On the structure of the correlation coefficients $S(E_e)$ and $U(E_e)$ of the neutron beta decay

A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov

1 Atom Institut, Technische Universität Wien, Stadionallee 2, A-1020 Wien, Austria
2 Department of Physics, New Mexico State University, Las Cruces, New Mexico 88003, USA
3 FH Campus Wien, University of Applied Sciences, Favoritenstrasse 226, 1100 Wien, Austria
4 Peter the Great St. Petersburg Polytechnic University, Polytechnicheskaya 29, 195251, Russian Federation

(Dated: March 24, 2021)

In the standard effective $V-A$ theory of low-energy weak interactions (i.e. in the Standard Model (SM)) we analyze the structure of the correlation coefficients $S(E_e)$ and $U(E_e)$, where $E_e$ is the electron energy. These correlation coefficients were introduced to the electron-energy and angular distribution of the neutron beta decay by Ebel and Feldman (Nucl. Phys. 4, 213 (1957)) in addition to the set of correlation coefficients proposed by Jackson et al. (Phys. Rev. 106, 517 (1957)). The correlation coefficients $S(E_e)$ and $U(E_e)$ are induced by simultaneous correlations of the neutron and electron spins and electron and antineutrino 3-momenta. These correlation structures do not violate discrete $P, C$ and $T$ symmetries. We analyze the contributions of the radiative corrections of order $O(\alpha/\pi)$, taken to leading order in the large nucleon mass $m_N$ expansion, and corrections of order $O(1/m_N)$, caused by weak magnetism and proton recoil. In addition to the obtained SM corrections we calculate the contributions of interactions beyond the SM (BSM contributions) in terms of the phenomenological coupling constants of BSM interactions by Jackson et al. (Phys. Rev. 106, 517 (1957)) and the second class currents by Weinberg (Phys. Rev. 112, 1375 (1958)).

PACS numbers: 12.15.Ff, 13.15.+g, 23.40.Bw, 26.65.+t

I. INTRODUCTION

The general form of the electron-energy and angular distribution of the neutron beta decay for polarized neutrons, polarized electrons and unpolarized protons were proposed by Jackson et al. and Ebel and Feldman. In the notations of Ref. it looks like

\[
\frac{d^3\lambda_n(E_e, \vec{k}_e, \vec{k}_\nu, \vec{\xi}_n, \vec{\xi}_e)}{dE_e d\Omega_e d\Omega_\nu} \propto \zeta(E_e) \left\{ 1 + b(E_e) \frac{m_e}{E_e} + a(E_e) \frac{\vec{k}_e \cdot \vec{k}_\nu}{E_e E_\nu} + A(E_e) \frac{\vec{\xi}_n \cdot \vec{\xi}_e}{E_e} + B(E_e) \right\}
\]

\[
+ K_n(E_e) \left( \frac{\vec{\xi}_n \cdot \vec{k}_e}{E_e} \right) + Q_n(E_e) \left( \frac{\vec{\xi}_n \cdot \vec{k}_\nu}{E_e E_\nu} \right) + D(E_e) \left( \frac{\vec{\xi}_n \cdot \vec{k}_e \times \vec{\xi}_e}{E_e E_\nu} \right) + G(E_e) \left( \frac{\vec{\xi}_n \cdot \vec{\xi}_e}{E_e} \right)
\]

\[
+ H(E_e) \left( \frac{\vec{\xi}_n \cdot \vec{k}_e}{E_\nu} \right) + N(E_e) \left( \frac{\vec{\xi}_n \cdot \vec{\xi}_e}{E_\nu} \right) + Q_e(E_e) \left( \frac{\vec{\xi}_e \cdot \vec{k}_e \times \vec{\xi}_n}{E_e + m_e} \right) + K_e(E_e) \left( \frac{\vec{\xi}_e \cdot \vec{k}_e \times \vec{\xi}_n}{E_e + m_e} \right)
\]

\[
+ R(E_e) \left( \frac{\vec{\xi}_e \cdot \vec{k}_e \times \vec{\xi}_n}{E_e} \right) + L(E_e) \left( \frac{\vec{\xi}_e \cdot \vec{k}_e \times \vec{\xi}_n}{E_e E_\nu} \right) + S(E_e) \left( \frac{\vec{\xi}_e \cdot \vec{k}_e \times \vec{\xi}_n}{E_e E_\nu} \right) + T(E_e) \left( \frac{\vec{\xi}_e \cdot \vec{k}_e \times \vec{\xi}_n}{E_e E_\nu} \right)
\]

\[
+ U(E_e) \left( \frac{\vec{\xi}_e \cdot \vec{k}_e \times \vec{\xi}_n}{E_e E_\nu} \right) + V(E_e) \left( \frac{\vec{\xi}_e \cdot \vec{k}_e \times \vec{\xi}_n}{E_e E_\nu} \right) \right\},
\]

where $\vec{\xi}_n$ and $\vec{\xi}_e$ are unit 3-vectors of spin-polarizations of the neutron and electron, $(E_e, \vec{k}_e)$ and $(E_\nu, \vec{k}_\nu)$ are energies and 3-momenta of the electron and antineutrino, $d\Omega_e$ and $d\Omega_\nu$ are infinitesimal solid angles in directions of 3-momenta of the electron and antineutrino, respectively.

*Electronic address: ivanov@kph.tuwien.ac.at
†Electronic address: roman.hoellwieser@gmail.com
‡Electronic address: natroitskaya@yandex.ru
§Electronic address: max.wellenzohn@gmail.com
¶Electronic address: berdnikov@spbstu.ru
The analysis of the distribution in Eq. (1) within the standard effective $V-A$ theory of low-energy weak interactions (4-5) (i.e. within the Standard Model (SM)), carried out to leading order in the large nucleon mass $m_N$ expansion, has shown that correlation coefficients $a(E_e)$, $A(E_e)$, $B(E_e)$, $G(E_e)$, $H(E_e)$, $N(E_e)$, $Q_e(E_e)$ and $K_e(E_e)$ of the electron-energy and angular distribution by Jackson et al. [1] and the correlation coefficient $T(E_e)$, introduced by Ebel and Feldman [2], survive and depend on the axial coupling constant $g_A$ only (8-11), which appears in the effective $V-A$ theory of low-energy weak interactions by renormalization of the hadronic axial-vector current by strong low-energy interactions [5, 11]. The function $\zeta(E_e)$ defines the contributions of different corrections to the neutron lifetime [16]. In the SM it is equal to unity in the leading order of the large nucleon mass $m_N$ expansion and at the neglect of radiative corrections [12, 21] (see also [16]). In Refs. [14-16] (see also [16]) the radiative corrections of order $O(\alpha/\pi)$ (or so-called outer model-independent radiative corrections [17]) were calculated to leading order in the large nucleon mass $m_N$ expansion to the neutron lifetime and correlation coefficients $a(E_e)$, caused by electron-antineutron 3-momentum correlations, $A(E_e)$ and $B(E_e)$, defining the electron- and antineutrino asymmetries, respectively. In turn, the outer radiative corrections of order $O(\alpha/\pi)$ were calculated to leading order in the large nucleon mass $m_N$ expansion to the correlation coefficients $G(E_e)$, $H(E_e)$, $N(E_e)$, $Q_e(E_e)$, and $K_e(E_e)$ in [18-19] and to the correlation coefficient $T(E_e)$ in [16].

These correlation coefficients are induced by correlations of the electron spin with a neutron spin and 3-moments of the electron and antineutrino. The corrections of order $O(E_e/m_N)$, caused by weak magnetism and proton recoil, were calculated i) to the neutron lifetime and correlation coefficients $a(E_e)$, $A(E_e)$ and $B(E_e)$ in [20, 21] (see also [14, 16]), ii) to the correlation coefficients $G(E_e)$, $H(E_e)$, $N(E_e)$, $Q_e(E_e)$ and $K_e(E_e)$ in [18, 19] and iii) to the correlation coefficient $T(E_e)$ in [16]. The correlation coefficients $D(E_e)$, $R(E_e)$ and $L(E_e)$, characterizing the strength of violation of time reversal invariance (T-odd effect) [22], are induced by the distortion of the Dirac wave function of the decay electron in the Coulomb field of the decay proton [23, 24] (see also [19]). The correlation coefficient $b(E_e)$ is the Fierz interference term [27]. It is assumed that the Fierz interference term is caused by interactions beyond the SM [27]. As regards the contemporary experimental and theoretical status of the Fierz interference term we refer to Refs. [28-34].

This paper is addressed to the analysis of the structure of the correlation coefficients $S(E_e)$ and $U(E_e)$, introduced by Ebel and Feldman [2]. As has been shown in [3] these correlation coefficients do not survive to leading order in the large nucleon mass $m_N$ expansion in contrast to the correlation coefficient $T(E_e)$.

The paper is organized as follows. In section II we aduce the analytical expressions for the correlation coefficients $S(E_e)$ and $U(E_e)$ in dependence of i) the radiative corrections of order $O(\alpha/\pi)$, calculated to leading order in the large nucleon mass $m_N$ expansion, and ii) the corrections of order $O(E_e/m_N)$, caused by weak magnetism and proton recoil. In section III we give the contributions of interactions beyond the SM, expressed in terms of the phenomenological coupling constants of the effective phenomenological BSM interactions by Jackson et al. [1] and the contributions of the second class currents by Weinberg [32]. In section IV we give the total expressions for the $S(E_e)$ and $U(E_e)$. We discuss the obtained results and the usage of these correlation coefficients for experimental searches of interactions beyond the SM. We point out that the obtained SM theoretical background of the correlation coefficients $S(E_e)$ and $U(E_e)$ at the level a few parts of $10^{-4}$ should be very useful for experimental searches of contributions of interactions beyond the SM in the experiments with transversally polarized decay electrons [33]. In Appendices A and B of the Supplemental Material within the SM we give in details the calculations of the correlation coefficients $S(E_e)$ and $U(E_e)$ and the analysis of the correlation structure of the neutron radiative beta decay for polarized neutrons, polarized electrons, unpolarized protons and unpolarized photons.

II. CORRELATION COEFFICIENTS $S(E_e)$ AND $U(E_e)$ IN THE STANDARD MODEL

In the SM with the account for the contributions of the radiative corrections of order $O(\alpha/\pi)$ and the corrections of order $O(E_e/m_N)$, caused by weak magnetism and proton recoil, the neutron beta decay can be described by the standard effective $V-A$ low-energy weak interaction [4, 5] and electromagnetic interaction with the Lagrangian

$$\mathcal{L}_{W\gamma}(x) = \mathcal{L}_W(x) + \mathcal{L}_{em}(x),$$

(2)

where $\mathcal{L}_W(x)$ and $\mathcal{L}_{em}(x)$ are the Lagrangian of the standard effective $V-A$ low-energy weak interactions [4, 5] (see also [16])

$$\mathcal{L}_W(x) = -G_V \left\{ \bar{\psi}_p(x) \gamma_\mu (1 - g_A \gamma^5) \psi_n(x) \right\} + \frac{\kappa}{2m_N} \partial^\nu \bar{\psi}_p(x) \sigma_{\mu\nu} \psi_n(x) \right\} [\bar{\psi}_e(x) \gamma^\mu (1 - \gamma^5) \psi_p(x)]$$

(3)
and the Lagrangian of electromagnetic interactions \( \mathcal{L}_{\text{em}}(x) = -e \left[ \left[ \bar{\psi}_p(x) \gamma_\mu \psi_p(x) \right] - \left[ \bar{\psi}_e(x) \gamma_\mu \psi_e(x) \right] \right] A_\mu(x), \)

respectively, where \( G_L \) is the vector weak coupling constant, including the Cabibbo-Kobayashi-Maskawa (CKM) matrix element \( V_{ud} \) \( [10] \), \( g_A \) is the real axial coupling constant \( \left[ 3, 12 \right] \), \( \psi_p(x), \psi_e(x) \) and \( \bar{\psi}_e(x) \) are the field operators of the proton, electron and antineutrino, respectively, \( \gamma^\mu = (\gamma^0, \vec{\gamma}) \), \( \gamma^5 \) and \( \sigma^{\mu\nu} = \frac{i}{2} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) \)

are the Dirac matrices \( \left[ 22 \right] \), \( \kappa = \kappa_p - \kappa_n = 3.7059 \) is the isovector anomalous magnetic moment of the nucleon, defined by the anomalous magnetic moments of the proton \( \kappa_p = 1.7929 \) and the neutron \( \kappa_n = -1.9130 \) and measured in nuclear magneton \( \left[ 10 \right] \), and \( m_N = (m_n + m_p)/2 \) is the average nucleon mass; \( e \) is the electric charge of the proton, and \( A_\mu(x) \) is a 4-vector electromagnetic potential.

For the calculation of the correlation coefficients under consideration we use the amplitude of the neutron beta decay, calculated in \( \left[ 16 \right] \) (see also \( \left[ 19 \right] \) and the Supplemental Material). The detailed calculation we have carried out in the Supplemental Material. Below we adduce only the obtained results.

### Analytical expressions for the correlation coefficients \( S(E_e) \) and \( U(E_e) \) in the Standard Model

In Eq. \( \left[ A-20 \right] \) of Appendix A in the Supplemental Material we have defined the general expression for the structure part of the electron-energy and angular distribution of the neutron beta decay for a polarized neutron, a polarized electron and an unpolarized proton. According to this expression, we have shown that the contributions of the radiative corrections of order \( O(\alpha/\pi) \), caused by one-virtual photon exchanges \( \left[ 12, 13 \right] \) (for the detailed calculations we refer to \( \left[ 16 \right] \) do not appear in the correlations coefficients \( S(E_e) \) and \( U(E_e) \), respectively. In Appendix B of the Supplemental Material we have shown that the neutron radiative beta decay \( n \to p + e^- + \bar{\nu}_e + \gamma \) does not contribute to the correlation coefficients \( S(E_e) \) and \( U(E_e) \). It is well-known \( \left[ 37, 41 \right] \) (see also \( \left[ 12, 13 \right] \) and \( \left[ 16 \right] \) that the contribution of the neutron radiative beta decay is extremely needed for cancellation of the infrared divergences in the radiative corrections of order \( O(\alpha/\pi) \), caused by one-virtual photon exchanges.

Thus (see Eq. \( \left[ A-20 \right] \)) the contributions, caused by the SM interactions, appear in the correlation coefficients \( S(E_e) \) and \( U(E_e) \) only due to weak magnetism and proton recoil. For the correlation coefficients \( \zeta(E_e)S(E_e) \) \( S(E_e) \) and \( \zeta(E_e)U(E_e) \) \( U(E_e) \) we have obtained the following analytical expressions

\[
\zeta(E_e)S(E_e) = \frac{1}{1 + 3g^2_A} \frac{m_e}{m_N} \left( -5g_A^2 - g_A(\kappa - 4) + (\kappa + 1) \right),
\]

\[
\zeta(E_e)U(E_e) = 0,
\]

where for the calculation of the contributions of order \( O(E_e/m_N) \), caused weak magnetism and proton recoil, we have taken into account the contribution of the phase-volume of the neutron beta decay (see Eq. \( \left[ A-20 \right] \)). The correlation function \( \zeta(E_e)S(E_e) \) was calculated in \( \left[ 12, 16 \right] \). It is equal to unity at the neglect of the contributions of radiative corrections and corrections, caused by weak magnetism and proton recoil. Hence, the correlation coefficients \( S(E_e) \) \( S(E_e) \) and \( U(E_e) \) \( U(E_e) \), including the SM contributions of order \( O(E_e/m_N) \), are equal to

\[
S(E_e) = \frac{1}{1 + 3g^2_A} \frac{m_e}{m_N} \left( -5g_A^2 - g_A(\kappa - 4) + (\kappa + 1) \right),
\]

\[
U(E_e) = 0.
\]

Now we may move on to calculating the contributions of interactions beyond the SM.

### III. Contributions of Interactions Beyond the Standard Model and Second Class Currents of the G-odd Correlations

For the calculation of the contributions of interactions beyond the SM we use the effective phenomenological Lagrangian of BSM interactions proposed by Jackson et al. \( \left[ 1 \right] \) (see also \( \left[ 12, 13 \right] \)). In turn, the account for the contributions of the second class currents or the G-odd correlations (see \( \left[ 14 \right] \)) we follow Weinberg \( \left[ 32 \right] \). Gardner and Plaster \( \left[ 16 \right] \) (see also \( \left[ 3, 19, 47 \right] \)). Skipping intermediate calculations we give the results

\[
S(E_e) = \frac{1}{1 + 3g^2_A} \text{Re}(C_T - \tilde{C}_T) - \text{Re}(g_2(0) - g_3(0)) \frac{2g_A}{1 + 3g^2_A} \frac{m_e}{m_N},
\]

\[
U(E_e) = -\frac{1}{1 + 3g^2_A} \text{Re}(C_T - \tilde{C}_T) + \text{Re}(g_2(0) - g_A g_3(0)) \frac{2}{1 + 3g^2_A} \frac{m_e}{m_N}.
\]
where $C_T$ and $\tilde{C}_T$ are the phenomenological tensor coupling constants of the effective phenomenological BSM interactions by Jackson et al. \cite{1}, and $\text{Re}f_3(0)$ and $\text{Re}g_2(0)$ are the phenomenological coupling constants of the induced scalar and tensor second class currents \cite{23,12,16} (see also \cite{28}), respectively. The contributions of the tensor BSM interactions by Jackson et al. \cite{1} are linear in the phenomenological tensor coupling constants $C_T$ and $\tilde{C}_T$. This agrees well with the result obtained by Ebel and Feldman \cite{2}. However, in addition to the result obtained by Ebel and Feldman \cite{2} we, following \cite{18,52} (see also \cite{16,13,15,17}), have taken the contributions of the phenomenological vector coupling constants $C_V$ and $\tilde{C}_V$ in the linear approximation, i.e. $C_V = 1 + \delta C_V$ and $\tilde{C}_V = -1 + \delta \tilde{C}_V$, where we have used the notations of \cite{3,16,18,19,47}.

IV. DISCUSSION

We have analyzed the structure of the correlation coefficients $S(E_e)$ and $U(E_e)$, introduced by Ebel and Feldman \cite{2} in addition to the set of correlation coefficients proposed by Jackson et al. \cite{1}. Summing up the SM contributions, caused by weak magnetism and proton recoil only, and contributions beyond the SM we obtain the following expressions

$$S(E_e) = \frac{1}{1 + 3g_A^2} \left\{ \frac{m_e}{m_N} \left( -5g_A^2 - g_A(\kappa - 4) + (\kappa + 1) \right) + \frac{1}{1 + 3g_A^2} \text{Re}(C_T - \tilde{C}_T) \right\}$$

and

$$U(E_e) = \frac{1}{1 + 3g_A^2} \text{Re}(C_T - \tilde{C}_T) + (\text{Re}g_2(0) - g_A\text{Re}f_3(0)) \frac{2}{1 + 3g_A^2} \frac{m_e}{m_N}.$$

For the axial coupling constant $g_A = 1.2764$ \cite{8} the correlation coefficients $S(E_e)$ and $U(E_e)$ are given by

$$S(E_e) = -2.83 \times 10^{-4} + 0.17 \left( \text{Re}(C_T - \tilde{C}_T) + 1.39 \times 10^{-3} \text{Re}f_3(0) \right) - 2.36 \times 10^{-4} \text{Re}g_2(0),$$

$$U(E_e) = -0.17 \left( \text{Re}(C_T - \tilde{C}_T) + 1.39 \times 10^{-3} \text{Re}f_3(0) \right) + 1.85 \times 10^{-4} \text{Re}g_2(0),$$

where we have also used $m_e = 0.5110$ MeV and $m_N = (m_u + m_p)/2 = 938.9188$ MeV \cite{10}.

We would like to notice that the correlation structures of the correlation coefficients $S(E_e)$ and $U(E_e)$ are in contrast to the correlation coefficient $T(E_e)$ of the absolute value of which is of about $\left| \text{Re}g_2(0) \right|$ \cite{1}. Summing up the SM contributions, the absolute value of which is of about $|T(E_e)| \sim 1$, the absolute values of the correlation coefficients $S(E_e)$ and $U(E_e)$ are of a few orders of magnitude smaller. It is also important to mention that unlike the correlation coefficient $T(E_e)$ the correlation coefficients $S(E_e)$ and $U(E_e)$ do not depend on the electron energy $E_e$.

The correlation coefficients $S(E_e)$ and $U(E_e)$ can, in principle, be investigated in experiments with both longitudinally and transversely polarized decay electrons \cite{8} (see also \cite{8}). However, a successful results for searches of interactions beyond the SM one may expect only from experiments with experimental uncertainties of about a few parts of $10^{-5}$. In this case any deviation of the correlation coefficient $S(E_e)$ from $2.83 \times 10^{-4}$, caused by weak magnetism and proton recoil, should testify a presence of interactions beyond the SM. Since most likely $f_3(0) = 0$ \cite{28,29} (see also \cite{53}) the contributions of the phenomenological tensor BSM interactions by Jackson et al. \cite{1}, proportional to $\text{Re}(C_T - \tilde{C}_T)$, can be distinguished from the contributions of the tensor second class currents, defined by the phenomenological tensor coupling constant $g_A\text{Re}f_3(0)$, only after the measurement of the contribution of the phenomenological tensor second class currents to the correlation coefficient $T(E_e)$ \cite{25}. In case of $f_3(0) \neq 0$ and if in the neutron beta decay the absolute value of the Fierz interference term $b$ could be of order $10^{-2}$ (see, for example, \cite{31,52}) the neutron measurements of the phenomenological tensor coupling constant $g_A\text{Re}f_3(0)$ from the correlation coefficient $T(E_e)$, the contribution of the scalar coupling constant $\text{Re}g_2(0)$ from the correlation coefficients $S(E_e)$ and $U(E_e)$ could be screened by the contributions of the phenomenological tensor coupling constants $\text{Re}(C_T - \tilde{C}_T)$ of the phenomenological tensor BSM interactions by Jackson et al. \cite{1}.

V. ACKNOWLEDGEMENTS

We thank Hartmut Abele for discussions stimulating this work. The work of A. N. Ivanov was supported by the Austrian “Fonds zur Förderung der Wissenschaftlichen Forschung” (FWF) under contracts P31702-N27 and P26636-N20, and “Deutsche Förderungsgemeinschaft” (DFG) AB 128/5-2. The work of R. Höllwieser was supported by the Deutsche Forschungsgemeinschaft in the SFB/TR 55. The work of M. Wellenzohn was supported by the MA 23 (FH-Call 16) under the project “Photonik - Stiftungsprofessur für Lehre". 


VI. THE SUPPLEMENTAL MATERIAL

Appendix A: The electron-energy and angular distribution of the neutron beta decay for polarized neutrons, polarized electrons and unpolarized protons

Following [16, 18, 19] (see also [3]) we define the electron-energy and angular distribution of the neutron beta decay for a polarized neutron, an unpolarized electron, and an unpolarized proton as follows

$$\frac{d^3\mathcal{L}_{e^-}\gamma}(E_e, k_e, k_{\nu}, \xi_n, \xi_{\nu}) = (1 + 3g_A^2) \frac{|G_V|^2}{16\pi^5} (E_0 - E_e)^2 \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1)$$

$$\times \Phi_n(k_e, k_{\nu}) \sum_{\text{pol}} \frac{|M(n \rightarrow p e^- \nu_e)|^2}{(1 + 3g_A^2)|G_V|264m_e^2 E_e E_{\nu}},$$

(A-1)

where the sum is over polarizations of massive fermions. Then, $F(E_e, Z = 1)$ is the relativistic Fermi function, describing the electron–proton final-state Coulomb interaction, is equal to (see, for example, [54] (see also [21] and a discussion in [18])

$$F(E_e, Z = 1) = \left(1 + \frac{1}{2}\right) \left(\frac{4(2\pi m_e\beta)^2}{1 - (\beta^2)}\right) e^{\gamma/\beta} \left[1 + \frac{\alpha}{\beta} \right]^2,$$

(A-2)

where $\beta = k_e/E_e = \sqrt{E_e^2 - m_e^2}/E_e$ is the electron velocity, $\gamma = \sqrt{1 - \alpha^2} - 1$, $r_p$ is the electric radius of the proton [55]. The function $\Phi_n(k_e, k_{\nu})$ defines the contribution of the phase-volume of the neutron beta decay [16, 56]. It is equal to [16, 56]

$$\Phi_n(k_e, k_{\nu}) = 1 + 3 \frac{E_e}{m_N} \left(1 - \frac{k_e \cdot k_{\nu}}{E_e E_{\nu}}\right),$$

(A-3)

taken to next-to-leading order in the large nucleon mass $m_N$ expansion. The amplitude of the neutron beta decay $M(n \rightarrow p e^- \nu_e)$, taking into account the contribution of the corrections, caused by one-virtual photon exchanges, weak magnetism and proton recoil, was calculated in [16] (see also [19]). It is given by

$$M(n \rightarrow p e^- \nu_e) = -2m_n G_V \left\{\left(1 + \frac{\alpha}{\pi} f_{A^-}(E_e, \mu)\right) |\varphi_p|^2 |\varphi_n| |u_e|^2 (1 - \delta^5) \nu_e \right.\
+ \frac{g_A}{2\pi} \left[1 + \frac{\alpha}{2\pi} f_{A^-}(E_e, \mu)\right] |\varphi_p|^2 |\varphi_n| |u_e|^2 (1 - \delta^5) \nu_e - \frac{\alpha}{2\pi} g_F(E_e) |\varphi_p|^2 |\varphi_n| |u_e|^2 (1 - \delta^5) \nu_e \right.\
- \frac{\alpha}{2\pi} g_A f_{A^-}(E_e) |\varphi_p|^2 |\varphi_n| |u_e|^2 (1 - \delta^5) \nu_e - \frac{m_e}{2m_N} |\varphi_p|^2 |\varphi_n| |u_e|^2 (1 - \delta^5) \nu_e \right.\
- \frac{\alpha}{2\pi} \frac{g_A}{2m_N} |\varphi_p|^2 |\varphi_n| |u_e|^2 (1 - \delta^5) \nu_e - i \frac{\alpha}{2m_N} |\varphi_p|^2 |\varphi_n| |u_e|^2 (1 - \delta^5) \nu_e \right\},$$

(A-4)

where $\varphi_p$ and $\varphi_n$ are Pauli spinorial wave functions of the proton and neutron, $u_e$ and $u_\nu$ are Dirac wave functions of the electron and electron antineutrino, $\bar{\sigma}$ are the Pauli $2 \times 2$ matrices, and $g_A = g_A(1 - E_0/2m_N), E_0 = (m_e^2 - m_\nu^2 + m_\nu^2)/2m_N = 1.2926$ MeV is the end-point energy of the electron-energy spectrum of the neutron beta decay [12, 15, 14], and $\tilde{k}_e = -k_e - k_{\nu}$ is the proton 3–momentum in the rest frame of the neutron. The functions $f_{A^-}(E_e, \mu)$ and $g_F(E_e)$ were calculated by Sirlin [12] (see also Eq.(D-51) of Ref. [16] and ), $\mu$ is a covariant infrared cut-off introduced as a finite virtual photon mass [12] (see also [37, 41]). The function $g_F(E_e)$ (see Eq.(D-44) of Ref. [16]) is equal to

$$g_F(E_e) = \frac{\sqrt{1 - \beta^2}}{\beta} \ln \left(\frac{1 + \beta}{1 - \beta}\right).$$

(A-5)

It is defined by the contributions of one-virtual-photon exchanges [12] (see also [16]). Using Eq.(A-1) for the square of the absolute value of the amplitude $M(n \rightarrow p e^- \nu_e)$, summed over polarizations of massive fermions, we obtain the following expression

$$\sum_{\text{pol}} \frac{|M(n \rightarrow p e^- \nu_e)|^2}{(1 + 3g_A^2)|G_V|264m_e^2 E_e E_{\nu}} = \frac{1}{(1 + 3g_A^2)|G_V|264m_e^2 E_e E_{\nu}} \left\{\left(1 + \frac{\alpha}{\pi} f_{A^-}(E_e, \mu)\right) \left(\bar{\xi}_n \cdot \bar{\sigma}\right) \left(\bar{\xi}_{\nu} \gamma_5 \bar{k}_e \gamma_0 \right) \right.\
\times (1 - \delta^5) + \frac{g_A}{2\pi} \left(\bar{\xi}_n \cdot \bar{\sigma}\right) \left(\bar{\xi}_{\nu} \gamma_5 \bar{k}_e \gamma_0 \right) \left(1 - \delta^5\right) + \frac{g_A}{2\pi} \left(\bar{\xi}_n \cdot \bar{\sigma}\right) \left(\bar{\xi}_{\nu} \gamma_5 \bar{k}_e \gamma_0 \right) \left(1 - \delta^5\right) \nu_e \right\}
\times \left\{\left(\bar{\xi}_e \gamma_5 \bar{k}_{\nu} \gamma_0 \right)(1 - \delta^5) \right\} - \left(\frac{\alpha}{2\pi} g_F(E_e) + \frac{m_e}{2m_N} \right) \left(\bar{\xi}_n \cdot \bar{\sigma}\right) \left(\bar{\xi}_{\nu} \gamma_5 \bar{k}_e \gamma_0 \right) \left(1 - \delta^5\right) \nu_e \right\}$$

(A-6)
The 4-vector \( \eta \)
\[
\eta = \left( \frac{\vec{k}_e \cdot \vec{E}}{m_e}, \frac{\vec{\gamma} \cdot \vec{B}_0}{m_e}, \vec{\gamma} \cdot \vec{k}_e \right),
\]
where \( \eta = \left( \frac{\vec{k}_e \cdot \vec{E}}{m_e}, \frac{\vec{\gamma} \cdot \vec{B}_0}{m_e}, \vec{\gamma} \cdot \vec{k}_e \right) \). Calculating the traces over the nucleon degrees of freedom and using the properties of the Dirac matrices \( \gamma_{\mu} \gamma_{\nu} \gamma^\mu \gamma^\nu = \gamma_{\mu} \eta^\mu + \gamma^\mu \eta_{\mu} + i \varepsilon_{\alpha \nu \mu \beta} \gamma_{\alpha} \gamma_{\beta} \), where \( \gamma^\mu \) is the metric tensor of the Minkowski space–time, \( \varepsilon_{\alpha \nu \mu \beta} \) is the Levi–Civita tensor defined by \( \varepsilon^0123 = 1 \) and \( \varepsilon_{\alpha \nu \mu \beta} = -\varepsilon_{\nu \mu \beta \alpha} \) \( \bar{a} \) \[ \sum \text{pol.} \frac{|M_m \rightarrow p e^{-}\bar{e}}{|G_V|^2 64\pi^2 m_e E_p E_0} = \frac{1 + 3\bar{g}_A^2}{(1 + 3\bar{g}_A^2)4E_0} \left( \left( 1 + \frac{\alpha}{\pi} f_{\beta \epsilon}(E, \mu) \right) \left( 1 + B_0 \vec{\xi}_a \cdot \vec{k}_e \right) \right) \text{tr}(\vec{k}_e + m_e \gamma^5 \vec{\zeta_e} \gamma(1 - \gamma))^5)
\]
\[
\frac{\vec{k}_e}{E_p} \cdot \text{tr}(\vec{(k}_e + m_e \gamma^5 \vec{\zeta_e}) \gamma^0 \gamma(1 - \gamma))^5) = \frac{\bar{g}_A}{1 + 3\bar{g}_A^2} \left( \frac{\alpha}{\pi} g_F(E_e) + \frac{m_e}{m_N} \right) \left( \frac{\vec{\zeta}_a \cdot \vec{k}_e}{E_p} \right) \text{tr}(\vec{(k}_e + m_e \gamma^5 \vec{\zeta_e}) \gamma^0 \gamma(1 - \gamma))^5)
\]
\[
\left( \vec{k}_e \cdot \vec{E}_p \right) \cdot \text{tr}(\vec{(k}_e + m_e \gamma^5 \vec{\zeta_e}) \gamma^0 \gamma(1 - \gamma))^5) = \frac{\bar{g}_A}{1 + 3\bar{g}_A^2} \left( \frac{\alpha}{\pi} g_F(E_e) + \frac{m_e}{m_N} \right) \left( \frac{\vec{\zeta}_a \cdot \vec{k}_e}{E_p} \right) \text{tr}(\vec{(k}_e + m_e \gamma^5 \vec{\zeta_e}) \gamma^0 \gamma(1 - \gamma))^5)
\]
where \( \bar{a}, \bar{A}_0 \) and \( \bar{B}_0 \) are defined in terms of the axial coupling constant \( \bar{g}_A \)
\[
\bar{a}_0 = \frac{1 - \bar{g}_A^2}{1 + 3\bar{g}_A^2}, \quad \bar{A}_0 = 2\bar{g}_A(1 - \bar{g}_A) \frac{1 + \bar{g}_A}{1 + 3\bar{g}_A^2}, \quad \bar{B}_0 = 2\bar{g}_A(1 + \bar{g}_A) \frac{1 + \bar{g}_A}{1 + 3\bar{g}_A^2}.
\]
Before the calculation of the traces over leptonic degrees of freedom one may see that the terms proportional two \( g_F(E_e) i(\vec{\zeta}_n \times \vec{k}_0) \cdot | \{ m_e + \vec{k}_e \gamma^5 \zeta_e \} \gamma^0 \gamma | \), which are responsible for contributions of the radiative corrections of order \( O(\alpha/\pi) \) to the correlation coefficients \( S(E_e) \) and \( U(E_e) \), cancel each other out. Hence, there are no contributions of the radiative corrections of order \( O(\alpha/\pi) \), caused by one-virtual-photon exchanges, to the correlation coefficients \( S(E_e) \) and \( U(E_e) \), respectively.

In Eq. (A-9), the second term on the third line from above, proportional to \( m_e/m_N \), and last four lines define the contributions of order \( O(E_e/m_N) \) of weak magnetism and proton recoil to the correlation coefficients of the neutron beta decay. Having calculated the traces over leptonic degrees of freedom, taking into account the contribution of the phase-volume Eq. (A-3) and keeping only the contributions with the correlation structures, inducing the correlation coefficients \( S(E_e) \) and \( U(E_e) \), we obtain the SM corrections, caused by weak magnetism and proton recoil only, which we give in Eq. (3).

Appendix B: The electron-photon-energy and angular distribution of the neutron radiative beta decay for polarized neutrons, polarized electrons and unpolarized protons and photons

Following [3, 16, 18, 19] we define the electron-photon-energy and angular distribution of the neutron radiative beta decay for a polarized neutron, a polarized electron, a polarized photon and an unpolarized proton as follows

\[
\frac{d^8 \lambda \gamma}{d\omega dE_{e}d\Omega_{e} d\Omega_{\gamma}} = (1 + 3g_{\lambda e}^2) \frac{G_{V}^2}{E_e} \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1) \left( E_0 - E_e - \omega \right)^{-2} \frac{1}{\omega} \times \sum_{\text{pol}} \frac{|M(n \rightarrow p e^{-} \bar{\nu}_e \gamma)|^2}{(1 + 3g_{\lambda e}^2)2E_eE_\nu},
\]

(B-1)

where we sum over polarizations of massive fermions. Since we calculate the contribution of the neutron radiative beta decay to leading order in the large nucleon mass \( m_N \) expansion, the contribution of the phase volume of the decay is equal to unity. The photon state is determined by the 4-momentum \( q^\mu = (\omega, \vec{q}) \) and the 4-vector of polarization \( \varepsilon^\mu(q) \), with \( \lambda = 1, 2 \), obeying the constraints \( \varepsilon^\mu(q) \varepsilon_\mu(q) = 4 \delta_{\lambda \lambda} \) and \( q \cdot \varepsilon_\lambda(q) = 0 \). In the tree approximation and to leading order in the large nucleon mass \( m_N \) expansion the amplitude of the neutron radiative beta decay is equal to \( \frac{1}{\omega} \)

\[
M(n \rightarrow p e^{-} \bar{\nu}_e \gamma)_{\lambda} = e G_{V} \frac{m_n}{E_e} \frac{1}{E_e - \vec{n} \cdot \vec{k}_e} \{ [\bar{u}_e Q\lambda \gamma^0 (1 - \gamma^5)v_\nu] + g_A [\bar{\nu}_e \gamma^\nu \gamma \cdot [\bar{u}_e Q\lambda \gamma^0 (1 - \gamma^5)v_\nu] \}.
\]

(B-2)

The hermitian conjugate amplitude is determined by

\[
M^\dagger(n \rightarrow p e^{-} \bar{\nu}_e \gamma)_{\lambda} = e G_{V} \frac{m_n}{E_e} \frac{1}{E_e - \vec{n} \cdot \vec{k}_e} \{ [\bar{u}_e Q\lambda \gamma^0 (1 - \gamma^5)v_\nu] + g_A [\bar{\nu}_e \gamma^\nu \gamma \cdot [\bar{u}_e Q\lambda \gamma^0 (1 - \gamma^5)v_\nu] \}.
\]

(B-3)

where \( \vec{n} = \vec{q}/\omega, Q = 2(\varepsilon \cdot k_e) + \vec{\varepsilon} \times \vec{k}_e \) and \( \bar{Q} = \gamma^0 Q\gamma^0 = 2(\varepsilon \cdot k_e) + \vec{\varepsilon} \times \vec{k}_e \). Then, \( \varphi_\nu \) and \( \varphi_\rho \) are the Pauli wave functions of the neutron and proton, \( u_e \) and \( v_\nu \) are the Dirac wave functions of the electron and antineutrino, respectively. The sum over polarizations of the massive fermions is equal to \( \sum_{\text{pol}} \).

\[
\sum_{\text{pol}} \frac{|M(n \rightarrow p e^{-} \bar{\nu}_e \gamma)|^2}{(1 + 3g_{\lambda e}^2)2E_eE_\nu} = \frac{1}{(E_e - \vec{n} \cdot \vec{k}_e)^2} \frac{1}{(1 + 3g_{\lambda e}^2)32E_eE_\nu} \left\{ (1 + \vec{\xi}_n \cdot \vec{\sigma}) \delta \{ (k_e + m_e\gamma^5 \zeta_e)Q\lambda \gamma^0 k_0 \gamma^0 Q\lambda \} \times (1 - \gamma^5) \right\} + g_A \left\{ (1 + \vec{\xi}_n \cdot \vec{\sigma}) \right\} \delta \{ (k_e + m_e\gamma^5 \zeta_e)Q\lambda \gamma^0 k_0 \gamma^0 Q\lambda \} \times (1 - \gamma^5) \right\} + g_A \left\{ (1 + \vec{\xi}_n \cdot \vec{\sigma}) \right\} \delta \{ (k_e + m_e\gamma^5 \zeta_e)Q\lambda \gamma^0 k_0 \gamma^0 Q\lambda \} \times (1 - \gamma^5) \right\} + g_A \left\{ (1 + \vec{\xi}_n \cdot \vec{\sigma}) \right\} \delta \{ (k_e + m_e\gamma^5 \zeta_e)Q\lambda \gamma^0 k_0 \gamma^0 Q\lambda \} \times (1 - \gamma^5) \right\} \}

\]

(B-4)

Having calculated the traces over the nucleon degrees of freedom and using the properties of the Dirac matrices Eq. (A-5) we transcribe the r.h.s. of Eq. (B-4) into the form

\[
\sum_{\text{pol}} \frac{|M(n \rightarrow p e^{-} \bar{\nu}_e \gamma)|^2}{(1 + 3g_{\lambda e}^2)2E_eE_\nu} = \frac{1}{(E_e - \vec{n} \cdot \vec{k}_e)^2} \frac{1}{16E_e} \left\{ (1 + B_0 \frac{\vec{\xi}_n \cdot \vec{\sigma}}{E_\nu}) \delta \{ (k_e + m_e\gamma^5 \zeta_e)Q\lambda \gamma^0 Q\lambda \} \times (1 - \gamma^5) \right\} + \left( A_0 \frac{\vec{\xi}_n}{E_\nu} + a_0 \frac{\vec{\xi}_n \cdot \vec{\sigma}}{E_\nu} \right) \cdot \delta \{ (k_e + m_e\gamma^5 \zeta_e)Q\lambda \gamma^0 Q\lambda \} \times (1 - \gamma^5) \right\} \}

\]

(B-5)
The traces over Dirac matrices in Eq. (B-5) were calculated in the covariant form in \[16, 19\]. The result is
\[
\begin{align*}
\frac{1}{16} \text{tr} \left[ a Q_{\lambda} \gamma^\mu Q_{\lambda} (1 - \gamma^5) \right] &= (\epsilon_+^\lambda \cdot k_e) (\epsilon_+ \cdot k_e) a^\mu + \frac{1}{2} \left( (\epsilon_+^\lambda \cdot k_e) (\epsilon_+ \cdot a) + (\epsilon_+^\lambda \cdot a) (\epsilon_+ \cdot k_e) - (\epsilon_+^\lambda \cdot \epsilon_+) (a \cdot q) \right) g^\mu
\end{align*}
\]

Plugging Eq. (B-9) into Eq. (B-1) we obtain the electron-energy and angular distribution for a polarized neutron, a

where \(a = k_e\) or \(a = \zeta\), respectively. As a result, for the r.h.s. of Eq. (B-6) we obtain the following expression
\[
\sum_{\text{pol}} \frac{|M(n \rightarrow p e^{-})|)^2}{(1 + 3 g_0^2) G_e^2} = \frac{1}{(E_e - n \cdot k_e)^2} \left[ (1 + B_0 \frac{E_n - k_e}{E_p}) \left\{ (\epsilon_+^\lambda \cdot k_e) (\epsilon_+ \cdot k_e) (1 + \frac{\omega}{E_e}) + \frac{1}{2} (\epsilon_+^\lambda \cdot k_e) (\epsilon_+ \cdot a) + (\epsilon_+^\lambda \cdot a) (\epsilon_+ \cdot k_e) - (\epsilon_+^\lambda \cdot \epsilon_+) (a \cdot q) \right] \right]
\]

(B-6)

In the physical gauge \(\epsilon_+ = (0, \epsilon_+^\lambda)\) \[16, 18, 19, 57\], where the polarization vector \(\epsilon_+^\lambda\) obeys the constraints
\[
\bar{q} \cdot \epsilon_+^\lambda = \bar{q} \cdot \epsilon_+ = 0, \quad \epsilon_+^\lambda \cdot \epsilon_+ = \delta_{\lambda \lambda'}, \quad \sum_{\lambda=1,2} \epsilon_+^\lambda \epsilon_+^\lambda = \delta_{ij} - \frac{k_i k_j}{\omega^2} = \delta_{ij} - n_i n_j, \quad \sum_{j=1,2,3} \epsilon_+^\lambda \epsilon_+^\lambda = 2. \quad (B-8)
\]

In the physical gauge \(\epsilon_+ = (0, \epsilon_+^\lambda)\) we obtain for the r.h.s. of Eq. (B-7) the following expression
\[
\sum_{\text{pol}} \frac{|M(n \rightarrow p e^{-})|)^2}{(1 + 3 g_0^2) G_e^2} = \frac{1}{(E_e - n \cdot k_e)^2} \left[ (1 + B_0 \frac{E_n - k_e}{E_p}) \left\{ (\epsilon_+^\lambda \cdot k_e) (\epsilon_+ \cdot k_e) (1 + \frac{\omega}{E_e}) + \frac{1}{2} (\epsilon_+^\lambda \cdot \epsilon_+) \right\} \right]
\]

(B-9)

Plugging Eq. (B-9) into Eq. (B-1) we obtain the electron-energy and angular distribution for a polarized neutron, a polarized electron, an unpolarized proton and a polarized photon:
\[
\frac{d^8 \lambda_{\gamma^5}}{d\omega dE_e dA \Omega_e} = (1 + 3 g_0^2) \frac{\alpha}{\pi} \frac{|G_e|^2}{2 (2\pi)^2} \sqrt{E_e^2 - m_e^2} F(E_e, Z = 1) (E_0 - E_e - \omega)^2 \frac{1}{\omega (E_e - n \cdot k_e)^2}
\]

\[
\times \left( \left\{ (1 + B_0 \frac{E_n - k_e}{E_p}) \left\{ (\epsilon_+^\lambda \cdot k_e) (\epsilon_+ \cdot k_e) (1 + \frac{\omega}{E_e}) + \frac{1}{2} (\epsilon_+^\lambda \cdot \epsilon_+) (k_e \cdot q) \right\} \right\} \right)
\]

(B-10)
Summing up over polarizations of the photon we get

\[
\frac{d^8\lambda_{\gamma^-\gamma}(E_e, \omega, k_\perp, \vec{q}, \vec{q}_\perp, \epsilon_e, \epsilon_\gamma)}{d\omega dE_e d\Omega_\gamma d\Omega_e} = (1 + 3g_\lambda^2) \frac{\alpha |G_V|^2}{16\pi^2} \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1)(E_0 - E_e - \omega)^2 \frac{1}{\omega}
\]

\[
\times \left\{ \left( 1 + B_0 \frac{\xi_\alpha \cdot k_\perp}{E_0} \right) \left( \frac{1}{E_0} \right)^2 \left[ \frac{(1 + \omega_\perp + 1/2 E_e^2)}{2 E_e^2} \right] \frac{1}{E_0} \left[ \frac{(1 + \omega_\perp + 1/2 E_e^2)}{2 E_e^2} \right] - m_e \left[ \frac{(1 + \omega_\perp + 1/2 E_e^2)}{E_0} \right] \left( \frac{(1 + \omega_\perp + 1/2 E_e^2)}{E_0} \right)
\right\} + \frac{m_e \left[ \frac{(1 + \omega_\perp + 1/2 E_e^2)}{E_0} \right] \left( \frac{(1 + \omega_\perp + 1/2 E_e^2)}{E_0} \right) - \omega^2 E_e E_0}{E_e^2 (1 - \omega_\perp - \epsilon_\gamma)^2} \right\} \left( \frac{1}{E_e^2 (1 - \omega_\perp - \epsilon_\gamma)^2} \right)
\]

\[
\left( \frac{1}{E_e^2 (1 - \omega_\perp - \epsilon_\gamma)^2} \right)
\]

where we have used that \( \vec{\beta} \cdot \vec{\epsilon} = \epsilon_\gamma^0 \). The next step is to average over directions of the 3-momentum \( \vec{q} = \omega \vec{n} \) of the real photon. This gives

\[
\frac{d^8\lambda_{\gamma^-\gamma}(E_e, \omega, k_\perp, \vec{q}, \vec{q}_\perp, \epsilon_e, \epsilon_\gamma)}{d\omega dE_e d\Omega_\gamma d\Omega_e} = (1 + 3g_\lambda^2) \frac{\alpha |G_V|^2}{16\pi^2} \sqrt{E_e^2 - m_e^2} E_e F(E_e, Z = 1)(E_0 - E_e - \omega)^2 \frac{1}{\omega}
\]

\[
\times \left\{ \left( 1 + B_0 \frac{\xi_\alpha \cdot k_\perp}{E_0} \right) \left( \frac{1}{E_0} \right)^2 \left[ \frac{(1 + \omega_\perp + 1/2 E_e^2)}{2 E_e^2} \right] \frac{1}{E_0} \left[ \frac{(1 + \omega_\perp + 1/2 E_e^2)}{2 E_e^2} \right] - m_e \left[ \frac{(1 + \omega_\perp + 1/2 E_e^2)}{E_0} \right] \left( \frac{(1 + \omega_\perp + 1/2 E_e^2)}{E_0} \right)
\right\} + \frac{m_e \left[ \frac{(1 + \omega_\perp + 1/2 E_e^2)}{E_0} \right] \left( \frac{(1 + \omega_\perp + 1/2 E_e^2)}{E_0} \right) - \omega^2 E_e E_0}{E_e^2 (1 - \omega_\perp - \epsilon_\gamma)^2} \right\} \left( \frac{1}{E_e^2 (1 - \omega_\perp - \epsilon_\gamma)^2} \right)
\]

\[
\left( \frac{1}{E_e^2 (1 - \omega_\perp - \epsilon_\gamma)^2} \right)
\]
\[ \times \left[ \frac{1}{\beta} \ln \left( \frac{1+\beta}{1-\beta} \right) - 2 \right] - 2 \right) \} + \left( -1 \right) a_0 \frac{\vec{\xi}_n \cdot \vec{\theta}_{\nu}}{(E_{e} + m_{e})E_{e}E_{\nu}} \left\{ \frac{1}{\omega} \left( 1 - \frac{\omega^2}{2\beta^2 E_{e}^2} \right) \frac{1}{\beta} \ln \left( \frac{1+\beta}{1-\beta} \right) - 2 \right] \]

\[ + (1 + \sqrt{1 - \beta^2}) \left[ \frac{1}{\beta^2 E_{e}} \left\{ \frac{1}{\beta} \ln \left( \frac{1+\beta}{1-\beta} \right) - 2 \right\} + \frac{1}{2\beta^2 E_{e}} \left( \frac{3 - \beta^2}{\beta^2} \frac{1}{\beta} \ln \left( \frac{1+\beta}{1-\beta} \right) - 2 \right) \right] \]

\[ - B_0 \frac{\vec{\xi}_n \cdot \vec{k}_{\nu}}{E_{e}E_{\nu}} \frac{1}{\omega} \left( 1 + \frac{1}{\beta^2 E_{e}} + \frac{1}{2\beta^2 E_{e}^2} \right) \left\{ \frac{1}{\beta} \ln \left( \frac{1+\beta}{1-\beta} \right) - 2 \right\} \}. \] (B-14)

It is seen that the terms with the correlation structures \((\vec{\xi}_n \cdot \vec{\theta}_{\nu})(\vec{k}_{e} \cdot \vec{k}_{\nu})\) and \((\vec{\xi}_n \cdot \vec{k}_{\nu})(\vec{k}_{e} \cdot \vec{\theta}_{\nu})\), inducing the correlation coefficients \(S(E_{e})\) and \(U(E_{e})\), respectively, do not appear in the electron-energy and angular distribution of the neutron radiative beta decay for polarized neutrons, polarized electrons, unpolarized protons and unpolarized photons. This confirms the results, obtained in Appendix A, that there are no contributions of the radiative corrections of order \(O(\alpha/\pi)\), caused by one-virtual photon exchanges, to the correlation coefficients \(S(E_{e})\) and \(U(E_{e})\), respectively.
Some considerations on the radiative corrections to muon and neutron decay
Framework for maximum likelihood analysis of neutron beta decay observables to resolve the

Charge conjugation, a new quantum number

Sharpening low-energy, Standard-Model tests via correlation coefficients in neutron beta decay

Tests of the standard electroweak model in nuclear beta decay

Tests of the standard model in neutron beta decay: 2020 critical survey, with implications for V_{ud} and CKM unitarity

Improved limits on Fierz interference using asymmetry measurements from the ultracold neutron asymmetry (UCNA) experiment (UCNA Collaboration)

Beta-decay correlations in the LHC era

Precision analysis of pseudoscalar interactions in neutron beta decays

Beta decay beyond the standard model

Tests of the standard electroweak model in nuclear beta decay

Radiative corrections to Fermi interactions

Some considerations on the radiative corrections to muon and neutron decay

Tests of the standard model in neutron beta decay with polarized neutron and electron and an unpolarized proton

Round Table: Resolving Physics BSM at Low Energies, PoS ConfinementX (2012) 024
[50] V. Cirigliano, M. González-Alonso, M. L. Graesser, *Non-standard charged current interactions: beta decays versus the LHC*, JHEP **2012**, 25 (2012); DOI: https://doi.org/10.1007/JHEP10(2012)025.

[51] V. Cirigliano and M. J. Ramsey-Musolf, *Low energy probes of physics beyond the Standard Model*, Prog. Part. Nucl. Phys. **71**, 2 (2013); DOI: https://doi.org/10.1016/j.ppnp.2013.03.002.

[52] V. Cirigliano, S. Gardner, and B. Holstein, *Beta decays and non-standard interactions in the LHC era*, Prog. Part. Nucl. Phys. **71**, 93 (2013); DOI: https://doi.org/10.1016/j.ppnp.2013.03.005.

[53] J. C. Hardy and I. S. Towner, *Superallowed 0^+ \rightarrow 0^+ nuclear beta decays: A new survey with precision tests of the conserved vector current hypothesis and the standard model*, Phys. Rev. C **79**, 055502 (2009); DOI: https://doi.org/10.1103/PhysRevC.79.055502.

[54] J. M. Blatt and V. F. Weisskopf, *Theoretical nuclear physics*, John Wiley & Sons, New York 1952.

[55] A. Antognini et al., *Proton structure from the measurement of 2S-2P transition frequencies of muonic hydrogen*, Science **339**, 417 (2013); DOI: 10.1126/science.1230016.

[56] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov, *Corrections of order O(E^2_e/m^2_N)*, caused by weak magnetism and proton recoil, to the neutron lifetime and correlation coefficients of the neutron beta decay, *Results in Physics* **21**, 103806 (2021); DOI: https://doi.org/10.1016/j.rinp.2020.103806; arXiv: 2010.14336 [hep-ph].

[57] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov, *Precision theoretical analysis of neutron radiative beta decay to order O(α^2/π^2)*, Phys. Rev. D **95**, 113006 (2017); DOI: https://doi.org/10.1103/PhysRevD.95.113006.