Natural Characteristics of The Herringbone Gear Transmission System

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Abstract. According to the structure characteristics of herringbone gear transmission, a more realistic dynamic model of the transmission system is built in consideration of the inner excitation, herringbone gears axial positioning and sliding bearing etc. The natural frequencies of the system are calculated, and the vibration mode is divided into symmetric vibration modes and asymmetric vibration modes. The time history of system dynamic force is obtained by solving the dynamic model. The effects of the connection stiffness of left and right sides of herringbone gears and axial support stiffness on natural characteristics are discussed.

1. Introduction

Gear transmission is one of the most extensively adopted transmission modes; herringbone gear is favorably characterized by high bearing capacity, steady operation and small axial force [[1]], having been extensively applied to heavy load device, noise is associated with comfort and reliability of equipment directly. Therefore, proper establishment of transmission system analysis model to accurately analyze its dynamic characteristic shall be the basis to improve transmission system performance.

Scholars have made remarkable achievement in terms of dynamic characteristic of gear transmission system. Feng set up herringbone gear pair analysis model by three-dimensional finite element method to carry out analysis of contact behavior and modality of gear pair [[2]]. Wang established dynamical model for bending-torsion-axis coupling of herringbone gear based on lumped parameter method, then the influence of various excitations and gear tooth corrections on dynamic characteristic of herringbone gear are analyzed [[3]]. Wang built a 12-DOF nonlinear herringbone gear transmission model, the effect of the meshing stiffness and corner mesh impact on vibration characteristics under multi-load were studied [[4]]. Zhao proposed a parabola modification method for herringbone gears, and examples show that path of contact and transmission errors of optimized gears are preferable distinctly [[5]]. Considering that axial component of herringbone gear is identical in size and opposite in direction in theory, therefore, some researches on building of analysis model have ignored axial vibration of herringbone gear, which is reduced to spur gear, however, for herringbone gear with larger width-diameter ratio, such reduction is likely to bring about bigger error in calculation.

The paper has built the dynamic model of herringbone gear transmission, in which the axial positioning of herringbone gear and inner excitation are considered. The natural characteristic of herringbone gear transmission system is analyzed.
2. Dynamical model building

2.1. Dynamical model for herringbone gear transmission system

In the process of dynamical model building, left hand and right hand gears in gear reducer are taken as helical gear, two gears adopt axis section connection with stiffness, with bending-torsion-axis coupling vibration model for herringbone gear built based on the relationship shown in assembly drawing of gear reducer, as shown in fig. 1, in which X and Y are transverse vibration respectively, Z is axial, p denotes input end, g indicates output end, \( k_{mi} \) in the fig represents meshing stiffness (in which i=1,2 indicates helical gear pair on left and right sides respectively). Every gear adopt sliding bearing support, in order to embody asymmetry of oil film stiffness\[6\], the paper adopts four stiffness coefficients to describe oil film stiffness of sliding bearing, in which \( k_{pix} \) and \( k_{piy} \) represent bearing support stiffness at input end; \( k_{gix} \) and \( k_{giy} \) represent bearing support stiffness at output end.

![Fig 1. The dynamics model of herringbone gear system](image)

The paper defines \( k_{p} \) and \( k_{g} \) as stiffness of intermediate connection axis section (namely intermediate tool withdrawal groove) of helical gear on left and right sides, which is abbreviated to intermediate connection stiffness. Since output end adopts axial positioning, therefore, elastic connection is added to \( g_{2} \) along axial direction, \( k_{z} \) indicates axial positioning stiffness, with respect to physical model, \( k_{z} \) is axial stiffness of thrust face on bearing position.

Every gear has four degrees of freedom, namely translational degree of freedom along X, Y and Z directions and torsional degree of freedom around its center Generalized displacement vector of the system is expressed as

\[
\{X\} = \begin{bmatrix} x_{p1} & y_{p1} & z_{p1} & \theta_{p1} & x_{g1} & y_{g1} & z_{g1} & \theta_{g1} \\
p2 & x_{p2} & y_{p2} & z_{p2} & \theta_{p2} & x_{g2} & y_{g2} & z_{g2} & \theta_{g2} \end{bmatrix}^T
\]

Mesh force and damping force of every gear pair are expressed as


\[
F_{K1} = k_m \left[ (R_{p1} \theta_{p1} + x_{p1} \sin \alpha - y_{p1} \cos \alpha) \cos \beta_b + z_{p1} \sin \beta_b 
+ (y_{g1} \cos \alpha - x_{g1} \sin \alpha - R_{g1} \theta_{g1}) \cos \beta_b - z_{g1} \sin \beta_b - e_1 \right]
\]

\[
F_{K2} = k_m \left[ (R_{p2} \theta_{p2} + x_{p2} \sin \alpha - y_{p2} \cos \alpha) \cos \beta_b + z_{p2} \sin \beta_b 
+ (y_{g2} \cos \alpha - x_{g2} \sin \alpha - R_{g2} \theta_{g2}) \cos \beta_b + z_{g2} \sin \beta_b - e_2 \right]
\]

(1)

Where: \( k_m \) is meshing stiffness of gear and \( c_m \) is meshing damping of gear

Dynamic mesh force of gear pair is

\[
F_{pg1} = F_{k1} + F_{g1} \quad F_{pg2} = F_{k2} + F_{g2}
\]

(2)

Model is converted into matrix form

\[
[M][\ddot{x}] + [C][\dot{x}] + [K(t)]\{x\} = \{P(t)\}
\]

(3)

Where: \( M, C, K \) — mass matrix, damping matrix and stiffness matrix

\( X \) — displacement vector

\( P \) — generalized force vector

2.2. Herringbone gear meshing stiffness

Herringbone gear pair is split into two symmetric helical gear pairs as shown in fig.2(a). One meshing period is divided into several parts, with finite element method adopted to calculate helical gear pair stiffness, in which meshing stiffness curve in one period is as shown in fig.2(b).

| Table 1. The system parameters |
|-------------------------------|
| Number of teeth | Modulus (mm) | Pressure angle (°) | Helical angle (°) | Face width B (mm) |
|-----------------|--------------|--------------------|-------------------|-------------------|
| 37              | 106          | 5                  | 20                | 24                | 92                |

![Gear pair model](a) ![Gear mesh stiffness](b)

Fig 2. The Gear pair model and mesh stiffness

Finite element method is adopted to calculate bending, torsional and axial stiffness of intermediate connecting axis to obtain that lateral bending stiffness of intermediate axis section of driving gear is \( k_{px}=1.92 \times 10^9 \) N/m, axial stiffness is \( k_{py}=3.1 \times 10^9 \) N/m, and torsional stiffness is \( k_{p\theta}=1.89 \times 10^{10} \) N·m/rad, while lateral bending stiffness of intermediate axis section of driven gear is \( k_{gx}=1.376 \times 10^9 \) N/m, axial stiffness is \( k_{gy}=1.1 \times 10^{10} \) N/m, torsional stiffness is \( k_{g\theta}=2.7 \times 10^{10} \) N·m/rad.

3. Dynamic characteristic of herringbone gear transmission system

3.1. Inherent characteristic of herringbone gear transmission system

Inherent frequency of system is obtained by solution, as shown in table 2.

| Table 2. The natural frequency of the gear system (Hz) |
|-------------------------------------------------------|

3
| Order | Inherent frequency | Order | Inherent frequency | Order | Inherent frequency |
|-------|-------------------|-------|-------------------|-------|-------------------|
| 1     | 269.59            | 6     | 653.46            | 11    | 1843.73           |
| 2     | 306.64            | 7     | 804.46            | 12    | 2108.46           |
| 3     | 359.60            | 8     | 967.76            | 13    | 2551.08           |
| 4     | 449.18            | 9     | 1507.08           | 14    | 9855.25           |
| 5     | 602.08            | 10    | 1714.18           |       |                   |

Mode of vibration of transmission system is as shown in Fig. 3, the abscissa in the figures corresponds to every degree of freedom of system in turn, namely degree of freedom in dynamical model \( \{X_i\} \), ordinate indicates vibration amplitude relevant to every degree of freedom. It is observed from Fig that mode of vibration of transmission system may be divided into two categories, one is symmetrical mode and the other one is asymmetrical mode, in which the first, third, fourth and sixth order mode of vibration is symmetrical mode of vibration for pinion gear \( p_1, p_2 \) and bull gear \( g_1, g_2 \), other modes of vibration are not fully symmetrical, in which the second order is axial vibration mode of pinion gear \( p_1 \) and \( p_2 \), the fifth order is axial vibration mode of bull gear, the seventh order is lateral vibration mode of bull gear \( g_1 \) and \( g_2 \), the ninth and eleventh orders are lateral vibration mode of pinion gear \( p_1 \) and \( p_2 \), and the thirteenth and fourteenth orders are torsional vibration mode of gear.

![Modal Analysis](image)

Fig. 3. The vibration mode of the transmission system

### 3.2. Dynamic load of system

Fig. 4 shows time domain course and frequency spectrum of dynamic load of gear box bearing along direction \( Y \), in which the mean value of dynamic load of bearing at input end is \( 8,192.3 \text{N} \), being on the contrary to positive direction defined in model and negative value; in addition, gravity action on gear at input end is consistent with dynamic load component on bearing along direction \( Y \), being mutual superposition. Gravity of gear at output end is opposite to dynamic load component on bearing along direction \( Y \), therefore they offset each other, with mean value smaller than that at input end, being \( 6385.4 \text{N} \).

For frequency spectrum, dynamic load on both ends generated larger peak value at mesh frequency and 2-multiple-frequency, since mesh frequency \( (1,233 \text{Hz}) \) of gear pair and inherent frequency \( (1,507 \text{Hz}) \) of lateral vibration of transmission system are similar, accordingly, fundamental harmonic is to directly transferred to bearing and become main part. The peak value at mesh frequency of bearing at input end is maximum, being \( 14.32 \text{N} \).
4. Analysis of influencing factor of dynamic characteristic of system

4.1. Influence of stiffness of intermediate connection of herringbone gear on dynamic characteristic

Influence of intermediate connection stiffness on inherent frequency of transmission system is as shown in fig.5.

Fig 4. The bearing dynamic load of the gear system

Fig 5. The natural frequency of the transmission system under different connection stiffness

Variation in intermediate connection stiffness has no influence on inherent characteristics of first order, third order, fourth order and sixth order, the reason for which is that the mode of vibration corresponding to inherent frequency of the four orders is symmetric vibration centering around intermediate connection axis, degree of freedom on both sides fail to subject to mutual coupling, accordingly, variation in intermediate connection stiffness has no influence on vibration characteristic. Other inherent frequencies are to show monotonous increase trend with the increase of intermediate connection stiffness.

4.2. Influence of axial positioning stiffness on dynamic characteristic

In herringbone gear transmission system, pinion gear is free of positioning in axial direction as a general rule; while, the entire system is subject to axial constraint by axial positioning on bearing thrust face of bull gear. The paper made a calculation of inherent frequency of transmission system in case that axial positioning stiffness is 0.5×kp, 0.75×kp, 1.5×kp and 2×kp, from which it is observed that axial positioning stiffness only exerts an influence on inherent frequency (axial vibration mode of pinion gear) at second order and inherent frequency (axial vibration mode of bull gear) at fifth order of the system, having no influence on other inherent frequencies. The inherent frequency at second order and fifth order of system varies with axial positioning stiffness as shown in fig.6, from which it is observed that inherent frequencies at two orders are on the increase with the increase of positioning stiffness and that the difference is that positioning stiffness is to constrain pinion gear along axial direction in a indirect way by meshing spring after constraining bull gear, therefore, variation in stiffness is to exert insignificant influence on pinion gear, inherent frequency of axial vibration of pinion gear is less dependent on variation of positioning stiffness. Relatively speaking, inherent
frequency at fifth order is the most sensitive to variation in axial positioning stiffness, with larger rate of variation.

Fig. 6: Relationship of the axial positioning stiffness in natural frequency

5. Conclusion
The paper set up dynamic model of the herringbone gear system, and natural characteristics of the gear transmission system are analyzed, some conclusions are found:
(1) the natural characteristic of transmission system may be divided into symmetrical modes and asymmetrical modes. The dynamic load of the bearing contains significant mesh frequency component.
(2) Variation in intermediate connection stiffness of herringbone gear is only to influence asymmetrical modes. With the increase of stiffness value, there is no influence on symmetrical modes, but other inherent frequencies are on the increase.
(3) Axial positioning stiffness of gear is only to influence axial mode of vibration in system. With the increase of axial positioning stiffness, fluctuation of dynamic load of axial support for gearbox is to gradually intensify.

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