Dominant NNLO Corrections to Four-Fermion Production at the $W\bar{W}$ Threshold

Stefano Actis *

Institut für Theoretische Physik E, RWTH Aachen University, D-52056 Aachen - Germany

The recent evaluation of the parametrically dominant next-to-next-to-leading order corrections to four-fermion production near the $W$-pair threshold in the framework of unstable-particle effective theory is briefly summarized.

1 Introduction

The production of $W$-boson pairs at electron-positron colliders is a process of crucial relevance for a precise determination of the $W$ mass. If the International Linear Collider will measure the total cross section at the per-mille level [2], a direct reconstruction of the $W$-decay products will allow to reach a 10 MeV accuracy on the determination of the $W$ mass [3]. A higher precision could be achieved through a dedicated threshold scan leading to a 6 MeV accuracy [4].

The aforementioned estimates rely on statistics and the performance of the future collider, and assume that the cross section for $W$-pair production is theoretically under control. In particular, in view of the 6 MeV precision goal, accurate predictions are needed for a final state containing the fermion pairs produced by $W$ decay, instead of on-shell $W$ bosons.

A full next-to-leading order (NLO) evaluation of four-fermion production in the complex-mass scheme has been performed by the authors of [5], extending the methods introduced in [6]. Recently, a compact analytic result for the threshold region has been derived in [7] (see [8] for reviews) using the method of unstable-particle effective field theory [9].

The work of [7] has concluded that collinear logarithms arising from initial-state radiation have to be re-summed at next-to-leading accuracy for reducing the threshold-scan error on the $W$ mass to less than 30 MeV. Furthermore, it has been shown that the NLO partonic evaluation in the effective-theory framework is affected by a residual error of $10-15$ MeV. Although a large part of the uncertainty at the partonic level can be removed using the full NLO result of [5], the evaluation of the dominant next-to-next-to-leading order (NNLO) corrections is mandatory to secure the 6 MeV threshold-scan accuracy goal.

In [10] we have evaluated the parametrically dominant NNLO corrections to the total cross section for the production process $e^-e^+ \rightarrow \mu^- \nu_\mu \bar{u}d + X$, where $X$ is an arbitrary flavor-singlet state. The result is expressed through a compact semi-analytic formula that can be easily added on top of both effective-theory [7] and full NLO [5] predictions.

In Section 2 of this note we show an overview of the NNLO corrections. Next, in Section 3 we discuss their numerical impact.

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2 Overview of the dominant NNLO corrections

The inclusive cross section for the process $e^- e^+ \to \mu^- \bar{\nu}_\mu \bar{u} d + X$ is computed in the context of the effective theory by means of a non-standard perturbative expansion in three small parameters of the same order $\delta$: 1) $\alpha_{ew} \equiv \alpha / \sin^2 \theta_w$, where $\alpha$ is the fine-structure constant and $\theta_w$ stands for the weak-mixing angle; 2) $(s-4M_W^2)/(4M_W^2) \sim v^2$, where $s \equiv (p_\mu^- + p_\mu^+)^2$, $M_W$ is the $W$ mass and $v$ is the non-relativistic velocity of the $W$; 3) $\Gamma_W/M_W$, with $\Gamma_W$ denoting the $W$ decay width.

The re-organized loop and kinematical expansion is performed through the method of regions and relies on the identification of different momentum scalings in the center-of-mass frame in order to exploit the hierarchy of scales around threshold. Denoting by $k$ an arbitrary loop-integration momentum, we deal with hard ($k_0 \sim |\vec{k}| \sim M_W$), potential ($k_0 \sim M_W \delta, |\vec{k}| \sim M_W \sqrt{\delta}$), soft ($k_0 \sim |\vec{k}| \sim M_W \delta$), collinear ($k_0 \sim M_W, k^2 \sim M_W^2 \delta$) and semi-soft ($k_0 \sim |\vec{k}| \sim M_W \sqrt{\delta}$) momentum scalings. Semi-soft modes are not relevant for the NLO evaluation, and start playing a role for the NNLO calculation.

After integrating hard modes out, the residual dynamical degrees of freedom contribute to genuine loop computations in the context of the effective theory. The different scaling properties lead to a peculiar half-integer power counting in the expansion parameter $\delta$ and to a straightforward identification of the parametrically dominant radiative corrections.

The total cross section for four-fermion production is computed from the cuts of the $e^- e^+$ forward-scattering amplitude, as shown in Figure for the leading order (LO) diagram (see Figure for the Standard Model counterpart). Here the LO operator $O_{\mu} \langle 0 \rangle$ accounts for the production (destruction) of a pair of non-relativistic $W$ bosons, denoted by $\Omega$. In Figure and Figure, we show also the NLO Coulomb- and soft-photon corrections evaluated in. The conventional Standard Model (SM) loop expansion of Figure treats virtual Coulomb effects ($\gamma_c$) and soft real-photon contributions ($\gamma_s$) as genuine NLO terms. In the framework of the effective theory, instead, a simple power-counting argument shows that Coulomb corrections at the $W$-pair threshold are suppressed by a factor $\delta^{1/2}$ with respect to the LO result, and can be classified as dominant NLO effects, whereas soft-photon diagrams, being weighted by one power of $\delta$, lead to sub-dominant NLO effects.

Relying on analogous observations, we have analyzed in the set of SM diagrams which
are suppressed in the effective-theory framework by a factor $\delta^{3/2}$ rather than $\delta^2$ with respect to the LO cross section, and can be classified as parametrically dominant NNLO corrections. They can be conveniently organized in three sub-sets: 1) mixed hard-Coulomb corrections; 2) interference effects of Coulomb and soft (collinear) photons; 3) radiative corrections to the Coulomb potential.

Mixed hard-Coulomb corrections, given by diagrams with a Coulomb photon and one insertion of a hard NLO correction, are illustrated in Figure 2. Here a hard correction has been inserted at the: a) production stage, replacing the LO production operator $O_p^{(0)}$ with the NLO expression $O_p^{(1)}$ as in Figure 2a; b) decay stage, as graphically shown in Figure 2b by the insertion of the black dot labeled $\delta_{\text{decay}}$, summarizing flavor-specific contributions to $W$ decay; c) propagation stage, as illustrated by the $\delta_{\text{residue}}$ insertion in Figure 2c. The last contribution is inherent to the inclusion of wave-function renormalization factors in the effective-theory matching coefficients. SM counterparts for all three cases are shown in Figure 2d, Figure 2e and Figure 2f.

Interference effects of Coulomb and soft ($\gamma_s$) or collinear ($\gamma_{\text{coll.}}$) photons are shown in Figure 3a and Figure 3b with their SM counterparts in Figure 3d and Figure 3e. As discussed in [10], they are naturally merged with the mixed hard-Coulomb corrections at the production stage of Figure 2a.

Radiative corrections to the Coulomb potential due to the insertion of a semi-soft fermion bubble ($f_{ss}$) are shown in Figure 3c and Figure 3f.

Figure 2: mixed hard-Coulomb corrections in the effective theory (first line) and in the Standard Model (second line).

Figure 3: diagrams involving Coulomb, soft and collinear photons and corrections to the Coulomb potential in the effective theory (first line) and in the Standard Model (second line).
3 Results

The NNLO total cross section follows from the convolution of the corrections shown in Figure 2 and Figure 3 with the electron structure functions provided in [12], in order to re-sum collinear logarithms from initial-state radiation.

Results for the NLO evaluation of [7] and the NNLO shifts of [10], ranging from 0.07% for \( \sqrt{s} = 161 \) GeV to 0.3% for \( \sqrt{s} = 170 \) GeV, are summarized in Table 1. Using the procedure of [10], we have found that the impact of the dominant NNLO corrections on the \( W \)-mass determination is about 3 MeV. The result is well below the 6 MeV error in the measurement from an energy scan in electron-positron collisions.

We conclude observing that, although a differential calculation in the effective theory is not currently feasible (see developments for top-antitop production in [13]), the analysis of [10] has shown that the inclusive NNLO result is adequate for practical applications.

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**Table 1:** NLO total cross section for \( e^- e^+ \rightarrow \mu^- \tau_\mu ud + X \) and NNLO shift.

| \( \sqrt{s} [\text{GeV}] \) | \( \sigma_{\text{NLO}} [\text{fb}] \) | \( \Delta \sigma_{\text{NNLO}} [\text{fb}] \) |
|---|---|---|
| 161 | 117.81(5) | 0.087 |
| 164 | 234.9(1) | 0.544 |
| 167 | 328.2(1) | 0.936 |
| 170 | 398.0(2) | 1.207 |