Dynamically broken Lorentz invariance from the Higgs sector?

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Abstract
The original Abelian $U(1)$ Higgs model in flat spacetime is enlarged by the addition of one real scalar with a particular potential. It is then shown that, while maintaining the original masses of the vector boson and Higgs scalar, there exists a time-dependent homogeneous solution of the classical field equations, which corresponds to dynamical breaking of Lorentz invariance (DBLI). The same DBLI mechanism holds for the standard model enlarged by the addition of a real isosinglet scalar with an appropriate potential. The resulting DBLI with an assumed TeV energy scale would manifest itself primarily in the interactions of the two scalar particles. In principle, this DBLI could feed into the neutrino sector and give rise to a superluminal maximum velocity.

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I. INTRODUCTION

The standard model (SM) of elementary particle physics is, most likely, an effective theory resulting from fundamental interactions at an energy scale far above the electroweak energy scale (taken as $E_{\text{ew}} = M_W \sim 10^2 \text{ GeV}$).

If this point of view is correct, it is possible that the Higgs sector contains further scalar particles and interactions, in addition to those of the minimal SM. Here, we explore certain effects of an additional real scalar field, which may lead to dynamical breaking of Lorentz invariance (DBLI).\(^1\) This DBLI would manifest itself primarily in the Higgs sector but perhaps also through nonstandard neutrino-propagation properties (e.g., superluminal maximum velocities), as will be explained later on.

In order to keep the number of references manageable, we only quote Ref. [1] for an early paper on spontaneously broken Lorentz invariance, Ref. [2] for the Higgs mechanism, and Ref. [3] for a textbook with the basics of the SM and references to the original articles. As far as the main idea of the present article is concerned, DBLI from scalar fields, we are not aware of a similar discussion in the literature (Ref. [4], for example, obtains bounds on certain types of Lorentz violation in the Higgs sector, but does not discuss the dynamic origin of the Lorentz violation).

It also needs to be emphasized that the present article considers only toy-models for DBLI by scalar fields, leaving aside all questions about naturalness and renormalization. The aim is really to see if it is at all possible to get an acceptable form of DBLI from the Higgs sector, more at the level of an existence proof than a fully realistic theory.

II. ABELIAN $U(1)$ HIGGS MODEL

A. Nonstatic background solution

The starting point is the original Abelian $U(1)$ Higgs model [2, 3] in terms of the real gauge field $A_\alpha(x)$ and the complex scalar field $\phi(x) \equiv [\phi_1(x) + i\phi_2(x)]/\sqrt{2}$. To this model is added a single real scalar $\xi(x)$ with a particular potential. The Lagrange density is taken as follows ($\hbar = c = 1$):

\[ \hat{\mathcal{L}} = -\frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} - (D_\alpha \phi)^* (D^\alpha \phi) - \frac{1}{2} (\partial_\alpha \xi) (\partial^\alpha \xi) - \hat{V}, \quad (2.1a) \]

\(^1\) In an earlier version of this article, the term ‘spontaneous breaking of Lorentz invariance’ (SBLI) was used. But, for the moment, the term ‘dynamical breaking of Lorentz invariance’ is more appropriate. Ultimately, the goal is to get rid of the global (and gravitational) effects of the condensate, with only flat Minkowski spacetime remaining and a nonzero order parameter. Then, it is perhaps also possible to get genuine spontaneous symmetry breaking from the Higgs sector, independent of the boundary conditions.
\[ \hat{V} = \lambda (|\phi|^2 - v^2/2)^2 + \frac{1}{2} m^2 \xi^2 + \zeta v^{-6} (|\phi|^2 - v^2/2)^4 \xi^2, \quad (2.1b) \]

employing the usual Minkowski metric for the standard global spacetime coordinates \( (x^\alpha) = (t, x^1, x^2, x^3), \)

\[ (\eta_{\alpha\beta}) = \text{diag}(-1, 1, 1, 1), \quad (2.2a) \]

and the usual Maxwell field strength tensor and covariant derivative,

\[ F_{\alpha\beta}(x) \equiv \partial_\alpha A_\beta(x) - \partial_\beta A_\alpha(x), \quad (2.2b) \]
\[ D_\alpha \phi(x) \equiv \left( \partial_\alpha - i e A_\alpha(x) \right) \phi(x), \quad (2.2c) \]

where the nonzero real gauge coupling constant of this model is denoted by \( e \) (no reference to the charge of the electron, just as the vector field \( A_\alpha \) has no reference to the photon field).

The theory \((2.1)\) has a local \( U(1) \) gauge invariance involving the complex scalar field, \( \phi(x) \to \exp[i e \omega(x)] \phi(x) \), and a global \( \mathbb{Z}_2 \) invariance of the real scalar field, \( \xi(x) \to \pm \xi(x) \).

The parameters \( v^2, \lambda, m^2, \) and \( \zeta \) in the potential \((2.1b)\) are real and bounded as follows:

\[ v^2 > 0, \quad \lambda > 0, \quad m^2 > 0, \quad \zeta \geq 0. \quad (2.3) \]

The last term in the potential \((2.1b)\) is nonrenormalizable and has mass-dimension 10. It is only added to provide an interaction between the two scalar fields and can, in principle, be omitted by setting \( \zeta = 0 \). Incidentally, a single quartic coupling \( |\phi|^2 \xi^2 \) would not be compatible with the generalized solution to be discussed shortly.

The classical field equations are:

\[ \partial^\alpha F_{\alpha\beta} = -2 e \text{ Im} (\phi^* D_\beta \phi), \quad (2.4a) \]
\[ D_\alpha D_\alpha \phi = 2\lambda (|\phi|^2 - v^2/2) \phi + 4 \zeta v^{-6} (|\phi|^2 - v^2/2)^3 \xi^2 \phi, \quad (2.4b) \]
\[ \partial^\alpha \partial_\alpha \xi = m^2 \xi + 2 \zeta v^{-6} (|\phi|^2 - v^2/2)^4 \xi. \quad (2.4c) \]

These classical field equations have the standard Higgs vacuum solution \([2]\):

\[ A_\alpha(x) = 0, \quad (2.5a) \]
\[ \phi(x) = v/\sqrt{2}, \quad (2.5b) \]
\[ \xi(x) = 0, \quad (2.5c) \]

up to a global phase of the field \( \phi \). This solution is static and homogenous: the constant scalar fields correspond to a Lorentz-invariant state.
The background solution (2.5) can be generalized to a nonstatic homogeneous solution, without changing the masses of the vector and scalar perturbation modes (see below). Specifically, this generalized solution is

\[ A_\alpha(x) = 0, \quad (2.6a) \]

\[ \phi(x) = v/\sqrt{2}, \quad (2.6b) \]

\[ \xi(x) = \xi_0 \cos(mt), \quad (2.6c) \]

for arbitrary real constant \( \xi_0 \). The solution is only given up to a global phase of \( \phi \) and up to time translation.

The nonstatic homogeneous background (2.6) with \( \xi_0 \neq 0 \) can, in principle, be selected by imposing appropriate initial boundary conditions on the fields and their derivatives. Remark that, for the case of vanishing coupling constant \( \zeta \) and for the class of homogeneous field configurations, generic initial boundary conditions \( \xi(t_{in}) \) and \( \partial_t \xi(t_{in}) \) give a solution of the type (2.6a) with \( \xi_0 \neq 0 \), whereas only the special values \( \xi(t_{in}) = 0 \) and \( \partial_t \xi(t_{in}) = 0 \) give a solution with \( \xi_0 = 0 \). What really needs to be explained is that these initial \( \xi \) fields are (approximately) homogeneous. The explanation of the extraordinarily smooth conditions just after the “big bang” is anyway the major unsolved problem of modern cosmology (the inflation mechanism may or may not play a role in the final answer \([5–7]\)). The present article, however, considers a fixed Minkowski spacetime and neglects gravity altogether (see also Sec. II C). For now, we simply assume that the initial boundary conditions select solution (2.6) with \( \xi_0 \neq 0 \).

**B. Localized perturbations**

It remains for us to calculate the particle spectrum for the nonstatic background (2.6). This can be done in unitary gauge \([3]\),

\[ A_\alpha(x) = A_\alpha(x), \quad (2.7a) \]

\[ \sqrt{2} \phi(x) = v + h(x), \quad (2.7b) \]

\[ \xi(x) = \xi_0 \cos(mt) + k(x). \quad (2.7c) \]
Localized perturbations allow for partial integrations without boundary terms and the Lagrange density up to quadratic order is found to be given by\(^2\)

\[
\mathcal{L}^\text{(lin.+ quadr.)} = -\frac{1}{4} (\partial_\alpha A_\beta - \partial_\beta A_\alpha)^2 - \frac{1}{2} \left[ (ev)^2 \right] A_\alpha A^\alpha
\]

\[
-\frac{1}{2} (\partial_\alpha h)^2 - \frac{1}{2} \left[ 2 \lambda v^2 \right] h^2 - \frac{1}{2} (\partial_k k)^2 - \frac{1}{2} \left[ m^2 \right] k^2 ,
\]

(2.8a)

where, as expected, all linear terms have dropped out and a zero-order time-dependent term has not been shown (this term will be discussed in the last two paragraphs of Sec. II C). The remainder of the Lagrange density includes cubic and higher-order interaction terms,

\[
\mathcal{L}^\text{(interact.)} = -\varepsilon^2 A_\alpha A^\alpha h \left( v + h/2 \right) - \lambda \left( v h^3 + h^4/4 \right)
\]

\[
-\left( \zeta/16 \right) v^{-6} \left( h^2 + 2 h v \right)^4 \left( k + \xi_0 \cos mt \right)^2 ,
\]

(2.8b)

where the \(\zeta\) term contains monomials \(h^a k^b\) for \(a = 4, \ldots, 8\) and \(b = 0, 1, 2\). Interestingly, the background parameter \(\xi_0\) only affects the interactions of the scalar perturbation modes (see Sec. III B for further discussion).

The scalar perturbations \(h\) and \(k\) in (2.8a) have positive mass-squares, denoted by square brackets, \((m_h)^2 = 2 \lambda v^2 > 0\) and \((m_k)^2 = m^2 > 0\). The same holds for the vector-field perturbation, \((m_A)^2 = (ev)^2 > 0\). Hence, the classical stability of the nonstatic background (2.6) is manifest and independent of the background parameter \(\xi_0\). Quantum-dissipative effects from the radiation of \(h\)-quanta can be made arbitrarily small by taking \(\zeta\) sufficiently small. For \(\zeta = 0\), the nonstatic background (2.6) is absolutely stable against localized perturbations.

C. Dynamical breaking of Lorentz invariance

It is possible to define the following tensor composite of scalar-field derivatives:

\[
\widehat{b}_{\alpha\beta} \equiv \frac{2}{m^4} \left( \partial_\alpha \xi \right) \left( \partial_\beta \xi \right) ,
\]

(2.9a)

whose average over time intervals very much larger than \(1/m\) does not vanish for the nonstatic classical background (2.6),

\[
\langle \widehat{b}_{\alpha\beta} \rangle_{\text{time-average}}^{\text{(nonstat. class. sol.)}} = \left( \xi_0/m \right)^2 \delta_{\alpha,0} \delta_{\beta,0} .
\]

(2.9b)

\(^2\) In an earlier version of this article, we considered a theory with a complex (charged) scalar \(\psi(x)\) instead of the real (neutral) scalar \(\xi(x)\) used here. However, the quadratic Lagrange density given previously missed two terms which may lead to instability. For this reason, we now restrict ourselves to having just an additional real scalar \(\xi(x)\) and make sure to have only higher-order couplings between the \(h(x)\) and \(k(x)\) modes.
In the quantum theory with $\zeta = 0$ (i.e., noninteracting $\xi$ fields), the corresponding time-averaged expectation value in a ground state with dynamical symmetry breaking (DSB) is given by

$$\hat{b}_{\alpha\beta} \bigg|_{\text{DSB, quant. th., time-average}} \equiv \frac{2}{m^4} \left( \langle \partial_\alpha \xi \rangle \langle \partial_\beta \xi \rangle \right)_{\text{DSB, time-average}} \sim \left( \frac{\xi_0}{m} \right)^2 \delta_{\alpha,0} \delta_{\beta,0} , \tag{2.9c}$$

now in terms of the renormalized quantities $\xi_0$ and $m^2$. Having a nonzero order parameter $(2.9b)$ or $(2.9c)$ signals the dynamical breaking of Lorentz invariance.

The composite operator $(2.9a)$ used as a diagnostic for broken Lorentz invariance is, of course, not unique. A mathematically attractive alternative would be

$$\bar{b}_{\alpha\beta} \equiv \frac{(\partial_\alpha \xi) (\partial_\beta \xi)}{\sqrt{[(\partial_\gamma \xi) (\partial_\gamma \xi)]^2 + \epsilon m^8}} , \tag{2.10}$$

for a positive infinitesimal $\epsilon$. The order parameter $\bar{b}_{00}$ would be nonzero and approximately constant also for $\xi(t)$ oscillations with a slowly decreasing amplitude (for example, due to quantum-dissipative effects if $\zeta$ is small but nonzero). But, for the moment, we restrict the discussion to the simpler type of composite $(2.9a)$.

Returning to the nonstatic background solution, two technical remarks are in order. First, given the fields equations $(2.4)$, there is no background solution equal to $(2.6)$ with the argument $mt$ of the cosine function replaced by, for example, $mx^1$. In this way, the dynamics singles out the time components of the order parameter $(2.9b)$. Second, the background solution $(2.6)$ is invariant under time reversal (T), charge conjugation (C), and parity reflection (P). Hence, there is spontaneous breaking of Lorentz invariance but not of CPT or any of the separate discrete symmetries.

Finally, there is an important issue, already alluded to in Ftn. 1, which has to do with gravity. In the standard approach, solution $(2.6)$ would give a homogeneous pressure $P_{\phi, \xi}$ and energy density $\rho_{\phi, \xi}$, so that Minkowski spacetime would no longer be a solution of the Einstein gravitational field equations (see the Appendix for further discussion). It is, of course, known that the Higgs mechanism has a potential conflict with gravity, for example, as regards the cosmological constant problem [8, 9], and it is possible that the pressure and energy density of the Higgs condensate gravitate unconventionally. A fortiori, unconventional gravitational properties may hold for the Lorentz-noninvariant state considered, with $P_{\phi, \xi} + \rho_{\phi, \xi} \neq 0$.

For the moment, we choose to completely neglect gravity and to postulate the flat Minkowski spacetime metric $(2.2a)$. Having fixed the metric $(2.2a)$, the ground state $(2.6)$ is a perfectly stable classical solution, although with inherent Lorentz breaking, as made clear by the time-dependent interaction terms in $(2.8b)$ and the nonzero order parameter $(2.9c)$. 

III. ENLARGED STANDARD MODEL

A. Nonstatic background and DBLI

Now let us turn to the SM and use the more or less standard notation of Ref. [3]. As the SM has only a single physical scalar particle, the previous DBLI mechanism cannot be implemented directly. It seems, therefore, necessary to extend the scalar content of the SM. One possibility is simply to add the two previous scalar fields \( \phi(x) \) and \( \xi(x) \), without coupling to the gauge field \( (e = 0) \) but keeping the previous potential term (2.1b). Then, however, there is no reason that the mass scales in this potential would be of the order of the electroweak scale \( E_{ew} \). Another possibility is to add only one neutral (sterile) scalar field, now denoted \( \Xi(x) \), and to couple it via the potential term to the SM isodoublet \( \Phi(x) \).

The Lagrange density is taken to be

\[
L = L_{\text{SM, vector, spinor}} + L_{\text{scalar}}, \tag{3.1a}
\]

with

\[
L_{\text{scalar}} = -\left( D_\alpha \Phi \right)^\dagger \left( D^\alpha \Phi \right) - \frac{1}{2} \left( \partial_\alpha \Xi \right) \left( \partial^\alpha \Xi \right) - V_{\text{scalar}}, \tag{3.1b}
\]

\[
V_{\text{scalar}} = \lambda \left( \Phi^\dagger \Phi - v^2/2 \right)^2 + \frac{1}{2} m^2 \Xi^2 + \zeta v^{-6} \left( \Phi^\dagger \Phi - v^2/2 \right)^4 \Xi^2, \tag{3.1c}
\]

where \( \Xi(x) \) is a real isosinglet and \( \Phi(x) \) a complex isodoublet with covariant derivative

\[
D_\alpha \Phi(x) \equiv \left[ \partial_\alpha + g \tau^a/(2i) W^a_\alpha(x) + g' \mathbb{1}_2/(2i) B_\alpha(x) \right] \Phi(x), \tag{3.1d}
\]

in terms of the \( SU(2) \) gauge fields \( W^a_\alpha(x) \) and the \( U(1) \) gauge field \( B_\alpha(x) \), the three matrices \( \tau^a \) being the usual \( 2 \times 2 \) Pauli matrices of isospin and \( \mathbb{1}_2 \) the \( 2 \times 2 \) unit matrix. The parameters of potential (3.1c) are taken as in (2.3). More specifically, it may be natural to have \( v^2 \sim m^2 \) and \( \lambda \sim \zeta \sim 1 \), with (3.1) considered to be an effective theory for energies up to the TeV scale. Remark that, with a vanishing scalar field \( \Xi(x) \equiv 0 \), the Lagrange density \( \mathcal{L} \) in (3.1a) equals the standard-model Lagrange density \([3], \mathcal{L}_{\text{SM}}\).

For later use, we explicitly give the \( SU(2) \) representations (isodoublet or isosinglet) of the basic SM (anti-)fermion fields of the three lepton families (label \( f = e, \mu, \tau \)) and the \( SU(2) \) representation (isodoublet) of the SM Higgs field:

\[
L_f(x) = \begin{pmatrix} \nu_{f,L}(x) \\ \bar{f}_{\ell}^-(x) \end{pmatrix}_{-1}, \quad R_f(x) = \begin{pmatrix} f^+_R(x) \end{pmatrix}_{+2}, \tag{3.2a}
\]

\[
\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}_{+1}, \quad \bar{\Phi} \equiv i \tau_2 \cdot \Phi^* \equiv \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix} \cdot \Phi^*, \tag{3.2b}
\]
where the suffix on the first three $SU(2)$ representations gives the value of the $U(1)$ hypercharge $Y$ (recall that the electric charge is given by $Q \equiv I_3 + Y/2$, for isospin $I_3$) and the asterisk in the last definition of (3.2b) denotes complex conjugation. The isodoublet in (3.2a) has lepton number $L = +1$ and the corresponding isosinglet $L = -1$. As said, we also add the $Y = 0$ isosinglet scalar $\Xi(x)$ with an appropriate potential (3.1c).

Turning to the bosonic fields of the enlarged-SM theory (3.1), the discussion is the same as for the Abelian $U(1)$ Higgs model of Sec. II A. Again, there is a nonstatic homogeneous Higgs-like solution,

\begin{align}
W^a_\alpha(x) &= B_\alpha(x) = 0, \\
\Phi(x) &= v/\sqrt{2} \begin{pmatrix} 0 \\
1 \end{pmatrix}, \\
\Xi(x) &= \Xi_0 \cos(mt),
\end{align}

up to global transformations of $\Phi$ and time translations. Also, as mentioned in Sec. II A, generic homogeneous boundary conditions favor having (3.3c) with $\Xi_0 \neq 0$. The discussion of the masses of the scalar and vector perturbation modes is essentially the same as in Sec. II B.

It is, again, possible to identify a tensor composite of scalar fields,

\begin{equation}
b_{\alpha\beta} = \frac{2}{m^4} \left( \partial_\alpha \Xi \right) \left( \partial_\beta \Xi \right),
\end{equation}

which has a nonzero time-averaged $b_{00}$ component for the nonstatic background (3.3),

\begin{equation}
\langle b_{\alpha\beta} \rangle_{\text{time-average}}^{\text{nonstat. class. sol.}} = \left( \frac{\Xi_0}{m} \right)^2 \delta_{\alpha,0} \delta_{\beta,0}.
\end{equation}

This background tensor will play an important role in Sec. IV.

**B. Nonstandard Higgs physics**

In our toy-model (3.1) with nonstatic background (3.3), the two neutral scalar particles (denoted $h$ and $k$, just as in Sec. II) have propagation properties and interactions which are described by action-density terms similar to those in, respectively, (2.8a) and (2.8b). The dispersion relations of these two scalars are entirely standard (Lorentz-invariant), only their interactions are nonstandard (i.e., spacetime-dependent). In this subsection, it is assumed that the coupling constant $\zeta$ is nonzero and that the experiments run over time intervals during which quantum-dissipative effects can be neglected.

The quartic coupling constant of the Higgs scalar $h$, for example, is given by

\begin{equation}
\lambda_{h-\text{quart.}} = \lambda + \zeta' \cos^2 mt,
\end{equation}
with $\zeta' \geq 0$ depending on the amplitude $\Xi_0$ of the background solution, $\zeta' \equiv (\Xi_0/v)^2 \zeta$. For low-energy processes ($\sqrt{s} \ll m \sim \text{TeV}$), the effective coupling constant would be

$$\lambda_{h-\text{quart.}}^{(\text{low-energy})} \sim \lambda + \frac{1}{2} \zeta'.$$

The rapid oscillations in (3.5a) would only show up for high-energy processes ($\sqrt{s} \sim m$). Concretely, the quartic Higgs coupling constant determined from high-energy-collision experiments (many identical experiments repeated over time) would show an inherent uncertainty with spread $\zeta'$, independent of the experimental uncertainties involved.

Two parenthetical remarks are as follows. First, the above discussion of the Higgs self-coupling (3.5) refers to experiments in the preferred frame defined by the background solution (3.3). In the laboratory frame, there are (small) changes due to the motion of the spinning Earth around the Sun and the motion of the solar system as a whole with respect to the preferred frame (see, e.g., Ref. [10] for further discussion). In addition to special-relativistic time-dilatation effects, there may also be gravitational time-dilatation effects due to the various solar-system masses. The proper analysis of these gravitational time-dilatation effects requires a complete solution of the combined field equations, which lies outside the scope of this article as it neglects gravity altogether (cf. the last paragraph of Sec. II C).

Second, spacetime-dependent coupling constants have been considered in other contexts (see, e.g., Ref. [11] for a review). The focus, here, is on rapidly variable couplings with ultrashort timescales, not cosmological timescales. Another possible source of small-scale modulations of the coupling constants may be a topologically nontrivial small-scale spacetime structure [12].

All in all, the scalars $h$ and $k$ from (2.8) display a rather ‘mild’ form of Lorentz violation or, more precisely, Poincaré violation. Still, this Lorentz/Poincaré violation with spacetime-dependent coupling constants has not been put in by hand but has arisen dynamically.\(^3\) Theoretically, this is a significant improvement. But important questions remain [assuming the toy-model (3.1) to have any physical significance at all], for example, the origin of the special initial boundary conditions needed to select the particular nonstatic homogeneous background solution (cf. the last paragraph of Sec. II A).

IV. SCALAR-NEUTRINO INTERACTIONS

A. Higher-derivative term

Let us discuss one further application of our dynamic Lorentz-symmetry-breaking mechanism, namely, as a model to describe the, as of yet unconfirmed, OPERA result [13] on a

\(^3\)Remark that, generally speaking, it does not make sense to consider ‘wild’ \textit{ad hoc} forms of Lorentz violation if they cannot be generated dynamically.
superluminal neutrino velocity.\textsuperscript{4} It was soon realized \cite{14-17} that SBLI, from fermion condensates in particular, could play a role in the explanation of the OPERA result. Ref. \cite{18} also mentions SBLI but really focusses on DBLI for an explanation, although not giving a comprehensive dynamical solution. With the background tensor (3.4), we can now propose a relatively simple DBLI model which appears to be phenomenologically attractive (apart from one outstanding problem as will be explained in Sec. IV B).

We use the enlarged SM of Sec. III A and, in particular, the fields (3.2) and the composite (3.4a). For now, we take for granted that the three neutrinos obtain masses $m_{\nu, n} \lesssim eV$, for $n = 1, 2, 3$. We, then, introduce a further gauge-invariant interaction term \cite{17} for the enlarged-SM fields:

$$\mathcal{L}_{11}(x) = \frac{2M}{v^2} \sum_f \left( L_f(x) \cdot \tilde{\Phi}(x) \right) \left( \frac{1}{M^2} b^{\alpha\beta} \partial_\alpha \partial_\beta \right) \left( \tilde{\Phi}^\dagger(x) \cdot L_f(x) \right) + \text{H.c.}, \quad (4.1)$$

where $\psi^c(x)$ denotes the charge conjugate of field $\psi(x)$ and the prefactor $2/v^2$ is chosen to cancel the $\Phi$ contributions from (3.3b). The interaction term (4.1) is non-renormalizable (with a suppression factor $1/M$ at low energies) and violates lepton-number conservation. Recalling the definition (3.4a) of $b^{\alpha\beta}$, it follows that the composite field operator on the right-hand side of (4.1) has mass dimension 11, hence the suffix on the left-hand side. Without the insertion $[M^{-2} b^{\alpha\beta} \partial_\alpha \partial_\beta]$ and replacing the prefactor $2M/v^2$ by $1/M_v$, the resulting dimension-5 interaction term is precisely the Majorana-mass-type term considered in Ref. \cite{19} and, many years later, in Ref. \cite{17}, where its potential role for neutrino-LV was emphasized.

We assume all mass scales entering (4.1) to be of the same order,

$$v \sim m \sim M \sim \text{TeV}. \quad (4.2a)$$

According to (3.4), the time-averaged tensor $b^{\alpha\beta}$ in (4.1) is of order unity for the nonstatic background (3.3) with $\Xi_0 \sim m$,

$$b^{\alpha\beta} \sim \delta^{\alpha, 0} \delta^{\beta, 0}. \quad (4.2b)$$

As mentioned in Sec. III C having a nonzero order parameter (4.2b) signals the dynamical breaking of Lorentz invariance. In principle, it is also possible to use in (4.1) the background tensor $\tilde{b}^{\alpha\beta}$ from (2.10) with $\xi(x)$ replaced by $\Xi(x)$.

### B. Superluminal neutrinos

In the nonstatic background (3.3) with $\Xi_0 \sim m$, the resulting Lorentz-violating interaction term (4.1) leads to modified dispersion relations of the neutrinos \cite{14-17}. For the three

\textsuperscript{4} A CERN press release (February 23, 2012) from the OPERA Collaboration states that two possible sources of error have been found and that new short-pulse measurements are scheduled for May, 2012.
neutrino mass states \((n = 1, 2, 3)\) and 3-momenta \(\mathbf{p}\) bounded by \(\max[(m_{\nu,n})^2] \ll |\mathbf{p}|^2 \ll \min(M^2, m^2)\), these dispersion relations are \((c = 1)\)

\[
(E_{\nu,n}(\mathbf{p}))^2 \sim |\mathbf{p}|^2 + (m_{\nu,n})^2 + (\delta^{00})^2 M^{-2} |\mathbf{p}|^4,
\]

\((4.3)\)

with \(\delta^{00} \sim 1\) according to \((4.2)\). Remark that the preferred frame of the Lorentz violation in \((4.3)\) traces back to the background solution \((3.3)\). Most importantly, the quartic term in \((4.3)\) is identical for all three neutrino mass states. It is, of course, also possible to have a higher-order Lorentz-violating term, for example, a term proportional to \(M^{-6} |\mathbf{p}|^8\) from the use of two operator insertions with square brackets in \((4.1)\). For large neutrino energies, \(|\mathbf{p}|^2 \gtrsim \min(M^2, m^2)\), the quartic momentum-dependence of the neutrino dispersion relations \((4.3)\) needs to be tempered, possibly by the introduction of further higher-derivative terms \([16]\).

Referring to the list of experimental “facts” given in Sec. I of Ref. \([16]\) (which contains further references in addition to those given here), the situation is as follows:

(i) The OPERA result \([13]\) \(v/c - 1 \sim 10^{-5}\) for the muon-neutrino time-of-flight velocity at an energy of order \(10\) GeV, assumed to be correct for the sake of argument, can be explained by the modified dispersion relations \((4.3)\) if the mass parameters are of the electroweak scale \((4.2)\), possibly \(\Xi_0 \sim m \sim v\) and \(M \sim 30\) TeV (it is already known that \(v \sim 250\) GeV). Moreover, a narrow initial pulse of muon-neutrinos at CERN would have negligible broadening after traveling the 730 km to the OPERA detector in the GranSasso Laboratory (see the last paragraph of Sec. 3 in Ref. \([16]\)). Incidentally, sterile-neutrino models in four or more spacetime dimensions (see Ref. \([15]\) and references therein) typically predict a substantial broadening of the detected muon-neutrino pulse profile, which is not what OPERA observes (Sec. 9 of Ref. \([13]\)).

(ii) The supernova SN1987a bound \([20]\) \(|v/c - 1| \lesssim 10^{-9}\) on the electron-antineutrino velocity at an energy of order \(10\) MeV is satisfied because of the quadratic energy dependence of the group velocity from \((4.3)\).

(iii) Coherent mass-difference neutrino oscillations remain unaffected \([21]\), because the Lorentz violation from \((4.3)\) operates equally for all three neutrino masses and, thereby, equally for all three neutrino flavors [the original Lagrange density \((4.1)\) has indeed identical terms for all three families].

(iv) Energy losses \([22]\) of the CERN–GranSasso neutrinos from the vacuum-Cherenkov process \(\nu_{\mu} \rightarrow \nu_{\mu} + Z^0 \rightarrow \nu_{\mu} + e^- + e^+\) are significantly reduced \([23]\), by a factor of approximately \((3)^{-5/2} \sim 1/16\), compared to the losses in the theory with an identical Lorentz-violating \(|\mathbf{p}|^2\) term in the three neutrino dispersion relations. The heuristic argument \([16]\) for the reduction factor \((1/\sqrt{3})^5\) relies on the effective-mass-square
concept applied to this muon-decay-type process, giving the vacuum-Cherenkov rate \( \Gamma \propto (G_F)^2 (m_{\text{eff}})^5 \).

(v) The leakage of Lorentz violation from the neutrino sector into the charged-lepton sector by quantum effects appears to be problematic, especially in view of the tight bounds on, e.g., the electron velocity. Obviously, replacing \( M \sim \text{TeV} \) in (4.1) by a very much larger value such as \( M \sim 10^{10} \text{TeV} \) would reduce OPERA-like effects from (4.3) by a factor \( 10^{-20} \), bringing the neutrino-sector Lorentz violation down to the level of the current electron bounds, at least, for low enough energies.

To conclude, it appears possible to have a scalar-DBLI model which can describe OPERA’s claimed result on a superluminal neutrino velocity and other experimental facts of neutrino physics, see items (i)–(iv) above. The difficulty is to connect to experimental facts outside the neutrino sector, see item (v) above. But it is precisely this difficulty which would make the relatively large OPERA value for \( v/c - 1 \), if it turns out to be correct, so significant.

V. DISCUSSION

In this article, we have shown (perhaps not for the first time) that extended Higgs models in Minkowski spacetime can have time-dependent homogeneous solutions of the classical field equations, which correspond to dynamical breaking of Lorentz invariance. This holds, in particular, for a simple enlargement of the standard-model Higgs sector with one extra real isosinglet scalar and an appropriate potential. The energy scales of this potential and the corresponding nonstatic background solution are assumed to be at the TeV scale.

The dynamical breaking of Lorentz invariance from a nonstatic scalar background may lead to new effects in the Higgs sector such as time-dependent couplings of the scalar particles. In addition, this DBLI may feed into the neutrino sector and give rise to superluminal maximum velocities, with a velocity excess controlled by a mass scale \( M \gtrsim \text{TeV} \).

At a more theoretical level, the fundamental problem is merging this DBLI mechanism of scalar fields with gravity (see also the Appendix). This must be done in such a way that the DBLI persists over cosmological time scales and also meshes with the solution of the cosmological constant problem. Incidentally, the solution of the cosmological constant problem is still outstanding: there have been many suggestions (for example, a dynamic adjustment mechanism) but there is not yet a definitive solution.

Possibly related phenomena have been observed in condensed-matter physics, in particular, Bose-Einstein-condensed states of coherent precession in superfluid \(^3\text{He} \); see Ref. and further references therein.
In the present article, we have simply side-stepped the problem of merging the scalar-
DBLI mechanism and gravity by considering only flat Minkowski spacetime. This is sufficient
for a preliminary investigation, but, ultimately, gravity needs to be included.

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Appendix A: Flat-spacetime solution

It has been remarked in the penultimate paragraph of Sec. II C that the nonstatic scalar-
field background of the Abelian $U(1)$ Higgs model considered does not allow for a Minkowski
spacetime solution of the Einstein equations, assuming a standard gravitational behavior of
the scalar condensate. In this appendix, we present an example of how, in principle, it may
be possible to get a flat-spacetime solution if further vacuum and matter contributions are
included.

Consider, in fact, the introduction of two additional real scalar fields. The first scalar
field $\kappa(x)$ has the same kinetic and mass terms as $\xi(x)$ in (2.1) but no further interaction
terms. The second scalar field $\chi(x)$ has a “wrong-sign” kinetic term and no potential term
at all (scalars with a wrong-sign kinetic term have already been considered in, for example,
ghost-condensation models for infrared modifications of gravity [29]). In addition, there are
fine-tuned initial boundary conditions and a fine-tuned cosmological constant $\Lambda$.

Specifically, we add to the Lagrange density $\hat{L}$ of (2.1a) the following four terms:

$$\hat{L}'[A, \phi, \xi, \kappa, \chi] = \hat{L}[A, \phi, \xi] - \frac{1}{2} \partial_\alpha \kappa \partial^\alpha \kappa - \frac{1}{2} m^2 \kappa^2 + \frac{1}{2} \partial_\alpha \chi \partial^\alpha \chi - \Lambda. \tag{A1}$$

The field equation of $\kappa(x)$ is given by (2.4c) with $\xi(x)$ replaced by $\kappa(x)$ but without the $\zeta$
interaction term. The field equation of $\chi(x)$ is $\partial_\alpha \partial^\alpha \chi = 0$ and the homogeneous solution
takes the form $\chi(x) = c_1 t + c_2$ with real dimensional constants $c_1$ and $c_2$.

Special boundary conditions are taken to select the following homogeneous classical solution:

$$A_\alpha(x) = 0, \tag{A2a}$$

$$\phi(x) = v/\sqrt{2}, \tag{A2b}$$

$$\xi(x) = m \cos(mt), \tag{A2c}$$

$$\kappa(x) = m \sin(mt), \tag{A2d}$$

$$\chi(x) = \pm m^2 t + \chi_0, \tag{A2e}$$
with an arbitrary sign of the linear term in $\chi(t)$ and an arbitrary additive constant $\chi_0$. The above solution has an equal amplitude for $\xi(t)$ and $\kappa(t)$, chosen as $\xi_0 = m$, and a nonzero phase difference, chosen as $\pi/2$. Moreover, the cosmological constant is fine-tuned to the following negative value:

$$\Lambda = -\frac{1}{2} m^4.$$

(A2f)

It is also possible that an effective cosmological constant with the precise value $\Lambda$ arises dynamically without fine-tuning \[26–28\]. But, here, we simply postulate the appropriate cosmological constant $\Lambda$. Observe that, with metric signature (2.2a), the contributions of the solutions (A2e) and (A2f) cancel in the action density (A1).

The homogeneous background fields (A2b), (A2c), and (A2d) give the following contributions to the pressure and the energy density:

$$P_{\phi, \xi, \kappa} = |\dot{\phi}|^2 + \frac{1}{2} (\dot{\xi})^2 + \frac{1}{2} (\dot{\kappa})^2 - \dot{V}(\phi, \xi) - \frac{1}{2} m^2 \kappa^2 = 0,$$

(A3a)

$$\rho_{\phi, \xi, \kappa} = |\dot{\phi}|^2 + \frac{1}{2} (\dot{\xi})^2 + \frac{1}{2} (\dot{\kappa})^2 + \dot{V}(\phi, \xi) + \frac{1}{2} m^2 \kappa^2 = m^4,$$

(A3b)

where the overdot stands for differentiation with respect to the coordinate time $t$. The wrong-sign scalar field (A2e) contributes

$$P_{\chi} = -\frac{1}{2} (\dot{\chi})^2 = -\frac{1}{2} m^4,$$

(A4a)

$$\rho_{\chi} = -\frac{1}{2} (\dot{\chi})^2 = -\frac{1}{2} m^4,$$

(A4b)

and the cosmological constant (A2f) gives

$$P_{\Lambda} = -\Lambda = +\frac{1}{2} m^4,$$

(A5a)

$$\rho_{\Lambda} = +\Lambda = -\frac{1}{2} m^4.$$

(A5b)

With vanishing gauge-field background (A2a), the total pressure and energy-density are nullified,

$$P_{\text{total}}^{(\text{background})} = P_{\phi, \xi, \kappa} + P_{\chi} + P_{\Lambda} = 0,$$

(A6a)

$$\rho_{\text{total}}^{(\text{background})} = \rho_{\phi, \xi, \kappa} + \rho_{\chi} + \rho_{\Lambda} = 0.$$  

(A6b)

Having only background fields contributing (i.e., no excitations), flat Minkowski spacetime is then a solution of the Einstein equations. Indeed, the Hubble parameter $H \equiv \dot{a}/a = 0$ of Minkowski spacetime solves the spatially-flat Friedmann equations $H^2 = (8\pi/3) G_N \rho_{\text{total}}$ and $2\dot{H} + 3H^2 = 8\pi G_N P_{\text{total}}$. But, even in a fictitious world without gravity ($G_N = 0$), the

An alternative formulation of the theory uses the complex (but neutral) scalar field $\theta(x) = \xi(x) + i\kappa(x)$.

Then, the initial boundary conditions select $\theta(x) = m \exp[i mt]$. 

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pressure condition \( (A6a) \) is needed to describe a self-sustained equilibrium state (cf. Secs. II A and IV C in Ref. \[21\]).

Classically and with localized perturbations of the original \( A(x) \), \( \phi(x) \), and \( \xi(x) \) fields, it is possible, in first approximation, to neglect the existence of the additional fields \( \kappa(x) \) and \( \chi(x) \), as these fields have no interactions with the other fields (apart from gravitational interactions). The background matter fields \( (A2d) \) and \( (A2e) \) ensure having a constant background pressure and energy density, and do not play a role in the local physics, as long as localized gravitational interactions can be neglected (or in the fictitious world without gravity, \( G_N = 0 \)).

Admittedly, the example of this appendix is completely \textit{ad hoc} and physically unconvincing. But the example does demonstrate that, in principle, it may be possible to recover the flat-spacetime solution of the standard Einstein equations even if there is a nonstatic scalar background \( (2.6) \).

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