Clustered star formation as a natural explanation of the Hα cutoff in disc galaxies

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Star formation is mainly determined by the observation of Hα radiation which is related to the presence of short lived massive stars. Disc galaxies show a strong cutoff in Hα radiation at a certain galactocentric distance which has led to the conclusion that star formation is suppressed in the outer regions of disc galaxies. This is seemingly in contradiction to recent UV observations1 that imply disc galaxies to have star formation beyond the Hα cutoff and that the star-formation-surface density is linearly related to the underlying gas surface density being shallower than derived from Hα luminosities2. In a galaxy-wide formulation the clustered nature of star formation has recently led to the insight that the total galactic Hα luminosity is non-linearly related to the galaxy-wide star formation rate3. Here we show that a local formulation of the concept of clustered star formation naturally leads to a steeper radial decrease of the Hα surface luminosity than the star-formation-rate surface density in quantitative agreement with the observations, and that the observed Hα cutoff arises naturally.

The integrated galactic initial mass function (IGIMF) describes the mass spectrum of all newly formed stars in a galaxy. The IGIMF is calculated by adding up all stars of all newly formed star clusters4,5 and falls off more steeply with increasing stellar masses for massive stars5 than the canonical initial mass function (IMF) in each star cluster due to the combination of two effects: the masses of the young star clusters are distributed according to the embedded cluster mass function (ECMF), for which the upper mass limit is a function of the total star formation rate6 (SFR), and the stellar upper mass limit of the IMF is a function of the total star cluster mass7. Consequently, the total fraction of massive stars and therefore the total Hα luminosity drops faster with decreasing SFR than the SFR3. The IGIMF theory has already been shown to naturally lead to the observed mass-metallicity relation of galaxies8 and has received recent empirical verification in a study of IMF variations among galaxies9.

In order to construct a quantitative local IGIMF-theory we introduce the local embedded cluster mass function (LECMF),

\[ \xi_{\text{LECMF}}(M_{\text{ecl}}, x, y) = \frac{dN_{\text{ecl}}}{dM_{\text{ecl}} dx dy}, \]

which defines the number of newly formed star clusters with mass \( M_{\text{ecl}} \) per unit area at the location \( x, y \) in a disc galaxy. Observations10 of Galactic star forming regions show that this function is a single part power law, \( \xi_{\text{LECMF}} \propto M_{\text{ecl}}^{-\beta} \), with an index of \( \beta = 2 \).
smallest cluster mass\(^5\), \(M_{\text{ecl,min}} = 5 \, M_\odot\), should form at any place in the galaxy, whereas the most massive star cluster, \(M_{\text{ecl,max,loc}}(x, y)\), which can form locally is expected to depend on the local gas density, i.e. how much material is locally available for star cluster formation. Observations\(^5,6\) show that the most massive star cluster, \(M_{\text{ecl,max}}\), of the whole galaxy is a function of the total galactic star formation rate. To express the upper limit of the LECMF in dependence on the local gas surface density we write

\[
M_{\text{ecl,max,loc}}(x, y) = M_{\text{ecl,max}} \left( \frac{\Sigma_{\text{gas}}(x, y)}{\Sigma_{\text{gas,0}}} \right)^\gamma,
\]

where \(\Sigma_{\text{gas}}(x, y)\) and \(\Sigma_{\text{gas,0}}\) are the gas density at the location \(x, y\) and at the origin, respectively. The local mass of all star clusters between the two mass limits is determined by the local star formation rate which is described by the Kennicut-Schmidt-law\(^2,11\)

\[
\Sigma_{\text{SFR}}(x, y) = A \Sigma_{\text{gas}}^N(x, y)
\]

with \(N = 1.4\) being the widely accepted value\(^2\), whereas \(N = 0.99\) follows from recent UV observations\(^1\). As UV emission is a star formation tracer that is much less sensitive to the presence of OB stars than \(H\alpha\) emission, the true exponent \(N\) must be much closer to the value derived from UV observations. Thus, we chose \(N = 1\). Interestingly, the high gas-density part of the Kennicut-Schmidt-plot\(^2\) based on FIR observations also has a flatter slope of \(N = 1.08\) and galaxy evolution models suggest \(N\) not to exceed unity in order to reproduce observed radial density profiles of disc galaxies\(^12\), confirming our choice. The mass spectrum of all newly formed stars per unit area (the local integrated galactic initial mass function, LIGIMF) is calculated by adding up all newly formed stars of all young star clusters, and the \(H\alpha\) surface density follows by adding up the \(H\alpha\) flux contributions of all newly formed stars. The newly formed stars in each young star cluster are distributed according to the invariant canonical initial mass function (IMF)\(^13,14\) with a fixed lower mass limit but an upper mass limit depending on the total cluster mass\(^7\). Young star clusters above \(\approx 3000 \, M_\odot\) have constant \(H\alpha\)-light-to-mass ratios whereas smaller clusters are increasingly \(H\alpha\) under-luminous\(^3\). With decreasing star formation rate surface density the upper mass limit of the LECMF decreases, and consequently the fraction of under-luminous star clusters increases. Thus UV and \(H\alpha\) scale differently with the star formation surface density, gas surface density or galactocentric radius, respectively. A detailed explanation how the \(H\alpha\) surface luminosity is calculated is given in the Supplementary Discussion.

The LIGIMF-theory is next applied to a sample of disc galaxies\(^15\) with measured gas surface densities and \(H\alpha\) surface luminosities of \(H\ II\) regions averaged over annuli at different galactocentric radii. It is known that ionising photons emitted by massive stars can escape from well defined \(H\ II\) regions and lead to recombinations and thus \(H\alpha\) radiation in the surrounding diffuse ionised gas (DIG)\(^16\). Using \(H\alpha\) emission as a star formation tracer this kind of photon leakage has to be taken into account to get an estimate of the true star formation rate. The study\(^16\) of NGC 247 and NGC 7793 allows to construct a correction procedure in order to obtain the total \(H\alpha\) surface luminosities from the surface luminosities of \(H\ II\) regions only (see Supplementary Discussion).

For a linear star-formation law \((N = 1)\) as derived from UV observations\(^1\) the LIGIMF-theory predicts an \(H\alpha\) surface luminosity as a function of the gas surface density which is
in full agreement with the observations (Fig. 1). Additionally, the radial Hα profile derived in the LIGIMF-theory matches the observations perfectly (Fig. 2). The concept of clustered star formation resolves the discrepancies between Hα and UV observations completely.

At first sight it might be objected that the LIGIMF-theory contradicts observations of the UV sources in the outer disc of galaxies: 5%–10% of all clusters in the outer discs of galaxies detected in UV have associated Hα emission\(^\text{17}\). The age estimates of the UV knots range up to 400 Myr. Clusters with Hα emission have ages ≤20 Myr as they are powered by short lived massive stars and therefore 5% of all observed UV knots are expected to have associated Hα emission in agreement with observations. The LIGIMF-theory predicts an overabundance of under-luminous star clusters beyond the Hα cutoff and a smaller number ratio of Hα to non-Hα-emitting UV knots is expected. But under-luminous does not mean no Hα emission. In the LIGIMF theory each young UV cluster is an Hα source, too, but UV and Hα luminosity scale differently with the cluster mass. Thus, this finding\(^\text{17}\) is entirely consistent with the LIGIMF-theory. The observed UV knots in the outer disc of M83 are at any time systematically smaller than their counterparts in the inner disc\(^\text{18}\) in agreement with the fundamental basics of the LIGIMF-theory. Even one outstanding massive young star cluster in the outer region of M83 does not contradict this principle but is expected from a statistical point of view (see Supplementary Discussion). Furthermore, the M83 FUV luminosity function of outer-disc stellar complexes is steeper than for the inner-disc population\(^\text{18}\). A similar trend is reported in NGC 628 for the Hα luminosity function of H II regions\(^\text{19}\). In the LIGIMF-theory inner disc LECMFs have higher upper mass limits than in the outer disc LECMFs. Integration of the LECMFs over the outer regions leads to a steeper ECMF of the outer disc than the resultant ECMF of the inner disc, indicating that outer disc star formation complexes are systematically smaller than the inner disc ones. This integration effect is fundamentally the same as the IGIMF being steeper for dwarf galaxies with low global star formation rates than for disc galaxies with high star formation rates\(^\text{3}\).

Previously, the Hα cutoff has been explained\(^\text{15}\) by a drop of the local gas density below a critical density determined by the stability condition of a thin isothermal disc\(^\text{20, 21}\) where no star formation can occur. In contradiction to this explanation recent UV observations\(^\text{1}\) reveal star formation outside the Hα cutoff, and dwarf galaxies\(^\text{22}\) show star formation although their average gas density is lower than the critical density. Indeed, it has been shown that in regions with densities lower than the critical value, star formation can be driven by other than thermal instabilities\(^\text{23}\). It has been argued that H II regions powered by the same massive stars are larger in a thin environment, i.e. at large galactocentric radii, than in a dense one and thus identical H II regions become fainter in the outer galaxy. Therefore, it has been concluded\(^\text{23}\) that the Hα surface luminosity should drop faster than the star formation rate surface density. Indeed, the surface brightness of individual H II regions should be fainter in the outer galaxy. But the Hα surface density considered in star formation laws refers to the total Hα luminosity per unit area of the galaxy and not to the cross section of the H II region. Identically powered H II regions contribute equally to the Hα surface luminosity independently of their location in a thin or a dense gas environment. Thus, the proposed solution\(^\text{23}\) does neither explain the Hα cutoff nor the different slopes of the
UV-based and Hα-based star formation laws. It has been shown recently that a required minimum column density for massive star formation might exist implying star formation with no massive stars in low density environments. However, this model predicts a top-heavy IMF for cloud column densities much larger than this threshold, for which no observational evidence exists, and allows no quantitative linkage of Hα luminosity and the star formation rate. Contrary to this previously existing work, the LIGIMF theory is in excellent agreement with the observed radial Hα- and UV-luminosity profiles (Fig. 2) and the Kennicutt-Schmidt star formation law (Fig. 1), and also allows the determination of star formation rates even in Hα-faint galaxy regions.

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**Supplementary Information** is linked to the online version of the paper at www.nature.com/nature.

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Figure 1 | \textbf{Hα-luminosity surface density versus total gas surface density.} The Hα-luminosity surface density versus the total gas surface density observed for seven disc galaxies\textsuperscript{15} averaged over annuli at different galactocentric radii is plotted (black squares) after correcting for photon leakage from \textsc{H} ii regions (see Supplementary Discussion). These galaxies have a mean star formation rate of SFR=6.9 $M_\odot$ yr\textsuperscript{-1} $(3.2 - 16.4 M_\odot$ yr\textsuperscript{-1})\textsuperscript{2,15}, a mean total gas mass of $M_{\text{gas}} = 2.1 \cdot 10^{10} M_\odot$ $(0.6 - 3.6 \cdot 10^{10} M_\odot)$\textsuperscript{2,15} and a mean scale length of $r_d = 4.4$ kpc $(3.9 - 5.2$ kpc)\textsuperscript{25–28}. These mean values define our model standard disc galaxy. For a choice of $\gamma = \frac{3}{2}$ the LIGIMF-theory predicts an $\Sigma_{\text{Hα}} - \Sigma_{\text{gas}}$ relation which matches the observations excellently (solid line). Note that the underlying true star-formation density as derived from UV observations\textsuperscript{1} is directly proportional to the gas surface density ($N = 1$) and is shown after converting it into an Hα surface luminosity using the wrong linear Kennicutt Hα-SFR relation\textsuperscript{2,29} (dashed line) and shows the expected $\Sigma_{\text{Hα}} - \Sigma_{\text{gas}}$ relation based on the classical picture which is in disagreement with the observations. Furthermore, the Hα-luminosity surface density in the high-luminosity part ($\Sigma_{\text{Hα}} \geq 10^{32.5}$ erg s\textsuperscript{-1} pc\textsuperscript{-2}) depends, for the correct LIGIMF-theory, on the gas surface density with a power of 1.4 (dotted line, extrapolated to low Hα surface luminosity) in agreement with the classical Kennicutt-Schmidt slope of $N = 1.4$. The LIGIMF-theory puts the hitherto inconsistent Hα and UV observations in perfect agreement with each other. Both the steeper high-luminosity slope of $N = 1.4$ and the Hα cutoff at low gas densities are two simultaneous outcomes of the LIGIMF-theory. The thick dotted line which coincides with the thick solid curve shows a fitting function of the LIGIMF-model (see Supplementary Discussion).
Figure 2 | **Star formation surface density versus scaled galactocentric radius.** Radial distribution of the star formation surface density of nine disc galaxies based on UV\(^1\) (thin dashed-dotted lines) and H\(\alpha\)\(^30\) (thin dotted lines) observations that rely on a wrong linear conversion\(^{29}\) between the corresponding H\(\alpha\)-luminosity surface density and star-formation-rate surface density after correction for photon leakage (see Supplementary Discussion). The galactocentric radius is in units of the H\(\alpha\) threshold radius\(^{30}\). Over-plotted is the true underlying star-formation-rate surface density of our standard disc galaxy (thick black line) as defined in Fig. 1 and the model H\(\alpha\) surface luminosity (thick dotted black line) converted into a star-formation surface density using the same linear conversion\(^{29,30}\). The LIGIMF-theory thus naturally accounts for the discrepant \(\Sigma_{SFR}\) value at a particular radius.
Supplementary Discussion

This Supplementary Discussion provides more information on i) how the H\(\alpha\) surface luminosity is calculated, ii) the correction for photon leakage from H\(\text{II}\) regions, iii) the local applicability of the IGIMF theory, iv) a discussion on the sizes of typical star forming regions, v) a discussion of the presence of star clusters more massive than \(M_{\text{ecl, max, loc}}\) and vi) the fitting formula for the Kennicutt-Schmidt law in the LIGIMF-model.

i) Calculation of the H\(\alpha\) surface luminosity

In order to construct a local IGIMF theory we start with the IGIMF theory\(^3,5\), developed for the entire galaxy, and transfer all quantities into their corresponding local surface densities.

Thus, we first define the local embedded cluster mass function (LECMF),

\[
\xi_{\text{LECMF}}(M_{\text{ecl}}, x, y) = \frac{dN_{\text{ecl}}}{dM_{\text{ecl}} \, dx \, dy},
\]

which is the number of newly formed star clusters with mass \(M_{\text{ecl}}\) per unit area at the location \(x, y\) in a disc galaxy. Observations\(^10\) of Galactic star forming regions show that the LECMF is a single part power law, \(\xi_{\text{LECMF}} \propto M_{\text{ecl}}^{-\beta_{\text{ecl}}}, \) with an index of \(\beta = 2\). The local mass surface density of newly formed stars is

\[
\delta t \, \Sigma_{\text{SFR}}(x, y) = \int_{M_{\text{ecl, min}}}^{M_{\text{ecl, max, loc}}(x, y)} \xi_{\text{LECMF}}(M_{\text{ecl}}) \, M_{\text{ecl}} \, dM_{\text{ecl}},
\]

where \(\Sigma_{\text{SFR}}\) is the star-formation surface density, \(\delta t \approx 10\) Myr is the time span required to populate the cluster mass function completely\(^6\), and \(M_{\text{ecl, min}} = 5 \, M_\odot\) is the smallest cluster mass\(^5\).

The observed most massive embedded star cluster in a galaxy is determined through the total star formation rate\(^5,6\),

\[
\frac{M_{\text{ecl, max}}}{M_\odot} = 84793 \left( \frac{\text{SFR}}{M_\odot \, \text{yr}^{-1}} \right)^{0.75}.
\]

To express the upper mass limit of the LECMF in dependence on the local gas surface density we write

\[
M_{\text{ecl, max, loc}}(x, y) = M_{\text{ecl, max}} \left( \frac{\Sigma_{\text{gas}}(x, y)}{\Sigma_{\text{gas, 0}}} \right)^\gamma.
\]

The maximal gas-mass surface density, associated with the position of the most massive embedded star cluster, in a disc galaxy, which has a total gas mass \(M_{\text{gas}}\) and a single-exponential gas disc,

\[
\Sigma_{\text{gas}}(x, y) = \Sigma_{\text{gas, 0}} \, e^{-r/r_d},
\]

where
with a scale length \(r_d\), is determined by

\[
\Sigma_{\text{gas},0} = \frac{M_{\text{gas}}}{2 \pi r_d^2}.
\] (8)

One may raise the objection if the crude assumption of an exponential gas disc model is sufficient to construct a standard galaxy as the outer galaxy discs may also be described by other expressions as for example a power law. The key issue of the theory developed here is that the gas surface density generally tends to decrease with increasing galactocentric radius. The main aim here is not the detailed modelling of individual galaxies but to explain the characteristic discrepancies between H\(\alpha\) and UV observation and their traditional interpretations: i) a distinct star formation cutoff inferred from the H\(\alpha\) cutoff contrary to the extended radial star formation inferred from UV. ii) A steeper star-formation-law slope inferred from H\(\alpha\) (N=1.4) observations than from UV (N=1.0) observations within the H\(\alpha\) cutoff. The explanation through the present LIGIMF theory relies on the description of the gas surface density beyond the H\(\alpha\) cutoff. If the very outer gas disc is better described by for example a power law rather than by a single exponential law then this would not change the outcome of the LIGIMF theory. The radial H\(\alpha\) profiles of the disc galaxies examined in UV\(^1\) and Fig. 2 in the main Text are from ref. 30. Figure 9 of ref. 30 shows the total gas surface densities of some of these galaxies. As this is a log-lin diagram it can easily be seen that the general trend of how the gas surface density scales with radius can be well described by an exponential law up to \(\approx 2\) H\(\alpha\) radii. If the aim is to construct detailed mass models of individual galaxies then an exponential law is inaccurate, but by far sufficient for our purpose here.

The star-formation surface density is described by the Kennicutt-Schmidt-law\(^2,11\),

\[
\Sigma_{\text{SFR}}(x,y) = A \Sigma_{\text{gas}}^N (x,y),
\] (9)

with \(N = 1.4\) being the widely accepted value\(^2\), whereas \(N = 0.99\) follows from UV observations\(^3\). As UV emission is a star formation tracer that is much less sensitive to the presence of OB stars than H\(\alpha\) emission, the true exponent \(N\) must be much closer to the value derived from UV observations. Thus, we chose \(N = 1\). Interestingly, the high gas-density part of the Kennicutt-Schmidt-plot\(^2\) based on FIR observations also has a flatter slope of \(N = 1.08\), and galaxy evolution models suggest \(N\) not to exceed unity in order to reproduce observed radial density profiles of disc galaxies\(^12\), confirming our choice.

The factor \(A\) is determined by integrating Supplementary equation (9) over the whole disc,

\[
A = \frac{\text{SFR} \ n^2}{2 \pi \Sigma_{\text{gas},0} r_d^2}.
\] (10)

In each embedded star cluster the newly formed stars are distributed according to the invariant canonical initial mass function (IMF)\(^13,14\) which is a two-part power law, \(m^{-\alpha_i}\), with
indices $\alpha_1 = 1.3$ between $m_{\text{low}} = 0.1$ and $m_1 = 0.5 \, M_\odot$, and $\alpha_2 = 2.35$ between $m_1 = 0.5 \, M_\odot$ and $m_{\text{max}}$, where $m_{\text{max}} = m_{\text{max}}(M_{\text{ecl}})$ is given by the maximum-stellar-mass–star-cluster-mass relation\(^7\). This relation is well defined empirically and is a result of feedback-driven star formation on a star-cluster spatial scale of ≤ few pc (Weidner, Kroupa & Goodwin, in prep.). The IMF in each star cluster, $\xi_{M_{\text{ecl}}}$, is normalised by the total star cluster mass,

$$M_{\text{ecl}} = \int_{m_{\text{low}}}^{m_{\text{max}}(M_{\text{ecl}})} m \, \xi_{M_{\text{ecl}}}(m) \, dm .$$

Finally, the local integrated galactic initial mass function (LIGIMF) can be calculated by locally adding up all stars in all newly formed star clusters,

$$\xi_{\text{LIGIMF}}(m, x, y) = \int_{M_{\text{ecl}, \text{min}}}^{M_{\text{ecl}, \text{max, loc}}(x,y)} \xi_{M_{\text{ecl}}}(m) \, \xi_{\text{LECMF}}(M_{\text{ecl}}, x, y) \, dM_{\text{ecl}} . (12)$$

The surface density of ionising photons emitted by all new stars at the position $x, y$ is

$$N_{\text{ion,}\delta t}(x, y) = \int_{m_{\text{low}}}^{m_{\text{max}}} \xi_{\text{LIGIMF}}(m, x, y) \, N_{\text{ion,}\delta t}(m) \, dm , (13)$$

where $N_{\text{ion,}\delta t}(m)$ is the total number of ionising photons\(^3\) emitted by a star with mass $m$ in time $\delta t$. The H$\alpha$-luminosity surface density then follows from

$$\Sigma_{\text{H}\alpha}(x, y) = 3.02 \times 10^{-12} \text{ erg } N_{\text{ion,}\delta t}(x, y)/\delta t . (14)$$

### ii) Photon leakage

In order to correct for the leakage of ionising radiation from H II regions we use a comparative study\(^{16}\) of the H$\alpha$ radiation of well defined H II regions and their embedding diffuse ionised gas (DIG). This study of the two galaxies NGC 247 and NGC 7793 shows that the radial and azimuthal surface luminosity of the DIG is highly correlated with bright H II regions and that the required power to sustain the DIG can only be met by the ionising radiation from massive star formation. In faint H$\alpha$ luminosity regions the luminosity contribution of the DIG can exceed the one by distinct H II regions. We use the published values of the H$\alpha$ surface luminosities of H II regions only and H II regions plus DIG in different regions of these galaxies to construct a correction formula. Supplementary Fig. 1 shows the fraction of the H$\alpha$ luminosity due to H II regions of the total (H II + DIG) H$\alpha$ luminosity as a function of the H$\alpha$ luminosity in H II regions. Two interpolating functions are used to describe the resultant relation. With

$$y = \log_{10} \left( \frac{\Sigma_{\text{HII}}}{\Sigma_{\text{HII} + \text{DIG}}} \right) , (15)$$

and

$$x = \log_{10} \Sigma_{\text{HII}} . (16)$$
the high luminosity region of NGC 7793 and all data points of NGC 247 can be easily interpolated (solid line) by

\[ y(x) = 0.22 (x - 32.3) . \tag{17} \]

NGC 7793 alone can be interpolated (dashed line) by

\[ y(x) = 0.22 (x - 32.3) - \exp(-(x - 28.2)) . \tag{18} \]

The total surface luminosities are then given by

\[ \log_{10}(\Sigma_{\text{HII} + \text{DIG}}) = x - y(x) . \tag{19} \]

Luminosities brighter than \(10^{32.3}\) erg s\(^{-1}\) pc\(^{-2}\) need no correction. The influence of photon leakage from H\(\Pi\) regions for the Kennicutt-Schmidt diagram of Fig. 1 is shown in Supplementary Fig. 2. The corrected values (small black squares) are obtained by applying Supplementary equation (19) on the original values (open circles). The steeper (\(N = 1.4\)) \(\Sigma_{\text{H}\alpha}-\Sigma_{\text{gas}}\) relation left of the \(\text{H}\alpha\) cutoff and the position of the cutoff still persist. The lines are as described in the main text. Correcting for photon leakage therefore does not affect our results in any way.

### iii) Local applicability of the IGIMF theory

Recently it has been demonstrated that the concept of clustered star formation has a significant influence on the appearance of galaxies in \(\text{H}\alpha\). The galaxy-wide \(\text{H}\alpha\) luminosity scales non-linearly with the underlying total star-formation rate (SFR). As a first consequence it has been shown that a sample of the Sculptor group of dwarf irregular galaxies have much higher SFRs than previously deduced and that their SFRs scales approximately linearly with their gas mass. The sample consists only of 11 galaxies and the validity of the obtained star formation relation might be questionable. But a subsequent investigation of a sample of 205 local star forming galaxies confirms this finding and reveals a strictly linear relation between the SFR of a galaxy and its total gas mass over five orders of magnitude in mass (Pflamm-Altenburg and Kroupa, in preparation). Furthermore, the local UV-based star formation law integrated over the disc of a galaxy is in perfect agreement with global \(\text{H}\alpha\) based and IGIMF corrected star formation law. Therefore, dwarf galaxies can be described very well with the IGIMF-theory, i.e. by the concept of clustered star formation. For a radius of 3 kpc of a dwarf galaxy the reference area is about 28.3 kpc\(^2\). In this context "local" regions in a disc galaxy are annuli. E.g. an annulus between 10 and 11 kpc has an area of 66 kpc\(^2\), much larger than a dwarf galaxy for which the IGIMF theory is seemingly applicable. This means local regions in the context of disc galaxies have test areas larger than dwarf galaxies which we have mentioned to be excellently described by the IGIMF theory, i.e. by the concept of clustered star formation.
iv) Sizes of outer disc star forming regions

The basis of the LIGIMF theory is that X-UV discs result from local ”stellar IMF biases”. The Hα-luminosity—star-cluster-mass relation becomes non-linear\(^3\) for cluster masses \(\leq 3000\, M_\odot\). The LIGIMF theory predicts that inner-disc star-forming complexes sample the LECMF up to higher upper star-cluster mass limits than outer-disc star-forming complexes. As UV is a star formation tracer much less sensitive to the LIGIMF-effect (i.e. the supression of OB-star numbers in low-SFR regions), the UV/Hα luminosity ratio in the inner disc is smaller than in the outer disc. Thus, a basic requirement of the LIGIMF theory is that star forming regions in the inner disc are typically larger or more massive than the outer disc ones. A colour-magnitude diagram of UV knots observed in M83 differentiates into outer and inner disc UV knots (Fig. 3 in ref. 18). UV knots younger than 100 Myr have FUV-NUV colours (Fig. 2 in ref. 17). It can clearly be seen that the typical young (\(\leq 20\, \text{Myr}\)) UV outer-disc knots of M83 have masses \(\leq 1000\, M_\odot\) and the main bulk lies at \(\approx 500\, M_\odot\) while the inner knots extend to much higher masses. Only these star forming clusters (\(\leq 20\, \text{Myr old}\)) are of interest as only they contribute to Hα emission. Only one outer-disc object stands out with \(10^4\, M_\odot\). The presence of such a massive star cluster in the outer region is not forbidden statistically (see next Supplementary Section). Young inner-disc objects populate the CMD densely up to \(5000\, M_\odot\) and have masses which are typically significantly larger than outer-disc UV knots. Outer-disc knots which are much larger than \(1000\, M_\odot\) are older than \(20\, \text{Myr}\) and can not be considered in the context of the Hα cutoff. But at these somewhat later ages the inner-disc knots are also more massive. Summarising: at any time the typical star forming regions are less massive in the outer disc than in the inner disc. Additionally, the median FUV luminosity of the inner-disc population is 3 times greater than in the outer region\(^{18}\). And outer-disc complexes appear systematically fainter than their inner-disc counterparts\(^{18}\). Furthermore, the M83 FUV luminosity function of outer-disc stellar complexes is steeper than for the inner-disc population. A similar trend is reported\(^{19}\) for the Hα luminosity function of H II regions in NGC 628. This is in full agreement with the concept of a varying upper mass limit of the LECMF [equation (2)]. If a galaxy is divided into two parts, an inner and an outer part, then the LECMF of the respective part has to be integrated over the whole part of the galaxy to get the ECMF of each part separately. The inner part consists of regions where the LECMF has high local upper cluster mass limits. The outer part consists of regions where the LECMF has only low or intermediate local upper cluster mass limits. Thus, solving the surface integral over the LECMF it follows that the ECMF in the outer part is steeper than in the inner part. This is the same integration effect as for galaxies with a low SFR having steeper IGIMFs than galaxies with a high SFR\(^3\). The same holds for the FUV-luminosity and H II-region luminosity function as observed. These observations are an indirect evidence that the upper mass limit of the LECMF decreases towards galactic regions with lower star formation surface densities.
Also the finding that 5\%-10\% of the UV knots in the extended disc of NGC 628 have associated H\(\alpha\) emission\(^1\) is in-line with the LIGIMF theory. This ratio of H\(\alpha\) emitting sources to non-emitting sources is entirely reasonable given the age estimates of \(\approx 20\) Myr for H\(\Pi\) regions and \(\approx 400\) Myr for GALEX knots, i.e. \(20/400 = 5\%\). As explained above, an H\(\alpha\)-under-luminous star cluster is not the same as an H\(\alpha\)-non-luminous cluster. Each young UV source (\(\leq 20\) Myr) can appear as an H\(\alpha\) source. The crucial point is that the UV and H\(\alpha\) luminosities of star clusters of mass \(\leq 3000\) \(M_\odot\) scale differently with the star cluster mass\(^3\). Therefore, these observations\(^1\) are in agreement with the LIGIMf theory.

v) Local maximum cluster mass

The presence, for example in outer-disc regions of galaxies, of outstanding star cluster masses is not forbidden, but expected. To demonstrate this we take the cluster mass probability distribution density, \(f(M) = k M^{-\beta}\), between the lower, \(M_1\), and an upper limit, \(M_u\), normalised by \(\int_M^{M_u} f(M) \, dM = 1\).

\[ f(M) = k M^{-\beta} \]

\(N\) clusters are drawn from this distribution function with \(\beta = 2\), \(M_1 = 5\) \(M_\odot\), and \(M_u = 10^7\) \(M_\odot\), the mass of the most massive star cluster of this set is stored. This experiment is repeated several times. We then get a distribution of the most massive star cluster of a set of \(N\) star clusters. In general, the distribution function of the \(i\)-th massive star cluster of a set of \(N\) star clusters can be constructed as follows: the probability that the \(i\)-th massive star cluster lies in the mass range from \(M\) to \(M + dM\) is \(f(M)\). There are \(\binom{N}{i}\) = \(N\) possible realisations to choose the \(i\)-th massive star cluster out of \(N\). The probability that \(i - 1\) more massive star clusters lie in the mass range from \(M\) to \(M_u\) is \(\left(\int_M^{M_u} f(M') \, dM'\right)^{i-1}\). There are \(\binom{N-1}{i-1}\) possible realisations to choose \(i - 1\) clusters out of the remaining \(N - 1\) clusters. Finally, the probability that \(N - i\) less massive star clusters lie in the mass range from \(M_1\) to \(M\) is \(\left(\int_{M_1}^{M} f(M') \, dM'\right)^{N-i}\). There are \(\binom{N-i}{N-i}\) = \(1\) possible realisations to choose \(N - i\) clusters out of the remaining \(N - i\). Thus, the total distribution of the \(i\)-th massive star cluster is given by

\[ p_{i,N}(M) = N \binom{N-1}{i-1} \left(\int_M^{M_u} f(M) \, dM\right)^{N-i} f(M) \left(\int_M^{M_u} f(M) \, dM\right)^{i-1}. \]

For the most-massive star cluster (\(i = 1\)) and a single part power law for the mass function the distribution simplifies to

\[ p_{1,N}(M) = N k \left(\frac{k}{1-\beta (M^{1-\beta} - M_1^{1-\beta})}\right)^{N-1} M^{-\beta}, \text{ for } \beta \neq 1. \]

The distribution of the most-massive star cluster is plotted in Supplementary Fig. 3 for \(N = 10, 100, 1000, 10000\) (drawing \(10^6\) times from \(f(M)\)). Shown are the distributions obtained from Monte-Carlo simulations (grey histograms) and the analytical distributions (black curves) calculated using Supplementary equation (22). If a test area in the inner region of the galaxy with a high star formation rate is specified one would expect, for example,
1000 clusters per galactic test area and the expected most-massive star cluster and thus the upper mass limit of the LECMF in a typical inner region would be about $10^4 M_\odot$. In a test area of equal size in the outer regions were the star formation rate is lower by a factor of ten the expected number of star clusters per test area is reduced by a factor of ten, too. The expected most-massive star cluster would have a mass of slightly less than 1000 $M_\odot$, while a $10^4 M_\odot$ star cluster is unlikely but possible. Thus, from the statistical point of view, the typical star forming regions in the outer galaxies have to be of lower mass than in the inner regions with a much higher star formation rate.

**vi) Fitting formula**

The relation between the theoretical H$\alpha$ surface luminosity and the gas surface density can be very well fitted (Fig. 1) by a polynomial of fifth order,

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5,$$

where

$$y = \log_{10} \left( \frac{\Sigma_{\text{gas}}}{M_\odot \text{pc}^{-2}} \right), \quad x = \log_{10} \left( \frac{\Sigma_{\text{H} \alpha}}{\text{erg s}^{-1} \text{pc}^{-2}} \right).$$

(23)

The coefficients are listed in Supplementary Table 1. The underlying theoretical star formation law [$N = 1$ in Supplementary equation (9)] is

$$\Sigma_{\text{SFR}} = 3.27 \cdot 10^{-10} \text{yr}^{-1} \Sigma_{\text{gas}}.$$

(25)

By combining these two equations the LIGIMF theory allows a convenient and accurate conversion of an observed H$\alpha$ luminosity surface density into a true star-formation-rate surface density.
Fit-coefficients

| \( a \) | Value          |
|--------|---------------|
| \( a_0 \) | -136.388      |
| \( a_1 \) | 29.3544       |
| \( a_2 \) | -2.52783      |
| \( a_3 \) | 0.108099      |
| \( a_4 \) | -2.29326e-03  |
| \( a_5 \) | 1.93394e-05   |

**Supplementary Table 1 | Coefficients for the \( \Sigma_{H\alpha} - \Sigma_{\text{gas}} \) fit.** Coefficients of the fitting function [Supplementary equation (23)] of the relation between the H\( \alpha \) surface luminosity and gas surface density (Fig. 1).
Supplementary Figure 1 | Estimating the photon leakage from HII regions. Ratio of the Hα-surface luminosity of H II regions only and the total Hα-surface luminosity (H II regions plus DIG) versus Hα-surface luminosity of H II regions only, based on a comparative study of the two disc galaxies NGC 7793 and NGC 247 in annuli at different galactocentric radii.
Supplementary Figure 2 | Influence of photon leakage on the derived Hα surface density. Hα-surface luminosity density corrected for photon leakage (small black squares) and uncorrected (open circles) versus gas surface density of the same galactic annuli.
Supplementary Figure 3 | Most-massive star cluster distribution. Distribution of the most massive star cluster of a set of $N$ clusters from Monte-Carlo simulations (grey histograms) and the analytical treatment (black lines).