A Hint of Poincaré Dodecahedral Topology in the WMAP First Year Sky Map

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Abstract. Several analyses of the cosmic microwave background map made by the satellite WMAP suggest that the global shape of a spatial section of the Universe is that of a Poincaré dodecahedral space. A summary of some of these analyses and a description of independent tests which should be able to either increase confidence in the hypothesis or else refute it to extremely high significance will be presented.

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COSMIC TOPOLOGY

The possibility of the Universe (or more carefully, of a comoving spatial section of the Universe) being multiply connected is first known to have been suggested by Schwarzschild [1, 2], prior to the theories of special and general relativity which showed that there is no mathematically fundamental requirement for the curvature of the Universe to be zero.

General relativity provided a physical theory tightly relating curvature to matter-energy density. However, no clear theory relating topology to other physical constituents of the Universe has so far been proposed (though some hints are starting to surface; see Uzan et al. [3] and Barrow and Levin [4] for the relation between multiple connectedness and the comoving reference frame; and Roukema et al. [5] for a link between multiple connectedness and dark energy). For this reason, most observational cosmology research into Universe geometry has concentrated on empirically constraining the parameters of the perturbed Friedman-Lemaître-Robertson-Walker (FLRW) solution to the Einstein field equations.

However, in the last decade, research has accelerated into investigating the (possibly not measurable) global shape of the Universe, i.e. its topology. For recent reviews of this work, see Lachièze-Rey and Luminet [6], Luminet [7], Starkman [8], Luminet and Roukema [9]; workshop proceedings: Starkman [8] and following articles, and Blanlœil and Roukema [10]; and for detection strategies including both two-dimensional methods (based on temperature fluctuations in the surface of last scattering) and three-dimensional methods (based on distributions of gravitationally collapsed objects distributed in three-dimensional comoving space) see Uzan et al. [11], Luminet and Roukema [9], Roukema [12], Rebouças and Gomero [13].

WMAP ANALYSES: THE POINCARÉ DODECAHEDRAL SPACE

The WMAP first-year observational all-sky map of the cosmic microwave background has generated considerable interest due to the weakness of large-scale correlations, i.e. the amplitudes of the low-$l$ spherical harmonic components of the map are unexpectedly low and they seem to be aligned (this has been referred to as “the axis of evil”).

Several different groups have now found that the multiply connected Poincaré dodecahedral space (PDS) better models the statistics of the observed WMAP map than simply connected, infinite, flat models: Luminet et al. [14], Aurich et al. [15, 16], Gundermann [17]. Both classes of models assume a standard FLRW solution to the Einstein field equations.

Roukema et al. [18], again for the FLRW metric, used the identified circles principle [13, 20] to test the PDS hypothesis in the WMAP data and found the most correlated circles appear for circle radii of $\alpha = 11 \pm 1^\circ$, for a left-handed screw motion when matching opposite circles, but not for a right-handed one, nor for an unphysical, control, zero rotation. The favoured six dodecahedral face centres and circle radii are listed in Table 1. These six pairs of circles independently each favour a circle angular radius of $11 \pm 1^\circ$. The temperature fluctuations along the matched circles are shown in Figs 13–17 of Roukema et al. [18] and are clearly highly correlated.
**TABLE 1.** Galactic sky coordinates of the six face centres for the (positively curved) dodecahedron which shows excess values of the correlation statistic $S$ in Roukema et al. [18], listing the face number $i$, galactic longitude, latitude and estimated circle radius $\alpha$ (all in degrees). The other 6 faces are directly opposite to the ones listed. The orientation of the $36^\circ$ screw motion used to match faces is left-handed.

| $i$ | $l^H$ in $^\circ$ | $b^H$ in $^\circ$ | $\alpha$ in $^\circ$ |
|-----|-------------------|-------------------|------------------------|
| 1   | 252.4             | 64.7              | 9.8                    |
| 2   | 50.6              | 50.8              | 10.7                   |
| 3   | 143.8             | 37.8              | 10.7                   |
| 4   | 207.5             | 9.5               | 10.7                   |
| 5   | 271.0             | 2.7               | 11.8                   |
| 6   | 332.8             | 25.0              | 10.7                   |

**INDEPENDENT TESTS**

**CMB data: phase tests**

The Roukema et al. solution can be tested by several consistency tests, including phase tests. If the solution is the correct one, then it should mostly likely be an optimum in phase space in several different directions. For example, allowing for an arbitrary (non-physical) phase of rotation between matched circles should imply an optimal phase of $-36^\circ$.

Fig.1 shows that the solution is clearly consistent with the optimal phase.

**Total density parameter $\Omega_{\text{tot}}$**

The tests by Luminet et al. [14], Aurich et al. [15, 16], Gundermann [17] require assumptions regarding the power spectrum of density perturbations and the gaussianity or non-gaussianity of the distributions of amplitudes of the perturbations.

Nevertheless, both these analyses and the Roukema et al. [18] analysis make strong, highly falsifiable predictions for $\Omega_{\text{tot}}$, which must be strictly greater than unity for the PDS hypothesis to be correct.

Roukema et al. [18] predict $\Omega_{\text{tot}} = 1.009 \pm 0.001$ (1$\sigma$) for $\Omega_{\text{m}} = 0.28 \pm 0.02$; the other groups predict slightly higher values.

Eisenstein et al. [21], using a standard ruler method from the Sloan Digital Sky Survey, as in Roukema et al. [22], found $\Omega_{\text{tot}} \approx 1.015 \pm 0.015$.

If future tests eventually found, for example, $\Omega_{\text{tot}} \approx 1.001 \pm 0.001$, then the PDS hypothesis (whether from statistical analyses or identified circles analysis) would be excluded to high significance.

**Other tests**

Other tests which avoid making assumptions on hypothetical statistical ensembles of universes include:

- separating naïve-SW, ISW and doppler components
- foreground “predictions”
- polarisation data — see Riazuelo et al. [23]
FIGURE 1. Mean temperature correlation in $\mu K^2$, for five of the six pairs of the Roukema et al. [18] PDS solution, as a function of offset phase between the circles, for the Integrated Linear Combination (ILC) map of the WMAP first-year data. Circle pair #4 is excluded here because large parts of the circles fall on the galactic plane, so that a phase test is of low significance. The expected phase of $-36^\circ$ is shown with a vertical line. Note that correlations are not constrained to be positive. It is reasonable that the random properties of the data lead to correlations at the non-optimal phases which mostly cancel to what happen to be small positive values. Only at the expected phase do these correlations add up to a large positive value.

GPL SOFTWARE

The reader may carry out circles tests by using and/or modifying and/or redistributing (under the terms of the GPL licence) the CIRCLES package available at: [http://cosmo.torun.pl/GPLdownload/dodec/](http://cosmo.torun.pl/GPLdownload/dodec/).

Installation and example usages include:

- `./configure && make && make install`
- `circles --help; info circles; man circles`
- `circles --statistics` (correlation calculations)
- `circles --circles` (plot the circles)
- `circles --plot-phase` (phase plots)
- short form: `circles -s -c -P -d /scratch/topowork`
  do everything and use data files in `/scratch/topowork/`.

The data files are available as:

- [http://lambda.gsfc.nasa.gov/data/map/ilc/map_ilc_yr1_v1.fits](http://lambda.gsfc.nasa.gov/data/map/ilc/map_ilc_yr1_v1.fits) — the WMAP ILC map
- [http://cosmo.torun.pl/WMAPdata](http://cosmo.torun.pl/WMAPdata) — secondary files for default installation (in principle, these should not be necessary, but as of circles-0.1.27, it will be easier if the user downloads these.)
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