Physics of the Top Quark at Future Lepton Colliders

Thomas Teubner
Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, U.K.
E-mail: thomas.teubner@liverpool.ac.uk

Abstract. The physics scenario of the top quark at future lepton colliders is discussed, concentrating on the theory and simulation of $t\bar{t}$ and $t\bar{t}H$ production close to threshold. The study of these processes would allow to determine the mass and couplings of the top quark with very high precision, including the direct measurement of the top Yukawa coupling.

1. Introduction
At this conference, impressive measurements of the top quark’s properties at hadron colliders have been discussed, and a lot more will come from the LHC, so why top quark physics at a lepton collider? With a Yukawa coupling $y_t$ of order one the top plays a special role in the electroweak symmetry breaking. Its mass and couplings are key parameters for electroweak precision tests of the Standard Model (SM) and its extensions, and the accuracy of these parameters is the limiting factor in many studies [1]. Similarly, when extrapolating masses and couplings within scenarios of unified theories via renormalisation group equations (‘running’) over many orders of magnitude in energy, meaningful constraints require a very high accuracy of the top couplings. For such precision measurements, a linear $e^+e^-$ (or a muon) collider would offer unique possibilities for several reasons: the centre-of mass energy is clearly defined and can be monitored, together with the sharp luminosity spectrum, with highest accuracy; the leptonic initial state has only electro-magnetic and no strong initial state radiation so no PDFs are required; the final states produced in lepton annihilation are of low multiplicity compared to hadron collisions, leading to a ‘clean’ environment with controllable backgrounds and measurable cross section normalisations. The top quark mass measurements at the Tevatron and the LHC have already achieved a very high relative accuracy, making this the best known quark in this respect. However, connecting a kinematical reconstruction of a coloured object to a mass parameter as used in theoretical calculations introduces systematic uncertainties of the order of the QCD scale $\Lambda_{QCD}$ [2]. A threshold scan of top pair production at an $e^+e^-$ collider offers the possibility to avoid this problem and is the only known way to determine the top quark mass with an accuracy of order 100 MeV. Similarly, studying $e^+e^- \rightarrow t\bar{t}H$ close to the kinematic threshold will, assuming that the LHC has found the Higgs at $\sim 125$ GeV, allow to directly measure the top Yukawa coupling. In the following these scenarios will be discussed in more detail.

2. Threshold production; theoretical description
Top quark production at threshold is strongly enhanced due to the attractive Coulomb-like gluonic interaction. The fast decays of the top and antitop prevent the formation of stable bound states, hence there will be no top quark spectroscopy. However, the remnant of the smeared-out
1S peak provides a very distinctive feature in the threshold cross section $\sigma_{tt} \equiv \sigma(e^+e^- \rightarrow t\bar{t})$, which can be related to the top quark mass without infrared ambiguities, therefore avoiding the aforementioned uncertainties of order $\Lambda_{\text{QCD}}$. Note that, contrary to a kinematic reconstruction of top decay products, the threshold scan for the total cross section is basically a counting experiment. The accuracy of the extraction of the top mass and couplings will therefore rely on the precise knowledge of the cross section, i.e. of the luminosity spectrum of the $e^+, e^-$ beams (not just the mean energy and luminosity, but see below) and, of course, on having enough statistics. In addition, the theoretical description must match the experimental accuracy.

2.1. Effective theories; observables

Due to its large mass and decay width, $t\bar{t}$ production can be described using perturbative QCD (pQCD). To achieve high accuracy of the theoretical prediction, higher order corrections for the threshold cross section are mandatory. These corrections are highly non-trivial as they (a) require the summation of certain classes of diagrams and, (b) involve several physical scales, i.e. the top mass, $m_t$, the top momentum, $p_t \sim m_t v$ (with $v$ the top’s non-relativistic velocity) and the ‘binding’ energy of the quasi-bound system, $E = \sqrt{s} - 2m_t \sim m_t v^2$. In addition the top width, $\Gamma_t$, has to be taken into account. As, close to threshold, the non-relativistic velocity $v$ is small, these scales have a strong hierarchy, $m_t >> p_t >> E$. This makes it possible to formulate the problem in form of effective field theories. Over the last two decades several forms of such effective field theories have been developed, called Non-Relativistic QCD (NRQCD). Within this framework, $\sigma_{tt}$ has been calculated up to next-to-next-to leading order (NNLO) in pQCD by several groups, see [3] for an early review and references therein. Close to threshold the perturbative expansion has to be organised as a series in both $\alpha_s$ and the small non-relativistic velocity $v$. Therefore the normalised cross section ratio $R$ in the threshold region takes the form $R = \sigma_{tt}/\sigma_{\mu^+\mu^-} \sim v \sum_n (2\alpha_v/n)^n$ [LO $\{1\}$, NLO $\{v, \alpha_s\}$, NNLO $\{v^2, \alpha_s v, \alpha_s^2\}$]. Already at leading order (LO) all terms of order $(\alpha_s/v)^n$ have to be summed; they correspond to ladder-type diagrams and build up the Coulombic QCD potential which is attractive for the $t\bar{t}$ colour singlet. Other potentials appear at higher orders and are taken into account using Green function techniques. The effective theory with its systematic power counting separates the dynamical from the non-dynamical degrees of freedom and allows to sum these order $(\alpha_s/v)^n$ contributions and, for certain versions of the effective field theory, also logarithmically enhanced contributions. These new theories have also improved the description of other quarkonium systems like $b\bar{b}$.

In Fig. 1 the $t\bar{t}$ production cross section is shown as a function of the centre-of-mass

![Figure 1](https://example.com/image1.png)

**Figure 1.** Dependence of the total cross section $e^+e^- \rightarrow Z^*, \gamma^* \rightarrow t\bar{t}$ as a function of $\sqrt{s}$ on (a) the strong coupling $\alpha_s$ and (b) the top width $\Gamma_t$. The solid (black) lines correspond to $\alpha_s = 0.118$ and $\Gamma_t = 1.43$ GeV, whereas the dashed (red) ones to variations of $\alpha_s$ and $\Gamma_t$ by 0.002 and 10%, respectively. The predictions are from [4] and based on vNRQCD.
energy \( \sqrt{s} \) in the threshold region. The results [4] are obtained within so-called velocity NNRQCD (vNRQCD) and are at next-to-next-leading logarithmic order (NNLL), see also [5, 6]. They include contributions due to the summation of leading logarithms of the ratios of the different scales \( m_t, p_T = m_t v \) and \( E \sim m_t v^2 \) through renormalisation group running [4, 5, 6]. This leads to a stabilisation of the perturbative series which then takes the form \( R = \sigma_{t\bar{t}}/\sigma_{\mu^+\mu^-} \sim v \sum_{n,k} (\frac{m_t}{v})^n (\alpha_s \log v)^k \left[ \text{LL } \{1\}, \text{NLL } \{v, \alpha_s\}, \text{NNLL } \{v^2, \alpha_s v, \alpha_s^2\} \right] \). Despite the large decay width, the remnant of the 1S peak is clearly visible. The dashed (red) lines indicate the dependence of \( \sigma_{t\bar{t}} \) on the parameters \( \alpha_s \) (left panel) and on the width \( \Gamma_t \) (right panel). In addition, \( \sigma_{t\bar{t}} \) also depends on the top Yukawa coupling \( y_t \) through virtual Higgs corrections and on its electroweak couplings. Although there is a correlation in the dependence on these parameters, it will be possible to measure not only \( m_t \), but also some of the other parameters with high accuracy through a multi-parameter fit of data from a threshold scan. To help to disentangle the correlated dependencies, observables other than the total cross section can be used, such as the top momentum distribution and the forward-backward asymmetry \( A_{FB} \) which stems from the interference of the leading \( S \) wave \( t\bar{t} \) production with the \( P \) wave contribution from production through a virtual \( Z \). While the momentum distribution is mainly sensitive to \( m_t \) and less to \( \alpha_s \), the asymmetry \( A_{FB} \) is strongly dependent on the width \( \Gamma_t \) which determines the overlap of the \( S \) and \( P \) wave contributions. Other observables, like the spectra of \( W \) decay leptons, will be instrumental to measure the top polarisation and to gain sensitivity e.g. on CP violation from possible BSM contributions to the top electric dipole moment, on \( V + A \) contributions or other anomalous couplings.

### 2.2. Status of higher order predictions

The first predictions for \( \sigma_{t\bar{t}} \) were already made more than ten years ago, see e.g. [3]. They showed large corrections not only from LO to NLO, but again from NLO to NNLO. While this can be understood from the way in which certain corrections appear in the perturbative series, it means that the stability of the predictions, estimated from either the size of the corrections or from variation of the renormalisation scales, is limited. As this may threaten the accuracy of future threshold analyses, several groups have worked on further improvements. One way is the already mentioned summation of large logarithms through renormalisation group running, done within effective theories called velocity (v) or potential (p) NRQCD. This has led to a stabilisation of the corrections and hence the predictions [4, 5, 6]. Another way is to go to even higher ‘fixed-order’ perturbation theory, i.e. next-to-next-to-next-to leading order (NNNLO). A complete result for the cross section is not yet available at this order, but results for the 1S peak position and normalisation indicate an improved stability at NNNLO, see [7, 8] for short reviews and [9, 10] for recent results. As a part of this programme, the recent results for the three-loop coefficient \( \alpha_3 \) of the QCD potential should be mentioned [11, 12]; more than 20000 Feynman diagrams contribute to this quantity.

For a reliable prediction of the top pair production cross section also electroweak corrections have to be taken into account. For the pure QCD corrections the top width \( \Gamma_t \), which is due to the weak interaction, has usually been included as an imaginary part of the non-relativistic energy. However, this approach is not consistent at higher order, where, in the power counting of the effective theory, the electroweak coupling \( \alpha_{EW} \sim \Gamma_t/m_t \sim \alpha_s^2 \sim v^2 \). In addition, electroweak interactions lead to single- and non-resonant \( W^+W^-bb \) final states which can, depending on cuts in the experimental analyses, contribute as irreducible backgrounds to the double-resonant \( t\bar{t} \) signal. Recently a lot of effort has been invested in understanding these electroweak effects in the framework of effective theories which include unstable particles [13, 14, 15]. Work done so far has shown that the corrections are of the order of several percent for the total cross section and will have to be included in future complete threshold analyses.
2.3. Mass definitions
To reach an accuracy of order 100 MeV in the determination of $m_t$, it is crucial to go beyond the usual definition of the top mass as a pole mass. This becomes obvious when using the pole mass in the prediction of $\sigma_{tt}$ in the threshold region: the position of the $1S$ peak, which is a physical quantity, moves significantly (by several 100 MeV) from order to order. The reason for this is understood. It is well known that, although infrared (IR) finite and gauge invariant, the pole mass itself is not an observable and suffers from ambiguities of order $\Lambda_{QCD}$ which can be related to IR renormalon effects. In contrast, for the $tt$ $1S$ peak, individual IR contributions are cancelling between the mass and potential in $E(1S) = 2m_{tt}^{pole} + V$. Hence the $1S$ peak position is IR safe and an observable from which a suitable ‘short-distance mass’ can be extracted. Several such mass schemes have been developed, such as the ‘potential subtracted mass’ [16] or the ‘$1S$ mass’ $m_{1S}^{t\bar{t}}$ [17] which is perturbatively defined in the $\Gamma_t \to 0$ limit, see [3] for a discussion of the application of these schemes. These masses can be translated, order-by-order in perturbation theory and without an IR ambiguity $\sim \Lambda_{QCD}$ to other masses, like the $\overline{MS}$ mass which is suitable for the high-energy regime (but not in the threshold region).

3. Simulation of the threshold scan
Several groups have studied the analysis of the top quark threshold from an experimental point of view. While backgrounds and event reconstruction should not pose a major problem for the planned machine and detector designs, the required understanding of the luminosity spectrum is challenging. The spectrum relevant for the experimentally measured threshold cross section is strongly influenced by the initial beam profile, beam-beam interactions in the interaction region and also by QED initial state radiation. It will be crucial to measure and monitor this spectrum as otherwise large effects would bias the top mass and other parameters determined via a fit of the threshold scan data. Figure 3 shows a recent simulation [18] done for CLIC and ILC designs. The left panel shows typical beam spectra, the right one the resulting $tt$ threshold cross section for an ILC of 350 GeV. The simulated data together with the theoretical curves used for the fit demonstrate that, while the relatively wide beam spectrum leads to a considerable smearing of the original theory curve as shown in Fig. 1, the measured threshold cross section will still be a highly sensitive probe of the top mass. This simulation of the threshold scan predicts that an experimental uncertainty of 21 MeV for $m_{1S}^{t\bar{t}}$ will be reachable at CLIC with a one-parameter fit using a fixed input for $\alpha_s$. With a two-dimensional fit $\Delta m_{1S}^{t\bar{t}} = 33$ MeV and $\Delta\alpha(M_Z^2) = 0.0009$. For an ILC the accuracy would be even better by 10 to 20%. Clearly, it will be crucial to know the beam spectrum with the best possible accuracy, see also [19] for related

![Figure 2. Luminosity spectrum and resulting $tt$ threshold cross section as simulated by [18].](image-url)
work on the simulation of beam effects in the $t\bar{t}$ threshold scan. The quoted fit results are in line with earlier studies by Martinez et al [20], who in addition predicted a relative accuracy of about 35\% for the simultaneous determination of the top Yukawa coupling for a light Higgs in the mass range found at the LHC. This is possible as a relatively light Higgs leads to significant corrections at threshold from the resulting Yukawa potential.

4. Studies for $t\bar{t}$ production in the continuum
Compared to these predictions for a threshold scan, similar recent simulations for the top mass reconstruction at $\sqrt{s} = 500$ GeV at CLIC [21] or ILC indicate that the achievable statistical uncertainty, though impressive taken face value, will be about three times worse, in addition to the theoretical problem of relating such a kinematic mass to a short distance mass. However, $t\bar{t}$ production in the continuum offers unique possibilities to measure certain top couplings. One example is the $Ztt$ coupling, which could be a sensitive probe of BSM physics but is not accessible at the LHC. Through measurements of the asymmetries $A_{FB}$ and $A_{LR}$ the $Wtb$ and $Ztt$ couplings can be determined at the 1\% level of accuracy [22, 23]. When parametrising SM and possible BSM contributions in a model independent way through general dimension four and six operators, many can only be tested at lepton colliders, while for others the ILC would improve markedly on what can be done at the LHC, see e.g. [24] for recent analyses.

5. $t\bar{t}H$
To establish if the particle found at the LHC is the SM Higgs or something different one needs to measure its couplings with the best possible accuracy. At the LHC the top Yukawa coupling $y_t$ is accessible indirectly through the cross section which is mainly induced by the $ggH$ coupling via a top loop. Total cross section measurements at hadron colliders are plagued by various uncertainties, and the accuracy of such a determination will be about 15\%. At a lepton collider, the measurement of $y_t$ in $t\bar{t}H$ production will be possible, but also challenging due to the relatively complicated final state, low rates and high backgrounds [25]. Early studies had indicated that the prospects for such a measurement are better at higher centre-of-mass energies. However, the situation has changed with the observation that, close to the $t\bar{t}H$ threshold the final state predominantly consists of a non-relativistic $t\bar{t}$ pair, back-to-back with a Higgs boson. Therefore threshold corrections similar to the case of the $t\bar{t}$ threshold enhance the cross section and have been calculated within NRQCD to NLL order [26]. Together with possible enhancements due to $e^-$ and possibly $e^+$ beam polarisation, this leads to a significantly enlarged cross section. These effects have previously been estimated to give enhancement factors of 2.4 and 2.1, respectively, and resulted in an estimated relative accuracy of 10\% for the measurement of $y_t$ [27]. Figure 5 shows the results of a recent simulation [28] of the $t\bar{t}$ mass spectrum in the

![Figure 3. Simulation of the $t\bar{t}$ mass distribution in $e^+e^-\rightarrow t\bar{t}H$ production close to threshold. (From [28].)
process $e^+e^- \rightarrow t\bar{t}H$ with and without the bound-state effects due to the QCD dynamics of the $t\bar{t}$ system. Even well above the nominal threshold they lead to a significant enhancement of the cross section. This recent analysis is based on a fastsim simulation and has confirmed the previously estimated improvement, giving a statistical uncertainty of 10% for the measurement of $y_t$, if a beam polarisation of $(P_{e^+}, P_{e^-} = (+0.3, -0.8)$ is taken into account and assuming an integrated luminosity of 1000 fb$^{-1}$ and a centre-of-mass energy $\sqrt{s} = 500$ GeV.

6. Outlook

At this conference we have seen impressive results on top quark physics from the Tevatron and the LHC, and a lot more is to come. At the same time, studies of top quark physics at a future lepton collider have moved forward tremendously compared to the first explorations of the physics scenario and have triggered important theoretical developments like effective field theories and short-distance mass definitions. The theoretical accuracy stands typically at next-to-next-to leading order or even higher, but only for inclusive quantities. To be fully prepared, more and better Monte Carlo tools will be needed, which have to include the recent theoretical developments. However, the physics case for a future linear collider running at the $t\bar{t}$ and the $t\bar{t}H$ thresholds has been clearly made: It would offer the possibility to determine the top mass and its couplings in a clean environment and with the highest accuracy, impossible to reach in hadron collisions. With the recent discovery of a Higgs-like particle at the LHC it is a good time to push for the realisation of an International Linear Collider.

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