Measuring the Hubble constant and spatial curvature from supernova apparent magnitude, baryon acoustic oscillation, and Hubble parameter data

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Abstract Cosmic microwave background (CMB) anisotropy (spatial inhomogeneity) data provide the tightest constraints on the Hubble constant, matter density, spatial curvature, and dark energy dynamics. Other data, sensitive to the evolution of only the spatially homogeneous part of the cosmological model, such as Type Ia supernova apparent magnitude, baryon acoustic oscillation distance, and Hubble parameter measurements, can be used in conjunction with the CMB data to more tightly constrain parameters. Recent joint analyses of CMB and such non-CMB data indicate that slightly closed spatial hypersurfaces are favored in non-flat untilted inflation models and that dark energy dynamics cannot be ruled out, and favor a smaller Hubble constant. We show that the constraints that follow from these non-CMB data alone are consistent with those that follow from the CMB data alone and so also consistent with, but weaker than, those that follow from the joint analyses of the CMB and non-CMB data.

Keywords Cosmological parameters · Large-scale structure of universe · Observations · Methods: statistical

1 Introduction

Establishing an accurate cosmological model that is consistent with observations is one of the primary goals of cosmology. Currently the spatially flat \( \Lambda \)CDM model is considered to be the standard cosmological model (Peebles 1984). In this model the current energy budget is dominated by the cosmological constant \( \Lambda \) with nonrelativistic cold dark matter (CDM) being the second biggest contributor. The former is responsible for the accelerated cosmological expansion at the present epoch, and the latter for initiating large-scale cosmological structure formation, and, along with nonrelativistic baryonic matter, for the earlier decelerated cosmological expansion.

The spatially flat \( \Lambda \)CDM model is consistent with the cosmic microwave background (CMB) anisotropy (Ade et al. 2016; Aghanim et al. 2018), Type Ia supernova apparent brightness (Scolnic et al. 2018), baryonic acoustic oscillation distance (Alam et al. 2017), and Hubble parameter (Farooq et al. 2017)\(^1\) measurements. While CMB anisotropy data most tightly constrain cosmological models, at present no single kind of cosmological data is that restrictive, and it is the combination of CMB and non-CMB data that results in powerful constraints (and breaks some of the degeneracy between correlated parameters).

Though the standard \( \Lambda \)CDM model assumes flat spatial hypersurfaces and a constant dark energy density, current observations do not require either. To include nonzero spatial curvature in the analysis of CMB (and other spatial inhomogeneity) data requires the use of a nonflat inflation model to generate a physically consistent power spectrum of spatial inhomogeneities (as touched upon below and elsewhere; this has previously been ignored, resulting in invalid constraints on spatial curvature based on physically inconsistent

\(^1\)Hubble parameter measurements provide evidence for the earlier nonrelativistic matter dominated cosmological expansion as well as the current dark energy powered accelerating cosmological expansion (Farooq and Ratra 2013; Farooq et al. 2013; Capozziello et al. 2014; Moresco et al. 2016; Farooq et al. 2017; Yu et al. 2018; Jesus et al. 2018; Haridasu et al. 2018b).
power spectra). As an alternative to the constant dark energy density of $\Lambda$CDM model, the XCDM model is based on a simple and widely used dynamical dark energy parameterization. In this model, the ratio of dark energy pressure and density—the equation of state parameter ($w = p_X/\rho_X$)—is constant. However, XCDM is not able to consistently describe the evolution of density inhomogeneities and so is not physically consistent. The $\phi$CDM model is a physically consistent dynamical dark energy model based on the evolution of a scalar field (Peebles and Ratra 1988; Ratra and Peebles 1988).

Recently, Ooba et al. (2018a,b,c,d) and Park and Ratra (2018a,b,c) reported that compilations of Planck 2015 CMB anisotropy data (Ade et al. 2016) and non-CMB data favor slightly closed spatial hypersurfaces in the nonflat $\Lambda$CDM, XCDM, and $\phi$CDM dark energy untilted inflation models, and noted that a dynamical dark energy density that varies both temporally and spatially cannot be ruled out. Most studies concentrate on using the most recent compilation of CMB and non-CMB data to estimate the cosmological parameters as precisely as possible. Here we want to examine the constraints on cosmological parameters that follow from the non-CMB observations alone, to avoid having to assume an energy density inhomogeneity power spectrum, and to examine whether the non-CMB data constraints are consistent with the CMB ones. In this paper, we constrain the flat and nonflat $\Lambda$CDM, XCDM, and $\phi$CDM dark energy models using an up-to-date collection of non-CMB data sets to constrain the spatially homogeneous cosmological models. We use Type Ia supernova apparent magnitude, baryon acoustic oscillation distance, and Hubble parameter data to measure the matter density, Hubble constant, spatial curvature, and parameters characterizing dark energy dynamics. We find that the conclusions obtained by jointly using CMB and non-CMB data sets, that favor slightly closed spatial hypersurfaces and a slightly smaller Hubble constant, and allow for mild dark energy dynamics, also hold for the non-CMB data, but with lower statistical significance.

In Sect. 2 the non-CMB data sets used in our analysis are briefly summarized. In Sect. 3 we summarize our analysis methods that use the flat and nonflat $\Lambda$CDM, XCDM, and $\phi$CDM models. The observational constraints on the parameters of the six cosmological models are presented in Sect. 4. We summarize our results in Sect. 5.

## 2 Data

We use Type Ia supernova apparent magnitude (SN), baryon acoustic oscillation distance (BAO), and Hubble parameter [$H(z)$] measurements to constrain the flat and nonflat $\Lambda$CDM, XCDM, and $\phi$CDM models.

We use the most recent SN data compilation, the Pantheon collection of 1048 Type Ia supernova apparent magnitude measurements over a redshift range of 0.01 < $z$ < 2.3 (Scolnic et al. 2018). This data set is a combination of Type Ia supernovae discovered by the Pan-STARRS1 Medium Deep Survey, the Sloan Digital Sky Survey, and the Supernova Legacy Survey, together with low-$z$ and Hubble Space Telescope SN samples. In our analyses here we account for the statistical and systematic uncertainties in the Pantheon measurements.

We use a compilation of BAO data from Alam et al. (2017), Beutler et al. (2011), Ross et al. (2015), Ata et al. (2018), Bautista et al. (2017), and Font-Ribera et al. (2014), which is summarized in Table 1. Here $D_M(z)$ is the comoving distance at redshift $z$, $D_H(z) = c/H(z)$, $D_V(z) = [czD_M^2(z)/H(z)]^{1/3}$, $D_A(z) = D_M(z)/(1+z)$, $r_d$ is the radius of the sound horizon at the drag epoch $z_d$, and $c$ is the speed of light (see Sect. 2.3 of Park and Ratra 2018a). Although Table 1 here is similar to Table 1 of Park and Ratra (2018a), here we exclude the growth rate ($f \, \sigma_8$) points from the Baryon Oscillation Spectroscopy Survey (BOSS) DR12 data (Alam et al. 2017). Since we exclude these $f \, \sigma_8$ points, the DR12 covariance matrix between measurement errors (Alam et al. 2017) we use here is

$$C_{\text{DR12}} = \begin{pmatrix}
624.7 & 23.73 & 325.3 & 8.350 & 157.4 & 3.578 \\
23.73 & 5.609 & 11.64 & 2.340 & 6.393 & 0.9681 \\
325.3 & 11.64 & 905.8 & 29.34 & 515.3 & 14.10 \\
8.350 & 2.340 & 29.34 & 5.423 & 16.14 & 2.853 \\
157.4 & 6.393 & 515.3 & 16.14 & 1375 & 40.43 \\
3.578 & 0.9681 & 14.10 & 2.853 & 40.43 & 6.259
\end{pmatrix}$$

(data are included in the mix we are unable to quantitatively determine the goodness-of-fit of the best-fit set of cosmological parameters to the measurements. This is in part due to the ambiguity in the number of degrees of freedom of the Planck CMB data (see discussion in Ooba et al. 2018a,b,c,d; Park and Ratra 2018a,b,c). We also emphasize that qualitatively the slightly closed models better fit the lower multipole number CMB temperature anisotropy data and the weak lensing constraints on density inhomogeneities (Abbott et al. 2018) while the flat models better fit the higher multipole number CMB temperature anisotropy data and the observed deuterium abundances (Penton et al. 2018).
References: [1] Alam et al. (2017), [2] Beutler et al. (2011), [3] Ross et al. (2015), [4] Ata et al. (2018), [5] Bautista et al. (2017), [6] Font-Ribera et al. (2014). Note: The sound horizon size (at the drag epoch) is 9.242 ± 0.35 Mpc in Ross et al. (2015) and 148.69 Mpc in Ross et al. (2015).

As in Park and Ratra (2018b,c) we also use the updated BAO data point of Ata et al. (2018). In actual parameter estimation we use the probability distributions of the BAO data points of Ross et al. (2015) and Font-Ribera et al. (2014), instead of the approximate Gaussian constraints shown in Table 1. See Sect. 2.3 of Park and Ratra (2018a) for more details about our procedure.

For $H(z)$ data, we use the collection of 31 Hubble parameter measurements over a large redshift range (0.070 ≤ $z$ ≤ 1.965) listed in Table 2 of Park and Ratra (2018a). See Moreasco et al. (2018) for a recent discussion of Hubble parameter measurement error bars.

### Methods

We measure the parameters of the flat and nonflat $\Lambda$CDM, XCDM, and $\phi$CDM models by comparing model predictions with the observed SN apparent magnitudes, BAO distances, and Hubble parameters over a large range of redshift.

The evolution of the spatially homogeneous background in the $\Lambda$CDM and XCDM models is usually described by the evolution of the Hubble parameter. For the nonrelativistic matter and dark energy dominated epochs, the Hubble parameter $H(a)$ as a function of the scale factor $a$ (normalized to be unity now) is

$$
(H/H_0)^2 = \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda
$$

for the $\Lambda$CDM model, and

$$
(H/H_0)^2 = \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_X a^{-(1+w)}
$$

for the XCDM parameterization. Here $H_0$ is the Hubble constant, the nonrelativistic matter density parameter present value is the sum of present baryonic matter and CDM density parameters, $\Omega_m = \Omega_b + \Omega_c$, $\Omega_k$ is the present value of the spatial curvature density parameter, and $\Omega_\Lambda$ and $\Omega_X$ are the present values of the dark energy density parameters in the $\Lambda$CDM and XCDM models, respectively. In the limit $w = -1$ the XCDM dark energy becomes the cosmological constant $\Lambda$.

In the $\phi$CDM model we consider a minimally coupled dark energy scalar field $\phi$ with an inverse power-law potential energy density

$$
V(\phi) = V_1 \phi^{-\alpha},
$$

where $\alpha$ is a positive constant parameter and $V_1$ is determined in terms of $\alpha$ (Peebles and Ratra 1988). In the limit $\alpha = 0$ the scalar field dark energy becomes the cosmological constant $\Lambda$.

For the background evolution of the $\phi$CDM model in the nonrelativistic matter and scalar field dominated epochs we use

$$
(H/H_0)^2 = \frac{1}{1 - \frac{1}{b}(\phi)^2} \left[ \Omega_m a^{-3} + \Omega_k a^{-2} + \frac{1}{3} \dot{V}(\phi) \right],
$$

where the evolution of the dark energy scalar field is governed by the equation of motion

$$
\dddot{\phi} + \left( 3 + \frac{H}{H^2} \right) \ddot{\phi} + \dot{V}_\phi \left( \frac{H_0}{H} \right)^2 = 0.
$$

Here $\dot{\phi} \equiv d\phi/d\ln a$, $H = \dot{a}/a$, $\dot{V}(\phi) \equiv V(\phi)/H_0^2$, $\ddot{V}_\phi = -\dot{V}_1 \alpha \phi^{-\alpha-1}$, $\dot{V}_1 \equiv V_1/H_0^2$, and an overdot denotes the time derivative $d/dt$. We have chosen units such that the Newtonian gravitational constant $G \equiv 1/8\pi$. We use the initial conditions of Peebles and Ratra (1988) at scale factor $a_i = 10^{-10}$. This places the homogeneous background scalar field on the attractor/tracker solution (Peebles and Ratra 1988; Ratra and Peebles 1988; Pavlov et al. 2013). For a given set of cosmological parameters and initial conditions for the scalar field, we numerically determine the value of $\dot{V}_1$ to satisfy the condition $H/H_0 = 1$ at the present epoch (when $a = 1$). The current value of the dark energy density parameter is $\Omega_\phi = (\phi_0)^2/6 + \dot{V}(\phi)/3$, where $\phi_0$ and $\dot{\phi}_0$ are the current values of $\phi$ and $\dot{\phi}$.

The version of Eqs. (2)–(6) we use in the actual computations also take into account the contribution of photons and massless and massive neutrinos. We assume that the present CMB temperature $T_0 = 2.7255$ K, that the effective number of neutrino species $N_{\text{eff}} = 3.046$, and one massive neutrino species (with mass $m_\nu = 0.06$ eV).
To obtain the likelihood distributions of the cosmological parameters, we use the Markov chain Monte Carlo (MCMC) method that randomly explores the probability space based on the probability function $P(m|d) \propto \exp(-\chi^2/2)$, where $m$ and $d$ denote model and data, respectively, and $\chi^2 = X_{\text{SN}}^2 + X_{\text{BAO}}^2 + X_{H(z)}^2$ is the sum of individual contributions from the SN, BAO, and $H(z)$ data. When comparing the Pantheon SN apparent magnitude data with model predictions we use the $X_{\text{SN}}^2$ defined in Appendix C (Eq. (C1)) of Conley et al. (2011). The SN covariance matrix $C_{\text{SN}}$ is the sum of the diagonal statistical uncertainty covariance matrix, $C_{\text{stat}} = \text{diag}(\sigma_{\text{SN}}^2)$, and the systematic uncertainty covariance matrix, $C_{\text{sys}}$: $C_{\text{SN}} = C_{\text{stat}} + C_{\text{sys}}$. For the BAO data, $X_{\text{BAO}}^2$ is the sum of contributions from each BAO measurement. For example for the BOSS DR12 BAO data we have $X_{\text{DR12}}^2 = X^T C_{\text{DR12}}^{-1} X$ where $X$ is a vector whose elements are the differences between model predictions and data points (the first six entries in Table 1). For Hubble parameter data, $X_{H(z)}^2 = \sum_{i=1}^{31} (H(z_i) - H_{\text{obs}}(z_i))^2/\sigma_H^2(z_i)$. We constrain the flat LCDM model with three cosmological parameters ($\Omega_b h^2$, $\Omega_c h^2$, $H_0$) and the nonflat $\Lambda$CDM model with four parameters ($\Omega_b h^2$, $\Omega_c h^2$, $H_0$, $\Omega_k$), where $h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$. We add one more free parameter, the equation of state parameter $w$, for the XCDM parameterization, and the scalar field potential parameter $\alpha$ for the $\phi$CDM model.

We modified the publicly available CAMB/COSMOMC package (version of November 2016, Challinor and Lasenby 1999; Lewis et al. 2000; Lewis and Bridle 2002) to constrain the flat and nonflat $\Lambda$CDM, XCDM, and $\phi$CDM models, at the spatially homogeneous background level, by using the SN + $H(z)$, SN + BAO, BAO + $H(z)$, and SN + BAO + $H(z)$ data combinations. For the SN + $H(z)$ data combination, the model predictions are not sensitive to baryonic density parameter variations. In this case $\Omega_b h^2$ is not constrained but instead taken to be $\Omega_b h^2 = 0.022277$, the best-fit value of the flat LCDM model constrained using Planck 2015 TT + lowP + lensing CMB data (Ade et al. 2016). In our analyses here we assume flat priors nonzero over $0.005 \leq \Omega_b h^2 \leq 0.1$, $0.001 \leq \Omega_c h^2 \leq 0.99$, $0.2 \leq h \leq 1.0$, $-0.5 \leq \Omega_k \leq 0.5$, $-3 \leq w \leq 0.2$, and $0 \leq \alpha \leq 10$.

### 4 Observational constraints

Figures 1, 2 and 3 show the likelihood distributions of model parameters of flat and nonflat $\Lambda$CDM, XCDM, and $\phi$CDM models, respectively. The mean and 68.3% confidence limits (or 95.4% upper limits) are summarized in Table 2.

The SN + BAO data do not tightly constrain $\Omega_b h^2$, $\Omega_c h^2$, and especially not $H_0$, in the flat and nonflat $\Lambda$CDM, XCDM, and $\phi$CDM models. However, the SN + BAO data provide the most restrictive constraints on the dynamical dark energy parameters $w$ and $\alpha$ in the XCDM and $\phi$CDM models.

The results obtained using BAO + $H(z)$ data are interesting. The Hubble constant measured using the flat and nonflat XCDM and $\phi$CDM models are lower than the recent local measurement of $H_0 = 73.48 \pm 1.66 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Riess et al. 2018) by between 2.6$\sigma$ and 3.1$\sigma$ (of the quadrature sum of the two error bars), while in the flat (nonflat) $\Lambda$CDM model it is lower by 1.9$\sigma$ (1.3$\sigma$). In the nonflat XCDM parameterization, the BAO + $H(z)$ data strongly favor dark energy dynamics with $w$ deviating from $-1$ towards $0$ by 4.0$\sigma$. For the XCDM parameterization and the full SN + BAO + $H(z)$ data set, the equation of state parameter $w$ in the flat model is measured to be consistent with that of the cosmological constant ($w = -1$) while it deviates from $w = -1$ by 2.0$\sigma$ in the nonflat case, which is still significant though smaller than the 4.0$\sigma$ of the BAO + $H(z)$ case. In the nonflat $\phi$CDM model, the BAO + $H(z)$ data constraint also favors dark energy dynamics with $\alpha = 3.1 \pm 1.5$ (a 2.1$\sigma$ deviation from $\alpha = 0$), but for the SN + BAO + $H(z)$ data combination $\alpha$ is consistent with zero and a cosmological constant. We note that in the nonflat $\phi$CDM model the CMB data alone (without lensing data) cannot tightly constrain $\alpha$, allowing large $\alpha \approx 10$ (Park and Ratra 2018c), while even the least effective combination for this here, BAO + $H(z)$, is able to bound $\alpha < 8$, see the bottom subplot in the right panel of Fig. 3.

For the full SN + BAO + $H(z)$ data combination, closed spatial hypersurfaces are favored at 1.1$\sigma$, 2.1$\sigma$, and 1.4$\sigma$ significance in the nonflat $\Lambda$CDM, XCDM, and $\phi$CDM models. The Planck 2015 CMB anisotropy measurements (Ade et al. 2016) also favor closed spatial hypersurfaces (Ooba et al. 2018a,b,c), at 1.8$\sigma$, 1.1$\sigma$, and 1.8$\sigma$ in the XCDM, XCDM, and $\phi$CDM untitled nonflat inflation cases, and when combined with the SN + BAO + $H(z)$ data, as well as with growth factor ($f\sigma_8$) observations, they favor closed hypersurfaces at 5.2$\sigma$, 3.4$\sigma$, and 3.1$\sigma$ significance, respectively (Park and Ratra 2018a,h,c). It is interesting, and possibly significant, that in Table 2 all three pairs of data combinations, SN + $H(z)$, SN + BAO, and BAO + $H(z)$, also favor closed geometries in the nonflat models, at between 1.0$\sigma$ and 2.9$\sigma$.

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5Although we use the parameter $\theta_{\text{MC}}$, the approximate angular size of the sound horizon at recombination (Ade et al. 2014), instead of $H_0$ in our $\Lambda$CDM and XCDM model analyses, we instead record the derived $H_0$ as one of the main cosmological parameters for these models. For the $\phi$CDM model, however, $H_0$ (not $\theta_{\text{MC}}$) is the active parameter in the MCMC analysis.

6For earlier discussions of constraints on spatial curvature, see Farooq et al. (2015), Chen et al. (2016), Yu and Wang (2016), LuHuillier and Shafileo (2017), Farooq et al. (2017), Wei and Wu (2017), Rana et al. (2017), Yu et al. (2018), Mitra et al. (2018, 2019), and Ryan et al. (2018, 2019).
Fig. 1 Left panel: two- and one-dimensional likelihood distributions of flat $\Lambda\text{CDM}$ model parameters ($\Omega_b h^2$, $\Omega_c h^2$, $H_0$) constrained using the SN + $H(z)$, SN + BAO, BAO + $H(z)$, and SN + BAO + $H(z)$ data combinations. Right panel: similar distributions of nonflat $\Lambda\text{CDM}$ model parameters ($\Omega_b h^2$, $\Omega_c h^2$, $H_0$, $\Omega_k$). Horizontal and vertical lines in the $H_0$-related plots indicate the recent local Hubble constant measurement (solid lines) and 68.3% confidence limits (dashed lines) of Riess et al. (2018), $H_0 = 73.48 \pm 1.66 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The dashed lines in $\Omega_k$-related plots demarcate the spatially-flat model. In both panels, the baryonic density parameter $\Omega_b h^2$ is not constrained by the SN + $H(z)$ data.

Fig. 2 Similar to Fig. 1, but for flat XCDM model parameters ($\Omega_b h^2$, $\Omega_c h^2$, $H_0$, $w$) in the left panel, and for nonflat XCDM model parameters ($\Omega_b h^2$, $\Omega_c h^2$, $H_0$, $\Omega_k$, $w$) in the right panel. The dashed lines in $w$-related plots indicate $w = -1$ (the cosmological constant).

Using the SN + BAO + $H(z)$ combination, $H_0$ is measured to be $69.0 \pm 1.7$ ($69.8 \pm 1.8$), $68.9 \pm 1.7$ ($70.1 \pm 1.9$), and $68.5 \pm 1.8$ ($69.6 \pm 1.9$) $\text{km s}^{-1} \text{ Mpc}^{-1}$ for the flat (nonflat) $\Lambda\text{CDM}$, XCDM, and $\phi\text{CDM}$ models, respectively, these are all very mutually consistent and are also consistent with the most recent median statistics estimate.
of $H_0 = 68 \pm 2.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Chen and Ratra 2011a), which is very consistent with earlier estimates based on median statistics (Gott et al. 2001; Chen et al. 2003).\(^7\) However, these values are a little lower than the recent local expansion rate measurement of $H_0 = 73.48 \pm 1.66 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Riess et al. 2018)\(^8\) by between 1.9σ and 2.0σ for the flat models and between 1.3σ and 1.5σ for the nonflat models (of the quadrature sum of the two error bars, in both cases), less discrepant than when the CMB anisotropy data is included in the mix (Park and Ratra 2018a,b,c).

It is also interesting to see the estimated values of the current matter density parameter (for the full non-CMB data), $\Omega_m = 0.302 \pm 0.014$ (0.321 ± 0.022), 0.297 ± 0.019 (0.325 ± 0.023), and 0.287 ± 0.018 (0.305 ± 0.025) for the flat (nonflat) $\Lambda$CDM, XCDM, and $\phi$CDM models, respectively. The flat models and both the $\phi$CDM cases are more consistent with the Dark Energy Survey (DES) constraint, $\Omega_m = 0.264_{-0.019}^{+0.032}$ (Abbott et al. 2018) while the nonflat $\Lambda$CDM and XCDM model results are 1.5σ (of the quadrature sum of the two error bars) larger than the DES measurement.

Table 3 summarizes the individual and total $\chi^2$ for the best-fit flat and nonflat $\Lambda$CDM, XCDM, and $\phi$CDM models. The best-fit set of parameters for each model has been determined by using Powell’s minimization method (built into the COSMOMC program) for finding the location of the maximum likelihood. The $\Delta\chi^2$ of the XCDM and $\phi$CDM models denotes the excess $\chi^2$ relative to the $\Lambda$CDM one for the same combination of data sets and spatial curvature sign. The last two columns list the number of degrees of freedom $v$ and the reduced chi-square $\chi^2/v$. The number of degrees of freedom is $v = N - n - 1$, where $N$ is the number of data points and $n$ is the number of parameters. For example, for the nonflat $\phi$CDM model constrained using SN + BAO + $H(z)$ data, $N = 1048 + 12 + 31 = 1091$ and $n = 5 + 5 = 10$, considering the five cosmological parameters ($\Omega_b h^2$, $\Omega_c h^2$, $H_0$, $\alpha$, $\Omega_k$) and the five nuisance parameters of the SN sample. Except for the case of the nonflat XCDM parameterization constrained using the SN + BAO + $H(z)$ data, the XCDM parameterizations fit the observations better than do the $\Lambda$CDM models. Furthermore, the $\phi$CDM models better fit the data than do the XCDM parameterizations, except for the flat $\phi$CDM case constrained using SN + $H(z)$ data.\(^9\) However, the $\Delta\chi^2$ values are not very statistically significant.

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\(^7\)The $H_0$ estimates here are consistent with many recent estimates based on non-CMB data (L’Huillier and Shafiello 2017; Chen et al. 2017; Wang et al. 2017; Lin and Ishak 2017; Abbott et al. 2017; Yu et al. 2018; Haridasu et al. 2018a; Zhang et al. 2018; Gómez-Valent and Amendola 2018; Haridasu et al. 2018b; da Silva and Cavalcanti 2018; Zhang 2018) as well as with those from CMB data (Aghanim et al. 2018; Park and Ratra 2018a,b,c).

\(^8\)Other local expansion rate measurements find slightly lower $H_0$ values and slightly larger error bars (Rigault et al. 2015; Zhang et al. 2017; Dhawan et al. 2017; Fernández Arenas et al. 2018); also see Roman et al. (2017), Kim et al. (2018), and Jones et al. (2018).

\(^9\)The main reason for the smaller $\chi^2$ value in the nonflat $\phi$CDM model is that it fits the BAO data much better than do the $\Lambda$CDM and XCDM models.

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![Fig. 3](image-url) Similar to Fig. 1, but for flat $\phi$CDM model parameters ($\Omega_b h^2$, $\Omega_c h^2$, $H_0$, $\alpha$) in the left panel, and for nonflat $\phi$CDM model parameters ($\Omega_b h^2$, $\Omega_c h^2$, $H_0$, $\Omega_k$, $\alpha$) in the right panel.
Comparing the results for SN + $H(z)$ and SN + BAO data in Figs. 1–3, we see that the BAO data are less restrictive than the Hubble parameter data in the parameter estimation. Especially, SN + BAO data do not provide a tight constraint on $H_0$, allowing extreme values of the Hubble constant $H_0 > 90$ km s$^{-1}$ Mpc$^{-1}$. This seems to be in contradiction to the recent estimation of Hubble constant using the inverse distance ladder method (Macaulay et al. 2019; Aubourg et al. 2015), where the Hubble constant has been tightly constrained by the SN and BAO data with a reasonable prior.

### Table 2 Flat and nonflat $\Lambda$CDM, XCDM, and $\phi$CDM model parameter constraints from SN, BAO, and $H(z)$ data (mean and 68.3% confidence limits)

| Parameter                  | SN + $H(z)$ | SN + BAO | BAO + $H(z)$ | SN + BAO + $H(z)$ |
|----------------------------|-------------|----------|--------------|-------------------|
| Flat $\Lambda$CDM model    |             |          |              |                   |
| $\Omega_{b}h^2$            | ...         | 0.043 ± 0.014 | 0.02310 ± 0.0030 | 0.0235 ± 0.0028   |
| $\Omega_{c}h^2$            | 0.1205 ± 0.0088 | 0.174 ± 0.042 | 0.1212 ± 0.0086 | 0.1197 ± 0.0074   |
| $H_0$ [km s$^{-1}$ Mpc$^{-1}$] | 69.1 ± 1.8 | > 61.4 (95.4% C.L.) | 68.8 ± 1.8 | 69.0 ± 1.7 |
| $\Omega_{m}$               | 0.301 ± 0.020 | 0.302 ± 0.015 | 0.306 ± 0.019 | 0.302 ± 0.014 |
| Nonflat $\Lambda$CDM model |             |          |              |                   |
| $\Omega_{b}h^2$            | ...         | 0.036 ± 0.015 | 0.0205 ± 0.0037 | 0.0207 ± 0.0036   |
| $\Omega_{c}h^2$            | 0.168 ± 0.027 | 0.187 ± 0.054 | 0.137 ± 0.017 | 0.135 ± 0.016 |
| $H_0$ [km s$^{-1}$ Mpc$^{-1}$] | 70.2 ± 2.0 | > 57.0 (95.4% C.L.) | 70.1 ± 2.1 | 69.8 ± 1.8 |
| $\Omega_{k}$               | −0.23 ± 0.12 | −0.066 ± 0.066 | −0.086 ± 0.078 | −0.072 ± 0.065 |
| $\Omega_{m}$               | 0.387 ± 0.049 | 0.319 ± 0.023 | 0.322 ± 0.023 | 0.321 ± 0.022 |
| Flat XCDM parameterization |             |          |              |                   |
| $\Omega_{b}h^2$            | ...         | 0.044 ± 0.016 | 0.0317 ± 0.0080 | 0.0246 ± 0.0036   |
| $\Omega_{c}h^2$            | 0.127 ± 0.022 | 0.164 ± 0.045 | 0.087 ± 0.027 | 0.116 ± 0.012   |
| $H_0$ [km s$^{-1}$ Mpc$^{-1}$] | 68.9 ± 1.8 | > 57.0 (95.4% C.L.) | 65.5 ± 2.5 | 68.9 ± 1.7 |
| $\omega$                   | −1.07 ± 0.15 | −0.963 ± 0.070 | −0.72 ± 0.16 | −0.973 ± 0.071 |
| $\Omega_{m}$               | 0.316 ± 0.048 | 0.295 ± 0.019 | 0.276 ± 0.035 | 0.297 ± 0.019 |
| Nonflat XCDM parameterization |           |          |              |                   |
| $\Omega_{b}h^2$            | ...         | 0.038 ± 0.014 | 0.0313 ± 0.0093 | 0.0212 ± 0.0037   |
| $\Omega_{c}h^2$            | 0.155 ± 0.029 | 0.193 ± 0.051 | 0.095 ± 0.033 | 0.138 ± 0.016 |
| $H_0$ [km s$^{-1}$ Mpc$^{-1}$] | 70.6 ± 2.1 | > 59.6 (95.4% C.L.) | 66.0 ± 2.4 | 70.1 ± 1.9 |
| $\omega$                   | −0.28 ± 0.13 | −0.24 ± 0.11 | −0.32 ± 0.11 | −0.23 ± 0.11 |
| $\omega$                   | −0.92 ± 0.12 | −0.841 ± 0.066 | −0.604 ± 0.099 | −0.856 ± 0.071 |
| $\Omega_{m}$               | 0.358 ± 0.056 | 0.322 ± 0.022 | 0.291 ± 0.044 | 0.325 ± 0.023 |
| Flat $\phi$CDM model        |             |          |              |                   |
| $\Omega_{b}h^2$            | ...         | 0.049 ± 0.017 | 0.039 ± 0.010 | 0.0264 ± 0.0038   |
| $\Omega_{c}h^2$            | 0.097 ± 0.023 | 0.157 ± 0.042 | 0.062 ± 0.032 | 0.108 ± 0.011   |
| $H_0$ [km s$^{-1}$ Mpc$^{-1}$] | 69.2 ± 1.8 | > 57.2 (95.4% C.L.) | 64.8 ± 2.2 | 68.5 ± 1.8 |
| $\omega$ [95.4% C.L.]      | < 2.2       | < 0.82     | < 6.0 [2.5 ± 1.6 (68.3% C.L.)] | < 0.73 |
| $\Omega_{m}$               | 0.250 ± 0.049 | 0.284 ± 0.019 | 0.241 ± 0.045 | 0.287 ± 0.018 |
| Nonflat $\phi$CDM model     |             |          |              |                   |
| $\Omega_{b}h^2$            | ...         | 0.044 ± 0.017 | 0.041 ± 0.011 | 0.0240 ± 0.0043   |
| $\Omega_{c}h^2$            | 0.116 ± 0.037 | 0.173 ± 0.049 | 0.060 ± 0.035 | 0.123 ± 0.018   |
| $H_0$ [km s$^{-1}$ Mpc$^{-1}$] | 70.2 ± 2.1 | > 57.4 (95.4% C.L.) | 65.8 ± 2.2 | 69.6 ± 1.9 |
| $\omega$ [95.4% C.L.]      | < 2.8       | < 1.5      | 3.1 ± 1.5 (68.3% C.L.) | < 1.3 |
| $\Omega_{m}$               | 0.283 ± 0.072 | 0.300 ± 0.025 | 0.235 ± 0.048 | 0.305 ± 0.025 |

Note: $\Omega_m$ is a derived parameter.
Table 3 Individual and total $\chi^2$ values for the best-fit flat and nonflat $\Lambda$CDM, XCDM, and $\phi$CDM models

| Data sets | $\chi^2_{\text{SN}}$ | $\chi^2_{\text{BAO}}$ | $\chi^2_{\text{H}(z)}$ | Total $\chi^2$ | $\Delta\chi^2$ | $v$ | $\chi^2/v$ |
|-----------|----------------------|------------------------|------------------------|----------------|--------------|----|-----------|
| Flat $\Lambda$CDM model |
| SN + $H(z)$ | 1035.98 | 14.61 | 1050.59 | 1070 | 0.9819 |
| SN + BAO | 1035.99 | 10.06 | 1046.05 | 1051 | 0.9953 |
| BAO + $H(z)$ | 10.03 | 14.58 | 24.61 | 39 | 0.6310 |
| SN + BAO + $H(z)$ | 1036.00 | 10.03 | 14.61 | 1060.64 | 1082 | 0.9803 |
| Nonflat $\Lambda$CDM model |
| SN + $H(z)$ | 1035.88 | 14.56 | 1050.44 | 1069 | 0.9826 |
| SN + BAO | 1036.10 | 10.04 | 1046.13 | 1050 | 0.9963 |
| BAO + $H(z)$ | 10.29 | 14.97 | 25.26 | 38 | 0.6647 |
| SN + BAO + $H(z)$ | 1036.06 | 10.02 | 14.58 | 1060.66 | 1081 | 0.9812 |
| Flat XCDM parameterization |
| SN + $H(z)$ | 1035.93 | 14.43 | 1050.37 | -0.22 | 1069 | 0.9826 |
| SN + BAO | 1036.12 | 9.73 | 1045.84 | -0.21 | 1050 | 0.9960 |
| BAO + $H(z)$ | 7.03 | 14.98 | 22.01 | -2.60 | 38 | 0.5792 |
| SN + BAO + $H(z)$ | 1036.25 | 9.57 | 14.67 | 1060.49 | -0.15 | 1081 | 0.9810 |
| Nonflat XCDM parameterization |
| SN + $H(z)$ | 1035.90 | 14.51 | 1050.41 | -0.03 | 1068 | 0.9835 |
| SN + BAO | 1036.03 | 9.98 | 1046.00 | -0.13 | 1049 | 0.9971 |
| BAO + $H(z)$ | 10.30 | 14.71 | 25.02 | -0.24 | 37 | 0.6762 |
| SN + BAO + $H(z)$ | 1036.15 | 9.81 | 14.81 | 1060.76 | +0.10 | 1080 | 0.9822 |
| Flat $\phi$CDM model |
| SN + $H(z)$ | 1035.98 | 14.61 | 1050.59 | 0.00 | 1069 | 0.9828 |
| SN + BAO | 1036.38 | 9.37 | 1045.74 | -0.31 | 1050 | 0.9959 |
| BAO + $H(z)$ | 7.00 | 14.89 | 21.88 | -2.73 | 38 | 0.5758 |
| SN + BAO + $H(z)$ | 1036.36 | 9.39 | 14.68 | 1060.43 | -0.21 | 1081 | 0.9810 |
| Nonflat $\phi$CDM model |
| SN + $H(z)$ | 1035.84 | 14.54 | 1050.38 | -0.06 | 1068 | 0.9835 |
| SN + BAO | 1036.07 | 8.02 | 1044.09 | -2.04 | 1049 | 0.9953 |
| BAO + $H(z)$ | 4.14 | 15.05 | 19.19 | -6.07 | 37 | 0.5186 |
| SN + BAO + $H(z)$ | 1036.26 | 7.86 | 15.14 | 1059.27 | -1.39 | 1080 | 0.9808 |

Note: $\Delta\chi^2$ of the XCDM and $\phi$CDM models represent the excess value relative to $\chi^2$ of the corresponding $\Lambda$CDM model for the same combination of data sets and spatial curvature sign.



on the sound horizon size at recombination ($r_s$) based on the CMB data. In our analysis, however, we do not assume any prior on the sound horizon size because we aim to see how the cosmological parameters of the dark energy models are constrained without relying on the CMB data. Figure 4 shows the relation between the Hubble constant ($H_0$) and the sound horizon size at recombination ($r_s$) in the six models considered here. As expected, the case of SN + BAO data shows strong correlation between $H_0$ and $r_s$. For a higher value of Hubble constant, the lower sound horizon size is favored. We note that such a low value of sound horizon (e.g., $r_s \simeq 100$ Mpc) is certainly unrealistic in most cosmological models. However, we emphasize that adding Hubble parameter measurements to our analysis provide a very tight constraint on the sound horizon size. For SN + BAO + $H(z)$ data set, $r_s = 143.7 \pm 2.8$ Mpc (142.0 \pm 3.0 Mpc) in flat (nonflat) $\Lambda$CDM model, $r_s = 143.8 \pm 2.8$ Mpc (141.1 \pm 3.1 Mpc) in flat (nonflat) XCDM model, and $r_s = 144.6 \pm 2.9$ Mpc (142.5 \pm 3.2 Mpc) in flat (nonflat) $\phi$CDM model. These values are consistent with $r_s = 144.43 \pm 0.26$ Mpc of the flat $\Lambda$CDM model constrained with the Planck 2018 data (TT, TE, EE + lowE + lensing, Aghanim et al. 2018).

5 Summary

We have used Type Ia supernova apparent magnitude, baryon acoustic oscillation distance, and Hubble parameter measurements to constrain parameters of the flat and nonflat $\Lambda$CDM, XCDM, and $\phi$CDM models.
Our main results, in summary, are:

- These data favor closed spatial hypersurfaces at 1.1σ to 2.1σ, depending on the nonflat model.
- These data do not rule out dark energy dynamics.
- These data favor a smaller Hubble constant than the recent local expansion rate measurement of $H_0 = 73.48 \pm 1.66 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Riess et al. 2018) at 1.3σ to 2.0σ, depending on model.

These results are consistent with those that follow from similar analyses of CMB anisotropy data in untilted nonflat inflation models, and consequently joint analyses of CMB and non-CMB data reinforce the above findings (Park and Ratra 2018a,b,c).

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