Original Paper

Algebra I Teachers’ Beliefs and Knowledge of Algebra For Teaching

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Abstract
Research indicates that teachers’ mathematical beliefs and mathematical knowledge for teaching impacts practices in the classroom. Research also suggests that success in Algebra I is the gatekeeper to higher-level mathematics. With the increased number of certification pathways in some states, it is important to identify those Algebra I teachers’ beliefs and knowledge of algebra for teaching. A study of current Algebra I teachers revealed that regardless of certification pathway, mathematical beliefs are not significantly different. Additionally, significant differences did exist in regards to the certification pathway and Knowledge of Algebra for Teaching (KAT) levels.

Keywords
Algebra I teachers, mathematics, teachers’ beliefs

1. Introduction
The teacher shortage crisis in public schools over the last two decades has created concern for many school systems and policymakers (Flynt & Morton, 2009; Ingersoll & Smith, 2003). Enrollment in teacher preparation programs in the United States decreased over 35 percent from 2009 – 2014 and enrollment only continues to decline with each passing year (Aragon, 2016). With the shortage of teachers to place in the classroom, schools are forced to lower their standards for teacher quality, especially in the areas of mathematics and science (Ingersoll & Smith, 2003; Murnane et al., 1991; Liu et al., 2008). This shortage has opened up a multitude of pathways for teachers to enter the mathematics classroom. Particularly in the state of Oklahoma where there are now seven different pathways to certification (Oklahoma State Department of Education, 2016).
This leads to questions about who the teachers are currently in classrooms and what practices they are using to instruct students. There are several factors that contribute to a teacher’s practices in the classroom. However, teacher beliefs and pedagogical content knowledge are considered to be two of those main factors (Pajares, 1992; Wilkins, 2008; Ball, Thames, & Phelps, 2008). With a push for mathematics to be taught with a more conceptual and problem-solving approach by researchers and educators alike, the overarching view of mathematics by teachers is still seen as procedural and full of algorithms to be memorized (Cai, 1994). Teachers with these beliefs are less likely to understand how to guide students’ construction of mathematical ideas (Battista, 1994). Along with teacher beliefs, pedagogical content knowledge connects content knowledge to teaching practices (Shulman, 1986; Ball & Bass, 2000).

Teacher practices are especially important in gatekeeper courses such as Algebra I (Stinson, 2004). The success of students in Algebra I has been linked to performance in college, career readiness, impact on career salary, and perceptions of higher mathematics (Eddy et al., 2015; Gaertner, Kim, DesJardins, & McClarty, 2014).

The research on teacher beliefs in this study are broken down into three main categories: beliefs about nature of mathematics, beliefs about learning mathematics, and beliefs about teaching mathematics. Mathematical Knowledge for Teaching (MKT) and teacher certification pathway are the two other main focuses of this study. Thus, a discussion of how the combination of teacher beliefs and MKT influence teacher practices follows.

1.1 Teacher Beliefs

The term beliefs refer to “psychologically held understandings, premises, or propositions about the world that are thought to be true” (Richardson, 1996, p. 259). Research indicates that teachers’ belief systems are highly individualized (Gudmundsdottir & Shulman, 1987; Lovat & Smith, 1995). Additionally, researchers found that these individual beliefs held by teachers greatly determine their teaching practices in the classroom regardless of their pedagogical knowledge or curriculum standards (Lepik & Pipere, 2011). Furthermore, Love & Kruger (2005) found teacher beliefs to impact student achievement when teachers believed in a sense of community in the classroom and flexible teaching strategies.

1.2 Teacher Beliefs about Nature of Mathematics

Mathematicians and mathematics educators have debated for years over how the nature of mathematics can be defined (Skemp, 1976; Hersh, 1979; Ernest, 1989). Describing the nature of mathematics, the definition given by Ernest (1989) provided the following three views commonly held by mathematics teachers: (1) Mathematics is a continually expanding field of human inquiry where it is an unfinished product and its results remain open to revision. (2) Mathematics is a static, but unified body of knowledge, consisting of truths which are discovered, not created. (3) Mathematics is a useful, but unrelated collection of facts, rules, and skills (p. 21).
Each of these views are known respectively as the dynamic problem-solving view, the Platonist view, and the static instrumentalist view. Teachers who hold a dynamic problem-solving view about the nature of mathematics show strong impact through the teaching instructions in the classroom and bring about more understanding and desire to learn mathematics (Francis, 2014). Similarly, Lerman (1990) recognized that children will be able to apply prior knowledge of mathematics in a creative manner when problem-solving views of mathematics are held.

1.3 Teacher Beliefs about Teaching of Mathematics

Researchers have found that typical, daily mathematics instruction is taught by the introduction of a new procedure with step-by-step instructions, then followed up by homework problems that are meant to mimic the procedure (Stipek et al., 2001). The opposing style of constructivist teaching focuses more on posing relevant problems to solve, learning the essence of primary concepts, and valuing the student’s viewpoints on solving the problems (Kim, 2005). Even when a professional development program was implemented to encourage the use of a more constructivist teaching style, researchers found that teachers assimilated new practices back to traditional practices due to their own experiences of how they were taught mathematics (Cohen & Ball, 1990; Raymond, 1997).

Similarly, Van Zoest, Jones, and Thornton (1994) studied two groups of pre-service mathematics teachers. One group was in a mentorship program heavily based on a constructivist approach to teaching mathematics. The second group of pre-service mathematics teachers were not in the mentorship program. After comparing beliefs of each group after the mentorship was completed, it initially seemed like the pre-service teachers in the program were persuaded to teach mathematics in a constructivist way. At the conclusion of the study, it was determined that both groups had resorted to a more traditional set of beliefs about teaching mathematics.

Stipek et al. (2001) found a direct correlation between teachers’ beliefs and their classroom practices. The study showed those teachers who held traditional beliefs about mathematics teaching tended to stress getting correct answers, achieving good grades, and speed of finding solutions, rather than teaching for conceptual understanding. These teachers were also found to assert mistakes as a negative in the classroom instead of using those mistakes as a learning opportunity.

A teachers’ belief of how students learn mathematics is a major factor in how teachers carry out their instruction in the classroom. Teachers need to be able to perceive the types of mathematics activities that will best develop the learning of the students (Ball, 2003).

In order to describe the relationship between beliefs and practice, Peterson, Fennema, Carpenter, and Loef (1989) found that teachers who believe their students learn mathematics through a problem-solving approach, used more word problems in instruction. Similarly, they found those same teachers spent more time developing number sense rather than teaching number facts. Conversely, another study conducted by Even and Tirosh (2002) found that teachers rarely base their practices on how they believe their students learn mathematics, but rather on how their students can immediately put the rules and information to use.
Teachers’ beliefs about mathematics, the teaching of mathematics, and the learning of mathematics can have implications on classroom practice. Although beliefs are not the only factor impacting practice, they can be considered highly influencing.

1.4 Mathematical Knowledge for Teaching

Researchers in mathematics education have now spent the last couple of decades assessing the types of mathematical knowledge that is necessary for effective teaching (Ball, 2003; Ball et al., 2008; Hill, Ball, & Schilling, 2008; Hill et al., 2005; Hurrell, 2013; Li, 2011). According to Ball et al. (2008), Mathematical Knowledge for Teaching (MKT) places an emphasis on both subject matter and pedagogical content knowledge. MKT moves away from just knowing mathematical content to being able to teach the mathematical content. Focusing specifically on secondary mathematics teachers, MKT research showed that simply taking a certain number of higher-level mathematics courses does not always guarantee a suitable level of mathematical knowledge to teach mathematics (Even, 1999).

A recent framework was developed to investigate mathematical knowledge for teaching algebra (McCrorry, Floden, Ferrini-Mundy, Reckase, & Senk, 2012). The researchers suggested three categories of knowledge were needed to effectively teach algebra. These categories include (a) school algebra – knowledge of concepts and ideas taught in high school algebra, (b) advanced mathematics – knowledge of college-level mathematics, and (c) algebra for teaching – pedagogical content knowledge of teaching algebra. McCrorry et al. (2012) found that secondary teachers are strictly measured for knowledge of teaching through content tests or by a number of mathematics courses taken, where neither of these measures gives a full understanding of a teachers’ knowledge of algebra for teaching.

This study aimed to portray a picture of who is teaching Algebra I in Oklahoma and highlight the pathways to certification taken by those teachers. Additionally, this study explored the algebra beliefs of these Algebra I teachers along with their own understanding of algebra and the teaching of algebra concepts. This study attempted to answer the following research questions: (1) Who is the Algebra I teaching force in Oklahoma? (2) Is there a significant difference between an algebra teacher’s certification pathway and the beliefs he or she holds? (3) Is there a significant difference between an algebra teacher’s certification pathway and his or her Knowledge of Algebra for Teaching (KAT)? (4) Is there an association between an Algebra I teachers’ Knowledge of Algebra Teaching (KAT) and their beliefs about algebra, about teaching algebra, and about learning algebra across certification pathways?

2. Method

This study used a survey research design to quantitatively describe the beliefs and Knowledge of Algebra for Teaching (KAT) of Algebra I teachers in the state of Oklahoma (Creswell, 2013). The sample of teachers in the study can be used to generalize all Algebra I teachers in Oklahoma.

2.1 Participants

After an open records request was made to the Oklahoma State Department of Education, all Oklahoma public school mathematics teachers (N = 2,488) were emailed a link to an online questionnaire. The
email addresses of specifically Algebra I teachers were not given, although the number of Algebra I teachers (N = 1,455) was given. The questionnaire was completed by 144 Algebra I teachers from across the state of Oklahoma, which resulted in a 9.9% response rate from Algebra I teachers.

The geographic regions in the state of Oklahoma were divided into eight different regions by the Oklahoma State Department of Education called the REAC³H regions. The data in Table 1 shows that the sample was representative of the state population of mathematics teachers according to geographic distribution, education level, teaching experience, and ethnicity.

For the purpose of this study, the sample was broken down into four different grouping variables based on the certification pathway. The four different pathways used were the following: (1) Mathematics Education (n = 67) – any teacher who completed a degree in mathematics or secondary education mathematics while completing a teacher education program leading to certification. (2) Mathematics (n = 16) – any teacher who holds a Bachelor’s degree in mathematics while becoming alternatively certified. (3) Elementary Education (n = 23) – any teacher who completed a degree in elementary education while completing a teacher education program leading to certification. (4) Other (n = 38) – any teacher who did not follow one of three previous paths mentioned above.

The mathematics education and elementary education grouping variables in this study follows the traditional pathway in Oklahoma. The mathematics grouping variable follow the alternative placement program in Oklahoma. Finally, the other grouping variable may follow one of the other five pathways to certification.

Table 1. Demographics of Oklahoma Mathematics Teacher Population and Study Sample

| Oklahoma Reac³h Region | Population Number | Population Percentage | Sample Number | Sample Percentage |
|------------------------|-------------------|-----------------------|--------------|------------------|
|                        |                   |                       |              |                  |
| 1                      | 113               | 4.23                  | 6            | 4.17             |
| 2                      | 191               | 7.15                  | 20           | 13.89            |
| 3                      | 550               | 20.60                 | 26           | 18.06            |
| 4                      | 341               | 12.73                 | 11           | 7.63             |
| 5                      | 185               | 6.93                  | 9            | 6.25             |
| 6                      | 229               | 8.58                  | 17           | 11.81            |
| 7                      | 205               | 7.68                  | 8            | 5.56             |
| 8                      | 856               | 32.06                 | 47           | 32.64            |

| Highest Education Level | Population Number | Population Percentage | Sample Number | Sample Percentage |
|-------------------------|--------------------|-----------------------|--------------|------------------|
| Bachelor’s              | 1853               | 69.40                 | 71           | 49.31            |
| Master’s                | 801                | 30.00                 | 72           | 50.00            |
| Doctorate               | 15                 | 0.56                  | 1            | 0.69             |
### Teaching Experience

#### Years

| Years       | Participants | Mean | SD  |
|-------------|--------------|------|-----|
| 1 to 5      | 701          | 26.25| 27  |
| 6 to 10     | 508          | 19.03| 30  |
| 11 to 15    | 457          | 17.12| 35  |
| 16 to 20    | 363          | 13.60| 16  |
| 21 to 25    | 252          | 9.44 | 14  |
| 26 to 30    | 203          | 7.60 | 15  |
| 31 to 35    | 92           | 3.45 | 3   |
| 36 to 40    | 61           | 2.29 | 2   |
| over 40     | 33           | 1.24 | 2   |

#### Ethnicity

| Ethnicity                             | Participants | Mean | SD  |
|---------------------------------------|--------------|------|-----|
| African American                      | 68           | 2.55 | 2   |
| American Indian or Alaskan Native     | 148          | 5.54 | 13  |
| Hispanic                              | 49           | 1.84 | 3   |
| Asian or Pacific Islander             | 25           | 0.94 | 0   |
| White                                 | 2,316        | 86.74| 122 |
| More than One                         | 64           | 2.40 | 4   |

1 The Oklahoma Reac^3h regions were used to determine the geographical representation of the state. A map of the Reac^3h regions can be found at http://ok.gov/sde/reac3h-network.

2.2 Measures

Three different instruments constituted the data sources in the online questionnaire. Participants were asked to provide demographic information, respond openly about their beliefs about algebra, and participate in a 20-question assessment that measures their Knowledge of Algebra for Teaching (KAT). Since the last three questions on the KAT were open-ended and the process of uploading solutions was time-consuming, several teachers did not complete that portion. Scoring on the KAT was adjusted to not include the last three open-ended questions. Therefore, those participants who completed all parts of the questionnaire excluding the three open-ended questions were still considered in this study.

2.3 Demographics

The online questionnaire collected information on the Algebra I teachers’ current grade being taught, school name, and district. This allowed the teachers to be filtered in the correct REAC' H region. Additionally, the teachers were asked to state their age, ethnicity, number of years they have taught mathematics, educational background, and certification pathway.
2.4 Algebra Beliefs Questionnaire

The algebra beliefs questionnaire used in this study is a modification of Raymond’s (1997) beliefs questionnaire by changing all mentions of “mathematics” to “algebra.” The questionnaire has three subscales – beliefs about the nature of algebra, beliefs about learning algebra, and beliefs about teaching algebra. While Raymond did not validate the instrument, two mathematics educators examined the revised instrument to ensure that the questions measured the individual beliefs specified. Cronbach’s alphas were calculated for each of the three subscales using the data from this study. Those Cronbach’s alphas for beliefs about the nature of algebra, learning algebra, and teaching algebra were .81, .75, and .54, respectively. Each subscale has a series of semantic differential ranging from 1 – 13 and a group of 5-point Likert-type questions. The 5-point Likert questions were scaled to match the 13-point range of the semantic differential questions. The beliefs about the nature of algebra subscale have eight questions of each type with a range of potential scores being from 16 to 176 with higher scores more indicative of a problem-solving view of algebra. For the beliefs about learning algebra, there were 7 semantic differential questions and 10 Likert-type questions with a range of scores being from 17 to 187 with higher scores more indicative of a discovery view of algebra. The beliefs about the teaching of algebra subscale have 8 semantic differential questions and 7 Likert-type questions with a range of potential scores being from 15 to 165 with higher scores more indicative of a discovery view of algebra.

2.5 Survey of Knowledge of Algebra for Teaching (KAT)

The Survey of Knowledge of Algebra for Teaching measures the knowledge most efficient in the teaching of algebra. During the validation study, this instrument had a Cronbach alpha reliability of .84. The KAT measures three dimensions of a teachers’ mathematical knowledge for teaching with the range of scores included – teaching knowledge (25.72 – 53.23), knowledge of school algebra (29.11 – 57.63), and advanced knowledge of mathematics (37.02 – 60.11). The teaching knowledge dimension includes five questions about mathematical knowledge specific to teaching algebra concepts. The knowledge of school algebra dimension involves questions typically taught in a middle or high school algebra course. The advanced knowledge of mathematics dimension includes questions typically taught in college-level mathematics courses. The instrument also provides a final score, incorporating all three dimensions together with a score range of 26.72 – 57.62.

2.6 Data Analysis

Results were analyzed using both descriptive and inferential statistics using SAS® software, version 9 of the SAS system (SAS Institute, 2013). Descriptive statistics were used to show information across certification pathways. Inferential statistics included the use of a one-way ANOVA to find any significant differences between the four certification pathways in terms of beliefs scores and KAT scores. All assumptions for one-way ANOVA’s were checked including the use of the Levene’s test to check the homogeneity of variances between groups. Where significant differences were found between groups, Tukey’s HSD test was then run to determine the differences between exact groups. Box and whisker plots were used to visualize data and make comparisons across certification pathways.
3. Result
In order to describe who Algebra I teachers in Oklahoma are, a variety of characteristics were used such as age, ethnicity, years of teaching experience, and highest education level (see Table 1). Of the 144 teachers sampled, the average age was nearly 43-years-old. The ethnicity of the teachers is predominantly White with the second-largest ethnicity being American Indian or Alaska Native. The years of teaching experience of those teachers were largely clumped between 1 – 15 years with just under 64 percent of Algebra I teachers falling in that category. Also, nearly 20 percent of those teachers are novice with only 1 – 5 years of teaching experience. The number of teachers who held a Bachelor’s degree and those who held a Master’s degree was 49 percent and 50 percent, respectively. Furthermore, 32 percent of teachers with a Master’s degree hold one in the area of mathematics education.

The certification pathways of Algebra I teachers in this study were broken down into four groups – traditional mathematics education certification, Bachelor’s degree in mathematics with alternative certification, traditional elementary education certification, or any other pathway different from the previous three. The percentage of teachers who followed a traditional mathematics education certification was 47 percent and teachers who followed a traditional elementary education pathway was 16 percent. This means that 63 percent of teachers in this study were certified through a traditional certification pathway. Teachers who hold a Bachelor’s degree in mathematics and were alternatively certified to teach makeup 11 percent of this sample. The remaining 26 percent of teachers hold non-traditional teacher certifications in non-mathematics areas.

The three different beliefs being measured in this study are beliefs about the nature of algebra, learning of algebra, and teaching of algebra, where descriptive statistics of each of the four certification pathways on beliefs are given in Table 2. Overall, Algebra I teachers in Oklahoma did not have mean belief scores that were considered to be problem-solving or constructivist views. In general, the means and standard deviations in each certification pathway were very similar and the teachers fell noticeably in the middle of each spectrum of the beliefs categories. Although, a consistency was found in that the mean belief scores of the nature of algebra were consistently the highest of the three types of beliefs regardless of certification pathway. After using one-way ANOVA, no significant differences were found at the = .05 level between any of the four certification pathways in any of the three areas of beliefs (see Table 3).
Table 2. One-Way ANOVA Results of Certification Pathway on Beliefs about Nature, Learning, and Teaching of Algebra

| Variable and Source          | df  | SS   | MS  | F    | p    |
|------------------------------|-----|------|-----|------|------|
| Nature of Algebra            |     |      |     |      |      |
| Between groups               | 3   | 1.97 | .66 | 1.80 | .1497|
| Within groups                | 140 | 51.04| .36 |      |      |
| Learning of Algebra          |     |      |     |      |      |
| Between groups               | 3   | 1.45 | .48 | 1.32 | .2691|
| Within groups                | 140 | 51.11| .37 |      |      |
| Teaching of Algebra          |     |      |     |      |      |
| Between groups               | 3   | 1.58 | .53 | .96  | .4121|
| Within groups                | 140 | 76.43| .55 |      |      |

Table 3. Descriptive Statistics of Knowledge of Algebra for Teaching (KAT) Scores

| Pathway                     | Tscore M (SD) | Sscore M (SD) | Ascore M (SD) | Final Score M (SD) |
|-----------------------------|---------------|---------------|---------------|--------------------|
| Mathematics Education       | 40.38 (5.85)  | 46.16 (6.51)  | 51.19 (6.85)  | 45.13 (5.97)       |
| Mathematics                 | 38.58 (6.18)  | 45.15 (5.38)  | 52.21 (7.67)  | 44.74 (5.08)       |
| Elementary Education        | 34.99 (5.40)  | 39.46 (4.71)  | 42.20 (4.78)  | 36.93 (3.76)       |
| Other                       | 37.64 (6.36)  | 41.54 (4.86)  | 45.38 (6.91)  | 39.86 (5.30)       |

Note. Tscore = Teaching Knowledge; Sscore = Knowledge of School Algebra; Ascore = Advanced Knowledge of Mathematics.

Range of Tscore = 25.72 – 53.23, Sscore = 29.11 – 57.63, and Ascore = 37.02 – 60.11

When analyzing the KAT scores for Algebra I teachers in this study, the knowledge of teaching (Tscore), knowledge of school algebra (Sscore), advanced knowledge of mathematics (Ascore), and Final Score mean scores were 38.58, 43.73, 48.29, and 42.35 respectively. Those Algebra I teachers who were certified through a traditional mathematics education pathway and those with a Bachelor’s in mathematics plus alternative certification scored at or above the overall means in every dimension of the KAT. The mathematics education group obtained the highest mean score on all of the KAT scores, excluding the Ascore (M = 51.19). The mathematics group had the highest mean score for the Ascore dimension of the KAT (M = 52.21). The elementary education group obtained the lowest mean score on all four of the KAT scores, including the final score (M = 36.93). It is important to notice that the elementary education group had the lowest standard deviations in all four of the scoring sections, emphasizing that their scores are not spread out much about the mean scores. It is also notable that those teachers who were certified through a mathematics education (M = 45.13, SD = 5.97) pathway...
and those were alternatively certified with a Bachelor’s in mathematics (M = 44.74, SD = 5.08) had final mean scores that only differed by .39. The full descriptive statistics are given for the four different KAT scores depending on certification pathway in Table 4. Also, a visual representation of all four dimensions of the KAT scores between certification pathways using box and whisker plots is provided in Figure 1. The plots show the differences in the range of scores between certification pathways along with showing which pathways scored higher and lower on each dimension of the KAT. From an initial look, it appears that elementary education and other pathways tend to have the lowest scores in every aspect of the KAT. Additionally, mathematics education and mathematics pathways appear to have the highest mean score in every aspect of the KAT. It was noticed that mathematics education and mathematics tend to score near the same in each dimension of the KAT.

**Table 4. One-Way ANOVA Results of Certification Pathway on KAT Tscore, Sscore, Ascore, and Final Score**

| Variable and Source | df | SS   | MS  | F     |
|---------------------|----|------|-----|-------|
| Tscore              |    |      |     |       |
| Between groups      | 3  | 549.09 | 183.03 | 5.15** |
| Within groups       | 140 | 4974.13 | 35.53 |
| Sscore              |    |      |     |       |
| Between groups      | 3  | 1031.01 | 343.67 | 10.49*** |
| Within groups       | 140 | 4584.68 | 32.75 |
| Ascore              |    |      |     |       |
| Between groups      | 3  | 1976.04 | 658.68 | 14.78*** |
| Within groups       | 140 | 6237.29 | 44.55 |
| Final Score         |    |      |     |       |
| Between groups      | 3  | 1520.99 | 506.99 | 17.35*** |
| Within groups       | 140 | 4091.61 | 29.23 |

*Note. Tscore = Teaching Knowledge; Sscore = Knowledge of School Algebra; Ascore = Advanced Knowledge of Mathematics
**significant at p < .01
***significant at p < .001*
Figure 1. Certification Pathway Scores on the Tscore, Sscore, Ascore, and Final Score Portions of the Knowledge of Algebra for Teaching (KAT)

With the use of a one-way ANOVA, significant differences were found between multiple different certification pathways when comparing all four dimensions of KAT levels (see Table 5). All dimensions were significant at the $\alpha=.001$ level, except Tscore, which was significant at $\alpha=.01$. Thus, post hoc comparisons using the Tukey HSD test were made on all four dimensions of the KAT scores with those test results for Tscore, Sscore, Ascore, and Final Score. The significant comparison results for each of the four dimensions are discussed below.

Table 5. Tukey HSD Comparison for KAT Scores across Certification Pathways

|          | ME      | M       | EE       | O       |
|----------|---------|---------|----------|---------|
| **Tscore** |         |         |          |         |
| ME $(M=40.38)$ | -       | 1.81    | 5.39*    | 2.75    |
| M $(M=38.58)$ | -       | -       | 3.59     | .94     |
| EE $(M=34.99)$ | -       | -       | -2.65    |         |
| O $(M=37.64)$ | -       | -       | -        |         |
| **Sscore** |         |         |          |         |
| ME $(M=46.16)$ | -       | 1.01    | 6.70*    | 4.62*   |
| M $(M=45.15)$ | -       | -       | 5.68*    | 3.60    |
| EE $(M=39.46)$ | -       | -       | -2.08    |         |
| O $(M=41.54)$ | -       | -       | -        |         |
| **Ascore** |         |         |          |         |
| ME $(M=51.19)$ | -       | -1.02   | 8.99*    | 5.81*   |
| M $(M=52.21)$ | -       | -       | 10.00*   | 6.83*   |
| EE $(M=42.20)$ | -       | -       | -3.81    |         |
The teaching knowledge scores of the KAT had two significant differences between pathways. Post hoc comparisons indicated that the mean score for teachers who completed a traditional mathematics education certification pathway (M = 40.38, SD = 5.85) was significantly different than those teachers who completed an elementary education certification pathway (M = 34.99, SD = 5.40). These results suggest that those teachers certified through a mathematics education pathway have a much higher teaching knowledge than those certified through elementary education.

### 3.2 Score

The knowledge of school algebra dimension of the KAT posed significant differences between in three different comparisons. Those Algebra I teachers who were certified through a mathematics education pathway (M = 46.16, SD = 6.51) scored significantly higher than those who were certified through an elementary education pathway (M = 39.46, SD = 4.71) or other pathway (M = 41.54, SD = 4.86). Results also suggested a significant difference between those alternatively certified through a mathematics only pathway (M = 45.15, SD = 5.38) scoring higher than those certified through an elementary education pathway. The only comparison that was not significantly different from elementary education was the pathway considered to be other.

### 3.3 A Score

The advanced knowledge of mathematics dimension of the KAT, along with the final score, showed the greatest number of significant differences between certification pathways. The only pathways that were not significantly different in A score were mathematics education versus mathematics and elementary education versus others. This implies that the mathematics education pathway scores (M = 51.19, SD = 6.85) and mathematics pathway scores (M = 52.21, SD = 7.76) were significantly higher than those of elementary education (M = 42.20, SD = 4.78) and other (M = 45.38, SD = 6.91) pathways. These results indicate that teachers who have an educational background with more depth of mathematical content score higher than those who do not have mathematics as an educational background.

### 3.4 Final Score

Similarly to the A score results, significant differences occurred between certification pathways. Those Algebra I teachers with certification through a mathematics education (M = 45.13, SD = 5.97) pathway
and mathematics (M = 44.74, SD = 5.08) pathway scored significantly higher than those Algebra I teachers who are certified to teach through an elementary education (M = 36.93, SD = 3.76) pathway or other (M = 39.86, SD = 5.30) pathway. Results suggest that those teachers who were certified through a mathematics education pathway or those alternatively certified with a mathematics degree have an overall higher KAT than those who are certified in elementary education or another pathway not in mathematics.

4. Discussion and Conclusion

The purpose of this study was to (a) paint a picture of who is teaching Algebra I in Oklahoma and (b) explore the algebra beliefs of these Algebra I teachers along with their own understanding of algebra and the teaching of algebra concepts. This study is filling a gap in the research literature by looking at associations between teachers’ beliefs and KAT determined by their certification pathway. Particularly focusing on Algebra I teachers allows the research to indicate the type of teachers that should be teaching this gatekeeper course.

Regarding to certification pathway of Algebra I teachers in the state of Oklahoma, this study contained a sample of 144 where 77 teachers did not receive their certification through a traditional mathematics education teacher preparation program. Of those 77 teachers, many of them did not complete a certification pathway with a strong mathematics content background, but were still placed in an Algebra I classroom to teach algebraic content.

The teaching experience of Algebra I teachers is another important characteristic to discuss. In Oklahoma, one out of every five Algebra I teachers are novice teachers with only 1 – 5 years of teaching experience. This brings up the ideas of whether novice teachers should be teaching Algebra I, which is a gatekeeper course. If novice teachers do teach this course, strict attention should be paid to the teacher to ensure support whenever needed and ensure the teachers have a strong knowledge of algebra and pedagogical strategies to teach algebraic concepts.

Findings in the current study indicate that Algebra I teachers in the state of Oklahoma hold similar beliefs about the nature, teaching, and learning of algebra. Previous research has shown that teachers’ prior school experience in a mathematics classroom tend to be the main influence in beliefs, which leads teachers to teach more traditionally and procedurally (Raymond, 1997; Prawat, 1992). Furthermore, even when teachers are taught and encouraged to teach in a more constructivist manner, they must have the opportunity to be surrounded by other educators and teachers who share similar constructivist beliefs in the teaching and learning of mathematics (Prawat, 1992).

On the other hand, the findings in this study indicate that certification pathway are linked to the teaching and content knowledge of Algebra I teachers. Findings from past research have shown that effective classroom instruction is strongest when the teacher holds high subject content knowledge, curricular knowledge, and pedagogical content knowledge (Ball et. al, 2008; Shulman, 1986). Those Algebra I teachers who were certified through a traditional mathematics education teacher preparation
program and those were alternatively certified after receiving a degree in mathematics consistently had a higher level of algebraic content and teaching knowledge than those who were certified through an elementary education teacher preparation programs or any other type of certification pathway. Previous research suggests that in order to effectively teach mathematics at the middle and secondary level, teachers need a deep knowledge of advanced mathematics including calculus, linear algebra, and other courses (McCormy et al., 2012). The current study suggests similar findings. Elementary education certified teachers and other non-mathematics based majors may not have the depth of mathematical content background to effectively teach algebra courses. With so many teachers in the Algebra I classroom with an educational background in non-mathematics content, the rigor and depth of the content on the regional certification exam should be carefully established before distributing mathematics certifications.

Implications of this study include, notably, that there should be a more strict and rigorous process to enter the mathematics classroom as a teacher. For those teachers who do not follow a traditional mathematics certification pathway, simply passing a content knowledge exam may not be enough to be designated a teacher of that subject area, especially in mathematics, unless this exam is rigorous and focuses on content well beyond Algebra I. Usiskin et al. (2001) argued that middle and secondary mathematics teachers should understand three major categories of mathematical understanding: “concept analysis – the phenomenology of mathematical concepts, problem analysis – the extended analysis of related problems, and connections and generalizations within and among the diverse branches of mathematics” (p. 3). These categories are a mixture of content and pedagogical content knowledge that would ensure middle level and secondary level mathematics teachers are prepared to teach content effectively to students.

Since Algebra I is considered the gatekeeper to higher-level mathematics, schools need to ensure that the highest quality teachers are instructing this course. Algebra I teachers should be able to bridge mathematics across different topics and concepts that will link those ideas of standard school algebra to more advanced mathematics (McCormy et al., 2012). There is a need for the state of Oklahoma to examine their Algebra I teacher workforce and the pathways to certify those teachers. Otherwise, how can we improve student achievement in Algebra I or prepare the students for other high school, or college mathematics courses?

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