Introduction

Biological fluids are the fluids in human body that relief in the transport of proteins and nutrients or expel waste products from the cells. Blood, semen, saliva, urine, and mucus are commonly known as biological fluids. Peristaltic movement is a natural phenomenon for the transport of biological fluids from lower pressure region to higher pressure region due to propagation of Sinusoidal wave on the walls. Many theoretical studies are reported for observing the peristaltically moving biological materials. Srivastava et al. 1 developed a peristaltic model for the peristaltic transport of psychological fluid in a non-uniform tube. The flow rates in the present study are compared with observed flow rates vas deferens of rhesus monkeys, the small intestine and the ductus efference of the male reproductive tract. A theoretical study is carried out by Tripathi et al. 2 to discuss the peristaltic flow of...
viscoelastic fluid in a channel with applications for the chyme movement in small intestine. Srivastava and Srivastava\textsuperscript{7} used Power law fluid model to discuss the peristaltic transport of biological material through vas deferens and small intestine under long wavelength and low Reynolds approximations. Gastric transport of biofluid using viscoelastic fluid model is discussed by Tripathi and Anwar Bég.\textsuperscript{5} A biological model proposed by Eytan et al.\textsuperscript{5} to discuss the peristaltic transport of embryo in uterine cavity. AbdelSalam et al.\textsuperscript{6} discussed the self-population of swimming sperms in the fallopian tube under biological assumptions. The outcome of present study is helpful to human cervical canal of female. Johnson-Segalman fluid model is used by Ashraf et al.\textsuperscript{7} to report the peristaltic-ciliary flow in human fallopian tube.

During the last few years, a lot of work has been reported to discuss the mechanism of different biological liquids which presence importance in biosciences, health engineering, and bio-technological processes. The importance of magnetic induction is preserved in different eras of engineering and health treatment. The bioliquids applications with interference of magnetic force is observed by various authors.\textsuperscript{8–10}

An interesting feature of magnetic hydrodynamic on the peristaltic flow is deal with Hall current. The occurrence of Hall current is when the electron-atom collision frequency is low under the influence of strong magnetic field. The Hall current has marked effects on the current density and magnetic force term. Hall current has many applications in magnetic field sensing equipments, Hall sensors and probes, proximity testers, and MRI. Effects of Hall current on the flow of viscous fluid in non-uniform regime is investigated by Javid et al.\textsuperscript{11} Important outcomes of this study can help in the design of robot capsule which can flow though small vessels during surgery without any hurdle. In another attempt, Javid et al.\textsuperscript{12} interpreted the onset of Hall current on the flow of viscoelastic fluid in a complex wavy non-uniform channel with large number of application in bioengineering. Krishna and Chakraborty\textsuperscript{13} analyzed the effects of Hall current and ionic slip on elastico-viscous fluid in rotating frame. Li et al.\textsuperscript{14} used the shooting method to discuss the effects of Hall current on the peristaltic flow and heat transfer features of Jeffery fluid in curved channel with different wave frames.

Various investigations are contributed by scientists on the thermal flow of nanofluids in recent days. Based on improved assessment of nanofluids properties, dynamic applications of such materials are noticed in thermal processes, industrial mechanisms, extrusion framework, energy generation, and other more. The nanomaterials report fine thermal determination and improved consequences. Some multidisciplinary applications are noticed for nanofluids flow in heating of various devices, cooling mechanisms, heat transmission systems, etc. The low case energy resources are also suggested on basis of nanofluid intersections. Different mechanism of treatment for diseases have also been introduced where nanomaterials play significant role. Choi\textsuperscript{15} led the novel idea of such materials with experimental approach. Buongiorno\textsuperscript{16} endorsed the Brownian aspect and thermophoresis object of nanofluid model. Khan et al.\textsuperscript{17} predicted the nanomaterials thermal capacitance referred to the disk flow where contribution of slip phenomenon was effective. A theoretical study is carried out by Nadeem et al.\textsuperscript{18} to analyze the peristaltic flow of hybrid nanofluid with SWCNTs and MWCNTs in a rectangular wavy duct. A numerical simulation is performed by Abbasi and Farooq\textsuperscript{19} to investigate the flow of water based hybrid nanofluid in a non-uniform channel. It is concluded that magnitude of heat transfer coefficient is large in non-uniform channel as compared to uniform channel. Javid et al.\textsuperscript{20} theoretically investigate the flow characteristics for the peristaltic flow of Powell Eyring hybrid nanofluid in a curved domain with ciliated walls. A comparison for different flow features is reported for both curved and straight channel as a productive application in drug delivery systems. Tripathi et al.\textsuperscript{21} numerically investigated the electro osmotic flow of coupled stress hybrid nanofluid through a micro channel. It is noted that thermal characteristics are strongly affected by joule heating parameter. Simultaneous effects of slip and Lorentz forces are addressed by Ali et al.\textsuperscript{22} in the peristaltic transport of Jeffery hybrid nanomaterial through asymmetric planner channel. In the above-mentioned literature, the viscosity of the nanofluids is taken to be constant but when the temperature rises due to the viscous dissipation, the physical properties may change significantly. It is necessary to take the variation in viscosity of hybrid nanofluid to predict the flow behavior at high temperature. Therefore, it is very interesting to the researchers to study the peristaltic flows and heat transfer with variable properties. Ali et al.\textsuperscript{23} exhibited the viscous particles flow space dependent viscosity induced by peristaltic waves. Ashwinkumar et al.\textsuperscript{24} highlighted the thermal influence of cross hybrid nanofluid confined by diverse surface. Ramana Reddy et al.\textsuperscript{25} numerically simulate the consequence of Carreau nanofluid properties with variable viscosity. Sucharitha et al.\textsuperscript{26} investigated the simultaneous effects of Joule heating and wall flexibility for nanofluid flow. The cross-diffusion effects of nanofluid in duct was reported by Sucharitha et al.\textsuperscript{27}

In human cardiovascular system veins and arteries are curved in nature and the blood is transported on the mechanism of peristalsis. Owning the applications of magnetic nanoparticles in biomedical science in this study peristaltic motion of blood-based copper and iron oxide hybrid nanofluid is investigated. The mathematical model is established using fundamental conservation laws and simplified using biological assumptions. The resulting equations are
numerically simulated to analyze the impact of physical parameters on different features of the blood motion.

**Flow configuration and fundamental equations**

The problem for hybrid nanofluid due to curved surface channel with sinusoidal wave is under consideration. The speed of peristaltic waves is expressed with \( c \). The width of channel is taken as \( 2d \). A constant magnetic field of strength \( B_0 \) in the presence of Hall current is applied in radial direction. The lower and upper wall of the channel with radius of curvature \( R^* \) having center \( O \) are maintained temperature \( T_1 \) and \( T_2 \) respectively. The orthogonal curvilinear coordinates \( (R, X) \) are chosen to model the problem so that \( X - \) axis is considered along the flow direction and \( R - \) axis has been normally impacted. The flow illustration of the problem is shown in the Figure 1 in which \( H_1 \) and \( H_2 \) depicts lower and upper walls and their mathematical relations are 28–33:

\[
\begin{align*}
H_1 &= -d - \bar{M} (X - cT) + \Phi_1 \sin \left( \frac{2\alpha \gamma}{\nu} (X - cT) + \sigma \epsilon \right) - \Phi_2 \sin \left( \frac{2\alpha \gamma}{\nu} (X - cT) + \omega \epsilon \right), \\
H_2 &= d + \bar{M} (X - cT) + \Phi_1 \sin \left( \frac{2\alpha \gamma}{\nu} (X - cT) + \Phi_2 \sin \left( \frac{2\alpha \gamma}{\nu} (X - cT) \right) \right).
\end{align*}
\]

In which \( \bar{M} \) represents the non-uniformity of the channel, \( \Phi_1, \Phi_2, \gamma \) are amplitudes and length of waves, \( \epsilon \) is phase difference, and \( \sigma \) and \( \omega \) are geometrical parameters satisfying the condition \( d \leq \sigma + \omega \).

The fundamental conservation laws of mass, momentum, and energy used in the analysis are

\[
\begin{align*}
\nabla \cdot \dot{\mathbf{V}} &= 0, & \text{Continuity equation} \\
\rho_{huf} \frac{d\mathbf{V}}{dt} &= \nabla \cdot \mathbf{J} + \mathbf{J} \times \mathbf{B}, & \text{Momentum equation} \\
(\rho C_p)_{huf} \frac{dT}{dt} &= K_{huf} \nabla^2 T + \mathbf{L} + \frac{\mathbf{J} \cdot \mathbf{J}}{\sigma_{huf}}, & \text{Energy equation}
\end{align*}
\]

Where \( \dot{\mathbf{V}} \) is velocity and \( \mathbf{J} = -P\mathbf{I} + \mu_{huf} (\dot{T}) \mathbf{A} \) is the Cauchy stress tensor. The term \( \mathbf{J} \times \mathbf{B} \) represents the Lorentz force, the mathematical relationship between velocity field and current density in the absence of ionic slip and electric field is

\[
\mathbf{J} + \frac{e_n \epsilon B_0}{\sigma_{huf}} (\mathbf{J} \times \mathbf{B}) = \sigma_{huf} \left[ \nabla \times \mathbf{B} \right].
\]

\( e_n \) is Hall parameter. While \( \rho_{huf} \) is density, \( (C_p)_{huf} \) is specific heat, \( K_{huf} \) is thermal conductivity, \( \sigma_{huf} \) is the electric conductivity of hybrid nanofluid. The expression of physical properties are given in Table 1.

In above table

\[
\left( \rho_f, \rho_s, \rho_{fs} \right) \cdot \left( (C_p)_f, (C_p)_s, (C_p)_{fs} \right) \cdot \left( K_f, K_s, K_{fs} \right)
\]

\( \left( \sigma_f, \sigma_s, \sigma_{fs} \right) \) are density, specific heat, thermal conductivity, electric conductivity of blood, \( Cu \), and \( Fe_3O_4 \) respectively \( \alpha_f, \alpha_s \) are volume fraction of \( Cu \) and \( Fe_3O_4 \) nanoparticles. All the physical quantities except the viscosity is assumed to be constant. The viscosity of the hybrid nanofluid depends upon the temperature and is approximated by Brinkman viscosity model \( \mu_T (T) = \mu_0 e^{-\alpha_T (T - T_0)} \).

**Governing equations in curvilinear-coordinate system**

The rheological equations to describe the unsteady hybrid nanofluid flow in the curvilinear system are 28–33:

\[
\frac{\partial}{\partial R} \left( R^* (R + R^*) \dot{\mathbf{V}} \right) + R^* \frac{\partial U}{\partial X} = 0,
\]

Figure 1. Geometry of the problem.
The relations which endorsed the properties of hybrid nanofluid are given as.  

\[
\rho_{\text{nf}} = (1 - \alpha_2) \rho_f \left( 1 - \alpha_1 \right) + \alpha_1 \left( \frac{\rho_s}{\rho_f} \right) + \alpha_2 \rho_s
\]

\[
\mu_{\text{nf}}(\bar{T}) = \frac{\mu_f(\bar{T})}{(1 - \alpha_2)^{2.5}} \left( 1 - \alpha_2 \right)^{3.5}
\]

\[
(pC_\rho)_{\text{nf}} = (1 - \alpha_2) \rho_f (C_p)_f \left( 1 - \alpha_1 \right) + \alpha_1 \left( \frac{C_s}{\rho_f} \right) + \alpha_2 C_s
\]

Thermal conductivity

\[
K_{\text{nf}} = \frac{1 - \alpha_2 + 2 \alpha_2 \left( \frac{K_s}{K_s - K_{\text{nf}}} \right) \ln \left( \frac{K_s + K_{\text{nf}}}{2K_{\text{nf}}} \right)}{1 - \alpha_2 + 2 \alpha_2 \left( \frac{K_{\text{nf}}}{K_s - K_{\text{nf}}} \right) \ln \left( \frac{K_s + K_{\text{nf}}}{2K_{\text{nf}}} \right)}
\]

\[
\frac{K_{\text{nf}}}{K_f} = \frac{1 - \alpha_2 + 2 \alpha_1 \left( \frac{K_s}{K_s - K_f} \right) \ln \left( \frac{K_s + K_f}{2K_f} \right)}{1 - \alpha_2 + 2 \alpha_1 \left( \frac{K_f}{K_s - K_f} \right) \ln \left( \frac{K_s + K_f}{2K_f} \right)}
\]

Electric conductivity

\[
\frac{\sigma_{\text{nf}}}{\sigma_f} = \frac{\sigma_s + 2 \sigma_f - 2 \sigma_f (\sigma_f - \sigma_s)}{\sigma_s + 2 \sigma_f + \alpha_2 (\sigma_f - \sigma_s)}
\]

where \( \sigma_{\text{nf}} = \frac{\sigma_s + 2 \sigma_f - 2 \alpha_1 (\sigma_f - \sigma_s)}{\sigma_s + 2 \sigma_f + \alpha_1 (\sigma_f - \sigma_s)} \sigma_f \)

\[
\rho_{\text{nf}} \left[ \frac{\partial \bar{V}}{\partial \bar{T}} + \frac{\bar{U} \partial \bar{V}}{\bar{R} + R^*} \frac{\partial \bar{V}}{\partial \bar{X}} - \frac{\partial \bar{P}}{\partial \bar{R}} + \mu_{\text{nf}}(\bar{T}) \right]
\]

\[
= - \frac{R^* \partial \bar{P}}{\bar{R} + R^*} \frac{\partial \bar{U}}{\partial \bar{S}} + \mu_{\text{nf}}(\bar{T}) \left[ \frac{1}{\bar{R} + R^*} \frac{\partial}{\partial \bar{R}} \left( \frac{\bar{R} + R^*}{\bar{R} + R^*} \right) \frac{\partial \bar{V}}{\partial \bar{X}} - \frac{\bar{V}}{(\bar{R} + R^*)^2} \left( \frac{\bar{R} + R^*}{\bar{R} + R^*} \right)^2 \right]
\]

\[
\rho_{\text{nf}} \left[ \frac{\partial \bar{U}}{\partial \bar{T}} + \frac{\bar{U} \partial \bar{U}}{\bar{R} + R^*} \frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V} \frac{\partial \bar{U}}{\partial \bar{R}} + \frac{\bar{U} \partial \bar{V}}{\partial \bar{R}} \right]
\]

\[
= - \frac{R^* \partial \bar{P}}{\bar{R} + R^*} \frac{\partial \bar{U}}{\partial \bar{S}} + \mu_{\text{nf}}(\bar{T}) \left[ \frac{1}{\bar{R} + R^*} \frac{\partial}{\partial \bar{R}} \left( \frac{\bar{R} + R^*}{\bar{R} + R^*} \right) \frac{\partial \bar{U}}{\partial \bar{X}} - \frac{\bar{U}}{(\bar{R} + R^*)^2} \left( \frac{\bar{R} + R^*}{\bar{R} + R^*} \right)^2 \right]
\]

\[
(pC_\rho)_{\text{nf}} \left[ \frac{\partial \bar{T}}{\partial \bar{T}} + \bar{V} \frac{\partial \bar{T}}{\partial \bar{R}} + \frac{\bar{U} \partial \bar{T}}{\bar{R} + R^*} \frac{\partial \bar{T}}{\partial \bar{X}} \right]
\]

\[
= K_{\text{nf}} \left[ \frac{1}{\bar{R} + R^*} \frac{\partial}{\partial \bar{R}} \left( \frac{\bar{R} + R^*}{\bar{R} + R^*} \right) \frac{\partial \bar{T}}{\partial \bar{X}} + \left( \frac{R^*}{\bar{R} + R^*} \right)^2 \frac{\partial^2 \bar{T}}{\partial \bar{X}^2} \right]
\]

\[
+ \mu_{\text{nf}}(\bar{T}) \left[ 2 \left( \frac{\partial \bar{V}}{\partial \bar{R}} \right)^2 + \left( \frac{\partial \bar{U}}{\bar{R} + R^*} \frac{\partial \bar{V}}{\partial \bar{X}} + \bar{V} \right)^2 + \left( \frac{\partial \bar{U}}{\partial \bar{R}} + \frac{R^*}{\bar{R} + R^*} \frac{\partial \bar{V}}{\partial \bar{X}} - \frac{\bar{U}}{\bar{R} + R^*} \right)^2 \right] + \frac{\sigma_{\text{nf}}B_0^2 R^*}{(\bar{R} + R^*)^2 + (mR^*)^2} \left( \frac{\bar{R} + R^*}{\bar{R} + R^*} \right)^2 \right]
\]
Table 2. Numerical values of thermo-physical properties.\textsuperscript{33}

| Material     | Blood | $C_u$ | $FeO_4$ |
|--------------|-------|-------|---------|
| $\rho$ (kg/m$^3$) | 1063  | 8933  | 5200   |
| $K$ (W/mK)    | 0.492 | 401   | 6       |
| $C$ (J/kgK)   | 3594  | 385   | 670     |
| $\sigma$ (N/m) | 0.8   | 5.96 $\times$ 10$^7$ | 25,000 |

The relations reported above of unsteady in fixed frame and can be transformed to time independent in wave frame. Both the frame of reference can be related by using the following linear transformations\textsuperscript{28-33}:

$$ \tilde{x} = \bar{x} - cT, \tilde{R} = \bar{R}, \tilde{v} = \bar{v}, \tilde{u} = \bar{u} - c, \tilde{p} = \bar{p}. $$

(7)

In wave frame of reference equations (3)–(6) takes the form

$$ \frac{\partial}{\partial \tilde{r}} \left[ (\tilde{r} + \tilde{R}) \tilde{v} \right] + R^* \frac{\partial \tilde{u}}{\partial \tilde{r}} = 0, $$

(8)

$$ \rho_{hof} \left[ -\frac{\partial \tilde{v}}{\partial \tilde{r}} + \frac{u + c}{\tilde{r} + \tilde{r}^*} \frac{\partial (\tilde{r} + \tilde{R})}{\partial \tilde{r}} \right] = -\frac{R^*}{\tilde{r} + \tilde{r}^*} \frac{\partial \tilde{p}}{\partial \tilde{r}} + \mu_{hof} (\tilde{T}), $$

(9)

$$ \left( \rho C_p \right)_{hof} \left[ -\frac{\partial \tilde{T}}{\partial \tilde{r}} + \frac{u + c}{\tilde{r} + \tilde{r}^*} \frac{\partial (\tilde{r} + \tilde{R})}{\partial \tilde{r}} \tilde{T} \right] = K_{hof} \left[ \frac{1}{\tilde{r} + \tilde{r}^*} \frac{\partial \tilde{v}}{\partial \tilde{r}} + \frac{R^*}{\tilde{r} + \tilde{r}^*} \frac{\partial \tilde{T}}{\partial \tilde{r}} \right] $$

(10)

$$ + \mu_{hof} (\tilde{T}) \left[ 2 \left( \frac{\partial \tilde{v}}{\partial \tilde{r}} \right)^2 + \frac{R^*}{\tilde{r} + \tilde{r}^*} \frac{\partial \tilde{v}}{\partial \tilde{r}} \right] + \frac{(u + c)}{(\tilde{r} + \tilde{r}^*)} \left( u + c \right) $$

$$ - \frac{R^*}{\tilde{r} + \tilde{r}^*} \frac{\partial (u + c)}{\partial \tilde{r}} \left[ \frac{1}{1 + \left( \frac{mR^*}{\tilde{r} + \tilde{r}^*} \right)^2} \right], $$

(11)

**Dimensionless formulation and resulting equations**

The above equations (8)–(11) can be normalized by defining the dimensionless variables\textsuperscript{28-32}:

$$ s = \frac{2\pi \bar{x}}{\gamma}, \eta = \frac{r}{d}, u_1 = \frac{\bar{v}}{c}, u_2 = \frac{\bar{R}}{c}, h = \frac{\bar{H}}{d}, p = \frac{2\pi d^2}{\rho_{hof}c} \bar{p}, $$

$$ \Phi_1 = \frac{\Phi_1}{d}, i = 1, 2, \theta(\eta) = \frac{\tilde{T} - T_i}{T_2 - T_1} $$

(12)

After using the above dimensionless variables and stream function related by $u_1 = \delta (k / \eta + k) \partial \psi / \partial s$ and $u_2 = -\partial \psi / \partial \eta$ in (8)–(11) and applying the assumptions of low Reynolds’s number and long wave length, the mass conservation equation is satisfied identically and momentum and energy conservation equations (9)–(11) takes the form

$$ \frac{\partial \tilde{p}}{\partial \tilde{n}} = 0, $$

(13)

$$ \frac{\partial \tilde{p}}{\partial \tilde{n}} = \frac{1}{k (k + \eta) \eta C_s} \frac{\partial}{\partial \eta} \left[ (\eta + k + 1) \left( \frac{\eta + k}{\eta} \frac{\partial \psi}{\partial \eta} \right) \right] $$

(14)

$$ - C_1 \left( \frac{H \eta k^2}{(\eta + k)^2} \right) \left( \frac{1}{\eta} \frac{\partial \psi}{\partial \eta} \right), $$

$$ - C_2 \left( \frac{\partial \psi}{\partial \eta} \right) $$
Figure 2. (a-b) Variation of axial velocity $u_2(\eta)$ against $\alpha_1$ and $\alpha_2$ with $q = -0.5, k = 3.0 = Ha, m = 2.0$ and $Br = 1.0$.

with axial component $s$, radial velocity $u_1$, radial component $\eta$, and axial velocity $u_2$. The definition of Reynolds number $Re$, curvature parameter $k$, Hartman number $Ha$, wave number $\delta$, Eckert number $Ec$, Prandtl number $Pr$, viscosity parameter $\omega_1$, Brinkman number $Br$, and constants $(C_1, C_2, C_3)$ are:

$$Re = \rho c d / \mu_0, k = R^2 / d, Ha = B_0 d \sqrt{|\sigma_f / \mu_0|, \delta = 2\pi d / \gamma},$$

$$Ec = c^2 / C_f (T_1 - T_i), Pr = \mu_0 c_f / K_f,$$

$$\omega_1 = \alpha_1 (T_2 - T_i), Br = Pr Ec, C_1 = \sigma_{l1d} / \sigma_t,$$

$$C_2 = K_{l1d} / K_t^{\prime}$$

The dimensionless flow constraints is28–30:

$$\psi = -q \frac{\partial \psi}{\partial \eta} = 1, \theta = 1 \text{ at } \eta = h_2$$

$$\psi = -q \frac{\partial \psi}{\partial \eta} = 1, \theta = 0 \text{ at } \eta = h_1$$

In laboratory frame, mean flow rate $Q$ is prescribed as31,32:

$$Q_i = q + 2, \quad (17)$$

Mathematical expressions of the walls in dimensionless form are

$$h_2 = 1 + m_s + \Phi_1 \sin (\sigma s) + \Phi_2 \sin (\alpha s)$$

$$h_1 = -1 - m_s - \Phi_1 \sin (\sigma s + \epsilon) - \Phi_2 \sin (\omega (s + \epsilon))$$

in which $m_s = \gamma \bar{M} / d$ is non-uniform parameter. The skin fraction and heat transfer coefficient at upper wall of the channel can be computed using relations

The thermo-physical values of nanoparticles and base fluid is presented in table 2.

Solution methodology

Eliminating the pressure from the equations (13)–(14) by cross differentiation, the resulting equation along with equation (15) subject to boundary conditions (16) are numerically solved using Mathematica built in function ND Solve for the particular values of involved parameters.

Results and discussion

The fundamental onset of flow parameters governs to formulated model is tested graphically. The investigation deals with different cases like for constant viscosity $\omega_1 = 0.0$ and temperature dependent viscosity $\omega_1 \neq 0.0$. The parameters related to the geometry are fixed. That is, $\Phi_1 = 0.1, \Phi_2 = 0.2, m_s = 0.2, \sigma = 1.0 = \omega$ and $\epsilon = \pi$. Figure 2 provides the physical description of copper nanoparticles volume fraction for the axial velocity component. The proposed results are prepared for both constant and variable viscosity. The lower trend in axial velocity in core region of channel is noticed. Moreover, this decreasing change in maximum for constant viscosity instead of variable viscosity. Similar response is noted by increasing the solid volume fraction of Fe$_3$O$_4$ nanoparticles presented in Figure 2(b). Figure 3(a) evaluated the description of velocity due to enhancing role of Hartman number Ha. A significant response is observed in the boundary layer for macro
and micro scales. The velocity $u_2(\eta)$ not decreases on the heat of the channel but also shifted toward the upper surface boundary. The restricted motion of particles is due to Lorentz force which appeared when magnetic force impacted. In Figure 3(b) rolled out the evaluation of Hartmann number on velocity for straight and curved channel. It is confirmed that Hall current enrolled major factor to control the Lorentz force on the transportation of the hybrid nanofluid. The response of axial velocity $u_2(\eta)$ against the increasing values of curvature parameter $k$ and non-uniform parameter $m_1$ are depicted in Figure 4. It is noted that the symmetry in axial velocity at the heart of the channel is reported for $k \rightarrow \infty$. This means that the velocity is symmetric for planner channel as compared to the curved channel. Furthermore, the velocity of the hybrid nanofluid enhances in upper part of channel and declined near the lower wall of channel as the curvature of the channel is stunned. The improved velocity determination is noted in view of non-uniform parameter for both constant viscosity model and variable viscosity model.

In Figure 5 the temperature distribution $\theta(\eta)$ is plotted against the solid volume fraction of both Cu and $Fe_5O_4$ nanoparticles. The observed sketches show that the change in the copper nanoparticles volume fraction, the heat transfer rate exclusive get large. On other hand, the volume fraction of $Fe_5O_4$ reduces the temperature of the hybrid nanofluid. Actually, the solid volume fraction of Cu nanoparticles rises the thermal conductivity of base fluid while volume fraction of $Fe_5O_4$ reduces the thermal conductivity of the nanofluid which rises or reduces the temperature of the blood. In Figure 6 the impact of Hartman number $Ha$ and Hall parameter $m$ on the temperature distribution is presented for constant viscosity ($\omega_1 = 0$) and temperature dependent viscosity ($\omega_1 = 0.1$). The temperature profile of the hybrid nanofluid rises with Hartman number while decreases with Hall parameter. The rise in Hartman number growths the Lorentz forces which rises the internal kinetic energy of the nanofluid and as a result the temperature rises. The effects of Hartman number is overcome by rising the Hall parameter. The temperature profile $\theta(\eta)$ is decreasing function of curvature parameter $k$ while increasing function of non-uniform parameter for both constant viscosity and temperature dependent viscosity. As for $k = 2.5$ the temperature is maximum and for $k \rightarrow \infty$ the temperature is minimum.
For curved channel the temperature is large and for straight channel the temperature is small. The temperature is minimum in uniform regime while in non-uniform region, the temperature is large. Figure 7(a-b).

Streamlines for different values of Hartman number $Ha$, Hall parameter $m$, and curvature parameter $k$ are presented for both constant and variable viscosity in Figures 8 to 10. The rise in Hartman number $Ha$ rises
the size of trapping bolus for both constant and variable viscosity in the upper half of the channel. For $Ha = 0.0$ the only increase in the size of trapping bolus is noticed for both $\omega_0 = 0.0$ and $\omega_1 = 0.1$ but for $Ha = 2.0$ both size and number of bolus increases in the flow sketch. By rising Hall parameter the splitting in the trapping bolus occur and for temperature dependent viscosity addition of trapping bolus is also noticed in the upper half of the channel. The symmetry in the number of bolus is noticed for $k = \infty$ for both constant and variable viscosity.

Figures 11 and 12 present the pressure gradient variation against the volume fraction of $Cu$ and $FeO_3$ nanoparticles $\alpha_1$ and $\alpha_2$, respectively, Hartman number $Ha$ and Hall parameter for constant as well as temperature dependent viscosity. From Figure 11(a) and (b) it is noted that the pressure gradient is increasing function of solid volume fractions of both $Cu$ and $FeO_3$ nanoparticles. Furthermore, due to the addition of variable viscosity the magnitude of pressure gradient declines. The pressure gradient shows decreasing trend by rising the Hartman number while rise is noticed for Hall parameter. Actually, the rise in Hall parameter overcomes the effects of Lorentz force.

The effects of Hartman number, Hall parameter and curvature parameter on both skin friction and heat transfer coefficient are presented in Tables 3 and 4 for two working nanofluids. That is, $Cu$ / Blood nanofluid and hybrid
nanofluid. Furthermore, both skin friction and heat transfer coefficient are calculated and presented for both constant and variable viscosity models. It is noted that the skin friction declines with Hartman number while rises against Hall parameter for both types of nanofluids. Also the skin friction is large for temperature dependent viscosity as compared to constant viscosity. The heat transfer coefficient has opposite trend to the skin friction for both Hartman number and Hall parameter. The coefficient of heating phenomenon is larger for Cu/blood nanofluid inconstant to hybrid nanofluid. The rise in curvature parameter cause a remarkable decline in skin friction for both constant and variable viscosity. The heat transfer is also decreasing function of curvature parameter both Cu/blood and hybrid nanofluid.

**Main findings**

From the above graphical and tabulated results the main outcomes of the present study are summarized as follow

- Velocity is decreasing function of solid volume fractions of Cu and Fe₃O₄ nanoparticles.
- Hartman number decreases the velocity in upper half while Hall parameter enhanced the velocity along the axial direction in upper regime of channel.

![Figure 9](image-url)
The symmetry in the velocity profile is noted for the increasing values of curvature parameter.
- A smaller velocity change is associated to the temperature dependent viscosity.
- The temperature is increasing function Hall parameter while decreasing function of Hall parameter.
- The temperature is large for variable viscosity when compared to constant viscosity.
- Accumulation of trapping bolus in the upper half of the curved channel is noticed for temperature dependent viscosity.
- For $k = \infty$ symmetry in the number of trapping bolus is observed in both regimes of channel.
- Solid volume fraction of both Cu and $\text{Fe}_3\text{O}_4$ nanoparticles rises the pressure gradient.
- The pressure gradient declines by prompting the variable viscosity.
- The heat transfer coefficient is decreasing function of Hartman number and curvature parameter.
- For temperature dependent viscosity the heat transfer coefficient enhanced as compared to constant viscosity.

Figure 10. (a-d) Variation of streamlines against curvature parameter $k$ with $\Phi_1 = 0.2, \Phi_2 = 0.3, \eta_1 = 0.1, q = 1.0, Ha = 1.0, \alpha_1 = 0.02, \alpha_2 = 0.03, m = 1.0$ and $Br = 0.5$. 
Table 3. Variation of skin friction against several parameters with $m_1 = 0.1, \Phi_1 = 0.1, \Phi_2 = 0.2, q = -0.2, \sigma = \omega = 1.0$ and $Br = 1.0$.

| Parameters | $\omega_1 = 0.0$ | $\omega_1 = 0.1$ | $\omega_1 = 0.0$ | $\omega_1 = 0.1$ |
|------------|------------------|------------------|------------------|------------------|
| $Ha$ | $m$ | $k$ | $C_w$ nanofluid $\alpha_1 = 0.05, \alpha_2 = 0.0$ | $\gamma_1 = 0.05, \gamma_2 = 0.05$ | $C_H$ nanofluid $\alpha_1 = 0.05, \alpha_2 = 0.0$ | $\gamma_1 = 0.05, \gamma_2 = 0.05$ |
| 0.0 | 2.0 | 3.0 | 0.77630334 | 0.77630334 | 0.77648369 | 0.77648291 |
| 1.0 | 0.77620103 | 0.77620710 | 0.77638146 | 0.77638674 |
| 2.0 | 0.77589401 | 0.77591833 | 0.77607468 | 0.77609819 |
| 1.0 | 0.77606729 | 0.77608131 | 0.77624783 | 0.77626105 |
| 1.5 | 0.77615050 | 0.77615958 | 0.77633926 | 0.77633926 |
| 3.0 | 0.77620103 | 0.77620710 | 0.77638146 | 0.77638674 |
| 5.0 | 0.76948891 | 0.76950197 | 0.76960027 | 0.76961284 |
| 10 | 0.76428459 | 0.76429810 | 0.76434148 | 0.76435474 |
| $\infty$ | 0.75885426 | 0.75886824 | 0.75885432 | 0.75886830 |
Table 4. Change in heat transfer coefficient with $m_1 = 0.1, \Phi_1 = 0.1, \Phi_2 = 0.2, q = -0.2, \sigma = \omega = 1.0$ and $\beta r = 1.0$.

| Parameters | $\alpha_1 = 0.0$ | | $\alpha_1 = 0.1$ | |
|------------|------------------| |------------------|----------|
| $Ha$ $m$ $k$ | $Cu$ nanofluid | Hybrid nanofluid | $Cu$ nanofluid | Hybrid nanofluid |
| 0.0 2.0 3.0 | 1.60760557 | 1.59587658 | 1.6076152 | 1.59592698 |
| 1.0 2.0 3.0 | 1.6074702 | 1.59602383 | 1.60780301 | 1.59607428 |
| 2.0 2.0 3.0 | 1.60817139 | 1.59646561 | 1.60822754 | 1.59651619 |
| 1.5 2.0 3.0 | 1.60791904 | 1.59622566 | 1.60799714 | 1.59627617 |
| 3.0 2.0 3.0 | 1.60774702 | 1.59609877 | 1.60787506 | 1.59614923 |
| 5.0 2.0 3.0 | 1.58654453 | 1.57697969 | 1.58676899 | 1.57701074 |
| 10 2.0 3.0 | 1.57347380 | 1.56184954 | 1.57349135 | 1.56186535 |
| $\infty$ 2.0 3.0 | 1.55764133 | 1.54605880 | 1.55764134 | 1.54605882 |

Refs. 34–38 are added for the future work, which altered the existed model to the peristaltic flow model.

Author’s Note

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