Manifestation of nuclear cluster structure in Coulomb sums

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Abstract

Experimental Coulomb sum values of \(^6\)Li and \(^7\)Li nuclei have been obtained, extending the earlier reported momentum transfer range of Coulomb sums for these nuclei up to \(q = 0.750 \div 1.625 \ \text{fm}^{-1}\). The dependence of the Coulomb sums on the momentum transfers of \(^6\)Li and \(^7\)Li is shown to differ substantially from similar dependences for all the other nuclei investigated. Relationship between the nuclear cluster structure and Coulomb sums has been considered. The momentum transfer value, above which the Coulomb sum becomes constant, is found to be related to the cluster isolation parameter \(x\), which characterizes the degree of nuclear clusterization.

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I. INTRODUCTION

In the double-differential cross-section for electron scattering by the nucleus, \( (d^2\sigma/d\Omega d\omega) \), the contributions from the electron-nucleus interaction may be separated by means of longitudinal and transverse components of the electromagnetic field. Accordingly, these contributions are called the longitudinal and transverse response functions \( R_L(q, \omega) \) and \( R_T(q, \omega) \), respectively. According to ref. [1], the double-differential cross-section is related to the response functions by the equation

\[
\frac{d^2\sigma}{d\Omega d\omega}(\theta, E_0, \omega) = \sigma_M(\theta, E_0) \times \left[ \frac{Q^4}{q^4} R_L(q, \omega) + \left( \frac{Q^2}{2q^2} + \tan^2\frac{\theta}{2} \right) R_T(q, \omega) \right], \tag{1}
\]

where \( \omega, q, Q = (q^2 - \omega^2)^{1/2} \) are, respectively, the energy, 3-momentum, 4-momentum transferred to the nucleus by the incident electron of initial energy \( E_0 \) and scattered by the angle \( \theta \); \( \sigma_M(\theta, E_0) = e^4 \cos^2(\theta/2)/[4E_0^2 \sin^4(\theta/2)] \) is the Mott cross-section; \( e \) is the electron charge.

In the treatment of the experimental data, one must take into account the influence of the nuclear electrostatic field on the incident electron. For this purpose, the correction \( \Delta E_0 \) is introduced into the definition of the 3-momentum transfer \( q = \{(4(E_0 + \Delta E_0)[(E_0 + \Delta E_0) - \omega] \sin^2(\theta/2) + \omega^2)^{1/2} \}. The correction \( \Delta E_0 \) is given by \( k(3/2)Ze^2/R \), where \( R \) is the radius of the equivalent homogeneous distribution. According to ref. [2], for electrons scattered by light nuclei to the continuum region the coefficient \( k \) is equal to 0.8.

The experimental data on the longitudinal functions \( R_L(q, \omega) \) are generally represented as Coulomb sums

\[
S_L(q) = \frac{1}{Z} \int_{\omega_0^+}^{\infty} \frac{R_L(q, \omega)}{\eta [G_E(Q^2)]^2} d\omega, \tag{2}
\]

where \( \tilde{G}_E(Q^2) \) is the charge form factors of the proton and the neutron, respectively; \( \eta = [1 + Q^2/(4M^2)] \times [1 + Q^2/(2M^2)]^{-1} \) is the correction for the relativistic effect of nucleon motion in the nucleus; \( M \) is the proton mass; \( G_E^n \) and \( G_E^p \) are the charge form factors of the proton and the neutron, respectively.

For all the nuclei studied, the behavior of \( S_L(q) \) with variations in the momentum transfer is similar in its character. With an increase in \( q \), the function \( S_L(q) \) increases until at a certain
momentum transfer value, denoted as $q_p$, the $S_L(q)$ takes on constant values forming the function $S_L(q)$ plateau. For almost all previously studied nuclei we have $q_p \approx 2\ \text{fm}^{-1}$. By way of illustration, Fig. 1 shows the experimental $S_L(q)$ values for the $^4\text{He}$ nucleus.

The authors of papers [6, 7] have determined $S_L(q)$ values for the $^6\text{Li}$ nucleus, and have found that the behavior of the function differs from the usual one (Fig. 1). It can be seen that the $S_L(q)$ function reaches the plateau at $q_p \approx 1.4\ \text{fm}^{-1}$, this being much earlier in $q$ than in the case with $^4\text{He}$ and other nuclei. In the $^7\text{Li}$ case, in the measurement range $q = 1.250 \div 1.625\ \text{fm}^{-1}$ (see ref. [8]), the function $S_L(q)$ is constant within the experimental error. It means that if the $S_L(q)$ value is lower at certain momentum transfers, then it will reach the plateau range at $q_p \leq 1.3\ \text{fm}^{-1}$. Thus, the data of ref. [8] do not specify $q_p$ for the $^7\text{Li}$ nucleus, but restrict its upper value. The authors of works [7, 8] have put forward the hypothesis that a comparatively low $q_p$ value in the $^6,^7\text{Li}$ case may be due to the Coulomb sum manifestation of clusterization peculiar to the nuclei under discussion.

However, on a more rigorous approach to the problem of relationship between the $q_p$ value and nuclear cluster structure it should be noted that this hypothesis is actually based only on the experimental $q_p$ value of the $^6\text{Li}$ nucleus. As regards the $q_p$ value of $^7\text{Li}$, from the data of [8] it follows that it is not higher than that of $^6\text{Li}$, and it is not improbable that it may be substantially lower. The last version would be in contrast with the proposed hypothesis, because if the $q_p$ value is related to the clusterization (and the nuclei $^6\text{Li}$ and
$^7\text{Li}$ are close in the degree of clusterization), then the $q_p$ values of these nuclei should also be little different from each other.

It follows from the above that for checking the hypothesis for the relationship between the nuclear cluster structure and the momentum transfer value $q_p$, it is necessary:

a) to determine the $q_p$ value for the $^7\text{Li}$ nucleus;

b) to define more exactly the $q_p$ value for the $^6\text{Li}$ nucleus;

c) to obtain the $q_p$ values for the previously investigated nuclei.

II. THE EXPERIMENT AND HANDLING OF THE MEASURED DATA

The measurements, from which the present $S_L(q)$ values were determined, were carried out at the experimental facility SP-95 with the use of the electron beam from the NSC KIPT electron linear accelerator LUE-300. The electron beam of monochromaticity between 0.4% and 0.6%, and of energies ranging from 104 to 259 MeV, was incident on the $^6\text{Li}$ (or $^7\text{Li}$) target, the isotopic enrichment of which in the nuclide of interest was determined to be 90.5% (or 93.8%), respectively. The measurements were performed at electron scattering angles from 34.2° up to 160°. For momentum analysis of scattered electrons we have used the spectrometer that had the second-order double focusing in vertical and horizontal planes [9]. Electrons in the focal plane of the spectrometer were registered by the 8-channel scintillation Cherenkov counter [10].

The experimental setup has been described a number of times in the literature, see e.g. [7, 8, 11, 12]. A detailed description of the measurements and the data processing is presented in refs. [3, 6, 7, 8].

The experiment was designed so that the response functions at several constant 3-momentum transfer values ranging from 0.750 to 1.625 fm$^{-1}$, and also, the Coulomb sums corresponding to these functions, could be obtained from the measurements. It should be mentioned that the most complicated and labor-consuming stage in these experiments is the processing of the measurement results for obtaining the response functions and the Coulomb sums. Taking into account the long duration of the processing, the work was planned so as to process first the data measured at the highest $q$ values, and then to process the data corresponding to lower momentum transfers. One of the advantages of this approach was
the point that if the processing of a part of the experimental data yielded the physical data of prime interest, they could be discussed and submitted for publication at once, without waiting for the final processing of the whole body of initial measured data.

At the previous stage of measured data processing, we have obtained in this way four $S_L(q)$ values for $^7\text{Li}$ at $q = 1.250 \div 1.625 \text{ fm}^{-1}$ [8], and five $S_L(q)$ values for $^6\text{Li}$ at $q = 1.125 \div 1.625 \text{ fm}^{-1}$ [7].

By the present time, in addition to the above-given values, we have obtained $S_L(q)$ values for $^7\text{Li}$ at $q = 0.750 \div 1.125 \text{ fm}^{-1}$ (Fig. 2), and preliminary $S_L(q)$ values for $^6\text{Li}$ at $q = 0.750 \div 1.000 \text{ fm}^{-1}$ (Fig. 3).

![Figure 2](image)

**FIG. 2.** Coulomb sum of $^7\text{Li}$. Full squares – $^7\text{Li}$ [8]; open squares – $^7\text{Li}$ (present data).

### III. NUCLEAR CLUSTER STRUCTURE AND THE COULOMB SUM

To analyse the relationship between the momentum transfer $q_p$ and the nuclear cluster structure, the $q_p$ value determination must be formalized using a certain simple procedure, which will be applied to the experimental $S_L(q)$ values of the nuclei under consideration. We define $q_p$ as the momentum transfer that corresponds to the point of intersection of two straight lines, one of which (horizontal) approximates the $S_L(q)$ values on the plateau of
FIG. 3. Coulomb sum of $^6$Li. Full circles – $^6$Li [7]; open circles – $^6$Li (present data).

$S_L(q)$, and the other line approximates the $S_L(q)$ values before reaching the plateau starting from $S_L \approx 2/3S_{L,p}$, where $S_{L,p}$ is the $S_L(q)$ value on the plateau. The given definition of the momentum transfer $q_p$ is exemplified by the $S_L(q)$ for the $^4$He nucleus (see Fig. 4).

We apply this definition of $q_p$ to all the nuclei having the atomic mass $A \geq 4$, for which a sufficient amount of experimental $S_L(q)$ data is known. These are the data of the present work and of our previous works on the nuclei $^6,^7$Li [6–8], $^4$He [3] and $^{12}$C [13]. Besides, from ref. [5], we have used the experimental $S_L(q)$ data obtained at the Saclay and Bates Laboratories for $^4$He, $^{12}$C, $^{40}$Ca, $^{48}$Ca, $^{56}$Fe. The momentum transfers $q_p$ derived from these data are shown in Fig. 5. It can be seen that the $q_p$ values of the nuclei $^4$He, $^{40}$Ca, $^{48}$Ca, $^{56}$Fe are grouped at $q_p = (1.9 \div 2.1)$ fm$^{-1}$, and in the case of $^6$Li and $^7$Li - at $q_p = (1.20 \div 1.35)$ fm$^{-1}$. For the $^{12}$C nucleus we have $q_p = 1.65$ fm$^{-1}$. The momentum $q_p$ grouping of the nuclei, observed in Fig. 5, corresponds to their distribution over the cluster isolation parameter $x$.\footnote{The parameter "x" defines the degree, to which the clusters are formed within the nucleus [14]. The x value varies from $x = 1$ (shell model, e.g., $^4$He) to $x = 0$ (limiting case of the cluster model).} The first-group nuclei are not clusterized, whereas the second-group nuclei are strongly clusterized. Thus, for the $^6$Li nucleus, the parameter $x$ varies between 0.3 and 0.4 [6, 14, 15], while for $^7$Li we have $x = 0.5$ [14]. With this approach, we arrive...
FIG. 4. Coulomb sum of $^4\text{He}$. Full circles – $^4\text{He}$ [3–5]; horizontal and inclined lines – data fitting; the intersection of the lines determines the $q_p$ value.

at understanding of the intermediate (between the two groups) value $q_p = 1.65$ fm$^{-1}$ of the $^{12}\text{C}$ nucleus, because this nucleus is though clusterized but to a less degree than the nuclei of the lithium isotopes. For the $^{12}\text{C}$ nucleus, the parameter $x$ ranges from 0.7 to 0.8 [14, 15].

Let us consider the momentum $q_p$ as a function of the parameter $x$. For this purpose we put the cluster isolation parameter of the nuclei $^4\text{He}$, $^{40}\text{Ca}$, $^{48}\text{Ca}$, $^{56}\text{Fe}$ to be equal to 1.0. As is obvious from Fig. 6, the $x$ dependence of $q_p$ is close to linear, this being in agreement with the result of fitting the straight line to all the data with the least $\chi^2$ value. Note that the observed dependence displays a high sensitivity of $q_p$ to the $x$ value. After refinement of $q_p(x)$, this feature of the function considered might be used for determination of $x$ from the $q_p$ value. However, because of the laborious procedure of obtaining experimental Coulomb sums, this method would be hardly applicable in practice.

IV. RESULTS AND CONCLUSIONS

The results of the present work can be summarized as follows.

2 In particular, more precise determination of $x$ values for the nuclei of lithium isotopes.
FIG. 5. Momentum transfers $q_p$ for different nuclei. The atomic mass $A$ is plotted on the axis of ordinates.

A. Experimental $S_L(q)$ values of the nuclei $^6\text{Li}$ and $^7\text{Li}$ have been obtained at momentum transfers $q = 0.750 \div 1.000$ fm$^{-1}$ and $q = 0.750 \div 1.125$ fm$^{-1}$, respectively. This has essentially extended the range of the measured $S_L(q)$ towards $q$ values lower than those investigated in refs. [7, 8].

B. Using the $S_L(q)$ data of the present work and of works [6–8], the momentum transfer $q_p$ has been determined for the $^7\text{Li}$ nucleus ($q_p = 1.20 \pm 0.10$ fm$^{-1}$), and has been redetermined more exactly for the $^6\text{Li}$ nucleus ($q_p = 1.35 \pm 0.10$ fm$^{-1}$). The analysis of the available literature data on the Coulomb sums for $^4\text{He}$, $^{12}\text{C}$, $^{40}\text{Ca}$, $^{48}\text{Ca}$, $^{56}\text{Fe}$ has yielded the $q_p$ values for the mentioned nuclei (see Fig. 5).

C. The momentum transfers $q_p$ of $^6,^7\text{Li}$ nuclei have been found to be much lower than those in the case of other nuclei.

The comparison of the present experimental data with the data obtained elsewhere for a number of nuclei has demonstrated the validity of the hypothesis of the manifestation of nuclear cluster structure in the Coulomb sum of the nucleus.
FIG. 6. Momentum transfer $q_p$ versus the cluster isolation parameter $x$ for different nuclei. The straight line represents the data fitting by the linear dependence.

The effect manifests itself in the observable proportionality of the momentum transfer $q_p$ to the cluster isolation parameter $x$, which characterizes the degree of nuclear clusterization. Besides, the hypothesis under discussion might be also supported provided that the Coulomb sums were measured for the $^9\text{Be}$ nucleus at $q = 0.8 \div 1.7$ fm$^{-1}$, from which the momentum $q_p$ of this nucleus can be derived. Since the parameter $x = 0.6$ [14], related to the $^9\text{Be}$ nucleus, lies between the $x$ values of the nuclei $^6\text{Li}$ and $^{12}\text{C}$, i.e., in the range from 1.3 fm$^{-1}$ to 1.6 fm$^{-1}$.

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