The Curious Case of Near-Identical Cosmic-Ray Accelerators

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A commonly-used, simplifying assumption when modeling the sources of ultra-high energy cosmic rays (UHECRs) is that all of them accelerate particles to the same maximum energy. Motivated by the fact that candidate astrophysical accelerators exhibit a vast diversity in terms of their relevant properties such as luminosity, Lorentz factor, and magnetic field strength, we study the compatibility of a population of sources with non-identical maximum cosmic-ray energies with the observed energy spectrum and composition of UHECRs at Earth. For this purpose, we compute the UHECR spectrum emerging from a population of sources with a power-law distribution of maximum energies applicable to a broad range of astrophysical scenarios. We find that the allowed source-to-source variance of the maximum energy must be small to describe the data. Even in the most extreme scenario, with a very sharp cutoff of individual source spectra and negative redshift evolution of the accelerators, the maximum energies of 90% of sources must be identical within a factor of three – in contrast to the variance expected for astrophysical sources.

I. INTRODUCTION

Ultra-high-energy cosmic rays (UHECRs) are charged particles that reach Earth with energies of up to several $10^{20}$ eV. The identification of the astrophysical sources capable of accelerating particles to these energies is one of the unsolved mysteries of high energy astrophysics (see e.g. [1, 2] for recent reviews). A correlation between astrophysical objects and the measured arrival directions of cosmic rays has not yet been established at high significance [3], but the properties of the sources are constrained by measurements of the diffuse particle flux and composition at Earth, see e.g. [4–19].

Most of these studies assume an acceleration mechanism that is universal in rigidity [1] up to a maximum rigidity of $R_{\text{max}}$ leading to consecutive flux suppressions of the elemental spectra at energies of $E_{\text{max}} = Z R_{\text{max}}$, where $Z$ denotes the cosmic-ray charge. Assuming such a “Peters Cycle” [20, 21] at the sources gives a good description of the flux and composition measured at Earth at ultra-high energies, see e.g. [11].

However, one major caveat of these studies is that the sources are typically assumed to be identical, a description that is unlikely to be an accurate reflection of reality. The most probable astrophysical candidates for the sources of UHECRs, e.g. active galactic nuclei (AGN), and gamma-ray bursts (GRBs), are generally not very similar – even within a single source class – but exhibit a large diversity in terms of key parameters like luminosity, size, magnetic field and jet power.

Only few studies have relaxed the assumption of identical sources in the past, by focusing on a low number of discrete local sources [22–24], or by considering the superposition of a few ($\leq 3$) sources classes, e.g. [15, 18, 25, 26].

The time variation of $R_{\text{max}}$ in AGN jets was studied in Ref. [27] and the effective spectrum produced by sources with non-identical spectral shapes and spectral indices has been discussed in the context of gamma-ray spectra [28] and Galactic cosmic-ray sources [29]. A population of sources with non-identical maximum cosmic-ray rigidities has previously been studied assuming a pure proton UHECR distribution [30], and in the context of Galactic cosmic-ray sources [31].

Here we present, for the first time, a rigorous exploration of the population variance of $R_{\text{max}}$ compatible with current observations of the spectrum and composition of UHECRs. This is achieved by convolving the distribution of source properties, parameterised by the maximum rigidity, with the individual source spectra to obtain an analytical description of the total population spectrum, as detailed in Sec. II. We simulate the propagation of UHECRs to Earth through the extragalactic photon fields and find the best source parameters by comparing the model predictions to UHECR data in Sec. III. From these model fits we then derive lower limits on the source variance allowed by the data, as described in Sec. IV. We conclude in Sec. V that only a very limited amount of population variance is permitted, and UHECR sources are required to be nearly identical in terms of maximum rigidity under realistic choices of the model parameters.

II. POPULATION SPECTRUM OF NON-IDENTICAL SOURCES

In this study we assume that the rigidity spectra of individual sources of UHECRs are well described by the aforementioned Peters Cycle, and thus we assume a power law with a high-rigidity cutoff,

$$\phi_{\text{src}} = \frac{d^2 N}{dR dt} = \sum_i \phi_0(Z_i) R^{-\gamma_{\text{src}}} f(R, R_{\text{max}}),$$

where $Z e$ and momentum $p$ is $R = pc/(Ze) \simeq E/Z$ (using natural units and with energy $E$).

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1 The rigidity of a particle with charge $Ze$ and momentum $p$ is $R = pc/(Ze) \simeq E/Z$ (using natural units and with energy $E$).
where the sum runs over all accelerated chemical elements with charge $Z_i$ and the spectral index is assumed to be universal\(^2\). The term $f(R, R_{\text{max}})$ describes the high-rigidity cutoff at maximum rigidity $R_{\text{max}}$. We refer to the sum of the spectra of all sources within a certain volume as the population spectrum $\phi_{\text{pop}}$.

In the limit of identical sources, the population spectrum, will of course have the same shape as the spectra of individual sources. A source-by-source variation of the normalisation factors $\phi_0(Z_i)$ does not lead to a qualitatively different population spectrum since it is equivalent to identical sources with the source-averaged normalisation $\phi_0$, and thus we will not consider it in the following. It should, however, be kept in mind, that the elemental fractions obtained from the fits presented later in this paper should be understood as source-averaged fractions. A phenomenologically more interesting source property is the maximum rigidity. If the probability for an individual accelerator to reach a certain maximum rigidity is distributed as $p(R_{\text{max}}) \equiv d\rho / dR_{\text{max}}$ and the source spectra follow $\phi_{\text{src}}(R, R_{\text{max}})$, then the combined spectrum of the entire population is given by the convolution

$$\phi_{\text{pop}}(R, R_0) = \int_0^\infty \phi_{\text{src}}(R, R_{\text{max}}) p(R_{\text{max}}) \, dR_{\text{max}}. \quad (2)$$

Here and in subsequent occurrences of $\phi_{\text{src}}$, the sum over all chemical elements of charge $Z_i$ (cf. Eq. (1)) is assumed implicitly. In the following we specify the functional forms of individual source spectra and probability distribution of $R_{\text{max}}$ that will be studied in this work.

### A. Source Spectra

In general, the rigidity cutoff of astrophysical accelerators depends on the acceleration mechanism and the source environment, in particular on the dominant energy-loss process, see e.g. [34, 35]. The simplest description of the shape is given by a sharp termination when particles exceed the maximum rigidity

$$\phi_{\text{src}}^{\text{hs}} = \phi_0 R^{-\gamma_{\text{src}}} \theta(R_{\text{max}} - R), \quad (3)$$

where $\theta(x)$ denotes the Heaviside step function. The population spectrum corresponding to this shape has been studied previously in Ref. [30]. Since $\phi_{\text{src}}^{\text{hs}}$ describes the sharpest-possible rigidity cutoff, it provides a useful extreme case that will allow for a maximum variation of source spectra.

A more commonly-used choice, expected under certain astrophysical conditions, see e.g. [35, 36], is given by an exponential cutoff

$$\phi_{\text{src}}^{\text{exp}} = \phi_0 R^{-\gamma_{\text{src}}} \exp \left( -\frac{R}{R_{\text{max}}} \right). \quad (4)$$

However, this function has the disadvantage that the effect of the cutoff already starts to become noticeable well below the maximum rigidity and the interpretation of the spectral index $\gamma_{\text{src}}$ is complicated. For this reason, some phenomenological studies assume a broken-exponential source spectrum, e.g. [37]

$$\phi_{\text{src}}^{\text{b-exp}} = \phi_0 R^{-\gamma_{\text{src}}} \begin{cases} 1, & R < R_{\text{max}} \\ \exp \left( 1 - \frac{R}{R_{\text{max}}} \right), & \text{otherwise} \end{cases} \quad (5)$$

which alleviates the issue but lacks a physical motivation.

Finally, we consider spectra with an exponential cutoff raised to the power of $\lambda_{\text{cut}}$,

$$\phi_{\text{src}}^{\lambda_{\text{cut}}-\text{exp}} = \phi_0 R^{-\gamma_{\text{src}}} \exp \left( -\frac{R}{R_{\text{max}}} \right)^{\lambda_{\text{cut}}}, \quad \lambda_{\text{cut}} > 0. \quad (6)$$

We refer to this description as a “super-exponential” cutoff. The function can be used to interpolate continuously between classical, exponential cutoffs ($\lambda_{\text{cut}} = 1$) and sharp, Heaviside-like terminations ($\lambda_{\text{cut}} = \infty$). Additionally, for $\lambda_{\text{cut}} < 1$ the cutoff shape becomes sub-exponential up to no cutoff when $\lambda_{\text{cut}} \to 0$. Super-exponential cutoff profiles were obtained e.g. in Ref. [36] with $\lambda_{\text{cut}} = 2$ for synchrotron losses during acceleration.

An illustration of the source spectrum for different choices of the cutoff is shown in the left panel of Fig. 1.

### B. Distribution of Maximum Rigidities

In this paper, we mainly consider the effect of a population of sources with a distribution of maximum rigidities that follows a power-law with spectral index $\beta_{\text{pop}}$ above a minimum allowed maximum rigidity $R_0$

$$p(R_{\text{max}}) = \begin{cases} 0, & R_{\text{max}} < R_0 \\ \frac{\beta_{\text{pop}} - 1}{R_0} \left( \frac{R_{\text{max}}}{R_0} \right)^{\beta_{\text{pop}}}, & R_{\text{max}} \geq R_0 \end{cases} \quad (7)$$

which is also known as a Pareto distribution. This $R_{\text{max}}$ distribution was previously considered in Ref. [30]. Because of the asymmetric nature of the power-law distribution, the standard deviation is of limited use to characterize the source variance, and instead we will report the one-sided 90% quantile of the distribution $R_{\text{max}}^{0.90}$ defined as

$$\int_{R_0}^{R_{\text{max}}^{0.90}} dR_{\text{max}} \, p(R_{\text{max}}) = 0.90. \quad (8)$$

For a power-law distribution of maximum rigidities, the quantile $q$ is given by the relation

$$R_{\text{max}}^q / R_0 = [1 - q]^{1/(1 - \beta_{\text{pop}})}. \quad (9)$$

For illustration, population diversity of more than a decade, i.e. $R_{\text{max}}^{0.90} / R_0 \geq 10$, is obtained if $\beta_{\text{pop}} \lesssim 2.3$.\(^2\)
Another, more general, functional form of the $R_{\text{max}}$ distribution is a broken power law in $R_{\text{max}}$. In Appendix B we show that under certain assumptions, the obtained population spectrum is equal to the one obtained with a single power law.

### C. Population Spectrum

Assuming a power-law distribution of $R_{\text{max}}$, it is possible to derive an analytical description of the population spectrum for all source spectra presented in Sec. II A. In the case of sources with a Heaviside termination in rigidity, the population spectrum is given by,

$$\phi_{\text{pop}}^\text{hs} = \phi_0 R^{-\gamma_{\text{src}}} \begin{cases} 1 & R < R_0 \\ \left(\frac{R}{R_0}\right)^{-\beta_{\text{pop}}+1} f\left(\frac{R}{R_0}, \beta_{\text{pop}}\right) & \text{otherwise.} \end{cases}$$

For source spectra with a broken exponential cutoff, the population spectrum is given by,

$$\phi_{\text{pop}}^\text{b-exp} = \phi_0 R^{-\gamma_{\text{src}}} \begin{cases} 1 & R < R_0 \\ \left(\frac{R}{R_0}\right)^{-\beta_{\text{pop}}+1} f\left(\frac{R}{R_0}, \beta_{\text{pop}}\right) & \text{otherwise} \end{cases}$$

with

$$f\left(\frac{R}{R_0}, \beta_{\text{pop}}\right) = 1 + e\left(\beta_{\text{pop}} - 1\right) \left[ \gamma\beta_{\text{pop}} - 1, \frac{R}{R_0}\right] - \gamma\beta_{\text{pop}} - 1, 1\right].$$

Finally, for sources with a (super-)exponential cutoff the population spectrum is given by,

$$\phi_{\text{pop}}^\text{exp} = \phi_0 R^{-\gamma_{\text{src}}} \left(\frac{R}{R_0}\right)^{-\beta_{\text{pop}}+1} \frac{\beta_{\text{pop}}-1}{\lambda_{\text{cut}}} \times \gamma\left(\frac{\beta_{\text{pop}}-\lambda_{\text{cut}}-1}{\lambda_{\text{cut}}} + 1, \left(\frac{R}{R_0}\right)^{\lambda_{\text{cut}}}\right).$$

For a standard exponential distribution with $\lambda_{\text{cut}} = 1$ this simplifies to,

$$\phi_{\text{pop}}^\text{exp} = \phi_0 R^{-\gamma_{\text{src}}} \left(\frac{R}{R_0}\right)^{-\beta_{\text{pop}}+1} \left(\beta_{\text{pop}}-1\right) \gamma\left(\beta_{\text{pop}}-1, \frac{R}{R_0}\right),$$

whereas for $\lambda_{\text{cut}} \to \infty$, Eq. 10 is recovered. Here $\gamma$ denotes the lower incomplete gamma function, not to be confused with the source spectral index $\gamma_{\text{src}}$.

Population spectra for a particular choice of parameters are shown in Fig. 1, right. The main impact of the cutoff parameter $\lambda_{\text{cut}}$ is a later onset and faster turnover of the spectral break around $R_0$. Interestingly, the limiting behavior of the population spectrum is independent of the source cutoff function. The asymptotic rigidity dependencies of the population spectra are

$$\lim_{R \to 0} \phi_{\text{pop}}(R) \propto R^{-\gamma_{\text{src}}}$$

and

$$\lim_{R \to \infty} \phi_{\text{pop}}(R) \propto R^{-\gamma_{\text{src}}-\beta_{\text{pop}}+1}.$$

### D. Relation to Astrophysical Quantities

The population spectra derived in the previous section provide simple analytic expressions that are well-suited for fits to UHECR observations at Earth, with which the key parameters $\beta_{\text{pop}}$ and $\gamma_{\text{src}}$ can be derived. The connection of these parameters to the properties of UHECR sources is discussed in the following for a few examples. We will show that the assumed power-law distribution in maximum rigidity can be attributed to different acceleration scenarios. The relevant parameters of these scenarios are summarised in Table I and the re-interpretation of the fitted parameters $\beta_{\text{pop}}$, $\gamma_{\text{src}}$ in terms of proposed underlying physical properties are listed in Table II. The process of reducing all the considered scenarios to the same population spectra that are obtained for a power-law distribution of $R_{\text{max}}$ is illustrated in Fig. 2.
TABLE I. Summary of parameters used for the three illustrative scenarios. See sections given in the first column for further details.

| Scenario        | Parameter | Description                                                                 | Equation                                                                 |
|-----------------|-----------|------------------------------------------------------------------------------|--------------------------------------------------------------------------|
| power law       | $\gamma_{\text{src}}$ | true spectral index of the sources.                                          | $\phi_{\text{src}} \propto E^{-\gamma_{\text{src}}}$                  |
| (Sec. II B)     | $\xi$     | effective spectral index of the sources (same as the true spectral index for a simple power-law distribution of $p(R_{\text{max}})$). | $\phi_{\text{src}} \propto E^{-\gamma_{\text{src}}}$                  |
|                 | $\alpha$  | spectral index of the $p(R_{\text{max}})$ distribution.                      | $p(R_{\text{max}}) \propto R_{\text{max}}^{-\beta_{\text{pop}}}$       |
|                 | $\beta_{\text{pop}}$ | spectral index of the distribution of maximum rigidities below (above) the break for a broken power-law in $p(R_{\text{max}})$. | $p(R_{\text{max}}) \propto R_{\text{max}}^{-\beta_{\text{pop}}}$       |
| Lorentz factor  | $\eta$    | spectral index of the power-law distribution of Lorentz-factors.             | $dp/d\Gamma \propto \Gamma^{-\eta}$                                    |
| (Sec. II D 1)   | $\alpha$  | energy boosting by the relativistic motion of the jet.                       | $E = E' \Gamma^\alpha$                                                  |
|                 | $\xi$     | time dilation caused by the relativistic motion of the source region.        | $t = t' \Gamma^\xi$                                                    |
| luminosity      | $y_1, y_2$ | spectral index of the broken power law luminosity distribution of sources   | $dp/\alpha \propto L^{-y_1} (L^{-y_2})$                                  |
| (Sec. II D 2)   |           | below (above) the break.                                                     |                                                                          |

1. Jet Lorentz Factor

In some scenarios, for sources with relativistic jets, the maximum rigidity is directly related to the bulk Lorentz-factor of the motion, $\Gamma_{\text{jet}}$. For instance, the Hillas criterion [38] for relativistic sources gives $R_{\text{max}} = R_0 \Gamma_{\text{jet}}$, with $R_0 \propto l B$ where $l$ is the size of the source and $B$ the magnetic field.

It is also possible that UHECRs are galactic cosmic rays that receive a “one-shot” boost of a factor of $\sim \Gamma_{\text{jet}}^2$, in the jet of their host galaxies, in which case $R_{\text{max}} \sim R_0 \Gamma_{\text{jet}}^2$, where $R_0$ is the maximum energy of the cosmic rays before re-acceleration. This is referred to as the *Espresso* mechanism [39–41]. Cosmic rays that do not enter the most relativistic parts of the jet are only partially boosted with $R_{\text{max}} \propto \Gamma_{\text{jet}}^{\alpha}$, $\alpha < 2$. Thus, here we investigate the general case of

$$R_{\text{max}} = R_0 \Gamma_{\text{jet}}^{\alpha},$$

where the aforementioned cases are described by $\alpha = 1$ (Hillas) and $\alpha \leq 2$ (Espresso).

Assuming, for example, a power-law distribution of the Lorentz factors, as found consistent with observations of jetted AGN in [42, 43],

$$dp/d\Gamma_{\text{jet}} = (\eta - 1) \Gamma_{\text{jet}}^{-\eta},$$

the distribution of maximum rigidities can be calculated as

$$p(R_{\text{max}}) = \frac{dp}{d\Gamma_{\text{jet}}} \left| \frac{d\Gamma_{\text{jet}}}{dR_{\text{max}}} \right| = \frac{\eta - 1}{\alpha} R_0^{-1} \left( \frac{R_{\text{max}}}{R_0} \right)^{1-\eta} \theta(R_{\text{max}} - R_0).$$

(17)

Of course, $R_0$ is also expected to vary from source to source, and therefore the distribution of $R_{\text{max}}$ should be broader and the above equation can be understood as a lower limit on the source-to-source variation of $R_{\text{max}}$.

The boosting of particle energies also affects the expected flux emitted by individual sources. This introduces additional terms into Eq. (17) but it is shown in Appendix A that the convolution of the source spectra and the $R_{\text{max}}$ distribution evaluate to the same functional forms as derived in the last section. However, the parameter $\beta_{\text{pop}}$ can now be related to physical properties of the source population, namely the spectral index $\gamma_{\text{src}}$, the Lorentz-boosting factor $R_{\text{max}} \propto \Gamma_{\text{jet}}^{\alpha}$, and the distribution of Lorentz-factors $p(\Gamma_{\text{jet}}) \propto \Gamma_{\text{jet}}^{-\eta}$ via

$$\beta_{\text{pop}} = \frac{\eta - 1}{\alpha} + 2 - \gamma_{\text{src}} + \xi/\alpha,$$

(18)

where the dilation factor $\xi = 1$ for accelerators co-moving with the jet and $\xi = 0$ for espresso-type re-acceleration.

2. Luminosity

Another plausible distribution of the maximum rigidity can be derived from the minimum luminosity requirement for particle acceleration in expanding flows, whereby the minimum luminosity, $L_0$, needed to accelerate CRs to maximum rigidity $R_0$ is given by [44–48],

$$L_0 \approx 5 \times 10^{42} \frac{\Gamma^2}{\beta} \left( \frac{R_0}{10^{20} \text{ EV}} \right)^2 \text{ erg s}^{-1},$$

(19)

where $\beta$ is the speed of the flow in units of $c$. In this scenario, we can relate $R_{\text{max}}$ of the population to $L$ via

$$R_{\text{max}} \sim \frac{R_0}{\Gamma} \sqrt{\frac{L}{L_0}}.$$

(20)

In what follows we consider only scenarios in which the outflow is mildly relativistic $\Gamma \sim 1$ or the luminosity fluctuations far exceed those of $\Gamma^2$ to simplify the expression.
We assume that the emitted flux of a single source scales with the luminosity as $\phi_{\text{src}} \propto L/L_0$. Noting Eq. (20), this introduces an additional dependency of the source flux of the maximum rigidity which can be absorbed into the $p(R_{\text{max}})$ distribution by adjusting the definition of the effective slope $\beta_{\text{pop}}$. The distribution of maximum rigidities then evaluates to

$$p(R_{\text{max}}) = \frac{2}{R_0} \left( \frac{R_{\text{max}}}{R_0} \right)^{2y_2+3},$$

which, except for an additional normalisation constant $\kappa$, reduces to the regular SPL description (Eq. (7)) after defining

$$\beta_{\text{pop}} = 2y_2 - 3 \quad \text{and} \quad \kappa = \frac{\beta_{\text{pop}} + 1}{\beta_{\text{pop}} - 1}$$

The situation is more complicated if sources have a broken power-law luminosity distribution $\text{d}p/\text{d}L(y_1, y_2)$ instead, but by redefining $\beta_1 = 2y_1 - 3$ and $\beta_2 = 2y_2 - 3$ it is possible to show that this will lead to a BPL in $p(R_{\text{max}})(\beta_1, \beta_2)$ which can in turn be approximated as a power-law distribution $p(R_{\text{max}})(\beta_{\text{pop}})$ (see Appendix B).

The resulting values of $\beta_{\text{pop}}$ (labeled $\beta_{\text{pop, max}}$) are listed in the last column of Table II. These can directly be compared to the fitted values of $\beta_{\text{pop}}$ discussed below. For the models investigated $\beta_{\text{pop, max}}$ is in general low, meaning that we would expect to observe the effect of the variance of the population in the UHECR data. It should be kept in mind that the estimates given in this section are only a lower limit on the source variance (upper limit on $\beta_{\text{pop}}$) as we focused only on the variation of a few key parameters and treated others as a constant (e.g. $R_0$) and therefore the real source variance will be larger and $\beta_{\text{pop}}$ be smaller than estimated here.

### III. METHODS

#### A. UHECR Data

We use the latest publicly available data from the Pierre Auger Observatory for comparison with our numerical simulations. These are the energy spectrum of UHECRs from Ref. [51], and the mean and standard deviation of the maximum depth of air showers [52, 53] that are sensitive to the composition of cosmic rays, see e.g. [54].

#### B. UHECR Propagation

UHECR injection and propagation are simulated with the numerical Monte-Carlo framework CRPropA3 [55], including the production of cosmogenic neutrinos and gamma rays. Upper limits and measurements of the...
TABLE II. Effective fit parameters, $\beta_{\text{pop}}$ - the spectral index of the maximum rigidity distribution of the UHECR source population - and $\eta_{\text{src}}$ - the assumed spectral index of the UHECR spectrum of individual sources -, and their interpretation in terms of source properties for various scenarios considered in this work. The scenarios are: (I) a distribution of $p(R_{\text{max}})$ that follows an ad-hoc single power-law (SPL) or broken power-law (BPL); (II) maximum rigidity that scales as $R_{\text{max}} \propto \Gamma^\alpha$ with $\Gamma$ the SPL-distributed bulk Lorentz-factor of the acceleration region (see Sec. II D 1) with $dp/d\Gamma \propto \Gamma^{-\gamma}$; and (III) $R_{\text{max}}$ as a function of source luminosity, $R_{\text{max}} \propto \sqrt{L}$, with SPL or BPL distribution of $dp/dL$ (see Sec. II D 2). For power-law distributions the parameter in brackets denotes the slope, e.g. $dp/d\Gamma(\eta) \propto \Gamma^{-\gamma}$, while for broken power-law distributions the parameters give the slope before and after the break respectively. Scenario I represents our baseline model that we use to compute the population spectra for different source spectral cutoff functions. Case II and III can be reduced to the former after re-interpretation of the source and population parameters ($\eta$, $\alpha$, $y_1$, $y_2$, $\beta_1$, $\beta_2$, $\gamma_{\text{src}}$) in terms of the parameters ($\beta_{\text{pop,max}}, \eta_{\text{src}}$).

| ID | Param. | Distribution | $\beta_{\text{pop}}$ | $\gamma_{\text{src}}$ | Sources | $\beta_{\text{pop,max}}$ |
|----|--------|--------------|----------------------|-----------------------|---------|-----------------|
| I.1 | $R_{\text{max}}$ | SPL, $p(R_{\text{max}})$ | $\beta_{\text{pop}}$ | $\gamma_{\text{src}}$ | AGN [43]: $\eta = 1.4$ | 3.4 - $\gamma_{\text{src}}$ |
| I.2 | $R_{\text{max}}$ | BPL, $p(R_{\text{max}})$ | $\beta_1 < 1$ | $\approx 2 \beta_2 - 1$ | Hillas: $\alpha = 1$, $\xi = 1$ | 2.2 - $\gamma_{\text{src}}$ |
|     |       |              | $\beta_1 > 1$ | $2 \beta_2 - 2 \beta_1 + 1$ | + Espresso: $\alpha = 2$, $\xi = 0$ | 2.34 |
| I.3 | $R_{\text{max}} \propto \Gamma^\alpha$ | SPL, $dp/d\Gamma(\eta)$ | $\langle \eta - 1 \rangle/\alpha + 2$ | $-\gamma_{\text{src}} + \xi/\alpha$ | BL Lacs only [49] *: $y_2 = 2.61$ | 2.22 |
| III.1 | $R_{\text{max}} \propto \sqrt{T}$ | SPL, $dp/dL(y_2)$ | $2y_2 - 3$ | $\gamma_{\text{src}}^{\text{orig}}$ | FSRQs only [49] *: $y_1 < -50$, $y_2 = 2.49$ | 1.98 |
| III.2 | $R_{\text{max}} \propto \sqrt{T}$ | BPL, $dp/dL(y_1, y_2)$ | $y_1 < 1$ | $\approx 2y_2 - 3$ | all Blazars [49] *: $y_1 = -0.87$, $y_2 = 2.73$ | 2.46 |
|     |       |              | $\approx 2y_2 - 3$ | $\gamma_{\text{src}}^{\text{orig}}$ | Seyferts [49] *: $y_1 = 0.80$, $y_2 = 2.67$ | 2.34 |

* Assuming the pure-luminosity-evolution (Μ)PLE model. Here, the values correspond to apparent as opposed to the intrinsic (unbeamed) luminosity function. However, since $\beta_1 < 1$, $y_1$ does not enter the calculation of $\beta_{\text{pop}}$, while $y_2$ is unaffected by relativistic beaming (see [50]).

latter are qualitatively taken into account in what follows. These are the Fermi-LAT isotropic diffuse gamma-ray background [56], the observed IceCube high-energy starting-event neutrino flux [57], and the IceCube 90% upper limits above $5 \times 10^6$ GeV [58]. The UHECR sources are simulated in the continuous-source approximation out to maximum redshift $z_{\text{max}} = 4$. For UHECRs in the energy range that we fit, the effective horizon is much closer at no more than $z \lesssim 1$, e.g. [59], but sources at larger distances can have a strong impact on the predicted flux of cosmogenic neutrinos.

All relevant interactions are taken into account during propagation [37, 55, 60]; these are (i) redshift energy loss, (ii) photo-pion production and (iii) electron-positron pair production on the cosmic microwave background (CMB) and the infrared background (IRB [61]), and for heavier cosmic rays also (iv) photo-disintegration on CMB & IRB and (v) nuclear decay.

We assume that UHECRs propagate in the ballistic regime and neglect the effects of extragalactic magnetic fields on trajectories of UHECRs. Based on the results of previous studies [62–66], these propagation effects are mainly important at low rigidities, where they can lead e.g. to an apparent hardening of the UHECR flux at Earth, but are not expected to alter our conclusions about the source variance of maximum rigidities.

### C. Model Fit

We compare the model-predicted UHECR spectrum and composition after propagation to observations by the Pierre Auger Observatory. For that purpose we convert the model composition into the air shower observables – the mean depth of the shower maximum $\langle X_{\text{max}} \rangle$ and its standard deviation $\sigma(X_{\text{max}})$ – following Ref. [67].

The agreement between simulation and observations is evaluated with a standard $\chi^2$-estimator plus additional penalty-terms

$$
\chi^2 = \sum_{E_i \geq E_{\text{min}}} \left( \frac{d_i - m(E_i, \mathbf{p})}{\sigma_{\text{stat}}(d_i)} \right)^2 + \chi_{UL}^2 + \chi_{\alpha}^2 + \chi_{\text{shift}}^2.
$$

and minimized adjusting the model parameters $\mathbf{p}$. The sum runs over all Auger data points at energies $E_i$ above the threshold energy. $d_i$ denotes the three measured quantities, i.e. the energy spectrum, average $X_{\text{max}}$ and standard deviation of $X_{\text{max}}$. We select a high value of the minimum observed UHECR energy $E_{\text{min}} = 10^{18.8}$ eV as the lower energy threshold of this analysis to reduce the impact of a possible low-energy cosmic-ray component, different from the one responsible for the highest energies (Hillas’ “component B” [68]). The smallest set
of free fit parameters \( p \) are the minimum rigidity \( R_0 \) and slope \( \beta_{\text{pop}} \) of the single power-law distribution of maximum rigidities, source spectral index \( \gamma_{\text{src}} \), total population emissivity \( L_0 \), and elemental injection fractions \( f^A_k \) which are defined as relative flux ratios at the same rigidity \( A \). A combination of five injection elements - \( ^1\text{H}, ^4\text{He}, ^{11}\text{N}, ^{28}\text{Si} \) and \( ^{56}\text{Fe} \) - is used as an effective approximation of mass groups in the cosmic-ray composition.

Spectral data points below the threshold are included as one-sided \( \chi^2 \) penalty terms that only provide a contribution to the overall goodness-of-fit if the predicted flux exceeds the observations. This component is denoted as \( \chi_{\text{UL}}^2 \) in Eq. (25) and is defined analogous to the first term but only evaluated if the model exceeds the data.

No cosmic rays were observed in the two highest-energy bins at \( E \geq 10^{20.2} \text{eV} \) and only 90% upper limits are given in [51]. The \( \chi^2 \)-penalty derived for this type of zero-event data points follows from the asymptotic \( \chi^2 \)-term assuming a Poissonian distribution of events [69], and is estimated as

\[
\chi^2_{\text{zero}} = \sum_{i=1}^{\text{ULs}} 2\eta_i^{\text{model}}, \tag{26}
\]

where \( \eta_i^{\text{model}} \) is the number of particles predicted by the simulation at the energy \( E(\text{UL}_i) \) after taking into account the detector exposure [51].

Finally, we consider the systematic uncertainties in the absolute scale of the energy, \( \langle X_{\text{max}} \rangle \) and \( \sigma(X_{\text{max}}) \). They are included in the fit as nuisance parameters with

\[
\chi^2_{\text{shifts}} = \sum_{k \in \{E, \langle X_{\text{max}} \rangle, \sigma(X_{\text{max}})\}} \left( \frac{\delta_k}{\sigma_k} \right)^2, \tag{27}
\]

where the energy uncertainty is assumed as \( \sigma_E \leq 14\% \) [70], and the shower depth uncertainties are taken directly from the dataset [52, 53]. The scale shifts can be fit freely within \( \delta_k \in [-\sigma_k, \sigma_k] \), but unless indicated otherwise, we fix the values of the systematic shifts \( \delta_k \) to fiducial values, as detailed below.

**IV. RESULTS AND DISCUSSION**

**A. Fiducial Model**

The observed variance of \( X_{\text{max}} \) consists of two separate contributions: (i) shower-to-shower variations, and (ii) intrinsic shower variability. To allow for stronger source diversity it is necessary that \( \sigma(X_{\text{max}}) \) is not already dominated by the latter. Air showers originating from light primary cosmic rays have a larger variability [67], and intrinsic shower variations can be minimised by shifting the observational data to the heaviest composition that is allowed within systematic uncertainties - corresponding to an adjustment of \( \langle X_{\text{max}} \rangle \) by about \(-8.5 \text{g cm}^{-2} \) at the ankle and \(-7 \text{g cm}^{-2} \) at the highest energies [52, 53]. In addition, we shift the observed variance of the shower maximum up by the systematic uncertainty to allow for the largest reasonable source variance. We adopt these shifts as our fiducial model to allow for maximum intrinsic source diversity. A shift of the energy scale is also possible but we found the impact to be primarily of quantitative nature and neglect it for simplicity.

We furthermore select Sibyll2.3c [71] as our default hadronic interaction model to convert the predicted UHECR composition to mean shower depth and variance. This is motivated by the fact that interpreting the \( X_{\text{max}} \) data of Auger with Sibyll yields a heavier composition than when using EPOS-LHC [72]. As default, we assume a flat redshift evolution of the source density and an exponential source cutoff. Alternative redshift evolutions are explored in Sec. IV B and different cutoff functions in Sec. IV D.

In line with our outlined considerations we find that the fiducial scale shifts allow for a larger amount of population variance compared to the un-shifted observations (Table III). This holds true independent of the choice of hadronic interaction model.

The predicted spectrum and composition at Earth for the best-fit source parameters of the fiducial model are shown in Fig. 3. The viable range of \( \beta_{\text{pop}} \) is sharply

![FIG. 3. Predicted spectrum and composition at Earth for the best-fit scenario of the fiducial model (Sibyll2.3c, \( \langle X_{\text{max}} \rangle \) - \( \sigma_{\text{sys}}, \sigma(X_{\text{max}}) + \sigma_{\text{sys}} \)). The coloured bands indicate the contributions of the separate mass-groups with \( [A_{\text{min}}, A_{\text{max}}] \), including the 68% uncertainties.](image-url)
bounded from below (Fig. 4), approximately as

$$\beta_{\text{pop}} \gtrsim -\gamma_{\text{src}} + 4 \quad (28)$$

and appreciable source diversity is only possible for soft source spectra \(\gamma_{\text{src}} \gtrsim 1\). Yet, when assuming acceleration following a Peters cycle, soft source spectra imply a significant amount of mixing between the different mass groups which leads to an increase in shower variance – a prediction that is in tension to the low values of \(\sigma(X_{\text{max}})\) measured by the Pierre Auger Collaboration, and the resulting agreement between simulation and observations is poor. This problem is even more pronounced for the soft spectra expected from diffusive shock acceleration and values above \(\gamma_{\text{src}} \approx 2\) are excluded.

Realisations of the source model where Eq. (28) is violated are characterised by a population spectrum with extreme UHE tail; a consequence of \(\lim_{R \to \infty} (\beta_{\text{pop}}) \propto R^{-\beta_{\text{pop}} - \gamma_{\text{src}} + 1}\). Such extremely-UHE cosmic rays experience strong interactions during propagation, resulting in the production of a large flux of light secondary cosmic rays up to GZK energies. In combination with the remaining non-disintegrated component of heavy primaries the predicted flux at Earth exhibits a large amount of mixing between the mass groups which leads to strong intrinsic shower variance in excess of observations and an overall bad fit to the observed spectral shape.

The parameter space where a good fit to the measured UHECR spectrum and composition is achieved can be divided into two different regimes, one that runs approximately parallel to the boundary with \(\beta_{\text{pop}} + \gamma_{\text{src}} \approx 4 - 6\) in the range \(\gamma_{\text{src}} \in [-1, 0.5]\), and a second that is effectively degenerate in the population variance \(\beta_{\text{pop}} \gtrsim 5\) with \(\gamma_{\text{src}} \in [0, 1]\). The former is associated with a sub-EV maximum rigidity threshold \(R_0\) and a heavy composition dominated by nitrogen-like nuclei with little contribution from lighter elements. The second regime allows for a lighter composition of up to 50% protons / helium with \(R_0 \sim 1\) EV. Only the second of these regions is also present in the scenario without fiducial scale shifts applied. In both regimes sources are effectively identical and a population variance of half a decade or more is required.

The surface plot shows the agreement between prediction and Auger observations in terms of the \(\chi^2\) estimator and the contour lines indicate the one (green), two (orange) and three (red) sigma confidence interval for two degrees of freedom. The best fit is marked with a white cross.

---

### TABLE III. Best fit parameters for several variations of the source model.

| model            | Sibyll2.3c | Sibyll2.3c | Epos-LHC |
|------------------|------------|------------|----------|
|                  | (no shifts) | (fid. shifts) | (fid. shifts) |
| \(R_0\) [EV]    | 1.73 \(\pm\) 0.18 | 0.57 \(\pm\) 0.11 | 1.6 \(\pm\) 0.6 |
| \(\beta_{\text{pop}}\) | 29.9 \(\pm\) 1.7 | 5.2 \(\pm\) 2.6 | 4.4 \(\pm\) 0.5 |
| \(\gamma_{\text{src}}\) | \(-0.23\) \(\pm\) 0.08 | \(-0.8\) \(\pm\) 0.7 | 0.1 \(\pm\) 0.4 |
| \(L_0\) [10^{44} \text{ erg}\] | 2.84 \(\pm\) 0.06 | 2.22 \(\pm\) 0.42 | 2.77 \(\pm\) 0.05 |
| \(N_{\text{max}}\) [n] | 1.083 \(\pm\) 0.005 | 1.72 \(\pm\) 0.14 | 1.97 \(\pm\) 0.22 |
| \(f_R\) [%] | \(\approx 0^\circ\) | \(0^\circ\) | \(0^\circ\) |
|                | 84.21 \(\pm\) 0.56 | 98.03 \(\pm\) 0.30 | 69.6 \(\pm\) 3.0 |
|                | 14.47 \(\pm\) 0.96 | 98.16 \(\pm\) 2.22 | 27.1 \(\pm\) 26.0 |
|                | 1.83 \(\pm\) 0.12 | 0.14 \(\pm\) 0.14 | 3.0 \(\pm\) 1.2 |
|                | 0.130 \(\pm\) 0.001 | 0.169 \(\pm\) 0.014 | 2.05 \(\pm\) 0.03 |
| \(\chi^2/\text{dof}\) | 45/26 | 40/26 | 56/26 |

---

5 A penalty factor \(S\) that takes into account the quality of the global best fit point \(\chi^2_{\text{min}}\) is included in this estimate. We adopt the form \(S^{-1} = \sqrt{2/\chi^2_{\text{min}}/\text{dof}}\) proposed in [73]. In essence, the penalty factor reduces the rejection strength of sub-optimal fit points if the overall best fit itself is poor.
B. Redshift Evolution of the Source Density

For simplicity we have so far assumed the distribution of sources to be flat in redshift. However, the most probable source classes of UHECRs do not exhibit this behaviour but have densities evolving as function of redshift. A common parametrisation of the evolution is given by

\[
n(z) = \begin{cases} 
(1 + z)^m & \text{for } m \leq 0, \\
(1 + z)^m & \text{for } m > 0 \text{ and } z < z_0, \\
z_0^{-m} & \text{for } m > 0 \text{ and } z_0 < z < z_{\text{max}}, \\
0 & \text{otherwise}
\end{cases}
\]

which captures the general trends for the expected source classes: a power-law increase or decrease in density up to some break-point and an approximate flattening above that. Positive redshift evolution \((m > 0)\) is observed e.g. for active galactic nuclei [74] and gamma-ray bursts [75, 76], and negative evolution for some BL Lac sub classes [77] and tidal disruption events [75, 78, 79]. Source densities following the star formation rate are approximately reproduced for \(m = 3.4\) [80].

We study the effect of source density evolution on the allowed level of population variance by evaluating the fiducial model also for redshift evolutions of \(m = -3, 3, 6\) with \(z_0 = 1.5\) and \(z_{\text{max}} = 4\). Results are shown in Table IV.

Best agreement with observations is found for predominantly local sources \((m = -3)\) and a continuous decrease in fit quality is identified for stronger density evolutions. The improved fit for small \(m\) is primarily driven by a better match of the observed composition, in particular \(\langle X_{\text{max}}\rangle\), but the difference in \(\chi^2\) only becomes large once \(m > 3\). Redshift evolutions stronger than \(m = 6\) could be excluded at more than 3σ based on their cosmogenic neutrino signature by future neutrino detectors such as Grand200k [81] or IceCube Gen2 [82].

We find a clear anti-correlation between source density evolution \(m\) and spectral index \(\gamma_{\text{src}}\), in agreement with previous studies, e.g. [10, 37, 59, 83]. This is caused by the, on average, larger source distance for stronger density evolutions and consequently increased amount of interactions during propagation. Since interactions lead to a softening of the spectrum, a harder injection spectrum is required at the sources. The same argument applies to the progressively heavier source composition at the best fit. For strong evolution, the viability of the \(\beta_{\text{pop}}\)-degenerate regime of the fiducial model is reduced and the \(\beta_{\text{pop}} + \gamma_{\text{src}} \approx 4\) regime is preferred more strongly but shifted to harder source spectrum \(\gamma_{\text{src}} < 0\). Extremely identical sources are disfavoured in this case because they would lead to a worse description of the observed spectral shape and an underestimation of the shower variance.

As established previously for the fiducial model, there exists an approximate boundary of \(\beta_{\text{pop}} > -\gamma_{\text{src}} + 4\) which dictates that larger population variance requires softer source spectra. Local source distributions allow for softer spectra and the source density redshift evolution and population variance are therefore positively correlated, in the sense that smaller values of \(m\) allow for smaller values of \(\beta_{\text{pop}}\).

To summarise, negative redshift evolutions of the source density provide many attractive benefits: (i) a quantitatively better fit to the observed UHECR spectrum and composition, (ii) tighter required injection composition, (iii) more natural spectral indices \(\gamma_{\text{src}} > 0\), and (iv) a potentially higher, but still not very large, population variance. This makes classes of astrophysical objects with a negative redshift evolution, such as tidal disruption events [75, 79] and high-spectral-peak BL Lacs [77], interesting as sources of ultra-high-energy cosmic rays.

C. Redshift Evolution of the Maximum Rigidity

In addition to the interactions with ambient photon fields, cosmic rays lose energy due to the adiabatic expansion of the Universe, with \(E_{\text{obs}} = E_{\text{inj}} (1 + z)\). For a population of sources this will lead to different effective maximum rigidities for sources at different distances and result in a naturally broadened population spectrum at Earth, even in the limit of identical sources.

We have previously assumed that the distribution of maximum rigidities \(p(R_{\text{max}})\) does not evolve as a function of distance. However, most classes of astrophysical objects exhibit larger luminosities at higher redshifts [74–76]. If the maximum rigidity of a cosmic-ray source is

| redshift evolution \(m\) | -3 | 0 | 3 | 6 |
|------------------------|----|---|---|---|
| \(R_0\) [EV]           |    |   |   |   |
| \(\beta_{\text{pop}}\)  |    |   |   |   |
| \(\gamma_{\text{src}}\) |    |   |   |   |
| \(L_0\) [10^{44} \text{ erg Mpc}^{-3} \text{yr}^{-1}]\) |    |   |   |   |
| \(\mathcal{I} [%]\)     |    |   |   |   |
| \(\chi^2/\text{dof}\)   |    |   |   |   |
FIG. 5. Results of the source parameter scan for the population model with redshift evolution of the distribution of maximum rigidities marginalised onto $\beta_{\text{pop}} - \gamma_{\text{src}}$ space. The agreement between prediction and Auger observations in terms of the $\chi^2$ estimator is displayed with different color levels and the contour lines indicate the one (green), two (orange) and three (red) sigma confidence interval for two degrees of freedom. The best fit is marked with a white cross. Red crosses indicate that these parameter combinations are excluded by neutrino multimessenger constraints.

Indeed connected to the source luminosity, as outlined in Sec. II D 2, then $R_{\text{max}}$ should also evolve as a function of redshift. We adapt our population model to this expectation by evolving the starting point of the $R_{\text{max}}$ distribution with redshift,

$$R_0(z) = R_0 (1 + z)^q, \quad q \in \mathbb{R}. \quad (30)$$

Different evolutions can be sampled by changing the value of the free parameter $q$. In the limit of $q = 0$ we obtain our default no-redshift-scaling case while for $q = 1$ adiabatic losses are exactly compensated and sources would have the same effective maximum rigidity at all redshifts. Overcompensation ($q > 1$), and even enhancement of local sources ($q < 0$) are also possible.

We find that the cosmic-ray fit has only moderate sensitivity to the value of $q$, and no appreciable correlation with $R_0$, $\beta_{\text{pop}}$ or $\gamma_{\text{src}}$ can be observed. Nevertheless, negative evolutions are slightly preferred, with the best fit at $q = -4.3^{+1.0}_{-1.8}$, and positive values of $q \geq 1$ excluded at $3\sigma$ confidence level. The difference is explained by a slightly better fit of the mean shower depth but more importantly by a better description of the observed spectral shape which is related to the stacking of contributions from different redshift shells.

Intuitively, the largest possible population rigidity variance should be allowed for a redshift scaling of $R_{\text{max}}(z) = R_{\text{max}}(1 + z)$ as this would compensate the intrinsic broadening of the maximum rigidity termination via adiabatic energy losses. This expectation is not reflected in the results (Fig. 5) and we find a lower limit of $\beta_{\text{pop}} \ll 4$ independent of $q$.

Perhaps the most interesting result of this scan is not the precise best fit but rather the realisation that strongly positive rigidity-redshift scalings lead to an important multimessenger signature in the form of a large flux of cosmogenic high-energy and ultra-high-energy neutrinos (Fig. 6). If the evolution is strong then the more-distant, high-$R_{\text{max}}$ sources are largely screened from our view in cosmic rays because of the interactions experienced during propagation. At the same time, these interactions produce a large flux of cosmogenic neutrinos that can reach us even from high redshifts. Around the peak at $E_\nu \approx 10^{17.4}$ eV more than 95% of the predicted neutrino flux is produced by UHECRs from sources at redshift $z > 1$. Based on this prediction, existing UHE limits of the IceCube [58] and Auger [84] are able to constrain the redshift evolution of maximum rigidities to $q \lesssim 2$. With the increased sensitivity of future detectors, such as IceCube Gen2 [82] or Grand200k [81], this upper limit can be reduced to $q \lesssim 1$ assuming a non-detection of UHE neutrinos.

It is important to remark that these limits are derived under the assumption of a source density that does not evolve as a function of distance. For sources that are more abundant in the local Universe the constraints are weakened while they are enhanced for most other realistic source classes (AGN [74], GRB [75, 76]).
D. Other Variations of the Source Model

We have considered additional variations of the source model to study the impact on the allowed level of population diversity in maximum rigidity (Table V). They are described briefly in the following. In all scenarios a small but generally non-zero level of population variance on the order of $\beta_{\text{pop}} \sim 4 - 6$ is recovered at the best fit to Auger observations. The largest amount of source diversity – the most extreme case – is obtained for negative redshift evolution of the source density and Heaviside cutoff of the source spectra, which yields $\beta_{\text{pop}} \sim 3 - 3.5$ depending on the choice of hadronic interaction model.

a. Minimum Source Distance Cosmic rays are attenuated during propagation depending on their source distance. UHECRs with energies around the cutoff, and with the observed heavy composition, are expected to reach us only from relatively local sources. To avoid artifacts in the simulations we have chosen $z_{\text{min}} = 10^{-3}$, or about 4.3 Mpc, as minimum source distance. Setting a larger minimum distance, i.e. 43 Mpc, we observe a decrease in fit quality for our fiducial model. This is because nearby sources primarily contribute at the highest energies. If these are removed, sources from larger distances must compensate the loss in flux. However, for a Peters cycle progression of maximum rigidities with preference for low maximum rigidities $\mathcal{O}(1 \text{ EeV})$ this compensation must come predominantly from heavier elements since only they can reach the required energies. Because of increased interaction due to the larger source distance this also leads to the production of a substantial flux of lighter secondary cosmic rays and a stronger mixing of the mass groups – in contrast with observations. The tension can be partially mitigated when sources are essentially identical and the high-rigidity tail is very small. This shift to larger values of $\beta_{\text{pop}}$ for the best fit is observed in our simulations; however, the viable range is not affected strongly and the lower limit remains approximately the same.

b. Source Cutoff Function Sources with an exponential UHE cutoff already include an intrinsic dispersion in the maximum rigidity of the produced cosmic rays even for a single source. This contribution is reduced for sharper-than-exponential cutoffs and becomes zero for a sudden, Heaviside-like limit. An increased level of population variance should therefore be expected for sources with steeper cutoff function.

As proposed in Eq. (6) and Sec. II C, a super-exponential cutoff can be assumed with adjustable exponent $\lambda_{\text{cut}}$. Exactly Heaviside-like sources are obtained only for $\lambda_{\text{cut}} \to \infty$ but effectively the population spectra become very similar already for $\lambda_{\text{cut}} \gtrsim 2$. Beyond that point, the difference presents mainly in the sharpness of the break at $R_0$.

Re-simulating the fiducial model for a range of exponents, $\lambda_{\text{cut}} \in [1, 50]$, we find the global best fit at $\lambda_{\text{cut}} = 5.4^{+1.2}_{-2.3}$ which indicates effectively Heaviside-like sources. Overall the fit exhibits only moderate sensitivity to the precise shape of the cutoff except when it is close to an ordinary exponential (Fig. 7). The latter is weakly disfavoured at a level of $2.3\sigma$. As expected from previous results, the population variance is poorly constrained for sources with exponential cutoff function, but even for close-to-Heaviside source cutoffs an upper limit cannot be placed – only at $\lambda_{\text{cut}} \gtrsim 8$ a significant preference of intermediate diversity $\beta_{\text{pop}} \sim 4$ emerges. This reveals substantial dependence of the population variance on the precise shape of the spectrum at the break. It is worthwhile noting that for approximately Heaviside-like sources the best fit shifts to softer source spectra and hard spectra of $\gamma_{\text{src}} \lesssim 1$ are strongly disfavoured. This is in better agreement with the expectation from diffusive shock acceleration. The correlation between the sharpness of the cutoff and rigidity variance is not as strong as expected, and even sources with effectively instantaneous limit do not allow for diversities much greater than $\beta_{\text{pop}} \sim 3.5$.

c. Fixed Injection Composition Although we are not able to find evidence for strong population variance in $R_{\text{max}}$ for any realisation of our source model, an argument can be made that even the identified preference for intermediate levels of source diversity is a direct consequence of not constraining the injection composition and therefore overestimating the parameter space. However, similar results are obtained after fixing the source fractions to theoretically motivated values.

A natural choice is to adapt the Galactic cosmic-ray (GCR) composition [87] to the UHE regime. We find...
TABLE V. Best-fit source spectral index $\gamma_{\text{src}}$ and maximum rigidity variance $\beta_{\text{pop}}$ plus corresponding $\chi^2$ for different variations of the source model. These are (fd) the fiducial model, (zr) redshift evolution of $p(R_{\text{max}})$, (zn) redshift evolution of the source density, (zm) larger minimum source distance, (sc) super-exponential source cutoff function, (fg) injection fractions fixed to the composition observed for Galactic cosmic rays, and (ex) the extreme scenario that yields the largest amount of population variance with negative redshift evolution of the source density ($m = -3$) and Heaviside rigidity cutoff at the source. The best-fit values of the additional free parameters are, $q = -4.3^{+1.0}_{-0.8}$ for (zr) and $\lambda = 5.4^{+1.2}_{-1.5}$ for (sc).

Confidence intervals that reach a limit of the scan range are marked with an asterisk.

| Model | Parameter | $\beta_{\text{pop}}$ | $\gamma_{\text{src}}$ | $\chi^2_{\text{dof}}$ |
|-------|-----------|---------------------|----------------------|---------------------|
| fd    |           | $5.3^{+2.6}_{-0.5}$ | $-0.8^{+1.4}_{-0.5}$ | 40.4                |
| zr    | $q \in [-5, 2]$ | $4.8^{+2.6}_{-0.5}$ | $-0.19^{+0.8}_{-0.18}$ | 33.7                |
| zn    | $m = -3$  | $4.4^{+2.9}_{-0.5}$ | $0.2^{+0.8}_{-0.4}$   | 37.3                |
| m = 3 |           | $6.4^{+0.3}_{-0.34}$| $-2.0^{+0.4}_{-0.5}$  | 42.5                |
| m = 6 |           | $6.4^{+0.36}_{-0.34}$| $-2.24^{+0.35}_{-0.18}$| 68.9                |
| zm    | $z_{\text{min}} = 0.01$ | $29.9^{+1.7}_{-1.22}$| $0.38^{+0.18}_{-0.2}$ | 46.2                |
| sc    | $\lambda \in [1, 50]$ | $4.0^{+3.4}_{-0.4}$ | $1.43^{+0.16}_{-0.16}$ | 33.6                |
| fg    | $f_R$     | $3.16^{+0.17}_{-0.16}$| $1.07^{+0.08}_{-0.08}$| 110.8               |
| ex    | EPOS-LHC  | $3.17^{+0.17}_{-0.18}$| $1.43^{+0.09}_{-0.09}$| 40.6                |
| SIBYLL2.3c | $3.5^{+0.6}_{-0.5}$ | $1.69^{+0.09}_{-0.09}$ | 34.7                |

that under this assumption the best agreement with observations is obtained for hard source spectra $\gamma_{\text{src}} \approx 1$ and population variance with $\beta_{\text{pop}} \approx 3.2$, which indicates significantly larger source diversity than any fit we have performed with free injection fractions except for the case of sources with negative redshift evolution and sudden spectral cutoff. However, the fit quality is relatively poor at $\chi^2_{\text{dof}} \approx 3$ and the $\Delta \chi^2$ with respect to the fiducial scenario is $\approx 70$, allowing a rejection of this scenario at a significance of $> 7\sigma$.

V. CONCLUSION

We have derived analytical expressions for the population spectrum of an ensemble of non-identical UHECR sources assuming a power-law distribution of maximum rigidities $p(R_{\text{max}}) \propto R_{\text{max}}^{-\beta_{\text{pop}}}$ and different choices of the spectral high-energy cutoff at the sources. For the first time, we have integrated this approach to a fit of energy spectrum and composition data to quantify the constraints on source similarity from existing observations by the Pierre Auger Observatory.

Our results show that sources are required to be effectively identical if only Auger data at the nominal energy and composition scale is considered. After adjusting the measured mean shower depth and variance within systematic uncertainties to the favourable directions, that result in the heaviest composition interpretation, we find that large yet finite values of $\beta_{\text{pop}} \approx 5$ are preferred, but the case of identical sources cannot be excluded except for sources with a very sudden cutoff function.

Increased levels of population diversity up to $\beta_{\text{pop}} \approx 3 - 4$ are possible for sources with sharp UHE cutoff and for source densities evolving negatively with redshift. Even then, maximum rigidities do not differ between sources by more than a factor of a few. In contrast, if sources are more abundant at larger redshifts they are required to be more identical. This is because the preferred source spectrum becomes harder with redshift due to increased interactions during propagation. Since the population spectrum behaves as $\lim_{R \to \infty}(\phi_{\text{pop}}) \propto R^{-\gamma_{\text{src}} - \beta_{\text{pop}} + 1}$ smaller source diversity (larger $\beta_{\text{pop}}$) is required to limit the strength of the UHE tail of the spectrum.

What could be the explanation for the small population variance of UHECR accelerators? Cosmic-ray sources that are essentially identical in their maximum acceleration energy are at odds with the typically high variances of intrinsic properties for the most commonly assumed astrophysical sources. Unless exotic mechanisms limit the maximum rigidities of accelerators to the same value, e.g. [88], the inferred small source variance could be an indicator that the observed flux of UHECRs is dominated by a single local source. Such a single-or few-source scenario seems however incompatible with the observed level of anisotropy of the cosmic-ray arrival directions at UHE unless deflections of cosmic rays in the Galactic and extragalactic magnetic fields are much larger than commonly expected. An analysis of the effect of cosmic variance is beyond the scope of this paper, but it should be noted that the fitted scenarios result in typical maximum rigidities that correspond to cosmic-ray energies below the onset of photo-nuclear interactions with the cosmic microwave background radiation. Thus the energy-loss lengths of nuclei are large and the volume of UHECR sources contributing to the flux at Earth can be $\mathcal{O}(\text{Gpc}^3)$. Further studies are needed to understand the curious case of near-identical cosmic-ray sources reported in this article.

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Appendix A: Transformation of the Emitted Flux

The apparent brightness of a highly relativistic source depends, in general, on the angle θ between the observer and the direction of motion and is affected by relativistic beaming (headlight effect) and the relativistic Doppler effect. However, for charged particles we can assume that their direction of motion is isotropised after emission from the source, and consequently, the effects of geometrical beaming are not relevant in our case and only the Lorentz-boost from the rest-frame of the acceleration region and the observer frame needs to be taken into account. For sources with a flux cutoff $f_{\text{cut}}$, the differential flux within the jet-frame is given by

$$\phi'(R') = \frac{d^2 N'}{dR' \, dt'} = \phi_0' R'^{-\gamma_{\text{src}}} f_{\text{cut}}(R'/R_0). \tag{A1}$$

To transform this flux into the observer frame the following transformations need to be taken into account

$$R = R' \, \Gamma^\alpha \quad \text{and} \quad t = t' \, \Gamma^\xi. \tag{A2}$$

Conservation of particle number implies $N = N'$. The first transformation is the boost in energy for particles; with $\alpha = 1$ for particles accelerated in the jet frame and emitted isotropically, and $\alpha = 2$ for the espresso mechanism. The second transformation is due to the time dilation and it also depends on the acceleration process. If a production of UHECR within the jet is considered then the relativistic motion of the source region will stretch the observed time by a factor of $\Gamma$ ($\xi = 1$). On the other hand, if an espresso-like mechanism is assumed, where the jet merely re-accelerates a pre-existing flux of cosmic rays $dN/dt$, then no dilation is expected, assuming that the rate of particles entering and exiting the jet is the same, i.e. $dN/dt_{\text{out}} = dN/dt_{\text{in}}$ and $\xi = 0$. The observed flux can then be written as

$$\phi(R) = \frac{d^2 N}{dR \, dt} = \frac{d^2 N(R(R))}{dR' \, dt'} \left| \frac{dR'}{dR} \frac{dt'}{dt} \right| = \phi_0' R^{-\gamma_{\text{src}}} \Gamma^{\alpha(\gamma_{\text{src}}-1)-\xi} f_{\text{cut}}(-\frac{R}{R_0})$$

$$= \phi_0' R^{-\gamma_{\text{src}}} \left( \frac{R_{\text{max}}}{R_0} \right)^{\gamma_{\text{src}}-1-\xi/\alpha} f_{\text{cut}}(-\frac{R}{R_{\text{max}}}), \tag{A3}$$

where in the last step Eq. (15) was used. To evaluate the convolution of source spectra and $R_{\text{max}}$ distributions, the product $\phi(R, R_{\text{max}}) \times p(R_{\text{max}})$ needs to be evaluated using the $R_{\text{max}}$ distribution from Eq. (17). The resulting product can be re-written in the ‘usual’ form used in Sec. II C,

$$\phi(R, R_{\text{max}}) \times p(R_{\text{max}}) = \phi_0 R^{-\gamma_{\text{src}}} f_{\text{cut}} \left( -\frac{R}{R_{\text{max}}} \right) \frac{\beta_{\text{pop}}-1}{\beta_{\text{pop}}} -\frac{1}{\beta_{\text{pop}}} \tag{A4}$$

with definitions

$$\phi_0 = \phi_0' \frac{\eta-1}{\eta+\alpha(1-\gamma_{\text{src}})-\xi-1} \tag{A5}$$

and

$$\beta_{\text{pop}} = \eta-1 + 2 - \gamma_{\text{src}} + \xi/\alpha. \tag{A6}$$

Therefore, the same analytical forms derived in Sec. II C can be used in this case, but the interpretation of $\beta_{\text{pop}}$ is slightly more complex as it depends on several source properties.

Appendix B: Broken Power-Law Maximum Rigidity Distribution

Source properties such as jet Lorentz factor (Sec. II D 1) or luminosity (Sec. II D 2) commonly follow broken power-law (BPL) distributions for the most likely UHECR source classes rather than simple power laws (SPL). However, it is possible to show that the resulting population spectra will behave similarly to the SPL scenario, although a redefinition of the parameters $\beta_{\text{pop}}$ and $\gamma_{\text{src}}$ is required if the slope below the break is too steep.

A normalised broken power-law distribution of maximum rigidities can be written as

$$p(R_{\text{max}}) = \frac{R_{\text{max}}^{-\beta_1} \left( \frac{R_{\text{max}}}{R_0} \right)^{-\beta_1-1}}{1 - \left( \frac{R_{\text{max}}}{R_0} \right)^{-\beta_1-1} + \frac{1}{\beta_2-1}} \times \begin{cases} 0 & R_{\text{max}} < R_0 \\ \left( \frac{R_{\text{max}}}{R_0} \right)^{-\beta_1} & R_0 \leq R_{\text{max}} < R_T \\ \left( \frac{R_{\text{max}}}{R_0} \right)^{-\beta_1} \left( \frac{R_{\text{max}}}{R_T} \right)^{-\beta_2} & R_{\text{max}} \geq R_T, \end{cases} \tag{B1}$$

with lower limit $R_0$, break at $R_T$, and slope $R_{\text{max}}^{-\beta_1} / R_{\text{max}}^{-\beta_2}$ below / above the break.

We can identify two fundamentally different regimes, depending on the slope $R_{\text{max}}^{-\beta_1}$ below the break. In both cases the impact of sources with $R_{\text{max}} < R_T$ is negligible for large rigidities.

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6 For a general Lorentz-boost of $\Gamma (1 - \beta \cos \theta)$ with isotropic emission angle $\theta$ the high-energy tail of the rigidity spectrum retains its spectral shape and we thus concentrate on the simpler case of a boost of $\Gamma$. 
1. $\beta_1 < 1$ : For increasing maximum rigidity, the growth in density of $R_{\text{max}}$-states, i.e. $1/dR_{\text{max}}$, exceeds the decrease in probability per state $dp(R_{\text{max}})$ and the distribution $dp/d\log(R_{\text{max}})$ will be peaked, with the preferred value of $R_{\text{max}}$ around the break at $RT$. The influence of sources with $R_{\text{max}} < RT$ is thus negligible even for small rigidities, and the population spectrum has the two limits of $\lim_{R \to 0} (\phi_{\text{pop}}) \propto R^{-\beta_{\text{pop}}} \gamma_{\text{src}}$ and $\lim_{R \to \infty} (\phi_{\text{pop}}) \propto R^{-\gamma_{\text{src}}-\beta_2+1}$, which is precisely the same behaviour as in the case of a single power-law distribution of $p(R_{\text{max}})$ for $\beta_{\text{pop}} = \beta_2$.

2. $\beta_1 > 1$ : The probability $p(R_{\text{max}})$ decreases faster than the growth in density of states for increasing maximum rigidity. Lower rigidities are therefore preferred and the population spectrum behaves as $\lim_{R \to 0} (\phi_{\text{pop}}) \propto R^{-\beta_{\text{pop}}+1}$. The high rigidity limit is unaffected by this. Via re-interpretation of $\gamma_{\text{src}} = \gamma_{\text{src}} - \beta + 1$ and $\beta_{\text{pop}} = \beta_2 - \beta_1 + 1$ the same functional form as for the single power-law scenario can be obtained. Since we require $\beta_1 > 1$ it is clear that the true high-rigidity slope $\beta_2$ is strictly steeper than the fitted, effective exponent $\beta_{\text{pop}}$ and the true source spectrum $\gamma_{\text{src}}$ is harder than the fitted spectral exponent $\gamma_{\text{src}}$.

In summary, the population spectrum behaves the same for a single power-law and broken power-law distribution of maximum rigidities for very small and very large rigidities, and it is therefore qualitatively sufficient to study the former scenario. This holds especially true since the behaviour in the low-rigidity limit is of no consequence to the study of UHECR – even more so if the break point is below the considered range of rigidities. However, if the distribution $p(R_{\text{max}})$ below the break is too steep a re-interpretation of the exponents is necessary, resulting in more-identical sources with harder spectra than predicted by the effective fit parameters. This can be understood as an approximation of the fundamental broken power-law distribution $\text{BPL}(\beta_1, \beta_2)$ by a single power-law $\text{SPL}(\beta_{\text{pop}})$ with effective slope $\beta_{\text{pop}}$. The latter provides an estimate of the total width of the $R_{\text{max}}$-distribution by combining the components below and above the break into a single aggregate parameter.

A BPL distribution will lead to a softening of the break in the population spectrum, and consequently an increase of the best-fit $\beta_{\text{pop}}$ and decrease of $\gamma_{\text{src}}$. This is equivalent to a reduction in the allowed source variance and leads us to conclude that the largest population variance in terms of $\beta_{\text{pop}}$ can be obtained for a single power-law distribution of source properties. BPL scenarios invariably require larger values of $\beta_{\text{pop}}$ except when the distribution below the break $RT$ is relatively flat or peaked. Yet, while the variance above the break is reduced, the existence of sources below the break again increases the total population diversity.

[1] L. A. Anchordoqui, Ultra-High-Energy Cosmic Rays, Phys. Rept. **801**, 1 (2019), arXiv:1807.09645 [astro-ph.HE].
[2] R. Alves Batista et al., Open Questions in Cosmic-Ray Research at Ultrahigh Energies, Front. Astron. Space Sci. **6**, 23 (2019), arXiv:1903.06714 [astro-ph.HE].
[3] A. di Matteo et al. (Telescope Array, Pierre Auger), UHECR arrival directions in the latest data from the original Auger and TA surface detectors and nearby galaxies, PoS **ICRC2021**, 308 (2021), arXiv:2111.12366 [astro-ph.HE].
[4] D. Allard, E. Parizot, and A. V. Olinto, On the transition from galactic to extragalactic cosmic-rays: spectral and composition features from two opposite scenarios, Astropart. Phys. **27**, 61 (2007), arXiv:astro-ph/0512345 [astro-ph].
[5] D. Hooper, S. Sarkar, and A. M. Taylor, The intergalactic propagation of ultrahigh energy cosmic ray nuclei, Astropart. Phys. **27**, 199 (2007), arXiv:astro-ph/0608085.
[6] D. Allard, A. V. Olinto, and E. Parizot, Signatures of the extragalactic cosmic-ray source composition from spectrum and shower depth measurements, Astron. Astrophys. **473**, 59 (2007), arXiv:astro-ph/0703633 [ASTRO-PH].
[7] D. Allard, N. G. Busca, G. Decerprit, A. V. Olinto, and E. Parizot, Implications of the cosmic ray spectrum for the mass composition at the highest energies, JCAP **0810**, 033, arXiv:0805.4779 [astro-ph].
[8] N. Globus, D. Allard, R. Mockovcivitch, and E. Parizot, UHECR acceleration at GRB internal shocks, Mon. Not. Roy. Astron. Soc. **451**, 751 (2015), arXiv:1409.1271 [astro-ph.HE].
[9] N. Globus, D. Allard, and E. Parizot, A complete model of the cosmic ray spectrum and composition across the Galactic to extragalactic transition, Phys. Rev. D **92**, 021302 (2015), arXiv:1505.01377 [astro-ph.HE].
[10] M. Unger, G. R. Farrar, and L. A. Anchordoqui, Origin of the ankle in the ultrahigh energy cosmic ray spectrum, and of the extragalactic protons below it, Phys. Rev. D **92**, 123001 (2015), arXiv:1505.02153 [astro-ph.HE].
[11] A. Aab et al. (Pierre Auger), Combined fit of spectrum and composition data as measured by the Pierre Auger Observatory, JCAP **04**, 038, [Erratum: JCAP 03, E02 (2018)], arXiv:1612.07155 [astro-ph.HE].
[12] K. Fang and K. Murase, Linking High-Energy Cos- mic Particles by Black Hole Jets Embedded in Large-Scale Structures, Phys. Lett. **14**, 396 (2018), [Nature Phys.14.no.4,396(2018)], arXiv:1704.00015 [astro-ph].
[13] M. Kachelrieß et al., Minimal model for extragalactic cosmic rays and neutrinos, Phys. Rev. D **96**, 083006 (2017), arXiv:1704.06893 [astro-ph.HE].
[14] D. Boncioli, D. Biehl, and W. Winter, On the common origin of cosmic rays across the ankle and diffuse neutrinos at the highest energies from low-luminosity Gamma-Ray Bursts, Astrophys. J. **872**, 110 (2019),
[51] A. Aab et al. (Pierre Auger), Measurement of the cosmic-ray energy spectrum above 2.5 \times 10^{18} \text{eV} using the Pierre Auger Observatory, Phys. Rev. D 102, 062005 (2020), arXiv:2008.06486 [astro-ph.HE].

[52] A. Aab et al. (Pierre Auger), Depth of Maximum of Air-Shower Profiles at the Pierre Auger Observatory: Measurements at Energies above 10^{17.8} \text{eV}, Phys. Rev. D 90, 122005 (2014), arXiv:1409.4800 [astro-ph.HE].

[53] A. Yushkov (Auger), Mass Composition of Cosmic Rays with Energies above 10^{17.2} \text{eV} from the Hybrid Data of the Pierre Auger Observatory, PoS ICRC2019, 482 (2020).

[54] K.-H. Kampert and M. Unger, Measurements of the Cosmic Ray Composition with Air Shower Experiments, Astropart. Phys. 35, 600 (2012), arXiv:1201.6018 [astro-ph.HE].

[55] M. G. Aartsen et al. (IceCube), Observation of Astrophysical Neutrinos in Six Years of IceCube Data, PoS ICRC2017, 981 (2018).

[56] M. Ackermann et al. (Fermi-LAT), The spectrum of isotropic diffuse gamma-ray emission between 100 MeV and 820 GeV, Astrophys. J. 799, 86 (2015), arXiv:1410.3096 [astro-ph.HE].

[57] C. Kopper (IceCube), The Cosmic Evolution of Fermi BL Lacertae Objects, Astrophys. J. 812, 33 (2015), arXiv:1509.01592 [astro-ph.HE].

[58] D. Wanderman and T. Piran, The luminosity function of the cosmic star formation history, Mon. Not. Roy. Astron. Soc. 461, 1944 (2010), arXiv:0912.0709 [astro-ph.HE].

[59] B. Dawson (Pierre Auger), The Energy Scale of the Pierre Auger Observatory, Nucl. Instrum. Meth. B 316, R95 (2005).

[60] J. M. González, S. Mollerach, and E. Parizot, Propagation of high-energy cosmic rays in extragalactic turbulent magnetic fields: resulting energy spectrum and composition, Astron. Astrophys. 479, 97 (2008), arXiv:0709.1541 [astro-ph].

[61] W. Winter, EPOS LHC: Test of collective hadronization with data measured at the CERN Large Hadron Collider, J. Phys. G 31, 437 (1984).

[62] D. Wittkowski, Reconstructed properties of the sources of UHECR and their dependence on the extragalactic magnetic field, PoS ICRC2017, 563 (2017).

[63] A. M. Hillas, Can diffusive shock acceleration in supernova remnants account for high-energy galactic cosmic rays?, J. Phys. G 31, R95 (2005).

[64] J. Heinze, A. Fedynitch, D. Boncioli, and W. Winter, A new view on Auger data and cosmogenic neutrinos in light of different nuclear disintegration and air-shower models, Astrophys. J. 873, 88 (2019), arXiv:1901.03338 [astro-ph.HE].

[65] K. Werner, EPOS LHC: Test of collective hadronization with data measured at the CERN Large Hadron Collider, Phys. Rev. D 92, 034006 (2015), arXiv:1306.0121 [hep-ph].

[66] R. Alves Batista et al., CRPropa 3 - a Public Astrophysical Simulation Framework for Propagating Extraterrestrial Ultra-High Energy Particles, JCAP 05, 038, arXiv:1603.07142 [astro-ph.IM].

[67] M. Ackermann et al. (Fermi-LAT), The spectrum of isotropic diffuse gamma-ray emission between 100 MeV and 820 GeV, Astrophys. J. 799, 86 (2015), arXiv:1410.3096 [astro-ph.HE].

[68] C. Kopper (IceCube), Observation of Astrophysical Neutrinos in Six Years of IceCube Data, PoS ICRC2017, 981 (2018).

[69] M. G. Aartsen et al. (IceCube), Differential limit on the extremely-high-energy cosmic neutrino flux in the presence of astrophysical background from nine years of IceCube data, Phys. Rev. D 98, 062003 (2018), arXiv:1807.01820 [astro-ph.HE].

[70] R. Alves Batista, R. M. de Almeida, B. Lago, and K. Kotera, Cosmogenic photon and neutrino fluxes in the Auger era, JCAP 01, 002, arXiv:1806.10879 [astro-ph.HE].

[71] R. Alves Batista, D. Boncioli, A. di Matteo, A. van Vliet, and D. Walz, Effects of uncertainties in simulations of extragalactic UHECR propagation, using CRPropa and SimProp, JCAP 10, 063, arXiv:1508.01824 [astro-ph.HE].

[72] C. Guépin, K. Kotera, E. Barausse, K. Fang, and A. D’Amico, Semi-analytic modelling of the extragalactic background light and consequences for extragalactic gamma-ray spectra, Mon. Not. Roy. Astron. Soc. 422, 3189 (2012).

[73] R. Aloisio and V. Berezinsky, Diffuse propagation of UHECR and the propagation theorem, Astrophys. J. 612, 900 (2004), arXiv:astro-ph/0403005.

[74] J. M. González, S. Mollerach, and E. Parizot, Magnetic diffusion and interaction effects on ultrahigh energy cosmic rays: Protons and nuclei, Phys. Rev. D 104, 063005 (2021), arXiv:2105.08138 [astro-ph.HE].

[75] R. C. Gilmore, R. S. Somerville, J. R. Primack, and A. Domínguez, Magnetic field models of the cosmic ray source and its connection to the cosmic magnetic field, Mon. Not. Roy. Astron. Soc. 461, 371 (2016), arXiv:1601.06787 [astro-ph.HE].

[76] A. M. Hopkins and J. F. Beacom, On the normalisation of the cosmic star formation history, Astrophys. J. 651, 142 (2006), arXiv:astro-ph/0601463.

[77] C. Guépin, K. Kotera, E. Barausse, K. Fang, and A. D’Amico, Extragalactic Energy Cosmic Rays and Neutrinos from Tidal Disruptions by Massive Black Holes, Astron. Astrophys. 616, A179 (2018), [Erratum: Astron. Astrophys. 636, C3 (2020)], arXiv:1711.1274 [astro-ph.HE].

[78] C. S. Kochanek, Tidal disruption event demographics, Mon. Not. Roy. Astron. Soc. 461, 371 (2016), arXiv:1601.06787 [astro-ph.HE].

[79] J. Heinze, A. Fedynitch, D. Boncioli, and W. Winter, A new view on Auger data and cosmogenic neutrinos in light of different nuclear disintegration and air-shower models, Astrophys. J. 873, 88 (2019), arXiv:1901.03338 [astro-ph.HE].
[84] F. Pedreira (Pierre Auger), Bounds on diffuse and point source fluxes of ultra-high energy neutrinos with the Pierre Auger Observatory, PoS ICRC2019, 979 (2021).
[85] P. Allison et al. (ARA), Performance of two Askaryan Radio Array stations and first results in the search for ultra-high energy neutrinos, Phys. Rev. D 93, 082003 (2016), arXiv:1507.08991 [astro-ph.HE].
[86] A. L. Cummings, R. Aloisio, and J. F. Krizmanic, Modeling of the Tau and Muon Neutrino-induced Optical Cherenkov Signals from Upward-moving Extensive Air Showers, Phys. Rev. D 103, 043017 (2021), arXiv:2011.09869 [astro-ph.HE].
[87] P. A. Zyla et al. (Particle Data Group), Review of Particle Physics, PTEP 2020, 083C01 (2020).
[88] L. A. Anchordoqui, The dark dimension, the Swampland, and the origin of cosmic rays beyond the GZK barrier (2022), arXiv:2205.13931 [hep-ph].