Can the $X(4350)$ narrow structure be a $1^{++}$ exotic state?

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Using the QCD sum rules we test if the new observed resonance structure, the $X(4350)$ recently observed by the Belle Collaboration, can be described as a $J^{PC} = 1^{++}$ exotic $D_{s}^{*}D_{s0}^{*}$ molecular state. We consider the contributions of condensates up to dimension eight, we work at leading order in $\alpha_{s}$ and we keep terms which are linear in the strange quark mass $m_{s}$. The mass obtained for such state is $m_{D_{s}^{*}D_{s0}^{*}} = (5.05 \pm 0.19)$ GeV. We also consider a molecular $1^{--}$, $D_{s}^{*}D_{0}^{*}$ current and we obtain $m_{D_{s}^{*}D_{0}^{*}} = (4.92 \pm 0.08)$ GeV. We conclude that it is not possible to describe the $X(4350)$ structure as a $1^{++} D_{s}^{*}D_{s0}^{*}$ molecular state.

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In the recent years, many new states were observed by BaBar, Belle and CDF Collaborations. All these states were observed in decays containing a $J/\psi$ or $\psi'$ in the final states and their masses are in the charmonium region. Therefore, they certainly contain a $c\bar{c}$ pair in their constituents. Although they are above the threshold for a decay into a pair of open charm mesons they decay into $J/\psi$ or $\psi'$ plus pions, which is unusual for $c\bar{c}$ states. Another common feature of these states is the fact that their masses and decay modes are not in agreement with the predictions from potential models. For these reasons they are considered as candidates for exotic states. Some of these new states have their masses very close to the meson-meson threshold, like the $X(3872)$ [1] the $Z^{+}(4340)$ [2] and the $Y(4140)$ [3]. Therefore, a molecular interpretation for these states seems natural.

Concerning the $Y(4140)$ structure, it was observed by CDF Collaboration in the decay $B^{+} \to Y(4140)K^{+} \to J/\psi K^{+}$. The mass and width of this structure is $M = (4143 \pm 2.9 \pm 1.2)$ MeV, $\Gamma = (11.7^{+8.3}_{-5.0} \pm 3.7)$ MeV [3]. Its interpretation as a conventional $c\bar{c}$ state is complicated because it lies well above the threshold for open charm decays and, therefore, a $c\bar{c}$ state with this mass would decay predominantly into an open charm pair with a large total width. This state was interpreted as a $J^{PC} = 0^{++}$ or $2^{++}$ $D_{s}^{*}D_{s}$ molecular state in different works [4,12]. In particular, using an effective lagrangian model, the authors of ref. [7] have suggested that a $D_{s}^{*+}D_{s}^{-}$ molecular state should be seen in the two-photon process. Following this suggestion the Belle Collaboration [13] searched for the $Y(4140)$ state in the $\gamma\gamma \to \phi J/\psi$ process. However, instead of the $Y(4140)$, the Belle Collaboration found evidence for a new narrow structure in the $\phi J/\psi$ mass spectrum at 4.35 GeV. The significance of the peak is 3.2 standard deviations and, if interpreted as a resonance, the mass and width of the state, called $X(4350)$ is $M = (4350.6^{+4.6}_{-3.5} \pm 0.7)$ MeV and $\Gamma = (13.3^{+7.9}_{-9.1} \pm 4.1)$ MeV [13].

The possible quantum numbers for a state decaying into $J/\psi\phi$ are $J^{PC} = 0^{++}$, $1^{--}$ and $2^{++}$. At these quantum numbers, $1^{--}$ is not consistent with the constituent quark model and it is considered exotic [4]. In ref. [13] it was noted that the mass of the $X(4350)$ is consistent with the prediction for a $c\bar{s}c\bar{s}$ tetraquark state with $J^{PC} = 2^{++}$ [14] and a $D_{s}^{*+}D_{s}^{*-}$ molecular state [15]. However, the state considered in ref. [13] has $J^{P} = 1^{-}$ with no definite charge conjugation. A molecular state with a vector and a scalar $D_{s}$ mesons with negative charge conjugation was studied by the first time in ref. [12], and the obtained mass was $(4.42 \pm 0.10)$ GeV, also consistent with the $X(4350)$ mass, but without consistent quantum numbers. A molecular state with a vector and a scalar $D_{s}$ mesons with positive charge conjugation can be constructed using the combination $D_{s}^{*+}D_{s0}^{*-} - D_{s}^{*-}D_{s0}^{*+}$.

There is already some interpretations for this state. In ref. [17] it was interpreted as an excited $P$-wave charmonium state $Z_{6}^{0}$ and in ref. [18] it was interpreted as a mixed charmonium-$D_{s}^{*+}D_{s}^{*}$ state. In this work, we use the QCD sum rules (QCDSR) [15,21], to study the two-point function based on a $D_{s}^{*}D_{s0}^{*}$ current with $J^{PC} = 1^{--}$, to test if the new observed resonance structure, $X(4350)$, can be interpreted as such molecular state, as suggested by Belle Coll. [13].

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The QCD sum rule approach is based on the two-point correlation function
\[ \Pi_{\mu\nu}(q) = i \int d^4 x \ e^{iq \cdot x} \langle 0 | T[j_\mu(x) j_\nu(0)] | 0 \rangle, \]
(1)
where a current that couples with a \( J^{PC} = 1^{-+} D_s^* D_{s0}^* \) state is given by:
\[ j_\mu = \frac{1}{\sqrt{2}} \left[ (\bar{s}_a \gamma_\mu c_a)(\bar{c}_b s_b) - (\bar{c}_a \gamma_\mu s_a)(\bar{s}_b c_b) \right], \]
(2)
where \( a \) and \( b \) are color indices.

Since the current in Eq. (2) is not conserved, we can write the correlation function in Eq. (1) in terms of two independent Lorentz structures:
\[ \Pi_{\mu\nu}(q) = -\Pi_1(q^2)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + \Pi_0(q^2) \frac{q_\mu q_\nu}{q^2}, \]
(3)
The two invariant functions, \( \Pi_1 \) and \( \Pi_0 \), appearing in Eq. (3), have respectively the quantum numbers of the spin 1 and 0 mesons. Therefore, we choose to work with the Lorentz structure \( g_{\mu\nu} \), since it gets contributions only from the \( 1^{-+} \) state.

The QCD sum rule is obtained by evaluating the correlation function in Eq. (1) in two ways: in the OPE side, we calculate the correlation function at the quark level in terms of quark and gluon fields. We work at leading order in \( \alpha_s \) in the operators, we consider the contributions from condensates up to dimension eight and we keep terms which are linear in the strange quark mass \( m_s \). In the phenomenological side, the correlation function is calculated by inserting intermediate states for the \( D_s^* D_{s0}^* \) molecular scalar state. Parametrizing the coupling of the exotic state, \( X = D_s^* D_{s0}^* \), to the current, \( j_\mu \), in Eq. (2) in terms of the parameter \( \lambda \):
\[ \langle 0 | j_\mu | X \rangle = \lambda \varepsilon_\mu, \]
(4)
the phenomenological side of Eq. (1), in the \( g_{\mu\nu} \) structure, can be written as
\[ \Pi_1^{\text{phen}}(q^2) = \frac{\lambda^2}{M_X^2 - q^2} + \int_0^\infty ds \frac{\rho^{\text{cont}}(s)}{s - q^2}, \]
(5)
where the second term in the RHS of Eq. (5) denotes higher resonance contributions.

The correlation function in the OPE side can be written as a dispersion relation:
\[ \Pi_1^{\text{OPE}}(q^2) = \int_{4m_s^2}^\infty ds \frac{\rho^{\text{OPE}}(s)}{s - q^2}, \]
(6)
where \( \rho^{\text{OPE}}(s) \) is given by the imaginary part of the correlation function: \( \pi \rho^{\text{OPE}}(s) = \text{Im}[\Pi_1^{\text{OPE}}(s)]. \)

As usual in the QCD sum rules method, it is assumed that the continuum contribution to the spectral density, \( \rho^{\text{cont}}(s) \) in Eq. (5), vanishes bellow a certain continuum threshold \( s_0 \). Above this threshold, it is given by the result obtained with the OPE. Therefore, one uses the ansatz [22]
\[ \rho^{\text{cont}}(s) = \rho^{\text{OPE}}(s) \Theta(s - s_0), \]
(7)
To improve the matching between the two sides of the sum rule, we perform a Borel transform. After transferring the continuum contribution to the OPE side, the sum rules for the exotic meson, described by a \( 1^{-+} D_s^* D_{s0}^* \) molecular current, up to dimension-eight condensates, using factorization hypothesis, can be written as:
\[ \lambda^2 e^{-m_{D_s^* D_{s0}^*}^2/M^2} = \int_{4m_s^2}^{s_0} ds \ e^{-s/M^2} \rho^{\text{OPE}}(s), \]
(8)
where
\[ \rho^{\text{OPE}}(s) = \rho^{\text{pert}}(s) + \rho^{(ss)}(s) + \rho^{(G^2)}(s) + \rho^{\text{mix}}(s) + \rho^{(s s)^2}(s) + \rho^{(8)}(s), \]
(9)
with

\[
\rho^{\text{pert}}(s) = \frac{1}{2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\alpha_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} \int_{\beta_{\min}}^{1-\alpha-\beta} (1-\alpha-\beta) F^3(\alpha, \beta) \left[ 3(1+\alpha+\beta) F(\alpha, \beta) + 2m_c^2 (1-\alpha-\beta)^2 \right],
\]

\[
\rho^{\text{mix}}(s) = -\frac{3m_s m_c}{2^9 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\alpha_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta)^2 F^3(\alpha, \beta),
\]

\[
\rho^{\text{mix}}(s) = -\frac{3m_s}{2^9 \pi^6} \int_{\alpha_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta)^2 F^3(\alpha, \beta),
\]

\[
\rho^{G^2}(s) = -\frac{(g^2G^2)}{3} \frac{1}{2^9 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\alpha_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} \left[ 6\alpha (1-2\alpha-2\beta) F^2(\alpha, \beta) - 3m_c^2 (1-\alpha-\beta) \left\{ 1+\alpha (1-2\alpha) + (\alpha+3\beta) \right\} F(\alpha, \beta) - m_c^4 \beta (1-\alpha-\beta)^3 \right],
\]

\[
\rho^{m_s \text{-mix}}(s) = \frac{m_s}{2^9 \pi^6} \int_{\alpha_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} \left\{ m_c^2 (3+5\alpha+4\beta) - \alpha \beta s \right\} - \frac{m_s}{2^9 \pi^6} \int_{\alpha_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} \left\{ m_c^2 (2+\alpha) - \alpha (1-\alpha) \right\} \left(2-7\alpha\right),
\]

\[
\rho^{ss}(s) = -\frac{(ss)^2}{2^{10} \pi^2} (8m_c^2 + s) \sqrt{1-4m_c^2/s},
\]

\[
\rho^{m_s \cdot (ss)}(s) = -\frac{m_s m_c (ss)^2}{2^9 \pi^2} \sqrt{1-4m_c^2/s},
\]

\[
\rho^{(8)}(s) = \frac{m_c^2 (ss) (\bar{s} g_{\sigma} G_s)}{2^9 \pi^2} \sqrt{1-4m_c^2/s},
\]

\[
\Pi^{(8)}(M^2) = \frac{m_c^2 (ss) (\bar{s} g_{\sigma} G_s)}{2^9 \pi^2} \int_0^1 \frac{d\alpha}{(1-\alpha)} e^{-\frac{m_c^2}{\alpha (1-\alpha) M^2}} \left[ 1-3\alpha + \frac{2m_c^2}{\alpha M^2} \right],
\]

\[
\Pi^{m_s \cdot (8)}(M^2) = \frac{m_s m_c (ss) (\bar{s} g_{\sigma} G_s)}{3} \int_0^1 \frac{d\alpha}{(1-\alpha)} e^{-\frac{m_c^2}{\alpha (1-\alpha) M^2}} \left[ \frac{m_c^2 (6-4\alpha-10\alpha^2)}{M^2 (\alpha(1-\alpha))} - (6-13\alpha+20\alpha^2) \right],
\]

where we use the following definitions:

\[
F(\alpha, \beta) = m_c^2 (\alpha+\beta) - \alpha \beta s,
\]

\[
H(\alpha) = m_c^2 - \alpha (1-\alpha) s.
\]

The integration limits are given by \( \alpha_{\min} = (1 - \sqrt{1-4m_c^2/s})/2 \), \( \alpha_{\max} = (1 + \sqrt{1-4m_c^2/s})/2 \), \( \beta_{\min} = \alpha m_c^2 / (s \alpha - m_c^2) \). We have neglected the contribution of the dimension-six condensate \( \langle g^2 G^3 \rangle \), since it is assumed to be suppressed by the loop factor \( 1/16\pi^2 \).

To extract the mass \( m_{D_s} D_s^* \) we take the derivative of Eq. (8) with respect to \( 1/M^2 \), and divide the result by Eq. (9).
For a consistent comparison with the results obtained for the other molecular states using the QCDSR approach, we have considered here the same values used for the quark masses and condensates as in refs. [16, 23–29]:

\[ m_c = (1.23 \pm 0.05) \text{ GeV}, \quad m_s = (0.13 \pm 0.03) \text{ GeV}, \quad \langle \bar{q}q \rangle = - (0.23 \pm 0.03)^3 \text{ GeV}^3, \]

\[ \langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle, \quad \langle \bar{s}g\sigma.Gs \rangle = m_0^2 \langle \bar{s}s \rangle \text{ with } m_0^2 = 0.8 \text{ GeV}^2, \quad \langle g^2 G^2 \rangle = 0.88 \text{ GeV}^4. \]

The Borel window is determined by analysing the OPE convergence, the Borel stability and the pole contribution. To determine the minimum value of the Borel mass we impose that the contribution of the dimension-8 condensate should be smaller than 20% of the total contribution.

In Fig. 1 we show the contribution of all the terms in the OPE side of the sum rule. From this figure we see that for \( M^2 \geq 2.8 \text{ GeV}^2 \) the contribution of the dimension-8 condensate is less than 10% of the total contribution, which indicates a good Borel convergence. However, from Fig. 2 we see that the Borel
stability is good only for $M^2 \geq 3.2 \text{ GeV}^2$. Therefore, we fix the lower value of $M^2$ in the sum rule window as $M_{\text{min}}^2 = 3.2 \text{ GeV}^2$.

The maximum value of the Borel mass is determined by imposing that the pole contribution must be bigger than the continuum contribution.

![Graph](image)

**FIG. 3:** The dashed line shows the relative pole contribution (the pole contribution divided by the total, pole plus continuum, contribution) and the solid line shows the relative continuum contribution for $\sqrt{s_0} = 5.3$ GeV.

From Fig. 3 we see that for $\sqrt{s_0} = 5.3$ GeV, the pole contribution is bigger than the continuum contribution for $M^2 \leq 3.74 \text{ GeV}^2$. We show in Table I the values of $M_{\text{max}}$ for other values of $\sqrt{s_0}$. For $\sqrt{s_0} \leq 5.1$ GeV there is no allowed Borel window.

**Table I:** Upper limits in the Borel window for the $!^{--}, D^*_s D^*_s$ current obtained from the sum rule for different values of $\sqrt{s_0}$.

| $\sqrt{s_0}$ (GeV) | $M_{\text{max}}^2 (\text{GeV}^2)$ |
|-------------------|-------------------------------|
| 5.2               | 3.42                          |
| 5.3               | 3.74                          |
| 5.4               | 3.95                          |
| 5.5               | 4.26                          |
| 5.6               | 4.47                          |

Using the Borel window, for each value of $s_0$, to evaluate the mass of the exotic meson and then varying the value of the continuum threshold in the range $5.3 \leq \sqrt{s_0} \leq 5.6$ GeV, we get $m_{D^*_s D^*_s} = (5.04 \pm 0.09)$ GeV.

Up to now we have kept the values of the quark masses and condensates fixed. To check the dependence of our results with these values we fix $\sqrt{s_0} = 5.45$ GeV and vary the other parameters in the ranges: $m_c = (1.23 \pm 0.05)$ GeV, $m_s = (0.13 \pm 0.03)$ GeV, $\langle \bar{q}q \rangle = -(0.23 \pm 0.03)^3 \text{ GeV}^3$, $m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2$.

In our calculation we have assumed the factorization hypothesis. However, it is important to check how a violation of the factorization hypothesis would modify our results. For this reason we multiply $\langle \bar{s}s \rangle^2$ and $\langle \bar{s}s \rangle (\bar{g}s \sigma Gs)$ in Eq. (10) by a factor $K$ and we vary $K$ in the range $0.5 \leq K \leq 2$. We notice that the results are more sensitive to the variations on the values of $\langle \bar{q}q \rangle$ and $K$.

Taking into account the uncertainties given above we get

$$m_{D^*_s D^*_s} = (5.05 \pm 0.19) \text{ GeV},$$

which is not compatible with the mass of the narrow structure $X(4350)$ observed by Belle. It is, however, very interesting to notice that the mass obtained for a state described with a $1^{--}, D^*_s D^*_s$ molecular current is $m_{1^{--}} = (4.42 \pm 0.10)$ GeV, much smaller than what we have obtained with the $1^{--}, D^*_s D^*_s$ molecular current. This may be interpreted as an indication that it is easier to form molecular states with not exotic quantum numbers.
From the above study it is very easy to get results for the $D^*D_0^*$ molecular type current with $J^{PC} = 1^{-+}$. For this we only have to take $m_s = 0$ and $\langle \bar{s}s \rangle = \langle \bar{q}q \rangle$ in Eq. (10). The OPE convergence in this case is very similar to the preliminary case, and we also get a good Borel stability only for $M^2 \geq 3.2$ GeV$^2$. Fixing $M_{\text{min}}^2 = 3.2$ GeV$^2$, the minimum allowed value for the continuum, threshold is $\sqrt{s_0} = 5.2$ GeV. We show, in Fig. 4 the result for the mass of such state using different values of the continuum threshold, with the upper and lower limits in the Borel region indicated.

Using the values of the continuum threshold in the range $5.2 \leq \sqrt{s_0} \leq 5.5$ GeV we get for the state described with a $1^{-+}$, $D^*D_0^*$ molecular current: $m_{D^*D_0^*} = (4.92 \pm 0.08)$ GeV. Approximately one hundred MeV below the value obtained for the similar strange state. In the case of the $D^*D_0^*$ molecular current with $J^{PC} = 1^{--}$, the mass obtained was [10]: $m_{1^{--}} = (4.27 \pm 0.10)$ GeV, again much smaller than for the exotic case.

In conclusion, we have presented a QCDSR analysis of the two-point function based on $D^*_sD_0^{*0}$ and $D^*D_0^*$ molecular type currents with $J^{PC} = 1^{-+}$. Our findings indicate that the $X(4350)$ narrow structure observed by the Belle Collaboration in the process $\gamma\gamma \rightarrow X(4350) \rightarrow J/\psi\phi$, cannot be described by using an exotic $1^{-+}$, $D^*_sD_0^{*0}$ current.

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