The topological RN-AdS black holes cannot be overcharged by the new version of gedanken experiment

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Abstract

In this paper, we test the weak cosmic censorship conjecture (WCCC) for \( n \geq 4 \) dimensional nearly extremal RN-AdS black holes with non-trivial topologies, namely plane and hyperbola, using the new version of gedanken experiment proposed by Sorce and Wald. Provided that the non-electromagnetic part of the stress tensor of matter fields satisfies the null energy condition and the linear stability condition holds, we find that the black holes cannot be overcharged under the second-order perturbation approximation, which includes the self-force and finite-size effects. As a result, we conclude that the violation of Hubeny type never occurs and the WCCC holds for the topological RN-AdS black hole.

Keywords: Weak cosmic censorship conjecture, RN-AdS black holes, non-trivial topologies

1. Introduction

The inevitability of singularities is guaranteed by the singularity theorem proposed by Penrose and Hawking [1, 2]. The existence of spacetime singularity poses a problem for the general relativity, in the sense that the theory does not apply to the spacetime region close to the singularity. To resolve the problem, Penrose proposed the weak cosmic censorship conjecture [3], stating that the singularities must be hidden from an observer at infinity by the event horizon of a black hole. Due to the difficulty of proving this conjecture universally, researchers hope to test the validity of this conjecture by considering physically reasonable processes. In this regard, Wald first proposed a gedanken experiment to see whether an extremal Kerr-Newman black hole can be destroyed by throwing a test particle into it [4]. As a result, he showed that the test particle that would cause the black hole to be destroyed cannot be captured, in support of WCCC. Nonetheless, Hubeny found that a near-extremal RN black hole can be destroyed if the infalling matter’s parameters are chosen appropriately [5], and some subsequent studies also claimed to have succeeded in overspining Kerr black holes [6, 7] or Kerr-Newman black holes [8, 9]. However, the back-reaction effects are not taken into account in the above investigations [5, 10, 11, 12, 13, 14, 15, 16]. Thus, if we want to make certain that these potential violations occur, we need a more comprehensive analysis of the back-reaction effects in these processes, which includes but is not limited to self-force and finite-size effects.

For this purpose, Sorce and Wald have recently proposed a new gedanken experiment to test WCCC [17]. Rather than analyze the motion of test particles to determine whether or not they fall into black holes, they assume all charged matter that is initially present is absorbed by the black holes. Assume that the non-electromagnetic part of the stress tensor of matter fields satisfies the null energy condition and the linear stability condition holds, they estimate the second-order correction to the black hole parameters, where the self-force and finite size effects are automatically taken into account. As a result, they show that the event horizon of nearly extremal Kerr-Newman black holes cannot be destroyed under the second-order perturbation approximation, although the black holes can be destroyed at the linear order. This demonstrates the back-reaction effects protect the validity of WCCC. With this new gedanken experiment, numerous physical processes involved in the investigation of WCCC have been examined or reexamined [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37], indicating that WCCC holds.

In [31, 32], the weak cosmic censorship conjecture for asymptotically AdS charged black holes with spherical topology have been investigated. However, the perturbation of matter fields considered therein is restricted to be...
radial. On the other hand, it is well-known that the presence of a negative cosmological constant allows for more diverse topologies for black holes\cite{38, 39, 40, 41, 42}. In particular, the AdS black hole horizon with the maximal symmetry can not only be spherical, but also be topological, namely planar or hyperbolic. Numerous studies have established that the topological black holes with planar and hyperbolic horizons exhibit new properties that are not observed in spherical black holes\cite{43}. With this in mind, we aim to apply the new gedanken experiment in a uniform manner to both spherical and topological charged AdS black holes, where the aforementioned perturbation of matter fields is relaxed to be as general as possible. As a result, the violation of Hubeny type never occurs in topological case as in spherical case.

The organization of the paper is as follows. In Section II, we study the perturbation of RN-AdS black holes under the matter field, where the spherical, planar and hyperbolic are treated in a uniform manner. In Section III, based on the null energy condition and linear stability condition, we deduce the first and second-order perturbation inequalities by using Iyer-Wald formalism. In Section III, we test WCCC for the RN-AdS black holes. Section IV includes some concluding remarks.

2. The perturbed RN-AdS black holes

Let us start with $n$-dimensional Einstein-Maxwell gravity with a negative cosmological constant, whose Lagrangian is written as

$$L = \frac{\epsilon}{16\pi} \left( R - 2\Lambda - F_{ab}F^{ab} \right) + L_{\text{matter}},$$  \hspace{1cm} (1)$$

where $\epsilon$ is the volume element, $R$ is the Ricci scalar, $F = dA$ is the electromagnetic field strength, with $A$ is the vector potential of electromagnetic field, $\Lambda$ is the negative cosmological constant and $L_{\text{matter}}$ denotes the Lagrangian of extra matter fields. The equations of motion (EOM) of this theory can be written as

$$8\pi T_{ab} = G_{ab} - \Lambda g_{ab} - 8\pi T_{ab}^{EM},$$  \hspace{1cm} (2)$$

$$\nabla_a F^{ab} = 4\pi j^b,$$  \hspace{1cm} (3)$$

with

$$T_{ab}^{EM} = \frac{1}{4\pi} \left( F_{ac}F^c_b - \frac{1}{4} g_{ab}F_{cd}F^{cd} \right),$$

where $T_{ab}$ corresponds to the non-electromagnetic part of the stress tensor and $j^a$ corresponds to the electromagnetic current.

When the extra matter fields vanish, the theory yields a sequence of maximally symmetric black hole solutions, denoted by

$$ds^2 = -f(r)dr^2 + \frac{2drd\nu + r^2d\Omega_{n-2}^{(k)}}{4\pi Q},$$  \hspace{1cm} (4)$$

where $k$ indicates the spatial curvature of the black hole, and

$$f(r) = k - \frac{p}{r^{n-3}} + \frac{q^2}{r^{n-6}} + \frac{r^2}{L^2},$$

$$d\Omega_{n-2}^{(k)} = \frac{1}{\Omega_{n-2}^{(k)}} (x) dx^1dx^n,$$

with $l$ is the AdS radius. Specifically, $k = \{1, 0, 0\}$ corresponds to spherical, planar and hyperbolic horizon topologies, respectively, and $\Omega_{n-2}^{(1,0,-1)}$ is the volume of $(n-2)$ dimensional “unit” sphere, plane or hyperbola, respectively, where the latter two (non-compact) have been compactified properly\cite{44, 45}. Furthermore, the parameters $p$ and $q$ are related to the mass and the charge of the black hole as

$$p = \frac{16\pi M}{(n-2)\Omega_{n-2}^{(k)}}, \hspace{1cm} q = \frac{4\pi Q}{\Omega_{n-2}^{(k)} \sqrt{(n-2)(n-3)Q}}.$$  \hspace{1cm} (5)$$

Following that, we consider the case $f(r)$ has at least two roots, and we refer to the maximal root of $f(r)$ as $r_h$, which corresponds to the black hole event horizon. Then the temperature, area and electric potential of the event horizon are given by

$$T = \frac{f'(r_h)}{4\pi}, \hspace{1cm} A = \Omega_{n-2}^{(k)} r_h^{n-2}, \hspace{1cm} \Phi_H = \frac{4\pi Q}{(n-3)\Omega_{n-2}^{(k)} r_h^{n-3}}.$$  \hspace{1cm} (6)$$

Furthermore, when $f(r)$ has no root, the solutions describe naked singularities.

With the new gedanken experiment, we assume that the background spacetime is linearly stable to perturbations, i.e., the linear perturbed RN-AdS black hole will evolve into another RN-AdS black hole at sufficiently late times, then the late spacetime geometry will be generally written as

$$ds^2 = -f(r, \lambda)dr^2 + 2drd\nu + r^2d\Omega_{n-2}^{(k)},$$  \hspace{1cm} (7)$$

where

$$f(r, \lambda) = k - \frac{p(\lambda)}{r^{n-3}} + \frac{q^2(\lambda)}{r^{n-6}} + \frac{r^2}{L^2}.$$  \hspace{1cm} (8)$$

3. Null energy conditions and perturbation inequalities

In this section, we will use the Iyer-Wald formalism\cite{46, 47} to derive perturbation inequalities. Consider a $n$-dimensional Einstein-Maxwell theory with a negative cosmological constant, whose $n$-form Lagrangian is written as

$$L = \frac{\epsilon}{16\pi} \left( R - 2\Lambda - F_{ab}F^{ab} \right).$$  \hspace{1cm} (9)$$

Then we focus on such a one-parameter family $\phi(\lambda)$ that satisfies the EOM (2) and describes the whole perturbation process, where $\phi$ denotes $(\gamma_{ab}, A_a)$ and extra matter fields. The first-order variation of the Lagrangian gives

$$\delta L = E_\phi \delta \phi + d\Theta(\phi, \delta \phi),$$  \hspace{1cm} (10)$$

Here $k$ indicates the spatial curvature of the black hole, and
When the EOM are satisfied, (10) reduces to

\[ E \Theta = -\epsilon \left( \frac{1}{2} T^{ab}_{\phi} \delta g_{ab} + j^a \delta A_a \right), \]

\[ \Theta (\phi, \delta \phi) = \Theta^{GR} (\phi, \delta \phi) + \Theta^{EM} (\phi, \delta \phi), \]

\[ \Theta^{GR}_{a_2 \cdots a_n} (\phi, \delta \phi) = \frac{1}{16 \pi} \epsilon_{a_2 \cdots a_n} g^{de} g^{fh} (\nabla_h \delta g_{ef} - \nabla_e \delta g_{fh}), \]

\[ \Theta^{EM}_{a_2 \cdots a_n} (\phi, \delta \phi) = -\frac{1}{4 \pi} \epsilon_{a_2 \cdots a_n} \epsilon^{de} \delta A_{de}, \]

where \( E \Theta = 0 \) is the EOM of this theory and \( \Theta \) is called symplectic potential.

The \((n - 1)\)-form symplectic current then is defined as

\[ \omega (\phi, \delta \phi, \delta \phi) = \delta_1 \Theta (\phi, \delta \phi) - \delta_2 \Theta (\phi, \delta \phi). \quad (8) \]

The Noether current, which is associated with arbitrary vector field \( \chi^a \), is defined as

\[ J_{\chi} = \Theta (\phi, L_{\chi} \phi) - \tau_{\chi} L (\phi), \quad (9) \]

where \( \tau_{\chi} L (\phi) \) denotes the vector \( \chi^a \) contracting with the first index of \( n \)-form Lagrangian. A simple calculation which combines (7) and (9) can show that the exterior derivative of \( J_{\chi} (\phi) \) gives

\[ dJ_{\chi} = -E \Theta L_{\chi} \phi. \quad (10) \]

When the EOM are satisfied, (10) reduces to \( dJ_{\chi} = 0 \), implying the Noether current defined by (9) satisfies the conserved equation.

On the other hand, it was shown in [47] that the \((n - 1)\)-form Noether current can also be written as

\[ J_{\chi} = C_{\chi} + dQ_{\chi}, \quad (11) \]

\[ (C_{\chi})_{a_2 \cdots a_n} = \chi^a \epsilon_{a_2 \cdots a_n} (T^b_{an} + A_a b), \]

\[ Q_{\chi} = Q^{GR}_{\chi} + Q^{EM}_{\chi}, \]

\[ (Q^{GR}_{\chi})_{a_3 \cdots a_n} = -\frac{1}{16 \pi} \epsilon_{a_3 \cdots a_n} \nabla^d \chi^e, \]

\[ (Q^{EM}_{\chi})_{a_3 \cdots a_n} = -\frac{1}{8 \pi} \epsilon_{a_3 \cdots a_n} F^{d e} \chi^e A_{de}, \]

where \( Q_{\chi} \) is called the Noether charge and \( C_{\chi} \) describes the constraints of the theory.

Comparing the variation of (9) and (11), we obtain the first-order variational identity

\[ d (\delta Q_{\chi} - \chi \cdot \Theta (\phi, \delta \phi)) = \omega (\phi, \delta \phi, L_{\chi} \phi) - \chi^a E \delta \phi = \delta C_{\chi}. \quad (12) \]

Varying (12) gives the second order variational identity

\[ d (\delta^2 Q_{\chi} - \chi \cdot \delta \Theta (\phi, \delta \phi)) = \omega (\phi, \delta \phi, L_{\chi} \delta \phi) - \chi^a (E \delta \phi) - \delta^2 C_{\chi}, \quad (13) \]

where we have assumed that \( \chi^a \) is a symmetry of \( \phi \), i.e., \( L_{\chi} \phi = 0 \).

In order to test WCCC, we assume that all the charged matter enters the black hole via a finite portion of the future event horizon. Based on this, we can always choose a hypersurface \( \Sigma = \bar{\mathcal{H}} \cup \Sigma_1 \) such that it begins at the unperturbed bifurcation surface \( B \) and continues up the future event horizon. Based on this, we can always choose a hypersurface \( \Sigma = \bar{\mathcal{H}} \cup \Sigma_1 \) such that it begins at the unperturbed bifurcation surface \( B \) and continues up the future event horizon. Based on this, we can always choose a hypersurface \( \Sigma = \bar{\mathcal{H}} \cup \Sigma_1 \) such that it begins at the unperturbed bifurcation surface \( B \) and continues up the future event horizon.
term gives

\[
\int_S \delta C_\xi = \int_H \delta C_\xi + \int_{\Sigma_1} \delta C_\xi
= \delta \int_H \xi^a \epsilon_{ba_2 \ldots a_n} (T_a^b + A_a J^b)
= -\left( \int_H \xi^a k_b \delta T^{ab} \epsilon_{a_2 \ldots a_n} \right)
+ (\xi^a A_a) \delta \int_H \epsilon_{ba_2 \ldots a_n} J^b
= -\left( \int_H \xi^a k_b \delta T^{ab} \epsilon_{a_2 \ldots a_n} \right) - \Phi_H \delta Q,
\]

where \( k^a \propto \xi^a \) is the future-directed tangent vector field to the horizon and \( \epsilon_{a_2 \ldots a_n} \) denotes the volume element on the horizon, which is defined as \( \epsilon_{ba_2 \ldots a_n} = -nk[\xi_{a_2 \ldots a_n}] \).

As a result, (14) can be rewritten as

\[
\delta M - \Phi_H \delta Q = \int_H \xi^a k_b \delta T^{ab} \epsilon_{a_2 \ldots a_n}.
\]  

(17)

Assume that the non-electromagnetic stress tensor satisfies the null energy condition on the horizon, i.e. \( \delta T^{ab} \xi_a \xi_b |_H \geq 0 \), we will obtain an inequality

\[
\delta M - \Phi_H \delta Q \geq 0,
\]  

(18)

which is referred to as the first-order perturbation inequality.

In this paper, we investigate whether WCCC holds for RN-AdS black holes in terms of the second-order perturbation approximation. For this purpose, we assume that the first-order perturbation has been done optimally, i.e. \( \delta T^{ab} \xi_a \xi_b |_H = 0 \), which indicates that the non-electromagnetic energy flux associated with the first-order perturbation through the horizon vanishes. Under this assumption, one can show that the first-order perturbation of expansion \( \delta \theta \) vanishes on the horizon \( H \) [48, 49].

With the optimal condition, we turn to derive the second-order perturbation inequality. Similar to the first-order perturbation, the integral of (13) on \( \Sigma \) is written as

\[
\int_{\partial \Sigma} \left[ \delta^2 Q_\xi - \xi \cdot \delta \Theta (\phi, \delta \phi) \right] = \varepsilon_H (\phi, \delta \phi) + \varepsilon_{\Sigma_1} (\phi, \delta \phi) - \int_{\Sigma} \delta \Theta (E \delta \phi) - \int_{\Sigma} \delta^2 C_\xi,
\]

(19)

where \( \varepsilon_H (\phi, \delta \phi) = \int_H \omega^{GR} (\phi, \delta \phi, \xi \delta \phi) + \int_H \omega^{EM} (\phi, \delta \phi, \xi \delta \phi) \), \( \varepsilon_{\Sigma_1} (\phi, \delta \phi) = \int_{\Sigma_1} \omega (\phi, \delta \phi, \xi \delta \phi) \).

Based on the assumption that the second-order perturbation vanishes at the bifurcation surface \( B \), one can show that the left hand side of (19) gives

\[
\int_{\partial \Sigma} \left[ \delta^2 Q_\xi - \xi \cdot \delta \Theta (\phi, \delta \phi) \right] = \delta^2 M.
\]  

(20)

For the gravitational part of the canonical energy \( \varepsilon_H (\phi, \delta \phi) \), we borrow the result from [49, 50, 51], which shows

\[
\int_H \omega^{GR} (\phi, \delta \phi, \xi \delta \phi) \geq 0.
\]  

(21)

Furthermore, the electromagnetic part of \( \varepsilon_H (\phi, \delta \phi) \), whose calculation is similar to [17], can be showed as

\[
\int_H \omega^{EM} (\phi, \delta \phi, \xi \delta \phi) = -\frac{1}{2\pi} \int_H \xi^\delta F^{ab} \delta F_{cb} \epsilon_{a_2 \ldots a_n}
= \int_H \delta^2 T^{ab} k^a \epsilon_{b} \xi \delta T^{\xi_a \xi_b} \epsilon_{a_2 \ldots a_n} \geq 0,
\]

(22)

which implies the total flux of electromagnetic energy into the black hole is nonnegative.

Furthermore, it is obvious that the third term of right-hand side of (19) vanishes, and the fourth term gives

\[
\int_{\Sigma} \delta^2 C_\xi = \int_{\Sigma_1} \delta^2 C_\xi + \int_H \delta^2 C_\xi
= \int_H \delta^2 C_\xi
= -\int H \hat{\epsilon} (\delta^2 T^{ab} \xi_a k_b) - \Phi_H \delta^2 Q.
\]  

(23)

Until now, the only term that has not been obtained yet is the canonical energy on \( \Sigma_1 \). In order to calculate this term, we consider another one-parameter family of field configuration \( \phi^{RA} (\lambda) \), which describes the RN-AdS black hole with parameters given by

\[
M^{RA} (\lambda) = M + \lambda \delta M, \quad Q^{RA} (\lambda) = Q + \lambda \delta Q,
\]  

(24)

where \( \delta M \) and \( \delta Q \) equal to the corresponding value of our previous first-order perturbation when we set \( \delta \phi^{RA} = \delta \phi \). For this family, we have \( \delta^2 M = \delta^2 Q = \delta^2 C_\xi = \delta E = \delta H (\phi, \delta \phi^{RA}) = 0 \), then (19) gives

\[
\varepsilon_{\Sigma_1} (\phi, \delta \phi) = \varepsilon_{\Sigma_1} (\phi, \delta \phi^{RA})
= -\int B \left[ \delta^2 Q_\xi - \xi \cdot \delta \Theta (\phi, \delta \phi^{RA}) \right]
= \frac{4\pi}{(n-3) r_h^{-n-3} \Omega_{n-2}} (\lambda Q)^2.
\]  

(25)

Putting these results together, we conclude that (19) can result in the following second-order perturbation inequality, which is written as

\[
\delta^2 M - \Phi_H \delta^2 Q - \frac{4\pi}{(n-3) r_h^{-n-3} \Omega_{n-2}} (\lambda Q)^2 \geq 0,
\]  

(26)

where we have assumed that the null energy condition for matter fields is satisfied, i.e. \( \delta^2 T^{ab} \geq 0 \).
4. Gedanken experiment to destroy a nearly extremal RN-AdS black hole

In this section, we attempt to overcharge a nearly extremal RN-AdS black hole using the previously mentioned gedanken experiment. All that we need to do is to determine whether the perturbed geometry at late times still describes a black hole, which is equivalent to investigating if the solution of \( f(r, \lambda) = 0 \) exists. For simplicity, we consider the function

\[
F(r, \lambda) = r^{n-3} f(r, \lambda) = kr^{n-3} - p(\lambda) + \frac{q^2(\lambda)}{r^{n-3}} + \frac{r^{n-1}}{l^2},
\]

whose largest root gives the location of the event horizon \( r_h \).

Moreover, we define

\[
g(\lambda) = F(r_m(\lambda), \lambda) = kr_m^{n-3} - p(\lambda) + \frac{q^2(\lambda)}{r_m^{n-3}} + \frac{r_m^{n-1}}{l^2},
\]

where \( r = r_m(\lambda) \) gives the minimal value of \( F(r, \lambda) \). So we have

\[
\partial_{\lambda} F(r_m(\lambda), \lambda) = 0
\]

and

\[
\delta [\partial_{\lambda} F(r_m(\lambda), \lambda)] = 0,
\]

which imply that

\[
\delta r_m = \frac{-2(3-n)r_m^{2-n}(\lambda)q(\lambda)\delta q}{\partial_{\lambda} F(r_m(\lambda), \lambda)}.
\]

(31)

Notice that the violation of the black hole happens when \( g(\lambda) > 0 \). For convenience, we will refer to \( r_m(0), r_h(0) \) as \( r_m, r_h \) respectively in the following.

Considering (29) and (30), we expand \( g(\lambda) \) to the second-order and obtain

\[
g(\lambda) = F(r_m, 0) + \lambda \left( -\delta p + \frac{2q\delta q}{r_m} \right) + \lambda^2 \left( -\delta^2 p + \frac{2q\delta^2 q}{r_m} \right) + O(\lambda^3).
\]

(32)

Let us consider a nearly extremal black hole as the background spacetime, whose event horizon has the relationship \( r_m = (1 - \epsilon) r_h \), with a small parameter \( \epsilon > 0 \). With above preparation, (29) gives

\[
\partial_{\lambda} F(r_h, 0) = \epsilon r_h \partial_{\lambda} F(r_h, 0) + O(\epsilon^2).
\]

(33)

Thus, for the first term of (32), we have

\[
F(r_m, 0) = F((1 - \epsilon) r_h, 0)
= -\epsilon r_h \partial_{\lambda} F(r_h, 0) + \frac{1}{2} \epsilon^2 r_h^2 \partial_{\lambda}^2 F(r_h, 0) + O(\epsilon^3)
= \frac{1}{2} \epsilon^2 r_h^2 \partial_{\lambda}^2 F(r_h, 0) + O(\epsilon^3).
\]

(34)

Combine (31) and (34), (32) gives

\[
g(\lambda) = \frac{1}{2} \epsilon^2 r_h^2 \partial_{\lambda}^2 F(r_h, 0) + \left( -\delta p + \frac{2q\delta q}{r_h^{n-3}} \right) \lambda
- \frac{2(3-n)q\delta q}{r_h^{n-3}} \epsilon \lambda + \left( -\delta^2 p + \frac{q\delta^2 q}{r_h^{n-3}} + \frac{q\delta q}{r_h} \right)^2 \lambda^2
- \frac{2}{\partial_{\lambda}^2 F(r_h, 0)} \left( (3-n)r_h^{2-n}q\delta q \right)^2 \lambda^2
+ O(\lambda^3, \epsilon^2, \epsilon^3, \lambda \epsilon^2, \lambda^2)\]

\[\leq \frac{1}{2} \epsilon^2 r_h^2 \partial_{\lambda}^2 F(r_h, 0) - \frac{2(3-n)q\delta q}{r_h^{n-3}} \epsilon \lambda
- \frac{2}{\partial_{\lambda}^2 F(r_h, 0)} \left( (3-n)r_h^{2-n}q\delta q \right)^2 \lambda^2
+ O(\lambda^3, \epsilon^2, \epsilon^3, \lambda \epsilon^2, \lambda^2)\]

\[= \left( \epsilon r_h \partial_{\lambda}^2 F(r_h, 0) + 2(3-n)r_h^{2-n}q\delta q \lambda \right)^2
+ O(\lambda^3, \epsilon^2, \epsilon^3, \lambda \epsilon^2, \lambda^2)\]
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