Linear space of spinor monomials and realization of the Nambu-Goldstone fermion in the Volkov-Akulov and Komargodski-Seiberg Lagrangians

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Abstract

The analytical algorithm previously proposed by the author for matching the Volkov-Akulov (VA) and Komargodski-Seiberg (KS) actions describing the Nambu-Goldstone (NG) fermion, is discussed. The essence of the algorithm is explained, its consistency is proved and the recent results obtained with computer assistance are reproduced, when the proper Fierz rearrangements for Majorana bispinors are taken into account. We reveal a linear space of composite spinorial monomials $\Delta_m$ which are the solutions of the scalar constraint $(\partial^m \bar{\psi} \Delta_m) = 0$. This space is used to clarify relations between the KS and VA realizations of the NG fermionic field $\psi$.

1 Introduction

The ideas of spontaneous supersymmetry breaking and the Nambu-Goldstone fermions [1] can play an important role in a new physics expected from LHC. In particular, it is of a great interest to study relevant phenomenological Lagrangians invariant under supersymmetry. Another topical problem is the relation between the linear and non-linear realizations of supersymmetry [2], the study of which resulted in new approaches [3–11]. Some of

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these achievements were taken into account and extended further in the recent paper by Komargodski and Seiberg [12], where the $D = 4 \mathcal{N} = 1$ supercurrent approach was proposed to construct the low-energy effective Lagrangian. This approach stimulated some new proposals concerning the role the NG fermions and their couplings in the minimal standard supersymmetric model, astroparticle physics and others (see e.g. [13], [14]).

In the recent paper [15] we started the study of the direct relation between the KS and VA Lagrangians and developed an analytic algorithm for proving their equivalency. This algorithm reduces the equivalency problem to the solution of three nonlinear equations [16] for the three Majorana bispinors which represent the KS fermion in terms of the VA one. We solved these equations using the factorization property of these algorithmic equations. This shows that the use of the Majorana bispinor representation might be more suitable for finding out a geometric principle which connects the KS and VA realizations of the NG fermion. On the contrary, working in the frame of the 2-spinor Weyl formalism hides deeper such a geometric origin. In some cases the use of the Weyl spinors greatly complicates the proof of the universality of the NG fermion action in practice (see e.g. [17], [18], [19]).

Here we clarify the essence of the analytical algorithm, prove its consistency and show how the recent results, obtained with computer assistance and the Weyl spinors [20], are reproduced in our approach, when the proper Fierz rearrangements for Majorana bispinors are taken into account. We reveal a linear space of composite spinorial monomials $\Delta_m$, which play an important role in the redefinition of the KS fermion via the VA fermion $\psi$. The spinorial monomials $\Delta_m$ appear as the solution of the scalar constraint $(\partial^m \bar{\psi} \Delta_m) = 0$.

### 2 The algorithm for matching the VA and KS actions

Let us remind the analytical and algorithmic procedure [15] for derivation of the relation between the VA and KS realizations of the NG fermions and their actions.

The VA Lagrangian [1], expressed in terms of the Majorana bispinors, is as follows

$$
\mathcal{L}_{VA} = \frac{1}{a} - \frac{i}{4} \bar{\psi}^m \gamma_m \psi - \frac{a}{32} \left[ (\bar{\psi}^m \gamma_m \psi)^2 - (\bar{\psi}^m \gamma_m \psi) (\bar{\psi}^n \gamma_n \psi) \right] + \frac{a^2}{3!} \sum_p (-)^p T^m_n T^q_l, \quad (1)
$$

where the sum $\sum_p$ in (1) corresponds to the sum in all the permutations of the subindices in the products of the tensors $T^m_n$ defined as

$$
T^m_n = -\frac{i}{4} \bar{\psi}^n \gamma_m \psi, \quad \bar{\psi}^n_a := \partial^a \bar{\psi}_a. \quad (2)
$$

The KS Lagrangian [12] expressed in the bispinor Majorana representation [15] is

$$
\mathcal{L}_{KS} = -f^2 - \frac{i}{2} \bar{G}^m \gamma_m G - \frac{1}{16 f^2} \left[ (\bar{G}^m G)^2 + (\bar{G}^m \gamma_5 G)^2 \right] - \frac{1}{(16 f^2)^2} \left[ (\bar{G} G)^2 + (\bar{G} \gamma_5 G)^2 \right] [ (\partial^2 (\bar{G} G))^2 + (\partial^2 (\bar{G} \gamma_5 G))^2 ],
$$

(3)
where the total derivative terms are omitted. For matching the Lagrangians (3) and (1) we redefine the KS field \( g := \sqrt{2}G \) and use arbitrariness in the relation between the interaction constants \(-1/f^2 := a/2\) resulting in the equivalent representation of the KS Lagrangian

\[
\mathcal{L}_{KS} = \frac{2}{a} - i\frac{4}{a^2} \bar{g}^m \gamma_m g + \frac{a}{2}[(\bar{g}^m g)^2 + (\bar{g}^m \gamma_5 g)^2]
- 2\left(\frac{a}{16}\right)^3[(\bar{g}g)^2 + (\bar{g} \gamma_5 g)^2][(\partial^2(\bar{g}g))^2 + ((\partial^2(\bar{g} \gamma_5 g))^2].
\]  

(4)

A congruent structure of the Lagrangians (1) and (4), observed in [15], was used there to relate the fields \( g \) and \( \psi \). It means that the Lagrangians contain the fermions and their derivatives in the form of the powers \((\partial \bar{\psi} \psi)^r\) and \((\partial \bar{g}g)^r\). The only difference consists in various distributions of the spinor indices, derivatives and \(\gamma\)-matrices which generate such Lorentz covariants. The dimensions of these monomials are measured by the degrees \( L^{-4r} \) that are inverse to the dimensions of \( a^r \). This key information was encoded in the general polynomial expansion for the Majorana bispinor \( g_a \) in degrees of \( \partial \bar{\psi} \psi \)

\[
g = \psi + a\chi + a^2\chi_2 + a^3\chi_3,
\]  

(5)

where \( \chi_1 \equiv \chi \). The maximal degree of the polynomial (5) is equal to three. Really, since the products \( a^r \chi_r \) have the same dimension as \( \psi \), the Grassmannian bispinors \( \chi, \chi_2, \chi_3 \) must have the general form \( \chi_r \sim \psi(\partial \bar{\psi} \psi)^r \) fixed by the above-discussed monomials. These monomials are nilpotent and their maximal degree \( r = 3 \), because \( \psi(\partial \bar{\psi} \psi)^3 \) contains the maximal power of the Grassmannian bispinor \( \psi \) equal to four for the considered case \( D = 4, N = 1 \). The substitution of (5) in the KS Lagrangian (4) and comparison of its terms with those in (1) with the same degree in the constant \( a \), yield the coupled chain of the three equations for \( \chi, \chi_2 \) and \( \chi_3 \), which was reduced [16] to the factorized form

\[
(\bar{\psi}^m \gamma_m \chi_r) = \bar{\psi}^m \mathcal{P}^{(r)}_m, \quad (r = 1, 2, 3).
\]  

(6)

The polynomials \( \mathcal{P}^{(r)}_m(\psi, \chi_{r-1}, \chi_{r-2}) \) (6), found from the previous steps of the algorithmic procedure, depend on \( \psi, \chi_{r-1}, \chi_{r-2} \), their derivatives and are considered to be known.

The presence of the derivative \( \bar{\psi}^m \) in the l.h.s and r.h.s. of Eqs. (6) was used in [15], [16] for their simplification by means of cancellation of \( \bar{\psi}^m \). These cancellations reveal representations for \( \gamma_m \chi_r \) that can be associated with particular solutions of the inhomogeneous Eqs. (6). Then the general solution of (6) is presented by the sum

\[
\gamma_m \chi_r = \mathcal{P}^{(r)}_m + \Delta^{(r)}_m, \quad (r = 1, 2, 3)
\]  

(7)

of the particular solutions and the general solution \( \Delta^{(r)}_m \) of the homogeneous system

\[
(\bar{\psi}^m \gamma_m \chi_r) = 0,
\]  

(8)
formed by the linear combination of the bispinors $\Delta^{(r)i}_m$ with the complex constants $\alpha^{(r)i}_i$

$$\Delta^{(r)}_m = \sum_i \alpha^{(r)i}_i \Delta^{(r)i}_m. \quad (9)$$

The bispinors $(\Delta^{(r)i}_m)_a$ in (9) form a linear space $\Delta^{(r)}$ of the monomials $\sim \psi(\partial \bar{\psi} \psi)^r$, similar to the monomials composing $\chi_r$, but satisfying the orthogonality constraints

$$(\bar{\psi}^m \Delta^{(r)i}_m) = 0, \quad (r = 1, 2, 3). \quad (10)$$

In general, the constraints (10) can be weakened to take into account that the KS and VA Lagrangians might be equal modulo total derivatives

$$(\bar{\psi}^m \Delta^{(r)i}_m) \sim \partial^n \phi^{(r)i}_n, \quad \phi^{(r)i}_n \sim \bar{\psi} \psi(\partial \bar{\psi} \psi)^r, \quad (11)$$

where $\phi^{(r)i}_n$ are composite Lorentz vectors built of $(r + 2)$ bispinors $\psi$ and of $r$ partial derivatives $\partial \psi$. Because of the nilpotency of these monomials the weakening concerns the monomials with $r < 3$. The conditions (11) can be written in the form generalizing (10)

$$(\bar{\psi}^m \Delta^{(r)i}_m) = 0 \ (\text{mod total derivatives}), \quad (12)$$

where the total derivatives in (12) are the Lorentz invariant monomials $\sim \partial[\bar{\psi} \psi(\partial \bar{\psi} \psi)^r]$ representable in the form of total derivatives (TD).

Taking into account the fact that the polynomials $P^{(r)}_m$ in (6) are fixed by the algorithmic procedure, one can conclude that the solution of Eqs. (7), presented in the form

$$\gamma_m \chi_r = P^{(r)}_m + \Delta^{(r)}_m, \quad (13)$$

is reduced to the construction of the three linear spaces $\Delta^{(r)}$ for each of $r=(1,2,3)$.

Below, we demonstrate the work of the algorithm by explicit calculation of $\chi$ from (5).

### 3 Explicit construction of the space $\Delta^{(1)}$

Here we build an explicit realization of the $\Delta^{(1)}$ space, introduced in [15], and find $\chi$.

In this case the system (6) is reduced to the only one equation

$$i(\bar{\psi}^m \gamma_m \chi) = \frac{1}{16}[(\bar{\psi}^m \psi)^2 + (\bar{\psi}^m \gamma_5 \psi)^2] + \frac{1}{16}[(\bar{\psi}^m \gamma_m \psi)^2 - (\bar{\psi}^m \gamma_m \psi)(\bar{\psi}^m \gamma_n \psi)] \quad (15)$$

which represents the equivalency conditions in the first order in $a$ and has the discussed factorizable form. The cancellation of the derivative $\bar{\psi}^m$ reduces Eq. (15) to the form

$$\gamma_m \chi = -\frac{i}{16}[\psi(\bar{\psi}_m \psi) + \gamma_5 \psi(\bar{\psi}_m \gamma_5 \psi)] - \frac{i}{16}[\gamma_m \psi(\bar{\psi}_m \gamma_n \psi) - \gamma_n \psi(\bar{\psi}_m \gamma_m \psi)] + \Delta_m, \quad (16)$$

$$(\bar{\psi}^m \Delta_m) = 0 \ (\text{mod TD}). \quad (17)$$
The system (16) consists of eight complex equations for two complex components of the bispinor $\chi$ and six complex coefficients $\alpha^i \equiv \alpha^{(1)i}$ from (9)

$$\Delta_m = \sum_i \alpha_i \Delta^i_m,$$  \hspace{1cm} (18)

associated with the linear space of the monomials $\Delta^i_m \equiv \Delta^{(1)i}_m$, $(i = 1, 2, \ldots, 6)$, which form $\Delta_m$ for the case $r = 1$. The bispinors $\Delta^i_m$ are built from the monomials $\sim \psi(\partial \bar{\psi}\psi)$ and may be split in two different subsets forming the linear space $\Delta^{(1)}$.

The first subset is presented by the three monomials $\nu^i_m, (i = 1, 2, 3)$ belonging to the exact solutions of (17) that are not accompanied with total derivatives in the r.h.s. of (17)

$$(\bar{\psi}^m \nu^i_m) = 0.$$  \hspace{1cm} (19)

The explicit form of these monomials $\nu^i_m$ is as follows

\begin{align*}
\nu^1_m &= \varepsilon_{mnpq} \gamma^n [\gamma^5 \psi(\bar{\psi}^p \gamma^q \gamma^5 \psi) - \psi(\bar{\psi}^p \gamma^q \gamma^5 \psi)], \\
\nu^2_m &= \varepsilon_{mnpq} [\gamma^5 \psi(\bar{\psi}^n \Sigma^p \psi) + \Sigma^p \psi(\bar{\psi}^n \gamma^5 \psi)], \\
\nu^3_m &= \psi(\bar{\psi}^m \Sigma mn \psi) - \Sigma mn \psi(\bar{\psi}^m \psi).
\end{align*}  \hspace{1cm} (20)

The second subset is presented by three solutions $\Delta^i_m, (i = 1, 2, 3)$ of Eq.(17) that are accompanied with the total derivative terms in the r.h.s. of (17) having the form

$$\begin{align*}
\Delta^1_m &= \frac{1}{2} \varepsilon_{mnpq} \gamma^n \partial^p [\psi(\bar{\psi}^n \gamma^5 \gamma^q \psi)], \\
\Delta^2_m &= \frac{1}{2} \varepsilon_{mnpq} \Sigma^p \partial^m [\psi(\bar{\psi}^n \gamma^5 \psi)], \\
\Delta^3_m &= \frac{1}{2} \Sigma mn \partial^n [\psi(\bar{\psi}^m \psi)].
\end{align*}$$  \hspace{1cm} (21)

The bispinors $\Delta^i_m$ (21) have a remarkable property to remain to be total derivatives after their contraction with the derivative $\bar{\psi}^m$ of the bispinor $\bar{\psi}$

$$\begin{align*}
(\bar{\psi}^m \Delta^1_m) &= \frac{1}{2} \varepsilon_{mnpq} \partial^p [(\bar{\psi}^m \gamma^n \gamma^q \psi)(\bar{\psi}^n \gamma^5 \gamma^q \psi)], \\
(\bar{\psi}^m \Delta^2_m) &= \frac{1}{2} \varepsilon_{mnpq} \partial^m [(\bar{\psi}^m \Sigma^p \psi)(\bar{\psi}^n \gamma^5 \gamma^q \psi)], \\
(\bar{\psi}^m \Delta^3_m) &= \frac{1}{2} \partial^n [(\bar{\psi}^m \Sigma mn \psi)(\bar{\psi}^n \psi)].
\end{align*}$$  \hspace{1cm} (22)

The explicit form of the bispinors (20,21), saturating the scalar constraint (17), results in the desired monomial representation of $\Delta_m$ (18)

$$\Delta_m = \alpha_1 \nu^1_m + \alpha_2 \nu^2_m + \alpha_3 \nu^3_m + \beta_1 \Delta^1_m + \beta_2 \Delta^2_m + \beta_3 \Delta^3_m.$$  \hspace{1cm} (23)

including six arbitrary complex constants $\alpha_i$ and $\beta_i$. The substitution of (23) in equations (16) fixes position of the arbitrary constants $\alpha_i, \beta_i$ in the r.h.s. of Eqs. (16)

$$\gamma_m \chi = -\frac{1}{16} [\psi(\bar{\psi}^m \psi) + \gamma_5 \psi(\bar{\psi}^m \gamma_5 \psi)] - \frac{1}{16} [\gamma_m \psi(\bar{\psi}^n \gamma_n \psi) - \gamma_n \psi(\bar{\psi}^n \gamma_m \psi)] + \sum \alpha_i \nu^i_m + \sum \beta_i \Delta^i_m.$$  \hspace{1cm} (24)

The general solution for $\chi, \alpha_i, \beta_i$ following from Eqs. (24) is considered below.
4 Fierz rearrangements and general solution

In the case of nonlinear realization of supersymmetry by the NG field $\psi$, the VA Lagrangian is invariant modulo total derivatives (see e.g. [2]). This points to possible presence of the monomials from the space $\Delta^{(1)}$ in the VA Lagrangian or in its contribution

$$\zeta_m := \gamma_m \psi(\bar{\psi}^n \gamma_n \psi) - \gamma_n \psi(\bar{\psi}^m \gamma_m \psi)$$

in Eqs. (24), denoted by a condensed bispinor $\zeta_m$. If the total derivatives, discovered in $\zeta_m$ (25), would coincide with the bispinors $\nu^i_m$ (20) or $\Delta^i_m$ (21), they could be unified with these bispinors resulting in a simplification of the system (24). Consequently, the next step is to verify possible presence of the monomials from $\Delta^{(1)}$-space in the bispinor $\zeta_m$ (25).

To this end consider the following Fierz rearrangement of $\zeta_m$ (25)

$$\zeta_m = -\frac{1}{4} \sum_A \{ \gamma_m \Gamma^A \psi(\bar{\psi}^n \gamma_n \Gamma_A \psi) - \Gamma^A \gamma_m \psi(\bar{\psi}^n \Gamma_A \gamma_n \psi) \}.$$  \hspace{1cm} (26)

The 16 Dirac matrices $\Gamma^A$ and their inverse $\Gamma_A = (\Gamma_A)^{-1}$, defined as

$$\Gamma^A := (1, \gamma^m, \Sigma^{mn}, \gamma^5, \gamma^5 \gamma^m),$$  \hspace{1cm} (27)

form the complete basis in the space of $4 \times 4$ matrices. The r.h.s. of (26) includes only the products of $\gamma^m \times \Gamma^A$ and is expressed via their (anti)commutators using the identities

$$\gamma^m \Gamma^A = \frac{1}{2} \{ \gamma^m, \Gamma^A \} + \frac{1}{2} [\gamma^m, \Gamma^A].$$  \hspace{1cm} (28)

This yields the following representation for $\zeta_m$

$$\zeta_m = -\frac{1}{8} \sum_A \{ [\gamma_m, \Gamma^A] \psi(\bar{\psi}^n \{ \gamma_n, \Gamma_A \} \psi) - \{ \Gamma^A, \gamma_m \} \psi(\bar{\psi}^n [\Gamma_A, \gamma_n] \psi) \}.$$  \hspace{1cm} (29)

including only the products of the commutators with the anticommutators.

The desired simplification of (29) is achieved by substitution in the relations

$$\{ \gamma_m, \gamma_n \} = -2 \eta_{mn}, \hspace{1cm} [\gamma_n, \Sigma_{pq}] = -2 (\eta_{np} \gamma_q - \eta_{nq} \gamma_p),$$

$$\{ \gamma_n, \Sigma_{pq} \} = -2 \varepsilon_{npq} \gamma^5, \hspace{1cm} \gamma^5 \Sigma_{mn} = -\frac{1}{2} \varepsilon_{mnpq} \Sigma^{pq},$$  \hspace{1cm} (30)

where $\Sigma^{mn} := \frac{1}{2} [\gamma_m, \gamma_n]$, accompanied with the use of the explicit form of $\nu^i_m$ (20).

As a result, we reveal the presence of the bispinors from $\Delta^{(1)}$ in $\zeta_m$ (29)

$$\zeta_m \equiv \gamma_m \psi(\bar{\psi}^n \gamma_n \psi) - \gamma_n \psi(\bar{\psi}^m \gamma_m \psi)$$

$$= -[\Sigma_{mn} \psi(\bar{\psi}^n \psi) + \gamma^5 \Sigma_{mn} \psi(\bar{\psi}^n \gamma^5 \psi)] + (\nu^1_m - \frac{1}{4} \nu^2_m - \frac{1}{2} \nu^3_m)$$

which coincide with the bispinors $\nu^i_m$ (20).
The substitution of (31) in Eqs. (24) transforms the latter into the equations
\[ \gamma_m \chi = -\frac{i}{16}[\psi(\bar{\psi}_m \psi) + \gamma^5 \psi(\bar{\psi}_m \gamma^5 \psi)] + \frac{i}{16}[\Sigma_{mn} \psi(\bar{\psi}_n \psi) + \gamma^5 \Sigma_{mn} \psi(\bar{\psi}_n \gamma^5 \psi)] + \tilde{\Delta}_m, \] (32)
where \( \tilde{\Delta}_m \) differ from \( \Delta_m \) (23) by shifts of the coefficients \( \alpha_i \)
\[ \tilde{\Delta}_m := (\alpha_1 - \frac{i}{16}) \nu^1_m + (\alpha_2 + \frac{i}{64}) \nu^2_m + (\alpha_3 + \frac{i}{32}) \nu^3_m + \sum \beta_i \Delta^i_m. \] (33)

The sum of the two first terms in the r.h.s. of (32) can be combined in a compact form
\[ -\frac{i}{16}[\psi(\bar{\psi}_m \psi) + \gamma^5 \psi(\bar{\psi}_m \gamma^5 \psi)] + \frac{i}{16}[\Sigma_{mn} \psi(\bar{\psi}_n \psi) + \gamma^5 \Sigma_{mn} \psi(\bar{\psi}_n \gamma^5 \psi)] \]
\[ = \frac{i}{16}(-\eta_{mn} + \Sigma_{mn})[\psi(\bar{\psi}_n \psi) + \gamma^5 \psi(\bar{\psi}_n \gamma^5 \psi)], \] (34)
simplifying Eqs. (32) to the equations
\[ \gamma_m \chi = \frac{i}{16}(-\eta_{mn} + \Sigma_{mn})[\psi(\bar{\psi}_n \psi) + \gamma^5 \psi(\bar{\psi}_n \gamma^5 \psi)] + \tilde{\Delta}_m. \] (35)

Contraction of Eqs. (35) with \( \gamma^m \) and the use of the relation \((-\eta_{mn} + \Sigma_{mn}) = \gamma^m \gamma^m \) result in the following representation of the sought-for bispinor \( \chi \)
\[ \chi = \frac{i}{16} \gamma^m [\psi(\bar{\psi}_n \psi) + \gamma^5 \psi(\bar{\psi}_n \gamma^5 \psi)] - \frac{1}{4} \gamma^m \tilde{\Delta}_m. \] (36)

The substitution of the representation (36) back in Eqs. (35) reveals homogeneous equations for still unknown coefficients \( \alpha_i \) and \( \beta_i \) included in \( \tilde{\Delta}_m \) (33)
\[ (\eta_{mn} + \frac{1}{3} \Sigma_{mn}) \tilde{\Delta}^n = 0. \] (37)

One can observe that the rescaled matrix \((-\eta_{mn} + \frac{1}{3} \Sigma_{mn}) \) from (37), acting on the vector and spinor indices of the bispinor \( \tilde{\Delta}_m \), has the property \( P^2 = P \)
\[ P^m_q P^m_q = P^m_m, \quad P^m_m := \frac{3}{4} (\delta^m_m + \frac{1}{3} \Sigma^m_m), \] (38)
because of the matrix relation \( \Sigma^{mq} \Sigma_{qn} = 3 \delta^m_n + 2 \Sigma^m_n \).

Then Eqs. (37) may be treated as a projection condition which requires a special form for spinor \( \tilde{\Delta}_m \) (33) with its Lorentz vector index generated by \( \gamma^m \)
\[ \tilde{\Delta}_m = \gamma_m \tilde{\rho}, \] (39)
where the bispinor \( \tilde{\rho} \) belongs to the monomial space \( \Delta^{(1)} \). The representation (39) is derived with the help of the relation \( \Sigma_{mn} \gamma^n = -3 \gamma^m \).

The explicit form of \( \tilde{\Delta}_m \), fixed by (33), (20), (21), shows that its vector index is generated by either \( \varepsilon_{mnq} \) or \( \Sigma_{mn} \). However, the representation (39) requires to transfer this function into the matrix \( \gamma^m \), and this requirement seems to be hardly realized by some
choice of the constants $\alpha_i$ and $\beta_i$ in (33). This hints that $\bar{\rho} = 0$, which implies that Eqs. (37) have the only solution $\bar{\Delta}_m = 0$. It is sufficient to prove the consistency of the analytical approach. The zero solution fixes the constants $\alpha_i$, $\beta_i$ as

$$\bar{\Delta}_m = 0 \implies \alpha_1 = \frac{i}{16}, \quad \alpha_2 = -\frac{i}{64}, \quad \alpha_3 = -\frac{i}{32}, \quad \beta_i = 0.$$ (40)

This choice of the constants $\alpha_i$, $\beta_i$ results in the explicit form of the bispinor $\Delta_m$ (23), introduced in [15] and belonging to the space $\Delta^{(1)}$, in terms of the monomials $\nu^i_m$

$$\Delta_m = \frac{i}{16} [\nu^1_m - \frac{1}{4} \nu^2_m - \frac{1}{2} \nu^3_m].$$ (41)

The use of (40) transforms the general solution (36) into the reduced form

$$\chi = \frac{i}{16} \gamma_n [\psi(\bar{\psi}^n \psi) + \gamma^5 \psi(\bar{\psi}^n \gamma^5 \psi)]$$ (42)

which coincides with the solution found in [20] with use of the Weyl 2-spinor representation.

The substitution of (42) into the system (13) and repetition of the above-considered steps using the equivalency equations [16], restores the bispinors $\chi_2$ and $\chi_3$ in the expansion (5). All that proves the consistency of the analytical algorithm proposed in [15].

5 Conclusion

We analysed in detail the analytical algorithm [15] based on the factorization mechanism and asserting the equivalency between the KS and VA Lagrangians. The consistency of the algorithm was analyzed and proved using the Majorana spinorial monomials $\Delta^{(r)}_m$.

These monomials have the form $\sim \psi(\partial \bar{\psi} \psi)^r$, $(r = 1, 2, 3)$ and satisfy the scalar constraints $(\bar{\psi}^m \Delta^{(r)}_m) = 0 (mod \ total \ derivatives)$. They form the linear spaces $\Delta^{(r)}$ constructed of the NG fermion $\psi$ and its derivatives. The appearance of these monomials, triggered by the factorization mechanism, sheds new light on the relation between the KS and VA realizations of the NG fermionic field and algebraic structures associated with it. It would be interesting to find out a geometrical principle lying in the base of this algebraic structure. We constructed an explicit realization of the space $\Delta^{(1)}$ and used it together with the Fierz rearrangements to show how the considered analytic approach reproduces the recent results obtained with computer assistance and using the Weyl spinors [20].

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