Accounting for polarization–related effects in the measurement of the bidirectional reflectance distribution function

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Abstract
When measuring the Bidirectional Reflectance Distribution Function (BRDF) to characterize typical materials, the effect of the polarization of the light source or the dependence of the detector on the polarization is not usually considered, even though many studies have proven their impact in the uncertainty of the BRDF of most materials, including diffuse reflectance standards. A methodology to assess polarization-related effects proposed by other authors has been tested in this work for the measurement of the BRDF of four typical materials used for realizing white diffuse reflectance standards (ceramic tile, barium sulfate, Spectralon and Russian opal glass). Relative systematic error due to polarizing conditions is calculated as a combination of the instrument polarization bias and sample-based coefficients, and the impact of the proposed methodology on the uncertainty is discussed.

Keywords: reflectance, BRDF, polarization, radiometry, gonio-spectrophotometry, PRD

(Some figures may appear in colour only in the online journal)

1. Introduction
In order to improve the measurement of materials reflectance, national metrology institutes (NMIs) and other research centers have developed in recent years complex robot–based goniospectrophotometers to measure the bidirectional reflectance of surfaces with as few geometrical restrictions as possible [1–10]. When measuring the Bidirectional Reflectance Distribution Function (BRDF) [11] to characterize typical materials, the effect of the polarization of the light source or the dependence of the detector on the polarization are usually not considered, even though many studies have proven their impact in the uncertainty of the BRDF of most materials, including diffuse reflectance standards [12–15].

The principal objective of this work was to assess polarization-related effects in the measurement of the BRDF, and to provide an approach to account for systematic errors. This approach was experimentally tested on four typical materials for realizing white diffuse reflectance standards (ceramic tile, barium sulfate, Spectralon and Russian opal glass), which also are used as $0^\circ$–$45^\circ$ bidirectional reflectance factor standards in BRDF measurements. Since they are not perfect reflecting diffusers (PRD, diffuser exhibiting isotropic diffuse reflection with a reflectance equal to one [16]), they need to be spectrophotometrically characterized in order to account for their deviation from ideality. The measurement of the BRDF at different measurement geometries (pairs of
The BRDF is a variable that describes the radiance distribution of a surface when it is directionally irradiated from a given incidence direction. It can be formally expressed as:

\[ f_i(\theta_i, \phi_i; \theta_r, \phi_r; \lambda) = \frac{L_i(\theta_i, \phi_i; \theta_i, \phi_i; \lambda)}{E_i(\theta_i, \lambda)} \]  

(1)

where \( \lambda \) is the wavelength, \( L_i \) is the surface radiance, \( E_i \) is the irradiance on the surface, \( \theta_i \) and \( \theta_r \) are the polar angles for irradiation and collection directions, respectively, and \( \phi_i \) and \( \phi_r \) are their azimuthal angles. In this work, \( \phi = 0^\circ \) is defined as the half-plane containing the irradiation direction, and, therefore, the value of \( \phi \) can be regarded as the difference between the azimuthal angles of the incidence and collection directions.

The radiance factor \( \beta \) is a variable very similar to the BRDF, defined as the ratio between the radiance of a PRD irradiated under exactly the same conditions. The relation between \( \beta \) for collimated illumination and BRDF is simply \( \beta = \Phi f_c \) [11]. Radiance factor is used in the results presented in this work in order to better interpret them (PRD’s \( \beta = 1 \)).

2. Methodology

When the BRDF of a sample is measured, it is assumed, unless otherwise it is explicitly said, that the results are given for unpolarized illumination and for detection responsivity independent of the polarization. If the measuring system does not fully fulfill these ideal conditions, a systematic error with respect to the assumed unpolarized BRDF is produced, which originates from both the polarization bias of the measuring system (described by the polarization parameters \( A \) and \( B \), as below explained) and the sample’s BRDF dependence on the polarization (variation of the radiance factor \( \beta \) at different polarization configurations). The following methodology should allow this systematic error as a function of these two variables to be estimated. A very important part of this formalism was published in [12], the novelty presented here being the experimental procedure to measure the value of \( A \) and \( B \).

2.1. Terminology

Here, the same terminology as that in [12] is followed. s- and p-plane polarizations (perpendicular and parallel respectively to the plane of incidence) and unbiased (or random) polarization are denoted by subscripts s, p and r respectively. Unbiased polarization means unpolarized light when referred to illumination, and detector responsivity independent of the state of polarization of the light when referred to detection. The radiance factor \( \beta \), the detector reading (denoted by \( R \)) or the reading when the standard for reference is used (denoted by \( R' \)) are specified with two subscripts describing polarization using these notations, the first referring to illumination and the second to detection.

2.2. Instrument polarization parameters: \( A \) and \( B \)

The parameter \( A \) represents the polarization bias of the irradiation on the sample. It is given by the ratio of the radiant fluxes with s-polarization and p-polarization:

\[ A = \frac{\Phi_s}{\Phi_p} \]  

(2)

In [12], \( A \) was measured as the ratio of the detector readings at s- and p-polarizations of the illumination, when the detector is completely free from polarization bias. It was achieved by placing several sheets of pot opal translucent glass between illumination and detector, and linear polarizers just after the source.

On the other hand, the parameter \( B \) represents the polarization bias of the detector. It is given by the ratio of the responsivities of the detector to s- and p-polarization (\( S_s \) and \( S_p \), respectively):

\[ B = \frac{S_s}{S_p} \]  

(3)

In [12], \( B \) was measured as the ratio of the detector readings at s- and p-polarizations in detection, when the illumination is completely free from polarization bias. It was achieved by placing several sheets of pot opal translucent glass between illumination and detector, and linear polarizers just before the detector.

We propose a slightly modified procedure to measure \( A \) and \( B \): To keep linear polarizers at source and detector and acquire the readings for a diffuse reflectance standard at geometry 0°: 15° with the four possible polarization configurations (ss, sp, ps, pp). According to a previous work [13], the polarization bias of the sample is very small for white diffusers at low incidence and collection angles. Therefore, in this work it is assumed that the diffuse reflectance standard at geometry 0°: 15° almost completely depolarizes the light, and that it can play the same role as the sheets of pot opal translucent glass in [12]. From the readings at the four polarization configurations, \( A \) and \( B \) are calculated as:

\[ A = \frac{R_{sp} + R_{ss}}{R_{ps} + R_{pp}} = \frac{R_{ss}}{R_{pp}} \]  

(4)

and

\[ B = \frac{R_{ps} + R_{ss}}{R_{pp} + R_{sp}} = \frac{R_{ss}}{R_{sp}} \]  

(5)

Notice that in both equations the sum of readings at s- and p-polarization has been made equal to an expected value for readings without polarization bias (subscript r).

2.3. Instrument degrees of polarization bias

The parameters \( A \) and \( B \) are used to calculate the degrees of polarization bias of the system \( P_1 \) and \( P_2 \), where, as in [12],
\( P_1 \) is the degree of polarization bias of the illumination with respect to the s-polarization plane:

\[
P_1 = \frac{\Phi_s - \Phi_p}{\Phi_s + \Phi_p}
\]

(6)

and similarly \( P_2 \) is the degree of polarization bias of the detector:

\[
P_2 = \frac{S_s - S_p}{S_s + S_p}
\]

(7)

Equations (2) and (3), can be used to express \( P_1 \) and \( P_2 \) as a function of \( A \) and \( B \):

\[P_1 = \frac{2A}{A+1} - 1.
\]

(8)

\[P_2 = \frac{2B}{B+1} - 1
\]

(9)

Notice that the values of \( P_1 \) and \( P_2 \) range from +1 (s-polarization bias) to -1 (p-polarization bias), without being no polarization bias (unpolarized light or detector responsivity independent of polarization).

2.4. Radiance factor

The radiance factor \( \beta \) is obtained as the ratio of the reading for the sample to the reading for a PRD, \( R^{PRD} \), as:

\[
\beta = \frac{R}{R^{PRD}}
\]

(10)

Then, if it was possible to use a PRD, the radiance factor for the different polarization configurations would be obtained as:

\[
\beta_{ss} = \frac{R_{ss}}{R_{ss}^{PRD}}
\]

(11)

\[
\beta_{sp} = \frac{R_{sp}}{R_{sp}^{PRD}}
\]

(12)

\[
\beta_{ps} = \frac{R_{ps}}{R_{ps}^{PRD}}
\]

(13)

\[
\beta_{pp} = \frac{R_{pp}}{R_{pp}^{PRD}}
\]

(14)

However, the PRD is an ideal artefact which cannot be realized in practice, and its detector readings (\( R^{PRD} \)) are not available. It is necessary to use white diffuse reflectance standards, whose reflectance is close to that of a PRD, but not exactly identical. Typically, the value of the standard’s bidirectional radiance factor at 0°: 45° or 45°: 0° for no polarization bias is known (hereafter \( \beta_{ct} ^{\circ} \)) and used to find the relation between the radiance factor \( \beta_{rr} \) to be measured and the detector reading, as:

\[
\beta_{rr} = \frac{R_{rr}}{R_{rr}^{PRD}}
\]

(15)

when there is no polarization bias in the instrument. When there is such a bias, the detector readings (\( R^{PRD} \)) can be replaced in equations (11)–(14), as explained in Ref: [12], as:

\[
\beta_{pp} = \frac{4\beta_{rr}^2R_{pp}^+}{\frac{R_{pp}^+}{A} + \frac{R_{pp}^-}{B} + R_{pp}^r}
\]

(16)

\[
\beta_{ps} = \frac{4\beta_{rr}^2R_{ps}^+}{\frac{R_{ps}^+}{A} + \frac{R_{ps}^-}{B} + R_{ps}^r}
\]

(17)

\[
\beta_{sp} = \frac{4\beta_{rr}^2R_{sp}^+}{\frac{R_{sp}^+}{A} + \frac{R_{sp}^-}{B} + R_{sp}^r}
\]

(18)

\[
\beta_{ss} = \frac{4\beta_{rr}^2R_{ss}^+}{R_{ss}^+ + R_{ss}^- + R_{ss}^r + R_{ss}^+AB}
\]

(19)

to obtain \( \beta_{rr} \) as:

\[
\beta_{rr} = \frac{(\beta_{ss} + \beta_{sp} + \beta_{ps} + \beta_{pp})}{4}
\]

(20)

The complete derivation of equations (16)–(19) is given in [12]. These equations are used to measure \( \beta_{ss} \), \( \beta_{sp} \), \( \beta_{ps} \) and \( \beta_{pp} \).

2.5. Sample-related coefficients \( C_1, C_2 \) and \( C_{12} \) for estimating systematic error

Although it is possible to measure the radiance factors according to equations (16)–(20), in many practical cases, as when using commercial instruments, \( \beta_{rr} \) is the measurand of interest and no linear polarizers are available in the measuring system. In such cases, \( P_1 \) and \( P_2 \) might be part of the system specifications, and, in addition, typical dependence on the polarization of a certain kind of samples can be known. Then, the systematic error due to polarization bias can be estimated as in the following.

It can be proven that the measured radiance factor, \( \beta_{m} \), for partially polarized conditions is given by [12]:

\[
\beta_{m} = \frac{1}{4} \left[ (1 + P_1)(1 + P_2)\beta_{ss} + (1 + P_1)(1 - P_2)\beta_{sp} + (1 - P_1)(1 + P_2)\beta_{ps} + (1 - P_1)(1 - P_2)\beta_{pp} \right]
\]

(21)

Since \( \beta_{m} \) can be regarded as the radiance factor measured with a system with degrees of polarization \( P_1 \) and \( P_2 \), it can
be used to estimate the relative difference between the measured radiance factor and the radiance factor $\beta_{rr}$, which is the expected value to be reported:

$$\Delta \beta = \frac{\beta_m - \beta_{rr}}{\beta_{rr}}$$  \hspace{1cm} (22)

$\Delta \beta$ denotes the relative systematic error introduced in the measurement of $\beta_{rr}$ when measured with a system with polarization bias. It can be expressed (see appendix) as:

$$\Delta \beta = P_1C_1 + P_2C_2 + P_1P_2C_{12}$$  \hspace{1cm} (23)

where the coefficients $C_1$, $C_2$ and $C_{12}$ specifically describe the dependence of the sample on polarization, and are given as:

$$C_1 = \frac{(\beta_{ss} + \beta_{sp} - \beta_{ps} - \beta_{pp})}{4\beta_{rr}},$$  \hspace{1cm} (24)

$$C_2 = \frac{(\beta_{ss} - \beta_{sp} + \beta_{ps} - \beta_{pp})}{4\beta_{rr}},$$  \hspace{1cm} (25)

$$C_{12} = \frac{(\beta_{ss} - \beta_{sp} - \beta_{ps} + \beta_{pp})}{4\beta_{rr}}$$  \hspace{1cm} (26)

It must be noticed that, in order to estimate the relative systematic error when measuring a given sample, typical values for $C_1$, $C_2$ and $C_{12}$ are required.

### 3. Experimental system and samples

The goniospectrophotometer GEFE at IO-CSIC [7, 17] was used for the measurements reported here. A sketch of the system is shown in figure 1. The irradiation system is fixed, whereas sample and detector systems are mobile. The sample is held on a 6-axis robot-arm able to realize any required orientation relative to the incoming beam, while the detector can revolve around the sample.

The system allows for angular, spectral (of illumination and collection beams) and spatial measurements, within a spectral range from 380 nm to 1700 nm, but only the features relevant for this work are described here.

The six-axis robot-arm (R6) positions the sample quickly at the desired orientation. The samples are held at the robot-arm

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**Figure 1.** Sketch of the goniospectrophotometer GEFE at IO-CSIC. (top) Top view; (bottom) Side view.
by means of a vacuum sucker. A wide–band Xenon arc lamp (S2), which emits in the spectral range from 185 nm to 2000 nm, is used as light source. In order to irradiate uniformly and with an almost collimated beam the samples, a Köhler optical system was used (see figure 1). It was formed by two 2-inch-diameter converging lenses (L1 and L2) made of UV Fused Silica. A diaphragm (P1) was placed after the first lens, which allows, by adjusting its diameter, the spot size on the sample (S) to be modified, since it is precisely the image of P1. A second diaphragm (P2) is located after the second lens L2. By modifying its diameter, the irradiation solid angle is adjusted, but also the irradiance on the sample plane varies. Between L1 and L2 there is a neutral-density-filter wheel (FW), used to produce different irradiance levels on the sample, depending on the particular measurement conditions. Before the filter wheel, an uncoated plate of fused silica (W) redirects around 10 % of the incoming beam towards a detector (Mon), whose role is monitoring the source’s intensity. After the Köhler system, a mirror (M45°) was placed at 45°, followed by a 50:50 UV fused-silica broadband-plate beamsplitter (BS), also at 45° (see figure 1). This periscope configuration makes it possible to perform retro-reflection measurements by placing the detection system behind the beamsplitter.

A spectroradiometer Konica-Minolta CS-2000 A is used to measure spectral radiance in the visible range between 380 nm and 780 nm (VIS detector), with a variable field of view of 0.1°, 0.2° or 1°. It is mounted onto a platform that travels along a 1.03 m diameter cogwheel, whose center coincides with the location of the sample’s reference system. The movement along the cogwheel is performed by means of a stepper motor with a step coder for position control.

Two linear polarizers were added to the setup for this work. The first was located just before the sample and after the beamsplitter, and the second in front of the objective lens of the spectroradiometer.

The relative expanded uncertainty (k = 2) of the measurements depends slightly on the geometry for these measurements, and it was estimated between 0.8 % and 1.1 % for the visible range (a detailed uncertainty budget is given at reference [7]).

The measurement geometries were restricted to the incidence plane. The spectral BRDF was measured for the geometries resulting from the combination of the following spherical coordinates: six polar angles for both irradiation and collection (θi and θc from 0° to 70°, with angular steps of 15°), and two azimuthal angles for collection (ϕc = 0° and 180°, within the incidence plane). Notice that measurements are said to be within the incidence plane (in–plane) when the azimuthal angle for collection is 0° or 180°, and out-of-plane otherwise. The incidence direction defines the origin of azimuthal angles.

The white diffuse reflectance standards available at IO-CSIC for which the spectral BRDF was measured are: Spectralon (sintered polytetrafluoroethylene, PTFE); matte ceramic reflection standard from CCS, pressed barium sulphate (BaSO4) powder, prepared in our laboratory prior to the measurement (a molding was used, where the previously sieved BaSO4 was pressed); and polished Russian opal glass, used by the National Institute of Standards and Technology (NIST) for its Multi-Angle White Reflectance Standards.

4. Results and discussion

4.1. Determination of A and B

A and B were spectrally determined with GEFE according to the procedure described in section 2.2, for the four available types of diffuse reflectance standards at 0°: 15°. The average A and B values, shown in figure 2, are used for the following calculations. B is very stable across the spectral range, and close to unity, much more than A. Therefore, it is concluded that, in GEFE, the polarization bias due to the illumination clearly prevails over the dependence on the polarization of the detector. For that prevailing bias, s-polarization is in average 20 % larger than p-polarization.

Whereas the polarization biases of source and detector are independent, the averaged values of A and B seem to be spectrally correlated, which might indicate a systematic issue in their determinations. It may be related to the non-perfect depolarization properties of the diffuse reflectance standards used in this study. Individually examined, they provide a very consistent result for A, but the spectral distributions obtained for B are more variable (this variation is shown in figure 2 with the dashed lines). When using polished Russian opal glass, there is no correlation between the spectral distribution of A and B, while when using the matte ceramic reflection standard, the correlation is the largest. Intermediate cases are pressed barium sulphate powder and Spectralon, the latter with very low correlation.

4.2. C1, C2 and C12 coefficients

Calculated spectrally-averaged C1, C2 and C12 coefficients (equations (24)–(26)) are shown in figures 3–6 for each sample, where the error bars represent the standard deviation across the different wavelengths. The wavelength dependence of these coefficients was determined, and the spectral average is only for a more convenient angular representation. Each figure is composed of 6 plots, corresponding to the 6 different incidence angles, θi, used in this work. The y axis represents the values of the coefficients, whereas the x axis represent the collection angles, θc. These in kind of plots, a negative value of θi means that ϕc = 0°, whereas a positive value means that ϕc = 180°. The larger the coefficients, the larger the systematic error of the measurement. The absolute value of the C12 coefficient is generally larger than those of C1 and C2, except at some geometries with large incidence and collection angles. The larger value of C12 does not mean a larger contribution to the systematic error, since it multiplies both P1 and P2 in equation (23). These coefficients are not exactly zero at low angles (as assumed in the characterization of A and B) because the complete depolarization of these materials is not always perfect. At low incidence angles, some symmetry is observed around collection at normal direction, which increasingly disappears toward larger incidence angles. Except for Russian
opal glass, where the absolute value of $C_2$ prevails over $C_1$, the two coefficients present very similar values.

4.3. Systematic error ($\Delta \beta$)

The spectrally-averaged relative systematic error of radiance factor, $\Delta \beta$ (equation (23)), expected when measuring the studied diffuse reflectance standards with our goniospectrophotometer are given in figures 7–10. Again, the error bars represent the standard deviation across the different wavelengths. In general, systematic errors lie between $10^{-3}$ and $10^{-2}$, and their behaviours are similar to the coefficient $C_1$, $C_2$ or $C_{12}$ with the largest absolute value.

Spectrally-averaged relative systematic errors at 0°: 45° for the four different materials are shown in table 1, for different values of $P_1$ and $P_2$, assuming that they are equal ($P_1 = P_2$). If the value is 0.1, this relative systematic error will be in the order of 0.1 %.

Regarding the dependence on the geometry, shown in figures 7–10, the behaviour of the relative systematic error is similar for ceramic tile, barium sulphate and Spectralon, but it differs for Russian opal glass. Whereas in the former three there is a notable increase of $\Delta \beta$ towards larger positive values of $\theta_r$, symmetry with respect to $\theta_r = 0^\circ$ is kept in the latter.

It is important to notice that, for the white diffuse reflectance standards, the systematic errors here estimated are lower than the measurement uncertainty (around 1 %, $k = 2$) at one polarization configuration for most of the measurement geometries. This measurement uncertainty is given by the reference value $\beta_{rr}^\prime$ (0.7 %), and other uncertainty sources (0.6 %), as detector readings, angular position, non-uniformity or non-linearity.

![Figure 2. $A$ and $B$ average values as a function of the wavelength. Values ± their standard deviations are plotted too.](image)

Table 1. Values of $\Delta \beta$ (%) at 0°: 45° for the different studied diffuse reflectance materials, and for the different polarization degrees ($P_1$ and $P_2$) when have equal value (second row).

| Material                | $\Delta \beta$ at 0°: 45° (%) | $P_0 = 0.5$ | $P_0 = 0.1$ | $P_0 = 0.01$ | $P_0 = 0.001$ |
|-------------------------|-------------------------------|-------------|-------------|--------------|---------------|
| Matt ceramic            | 1.5                           | 0.08        | 0.004       | 0.0003       |
| Barium sulphate         | 0.4                           | 0.08        | 0.01        | 0.001        |
| Spectralon              | 0.2                           | 0.06        | 0.01        | 0.001        |
| Russian opal glass      | 1.1                           | 0.1         | 0.01        | 0.001        |
Figure 3. Calculated spectrally-averaged $C_1$, $C_2$ and $C_{12}$ coefficients for a diffuse reflectance standard ceramic tile.

Figure 4. Calculated spectrally-averaged $C_1$, $C_2$ and $C_{12}$ coefficients for a diffuse reflectance standard made of barium sulphate pressed powder.

In those cases where the measurement uncertainty is larger than the estimated systematic error, the measurement of $\beta_{rr}$ by equation (20) is not recommended. It would increase the uncertainty measurement because four instead of only one polarization configuration must be evaluated, and because the additional contribution of the uncertainty of $A$ and $B$
coefficients. In the presented measures, the estimated uncertainty of these parameters generally is lower than 1 %, but it reaches 3 % ($k = 2$) for some wavelengths, mainly due to the imperfect depolarization of the white diffuse reflectance references. This uncertainty source scarcely affects the data here provided for specific geometries, since they are spectrally averaged.
averaged values. The uncertainty of the $C_1$, $C_2$ and $C_{12}$ coefficients is estimated to be lower than 3 % too, whereas the uncertainty for the systematic error is slightly larger (4 %).

Other kind of samples, as coloured, grey specimens made of the same materials or less isotropic materials, are more sensitive to polarization, so that the effect described here should be
much more pronounced. In those cases, the measurement of $\beta_\mu$ by equation (20) is recommended.

Estimated values of the $C_1$, $C_2$ and $C_{12}$ coefficients for a given sample are required to calculate the relative systematic error due to polarization bias from equation (23). Therefore, representative values for different kind of materials should be obtained and reported, in a similar way as done in this work for white diffuse materials. That would allow the uncertainty
budget in the bidirectional reflectance measurements to be improved, and to recommend tolerances on polarization bias for commercial instruments, as a function of the kind of material intended to be measured. In addition, it would be convenient if any instrument degrees of polarization bias \((P_1\) and \(P_2\)) are reported as a part of its specifications.

5. Conclusions

Bidirectional radiance factors of four typical white diffuse reflectance materials have been measured at four linear polarization configurations \(pp, ps, sp\) and \(ss\), and in different measurement geometries. A methodology for this kind of measurement has been proposed, which allows the relative systematic error due to the polarizing conditions to be estimated from sample-related coefficients and instrument degrees of polarization. The calculation of the sample-related coefficients \((C_1, C_2 \text{ and } C_{12})\) has been explained. Representative values for different kinds of materials should be obtained and reported, in a similar way as done in this work for white diffuse materials. That would allow the uncertainty budget in the bidirectional reflectance measurements to be improved, and to recommend tolerances on polarization bias for commercial instruments, as a function of the kind of material intended to be measured. Finally, the impact of the proposed methodology on the uncertainty has been discussed.

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Appendix: Derivation of the expressions for \(C_1, C_2 \text{ and } C_{12}\)

The equation derived by Clarke et al [see equation (22) in [12]]:

\[
\beta_m = \frac{1}{4} \left( (1 + P_1)(1 + P_2)\beta_{ss} + (1 + P_1)(1 - P_2)\beta_{sp} + 
(1 - P_1)(1 + P_2)\beta_{ps} + (1 - P_1)(1 - P_2)\beta_{pp} \right) \tag{A1}
\]

allows the radiance factor for any partially polarizing conditions \((P_1\) and \(P_2\)) to be calculated from \(\beta_{ss}, \beta_{sp}, \beta_{ps}\) and \(\beta_{pp}\).

Using equation (20), this equation can be rearranged as:

\[
\beta_m = \beta_{rr} + \frac{1}{4} \left( \beta_{ss}(P_1 + P_2 + P_1P_2) + \beta_{sp}(-P_2 + P_1P_2) + 
\beta_{ps}(P_2 - P_1 - P_1P_2) + \beta_{pp}(1 - P_1 - P_2 + P_1P_2) \right) \tag{A2}
\]

The systematic error due to plane polarizing condition \((P_1\) and \(P_2\)) can be estimated as the relative deviation between \(\beta_m\) and the radiance factor intended to be measured, \(\beta_{rr}\):

\[
\Delta \beta = \frac{\beta_m - \beta_{rr}}{\beta_{rr}} \tag{A3}
\]

Equations (A2) and (A3) can be combined to obtain:

\[
\Delta \beta = \frac{1}{4\beta_{rr}} \left[ \beta_{ss}(P_1 + P_2 + P_1P_2) + \beta_{sp}(-P_2 + P_1P_2) + 
\beta_{ps}(P_2 - P_1 - P_1P_2) + \beta_{pp}(1 - P_1 - P_2 + P_1P_2) \right] \tag{A4}
\]

Finally, this last equation can be rearranged as:

\[
\Delta \beta = P_1 \left( \frac{\beta_{ss} + \beta_{sp} - \beta_{ps} - \beta_{pp}}{4\beta_{rr}} \right) + P_2 \left( \frac{\beta_{ss} - \beta_{sp} + \beta_{ps} - \beta_{pp}}{4\beta_{rr}} \right) + 
P_1P_2 \left( \frac{\beta_{ss} - \beta_{sp} - \beta_{ps} + \beta_{pp}}{4\beta_{rr}} \right) \tag{A5}
\]

\[
\Delta \beta = P_1C_1 + P_2C_2 + P_1P_2C_{12} \tag{A6}
\]

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