Conservative Dynamics of Binary Systems of Compact Objects 
at the Fourth Post-Newtonian Order

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We review our recent derivation of a Fokker action describing the conservative dynamics of a compact binary system at the fourth post-Newtonian (4PN) approximation of general relativity. The two bodies are modeled by point particles, which induces ultraviolet (UV) divergences that are cured by means of dimensional regularization combined with a renormalization of the particle’s wordlines. Associated with the propagation of wave tails at infinity is the appearance of a non-local-in-time conservative tail effect at the 4PN order in the Lagrangian. In turn this implies the appearance of infrared (IR) divergent integrals which are also regularized by means of dimensional regularization. We compute the Noetherian conserved energy and periastron advance for circular orbits at 4PN order, paying special attention to the treatment of the non-local terms. One ambiguity parameter remaining in the current formalism is determined by comparing those quantities, expressed as functions of the orbital frequency, with self-force results valid in the small mass ratio limit.

1 Introduction

Inspiraling and merging black-hole binary systems are the most common sources of gravitational waves detectable by ground or space-based laser interferometric detectors. Banks of extremely accurate replica of theoretical templates are a compulsory ingredient of a successful data analysis for these detectors — both on-line and off-line. In the early inspiral phase, the post-Newtonian (PN) approximation of general relativity should be pushed to extremely high order. Furthermore, high accuracy comparison and matching of PN results are performed with numerical relativity computations appropriate for the final merger and ringdown phases. In this context, we have undertaken the derivation of the equations of motion for binary systems of compact (non-spinning) objects at the 4PN order. Solving this problem is of great importance for various applications, most notably numerical/analytical self-force comparisons and effective-one-body calculations, and paves the way to the determination of physical observables in the radiation field such as the orbital phase at the 4PN order beyond the Einstein quadrupole formalism.

After the introduction by Lorentz & Droste of the perturbative PN scheme for solving the Einstein field equations (EFE) for weakly gravitating, slowly moving sources, it was further explored in several historical works, including the famous paper on the motion of N planets at the 1PN order by Einstein, Infeld & Hoffmann. In the 1980s, the PN scheme was successfully
applied to the derivation of the equations of motion of compact binaries up to the 2.5PN order, where radiation reaction effects first appear,\(^6\) which put an end to the radiation reaction controversy raging at the time.\(^7\) The 3PN dynamics was tackled in the 2000s with the help of various methods, and the 4PN order has been investigated since the early 2010s.

After first partial results obtained by means of the effective field theory (EFT)\(^8\),\(^9\) and the Arnowitt-Deser-Misner (ADM) formalism,\(^10\) the important effect of gravitational wave tails at the time.\(^11\),\(^12\) This allowed a better control of the IR divergences and the completion of the full 4PN dynamics, in spite of the appearance of one unified numerical constant which could only be set by comparison with self-force calculations. We report here on our alternative approach,\(^13\),\(^14\),\(^15\) based on the construction of a Fokker Lagrangian in harmonic coordinates, and whose end result is physically equivalent to the one of the ADM Hamiltonian formalism.\(^10\),\(^11\),\(^12\)

2 Fokker action for post-Newtonian sources

The two compact objects are represented by means of non-spinning, structureless particles, with masses \(m_A\) and trajectories \(y_A^A(t) = (ct, y_A(t))\) (with \(A = 1, 2\)), where \(c\) represents the speed of light. The corresponding matter action reads

\[
S_m = -\sum_A m_A c^2 \int dt \sqrt{-g_{\mu\nu}} A^\mu A^\nu / c^2 ,
\]

(1)

with \(A^\mu = dy^A^\mu / dt = (c, v_A)\). The time-dependent tensor \((g_{\mu\nu})_A\) stands for the metric evaluated at the location of the particle \(A\). We deal with the divergences arising there by means of dimensional regularization. On the other hand, the gravitational sector is described by the Einstein-Hilbert action in Landau-Lifshitz form with the usual harmonic gauge-fixing term

\[
S_g = \frac{c^3}{16\pi G} \int d^4 x \sqrt{-g} \left[ g^{\mu\nu} \left( \Gamma^\rho_{\mu\lambda} \Gamma^\lambda_{\rho\nu} - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\mu\rho} \right) - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \Gamma^\mu_{\alpha\beta} \Gamma^\nu_{\rho\sigma} \right] ,
\]

(2)

where \(G\) is the Newton constant, \(g = \det g_{\mu\nu}\), and \(\Gamma^\mu_{\alpha\beta}\) stands for the Christoffel symbols.

The gravitational action can be written in terms of the deviation of the metric from the inverse flat metric \(\eta^{\mu\nu} = \text{diag}(-1, 1, 1, 1)\), namely \(h^{\mu\nu} = \sqrt{-g} g^{\mu\nu} - \eta^{\mu\nu}\). The action appears then as an infinite non-linear power series in \(h\), in which indices on \(h\) and on partial derivatives \(\partial\) are lowered and raised with the Minkowski metric \(\eta\). The Lagrangian density \(\mathcal{L}_g\) can take various forms, obtained from each other by integrations by parts. For our purpose, we adopt the form that starts at quadratic order by terms like \(\sim h^2\), i.e., the “propagator” form, with \(\square = \eta^{\rho\sigma} \partial_\rho \partial_\sigma\) denoting the flat d’Alembertian operator. Therefore, the general structure of our Lagrangian density is \(\mathcal{L}_g \sim h\square h + h\partial h\partial h + h\partial^2 h + \cdots\).

The harmonic gauge fixed action yields the following “relaxed” EFE:

\[
\square h^{\mu\nu} = \frac{16\pi G}{c^4} T^{\mu\nu} , \quad T^{\mu\nu} \equiv [g|T^{\mu\nu} + \frac{c^4}{16\pi G} \Sigma^{\mu\nu}[h, \partial h, \partial^2 h] .
\]

(3)

The quantity \(T^{\mu\nu}\) denotes the pseudo-stress-energy tensor of the matter and gravitational fields, with \(T^{\mu\nu} = \frac{2}{\sqrt{-g}} \delta S_m / \delta g_{\mu\nu}\). The gravitational source term \(\Sigma^{\mu\nu}\) is at least quadratic in \(h\) or its first and second derivatives. Those wave-like equations have the same Green function as in harmonic gauge, although the harmonicity conditions \(\partial_\mu h^{\mu\nu} = 0\) do not hold unless the evolution equations for the matter are also satisfied.

The Fokker action is obtained by inserting back into (1)–(2) an explicit PN iterated solution of the field equations (3) given as a functional of the particle’s trajectories, i.e., an explicit PN metric \(g_{\mu\nu}(x; y_B(t), v_B(t), \cdots)\) at point \(x\). The extra variables indicated by ellipsis are higher derivatives such as accelerations \(a_B(t)\) or derivatives of accelerations \(b_B(t)\). Their presence is
due to the fact that we solve Eqs. (3) without replacing accelerations because we are off-shell at this stage. Thus, the Fokker generalized PN action, depending not only on positions and velocities but also on accelerations and their derivatives, reads

$$S_F [y_B(t), v_B(t), ...] = \int d^4x \mathcal{L}_g [x; y_B(t), v_B(t), ...] - \sum_A m_A c^2 \int dt \sqrt{-g_{\mu\nu} (y_A(t))} v_A^\mu v_A^\nu/c^2.$$  (4)

Now, by the stationarity of the total action $S = S_g + S_m$ for the PN iterated solution, the PN equations of motion are nothing but the Euler-Lagrange equations of $S_F$ for the particles. Once they have been obtained, they may then be order reduced as usual, by replacing all accelerations by the PN equations of motion themselves. The classical Fokker action is completely equivalent, in the “tree-level” approximation, to the effective action used in the EFT. 8,9,16,17

In (4), the gravitational term integrates over the whole space a PN solution of the EFE that is valid only in the near zone of the source. Denoting by $\mathbf{h}$ the PN expansion of the full-fledged gravitational field $h = h(x; y_A(t), v_A(t), ...)$, solution of the EFE (3), we have the equality $h = \mathbf{h}$ in the near zone of the matter system. By contrast, outside the near zone, $\mathbf{h}$ is not expected to agree with $h$ and typically diverges at infinity. On the other hand, the multipole expansion of the metric perturbation, denoted $\mathcal{M}(h)$, agrees with $h$ in all the exterior region of the source, but blows up when formally extended inside the near zone as $r \rightarrow 0$. 18 To properly define the Fokker action, we initially introduced a Hadamard regularization (HR). With that regularization, we demonstrated that the gravitational part of the Fokker Lagrangian, say $L_g^{\text{HR}}$, can be written as a space integral over the looked-for PN Lagrangian density, plus an extra contribution involving the multipole expansion:

$$L_g^{\text{HR}} = \text{FP} \int d^3x \left( \frac{r}{r_0} \right)^B \mathcal{R}_g + \text{FP} \int d^3x \left( \frac{r}{r_0} \right)^B \mathcal{M}(\mathcal{L}_g).$$  (5)

Here, we have introduced a regulator $(r/r_0)^B$, with $B$ being a complex number, and a finite part (FP) at $B = 0$ in order to cure the divergences of the PN expansion when $r \equiv |x| \rightarrow +\infty$ in the first term while dealing with the singular behaviour of the multipole expansion when $r \rightarrow 0$ in the second one. The constant $r_0$, representing an IR scale in the first term and a UV scale in the second, cancels out between the two contributions. We have proved, though, that the second term in (5) does not contribute to $S_F$ below the 5.5PN order, hence we consider at 4PN order:

$$L_g^{\text{HR}} = \text{FP} \int d^3x \left( \frac{r}{r_0} \right)^B \mathcal{L}_g.$$  (6)

### 3 The 4PN conservative dynamics

The general PN solution that matches an exterior solution with retarded boundary conditions at infinity may be decomposed in two pieces. The first one consists of the naive near zone expansion of the retarded integral of the PN source, each term being regularized by means of the same FP procedure as in Eq. (6). The second piece is a homogeneous multipolar solution regular inside the source, expanded in the near zone, 19,20

$$\mathcal{R}^{\mu\nu} = \frac{16\pi G}{c^4} \mathcal{R}^{\mu\nu}_{\text{ret}} - \frac{2G}{c^4} \sum_{\ell=0}^{+\infty} \frac{(-)^\ell}{\ell!} \partial_L \left\{ \frac{\mathcal{R}^{\mu\nu}_L (t - r/c)}{r} - \mathcal{R}^{\mu\nu}_L (t + r/c) \right\}.$$  (7)

The multipole moments $\mathcal{R}^{\mu\nu}_L$ in (7) are functionals of the multipole expansion of the effective gravitational source in the EFE, $\mathcal{M}(\tau^{\mu\nu})$, thus depending on the boundary conditions imposed at infinity. Most importantly, the functions $\mathcal{R}^{\mu\nu}_L$ are responsible for the tail effects in the near
zone metric. At the 4PN order, where they first appear in $\mathcal{R}^{\mu\nu}$, it is sufficient to consider only quadrupolar tail terms corresponding to the interaction between the total ADM mass $M$ of the source and its Symmetric Trace-Free (STF) quadrupole moment $I_{ij}$. Inserting them into the original Fokker action, we obtain, after redefining the matter variables $y_A$ with the help of an appropriate non-local-in-time 4PN shift, a net contribution

$$
S_{\text{F}}^{\text{tail}} = \frac{G^2 M}{5\varepsilon^3} \text{Pf}_{2s_0/c} \int \int \frac{dt dt'}{|t-t'|} I_{ij}^{(3)}(t) I_{ij}^{(3)}(t'),
$$

where the upper indices $(3)$ represent third order time differentiation. With HR, a dependence on the scale $s_0$ [a priori different from $r_0$ in (6)] occurs through the definition of the Hadamard partie finie Pf.\footnote{For any regular function $f(t)$ tending to zero sufficiently rapidly when $t \to \pm \infty$, we have} Varying the action (8) with respect to the particle’s worldlines, we recover the conservative part of the known 4PN tail effect.\footnote{Following previous works on the 3PN equations of motion\textsuperscript{22,23} we shall proceed in several steps. First, we parametrize the particular solution $\mathbb{L}_{\text{ret}}^{-1}$ in the metric by means of specific PN potentials. Next, those potentials are computed at any point in three-dimensional space and inserted into the action. To deal with quadratic source terms, we extensively make use of the important Fock function $g = \ln(r_1 + r_2 + r_{12})$, such that $\Delta g = r_1^{-1}r_2^{-1}$ with $r_A = |\mathbf{x} - \mathbf{y}_A|$ and $r_{12} = |\mathbf{y}_1 - \mathbf{y}_2|$. We also need to integrate a cubic source term for which we resort to more complicated elementary solutions.\textsuperscript{24}}

The ensuing Fokker Lagrangian is then implemented by means of a Hadamard regularization, later corrected to a dimensional regularization (DR) for the UV divergences. The UV poles $\propto 1/(d - 3)$ are then renormalized through a redefinition of the particle’s worldlines.\textsuperscript{23}

## 4 IR divergences and ambiguity parameters at the 4PN order

The ensuing Fokker Lagrangian\textsuperscript{13} depends on the IR length $r_0$ and on the scale $s_0$ in the tail term (8). However, we have shown that these two scales combine into a single undetermined constant $\alpha = \ln(r_0/s_0)$ after suitable shifts of the particle’s worldlines. Then, the tail integral (8) simply involves the separation distance $r_{12}$ as “partie finie” scale. The constant $\alpha$ cannot be eliminated and is considered to be an ambiguity parameter, equivalent to the ambiguity parameter called $C$ in the ADM Hamiltonian formalism.\textsuperscript{11,12}

The ambiguity parameters are associated with IR divergences, which are in turn linked to the presence of the tail effect at the 4PN order.\textsuperscript{21} It is thus important to check the “stability” of the calculation under a change of regularization procedure for the IR divergences, and eventually to determine which regularization should be used. We argued\textsuperscript{14} that the Fokker Lagrangian derived by resorting to dimensional regularization for both IR and UV divergences is not dynamically equivalent to the HR Lagrangian obtained via our original approach, the difference being composed of two and only two types of terms (modulo some irrelevant shifts of the trajectories):

$$
L^{\text{DR}} = L^{\text{HR}} + \frac{G^4 m_1^2 m_2^2}{c^5 r_{12}^4} \left( \delta_1 (n_{12}n_{12})^2 + \delta_2 v_{12}^2 \right),
$$

where $(n_{12}n_{12})$ denotes the scalar product between the unit separation vector $n_{12} = (\mathbf{y}_1 - \mathbf{y}_2)/r_{12}$ and the relative velocity $v_{12} = \mathbf{v}_1 - \mathbf{v}_2$, while $v_{12}^2 = (v_{12}v_{12})$ and $m = m_1 + m_2$. A pragmatic way to circumvent the problem is to acknowledge our (provisional) ignorance about the real values of $\delta_1$, $\delta_2$ and regard them as ambiguity parameters. Moreover, the terms containing $\alpha$ in...
$L^{\text{HR}}$ can be put precisely in the form of the extra terms in (9) so that $\alpha$ can be absorbed into a redefinition of the two ambiguity parameters $\delta_1$ and $\delta_2$ without loss of generality.

Now, it turns out that the two ambiguity parameters are uniquely fixed by making our dynamics compatible with existing gravitational self-force (GSF) calculations of the conserved energy and periastron advance for circular orbits in the small mass-ratio limit $\nu = m_1 m_2 / m_2^2 \to 0$ (see the next section). Nonetheless, it is important to determine them from first principles, i.e., without resorting to external calculations. A recent progress has been made in that direction: We have replaced the HR prescription for Eq. (6) above by a full DR evaluation based on

$$L^{\text{DR}}_g = \int d^d x \mathcal{L}_g,$$  \hspace{1cm} (10)

for the instantaneous terms, and computed the analogue of the tail term (8) in $d = 3 + \varepsilon$ dimensions. Notably, the computation of the difference between the two prescriptions for the instantaneous terms, i.e., $L^{\text{DR}}_g - L^{\text{HR}}_g$, is quite lengthy as it depends on the detailed structure of the expansion of the integrand at infinity. We proved that, in DR, the instantaneous terms develop an IR pole, but that it is exactly cancelled by a corresponding UV pole coming from the tail term in $d$ dimensions (related cancellation of poles has been discussed in the EFT formalism).

Finally, with our full DR calculation, we found that the two ambiguity parameters can be expressed with a single parameter $\kappa$ as

$$\delta_1 = \frac{1733}{1575} - \frac{176}{15} \kappa, \quad \delta_2 = -\frac{1712}{525} + \frac{64}{5} \kappa.$$  \hspace{1cm} (11)

This parameter $\kappa$ comes from our computation of the tails and is (provisionally) left undetermined. It is equivalent to our former parameter $\alpha$ or to the ambiguity parameter $C$ in the Hamiltonian formalism. The computation of $\kappa$ from first principles is in progress. Let us now see how to compute it thanks to the circular orbit limit of the invariants of the motion.

5 Conserved energy and periastron advance at 4PN order

To investigate the notions of conserved energy and angular momentum in the case of a non-local-in-time dynamics, we adopt the Hamiltonian formalism where the two-body system is described by the canonical conjugate variables $y_A$ and $p_A$. The Hamiltonian is made of a local instantaneous piece (containing many instantaneous terms up to 4PN order) and the non-local-in-time tail part which is the analogue of Eq. (8), namely

$$H^{\text{tail}}[x_A, p_A] = -\frac{G^2 M}{5 c^8} \hat{I}^{(3)}_{ij}(t) \text{ Pf} \left[ \frac{2 \pi c}{20} \right] \int_{-\infty}^{+\infty} d\tau \frac{\hat{I}^{(3)}_{ij}(t + \tau)}{\tau}.$$  \hspace{1cm} (12)

The hat over the quadrupole moment means that all time derivatives must be explicitly evaluated by means of the Newtonian equations of motion. It is crucial to realize that, in Hamilton’s equations, the tail part of the Hamiltonian is to be differentiated in the sense of functional derivatives, e.g.,

$$\frac{\delta H^{\text{tail}}}{\delta y_A^j} = -\frac{2G^2 M}{5 c^8} \frac{\partial \hat{I}^{(3)}_{jk}}{\partial y_A^j} \text{ Pf} \left[ \frac{2 \pi c}{20} \right] \int_{-\infty}^{+\infty} d\tau \frac{\hat{I}^{(3)}_{jk}(t + \tau)}{\tau}.$$  \hspace{1cm} (13)

Since the time derivative of $H$ computed on shell is linked to the partial derivatives of $H$, through the chain rule, the usual cancellations implying $dH(t)/dt = 0$ do not occur when non-local-in-time contributions are present. We find instead a more complicated “non-conservation” law $dH/dt = P^{\text{tail}}$, where $P^{\text{tail}}$ involves non-local integrals constructed from the tail term (12). From that law, we can derive explicitly the conserved energy $E$ associated with the non-local Hamiltonian. We start by performing a Taylor expansion of $\hat{I}_{ij}(t + \tau)$ when $\tau \to 0$. To remedy the appearance of divergent integrals, we introduce in the integrand an exponential...
It is worth mentioning that the GSF periastron advance (analytical or numerical) is not computed directly but indirectly deduced from the so-called redshift variable via the first law of binary mechanics, but the latter has been checked to hold even at the 4PN order for the non-local-in-time dynamics.  

\[ \Delta H^{\text{tail}} = -\frac{2G^2M\omega^6}{5c^8} \left[ \sum_p |\mathcal{I}_{ij}|^2 p^6 - \frac{1}{2} \sum_{p+q \neq 0} \mathcal{I}_{ij} \mathcal{I}_{pq} p^3 q^3 (p-q) \ln \left| \frac{p}{q} \right| \right], \]  

(14) where \( \omega \) represents the orbital frequency. Remarkably, this expression contains a constant (DC) contribution [first term in Eq. (14)] proportional to the gravitational wave energy flux, \( F_{GW} = \frac{c^2}{8\pi E} (I_{ij}^{(b)})^2 \). The remaining (AC) terms average to zero, and are strictly zero in the case of circular orbits. A similar procedure allows us to construct the conserved angular momentum.  

The complete expression of the energy through 4PN order in the strictly non-circular orbit is the sum of the instantaneous part of the 4PN dynamics, composed of many different terms, and of the tail part, composed of (12) plus the crucial DC contribution in (14). After reducing to the frame of the center of mass and specializing to circular orbits, we obtain  

\[ E = -\frac{m \nu c^2 x}{2} \left( 1 + \left( \frac{3}{4} - \frac{\nu}{12} \right) x + \left( \frac{27}{8} + \frac{19}{8} \nu - \frac{\nu^2}{24} \right) x^2 + \left( -\frac{675}{64} + \frac{34445}{576} - \frac{205}{96} \nu^2 \right) \right) \nu \]  

\[ + \left( \frac{155}{96} \nu^2 - \frac{35}{5184} \nu^3 \right) x^3 + \left( -\frac{3696}{128} + \frac{123671}{5760} + \frac{9037}{1536} \nu^2 + \frac{896}{15} \gamma_E + \frac{448}{15} \ln(16x) \right) \nu \]  

\[ + \left( -\frac{498449}{3456} + \frac{3571}{576} \nu^2 + \frac{301}{1728} \nu^3 + \frac{77}{31104} \nu^4 \right) x^4 \].  

(15) The PN parameter reads  

\[ x = (G \omega / c^3)^{2/3} \]  

and we have used the approximation  

\[ M = m_1 + m_2 \]  

the tail terms. We adjust the remaining ambiguity parameter in (11) by comparing to the circular energy obtained in the GSF framework, at first order in the perturbative expansion in the small mass ratio limit.  

The correct value is  

\[ \kappa = \frac{41}{60} \]  

such value agrees with the one found in the computation of the tail term in \( d \) dimensions (including both conservative and dissipative effects) by means of EFT methods.  

Finally we report the complete expression of the periastron advance at the 4PN order, for a slightly non-circular orbit, in the limit where the eccentricity goes to zero:  

\[ K = 1 + 3x + \left( \frac{27}{2} - 7\nu \right) x^2 + \left( \frac{135}{2} + \left[ -\frac{649}{4} + \frac{123}{32} \pi^2 \right] \nu + 7\nu^2 \right) x^3 + \left( \frac{2835}{8} + \left[ -\frac{275941}{360} \right] \nu^2 - \frac{98}{27} \nu^3 \right) x^4. \]  

(16) Note that, for the previous value of \( \kappa \), the result agrees directly with GSF calculations. The GSF contribution to the periastron is generally described by means of the function \( \rho(x) \) such that  

\[ K^{-2} = 1 - 6x + \nu \rho(x) + O(\nu^2), \]  

\[ \rho = 14x^2 + \left( \frac{397}{2} - \frac{123}{16} \pi^2 \right) x^3 \]  

\[ + \left( -\frac{215729}{180} + \frac{58265}{1536} \pi^2 + \frac{1184}{15} \ln 2 + \frac{2916}{5} \ln 3 + \frac{5024}{15} \gamma_E + \frac{2512}{15} \ln x \right) x^4. \]  

(17) The 4PN coefficient  

\[ a^{\text{PN}} = a^{4\text{PN}} + b^{4\text{PN}} \ln x, \]  

in particular the coefficient  

\[ a^{4\text{PN}} \approx 64.6406, \]  

is in perfect agreement with GSF numerical results. It is worth mentioning that the GSF periastron advance (analytical or numerical) is not computed directly but indirectly deduced from the so-called redshift variable via the first law of binary mechanics, but the latter has been checked to hold even at the 4PN order for the non-local-in-time dynamics.  

\[ \text{cut-off factor } e^{-\epsilon|\tau|} \text{ for some } \epsilon > 0, \text{ and } \epsilon \text{ tend to zero at the end of our calculation.} \]
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