Shell effect in Pb isotopes near the proton drip line

C. Samanta\textsuperscript{ab}, S. Adhikari\textsuperscript{a}

\textsuperscript{a}Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Calcutta 700064
\textsuperscript{b}Physics Department, Virginia Commonwealth University, Richmond, Virginia 23284

A mass formula (BWM) without shell effect is employed to study the variation of the shell effect in Pb isotopes through comparison with the experimental data. Unlike other macroscopic formulae, the BWM reproduces the general trend of the binding energy versus neutron number curves of all the nuclei from Li to Bi. The shell effect in Pb-isotopes reduces to \( \sim 56 \) keV at \( N = 106 \) but, increases gradually for \( N < 106 \), indicating increasing shell effect in Pb near the proton drip line.

Quenching of the \( N = 82 \) magic neutron shell below \( ^{132}\text{Sn} \) \cite{1,2} was suggested in response to an astrophysical quest to properly reproduce the isotopic solar r-process abundances in the \( A \approx 120 \) mass region. As the shell effect is not a directly measurable quantity, one tries to estimate it from various theoretical prescriptions and looks for its signature through comparison with the experimental data. When the normalised mass deviations are plotted, the "unquenched" FRDM model \cite{3} delineates similar deviation from the experimental data for both Cd (\( N > 78 \)) and Pb (\( N = 105 - 120 \)) isotopes, indicating possible quenching of \( N = 82 \) and \( Z = 82 \) shell gaps. While experimental signature of \( N = 82 \) shell quenching in \( ^{130}\text{Cd} \) is found \cite{4,5}, the \( Z = 82 \) shell quenching still remains controversial. Earlier, from the analysis of a few \( \alpha \)-decay experiments it was predicted that the shell closure effect at \( Z = 82 \) may disappear in the vicinity of \( N = 112 - 114 \) \cite{6,7,8}. But the observation of large variation in the hindrance factors of \( l = 0 \) \( \alpha \)-decay to the excited \( 0^+ \) state in even-even Po, Pb, Hg and Pt nuclei indicates persistence of the \( Z = 82 \) "shell gap" at the neutron-deficient side \cite{9}.

The "shell effect" in nuclei in principle contains the deformation and shell closure effects. Extraction of the shell effect through comparison of the experimental mass with the liquid drop mass formula of Bethe-Weizsäcker (BW) is quite well known. The BW formula, originally designed for medium and heavy mass nuclei, fails for light nuclei especially, near the drip lines \cite{10}. The improved liquid drop model (ILDM) \cite{11,12} also fails to give the correct shape of the binding energy versus neutron number curves of light nuclei. In search of a more complete macroscopic formula, we formulated a mass formula, called BWM \cite{10}, modifying the asymmetry and the pairing energy terms of BW. The parameters of BWM were optimised to fit the gross properties of the binding energy versus neutron number curves from Li to Bi. Fitting to such large number of nuclei over a wider
mass range puts stringent constraints on the choice of parameters. In fact, as both heavy and light nuclei are fitted with a single set of parameters, BWM delineates shell effect more accurately than BW and ILDM. In the modified-Bethe-Weizsäcker mass formula (BWM) the expression for the binding energy (BE) is [10],

\[
BE(A, Z) = 15.777A - 18.34A^{2/3} - 0.71 \frac{Z(Z-1)}{A^{1/3}} - 23.21 \frac{(A - 2Z)^2}{A \times (1 + e^{-A/17})} + (1 - e^{-A/30})\delta,
\]

(1)

where the term \(\delta = +12 A^{-1/2}\) for even Z-even N nuclei, and \(-12 A^{-1/2}\) for odd Z-odd N nuclei and 0 for odd A nuclei. This formula is applicable only for the spherical nuclei having negligible shell effects and it shows marked deviation for nuclei with shell effects.

Figure 1. Variation of \(G_{2p}\) with \(N\) at \(Z=82\) from experimental data [13, 14, 15] and predictions of different mass formulae (SN [18], Koura [17], Möller [3] and BWM [10]).

Figure 1 shows plot of \(G_{2p} = 2 \times BE(A,Z) - BE(A-2,Z-2) - BE(A+2,Z+2)\) for \(Z=82\) computed from the masses of Hg, Pb and Po isotopes [13, 14, 15]. The BWM having no shell effect gives a smooth straight line. At \(N = 106\) the difference between the BWM and experimental data almost disappears. The difference rises again at \(N < 106\). Bender et al. [16] argued that \(G_{2p}\) does not reflect the \(Z = 82\) shell gap as, most of the Hg and Po nuclei have deformed ground states. Interestingly, none of the "unquenched" mass formulae of refs. [3, 17, 18] can explain the actual shape of the experimental data (Figure 1) although they implicitly contain deformation and shell effects.

Figure 2 shows the single-proton separation energy \(S_p\) from Pb and nearby elements. For \(Z = 82\), a large discrepancy between the experimental value and the BWM prediction
Figure 2. Plots of $S_p$ versus $Z$ for $N=104$, 106 and 126 from experimental mass data [13, 14, 15] and BWM predictions. Figure 3. Variation of "Shell effect" $\Delta B$ with proton number $Z$ for (a) $N=114-132$ and (b) $N=100-114$.

is seen at $N = 126$, as expected. The discrepancy reduces greatly at $N = 106$ but increases again at $N = 104$ where the break at $Z=82$ reappears in the experimental data. In Figure 3, the "shell effect" $\Delta B$ i.e., the difference between the binding energies computed from the experimental masses and the BWM is plotted for several nuclei around $Z = 82$. The shell effect at $Z = 82$ reduces for both $N < 126$ and $N > 126$. Interestingly, after reaching to a low value ($\sim 56$ keV) at $N = 106$, the shell effect increases again for $N < 106$. This rise occurs due to mutual support of magicity coming from the approaching $N = 82$ magic number.

In summary, the $G_{2p}$, $S_p$ and $\Delta B$ values are computed from the experimental mass defect data and a mass formula (BWM) which has no shell effect incorporated [10]. As the shell effect in a nucleus decreases, the discrepancy between the experimental data and the predictions of BWM diminishes. For all the above three quantities a close agreement between the experimental data and the BWM predictions is observed for Pb isotope with $N=106$. Unlike the $G_{2p}$ and $S_p$ values the "shell effect" $\Delta B$ computed here is not affected by the possible deformations of nearby elements.
As all the even-even isotopes of Pb have spherical ground states, one of the reasons for reduction of shell effect near \( N = 106 \) and its increase approaching the proton drip line might be the change in the \( Z = 82 \) shell closure effect. From a self consistent calculation Bender et al. [16] also found an increase of \( Z = 82 \) shell gap approaching the proton drip line, but the \( Z = 82 \) shell gap was shown to remain large all along. It is pertinent to note that in moving away from the neutron shell closure at \( N = 126 \), the number of valence neutrons increases up to the mid-shell configuration and this brings in an important energy correlation that contributes to nuclear masses. The large amount of valence neutrons will induce a polarization of the closed proton core. This is clearly present in the mass behaviour of intruder states and should also have important effects on the binding energy [19]. In a microscopic theory these specific and important correlations should first be removed in order to be able to deduce results on a changing spherical \( Z = 82 \) shell gap, but such a microscopic calculation is beyond the scope of this work.

On the experimental side, measurements of knockout and pickup cross sections from light Pb isotopes are needed to extract the spectroscopic factors and to confirm either the quenching or, the persistence of the \( Z = 82 \) shell gap.

REFERENCES

1. K.L.Kratz et al., Ap. J. \textbf{403}, (1993) 216
2. B.Chen et al, Phys. Lett. \textbf{B355}, (1995) 37
3. P.Möller et al., At. Data Nucl. Data Tables \textbf{59}, (1995) 185
4. T.Kautzsch et al., Eur. Phys. J. \textbf{A9}, (2001) 201
5. I. Dillmann et al., Phys. Rev. Lett., \textbf{91}, (2003) 162503
6. K.Toth et al., Phys. Rev. Lett. \textbf{53}, (1984) 1623
7. B.A.Brown, Phys. Rev. \textbf{C46}, (1992) 811
8. K.H.Schmidt, W.Faust and H.Münzenberg, Nucl. Phys. \textbf{A318}, (1979) 253
9. J.Wauters et al., Phys. Rev. Lett. \textbf{72}, (1994) 1329
10. C.Samanta and S.Adhikari, Phys.Rev.\textbf{C65}, (2002) 037301
11. W.D.Myers and W.J.Swiatecki, Nucl. Phys. \textbf{81}, (1966) 1
12. S.R.Souza et al., Phys.Rev.\textbf{C67}, (2003) 051602(R)
13. Nuclear Wallet Cards, Brookhaven National Laboratory, 2000
14. S.Schwarz et al., Nucl. Phys. \textbf{A693}, (2001) 533
15. Yu.N.Novikov et al., Nucl. Phys. \textbf{A697}, (2002) 92
16. M.Bender et al.,Eur. Phys. J. \textbf{A14}, (2002) 23
17. H.Koura, M.Uno, T.Tachibana and M.Yamada, Nucl. Phys. \textbf{A674}, (2000) 47
18. L.Satpathy and R.Nayak, Phys. Rev. Lett. \textbf{51}, (1983) 1243
19. R.Fossion et al., Phys.Rev. \textbf{C67}, (2003) 024306; see references therein