A NEW LUMINOSITY RELATION FOR GAMMA-RAY BURSTS AND ITS IMPLICATION

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ABSTRACT

Gamma-ray bursts (GRBs) are the most luminous astrophysical events observed so far. They are conventionally classified into long and short ones depending on their time duration, $T_{90}$. Because of the advantage that their high redshifts offer, many efforts have been made to apply GRBs to cosmology. The key to this is to find correlations between some measurable properties of GRBs and the energy or the luminosity of GRBs. These correlations are usually referred to as luminosity relations and are helpful in understanding the GRBs themselves. In this paper, we explored such correlations in the X-ray emission of GRBs. The X-ray emission of GRBs observed by Swift has the exponential functional form in the prompt phase and relaxes to a power-law decay at time $T_p$. We have assumed a linear relation between log $L_{X,p}$ (with $L_{X,p}$ being the X-ray luminosity at $T_p$) and log$[T_p/(1 + z)]$, but there is some evidence for curvature in the data and the true relationship between $L_{X,p}$ and $T_p/(1 + z)$ may be a broken power law. The limited GRB sample used in our analysis is still not sufficient for us to conclude whether the break is real or just an illusion caused by outliers. We considered both cases in our analysis and discussed the implications of the luminosity relation, especially on the time duration of GRBs and their classification.

Key words: gamma-ray burst: general

Online-only material: color figures

1. INTRODUCTION

Gamma-ray bursts (GRBs), which can last from milliseconds to nearly an hour, are the most luminous astrophysical events observed so far. The parameter of $T_{90}$, which is defined as the time interval during which the background-subtracted cumulative counts increase from 5% to 95%, is usually used to denote the time duration of GRBs. Those with $T_{90} > 2\ s$ are conventionally described as long/soft GRBs and those with $T_{90} < 2\ s$ as short/hard GRBs (Kouveliotou et al. 1993).

In Willingale et al. (2007), it was demonstrated that the X-ray decay curves of GRBs measured by Swift can be fitted using one or two components—the prompt component and the optional afterglow component—both of which have exactly the same functional form comprised of an early falling exponential phase and a following power-law decay. The prompt component contains the prompt gamma-ray emission and the initial X-ray decay. The transition time $T_p$ from the exponential phase to the power-law decay in the prompt component defines an alternative estimate of the GRB duration, which is comparable with $T_{90}$ for most GRBs (O’Brien et al. 2006a, 2006b). O’Brien & Willingale (2007) proposed the classification of GRBs into long and short ones at $T_p = 10\ s$ instead.

GRBs can be observed at very high redshifts due to their high luminosities. For example, the recently observed GRB 090423 has a redshift of $z \approx 8.2$ (Tanvir et al. 2009; Salvaterra et al. 2009). It may be possible to calibrate GRBs as standard candles (see, for example, Dai et al. 2004; Ghirlanda et al. 2004b; Firmani et al. 2005; Lamb et al. 2005; Liang & Zhang 2005; Xu et al. 2005; Wang & Dai 2006; Ghirlanda et al. 2006; Schaefer 2007; Wang et al. 2007; Li et al. 2008; Amati et al. 2008; Basilakos & Perivolaropoulos 2008; Qi et al. 2008a, 2008b; Kodama et al. 2008; Liang et al. 2008; Wang 2008; Qi et al. 2009). The key to the calibration of GRBs is to find correlations between some measurable properties (the luminosity indicators) of GRBs and the energy (the isotropic energy $E_{\gamma,\text{iso}}$ or the collimation-corrected energy $E_{\gamma}$) or the luminosity (e.g., the peak luminosity $L$) of GRBs. These correlations are usually referred to as luminosity relations, which are useful both in applying GRBs to cosmology and understanding GRBs themselves. The GRB luminosity relations used in cosmological studies in the literature include the relations of $\tau_{\text{lag}}$ (the spectral lag, i.e., the time shift between the hard and soft light curves)—$L$ (Norris et al. 2000), $\tau_p$ (the variability, a quantitative measurement on the spikiness of the light curve; there exist several definitions of $\tau_p$, mainly depending on the smoothing time intervals the reference curve is built upon, and on the normalization as well)—$L$ (Fenimore & Ramirez-Ruiz 2000; Reichart et al. 2001), $E_{\text{peak}}$ (peak energy of the spectrum)—$E_{\gamma,\text{iso}}$ (Amati et al. 2002), $E_{\text{peak}} - E_{\gamma}$ (Ghirlanda et al. 2004a), $E_{\text{peak}} - L$ (Schaefer 2003), and $\tau_{\text{RT}}$ (minimum rise time of the light curve)—$L$ (Schaefer 2007). For each of the above luminosity relations, there is only one luminosity indicator involved. More complicated luminosity relations, which include two luminosity indicators, are also discussed in the literature; see, for example, Yu et al. (2009) and references therein.

In this paper, we explore the correlation between $T_p$ and the X-ray luminosity of GRBs at $T_p$ and discuss its implications, especially on the time duration of GRBs and their classification.

2. METHODOLOGY

In Willingale et al. (2007), the X-ray light curves of GRBs were constructed from the combination of Burst Alert Telescope (BAT) and X-ray telescope (XRT) data in the way described by O’Brien et al. (2006b, the BAT data is extrapolated to the XRT band) and fitted using one or two components—the prompt component and the optional afterglow component—both of
which have the same functional form:

\[
f_c(t) = \begin{cases} 
F_c \exp \left( \alpha_c - \frac{t}{T_c} \right) \exp \left( \frac{-t}{t_c} \right), & t < T_c, \\
F_c \left( \frac{t}{T_c} \right)^{-\alpha_c} \exp \left( -\frac{t}{t_c} \right), & t \geq T_c.
\end{cases}
\]

(1)

The transition from the exponential to the power law occurs at the point \(T_c\), \(F_c\). The subscript \(c\) is replaced by \(p\) for the prompt component and by \(a\) for the afterglow component, and the fitted X-ray flux is the sum of the two components, i.e.,

\[f(t) = f_p(t) + f_a(t).\]

In the derivation of the parameters, an initial fit was done to find the peak position of the prompt emission; the peak time was then used as time zero. A second fit was done with \(t_p = 0\). Thus, the derived parameters are all referenced with respect to the estimated peak time rather than the somewhat arbitrary BAT trigger time. In addition, large flares have been masked out of the fitting procedure (see Lazzati et al. 2008 for a discussion on the average luminosity of X-ray flares as a function of time).

We investigated the correlation between the transition time \(T_p\) for the prompt component and the X-ray luminosity of GRBs at \(T_p\) (see also Dainotti et al. 2008 for a discussion on the correlation between \(T_p\) and the luminosity at \(T_p\)). The fit of the GRB light curves directly gives the values of \(T_p\). The X-ray luminosity of GRBs at a given time \(t\) is calculated by

\[L_X(t) = 4\pi d_L^2(z)F_X(t),\]

(2)

where \(d_L(z)\) is the luminosity distance, for which we have used the flat ΛCDM cosmological model with \(\Omega_m = 0.27\) and \(H_0 = 71\) km s\(^{-1}\) Mpc\(^{-1}\), and \(F_X(t)\) is the \(K\)-corrected flux given by

\[F_X(t) = f(t) \times \int_{E_{\text{min}}/(1+z)}^{E_{\text{max}}/(1+z)} E^{-\beta} dE \int_{E_{\text{min}}}^{E_{\text{max}}} E^{-\beta} dE = f(t) \times (1 + z)^{\beta - 1},\]

(3)

where \(\beta\) is the spectral index and, in general, it is time dependent. Willingale et al. (2007) presented in Table 3 of their paper the spectral index in the prompt phase (\(\beta_p\); from the BAT data), in the prompt decay (\(\beta_{ad}\); from the XRT data), on the plateau of the afterglow component (\(\beta_a\); XRT data), and in the final decay (\(\beta_{at}\); XRT data). Since the BAT data have been extrapolated to the XRT band in the combination of the BAT data and XRT data, \((E_{\text{min}}, E_{\text{max}}) = (0.3, 10)\) keV should be used here and for this limited energy range, the simple power-law \(E^{-\beta}\) is sufficient for the GRB spectrum. So, the X-ray luminosity of GRBs at the transition time \(T_p\) is

\[L_{X,p} = 4\pi d_L^2(z)F_{X,p} = 4\pi d_L^2(z)F_p(1 + z)^{\beta_p - 1},\]

(4)

where we have used \(\beta = \beta_p\) at \(t = T_p\).

For the investigation of the correlation, we mainly fit the data to the relation

\[\log L_{X,p} = a + b \log \left[ \frac{T_p}{1 + z} \right],\]

(5)

where \(T_p/(1 + z)\) is the corresponding transition time in the burst frame. Like many other luminosity relations, this relation is by no means an accurate one. Due to the complexity of GRBs, it is hardly possible that \(\log L_{X,p}\) is fully determined by only \(\log[T_p/(1 + z)]\). As usual, intrinsic scatter \(\sigma_{\text{int}}\) is introduced here, i.e., lacking further knowledge, an extra variability that follows the normal distribution \(\mathcal{N}(0, \sigma_{\text{int}}^2)\) is added to take into account the contributions to \(\log L_{X,p}\) from hidden variables. For the fit, we used the techniques presented in D’Agostini (2005). Let \(x = \log[T_p/(1 + z)]\) and \(y = \log L_{X,p}\); according to D’Agostini (2005), the joint likelihood function for the coefficients \(a\) and \(b\) and the intrinsic scatter \(\sigma_{\text{int}}\) is

\[L(a, b, \sigma_{\text{int}}) \propto \prod_i \frac{1}{\sqrt{2\pi \sigma_{\text{int}}^2 + \sigma_i^2 + b^2\sigma_i^2}} \exp \left[ -\frac{(y_i - a - bx_i)^2}{2(\sigma_{\text{int}}^2 + \sigma_i^2 + b^2\sigma_i^2)} \right],\]

(6)

where \(y_i\) and \(x_i\) are corresponding observational data for the \(i\)th GRB.

For GRB data, we used the samples compiled in Willingale et al. (2007), that is, the 107 GRBs detected by both BAT and XRT on Swift up to 2006 August 1. The \(T_{90}\) duration for these 107 GRBs were obtained from http://swift.gsfc.nasa.gov/. The uncertainties of the fitted parameters in Willingale et al. (2007) are given in 90% confidence level. We symmetrize the errors and derive the corresponding 1σ uncertainties by just dividing the 90% confidence level errors by 1.645. Unless stated explicitly, the errors in this paper are for the 1σ confidence level. In our analysis, when not all of the GRBs have the needed parameters available, we use the maximum subset of the 107 GRBs satisfying the requirement. For example, in Equation (4), the calculation of \(L_{X,p}\) needs the observed redshift \(z\), in addition to \(F_p\) and \(\beta_p\), which are derived from the fit of GRB data.

3. RESULTS AND DISCUSSION

We plot the logarithm of \(L_{X,p}\) versus the logarithm of \(T_p/(1 + z)\) in Figure 1, which includes 47 GRBs. We can see that most of the GRBs (34 GRBs) lie in the range of \(2s < T_p/(1 + z) < 100\) s. There is obviously a correlation between \(L_{X,p}\) and \(T_p/(1 + z)\). However, when fitting the GRBs to the relations of Equation (5), we have different options depending on how we view the three GRBs with the largest \(T_p/(1 + z)\), i.e., those with \(T_p/(1 + z) > 100\) s. For the first choice, we can simply include all the data points in the fit (see the top panel of Figure 2), which leads to a result that the GRB with the largest \(T_p/(1 + z)\) lies outside the 2σ confidence region of the fit. Alternatively, it is also possible that the GRBs with the largest \(T_p/(1 + z)\), instead of just being outliers, may indeed reveal some trend of the luminosity relation at large \(T_p/(1 + z)\). In this case, we cannot simply ignore the GRBs with large \(T_p/(1 + z)\) just because their quantity is small and, if they are taken more seriously, it seems that the samples are split into two groups at some value of \(T_p/(1 + z)\) based on the slope of the luminosity relation of Equation (5). To show this, we perform a fit using only the GRBs with \(T_p/(1 + z) > 2s\) (see the bottom panel of Figure 2), which gives a result quite different from the prior fit. From the bottom panel of Figure 2, we can see that if the best fit line is extended to the range of \(T_p/(1 + z) < 2s\), all the GRBs with \(T_p/(1 + z) < 2s\) lie below the line. As a comparison, we also fit the GRBs with \(2s < T_p/(1 + z) < 100\) s (see Figure 3), which show that the difference is indeed introduced by the three GRBs with the largest \(T_p/(1 + z)\) when compared with the fit.
of GRBs with $T_p/(1+z) > 2\text{ s}$. We tabulate the fit results in Table 1. From the table, we can see that the values of $b$ for the first two cases are quite different. In addition, it is also interesting to note that, for the first case, the slope $b$ is close to the slope $(-0.74^{+0.20}_{-0.19})$ of a similar luminosity relation about $T_a$ and luminosity at $T_a$ presented in Dainotti et al. (2008). For the second case, the slope $b$ is close to the index $(-1.5 \pm 0.16)$ of the power-law declination of the average luminosity of X-ray flares as a function of time presented in Lazzati et al. (2008). These coincidences may be worth noting in future studies with a bigger sample of GRBs.

Generally speaking, in statistical analysis, except for the different measurement precisions, we should treat all the data points equally and should not give more attention to some data points over the others. However, since our sample of GRBs here is very limited, and there are only a few GRBs with very large $T_p/(1+z)$, some selection rules may have already been imposed implicitly on the sample itself. Considering this, it may be unfair to the GRBs with large $T_p/(1+z)$ to be treated as outliers just because their quantity is small, especially when they may reveal some trend in the luminosity relation. This is why we consider both cases in the analysis, i.e., whether the few GRBs with large $T_p/(1+z)$ should be treated as just outliers or taken more seriously. Correspondingly, the relation between $L_{X,p}$ and $T_p/(1+z)$ could be a simple power law or a broken power law with a change in the slope of Equation (5) at some characteristic value of $T_p/(1+z)$. For the present sample of GRBs used in our analysis, it is not sufficient for us to conclude which case is real and, for the later one, to determine the exact value of $T_p/(1+z)$ where the slope of the luminosity relation changes. Here, we leave it open to future studies with more GRBs and discuss

\begin{table}
\centering
\caption{Results of the Fit to the Luminosity Relation of Equation (5)}
\begin{tabular}{llll}
\hline
GRB Set & $a$ & $b$ & $\sigma_{\text{int}}$
\hline
All available GRBs & 50.91 $\pm$ 0.23 & $-0.89 \pm 0.19$ & 1.06 $\pm$ 0.13
$T_p/(1+z) > 2\text{ s}$ & 51.96 $\pm$ 0.32 & $-1.73 \pm 0.25$ & 0.78 $\pm$ 0.11
$2\text{ s} < T_p/(1+z) < 100\text{ s}$ & 51.09 $\pm$ 0.32 & $-0.74 \pm 0.30$ & 0.63 $\pm$ 0.09
\hline
\end{tabular}
\end{table}
in the following the implications of the luminosity relation, especially in the situation where there is a change in the slope at some value of \( T_p/(1+z) \).

First of all, we emphasize that the luminosity relation is between the luminosity and \( T_p \), though \( T_p \) and \( T_{90} \) can both act as an estimate of the GRB duration and are comparable to each other for most GRBs, as can be seen from Figure 4. A similar relation seems not to exist between the luminosity and the \( T_{90} \). See Figure 5; the corresponding data points turn out to be very dispersive. Despite the fact that most GRBs in Figure 4 are distributed around the line on which \( T_p \) and \( T_{90} \) are equal, there are some of them that have considerably different values of \( T_p \) and \( T_{90} \). In fact, the two quantities differ from each other significantly from their derivations. \( T_{90} \) is calculated by directly using the BAT data in the 15–150 keV band, while for the calculation of \( T_p \), the BAT data are first extrapolated to the XRT band of 0.3–10 keV in order to be combined with the XRT data. For \( T_{90} \), the emphasis is on the percent of the fluence of a burst, while for \( T_p \), the emphasis is on the transition in a GRB light curve from the exponential decay in the prompt phase to the initial power-law decay. In the time interval of \( T_{90} \), 90% of the total fluence is observed, while the ratio between the observed fluence in the time interval of \( T_p \) and the total fluence depends not only on the temporal decay index of the initial power-law decay duration in the prompt component, but also on the shape of the light curve in the afterglow component. In addition, we must remember that large flares have been masked out from the light curves before performing the fit that allows us to derive \( T_p \) (Willingale et al. 2007).

As stated above, because the data are limited, there are two possible models for the luminosity relation of Equation (5), i.e., the luminosity relation, using one set of values for the parameters (the intercept \( a \) and the slope \( b \)), may be applicable to all the GRBs except for some outliers, or there may be a change in the slope \( b \) of the luminosity relation at some value of \( T_p/(1+z) \). If there is a change in slope this may suggest that GRBs could be classified into two groups based on their values of \( T_p/(1+z) \). Since \( T_p/(1+z) \) is an estimate of the GRB duration, this is in fact an indication of how we should classify GRBs into long and short ones and is actually the same as the proposal by O’Brien & Willingale (2007) to use \( T_p \) as the criterion for the classification of long and short GRBs. In principle, we should use the quantity in the burst frame \( (T_p/(1+z)) \) instead of that in the observer frame \( (T_p) \). However, for a large portion of the observed GRBs, the redshifts are not available. Generally speaking, due to the diversity of GRBs, the classification of a GRB is unlikely to be completely determined by only its time duration, not to mention that the time duration of long and short GRBs overlaps near the demarcation point. Let us assume the demarcation point in the burst frame for the long and short GRBs to be

\[
T_p/(1+z) = T_{90}/(1+z).
\]  

Then, as an approximate method, an effective redshift \( z_{\text{eff}} \) can be defined, such that GRBs can be classified into long and short ones in the observer frame at

\[
T_p = (1+z_{\text{eff}})T_{90}/s.
\]  

In addition, since a similar relation does not hold if we replace \( T_p \) with \( T_{90} \) as mentioned previously, the change in the slope \( b \), which may be a reflection of different mechanisms, if confirmed, would favor \( T_p \) over \( T_{90} \) as a criterion for the classification of long and short GRBs.

4. SUMMARY

We investigated the correlation between \( T_p \) and the X-ray luminosity of GRBs at \( T_p \) and found a (broken) linear relation between \( \log L_{X,p} \) and \( \log(T_p/(1+z)) \). There may be a change in the slope of the relation at some value of \( T_p/(1+z) \) mainly because of the presence of the few GRBs with large \( T_p/(1+z) \). The limited GRB sample used in our analysis is still not sufficient for us to conclude whether the change in the slope is real or just an illusion caused by outliers. We considered both the cases in our analysis. If the change is real, the different slopes may be a reflection of different mechanisms for GRBs, which may suggest that using \( T_p \) instead of \( T_{90} \) (considering that a similar relation does not hold if we replace \( T_p \) with \( T_{90} \), though \( T_{90} \) and \( T_p \) are both estimates of the GRB duration) as a criterion for the classification of long and short GRBs.
Shi Qi thanks Xue-Wen Liu, Lang Shao, Bo Yu, and Xue-Feng Wu for helpful conversations. We also thank the anonymous referee for many helpful suggestions and comments. We acknowledge the use of public data from the *Swift* data archive. This research was supported by the National Natural Science Foundation of China under grant No. 10973039 and the Jiangsu Planned Projects for Postdoctoral Research Funds 0901059C (for S.Q.).

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