ON THE SPECTRAL SHAPE OF RADIATION DUE TO INVERSE COMPTON SCATTERING CLOSE TO THE MAXIMUM CUTOFF

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ABSTRACT

The spectral shape of radiation due to inverse Compton scattering is analyzed in the Thomson and the Klein–Nishina regime for electron distributions with exponential cutoff. We derive analytical, asymptotic expressions for the spectrum close to the maximum cutoff region. We consider monoenergetic, Planckian, and synchrotron photons as target photon fields. These approximations provide a direct link between the distribution of parent electrons and the upscattered spectrum at the cutoff region.

Key words: gamma rays: galaxies – radiation mechanisms: non-thermal – scattering

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1. INTRODUCTION

The interaction of relativistic electrons with low-energy radiation through inverse Compton scattering (ICS) provides one of the principal γ-ray production processes in astrophysics. In a variety of astrophysical environments, from very compact objects like pulsars and active galactic nuclei to extended sources like supernova remnants and clusters of galaxies, low-energy photons are effectively boosted to high energies through this mechanism.

The basic features of the ICS have been analyzed by Jones (1968) and Blumenthal & Gould (1970). The case of anisotropic electrons and/or photons has been studied by Aharonian & Atoyan (1981), Narginen & Putanen (1993), Brunetti (2000), and Sazonov & Sunyaev (2000). The impact of the Klein–Nishina effect on the formation of the energy distribution of electrons was first realized by Blumenthal (1971). Its importance in astrophysics has been discussed in the literature in the context of different non-thermal phenomena, in particular, by Aharonian & Ambartsumyan (1985), Zdziarski et al. (1989), Dermer & Atoyan (2002), Moderski et al. (2005), Khangulyan & Aharonian (2005), Kusunose & Takahara (2005), and Stawarz et al. (2006, 2010).

Generally, the energy spectrum as well as the effects related to ICS is numerically calculated using the exact expression for the Compton cross-section. On the other hand, compact, analytical approximations can serve as useful tools for a deeper understanding of the properties of Compton radiation and the implications of the complex numerical calculations. In particular, inferring the energy distribution of the parent particles from the observed spectrum is a much more efficient procedure when analytical approximations are available. For example, a power-law distribution of electrons normally results in power-law photon spectra. If the observed photon index is Γ (in a dNγ/dEγ ∝ E−Γ representation), then we can obtain the power-law index of the electron distribution dN_e/dE_e ∝ E−Γ_e from the relation Γ_e = 2Γ − 1 for the Thomson regime and Γ_e ≈ Γ − 1 if the scattering occurs in the Klein–Nishina regime.

This, however, only applies to the energy interval far from the cutoff, the “main” part of the electron distribution. At the highest (and the lowest) energies, there should be a break/cutoff in the electron distribution, and in fact, the corresponding break at the radiation spectrum contains a lot of interesting information on the parent electrons. In particular, the peaks in the spectral energy distribution (SED) appear at this energy range in the majority of cases, indicating that the source luminosity is mostly radiated at the maximum cutoff. Moreover, as the main, power-law part of the distribution, the shape of the cutoff carries as well important implications for the acceleration of the particles and in general the mechanisms acting in the source. Although the shape of the spectrum close to the highest energy cutoff is critical, this topic has not yet been adequately addressed. In this paper, we examine the shape of the Compton spectrum close to the maximum cutoff and we derive convenient analytical formulas that allow us to approximate the radiated flux in this specific energy range.

In general, the shape of the electron distribution around the cutoff can be expressed as an exponential, exp[−(E_e/E_c)^Γ]. This allows us to describe a quite broad range of distributions, even very sharp, abrupt, step-function like cutoffs for β ≫ 1. Apart from the convenience of such a mathematical description, exponential cutoffs naturally arise in theoretical considerations. For example, in diffusive shock acceleration, power-law particle distributions with exponential cutoff are formed when (synchrotron) energy losses are taken into account (Webb et al. 1984) and the cutoff index is very important for investigating the acceleration mechanism. Analytic solutions show that in the case of Bohm diffusion a simple exponential cutoff exp[−E_e/E_c] arises, whereas the index approaches β = 2 if E_e ≈ E_c type energy losses are taken into account, e.g., synchrotron or Thomson losses (see Zirakashvili & Aharonian 2007).

In stochastic acceleration scenarios, where pile-up particle distributions are formed when acceleration is balanced by synchrotron-type losses, the shape of the electron cutoff is directly related to the assumed turbulent wave spectrum (Schlickeiser 1985; Aharonian et al. 1986), e.g., β = 5/3 for Kolmogorov, β = 3/2 for Kraichnan-like, or β = 2 for the hard sphere approximation. Of course, if more complicated energy losses dominate, like in the case of Klein–Nishina losses in radiation-dominated environments, more complex shapes for...
the electron distribution cutoff may be expected in both stochastic and diffusive shock acceleration scenarios (e.g., Stawarz & Petrov 2008; Vannoni et al. 2009).

Nevertheless, it seems reasonable to consider particle distributions that exhibit exponential cutoff in a general form, for investigating and modeling the radiated spectra. In Fritz (1989) and Zirakashvili & Aharonian (2007), the shape of the synchrotron spectrum close to the high-energy cutoff has been discussed. They found that when the electron distribution possesses an exponential cutoff of index $\beta$, $\exp[-(E_e/E_c)\beta]$, then the radiated synchrotron spectrum exhibits a smoother cutoff, of index $\beta/2$. Apart from the practical importance, this analytic result demonstrates that a $\delta$-function approximation for the synchrotron radiation emissivity does not give the correct result.

Here we examine the corresponding Compton spectrum, in the Thomson and Klein–Nishina regimes, considering different target photon fields so that both synchrotron self-Compton (SSC) and external Compton (EC) scattering can be addressed. We derive analytically the asymptotic behavior of the upscattered photon distribution close to the cutoff region. We consider a general electron energy distribution of the form

$$\frac{dN_e}{dE_e} = F_e(E_e) = AE_e^\alpha e^{-\frac{E_e}{E_{\gamma}}}$$ (1)

where $E_e = \gamma mc^2$ is the electron energy and $E_e = \gamma mc^2$ is the cutoff energy. This presentation allows us to consider either a power-law distribution (for $\alpha = -2\alpha$) with exponential cutoff or a relativistic Maxwell-like distribution (for $\alpha = 2$) that may be formed in stochastic acceleration scenarios. We consider monochromatic, Planckian, and synchrotron photons as target photon fields. The resulting inverse Compton (IC) spectral shape is discussed for the Thomson and the Klein–Nishina regime and we show that is not always identical to the synchrotron spectrum, as is often silently assumed. Finally, we discuss basic features and physical properties of the radiated spectrum.

2. COMPTON SPECTRUM FOR MONOCHROMATIC PHOTONS

In this section, we calculate the asymptotic behavior that the Compton spectrum exhibits close to the maximum cutoff, when monochromatic photons (with isotropic angular distribution) are upscattered. Let us consider the general function of Equation 1 that describes the differential number of electrons. Electrons are considered to be isotropically and homogeneously distributed in space. Then the spectrum of photons generated per unit time due to ICS is (see, e.g., Blumenthal & Gould 1970)

$$\frac{d\dot{N}_\gamma}{dE_\gamma} = \int_0^\infty \int_{E_{\gamma}}^\infty W(E_e, \epsilon, E_\gamma) F_e(E_e) n_{ph}(\epsilon) dE_e d\epsilon,$$

where

$$E_{\gamma\min} = \frac{1}{2} \frac{E_\gamma}{\epsilon_\gamma} \left(1 + \sqrt{1 + \frac{m^2c^4}{\epsilon_\gamma E_\gamma}}\right),$$

$$W(E_e, \epsilon, E_\gamma) = \frac{8\pi \gamma^2 c}{E_\gamma \eta} \times \left[2q \ln q + (1 - q) \left(1 + 2q + \frac{\eta^2 q^2}{2(1 + \eta q)}\right)\right]$$

and

$$\eta = \frac{4\epsilon_e E_e}{m^2c^2}, \quad \frac{q}{\eta} = \frac{E_\gamma}{(E_e - E_\gamma)},$$

Here the function $W(E_e, \epsilon, E_\gamma)$ in Equation 4 describes the total scattering probability, taking into account Klein–Nishina effects. The parameter $\eta$ in Equation 5 defines the domain of the scattering. For $\eta \ll 1$ the Thomson regime applies whereas for $\eta \gg 1$ we are in the Klein–Nishina regime. In the case of monochromatic photons, the number density is $n_{ph}(\epsilon_\gamma) = n_0(\epsilon_\gamma - \epsilon_0)$ and we will set $n_0$ equal to 1 for this case. From now on we set $mc^2 = 1$ throughout the calculations for simplicity, apart from the formulas at which the results are demonstrated. The case of $\alpha < 0$ will be referred to as power-law distribution, whereas $\alpha = 2$ will be referred to as Maxwellian distribution.

2.1. Thomson Regime

In the limiting case of $4\epsilon_0 E_e \ll 1$, when all the scatterings occur in the Thomson regime, the photons take a small fraction of the electron energy. Thus, from the previous relation, it follows that $E_\gamma \ll 1/(4\epsilon_0)$ and the lower limit of the integration becomes

$$E_\gamma \ll E_{\gamma\min} = \sqrt{E_\gamma/(4\epsilon_0)} \gg E_\gamma.$$ (6)

Therefore, in this case $\eta q = E_\gamma/E_e \ll 1$ and Equation 4 can be written as

$$W(E_e, \epsilon_0, E_\gamma) = \frac{8\pi \gamma^2 c}{E_\gamma \max} \left[2q \ln q + (1 - q)(1 + 2q)\right],$$

and in this approximation $q = E_\gamma/E_\gamma \max$, where $E_\gamma \max = 4\epsilon_0 E_e^2$. Using the electron distribution of Equation 1 and changing the integration variable from $E_e$ to $q$, the integral of Equation 2 for the Compton spectrum becomes

$$\frac{d\dot{N}_\gamma}{dE_\gamma} = \int_0^1 \frac{1}{2\pi} \frac{r^2 c}{\epsilon_0^2} \frac{e^{-\frac{\eta}{2\epsilon_\gamma}} \eta}{E_\gamma} \frac{\eta}{E_\gamma} e^{-\frac{\eta}{E_\gamma}} q^{-\frac{\eta}{E_\gamma}} \times \left[2q \ln q + (1 - q)(1 + 2q)\right] dq,$$

where

$$\xi = \left(\frac{E_\gamma}{4\epsilon_0 E_e^2}\right)^{\frac{\beta}{2}}.$$ (9)

We are interested in the behavior of the spectrum near the exponential cutoff, i.e., $E_\gamma \gg 4\epsilon_0 E_e^2$ or $\xi \gg 1$. Then the integrand is dominated by values of $q$ very close to unity, and in order to perform the above integration it is convenient to change again variables to $\tau = q^{-\beta/2}$ so that

$$\frac{d\dot{N}_\gamma}{dE_\gamma} = \int_1^\infty \frac{2\pi r^2 c}{2\pi} \frac{e^{-\eta q}}{\epsilon_0^2} \frac{E_\gamma}{E_\gamma} f(\tau) e^{-\xi} d\tau,$$

where

$$f(\tau) = 2\frac{4\pi \tau^{-\frac{\beta+1}{2}} \ln \tau - \beta \tau^{\frac{\beta-1}{2}} - \beta \tau^{\frac{\beta-3}{2}} + 2\beta \tau^{\frac{\beta+1}{2}}}{\beta^2 \tau^2}.$$ (11)

The function $f(\tau)$ is also dominated by values of $\tau$ close to unity. Thus, we expand $f(\tau)$ in series around $\tau = 1$. By keeping terms up to first order, the resulting spectrum reads

$$\frac{d\dot{N}_\gamma}{dE_\gamma} = \frac{8\pi r^2 c A(m^2 c^2)^{\frac{\beta+1}{2}}}{\beta^2 \xi^2} \frac{\epsilon_0}{\epsilon_\gamma} \frac{E_\gamma}{E_e} e^{-\xi},$$

for $E_\gamma \gg \epsilon_\gamma E_e^2$.

$$2.$$
Equation (10) directly to obtain the result
\[
\frac{d\hat{N}_\gamma}{dE_\gamma} \bigg|_T \propto \exp \left[ -\left( \frac{E_\gamma (mc^2)^2}{4\epsilon_0 E_e^2} \right)^{\beta/2} \right].
\]

As expected, the cutoff in the photon spectrum, \(4\epsilon_0 E_e^2/(mc^2)^2\), corresponds to the maximum photon energy that an electron of energy \(E_e\) can radiate in the Thomson regime. We also note that in this case, an abrupt cutoff (\(\beta \to \infty\)) of the electron energy distribution would correspond to an abrupt cutoff of the photon spectrum. In Figures 1 and 2 the analytic formula of Equation (12) and the full, numerical spectrum are plotted, for a Maxwellian and a power-law distribution, respectively. Asymptotics are better for \(\alpha = 2\). For the power-law distribution, the numerical and analytical solution converge for very large values of the parameter \(\xi\). In both cases, very close to the cutoff energy, the numerical spectrum is smoother than the approximated one.

Finally, for a “pure” power-law distribution without exponential cutoff (\(\alpha\) negative and \(\beta = 0\)) one can integrate Equation (10) directly to obtain the result
\[
\frac{d\hat{N}_\gamma}{dE_\gamma} \bigg|_T = \frac{2\pi r^2 c A(m^2c^2)^3}{2\xi_0} \frac{1}{2\pi} \frac{4(\alpha^2 - 4\alpha + 11)}{(\alpha - 3)^2(\alpha^2 - 6\alpha + 5)}, \quad \text{if } \alpha < 3,
\]

that demonstrates that in the Thomson regime the radiated spectrum follows a power law of the form \(d\hat{N}_\gamma/dE_\gamma \propto E_\gamma^{\frac{-\alpha}{2}}\).

2.2. Klein–Nishina Regime

When \(\eta = 4\epsilon_0 E_e \gg 1\), photons take almost all the energy of the electrons in one scattering. Then, we may define the parameter \(\xi \equiv 4\epsilon_0 E_e\) and the low limit of the integration in Equation (3) becomes \(E_{\text{emin}} \approx E_\gamma\). Let us change variables to \(x = E_\gamma / E_e\). As \(\xi \gg 1\), we keep only the leading terms
\[
\frac{d\hat{N}_\gamma}{dE_\gamma} = \frac{8\pi r^2 c}{\xi} \int_0^{1-1/\xi} \left[ 1 + x^2 \frac{\beta}{2(1-x)} \right] F_\gamma(E_\gamma/x) dx.
\]

Here we have approximated the upper limit of the integration by \(1 - 1/\xi\) because the integral in Equation (15) diverges logarithmically at \(x = 1\). For an electron distribution with exponential cutoff, the main contribution to the integral comes from regions of \(x\) close to unity, so that \(1 \ll x^{-1/\alpha} - 1\) and we can neglect the first term from the expression in the brackets,
\[
\frac{d\hat{N}_\gamma}{dE_\gamma} = \frac{4\pi r^2 c}{\xi} \left( \frac{4\epsilon_0 E_e}{(mc^2)^2} \right)^{1/2} \int_0^{1-1/\xi} \frac{\chi^2 dx}{\left( 1 - x \right)^{1/2}} F_\gamma \left( \frac{E_\gamma}{x} \right).
\]

Now the integral of Equation (16) can be calculated for energies close to the cutoff region, \(E_\gamma \gg E_e\), leading to a spectrum of the form
\[
\frac{d\hat{N}_\gamma}{dE_\gamma} \bigg|_{\text{KN}} = \frac{\pi r^2 c (mc^2)^2}{\epsilon_0 E_e} \left( \frac{4\epsilon_0 E_e}{(mc^2)^2} \right)^{1/2} \left[ \ln \left( \frac{4\epsilon_0 E_e}{(mc^2)^2} \right) - \ln \beta - \ln \left( \frac{E_\gamma}{E_e} \right) - \gamma \right] \times F_\gamma(E_\gamma), \quad \text{for } E_\gamma \gg E_e,
\]

where \(\gamma = 0.5772\) is the Euler’s constant. In the Klein–Nishina regime where electrons lose almost all their energy in each scattering, the Compton spectrum practically reflects the behavior of the electron distribution. Thus, in this case, the exponential...
cutoff maintains the index $\beta$ and is steeper than in the Thomson case,

$$
\frac{dN_\gamma}{dE_\gamma} \bigg|_{\text{KN}} \propto \exp \left[ -\left( \frac{E_\gamma}{E_c} \right)^{\beta} \right].
$$

As before, the photon cutoff energy corresponds to the maximum photon energy that electrons of energy $E_c$ radiate in the Klein–Nishina regime. Moreover, an abrupt electron distribution cutoff would result in an abrupt photon spectrum cutoff. The asymptotics of Equation (17) are plotted in Figures 3 and 4; as can be seen, they provide a good approximation just after the peak of the SED.

The fact that the exponential index becomes $\beta/2$ and $\beta$ in the Thomson and the Klein–Nishina regime, respectively, indicates that using a $\delta$-function approximation for the cross-section provides a correct result for the calculated spectrum in these two regimes (for an extended discussion on the applicability of $\delta$-function approximation; see Coppi & Blandford 1990). Obviously this is not true for values of $\eta_c$ close to unity (see Figure 5), where $\eta_c = 4\epsilon_0 E_c$ refers to the electron cutoff energy. As we are interested in the highest energy part of the spectrum, the approximation in the Klein–Nishina regime is satisfactory even for values of $\eta_c$ that do not significantly exceed unity. This happens because since $\eta_c > 1$ for the electron cutoff energy it holds for all the energies $E_\gamma > E_c$ that actually form the shape of the exponential cutoff. On the contrary, for the Thomson regime one needs all the radiated photons above the cutoff to be emitted at this regime, which indicates rather small values of $\eta_c$ for the approximation to be good, especially for low $\beta$ factors.

A similar calculation can be performed for a “pure” power-law electron distribution, with $\beta = 0$. If we rewrite the second term of Equation (15) in the form

$$
\int_0^{1-1/\xi} x^2 \frac{x^{\alpha}}{1-x} F_\epsilon(E_\gamma/x) \, dx = F_\epsilon(E_\gamma) \times \int_0^{1-1/\xi} x^2 \, dx + F_\epsilon(E_\gamma) \int_0^1 \frac{(x^{2\alpha} - 1) \, dx}{1-x},
$$

we can perform the above integration resulting in an emitted spectrum of the form

$$
\frac{dN_\gamma}{dE_\gamma} \bigg|_{\text{KN}} = \frac{\pi r^2 c A (m c^2)^2}{\epsilon_0} E_\gamma^{\alpha-1} \times \left[ \ln \left( \frac{4\epsilon_0 E_\gamma}{(m c^2)^2} \right) - 3\alpha^2 + 15\alpha + 14 \right. \left. \frac{4\alpha(\alpha + 1)(\alpha + 2)}{4(\alpha + 1)(\alpha + 2)} - \Psi(\alpha) \right],
$$

where $\Psi(\alpha)$ is the digamma function defined as the logarithmic derivative of the $\Gamma$ function, $\Psi(\alpha) = \Gamma'(\alpha)/\Gamma(\alpha)$. This formula shows that the emitted Compton spectrum is much steeper in this than in the Thomson regime due to the suppression of the cross-section. The functional dependence of Equation (20)

$$
\frac{dN_\gamma}{dE_\gamma} \bigg|_{\text{KN}} \propto E^{\alpha-1}
$$

has been obtained in Blumenthal & Gould (1970), see, e.g., their Equation (2.87), and in Aharonian & Atoyan (1981), see their Equation (32). These formulas differ in the term related to the power-law index $\alpha$, due to the different approach used for the calculation of the asymptotics. This difference is negligible.

### 3. Compton Spectrum for a Broad Photon Distribution

Once we have calculated the radiated spectrum for monochromatic photons, we can examine the behavior of the Compton spectrum for various photon fields. The case of monochromatic photons is important for understanding the scattering mechanism and a necessary step for further calculations. However, in nature the photon fields are usually broader than the monochromatic one, except if we deal with emission lines. Here we will consider a Planckian photon distribution which is often the case in EC scenarios and we will examine as well the case of synchrotron photons from the same parent electron distribution that are used as the target photon field in SSC models.
3.1. Planckian Photon Field

Let us assume a Planckian distribution for the photon field so that the differential number density is given by

$$n_{\text{ph}}(\epsilon_{\gamma}) = \frac{1}{\pi^{2/3} \hbar^{3/2} c} \frac{\epsilon_{\gamma}^{2}}{\epsilon_{\gamma}^{3} + \frac{\epsilon_{\gamma}^{2}}{kT} - 1}. \quad (22)$$

For the Thomson regime, where now we demand $4kT_e \ll 1$, we can use the Wien limit ($\epsilon_{\gamma} \gg kT$) at which

$$n_{\text{ph}}(\epsilon_{\gamma}) = \frac{1}{\pi^{2/3} \hbar^{3/2} c} \epsilon_{\gamma}^{2} e^{-\epsilon_{\gamma}/kT}. \quad (23)$$

This is acceptable as the asymptotic behavior of the Compton spectrum at high energies is mostly defined by the soft photons with energy around and greater than $kT$.

In that case the integration of the spectrum equation (12) over the photon energies can be evaluated by the saddle point method of integration. After replacing $\epsilon_{\gamma}$ with $\epsilon_{\gamma}$ in Equation (12), the integral is written as

$$\frac{d\dot{N}_{\gamma}}{dE_{\gamma}} \bigg|_{\text{BB}} = \int_{0}^{\infty} \frac{d\dot{N}_{\gamma}}{dE_{\gamma}} \bigg|_{\text{BB}} n_{\text{ph}}(\epsilon_{\gamma}) d\epsilon_{\gamma}$$

$$= \int_{0}^{\infty} g(E_{\gamma}, \epsilon_{\gamma}) e^{S(E_{\gamma}, \epsilon_{\gamma})} d\epsilon_{\gamma}, \quad (24)$$

where

$$g(E_{\gamma}, \epsilon_{\gamma}) = \frac{8\pi c^{2} A}{2\pi^{2} \hbar^{3} c^{2}} \frac{E_{\gamma}^{\nu+1} \epsilon_{\gamma}^{-\nu}}{\beta^{2} \xi^{2}}. \quad (25)$$
and

\[ S(E_\gamma, \epsilon_\gamma) = -\left( \frac{E_\gamma}{4\epsilon_\gamma E_c^2} \right)^{\beta/2} \frac{\epsilon_\gamma}{kT} \]  

(26)

Here BB stands for blackbody. Let us define the parameter \( \xi_1 \) that is related to the cutoff of the upscattered photon spectrum for the case of monochromatic photons, if we replace \( \epsilon_\gamma \) with \( kT \),

\[ \xi_1 \equiv \frac{E_\gamma}{4kT E_c^2}. \]  

(27)

This parameter simply describes the outgoing photon energy normalized to the maximum energy which \( E_c \) electrons radiate when they upscatter photons of energy \( kT \). One can use the saddle point method for the above integration because the integral at large energies, \( \xi_1 \gg 1 \), is determined by the soft photon energy interval around the energy \( x_0 \) which maximizes/minimizes the function \( S(E_\gamma, \epsilon_\gamma) \). The saddle point \( x_0 \) is at

\[ x_0 = \frac{\beta}{2} \left( \frac{2\xi_1}{\beta} \right)^{\frac{\beta}{2}} kT. \]  

(28)

Then, the Thomson spectrum for a Planckian photon distribution is calculated to be \( dN_\gamma/dE_\gamma = g(E_\gamma, \epsilon_\gamma) \exp[-S(E_\gamma, x_0)] \sqrt{2\pi/ - S''(E_\gamma, x_0)} \) where \( S'' \) is the second derivative of \( S \) at the saddle point \( x_0 \). After rearranging the terms, we retrieve the following expression:

\[
\left. \frac{dN_\gamma}{dE_\gamma} \right|_{\text{BB}} = \frac{4\pi r_e^2 A(kT)^{\frac{\beta}{2}}(mc^2)^{\frac{\beta+1}{2}}}{2^{\frac{\beta+1}{2}}\pi^{\frac{\beta+1}{2}}c^2} \\
\times \sqrt{\frac{\pi}{\beta+2} x_0^{-\frac{\beta+1}{2}} E_\gamma^{-\frac{\beta+1}{2}} \epsilon_\gamma^{\frac{\beta+1}{2}} e^{-\frac{\beta+1}{2} (\frac{\eta_0}{\pi})^2}}, \\
\text{for } E_\gamma \gg \frac{4kT E_c^2}{(mc^2)^2},
\]

(29)

Therefore, when the target photon field is a blackbody, the shape of the cutoff is affected by the soft photon distribution and the exponential cutoff in the Thomson spectrum is smoother in comparison to the monochromatic photons case. The index now becomes \( \beta/(\beta + 2) \) as

\[
\frac{dN_\gamma}{dE_\gamma} \bigg|_{\text{BB}} \propto \exp \left[ \frac{-\beta + 2}{2} \left( \frac{2E_\gamma (mc^2)^2}{kT E_c^2} \right)^{\frac{\beta}{2}} \right]. \]  

(30)

This exponential cutoff is always smooth (less than unity) and it becomes unity in the case of an abrupt electron distribution cutoff, as \( \lim_{\beta \to \infty} \beta/(\beta + 2) = 1 \). Interestingly, the Thomson spectrum for Planckian photons at high energies exhibits the same exponential cutoff shape as the synchrotron spectrum. This is the only case where the two components of the spectrum show the same behavior for arbitrary index \( \beta \). For the Maxwellian and power-law type distributions of electrons, Equation (30) is presented in Figure 6 and Figure 7, respectively.

In the Klein–Nishina regime, the shape of the exponential cutoff does not depend on the upscattered photon distribution but, as mentioned above, preserves the electron index \( \beta \). Integration of Equation (17) over photon energies (after replacing \( \epsilon_0 \) with \( \epsilon_\gamma \)) requires the calculation of the following integrals:

\[
\int_0^\infty \frac{1}{\epsilon_\gamma} n(\epsilon_\gamma)d\epsilon_\gamma = \frac{(kT)^2}{6\hbar c^3}, \]

(31)

\[
\int_0^\infty \frac{1}{\epsilon_\gamma} \ln(4\epsilon_\gamma E_c)d\epsilon_\gamma = \frac{(kT)^2}{6\hbar c^3}(\ln(4kT E_c) - 0.1472). \]

(32)
Then, the asymptotic behavior of the upscattered spectrum close to the cutoff follows the formula

\[
\frac{dN_{\gamma}}{dE_{\gamma}} \bigg|_{BB}^{KN} = \frac{\pi^3 r_e^2 \Lambda(KT)^2 (mc^2)^2}{\hbar^3 c^2} \frac{F_e(E_{\gamma})}{E_{\gamma}} \times \left[ \ln \frac{4KT E_{\gamma}}{(mc^2)^2} - \ln \beta - \beta \ln \frac{E_{\gamma}}{E_c} - 0.724 \right],
\]

for \( E_{\gamma} \gg E_c \).

(33)

Thus, in the Klein–Nishina regime the cutoff is always much sharper than in the Thomson regime. The spectra for Maxwellian and power-law distributions of electrons are shown in Figure 8 and Figure 9, respectively. For values of the index \( \beta = 1, 2, 3 \), the corresponding shape in Thomson regime becomes 1/3, 1/2, and 3/5, respectively, always less than unity. For an abrupt cutoff \( (\beta \to \infty) \), the Klein–Nishina spectrum appears sharp as well, while the Thomson spectrum exhibits a simple exponential cutoff.

### 3.2. Synchrotron Photon Field

In SSC models, the electrons upscatter the photon which they produced via synchrotron radiation. In contrast to EC models, we do not have an analytic expression for the target photon density. However, we can use an approximation for the synchrotron spectrum at energies around the synchrotron cutoff, i.e., for \( \epsilon_{\gamma} \gg bE_c^2 \), given that the main contribution to the scattering process at high energies comes from this energy range. Here \( b = 3qBh/4\pi mc^3 \).

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**Figure 6.** Asymptotic behavior of the Thomson spectrum at the cutoff region for Maxwell-like electrons upscattering Planckian photons. Parameters used are \( E_c = 10^2 \), \( KT = 10^{-6} \) so that \( \eta_c = 4KT E_c = 0.0004 \ll 1 \). The exponential cutoff possesses an index \( \beta/(\beta + 2) \).

(A color version of this figure is available in the online journal.)

**Figure 7.** Same as Figure 6, but for a power-law electron distribution.

(A color version of this figure is available in the online journal.)
Figure 8. Compton spectrum in the Klein–Nishina regime for Maxwellian electron and Planckian photons. $E_e = 10^8$, $Kk = 10^{-4}$, and $\eta_c = 4kT E_e = 400 \gg 1$. The spectrum shows a cutoff of index $\beta$, same as the electron energy distribution. (A color version of this figure is available in the online journal.)

Figure 9. Same as Figure 8, but for power-law electrons. (A color version of this figure is available in the online journal.)

3.3. Synchrotron Spectrum

In the case of chaotic magnetic fields, the synchrotron emissivity of an electron with energy $E_e$ is described by the equation

$$\frac{dN_\gamma}{d\epsilon_\gamma} \approx \frac{\sqrt{3}q^3 B}{mc^2 \epsilon_\gamma} G\left(\frac{\epsilon_\gamma}{\epsilon_s}\right),$$

(34)

where

$$\epsilon_s = b \gamma^2 \frac{3q B h}{4\pi mc} \frac{E_e^2}{(mc^2)^2}$$

(35)
is the “critical” energy for synchrotron emission. The synchrotron power $\epsilon_\gamma dN_\gamma/d\epsilon_\gamma$ peaks at $0.29\epsilon_s$ (e.g., Rybicki & Lightman 1979). The function $G(y)$ can be well approximated, with an accuracy better than 0.2% over the entire range of variable $y$, by the formula (Aharonian et al. 2010)

$$\tilde{G}(y) = G(y)e^{-y} = \frac{1.808 y^{1/3}}{\sqrt{1 + 3.4 y^{2/3}}} \times \frac{1 + 2.21 y^{2/3} + 0.347 y^{4/3}}{1 + 1.353 y^{2/3} + 0.217 y^{4/3}} e^{-y}.$$  

(36)

For large $y \gg 1$, the function $G(y)$ is approximately $G(y) \approx \sqrt{\pi/2}$. Let us define the parameter $\xi_2$ that defines the synchrotron photon energy normalized to the energy $\epsilon_s$ (Equation (35)) for the electrons with energy $E_e$,

$$\xi_2 = \epsilon_\gamma/\epsilon_0 = \frac{\epsilon_\gamma}{b E_e^2}.$$  

(37)
In order to calculate the emitted synchrotron spectrum, we need to integrate Equation (34) over the electron distribution given by Equation (1). By changing variables from $E_e$ to $y = \epsilon_\gamma/\epsilon_s$, we find

$$\frac{d\hat{N}_\gamma}{d\epsilon_\gamma}\bigg|_{\text{SYN}} = \int_0^\infty \frac{d\hat{N}_\gamma}{d\epsilon_\gamma} F_e(E_e)dE_e$$

$$= \frac{\sqrt{3}q^3 BA}{2mc^2\hbar \epsilon_\gamma} \left(\frac{\epsilon_\gamma}{b}\right)^{\frac{\beta+2}{\beta}} \int_0^\infty y^{\frac{\beta}{2} - \frac{1}{2}} G(y) e^{-y - \left(\frac{y}{\epsilon_0}\right)^{\beta/2}} dy.$$  \hspace{1cm} (38)

For $\xi_2 > 1$ the integration over $y$ reveals a saddle point at

$$y_0 = \frac{\beta}{2} \left(\frac{2\xi_2}{\beta}\right)^{\frac{\beta}{2}},$$ \hspace{1cm} (39)

so that finally the emitted synchrotron spectrum can be expressed as

$$\frac{\epsilon_\gamma d\hat{N}_\gamma}{d\epsilon_\gamma}\bigg|_{\text{SYN}} = \frac{\sqrt{3}q^3 B}{mc^2\hbar} \left(\frac{\epsilon_\gamma}{b}\right)^{\frac{\beta+2}{\beta}} A$$

$$\times \left(\frac{\epsilon_\gamma}{b}\right)^{\frac{\beta}{2}} y_0^{\frac{\beta}{2} - 1} G(y_0) e^{-2y_0^{\frac{\beta}{2}} \gamma_k}. \hspace{1cm} (40)$$

This formula indicates the shape of the exponential cutoff for synchrotron radiation,

$$\frac{\epsilon_\gamma d\hat{N}_\gamma}{d\epsilon_\gamma}\bigg|_{\text{SYN}} \propto \exp \left[-\frac{\beta + 2}{2} \left(\frac{2\epsilon_\gamma}{\beta \epsilon_0}\right)^{\beta/(\beta+2)}\right]. \hspace{1cm} (41)$$

We note that the $\beta/(\beta+2)$ index for the synchrotron exponential cutoff has already been found in Fritz (1989) and Zirakashvili & Aharonian (2007). With the above calculations we can moreover estimate critical aspects of the emitted spectrum, and in particular the cutoff energy of the emitted synchrotron spectrum (see Section 4). The above equations correspond to optically thin synchrotron sources when the synchrotron self-absorption can be ignored. This could be the case of even very compact and highly magnetized sources, as long as the synchrotron cutoff appears at optical and higher frequencies. One should also mention that the synchrotron spectrum is sensitive to inhomogeneities of the magnetic field (e.g., Katz-Stone & Rudnick 1994; Eilek et al. 1997). Obviously, the fluctuations of the magnetic field should have an impact on the synchrotron spectrum, namely, they will make the cutoff smoother and shifted toward higher frequencies (see, e.g., Eilek & Arendt 1996). In this regard, the ICS $\gamma$-ray spectrum is free of uncertainties related to the magnetic field distribution, except for realization of the SSC scenario in the Thomson limit.

Last, we should mention that using a $\delta$-function approximation for the synchrotron emissivity would result in an exponential cutoff of index $\beta/2$, whereas the correct value is $\beta/(2 + \beta)$. The synchrotron asymptotics are plotted in Figures 10 and 11, for Maxwellian and power-law electron distributions, respectively. For comparison, the Thomson spectrum for Planckian photons is shown as well.

### 3.4. SSC Spectrum

Now we can integrate the Compton spectrum for monochromatic photons over the photon energies for the synchrotron distribution. The differential photon number density (for a spherical source) is

$$n_{\text{ph}}(\epsilon_\gamma) = \frac{3}{4\pi R^3} \frac{R}{c} \frac{dN_{\gamma}}{d\epsilon_\gamma}.$$ \hspace{1cm} (42)

where $R$ is the source size. Let us first perform the calculations for the Thomson regime described by Equation (12) with $\epsilon_0 \rightarrow \epsilon_\gamma$. In that case the exponential factor in the integrant is $\exp[-S]$, where $S = \xi + (2 - \beta)\gamma_k/\beta$. The function $S$ has an extremum at the saddle point

$$z_0 = \frac{\beta}{2} \left(\frac{2}{\beta \epsilon_0}\right)^{\frac{\beta}{2}} \epsilon_0.$$ \hspace{1cm} (43)
where \( E_0 = 4bE_c^4 \) is the maximum photon energy that results from electrons with energy \( E_c \) when they upscatter synchrotron photons with energy \( \epsilon_0 \). The analogy with the case of Planckian soft photons is direct. Replacing the synchrotron characteristic energy \( bE_c^2 \) with \( kT \) results in the same saddle point.

Here, the second derivative of the exponential argument at the saddle point has a simple form

\[
S''(z_0) = \frac{\beta E_0}{2E_c \epsilon_0},
\]

so that the integration over the synchrotron number density gives

\[
\text{d}N_y \bigg|_{\text{SSC}} = \frac{3}{4\pi c R^2} \frac{\gamma^2 c A^2 \gamma (mc^2)^{\alpha + 1} \sqrt{3q^3 B}}{2^{a\sqrt{\beta(\beta + 2)}} h} \times \left( \frac{E_y}{b} \right)^{\frac{\gamma}{2}} \frac{G(\tilde{y}_0)}{\tilde{y}_0} - \tau - 3 e^{\frac{\gamma}{2\alpha} \tilde{y}_0},
\]

where \( \tilde{y}_0 \) is calculated at the saddle point

\[
\tilde{y}_0 = \frac{\beta}{2} \left[ \frac{2E_y}{bE_0} \right]^{\frac{1}{2\gamma}}, \quad E_0 = \frac{4bE_c^4}{(mc^2)^2}.
\]

In the case of SSC radiation, the electron distribution upscatters the synchrotron photon distribution with an exponential cutoff

\[
\exp\left[ -\left( \frac{E_y}{\epsilon_0} \right)^{\frac{1}{2\beta}} \right].
\]

The corresponding Compton flux at high energies exhibits a cutoff index \( \beta + 4 \) which is smoother than the seed synchrotron distribution,

\[
\frac{E_y dN_y}{dE_y} \bigg|_{\text{SSC}} \propto \exp\left[ -\frac{\beta + 4}{2} \left( \frac{2E_y}{bE_0} \right)^{\frac{1}{2\gamma}} \right].
\]

For \( \beta = 1, 2, 3 \), the corresponding values for the Thomson exponential index is \( 1/5, 1/3, 3/7 \), significantly less than the electron distribution index and different than the synchrotron case. If \( \beta \rightarrow \infty \), then the SSC spectrum shows a simple cutoff, like in the case of upscattering Planckian photons. The asymptotic formula of Equation (45) is plotted in Figures 12 and 13 for Maxwellian and power-law electrons.

In the Klein–Nishina regime, the integration over the synchrotron photon density cannot be performed analytically for arbitrary values of the indexes \( \alpha \) and \( \beta \). In this regime, however, the soft photon field does not play an important role in the shape of the upscattered spectrum close to the maximum cutoff. Thus, Equation (17) for monochromatic photons offers a rather good description of the asymptotic behavior of the SSC spectrum at the Klein–Nishina regime. See, e.g., Figures 14 and 15 for Maxwellian and power-law electrons, respectively. The analytic formula describing the asymptotes is Equation (17) normalized to the numerical solution, where the soft photon energy \( \epsilon_y \) has been replaced by \( \epsilon_z = bE_c^2 \).

4. DISCUSSION

The main results of this paper are summarized in Table 1. In the Klein–Nishina regime the upscattered Compton spectrum exhibits the same exponential cutoff index \( \beta_C \), as the electron distribution index \( \beta \), and does not depend strongly on the target photon field. This implies that from the \( \gamma \)-ray spectrum we practically “observe” the electron cutoff shape. In particular, an abrupt electron distribution cutoff \( (\beta \rightarrow \infty) \) would result in an abrupt cutoff for the photon spectrum \( (\beta \rightarrow \infty) \). The case of upscattering monochromatic photons is shown in Figures 3 and 4. The case of Planckian photons is plotted in Figures 8 and 9, whereas Figures 14 and 15 for Maxwellian and power-law electrons, respectively. The analytic formula of Equation (45) is plotted in Figures 12 and 13 for Maxwellian and power-law electrons.

On the contrary, in the Thomson regime the upscattered photon exponential shape is always smoother than the electron distribution cutoff shape. For monochromatic target photons, this is \( \beta_C = \beta/2 \), as shown in Figures 1 and 2. In this case, the shape of the cutoff in both the Thomson and Klein–Nishina regimes (Equations (13) and (18)) shows that using a \( \delta \)-function for the Compton emissivity provides the correct result. Here the asymptotic analytic expression of Equation (12) approaches better the numerical solution for a Maxwellian electron distribution.
Figure 12. SSC radiation at the cutoff region in Thomson regime. A Maxwellian electron distribution has been used, with parameters $E_c = 10^7$ and $B = 1$ G for the magnetic field. The exponential cutoff that arises is very smooth, of index $\beta/(\beta + 4)$.

(A color version of this figure is available in the online journal.)

Figure 13. Same as Figure 12 but power-law electrons. The asymptotics approach the numerical solution only for $E_\gamma \gg 4bE_{\text{cut}}^4$. Very close to the photon cutoff energy, the numerical spectrum is smoother than the approximated one.

(A color version of this figure is available in the online journal.)

Table 1

| Scattering regime | Thomson | Klein-Nishina | Thomson | Klein-Nishina |
|-------------------|---------|---------------|---------|---------------|
| Radiation field electrons | $\beta$ | $\beta$ | abrupt cutoff | abrupt cutoff |
| Monochromatic photons | $\beta/(\beta + 2)$ | $\beta$ | $\infty$ | $\infty$ |
| Planckian photons | $\beta/(\beta + 2)$ | $\beta$ | $1$ | $\infty$ |
| Synchrotron photons | $\beta/(\beta + 4)$ | $\beta$ | $1$ | $\infty$ |

Note. The index $\beta$ characterizes the exponential cutoff in the electron energy distribution given by Equation 1.

Interestingly, for Planckian photons we find a different relation between $\beta_C$ and $\beta$, which is the same as for synchrotron radiation, $\beta_C = \beta/(\beta + 2)$. As can be seen from Figures 6 and 7, the approximation is very good. Finally, for synchrotron photons it holds that $\beta_C = \beta/(\beta + 4)$. As in the case of monochromatic photons, the SSC asymptotics in the Thomson regime are better for Maxwellian electrons.

In general, although in the Klein–Nishina regime the Compton spectrum preserves the electron distribution index $\beta_C = \beta$, in the Thomson regime the upscatter photon cutoff index is...
always smaller than the electron distribution cutoff, $\beta_C < \beta$. The only exception is for monochromatic photons upscattered by electrons with $\beta \rightarrow \infty$. In this case the Compton spectrum should also exhibit an abrupt cutoff. For Planckian and synchrotron photons $\beta \rightarrow \infty$ for electrons means a simple exponential cutoff for the Compton SED, $\beta_C = 1$.

The Thomson spectrum for Planckian photons exhibits a cutoff index $\beta_T = \beta/((\beta + 2)$, same as the synchrotron spectrum. Thus, if the SED of the observed object is considered to consists of these two components, synchrotron radiation for the low energies and upscattering of Planckian photons in the Thomson regime for high energies, then the exponential part of the two “bumps” is very similar (see Figures 10 and 11). If, on the other hand, the upscattering occurs in the Klein–Nishina regime in the energy band of the cutoff, then this is not true. While $\beta_T = \beta/((\beta + 2)$ for the synchrotron component cutoff, $\beta_C = \beta$ for the high-energy component. Even if we consider an abrupt cutoff for the electron distribution, the two bumps would be different ($\beta_T = 1$ and $\beta_C \rightarrow \infty$, respectively). For an SSC model, the two components do not show the same exponential cutoff shape neither in the Thomson (where we get $\beta_T = \beta/((\beta + 2)$ and $\beta_C = \beta/((\beta + 4)$ for low and high energies, respectively), nor in the Klein–Nishina regime ($\beta_T = \beta/((\beta + 2)$ and $\beta_C = \beta$). Only if the electron distribution has an abrupt cutoff, then in the Thomson regime we can get $\beta_C = \beta_T = 1$, while in the Klein–Nishina regime $\beta_C \rightarrow \infty$. Thus, the two components of the SED do not show in general the same shape at the cutoff region.
Apart from the shape of the upscattered, photon spectrum at the cutoff region, another interesting point concerns the cutoff energy itself, $E_{\gamma,\text{cut}}$. We have shown that in the Klein–Nishina regime this remains always at $E_{\gamma,\text{cut}} = E_\gamma$, same as the electron distribution cutoff energy, independently of the soft photon field. On the contrary, at the Thomson regime, the cutoff photon energy depends on the target photon field. For the monochromatic target photons the value of $E_{\gamma,\text{cut}}$ is obvious; it is simply equal to the cutoff energy of electrons. For monochromatic photons, the resulting value of $E_{\gamma,\text{cut}}$ is rather obvious, simply because there is a maximum upscattered photon energy for the fixed electron energy. Consequently, the cutoff energy of the IC spectrum must be equal to the maximum upscattered photon energy by electrons of energy $E_e$, as it follows from (Equation 13),

$$E_{\gamma,\text{cut}} = 4E_e^2\varepsilon_0/(mc^2)^2.$$  \hspace{1cm} (48)

On the other hand, in the case of a broad distribution of target photons, the relation between $E_e$ and $E_{\gamma,\text{cut}}$ depends on the seed photon spectrum and the index $\beta$ of the electron distribution. For the Planckian photon distribution, we find (see Equation (30))

$$E_{\gamma,\text{cut}} = 4E_e^2kT/(mc^2)^2 \left( \frac{2}{\beta + 2} \right)^{\frac{2}{\beta}} \approx \frac{bE_e^2kT}{(mc^2)^2} \left( \frac{2}{\beta + 2} \right)^{\beta/2}. \hspace{1cm} (49)$$

In analogy to the monoenergetic photons case, the maximum energy at which electrons of energy $E_e$ can radiate, when they upscatter photons of energy $kT$, is $4E_e^2kT/(mc^2)^2$. In respect to this, the Thomson spectrum energy cutoff is smaller by a factor of $(\beta/2)^2/(\beta + 2)^2(\beta + 4)^2/2$. This factor is not negligible especially for small $\beta$, e.g., it takes values of $\sim 0.15, 0.25, 0.33$ for $\beta = 1, 2, 3$, respectively (almost one order of magnitude for a simple exponential cutoff). As expected, it tends to unity for $\beta \to \infty$. It does not depend, however, on index $\alpha$ of the electron distribution.

For synchrotron radiation, from Equation (41), we find the cutoff energy

$$\varepsilon_{\gamma,\text{cut}}^{\text{SYN}} = bE_e^2 \left( \frac{2}{\beta + 2} \right)^{\beta/2}. \hspace{1cm} (50)$$

which reveals exactly the same factor as in the case of the Thomson spectrum for Planckian photons, but now in respect to the characteristic energy $bE_e^2$.

Finally, for the case of SSC, Equation (47) gives

$$E_{\gamma,\text{cut}}^{\text{SSC}} = \frac{4bE_e^4}{(mc^2)^2} \left( \frac{2}{\beta + 4} \right)^{\frac{2}{\beta}} \approx \frac{4E_e^2\varepsilon_{\gamma,\text{cut}}^{\text{SYN}}}{(mc^2)^2} \left( \frac{2}{\beta + 2} \right)^{\beta/2}. \hspace{1cm} (51)$$

If we compare the cutoff energy with, e.g., $4bE_\gamma^2\varepsilon_{\gamma,\text{cut}}^{\text{SYN}}/(mc^2)^2$, then the factor related to the index $\beta$ takes values of $\sim 0.035, 0.15, 0.25$ for $\beta = 1, 2, 3$, slightly less than in the case of a Planckian target photon field. This is due to the fact that the cutoff shape is now smoother. These analytic results are useful when modeling the observed spectrum, as one may infer the electron distribution cutoff energy from the photon spectrum cutoff energy with only the uncertainty introduced by a (possible) Doppler boosting. The position and the amplitude of the synchrotron and IC peaks in the SED contain very important information about physical parameters of non-thermal sources, like the strength of the average magnetic field and the energy density of relativistic electrons. The shape of the SED, especially in the region of the cutoffs of the synchrotron and IC components of radiation, provide additional, more detailed information about the distributions of electrons and magnetic fields. For example, the spectral cutoff in the IC component formed in the Klein–Nishina regime provides direct, model-independent information about the energy spectrum of the highest energy electrons. This is a critical issue for understanding of particle acceleration mechanisms. Furthermore, combined with the shape of the synchrotron cutoff, it can allow us to extract information about the distribution of the magnetic field. This can be demonstrated by the following simple example. Let as assume that we have observed a smooth synchrotron cutoff which can be interpreted as the result of an electron distribution with an exponential index, e.g., $\beta \approx 1$. This hypothesis can be checked by the shape of the cutoff of the IC component. If the latter is formed in the Klein–Nishina regime, and exhibits a sharp cutoff behavior indicating to the electron distribution with $\beta > 1$, then one should attribute the smoothness of the synchrotron cutoff to magnetic field inhomogeneities rather than to the actual shape of the electron distribution.

5. SUMMARY

In this paper, we have examined the asymptotic behavior of the Compton spectrum close to the maximum cutoff. We assumed that the electron distribution follows the general formula $E_e^\gamma \exp[-(E_e/E_e^\gamma)_\beta]$ so that our analysis may account for a relativistic Maxwell-type distribution, as well as for a power-law distribution with exponential cutoff. The exponential cutoff of the electron energy spectrum results in an exponential cutoff in the Compton spectrum, of the form $\exp[-(E_{\gamma,\text{cut}}/E_{\gamma,\text{cut}}^\beta)]$, with $\beta$ and $E_{\gamma,\text{cut}}$ the corresponding cutoff index and energy, respectively. We show that in the Klein–Nishina regime, the cutoff index remains unchanged, $\beta_e = \beta$. The shape of the upscattered spectrum close to the maximum cutoff basically “reflects” the electron distribution and does not depend strongly on the target photon field. The cutoff energy also correspond to the electron distribution cutoff energy, $E_{\gamma,\text{cut}} = E_e$.

In the Thomson regime, the resulting spectrum close to the cutoff is very different. First of all, it strongly depends on the upscattered photon field. Monoenergetic photons lead to a cutoff of index $\beta_e = \beta/2$, whereas Planckian photons result in $\beta_e = \beta/(\beta + 2)$. When the upscattered photon field is the synchrotron photon field, as in SSC models, then the cutoff appears extremely smooth, with an index $\beta_e = \beta/(\beta + 4)$. In contrast to the Klein–Nishina regime, the Thomson spectrum energy cutoff $E_{\gamma,\text{cut}}$ depends not only on the electron distribution cutoff energy, but also on the target photon field and as well on the index $\beta$.

The obtained analytic expressions are useful for deriving the electron spectral shape at the cutoff region directly from the observed high-energy flux. These two parameters may give important insight into the acceleration and radiation mechanisms acting in the source. Furthermore, one may use the higher energy part of the observed Compton spectrum as a “diagnostic tool” to distinguish between EC and SSC models, as different photon fields lead to different cutoff shapes.

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