The Solar $pp$ and $hep$ Processes in Effective Field Theory

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Abstract. The strategy of modern effective field theory is exploited to pin down accurately the flux $S$ factors for the $pp$ and $hep$ processes in the Sun. The technique used is to combine the high accuracy established in few-nucleon systems of the “standard nuclear physics approach” (SNPA) and the systematic power counting of chiral perturbation theory (ChPT) into a consistent effective field theory framework. Using highly accurate wave functions obtained in the SNPA and working to $N^3$LO in the chiral counting for the current, we make totally parameter-free and error-controlled predictions for the $pp$ and $hep$ processes in the Sun.

In this talk, we report on the result of the program sketched in [2] which was made possible by the collaboration with L.E. Marcucci, R. Schiavilla, M. Viviani, A. Kievsky and S. Rosati, recently summarized in [3].

INTRODUCTION

One of the ultimate goals of quantum chromodynamics (QCD) in nuclear physics is to make precise model-independent predictions for certain processes that figure importantly in astrophysics and cosmology. The only technique presently available to achieve such goals in the low-energy domain is effective field theory (EFT) [1]. In this talk, we apply a variant of nuclear EFT developed by the authors to the solar $pp$ and $hep$ processes within the framework sketched in [2]. In doing so, we rely on the accurate wave functions constructed by Marcucci et al [4] as the essential ingredient of EFT coming from the standard nuclear physics approach (SNPA). The power of the proposed scheme is the ability to correlate the beta decay processes of $A=2, 3, 4$ nuclei that allows us to fix one unknown constant in the theory, rendering possible a totally parameter-free prediction of the two-nucleon and four-nucleon processes.

The most abundant source of solar neutrinos (carrying 91% of the total flux) is the $pp$ process

$$p + p ightarrow d + e^+ + \nu.$$  \hspace{1cm} (1)

This process has been carefully studied [5, 6, 7, 8], and the calculated transition strength, governed by the leading-order Gamow-Teller (GT) operator, is believed to be reasonably reliable. However, given its extremely important role in the solar burning process, the $pp$ process invites further elaborate studies. Meanwhile, the $hep$ process

$$^3\text{He} + p ightarrow ^4\text{He} + e^+ + \nu.$$  \hspace{1cm} (2)

produces the highest energy solar neutrinos, $E_{\nu}^{\text{max}}(\text{hep}) = 20$ MeV. While the $hep$ neutrino flux is estimated to be much smaller than the $^8\text{B}$ neutrino flux, there can be significant distortion of the $^8\text{B}$ neutrino spectrum at its higher end if the $hep$ $S$-factor is much larger than the existing estimates. This change can influence the interpretation of the results of a recent Super-Kamiokande experiment that have raised many important issues concerning the solar neutrino problem and neutrino oscillations [9, 10]. To address these issues, a reliable estimate of the $hep$ cross section is indispensable. Its accurate evaluation, however, has been a long-standing challenge for nuclear and hadron physics [11]. The difficulty involved is reflected in the pronounced variance in the documented estimates of the $hep$ $S$-factor. For example, the first estimate given by Salpeter [12] was $S(\text{hep}) = 630 \times 10^{-20}$ keV-b, which was eventually
replaced by the much smaller (so-called “standard”) value, \( \simeq 2 \times 10^{-20} \) keV-b [13, 14]; the latest, most elaborate estimation gives \( 9.64 \times 10^{-20} \) keV-b [4]. The reason for the difficulty in making a precise estimation of the hep S-factor is multifold. First, the one-body (1B) GT matrix element for the hep process is strongly suppressed due to the symmetry properties of the orbital wave functions of the initial and final states. The main orbital wave function for \(^4\)He has [4] symmetry (totally symmetric) under the particle exchanges, whereas the dominant \(^3\)He + \( p \) orbital wavefunction has [31] symmetry; the [4] component is forbidden by the Pauli principle when there are three protons. Then, the main components of the initial and final states – with different symmetry properties in orbital space – cannot be connected by the leading-order (LO) GT operator, which does not contain orbital variables. This means that the non-vanishing 1B GT matrix element (for the hep process) is due to either minor components of the wavefunctions or higher order corrections of the GT operators. Since these are all quite small, the 1B matrix element becomes comparable to multi-body corrections, e.g., meson-exchange-current (MEC) contributions. A further complicating feature is that there is a substantial cancellation between the 1B and two-body (MEC) contributions, which can amplify the errors. Finally, with the “chiral-filter mechanism” rendered ineffective, at non-vanishing leading order of effective field theory (see below), the many-body corrections contain short-ranged operators the strengths of which are not known a priori and hence difficult to control.

The objective of our present work is to make accurate effective field theory (EFT) predictions on the pp and hep processes within a single framework. For this purpose, we adopt the strategy that exploits the power of both SNPA and heavy-baryon chiral perturbation theory (HBChPT), which is a well-studied low-energy EFT. In this approach, EFT enters in the calculation of relevant transition operators. We will calculate them up to next-to-next-to-next-to-leading order (N\(^3\)LO); all the operators that appear up to N\(^3\)LO will be considered. To obtain the corresponding nuclear matrix elements, we need highly accurate nuclear wave functions. Although it is – at least in principle – possible to also derive nuclear wave functions to the appropriate order from HBChPT, we choose not to do so. Instead, we use realistic wave functions obtained in the standard nuclear physics approach (SNPA). The potentials that generate such wave functions are supposed to contain high orders in the chiral counting, presumably much higher than what can be accounted for in the irreducible vertex for the current operators. For a review of SNPA, see Ref. [15]. Such an approach – which is close in spirit to Weinberg’s original scheme [16] based on the chiral expansion of “irreducible terms” – has been found to have an amazing predictive power for the \( n + p \rightarrow d + \gamma \) process [17, 18, 19].

The basic advantage of EFT is that the SNPA and HBChPT can be combined into a model-independent framework based on the first principle. A systematic expansion scheme of EFT reveals that, for the GT transition for which the “chiral-filter mechanism” [2] is rendered inoperative, the one-pion-exchange (OPE) operators and the leading short-ranged operators have the same chiral order, and hence their contributions should be comparable. This nullifies the intuitive argument that the long-ranged OPE contribution should dominate the MEC corrections. The existing SNPA calculation, however, lacks this leading short-ranged operator while containing instead some higher order short-ranged contributions. What our EFT manages to do is to account for this short-ranged contribution in a way consistent with renormalization group invariance. This is a novel way of understanding the so-called “short-range correlation” in SNPA. We will see that this aspect indeed plays an essential role both for pp and hep processes.

Briefly, our approach to HBChPT is as follows. We take only pions and nucleons as pertinent degrees of freedom. All others have been integrated out, and their dynamical roles are embedded in the higher-order operators. In the scheme relevant to us, it suffices to focus on “irreducible graphs” according to Weinberg’s classification [16]. Graphs are classified by the chiral power index \( v \) given by \( v = 2(A - C) + 2L + \sum \nu_i, \) where \( A \) is the number of nucleons involved in the process, \( C \) the number of disconnected parts, and \( L \) the number of loops. The chiral index, \( \nu_i, \) of the \( i \)-th vertex is given by \( \nu_i = d_i + e_i + n_i/2 - 2, \) where \( d_i, e_i \) and \( n_i \) are respectively the numbers of derivatives, external fields and nucleon lines belonging to the vertex. The Feynman diagrams with a chiral index \( v \) are suppressed by \( (Q/\Lambda_\chi)^v \) compared with the leading-order one-body GT operator, with \( Q \) standing for the typical three-momentum scale and/or the pion mass, and \( \Lambda_\chi \sim m_N \sim 4\pi f_\pi \) is the chiral scale. The physical amplitude is then expanded with respect to \( v. \)

**THEORY: GT OPERATORS UP TO N\(^3\)LO**

The LO and N\(^2\)LO contributions come from the well-known one-body currents; the GT operator for the \( a \)-th isospin component reads

\[
A_{1B}^a = g_A \sum_{l=1,2} \sum_{j=1}^4 \frac{\bar{v}_j}{2} \left[ \sigma_i + \frac{\bar{p}_j \sigma_j - \sigma_j \bar{p}_j}{2m_N^2} \right],
\]

(3)
with \( \mathbf{p}_f \equiv (\mathbf{p}_i + \mathbf{p}_f')/2 \) and \( g_A \simeq 1.267 \). Corrections to the above 1B operators are due to MECs, which start at \( N^3\text{LO} \). In our work we include all the contributions up to \( N^4\text{LO} \). We emphasize in particular that, up to \( N^3\text{LO} \), only two-body (2B) currents enter, three-body currents appearing only from \( N^4\text{LO} \); thus there is no arbitrary truncation involved here. The \( N^3\text{LO} \) 2B currents consist of the OPE and contact-term (CT) parts,

\[
A_{2B}^\alpha = A_{2B}^{\alpha\text{OPE}} + A_{2B}^{\alpha\text{CT}}.
\]

(4)

The OPE part is given as

\[
A_{2B}^{\alpha\text{OPE}} = -\frac{g_A}{2m_N f_N^2} \frac{1}{m_N^2 + q^2} \left[ -\frac{i}{2} \tau_3^\alpha \mathbf{p} \mathbf{q} + 2 \hat{c}_3 \mathbf{q} \cdot \mathbf{q} \right] + \left( \hat{c}_4 + \frac{1}{4} \right) \tau_3^\alpha \mathbf{q} \times [\sigma \times \mathbf{q}]
\]

(5)

where \( \tau_3^\alpha \equiv (\tau_1 \otimes \tau_2)^\alpha \), with \( \otimes = \times, - \), and similarly for \( \sigma \). Since the couplings \( \hat{c}_{3,4} \) are determined from the \( \pi N \) data [20], \( \hat{c}_3 = -3.66 \pm 0.08 \) and \( \hat{c}_4 = 2.11 \pm 0.08 \), there is no unknown parameter here. Now the CT part is given as

\[
A_{2B}^{\alpha\text{CT}} = -\frac{g_A}{m_N f_N^2} \left[ d_1 (\tau_1^3 \sigma_1 + \tau_2^3 \sigma_2) + d_2 \tau_3^\alpha \sigma \times \right],
\]

(6)

which contains two low-energy constants. However, the Pauli principle (or the “\( L+S+T = \text{odd} \)” rule) combined with the fact that the CT term is effective only for s-wave \( (L = 0) \) implies that we need only work with one unknown constant, \( d^R \), defined by

\[
d^R \equiv d_1 + 2d_2 + \frac{1}{3} \hat{c}_3 + \frac{2}{3} \hat{c}_4 + \frac{1}{6}.
\]

(7)

Furthermore, the same combination also enters into tritium \( \beta \)-decay, \( \mu \)-capture on deuteron, and \( \nu - d \) scattering. Although \( d^R \) is in principle calculable from QCD for a given scale \( \Lambda \), this calculation is not available at present; we therefore need to fix \( d^R \) by fitting the tritium \( \beta \)-decay rate, \( \Gamma_\beta \), which is accurately known experimentally [7]. Once \( d^R \) is fixed, our calculation involves no unknown parameters.

We calculate the matrix elements of the transition operators with state-of-the-art realistic nuclear wave functions for \( A = 2, 3, 4 \). We employ the correlated-hyperspherical-harmonics (CHH) wave functions, obtained with the Argonne \( v_{18} \) (Av18) potential (supplemented with the Urbana-IX three-nucleon potential for the \( A \geq 3 \) nuclei). To control short-range physics in a consistent manner, we apply the same regularization method to all the nuclear systems in question. Specifically, in performing Fourier transformation to derive the \( r \)-space representation of transition operators, we use the Gaussian regularization. This is equivalent to replacing the delta and Yukawa functions with the regularized ones,

\[
\left( \delta^{(3)}(r), \phi^{(3)} \right) \equiv \int \frac{d^3q}{(2\pi)^3} S^{\Lambda}(q^2) e^{iq} \left( 1, \frac{1}{q^2 + m_N^2} \right),
\]

where the cut-off function \( S^{\Lambda}(q^2) \) is defined as

\[
S^{\Lambda}(q^2) = \exp \left( -\frac{q^2}{2\Lambda^2} \right).
\]

(8)

The cutoff parameter \( \Lambda \) characterizes the energy-momentum scale of our EFT.

RESULTS

The value of \( d^R \) determined from the experimental value of \( \Gamma_\beta \) is

\[
d^R = (1.00 \pm 0.07, 1.78 \pm 0.08, 3.90 \pm 0.10)
\]

(9)

for the choice of \( \Lambda = (500, 600, 800) \) MeV, respectively. We list in Table 1 the GT matrix elements for the pp and hep processes (in arbitrary units) as a function of \( \Lambda \).
TABLE 1. GT matrix element for the $pp$ and $hep$ processes, calculated for representative values of $\Lambda$. The 2B contribution is the sum of the OPE part ($\delta^R$-independent) and the CT part (linear in $\delta^R$), for each case.

| $\Lambda$ (MeV) | $(1B)_{pp}$ | $(2B)_{pp}$ | $(1B)_{hep}$ | $(2B)_{hep}$ |
|---------------|------------|-------------|--------------|-------------|
| 500           | 4.82       | 0.076 - 0.035 $\delta^R \simeq 0.041$ | -0.81        | 0.93 - 0.44 $\delta^R \simeq 0.49$ |
| 600           | 4.82       | 0.097 - 0.031 $\delta^R \simeq 0.042$ | -0.81        | 1.22 - 0.39 $\delta^R \simeq 0.52$ |
| 800           | 4.82       | 0.129 - 0.022 $\delta^R \simeq 0.042$ | -0.81        | 1.66 - 0.27 $\delta^R \simeq 0.59$ |

The $pp$ results

We observe that, while the OPE part by itself has a sizable $\Lambda$-dependence, the net amplitude is completely $\Lambda$-independent. In other words, the $\Lambda$-dependence of the OPE part has been perfectly removed by that of CT part. The $\Lambda$-independence of the physical quantity, $\langle 1B \rangle + \langle 2B \rangle$, which is in conformity with the general tenet of EFT, is a crucial feature of the result in our present study.

The relative strength of the two-body contribution as compared with the one-body contribution is

$$\delta_{2B}^{pp} \equiv 2\langle B \rangle_{pp}/\langle 1B \rangle_{pp} = (0.86 \pm 0.05)\%.$$  \hspace{1cm} (10)

This ratio is consistent with the latest SNPA calculation \cite{7}, $\delta_{2B}^{pp} = (0.5 \sim 0.8)\%$. The resulting $pp$ $S$-factor is

$$S(pp) = 3.94 \times (1 \pm 0.15\% \pm 0.1\%) \times 10^{-25} \text{ MeV-barn},$$  \hspace{1cm} (11)

where the first and the second uncertainties come from one- and two-body contributions, respectively.

The $hep$ results

The general tendency here is quite similar to the $pp$ case; the variation of the two-body GT amplitude is only $\sim 10\%$ for the entire range of $\Lambda$ under study. The $\Lambda$-dependence in the total GT amplitude becomes more pronounced by a strong cancellation between the 1B and 2B terms, but this amplified $\Lambda$-dependence still lies within acceptable levels.

Table 2 shows the contribution to the $S$-factor from each initial channel, at zero c.m. energy. For comparison we have also listed the latest results based on SNPA \cite{4} (which we refer to as MSVKRB). It is noteworthy that for all the channels other than $^3S_1$, the $\Lambda$-dependence is very small ($< 2\%$). While the GT terms are dominant, the contribution of the axial-charge term in the $^3S_1$ channel is sizable even though it is kinematically suppressed by the factor $q$.

The results given in Table 2 lead to a much improved estimate of the $hep S$-factor:

$$S(hep) = (8.6 \pm 1.3) \times 10^{-20} \text{ keV-b},$$  \hspace{1cm} (12)

where the “error” spans the range of the $\Lambda$-dependence for $\Lambda = 500 – 800$ MeV. This result is to be compared to that obtained by MSVKRB \cite{4}, $S = 9.64 \times 10^{-20}$ keV-b. To decrease the uncertainty in Eq.(12), we need to reduce the $\Lambda$-dependence in the two-body GT term. According to a general tenet of EFT, the $\Lambda$-dependence should diminish when higher order terms are included. A preliminary study indicates that it is indeed possible to reduce the $\Lambda$-dependence significantly by including N$^\infty$LO corrections.
DISCUSSION

By determining the only parameter of the theory \( \hat{d} R \) from the experimental data on triton beta decay, we have succeeded in making rather accurate EFT predictions (up to \( N^3 \)LO) in a parameter-free manner for both \( pp \) and \( hep \) processes. These predictions turn out to give support to the latest SNPA results.

The prediction for the \( pp \) prediction comes out to be independent of the cutoff scale \( \Lambda \), which means that it is fully consistent with the tenet of EFT. On the other hand, there remains some \( \Lambda \)-dependence for the \( hep \) process, which could be due to many-body nature absent in the \( pp \) case. Even so, it is remarkable that the theoretical uncertainty can be reduced from “orders of magnitude” to \( \sim 20 \% \). This uncertainty can be further reduced if \( N^4 \)LO terms – which involve no additional unknowns – are taken into account.

We should note that by using the “exact” wave functions, we are sacrificing the strict adherence to chiral order counting in favor of predictivity. The counting error committed therein comes at one order higher than that accounted for in the irreducible vertex for the current, i.e., at \( N^4 \)LO in the present calculation, and should be small for the whole scheme to make sense. This can be checked by an \( N^4 \)LO calculation for which the counting error would come at \( N^5 \)LO or higher. Furthermore, the notion implicit in our approach that the 1B matrix element calculated with the “exact” wave functions should be taken as “empirical” could be checked by looking at the \( hen \) process:

\[ ^3\text{He} + n \rightarrow ^4\text{He} + \gamma \]

in which the 1B matrix element is expected to suffer the same suppression due to the symmetry properties of the initial and final states.

It seems likely that an EFT calculation of the \( hep \) process that adheres to the strict power counting – such as the pionless theory [21] – would be obstructed by a plethora of unknown parameters that are difficult to completely control. If such a calculation is feasible, however, it would be interesting to see if and how the symmetry suppression and the sensitive cancellation encountered in our version of EFT – where an accurate 1B matrix element plays a key role – could either be circumvented or manifest themselves in the description.

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