Pseudo-Random Sequence (PRS)
(Space)Time-Modulated Metasurfaces

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Abstract
This paper presents a novel class of (space)time-modulated metasurfaces, namely (space)time meta-
surfaces that are modulated by pseudo-random sequence (PRS) waveforms. In contrast to their harmon-
ically or quasi-harmonically modulated counterparts, these metasurfaces massively alter the temporal
spectrum of the waves that they process; as a result, they exhibit distinct properties and offer com-
plementary applications. These metasurfaces are assumed here to operate in the 'slow-modulation'
regime, where the fixed-state time between state-transition is much larger than the transient time asso-
ciated with the dispersion of the media involved, which allows safe separation of the time-variance
and frequency-dispersive effects of the system. Thanks to the special properties of their modulation, which
are generally assumed to have a staircase shape and to be periodic in addition to being pseudo-random,
the PRS (space)time-modulated metasurfaces can perform a number of unique operations, such as spec-
trum spreading, interference suppression, and row/cell selection. These properties, combined with mod-
ern microwave CMOS technologies, lead to applications with unique performance or/and features, such
as electromagnetic stealth, secured communication, direction of arrival estimation, and spatial multi-
plexing.

1 Introduction

After hardly a decade of intensive worldwide research, metasurfaces have become a revolutionary advance
in microwave, terahertz and optical technologies\textsuperscript{(1)}, and this area is far from being exhausted at the time
of this writing. Metasurfaces may be considered as dramatic generalizations of Fresnel-zone plate reflec-
tors\textsuperscript{(2)}, reflect/transmit arrays\textsuperscript{(3)}, frequency/polarization selective surfaces\textsuperscript{(4)}, diffraction gratings\textsuperscript{(5)}
and spatial light modulators\textsuperscript{(6)}, given the unprecedented versatility provided by their 36 bianisotropic de-
grees of freedom\textsuperscript{(1)}; alternately, they may be considered as the two-dimensional reductions of voluminal
metamaterials\textsuperscript{(9)}, with the benefits of lower form factor, lower loss, along with – amazingly! – far greater
wave transformation capabilities. They have already led to a myriad of applications with unprecedented
functionality or/and performance, including refraction and wavefront transformation\textsuperscript{(10; 11; 12; 13; 14)}, ab-
sorption\textsuperscript{(15; 16; 17)}, polarization transformation\textsuperscript{(18)}, holography\textsuperscript{(19; 20)}, analog computing\textsuperscript{(21)}, optical
force carving\textsuperscript{(22; 23)}, and cloaking\textsuperscript{(24; 25; 26; 27)}.

While the vast majority of the metasurfaces reported until 2018 were static, i.e., invariant in time, the past
lustrum has witnessed an explosion of interest for time-modulated and spacetime-modulated metasur-
faces. In these metasurfaces, the time dimension is introduced as an extra degree of freedom to manip-
ulate electromagnetic waves, both by breaking the fundamental bounds of linear time-invariant systems
and by bringing about the possibility to engineer the temporal spectrum of waves in addition to just their
spatial spectrum\textsuperscript{(28; 29)}. These time/spacetime-modulated metasurfaces have quickly given rise to their
own range of applications, extending those of static metasurfaces, with the main applications including
simplified front-end wireless communication\textsuperscript{(30; 31; 32)}, spatial frequency conversion\textsuperscript{(33; 34; 35)}, and
generalized nonreciprocity\textsuperscript{(36; 37; 38; 39; 40; 41)}, and this field is currently evolving at a spectacular
pace.

\textsuperscript{1}Bianisotropy is a weak form of spatial dispersion (or spatial nonlocality) occurring in complex media, whereby the response
of the medium at a point of space does not depend on the excitation only at this point but also in the neighbourhood of it\textsuperscript{(1; 7; 8)}. It is the dominant form of spatial dispersion in subwavelength structures, such as typical metamaterials and metasurfaces, but
mesoscopic structures, whose unit cells are not much smaller than the wavelength, involve higher-order spatial dispersion, with even
more degrees of freedom that could be leveraged for further engineering opportunities\textsuperscript{(1)}.
The time/spacetime-modulated metasurfaces mentioned in the applications of the previous paragraph are based on modulation waveforms that are fully deterministic. Specifically, these waveforms are either sinusoidal or exhibiting shapes that are causally related by the required metasurface operation. Such metasurfaces induce relatively light alterations, typically limited to harmonic generation and frequency conversion, of the spectrum of the waves they process. We present here an overview of a different class of time/spacetime-modulated, namely metasurfaces whose modulation is a Pseudo-Random Sequence (PRS), typically of staircase shape and periodic nature. In contrast to their deterministic-modulation counterparts, these metasurfaces induce major alterations of the temporal spectrum of the waves that they process. As a result, they offer complementary applications, such as electromagnetic stealth (42), direction of arrival (DoA) estimation (43), and spatial multiplexing (44).

The paper is organized as follows. Section 2 presents a general description of time-modulated metasurfaces, their operation principle, and their modulation regimes. Then, Sec. 3 establishes the key properties of PRS modulation waveforms in terms of autocorrelation, power spectral density, and orthogonality. Next, Sec. 4 overviews the fundamental operations of PRS time-modulated metasurfaces, namely spectrum spreading, interference suppression, and signal selection. This concludes the theoretical part. From that point, the paper goes on with a practical part. Section 5 discusses technological aspects for microwave implementations, specifically the realization of a binary metaparticle and the design of a fully addressable modulation array. Then, Sec. 6 presents four applications, namely electromagnetic stealth, secured communication, direction of arrival estimation, and spatial multiplexing. Finally, Sec. 7 provides conclusions and invokes possible future developments of the field.

2 Time-Modulated Metasurfaces

Figure 1 depicts a generic (space)time-modulated metasurface, assuming a staircase modulation, and its operation principle. A time-harmonic wave, \( \psi(t) \), of frequency \( f_0 = \omega_0/(2\pi) \), and hence of period \( T_0 = 1/f_0 \), impinging on the metasurface, which is typically modeled by a bianisotropic susceptibility tensor \( \chi \) during a fixed-state time interval of the staircase modulation. In order to efficiently process this wave, the metasurface must strongly interact with it; it is therefore designed to be at all times resonant near the frequency of the incident wave \( (f_0) \). As a result, its impulse response during the \( n^{th} \) fixed-state time interval, \( T_{1,n} \), of the modulation, \( \chi T_{1,n}(t') \), rings at the frequency \( \nu_{0,n} = [\omega_{0,n}^2 - (\gamma_n/2)^2]^{1/2} \), where \( \omega_{0,n} \approx \omega_0 \) and \( \gamma_n \) are respectively the resonance frequency and the damping of the metasurface during the interval \( T_{1,n} \), and features a time constant of \( \tau_{dn} = 2/\gamma_n \) and hence a memory time of \( T_{d,n} = 3\tau_{dn} = 6/\gamma_n \).

The metasurface is modulated, via a proper electronic control of its scattering particles, by the modulation waveform \( m(t) \) (Fig. 1). This waveform is typically periodic, with \( N \) time intervals of duration \( T_{1,n} \) per period and with period \( T_n = \sum_{n=1}^{N} T_{1,n} = T_m \). This wave modulates the metasurface by varying its dispersive \( (\omega) \) susceptibility in time \( (t) \), in possible addition to space \( (\rho) \), i.e., as \( \chi = \chi(\rho, t; \omega) \). This modulation transforms the incident wave into a scattered wave, \( \psi_s(t) \), which may be either a wave that is transmitted through the metasurface (as in the figure) or a wave that is reflected from it. The scattered wave exhibits a transient response during the time \( T_{1,n} \), with \( T_{1,n} \gg T_0 \), following the transition from the \( (n - 1)^{th} \) to the \( n^{th} \) modulation intervals, as illustrated in Fig. 1. This transient time is proportional to the dispersion difference between the modulation states \( (n - 1) \) and \( n \), and is limited to \( T_{d,n} \). If the modulation has a smooth (rather than a staircase) form, which is also quite common, we have \( T_{1,n} \to 0 \), and \( T_{1,n} \to 0 \) since the difference between the modulation states \( (n - 1) \) and \( n \) vanishes.

One may distinguish two modulation regimes depending on the ratio between the time scales of the modulation, which is related to the fixed-state time \( (T_{1,n}) \) if the modulation is of staircase form, and of the dispersion of the metasurface, which is related to the transient time \( (T_{1,n}) \). In this paper, we are mostly concerned with the regime \( (T_0 \ll T_{1,n} \ll T_{1,n}) \) (case of Fig. 1), which we shall refer to as the slow-modulation regime. In this regime, the transient part of the response can be neglected, since it is much shorter than the steady-state part of the modulation states (see \( \psi_s(t) \) in Fig. 1), and the polarization density response of the metasurface can hence be safely written in terms of a multiplication product in the time domain, as \( P(t) = \epsilon_0 \chi(\rho, t; \omega) \cdot E(t) \) (temporal locality), where the transients due to dispersion \( (\omega) \) may be simply ignored. In contrast, for \( T_{1,n} \sim T_{1,n} \), or \( T_{1,n} > T_{1,n} \), which we shall refer to as the fast-modulation regime, the temporal effects due to the modulation and to the dispersion (or memory) of the system are intertwined.

2 The spatial modulation may be either intrinsic to the metasurface, i.e., consisting in different metaparticle shapes, or induced by the modulation, for instance via different varactor capacitance levels, in which case the modulation would be indexed as \( m_{ij}(t) \), with \( i \) and \( j \) referring to the rows and columns of the structure, assuming a rectangular lattice.
Figure 1: (Space)time-modulated metasurface, processing a harmonic incident electromagnetic wave using a modulation $m(t)$, which will be a staircase periodic Pseudo-Random Sequence (PRS) in this paper. The metasurface is generally dispersive ($\omega$-dependent) and space-varying ($\rho$-dependent) in addition to time-varying ($t$-dependent), and may be characterized by the susceptibility tensor function $\chi(\rho, t; \omega)$ in the slow modulation regime.

and the polarization response cannot anymore be expressed in terms of a susceptibility function (28). In the case of a smooth modulation, the situation is even more complicated. If the modulation is periodic, as in Fig. 1, the slow- and fast-modulation regimes would rather correspond to the regimes $T_m \ll T_d$ and $T_m \sim T_d$, respectively, where $T_d$ would now refer to the maximal dispersion time within one period\(^3\).

Note that the regime $T_m \sim T_0$, which belongs the fast regime, is the regime that typically prevails in parametric amplifiers (47; 48).

3 Pseudo-Random Sequence (PRS) Modulation

The term ‘pseudo-random sequence’ (PRS) refers to a sequence of values that are generated by an algorithm whose properties approximate the properties of a sequence of random numbers. Such a sequence has therefore, per se, a white-noise spectrum. However, we shall consider here that the PRS function is **periodic**. This assumption does not represent a restriction of generality insofar as an aperiodic function can always be treated as the limit of a periodic function with infinite period. However, periodicity endows the PRS with a rich set of temporal and spectral properties that are crucial in many applications. We shall establish here the characteristics and properties of this modulation, with the help of Fig. 2.

In this paper, we will restrict our attention to **binary polar staircase periodic PRS modulations**, $m(t)$, of the type plotted in Fig. 2(a). This PRS oscillates between the levels $\pm 1$. It has bits of duration $T_b$ and $N$ bits per period, corresponding to the period $T_p = NT_b$, with $N$ being generally odd for technological reasons (linear feedback shift register generation) (49).

The autocorrelation of a function is a measurement of the similarity of this function with delayed copies of

\(^{3}\text{One may also distinguish the ultra-slow regime } T_m \ll T_d \text{, which corresponds to a simple regime of reconfigurability (between different operation modes), and, at the other extreme, the ultra-fast regime } T_m \gg T_d, \text{ which prevails in attophysics (46), but these regimes are not of interest in this paper.}
Figure 2: Characteristics of the periodic Pseudo-Random Sequence (PRS) used as the modulation, \( m(t) \), in the time-modulated metasurface of Fig. 1. (a) Waveform. (b) Autocorrelation function, obtained by Eq. (1). (c) Power spectral density, obtained by Eq. (2).

itself versus the delay. In the case of the periodic PRS function \( m(t) \) in Fig. 2(a), it is found as (49)

\[
s_m(t) = \frac{1}{T_p} \int_{t_0}^{t_0+T_p} m(\tau)m(t + \tau) \, d\tau
\]

\[
= -\frac{1}{N} + \frac{N+1}{N} \sum_{n=-\infty}^{+\infty} \Lambda \left( \frac{t-nNT_b}{T_b} \right), \tag{1a}
\]

where \( t_0 \) is an arbitrary time and \( \Lambda(\cdot) \) is the triangular function

\[
\Lambda(t) = \begin{cases} 
1-|t| & \text{if } t \leq 1, \\
0 & \text{if } t > 1,
\end{cases} \tag{1b}
\]

This function is plotted in Fig. 2(b). It shares the periodicity \( T_p \) of \( m(t) \), since copies of the PRS that are delayed by an integer number of the period perfectly superimpose with each other. Moreover, it is composed of triangular pulses of width \( 2T_b \), corresponding to the correlation of rectangular pulses of width \( T_b \), with a small negative offset of \(-1/N\) that is due to the excess, by one unity, of the number of \(-1\)'s compared to the number of \(+1\)'s, due to the odd nature of \( N \). Note that the larger is \( N \), and hence the
more random is \( m(t) \), the larger is the spacing between the triangles \((T_p)\), and of course the smaller is the offset \( |\frac{1}{N}| \); in the limit \( N \to \infty \) of an aperiodic PRS, \( s_m(t) \) reduces to the triangle at the origin since there is then no delay anymore that leads to perfect superposition, and in the further limit \( T_b \to 0 \), \( s_m(t) \) reduces to a unit impulse at the origin, corresponding to a perfectly random wave, which superimposes only with the non-delayed copy of itself.

The Fourier transform of the autocorrelation function is called the Power Spectral Density (PSD). In the case of the PRS \( m(t) \) in Fig. 2(a), the PSD is (49)

\[
s_m(f) = \frac{1}{N^2} \delta(f) + \frac{1}{N^2} \sum_{n=-\infty}^{\infty} \sin^2 \left( \frac{n}{N} f \right) \delta \left( f - \frac{n}{N} f_b \right),
\]

and is plotted in Fig. 2(c). This function has a sinc\(^2(\cdot)\)-form envelope, corresponding to the Fourier transform of the triangular components of \( s_m(t) \), whose magnitude, \( (N+1)/N^2 \), depends only on \( N \). Due to the periodicity of \( s_m(t) \), with period \( T_p \), \( s_m(f) \) is a discrete function, with spacing of \( f_p = 1/T_p \) between its impulses; note the small DC component, of magnitude \( 1/N^2 \), due to the imperfect balance between the +1’s and −1’s of \( m(t) \), which quickly falls to zero as the length of the PRS increases. Finally, the bandwidth of \( s_m(t) \), defined as the width of the main beam, is \( f_b = 1/T_b \), and includes \( f_b/f_p = T_p/T_b = NT_b/T_b = N \) impulses.

Some PRS-modulated metasurface applications involve, as we shall see, a set of PRSs, \( \{ m_k(t) \} \), with \( k = 1, 2, \ldots, K \), where each of the \( K \) PRSs, \( m_k(t) \), is of the type of \( m(t) \) in Fig. 2(a) while composed of a distinct sequence of +1’s and −1’s. This set can be designed to be approximately orthogonal as

\[
\frac{1}{T_p} \int_{0}^{T_p} m_p(t)m_q(t)dt = \begin{cases} 1 & \text{if } p = q, \\ -\frac{1}{N} & \text{if } p \neq q, \end{cases}
\]

where \( m_p(t) \) and \( m_q(t) \) are an arbitrary pair of sequences in the set, and the −1/\( N \) result is due to the same reason as before.

4 Operations of PRS Time-Modulated Metasurfaces

PRS-modulated metasurfaces may perform three fundamental operations, which underpin their applications (Sec. 6): A. spectrum spreading, B. interference suppression, and C. row/cell selection. We shall demonstrate here these operations with the help of Fig. 3.

4.1 Spectrum Spreading

The spectrum spreading operation is depicted in Fig. 3(a). The incident wave, \( \psi(t) \), is assumed to have a narrow spectrum, \( \Delta f_i \), centered at \( f_0 \), and it is modulated by the binary \( \pm 1 \) PRS waveform \( m(t) \) described in Sec. 3. In the slow-modulation regime \( (T_0 \ll T_{i,n} \ll T_{i,n}) \), which is assumed throughout the paper, temporal locality (Sec. 4) allows to write the scattered wave as

\[
\psi_s(t) = m(t) \cdot \psi(t),
\]

where the symbol `·` denotes the simple multiplication product, which will be from now on omitted.

The Fourier transform of Eq. (4) expresses then the Fourier transform of the scattered wave as the convolution of the Fourier transforms of the modulation and of the input wave, namely

\[
\hat{\psi}_s(f) = \hat{m}(f) * \hat{\psi}(f).
\]

Given the assumed narrow width of \( \hat{\psi}(f) \), \( \Delta f_i \), and the much broader width of \( \hat{m}(f) \), \( 2f_b \), due to the pseudo-random nature of \( m(t) \) [Fig. 2(a)], this operation results in a response \( \hat{\psi}_s(f) \) that is much broader than \( \hat{\psi}(f) \) [Fig. 3(a)]. In other words, the PRS-modulated metasurface spreads out the spectrum of the wave that it processes by the factor \( 2f_b/\Delta f_i \), which can be extremely large in practice.

Note that the spreading of the spectrum by the modulation does not decrease the envelope of the power spectral density of the scattered wave, because the spectrum envelope is essentially a translation from zero to the frequency of the incident wave \( (f_0) \) of the modulation spectrum [Fig. 3(a)], whose envelope was shown in Sec. 3 to depend only on the length \( (N) \) of the PRS. However, the spectrum spreading is
accompanied by a dramatic reduction of the spectral power level of the wave, particularly at $f_0$ where the reduction is by the factor $1/N^2$, but also elsewhere, where the reduction is in the form of the $\text{sinc}^2(\cdot)$ function with maximum of $(N + 1)/N^2$, which represents an overall major reduction of the power spectral level of the processed wave.
The spectrum spreading ratio between the scattered wave and the incident wave, $2f_b/\Delta f$, is particularly strong – in fact infinite! – when $\psi(t)$ tends to pure single-tone harmonic wave, i.e., $\psi(t) = e^{j2\pi f_0 t}$ and $\tilde{\psi}_m(f) = \delta(f - f_0)$. In this case, Eq. (5) reduces indeed to

$$\tilde{\psi}_m(f) = \tilde{m}(f - f_0),$$

(6)

which is simply the Fourier transform of $m(t)$ shifted to the frequency of the incident wave, and the scattered wave has therefore exactly the same spectral width as $\tilde{m}(f)$.

### 4.2 Interference Suppression

The interference suppression operation is depicted in Fig. 3(b). It corresponds to a scenario where an interfering wave, $\psi_{\text{int}}(t)$, which may typically be a harmonic wave or a narrow-band burst wave, escapes the metasurface and reaches unimpeded a point of space where the signal passed through the metasurface, $\psi(t)$, is to be demodulated.

The total wave reaching the point of interest is then

$$\psi(t) = \psi(t) + \psi_{\text{int}}(t) = m(t)\psi(t) + \psi_{\text{int}}(t),$$

(7)

and a receiver possessing there the information of $m(t)$ can process this wave as

$$\tilde{\psi}_d(f) = \tilde{\psi}(f)m(t) = \tilde{\psi}(f)m^2(t) + \tilde{\psi}(f)m(t)$$

$$= \tilde{\psi}(f) + \tilde{\psi}_{\text{int}}(f)m(t),$$

(8)

where we used the fact that $m^2(t) = 1$ for the assumed unitary bipolar symmetric sequence $m(t)$ [Fig. 2(a)]. The input wave has thus been demodulated by the receiver [Fig. 3(b)]. Moreover, Fourier-transforming the final result of Eq. (8) yields

$$\tilde{\psi}_d(f) = \tilde{\psi}(f) + \tilde{\psi}_{\text{int}}(f) \ast \tilde{m}(f),$$

(9)

which shows that the spectrum of the interfering wave has been spread out, just as the wave incident on the metasurface in the previous subsection, so that its power spectral level has decreased to a very small level Fig. 3(b). The effect of the interference can then be further reduced, if necessary, to quasi total suppression by applying a band-filter at $f_0$ so as to eliminate all of its energy contents distributed over the other spectral impulses.

### 4.3 Row/Cell Selection

The row/cell selection operation is depicted in Fig. 3(c). This pertains yet to another scenario, where the goal is to select out, at a given point of space where this selection is to be performed, the parts of the incident wave, $\psi(t)$, that impinged on the different rows/cells of the metasurface. We shall call the so-defined $k$th part of the wave $\psi_{t_k}(t)$, where $k = 1, 2, \ldots, K$, for a metasurface modulated in its $K$ columns. Here, the incident wave may generally be a combination of different waves coming from arbitrary directions of space. The row/cell selection operation can be performed using an orthogonal set of modulation PRSs, $\{m_k(t)\}, k = 1, 2, \ldots, K$, i.e., a set of PRSs whose any pair satisfies the condition (3).

Note that such a system is actually a space-time modulated system. Indeed, the different rows/cells, even if they happen to have all the same shape, are distinctly modulated, so that the incident wave actually "sees" a spatial modulation in addition to the temporal modulation when impinging on the metasurface.

If the $K$ parts of the incident wave have distinct magnitudes and phases, they may be generically expressed as $\psi_{t_k}(t) = A_k(t)e^{j(\omega_0 t + \phi_k(t))}$, and the global signal scattered by the metasurface is then

$$\psi_i(t) = \sum_{k=1}^{K} m_k(t)\psi_{t_k}(t) = \sum_{k=1}^{K} m_k(t)A_k(t)e^{j(\omega_0 t + \phi_k(t))},$$

(10)

which represents a row/cell encoded mixture of the incident wave. This scattered wave may then down-
converted to base-band as

\[ \psi_0(t) = \psi_k(t)e^{-j\omega_t} = \sum_{k=1}^{K} m_k(t)A_k(t)e^{j\phi_k(t)}. \] (11)

If the PRS set \( \{m_k(t)\} \) is known, and assuming that \( A_k(t) \) and \( \phi_k(t) \) are slowly varying function of time with respect to the time scale of the modulation period \( T_p \) (Figs. 1 and 2), this wave may be processed as follows at the point of space where the selection is to be performed:

\[
\psi_{dr} = \frac{1}{T_p} \int_0^{T_p} m_r(t)\psi_0(t)dt = \frac{1}{T_p} \int_0^{T_p} m_r(t) \sum_{k=1}^{K} m_k(t)A_k e^{j\phi_k} dt
\]

\[
= A_r e^{j\phi_r} \int_0^{T_p} m_r^2(t)dt.
\]

\[
+ \frac{1}{T_p} \sum_{k=1}^{K} A_k e^{j\phi_k} \int_0^{T_p} m_r(t)m_k(t)dt
\]

\[
= A_r e^{j\phi_r} \frac{1}{N} \sum_{k=1}^{K} A_k e^{j\phi_k} \approx A_r e^{j\phi_r}.
\] (12)

where the orthogonality relation (3) was used in the last-but-one equality and the approximation \( N \gg K \) was used in the last equality. This result shows that integrating an \( m_r(t) \)-weighted version of the wave \( \psi_0(t) \) selects out the magnitude and phase of the part \( \psi_{dr}(t) \) of the incident wave, i.e., the part of this wave that impinged on the \( r \)-th row/cell of the metasurface.

5 Design

The design of a time-modulated metasurface system involves a mixture of electromagnetics engineering, for the metasurface structure part, and of electronics engineering, for the time modulation part. While the former is equally mature in the microwave and optical regimes (1), the latter still represents an insurmountable task in the optical regime at the present time. Therefore, we shall discuss here only microwave-regime designs, with the help of the representative example described in Fig. 4.

Figure 4(a) depicts a unit-cell design for the assumed binary-polar PRS modulation and for individual cell addressing, with polarity reversal following from switching between the opposite sides of the receiving (resonant, and hence \( \lambda/2 \)-long) slotted-patch unit-cell particle. Figure 4(b) shows a simplified version of (a), and plots the corresponding scattering parameters, which exhibit an excellent broadband equal-magnitude and opposite-phase response. Figure 4(c) shows a modulation array for individual cell addressing, inspired by the Thin-Film Transistor (TFT) matrices used in screen displays (50) and implementable in standard CMOS technologies (51), with Field Programmable Gate Array (FPGA) control. Finally, Fig. 4(d) shows the individual-cell addressing scheme of this array, where the array refresh time, \( T_r \), is assumed to be much larger (up to the bit duration \( T_b \), see Fig. 2) than \( M/T_r \), with \( M \) being the number of rows and \( T_r \) the row selection time, so that the sequential addressing cycle of all the rows of the array can be, practically, considered simultaneous.

6 Applications

Figure 5 presents four applications of PRS (space)time-modulated metasurfaces. Figure 5(a) depicts the application of electromagnetic stealth (42). Instead of altering the shape of the object to conceal, as typical stealth techniques, this technology covers the object by the PRS time-modulated metasurface and alters its spectrum. The narrow-band signal of the interrogating radar is spread out upon reflection and its spectral power level is reduced below the noise floor of that radar, according to the spectrum spreading principles established in Sec. 4.1, which makes the object undetectable. Such temporal spectrum spreading can be enhanced by spatial dispersion upon adding spatial modulation in the form of alternating PEC and PMC metasurface cells (53).

Figure 5(b) shows the application of secured communication. Here, the spectrum spreading (Sec. 4.1) induced by PRS modulation acts as an encryption mechanism of the message. Whereas an eavesdropper
receives a jammed, undecipherable version of the message, plus possible interfering burst noise, the intended receiver can safely demodulate the message using the modulation key in his possession while eliminating the interfering noise, according to the interference suppression principles established in Sec. 4.2. If the position of the receiver is known, the metasurface may additionally use spatial modulation in the form of an appropriate phase gradient, possibly dynamic for motion tracking, to specifically radiate in the direction of the receiver, and hence further enhance the safety of the communication link.

Figure 5(c) describes the application of direction of arrival (DoA) estimation (43). In contrast to conventional DoA systems, which require an array of independent receive antennas with individual phase detectors, this DoA system offers the advantage of requiring only one pick-up antenna, thanks to the row/cell selection principles established in Sec. 4.3. The wave impinging on the metasurface, under an angle \( \theta \) (to determine), is modulated at each row of the metasurface by a different PRS from a set of mutually orthogonal PRSs. Then, the wave scattered by the metasurface is picked up by an antenna, down-converted to base-band and stored in a memory. Next, the phase of the waveform part originating from any \( r \)th row of the metasurface is determined from the operation corresponding to Eq. (12), and \( \theta \) is finally obtained from the so obtained phases between waveforms from adjacent rows, \( \phi_{r+1} \) and \( \phi_{r+1} \), as \( \theta = \sin^{-1}[e(\phi_{r+1} - \phi_{r+1})/(\omega \Delta d)] \), where \( \Delta d \) is the distance between adjacent rows of the metasurface.

Finally, Fig. 5(d) shows the application of spatial multiplexing (44), where the metasurface can operate either as a multiplexer or as a demultiplexer. In the demultiplexing mode, it receives a mixture of \( Q \) precoded...

Figure 4: Representative design example for a microwave PRS (space)time-modulated metasurface. (a) Unit-cell design, with binary-polar state switching principle and double-plate enclosure to support and isolate the electronics and bias lines from the scattering/radiating top and bottom parts. (b) Simplified version of (a), reported in (52), for only row addressing, with biasing on the top plane, and corresponding measured scattering parameters. (c) Overall modulation array for individual cell addressing corresponding to (a). (d) Detail of the individual cell addressing scheme in 2 x 2 section of (c), with state remanence ensured by a capacitance discharge time being much longer than the refresh time \( T_r \), for the setup \{11, 12; 21, 22\} = \{(on,off), (off,on); (off, on), (on, off)\}.
messages destined to different users, \( \hat{\psi}_i(t) = \sum_{q=1}^{Q} \hat{\psi}_{iq}(t) \), where \( \hat{\psi}_{iq}(t) = \psi_{iq}(t) \sum_{m=1}^{N} e^{j\phi_m} m(t) \) with \( \phi_m = -k_0 \Delta d \sin \theta_q \) representing the metasurface’s inter-row phase gradient that encodes the destination direction, \( \theta_q \), of the \( q \)th message, \( \psi_{iq}(t) \), and \( \{m_k(t)\} \) representing a set of orthogonal PRSs. The metasurface modulates the signal \( \hat{\psi}_i(t) \) by the same PRS sequences as those used to precode the messages, which results into the scattered wave \( \hat{\psi}_s(t, \theta) = \sum_{q=1}^{Q} m(t) \hat{\psi}_{iq}(t) e^{j\phi_m} \), where \( \phi_m = k_0 \Delta d \sin \theta_q \). It may be easily verified that substituting in this expression for \( \hat{\psi}_s(t, \theta) \), the expression for \( \hat{\psi}_i(t) \) and applying the PRS orthogonality relation (3) leads to \( \psi_s(t, \theta) = \sum_{q=1}^{Q} \psi_{iq}(t) \sum_{m=1}^{N} e^{j\phi_m} \Delta \phi_0 \sin \theta_q \). Thus, according to basic antenna array theory (54), each message \( \psi_{iq}(t) \) is properly radiated into its intended direction, \( \theta_q \), and the multiplexing operation is therefore accomplished as expected. The reciprocal, multiplexing operation of the system is essentially identical to that of the DoA system presented in the previous paragraph, with multiple angles corresponding to the multiple messages. Using a unique antenna, this metasurface multiplexing system clearly represents a major interest for massive Multiple-Input Multiple-Output (MIMO) systems.

7 Conclusion

We have presented an overview of a novel class of (space)time-modulated metasurfaces, namely (space)time-modulated metasurfaces that are modulated by pseudo-random sequence (PRS) waveforms. These metasurfaces exhibit very distinct and complementary properties to those of other (space)time-modulated metasurfaces, and offer thereby a cornucopia of new application opportunities. Many future research directions may be envisioned in this area, such as for instance the exploitation of other modulation sequences (e.g., Walsh-Hadamard, Gold and \( M \)-ary sequences), the combination of PRS time modulation
with the versatile electromagnetic transformations allowed by more sophisticated metaparticles (bianisotropic and higher-order spatially dispersive structures), and the investigation of potential of the so far quasi-unexplored fast-modulation regime.

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