interesting era. Measurements of rare B mesons decays such as \( B \to X_s \gamma \), \( B \to X_s \ell^+\ell^- (\ell = e, \mu) \) and \( B_s \to \mu^+\mu^- \), \( B_s \to \ell^+\ell^-\gamma \) are likely to provide sensitive test of the SM. In fact, these decays, induced by the flavor changing neutral currents which occur in the SM only at loop level, play an important role in testing higher order effects of SM and in searching for the physics beyond the SM [1, 2]. Nevertheless, these processes are also important in determining the parameters of the SM and some hadronic parameters in QCD, such as the CKM matrix elements, the meson decay constant \( f_{B_s} \), providing information on heavy meson wave functions [3].

The rare B inclusive radiative decays \( B \to X_s \gamma \) and \( B \to X_s \ell^+\ell^- (\ell = e, \mu) \) as well as the exclusive decays \( B_s \to \mu^+\mu^- \) have been studied extremely at the leading logarithm order [4] and high order in the SM [5] and various new physics models. In previous works, prediction for the exclusive decays \( B_s \to \ell^+\ell^-\gamma \) have been carried out by using the light cone sum rule [1, 2], the simple constituent quark model [6], and the B meson distribution amplitude extracted from non-leptonic B decays [7].

At parton level, \( B_s \to \ell^+\ell^-\gamma \) decays have been thought to be obtained from decay \( b \to s\ell^+\ell^-\gamma \), and further, from \( b \to s\ell^+\ell^- \) directly. To achieve this, a necessary work is attaching real photon to any charged internal and external lines in the Feynman diagrams of \( b \to s\ell^+\ell^- \) with two statements: i) Contributions from the attachment of photon to any charged internal propagator are regarded as to be strongly suppressed and can be neglected safely [1, 2, 6, 7]; ii) Contributions from the attachment of real photon with magnetic-penguin vertex to any charged external lines are always neglected [1, 2] or stated to be negligibly small [7]. Here we would like to address that the conclusion of the first statement is correct, but the explanation is not as what it is described [11]. The second statement seems to be questionable, for that the pole of propagator of the charged line attached by photon may enhance the decay rate greatly which make some diagrams can not be neglected in the calculation. Since the weak radiative B-meson decay is well known to be a sensitive
probe of new physics, it is essential to calculate the Standard Model value of its branching ratio as precisely as possible. Although the second contribution has been calculated in Ref. [8], it mainly concentrated on the long distance effects of the meson resonances, whereas the short distance contribution which was incompletely analyzed.

In this letter, we will concentrate on the short distance contribution to $B_s \to \ell^+\ell^−\gamma$ and check whether the contribution from magnetic-penguin operator with real photon to $B_s \to \ell^+\ell^−$ is negligible or not, and give some remarks including a comparison with other works. The paper is organized as follows. In sec. II we present the detailed calculation of exclusive decays $B_s \to \ell^+\ell^−\gamma$, including full contribution from magnetic-penguin operator with real photon. Sec. III contains the numerical results and comparison with previous works, and the conclusions are given in sec. IV.

II. THE CALCULATION

In order to simplify the decay amplitude for $B_s \to \ell^+\ell^−\gamma$, we have to utilize the $B_s$ meson wave function, which is not known from the first principal. Fortunately, many studies on non-leptonic $B$ [12, 13] and $B_s$ decays [14] have constrained the wave function strictly. It was found that the wave function has form

$$\Phi_{B_s} = (\not{p}_{B_s} + m_{B_s})\gamma_5 \phi_{B_s}(x),$$

(1)

where the distribution amplitude $\phi_{B_s}(x)$ can be expressed as [15]:

$$\phi_{B_s}(x) = N_{B_s} x^2 (1 - x)^2 \exp \left( -\frac{m_{B_s}^2 x^2}{2\omega_{B_s}^2} \right)$$

(2)

with $x$ being the momentum fractions shared by $s$ quark in $B_s$ meson. The normalization constant $N_{B_s}$ can be determined by comparing

$$\langle 0 | \bar{s} \gamma^\mu \gamma_5 b | B_s \rangle = i \int_0^1 \phi_{B_s}(x) dx \text{Tr} [\gamma^\mu \gamma_5 (\not{p}_{B_s} + m_{B_s})\gamma_5] dx = -4i p_{B_s}^\mu \int_0^1 \phi_{B_s}(x) dx$$

(3)

with

$$\langle 0 | \bar{s} \gamma^\mu \gamma_5 b | B_s \rangle = -i f_{B_s} p_{B_s}^\mu,$$

(4)

the $B$ meson decay constant $f_{B_s}$ is thus determined by the condition

$$\int_0^1 \phi_{B_s}(x) dx = \frac{1}{4} f_{B_s}.$$
Let us start with the quark level processes $b \to s\ell^+\ell^-$. They are subject to the QCD corrected effective weak Hamiltonian, obtained by integrating out heavy particles, i.e., top quark, higgs, and $W^\pm$, $Z$ bosons:

\[
H_{\text{eff}}(b \to s\ell^+\ell^-) = \frac{-\alpha_{\text{em}}G_F}{\sqrt{2}\pi}V_{tb}V_{ts}^\ast \left\{ -\frac{2C_7^{\text{eff}}m_b}{q^2}\bar{s}i\sigma^\mu\nu q_\mu P_R b + C_9^{\text{eff}}\bar{s}\gamma^\mu P_L b \right\}\bar{\ell}\gamma_\mu\ell
\]
\[+C_{10}(\bar{s}\gamma^\mu P_L b)\bar{\ell}\gamma_\mu\gamma_5\ell,\]

where $P_{L,R} = (1 \mp \gamma_5)/2$, $q^2$ is the dilepton invariant mass squared. The QCD corrected Wilson coefficients $C_7^{\text{eff}}$, $C_9^{\text{eff}}$ and $C_{10}$ at $\mu = m_b$ scale can be found in Ref. \cite{16}.

If an additional photon line is attached to any of the charged lines in diagrams contributing to the Hamiltonian above, we will have the radiative leptonic decays $b \to s\ell^+\ell^-\gamma$. Therefore, there are two kinds of diagrams: photon connecting to the internal propagators, and photon connecting to the external line. As addressed in the introduction, the contribution from the first kind of diagrams is neglected safely. Now we will only consider the second category of diagrams which are displayed in Fig. 1. At first, we recalculate the diagrams (a)-(d) in Fig. 1 with photon emitted from the external quark lines $b$ or $s$ by using the $B_s$ meson wave function extracted from non-leptonic $B_s$ decays. Note that these diagrams are
already studied in previous literatures and considered as giving the dominant contribution to $B_s \to \ell^+\ell^-\gamma$. At parton level, the amplitudes for transition $b \to s\ell^+\ell^-\gamma$ can be calculated directly using from the Hamiltonian of $b \to s\ell^+\ell^-$ in [9]. For example, contribution from the magnetic-penguin operator with virtual photon shown in Fig. 1(a) reads:

$$A_a = -iG_{ee} d \frac{m_b m_{B_s}}{q^2} \frac{1}{p_{B_s} \cdot k} \int_0^1 \left[ \frac{\delta_{B_s}(x)}{x} \int_0^1 \left( \phi_{B_s}(x) dx \right) [k_\mu q \cdot \epsilon - \epsilon_\mu k \cdot q - i \epsilon_{\mu \nu \alpha \beta} \ell_\nu k^\alpha q^\beta] \prod_{\mu} \gamma_\mu \ell \right],$$

(7)

where $p_{b,s}, k$ denotes the momentum of quarks and photon respectively, $\epsilon$ is the vector polarization of photon and $G = \alpha_e m G_F V_u V_d^*/(\sqrt{2})$, $e_d = -1/3$ is the number of electrical charge of the external quarks. In deriving above equation, we have used motion equation for quarks and $q^\mu \ell_\mu = 0$. Using Eqs. (1) and (7), we write the amplitude of $B_s \to \ell^+\ell^-\gamma$ at meson level as:

$$A_a = 2iG_{ee} d \frac{m_b m_{B_s}}{q^2} C_{eff} \frac{1}{p_{B_s} \cdot k} \int_0^1 \phi_{B_s}(x) dx \left[ k_\mu q \cdot \epsilon - \epsilon_\mu k \cdot q - i \epsilon_{\mu \nu \alpha \beta} \ell_\nu k^\alpha q^\beta \right] \prod_{\mu} \gamma_\mu \ell,$$

(8)

where $x, y = 1 - x$ are the momentum fractions shared by $s, b$ quark in $B_s$. By doing the similar calculation to diagrams (b)-(d), the decay amplitude is then obtained as:

$$A_{a+b+c+d} = iG_{ee} d \frac{1}{p_{B_s} \cdot k} \left[ C_1 i \epsilon_{\alpha\beta\mu\nu} p_{B_s}^\alpha \epsilon^\beta k^\nu + C_2 p_{B_s}^\nu (\epsilon_\mu k_\nu - k_\mu \epsilon_\nu) \right] \prod_{\mu} \gamma_\mu \ell$$

$$+ C_{10} \left[ C_1 + i \epsilon_{\alpha\beta\mu\nu} p_{B_s}^\alpha \epsilon^\beta k^\nu + C_2 p_{B_s}^\nu (\epsilon_\mu k_\nu - k_\mu \epsilon_\nu) \right] \prod_{\mu} \gamma_\mu \gamma_5 \ell.$$}

(9)

The form factors in Eq. (9) are found to be:

$$C_1 = C_+ \left( C_{eff}^0 - 2 \frac{m_b m_{B_s}}{q^2} C_{eff}^0 \right),$$

$$C_2 = C_{eff}^0 C_- - 2 \frac{m_b m_{B_s}}{q^2} C_{eff}^0 C_+,$$

(10)

where

$$C_\pm = \int_0^1 \left( \frac{1}{x} \pm \frac{1}{y} \right) \phi_{B_s}(x) dx.$$

(11)

The expression in (9) can be compared with Ref. [9].

Now we will focus attention on calculating the diagrams (e) and (f) in Fig. 1 which are always neglected in other works. In these two diagrams, photon of the magnetic-penguin operator is real, thus its contribution to $B_s \to \ell^+\ell^-\gamma$ is different from that of magnetic-penguin operator with virtual photon in diagram (a) and (c). We get the amplitude:

$$A_{e+f} = i2G_{ee} d C_{eff} \frac{m_b m_{B_s}}{q^2} \frac{1}{p_{B_s} \cdot q} \prod_{\mu} \gamma_\mu \ell,$$

(12)
with coefficients \( \overline{C}_+ \) obtained by a replacement:

\[
\overline{C}_+ = C_+(x \rightarrow \bar{x} = x - z - i\epsilon; \ y \rightarrow \bar{y} = y - z - i\epsilon) = N_B \int_0^1 dx \left( \frac{1}{x - z - i\epsilon} + \frac{1}{1 - x - z - i\epsilon} \right) x^2 (1 - x)^2 \exp \left[ -\frac{m_{B_s}^2}{2\omega_{B_s}^2} x^2 \right],
\]

where \( z = \frac{q^2}{2 p_{B_s} \cdot q} \). Note that pole in \( \overline{C}_+ \) corresponds to the pole of the quark propagator when it is connected by the off-shell photon propagator. Thus the \( \overline{C}_+ \) term may enhance the decay rate of \( B_s \rightarrow \ell^+ \ell^- \gamma \) and its analytic expression reads

\[
\overline{C}_+ = 2 N_{B_s} \pi i z^2 (1 - z)^2 \exp \left[ -\frac{m_{B_s}^2}{2\omega_{B_s}^2} z^2 \right]
\]

\[
+ N_{B_s} \int_0^1 dx \left( \frac{1}{x + z} - \frac{1}{1 + x - z} \right) x^2 (1 + x)^2 \exp \left[ -\frac{m_{B_s}^2}{2\omega_{B_s}^2} x^2 \right]
\]

\[
- N_{B_s} \int_{-1}^1 \left( \frac{1}{x - z} + \frac{1}{1 - x - z} \right) \frac{dx}{x^4} (1 - \frac{1}{x})^2 \exp \left[ -\frac{m_{B_s}^2}{2\omega_{B_s}^2} \frac{1}{x^2} \right].
\]

As the contribution from the Fig.1 (g) and (h) with photon attached to external lepton lines, considering the fact that (i) being a pseudoscalar meson, \( B_s \) meson can only decay through axial current, so the magnetic penguin operator \( O_7 \)'s contribution vanishes; (ii) the contribution from operators \( O_9, O_{10} \) has the helicity suppression factor \( m_\ell / m_{B_s} \), so for light lepton electron and muon, we can neglect their contribution safely.

The total matrix element for the decay \( B_s \rightarrow \ell^+ \ell^- \gamma \) is obtained a sum of the \( A_a+b+c+d \) and \( A_e+f \). After summing over the spins of leptons and polarization of the photon, and then performing the phase space integration over one of the two Dalitz variables, we get the differential decay width versus the photon energy \( E_\gamma \),

\[
\frac{d\Gamma}{dE_\gamma} = \frac{\alpha^3 G_F^2}{108\pi^4} \left| V_{tb} V^*_{ts} \right|^2 (m_{B_s} - 2E_\gamma) E_\gamma \left[ |\overline{C}_1|^2 + |\overline{C}_2|^2 + C_{10}^2 (|C_+|^2 + |C_-|^2) \right].
\]

The coefficients \( \overline{C}_i \) \((i = 1, 2)\) can be obtained by a shift:

\[
\overline{C}_i = C_i - \frac{2m_b m_{B_s} p_{B_s} \cdot k}{q^2} \frac{C_{10}^\text{eff}}{C_+ C_-}
\]

\[
\text{III. RESULTS AND DISCUSSIONS}
\]

The decay branching ratios can be easily obtained by integrating over photon energy. In numerical calculations, we use the following parameters [18]:

\[
\alpha_{em} = \frac{1}{137}, \ G_F = 1.166 \times 10^{-5} \text{GeV}^{-2}, \ m_b = 4.2 \text{GeV},
\]
\[ |V_{tb}| = 0.88, \ |V_{ts}| = 0.0387, \ |V_{td}| = 0.0084 \]

\[ m_{B_s} = 5.37 \text{GeV}, \ \omega_{B_s} = 0.5, \ f_{B_s} = 0.24 \text{GeV}, \ \tau_{B_s} = 1.47 \times 10^{-12} \text{s}. \]

\[ m_{B_d^0} = 5.28 \text{GeV}, \ \omega_{B_d} = 0.4, \ f_{B_d} = 0.19 \text{GeV}, \ \tau_{B_d} = 1.53 \times 10^{-12} \text{s}. \]

The ratios of \( B_s \to \ell^+ \ell^- \gamma \) with and without the contribution from magnetic-penguin operator with real photon are shown in Table I together with results of \( B_{d,s} \to \ell^+ \ell^- \gamma \) from this work and other models for comparison. The errors shown in the Table I comes from the heavy meson wave function, by varying the parameter \( \omega_{B_d} = 0.4 \pm 0.1 \), and \( \omega_{B_s} = 0.5 \pm 0.1 \) \cite{7}. Note that, the predicted branching ratios receive errors from many parameters, such as meson decay constant, meson and quark masses etc.

### TABLE I: Comparison of branching ratios with other model calculations

| Branching Ratios (\( \times 10^{-9} \)) | Our Results | Quark Model | light cone |
|----------------------------------------|-------------|-------------|-----------|
|                                        | Excluded Fig.1(e),(f) | Included Fig.1(e),(f) | Ref.[7] | Ref.[6] | Ref.[1] |
| \( B_s \to \ell^+ \ell^- \gamma \)    | 3.74\(^{+1.76}_{-1.00} \) | 7.45\(^{+2.98}_{-1.82} \) | 1.90 | 6.20 | 2.35 |
| \( B_d^0 \to \ell^+ \ell^- \gamma \) | 0.16\(^{+0.11}_{-0.05} \) | 0.31\(^{+0.20}_{-0.10} \) | 0.08 | 0.82 | 0.15 |

A couple of remarks on the \( B_s \) rare exclusive radiative decays are follows:

1. As pointed out in Ref. [3, 4], the branching ratios are proportional to the heavy meson wave function squared, the radiative leptonic decays are very sensitive probes in extracting the heavy meson wave functions;

2. The contributions from magnetic-penguin operator with real photon to the exclusive decay is large, and the branching of \( B_s \to \ell^+ \ell^- \gamma \) is enhanced nearly by a factor 2 compared with that only contribution from magnetic-penguin operator with virtual photon and nearly up to \( 10^{-8} \), implying the search of \( B_s \to \ell^+ \ell^- \gamma \) can be achieved in near future.

3. Due to the large contributions from magnetic-penguin operator with real photon, the form factors for matrix elements \( \langle \gamma | s \gamma^\mu (1 \pm \gamma_5) b | B_s \rangle \) and \( \langle \gamma | s \sigma_{\mu\nu} (1 \pm \gamma_5) q^\nu b | B_s \rangle \) as a function of dilepton mass squared \( q^2 \) are not as simple as \( 1/(q^2 - q_0^2) \) where \( q_0^2 \) is constant \cite{17}. The \( B_s \to \gamma \) transition form factors predicted in this works have
also some differences from those in Ref. [8–10]. For instance, Ref. [9] predicted the form factors $F_{TV}(q^2, 0)$, $F_{TA}(q^2, 0)$ induced by tensor and pseudotensor currents with emission of the virtual photon, as shown in diagrams (a) and (c) of FIG. are only equal at maximum photon energy, whereas the corresponding formula in this work have the same expression as $-\frac{e q m_{B_s}}{p_{B_s} \cdot k} C_+ \propto 1/(q^2 - q_0^2)$ in Eq. (9). The research of $B_s \to \ell^+ \ell^- \gamma$ may give some hints on these form factors.

At this stage, we think it is necessary to present a few more comments about the calculation of Ref. [8]. In order to estimate the contribution of emission of the real photon from the magnetic-penguin operator, the authors of Ref. [8] calculated the form factors $F_{TA,TV}(0, q^2)$ by including the short distance contribution in $q^2 \to 0$ limit and additional long distance contribution from the resonances of vector mesons such as $\rho^0$, $\omega$ for $B_d$ decay and $\phi$ for $B_s$ decay. Obviously, this means the pole mass enhancement of the valence quark were not appropriately taken into account. Moreover, if $F_{TA,TV}(0, q^2) = F_{TA,TV}(0, 0)$ stands for the short distance contribution, it seems double counting since in this case photons which emits from magnetic-penguin vertex and quark lines directly are not able to be distinguished.

IV. CONCLUSION

We evaluated the rare decays $B_s \to \gamma \ell^+ \ell^-$ in the SM, including two kinds of contributions from magnetic-penguin operator with virtual and real photon. In contrast to the previous works which treated contribution from magnetic-penguins operators with real photon to the decays as negligible small, we found that the contributions is large, leading to the branching of $B_s \to \ell^+ \ell^- \gamma$ being nearly enhanced by a factor 2. In the current early phase of the LHC era, the exclusive modes with muons in the final states are among the most promising decays. The decay $B_s \to \mu^+ \mu^-$ is likely to be confirmed before the end of 2012 [19]. Although there are some theoretical challenges in calculation of the hadronic form factors and non-factorable corrections, with the predicted branching ratios at order of $10^{-8}$, $B_s \to \ell^+ \ell^- \gamma$ can be expected as the next goal once $B_s \to \mu^+ \mu^-$ measurement is finished since the final states can be identified easily and branching ratios are large. Our predictions for such processes can be tested in the LHC-b and B factories in near future.
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