A MODEL FOR STRENGTH AND STRAIN ANALYSIS OF STEEL FIBER REINFORCED CONCRETE

Gediminas Marčiukaitis¹, Remigijus Šalna², Bronius Jonaitis³, Juozas Valivonis⁴

¹Department of Reinforced Concrete and Masonry Structures, Vilnius Gediminas Technical University, Saulėtekio al. 11, LT-10223 Vilnius, Lithuania
²E-mails: gelez@vgtu.lt; Remigijus.Salna@vgtu.lt (corresponding author); Bronius.Jonaitis@vgtu.lt; Juozas.Valivonis@vgtu.lt

1. Introduction

Concrete reinforced with steel fibers is a composite material the properties of which differ from those of concrete and steel fibers when taken separately. Concrete properties are mainly changed by steel fibers: strength and strain at tension, flexion, and elasticity modulus are increased and other mechanical properties are enhanced.

The areas of using fiber concrete may be both non-structural and structural. Fibers enable to control plastic strain and moister movements of concrete, and consequently the process of cracking. In structural sense, fibers can be substituted for complicated reinforcement with bars and in the case of a combined stress state, enable to avoid sudden failure due to an action of static and dynamic loads etc.

The application of SFRC for various stiff joints of reinforced concrete structures was well known long ago as an effective choice for additional reinforcement in the case of a combined stress state due to its distribution to various chaotic directions (Li 2002; Šalna and Marčiukaitis 2007; Szmigiera 2007; Özcan et al. 2009; Chiaia et al. 2009; Brandt 2008). However, the application of steel fibers to load bearing structures has been strictly limited for a long time due to a lack of the regulated methods of analysis.

These factors encouraged various countries (USA, Japan, Russia etc.) to work out documents regulating the use of steel fibers in the form of additional supplements to design codes. Furthermore, it is implicitly stressed in modern research work that the application of steel fibers to stiff joints, such as a connection between a column and a slab, is expedient. In such case, not only the strength of the structure is increased but also failure becomes predictive – the brittle failure mode is superseded by the plastic one. The application of such concrete is expedient for pavements on bridges, airports, tunnels and the like because of roughness, resistance to cracking and abrasion of their surface (Li 2002; Johnston and Zemp 1991; Meddah and Bencheikh 2009; Maleki and Mahoutian 2009; Kasper et al. 2008; Chiaia et al. 2009).

Steel fibers provide a possibility of producing and applying thin slabs with various shapes of the surface in accordance with architectural solutions to buildings.

The extensive use of fiber concrete for load bearing structures is confined by different existing methods for determining properties and especially by the absence of any method defining the properties of such concrete in the elastic-plastic state of stress.

It should be emphasized that when reinforcing concrete with fibers under compression, plastic deformations originate at the stress of 0.4σ_uy. Concrete ultimate stress σ_uy corresponds to stress in steel fibers only of (0.1...0.3)σ_y. In such case, for relationship (σ–ε), there will be limits within which both components will deform linearly; however, above the limit, one component deforms elastically while the other – plastically (Fig. 1). According to the classical theory of composites, limit stress in the composite is determined by criterion:
where \( F \) – force acting the composite.

Then, according to the additive law, we obtain:

\[
\frac{dF}{d\varepsilon} = \left( \frac{dF_f}{d\varepsilon} \right) V_f + \left( \frac{dF_c}{d\varepsilon} \right) (1 - V_f).
\]

This condition can be accurately solved when complete diagrams of \( \sigma - \varepsilon \) for components are known. Nevertheless, in the majority of cases, the linear relationship of \( \varepsilon = \sigma / E \) is assumed and the utilization of component properties is evaluated using empirical coefficients. Diagrams in Fig. 1 indicate that their accurate description is complicated. For common concrete, as suggests EC 2, \( \sigma - \varepsilon \) diagram may be approximated by broken lines, i.e. \( \sigma - \varepsilon \) diagram is resolved into a geometrical regular parabolic, triangular, trapezium and even rectangular diagrams or into a combination of those. For usual reinforced concrete, three shapes are used: 1) parabola and rectangular, 2) triangular and rectangular and 3) rectangular only. Our investigations showed that the second form was the simplest and the closest one to the real diagram. Moreover, considering fiber concrete, some theoretical attempts to assume the simplified diagram were made. Maalej and Li (1994), Kanda et al. (2000), Soranakom and Mobasher (2008), Bareišis and Kleiza (2004) proposed employing idealized \( \sigma - \varepsilon \) diagrams for SFRC. Various methods of examining SFRC were created (Pupurs et al. 2006; Li and Wang 2002; Nelson et al. 2002; Li 1992; Wang et al. 1989; Zhang and Li 2002; Kanda and Li 1999; Kanda et al. 2000; Leung and Li 1991; Maalej et al. 1995; Stang et al. 1995; Volkov et al. 2007; Kang et al. 2010; Olivito and Zuccarello 2010, Fantilli et al. 2009) the analysis of which shows that three groups of models can be distinguished:

1. strength of SFRC is determined on the basis of the additive law;
2. strength of SFRC is determined using the principles of the mechanics of failure;
3. strength of SFRC is determined using empirical relationships.

The main principle of the majority examined methods for the analyses of SFRC is the additive law (Malmeysteg et al. 1980); however, they differ in the assumptions of determining correction coefficients. Some authors reduce a chaotic distribution of steel fibers to the regularly orientated one (Филлинс и Харис 1980; Рабинович 2004; Марчукайтис 1998), some of those create models according to probabilistic principles (Wang and Becker 1989) while others determine experimentally or do not reduce chaotic distribution to regular one but introduce empirical coefficients (Li 1992; Stang et al. 1995; Kanda and Li 1999; Kanda et al. 2000; Zhang and Li 2002).

The principles of determining the strength of fiber concrete using the principles of failure mechanics were analyzed by Li (1992), Maalej et al. (1995), Kanda et al. (2000), Zhang and Li (2002), Zhang et al. (2001). An agreement with experimental results is good but the theoretical apparatus is complicated and practically almost was not used.

The methods of analyses based on empirical data belong to the third group (Harajli et al. 1995; Narayanan and Darwish 1987). Though this method is frequently applied, still, the actual performance of the composite components is almost not evaluated. The analysis of investigation results presented by the above mentioned as well as by other authors shows that the most precise results are obtained using the additive law. Great differences in relation to experimental results are obtained because plastic deformations in tension and compression concrete have not been taken into account.

2. The Proposed Model for Strength and Strain Analysis of SFRC

A comparison and analysis of models for strength analysis of SFRC have showed that there is no united opinion how strength and strains are to be determined. In the methods proposed by the majority of authors and in some standards for designing composites, using either \( \sigma - \varepsilon \) composite diagram determined experimentally (ASTM C1018; JSCE-SF4; NBP No.7; NFP 18-409; TR-34) or employing general principals of designing composites using additional (mostly empirical) service coefficients for components is recommended.

The suggested model for strength and strain analysis of SFRC is based on general principles of creating and modelling composites (additive law) with a direct evaluation of elastic and plastic characteristics of composite component materials.

Following classical assumptions of creating and designing composites one can write:

\[
\sigma_{sfrc} = \sigma_f V_f \psi_f + \sigma_c (1 - V_f),
\]

elasticity modulus of the composite in the general case is

\[
\text{Fig. 1. Schematic } \sigma - \varepsilon: \text{ for steel fiber (1), actual for concrete (2), for concrete according to EC 2 (3), for steel fiber reinforced concrete (4)}
\]
where \( \psi_f \) – the coefficient of service for materials depending on their joint action, strain properties, the quantity and orientation of inclusions, anchorage properties etc. It can be mostly determined by modelling; at a later stage, correction is made conducting experiments.

Formulas 3 and 4 show that for describing composite stress, to strain relationships expressed via variation in the elasticity modulus of materials is required.

For describing stress to strain relationships with materials, the following assumptions were applied:

1. for steel inclusions – linear \( \sigma - \varepsilon \) relationship (since elastic steel strains are much less in comparison with ultimate concrete strains);
2. for concrete – trapezoidal – the expression of elastic strains is \( \varepsilon_{c,el} = f_c / E_c \), plastic ones – \( \varepsilon_{c,pl} = f_c / E_c (1 - \alpha_c) \) and ultimate strains – \( \varepsilon_{c,u} = f_c / E_c (1 - \alpha_{cu}) \).

In this case, plasticity coefficients are as follows:

\[
\begin{align*}
\alpha_c & = \varepsilon_{c,pl} - f_c / E_c, \\
\alpha_{cu} & = \varepsilon_{c,u} - f_c / E_c.
\end{align*}
\]

The performed tests revealed that the plasticity coefficient of concrete can be defined in the following way:

\[
\lambda_c = 1 - 0.061 f_c^{0.5}.
\]

3. For composite – trapezoidal, strains are expressed as follows: elastic \( \varepsilon_{sfrc,el} = f_{sfrc} / E_{sfrc} \), plastic – \( \varepsilon_{sfrc,pl} = f_{sfrc} / E_{sfrc} (1 - \lambda_{sfrc}) \), ultimate – \( \varepsilon_{sfrc,u} = f_{sfrc} / E_{sfrc} (1 - \lambda_{sfrc,u}) \).

Under load composite, inclusion and matrix deform together, and therefore it can be expressed as

\[
\varepsilon_{sfrc} = \varepsilon_c = \varepsilon_f.
\]

The relation between strains and stress at the elastic stage when \( \varepsilon_c \leq f_c / E_c \) is

\[
\begin{align*}
\sigma_c & = \varepsilon_c E_c, \\
\sigma_f & = \varepsilon_f E_f.
\end{align*}
\]

Eqs 7 and 8 show that the strain of the composite equals to:

\[
\varepsilon_{sfrc} = \sigma_c / E_c = \sigma_f / E_f.
\]

Using the law of mixtures and formula (9), composite stress equals to

\[
\sigma_{sfrc} = \frac{\sigma_f}{E_f} \left( E_c \varepsilon_c + E_f V_f \right) = \\
\sigma_f \left( \frac{E_c \varepsilon_c + E_f V_f}{E_f} \right) = \sigma_f \left( \frac{\alpha_f V_c + V_f}{E_f} \right),
\]

where \( \alpha_f = E_c / E_f \) when \( \varepsilon_{sfrc} \leq f_c / E_c \); and

\[
\alpha_f = E_c (1 - \lambda_c) / E_f \text{ when } f_c / E_c < \varepsilon_{sfrc} \leq \varepsilon_{cu}.
\]

Formulas (9) and (10) show that the strength of the composite depends on two main parameters – the value of inclusion stress \( \sigma_f \) and composite strain \( \varepsilon_{sfrc} \), while ultimate composite strength – on ultimate inclusion stress \( \sigma_{f,u} \) and ultimate composite strain \( \varepsilon_{sfrc,u} \). Formula (10) is valid when \( \varepsilon_{sfrc} \leq \varepsilon_{c,u} \). However, ultimate composite strain \( \varepsilon_{sfrc,u} \) when inclusion strength is greater than matrix strength \( f_c / f_e \geq 1 \), exceeds concrete matrix strain \( \varepsilon_{c,u} \) (Nelson et al. 2002; Li and Wang 2002; Li 1992; Wang et al. 1989; Рабинович 2004; Zhang and Li 2002; Kanda and et al. 1995; Leung and Li 1991; Stang et al. 2000; Leung and Li 1991; Stang et al. 1995) since inclusion can resist acting stress (especially tensile one). Using trapezoidal \( \sigma - \varepsilon \) diagram for the composite (Fig. 2), the ultimate strain of composite \( \varepsilon_{sfrc,u} \) may be expressed via composite elasticity modulus:

\[
\varepsilon_{sfrc,u} = \frac{\sigma_{sfrc}}{E_{sfrc} \nu_{sfrc,u}} = \frac{\sigma_{sfrc}}{E_{sfrc} (1 - \lambda_{sfrc,u})},
\]

where

\[
\nu_{sfrc,u} = \frac{\sigma_{sfrc}}{E_{sfrc}} = \frac{E_c \varepsilon_c}{E_{c,el}}.
\]

Elasticity coefficient for composite \( \nu_{sfrc,u} \) can be expressed via the ultimate strains of the concrete matrix assuming that \( \varepsilon_{c,u} = \varepsilon_{sfrc} \) and \( \nu_{sfrc,u} = \nu_{sfrc} \) and using the iteration method, a more accurate value of \( \nu_{sfrc,u} \) can be obtained. According to this assumption and equating strains of the composite and concrete, one can write:

\[
\frac{\sigma_{sfrc}}{E_{sfrc} \nu_{sfrc}} = \frac{\sigma_c}{E_c \nu_c}.
\]

Using the law of mixtures for determining composite stress \( \sigma_{sfrc} \) and elasticity modulus \( E_{sfrc} \) and making mathematical rearrangements from relation (12), the coefficient of elasticity for the composite is equal to

\[
\sigma_{sfrc} = \sigma_c \left( \frac{1}{E_c} + \frac{V_f}{E_f} \right) = \\
\sigma_f \left( \frac{\alpha_f V_c + V_f}{E_f} \right),
\]
can be written in the form of:

\[
\begin{bmatrix}
V_c + \frac{\sigma_f}{\sigma_c} V_f
\end{bmatrix}
\]

and the application of \( \frac{E_f}{E_c} \), then the \( \lambda = \nu \) is expressed in the form

\[
\begin{bmatrix}
1 - V_f + \frac{\alpha_f - V_f (\alpha_f - 1)}{1 + V_f (\alpha_f - 1)} V_f
\end{bmatrix} V_c.
\]

Putting expression (17) into expression (13), the elasticity coefficient of the composite at ultimate strain can be expressed by:

\[
\nu_{\text{sfrc}} = \begin{bmatrix}
1 - V_f + \frac{\alpha_f - V_f (\alpha_f - 1)}{1 + V_f (\alpha_f - 1)} V_f
\end{bmatrix} V_c.
\]

Relation (13) clearly shows that the coefficients of elasticity and plasticity for the composite along with ultimate strain depend on the ratio of stresses \( \sigma_f / \sigma_c \) acting in the inclusion and concrete matrix. This ratio is determined using the general principles of work produced by external and internal forces. In the same way, inclusion stress \( \sigma_f \) may be expressed via reduced cross-section of concrete:

\[
\sigma_f = \frac{N}{A_{\text{eff}}} = \frac{N}{A_c + \frac{E_f}{E_c} A_f}.
\]

From (14) and (15), it is obvious that the ratio of \( \sigma_f / \sigma_c \) equals to:

\[
\frac{\sigma_f}{\sigma_c} = \frac{A_c \alpha_f + A_f}{A_c + A_f \alpha_f},
\]

where \( \alpha_f = \frac{E_f}{E_c} \).

Since \( (A_f + A_c) \parallel 1 \) or \( V_f + V_c = 1 \), then after mathematical rearrangements, equation (16) can be put in this way:

\[
\frac{\sigma_f}{\sigma_c} = \frac{\alpha_f - V_f (\alpha_f - 1)}{1 + V_f (\alpha_f - 1)}.
\]

The values of elasticity coefficient \( \nu_{\text{sfrc}} \) determined by equation (18) and those of ultimate composite strain \( \varepsilon_{\text{sfrc}} \) determined by equation (11) are compared with the experimental ones.

The second parameter from formulas (9) and (10) to be considered is stress \( \sigma_f \) in the inclusion of the composite. Formula (10) indicates that composite strength \( \sigma_{\text{sfrc}} \) practically depends on the value of stress \( \sigma_f \) in inclusion. The ultimate stress value in inclusion, i.e., strength \( f \) of inclusion, can be reached when inclusion is properly anchored (Laranjeira et al. 2010; Šalna and Marčiukaitis 2010). According to the classical theory of reinforced concrete, anchor strength depends on bond stress \( \tau_f \) between the matrix and inclusion and the area of the bond. Then, in the case of full anchorage, the following condition has to be satisfied:

\[
\tau_f d f_{\text{f,an}} \mu = f_y A_f.
\]

From equation (19), required anchorage length \( l_{f,\text{an}} \) is determined or taking analogous \( l_{f,\text{an}} \) ultimate bond stress can be obtained. However, when steel fiber is bent, the bond stress is supplemented with additional tangential stress at the bend. When the bond stress is noted by \( \tau_1 \) and tangential stress at the bend \( \tau_2 \), then the total bond stress \( \tau_f \) can be written in the form of:

\[
\tau_f = \tau_1 + \tau_2.
\]

When tangential stress \( \tau_2 \) is expressed in the form of product \( \tau_2 = \tau_1 k_{\text{at}} \), then (20) can be presented by:

\[
\tau_f = \tau_1 + \tau_1 k_{\text{at}} = \tau_1 (1 + k_{\text{at}}).
\]

The expression of the average normal stress \( \sigma_f \) in steel fibers via tangential ones \( \tau_f \) and the application of formula (22) give expression for determining normal stress according to the geometrical parameters of steel fibers and evaluation of different influence of the bend on the anchorage:

\[
\sigma_f = \tau_f \frac{l_f}{d_f} = \tau_1 (1 + k_{\text{at}}) \frac{l_f}{d_f}.
\]

The coefficient of effectiveness \( k_{\text{at}} \) for the bend depends on the type of steel fibers, bending shape and failure type at pull-out and has to be determined conducting tests. Coefficient \( k_{\text{at}} \) values determined performing experiments (Šalna and Marčiukaitis 2010) are presented in Table 1.

Table 1. The average tangential stress at the bend and the values of coefficient \( k_{\text{at}} \)

| Steel fiber type | \( \tau_2 \) MPa | \( k_{\text{at}} \) | Coefficient dispersion \( k_{\text{at}} \) | Variation in the coefficient \( k_{\text{at}} \) |
|-----------------|-----------------|------------------|-----------------|-----------------|
| MPZ 60          | 9.13            | 2.20             | 0.11            | 0.05            |
| MPZ 50          | 10.75           | 2.59             | 0.06            | 0.02            |
| MPS 50          | 15.60           | 3.76             | 0.48            | 0.13            |
| MPD 50          | 9.71            | 3.24             | 0.51            | 0.22            |
| MPG 32          | 10.71           | 2.58             | 0.24            | 0.09            |
When the stress of steel fibers only (22) is known, it is possible, according to (4), to determine the strength of the composite applying to regularly orientated inclusions. The analysis of the models proposed by the majority of authors and comparison with test results have showed that when reducing chaotic reinforcement by inclusions into the uniaxial one, the most accurate results are obtained when reducing chaotic reinforcement by inclusions into the uniaxial one, the most accurate results are obtained using the model suggested by Рабинович (2004) by means of a product of coefficients allowing for the probability of steel fibers to get into design plane \( \lambda_{op} \) and coefficient \( \lambda_p \) evaluating the orientation of the introduced reinforcement in relation to design plane:

\[
\lambda_{op} \lambda_p = 0.41.
\]  

(23)

After comparing formulas (3, 10, 22, 23), the strength of the composite equals to:

\[
\sigma_{sfrc} = 0.41 \sigma_f \left( \frac{E_c(1-\lambda_p)}{E_f} V_c + V_f \right) = 0.41(1+k_{uf}) \left( \frac{1}{l_f} \left( \frac{E_c(1-\lambda_p)}{E_f} V_c + V_f \right) \right).
\]  

(24)

On the basis of the adequacy of formulas (3, 4, 24), the coefficient allowing conditions for service \( \psi_f \) may be expressed via the ultimate stress of steel fibers. Then, formulas (3) and (4) can be rewritten in the forms:

\[
\sigma_{sfrc} = 0.41 \frac{\sigma_f}{\sigma_{fu}} \sigma_{fu} V_f + \sigma_c \left(1-V_f \right),
\]  

(25)

\[
E_{sfrc} = 0.41 \frac{\sigma_f}{\sigma_{fu}} E_{fu} V_f + E_c \left(1-V_f \right).
\]  

(26)

Thus, in the developed formulas (25) and (26), the strength and elasticity modulus of SFRC are evaluated and depend on the plasticity coefficient value (5). The ratio of the elasticity modulus of steel fibers to that of concrete gives an opportunity to determine composite stress in relation to the value of plastic strain.

3. Experimental Investigation into the Strength and Strain Properties of SFRC and Comparison with the Model

Test specimens made three main series of different strength of SFRC. The first SFRC series was intended for investigating the anchorage of steel fibers in the concrete matrix using steel fibers of type MZP 50. A concrete mix was produced under laboratory conditions. The quantity of steel fibers corresponding to 1, 1.5 and 2 percent of the volume mass was interblended into the concrete mix in the laboratory. The composition of steel fibers and the concrete mix is presented in Table 2.

The compression strength of SFRC was determined testing under the standard of 150×150×150 mm cubes and that of 100×100×400 mm prisms. The tension strength of SFRC was determined by bending 100×100×400 mm prisms. The elasticity (and strain) moduli of SFRC in the tension and compression processes were determined by means of measuring tension and compression strains of experimental specimens employing electrical resistance strain gauges. Totally, four specimens in each series were tested.

Table 2. A composition of concrete for the experimental program

| Material name | Material quantity kg/m³ |
|---------------|-------------------------|
| Cement(42.5R) | 320, 308.67, 312.14     |
| Sand (0–4 mm) | 773, 947.33, 933.53      |
| Gravel(4–16 mm)| 1180, 924.00, 912.54     |
| Water         | 163, 124.67, 126.9       |
| Plasticizer   | 0, 1.36, 1.37            |
| Steel fibers  | 0, 78.5; 0, 78.5; 0, 78.5; |
| MZP 50 type   | 117.75; 157; 117.75; 157 |

Strains were measured at the geometrical centres of prism sides in transverse and longitudinal directions using glued electric resistance gauges of 50 mm base length (Fig. 3). The load was increased up to failure in the steps of 20 kN and sustained for 5 min at each step. Load increasing speed was 0.05 kN/s. Apparent elastic limit assumed at the load limit equals to 0.4\( F_{u} \).

For assessing experimental plasticity, the value of coefficient \( \lambda_{sfrc} \) for SFRC at tension and compression as well as the elasticity modulus of concrete for compression were determined by the compression of standard 100×100×140 mm prisms and that for tension – by bending the above introduced prisms. The elasticity modulus of the specimens subjected to bending was determined by means of measuring strains of the tension layer in the zone of pure bending with two electrical resistance strain gauges and one inductance strain gauge. Electrical resistance strain gauges provide a possibility of measuring the ultimate strain only for concrete prisms subjected to bending. In SFRC prisms, cracks opened and the ultimate strain and crack width were determined with inductance strain gauge only.

When the ultimate strain and ultimate stress are obtained, experimental plasticity coefficients \( \lambda_{sfrc,c,obs} \) and \( \lambda_{sfrc,t,obs} \) for SFRC at compression and tension are determined according to formula (5). The values of theoretical

![Image](306x99 to 420x230)

![Image](427x99 to 543x230)

Fig. 3. An experiment on determining the elasticity modulus of SFRC (a); a series of prisms prepared for the test (b)
plasticity coefficient $\lambda_{sfrc,c,cal}$, $\lambda_{sfrc,t,cal}$ are obtained from analysis according to the proposed model. A comparison of experimental and theoretical values is presented in Table 4. The values of theoretical plasticity coefficient $\lambda_{sfrc,c,obs}$, $\lambda_{sfrc,t,obs}$ show a good agreement with the experimental ones. The values of a very small coefficient (0.01 and 0.02, Table 4) of variation demonstrate that the characters of plasticity coefficients both at tension and compression coincide closely.

The average experimental and theoretical values of modulus of deformation, compression and tension strength of SFRC are presented in Table 3, 5, respectively.

A comparison of experimental and theoretical values shows that compression strengths and modulus of deformation of SFRC coincide strongly (0.98 ± 1.06). An agreement between theoretical and experimental tension strength values is slightly worse (0.95 ± 1.18). After the regression analysis of experimental investigations, the plasticity coefficient of SFRC was determined. It equals to

$$\lambda_{sfrc,c} = \lambda_c \left(1 + 0.008 f_f^{1.5}\right).$$

### Table 3. A comparison of the values of experimental and theoretical elasticity modulus of SFRC

| Series | $V_f$, % | $E_{c,obs}$, GPa | $E_{sfrc,obs}$, GPa | $E_{sfrc,cal}$, GPa | $E_{sfrc,obs}$ | $E_{sfrc,cal}$ |
|--------|---------|-----------------|-----------------|-----------------|----------------|----------------|
| C      | 0       | 37.3            | –               | –               | –              | –              |
| FRC1   | 1.0     | –               | 38.2            | 37.38           | 1.02           |
| FRC1.5 | 1.5     | –               | 39.3            | 37.42           | 1.05           |
| FRC2   | 2.0     | –               | 39.8            | 37.46           | 1.06           |
| I      | 0       | 32.95           | –               | –               | –              | –              |
|        | 1.0     | –               | 33.76           | 33.07           | 1.02           |
|        | 1.5     | –               | 34.67           | 33.14           | 1.05           |
| II     | 0       | 35.50           | –               | –               | –              | –              |
|        | 2.0     | –               | 35.93           | 35.7            | 1.05           |

### Table 4. A comparison of the values of experimental and theoretical plasticity coefficient of SFRC

| Series | $V_f$, % | $\lambda_{sfrc,c,obs}$ | $\lambda_{sfrc,t,obs}$ | $\lambda_{sfrc,c,cal}$ | $\lambda_{sfrc,t,cal}$ | $\lambda_{sfrc,c,obs}$ | $\lambda_{sfrc,t,cal}$ |
|--------|---------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| C      | 0       | 0.706                   | 0.61                    | –                       | –                       | –                       | –                       |
| FRC1   | 1.0     | 0.71                    | 0.981                   | 0.649                   | 0.8952                  | 1.09                    | 1.10                    |
| FRC1.5 | 1.5     | 0.717                   | 0.982                   | 0.6503                  | 0.8955                  | 1.10                    | 1.10                    |
| FRC2   | 2.0     | 0.721                   | 0.983                   | 0.6517                  | 0.8959                  | 1.11                    | 1.10                    |
| I      | 0       | 0.694                   | 0.562                   | –                       | –                       | –                       | –                       |
|        | 1.0     | 0.702                   | 0.98                    | 0.6655                  | 0.9048                  | 1.05                    | 1.08                    |
|        | 1.5     | 0.73                    | 0.982                   | 0.668                   | 0.9052                  | 1.09                    | 1.08                    |
| II     | 0       | 0.712                   | 0.58                    | –                       | –                       | –                       | –                       |
|        | 2.0     | 0.719                   | 0.984                   | 0.652                   | 0.9016                  | 1.10                    | 1.09                    |
|        |         |                         |                         |                         |                         | Average                 | 1.09                    |
|        |         |                         |                         |                         |                         | Square deviation        | 0.02                    |
|        |         |                         |                         |                         |                         | Coefficient of variation| 0.02                    |

### Table 5. The average values of experimental and theoretical compression and tension strength regarding SFRC

| Series | $f_{sfrc,cube,obs}$, MPa | $f_{sfrc,obs}$, MPa | $f_{sfrc,cube,cal}$, MPa | $f_{sfrc,cal}$, MPa | $f_{sfrc,cube,obs}$ | $f_{sfrc,obs}$ | $f_{sfrc,cube,cal}$ | $f_{sfrc,cal}$ |
|--------|--------------------------|---------------------|--------------------------|---------------------|---------------------|----------------|---------------------|----------------|
| C      | 50.88                    | 5.71                | –                        | –                   | –                   | –              | –                   | –              |
| FRC1   | 53.78                    | 6.78                | 52.81                    | 5.73                | 1.02                | 1.18           |
| FRC1.5 | 54.03                    | 8.21                | 53.77                    | 6.93                | 1.00                | 1.18           |
| FRC2   | 56.37                    | 8.95                | 54.73                    | 8.13                | 1.03                | 1.10           |
| I      | 37.94                    | 4.41                | –                        | –                   | –                   | –              | –                   | –              |
|        | 40.10                    | 5.1                 | 40                       | 4.86                | 1.00                | 1.05           |
|        | 40.29                    | 5.75                | 41.02                    | 6.07                | 0.98                | 0.95           |
|        | 41.56                    | 5.13                | –                        | –                   | –                   | –              | –                   | –              |
|        | 46.04                    | 7.3                 | 45.6                     | 7.49                | 1.01                | 0.97           |
Empirical expression (27) clearly points out that variation in the plasticity coefficient of SFRC and the quantity of steel fibers is not significant and mostly depends on the plasticity coefficient of concrete itself. It agrees with the proposed model (18). According to (27), the values of the experimental plasticity coefficient agree well with the theoretical ones determined by (18): when the amount of steel fibers increases from $V_f=1\%$ to $V_f=2\%$, the ratio between theoretical and experimental values remains almost constant and makes $\lambda_{obs}/\lambda_{theor} = 1.06...1.08$. Expression (6) for the plasticity coefficient of non-reinforced concrete in relation to the experimental value also slightly differs – up to 1.06 times.

The regression analysis of the performed experiments has shown that the plasticity coefficient (in relation to the quantity of steel fibers) can be described by the following empirical relationship:

$$\lambda_{sfrc,t} = \lambda_t \left(1.63 + 0.008 V_f^{1.5}\right). \tag{28}$$

The nature of the plasticity coefficient for concrete in tension differs from that in compression (formulas 27 and 28): the plasticity coefficient value is close to 0.90. This difference can be explained by the fact that the coefficients of elasticity and plasticity for concrete in tension approximately are equal $\nu_t \approx \lambda_t \approx 0.5$ (Залесов et al. 1988), i.e., elastic and plastic strains are of a similar value; moreover, they vary insignificantly depending on concrete class. Nonetheless, the plastic strain of SFRC tension is very heavy, and therefore $\nu_t \ll \lambda_t = 1$.

Experimental results presented in Table 4 show that the values of experimental and theoretical plasticity coefficient $\lambda_{obs}/\lambda_{cal}$ differ 1.08–1.10 times. Unfortunately, the ratio between theoretical and experimental values differs insignificantly and varies within the limits of 1.06–1.10.

The executed regression analysis of the results of experimental investigation offers a possibility of presenting simplified empirical formulas for determining the plasticity coefficient of SFRC at compression and tension without deviation from the main additive law in the theory of composites.

5. This model may be used for the analysis of flexural SFRC members assuming normal stress distribution diagrams in tension and compression zones. For practical use, to verify the coefficient of plasticity using test results is recommended.

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BETONO, ARMUOTO PLIENINE DISPERSINE ARMATŪRA, STIPRIO IR DEFORMACIŲ APSKAICIAVIMO MODELIS

G. Marčiukaitis, R. Šalna, B. Jonaitis, J. Valivonis

Santrauka

Straipsnyje pasiūlytas betono, armuoto plienine dispersine armatūra, stiprio ir deformacijų skaičiavimo modelis, pagrįstas bendrais statybinių kompozitų kūrimo ir modeliavimo principais bei gelžbetonio normomis. Šiame modelyje skirtingai nuo daugelio kitų yra tiesiogiai įvertinamos tampriosios ir plastinės kompozito komponentų (betono ir plieninės dispersinės armatūros) savybės. Modelis leidžia apskaičiuoti betono, armuoto plienine dispersine armatūra, tampiamajį ir gniuždomajį stiprius, tamprumo modulį ir pagrindinius jo deformatyvumo parametrus – tamprumo ir plastiškumo koefficientus. Siūlomo modelio palyginimas su šio straipsnio ir kitų autorių atliktais eksperimentų duomenimis parodė, kad rezultatai sutampa. Teorinių ir eksperimentinių reikšmių santykiai skiriasi nedaug ir kinta nuo 1,06 iki 1,10. Šis modelis gali būti taikomas priimant įtempių pasiskirstymo diagramas apskaičiuojant lenkiamuosius, plienine dispersine armatūra armuotos betoninius elementus.

Reikšminiai žodžiai: kompozitas, betonas, plieninė dispersinė armatūra, betonas, armuotas plienine dispersine armatūra, deformacijų modulis, tampriosios ir plastinės deformacijos.

Gediminas MARČIUKAITIS. Prof., Dr Habil at the Department of Reinforced Concrete and Masonry Structures, Vilnius Gediminas Technical University (VGTU). PhD from Kaunas Polytechnic Institute in 1963. Research visit to the University of Illinois (1969). A Habilitated Doctor from Moscow Civil Engineering University in 1980. Professor (1982). The author and co-author of 5 monographs, 8 textbooks, 5 coursebooks and more than 300 scientific articles. Research interests: mechanics of reinforced concrete, masonry and layered structures, new composite materials, investigation and renovation of buildings.

Remigijus ŠALNA. A Doctor at the Department of Reinforced Concrete and Masonry Structures, Vilnius Gediminas Technical University, Lithuania. Research interests: punching shear strength of RC and SFRC slabs, investigation of buildings.

Bronius JONAITIS. Assoc. Prof., Dr at the Department of Reinforced Concrete and Masonry Structures, Vilnius Gediminas Technical University, Lithuania. The author and co-author of more than 50 scientific publications, 1 textbook, 3 coursebooks, 3 patented investigations. Research interests: theory of reinforced concrete behavior, masonry structures, strengthening of structures.

Juozas VALIVONIS. Prof., Dr at the Department of Reinforced Concrete and Masonry Structures, Vilnius Gediminas Technical University (VGTU). Publications: the author and co-author of more than 55 scientific publications, 4 textbooks, 5 coursebooks. Research interests: theory of reinforced concrete behavior, composite structures, strengthened concrete bridges.