Modelling tree height-diameter relationship of *Quercus acutissima* in a coniferous and broad-leaved mixed forest in Mount Tai, China

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Abstract. The aim of this study was to establish the relationships between height and diameter (DBH) of *Quercus acutissima* in a broadleaved-conifer mixed forests in Mountain Tai in China. Based on the data of *Q. acutissima* from a 0.6 hm² permanent forest plot, the relationships between height and DBH were simulated with six empirical models, including Linear, Power, Chapman-Richards, Logistic, Korf and Weibull models, to select the optimization model. There is no significant difference in the predictive power of each growth model. The fitting and test results of each model in this study show that these six model forms have good prediction effects. The results showed that the logistic fitting effect is the best model for *Q. acutissima*. The Logistic model can be applied to the simulation of tree height and diameter of coniferous and broad-leaved mixed forests in this area.

1. Introduction

Tree height and diameter are the two most important factors in forest resource surveys. They are not only the basic variables for constructing forest growth and yield models, but also have an important significance in predicting forest stand structure, timber distribution, and stock volume [1]. The measurement of diameter at breast height (DBH) is relatively easier and the cost is lower, while the measurement of tree height is relatively difficult and costly due to the limitation of measurement time, the obstacle of the canopy and the measurement error. Therefore, the use of DBH to predict tree height by quantifying the relationship between tree height and DBH is an important way to reduce the difficulty and cost of forest survey [2,3]. Until now, there have been a large number of tree height curve models in forestry research, and they have been widely used in various regions and different tree species.

In different forest stands, the relationship between tree height and DBH is often different [4]. Even in the same forest stand, the relationships may change over time. Compared with even-aged pure forest, individual trees in natural forests may have different ages, canopy types, and tolerance to shade, the relationship between tree height and diameter is more complicated. The relationship between tree height and diameter is often non-linear in mature forests [5], so nonlinear models are usually used to describe the relationship between tree height and diameter. Researchers study the difference in the relationship between tree height and diameter from different aspects. They believe that the relationship between tree height and diameter of trees is related to the type of habitat and forest density in which they are located, and is also related to the aspect of the slope [6]. These differences can help us understand how plants adapt to their living environment or how they compete for resources.

In the past research, few studies have explored the influence of different altitudes on the relationship between tree height and DBH in mixed forests. Thus, this study proposed the hypothesis that the
difference in altitude will affect the tree height-diameter relationship of trees. Therefore, the model fitting results of species at different altitudes may be different. This study chose six widely used height-diameter models that have a high degree of goodness of fit. Models were established for *Q. acutissima* to describe the relationship between tree height and diameter, and the tree height-diameter models were established for individuals at different altitudes to improve the model’s performance. The research will provide basis for the management of natural mixed forests.

2. Materials and methods

2.1. Study site

The research site is located in Mount Tai Forest Farm in Shandong Province. With a warm temperate continental monsoon climate, the average annual temperature is 13.4°C, and the average annual temperature is 6.0°C. The average annual precipitation is 678.5 mm. The most precipitation season is summer. The month with the highest average precipitation is July, and the lowest is January. The geology of Mount Tai is dominated by gneiss, granite and transitional rocks. The soil types are mainly brown soil and mountain meadow soil. The thickness of the soil layer is 30~40cm, and the slope is 15°~25°.

According to the CTFS (Center for Tropical Forest Science) sample plot construction technical standards, the permanent dynamic monitoring sample plot of 0.6 hm² (100 m×60 m) was established in October 2016. The research plot is located in the Luohanya Forest Farm. The longitude is 117°07′09.4″, and the latitude is 36°13′21.0″, and the altitude is 391~435m. The sample plot is dominated by *Platycladus orientalis* and *Q. acutissima*. *P. orientalis* occupies an absolute predominance in the canopy layer. The main associated species are *Pinus densiflora*, *Robinia pseudoacacia*, *Pistacia chinensis Bunge*, *Diospyros lotus L*, *Albizia julibrissin Durazz*, and *Broussonetia papyrifera*. Shrubs and herbs mainly include *Vitex negundo Linn* and *Grewia biloba*.

2.2. Statistical method

The species, diameter at breast height (DBH), tree height, crown width and coordinates of all woody plants with DBH≥1cm in each plot were surveyed. This research is based on a scatter diagram that reveals the relationship between tree height and diameter. The linear model and five widely used nonlinear models, including power function model, Chapman-Richards model, Logistic model, Korf model and Weibull model, were selected to analyze the relationship between tree height and diameter to determine the optimal model (Table 1).

| Table 1. Models of height-diameter relationships |
|-----------------------------------------------|
| **Model Name** | **Equation** |
| Linear | $\hat{y} = a \times DBH + b$ |
| Power | $\hat{y} = a \times DBH^b$ |
| Chapman-Richards | $\hat{y} = a(1 - e^{-b\times DBH})$ |
| Logistic | $\hat{y} = a/(1 + e^{b-c\times DBH})$ |
| Korf | $\hat{y} = ae^{-b/DBH^c}$ |
| Weibull | $\hat{y} = a(1 - e^{-b\times DBH^c})$ |

The non-linear weighted least squares method was used for model fitting. We used adjusted coefficient of determination (R²), root mean square error (RMSE), mean error (ME), mean absolute error (MAE), Akaike Information Criterion (AIC) and Bayesian Information Criteria (BIC) for model comparison and evaluation. Except for the adjusted R² is the bigger the better, the other indicators are the smaller the better. The optimal model of each tree species was selected from the models whose parameters were significant (p<0.05). The specific calculation formulas of R², RMSE, ME, MAE, are as follows:
In the formulas above: \( y_i \) is the tree height of the \( i \)-th individual. \( \hat{y}_i \) is the predicted tree height of the \( i \)-th individual. \( \bar{y} \) is the average tree height; \( N \) is the number of samples; \( p \) is the number of parameters.

All the data statistics and graphics making was analyzed with R4.0.3 statistical software.

3. Results and discussion

A total of 238 \( Q. \) acutissima individuals were survey in the plot. The mean DBH is 12.9 cm, and the mean height is 6.93 m. From Table 4, we can see that the power and logistic models have very significant fitting effects on the relationship between the DBH and tree height of \( Q. \) acutissima. The R square of Logistic is higher, the explanation proportion is 0.309. RSME is lower at 1.166; the ME, MAE, AIC, and BIC values of the two models are relatively closer. Based on the above results, it is determined that the Logistic model is the model with the highest goodness of fit, which can be used as a basic model for the growth of tree height and diameter for \( Q. \) acutissima.

There are many different forms of tree height-diameter models to choose from in the forest resource survey [7]. In order to improve the prediction accuracy of unobserved standing tree heights in the sample plots, six biologically meaningful tree height-diameter relationship models were selected in this study. The fitting and test results of each model in this study showed that these six model forms have good prediction effects (Figure 1). This study explored the influence of altitude and the composition of mixed forests on the relationship between tree height and DBH, and it is preliminarily found that the fitting effect of each growth equation is better when the altitude is lower. In addition, different tree species composition and competition intensity affect the choice of model.

| Model          | a    | b    | c    | \( R^2 \) | RSME | ME   | MAE  | AIC     | BIC     |
|----------------|------|------|------|----------|------|------|------|---------|---------|
| Linear         | 0.174| 4.686| /    | 0.304    | 1.17 | 5.974| 5.984| 740.456 | 750.809 |
| Power          | 5.085***| 0.024***| /    | 0.298    | 1.175| 5.973| 5.986| 742.39  | 752.743 |
| Chapman-Richards| 15.941| -0.008| 0.353.| 0.309    | 1.166| 5.974| 5.976| 740.722 | 754.526 |
| Logistic       | 9.573***| 0.309* | 0.102*| 0.309    | 1.166| 5.974| 5.979| 740.703 | 754.507 |
| Korf           | 20933.146| 8.902| 0.042| 0.309    | 1.166| 5.974| 5.976| 740.727 | 754.531 |
| Weibull        | 34.24  | 0.088| 0.376| 0.309    | 1.166| 5.974| 5.976| 740.726 | 754.535 |

Note: \( a, b, c \) are simulation parameters
In the process of tree growth, the intervention of many factors makes the relationship between DBH and tree height not constant [8-9], which leads to greater uncertainty in the relationship model between tree height and DBH. In Peng et al.’s study of 9 main tree species in Canadian forests, the best model obtained is the Chapman-Richards model [10]. Ahmadi et al. believe that Richards equation is a theoretical prediction model with greater flexibility and accuracy [11]. Research by Zhang et al. showed that the Weibull equation has higher prediction accuracy in describing the relationship between tree height and DBH [12]. From the results of this paper, we can see that for the Taishan coniferous and broad-leaved mixed forest Q. acutissima, the images obtained from the fitting of each model show that the predictive capabilities of these empirical models are not very different (Figure 1). However, in comparison, the Logistics model has a better fitting effect and is a predictive model with higher accuracy. This shows that there are differences in the fitting results of the relationship between the DBH of different tree species in different regions, and these differences depend on a variety of factors that affect the growth of the forest.

Some scholars believe that tree growth will be restricted by various factors such as forest stand characteristics, tree competition, site conditions, and management level. These factors will cause the diversity of the relationship between tree height and DBH. Marziliano's research showed that forest age and density are the main factors affecting the relationship model between tree height and DBH [13]. Other scholars have discussed stand density [14], dominant height and stand area [15], spatial location [16], species composition and secondary average diameter [17] and other variables or combinations of variables are explored to improve the prediction accuracy and applicability of the model [18].

4. Conclusion
In this study, six empirical models were used to fit the tree height-diameter of Q. acutissima in Mount Tai, China. It fits the relationship between tree height and diameter well and can be applied to the simulation of tree height and diameter of this species in this area.

The research method of this study can provide a reference for accurately describing the relationship between height and diameter of different tree species in mixed forests. The conclusions drawn can be used for reference in the prediction of forest correlations in other forest areas, and provide theoretical support for the improvement of forestation and afforestation programs in various regions. This method has strong applicability, but the sample size is small and the consideration factors are still incomplete. In order to obtain a model with higher goodness of fit and higher prediction accuracy, the next step is to increase the sample size and quantitatively analyze and evaluate the competition intensity of various tree species in different mixed forests.
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