Tension remodeling controls topological transitions and fluidity in epithelial tissues

Fernanda Pérez-Verdugo¹ and Shiladitya Banerjee¹,*

¹Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213, USA

Cell neighbor exchanges promote tissue fluidity during epithelial morphogenesis and repair. In vivo, cell neighbor exchanges are often stalled by the formation of stable multicellular junctions, which do not arise in existing theoretical models. We present a dynamic vertex model of epithelial tissues where cellular tensions remodel in response to active fluctuations and local strains. We show that an increase in tension upon contraction and reduction upon extension can stall cell rearrangements, while increasing the rate of tension remodeling beyond a critical value drives tissue fluidization.

Fluidization of epithelial tissues plays a vital role in coordinating large-scale structural changes in early development [1, 2], wound healing [3], and collective cell migration [4–6]. While multiple cell-level mechanisms contribute to tissue fluidity, including cell migration, division, and death [7], cell neighbor exchanges are one of the most common drivers of tissue fluidity during morphogenesis [8, 9]. During a neighbor exchange process occurring via a T1 transition, two cells in contact shrink their shared junction to a single point, forming a 4-fold vertex. This 4-fold vertex then extends into a new intercellular junction in a direction orthogonal to the contracting junction. While neighbor exchange processes rely on the instability of 4-fold vertices, in vivo experiments showed that 4-fold vertices can be stable for long times in developing tissues [10–12]. In particular, during axis elongation in Drosophila, vertices shared by 4 or more cells (termed as rosettes), could persist for up to 15–40 min [10, 12], stalling cell neighbor exchanges.

Experimental observations of controlled cell neighbor exchanges contrast with existing vertex models of epithelial tissues [13, 14], where stationary 4-fold vertices do not naturally arise and are energetically unstable [15]. Furthermore, experiments showed that 4-fold and higher-order vertices often restore the original cell junction, resulting in a reversible T1 process [11, 12, 16]. By contrast, most theoretical studies treat the creation and resolution of 4-fold vertices as an instantaneous and unidirectional event triggered by junctions contracting below a length threshold [17] or if neighbor exchange is energetically favorable [18, 19]. Others have engineered the formation of higher-order vertices by ad-hoc rules. For instance, Farhadifar et al. [20, 21] enforced the creation of 4-fold vertices by joining proximal three-fold vertices, and stalling their subsequent resolution. On the other hand, there have been recent theoretical efforts to understand the impact of non-instantaneous resolution of 4-fold vertices and probabilistic T1 events [12, 22–24]. However, in all these studies, stalling of T1 events were imposed by ad-hoc rules, and they did not naturally arise from the system mechanics.

To explain the mechanistic origin of vertex stability and topological transitions in epithelial tissues, we extended the existing framework of vertex models [13, 14, 20] to incorporate dynamic tension remodeling on cell edges. In particular, the tension in intercellular junctions is not static but evolves in time due to changes in junctional strain or active fluctuations. We show that tension remodeling naturally leads to controlled cell neighbor exchanges, such that T1 transitions are stalled when they are not energetically favorable. Furthermore, by tuning the rates of tension remodeling in the system, the model tissue can transition between solid and fluid phases.

Model – Each cell in the tissue is modeled by a two-dimensional polygon, with edges representing the cell-cell junctions and the vertices representing multi-cellular junctions. The mechanical energy $E$ of the tissue is given by

$$E = \sum_\alpha \frac{K}{2} (A_\alpha - A_\alpha^0)^2 + \sum_{ij} (\Lambda_{ij} + \Delta \Lambda_{ij}) l_{ij} + \sum_{ij} \frac{\Gamma_{ij}}{2} l_{ij}^2,$$

where the first term represents the energy cost for cell compressibility, penalizing changes in the area $A_\alpha$ of cell $\alpha$, with respect to its target value $A_\alpha^0$, and $K$ is the elastic constant. The second term represents the energy due to interfacial tension $\Lambda_{ij}$ on edge $ij$, connecting vertices $i$ and $j$ with length $l_{ij}$. $\Delta \Lambda_{ij}$ represents stochastic fluctuations in junctional tension, as described later. Finally, the third term in the energy function arises from actomyosin contractility, with $\Gamma_{ij}$ representing the active stress generated by myosin motors on each edge. The mechanical force acting on each vertex $i$ results from the gradient in the mechanical energy $\mu dr_i/\partial r_i = -\partial E/\partial r_i$, where $r_i$ is the vertex position and $\mu$ is the vertex friction coefficient.

The dynamics of tension $\Lambda_{ij}$ is dependent on the junctional strain $\epsilon_{ij} = (l_{ij} - l_{ij}^0)/l_{ij}^0$, where $l_{ij}^0$ is the junction rest length. Junction tension evolves as

$$\frac{d\Lambda_{ij}}{dt} = -\alpha(\epsilon_{ij})(l_{ij} - l_{ij}^0) - \frac{1}{\tau_\alpha}(\Lambda_{ij} - \Lambda_0),$$

where the first term describes strain-dependent tension remodeling, as recently introduced [25] and validated experimentally [26, 27], and the second term describes tension relaxation to a mean value $\Lambda_0$, which occurs over a longer characteristic timescale $\tau_\alpha$. The rate of tension remodeling, $\alpha$, is defined as: $\alpha(\epsilon_{ij} < -\epsilon_c) = k_C$, $\alpha(\epsilon_{ij} > \epsilon_c) = k_E$, and $\alpha = 0$ otherwise, where $\epsilon_c$ is a threshold strain. As a result, tension increases upon contraction at a rate $k_C$ and reduces upon stretch at a rate $k_E$, consistent with recent experimental observations [26, 28, 29]. Tension remodeling above a critical strain threshold allows for irreversible junction deformation for sufficiently strong or sustained force [26]. Additionally, the system continuously relaxes strain at a rate $k_L$ such that the rest length approaches current junction length.


\[ \frac{d\Delta_{ij}}{dt} = -\frac{1}{\tau} \Delta_{ij} + \sqrt{2\sigma^2/\tau} \xi_{ij}(t), \]

where \( \sigma \) is the fluctuation amplitude, \( \xi_{ij}(t) \) is a white Gaussian noise satisfying \( \langle \xi_{ij}(t) \xi_{m,n}(t') \rangle = \delta(t-t')\delta_{im}\delta_{jn} \), and \( \tau \) is the persistence time of tension fluctuations.

With the model mechanics defined above, we now turn to describing the dynamics governing topological T1 transitions. To simulate a T1 transition, when a junction connected by two three-fold vertices becomes shorter than a threshold length \( l_{T1} \), one of the vertices is removed while the other is transformed into a 4-fold vertex, sustained by four shoulder junctions. During this process, each shoulder junction gains one-fourth of the deleted junction tension, and conserve it until the 4-fold vertex is resolved [30]. The latter is motivated by experimental observations of Myosin-II accumulation around junctions proximal to 4-fold vertices [16]. We then create a new junction of length 1.5\( l_{T1} \), tension \( \Lambda_{birth} \sim \Lambda_0 + 1.5\Gamma_{a} l_{T1} \), and attempt to resolve the 4-fold vertex in two different directions – one along the original contracting junction (resulting in reversible T1) and the other orthogonal to it (leading to neighbor exchange). If the effective force between the vertices of the newly created junction is attractive, then the 4-fold is considered stable and the T1 transition is stalled. Otherwise, the 4-fold vertex resolves along the direction with the largest repulsive force [16, 31], resulting in reversible or irreversible T1 transitions (see Supplemental Methods for details).

**Tension remodeling controls T1 transitions** – To characterize the role of tension remodeling on T1 transitions, we first simulated a disordered tissue comprising \( \sim 500 \) cells in a box with periodic boundary conditions. In simulations, we non-dimensionalized force scales by \( K(A_{ij}^0)^{1/2} \), length scales by \( \sqrt{A_{ij}^0} \), time scales by \( \mu/KA_{ij}^0 \), setting \( K = 1 \), \( \langle A_{ij}^0 \rangle = 1 \), and \( \mu = 0.2 \) (\( \sim 28 \) s), where \( \langle \ldots \rangle \) represents population average. The initial state of the simulations is characterized by \( i_{ij} = i_{ij}^0 \), \( \langle i_{ij}^0 \rangle \sim 0.62 \) and \( \langle \Lambda_{ij} \rangle = \Lambda_0 = 0.1 \) (see Supplemental material, Fig S1). We let the tissue evolve from an energy relaxed state with chosen values of active contractility \( \Gamma_a \), active fluctuations of amplitude \( \sigma \), tension remodeling rates \( k_E \), and a fixed value of \( k_C \). A representative simulation snapshot is shown in Fig. 1A, for \( k_E/k_L = 0.2 \) and \( k_E/k_L = 0.17 \), which displays multiple (transiently stable) 4-fold vertices representing stalled T1 transitions (Movie 1).

Four different types of dynamics are observed during T1 processes (Fig. 1B, Movie 1): instantaneous irreversible T1, delayed irreversible T1 with a stalled 4-fold vertex, instantaneous reversible, and delayed reversible T1 events. In all these cases, tension increases during contraction prior to 4-fold vertex formation, and decreases via remodeling after the 4-fold vertex resolves into an extending junction (Fig. 1B). Tension remodeling can decrease the local tensions in stretched shoulder junctions, hence decreasing the net repulsive force \( f_{rep} \) acting between the vertices of the newly created junction, promoting 4-fold stabilization. Specifically, a stalled 4-fold vertex arises when \( f_{rep} < 2\Lambda_{birth} \) (Fig. 1D), where \( f_{rep} \) comes from tensions in the shoulder junctions as well as pressures in the neighboring cells resisting changes in cell area. When the local tension increases (due to remodeling, fluctuations, or contractility), vertex stability is lost, resulting in delayed T1 or a delayed reversible event (Fig. 1B). Fig. 1C shows the distribution of stalling times, for both reversible and irreversible T1 events, suggesting that some 4-fold vertices can be resolved near-instantaneously, while others can remain stalled for longer periods. Without tension remodeling (\( k_E = k_C = 0 \)), we recover the standard vertex model dynamics where T1 transitions occur instantaneously (Movie 2).

Dynamics of the model tissue with tension remodeling, as characterized in Fig. 1 (Movie 1), settle into a fluctuating steady-state with an asymmetric distribution of junction length (Fig. 2A), as observed in vivo [16]. Furthermore, a negative correlation is observed between junction length and negative correlation is observed between junction length and...
tension in the fluctuating steady-state (Fig. 2B), analogous to the negative correlation between junction length and myosin intensity seen experimentally [16]. By contrast, without tension remodeling, junction length distribution is symmetric and independent of tension (Fig. 2A-B).

**Stability of 4-fold vertices** – During a T1 transition, 4-fold vertex can be transiently stable if the tension in the extending shoulder junctions are small compared to the tension in the newly created junction (Fig. 1D). This can be achieved via tension remodeling in the extending shoulder junctions, controlled by the rate $k_E$. To understand how $k_E$ affects vertex stability, we studied an effective medium model consisting of symmetric cell junctions embedded in an effective elastic medium (see Supplemental Material). We activated contraction in chosen junctions by increasing $\Gamma_a$. During this process, the contracting (extending) junctions increase (decrease) their tension at a rate $k_C$ ($k_E$). We found that if $\beta k_E > k_C$, the global tissue tension decreases, promoting mechanical stability of the 4-fold vertex, where $\beta$ is the ratio of the total length gained by the extending junctions to the total length lost by the contracting junctions.

To further investigate the role of tension remodeling on topological transitions, we performed numerical simulations using different values of the tension remodeling rates, $k_C/k_L$ and $k_E/k_L$ in the range [0.02,0.23] (Fig. 3). From fits to experimental data on single junction deformations, it was determined that $k_C/k_L \approx 0.14$ [25, 26] and $k_E/k_L \approx 0.12$ [29]. We found that the tension remodeling rates controlled the rate of T1 transitions as well as the probability of delayed T1 transitions. For very small values of $k_E$ and $k_C$, the tissue is in a quiescent state, where T1 events are scarce ($<10^{-4}$ per junction per minute) and occur instantaneously (Fig. 3A). Due to the lack of appreciable tension remodeling in the quiescent state, tensions in the shoulders of an intercalating junction $\sim A_0$, with negligible resistive pressure in the surrounding cells since $A_g \sim A_0^2$. As a result, the net force ($f_{rep}$) pulling the vertices of the newly created junction remains larger than

![Figure 2](image1.png)

**FIG. 2.** Tension remodeling promotes asymmetric length distribution. (A) Histogram of junction length at steady-state, for a tissue with remodeling (blue, $k_C/k_L = 0.2$ and $k_E/k_L = 0.17$) and without remodeling (black, $k_C = k_E = 0$). (B) Correlation between junction tension (normalized) and junction length (normalized) in tissues with and without junction remodeling. Error bars represent ±1 standard error of mean.

![Figure 3](image2.png)

**FIG. 3.** Junction tension remodeling regulates the rates of T1 transitions. (A) Rates of T1 (left) and reversible (right) transitions for different values of $k_E/k_L$ and $k_C/k_L$. Solid lines represent $10^{-3}$ T1 events per junction per minute. (B) Probability of stalled/delayed T1 transitions (left) and reversible T1 events (right), for different values of $k_E/k_L$ and $k_C/k_L$. Dashed lines represent 0.05 probability. See Supplemental Material for a list of parameters.

$\text{2A}_{\text{birth}}$, making the 4-fold vertex unstable. For larger values of $k_C$ and $k_E$, irreversible and reversible T1 events are mainly driven by tension remodeling, inducing wider pressure distributions (Fig S3), and higher rates of T1 events (Fig. 3A). In this parameter regime, $f_{rep}$ depends on both tensions and pressures in the surrounding cells. Since tension remodeling dynamics is fast compared to pressure relaxation, pressure-like forces make $f_{rep}$ larger in the original direction of contraction, turning the reversible T1 events more probable. In the presence of tension remodeling, T1 events either occur instantaneously or are delayed, with probabilities given in Fig. 3B. The probabilities of delayed events, and hence the presence of stalled 4-fold vertices, depend strongly on $k_E$, as predicted analytically. 4-fold stability increases for large $k_E$ and small $k_C$, reaching stalling times of 5 min on average (Fig. S4), consistent with experimental data [12]. T1 stalling time increases with reduced tension fluctuations, which can be controlled by the noise magnitude $\sigma$. Increasing $\sigma$ increases the rate of T1 events [3, 16, 32], while reducing T1 stalling times (Fig. S7). In the absence of noise ($\sigma = 0$), tension fluctuations are diminished with more unresolved T1 transitions (Movie 3).

**Mechanical properties of remodeling tissues** – Since reduced junction tension promotes the stability of 4-fold vertices and delayed T1 transitions, we computed the mean change in tissue tension from its initial state, as functions of the tension remodeling rates (Fig 4A). We found that the mean tension change is negative in the parameter space with non-zero probabilities of delayed T1s (Fig 3B), suggesting a
loss of tissue rigidity. The white solid line in Fig. 4A represents the phase boundary obtained from simulations, where the mean tissue tension does not change. For small \( k_E, k_C \), the phase boundary follows the line \( k_E = k_C \) (solid black), as predicted by the effective medium model in a system conserving the total junction length (see Supplemental Material). For large \( k_E, k_C \), the phase boundary increases in slope, as predicted in a system increasing its total junction length. In a two-dimensional effective medium model consisting of five symmetric cell junctions (see Supplemental Material), the predicted phase boundary is \( 2k_E = k_C \) (dashed black line).

From the measurements of changes in tension, we found that tissues with smaller values of \( k_E \) and \( k_C \) maintained a constant mean tension (Fig. 4A), with very low rates of neighbor exchange (Fig. 3A), characteristic of an arrested state. To characterize the mechanical properties of the remodeling tissue, we measured the mean-squared displacements of the cell centers (Supplemental Material) to compute the average diffusivity \( (D) \) of cells (Fig. 4B). We found that cells with smaller values of \( (k_E, k_C) \) do not diffuse significantly \( (D < 10^{-3}) \), representing solid-like tissues with mostly hexagonal cell shapes.

Diffusivity increases with \( k_E \) and \( k_C \), such that the tissue is liquid-like when \( (k_E + k_C)/k_L \) is larger than a critical value (Fig. 4B). Increasing \( k_E \) in a solid tissue increases the number of hexagonal cell shapes relative to pentagons and heptagons, while increasing \( k_E \) in a fluid tissue leads to a relative increase in the number of pentagons, along with the presence of triangles, squares, and octagonal cell shapes (Fig. S5). Interestingly, tissues possessing higher-order vertices are fluid-like with a high diffusivity of cell centers (Fig. 4B).

In vertex models, fluidity is related to the observed cell shape index \( q \) [4, 33, 34], defined as the mean ratio between the perimeter and the square root of the cell area. A fluid-solid phase transition occurs at \( q = 3.81 \), such that the tissue is solid-like for \( q < 3.81 \). The rigidity transition is related to the mechanical stability of junctions, which occurs in the effective medium theory when \( (k_C + k_E)/2k_L \) is smaller than the effective medium stiffness \( k \) (dashed line in Figs. 4B-C, see Supplemental Material). From our simulations, we obtained an excellent agreement between the contours \( D = 10^{-4} \) and \( q = 3.81 \) (white curves in Figs. 4B,C). Our theory thus relates the fluidity of confluent tissues and their emergent topology to the rates of tension remodeling, \( k_C \) and \( k_E \) (Fig. 4D). In particular, we find that 4-fold and higher-order vertices can become stable in fluid tissues if \( 1 > k_E > k_C \) (\( \beta > 1 \)), such that asymmetric tension remodeling reduces the mean tension in the tissue.

Interestingly, an increase in the tissue tension does not necessarily imply a more solid-like tissue. Instead, high-tension tissues display a higher rate of instantaneous T1 events, inducing cellular motion through cell neighbor exchanges. This results in increased diffusion of cells, making the tissue more fluid-like. It has been previously reported that the presence of higher-order vertices leads to rigidification of tissues [22]. While the presence of stalled 4-fold vertices reduces the rate of cell neighbor exchanges (Fig. S6), we find that the mechanical stability of 4-fold vertices demands a liquid-like tissue with low overall tension. These requirements imply that there are many T1 events occurring in the presence of stable 4-fold vertices, as observed during Drosophila axis elongation [12].

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* shiladtb@andrew.cmu.edu

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Supplemental Material
Tension remodeling controls topological transitions and fluidity in epithelial tissues

Fernanda Pérez-Verdugo and Shiladitya Banerjee

1 Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213, USA

1 Simulation Methods

1.1 Initial configuration
We created a disordered tissue composed of 494 cells in a box of dimensions $L_x \times L_y$ (Table 1), with periodic boundary conditions. The disordered tissue was built via Voronoi tessellation, where the positions of the cell centers were generated by a Monte Carlo simulation of hard disks, with an area fraction equal to 0.71 [1]. On this disordered tissue comprising variable-sided polygons, we obtained the energy relaxed state by evolving the system using the standard vertex model Hamiltonian:

$$E = \frac{1}{2} K \sum_\alpha (A_\alpha - A_\alpha^0)^2 + \frac{1}{2} K_P \sum_\alpha (P_\alpha - P_\alpha^0)^2,$$

where $P_\alpha$ and $P_\alpha^0 = P_0$ define the cellular perimeters and their target values, respectively. We used $K = 1, K_P = 0.2, P_0 = 3.5,$ and $A_\alpha^0$ was drawn from a normal distribution with unit mean and standard deviation 0.1. During the initial energy relaxation process, we allowed T1 rearrangements when cellular edges became smaller than a length threshold $l_{T1} = 0.05$ and if the topology change decreased the overall tissue energy. This procedure ensured that the relaxed tissue configuration only contained tricellular vertices. Using the configuration of the resulting tissue, we initiated a second round of energy relaxation using the hamiltonian defined in Eq. (1) of the main text, with active terms set to zero. During this process, we preserved the values of the target areas, and assumed that initial rest length $l_{0ij}$ is equal to the initial junction length $l_{ij}$. Further, cell junctions were assigned tension values $\Lambda_{ij}$, drawn from a uniform distribution with mean 0.1 and standard deviation 0.01. The parameter values are given in Table 1. We let this tissue relax its energy by setting $k_E = k_C = 0, \sigma = 0, \Gamma_a = 0$. As a result, the tensions $\Lambda_{ij}$ did not change, while $l_{0ij}$ and $l_{ij}$ reach approximately normal distributions with mean 0.62 (side of a regular hexagon with unit area), with standard deviation ~ 0.2. The relaxed state, defining the initial configuration of our simulations, contained only tricellular vertices, forming polygonal shapes from squares to octagons (Fig. S1).

1.2 Rules for T1 transitions
1. If a junction shared by two three-fold vertices $i$ and $j$, with total tension $\Lambda_{ij}$, shrinks below a threshold length $l_{ij} < l_{T1}$, then we remove one of the vertices ($j$, for example), while transforming the other ($i$) into a four-fold vertex. During this process, each shoulder junction sustaining the four-fold vertex gains one-fourth of the tension ($\Lambda_{ij}$) in the deleted junction, which remains in the system until the four-fold is resolved. We save the information of the cells that shared the deleted junction ($A$ and $C$, for example). In clockwise order, the four-fold vertex $i$ belongs to four cells, which we will call $A, B, C, D$.

Note: If a junction shared by two vertices $i$ and $j$ shrinks below a threshold $l_{ij} < l_{T1}$, and at least one of them is a four-fold vertex, no higher-order vertex is created, and the tension remodeling rate $k_C$ is set to zero until $l_{ij}$ reaches the threshold $l_{T1}$. 

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| Parameter                                               | Symbol | Value                  |
|--------------------------------------------------------|--------|------------------------|
| Area elastic modulus                                    | $K$    | 1                      |
| Mean preferred area                                     | $\langle A_0^\alpha \rangle$ | 1                      |
| Friction coefficient                                    | $\mu$  | 0.2 (28s)              |
| Mean initial tension                                    | $\Lambda_0$ | 0.1                    |
| Tension in newly created junction                       | $\Lambda_{\text{birth}}$ | 0.1                    |
| Active contractility                                    | $\Gamma_a$ | 0.03                    |
| Noise amplitude                                         | $\sigma$ | 0.02                    |
| Simulation box length                                   | $L_x$  | $\sim$ 20              |
| Simulation box width                                    | $L_y$  | $\sim$ 24              |
| Strain relaxation rate                                  | $k_L$  | 1                      |
| Tension remodeling rate under contraction               | $k_C$  | $\in [0.02, 0.23]$     |
| Tension remodeling rate under extension                 | $k_E$  | $\in [0.02, 0.23]$     |
| Critical strain threshold                               | $\varepsilon_c$ | 0.1                    |
| Length threshold for attempting T1 transitions          | $l_{T1}$ | 0.05                    |
| Time between attempts to resolve 4-fold vertices        | $\tau_{\text{test}}$ | 0.04                    |
| Tension relaxation timescale                            | $\tau_\Lambda$ | 10                    |
| Relaxation timescale for tension fluctuations           | $\tau$ | 0.4                    |
| Integration time step                                   | $\Delta t$ | 0.004                  |

Table 1: Model parameters.

Fig. S1: **Initial configuration of the tissue.** (A) Representative section of the energy relaxed tissue. (B) Histograms of cell target area, junction rest length, and junction tension at the relaxed initial state. Rest length is expressed in units of $a$, which corresponds to the side of a regular hexagon with area $\langle A_0 \rangle \left(a = 0.62\sqrt{\langle A_0 \rangle}\right)$. (C) Histogram of polygon sidedness in the initial configuration.
2. Every $\tau_{\text{test}}$ timesteps, we attempt to resolve the four-fold vertices present in the tissue via two configurations. First, we create a junction of length $1.5l_T$ in the direction given by the centers of cells $B$ and $D$, thereby recovering the original topology (reversible T1 process). Second, we create the same length junction but now in the direction given by the centers of cells $A$ and $C$, thereby changing the topology (T1 transition). In both arrangements, we assign a tension value $\Lambda_{ij}$ for the newly created junction drawn from a normal distribution with mean value $\Lambda_0$ and standard deviation $0.1\Lambda_0$. Then, the total tension in the new junction is given by $\Lambda_{\text{birth}} = \Lambda_{ij} + 1.5\Gamma al_T$. The rest length $l_{0ij}$ of the new junction is drawn from a truncated (only positive values) normal distribution with mean value $l_T$ and standard deviation equals $0.1l_T$. During this process, each shoulder junction sustaining the newly created junction loses one-fourth of $\Lambda_{\text{birth}}$. In both arrangements, we calculate the forces at vertices $i$ and $j$.

3. If the effective force between $i$ and $j$ is attractive in both previously tested configurations, then the four-fold vertex is considered stable, and we proceed to delete $j$ again. However, if the effective force between $i$ and $j$ is repulsive, at least in one case, then the four-fold vertex is unstable. Therefore, we choose the configuration with the largest repulsive force between the two three-fold vertices.

2 Effective medium model

To analytically predict the mechanical stability of cell junctions under contraction, we consider an effective medium theory of the system consisting of two cell junctions in series, connected in parallel to an effective elastic medium of spring constant $k$ (Fig. S2A). Tension in each junction $i$ ($i = 1, 2$) is given by $\Lambda_i$, with length $l_i$, rest length $l_{0i}$, deforming against an overdamped medium with friction coefficient $\mu$. We choose to activate junction 1 with contractility $\Gamma_a > 0$, neglect tension fluctuations, and set the critical strain threshold to 0 for simplicity. Junction tensions and rest lengths evolve following the dynamics defined in main text,

$$\frac{d\Lambda_1}{dt} = -k_C(l_1 - l_{01}),$$

$$\frac{dl_{01}}{dt} = -k_L(l_{01} - l_1),$$

$$\frac{d\Lambda_2}{dt} = -k_E(l_2 - l_{02}),$$

$$\frac{dl_{02}}{dt} = -k_L(l_{02} - l_2).$$

Fig. S2: Schematic of the effective medium models. (A) Effective system composed of two junctions of natural length $2L$, under fixed boundary conditions. Each junction is composed of an elastic element with spring constant $k$, a dashpot with friction coefficient $\mu$, and a tension $\Lambda$ that remodels at a rate $k_C$ under contraction, and $k_E$ under stretch. Junction rest length remodels at a rate $k_L$. (B) Schematic of an effective five-junction-system in two dimensions, as part of a hexagonal lattice. The central junction is activated by contraction, sustained by four shoulder junctions. (Left) Shoulder junctions are under a fixed boundary condition. (Right) The four shoulder junctions are free to move vertically.
Assuming the system conserve its total length, we have the constraint $2L = l_1 + l_2$. Dynamics of junction length then follows from considering the force-balance equation at the vertex between the two junctions,

$$2\mu \frac{dl_1}{dt} = k(l_2 - L) + k(L - l_1) + \Lambda_2 - \Lambda_1 - \Gamma_a l_1,$$

$$= k(2L - l_1 - L) + k(L - l_1) + \Lambda_2 - \Lambda_1 - \Gamma_a l_1,$$

$$= 2k(L - l_1) + \Lambda_2 - \Lambda_1 - \Gamma_a l_1. \quad (6)$$

### 2.1 Mechanical stability of cell junctions

The activated junction (junction 1) will collapse to zero length if the system is unstable to contraction. Stability is thus defined by the junction reaching a non-zero length $l_1 > 0$ at steady-state. At steady-state, solution to Eqs. (2)-(6) are given by,

$$0 = 2kL - (2k + \Gamma_a)l_1^{EQ} + \Lambda_2^{EQ} - \Lambda_1^{EQ}, \quad (7)$$

$$l_1^{EQ} = l_0^{EQ}, \quad (8)$$

$$l_2^{EQ} = 2L - l_1^{EQ}. \quad (9)$$

Since $d\Lambda_1/dt = -(k_C/k_L)dl_0^{EQ}/dt$, and $d\Lambda_2/dt = -(k_C/k_L)dl_0^{EQ}/dt$, we can integrate these equations from $t = 0$ to the time at which steady-state is reached, to calculate the steady-state tension values $\Lambda_1^{EQ}$ and $\Lambda_2^{EQ}$. Considering that initial lengths and tensions were given by $L$, and $T_0$, respectively, we obtain

$$\Lambda_1^{EQ} = T_0 - \frac{k_C}{k_L} (l_0^{EQ} - L), \quad (10)$$

$$\Lambda_2^{EQ} = T_0 - \frac{k_E}{k_L} (l_0^{EQ} - L). \quad (11)$$

Using relations in Eqs. (8)-(11) in Eq. (7) we get,

$$0 = 2kL - (2k + \Gamma_a)l_1^{EQ} + T_0 - \frac{k_E}{k_L} (l_0^{EQ} - L) - T_0 + \frac{k_C}{k_L} (l_0^{EQ} - L),$$

$$0 = 2kL - (2k + \Gamma_a)l_1^{EQ} - \frac{k_E}{k_L} (2L - l_1^{EQ} - L) + \frac{k_C}{k_L} (l_1^{EQ} - L),$$

$$0 = 2kL - (2k + \Gamma_a)l_1^{EQ} - \frac{k_E}{k_L} (L - l_1^{EQ}) - \frac{k_C}{k_L} (L - l_1^{EQ}),$$

$$0 = 2kL - (2k + \Gamma_a)l_1^{EQ} - \left(\frac{k_E + k_C}{k_L}\right) L + \left(\frac{k_E + k_C}{k_L}\right) l_1^{EQ},$$

$$0 = L \left(2k - \frac{k_E + k_C}{k_L}\right) - \left(2k + \Gamma_a - \frac{k_E + k_C}{k_L}\right) l_1^{EQ}. \quad (12)$$

The above equation gives,

$$l_1^{EQ} = \frac{2k - \frac{k_E + k_C}{k_L}}{2k + \Gamma_a - \frac{k_E + k_C}{k_L}}, \quad (13)$$

Thus, in order to have $L > l_1^{EQ} > 0$, $(k_E + k_C)/k_L$ has to be smaller than $2k$.

### 2.2 Tension change due to remodeling

In the main text, we presented results relating the stability of four-fold vertices to reduction in tension in the tissue. Here we derive the condition for reduction in tension using the effective medium model. We can write Eqs. (10) and (11) in a
more general form, with the initial lengths of a junction under contraction (stretch) given by \( L_{\text{ini}}^C \) (\( L_{\text{ini}}^E \)),

\[
\Lambda_C^{\text{EQ}} = T_0 - \frac{k_C}{k_L} (L_{\text{EQ}}^C - L_{\text{ini}}^C), \tag{14}
\]

\[
\Lambda_E^{\text{EQ}} = T_0 - \frac{k_E}{k_L} (L_{\text{EQ}}^E - L_{\text{ini}}^E). \tag{15}
\]

Assuming that we have \( A \) junctions under contraction and \( B \) junction under stretch, all of them with initial tension \( T_0 \), the global change in tissue tension from an undeformed state is given by

\[
\Delta \Lambda = \left[ A T_0 - \frac{k_C}{k_L} \sum_{C=1}^{A} (L_{\text{EQ}}^C - L_{\text{ini}}^C) + B T_0 - \frac{k_E}{k_L} \sum_{E=1}^{B} (L_{\text{EQ}}^E - L_{\text{ini}}^E) \right] - (A + B)T_0,
\]

\[
= \left[ -\frac{k_C}{k_L} \sum_{C=1}^{A} (L_{\text{EQ}}^C - L_{\text{ini}}^C) - \frac{k_E}{k_L} \sum_{E=1}^{B} (L_{\text{EQ}}^E - L_{\text{ini}}^E) \right],
\]

\[
= -\left[ \frac{k_E}{k_L} \sum_{E=1}^{B} (L_{\text{EQ}}^E - L_{\text{ini}}^E) - \frac{k_C}{k_L} \sum_{C=1}^{A} (L_{\text{EQ}}^C - L_{\text{ini}}^C) \right]. \tag{16}
\]

We then define \( \delta L^+ = \sum_{E=1}^{B} (L_{\text{EQ}}^E - L_{\text{ini}}^E) \), and \( \delta L^- = \sum_{C=1}^{A} (L_{\text{EQ}}^C - L_{\text{ini}}^C) \), where \( \delta L^+ \) represents the net elongation of the junctions under stretch, while \( \delta L^- \) represents the net contraction of the junctions under contraction. If \( \delta L^+ = \beta \delta L^- \), we then have

\[
\Delta \Lambda = -\left( \frac{k_E}{k_L} \delta L^+ - \frac{k_C}{k_L} \delta L^- \right),
\]

\[
= -\left( \frac{k_E}{k_L} \beta \delta L^- - \frac{k_C}{k_L} \delta L^- \right),
\]

\[
= -\frac{\delta L^-}{k_L} (\beta k_E - k_C). \tag{17}
\]

From the last equation, we can see that the condition for reducing the global tension is \( \beta k_E > k_C \), with \( \beta \) depending on the increase or decrease of the total junction length after activation. In a system that conserves its total junction length (\( \delta L^+ = \delta L^- \)), the condition for reducing the total tension is given by \( k_E > k_C \). However, if after the activation, the system increases its total junction length (\( \delta L^+ > \delta L^- \)), then the condition for reducing tension is given by \( \beta k_E > k_C \), with \( \beta > 1 \). In the context of a confluent tissue simulated with the vertex model, a solid tissue tries to maintain the initial steady-state configuration, and we expect the condition \( k_E > k_C \) to hold for global tension reduction. Instead, fluid tissues increase their cell shape index, and then we expect the condition \( \beta k_E > k_C \), with \( \beta > 1 \).

If we now consider a two-dimensional system as in Fig. S2B-(left) and activate the red junction, the system will decrease its total junction length (\( \beta < 1 \)). Instead, if we consider a two-dimensional system as in Fig. S2B-(right), with the condition that outer ends of the shoulder junctions can move vertically while keeping \( 2l_2 + l_1 = 3L \) fixed, we get

\[
\delta L^- = L - l_1, \tag{18}
\]

\[
\delta L^+ = 4(l_2 - L) = 4(3L/2 - l_1/2 - L) = 2(L - l_1). \tag{19}
\]

Thus, \( \beta = \delta L^+ / \delta L^- = 2 \).

3 Additional characterizations of tissue mechanics from the vertex model simulations

Effect of tension remodeling on cellular pressure. Figure S3 shows the distribution of cellular pressure in the tissue, which is given for each cell as \( K (A_d - A_d^0) \). The pressure distribution becomes wider and larger in magnitude for higher rates of tension remodeling. This suggests that pressure-like forces play an important role in regulating tissue topology.
and the stability of four-fold vertices.

**Effect of tension remodeling on T1 stalling times.** Figure S4 shows the role of tension remodeling on the mean stalling time (in minutes) for T1 transitions. Four-fold vertices are present for longer times for large $k_E$ and small $k_C$, reaching mean stalling times of 4.5 min and 6 min, for T1 and reversible T1 events, respectively.

**Mean-squared displacement of cell centers.** In the main text, Fig. 4B showed the role of tension remodeling on the diffusion coefficient ($D$) of cell centers. The diffusion coefficient was calculated as $(1/4)$ times the rate of change of the mean-squared displacement (MSD), during the last half of the simulations. Fig. S5-(left) shows the MSD for two different values of $k_C/k_L$: 0.08 (Fig. S5A) and 0.20 (Fig. S5B), and several values of $k_E/k_L$ in the range 0.05 – 0.23. For $k_C/k_L = 0.08$, increasing $k_E/k_L$ from 0.05 to 0.23 drives a phase transition from a solid state (low $D$) to a fluid state (high $D$, Fig. 4 of the main text), where the fluid state is characterized by a lower average tension and stable four-fold vertices. Instead, tissues with $k_C/k_L = 0.20$ are fluid-like for all the considered values of $k_E/k_L$, but there is a transition from a high tension state (with no stable four-fold vertices) to a low tension state (with stable four-fold vertices).

**Polygon sidedness.** Fig. S5-(right) shows the distribution of number of polygon sides for different combination values of $(k_C/k_L, k_E/k_L)$, along with the polygon sidedness for tissues with no tension remodeling (white bars). We obtained that increasing $k_E$ in a solid tissue increases the number of hexagons while decreasing the relative numbers of pentagons and heptagons. The behavior is similar to what was obtained in Ref. [2], when increasing the mean line tension in the cell edges. On the other hand, when increasing $k_E$ in a fluid tissue, the number of hexagons and heptagons decrease, while pentagons increase in number. In fluid tissues, triangles, squares, and octagons are observed as $k_E$ is increased. Experimental data show the presence of triangles, squares, and octagons in the third instar larval wing disc of *Drosophila* [3].

**Correlation between T1 stalling time and T1 rates.** Figure S6 shows the correlation between the rate of T1 transitions per junction and the mean T1 stalling time. The initial increase in the rate of T1 with T1 stalling times indicates that stable four-fold vertices are present in fluid-like tissues with a high rate of T1 events. A negative correlation between the mean stalling time and the rate of T1 events emerges for higher stalling times. The presence of four-fold vertices for longer times implies a decrease in the events of cellular rearrangements.

**Role of tension noise amplitude $\sigma$.** In the main text, all the simulations considered a fixed amplitude of tension fluctuations, $\sigma = 0.02$. Fig. S7A shows how the rate of instantaneous and delayed events change when using different values of the tension noise amplitude $\sigma$. We find that tension fluctuations increase the number of T1 events. Additionally, we obtain that T1 stalling time decreases with increasing tension fluctuations (Fig. S7B). Stable four-fold vertices can be present for more than 40 minutes if fluctuations are minimal. The resolution of four-fold vertices is achieved due to tension remodeling and fluctuations.

**Effects of tension relaxation on tissue mechanics.** In the main text, all simulations were run considering a fixed value for the tension relaxation timescale $\tau_\Lambda = 10$. Fig. S8 shows the temporal evolution of tissue mechanical energy for $(k_C, k_E) = (0.02, 0.23)$ (left), and $(k_C, k_E) = (0.23, 0.02)$ (right), while varying $\tau_\Lambda$. We find that $\tau_\Lambda$ sets the steady-state energy of the tissue. Furthermore, increasing $\tau_\Lambda$ decreases the threshold for transition from solid-to-fluid phase. Fig. S9 shows the comparison between the phase diagrams obtained for $\tau_\Lambda = 10$ and $\tau_\Lambda = 20$. 
Fig. S3: **Role of tension remodeling on the distribution of cellular pressure.** (A) Histogram of cellular pressure for low \((k_C/k_L = k_E/k_L = 0.02)\) and higher \((k_C/k_L = k_E/k_L = 0.20)\) rates of tension remodeling. (B) Snapshots of the tissues considered in (A) show the spatial distribution of cellular pressure, for low (left) and high (right) rates of tension remodeling. Red circles represent four-fold vertices.

Fig. S4: Mean T1 stalling time (colorscale) as functions of normalized tension remodeling rates, \(k_E/k_L\) (x-axes) and \(k_C/k_L\) (y-axes). Left: Mean stalling time for irreversible T1 transitions. Right: Mean stalling time for reversible T1 transitions.
Fig. S5: **Role of tension remodeling on the mean-squared displacement of cell centers and distribution of polygon sidedness.** (A) Mean-squared displacement (left) and the distribution of polygon sidedness (right), for $k_{C}/k_{L} = 0.08$, and different values of $k_{E}/k_{L}$. (B) Mean-squared displacement (left) and the distribution of polygon sidedness (right), for $k_{C}/k_{L} = 0.20$. White histogram bars correspond to the case of no tension remodeling ($k_{E} = k_{C} = 0$).

Fig. S6: Correlation between the rate of T1 transitions and the mean T1 stalling times. A negative correlation emerges at larger stalling times. Error bars represent ±1 standard error of mean.
Fig. S7: Role of tension fluctuations on the rate of T1 events and T1 stalling times. (A) Rate of irreversible and reversible T1 events (left: all T1 events, right: only delayed T1 events), for \( k_C/k_L = 0.1, k_E/k_L = 0.2 \) (fluid tissue) and different values of the tension noise amplitude \( \sigma \). (B) Histogram of T1 stalling times for delayed irreversible T1 events (left) and delayed reversible T1 events (right), for different values of the tension noise amplitude \( \sigma \).
Fig. S8: **Role of the tension relaxation timescale on the mechanical energy of the tissue.** Temporal evolution of the tissue mechanical energy for various values of the tension relaxation timescale $\tau$, corresponding to two different combinations of the tension remodeling rates, $(k_C/k_L = 0.02,k_E/k_L = 0.23)$ (left) and $(k_C/k_L = 0.23,k_E/k_L = 0.02)$ (right). Dashed black curves correspond to simulations with no tension relaxation, i.e. $\tau \rightarrow \infty$.

![Graph showing mechanical energy over time for various $\tau$ values.]

Fig. S9: **Role of the tension relaxation timescale on solid-fluid phase transitions.** Phase diagram showing transitions between solid (blue) and fluid (red) states of the tissue, with solid white curve representing the phase boundary. Left: $\tau = 10$. Right: $\tau = 20$. Below the dashed white curve in the fluid phase, stable 4-fold vertices are prevalent.

![Phase diagram showing solid-fluid transitions.]

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*Note: The graphs and figures are placeholders as the actual images are not provided in the text.*
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