PAPER

Leptoquark effects on $b \to s \nu \bar{\nu}$ and $B \to K l^+ l^-$ decay processes

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Abstract

We study the rare semileptonic decays of $B$ mesons induced by $b \to s \nu \bar{\nu}$ as well as $b \to s l^+ l^-$ transitions in the scalar leptoquark (LQ) model, where the LQs have the representation $(3, 2, 7/6)$ and $(3, 2, 1/6)$ under the standard model gauge group. The LQ parameter space is constrained using the most recent experimental results on $\text{Br}(B_s \to \mu^+ \mu^-)$ and $\text{Br}(B_l \to X_s \mu^+ \mu^-)$ processes. Considering only the baryon number conserving LQ interactions, we estimate the branching ratios for the exclusive $B \to R(\ast) \nu \bar{\nu}$ and inclusive $B \to X_s \nu \bar{\nu}$ decay processes by using the constraint parameters. We also obtain the low recoil (large lepton invariant mass, i.e., $q^2 \sim m_B^2$) predictions for the angular distribution of $B \to K l^+ l^-$ process and several other observables including the flat term and lepton flavour non-universality factor in this model.

1. introduction

It is well-known that the study of $B$ physics plays an important role to critically test the standard model (SM) predictions and to look for possible signature of new physics beyond it. In particular, the rare decays of $B$ mesons which are mediated by flavour changing neutral current (FCNC) transitions are well-suited for searching the effects of possible new interactions beyond the SM. This is due to the fact that the FCNC transitions $b \to s, d$ are highly suppressed in the SM as they occur only at one-loop level and hence, they are very sensitive to new physics. Recently the decay modes $B \to K^{\ast} l^+ l^-$, which are mediated by the quark level transition $b \to s l^+ l^-$ have attracted a lot of attention, as several anomalies at the level of few sigma are observed in the LHCb experiment [1–3]. Furthermore, the deviation in the ratio of rates of $B \to K \mu \mu$ over $B \to K e e$ is a hint of violation of lepton universality [4]. This in turn requires the careful analyses of the angular observables for these processes both in the low and high $q^2$ regime.

Recently various $B$ physics experiments such as BaBar, Belle, CDF and LHCb have provided data on the angular distributions of $B \to K^{\ast} l^+ l^-$ and $B \to K l^+ l^-$ decay processes both in the low and the large recoil region except the intermediate region around $q^2 \sim m_B^2/\psi$ and $m^2_\psi$. The intermediate region is dominated by the pronounced charmonium resonance background induced by the decays $B \to K (\bar{c}c) \to K l^+ l^-$, where $\bar{c}c = J/\psi$, $\psi'$. Using QCD factorization method the physical observables in the high recoil region can be calculated and the angular distribution of $B \to K l^+ l^-$ at low recoil can be computed using simultaneous heavy quark effective theory and operator product expansions in $1/Q$, with $Q = (m_B, \sqrt{q^2})$ i.e. $\sqrt{q^2}$ is of the order of the $b$-quark mass [5, 6]. In this work, we are interested to study the decay process $B \to K l l$ in the region of low hadronic recoil i.e. above the $\psi'$ peak in the scalar leptoquark (LQ) model. We have studied the $B \to K \mu^+ \mu^-$ in the large recoil limit in [7] and found that the various anomalies associated with the isospin asymmetry parameter and the lepton flavour non-universality factor ($R_{K\ell}$) for this process can be explained in this model. Similarly the rare semileptonic decays of $B$ mesons with $\nu \bar{\nu}$ pair in the final state, i.e., $B \to K^{\ast}(\nu \bar{\nu})$ are also significantly suppressed in the SM and their long distance contributions are generally subleading. These decays are theoretically very clean due to the absence of photonic penguin contributions and strong suppression of light quarks. The experimental measurement of the inclusive decay rate probably be un-achievable due to the missing neutrinos, however, the exclusive channels like $B \to K^{\ast} \nu \bar{\nu}$ and $B \to K \nu \bar{\nu}$ are more promising as far as the
measurement of branching ratios and other related observables are concerned. Theoretically, study of these decays requires calculation of relevant form factors by non-perturbative methods.

In recent times, there are many interesting papers which are contemplated to explain the anomalies associated with the \( b \to s l^+l^- \) processes, observed at LHCb experiment [1–4], both in the context of various new physics models as well as in model independent ways [8–11]. In this paper, we intend to study the effect of scalar LQs, i.e., \( \Delta^7/10 \) (3, 2, 7/6) and \( \Delta^{1/6} \) (3, 2, 1/6) on the branching ratio as well as on other asymmetry parameters in the low-recoil region of \( B \to K^\pm l^\mp \) process. We also consider the processes \( B \to K^{\mu\nu}l^\pm \) involving the quark level transitions \( b \to s l^\pm \) in the full physical regime. It is well-known that LQs are scalar or vector colour triplet bosonic particles which make leptons couple directly to quarks and vice versa and carry both lepton as well as baryon quantum numbers and fractional electric charge. Leptoquarks can be included in the low energy theory as a relic of a more fundamental theory at some high energy scale in the extended SM [12], such as grand unified theories [12, 13], Pati–Salam models, models of extended technicolor [14] and composite models [15]. Leptoquarks are classified by their fermion number (\( F = 3B + L \)), spin and charge. Usually they have a mass near the unification scale to avoid rapid proton decay, even so LQs may exist at a mass accessible to present collider, if baryon and lepton numbers would conserve separately. The LQ properties and the additional new physics contribution to the SM have been very well studied in the literature [7, 16–20].

The plan of the paper is follows. In section 2 we present the effective Hamiltonian responsible for \( b \to s l^+l^- \) processes. We also discuss the new physics contributions due to the exchange of scalar LQs. In section 3 we discuss the constraints on LQ parameter space by using the recently measured branching ratios of the rare decay modes \( B_l \to \mu^+\mu^- \) and \( B_l \to X_s\mu^+\mu^- \). The branching ratio, the flat term and the lepton non-universality factor \( (R_{Kl}) \) for the decay mode \( B \to K^\pm l^\mp \), where \( l = e, \mu, \tau \) at low recoil limit are computed in section 4. In section 5 we work out the branching ratio of \( B \to K^{\mu\nu}l^\pm \) process in the full kinematically accessible physical region. The branching ratio, polarization and other asymmetries in \( B \to K^{\mu\nu}l^\pm \) process have been computed in section 6. The inclusive decay process \( B \to X_s l^\pm \) is discussed in sections 7 and 8 contains the summary and conclusion.

2. The effective Hamiltonian for \( b \to s l^+l^- \) process

The effective Hamiltonian describing the processes induced by the FCNC \( b \to s l^+l^- \) transitions is given by [21]

\[
\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^* \sum_{i=1}^{8} \left( C_i(\mu)O_i + \sum_{i=7,9,10} (C_i(\mu)O_i + C_i'(\mu)O_i') \right),
\]

which consists of the tree level current–current operators \( (O_{1,2}) \), QCD penguin operators \( (O_{3,4,5}) \) along with the magnetic \( O_{7,8}^{(1)} \) and semileptonic electroweak penguin operators \( O_{7,8}^{(2)} \). The magnetic and electroweak penguin operators can be expressed as

\[
O_{7,8}^{(1)} = \frac{e}{16\pi^2} (\overline{\psi}_s \gamma^\mu (m_s P_{LR} + m_b P_{RL}(b)) \psi_b) F_{\mu
u},
\]
\[
O_{7,8}^{(2)} = \alpha \frac{1}{4\pi} (\overline{\psi}_s \gamma^\mu P_{LR}(b)(\bar{\gamma}_{\nu} l), \quad O_{7,8}^{(0)} = \alpha \frac{1}{4\pi} (\overline{\psi}_s \gamma^\mu P_{LR}(b)(\bar{\gamma}_{\nu} \gamma_5 l).
\]

It should be noted that the primed operators are absent in the SM. The values of Wilson coefficients \( C_{i=1, \ldots, 10} \), which are evaluated in the next-to-next leading order at the renormalization scale \( \mu = m_b \) are taken from [22].

Here \( V_{q\bar{q}} \) denotes the CKM matrix element, \( G_F \) is the Fermi constant, \( \alpha \) is the fine-structure constant and \( P_{LR} \equiv (1 \mp \gamma_5)/2 \) are the chiral projectors. Due to the negligible contribution of the CKM-suppressed factor \( V_{tb} V_{ts}^* \), there is no CP violation in the decay amplitude in the SM. These processes will receive additional contributions due to the exchange of scalar LQs. In particular there will be new contributions to the electroweak penguin operators \( O_8 \) and \( O_{10} \) as well their right-handed counterparts \( O_8^\prime \) and \( O_{10}^\prime \). In the following subsection we will present these additional contributions to the SM effective Hamiltonian due to the exchange of such LQs.

2.1. Scalar LQ contributions to \( b \to s l^+l^- \) effective Hamiltonian

There are ten different types of LQs under the \( SU(3) \times SU(2) \times U(1) \) gauge group [23], half of them have scalar nature and other halves have vector nature under the Lorentz transformation. The scalar LQs have spin zero and could potentially contribute to the quark level transition \( b \to s l^+l^- \). Here we would like to consider the minimal renormalizable scalar LQ model [17], containing one single additional representation of \( SU(3) \times SU(2) \times U(1) \), which does not allow proton decay. There are only two such models with representations under the SM gauge group as \( \Delta^7/10 \equiv (3, 2, 7/6) \) and \( \Delta^{1/6} \equiv (3, 2, 1/6) \) [17], which have sizeable Yukawa couplings to matter fields. These scalar LQs do not have baryon number violation in the perturbation theory and could be...
light enough to be accessible in accelerator searches. The interaction Lagrangian of the scalar LQ \( \Delta^{7/6} \) with the fermion bilinear is given as

\[
\mathcal{L}^{7/6} = g_{6} \bar{Q}_{L} \Delta^{7/6} l_{R} + \text{h.c.,}
\]

where \( Q_{L} \) is the left-handed quark doublet and \( l_{R} \) is the right-handed charged lepton singlet. After performing the Fierz transformation and comparing with the SM effective Hamiltonian \((1)\), one can obtain the new Wilson coefficients as discussed in \((18)\)

\[
C_{9}^{NP} = C_{10}^{NP} = -\frac{\pi}{2\sqrt{2} G_{F} \alpha V_{td} V_{ts}^{*}} \frac{(g_{6})_{l}(g_{6})_{M}^{*}}{M_{\Delta^{7/6}}^{2}}.
\]

Similarly, the Lagrangian for the coupling of scalar LQ \( \Delta^{1/6} \) to the SM fermions is given by

\[
\mathcal{L}^{1/6} = g_{6} \bar{d}_{R} \Delta^{1/6} l + \text{h.c.,}
\]

where \( \tau_{S} \) is the Pauli matrix and consists of operators with right-handed quark currents. Proceeding like the previous case one can obtain the new Wilson coefficients as

\[
C_{9}^{NP} = -C_{10}^{NP} = -\frac{\pi}{2\sqrt{2} G_{F} \alpha V_{td} V_{ts}^{*}} \frac{(g_{6})_{l}(g_{6})_{M}^{*}}{M_{\Delta^{1/6}}^{2}},
\]

which are associated with the right-handed semileptonic electroweak penguin operators \( O_{9} \) and \( O_{10} \).

### 3. Constraint on the LQ parameters

After having the idea of possible scalar LQ contributions to the \( b \to s l \bar{l} \) processes we now proceed to constraint the LQ couplings using the theoretical \((24)\) and experimental branching ratio \((25, 26)\) of \( B_{s} \to \mu^{+} \mu^{-} \) process. This process is mediated by \( b \to s \mu \mu \) transition and hence well-suited for constraining the LQ parameter space. In the SM the branching ratio for this process depends only on the Wilson coefficient \( C_{10} \). However, in the scalar LQ model there will be additional contributions due to the LQ exchange which are characterized by the new Wilson coefficients \( C_{9}^{NP} \) and \( C_{10}^{NP} \) depending on the nature of the LQs. Thus, in this model the branching ratio has the form \((7, 19)\)

\[
\text{Br}(B_{s} \to \mu^{+} \mu^{-}) = \frac{G_{F}^{2}}{16 \pi^{3}} \tau_{B_{s}} \alpha^{2} f_{B_{s}}^{2} M_{B_{s}} m_{\mu}^{2} |V_{td} V_{ts}^{*}|^{2} \left| C_{10}^{SM} + C_{10}^{NP} \right|^{2} \left( 1 - \frac{4m_{\mu}^{2}}{M_{B_{s}}^{2}} \right),
\]

which can be expressed as

\[
\text{Br}(B_{s} \to \mu^{+} \mu^{-}) = \text{Br}^{SM} \left[ 1 + \frac{C_{10}^{NP}}{C_{10}^{SM}} \right]^{2} \equiv \text{Br}^{SM} \left[ 1 + r e^{i\phi^{NP}} \right]^{2},
\]

where \( \text{Br}^{SM} \) is the SM branching ratio and we define the parameters \( r \) and \( \phi^{NP} \) as

\[
re^{i\phi^{NP}} = \frac{C_{10}^{NP}}{C_{10}^{SM}}.
\]

Now comparing the SM theoretical prediction of \( \text{Br}(B_{s} \to \mu \mu) \) \((24)\)

\[
\text{Br}(B_{s} \to \mu^{+} \mu^{-}) = (3.65 \pm 0.23) \times 10^{-9},
\]

with the corresponding experimental value

\[
\text{Br}(B_{s} \to \mu^{+} \mu^{-}) = (2.9 \pm 0.7) \times 10^{-9},
\]

one can obtain the constraint on the new physics parameters \( r \) and \( \phi^{NP} \). The constraint on the LQ parameter space has been extracted in \((7, 19)\) from this process, therefore, here we will simply quote the results. The allowed parameter space in \( r - \phi^{NP} \) plane which is compatible with the 1\,\sigma range of the experimental data is \( 0 \leq r \leq 0.1 \) for the entire range of \( \phi^{NP} \), i.e.

\[
0 \leq r \leq 0.1, \quad \text{for} \quad 0 \leq \phi^{NP} \leq 2\pi.
\]

However, in this analysis we will use relatively mild constraint, consistent with both measurement of \( \text{Br}(B_{s} \to \mu^{+} \mu^{-}) \) and \( \text{Br}(B_{d}^{0} \to \mu^{+} \mu^{-}) \) \((7)\) as

\[
0 \leq r \leq 0.35, \quad \text{with} \quad \pi/2 \leq \phi^{NP} \leq 3\pi/2.
\]
It should be noted that the use of this limited range of CP phase, i.e., \((\pi/2 \leq \phi_{NP} \leq 3\pi/2)\) is an assumption to have a relatively larger value of \(\tau\). These bounds can be translated to obtain the bounds for the LQ couplings as
\[
0 \leq \frac{|(g_{R}^{\mu}) (g_{R}^{\nu})^{*}|}{\mathcal{M}_{A}} \leq 5 \times 10^{-9} \text{ GeV}^{-2} \quad \text{for} \quad \pi/2 \leq \phi_{NP} \leq 3\pi/2. \tag{14}
\]
After obtaining the bounds on LQ couplings, we now proceed to study the decay processes \(B \to Kl l\) and \(B \to K^{(*)}(X_{s})\nu\bar{\nu}\) and the associated observables in the following sections.

4. \(\bar{B} \to K^{*+}l^{-}\) process in the low-recoil limit

The transition amplitude for the \(B \to K^{*+}l^{-}\) decay process can be obtained using the effective Hamiltonian presented in equation (1). The matrix elements of the various hadronic currents between the initial \(B\) meson and the final \(K\) meson can be parameterized in terms of the form factors \(f_{0}, f_{2}\) and \(f_{+}\) as [28]
\[
\langle \bar{K}(k)|\gamma^\mu b|\bar{B}(p)\rangle = f_{+}(q^{2})(p + k)^{\mu} + [f_{0}(q^{2}) - f_{+}(q^{2})] \frac{m_{B}^{2} - m_{K}^{2}}{q^{2}} q^{\mu}, \tag{15}
\]
\[
\langle \bar{K}(k)|\bar{b}\sigma^{\mu\nu}l|\bar{B}(p)\rangle = i\frac{f_{2}(q^{2})}{m_{B} + m_{K}}[(p + k)^{\mu}q^{\nu} - q^{\mu}(p + k)^{\nu}], \tag{16}
\]
where \(p, k\) are the four-momentum of the \(B\)-meson and Kaon respectively and \(q = p - k\) is the four-momentum transferred to the dilepton system. Furthermore, using the QCD operator identity [5, 29, 30],
\[
i\partial^{\mu}(i\sigma^{\mu\nu}b) = -m_{b}(\gamma_{\nu}b) + i\partial_{\nu}(sb) - 2(s\bar{D}_{\nu}b), \tag{17}
\]
an improved Isgur–Wise relation between \(f_{2}\) and \(f_{+}\) can be obtained as
\[
f_{2}(q^{2}, \mu) = \frac{m_{B}(m_{B} + m_{K})}{q^{2}} \kappa(\mu)f_{+}(q^{2}) + O\left(\frac{\Lambda}{m_{b}}\right), \tag{18}
\]
where strange quark mass has been neglected. Thus, one can obtain the amplitude for the \(\bar{B} \to K^{*+}l^{-}\) process in low recoil limit [28, 31], after applying form factor relation (18) as
\[
A(\bar{B} \to K^{*+}l^{-}) = \frac{G_{F}C_{10}^{tot}}{\sqrt{2}\pi} V_{ub}^{*}V_{cs}(q^{2})[F_{V}p^{\mu}(\bar{\gamma}_{\mu}l) + F_{A}p^{\mu}(\bar{\gamma}_{\nu}\gamma_{5}l) + F_{P}(\bar{\gamma}_{\nu}l)], \tag{19}
\]
where
\[
F_{V} = C_{10}^{tot}, \quad F_{A} = C_{9}^{tot} + \frac{2m_{b}m_{B}}{q^{2}} C_{7}^{eff}, \quad F_{P} = -m_{b} \left[ 1 + \frac{m_{B}^{2} - m_{K}^{2}}{q^{2}} \left( 1 - \frac{f_{0}}{f_{+}} \right) \right] C_{10}^{tot}. \tag{20}
\]
In equation (20), \(C_{9}^{tot} = C_{9}^{eff} + C_{9}^{NP}\) and \(C_{10}^{tot} = C_{10}^{SM} + C_{10}^{NP} = C_{10}^{NP}\), where \(C_{9}^{NP}\) and \(C_{10}^{NP}\) are the new contributions to the Wilson coefficients arising due to the exchange of LQs and the effective Wilson coefficients \(C_{7,9}^{eff}\) are given in [32]. The corresponding differential decay distributions is given by
\[
\frac{d\Gamma}{dq^{2}d\cos\theta_{l}}[\bar{B} \to K^{*+}l^{-}] = a_{1}(q^{2}) + c_{1}(q^{2}) \cos^{2}\theta_{l}, \tag{21}
\]
where \(\theta_{l}\) is the angle between the directions of \(\bar{B}\) meson and the \(l^{-}\) in the dilepton rest frame. The expressions for the \(q^{2}\) dependent parameters \(a_{1}, c_{1}\) are presented in appendix A. Thus, the decay rate for the process \(\bar{B} \to K^{*+}l^{-}\) can be written as
\[
\Gamma_{l} = 2 \int_{0}^{\pi/2} d\theta_{l} \left[ a_{1} + \frac{1}{2} c_{1} \right]. \tag{22}
\]
Another useful observable known as the flat term is defined as
\[
F_{H}^{l} = 2 \int_{0}^{\pi/2} d\theta_{l} \left[ a_{1} + c_{1} \right], \tag{23}
\]
where the hadronic uncertainties are reduced due to cancellation between the numerator and denominator. It should be noted that the lepton mass suppression of \((a_{1} + c_{1})\) follows as \((F_{H}^{l})^{SM} \propto m_{l}^{2}\), hence, it vanishes in the limit \(m_{l} \to 0\).

After obtaining the expressions for branching ratio and the observable \(F_{H}^{l}\), we now proceed for numerical estimation for \(B \to K^{*+}l^{-}\) process in the low recoil region. In our analysis we use the following parametrization
for the $q^2$ dependence of form factors $f_i (i = +, T, 0)$ as [28, 33]

$$f_i (s) = \frac{f_i (0)}{1 - s/m^2_{\text{res}}} \left[ 1 + b_i^t \left( z (s) - z (0) + \frac{1}{2} (z (s)^2 - z (0)^2) \right) \right],$$

(24)

where we have used the notation $q^2 \equiv s$. The $z(s)$ functions are given as

$$z (s) = \frac{\sqrt{s} - \sqrt{T_+ - \tau_0}}{\sqrt{s} + \sqrt{T_+ - \tau_0}}, \quad \tau_0 = \sqrt{T_+ (\sqrt{T_+ - \sqrt{T_- - \tau_0}}), \quad \tau_{\pm} = (m_B \pm m_K)^2.$$

The values of $f_i (0)$ and $b_i^t$ are taken from [28].

For numerical evaluation, we have used the particle masses and the lifetimes of $B$ meson from [34]. For the CKM matrix elements, we have used the Wolfenstein parametrization with values $A = 0.814 \pm 0.025$, $\lambda = 0.22537 \pm 0.00061$, $\bar{\rho} = 0.117 \pm 0.021$ and $\bar{\eta} = 0.353 \pm 0.013$ and the fine structure coupling constant $\alpha = 1/137$. With these input parameters, the differential branching ratios for $B_d^0 \rightarrow \bar{K}^0 e^+ e^-$ (left panel), $\bar{B} \rightarrow K^0 \mu^+ \mu^-$ (right panel) and $\bar{B} \rightarrow K^{*0} \tau^+ \tau^-$ (lower panel) processes with respect to high $q^2$, both in the SM and in the LQ model are shown in figure 1 for $\Delta^{(7/6)}$ LQ and in figure 2 for $\Delta^{(1/6)}$. The grey bands in these plots correspond to the uncertainties arising in the SM due to the uncertainties associated with the CKM matrix elements and the hadronic form factors. The green bands correspond to the LQ contributions. For $B \rightarrow K_{\mu X}$ process, we vary the values of the LQ couplings as given in equation (14) and for $B \rightarrow K_{ee}$ and $B \rightarrow K_{\tau \tau}$ processes we use the limits on the LQ couplings extracted from $B_d \rightarrow X_c e^+ e^-$ and $B_s \rightarrow \tau^+ \tau^-$ processes [7] as

$$0 \leq \left| \frac{g_{K_{ee}}}{M^2_{\Delta}} + \frac{g_{K_{\tau \tau}}}{M^2_{\Delta}} \right| \leq 1.0 \times 10^{-8} \text{ GeV}^{-2},$$

(25)
Since the LQ couplings are more tightly constrained in $b \to s\mu\mu$ transitions, the deviations of the branching ratios in the LQ model from the corresponding SM values are found to be small. For $B \to K\mu\mu$ and $B \to K\tau\tau$ these deviations are found to be significantly large. The bin-wise experimental values are shown in black in $B \to K\mu\mu$ process. From these figures it can be seen that the observed experimental data can be explained in the scalar LQ model but the deviation from the SM branching ratios are more in the $\Delta^1/60$ model. For the other observables in $B \to Kl\ell$ processes we will show the results only for $\Delta^{1/60}$ LQ model. In figure 3, we have shown the lepton non-universality factors $R_{K^0}^{\mu}$ (left panel) (i.e. the ratio of branching ratios of $B \to K\mu^+\mu^-$ and $\bar{B} \to \bar{K}e^+e^-$), $R_{K^0}^{e}$ (right panel) and $R_{K^0}^{\ell}$ (lower panel) variation with high $q^2$. From the figure one can see that there is significant deviations in the lepton-flavour non universality factor from their corresponding SM values in all the above three cases. The flat term for the $R_{K^0}^{\mu} \to \bar{K}^0\mu^+\mu^-$ (left panel) and $R_{K^0}^{e} \to \bar{K}^0\tau^+\tau^-$ (right panel) decay processes in the low recoil region are presented in figure 4 for $\Delta^{1/60}$. In this case there is practically no deviation in $B \to K\mu\mu$ whereas there is significant deviation in $B \to K\tau\tau$ process. The integrated branching ratios, flat terms and the lepton flavour non-universality factors for the $B \to Kl\ell$ processes over the range $q^2 \in [14.18, 22.84]$ are given in table 1. The flat term for $B \to K\ell\ell$ process has been found to be negligibly small ($F_{M} \sim \mathcal{O}(10^{-7})$) due to tiny electron mass. In the low recoil region, the process having tau lepton in the final state has significant deviation from the SM.

The integrated branching ratio for $B^0 \to K\mu\mu$ process in the range $q^2 \in [15, 22] \text{ GeV}^2$ has been measured by the LHCb Collaboration [1] and is given as

$$\text{Br}(B^0 \to K\mu\mu) = (6.7 \pm 1.1 \pm 0.4) \times 10^{-8}. \quad (27)$$
Our predicted value in this range of $q^2$ is found to be

$$\text{Br}(B^0 \to K^0 \mu^+\mu^-) = (8.35 \pm 0.5) \times 10^{-8}, \quad \text{(SM)}$$

$$= (8.34 - 9.26) \times 10^{-8}, \quad \text{(}\Delta^{7/6} \text{ LQ model)}$$

$$= (8.34 - 15.6) \times 10^{-8}. \quad \text{(}\Delta^{1/6} \text{ LQ model)} \quad (28)$$

Figure 3. The variation of lepton non-universality $R_{K}^{e}$ (left panel), $R_{K}^{\mu}$ (right panel) and $R_{K}^{\tau}$ (bottom panel) in low recoil region due to $\Delta^{7/6}$ LQ exchange.

Figure 4. The variation of flat term for $B^0 \to K^0 \mu^+\mu^-$ (left panel) and $B^0 \to K^0 \tau^+\tau^-$ (right panel) with high $q^2$ for $\Delta^{7/6}$ LQ.
The predicted values of the branching ratios are slightly higher than the central measured value but consistent with its 1-σ range.

5. $B \to K \nu \bar \nu$ process

The $B \to K \nu \bar \nu$ process is mediated by the quark level transition $b \to s \nu \bar \nu$ and the effective Hamiltonian describing such transition is given as [35]

$$H_{\text{eff}} = \frac{-4G_f}{\sqrt{2}} V_{tb}^* V_{ts} (C_L^{\nu} O_{L}^{\nu} + C_R^{\nu} O_{R}^{\nu}) + \text{h.c.,}$$

where

$$O_{L}^{\nu} = \frac{e^2}{16\pi^2} (\bar{\nu} \gamma_\mu P_L b) (\bar{\nu} \gamma_\mu (1 - \gamma_5) \nu),$$
$$O_{R}^{\nu} = \frac{e^2}{16\pi^2} (\bar{\nu} \gamma_\mu P_R b) (\bar{\nu} \gamma_\mu (1 - \gamma_5) \nu),$$

are the dimension-six operators and $C_L^{\nu}, C_R^{\nu}$ are their corresponding Wilson coefficients. The coefficient $C_R^{\nu}$ has negligible value within the SM while $C_L^{\nu}$ can be calculated by using the loop function and is given by

$$C_L^{\nu} = -X(x_\nu)/\sin^2 \theta_{\nu}. $$

The necessary loop functions are presented in appendix B. The decay distribution with respect to the di-neutrino invariant mass can be expressed as [36]

$$\frac{d\Gamma}{ds_\nu} = \frac{G_f^2 \alpha^2}{256 \pi} |V_{tb}^* V_{ts}|^2 m_b^5 \lambda^{3/2}(s_\nu, m_b^2, m_{\tilde{m}}^2, 1) |f^K_+ (s_\nu)|^2 |C_L^{\nu} + C_R^{\nu}|^2.$$

where $m_{\tilde{m}} = m_f/m_b$ and $s_\nu = s/m_{\tilde{m}}^2$. The decay rate has been multiplied with an extra factor 3 due to the sum over all neutrino flavours. It should be noted that in equation (32) $C_L^{\nu}$ is the new Wilson coefficient arises due to the exchange of the LQ $\Delta^{1/6}$ in $\Delta^{1/6}$ process. In order to find out its value, we consider the new contribution to the effective Hamiltonian due to the exchange of such LQ which is given as

$$H_{\text{LQ}} = \frac{(g_{L})_{\nu \nu} (g_{L})_{\nu \nu}^*}{4M^2_{\Delta^{1/6}}} (\bar{\nu} \gamma_\mu P_R b) (\bar{\nu} \gamma_\mu (1 - \gamma_5) \nu). $$

Comparing equations (29) and (33), one can obtain the new Wilson coefficient as

$$C_L^{\nu}_{\text{LQ}} = -\frac{\pi}{2\sqrt{2} G_f \alpha V_{tb}^* V_{ts}} \frac{(g_{L})_{\nu \nu} (g_{L})_{\nu \nu}^*}{M^2_{\Delta^{1/6}}}.$$

For numerical estimation, we use the $B \to K$ form factor $f^K_+$ evaluated in the light cone sum rule approach [37] as

$$f^K_+(q^2) = \frac{n_1}{1 - q^2/m_b^2} + \frac{n_2}{(1 - q^2/m_{\tilde{m}}^2)^2},$$

which is valid in the full physical region. Furthermore, in contrast to $B \to K^{*+}l^- \bar \nu$ process, which has dominant charmonium resonance background from $B \to K (f/\psi) \to K^{*+}l^- \bar \nu$, there are no such analogous long-distance QCD contributions in this case as there are no intermediate states which can decay into two neutrinos. For the $b \to s \nu \bar \nu$ LQ couplings we use the values as we used for $b \to s \mu \bar \mu$ as these two processes are related by $SU(2)_L$ symmetry. The variation of branching ratio with respect to $s_\nu$ in the full physical regime $0 \leq s_\nu \leq (1 - m_{\tilde{m}}^2)^2$ is shown in figure 5 and the predicted branching ratio is given in table 2, which is well below the present upper limit $Br(B_d^0 \to K\nu\bar\nu) < 4.9 \times 10^{-3}$ [34].
6. \( B \rightarrow K^\ast \nu\bar{\nu} \) process

The study of \( B \rightarrow K^\ast \nu\bar{\nu} \) is also quite important as this process is related to \( B \rightarrow K^\ast \mu\mu \) process by \( SU(2)_L \) and therefore, the recent LHCb anomalies in \( B \rightarrow K^\ast \mu\mu \) would in principle also show up in \( B \rightarrow K^\ast \nu\bar{\nu} \). The experimental information about this exclusive decay process can be described by the double differential decay distribution. In order to compute the decay rate, we must have the idea about the matrix element of the effective Hamiltonian \((29)\) between the initial \( B \) meson and the final particles. Due to the non-detection of the two neutrinos, experimentally we can not distinguish between the transverse polarization, so the decay rate will be the addition of both longitudinal and transverse polarizations. The double differential decay rate with respect to \( s_B \) and \( q \cos \theta \) is given by \([35,38]\)

\[
\frac{d^2\Gamma}{d s_B d \cos \theta} = \frac{3}{4} \frac{d^2 \Gamma^L}{d s_B} \sin^2 \theta + \frac{3}{2} \frac{d^2 \Gamma^T}{d s_B} \cos^2 \theta,
\]

where the longitudinal and transverse decay rate are

\[
\frac{d\Gamma^L}{d s_B} = 3m_B^2 |A_0|^2, \quad \frac{d\Gamma^T}{d s_B} = 3m_B^2 (|A_L|^2 + |A_T|^2).
\]

The transversality amplitudes \( A_{L,T} \, |0 \rangle \) in terms of the form factors and Wilson coefficients are listed in appendix C.

The fractions of \( K^\ast \) longitudinal and transverse polarizations are given as

\[
F_{L,T} = \frac{d\Gamma_{L,T}}{d s_B},
\]

and the \( K^\ast \) polarization factor is

\[
\alpha_{K^\ast} = \frac{2F_L}{F_T} - 1.
\]

The transverse asymmetry parameters are given as \([39,40]\)

\[
A_T^{(1)} = \frac{-2Re(A_L A_T^*)}{|A_L|^2 + |A_T|^2}, \quad A_T^{(2)} = \frac{|A_T|^2 - |A_L|^2}{|A_L|^2 + |A_T|^2}.
\]

However, one can not extract \( A_T^{(1)} \) from the full angular distribution of \( B \rightarrow K^\ast \nu\bar{\nu} \), as it is not invariant under the symmetry of the distribution function and requires measurement of the neutrino polarization. So it can not

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**Table 2.** The predicted branching ratios for \( B \rightarrow (K^\ast, K^\ast, X_1)\nu\bar{\nu} \) processes and \( R_{K,K^\ast} \) for \( B \rightarrow X_1\nu\bar{\nu} \) in their respective full physical ranges.

| Observables | SM prediction | Values in \( \Delta^{1/4} \) LQ model |
|-------------|---------------|-------------------------------------|
| \( Br(B^0 \rightarrow K^0\nu\bar{\nu}) \) | \((4.9 \pm 0.29) \times 10^{-6}\) | \((3.6 - 5.2) \times 10^{-6}\) |
| \( Br(B_S^+ \rightarrow K^+\nu\bar{\nu}) \) | \((9.54 \pm 0.57) \times 10^{-6}\) | \((7.02 - 10.15) \times 10^{-6}\) |
| \( Br(B \rightarrow X_1\nu\bar{\nu}) \) | \((2.98 \pm 0.18) \times 10^{-5}\) | \((2.2 - 3.17) \times 10^{-5}\) |
| \( R_K \) | 0.164 | \((1.01 - 1.3)\) |
| \( R_{K^\ast} \) | 0.32 | 0.32 |

**Figure 5.** The variation of branching ratio of \( B \rightarrow K^\ast\nu\bar{\nu} \) with respect to the normalized invariant masses squared \( s_B \) in the SM and \( \Delta^{1/4} \) LQ model. The grey band corresponds to the uncertainties arising in the SM.
**Figure 6.** The variation of branching ratio of $B \to K^*\phi$ with respect to the $s_B$ in the SM and $\Delta^{(1/6)} LQ$ model. The grey band corresponds to the uncertainties arising in the SM.

**Figure 7.** The variation of longitudinal (left panel) and transverse (right panel) polarization of $K^*$ with $s_B$.

**Figure 8.** The variation of $K^*$ polarization factor (left panel) and the transverse asymmetry (right panel) with respect to $s_B$. 
be measured experimentally at $B$ factories or in LHCb. The transverse asymmetry $A_T^{AT^2}$ is theoretically clean and could be measurable in Belle-II.

For numerical evaluation we use the $q^2$ dependence of the $B \to K^*\nu\bar{\nu}$ form factors $V(q^2)$, $A_1(q^2)$, $A_2(q^2)$ from [41, 42]. The variation of the branching ratio of $B \to K^*\nu\bar{\nu}$ with respect to the neutrino invariant mass, $s_B$ is shown in figure 6. Figure 7 contains the longitudinal and transverse polarizations of $K^*$ versus $s_B$. The polarization factor and the transverse asymmetry variation with respect to $s_B$ in the full region are shown in figure 8. Although there is certain deviation found between the SM and LQ model predictions for the branching fraction, but no such noticeable deviations found between the SM and LQ predictions for the longitudinal/transverse polarizations, transverse asymmetry parameters $A_T^{AT^2}$. The integrated values of branching ratio over the range $s_B \in [0, 0.69]$ are presented in table 2, which are well below the the present upper limit $\text{Br}(B_d^0 \to K^*\nu\bar{\nu}) < 5.5 \times 10^{-5}$ [34].

7. $B \to X_s\nu\bar{\nu}$

The inclusive decay $B \to X_s\nu\bar{\nu}$ is dominated by the $Z$-exchange and can be searched through the large missing energy associated with the two neutrinos. This decay mode is theoretically very clean, since both the perturbative $\alpha_s$ and the non-perturbative corrections are small. So these decays do not suffer from the form factor uncertainties and thus, are very sensitive to the search for new physics beyond the SM. The decay distribution with respect to $s_B = s/m_{\nu\bar{\nu}}^2$ can be written as

$$\frac{d\Gamma}{ds_B} = m_B^3 \frac{\alpha_s^2 G_F^2}{128\pi^3} |V_{ub}|^2 |V_{cb}|^2 \kappa(0) (|C_L|^2 + |C_R|^2) \lambda^{1/2} \left(1, \bar{m}_N^2, s_B\right)$$

$$\times \left[3s_B(1 + \bar{m}_N^2 - s_B - 4\bar{m}_N \frac{Re(C_L^{-1}C_R)}{|C_L|^2 + |C_R|^2}) + \lambda(1, \bar{m}_N^2, s_B)\right]$$

(41)

where $\bar{m}_N = m_\nu/m_\mu$ and $\kappa(0) = 0.83$ is the QCD correction to the $b \to s\nu\bar{\nu}$ matrix element [43]. The full kinematically accessible physical region is $0 \leq s_B \leq (1 - \bar{m}_N^2)^2$. In figure 9, we have shown the variation of the branching ratio with respect to $s_B$ and the integrated branching ratio values over the range $s_B \in [0, 0.96]$ both in the SM and in the LQ model are presented in table 2.

In this case also no deviation found between the SM and LQ predictions.
8. Conclusion

In this paper we have studied the effect of scalar LQs on the rare semileptonic decays of $B$ meson. In particular, we focus on the decay processes $B \rightarrow K^\pm l^\mp$ in low recoil limit and the di-neutrino decay channels $B \rightarrow K^* l l^{-}$ and $B \rightarrow X_s l^{-} l^{-}$ processes. Using the allowed parameter space we predicted the branching ratio, lepton non-universality factors and the flat terms for the $B \rightarrow K^\pm l^\mp$ process in the low recoil region. We found that the measured branching ratio can be accommodated in the scalar LQ model. We have also calculated the branching ratios of $B \rightarrow K^* l l^{-}$ and $B \rightarrow X_s l^{-} l^{-}$ processes. The predicted branching ratios for $B \rightarrow K^* l l^{-}$ processes are well below the present upper limits. The polarization of $K^*$ and transverse asymmetry for $B \rightarrow K^* l l^{-}$ are also computed using the constraint LQ parameters. However, we found no deviation between the SM prediction and the LQ results for different polarization variables and the transverse asymmetry parameter.

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Appendix A. $a_l$ and $c_l$ functions in $B \rightarrow K l l$ process

The $a_l$ and $c_l$ parameters in the decay distribution of the $B \rightarrow K l l$ processes (21) can be expressed as

$$\frac{a_l}{\Gamma_0 \sqrt{\lambda} \beta_l f_+^2} = \frac{\lambda}{4} (|F_a|^2 + |F_V|^2) + 2m_t (m_b^2 - m_K^2 + q^2) Re (F_a F_V^*)$$

$$+ 4m_t^2 m_b^2 |F_a|^2 + q^2 |F_V|^2,$$

(A1)

$$\frac{c_l}{\Gamma_0 \sqrt{\lambda} \beta_l f_+^2} = -\beta_l^2 \frac{\lambda}{4} (|F_a|^2 + |F_V|^2),$$

(A2)

with

$$\Gamma_0 = \frac{G_F^2 \alpha^2 |V_{tb} V_{ts}^*|^2}{2\pi^3 m_b^3}, \quad \beta_l = \sqrt{1 - \frac{4m_t^2}{q^2}},$$

(A3)

and

$$\lambda = m_b^4 + m_K^4 + q^4 - 2(m_b^2 m_K^2 + m_b^2 q^2 + m_K^2 q^2).$$
Appendix B. Loop functions

The loop function $X(x_t)$ in equation (31), including correction $O(\alpha_s)$ at the next-to-leading order in QCD, is given by [44, 45]

$$X(x_t) = X_0(x_t) + \frac{\alpha_s}{4\pi}X_1(x_t), \quad (B1)$$

where

$$X_0(x_t) = \frac{x_t}{8} \left[ \frac{2 + x_t}{1 - x_t} + \frac{3x_t - 6}{(1-x_t)^2} \ln x_t \right], \quad (B2)$$

and

$$X_1(x_t) = -\frac{29x_t - x_t^3 - 4x_t^5}{3(1-x_t)^2} - \frac{x_t + 9x_t^7 - x_t^4 - x_t^3}{(1-x_t)^3} \ln x_t$$

$$+ \frac{8x_t + 4x_t^2 + x_t^3 - x_t^4}{2(1-x_t)^3} \ln^2 x_t$$

$$+ 8x_t \frac{\partial X_0(x_t)}{\partial x_t} \ln x_t. \quad (B3)$$

In equations (B1)–(B3), the parameters used are defined as $x_t = m_t^2/m_W^2$, $x_{\mu} = \mu^2/m_W^2$ with $\mu = O(m_t)$ and $L_2(1-x_t) = \int_1^0 \frac{dx}{1-x}$.

Appendix C. Transversity amplitudes for $B \to K^*\ell\nu$ process

The transversality amplitudes $A_{-\ell\perp,0}$ for $B \to K^*\ell\nu$ process are given as

$$A_{-\ell\perp,0}(s_B) = 2N\sqrt{2} \lambda^{1/2}(1, m_B^2, s_B)(C_1' + C_3') \frac{V(s_B)}{(1 + m_{K^*})}, \quad (C1)$$

$$A_{-\ell\perp,0}(s_B) = -2N\sqrt{2} (1 + m_{K^*}) (C_1' - C_3') A_{-\ell\perp,0}(s_B), \quad (C2)$$

$$A_{0}(s_B) = -\frac{N(C_1' - C_3')}{m_{K^*} + s_B} \left[ (1 - m_B^2 - s_B)(1 + m_{K^*})A_{-\ell\perp,0}(s_B) - \lambda(1, m_B^2, s_B) \frac{A_2(s_B)}{1 + m_{K^*}} \right], \quad (C3)$$

with

$$N = V_{ub} \mathcal{V}_{ub}^{\text{eff}} \frac{G_F^2 m_B^3}{3 - 2m_{\pi}^2 s_B} \lambda^{1/2}(1, m_B^2, s_B)^{1/2}. \quad (C4)$$

The various form factors $V(s_B)$, $A_1(s_B)$, $A_2(s_B)$ associated with $B \to K^*$ transition in equations (C1)–(C3) are defined as

$$\langle K^*(p_{K^*})|\gamma_{\mu},P_{L,R}b|B(p)\rangle = i\epsilon_{\mu\nu\alpha\beta} \epsilon^{\nu\rho\sigma} q^{\rho} \frac{V(s_B)}{m_B^2 + m_{K^*}} \pm \frac{1}{2} ((m_B + m_{K^*}) \epsilon_{\mu} A_1(s_B)$$

$$- (\epsilon^* \cdot q)(2p - q)_{\mu} \frac{A_2(s_B)}{m_B + m_{K^*}} - 2m_{K^*} \epsilon_{\mu} \epsilon_{\nu} A_3(s_B) - A_0(s_B) \frac{A_2(s_B)}{m_B + m_{K^*}} - A_0(s_B) \right) \frac{q_{\nu}}{s} (\epsilon^* \cdot q) A_3(s_B) - A_0(s_B) \right), \quad (C5)$$

where $q = p_{L^*} + p_{R^*}$ and $\epsilon^*$ is the polarization vector of $K^*$.

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