Maximum Synchrotron Frequency for Shock Accelerated Particles

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ABSTRACT

It is widely believed that the maximum energy of synchrotron photons when electrons are accelerated in shocks via the Fermi process is about 50 MeV (in plasma comoving frame). We show that under certain conditions, which are expected to be realized in relativistic shocks of gamma-ray bursts, synchrotron photons of energy much larger than 50 MeV (comoving frame) can be produced. The requirement is that magnetic field should decay downstream of the highest energy electrons before they lose half their energy; photons of energy much larger than 50 MeV are produced close to the shock front whereas the highest Lorentz factor that electrons can attain is controlled by the much weaker field that occupies most of the volume of the shocked plasma.

Key words: radiation mechanisms: non-thermal - methods: analytical - gamma-rays burst: general.

1 INTRODUCTION

High energy radiation from many astrophysical objects is believed to be generated in shock heated plasma where charged particles are accelerated by the Fermi mechanism, eg. Fermi (1949), Axford et al. (1977), Bell (1978), Blandford & Ostriker (1978), Achterberg et al. (2001), Sironi & Spitkovsky (2011). The maximum particle energy is set by the condition that radiative losses between acceleration episodes (which is the time it takes to travel from one side of the shock front to the other) should be smaller than the energy gain. This upper bound to electron energy means that the emergent synchrotron radiation falls off to zero above a certain cut-off frequency which is about 50 MeV in the plasma rest frame independent of the magnetic field strength in the shock heated fluid (de Jager & Harding, 1992; Lyutikov, 2009; Piran & Nakar, 2010; the derivation is provided in the next section).

Many astrophysical objects, such as supernova (SN) remnants, pulsars, AGNs and gamma-ray bursts (GRBs), emit photons of energy larger than ~10^7 MeV, and this upper limit has been used to eliminate the electron synchrotron process in shock-heated plasma as the radiation mechanism for these objects. However, we show in this Letter that the ~50 MeV upper limit can be violated under certain astrophysically realistic conditions.

The upper limit of 50 MeV for synchrotron radiation is arrived at by assuming a uniform magnetic field throughout shock heated plasma. We show that this upper limit can be considerably raised when the magnetic field is not uniform. A particular case we analyze in some detail is where the magnetic field decays with distance downstream of the shock front, and find that for a range of parameters there can be a significant synchrotron flux even at several GeV (in fluid comoving frame). This is described in section 2. The calculations presented in section 2 can be easily generalized to consider a situation where magnetic field fluctuates randomly instead of decaying with distance from the shock front. The application to a few astrophysical systems are presented in §3.

2 MAXIMUM SYNCHROTRON FREQUENCY FOR SHOCK HEATED PLASMA

The argument that the maximum photon energy for synchrotron emission for shock accelerated electrons is ~50 MeV is straightforward and goes as follows.

Electrons gain energy by a factor ~2 whenever they cross a relativistic shock front and are scattered back to the other side. The time it takes for a charge particle of Lorentz factor γ_e to make this trip from one side of the shock front to the other is of order of the Larmor time, \( t_L = \frac{m_e \gamma_e}{qB} \); where \( B \) is the comoving frame magnetic field strength, \( m \) is the particle mass, and \( q \) is its charge. The energy lost to synchrotron radiation during this time is \( \delta E \sim t_L \sigma_T B^2 \gamma_e^2 c/6\pi \sim \sigma_T B^2 \gamma_e^2 m c^2/(6\pi q). \) Particle acceleration ceases when \( \delta E \sim m c^2 \gamma_e/2, \) and therefore the maximum Lorentz factor a particle can attain is given by

\[
\frac{\sigma_T B^2 \gamma_e^2}{3 \pi q} \sim 1 \quad \text{or} \quad \gamma_{\text{max}}^2 \sim \frac{9 m^2 c^4}{8 B q^3},
\]  

(1)
where \( \sigma_T = 8\pi e^4/(3m^2c^3) \) is the Thomson cross-section. The synchrotron photon energy corresponding to \( \gamma_{max} \) is

\[
h\nu_{max} \sim \frac{qB_{max}^2h}{2\pi mc} \sim \frac{9m^3c^3h}{16\pi q^2} \tag{2}
\]

Thus, the maximum photon energy for electrons is \( \sim 50 \) MeV and for protons \( \sim 10^2 \) GeV. These numbers might be overestimated by a factor 5-10 due to uncertainty regarding the time it takes particles to travel from one side of the shock front to the other (which could be an order of magnitude larger than the Larmor time assumed in these calculations, e.g. Achterberg et al. 2001, Lemoine & Revenu 2006, Sagi & Nakar 2012, Uchiyama et al. 2007). All calculations in this paper are affected by this uncertainty. However, the ratio of \( \nu_{max} \) for the case of a uniform magnetic field to that for an inhomogeneous field geometry, discussed below, should be fairly secure.

### 2.1 Inhomogeneous magnetic field and the maximum synchrotron frequency

Let us consider that the magnetic field decays with distance downstream of the shock front as

\[
B(x) = B_w(x/L_p)^{-\eta} + B_0 \tag{3}
\]

where \( L_p \) is the length scale over which the field decays, \( B_w \) & \( B_0 \) are the strongest and weakest magnetic field strengths in the shocked fluid, and \( x \geq L_p \). If magnetic field is generated by the Weibel mechanism then \( L_p \) is of order the plasma length scale (Medvedev & Loeb, 1999), i.e.

\[
L_p = \left[ n_p \Gamma e^2/(4\pi n_e q^2) \right]^{1/2} = 2.2 \times 10^7 \text{cm}(\Gamma/\eta)^{1/2} \tag{4}
\]

where \( n_p \) is proton mass, \( \Gamma \) is the Lorentz factor of the shock front wrt the unshocked fluid, and \( n_e \) is the number density of electrons in the shocked fluid comoving frame.

Particles accelerated in a shock influence the generation of magnetic fields and can substantially increase the length scale for field decay (Medvedev et al. 2005; Medvedev & Zakutnyaya, 2009; Keshet et al. 2009). A larger coherence length magnetic field produced by high energy particles – which have larger plasma scale – decays on a longer time scale. However, these fields are also likely to be weaker (Medvedev & Zakutnyaya, 2009), although their true strength is quite uncertain. We, therefore, use measurements of magnetic fields in GRB afterglows for guidance. The volume averaged magnetic field energy density in GRB external shocks, while they are relativistic, is found to be a factor \( \sim 10^4 \) smaller than the equipartition value for a good fraction of bursts (Panaitescu & Kumar, 2001, 2002; Bjornsson et al. 2004; Wei et al. 2006; Rykoff et al. 2006; Rol et al. 2007; Chandra et al. 2008; Liang et al. 2008; Gao et al. 2009; Xu et al. 2009; Cenko et al. 2010; Rossi et al. 2011; Kumar & Barniol Duran, 2010). This suggests that the field decays with distance downstream by a factor \( \sim 10^2 \) since the strength close to the shock front, due to the Weibel instability, is of order of the equipartition value. The length scale for this decay, however, is uncertain. For the sake of concreteness we take \( L_p \) to be plasma scale for our calculations. Fortunately, the results presented in this paper don’t depend on the precise length-scale for field decay as long as it is much smaller than the distance traveled by the highest energy electrons before they lose their energy to radiation.

The Larmor radius of an electron, \( R_L(\gamma_e) = m_e c^2 \gamma_e/(qB) \), increases with distance from the shock front as the magnetic field gets weaker (eq. 3), and an electron traveling down-stream is likely to be sent back upstream when \( R_L \ll x \) (\( x \) is the distance from the shock-front). Therefore, an absolute upper limit on \( \gamma_e \) is set by the requirement that \( R_L \) is smaller than the width of the shock heated plasma.

A more stringent upper limit on \( \gamma_e \) is obtained by the requirement that energy lost by an electron to radiation while traveling from upstream to downstream (in between two consecutive episodes of energy gain) should be smaller than \( m_e \gamma_e c^2/2 \).

The energy loss rate for an electron of LF \( \gamma_e \) due to synchrotron radiation while traveling down-stream of the shock front is

\[
\frac{d(m_e c^2 \gamma_e)}{dt} = \frac{\sigma_T}{6\pi} B^2 \gamma_e^2 c = -\frac{\sigma_T}{6\pi} \left( \frac{B_w}{x} \right)^{\eta} \left[ \frac{B_w}{L_p} \right]^{\eta} + B_0^2 \tag{5}
\]

We are interested in the case where \( B_w \gg B_0 \). However, since \( L_p \) – the length scale over which the magnetic field decays – is much smaller than the thickness of the shocked plasma, most of the synchrotron loss occurs in the region of low magnetic field \( (x \gg L_p) \), which also controls the maximum Lorentz factor of electrons, provided that

\[
B_w^2 \left[ 1 - (L_p/x_0)^{2\eta} \right] / (2\eta - 1) < B_0^2 \left( R_L/L_p \right), \tag{6}
\]

where \( x_0/L_p \equiv (B_w/B_0)^{1/\eta} \); the above equation is obtained by calculating energy loss in the region of low magnetic field \( (B) \) where the electron travels a distance \( \sim R_L \), and the loss in the region \( L_p \lesssim x < x_0 \), and requiring the former to be larger. Since \( x_0/L_p \gg 1 \) for the case of interest, the above condition simplifies to

\[
(B_w/B_0)^2 \lesssim R_L/L_p \tag{7}
\]

when \( \eta > 1/2 \). Thus, one of the requirements for exceeding the \( \sim 50 \) MeV limit is that the width of the region occupied by high magnetic fields \( (x_0) \) is much smaller than \( R_L \). The above condition (eq. 13) also guarantees that the Larmor radius of electron in the region of high magnetic field is much larger than \( L_p \), and thus the deflection of electron orbit while passing through this region is small; electrons are turned around in the region of low magnetic field which occupies most of the shocked plasma volume.

The maximum LF of an electron, \( \gamma_{max} \), is obtained by requiring that the energy loss due to synchrotron radiation in a Larmor time \( (R_L/c) \) is less than \( m_e \gamma_{max} c^2/2 \). The case of interest is where electrons lose their energy while traveling through the region of weak magnetic field. In this case \( \gamma_{max} \) is same as given in equation (1), i.e.

\[
\gamma_{max}^2 \sim \frac{9m^2c^4}{8\sigma_T q^2 B_0}. \tag{8}
\]

This Lorentz factor must satisfy the condition that \( R_L(\gamma_{max}) \) is smaller than the comoving radial width of the shocked fluid (the width is \( R/\Gamma \) by causality argument), i.e. the electron is confined to the system. This requires

\[
B_0 \gtrsim \left[ \frac{3\pi}{\sigma_T q} \right]^{1/3} \left( m_e c^2 \Gamma / R \right)^{2/3} \approx 0.1 \text{mG} \left( \Gamma / R_{17} \right)^{2/3}, \tag{9}
\]

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where $\Gamma$ is the Lorentz factor of the shocked fluid, $R$ is the distance of the shock front from the center of explosion, and we have adopted the widely used convenient notation $x_n \equiv x/10^n$.

Electrons also suffer IC loss of energy which is ignored here; considering that $\gamma_{\text{max}} \sim 10^8 B_0^{-1/2}$, IC scatterings are in Klein-Nishina regime and thus the IC loss is highly suppressed (Barniol Duran & Kumar, 2011).

The maximum synchrotron frequency, in shock comoving frame, is given by

$$\nu_{\text{max}} \sim \frac{q \gamma_{\text{max}} B_w}{2\pi m_e c} \sim \frac{9 m_e c^3}{16 \pi q^2} \left( \frac{B_w}{B_0} \right) \sim 50 \text{ MeV} (B_w/B_0),$$

which is larger than the case of a uniform magnetic field by a factor $B_w/B_0$.

We next estimate the synchrotron flux at $\nu_{\text{max}}$ to see if it lies on the powerlaw extension of flux at lower frequencies (< 50 MeV) or not.

Let us consider electrons of LF $\gamma > \gamma_{\text{max}}/2^{3/2}$ which produce synchrotron photons of frequency $\nu > \nu_{\text{max}}/2$. By the definition of $\gamma_{\text{max}}$ these electrons will lose half of their energy while traveling down-stream of the shock front. The energy fraction of these electrons lost to synchrotron radiation ($\xi$) while they travel through the region where the magnetic field strength is $\sim B_w$ is

$$\xi \sim \sum \frac{B_w^2 L_p}{B_0^2 R_L(\gamma_{\text{max}})} \sim 2 \times 10^{-4} \left( \frac{B_w}{B_0} \right)^2 B_w^3/(\Gamma_s/n_e)^{1/2},$$

where $R_L(\gamma_{\text{max}})$ is the Larmor radius, and we made use of equations (4) & (5) to substitute for $L_p$ & $\gamma_{\text{max}}$. The frequency of radiation produced in the region of high magnetic field (Bw) is $\sim \nu_{\text{max}}$ which is given by equation (10). Much of the rest of the electron energy is lost to radiation in the region occupied by the weaker field $B_0$, and the synchrotron frequency of the emergent radiation from this region is $\nu_{\text{max}}(B_0/B_w) \equiv \nu_{\text{low}}$, where $\nu_{\text{low}} \sim 50$ MeV is the maximum synchrotron frequency when the magnetic field is uniform.

The specific flux at $\nu_{\text{max}}$ is obtained from the equation $\nu_{\text{max}} f(\nu_{\text{max}}) \sim \xi \nu f(\nu_{\text{low}}).$ Therefore, the spectral index between $\nu_{\text{low}}$ and $\nu_{\text{max}}$ defined as $f \propto \nu^{-\beta}$ is

$$\beta = -1 + \ln(\xi) - \ln(\nu_{\text{max}}/\nu_{\text{low}}) = 1 + \frac{1.5 \ln(\nu_{\text{low}})}{\ln(\nu)} + \frac{0.5 \ln(\Gamma_s/n_e)}{\ln(\nu)}.\label{beta}$$

The second equality is obtained by substituting for $\xi$ from equation (11). It can be shown, after some algebra, that $\beta$ cannot be larger than $-p/2$, where $p$ is the index for electron distribution.

We note that the synchrotron radiation formula can only be used provided that the magnetic field is coherent on scale $\sim R_L/\gamma_{\text{max}} \sim m_e c^2/(q B_w) \sim 1.7 \times 10^3 B_w^{-1} \text{ cm}$. The coherence scale for magnetic field close to the shock-front is $\sim L_p \gtrsim 2 \times 10^7 (\Gamma_s/n_e)^{1/2} \text{ cm}$. Thus, we see that the coherence length for magnetic fields is much larger than $R_L/\gamma_{\text{max}}$ as long as $B_w \gtrsim 0.1 \text{ mG}$. The coherence scale grows with increasing distance from the shock front (since smaller scale fields have time to decay), and so does the Larmor radius. However, as long as $B_0 \gtrsim 1 \text{ mG}$, the magnetic field is coherent on length scales that are much larger than $R_L/\gamma_{\text{max}}$ everywhere down-stream of the shock front. Therefore, it is safe to use the synchrotron radiation results as we have in this section.

3. APPLICATIONS TO GRBS AND SUPERNOVA REMNANTS

We consider two particular applications of the results found in the previous section, one of which is the application to GRB afterglow radiation, and the other is the application to synchrotron radiation from SN remnants.

3.1 Gamma-ray Bursts

The relativistic jet produced in GRBs drives a shock wave into the medium in the vicinity of the burst, and synchrotron radiation from shock heated plasma is responsible for the long lived afterglow radiation from radio to $\gamma$-ray frequencies (e.g. Piran, 2004; Meszaros, 2006; Gehrels et al. 2009; Zhang, 2007). Roughly half of the GRB jet energy is imparted to the surrounding medium at the deceleration radius, $R_d$, which is given by, e.g. Piran (2004), Zhang (2007)

$$R_d = \left[ \frac{3 E}{4 \pi n_0 m_p c^2 \Gamma_0^2} \right]^{1/3} \sim 1.2 \times 10^{17} \text{ cm} (E_{50}/n_0)^{1/3} \Gamma_{0.2}^{2/3},$$

where $E$ is the isotropic equivalent of energy carried by the jet, $n_0$ is the average number density of protons in the unshocked medium within the region of radius $R_d$, and $\Gamma_0$ is the initial Lorentz factor of the jet. The Lorentz factor of the jet decreases with distance as $R^{-3/2}$. The comoving radial width of the shocked plasma is $\delta R \sim R/\Gamma$, and therefore $\delta R/L_p \sim 10^3 R_1 \Gamma_2^{-1} (n_e/\Gamma_1)^{1/2}$.

The minimum magnetic field expected in the shocked fluid is $B_0 = 4 \Gamma L_{\text{sh}}$ which is simply the field of the GRB circum-stellar-medium (CSM) compressed by the relativistic shock by a factor $4 \Gamma$; where $L_{\text{sh}}$ is the field in the CSM at a distance $\sim R_4$ from the center of explosion and is expected to be of order a few $\mu G$. Thus, the condition that electrons of Lorentz factor $\gamma_{\text{max}}$ are confined to the shocked fluid by the weak magnetic field (see eq. 5) is satisfied for GRB external shocks as long as the shock is highly relativistic ($\Gamma \gtrsim 10$).

The field is likely amplified by a large factor immediately down-stream of the shock front by the two-stream instability, such that the energy density in the magnetic field is a fraction, $\epsilon_B$, of the thermal energy density of the shocked fluid (Medvedev & Loeb, 1999). In this case

$$B_w \sim (32 \pi B R_0 n_0 m_p c^2 \Gamma_2^2)^{1/2},$$

with $\epsilon_B \sim 10^{-1}$ (Medvedev & Loeb, 1999); we have used here the relativistic self-similar shock solution (Blandford & McKee, 1976) according to which the thermal energy density behind the shock front is $4 \pi n_0 m_p c^2 \Gamma_2^2$. This magnetic field has a coherence length of order the plasma scale and decays down-stream of the shock front (Gruzinov, 2001; Sironi & Spitkovsky, 2011; but see also Medvedev et al. 2005). It is unclear what is the exact value of the field very far from the shock front. According to recent numerical simulations of Sironi & Spitkovsky (2011) the far field might be of order the shocked compressed ISM field; empirically we know from GRB afterglows that $\epsilon_B \lesssim 10^{-4}$ for a large fraction of bursts (see §2.1 for references to a number of papers on this topic). Therefore, for GRB afterglows we expect the ratio between $B_w$ and $B_0$ to be larger than $\sim 10^6$ and possibly of order

$$\frac{B_w}{B_0} \sim \frac{\left( 2 \pi B_R n_0 m_p \right)^{1/2} c}{B_{\text{sh}}} \sim 9.5 \times 10^3 \left( \epsilon_B n_0 \right)^{1/2} B_{\text{sh}}^{-1}$$

where $B_{\text{sh}}$ is the ISM magnetic field in units of $10 \mu$G; the precise value of $B_w/B_0$ is not important as long as it is larger than

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than 50 MeV
certain range of parameters the flux at photon energies much larger
in Barniol Duran & Kumar (2011) and Piran & Nakar (2010). For
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low
Photons of energy
ergy electrons travel before losing half their energy. Substituting
from one side of the shock front to another is larger than
the system is larger than
η = 0.6, 0.7 and 0.9 for the red, black and blue lines, respectively. For
to that at 100 MeV would be much larger than the observed value (large
cooling frequency, ~ 10 MeV, implies small magnetic field).

Figure 1. Numerical spectra for different values of $R_L/L_p$ and $\eta$. The
solid, dashed and dotted lines correspond to $R_L/L_p = 10^7$, $3 \times 10^6$
and $10^5$, respectively (and $\eta = 0.7$). Similarly, $R_L/L_p = 10^7$ and
$\eta = 0.6, 0.7$ and 0.9 for the red, black and blue lines, respectively. For
all these cases, $\gamma_{\text{max}}/\gamma_{\text{min}} = 10^8$ and $B_0/B_p = 10^3$, and the size of
the system is larger than $R_L$ by a factor of ~ 10. The spectrum above
$v_{\text{low}} \equiv 9m_e c^3/(16\pi q^2) \sim 50$ MeV, has a power-law slope roughly
equal to the spectrum at lower frequencies and it extends to well beyond
$v_{\text{low}}$. The flux at $v_{\text{low}}$ can exhibit a sharp decline by almost an order of
magnitude for certain parameters, however, the flux at $\gtrsim 1$ GeV even in the
worst case lies on a slightly steeper power-law extension of the lower energy
(\lesssim 50$ MeV) spectrum.

\begin{equation}
\sim 10, \text{ and the field decays on a length-scale that is short com-
pared with the Larmor radius ($R_L$) or the distance the highest en-
ergy electrons travel before losing half their energy. Substituting
this into equation (10) we find

$$h\nu_{\text{max}} \sim 0.5 \text{ TeV (eB}_0 n_0^{1/2} B_{L,15}^{-1}. (16)$$

The maximum energy for synchrotron photons in the observer
frame is $T \nu_{\text{max}}/(1+z)$. If the time it takes for an electron to travel from
one side of the shock front to another is larger than $R_L/c$ by a
factor 5–10 then $\nu_{\text{max}}$ would be smaller than given by the above
equation by the same factor.

Using equations (12) & (15) one can obtain an approximate
analytical power-law spectral index for $v_{\text{low}} < v < \nu_{\text{max}}$; how-
ever, we have calculated the spectrum numerically for different
choices of parameters and present the results in Figure 1.

The spectrum in the observer frame should extend to $\sim$ TeV
energies provided that IC losses don’t prevent electrons from ac-
celerating to $\gamma_{\text{max}} \sim 10^6$ (eq. 8), and these high energy photons are
not converted to pairs while traveling from the emission site to us4.
The effect of IC loss on $\gamma_{\text{max}}$, in the context of GRBs, is discussed in
Barniol Duran & Kumar (2011) and Piran & Nakar (2010). For
certain range of parameters the flux at photon energies much larger
than 50 MeV $\times$ T lies below the power-law extrapolation of flux be-
low $v_{\text{low}}$ (Fig. 1), and in those cases the flux at very high energies
is likely to be lower than the sensitivity limit of current detectors.

Several bursts detected by the Fermi satellite show emission
above 10 GeV in the GRB host galaxy rest frame: Abdo et al.
(2009a,b, 2010), Ackermann et al. (2010, 2011). These bursts
violate the conventional upper limit of $\sim 50\Gamma/(1+z)$ MeV $\sim 5

4 Photons of energy $\gtrsim$ TeV from a source at cosmological distance are con-
verted to electron-position pairs due to interaction with infrared background
radiation.

$\gamma_{\text{max}} \sim 10^8$ for synchrotron radiation from shock heated plasma, eg. Piran
& Nakar (2010). However, the $>10^2$ MeV data for these bursts is
in excellent agreement with synchrotron radiation from the exter-
nal shock, which also provides a very nice fit to the late time x-ray,
optical and radio data using the same exact parameters that provide
a fit to the early $>10^2$ MeV data (Kumar & Barniol Duran, 2009, 2010).

This conflict between the excellent fit to the high energy data
on one hand and the conventional upper limit on synchrotron pho-
ton energy of $\sim$ 5 GeV on the other hand can be resolved provided
that the magnetic field just behind the shock front is much larger
than for down-stream; photons of energy $\gtrsim$ 5 GeV are produced
by electrons of LF $\sim \gamma_{\text{max}}$ while they travel through regions of
stronger magnetic field close to the shock front.

We note that there is a large uncertainty regarding the length
scale over which fields decay, $B_a$ & $B_o$. However, the comoving
frame 50 MeV limit on synchrotron photons is violated whenever
$B_a/B_o \gtrsim 10$, and the distance electrons of LF $\gamma_{\text{max}}$ travel before
losing their energy to radiation is large compared to the length-
scale for the decay of magnetic field. The data for GRBs with $\gtrsim 10$
GeV emission show that the volume averaged magnetic field in the
shocked fluid is a factor 10 smaller than the equipartition value4
(Kumar & Barniol Duran, 2010) whereas the Weibel generated field
strength in the immediate vicinity of the shock front is expected to be
$\sim$ 30% of the equipartition value (Medvedev & Loeb, 1999),
and therefore $B_a/B_o \gtrsim 10^2$ for these bursts.

3.2 Supernova remnants

The deceleration radius for supernova remnants is

$$R_d \approx \left[\frac{3E}{4\pi n_0 m_{\text{p}0}^2}\right]^{1/3} \approx 5 \times 10^{18} (E_{51}/n_0)^{1/3} v_{0.9}^{-2/3} \text{ cm, (17)}$$

where $E$ is the kinetic energy of the remnant, and $v_{0.9}$ is its
initial speed in unit of $10^9$ cm/s. The radius increases with time as
$R_d \approx \delta R/L_p \sim 2 \times 10^9 n_0^{-1/2}$ cm. Therefore, $R_d/L_p \gtrsim 10^{11}$.

The equipartition magnetic field for the remnant is

$$B_a \sim (40 \pi m_{\text{p}0} n_0)^{1/2} v \sim 1 \text{ mG n}_0^{1/2} v_8, \text{ (18)}$$

where $v$ is the remnant speed. The mean magnetic field of the remnant
should be larger than, or equal to, the shock compressed magnetic field
of the CSM, i.e. $B_a \gtrsim 5 B_{\text{iso}} \sim 10 \mu G$. Therefore, $B_a/B_o \gtrsim 10^2$.

The maximum Lorentz factor for electron for this mean mag-
netic field is $\gamma_{\text{max}} \sim [3\pi q/(\sigma_T B_0)]^{1/2} \sim 3 \times 10^{10}$ (see eq. 8), and
and its Larmor radius is $5 \times 10^{10}$ which is smaller than the shell
width when the velocity drops below 10 cm/s.

The synchrotron cooling time for electrons with Lorentz factor
$\gamma_{\text{max}} \sim 10^{10}$ traveling in a field of strength $B_0$, which occupies
much of the volume of the remnant, is $t_c \approx 6 \pi m_e c/\sigma_T B_0^2 \gamma_{\text{max}}$.
In other words, the distance electrons travel before cooling down is

$\gamma_{\text{max}} \sim 10^{10}$ for synchrotron radiation from shock heated plasma, eg. Piran
& Nakar (2010). However, the $>10^2$ MeV data for these bursts is
\( L_c \sim cL_L \sim 8 \times 10^{18} \text{ cm} \) which is of order of the remnant width when the remnant velocity drops below \( 10^3 \text{ cms}^{-1} \).

Thus, \( L_c / L_L \sim 10^{11} \), and so \( L_c / L_p \gg (B_L / B_0)^2 \sim 10^4 \). Therefore, the specific flux at 50 MeV \( (B_W / B_0) \sim 5 \text{ GeV} \) is a factor \( \xi \sim 10^7 \) smaller than the flux at 50 MeV, which is perhaps too small to be of practical consequence. However, if the magnetic field in the remnant decays not on the plasma length scale but on \( \sim 10^4 \) times the plasma scale then the flux at \( \nu_{\max} \sim 5 \text{ GeV} \) would lie on the extension of the powerlaw spectrum at lower energies.

4 CONCLUSIONS

We have shown that photons of \( \gtrsim 10 \text{ GeV} \) energy observed from several GRBs by the Fermi satellite can be produced via the synchrotron process in the shock heated circum-stellar medium. The conventional wisdom that the maximum energy of photons in this situation should not exceed \( \sim 50 \Gamma^2 (1+z) \text{ MeV} \) \( \sim 5 \text{ GeV} \) is violated because this limit is obtained by assuming a uniform magnetic field downstream of the shock front. However, when the field is much stronger close to the shock front and decays downstream (such as when magnetic fields are generated by Weibel-type instability\(^6\)), the maximum photon energy is larger than this limit by a factor of the ratio of field just behind the shock front and far downstream; photons of energy \( \gtrsim 10^2 \text{ GeV} \) (in observer frame) can be generated this way in GRB early afterglows.

The upper limit of \( \sim 50 \text{ MeV} \) is difficult to violate for synchrotron radiation from supernova remnants unless the field were to decay on a length scale much larger than proton plasma length but smaller than the distance highest energy electrons travel before losing half their energy.

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\(^6\) Large variations in magnetic field strength across the shocked plasma could also arise as a result of field in the circum-stellar medium varying with distance from the progenitor star which is compressed and amplified near the shock front. Long-duration GRBs and SNe are produced when a massive star dies, and the medium within a few parsecs of the progenitor star is carved out by its wind within the last \( \sim 10^5 \) years of the collapse. Although little is known about the wind and its associated magnetic field in the last \( \sim 10^5 \) years of a star’s life, it is possible that the magnetic field in the wind could undergo large variation with time due to the magnetic cycle of the star driven by its rapid rotation and sub-surface convection.

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