The *spin-charge-family* theory is explaining the origin of families, of the Higgs and the Yukawa couplings

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**Abstract**

The *spin-charge-family* theory offers a possible explanation for the assumptions of the *standard model* – for the charges of a family members, for the gauge fields, for the appearance of families, for the the scalar fields – interpreting the *standard model* as its low energy effective manifestation. The *spin-charge-family* theory predicts at the low energy regime two decoupled groups of four families of quarks and leptons. The predicted fourth family waits to be observed, while the stable fifth family is the candidate to form the dark matter. The Higgs and Yukawa couplings are the low energy effective manifestation of several scalar fields, all with the bosonic (adjoint) representations with respect to all the charge groups, with the family groups included. Properties of the families are analysed and relations among coherent contributions of the loop corrections to fermion properties discussed, including the one which enables the existence of the Majorana neutrinos. The appearance of several scalar fields is presented, their properties discussed, it is explained how these scalar fields can effectively be interpreted as the *standard model* Higgs (with the fermion kind of charges) and the Yukawa couplings, and a possible explanation why the Higgs has not yet been observed offered. The relation to proposals that the Yukawas follow from the $SU(3)$ family (flavour) group, having the family charges in the fundamental representations of these groups, is discussed. The *spin-charge-family* theory predicts that there are no supersymmetric partners of the observed fermions and bosons.

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I. INTRODUCTION

The standard model offered more than 35 years ago an elegant next step in understanding the origin of fermions and bosons. It is built on several assumptions leaving many open questions to be answered in the next step of theoretical interpretations. A lot of proofs and calculations have been done which support the standard model.

The measurements so far offer no sign which would help to make the next step beyond the standard model.

To explain the assumptions, which are the building blocks of the standard model, and to make successful new step beyond it any new proposal, model, theory must in my opinion at least i.) explain the origin of families and their mass matrices, predict the number of families, and accordingly explain the Yukawa couplings, mixing matrices and masses of family members, ii.) explain the origin of the standard model scalar field, the Higgs, and its connections to fermion masses and Yukawa couplings, and iii.) explain the origin of the dark matter. These three open questions are to my understanding so tightly connected that they call for common explanation.

There are many proposals in the literature [1–9] extending the standard model. No one explains, to my knowledge, the origin of families. Without a theory which is offering the answers to at least the above questions the suggestions for what and how should experiments search for new events can hardly be successful.

The theory unifying spin and charges and predicting families [10–14, 19], to be called the spin-charge-family theory, seems promising in answering these, and several other questions, which the standard model leaves unanswered.

The spin-charge-family theory assumes in $d = (1 + (d - 1))$, $d = 14$ (or larger), a simple starting action for spinors and the gauge fields: Spinors carry only two kinds of the spins (no charges), namely the one postulated by Dirac 80 years ago and the second kind proposed by the author of this paper. There is no third kind of a spin. Spinors interact with only vielbeins and the two kinds of the corresponding spin connection fields.

After the breaks of the starting symmetry, leading to the low energy regime, the simple starting action (Eq.(4)) manifests two decoupled groups of four families of quarks and leptons, with only the left handed (with respect to $d = (1+3)$) members of each family carrying the weak charge while the right handed ones are weak chargeless. The fourth family is pre-
dicted to be possibly observed at the LHC or at somewhat higher energies, while the stable fifth family members, forming neutral (with respect to the colour and electromagnetic charge) baryons and the fifth family neutrinos are predicted to explain the origin of the dark matter.

The spin connections, associated with the two kinds of spins, together with vielbeins, all behaving as scalar fields with respect to \( d = (1 + 3) \), are with their vacuum expectation values at the two \( SU(2) \) breaks responsible for the nonzero mass matrices of fermions and also for the masses of the gauge fields. The spin connections with the indices of vector fields with respect to \( d = (1 + 3) \), manifest after the break of symmetries as the known gauge fields.

Although the properties of the scalar fields, that is their vacuum expectation values, coupling constants and masses, can not be calculated without the detailed knowledge of the mechanism of breaking the symmetries, and have been so far only roughly estimated, yet one can see, assuming breaks which lead to observable phenomena at the low energy regime, how properties of the scalar fields determine the fermion mass matrices, manifesting effectively as the standard model Higgs and its Yukawa couplings.

According to the spin-charge-family theory the break of the starting symmetry is caused by nonzero expectation values of vielbeins and both kinds of the spin connection fields which are scalars with respect to \( SO(1, 3) \) symmetry. At the symmetry of \( SO(1, 7) \times U(1)_{II} \times SU(3) \) there are \( (2^{1+7-1}) \) massless left handed (with respect to \( SO(1, 7) \)) families, which stay massless until the symmetry \( SO(1, 3) \times SU(2)_{I} \times SU(2)_{II} \times U(1)_{II} \times SU(3) \) breaks. It is namely assumed that at the symmetry of \( SO(1, 7) \times U(1)_{II} \times SU(3) \) we indeed are left with the symmetry \( SO(1, 7) \gamma \times U(1)_{II} \gamma \times SU(3) \gamma \times SO(1, 7) \tilde{\gamma} \), while all possible other families which we started with become massive during the breaks up to this point.

Four of the eight families are doublets with respect to the generators (Eqs. \([14,15,16]\)) \( \tilde{\tau}^2 \) and with respect to \( \tilde{N}_R \), while they are singlets with respect to \( \tilde{\tau}^1 \) and \( \tilde{N}_L \). The other four families are singlets with respect to the first two kinds of generators and doublets with respect to the second two kinds (\( \tilde{\tau}^1 \) and \( \tilde{N}_L \)) of generators.

Each member of these eight massless families carries before the two successive breaks, from \( SO(1, 3) \times SU(2)_{I} \times SU(2)_{II} \times U(1)_{II} \times SU(3) \) (first to \( SO(1, 3) \times SU(2)_{I} \times U(1)_{I} \times SU(3) \) and then to \( SO(1, 3) \times U(1) \times SU(3) \)) the quantum numbers of the two \( SU(2) \), one \( U(1) \) and one \( SU(3) \) charges and the quantum numbers of the subgroups to which each of the
four families belong with respect to $\tilde{\tau}^{(2,1)}$ and $\tilde{N}_{(R,L)}$.

Analysing properties of each family member with respect to the quantum numbers of the spin and the charges subgroups, $SO(1, 3)$, $SU(2)_I$, $SU(2)_{II}$, $U(1)_{II}$ and $SU(3)$, we find that each family includes left handed (with respect to $SO(1, 3)$) weak $SU(2)_I$ charged quarks and leptons and right handed (again with respect to $SO(1, 3)$) weak $SU(2)_I$ chargeless quarks and leptons. While the right handed members (with respect to $SO(1, 3)$) are doublets with respect to $SU(2)_{II}$, the left handed fermions are singlets with respect to $SU(2)_{II}$.

Each member of eight massless families of quarks and leptons carries the $SU(2)_{II}$ charge, in addition to the family quantum number and the quantum numbers of the standard model. This quantum number determines, together with the $U(1)_{II}$ charge, after the first of the two breaks the standard model hyper charge and the fermion quantum number ($-\frac{1}{2}$ for leptons and $\frac{1}{6}$ for quarks).

Each break is triggered by the vielbeins and by one or both kinds of the spin connection fields. To be in agreement in the low energy regime with the standard model assumptions supported by the experimental data, the gauge fields which are scalars with respect to $SO(1, 3)$ and have appropriate symmetries are assumed to contribute. In the break of $SU(2)_I \times SU(2)_{II} \times U(1)_{II}$ to $SU(2)_I \times U(1)_I$ the scalar (with respect to $d = (1 + 3)$) fields originating in vielbeins and in the second kind of spin connection fields belonging to a triplet with respect to the $SU(2)_{II}$ symmetry in the $\tilde{S}^{ab}$ sector (with the generators $\tilde{\tau}^{2i} = c_{\tau_{2i}}^{ab}\tilde{S}^{ab}$, $\{\tilde{\tau}^{2i}, \tilde{\tau}^{2j}\}_- = \varepsilon^{ijk}\tilde{\tau}^{2k}$) and with respect to one of the two $SU(2)$ from $SO(1, 3)$ again in the $\tilde{S}^{ab}$ sector (with the generators $\tilde{N}^{ij}_R = c_{\tau_{ij}}^{Rab}\tilde{S}^{ab}$, $\{\tilde{N}^{ij}_R, \tilde{N}^{ij}_R\}_- = \varepsilon^{ijk}\tilde{N}^{jk}_R$) gain nonzero vacuum expectation values.

The upper four families, which are doublets with respect to these two $SU(2)$ groups, become massive and so does the $SU(2)_{II}$ gauge vector field (in the adjoint representation of $SU(2)_{II}$ in the $S^{ab}$ sector with $\tau^{2i} = c_{\tau_{2i}}^{ab}S^{ab}$, $\{\tau^{2i}, \tau^{2j}\}_- = \varepsilon^{ijk}\tau^{2k}$). The lower four families, which are singlets with respect to $\tilde{\tau}^{2i}$ and $\tilde{N}^{ij}_R$, the two $SU(2)$ subgroups in the $\tilde{S}^{ab}$ sector, stay massless.

This is assumed to happen below the energy scale of $10^{13}$ GeV, that is below the unification scale of all the three charges, and also pretty much above the electroweak break.

The lower four families become massive at the electroweak break, when $SU(2)_I \times U(1)_I$ breaks into $U(1)$. To this break the vielbeins and the scalar part of both kinds of the spin connection fields contribute, those which are triplets with respect to the two remaining
invariant $SU(2)$ subgroups in the $\tilde{S}^{ab}$ sector, $\tilde{\tau}^{1i} (\tilde{\tau}^{1i} = c^{1i}_{ab} \tilde{S}^{ab})$ and $\tilde{N}^{i}_{L} (\tilde{N}^{i}_{L} = c^{Li}_{ab} \tilde{S}^{ab})$, as well as the scalar gauge fields of $Q, Q'$ and $Y'$ (all expressible with $S^{ab}$). In this break also the $SU(2)_{I}$ weak gauge vector field becomes massive [28].

Although the estimations of the properties of families done so far are very approximate [13, 14], yet the predictions give a hope that the starting assumptions of the spin-charge-family theory are the right ones:

i. Both existing Clifford algebra operators determine properties of fermions. The Dirac $\gamma^{a}$'s manifest in the low energy regime the spin and all the charges of fermions (like in the Kaluza-Klein[like] theories [29]). The second kind of the spin, forming the equivalent representations with respect to the Dirac one, manifests families of fermions.

ii. Fermions carrying only the corresponding two kinds of the spin (no charges) interact with the gravitational fields – the vielbeins and (the two kinds of) the spin connections. The spin connections originating in the Dirac’s gammas manifest at the low energy regime the known gauge fields. The spin connections originating in the second kind of gammas are responsible, together with the vielbeins and the spin connections of the first kind, for the masses of gauge fields and fermions.

iii. The assumed starting action for spinors and gauge fields in $d$-dimensional space is simple: In $d$-dimensional space all the fermions are massless and interact with the corresponding gauge fields of the Poincaré group and the second kind of the spin connections, the corresponding Lagrange densities for the gauge fields are linear in the two Riemann scalars.

The project to come from the starting action through breaks of symmetries to the effective action at low (measurable) energy regime is very demanding. Although one easily sees that a part of the starting action manifests, after the breaks of symmetries, at the tree level the mass matrices of the families and that a part of the vielbeins together with the two kinds of the spin connection fields manifest as scalar fields, yet several proofs are still needed besides those done so far [16, 17] to guarantee that the spin-charge-family theory does lead to the measured effective action of the standard model. Very demanding calculations in addition to rough estimations [11, 13, 14] done so far are needed to show that predictions agree also with the measured values of masses and mixing matrices of the so far observed fermion families, explaining where do large differences among masses of quarks and leptons, as well as among their mixing matrices originate.

Let us point out that in the spin-charge-family theory the scalar (with respect to $(1 + 3)$)
spin connection fields, originating in the Dirac kind of spin, couple only to the charges and spin, contributing on the tree level equally to all the families, distinguishing only among the members of one family (among the u-quark, d-quark, neutrino and electron, the left and right handed), the other scalar spin connection fields, originating in the second kind of spin, couple only to the family quantum numbers. Both kinds start to contribute coherently only beyond the tree level and a detailed study should manifest the drastic differences in properties of quarks and leptons: in their masses and mixing matrices \[10, 19\]. It is a hope that the loop corrections will help to understand the differences in properties of fermions, with neutrinos included and the calculations will show to which extent are the Majorana terms responsible for the great difference in the properties of neutrinos and the rest of the family members.

In this work the mass matrices of the two groups of four families, the two groups of the scalar fields giving masses to the two groups of four families and to the gauge fields to which they couple, and the gauge fields are studied and their properties discussed, as they follow from the \textit{spin-charge-family} theory. Many an assumption, presented above, allowed by the \textit{spin-charge-family} theory, is made in order that the low energy manifestation of the theory agrees with the observed phenomena, but not (yet) proved that it follows dynamically from the theory.

This paper manifests that there is a chance that the properties of the observed three families naturally follow from the \textit{spin-charge-family} theory when going beyond the tree level, although on the tree level the mass matrices of leptons and quarks are (too) strongly related. A possible explanation is made why the observed family members differ so much in their properties. It is also explained why does the \textit{spin-charge-family} theory predict two stable families, and why and how much do the fifth family hadrons differ in their properties from the first family ones, offering the explanation for the existence of the dark matter.

In the refs. \[11, 13\] we studied the properties of the lower four families under the assumption that the loop corrections would not change much the symmetries of the mass matrices of the family members, as they follow from the \textit{spin-charge-family} theory on the tree level, but would take care of differences in properties among members. Relaxing strong connections between the mass matrices of the \textit{u}-quark and neutrino and the \textit{d}-quark and electron, we were able to predict some properties of the fourth family members and their mixing matrix elements with the three so far measured families. This paper is bringing a
possible justification for these relaxation. The concrete evaluations of the properties of the mass matrices beyond the tree level are in progress. First steps are done in the contribution with A. Hernández-Galeana [15], while more detailed analyses of the mass matrices with numerical results are in preparation.

The \textit{standard model} is presented as a low energy effective theory of the \textit{spin-charge-family} theory. Also some attempts in the literature to understand families as the $SU(3)$ flavour extension of the \textit{standard model} are commented.

The \textit{spin-charge-family} does not at support the existence of the supersymmetric partners of the so far observed fields.

\section{II. THE SPIN-CHARGE-FAMILY THEORY FROM THE STARTING ACTION TO THE STANDARD MODEL ACTION}

Let us in this section add to the introduction into the \textit{spin-charge-family} theory, made in the previous section, the mathematical part. The theory assumes that the spinor carries in $d=(1+13)$-dimensional space two kinds of the spin, no charges \textbf{[10]}: \textbf{\textit{i.}} The Dirac spin, described by $\gamma^a$’s, defines the spinor representations in $d=(1+13)$, and correspondingly in the low energy regime after several breaks of symmetries and before the electroweak break, the spin $(SO(1,3))$ and all the charges (the colour $SU(3)$, the weak $SU(2)$, the hyper charge $U(1)$) of quarks and leptons, left handed weak charged and right handed weakless, the left and the right handed distinguishing also in the weak charge, as assumed by the \textit{standard model}. \textbf{\textit{ii.}} The second kind of the spin \textbf{[18]}, described by $\tilde{\gamma}^a$’s ($\{\tilde{\gamma}^a,\tilde{\gamma}^b\}+ = 2\eta^{ab}$) and anticommuting with the Dirac $\gamma^a$ ($\{\gamma^a,\tilde{\gamma}^b\}+ = 0$), defines the families of spinors, which at the symmetries of $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ manifest two groups of four massless families.

There is no third kind of the Clifford algebra objects. The appearance of the two kinds of the Clifford algebra objects can be understood as follows: If the Dirac one corresponds to the multiplication of any spinor object $B$ (any product of the Dirac $\gamma^a$’s, which represents a spinor state when being applied on a spinor vacuum state $|\psi_0>$) from the left hand side, the second kind of the Clifford objects can be understood (up to a factor, determining the Clifford evenness ($n_B=2k$) or oddness ($n_B=2k+1$) of the object $B$ as the multiplication
of the object from the right hand side
\[ \tilde{\gamma}^a B |\psi_0 > := i(-)^{n_B} B \gamma^a |\psi_0 >, \] (1)
with \(|\psi_0 >\) determining the spinor vacuum state. Accordingly we have
\[ \{ \gamma^a, \gamma^b \} _+ = 2\eta^{ab} = \{ \tilde{\gamma}^a, \tilde{\gamma}^b \} _+ , \quad \{ \gamma^a, \tilde{\gamma}^b \} _+ = 0, \]
\[ S^{ab} := (i/4)(\gamma^a \gamma^b - \gamma^b \gamma^a) , \quad \tilde{S}^{ab} := (i/4)(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a), \quad \{ S^{ab}, \tilde{S}^{cd} \} _- = 0. \] (2)

More detailed explanation can be found in appendix. The spin-charge-family theory proposes in \( d = (1 + 13) \) a simple action for a Weyl spinor and for the corresponding gauge fields
\[ S = \int d^d x \ E \mathcal{L}_f + \int d^d x \ E (\alpha \tilde{R} + \tilde{\alpha} R) , \] (3)
\[ \mathcal{L}_f = \frac{1}{2} (\bar{\psi} \gamma^a p_0^a \psi) + h.c., \]
\[ p_0^a = f^a_{\ alpha} p_{\ alpha} + \frac{1}{2E} \{ p_{\ alpha}, Ef^a_{\ \alpha} \} _-, \]
\[ P_{\ alpha} = p_{\ alpha} - \frac{1}{2} S^{ab} \omega_{aba} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{aba}, \]
\[ R = \frac{1}{2} \{ f^{\alpha[a} f^{\beta b]} (\omega_{aba,\beta} - \omega_{ca \alpha} \omega_{c \ a \beta}) \} + h.c. , \]
\[ \tilde{R} = \frac{1}{2} f^{\alpha[a} f^{\beta b]} (\tilde{\omega}_{aba,\beta} - \tilde{\omega}_{ca \alpha} \tilde{\omega}_{c \ a \beta}) + h.c.. \] (4)

Here \[ f^{\alpha[a} f^{\beta b]} = f^{\alpha a} f^{\beta b} - f^{\alpha b} f^{\beta a} . \] To see that the action (Eq.(4)) manifests after the break of symmetries \[ 11, 13, 19 \] all the known gauge fields and the scalar fields and the mass matrices of the observed families, let us rewrite formally the action for a Weyl spinor of (Eq.4) as follows
\[ \mathcal{L}_f = \bar{\psi} \gamma^a p_0^a \psi - \sum_{A,i} g^{A} A^{Ai}_{A^i} A_{n} + \]
\[ \{ \sum_{s=7,8} \bar{\psi} \gamma^s p_0^s \psi \} + \]
the rest, \hspace{1cm} (5)

where \( n = 0, 1, 2, 3 \) with
\[ \tau^{Ai} = \sum_{a,b} C^{Ai}_{ab} S^{ab}, \]
\[ \{ \tau^{Ai}, \tau^{Bj} \} _- = i\delta^{AB} f^{-Aijk} \tau^{Ak}. \] (6)
All the charge ($\tau^A_i$ (Eqs. (6), (15), (16))) and the spin (Eq. (14)) operators are expressible with $S^{ab}$, which determine all the internal degrees of freedom of one family.

Index $A$ enumerates all possible spinor charges and $g^A$ is the coupling constant to a particular gauge vector field $A^A_i$. Before the break from $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ to $SO(1,3) \times SU(2)_I \times U(1)_I \times SU(3)$, $\tau^3_i$ describe the colour charge ($SU(3)$), $\tau^1_i$ the weak charge ($SU(2)_I$), $\tau^2_i$ the second $SU(2)_{II}$ charge and $\tau^4_i$ determines the $U(1)_{II}$ charge. After the break of $SU(2)_{II} \times U(1)_{II}$ to $U(1)_I$ stays $A = 2$ for the $U(1)_I$ hyper charge $Y$ and after the second break of $SU(2)_I \times U(1)_I$ to $U(1)$ stays $A = 2$ for the electromagnetic charge $Q$, while instead of the weak charge $Q'$ and $\tau^\pm$ of the standard model manifest.

The breaks of the starting symmetry from $SO(1,13)$ to the symmetry $SO(1,7) \times SU(3) \times U(1)_{II}$ and further to $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ are assumed to leave the low lying eight families of spinors massless [31]. After the break of $SO(1,13)$ to $SO(1,7) \times SU(3) \times U(1)$ there are eight such families ($2^{8/2-1}$), all left handed with respect to $SO(1,13)$.

Accordingly the first row of the action in Eq. (5) manifests the dynamical fermion part of the action, while the second part manifests, when $\omega_{ab\sigma}$ and $\tilde{\omega}_{ab\sigma}$ ($\sigma \in (7,8)$) fields gain nonzero vacuum expectation values, the mass matrices of fermions on the tree level. Scalar fields contribute also to masses of those gauge fields, which at a particular break lose symmetries. It is assumed that the symmetries in the $\tilde{S}^{ab}_{\sigma \rho \tau \mu}$ and in the $S^{ab}_{\rho \tau \mu \nu}$ part break in a correlated way, triggered by particular superposition of scalar (with respect to the rest of symmetry) vielbeins and spin connections of both kinds ($\omega_{abc}$ and $\tilde{\omega}_{abc}$). I comment this part in sections [II B, III]. The Majorana term, manifesting the Majorana neutrinos, is contained in the second row as well (III A 1). The third row in Eq. (5) stays for all the rest, which is expected to be at low energies negligible or might slightly influence the mass matrices beyond the tree level.

The generators $\tilde{S}^{ab}$ (Eqs. (14), (15), (16)) transform each member of one family into the corresponding member (the same family member) of another family, due to the fact that $\{S^{ab}, \tilde{S}^{cd}\}_{-} = 0$ (Eq. (2A.18)).

Correspondingly the action for the vielbeins and the spin connections of $S^{ab}$, with the Lagrange density $\alpha E R$, manifests at the low energy regime, after breaks of the starting symmetry, as the known vector gauge fields – the gauge fields of $U(1)$, $SU(2)$, $SU(3)$ and the ordinary gravity, contributing also to the break of symmetries and correspondingly to the masses of the gauge fields and fermions, while $\tilde{\alpha} E \tilde{R}$ are responsible for off diagonal
mass matrices of the fermion members and also to the masses of the gauge fields. Beyond the tree level all the massive fields contribute coherently to the mass matrices.

After the electroweak break the effective Lagrange density for spinors looks like

$$\mathcal{L}_f = \bar{\psi} (\gamma^m p_{0m} - M) \psi,$$

$$p_{0m} = p_m - \{ e Q A_m + g^1 \cos \theta_1 Q' Z_m^Q' + \frac{g^1}{\sqrt{2}} (\tau^{1+} W_m^{1+} + \tau^{1-} W_m^{1-}) +$$

$$g^2 \cos \theta_2 Y' A_m^Y' + \frac{g^2}{\sqrt{2}} (\tau^{2+} A_m^{2+} + \tau^{2-} A_m^{2-}) ,$$

$$\bar{\psi} M \psi = \bar{\psi} \gamma^s p_{0s} \psi$$

$$p_{0s} = p_s - \{ \bar{g} \tilde{N}_R \tilde{N}_R \tilde{A}_s^{\tilde{N}_R} + \bar{g} \tilde{Y}' \tilde{Y}' \tilde{A}_s^{\tilde{Y}' \tilde{Y}'} + \frac{g^1}{\sqrt{2}} (\tilde{\tau}^{1+} \tilde{A}_s^{1+} + \tilde{\tau}^{1-} \tilde{A}_s^{1-})$$

$$\bar{g} \tilde{N}_L \tilde{N}_L \tilde{A}_s^{\tilde{N}_L} + \frac{g^2}{\sqrt{2}} (\tilde{\tau}^{2+} \tilde{A}_s^{2+} + \tilde{\tau}^{2-} \tilde{A}_s^{2-}) \} + e Q A_s + g^1 \cos \theta_1 Q' Z_s^Q' + g^2 \cos \theta_2 Y' A_s^{Y'} \}, \ s \in \{ 7, 8 \} .$$

(7)

The term $\bar{\psi} M \psi$ determines the tree level mass matrices of quarks and leptons. The contributions to the mass matrices appear at two very different energy scales due to two separate breaks. Before the break of $SU(2)_{II} \times U(1)_{II}$ to $U(1)_I$ the vacuum expectation values of the scalar fields appearing in $p_{0s}$ are all zero. The corresponding dynamical scalar fields are massless. All the eight families are massless and the vector gauge fields $A_m^A \, ; \, A = 2$; in Eq. (5) are massless as well. To the break of $SU(2)_{II} \times U(1)_{II}$ to $U(1)_I$ the scalar fields from the first row in the covariant momentum $p_{0s}$, that is the two triplets $\tilde{A}_s^{\tilde{N}_R}$ and $\tilde{A}_s^2$ are assumed to contribute, gaining non zero vacuum expectation values. The upper four families, which are doublets with respect to the infinitesimal generators of the corresponding groups, namely $\tilde{N}_R$ and $\tilde{N}_L$, become massive. No scalar fields of the kind $\omega_{abs}$ is assumed to contribute in this break. Therefore, the lower four families, which are singlets with respect to $\tilde{N}_R$ and $\tilde{N}_L$, stay massless. Due to the break of $SU(2)_{II} \times U(1)_{II}$ symmetries in the space of $\tilde{S}^{ab}$ and $S^{ab}$, the gauge fields $\tilde{A}_m^2$ become massive. The gauge vector fields $\tilde{A}_m^1$ and $\tilde{A}_m^\gamma$ stay massless at this break.

To the break of $SU(2)_I \times U(1)_I$ to $U(1)$ the scalar fields from the second row in the covariant momentum $p_{0s}$, that is the triplets $\tilde{A}_s^{\tilde{N}_L}$ and $\tilde{A}_s^1$ and the singlet $\tilde{A}_s^4$, as well as the ones from the third row originating in $\omega_{abc}$, that is $(A_s, Z_s^Q', A_s^{Y'})$, are assumed to contribute, by gaining non zero vacuum expectation values.

This electroweak break causes non zero mass matrices of the lower four families. Also
the gauge fields $Z'_m, W^1_+^m$ and $W^1_-^m$ gain masses. The electroweak break influences slightly the mass matrices of the upper four families, due to the contribution of $A_s, Z'_s, A'_s$ and $\tilde{A}^1_s$ and in loop corrections also $Z'_m$ and $W^\pm_m$.

To loops corrections of both groups of families the massive vector gauge fields contribute. The dynamical massive scalar fields contribute only to families of the group to which they couple.

The detailed explanation of the two phase transitions which manifest in Eq. (7) is presented in what follows.

A. Spinor action through breaks

In this subsection properties of quarks, $u$ and $d$, and leptons, $\nu$ and $e$, of two groups of four families are presented at the stage of

$$SO(1, 3)_\gamma \times SO(1, 3)_{\tilde{\gamma}} \times SU(2)_I \gamma \times SU(2)_{I\tilde{\gamma}}$$
$$\times SU(2)_{II} \gamma \times SU(2)_{II\tilde{\gamma}} \times U(1)_{II} \gamma \times U(1)_{II\tilde{\gamma}}$$
$$\times SU(3)_\gamma,$$ (8)

when eight families are massless, and then when in the two successive breaks, in which first four and then the last four families gain masses. Half of the eight massless families are doublets with respect to the subgroup $SU(2)_{\tilde{\gamma} R}$ of $SO(1, 3)_{\tilde{\gamma}}$ and with respect to $SU(2)_{II\tilde{\gamma}}$ and singlets with respect to the $SU(2)_{\tilde{\gamma} L}$ subgroup of $SO(1, 3)_{\tilde{\gamma}}$ and with respect to $SU(2)_{I\tilde{\gamma}}$, the rest four families are singlets with respect to the subgroup $SU(2)_{\tilde{\gamma} R}$ and with respect to $SU(2)_{II\tilde{\gamma}}$ while they are doublets with respect to the $SU(2)_{\tilde{\gamma} L}$ and with respect to $SU(2)_{I\tilde{\gamma}}$. The two indices $\gamma$ and $\tilde{\gamma}$ are to point out that there are two kinds of subgroups of $SO(1, 7)$, those in the $S^{ab}$ (taking care of the spin and charges) and those in the $\tilde{S}^{ab}$ (taking care of the families) sector. We shall in what follows omit these two indices, keeping in mind that there are two kinds of groups and subgroups.

At the break of $SO(1, 3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ to $SO(1, 3) \times SU(2)_I \times U(1)_I \times SU(3)$ the four families coupled to the scalar fields which gain at this break nonzero vacuum expectation values become massive, while the four families which do not couple to these scalar fields stay massless, representing four families of left handed weak charged colour triplets quarks ($u_L, d_L$), right handed weak chargeless colour triplets quarks ($u_R, d_R$),
left handed weak charged colour singlets leptons ($\nu_L, e_L$) and right handed weak chargeless colour singlets leptons ($\nu_R, e_R$). After the second break the members of the lowest of the upper four families, sharing after the second break the charges and spin of the lowest four families, are the candidates to form the dark matter.

After the second break from $SO(1, 3) \times SU(2)_I \times U(1)_I \times SU(3)$ to $SO(1, 3) \times U(1) \times SU(3)$ the last four families become massive due to the nonzero vacuum expectation values of the rest of scalar fields. Three of the families represent the observed families of quarks and leptons, so far included into the standard model, if we do not count the right handed $\nu$s (which carry the additional charge $Y'$, not assumed in the standard model, as also all the other members do).

The technique [18], which offers an easy way to keep a track of the symmetry properties of spinors, is used as a tool to clearly demonstrate properties of spinors. This technique is explained in more details in appendix. In this subsection only a short introduction, needed to follow the explanation, is presented. Mass matrices of each groups of four families, on the tree and below the tree level, originated in the scalar gauge fields, which at each of the two breaks gain a nonzero vacuum expectation values, will be discussed in section III.

Following the refs. [18] we define nilpotents ($ab^2 = 0$) and projectors ($ab^2 ab^2 = ab^2$) (Eq. (A.6) in appendix)

\[
\begin{align*}
\gamma^a ab^2 (\pm i) & = \frac{1}{2}(\gamma^a \mp i\gamma^b), \\
\gamma^a ab^2 (\pm) & = \frac{1}{2}(\gamma^a \mp i\gamma^b), \\
\eta^a ab^2 = -1, \\
\eta^a ab^2 = 1,
\end{align*}
\]

as eigenvectors of $S^{ab}$ as well as of $\tilde{S}^{ab}$ (Eq. (A.7) in appendix)

\[
S^{ab} ab^2 (\pm i) = \frac{k}{2} ab^2 (\pm i), \\
S^{ab} ab^2 [\pm i] = \frac{k}{2} ab^2 [\pm i], \\
\tilde{S}^{ab} ab^2 (\pm) = \frac{k}{2} ab^2 (\pm), \\
\tilde{S}^{ab} ab^2 [\pm] = -\frac{k}{2} ab^2 [\pm].
\]

One can easily verify that $\gamma^a$ transform $ab^2 (\pm)$ into $ab^2 [\pm]$, while $\tilde{\gamma}^a$ transform $ab^2 (\pm)$ into $ab^2 [\pm]$ (Eq. (A.8) in appendix)

\[
\begin{align*}
\gamma^a ab^2 (\pm) & = \eta^a ab^2 [\pm], \\
\gamma^b ab^2 (\pm) & = -ik ab^2 [\pm], \\
\gamma^a ab^2 [\pm] & = ab^2 (-\pm), \\
\gamma^b ab^2 [\pm] & = -ik \eta^a ab^2 (-\pm),
\end{align*}
\]

\[
\begin{align*}
\tilde{\gamma}^a ab^2 (\pm) & = -i\eta^a ab^2 [\pm], \\
\tilde{\gamma}^b ab^2 (\pm) & = -k ab^2 [\pm], \\
\tilde{\gamma}^a ab^2 [\pm] & = i ab^2 (\pm), \\
\tilde{\gamma}^b ab^2 [\pm] & = -k \eta^a ab^2 (\pm).
\end{align*}
\]

Correspondingly, $\tilde{S}^{ab}$ generate the equivalent representations to representations of $S^{ab}$, and opposite. Defining the basis vectors in the internal space of spin degrees of freedom in
\[ d = (1 + 13) \] as products of projectors and nilpotents from Eq. (9) on the spinor vacuum state \(|\psi_0 >\), the representation of one Weyl spinor with respect to \(S^{ab}\) manifests after the breaks the spin and all the charges of one family members, and the gauge fields of \(S^{ab}\) manifest as all the observed gauge fields. \(\tilde{S}^{ab}\) determine families and correspondingly the family quantum numbers, while scalar gauge fields of \(\tilde{S}^{ab}\) determine, together with particular scalar gauge fields of \(S^{ab}\), mass matrices, manifesting effectively as Yukawa fields and Higgs.

Expressing the operators \(\gamma^7\) and \(\gamma^8\) in terms of the nilpotents \(78\) \((\pm)\), the mass term in Eqs. (5, 7) can be rewritten as follows

\[
\bar{\psi} M \psi = \sum_{s=7,8} \bar{\psi} \gamma^s p_{0s} \psi = \psi^+ (\gamma_0 \gamma^s) \psi \quad \text{and} \quad p_{0s} = (p_{07} \pm i p_{08}).
\]

After the breaks of the starting symmetry (from \(SO(1,13)\) through \(SO(1,7) \times U(1)_{II} \times SU(3)\)) to \(SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)\) there are eight \((2 \cdot 2 - 1)\) massless families of spinors. (Some support for this assumption is made when studying toy models [16, 17].)

Family members \(\alpha \in (u,d,\nu,e)\) carry the \(U(1)_{II}\) charge (the generator of the infinitesimal transformations of the group is \(\tau^4\), presented in Eq. (16)), the \(SU(3)\) charge (the generators are \(\tilde{\tau}^3\), presented in Eq. (16)) and the two \(SU(2)\) charges, \(SU(2)_{II}\) and \(SU(2)_I\) (the generators are presented in Eq. (15) as \(\tilde{\tau}^2\) and \(\tilde{\tau}^1\), respectively). Family members are in the representations, in which the left handed (with respect to \(SO(1,3)\)) carry the \(SU(2)_{II}\) (weak) charge (with the corresponding generators \(\tilde{\tau}^1\)), while the right handed carry the \(SU(2)_I\) charge (with the corresponding generators \(\tilde{\tau}^2\)).

Each family member carries also the family quantum number, which concern \(\tilde{S}^{ab}\) and is determined by the quantum numbers of the two \(SU(2)\) from \(SO(1,3)\) (with the generators \(\tilde{N}_{(L,R)}\), Eq. (16)) and the two \(SU(2)\) from \(SO(4)\) (with the generators \(\tilde{\tau}^{(1,2)}\), Eq. (15)).

Properties of families of spinors can transparently be analysed if using our technique. We arrange products of nilpotents and projectors to be eigenvectors of the Cartan subalgebra \(S^{03}, S^{12}, S^{56}, S^{78}, S^{910}, S^{1112}, S^{1314}\) and, at the same time, they are also the eigenvectors of the corresponding \(\tilde{S}^{ab}\), that is of \(\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \tilde{S}^{910}, \tilde{S}^{1112}, \tilde{S}^{1314}\).

Below the generators of the infinitesimal transformations of the subgroups of the group
SO(1, 13) in the $S^{ab}$ and $\tilde{S}^{ab}$ sectors, responsible for the properties of spinors in the low energy regime, are presented.

\[
\vec{N}_{\pm} = \vec{N}_{(L,R)} : = \frac{1}{2} (S^{23} \pm i S^{01}, S^{31} \pm i S^{02}, S^{12} \pm i S^{03}),
\]
\[
\vec{\tilde{N}}_{\pm} = \vec{\tilde{N}}_{(L,R)} : = \frac{1}{2} (\tilde{S}^{23} \pm i \tilde{S}^{01}, \tilde{S}^{31} \pm i \tilde{S}^{02}, \tilde{S}^{12} \pm i \tilde{S}^{03})
\]

(14)

determine representations of the two SU(2) subgroups of SO(1, 3),

\[
\vec{\tau}_1 : = \frac{1}{2} (S^{58} - S^{67}, S^{57} + S^{68}, S^{56} - S^{78}), \quad \vec{\tau}_2 : = \frac{1}{2} (S^{58} + S^{67}, S^{57} - S^{68}, S^{56} + S^{78}),
\]
\[
\vec{\tilde{\tau}}_1 : = \frac{1}{2} (\tilde{S}^{58} - \tilde{S}^{67}, \tilde{S}^{57} + \tilde{S}^{68}, \tilde{S}^{56} - \tilde{S}^{78}), \quad \vec{\tilde{\tau}}_2 : = \frac{1}{2} (\tilde{S}^{58} + \tilde{S}^{67}, \tilde{S}^{57} - \tilde{S}^{68}, \tilde{S}^{56} + \tilde{S}^{78})
\]

(15)

determine representations of SU(2)$_I \times$ SU(2)$_{II}$ of SO(4) and

\[
\vec{\tau}^3 : = \frac{1}{2} \left\{ S^{9 \ 12} - S^{10 \ 11}, S^{9 \ 11} + S^{10 \ 12}, S^{9 \ 10} - S^{11 \ 12}, S^{9 \ 14} - S^{10 \ 13}, S^{9 \ 13} + S^{10 \ 14}, S^{11 \ 14} - S^{12 \ 13}, S^{11 \ 13} + S^{12 \ 14}, \frac{1}{\sqrt{3}} (S^{9 \ 10} + S^{11 \ 12} - 2 S^{13 \ 14}) \right\},
\]
\[
\tau^4 : = \frac{1}{3} (S^{9 \ 10} + S^{11 \ 12} + S^{13 \ 14}), \quad \tilde{\tau}^4 : = -\frac{1}{3} (\tilde{S}^{9 \ 10} + \tilde{S}^{11 \ 12} + \tilde{S}^{13 \ 14}),
\]

(16)

determine representations of SU(3) $\times$ U(1), originating in SO(6).

It is assumed that at the break of SO(1, 13) to SO(1, 7) $\times$ U(1)$_{II} \times$ SU(3) all spinors but one become massive, which then manifests eight massless families generated by those generators of the infinitesimal transformations $\tilde{S}^{ab}$ which belong to the subgroup SO(1, 7). Some justification for such an assumption can be found in the refs. [16, 17].

At the stage of the symmetry $SO(1, 3) \times SU(2)_I \times SU(2)_{II} \times U(1)_I \times U(1)_{II} \times SU(3)$ each member of a family appears in eight massless families. Each family manifests at this symmetry eightplets of $u$ and $d$ quarks, left handed weak charged and right handed weak chargeless (of spin $(\pm \frac{1}{2})$) in three colours, and the colourless eightplet of $\nu$ and $e$ leptons, left handed weak charged and right handed weak chargeless (of spin $(\pm \frac{1}{2})$).

In Table I the eightplet of quarks of a particular colour charge ($\tau^{33} = 1/2, \tau^{38} = 1/(2\sqrt{3})$) and the $U(1)_{II}$ charge ($\tau^4 = 1/6$) is presented in our technique [18], as products of nilpotents and projectors.

In Table II the eightplet of the colourless leptons of the $U(1)_{II}$ charge ($\tau^4 = -1/2$) is presented in the same technique.
and ˜SU

Table I by changing the colour part of the states \([19]\), that is by applying members of the Cartan subalgebra, on the states of Table I.

\[
\begin{array}{|c|ccc|cccc|c|c|}
\hline
i & |a\psi_i> & \Gamma^{(1,3)} & S^{12} & \Gamma^{(4)} & \tau^{13} & \tau^{23} & Y & Q \\
\hline
1 & \(d^3_R\) & (\(+i\) & \(+\) \[\begin{array}{|c|c|}
\hline
\tau & \tau \\hline
\end{array}\]) & 1 & 1 & 0 & 1/2 & 0 & 1/2 & 0 & 2/3 & 2/3 \\
2 & \(d^3_R\) & (\(-i\) \[\begin{array}{|c|c|}
\hline
\tau & \tau \\hline
\end{array}\]) & 1 & -1/2 & 1 & 0 & 1/2 & 0 & 2/3 & 2/3 \\
3 & \(d^3_L\) & (\(+i\) \[\begin{array}{|c|c|}
\hline
\tau & \tau \\hline
\end{array}\]) & 1 & 0 & -1/2 & 1/2 & 0 & -1/2 & 0 & 1/3 & -1/3 \\
4 & \(d^3_L\) & (\(-i\) \[\begin{array}{|c|c|}
\hline
\tau & \tau \\hline
\end{array}\]) & 1 & 1/2 & 0 & -1/2 & 0 & 1/3 & -1/3 \\
5 & \(d^3_L\) & (\(+i\) \[\begin{array}{|c|c|}
\hline
\tau & \tau \\hline
\end{array}\]) & -1 & -1/2 & 0 & 1/2 & 0 & 1/3 & -1/3 \\
6 & \(d^3_L\) & (\(-i\) \[\begin{array}{|c|c|}
\hline
\tau & \tau \\hline
\end{array}\]) & -1 & 1/2 & 0 & 1/2 & 0 & 1/3 & -1/3 \\
7 & \(d^3_L\) & (\(+i\) \[\begin{array}{|c|c|}
\hline
\tau & \tau \\hline
\end{array}\]) & -1 & -1/2 & 0 & 1/2 & 0 & 1/3 & -1/3 \\
8 & \(d^3_L\) & (\(-i\) \[\begin{array}{|c|c|}
\hline
\tau & \tau \\hline
\end{array}\]) & -1 & 1/2 & 0 & 1/2 & 0 & 1/3 & -1/3 \\
\hline
\end{array}
\]

TABLE I: The 8-plet of quarks - the members of \(SO(1,7)\) subgroup of the group \(SO(1,13)\), belonging to one Weyl left handed (\(\Gamma^{(1,13)} = -1 = \Gamma^{(1,7)} \times \Gamma^{(6)}\)) spinor representation of \(SO(1,13)\) is presented in the technique \([18]\). It contains the left handed weak charged quarks and the right handed weak chargeless quarks of a particular colour (1/2, 1/(2\(\sqrt{3}\))). Here \(\Gamma^{(1,3)}\) defines the handedness in (1 + 3) space, \(S^{12}\) defines the ordinary spin (which can also be read directly from the basic vector, both vectors with both spins, \(\pm \frac{1}{2}\), are presented), \(\tau^{13}\) defines the third component of the weak charge, \(\tau^{23}\) the third component of the \(SU(2)_{II}\) charge, \(\tau^{4}\) (the \(U(1)\) charge) defines together with \(\tau^{23}\) the hyper charge (\(Y = \tau^{4} + \tau^{23}\)), \(Q = Y + \tau^{13}\) is the electromagnetic charge. The vacuum state \(|\psi_0>\), on which the nilpotents and projectors operate, is not shown. The basis is the massless one. The reader can find the whole Weyl representation in the ref. \([19]\).

In both tables the vectors are chosen to be the eigenvectors of the operators of handedness \(\Gamma^{(n)}\) and \(\bar{\Gamma}^{(n)}\), the generators \(\tau^{13}\) (the member of the weak \(SU(2)_I\) generators), \(\tau^{23}\) (the member of \(SU(2)_{II}\) generators), \(\tau^{33}\) and \(\tau^{38}\) (the members of \(SU(3)\)), \(Y (= \tau^{4} + \tau^{23})\) and \(Q (= Y + \tau^{13})\). They are also eigenvectors of the corresponding \(\bar{S}_{ab}, \bar{\tau}^{Ai}, A = 1, 2, 4\) and \(\bar{Y}\) and \(\bar{Q}\). The tables for the two additional choices of the colour charge of quarks follow from Table I by changing the colour part of the states \([19]\), that is by applying \(\tau^{3i}\), which are not members of the Cartan subalgebra, on the states of Table I.
Looking at Tables (I, II) and taking into account the relation $\psi = \bar{\psi}^* \Gamma^{(1,13), (6)}$ from Eq. (11) and the relation $\psi = \bar{\psi}^* \Gamma^{(1,13), (6)}$ from Eq. (11) one notices that the operator $\psi = \bar{\psi}^* \Gamma^{(1,13), (6)}$ (Eq. (13)) transforms the right handed $\nu_R$ from the first row of Table II into the left handed $\nu_L$ of the same spin and charge from the seventh row of the same table, and that it transforms the right handed $\nu_L$ from the first row of Table II into the left handed $\nu_R$ presented in the seventh row of the same table, just what the Higgs and $\gamma^0$ do in the standard model. Equivalently one finds that the operator $\psi = \bar{\psi}^* \Gamma^{(1,13), (6)}$ (Eq. (13)) transforms the right handed $\nu_R$ from the third row of Table II into the left handed one (of the same spin and colour) presented in the fifth row of Table II and that it transforms the right handed $e_R$ from the third row of Table II into the left handed one (of the same spin) presented in the fifth row of Table II.

The superposition of generators $\tilde{S}^{ab}$ forming eight generators $(\tilde{N}_{R,L}, \tilde{\tau}^{(2,1,1)})$ presented in appendix A, Eq. (A.19), generate families, transforming each member of one family into the

| i  | $|a\psi_i>$ | $\Gamma^{(1,3)}$ | $S^{12}$ | $\Gamma^{(4)}$ | $\gamma^{13}$ | $\gamma^{23}$ | $Y$ | $Q$ |
|----|-------------|----------------|--------|--------------|-------------|-------------|----|----|
| 1  | $\nu_R$    | $3_1^{12}$    | 3_12   | 1            | $1/2$       | 0           | 1  | 0  |
| 2  | $\nu_R$    | $-1_1^{12}$   | 1_12   | $1/2$        | 0           | 1           | 0  | 0  |
| 3  | $\nu_R$    | $-1_1^{12}$   | 1_12   | $1/2$        | 1           | $1/2$       | 1  | 1  |
| 4  | $\nu_R$    | $1_1^{12}$    | 1_12   | $1/2$        | 1           | $1/2$       | 1  | 1  |
| 5  | $\nu_R$    | $1_1^{12}$    | 1_12   | $1/2$        | 1           | $1/2$       | 1  | 1  |
| 6  | $\nu_R$    | $1_1^{12}$    | 1_12   | $1/2$        | 1           | $1/2$       | 1  | 1  |
| 7  | $\nu_R$    | $1_1^{12}$    | 1_12   | $1/2$        | 1           | $1/2$       | 1  | 1  |
| 8  | $\nu_R$    | $1_1^{12}$    | 1_12   | $1/2$        | 1           | $1/2$       | 1  | 1  |

TABLE II: The 8-plet of leptons - the members of $SO(1, 7)$ subgroup of the group $SO(1, 13)$ belonging to one Weyl left handed ($\Gamma^{(1,13)} = -1 = \Gamma^{(1,7)} \times \Gamma^{(6)}$) spinor representation of $SO(1, 13)$ is presented in the massless basis. It contains the colour chargeless left handed weak charged leptons and the right handed weak chargeless leptons. The rest of notation is the same as in Table I.
are singlets with respect to $\vec{f}$ families. One group of families contains doublets with respect to $\vec{f}$ to leave all the eight families massless, allows to divide eight families into two groups of four families. 

The application of the operators (\tilde{\tau} in the left and in the right column, respectively. All the families follow from the starting one by (Eq. (A.19)) transform $u^1_R$ of spin $\frac{1}{2}$ and the chosen colour $c^1$ to all the members of one family of the same colour. The same generators transform equivalently the right handed neutrino $\nu_R$ of spin $\frac{1}{2}$ to all the colourless members of the same family.

| $I_R$ | $u^1_R$ | $\nu_R$ |
|-------|---------|---------|
| 03 12 56 78 9 10 11 12 13 14 | $\nu_R$ |
| 03 12 56 78 9 10 11 12 13 14 |

TABLE III: Eight families of the right handed $u^1_R$ quark with spin $\frac{1}{2}$, the colour charge ($c^1 = (\tau^{33} = 1/2, \tau^{38} = 1/(2\sqrt{3})$), and of the colourless right handed neutrino $\nu_R$ of spin $\frac{1}{2}$ are presented in the left and in the right column, respectively. All the families follow from the starting one by the application of the operators ($\tilde{N}^{\pm}_{R,L}, \tilde{\tau}^{(2,1)\pm}$) from Eq. (A.19). The generators ($N^{\pm}_{R,L}, \tau^{(2,1)\pm}$) (Eq. (A.19)) transform $u^1_R$ of spin $\frac{1}{2}$ and the chosen colour $c^1$ to all the members of one family of the same colour. The same generators transform equivalently the right handed neutrino $\nu_R$ of spin $\frac{1}{2}$ to all the colourless members of the same family.

same member of another family, due to the fact that $\{S^{ab}, S^{cd}\}_- = 0$ (Eq. (2)). The eight families of the first member of the eightplet of quarks from Table I for example, that is of the right handed $u^1_R$-quark with spin $\frac{1}{2}$, are presented in the left column of Table III. The generators ($\tilde{N}^{\pm}_{R,L}, \tilde{\tau}^{(2,1)\pm}$) (Eq. (A.19)) transform the first member of the eightplet from Table I that is the right handed neutrino $\nu_R$ with spin $\frac{1}{2}$, into the eight-plet of right handed neutrinos with spin up, belonging to eight different families. These families are presented in the right column of the same table. All the other members of any of the eight families of quarks or leptons follow from any member of a particular family by the application of the operators ($N^{\pm}_{R,L}, \tau^{(2,1)\pm}$) on this particular member.

Let us point out that the break of $SO(1, 7)$ into $SO(1, 3) \times SU(2)_I \times SU(2)_I$, assumed to leave all the eight families massless, allows to divide eight families into two groups of four families. One group of families contains doublets with respect to $\tilde{N}_R$ and $\tilde{\tau}_2$, these families are singlets with respect to $\tilde{N}_L$ and $\tilde{\tau}_1$. Another group of families contains doublets with
respect to $\vec{N}_L$ and $\vec{\tau}^1$, these families are singlets with respect to $\vec{N}_R$ and $\vec{\tau}^2$. The scalar fields which are the gauge scalars of $\vec{N}_R$ and $\vec{\tau}^2$ couple only to the four families which are doublets with respect to this two groups. When gaining non zero vacuum expectation values, these scalar fields determine nonzero mass matrices of the four families, to which they couple. These happens at some scale, assumed that it is much higher than the electroweak scale.

The group of four families, which are singlets with respect to $\vec{N}_R$ and $\vec{\tau}^2$, stay massless unless the gauge scalar fields of $\vec{N}_L$ and $\vec{\tau}^1$, together with the gauge scalars of $Q, Q'$ and $Y'$, gain a nonzero vacuum expectation values at the electroweak break. Correspondingly the decoupled twice four families, that means that the matrix elements between these two groups of four families are equal to zero, appear at two different scales, determined by two different breaks.

To have an overview over the properties of the members of one (any one of the eight) family let us present in Table IV quantum numbers of particular members of any of the eight families: The handedness $\tilde{\Gamma}^{(1+3)}(= -4iS_{03}S_{12})$, $S_{03}^{L}, S_{12}^{L}, S_{03}^{R}, S_{12}^{R}, \tau^{13}$ (of the weak $SU(2)_I), \tau^{23}$ (of $SU(2)_{II}$), the hyper charge $Y = \tau^4 + \tau^{23}$, the electromagnetic charge $Q$, the $SU(3)$ status, that is, whether the member is a member of a triplet (the quark with the one of the charges $\{(\frac{1}{2}, \frac{1}{2\sqrt{3}}), (-\frac{1}{2}, \frac{1}{2\sqrt{3}}), (0, -\frac{1}{\sqrt{3}})\}$) or the colourless lepton, and $Y'$ after the break of $SU(2)_{II} \times U(1)_{III}$ into $U(1)_I$.

Before the break of $SU(2)_{II} \times U(1)_{III}$ into $U(1)_I$ the members of one family from Tables II and III share the family quantum numbers presented in Table IV. The ”tilde handedness” of the families $\tilde{\Gamma}^{(1+3)}(= -4i\tilde{S}_{03}\tilde{S}_{12})$, $\tilde{S}_{03}^{L}, \tilde{S}_{12}^{L}, \tilde{S}_{03}^{R}, \tilde{S}_{12}^{R}$ (the diagonal matrices of $SO(1, 3)$), $\tilde{\tau}^{13}$ (of one of the two $SU(2)_I), \tilde{\tau}^{23}$ (of the second $SU(2)_{II}$).

We see in Table IV that the first four of the eight families are singlets with respect to subgroups determined by $\vec{N}_R$ and $\vec{\tau}^2$, and doublets with respect to $\vec{N}_L$ and $\vec{\tau}^1$, while the rest four families are doublets with respect to $\vec{N}_R$ and $\vec{\tau}^2$, and singlets with respect to $\vec{N}_L$ and $\vec{\tau}^1$.

When the break from $SU(2)_I \times SU(2)_{II} \times U(1)_{III}$ to $SU(2)_I \times U(1)_I$ appears, the scalar fields, the superposition of $\tilde{\omega}_{ala}$, which are triplets with respect to $\vec{N}_R$ and $\vec{\tau}^2$ (are assumed to) gain a nonzero vacuum expectation values. As one can read from Eq. [3] these scalar fields cause nonzero mass matrices of the families which are doublets with respect to $\vec{N}_R$ and $\vec{\tau}^2$ and correspondingly couple to these scalar fields with nonzero vacuum expectation values. The four families which do not couple to these scalar fields stay massless. The
nonzero vacuum expectation values. The scalar fields $\vec{\tau}$ of the rest of eight families, gain nonzero vacuum expectation values. Together with them also the vector gauge fields of the last four massless families. At this break also the vector gauge fields become massive.

In the successive (electroweak) break the scalar gauge fields of $\vec{N}_L$ and $\vec{\tau}^1$, coupled to the rest of eight families, gain nonzero vacuum expectation values. Together with them also the scalar gauge fields $A^\nu_s$, $A^Q_s$ and $A^Q_s$ (the superposition of $\omega_{rst}$, spin connection fields) gain nonzero vacuum expectation values. The scalar fields $\vec{A}^1_s$, $\vec{A}^N_s$, $A^Q_s$ and $A^Q_s$ determine mass matrices of the last four massless families. At this break also the vector gauge fields of $\vec{\tau}^1$ become massive.

The second break, which (is assumed to) occurs at much lower energy scale, influences slightly also properties of the upper four families.

There is the contribution which appears in the loop corrections as the term bringing nonzero contribution only to the mass matrix of neutrinos, transforming the right handed neutrinos to the left handed charged conjugated ones. It looks like that (for a particular

| $u_{Li}$ | $-1$ | $\frac{1}{2}$ | $\pm \frac{1}{2}$ | $0$ | $0$ | $\frac{1}{2}$ | $0$ | $\frac{1}{2}$ | triplet | $\frac{1}{2} \tan^2 \theta_2$ |
| $d_{Li}$ | $-1$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $0$ | $0$ | $-\frac{1}{2}$ | $0$ | $\frac{1}{2}$ | triplet | $-\frac{1}{2} \tan^2 \theta_2$ |
| $\nu_{Li}$ | $-1$ | $\frac{1}{2}$ | $\pm \frac{1}{2}$ | $0$ | $0$ | $\frac{1}{2}$ | $0$ | $-\frac{1}{2}$ | singlet | $\frac{1}{2} \tan^2 \theta_2$ |
| $e_{Li}$ | $-1$ | $\pm \frac{1}{2}$ | $\frac{1}{2}$ | $0$ | $-\frac{1}{2}$ | $0$ | $-\frac{1}{2}$ | $-1$ | singlet | $\frac{1}{2} \tan^2 \theta_2$ |
| $u_{Ri}$ | $1$ | $0$ | $0$ | $\pm \frac{1}{2}$ | $\pm \frac{1}{2}$ | $0$ | $\frac{1}{2}$ | $\frac{1}{2}$ | triplet | $\frac{1}{2} (1 - \frac{1}{3} \tan^2 \theta_2)$ |
| $d_{Ri}$ | $1$ | $0$ | $0$ | $\pm \frac{1}{2}$ | $\pm \frac{1}{2}$ | $0$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | triplet | $-\frac{1}{2} (1 + \frac{1}{3} \tan^2 \theta_2)$ |
| $\nu_{Ri}$ | $1$ | $0$ | $0$ | $\pm \frac{1}{2}$ | $\frac{1}{2}$ | $0$ | $\frac{1}{2}$ | $0$ | singlet | $\frac{1}{2} (1 + \tan^2 \theta_2)$ |
| $e_{Ri}$ | $1$ | $0$ | $0$ | $\pm \frac{1}{2}$ | $\frac{1}{2}$ | $0$ | $-\frac{1}{2}$ | $-1$ | singlet | $-\frac{1}{2} (1 - \tan^2 \theta_2)$ |

TABLE IV: The quantum numbers of the members – quarks and leptons, left and right handed – of any of the eight families ($i \in \{I, \cdots, VIII\}$) from Table III are presented: The handedness $\Gamma^{(1+3)} = -4i S^0 I^{12}, S^0 \bar{S}^0, S^1 I^{12}, S^1 \bar{S}^1, \tau^{13}$ of the weak $SU(2)_I$, $\tau^{23}$ of the second $SU(2)_{II}$, the hyper charge $Y (= \tau^4 + \tau^{23})$, the electromagnetic charge $Q (= Y + \tau^{23})$, the $SU(3)$ status, that is, whether the member is a triplet – the quark with the one of the charges determined by $\tau^{33}$ and $\tau^{38}$, that is one of $\{(\frac{1}{2}, \frac{1}{2\sqrt{3}}), (\frac{1}{2}, \frac{1}{2\sqrt{3}}), (0, -\frac{1}{\sqrt{3}})\}$ – or a singlet, and the charge $Y' (= \tau^{23} - \tau^4 \tan^2 \theta_2)$. vacuum expectation value of $\vec{A}_\pm^4 = 0$ is assumed to stay zero at the first break. In this break also the vector (with respect to (1+3)) gauge fields of $\vec{\tau}^2$ (the generators of $SU(2)_{II}$) become massive.
TABLE V: Quantum numbers of a member of the eight families from Table III, the same for all the members of one family, are presented: The “tilde handedness” of the families $\tilde{\Gamma}^{(1+3)} = -4i\tilde{S}^{03}\tilde{S}^{12}$, the left and right handed $SO(1,3)$ quantum numbers (Eq. (14), $\tilde{S}^{03}_{L}, \tilde{S}^{12}_{L}, \tilde{S}^{03}_{R}, \tilde{S}^{12}_{R}$ of $SO(1,3)$ group in the $\tilde{S}^{mn}$ sector), $\tilde{\tau}^{13}$ of $SU(2)_{I}$, $\tilde{\tau}^{23}$ of the second $SU(2)_{II}$, $\tilde{\tau}^{4}$ (Eq. (16)), $\tilde{Y}' (= \tilde{\tau}^{23} - \tilde{\tau}^{4} \tan \tilde{\theta}_{2})$, taking $\tilde{\theta}^{2} = 0$, $\tilde{Y} (= \tilde{\tau}^{4} + \tilde{\tau}^{23})$, $\tilde{Q} = (\tilde{\tau}^{4} + \tilde{S}^{56})$.

choice of operators and parameters) such a Majorana mass term appears for the lower four families only. We discuss the Majorana neutrino like contribution in subsect. III A 1.

Let us end this subsection by admitting that it is assumed (not yet showed or proved) that there is no contributions to the mass matrices from $\psi_{L}^{\dagger} \gamma^{0}\gamma^{s} p_{0s} \psi_{R}$, with $s = 5, 6$. Such a contribution to the mass term would namely mix states with different electromagnetic charges ($\nu_{R}$ and $e_{L}$, $u_{R}$ and $d_{L}$), in disagreement with what is observed.

B. Scalar and gauge fields in $d = (1 + 3)$ through breaks

In the spin-charge-family theory there are the vielbeins $e^{s}_{\alpha}$

$$e^{s}_{\alpha} = \begin{pmatrix} \delta^{m}_{\mu} & 0 \\ 0 & e^{s}_{\sigma} \end{pmatrix}$$

in a strong correlation with the spin connection fields of both kinds, with $\tilde{\omega}_{st\sigma}$ and with $\omega_{ab\sigma}$, with indices $s, t, \sigma \in \{5, 6, 7, 8\}$, which manifest in $d = (1 + 3)$-dimensional space as scalar fields after particular breaks of a starting symmetry. Phase transitions are (assumed to be)
triggered by the nonzero vacuum expectation values of the fields $f^\alpha_s \tilde{\omega}_{ab\alpha}$ and $f^\alpha_s \omega_{ab\alpha}$.

The gauge fields then correspondingly appear as

$$e^\alpha_a = \begin{pmatrix} \delta^m_\mu & 0 \\ e^s_\mu = e^s_\sigma E^{\sigma Ai} A^A_i & e^s_\sigma \end{pmatrix},$$

with $E^{\sigma Ai} = \tau^{Ai} x^\sigma$, where $A^A_i$ are the gauge fields, corresponding to (all possible) Kaluza-Klein charges $\tau^{Ai}$, manifesting in $d = (1 + 3)$. Since the space symmetries include only $S^{ab}$ ($M^{ab} = L^{ab} + S^{ab}$) and not $\tilde{S}^{ab}$, there are no vector gauge fields of the type $e^s_\sigma \tilde{E}^{\sigma Ai} \tilde{A}^{A^I_i}$, with $\tilde{E}^{\sigma Ai} = \tilde{\tau}^{Ai} x^\sigma$. The gauge fields of $\tilde{S}_{ab}$ manifest in $d = (1 + 3)$ only as scalar fields.

The vielbeins and spin connection fields from Eq. (4) \( \int d^dx E(\alpha R + \alpha \tilde{R}) \) are manifesting in $d = (1 + 3)$ in the effective action, if no gravity is assumed in $d = (1 + 3)$ \( e^m_\mu = \delta^m_\mu \)

$$S_b = \int d^{(1+3)} x \left\{ -\frac{\varepsilon^A}{4} F^{A mn} F^{A}mn + \frac{1}{2} (m^A_i)^2 A^A_i A^{A}i + \text{contributions of scalar fields} \right\}. \tag{17}$$

Masses of gauge fields of the charges $\tau^{Ai}$, which symmetries are unbroken, are zero, nonzero masses correspond to the broken symmetries.

In the breaking procedures, when $SO(1, 7) \times U(1)_{II} \times SU(3)$ breaks into $SO(1, 3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$, there are eight massless families of quarks and leptons (as discussed above) and massless gauge fields $SU(2)_I$, $SU(2)_{II}$, $U(1)_{II}$ and $SU(3)$. Gravity in $(1 + 3)$ is not discussed.

In the break from $SO(1, 3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$ to $SO(1, 3) \times SU(2)_I \times U(1)_I \times SU(3)$ the scalar fields originating in $f^\alpha_s \tilde{\omega}_{ab\alpha} = \tilde{\omega}_{ab\alpha}$ gain nonzero vacuum expectation values causing the break of symmetries, which manifests on the tree level in masses of the superposition of gauge fields $\tilde{A}^2_m$ and $A^4_m$, as well as in mass matrices of the upper four families.

To the break from $SO(1, 3) \times SU(2)_I \times U(1)_I \times SU(3)$ to $SO(1, 3) \times U(1) \times SU(3)$ both kinds of scalar fields, a superposition of $f^\alpha_s \tilde{\omega}_{ab\alpha} = \tilde{\omega}_{ab\alpha}$ and $f^\alpha_s \omega_{s'ta} = \omega_{s'ta}$, with $(s', t) = \{(5, 6), (7, 8)\}; s = \{7, 8\}$ and $A^4_m$, contribute which manifests in the masses of $W^\pm_m, Z_m$ and in mass matrices of the lower four families.

Detailed studies of the appearance of breaks of symmetries as follow from the starting action, the corresponding manifestation of masses of the gauge fields involved in these breaks, as well as the appearance of the nonzero vacuum expectation values of the scalar (with
respect to (1+3)) fields which manifest in mass matrices of the families involved in particular breaks are under consideration. We study in the refs. [16, 17] on toy models possibilities that a break (such a break is in the discussed cases the one from $SO(1,13) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3)$, via $SO(1,7) \times U(1)_{II} \times SU(3)$) can end up with massless fermions. We found in the ref. [16] for a toy model scalar vielbein and spin connection fields which enable massless fermions after the break. We were not been able yet, even not for this toy model, to solve the problem, how do particular scalar fields causing a break of symmetries appear and what fermion sources are responsible for their appearance.

In this paper it is (just) assumed that there occur nonzero vacuum expectation values of particular scalar fields, which then cause breaks of particular symmetries, and change properties of gauge fields and of fermion fields.

Although the symmetries of the vacuum expectation values of the scalar fields are known when the break of symmetries is assumed, yet their values (numbers) are not known. Masses and potentials determining the dynamics of these scalar fields are also not known, and also the way how do scalar fields contribute to masses of the gauge fields, on the tree and below the tree level, waits to be studied.

Let us repeat that all the gauge fields, scalar or vectors, either originating in $\omega_{abc}$ or in $\tilde{\omega}_{abc}$ are after breaks in the adjoint representations with respect to all the groups, to which the starting groups break.

1. **Scalar and gauge fields after the break from $SU(2)_{II} \times U(1)_{II}$ to $U(1)_I$**

Before the break of $SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1) \times SU(3)$ to $SO(1,3) \times SU(2)_I \times U(1) \times SU(3)$ the gauge fields $\bar{A}^{22}_m$ ($A^{21}_m = \omega_{58m} + \omega_{67m}$, $A^{22}_m = \omega_{57m} - \omega_{68m}$, $A^{23}_m = \omega_{56m} + \omega_{78m}$), $\bar{A}^{11}_m$ ($A^{11}_m = \omega_{58m} - \omega_{67m}$, $A^{12}_m = \omega_{57m} + \omega_{68m}$, $A^{13}_m = \omega_{56m} - \omega_{78m}$) and $A^4_m$ are all massless.

After the break the gauge fields $A^{2\pm}_m$, as well as one superposition of $A^{23}_m$ and $A^4_m$, become massive, while another superposition ($A^\nu_m$) and the gauge fields $\bar{A}^{11}_m$ stay massless, due to the (assumed) break of symmetries.

The fields $A^\nu_m$ and $A^{2\pm}_m$, manifesting as massive fields, and $A^\nu_m$ which stay massless, are
defined as the superposition of the old ones as follows

\[ A_m^{23} = A_m^Y \sin \theta_2 + A_m^{Y'} \cos \theta_2, \]
\[ A_m^4 = A_m^Y \cos \theta_2 - A_m^{Y'} \sin \theta_2, \]
\[ A_m^{2\pm} = \frac{1}{\sqrt{2}}(A_m^{21} \mp iA_m^{22}), \] (18)

for \( m = 0, 1, 2, 3 \) and a particular value of \( \theta_2 \). The scalar fields \( A_s^{Y'}, A_s^{2\pm}, A_s^{Y''} \), which do not gain in this break any vacuum expectation values, stay masses. This assumption guarantees that they do not contribute to masses of the lower four families on the tree level.

The corresponding operators for the new charges which couple these new gauge fields to fermions are

\[ Y = \tau^4 + \tau^{23}, \quad Y' = \tau^{23} - \tau^4 \tan^2 \theta_2, \quad \tau^{2\pm} = \tau^{21} \pm i\tau^{22}. \] (19)

The new coupling constants become \( g^Y = g^4 \cos \theta_2, g^{Y'} = g^2 \cos \theta_2 \), while \( A_m^{2\pm} \) have a coupling constant \( g^2/\sqrt{2} \).

In the break also the scalar fields originating in \( \tilde{\omega}_{\text{abs}} \) contribute, and symmetries in both sectors, \( \tilde{S}^{ab} \) and \( S^{ab} \), are broken simultaneously. The scalar fields \( \tilde{A}_s^2 \) (which are the superposition of \( \tilde{\omega}_{\text{abs}}, \tilde{A}_s^{21} = \tilde{\omega}_{5\ell s} + \tilde{\omega}_{6\ell m}, \tilde{A}_s^{22} = \tilde{\omega}_{57s} - \tilde{\omega}_{68s}, \tilde{A}_s^{23} = \tilde{\omega}_{56s} + \tilde{\omega}_{78s} \)) gain a nonzero vacuum expectation values.

We have for the scalar fields correspondingly

\[ \tilde{A}_s^{23} = \tilde{A}_s^Y \sin \tilde{\theta}_2 + \tilde{A}_s^{Y'} \cos \tilde{\theta}_2, \]
\[ \tilde{A}_s^4 = \tilde{A}_s^Y \cos \tilde{\theta}_2 - \tilde{A}_s^{Y'} \sin \tilde{\theta}_2, \]
\[ \tilde{A}_s^{2\pm} = \frac{1}{\sqrt{2}}(\tilde{A}_s^{21} \mp i\tilde{A}_s^{22}), \] (20)

for \( s = 7, 8 \) and a particular value of \( \tilde{\theta}_2 \). These scalar fields, having a nonzero vacuum expectation values, define according to Eq. (7) mass matrices of the upper four families, which are doublets with respect to \( \tilde{\tau}^2 \) and \( \tilde{N}_R \).

To this break and correspondingly to the mass matrices of the upper four families also the scalar fields which couple to the upper four families

\[ \tilde{A}_s^{\tilde{N}R} = (\tilde{\omega}_{23s} - i \tilde{\omega}_{01s}, \tilde{\omega}_{31s} - i \tilde{\omega}_{02s}, \tilde{\omega}_{12s} - i \tilde{\omega}_{03s}) \] (21)

contribute. The lower four families, which are singlets with respect to both groups, stay correspondingly massless.
The corresponding new operators are then
\[
\tilde{Y} = \tau^4 + \tau^{23}, \quad \tilde{Y}' = \tau^{23} - \tau^4 \tan^2 \tilde{\theta}_2, \quad \tau^{2\pm} = \tau^{21} \pm i \tau^{22}, \quad \tilde{N}_R.
\] (22)

New coupling constants are correspondingly \(\tilde{g} \tilde{Y} = g \cos \tilde{\theta}_2, \tilde{g} \tilde{Y}' = g \cos \tilde{\theta}_2, \tilde{A}_s^{2\pm}\) have a coupling constant \(\frac{\tilde{g}^2}{\sqrt{2}}\), and \(\tilde{A}_{sN_L} \tilde{g} \tilde{N}_{R}\).

2. Scalar and gauge fields after the break from \(SU(2)_I \times U(1)_I\) to \(U(1)\)

To the electroweak break, when \(SO(1, 3) \times SU(2)_I \times U(1)_I \times SU(3)\) breaks into \(SO(1, 3) \times U(1) \times SU(3)\), both kinds of the scalar spin connection fields are assumed to contribute, that is a superposition of \(\tilde{\omega}_{abs}\), which is orthogonal to the one trigging the first break

\[
\tilde{A}_{s}^{13} = \tilde{A}_s \sin \tilde{\theta}_1 + \tilde{Z}_s \cos \tilde{\theta}_1, \\
\tilde{A}_{s}^Y = \tilde{A}_s \cos \tilde{\theta}_1 - \tilde{Z}_s \sin \tilde{\theta}_1, \\
\tilde{W}_s^{\pm} = \frac{1}{\sqrt{2}} (\tilde{A}_s^{11} \mp i \tilde{A}_s^{12}),
\] (23)

and

\[
\tilde{A}_{sN_L}
\] (24)

and a superposition of \(\omega_{sts'}\)

\[
A_s^Q = \sin \theta_1 A_s^{13} + \cos \theta_1 A_s^Y, \quad A_s'^Q = \cos \theta_1 A_s^{13} - \sin \theta_1 A_s^Y,
\]
\[
A_s^Y' = \cos \theta_2 A_s^{23} - \sin \theta_2 A_s^4.
\] (25)

\(s \in (7, 8)\). While the superposition of Eqs. (23, 24) couple to the lower four families only, since the lower four families are doublets with respect to \(\tilde{\tau}^1\) and \(\tilde{N}_N\), and the upper four families are singlets with respect to \(\tilde{\tau}^1\) and \(\tilde{N}_N\), the scalar fields \(A_s^Q, A_s'^Q\) and \(A_s^Y'\) (they are a superposition of \(\omega_{sts'}\); \(s, t \in (5, \cdots, 14); s' = 7, 8\)) couple to all the eight families, distinguishing among the family members.

Correspondingly a superposition of the vector fields \(\tilde{A}_m^{11}\) and \(A_m^4\),

\[
A_{m}^{13} = A_m \sin \theta_1 + Z_m \cos \theta_1, \\
A_{m}^Y = A_m \cos \theta_1 - Z_m \sin \theta_1, \\
W_{m}^{\pm} = \frac{1}{\sqrt{2}} (A_{m}^{11} \mp i A_{m}^{12}),
\] (26)
that is $W^\pm_m$ and $Z_m$, become massive, while $A_m$ stays with $m = 0$. The new operators for charges are

\[
Q = \tau^{13} + Y = S^{56} + \tau^4,
\]
\[
Q' = -Y \tan^2 \theta_1 + \tau^{13},
\]
\[
\tau^{1\pm} = \tau^{11} \pm i\tau^{12},
\]

(27)

and the new coupling constants are correspondingly $e = g^Y \cos \theta_1, g' = g^1 \cos \theta_1$ and $\tan \theta_1 = \frac{g^Y}{g^1}$, in agreement with the standard model. We assume for simplicity that in the scalar sector of $\omega_{stc} - \omega_{s,t,s}'$ – the same $\theta_1$ determines properties of the coupling constants as it does in the vector one – $\omega_{s,t,m}$.

In the sector of the $\tilde{\omega}_{abs}$ scalars the corresponding new operators are

\[
\tilde{Q} = \tilde{\tau}^{13} + \tilde{Y} = \tilde{S}^{56} + \tilde{\tau}^4,
\]
\[
\tilde{Q}' = -\tilde{Y} \tan^2 \tilde{\theta}_1 + \tilde{\tau}^{13},
\]
\[
\tilde{\tau}^{1\pm} = \tilde{\tau}^{11} \pm i\tilde{\tau}^{12},
\]

(28)

with the new coupling constants $\tilde{e} = \tilde{g}^Y \cos \tilde{\theta}_1, \tilde{g}' = \tilde{g}^1 \cos \tilde{\theta}_1$ and $\tan \tilde{\theta}_1 = \frac{\tilde{g}^Y}{\tilde{g}^1}$.

To this break and correspondingly to the mass matrices of the lower four families also the scalar fields $\tilde{A}_s^{N_L}$ (orthogonal to $\tilde{A}_s^{N_R}$) contribute.

All the scalar fields presented in this and the previous subsection are massive dynamical fields, coupled to fermions and governed by the corresponding scalar potentials, for which we assume that they behave as normalizable ones (at least up to some reasonable accuracy).

III. MASS MATRICES ON THE TREE LEVEL AND BEYOND IN THE SPIN-CHARGE-FAMILY THEORY

In the two subsections (II B 1, II B 2) of section II B properties of scalar and gauge fields after each of the two successive breaks are discussed. The appearance of the vacuum expectation values of some superposition of two kinds of spin connection fields and vielbeins, all scalars with respect to $(1 + 3)$, is assumed. These scalar fields determine in the spin-charge-family theory mass matrices of fermions and masses of vector gauge fields on the tree level. It is the purpose of this section to discuss properties of families of fermions after these two breaks, on the tree level and beyond the tree level. Properties of the family members within
the pairs \((u, \nu)\) and \((d, e)\) are, namely, on the tree level very much related and it is expected that hopefully loop corrections (in all orders) make properties of the lowest three families in agreement with the observations.

The starting fermion action (Eq. 5) manifests after the two successive breaks of symmetries in the effective low energy action presented in Eq. (7). The mass term (Eq. (13)) manifests correspondingly in the fermion mass matrices.

Let us repeat the assumptions made to come from the starting action to the low energy effective action:  

**i.** In the break from \(SU(2)_I \times SU(2)_{II} \times U(1)_{II}\) to \(SU(2)_I \times U(1)_I\), the superposition of the \(\tilde{\omega}_{abs}\) scalar fields which are the gauge fields of \(\tilde{\tau}_2\) and \(\tilde{N}_R\), with the index \(s \in (7,8)\), gain non zero vacuum expectation values.  

**ii.** In the electroweak break the superposition of the \(\tilde{\omega}_{abs}\) scalar fields which are the gauge fields of \(\tilde{\tau}_1\) and \(\tilde{N}_L\), and the superposition of scalar fields \(\omega_{s'ts}(s,s',t \in (7,8))\) which are the gauge fields of \(Q, Q'\) and \(Y'\), gain nonzero vacuum expectation values.

The first break leaves the lower four families, which are singlets with respect to the groups \((\tilde{\tau}_2, \tilde{\tau}_2)\) involved in the break, massless. At the electroweak break all the families become massive. While the scalar fields coupled with \(\tilde{\tau}_1\) and \(\tilde{N}_L\) to fermions influence only the lower four families, the scalar gauge fields coupled with \(Q, Q'\) and \(Y'\) to fermions influence mass matrices of all the eight families.

To loop corrections the gauge vector fields, the scalar dynamical fields originating in \(\omega_{s'ts}\) and in \(\tilde{\omega}_{abs}\) contribute, those to which a particular group of families couple. Let us tell that there is also a contribution to loop corrections, manifesting as a very special products of superposition of \(\omega_{abs}, s = 5, 6, 9, \cdots, 14\) and \(\tilde{\omega}_{abs}, s = 5, 6, 7, 8\) fields, which couple only to the right handed neutrinos and their charge conjugated states of the lower four families. This term might strongly influence properties of neutrinos of the lower four families.

Table [VI] represents the mass matrix elements on the tree level for the upper four families after the first break, originating in the vacuum expectation values of two superposition of \(\tilde{\omega}_{abs}\) scalar fields, the two triplets of \(\tilde{\tau}_2\) and \(\tilde{N}_R\). The notation \(\tilde{A}_\pm A^\dagger_\pm = -g\tilde{A}^\dagger A\) is used. The sign \(\pm\) distinguishes between the values of the two pairs \((u\text{-quarks, } \nu\text{-lepton})\) and \((d\text{-quark, } e\text{-lepton})\), respectively. The lower four families, which are singlets with respect to the two groups \((\tilde{\tau}_2, \tilde{N}_R)\), as can be seen in Table [IV], stay massless after the first break.

Masses of the lowest of the higher four family were evaluated in the ref. [14] from the cosmological and direct measurements, when assuming that baryons of this stable family...
TABLE VI: The mass matrix for the eight families of quarks and leptons after the break of $SO(1,3) \times SU(2)_{I} \times SU(2)_{II} \times U(1)_{I} \times SU(3)$ to $SO(1,3) \times SU(2)_{I} \times U(1)_{I} \times SU(3)$. The notation $\tilde{a}_{i}$ stays for $-\tilde{g}^{i} \tilde{A}_{i}^{A}$, $(\mp)$ distinguishes $u_i$ from $d_i$ and $\nu_i$ from $e_i$, index $i$ determines families.

(with no mixing matrix to the lower four families) constitute the dark matter.

The lower four families obtain masses when the second $SU(2)_{I} \times U(1)_{I}$ break occurs, at the electroweak scale, manifesting in nonzero vacuum expectation values of the two triplet scalar fields $\tilde{A}_{i}^{A}$, $\tilde{A}_{L}^{A}$, and the $U(1)$ scalar fields $\tilde{A}_{i}^{i}$, as well as $A_{i}^{Q}$, $A_{i}^{Q'}$ and $A_{i}^{Y'}$, and also in nonzero masses of the gauge fields $W^{\pm}_{m}$ and $Z_{m}$.

Like in the case of the upper four families, also here is the mass matrix contribution from the nonzero vacuum expectation values of $f^{a}_{s} \bar{\omega}_{a\sigma}$ on the tree level the same for $(u$-quarks and $\nu$-leptons) and the same for $(d$-quarks and $e$-leptons), while $(\mp)$ distinguishes between the values of the $u$-quarks and $d$-quarks and correspondingly between the values of $\nu$ and $e$. The contributions from $A_{i}^{Q}$, $A_{i}^{Q'}$ and $A_{i}^{Y'}$ to mass matrices are different for different family members and the same for all the families of a particular family member.

Beyond the tree level all mass matrix elements of a family member become dependent on the family member quantum number, through coherent contributions of the vector and all the scalar dynamical fields.

Table VII represents the contribution of $\tilde{g}^{i} \tilde{A}_{i}^{A}$ and $\tilde{g}^{N_{L}} \tilde{N}_{L}^{A}$, to the mass matrix elements on the tree level for the lower four families after the electroweak break. The
TABLE VII: The mass matrix on the tree level for the lower four families of quarks and leptons after the electroweak break. Only the contributions coming from the terms $\tilde{S}^{ab} \tilde{\omega}_{abs}$ in $p_0$s in Eq. (7) are presented. The notation $\tilde{a}_{\pm}^{\tilde{A}_i}$ stays for $-\tilde{g}_{\tilde{A}_i}^{\tilde{A}_j}$, where $(\mp)$ distinguishes between the values of the ($u$-quarks and $d$-quarks) and between the values of ($\nu$ and $e$). The terms coming from $S^{ss'} \tilde{\omega}_{ss'} t$ are not presented here. They are the same for all the families, but distinguish among the family members.

The absolute values of the vacuum expectation values of the scalar fields contributing to the first break are expected to be much larger than those contributing to the second break ($|\tilde{A}_i| / |\tilde{A}_2| \ll 1$).

The mass matrices of the lower four families were studied and evaluated in the ref. [13] under the assumption that if going beyond the tree level the differences in the mass matrices of different family members start to manifest. In this ref. we assumed the symmetry properties of the mass matrices from Table VII and fitted the matrix elements to the experimental data for the three observed families within the accuracy of the experimental data. We were not able to determine masses of the fourth family. Taking the fourth family masses as parameters we were able to calculate matrix elements of mass matrices, predicting mixing matrices for all the members of the four lowest families.

In Table VIII we present quantum numbers of all members of a family, any one, after the electroweak break. It is easy to show that the contribution of complex conjugate to $\psi_L^\dagger \gamma^0 (-) p_\mp \psi_R$ gives the same value.
\[
\begin{align*}
Y & \quad Y' \quad Q \quad Q' \\
\begin{array}{cccc}
u_R & \frac{1}{2} & \frac{1}{2} (1 - \frac{1}{3} \tan^2 \theta_1) & \frac{1}{2} & -\frac{1}{2} \tan^2 \theta_1 \\
d_R & -\frac{1}{2} & \frac{1}{2} (1 + \frac{1}{3} \tan^2 \theta_2) & \frac{1}{2} & \frac{1}{2} \tan^2 \theta_1 \\
v_L & 0 & \frac{1}{2} (1 + \tan^2 \theta_2) & 0 & 0 \\
e_L & -1 & \frac{1}{2} (-1 + \tan^2 \theta_2) & -1 & \tan^2 \theta_1 \\
u_L & -\frac{1}{2} & \frac{1}{2} \tan^2 \theta_2 & 0 & 0 \\
d_L & \frac{1}{2} & \frac{1}{2} \tan^2 \theta_2 & -1 & \frac{1}{2} (1 - \tan^2 \theta_1) \\
\end{array}
\end{align*}
\]

TABLE VIII: The quantum numbers \(Y, Y', Q, Q'\) of the members of a family.

\[
\begin{align*}
\Sigma = I/i & \quad \tilde{\tau}^{23} & \tilde{\tau}^{13} & \tilde{N}_R^3 & \tilde{N}_L^3 \\
\Sigma = II/i & \quad \tilde{\tau}^{23} & \tilde{N}_R^3 & \tilde{N}_L^3 \\
1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\
2 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\
3 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\
4 & 0 & 0 & \frac{1}{2} & \frac{1}{2}
\end{align*}
\]

TABLE IX: The quantum numbers \(\tilde{\tau}^{23}, \tilde{N}_R^3, \tilde{\tau}^{13}\) and \(\tilde{N}_L^3\) for the two groups of four families are presented.

Table IX presents the quantum numbers \(\tilde{\tau}^{23}, \tilde{N}_R^3, \tilde{\tau}^{13}\) and \(\tilde{N}_L^3\) for all eight families. The first four families are singlets with respect to \(\tilde{\tau}^{21}\) and \(\tilde{N}_R^1\), while they are doublets with respect to \(\tilde{\tau}^{11}\) and \(\tilde{N}_L^1\) (all before the break of symmetries). The upper four families are correspondingly doublets with respect to \(\tilde{\tau}^{21}\) and \(\tilde{N}_R^1\) and are singlets with respect to \(\tilde{\tau}^{11}\) and \(\tilde{N}_L^1\).

A. Mass matrices beyond the tree level

While the mass matrices of \((u, v)\) have on the tree level the same off diagonal elements and differ only in diagonal elements due to the contribution of \(eQA_s, gQ'QA'_s\) and \(gQ'YA'_{s'}\) and the same is true for \((d, e)\), loop corrections, to which massive gauge fields and dynamical scalar fields of both origins \((\tilde{\omega}_{abs} and \omega_{s'ts})\) contribute coherently, are expected to change mass matrices of the lower four families drastically. For the upper four families, for which the diagonal terms from \(eQQA_s', gQ'QA_s'\) and \(gQ'YA'_{s'}\) are almost negligible, since they are the same for all eight families, loop corrections are not expected.
to bring drastic changes in mass matrices between different family members. On the tree
level the mass matrices demonstrate twice four by diagonal matrices (this structure stays
unchanged also after taking into account loop corrections in all orders)

\[ M^\alpha_{(o)} = \begin{pmatrix} M^\alpha_{II(0)} & 0 \\ 0 & M^\alpha_{I(0)} \end{pmatrix}, \quad (29) \]

where \( M^\alpha_{II(0)} \) and \( M^\alpha_{I(0)} \) have the structure

\[ M_{(o)} = \begin{pmatrix} -a_1 & b & 0 & c \\ b & -a_2 & c & 0 \\ 0 & c & a_1 & b \\ c & 0 & b & a_2 \end{pmatrix}, \quad (30) \]

with the matrix elements \( a_1 \equiv a^\Sigma_{\pm 1}, \ a_2 \equiv a^\Sigma_{\pm 2}, \ b \equiv b^\Sigma_{\pm} \) and \( e \equiv e^\Sigma_{\pm} \). The values \( a_1, a_2, b \) and \( c \)
are different for the upper (\( \Sigma = II \)) and the lower (\( \Sigma = I \)) four families, due to two different
scales of two different breaks. One has

\[
\begin{align*}
  a_1 &= \frac{1}{2}(\tilde{a}_{\pm}^{(1,2)3} - \tilde{N}_{(R,L)}^{(R,L)3}) \\
  a_2 &= \frac{1}{2}(\tilde{a}_{\pm}^{(1,2)3} + \tilde{N}_{(R,L)}^{(R,L)3}) \\
  b &= \tilde{a}_{\pm}^{N_{(R,L)}+} = \tilde{a}_{\pm}^{N_{(R,L)}-} \\
  c &= \tilde{a}_{\pm}^{(1,2)+} = \tilde{a}_{\pm}^{(1,2)-}.
\end{align*}
\]

(31)

For the upper four families (\( \Sigma = II \)) we have correspondingly \( \tilde{a}_3^\pm = \tilde{a}_3^+ = \tilde{a}_3^- = \tilde{a}_3^{R3} \),
\( \tilde{a}_3^\pm = a_3^1 \pm i a_3^2, \ \tilde{a}_3^\pm = \tilde{a}_3^{R1} \pm i \tilde{a}_3^{R2} \) and for the lower four families (\( \Sigma = I \)) we must take
\( \tilde{a}_3^\pm = a_3^3, \ \tilde{a}_3^\pm = \tilde{a}_3^{L3} \), \( \tilde{a}_3^\pm = a_3^{11} \pm i a_3^{12}, \ \tilde{a}_3^\pm = \tilde{a}_3^{L1} \pm i \tilde{a}_3^{L2} \).

To the tree level contributions of the scalar \( \tilde{\omega}_{ab\pm} \) fields, diagonal matrices \( a_\pm \) have to
be added, the same for all the eight families and different for each of the family member
(\( u, d, \nu, e \)), \( (\tilde{a}_\pm \equiv a^\alpha_\pm) \) \( \psi \), which are the tree level contributions of the scalar \( \omega_{sts'} \) fields

\[ \dot{\hat{a}}_\pm = e \dot{\hat{Q}} A_\pm + g^1 \cos \theta_1 \dot{\hat{Q}}' Z^Q_\pm + g^2 \cos \theta_2 \dot{\hat{Y}}' Y^Y_\pm. \quad (32) \]

Since the upper and the lower four family mass matrices appear at two completely different
scales, determined by two orthogonal sets of scalar fields, the two tree level mass matrices
\( M^\alpha_{(o)}^{\Sigma} \) have very little in common, besides the symmetries and the contributions from
Eq. (32).

Let us introduce the notation, which would help to make clear the loop corrections contributions. We have before the two breaks two times (\( \Sigma \in \{ II, I \} \), \( II \) denoting the upper
four and \( I \) the lower four families) four massless vectors \( \psi_{\Sigma(L,R)}^\alpha \) for each member of a family \( \alpha \in \{u, d, \nu, e\} \). Let \( i, i' \in \{1, 2, 3, 4\} \) denote one of the four family members of each of the two groups of massless families

\[
\psi_{\Sigma(L,R)}^\alpha = (\psi_{\Sigma 1}^\alpha, \psi_{\Sigma 2}^\alpha, \psi_{\Sigma 3}^\alpha, \psi_{\Sigma 4}^\alpha)_{(L,R)} .
\]

Let \( \Psi_{\Sigma(L,R)}^\alpha \) be the final massive four vectors for each of the two groups of families, with all loop corrections included

\[
\psi_{\Sigma(L,R)}^\alpha = V_{\Sigma}^\alpha \Psi_{\Sigma(L,R)}^\alpha ;
\]

\[
V_{\Sigma}^\alpha = V_{\Sigma(o)}^\alpha V_{\Sigma(1)}^\alpha \cdots V_{\Sigma(k)}^\alpha .
\]

Then \( \Psi_{\Sigma(L,R)}^\alpha(k) \), which include up to \( (k) \) loops corrections, read

\[
V_{\Sigma(o)}^\alpha V_{\Sigma(1)}^\alpha \cdots V_{\Sigma(k)}^\alpha \Psi_{\Sigma(L,R)}^\alpha(k) = \psi_{\Sigma(L,R)}^\alpha .
\]

Correspondingly we have

\[
< \psi_{\Sigma}^\alpha | \gamma^0 (M_{(o)}^\Sigma + \cdots + M_{(1)}^\Sigma) | \psi_{\Sigma}^\alpha > =
\]

\[
< \Psi_{\Sigma(L,R)}^\alpha(k) | \gamma^0 (V_{\Sigma(o)}^\alpha V_{\Sigma(1)}^\alpha \cdots V_{\Sigma(k)}^\alpha)^\dagger (M_{(o)}^\Sigma + \cdots + M_{(1)}^\Sigma) + \cdots + M_{(1)}^\Sigma) V_{\Sigma(1)}^\alpha \cdots V_{\Sigma(k)}^\alpha | \Psi_{\Sigma}^\alpha > .
\]

Let us repeat that to the loop corrections two kinds of the scalar dynamical fields contribute, those originating in \( \tilde{\omega}_{abs} (\tilde{g}_l \tilde{Y}^l \tilde{Y}^j \tilde{A}_s^j, \tilde{A}^2_\pm \tilde{A}^2_\mp \tilde{A}^2_\pm, \tilde{g}^\tilde{N}_{L,R} \tilde{N}_{L,R} \tilde{A}^\tilde{N}_{L,R}, \tilde{g}^\tilde{A}^\tilde{A}^\prime \tilde{A}^\tilde{A}^\prime, \tilde{g}_l \tilde{A}^2_\pm \tilde{A}^2_\pm) \) and those originating in \( \omega_{abs} (e \tilde{Q} A_s, g^1 \cos \theta_1 \tilde{Q}_m \tilde{Z}_s^m, g^1 \cos \theta_2 \tilde{Y}^i \tilde{A}_s^m) \) the massive fields \( g^2 \cos \theta_2 \tilde{Y}^i \tilde{A}_s^m, g^1 \cos \theta_1 \tilde{Q}_m \tilde{Z}_s^m \) as it follow from Eq. (7).

In the ref. [15] the loop diagrams for these contributions to loop corrections are presented and numerical results discussed for both groups of four families. The masses and coupling constants of dynamical scalar fields and of the massive vector fields are taken as an input and the influence of loop corrections on properties of fermions studied.

Let us arrange mass matrices, after the electroweak break and when all the loop corrections are taken into account, as a sum of matrices as follows

\[
M_{\Sigma}^\alpha = \sum_{k=0,k'=0,k''=0}^{\infty} (Q^\alpha)^{k}(Q'^{\alpha})^{k'} (Y^\alpha)^{k''} M_{QQ'}^{\alpha\alpha} M_{kk'k''} .
\]
To each family member there corresponds its own matrix $M^{\alpha \Sigma}$. It is a hope, however, that the matrices $M^{\alpha \Sigma}_{\overline{Q}'Y'k'k''} = M^{\Sigma}_{\overline{Q}'Y'k'k''}$ might depend only slightly on the family member index $\alpha$. To masses of neutrinos only the terms $(Q^{0})^{\alpha} (Q'^{0})^{\alpha} (Y'^{0})^{k''} M^{\alpha \Sigma}_{\overline{Q}'Y'k'k''}$ contribute.

There is an additional term, however, which does not really speak for the suggestion of Eq. (37). Namely the term in loop corrections which transforms the right handed neutrinos into their left handed charged conjugated ones and which manifests accordingly the Majorana neutrinos. These contribution is presented in Eq. (38) of the subsection III A 1. It concerns only the lower group of four families and might contribute a lot, in addition to the "Dirac masses" of Eq. (37), to the extremely small masses of the observed families of neutrinos. This term needs, as also all the loop corrections to the tree level mass matrices for all the family members, additional studies.

More about the mass matrices below the tree level can be found in the ref. [15].

1. Majorana mass terms in the spin-charge-family theory

There are mass terms within the spin-charge-family theory, which transform the right handed neutrino to its charged conjugated one, contributing to the right handed neutrino Majorana masses

\[ \psi^{\dagger} \gamma^{0} \begin{pmatrix} 78 \\ - \end{pmatrix} p_{0-}, \]

\[ p_{0-} = - (\bar{\tau}^{1+} \bar{A}_{1+}^{1} + \bar{\tau}^{1-} \bar{A}_{1-}^{1}) \mathcal{O}^{[+]}, \]

\[ \mathcal{O}^{[+]} = \begin{pmatrix} 78 \\ 56 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \end{pmatrix}, \]

(38)

One easily checks, using the technique with the Clifford objects, that $\gamma^{0} \begin{pmatrix} 78 \\ - \end{pmatrix} p_{0-}$ transforms a right handed neutrino of one of the lower four families into the charged conjugated one, belonging to the same group of families. It does not contribute to masses of other leptons and quarks, right or left handed. Although the operator $\mathcal{O}^{[+]}$ appears in a quite complicated way, that is in the higher order corrections, yet it might be helpful when explaining the properties of neutrinos. The operator $- (\bar{\tau}^{1+} \bar{A}_{1+}^{1} + \bar{\tau}^{1-} \bar{A}_{1-}^{1}) \mathcal{O}^{[+]}, \mathcal{A}^{[+]}_{\mathcal{O}}$ gives zero, when it applies on the upper four families, since the upper four families are singlets with respect to $\bar{\tau}^{1\pm}$. 

32
This term needs further studies.

IV. SCALAR FIELDS OF THE SPIN-CHARGE-FAMILY THEORY MANIFESTING EFFECTIVELY AS THE STANDARD MODEL HIGGS

Before starting to interpret the standard model as an effective approach of the spin-charge-family theory let me remind the reader that effective interactions are commonly used in many body systems. The Heisenberg model, for example, uses the ”spin-spin” interactions for describing ferromagnetic properties of materials, replacing the complicated electromagnetic interactions among many particles involved.

The right handed neutrino is a regular family member in the spin-charge-family theory. In the standard model the right handed neutrinos were left out only because of historical reasons: neutrinos were assumed to be massless and the right handed neutrinos have in the standard model all the charges equal to zero. Accordingly there was no need to postulate the existence of an additional gauge field to which the right handed neutrinos, together with all the quarks and leptons, would couple.

Let us point out the starting assumptions of the standard model from the point of view of symmetries. The infinitesimal generators of the symmetry groups assumed by the standard model

\[ [SO(1,3) \times SU(3) \times SU(2) \times U(1)]_{(v,s)} \]  

(39)

determine singlet (scalar), spinor (fundamental), vector and \( \cdots \) (staying for infinite many) representations, all corresponding to the same algebra of the infinitesimal generators of the group \( SO(1,3); \) singlet, spinor, vector (adjoint), \( \cdots \) representations determined by the algebra of the \( SU(3), \) of \( SU(2) \) and of \( U(1) \) infinitesimal generators. Index \( (v,s) \) is meant to point out that spinors are coupled to the vector fields according to the action of the standard model: Massless spinors with the charges in the fundamental representations of the groups couple to massless vectors with the charges in the adjoint representations of the same groups as described by the Lagrange density presented in the first line of Eq. (5) – if the assumptions are made that only the left handed spinors carry the weak charge while the right handed ones are weakless and that the appropriate values for the family members with respect to the \( U(1) \) charge group are chosen. While in the spin-charge-family theory it
is straightforward to see that the $U(1)_{II,1}$ charges representations, as well as colour singlets, are of the spinor kind, since they all originate in the starting spinor multiplet, in the standard model the spinor representations of all the charges can only be assumed.

After assuming that triplets and singlets of the $SU(3)$ charge, doublets and singlets of the $SU(2)$ charge and singlets of $U(1)$ charge are of the spinor origin, then it seems natural from the point of view of representations that all of them constitute members of one family. The standard model makes in addition the assumption about the relation between the spinor representation of $SO(1,3)$ and the representations of the charges. (It turns out that these choices lead to also other useful properties.) That each vector carries only the charge through which it couples to fermions seems an elegant (the simplest one) assumption. There are families of spinors assumed in addition, all massless, all equal with respect to the groups (Eq. (39)).

To make spinors and weak bosons massive the standard model assumes the Higgs (the scalar representation of $SO(1,3)$) with the spinor charges with respect to the rest of the groups of Eq. (39) and with the values of $U(1)$ group which ensure with the Higgs ”dressed” right handed members of any family to have the charges of the left handed partners: $u$ and $\nu$ are ”dressed” with anti-Higgs, $d$ and $e$ with Higgs. The action added to the massless ones (for the massless spinors and vectors) takes care of the interaction of the vector fields with this scalar field and of the scalar potential. Mass matrices for each of the family members are ”put by hand” (just assumed) in addition.

To include into Eq. (39) the scalar field and to point out all the relations – interactions – assumed by the standard model we could correspondingly rewrite Eq. (39) as follows

\[
\{[SO(1,3) \times SU(3) \times SU(2) \times U(1) \times SU(3)_{fa}]_{(v,s)}^* \times [SO(1,3) \times SU(3) \times SU(2) \times U(1)]_{(v,sc)}^{**} \}^{***},
\]

(40)

with the indices in Eq. (40) which tell us the limitation of the choice: i. The index $(v, s)^*$ tells that spinor charges are in relation with the handedness and that no vector field is assumed for the family charge $SU(3)_{fa}$. ii. The index $(v, sc)^{**}$ tells us that for the scalar field a very particular representations of the charge groups is allowed (Higgs is a colour singlet, weak doublet and of particular $U(1)$ charge). iii. The index $^{***}$ tells us that the standard model is offering no explanation for the appearance of the families and the Yukawa couplings.
The *spin-charge-family* theory starting symmetry group is

\[ \left[ SO(1,13)_\gamma \times S0(1,13)\tilde{\gamma} \right]_{v,s} \]  

(41)

with the action (Eq. (41)) which couples vielbeins and spin connection fields with spinors. Since there are two kinds of spins \((S^{ab} \text{ and } \tilde{S}^{ab})\) there are also two kinds of the spin connection fields. To observe one kind of spin as the spin and all the charges of fermions, the break of the starting symmetry must occur. Accordingly it is meaningful to replace Eq. (41) with

\[ \left[ SO(1,13)_\gamma \times S0(1,13)\tilde{\gamma} \right]_{(v,s)} \]  

(42)

where the index \((v,s)\) points out that several particular breaks of the starting symmetry (phase transitions) happen. At the stage when the starting symmetry has broken to the symmetry \((SO(1,3)_\gamma \times SO(1,3)\tilde{\gamma} \times SU(2)_\gamma \times SU(2)\tilde{\gamma} \times U(1)_\gamma \times U(1)\tilde{\gamma} \times SU(3)_\gamma)\), that is before the electroweak break, there are four massless families with the members: coloured quarks and colourless leptons, the left handed members all weak charged and the right handed members all weak chargeless, of a very determined \(U(1)\) charge (just the ones assumed by the *standard model*), all in the spinor representations, since all the members of one family follow from the starting left handed massless spinor representation. All the families are equivalent. No assumption, except the one that each phase transition is connected with a particular break of a symmetry, need to be made. The way of breaking the starting symmetry determines also that the number of massless families is before the electroweak break equal to four, rather than to the observed three.

The mass term appears after the break by itself. There are several scalar fields, all with the charges in the adjoint representations, which determine after the phase transition triggered by their nonzero vacuum expectation values the properties of families of quarks and leptons.

Mass matrices of fermions of the lower four families are in the *spin-charge-family* theory, according to Eq.(7), on the tree level determined by the scalar fields through the operator

\[ \hat{\Phi}_+^I = \frac{78}{(+)} \left\{ \frac{g}{2} \tilde{N}_L \tilde{N}_L \tilde{A}_+ + \frac{g}{2} \frac{1}{2} \tilde{A}_+ + e Q A_+^Q + g Q' Q' Z_+^{Q'Y'} + g Y' Y' A_+^{Y'} \right\} . \]  

(43)

The operator \((\frac{78}{+})\), appearing in Eq. (43) on the left hand side, transforms all the quantum numbers of the right handed quarks and leptons to those of the left handed ones, except the handedness, for the transformation of which in the *standard model* as well as in the
spin-charge-family theory $\gamma^0$ takes care. One can formally replace the operator $(\mp)$ with the operator $\sum_{\alpha,i} |\psi_{Lj}^\alpha > < \psi_{Ri}^\alpha|,$

$$ (\mp) \Rightarrow \sum_{\alpha,i} |\psi_{Lj}^\alpha > < \psi_{Ri}^\alpha |, \tag{44} $$

$\alpha \in \{u, d, \nu, e\}$ and $i \in \{1, 2, 3, 4\}$. Both operators of Eq. (44) do the same: Transform the right handed member of any family with the left handed one of the same family, doing what the Higgs does (up to its vacuum expectation value) when "dressing" right handed quarks and leptons. The great difference among these three operators is that the operator $\gamma^0 (\mp)$ follows from the simple starting action, while the Higgs and the operator $\sum_{\alpha,i} |\psi_{Lj}^\alpha > < \psi_{Ri}^\alpha|,$ are put by hand.

The product of the operators $\gamma^0$ and $(\mp)$ transforms the right handed quarks and leptons into the left handed ones, as explained in section III and can be read in Tables III and IV. The part of the operator $\hat{\Phi}^I$, that is $\{ \tilde{g}^{N_L} \tilde{N}_L \tilde{A}_+^{N_L} + \tilde{g}_1 \tilde{\tau}_1 \tilde{A}_+^{L_1} + e Q A^Q_+ + g^Q Q' Z^Q_+ + g^Y Y' A^Y_+ \}$, takes care of the mass matrices of quarks and leptons. The application of $\{ \tilde{g}^{N_R} \tilde{N}_R \tilde{A}_+^{N_R} + \tilde{g}_2 \tilde{\tau}_2 \tilde{A}_+^{L_2} \}$ on the lower four families is zero, since the lower four families are singlets with respect to $\tilde{N}_R$ and $\tilde{\tau}_2$. In the loop corrections besides the massive scalar fields $- \tilde{N}_L, \tilde{N}_R, A^Q_+, Z^Q_+ \text{ and } A^Y_+$ - also the massive gauge vector fields - $Z^Q_m, A^\pm_m = W^\pm_m, A^Y_m$ and $A^2^{\pm}_m$ - start to contribute coherently.

We do not (yet) know the properties of the scalar fields, their vacuum expectation values, masses and coupling constants. We may expect that they behave similarly as the Higgs field in the standard model, that is that their dynamics is determined by potentials which make contributions of the scalar fields renormalizable. Starting from the spin connections and vielbeins we only can hope that at least effectively at the low energy regime, that is in the weak field regime, the effective theory is behaving as a renormalizable one.

Yet we can estimate masses of gauge bosons under the assumption that the scalar fields, which determine mass matrices of fermion family members, are determining effectively also masses of gauge fields, like this is assumed in the standard model. If breaking of symmetries occurs in both sectors in a correlated way (I have assumed so far that this is the case) then symmetries of the vielbeins $f^a_\alpha$ and the two spin connection fields, $\omega_{abc} = f^a_\alpha \omega_{aba}$ and $\tilde{\omega}_{abc} = f^a_c \tilde{\omega}_{aba}$, change simultaneously.

To see the standard model as an effective theory of the spin-charge-family theory let us
assume the existence of the scalar fields, which by "dressing" right handed family members, ensure them the weak and the hyper charge of their left handed partners. Therefore, we shall replace the part \((\frac{-78}{2})\) of the operator \(\hat{\Phi}^I_+\) in Eq. (43), which transforms the weak chargeless right handed \(u_R\) quark of a particular hyper charge \(Y\left(\frac{2}{3}\right)\) of any family into the weak charged \(u_R\) quark with \(Y\) of the left handed \(u_L\left(\frac{1}{6}\right)\), while \(\gamma^0\) changes its right handedness into the left one, with the scalar field of Table [X] which has the appropriate weak and hyper charge. Although all the scalar fields, as all the gauge fields, of the spin-charge-family theory are bosons manifesting their properties – the charges of all origins – in the adjoint representations, we replace, in order to mimic the standard model, the part \((\frac{-78}{2})\) by the scalar field with the charges originating in \(S^{ab}\) in the fundamental representation. This scalar field must be a colourless weak doublet with the hyper charges \(Y = -\frac{1}{2}\) for \((u_R\) and \(\nu_R\) and \(Y = \frac{1}{2}\) for \((d_R\) and \(e_R\), while only the components with the electromagnetic charge \(Q = (\tau^{13} + Y)\) equal to zero are allowed to have nonzero vacuum expectation values. This scalar field must be a dynamical field, with a nonzero vacuum expectation value, massive, and governed by a hopefully renormalizable potential. This means that averaging over all the scalar fields appearing in Eq. (43) manifests as the assumed scalar field and the Yukawa couplings of the standard model.

We can simulate the part \((\frac{-78}{2})\) with the scalar field \(\Phi^I_L\), presented in Table [X] which "dresses" \(u_R\) and \(\nu_R\) in the way assumed by the standard model, and we simulate the part \((\frac{+78}{2})\) with the scalar field \(\Phi^I_+\) from Table [X] which "dresses" \(d_R\) and \(e_R\).

In the spin-charge-family theory mass matrices are determined on the tree level by
\[
\{\tilde{g}^{N}\tilde{N}_L \tilde{A}_L^i \bar{A}^{\dagger}_L^i + \tilde{g}^1 \tau^{1i} \bar{A}^{\dagger}_L^i + e Q \bar{A}_L^Q + g' Q' Z'_+ + g'' Y' A'_Y\} \text{ (Eq. (43))}
\]
Let this operator be called \(\hat{\Phi}_+^{\dagger}\)

\[
\hat{\Phi}_+^{\dagger} = \{\tilde{g}^{N}\tilde{N}_L \tilde{A}_L^i \bar{A}^{\dagger}_L^i + \tilde{g}^1 \tau^{1i} \bar{A}^{\dagger}_L^i + e Q \bar{A}_L^Q + g' Q' Z'_+ + g'' Y' A'_Y\}.
\] (45)

In the attempt to see the standard model as an effective theory of the spin-charge-family theory the standard model Higgs together with the Yukawa couplings can be presented as the product of \(\Phi^I_+\) and \(\hat{\Phi}^{\dagger}_+\). The role of \(\Phi^I_+\) in this product is to "dress" the right handed quarks and leptons with the weak charge and the appropriate hyper charge, while \(\hat{\Phi}^{\dagger}_+\) effectively manifests on the tree level as the Higgs and the Yukawa couplings together.

Masses of the vector gauge fields as well as the properties of the scalar fields should in the
TABLE X: One possible choice of the weak and hyper charge components of the scalar fields carrying the quantum numbers of the standard model Higgs, presented in the technique \cite{10, 18}, chosen to play the role of the standard model Higgs. Both states on the table are colour singlets, with the weak and hyper charge, which if used in the standard model way ”dress” the right handed quarks and leptons so that they carry quantum numbers of the left handed partners. The state $(\Phi^I u_R)$, for example, carries the weak and the hyper charge of $u_L$. ”Dressing” the right handed family members with the $\Phi^I$ manifests effectively as the application of the operators \cite{78} (Eq. (43)) on the right handed family members.

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
& $\Phi^I$ & $\tau^1$ & $\tau^{23}$ & $\tau^4$ & $Y$ & $Q$ & colour charge \\
\hline
$\Phi^I_1$ & $[+][-]$ & $9\ 10\ 11\ 12\ 13\ 14$ & $\frac{1}{2}$ & 0 & $-\frac{1}{2}$ & 0 & colourless \\
\hline
$\Phi^I_2$ & $[-][+][+]$ & $9\ 10\ 11\ 12\ 13\ 14$ & $-\frac{1}{2}$ & 0 & $\frac{1}{2}$ & 0 & colourless \\
\hline
\end{tabular}
\end{table}

spin-charge-family theory be determined by studying the break of symmetries. We discuss in subsect. II B the break, but the detailed calculations are very demanding and we have not (yet) been able to perform them.

One can extract some information about properties of the scalar fields in Eq. (45) from the masses of the so far observed quarks and leptons and the weak boson masses. From the covariant momentum after the electroweak break

$$p_0 m = p_m - \frac{g_1^2}{\sqrt{2}} \left[ \tau^{1+} A^{1+}_m + \tau^{1-} A^{1-}_m \right] + g^1 \sin \theta_1 Q A^Q_m + g^1 \cos \theta_1 Q' A^{Q'}_m,$$

with $\theta_1$ equal to $\theta_W$, with the electromagnetic coupling constant $e = \sin \theta_W$, the charge operators $Q = \tau^{13} + Y$, $Q' = \tau^{13} - \tan^2 \theta_W Y$, and with the gauge fields $W^\pm_m = A^{1\pm}_m = \frac{1}{\sqrt{2}} (A^{11}_m \mp A^{12}_m)$, $A_m = A^Q_m = A^{13}_m \sin \theta_W + A^Y_m \cos \theta_W$ and $Z_m = A^{Q'}_m = A^{13}_m \cos \theta_W - A^Y_m \sin \theta_W$, we estimate

$$\langle p_0 m | \Phi^{I\dagger} \rangle \langle \Phi^{I\dagger} \rangle = \left\{ \frac{(g_1^1)^2}{2} A^{1+}_m A^{1-}_m + \left( \frac{g^1}{2 \cos \theta_1} \right)^2 A^Q_m A^{Q'}_m \right\} Tr(\Phi^{I\dagger} \Phi^{I\dagger}).$$

$\Phi^{I\dagger}$ are determined in Eq. (45), while the states $\Phi^{I\dagger}$ from Table X are normalized to unity as explained in the refs. \cite{18} and in the appendix. Assuming, like in the standard model, that $Tr(\Phi^{I\dagger} \Phi^{I\dagger}) = \frac{v^2}{2}$, we extract from the masses of gauge bosons one information about the vacuum expectation values of the scalar fields, their coupling constants and their masses. Mass matrices of quarks and leptons offer additional information about the scalar fields of the standard model.
spin-charge-family theory. Measuring charged and neutral currents, decay rates of hadrons, the scalar fields productions in the fermion scattering events and their decay properties provides us with additional information.

Studying neutral and charged currents and possible scalar field productions and decays are important next step to be done.

Let me conclude this section by the observation that the colourless scalar with the weak charges in the fundamental representation of the $SU(2)$ group is a strange object from the point of view of the fact that all the known fields are either fermions in the fundamental representations with respect to the charge groups or they are (vector) bosons in the adjoint representations with respect to the charge groups. In the spin-charge-family theory the scalar fields carry all the charges (with the family quantum numbers included) in the adjoint representations. It is challenging to prove or disprove whether or not the standard model can be interpreted as an effective low energy manifestation of the spin-charge-family theory. And that several scalar fields of the spin-charge-family theory with all the charges in the adjoint representations of the corresponding groups effectively manifest as the Higgs with the charges in the fundamental representations and the Yukawa couplings.

V. MODELS WITH THE $SU(3)$ FLAVOUR GROUPS AND THE SPIN-CHARGE-FAMILY THEORY

In section IV we look at the standard model assumptions from the point of view of the spin-charge-family theory. There are many attempts in the literature to connect families of quarks and leptons with the fundamental representations of the $SU(3)$ gauge group. Let me comment a quite simplified version of the assumptions presented in the refs. [23, 25] from the point of view of the spin-charge-family theory.

Let us therefore assume that the three so far observed families of quarks and leptons, neutrinos will be treated as ordinary family members, if all massless, manifest the "flavour" symmetry

\[
\begin{align*}
\{&[SO(1,3) \times SU(3) \times SU(2) \times U(1) \times SU(3)_{fa}(v,s)\} \\
&\times \{SO(1,3) \times SU(3) \times SU(2) \times U(1)_{(v,sc)}\}\} \\
&\{\times [SO(1,3) \times SU(3) \times SU(2) \times U(1)] \times SU3_{fasc}\}\},
\end{align*}
\]

(48)
with the indices in Eq. (48) which tell us the limitations of the choice: i. The index \((v, s)^*\) tells that spinor charges are in relation with the handedness, that no vector gauge field is assumed for the family charge \(SU(3)_{fa}\), and that family (flavour) groups are different for different family members; The left handed quarks have different \(SU(3)\) family charge than the right handed ones and the right handed \(u\) and the right handed \(d\) have each their own \(SU(3)\) family charge. Similarly the left handed leptons have their own \(SU(3)\) family charge which differs from the \(SU(3)\) family charges of the right handed \(\nu\) and the right handed \(e\), and the right handed \(SU(3)\) family charge of \(\nu\) is different than the \(SU(3)\) family charge of \(e\). ii. The index \((v, sc)^\ast\) tells us, as before, that for the scalar field a very particular choice of representations of the charge groups is allowed (Higgs is a colour singlet, weak doublet and of particular \(U(1)\) charge). iii. The index \(\cdot\) tells us that there are scalar fields which are singlets with respect to the groups \([SO(1, 3) \times SU(3) \times SU(2) \times U(1)]\) (colourless, weakless, with zero hyper charge scalars) which accordingly do not couple to the gauge vector fields of the charges \(SU(3), SU(2), U(1)\). They carry family charges in the fundamental, anti-fundamental or singlet representations of the groups, depending to which family members do they couple. To \(u\) quarks the scalar which is a triplet with respect to the family \(SU(3)\) charge of \(u_L\) and \(d_L\), anti-triplet with respect to the family \(SU(3)\) charge of \(u_R\) and chargeless with respect to the family \(SU(3)\) charge of the right handed \(d_R\) quarks. Equivalently there is the scalar which is again a triplet with respect to the family \(SU(3)\) charge of \(u_L\) and \(d_L\) anti-triplet with respect to the family \(SU(3)\) charge of \(d_R\) and \(SU(3)\) family chargeless with respect to the family \(SU(3)\) charge of the right handed \(u_R\) quarks. These scalars have no couplings to the leptons. In the lepton sector goes equivalently.

It is assumed in addition that Yukawa scalar fields are, like the Higgs, the dynamical fields, which by gaining nonzero vacuum expectation values, break the family (flavour) symmetry.

In the spin-charge-family theory the families appear as representations of the group with the infinitesimal generators \(\tilde{S}^{ab}\), forming the equivalent representations with respect to the group with the generators \(S^{ab}\). These latter generators determine spin of fermions and their charges. The fields gauging the group \(S^{ab}\) determine after the break of symmetries in the low energy regime all the known gauge fields, which are vectors in \((1 + 3)\) with the charges in the adjoint representations.

The scalar fields which are gauge fields of the \(\tilde{\tau}^i = \tilde{c}^{1i}_{ab} \tilde{S}^{ab}\) and \(\tilde{\bar{\bar{N}}}^i_L = \tilde{\bar{\bar{c}}}^{\bar{\bar{N}}}_{iab} \tilde{S}^{ab}\) in the adjoint representations of the two \(SU(2)\) groups determine together with the singlet
scalar fields which are the gauge fields of $Q, Q'$ and $Y'$ (all three are expressible with $S^{ab}$) after the electroweak break the mass matrices of four families of quarks and leptons, and correspondingly the Yukawa couplings, masses and mixing matrices of the (lowest) four families of quarks and leptons. The scalar fields, with their nonzero vacuum expectation values, determine mass matrices of quarks and leptons on the tree level and contribute to masses of weak bosons. Below the tree level the dynamical scalar fields of both origins and the massive gauge fields bring coherent contributions to the tree level mass matrices.

While on the tree level the off diagonal matrix elements of the $u$-quark mass matrix are equal to the off diagonal matrix elements of the $\nu$-lepton mass matrix, and the off diagonal matrix elements of the $d$-quark mass matrix equal to the off diagonal matrix elements of the $e$-lepton mass matrix, the loop corrections change this picture drastically, hopefully reproducing the experimentally observed properties of fermions.

In the spin-charge-family theory there is no scalar field in the fundamental representation of the weak charge, the "duty" of which is in the standard model to take care of the weak and the hyper charge of the right handed family members.

All the fermion charges, with the family quantum number included, are described by the fundamental representations of the corresponding groups, and all the bosonic fields, either vectors or scalars, have their charges in the adjoint representations.

The refs. [23–25] try to explain the appearance of mass matrices of the standard model, which manifest in the Higgs fields and the Yukawa matrices, by taking the Yukawas as dynamical fields. Yukawa scalar fields with their bi-fundamental representations of the $SU(3)$ flavour group are an attempt to continue with the assumption of the standard model that there exist scalar fields with charges in the fundamental representations. To do the job Yukawa scalar field are assumed to be in the bi-fundamental representations. It seems a very nontrivial task to make use of the analyses of the experimental data presented as a general extension of the standard model in the refs. [24, 25] for the spin-charge-family theory as it manifests in the low energy region.

VI. CONCLUSIONS

The spin-charge-family theory [10–14, 19] is offering the way beyond the standard model by proposing the mechanism for generating families of quarks and leptons and consequently
predicting the number of families at low (sooner or later) observable energies and the mass matrices for each of the family member (and correspondingly the masses and the mixing matrices of families of quarks and leptons).

The spin-charge-family theory predicts the fourth family to be possibly measured at the LHC or at some higher energies and the fifth family which is, since it is decoupled in the mixing matrices from the lower four families and it is correspondingly stable, the candidate to form the dark matter [14].

The proposed theory also predicts that there are several scalar fields, taking care of mass matrices of the two times four families and of the masses of weak gauge bosons. At low energies these scalar dynamical fields manifest effectively pretty much as the standard model Higgs field together with Yukawa couplings, predicting at the same time that observation of these scalar fields is expected to deviate from what for the Higgs the standard model predicts.

To the mass matrices of fermions two kinds of scalar fields contribute, the one interacting with fermions through the Dirac spin and the one interacting with fermions through the second kind of the Clifford operators (anticommuting with the Dirac ones, there is no third kind of the Clifford algebra object). The first one distinguishes among the family members, the second one among the families. Beyond the tree level these two kinds of scalar fields and the vector massive fields start to contribute coherently, leading hopefully to the measured properties of the so far observed three families of fermions and to the observed weak gauge fields.

In the ref. [11, 13] we made a rough estimation of properties of quarks and leptons of the lower four families as predicted by the spin-charge-family theory. The mass matrices of quarks and leptons turns out to be strongly related on the tree level. Assuming that loop corrections change elements of mass matrices considerably, but keep the symmetry of mass matrices, we took mass matrix element of the lower four families as free parameters. We fitted the matrix elements to the existing experimental data for the observed three families within the experimental accuracy and for a chosen mass of each of the fourth family member. We predicted then elements of the mixing matrices for the fourth family members as well as the weakly measured matrix elements of the three observed families.

In the ref. [14] we evaluated the masses of the stable fifth family (belonging to the upper four families) under the assumption that neutrons and neutrinos of this stable fifth family
form the dark matter. We study the properties of the fifth family neutrons, their freezing out of the cosmic plasma during the evolution of the universe, as well as their interaction among themselves and with the ordinary matter in the direct experiments.

In this paper we study properties of the gauge vector and scalar fields and their influence on the properties of eight families of quarks and leptons as they follow from the spin-charge-family theory on the tree and below the tree level after the two successive breaks, from \( SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3) \) to \( SO(1,3) \times SU(2)_I \times U(1)_I \times SU(3) \) and further to \( SO(1,3) \times U(1) \times SU(3) \), trying to understand better what happens during these two breaks and after them.

We made assumptions about succesive breaks of the starting symmetry since we are not (yet) able to evaluate how do the breaks occur and what does trigger them.

In the break from \( SO(1,3) \times SU(2)_I \times SU(2)_{II} \times U(1)_{II} \times SU(3) \) to \( SO(1,3) \times SU(2)_I \times U(1)_I \times SU(3) \) several scalar fields (the superposition of \( f^s\tilde{\omega}_{ab\sigma} \), \( s = (7,8) \)) contribute to the break as gauge triplet fields of \( \vec{\tilde{N}}_R \) and of \( \vec{\tau}^2 \), gaining nonzero vacuum expectation values. Correspondingly they cause nonzero mass matrices of the upper four families to which they couple and nonzero masses of vector fields, the superposition of the gauge triplet fields of \( \vec{\tau}^2 \) and of the gauge singlet field of \( \tau^4 \). Since these scalar fields do not couple to the lower four families (they are singlets with respect to \( \vec{\tilde{N}}_R \) and \( \vec{\tau}^2 \)) the lower four families stay massless at this break.

At the successive break, that is at the electroweak break, several other combinations of \( f^s\tilde{\omega}_{ab\sigma} \), the gauge triplets of \( \vec{N}_L \) and of \( \vec{\tau}^1 \) (which are orthogonal to previous triplets), together with some combinations of scalar fields \( f^s\omega_{\sigma\tau} \), the gauge fields of \( Q, Q' \) and \( Y' \), gain nonzero vacuum expectation values, contributing correspondingly to mass matrices of the lower four families and to masses of the gauge fields \( W^\pm_m \) and \( Z_m \), influencing slightly, together with the massive vector gauge fields, also mass matrices of the upper four families.

Although mass matrices of the family members are in each of the two groups of four families very much related on the tree level (\( u \)-quarks are related to \( \nu \)-leptons and \( d \)-quarks to \( e \)-leptons), the loop corrections, in which the scalar fields of both kinds contribute, those distinguishing among the families and those distinguishing among the family members (\( u, d, \nu, e \)), together with the massive vector gauge fields which distinguish only among family members, start to hopefully (as so far done calculations [15] manifest) explain why are properties of the so far observed quarks and leptons so different. Numerical evaluations
of the loop corrections to the tree level are in preparation (the ref. [15]).

It might be, however, that the influence of a very special term in higher loop corrections, which influences only the neutrinos, since it transforms the right handed neutrinos into the left handed charged conjugated ones, is very strong and might be responsible for the properties of neutrinos of the lower three families.

To simulate the standard model the effective low energy model of the spin-charge-family theory is made in which the operator, which in the spin-charge-family theory transforms the weak and hyper charges of right handed quarks and leptons into those of their left handed partners, is replaced by a weak doublet scalar, colour singlet and of an appropriate hyper charge, while the scalar dynamical fields of the spin-charge-family theory determine the Yukawa couplings. This weak doublet scalar ”dresses” the right handed family members with the appropriate weak charge and hyper charge behaving as the Higgs of the standard model. It is further tried to understand to which extent can the scalar fields originating in $\tilde{\omega}_{abs}$ and $\omega_{abs}$ spin connection dynamical fields (all in the adjoint representations with respect to all the gauge groups) be replaced by a kind of a ”bi-fundamental” (with respect to their several family groups) Yukawa scalar dynamical fields of the models presented in the refs. [24, 25], in which fermion families are assumed to be members of several $SU(3)$ family (flavour) $SU(3)$ groups. It seems so far that it is hard to learn something from such, from the point of view of the spin-charge-family theory, very complicated models extending further the standard model assumption that the scalar (the Higgs) has charges in the fundamental representations. The so far very successful Higgs is in the spin-charge-family theory seen as an effective object, which can not very easily be extended to Yukawas.

Let me repeat that the spin-charge-family theory does not support the existence of the supersymmetric partners of the so far observed fermions and gauge bosons (assuming that there exist fermions with the charges in the adjoint representations and bosons with the charges in the fundamental representations). The supersymmetry does not show up at least up to the unification scale of all the charges.

Let me add that if the spin-charge-family theory offers the right explanation for the families of fermions and their quantum numbers as well as for the gauge and scalar dynamical fields, then the scalar dynamical fields represent new forces, as do already – in a hidden way – the Yukawas of the standard model.

Let me point out at the end that the spin-charge-family theory, offering explanation for
the appearance of spin, charges and families of fermions, and for the appearance of gauge vector and scalar boson fields at low energy regime, still needs careful studies, numerical ones and also proofs, to demonstrate that/whether this is the right next step beyond the standard model.

Appendix: Short presentation of technique [10, 18]

I make in this appendix a short review of the technique [18], initiated and developed by me when proposing the spin-charge-family theory [10–14, 19] assuming that all the internal degrees of freedom of spinors, with family quantum number included, are describable in the space of $d$-anticommuting (Grassmann) coordinates [18], if the dimension of ordinary space is also $d$. There are two kinds of operators in the Grassmann space, fulfilling the Clifford algebra which anticommute with one another. The technique was further developed in the present shape together with H.B. Nielsen [18] by identifying one kind of the Clifford objects with $\gamma^a$’s and another kind with $\tilde{\gamma}^a$’s. In this last stage we constructed a spinor basis as products of nilpotents and projections formed as odd and even objects of $\gamma^a$’s, respectively, and chosen to be orientates of a Cartan subalgebra of the Lorentz groups defined by $\gamma^a$’s and $\tilde{\gamma}^a$’s. The technique can be used to construct a spinor basis for any dimension $d$ and any signature in an easy and transparent way. Equipped with the graphic presentation of basic states, the technique offers an elegant way to see all the quantum numbers of states with respect to the two Lorentz groups, as well as transformation properties of the states under any Clifford algebra object.

The objects $\gamma^a$ and $\tilde{\gamma}^a$ have properties [2],

$$\{ \gamma^a, \gamma^b \}_+ = 2\eta^{ab}, \quad \{ \tilde{\gamma}^a, \tilde{\gamma}^b \}_+ = 2\eta^{ab}, \quad \{ \gamma^a, \tilde{\gamma}^b \}_+ = 0, \quad (A.1)$$

for any $d$, even or odd. $I$ is the unit element in the Clifford algebra.

The Clifford algebra objects $S^{ab}$ and $\tilde{S}^{ab}$ close the algebra of the Lorentz group

$$S^{ab} := (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a),$$

$$\tilde{S}^{ab} := (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a),$$

$$\{ S^{ab}, \tilde{S}^{cd} \}_- = 0,$$

$$\{ S^{ab}, S^{cd} \}_- = i(\eta^{ad}S^{bc} + \eta^{bd}S^{ac} - \eta^{ac}S^{bd} - \eta^{bd}S^{ac}),$$

$$\{ \tilde{S}^{ab}, \tilde{S}^{cd} \}_- = i(\eta^{ad}\tilde{S}^{bc} + \eta^{bd}\tilde{S}^{ac} - \eta^{ac}\tilde{S}^{bd} - \eta^{bd}\tilde{S}^{ac}), \quad (A.2)$$
We assume the “Hermiticity” property for $\gamma^a$’s and $\tilde{\gamma}^a$’s
\[ \gamma^a \dagger = \eta^{aa} \gamma^a, \quad \tilde{\gamma}^a \dagger = \eta^{aa} \tilde{\gamma}^a, \quad (A.3) \]
in order that $\gamma^a$ and $\tilde{\gamma}^a$ are compatible with (A.1) and formally unitary, i.e. $\gamma^a \dagger \gamma^a = I$ and $\tilde{\gamma}^a \dagger \tilde{\gamma}^a = I$.

One finds from Eq.(A.3) that $\left( S_{ab} \right)^\dagger = \eta^{aa} \eta^{bb} S_{ab}$.

Recognizing from Eq.(A.2) that two Clifford algebra objects $S_{ab}, S_{cd}$ with all indices different commute, and equivalently for $\tilde{S}_{ab}, \tilde{S}_{cd}$, we select the Cartan subalgebra of the algebra of the two groups, which form equivalent representations with respect to one another
\[ S^{03}, S^{12}, S^{56}, \ldots, S^{d-1}d, \quad \text{if} \quad d = 2n \geq 4, \]
\[ S^{03}, S^{12}, \ldots, S^{d-2}d-1, \quad \text{if} \quad d = (2n + 1) > 4, \]
\[ \tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \ldots, \tilde{S}^{d-1}d, \quad \text{if} \quad d = 2n \geq 4, \]
\[ \tilde{S}^{03}, \tilde{S}^{12}, \ldots, \tilde{S}^{d-2}d-1, \quad \text{if} \quad d = (2n + 1) > 4. \quad (A.4) \]

The choice for the Cartan subalgebra in $d < 4$ is straightforward. It is useful to define one of the Casimirs of the Lorentz group - the handedness $\Gamma \left( \{ \Gamma, S_{ab} \}_- = 0 \right)$ in any $d$
\[ \Gamma^{(d)} : = (i)^{d/2} \prod_a \left( \sqrt{\eta^{aa}} \gamma^a \right), \quad \text{if} \quad d = 2n, \]
\[ \Gamma^{(d)} : = (i)^{(d-1)/2} \prod_a \left( \sqrt{\eta^{aa}} \gamma^a \right), \quad \text{if} \quad d = 2n + 1. \quad (A.5) \]

One can proceed equivalently for $\tilde{\gamma}^a$’s. We understand the product of $\gamma^a$’s in the ascending order with respect to the index $a$: $\gamma^0 \gamma^1 \cdots \gamma^d$. It follows from Eq.(A.3) for any choice of the signature $\eta^{aa}$ that $\Gamma^\dagger = \Gamma$, $\Gamma^2 = I$. We also find that for $d$ even the handedness anticommutes with the Clifford algebra objects $\gamma^a$ ($\{ \gamma^a, \Gamma \}_+ = 0$), while for $d$ odd it commutes with $\gamma^a$ ($\{ \gamma^a, \Gamma \}_- = 0$).

To make the technique simple we introduce the graphic presentation as follows (Eq. (9))
\[ ab \left( k \right) : = \frac{1}{2}(\gamma^a + \eta^{aa} \gamma^b), \quad ab \left[ k \right] : = \frac{1}{2}(1 + i \frac{k}{k} \gamma^a \gamma^b), \]
\[ \dagger : = \frac{1}{2}(1 + \Gamma), \quad \bullet : = \frac{1}{2}(1 - \Gamma), \quad (A.6) \]
where $k^2 = \eta^{aa} \eta^{bb}$. One can easily check by taking into account the Clifford algebra relation (Eq[A.1]) and the definition of $S_{ab}$ and $\tilde{S}_{ab}$ (Eq[A.2]) that if one multiplies from the left hand
side by $S^{ab}$ or $\tilde{S}^{ab}$ the Clifford algebra objects $(k)$ and $[k]$, it follows that

$$
S^{ab}(k) = \frac{1}{2} k \left( \begin{array}{c}
\end{array} \right)^{ab}, \quad S^{ab}[k] = \frac{1}{2} k \left( \begin{array}{c}
\end{array} \right)^{ab},
$$

$$
\tilde{S}^{ab}(k) = \frac{1}{2} k \left( \begin{array}{c}
\end{array} \right)^{ab}, \quad \tilde{S}^{ab}[k] = -\frac{1}{2} k \left( \begin{array}{c}
\end{array} \right)^{ab},
$$

which means that we get the same objects back multiplied by the constant $\frac{1}{2} k$ in the case of $S^{ab}$, while $\tilde{S}^{ab}$ multiply $(k)$ by $k$ and $[k]$ by $(-k)$ rather than $(k)$. This also means that when $(k)$ and $[k]$ act from the left hand side on a vacuum state $|\psi_0\rangle$ the obtained states are the eigenvectors of $S^{ab}$. We further recognize (Eq. 11.12) that $\gamma^a$ transform $ab \left( \begin{array}{c}
\end{array} \right)^{ab}$, never to $[k]$, while $\tilde{\gamma}^a$ transform $(k)$ into $[k]$, never to $ab \left( \begin{array}{c}
\end{array} \right)^{ab}$

$$
\gamma^a (k) = \eta^{aa} [k], \quad \gamma^b (k) = -i k (k), \quad \gamma^a [k] = (k), \quad \gamma^b [k] = -i k \eta^{aa} (k),
$$

$$
\tilde{\gamma}^a (k) = -i \eta^{aa} [k], \quad \tilde{\gamma}^b (k) = -k [k], \quad \tilde{\gamma}^a [k] = i (k), \quad \tilde{\gamma}^b [k] = -k \eta^{aa} (k).
$$

From Eq. (A.8) it follows

$$
S^{ac} (k)(k) = -\frac{i}{2} \eta^{aa} \eta^{cc} [k][k], \quad \tilde{S}^{ac} (k)(k) = \frac{i}{2} \eta^{aa} \eta^{cc} [k][k],
$$

$$
S^{ac} [k][k] = \frac{i}{2} (-k)(k), \quad \tilde{S}^{ac} [k][k] = -\frac{i}{2} (k)(k),
$$

$$
S^{ac} (k)[k] = -\frac{i}{2} \eta^{aa} [k][k], \quad \tilde{S}^{ac} (k)[k] = -\frac{i}{2} \eta^{aa} [k][k],
$$

$$
S^{ac} [k][k] = \frac{i}{2} \eta^{cc} (k)(k), \quad \tilde{S}^{ac} [k][k] = \frac{i}{2} \eta^{cc} (k)(k).
$$

From Eqs. (A.9) we conclude that $\tilde{S}^{ab}$ generate the equivalent representations with respect to $S^{ab}$ and opposite.

Let us deduce some useful relations

$$
(k)(k) = 0, \quad (k)(-k) = \eta^{aa} [k], \quad (k)(k) = \eta^{aa} [-k], \quad (k)(-k) = 0,
$$

$$
[k][k] = [k], \quad [k][-k] = 0, \quad [k][k] = 0, \quad [-k][k] = [k], \quad [-k][-k] = [-k],
$$

$$
(k)[k] = 0, \quad (k)[-k] = (k), \quad (k)[-k] = (k), \quad (k)[-k] = 0,
$$

$$
(k)[-k] = (k), \quad (k)[-k] = 0, \quad [-k][k] = 0, \quad [-k][-k] = [k].
$$

We recognize in the first equation of the first row and the first equation of the second row the demonstration of the nilpotent and the projector character of the Clifford algebra objects.
\( ab(k) \) and \([k]\), respectively. Defining
\[
\left(\pm i\right) = \frac{1}{2} (\tilde{\gamma}^a \mp \tilde{\gamma}^b), \quad \left(\pm 1\right) = \frac{1}{2} (\tilde{\gamma}^a \pm i\tilde{\gamma}^b),
\]
(A.11)

one recognizes that
\[
\begin{align*}
\left(\pm i\right) \left(\pm 1\right) & = 12 \left(\tilde{\gamma}^a \mp i\tilde{\gamma}^b\right), \\
\left(\pm 1\right) \left(\pm 1\right) & = 12 \left(\tilde{\gamma}^a \pm i\tilde{\gamma}^b\right), \\
\left(\pm 1\right) \left(\pm 1\right) & = 12 \left(\tilde{\gamma}^a \pm i\tilde{\gamma}^b\right),
\end{align*}
\]
(A.12)

Recognizing that
\[
\begin{align*}
\left(\pm i\right) \left(\pm 1\right) & = 12 \left(\tilde{\gamma}^a \mp i\tilde{\gamma}^b\right), \\
\left(\pm 1\right) \left(\pm 1\right) & = 12 \left(\tilde{\gamma}^a \pm i\tilde{\gamma}^b\right), \\
\left(\pm 1\right) \left(\pm 1\right) & = 12 \left(\tilde{\gamma}^a \pm i\tilde{\gamma}^b\right),
\end{align*}
\]
(A.13)

we define a vacuum state \(|\psi_0\rangle\) so that one finds
\[
\begin{align*}
\langle (k) (k) \rangle & = 1, \\
\langle [k] [k] \rangle & = 1.
\end{align*}
\]
(A.14)

Taking into account the above equations it is easy to find a Weyl spinor reducible representation for \(d\)-dimensional space, with \(d\) even or odd.

For \(d\) even we simply make a starting state as a product of \(d/2\), let us say, only nilpotents \( ab(k) \), one for each \( S^{ab} \) of the Cartan subalgebra elements (Eq.(A.4)), applying it on an (unimportant) vacuum state. For \(d\) odd the basic states are products of \((d - 1)/2\) nilpotents and a factor \((1 \pm \Gamma)\). Then the generators \( S^{ab} \), which do not belong to the Cartan subalgebra, being applied on the starting state from the left, generate all the members of one Weyl spinor.

\[
\begin{align*}
\psi_0 & = \left(\begin{array}{c}
0d & 12 & 35 & \cdots & d-1 & d-2 \\
(k_{0d})(k_{12})(k_{35}) & \cdots & (k_{d-1})(d-2) & \psi_0 \\
0d & 12 & 35 & \cdots & d-1 & d-2 \\
[-k_{0d}][-k_{12}](k_{35}) & \cdots & (k_{d-1})(d-2) & \psi_0 \\
0d & 12 & 35 & \cdots & d-1 & d-2 \\
[-k_{0d}](k_{12})[-k_{35}] & \cdots & (k_{d-1})(d-2) & \psi_0 \\
\vdots & & & & \\
0d & 12 & 35 & \cdots & d-1 & d-2 \\
[-k_{0d}][k_{12}][k_{35}] & \cdots & [-k_{d-1}d-2] & \psi_0 \\
0d & 12 & 35 & \cdots & d-1 & d-2 \\
(k_{0d})[-k_{12}][-k_{35}] & \cdots & (k_{d-1})(d-2) & \psi_0 \\
\vdots & & & & 
\end{array}\right)
\end{align*}
\]
(A.15)
All the states have the handedness $\Gamma$, since \( \{ \Gamma, S^{ab} \} = 0 \). States, belonging to one multiplet with respect to the group $SO(q, d - q)$, that is to one irreducible representation of spinors (one Weyl spinor), can have any phase. We made a choice of the simplest one, taking all phases equal to one.

The above graphic representation demonstrate that for $d$ even all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of nilpotents $k_{ab}$, by transforming all possible pairs of $(k_{ab})(k_{mn})$ into $[-k_{ab}][-k_{mn}]$. There are $S^{am}, S^{an}, S^{bm}, S^{bn}$, which do this. The procedure gives $2^{(d/2 - 1)}$ states. A Clifford algebra object $\gamma^a$ being applied from the left hand side, transforms a Weyl spinor of one handedness into a Weyl spinor of the opposite handedness. Both Weyl spinors form a Dirac spinor.

For $d$ odd a Weyl spinor has besides a product of $(d - 1)/2$ nilpotents or projectors also either the factor $\frac{\Gamma}{2} := \frac{1}{2}(1 + \Gamma)$ or the factor $\frac{\bar{\Gamma}}{2} := \frac{1}{2}(1 - \Gamma)$. As in the case of $d$ even, all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of $(1 + \Gamma)$ and $(d - 1)/2$ nilpotents $(k_{ab})$, by transforming all possible pairs of $(k_{ab})(k_{mn})$ into $[-k_{ab}][-k_{mn}]$. But $\gamma^a$’s, being applied from the left hand side, do not change the handedness of the Weyl spinor, since $\{ \Gamma, \gamma^a \}_- = 0$ for $d$ odd. A Dirac and a Weyl spinor are for $d$ odd identical and a ”family” has accordingly $2^{(d-1)/2}$ members of basic states of a definite handedness.

We shall speak about left handedness when $\Gamma = -1$ and about right handedness when $\Gamma = 1$ for either $d$ even or odd.

While $S^{ab}$ which do not belong to the Cartan subalgebra (Eq. (A.4)) generate all the states of one representation, generate $\tilde{S}^{ab}$ which do not belong to the Cartan subalgebra (Eq. (A.4)) the states of $2^{d/2 - 1}$ equivalent representations.

Making a choice of the Cartan subalgebra set of the algebra $S^{ab}$ and $\tilde{S}^{ab}$

\[
S^{03}, S^{12}, S^{56}, S^{78}, S^{09^{10}}, S^{11^{12}}, S^{13^{14}}, \quad \tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \tilde{S}^{09^{10}}, \tilde{S}^{11^{12}}, \tilde{S}^{13^{14}},
\]

\[\text{(A.16)}\]

a left handed $(\Gamma^{(1,13)} = -1)$ eigen state of all the members of the Cartan subalgebra, representing a weak chargeless $u_R$-quark with spin up, hyper charge $(2/3)$ and colour
\((1/2, 1/(2\sqrt{3}))\), for example, can be written as

\[
\frac{\psi}{\langle \gamma^0 - \gamma^3 (\gamma^1 + i\gamma^2) | (\gamma^5 + i\gamma^6) (\gamma^7 + i\gamma^8) | \rangle} =
\frac{1}{27} (\gamma^9 + i\gamma^{10}) (\gamma^{11} - i\gamma^{12}) (\gamma^{13} - i\gamma^{14}) | \psi \rangle.
\]

This state is an eigen state of all \(S^{ab}\) and \(\tilde{S}^{ab}\) which are members of the Cartan subalgebra (Eq. (A.16)).

The operators \(\tilde{S}^{ab}\), which do not belong to the Cartan subalgebra (Eq. (A.16)), generate families from the starting \(u_R\) quark, transforming \(u_R\) quark from Eq. (A.17) to the \(u_R\) of another family, keeping all the properties with respect to \(S^{ab}\) unchanged. In particular \(\tilde{S}^{01}\) applied on a right handed \(u_R\)-quark, weak chargeless, with spin up, hyper charge \(2/3\) and the colour charge \((1/2, 1/(2\sqrt{3}))\) from Eq. (A.17) generates a state which is again a right handed \(u_R\)-quark, weak chargeless, with spin up, hyper charge \(2/3\) and the colour charge \((1/2, 1/(2\sqrt{3}))\)

\[
\tilde{S}^{01} (\psi) = \frac{\psi}{\langle \gamma^0 - \gamma^3 (\gamma^1 + i\gamma^2) | (\gamma^5 + i\gamma^6) (\gamma^7 + i\gamma^8) | \rangle} =
\frac{i}{2} (\gamma^9 + i\gamma^{10}) (\gamma^{11} - i\gamma^{12}) (\gamma^{13} - i\gamma^{14}) | \psi \rangle.
\]

Below some useful relations are presented

\[
N^\pm = N_1^+ \pm i N_2^+ = - (\mp i)(\pm), \quad N_1^- = N_1^- \pm i N_2^- = (\pm i)(\pm),
\]

\[
\tilde{N}^\pm = - (\mp i)(\pm), \quad \tilde{N}_1^- = (\pm i)(\pm),
\]

\[
\tau^1^\pm = (\mp)(\mp), \quad \tilde{\tau}^2^\mp = (\mp)(\mp),
\]

\[
\tau^1^\pm = (\mp)(\mp), \quad \tilde{\tau}^2^\mp = (\mp)(\mp).
\]

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[27] We studied in the refs. [16, 17] a toy model, in which fermions, gauge and scalar fields live in $M^{1+5}$, which is broken into $M^{1+3} \times \text{an infinite disc}$. One can find several conditions, under which only left handed family members stay massless.

[28] The fourth family is not in contradiction with the measurements [20].

[29] The Kaluza-Klein[like] theories [21, 22] have difficulties with (almost) masslessness of the spinor fields at the low energy regime. In the refs. [16, 17] we are proposing possible solutions to these kind of difficulties.

[30] $f^\alpha_a$ are inverted vielbeins to $e^a_\alpha$ with the properties $e^a_\alpha f^\alpha_b = \delta^b_b$, $e^a_\alpha f^\beta_a = \delta^\beta_\alpha$. Latin indices $a, b, \ldots, m, n, \ldots, s, t, \ldots$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, \ldots, \mu, \nu, \ldots, \sigma, \tau, \ldots$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, \ldots$ and $\alpha, \beta, \gamma, \ldots$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, \ldots$ and $\mu, \nu, \ldots$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, \ldots$ and $\sigma, \tau, \ldots$). We assume the signature $\eta^{ab} = \text{diag}\{1, -1, -1, \ldots, -1\}$.

[31] We proved that it is possible to have massless fermions after a (particular) break if we start with massless fermions and assume particular boundary conditions after the break or the "effective two dimensionality" cases [16-18].