The covalent bond in Particle Spectroscopy

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Abstract
It is proposed that meson resonances are linear combinations of \( q\bar{q} \) and meson-meson (MM); baryon resonances are combinations of \( qqq \) and meson-baryon (MB). Mixing between these combinations arises via decays of confined states to meson-meson or meson-baryon. There is a precise analogy with the covalent bond in molecular physics; it helps to visualise what is happening physically. One eigenstate is lowered by the mixing; the other moves up and normally increases in width. Cusps arise at thresholds. At sharp thresholds due to S-wave 2-particle decays, these cusps play a conspicuous role in many sets of data.

PACS numbers: 12.39.Ki, 12.40.Yx, 13.25.Ft, 14.20.Gk

1 Principles
The Hamiltonian for a \( q\bar{q} \) state decaying to meson-meson obeys

\[
H\Psi = \begin{pmatrix} H_{11} & V \\ V & H_{22} \end{pmatrix} \Psi; \tag{1}
\]

\( H_{11} \) describes short-range configurations and \( H_{22} \) refers to ingoing and outgoing states; \( V \) accounts for the coupling between them due to \( s \)-channel decays. Then \( H_{22} \) describes \( s \)-channel loop diagrams and also \( t \) and \( u \)-channel exchanges. Several authors adopt this principle, notably Jaffe [1]; he gives the equations and discusses many of the implications.

The central premise of the present paper is that both \( H_{11} \) and \( H_{22} \) play essential roles in all cases. This is different from approaches where only one of the two components in the Hamiltonian contributes, for example the approach based on four-quark mesonic states. It requires a review of all points of view and an appeal to data to support the contention that both short and long-range configurations are needed.

The wave function \( \Psi \) is a linear combination of \( q\bar{q} \) (or \( qqq \)) and unconfined MM (or MB). The crucial point is that two attractive components \( H_{11} \) and \( H_{22} \) lower the eigenstate via the mixing. This is a purely quantum mechanical effect. There is a direct analogy with the covalent bond in chemistry. The solution of Eq. (1) is given by the Breit-Rabi equation:

\[
E = (E_1 + E_2)/2 \pm \sqrt{(E_1 - E_2)^2 + |V|^2}, \tag{2}
\]

where \( E_1 \) and \( E_2 \) are eigenvalues of separate \( H_{11} \) and \( H_{22} \). One linear combination is pulled down in energy.

In chemistry, \( H \) is in principle known exactly. The discussion of the hydrogen molecule (and more complex ones) is given in many textbooks on Physical Chemistry, for example the one of Atkins [2]. In the valence bond approximation, \( \Psi \) is described by two molecular ions, equivalent

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to $q\bar{q}$ and $MM$, with a pair of electrons concentrated between the ions in the ground state or repelled from this region in the upper state. Two key points are that (a) the extension of the wave function into the overlap region lowers momentum, hence kinetic energy, (b) the whole system shrinks slightly and the binding of electrons to both nuclei increases. This is the source of binding. The analogue in Particle Physics is that the $MM$ wave function for the ground state is sucked into the region of overlap and repelled from it in the upper state, hence affecting zero-point energy. The second effect is that the increased binding draws the quarks slightly down the Coulombic part of the QCD potential, shrinking the radius of the state.

There is, however, a further vital element in Particle Physics. Resonances such as $f_0(980)$ are attracted to thresholds. For $f_0(980)$, the amplitude for $\pi\pi \rightarrow KK$ is given to first approximation by the Flatté formula [3]:

$$A_{12} = T_{12}\sqrt{\rho_1\rho_2} = \frac{G_1G_2\sqrt{\rho_1\rho_2}}{M^2 - s - i[G_1^2\rho_1(s) + G_2^2\rho_2(s)]} = \frac{N(s)}{D(s)}$$

where phase space is factored out of $T$. Here $G_i = g_iF_i(s)$ and $g_i$ are coupling constants, $F_i$ are form factors. Writing $D(s) = M^2 - s - i\Pi(s)$, a more exact form for $D(s)$ is $M^2 - s - Re\Pi(s) - i\Pi(s)$, where

$$Re\Pi_{KK}(s) = \frac{1}{\pi}\int_{4m^2}^{\infty} ds' G^2_{KK}(s')\rho_{KK}(s')$$

(4)

Fig. 1 illustrates $Re\Pi(s)$ for $f_0(980)$ using $F_{KK} = \exp(-3k_{KK}^2)$, where $k_{KK}$ is $KK$ momentum in GeV/c. $Re\Pi$ acts as an effective attraction [4]. Parameters of $f_0(980)$ are known. Ref. [4] gives tables of pole positions when $M$, $g_1$ and $g_2$ are varied. If $M$ is as low as 500 MeV, there is still a pole at $806 - i78$ MeV; for $M$ in the range 850–1100 MeV, there is a pole within 23 MeV of the $KK$ threshold. The moral is that a strong threshold can move a resonance a surprisingly long way. For $f_0(980)$, $g_{KK}^2 \sim 0.7$ GeV$^2$.

![Figure 1: $Re\Pi_{KK}(s)$ and $G^2_{KK}\rho_{KK}(s)$ for $f_0(980)$, normalised to 1 at the peak of $G^2_{KK}\rho_{KK}$.

Since this article is written as much for experimentalists as theorists, it is worth pausing briefly to describe what lies behind Eq. (4). There is a wide class of differential equations which obey the principle of analyticity. We do not know the exact equation, but in all of these equations, real and imaginary parts of the amplitude are related. It is possible to do experiments which measure the amplitude for $\pi\pi \rightarrow KK$ at all energies. Eq. (4) relates real and imaginary
parts of the amplitude. Another way of looking at this is that a KK pair at one value of $s$ can change its energy for as long as the uncertainty principle allows and become ‘virtual’ particles which explore the whole range of $s$. For this brief instant of time, the KK pair describes a virtual loop to other values of $s$, a loop diagram. Evaluating Eq. (4) is equivalent to evaluating loop diagrams to all orders or solving the Bethe-Salpeter equation. Confined $q\bar{q}$ configurations do decay and the logic behind this paper is that we must therefore allow for effects due to the meson pairs into which they decay, and the associated singularities in $t$ and $u$ channels.

Barnes and Swanson [5] consider meson loops due to $D, D^*, D_s$ and $D_s^*$ meson pairs. For 1S, 1P and 2P charmonium states, they find large mass shifts which may be ‘hidden’ in the valence quark model by a change of parameters. The important conclusion from their work is that two-meson continuum components of charmonium states can be quite large, with the result that the naive constituent quark model may be misleading, particularly near the thresholds of opening channels. Lee et al. [6] give an illuminating discussion of $X(3872)$, discussed below in Section 2.1.

Eq. (4) has two virtues. Firstly, it is easily evaluated, secondly it illustrates graphically the effect of the form factor $F$. $\text{Re } \Pi(s)$ goes negative close to the peak of $G^2\rho$ and subsequently has a minimum at $\sim 1.7 \text{ GeV}$; thereafter it slowly rises to 0 as $M \to \infty$. Here an exponential form factor is used: $F = \exp(-3k^2)$, with $k$ the centre of mass kaon momentum in GeV/c.

A difficulty at present is that the form factor is not known precisely. Some authors use a different formula for the form factor. The usual Flatté formula, Eq. (3), serves as an approximate fitting function where $M$ and $g^2$ are fitted empirically. A second point is that the cusp changes slope abruptly at the $KK$ threshold. Analysis of data then requires a precise knowledge of experimental resolution if the cusp is included in the fit. This is illustrated for $a_0(980)$ in Ref. [7], where Crystal Barrel data on $\bar{p}p \to \eta\pi^0\pi^0$ are fitted including the cusp. The mass resolution (9.5 MeV) is known accurately in this case, but seriously smears out the cusp. A further detail is that separate thresholds for $K^+K^-$ and $K^0\bar{K}^0$ are not used in Fig. 1, for simplicity; these two thresholds may be taken into account straightforwardly [8].

Oset and collaborators demonstrate in a series of papers that resonances may be understood as ‘dynamically driven states’ [9] [10] [11] [12] [13] [14] [15] [16] due to s, t and u-channel exchanges. They take the view that $\bar{q}q$ and $qqq$ components are not needed at all in these cases. However, the well known $J/\Psi, ^3P_0(3415), ^3P_1(3510), ^3P_2(3556), \Psi(2S) \text{ and } \Psi(3770)$ are interpreted naturally as $c\bar{c}$ states (with tiny admixtures of the mesonic states to which they decay). Therefore it is logical to included the $q\bar{q}$ component for all other resonances unless there is a good reason why not.

How is it that Oset et al. are able to reproduce known resonances (approximately) with meson exchanges alone? They use S-wave form factors which are adjusted to get one predicted resonance of each paper at the right mass. Resulting amplitudes are strong. The form factors may be mocking up short range $q\bar{q}$ components. The importance of the work of Oset et al. is the demonstration that components derived from meson loops are large, and should be taken into account by theorists. However, one should not be deterred from invoking $q\bar{q}$ and $qqq$ components to get all resonances with their correct masses and widths.

There is little evidence for resonance decays to $I = 2 \pi\pi$ pairs or $I = 3/2 K\pi$, where interactions are repulsive [17]. The commonly observed SU(3) octets and decuplets are those whose decays do not lead to such repulsive final states. Higher representations such as $27, 10$ and $10$ do lead to such repulsive final states. The natural interpretation is that repulsive final states
actively inhibit formation of representations higher than octets and decuplets. The Variational Principle arranges that the configurations which are produced are those where repulsive final states are suppressed.

An approach which has recently been popular is to suppose that 'molecules' are formed from $\bar{q}qq$ configurations \[18\] \[19\] \[20\] \[21\] \[22\]. The well known problem with this approach is why so few tetraquarks have been observed. Vijande et al. \[23\] throw light on this issue. They study the stability of pure $c\bar{c}n\bar{n}$ and $cc\bar{n}\bar{n}$ states excluding diquark interactions. They find that all 12 $c\bar{c}n\bar{n}$ states with $J = 0, 1$ or 2 are unstable. The calculation points to the conclusion that such molecules are rare unless either (i) there are attractive diquark interactions, or (ii) coupling to meson-meson final states contributes, as proposed here.

Jaffe has proposed \[24\] that $\sigma$, $\kappa$, $a_0(980)$ and $f_0(980)$ are colourless 4-quark states made from a coloured SU(3) $3$ combination of $qq$ and a $\bar{3}$ combination of $\bar{q}\bar{q}$. This naturally leads to a light $\sigma$, an intermediate $\kappa$ and the highest (degenerate) masses for $a_0(980)$ and $f_0(980)$, in agreement with experiment. Note, however that meson-meson configurations lead to a similar spectrum except that the $a_0(980)$ might lie at the $\eta\pi$ threshold. This does not happen because of the nearby Adler zero at $s = m_\eta^2 - m_\pi^2/2$. The $a_0(980)$ migrates to the $KK$ threshold because the Adler zero in this case is distant, at $s = m_K^2/2$ \[25\]. Jaffe’s model does not agree well with the observed decay branching ratio $(\sigma \to KK) / (\sigma \to \pi\pi)$ near 1 GeV \[26\]. A serious problem is that, from the width of the $\sigma$ pole, the $\kappa$ width is predicted to be $(236 \pm 39)$ MeV, much less than the latest value: $(758 \pm 10$(stat) $\pm 44$(syst)) MeV \[17\].

A further point is that Maiani et al. \[18\] extend Jaffe’s scheme to $[c\bar{q}]|\bar{q}q]$ configurations $6 \otimes 3$. They give a firm prediction for the observation of analogues of $a_0(980)$ in $c\bar{c}n\bar{n}$ with $I = 1$ and charges 0, +1 and +2. There is no evidence for such states as yet.

The $\sigma$ and $\kappa$ poles are well predicted in both mass and width by the Roy equations \[27\] \[28\], which are based on $t$ and $u$-channel exchanges. Exchange of $\rho(770)$ and $K^*(890)$ make strong contributions. The Julich group of Janssen et al. \[29\] showed that meson exchanges account for $f_0(980)$ and $a_0(980)$. It seems unavoidable that all four states $\sigma$, $\kappa$, $a_0(980)$ and $f_0(980)$ are strongly driven by meson exchanges. The Roy equations do however impose the Adler zero coming from chiral symmetry breaking, a short-range effect.

A final comment concerns what happens to the upper energy combinations appearing in Eq. (2). As Jaffe remarks, they become broad and are likely to fall apart, creating a broad high mass background. The high mass tail of the $\sigma$ does behave in this sort of way above 1 GeV, due to coupling to $4\pi$ \[30\]; the precise form of this high mass behaviour is poorly known because of lack of data on $\pi\pi \to 4\pi$. For $J^P = 0^-$, there is also a conspicuous slowly varying component which appears in data on $J/\Psi \to \gamma 4\pi$ \[31\]; an alternative explanation in that case is that the broad component originates from $J/\Psi \to \gamma GG$, where $G$ are gluons which couple strongly to $\rho\rho$ with $L = 1$.

The conclusion from this review (and what follows in Section 2) is that meson loops play a strong role as well as confined $q\bar{q}$ and $qqq$ states. There is as yet no conclusive evidence for anything going beyond these two components, although there might be small perturbations from diquarks. This scenario will now be assumed and confronted with data.
Applications

2.1 \( X(3872) \)

It seems generally agreed now that the \( X(3872) \) is a linear combination of \( \bar{c}c \) and \( \bar{D}_0 D_0^* \), locked close to the \( \bar{D}_0 D_0^* \) threshold. A non-resonant cusp alone is too broad to fit the data. The coupling to \( \bar{D}D^* \) is weaker than that of \( f_0(980) \) to \( KK \), but the shape of the dispersion curve is similar; it is evident from the width of the cusp in \( Re \Pi \) of Fig. 1 that a cusp alone fails to fit the \( \sim 3 \) MeV width of \( X(3872) \). Lee et al. [6] make a detailed fit to existing data, solving the Bethe-Salpeter equation - equivalent to evaluating the dispersion integral of Eq. (4). The binding energy of \( X(3872) \) arises essentially from \( \bar{D}D^* \) loop diagrams. The magnitude of \( \pi \) exchange is known from the width of the decay \( D^* \to D\pi \); other exchanges are modelled. However, meson exchanges are not the essential source of binding. They simply need to be attractive, so that \( \bar{D} \) and \( D^* \) approach one another near threshold. Both \( \bar{D}_0 D_0^* \) and \( D^+ D^- \) channels contribute, though the \( X(3872) \) appears at the lower threshold. The binding energy is controlled sensitively by the form factor.

Kalashnikova and Nefediev point out [32] that there is decisive evidence for the role of the \( \bar{c}c \) component from the decay width to \( \gamma \Psi' \); this cannot be described by a pure molecular model. The branching fraction for \( B \to KX \) is also too large to be explained by a molecule alone.

The narrow width of \( X(3872) \) arises because its decay modes to \( \pi^+ \pi^- J/\Psi, \omega J/\Psi \) and possibly \( \chi(3510)\sigma \) are OZI suppressed, therefore weak. Its coupling to \( \bar{D}^0 D^{*0} \) over the width of the resonance is also weak. However, the coupling to \( \bar{D}D^* \) rises rapidly above threshold and produces the binding via virtual loop diagrams. Kalashnikova and Nefediev conclude that the \( \bar{c}c \) state is attracted to the \( \bar{D}D^* \) threshold. Ortega et al. [33] reach a similar conclusion that \( X(3872) \) must have a large \( \bar{D}D^* \) component.

Gamermann et al. [34] conclude that a molecular state is consistent with the decays to \( \pi^+ \pi^- J/\Psi \) and \( \omega J/\Psi \), but do not consider the other evidence for a \( \bar{c}c \) component.

2.2 Not all cusps are resonances

There is a cusp at the \( \pi d \) threshold [35], but no resonance. The exotic \( Z^+(4430) \) of Belle [36] is at the threshold for \( D^*(2007)D_1(2420) \) and has a width close to that of \( D_1(2420) \). The data can be fitted as a resonance, but can also be fitted successfully by a non-resonant cusp, see Fig. 6 of [4]. Additionally, Babar do not confirm the existence of the \( Z(4430) \).

2.3 Light Vector Mesons

As an introduction to the vector mesons amongst light quarks, it is useful to read my review of Crystal Barrel data in flight [37]. The first three figures of this paper show plots of resonance masses as a function of \( s = \) mass squared. Fig. 2 below shows two examples. They resemble Regge trajectories, except they are drawn for one set of quantum numbers at a time. There is a striking agreement between all trajectories, with a common slope of \( 1.143 \pm 0.013 \) GeV\(^2\) for each unit of excitation. A similar regularity is observed for baryon resonances with similar slope [38].
The $\rho(1900)$ lies close to the $N\bar{N}$ threshold and it is well known that the $\bar{p}p$ interaction is strongly attractive. It is natural to interpret $\rho(1900)$ as the $n = 3\,^3S_1$ $n\bar{n}$ state mixed with $\bar{p}p$, i.e. attracted to the threshold. Then other $\rho$ states fall into place as follows:

(ii) $\rho(2000) = ^3D_1, n = 2$. It is observed in three sets of data: $\pi^+\pi^-, \pi\omega$ and $a_0\omega$. There are extensive differential cross section and polarisation data on $\bar{p}p \rightarrow \pi^+\pi^-$ from the PS 172 experiment down to a mass of 1910 MeV (a beam momentum of 360 MeV/c) [39]. There are further similar data above a beam momentum of 1 GeV/c from an experiment at the Cern PS of Eisenhandler et al [40]. The polarisation data determine the ratio of decay amplitudes to $^3D_1$ and $^3S_1\bar{p}p$ configurations: $r_{D/S} = g_{\bar{p}p}(^3D_1)/g_{\bar{p}p}(^3S_1) = 0.70 \pm 0.32$; for the low available momentum in $\bar{p}p$, this is a rather large $^3D_1$ component.

(iii) $\rho(2150) = ^3S_1, n = 4$. It is seen in $\pi^+\pi^-$ data of [39] and [40] and in Crystal Barrel data for $a_0(980)\pi$ and in GAMS and Babar data [41]; (the Particle Data Group incorrectly lists the 1988 MeV state of Hasan [42] under $\rho(2150)$, but it is the $\rho(2000)$). For $\rho(2150)$, $r_{D/S} = -0.05 \pm 0.42$.

(iv) $\rho(2265) = ^3D_1, n = 3$. It is observed only in two sets of data, $\pi^+\pi^-$ and in Crystal Barrel data for $a_2\omega$ and therefore needs confirmation; it has a large error for $r_{D/S}$. The $\rho(1700)$, $\rho(2000)$ and $\rho(2265)$ are consistent within errors with a straight trajectory with the same slope as other states, see Fig. 2(b).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Trajectories of (a) $I = 0, J^{PC} = 2^{++}$ and (b) $I = 1, J^{PC} = 1^{--}$ $n\bar{n}$ resonances.}
\end{figure}

Confirmation of these states and clarification of the mass range below 1700 MeV would be welcome, particularly the separation into $^3D_1$ and $^3S_1$. This could be done at VEPP 2 ($e^+e^-$ up to 2 GeV/c) and VEPP 4 (2–4 GeV/c) in Novosibirsk using large acceptance spectrometers available there. If an electron beam with transverse polarisation is used, the initial state is a 50:50 combination of $^3S_1$ and $^3D_1$, since the electrons are highly relativistic. Decays to the final state are not highly relativistic and $^3D_1$ decays contain a conspicuous dependence on the azimuthal angle $\phi$ with respect to the plane defined by the beam and initial polarisation. Both $\cos 2\phi$ and $\sin \phi$ terms appear, and should provide a clean separation of the magnitude and phase of the $^3D_1$ decay amplitude.

(v) The $Y(2175)$ [41] observed by BES 2 and Babar in $\phi f_0(980)$ and $K^+K^-f_0(980)$ makes a natural $s\bar{s}$ partner for $\rho(2000)$. Note that there is sufficient momentum in the final state to allow a $^3D_1$ state.
2.4 \(J^P = 2^+\) light mesons

The \(f_2(1565)\) lies at the \(\omega\omega\) threshold. It is distinctly lower in mass than \(a_2(1700)\) and has clearly been attracted to the \(\omega\omega\) threshold. The mass shift approaches 150 MeV and demonstrates the importance of the molecular component. The \(f_2(1565)\) also appears clearly in \(\pi\pi\), as observed by several groups \[41\]. It should appear in \(\rho\rho\) with \(g_{\rho\rho}^2 = 3g_{\omega\omega}^2\) by SU(2) symmetry, which predicts \(g(\rho^0\rho^0) = -g(\omega\omega)\) because of the similar masses of light quarks and the close masses of \(\rho(770)\) and \(\omega(782)\).

The \(f_2(1640) \to \omega\omega\) observed by GAMS and VES \[41\] may be fitted by folding the line-shape of \(f_2(1565)\) with \(\omega\omega\) phase space and a reasonable form factor \[43\], together with the dispersive term \(Re\Pi(s)\). There is no need for separate \(f_2(1565)\) and \(f_2(1640)\). This has confused a number of theoretical predictions of the sequence of \(2^+\) states.

Fig. 2(a) shows trajectories for \(2^{++}\) states, including those above the \(\bar{p}p\) threshold from Crystal Barrel data in flight, using trajectories with a slope of 1.14 GeV\(^2\).

The PDG makes errors in reporting the Crystal Barrel publications. It lists \(f_2(2240)\) under \(f_2(2300)\), which is observed in \(\phi\phi\) and \(KK\) by all other groups. The \(f_2(2300)\) is naturally interpreted as an \(ss\) state. Both \(f_2(2240)\) and \(f_2(2293)\) are observed in a combined analysis of ten sets of data: four sets of PS172 and Eisenhandler et al., together with Crystal Barrel data for \(\eta\pi^0\pi^0, \eta'\pi^0\pi^0, \eta\eta, \eta'\eta\) and \(\eta\eta'\). The data from the last 3 channels are fitted to a linear combination \(\cos\phi|n\bar{n}\rangle + \sin\phi|s\bar{s}\rangle\) and the mixing angle is determined to be \(\phi = 7.5^\circ\) for \(f_0(2240)\) and \(\phi = -14.8^\circ\) for \(f_2(2293)\) \[44\]. So the \(f_2(2240)\) is certainly not an \(ss\) state. The \(f_2(2240)\) is dominantly \(3P_2\) with \(r_{F/P} = 0.46 \pm 0.09\) (defined like \(r_{D/S}\)) and the \(f_2(2300)\) is largely \(3F_2\) with \(r_{F/P} = -2.2 \pm 0.6\). The PDG fails to list the \(f_2(2293)\) at all.

A further comment on PDG listings is that the \(f_2(2150)\) is conspicuous by its absence from Crystal Barrel data in flight. All \(ss\) states such as \(f_2(1525)\) are produced very weakly in \(\bar{p}p\) interactions. The \(f_2(2150)\) is observed elsewhere mostly in \(KK\) and \(\eta\eta\) channels. It is therefore naturally interpreted as an \(ss\) state, the partner of \(f_2(1905)\). The observation of an \(f_2(2135)\) by Adomeit et al. \[45\] was qualified in the publication by the warning that it could be due to the overlap of two or more \(f_2\) states. That was subsequently shown to be the case \[46\] in a later analysis of data with statistics a factor 7 larger and at 9 momenta instead of the two used by Adomeit et al. The later publication withdrew the claim for an \(f_2(2135)\) and the PDG was informed.

An important systematic observation is that \(\bar{p}p\) states tend to decay with the same \(L\) as the initial \(\bar{p}p\) state. There is a simple explanation, namely good overlap of the initial and final states in impact parameter. This observation may be useful to those calculating decays, hence mesonic contributions to eqns. (1)–(4).

2.5 Light \(0^+\) mesons

In BES 2 data for \(J/\Psi \to \omega K^+K^-\), there is a clear \(f_0(1710) \to KK\) \[47\]. In high statistics data for \(J/\Psi \to \omega\pi^+\pi^-\), \[48\] there is no visible \(f_0(1710)\), setting a limit on branching ratios: \(BR(f_0(1710) \to \pi\pi)/BR(f_0(1710) \to KK) < 0.11\) with 95% confidence. Thirdly, in \(J/\Psi \to \phi\pi^+\pi^-\), there is a \(\pi\pi\) peak requiring an additional \(f_0(1790)\) decaying to \(\pi\pi\) but weakly to \(KK\) \[49\]. It is readily explained as the radial excitation of \(f_0(1370)\). There is ample independent evidence for it in \(J/\Psi \to \gamma 4\pi\) \[50\] \[51\] \[31\] and \(\bar{p}p \to \eta\eta\pi^0\) in flight \[52\]. BES 2 also report an
ωφ peak of 95 events at 1812 MeV; \( J^P = 0^+ \) is favoured. It is confirmed by VES data at the Hadron09 conference. The BES 1812 MeV peak may be fitted well with the \( f_0(1790) \) line-shape folded with \( \omega \phi \) phase space and a form factor \( \exp \left(-3k^2_{\omega \phi} \right) \). The exotic signal is naturally interpreted as a \( 0^+ \) glueball component overlapping this mass range and mixing into \( f_0(1790) \):

\[
 gg \rightarrow (u\bar{u} + d\bar{d} + s\bar{s})(u\bar{u} + d\bar{d} + s\bar{s}) \rightarrow 4s\bar{s}n\bar{n}
\]

A comment is needed on \( f_2(1810) \) of the Particle Data Tables. It does not fit in naturally in Fig. 2(b). The spin analysis of the GAMS group finds a very marginal difference between spin 0 and spin 2. It depends on a fine distinction based on a difference in angular distribution depending strongly on experimental acceptance; however, no Monte Carlo of the acceptance is shown. With the benefit of hindsight, it seems possible that this was in fact the first observation of \( f_0(1790) \).

2.6 Glueballs

Morningstar and Peardon predict glueball masses in the quenched approximation where \( q\bar{q} \) are omitted. When mixing with \( q\bar{q} \) is included, mixing is likely to lower glueball masses.

2.7 Broad Thresholds

Broad thresholds do play a role. For \( f_0(1370) \), including dispersive effects, the Argand diagram still follows a circle closely. Experimentalists can then safely omit dispersive effects for such cases as a first approximation. However, phase shifts depart significantly from a Breit-Wigner of constant width; at low mass, where \( 4\pi \) phase space is small, the phase shift varies more rapidly and at high mass more slowly. A further point is that it is vital to include the effect of rapid changes in phase space in the Breit-Wigner numerator. The peak of \( f_0(1370) \rightarrow 4\pi \) is \( \sim 110 \) MeV higher than that in \( f_0(1370) \rightarrow 2\pi \) because of rapidly increasing \( 4\pi \) phase space. Likewise \( \eta(1405) \) and \( \eta(1475) \) may be fitted as two decay modes of a single \( \eta(1440) \). The \( \eta(1475) \) is seen only in \( KK^*(890) \), where phase space rises from threshold near 1385 MeV as momentum cubed in the final state; the \( \eta(1405) \) is seen in \( \kappa K \) S-waves and \( \eta\pi\pi \) where phase space changes slowly.

3 Remarks on further experiments

There has been important progress in \( c\bar{c} \) spectroscopy of narrow states and low mass broad states. This will probably continue for \( b\bar{b} \) states. However, the chances of sorting out the large number of broad states at high mass look remote.

Further progress towards a complete spectroscopy of light mesons and baryons is important for an understanding of confinement - one of the key phase transitions in physics. Progress is possible by measuring transverse polarisation in formation processes. Consider \( pp \) as an example. The high spin states appear clearly as peaks, e.g. \( f_4(2050) \) and \( f_4(2300) \). These serve as interferometers for lower states. However, differential cross sections measure only real parts of interferences. This leaves the door open to two-fold ambiguities in relative phases and large
errors if resonances happen to be orthogonal. A measurement of transverse polarisation normal to the plane of scattering measures $Tr < A^\ast \sigma_y A >$, where $A$ is the amplitude. This measures the imaginary part of interferences. The phase sensitivity is important in eliminating ambiguities between amplitudes. What appears to be less well known is that transverse polarisation in the sideways (S) direction gives additional information for three and four-body final states with a decay plane different to the plane defined by the beam and initial state polarisation. This depends on $Tr < A^\ast \sigma_x A >$, and measures the real parts of exactly the same interferences as appear from the $\sigma_y$ operator. Longitudinal polarisation depends only on differences of two intensities and is less useful.

An example of a simple experiment which would pay a rich dividend is to measure such polarisations with the Crystal Barrel detector at the forthcoming GSI $\bar{p}$ source, over the same mass range as used at LEAR. An extracted beam with these momenta will be available at the FLAIR ring. Such measurements could indeed have been made at LEAR if it had not been sacrificed to the funding of the LHC. The present situation is that the amplitudes for $I = 0, C = +1$ states are unique for all expected $J^P$. For $I = 1, C = -1$, they are nearly complete, but there are some weaknesses for low spin states, notably $^3S_1$ which leads to a flat decay angular distribution. For $I = 1, C = +1$ there is a two-fold ambiguity for $\eta\pi$ final states and crucial $J^P = 0^+$ states are missing. For $I = 0, C = -1$ there are many missing states.

The final states to be measured are (i) $\eta\pi$ and $\eta\eta\pi$ ($I = 1, C = +1$), (ii) $\omega\pi$ ($I = 1, C = -1$), (iii) $\omega\eta$ and $\omega\eta\pi^0$ ($I = 0, C = -1$). A measurement of $\eta\pi^0\pi^0$ would also cross-check the existing solution and provide information on interferences between singlet and triplet $\bar{p}p$ states. All of these channels can be measured simultaneously with the existing Crystal Barrel detector.

A Monte Carlo simulation of results extrapolated from existing analyses predicts a unique set of amplitudes for all quantum numbers. Data are required from 2 GeV/c down to the lowest possible momentum $\sim 360$ MeV/c. This cannot be done by the PANDA experiment which will not go significantly below 2 GeV/c; the two experiments should be viewed as complementary.

Seven of the nine momenta studied at LEAR were run in 3 months of beam-time, so it is not a long experiment, nor does it demand beam intensities above $10^5 \bar{p}/s$. A Monte Carlo study shows that backgrounds from heavy nuclei in the polarised target (and its cryostat) should be at or below an average level of 10%; this is comparable with cross-talk between final states and is easily measured from a dummy target. A frozen spin target is required, but the technology already exists in Bonn. The essential cost is to move the detector to and from Bonn.

Baryon spectroscopy would also benefit from similar $\pi^\pm p$ measurements, although they are more difficult. The essential problem is incompatibility between the uniform holding field required for the polarised target and the use of a magnetic field over the detector to measure charged particles. The feasibility of such an experiment is discussed in Ref. [58], using a Monte Carlo simulation based on the geometry and performance of the Crystal Barrel detector. The CLEO C detector, now idle, though still buried under concrete shielding, is an obvious alternative. For $\pi^- p$, all neutral final states can be measured with 3C kinematics if the neutron is detected in the calorimeter. For $\pi^+ p$, several important final states are accessible with 2C kinematics, e.g. $N(\pi\pi)$ and $\Delta(\pi\pi)$. Rates are enormous, so running time is governed essentially by down-time required for polarising the target and changing momenta. Data at 30 MeV steps of mass appear sufficient, except close to 2-body thresholds such as $\omega N$. 
4 Conclusions

The objective of this paper has been to make a case for what appears logically necessary, namely that both quark combinations at short range and decay channels at large range contribute to the eigenstates. The $X(3872)$ is a prime example of mixing between $c\bar{c}$ and meson-meson in the form of $D\bar{D}^*$. In view of the calculations of Oset et al. and Barnes and Swanson, it seems likely that many resonances contain large mesonic contributions.

The data on $a_0(980)$, $f_0(980)$, $f_2(1565)$ and $\rho(1900)$ fit naturally into this picture. There must be a large mesonic contribution to the nonet of $\sigma$, $\kappa$, $a_0(980)$ and $f_0(980)$, but there could be a modest diquark component as well. It is likely that there will be a small $q\bar{q}$ component, but this is suppressed by the $L = 1$ centrifugal barrier for $^3P_0$ combinations; it appears strongly in the mass range of $f_0(1370)$ and higher states.

Experimentalists must take care to fit the $s$-dependence of the numerator of Breit-Wigner resonances due to phase space, e.g. $f_0(1370) \to 2\pi$ has a very different line-shape to $f_0(1370) \to 4\pi$. The denominator may be fitted as a first approximation with a Breit-Wigner resonance of constant width; however, for high quality data, the effect of the dispersive component in the real part of the denominator matters. For sharp thresholds, e.g. $f_0 \to KK$, the Flatté formula is an approximation; with high quality data, the correction due to the cusp in Re $\Pi(s)$ is important, but requires precise information on experimental resolution.

Further experiments on transverse polarisation in inelastic processes are needed and appear to be practicable without large cost.

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