Sign Change in the Odd Parity Superconducting State of Iron-Based Superconductors

Ningning Hao¹,² and Jiangping Hu¹,²

¹Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100080, China
²Department of Physics, Purdue University, West Lafayette, Indiana 47907, USA

We discuss the sign change of superconducting order parameters in both real and reciprocal spaces in the odd parity state of iron-based superconductors proposed recently in [1]. In the real space, the odd parity state can be viewed as a s-wave or d-wave state depending on the choice of locations. In a 2-Fe Brillouin zone (BZ), sign change exists between two hole pockets and between two electron pockets. In a 1-Fe BZ which includes two 2-Fe BZs, the sign change is between two 2-Fe BZs, which leads to a d-wave type sign distribution on the electron pockets. However, there is no symmetry protected gapless node on the electron pockets. This sign change character consistently explains experimental results related to sign change properties measured on both iron-pnictides and iron-chalcogenides.

The d-wave pairing symmetry in high \( T_c \) superconductors [2], cuprates, is a unique symmetry character to distinguish the cuprates from conventional s-wave superconductors. The d-wave superconducting state exhibits symmetry protected sign change and nodes on Fermi surfaces and has strong implication to high \( T_c \) mechanism.

Is a sign-changed superconducting order a necessity for high \( T_c \)? The discovery of iron-based superconductors [3–5] in 2008 provided an opportunity to find out the answer. In the past five years, strong experimental evidence for the existence of sign change in the superconducting states of these new high \( T_c \) superconductors have been accumulated. Similar to cuprates, magnetic resonance modes, which imply a sign change of superconducting order parameters between two Fermi surfaces linked by resonance wavevectors, were observed in neutron scattering experiments in both iron-pnictides [6–13] and iron-chalcogenides [14–16]. No noticeable coherent peak, i.e. the Hebel-Slichter peak, was observed in the spin relaxation rate, \( T_1 \) [17–22], measured by NMR in all iron-based superconductors, which strongly implies a symmetry protected sign change. The magnetic field dependence of the impurity scattering pattern measured by STM in \( FeSe \), \( T_{c_{\text{1-Fe}}} \) also suggests a sign change between electron and hole pockets [23]. Finally, half-integer flux was observed in phase-sensitive junction loops constructed by composite niobium-\( NdFeAsO \) [24]. However, differing from the d-wave symmetry pairing in cuprates, the sign change in iron-based superconductors appears not to produce symmetry protected nodes or node lines [25]. In many materials near optimal doping, the superconducting gap structure is fully gapped, namely, a s-wave type in conventional wisdom.

Theoretical studies in the past focused on the \( s^± \)-pairing symmetry which is characterized by the sign change of superconducting order parameters between hole pockets at \( \Gamma \) and electron pockets at \( M \) in reciprocal space [25–35]. While the \( s^± \)-pairing symmetry was successful in explaining some sign change properties of iron-pnictides, it faces several fundamental challenges. First, the \( s^± \) state belongs to \( A_{1g} \) irreducible representation. Unlike the d-wave pairing symmetry in cuprates, the sign change in the \( s^± \) state is not symmetry protected, which essentially fails to explain the absence of the Hebel-Slichter peak in a clean sample [25, 36]. Second, the \( s^± \) state fails to explain the resonance mode observed in iron-chalcogenides [15]. Finally, the sign change in this state predicted for an in-plane corner junction was not observed yet [37–39].

In this Letter, we present a clear picture of the sign change in the odd parity state proposed recently by one of us for iron-based superconductors [1]. We show that the odd parity state overcomes all the above challenges and its sign change on different Fermi surfaces naturally explains sign change related experimental results in both iron-pnictides and iron-chalcogenides. The main results are shown in Fig. 1 in which the sign distribution of the odd parity state is shown in Fig. 1(a,b) and as a comparison, the sign distribution in the d-wave of the cuprates (see Fig. 1(c)) and the \( s^± \) state (see Fig. 1(d)) are also plotted. In the Brillouin zone (BZ) of a 2-Fe unit cell (see Fig. 1(b)), there are sign changes between hole pockets at \( \Gamma \) and electron pockets at \( M \), as well as sign changes between two hole pockets or two electron pockets. Viewed in

FIG. 1: (Color Online) Sign change of superconducting order parameters in reciprocal space (sign difference is indicated by blue and red colors): (a) the odd parity state viewed in the BZ of a 1-Fe unit cell; (b) the odd parity state viewed in the BZ of a 2-Fe unit cell; (c) the d-wave state in cuprates; (d) the \( s^± \)-pairing symmetry (even parity).
the BZ of a 1-Fe unit cell which includes two BZs of a 2-Fe unit cell, the sign changes are essentially between the two BZs (see Fig. 1(a)). In this case, the sign distribution on the electron pockets at M is a d-wave type but without symmetry protected gapless nodes because of the mixture of the $\eta$ pairing in the odd parity state. Microscopic mechanism related to the sign change is discussed.

We briefly review the odd parity state proposed in [1]. In a single FeAs(Se) layer structure, a unit cell of the lattice includes 2-Fe and 2-As(Se) atoms. The space group, which is non-symmetric, can be written as a product of $D_{3d} \otimes Z_2$ where $D_{3d}$ is a point group defined at Fe sites and $Z_2$ includes the space inversion which is defined with respect to the center of the nearest neighbor (NN) Fe-Fe link (see Fig. 2). Because the space inversion center does not locate at Fe sites, Pauli exclusion principle does not place a constraint on the parity of superconducting pairing order parameters even if only spin-singlet pairing is considered. A parity odd spin singlet pairing state becomes a natural choice in iron-based superconductors. A parity odd state has a real space sign inversion between the top and bottom As/Se layer. Moreover, provided that the d-p hybridization is essential in driving superconductivity, in a simplified effective model based on d-orbitals of Fe atoms, the uniform pairing between two Fe sublattices (inter-sublattice pairing) should be considered as an odd parity pairing. Thus, in an effective model with 1-Fe unit cell, we must divide the iron square lattice into two sublattices and the odd parity state is a combination of an uniform inter-sublattice pairing and an $\eta$-pairing, which is an intra-sublattice pairing with opposite sign between two sublattices. A real space picture of the odd parity state is shown in Fig. 2. In the real space, the state has a s-wave symmetry at Fe sites but exhibits a d-wave symmetry at the center of an iron square.

**Meanfield Hamiltonian for odd parity states** The meanfield Hamiltonian for an odd parity state [1] in an effective d-orbital model with 1-Fe unit cell can be generally written as

$$\hat{H} = \hat{H}_0 + \sum_{\alpha,\beta,\kappa} (\Delta_{\alpha,\eta,\beta} \hat{d}_{\alpha,\eta,\beta}(\hat{k}) + \eta_{\alpha,\eta,\beta} \hat{d}_{\alpha,\eta,\beta}^\dagger(\hat{k}) + h.c.)$$ (1)

where $\alpha, \beta$ label d-orbital characters, $\hat{H}_0$ is the effective band structure which has been constructed in [27, 29, 40, 42] and

$$\Delta_{\alpha,\eta,\beta} = \hat{d}_{\alpha,\eta,\beta}(\hat{k}) \hat{d}_{\beta,\eta,\beta}(-\hat{k}) - \hat{d}_{\beta,\eta,\beta}(\hat{k}) \hat{d}_{\alpha,\eta,\beta}(-\hat{k})$$ (2)

$$\Delta_{\eta,\eta,\beta} = \hat{d}_{\alpha,\eta,\beta}(\hat{k}) \hat{d}_{\eta,\eta,\beta}(-\hat{k} + \hat{Q}) - \hat{d}_{\eta,\eta,\beta}(\hat{k}) \hat{d}_{\alpha,\eta,\beta}(-\hat{k} + \hat{Q})$$ (3)

where $Q = (\pi, \pi)$. In general, the normal and $\eta$ pairing order parameters satisfy

$$\Delta_{\alpha,\eta,\beta}(\hat{k}) = -\Delta_{\eta,\eta,\beta}(\hat{k} + \hat{Q}), \Delta_{\eta,\eta,\beta}(-\hat{k}) = \Delta_{\eta,\eta,\beta}(-\hat{k} + \hat{Q}).$$ (4)

The parity symmetry becomes more transparent if we consider the model in 2-Fe unit cell. We divide the iron square lattice into two sublattices, A and B. We label d-orbitals in each sublattice by $\hat{d}^{\pm,\alpha,\beta}_{\alpha,\beta}(\hat{q})$ where $\hat{q}$ labels the momentum in the 2-Fe BZ. The d-orbital operators in 1-Fe unit cell can be defined as

$$\hat{d}_{\alpha,\beta}(\hat{k}) = \hat{d}^{\pm,\alpha,\beta}_{\alpha,\beta}(\hat{q}) + \hat{d}^{\pm,\alpha,\beta}_{\beta,\alpha}(\hat{q}) + \hat{d}^{\pm,\alpha,\beta}_A(\hat{k} + \hat{Q})$$ (5)

with $\hat{k} = \frac{1}{2}(q_x - q_y, q_x + q_y)$ for $d_{x^2-y^2}$ orbitals and $\hat{k} + \hat{Q} = \frac{1}{2}(q_x - q_y, q_x + q_y)$ for $d_{xy}, d_{x^2-y^2}$ and $d_{z^2}$ orbitals if we use a natural gauge setting for the d-orbitals.

Now the pairing order parameters can be defined as

$$\Delta_{\alpha,\beta}^{\pm,\alpha,\beta}(\hat{q}) = \hat{d}^{\pm,\alpha,\beta}_{\alpha,\beta}(\hat{q}) \hat{d}^{\pm,\alpha,\beta}_{\beta,\alpha}(\hat{q}) - \hat{d}^{\pm,\alpha,\beta}_{\alpha,\beta}(\hat{q}) \hat{d}^{\pm,\alpha,\beta}_{\beta,\alpha}(\hat{q})$$ (6)

where $\alpha$ and $\beta$ label sublattices. For an odd parity state, the Eq. can be specified as

$$\Delta_{\alpha,\beta}^{\pm,\alpha,\beta}(\hat{q}) = -\Delta_{\alpha,\beta}^{\pm,\beta,\alpha}(\hat{q})$$ (7)

The hopping Hamiltonian in 2-Fe unit cell can be written as

$$\hat{H}_0 = \sum_{\alpha,\beta,\kappa} \epsilon_{\alpha,\beta}(\hat{q}) \hat{d}^{\pm,\alpha,\beta}_{\alpha,\beta}(\hat{q}) \hat{d}^{\pm,\alpha,\beta}_{\beta,\alpha}(\hat{q}) + \epsilon_{\alpha,\beta}(\hat{q}) \hat{d}^{\pm,\alpha,\beta}_{\alpha,\beta}(\hat{q}) \hat{d}^{\pm,\alpha,\beta}_{\beta,\alpha}(\hat{q}) + \gamma_{\alpha,\beta}(\hat{q}) \hat{d}^{\pm,\alpha,\beta}_{\alpha,\beta}(\hat{q}) \hat{d}^{\pm,\alpha,\beta}_{\beta,\alpha}(\hat{q}) + \gamma_{\alpha,\beta}(\hat{q}) \hat{d}^{\pm,\alpha,\beta}_{\alpha,\beta}(\hat{q}) \hat{d}^{\pm,\alpha,\beta}_{\beta,\alpha}(\hat{q})$$ (8)

As $\hat{H}_0$ must be Hermitian and satisfies both space inversion and time reversal symmetry, we have $\epsilon_{\alpha,\beta}(\hat{q}) = \epsilon_{\beta,\alpha}(\hat{q}), \epsilon_{\alpha,\beta}(\hat{q}) = \epsilon_{\beta,\alpha}(\hat{q})$ and $\gamma_{\alpha,\beta}(\hat{q}) = \gamma_{\beta,\alpha}(\hat{q})$. If we use 1-Fe unit cell, the corresponding $\hat{H}_0 = \sum_{\alpha,\beta,\kappa} \epsilon_{\alpha,\beta}(\hat{k}) \hat{d}^{\pm,\alpha,\beta}_{\alpha,\beta}(\hat{k}) \hat{d}^{\pm,\alpha,\beta}_{\beta,\alpha}(\hat{k})$ with $\epsilon_{\alpha,\beta}(\hat{k}) = \epsilon_{\alpha,\beta}(\hat{k}) + \gamma_{\alpha,\beta}(\hat{k})$. $\gamma_{\alpha,\beta}(\hat{k}) = -\gamma_{\alpha,\beta}(\hat{k})$ when $\alpha$ and $\beta$ are d$_{x^2-y^2}$ orbitals and otherwise $\gamma_{\alpha,\beta}(\hat{k}) = \gamma_{\alpha,\beta}(\hat{k})$. It is also easy to see that $\epsilon_{\alpha,\beta}(\hat{k}) = \epsilon_{\alpha,\beta}(\hat{k} + \hat{Q})$ and $\gamma_{\alpha,\beta}(\hat{k}) = -\gamma_{\alpha,\beta}(\hat{k} + \hat{Q})$. Eq. can be written as

$$\hat{H} = \sum_{\hat{q}} \phi^*(\hat{q}) A(\hat{q}) \phi(\hat{q})$$ (9)

where

$$A(\hat{q}) = \begin{pmatrix}
\epsilon(\hat{q}) & \gamma(\hat{q}) & \Delta^{AA}(\hat{q}) & \Delta^{AB}(\hat{q}) \\
\gamma^*(\hat{q}) & \epsilon(\hat{q}) & \Delta^{AB}(\hat{q}) & -\Delta^{AA}(\hat{q}) \\
\Delta^{AA}(\hat{q}) & \Delta^{AB}(\hat{q}) & -\epsilon(\hat{q}) & -\gamma^*(\hat{q}) \\
\Delta^{AB}(\hat{q}) & -\Delta^{AA}(\hat{q}) & -\gamma(\hat{q}) & -\epsilon(\hat{q})
\end{pmatrix}$$ (10)

is a $20 \times 20$ matrix if all five d-orbitals are used and $\phi(\hat{q}) = (\{|\alpha\rangle, |\beta\rangle, |\alpha\rangle, |\beta\rangle, |\alpha\rangle, |\beta\rangle\}$.

FIG. 2: (Color online) A sketch of the odd parity state in the real space: (a) the view of pairing with As/Se atoms and symmetry characters with respect to different centers; (b) the view of pairing in an effective iron square lattice. Different colors represent the different signs of pairing.
Sign changes in reciprocal space We take a 5 orbital effective model to describe the band structure[27,29,40,42]. We set the orbital index as 1 → d_{xz}, 2 → d_{yz}, 3 → d_{x^2−y^2}, 4 → d_{xy}, and 5 → d_z. As discussed in [11], the odd parity state is in the A1 representation of D_{2d} at iron sites in order to avoid gapless nodes. In this case the inter-orbital pairings can be ignored in both normal and η-pairing channels. As the Fermi surfaces are dominated by t_{2g} d-orbitals, the leading order of normal pairing for d_{xz} and d_{xy} is given by

\[ Δ_{11}^{AB} = \frac{1}{2} (Δ_{11}^{N} cos \frac{q_x + q_y}{2} + Δ_{11}^{N} cos \frac{q_x - q_y}{2}) \] (11)

\[ Δ_{44}^{AB} = Δ_{44}^{N} (cos \frac{q_x}{2} cos \frac{q_y}{2}). \] (12)

The leading order of the η pairing or the intra-sublattice pairing is given by

\[ Δ_{\eta \eta}^{AA} = \frac{1}{2} Δ_{\eta \eta}^{NN} (cos q_x + cos q_y). \] (13)

In the presence of hole pockets at Γ point, the superconducting gaps on the hole pockets are determined by the normal pairing or the inter-sublattice pairing. Typically, in iron-pnictides, there are three hole pockets at Γ point in the 2-iron BZ. Two of them denoted as α and β pockets are mainly attributed to d_{xz} and d_{yz} orbitals and the other denoted as γ pockets are attributed to d_{xy} orbitals. There are two electron pockets at M points, which are denoted as λ and δ pockets. The orbital characters on the electron pockets are mixed. We take seven representative points on Fermi surfaces around Γ, M and M’ points as shown in Fig.3 one for each pocket ordered as 1 on α (xz), 2 on β (yz), 3 on γ (xy), 4 on λ at M (yz+xy), 5 on δ at M (xy+zx), 6 on λ at M’ (zx+xy) and 7 on δ at M’ (xy+yz) where their orbital characters are specified in parenthesis with the first one being the primary orbital character. One can easily check that the signs of superconducting order parameters at these points, sign(Δ), are determined by normal pairing parameters. To see this, we take a simple case that \( Δ_{11,4}^{N} = Δ_{11,3}^{N} \). In this case, without the η pairing, the superconducting gap vanishes along M − M’ directions. Therefore, there are nodes on electron pockets if only normal pairing is present. In the presence of the η pairing, the η pairing creates interband pairing between two degenerate bands along M − M’ and removes nodes. However, the interband pairing does not change the sign of order parameters given by normal pairing. This analysis is still valid even if \( Δ_{11,4}^{N} ≠ Δ_{11,3}^{N} \).

Now, if we fix the amplitudes of the order parameters in Eq.[12] the superconducting gaps on Fermi surfaces are larger if sign(Δ_{11,4}^{N}) = −sign(Δ_{44}^{N}) than if sign(Δ_{11,1}^{N}) = sign(Δ_{44}^{N}). Therefore, to gain the maximum superconducting condensation energy, sign(Δ_{11,1}^{N}) = −sign(Δ_{44}^{N}). Namely, we have to take opposite signs for the normal pairing for d_{xz} and d_{xy} orbitals in Eq.[12] These can be explicitly verified in numerical calculations by taking a ten-orbital effective model as shown in Fig.3 Taking sign(Δ_{11,1}^{N}) = −sign(Δ_{44}^{N}) we obtain the sign distribution at the representative points as

\[ sign(Δ_1) = sign(Δ_2) = −sign(Δ_3) = −sign(Δ_4) = sign(Δ_5) = −sign(Δ_6) = sign(Δ_7) \] (14)

which produces the sign distribution shown in Fig.3(b).

In fact, the above results can be analytically understood if we use 1-Fe unit cell. The above sign change is a generic consequence of the band structure and the normal pairing form factor in reciprocal space. In the BZ of 1-Fe unit cell, we have \( Δ_{\eta \eta}(\hat{k}) = −Δ_{\eta \eta}^{N}(\hat{k} + \hat{Q}) \) and \( d_{xy}(\hat{k}) \) is coupled to \( id_{xz,xy}(\hat{k} + \hat{Q}) \) in the band structure with the natural gauge setting. Therefore, in order to maximize superconducting condensation energy, we must have \( sign(Δ_{\eta \eta}^{N}(\hat{k})) = sign(Δ_{xz,xy}^{N}(\hat{k})) \). The γ pocket is located at (π, π) rather than Γ point in 1-Fe unit cell. Therefore, there is a sign change between hole pockets of d_{xz,xy} and the hole pocket of d_{xy}. In the 1-Fe BZ, on the electron pockets at M or M’, the normal pairing is d-wave like. The overall sign change is produced between two BZs of 2-Fe unit cell as shown in Fig.1(a).

Comparison between the even parity s^+ and the odd parity state We can show that the odd parity state captures all essential sign change features required to explain observed magnetic resonances. We can see that the odd parity state has similar sign change feature between hole pockets and electron pockets as the s^+ state which is shown in Fig.3(d). In fact, the odd parity state has an intra-orbital sign change for each orbit changes between hole pockets at Γ and electron pockets at M or M’. Therefore, in the presence of hole pockets, the odd

**FIG. 3:** (Color online) (a) the Fermi surface of normal states for iron-based superconductors obtained by modifying the model in[29] with \( ϵ_{12} = −0.0013, ϵ_4 = 0.2878 \) and a new chemical potential \( μ = −0.15 \) (k, dispersion is also ignored). The quasi-particle spectra in superconducting states is shown in (b) with only normal pairing and (d) with both normal and η pairing. In (b), the black (deep) lines corresponds to \( Δ_{\eta \eta}^{N} = Δ_{\eta \eta}^{AA} = Δ_{\eta \eta}^{NN} = 0.1 \) and the red (light) lines corresponds to \( Δ_{\eta \eta}^{N} = Δ_{\eta \eta}^{AA} = Δ_{\eta \eta}^{NN} = 0.1 \). In (d), the normal pairing parameters are same as that in (b) except the non-zero η pairing parameters \( Δ_{\eta \eta}^{N} = Δ_{\eta \eta}^{NN} = −Δ_{\eta \eta}^{AA} = 0.05 \). (c) the signs of the pairing along the Fermi surface are shown.
parity state will result in a magnetic resonance around \((0, \pi)\) and \((\pi, 0)\) wavevectors, which has been almost universally observed in iron-pnictides\cite{6, 13}.

The \(s^+\) can not explain the intrinsic sign change in 122 iron-chalcogenides where there is no hole pocket at \(\Gamma\). In the odd parity state, as shown in Fig.4\cite{5} we can see that the 4th (5th) representative point on Fermi surfaces around \(M\) and 7th (6th) representative point at \(M'\) have opposite superconducting sign. As we have specified earlier, the orbital character of the 4th point is \(d_{xy} + d_{yz}\) and that of the 7th point is \(d_{xy} + d_{xc}\). The orbital character of the 5th point is \(d_{xy} + d_{yc}\) and that of the 7th point is \(d_{xc} + d_{yc}\). The mixture between different orbitals becomes larger if the electron pocket is larger. Therefore, we can conclude that the sign change exists within the same orbitals between electron pockets near wave vector \((\pi, \delta)\), where \(\delta\) is determined by the size of Fermi pockets. Therefore, the sign change in the odd parity state in the absence of hole pockets is very similar to the sign change of a d-wave\cite{45}. Thus, the odd parity state is consistent with the neutron observation of the magnetic resonance at \((\pi, \pi/2)\) or \((\pi/2, \pi)\)\cite{15}.

**Mechanism for the odd parity state:** The superconducting order parameters in the odd parity state also imply a possible microscopic mechanism. In the ref.\cite{1}, it has been shown that the normal pairing between two sublattices in an effective model can be interpreted as both even and odd parity. In most previous studies, even parity was automatically assumed. If we take the assumption that the superconducting pairing is related to antiferromagnetic exchange coupling, as shown in \cite{32, 35}, the \(s^+\) pairing state can be naturally obtained from the next NN antiferromagnetic exchange coupling \(J_1\)\cite{44}. A decoupling of the \(J_2\) term in the pairing channel results in a \(\cos k, \cos k\) momentum dependence\cite{32}, which is characterized by opposite signs between hole pockets at \(\Gamma\) and electron pockets at \(M\) or \(M'\). However, there is no sign change between two electron pockets. In this case, as shown in several calculations\cite{32, 35}, the NN AFM coupling \(J_1\) actually competes with \(J_2\). If \(J_2\) is dominated, the effect of \(J_1\) is completely suppressed. This is the reason behind the argument for no sign changed s-wave in iron chalcogenides without hole pockets\cite{34}. Intuitively, the competing nature between these two terms can be understood from the sign distribution of order parameters produced by the \(J_1\) and \(J_2\) terms. It is impossible for them to collaboratively enhance superconducting gaps on all Fermi surfaces, a principle proposed recently in \cite{35}.

However, the situation is different if we interpret the state as an odd parity state. In the odd parity state, \(J_1\) and \(J_2\) terms do not compete. In fact, they are collaborative since the \(J_2\) term contributes to superconductivity in the \(\eta\) pairing channel while \(J_1\) is in the normal pairing channel. As shown in above analysis as well as in Fig.4\cite{5} the \(\eta\) pairing enhances superconducting gaps on Fermi surfaces where superconducting gaps induced by normal pairing from \(J_1\) are minimal. Considering the sign change can minimize the cost of repulsive interaction in a superconducting state, an odd parity state thus can be favored. A more detailed study of the collaborative nature of \(J_1\) and \(J_2\) will be explored in future.

**Discussion and Summary** As discussed in \cite{1}, the sign change in the odd parity state in real space is characterized by the sign change between top and bottom \(\text{As(Se)}\) layers. The sign distribution in reciprocal space revealed here suggests that a \(\pi\)-junction in the \(a-b\) plane is almost impossible to be made. This is consistent with the fact that sign change in \(a-b\) corner junctions was not observed in single crystals. Combining the absence of sign change in \(a-b\) plane and the fact that the composite \(\text{Nb-NdFeAsO}\) used in the phase sensitive experiment where the half-integer flux was observed\cite{24} is a multi-crystal, we actually can conclude that the phase change must be generated along \(c\)-axis, a strong support for an odd parity state.

An even parity state is translationally invariant with respect to 1-Fe unit cell while an odd parity is not. This difference results in a fundamental difference on superconductivity properties related to electron pockets. In an even parity state, we can essentially view electron pockets as one pocket while in the odd parity state, we must consider them as two electron pockets. Therefore, in the absence of hole pockets, an odd parity superconducting state can still exhibit two gap features. The local density measured in the single FeSe layer clearly exhibits a two-gap feature\cite{46}.

The sign change revealed here for the odd parity state without hole pockets is similar to what has been called as an incipient \(s^+\) state\cite{47}. However, we want to make it clear that previous proposals did not fundamentally understand the parity issue as well as the nature of coexistence of both normal pairing and \(\eta\)-pairing in an odd parity state. Realizing the odd parity allows to connect the sign change between real and momentum spaces.

We also want to point out the sign change between hole pockets from different orbitals was obtained in an exactly diagonalization study of a four site problem\cite{35}. However, the parity issue can not be addressed in a four site problem.

The normal pairing for \(d_{xc}\) and \(d_{yc}\) is determined by two independent parameters \(\Delta_{1,1,1}^{N,1}\) and \(\Delta_{1,1,1}^{N,1}\). Under the assumption that the superconductivity is induced by local AFM exchange coupling, a large difference between these two parameters can suggest the NN AFM coupling \(J_1\) is highly anisotropic for each orbital. Therefore, a superconducting gap structure on electron pockets which are characterized by highly mixture of different orbitals may help to determine microscopic AFM interactions.

In summary, we present the sign change in reciprocal space for the odd parity superconducting state. The sign change character consistently explains experimental results related to sign change properties measured on both iron-pnictides and iron-chalcogenides.

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