Research on Equivalent Magnetic Circuit Modeling and Electromagnetic Design of Vehicle Inverted-Triangle Interior Permanent Magnet Synchronous Motor

Qingjiang Han*, Jian Luoa, Minqi Hua b and Xun Wangc

School of Mechatronics Engineering and Automation, Shanghai University, Shanghai, China

*Corresponding author e-mail: h184gh@163.com, aluojian@shu.edu.cn, bhuaminqi@shu.edu.cn, cwangxun950208@163.com

Abstract. The inverted-triangle permanent magnet interior rotor is a new type of motor rotor structure for vehicles. First, this paper introduces and qualitatively analyzes the inverted-triangle interior permanent magnet synchronous motor (IPMSM), and establishes the equivalent magnetic circuit model of the inverted triangle IPMSM under no-load and load conditions. Secondly, based on the equivalent magnetic circuit model, a preliminary design of an inverted-triangle IPMSM for an 8-pole 48-slot vehicle was carried out. Finally, the finite element software is used to optimize the rotor of the motor to obtain ideal electromagnetic performance. The simulation results show that with the same torque performance, the amount of permanent magnets of the inverted-triangle IPMSM is significantly less than that of the V-shaped IPMSM, which is beneficial to reducing the cost of motors for automobiles.

1. Introduction

At present, the exhaust and noise pollution of fuel vehicles in cities will have a negative impact on the health and appearance of citizens; on the other hand, the world is suffering from the impact of the energy crisis. Based on the above, governments, businesses and academia in various countries have paid more attention to the development of electric vehicles [1].

Electric vehicles use electric motors instead of internal combustion engines to provide power to cars, so driving motors are one of the key components of electric vehicles. The selection of automotive motors is mainly induction motors [2], interior permanent magnet synchronous motors (IPMSM) [3], and axial flux permanent magnet synchronous motors (AFPMSM) [4, 5]. Among them, the IPMSM is the most popular in the world due to its high efficiency, wide speed regulation range, and mature production technology, and it is also widely studied [6].

The structure of traditional automotive IPMSM is mainly straight-shaped and V-shaped, as shown in Fig. 1. Many scholars have studied IPMSM of these two structures. The authors of [7] analyzed the influence of the mounting position of the straight-shaped permanent magnets on the cogging torque using analytical methods and finite element simulation. The authors of [8] compared the effects of a straight-shaped rotor structure and a V-shaped rotor structure on motor performance. The authors of [9] used an asymmetric V-shaped rotor structure to reduce the torque ripple of the motor.
With the continuous improvement of the driving performance and cost performance of electric vehicles, many auto companies have adopted various new motor structures to improve the performance of motors for vehicles and reduce their costs. As shown in Fig. 2, the vehicle motor in BMW i3 uses a double straight-shaped IPMSM rotor structure, and Toyota's 2017 Prius vehicle motor uses an inverted triangle-shaped permanent magnet interior rotor structure. These new structures achieve higher torque density and power density, while reducing the use of permanent magnets. Very few studies have been published on these new rotor structures. This article takes the inverted-triangle shape permanent magnet synchronous motor as the research object.

The current common motor design methods are equivalent magnetic circuit method [5, 10], electromagnetic field analysis method [11], and electromagnetic field numerical calculation method [12]. The advantages of the equivalent magnetic circuit method are simple calculations, and the disadvantages are low accuracy and inability to account for complex situations such as core saturation. It is suitable for performance analysis and main parameter calculation in the early stages of motor research and design. The electromagnetic field analysis method is based on the theory of electrodynamics. The parameters of the analysis object are obtained by establishing and solving differential equations. This method has high calculation accuracy, but is not suitable for analyzing objects with complex structures. The electromagnetic field numerical calculation method divides the analysis object into many units, and analyzes the electromagnetic field in the motor by solving the equations of each unit. This method has the most accurate results, and there are many commercial software that can perform electromagnetic field calculations, such as Maxwell, Jmag, and Opera. Therefore, this paper uses the equivalent magnetic circuit method to preliminary design the inverted triangle IPMSM, and uses Maxwell to optimize the motor design.
This paper uses the equivalent magnetic circuit model to determine the initial critical dimensions of an IPMSM with 8-pole and 48-slot. Based on Maxwell software, the width and thickness of permanent magnets and the position and size of permanent magnet slots were optimized to obtain the ideal electromagnetic performance of the motor. Finally, the electromagnetic simulation results of the motor are given and compared with V-shaped IPMSM with the same torque performance. The validity of the design method and the superiority of the inverted triangle rotor structure are verified.

2. Basic content of inverted-triangle shape IPMSM magnetic circuit method

2.1. Qualitative Analysis of Inverted-Triangle IPMSM

The cross-sectional view of the inverted-triangle IPMSM is shown in Fig. 3. Each pole in the rotor is composed of a straight-shaped permanent magnet and a V-shaped permanent magnet. The polar arc coefficient of the straight-shaped permanent magnet is smaller than that of the V-shaped permanent magnet. The three permanent magnets present an inverted triangle.

As shown by the red line in Fig. 4, each pole of the inverted-triangle IPMSM has 3 q-axis magnetic flux circuits, which can generate a large reluctance torque [13]. In contrast, straight IPMSM and V-shaped IPMSM have only 2 q-axis magnetic flux loops under each pole. Therefore, the inverted-triangle permanent magnet interior structure has higher torque density than straight and V-shaped. Under the same peak torque performance index, the former saves the permanent magnet more than the latter two, which can effectively reduce costs.
2.2. Basics of Motor Magnetic Circuit Method
When using the magnetic circuit method to analyze the motor, the main considerations are the permanent magnet magnetic potential, the armature winding magnetic potential, the stator and rotor core, and the air gap's magnetic resistance in the magnetic circuit [14]. The magnetic circuit of the permanent magnet and the armature winding's magnetic potential can be divided into the main magnetic circuit and the leakage magnetic circuit. A permanent magnet can be equivalent to a magnetic flux source and an internal permeance in parallel, or a magnetomotive force source and an internal magnetoresistance in series. Based on this, the equivalent magnetic flux permeability model and equivalent magnetic potential reluctance model of the motor can be established, as shown in Fig. 5. Among them, $\Phi_r$ is the virtual internal magnetic flux (Wb) of the permanent magnet, $\Phi_o$ is the virtual internal magnetic leakage flux (Wb) of the permanent magnet, $\Phi_m$ is the magnetic flux per pole (Wb) provided by the permanent magnet to the external magnetic circuit, and $\Phi_s$ is the motor magnetic circuit leakage flux (Wb), $F_m$ is the magnetomotive force (A) provided by the two ends of the permanent magnet to the external magnetic circuit, and $F_a$ is the composite magnetic motion generated by the armature winding Potential (A), $F_c$ is the magnetomotive force provided by the permanent magnet (A), $A_o$ is the magnetic flux in the permanent magnet, $A_s$ is the magnetic flux leakage path of the motor, $A_{o6}$ is the magnetic flux path of the motor, and $R_o$ is the magnet resistance, $R_c$ is the magnetic resistance of the motor leakage circuit, and $R_{s6}$ is the magnetic resistance of the main magnetic circuit of the motor.

![Equivalent Magnetic Circuit Models of Permanent Magnet Motor.](image)

The above two models are the simplest models of the equivalent magnetic circuit method of the motor. Other magnetic circuit models are based on these two models. In order to reduce the parallel connection in the magnetic circuit, the magnetic circuit modeling of this paper is carried out based on the equivalent magnetomotive reluctance model.

2.3. Magnetic circuit analysis of inverted-triangle IPMSM
The pole slot of the motor in this paper is matched with 8 poles and 48 slots. The stator and rotor magnetic circuit unit of the inverted-triangle IPMSM is shown in Fig. 6. The magnetic field lines of the other parts of the motor are consistent with this unit. In Fig. 6, the blue line represents the path of the leakage magnetic flux, and the red line represents the path of the main magnetic flux. The leakage magnetic flux can be divided into 3 parts. The first part is the leakage flux of the straight permanent magnet in the rotor core. The second part is the leakage flux of the V-shaped permanent magnet along the magnetically isolated bridge along the outer peripheral surface of the rotor. The third part is the leakage flux of the V-shaped permanent magnet along the magnetic isolation bridge at the bottom of the V-shape. The main magnetic flux is composed of two parts in parallel. The first part is the part passed by the inner red line in Fig. 6, which includes two V-shaped permanent magnets, air gaps and stator and rotor cores. The second part is the part passed by the red line on the outside in Fig. 6, which includes two V-shaped permanent magnets, two straight permanent magnets, air gaps, and stator and rotor cores. It can be seen that the two parts that make up the main magnetic flux in parallel are in
series structure. Therefore, the main magnetic flux of the inverted-triangle IPMSM includes both a parallel structure and a series structure.

Figure 6. Magnetic Circuit Diagram of inverted-triangle shape IPMSM.

3. No-load equivalent magnetic circuit model of motor
According to the analysis in the previous section, an equivalent magnetic circuit model can be established under no-load of the motor, as shown in Fig. 7, where $R$ represents the magnetic resistance and $F$ represents the magneto motive force; $R_s$ and $R_r$ are the magnetic resistance between the stator yoke magnetic resistance and the two magnetic poles in the rotor, respectively; $R_{st1}$, $R_{st2}$, $R_{st3}$, and $R_{st4}$ are the magnetic resistance of the stator teeth; $F_{one1}$, $F_{one2}$ are the magneto motive forces of two linear permanent magnets in the magnetic circuit; $F_{v1}$, $F_{v2}$, $F_{v3}$, and $F_{v4}$ are the magneto motive forces of two V-shaped permanent magnets; $R_{oner1}$ and $R_{oner2}$ are the magnetic resistance of the magnetic flux leakage circuit of the straight permanent magnet in the rotor core; $R_{rml}$, $R_{rml2}$, $R_{rml3}$, and $R_{rml4}$ are the reluctances of the leakage circuit of two V-shaped permanent magnets in the rotor core; $R_{onem1}$, $R_{onem2}$ are internal reluctances of straight permanent magnets; $R_{vm1}$, $R_{vm2}$, $R_{vm3}$, $R_{vm4}$ are V-shaped permanent magnets, each of which is divided into four internal magnetic resistances. $R_{g1}$, $R_{g2}$, $R_{g3}$, $R_{g4}$ are air gap magnetic resistance.

Figure 7. Equivalent magnetic circuit model of motor under no load.

The calculation formula for a single interior permanent magnet magneto motive force $F_{pm}$ is as follows:
Where $H_c$ is the coercive force of the permanent magnet, $h_{pm}$ is the magnetization length of the permanent magnet. For a parallel magnetized permanent magnet, it is the thickness of the permanent magnet.

The calculation formula of the magnetic flux leakage resistance $R_{ir}$ of a single permanent magnet in the rotor core is as follows:

$$R_{ir} = \frac{\mu_0 l_{pm}}{h_{pm}(1 + \frac{\pi}{2})} \int_{a_1}^{b_1} dx$$

(2)

Where $\mu_0$ is the vacuum permeability, $l_{pm}$ is the axial length of the permanent magnet, and $a_1$ and $b_1$ are determined by the magnetic flux leakage path.

The calculation formula of the internal reluctance $R_{m0}$ of a single permanent magnet is as follows:

$$R_{m0} = \frac{h_{pm}}{\mu_0 \mu_r S_{pm}}$$

(3)

Where $\mu_r$ is the relative permeability of the permanent magnet, and $S_{pm}$ is the equivalent cross-sectional area of the permanent magnet.

The calculation formula for a single air gap magnetic resistance $R_g$ is as follows:

$$R_g = \frac{h_g}{\mu_0 S_g}$$

(4)

Where $h_g$ is the equivalent air gap length, and $S_g$ is the equivalent cross-sectional area of the air gap at the permanent magnet. Its formula is:

$$S_g = \frac{D_g \theta_g}{2}$$

(5)

Where $D_g$ is the air gap equivalent diameter, $\theta$ is the mechanical angle (rad) occupied by a single permanent magnet, and $l_g$ is the axial length.

The magnetic permeability of the iron core is between 10,000 and 100,000 times that of the vacuum, and the leakage magnetic flux also accounts for a small proportion of the total magnetic flux, so the magnetic resistance of the iron core and all magnetic leakage can be ignored. The simplified equivalent magnetic circuit model is obtained as shown in Fig. 8.
Figure 8. Simplified magnetic circuit model of motor under no load.

4. Motor load equivalent magnetic circuit model
Under ideal load conditions, the three-phase stator windings of the motor pass a sinusoidal phase current with a phase difference of 120 degree, and the rotating magneto motive force with constant amplitude can be obtained in the motor. The d-axis magneto motive force $F_d$ and q-axis magneto motive force $F_q$ can be obtained through coordinate transformation of the magneto motive force. Fig. 9 and Fig. 10 show the d-axis equivalent magnetic circuit model and the q-axis equivalent magnetic circuit model.

Figure 9. D-axis equivalent magnetic circuit model.

Figure 10. Q-axis equivalent magnetic circuit model.

Where $R_{qr1}$, $R_{qr2}$, and $R_{qr3}$ are the magnetic resistance of the path of the q-axis magnetic circuit in the rotor core, $R_{st5}$ to $R_{st10}$ are the magnetic resistance of the stator teeth in the q-axis magnetic circuit, and $R_{g5}$ to $R_{g10}$ are air gap magnetic resistances in the q-axis magnetic circuit.

The calculation formula for the air gap magnetic resistance $R_{gq}$ in the q-axis magnetic circuit is as follows:
Where $h_{sp}$ is the equivalent air gap length of the q axis, and $S_{sp}$ is the air gap area of each q axis magnetic circuit. The calculation formula is as follows:

$$S_{sp} = \frac{D_g (\pi/2 - \theta_q) l_g}{2}$$  \hspace{1cm} (7)

Where $\theta_q$ is the mechanical angle occupied by a single q-axis.

5. Electromagnetic Design and Results of IPMSM

5.1. Preliminary motor design

The target electromagnetic performance parameters of the motor designed in this paper are shown in Table 1. The motor has an outer diameter of 220mm and an axial length of 140mm. The motor pole slot is matched with 8 poles and 48 slots, and a single layer winding is used. The core material of the motor is B35AH230 of Baoshan Iron and Steel Plant, and the permanent magnet is N35EH. The cogging part adopts a groove parallel structure.

| Parameter | Value |
|-----------|-------|
| Rated/peak power | 75/160kw |
| Rated/ peak torque | 143/320N.m |
| DC bus voltage | 370 VDC |
| Phase current | ≤500Arms |
| Rated/peak speed | 5000/16000 rpm |
| Effectiveness | ≥97.5 ± 0.5% |
| Cogging torque | -2~ +2N.m |
| Peak no-load back-EMF at maximum speed | ≤700V |
| Proportion of regions with efficiency greater than 90% | >90% |

According to the equivalent magnetic circuit models in Sections 2 and 3, the key topology size of the motor can be calculated. The preliminary values of the key dimensions of the stator are shown in Table 2. Due to the complicated structure of the rotor, the initial values of key dimensions are shown in Fig. 11 and Fig. 12.

| Parameter | Value |
|-----------|-------|
| Stator outer diameter | 220mm |
| Stator inner diameter | 151mm |
| Air gap length | 0.7mm |
| Stator yoke thickness | 14.5mm |
| Tooth minimum width | 5.9mm |
| Slot width | 2.5mm |
5.2. Preliminary motor design

After the initial design of the motor, the motor was optimized based on the finite element software Maxwell. After optimizing the shape of the permanent magnet slot, the size of the permanent magnet, the angle of the V-shaped permanent magnet, and the slot width, the better electromagnetic performance of the motor is obtained. The optimized motor is shown in Fig. 13. The optimized key dimensions of the stator are shown in Table 3. The optimized values of the critical dimensions of the rotor and permanent magnet are shown in Fig. 14 and Fig. 15. As shown in Fig. 16, in order to reduce the torque pulsation, auxiliary grooves are added to the outer peripheral surface of the rotor, the depth of which is 0.2 mm and the width is 2 mm.

Table 3. Optimal value of stator key size.

| Parameter               | Value  |
|-------------------------|--------|
| Stator outer diameter   | 220mm  |
| Stator inner diameter   | 151mm  |
| Air gap length          | 0.7mm  |
| Stator yoke thickness   | 15.5mm |
| Tooth minimum width     | 5.8mm  |
| Slot width              | 1.7mm  |
Figure 13. Optimized motor.

Figure 14. Optimization of key parameters of permanent magnet.

(a)V-shaped permanent magnet slot               (b)Straight-shaped permanent magnet slot

Figure 15. Optimization of key parameters of permanent magnet slots.

Figure 16. Schematic diagram of auxiliary grooves on outer surface of rotor.
The electromagnetic performance results of the motor after optimization are shown in Figure 17-21. Under the peak torque condition, the torque of the motor is 341.6Nm. Under peak speed conditions, the maximum value of induced electromotive force (line voltage) is 670V. The stator winding current under rated torque is 240A. The cogging torque of the motor is $+1.12 \sim -1.12$Nm. The highest efficiency of the motor is 98.4%. All electromagnetic properties are in line with the target performance.

Figure 17. Torque curve under peak torque condition.

Figure 18. Induced potential curve under peak speed condition (16000rpm).

Figure 19. Torque curve under rated condition.

Figure 20. Torque ripple curve.
The magnetic flux density cloud diagram of the motor under peak torque conditions is shown in Fig. 22. The magnetic flux density of the stator tooth tip, the magnetic isolation bridge and the vicinity of the magnetic isolation bridge is relatively large, exceeding 2.2T, and the magnetic flux density of the rest is about 1.8T at the highest.

5.3. **Comparison of inverted-triangle IPMSM and V-shaped IPMSM**

This paper uses the same method to design a V-shaped IPMSM with the same torque performance, and compares it with the inverted-triangle IPMSM designed in this paper, as shown in Figure 23. According to Figure 24, the amount of permanent magnets of the inverted-triangle IPMSM is only 62.6% of the V-shaped IPMSM.
6. Conclusion

In this paper, through theoretical research and simulation, the following conclusions are obtained: 1. There is a series relationship and a parallel relationship in the main magnetic circuit of the inverted-triangle IPMSM. 2. The effectiveness of the equivalent magnetic circuit method in the preliminary design of the inverted-triangle IPMSM is verified. 3. During the optimization design of the motor, it was found that the shape of the permanent magnet slot has a significant effect on the torque ripple and the magnitude of the induced electromotive force. 4. Inverted-triangle IPMSM has 3 q-axis circuits under each magnetic pole, which can provide large reluctance torque. This advantage makes it possible to reduce the amount of permanent magnets of inverted-triangle IPMSM with the same torque performance. The shortcomings of this paper are that the research scope is relatively one-sided, and the temperature rise, noise, and mechanical strength of the rotor are not considered. It needs to be further studied in the future.

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