SMALL ANGLE POLARIZATION IN HIGH ENERGY P–P SCATTERING THROUGH NONPERTURBATIVE CHIRAL SYMMETRY BREAKING

Mauro Anselmino\textsuperscript{a,b} and Stefano Forte\textsuperscript{b}

\textit{Dipartimento di Fisica Teorica, Università di Torino}\textsuperscript{a}
\textit{and}
\textit{I.N.F.N., Sezione di Torino}\textsuperscript{b}
\textit{via P. Giuria 1, I-10125 Torino, Italy}

\begin{abstract}
We show that a large anomalous contribution due to nonperturbative instanton-like gluonic field configurations to the axial charge of the proton implies high-energy spin effects in $p – p$ elastic scattering. This is the same mechanism which is responsible for anomalous baryon number violation at high energy in the standard model. We compute the proton polarization due to these effects and we show that it is proportional to the center-of-mass scattering angle with a universal (energy-independent) slope of order unity.
\end{abstract}

Submitted to: \textit{Physical Review Letters}
There are many indications, both experimental and theoretical, that perturbative techniques may be insufficient to account for spin effects, even at high energy. It has been suggested that the so-called “proton spin” problem may be such an instance. Namely, the observed smallness of the axial charge of the proton may be due to a cancellation between the axial charge of quarks, which are expected to carry most of the proton’s helicity, and a chirality-violating contribution triggered by the axial anomaly through the nonperturbative gluon configurations required to solve the U(1) problem. This implies that, even at high energy, chiral symmetry can be violated in the strong interactions, and entails the possibility of unusual polarization effects.

It is the purpose of this paper to describe one such effect. We shall compute the effective pseudoscalar (helicity-flipping) nucleon-nucleon-$n$ gluon interaction which is induced by the coupling of the nucleon to instanton vacuum configurations, analogously to what is done in treatments of baryon number violation in the weak interactions. We shall fix its magnitude by assuming it to account for the discrepancy between the experimental and the quark model value of the nucleon axial charge. We shall show that interference between this interaction and that which dominates elastic scattering leads to a nonvanishing value of the polarization $P(t)$ at small scattering angles.

We shall compute $P(t)$ for $t \to 0$, and show that it satisfies a scaling law, in that its forward limit depends only on the center of mass scattering angle $\theta$, and not on the energy. Indeed, we shall show that $P(t)$ saturates the kinematical bound which constrains it to vanish in the forward direction, i.e., $P(t) \sim \sin \frac{\theta}{2}$; we shall compute the slope of the $\sin \frac{\theta}{2}$ dependence and show it to be of order unity and energy independent. We shall see that this is consistent with currently available data.
Let us first briefly recall the import and meaning of the “proton spin” problem.\(^4\) Polarized deep-inelastic scattering experiments provide a measurement of the matrix element of the isosinglet axial current in a nucleon (with momentum \(p\) and helicity \(\lambda\)) in the limit of vanishing momentum transfer \(q\). Because of the absence of a singlet pseudoscalar Goldstone boson, this equals (in the helicity-nonflip channel)

\[
\langle p, \lambda | j_5^\mu | p, \lambda \rangle = \lim_{q \to 0} G_A(q^2) s^\mu(p, \lambda)
\]

where \(s^\mu(p, \lambda)\) is the spin four-vector associated to the given momentum and helicity.

The quark-parton model expectation that quarks carry most of the nucleon’s helicity leads to a value \(G_A(0) \sim 0.6\), while the experimental value is compatible with \(G_A(0) \sim 0.4\).\(^4\) In QCD, however, the chiral symmetry is broken by quantum effects, hence, the singlet axial current is not conserved and its matrix elements may differ from the quark expectation due to the interaction. This can be seen explicitly\(^2\) by noticing that the charge operator \(Q_5 = \int d^3x j_5^0\) is the sum of the canonical fermion helicity operator \(Q_5^q\), plus a gluonic operator \(Q_5^g\) which may in principle provide a large contribution to \(G_A(0)\).

An explicit mechanism which produces a value of the matrix elements of \(Q_5^g\) large and anticorrelated to that of \(Q_5^q\) has been suggested in Ref. \(^3\). This is based on the fact that the nonperturbative vacuum structure of QCD induces an effective helicity-flipping quark-quark interaction.\(^6\)\(^-\)\(^12\) That is, the vacuum can be approximated\(^6\)\(^9\) by a semiclassical superposition of gauge fields which tunnel between vacua connected by topologically nontrivial gauge transformations (instantons). These generate an effective fermion-fermion interaction\(^10\) that leads to processes where \(2N_f\) units of chirality are created. More generally, since the vacuum is not an eigenstate of chirality, \(2n\) units of chirality may be
created, while the remaining \(2(N_f - n)\) go into the vacuum mean-fields. This may provide a contribution to the axial form factor \(G_A(0)\).}

Whereas such contribution may be computed exactly only in simplified models, as that of Ref.\(^3\), let us investigate the consequence of assuming that such a contribution is sizable. Due to the anomaly equation\(^4\) satisfied by \(\partial \mu j_5^\mu\), Eq.\((1)\) implies

\[
\lim_{q \to 0} iG_A(q^2)q_\mu \bar{u}_{\lambda'}(p') \gamma_\mu \gamma_5 u_\lambda(p) = \langle p, \lambda' | \left( \frac{N_f}{8\pi^2} g^2 \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \sum_{\text{flavors}} 2i \imath \bar{\psi}_i \gamma_5 \psi_i \right) | p, \lambda \rangle. \tag{2}
\]

The anomaly density on the r.h.s. of Eq.\((2)\) is proportional to the instanton density \(Q(x)\): \(\frac{N_f}{8\pi^2} g^2 \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(x) = -2Q(x)\). Hence, the first term on the r.h.s. of Eq.\((2)\) receives a direct contribution from the instanton-nucleon-nucleon coupling; indeed, in the semiclassical approximation we may view the first term on the r.h.s. of Eq.\((2)\) as the matrix element for an instanton-nucleon-nucleon transition. If we were able to single out the instanton contribution to the r.h.s. of Eq.\((2)\), then we could view Eq.\((2)\) as an expression for the effective instanton-nucleon-nucleon coupling, which has the form

\[
\langle p, \lambda' | Q_{\text{Inst}}(x) | p, \lambda \rangle = -\lim_{q \to 0} iM_N G_A^{\text{Inst}}(q^2) \bar{u}_{\lambda'}(p') \gamma_5 u_\lambda(p) = -\lim_{q \to 0} iM_N G_A^{\text{Inst}}(0) \bar{u}_{\lambda'}(p') \gamma_5 u_\lambda(p), \tag{3}
\]

where \(M_N\) is the nucleon mass. Notice that the induced effective coupling is purely helicity-flipping (i.e. it is nonzero only if \(\lambda = -\lambda'\)) and chirality-flipping, even when \(M_N \neq 0\) and \(q \neq 0\). The energy dependence of the forward axial form factor \(G_A^{\text{Inst}}(0)\) which fixes the strength of the effective coupling \(3\) is entirely fixed by the fact that, due to the topological quantization condition on the instanton density, the instanton contribution to the axial charge \(\Pi\) is scale independent.\(^{13}\) It follows that \(G_A^{\text{Inst}}(0)\) is a universal, energy-independent coupling.
However, the r.h.s of Eq.(2) also receives contributions from non-instantonic field configurations, thus, there is no way to disentangle the instanton contribution to Eq.(2), except by looking at other processes. As is well known, in the semiclassical limit the non-vanishing chirality-flipping amplitude for instanton-quark-quark processes also implies the non-vanishing of processes with the same number of quarks and \(n\) extra gluons. That is, if \(G^{\text{Inst}}_A(0) \neq 0\), then Eq.(3) implies the existence of the effective nucleon-nucleon-\(n\) gluon coupling:

\[
\langle p, \lambda'; g_1, \ldots, g_n | p, \lambda \rangle = \lim_{q \to 0} iM_N G^{\text{Inst}}_A(q^2) \bar{u}_{\lambda'}(p) \gamma_5 u_\lambda(p) \prod_{i=1}^{n} \left( \frac{2}{g_s} \eta^{a_i}_{\mu_i \nu_i} k_{i \mu_i}^2 \right) ,
\]

where \(g_s\) is the strong coupling, \(\eta^{a_i}_{\mu_i \nu_i}\) are the 't Hooft symbols, \(a_i\), \(k_{i \mu_i}\), and \(\nu_i\) are respectively the color indices, four-momenta, and Lorentz indices of the \(i\)-th gluon, and the external gluon propagators have been amputated but there are no wave functions on the gluon legs. The characteristic parameter of the background field \(\rho\) (instanton radius) should be integrated over; we shall replace it with its mean value \(\rho_0\), determined phenomenologically.

A nonzero value of \(G^{\text{Inst}}_A(0)\) can be tested by looking at processes induced by the effective coupling with a distinct signature. The peculiar helicity structure of the nucleon line in Eq.(4) suggests to look at single-spin effects. These are generated by interference between the (elastic) helicity-nonflip amplitudes and the amplitude where only one of the nucleons’ helicities is flipped (and are accordingly quite hard to reproduce using standard QCD techniques).

We shall model the elastic process with Landshoff’s nonperturbative Pomeron, where \(p-p\) scattering at small angles is given by a single diagram which describes \(t\)-channel exchange of a coherent, color-singlet state of gluons (Pomeron). This model is
phenomenologically quite successful. A single-helicity-flip amplitude (Fig. 1) is obtained by assuming that the gluons emitted by the effective instanton-induced interaction (4) “hadronize” into the Pomeron, which is essentially a two-gluon state. Typical gluon multiplicities associated to semiclassical instanton-induced gluon emission are expected to be \(< 1/\alpha_s\); hence, we assume that at small momentum transfer the color-singlet component of the effective coupling (4), in the case \(n = 2\), “hadronizes” into the Pomeron state with a probability of order unity. The same assumption for the inverse process — namely that a Pomeron emitted by a proton in the \(t\)-channel at small \(t\) fragments with unit probability into a two-gluon state — leads to predictions in good agreement with experiment for soft diffractive nucleon-nucleon scattering.

We compute the amplitude displayed in Fig. 1 by projecting the color-singlet component of a two-gluon state generated via Eq.(4) on the Pomeron with a hadronization constant \(C_H\), of order unity. We get

\[
M_{++;+-} = iM_n G^\text{Inst}_A(t) \bar{u}_+(p_1') \gamma_5 u_-(p_1) \left( \frac{2}{g_s} \right)^2 \times C_H \langle f^\mu | k^2 \rangle \rho_0^4(3\beta) F_1(t) D^2(t) \bar{u}_+(p_2') \gamma_\mu u_+(p_2)
\]

where \(f^\mu\) is a Pomeron wave function. The parameter \(\beta = 1.8\ \text{GeV}^{-1}\), the elastic form factor \(F_1(t)\), and the Pomeron propagator \(D(t)\) are characteristic of the helicity-nonflip amplitude which dominates small-angle elastic scattering:

\[
M_{++;++} = \bar{u}_+(p_1') \gamma_\mu u_+(p_1) \left[ (3\beta) D(t) F_1(t) \right]^2 \bar{u}_+(p_2') \gamma_\mu u_+(p_2).
\]

This is known to be phenomenologically rather accurate when \(-t \lesssim 3\ \text{GeV}^2\).

The polarization is:

\[
P(\theta) = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow} = -2 \frac{\text{Im} \left[ \phi_5 (\phi_1 + \phi_2 + \phi_3 - \phi_4) \right]}{2 |\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + |\phi_5|^2}.
\]
where, assuming that at small $t$ and large enough $s$ the helicity flipping processes are controlled by the nonperturbative vertex (4), the five independent amplitudes $\phi_i$ are either expressed in terms of Eqs. (5), (6), or else they are negligible at small $t$:

$$\phi_1 = \phi_3 \approx M_{++;++}$$

$$\phi_2 \approx \phi_4 \approx M_{++;--} \propto \sin^2 \frac{\theta}{2}$$

$$\phi_5 \approx \frac{1}{2} (M_{++;+-} - M_{++;+-}) = M_{++;+-}. \tag{8}$$

Here $\theta$ is the center-of-mass scattering angle, and we neglected the contribution where the final state particles are exchanged. This obtains

$$P(\theta) = \sin \frac{\theta}{2} \frac{4\sqrt{2}}{3} G_A^{\text{Inst}}(-t) M_N \rho_0^4(k^2) \frac{\beta F_1(t) g_s^2}{\beta F_1(t) g_s^2} + O(\sin^2 \frac{\theta}{2}), \tag{9}$$

where $\langle k^2 \rangle$ is the average square momentum of the gluons in the Pomeron.

The form factor $F_1(t)$ is constant to good approximation if $-t \lessapprox 0.1 \text{GeV}^2$; the detailed $t$ dependence of the form factor $G_A^{\text{Inst}}(-t)$ Eq.(3), instead, is unknown. If we assume it to be controlled by the instanton density, as the model computation of Ref.(3) suggests, then its slope in $\sqrt{-t}$ is of the order of $V_4 n$ where $n$ is the instanton density, and the proton four-volume is roughly $V_4 \approx \frac{1}{\sqrt{-t}} \text{Fm}^3$. This gives a negligible variation in the $t$ range we are interested in, for reasonable values of the instanton density $n \sim 1 \text{ GeV}^4$. Notice that the slope of the full axial form factor $G_A(-t)$ Eq.(1) is expected from current algebra to be about the same as that of $F_1(t)$ Eq.(3). Thus, for small enough $|t|$ the polarization is

$$P(\theta) = P_0 \sin \frac{\theta}{2} \tag{10}$$

where the slope parameter $P_0$ is a universal constant, given by

$$P_0 = \frac{4\sqrt{2}}{3} G_A^{\text{Inst}}(0) M_N \rho_0^4(k^2) \frac{\beta g_s^2}{\beta g_s^2}. \tag{11}$$
This is our main result.

It should be noticed that whereas the magnitude of the polarization follows from our computation, its sign is not well determined: the relative phase between the two amplitudes \((5),(6)\) which enter the expression of the polarization \((7)\) through Eq.\((8)\) is fixed by hermiticity and TCP invariance, but their relative sign cannot be established with certainty in our model. We can get a feeling for the size of \(P_0\) by taking the instanton radius to be \(\rho_0 \approx \frac{1}{2} \text{Fm}\), the strong coupling \(g_s \sim 2\), and estimating the average virtuality of gluons in the Pomeron to be \((k^2) \approx (0.4 \text{ GeV})^2\) (which corresponds to the given value of \(g_s(k^2)\)). Finally, we assume \(G_A \approx 0.5\) — that is, we assume that the instanton contribution reduces by that amount the quark model value \(G_A \approx 0.6\). With these values we get \(P_0 \approx 0.8\).

Because angular momentum conservation forces the polarization to vanish at least as \(P(t) \sim \sin \frac{\theta}{2}\), Eq.\((10)\) shows that we predict that the kinematical bound is saturated. Furthermore, the coefficient of the \(\theta\) slope as \(\theta \to 0\) is a universal energy-independent constant. This is to be contrasted with the behavior that one would predict naively on the basis of the expectation that chiral symmetry be restored at high energy (thereby forbidding the helicity flipping amplitudes like \(M_{++,-+}\)), namely, that the slope decreases as \(\frac{M_N}{\sqrt{s}}\) (or some power of it).

Eq.\((10)\) is valid for small enough \(|t|\) and large enough \(s\). Effects which spoil its validity are: The \(t\)-dependence of the form factors, which sets in when \(-t \gtrsim 0.2 \text{ GeV}^2\); The breakdown of the simple-Pomeron exchange mechanism for elastic scattering, which occurs at somewhat larger values \(-t \gtrsim 0.3 \text{ GeV}^2\); The set-in of different helicity-flipping mechanisms (not based upon the anomaly) which are suppressed by the restoration of chiral symmetry,
i.e. by powers of $\frac{M^2}{\sqrt{s}}$, and are thus non-negligible only for $s \lesssim 10 \text{ GeV}^2$; The need to retain higher order terms in $\sin \frac{\theta}{2}$, required above $\frac{-t}{s} \sim 0.1$. The exchange contributions to Eq.(8) is negligible within these bounds. In sum, the range of applicability of Eq.(10) is $s \gtrsim 10 \text{ GeV}^2$, $-t \lesssim 0.1 \text{ GeV}^2$.

Let us review the reliability and robustness of our results. Our main assumption is that the instanton-induced helicity-flipping coupling (3) suggested in Refs.(2,3) may be treated semiclassically in an instanton model in order to relate it to the amplitude with gluon emission (4). Whereas the details of the computation leading to Eq.(10) depend on this assumption, the main feature of the result (10), namely the scaling law which it displays, does not. Indeed, a nonvanishing amplitude $\phi_5$ Eq.(8) which does not decrease as $M/\sqrt{s}$ is possible only if chiral symmetry is broken at arbitrarily high energy, thus it is a signal of an effect induced through the anomaly by nonperturbative field configurations.

The relative phase which is required in order to get nonvanishing polarization according to Eq.(7) is necessarily present, because the anomaly equation implies that such a coupling must be pseudoscalar (Eq.(4)), and this has the required phase by hermiticity and TCP invariance. Thus, whereas the instanton provides a well-defined model to perform computations, a scaling law for the polarization Eq.(10) is the signal that the much more general\footnote{The phenomenon of nonperturbative vacuum tunneling is at work. On the contrary, the numerical value of the coefficient $P_0$ should be taken as an order-of-magnitude estimate, since it is sensitive to poorly known theoretical parameters such as $\rho$ and $\langle k^2 \rangle$.}

Let us finally turn to the experimental situation. In order to test Eq.(10), ideally one would need to plot a few values of the polarization for $-t \lesssim 0.1 \text{ GeV}^2$ at various energies $s \gtrsim 10 \text{ GeV}^2$, and check that they all lie on the same line. A few data\footnote{are available\cite{21-25},}
despite the small values of the scattering angle involved. In Fig. 2 we plot the slope parameter $P_0$ extracted from these data sets by taking the point at the lowest available value of $t$ for each set, and assuming that it obeys the law Eq. (10). Clearly, the data do show a large value of $P_0$, which seems energy-independent to very good approximation. Notice that the data point with the lowest value $s = 10 \text{ GeV}^2$ is at the extreme of the allowed $s$ range. A more stringent test of our prediction would require the availability of several values of the polarization at different small angles. These currently are not available, perhaps also due to lack of theoretical motivation. If experimentally confirmed, this would provide the first direct evidence for non-perturbative QCD effects.

In conclusion, it is interesting to observe that due to the universality of both the Pomeron and the anomalous coupling (13) these effects should be present also in different related processes, such as $p-\bar{p}$ or $\pi-p$ scattering, which are currently under investigation.

**Acknowledgment:** We thank E. Predazzi for discussions.
References

[1] See e.g. E. Leader, in “Spin and Polarization Dynamics in Nuclear and Particle Physics”, A. O. Barut, Y. Onell and A. Penzo, Eds. (World Scientific, Singapore, 1990)

[2] S. Forte, Phys. Lett. B224, 189 (1989); Nucl. Phys. B331, 1 (1990)

[3] S. Forte and E. V. Shuryak, Nucl. Phys. B357, 153 (1991)

[4] For a review, see G. Altarelli, in “Proceedings of the 1989 Erice school” (Plenum, New York (1990). See also R. L. Jaffe and A. Manohar, Nucl. Phys., B337, 509 (1990)

[5] See R. Jackiw, in S. B. Treiman, R. Jackiw, B. Zumino and E. Witten, “Current Algebra and Anomalies” (World Scientific, Singapore, 1985)

[6] See R. Jackiw, Rev. Mod. Phys. 49, 681 (1977)

[7] G. ’t Hooft, Phys. Rep. 142, 357 (1986)

[8] A. Ringwald, Nucl. Phys B330, 1 (1990); for a review see M. Mattis, Phys. Rep., B214, 159 (1992)

[9] See e.g. C. Bourrely, J. Soffer and E. Leader, Phys. Rep. 59, 95 (1980)

[10] G. ’t Hooft, Phys. Rev. D14, 3432 (1976)

[11] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B163, 46 (1980)

[12] See E. V. Shuryak, Phys. Rep. 115, 151 (1984)

[13] S. Forte, Acta Phys. Pol. B22, 1065 (1991). In parton language, the instanton contribution should thus be identified with the “quark” component of the proton spin, which (as discussed by Altarelli, Ref.[4]) is defined as the scale invariant eigenstate of the Altarelli-Parisi equation satisfied by the first moments of the polarized quark and gluon distributions.

[14] Notice that the naive identification of the two terms on the r.h.s. of Eq.(2) with a gluon and a quark contribution, respectively, is incorrect. See G. M. Shore and G. Veneziano Phys. Lett. B244, 75 (1990); Nucl. Phys B381, 3 (1992)

[15] Eq.(4) is written for simplicity in the case of gauge group SU(2); in the physically relevant case of QCD this must be embedded into SU(3), as discussed in Ref.(11).

[16] P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. B32, 54 (1971); G. A. Jaroskiewicz and P. V. Landshoff, Phys. Rev. D10, 170 (1974).

[17] P. V. Landshoff and O. Nachtmann, Z. Phys. C35, 405 (1987)

[18] G. Ingelman and P. E. Schlein, Phys. Lett. B152, 256 (1985)

[19] A. Donnachie and P. V. Landshoff, Nucl. Phys. B244, 322 (1984) and B303, 634 (1988);
A. Schäfer, O. Nachtmann, and R. Schöpf, Phys. Lett. B249, 331 (1990)

[20] E. M. Levin and M. G. Ryskin, Sov. J. Nucl. Phys. 34, 619 (1982)

[21] M. Borghini et al. Phys. Lett. B36, 501 (1971)

[22] D. G. Crabb et al. Nucl. Phys. B121, 231 (1977)
[23] S. L. Kramer et al. Phys. Rev. D17, 1709 (1978)
[24] A. Gaidot et al. Phys. Lett. B61, 103 (1976)
[25] J. H. Snyder et al. Phys. Rev. Lett. 41, 781 (1978)
FIGURE CAPTIONS

[Fig. 1] The helicity-flipping amplitude Eq.(3)

[Fig. 2] The value of the forward polarization slope $P_0$ [Eq.(10)] versus $s$ (in GeV$^2$). The cross indicate data from Ref.(21), the diamond data from Ref.(22), the square the data from Ref. (23), the dash data from Ref.(24), and the star data from Ref.(25). The solid line is drawn through the mean value $P_0 = 0.83$. 
