Supplementary Information for

Phase diagram of superconductivity in the integer quantum Hall regime

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This PDF file includes:
Supplementary text
Figs. S1 to S3 (not allowed for Brief Reports)
Supporting Information Text

In the Supplementary Information (SI) we explain our choice of gauge and discuss self-consistent solutions for the excited states.

A. An example of periodic gauge choice. In Fig. S1, we illustrate the gauge choice discussed in the main text by giving an example for a small magnetic unit cell (MUC). Here the MUC is of size $3 \times 3$ and the flux through each plaquette is $\Phi = \Phi_0/9$ ($\Phi_0 = \hbar/e$ is the flux quantum), except for the lower rightmost plaquette (marked by a star) where an additional point flux has been inserted resulting in a net flux of $\Phi_0/9 - \Phi_0 = -8\Phi_0/9$. The values of the hopping phases $A_j + \delta_j$ are given for the horizontal edges of the lattice within the MUC. Hopping phases along edges connected by red arrows are set equal – this is the condition $A_{j+\hat{x},j} + A_{j+\hat{y},j} = A_{j+\hat{x},j}$ (see main text). The phases repeat periodically throughout the system.

B. Excited states. In addition to the ground state solutions discussed in the main text, we find higher energy self-consistent vortex lattice and skyrmion lattice solutions.

The excited vortex lattice solution is found to have $p_x - ip_y$ symmetry, instead of $p_x + ip_y$ which is found for the ground state vortex and dimer vortex lattices. The magnitudes of the two components of the order parameter, $|\Delta_{j,x}|$ and $|\Delta_{j,y}|$, the particle density, and the low energy BdG quasiparticle band structure are shown in Fig. S2 (a)-(d). The profile of each vortex is less isotropic than that of $p_x + ip_y$ vortices as can be seen in Fig. S2 (a) and (b) (also cf. Fig. 2 (e) and (f) from the main text). The spectrum displays low energy hybridized Majorana bands, and the SC Chern number is $C = 0, -2$.

Furthermore, we find skyrmions, which have their chirality inverted relative to the ground state skyrmions, among the excited states. The magnitudes of the two components of the order parameter, $|\Delta_{j,x}|$ and $|\Delta_{j,y}|$, the particle density, and the low energy BdG quasiparticle band structure are shown in (e)-(h). The order parameters and the particle density look qualitatively different from the ground state skyrmions but the quasiparticle spectrum is still gapped. The SC Chern number in this phase is $C = -1$. Fig. S3 shows the cosine and sine of the gauge invariant relative phase. The cosine (Fig. S3a) shows the characteristic alternating sign along the edge of the skyrmion. Inside the skyrmion, the pairing symmetry is $p_x - ip_y$ and outside the skyrmion the pairing symmetry is $p_x + ip_y$, as seen in Fig. S3b, which is the opposite of that of the ground state skyrmions discussed in the main text.
Fig. S1. An example of the gauge choice for a $3 \times 3$ MUC, with flux $\Phi_0/9$ passing through each plaquette. The MUC labels are given on each site. The hopping phases $A_{j,j+1}$ are given for the horizontal edges within the MUC; edges connected by red arrows have the same hopping phases (note the directionality of the edges). The dashed edges on the right are the periodic images of the edges on the far left. Similarly, the dashed edges on the top are periodic images of the edges on the very bottom. The opposing $\Phi_0$ flux is inserted into the lower rightmost square plaquette (the one marked by the star), which has a flux of $-8\Phi_0/9$ passing through it. The sum of the hopping phases around any closed path enclosing flux $\Phi$ is equal to $2\pi \Phi/\Phi_0$. 

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Fig. S2. (a-b) The magnitude of the components of the pairing potential $|\Delta_{j,x}|$ and $|\Delta_{j,y}|$, (c) the particle density, (d) the low energy quasiparticle bands for the excited state vortex lattice solution with $U = 5.5$ and $\mu = -3.8$. Panels (e)-(h) show the same but for the excited state skyrmion lattice solution with $U = 5$ and $\mu = -3.75$. The red dashed lines denote the boundary of the MUC.
Fig. S3. (a) The cosine of the gauge invariant phase difference between the $x$ and $y$ components of the order parameter for the excited state skyrmion phase. Along the boundary of the skyrmion, the pairing symmetry oscillates between $p_x + p_y$ and $p_x - p_y$. (b) The sine of the gauge invariant phase difference between the $x$ and $y$ components of the order parameter for the excited state skyrmion phase. Inside the skyrmion, the pairing symmetry is $p_x - ip_y$, and outside, the pairing symmetry is $p_x + ip_y$. 