Chapter 3

QUANTUM MECHANICS, QUANTUM GRAVITY, AND APPROXIMATE LORENTZ INVARIANCE FROM A CLASSICAL PHASE-BOUNDARY UNIVERSE

Michael Grady
Department of Physics, SUNY Fredonia, Fredonia NY 14063 USA

Abstract

A classical dynamical system in a four-dimensional Euclidean space with universal time is considered. The space is hypothesized to be originally occupied by a uniform substance, pictured as a liquid, which at some time became supercooled. Our universe began as a nucleation event initiating a liquid to solid transition. The universe we inhabit and are directly aware of consists of only the three-dimensional expanding phase boundary - a crystalline surface. Random energy transfers to the boundary from thermal fluctuations in the adjacent bulk phases are interpreted by us as quantum fluctuations, and give a physical realization to the stochastic quantization technique. Fermionic matter is modeled as screw dislocations; gauge bosons as surface acoustic waves. Minkowski space emerges dynamically through redefining local time to be proportional to the spatial coordinate perpendicular to the boundary. Lorentz invariance is only approximate, and the photon spectrum (now a phonon spectrum) has a maximum energy. Other features include a geometrical quantum gravitational theory based on elasticity theory, and a simple explanation of the quantum measurement process as a spontaneous symmetry breaking. Present, past and future are physically distinct regions, the present being a unique surface where our universe is being continually constructed.

1 Introduction

In the following, a new picture of the big bang and the underlying structure of the universe is proposed, based on a classical field theory in a four-dimensional Euclidean space with a
universal time (a 4+1 dimensional theory). The big bang is treated as a nucleation event for a first-order phase transition (pictured as a liquid to solid transition) and our universe is the three-dimensional phase boundary between the expanding solid and preexisting liquid phases. This classical brane-theory appears to have the potential to explain a diverse set of phenomena – Lorentz invariance, quantum fluctuations and zero-point energy, quantum superposition and measurement, elementary fermions and bosons, gauge forces, gravity, the big bang and a non-decelerating expansion of the universe. It is possibly rich enough to give a “theory of everything” from a relatively simple base-theory consisting of a small number of elementary atoms or molecules and basic elastic forces holding them together. In this model, all of the forces and particles of standard particle theory are secondary effects, consisting of the collective excitations and dislocations of the base-theory, just as in condensed matter physics where such excitations play a pivotal role, reducing the elementary degrees of freedom to a mere background for the more interesting and important collective excitations.

We begin by assuming a four-dimensional Euclidean space, filled with a uniform fluid at some temperature, undergoing thermal fluctuations. In addition to the four spatial dimensions, there is also a universal time. Another possibility would be to start already with a five-dimensional Minkowski space, however this does not seem to be necessary. This liquid was cooling, became supercooled, and at some point a solid crystal nucleated. This was the big bang. The universe begins as a fluctuation, already at a finite size, because in order to grow rather than shrink, the initial crystal must be large enough that the positive surface energy is less than the negative volume energy relative to the liquid. In such a model there is no physical singularity at the beginning and there is no reason for the universe to be particularly hot or dense at this time either (more on this later). The surface of the solid, the phase boundary, is an expanding three-dimensional space, our universe. This differs from other “bubble universe” pictures, where the universe is the interior of a 3-d bubble. In fact, it bears an uncanny resemblance to the simple “expanding balloon” model which is often used as an example of a uniformly expanding curved space. However the present model differs in that the interior and exterior of the balloon are real spaces, though not directly observed by us. We are directly aware only of the phase boundary separating the phases, which we refer to as the “present”. As the crystal grows, this hypersurface, our universe, expands. Already there is a variance with the usual Λ = 0 Friedmann universes. Namely, our universe is closed, but will expand forever. The pressure on the surface caused by the energy difference of the two phases acts something like a repulsive cosmological constant. This universe actually expands faster as time goes on, not slower. If, as is likely, dissipation is present, it will eventually approach a constant rate. (This assumes a constant amount of supercooling – if the base liquid cools more, the expansion rate could continue to increase as the degree of supercooling increases. Without dissipation, the expansion rate increases exponentially). Recent astrophysical evidence shows that the expansion rate is not slowing, but may even be speeding up[5] which is consistent with this scenario.

In the following, the emergence of a quantum field theory on the surface and the origin of quantum fluctuations is discussed in section two. The relation between real and
imaginary time path integrals is clarified as a difference between non-equilibrium and equilibrium statistical mechanics. Section three deals with the dynamical realization of Lorentz invariance and special relativity, including possible tests of the theory, and consequences for cosmic ray physics. In Section four, the description of photons as surface acoustic waves is explored. The Planck relation, \( E = h\nu \), and zero point energy are derived, with Planck’s constant being essentially the four-dimensional temperature. Section five describes the interpretation and realization of quantum superpositions and quantum measurements. Section six discusses four-dimensional dislocations as candidates for elementary fermions. The possibility of modeling quarks as partial dislocations, which, in ordinary crystals are naturally confined, is explored. Section seven outlines the likely gravitational theory that results from the relationship between the curvature of the surface and the presence of dislocations and interstitials, following previous analogies drawn by many authors between elasticity theory and general relativity. Modeling fermions as screw dislocations introduces a natural relation between spin and torsion, as in the Einstein-Cartan theory, which may be a good continuum approximation to the underlying lattice theory. Whatever gravitational theory that results is automatically a quantum theory of gravity since the 4-D thermal fluctuations are present in the surface. Section eight discusses the cosmology of the model, including possible difficulties in fitting observations. Section nine discusses the rather different nature of time that the model presents and relates it to Whitehead’s conception of time. The different causality structure due to the model having a preferred frame is discussed (special relativity is only approximately realized).

2 Quantum Field Theory from a Classical Field Theory

The basic theory needed to describe this expanding phase boundary is non-equilibrium classical statistical mechanics. The boundary itself may be describable in terms of dynamical critical phenomena. The solid, in some sense, lies in the past, since we have been there earlier, although it still exists in the present when observed from the higher dimension. The liquid represents the future, since that is where we are going, but it also exists now, as an undifferentiated, fluctuating medium. To distinguish the current states of the solid and the liquid from our own past and future, they may be called the “current past” and “current future”. They differ from our past and future because changes may have occurred after the solid was formed, and the future certainly will be different when we arrive there. To the extent that the solid is frozen, however, our past may be accurately preserved within it. We may not be aware of the existence of the liquid due to its uniformity. However, the boundary which we inhabit is in thermal contact with both the liquid and solid phases, and can certainly exchange energy with them. Actually, since the surface is continuously colonizing new parts of the liquid, the mountain, in this case, is moving to Mohammed. Energy fluctuations that were present in the adjacent liquid will be incorporated into the “new surface” an instant later. These will interact with propagating surface modes which are passed from the “old surface” to the “new surface” as each layer is added. Thus waves riding the interface will experience random energy fluctuations from this thermal contact. These random 4-d thermal fluctuations could explain quantum fluctuations.
It is well known that in ordinary quantum theory, if Minkowski space is analytically continued to Euclidean space, quantum fluctuations behave as higher-dimensional thermal fluctuations, i.e. the Feynman path integral becomes an ordinary statistical mechanical partition function in 4 (+1) dimensions (in equilibrium statistical mechanics there is an implied time dimension). Planck’s constant is proportional to the temperature of the four-dimensional Euclidean space. The existence of a 1-1 mapping between quantum field theory and statistical mechanics in one more dimension opens the possibility that the physical reality that quantum theories are describing actually corresponds to a higher dimensional classical theory, one for which, if all degrees of freedom were accounted for, would constitute a dynamical system of some kind. Aside from the important new feature of an extra dimension, this is essentially the point of view of Nelson[7], whose stochastic quantization technique attempts to explain the fluctuations of quantum mechanics through interaction with an otherwise unobservable fluctuating background field. Stochastic quantization was extended to field theory by Parisi and Wu[8], who showed the equivalence of the Euclidean path integral to a stochastic process controlled by a Langevin equation, which operated in a fictitious new time, completely unrelated to ordinary time. Whereas this can be seen as simply a mathematical tool, some have speculated that the reformulation could be closer to reality. Of course, to the extent that mathematical formulations are equivalent, it does not really matter to the physicist which is “more real”, however if our current theories are only approximations, then such considerations make sense in trying to find a more correct and accurate theory. If a stochastic differential equation explains quantum fluctuations, then this would likely be the case, since in most cases one can picture such equations as approximations resulting from more detailed deterministic dynamical systems for which some degrees of freedom have been averaged over.

The main problem in making sense of this connection between quantum systems and classical systems in one higher dimension is the analytic continuation to imaginary time, and the lack of any apparent connection between the “Langevin time” of a Langevin simulation and real time. However, if one considers the behavior of fields that live on an expanding phase boundary in a 4-d Euclidean space, such a connection can be made. If one accepts the Langevin time itself as real time, then there will be a connection between it and the fourth spatial coordinate at the surface (the coordinate perpendicular to the surface), due to the motion of the surface. For the sake of simplicity it will be assumed to travel at constant speed. For observers riding the surface, the fourth spatial coordinate will be nearly indistinguishable from time, since they increase in lockstep. In a following section it will be argued that this identification leads to a “spatialization” of time from which all of the properties of special relativity arise - in particular it will be seen that clocks constructed from dislocations and surface modes do not keep universal time, but rather the local time of special relativity.

The remaining question is why quantum field theory is given in terms of a real-time path integral with an oscillating exponential rather than the imaginary-time version with a real exponential. It is perhaps not a question of real or imaginary time which is a mathematical transformation with no apparent physical basis, but the rather less exotic notion
of real vs. imaginary frequency describing oscillatory vs. overdamped motions. This can also be seen as the difference between non-equilibrium and equilibrium statistical mechanics. If the universe were a single phase in equilibrium then it could be described by an equilibrium statistical mechanical ensemble. Correlation functions would be decaying real exponentials. The corresponding Langevin equation would be dominated by dissipative forces and the corresponding path integral would be Euclidean (i.e. the imaginary time version). However, an expanding phase boundary is a decidedly non-equilibrium object. It breaks time translation invariance and at least one spatial translational invariance. One may also have propagating modes present on the surface, due to conservation laws. Such propagating modes exhibit oscillatory rather than dissipative behavior, and occur in many 3-d systems [6]. They lead to various complications in the theory of dynamical critical phenomena, and are a crucial feature in the dynamical theory of phase transitions. In many cases these systems are still describable by a stochastic differential equation - a complex Langevin equation, where non-dissipative forces play a crucial role [6, 12]. Solutions are oscillating but contain random phase and amplitude fluctuations. The Fourier transforms of correlation functions contain real-axis poles.

A number of authors have shown that the Parisi-Wu stochastic quantization can be performed directly in Minkowski space, the result being a complex Langevin equation which will be exhibited shortly [9, 10]. This completes the logical connection. To sum, fields which represent dislocations or collective modes on a moving phase boundary in a 4-d Euclidean space are likely describable by a complex Langevin equation, which approximates the behavior of the larger deterministic dynamical system which fills the entire Euclidean space. This complex Langevin equation has an equivalent path-integral representation (meaning the two systems have the same correlation functions), which resembles the Minkowski space path integral of quantum field theory. Some details will likely be different, however. For instance, it does not seem likely that dissipation will be entirely absent from the surface. This could be countered by an energy input, resulting in a steady-state rather than an isolated system. Such a system lacks time-reversal invariance at some level, which could have observable consequences (and perhaps help to explain CP non-conservation in the $K^0 - \bar{K}^0$ system).

The Langevin equation is a first-order differential equation with a fluctuating random force. It was first applied to the case of Brownian motion of a small particle in a background of randomly moving molecules colliding with it. If $v$ represents the velocity of the particle, then the Langevin equation is

$$\dot{v} = -\gamma v + F + \eta(t)$$

(1)

The $\gamma v$ term is the frictional force exerted by the fluid, $F$ is an applied external force (if present) such as an electric field, and $\eta(t)$ is the fluctuating force designed to mimic the many collisions between the fluid which is assumed to be in thermodynamic equilibrium at some temperature and the particle. In the absence of force $F$, the particle exhibits a random walk in position. Without the damping term it would also perform a random walk in velocity, and the kinetic energy would increase without bound. However, any amount of dissipation is sufficient to stabilize it and the particle’s average kinetic energy will become
equal to $\frac{1}{2}kTd$, where $d$ is the number of spatial dimensions, $T$ is the fluid temperature, and $k$ is Boltzmann’s constant. If one wants to extend this treatment to an oscillator, a problem arises in that a position dependent force cannot be incorporated into a first order equation. The Hamilton equations are, of course, first order, but there are two of them. By introducing a complex variable $b = (p + ix)/\sqrt{2}$, $b^* = (p - ix)/\sqrt{2}$, one can write the Hamilton equations for one degree of freedom as a single complex equation:

$$\dot{b} = i\frac{\partial H}{\partial b^*}. \quad (2)$$

To explore these ideas in more detail, consider the system of a single harmonic oscillator interacting with a bath of other harmonic oscillators[12]. The simple harmonic oscillator in coordinates $x = \sqrt{m\omega}x'$, $p = p'/\sqrt{m\omega}$, where $x'$ and $p'$ are the usual coordinate and momentum has the Hamiltonian

$$H = \omega b^* b \quad (3)$$

Here, $k$ is the spring constant and $\omega \equiv \sqrt{k/m}$. The Hamilton equation (2) becomes

$$\dot{b} = i\omega b. \quad (4)$$

Interestingly, this formalism can be easily extended to the damped oscillator[12, 13] by allowing $\omega$ to become complex. Replacing $\omega$ with $\omega + i\gamma$ gives the equation of motion for the damped oscillator,

$$\dot{b} = i\omega b - \gamma b. \quad (5)$$

Here, the complex formalism goes beyond the real formalism, since the Hamiltonian does not technically exist for the damped oscillator unless auxiliary fields are added[13]. Note that this is not a fully complex Hamiltonian function which would result in doubling the number of equations of motion and producing an overdetermined system. Rather, the Hamiltonian takes values along a ray other than the real axis. If we add a fluctuating force, one obtains a complex Langevin equation,

$$\dot{b} = i\omega b - \gamma b + \eta(t). \quad (6)$$

This equation can be derived as the equation of motion of a tagged oscillator interacting with a collection of “bath” oscillators whose behavior is averaged over[12]. The bath provides both the random force and the damping. It can be used, for instance, to describe the behavior of a single-mode laser interacting with a thermal medium and thermal mirror fluctuations[12, 14]. Similarly, it can also be used to describe propagating modes in dynamical critical phenomena[6]. Thus the complex Langevin equation is a well-established equation for describing oscillating or propagating modes in a random medium.

If the Parisi-Wu quantization is applied to the Minkowski field theory directly, it has been shown[9, 10] that the correlation functions derived from the path integral

$$\int D\phi \exp(iS(\phi)/\hbar) \quad (7)$$
can be obtained from the long-time behavior of the Langevin equation
\[ \dot{\phi} = i\delta S/\delta \phi^* - \epsilon \phi + \eta(x, t) \] (8)
where \( t \) is a fictitious “Langevin time” unrelated to the real time in the path integral, \( x \) represents the four space-time variables, \( x_i \), with \( i = 1..4 \), and the Gaussian noise term has the following correlation function:
\[ < \eta^*(x, t) \eta(x', t') > = 2\hbar \delta^4(x - x') \delta(t - t'). \] (9)

Field correlations are computed at equal Langevin times and the damping, \( \epsilon \), is taken to zero after correlation functions are calculated. Equation (8) appears to be a multivariate version of equation (6) (the first term being generalized to the RHS of equation (2) with the Minkowski action \( S \) playing the role of a Hamiltonian. For example, for the complex scalar field,
\[ S = \int (|\partial_\mu \phi|^2 - m^2 \phi^* \phi) d^4x \] (10)
one obtains the complex Langevin equation
\[ \dot{\phi} = i(-\partial^2 \phi/\partial x_4^2 + \nabla^2 \phi - m^2 \phi) - \epsilon \phi + \eta(t) \] (11)

One can understand the difference in sign between the spatial and local-temporal \((x_4)\) derivative terms in relation to the different dynamic behavior of the interface in these directions. If one thinks of \( \phi \) as a displacement field of elementary atoms from their quiescent-crystal locations, one expects oscillatory behavior in the spatial directions. The sign of the \( \nabla^2 \) term is such as to provide the usual restoring force from neighboring atoms making this possible. A negative restoring force, as exists in the time direction, leads to an instability, as occurs in a \( \phi^4 \) theory with a negative mass-squared term, for example. This will result in translational motion (a soft mode). If we think of the membrane as the physically relevant object, it is in translational motion in the temporal direction. Therefore the “Minkowski signature” of the D’Alembertian operator would appear to be directly related to the dynamics of the phase boundary, which is itself, of course, controlled by the Lagrangian of the “base-theory” of the elementary atoms. The fact that the instability that resulted in the motion of the phase boundary is a phase transition of the base-theory, which is likely driven by a spontaneous symmetry breaking, suggests that the Minkowski space we are familiar with is due to a spontaneous symmetry breaking from original space-time symmetry of the base-theory. Such a dynamical origin for Minkowski space, and the consequences of special relativity that result, is in rather distinct contrast to the kinematical origin postulated by Einstein. Indeed it is more like the view held by Lorentz and others who clung to the idea of a cosmic ether, even if invisible. The crystal and liquid in the picture presented here is a form of ether, which, however, is only invisible at low energies. When photon wavelengths get close to the elementary lattice spacings, then the deviation from linearity of their phonon-like dispersion relations in this theory will become apparent, and the existence of the crystal will have observable effects. These ideas are expanded in secs. III & IV.
Getting back to the Langevin equation under discussion, we now consider the consequences of our somewhat different interpretation of the Langevin time coordinate. The usual treatment calculates correlation functions at equal Langevin times, whereas we are essentially locking the Langevin time to the ordinary time through the presumed uniform motion of the phase boundary. It is important to see whether this will make any difference in the relationship to the quantum field theory. One notices a peculiarity in equation 11 when subjected to dimensional analysis. Taking the usual dimension of \[\ell^{-1}\] for the \(\phi\) field and \[\ell\] for \(x_i\) and \(t\) variables leads to different dimensions for the \(\dot{\phi}\) and \(\Box \phi\) terms. One common solution is to let the fictitious time have dimensions \[\ell^2\] \[9, 11\]. Then dimensional consistency is obtained and \(\bar{\hbar}\) comes out dimensionless. However, since we want the fictitious time to become the real time, another solution must be taken. Introducing a parameter \(a\) with dimensions of length, which can be taken to be the lattice spacing, rewrite equation 11 as

\[
\dot{\phi} = ia (-\partial^2 \phi / \partial x^2 + \nabla^2 \phi - m^2 \phi) - a\epsilon \phi + \eta(x, t)
\]

(12)

where

\[
< \eta^*(x, t) \eta(x', t') > = 2a\hbar \delta^4(x - x') \delta(t - t').
\]

(13)

The two times now have the same dimensionality, the equation is dimensionally consistent and the two factors of \(a\), one multiplying \(S\) and one multiplying \(\bar{\hbar}\) will cancel in the path integral (\(\hbar\) will now be set to unity). For the free field theory we are considering here, the Langevin equation can be solved \[8, 9, 11\], with a long time stationary correlation function

\[
D(x - x', t - t') \equiv \lim_{t, t' \to \infty} < \phi^*(x, t) \phi(x', t') >
\]

(14)

(with \(t - t'\) fixed) given by

\[
D(x - x', t - t') = \frac{2a}{(2\pi)^5} \int d^4k \int d\omega \frac{e^{-ik(x-x')-i\omega(t-t')}}{\omega^2 + a^2(k^2 - m^2 + i\epsilon)^2}.
\]

(15)

Setting \(t - t' = x_4 - x'_4\), we get a free propagator of

\[
D(x - x') = \frac{1}{(2\pi)^4} \int d^4k \frac{e^{-ik(x-x')}e^{-a[(k^2-m^2)(x_4-x'_4)]}}{k^2 - m^2 + i\epsilon}.
\]

(16)

This is slightly modified from the usual field-theory propagator which results from taking equal Langevin times, \(t = t'\). However, the extra exponential factor affects only the off mass-shell propagator, and even for that is highly suppressed by the factor of the lattice spacing, a reasonable guess for which might be around \(10^{-16}\) \((\text{eV})^{-1}\). It thus seems unlikely that this extra factor would affect calculations at today’s accelerator energies. It breaks Lorentz invariance explicitly. As mentioned before, this theory is only approximately Lorentz invariant. Lorentz invariance is good at energies small compared to the inverse lattice spacing. From a more fundamental point of view, the rest frame of the crystal is a preferred frame and calculations should be performed in that frame. However, observable effects of this frame dependence are limited to very high energies. These are possibly accessible through studies of cosmic rays (see sec. III).
To sum, building on the known equivalence of the Minkowski path integral to a stochastic process involving a complex Langevin equation, it has been shown that ordinary quantum field theory may result from the dynamical critical behavior of an expanding phase boundary in a four-dimensional Euclidean space. In this picture, quantum fluctuations are actually thermal fluctuations in the higher dimensional space.

3 Special Relativity Realized Dynamically

The underlying theory pictured above is a classical dynamical system lying in the 4-D Euclidean space, governed by a universal Newtonian time. It is proposed that Minkowski space is the result of restricting attention to the hypersurface representing the phase boundary, and choosing local time to be the spatial coordinate perpendicular to the moving boundary. In calling it a Minkowski space, we are considering only a small portion of the surface which can be taken to be approximately flat. Globally, the spatial geometry is hyperspherical, and the space is a positively curved pseudo-Riemannian space similar to the positive-curvature case of the Robertson-Walker metric of standard General-Relativity-based big-bang cosmology.

To show the emergence of Minkowski space locally, a more detailed model is needed. If the phase boundary is considered the boundary between a liquid and crystalline solid, with the solid growing into the liquid, then a reasonable model for the photon is the surface acoustic wave, and for elementary fermions, screw dislocations in the crystal. The surface acoustic wave is a propagating solution within the surface that decays exponentially away from the surface. It obeys a phonon-like dispersion relation, with a speed somewhat below that of shear bulk waves. It is well known from the 1938 work of Frenkel and Kontorova [15] and of Frank and Eshelby in 1949 [16] that screw dislocations obey the Lorentz contraction formula with the speed of light replaced by the speed of transverse sound. In other words, the pattern of crystal distortion that surrounds the dislocation becomes elliptical for a moving dislocation, with the strain pattern in the direction of motion shrinking according to the Lorentz contraction formula. An “object” made from an array of such dislocations really does shrink in the direction of motion. In addition, the effective mass of the dislocation grows with velocity according to the relativistic formula (more precisely the energy and crystal-momentum transform according to the Lorentz transformation)[16][17][18]. Therefore, screw dislocations are prohibited from being accelerated beyond the velocity of transverse sound in a crystal, because the kinetic energy becomes infinite in that limit. In a real crystal, however, this limit can be exceeded by introducing a moving dislocation from an adjacent compatible medium where the sound speed is higher. The supersonic dislocation rapidly decelerates to subsonic velocities by emitting “vacuum Cerenkov radiation” [17][18]. It is also conceivable to exceed the limit by violating the approximations of continuum linear elasticity theory on which these results are based. This relativistic behavior appears to be followed for any reasonable dislocation model for which perturbations are subject to continuum linear elasticity theory[18]. It is not immediately clear what the minimum requirements are[19], but coupling to a single type of phonon with a relativistic
dispersion relation is necessary, and may be sufficient. Coupling to other types of phonons is possible only if these either have the same velocity or have an energy gap. The main point here is there can not be more than one limiting velocity for low-energy excitations. For instance, ordinary edge dislocations obey a more complicated set of contraction equations involving both the longitudinal and transverse sound velocities\[16, 17\].

Considering again the phase boundary universe model, if all matter is made up of screw dislocations then the above considerations strongly suggest that measuring rods constructed from “dislocation arrays” will obey the Lorentz contraction. For now, consider only observations made from the rest frame of the crystal. A measuring rod will physically shrink if moving with respect to this frame along the rod’s direction. The Lorentz transformation also involves time, however. The Lorentz contraction and mass increase certainly will have physical effects on clocks that are constructed from moving dislocations. Günther \[20\] has investigated using the breather solution of the sine-Gordon equation as a clock (sine-Gordon soliton kinks are a lower-dimensional dislocation model). He finds such a clock slows with velocity in accordance with the usual time-dilation formula. If length and time standards are both based on solitons, full Lorentz invariance ensues.

For our case, assuming only length contraction and observing from the crystal rest frame, a simple light-clock where a flash of light is given off and bounces off a mirror held by a rigid frame to the light source, then back to a detector near the source, in either transverse or longitudinal orientations, exhibits time dilation following the usual treatment in special relativity. However, although the argument is the same, the assumptions are different. At this point we have not assumed anything about moving frames of reference. We are simply observing a moving rod and a moving clock from the rest frame of the crystal, where we know the speed of sound (light), and know it is isotropic (we are always assuming an isotropic crystal). This is all that is needed to demonstrate time dilation from Lorentz contraction of the light-clock. We notice that when observed from this frame, rods shrink, and clocks slow down due to physical, dynamical effects. Energy and crystal-momentum of dislocations also obey relativistic equations\[16, 17, 18\].

Now we ask what coordinate system is a reasonable one for a moving observer to use? Of course, the moving observer will use the shrunken rod to measure distance and the slow clock to measure time - what other reasonable choice does s/he have? It is also natural for moving observers to choose their local time coordinate to be along their own world line, and spatial hypersurfaces to consist of points all with the same time coordinate, with synchronization performed using light signals. The full forward Lorentz transformation, which consists not only of scale changes inherent in Lorentz contraction and time dilation, but also in the aforementioned axis rotations, ensues. This now allows us to transform coordinates between the crystal rest frame, and the natural frame of a moving observer. Inverting this transformation is simply a matter of mathematics. As is well known but seems to have been initially unappreciated by Lorentz, this inverse Lorentz transformation has the same form as the forward transformation, with the relative frame velocity negated. The point is that once the full forward Lorentz transformation is realized, fully reciprocal special relativity results simply due to the mathematics of the Lorentz transformation. In Einstein’s special
relativity, this is due to the symmetry of the underlying Minkowski space - a kinematical symmetry. All frames are exactly equivalent. Although our result is the same, conceptually it is very different, since the Minkowski space has resulted from a dynamical symmetry of the moving boundary solution. Unlike in the Einstein picture, the Lorentz contraction and time dilation have different causes in different frames in this picture. From the crystal rest frame, the shrinking of a moving rod and slowing of a clock are physical effects, caused by motion within the stationary crystal. From the moving frame, the observation that a rod and clock in the crystal rest frame also appear to be shortened and slowed are more of an illusion, created by using moving instruments, and a bent reference frame, with its different notion of simultaneity. Because these points of view are conceptually different (kinematic vs. dynamic symmetry), Lorentz, Larmor, Langevin and others held on to the latter view for some time after special relativity won acceptance\[21\]. In fact, the view of relativity given above is very similar to that of Lorentz, who introduced the concept of local time given above. The unobservability of the ether in this continuum theory eventually led to the demise of this viewpoint. However, if the underlying medium is not a continuum, but a lattice (which itself may lie in a continuum), then at high enough energies differences between the stationary and moving observer must eventually show up. This is because unlike the photon, the phonon dispersion relation is not a straight line. For a linear isotropic material in three dimensions it is given by

\[
\omega^2(k) = (2c/a)^2\left(\sum_{i=1}^{3} \sin^2(k_i a/2)\right).
\] (17)

Surface phonons follow a similar dispersion relation. One has to get to within about 20% of the maximum frequency before the phonon curve differs from the photon curve by more than 1%. Above this point significant dispersion occurs. A very fast-moving light clock which blue-shifted the light into this frequency region would show measurable deviations. The crystal rest frame will be the only frame in which the speed of light at these high frequencies remains isotropic. It therefore becomes an observable preferred frame - the ether is detectable.

These considerations suggest a number of ways that this theory could be checked experimentally. Of course, the lattice spacing could always be made impossibly small, erasing all observable effects. Observations of very-high-energy cosmic rays can put a lower bound on the lattice spacing. Assuming the Plank relation \( E = \hbar \omega \) (a possible origin of which is given in the next section) and setting \( \hbar = 1 \), the definite identification of cosmic ray photons at energies of a few times 10^{15} eV \[22\] means the high-energy cutoff of the photon dispersion relation must lie above this, so probably \( a < 10^{-14} \) (eV)^{-1}. An interesting enigma in Cosmic Ray physics is the presence of an “ankle” in the spectrum around 10^{18} eV, where the drop in intensity with energy becomes less steep, along with the apparent absence of the expected cutoff due to interactions with the cosmic microwave background radiation (CMB)\[23\]. This Greisen-Zatsepin-Kuzmin (GZK) cutoff \[24\] is due to pion photo-production from interactions between the cosmic ray particle (assumed a proton or light nucleus) and CMB photons. This effectively limits cosmic rays of energy above
$5 \times 10^{19}$ eV to a relatively short travel distance - within the local supercluster (photons and heavy nuclei are also limited by similar mechanisms involving starlight). However, the number of cosmic rays at this energy and higher, although small in absolute event counts, does not show any diminution from the earlier trend. In other words, there is no observational evidence for the GZK cutoff. Another puzzle is that if the very high energy cosmic rays do come from nearby sources, then, they would be expected to point to within a few degrees of their sources, despite the deflection of magnetic fields, due to the high momentum of the particles. However, there appears to be no correlation with possible nearby sources. A photon energy cutoff in the range $10^{16}$ to $10^{19}$ eV could invalidate the Lorentz transformation which is used to derive the GZK cutoff from the known behavior in the center-of-mass frame\cite{25}. It would also affect the decays of other high-energy particles, such as neutral pions. A $10^{20}$ eV neutral pion could not decay into two photons, but at minimum into 10,000 photons for a photon energy cutoff of $10^{16}$ eV. This would be highly suppressed due to the large power of the fine structure constant required. If the weak bosons had similar cutoffs, then it seems the neutral pion could be made almost stable above a certain energy. Decay of the neutron could be similarly suppressed. If one or more of these neutral particles could travel cosmological distances above a certain energy threshold, it could possibly explain the ankle, due to the addition of a new species to the particle mix. High-energy neutral particles should point toward their sources even at great distances, since they are not much affected by magnetic fields (there is still some effect through magnetic moments). Interestingly, Farrar and Biermann have shown that the observed directions of some of the highest energy events can be correlated with distant quasars\cite{26}. This would be consistent with the scenario suggested here.

4 Photons as Surface Acoustic Waves

Generically, surface acoustic waves (SAW’s) traveling in the $x$-direction on a solid surface at $z = 0$, with the solid occupying the half-space $z < 0$ takes the form\cite{27,28}

$$u_j = u_{0j}e^{i(kx - \omega t) + \kappa z}$$

(18)

with $k$, $\omega$, and $\kappa$ real, and $\kappa \propto k$ at least for small $k$. Here $u_j$ is the elastic displacement field for the solid, which is defined only for $z \leq 0$. Typically most of the energy in surface waves is confined to a region within a few wavelengths of the surface. The most common SAW is the Rayleigh wave, first described by Lord Rayleigh in 1885\cite{27,28}. It is polarized in the sagittal plane (perpendicular to the surface), and consists of motion that is both transverse and longitudinal. It is dispersionless in the continuum version and has a typical phonon dispersion law on the lattice\cite{29}. This wave does not seem to be a promising one to model the photon after, however, since it has only one polarization, regardless of the dimensionality of the surface. The Rayleigh wave is the only type of surface wave for the simplest case of a flat linear elastic half-space. However, if the surface is allowed to have properties different from the bulk, such as a different density, elastic constant, surface tension, curvature, roughness, piezoelectricity, magnetoelasticity, etc. then another surface wave will usually
exist, a Love wave\cite{30,31,32,33}. This wave, originally derived for a finite slab of different material deposited on the half-space\cite{34}, has a shear-horizontal (SH) polarization, thus for a three dimensional surface would have two transverse polarizations. The Love wave also exists for a thin surface layer such as a thermodynamic phase boundary\cite{31,35,36}. It can be seen as a perturbation of the SH surface skimming bulk wave (SSBW) that exists even for the simple half-space. The SSBW is a wave that does not decay below the surface. This solution is unstable with respect to virtually any surface property that retards the wave speed near the surface, which will turn it into an SH surface wave with exponential decay away from the surface, i.e. a Love wave\cite{30,31}. The Love wave is somewhat dispersive, due to the introduction of a quantity with dimensions of length that characterizes the surface-layer thickness. However if this is no more than a few lattice spacings, then the dispersion is similar to that of an ordinary phonon. Adding a liquid to the external space complicates but does not significantly change the situation. However, the case being envisioned here has a more complicated boundary condition than has been considered in the surface-wave literature, since the boundary is growing, perhaps rapidly. This can perhaps be treated by the method of virtual power\cite{37}, and is briefly considered by Maugin\cite{31} and also by Kosevich and Tutov\cite{36}. The transfer of momentum to the “new surface” of the growing crystal will modify the usual traction-free boundary condition of the Raleigh-wave solution. It is this latter boundary condition which prevents the SH polarization from existing in the simple half-space\cite{28}. The violation of this boundary condition by the growing crystal is further evidence that SH waves probably do exist in this case.

Since the weak interactions also need to be accounted for, probably more structure needs to be incorporated into the model. If we imagine the elementary molecules to be non-spherical, then they have their own non-trivial symmetry group, compatible with but distinct from that of the crystal. This basis symmetry group could account for internal symmetries. For instance, if the molecule can be represented by a 4-d vector with nearest-neighbor Heisenberg-like interactions, then an SO(4) symmetry (isomorphic to SU(2) \times SU(2)), which may be partially broken by other interactions, will exist. The spontaneous breaking of this symmetry will result in surface magnons\cite{38}. These come in both acoustic and optical varieties. The surface magnons can also mix with surface elastic waves through the magneto-elastic effect, reminiscent of electroweak unification. These possibilities need to be examined in detail - they are mentioned here to indicate the rich possibilities for model building that occur in surface modes. It is also worth noting that a promising approach to incorporating chiral fermions on the lattice, necessary for a lattice approach to the weak interactions, incorporates a fourth spatial dimension, with the chiral fermions living on a domain wall\cite{2,39}. The picture of the universe presented here seems ideal for the realization of this mechanism.

A universal property of all surface modes is the exponential decay as the bulk is entered, characterized by a decay length proportional to the wavelength. This property can be used to derive the Plank relation $E = \hbar \omega$, perhaps the most fundamental equation of quantum mechanics, from the equipartition theorem. If we assume that all elementary degrees of freedom are thermally excited (actually not a completely good assumption due to
conservation laws and partial non-ergodicity - see discussion below), then the equipartition theorem will give equal energy to each harmonic degree of freedom of amount $kT$, where $k$ is Boltzmann’s constant and $T$ is the 4-d temperature. For a surface wave with decay length $\kappa = bk$, where $b$ is a constant, taking into account the energy of a wave being proportional to its square, one has an energy depth profile (1-d energy density)

$$E = E_0 e^{2\kappa x_4}$$  \hspace{1cm} (19)

where $x_4$ is taken to be zero at the surface, and becomes negative inside the medium. The energy in the monatomic surface layer itself is given by $E_0a$. The total energy can be computed by

$$E_{\text{tot}} = \int_{-\infty}^{0} Edx_4 = E_0/(2\kappa) \hspace{1cm} (20)$$

Setting $\kappa = bk$ (proportionality of decay length to wavelength), $\omega = ck$, and the total energy to $kT$, one can solve for the surface layer energy, $E_0a$, now denoted $E_3$

$$E_3 = (2bkT/c)\omega \hspace{1cm} (21)$$

which is the Plank relation if $\hbar = 2bkT/c$. This is consistent with the thermal explanation of quantum mechanics given above, namely that $\hbar$ is essentially the 4-d temperature, with the necessary factors of $a$ and $c$ to fix the dimensions. The essential feature which gives higher frequency excitations higher energy on the surface is the higher concentration of SAW energy near the surface, compared to lower frequency excitations which are more spread out in the fourth dimension. Thus equal sharing of energy in four dimensions naturally leads to unequal energies on the 3-d surface, as embodied in the Plank relation.

Not all surface modes will necessarily become excited for two reasons. First is the effect of global conservation laws, and second is the probable lack of full thermodynamic equilibrium. Consider the liquid degrees of freedom directly above the growing surface. These are presumably in thermal equilibrium in their liquid environment. When the surface arrives, they are rather suddenly thrust into a new environment with modified interactions due to the translational symmetry breaking of the crystallization. They therefore do not have much time to adjust to these new conditions by the time they can be considered part of the new crystal surface. Eventually they reach a new equilibrium state well after the surface has passed and they become part of the bulk. Thus the surface degrees of freedom are in a transitional state. With the arrival of crystalline order comes a new conserved quantity, the crystal momentum. It is a consequence of the remaining discrete translational invariance but technically is a permutation invariance of the atomic position variables, resulting in conservation of wave number for phonons\[40\]. Since this is only a single global constraint (or three constraints in three dimensions) it would not appear to limit the allowed random excitations much. However, satisfying global conservation laws requires global correlations, and these take a long time to establish. Therefore one expects to have to satisfy conservation laws locally. This means that one should not expect widely separated thermal excited phonons whose wave vectors happen to add to zero. Rather one expects standing
waves or standing wave packets where the zero net wave vector requirement is met pairwise and locally. Thus the random vibrational thermal energy of the liquid will re-organize on the surface primarily as such standing waves, with the amplitudes of constituent travelling waves locked, and phases randomly fluctuating. This may represent the zero-point energy of the photon field. Since each travelling wave mode is not independently excited, the net energy assigned to each is one-half that of the standing wave, i.e. $\frac{1}{2}\hbar\omega$.

However, if a travelling propagating wave already exists on the surface (perhaps excited by dislocation interactions etc.) then it will be preserved by the crystal momentum law, and, since it is now an allowed excitation can exist independently of the standing wave and be given the full equipartition energy of $\hbar\omega$, on top of what it gets from the zero-point excitation. One wonders how multiple photon excitations can arise in such a picture, which will lead to a discussion of resonances in coupled oscillator systems. Before embarking on that, it is worth noting that all of the discussion here concerning zero-point energies and photon excitations is in one sense unnecessary, since once the formal equivalence of the stochastic evolution of the phase boundary and the quantum system is accepted, one can merely plug in the QED or standard model Lagrangian and obtain the equivalent Langevin equation for the phase boundary evolution, which will, due to the above equivalence, necessarily include all of the known particle excitations and quantum effects. The discussion here is therefore not to prove that each feature of quantum mechanics is included, but rather to illustrate how each quantum feature might be manifested in the phase boundary evolution. In a similar sense, energy quantization is not as apparent in the path integral formulation of quantum mechanics as it is in the canonical formulation, but it has to be there, and can be seen from multiple poles of the propagator.

A collection of coupled oscillators, even if somewhat non-linear, is generally not ergodic. This was demonstrated by the famous computer simulation of Fermi, Pasta and Ulam in which they coupled 64 harmonic oscillators with non-linear couplings, expecting to see the approach to equilibrium\[41, 42, 43]. Instead they found that most modes remained unexcited with energy pouring back and forth between a few modes as in the Wilberforce pendulum - in other words a limit cycle as opposed to chaos. The only modes that participated were those that met or were very close to the resonance condition

$$\sum n_i \omega_i = 0$$

where the $n_i$ are integers\[42, 43]. This was later understood in terms of the KAM theorem (Kolmogorov, Arnol’d, Moser), which essentially states that for small non-linearities only regions near resonant surfaces in phase space will get occupied. Full chaos only ensues when these resonant regions grow large enough to be overlapping\[44]. Phenomena such as down-conversion or harmonic generation in the presence of small non-linearities can be understood in terms of the resonance condition. The n-photon state takes the form of an $n^{th}$ order resonance from this point of view. The integers in the resonance condition are the correspondents to energy quantization. The degree of excitation is consistent with that of a single $n^{th}$ harmonic photon with which the state is resonantly linked. Therefore it seems plausible that the Plank relation and full photon spectrum, including zero point energy, does
have a realization in the propagating modes of the phase boundary as it moves through the random medium.

5 Quantum Superposition and Measurement - Zitterbewegung and Spontaneous Symmetry Breaking

The picture described above treats quantum fluctuations as thermal fluctuations in the 4+1 dimensional space. In such a picture quantum tunneling is explained classically as thermal activation, i.e. due to a random kick of extra energy which results from thermal contact with the liquid and solid phases. Due to such thermal fluctuations, energy is not conserved over short time periods; it is conserved only in the average over time. Thermal fluctuations may create a kind of zitterbewegung – very rapid variation at small scales, that enforces the uncertainty principle and allows for superpositions. The ensemble average implied in a quantum expectation value is replaced by a time average. For rapid fluctuations which cover the ergodic subspace in times short compared to the time between measurements, these should yield identical results. Over very short time periods, additional correlations may appear in the time-averaged case, since the classical system is in a particular state at any one time, so subsequent states will retain some memory of previous states.

This more classical evolution affords the opportunity to explain the quantum measurement process as a spontaneous symmetry breaking event. Anderson has suggested that measuring devices incorporate spontaneous symmetry breaking in their operation [45]. Ne’eman has also espoused this viewpoint. In addition, he has shown that EPR type correlations can occur in classical systems with gauge symmetries, with the gauge connection enforcing long-distance correlations among fluctuating variables [46]. More detailed models have been considered in [47] and [48].

When a classical statistical mechanical system undergoes a spontaneous symmetry breaking, the ergodic phase space splits into non-communicating subspaces. From that point on, the system remains trapped in one of the subspaces. Which subspace is chosen is simply determined by the subspace the system happened to be in at the time of symmetry breaking. A measuring device is postulated to be any device that can couple its order parameter to a quantum system and that includes a control that can initiate spontaneous symmetry breaking of that order parameter. The measuring device, originally with an unbroken symmetry, couples to the system under study becoming strongly correlated with it. Then an adjustment is made to the potential of the measuring device which initiates spontaneous symmetry breaking. The measurement takes place at this time, when the ensemble of possible future states of the combined system splits into non-ergodic subensembles corresponding to the possible values of the order parameter, also corresponding to possible values of the measured quantity. Future evolution is confined to a single subensemble in the usual manner of a classical symmetry-breaking phase transition. In this picture measurements are well defined, the collapse is a physical event, and a clear distinction exists between what constitutes a measuring device and what does not. The symmetry breaking barrier does not even have to be infinitely high - all that is required is that the tunneling
time of the post measurement state to be long compared to the time scale of the experiment. This is in contrast to what occurs if the same concept of spontaneous symmetry breaking is applied to explain measurement in standard quantum mechanics. Here it is difficult to see how even spontaneous symmetry breaking can break the superposition, especially if measuring devices are finite so tunneling probabilities are not quite zero (e.g. ref. [47] still uses the Everett interpretation to deal with the “collapse”). Nevertheless it is assumed that when the universe undergoes a cosmological phase transition it does not end up in a superposition of the possible outcomes but rather “measures itself” so as to fall into a single vacuum. In the new picture given here, an event like this is in the same category as a measurement and the result of both is a physical collapse of the available future phase space.

Because the “current past” is continuously undergoing 4-d thermal fluctuations, it is only frozen to the extent that the ensemble is limited due to spontaneous symmetry breaking. Thus questions such as “which slit did the electron go through” or “which direction was the spin pointing” are as meaningless here as they are in standard quantum mechanics. This is because the details of history are continuously being rewritten as both the current past and present fluctuate. Only to the extent that the ensemble is limited by spontaneous symmetry breaking can one make definite statements about past events. EPR (Einstein-Podolsky-Rosen) states, which consist of two separated spins in a net spin-0 state, can only undergo correlated fluctuations which obey the global angular-momentum conservation law. The spin direction of each particle will fluctuate in such a way that its partner fluctuates oppositely. Measurement of either spin is performed by spontaneously breaking the spin direction symmetry, after which a barrier will exist preventing further fluctuations of either particle. Such a process was envisioned in [46]. Such non-local correlations may seem odd, but they are formed by the causal process of separating the particles, and do not violate causality (causality is arbitrated from the crystal rest frame, where universal and local time are equivalent).

### 6 Dislocations as Candidates for Elementary Fermions

Dislocations, particularly screw dislocations and their variants, provide a rich building ground for models of elementary particles. In this section a detailed model will not be attempted, but rather the general problem of extending the screw dislocation into four dimensions will be discussed, which will result in a four-dimensional string.

The idea of representing elementary particles as dislocations in a medium is a rather old one. Burton talked of “strain-figures” that could move through a medium and interact[49]. Although most 19th century physicists considered matter to be separate from the ether, Larmor suggested the possibility of matter particles being singularities in the ether itself and sought a unified theory of matter and radiation through the properties of a single medium[50][51]. In more recent times the modeling of elementary particles as topological solitons (a type of dislocation) has intrigued many, with the Skyrmion picture of the nucleon being perhaps the most successful.

The screw dislocation in three dimensions has a number of features that liken it to an
elementary fermion. The left and right handed versions can be pair-produced or annihilated, and their elastic interactions have a number of electromagnetic analogies, the most often cited being to magnetostatics\cite{18,52,53,54}. Although double dislocations are not totally prohibited, they are very unfavorable energetically. Screw dislocations are, of course, line defects, so cannot be compared directly to point particles. One is tempted to interpret the line defect as a world-line. However, this implies an extension to four dimensions. An isolated screw dislocation is unfortunately not an option in four dimensions. This can be seen in a number of ways. If one circles a screw dislocation in three dimensions, then one finds after a single loop that one has advanced one lattice spacing to the next sheet of atoms in the direction of the dislocation. The degree of non-closure of the loop is represented by the Burgers vector of the dislocation, which for a screw dislocation on a cubic lattice is one lattice spacing long and in the same direction as the dislocation, or opposite for an oppositely-handed dislocation. Although the transition is gradual, the point on the loop at which one can be deemed to be on the next level can be arbitrarily defined - the set of these points for all possible loops is called the Volterra surface. The freedom of choice of the Volterra surface can be thought of as a form of gauge invariance. There are many atoms far from the dislocation which have moved some fraction of a lattice spacing from their original lattice positions, but there is not much stress associated with this since all of the neighboring atoms have moved a similar amount. Stresses are concentrated only around the dislocation line. If one tries to embed this structure into a non-dislocated 4-d lattice, then those atoms far from the dislocation which are shifted from their original lattice positions by near 1/2 of a lattice spacing will fit badly the undislocated lattices adjacent to them in the fourth dimension, where the atoms are all at their original undislocated positions. The energy of such a structure is proportional to the four-volume - it is no longer a one-dimensional dislocation. The other way one can see there is something wrong in simply promoting the screw dislocation to four dimensions, is that a loop apparently surrounding the dislocation can be moved into the fourth dimension, where the dislocation does not exist, and shrunk to a point. Thus there is no longer a consistent topological classification of this object.

The screw dislocation can be extended into the fourth dimension by copying it onto each successive 3-lattice as the fourth coordinate is changed. This produces a wall of identical dislocations. The solution is translational invariant in the fourth dimension and involves no new stresses, since each atom is exactly one lattice spacing away from its neighbor in the fourth direction. However, we now have a domain wall in 4-d or line in each 3-d slice. For an elementary particle we want something closer to a point in 3-d. An obvious solution would be to wrap the domain wall around onto itself into a small tube, the 3-d cross-section of which would be a string. The bending of the wall introduces stresses which favor a larger string, but this is opposed to the ordinary screw stress proportional to the string length, so there is the possibility of a stable equilibrium size. Going back to the 3-d cross-section, the Volterra surface is any surface bounded by the string. A loop that threads the string will pass through the Volterra surface and detect the dislocation.

Assuming a planar loop in the 3-d cross-section introduces a direction, the spatial normal to this plane (the temporal direction is also normal). This suggests the possible inter-
interpretation of a spin direction. Another strong possibility is that the constituent screw dislocations are not straight but form spiral helices. Ordinary screw dislocations often take helical form through a process that involves absorption of interstitials or vacancies[18,53,54]. The plane of the helix introduces another spatial direction which could be related to spin or spin precession. Interstitials are important in that they introduce curvature into the crystal[55]. Such curvature is absolutely necessary to produce the large-scale hyper-spherical spatial geometry inherent in the cosmological scenario outlined above. It also allows a connection between particle properties and gravitation.

One additional property of crystal dislocations that may provide an intriguing parallel to the strong interactions is the existence of partial dislocations[18,53]. Under favorable conditions, a dislocation may split into two or more partial dislocations with fractional Burgers vectors. Such objects cannot exist in isolation since they would involve dislocating the entire lattice - resulting in infinite energy. These partial dislocations are linked by a sheet containing a stacking fault, which produces an attractive force proportional to the sheet area (the partial dislocations also repel each other through other elastic forces, resulting in an equilibrium separation). The possible analogy between quarks and partial dislocations, with gluons being related to the associated stacking faults is compelling. Confinement and fractional charge are inherent and linked properties of these configurations. Another common feature of dislocations in ordinary crystals is the formation of dislocation networks. These are ordered or disordered collections of either partial or full dislocations and anti-dislocations, with zero net Burgers vector. New kinds of dislocations can be defined from defects in an otherwise ordered dislocation network. For instance the chiral condensate could be modeled as a network of partial dislocations and associated stacking faults. Nucleons and mesons could then be modeled as dislocations and excitations of this underlying network, which is reminiscent of the Skyrmion approach. Ordered dislocation networks can have dislocations which can form an ordered network which can itself have higher-order dislocations, producing a possible hierarchy of dislocations several levels deep.

7 Gravity as Elasticity of Space

The similarities between the General Theory of Relativity and the theory of elasticity have been remarked upon by many authors. Sakharov spoke of relating General Relativity to a “metrical elasticity of space”[56]. Kokarev has likened space-time to a “strongly-bent plate”[57]. Several authors have developed three-dimensional continuum models of dislocations that resemble three-dimensional gravity[58].

Screw dislocations themselves do not result in curvature - rather they introduce torsion into the lattice, since an observer circling a screw dislocation finds themselves transported forward, along the dislocation direction. Two sources of curvature have been put forward - disclinations and extra matter (primarily interstitials). Disclinations are large angular defects. For example, the pentagons in a geodesic dome can be thought of as disclinations in an otherwise flat hexagonal tiling, and produces obvious curvature in the surface. The problem with disclinations is that they produce curvature only in large finite chunks, rather
than building up from many small pieces. So, whereas a disclination is a good model for a cosmic string[59], it is not a good candidate for an elementary particle. We are therefore left with the extra matter concept, which has been championed by Kroner[55]. In Kroner’s theory, the geometry of the resulting continuum model is characterized by both curvature, the source of which is extra matter, and torsion, which is caused by dislocations[54, 55, 60]. The obvious four-dimensional generalization would be the Einstein-Cartan-Sciama-Kibble theory of gravity, which supplements the usual Einstein equations with an equation relating spin density to the torsion tensor[61, 62]. Torsion effects are too small to be detected experimentally, so this theory is, so far, experimentally indistinguishable from General Relativity. In order to satisfy the equivalence principle, the absorption of interstitials by dislocations mentioned above would have to be a universal property, with the degree of absorption proportional to the energy, so that curvature could couple to the energy-momentum tensor. It is not clear that this would necessarily happen, however it could be forced by symmetries, since due to the Bianchi identity the Einstein tensor can only couple to a conserved quantity. Not all interstitials are necessarily absorbed. Unabsorbed interstitials are an intriguing dark-matter candidate. Unlike ordinary particles they do not persist, since they are true 4-d point defects. Their behavior is more like that of instantons. Their fleeting existence could make their detection difficult other than through their gravitational effects.

Regardless of the details of the gravitational theory that results, it will necessarily be a quantum theory of gravitation. This is because the evolution of the spatial hypersurface is influenced by the thermal fluctuations in the surrounding medium which are the source of quantum fluctuations in this picture. One certainly expects thermally induced curvature fluctuations. However, if the elementary lattice spacing is much larger than the Plank length, it is likely that such curvature fluctuations would be small, and gravitation would remain, in practical terms, a largely classical theory. As distances approached the lattice spacing, then the continuum theory (presumably a generalization of General Relativity) would have to be replaced with an appropriate lattice theory, just as continuum elasticity theory can be used for a crystal only for distances large compared to the lattice spacing. Of course, just as in ordinary crystallography, the lattice theory itself may be based on an underlying continuous space. The resolution of singularity problems in general relativity are more likely to come from the transition to an appropriate lattice theory than from the incorporation of quantum effects, unless the lattice spacing is of order the Plank length or smaller.

8 Cosmological Consequences

The model of an expanding phase boundary provides good explanations for some cosmological puzzles but introduces additional problems as well. Phase nucleation is a common way for structure to arise from chaos spontaneously. It naturally creates an expanding universe starting from a very small but not infinitesimal seed. Only if the initial fluctuation is above a certain minimum size, will the crystal grow - otherwise surface tension effects will remelt it back into the liquid. There would not seem to be a horizon problem because there is plenty of time before the big bang to establish causal contact, thermodynamic equilibrium
etc. Also there is the “flatness problem” which, in a non-inflationary universe, requires a careful fine-tuning of parameters to create a universe as long lasting as ours which nevertheless has a reasonable matter density and is close to being spatially flat in the present era. Phase transitions only occur when there is a fine tuning between various terms in the Hamiltonian, so a system undergoing a phase transition is already naturally fine tuned between forces that favor the transition and those that don’t. The other ingredient this model likely has which could reduce the need for fine-tuning would be dissipation, which could tame runaway solutions like inflation. In general, surface growth which is not diffusion-limited is controlled by the volume energy (which results in the liberation of latent heat), surface tension, and dissipation. The outward pressure from the volume energy takes the form of a repulsive cosmological constant and the 3-d surface tension may act like the ordinary spatial curvature term, but it is not immediately clear how to take dissipation into account within the standard Friedmann models. Comparison to ordinary phase transitions would suggest a period of slow growth at first, which accelerated as the surface term became less important, finally approaching a steady state constant growth rate. One can also consider the possibility that the background conditions responsible for the supercooling could vary over time. If this is allowed then a more complex growth-rate history could be accommodated.

An intriguing possibility for matter generation would be collisions between different crystal universes. Where crystals join, a lot of dislocations are formed. The join-boundary of two 3-d surfaces is a 2-d surface. Therefore, dislocations produced in such collisions would be distributed on 2-d surfaces within the combined 3-d surface of the joined crystals. Interestingly, matter in the universe is primarily distributed on a network of 2-d surfaces surrounding large voids. One can imagine this resulting from the twisting and folding of the join-boundary of a single cosmic collision or from a number of such events.

This scenario may have difficulty explaining both the uniformity of element abundances and of the cosmic background radiation. Helium could be produced in the cosmic collisions referred to above in much the same way as in the hot big-bang, but conditions would likely vary somewhat from place to place. A single large collision might be able to produce a fairly uniform result. Cosmic collisions, in addition to creating matter in the form of dislocations would also produce a lot of thermal radiation. Again, this could be fairly uniform for the case of a single large collision. This scenario shares some features with the colliding-branes string-model picture recently proposed by Khoury et. al.[63], although the geometry is rather different.

9 Discussion

At first glance, the idea that space could be crystalline would seem at odds with the notion of spatial isotropy. Wouldn’t the axis directions create preferred directions in space? For distances large compared to the lattice spacing, this is not necessarily so. For instance, the long distance behavior of an isotropic crystal (one with isotropic elastic constants) is well approximated by isotropic linear elasticity theory which has full rotational invariance. Another example is lattice gauge theory. Here forces along axis directions differ from
those along non-axis directions at short distances, but full rotational symmetry emerges at distances large compared to the lattice spacing. The longer lattice paths in diagonal directions are exactly compensated by the larger multiplicity of such paths. Also the surface of a growing crystal is more labile than the interior, resulting in features that are less “solid”. For instance, even sessile dislocations can move through growth, via formation of kinks and jogs, though they are essentially locked in place once formed. Glissile dislocations (those that can move freely through the crystal) may themselves essentially stop in the bulk by transferring all of their momentum to the “growth tip” through a mechanism similar to a Newton’s cradle onto which balls are added continuously, or a whip with a growing tip.

The similarities between condensed matter physics and particle physics are many. Phonons are surprisingly similar to photons. They can be thought of as Goldstone bosons resulting from the breaking of translation invariance, or as gauge fields relating to the remaining discrete translational invariance, which due to lattice periodicity, may be represented by an angular order parameter[45]. The counterpart to the Higgs mechanism is the plasma mechanism[64]. Even the chiral properties of the weak interaction may have an analog in the behavior of $^3$He-A[65]. Several gauge theories of dislocations have been proposed[66, 67]. What is being proposed here can be thought of as going all the way with this program, namely hypothesizing that particle physics is condensed matter physics. The main experimental signature of such a proposal, regardless of the details, would be the effects of a finite lattice spacing. Besides the dramatic cutoff of gauge boson spectra above a certain energy, one can look for effects of dispersion near the cutoff. The lattice also makes all ultraviolet divergences finite, which will introduce small effects in higher order corrections. This also adds impetus to proposals that a serious effort be made to search experimentally for violations of Lorentz invariance[68]. The effects of living on a physical lattice are somewhat different from string-inspired Lorentz-invariance violations. The other experimental signature this scenario has in common with other extra-dimension scenarios is the possibility of energy conservation violation beyond the statistical violation already discussed and interpreted as quantum fluctuations[2]. One can imagine the possibility of a high-energy interaction radiating a longitudinal phonon into the bulk, for instance, which would look like a missing-energy event. This can be made rare by either a very weak coupling to these modes or by giving them a low frequency cutoff (a mass). Radiation forward into the liquid could be prevented by having the surface growing at a rate exceeding the sound speed in the liquid. Another possible experimental signature to look for would be effects of dissipation including lack of time reversal invariance. Although conservation laws may prevent dissipation on the surface itself, the bulk phases undoubtedly are dissipative. The moving phase boundary breaks time reversal invariance spontaneously. Both T and CPT invariance could be broken.

A final note concerning time in this theory is the special role played by the present. The edifice of the universe is constructed at the present surface from material provided by the undifferentiated current-future (liquid) state. The past, being a solid, is more fixed, though still can undergo some fluctuations. Present, past and future are different, distinguishable phases. This would seem to conform with our personal experience better than the picture
presented in special relativity, where the present is not distinguished, and the future seems as well-formed as the past. Indeed, according to Einstein, “For us believing physicists, the distinction between past, present, and future is only an illusion, even if a stubborn one.” Davies states, “The four-dimensional space-time of physics makes no provision whatever for either a ‘present-moment’ or a ‘movement’ of time.” Quantum mechanics could play a possible role in blurring the future in the standard picture, but this depends on a definite resolution to the measurement problem. The phase boundary scenario, in contrast, matches well with the “process philosophy” concept of time as advanced by Whitehead, who talks of a “concrescence” unfolding at the present where the indefinite future is molded into a definite past.

The preferred frame offered by the crystal rest frame also gives a different point of view for causality arguments. One can imagine the possibility of interactions that occur by faster-than-light mechanisms, just as a bullet can exceed the speed of sound in an ordinary crystal. Although in some frames of reference, cause may appear to precede effect, this will never occur in the crystal rest frame, regardless of interaction speed. Since Lorentz invariance is only approximate, all frames are not equivalent. The correct result is that observed in the preferred frame. Thus there is no longer a paradox created by faster-than-light interactions by which one could travel backwards in time and kill one’s grandfather, for instance. Time always goes forward and effect follows cause in the crystal rest frame. Of course there is no evidence that any interaction or particle can exceed the speed of light, and ordinary dislocations probably can not, as previously discussed, but the removal of this causality paradox opens the door to such a possibility a crack wider.

10 Conclusion

At first glance this theory appears to be an anachronism - a neo-Lorentzian classical ether theory. Modifying special relativity and reintroducing an ether are probably the last thing that would enter the mind of a 20th or 21st century physicist, followed perhaps by a classical explanation of quantum mechanics. However, there are many ways in which this theory fits with modern ideas. The idea that we live on a membrane is becoming popular in string theory, and also for introducing chiral fermions into lattice gauge theory. Stochastic quantization, though never fully accepted in the realm of quantum mechanics, came very close to giving a classical statistical-mechanical explanation of quantum fluctuations. The very many analogies between particle physics and condensed-matter physics, especially in the realm of gauge field theories and spontaneous symmetry breaking, has led to tremendous sharing of ideas from one field to the other. The main difference between these is simply between relativistic and non-relativistic spacetime symmetries. One other difference is that, whereas in elementary particle physics gauge symmetries and Goldstone bosons are usually considered to be essentially separate mechanisms for producing massless particles (which conflict in the Higgs mechanism to give a mass), in condensed matter physics the gauge particles (phonons) can themselves be pictured as a type of Goldstone boson associated with the breaking of translational symmetry. A vector order parameter yields a vector
Goldstone boson and a tensor order parameter (associated with breaking of rotational invariance) should produce a tensor Goldstone boson (the graviton). Gauge invariance is actually born from the ambiguities in defining the unperturbed lattice[45]. This economy of ideas (gauge bosons as Goldstone bosons) is appealing and may help to explain why some symmetries are gauged and others are not. To benefit from this analogy, however, an ether-type background would appear to be necessary to provide the required translational symmetry breaking.

The idea of a moving, expanding, phase boundary, where relativistic space-time emerges as a dynamical symmetry, integrates these ideas into a coherent picture; the big-bang and gravity, from the flexible geometry of the interface, are incorporated almost for free. The quantum measurement process is also vastly clarified in this picture as a consequence of spontaneous symmetry breaking. Ideas from chaos and ergodic theory, nonequilibrium statistical mechanics and dynamical critical phenomena play an important role. When these are added, a classical theory doesn’t look so classical anymore.

Not much has been mentioned in this paper concerning the base-theory. What new sets of even more elementary particles (called elementary atoms above) and forces must be postulated for the underlying base-theory, upon which the dislocations and surface waves (our current set of elementary particles in this picture) can be built? Hopefully it is a simpler set than we currently have in the standard model. It seems possible that one or two types of elementary atoms, combined with a short-range force, repulsive at small distances and attractive at long, could be enough. Some model building seems to be in order, starting with extending simple crystal models to four dimensions.

In closing, one cannot help but speculate whether Einstein would have liked this idea. Certainly he may have disagreed with the reintroduction of the ether which he so strongly fought against, as well as the notion of a preferred frame. However from the point of view of the more fundamental base-theory there is no preferred frame – it is introduced through a spontaneous symmetry breaking, so the principle of relativity is safe there. Einstein’s discomfort with the inherently probabilistic nature of quantum mechanics is well known, so perhaps he would be pleased with the application of his ideas on Brownian motion toward the explanation of quantum fluctuations, as well as the replacement of quantum mechanics with a deterministic (though chaotic) theory. Finally, there is the essentially geometric basis for electromagnetism and possibly all interactions through the picture of a dislocated crystal. This bears a rather strong resemblance to unified field theories that he worked on in his later years. On balance, this theory seems relatively in concert with the ideas of Einstein.

References

[1] These ideas were introduced in M. Grady, gr-qc/9805076; see also M. Chown, New Scientist, 161, 42, 1999.

[2] The idea of the universe as a domain wall in a larger universe was discussed in V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. 125B, 136, 1983.
[3] Another early domain-wall scenario is described in K. Akama, in Lecture Notes in Physics, 176, Gauge Theory and Gravitation, Proceedings, Nara, 1982; K. Kikawa, N. Nakanishi, and H. Nariai, Eds.; Springer-Verlag: Berlin, 1983, p 267, hep-th/0001113.

[4] String inspired domain wall universes are discussed in, e.g., A. Lukas et. al., Phys. Rev. D 59, 086001, 1999; L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 4690, 1999.

[5] A. Reiss et. al., Astron. J., 116, 1009, 1998; S. Perlmutter et. al., Nature, 39, 51, 1998; P.M. Garnavich et. al., Ap. J., 493, L53, 1998.

[6] P.C. Hohenberg and B.I. Halperin, Rev. Mod. Phys. 49, 435, 1977; J. Zinn-Justin, Quantum Field Theory and Critical Phenomena, 3rd ed.; Clarendon Press: Oxford, 1996, pp 764-777.

[7] E. Nelson, Quantum Fluctuations; Princeton University Press: Princeton, 1985; E. Nelson, Phys. Rev. 150, 1079, 1966; F. Guerra, Phys. Rep. 77, 263, 1981; Ph. Blanchard, Ph. Comb and W. Zheng, Mathematical and Physical Aspects of Stochastic Mechanics; Springer-Verlag: Berlin, 1987.

[8] G. Parisi and Y. Wu, Scientia Sinica 24, 483, 1981.

[9] H. H"uffel and H. Rumpf, Phys. Lett. 148B, 104, 1984.

[10] E. Gozzi, Phys. Lett. 150B, 119, 1985; H. Nakazato and Y. Yamanaka, Phys. Rev. D 34, 492, 1986; H. Nakazato, Prog. Theor. Phys. 77, 20, 1987.

[11] M. Namiki and S. Tanaka, in Modern Problems of Theoretical Physics - Feschrift for Professor D. Ivanenko; P.I. Pronin and Yu.N. Obukhov, Eds.; World Scientific: Singapore, 1991.

[12] H. Haken Synergetics; Springer-Verlag: Berlin, 1977, especially Ch. 6-8; Rev. Mod. Phys. 47, 67, 1975.

[13] H. Dekker, Physica 95A, 311, 1979; Phys. Rep. 80, 1, 1981.

[14] H. Haken in Encyclopedia of Physics, vol. XXV/2c: Laser Theory; S. Flügge, Ed.; Springer-Verlag: Berlin, 1970.

[15] J. Frenkel and T. Kontorowa, Phys. Z. Sowjet 13, 1 (1938).

[16] F.C. Frank, Proc. Phys. Soc. A62, 131, 1949; J. Eshelby, Proc. Phys. Soc. A62, 307, 1949.

[17] J. Weertman and J.R. Weertman, in Dislocations in Solids - vol. 3: Moving Dislocations; F.R.N. Nabarro, Ed.; North-Holland Publishing Co.: Amsterdam, 1980.

[18] F.R.N. Nabarro, Theory of Crystal Dislocations; Oxford University Press: London, 1967.
[19] This issue is addressed in A. Unzicker, gr-qc/0011064.

[20] H. Günther, *Phys. Stat. Sol.* b149, 101, 1988; H. Günther in *Proceedings of the Eighth International Symposium on Continuum Models and Discrete Systems*, Varna, Bulgaria, 1995; K.Z. Markov, Ed.; World Scientific: Singapore, 1996, p 507.

[21] A.I. Miller, in *Some Strangeness in the Proportion*; H. Woolf, Ed.; Addison-Wesley: Reading MA, 1980, p 66.

[22] M. Catanese and T.C. Weeks, *Publ. Astron. Soc. Pac.* 111, 1193, 1999; B. Degrange and M. Punch, *C.R. Acad. Sci. Paris*, 1, 189, 2000.

[23] A.V. Olinto, *Phys. Rep.* 333-334, 329, 2000; A.A. Watson, *Phys. Rep.* 333-334, 309, 2000; M. Boratav and A.A. Watson, *C.R. Acad. Sci. Paris*, 1, 207, 2000.

[24] K. Greisen, *Phys. Rev. Lett.* 16, 748, 1966; G.T. Zatsepin and V.A. Kuzmin, *Sov. Phys. JETP* Lett. 4, 78, 1966.

[25] G. Amelino-Camelia and T. Piran, *Phys. Rev. D* 64, 036005, 2001 and references therein.

[26] G.R. Farrar and P.L. Biermann, *Phys. Rev. Lett.* 81, 3579, 1998.

[27] Lord Rayleigh (J.W. Strutt), *London Math. Soc. Proc.* 17, 4, 1885, available in *Scientific Papers of Lord Rayleigh, vol II*; Dover: New York, 1964, p 441.

[28] G.W. Farnell, in *Physical Acoustics vol. VI*; Academic Press: New York, 1970, p 109; L.D. Landau and E.M. Lifshitz, *Theory of Elasticity*; Pergamon Press: London, 1959, p 105; A.A. Maradudin in *Nonequilibrium Phonon Dynamics*; W.E. Bron, Ed.; Plenum Press: New York, 1985, p 395.

[29] I.M. Lifshitz and A.M. Kosevich, in *Lattice Dynamics*; W.A. Benjamin: New York, 1969, p 53.

[30] I.A. Viktorov, *Sov. Phys. Acoustics*, 25, 1, 1979.

[31] G.A. Maugin in *Recent Developments in Surface Acoustic Waves*; Springer-Verlag: Berlin, 1988, p. 158.

[32] A.I. Murdoch, *J. Mech. Phys. Solids*, 24, 137, 1976.

[33] G.W. Farnell and E.L. Adler in *Physical Acoustics, Vol. IX*; W.P. Mason and R.N. Thurston, Eds.; Academic Press: New York, 1972.

[34] A.E.H. Love, *Some Problems of Geodynamics*; Cambridge University Press: London, 1911.

[35] H.F. Tiersten, *J. Appl. Phys.* 40, 770, 1969; H.F. Tiersten, B.K. Sinha, and T.R. Mecker, *J. Appl. Phys.* 52, 5614, 1981.
[36] A.M. Kosevich and A.V. Tutov in *Continuum Models and Discrete Systems - Proceedings of the 8th International Symposium*, June 11-16, 1995, Varna, Bulgaria; World Scientific: Singapore, 1996, p 444.

[37] N. Daher and G.A. Maugin, *Acta Mech.* **60**, 217, 1986. For additional information on excitations on moving interfaces see W. Kosiński, *Field Singularities and Wave Analysis in Continuum Mechanics*; Wiley: New York, 1986, and references therein.

[38] M.G. Cottam and D.R. Tilley, *Introduction to Surface and Superlattice Excitations*; Cambridge University Press: Cambridge, 1989.

[39] D.B. Kaplan, *Phys. Lett. B* **288**, 342, 1992; M. Creutz, *Rev. Mod. Phys.* **73**, 119, 2001, and references therein.

[40] N.W. Ashcroft and N.D. Mermin, *Solid State Physics*; Saunders College Publishing: Philadelphia, 1976, pp 784-789.

[41] E. Fermi, J. Pasta and S. Ulam, in *The Collected Papers of Enrico Fermi, vol. 2*; University of Chicago Press: Chicago, 1966, p 977 (original report dated 1955).

[42] M. Rasetti, *Modern Methods in Equilibrium Statistical Mechanics*; World Scientific: Singapore, 1986.

[43] H.S. Robertson, *Statistical Thermophysics*; Prentice Hall: Englewood Cliffs, NJ, 1993, pp 43-48.

[44] J. Ford and G.H. Lunsford, *Phys. Rev. A* **1**, 59, 1970.

[45] P.W. Anderson, *Basic Notions of Condensed Matter Physics*; Addison-Wesley: Reading, MA, 1984, pp 30-69.

[46] Y. Ne’eman, *Proc. Natl. Acad. Sci. USA* **80**, 7051, 1983; *Found. Phys.* **16**, 361, 1986.

[47] G.T. Zimányi and K. Vladár, *Phys. Rev. A* **34**, 3496, 1986; *Found. Phys. Lett.* **1**, 175, 1988.

[48] M. Grady, [hep-th/9409049](https://arxiv.org/abs/hep-th/9409049)

[49] C.V. Burton, London, Edinburgh, and Dublin *Philosophical Magazine and Journal of Science* **33**, 191, 1892.

[50] J. Larmor, *Aether and Matter*; Cambridge University Press: Cambridge, 1900; J. Larmor, *Phil. Trans. Roy. Soc. London* **A190**, 205, 1897.

[51] K.F. Schaffner, *Nineteenth Century Aether Theories*; Pergamon Press: Oxford, 1972.

[52] J.D. Eshelby, *Phys. Rev.* **90**, 248, 1953.

[53] J.P. Hirth and J. Lothe, *Theory of Dislocations*; McGraw Hill: New York, 1968.
[54] R. de Wit, in Solid State Physics, vol. 10; F. Sietz and D. Turnbull, Eds.; Academic Press: New York, 1960, p 249.

[55] E. Kröner, Int. J. Theor. Phys. 29, 1219, 1990; E. Kröner in Les Houches XXXV - Physics of Defects; R. Balian, M. Kl’eman and J-P. Poirier, Eds.; North-Holland Publishing Company: Amsterdam, 1981, p 215.

[56] A.I. Sakharov, Dokl. Akad. Nauk SSSR, 177, 70, 1967 (Sov. Phys. Doklady 12, 1040, 1968).

[57] S.S. Kokarev, Nuovo Cim. 113B, 1339, 1998; Nuovo Cim. 114B, 903, 1999.

[58] See, e.g. C. Malyhev, Ann. Phys. 286, 249, 2000, and references therein; also numerous articles in RAAG Memoirs of the Unifying Study of Basic Problems in Engineering and Physical Sciences by Means of Geometry, vol. I-IV; K. Kondo, Ed.; Gakujutsu Bunken Fukyu-Kai: Tokyo, 1968.

[59] A. Vilenkin and E.P.S. Shellard, Cosmic Strings and Other Topological Defects; Cambridge University Press: Cambridge, 1994.

[60] A. Trzesowski, Int. J. Theor. Phys. 33, 931, 1994.

[61] F.W. Hehl, P. von der Hyde and G.D. Kerlick, Rev. Mod. Phys. 48, 393, 1976; F.W. Hehl, J.D. McCrea, E.W. Mielke, and Y. Ne’eman, Phys. Rep. 258, 1, 1995.

[62] M. Göckeler and T. Schücker, Differential Geometry, Gauge Theories, and Gravity; Cambridge University Press: Cambridge, 1987, ch. 5.

[63] J. Khoury, B.A. Ovrut, P.J. Steinhardt, and N. Turok, Phys. Rev. D 64, 123522, 2001.

[64] P.W. Anderson, Phys. Rev. 130, 439, 1963.

[65] G.E. Volovik, in Topological Defects and Non-Equilibrium Dynamics of Symmetry Breaking Phase Transitions; Y.M. Bunkov and H. Godfrin, Eds.; Kluwer Academic Publishers: Dordrecht, 2000, pp 353-387, cond-mat/9902171.

[66] A. Kadić and D.G.B. Edelen, A Gauge Theory of Dislocations and Disclinations; Springer-Verlag: Berlin, 1983.

[67] H. Kleinert, Gauge Fields in Condensed Matter, Vol. II - Stresses and Defects; World Scientific: Singapore, 1989.

[68] S. Coleman and S.L. Glashow, Phys. Rev. D 59, 116008, 1999; D. Colladay and V.A. Kostelecký, Phys. Rev. D 55, 6760, 1997; 58, 16002, 1998.

[69] B. Hoffman (with H. Dukas), Albert Einstein: Creator and Rebel; Viking: New York, 1972, p 258.
[70] P.C.W. Davies, *The Physics of Time Asymmetry*; University of California Press: Berkely, 1976, p 21.

[71] A. N. Whitehead, *Process and Reality*; corrected edition, D.R. Griffin and D.W. Sherburne Eds.; Free Press: New York, 1978; *Physics and the Ultimate Significance of Time*; D. R. Griffin, Ed.; SUNY Press: Albany, 1986.