Spin configuration of magnetic multi-layers: effect of exchange, dipolar and Dzyalozhinski–Moriya interactions

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Received 20 February 2013, in final form 22 May 2013
Published 10 July 2013
Online at stacks.iop.org/JPhysCM/25/316003

Abstract
We investigate the effect of coupling (intensity and nature), applied field, and anisotropy on the spin dynamics of a multi-layer system composed of a hard magnetic layer coupled to a soft magnetic layer through a nonmagnetic spacer. The soft layer is modeled as a stack of several atomic planes while the hard layer, of a different material, is either considered as a pinned macroscopic magnetic moment or again as a stack of atomic planes. We compute the magnetization profile and hysteresis loop of the whole multi-layer system by solving the Landau–Lifshitz equations for the net magnetic moment of each (atomic) plane. We study the competition between the intra-layer anisotropy and exchange interaction, applied magnetic field, and the interface exchange, dipolar or Dzyalozhinski–Moriya interaction. Compared with the exchange coupling, the latter two couplings present peculiar features in the magnetization profile and hysteresis loop that may help identify the nature of the interface coupling in multi-layer magnetic systems.

(Some figures may appear in colour only in the online journal)

1. Introduction

Although the ‘nano-rush’ tends to dominate the realm of technological applications, especially magnetic recording, multi-layer magnetic systems benefit from a growing interest in this area mainly due to their high performance [1–6]. Also, magnetic multi-layer systems and thin films benefit from the acquired long-standing experience and know-how both in growth and characterization leading to a good control of the relevant intrinsic parameters (dimensionality and anisotropy). Moreover, there are many well-established techniques for precise measurements, such as FMR [7–11], BLS [12, 13], and the ever-developing optical techniques [14–17], to cite a few.

The magnetization dynamics of laterally confined elements of alternating magnetic and nonmagnetic layers exhibits a large variety of interesting phenomena for both applications and fundamental research [18]. From the point of view of applications, multi-layer structures turn out to be much more interesting than single-layer elements because almost any technical device makes use of multi-layers. For instance, a stack consisting of two ferromagnetic layers separated by a nonmagnetic spacer has been proposed as a prototype for magnetoresistive cells [19]. The giant magnetoresistance [1, 3, 20, 21] accompanying the realignment of the magnetic configurations by an external magnetic field in such multi-layer systems is routinely used in magnetic read heads and is essential for high-density nonvolatile MRAMs.

In this context, theory has to play its usual role of providing reasonable models for interpreting the observed phenomena and suggesting new experiments. An issue of particular interest in this regard concerns the coupling in multi-layer systems [1, 3, 22]. To a large extent the interface coupling determines the mechanisms of transport and propagation of a stimulus applied at one end of the structure and also the mechanism of adjustment of magnetic configurations in metallic multi-layer systems by an external
magnetic field. In the context of perpendicular magnetic recording on exchange-spring media, improved writeability is achieved by appropriately tuning the coupling between the soft and hard magnetic layers [4, 23–27]. Therefore, it is of paramount importance to fathom the nature of interaction acting at the interface and the role it plays in conveying any perturbation through the multi-layer system. There is a great amount of published work investigating the interface coupling and its effects on the magnetic properties [11, 28–35]. For instance, in [11] the authors investigated the magnetization dynamics due to spin currents in magnetic double layers and argued that transport in this structure is governed by a kind of long-range interaction called dynamic exchange coupling. The microscopic mechanism underlying this effective coupling and the way it affects the collective behavior of magnetic hybrid structures require further investigation.

In addition to the Ruderman–Kittel–Karuyama–Yosida (RKKY) oscillatory coupling, several other kinds of interlayer coupling have been investigated and their effects studied in various situations. These are the effective exchange coupling [33], the dipolar coupling [28, 35, 36] and the Dzyaloshinski–Moriya (DM) coupling [30, 37–39].

While the RKKY coupling provides an interpretation of experiments on very thin conducting spacers assuming a high degree of perfection [40], rough surfaces may induce strong stray fields and thereby dipolar coupling in multi-layer systems [41]. For a magnetic bi-layer several configurations are considered in [29, 42–44]. In particular, in [29] the calculations confirm the fact that in the absence of roughness the dipolar coupling between two perfectly flat infinite planes vanishes and that for planes of finite dimensions the dipolar coupling may give rise to a ferromagnetic or an anti-ferromagnetic coupling. On the other hand, one cannot exclude the DM interaction, which is an anti-symmetrical exchange interaction and mainly stems from a combination of low symmetry and spin–orbit coupling [45, 46], and defects (see e.g. [47] and references therein). In the presence of disorder, especially at the interface of thin films or multi-layer systems, the DM interaction has been shown to play an important role since local symmetry is broken by surface effects and lattice defects. Indeed, it leads to large anisotropy (∼0.1 $J_{\text{exch}}$) and may even change the chirality of magnetic heterostructures and magnetic order, see [45–49].

In particular, it has been shown that the DM interaction is induced by spin–orbit coupling between two ferromagnetic layers separated by a paramagnetic plane [30].

As mentioned earlier, it is necessary to gain more insight into the nature and role of the interface coupling by investigating its origin and the way it affects the global system behavior. In particular, from the theoretical point of view it is useful to study different kinds of coupling and to compare their effects. This should make it possible to establish a means, which by comparison with experiments, should help discriminate between the various couplings at play in magnetic multi-layers of given materials. In this context, we investigated the effect of three interface couplings, namely exchange, dipolar, and DM. Accordingly, in [50] we started with a simpler model of two coupled macroscopic magnetic moments, the so-called magnetic dimer. We analytically studied the switching mechanisms and computed the related thermally assisted switching rates for various configurations of the anisotropy axes and for the three types of coupling, without a magnetic field. There, the task was to determine the coupling and anisotropy configuration that leads to the most efficient (e.g. the fastest) switching of the dimer’s magnetization. For a multi-layer system it is not straightforward to perform the same (semi-)analytical calculations. In particular, the analytical calculation of the relaxation rate is rather involved, if not impossible, in the general situation. However, before trying to do so it is important to investigate the effect of interface coupling and intra-layer anisotropies on the equilibrium properties of such systems, especially the spin configuration and hysteresis loop. For the exchange interface coupling we investigated in [51] the spin configuration and magnetization profile in Fe/FePt double layers. In these calculations there were considered two situations—with the FePt plane being either as a stack of several atomic planes or as a single pinned macroscopic magnetic moment. The two situations are referred to here respectively as relaxed interface or rigid interface. In the latter case an exact analytical solution was obtained for the magnetization profile. In the former case, the results of the numerical calculations agree very well with the experimental observations of [52].

Two important extensions of these works are worth developing: (i) for the kindred multi-layers, other interface couplings have to be considered and their effects compared on the spin structure and hysteresis, for instance, (ii) thermal effects have to be taken into account and the activation rate computed. The first extension is the task of the present work and the second is left for subsequent investigations.

Consequently, in this work we bring a new contribution that attempts to further clarify the role of three interface couplings, namely exchange, dipolar, and DM, in the spin configuration and magnetization profile (MP). It is also essential to compare these interactions with respect to their efficiency in the realignment of the spin configurations and eventually the magnetization reversal. More precisely, we investigate the effect of coupling (intensity and nature), applied field, and anisotropy on the spin configuration of a multi-layer system composed of a hard magnetic layer coupled to a soft magnetic layer through a nonmagnetic spacer. The soft layer is modeled as an array of several atomic layers while the hard plane, of a different material, is either regarded as a pinned macroscopic magnetic moment or as an array of atomic planes. We compute the magnetization profile and hysteresis loop of the whole system by solving the (coupled) Landau–Lifshitz equations for the net magnetic moment of each (atomic) plane.

This paper is organized as follows. In section 2 we define the system studied and fix the notation. In section 3 we present and discuss the results for the magnetization profile and examine the effects of the applied field, anisotropy, and interface coupling. Section 4 deals with the hysteresis loops and compares the results for the three interface couplings. We finally summarize the main results in the conclusion and give a few perspectives for future investigations.
2. Statement of the problem and system studied

In figure 1 (left) we sketch the system studied in this work. It consists of three layers (from bottom to top): (i) (in blue) a hard magnetic layer (HML) with strong out-of-plane anisotropy, (ii) (in gray) a nonmagnetic layer (NML), and (iii) (in green) a soft magnetic layer (SML) with in-plane anisotropy. Both SML and HML are pictured as stacks of atomic planes of different materials, e.g. Fe for SML and FePt for HML. Next, each atomic plane is represented by its normalized net magnetic moment (red arrows). A discussion of this approximation can be found in the textbook [18].

The two magnetic layers (SML and HML) are supposed to be coupled through the NML which is represented by a spring in figure 1(b). The whole system is then pictured as being composed of two spin chains coupled by an effective interaction that is assumed to be of one of the following origins: exchange interaction (EI), dipole–dipole interaction (DDI), or Dzyalozhinski–Moriya interaction (DMI). The magnetic moments in each chain (SML and HML) are coupled to each other by an (effective) intra-chain exchange coupling which is, in fact, the effective exchange interaction between adjacent planes. The effect of a varying thickness of the NML can be recognized as the variation of the magnitude of each of the three interactions, and it can affect the orientation of the DMI by changing the symmetry of the interface.

In this work, we investigate the effect on the spin structure (and thereby the magnetization profile) and hysteresis loop of the effective coupling \( \lambda \) between SML and HML, in addition to the intra-chain exchange coupling and on-site anisotropy. Let us now define in detail the model used.

The SML layer is modeled by a chain of \( N_s \) magnetic moments \( s_i, i = 1, \ldots, N_s \), interacting via the (effective) exchange coupling \( J_s \). Similarly, the HML layer is modeled by a chain of \( N_h \) magnetic moments \( \sigma_k, k = 1, \ldots, N_h \), with effective exchange coupling \( J_h \). The single site anisotropy is in-plane and has the intensity \( D_s \) and direction \( e_s = \mathbf{e}_s \) in the SML layer while it is out-of-plane with intensity \( D_h \) and direction \( e_h = \mathbf{e}_h \) in the HML layer. Within a given chain the anisotropy is uniaxial with a common easy axis for all magnetic moments belonging to the chain. A uniform magnetic field \( H = H_{\parallel} \) is applied to the whole system at an angle \( \theta_H \) from the \( z \)-axis, taken parallel to the easy axis of the HML layer. Therefore, the energy of the whole system reads

\[
H = H_{\text{SML}} + H_{\text{HML}} + H_{\text{Int}},
\]

with

\[
H_{\text{SML}} = -\mu_s H \sum_{i=1}^{N_s} s_i \cdot \mathbf{e}_f - D_s \sum_{i=1}^{N_s} (s_i^z)^2 - J_s \sum_{i=1}^{N_s-1} s_i \cdot s_{i+1}
\]

being the contribution of the SML chain and

\[
H_{\text{HML}} = -\mu_h H \sum_{k=1}^{N_h} \sigma_k \cdot \mathbf{e}_f - D_h \sum_{k=1}^{N_h} (\sigma_k^z)^2 - J_h \sum_{k=1}^{N_h-1} \sigma_k \cdot \sigma_{k+1}
\]

that of the HML chain. Here \( \mu_s \) and \( \mu_h \) are the magnetic moments of the soft and hard materials, respectively. The interaction contribution \( H_{\text{Int}} \) in equation (1) is a function of \( s_1 \) and \( \sigma_{N_h} \) (the lowest plane of SML and the top plane of HML) and its form depends on the nature of the interface coupling. Hence, we write

\[
\mathcal{E}_{\text{Int}} \equiv H_{\text{Int}}/J_s = \lambda \mathcal{F}(s_1, \sigma_{N_h}).
\]

For the three couplings the latter explicitly reads (with the corresponding dimensionless parameters)

\[
\mathcal{E}_{\text{Int}} = -\lambda_{\text{EI}} \sigma_{N_h} \cdot s_1, \quad \lambda_{\text{EI}} \equiv \frac{J_{\text{EI}}}{J_s},
\]

for the exchange coupling

\[
\mathcal{E}_{\text{Int}} = -\lambda_{\text{DMI}} e_{\text{DMI}} \cdot (\sigma_{N_h} \times s_1), \quad \lambda_{\text{DMI}} \equiv \frac{D_{\text{DMI}}}{J_s}.
\]

Figure 1. Scheme of a hard/soft coupled bi-layer system. (a) Magnetic multi-layer and (b) multi-layer models.
Table 1. Summary of the notations used for the different vectors and parameters present in the system.

| Notation | Description                 |
|----------|-----------------------------|
| $\mathbf{s}_i$ | $i$th magnetic moment of the SML |
| $\mathbf{a}_i$ | $i$th magnetic moment of the HML |
| $N^\text{S}$ | Number of atomic layers of the SML |
| $N^\text{H}$ | Number of atomic layers of the HML |
| $N^\text{S}$ | Single site anisotropy magnitude of the SML |
| $N^\text{H}$ | Single site anisotropy magnitude of the HML |
| $\mathbf{e}_i$ | Single site anisotropy direction of the SML |
| $\mathbf{e}_i$ | Single site anisotropy direction of the HML |
| $H$ | Magnitude of the applied magnetic field |
| $J$ | Direction of the applied magnetic field |
| $J_\text{int}^{1}$ | Angle deviation of the applied field from the $z$-axis |
| $\mu_\text{S}$ | Magnetic moment of the soft magnetic material |
| $\mu_\text{H}$ | Magnetic moment of the hard magnetic material |
| $J_\text{int}^{2}$ | Angle deviation of the magnetic moment of the bottom SML plane from the $z$-axis |
| $d$ | Approximated thickness of the NML |
| $D$ | Magnitude of the DM vector |
| $\xi_{int}^0$ | Angle deviation of the magnetic moment of the bottom HML plane from the $z$-axis |
| $\xi_{int}^1$ | Angle deviation of the magnetic moment of the top SML plane from the $z$-axis |

Table 2. Summary of the definitions of the reduced parameters of the system.

| Notation | Definition | Description |
|----------|------------|-------------|
| $d_s$ | $D_s/J_s$ | Reduced single site anisotropy of the SML |
| $d_h$ | $D_h/J_s$ | Reduced single site anisotropy of the HML |
| $h$ | $\mu_\text{S}H/J_s$ | Reduced applied magnetic field |
| $j_h$ | $J_h/J_s$ | HML reduced intra-chain exchange interaction |
| $\lambda_{\text{int}}$ | $J_{\text{int}}^{1}/J_s$ | Reduced inter-chain exchange interaction |
| $\lambda_{\text{DDI}}$ | $\mu_0\mu_s^2/4\pi d^3 J_s$ | Reduced DDI strength |
| $\lambda_{\text{DMI}}$ | $D/J_s$ | Reduced magnitude of the DM vector |

for DM, and

$$\xi_{\text{int}} = \lambda_{\text{DDI}} \mathbf{a}_s \cdot D_{12} \mathbf{s}_1, \quad \lambda_{\text{DDI}} = \frac{\mu_0 \mu_s^2}{4\pi} \frac{1}{d^3 J_s} (6)$$

for DDI with the tensor

$$D_{12} \equiv 3 (\rightarrow e_x e_y e_z) - 1. \quad (7)$$

We also introduce the reduced magnetic field $h = \mu_\text{S}H/J_s$, the HML and SML reduced anisotropies $d_h \equiv D_h/J_s$, $d_s \equiv D_s/J_s$, the HML reduced intra-chain exchange $j_h \equiv J_h/J_s$, and the inter-chain exchange coupling $j_{\text{int}}^{1}$. $D$ is the magnitude of the DM vector, and $d$ is roughly the thickness of the NML. Tables 1 and 2 summarize all the vectors, parameters, and reduced parameters used throughout this paper.

The equilibrium properties are obtained by solving the set of coupled (damped) Landau–Lifshitz equations for the magnetic moments of the chains. This numerical method has been successfully checked against the analytical results obtained in [51].

As discussed earlier, the present work is concerned with the spin structure and magnetization profile of the whole system, with the main objective to investigate how the magnetic structure/state of the SML adapts to a change in the physical parameters, especially the kind and intensity of the coupling at the interface between the two layers. In particular, we obtain the magnetization profile by plotting the projection of each magnetic moment of the whole chain on the $z$-axis, see [51] for details. In fact, we compute and plot the angle deviation $\xi_{n}$, with $n = 0, \ldots, N^\text{H} + N^\text{S} - 1$, from the $z$-axis of each magnetic moment as we move through the whole chain from the first magnetic moment ($\mathbf{a}_1$) in the HML layer to the last magnetic moment ($\mathbf{s}_{N^\text{S}}$) in the SML layer. It has been found that there is a minimal number of sub-layers of the SMS slab, $N^\text{Smin}$, or length of the chain, $L^\text{Smin}$, necessary for an onset of noncolinearities of these magnetic moments. For instance, in [51], $N^\text{Smin}$ was found to be given by

$$N^\text{Smin} = \frac{\pi}{2\sqrt{2d_s}} \quad (8)$$

This yields, for instance, $N^\text{Smin} \approx 11$ for $d_s = 0.01$.

We also compute separately the angle deviation of the first magnetic moment in HML and that of the last magnetic moment in SML layer as functions of the applied field. These are respectively denoted by $\xi_{1}^{\text{H}}$ and $\xi_{N^\text{S}}^{\text{S}}$.

The parameters used for the calculations are $d_h = 0.02$, $j_h = 1.44$ and $N^\text{H} = 10$, $N^\text{S} = 41$ for the calculations presented in section 3.1 and $N^\text{S} = 11$ for those presented in sections 3.2.1–3.2.3. The values of the parameters considered here are typical of the Fe/FePt multi-layer systems (see [51] and references therein). The parameters values used here may not apply for all the systems studied in the literature but they do correspond to the typical (relative) orders of magnitude. Moreover, the present work is meant to be more of a qualitative general study of the role of interface coupling on the spin configuration in multi-layers than a quantitative calculation for a specific material.

3. Magnetization profile

In order to obtain the magnetization profile for the system described above, we solve the set of (coupled) Landau–Lifshitz equations (LLE)

$$\frac{1}{\gamma} \frac{ds_i}{dt} = \mathbf{s}_i \times \mathbf{h}_i^{\text{eff}} - \alpha \mathbf{s}_i \times (\mathbf{s}_i \times \mathbf{h}_i^{\text{eff}}), \quad (9)$$

where $\mathbf{h}_i^{\text{eff}} = -\frac{\partial \mathbf{s}_i}{\partial s_i}$ is the (dimensionless) effective field comprising the anisotropy and exchange contributions, together with the interaction at the interface. $\alpha$ is the phenomenological damping parameter, set here to 0.01. In these zero-temperature calculations, damping is used to drive the system into the equilibrium state, i.e. the state of minimal energy. We start from a state of homogeneous magnetization along the HML anisotropy axis and allow the system to relax towards a minimum that yields the magnetization profile of the system.
3.1. Effects of the applied field and in-plane anisotropy

The angle deviations, $\xi_{h}^1$ for the HML bottom plane and $\xi_{s}^L$ for the SML top plane, are computed for various values of the applied field $h$ with a direction $\theta_H = 0$ and $\lambda_{EI} = 1.44$. The results are shown in figure 2.

It is clear that the external field competes with the anisotropy and tends to align all the magnetic moments of the system parallel to its direction. Indeed, with enough planes in the SML layer, $\xi_{s}^L$ can be obtained from the Stoner–Wohlfarth equation

$$h \sin (\theta - \theta_h) - d_s \sin 2\theta = 0. \quad (10)$$

In the specific case of $\theta_h = 0$, we get $\theta = \arccos (h/2d_s)$, in agreement with the asymptotes (dashed lines) in figure 2(b).

In cases where the system has fewer layers than the necessary number to attain the asymptotic value of $\xi_{h}^1$ and $\xi_{s}^L$ given by the Stoner–Wohlfarth equation (10), the magnitude of $h_c$ should decrease. Figures 2(c) and (d) show that there is a critical value of the field at which all magnetic moments are aligned along the direction of the field, i.e. $\theta_H = 0$. In this case, as the applied field is along the HML anisotropy axis, and the SML magnetic moments interact with those of the HML via exchange interaction; $h_c$ is lower than the value given by the Stoner–Wohlfarth equation, i.e. $h_c = 2d_s$. This indicates that the approximation given in equation (10) becomes less valid as the field approaches $h = 2d_s$, and more planes are necessary in the SML to preserve the validity of the approximation. For a system with a sufficient number of layers in the SML layer, $h_c$ will depend only on $d_s$. In general, however, $h_c$ depends on $d_h$, $d_s$, and the applied field orientation.

Figure 3 shows the same plots for a variable field orientation and the SML intra-plane anisotropy $d_s$, for $h = 0.01$ and $\lambda_{EI} = 1.44$. We see that as the field is turned towards the SML easy axis, the magnetization profile (MP) obviously shifts upwards with no noticeable change in shape, reaching a deviation at the top plane that is again given by the numerical solution of the Stoner–Wohlfarth equation (figure 3(b)—dashed lines). Figures 3(c) and (d) suggest that beyond a given $\theta_H$ the increase in the deviation is almost linear with a slope that depends on the anisotropy $d_s$. This implies a constant rate of change of $\xi_{h}^1$ as it is mainly affected by
3.2. Effect of interface coupling

Now, we investigate the effect of the interface coupling, considering successively EI, DDI, and DMI. In the end we compare their effects on the magnetization profile.

3.2.1. Exchange interaction. The results in figure 4(a) show that due to the strong anisotropy of the HML layer, varying the interface exchange coupling only affects the magnetic moments within the SML layer that starts here at $n = 11$. On the other hand, the interface exchange coupling competes with the intra-layer exchange coupling $J_s$ and anisotropy $D_s$ of the SML layer. Thus the weaker the $\lambda_{EI}$ the stronger is the deviation.

Figures 4(b) and (c) show the deviations $\xi^h_1$ and $\xi^s_L$ as functions of $\lambda_{EI}$, respectively. As could be expected, with increasing EI we achieve higher $\xi^h_1$ and lower $\xi^s_L$ deviations. As we effectively increase the rigidity of the interface, the HML deviation at the interface thereby increases, whereas that of the SML decreases. This change in deviation is then conveyed through all the planes by means of the intra-layer EI, leading to the observed changes in $\xi^h_1$ and $\xi^s_L$. For low values of $d_s$, the $\xi^h_1$ increase is much slower than the $\xi^s_L$ decrease. However, as $d_s$ approaches $d_h = 0.02$, the two become comparable as the two systems are close to being identical.

3.2.2. Dipolar interaction. In the present work we consider the system setup where the dipolar coupling is assumed to induce a ferromagnetic coupling between the two magnetic layers.

In figure 5(b) are plotted the MP for different values of the DDI interface coupling $\lambda_{DDI}$. Apart from the obvious shift upwards as $\lambda_{DDI}$ decreases, there is an abrupt change at the interface especially for small $\lambda_{DDI}$, mainly due to the fact that the in-plane anisotropy $d_s$ has a stronger effect than DDI.

Figures 5(c) and (d) show that the system tends towards an asymptote as $\lambda_{DDI}$ increases. In our case, since the DDI vector is parallel to the HML anisotropy axis (along...
the $z$-axis), a strong enough interaction aligns the magnetic moments at the interface (and all the magnetic moments in the HML) in the direction $\theta = 0$, thus driving the system into an effective rigid-interface state. As such, the asymptote can be found by computing $\xi_s L$ using the analytical expressions obtained by de Sousa et al [51]. Indeed, figure 5(e), where the MP is plotted for different values of $N_s$ with very strong DDI ($\lambda_{DDI} \sim 5$), shows a perfect agreement. Furthermore, if we examine the analytic curve (black line) near the plane of index 20, i.e. $N_s = 10$, we observe a regime where a slight change of $N_s$ induces a large change in $\xi_s L$, indicating that $N_s = 10$ is approximately the critical length of the SML. A similar regime starts to be seen for stronger values of DDI in figure 5(b). Then, if we take into account the fact that this behavior is not observed for weak DDI, it suggests that the critical length of the chain increases with increasing DDI.

DDI is a long-ranged interaction that can penetrate into the SML leading a priori to a coupling of the HML top plane to every SML plane, and vice versa. We also show in figure 5 that with $\lambda_{DDI}$ at the interface, with a bond $e_{DDI}$ along the $z$-axis, we can approximate the configuration of our system by a rigid interface. Upon taking long-distance interaction into account, the HML top plane will be coupled to every SML plane, but the SML lowest plane will only be coupled to the HML plane at the interface, due to the latter being in a rigid-interface configuration. We call this a long-distance (LD) configuration. Figure 6 shows a comparison between the MP for $\lambda_{DDI} = 0.15$ only at the interface and LD configuration. The effect of the coupling penetration is a global decrease in the deviation of the magnetic moments of the SML planes. A similar behavior can be obtained with the effective couplings $\lambda_{DDI}^{\text{eff}} = 0.201$ and $J_s^{\text{eff}} = 1.095$ with interaction only at the interface. This means that the effect of LD configuration is equivalent to that of an interaction which is limited to the interface, but with the re-normalized couplings $\lambda_{DDI}^{\text{eff}}$ and $J_s^{\text{eff}}$. Figures 6(a)–(c) show that $\lambda_{DDI}^{\text{eff}}$ and $J_s^{\text{eff}}$ do not change with the number of SML planes. However, figure 7 shows that they do depend on $\lambda_{DDI}$.

3.2.3. DM interaction. As discussed in the Introduction, it is relevant in the present study to investigate the effect of DMI on the dynamics of the magnetic dimer, on the same footing as the (symmetrical) effective exchange coupling and anisotropic dipolar coupling. DM interaction may be induced by spin–orbit coupling between two ferromagnetic
layers separated by a paramagnetic plane [30, 48]. It plays an important role in the presence of roughness and disorder at the interface of multi-layer systems. For a simple cubic lattice, the (1 0 0) surface the DMI vector $D$ lies in the plane and thus induces a perpendicular anisotropy. In the present work, we consider two situations where the DMI vector $D$ lies.
Figure 6. Comparison between different magnetization profile for a rigid-interface system with dipolar interaction at the interface only and with an effective value for both $\lambda_{DDI}$ and $j_{s}$. $\lambda_{DDI} = 0.15$, $\lambda_{eff}^{DDI} = 0.201$, $j_{s}^{eff} = 1.095$. (a) $N_{s} = 10$, (b) $N_{s} = 20$ and (c) $N_{s} = 30$.

Figure 7. Comparison of the magnetization profile for a rigid-interface system with dipolar interaction at the interface only, for the long-distance regime, and for the case with effective values for $\lambda_{DDI}$ and $j_{s}$ yielding the same magnetization profile as for the long-distance regime. $N_{s} = 10$. (a) $\lambda_{DDI} = 0.1$, $\lambda_{eff}^{DDI} = 0.1285$, $j_{s}^{eff} = 1.0778$. (b) $\lambda_{DDI} = 0.2$, $\lambda_{eff}^{DDI} = 0.2815$, $j_{s}^{eff} = 1.104$.

either along the x direction (thus in the xy plane) or along the z direction (i.e. normal to the xy plane).

DM vector in the x direction. According to equation (5), the DMI tends to orientate the magnetic moments in such a way that they are normal to each other and perpendicular to the vector $\mathbf{D}$. Figure 8(b) shows that with $\mathbf{D}$ along the x-axis, the HML magnetic moments at the interface align along their easy axis $e_{z}$, as it is perpendicular to the vector $\mathbf{D}$ and there are no
other fields that could lead to a deviation from it. Therefore, all magnetic moments of the HML layer will stick up along the direction $\theta = 0$. On the other hand, the SML magnetic moment at the interface, because it has to be perpendicular both to the HML magnetic moment ($z$-axis) and the vector $\mathbf{D}$ ($x$-axis), lies in the $xy$ plane. However, in the SML layer there is a competition between the DMI and the anisotropy $d_s$ in the $xy$ plane leading to a variable azimuthal angle $\phi_s$ as can be seen in figure 8(c). With weaker DMI the anisotropy has a stronger effect and the SML magnetization at the interface...
lies near the x-axis. As DMI increases the magnetization turns towards the y-axis as it has now to be perpendicular to the vector D.

In figure 8(d) we see that the HML magnetization stays along the z-axis ($\xi^h_z = 0$) while that of the SML layer remains in the xy plane ($\xi^s_\perp = \pi/2$) for values of $d_s = 0.005, 0.01, 0.015$ and 0.02. This means that the HML magnetic moment at the interface is not affected by a varying SML anisotropy in this setup. However, as is seen in figure 8(e) the $\varphi^s$ dependence on the SML–HML coupling is affected by the anisotropy, exhibiting higher deviations for lower values of the anisotropy. Indeed, it is more favorable for the interaction to win the competition and drive the magnetic moment of SML at the interface onto the y-axis. For a strong enough interaction the SML magnetization at the interface is pinned along the y-axis and the system can be modeled as a rigid magnet where the magnetization of the SML varies in-plane with no out-of-plane component. In this situation, the top SML magnetization can be calculated using [51], with the angle variation $\xi^s_\parallel$ now referring to $\varphi^s$ instead, because $\xi^s$ is fixed at $\pi/2$.

Figure 9 shows the MP similar to the one shown in figures 8(b) and (c), but in this case we force the HML magnetic moments to have a deviation $\xi^h = \pi$, opposite to that of figure 8(b), and we let the system evolve. This causes the orientation of the SML magnetic moments to switch as well, thus demonstrating that for a system with this specific setup, it can be used as a ‘magnetic switch’ since switching only the magnetic moment of the HML layer induces a switching of the SML magnetic moment. This is also true the other way round as is the case in exchange-spring systems where one attempts to achieve the reversal of the hard plane by smaller DC fields upon acting on the soft plane.

**DM vector in the z direction.** Similarly to the previous case, the DMI will align the magnetic moment of one of the layers at the interface along the anisotropy axis of the corresponding layer, whereby its magnetization will not vary with the interaction. In the present case of $D \parallel e_z$, however, it is the SML magnetic moment that remains constant because its anisotropy axis is normal to the vector D. On the other hand, the HML (net) magnetic moment becomes pinned in the yz plane, implying that the angles $\varphi$ for both layers will remain constant and equal to $\varphi^h = \pi/2$ and $\varphi^s = 0$. As such, the deviation at the interface for the HML layer will vary according to the competition between the HML anisotropy and the DMI, see figure 10(b). Hence, changes in the SML anisotropy will not affect the MP of the system. Indeed, calculations of the $\xi^h_\parallel$ and $\xi^s_\parallel$ for various values of $d_s$ were performed and the same results, shown in figure 10(c), were obtained for all of them, thus confirming that the system is not affected by varying the value of the SML anisotropy.

For larger values of DMI the system can be viewed as an ‘inverted’ rigid magnet, where the SML and the HML (net) magnetic moments at the interface are pinned in-plane. The out-of-plane variation of the HML magnetic moment results from the competition between the HML intra-layer exchange interaction and anisotropy. In this situation, the variation in the HML magnetization can be calculated by using [51], upon substituting $d_h$ for $d_s$.

### 3.2.4. Comparison between the interface couplings

In figure 11 we present different magnetization profiles for typical values of the EI ($\lambda_{EI} = 1.44$), DDI ($\lambda_{DDI}/\lambda_{EI} \approx 10^{-2}$) and DMI ($\lambda_{DMI}/\lambda_{EI} \approx 0.1$), as can be found in [48, 51, 53]. Figures 11 (left, right) present the MP in the polar (ξ) and azimuthal (ϕ) directions, respectively. In the polar MP the black curve with circles is the MP with only EI at the interface and serves as a reference. The red curve with squares is the MP obtained as we add DDI. Compared to the EI strength, the DDI is two orders of magnitude weaker and thereby its contribution to the alignment of the magnetic moments at the interface is very small. Nonetheless, it tends to align the interface along the z-axis and this effect propagates throughout all sub-layers by the in-plane exchange interaction, causing a global decrease in the MP. The dark blue curve (triangles up) represents the MP for DDI only. We see that the weak DDI is not sufficient to overcome the anisotropy of each sub-layer, leading only to a slight deviation.

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**Figure 9.** Magnetic ‘switch’ behavior in the (a) out-of-plane ($\xi$) and (b) in-plane ($\varphi$) directions of the system with Dzyaloshinski–Moriya coupling at the interface and vector $\mathbf{D}$ along the $x$ axis. $N_h = 10$ and $N_s = 11$. 

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**Equations:**

- $\xi^h_z = 0$
- $\xi^s_\perp = \pi/2$
- $\varphi^s$
- $\lambda_{DDI}/\lambda_{EI} \approx 10^{-2}$
- $\lambda_{DMI}/\lambda_{EI} \approx 0.1$
- $d_h$
- $d_s$
- $\lambda_{EI} = 1.44$
Figure 10. (a) System setup with interface Dzyaloshinski–Moriya interaction with a vector $\mathbf{D}$ parallel to the easy axis of the hard layer; (b) magnetization profile; (c) deviations of the magnetic moments at the lowest plane of the hard layer ($\xi^h_1$) and of the top plane of the soft layer ($\xi^s_L$) with the Dzyaloshinski–Moriya interaction at the interface $\lambda_{\text{DMI}}$. $N_h = 10$ and $N_s = 11$.

Figure 11. Effect of exchange, dipolar and Dzyaloshinski–Moriya interaction ($x$ direction) on the magnetic profile of a exchange spring, with $j = 1.44$, $\lambda_{\text{DDI}} = 0.0144$ and $\delta = 0.144$, $N_h = 10$ and $N_s = 11$. 
near the interface but away from the latter each layer remains parallel to its easy axis. We observe an increased (induced) deviation in the SML as compared to that of the HML. This is due to the stronger anisotropy of the HML enhanced by the DDI-induced anisotropy in the z direction, making the out-of-plane direction more favorable at the interface than the in-plane direction. On the other hand, for the reasons given earlier, the DMI with its vector \( \mathbf{D} \) along the \( x \)-axis, leads to a similar result with each layer aligned along its own easy axis even near the interface.

The MP in the azimuthal direction shown in figure 11 (right) compares the three interface couplings. The EI and DDI are two interactions that do not induce an azimuthal rotation and the corresponding MP always lies in the same plane (\( \varphi = 0 \)). The green curve (with diamonds) is the MP obtained after adding DMI with its vector \( \mathbf{D} \) in the \( x \) direction. In figure 11 we see that the DMI induces a slight decrease in the deviation \( \varphi^b \) because the DMI tends to align the magnetic moments of the SML layer along the \( y \) axis, as is seen previously in figure 8(e). Even though the magnitude of this interaction was not high enough to overcome the anisotropy and internal exchange, it induced a deviation at the interface in the azimuthal direction. This deviation is, however, constrained by the EI at the interface and the anisotropies of the layers. The light blue curve (triangles down) represents the MP for DMI with no exchange at the interface. As could be expected, the magnetic moments of the HML are all aligned along the \( z \)-axis (recall that \( \mathbf{D} \) is along the \( x \)-axis), while those of the SML layer are all in the \( xy \) plane with a higher \( \varphi \) deviation at the interface than in the previous case. This is obviously due to the fact that now the DMI competes only with the anisotropy of the SML layer.

3.2.5. Hysteresis loops. Here we present a succinct study of the hysteresis loop for different values of the interaction. It helps us to understand how the switching mechanism of the multi-layer system changes with the nature and strength of the interface coupling. All the hysteresis curves presented are plots of the normalized magnetization \( M_{\alpha}/M_{\alpha} \) versus reduced applied magnetic field \( h \), along the HML anisotropy axis (\( z \)-axis), \( N^h_B = 12 \) and \( N^s_B = 16 \). We start in a state of zero magnetization (i.e. the magnetic moments of all sub-layers are on the plane with no applied field) and iterate the Landau–Lifshitz equation until we reach the state of minimal energy. Next, we increase the applied field slightly and wait for the system to reach the new equilibrium state. We keep increasing the applied field until we reach saturation of the magnetization. Then, we ramp the field down and up again thus closing the hysteresis loop. Obviously, for each value of \( h \), we wait until the state of minimal energy is reached.

Figure 12(a) shows hysteresis cycles for different values of EI. Let us denote by \( h^c_B \) the SML switching field, by \( h^c_C \) the HML switching field, and by \( h_{sat} \) the saturation field of the whole multi-layer system. With no interaction (\( \lambda_{EI} = 0 \)) and for \( h = 0 \) the equilibrium state is that where the magnetic moments (of HML and SML) are oriented along their respective easy axes, i.e. \( \theta^i_k = 0 \) and \( \theta^i_i = \pi/2 \) for all \( k \in \text{HML}, i \in \text{SML} \). The net magnetization of the system is that given by the HML net magnetic moment projected on the \( z \)-axis. When \( h \) is increased the magnetic moments of the SML start to rotate towards the applied field in a reversible process until the system reaches saturation (\( \theta^i_k = \theta^i_i = 0 \) for all \( k, i \) at \( h = h_{sat} \). As we ramp down the field across zero until it reaches the SML anisotropy field (\( h^c_C = 2d_0 \approx 0.02 \)), the SML magnetic moments switch towards the field direction \( \theta^i_i = \pi \) for all \( i \in \text{SML} \). Further increase of the applied field (in the opposite direction) induces a slight reversible deviation of the HML magnetization. In figure 12(a) this corresponds to the plateau (on the \( \lambda_{EI} = 0 \) curve) from \( h = -0.02 \) to \( h \approx -0.04 \), until \( h \) reaches the HML anisotropy field \( h^c_B = h_{sat} \approx 2d_0 = 0.04 \). At this value of the field, the HML magnetic moments coherently switch from \( \theta^i_k \approx 0 \) to \( \theta^i_k = \pi \), thus achieving negative saturation. Along the lower branch the system follows the same switching process from \( \theta = \pi \) to 0.

For nonzero but weak coupling, \( \lambda_{EI} = 0.04 \), the system follows the same behavior, but neither the HML nor the SML goes through a coherent rotation because of the interface interaction. The SML switching field \( h^c_B \) is now higher because its magnetization is stabilized by the interaction with the HML. On the other hand, \( h^c_C = h_{sat} \) decreases because the molecular field of the already reversed SML acts against the HML anisotropy field. For stronger EI (\( \lambda_{EI} = 0.11, 1 \) the plateau disappears completely (\( h^c_C = h^c_B = h_{sat} \)), indicating that once the SML magnetic moments reach a certain deviation, the EI induces a cascade effect in the HML that causes the switching of the latter. The same observations of nonuniform magnetization switching leading to a smaller coercive field were made for magnetic recording exchange-spring media [23, 24]. Finally, an increase of the EI increases the remnant magnetization. The coercive field increases as long as \( h_{sat} > h^c_C \). As soon as the EI becomes strong enough to observe the cascade effect (\( h_{sat} = h^c_B \)), the coercive field becomes a decreasing function of EI. Now, since the interlayer exchange coupling decreases for an increasing spacer thickness, the coercive field passes by a minimum when the latter increases, again in agreement with what has been observed in [23, 24].

Weak (\( \lambda_{DDI} = 0, 0.03 \)) or medium (\( \lambda_{DDI} = 0.1, 0.3 \)) DDI has a similar behavior to that of the EI, as can be seen in figure 12(b). There is some difference, however, between the EI and DDI regarding the evolution of the hysteresis cycle as the interface coupling is varied. While increasing EI decreases \( h_{sat} \), the DDI induces an increase of \( h_{sat} \) with increasing \( \lambda_{DDI} \). Indeed, DDI whose bond is along the \( z \)-axis induces an additional anisotropy at the interface along this axis and thereby tends to stabilize the magnetization in this direction. For \( \lambda_{DDI} = 1.0 \) the induced-anisotropy field is stronger than the HML anisotropy field. This implies that it is possible for the magnetic moments of the HML that are far from the interface to switch before those at the interface. When the applied field becomes in excess of the induced-anisotropy field a complete switching is achieved. Finally, both the remnant magnetization and coercive field increase with DDI.

The effect of the DMI along the \( z \)-axis on the hysteresis cycle is shown in Figure 12(c). A similar behavior to those
Figure 12. Hysteresis loop for the whole multi-layer with (a) exchange, (b) dipolar, and (c) Dzyalozhinski–Moriya (along the \(z\)-axis) coupling at the interface between the soft and hard layers. \(N_h = 12\) and \(N_s = 16\).

exhibited by EI and DDI for weak interaction (\(\lambda_{\text{DMI}} = 0.09\)) is observed. Again, we observe a progressive rotation of the SML magnetization before the applied field becomes strong enough to induce a reversal of the HML magnetization. As we discussed earlier, with no applied field the DMI along the \(z\)-axis induces a variation in the deviation of the magnetic moments of the HML, while pinning the SML (net) magnetic moment along the \(x\)-axis. The slight deviation induced in the magnetic moments of the HML leads to a reduced HML switching field \(h_{cH} = h_{\text{sat}}\). Further increase of the interaction (\(\lambda_{\text{DMI}} = 0.3\)) induces again a cascade effect in the HML that causes both layers to switch in the same field (\(h_{cs} = h_{cH}\)). However, we have to note that unlike EI and DDI, the HML/SML switching field is not equal to \(h_{\text{sat}}\). This is again due to the deviation induced in the HML magnetic moments by DMI. Both the remanent magnetization and the coercive field decrease with increasing DMI.

We can see that each kind of interaction induces a specific ‘single magnetic moment’-like behavior for strong coupling \(\lambda\). A strong EI tends towards a system with no anisotropy, whereby the area of the loop tends to vanish, reaching saturation with very low applied fields in a switch-like behavior. DDI on the other hand tends to induce a square loop, typical of a very high uniaxial anisotropy with easy parallel to the applied field. Finally, the DMI tends to narrow the cycle and the magnetization follows the applied field, typical of a system with uniaxial anisotropy perpendicular to the field. Obviously, the perfect ‘single magnetic moment’ behavior is never reached because the interface interaction is present only at the interface and the deviation of the outer sub-layers is limited only by the intra-layer exchange interaction and the applied field.

4. Conclusion

We have studied a magnetic multi-layer system composed of a hard magnetic layer with out-of-plane uniaxial anisotropy and a soft magnetic layer with in-plane uniaxial anisotropy, separated by a nonmagnetic spacer. We have considered three cases of interface coupling, namely exchange, dipolar or Dzyalozhinski–Moriya interactions. The soft magnetic layer has been modeled as a stack of atomic (e.g. Fe) planes, while the hard magnetic layer has been modeled either as a macroscopic magnetic moment or again as a stack of atomic (e.g. FePt) planes. Each atomic plane is modeled as a macroscopic magnetic moment representing its net magnetic moment, and is coupled to the adjacent planes by exchange coupling.
We have investigated the effect of the external magnetic field, the in-plane or out-of-plane anisotropy, and the three interactions on the deviation angle (relative to the hard layer anisotropy easy axis). We have computed the magnetization profile through the whole multi-layer system.

For the effect of the applied magnetic field, we found that with a sufficient number of planes in the soft layer, the system behaves according to the Stoner–Wohlfarth regime and that there exists a critical field at which the whole system aligns along the applied field.

For exchange and dipolar interactions, there is an asymptotic value that depends on the anisotropy and the intra-layer exchange. For the dipolar interaction with a bond along the hard layer anisotropy easy axis, this asymptotic value is given by the analytical expression for rigid interface, where the hard layer is modeled as a single pinned magnetic moment, and only the variation in the soft layer is relevant. The dipolar interaction was next extended through the whole soft layer with a rigid interface. The ensuing effect on the magnetization profile of the soft layer has been recovered by an effective dipolar coupling at the interface only, upon re-scaling the intra-layer exchange coupling. The two corresponding effective coupling parameters depend on the initial dipolar interaction, but not on the number of atomic planes in the soft magnetic layer.

For the Dzyalozhinski–Moriya interaction, we found that the magnetic moments of one of the layers are pinned in a given direction, whereas those of the remaining layer rotate in either the polar angle or the azimuthal angle, depending on the direction of the vector \( \mathbf{D} \). Large values of the Dzyalozhinski–Moriya coupling lead to a system with rigid interface, with the magnetic moments at the interface being perpendicular to each other and to the vector \( \mathbf{D} \). In addition, a switch-like mechanism can be achieved with this interaction. Indeed, an indirect reversal of the magnetization of the soft magnetic plane can be achieved by directly forcing a reversal of the magnetization of the hard magnetic plane with the help of a magnetic field. It is obviously also possible to obtain the desired effect for the exchange-spring system by achieving the reversal of the hard magnetic plane via the switching of the magnetization of the soft magnetic plane with the help of a smaller magnetic field.

A comparison between the three interactions with typical orders of magnitude has been given. The exchange coupling shows the strongest effect, and when added, the dipolar and Dzyalozhinski–Moriya interactions induce a slight (but non negligible) deviation in either the polar or azimuthal magnetization profile.

Finally, hysteresis cycles for the three different interactions are computed. A typical exchange-spring behavior, where the soft layer switches first, followed by the hard layer at a stronger field, is observed for weak interaction in all cases. Strong coupling causes both layers to switch under the same field. Exchange and Dzyalozhinski–Moriya interactions tend to narrow the cycle, while the dipolar interaction leads to squared cycles. For application purposes such as magnetic recording using vertical exchange-spring media, the Dzyalozhinski–Moriya interface coupling strongly reduces the coercive field while keeping high values of the anisotropy which thus ensure good thermal stability.

A work in progress consists in treating each atomic plane as a two-dimensional lattice with the aim to compute spin correlations as functions of the various energy parameters and to determine the spin-wave spectrum. In the near future, this zero-temperature study will be extended to finite temperature with the aim to investigate thermal effects with special emphasis on the calculation of activation rates of such multi-layer systems and thereby assess their thermal stability.

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