Angular Velocity Perturbations Inducing the Papaloizou-Pringle Instability and QPOs in the Torus around the Black Hole

Orhan Donmez
College of Engineering and Technology, American University of the Middle East (AUM), Egbaila, Kuwait

Abstract
A numerical study of the dynamic of the nonselfgravitating, unmagnetized, nonaxisymmetric, and rotating the torus around the non-rotating black hole is presented. We investigate the instability of the rotating torus subject to perturbations presented by increasing or decreasing the angular velocity of the stable torus. We have done, for the first time, an extensive analysis of the torus dynamic response to the perturbation of the angular velocity of the stable torus. We show how the high, moderate, and low values of the perturbations affect the torus dynamic and help us to understand the properties of the instability and Quasi-Periodic Oscillation (QPO). Our numerical simulations indicate the presence of Papaloizou-Pringle instability (PPI) with global $m = 1$ mode and QPOs for the moderate and lower values of the perturbations on the angular velocity of the stable torus. Furthermore, with the lower values of the perturbations, the torus can lead to a wiggling initially and then PPI is produced in it. Finally, the matter of the torus would be dissipated due to the presence of a strong torque.

Keywords: numerical relativity – torus-black hole – angular velocity perturbation – Papaloizou-Pringle instability – quasi-periodic oscillation

1 Introduction
The accretion tori around the stellar and massive black holes are the one of the interest to explain the observed high energies ($X$- and $\gamma$-rays) in different astrophysical scenarios (Lee & Ramirez-Ruiz 2007; Meszaros & Gehrels 2012) such as active galactic nuclei which may form during the collapse of super-massive stars (Rees 1984; Shibata & Shapiro 2002), coalescence of the black hole and neutron star (Rezzolla et al. 2010; Kyutoku et al. 2010), etc.

The analytic and numerical investigations of the oscillatory modes of accreted tori have been focused for a few decades. The oscillating relativistic tori in a strong relativistic region were numerically developed by Zanotti et al. (2003, 2005) and reference therein. The axisymmetric modes of high frequency quasi-periodic oscillations (HF QPOs) were investigated around the non-rotating and rotating black holes using a fixed spacetime matrix. In order to examine the disk oscillation under the influence of the radial perturbation and their nonlinear coupling with other modes, Lee et al. (2004) studied the numerical simulations of the relativistic torus around the black hole implementing a pseudo potential. They found vertical epicyclic frequencies. We have previously found (Dönmez 2014a) the numerical study of the dynamical instability of the torus on the equatorial plane due to perturbation which was represented injecting gas from the outer boundary of the domain. It was shown that the mass accretion rate in the perturbed torus strongly depended on the cusp location and dynamical changes of the torus triggered the PPI. Recently, the evolution of the perturbed torus implemented by adding the subsonic velocity to radial, vertical, and diagonal velocities of the torus had been carried out by Parthasarathy et al. (2017). It was seen that the both $X$-ray and vertical epicyclic modes might be strongly excited in tori.

The long term oscillatory behavior of the black hole torus system is known as runaway instability (Abramowicz et al. 1983) in case of axisymmetric perturbation but it is called PPI (Papaloizou & Pringle 1984) if we apply a nonaxisymmetric perturbation on the rotating torus around the black hole. It is seen in recent numerical simulations that even though the runaway instability does not have a significant impact on dynamic of the rotating torus (Montero et al. 2010), the...
nonaxisymmetric perturbations on the rotating torus may have a critical impact on the disk instability and in the variation of high energetic astrophysical observations (Narayan et al. 1987). Based on the above insight, the aim of this paper is to extend the understanding of the dynamical instability of the relativistic torus in the case of the nonaxisymmetric perturbation of the angular velocity of the stable torus carried out by Dönmez (2017). All numerical simulations had been done by perturbing the torus which had a constant specific angular momentum rotating around the non-rotating black hole.

The PPI which created due to interactions of the propagation waves across the corotating radius is mostly explained by nonaxisymmetric perturbation of the torus around the black hole (Papaloizou & Pringle 1984). The instability was occurring when the waves inside the radius are transferring their energy to the waves outside of the corotation radius (Blaes & Glatzel 1986). Besides, the emerging the hydrodynamical instabilities were strongly depended on the physical parameters of the system such as the torus size, angular momentum of the black hole, angular momentum of the torus, and the type of the instability (Villiers & Hawley 2002; Dönmez 2014a, 2017). The observational evidence of the PPI in Seyfert galaxy NGC 1068 was found by Garcia-Burillo et al. (2017). The noncircular dynamics of the gas torus were seen. It enhanced the amplitude of the instability and produced the lopsided morphology on the gas torus. Hence, it was believed that, the complex kinematic and the lopsided morphology of the gas torus could be signature of the PPI.

The power spectrums of the observational data taken by various ground and space base detectors indicate the possibility of QPOs from the various astrophysical phenomena. These QPOs are the result of the consistent mechanism which is varying significantly with time. Studying the QPOs behavior of the tori around the black holes would help determine the physical properties of the black holes such as spin and mass (Silbergleit et al. 2001). The frequencies numerically determined from oscillating tori around the black hole create certain ratios which may be used to identify the properties of black holes. For example, the ratio $3:2$ is suggested to be a resonance between the fundamental frequency (orbital) and its overtones in a strong gravitational region.

The paper is organized as follows: equations, models, and initial setups of the relativistic torus are given in Section 2. In Section 3, the numerical results from our simulations are given and discussed in detail. The perturbations which produce QPOs and PPI are revealed. The results found our numerical calculations are summarized in Section 4. The geometrized unit, $c = G = 1$ and the space-time signature $(-, +, +, +)$ are used throughout the paper.

2 Equations, Models, and Initial Setups
A numerical study of the nonselfgravitating, unmagnetized, nonaxisymmetric and rotating torus around the non-rotating black hole is considered to model the instabilities and QPOs in case of the nonaxisymmetric perturbation. In order to understand the dynamics of the torus and instabilities created due to the interaction of matter with black hole, we have numerically solved the General Relativistic Hydrodynamical (GRH) equations on equatorial plane by using the fixed Schwarzschild spacetime metric (see Dönmez (2004); Dönmez & Kayali (2006) for details regarding the conserved form of the hydrodynamical equations, all formulations and their numerical solutions). The relation among the pressure, rest mass density, and internal energy is defined using the perfect fluid equation of state $P = (\Gamma - 1) \rho c^2$ with $\Gamma = 4/3$. Initially a steady state accreted torus is produced using the appropriate values of the highest density of the torus $\rho_0 = 1.140 \times 10^{-4}$, the mass ratio of the black hole:torus $M_{BH}/M_{BH} = 0.1$, the polytropic constant $K = 4.969 \times 10^{-2}$, the constant specific angular momentum $\ell_0 = 3.80$, inner $r_{in} = 4.57$ and outer, $r_{out} = 15.889$, radii of the torus, cusp location $r_{cusp} = 4.57$, and orbital period $t_{orb} = 151.6$ at $r_c = 8.35$ (Dönmez 2014a,b, 2015).

The negligible values are used in the rest of the computational domain after setting up the initial stable torus. They are: atmosphere density $\rho_{atm} = 10^{-8} \rho_c$, pressure $p_{atm} = 10^{-8} p_c$, radial velocity $v^r = 0.0$, and angular velocity $v^\phi = 0.0$ (Dönmez 2014a). The general form of the Schwarzschild metric is used to define the non-rotating black hole at the center of computational domain. The inner and outer boundaries along the radial direction are located at $r_{min} = 2.8M$, inside the apparent horizon, and $r_{max} = 200M$, respectively. The angular direction goes from 0 and $2\pi$. The computation domain is defined on equatorial of the spherical coordinate which covers the interval $(N_rXN_\phi) = (3072 X 256)$. The outflow boundary condition is set up close to or far away from the black hole to avoid the unwanted oscillations. The more details about initial setups as well as boundaries used in our numerical simulation can be found in Dönmez (2014a,b, 2015).

The stable initial tori are perturbed by increasing or decreasing the angular velocity of the torus everywhere to the computational domain in each model for different cases, seen in Table.1. So we have a chance to find out an instability and QPOs from to the perturbed torus. The instabilities and QPOs might be used to explain the observed X—rays from different astrophysical phenomena.
The evolution of the perturbed torus shows a chaotic behavior. In order to measure the instabilities produced during this chaotic behavior, we measure the mass accretion rate. The mass accretion rate computed at the equatorial plane can be handled by using the following expression

\[
\frac{dM}{dt} = \int_0^{2\pi} \alpha \sqrt{\gamma} \rho u^r d\phi,
\]

where \(\rho\), \(u^r\), \(\alpha\), and \(\gamma\) are the rest-mass density of the torus, four-velocity along the radial coordinate, the lapse function, and the determinant of the three-matrix, respectively.

The angular velocity perturbation can trigger the non-axisymmetric instability in the radial direction on the torus and produce a restoring force. As a consequence, the oscillation modes are produced on the non-axisymmetric disk around the black hole. To define this instability, we compute the Fourier power of the density. The mode power allows us to compute the saturation point of the instability and its growth rate. The detailed explanations of how to compute the power modes \(m_1\), \(m_2\), etc. are given in Dönmez (2014a); Villiers & Hawley (2002).

The simulations reported in this paper are performed for a long time at least 10 to 25 orbital periods to observe the instabilities and their persistent mechanism. Otherwise, they run until the code crashes. Thus we can measure the saturation, postsaturation, growth rates of the PPI, and QPOs observed from the continues emission of the system.

### 3.1 Perturbation of the Stable Torus: Case I

We have started by performing the nonaxisymmetric perturbation of the stable-rotating torus around the non-rotating black hole using the high values of the angular velocity. Initially, we perturb the torus by increasing or decreasing the angular velocity in an amount 20% or more seen in Table 1 and called Case I. Models \(MH_1\) and \(MH_2\) both represent the perturbations in order to investigate the dynamical behavior of the torus by increasing the angular velocity of the stable torus. During the evolution, the angular momentum of the torus is transported inwards through the unstable point therefore less matter falls into the black hole as seen in Fig.1. It is also noted in Fig.2 that as a consequence of increasing the angular velocity of the initial torus, the torus moves outward from the black hole and tightens more which causes an increase in the maximum density of initial stable torus. Later, the instability is created and triggers the distribution of matter over the computational domain. By contrast, Model \(MH_3\) is constructed by decreasing the angular velocity of the torus by amount 30%. As it is seen in Fig.1 that the angular momentum is transported outward, more matter starts falling into the black hole, and the torus is destroyed in less than a dynamical time step. The simulations in Case I do not produce any instability or show a prominent burst in a short time scale. The rapid changes of the torus dynamic in less than an orbital period produce an erratic behavior and it creates a strong shock. Therefore, the code crashes.

### 3.2 Perturbation of the Stable Torus: Case II

The oscillating the relativistic tori along the radial and angular directions can be handled by perturbing the torus angular velocity. In this section, we discuss the numerical results in the presence of moderate amount of perturbation on angular velocity of a rotating torus around a non-rotating black hole. Fig.3 shows the evolution of the mass accretion rates computed at the inner boundary of the computational domain \(r = 2.8M\) for Case II given in Table 1. It is revealed in this figure that the instability grows initially due to a sudden change in angular momentum of the torus. It causes the expansion of the torus around the black hole and the cusp location moves outward. During this expansion, \(60M < t < 400M\), less amount of matter would be accreted onto the black hole. Later, the torus starts falling toward to the black hole and the accreted mass increases exponentially. They reach their peak values and then start to oscillate approximately non-zero constant values for Models \(MM_1\), \(MM_2\), and \(MM_3\) in Case II. But the non-zero value could not be achieved.
Figure 1. The accretion rates computed from the perturbed torus around the non-rotating black hole plotted at the inner radius of the computational domain \( r_{in} = 2.8M \). The perturbation is represented by increasing or decreasing the angular velocity of the stable torus, 20% or more (Case I).

Figure 2. Density of the perturbed torus at different snapshots. The perturbation is presented by increasing the angular velocity of the torus, 20% (Model \( MH_1 \)).

Figure 3. The same as Fig.1 but it is for Case II. The torus is perturbed by increasing or decreasing the angular velocity of the stable torus with a moderate value, 20% \( > v_\phi^0 \geq 10\% \) (Case II).

Figure 4. The evolution of the torus for Case II. The radial location of the cusp (top panel) and the maximum density of the torus (bottom panel) are given. They are normalized to their values at \( t = 0M \).

by the initial perturbation given in Model \( MM_4 \). These results imply that the moderate perturbation of the angular velocity, 20% \( > v_\phi^0 \geq 10\% \), would trigger the instability with the non-zero constant values of the physical parameters of the torus (i.e. the maximum density, the cusp location, etc.) around the non-rotating black holes, seen in Fig.4. It is fair the stressing in Fig.4 that the maximum density and cusp locations for the Models \( MM_1, MM_2 \), and \( MM_3 \) in Case II survive with appreciable amplitudes after dynamics of the torus reach to a quasi-steady state. So we expect to find some PPI instabilities around the black holes.

Transporting the angular momentum of the torus outward can be activated by PPI through the corotation point (Narayan et al. 1987). It means that redistributing the angular momentum of the torus due to perturbation can trigger PPI. Fig.5 shows the mode power of the \( m = 1 \) nonaxisymmetric structure and saturation points for Models \( MM_1, MM_2 \) and \( MM_3 \). We observe a saturation point in the early time of the simulation when \( t \sim 70M \). It is created due to the first kick on the torus. Later, the torus starts falling towards to the black holes.
hole and forms a new cusp location and maximum density. The torus becomes subject to PPI because the cusp is located at a larger radius which can vary as seen in Fig.4. As shown in Fig.5, after the saturation points are created around \( t = 850M \), \( t = 1010M \), and \( t = 1220M \) for Models \( MM_1 \), \( MM_2 \) and \( MM_3 \), respectively, the non-zero values of \( m = 1 \) growth mode show a persistent structure during the evolution. This mechanism is a clear indication of PPI and leads to emission of X-rays in the observed black hole torus systems, such as Seyfert galaxy NGC 1068 (Garcia-Burillo et al. 2017).

Figure 6 shows ten different snapshots of the logarithmic rest-mass density of the torus for Model \( MM_1 \) of Case II. The snapshot \( t = 0M \) shows the stable torus dynamic and the others indicate the response of the torus dynamic to the perturbation. It is seen in Fig. 6 that the cusp location of the torus moves away from the black hole and the size of the torus expands through the domain. Later, the torus starts falling back toward to the black hole and forms a new cusp location which is almost 2 times bigger than the cusp location of the stable torus. As noted from the snapshot 4(four) at \( t = 709M \), the matter falling back heats the gas and spiral shock wave is created. This spiral pattern creates a persistent mechanics around the black hole, seen in the rests of the snapshots of Fig.6. It is important to note that we can even see the rotating spiral shock waves around the black hole after \( t = 26 \) orbital periods. The PPI and \( m = 1 \) mode structure survive for a long time after the saturation of the PPI is reached.

Formation of the QPOs around the black hole as a consequence of the torus-black hole interaction due to any type of perturbation is a common phenomena (Dönmez 2014a, 2017) and these QPOs can be used to define the physical properties of the black hole such as mass and spin. In this paper, to reveal the QPO frequencies occurring in the torus around the black hole, we compute the power spectrum of the mass accretion rate for Model \( MM_3 \), seen in Fig.7. It is indicated that the power law distribution of the oscillating torus presents a fundamental frequency \( f = 56Hz \) and their overtones, \( o_1 = 26Hz \), \( o_2 = 75Hz \), \( o_3 = 100Hz \), \( o_4 = 116Hz \), and \( o_5 = 200Hz \) and \( o_2 = 236Hz \). We see many harmonic structures from the computed frequencies 1:2:3:...

### 3.3 Perturbation of the Stable Torus: Case III

The results from representative models with the low values of angular velocity perturbation are shown in Figs. 8 - 11. The mass accretion rates for all models in CaseIII represent the clear indication of PPI instability on the torus, seen in Fig.8. The small amount of perturbation on the angular velocity of the torus triggers the instability but, as seen in upper panel of Fig.9 that, the amplitude of the oscillation is too small initially due to pushing the torus cusp location either outward or inward in a small amount. The torus oscillates around a location of the cusp of the initial stable torus. The torus feels the initial kick even in the first time step and later it indicates some wiggling in a quasi-periodic way (i.e. the torus starts to shake in the early time of the simulation). Finally, the instability and the erratic behavior of the torus matter are observed during the evolution, seen in Figs. 8, 9, and 11. These chaotic motion creates strong torques. During this process, the spiral density waves, result of the growth of \( m = 1 \), provide a channel with strong torque. As seen in the lower panel of Fig.9, the matter falls into black hole for Models \( ML_1 \) and \( ML_2 \). On the other hand, it is noted in Fig.9 that Model \( ML_3 \) did not have enough time to create the strong torque to destroy the disk dynamic and spiral structure. The recovery time of the instability created on the torus is much larger in case of lower value of the angular velocity perturbation, shown in Fig.8.

The power mode analysis of the models given in Case III shows that the growth of the PPI mode appears after the saturation points are created. These points are varying for different models. While the saturation points for Model \( ML_1 \) and \( ML_2 \) are created at \( t = 1228M \) and \( t = 1560M \), respectively, seen in Fig.10, this point does not appear for the Model \( ML_3 \) before the end of the simulation \( t = 2500M \) (\( \sim \) orbital periods). It is also clear in Fig.8 that PPI growth coincides with a significant oscillation in the mass accretion rate due to erratic behavior of the spiral shock waves generated in the torus. In spite of very significant changes of the dynamic around the black hole, PPI is not able to
Figure 6. The logarithmic change of the rest mass density at 10 different snapshots of the evolution for model MM1 of Case 11. The outer boundary of the domain seen in all panels is at \([X_{\text{min}}, Y_{\text{min}}] \rightarrow [X_{\text{max}}, Y_{\text{max}}] = [-55M, -55M] \rightarrow [55M, 55M]\) The spiral density waves are formed after the quasi-rotation motion is appeared in the time evolution of the simulation. So the growth of \(m=1\) mode and the appearance of the PPI are noticeable.

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survive long enough to create a persistent mechanism. Mode power starts to decrease and it causes the code to crush due to strong shock generated inside the torus.

Fig. 11 also shows the dynamical change of the logarithmic rest-mass density of the torus at $t = 0 M$ (the stable torus) and later times after the perturbation is applied, for Model $ML_1$. The formation of the instability and inflowing spiral waves appear in this figure. It is also seen how the spiral pattern spreads through the computation domain. It redistributes the disk material over the domain and is radially extended through the equatorial plane.

4 Conclusion

We have numerically done an extensive analysis of the oscillation properties of the perturbed torus in a strong general relativistic region by applying the perturbation to the angular velocity of the initially stable torus. Any increase or decrease in the angular velocity of the stable torus triggers the stability properties due to the modified centrifugal forces which are dominant in the relativistic region close to the black hole.
Figure 11. The same as Fig. 11 but for Model $ML_1$ in Case III.
The nonaxisymmetric perturbation due to the sudden change of the angular velocity of the stable torus in a high amount triggers the instability. The torus starts to extend outward if the angular velocity increases. Otherwise, driven instability can cause an increase in the mass flux exponentially and the matter rotating around the black hole quickly falls into the black hole and the torus would disappear less than in a dynamical timescale. In the any case of the decreasing the initial angular velocity of the stable torus, the angular momentum is transported outward, more matter starts falling into the black hole, and the torus would be destroyed again in less than an orbital period.

On the other hand, it is seen from our numerical simulations that the increasing in the angular velocity of the stable torus in an amount $18\% \geq v^\phi_n \geq 10\%$ would lead to a PPI. The transporting of the angular momentum of the torus outward can be activated by PPI through the corotation point. For the simulations in Case II, which consist of the moderate values of the perturbation for the angular velocities, the saturation points occur at different times for various models and then the non-zero values of $m = 1$ growth modes show a persistent structure during the evolution. This mechanism is a clear indication of PPI and leads to an emission of $X$- or $\gamma$-ray in the observed torus-black hole system, such as Seyfert galaxy NGC 1068 (Garcia-Burillo et al. 2017). Our results also demonstrate that PPI produces a QPO in the torus-black hole system. Exploring the QPOs allow us to estimate the black hole properties such as spin and mass.

We have also revealed the presence of the PPI and QPOs in our simulations in case of moderately low perturbation in the angular velocity. The torus feels the initial kick even in the first time step and later it indicates some wiggling in a quasi-periodic way. Finally, the instability and the erratic behavior of the torus matter are observed during the evolution. These chaotic motions create a strong torque. During these process, the spiral density waves result of the growth of $m = 1$ provide a channel with a strong torque which dissipates the matter of the torus. In the early times of the simulation the spiral wave has a small amplitude that it does not indeed have a strong interaction with surrounding. Once the amplitude of the spiral wave becomes larger, the nonlinear behavior is seen around the black hole. As a last, it is important to notice that the recovery time of the instability created in the torus is much larger in case of lower values of the perturbations in the angular velocities.

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