Modular Compact Modeling of Magnetic Tunnel Junction Devices

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Abstract—This paper describes a robust, modular, and physics-based circuit framework to model conventional and emerging Magnetic Tunnel Junction (MTJ) devices. Magnetization dynamics are described by the stochastic Landau-Lifshitz-Gilbert (sLLG) equation whose results are rigorously benchmarked with a Fokker-Planck Equation (FPE) description of magnet dynamics. We then show how sLLG is coupled to transport equations of MTJ-based devices in a unified circuit platform. Step by step, we illustrate how the physics-based MTJ model can be extended to include different spintronics phenomena, including spin-transfer-torque (STT), voltage-control of magnetic anisotropy (VCMA) and spin-orbit torque (SOT) phenomena by experimentally benchmarked examples. To demonstrate how our approach can be used in the exploration of novel MTJ-based devices, we also present a recently proposed MEMS resonator-driven spin-torque nano oscillator (STNO) that can reduce the phase noise of STNOs. We briefly elaborate on the use of our framework beyond conventional devices.

Index Terms—Compact modeling, giant spin Hall effect (GSHE), magnetic tunnel junctions (MTJs), spin circuits, spin-torque nano-oscillator (STNO), spin-transfer torque (STT)-MRAM, voltage-controlled magnetic anisotropy (VCMA).

I. INTRODUCTION

The recent progress of spintronics in the past two decades has led to the commercial development of the STT-MRAM technology as a low power and high-speed memory device [1], [2]. Integration of MTJs with existing CMOS devices has necessitated the development of experimentally-informed and physics-based compact models. So far, many MTJ models with varying degrees of sophistication have been developed for circuit simulators such as SPICE and Verilog-A [3]–[16].

The objective of this paper is to present a compact, physics-based SPICE framework for MTJ-based devices that can incorporate emerging materials and phenomena in a modular fashion. Specifically, our approach has the following distinguishing features:

- Each individual module is carefully benchmarked by a physical theory (e.g. spin diffusion equations) and/or by an experiment with stated assumptions.
- The framework can easily be extended to build new devices using existing modules, for example a GSHE (giant spin hall effect)-driven MTJ is modeled by combining the charge-driven MTJ module with a GSHE module without requiring a new model.

We illustrate the modular circuit framework with different examples: We first validate the nanomagnet dynamics captured by the sLLG by benchmarking it against a Fokker-Planck Equation (FPE) for magnets (Section II). Next, we briefly describe the individual transport modules (Section III) and present an experimentally benchmarked STT-MRAM cell (Section IV). We show how the bias-dependent MTJ model can be extended to capture the voltage-control of magnetic anisotropy (VCMA) effect. We then switch to 3-Terminal MTJ devices (Section VI) and combine the existing MTJ models with benchmarked spin-orbit modules. Lastly, we demonstrate how these modules can be combined to describe exploratory and hybrid MTJ devices using a recently proposed MEMS resonator-driven spin-torque nano oscillators (STNO) (Section VII).

In Fig. 1 we show a spin-circuit that is used to model the hybrid MEMS-STNO device discussed in Section VII (Fig. 9). This example combines most of the modules discussed in this paper and we use it to show explicit circuit schematics of each module. Charge currents and voltages (Black lines) are solved according to ordinary circuit theory, while spin-currents (Blue lines) and magnetization vectors (Green lines) are provided to and from the sLLG solver respectively. It is important to note that the time scales for magnetization dynamics are typically much shorter than typical transport times, therefore the charge circuit can be treated as a lumped model for each magnetization vector at a given time, allowing well-defined transient simulations. Each example in this paper has been generated from the circuit models shown in Fig. 1 using the parameters defined in Table I.

II. VALIDATION OF SPICE-BASED sLLG MODEL

In this section, we show how the SPICE-based sLLG equation is benchmarked by the FPE, closely following the treatment outlined in [19]. Fig. 2 shows how the sLLG equation rigorously reproduces the FPE results. The sLLG equation is solved by the transient noise feature (.trannoise) of the HSPICE simulator, however the equations and circuits are simulator-independent and can be implemented by powerful
where the gyromagnetic ratio, $\gamma$ maintains the single domain approximation. The magnetic fields and currents are solved according to ordinary circuit theory, while spin-currents (Blue lines) and magnetization (Green lines) are provided to and from the sLLG solver respectively. sLLG and transport models are described in Sections II and III, respectively. GSHE and NM/FM modules stand for Giant Spin Hall Effect and normal metal/ferromagnet interface. $R_s$ and $R_{sh}$ stand for series and shunt resistances. Butterworth-Van Dyke (BVD) equivalent circuit is used to model MEMS resonators [17]. Predictive Technology Models (PTM) are used for CMOS circuits [18].

This figure shows explicit circuit schematics for all modules that are used in this paper and the results of FPE and sLLG are compared for a spin-current input to the dynamics of a probability density while the sLLG tracks the dynamics of a single nanomagnet and requires ensemble averaging to be compared with FPE. 1D FPE reads:

$$\frac{\partial \rho(m_z, \tau)}{\partial \tau} = \frac{\partial}{\partial m_z} \left[ (i - h - m_z)(1 - m_z^2)\rho + \frac{1 - m_z^2}{2\Delta} \frac{\partial \rho}{\partial m_z} \right]$$

where $\tau$ is a normalized time in terms of the damping coefficient $\alpha$, the uniaxial anisotropy constant $H_k$ and the gyromagnetic ratio $\gamma$ such that $t = \tau(1 + \alpha^2)/(\alpha \gamma H_k)$ and $t$ denotes real time. $\Delta$ represents the normalized energy barrier of the magnet $(\Delta)kT$ representing the total energy barrier. When the results of FPE and sLLG are compared for a spin-current input $i$, the corresponding current $i \times I_{sc}$ must be used for sLLG.

Since Eq. 3 is a 1D Boundary Value Problem, we can solve it using the bvp4c function of MATLAB subject to the boundary conditions $\partial \rho(m_z, \tau)/\partial m_z = 0$ at $m_z = \pm 1$, and subject to the initial condition:

$$\rho(m_z, \tau = 0) = \frac{1}{Z} \exp \left[-\Delta(1 - m_z^2)\right] \Theta(m_z)$$

where $Z$ is a normalization constant ensuring $\int dm_z \rho(m_z) = 1$ in the range $(-1, +1)$ and $\Theta(m_z)$ is the Heaviside function so that the symmetric magnetization probability is broken to start from a given distribution, in this case chosen to be close to $m_z = 1$. Note that the full FPE equation requires a more general PDE solver (2D), for example, in the case of an in-circuit platforms such as MAPP [20]. Our implementation uses spherical coordinates to solve the LLG equation, but the noise field is input in Cartesian coordinates and added to any other external magnetic field before being expressed in spherical coordinates for the LLG solver. The LLG equation in the monodomain approximation reads:

$$(1 + \alpha^2)\frac{d\hat{m}_i}{dt} = -\gamma|d\hat{m}_i \times \vec{H}_{eff} + \alpha\gamma|\hat{m}_i \times \hat{m}_i \times \vec{H}_{eff}) + \frac{1}{qN_s}(\hat{m}_i \times \hat{I}_s \times \hat{m}_i) + \frac{\alpha}{qN_s}(\hat{m}_i \times \hat{I}_s)$$

where $\hat{m}_i$ is the unit vector along the magnetization, $\gamma$ is the gyromagnetic ratio, $\alpha$ is the damping constant, and $\hat{I}_s$ is the total spin current. $N_s$ is the total number of spins given by $N_s = M_s V_{free}/\mu_B$ where $M_s$ is the saturation magnetization, $V_{free}$ is the volume of the free layer, $\mu_B$ is the Bohr magneton, and $\vec{H}_{eff}$ is the effective magnetic field including the uniaxial, shape anisotropy and magnetic thermal noise terms. The magnetic thermal noise ($\vec{H}_{n}$) enters as an additional magnetic field in three dimensions with the following mean and variance, uncorrelated in all three directions:

$$\langle H_n^{x,y,z} \rangle = 0 \quad \text{and} \quad \langle (H_n^{x,y,z})^2 \rangle = \frac{2\alpha kT}{\gamma M_s V}$$

where $k$ is the Boltzmann constant and $T$ is the temperature. The model becomes more accurate as devices are scaled down, maintaining the single domain approximation.

As shown in Ref. [19], FPE reduces to a simple boundary value problem for magnets with perpendicular anisotropy, as long as all the external fields and currents are confined to the direction of the easy-axis ($\pm z$ direction). FPE describes the dynamics of a probability density while the sLLG tracks the dynamics of a single nanomagnet and requires ensemble averaging to be compared with FPE. 1D FPE reads:

$$\frac{\partial \rho(m_z, \tau)}{\partial \tau} = \frac{\partial}{\partial m_z} \left[ (i - h - m_z)(1 - m_z^2)\rho + \frac{1 - m_z^2}{2\Delta} \frac{\partial \rho}{\partial m_z} \right]$$

where $\tau$ is a normalized time in terms of the damping coefficient $\alpha$, the uniaxial anisotropy constant $H_k$ and the gyromagnetic ratio $\gamma$ such that $t = \tau(1 + \alpha^2)/(\alpha \gamma H_k)$ and $t$ denotes real time. $\Delta$ represents the normalized energy barrier of the magnet $(\Delta)kT$ representing the total energy barrier. The input spin current, $i$ and the external magnetic field, $h$ are both in the $\pm z$ direction, and they are normalized by $I_{sc}$ and $H_k$ respectively, where $I_{sc}$ is given by $4q/ha\Delta(kT)$. When the results of FPE and sLLG are compared for a spin-current input $i$, the corresponding current $i \times I_{sc}$ must be used for sLLG.
The procedure of comparing FPE with sLLG is as follows:

- Prepare 1000 identical samples that start at \( m_z = +1 \) at \( t = 0 \) for the sLLG simulation.
- Wait for 5 ns for identical samples to thermalize and approximately form a Boltzmann distributed initial ensemble.
- Apply spin-current in the \(-z\) direction (\( I_s = i \times I_{sc}, i = 2 \)) at 5 ns.
- Measure the last \( m_z \) value at each time point for each ensemble.
- Obtain \( P_{NS}(\tau) \) by counting the number of samples with \( m_z \) values that are greater than 0 (Not-switched) and normalizing to sample size, \( N=1000 \) (Each yellow square in Fig. 2).
- Solve FPE numerically as a function of \( \tau \) to obtain \( \rho(m_z, \tau) \).
- Obtain \( P_{NS}(\tau) \) from the integral:
  \[
  P_{NS}(\tau) = \int_{m_z=0}^{m_z=+1} dm_z \rho(m_z, \tau)
  \]  

- Compare \( P_{NS}\)-FPE with \( P_{NS}\)-sLLG (Fig. 2).

Note how in the second step, the requirement of a Boltzmann distributed initial ensemble is obtained naturally by running the sLLG equation for a few nanoseconds, so that each sample receives a random initial angle sampled from the correct equilibrium distribution, in accordance with the initial condition used for the FPE.

This precise agreement between the FPE method and the sLLG (Fig. 2) despite the elaborate comparison of the two completely different methods establishes the validity of our HSPICE-based sLLG model. This result is consistent with careful studies that have investigated the numerical solution of the sLLG equation [22]–[24].

III. DESCRIPTION OF TRANSPORT MODELS

In this section, we describe the transport models that are solved self-consistently with the sLLG model that takes spin-currents, either due to spin-transfer torques (STT) or spin-orbit-torques (SOT), as an input.

**Giant Spin Hall Effect:** To model the GSHE induced spin-orbit currents, we use a spin-circuit description whose results are shown to be equivalent to the spin-diffusion equations [25]. Under short-circuit (spin-sink) conditions, this model produces a spin-current that is proportional to a charge current \( I_c \):

\[
\beta = \frac{l_{HM}}{l_{c}} = \frac{\theta l_{HM}}{l_{c}} \left( 1 - \frac{t_{HM}}{\lambda_{sf}} \right)
\]  

where \( \theta \) is the spin Hall angle, \( l_{HM} \) and \( t_{HM} \) are the length and thickness, and \( \lambda_{sf} \) is the spin-flip length of the heavy metal. The GSHE module is implemented as two coupled circuits, one describing the charge transport (in the longitudinal direction) and the other describing the spin transport (in the transverse direction) in terms of 3-dimensional spin currents and voltages [25].

**Normal Metal (NM)/Ferromagnet (FM) interface:** This module represents the interface between a normal metal and a ferromagnet in terms of the microscopic “mixing conductances” that describe the transmission and reflection of spin-currents incident to an FM from a metal [26], [27]. For sufficiently large mixing conductances the incident spin-current is transferred to the ferromagnet with 100% efficiency (spin-sink). Combined with the GSHE module, the interface circuit can reduce the spin-current efficiency to less than 100% and can capture subtle physical effects such as spin-hall magnetoresistance [25]. The details of the interface between a heavy metal and the ferromagnet is believed to play an important role as recent studies suggest [28], and the NM/FM interface circuit can be modified to include these phenomena in the future.

**MTJ:** The bias dependence of the MTJ is captured by assuming voltage dependent interface polarizations \( P(V) \). This observation is supported by a microscopic Non-Equilibrium Green’s function based quantum transport model [29], [30]. This physically motivated assumption may not precisely fit the bias-dependence of MTJs for any given device, however, we believe that it can be useful in modeling more complicated junctions involving multiple interfaces [31].

In this picture, the MTJ conductance is expressed as:

\[
G_{MTJ}(V) = G_0 \left[ 1 + P_1(V)P_2(-V)\cos(\theta) \right]
\]  

where \( G_0 \) is average MTJ conductance \((G_{IP}+G_{AP})/2\), \( V \) is the voltage drop across the junction and \( P_1(V) = P_2(-V) \) are the voltage dependent interface polarizations for the free and fixed layers respectively, assuming a symmetric junction. \( \theta \) describes the angle between the fixed layer and the free layer. We postulate the voltage dependent polarizations as:

\[
P_{1,2}(V) = \frac{1}{1 + P_0 \exp(-V/V_0)}
\]  

where the parameter \( P_0 \) is determined by the low-bias Tunnelling Magnetoresistance (TMR) and \( V_0 \) is determined by the high-bias features of TMR (Fig. 3). \( V_0 \) is assumed to be the same for fixed and free layer interfaces assuming
In this section, we present an experimental benchmarking of the STT-MRAM cell [33]. Due to missing material information in this experiment, we assume CoFeB parameters for free and fixed layers and use typical values for $M_s$, $H_{da}$, and $\alpha$ as shown in Table I, obtaining reasonable agreement with the experiment. Our model captures the bias-dependence of TMR as well as the in-plane torque asymmetry by using the voltage dependent interface polarizations $P_0$ and $V_0$ and magnet parameters with no additional fitting parameters. Fig. 3(a-b) presents the spin-circuit model and experimental benchmarking of the STT-MRAM cell in [33]. For all MTJs described in this paper, our methodology to reproduce the MTJ characteristics is as follows:

- Use low-bias TMR to obtain the parameter $P_0$ from Juliere’s formula $TMR = 2P^2(V = 0)/[1 - P^2(V = 0)]$.
- Use high-bias TMR to obtain the parameter $V_0$.
- Predict switching characteristics only using $P_0$, $V_0$ and magnet parameters with no additional fitting parameters.

Using Predictive Technology Models [18] with no significant difference in the final results.

V. VOLTAGE CONTROLLED MAGNETIC ANISOTROPY

In recent years, enormous progress has been made in the voltage control of magnetism. A notable example is the voltage controlled magnetic anisotropy (VCMA) effect where the electric field across a 2-Terminal MTJ, in particular with PMA magnets, induces a change in the magnetic anisotropy. This change in anisotropy is modeled as a magnetic field that is dependent on instantaneous magnetization $\Delta H_{\alpha m}$.

- Experimental modeling of the VCMA effect. LLG solver receives the voltage across the MTJ as an additional input to modulate the magnetic anisotropy. This change in anisotropy is modeled as a magnetic field that is dependent on instantaneous magnetization $\Delta H_{\alpha m}$. Experimental benchmarking of the VCMA effect at different voltages. A 25 mT constant dipolar shift is added. The shifts in the central dipolar fields at different voltages are due to the spin-current flowing in the MTJ. Experimental figure is reprinted with permission from [34].
The coercivity of the PMA free layer under low-bias conditions is likely to depend on the applied pulse width and thermal activation. For simplicity, we assume this coercivity is equal to the uniaxial anisotropy ($H_k$) in a monodomain approximation while noting that this assumption could underestimate the extracted VCMA coefficient. We assume a linear modification of the free layer anisotropy with respect to voltage:

$$H_k(V) = H_{k0} + \frac{V}{t_{ox}} \left( \frac{\eta}{t_{fm} M_s} \right)$$

(9)

where $t_{fm}, t_{ox}$ are the oxide and free layer thickness and $\eta$ is the VCMA coefficient (in units $\mu$J/$m^2$ per V/nm) and $V$ is applied voltage. Combined with the bias-dependent MTJ, our VCMA model seems to capture three distinct effects as seen in Fig. 4: a) the $H_k$ modulation of the free layer, b) the bias dependence of the MTJ where $\pm0.8$ V decreases the TMR, and c) the shift of the hysteresis loop that arises due to a spin-current flowing from the fixed layer to the free layer in opposite directions for opposite voltage polarity. The asymmetric shift for $\pm0.8$ V seems consistent with the asymmetry in the magnitude of the spin-current captured by the voltage dependent polarizations. Note that only the first effect is added explicitly, the rest follows from the MTJ model discussed in Section IV.

VI. GSHE-DRIVEN MTJ

An SOT-MRAM cell consists of a 3-Terminal MTJ and access transistors where spin current is generated by passing a charge current through a heavy metal for writing information. Thus, SOT-MRAM achieves read and write operations using separate paths, improving a feature of STT-MRAMs that use same paths for read and write. In this section, we experimentally benchmark a GSHE-driven MTJ (without the transistor periphery) that consists of an in-plane CoFeB for fixed and free layers and Tantalum for the heavy metal (HM). Fig. 5 (a) shows spin-circuit modeling and physical implementation of the GSHE-driven MTJ in [36]. We follow the same methodology to reproduce the MTJ characteristics explained in the section IV using identical material and device parameters as in the experiment. Fig. 5 (b) compares our results with the experiment that seem to be in good quantitative agreement without any additional fitting parameters. We note, however, the switching currents in the experiment could also be influenced by thermal activation mechanisms and multi-domain features that are not included in our model. The symmetric switching currents as well as the roll-off due to high bias MTJ properties are captured, although the pronounced (and opposite) roll-off in the parallel TMR branch suggests mechanisms beyond what is in our model, possibly related to multi-domain features of the nanomagnets.

VII. MEMS RESONATOR DRIVEN STNO

In this section, our purpose is to show how the circuit framework that we present can be used to evaluate exploratory devices, using a recently proposed hybrid MTJ-based device as an example. It is well-established that the free layer of an MTJ can be driven to function as a nano oscillator by STT and SOT mechanisms [37]. However, these spin-torque-nano oscillators (STNO) suffer from very poor phase noise characteristics [38], limiting their potential as nanoscale, tunable oscillators.

A recent proposal suggested the coupling of GSHE-driven STNOs with very high quality MEMS-resonators as a CMOS-compatible solution to improve the phase noise characteristics of STNOs [39]. This concept is tested by combining a high overtone bulk acoustic wave resonator (HBAR) and 3-Terminal MTJ-based STNO. The 2-port HBAR is described by the Butterworth-Van Dyke (BVD) equivalent circuit [17], coupled to the GSHE-driven MTJ with current feedback. The full spin-circuit corresponding to this example is shown in Fig. 1. We use similar parameters for the GSHE-driven MTJ that is described in Section VI. Fig. 6 presents the hybrid model and compares FFT spectra of a free running STNO and an HBAR-driven STNO with current feedback: a) Spin-circuit modeling of a MEMS-resonator driven STNO. The frequency spectrum is obtained by taking the FFT of the time domain signal. (b) FFT spectra of free running STNO and HBAR driven STNO. A Lorentzian function is used to fit the FFT data to extract the linewidth of the oscillators. HBAR driven STNO shows ~15X enhancement in the quality factor.
driven STNO, where HBAR-driven STNO exhibits a significant enhancement in the oscillator linewidth. Furthermore, the original tunability of the STNO is maintained by locking it to the nearest high Q peak of the HBAR. This example demonstrates how our modular framework can be combined with established circuit models to propose and evaluate hybrid devices involving MTJs.

### TABLE I

| STI-MRAM | GSHE-driven MTJ |
|----------|-----------------|
| $V_{free}$ | 100 nm $\times$ 40 nm $\times$ 1.5 nm |
| $M_s$, $H_K$, $H_d$ | 1100 emu/cc, 0.01, 150 Oe, 1.3 T |
| TMR, $P_{0}$, $V_0$ | 1100%, 0.6887, 1.81 |
| CMOS models, Size | 14 nm HP-FinFET, nfin=50 |
| $t_{sim}$, $t_{step}$, (HSPIEC, noise:off) | 5 ps, 2 ps |
| VCMAs [4] | |
| $V_{free}$ | 70 nm $\times$ 70 nm $\times$ 1.8 nm |
| $M_s$, $H_K$, $H_d$ | 1100 emu/cc, 0.075, 120 Oe |
| TMR, $P_0$, $V_0$ | 1500%, 0.5253, 0.33 |
| $t_{ox}$, $t_{η}$ | 1.4 nm, 6.5 $\mu$m$^2$/N/nm |
| $t_{sim}$, $t_{step}$, (HSPIEC, noise:off) | 25 ps, 5 ps |
| GSHE-driven MTJ [5] | |
| $M_s$, $H_K$, $H_d$ | 350 nm $\times$ 100 nm $\times$ 1.6 nm |
| TMR, $P_0$, $V_0$, Ref[Gmax] | 1100 emu/cc, 0.021, 40 Oe, 0.76 T |
| $θ_{HM}$, $λ_{HM}$, $P_{HM}$ (HM=Ta) | 50%, 1.13, 0.083, 10$^{15}$ S/m$^2$ |
| $t_{HM}$, $P_{HM}$, $λ_{HM}$ | 0.12, 1.5 nm, 190 $\mu$m $\Omega$ $\cdot$ cm |
| $t_{sim}$, $t_{step}$, (HSPIEC, noise:off) | 10 µs, 5 µm, 6.2 nm |

### VIII. BEYOND CONVENTIONAL DEVICES

The modular framework we present in this paper can be used beyond compact modeling MTJ devices that are envisioned in the context of memory applications [2]. For example, materials such as Topological Insulators, exhibiting spin-momentum locking (SML) have been expressed in terms of general spin-circuit models recently [40, 41]. In conjunction with the nanomagnet modules discussed in this paper, such SML models have been used in an exploratory mode to propose new memory devices [42] and to model novel devices exhibiting three resistance states [40] that have subsequently received experimental confirmation [43]. In addition, the benchmarked stochastic LLG solver coupled with MTJ modules has been used to model a special type of superparamagnetic MTJs [44] functioning as three-terminal tunable random number generators [45], or probabilistic bits (p-bits), and p-circuits built out of such p-bits have been shown to be useful for a wide range of applications such as Ising computing [46], Bayesian inference [47] and invertible logic [48].

### IX. CONCLUSION

We present a modular, extensible and physics-based circuit framework for MTJ devices that can capture a wide range of spintronic phenomena. We start by a rigorous validation of the stochastic LLG model and then demonstrate our approach with a step by step extension of our framework. The same basic models are used to capture bias-dependence of MTJs to model STT-MRAM, GSHE driven-MTJs, VCMAs-based MTJs as well as hybrid circuits such as MEMS resonator-driven STNOs. We believe that the modular framework presented here can be an important compact modeling toolbox in the exploration of new MTJ devices as the fields of spintronics and magnetism progress with new materials and phenomena.

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