Non-Fermi liquid behaviour from dynamical effects of impurity scattering in correlated Fermi liquids

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The interplay of disorder and interactions is a subject of perennial interest. In this work, we have investigated the effect of disorder due to chemical substitution on the dynamics and transport properties of correlated Fermi liquids. A low frequency analysis in the concentrated and dilute limits shows that the dynamical local potentials arising through disorder averaging generate a linear (in frequency) term in the scattering rate. Such non-Fermi liquid behaviour (nFL), present for any disorder concentration, is investigated in detail within dynamical mean field theory. We show that the nFL behaviour has features that are distinct from those arising through Griffiths singularities or distribution of Kondo scales. Relevance of our findings to experiments on alloyed correlated systems is pointed out.

PACS numbers: 71.27.+a Strongly correlated electron systems, 75.20.Hr Heavy-fermion, 72.80.Ng Disordered Crystal alloy, Electrical conductivity, 71.23.-k Condensed matter disorder solids, 71.10.Hf Non-Fermi-liquid ground state.

There exist a number of metallic doped systems for which the Fermi liquid (FL) theory is clearly violated in transport and thermodynamic properties [1, 2]. Most theoretical scenarios for non-Fermi liquid (nFL) behaviour require the proximity of some kind of singularity in the phase diagram, such as a quantum critical point (QCP) [3–5], Griffiths singularities (GS) close to magnetic instabilities [6] or van Hove singularities [7]. Other proposals for nFL include distribution of Kondo scales [8] and multichannel Kondo effect [9]. The nFL behaviour in a wide range of materials has been explained through these proposals.

Nevertheless, there is an expanding set of correlated systems for which the nFL behaviour does not fit the existing scenarios [1, 2, 10, 12]. For example, in a recent study of the heavy fermion material Ce$_1$–$p$La$_p$B$_6$ [11], a resistivity of the form $\rho(T) = \rho_0 + AT^\alpha$ was found for $p \gtrsim 0.4$, where the non-universal and fractional exponent $\alpha$, was found to be $p$-dependent. In the FL regime ($p \lesssim 0.4$), the coefficient of the quadratic term of the resistivity apparently diverges at the crossover from FL to nFL regime. A consistent theoretical explanation for such unusual nFL behaviour has not been found yet.

The $T = 0$ scattering rate in clean Fermi liquids has the form $\Gamma(\omega) \propto a_0 \omega^2$, The static effects of scattering by quenched substitutional disorder in Fermi-liquids contribute a constant term $a_0$ to $\Gamma(\omega)$ [13]. In this work, we show that dynamical effects of impurity scattering can give rise to the type of aforementioned unusual nFL behaviour. The $\Gamma(\omega)$ is shown to acquire an additional marginal-FL like term, $a_1 \omega$. We also demonstrate that an excellent power-law description of $\Gamma(\omega)$ is possible over two decades in $\omega$, that yields a doping dependent exponent.

In heavy fermion systems such as CeCu$_6$, the substitution of Ce by La is of the Kondo hole (KH) type while that of Cu with Au is referred to as ligand-field type [13]. In this work, we will explicitly consider the KH type of substitution where the orbital energy of the $f$-level of the substituted non-magnetic atom is high enough that it decouples from the conduction band. Any type of random substitution breaks translational invariance, and leads to site and bond disorder. Random chemical substitution of the type $A_1$–$pB_p$ is usually incorporated through a probability distribution of the parameters in the Hamiltonian. Although we have explicitly considered site disorder with a binary distribution, our arguments will be shown to be applicable to other types of disorder and distributions.

The Hamiltonian for the periodic Anderson model (PAM), which is appropriate for heavy fermion systems, may be expressed in standard second-quantized notation as

$$H_{\text{PAM}} = -\sum_{(ij)\sigma} t_{ij} \left( c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.} \right) + \sum_i H_i$$

where $t_{ij}$ represents nearest neighbour hopping and $c_{i\sigma}$ is the conduction ($c$)-electron annihilation operator for site $i$ and spin $\sigma$. Within dynamical mean field theory (DMFT) [14, 15], which is exact in the limit of infinite dimensions $D$, the hopping $t_{ij}$ must be rescaled as $t_{ij} \propto t_s/\sqrt{D}$. The site-diagonal part, given by $H_i = \epsilon_i \sum_{\sigma} c_{i\sigma}^\dagger c_{i\sigma} + \epsilon_f \sum_{\sigma} f_{i\sigma}^\dagger f_{i\sigma} + V(\sum_{\sigma} f_{i\sigma}^\dagger c_{i\sigma} + \text{h.c.}) + \sum_{\sigma} \sum_{ij} U_{ij} f_{i\uparrow} c_{j\sigma}^\dagger$, represents the orbital energies, hybridization between $c$ and $f$ electrons and the cost of double occupancy of the $f$-orbital respectively.

The coherent potential approximation is the best local approach to study the interplay of disorder with interactions in strongly correlated systems [13, 19]. Even though the CPA ignores inter-site coherence and coherent back scattering effects (and hence Anderson localization), the
effects of configurational averaging are well accounted. A dynamical CPA within DMFT can be formulated that takes into account the dynamical effects of impurity scattering \[13\]. In this approach, the arithmetically averaged local c-electron Green’s function called \(G_c^{CPA}\) is given by

\[
G_c^{CPA}(\omega) = \frac{1 - p}{\omega^+ - \epsilon_c - \Sigma_c} + \frac{p}{\omega^+ - \epsilon_c - S(\omega)}
\]

(2)

where the self-energy is \(\Sigma_c(\omega) = V^2(\omega^+ - \epsilon_f - \Sigma_f(\omega))^{-1}\) and \(\Sigma_f(\omega)\) is the f-self-energy. The dopant concentration is denoted by \(p\), and \(S(\omega)\) is Feenberg self-energy \[17\] that represents the hybridization with the host. In the clean case, the \(S(\omega)\) is a functional of the local c-Green’s function \[16\]. In the disordered case, the sum over all neighbouring sites in the Feenberg renormalized perturbation series \[17\] can be reduced, within DMFT, to a probabilistic averaging of the magnetic and Kondo hole Green’s functions. Thus \(S(\omega)\) becomes a functional of the averaged (CPA) Green’s function, \(S(\omega) = S[G_c^{CPA}]\). It is given by the condition that the CPA restores translational invariance, hence \(G_c^{CPA}\) is given by

\[
G_c^{CPA}(\omega) = \frac{1}{N} \sum_k \frac{1}{\omega^+ - \epsilon_k - \Sigma_c} = \mathcal{H} [\gamma(\omega)] = \frac{1}{\gamma(\omega) - S(\omega)}
\]

(3)

where \(\gamma = \omega^+ - \epsilon_c - \Sigma_c^{CPA}\) and \(\mathcal{H}[z] = \int_{-\infty}^{\infty} dz \rho_0(\epsilon) (z - \epsilon)^{-1}\) is the Hilbert transform of \(z\) with respect to the non-interacting density of states, \(\rho_0(\epsilon)\). For the semi-elliptical density of states, \(\rho_0(\epsilon) = \sqrt{1 - \epsilon^2/t_\epsilon^2}/(2\pi t_\epsilon)\), corresponding to the Bethe lattice, \(S(\omega) = t_\epsilon^2 G_c^{CPA}(\omega)/4\). This allows us to relate the CPA self-energy directly to the \(\Sigma_f\) through the following cubic equation (for \(t_\epsilon = 2\))(see equation (S7) of Supplementary Information (SI)):

\[
\Sigma_c^{CPA} (\Sigma_c - \Sigma_c^{CPA}) [(\omega^+ - \epsilon_c) (\Sigma_c^{CPA} - (1 - p)\Sigma_c) - p\Sigma_c - \Sigma_c^{CPA}] - (\Sigma_c^{CPA} - (1 - p)\Sigma_c)^2 = 0
\]

(4)

We have carried out a low frequency analysis of equation 4 in the concentrated (\(p \to 0\)) and dilute limits (\(p \to 1\)). Our basic premise is that the \(f\)-moments are completely screened, and hence have a local FL form of the self-energy, \(\Sigma_f(\omega) \to \Sigma_f(0) + \omega (1 - 1/Z) - i\epsilon_f' \omega^2\). Using this, the \(c\)-self-energy \(\Sigma_c(\omega) = V^2[\omega^+ - \epsilon_f - \Sigma_f(\omega)]^{-1}\) can be Taylor expanded as

\[
\Sigma_c(\omega) = \frac{V^2}{\omega - \epsilon_f - \Sigma_f(\omega)} \to \frac{V^2}{\epsilon_f'} \left(1 + \frac{\omega}{Z\epsilon_f'} + i\frac{\lambda'}{Z\epsilon_f'} \omega^2\right)
\]

(5)

where the renormalized \(f\)-level, \(\epsilon_f' = \epsilon_f + \Sigma_f(0) \in \mathbb{R} \forall p\). The above may be used in equation 4 to find the low frequency form for \(\Sigma_c^{CPA}(\omega)\).

We find that

\[
\Sigma_c^{CPA}(\omega) \to 0 \quad S_0 + S_1 \omega + S_2 \omega^2
\]

(6)

where \(\{S_i\}\) have finite imaginary parts that depend on \(p\). The above expression, which shows that the CPA self-energy has a distinctly nFL form, is the central result of our work. A finite linear term in the \(\text{Im}\Sigma_c^{CPA}\) in addition to the well-known residual (\(\omega = 0\)) and quadratic terms has broad consequences. First, since \(\Sigma_c^{CPA} = V^2(\omega^+ - \epsilon_f - \Sigma_f^{CPA})^{-1}\), the CPA self-energy of the \(f\)-electrons will also have an nFL form, even though the local self-energy has a standard FL form. It is easy to show using equation 2 that \[13\]

\[
\text{Im}\Sigma_f^{CPA}(\omega) = \frac{1}{1 - p} \text{Im}\Sigma_f(\omega) + \frac{p}{1 - p} \frac{V^2}{\omega^+ - \epsilon_c - S(\omega)}.
\]

(7)

The above relation implies that the nFL part in the CPA self-energy arises through the contribution from the imaginary part of the self-consistently determined dynamical hybridization or the local potential. Second, the scattering rate, \(\Gamma(\omega) \propto -\text{Im}\Sigma_c^{CPA}\) will also have a linear non-Fermi liquid type term in addition to the static potential scattering and the quadratic electron-electron scattering terms. As an inevitable consequence, transport quantities would display an nFL form. For example, the resistivity would have the low temperature form \(\rho(T) = \rho(0) + A_K T + B_K T^2\). Thermodynamics quantities like the specific heat \((C)\) will also be affected. A linear term in the imaginary part of the self-energy naturally leads to a \(\omega \ln \omega\) term in the real part. From the expression of specific heat \([20]\), it is easy to see that a \(\ln T\) contribution would arise in \(C/T\).

A straightforward generalization of equation 2 to the case of ligand-field substitution may be carried out \[13\]. A low frequency analysis similar to the one done above for KH disorder shows that the linear term would arise even for this case. Similarly, substituional disorder in the Hubbard model \[21\] will also yield similar results, since the dynamical CPA equations for the local Green’s function are exactly the same as equations 2 and 3 with the \(\Sigma_c\) being replaced by the local self-energy of the interacting electrons. The CPA self-energy will thus have contributions from the FL self-energy and the dynamical local potentials which will again lead to a linear (in frequency) term in the scattering rate. Thus our findings have implication for transition metal oxides and other systems for which the Hubbard model is appropriate. Here, we have considered a binary distribution of site energies. A generalization to other discrete or continuous distributions will yield similar consequences as for the binary distribution.

The explicit dependence of \(\{S_i\}\) (equation 6) on \(p\) is given below for a symmetric conduction band (\(\epsilon_c = 0\)). In the concentrated limit, \(p \to 0\), the static part of \(\Sigma_c^{CPA}\)
is given by (see equation (S22) of SI)
\[
\text{Re}(S_0) = (1-p)\Sigma_c(0) + \frac{p(1-p)^3}{2}\Sigma_c(0)^3
\]
\[
\text{Im}(S_0) = -p(1-p)\Sigma_c^2(0) \times \left[1 - \frac{(1-p)^2}{4}\Sigma_c^2(0)\right]^{1/2}.
\]
(8)

Where \(\Sigma_c(0) = -V^2/\epsilon_f^*\). The \(\text{Im}(S_0)\) leads to a finite \(T = 0\) residual resistivity. The \(\text{Im}(S_1)\) is given by (see equation (S29) of SI)
\[
\text{Im}(S_1) = \frac{p}{Z\epsilon_f^*}\text{Im}\left[\frac{(\delta\Sigma(0))^2(\Sigma_c(0) - \delta\Sigma)}{(\delta\Sigma(0))^2 - 2(\delta\Sigma(0) - \Sigma_c(0))^2}\right]
\]
(9)

where \(\delta\Sigma(0) = -\frac{(1-p)\Sigma_c^2(0)}{2}[(1-p)\Sigma_c(0) + 2i]\). Thus, the nFL contribution is seen to be proportional to \(p\) at low dopant concentrations. It is also important to note that the quasiparticle weight \(Z\) does not appear in the static part (equation 8), but does appear in the dynamics, and arises purely because of the linear term in the FL form of the local self-energy, \(\Sigma_f(\omega)\). Hence, the dynamical effects of potential scattering are responsible for nFL behaviour due to substitutional doping.

In the dilute limit \((p \rightarrow 1)\), the hybridization is determined through the non-interacting density of states, \(\rho_0(\epsilon)\). Hence we get (see equation (S33) of the SI)
\[
\Sigma_c^{CPA}(\omega) = \frac{V^2(1-p)}{\omega^+ - \epsilon_f + \Sigma_f(\omega) - p\Delta_0(\omega)}
\]
(10)

where \(\text{Im}\Delta_0(\omega) = -\pi V^2\rho_0(\omega - \epsilon_c)\). In the strong coupling limit, and for low frequencies \(\omega \lesssim \omega_L = ZV^2/t_s\), the \(\rho_0(\omega)\) may be taken to be a constant, yielding the low frequency form of the CPA self-energy as:
\[
\Sigma_c^{CPA} \rightarrow 0 \quad \frac{V^2(1-p)}{\omega^+ - \epsilon_f^* + p\Delta_0 + p^2} \quad \omega \rightarrow 0
\]
(11)

where \(\epsilon_f^* = \epsilon_f + \Sigma_f(0) + p\text{Re}\Delta(0)\), and \(\Delta_0 = \pi V^2\rho_0(-\epsilon_c)\). From the above expression, it is easy to see that in the dilute limit, all the three coefficients of equation 6 (divided by the number of magnetic atoms \((1-p)\)) obtained through a Taylor expansion of equation 11 around \(\omega = 0\) will remain non-zero. Thus for \(p \rightarrow 1\), if either the effective \(f\)-level or the conduction band centre \((\epsilon_c)\) are non-zero, a linear term can be obtained in the temperature dependence of the resistivity.

In order to explore the nFL behaviour quantitatively in the full range of dopant concentration, we have carried out detailed calculations for the Kondo hole disordered PAM within DMFT. The local moment approach (LMA) has been used to solve the effective impurity problem arising within DMFT. Within the LMA, which is a diagrammatic perturbation theory based approach, the \(f\)-self energy is ensured to have a FL form, since adiabatic continuity to the noninteracting limit is imposed as a constraint. The reader is referred to our earlier work [16] for the detailed implementation of the LMA within DMFT for the clean PAM [22]. The coherence peak in the resistivity of clean heavy fermions is found to be at \(T \sim \omega_L\) [16], where \(\omega_L = ZV^2/t_s\) is the low energy scale of the local Fermi liquid. We focus on the frequency region \(\omega \lesssim \omega_L\), since the nFL behaviour is found experimentally at temperatures much below the coherence peak [1, 2].

In the main panel of figure 1, we show (through fitting) that the \(\text{Im}\Sigma_c^{CPA}(\omega)\) does indeed have the quadratic polynomial form of equation 6 for all \(p \in [0, 1]\). Nevertheless, we observe that a power law of the form \(C + D\omega^\alpha\) may also be fit (for \(\omega > 0\)) up to a certain upper cut-off \(\omega_c \sim \mathcal{O}(0.1\omega_L)\). This is shown in the inset of figure 1. Hence the low temperature resistivity, as obtained through the scattering rate, is susceptible to a power law interpretation, with the exponent being a function of disorder (see below). In the figure (S1) of the SI, we show that the quadratic and power law fits are equally good for the entire range of doping.

We now show the dependence of the fitting parameters on \(p\). The \(\text{Im}\{-S_i\}\) (equation 6) per magnetic atom, as a function of \(p\) are shown in the main panel of figure 2. The first two \((i = 0, 1)\) vanish for \(p \rightarrow 0\), but remain finite for all other \(p\) including \(p \rightarrow 1\), which is understood through the presence of finite potential scattering effects. The static part follows a Nordheim rule like behaviour, while the linear and quadratic coefficients show an apparent divergence before switching sign abruptly at \(p \sim 1 - n_c\). Many theoretical studies have pointed out the crossover of collective to single-impurity behaviour at \(p \sim 1 - n_c\) [23–25], and we see that indeed dramatic
changes could happen in the CPA quantities as $p$ is tuned through this crossover. The inset shows the power law exponent $\alpha$ as a function of $p$, obtained through a power law fitting of the low frequency $\text{Im}(\Sigma^{\text{CPA}}(\omega))$ for cutoff equal to $\sim 0.1\omega_L$ (circles) and $\sim 0.2\omega_L$ (squares). The power law fit depends sensitively on the cutoff $\omega_c$, and is thus ambiguous. The exponent $\alpha$ is seen to be $p$-dependent and close to 1 over a large range of $p \in [0, 1]$.

Although the CPA quantities display remarkable non-monotonic behaviour on varying $p$, the local quantities are either monotonic with $p$ or remain almost unchanged. This is shown in the figure 3, where we compare the occupancy (top panel) and the quasiparticle weight $Z$ (bottom panel) computed through local $\Sigma_f(\omega)$ and $G_{c/f}(\omega)$ (denoted by open squares and circles) with the corresponding CPA quantities (filled squares and circles). The local quasiparticle weight increases steadily with increasing Kondo hole concentration, while the CPA quantity shows non-monotonic behaviour in the form of a dip that appears at $p \sim 1 - n_c$. Such a dip would manifest as a peak in specific heat coefficient or the effective mass. The occupancy, calculated locally remains almost unchanged, while the $n_{tot}^{CPA}$, given by $n_f + n_c - p$, decreases linearly as expected [13].

There have been many recent theoretical studies of substitutional disorder in the Kondo lattice model or the PAM [13, 23–25]. It was shown by Kaul and Vojta [23] that Griffiths singularities (GS) appear in a wide range of concentrations leading to nFL behaviour. The GS induced nFL has specific ‘universal’ signatures, albeit dependent on a non-universal exponent $\lambda$ [6]. The authors also observe unscreened spins, which would imply that certain sites have a vanishing Kondo scale. Such a probability distribution of Kondo scales, where $P(T_K = 0)$ is finite, is also known to yield nFL behaviour [8]. In a recent work, a Lifshitz transition [24] is predicted to occur as a function of $p$, which could lead to nFL behaviour in the vicinity of the transition. The interaction of spin fluctuations with disorder close to a quantum critical point is also known to lead to power law behaviour, with a disorder-dependent exponent [4]. While inhomogeneities are natural and must be expected in any disordered system, instabilities such as a QCP and singularities such as GS are necessarily non-generic, i.e they must occur only in specific regions of the phase diagram.

While such singularities do give rise to specific nFL behaviour, our work shows that nFL behaviour can be far more generic and can arise simply as a consequence of disorder averaging. Hence, attributing the deviations from FL to a specific cause in disordered systems needs care. In Ce$_{1-p}$La$_p$B$_6$ for example, we suggest that a quadratic polynomial fit must be carried out for all $p$ instead of a partial power law fit. We predict that the fit parameters would follow the behaviour shown in figure 2.

A distinct signature of the nFL behaviour we find is that it is a macroscopic effect, hence local probes such as scanning tunneling microscopy should find local Fermi liquid behaviour while macroscopic response functions would show nFL signatures. However, large area scans would be necessary to rule out unscreened spins or Griffiths singularities.

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[1] G. R. Stewart, Rev. Mod. Phys. 73, 797 (2001).
[2] M. B. Maple et al., J. Low Temp. Phys. 161, 4-54
(2010).
[3] P. Coleman, Physica B 259-261, 353-358 (1999).
[4] A. Rosch, Phys. Rev. Lett. 82, 4280 (1999).
[5] N. S. Vidhyadhiraja et al., Phys. Rev. Lett. 102, 206407 (2009).
[6] David Nozadze and Thomas Vojta, Journal of Physics: Conference Series 391, 012162 (2011).
[7] Sebastian Schmitt, Phys. Rev. B 82, 155126 (2010).
[8] E. Miranda and V. Dobrosavljevic, “Conductor Insulator Quantum Phase Transitions”, Oxford University Press, (2013).
[9] A. M. Chang and J. C. Chen, Rep. Prog. Phys. 72, 096501 (2009).
[10] J. Paglione et al., Nature physics 3, 703 (2007).
[11] S. Nakamura et al., Phys. Rev. Lett. 97, 237204 (2006).
[12] C. M. Varma, Z. Nussinov and Wim van Saarloos, Physics Reports 361, 267417 (2002).
[13] C. Grenzebach et al., Phys. Rev. B 77, 115125 (2008).
[14] A. Georges et al., Rev. Mod. Phys. 68, 13 (1996).
[15] T. Pruschke, M. Jarrell and J. K. Freericks, Adv. Phys. 44, 187 (1995).
[16] Pramod Kumar and N. S. Vidhyadhiraja, J. Phys. Condens. Matter 23, 485601 (2011).
[17] E. N. Economou, Green’s Functions in Quantum Mechanics (Berlin: Springer) (1983).
[18] D. E. Logan, M. P. Eastwood and M. A. Tusch J. Phys. Condens. Matter 10, 2673 (1998).
[19] M. Potthoff and M. Balzer, Phys. Rev. B 75, 125112 (2007).
[20] N. S. Vidhyadhiraja, Europhys. Lett. 77, 3601 (2007).
[21] M. Laad, L. Craco, E. Muller-Hartmann, Phys. Rev. B 64, 195114 (2001).
[22] The calculations have been carried out for a hypercubic lattice, for which \( \rho_0(\epsilon) = \exp(-\epsilon^2/t^2_*)/\sqrt{\pi t_*} \). The unit of energy is \( t_* = 1 \).
[23] R. K. Kaul and M. Vojta, Phys. Rev. B 75, 132407 (2007).
[24] S. Burdin and C. Lacroix, arXiv:1210.5105 (2012).
[25] H. Watanabe and M. Ogata, Phys. Rev. B 81, 113111 (2010).
Supplemental Material for “Non-Fermi liquid behaviour from dynamical effects of impurity scattering in correlated Fermi liquids”

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CPA SELF-ENERGY

The CPA Green’s Function is given by

\[ G_{c}^{CPA}(\omega) = \frac{1}{\Gamma - \Sigma_{c}^{CPA}} = \frac{1 - p}{\Gamma - \Sigma_{c}} + \frac{p}{\Gamma}, \]  

(S1)

where

\[ \Gamma = \omega^+ - \epsilon_c - S(\omega), \]  

(S2)

and \( S(\omega) \) is the Feenberg self-energy. From Equation (S1), the CPA self-energy can be extracted as

\[ \Sigma_{c}^{CPA} = \frac{\Gamma \Sigma_{c}(1 - p)}{\Gamma - p \Sigma_{c}}. \]  

(S3)

The above can be inverted to get \( \Gamma \) in terms of \( p, \Sigma_{c} \) and \( \Sigma_{c}^{CPA} \) as

\[ \Gamma = \frac{p \Sigma_{c} \Sigma_{c}^{CPA}}{\Sigma_{c}^{CPA} - \Sigma_{c}(1 - p)}. \]  

(S4)

For a Bethe lattice,

\[ S(\omega) = \frac{t^2}{4} G^{CPA}(\omega) = \frac{t^2}{4} \frac{1}{\Gamma - \Sigma_{c}^{CPA}}, \]  

and for \( t_s = 2 \)

\[ S(\omega) = \frac{1}{\Gamma - \Sigma_{c}^{CPA}}. \]  

(S5)

Using equation (S2) and (S5), we get

\[ \omega^+ - \epsilon_c - \Gamma = \frac{1}{\Gamma - \Sigma_{c}^{CPA}}. \]  

(S6)

In equation (S6), if we substitute for \( \Gamma \), using (S1), we will get an equation relating \( \Sigma_{c}^{CPA} \) to \( \Sigma_{c} \). Alternatively, if we substitute for \( \Sigma_{c}^{CPA} \), using equation (S3), we will get an equation for \( \Gamma \) in terms of \( \Sigma_{c} \). In equation (S6), substituting \( \Gamma \) using (S4), we get

\[ \Sigma_{c}^{CPA} (\Sigma_{c} - \Sigma_{c}^{CPA}) \left[ (\omega^+ - \epsilon_c) (\Sigma_{c}^{CPA} - \Sigma_{c}(1 - p)) - p \Sigma_{c} \Sigma_{c}^{CPA} \right] - (\Sigma_{c}^{CPA} - \Sigma_{c}(1 - p))^2 = 0 \]  

(S7)

Alternatively by substituting equation (S3) in equation (S6), we get

\[ \Gamma(\Gamma - \Sigma_{c})(\omega^+ - \epsilon_c - \Gamma) - (\Gamma - p \Sigma_{c}) = 0 \]  

(S8)

This is again a cubic equation for \( \Gamma \).

CO-EFFICIENTS OF IMAGINARY PART \( \Sigma_{c}^{CPA} \)

Now the claim is that a FL expansion of \( \Sigma_{c} \) just to linear order, implying a real \( \Sigma_{c} \) will lead to an imaginary part of \( \Sigma_{c}^{CPA} \) that has a linear in \( \omega \) term. Thus we should get

\[ \text{Im}\Sigma_{c}^{CPA}(\omega) \cong \text{Im}\Sigma_{c}^{CPA}(0) + A\omega + B\omega^2 + .. \]  

(S9)

Expanding \( \Sigma_{c} = \frac{V^2}{\omega^+ - \epsilon_f - \Sigma(\omega)} \) as

\[ \frac{V^2}{\omega^+ - \epsilon_f - \Sigma(\omega)} \]  

for \( \omega \rightarrow 0 \)

\[ = \frac{V^2}{\epsilon_f}(1 + \frac{\omega}{Z\epsilon_f}) = \Sigma_{c0}(1 + \alpha \omega) \]  

(S10)

Similarly, expanding \( \Sigma_{c}^{CPA} = \Sigma_{c}^{CPA}(0) + A\omega \), and using in equation (S1) with \( (\omega^+ - \epsilon_c \rightarrow -\epsilon_c) \) for \( \omega \rightarrow 0 \)

\[ (\Sigma_{c}^{CPA}(0) + A\omega) (\Sigma_{c0}(1 + \alpha \omega) - \Sigma_{c}^{CPA}(0) - A\omega) \]  

\[ = (-\epsilon_c) \]  

\[ (\Sigma_{c}^{CPA}(0) + A\omega - \Sigma_{c0}(1 + \alpha \omega)(1 - p)) - p \Sigma_{c0} (1 + \alpha \omega) \]

\[ (\Sigma_{c}^{CPA}(0) + A\omega) \]  

\[ - \left[ \Sigma_{c}^{CPA}(0) + A\omega - \Sigma_{c0}(1 + \alpha \omega) \right]^2 = 0 \]  

(S11)

Substituting \( \omega = 0 \) will yield an equation for \( \Sigma_{c}^{CPA}(0) \). For finding \( A \) we need to collect the linear terms.

\[ A (\Sigma_{c0} - \Sigma_{c}^{CPA}) \left[ - \epsilon_c (\Sigma_{c}^{CPA}(0) - (1 - p) \Sigma_{c0}) \right] \]

\[ - p \Sigma_{c0} \Sigma_{c}^{CPA}(0) \left[ - \epsilon_c (\Sigma_{c}^{CPA}(0) - (1 - p) \Sigma_{c0}) \right] \]

\[ - p \Sigma_{c0} \Sigma_{c}^{CPA}(0) \left[ - \epsilon_c (A - \alpha \Sigma_{c0}(1 - p)) + (p \Sigma_{c0} \alpha \Sigma_{c}^{CPA}(0) - p \Sigma_{c0} A) \right] \]

\[ - 2 (\Sigma_{c}^{CPA}(0) - \Sigma_{c0}) (A - \Sigma_{c0} \alpha) = 0 \]  

(S12)

This is a linear equation for \( A \) and is hence trivial, although lengthy. But it is easy to see that since \( \Sigma_{c}^{CPA}(0) \) is expected to have an imaginary part, \( A \) will also have.

The other thing to notice is that the quasi particle weight, \( Z \) enters the \( A \) expression. This implies that interactions have little role to play in deciding the static part but play
Further, we get
\[ \epsilon p (\Sigma_c - \Sigma_c^{CPA}(0)) [- \epsilon_c (\Sigma_c^{CPA}(0) - (1-p)\Sigma_c(0)) - \epsilon_c (\Sigma_c^{CPA}(0) - (1-p)\Sigma_c(0))] \]
\[ -\epsilon (\Sigma_c^{CPA}(0) - (1-p)\Sigma_c(0)) - \epsilon (\Sigma_c^{CPA}(0) - (1-p)\Sigma_c(0)) \]
\[ = -\epsilon (\Sigma_c^{CPA}(0) - (1-p)\Sigma_c(0)) \]
\[ (\epsilon_c\Sigma_c(1-p) - \epsilon_c\Sigma_c^{CPA}(0)) + 2 (\Sigma_c^{CPA}(0) - \Sigma_c(0))\Sigma_c(0) \]
\[ \text{(S13)} \]

We need to find the dependence of \( \Sigma_c^{CPA}(0) \) for \( p \to 0 \) and \( p \to 1 \). We consider the concentrated limit, \( p \to 0 \) first.

**CONCENTRATED LIMIT \( p \to 0 \)**

**Static term : \( \omega = 0 \)**

Starting from equation \( \text{(S10)} \) and substituting \( \omega = 0 \), we get
\[ \Sigma_c^{CPA}(\Sigma_c - \Sigma_c^{CPA}) [- \epsilon_c (\Sigma_c^{CPA} - \Sigma_c(1-p)) - \epsilon (\Sigma_c^{CPA} - \Sigma_c(1-p))^2 = 0 \]. \]
\[ \text{(S14)} \]

For a symmetric bare conduction band, \( \epsilon_c = 0 \), hence
\[ p (\Sigma_c^{CPA})^2 (\Sigma_c - \Sigma_c^{CPA}) \Sigma_c(0) + (\Sigma_c^{CPA} - \Sigma_c(1-p))^2 = 0 \]. \]
\[ \text{(S15)} \]

Assuming \( \Sigma_c^{CPA} = \Sigma_c(1-p) + \delta \Sigma \) and neglecting \( \mathcal{O}(\delta \Sigma)^3 \) terms
\[ p [\Sigma_c(1-p) + \delta \Sigma]^2 (p \Sigma_c(0) - \delta \Sigma) \Sigma_c(0) + (\delta \Sigma)^2 = 0 \]. \]
\[ \text{(S16)} \]

Further, we get
\[ (\delta \Sigma)^2 [1 + p^2 \Sigma_c^2(1-p)] + 2(\delta \Sigma) [p^2 \Sigma_c^3(1-p) - \frac{1}{2} p(1-p)^2 \Sigma_c^3] + 2p^2 (1-p)^2 \Sigma_c^4 = 0 \]. \]
\[ \text{(S17)} \]

If \( (\delta \Sigma) \sim \mathcal{O}(p) \), then the above equation simplifies to
\[ (\delta \Sigma)^2 - (\delta \Sigma) p(1-p)^2 \Sigma_c^3 + p^2 (1-p) \Sigma_c^4 = 0 \]. \]
\[ \text{(S18)} \]

This can be easily solved to get
\[ (\delta \Sigma) = \frac{p(1-p)^2 \Sigma_c^3 \pm \sqrt{p^2(1-p)^4 \Sigma_c^6 - 4p^4(1-p)^2 \Sigma_c^6}}{2 \Sigma_c^3} \]
\[ \text{(S19)} \]

The imaginary part can only arise through from the square root term, hence
\[ \text{Im}(\delta \Sigma) \simeq -p(1-p) \Sigma_c^2 \left[ 1 - \frac{(1-p)^2 \Sigma_c^2}{4} \right]^{\frac{1}{2}}, \]
\[ \text{(S20)} \]

while
\[ \text{Re}(\delta \Sigma) = \frac{p(1-p)^2 \Sigma_c^3}{2}. \]
\[ \text{(S21)} \]

Thus for a BL and \( \epsilon_c = 0 \), the static CPA self-energy is given in \( p \to 0 \) limit by
\[ \Sigma_c^{CPA}(0) = \Sigma_c(1-p) + \frac{p(1-p)^2 \Sigma_c^3}{2} \]
\[ - ip(1-p) \Sigma_c^2 \left[ 1 - \frac{(1-p)^2 \Sigma_c^2}{4} \right]^{\frac{1}{2}}. \]
\[ \text{(S22)} \]

**Dynamics; Linear term**

Using \( \epsilon_c = 0 \) and \( p \to 0 \) in equation \( \text{(S13)} \)
\[ \tilde{A} \left[ p \Sigma_c^{CPA} - 2 \Sigma_c^{CPA}(0) - \Sigma_c(0) \right] \]
\[ = - \left[ - p \Sigma_c^{CPA} - 2 \Sigma_c^{CPA}(0) - p \Sigma_c^{CPA}(0) \right] \]
\[ (\Sigma_c - \Sigma_c^{CPA}(0)) + 2 \Sigma_c(0) \Sigma_c^{CPA}(0) - \Sigma_c(0) \text{).} \]
\[ \text{(S23)} \]

Using Eq \( \text{(S16)} \)
\[ \tilde{A} \left[ \frac{- (\delta \Sigma)^2}{\Sigma_c - \Sigma_c^{CPA}} - 2 \Sigma_c^{CPA}(0) - \Sigma_c(0) \right] \]
\[ = - \left[ (\delta \Sigma)^2 + \frac{(\delta \Sigma)^2 \Sigma_c(0)}{\Sigma_c - \Sigma_c^{CPA}(0)} + 2 \Sigma_c^{CPA}(0) - \Sigma_c(0) \right] \]
\[ \text{(S24)} \]

with \( \Sigma_c^{CPA}(0) = \Sigma_c(1-p) + \delta \Sigma \)

and \( \delta \Sigma \simeq -p(1-p) \Sigma_c^2 [(1-p) \Sigma_c(0) + 2i] \). \]
\[ \text{(S25)} \]

From Eq \( \text{(S20)} \) and \( \text{(S21)} \), we get
\[ \Sigma_c^{CPA}(0) - \Sigma_c(0) \simeq -p \Sigma_c(0) \left[ 1 - \frac{(1-p)^2 \Sigma_c^2}{2} + i(1-p) \Sigma_c^2 \right] \]. \]
\[ \text{(S26)} \]

The \( p \) dependence of \( \tilde{A} \) can be extracted as
\[ \tilde{A} \left[ \frac{- p^2 (\delta \Sigma)^2}{p \Sigma_c(0) - \delta \Sigma} - 2p(\delta \Sigma - \Sigma_c(0)) \right] \]
\[ = - \left[ p^2 (\delta \Sigma)^2 + \frac{- p^2 (\delta \Sigma)^2 \Sigma_c(0)}{p \Sigma_c(0) - \delta \Sigma} + 2p \Sigma_c(0) (\delta \Sigma - \Sigma_c(0)) \right] \]
\[ \text{(S27)} \]
Simplifying, we get
\[ \tilde{A} = \Sigma_{c0} + p \frac{\delta\Sigma^2 (\Sigma_{c0} - \delta\Sigma)}{(\delta\Sigma^2) - 2(\delta\Sigma - \Sigma_{c0})^2}. \]  
(S28)

Thus for \( \Sigma_{c}^{CPA}(\omega) = \Sigma_{c}^{CPA}(0) + A\omega \)

\[
\text{Im}(A) \simeq \frac{p}{Z\epsilon_f^*} \text{Im} \left[ \frac{\delta\Sigma^2 (\Sigma_{c0} - \delta\Sigma)}{(\delta\Sigma^2) - 2(\delta\Sigma - \Sigma_{c0})^2} \right],
\]  
(S29)

and

\[
\text{Re}(A) = \frac{1}{Z\epsilon_f^*} \left[ \Sigma_{c0} + p \text{Re} \left[ \frac{\delta\Sigma^2 (\Sigma_{c0} - \delta\Sigma)}{(\delta\Sigma^2) - 2(\delta\Sigma - \Sigma_{c0})^2} \right] \right].
\]  
(S30)

\[ f-\text{ CPA SELF-ENERGY IN } p \to 1 \]

In the concentrated \( (p \to 0) \) limit, the CPA selfenergies must coincide with the local self-energy. Thus the coefficient of the \( w^2 \) term in the \( p \to 0 \) limit arises purely through \( \text{Im} \Sigma_f(\omega) \) and hence must be negative. While in the other extreme \( (p \to 1) \), the CPA self-energy for f-electron is given by

\[
\omega - \epsilon_f - \Sigma_{f}^{CPA} = \frac{1}{1 - p} (\omega - \epsilon_f - \Sigma_f) - \frac{pV^2}{1 - p} g_0(\omega) \]  
(S31)

where \( g_0(\omega) = \frac{1}{\omega + \epsilon_c - S(\omega)} = H(\omega^+ - \epsilon_c) \) and \( H(z) \) is Hilbert transform of \( z \) with respect to the non-interacting density of state \( \rho_0(\epsilon) \). Thus the CPA self-energy for c-electrons will be

\[
\Sigma_{c}^{CPA} = \frac{V^2(1-p)}{\omega^+ - \epsilon_f - \Sigma_f(\omega) - p\Delta_0}, \]  
(S32)

where \( \Delta(\omega) = V^2g_0(\omega) \). Expanding \( \Sigma_f(\omega) \), we can write

\[
\Sigma_{c}^{CPA} \simeq \frac{V^2(1-p)}{\omega^+ - \epsilon_f^* - p\Delta_0}, \]  
(S33)

where \( \epsilon_f^* = \epsilon_f + \Sigma_f(0) + p\Delta_R(0) \) and \( \Delta_0 = \pi V^2\rho_0(\omega - \epsilon_c) \).

Using a Taylor expansion of \( \Sigma_{c}^{CPA} \) from equation \( S33 \), static, linear and quadratic terms may be easily found.

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**NFL BEHAVIOUR IN CPA SELF-ENERGY**

FIG. S1. (color online) Top panel: The imaginary part of the CPA self-energy as a function of scaled frequency \( \omega' = \frac{\omega}{\omega_L} \) has been shown with solid line for different \( p \) values. The dashed line is the quadratic polynomial fit up to a cutoff, \( \omega_c \sim 0.1\omega_L \). Bottom panel: Solid line represents the absolute value of the imaginary part of the CPA self-energy with \( \omega = 0 \) part subtracted for different \( p \) and \( \omega > 0 \). The dashed line is a power law fit \( D\omega^\alpha \) for respective \( p \). The model parameters for this calculation are \( U = 5.30; V^2 = 0.6 \) and the occupancies are \( n_f \simeq 0.98 \) and \( n_c \simeq 0.57 \).