FAILURE OF LOCAL DUALITY IN INCLUSIVE NON-LEPTONIC HEAVY FLAVOUR DECAYS

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ABSTRACT

We argue that there is strong experimental evidence in the data of $b$- and $c$-decays that the pattern of power suppressed corrections predicted by the short distance expansion, the heavy quark effective theory and the assumption of local duality is not correct for the non-leptonic inclusive widths. The data indicate instead the presence of $1/m$ corrections that should be absent in the above theoretical framework. These corrections can be simply described by replacing the heavy quark mass by the mass of the decaying hadron in the $m^5$ factor in front of all the non-leptonic widths.

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1. Introduction

Since the discovery of charm all attempts of constructing a satisfactory theory of heavy flavour inclusive decay properties (lifetimes and semileptonic branching ratios) have met considerable difficulties [1]. With the advent of beauty it was hoped that the substantially increased mass of these new states would finally lead to an understanding of their inclusive decays in terms of some adequately improved form of the QCD parton model. But even for beauty decays, with the steady progress of experimental information and a lot of accumulated theoretical insight, a number of problems remains unsolved [1, 2]. The main examples are the experimental value of the average semileptonic (SL) beauty meson branching ratio which appears to be somewhat smaller than the theoretical predictions and the observed difference of the lifetimes of the \( \Lambda_b \) baryon and of the \( B \) mesons which is larger than expected. This situation is especially deceiving in that an appealing theoretical framework has been developed [3] for power corrections in terms of a short distance operator expansion and the formalism of the heavy quark effective theory [4]. The result of this approach is puzzling because it predicts that all corrections to the leading QCD improved parton terms appear at the order \( 1/m^2 \) and beyond, where \( m \) is the heavy-quark mass, while the experimental findings suggest much larger corrections. However the above method relies on the use of the operator expansion in the timelike region, namely on the physical cut, so that, in principle, some smearing should be applied, in the spirit of ref. [5], in order to avoid the infrared sensitivity implied by the vicinity of the cut. One usually invokes what is called the assumption of either global or local duality to justify the neglect of this problem [2]. Global duality, the weaker form of the assumption, applies to the case of the SL width, where the integration over the lepton spectrum is equivalent to an average over the invariant mass of the final state hadronic system, thus providing an intrinsic source of smearing. The success of the improved parton model in inclusive hadronic \( \tau \) decay is an empirical argument in support of global duality (for a recent confirmation see ref. [6]) even at relatively small energies. The stronger assumption of local duality is instead necessary for inclusive non-leptonic (NL) decays, where the dynamics is even more complicated because of the presence in the basic interaction of two hadronic currents instead of one as in the SL case. Recently arguments against the validity, in general, of either form of duality have been given in ref. [7].

In the present note we argue that, in spite of the complexity of the problem, the charm and beauty data appear to indicate a simple phenomenological recipe that considerably improves the situation. We find that the validity of the usual approach for the SL widths is perhaps consistent with the data. In particular the SL widths have been determined experimentally for three charmed hadrons, the \( D^+ \), the \( D^0 \) and the \( \Lambda_c \), and, in spite of the large differences in the corresponding lifetimes, they are close together, with corrections that presumably could be described by the usual theory. Furthermore, the value of \( |V_{cb}| \) extracted from the inclusive SL \( B \)-meson width is in good agreement, for a reasonable value of the \( b \)-quark pole mass, with the corresponding determination from \( B \to D^* \ell \nu \) [2]. In the usual approach, all widths are predicted to be proportional to the fifth power of the quark mass apart from corrections of order \( 1/m^2 \) or smaller. On the contrary we will argue that for the NL widths the presence of unexpected corrections of order \( 1/m \) is strongly indicated by the data. Not only that but we find that these \( 1/m \) corrections are well described by the simple ansatz that replaces the quark
mass with the decaying hadron mass in the $m^5$ factor in front of the NL width [3].

This replacement provides a much better description of the NL widths. We show that, for beauty, both the problems of the SL branching ratio and of the difference in the lifetimes of the $\Lambda_b$ baryon and the $B$ mesons are quantitatively solved. For charm a much better fit to the seven known lifetimes is obtained in terms of four parameters of reasonable size: one lifetime, one interference contribution for $D^+$, one for $\Xi^+$ and a smaller W-exchange term for $D^s$.

In the following we present our analysis in comparison with the standard one. We first discuss $b$-decays, then $c$-decays and finally we present our conclusions.

2. Beauty Decays

The experimental value of the average SL width of the $B$ mesons can be obtained from the measured values of the average SL branching ratio and lifetime [8]. The result is in good agreement with the theoretical prediction [1, 2]. This statement is based on the equality (within errors) of the extracted value of agreement with the theoretical prediction [1, 2]. This statement is based on the equality (within errors) of the extracted value of $|V_{cb}|$ compared with its independent determination from the exclusive decay $B \to D^* \nu$. The value of $|V_{cb}|$ is obtained from the inclusive SL decay rate of $B$ mesons using the relation

$$\Gamma_{SL}(B) = \Gamma_0 \eta_{QCD} \left[ \left( 1 + \frac{\lambda_1 + 3\lambda_2}{2m_b^2} \right) I_0(x, 0, 0) - \frac{6\lambda_2}{m_b^2} (1-x)^4 + O(1/m_b^3) \right]$$

(1)

where $\Gamma_0 = (G_F^2 m_b^5/192\pi^3)|V_{cb}|^2$, $I_0(x, 0, 0)$ is a phase space factor

$$I_0(x, 0, 0) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$

(2)

with $x = (m_c/m_b)^2$ (all lepton masses are neglected) and $\eta_{QCD}$ is the perturbative QCD correction (precisely this correction is only appropriate for the first term in curly brackets, but the second is quite small, and the completely factorised form is particularly useful for our purposes). The power suppressed terms $\lambda_1$ and $\lambda_2$ arise from the kinetic-energy and the chromo-magnetic dimension 5 operators [1]–[3]. We have $-\lambda_1/2m_b = (B|\bar{h}(i\bar{D})^2h|B)|/2m_b$, the average kinetic energy of the heavy quark in the hadron, while $\lambda_2$ is related to the mass splitting between vector and pseudoscalar mesons $\lambda_2 = (m_V^2 - m_P^2)/4$. For $B$-mesons, current estimates give $\lambda_1 \sim -0.4$ GeV$^2$ [1]; $\lambda_2 \sim 0.12$ GeV$^2$ is instead obtained from the experimental squared mass-difference $m_{B^*}^2 - m_B^2$. The value of the QCD correction $\eta_{QCD}$ is affected by considerable uncertainties [12]. Another main source of uncertainty arises from the value of $m_b$, the $b$-quark pole mass. Perturbatively different definitions of $m_b$ result in a change of $\eta_{QCD}$. As a consequence, here we prefer to make use of the above expression to obtain $m_b^{1/5}\eta_{QCD}$ from the experimental value of $\Gamma_{SL}(B) = B_{SL}(B)/\tau_B$ and from $|V_{cb}|$ as derived from exclusive decays ($|V_{cb}| = (38.6 \pm 2.6) 10^{-3}$ [2]). Using for the average SL branching ratio, $B_{SL}(B) = (10.77 \pm 0.43)%$ [2, 13], and for the average $B$ meson lifetime, $\tau_B = (1.55 \pm 0.02)$ ps [14], we find

$$m_b^{1/5}\eta_{QCD} = 4.9 \pm 0.2 \text{ GeV}$$

(3)

The main uncertainties arise from the $x$ value (taken between 0.08 and 0.12 as suggested by the relation $x = (1 - \Delta m/m_b)^2$ with $\Delta m = m_b - m_c \sim 3.4$ GeV as found in ref. [2]), the
experimental error on $|V_{cb}|$, and, to a lesser extent, from the experimental error on $\Gamma_{SL}(B)$. The errors on $\lambda_1$ and $\lambda_2$ are practically irrelevant. For the indicative value $\eta_{QCD} = 0.8$ (in ref. [12] the authors estimated $\eta_{QCD} = 0.77 \pm 0.05$) one obtains about 5.1 GeV for $m_b$, using eq. (3). This value is somewhat large, although in agreement with the lattice results of ref. [13], and compatible with the results obtained from the QCD sum rules [14] or from the analysis of ref. [12]. The value $m_b = 5.1$ GeV corresponds to $m_c \sim 1.7$ GeV (using the quoted value for $\Delta m$) which is also well consistent with the pole mass result for $m_c$ derived from ref. [17], (see below).

The prediction for the SL width of $\Lambda_b$ mainly differs from eq. (1) in that $\lambda_2 = 0$. The kinetic energy term $\lambda_1$ in principle is also different, but in practice its estimate for $B$ and $\Lambda_b$ are identical within errors [1, 2]. Some presumably small additional difference arises from the neglected $1/m^3$ terms. The vanishing of $\lambda_2$ produces a 3.5% increase of $\Gamma_{SL}(\Lambda_b)$ with respect to $\Gamma_{SL}(B)$.

We now consider the NL widths. It is well known that the observed ratio of the $\Lambda_b$ to $B$ lifetimes appears too large to be explained by corrections of order $1/m^2$ or $1/m^3$. This is confirmed by a number of recent analyses [18, 19]. Here we show that the assumption that the NL widths scale as the fifth power of the decaying hadron mass (apart from corrections of order $1/m^2$ and beyond) gives a very good agreement with experiment. In fact this assumption leads for the ratio of lifetimes to the expression

$$\frac{\tau_B}{\tau_{\Lambda_b}} = \left(\frac{m_{\Lambda_b}}{m_B}\right)^5 [1 - 2.24 B_{SL}(B)] + 2.24 B_{SL}(B) + O(1/m^2)$$

(4)

Here the factor 2.24 arises from taking the electron, the muon and the tau SL modes in the ratio 1:1:0.24 [4], and the difference in the SL rates of the $\Lambda_b$ and $B$ has been neglected. From $m_{\Lambda_b} = 5623 \pm 6$ MeV [14] and $m_B = 5279 \pm 2$ MeV [8], by using the already quoted value for $B_{SL}(B)$, we obtain

$$\frac{\tau_B}{\tau_{\Lambda_b}} = 1.29 \pm 0.05$$

(5)

where the error is dominated by the uncertainty on the power suppressed corrections of order $1/m^2$ or $1/m^3$. In comparison, from the most recent data we have [14] $\tau_B = 1.55 \pm 0.02$ ps for the average $B$ lifetime and $\tau_{\Lambda_b} = 1.19 \pm 0.06$ ps, a value that includes the LEP data and the recent preliminary result of CDF. From these values we find

$$\left(\frac{\tau_B}{\tau_{\Lambda_b}}\right)_{EXP} = 1.30 \pm 0.07$$

(6)

in perfect agreement with the above prediction. Clearly it would be very interesting to measure the SL branching ratio of the $\Lambda_b$ in order to check whether the SL width is within a few percent equal to that of the $B$ mesons. Notice that, by neglecting terms of $O(1/m^3)$, the standard prediction from the heavy quark effective theory gives $\tau_B/\tau_{\Lambda_b} = 1.02$ [19]. Moreover it is very unlikely that the inclusion of the corrections of $O(1/m^3)$ is sufficient to remove the discrepancy [19].

If we repeat the same exercise by applying eq. (4) to the $B_s$ and $B$ lifetimes, we find

$$\frac{\tau_B}{\tau_{B_s}} = 1.07 \pm 0.03$$

(7)
where the value $m_{B_s} = 5369.6 \pm 2.3$ MeV from LEP and CDF was used \cite{14}. The error from the power suppressed terms can now be taken smaller than in eq. (5) because of the much closer similarity of the two mesons involved. The present value of the $B_s$ lifetime, also including the new preliminary data from CDF is given by $\tau_{B_s} = 1.49 \pm 0.07$ ps \cite{14}. For the ratio we then obtain the experimental value

$$\left( \frac{\tau_B}{\tau_{B_s}} \right)_{EXP} = 1.04 \pm 0.05$$  \hspace{1cm} (8)

At present the data are not sufficiently precise to check the assumed dependence on the hadronic mass, but this test could become significant in a near future.

We now discuss the problem of $B_{SL}(B)$. As well known, the theoretical prediction for $B_{SL}(B)$ is somewhat larger than the experimental value \cite{20}. A possible explanation of this fact could be a failure of the improved parton model for the $b \to c\bar{c}s$ mode due to the restricted phase space for the final state \cite{21}. If the rate for this mode would be sufficiently larger than the predicted value the corresponding increase of the NL width could reconcile the value of $B_{SL}(B)$ with the observed result. The problems with this explanation are, on the one hand, that the observed average number of charm quarks in the final state of $b$-decay is lower than required. The present experimental result for the charm counting is given by $n_c = 1.16 \pm 0.05$ \cite{22}, while the required amount would be at least $n_c = 1.3$, see the Erratum of ref. \cite{23}. On the other hand, the same mechanism clearly cannot be invoked to explain the ratio of the $\Lambda_b$ and the $B$ lifetimes. At lowest order in $1/m$, a different, larger $b \to c\bar{c}s$ rate would indeed modify identically the $\Lambda_b$ and $B$ lifetimes. On the contrary a modest increase of the effective $m^5$ factor in front of the NL channels with respect to that of the SL width decreases the value of $B_{SL}(B)$ to the observed value. A recent accurate analysis in the conventional approach of $B_{SL}(B)$ leads \cite{23} to a predicted value $B_{SL}^{th}(B) = (12.0 \pm 1.4) \%$, when the $b$-quark pole mass is used in the $m^5$ factor. If the pole mass is replaced by the $B$-mass in the $m^5$ factor, the central value for $B_{SL}^{th}(B)$ is changed into the new figure $\tilde{B}_{SL}^{th}(B)$ given by

$$\left( \tilde{B}_{SL}^{th}(B) \right)^{-1} = 2.24 + r \left[ \left( B_{SL}^{th}(B) \right)^{-1} - 2.24 \right]$$  \hspace{1cm} (9)

Inserting $r = (5.279/5.1)^5 = 1.188$, from $B_{SL}^{th}(B) = 12\%$ we find $\tilde{B}_{SL}^{th}(B) = 0.105$. Note that the value 5.1 GeV for the pole mass, as inferred from the SL width, being on the upper side of the error band for this quantity, leaves space for a larger adjustment if the preferred value of $B_{SL}^{th}(B)$ is larger. In fact, in the analysis of ref. \cite{23}, the value of $B_{SL}^{th}(B) = 12.0 \pm 1.4\%$ given before corresponds to $n_c = 1.24 \pm 0.05$, which is still too large with respect to the experimental value. For example, for $n_c \sim 1.16$ and $\tilde{B}_{SL}^{th}$ equal to the experimental value $\tilde{B}_{SL}(B) = 10.8\%$ one obtains $B_{SL}^{th}(B) \sim 13\%$ from ref. \cite{23} which leads to $m_b \sim 5$ GeV.

In conclusion the problems for the inclusive $b$-decay phenomenology seem to be solved with the replacement of the quark with the hadron mass in the $m^5$ factor in front of the NL width. As we shall see this is further confirmed by the analysis of charm decays.
neglect at this stage any other mass correction and we write $\Gamma_{NL} = \Gamma_{SL} = \Gamma_{SL}/\tau$ (ps$^{-1}$).

| Hadron | Mass (MeV/$c^2$) | $\tau$ (ps) | $B_{SL}(\%)$ | $\Gamma_{SL}$ |
|--------|------------------|-------------|--------------|--------------|
| $D^\pm$ | 1869.4 ± 0.4 | 1.057 ± 0.015 | 17.2 ± 1.9 | 16.3 ± 1.8 |
| $D^0$   | 1864.6 ± 0.5 | 0.415 ± 0.004 | 8.1 ± 1.1 | 19.5 ± 2.6 |
| $D_s^0$ | 1968.5 ± 0.7 | 0.467 ± 0.017 | 4.5 ± 1.7 | 22.5 ± 8.5 |
| $\Lambda_c^0$ | 2285.1 ± 0.6 | 0.200 ± 0.011 |             |              |
| $\Xi_c^0$ | 2470.3 ± 1.8 | 0.098 ± 0.019 |             |              |
| $\Xi_c^{\pm}$ | 2465.1 ± 1.6 | 0.350 ± 0.055 |             |              |
| $\Omega_c^0$ | 2704 ± 4 | 0.055 ± 0.023 |             |              |

Table 1: Properties of charmed mesons and baryons; the $\Omega^0$ values are our average of the data quoted in ref. [24].

3. Charm Decays

Up to date, seven charmed particle lifetimes have been measured and in three cases also the SL branching ratio is known, so that the corresponding SL width can be extracted. All the available data are collected in table 1. We start with the simplest case of the SL width. Up to terms of order $1/m^3$, which could be important but are more difficult to estimate [23], we have in the conventional theory (omitting, for simplicity, Cabibbo suppressed channels) an expression which is completely analogous to eq. (1), with the obvious replacements of $m_b$, $V_{cb}$, $x = (m_c/m_b)^2$ with $m_c$, $V_{cs}$, $x = (m_s/m_c)^2$. The value of $\lambda_2$ vanishes for $\Lambda_c$ (and $\Xi_c$) [1, 2]. In the calculation of the inclusive widths of the $D^+$ and $D^0$ we have used $\lambda_2 = 0.14$ GeV$^2$ obtained from the experimental value of the difference $\lambda_2 = (m_{D_s^0}^2 - m_{D_s^0}^2)/4$. For $\lambda_1$ a value around $-0.4$ GeV$^2$ has been used for both $D^0, +$ and $\Lambda_c$. The quantity $x$ is very small and we have taken $I_0 \sim 0.91$.

For D mesons, there is a strong cancellation between the term containing $\lambda_1 + 3\lambda_2$ and the one, proportional to $\lambda_2$ in eq. (1) (note that, in this case, it is appropriate to restore the factor $\eta_{QCD}$ at its place in front of $I_0$). This makes the prediction very unstable, with a central value around $\Gamma_{SL}(D) = 0.29 \Gamma_0$ for $\eta_{QCD} = 0.7$. Also, the smallness of the coefficient with respect to unity makes the neglect of the $1/m^3$ terms, which we know could be large especially for mesons, totally unjustified. For $\Lambda_c$ the prediction is much more stable, and within a $\pm 10\%$ accuracy, one finds $\Gamma_{SL}(\Lambda_c) = 0.59 \Gamma_0$. The value of $m_c\eta_{QCD}$ required to reproduce the experimental result for $\Gamma_{SL}(\Lambda_c)$ is around $m_c\eta_{QCD}^{1/5} = 1.5$ GeV, which is slightly large but not unreasonable. For example, by taking the $\overline{MS}$ charm-quark mass computed in lattice simulations, $m_c^{\overline{MS}}(\mu = 2$ GeV) = 1.48 ± 0.28 GeV [17], we get for the pole mass $m_c \sim 1.6 - 1.7$ GeV in agreement with $m_c\eta_{QCD}^{1/5} = 1.5$ GeV if we take $\eta_{QCD} = 0.7$. In conclusion, the large uncertainties present for charm and the limited number of the existing data on $\Gamma_{SL}$ prevent a stringent test of the theory, which is however consistent with the existing information (given in table 1).

We now consider the lifetimes of charmed particles. At lowest order in the $1/m$ expansion, a much better agreement with the experimental results for the lifetimes is obtained by replacing the heavy-quark mass by the hadron masses in the $m^5$ term of the expression for $\Gamma_{NL}$. We neglect at this stage any other mass correction and we write $\Gamma_{NL}(m) = \Gamma_{tot}(m) - 2\Gamma_{SL}$, where
for $\Gamma_{SL}$ we insert a universal value chosen as the average of the experimental values for $D^+$, $D^0$ and $\Lambda_c$, or $\Gamma_{SL} = (0.174 \pm 0.015) \text{ps}^{-1}$ [8]. The dependence on the hadron mass of $\Gamma_{NL}(m)$ will be taken according to $\Gamma_{NL}(m) = (m/m_0)^n \Gamma_{NL}(m_0)$ with $n = 5$, where $m_0$ is around the average mass of the relevant hadron. We then have

$$\Gamma_{tot}(m) = \tau^{-1}(m) = \tau^{-1}(m_0)(\frac{m}{m_0})^n + 2\Gamma_{SL}(1 - (\frac{m}{m_0})^n)$$ (10)

We first fix $n = 5, m_0 = 2.3 \text{ GeV}$ and $\Gamma_{SL} = 0.174 \text{ ps}^{-1}$ and fit all seven known lifetimes in terms of $\tau(m_0)$. We obtain $\tau(m_0) = 0.181 \text{ ps}$. The corresponding fit is shown in fig. 1 (dashed curve). We see that four out of seven lifetimes are in very good agreement with the fitted curve. The lifetimes of $D^+$, of $\Xi^+$ and, to a lesser extent, of $D_s$ are clearly out. We attribute the discrepancies for $D^+$ and $\Xi^+$ to the interference effect [1]. Note that $D^+$ is the only meson that can have interference at the Cabibbo allowed level and $\Xi^+$ is the only baryon that can have double interference, in the sense that $\Xi^+ = cus$ and both $u$ and $s$ can interfere with the
corresponding quarks from $c \to su\bar{d}$. For $D_s$ the observed smaller difference is attributed to the possibility of W-exchange [1]. All of these effects are of order $f_D^2/m^2$ or $1/m^3$. The solid line in fig. 1 has been obtained from a modified fit where only the $D^0$, $\Lambda_c$, $\Xi^0$ and $\Omega_c$ lifetimes have been considered. In this case, we obtain $\tau(m_0) = 0.161$ ps, with the respectable value of the $\chi^2/d.o.f.$ given by $\sim 3.5$. For comparison, the fit of the quark mass to constant lifetimes results in a $\chi^2/d.o.f. \sim 251$. Finally, for the same four lifetimes, we fit the power $n$ in eq.(10), keeping fixed the value of $m_0$ and $\tau(m_0)$ at the observed values for the $D^0$ meson. In this way we check whether the best power for $n$ is close to 5. We find $n = 4.5 \pm 0.5$, where the error arises from the experimental errors on the lifetimes. Moreover, if we write for $D^+, \Xi^+$ and $D_s$ the expression $\tau^{-1} = \tau^{-1}(m_H)[1 - (\mu/m_H)^3]$, where $\tau$ is the experimental number given in table 1 and $\tau(m_H)$ is taken from the previous fit to the four remaining lifetimes (with $n=5$), we find $\mu = 1.6, 2.2$ and $1.3$ GeV for $D^+, \Xi^+$ and $D_s$, respectively. We see that the resulting values of this correction are large, as it is obvious from fig. 1, but not unreasonable.

4. Conclusion

We have presented a number of experimental facts that, in our opinion, make rather clear that $\Gamma_{NL}$ for charm and beauty decay approximately scale with the fifth power of hadron masses apart from corrections of order $1/m^2$ or smaller. These facts are the ratio of the $\Lambda_b$ and $B$ lifetimes, the value of $B_{SL}(B)$ and the charm lifetimes. This conclusion is at variance with the predictions of the short distance operator expansion approach augmented by the heavy quark effective theory. In fact, according to this theory, the relevant mass in the rate should be a universal quark mass and no corrections of order $1/m$ should be present once this mass is used. On the contrary the hadron mass differs from the quark mass by non-universal terms of order $1/m$: $m_H = m_q(1 + \Lambda_H/m_q + O(1/m_q^2))$. We recall once more that in principle the validity of the operator expansion in the timelike region, in the vicinity of the physical cut, is not at all guaranteed [3, 7]. We therefore attribute the failure of the short distance approach to a violation of the local duality property that has to be assumed for NL widths. Apparently the conventional theory for SL widths is not inconsistent with the data. The experimental evidence for NL widths calls for a reexamination of the underlying theoretical framework.

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