Supersymmetry Breaking: constraint on U(1) R-charge

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Abstract

Holomorphy of the superpotential promotes any continuous symmetry group \( G \) to a complexified symmetry group \( G_C \) of the superpotential [1, 2]. For U(1) symmetry this means that the superpotential is not only invariant under U(1) phase rotation but also under some scaling. We use complexified R-symmetry to study the connection between choices of U(1) R-charges and existence of runaway directions as well as supersymmetry breaking global minimum in generic and calculable models.

1 Introduction

R-symmetries play a crucial role in spontaneous F-term supersymmetry (SUSY) breaking [3, 4]. It was shown by Nelson and Seiberg [3] that the existence of a U(1) R-symmetry is a necessary condition and spontaneously broken U(1) R-symmetry is a sufficient condition for SUSY breaking in generic and calculable models. Generic and calculable models are the dynamical supersymmetry breaking models whose low energy theories are described by effective and generic (superpotentials contain all the terms allowed by symmetries of the theory) supersymmetric Wess-Zumino (WZ) Lagrangians. In some generic and renormalizable WZ models, it is found that there is a connection between existence of runaway directions and choice of U(1) R-charges [5]. Along a runaway direction, scalar potential keeps on decreasing as we move away from the origin in the field space. We will study this issue in generic but effective WZ models. We will also study the connection of choices of U(1) R-charges with existence of a supersymmetry breaking global minimum in such theories.

Holomorphy and genericity give us power to find exact form of superpotentials [6, 7]. Holomorphy of the superpotential also promotes any continuous symmetry group \( G \) to a complexified symmetry group \( G_C \) of the superpotential [1, 2]. For example, if there is a U(1) symmetry, then superpotential will remain invariant not only under phase rotation but also under some scaling. Complexified R-symmetry \( U(1)_R^C \) was used to find connection between U(1) R-charge and existence of runaway directions in some generic and renormalizable WZ Models [5]. As this promotion of U(1) R-symmetry to \( U(1)_R^C \) has nothing to do with renormalizability of the superpotentials, we can also use it to find the connection between existence of runaway directions and choices of U(1) R-charges in generic but effective theories. After discussing the
frame work in sec. 2 we study the connection of U(1) R-charges of fields with the existence of runaway directions in sec. 3 and with the existence of SUSY breaking global minimum in sec. 4.

2 Frame-work

We consider generic and effective WZ models with a U(1) R-symmetry \((U(1)_R)\) and some \(Z_n\) internal symmetries. We denote R-charges of the fields \(\phi_i\) are \(R(\phi_i) = r_i\) in the normalization \(R(\theta) = 1\).

Let's assume that some of the fields with non-zero \(r_i\) get vacuum expectation values (VEV) in a vacuum and that one of these fields is \(\phi_1\). We can re-write the superpotential as follows.

\[
W = \frac{1}{2} \phi_1^2 f(U_2, U_3, \ldots, U_n), \quad \text{with} \quad U_i = \frac{\phi_i}{\phi_1^2},
\]

where \(U_i\)’s and the holomorphic function \(f\) are invariants of both the \(U(1)_R\) and \(U(1)_R^C\). To comment on SUSY breaking, we have to examine the following F-terms:

\[
\begin{align*}
\frac{\partial W}{\partial \phi_1} &= \frac{2}{r_1} \phi_1^{2/r_1 - 1} \left(f - \frac{r_1}{2} U_i \frac{\partial f}{\partial U_i}\right), \\
\frac{\partial W}{\partial \phi_i} &= \phi_i^{2/r_i - 1} \frac{\partial f}{\partial U_i} \quad \text{for } i \geq 2.
\end{align*}
\]

We see that, if we can solve the following equations

\[
\frac{\partial f}{\partial U_i} = 0, \quad \text{for } i \geq 2,
\]

then all the F-terms except that of the field \(\phi_1\) vanish. If we assume that \(f\) is also a generic function of \(U_i\)’s, then each of the above equations will contain all the variables \(U_i\). Therefore, number of independent equations will never be greater than the number of variables. Hence these equations can always be solved. Another important property of the above equations is that these are invariant under both \(U(1)_R\) and \(U(1)_R^C\). So, we can use the \(U(1)_R^C\) to increase or decrease \(|\phi_1|\) without altering these equations.

At the solution of these equations, the scalar potential takes the following form:

\[
V = \left(\frac{\partial^2 K}{\partial \phi_1 \partial \phi_1}\right)^{-1} \frac{4}{r_1^2} |\phi_1|^{4/r_1 - 2} |f|^2,
\]

and for canonical Kähler potential it looks simpler:

\[
V_{\text{can}} = \frac{4}{r_1^2} |\phi_1|^{4/r_1 - 2} |f|^2.
\]

Let’s now prove why genericity of the superpotentials \(W\) do not necessarily mean that \(f\)’s are also generic functions of \(U\)’s. We consider a model with four fields \(X, Y, Z\) and \(\phi_0\) where \(R(X) = R(Y) = 2, R(Z) = 1\) and \(R(\phi_0) = 0\), and under a \(Z_2\) internal symmetry only \(Y\) is odd. We also consider that the superpotential is
non-singular for any finite values of the fields. Then we can re-write the generic superpotential as follows:

\[ W = Xh_1(\phi_0) + Yh_2(\phi_0) + Z^2h_3(\phi_0) \]
\[ = X\{h_1(\phi_0) + \tilde{U}_1h_2(\phi_0) + \tilde{U}_2h_3(\phi_0)\}, \] (7)

where \( h_2 \) is odd under \( Z_2 \), \( \tilde{U}_1 = \frac{Y}{X} \) and \( \tilde{U}_2 = \frac{Z^2}{X} \). Clearly \( f = h_1(\phi_0) + \tilde{U}_1h_2(\phi_0) + \tilde{U}_2h_3(\phi_0) \) is not a generic function of \( \tilde{U}_1 \)'s because there is no reason from symmetry ground why the terms like \( \sum_{i,j\geq 2}(\tilde{U}_1)^i(\tilde{U}_2)^j h_{ij}(\phi_0) \) where \( h_{ij} \)'s transform as \((-1)^i h_{ij} \) under \( Z_2 \), are absent.

3 Runaway directions

For \( r_1 \notin [0,2] \), the exponent of \(|\phi_1|\) in the Eq. (6) is negative. So, the scalar potential \( V_{\text{can}} \) monotonically decreases as \(|\phi_1|\) increases and tends to zero when \(|\phi_1| \to \infty\).

Hence the scalar potential has a runaway direction if there is a field with R-charge \( \notin [0,2] \). If we have some U(1) internal symmetries in the theory then definition of R-charges become arbitrary. For this case, using this arbitrariness if we can make \( r_1 \notin [0,2] \), then there will be a runaway direction of the potential. This demand is true even for non-canonical Kähler potential \( K(\phi^\dagger, \phi) \), if \( \frac{\partial^2 K}{\partial\phi^\dagger\partial\phi} \) decreases more slowly than \(|\phi_1|^{\frac{4}{r_1^2}} - 2\).

This result is different from what is obtained by Ferretti [5]. He has shown that, in a particular class of generic and renormalizable models, if there is a field with R-charge not equal to 0,1,2 then there will always be a runaway direction. The R-charges which are excluded in our result for existence of runaway directions are also excluded in Ferretti’s result. Some extra R-charges are also excluded in his case and this might be characteristic of this class of models.

Now we are going to discuss how our result will change if the theory has some bigger symmetry, say SU(N), with the help of the famous ISS model [6]. Fields and their representations under the global symmetries of this model are given below:

| Field | SU(N) | SU(F) | U(1) | U(1)_R |
|-------|-------|-------|------|--------|
| \( \Phi \) | 1     | Adj + 1 | 0    | 2      |
| \( \phi \) | \( N \) | \( F \) | 1    | 0      |
| \( \tilde{\phi} \) | \( N \) | \( F \) | -1   | 0      |

where \( F > N \). In this model, Kähler potential is taken to be canonical and the superpotential is as follows:

\[ W = h\text{Tr}\phi\Phi\tilde{\phi} - h\mu^2\text{Tr}\Phi \] (8)

It is shown in Ref. [5] that there is SUSY breaking. Now, though we have assigned R-charges of \( \phi \) and \( \tilde{\phi} \) to zero, using the extra U(1) symmetry we can make them arbitrary as \( R(\phi) = -R(\tilde{\phi}) = q \).

So, this model illustrates the fact that though R-charges of some fields do not belong to the interval \([0,2]\), yet there is no runaway direction. However, if we rewrite the ISS superpotential in terms of SU(N) and SU(F) invariant operators as follows:

\[ W = hA - h\mu^2B, \] (9)
where $A = \text{Tr}\phi\Phi\phi$ and $B = \text{Tr}\Phi$, then we see that the operators $A$ and $B$ carry R-charge 2.

To examine existence of runaway directions in WZ models with bigger symmetries, we have to rewrite Kähler potentials and superpotentials in terms of invariant operators first and then we have to apply the procedure which we have discussed throughout this section.

4 Supersymmetry breaking

In this section we will deal with canonical Kähler potential only. As there exists a runaway direction if any field carries R-charge outside the interval $[0, 2]$, there cannot be a SUSY breaking global minimum for these choices of R-charges. Now the question is, can we get SUSY breaking global minimum for all other choices of R-charges? For $r_1 = 0$, we cannot re-write the superpotentials in the form given in Eq. (1) and we cannot perform rest of the analysis. For $r_1 = 2$, the exponent of $|\phi_1|$ in Eq. (6) is zero and there may be supersymmetry breaking depending on value of $f$ and on R-charges of other fields present in the theory. Now, for the case where $r_1 \in ]0, 2[$ (i.e. $0 < r_1 < 2$) the exponent of $|\phi_1|$ is positive. Hence $V_{\text{can}}$ decreases if we decrease $|\phi_1|$ using $U(1)_R$. We can make $|\phi_1|$ arbitrarily small but not zero because then $U_i$’s will be ill-defined. But if $V_{\text{can}}$ is non-zero for $|\phi_1| = 0$, then the scalar potential is discontinuous at $|\phi_1| = 0$.

Thus, we have seen that if there is any field whose R-charge is not equal to 0 or 2 then either there is no SUSY breaking or the scalar potential is discontinuous at the origin of the fields space.

5 Conclusion

We have seen that if we assume both $W$ and $f$ are generic functions, then we can solve all the F-term equations except the F-term equation corresponding to $\phi_1$ and we can write the scalar potential in a simpler form. On the other hand holomorphy of the superpotential promotes $U(1)$ R-symmetry to a complexified $U(1)$ R-symmetry of the superpotential and this enhancement of symmetries has nothing to do with renormalizability of the superpotentials. Using complexified $U(1)$ R-symmetry we find that, if the theory has a canonical Kähler potential and some $U(1)$ and $Z_n$ internal symmetries, then the scalar potential has a runaway direction if there is a field with R-charge $\notin [0, 2]$. So there is no SUSY breaking global minimum for these choices of fields. We have also found that if there is a field belongs to the open interval $]0, 2[$ then either there is no SUSY breaking or scalar potential is discontinuous at the origin of the fields space.

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