CAN COUPLED DARK ENERGY SPEED UP THE BULLET CLUSTER?

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ABSTRACT

It has been recently shown that the observed morphological properties of the Bullet Cluster can be accurately reproduced in hydrodynamical simulations only when the infall pairwise velocity $V_c$ of the system exceeds 3000 km s$^{-1}$ (or at least possibly 2500 km s$^{-1}$) at the pair separation of $2R_{\text{vir}}$, where $R_{\text{vir}}$ is the virial radius of the main cluster, and that the probability of finding such a bullet-like system is extremely low in the standard Λ cold dark matter (ΛCDM) cosmology. We suggest here the fifth force mediated by coupled dark energy (cDE) as a possible velocity-enhancing mechanism and investigate its effect on the infall velocities of bullet-like systems from the Coupled Dark Energy Cosmological Simulations public database. Five different cDE models are considered: three with constant coupling and exponential potential, one with exponential coupling and constant potential, and one with constant coupling and supergravity potential. For each model, after identifying the bullet-like systems, we determine the probability density distribution of their infall velocities at pair separations of $(2–3)R_{\text{vir}}$. Approximating each probability density distribution as a Gaussian, we calculate the cumulative probability of finding a bullet-like system with $V_c \geq 3000$ km s$^{-1}$ or $V_c \geq 2500$ km s$^{-1}$. Our results show that in all of the five cDE models the cumulative probabilities increase compared to the ΛCDM case and that in the model with exponential coupling $P(V_c \geq 2500$ km s$^{-1})$ exceeds $10^{-4}$. The physical interpretations and cosmological implications of our results are provided.

Key words: cosmology: theory – large-scale structure of universe – methods: statistical

Online-only material: color figures

1. INTRODUCTION

The splendid success of the standard Λ cold dark matter (ΛCDM) cosmology that has been witnessed for the past two decades seems to be overshadowed by the recent discoveries of several possible anomalies (e.g., see Perivolaropoulos 2008, for a review). The so-called Bullet Cluster (1E0657-56) is one of those observational challenges that the standard ΛCDM cosmology has to face (Mastropietro & Burkert 2008; Lee & Komatsu 2010; Thompson & Nagamine 2011; Akahori & Yoshikawa 2011). When Tucker et al. (1995) first observed it while looking for a failed cluster, it appeared as a large cloud of hot gas. Later, the Chandra observation revealed that it is in fact a very rare system composed of two head-on colliding massive clusters at $z = 0.296$ in which a bullet-like subcluster is in the middle of escaping its main cluster’s potential well at a bow-shock speed of $\sim 4700$ km s$^{-1}$ after the first core passage (Markevitch et al. 2002, 2004; Clowe et al. 2004, 2006; Markevitch 2006).

According to the results of Mastropietro & Burkert (2008)—based on high-resolution hydrodynamical simulations—the pairwise infall velocity $V_c$ of a “bullet” satellite onto the main cluster at a separation of $2R_{\text{vir}}$ (where $R_{\text{vir}}$ is the virial radius of the main halo) has to exceed 3000 km s$^{-1}$, or possibly at least 2500 km s$^{-1}$, in order to reproduce the peculiar morphological properties of the Bullet Cluster, such as the mass ratio between the two colliding clusters, the large separation between the gas and CDM distributions, and the high shock speed inferred from X-ray temperature measurements (see also Springel & Farrar 2007). In light of this result, Lee & Komatsu (2010) investigated how probable it is for a “bullet-like” system to have an infall velocity as high as 3000 km s$^{-1}$, using the cluster catalogs from the large-volume MICE simulations (Crocce et al. 2010). They found that the probability of finding a bullet-like system with $V_c \geq 3000$ km s$^{-1}$ in a WMAP7 universe (Komatsu et al. 2011) is between $\sim 10^{-9}$ and $\sim 10^{-11}$, which led them to conclude that the existence of the Bullet Cluster is incompatible with the ΛCDM cosmology. Very recently, Thompson & Nagamine (2011) and Akahori & Yoshikawa (2011) have confirmed the results of Lee & Komatsu (2010) and of Mastropietro & Burkert (2008) using the data from different N-body and hydrodynamical simulations, respectively.

Both a conservative and a radical approach to the Bullet Cluster problem have been recently proposed. The conservative approach sought for a lower infall velocity solution in a ΛCDM cosmology under the suspicion that the results from the hydrodynamical simulations or the high shock velocity inferred from the X-ray temperatures may not be trustworthy (e.g., Forero-Romero et al. 2010). In contrast, the radical approach attempted to figure out a mechanism capable of enhancing the infall velocities of bullet-like systems in non-standard cosmologies. For example, Wyman & Khoury (2010) claimed that in models with cascading gravity the probability of a bullet-like system with $V_c \geq 3000$ km is four orders of magnitude higher than in the ΛCDM model (see also Moffat & Toth 2010). Yet, their result was based on pure analytical speculation without any numerical backup.

Taking the radical direction, we examine here the possibility that the morphological properties and the required high infall velocity of the Bullet Cluster can be attributed to the presence of coupled dark energy (cDE). In cDE models, the dark energy (DE) is a scalar field $\phi$ with potential $U(\phi)$, which interacts...
with the CDM particles obeying the following two equations (Wetterich 1995; Amendola 2000, 2004):

\[
\ddot{\phi} + 3H\dot{\phi} + \frac{3}{\sqrt{\beta(\phi)}} \frac{dU}{d\phi} = \frac{2}{\sqrt{\beta(\phi)}} \frac{\rho_c}{M_{\text{Pl}}},
\]

\[
\ddot{\rho}_c + 3H \dot{\rho}_c = -\frac{2}{\sqrt{\beta(\phi)}} \frac{\rho_c \dot{\phi}}{M_{\text{Pl}}},
\]

(1)

where an overdot represents a derivative with respect to the cosmic time \( t \), \( H \equiv \dot{a}/a \) is the Hubble function, \( M_{\text{Pl}} \equiv 1/\sqrt{8\pi G} \) is the reduced Planck mass, and \( \rho_{\text{CDM}} \) is the CDM density. The interaction between the DE scalar field \( \phi \) and the CDM fluid determines an exchange of energy momentum, and a consequent time variation of the CDM particle mass \( m_{\text{CDM}} \). In Equations (1) the strength of the coupling is fully determined by the coupling function \( \beta(\phi) \), defined as \( \beta(\phi) \equiv -d \ln m_{\text{CDM}}/d\phi \).

An enhancement of the infall velocity of the Bullet Cluster is naturally expected in cDE models since a long-range fifth force generated by the coupling between \( \phi \) and CDM particles has been found to play the role of accelerating structure formation processes (e.g., Mangano et al. 2003; Maccio et al. 2004; Mainini & Bonometto 2006; Pettorino & Baccigalupi 2008; Baldi et al. 2010; Wintergerst & Pettorino 2010, and references therein). The level of enhancement of the infall velocity of the Bullet Cluster will however significantly depend on the shape of the scalar self-interaction potential \( U(\phi) \) as well as the coupling function \( \beta(\phi) \). We refer to the wide literature on the subject (see, e.g., Amendola 2000, 2004; Pettorino & Baccigalupi 2008; Baldi et al. 2010, 2011; Baldi 2011a, and references therein) for a more thorough description of the main basic features of cDE cosmologies. In particular, the specific models discussed in the present paper have been fully defined and presented by Baldi (2011c, 2011d).

The goal of this paper is to study comprehensively the effect of cDE models on the infall velocities of bullet-like systems by analyzing the public cluster catalogs of the Coupled Dark Energy Cosmological Simulations (CoDECS) project (Baldi 2011d)—the largest suite of cosmological N-body simulations for cDE models to date—which includes various choices for the scalar self-interaction potential \( U(\phi) \) and the coupling function \( \beta(\phi) \). The paper is organized as follows. In Section 2, we provide a brief overview of the N-body simulations for the cDE models and describe how to select bullet-like systems from the halo catalogs. In Section 3, we determine the probability density distribution of the bullet-like systems and calculate the probability of finding a bullet cluster with infall velocity larger than 3000 km s\(^{-1}\) (and 2500 km s\(^{-1}\) as well) for each cDE model. In Section 4, we provide the physical interpretation of our results and discuss its cosmological implications.

## 2. DATA AND ANALYSIS

### 2.1. A Brief Overview of the CoDECS Project

The CoDECS project is the largest suite of N-body simulations for cDE models ever performed, and all its numerical outputs, including halo and sub-halo catalogs, have been recently made publicly available (Baldi 2011d). Using the specific modified version by Baldi et al. (2010) of the widely used parallel Tree-PM N-body code GADGET (Springel 2005), the different runs of the L-CoDECS suite follow the evolution of 1024\(^3\) CDM and 1024\(^3\) baryonic particles in a periodic cosmological box of linear comoving size 1 h\(^{-1}\) Gpc from \( z_i = 99 \) to \( z = 0 \) for different realizations of the cDE scenario. The individual CDM and baryonic particles have a mass of 5.84 \times 10^{10} h^{-1} M_\odot and 1.17 \times 10^{10} h^{-1} M_\odot, respectively, at \( z = 0 \), and the gravitational softening was set at a twenty-fifth of the mean linear interparticle spacing, \( \epsilon_s = 20 h^{-1} \) kpc.

The CoDECS suite includes—besides the standard “fiducial” \( \Lambda \)CDM scenario—five different cDE cosmologies: four models with an exponential self-interaction potential (Lucchin & Matarrese 1985; Ratra & Peebles 1988; Wetterich 1988)

\[
U(\phi) = A e^{-a\phi},
\]

(2)

and one model with a SUGRA potential (Brax & Martin 1999)

\[
U(\phi) = A \phi^{-\alpha} e^{\phi/2},
\]

(3)

where for simplicity the scalar field has been redefined in units of the reduced Planck mass \( M_{\text{Pl}} \). All of the first four models have the same value for the potential slope \( \alpha = 0.08 \) but differ from one another in the coupling function: three models (named EXP001, EXP002, and EXP003) are characterized by a constant coupling \( \beta(\phi) = \text{constant} \) with values 0.05, 0.1, and 0.15, respectively, consistent with present observational bounds on the DE–CDM interaction (see, e.g., Bean et al. 2008; Xia 2009; Baldi & Viel 2010), while the remaining one (named EXP008e3) has an exponential coupling of the form \( \beta(\phi) = 0.4e^{0.3\phi} \). For the latter, instead, a negative constant coupling \( \beta = -0.15 \) is assumed, allowing for a non-standard evolution of the resulting cDE model as proposed and explained in detail by Baldi (2011c).

The combination of a SUGRA potential and negative coupling determines a peculiar dynamics of the scalar field that changes its direction of motion at some intermediate redshift between \( z_{\text{CMB}} \approx 1100 \) and the present time. As a consequence of this inversion of motion, which for the specific model presented here (named SUGRA003) occurs at \( z_{\text{inv}} \approx 6.8 \), the DE equation of state parameter \( \omega_\phi \) shows a “bounce” on the cosmological constant barrier \( \omega_\phi = -1 \). For this reason this new class of cDE models has been called the “bouncing cDE scenario” (Baldi 2011c).

All the models included in the CoDECS suite have been normalized according to the latest results from WMAP7 (Komatsu et al. 2011) for their background cosmological parameters at \( z = 0 \) and the amplitude of linear density perturbations at the last scattering surface \( z_{\text{CMB}} \approx 1100 \). The CoDECS runs are therefore fully consistent with present bounds on the perturbations amplitude at \( z_{\text{CMB}} \) and allow us to investigate the effects that a specific cDE scenario imprints on structure formation processes from \( z_{\text{CMB}} \) to the present. Table 1 lists the six cosmological models included in the CoDECS suite, with the corresponding coupling function, potential type, and the expected value of \( \sigma_8 \) that are derived for each scenario with the help of linear perturbation

| Model   | \( U(\phi) \) | \( \alpha \) | \( \beta(\phi) \) | \( \sigma_8 \) |
|---------|---------------|-------------|-----------------|-------------|
| \( \Lambda \)CDM | Const. | \( e^{-a\phi} \) | \( e^{-a\phi} \) | \( e^{-a\phi} \) | 0.809 |
| EXP001  | \( e^{-a\phi} \) | 0.08 | 0.05 | 0.825 |
| EXP002  | \( e^{-a\phi} \) | 0.08 | 0.1 | 0.875 |
| EXP003  | \( e^{-a\phi} \) | 0.08 | 0.15 | 0.967 |
| EXP008e3 | \( e^{-a\phi} \) | 0.08 | 0.4e^{0.3\phi} | 0.895 |
| SUGRA003 | \( \phi^{-\alpha} e^{\phi/2} \) | 2.15 | -0.15 | 0.806 |

Note. See Baldi (2011d) for detailed descriptions of the models.
theory. For a more detailed introduction to the CoDECS project and its related public database, see Baldi (2011d).

It is worth mentioning here that all the cDE models considered here, with the exception of the SUGRA003 model, could be directly constrained by local measurements of the linear perturbations amplitude \( \sigma_8 \) at low redshifts. In particular, the EXP003 model might already be in tension with presently available constraints on the power spectrum amplitude, \( 0.78 \leq \sigma_8 \leq 0.86 \), which have been determined by several independent low-z observations such as galaxy–galaxy correlation function, cosmic shear statistics, X-ray cluster abundance, and \( \Lambda \alpha \) forest (e.g., McDonald et al. 2005; Hetterscheidt et al. 2007; Henry et al. 2009; Wen et al. 2010, and references therein). On the other hand, the EXP002 and EXP008e3 models still appear marginally consistent with this current limit on \( \sigma_8 \).

2.2. Selecting the Bullet-like Systems

Using the CoDECS public halo catalogs at \( z = 0 \), we first construct a mass-limited (\( M \geq 10^{15} \, M_\odot \, h^{-1} \)) sample of the cluster-sized halos identified in the simulations of each cosmological model. The CoDECS halo catalogs have been produced by means of a friends-of-friends (FoF) algorithm (Davis et al. 1985) with linking length \( b = b \times d \), where \( d \) is the mean interparticle separation and the linking parameter \( b = 0.2 \) (Baldi 2011d). Reapplying the FoF algorithm with a linking parameter of \( b = 0.33 \) to the mass-limited sample from the CoDECS halo catalogs, we find the clusters of clusters for each of which the most massive cluster is identified as the main cluster while the others are considered as satellites, as done in Lee & Komatsu (2010).

The bullet-like systems are selected from the clusters of clusters identified according to the same four criteria that were used by Lee & Komatsu (2010): (1) the main cluster’s mass \( M_h \) exceeds a certain cutoff value; (2) the main cluster and at least one of its satellites are on track for a head-on collision (\( |\cos \alpha| \leq 0.9 \), where \( \alpha \) is the angle between the velocities of the main cluster and the colliding satellite); (3) the mass ratio between the satellite and main cluster is smaller than one-fifth (\( M_s / M_h \leq 1/5 \)); (4) the satellite is located within a distance of \( (2-3)R_{200} \) from its main cluster where \( R_{200} \) is the virial radius of its main cluster. The bullet-like systems must satisfy the first three criteria to match the observed properties of the Bullet Cluster (IE0657-57) while the last criterion is used to find the infall pairwise velocities of the main–satellite cluster pairs before the collisions. For the detailed explanation of these criteria, we refer the reader to Lee & Komatsu (2010).

Regarding the cutoff value of the main cluster’s mass, it is worth mentioning here that the main cluster in the observed Bullet Cluster (IE0657-57) is a very massive one with mass \( M_h \geq 10^{15} \, h^{-1} \, M_\odot \). In the current CoDECS samples, however, there are too few (less than 50) bullet-like systems satisfying \( M_h \geq 10^{15} \, h^{-1} \, M_\odot \). To avoid poor number statistics, we use lower cutoff values, \( M_h \geq 0.5 \times 10^{15} \, h^{-1} \, M_\odot \) and \( M_h \geq 0.7 \times 10^{15} \, h^{-1} \, M_\odot \). We also focus on the \( z = 0 \) sample for the same reason (at higher redshifts there are too few bullet-like systems).

Table 2 lists the number of clusters (\( N_c \)) in the mass-limited sample, the number of identified clusters of clusters (\( N_{ac} \)), and the number and mean pairwise infall velocity of the selected bullet-like systems (\( N_{bullet} \) and \( \bar{V}_c \)) for the two different cases of the main cluster’s cutoff mass. As can be seen, the cDE models have systematically larger values of \( N_c \), \( N_{ac} \), and \( N_{bullet} \) than the \( \Lambda \)CDM model, which is consistent with the picture that the dark sector coupling leads to a faster growth of the large-scale structures. Regarding the mean infall velocities, all of the cDE models except the SUGRA003 model yield higher values of \( \bar{V}_c \). The larger the coupling is, the higher is the mean infall velocity of the bullet-like system. In the SUGRA003 model, however, the value of \( \bar{V}_c \) is slightly lower than the \( \Lambda \)CDM case when \( M_h \geq 0.5 \times 10^{15} \, h^{-1} \, M_\odot \), which should be attributed to the negative value of the coupling constant (see Section 4 for further discussion).

3. INFALL VELOCITIES OF BULLET-LIKE SYSTEMS IN cDE MODELS

For each selected bullet-like system, we measure the relative pairwise velocities, \( V_c = V_h - V_s \), where \( V_h \) and \( V_s \) denote the velocities of the main and satellite clusters, respectively. Then, we bin the values of \( V_c \) and count the number of those bullet-like systems with log \( V_c \) belonging to a given differential bin, \([\log V_c, \log V_c + d \log V_c] \), to derive the probability density distribution of \( \log V_c \) for each cDE model and for the \( \Lambda \)CDM model as well, as done by Lee & Komatsu (2010). Figure 1 plots \( p(\log V_c) \) at \( z = 0 \) for the case of the standard \( \Lambda \)CDM cosmology (solid black histogram) and for the cases of the three cDE models with constant coupling and exponential potential (EXP001, EXP002, and EXP003 as dotted cyan, dashed green, and dot-dashed blue histograms, respectively). As can be seen, the distribution \( p(\log V_c) \) shows a tendency to develop a longer high-velocity tail in the cDE models compared with \( \Lambda \)CDM. More specifically, the EXP002 model with \( \beta = 0.1 \) exhibits the longest high-velocity tail among the three models.

To quantify the difference between the four distributions and to calculate the cumulative probability of \( V_c \), we fit \( p(\log V_c) \) to a Gaussian distribution by adjusting, for each model, the mean and standard deviation, \((\nu, \sigma)\), as done in Lee & Komatsu (2010). Figure 2 plots the fitting results. As can be seen, the Gaussian distribution gives a reasonably good fit to \( p(\log V_c) \) for each case. Table 3 lists the best-fit values of \((\nu, \sigma)\) for the three models, showing that the Gaussian fit to the infall velocity distribution \( p(\log V_c) \) has a larger mean and a larger standard deviation in the constant coupling cDE models than in the \( \Lambda \)CDM cosmology. The infall velocity distribution with the highest mean is found, as expected, for the case of the EXP003 model, while EXP002 has the largest standard deviation.

Integrating the best-fit Gaussian distributions, we also compute for each model the cumulative probabilities, \( P(V_c \geq 3000 \, \text{km} \, \text{s}^{-1}) \) and \( P(V_c \geq 2500 \, \text{km} \, \text{s}^{-1}) \), which are

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**Table 2**

| Model       | \( N_c \) | \( N_{ac} \) | \( N_{bullet} \) | \( \bar{V}_c^a \) (\( \text{km s}^{-1} \)) | \( \bar{V}_c^b \) (\( \text{km s}^{-1} \)) |
|-------------|-----------|--------------|------------------|----------------------------------|----------------------------------|
| \( \Lambda \)CDM | 121936    | 22019        | 184              | 999.56                           | 1073.33                          |
| EXP001      | 124467    | 22448        | 195              | 1037.38                          | 1118.27                          |
| EXP002      | 131880    | 23648        | 231              | 1069.03                          | 1247.71                          |
| EXP003      | 141915    | 25223        | 357              | 1112.55                          | 1197.93                          |
| SUGRA003    | 132992    | 23795        | 233              | 1146.49                          | 1221.66                          |

Notes.

\(^a\) For \( M_h \geq 0.5 \times 10^{15} \, h^{-1} \, M_\odot \).

\(^b\) For \( M_h \geq 0.7 \times 10^{15} \, h^{-1} \, M_\odot \).
Figure 1. Probability density distribution of the infall velocities, log $V_c$, of the bullet-cluster-like systems measured within $2 \leq r/R_{200} \leq 3$ at $z = 0$ for the four different dark energy models: $\Lambda$CDM, EXP001, EXP002, and EXP003 as dashed, dotted, solid, and dot-dashed histograms, respectively. The main cluster masses are $M_{\text{main}} \geq 0.7 \times 10^{15} h^{-1} M_\odot$, for each case. (A color version of this figure is available in the online journal.)

plotted in Figure 3 as a function of the constant coupling $\beta$, where $\Lambda$CDM corresponds to $\beta = 0$. As can be seen, the highest values of the two cumulative probabilities are found for the case of the EXP002 model. The values of $P(V_c \geq 3000 \text{ km s}^{-1})$ and $P(V_c \geq 2500 \text{ km s}^{-1})$ increase by a factor of $10^3$ and $10^4$ in the EXP002 model compared to the $\Lambda$CDM case. It is interesting to note that although the EXP003 model has a stronger coupling, its cumulative probabilities are not higher than those of the EXP002 model.

The same type of analysis has been carried out for the other two cDE models included in the CoDECS suite, namely EXP008e3 and SUGRA003. The results are shown in Figure 4, where the probability density distribution $p(\log V_c)$ computed for these models is plotted and compared to the $\Lambda$CDM case, while Figure 5 plots the Gaussian fitting functions whose best-fit means and best-fit standard deviations are listed in Table 3. As can be seen, the probability density distribution $p(\log V_c)$

### Table 3

| Model    | $(\nu, \sigma_\nu)^a$ | $(\nu, \sigma_\nu)^b$ |
|----------|------------------------|------------------------|
| $\Lambda$CDM | (2.99, 0.076)          | (3.03, 0.066)          |
| EXP001   | (3.00, 0.081)          | (3.04, 0.071)          |
| EXP002   | (3.01, 0.091)          | (3.06, 0.081)          |
| EXP003   | (3.04, 0.086)          | (3.08, 0.076)          |
| EXP008e3 | (3.05, 0.091)          | (3.08, 0.091)          |
| SUGRA003 | (2.98, 0.081)          | (3.02, 0.086)          |

Notes.

$^a$ For $M_h \geq 0.5 \times 10^{15} h^{-1} M_\odot$.

$^b$ For $M_h \geq 0.7 \times 10^{15} h^{-1} M_\odot$. 

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Figure 2. Gaussian fit (thin solid line) to the probability density distribution of the infall velocities, log $V_c$, of the bullet-cluster-like systems measured within $2 \leq r/R_{200} \leq 3$ at $z = 0$ (thick solid histogram) for the four different dark energy models: $\Lambda$CDM, EXP001, EXP002, and EXP003 in the top-left, top-right, bottom-left, and bottom-right panels, respectively. The main cluster masses are $M_{\text{main}} \geq 0.7 \times 10^{15} h^{-1} M_\odot$, for each case.
Figure 3. Cumulative probabilities of the infall velocities of the bullet clusters vs. the cDE coupling constant, $\beta$, for the $\Lambda$CDM, EXP001, EXPOO2, and EXP003 models.

for the case of the EXP008e3 model has both the largest mean and standard deviation, while the SUGRA003 model yields a smaller mean but a larger standard deviation of $p(\log V_c)$ than $\Lambda$CDM.

The cumulative probabilities, $P(V_c \geq 3000\, \text{km s}^{-1})$ and $P(V_c \geq 2500\, \text{km s}^{-1})$, are plotted in the top and bottom panels of Figure 6, respectively. As can be seen, both the SUGRA003 and EXP008e3 models have larger cumulative probabilities than $\Lambda$CDM. For the case of SUGRA003, when the lower mass limit for the main cluster is set at $0.7 \times 10^{15}\, M_\odot\, h^{-1}$, the cumulative probabilities, $P(V_c \geq 3000\, \text{km s}^{-1})$ and $P(V_c \geq 2500\, \text{km s}^{-1})$, increase by a factor of 10 and $10^2$, respectively, compared to $\Lambda$CDM. However, when the mass cutoff is set at $0.5 \times 10^{15}\, M_\odot\, h^{-1}$, the SUGRA003 model shows almost no enhancement. In contrast, EXP008e3 exhibits the maximal enhancement, no matter which lower mass limit of the main cluster is used. The cumulative probabilities, $P(V_c \geq 3000\, \text{km s}^{-1})$ and $P(V_c \geq 2500\, \text{km s}^{-1})$, increase by a factor of $10^3$ and $10^4$, respectively, compared to the $\Lambda$CDM result. Note that in this model $P(V_c \geq 2500\, \text{km s}^{-1})$ reaches up to above $10^{-4}$, which indicates that if the required infall velocity of a bullet-like system can be possibly reduced to $2500\, \text{km s}^{-1}$ (Akahori & Yoshikawa 2011), then the existence of the Bullet Cluster is not such an extremely rare event in a $\Lambda$CDM model like the exponential coupling scenario EXP008e3.

Furthermore, it is worth mentioning here that the above cumulative probabilities are likely to be underestimated due to the limited volume of the CoDECS runs. As shown in Forero-Romero et al. (2010), the probability of finding a bullet cluster increases almost linearly with the simulation box volume. In other words, the larger volume a simulation box has, the more probable it is to find a bullet-like system with the required infall velocity. In fact, we find for the case of the $\Lambda$CDM model that $P(V_c \geq 3000\, \text{km s}^{-1}) \sim 10^{-12}$, which is an order of magnitude lower than what Lee & Komatsu (2010) found, $P(V_c \geq 3000\, \text{km s}^{-1}) \sim 10^{-11}$, from the MICE simulation of volume $27\, h^{-3}\, \text{Gpc}^3$. Given this, we suspect that the real values of $P(V_c \geq 3000\, \text{km s}^{-1})$ and $P(V_c \geq 2500\, \text{km s}^{-1})$ would be higher than our estimates of $10^{-6}$ and $10^{-4}$, respectively.

4. DISCUSSION AND CONCLUSION

We have carried out a systematic investigation of how the probability of finding a bullet-like system with the required high infall velocity ($3000\, \text{km s}^{-1}$ or possibly at least $2500\, \text{km s}^{-1}$) changes in the context of cDE models compared to the standard $\Lambda$CDM cosmology, by analyzing the public numerical data from CoDECS, the largest $N$-body simulations of cDE models to date, with a box size of 1 comoving $h^{-3}\, \text{Gpc}^3$. Three models with constant coupling and exponential potential (EXP001, EXP002, EXP003), one bouncing cDE model with constant coupling and SUGRA potential (SUGRA003), and one model with exponential coupling and exponential potential (EXP008e3) are considered in the CoDECS suite, besides the fiducial $\Lambda$CDM cosmology.

In all of the five cDE models the cumulative probabilities have been found to be higher than in $\Lambda$CDM. It has also been shown that the maximal four and five orders of magnitude enhancements of the cumulative probabilities are yielded by the EXP008e3 model where $P(V_c \geq 2500\, \text{km s}^{-1})$ and $P(V_c \geq 3000\, \text{km s}^{-1})$ reach up to above $10^{-4}$ and $10^{-6}$, respectively. This is a remarkable result, considering that all the CoDECS simulations share the same random phases in the initial conditions and the same amplitude of density perturbations at the last scattering surface. Furthermore, given the possible underestimate of the cumulative probabilities due to the limited volume of CoDECS (Forero-Romero et al. 2010), we argue that our results should be considered conservative and that therefore the existence of the Bullet Cluster should not be regarded as an extremely rare event in the EXP008e3 model. Yet, in the other cDE models it should still be quite unlikely to find a bullet cluster with the observed morphology.

The different amplitudes of the bullet velocity enhancement in the different cDE models can be qualitatively understood by considering the time evolution of the linear density and velocity.
perturbations of each scenario. By taking into account only the coupling strength and its integrated effect on the growth of density perturbations, in fact, our finding that the maximal velocity enhancement is obtained for the exponential coupling model (EXP008e3) might look surprising. As shown by Baldi (2011d, see the right panel of Figure 2 in this reference), the maximal effect on the density perturbation amplitude is obtained for the EXP003 model that reaches at $z=0$ an enhancement of $\delta(z)$ of about 20% relative to $\Lambda$CDM, while the EXP008e3 model shows a 10% enhancement (as also described by the different $\sigma_8$ values for the models listed in Table 1).

However, if we also consider the velocity perturbation variable $\theta$, defined as

$$\theta(z) \equiv \frac{1}{\Omega_c + \Omega_b} (\theta_b \Omega_c + \theta_c \Omega_b),$$

where $\theta_{b,c} \equiv \vec{\nabla} \cdot \vec{u}_{b,c}$ are the individual velocity perturbation variables, $\vec{u}_{b,c}$ are the peculiar velocities, and $\Omega_{b,c}$ are the relative density parameters for baryons and CDM, respectively, we can separately solve for $\theta(z)$ in the different cosmologies, and compare the time evolution of the velocity perturbations to the $\Lambda$CDM case. In the linear regime, the density and velocity perturbations $\delta_{b,c}$ and $\theta_{b,c}$ for baryons and CDM in a cDE cosmology evolve according to the following system of coupled differential equations:

$$\delta_c' = \theta_c - f(\beta_c) \delta_c$$

$$\theta_c' = \left[ \frac{1}{2} \left( 3 w_\phi \Omega_\phi + \Omega_r - 1 \right) + g(\beta_c, \phi') \right] \theta_c + \frac{3}{2} \left[ \Omega_b \delta_b + \Omega_c \delta_c \Gamma_c \right]$$

$$\delta_b' = \theta_b$$

$$\theta_b' = \left[ \frac{1}{2} \left( 3 w_\phi \Omega_\phi + \Omega_r - 1 \right) \right] \theta_b + \frac{3}{2} \left[ \Omega_b \delta_b + \Omega_c \delta_c \right],$$

where a prime denotes a derivative with respect to the $e$-folding time $\alpha \equiv \ln a$, $\Omega_\phi$ and $\Omega_r$ are the DE and radiation fractional densities, respectively, $w_\phi$ is the DE equation of state parameter, and the three functions $f(\beta_c)$, $g(\beta_c, \phi')$, and $\Gamma_c(\phi) \equiv 1 + 4\beta_c^2(\phi)/3$ encode the deviation from the standard $\Lambda$CDM cosmology due to the DE interaction, being $(f, g, \Gamma) = (0, 0, 1)$ for an uncoupled DE field (see, e.g., Amendola 2004 for a more detailed description of the perturbation equations and a definition of the functions $f$ and $g$).

By solving system (5)–(8), we can separately compute the time evolution of the individual density and velocity perturbations for baryons and CDM and derive the total velocity perturbation $\theta$ given by Equation (4). In Figure 7 we plot the ratio of the velocity perturbation $\theta$ to the $\Lambda$CDM case for all the models considered in the present work. As one can see from the figure, the situation is significantly different with respect to the evolution of the density perturbation $\delta$. In particular, we display here for the first time the redshift evolution of the velocity perturbation $\theta$ in a significant number of cDE scenarios, and our results show how a given cDE model can have a significantly different relative impact on the density and velocity perturbations. Figure 7 clearly shows that the maximal enhancement of velocity perturbations with respect to $\Lambda$CDM is realized by the EXP008e3 model (orange, long-dashed line)—consistent
with our findings on the infall velocity in the bullet-like systems identified in the CoDECS simulations—even if the corresponding enhancement of the density perturbations amplitude (i.e., of $\sigma_8$) is significantly smaller than in the EXP003 model (blue, dot-dashed line).

Although the solution for $\theta$ displayed in Figure 7 is strictly valid only in the linear regime (as for the case of the density perturbation $\delta$), the relative impact of the different cDE models on the velocity perturbations can be considered reliable also in the nonlinear regime for situations where the velocity vector is aligned with the gradient of the gravitational potential, since in such a case the extra friction and the fifth-force contributions arising as a consequence of the DE interaction are also aligned with each other and keep behaving as for the linear case. This is, in particular, the situation for the bullet-like systems that we are considering in the present work, due to the head-on collision between the two clusters. Therefore, we can consider the results obtained for the linear solution of the velocity perturbation $\theta$ as indicative of the impact of the coupling even in these highly nonlinear systems. See Baldi (2011b) for a discussion on the linear and nonlinear effects of the friction term.

Particularly interesting also is the case of the bouncing cDE model SUGRA003: for this scenario, in fact, the velocity perturbation $\theta$ follows a similar path as for the EXP003 model at high redshifts, but is suddenly slowed down as a result of the bounce of the DE scalar field at $z_{\text{inv}} \approx 6.8$, and is the only model to have a smaller velocity perturbation $\theta$ as compared to $\Lambda$CDM at low redshifts. This is consistent with the background dynamics of the bouncing cDE scenario that feature an inversion of the scalar field motion at $z_{\text{inv}}$ with the consequent change of sign of the friction term $\phi \beta(\phi)$ that accelerates particles in their direction of motion before $z_{\text{inv}}$, while it decelerates particles after $z_{\text{inv}}$. The suppression of the velocity perturbation variable $\theta$ after $z_{\text{inv}}$ is therefore an expected effect due to the peculiar dynamics of the bouncing cDE scenario. This is reflected in the lower value of the mean pairwise infall velocity detected in our sample for the bouncing cDE model (see Table 3), unique among the cDE models considered in our analysis.

To see whether or not the enhanced velocities of the bullet-like systems in cDE models are really related to the enhanced values of the linear velocity perturbations, we compare the ratio of the mean infall velocity of the bullet-like systems for each cDE model to that for the $\Lambda$CDM model, $\langle V_\text{c}/V_{\theta,\Lambda\text{CDM}} \rangle$ (from Table 2) with the values of $\theta/\theta_{\Lambda\text{CDM}}$ at $z = 0$ (from Figure 7). The results are plotted in Figures 8 and 9 for the cases of $M_h \geq 0.5 \times 10^{15} h^{-1} M_\odot$ and $M_h \geq 0.7 \times 10^{15} h^{-1} M_\odot$, respectively. In each figure, the dotted line corresponds to the case that the two ratios have the same values. As can be seen, the enhancements of $V_\text{c}$ relative to the $\Lambda$CDM case indeed follow the pattern of the enhancements of $\theta$ relative to $\theta_{\Lambda\text{CDM}}$. In the SUGRA003 model, however, $V_\text{c}/V_{\theta,\Lambda\text{CDM}}$ is significantly larger than $\theta/\theta_{\Lambda\text{CDM}}$ at $z = 0$, which reflects the peculiar evolution of the SUGRA003 velocity perturbations shown in Figure 7.

To conclude, we have performed a comprehensive investigation of the effect of cDE models on the pairwise infall velocity of the bullet-like systems identified in the public catalogs of the CoDECS N-body simulations. Since our results have clearly demonstrated that cDE has the effect of speeding up bullet-like systems and that the strength of the effect significantly depends...
on the shape of the scalar potential as well as of the coupling function, we conclude that the Bullet Cluster will become a valuable constraint on the cDE scenario.

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