Calculation of profiled flooring using the mid-surface equation of the orthotropic plate

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Abstract. Steel profiled flooring is a structurally orthotropic material, since its orthotropy is not due to the material properties, but to the cross-sectional shape obtained at the production stage. Taking into account the orthotropic properties of the profiled flooring during the calculation is extremely rare, since it is a laborious process. A method is proposed for calculating profiled flooring taking into account its orthotropic properties, which is implemented in various computational software systems. The results of calculations in the Matlab package and the PC Lira-CAD are presented.

1. Introduction

When designing buildings and structures, one often has to deal with structures made of folded sheet sections, or, in other words, structures made of profiled flooring. Profiled flooring is a versatile material. It is used both as enclosing [1] and supporting structures [2], independently [3, 4] and as part of other structures [5], [6], [9], [10]. Such a wide area of application of profiled flooring is due to its advantages, which include: strength, low cost, ease of installation, durability (profiled flooring with a polymer coating has high corrosion resistance), and many others.

However, profiled flooring has its own characteristics, such as the structural orthotropy of the profiled sheet, which is practically not taken into account in the calculations, low resistance to fire, as well as local accumulations of permanent deformations.

The profiled flooring is not an isotropic material, due to its shape obtained at the manufacturing stage, it has different bending stiffnesses in two mutually perpendicular directions. In the direction across the corrugation, its rigidity is provided by the shape obtained during production, in the longitudinal direction, rigidity is provided only by the thickness of the sheet from which it is made (Fig. 1).

As a result, profiled flooring is used as an element that works according to the beam scheme, and its orthotropy is not taken into account. For example, in [9, 10], single- and multi-span beam design schemes with values of maximum loads for profiled sheets of various types are given.

Taking into account the orthotropic properties of profiled flooring is a time-consuming process, since in this case it must be modeled in a software package based on the finite element method (FEM).
Thus, the development of an engineering technique for calculating profiled decks taking into account their orthotropic properties is an urgent task.

2. Equation of the middle surface of an orthotropic plate
Let us use the equation of the mid-surface for an orthotropic plate, in which the directions of the x and y axes are aligned with the main directions of elasticity, the equation for determining the deflection functions \( v(x, y) \) has the form [11]:

\[
D_1 \frac{\partial^4 v}{\partial x^4} + 2D_3 \frac{\partial^4 v}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 v}{\partial y^4} = q, \quad (x, y) \in \Pi,
\]

where
- \( \Pi = [0, L_1] \times [0, L_2] \), \( L_1, L_2 \) - dimensions of the plate along the x and y axes, respectively, mm
- \( D_1 = \frac{E_1 h^3}{12(1-\mu_1 \mu_2)} \) - bending stiffness of the plate in the direction of the x-axis, N·mm;
- \( D_2 = \frac{E_2 h^3}{12(1-\mu_1 \mu_2)} \) - bending stiffness of the plate in the direction of the y-axis, N·mm;
- \( D_3 = D_1 \mu_2 + 2D_k \) is the main stiffness of the plate, N·mm;
- \( D_k = \frac{Gh^3}{12} \) - torsional rigidity for the main directions, N·mm;
- \( \mu_1, \mu_2 \) - Poisson's ratio, which characterizes the contraction in the direction of the y-axis when stretched in the direction of the x-and y-axes, respectively;
- \( E_1, E_2 \) - the elastic modulus of the plate material in the direction of the x and y axes, respectively, MPa;
- \( G \) the shear modulus of the plate material, MPa;
- \( h \) - plate thickness, mm.

Taking into account the orthotropic properties of the profiled flooring is complicated by the fact that it is made of steel, which has a constant elastic modulus, and the geometric characteristics are given in an assortment for a sheet width of one meter [12].

Thus, to calculate the flexural stiffness of the plate in two mutually perpendicular planes, it is necessary to obtain the conversion coefficients introduced to the values of the elastic modulus.

3. Solving the problem. Determination of conversion factors
The section of the profiled sheet in the direction across the corrugation is a rectangle with a height equal to the thickness of the profiled flooring (\( h \)). Therefore, the moment of inertia of such a section \( (I_2) \) is determined by the well-known formulas for the resistance of materials as for a solid elastic body. The moment of inertia of the section along the corrugation of the profiled sheet \( (I_1) \) is given in [10]. Thus, the conversion factor for the elastic modulus \( E_1 \) will be determined by the formula:
\[ k = I_1 / I_2 \]

Table 1 shows the values of the conversion factor for some types of profiled sheets [12].

**Table 1. Geometrical characteristics of sections of blanks with a width of 1 m**

| Profile No. | Sheet thickness \( \text{mm} \) | Moment of inertia of section \( I_1 \), \( \text{mm}^4/\text{m} \) | Section height \( H \), \( \text{mm} \) | Moment of inertia of section \( I_2 \), \( \text{mm}^4/\text{m} \) | \( k = I_1 / I_2 \) | \( E_1 = E_2 \cdot k \), MPa | \( E_2 \), MPa |
|-------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 57         | 0.6             | 462000          | 57              | 18.0            | 2566.7          | 5700            | 5.14E + 09      | 200000          |
| 57         | 0.7             | 538000          | 57              | 28.6            | 18822.2         | 18800           | 3.76E + 09      | 200000          |
| 57         | 0.8             | 612000          | 57              | 42.7            | 14343.8         | 14300           | 2.86E + 09      | 200000          |
| 60         | 0.7             | 621000          | 60              | 28.6            | 21725.9         | 21700           | 4.34E + 09      | 200000          |
| 60         | 0.8             | 706000          | 60              | 42.7            | 16546.9         | 16500           | 3.30E + 09      | 200000          |
| 60         | 0.9             | 790000          | 60              | 60.8            | 13004.1         | 13000           | 2.60E + 09      | 200000          |

4. Calculation of profiled flooring

To solve equation (1), a finite difference method (FCD) is considered, i.e. a finite-difference scheme of the second order of approximation on a uniform grid with steps \( \Delta x \), \( \Delta y \) along the x-axis and along the y-axis, respectively, is proposed [13], considered in a rectangular region \( \Pi \):

\[
D_1 \Lambda_1 v + 2D_3 \Lambda_{12} v + D_2 \Lambda_2 v = q, \quad (2)
\]

where

\[
\Lambda_1 v = \frac{v_{i+2,j} - 4v_{i+1,j} + 6v_{i,j} - 4v_{i-1,j} + v_{i-2,j}}{(\Delta x)^4},
\]

\[
\Lambda_2 v = \frac{v_{i,j+2} - 4v_{i,j+1} + 6v_{i,j} - 4v_{i,j-1} + v_{i,j-2}}{(\Delta y)^4},
\]

\[
\Lambda_{12} v = \frac{4v_{i,j} - 2(v_{i+1,j} + v_{i-1,j} + v_{i,j+1} + v_{i,j-1}) + v_{i+1,j+1} + v_{i+1,j-1} + v_{i-1,j+1} + v_{i-1,j-1}}{(\Delta x)^2(\Delta y)^2},
\]

\[
v_{i,j} = v(x_i, y_j), 0 \leq i \leq N_1, 0 \leq j \leq N_2, \Delta x = L_1/(N_1 - 1), \Delta y = L_2/(N_2 - 1).
\]

To write down the main boundary conditions and conditions in supports or hinges, we write out the corresponding matching conditions [13, 14] on one of the boundaries:

(i) condition of rigid termination (with fixed \( y \) and \( x = x_0 \)):

\[
v(x_0, y) = \frac{\partial^2 v(x_0, y)}{\partial x^2} = 0, \quad (3)
\]

(ii) pivotal support condition:

\[
v(x_0, y) = \frac{\partial^2 v(x_0, y)}{\partial x^2} = 0, \quad (4)
\]
(iii) free edge condition:

\[ \frac{\partial^2 v(x_0, y)}{\partial x^2} = \frac{\partial^3 v(x_0, y)}{\partial x^3} = 0. \]  \hspace{1cm} (5)

The derivatives occurring in formulas (3) - (5) are approximated with the second order at the points \( x_i \) for a fixed \( y_j \) by difference relations:

\[ \frac{\partial v(x_i, y_j)}{\partial x} \approx -3v_{i,j} + 4v_{i+1,j} + v_{i,j+2}, \]  \hspace{1cm} (6)

\[ \frac{\partial^2 v(x_i, y_j)}{\partial x^2} \approx 2v_{i,j} - 5v_{i+1,j} + 4v_{i,j+2} - v_{i+3,j}, \]  \hspace{1cm} (7)

\[ \frac{\partial^3 v(x_i, y_j)}{\partial x^3} \approx -5v_{i,j} + 18v_{i+1,j} - 24v_{i,j+2} + 14v_{i+3,j} - 3v_{i+4,j}. \]  \hspace{1cm} (8)

When constructing the matrix of the system, if there is a support or a hinge at the points \( x = x_i \) for a fixed \( y \), then the equations of system (2) with numbers \( i-1, i \) and \( i+1 \) are replaced by the corresponding equations (6)-(8).

5. Results of numerical experiments

In the Matlab package and in parallel in the PC Lira-SAPR, calculations were performed for the profiled flooring of the N60-0.9 brand, 2000 mm long, operating according to a single-span scheme (Fig. 2a), 4000 mm long, operating according to a two-span scheme with equal spans along 2000 mm (Fig. 2b), 6000 mm long, operating according to a three-span scheme with equal spans of 2000 mm each (Fig. 2c). The support of the profiled flooring is hinged. The calculation was made for the action of equivalent uniformly distributed loads with an intensity of 10 kN/m². The calculation results are shown in Figures 3 - 5.

![Figure 2. Calculation schemes of profiled flooring](image-url)
The advantages of the proposed method include its simplicity, as well as the fact that it can be implemented in various computational software systems, including those based on FEM. The calculation results are shown in Figures 7 - 9.
Figure 6. Calculation results of profiled flooring operating according to scheme 2a in PC Lira-CAD

Figure 7. Calculation results of profiled flooring operating according to scheme 2b in PC Lira-CAD

Figure 8. Calculation results of profiled flooring operating according to scheme 2c in PC Lira-CAD
Table 2 shows a comparative analysis of deflections and bending moments in profiled decking of grade N60-0.9, determined by the FEM in PK Lira and FCD. Note that the values of deflections and bending moments in the table are given in modulus.

| Scheme | FEM (PC Lira-CAD) | FCD | % |
|--------|-------------------|-----|----|
| 2a     | 9.17              | 9.16| 0.11|
| 2b     | 3.82              | 3.81| 0.26|
| 2c     | 3.14              | 3.23| 2.87|

As a result of a comparative analysis of the calculations, it was found that the discrepancy between the results obtained by the FEM and FCD differs by no more than 3%.

6. Conclusion
The proposed technique makes it possible to carry out calculations of structurally orthotropic plates, including profiled decks, taking into account their orthotropic properties, operating according to single and multi-span schemes with spans of arbitrary lengths.

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