An upper limit to the central density of dark matter haloes from consistency with the presence of massive central black holes

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ABSTRACT

We study the growth rates of massive black holes in the centres of galaxies from accretion of dark matter from their surrounding haloes. By considering only the accretion due to dark matter particles on orbits unbound to the central black hole, we obtain a firm lower limit to the resulting accretion rate. We find that a runaway accretion regime occurs on a timescale which depends on the three characteristic parameters of the problem: the initial mass of the black hole, and the volume density and velocity dispersion of the dark matter particles in its vicinity. An analytical treatment of the accretion rate yields results implying that for the largest black hole masses inferred from QSO studies ($>10^9 M_\odot$), the runaway regime would be reached on timescales which are shorter than the lifetimes of the haloes in question for central dark matter densities in excess of $250 M_\odot pc^{-3}$. Since reaching runaway accretion would strongly distort the host dark matter halo, the inferences of QSO black holes in this mass range lead to an upper limit on the central dark matter densities of their host haloes of $\rho_0 < 250 M_\odot pc^{-3}$. This limit scales inversely with the assumed central black hole mass. However, thinking of dark matter profiles as universal across galactic populations, as cosmological studies imply, we obtain a firm upper limit for the central density of dark matter in such structures.

Key words: galaxies: haloes — galaxies: evolution — dark matter — gravitation — accretion

1 INTRODUCTION

The question of what is the central density profile of galactic dark matter haloes has been much debated in the literature over many years. Since the work of Navarro et. al (1997), cosmological N-body simulations have consistently agreed in yielding central density profiles which can be accurately fitted by functional forms characterised by centrally divergent density cusps. Although the details vary and the innermost slope and radius to which said fits are to be trusted are still discussed, a broad agreement has been reached in that cosmologically simulated dark haloes exhibit what are termed 'cuspy density profiles', e.g. Merritt et al. (2006).

On the other hand, observational inferences of dark halo density profiles through rotation curve decomposition, have tended to favour those showing density profiles which tend to constant values towards the centre. Recent examples include de Blok et al. (2008), Kuzio de Naray et al. (2009) and Gebhardt & Thomas (2009). The issue is complicated by the necessity to estimate the dynamical relevance of the baryonic component, a function of the assumed mass to light ratio, the relevance of observational uncertainties such as beam smearing, and the importance of non circular motions and non centrifugal support of asymmetric drift and hydrodynamic pressure terms (e.g. Valenzuela et al. 2007).

An interesting independent clue to the puzzle might come from the presence of massive black holes in the centre of dark matter haloes. The relevance of such black holes is well established from their role as the central engines for quasars and active galaxies (Rees 1984) as well as in quiescent systems (Kormendy & Richstone 1995). Although mostly seen as active nuclei in the distant universe, it is commonly believed that all large galaxies host such objects in their centres. Recent empirical determinations of central QSO black hole masses have established their existence with masses in the range $10^7 - 10^9 M_\odot$ at redshifts beyond $z \approx 3$ (Kelly et al. 2008, Graham 2008).

We therefore must conclude that the central regions of large dark haloes have coexisted with massive black holes.
over most of the history of the universe. Given the existence of event horizons associated to black holes, and the assumption of standard cold dark matter subject only to gravitational interactions, it follows that central black holes have grown over the history of galactic dark haloes, through the capture of dark matter particles.

The problem of the growth of single central galactic black holes through accretion was addressed by Lynden-Bell & Rees (1971) and more recently by Gnedin & Primack (2004) and Zhao et al. (2002), but only in the accretion of particles on capture orbits. While readily accreted, they constitute only a minor fraction of those available in the distribution function of halo particles. Further, once absorbed, one has to wait over a comparatively long halo relaxation timescale for it to be re-populated with dark matter particles. Here we consider only the accretion of unbound particles, through the absorption cross section presented by the black hole through its event horizon, enhanced by gravitational focusing. In this case, the accretion is slower at first, but proceeds at an ever increasing rate as the growing black hole mass leads to an increase in its area, with little accompanying depletion of the overall dark matter halo distribution function. This, as accretion does not take place over a highly specific fraction of the distribution function, while the black hole mass constitutes only a fraction of the overall dark halo mass.

We find the process to be characterised by the onset of a rapid runaway growth phase after a critical timescale. This timescale is a function of the mass of the black hole and the local density of dark matter. By requiring that the runaway phase does not occur, as then the swallowing up of the halo by the black hole would seriously distort the former, we can obtain upper limits to the maximum allowed density of dark matter at the centres of haloes.

2 CENTRAL BLACK HOLE GROWTH RATES

In the case of cuspy dark matter profiles, the scales over which the density varies by a significant factor, even in the central regions, are typically of order 10 pc or above (Merritt et al. 2006, Stadel et al. 2009). This is many orders of magnitude greater than the typical length scales over which the accretion onto the central black hole of mass \( M \) takes place, the Schwarzschild radius, \( R_{\text{Sch}} = 5 \times 10^{-4}\text{pc}(M/5 \times 10^9M_\odot) \). The range of scales involved, from \( R_{\text{Sch}} \) to the scale of the dark matter halo, make a full simulation of the whole problem unfeasible. To first approximation, we will therefore treat the central region of the dark matter halo over which the black hole finds itself as one of constant density.

The effects of central black holes, mostly binary ones, on the stellar population of a galaxy have been treated before, e.g. Quinlan (1996). The problem here is different because the total mass of the dark halo, and the range of radii covered by the dark matter particles are both much larger than equivalent quantities in the case of bulge stars affected by central black holes. Since the mass of the black hole is still several orders of magnitude smaller than the total mass of the dark halo, we shall treat the presence of the black hole as a perturbation on the distribution function of the dark matter particles.

In Hernandez & Lee (2008), we calculated the density response of a constant density, isothermal dark matter distribution, to the presence of a point mass \( M \). A highly localised cusp appears, with the density profile changing from \( \rho_0 \) to \( \rho_0 + \rho_1(r) \), where

\[
\rho_1(r) = \frac{GM}{r^2} \rho_0. \tag{1}
\]

In the above, \( \sigma \) is the velocity dispersion of the halo particles, and \( r \) is the distance to the mass \( M \). In estimating the growth rate of the central black hole, of initial mass \( M_0 \), we shall begin by assuming that its presence will elicit a response in the otherwise unperturbed dark halo particles, as described by eq. (1). In Hernandez & Lee (2008) we showed through direct comparison with high resolution N-body simulations, that the analytic expression in eq. (1) accurately describes the response of a constant density isothermal region to the presence of a point mass. A first correction due to relativistic effects, the substitution of a Newtonian potential for the expression of Paczyński & Wiita (1980)\(^1\), will result only in the substitution of \( r - R_{\text{Sch}} \) for the current \( r \) in the denominator of eq. (1).

We can now estimate the growth rate of the central black hole dimensionally as:

\[
\dot{M} = C_0 \rho A \sigma, \tag{2}
\]

where \( \rho, A \) and \( \sigma \) are a characteristic density, area and velocity for the spherical accretion in question, and \( C_0 \) is a dimensionless constant which one would expect to be of order unity. From the preceding discussion regarding the reaction of the halo to the presence of the black hole, and from the fact that accretion of unbound particles will occur on crossing the event horizon at \( R = R_{\text{Sch}} \), we can estimate \( \dot{M} \) from taking \( \rho = \rho_1(R_{\text{Sch}}) \), \( A = 4\pi R_{\text{Sch}}^2 \), and \( \sigma \) as the velocity dispersion of the dark matter particles in the halo. We have ignored \( \rho_0 \) in favour of \( \rho_1 \) in the above considerations, as at distances of order \( R_{\text{Sch}} \) the latter dominates over the former by a factor of order \( (c/\sigma)^2 \). The above yields:

\[
\dot{M} = C_0 8\pi \frac{G^2 M^2 \rho_0}{\sigma c^2}. \tag{3}
\]

Gravitational focusing effectively increases the cross section of the black hole by trapping particles which would otherwise fail to be accreted, and enters into the computation of \( C_0 \). In fact, a fully relativistic calculation for the accretion rate of a black hole immersed in an isothermal distribution of non-relativistic particles leads to the result:

\[
\dot{M} = 16 (6\pi)^{1/2} \frac{G^2 M^2 \rho_0}{\sigma c^2}, \tag{4}
\]

as derived in Shapiro & Teukolsky (1983), eq. 14.2.26, for particles with positive energies. Those on bound orbits can be engulfed rapidly in a short initial transient phase, and will re-appear as the corresponding phase space is repopulated by the distribution function on the very long relaxation timescales of the full halo (Shapiro & Teukolsky 1983). Here again, by considering only the accretion rate of unbound particles, we are confident in having a secure lower limit on the accretion rate. Comparing with the dimensional analysis of eq. (3) we see that the result is exact.

\(^1\) In the P-W expression a point mass produces a gravitational field \( \Phi(r) = -GM/(r - R_{\text{Sch}}) \) instead of the usual \( \Phi = -GM/r \).
for \( C_0 = 2(6/\pi)^{1/2} \), a factor of less than 3. In what follows we shall use the exact result of eq. (4), with eq. (3) serving only in allowing a physical interpretation of the relativistic result.

Introducing dimensionless quantities \( \Sigma = \sigma/c, \tau = t/t_{\text{ff}} \) and \( M = M/M_J \), where \( t_{\text{ff}} = 1/(G\rho_0)^{1/2} \) is the free fall timescale of the unperturbed background density, and \( M_J = \sigma^3/(G^{3/2} \rho_0^{1/2}) \) is the Jeans mass of the unperturbed halo, we obtain the dimensionless growth rate:

\[
\frac{dM}{d\tau} = (6\pi)^{1/2}(4M\Sigma)^2.
\]

We note that in cosmological N-body simulations, the distribution function of dark matter particles exhibits a large degree of orbital anisotropy and is dominated by highly radial orbits (e.g. Ascasibar & Gottlober 2008). The increased fraction of the dark halo hence available for interaction with the central black hole, and the reduced angular momentum of the dark matter particles, compared to the isothermal case, will all tend to yield faster growth rates than those calculated here. The upper limits on central density derived below are hence safe upper estimates. More detailed calculations accounting for an intrinsically cusped dark halo profile with radially dominated distribution functions would yield even lower limit densities.

### 3 CENTRAL DARK MATTER DENSITY LIMITS

From eq. (4) we obtain for the black hole mass:

\[
M(t) = \frac{M_0 \sigma^3}{c^2 \sigma - 16(6\pi)^{1/2} G^2 \rho_0 M_\odot}.
\]

There is a strong divergence for \( t \to t_{\text{div}} = c^2 \sigma/(16(6\pi)^{1/2} G^2 \rho_0 M_0) \). The time for it to appear decreases as the initial black hole mass rises, as the central dark matter density increases, and increases as the velocity dispersion of the halo particles rises. The divergence is so abrupt that the time it takes for the black hole mass to increase by one order of magnitude, \( T_{10} \), is only 9/10 \( t_{\text{div}} \). The evolution we calculate will still be accurate up to \( t = T_{10} \), as the total mass of the dark halo (several times \( 10^{12} M_\odot \) for large galactic haloes), will still be over an order of magnitude larger than that of the central black hole, even for the largest initial black hole masses considered, of a few times \( 10^9 M_\odot \). We shall therefore define:

\[
T_{10} = \left( \frac{9}{10} \right) \frac{c^2 \sigma}{16(6\pi)^{1/2} G^2 \rho_0 M_0},
\]

as a characteristic timescale after which the accretion process results in substantial dynamical alterations to the overall dark halo. With the same dimensionless quantities as defined previously, we obtain the corresponding expressions:

\[
M(\tau) = \frac{M_0}{1 - C_1 M_0 \Sigma^2 \tau}, \quad \tau_{\text{div}} = \left( C_1 M_0 \Sigma^2 \right)^{-1},
\]

where \( C_1 = 16(6\pi)^{1/2} \). We can now calculate the evolution of eq. (6) for any value of the central black hole mass. We begin with parameters as appropriate for the largest inferred QSO central black holes, \( M_0 = 5 \times 10^9 M_\odot \) (Kelly et al. 2008, Graham 2008), as this case will lead to the most restrictive dark matter density limits. Although the dark haloes of QSOs cannot be observationally inferred, given the scalings observed at low redshift between black hole masses and galactic properties (e.g. Kormendy & Richstone 1995, Gehardt et al. 2000, Ferrarese & Merritt 2000, Tremaine et al. 2002, Gultekin et al. 2009), between black hole masses and halo properties (e.g. Volonteri et al. 2003, Bolton et al. 2008, Croton 2009, Bandara et al. 2009) and between galactic properties and dark halo masses, such as the Tully-Fisher relation (Tully & Fisher 1977), it is reasonable to assume that QSOs hosting the largest inferred black hole masses will be hosted by large galactic haloes. Consequently, we take a large value of \( \sigma = 200 \text{ km/s} \). Given the usual scaling between \( \sigma \) and the flat rotation curve velocity of a galactic halo of \( V_{\text{rot}} = 21/2 \sigma \), this choice corresponds to \( V_{\text{rot}} = 280 \text{ km/s} \), a value in the extreme range for any type of galactic system. Notice that as \( t_{\text{div}} \) scales with \( \sigma \), taking a large value for this parameter will again result in conservative upper limits on the final inferred limit central halo densities (see below). Also, given the ‘inside out’ and ‘downsizing’ aspects of current cosmological structure formation models, (e.g. Naab et al. 2009) the dynamical stability of the central regions of the most massive galactic systems over the lifetimes we have assumed appears reasonable.

Figure 1 shows the growth of a central black hole as a function of time, for our fiducial case with \( M_0 = 5 \times 10^9 M_\odot \) and \( \sigma = 200 \text{ km/s} \), and a range of values for the assumed central dark matter density \( \rho_0 = 400, 350, 300, 250, 200, 150 \) and \( 100 M_\odot \text{ pc}^{-3} \) (from top to bottom, respectively). The case where \( T_{10} = 10 \text{ Gyr} \) corresponds to the middle curve, where the central dark matter density is \( \rho_0 = 250 M_\odot \text{ pc}^{-3} \). The mass of the black hole increases by a factor of 10 in 10 Gyr, a conservative estimate of the lifetime of the systems in question, namely, dark haloes of QSOs observed at high redshift. We see that for central dark matter densities above this threshold of \( 250 M_\odot \text{ pc}^{-3} \), the mass of the central black hole enters the runaway accretion regime and diverges on timescales shorter than the lifetimes of the systems being treated, as given by the three upper curves in Figure 1. On the other hand, for values below this threshold, the growth of the central black hole is of only a factor of order unity over 10 Gyr, as shown by the three lower curves.

For this particular initial mass, \( M_0 = 5 \times 10^9 M_\odot \), we can hence identify \( 250 M_\odot \text{ pc}^{-3} \) as a maximum central halo dark matter density above which the inferences of black hole masses in high redshift QSOs would imply growth rates for the central black holes resulting in substantial dynamical distortions, leading today not to quiescent black holes in the centres of normal galaxies, but to exotic objects dynamically dominated by extreme super massive black holes. Consistency arguments of this type can be found e.g. in Gnedin & Ostriker (2001), who calibrate the physical parameters of self-interacting dark matter by requiring that galactic dark haloes should not have evaporated by now into galaxy cluster dark matter haloes. If we were to take larger values for the black hole mass, such as those given by Graham (2008), reaching \( 10^{10} M_\odot \), or upwards of \( 10^{10} M_\odot \) reported for some objects by Kelly et al. (2008), the threshold density we identify would go down by a factor of a few.

In Figure 2 we show the constraints on the central dark halo densities, \( \rho_M \), as a function of the assumed central black hole mass and for a fixed value of the velocity dispersion, \( \sigma = 200 \text{ km/s} \). The curves correspond to various values of
initial mass $M_0 = 5 \times 10^9 M_\odot$ in a dark halo with dark matter particles of isotropic velocity dispersion 200 km/s, for varying central region dark matter density: $\rho_0 = 400, 350, 300, 250, 200, 150$ and $100 \ M_\odot \ pc^{-3}$, top to bottom, respectively.

$T_{10}$. By taking a conservative measure of the lifetimes of the systems in question as 10 Gyr, the region containing the dotted curves above the thick black line at $T_{10} = 10$ Gyr is excluded from consistency arguments, while the allowed region of parameter space lies in the half plane below it. The choice of values for the lifetimes of high redshift QSOs larger than the 10 Gyr previously assumed, shifts the maximum central dark matter density values downwards onto the various thin solid curves. The most stringent limits apply to the highest black hole masses at $M \geq 5 \times 10^9 M_\odot$. However, the expectation of universality for the cosmological dark matter density profiles leads one to expect these limits will apply to all dark matter haloes.

We note that once the mass of the central black hole grows substantially, processes not included here would begin to become relevant, and would invalidate the simple physical hypothesis leading to eq. (4). Some include: the adiabatic contraction of the dark halo in response to the concentration of mass into the central black hole, resulting in higher central dark matter densities and hence even higher accretion rates; the accretion of a fraction of the baryons into the central black hole, which is known to occur; or the enhanced gravitational focusing of matter of all types into the black hole, once the approximation of the black hole mass being small compared to the total halo mass which we are working under begins to break down. All of these make it reasonable to assume that the first corrections to eq. (4) will lead to even larger accretion rates, hence leaving our conclusions, in terms of limit densities, unchanged.

Regarding the accretion of baryons and dark halo particles, it has been proposed that this can be partly responsible for the appearance of the observed scaling laws between central black hole masses and bulge and galactic properties, e.g., in the analytical work of Zhao et al. (2002) and Gnedin & Primack (2004), the large scale simulations of Di Matteo et al. (2008), or the luminosity function consistency arguments in Yu & Tremaine (2002) and Hopkins et al. (2007). Indeed, Hennawi & Ostriker (2002) constrain the velocity dependence of the interaction cross-section of hypothetical self-interacting dark matter, by requiring that the accretion of dark matter onto central black holes leads to the observed $M - \sigma$ relation. Also for the case of self-interacting dark matter, Balberg & Shapiro (2002) explore the formation of supermassive black holes through gravothermal core-collapse of the central regions of galactic dark haloes.

Comparing the upper limiting central dark matter density of $250 M_\odot \ pc^{-3}$ with the dynamically inferred structure of galactic dark haloes, it is reassuring that when a constant density core is used to model observations, the inferred central dark matter densities always lie below this limit, typically at $\sim 1 M_\odot \ pc^{-3}$, or below. Recent examples are given by Gilmore et al. (2007) for local dwarf spheroidal galaxies, and de Blok et al. (2008) for late type galaxies. Hence no conflict appears, in that the runaway accretion regime for the central black hole will not be reached in 10 Gyr for any directly inferred values of the central dark matter density, for any inferred central black hole masses.

From the point of view of cuspy dark matter haloes, the limits we derive here establish an inner boundary, exterior to which the globally fitted centrally divergent dark halo profiles can be valid. At smaller radii, this solution must be modified to avoid the divergent black hole growth rates found here. Although our high limit densities are not reached by current cosmological N-body simulations, which typically stop at volume densities of order $1 M_\odot \ pc^{-3}$ at their resolution limit, even for the most recent highest resolution experiments (Cuesta et al. 2008, Stadel et al. 2009), the logarithmic slopes in these regions are still such that...
volume densities above the limits we derive here would be reached orders of magnitude in the radial coordinate before reaching the scale of the super massive central black holes, even for central black hole masses below the upper ranges of these values. Studies of the origin of cuspy cosmological density profiles within the secondary infall scenario have traced the cusp to the close to scale free initial perturbation spectrum (e.g. Williams et al. 2004, Salvador-Solé et al. 2005, Del Popolo 2009). Such studies explain the steep negative logarithmic slopes of density profiles from N-body simulations, predicting them to extend into the very centres. Although these considerations apply only to cosmological dark matter haloes in the absence of any central black holes, the inclusion of single central black holes, currently beyond the reach of fully self consistent simulations, will result in even steeper central dark matter profiles, strengthening the consistency arguments made here. A solution might include dark matter physics not ordinarily considered, such as self-interacting dark matter (e.g. Firmani et al. 2000, de la Macorra 2009), warm dark matter, or changes to the initial fluctuation spectrum (e.g. Alam et al. 2002).

4 CONCLUSIONS

We study the mass growth rates of central black holes through accretion of dark matter particles on orbits unbound to the central black hole. As the black hole mass grows, a runaway accretion regime ensues. Requiring that no such runaway regime has been reached over lifetimes of galactic dark haloes of 10 Gyr leads to the identification of no such runaway regime has been reached over lifetimes of the assumed value of the central black hole mass. As the black hole mass through accretion of dark matter particles on orbits unbound to the central black hole. As the black hole mass through accretion of dark matter particles on orbits unbound to the central black hole.

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