Research on the handling stability of four-wheel steering vehicle

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Abstract. Now days, people have higher and higher requirements on automobile performance. As an important performance of a vehicle, the handling stability not only affects the driver’s driving experience, but also relates to the safety of the occupants. In order to improve the handling stability of vehicles, a simulation of the four-wheel-steering system in the linear range based on the time domain was conducted in this paper. Sideslip angle and yaw rate were taken as control targets. Proportional feedback control and PID control were integrated in the control system. The comparison of the front-wheel-steering system and the four-wheel-steering system indicated that the four-wheel-steering system had obvious effect on improving the handling stability of the vehicle.

1. Introduction
At present, most cars use two front wheels for steering which has reached its limit for improving the handling stability of the vehicle. Therefore, the concept of four-wheel steering came into being[1]. 4WS first appeared in the 1920s. Along with the development of activity safety technology and related disciplines, it is now getting more and more attention. 4WS vehicles are equipped with two sets of steering mechanisms on two axels respectively to make the front and rear wheels participate in the steering simultaneously which enables the steering state of the vehicle to meet the requirements of Ackermann geometry to the maximum extent. Then the lateral motion characteristics of the vehicle can be controlled effectively and the need of good control stability of the vehicle under any working condition can be met[2-3]. In this paper, based on the two-DOF theoretical model of the automobile, the model of 4WS system was established by combining proportion control and PID control, and its influence on improving handling stability was analysed which provided more ideas and experience in this field.

2. Dynamical model
The 2-DOF model based on linear range is the basic model to study vehicle handling stability. Research shows that the precision of the 2-DOF model in the linear range is sufficient to reflect the basic characteristics of vehicle handling stability in most daily driving conditions, namely, small steering.
wheel angle, normal driving speed and small lateral acceleration[4-5]. The steering principle of the 2-DOF vehicle model is shown in figure 1.

![Figure 1. Steering principle of 4WS vehicle.](image)

According to figure 1 and Newton's second law, 2-DOF vehicle differential equation of motion in 4WS vehicle can be derived as equation (1).

\[
\begin{align*}
\sum F_Y &= (k_1 + k_2)\beta + \frac{1}{u}(a k_1 - b k_2)\omega_r - (k_1 \delta_1 + k_2 \delta_2) = m(\dot{v} + u \dot{\omega}_r) \\
\sum M_Z &= (a k_1 - b k_2)\beta + \frac{1}{u}(a^2 k_1 + b^2 k_2)\omega_r + b k_2 \delta_2 - a k_1 \delta_1 = I_z \dot{\omega}_r
\end{align*}
\]

(1)

Where, \(m\) is the mass of the car. \(I_z\) is the moment of inertia of the car around the Z axis. \(\omega_r\) is yaw rate, \(\beta\) is the sideslip angle. \(u\) is the driving speed. \(a, b\) are the distance from the front and rear axles to the center of mass. \(\delta_1, \delta_2\) are steering angle of front and rear wheels. \(k_1, k_2\) are cornering stiffness of front and rear wheels. \(F_Y\) is the resultant force in the Y direction. \(M_Z\) is the resultant moment of force in the Z direction.

For the convenience of modeling, equation (1) is arranged into the form of state equation. State variable is \(X=[\beta \ \omega_r \ \omega]^{T}\), input vector is \(U=[\delta_1 \ \delta_2]^{T}\), the system output vector is \(Y=[\beta \ \omega_r \ \omega_\gamma]^{T}\). Assuming that the car is traveling at a constant speed and the state equation can be obtained as

\[
\begin{align*}
\dot{\beta} &= \frac{k_1 + k_2}{mu} \beta + \frac{a k_1 - b k_2}{mu^2} \omega_r - \frac{k_1}{mu} \delta_1 - \frac{k_2}{mu} \delta_2 \\
\dot{\omega}_r &= \frac{1}{a k_1 - b k_2} \beta + \frac{a^2 k_1 + b^2 k_2}{a^2 k_1 + b^2 k_2} \omega_r \\
\dot{\omega} &= \frac{k_1 + k_2}{m} \beta + \frac{a k_1 - b k_2}{m} \omega_r
\end{align*}
\]

(2)

3. 4WS control system

The main parameters used in the simulation are as follows.

| m   | a       | b       | k_1          | k_2          | I_z        |
|-----|---------|---------|--------------|--------------|-----------|
| 1640kg | 1.105m  | 1.345m | -33020 Nrad\(^{-1}\) | -55830 Nrad\(^{-1}\) | 2720kgm\(^2\) |

In order to obtain better handling stability, two control targets are chosen in this paper. One is that the sideslip angle is zero, \(\beta=0\), which has three classic control strategies: front wheel angle proportional...
feed forward control (#1), yaw rate proportional feedback control (#2) and integrated control(#3)[6].

The simulation of these three control methods was carried out to observe the control effect on the sideslip angle of the target vehicle. The results are shown in figure 2.

![Figure 2. Response of under three control methods.](image)

In terms of the response of $\beta$, feed forward and integrated control cannot meet the control purpose which have a longer response time in the dynamic process and a large value of in the steady state. Only feedback control is suitable for the target vehicle. Therefore, the yaw rate proportion feedback was selected as the control strategy of sideslip angle and the principle of this control strategy can be expressed as[7]

$$\delta_i = k\omega_r$$

$$k = \frac{a(a_k - b_k) - (a^2k_1 + b^2k_2) - mu^2a}{k_2u(a + b)}$$

(3)

(4)

The other control target is to control the yaw rate to track the ideal yaw rate model, $\omega \rightarrow \omega_{\text{ideal}}$. In this paper, the yaw rate can reach the steady state rapidly is ideal, so it is written as a first-order inertial transfer function[8]

$$\omega(s) = \frac{g}{1 + \tau s} \delta_i(s)$$

(5)

Detailed equations are acquired by simultaneous equations (2) and (5)

$$g = \frac{u}{(1 + K_iu^2)(a+b)}$$

(6)

$$t = \left[ \frac{(a+b)^2k_2^2}{mu^2(1 + K_iu^2)} \right]^{1/2}$$

(7)

$$K_i = \frac{m}{(a+b)^2} \left( \frac{a}{k_2} - \frac{b}{k_1} \right)$$

(8)

The control strategy of yaw rate chooses the more practical PID control, the control effect of which is based on its three parameters: proportion coefficient, integral time constant and differential time constant. Here, trial and error method was selected for parameter setting and PID controller turned out to have a good control effect[9-10].

According to the mathematical expression of 4WS system and control strategy, a complete model of 4WS vehicle system was built based on Matlab/Simulink in figure 3.
4. Results analysis

The 2WS system was also constructed so as to demonstrate the superiority of the 4WS system by comparison. In the simulation, two working conditions which are 30km/h and 100km/h were considered and the input angle of the front wheel is 5°. The response of 2WS and 4WS system in the time domain under step input are as follows.

According to figure 4, when 2WS vehicle reaches steady state, the value of sideslip angle is not equal to zero. Especially in high speed condition, the sideslip angle is too big to stabilize the vehicle. There are risks of sideslip and drift which will threaten the safety of the occupants. However, 4WS vehicle can make sure the sideslip is equal to zero when it reaches steady state. Besides, the response time of sideslip is shortened in high speed so the 4WS turns more quickly which means 4WS has better manoeuvrability.
As can be seen from the above figure 5, under the low speed condition, the steady-state value of yaw rate in 4WS system is larger than that of 2WS. That is, when two cars pass the same curve, the steering wheel input of the 4WS is smaller, making the steering more flexible. From the formula: \( r = \frac{u}{\omega} \), we also know that with a certain speed, increasing \( \omega \) can reduce the steering radius \( r \), so the mobility of 4WS during low-speed steering can be improved. When the car moves at high speed, the change in \( \omega \) is the opposite of that at low speed. With the same wheel angle input, \( \omega \) of 4WS decreases as it reaches equilibrium, resulting in the decrease of the yaw rate gain of 4WS which causes a certain understeer, so the risk of drift at high speed could be reduced. In addition, the increase of high-speed steering radius ensures that the car body is consistent with the speed direction to the maximum extent, improving the driving safety. With respect to dynamics process, the adjustment time and the overshoot of 4WS system are greatly reduced in high speed condition, making the steering to achieve steady state aster and smoother.

According to the lateral acceleration formula: \( a_y = \dot{v} + u \omega \), the trend of \( a_y \) is roughly the same with the yaw rate’s in steady state which is validated by the response curve. In terms of dynamic process, the overshoot of 4WS system is lower than that of 2WS under two working conditions, showing that 4WS
system runs smoothly. Moreover, the stabilization time of 4WS system is obviously low at high speed so the system responds faster.

5. Conclusion
In this paper, based on the front wheel steering system, the model of 4WS vehicle was established by adding the proportional feedback of yaw and PID control system. Take sideslip angle, yaw rate and lateral acceleration as response indicators, the handling stability of 2WS and 4WS systems were compared through simulation comparison. It turned out that 4WS vehicle has better handling stability which can increase the mobility of the car during low speed steering and improve the driving safety during high speed steering. So the control strategy in this paper has some reference significance to the research of 4WS system.

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