CP violation at one loop in the polarization-independent chargino production in $e^+e^-$ collisions

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Abstract

Recently Osland and Vereshagin noticed, based on sample calculations of some box diagrams, that in unpolarised $e^+e^-$ collisions CP-odd effects in the non-diagonal chargino-pair production process are generated at one-loop. Here we perform a full one-loop analysis of these effects and point out that in some cases the neglected vertex and self-energy contributions may play a dominant role. We also show that CP asymmetries in chargino production are sensitive not only to the phase of $\mu$ parameter in the chargino sector but also to the phase of stop trilinear coupling $A_t$. 
1 Introduction

The electroweak sector of the Standard Model (SM) contains only one CP-violating phase which arises in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. Adding right-handed neutrinos to account for non-zero neutrino masses and their mixing opens up a possibility of new CP-violating phases in the Maki-Nakagawa-Sakata (MNS) lepton mixing matrix. While the observed amount of CP violation in the $K$ and $B$ can be accommodated within the SM, another (indirect) piece of evidence of CP violation, the baryon asymmetry in the universe, requires a new source of CP violation \[1\]. Thus new CP-violating phases must exist in nature.

Supersymmetric extensions of the SM introduce a plethora of CP phases in soft supersymmetry breaking terms. This poses a SUSY CP problem, since if the phases are large $\mathcal{O}(1)$, SUSY contributions to the lepton and neutron EDMs can be too large to satisfy current experimental constraints \[2\]. Many models have been proposed \[3\] to overcome this problem: fine tune phases to be small, push sparticle spectra (especially squarks and sleptons) above a TeV scale to suppress effects of large phases on the EDM, arrange for internal cancelations etc.

In the absence of any reliable theory that forces in a natural way the phases to be vanishing or small, it is mandatory to consider scenarios with some of the phases large and arranged consistent with experimental EDM data. In such CP-violating scenarios charginos and neutralinos (denoted generically by $\tilde{\chi}$) might be light enough to be produced at $e^+e^-$ colliders, and many phenomena will be affected by non-vanishing phases: sparticle masses, their decay rates and production cross sections, SUSY contributions to SM processes etc. However, most unambiguous way to study the presence of CP-violating phases would be in some CP-odd observables measurable at future accelerators.

To build a CP-odd observable in a two-fermion $\to$ two-fermion process, e.g. $e^+e^- \to \tilde{\chi}_i\tilde{\chi}_j$, typically one uses spin information of one of the particles involved. For example, a measurement of the fermion polarization $s$ transverse to the production plane \[4\] allows to build a CP-odd observable $s \cdot (p_e \times p_{\tilde{\chi}})$. This requires either transverse beam polarization and/or spin-analyser of produced $\tilde{\chi}$’s via angular distributions of their decay products \[5\]. Another possibility is to look into triple products involving momenta of the decay products of charginos in case of longitudinal polarization of the beams \[6\].

However, CP-odd effects can also be detected in simple event-counting experiments if several processes are measured. One example is provided by non-diagonal neutralino-pair production in $e^+e^-$ annihilation with unpolarized beams: observing the $\tilde{\chi}_i^0\tilde{\chi}_j^0$, $\tilde{\chi}_i^0\tilde{\chi}_k^0$ and $\tilde{\chi}_j^0\tilde{\chi}_k^0$ pairs to be excited all in S-wave near respective thresholds signals CP-violation in the neutralino sector at tree-level \[7\]. Alternatively, unambiguous evidence for CP-violation in the neutralino system is provided by the observation of simultaneous sharp S-wave excitations of both the production of any non-diagonal neutralino pair $\tilde{\chi}_i^0\tilde{\chi}_j^0$ near threshold and the $f\bar{f}$ invariant mass distribution of
the decay $\tilde{\chi}_j^0 \to \tilde{\chi}_i^0 f\bar{f}$ near the end point $[8].$

Recently Osland and Vereshagin pointed out that in the non-diagonal chargino pair production process $e^+e^- \to \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp$ a CP-odd observable can be constructed from unpolarized cross sections at one-loop $[9].$ Their simplified numerical analysis based on only some of the box diagrams shows that, indeed, the CP-violation induced by the complex higgsino mass parameter $\mu$ may in principle be observed in this reaction without any spin detection and with unpolarized initial beams.

In this note we perform a full one-loop analysis of the non-diagonal chargino pair production. First we recapitulate the MSSM chargino sector at tree level and show explicitly that such a CP asymmetry vanishes. In Section 3 we discuss the CP asymmetry at one loop. We note that a non-zero asymmetry requires not only complex couplings but also absorptive parts of Feynman diagrams. In Section 4 we present numerical results for the CP asymmetry and discuss relative weights of various contributions. We consider effects of both the complex higgsino mass parameter and the complex trilinear scalar coupling in the top squark sector. Section 5 summarizes and concludes our analysis.

2 MSSM chargino sector at tree level

In the minimal supersymmetric extension of the Standard Model (MSSM), the tree-level mass matrix of the spin-1/2 partners of the charged gauge and Higgs bosons, $\tilde{W}^-$ and $\tilde{H}^−$, takes the form

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2m_W \cos \beta} \\ \sqrt{2m_W \sin \beta} & \mu \end{pmatrix},$$ (1)

where $M_2$ is the SU(2) gaugino mass, $\mu$ is the higgsino mass parameter, and $\tan \beta$ is the ratio $v_2/v_1$ of the vacuum expectation values of the two neutral Higgs fields. By reparametrization of the fields, $M_2$ can be taken real and positive, while $\mu$ can be complex $\mu = |\mu| e^{i \Phi_\mu}$. Since the chargino mass matrix $\mathcal{M}_C$ is not symmetric, two different unitary matrices acting on the left- and right-chiral $(\tilde{W}, \tilde{H})_{L,R}$ two-component states

$$U_{L,R} \begin{pmatrix} \tilde{W}^- \\ \tilde{H}^- \end{pmatrix}_{L,R} = \begin{pmatrix} \tilde{\chi}_1^- \\ \tilde{\chi}_2^- \end{pmatrix}_{L,R}$$ (2)

are needed to diagonalize it. The unitary matrices $U_L$ and $U_R$ can be parameterized in the following way $[10]$ ($s_\alpha = \sin \alpha, c_\alpha = \cos \alpha$):

$$U_L = \begin{pmatrix} c_{\phi_L} & e^{-i\beta_L} s_{\phi_L} \\ -e^{i\beta_L} s_{\phi_L} & c_{\phi_L} \end{pmatrix}, \quad U_R = \begin{pmatrix} e^{i\gamma_1} & 0 \\ 0 & e^{i\gamma_2} \end{pmatrix} \begin{pmatrix} c_{\phi_R} & e^{-i\beta_R} s_{\phi_R} \\ -e^{i\beta_R} s_{\phi_R} & c_{\phi_R} \end{pmatrix}. \quad (3)$$
As far as $\Phi_\mu$ dependence is concerned, the mass eigenvalues and rotation angles $\cos 2\phi_{L,R}$, $\sin 2\phi_{L,R}$, being CP-even, are functions of $\cos \Phi_\mu$ only. On the other hand, the four phases $\beta_{L,R}$ and $\gamma_{1,2}$ are CP-odd since their tangents depend linearly on $\sin \Phi_\mu$; all four phases vanish in CP-invariant case for which $\Phi_\mu = 0$ or $\pi$.

Charginos can copiously be produced at prospective $e^+e^-$ linear colliders \[11\]. At tree-level they are produced via the s-channel $\gamma, Z$ exchange and t-channel electron sneutrino exchange. Photon exchange contributes only to the production of diagonal pairs $\tilde{\chi}_1^+ \tilde{\chi}_1^-$ and $\tilde{\chi}_2^+ \tilde{\chi}_2^-$. The production amplitude, after a Fierz transformation of the t-channel contribution,

$$\mathcal{A}[e^+e^- \rightarrow \tilde{\chi}_1^- \tilde{\chi}_1^+] = \frac{e^2}{s} Q_{ij}^{\alpha \beta} \left[ \bar{v}(e^+) \gamma_{\mu} P_\alpha u(e^-) \right] \left[ \bar{u}(\tilde{\chi}_1^-) \gamma_{\mu} P_\beta v(\tilde{\chi}_1^+) \right],$$

is expressed in terms of four bilinear charges $Q_{ij}^{\alpha \beta}$, defined by the chiralities $\alpha, \beta = L, R$ of the lepton and chargino currents. The charges take the form

$$Q_{RL}^{ij} = \delta_{ij} D_R + C_{ij}^L F_R, \quad Q_{LL}^{ij} = \delta_{ij} D_L + C_{ij}^L F_L, \quad Q_{RR}^{ij} = \delta_{ij} D_R + C_{ij}^R F_R, \quad Q_{LR}^{ij} = \delta_{ij} D_L + C_{ij}^R F_L + \frac{D_\gamma}{4s_{\gamma}} (\delta_{ij} - C_{ij}^R),$$

with s-, t-channel propagators $D_L = 1 + \frac{D_\gamma}{c_{\beta} s_{\beta}} (s_{W}^2 - \frac{1}{2}) (s_{W}^2 - \frac{3}{4})$, $F_L = \frac{D_\gamma}{4s_{\gamma}} (s_{W}^2 - \frac{1}{2})$, $D_R = 1 + \frac{D_\gamma}{c_{\beta} s_{\beta}} (s_{W}^2 - \frac{3}{4})$, $F_R = \frac{D_\gamma}{4s_{\gamma}}$, $D_Z = s/(s - m_Z^2)$, $D_\phi = s/(s - m_\phi^2)$; the $\bar{\nu}_e$ exchange contributes only to the LR amplitude. The coefficients $C_{ij}^L$ are functions of $U_L$ as follows

$$C_{11}^L = - \cos 2\phi_L, \quad C_{22}^L = \cos 2\phi_L, \quad C_{12}^L = e^{-i\beta L} \sin 2\phi_L, \quad C_{21}^L = e^{i\beta L} \sin 2\phi_L,$$

for $C^R$ replace $\phi_L \rightarrow \phi_R$ and $\beta_L \rightarrow \beta_R - \gamma_1 + \gamma_2$.

Note that the phases $\beta_L, \beta_R, \gamma_1, \gamma_2$ enter only non-diagonal \{12\} and \{21\} amplitudes. However, after summing over chargino helicities, the dependence on these phases disappears in the polarized differential cross section for the $e^+e^- \rightarrow \tilde{\chi}_1^- \tilde{\chi}_1^+$. Defining the polar angle $\theta$ and the azimuthal angle $\phi$ of $\tilde{\chi}_1^+$ with respect to the $e^-$ momentum direction and the $e^-$ transverse polarization vector, respectively, the polarized differential cross section is given by \[10\]

$$d\sigma^{ij} = \frac{d\sigma^{(ij)}}{d\cos \theta d\phi} = \frac{\alpha^2}{16} \lambda^{1/2} \left[ (1 - P_L \bar{P}_L) \Sigma_{\text{unp}} + (P_L - \bar{P}_L) \Sigma_L + P_T \bar{P}_T \cos(2\phi - \eta) \Sigma_T \right]$$

where $P=(P_T, 0, P_L)$ \linebreak $\bar{P}=(\bar{P}_T \cos \eta, \bar{P}_T \sin \eta, -\bar{P}_L)$] is the electron [positron] polarization vector; $\lambda = [1 - (\mu_i + \mu_j)^2][1 - (\mu_i - \mu_j)^2]$ with $\mu_i = m_i/\sqrt{s}$. The distributions $\Sigma_{\text{unp}}, \Sigma_L$ and $\Sigma_T$ depend only on the polar angle $\theta$ and can be expressed as (the superscripts \{ij\} labeling the produced chargino pair are understood)

$$\Sigma_{\text{unp}} = 4 \{ [1 - (\mu_i^2 - \mu_j^2)^2 + \lambda \cos^2 \theta] Q_1 + 4\mu_i \mu_j Q_2 + 2\lambda^{1/2} Q_3 \cos \theta \},$$
$$\Sigma_{LL} = 4 \{ [1 - (\mu_i^2 - \mu_j^2)^2 + \lambda \cos^2 \theta] Q'_1 + 4\mu_i \mu_j Q'_2 + 2\lambda^{1/2} Q'_3 \cos \theta \},$$
$$\Sigma_{TT} = -4 \lambda \sin^2 \theta Q_5.$$
The eight quartic charges for each of the production processes of the diagonal and mixed chargino pairs, expressed in terms of bilinear charges, are collected in Table 1, including the transformation properties under P and CP.

**Table 1: The quartic charges of the chargino system.**

| P   | CP   | Quartic charges                                           |
|-----|------|----------------------------------------------------------|
| even| even | \( Q_1 = \frac{1}{4} \left[ |Q_{RR}|^2 + |Q_{LL}|^2 + |Q_{RL}|^2 + |Q_{LR}|^2 \right] \) |
|     |      | \( Q_2 = \frac{1}{2} \text{Re} [Q_{RR}Q_{RL} + Q_{LL}Q_{LR}^*] \) |
|     |      | \( Q_3 = \frac{1}{4} \left[ |Q_{RR}|^2 + |Q_{LL}|^2 - |Q_{RL}|^2 - |Q_{LR}|^2 \right] \) |
|     |      | \( Q_5 = \frac{1}{2} \text{Re} [Q_{LR}Q_{RR}^* + Q_{LL}Q_{RL}^*] \) |

| odd | even | \( Q_4' = \frac{1}{4} \left[ |Q_{RR}|^2 + |Q_{RL}|^2 - |Q_{LR}|^2 - |Q_{LL}|^2 \right] \) |
|     |      | \( Q_2' = \frac{1}{2} \text{Re} [Q_{RR}Q_{RL} - Q_{LL}Q_{LR}^*] \) |
|     |      | \( Q_3' = \frac{1}{4} \left[ |Q_{RR}|^2 + |Q_{LR}|^2 - |Q_{RL}|^2 - |Q_{LL}|^2 \right] \) |

The charges \( Q_1 \) to \( Q_5 \) are manifestly parity-even, \( Q_4' \) to \( Q_3' \) are parity-odd. The charges \( Q_1 \) to \( Q_3 \), \( Q_5 \), and \( Q_4' \) to \( Q_3' \) are CP-invariant. Only \( Q_4 \) changes sign under CP transformations.

From above expressions it is evident that even for transverse beam polarization the differential cross section \( \sim \Sigma_{TT} \) is CP-even. Also the differential distributions for non-diagonal chargino pairs \( \tilde{\chi}_1^- \tilde{\chi}_2^+ \) and \( \tilde{\chi}_2^- \tilde{\chi}_1^+ \) are equal, so the asymmetry

\[
A_{12} = \frac{\int_{-1}^{1} (d\sigma^{12} - d\sigma^{21}) d\cos\theta}{\int_{-1}^{1} (d\sigma^{12} + d\sigma^{21}) d\cos\theta}
\]

at tree level vanishes in CP-noninvariant theories. The CP-odd quartic charge \( Q_4 \) can only be probed by observables sensitive to the chargino polarization component normal to the production plane in mixed \( e^+e^- \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp \) processes [10]. Thus, at tree-level one cannot build a CP-odd observable from chargino polarized cross sections alone.

Due to Poincaré invariance the unpolarized differential cross section \( \sim \Sigma_{\text{unp}} \) may depend only on masses \( m_i, m_j \) and on two independent scalar variables \( s \) and \( t \). As a result, the unpolarized differential cross-sections for equal-mass fermions \( m_i = m_j \) in the final state are always CP-even. However, if the chargino species are different, beyond tree level the CP-violating terms can arise even in the unpolarized cross-section [9].
3 CP-odd asymmetry at one-loop

Radiative corrections to the chargino pair production include the following generic one-loop Feynman diagrams: the virtual vertex corrections Fig. 1, the self-energy corrections to the $\tilde{\nu}$, $Z$ and $\gamma$ propagators, and the box diagrams contributions Fig. 2. We also have to include corrections on external chargino legs. Generation and calculation of one-loop graphs is performed using FeynArts 3.2 and FormCalc 5.2 packages \[12\]. For numerical evaluation of loop integrals we use LoopTools 2.2 \[13\].

In the Ref. \[9\] sample calculations of box diagrams with only photon, $Z$ and $W$ boson exchanges (c.f. diagrams 5 and 10 in Fig. 2) and neglecting all sfermion contributions have been performed to demonstrate non-zero asymmetry $A_{12}$ at one-loop. Here we present the full calculation, including all possible contributions at the one-loop level taking into account CP-violating phases. Full calculation of radiative corrections to chargino pair production without CP-violating phases can be found in \[14\].

One-loop corrected matrix element squared is given by

\[ |M_{\text{loop}}|^2 = |M_{\text{tree}}|^2 + 2\text{Re}(M_{\text{tree}}^\ast M_{\text{loop}}). \] (10)

Accordingly, the one-loop CP asymmetry for the non-diagonal chargino pair is defined as

\[ A_{12} = \frac{\int_{-1}^{1} (d\sigma_{\text{loop}}^{12} - d\sigma_{\text{loop}}^{21})d\cos\theta}{\int_{-1}^{1} (d\sigma_{\text{tree}}^{12} + d\sigma_{\text{tree}}^{21})d\cos\theta}. \] (11)

Since, as mentioned in the previous section, the CP-odd contribution vanishes at tree level, it has to be UV-finite. In fact one can note, that the structure of counterterms is the same as the tree level graphs, so using the same arguments as in Sec. 2 it can be shown that renormalization procedure will not give rise to the asymmetry. Nevertheless self-energy and vertex corrections are UV-divergent, and proper treatment of divergences is needed. We choose to work in the dimensional reduction scheme \[15\], which preserves supersymmetry.

Loop diagrams with internal photon line also introduce infrared singularities. They can be removed by adding emission of soft photons from external charged particles. The sum of both contributions is then IR finite, however it depends on the soft photon cut. On the other hand soft photon emission part has the form of tree-level amplitude multiplied by soft photon factor \[16\]. Therefore, as it was explained in Sec. 2 the terms arising due to soft photon bremsstrahlung do not affect the asymmetry $A_{12}$. Similar arguments apply for hard photon emission from external fermions.

At this point we want to make some remarks about the origin of the CP-odd asymmetry $A_{12}$. In order to obtain a non-zero asymmetry in the chargino production it is not enough to have CP-violating phases in the MSSM lagrangian. In addition it requires a non-trivial imaginary
Figure 1: Generic triangle graphs contributing to chargino pair $\tilde{\chi}_2^- \tilde{\chi}_1^+$ production in $e^+e^-$ collisions.

part from Feynman diagrams – the absorptive part. Such contributions appear when some of the intermediate state particles in loop diagrams go on-shell. CP-odd asymmetry is generated due to the interference between imaginary part of loop integrals and imaginary parts of the couplings. As one can see from Eq. (6), the contributions for the production of non-diagonal chargino pairs $\{12\}$ and $\{21\}$ differ by the opposite sign of the imaginary part. Since the absorptive parts of loop integrals are the same for both processes, we clearly see that the final real contribution to the matrix element squared will be different in each of these final states.
Figure 2: Generic box graphs contributing to chargino pair $\tilde{\chi}_2^-\tilde{\chi}_1^+$ production in $e^+e^-$ collisions.

4 Numerical analysis

To illustrate relative weights of various contributions to the CP asymmetry, we consider two scenarios: (A) which is close to the SPS1a' point that has been studied particularly widely [17]; (B) for comparison with Ref. [9]. In both scenarios the value of the ratio of the vacuum expectation values for Higgs fields is taken to be $\tan\beta = 10$ and the parameters defined below are low-scale parameters.

In scenario (A) we take the following values for the gaugino and higgsino mass parameters:

$$|M_1| = 100 \text{ GeV}, \quad M_2 = 200 \text{ GeV}, \quad |\mu| = 400 \text{ GeV}.$$ 

For the sfermion mass parameters we assume

$$m_{\tilde{q}} \equiv M_{\tilde{q}_{1,2}} = M_{\tilde{u}_{1,2}} = M_{\tilde{d}_{1,2}} = 450 \text{ GeV},$$

$$M_{\tilde{Q}} \equiv M_{\tilde{Q}_3} = M_{\tilde{U}_3} = M_{\tilde{D}_3} = 300 \text{ GeV},$$

$$m_{\tilde{l}} \equiv M_{\tilde{L}_{1,2,3}} = M_{\tilde{E}_{1,2,3}} = 150 \text{ GeV},$$
and for the trilinear coupling

\[ |A_t| = -A_b = -A_r = A = 400 \text{ GeV}. \]

Moreover, we allow non-zero phases \( \Phi_\mu \) of the \( \mu \) parameter, \( \Phi_1 \) of the bino mass parameter \( M_1 \) and \( \Phi_t \) of the trilinear coupling in the stop sector \( A_t \).

In scenario (B) we take, as in Ref. [9], the gaugino/higgsino masses

\[ |M_1| = 250 \text{ GeV}, \quad M_2 = 200 \text{ GeV}, \quad |\mu| = 300 \text{ GeV}. \]

For comparison with Ref. [9] we set the universal scalar mass

\[ M_S = M_Q = M_U = M_L = M_E = 10 \text{ TeV}, \]

so the contributions from diagrams with exchanges of supersymmetric scalars are negligibly small. We also investigate the \( M_S \) dependence of the asymmetry.

In addition, the following values of the SM parameters are used:

\[
\begin{align*}
m_W &= 80.45 \text{ GeV}, \quad m_Z = 91.1875 \text{ GeV}, \quad \cos \theta_W = m_W/m_Z, \\
m_t &= 171 \text{ GeV}, \quad \alpha = 1/127.9. \quad \tag{12}
\end{align*}
\]

The masses of relevant particles are given in Table 2.

| masses | \( m_{\tilde{\chi}^\pm_1} \) | \( m_{\tilde{\chi}^\pm_2} \) | \( m_{\tilde{\chi}^0_1} \) | \( m_{\tilde{\chi}^0_2} \) | \( m_{\tilde{\chi}^0_3} \) | \( m_{\tilde{\chi}^0_4} \) |
|--------|-----------------|-----------------|--------------|--------------|--------------|--------------|
| scenario (A) | 186.7 GeV | 421.8 GeV | 97.5 GeV | 187.0 GeV | 405.8 GeV | 421.2 GeV |
| scenario (B) | 175.6 GeV | 334.5 GeV | 172.8 GeV | 242.4 GeV | 306.5 GeV | 341.4 GeV |

The masses of stop squarks in scenario (A) are \( m_{\tilde{t}_1} = 204.9 \text{ GeV} \) and \( m_{\tilde{t}_2} = 438.6 \text{ GeV} \). The threshold for non-diagonal chargino pair production is 608.5 GeV in scenario (A) and 510.1 GeV in scenario (B). Therefore for all plots in the present analysis we take the center of mass energy \( \sqrt{s} = 700 \text{ GeV} \).

First we consider scenario (A). The dependence of the CP asymmetry on the phase \( \Phi_\mu \) of the higgsino mass parameter \( \mu \) is shown in the left panel of Fig. [5]. Contributions due to box corrections, vertex corrections and self energy corrections have been plotted in addition to the full result. The asymmetry can reach values as large as 1%. Box and self-energy diagrams can give the asymmetry of the order 1.5% – 2%, but since they are of opposite signs the total asymmetry tends to be smaller. Moreover, in this scenario the constraints from EDMs restrict the phase
Φµ to be close to nπ. For such values the predicted asymmetry is very small and probably unmeasurable even at high luminosity e+e− linear colliders.

For the case of CP asymmetry induced by the phase Φt of the trilinear coupling in the top squark sector A_t, the situation is quite different, as illustrated in the right panel of Fig. 3. The box diagrams do not give rise to the CP asymmetry in this case, since there are no box diagrams with stop exchanges. Diagrams with top squark exchanges appear only in vertex and self energy corrections. As for vertex corrections, only diagrams of class 1 (FFS = bb̄ti) and class 2 (SSF = t̄itīb) from Fig. 1 contribute. The contributions from vertex and self-energies are of the same sign and add coherently to give the full asymmetry of the order 6% – large enough to be measurable in future experiments.

Note that the triangle graphs induce a coupling of the photon field to fermions of different mass. To this coupling the diagrams of class 1, 2, 3 and 6 with V = γ from Fig. 1 contribute. These diagrams give rise to the CP asymmetry of the order 0.1% both for Φµ and Φt.

In the left panel of Fig. 4, the dependence of the differential asymmetry a_{12} (defined as in Eq. (11) but with integrals removed) as a function of the production angle cos θ. For comparison we show the plots for two choices of phases: Φµ = 3π/2 (full line) and and Φt = π/2 (dotted line). Apart from the difference in magnitude, these asymmetries have different dependence on the cos θ: Φµ asymmetry decreases with cos θ, whereas Φt asymmetry increases from 5.7% to 7%.

It is also interesting to investigate the tan β dependence of the various contributions to the asymmetry. It is shown in the right panel of Fig. 4 for the range tan β ∈ [1, 20] with Φµ = π/2, Φ1 = Φ2 = 0. As tan β increases from 1, the absolute values of box and vertex contributions to the CP asymmetry increase reaching maxima around tan β = 2 and then drop down. This
behavior follows mainly from the structure of the chargino mass matrix and consequently the tan β dependence of the chargino mixing angles and phases – imaginary parts of coefficients $C_{12}^{R,L}$ in Eq. (6) rapidly go down for small and large values of tan β. For tan β = 1 the full asymmetry is close to 0, although again it is a result of cancelations between various contributions.

We now turn to scenario (B) as discussed in Ref. [9]. In the left panel of Fig. 5 we show the full asymmetry and contributions from box, vertex and self-energy diagrams. The asymmetry at its maximum reaches almost 0.5%, and is significantly smaller than in scenario (A). Because sfermions are very heavy at this parameter point, the main contribution to the asymmetry is due to box diagrams with exchanges of vector bosons $\gamma$, $Z$, $W$, namely diagrams of class 5 and 10 with $FFVV = e\tilde{\chi}_i\gamma Z$, $e\tilde{\chi}_i Z\gamma$, $e\tilde{\chi}_i ZZ$, and diagram of class 5 with $FFVV = \nu\tilde{\chi}_0^i WW$ of Fig. 2. Contributions from vertex and self-energy diagrams are significantly smaller and opposite in sign and almost cancel each other. This is the reason why our results are consistent with results obtained by [9]. In addition, in the right panel of Fig. 5 we show the dependence of the full CP asymmetry on the universal soft SUSY-breaking scalar mass $M_S = M_{\tilde{Q}} = M_{\tilde{U}} = M_{\tilde{D}} = M_{\tilde{L}} = M_{\tilde{E}}$ and compare it to the approximate result obtained by Osland and Vereshagin, i.e. when only box contributions without sfermion exchanges are included. With increasing $M_S$ the full result approaches a constant value, which is slightly lower than the approximate result. This small difference (which depends on $\sqrt{s}$) is due to vertex and self-energy corrections.

We have shown the influence of the phases $\Phi_\mu$ and $\Phi_t$ on the CP asymmetry Eq. (11). However one can also introduce CP violating phases for the trilinear couplings for other sfermions, e.g.
Figure 5: Left panel: CP asymmetry in chargino production in scenario (B) as a function of $\Phi_\mu$ phase: full asymmetry (full line) and contributions from box (dashed), vertex (dotted), self energy (dashed-dotted) diagrams. Right panel: CP asymmetry Eq. (11) as a function of $M_S = M_{\tilde{Q}} = M_{\tilde{U}} = M_{\tilde{D}} = M_{\tilde{L}} = M_{\tilde{E}}$: Full line is for the full result and a dotted line is for box contributions only, as in [9]. Other parameters are taken as in scenario (B) with $\Phi_\mu = \pi/2$.

for bottom squarks $A_b$ and for tau sleptons $A_\tau$, as well as for the bino mass parameter $M_1$ in the neutralino sector. Indeed these can give rise to the CP asymmetry. However calculations show that CP asymmetries due to these phases are typically very small, e.g. for $\Phi_1$ of the order 0.1%, so we do not include them here.

5 Conclusions

In this note we have investigated the non-diagonal chargino pair production $e^+ e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-$ at one loop and calculated the loop-generated CP asymmetry. The CP-odd observable can be constructed from unpolarized production cross sections alone without the need of measuring chargino polarizations in the final state. Our numerical analyses show that not only the box diagrams but also the vertex and self-energy diagrams can contribute to the CP-violation if it is induced by the complex higgsino mass parameter. For the case of CP-violation in the top squark sector the box diagrams do not contribute at one loop and the asymmetry comes entirely from vertex and self-energy diagrams. The asymmetries can be of the order of a few percent and in principle measurable allowing to discover the CP-violating phases via simple event counting experiments.

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