Schwinger-Boson Mean Field Solution of Spin-1 (or 1/2) 2D Anisotropic Heisenberg Antiferromagnet

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Abstract

Using the Resonant Valence Bond (RVB) concept and the Schwinger-boson mean field approximation, we investigate a two dimensional anisotropic Heisenberg antiferromagnet. We find that increasing the coupling ratio ($\alpha = J_y/J_x$), a disordered-ordered phase transition appears beyond a critical value ($\alpha_c$). When $\alpha > \alpha_c$ there exists a finite $T_N$ beyond which an ordered-disordered phase transition occurs, while for $\alpha < \alpha_c$ there always exists a gap. Also we find that a decoupling temperature $T_D$ exists for 1D and 2D isotropic case, when $T$ approaches $T_D$ the RVB order parameter decreases rapidly to zero.

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I. INTRODUCTION

Low dimensional Heisenberg antiferromagnet (HAF) system has always been an interesting problem to theoretical physicists. Many new concepts and techniques are developed in this field. In the 1950’s P.W.Anderson and Kubo developed the spin-wave theory (SWT) \[^{11}\]. Spin-wave theory predicted for the existence of an ordered ground state and gaplessed spin-wave excitations above it for higher dimensional magnets. In 1973 \[^{2}\], P.W.Anderson proposed his well known Resonant Valence Bond (RVB) model to describe some spin-1/2 systems. Then in 1983 \[^{3}\], Haldane proposed his conjecture, it pronounced that the excitation spectrum of a linear-chain Heisenberg antiferromagnet (LCHA) with integer spin has a finite energy gap \(E_H\) above its singlet ground state, while a LCHA with half-odd-integer spin has a gapless spectrum. Later Affleck, Kennedy, Lieb and Tasaki (AKLT) \[^{4}\] generalized the RVB model to spin-1 LCHA by proposing the Valence-Bond-Solid (VBS) concept to treat the conjecture. They said that these valence bonds are formed by two 1/2 spins as a singlet \(\uparrow\downarrow - \downarrow\uparrow\) on the nearest neighbors, while the two 1/2 spins on the same site should be symmetrized to form a triplet state \(S = 1\). Since then many theoretical and experimental works \[^{5, 6, 7}\] are made and now the existence of the gap in integer spin systems is generally accepted.

Most recently the two dimensional (2D) anisotropic Heisenberg antiferromagnet problem has stired many people’s interests. Many theoretical techniques (spin-wave theory \[^{8}\], Schwinger-boson mean field theory \[^{9}\] and nonlinear \(\sigma\) model \[^{10}\]) are used to study this problem. It seems that there exists complex phase transition in these anisotropic systems due to the coupling ratio \(\alpha = J_y/J_x\) and temperatue \(\beta = k_B T\). In this article, we will apply the Schwinger boson mean field theory (SBMFT) to treat the problem. And a comparation to other methods will be made. In Section II, we present the self consistent equations and make a brief discussion. In Section III, we calculate the self consistent equations at ground state numerically and find a disordered-ordered phase transition in the spin-anisotropy phase diagram. In Section IV, we discuss the Néel transition (AF ordered-disordered phase) due to the temperature, and a comparation to experiments \[^{5, 6}\] on \(CsNiCl_3\) will be made. Also, we find that a decoupling temperature \(T_D\) exists for 1D and 2D isotropic case, when \(T > T_D\) the valence bond will be decoupled, and the self consistent equations have no solution in this region.
II. DERIVATION OF SELF-CONSISTENT EQUATIONS

A. SBMFT Derivation

The 2D square lattice Heisenberg antiferromagnet can be given by the following Hamiltonian [9],

\[ H = J_x \sum_{<i,j>} S_i \cdot S_j + J_y \sum_{<l,m>} S_l \cdot S_m, \]  

(1)

where the sums \(<i,j>\) and \(<l,m>\) are defined for the nearest neighbors along the \(x\) and the \(y\) directions. \(J_x > 0\) is the exchange coupling along the \(x\) direction and \(J_y > 0\) along the \(y\) direction. Without losing the generality, we suppose the coupling ratio \(\alpha = J_y/J_x\) is not larger than 1. If we take the limit \(\alpha \ll 1\), the problem degenerates into the 1D case. When \(\alpha\) increases, we can expect a 1D to 2D crossover.

We employ the Schwinger-boson mean field theory (SBMFT) developed by Arovas and Auerbach [11] to treat the Hamiltonian(1). Unlike the Holstein-Primakoff transformation in the spin-wave theory, we use the Schwinger-bosons to represent the spin operator.

With the Schwinger-boson operators: \(s_\uparrow = a\), \(s_\downarrow = b\), the spin \(S\) can be written as:

\[ S^+ = a^+ b, S^- = b^+ a \]  

(2a)

\[ S^z = \frac{1}{2} (a^+ a - b^+ b) \]  

(2b)

\[ S = \frac{1}{2} (a^+ a + b^+ b) \]  

(2c)

It’s easy to establish that these operators fulfill the restriction: \([S^z, S^+] = S^+, [S^z, S^-] = -S^-\).

Since the relation(2), we can rewrite the Hamiltonian(1) as:

\[ H = J_x \sum_{<i,j>} S_i \cdot S_j + J_y \sum_{<l,m>} S_l \cdot S_m + \mu \sum_q \left( a_q^+ a_q + b_q^+ b_q - 2S \right) \]  

(3)

Where the index \(q\) runs over the whole lattice, \(a_q^+ a_q\) and \(b_q^+ b_q\) are the number operators of spin-up and spin-down bosons. And \(\mu\) is the lagrange multiplier to conform that there are \(2S\) Schwinger-bosons per site. Since the spin values are all the same, the lagrange multipliers for different sites are equal. The Hamiltonian(3) has the translational symmetry and we can apply the Fourier transformation (FT) to it.
To simplify the discussion, let’s study the $x$ direction only, the $H_x$ term can be written as:

$$H_x = J_x \sum_{<i,j>} (S_i \cdot S_j - \frac{1}{4} n_i n_j) + J_x \sum_{<i,j>} \frac{1}{4} n_i n_j + \frac{\mu}{2} \sum_q (a_q^+ a_q + b_q^+ b_q - 2S) \quad (4)$$

Now let’s introduce the RVB order parameter $\Delta$ (or the valence bond for spin-1 case)\[12\]:

$$\Delta_z = \frac{1}{2} (b_i a_{i+z} - a_i b_{i+z}) \quad (6)$$

$\Delta_z$ is the order parameter along $z(x\ or\ y)$-direction, and $z$ is the unit vector along $z(x\ or\ y)$-direction. Then the Hamiltonian has the following form:

$$H_x = C - 2 (\frac{\mu}{2} + J_x S) NS - J_x \sum_{<i,j>} \left\{ (a_i^+ b_i^+ - b_{i+x} a_i^+) \Delta_x + \text{h.c.} \right\} + J_x \sum_i \left( \frac{\mu}{2} + J_x S \right) n_i \quad (5)$$

where $C = 2 J_x N |\Delta_x|^2 + J_x S^2 N$, $N$: number of lattice sites. To simplify the derivation we ignore the $C$ term in Hamiltonian(5), for it does not involve $\mu$. It only shifts the whole system’s energy with a certain value, and does not influence the excitation’s spectrum. Thus, the Hamiltonian(5) can be rewritten in momentum space as:

$$H_x = \sum_k \left\{ \lambda_x \left( a_k^+ a_k + b_{-k}^+ b_{-k} \right) + \gamma_{kx} a_k^+ b_{-k}^+ + \gamma_{ky}^* a_k b_{-k} \right\} - 2NS \lambda_x \quad (6)$$

Hence the Hamiltonian(3) can be expressed as:

$$H = H_x + H_y = \sum_k \left\{ \lambda \left( a_k^+ a_k + b_{-k}^+ b_{-k} \right) + \gamma_{kx} a_k^+ b_{-k}^+ + \gamma_{ky}^* a_k b_{-k} \right\} - 2NS \lambda, \quad (7)$$

with: $\lambda_x = \frac{(\mu + 2J_x S)}{2}$, $\lambda_y = \frac{(\mu + 2J_y S)}{2}$, $\lambda = \lambda_x + \lambda_y$; $\gamma_{kx} = 2i J_x \Delta_x \sin k_x$, $\gamma_{ky} = 2i J_y \Delta_y \sin k_y$, $\gamma_k = \gamma_{kx} + \gamma_{ky}$.

Using the following Bogliubov transformation\[12\]:

$$a_k = \left( \frac{\lambda + E_k}{2E_k} \right)^{1/2} e^{i\theta_k/2} \alpha_k - \left( \frac{\lambda - E_k}{2E_k} \right)^{1/2} e^{i\theta_k/2} \beta_k^+ \quad (8a)$$

$$b_k = - \left( \frac{\lambda - E_k}{2E_k} \right)^{1/2} e^{i\theta_k/2} \alpha_k^+ + \left( \frac{\lambda + E_k}{2E_k} \right)^{1/2} e^{i\theta_k/2} \beta_k^, \quad (8b)$$

the Hamiltonian can be diagonalized as:

$$H_{MF} = \sum_k \left\{ E_k \left( \alpha_k^+ \alpha_k + \frac{1}{2} \right) + E_k \left( \beta_k^+ \beta_k + \frac{1}{2} \right) \right\} - (2S + 1) N \lambda \quad (9)$$
with the energy spectrum: \( E_k = \sqrt{\lambda^2 - \gamma_0^2 \gamma_k} \).

Using the diagonalized Hamiltonian (9), we obtain the free energy:

\[
F = -\beta^{-1} \ln \{Tr \exp (-\beta H)\} = \frac{2}{\beta} \sum_k \ln \left\{ \sinh \left( \frac{\beta E_k}{2} \right) \right\} - (2S + 1) N \lambda. \tag{10}
\]

Thus we get the self consistent equations for \( \Delta_x \) and \( \Delta_y \):

\[
\Delta_x = \sum_k \left( \coth \frac{\beta E_k}{2} \right) \left( \sin \frac{k_x E_k}{k_N} \right) \left( J_x \Delta_x \sin k_x + J_y \Delta_y \sin k_y \right) \tag{11a}
\]

\[
\Delta_y = \sum_k \left( \coth \frac{\beta E_k}{2} \right) \left( \sin \frac{k_y E_k}{k_N} \right) \left( J_x \Delta_x \sin k_x + J_y \Delta_y \sin k_y \right). \tag{11b}
\]

From the condition \( \frac{\delta F}{\delta \mu} = \frac{\delta F}{\delta \lambda} = 0 \) \[11\] or \( \sum_i n_i = 2SN \) \[12\], we obtain the third self-consistent equation:

\[
\frac{2S+1}{\lambda} = \sum_k (E_k N)^{-1} \left( \coth \frac{\beta E_k}{2} \right). \tag{12c}
\]

where the self consistent equations integrate from \(-\pi\) to \(\pi\), and \( E_k = \sqrt{\lambda^2 - 4 (J_x \Delta_x \sin k_x + J_y \Delta_y \sin k_y)^2} \).

B. Spin-spin correlation

Since \( S_i \cdot S_j \) can be expanded as:

\[
S_i \cdot S_j = -\frac{1}{4} (a_j^+ b_i^+ - b_j^+ a_i^+) \left( b_i a_j - a_i b_j \right) + \frac{1}{4} (a_i^+ a_j - b_i^+ b_j) \left( a_j^+ a_i - b_j^+ b_i \right), \tag{13}
\]

where \( R_j = R_i + R, R = R_x + R_y \). Using expression(13), the spin-spin correlation can be expressed as \[11\]:

\[
\langle S_i \cdot S_j \rangle = \langle S_0 \cdot S_R \rangle = |f (R)|^2 - |g (R)|^2, \tag{14}
\]
TABLE I: renormalized factor $F_R$ for different spin values: the SBMFT solution of large spin system (such as spin-1) is more believable than the small spin system (spin-1).

| spin | $1/2$ | 1 | $S$ | $\infty$ |
|------|-------|---|-----|--------|
| $F_R$| $3/4$ | $8/9$ | $4S(S + 1)/(2S + 1)^2$ | 1 |

with:

\[
f(R) = \frac{1}{2} \int \left( \frac{dk}{2\pi} \right)^2 e^{ik\cdot R} \frac{\lambda}{E_k} \coth \frac{\beta E_k}{2}
\]

\[
g(R) = \frac{1}{2} \int \left( \frac{dk}{2\pi} \right)^2 e^{ik\cdot R} \frac{\gamma_k}{E_k} \coth \frac{\beta E_k}{2}.
\]

Consider $R = 0$, $\langle S_i \cdot S_j \rangle = \langle S_i^2 \rangle = S(S + 1)$ ($\hbar = 1$), we can renormalize the spin-spin correlation function as: $\langle S_0 \cdot S_R \rangle = F_R \left( |f(R)|^2 - |g(R)|^2 \right)$, $F_R$ is the renormalized factor.

From equation (14) it’s easy to conclude that if $(i, j)$ belongs to the same sublattice $g(R)$ vanishes, and if $(i, j)$ belongs to the different sublattice $f(R)$ vanishes. That means there exists antiferromagnet (AF) order in this system \[9, 11\]. When we take the limit $R \to \infty$, both $f(R)$ and $g(R)$ vanish. This indicates the AF order is short ranged and decreases exponentially \[9, 11\].

III. GROUND STATE PROPERTIES

A. 1D solution

First, we shall study the $\alpha = 0$ 1D limit and $\alpha = 1$ 2D isotropic limit. In the 1D case the self consistent eqs. can be written as:

\[
\Delta_x = \int \frac{dk \coth \frac{\beta E_k}{2} J_x \Delta_x \sin^2 k_x}{2\pi} = \frac{2S + 1}{\lambda} = \int \frac{dk \coth \frac{\beta E_k}{2}}{2\pi} E_k.
\]

with: $E_k = \sqrt{\lambda^2 - 4 (J_x \Delta_x \sin k_x)^2}$.

Consider the ground state $T \to 0$, i.e., $\coth \frac{\beta E_k}{2} \to 1$. We can calculate $\Delta_x$ and $\lambda$ numerically for different spin values.
TABLE II: 1D SBMFT solution for different spin values at ground state: Spin-0 case only calculated as a reference, and itself has no specific physical meaning. Both spin-1/2 and spin-1 showed an energy gap (2S factor in $E_H$ formula indicates that there are 2S bosons excited for a physical excitations).

| spin | $\Delta_x$ | $\lambda$ | $E_H = 2S\omega_m in$ |
|------|-------------|------------|------------------|
| 0    | 0           | 0.5$J_x$   | 0                |
| 1/2  | 0.6792      | 1.38$J_x$  | 0.2425$J_x$      |
| 1    | 1.1816      | 2.3647$J_x$| 0.1701$J_x$      |
| $\infty$ | $\infty$  | 2$\Delta_x J_x$ | 0               |

Experiments showed that only integer spin case has a finite energy gap (the Haldane gap). This implies that the SBMFT does not fit for the 1D half-odd-integer spin problems. Theoretically this is because the topological term in the long-wavelength effective action. A formalism based on fermionic representation \[11\] for the spin operators is relevant for the half integer case. Numerical result for spin-1 case is agree with other authors’ result. Experimently the gap (0.41$J_x$) \[7\] is larger than the SBMFT result, this is because our model is too simple. The model for a real spin-1 system contains a single-ion anisotropy $D \sum_i (S^z_i)^2$.

**B. 2D isotropic solution**

Similarly we can study the self-consistent eqs. of 2D isotropic case \[12\]. We assume $J_x = J_y = J$ and obtain the 2D isotropic self consistent eqs.:

\[
\Delta_x = \Delta_y = \Delta = \int \left( \frac{dk}{2\pi} \right)^2 \frac{\coth \frac{\beta E_k}{2} J \Delta \sin k_x (\sin k_x + \sin k_y)}{E_k} \frac{E_k}{2} \coth \frac{\beta E_k}{2},
\]

(17a)

\[
\frac{2S + 1}{\lambda} = \int \left( \frac{dk}{2\pi} \right)^2 \coth \frac{\beta E_k}{2} \frac{E_k}{E_k},
\]

(17b)

with: $E_k = \sqrt{\lambda^2 - 4J^2 \Delta^2 (\sin k_x + \sin k_y)^2}$. For the ground state $T \to 0$ and $\coth \frac{\beta E_k}{2} \to 1$, there exists an finite maximum value of $2S + 1$ for the 2D integration(17). So there exists
an critical spin value $S_c$:

$$(2S + 1)_{\text{max}} = 2S_c + 1 = \int \left(\frac{dk}{2\pi}\right)^2 \left(1/E_k\right)$$

(18)

If $S < S_c$, there exists a gap, and if $S > S_c$ gapless, a Bose-Einstein condensation occurs at the zero energy point [11, 12]. Numerical calculation shows that $S_c = 0.1956$. This means that 2D isotropic Heisenberg antiferromagnets on a square lattice with nearest coupling always have ordered ground states, and gapless. D. Yoshioka has studied the problem for a spin-1/2 isotropic square lattice using the slave-fermion mean field theory. Likewise, we can generate to an anisotropic case. Since the spectrum is gapless, the Bogoliubov transformation(8) lost its meaning at certain $k_i$ in which $E(k_i) = 0$. D. Yoshioka introduces a special transformation to diagonalize the mean field Hamiltonian(7) at $K_i$.

$$a_{k_i} = \frac{1}{\sqrt{2}} (\zeta_{k_i} + \xi_{k_i})$$
$$b_{k_i} = \frac{1}{\sqrt{2}} e^{i\theta_{k_i}} (\zeta_{k_i}^+ - \xi_{k_i}^+).$$

(19a)
(19b)

and $\zeta_{k_i}, \xi_{k_i}$ fulfill the following commutate relations:

$$[\zeta_{k_i}, \zeta_{k_i}^+] = [\xi_{k_i}, \xi_{k_i}^+] = 1$$
$$[\zeta_{k_i}, \xi_{k_i}^+] = [\xi_{k_i}, \zeta_{k_i}^+] = 0.$$

(20a)
(20b)

Hence we obtain:

$$H = H (k) (1 - \delta_{k,k_i}) + H (k_i) \delta_{k,k_i}$$
$$H (k_i) = 2\lambda \left(\zeta_{k_i}^+ \zeta_{k_i} - \frac{1}{2}\right)$$
$$[\zeta_{k_i}, H] = [\zeta_{k_i}^+, H] = 0.$$

(21a)
(21b)
(21c)
Now the self consistent eqs. should be rewritten as:

\[ 2S + 1 = \frac{1}{N} \sum_k \left[ \frac{\lambda \coth \beta E_k}{E_k} \left( 1 - \delta_{k,k_i} \right) + n_0 \delta_{k,k_i} \right] \]  (22a)

\[ \Delta_x = \frac{1}{N} \sum_k \sin k_x \left( J_x \Delta_x \sin k_x + J_y \Delta_y \sin k_y \right) \left[ \frac{\lambda \coth \beta E_k}{E_k} \left( 1 - \delta_{k,k_i} \right) + n_0 \delta_{k,k_i} \right] \]  (22b)

\[ \Delta_y = \frac{1}{N} \sum_k \sin k_y \left( J_x \Delta_x \sin k_x + J_y \Delta_y \sin k_y \right) \left[ \frac{\lambda \coth \beta E_k}{E_k} \left( 1 - \delta_{k,k_i} \right) + n_0 \delta_{k,k_i} \right] , \]  (22c)

and the spin-spin correlation for the gaplessed spectrum:

\[ \langle S_0 \cdot S_R \rangle = F_R \left( |f(R)|^2 - |g(R)|^2 \right) \]  (23a)

\[ f(R) = \frac{1}{2N} \sum_k e^{ik \cdot R} \left[ \frac{\lambda \coth \beta E_k}{E_k} \left( 1 - \delta_{k,k_i} \right) + n_0 \delta_{k,k_i} \right] \]  (23b)

\[ g(R) = \frac{1}{2N} \sum_k e^{ik \cdot R} \frac{\gamma_k}{\lambda} \left[ \frac{\lambda \coth \beta E_k}{E_k} \left( 1 - \delta_{k,k_i} \right) + n_0 \delta_{k,k_i} \right] \]  (23c)

Compared eqs. (23) with eqs. (15), we can conclude that if there is no gap then there is Néel long range order, and if there is a gap there only exists short range order and long range disordered.

C. 2D anisotropic solution

As we have discussed in section III.A there exist significant difference between 1D and 2D isotropic Heisenberg antiferromagnet systems. There should exist a critical coupling \( \alpha_c \), when \( \alpha < \alpha_c \), the system behaves like a 1D chain (quasi 1D spin system) and when \( \alpha > \alpha_c \) it has a Néel order and gaplessed spectrum (2D spin system). This prediction has been made by T.Sakai et.al. [8] using the spin wave theory and M.Azzour [9] using the SBMFT. Recently A.Parola et.al. [13] declare that for spin-1/2 system there exists a disordered transition induced by anisotropy at about \( \alpha < 0.1 \), they find that the disordered phase is gapless and its wavelength can be interpreted in terms of a decoupled 1D chains.

In M.Azzour et.al.’s article [9], they provide a solution of SBMFT for large spin approx-
TABLE III: critical coupling $\alpha_c$ for different spin values at ground state $T = 0$.

| $\alpha_c$ | This article | T.Sakai [8] | M.Azzour [9] | D.Sénéchal [10] |
|------------|--------------|-------------|-------------|------------------|
| Method     | SBMFT        | SWT         | SBMFT       | NL$\sigma$ model |
| spin-1/2   | 0.13         | 0.03367     | 0.02        | 0.55             |
| spin-1     | 0.009        | 0.0013      | 0.0019      | 0.03             |

Imitation:

$$\alpha_c = Se^{-2\pi S}$$  \hspace{1cm} (24a)

$$\frac{\Delta_y}{\Delta_x} = e^{-2\pi S}.$$  \hspace{1cm} (24b)

And D.Sénéchal et al. [10] work out a nonlinear $\sigma$ model solution:

$$\alpha_c = 4 \exp \left[ 1 - \frac{8\pi^2}{3(a\eta)^2} \right],$$  \hspace{1cm} (25)

where they choose $a\eta = 2.1$.

Employing the condition $E_H = 0$ and $T \to 0$. We discuss the ordered-disordered phase transition of ground state by calculating the self consistent eqs.(12). Compared with other authors’ results, our numerical results are listed in table III. The ordered-disordered phase diagram for different spin value and anisotropy is given in Fig.I.

IV. EXCITED STATE PROPERTIES

A. The Néel phase transition

Experimentally 1D spin systems are realized in quasi-1D compounds in which the coupling $J_x$ is much higher than the coupling $J_y$. It is the coupling ratio $\alpha = J_y/J_x$ determines the degree of “quasi-one-dimensionality” of the materials. For instance, $\alpha$ is estimated to be 0.02 in $CsNiCl_3$, and NENP [$Ni(C_2H_8N_2)_2VO_2(ClO_4)$] 0.0006. And experiments indicate that there exists notable difference between them. In $CsNiCl_3$ 1D behavior, the existence of gap is observed above a critical temperature (the Néel temperature $T_N$) of about 5K [5, 6]. While in NENP 1D behavior, the existence of gap is observed at temperature as low as can
be reached. This indicates that for spin-1 system $\alpha$ is not larger than 0.02. The result from nonlinear $\sigma$ model in table III seems too large \[^{10}\], while SBMFT solution and SWT solution are permitted.

Calculating the self consistent eqs.(12) numerically, we can find the $T_N$ for $CsNiCl_3$. It approximates 1K, much lower than the experimental result. Perhaps it is due to the compound $CsNiCl_3$'s structure is $ABX_3$ type, not cubic \[^{6}\]. And for a real system, the Hamiltonian should include the anisotropic term $D \langle S_z^2 \rangle$ \[^{5, 6}\]. $D$ is produced by the coupling of a spin to the anisotropic orbital motion.

R.Botet et.al. \[^{14}\] found that a gap exists for $1.6 \geq \frac{D}{J} \geq -0.5$, and the gap decreased rapidly for negative $D$. Experiments indicate that the anisotropy constant is negative $\frac{D}{J} = -0.038$ \[^{5}\]. So the calculated $T_N (D = 0)$ should be smaller than the $T_N$ for a real system ($D = -0.038J$). The ordered-disordered phase diagram for different spin values and temperature with $\alpha = 0.02$ is given in Fig.2.

**B. The temperature-anisotropy phase diagram of spin-1 system**

Numerical solution of eqs.(12) indicate that when $\beta \to 0.46$ the RVB order parameter $\Delta$ decreases rapidly to zero for both 1D and 2D isotropic spin-1 systems. Suppose $\Delta_x = \Delta_y = 0$, we can calculate the critical temperature $T_D$, denoting that when $T \geq T_D$ the boson pairs on the nearest neighbors will be decoupled.

$$\coth \frac{\lambda}{2T_D} = 2S + 1$$ (26)

Solving the equation (26), we find that $T_D = 2.16$ for spin-1 case of $\alpha = 1$ and $\alpha = 0$. But, for the anisotropic case $0 < \alpha < 1$, the self consistent eqs.(12) has no solution under the condition $\Delta_x = \Delta_y = 0$ or $\Delta_x \neq 0, \Delta_y = 0$. This indicates that when increasing the temperature, the Valence-Bond-Solid concept lost its validity before the boson pairs on the nearest sites decoupled. The phase diagram of spin-1 Heisenberg antiferromagnet can be divided into three regions: the ordered phase, the disordered phase, and the decoupled phase. Valence-Bond-Solid concept lost its validity in the decoupled phase, we must consider other order parameters. The phase diagram for spin-1 2D anisotropic Heisenberg antiferromagnet is given in Fig.3.
For spin-1/2 systems such as $KCuF_3$ and Cu-O layers etc., $T_D$ is calculated out to be 0.910.

V. SUMMARY

Using the the Schwinger-boson mean field theory, we have investigated the phase transition in anisotropic 2D Heisenberg antiferromagnet for various spin values. We find that the anisotropic HAF system can be characterized as two types according to different coupling ratio $\alpha = J_y/J_x$. When $\alpha < \alpha_c$, such as NENP for spin-1 system, it belongs to the quasi 1D spin systems. For the quasi 1D spin systems, the disordered phase always exists. When $\alpha > \alpha_c$, such as $CsNiCl_3$ for spin-1 system, it belongs to the usual 2D spin system. For the usual 2D spin system, the disordered phase only exists when $T > T_N$. So we can use $CsNiCl_3$ and other $ABX_3$ type compounds (such as $RbNiCl_3$, $CsNiF_3$ et.al.) for testing anisotropic 2D HAF problem and use NENP for testing Haldane conjecture and other 1D spin-chain HAF problems. Also we find that for the 1D and 2D isotropic HAF system there exists another critical temperature $T_D$, when $T \geq T_D$, $\Delta_x = \Delta_y = 0$, the boson pairs will be decoupled. But for the anisotropic case ($0 < \alpha < 1$), there has not such a decoupled solution for $\Delta_x = \Delta_y = 0$ or $\Delta_x \neq 0, \Delta_y = 0$. This implies order parameters other than RVB should be considered in the high temperature region.

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FIG. 1: Spin value - anisotropy $\alpha$ phase diagram at ground state $T = 0$. 
FIG. 2: Spin value - temperature $J_x/k_B T$ phase diagram with $\alpha = 0.02$. 
FIG. 3: Temperature $J_x/k_B T$ - anisotropy $\alpha$ phase diagram for spin-1 2D anisotropic Heisenberg antiferromagnet.