Measuring the stellar luminosity function and spatial density profile of the inner 0.5 pc of the Milky Way nuclear star cluster

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Abstract. We report on measurements of the luminosity function of early (young) and late-type (old) stars in the central 0.5 pc of the Milky Way nuclear star cluster as well as the density profiles of both components. The young (∼6 Myr) and old stars (>1 Gyr) in this region provide different physical probes of the environment around a supermassive black hole: the luminosity function of the young stars offers us a way to measure the initial mass function from star formation in an extreme environment, while the density profile of the old stars offers us a probe of the dynamical interaction of a star cluster with a massive black hole. The two stellar populations are separated through a near-infrared spectroscopic survey using the integral-field spectrograph OSIRIS on Keck II behind the laser guide star adaptive optics system. This spectroscopic survey is able to separate early-type (young) and late-type (old) stars with a completeness of 50% at K' = 15.5. We describe our method of completeness correction using a combination of star planting simulations and Bayesian inference. The completeness corrected luminosity function of the early-type stars contains significantly more young stars at faint magnitudes compared to previous surveys with similar depth. In addition, by using proper motion and radial velocity measurements along with anisotropic spherical Jeans modeling of the cluster, it is possible to measure the spatial density profile of the old stars, which has been difficult to constrain with number counts alone. The most probable model shows that the spatial density profile, n(r) ∝ r^γ, to be shallow with γ = 0.4 ± 0.2, which is much flatter than the dynamically relaxed case of γ = 3/2 to 7/4, but does rule out a ‘hole’ in the distribution of old stars. We show, for the first time, that the spatial density profile, the black hole mass, and velocity anisotropy can be fit simultaneously to obtain a black hole mass that is consistent with that derived from individual orbits of stars at distances < 1000 AU from the Galactic center.

1. Introduction
The proximity of the Galactic center offers us the unique opportunity to study the nucleus of a typical spiral galaxy. In particular, the combination of high angular resolution imaging and spectroscopy enabled by ground-based adaptive optics systems have helped to realize this potential with detailed observations of the kinematics and photometry of individual stars within the central parsec. Recent findings include: measurement of the mass of the Milky Way supermassive black hole, Sgr A* [1, 2], and the spectroscopic separation of the young and old stars allowing study of their very different kinematic and spatial...
distributions [e.g. 3, 4]. These observations challenge some long-standing theoretical expectations for a cluster with a massive black hole. In particular, the projected surface density radial profile of the old stars is very flat and inconsistent with the prediction of a steep stellar cusp around massive black holes within dynamically relaxed star clusters [e.g. 3, 5–7]. In addition, not only are there numerous young stars (≈ 6 Myr old) in an environment inhospitable to star formation, there may be evidence that the young stars at the Galactic center formed with a very top heavy initial mass function (IMF) compared to the standard IMF [4], and that these stars may be distributed in one or more kinematically distinct structures [8, 9].

These recent findings have motivated us to extend our integral-field spectroscopic survey in Do et al. [3] by a factor of 3 radially out to 0.5 pc from Sgr A* along the major axis of the projected disk plane of the clockwise disk of young stars [e.g. 9]. We obtain spectra for ≈ 500 stars with $K' < 15.5$ mag, which allow us to investigate the radial profile and luminosity functions of both the early-type (young) and late-type (old) populations. In Section 2, we discuss our new observations while in Section 3 we review our method for spectral typing. In Section 4, we adapt a method from Bayesian inference with star-planting simulations to spectral type the sources and establish the spectral completeness of our survey. In Section 5, we present the resulting surface number density profiles and $K'$ luminosity functions of the early and late-type stars and in Section 5.2, we discuss the implications of these results for the mass function and origins of the young stars. We also use anisotropic spherical Jeans modeling with three-dimensional velocity measurements for the late-type stars to simultaneously derive the spatial density profile, black hole mass, and velocity anisotropy (Section 6).

2. Observations and data reduction
Near-IR integral-field spectra of the Galactic center were obtained between 2007 and 2010 using the OH-Suppressing Infra-Red Imaging Spectrograph (OSIRIS) in conjunction with the laser guide star adaptive optics (LGS AO) system on the Keck II telescope. The laser guide star was fixed at the center of the field of view, while low-order tip-tilt corrections were made on the $R = 13.7$ mag star, USNO 0600-28577051, which is located ≈ 19″ from Sgr A*. The primary observations for this work constitute a survey through the narrow-band filter Kn3 (2.121 to 2.220 μm) at a spectral resolution of $R = 3800$. This includes both observations initially reported in [3] and [10] and new 2010 observations, which extends from 4″ to 14″ (0.16 pc to 0.56 pc). We refer to this new extension of our survey as the Galactic Center OSIRIS Wide-field Survey (GCOWS). While our initial work, which covered only a $8' \times 6''$ region centered on Sgr A*, used OSIRIS’s 35 mas plate scale (field of view $1.58'' \times 2.24''$), the new GCOWS observations, which are at larger Galactic center radii where the stellar densities are lower, were obtained with 50 mas plate scale (field of view $2.25'' \times 3.2''$). This provided a good compromise between spatial resolution and field of view in this region. GCOWS covers a region of approximately $10'' \times 7.2''$ east of the survey in [10], along the projected disk plane of the clockwise disk of young stars [9]. Each of the new fields is observed with a six point dither pattern of 900 s per frame, in which the dithered frames have small ($\sim 0''.01$) offsets from one another. The larger plate scale allows us to reach a sensitivity comparable to the previous 35 mas plate scale observations, which have about 9 dithers per field. We also observe skies and an A0V and G2V calibration star after each night’s Galactic center observations.

3. Spectral identification
Within the wavelength region (2.121 to 2.220 μm) chosen for this survey, the two prominent features useful for differentiating between the cool late-type giants and the hot OB stars are the Br $\gamma$ and Na I doublet features. The old stars show strong Na I lines at 2.062 and 2.090 μm, but weak Br $\gamma$ lines at 2.1661 μm. The O and B stars have no Na I lines, and show variable amounts of Br $\gamma$ absorption depending on spectral type, with Br $\gamma$ preferentially becoming stronger for B main sequence stars [See 11]. We group the stellar spectra by eye into three groups: 1) early-type, 2) late-type, or 3) unknown, as in Do et al. [3]. More details are given in Do et al. (in prep.). This sample of manually spectral-type
sources is complete to about 50% at $K' = 15.5$, when compared to sources detected from imaging (which are > 96% complete at this magnitude, see Lu et al. in prep.).

4. Completeness correction

In order to determine the luminosity function of the two populations of stars, we need to correct for the incompleteness of our spectral identification. As the luminosity function is an aggregate measurement of all the stars, we can correct for the incompleteness statistically by determining the probability that a given star is early or late-type. In order to assign probabilities for each star, we use a combination of star planting simulations as well as Bayesian inference based on the properties of stars that have been manually spectral-typed. The star planting simulations are used to sample the spatially varying sensitivity to early and late-type stars, which we use in our Bayesian inference to determine the probability of the type of star. While Do et al. (in prep) gives a detailed description, here we give a brief summary of our method:

- An automated method is developed to measure equivalent widths of key lines - in particular, the Br $\gamma$ and Na I lines.
- We then use a faint ($K' > 14.0$) subset of the sample of manually typed stars to characterize the observed distribution of equivalent widths of the spectral features (Br $\gamma$ and Na I) seen in the early-type stars compared to the late-type stars.
- The Bayes factor is computed to establish the relative strength of the evidence that an untyped star is either early-type or late-type by using the equivalent width of its spectral features with the distributions in equivalent widths in the previous step as priors.
- In order to translate the Bayes factor into a probability that a star is early or late-type, we plant either early and late-type spectra next to each untyped source. The simulations are repeated 100 times for each type of star. The simulated spectra are extracted and the Bayes factor is computed for all the simulated sources. Using these distributions, we determine our sensitivity for observing early type compared to late-type stars. We find that we are about twice as sensitive to late-type stars as compared to early-type stars at $K' = 15.5$. A half-sample boot-strap is used to determine the errors on these probabilities, which are small compared to Poisson errors.
- In order to obtain the final probabilities, we also include the fact that the two types of stars have different radial surface density profiles. For example, a random star far from Sgr A* would have a lower probability of being an early-type star compared to one that is found close to Sgr A*, since the surface density profile of early-type stars drop rapidly while those of late-type stars is flat [e.g. 3, 7]. This effect is included as a prior on the relative fraction of both types of stars as a function of radial distance from Sgr A*.

Using the above method, we assign a probability for all untyped sources; with these probabilities and the manually typed sources (which have a probability of 1), we have a complete statistical sample of the stars to measure their luminosity function and surface density profiles.

5. Results

5.1. K-luminosity function

We construct the luminosity function of the two populations by star counts in 0.5 magnitude bins. We use all detected stars in our luminosity function, which includes stars that were manually spectral-typed and the rest which have probabilities of being early or late-type. The manually typed population is about 50% complete in the $K' = 15.0 - 15.5$ magnitude bin. The manually typed sources are assigned a probability of 1 while the stars not manually spectral-typed contribute to the luminosity function based on their probabilities. The error bar in each luminosity bin is slightly larger than Poisson errors as they include errors in assigning probabilities (as determined by a half-sample bootstrap of the simulated sources). We also correct for variable extinction using the extinction map from [12] to a uniform $A_K$ value of 2.74. Figure 1 shows the extinction and completeness corrected $K'$ luminosity functions for both types of stars.
5.2. Implications for the mass function of the young stars

It is not trivial to derive an IMF from a luminosity function as there are many variables that can affect the luminosity beyond the distribution of mass. For example, the age, star formation history, and metallicity will all affect the transformation from mass to luminosity. In Lu et al. (in prep), a detailed analysis is performed using the data presented in this paper and a combination of stellar evolution and stellar atmosphere models. Nevertheless, it is illuminating to compare our luminosity function with those determined from previous observations. Compared to a similar study by [4], the luminosity function of the young stars presented here contains significantly more faint B-stars, resulting in a steeper slope for the luminosity function (see Figure 2). This increase in the number of fainter stars should translate to a steeper mass function. The difference in luminosity functions are important to resolve as the observations of [4] are one of only a few that show a significant departure from a standard IMF, with $dN/dm \propto m^{-0.45 \pm 0.3}$, which is very top-heavy compared to a Salpeter IMF of $dN/dm \propto m^{-2.35}$ [for a review of variations in IMF see 13]. There are two major difference between our study and that of [4]. First, we accounted for incompleteness by using a combination of star planting simulations around each untyped source and Bayesian inference using their spectra and location. In contrast, the authors of [4] planted early-type stars in data cubes and determined the recovery rate by measuring the CO index and applying the fraction recovered as the completeness correction to the young star counts. However, the difference between the number of early-type stars in the faintest bin at $K' = 15.5$ is so large that our result would only be consistent with that of [4] only if we do not apply any completeness correction. It is unclear at the present time how the different methods of completeness correction affect the stars, and more work will have to be done to compare the two methods. The second major difference between the two studies is that the present survey is conducted mainly along the disk plane of the clock-wise disk of young stars, whereas most of the Bartko et al. survey [4] is perpendicular to the disk plane. The differences in luminosity function may be due to differences in the mass function of young stars on the stellar disk and those outside. Explicitly comparing the luminosity function of the on-disk versus off-disk population may help to resolve this issue (see Yelda et al. in prep).

5.3. Surface density profile

We construct the radial surface density profile by averaging the density azimuthally in radial bins centered at Sgr A*. The radial width of the bins grow with radius so that they become larger to accommodate the falling radial density profile of the young stars. The radial surface density profile separated by early and late-type stars corrected for completeness is shown in Figure 3. The surface density profile of the late-type stars is very flat out to the edge of our survey. The completeness corrected surface density profile as a function of projected distance, $R$, from Sgr A* has a best fit power law, $\Sigma(R) \propto R^{-\Gamma}$, with $\Gamma_L = 0.02 \pm 0.14$ for late-type stars, and $\Gamma_E = 0.87 \pm 0.14$ for early-type stars.
A comparison between the observed (dashed purple) and completeness corrected $K'$ luminosity function of early-type stars (blue) with the completeness corrected $K'$ luminosity function from Bartko et al. [4] (green) for stars located at a projected distance $R > 0.78$ from Sgr A*. The number counts are normalized between our observations and those from Bartko et al. [4] in the $K' = 11.5 - 12.5$ bin in order to compare the relative difference in the slopes. The points for the observed $K'$ luminosity function are shifted slightly for clarity. We find a significantly greater fraction of faint young stars than in [4].

The azimuthally averaged and completeness corrected surface density profile of young stars (blue) and old stars (red). The completeness correction includes our relative sensitivity to both types of stars from star planting simulations, as well as the distance prior, and imaging completeness.

6. Jeans modeling of the nuclear star cluster

While the projected surface density profile of the late-type stars rules out a Bahcall-Wolf cusp, the true physical distribution of stars is difficult to constrain with only positional measurements due to the effects of projection onto the plane of the sky. In order to overcome projection effects, we utilize the kinematic measurements of the old late-type stars to constrain the cluster density profile. Similar to the methodologies described in the literature [14, 15], we use the Jeans equation to relate the velocity dispersion and structure of the cluster to the shape of the gravitational potential. The Galactic center is unique in that we have measurements of three velocity components for each individual star as well as two dimensions in position. Thus, we extend the above methodologies to include the full three dimensional velocity phase space [16].

As mentioned above, in order to relate the velocity and positional measurements, we will use the spherically symmetric Jeans equation that allows for anisotropy:

$$ \frac{d(\rho_\star \sigma_r^2)}{dr} + 2\beta(r) \rho_\star \sigma_r^2 \frac{1}{r} = -G\rho_\star M(r) \frac{1}{r^2}, $$

(1)

where $\sigma_r$ is the dispersion in the radial direction (in the sense of spherical coordinates, not the radial velocity). The anisotropy, $\beta(r)$ is parameterized as $\beta(r) = \frac{\beta_0 + (r/r_\theta)^\eta}{1 + (r/r_\theta)^\eta}$, where $\beta_0$ is the inner anisotropy, $\beta_\infty$ is the outer anisotropy, $r_\theta$ is the location of the transition, and $\eta$ is the sharpness of the transition. The stellar spatial density profile, $\rho_\star(r)$ is defined to be a broken power law:

$$ \rho_\star \propto r^{-\gamma} \left(1 + (r/r_b)^\delta \right)^{(\gamma - \alpha)/\delta}, $$

(2)

where $\gamma$ is the inner power law slope, $\alpha$ is the outer slope, $r_b$ is the break radius, and $\delta$ is the sharpness of the transition between the two slopes. The mass profile $M(r)$ is defined to be a point source with the
mass of the black hole $M(r) = M_{BH}$; we find that including an extended stellar mass component does not significantly any of the resulting cluster parameters or the measured black hole mass, because our observations ($R < 0.5$ pc) are well within the sphere of influence of the black hole ($r_{inf} = 2$ pc). The total set of model parameters, including the bulk cluster velocity in cylindrical coordinates $\bar{v}_T$, $\bar{v}_R$, $\bar{v}_z$, and the normalization for the surface density profile $A$, are thus:

$$ \mathcal{M} = \{\bar{v}_T, \bar{v}_R, \bar{v}_z, A, r_b, \alpha, \delta, \gamma, r_\beta, \beta_\alpha, \eta, \beta_\beta, M_{BH}\}. $$

(3)

Given this set of model parameters, with the distance to the Galactic center fixed at $R_o = 8$ kpc, we use the Jeans equation to compute the radial dispersion and project it on the sky to compare to the observed velocity dispersions and their covariances. In order to avoid having to bin the data either radially or by velocity, thus losing information, we choose to compute the likelihoods for each source having an observed velocity vector $v$ and a projected distance from Sgr A* of $R$, given the set of model parameters $\mathcal{M}$. The probability density function (PDF) for each individual star is defined as:

$$ P(v, R | \mathcal{M}) \propto \frac{1}{\sqrt{|C(R)|}} \exp \left[ -\frac{(v - \bar{v}) \cdot C(R)^{-1} \cdot (v - \bar{v})}{2} \right]. $$

(4)

Where $C(R)$ is the covariance matrix for the velocity moments, including the errors for the three velocity components. The combined likelihood of the whole sample is thus:

$$ \mathcal{L}(\mathcal{M}) = \prod_i P(v_i, R_i | \mathcal{M}) $$

(5)

over the data points $\{v_i, R_i\}$. The $\mathcal{M}$ parameter space is explored in a Bayesian fashion utilizing a technique based on the MultiNest algorithm [17, 18]. Here, we assume the typical non-informative uniform priors on every parameter considered above.

6.1. Measuring positions and velocities

Since we would like to use the sample of stars with all three dimensions in velocity, we limit the kinematic analysis to only late-type stars with spectroscopy and radial velocity measurements with precision of $\leq 100$ km/s. This includes a sample of 248 stars. The proper motions ($v_x, v_y$) and positions ($x, y$) are from the results of Yelda et al. [19], which are determined from LGS AO images of the central $20'' \times 20''$ taken with NIRC2 instrument; the positions and velocities are measured in a reference frame defined by the radio masers [19]. The line of sight velocity measurements ($v_z$) are obtained by cross-correlation of the spectra with an M3III stellar template (HD40239) from the SPEX telescope [20]. These radial velocities are then corrected for the local standard of rest.

6.2. Results: spatial density profile, black hole mass, and anisotropy

Given the assumption of spherical symmetry, no rotation, and $R_o = 8$ kpc, we determine the posterior PDFs given the observed positions and velocities. The resulting PDFs for the inner slope, black hole mass, and inner anisotropy is shown in Figure 4. The 1 $\sigma$ central confidence intervals for the three parameters are: $\gamma = 0.4^{+0.2}_{-0.2}$, $M_{BH} = 4.0^{+0.5}_{-0.4} \times 10^6$ $M_\odot$, and $\beta_\alpha = -0.2^{+0.3}_{-0.5}$. With the kinematic measurements, $\gamma$ has been much better constrained than with the number counts alone (which was only able to place an upper limit of $\gamma < 1.0$ at the 3 $\sigma$ level). The kinematic modeling also shows that the black hole mass is consistent with that derived by using the stellar orbit measurement of the star S0-2 [e.g. 1, 2]. Because the data are limited to only 0.5 pc, a few parameters such as the break radius of the density profile and the outer density slope are only weakly constrained. The anisotropy indicates that the cluster is tangentially anisotropic within the radial range of our data set. In Do et al. (in prep.), we will explore these issues further. Here we limit our discussion to the implications of our spatial density measurement.
Figure 4. The posterior distribution for the inner slope (left), black hole mass (center), and the inner anisotropy. The line is the location of the central interval with the shaded regions encompassing 68\% or 1\%. The 1\% central confidence intervals for the three parameters are: \(\gamma = 0.4^{+0.2}_{-0.2}\), \(M_{BH} = 4.0^{+0.5}_{-0.4} \times 10^6\) M\(_\odot\), and \(\beta_o = -0.2^{+0.3}_{-0.3}\).

7. Implications for the dynamical history of the MW NSC and black hole mass measurements

Having a measured spatial density profile is an important step in understanding the current dynamical state of the inner regions of the MW nuclear star cluster. Merritt [21] showed that the elimination of highly-bound stellar orbits generically leads to a density profile \(\rho \sim r^{-0.5}\) at small radii if the distribution function is isotropic. Tightly bound orbits may be eliminated from the Galactic Center through the inspiral of an intermediate or a second supermassive black hole. A collision-dominated nucleus may also show density slopes as flat as \(\gamma = 0.5\) [22]. These predicted density-profile slopes are close to our central value of \(\gamma = 0.4\), and \(\beta = 0\) is within the 68\% credible interval. Thus, our results are consistent with the existence of a major dynamical event in the recent history of the Galactic Center.

Previous attempts at measuring the black hole mass from the star cluster have yielded systematically low black-hole mass estimates compared to the stellar orbit measurements [23–25]. These previous analyses did not always separate the old stars from the young, sometimes enforced isotropy of the stellar orbits, and often fixed the old-star density profile to \(\rho \sim r^{-1.8}\) or the entire stellar population to \(\rho \sim r^{-2.5}\). These effects drive the black-hole mass estimates to be too low (Do et al., in prep.). For example, Schödel et al. [26] showed that by fixing the stellar density profile to the observed old stars, they were able to obtain a black hole mass consistent with the individual stellar orbits using proper motion measurements. For the first time, our results show that given a uniform sample of stars, it is unnecessary to fix either the cluster density profile or the black hole mass to obtain a simultaneous fit for the cluster parameters and the black hole mass through Jeans modeling.

Acknowledgments

A.H.G.P is supported by a Gary McCue Fellowship through the Center for Cosmology at UC Irvine. The infrared data presented herein were obtained at the W. M. Keck Observatory, which is operated as a scientific partnership among the California Institute of Technology, the University of California and the National Aeronautics and Space Administration. The Observatory was made possible by the generous financial support of the W. M. Keck Foundation. The authors wish to recognize and acknowledge the very significant cultural role that the summit of Mauna Kea has always had within the indigenous Hawaiian community. We are most fortunate to have the opportunity to conduct observations from this mountain.

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