QCD Corrections to Photon Production in Association with Hadrons in $e^+e^-$ Annihilation

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Abstract

A detailed investigation of the theoretical ambiguities present in the QCD description of photon production in $e^+e^-$ annihilation is given. It is pointed out that in a well-defined perturbative analysis it is necessary to subtract the quark-photon collinear singularities. This subtraction requires the introduction of an unphysical parameter in the perturbative part of the cross section. The subtracted term is factored into non-perturbative fragmentation function. The dependence on the unphysical parameter cancels in the sum of non-perturbative and perturbative parts. It is pointed out that for $E_\gamma \leq \sqrt{s}/(2(1+\epsilon_c))$ the non-perturbative contributions are suppressed. Using a general purpose next-to-leading order Monte Carlo program, we calculate various physical quantities that were measured in LEP experiments recently.

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1 Introduction

The production of a photon (or an isolated photon) in association with hadrons in $e^+e^-$ annihilation is a useful process to learn about the differences in the properties of $q\bar{q}\gamma$ and $q\bar{q}g$ final states, to measure the parton-photon fragmentation function and to test QCD predictions in a channel crossed to photon-photon annihilation. The corresponding theoretical problems are well understood in the case of prompt photon production at hadron colliders, photo-production of jets and heavy flavor and photon-photon scattering. It is an important development that experiments at LEP give us high statistics data and open ground to study even photon plus multijet final states [1, 2]. The better data call for a quantitative QCD description.

The QCD description of inclusive photon production has a simple, but important feature: the photon has hadronic component. In the perturbative treatment this fact is reflected by the appearance of collinear photon-quark singularities. In order to obtain well defined cross sections in perturbative QCD in all orders of the running coupling $\alpha_s$, these singularities are to be subtracted and absorbed into the photon fragmentation functions (factorization theorem) [3, 4]. The fragmentation functions of the photon satisfy inhomogeneous evolution equation; it grows with $Q^2$ therefore, it is called “anomalous” [5, 6, 7].

It is also interesting to study the case of isolated photon. Physical isolation means that we isolate the photons from hadrons and so we cannot make distinction between quarks and gluons. Gluons, however, cannot be isolated completely from the photon without destroying the cancellation of soft gluon singularities between the virtual and real gluon corrections. Therefore, a physical isolation cannot eliminate completely the collinear photon-quark singularities, and so, even in the case of isolated photon production the cross section contains “anomalous” (non-perturbative) piece. This problem has been recognized clearly in the next-to-leading order QCD study of isolated photon production at hadron colliders [8, 9]. The theoretical subtleties of defining isolated photon cross section in perturbative QCD, however, have not been clearly formulated in previous studies in the case of $e^+e^-$ annihilation [10, 11].

In section 2 we review the next-to-leading order description of the inclusive (non-isolated) photon production. In section 3 we outline the change in the formalism due to the introduction of isolation cuts for the photon production. We point out that isolation cannot completely eliminate the non-perturbative fragmentation contribution, although it can reduce its size. In section 4 a detailed perturbative study is given for the cross section of isolated photon plus jet production up to order $\mathcal{O}(\alpha\alpha_s)$. We review the mechanisms of the cancellation of the infrared singularities and point out that in perturbation theory for processes containing a photon in the final state the definition of a finite hard scattering cross requires a counter term which necessarily introduces an unphysical parameter. Section 5 contains our numerical results for the isolated photon plus $n$-jet production at LEP. To demonstrate the flexibility of our numerical program to calculate any jet shape parameters, we calculate the distribution of the photon transverse momentum with respect
to the thrust axis as well. The last section contains our conclusions.

2 Inclusive photon production in $e^+e^-$ annihilation

According to the factorization theorem, the physical cross section of inclusive photon production is obtained by folding the fragmentation functions $D_{\gamma/a}(x, \mu_f)$ with the finite hard-scattering cross sections $d\sigma_a$:

$$\frac{d\sigma_{\gamma}}{dE_{\gamma}} = \sum_a \int_0^{\sqrt{s}/2} dE_a \int_0^1 dx \frac{d\sigma_{\gamma/a}(x, \mu_f)}{dE_a} \frac{d\sigma_a(E_a, \mu, \mu_f, \alpha_s(\mu))}{dE_a} \delta(E_{\gamma} - xE_a),$$

where $\alpha_s(\mu)$ is the strong coupling constant at the ultraviolet renormalization scale $\mu$ and $\mu_f$ is the factorization scale.

It is instructive to investigate the decomposition of this generally valid expression up to next-to-leading order. First we remark that $D_{\gamma/\gamma}(x) = \delta(1 - x) + O(\alpha^2)$,

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therefore, to leading order in the electromagnetic coupling, the term in eq. (2) given by $a = \gamma$ is a purely perturbative contribution. We use this equation to eliminate $D_{\gamma/\gamma}(x)$ from eq. (1). The hard scattering cross section $d\hat{\sigma}_{\gamma/\gamma}/dE_{\gamma}$ is of order $\alpha$ in comparison to the leading order annihilation cross section $\sigma_0$. The leading non-perturbative part given by the fragmentation function, however, is of order $\alpha/\alpha_s$. This contribution is the “anomalous” photon component. Its enhanced order is due to the fact that the scale dependence of the fragmentation functions $D_{\gamma/a}(x, \mu_f)$, $a = q, \bar{q}, g$ are given by the inhomogeneous renormalization group equations [12, 14]:

$$\mu \frac{\partial}{\partial \mu} D_{\gamma/a}(x, \mu) = \frac{\alpha}{\pi} P_{\gamma/a}(x) + \frac{\alpha_s}{\pi} \sum b \int \frac{dy}{y} D_{\gamma/b}\left(\frac{x}{y}, \mu\right) P_{b/a}(y),$$

where $P_{b/a}(x)$ denote the Altarelli-Parisi splitting functions. To order $\alpha \alpha_s$ the inhomogeneous terms have the expressions [4]

$$P_{\gamma/a}(x) = P_{\gamma/a}^{(0)}(x) + \frac{\alpha_s}{2\pi} P_{\gamma/a}^{(1)}(x),$$

where

$$P_{\gamma/q}^{(0)}(x) = \frac{1}{2} + \frac{(1 - x)^2}{x}, \quad P_{\gamma/g}^{(0)}(x) = 0,$$

and after trivial replacement of the color factors in eq. (12) of ref. [4], we have

$$P_{\gamma/q}^{(1)}(x) =$$

$$\epsilon_q^2 C_F \left\{-\frac{1}{2} + \frac{9}{2} x + \left(-8 + \frac{1}{2} x\right) \log x + 2x \log (1 - x) + \left(1 - \frac{1}{2} x\right) \log^2 x\right\} + \frac{\log^2 (1 - x) + 4 \log x \log (1 - x) + 8 \text{Li}_2(1 - x) - \frac{4}{3} \pi^2}{\text{Li}_2(1 - x)} P_{\gamma/q(q)}^{(0)}(x),$$

1 In the following analysis, when the order of a contribution is given, it is always understood in comparison to the leading order annihilation cross section $\sigma_0$. 

2
\[ P_{\gamma/y}^{(1)}(x) = \langle e_q^2 \rangle_T R \left\{ -4 + 12x - \frac{164}{9}x^2 + \frac{92}{9}x^{-1} + \left( 10 + 14x + \frac{16}{3}x^2 + \frac{16}{3}x^{-1} \right) \log x + 2(1 + x) \log^2 x \right\} . \]

In the last equation,
\[ \langle e_q^2 \rangle \equiv \sum_{q=1}^{N_F} e_q^2, \tag{7} \]
where \( N_F \) is the number of flavors.\(^2\)

The unique solution of these inhomogeneous equations requires non-perturbative input\(^3\) at a certain initial scale \( \mu \). At asymptotically large values of \( \mu \), however, the solutions are independent of the initial values and one obtains
\[ \lim_{\mu \to \infty} D_{\gamma/q}(x, \mu) = \frac{\alpha}{2\pi} \log \frac{\mu^2}{\Lambda^2} a_{\gamma/q}(x), \tag{8} \]
\[ \lim_{\mu \to \infty} D_{\gamma/g}(x, \mu) = \frac{\alpha}{2\pi} \log \frac{\mu^2}{\Lambda^2} a_{\gamma/g}(x). \tag{9} \]

Exact analytic expressions for the Mellin transforms of the \( a_{\alpha/\gamma}(x) \) functions have been found in refs. \([5, 6]\). These are related to the \( a_{\gamma/q} \) functions via crossing. It is useful, however, to have a parametrization in \( x \)-space. Formulas which accurately reproduce the exact leading logarithmic solutions were given in ref. \([15]\):
\[ a_{\gamma/q}(x) = e_q^2 \frac{1}{x} \left[ 2.21 - 1.28x + 1.29x^2 \right] \left[ \frac{1}{1 - 1.63 \log(1 - x)} \right]^{0.049} + 0.002(1 - x)^2x^{-1.54}], \tag{10} \]
\[ a_{\gamma/g}(x) = \frac{1}{x} [0.0243(1 - x)^{1.03} - 0.97]. \tag{11} \]

A new parametrization of the photon fragmentation functions is described in ref. \([20]\). The most striking feature of these solution is that they increase as \( 1/\alpha_s \) with increasing the evolution scale. Therefore, at high energy the contribution from the quark fragmentation into a photon gives the leading order \( (\alpha/\alpha_s) \) term
\[ \frac{d\sigma^{(0)}_{\gamma}}{dE_{\gamma}} = \sigma_0 \frac{4}{\sqrt{s}} \sum_q e_q^2 \langle e_q^2 \rangle D_{\gamma/q} \left( \frac{2E_{\gamma}}{\sqrt{s}}, \mu \right) + O(\alpha), \tag{12} \]
In next-to-leading order, the \( \mu \) dependence of \( D_{\gamma/q} \) has to be calculated with the next-to-leading order evolution equation and we should also add the order \( \alpha \) hard scattering cross section
\[ \frac{d\sigma_{\gamma}}{dE_{\gamma}} = \sigma_0 \frac{4}{\sqrt{s}} \sum_q e_q^2 \langle e_q^2 \rangle D_{\gamma/q} \left( \frac{2E_{\gamma}}{\sqrt{s}}, \mu \right) \]
\(^2\)We assume \( e^+e^- \) annihilation via virtual photon. In order to obtain formulas valid at the \( Z^0 \) peak, trivial modifications of charge factors are required.
\(^3\)In the literature it is usually called Vector Meson Dominance (VMD) contribution \([13, 14, 3]\).
\[
+ \sum_a \int_0^{\sqrt{s}/2} dE_a \int_0^1 dx \, D_{\gamma/a}(x, \mu_f) \frac{d\hat{\sigma}_a^{(1)}}{dE_a} (E_a, \mu, \mu_f, \alpha_s(\mu)) \delta(E_\gamma - x E_a) \\
+ \frac{d\hat{\sigma}_\gamma^{(0)}}{dE_\gamma}(E_\gamma, \mu_f, \alpha_s(\mu)) + \mathcal{O}(\alpha \alpha_s),
\]

where \(d\hat{\sigma}_a^{(1)}/dE_a\) denotes the order \(\alpha_s\) cross section of quark and gluon production.

The \(\mathcal{O}(\alpha)\) hard-scattering cross section \(d\hat{\sigma}_\gamma^{(0)}/dE_\gamma\) is defined by subtracting the photon-quark collinear singularity in the \(\overline{\text{MS}}\) scheme

\[
\frac{d\hat{\sigma}_\gamma^{(0)}}{dE_\gamma} = \lim_{\varepsilon \to 0} \left( \frac{d\hat{\sigma}_\gamma^{(0)}}{dE_\gamma} + \frac{d\hat{\sigma}_{\gamma F}^{(0)}}{dE_\gamma} \right),
\]

where the first term on the right hand side is the partonic cross section in \(4-2\varepsilon\) dimensions as defined by Feynman diagrams

\[
\frac{d\hat{\sigma}_\gamma^{(0)}}{dE_\gamma} = \sigma_0 \sum_q \frac{\epsilon_q^4}{\langle \epsilon_q^2 \rangle} \frac{2}{2\pi} \varepsilon \frac{1}{\Gamma(1-\varepsilon)} \int dy_{12} dy_{13} dy_{23} \theta(1-y_{13}-y_{23})(y_{12} y_{13} y_{23})^{-\varepsilon} \\
\times \left[ (1-\varepsilon) \left( \frac{y_{23}}{y_{13}} + \frac{y_{13}}{y_{23}} \right) + \frac{2y_{12} - \varepsilon y_{13} y_{23}}{y_{13} y_{23}} \right] \delta(1-y_{12} - y_{13} - y_{23}) \delta \left( 1 - y_{12} - \frac{2E_\gamma}{\sqrt{s}} \right),
\]

where \(H = 1 + \mathcal{O}(\varepsilon)\), while the second term is the \(\overline{\text{MS}}\) counter-term

\[
\frac{d\hat{\sigma}_{\gamma F}^{(0)}}{dE_\gamma} = \frac{\alpha}{2\pi} \frac{(4\pi)^\varepsilon}{\varepsilon \Gamma(1-\varepsilon)} \sum_q \int_0^{\sqrt{s}/2} dE_q \int_0^1 dx \, P_{\gamma/q}^{(0)}(x) \frac{d\hat{\sigma}_q^{(0)}}{dE_q}(E_q) \delta(E_\gamma - x E_q).
\]

The integrations in eqs. (13), (14) are easily performed. The collinear poles cancel in their sum. Setting \(\varepsilon = 0\), one obtains

\[
\frac{d\hat{\sigma}_\gamma^{(0)}}{dE_\gamma} = \sigma_0 \frac{\alpha}{\sqrt{s}} \frac{2}{2\pi} \sum_{q=1}^{2N_F} \frac{\epsilon_q^2}{\langle \epsilon_q^2 \rangle} P_{\gamma/q}^{(0)}(x_\gamma) \log \left( \frac{s(1-x_\gamma)x_\gamma^2}{\mu^2} \right),
\]

where \(x_\gamma = 2E_\gamma/\sqrt{s}\).

The \(\mathcal{O}(\alpha_s)\) corrections to the \(d\hat{\sigma}_{q,g}\) hard-scattering cross sections are defined by the Feynman diagrams of fig. 1. First we note that \(d\hat{\sigma}_g^{(1)}\) can be obtained from \(d\hat{\sigma}_\gamma^{(0)}\) by modifying the charge factors:

\[
\frac{d\hat{\sigma}_g^{(1)}}{dE_g} = C_F \sigma_0 \frac{\alpha_s}{2\pi} \frac{4}{\sqrt{s}} N_F P_{g/q}^{(0)}(x_g) \log \left( \frac{s(1-x_g)x_g^2}{\mu^2} \right).
\]

The cross sections \(d\hat{\sigma}_q^{(1)}\) receives both real and virtual corrections. The loop correction can be written as

\[
\frac{d\hat{\sigma}_{\text{loop}}^{(1)}}{dE_q} = C_F \sigma_0 \frac{\alpha_s}{2\pi} \frac{2}{\sqrt{s}} H \left( \frac{\mu^2}{s} \right) \frac{\varepsilon}{\varepsilon \Gamma(1-\varepsilon)} \left[ -\frac{3}{2} - 3 + (\pi^2 - 8) \varepsilon \right] \delta \left( \frac{\sqrt{s}}{2} - E_q \right).
\]
The Bremsstrahlung contribution has an expression similar to $d\tilde{\sigma}^{(0)}/dE_\gamma$ (15):

$$\frac{d\sigma_{\text{real}}}{dE_q} = C_F \sigma_0 \frac{e_q^2}{\langle e_q^2 \rangle} \frac{2}{2\pi} \sqrt{s} H \left( \frac{4\pi \mu^2}{s} \right)^\varepsilon \frac{1}{\Gamma(1-\varepsilon)} \int dy_{12} dy_{13} dy_{23} \theta(1-y_{13} - y_{23})(y_{12}y_{13}y_{23})^{-\varepsilon}$$

$$\times \left[ (1-\varepsilon) \left( \frac{y_{23}}{y_{13}} + \frac{y_{13}}{y_{23}} \right) + \frac{2y_{12} - \varepsilon y_{13}y_{23}}{y_{13}y_{23}} \right] \delta(1+y_{12} - y_{13} - y_{23}) \delta \left( 1-y_{23} - \frac{2E_\gamma}{\sqrt{s}} \right).$$

The sum of the loop and Bremsstrahlung contributions has the simple expression

$$\frac{d\tilde{\sigma}^{(1)}}{dE_q} = C_F \sigma_0 \frac{\alpha_s}{2\pi} \frac{2}{\sqrt{s}} \sqrt{s} \left( \frac{4\pi}{\varepsilon} \right)^\varepsilon \frac{\Gamma(1-\varepsilon)}{\varepsilon \Gamma(1-\varepsilon)}$$

$$\times \left\{ -P_{q/q}^{(0)}(x_q) + \varepsilon \left[ P_{q/q}^{(0)}(x_q) \log \left( \frac{s}{\mu^2} \right) + \left( \frac{2}{3} \pi^2 - \frac{9}{2} \right) \delta(1-x_q) + 2 \log x_q \frac{1 + x^2_q}{1-x_q}$$

$$+ (1 + x^2_q) \left( \log \frac{1-x_q}{1-x_q} \right) - \frac{3}{2} \left( \frac{1}{1-x_q} \right) - \frac{3}{2} x_q + \frac{5}{2} \right\},$$

where the index + denotes the usual “+ prescription” of regularizing singular behavior at $x_q = 1$. The remaining single pole is cancelled when one adds the $\overline{\text{MS}}$ counterterm $d\sigma_{\text{CT}}^{(0)}$ which is defined as

$$\frac{d\sigma_{\text{CT}}^{(0)}}{dE_q} = C_F \sigma_0 \frac{\alpha_s}{2\pi} \frac{2}{\sqrt{s}} \sqrt{s} \left( \frac{4\pi}{\varepsilon} \right)^\varepsilon P_{q/q}^{(0)} \left( \frac{2E_\gamma}{\sqrt{s}} \right).$$

The final result is obtained after setting $\varepsilon = 0$:

$$\frac{d\tilde{\sigma}^{(1)}}{dE_q} = C_F \sigma_0 \frac{\alpha_s}{2\pi} \frac{2}{\sqrt{s}}$$

$$\times \left\{ P_{q/q}^{(0)}(x_q) \log \left( \frac{s}{\mu^2} \right) + \left( \frac{2}{3} \pi^2 - \frac{9}{2} \right) \delta(1-x_q) + 2 \log x_q \frac{1 + x^2_q}{1-x_q}$$

$$+ (1 + x^2_q) \left( \log \frac{1-x_q}{1-x_q} \right) - \frac{3}{2} \left( \frac{1}{1-x_q} \right) - \frac{3}{2} x_q + \frac{5}{2} \right\},$$

where $x_q = 2E_q/\sqrt{s}$. This result can also be deduced after replacing trivial color factors from the coefficient functions of inclusive single hadron production calculated in ref. [3].

The theoretical input described in this section is sufficient to extract the photon fragmentation functions from experimental data in next-to-leading order accuracy. A complete analysis requires the measurement of the inclusive photon production cross section at various energies. The recent LEP data give information at the $Z$-pole. Unfortunately, the data obtained at PETRA, LEP and TRISTAN suffer from low statistics. Needless to say that such an experimental study would give very important complementary information on the fragmentation functions of the photon obtained at hadron colliders.
3 Inclusive isolated photon production in $e^+e^-$ annihilation

Let us now consider the inclusive photon cross section with photon isolation. One can argue that due to isolation cuts the fragmentation contribution is suppressed. As a consequence, isolation changes the relative importance of the different contributions. It is reasonable to consider the effect of isolation typically as an order $\alpha_s$ effect. After imposing the isolation cuts, the fragmentation contribution will be of order $\alpha$, i.e., the same order as the order of the pointlike perturbative cross section $d\hat{\sigma}^{(0)}_{\gamma}/dE_{\gamma}$. Isolation in practice can only be made with finite energy resolution. Therefore, we require that in a cone of half angle $\delta_c$ around the photon three momentum the deposited energy be less than a fraction $\epsilon_c$ of the photon energy. In experiments this parameter $\epsilon_c$ has a value typically about 0.1. Calculating $d\hat{\sigma}^{(0)}_{\gamma,\text{iso}}/dE_{\gamma}$, we should insert a combination of $\theta$ functions in the phase space integrals as follows

$$S(\epsilon_c, \delta_c) = \theta(\vartheta_{q\gamma} - \delta_c)\theta(\vartheta_{q\gamma} - \delta_c) + \theta(\vartheta_{\bar{q}\gamma} - \delta_c)\theta(\delta_c - \vartheta_{q\gamma})\theta(\epsilon_c E_{\gamma} - E_{\bar{q}})$$ \hspace{1cm} (24)

Let us require that

$$\epsilon_c < \frac{1}{2} \quad \text{and} \quad \sin^2 \frac{\delta_c}{2} < \frac{1}{2}$$

and choose integration variables

$$x_{\gamma} = \frac{2E_{\gamma}}{\sqrt{s}}, \quad y = \frac{y_{13}}{x_{\gamma}}.$$

We define the hard scattering cross section again with a collinear counter-term

$$d\hat{\sigma}_{\gamma,\text{iso}}^{(0)} = \lim_{\epsilon \to 0} \left( \frac{d\hat{\sigma}_{\text{iso}}^{(0)}}{dx_{\gamma}} + \frac{d\sigma_{\text{CT, iso}}^{(0)}}{dx_{\gamma}} \right),$$ \hspace{1cm} (25)

where the first term in the right hand side is calculated as given by Feynman diagrams in $4 - 2\varepsilon$ dimensions and for the counter-term, we use the MS-type expression

$$\frac{d\sigma_{\text{CT, iso}}^{(0)}}{dx_{\gamma}} = 2\sigma_0 \frac{\alpha}{2\pi} \sum_q \frac{e^2_q}{\langle e^2_q \rangle} \epsilon \Gamma(1 - \epsilon) P^{(0)}_{\gamma/q}(x_{\gamma}) \theta(x_{\gamma} - \frac{1}{1 + \epsilon_c}).$$ \hspace{1cm} (26)

After performing the integration over $y$ and setting $\varepsilon = 0$, one obtains

$$\frac{d\sigma_{\gamma,\text{iso}}^{(0)}}{dx_{\gamma}} = 2\sigma_0 \frac{\alpha}{2\pi} \sum_q \frac{e^2_q}{\langle e^2_q \rangle} \left( \left[ P^{(0)}_{\gamma/q}(x_{\gamma}) \log \frac{s(1 - x_{\gamma})x_{\gamma}^2y_m}{\mu^2(1 - y_m)} + e^2_q x_{\gamma} (1 - 2y_m) \right] \theta(x_{\gamma} - \frac{1}{1 + \epsilon_c}) \right).$$ \hspace{1cm} (27)
where $y_c$ and $y_m$ are defined as follows

$$y_c = \frac{1 - x_\gamma}{1 - x_\gamma \sin^2 \frac{\delta_c}{2}} \frac{\sin^2 \frac{\delta_c}{2}}{2}, \quad y_m = \min\left\{y_c, 1 + \epsilon_c - \frac{1}{x_\gamma}\right\}.$$ 

One can make several comments on this result.

- The unisolated case can be recovered in the limit $\epsilon_c \to \infty$ (cf. eq. (17)).
- Imperfect isolation allows for a contribution from the fragmentation: the photon looks isolated since the relatively soft fragments surrounding it are not counted.
- Assuming perfect energy resolution ($\epsilon_c = 0$) we obtain vanishing counter term. In higher order, however, we cannot isolate the photon from the soft gluons completely (we shall discuss this point in great detail in the next section), therefore, one cannot set the value of $\epsilon_c$ to zero.
- In the leading logarithmic approximation one can define a fragmentation function with isolation satisfying a modified inhomogeneous evolution equation

$$\mu \frac{\partial}{\partial \mu} D_{\gamma/a}(x, \mu, \epsilon_c) =$$

$$+ \frac{\alpha_s}{\pi} \int \frac{dy}{y} D_{\gamma/b}(x, \mu, \epsilon_c) P_{b/a}(y).$$

Clearly, if $D_{\gamma/a}(x, \mu)$ is a solution of the evolution equation without isolation then

$$D_{\gamma/a}(x, \mu, \epsilon_c) = D_{\gamma/a}(x, \mu) \theta\left(x - \frac{1}{1 + \epsilon_c}\right)$$

will be the solution of the evolution equation with isolation. In next-to-leading logarithmic approximation and/or choosing a different counter-term (for example completely subtracting the contribution of the singular region as defined by the third term of eq. (24)), the isolated fragmentation can also be dependent on $\delta_c$ therefore, in general, one cannot simply identify the isolated fragmentation with the non-isolated fragmentation in the high-x region.

In next-to-leading order, the physical cross section of isolated photon production is given by the terms as follows

$$\frac{d\sigma_{iso}}{dE_\gamma}(\epsilon_c, \delta_c) = \frac{d\sigma_{iso}^{(0)}}{dE_\gamma} + \frac{d\sigma_{iso}^{(1)}}{dE_\gamma} + 2\alpha_0 \frac{\alpha}{2\pi} D_{\gamma/q}(\frac{2E_\gamma}{\sqrt{s}}, \mu) \theta\left(\frac{2E_\gamma}{\sqrt{s}} - \frac{1}{1 + \epsilon_c}\right)$$

$$+ \sum_a \int_0^{\sqrt{s}/2} dE_a \int_{\frac{1}{1 + \epsilon_c}}^{\epsilon_c} \frac{d\sigma_{iso}}{dE_a}(x, \mu, \mu_f) \frac{d\sigma_{iso}}{dE_a}(E_a, \mu, \mu_f, \alpha_s(\mu)) \delta(E_\gamma - xE_a).$$

(30)
This decomposition is scheme dependent. The first term on the right hand side of this equation has been calculated in the \( \overline{\text{MS}} \) scheme (see eq. (27)). It also appears useful to calculate the next-to-leading order perturbative cross section \( d\hat{\sigma}^{(1)} / dE_\gamma \) in the \( \overline{\text{MS}} \) scheme. This requires the calculation of the next-to-leading order splitting function \( P^{(1)}_{\gamma/q}(x) \) in the presence of isolation cuts and a corresponding local subtraction term has to be found. This is a complex but feasible calculation. Since such a result is not yet available, in the next section we carry out the calculation of \( d\hat{\sigma}^{(1)}_\gamma \) in a different subtraction scheme where the photon is completely isolated from the quarks but not from soft gluons (“cone subtraction”). In this scheme, in leading order, the counter-term is vanishing and the cross section becomes independent of \( \epsilon_c \):

\[
\frac{d\hat{\sigma}^{(0)}}{dE_\gamma} = 2\sigma_0 \frac{\alpha}{2\pi} \sum_q \frac{e_q^2}{\langle e_q^2 \rangle} \left\{ P^{(0)}_{\gamma/q}(x_\gamma) \ln \frac{1 - y_c}{y_c} - x_\gamma (1 - 2y_c) \right\}. \tag{31}
\]

We note that the logarithmic divergence at \( x_\gamma = 1 \) is the usual soft singularity. Contrary to the case of the \( \overline{\text{MS}} \) scheme, with cone subtraction the cross section is continuous and always positive (see fig. 2 for comparison). One may argue that in this scheme the perturbative part is separated more efficiently, consequently the contributions of the non-perturbative terms (proportional to \( D^{iso}_{\gamma/a} \)) become relatively smaller.

In general we find that the non-perturbative terms contribute mainly in the region \( x_\gamma > 1/(1 + \epsilon_c) \) thus we conclude that the perturbative predictions appear to be reliable for \( E_\gamma < \sqrt{s}/(2(1 + \epsilon_c)) \).

In the next section we present the results of our next-to-leading order perturbative calculation of \( d\hat{\sigma}^{(1)}_{iso} \) for isolated photon plus \( n \)-jet production. We conjecture that a jet algorithm applied to the isolated photon hard scattering cross section (eq. (30)) provides an infrared safe isolated photon plus \( n \)-jet cross section. This is supported by the fact that our isolation prescription does not influence the soft-gluon structure of the cross section. If we can define a jet algorithm,

\[
\frac{d\hat{\sigma}^{iso}}{dE_\gamma} (\delta_c, \epsilon_c) = \frac{d\hat{\sigma}^{iso}}{dE_\gamma} (\delta_c, \epsilon_c) + \frac{d\hat{\sigma}^{iso}}{dE_\gamma} (\delta_c, \epsilon_c) + \frac{d\hat{\sigma}^{iso}}{dE_\gamma} (\delta_c, \epsilon_c) + \ldots, \tag{32}
\]

such that every term on the right hand side is finite and we count every particle only once, then isolated photon plus \( n \)-jet cross section appears to be infrared safe.

We shall see that the non-perturbative (“anomalous”) contributions are important only in the case of photon plus 1 jet production when the cross section is dominated by the \( x_\gamma > 1/(1 + \epsilon_c) \) region.

### 4 Isolated photon plus \( n \)-jet production

In QCD, the differential cross section at \( \mathcal{O}(\alpha_s^2) \) is a sum of the real and virtual corrections:

\[
d\sigma = |M_1|^2 dS^{(4)} + |M_3|^2 dS^{(3)}, \tag{33}
\]
where $dS^{(n)}$ is the $n$-body phase space element with the flux factor included. For infrared safe quantities both terms on the right hand side are separately divergent, but the sum is finite. It is very difficult to handle numerically this cancellation. Fortunately, at least at one loop, the divergencies can be cancelled analytically. There are two commonly used algorithms to achieve such a cancellation — the subtraction method [16, 17] and the phase space slicing method [18, 19]. They both rely on the fact that after partial decomposition $|M_4|^2$ can be written as a sum of terms with single pole singularity. Focusing our attention to the case of $q\bar{q}\gamma g$ final state, we find four such terms:

$$
|M_4|^2 = C_F\alpha_s \left( \frac{M_{gq}}{y_{gq}} + \frac{M_{g\bar{q}}}{y_{g\bar{q}}} + \frac{M_{\gamma q}}{y_{\gamma q}} + \frac{M_{\gamma \bar{q}}}{y_{\gamma \bar{q}}} \right),
$$

(34)

where

$$y_{ij} = \frac{(p_i + p_j)^2}{s}, \quad (s = M_Z^2)$$

(35)

The pole part of each term is defined as

$$P_{ij} \frac{y_{ij}}{y_{ij}}, \quad \text{where} \quad P_{ij} = \lim_{y_{ij} \to 0} M_{ij}.$$  

(36)

It can be integrated analytically over either the whole or a part of the phase space. In this way, in general, we obtain analytical expressions for the regularized divergencies of $d\sigma^{(4)}$ which cancel against the divergencies of the virtual corrections, $d\sigma^{(3)}$ (KLN theorem). When a photon in the final state is observed, the cancellation mechanism described above does not apply to the $y_{\gamma(q\bar{q})}$ poles. The reason for this is that the process is exclusive in the photon and the virtual corrections with the photon in the loop cannot contribute for kinematical reasons.

To make the discussion more transparent, let us consider contributions from the region where only $y_{gq}$ is small. The virtual corrections can also be split into three terms

$$|M_3|^2 = C_F\alpha_s \left( M_{gq}^{(3)} + M_{g\bar{q}}^{(3)} + M_f^{(3)} \right),$$

(37)

such that $M_{gq}^{(3)}$ contains one half of the singularities, the second term contains the other half and the third is the finite part. (Notice that there are no $M_{\gamma q}^{(3)}$, $M_{\gamma \bar{q}}^{(3)}$ terms.) Then we shall concentrate on

$$\frac{M_{ij}}{y_{ij}} dS^{(4)} + M_{ij}^{(3)} dS^{(3)}$$

(38)

parts of the cross section.

In the subtraction method, one considers the combination

$$\frac{M_{gq}}{y_{gq}} dS^{(4)} - P_{gq} dS^{(4\to3)} + \left( \int \frac{P_{gq}}{y_{gq}} dS^{(g)} \right) dS^{(3)} + M_{gq}^{(3)} dS^{(3)},$$

(39)

4For the reader’s convenience, we give the explicit expressions for $M_{ij}$, $M_{kl}^{(3)}$ and $M_f^{(3)}$ in the appendix.
where the integration over $dS^{(g)}$ is meant to be an integral over the gluon variables. $dS^{(4\rightarrow 3)}$ means the factorized four-body phase space element in the limit when the gluon is soft or collinear to the quark: $dS^{(4\rightarrow 3)} = dS^{(g)}dS^{(3)}$.

In the phase space slicing method, formula (38) is written as

$$M_{gq} \frac{\theta(y_{gq} - y_0)}{y_{gq}} dS^{(4)} + M_{gq} \frac{\theta(y_0 - y_{gq})}{y_{gq}} dS^{(4)} + M_{gq} dS^{(3)}. \quad (40)$$

If $y_0$ is chosen small enough ($y_0 \leq 10^{-4}$), then

$$M_{gq} \frac{\theta(y_{gq} - y_0)}{y_{gq}} dS^{(4)} + P_{gq} \frac{\theta(y_0 - y_{gq})}{y_{gq}} dS^{(4\rightarrow 3)} + M_{gq} dS^{(3)} \quad (41)$$

is a good approximation. The first and second terms depend on $y_0$ strongly, but their sum is independent of this unphysical parameter. The strong $y_0$ dependence originates mainly from the slicing of the soft gluon region.

If one wishes to calculate isolated photon production, one has to make sure that the restriction of the phase space does not disturb the cancellation mechanism of soft and collinear gluons. At hadron level the meaning of photon isolation is well-defined. At parton level however, one has to be careful because the isolation prescription is different for states with different number of partons.\footnote{One may object isolation at parton level arguing that the fragmentation process inevitably scatters hadronic matter into the isolation cone. For a purely perturbative analysis, this objection is not valid. To understand the reason for this, let us consider the same process at higher energies, say $\sqrt{s} = 10 \text{ TeV}$, in which energy region perturbation theory is expected to give even better description. Clearly, at this energy, fragmentation does not alter the energy flow, therefore isolation at parton level corresponds to isolation at hadron level.} If the photon is isolated from the partons with $y_\gamma$, we should include isolation cuts with respect to all partons:

$$\theta(y_{\gamma q} - y_\gamma) \theta(y_{\gamma \bar{q}} - y_\gamma) \theta(y_{\gamma g} - y_\gamma). \quad (42)$$

The isolation from the gluon can be implemented only in the first term of formula (39). However, if we cut the soft gluons in the first term, then the cancellation of singularities between the first and second terms breaks down. One can maintain the cancellation introducing an energy resolution parameter $\epsilon$ such that a gluon is isolated from the photon only if its energy is greater then $\epsilon E_\gamma$. Accordingly, isolation for the first term means multiplication with

$$\theta(y_{\gamma q} - y_\gamma) \theta(y_{\gamma \bar{q}} - y_\gamma) (1 - \theta(y_\gamma - y_{\gamma g}) \theta(E_g - \epsilon E_\gamma)). \quad (43)$$

Clearly, this criterion is “not physical” in the sense that one cannot implement it at hadron level since we apply different cuts to quarks and gluons.

If we introduce photon isolation in the slicing method, from formula (41) we obtain

$$\frac{M_{gq}}{y_{gq}} \theta(y_{gq} - y_0) dS^{(4)} \theta(y_{\gamma q} - y_\gamma) \theta(y_{\gamma \bar{q}} - y_\gamma) \theta(y_{\gamma g} - y_\gamma) + \quad (44)$$
$$\left( \frac{P_{gq}}{y_{gq}} \theta(y_0 - y_{gq}) dS^{(4 \to 3)} + M_{gq}^{(3)} dS^{(3)} \right) \theta(y_{\gamma g} - y_0) \theta(y_{\gamma g} - y_\gamma).$$

Usually, $y_\gamma \gg y_0$. This means that when changing $y_0$ at fixed $y_\gamma$, the contribution from the soft gluons will be cut independently of $y_0$ and consequently, the $y_0$ dependence is damped in the first term. On the other hand, in the second term the $y_0$ dependence is not damped by the gluon-photon isolation condition. The conclusion is that the $y_0$ dependence will be different in the two terms and, therefore, the cross section will depend on the unphysical parameter $y_0$. It is important to notice that if $y_{gq} > y_0$, then there exist an $\epsilon'$ such that if $\epsilon < \epsilon'$ then

$$1 - \theta(y_\gamma - y_{\gamma g}) \theta(E_g - \epsilon E_\gamma) = \theta(y_{\gamma g} - y_\gamma),$$

therefore (44) defines a finite cross section, but $y_0$ plays in a sense the role of $\epsilon$ used in formula (43).

To demonstrate the $y_0$ dependence of the isolated photon cross section explicitly, we calculated the isolated photon plus 1- and 2-jet cross sections using the isolation criterium given by formula (44). First we make two technical remarks about the slicing method.

Choosing large $y_0$, the pole approximation is not precise enough in the singular region; one has to take into account the non-singular terms in the same region, i.e., one should add the

$$\left( \frac{M_{gq}}{y_{gq}} dS^{(4)} - \frac{P_{gq}}{y_{gq}} dS^{(4 \to 3)} \right) \theta(y_0 - y_{gq})$$

(46)
correction term. The calculation becomes analogous to the subtraction method and one has to introduce the $\epsilon$ energy resolution parameter.

It is a practical question to establish at what value of $y_0$ the correction term (46) becomes important. The most straightforward way to calculate the finite terms in (44) is to perform the integration by a Monte Carlo method, which leaves sufficient flexibility to calculate any jet shape parameter one wishes to obtain. The Monte Carlo calculation has a finite statistical error which of course, can be reduced by generating more points. Then the criterium which determines the importance of the correction term (46) is to require that the systematic error introduced by neglecting (46) has to be smaller than the statistical one. Clearly, this critical value of $y_0$ depends on the jet resolution parameter $y_J$ as well as on $y_\gamma$. For the case of 3-jet production without photon in the final state, an analysis was carried out in ref. [13] to determine the critical value of $y_0$ above which the systematic error dominates. They found that choosing $y_0/y_J \leq 0.01$ removes the systematic error.

The number of isolated photon plus $n$-jet events can be conveniently parametrized in the form

$$\frac{1}{\sigma_0} \sigma_{\gamma+n \text{jets}}(y_J, y_0) = \frac{\alpha}{2\pi} \sum_q \frac{e_q^4}{\langle e_q^2 \rangle} \left( g_n^{(0)}(y_J, y_0) + \frac{\alpha_s}{2\pi} g_n^{(1)}(y_J, y_0) \right)$$

(47)

$$\equiv \frac{\alpha}{2\pi} \sum_q \frac{e_q^4}{\langle e_q^2 \rangle} g_n^{(0)}(y_J, y_0)(1 + \alpha_s R_n(y_J, y_0)),\)
where $\sigma_0$ is the leading order cross section of the reaction $e^+e^- \to$ hadrons and the $R_n(y_J,y_0)$ functions are defined by the equation. In figs. 3 and 4, we show the $y_0$ dependence of the $\mathcal{O}(\alpha_s)$ QCD corrections, $R_n(y_J,y_0)$, to the isolated photon plus 1-jet and the isolated photon plus 2-jet cross sections. To obtain the corrections, we used the following algorithm:

1. select isolated $\gamma + n$-jet events by requiring the invariant mass of the photon with any particle in the event to be larger than $y_\gamma$ (see formula (44));
2. apply E0 cluster algorithm to the hadronic part of the event;
3. separate $\gamma + 1$-, 2-, and 3-jet events by the number of remaining clusters of hadrons.

We used $y_\gamma = y_J$. As expected, the $y_0$ dependence in $R_n(y_J,y_0)$ is strong up to $y_0 = y_\gamma$. As explained before, in formula (44) the $y_0$ cut plays the role of the $\epsilon$ parameter of formula (43). Therefore, the (apparently) physical cut, (44) is in fact unphysical because $y_0$ is no longer a dummy variable of the cross section.

In order to demonstrate that we control the numerical evaluation of the integrals at small $y_0$ values, we calculated $R_n(y_J,y_0)$ with $\theta(y_{\gamma g} - y_{\gamma})$ in (44) removed. We denote the corresponding quantity with $\tilde{R}_n(y_J,y_0)$. According to the discussion after formula (44), this alteration should remove the $y_0$ dependence. The explicit calculation shows that this indeed happens. We see that in order to obtain a $y_0$ independent result, we have to use an unphysical cut: different cuts are applied to quarks and gluons.

We conclude from this discussion that if we want to define a finite isolated photon plus $n$-jet cross section we have to make a subtraction which depends on some unphysical parameter no matter which algorithm — the subtraction or slicing one — is used (see formulas (43), (44) and (45)). In other words, the physical isolated photon plus $n$-jet cross section always contains some non-perturbative ("anomalous") contribution which is expected to give contributions comparable or somewhat smaller than the $\mathcal{O}(\alpha_s)$ QCD corrections. For the separation of perturbative and non-perturbative parts of the cross section, one must introduce an unphysical (non-zero) parameter. Of course, the sum of the perturbative and non-perturbative pieces is independent of this parameter. In the previous section we pointed out that the non-perturbative contribution is expected to be small for $x_\gamma < 1/(1 + \epsilon_c)$.

5 Numerical results

As advocated in section 4, we carry out our calculation with the subtraction method. The event definition is the following:

\footnote{In fact, we can see weak $y_0$ dependence in $\tilde{R}_n(y_J,y_0)$. The origin of this dependence is the use of the pole approximation. It can be observed for values $y_0 > 10^{-3}$ (this value depends on $y_J$) in accordance with the observation made in ref. [13]. The results are shown for $y_J = 0.06$. For other values of $y_J$ the dependence is similar.}
1. isolate the photon;
2. apply E0 cluster algorithm to the hadronic part of the event;
3. separate $\gamma + 1$, 2-, and 3-jet events by the number of remaining clusters of hadrons.

Photon isolation can be achieved either by isolating the photon in a cone (cone isolation) or by requiring the invariant mass of the photon with any particle in the event to be larger than an invariant mass cut $y_{\gamma}$. From experimental point of view the cone isolation is more natural. Unfortunately, the results by OPAL [2] are corrected experimental values in order to compare the measured rates with the matrix element calculation of [10] where invariant mass isolation was used (with $y_{\gamma} = y_J$). We give results for both. Since the QCD corrections are very sensitive to the event definition we give explicitly how formula (39) is modified in the case of cone isolation:

$$
\theta(\vartheta_{q\gamma} - \delta_c)\theta(\vartheta_{\bar{q}\gamma} - \delta_c) \times \left\{ (1 - \theta(\delta_c - \vartheta_{g\gamma})\theta(E_g - \epsilon_c E_\gamma)) \frac{M_{gq}}{y_{gq}} dS^{(4)} - \frac{P_{gq}}{y_{gq}} dS^{(4 \rightarrow 3)} + \left( \int \frac{P_{gq}}{y_{gq}} dS^{(g)} \right) dS^{(3)} + M_{gq}^{(3)} \right\};
$$

and in the case of invariant mass isolation:

$$
\theta(y_{q\gamma} - y_J)\theta(y_{\bar{q}\gamma} - y_J) \times \left\{ (1 - \theta(y_J - y_{g\gamma})\theta(E_g - \epsilon_c E_\gamma)) \frac{M_{gq}}{y_{gq}} dS^{(4)} - \frac{P_{gq}}{y_{gq}} dS^{(4 \rightarrow 3)} + \left( \int \frac{P_{gq}}{y_{gq}} dS^{(g)} \right) dS^{(3)} + M_{gq}^{(3)} \right\}.
$$

To obtain the isolated photon plus $n$-jet rates, the formulas above are multiplied with $\theta$ functions as follows:

- One photon plus 3-jet:
  $$
  \theta(y_{gq} - y_J)\theta(y_{g\bar{q}} - y_J)\theta(y_{q\gamma} - y_J)\theta(y_{\bar{q}\gamma} - y_J)\theta(y_{g\gamma} - y_J)\theta(y_{g\gamma} - y_J).
  $$

- One photon plus 2-jet:
  Denote $i$ and $j$ the partons which when combined have the smallest invariant mass in the hadronic part of the event, so they are combined into pseudoparticle $c$. Denote $k$ the third parton. In the three-body phase space the momentum of the $j$ particle is identically zero. Then we use
  $$
  \theta(y_J - y_{ij})\theta(y_{ck} - y_J)\theta(y_{c\gamma} - y_J)\theta(y_{k\gamma} - y_J).
  $$
• One photon plus 1-jet:

$$\theta(y_J - y_{qg})\theta(y_J - y_{\bar{q}g})\theta(y_J - y_{q\bar{q}})\theta(y_J - y_{c\bar{c}}).$$  (52)

In the case of cone isolation, we also required that the energy of the photon has to be larger than 7.5 GeV. The half-opening angle of the cone is 15°.

We shall give the results of our calculation for the partial widths $\Gamma(Z \rightarrow \gamma + n \text{jets})$ as ratios to the hadronic width:

$$\frac{\Gamma(Z \rightarrow \gamma + n \text{jets})}{\Gamma(Z \rightarrow \text{hadrons})} = \left( \frac{8}{9}c_u + \frac{1}{3}c_d \right) \frac{\alpha}{2\pi} \left( \frac{1 + \alpha_s}{\pi} + 1.42 \left( \frac{\alpha_s}{\pi} \right)^2 \right) g_n(y_J),$$  (53)

where

$$c_f = v_f^2 + a_f^2$$  (54)

and $v_f$ and $a_f$ are the weak vector and axial vector couplings:

$$v_f = 2I_{3,f} - 4e_f \sin^2 \theta_W$$  (55)

$$a_f = 2I_{3,f},$$  (56)

so with $\sin^2 \theta_W = 0.23$, $v_u = 0.39$, $v_d = -0.69$, $a_u = +1$ and $a_d = -1$. The $g_n(y_J)$ functions can be expanded in $\alpha_s$:

$$g_n(y_J) = g_n^{(0)}(y_J) + \frac{\alpha_s}{2\pi} g_n^{(0)}(y_J) \equiv g_n^{(0)}(y_J) (1 + \alpha_s R_n(y_J)),$$  (57)

and our aim is to compute the $g_n^{(i)}$ functions (of course, $g_3^{(0)}(y_J) = 0$.)

It is interesting to study the photon energy spectrum of the jet cross sections

$$\frac{d\sigma_{\gamma + 1\text{jet}}^{\text{iso}}}{dE_\gamma}(\delta_c, \epsilon_c), \quad \frac{d\sigma_{\gamma + 2\text{jet}}^{\text{iso}}}{dE_\gamma}(\delta_c, \epsilon_c)$$  (58)

at some realistic values of the isolation parameters $\delta_c, \epsilon_c$. In fig. 5 the photon energy distributions of one jet and two jet production are shown in the Born approximation, while in fig. 6 the same curves are plotted but including the next-to-leading order corrections. Due to obvious kinematical reasons one jet production is completely dominated by the hard photon region $x_\gamma > 1/(1 + \epsilon_c)$. In this region as we pointed out one may get substantial (but not overwhelmingly large) “anomalous” photon contribution. It is difficult to estimate the “anomalous” contribution since we do not have yet enough phenomenological input. Certainly a combined study of the hadron collider and LEP data would help to understand its size better. We note that in the high $x$ region the application of perturbative QCD by itself requires some care due to the appearance of large logarithms of type $\log(1 - x)$. Indeed the QCD corrections are larger for one jet than for two jet production. It is interesting to compare the one jet data to the perturbative QCD prediction, but one should not
be surprised if one does not find perfect agreement. The non-perturbative corrections appear, however, negligible in the case of 2-jet production since it is dominated by the complementary region $x_\gamma < 1/(1 + \epsilon_c)$. Requiring that $x_\gamma < 1/(1 + \epsilon_c)$, the 2-jet results remain practically unaffected, while this cut largely eliminates the 1-jet production. This is illustrated by the numbers given in Table 1. There is a tendency that if we shrink the isolation region the perturbative contribution increases. From figs. 5, 6 and, we can see also that the total one jet and two jet rates should depend weekly on $\epsilon_c$. The reason for this is that the photon energy distribution changes weakly if we change $\epsilon_c$ in the physically interesting region of 0.06–0.2.

In addition to the ambiguities due to “anomalous” photon production there are also the usual scale ambiguities. In fig. 7 we present the predicted values of the $\Gamma(Z \rightarrow \gamma + n \text{jets})$ ($n = 1, 2$) partial widths for the cone isolation with $\epsilon_c = 0.1$. The bands between the dashed lines represent the scale dependence between the scales $M_Z/2$ and $2M_Z$. We used $\alpha_s(M_Z) = 0.12$ and $\alpha = 1/137$. The $\epsilon_c$ dependence is so weak for experimentally feasible values that the uncertainty introduced by the $\epsilon_c$ dependendce is much smaller than the scale dependence and therefore we did not show it. The scale dependence of the 1-jet rate is rather large. This is a reflection of the fact that the QCD corrections are large. In figs. 8 and 9 the same curves as in fig. 7 are depicted in the case of invariant mass isolation with $y_\gamma = y_J$ for the 1-jet and 2-jet rates, respectively. In the same figures, we show the enhancement induced by the choice of smaller isolation region. In accordance with our previous discussion, the enhancement is larger for the 1-jet rate than for the 2-jet rate.

As mentioned in the section 4, the Monte Carlo approach is useful because it leaves sufficient flexibility to calculate any jet shape parameter. To demonstrate this feature of our work we present the result of matrix element calculation for the distribution of the photon transverse momentum with respect to the thrust axis (fig. 10). The thrust axis has been calculated all particles taken into account, including the photon. We used invariant mass isolation (with $y_\gamma = 0.005$ and 0.06) to isolate the photon from the partons. We also required the photon to be more energetic than 7.5 GeV. For small $p_T$, configurations with thrust value close to one may occur. The histogram is normalized to one, therefore the uncertainty in the small $p_T$ region influences the behaviour in the large $p_T$ region. We note, however, that requiring $x_\gamma < 1/(1 + \epsilon_c)$ the small $p_T$ region will be suppressed.

Finally, in fig. 11, we present the predicted values of the $\Gamma(Z \rightarrow \gamma + n \text{jets})$ ($n = 1, 2$) partial widths for the cone isolation with $\epsilon_c = 0.1$ when Durham clustering algorithm is
used \cite{21}. In this algorithm, two jets are combined into a single jet if

\[ y_{Dij} = \frac{2\text{min}(E_i^2, E_j^2)(1 - \cos \theta_{ij})}{s} \]

(59)

is smaller than the jet resolution parameter \( y_J \). For pure QCD events, this algorithm tends to emphasize 2-jet events as compared to other algorithms and suited better for resummation purposes \cite{23}. When a photon in the final state is observed, we find higher 1-jet rate and lower 2-jet rate and the QCD corrections are smaller as compared to the E0 cluster algorithm.

6 Conclusions

Photon production in association with hadrons in \( e^+e^- \) annihilation provides us interesting information on the non-perturbative component of the photon and new possibilities to test the underlying structure of perturbative QCD.

In this paper we paid special attention to the importance of the correct treatment of the collinear photon-quark region. It was shown that next-to-leading and higher orders the perturbative part can only be defined using some non-physical parameter, no matter whether non-isolated or isolated photon production is considered. The physical cross section defined as the sum of the perturbative and non-perturbative part is, of course, independent of such a parameter.

We briefly reviewed the theoretical description of the inclusive non-isolated photon production in \( e^+e^- \) annihilation. It was pointed out that the LEP data can be used to constrain the parametrization of the fragmentation functions of the photon, \( D_{\gamma/q}(x, \mu) \), \( D_{\gamma,g}(x, \mu) \). The measurement of these fragmentation functions would give important input information for the other inclusive photon production measurements at hadron colliders and at HERA. Furthermore, one could test the anomalous \( \mu \)-dependence at asymptotically large \( \mu \) values predicted by perturbative QCD.

The case of isolated photon production was studied as well. Under well defined circumstances, isolation can suppress the numerical contribution of the non-perturbative contributions. We pointed out that the non-perturbative (“anomalous”) contribution can be sizable only for \( E_\gamma > \sqrt{s}/2/(1 + \epsilon_c) \), where \( \epsilon_c \) is the energy fraction in the isolation cone with respect to the photon energy. When a jet algorithm is used, then the non-perturbative contribution is expected to be further suppressed for isolated photon plus \( n \)-jet cross section for \( n > 1 \), but not for \( n = 1 \).

We demonstrated the difficulty due to the quark photon collinear singularity with careful calculation of the next-to-leading order QCD corrections to isolated photon plus one or two jets. We argued that in the case of isolated photon plus 2-jet production indeed, as suggested by Kramer and Lampe, the perturbative contribution dominates the physical cross section. The next-to-leading order corrections are calculated by developing a Monte Carlo program which can be used to calculate the perturbative corrections to any physical quantity.
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Appendix A

In this appendix we give the explicit expressions for the $M_{ij}$ and $M^{(3)}_{ij}$ expressions used in formulas (34) and (37) respectively. We shall make the following renaming:

$$
q \rightarrow \text{particle 1} \\
\bar{q} \rightarrow \text{particle 2} \\
g \rightarrow \text{particle 3} \\
\gamma \rightarrow \text{particle 4}.
$$

Then the following relations are valid:

$$M_{23} = M_{13}(1 \leftrightarrow 2), \quad M_{14} = M_{13}(3 \leftrightarrow 4), \quad M_{24} = M_{13}(1 \leftrightarrow 2, \ 3 \leftrightarrow 4),$$

therefore, it is sufficient to spell out $M_{13}$. The corresponding expression can be obtained from the four-parton matrix element given in Appendix B of ref. [16] by setting $N_C = 0$ and $T_R = 0$. After performing partial fractioning one obtains

$$M_{13} = \frac{2}{4\pi^2} \left[ \frac{2y_{12}y_{13}(1 + y_{34})}{y_{134}y_{234}(y_{13} + y_{23})} + \frac{2y_{14}(1 - y_{24})}{y_{234}(y_{13} + y_{24})} + \frac{2(1 - y_{13})y_{23}}{y_{134}(y_{13} + y_{24})} \right. \\
+ \frac{1}{y_{134}} \left( \frac{y_{24}y_{34} + y_{12}y_{34} + y_{13}y_{24} - y_{14}y_{23} + y_{12}y_{13}}{y_{234}(y_{13} + y_{24})} \right) \\
+ \frac{y_{12}y_{13}y_{34} + y_{12}y_{14}y_{34} - y_{13}y_{24} + y_{13}y_{14}y_{24}}{y_{134}(y_{13} + y_{14} + y_{23})} \left( \frac{1}{y_{13} + y_{24}} + \frac{1}{y_{13} + y_{14}} \right) \\
+ \frac{y_{12}y_{24}y_{34} + y_{12}y_{14}y_{34} - y_{13}y_{24} + y_{13}y_{14}y_{24}}{y_{234}(y_{13} + y_{24})} \left( \frac{1}{y_{13} + y_{24}} + \frac{1}{y_{13} + y_{24}} \right) \\
+ \frac{y_{12}y_{23}y_{34} + y_{12}y_{13}y_{34} - y_{14}y_{23} + y_{13}y_{14}y_{23} + 2y_{12}y_{13}y_{24}}{y_{134}(y_{13} + y_{14} + y_{24})} \left( \frac{1}{y_{13} + y_{14}} + \frac{1}{y_{13} + y_{24}} \right) \\
+ \left. \frac{2y_{12}y_{13}y_{14}y_{24}}{y_{13} + y_{23} + y_{14} + y_{24}} \left( \frac{1}{y_{13} + y_{24}} + \frac{1}{y_{13} + y_{14}} + \frac{1}{y_{13} + y_{23}} \right) \right].
$$
Due to partial fractioning, this expression is finite if a single $y_{ij} \to 0$ (and for the same reason the expression is lengthy.)

The virtual corrections can also be obtained easily from eq. (2.20) of ref. [16] by setting $N_C = 0$ and $T_R = 0$. In our decomposition

$$M^{(3)}_{gg} = M^{(3)}_{g\bar{q}} = \frac{1}{4\pi^2} \left[ -\frac{1}{\varepsilon^2} - \frac{1}{2\varepsilon} \left( 3 - 2 \log y_{12} \right) \right] ,$$

and the finite part is

$$M^{(3)}_f = \frac{1}{4\pi^2} \left[ \frac{y_{12}}{y_{12} + y_{14}} + \frac{y_{12}}{y_{12} + y_{24}} + \frac{y_{12} + y_{24}}{y_{14}} + \frac{y_{12} + y_{14}}{y_{24}} ight. \left. + \log y_{14} \left[ \frac{4y_{12}^2 + 2y_{12}y_{14} + 4y_{12}y_{24} + y_{14}y_{24}}{(y_{12} + y_{24})^2} \right] \right. \left. + \log y_{24} \left[ \frac{4y_{12}^2 + 2y_{12}y_{24} + 4y_{12}y_{14} + y_{14}y_{24}}{(y_{12} + y_{14})^2} \right] \right. \left. - 2 \left[ \frac{y_{12}^2 + (y_{12} + y_{14})^2}{y_{14}y_{24}} R(y_{12}, y_{24}) + \frac{y_{12}^2 + (y_{12} + y_{24})^2}{y_{14}y_{24}} R(y_{12}, y_{14}) \right. \right. \left. + \frac{y_{12}^2 + y_{24}^2}{y_{14}y_{24}(y_{14} + y_{24})} - 2 \log y_{12} \left( \frac{y_{12}^2}{(y_{14} + y_{24})^2} + \frac{2y_{12}}{y_{14} + y_{24}} \right) \right. \left. + \left( \frac{y_{24}}{y_{14}} + \frac{y_{14}}{y_{24}} + \frac{2y_{12}}{y_{14}y_{24}} \right) \left( \frac{2}{3} \pi^2 - \log^2 y_{12} - 8 \right) \right] ,$$

where

$$R(x, y) = \log x \log y - \log x \log(1 - y) - \log y \log(1 - x) + \frac{1}{6} \pi^2 - \text{Li}_2(x) - \text{Li}_2(y)$$

and

$$\text{Li}_2(x) = - \int_0^x dz \frac{\log(1 - z)}{z} .$$
References

[1] ALEPH Collaboration, D. Decamp et. al., Phys. Lett. B264 (1991) 476;
DELPHI Collaboration, P. Abreu et. al., CERN Preprint CERN-PPE/91-174;
L3 Collaboration, B. Adeva et. al., L3 Internal note 971, July 1991;
OPAL Collaboration, G. Akrawy et. al., Phys. Lett. B246 (1990) 285;
OPAL Collaboration, G. Alexander et. al., Phys. Lett. B264 (1991) 219.

[2] OPAL Collaboration, P. D. Acton et al., CERN Preprint CERN-PPE/91-189.

[3] G. Altarelli, R. K. Ellis, G. Martinelli and S. Y. Pi, Nucl. Phys. B160 (1979) 301.

[4] W. Furmanski and R. Petronzio, Phys. Lett. 97B (1980) 437.
G. Curci, W. Furmanski and R. Petronzio, Nucl. Phys. B175 (1980) 27.

[5] E. Witten, Nucl. Phys. B120 (1977) 189.

[6] R. J. DeWitt, L. M. Jones, J. D. Sullivan, D. E. Willen and H. W. Wyld, Jr., Phys. Rev. D19 (1979) 2046; Erratum Phys. Rev. D20 (1979) 1751.

[7] T. F. Walsh and P. M. Zerwas, Phys. Lett. 44B (1973) 195;
R. L. Kingsley, Nucl. Phys. B60 (1973) 45.

[8] P. Aurenche, R. Baier and M. Fontannaz, Phys. Rev. D42 (1990) 1440.

[9] E. L. Berger and J. Qiu, Phys. Rev. D44 (1991) 2002.

[10] G. Kramer and B. Lampe, Phys. Lett. B269 (1991) 401.

[11] G. Kramer and H. Spiesberger, DESY Preprint DESY 92-022.

[12] K. Koller, T. F. Walsh and P. M. Zerwas, Zeit. Phys. C2 (1979) 197.

[13] M. Glück, K. Grassie and E. Reya, Phys. Rev. D30 (1984) 1447.

[14] J. F. Owens, Rev. Mod. Phys. 59 (1987) 465.

[15] D. W. Duke and J. F. Owens, Phys. Rev. D26 (1982) 1600.

[16] R. K. Ellis, D. A. Ross and A. E. Terrano, Nucl. Phys. B178 (1981) 421.

[17] Z. Kunszt, P. Nason, G. Marchesini and B. Webber, CERN Yellow Report 89-08, Vol. 1, eds. G. Altarelli et. al..

[18] K. Fabricius, I. Schmitt, G. Kramer and G. Schierholz, Zeit. Phys. C11 (1982) 318.

[19] W. T. Giele and E. W. Glover, FERMILAB Preprint FERMILAB-Pub-91/100-T.

[20] P. Aurenche et. al., Preprint LPTHE-Orsay 92/30.
[21] Yu. L. Dokshitzer, contribution to the Workshop on jets at LEP and HERA, Durham, December 1990.

[22] S. Bethke, Z. Kunszt, D. E. Soper and W. J. Stirling, *Nucl. Phys.* B370 (1992) 310.

[23] S. Catani, Yu. Dokshitzer and B. R. Webber, CERN Preprint CERN-TH-6473-92.
Figure captions

Figure 1 Typical Feynman diagrams contributing to the calculation of the $O(\alpha_s)$ corrections to the inclusive quark production in $e^+e^-$ annihilation.

Figure 2 Leading order hard-scattering cross section for inclusive isolated photon production calculated in the $\overline{\text{MS}}$ subtraction scheme (eq. [22]) and in the cone subtraction scheme (eq. [34]). $\delta_c = 15^\circ$ and $\epsilon_c = 0.1$ isolation parameters were used.

Figure 3 The dependence of the QCD corrections to the isolated photon plus 1-jet production on the unphysical parameter $y_0$ when physical cuts are applied (see formula (44)) — solid curves — and with unphysical cuts (only quarks are cut) — dashed curves. The slicing method in the pole approximation was used with $y_\gamma = y_J$.

Figure 4 The dependence of the QCD corrections to the isolated photon plus 2-jet production on the unphysical parameter $y_0$ when physical cuts are applied (see formula (44)) — solid curves — and with unphysical cuts (only quarks are cut) — dashed curves. The slicing method in the pole approximation was used with $y_\gamma = y_J$.

Figure 5 Leading order photon energy spectrum of the partial widths $\Gamma(Z \to \gamma+n\text{jets})$, $(n = 1, 2)$ normalized to $10^{-3}\Gamma(Z \to \text{hadrons})$ at $y_J = 0.1$, $\delta_c = 15^\circ$ and $\epsilon_c = 0.1$.

Figure 6 Next-to-leading order photon energy spectrum of the partial widths $\Gamma(Z \to \gamma+n\text{jets})$, $(n = 1, 2)$ normalized to $10^{-3}\Gamma(Z \to \text{hadrons})$ at $y_J = 0.1$, $\delta_c = 15^\circ$ and $\epsilon_c = 0.1$.

Figure 7 Partial widths $\Gamma(Z \to \gamma+n\text{jets})$, $(n = 1, 2)$ as a function of $y_J$ normalized to $10^{-3}\Gamma(Z \to \text{hadrons})$ when cone isolation of the photon is used (solid lines) with $\alpha_s(M_Z) = 0.12$, $\alpha = 1/137$. $\delta_c = 15^\circ$ and $\epsilon_c = 0.1$. The dashed curves represent the scale dependence between scales $\mu = M_Z/2$ and $\mu = 2M_Z$.

Figure 8 Partial width $\Gamma(Z \to \gamma+1\text{jet})$, as a function of $y_J$ normalized to $10^{-3}\Gamma(Z \to \text{hadrons})$ (solid line) when invariant mass isolation is used with $y_\gamma = y_J$, $\epsilon_c = 0.1$, $\alpha_s(M_Z) = 0.12$, $\alpha = 1/137$. The dashed curves represent the scale dependence between scales $\mu = M_Z/2$ and $\mu = 2M_Z$. The dashed dotted curve is the partial width calculated with $y_\gamma = 0.005$ and the long-dashed short-dashed curve is that with $y_\gamma = 0.001$.

Figure 9 Partial width $\Gamma(Z \to \gamma+2\text{jets})$, as a function of $y_J$ normalized to $10^{-3}\Gamma(Z \to \text{hadrons})$ (solid line) when invariant mass isolation is used with $y_\gamma = y_J$, $\epsilon_c = 0.1$, $\alpha_s(M_Z) = 0.12$, $\alpha = 1/137$. The dashed curves represent the scale dependence between scales $\mu = M_Z/2$ and $\mu = 2M_Z$. The long-dashed short-dashed curve is the partial width calculated with $y_\gamma = 0.001$. 

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Figure 10 Distribution of the photon transverse momentum with respect to the thrust axis. The photon was isolated using invariant mass isolation with $y_\gamma = 0.005$ and 0.06. The dotted histograms show the scale dependence between scales $\mu = M_Z/2$ and $\mu = 2M_Z$. The width of one bin is $M_z/100$.

Figure 11 Partial widths $\Gamma(Z \rightarrow \gamma + n \text{jets})$, ($n = 1, 2$) as a function of $y_D$ normalized to $10^{-3}\Gamma(Z \rightarrow \text{hadrons})$ when Durham clustering algorithm and cone isolation of the photon is used (solid lines) with $\alpha_s(M_Z) = 0.12$, $\alpha = 1/137$. The dashed curves represent the scale dependence between scales $\mu = M_Z/2$ and $\mu = 2M_Z$. 