An efficient branch and bound algorithm for direct model predictive control of boost converter

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Abstract: The algorithmic efficiency offered by sphere decoding when compared with enumeration technique makes it a better choice for direct model predictive control of power converters. However, the application of sphere decoding algorithm requires that the converter’s model should have the same state transition and input matrices irrespective of switch position and continuous/discontinuous conduction mode. Whereas, in power electronics we usually encounter such models, e.g. the model of the dc-dc boost converter. In this paper, we propose a branch and bound control algorithm inspired from sphere decoding for the current control of boost converter. The simulation results, verified by experimental results show that the proposed control algorithm produces same results as enumeration algorithm while being computationally efficient.

Keywords: current control, dc-dc power converters, finite control set, model predictive control (MPC), boost converter, sphere decoding

Classification: Power devices and circuits

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1 Introduction

Power electronic converters are being extensively used in various applications. The control method employed in a converter has an important implication on its performance. Direct model predictive control (DMPC) is a recent and one of the most efficient control techniques for power converters [1, 2]. In DMPC, instead of computing a duty ratio, the optimal switch position is directly computed and applied to the converter. This avoids the use of a modulator. Moreover, constraints, such as peak inductor current, can also be incorporated explicitly in DMPC. Since the number of possible switch positions are finite, the optimization problem in DMPC could be solved by enumeration. However, a major disadvantage of enumeration is its high computational cost, especially in the case of long prediction horizon, that could limit the practical use of DMPC [3].

Reducing the computational complexity of enumeration based DMPC has been the focus of several research articles [4, 5, 11]. Sphere decoding algorithm (SDA) has been shown to significantly reduce the computations while providing the same control action as the enumeration algorithm [6, 12]. Sphere decoding has been applied to several converters, including the neutral point clamped inverter [6], grid connected converter [7], and cascaded H-Bridge inverter [8]. However, a limitation of the sphere decoding based DMPC is that it is only applicable to converters that can be modeled with a linear state space model with the same state transition matrix.
and input matrix across all possible switching states, such as the models of the converters in [6, 7, 8]. Whereas, in power electronics we commonly encounter switched linear models, in which the state transition and input matrix are different for different switch positions.

The discrete-time model of dc-dc boost converter has three modes of operation, depending on the switch position and continuous conduction mode (CCM) or discontinuous conduction mode (DCM) [9]. The state space matrices for each mode are different [9]. Therefore, sphere decoding algorithm cannot be applied with this model.

This paper proposes a branch and bound control algorithm inspired from sphere decoding algorithm for the DMPC of the boost converter. It has been achieved by a novel state space model and a feedback control law. The modified state space model has the same matrices for all the three modes of operation of the converter. Moreover, the state space model together with control law can represent all modes of operation. The details are provided in Section 2.

In Section 3, the DMPC problem is formulated for the state-space model. The proposed control algorithm is discussed in Section 4. The simulation and experimental results are given in Section 5.

2 Proposed model of the boost converter

![Boost converter topology](image)

The dc-dc boost converter is shown in Fig. 1, where $S$ is the controllable switch, $D$ is a diode, $R_L$ is the resistance of inductance $L$, $C_o$ is the output capacitor, $R$ is the load resistance, $i_L$ is the inductor current, and $v_o$ is the output voltage. A continuous-time and discrete-time model of the boost converter is given in [9]. The discrete-time model, with a sample time $T_s$, which caters for the three modes of operation of the converter as summarized in Table I.

To allow the application of proposed algorithm to DMPC for the current control of dc-dc boost converter, we propose a model with the same matrices in all the modes. However, the presented model doesn’t represent the dc-dc boost converter in open loop. The model only represents the converter in closed loop with the feedback control law that we have proposed.

Consider the continuous-time state space equations:

\[
\dot{x}(t) = Ax(t) + Bu(t),
\]

\[
y(t) = Cx(t),
\]
\[ A = \begin{bmatrix} -R_L/L & 0 \\ 0 & -1/RC \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -1/L & 1/L \\ 1/RC & 0 & 0 \end{bmatrix}, \quad C = [1 \ 0], \]

\[ x(t) \triangleq [i_L(t) \ v_o(t)]^T \] is the state vector, \( u(t) \in \mathbb{R}^3 \) is the input, and \( y(t) \) is the output. The output \( y(t) \) is the inductor current, which needs to be controlled. The voltage of the boost converter can also be controlled indirectly by setting a reference of the inductor current based on the voltage reference \[9\]. The above continuous-time equations can be discretized by the forward Euler approximation to obtain

\[ x_{k+1} = Ex_k + Fu_k, \quad (2a) \]
\[ y_k = Gx_k, \quad (2b) \]

where \( E = I + AT_s \), \( I \) is the identity matrix, \( F = BT_s \), and \( G = C \).

Now consider the following feedback control law:

\[ u_k = ccm_k \begin{bmatrix} (1 - s_k)x_k \\ v_g \end{bmatrix}, \quad (3) \]

where \( s_k \) is the position of the switch \( S \) at time instant \( k \), which is 1 or 0 if the switch \( S \) is ON or OFF, respectively, and value of \( ccm_k \) represents whether the converter is operating in CCM or DCM. The value of \( ccm_k \) can be determined using the switch position \( s_k \) and the state of the converter \( x_k \) \[9\].

The discrete-time state space equations (2) along with the feedback control law (3) can represent the three modes of operation of the dc-dc boost converter. In mode 1, \( s_k = 1 \) and \( ccm_k = 1 \); in mode 2, \( s_k = 0 \) and \( ccm_k = 1 \); and in mode 3, \( s_k = 0 \) and \( ccm_k = 0 \).

### Table I. Modes of operation of the dc-dc boost converter

| Mode | Description                  |
|------|------------------------------|
| 1    | \( S \) is ON and converter is in CCM |
| 2    | \( S \) is OFF and converter is in CCM |
| 3    | \( S \) is OFF and converter is in DCM |

3 Model predictive control

With the state-space model (2) and the feedback control law (3), the optimization problem to be solved in direct model predictive control \[9\] can be stated as follows:

\[ S_{opt,k} = \arg\min_{S_k} \sum_{i=k}^{k+N-1} \| i_{o,i+1} \|^2 + \lambda_u \| \Delta s_i \|^2, \quad (4a) \]

subject to \( x_{k+1} = Ex_k + Fu_k, \quad (4b) \)

\[ u_k = ccm_k \begin{bmatrix} (1 - s_k)x_k \\ v_g \end{bmatrix}, \quad (4c) \]

\[ s_k \in \{0, 1\}, \quad (4d) \]

where \( i_c \triangleq i_{L,ref} - i_L \) is the current error, \( \Delta s_i \triangleq s_i - s_{i-1} \) represents change in switch positions, \( \lambda_u \) is the weighting factor, and \( S_k \triangleq \{ s_k, s_{k+1}, \ldots, s_{k+N-1} \} \) is the sequence.
of switch positions. The reference value of inductor current $i_{L,\text{ref}}$ is computed using the formula that relates the desired output voltage to reference current value, thus, establishing indirect voltage control [9].

The weighting factor $\lambda_u$ in DMPC is often kept to be very small. If we assume that $\lambda_u$ is zero, the optimization problem (4) can be stated as follows [6]:

$$S_{\text{opt},k} = \text{argmin}_{S_k} \|HU_k - U_{\text{unc},k}\|_2^2$$

subject to $u_k = ccm_k \left[ (1 - s_k)v_k \right]$, (5b)

$$s_k \in \{0, 1\}, \ (5c)$$

where $H \in \mathbb{R}^{3N \times 3N}$ denotes a lower triangular matrix (details in [6]), $U_{\text{unc},k} \in \mathbb{R}^{3N}$ denotes the unconstrained solution to the optimal control problem, and $U_k \triangleq [u_k^T \ u_{k+1}^T \ldots u_{k+N-1}^T]^T$. Other constraints if any, such as a limit on the peak value of inductor current, can also be added to problem (5a) provided they can be expressed in terms of $u_k$.

4 Proposed control algorithm

The proposed control algorithm is a branch and bound technique inspired from sphere decoding technique [10]. Besides some difference outlined in the next paragraph, the optimal control problem formulation follows similar steps as in sphere decoding mentioned in [6]. The control algorithm solves the optimal control problem by performing depth first search, allowing only those input sequences that lie inside a hypersphere centered at $U_{\text{unc},k}$ and of radius $\rho$. A possible choice of the initial radius is $\rho = \|HU_{\text{ini},k} - \tilde{U}_{\text{unc},k}\|_2$, where $U_{\text{ini},k}$ is an initial guess that can be obtained by a warm start strategy [6].

Algorithm 1 Proposed Control Algorithm

1: function $S_{\text{opt},k} = \text{PCA}(S_k, d^2, i, \rho^2, U_{\text{unc},k}, x_{k+1})$
2: for each $s \in \{0, 1\}$ do
3: if $s_{k+1} == 1$ then
4: $ccm_{k+1} = 1$
5: else
6: calculate $ccm_{k+1}$; details in [9]
7: end if
8: calculate $u_{k+1}$ according to eq. (3)
9: $d^2 = \|U_{\text{unc},k}[3i : 3i + 2] - H[3i : 3i + 2, 0 : 3i - 1][U[0 : 3i - 1]]\|_2^2 + d^2$
10: if $d^2 \leq \rho^2$ then
11: if $i < N - 1$ then
12: calculate $x_{k+1}$ according to eq. (2a)
13: PCA($S_k, d^2, i + 1, \rho^2, U_{\text{unc},k}, x_{k+1}$)
14: else
15: $S_{\text{opt},k} = S_k$
16: $\rho^2 = d^2$
17: end if
18: end if
19: end for
20: end function
The optimization problem (5) is different from the one reported in [6] due to the reason that the control input is a function of the state. Therefore, some changes are required in the algorithm. The proposed algorithm is given in Algorithm 1. It is invoked by

\[ S_{\text{opt},k} = \text{PCA}(\cdot, 0, 0, \rho^{2}, U_{\text{unc},k}, x_{k}) \].

The main computational advantage of proposed algorithm is that a much lower number of nodes are visited in comparison to \((2^{N+1} - 2)\) nodes in enumeration; thus making it more efficient. Moreover, the calculation of input and prediction of state also has to be done in enumeration for each of the \((2^{N+1} - 2)\) nodes.

A major difference in the proposed algorithm, as compared to sphere decoding, is the dependence of control input \(u_{k}\) on the state. Consequently, a lattice in the input space will not exist, which is fundamental in sphere decoding. However, by proposing a model of the boost converter that has constant state matrices in all the three modes, we are able to formulate the optimization problem (5) with a triangular \(H\). The triangular structure reduces the computational complexity and facilitates the implementation of branch and bound technique.

It is important to note that the proposed technique and the technique in [9] solve the same DMPC problem. However, our reformulation and algorithm helps to reduce the computational complexity, which is an important factor in DMPC.

5 Simulation and experimental results

In this section we provide the simulation and experimental results of the proposed technique for the boost converter. The parameter values used for the boost converter are shown in Table II. The value of voltage error weight \(h\) determines the transient response of the system [9].

The simulation result of the capacitor voltage and the inductor current in Fig. 2 shows the behavior of the converter for start-up, step-up change in output voltage reference, and step-down change in voltage reference. As mentioned earlier, both the proposed technique and the enumeration algorithm [9] solve the same optimal control problem. Therefore, for both techniques we have the same simulation results. However, the difference is in the computational complexity that is evident from Table III and Table IV.

Table III compares the number of nodes visited for enumeration and proposed algorithm, while Table IV compares the number of complete switching sequences explored for various values of \(N\). The values for both tables are obtained for parameters shown in Table II by initializing the radius using warm-start strategy [6, 9]. In case of enumeration, a fixed number of nodes are visited and sequences are explored at each time sample. Whereas, they vary in the proposed algorithm. For all values of \(N\) we observe that the average and maximum number of nodes visited and switching sequences explored by the proposed algorithm are less than the enumeration technique. The computational advantage of the proposed algorithm is more significant for higher values of \(N\). For \(N = 15\) the proposed algorithm only visits 2.8% of the nodes on average as compared to enumeration.

In Tables III and IV, we have compared the computational complexities of the two algorithms for different values of the prediction horizon \(N\). It is well known that
the dynamic performance of the controller is improved for larger values of $N$ [14, 15]. The proposed technique will result in an improved dynamic performance by allowing implementation of larger values of $N$ on the same computational hardware. Alternatively, as compared to enumeration a cheaper computational hardware may be used to implement the same value of $N$. Further details of the impact of $N$ on the performance of converters can be found in [14, 15].

To verify the simulation results, experimental results were also obtained for the proposed algorithm. The boost converter was built using the IRF540 MOSFET and

![Fig. 2. Simulation results for start-up and step-up/step-down change in reference output voltage. Voltage (blue), current (red), and reference voltage (black).](image)

**Table II.** System parameters

| Variable | Description | Value |
|----------|-------------|-------|
| $V_g$    | source voltage (V) | 12    |
| $V_{ref}$| output reference voltage (V) | 17    |
| $L$      | inductance ($\mu$H) | 625   |
| $R_L$    | resistance of inductor (Ω) | 1.2   |
| $R$      | load resistance (Ω) | 82    |
| $C$      | capacitance (µF) | 220   |
| $T_s$    | sampling time (µs) | 10    |
| $N$      | horizon length | 3     |
| $\lambda_u$ | weighting factor | 0     |
| $h$      | voltage error weight | 0.3   |

**Table III.** Comparison of nodes for different values of prediction horizon

| $N$ | Number of nodes visited using enumeration | Maximum number of nodes visited using proposed algorithm | Average number of nodes visited using proposed algorithm |
|-----|------------------------------------------|--------------------------------------------------------|-------------------------------------------------------|
| 3   | 14                                       | 14                                                     | 11.66                                                 |
| 5   | 62                                       | 48                                                     | 39.68                                                 |
| 8   | 510                                      | 208                                                   | 174.21                                                |
| 10  | 2046                                     | 454                                                   | 375.27                                                |
| 15  | 65534                                    | 2168                                                  | 1864.30                                               |

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SR5100 diode number. The rest of the circuit parameters are the same as given in Table II. For the processing, we used the National Instrument’s sbRIO GPIC board. The readings were taken on the Yokogawa DL9140 oscilloscope. The experimental setup is shown in Fig. 3 and Fig. 4. The experimental results for a step-up change in output voltage reference from $v_o = 17$ to $v_o = 23$ V are shown in Fig. 5. Moreover, Fig. 6 shows the experimental results for a step-down change in output voltage reference from $v_o = 23$ to $v_o = 17$ V. The average number of nodes visited for proposed algorithm were 11.66 as compared to 14 in the enumeration algorithm. It can be seen that the experimental results are similar to the simulation results.

| N  | Number of complete switching sequences explored using enumeration | Maximum number of complete switching sequences explored using proposed algorithm | Average number of complete switching sequences explored using proposed algorithm |
|----|-----------------------------------------------------------------|---------------------------------------------------------------------------------|--------------------------------------------------------------------------------|
| 3  | 8                                                               | 7                                                                               | 5.13                                                                           |
| 5  | 32                                                              | 15                                                                              | 11.76                                                                          |
| 8  | 256                                                             | 40                                                                              | 32.96                                                                          |
| 10 | 1024                                                            | 69                                                                              | 57.10                                                                          |
| 15 | 32768                                                           | 205                                                                             | 173.07                                                                         |

Fig. 3. Experimental setup.

Fig. 4. A closer look at the processing board and boost converter.

Table IV. Comparison of switching sequences for different values of prediction horizon
In this paper, we have proposed an efficient algorithm for the DMPC of boost converter. Therefore, we have emphasized on the comparison of computational complexities of two approaches to DMPC i.e. the proposed technique and enumeration. For the comparison of DMPC with other control techniques we refer the readers to [9, 13] and references therein. In particular, the comparison of DMPC with PI controller for indirect voltage control of boost converter [9] reveals that DMPC offers a lower overshoot, faster settling time, improved response time to disturbances, and higher converter efficiency.

6 Conclusion

By introducing a feedback control law, we have been able to obtain the state-space model with constant matrices that can represent the three modes of operation of the dc-dc boost converter. Thus, allowing us to apply the sphere decoding inspired
branch and bound algorithm for the DMPC of the converter and take benefit of the algorithm’s computational efficiency.

Switched models are ubiquitous in converters. The proposed approach for the dc-dc boost converter may also provide insight for efficient DMPC of other converters.