Rapid Gradient Projection Algorithm for Remote Sensing Image Reconstruction

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Abstract. Remote sensing reconstruction technology has become an important research topic due to the rapid development of aerospace, remote sensing, and communication technologies. In order to improve remote sensing reconstruction image quality, we design of a rapid Gradient Projection method. In the traditional Gradient Projection method, the searching direction changes each step. Now we propose a rapid searching method by running with fix step. Experiment results show that the rapid Gradient Projection Algorithm get the better reconstruction image. The running results are shown by two remote sensing pictures. Further research is required in remote sensing edge intelligence technology to improve its depth and breadth of application.

1. Introduction
The higher resolution remote sensing image is an important application value in environmental monitoring, urban planning [1,2]. There are more information in higher resolution remote sensing image. Because of the effect of long distance transmission, there are more noise in the received image. It will be difficult of analysis the remote sensing image. The noise reduction researching is always the key researching area of remote sensing researching [3~5]. The purpose of remote sensing image noise reduction is protecting the important information of remote sensing image [6~8]. Now the resolution ratio of digital image is higher and higher. The method of improving the image resolution is that increase the array density of collecting sensor, and reduce the noise of image.

In 2015-2020 years, with the research and rapid development of Compressed Sensing theory, the sparse sign method are used on the reconstruction of image field, this method is constrained by sparse sign, and reconstructed by sparse dictionaries. In 2004 Candes and Tao proposed the CS theory. And the Compressed Sensing theory is used in many fields, such as the remote sensing image, medical imaging, video monitor, biological information processing, Computer imaging processing etc.

Remote sensing reconstruction technology has become an important research topic due to the rapid development of aerospace, remote sensing, and communication technologies. In order to improve remote sensing reconstruction image quality, we design of a rapid Gradient Projection method. In the traditional Gradient Projection method, the searching direction changes every step. Now we propose a rapid searching method in fix step. Experiment results show that the rapid Gradient Projection Algorithm get the better reconstruction image. The running results are shown by two remote sensing pictures. Further research is required in remote sensing edge intelligence technology to improve its depth and breadth of application.
2. The CS Theory Processing Procedure

All the signals or images, which will running in CS theory, the signals or images must be sparse. If the signals don’t meet the sparse item, we have to translate the signals by following formula.

Firstly, the input signals (data or image) are sparse. If the signals (data or image) are compact, this classic Compressed Sensing theory will not worked. After that the signals are represented by formula or transform.

In formula one the signals are shown. The $xx$ is the input signal.

$$s = Wxx \quad (1)$$

Formula one must be shown in norm as formula two.

$$\|xx\|_0 \leq K \quad (2)$$

We have to show non-zero norm $\|xx\|_0$ as the formula three.

$$y = \Phi s = \Phi Wxx = AAxx \quad (3)$$

In order to get the results of CS theory, we have to show the formula three in formula four.

$$\text{minimum} \|xx\|_0 \text{ s.t. } y = AAxx \quad (4)$$

3. Our Improved Method of CS Theory

The random projection process is shown in formula five, $y = \Phi s , \Phi \in R^{m,n}$.

$$\hat{\alpha} = \text{arg} \text{minimum}_\alpha \frac{\|y - \Phi s\|^2_2 + \tau \|\alpha\|}{2} \quad (5)$$

We improved the formula five by formula six. $\alpha = \Psi^T s$

$$\hat{\alpha} = \text{arg} \text{minimum}_\alpha \frac{\|y - \Phi \Psi^T \alpha\|^2_2 + \tau \|\alpha\|}{2} \quad (6)$$

In formula seven, we use the $A = \Phi \Psi^T , x = \alpha$ symbol.

$$\hat{x} = \text{arg} \text{minimum}_xx \left\{ \frac{\|y - Ax\|^2_2 + \tau \|xx\|_0}{2} \right\} \quad (7)$$

CS theory, we use formula eight replace formula seven.

$$\text{minimum}_{uu,vv} \frac{\|y - A(uu - vv)\|^2_2 + \tau I_n^T uu + \tau I_n^T vv}{2} \quad (8)$$

s.t. $uu \geq 0, vv \geq 0$

In two-times constraint form, the formula nine

$$\text{minimum} c^T z + \frac{1}{2} z^T Bz = F(z) \quad (9)$$

Here, $z = \begin{bmatrix} uu \\ vv \end{bmatrix} , b = A^T y , c = \tau 1_{2n} + \begin{bmatrix} -b \\ b \end{bmatrix}$, $B = \begin{bmatrix} A^T A, -A^T A \\ -A^T A, A^T A \end{bmatrix}$.

The gradient project is shown in formula ten, formula eleven, and formula twelve.
\[ \nabla F(Z) = c + Bz \]  

(10)

\[ w^{(kk)} = (z^{(kk)} - \alpha^{(kk)} \nabla f(z^{(kk)})) \]  

(11)

\[ z^{(kk+1)} = z^{(kk)} - \chi^{(kk)} (w^{(kk)} - z^{(kk)}) \]  

(12)

We get the \( \alpha_0 \) in formula thirteen.

\[ \alpha_0 = \frac{g^{(kk)T}g^{(kk)}}{(g^{(kk)T})Bg^{(kk)}} \]  

(13)

We get the \( \chi^{(kk)} \) by formula fourteen.

\[ \delta^{(kk)} = (z^{(kk)} - \Delta f(z^{(kk)}) + \alpha^{(kk-1)} \delta^{(kk-1)} - z^{(kk)}) \]  

(14)

The searching step is computed by formula fifteen.

\[ d_{kk} = \psi_{kk}^T (y - \hat{H}_{kk}) \frac{\tau d}{d\hat{H}_{kk}} = \psi_{kk}^T r^{(kk-1)} / 2 - \tau w \]  

(15)

Where \( w = \begin{cases} 1 & \text{where } \hat{H}_{kk} > 0 \\ -1 & \text{where } \hat{H}_{kk} < 0 \end{cases} \).

The formula fifteen is shown in to formula sixteen.

\[ \alpha_0 = \frac{\langle g^{(kk)T}g^{(kk)} \rangle}{\langle (g^{(kk)T})Bg^{(kk)} \rangle} = \frac{\| g^{(kk)} \|^2}{\| \Psi^{(kk)} g^{(kk)} \|^2} \]  

(16)

4. Image Running Results and Analysis Method

We estimate the method by two signs PSNR and MSE. The image size is \( M \times N \), \( I_1(m,n) \) is the pixel grey value of original signal, \( I_2(m,n) \) is the grey pixel value of reconstruction signal, MSE of reconstruction signal is computed by formula seventeen.

\[ \text{Mse} = \frac{\sum_{M \times N} [I_1(m,n) - I_2(m,n)]^2}{M \times N} \]  

(17)

The PSNR (Peak Signal to Noise Ratio) is computed by by formula eighteen.

\[ \text{PSNR} = 10 \log_{10} \frac{\text{MAX}_i^2}{\text{MSE}} \]  

(18)

In this article, we test the improved method and traditional method with two remote sensing images. Picture one is the airport remote sensing image, Picture two is the building remote sensing image.
There are four images in Fig.1, Image(a) is the original airport remote sensing picture, Image(b) is the noise picture airport remote sensing picture, Image(c) is the reconstruction result of FXHM method, Image(d) is the reconstruction result of classical method of Compressed Sensing theory. We can see that the effect of FXHM method is better than the classical method. The PSNR of FXHM and classical method are 65.63 and 63.68 respectively, the PSNR is improved 1.95 points.

**Figure 1.** Original image noise image and test results image of airport remote image
There are four images in Fig.2. Image(a) is the original building remote sensing picture, Image(b) is the noise picture building remote sensing picture, Image(c) is the reconstruction result of FXHM method, Image(d) is the reconstruction result of classical method of Compressed Sensing theory. We can see that the effect of FXHM method is better than the classical method. The PSNR of FXHM and classical method are 65.41 and 64.35 respectively, the PSNR is improved 0.91 points.

5. Conclusions
The reconstruction of Remote sensing image is the key researching field. Because the long distance transmission will get more noise on remote sensing image. In order to resolve this problem and improve the quality of reconstruction image, we propose the new method fix step hunt method (FSHM) on the basis of Compressed Sensing theory, experiment results show that the fix step hunt method (FSHM) can reduce more noise of image than the classical Mallat method, and the same time the fix step hunt method (FSHM) can get higher PSNR. The fix step hunt method (FSHM) can be used on more fields, not only the remote sensing reconstruction filed. The fix step hunt method (FSHM) gets the goal of improving the quality of reconstruction image. The algorithm has good practical application value.
6. References

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