**BIG STEP GREEDY ALGORITHM FOR MAXIMUM K-COVERAGE PROBLEM**

DRONA PRATAP CHANDU  
Indian Institute of Technology Roorkee, India  
pratap.rrke@gmail.com

**Abstract:** This paper proposes a greedy heuristic named as Big step greedy heuristic and investigates the application of Big step greedy heuristic for maximum \(k\)-coverage problem. Greedy algorithms construct the solution in multiple steps, the classical greedy algorithm for maximum \(k\)-coverage problem, in each step selects one set that contains the greatest number of uncovered elements. The Big step greedy heuristic, in each step selects \(p\) (1 \( \leq p \leq k\)) sets such that the union of selected \(p\) sets contains the greatest number of uncovered elements by evaluating all possible \(p\)-combinations of given sets. When \(p=k\) Big step greedy algorithm behaves like exact algorithm that computes optimal solution by evaluating all possible \(k\)-combinations of given sets. When \(p=1\) it behaves like the classical greedy algorithm.

**Keywords:** Big step, Greedy, Heuristic, Maximum k-cover, Algorithm, Approximation

1. **INTRODUCTION**

Maximum \(k\)-coverage problem is to select \(k\) sets \(\{Sx1,Sx2,Sx3,\ldots, Sxk\}\) from given collection of sets \(S= \{S1,S2,\ldots, Sn\}\) such that the number of elements in the union of selected \(k\) sets \(|Sx1\cup Sx2\cup \ldots \cup Sxk|\) is maximum. Maximum \(k\)-coverage problem is a NP-hard problem[7].

Greedy algorithms construct the solution in multiple steps by making a locally optimal decision in each step. According to David[2], the advantage of greedy algorithms is that they are typically very easy to implement and hence greedy algorithms are a commonly used heuristic, even when they have no performance guarantee. The classical greedy algorithm for maximum \(k\)-coverage problem, in each step selects one set that contains the greatest number of uncovered elements[3]. The proposed algorithm named
as Big step greedy algorithm, in each step selects \( p \) \((1 <= p <= k)\) sets such that the union of selected \( p \) sets contains the greatest number of uncovered elements by evaluating all possible \( p \)-combinations of given sets.

2. THE CLASSICAL GREEDY ALGORITHM

The classical greedy algorithm for maximum \( k \)-coverage problem is shown in Figure 1. The classical greedy algorithm starts with empty set cover, and in each step it selects one set that contains the greatest number of remaining elements that are uncovered by current partial solution and adds the selected set to partial solution. The process of adding a set to partial solution is repeated \( k \) times to select \( k \) sets. Hochbaum and Pathria[6] provides analysis of the classical greedy approach for maximum \( k \)-coverage problem. The earlier approximation algorithms[8,9,10] use greedy heuristic for set covering problem.

Algorithm GMC(S,k)

\( S \) : A collection of sets \( \{S1,S2, ... Sn\} \)

\( k \) : Number of sets to be selected from \( S \)

begin

\( C \leftarrow \emptyset \)

\( W \leftarrow S1 \cup S2 \cup ....... \cup Sn \)

\( S' \leftarrow S \)

while (\( |C| < k \))

Select \( T \in S' \) that maximizes \( |T \cap W| \)

\( S' \leftarrow S' \setminus \{T\} \)

\( C \leftarrow C \cup \{T\} \)

\( W \leftarrow W \setminus T \)

end while

return \( C \)

end

Figure 1. Greedy Algorithm for Maximum \( k \)-Coverage Problem

Example 1 explains greedy method with help of a small set collection. The same set collection is used in Example 2 to explain Big step greedy algorithm with step-size \( p = 2 \) and it can be seen that the union of \( k \) sets selected by Big step greedy algorithm is bigger than the union of \( k \) sets selected by the classical greedy algorithm.
Example 1. Let \( S = \{ \{a,b,c,d,e,f\}, \{a,b,c,g,h\}, \{d,e,f,i,j\}, \{g,h,i\}, \{k,l\} \} \) be the given collection of sets and \( K = 3 \).

Assume labels for given sets \( S1 = \{a,b,c,d,e,f\} \), \( S2 = \{a,b,c,g,h\} \), \( S3 = \{d,e,f,i,j\} \), \( S4 = \{g,h,i\} \), \( S5 = \{k,l\} \). Initially partial cover \( C = \{\} \).

In the first step of algorithm, among the five sets \( S1 \) has six uncovered elements \( \{a,b,c,d,e,f\} \) and is better than the coverage of sets \( S2, S3, S4 \), and \( S5 \). So first step selects \( S1 \) and now partial cover \( C = \{ \{a,b,c,d,e,f\}\} \).

In second step, \( S4 \) has three uncovered elements \( \{g,h,i\} \), \( S2 \) has two uncovered elements \( \{g,h\} \), \( S3 \) has two uncovered elements \( \{i,j\} \) and \( S5 \) has two uncovered elements \( \{k,l\} \). So second step selects \( S4 \) and now partial cover \( C = \{ \{a,b,c,d,e,f\}, \{g,h,i\}\} \).

In third step, \( S5 \) has two uncovered elements \( \{k,l\} \), \( S2 \) has no uncovered elements and \( S3 \) has one element \( \{j\} \). So third step selects \( S5 \) and \( C = \{ \{a,b,c,d,e,f\}, \{g,h,i\}, \{k,l\}\} \). Now \( |C| = 3 \) and \( C \) covers 11 elements.

3. BIG STEP GREEDY HEURISTIC

The proposed Big step greedy heuristic starts with empty set cover, in each step it selects \( p \) (\( 1 <= p <= k \)) sets such that the union of selected \( p \) sets contains the greatest number of uncovered elements by evaluating all possible \( p \)-combinations of given sets and adds the \( p \) selected sets to partial set cover. The process of adding \( p \) subsets is repeated \( k/p \) times. The last step of the algorithm selects less than \( p \) sets when \( k \) is not a multiple of \( p \).

The Big step greedy algorithm is shown in Figure 2. The \( k \) sets selected by Big step greedy heuristic depends on its implementation in certain cases, particularly on the order in which candidates \( T = \{T1, T2, ..., Tq\} : T \subseteq S \) are iterated when there are multiple candidates having the best possible coverage and is same with the classical greedy algorithm.

When \( p = k \) big step greedy algorithm behaves as exact algorithm that computes optimal solution by evaluating all possible \( k \)-combinations of given sets. When \( p = 1 \) it behaves as the classical greedy algorithm.

Example 2. Let \( S = \{ \{a,b,c,d,e,f\}, \{a,b,c,g,h\}, \{d,e,f,i,j\}, \{g,h,i\}, \{k,l\}\} \) be the given collection of sets, \( K = 3 \) and step-size of algorithm is \( p = 2 \). Assume labels for given sets \( S1 = \{a,b,c,d,e,f\} \), \( S2 = \{a,b,c,g,h\} \), \( S3 = \{d,e,f,i,j\} \), \( S4 = \{g,h,i\} \), and \( S5 = \{k,l\} \). As step-size \( p = 2 \), every step of the algorithm choose two sets such that union of the two selected sets contains the greatest number of uncovered elements.

Initially partial cover \( C = \{\} \).

In the first step of algorithm, candidates are \( (S1,S2), (S1,S3), (S1,S4), (S1,S5), (S2,S3), (S2,S4), (S3,S4), (S3,S5), (S4,S5) \) among the ten candidates \( (S2,S3) \) is better than all other candidates as \( S2 \cup S3 \) has ten uncovered elements and is greater than that of other candidates. So the first step selects \( (S2,S3) \) and now partial cover \( C = \{ \{a,b,c,g,h\}, \{d,e,f,i,j\}\} \).
In second step, it selects only one set instead of two sets because \( K=3 \) and two sets \( S2, S3 \) are already selected by first step. Candidates are \( S1, S4, \) and \( S5 \). \( S5 \) has two uncovered elements \( \{k, l\} \), \( S1 \) has no uncovered element and \( S4 \) has no uncovered elements. So second step selects \( S5 \) and now finally solution \( C = \{\{a, b, c, g, h\}, \{d, e, f, i, j\}, \{k, l\}\} \) and \( C \) covers 12 elements. This is better than coverage of the sets selected by the classical greedy algorithm in Example 1.

**Algorithm** BSGMKC\((S, k, p)\)

- \( S \): A collection of sets \( \{S1, S2, \ldots, Sn\} \)
- \( k \): Number of sets to be selected
- \( p \): step-size of the algorithm

**begin**

- \( C \leftarrow \emptyset \)
- \( W \leftarrow S1 \cup S2 \cup \ldots \cup Sn \)

**while** \((|C| < k)\)

- **if** \(( (k - |C|) < p)\) **then**
  - \( q \leftarrow k - |C| \)
- **else**
  - \( q \leftarrow p \)

**end if**

- Select \( T={T1, T2, \ldots, Tq} \), \( T \subseteq S \setminus C \) that maximizes \(|W \cap (T1 \cup T2 \cup \ldots \cup Tq)|\)
- \( W \leftarrow W \setminus (T1 \cup T2 \cup \ldots \cup Tq) \)
- \( C \leftarrow C \cup \{T1, T2, \ldots, Tq\} \)

**end while**

- return \( C \)

**end**

**Figure 2. Big Step Greedy Algorithm for Maximum \( k \)-Coverage Problem**

### 3. EXPERIMENTAL RESULTS

The classical greedy algorithm and Big step greedy algorithm for maximum \( k \)-coverage problem were implemented using Java programming language. Large number of problem
instances were generated by using a computer program. For selected values of $|X|$ the universal set size and $n$ the number of sets in set collection $S$, all possible problems instances were generated.

The methodology used to generate the problem instances is described below

1. Select $|X|$ the size of universal set and $n$ the number of sets in collection $S$ of problem instances.
2. Assume $X = \{1,2,3,\ldots,|X|\}$, generate all the $2^n$ subsets of $X$ and exclude $\emptyset$.
3. Generate all $n$-combinations of subsets of $X$.

Example 3. To generate problem instances with $|X| = 2$ and $n = 2$

1. Let $X = \{1,2\}$ that satisfies $|X| = 2$.
2. All subsets of $X$ are $\emptyset$, $\{1\}$, $\{2\}$ and $\{1,2\}$. The empty set $\emptyset$ is ignored in next step.
3. Now required 2-combinations are $\{\{1\},\{2\}\}$, $\{\{1\},\{1,2\}\}$ and $\{\{2\},\{1,2\}\}$.

Generated combinations of the sets were used with different $k$ values to generate instances of maximum $k$-coverage problem. Java implementations of both algorithms were used to solve all the generated problems.

| $|X|$ | Used Subsets | Collection Size | $k$ | Number of Problems | BSGH is better | Greedy is better |
|-----|-------------|-----------------|----|-------------------|---------------|-----------------|
| 6   | 63          | 3               | 2  | 39711             | 735           | 0               |
| 6   | 63          | 4               | 2  | 595665            | 20860         | 0               |
| 6   | 63          | 4               | 3  | 595665            | 1363          | 0               |
| 6   | 63          | 5               | 2  | 7028847           | 314998        | 0               |
| 6   | 63          | 5               | 3  | 7028847           | 21264         | 0               |
| 6   | 63          | 6               | 2  | 67945521          | 3260866       | 0               |
| 6   | 63          | 6               | 3  | 67945521          | 162916        | 0               |
| 6   | 63          | 6               | 4  | 67945521          | 856           | 0               |
| 7   | 127         | 3               | 2  | 333375            | 7500          | 0               |
| 7   | 127         | 4               | 2  | 10334625          | 434135        | 0               |
| 7   | 127         | 4               | 3  | 10334625          | 44100         | 856             |
| 7   | 127         | 5               | 2  | 254231775         | 13940031      | 0               |
| 7   | 127         | 5               | 3  | 254231775         | 1536736       | 35357           |
| 7   | 127         | 5               | 4  | 254231775         | 19021         | 0               |
| 8   | 255         | 3               | 2  | 2731135           | 70503         | 0               |
| 8   | 255         | 4               | 2  | 172061505         | 8211951       | 0               |
| 8   | 255         | 4               | 3  | 172061505         | 1157243       | 48818           |
| 9   | 511         | 3               | 2  | 22108415          | 633540        | 0               |

Table I provides details of experiments and performance of the classical greedy algorithm and Big step greedy heuristic with $p=2$. In the Table I, column labeled as “$|X|$” is the number of elements in the universal set, column labeled as “Used Subsets” is the number of subsets of universal set $X$ that are used to create problem instances, column labeled as “Collection Size” is the number of sets in the set collection $S$ of problem
instance, column labeled as “k” is number of sets to be selected from given collection, column labeled as “Number of Problems” is the number of problems used for performance comparison, column labeled “BSGH is better” is the number of problem instances for which Big step greedy heuristic with $p=2$ is selecting better $k$-combination of sets than the $k$-combination of sets selected by the classical greedy algorithm and column labeled as “Greedy is better” is the number problem instances for which the classical greedy algorithm is selecting better $k$-combination of sets than the $k$-combination of sets selected by Big step greedy heuristic with $p=2$.

Table II provides details of performance comparison experiments on randomly generated problem instances. The problem instances were generated with probability of any set in the set collection containing any element from the universal set is 0.2. Table III provides details of performance comparison experiments on randomly generated problem instances for the same values of problem parameters shown in Table II, but the problem instances were generated with probability of any set in the set collection containing any element from the universal set is 0.5.

### Table II. Greedy Vs Big step greedy with $p=2$ on random problem instances

| X | Collection Size | k | Number of Problems | BSGH is better | Greedy is better |
|---|-----------------|---|--------------------|----------------|-----------------|
| 100 | 7 | 3 | 1000000 | 41369 | 8240 |
| 100 | 10 | 5 | 1000000 | 86935 | 22960 |
| 100 | 10 | 7 | 1000000 | 71021 | 23043 |
| 100 | 20 | 5 | 1000000 | 160477 | 45552 |
| 100 | 20 | 7 | 1000000 | 186349 | 66241 |
| 100 | 30 | 5 | 1000000 | 194323 | 57014 |
| 100 | 40 | 10 | 1000000 | 135049 | 50102 |

### Table III. Greedy Vs Big step greedy with $p=2$ on random problem instances

| X | Collection Size | k | Number of Problems | BSGH is better | Greedy is better |
|---|-----------------|---|--------------------|----------------|-----------------|
| 100 | 7 | 3 | 1000000 | 121702 | 34205 |
| 100 | 10 | 5 | 1000000 | 114687 | 35886 |
| 100 | 10 | 7 | 1000000 | 877 | 211 |
| 100 | 20 | 5 | 1000000 | 10804 | 1619 |
| 100 | 20 | 7 | 1000000 | 0 | 0 |
| 100 | 30 | 5 | 1000000 | 222 | 5 |
| 100 | 40 | 10 | 1000000 | 0 | 0 |

It can be seen that Big step greedy heuristic with $p=2$ is selecting better $k$-set combination than the $k$-set combination selected by the classical greedy algorithm in many cases.

### 4. CONCLUSION

Experimental results provide sufficient evidence that the proposed Big step greedy heuristic computes better approximate solution for maximum $k$-coverage problem than the classical greedy algorithm. When step-size $p$ is small enough Big step greedy algorithm runs in polynomial time. Experiments on many instances of maximum $k$-coverage problem show that Big step greedy algorithm with $p = 2$ performs better than
the classical greedy algorithm in many cases. Big step greedy heuristic is preferable than
the classical greedy algorithm in scenarios where small improvement in approximate
solution is valuable and some increment in running time is acceptable. The proposed Big
step greedy heuristic can be used for some other combinatorial optimization problems.

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