Measurement of the CP-violation parameter of $B^0$ mixing and decay with $p\bar{p}\rightarrow \mu\mu X$ data

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We measure the dimuon charge asymmetry $A$ in $p\bar{p}$ collisions at a center of mass energy $\sqrt{s} = 1960$ GeV. The data was recorded with the D0 detector and corresponds to an integrated luminosity of approximately $1.0$ fb$^{-1}$. Assuming that the asymmetry $A$ is due to asymmetric $B^0 \leftrightarrow \bar{B}^0$ mixing and decay, we extract the CP-violation parameter of $B^0$ mixing and decay:

$$\Re(\epsilon_{B^0}) \frac{1}{1 + |\epsilon_{B^0}|^2} = \frac{A_{B^0}}{4} = -0.0023 \pm 0.0011 \text{ (stat)} \pm 0.0008 \text{ (syst)}.$$
\[ A_{B^0} \] is the dimuon charge asymmetry from decays of \( B^0 \bar{B}^0 \) pairs. The general case, with CP violation in both \( B^0 \) and \( B_s^0 \) systems, is also considered. Finally we obtain the forward-backward asymmetry that quantifies the tendency of \( \mu^+ \) to go in the proton direction and \( \mu^- \) to go in the anti-proton direction. The results are consistent with the standard model and constrain new physics.

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I. INTRODUCTION

We measure the dimuon charge asymmetry:

\[ A = \frac{N^{++} - N^{--}}{N^{++} + N^{--}} \]  (1)

in pp collisions at a center of mass energy \( \sqrt{s} = 1960 \) GeV. \( N^{++} \) (\( N^{--} \)) is the number of events with two positive (negative) muon candidates passing selection cuts. The data was recorded with the D0 detector at the Fermilab Tevatron between 2002 and 2005. The exposed integrated luminosity is approximately 1.0 fb\(^{-1}\). Assuming that the asymmetry \( A \) is due to asymmetric \( B^0 \leftrightarrow \bar{B}^0 \) mixing and decay, we extract the CP-violation parameter of \( B^0 \) mixing and decay: 

\[ \frac{\Re(\epsilon_{B^0})}{1 + |\epsilon_{B^0}|} = \Im \left\{ \frac{\Gamma_{12}}{4M_{12}} \right\} = \frac{A_{B^0}}{4} \equiv f \cdot A. \]  (2)

\( M_{12} \) \((\Gamma_{12})\) is the real (imaginary) part of the transition matrix element of the Hamiltonian corresponding to \( (B^0, \bar{B}^0) \) mixing and decay. Throughout this article we use the Particle Data Group notation: \( B^0 = db \), \( B_s^0 = sb \). \( A_{B^0} \) is the dimuon charge asymmetry from direct-direct decays of \( B^0 \bar{B}^0 \) (we define “direct decay” as \( b \rightarrow \mu^+ X \), and “sequential decay” as \( b \rightarrow c \rightarrow \mu^+ X \)).

The dimuon charge asymmetry \( A \) in Eq. (2) excludes events with a muon from \( K^\pm \) decay. Equation (2) defines the factor \( f \), to be obtained below, which accounts for other processes contributing to dimuon events. As a sensitive cross check, we also measure the mean mixing probability \( \chi_0 \) of \( B \leftrightarrow \bar{B} \) hadrons, averaged over the mix of hadrons with a b quark. Finally we obtain the forward-backward asymmetry that quantifies the tendency of \( \mu^+ \) to go in the proton direction and \( \mu^- \) to go in the anti-proton direction.

The general case, with CP violation in both \( B^0 \) and \( B_s^0 \) systems, is considered in the last section of this article. In this general case, the dimuon charge asymmetry \( A \) has contributions from both \( B^0 \) and \( B_s^0 \). Therefore, this measurement at the Fermilab Tevatron pp collider is complementary to similar measurements at B factories that are sensitive only to \( A_{B^0} \), not \( A_{B_s^0} \).

The CP-violation parameter, defined in Eq. (2), is sensitive to several extensions of the standard model because new particles may contribute to the box diagrams of \( M_{12} \) [4]. Reference [2] concludes that “It is possible that the dilepton asymmetry could be one of the first indications of physics beyond the standard model”.

The D0 detector has an excellent muon system in Run II [2], with large \( (\eta, \phi) \) coverage, good scintillator-based triggering and cosmic ray rejection, low punch-through rate, and precision tracking. The muon is the particle with cleanest identification. The like-sign dimuon channel is particularly clean: few processes contribute to it and fewer still contribute to an asymmetry. The D0 detector is well suited for this precision measurement.

The outline of the paper is as follows. The D0 detector is described in Section II. In Section III we consider the event selection. Physics and detector asymmetries are studied in Section IV. The processes contributing to the asymmetry \( A \) are presented in Section V, and their weights are summarized in Section VI. The breakdown of systematic uncertainties of \( A \) is discussed in Section VII. Cross-checks are listed in Section VIII. Final results are summarized in Section IX.

II. THE D0 DETECTOR

The D0 detector consists of a magnetic central-tracking system, comprised of a silicon microstrip tracker (SMT) and a central fiber tracker (CFT), both located within a 2 T superconducting solenoidal magnet [2]. The SMT has \( \approx 800,000 \) individual strips, with typical pitch of 50–80 \( \mu \)m, and a design optimized for tracking and vertexing capability at pseudorapidities of \(|\eta| < 2.5\). The system has a six-barrel longitudinal structure, each with a set of four layers arranged axially around the beam pipe, and interspersed with 16 radial disks. The CFT has eight thin coaxial barrels, each supporting two doublets of overlapping scintillating fibers of 0.835 mm diameter, one doublet being parallel to the collision axis, and the other alternating by \(+3^\circ\) relative to the axis. Light signals are transferred via clear fibers to solid-state photon counters (VLPC) that have \( \approx 80\% \) quantum efficiency.

Central and forward preshower detectors located just outside of the superconducting coil (in front of the calorimetry) are constructed of several layers of extruded triangular scintillator strips that are read out using wavelength-shifting fibers and VLPCs. The next layer of detection involves three liquid-argon/uranium calorimeters: a central section (CC) covering \(|\eta| \) up to \( \approx 1.1 \), and two endcap calorimeters (EC) that extend coverage to \(|\eta| \approx 4.2\), all housed in separate cryostats [7]. In addition to the preshower detectors, scintillators between the CC and EC cryostats that have \( \approx 80\% \) quantum efficiency.

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A muon system [3] is located beyond the calorimetry, and consists of a layer of tracking detectors and scintillation trigger counters before 1.8 T iron toroids, followed by two similar layers after the toroids. Tracking at \(|\eta| < 1 \)
relies on 10 cm wide drift tubes[^1], while 1 cm mini-drift tubes are used at $1 < |\eta| < 2$.

A muon originating in a $p\bar{p}$ collision traverses the silicon microstrip tracker and the scintillating fiber tracker in the 2 T solenoidal magnetic field, the calorimeter, layer $A$ of the muon spectrometer, the 1.8 T magnetized iron toroid, and layers $B$ and $C$ of the spectrometer.

Luminosity is measured using plastic scintillator arrays located in front of the EC cryostats, covering $2.7 < |\eta| < 4.4$.

Trigger and data acquisition systems are designed to accommodate the high luminosities of Run II. Based on preliminary information from tracking, calorimetry, and muon systems, the output of the first level of the trigger is used to limit the rate for accepted events to $\approx 2$ kHz. At the next trigger stage, with more refined information, the rate is reduced further to $\approx 1$ kHz. These first two levels of triggering rely mainly on hardware and firmware. The third and final level of the trigger, with access to all the event information, uses software algorithms and a computing farm, and reduces the output rate to $\approx 50$ Hz, which is written to tape.

The polarities of the toroid and solenoid magnetic fields are reversed roughly every two weeks so that the four solenoid-toroid polarity combinations are exposed to approximately the same integrated luminosity. This allows cancellation of first-order effects of the detector geometry.

### III. EVENT SELECTION

Our standard cuts require global muons, i.e., local muon candidates (reconstructed from hits in layers $A$, $B$ and $C$) with a matching central track (reconstructed from hits in the silicon and fiber trackers). To reduce punch-through of hadrons we only consider muons that traverse the iron toroid. To select muons that emerge from the toroid with momentum $p \gtrsim 0.2$ GeV/$c$, we require $p_T > 4.2$ GeV/$c$ or $|p_z| > 6.4$ GeV/$c$, where $p_T$ is the momentum transverse to the beam measured by the central tracking system, and $p_z$ is the component of the momentum in the direction of the proton beam. We require at least two wire chamber hits in the $A$ layer and at least three wire chamber hits in layers $B$ or $C$. We require local and global track fits with good $\chi^2$. To reduce cosmic ray background, we require at least one scintillator hit associated with the muon to be within a time window of $\pm 5$ ns with respect to the expected time. To reduce muons from $K^\pm$ and $\pi^\pm$ decay, we require $p_T > 3.0$ GeV/$c$. The track is required to have a distance of closest approach to the beam less than 0.3 cm. We use the full pseudorapidity range $|\eta| < 2.2$. We apply a cut of $p_T < 15.0$ GeV/$c$ to reduce the number of muons reconstructed with wrong sign, and to reduce the background from $W^\pm$ and $Z$ boson decay. This list completes the single muon cuts.

The dimuon cuts are as follows. We require that both muon candidates pass within 2.0 cm of each other in the direction along the beam line at the point of closest approach to the beam. To further reduce cosmic rays and repeated reconstructions of the same track (with different hits), we require the 3-dimensional opening angle between the muons to be between $10^\circ$ and $170^\circ$. We also require that the two muons have different $A$ layer positions (by at least 5 cm), different local momentum vectors (by at least 0.2 GeV/$c$), and different central track momentum vectors (by at least 0.2 GeV/$c$). This completes the set of standard cuts.

To avoid a bias due to mismatched central tracks (which are measured to be charge-asymmetric) we use the local muon charge, instead of the matching central track charge, to obtain the asymmetries. The positive charge asymmetry of central tracks is due to secondary particles that emerge from interactions with detector material. The measured charge asymmetry of central tracks with $p_T > 3.0$ GeV/$c$ is $(N^+ - N^-)/(N^+ + N^-) = 0.0049 \pm 0.0005$.

The measurement of the dimuon charge asymmetry $A$ is based on ratios of muon counts. To minimize the statistical error we use all recorded events regardless of trigger. If an event passes cuts, it is valid to accept it regardless of trigger, since an event with opposite muon charges can also be accepted by that trigger.

Histograms of $p_T$, $\eta$, and $\phi$ for standard cuts are shown in Figs. 1 and 2. Each plot superposes two histograms in Figs. 1 and 2. Therefore, to cancel detector introduce a charge asymmetry? It would have to have different acceptance $\times$ efficiency for positive and negative muons, i.e., for tracks bending north and south in the magnetized iron toroid, see Fig. 3. How can the detector introduce a charge asymmetry? It would have to have different acceptance $\times$ efficiency for positive and negative muons, i.e., for tracks bending north and south in the magnetized iron toroid, see Fig. 3. Such a difference may be due to an offset of the mean beam spot, to mechanical asymmetries, and to differences in wire chamber and scintillator efficiencies. We find that the detector, operating with a given toroid and solenoid polarity, introduces an apparent dimuon charge asymmetry of approximately 0.006 in absolute value (to be discussed later in this Section). This detector effect changes sign when the toroid and solenoid polarities are reversed (since the exact same track that is called “positive” with one polarity is called “negative” with the opposite polarity). This effect can be seen by comparing the $\eta$ histograms in Figs. 1 and 2. Therefore, to cancel detector geometry effects to first order, we always consider data sets that have equal event counts for each toroid-solenoid magnet polarity (or weight the events appropriately). We combine events with one solenoid polarity and...
toroidal polarity, with events with the opposite solenoid polarity and toroidal polarity. The analysis is done separately for solenoid polarity equal to the toroidal polarity, and solenoid polarity opposite to the toroidal polarity.

Let \( n_{\alpha\beta\gamma} \) be the number of muons passing cuts with charge \( \alpha = \pm 1 \), toroidal polarity \( \beta = \pm 1 \), and \( \gamma = +1 \) if \( \eta > 0 \) and \( \gamma = -1 \) if \( \eta < 0 \).

We model the physics and the detector as follows:

\[
\begin{align*}
n_{\alpha\beta\gamma} &= \frac{1}{4} N \epsilon^\beta (1 + \alpha A)(1 + \alpha \gamma A_{fb})(1 + \gamma A_{det}) \\
&\times (1 + \alpha \beta \gamma A_{ro})(1 + \beta \gamma A_{det})(1 + \alpha \beta A_{ro}),
\end{align*}
\]

where \( \epsilon^+ + \epsilon^- = 1 \). The eight equations define eight parameters in terms of the eight numbers \( n_{\alpha\beta\gamma} \). The parameters are \( N, \epsilon^+ \), and six asymmetries. \( N \epsilon^\beta \) is approximately equal to the number of muons passing cuts with toroidal polarity \( \beta \). \( A \) is the dimuon charge asymmetry, \( A_{fb} \) is the forward-backward asymmetry (that quantifies the tendency of \( \mu^+ \) to go in the proton direction and \( \mu^- \) to go in the anti-proton direction), \( A_{det} \) measures the north-south asymmetry of the detector (“north” has \( \eta < 0 \)), and \( A_{ro} \) is the range-out asymmetry (that quantifies the change in acceptance and range-out of muon tracks that bend toward, or away from, the beam line, see Fig. 3). \( A_{\alpha\beta} \) is a detector asymmetry between tracks bending north and tracks bending south. \( A_{fb} \) is a physics asymmetry that we want to measure, and \( A_{det}, A_{ro} \) and \( A_{\alpha\beta} \) are detector asymmetries. \( A_{\beta\gamma} \) is a second-order asymmetry that is different from zero only if \( A_{ro} \) and \( A_{\alpha\beta} \) are different from zero. If the selection of events includes single muon and dimuon cuts, we use capital \( A \) for the asymmetries. If only single muon cuts are required, we use lower case \( a \). In Tables I and II we show the numbers \( n_{\alpha\beta\gamma} \) for our standard cuts. The measured asymmetries are presented in Table III.

We can understand the detector asymmetry \( A_{\alpha\beta} \) in more detail. Let \( A^p \) be the dimuon charge asymmetry of events with toroidal polarity \( \beta \). From Eq. 1 we obtain, to first order in the asymmetries, \( A^p = A + \beta A_{\alpha\beta} \). Therefore \( \beta A_{\alpha\beta} \) is the dimuon charge asymmetry due to detector

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**FIG. 1:** Distributions of single muon \( p_T \) (a) and (d), \( \eta \) (b) and (e), and \( \phi \) (c) and (f), for events with opposite toroid and solenoid polarities passing standard single muon and dimuon cuts. The charge times toroid polarity, \( \alpha \beta \), is indicated in the labels. Each plot superposes two histograms with opposite charge and opposite polarities. Positive (negative) charge corresponds to solid (dotted) lines (that can generally not be distinguished). Histograms with negative toroid polarity are scaled by the ratio of muon counts \( e = 1.0300 \), see Section IV.
FIG. 2: Same as Fig. 1 but for equal toroid and solenoid polarities. Here $e = 0.9292$.

FIG. 3: Schematic drawing of the magnetized iron toroids of the D0 detector, and muon tracks related by toroid polarity reversal, CP conjugation and forward-backward reflection.

g eo m e t r y e f f e c t s. It c h a n g e s s i g n w h e n t h e m a g n e t p o l a r i t i e s a r e r e v e r s e d. T h i s i s t h e d e t e c t o r g e o m e t r y e f f e c t t h a t i s c a n c e l l e d b y t a k i n g t h e w e i g h t e d a v e r a g e o f $A^+$ and $A^-$. T h e m a g n i t u d e o f t h i s e f f e c t i s $A_{\alpha\beta} \approx -0.006$, s e e T a b l e III.

Solving equations (3) up to second-order terms in the asymmetries, $A$ is obtained by taking the weighted average $\frac{1}{2}(A^+ + eA^-)$:

$$
\frac{(n_{++} + n_{+-} - n_{+} - n_{-} + n_{-} + n_{+}) + e(n_{++} + n_{+-} - n_{-} - n_{+} + n_{-} + n_{+})}{(n_{++} + n_{+-} + n_{-} + n_{+} + n_{-} + n_{+})} = A + A_{fb}A_{det},$$

where $e \equiv \epsilon^+ / \epsilon^-$. T o t h e r e q u i r e d a c c u r a c y, $e$ i s t h e r a t i o o f t h e n u m b e r o f e v e n t s p a s s i n g c u t s w i t h t o r o i d p o l a r i t y $\beta = +1$ over t h e c o r r e s p o n d i n g n u m b e r w i t h $\beta = -1$. T h e r a t i o $e$ i s d e t e r m i n e d b y c o u n t i n g s i n g l e m u o n s f o r t h e s i n g l e m u o n a s y m m e t r i e s, o r b y c o u n t i n g d i m u o n s f o r t h e d i m u o n a n a l y s i s. T h i s p r o c e d u r e i n t r o d u c e s n o b i a s s i n c e w e c o u n t a l l m u o n s o r d i m u o n s r e g a r d l e s s o f t h e c h a r g e s o f e a c h m u o n. T h e l e f t-h a n d-s i d e i s t h e m e a s u r e d a s y m m e t r y, $A$ i s t h e c o r r e c t e d a s y m m e t r y, a n d $-A_{fb}A_{det}$ i s t h e c o r r e c t i o n d e d u e t o t h e f o r w a r d-b a c k w a r d a n d d e t e c t o r a s y m m e t r i e s. W e d o n o t a p p l y t h i s c o r r e c t i o n b e c a u s e i t t u r n s o u t t o b e n e g l i g i b l e a n d c o m p a t i b l e w i t h z e r o, s e e T a b l e III. W e u s e t h e c o r r e c t i o n t o e s t i m a t e t h e c o r r e s p o n d i n g s y s t e m a t i c u n c e r t a i n t y. W e c a n u n d e r s t a n d t h e l a s t t e r m i n Eq. (4): i f p o s i t i v e (n e g a t i v e) m u o n s p r e f e r t o g o i n t h e p r o t o n (a n t i p r o t o n) d i r e c t i o n a n d t h e d e t e c t o r i s n o r t h-s o u t h a s y m m e t r i c, t h e n w e o b t a i n a n a p p e a r a n t c h a r g e a s y m m e t r y.
TABLE I: Numbers \( n_\alpha^\beta \) of muons passing standard single and dimuon cuts with charge \( \alpha \), toroid magnet polarity \( \beta \), and \( \eta < 0 \) (\( \gamma = -1 \)) or \( > 0 \) (\( \gamma = +1 \)). There are two entries per event. The solenoid and toroid polarities are opposite. In total there are 154667 positive-positive, 154482 negative-negative, and 1075192 positive-negative dimuon events. No cuts on the dimuon mass are imposed at this stage.

| charge | toroid polarity | \( -2.2 < \eta < 0 \) | \( 0.0 < \eta < 2.2 \) |  
|--------|-----------------|-------------------------|-------------------------|  
| \( \alpha \) | \( \beta \) | \( \gamma = -1 \) | \( \gamma = +1 \) |  
| +1     | +1             | 367376                  | 335700                  |  
| -1     | +1             | 348295                  | 353453                  |  
| +1     | -1             | 337697                  | 343753                  |  
| -1     | -1             | 356891                  | 325517                  |  

TABLE II: Same as Table I but for equal solenoid and toroid polarities. In total there are 136422 positive-positive, 262247 negative-negative, and 944013 positive-negative dimuon events.

| charge | toroid polarity | \( -2.2 < \eta < 0 \) | \( 0.0 < \eta < 2.2 \) |  
|--------|-----------------|-------------------------|-------------------------|  
| \( \alpha \) | \( \beta \) | \( \gamma = -1 \) | \( \gamma = +1 \) |  
| +1     | +1             | 395694                  | 279508                  |  
| -1     | +1             | 290227                  | 296264                  |  
| +1     | -1             | 311153                  | 319602                  |  
| -1     | -1             | 329285                  | 301908                  |  

The forward-backward asymmetry \( A_{fb} \) (up to second-order terms) is:

\[
\frac{(n_{++}^+ + n_{--}^- - n_{+-}^+ - n_{-+}^-) + c(n_{+-}^+ + n_{-+}^- - n_{++}^- - n_{--}^+)}{(n_{++}^+ + n_{--}^- + n_{+-}^+ + n_{-+}^-) + c(n_{+-}^+ + n_{-+}^- + n_{++}^- + n_{--}^+)} = A_{fb} + AA_{det}(5)
\]

We have repeated the study of detector asymmetries for the central (\( |\eta| < 0.95 \)) and forward (\( 0.95 < |\eta| < 2.2 \)) muon systems separately.

Also shown in Table III is the dimuon charge asymmetry for flavor creation \( A_{fc} \) (defined as \( \Delta \psi \geq 90^\circ \)) and flavor excitation \( A_{fe} \) (defined as \( \Delta \psi < 90^\circ \)), where \( \Delta \psi \) is the 3-dimensional angle between the two muons. Flavor creation corresponds to the \( b \) and \( \bar{b} \) quarks in opposite jets, while flavor excitation corresponds generally to a \( bb \) pair produced in the hadronization of one parton, including gluon splitting. The accepted cross sections for flavor creation and flavor excitation are nearly equal (as indicated by their statistical errors in Table III). In Table III we also show the ratio \( R = (N^{++} + N^{--})/N^{++} \) of like-sign to opposite-sign dimuon events in either the mass window 5.0 to 8.7 GeV/c\(^2\) or 11.5 to 30 GeV/c\(^2\) for events passing a dimuon trigger. This ratio is used for mixing studies. These mass windows reduce backgrounds from same-side direct-sequential muon pairs (process \( P_3 \) in Table IV), and decays of \( J/\psi \)'s, \( \Upsilon \)'s and their resonances (part of process \( P_k \)), defined later.

TABLE III: Asymmetries described in Section IV are shown for central (c) muons, \( |\eta| < 0.95 \), and forward (f) muons, \( 0.95 < |\eta| < 2.2 \). All errors are statistical. The average for opposite and equal toroid and solenoid polarities (torpol*solpol = \(-1\) and \( 1 \)) is \( A \approx -0.0005 \pm 0.0013 \) (all).

| torpol*solpol | \( A \) |  
|---------------|--------|  
| \( e \)       | 1.0300 | 0.9929 |  
| \( a \) (all) | 0.0001 ± 0.0005 | -0.0012 ± 0.0005 |  
| \( a_{fb} \) (all) | 0.0201 ± 0.0005 | 0.0011 ± 0.0005 |  
| \( a_{det} \) (all) | -0.0074 ± 0.0005 | -0.0045 ± 0.0005 |  
| \( a_{ro} \) (all) | -0.0268 ± 0.0005 | -0.0296 ± 0.0005 |  
| \( a \) (c) | -0.0014 ± 0.0007 | -0.0035 ± 0.0008 |  
| \( a_{fb} \) (c) | 0.0004 ± 0.0007 | -0.0010 ± 0.0008 |  
| \( a_{det} \) (c) | -0.0069 ± 0.0007 | -0.0068 ± 0.0008 |  
| \( a_{ro} \) (c) | -0.0867 ± 0.0007 | -0.0891 ± 0.0008 |  
| \( a \) (f) | 0.0010 ± 0.0006 | 0.0003 ± 0.0006 |  
| \( a_{fb} \) (f) | 0.0033 ± 0.0006 | 0.0025 ± 0.0006 |  
| \( a_{det} \) (f) | -0.0078 ± 0.0006 | -0.0029 ± 0.0006 |  
| \( a_{ro} \) (f) | 0.0122 ± 0.0006 | 0.0089 ± 0.0006 |  
| \( A_{fb} \) (all) | 0.0006 ± 0.0006 | 0.0001 ± 0.0007 |  
| \( A_{det} \) (all) | -0.0187 ± 0.0006 | -0.0165 ± 0.0007 |  
| \( A_{ro} \) (all) | -0.0268 ± 0.0006 | -0.0283 ± 0.0007 |  
| \( A_{ro} \) (all) | -0.0059 ± 0.0006 | -0.0069 ± 0.0007 |  
| \( A_{gb} \) (all) | -0.0002 ± 0.0006 | -0.0015 ± 0.0007 |  
| \( R \) | 0.4603 ± 0.0013 | 0.4610 ± 0.0013 |  
| \( A_{c} \) (all) | -0.0008 ± 0.0026 | -0.0023 ± 0.0027 |  
| \( A_{f} \) (all) | 0.0019 ± 0.0026 | 0.0008 ± 0.0028 |  
| \( A \) (cc) | 0.0015 ± 0.0044 | -0.0021 ± 0.0048 |  
| \( A \) (ff) | 0.0024 ± 0.0031 | 0.0016 ± 0.0033 |  
| \( A \) (all) | 0.0005 ± 0.0018 | -0.0016 ± 0.0019 |  

V. DIMUON PROCESSES

In this Section we obtain the factor \( f \) defined in Eq. 23. We consider the processes \( P_1 - P_{13} \) listed in Table I.

\( P_1 \) is direct-direct \( \bar{b}b \) decay. \( P_2 \) is opposite-side direct-sequential decay. \( P_3 \) is sequential-sequential decay. \( P_4 \) is same-side direct-sequential decay. \( P_5 \) corresponds to cosmic ray muons that traverse the D0 detector and are reconstructed twice: once upon entry and once upon exit. \( P_6 \) corresponds to muons from \( K^\pm \) decay, in coincidence with a prompt muon from the collision. This process is discussed in detail in Sections VI and VII. \( P_8 \) corresponds to a cosmic ray muon, in coincidence with a prompt muon. \( P_{10} \) corresponds to a hadron that traverses the calorimeter and iron toroid and is reconstructed as a muon, in coincidence with a prompt muon. \( P_{18} \) corresponds to a combinatoric background faking a muon, in coincidence with a prompt muon. Examples of “other” processes \( P_{12} \) are dimuons with the following parents: \( B^\pm \) and \( \pi^\pm \), \( B^0 \) and \( \tau^\pm \), \( B^0 \) and \( J/\psi \), \( B^\pm \) and \( \tau^\pm \), \( b \) and unrelated \( c \). \( P_{13} \) are events that have one of the muons reconstructed with the wrong sign. Contributions from both muons coming from hadron misidentification or combinatoric background are negligible.

Let \( \chi_d \) be the probability that a \( B^0(b\bar{d}) \) meson that decays to a flavor specific final state, mixes and decays
as a $B^0(\bar{d}b)$. Similarly, $\bar{\chi}_d$ is the probability that a $B^0$ meson mixes and decays as a $B^0$. Here we consider the possibility that $\chi_d \neq \bar{\chi}_d$ (the general case with $\chi_d \neq \bar{\chi}_s$ and $\chi_s \neq \bar{\chi}_s$ is considered in Section IX). The probability that a $b$ quark in the sample decays as a $b$ is

$$\chi = f_d \frac{\beta_d}{(\beta)} \chi_d + f_s \frac{\beta_s}{(\beta)} \chi_s,$$

where $f_d$ and $f_s$ are the fractions of $b$ quarks that hadronize to $B^0$ or $B^0$ and $B^0$ respectively, and $\beta_d$, $\beta_s$ and $(\beta)$ are the branching fractions for $B^0$, $B^0$ and the $b$-hadron admixture respectively decaying to $\mu X$ with $\mu$ passing cuts. Similarly,

$$\bar{\chi} = f_d \frac{\beta_d}{(\beta)} \bar{\chi}_d + f_s \frac{\beta_s}{(\beta)} \bar{\chi}_s$$

is the probability that a $\bar{b}$ in the sample decays as a $b$. From Ref. [1] we take $f_d = 0.397 \pm 0.010$, $f_s = 0.107 \pm 0.011$, $\chi_{d0} \equiv \frac{1}{2}(\chi_d + \bar{\chi}_d) = 0.186 \pm 0.004$, and $\chi_{s0} \equiv \frac{1}{2}(\chi_s + \bar{\chi}_s) > 0.49883$. We take $\beta_d = \beta_s = (\beta)$. To abbreviate, we define $\chi_0 \equiv \frac{1}{2}(\chi + \bar{\chi})$, and $\xi \equiv 2\chi_0 (1 - \chi_0)$. We assume CPT symmetry. The probability that a $B (\bar{B})$ hadron that decays to a flavor specific final state, decays as a $B (\bar{B})$ is then $1 - \chi_0$. From Table [IV] we obtain the dimuon charge asymmetry $A$ after correcting for asymmetric kaon decay, (i.e., after subtracting a term $0.5aP_8$ in the numerator),

$$A = \frac{(\chi - \bar{\chi})[(1 - \chi_0)(P_1 - P_3) + 0.25\rho' P'_{8}]}{A_{den}},$$

and the factor $f$ in Eq. (2):

$$f = 8f_d\chi_{d0}[(1 - \chi_0)(P_1 - P_3) + 0.25\rho' P'_{8}].$$

where $A_{den} = \xi(P_1 + P_3) + (1 - \xi)P_2 + 0.28P_0 + 0.5P_8' + 0.78P_{13}$ and $P_6' = P_6 + P_0 + P_0 + P_1 + P_2$. The fraction of prompt muons from $b$ decay is $\rho' = 0.6 \pm 0.15$.

VI. WEIGHTS $P_i$ OF DIMUON PROCESSES

The weights $P_2$ through $P_{13}$, normalized to direct-decay $b\bar{b}$ decay $P_1 \equiv 1$, are summarized in Table [V]. The weights $P_2$, $P_4$, $P_5$, $P_8$ and $P_{12}$ were obtained from Monte Carlo simulations (using the PYTHIA generator [9]) with full detector simulation (based on the GEANT program [10]) and event reconstruction and selection. A cross-check for weight $P_2$ is the measurement of the average mixing probability of $B$ hadrons to be described below. Weights $P_4$, $P_5$ and $P_6$ do not contribute like-sign dimuons, and so do not enter into the measurement of the CP-violation parameter. Weight $P_3$ was obtained from $P_3 \approx P_2^2/(4P_1)$. Weight $P_7$ was obtained by two methods: (i) from the data of a cosmic ray run, and (ii) extrapolating the out-of-time muon background (as measured by the scintillators) into the acceptance window of $\pm 5$ ns. Weight $P_9$ was obtained using the data of the cosmic ray run. Weight $P_{10}$ was obtained by counting the number of tracks that had enough momentum to traverse the calorimeter and iron toroid, and multiplying by the probability $\exp(-14)$ that they do not interact (the calorimeter has $\approx 7$ nuclear interaction lengths, and the iron toroid has $\approx 7$ nuclear interaction lengths). Weight $P_{11}$ was estimated by relaxing the number of required wire chamber hits (one less hit in layer A and/or 1 less hit in layers B or C). Weight $P_{13}$ was estimated using the measured resolution of the local muon spectrometer. In our data set, passing standard single and dimuon cuts, we expect $\approx 1$ dimuon event from $Z$ boson decay, and less than one event from prompt muons plus $W \pm$ boson decay.

Let us consider the weight $P_8$ in some detail. This weight corresponds to prompt muons from $b$ or $c$ or $s$ decay plus $K^\pm$ decay. This is an important background because kaon interactions are charge asymmetric, and dominate the systematic uncertainty of the measurement of the CP-violation parameter. The inelastic interaction length of $K^+$ in the calorimeter is greater than the inelastic interaction length of $K^-$. This difference is due to the existence of hyperons $Y$ (strangeness $-1$ baryons: $\Lambda$, $\Sigma$, $Y^*$). Reactions $K^-N \rightarrow Y \pi$ have no $K^+N$ analog. Therefore $K^+$ has more time to decay than $K^-$. The result is a charge asymmetry from $K^\pm$ decay. The single muon charge asymmetry from $K^\pm$ decay is obtained from the inelastic cross sections for $K^-d$ and $K^+d$ and the geometry and materials of the D0 detector: $a \equiv (n^+ - n^-)/(n^+ + n^-) = 0.026 \pm 0.005$.

For $P_8$, we make two complementary estimates based on data. In the first, we measure the exclusive decay $B^0 \rightarrow D^{*}(2010)^-\mu^+\nu_\mu$, $D^{*}(2010)^- \rightarrow D^0\pi^-$, $D^0 \rightarrow K^+\pi^-$, and its charge conjugate. We apply standard single and dimuon cuts, and count events with and without a muon matching the kaon track. We subtract the background by two methods: using a side-band of $m_{D^0} - m_{D^0}$, or using the wrong relative sign of the muon from $B^0$ decay and the pion from $D^+ \rightarrow D^+ \mu^+ \nu_\mu \rightarrow \mu^+ \nu_\mu \rightarrow \mu^+(\rightarrow \nu_\mu\overline{\nu_\mu}) + X$. The result is $P_8 = 0.078 \pm 0.013$ (stat) $\pm 0.019$ (syst). From studies with this exclusive decay, we learn that the global track $\chi^2$-cut is not very effective in reducing $K^\pm$ decay kinks (for the high momentum muons passing cuts). Therefore we must correct $A$ for $K^\pm$ decay as discussed in Section VII.

We use the following alternative procedure to estimate the background weight $P_8$ from data. Instead of $K^+ \rightarrow \mu^+\nu_\mu$, we study $K^0_S \rightarrow \pi^+\pi^-$. For this estimate, we assume that the production and decay kinematics of $K^+$ and $K^0_S$ are approximately the same. The branching fractions are similar. To account for the smaller decay length of $K^0_S$ compared to $K^\pm$, we scale the volume in which the $K^\pm$ can decay (719 mm in radius and 1360 mm in half-length) by the fraction of lifetimes. 719 mm is the sum of the inner radius of the calorimeter, plus the transverse interaction length of $K^+$ in the calorimeter. We analyze single muon (instead of dimuon) data to account
TABLE IV: Processes contributing to dimuon events. Each row includes processes related by CP conjugation and $B \leftrightarrow \bar{B}$ mixing. The weights are normalized to direct-direct $b\bar{b}$ decay $P_b \equiv 1$. $\xi \equiv 2\chi_0(1 - \chi_0)$, $\chi = f_d x_d + f_s x_s$ is the probability that $b$ quarks decay as $b$, $\bar{\chi} = f_{\bar{d}} x_{\bar{d}} + f_{\bar{s}} x_{\bar{s}}$ is the probability that $b$ anti-quarks decay as $b$. The fraction of prompt muons from $b$ decay is $\rho' \equiv 0.6 \pm 0.15 \frac{K}{\rho}$. $\rho \equiv \frac{1}{2} \rho' (\chi - \bar{\chi})$. $a = 0.026 \pm 0.005$ is the charge asymmetry of $K^\pm$ decay, see the text. CPT symmetry is assumed. For example, the number of direct-direct decays $b\bar{b} \rightarrow \mu^+ \mu^- X$ is $\propto P_b(1 - \chi_0)$. 

| process | weight | $N^{+\pm}$ | $N^{-\pm}$ |
|---------|--------|------------|------------|
| $b \rightarrow \mu^- \bar{b} \rightarrow \mu^+$ | $P_b \equiv 1$ | $\chi(1 - \chi_0)$ | $(1 - \chi_0) \bar{\chi}$ |
| $b \rightarrow \mu^- \bar{b} \rightarrow \bar{\mu}^-$ | $P_b$ | $\frac{1}{2} (1 - \xi)$ | $\frac{1}{2}(1 - \xi)$ |
| $b \rightarrow c \rightarrow \mu^+ \bar{b} \rightarrow \bar{c} \rightarrow \mu^-$ | $P_b$ | $\bar{\chi}(1 - \chi_0)$ | $\chi(1 - \chi_0)$ |
| $b \rightarrow \mu^- \bar{c} \rightarrow \bar{\mu}^-$ | $P_c$ | 0 | 0 |
| Drell-Yan, $J/\psi$, $\Upsilon$ | $P_q$ | 0 | 0 |
| dimuon cosmic rays | $P_q$ | $\approx 0.14$ | $\approx 0.14$ |
| $\mu + K^\pm$ decay | $P_{10}$ | $0.25 \cdot (1 + a + \rho)$ | $0.25 \cdot (1 - a - \rho)$ |
| $\mu +$ cosmic | $P_{11}$ | $0.25 \cdot (1 + \rho)$ | $0.25 \cdot (1 - \rho)$ |
| $\mu +$ punch-through | $P_{12}$ | $0.25 \cdot (1 + \rho)$ | $0.25 \cdot (1 - \rho)$ |
| $\mu +$ combinatoric | $P_{13}$ | $0.25 \cdot (1 + \rho)$ | $0.25 \cdot (1 - \rho)$ |
| Dimuon w. wrong sign | $P_{14}$ | 0.39 | 0.39 |

for correlations. We compare two histograms: one is $p_T$ of pions from $K_0^0$’s decaying in the scaled-down volume, and the other histogram is $p_T$ of the second (in order of decreasing $p_T$) muon passing cuts. By this indirect method we obtain $P_S \approx 0.041 \pm 0.010$ (stat) $\pm 0.041$ (syst).

Within errors, the two measurements of $P_S$ agree and we use the result from the first method.

The systematic uncertainty of $P_S$, $\pm 0.019$, was taken as half the difference of 0.078 and 0.041. This value is reasonable in view of the variation of the measured $P_S$ with different cuts and data sub-sets.

From Monte Carlo simulations we obtain $P_S = 0.047 \pm 0.034$ (stat), consistent with the above.

VII. SYSTEMATIC UNCERTAINTIES OF $A$

We add in quadrature the following systematic uncertainties. A summary is presented in Table VI.

Detector effects. Before averaging over magnetic field polarities, the detector introduces a dimuon charge asymmetry of $\approx 0.006$ in absolute value, as discussed in Section IV. After averaging over magnetic field polarities (with appropriate weights), the uncertainty of the dimuon charge asymmetry due to detector geometry effects is $|A_{fb} \cdot A_{det}|$ (see Eq. [1]). As a measure of this uncertainty, we have used the largest deviation from zero, $|A_{fb} \cdot A_{det}| = 0.0049 - 0.030 = 0.00015$, obtained after any of the 54 sets of dimuon cuts, for any of the three detector regions (central, forward, or all).

Inaccuracy of $e \equiv e^+ / e^-$. We have obtained the ratio $e$ by counting dimuon events with toroid polarity $\beta = 1$ and dividing by the corresponding number for $\beta = -1$. This procedure introduces no bias since we count all dimuons regardless of the charges of each muon. We take $\Delta e = 0.03$ from the largest difference between any cuts. Multiplying by the detector dimuon charge asymmetry before averaging over magnet polarities, $\approx 0.006$ in absolute value, we obtain $\Delta A \approx 0.00018$.

Prompt $\mu + K^\pm$ decay. The single muon charge asymmetry of kaon decay is $a = 0.026 \pm 0.005$ as explained in Section VI. We take $P_S = 0.078 \pm 0.023$ from Table V. The corresponding correction to $A$, explained in Section V, is $\delta A = -0.5 a P_S / A_{det} = -0.5 \times (0.026 \pm 0.005) \times (0.078 \pm 0.023) / A_{det} = -0.0023 \pm 0.0008$ ($A_{det} \approx 0.436$ is the denominator of Eq. [3]). This uncertainty on $\delta A$ is by far the dominating systematic uncertainty of the entire measurement.

Dimuon cosmic rays. These are cosmic rays detected twice, once as they enter and once as they exit the D0 detector. We take $P_T < 0.007$ from Table V. From cuts that
The measured charge asymmetry of tracks with $p_{3}$ cross-checked with forward muons relative to the sign of central muons was measured to be equal to within 0.1%. From cuts that select cosmic rays, we obtain the apparent dimuon charge asymmetry $A = -0.0095 \pm 0.0117$. Then the corresponding uncertainty of $A$ is $< 0.007 \times 0.28 \times 0.021/A_{\text{den}} = 0.0001$ (see Table VI, $0.021 = | - 0.0095 | + 0.0117$).

**Prompt $\mu +$ single cosmic ray.** We take $P_{9} < 0.0002$, see Table VI. From cuts that select cosmic rays, we obtain an apparent single muon charge asymmetry $a = 0.026 \pm 0.002$. Then the uncertainty in $A$ is $< 0.0002 \times 0.5 \times 0.028/A_{\text{den}} = 6 \times 10^{-6}$ (see Table VI, $0.028 = 0.026 + 0.002$).

**Wrong local muon sign.** $P_{13} < 0.001$, see Table VI. Even if the asymmetry of wrong tracks is 0.1 (overestimate), the corresponding uncertainty of $A$ is small: $< 0.001 \times 0.78 \times 0.1/A_{\text{den}} = 0.00018$ (see Table VI).

**Punch-through.** We take $P_{10} < 0.002$, see Table VI. The measured charge asymmetry of tracks with $p_{T} > 3.0 \text{ GeV/c}$ is $0.0049 \pm 0.0005$ due to showers on matter instead of antimatter. Then the error in $A$ is $< 0.002 \times 0.5 \times 0.0054/A_{\text{den}} = 1 \times 10^{-5}$ (see Table VI, $0.0054 = 0.0049 + 0.0005$).

**VIII. OTHER CROSS-CHECKS**

The sign of central muons was cross-checked using cosmic rays (which are charge asymmetric). The sign of forward muons relative to the sign of central muons was cross-checked with $J/\psi$'s.

The direct and reverse magnetic fields in the iron toroid were measured to be equal to within 0.1%.

We find the dimuon charge asymmetry $A$ stable (within statistical errors) for all recorded events or events passing a set of dimuon triggers, and across the different cuts (54 sets were studied), data subsets, opposite or equal toroid and solenoid polarities, central or forward muons, or flavor creation or flavor excitation events.

**IX. RESULTS**

We obtain $A = 0.0005 \pm 0.0018$ (stat) for opposite solenoid and toroid polarities, and $A = -0.0016 \pm 0.0019$ (stat) for equal solenoid and toroid polarities (see last line in Table III). Combining these measurements we obtain

$$A = -0.0005 \pm 0.0013 \text{ (stat)}. \quad (10)$$

We add a correction $\delta A = -0.0023 \pm 0.0008$ due to asymmetric $K^{\pm}$ decay (this effect is explained in Sections VI and VII). The uncertainty of this correction dominates the systematic uncertainties of the CP-violation parameter. The final corrected value of the dimuon charge asymmetry is

$$A = -0.0028 \pm 0.0013 \text{ (stat) } \pm 0.0009 \text{ (syst)}. \quad (11)$$

The breakdown of systematic uncertainties of $A$ is presented in Table VII

From the dimuon charge asymmetry $A$ we obtain

$$\frac{\Re(\epsilon_{B^0})}{1 + |\epsilon_{B^0}|^2} = \frac{A_{B^0}}{4} \equiv f \cdot A$$

$$= -0.0023 \pm 0.0011 \text{ (stat) } \pm 0.0008 \text{ (syst)},$$

where

$$f = 0.814 \pm 0.105 \text{ (syst)}. \quad (13)$$

The breakdown of systematic uncertainties of $f$, calculated from information provided in preceding sections, is listed in Table VII. In comparison, the Particle Data Group average of 2004 is $\Re(\epsilon_{B^0})/(1 + |\epsilon_{B^0}|^2) = 0.0005 \pm 0.0031$.

All preceding equations correspond to the case $\chi_{s} = \bar{\chi}_{s}$. From (8) we obtain, for the general case $\chi_{d} \neq \bar{\chi}_{d}$ and $\chi_{s} \neq \bar{\chi}_{s}$,

$$A = \frac{1}{4f} \left[ A_{B^0} + \frac{f_{s} \chi_{s0}}{f_{d} \chi_{d0}} A_{B^0} \right]. \quad (14)$$

We measure the ratio $R$ of like-sign to opposite-sign dimuons. For this measurement we consider the invariant mass of the two muons to be either in the window $5.0 \pm 8.7 \text{ GeV/c}^2$ or $11.5 \pm 30 \text{ GeV/c}^2$. These mass cuts are designed to reduce backgrounds from same-side direct-sequential decay and backgrounds from $\psi$ and $Y$ meson decays to allow a measurement of $B \leftrightarrow \bar{B}$ mixing. For this reason we set $P_{9} = P_{10} = 0$ for the mixing analysis. We also require dimuon triggers from a
TABLE VIII: Systematic uncertainties of $\xi = 2\chi_0(1 - \chi_0)$. $P'_4 \equiv P_4 + P_5 + P_6$.

| Source of error | $\Delta\xi$ |
|-----------------|-------------|
| $P_2$           | 0.0074      |
| $P'_2$          | 0.028       |
| $P'_3$          | 0.015       |
| $P'_5$          | 0.0001      |
| $P'_7$          | 0.015       |
| $P'_8$          | 0.0002      |
| **Total**       | **0.036**   |

list that excludes triggers requiring opposite sign muons. We obtain $R = 0.461 \pm 0.001$ (stat) $\pm 0.010$ (syst), and $\xi = 0.229 \pm 0.001$ (stat) $\pm 0.036$ (syst). The breakdown of systematic uncertainties, calculated from information provided in preceding sections, is shown in Table VIII. The final result for the mixing probability, averaged over the mix of hadrons with a $b$ quark, is

$$\chi_0 = 0.132 \pm 0.001 \text{ (stat) } \pm 0.024 \text{ (syst)}.$$  \hspace{1cm} (15)

The agreement with the world average, $0.127 \pm 0.006$ [1], is a sensitive test for $P_2$ and $f$, since the largest systematic uncertainty of $\xi$ and $f$ are due to the same weight $P_2$.

Finally, we measure the tendency of $\mu^+$ ($\mu^-$) to go in the proton (antiproton) direction. We obtain the forwardbackward asymmetry for events passing standard single and dimuon cuts:

$$A_{fb} = 0.0004 \pm 0.0005 \text{ (stat) } \pm 0.0002 \text{ (syst)}.$$ \hspace{1cm} (16)

$A_{fb}$ is defined in Section IV. The systematic uncertainty is $[A||A_{det}] < 0.0044 \times 0.30$. As indicated earlier, the fraction of muons from $W$ decay in this sample is negligible.

In conclusion, the results (11) through (16), are consistent with the standard model [1], and constrain some of its extensions [3, 4]. The general result (14) complements measurements at $B$ factories, which are sensitive only to $A_{B^0}$, not $A_{B_s}$.

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