A Bayesian spatial voting model to characterize the legislative behavior of the Colombian Senate 2010–2014

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ABSTRACT
This paper characterizes the legislators voting behavior in the Colombian Senate 2010–2014, by implementing a one-dimensional standard Bayesian ideal point estimator via Markov chain Monte Carlo algorithms. Our main goal is to retrieve the political preferences of legislators from their roll-call voting records, which individualizes the electoral behavior of the legislative chamber. Furthermore, we conclude about the nature of the latent trait underlying the deputies voting decisions and the legislators locations in political space. Finally, we also offer several methodological and theoretical tools to guide the analysis of nominal voting data in the context of unbalanced parliaments (multi-party systems), taking as reference the particular case of the Colombian Senate.

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1. Introduction

Spatial voting models allow studying the competitive behavior of political agents. Their implementation has played a prominent role in Political Science since the 1950s [6,28]. Many publications since then have increased the visibility and highlighted the usefulness of these models to provide empirical evidence supporting the theoretical explanation of electoral behavior in different contexts, particularly in legislatures. The Bayesian ideal point estimator [10,23], whose specification and implementation date back to the last decade, stands out among the spatial voting models. The literature shows that this model is a fundamental tool for research in Political Science [54] and also evidences that it is a dominant methodological and theoretical tool for analyzing modern voting patterns [32,55].

Studies on legislative electoral behavior in Colombia point to an important field of research (see [4,5,8,9,35–37], among others). An exhaustive literature review shows that quantitative research about the Colombian parliament primarily reveals patterns and determinants of legislative behavior at the party, coalition, and committee levels (e.g. [9,36]), typically leaving the political behavior of each legislator behind. The subject-specific electoral analysis in Colombia is relevant because the political landscape is personalized [35,37]. In addition, studying parliamentary electoral behavior based on individualized political preferences (latent features of legislators) makes it possible to analyze the electoral conduct
of Senate members belonging to factions with low representation in Congress. Political parties with few seats (e.g. indigenous minorities) are typically excluded from studies taking political groups as observational units (e.g. [9]). Finally, inquiring about the individual electoral behavior from a Bayesian perspective allows us to make probability statements about the deputies location in the political space [10].

Also, most studies about Colombian legislative behavior are merely descriptive (e.g. [35,37]). In particular, very few studies apply any spatial voting model to analyze parliamentary voting behavior. To the best of our knowledge, there exists only one peer-reviewed article [9] that applies scaling methods [17] as well as classification methods [39] to account for patterns of party-specific legislative voting behavior through roll-call votes (votes either in favor of or against a particular motion). Specifically, no available research uses ideal points to characterize legislators political preferences from roll-call data.

The Bayesian ideal point estimator has been widely applied in the United States. In this scenario, several studies reveal technical conditions to guarantee its functionality (e.g. the election of deputies to ensure identifiability, known as anchor legislators, [10]). However, these investigations only exemplify operativity conditions framed in the North American parliament. The United States Congress is known for having two dominant political parties, Democrats and Republicans, with roughly the same size within parliament, a bipartisan structure that we refer to as balanced. However, this type of parliament is quite different from other legislative bodies in terms of composition. In the Colombian case, handing out seats among more than two political parties is common, with typically unequal sizes, a multi-party structure we refer to as unbalanced. For instance, in the 2010–2014 Senate, the total number of seats was divided among ten parties, with participation percentages ranging between 27.45% and 0.99%. In addition, parties usually make up other political groups within the chamber (not necessarily politically opposed), such as the government coalition (four parties, 73.53%), independents (three parties, 16.66%), indigenous minorities (two parties, 1.98%), and opposition (one party, 7.84%).

The Colombian Senate composition raises concerns about implementing the one-dimensional Bayesian ideal point estimator. For instance, studies on the United States Congress handle model identifiability by selecting an anchor legislator on each side of the political spectrum, typically the party leader [10]. However, such a choice is neither transparent nor immediate in unbalanced parliaments (e.g. there are more than two political groups out of which we need to select the anchor legislators, and it is not always clear who might be the leader of a given political party).

Some authors apply the one-dimensional model in scenarios other than the North American one (e.g. [41]). However, we are not aware of any study that evaluates the implementation of the one-dimensional standard Bayesian ideal point estimator when applied to nominal vote data in unbalanced parliaments. Some analysts argue that adopting methodologies applied to the United States Congress is common when investigating foreign legislative bodies. However, they emphasize that it is essential to assess the validity of these methods in contexts other than the one in North America (e.g. [24,38,57]).

In this way, we characterize the legislators voting behavior in the Colombian Senate 2010–2014 by implementing a one-dimensional standard Bayesian ideal point estimator. Our main goal is to retrieve the political preferences of legislators from their voting records. In addition, from point estimates, we reveal latent characteristics along with individual and grouping patterns that underlie the legislative voting decisions of the deputies. We also
discussed the dimension of political space. Moreover, we carry out an exhaustive simulation study to evaluate the model performance under different configurations of anchor legislators, link functions, missing data rates, and prior beliefs in unbalanced parliaments. The experiments in this regard support our Colombian Senate findings and provide a general guideline to implement the model in unbalanced parliaments with a compositional structure different from the North American one.

This document is structured as follows: Section 2 presents a synthesis of the quantitative studies on legislative behavior in Colombia; Section 3 shows the theoretical aspects of the Bayesian ideal point estimator, together with the particularities of the dimension of the political space and the identification of pivot legislators; Section 4 extends the details of the implementation of the one-dimensional standard Bayesian ideal point estimator; Section 5 states the results of the simulation study; Section 6 analyzes the data for the Colombian Senate 2010–2014; and finally, Section 7 discusses the results of the work and some alternatives for future research.

2. Quantitative studies on legislative electoral behavior in Colombia

Work specifically focused on analyzing legislative chambers in Colombia dating back to the 1970s [18,20,26,27]. These studies characterize the legislature from a descriptive point of view (e.g. [27]) and in a context prior to the 1991 constitutional reform, which substantially modifies the regulation of the Congress of the Republic as well as the policy-making process in the country [8,37]. These researches do not use voting records to analyze electoral behavior within legislative bodies. The employment of voting data for the characterization of parliamentary electoral behavior in Colombia is quite recent since such data have been systematically collected as of 2006 [9].

After the reform, a contemporary field of research in relation to quantitative studies on legislative behavior was identified. Studies such as the ones developed in Archer and Shugart [5], Cárdenas et al. [8], Alemán and Pachón [4], Pachón [35], Carroll and Pachón [9], Pachón and Johnson [36], Pachón and Muñoz [37], among others, offer a look at the Colombian legislature from its current context. Some of them provide empirical evidence regarding the programmatic nature, nominal voting patterns, and legislative activity trends of political groups [9,36].

Although some authors have used sophisticated statistical techniques (e.g. frequentist probit regression models and optimal classification methods to analyze legislative electoral behavior, [9]), it is noteworthy that no study uses data from nominal voting to date to recover individual political preferences. Instead, roll calls have been used to examine aggregate voting patterns (by party, coalition, and committee), considering only the political groups with the largest seats in Congress [9]. To the best of our knowledge, the literature does not offer studies focused exclusively on the electoral conduct of the Senate of the Republic of Colombia.

3. Bayesian spatial voting modeling

Roll call data are generated when $n$ legislators vote on $m$ motions (bills, legislative initiatives, etc.); each legislator $i \in \{1, \ldots, n\}$ must take a position in favor of or against motion $j \in \{1, \ldots, m\}$. In order to operationalize the legislative voting behavior, the votes in favor
of or against motion $j$ are conceptualized as points in a $d$-dimensional Euclidean space (known as political space), which in turn are denoted by $\psi_j$ and $\zeta_j$, respectively.

Let $y_{ij} \in \{0, 1\}$ be the vote cast by legislator $i$ on motion $j$, with $y_{ij} = 1$ if such a vote turns out in favor of the motion, and $y_{ij} = 0$ otherwise. It is assumed that all the legislators have a political preference, i.e. each legislator $i$ has a latent (unobserved) factor $\beta_i \in \mathbb{R}^d$ known as ‘ideal point’. Thus, considering both voting alternatives and ideal points, decisions are made based on a quadratic utility function defined over the political space given by

$$U_i(\psi_j) = -\| \psi_j - \beta_i \|^2 + \eta_{ij} \quad \text{and} \quad U_i(\zeta_j) = -\| \zeta_j - \beta_i \|^2 + \nu_{ij}$$

(1)

where $U_i(\psi_j)$ and $U_i(\zeta_j)$ are the corresponding profits associated with legislator $i$ voting in favor of or against motion $j$, respectively, $\| \cdot \|$ is the Euclidean norm in $\mathbb{R}^d$, and finally, $\eta_{ij}$ and $\nu_{ij}$ are independent stochastic deviations (random shocks) resulting from the uncertainty involved in the voting processes, whose joint probabilistic distribution is such that $\mathbb{E}(\eta_{ij} - \nu_{ij}) = 0$ and $\text{Var}(\eta_{ij} - \nu_{ij}) = \sigma_j^2$. Even though there exist other meaningful ways to define both the political space and the utility function, they fall outside the scope of this this work and will be pursued elsewhere.

Rational choice theory (e.g. [10,55]) states that under the previous setting, legislator $i$ votes in favor of motion $j$ if and only if $U_i(\psi_j) > U_i(\zeta_j)$, i.e.,

$$y_{ij} \mid \zeta_j, \psi_j, \sigma_j, \beta_i = \begin{cases} 1, & \text{if } U_i(\psi_j) - U_i(\zeta_j) > 0 \\ 0, & \text{otherwise} \end{cases}$$

and therefore,

$$\text{Pr}(y_{ij} = 1 \mid \zeta_j, \psi_j, \sigma_j, \beta_i) = \text{Pr}(\epsilon_{ij} < \mu_j + \alpha_j^T \beta_i) = G(\mu_j + \alpha_j^T \beta_i)$$

where $\epsilon_{ij} = (\nu_{ij} - \eta_{ij})/\sigma_j$ is the random quantity associated with the voting process, $\mu_j = (\zeta_j - \psi_j^T \beta_i)/\sigma_j$ is the ‘approval’ parameter representing the basal probability of a vote in favor of motion $j$, $\alpha_j = 2(\psi_j - \zeta_j)/\sigma_j$ is the ‘discrimination’ parameter representing the effect that ideal points have upon the probability of a vote in favor of motion $j$ across political space dimensions, and finally, $G(\cdot)$ is an appropriate link function. If $\epsilon_{ij} \sim \text{N}(0, 1)$, then $G$ is a probit link, i.e., $G(\mu_j + \alpha_j^T \beta_i) = \Phi(\mu_j + \alpha_j^T \beta_i)$, where $\Phi(\cdot)$ is the cumulative distribution function of the standard Normal distribution. On the other hand, if $\epsilon_{ij}$ follows a Standard Logistic distribution, then $G$ is a logit link, i.e., $G(\mu_j + \alpha_j^T \beta_i) = \expit(\mu_j + \alpha_j^T \beta_i)$, where $\expit(x) = 1/(1 + \exp(-x))$ is the inverse function of the logit function $\logit(x) = \log\frac{x}{1-x}$.

The previous formulation leads to a latent factor model for binary data, which fully characterizes the probability of obtaining a positive vote, since

$$y_{ij} \mid \mu_j, \alpha_j, \beta_i \sim \text{Ber}(G(\mu_j + \alpha_j^T \beta_i))$$

and therefore, the likelihood of the model is

$$p(Y \mid \mu, A, B) = \prod_{i=1}^{n} \prod_{j=1}^{m} G(\mu_j + \alpha_j^T \beta_i)^{y_{ij}} \left[1 - G(\mu_j + \alpha_j^T \beta_i)\right]^{1-y_{ij}}$$

(2)
where $Y = [y_{ij}]$ is a binary matrix of size $n \times m$, $\mu = (\mu_1, \ldots, \mu_m)$ is a column vector of size $d \times 1$, and $A = [\alpha_1, \ldots, \alpha_m]^T$ and $B = [\beta_1, \ldots, \beta_n]^T$ are rectangular matrices of size $m \times d$ and $n \times d$, respectively.

In order to carry out a fully Bayesian specification of the model, it is mandatory to specify a joint prior distribution on the model parameters $(\mu, A, B)$. A simple but powerful alternative that works well in practice relies on assuming Normal independent priors, which leads to a semi-conjugate analysis and higher computational efficiency [23,54,55]. Thus, it follows that

$$ (\mu_j, \alpha_j) \mid a_0, A_0 \sim \text{iid } N(a_0, A_0) \quad \text{and} \quad \beta_j \mid b_i, B_i \sim \text{iid } N(b_i, B_i) $$

where $a_0, A_0, b_i, \text{ and } B_i$ are the (known) hyperparameters of the model. Note that $a_0$ and $b_i$ are mean column vectors of size $(d + 1) \times 1$ and $d \times 1$, respectively, and $A_0$ and $B_i$ are covariance matrices of size $(d + 1) \times (d + 1)$ and $d \times d$, respectively. As a final remark, some authors (e.g. [10]) propose setting $a_0 = 0_{(d+1)}$ and $A_0 = \sigma^2 I_{(d+1)}$ with $\sigma^2$ an arbitrarily large constant in order to assign a zero-centered non-informative prior to $\mu_j$ and $\alpha_j$, as well as setting $b_i = 0_d$ and $B_i = I_d$ for each $\beta_j$, which imposes a identification restriction on the ideal points based on the prior distribution (see Section 3.2 for more details).

### 3.1. Posterior inference

A $d$-dimensional Euclidean spatial voting model involves many parameters. Specifically, considering data from $n$ legislators on $m$ voting proposals, the model requires $dn + m(d+1)$ parameters, out of which $nd$ correspond to ideal points, whereas $m(d+1)$ to voting-proposal-specific parameters. Such a multi-parameter space makes the posterior distribution $p(\mu, A, B \mid Y)$ particularly difficult to obtain since it constitutes an analytically-intractable distribution framed in a high dimension.

We consider Markov chain Monte Carlo algorithms (MCMC; e.g. [14]) to approximate the posterior distribution since the prior specification allows us to structure tractable simulation-based algorithms. In particular, the Gibbs sampler ensures that a sequence of dependent but approximately independent draws from the posterior distribution can be generated, through iterative sampling from the full conditional distributions of the model parameters $\mu_j, \alpha_j, \text{ and } \beta_i$. Thus, point and interval estimates can be approximated from the corresponding empirical distributions. Details about the MCMC algorithms implemented here can be found in Appendix 1.

### 3.2. Identifiability

The model parameters in (2) and (3) are not identifiable. Specifically, note that Euclidean distances among ideal points $\beta_i$ and the voting alternatives $\psi_j$ and $\zeta_j$ remain invariant under any change of scale, translation, rotation, or reflection of the political space. Such a geometric occurrence ensures that both discrimination parameters and ideal points are not distinguishable for any voting pattern $Y$ [10,23]. For instance, consider a rotation of the political space through an $d \times d$ orthogonal matrix $Q$ (i.e. $Q^T Q = I_d$). Then, $(Q \alpha_j)^T (Q \beta_j) = \alpha_j^T \beta_j$, and therefore, $Pr(\psi_{ij} = 1 \mid \mu_j, Q \alpha_j, Q \beta_j) = Pr(\psi_{ij} = 1 \mid \mu_j, \alpha_j, \beta_j)$ for all $i$ and all $j$. Such a phenomenon is also typical of latent position models for social networks (e.g. [48]).
Identifiability does not impede model fitting. However, it is required to support inferences about ideal points of political agents and discrimination parameters. That is why the literature has stated necessary and sufficient conditions for identifying spatial voting models based on parameter restrictions. For example, it is typical just setting $\sigma_j = 1$ because random shock scales cannot be identified separately from the scale associated with the political space [54]. On the other hand, it is common imposing restrictions on the mean and variance of the ideal points in order to make the political space invariant to translations, rotations, reflections, and re-scaling [23]. Specifically, $b_i = 0$ and $B_i = I_d$ is fruitful to overcome simultaneously translation and scale issues. Finally, it is also convenient fixing the position of $d + 1$ legislators (also known as anchor legislators or simply anchors) with known (but distinctive!) political patterns in the political space, since it allows the model to differentiate legislative tendencies [10,42]

### 3.3. Dimension of the political space

The choice of the dimension of the political space is a popular topic in the political science literature (e.g. [2,13,22,32,40,43,50]). From a technical point of view, the dimension of the political space corresponds to the number of latent features necessary to characterize accurately the legislators voting behavior. Such a choice necessarily leads to a model selection problem, in which a trade-off between model fit and complexity needs to be faced [32]. A large portion of the literature in this regard mostly discusses epistemological considerations [7,12] as well as specific quantitative mechanisms based on optimality criteria [22,30,32]. However, other alternatives are available. For instance, Jackman [22] shows how discrimination parameters can be used to make a reasonable model choice and assess the relevance of moving to higher dimensions.

In the Latin American case, research papers determining the dimension of the political space are quite scarce. Most studies assume one or two dimensions underlying the nominal voting behavior, depending on the political context of the corresponding legislatures under analysis (e.g. [57,58]). Along this road, some authors point out that the presence of religious, linguistic, or ethnic parties underlies the existence of an ideological dimension [58]. In contrast, coalitions, electoral districts, regional or provincial divisions, and others are indicators of non-ideological dimensions [57,58]. Likewise, some argue that legislative policy-making is typically one-dimensional in most countries across the region [44]. In the Colombian case, previous studies have assumed one dimension equating ideology with a non-ideological characteristic (left/opposition-right/government, [9]).

A single Latin American work empirically explores the number of latent traits underlying legislative voting patterns. Thus, inspired by the work of Jackman [22], Jones and Hwang [24] determine the dimension of the political space in the Argentinian parliament through the analysis of discrimination parameters. The authors provide empirical evidence to ensure that there was only one dimension underlying the political space in the case of the roll call voting in the Argentinian House of Representatives 1989–2003.

In the Colombian case, there exists some research that raises discussions about the dimension of political space (e.g. [19,21]). For example, Hoskin and Swanson [21] point out that there are two outstanding dimensions involved in the Colombian party system, an opposition-government feature as well as a left-right ideological feature. These authors argue that the conflict around government control is definitely more forceful than the
corresponding ideological conflict. However, to the best of our knowledge, no research provides empirical evidence about the dimension of the political space in the particular setting of the Colombian Senate.

### 3.4. Identifying pivot legislators

Pivot legislators (or pivots, not to be confused with anchor legislators above) are those parliamentarians whose position in political space is considered relevant to understanding what happens within the legislature. Therefore, research in this direction includes all sorts of work focused on determining the identity and position of those legislators (e.g. extremists, minority legislators, among others) that allow us to characterize a given camera completely.

Clinton et al. [10] state that pivot legislators are crucial to support some theories of parliamentary behavior in the North American context. The position of these legislators in the political space can be used to characterize and predict processes of policy-making [29]. However, from a political perspective, we are not aware of whether or not pivotal theory has meaningful applications in the context of unbalanced parliaments such as the Colombian one (substantial research is required in this direction). For this reason, we limit ourselves in this work to individualize those deputies who are more likely to have either a centrist or extremist position within the parliament, as opposed to making inferences on other quantities that can be obtained from ideal point estimates.

### 4. Prior elicitation and computation

Now we discuss our prior choice in the case of a one-dimensional setting, i.e., $d = 1$. Along the lines of Clinton et al. [10], we let $a_0 = 0$ and $A_0 = 25$ for the prior distribution of $\mu_j$ and $\alpha_j$, aiming to emulate roughly the behavior of a diffuse prior distribution. This choice is quite reminiscent of the hyperparameter elicitation in a standard linear regression model when there is no need for informative or empirical alternatives, such as the unit information prior [25] or the $g$-prior [56]. In addition, in the same spirit of Jackman [23] and Lofland et al. [30], we let $b_i = 0$ and $B_i = 1$ for the prior distribution of $\beta_i$. Unlike discrimination and approval parameters, it is not required to set a large variance a priori for the ideal points. All what required is a prior notion of scale for this set of parameters.

Parameter identifiability can be carried out as in Rivers [42] and Clinton et al. [10], by selecting $d + 1$ anchors (i.e., fixing the position of $d + 1$ legislators in the political space). For instance, in our case study, just two legislators need to be anchored since we are fitting a one-dimensional model (see also [9]). Specifically, we delimit the positions of Jorge Enrique Robledo Castillo (PDA member) and Roy Leonardo Barreras Montealegre (PU member) at $-1$ and $1$, respectively (the selection of these legislators is motivated by the reasons provided in Section 5.1). Thus, having fixed the location of two deputies, in a one-dimensional case like ours with $n = 91$ parliamentarians and $m = 417$ voting lists, leave us with a total of $2m + n - 2 = 923$ model parameters to estimate.

On the other hand, missing data are removed from the analysis (abstentions and absences, a rate of 40%) since they are not missing completely at random (MCA), so data imputation through naive sampling from the sampling distribution is not recommended (a discussion on this matter can be found in [46]). Although there are available imputation
methods for missing data with non-random behavior (e.g. [47]) and even extensions of the Bayesian spatial voting model that incorporate them (e.g. [45]), these methodologies are beyond the scope of this work (such methodologies are still not completely understood for unbalanced parliaments). However, the simulation study presented in Section 5.3 shows substantial evidence that our findings are robust to different missing data rates.

Model fitting is carried out using MCMC algorithms (see Appendix 1 for details) on a model based on a logit link function (see Section 3 for details). Inferences on model parameters are based on 80,000 samples of the posterior distribution obtained after thinning the original chains every 5 observations and a burn-in period of 24,000 iterations. In addition, before using the MCMC samples with inferential purposes, we determine first if there is any evidence of lack of convergence of any chain to its stationary distribution. Convergence diagnostics (not shown here) show that effective sample sizes are large enough to perform adequate inductive processes. Our code will be available for those readers that explicitly ask it from the corresponding author.

5. Simulation study

Here we provide an exhaustive simulation study to assess the robustness of a one-dimensional model to several features. Each synthetic dataset is generated with \( n = 91 \) legislators and \( m = 417 \) voting lists in order to assess scenarios as close to our case study as possible. Voting matrices are generated by setting the model parameters to random values. Specifically, the parameters \( \alpha_j \) and \( \mu_j \) are simulated from a Normal distribution with mean 0 and variance 3, whereas the ideal points \( \beta_i \) are simulated from a Uniform distribution with parameters \( a \) and \( b \). We set \( a = -3 \) and \( b = 3 \) for balanced parliaments (two parties, each with 50% of the deputies), and \( a = -3 \) and \( b = 4 \) for unbalanced parliaments (four parties, distributed as 75%, group 1; 15%, group 2; 2%, group 3; and 8%, group 4). Note that the unbalanced parliament emulates groups and proportions in accordance with our case study (see Section 6 for details).

We intentionally generate the ideal points so that the two groups of the balanced parliament and the three of the unbalanced parliament lie in a specific region of the political spectrum. Concurrently, we simulate group 3 of the unbalanced parliament so that its legislators show heterogeneous ideal points. In particular, one of the legislators lies between groups 1 and 4, and the other between groups 2 and 4. This way of reproducing the ideal points seeks to prove that the proposed model recovers the location of the groups in the political space, regardless of low or high within-variability.

5.1. Sensitivity analysis to anchor legislators

In order to assess our strategy about fixing anchor legislators to \(-1\) and 1, we select such anchors from different positions across the political space (we know all the true ideal points when generating synthetic data). Thus, we consider five scenarios, namely, opposite and close to zero (Scenario 1), left–center (Scenario 2), center–right (Scenario 3), opposite and at dissimilar distances from the center (Scenario 4), and extremists (Scenario 5). We evaluate these possibilities for balanced and unbalanced parliaments, using a 40% missing data rate and a logit link function. Particularly, Scenario 4 seems to emulate better the characteristics of the legislators that we decide to take as anchors in our case study.
Table 1. Information criteria for contrasting different anchor legislator alternatives.

| Scenario | Anchor legislators                  | Balanced DIC | Balanced WAIC | Unbalanced DIC | Unbalanced WAIC |
|----------|------------------------------------|--------------|---------------|----------------|-----------------|
| 1        | Opposite and close to center        | 11,886.95    | 12,096.09     | 8688.34        | 8929.50         |
| 2        | Left-center                        | 11,798.25    | 11,994.53     | 8682.17        | 8913.65         |
| 3        | Center-right                       | 11,820.27    | 12,019.34     | 8573.97        | 8802.72         |
| 4        | Different distances from the center | 11,782.73    | 11,972.11     | 8563.98        | 8775.22         |
| 5        | Extremists                          | 11,809.88    | 11,993.12     | 8582.86        | 8790.65         |

Figure 1. Scatter plots of $\beta_i$ vs. $\hat{\beta}_i$ under Scenario 4 (different distances from the center). Red points indicate the location of legislators used as anchors, whereas blue lines represent the line of reference $\beta = \hat{\beta}$. Finally, $r$ is the Pearson’s correlation coefficient, and $m$ is the slope associated with the corresponding regression line. (a) $\hat{\beta}_i$ vs. $\beta_i$, balanced parliament and (b) $\hat{\beta}_i$ vs. $\beta_i$, unbalanced parliament.

Table 1 presents the deviance information criterion (DIC, [49]) and the Watanabe-Akaike information criterion (WAIC, [52]) for every scenario. We see similar information criteria values within each legislative body and expect discrepancies between parliaments as they represent two entirely different chambers. In both cases, scenario 4 shows the lowest DIC and WAIC. However, these values do not show large discrepancies compared to the other scenarios.

Given that the information criteria values within each parliament are quite similar (see Table 1), this strongly suggests that the performance of the model is not affected by the political position of the legislators used as anchors, regardless of the number of groups and the sizes of the groups partitioning the parliament. However, we recommend choosing as anchor legislators those parliamentarians known to be on opposite sides of the political spectrum (as in Scenario 4) to ease interpretation and avoid selecting two legislators who share the same ideal point. Even though the existing literature supports this finding (e.g. [30]), this is the first work that provides empirical evidence regarding the choice of anchor legislators in unbalanced parliaments considering different positions in the political space.

Figure 1 makes it clear that the deputies ideal points are not accurately recovered because the model is not identifiable, and also, parameter estimation is carried out on a different scale (see Sections 3.2 and 4 for details). Also, we see a greater scaling factor effect on extreme ideal points, as opposed to those located around the center. Extreme values of discrimination and approval parameters (not shown here) are also more variable than the ideal points due to the scale effect. Finally, notice that scales behave proportionally with a
Figure 2. Estimates under the election of opposite anchor legislators and at different distances from the center (scenario 4). In each case, filled circles represent point estimates, and horizontal lines the corresponding 95% credible interval based on percentiles. (a) Ideal points, balanced parliament and (b) Ideal points, unbalanced parliament.

constant rate of change that can be quantified by the slope of the corresponding regression line.

Despite the scale change, by anchoring two legislators at −1 and 1, we retrieve the groups as they were generated, regardless of the scenario. Such behavior is particularly evident in Figure 2 where we show our findings for Scenario 4. Specifically, we see that for both parliaments, ideal point estimates between −1 and 1 reveal shorter credibility intervals. As legislators move away from the center towards the extremes of the political spectrum, the uncertainty about the ideal point estimates grows. This implies that the scaling effect is mostly evident through the ideal points margin of error. Also, legislators from different groups, whose ideal points are similar, are the most likely to be mixed with each other. The exchanges that are evident among members of different groups (e.g. around zero in Figure 2(a)) derive from the inherent uncertainty in the estimation of the parameters.

5.2. Sensitivity analysis to link functions

In order to investigate the robustness of the analysis to the choice of link functions (see Section 3 for details), we consider an unbalanced parliament with a 40% missing data rate and generate voting data using a logit link. Then, we fit the model anchoring two opposite legislators at different distances from the center to −1 and 1, using a logit link (as in Scenario 4 above) as well as a probit link (Scenario 6). All the different combinations of data generating process/model fitting in link functions, namely, logit–logit, logit–probit, probit–logit, probit–probit, yield analogous results.

The information criteria in Scenario 4 (DIC = 8563.98, WAIC = 8775.22) and Scenario 6 (DIC = 8420.61, WAIC = 8670.81) reflect similar results in predictive terms. Although the probit model exhibits lower information criteria than the logit model, such a difference
5.3. Sensitivity analysis to missing data rates

In order to evaluate the performance of the model at different missing data rates, we consider three scenarios, namely, with a missing rate of 10% (Scenario 7), 40% (as in Scenario 4 above), and 60% (Scenario 8). For this comparison, we use an unbalanced parliament taking as anchors legislators at different distances from the center as well as a logit link function.

Figure 3 shows that an increase in the percentage of missing data implies an increase in the width of the credibility intervals of the ideal points (the same pattern is revealed with the discrimination and approval parameters when contrasting the amplitude with the percentage of missing data per voting list). This observation allows us to conclude that the higher the missing data rate is, the greater the uncertainty in the estimates of all the model parameters is too. Furthermore, the effect of missing data seems to be larger for those estimates exhibiting values at the extremes of the political spectrum. Interestingly, we see that different missing data rates do not generate erroneous conclusions in terms of inference about the model parameters. In particular, it does not distort the clustering of the ideal points. In fact, they continue showing the same pattern as in Figure 2(b).

5.4. Sensitivity analysis to prior distributions of ideal points

Finally, we investigate the robustness of the analysis to the choice of prior distribution and hyperparameters for the ideal points. Such a task is essential because later analyzes focus on
these parameters. Thus, we compare the scenarios proposed below, considering an unbalanced parliament with a 40% missing data rate taking as anchors legislators at different distances from the center as well as a logit link function.

Three new scenarios are contrasted, namely, prior distribution without hierarchies (as in Scenario 4 above), i.e. $b_i = 0$ and $B_i = 1$; hierarchical prior with two stages for the variance of ideal points (Scenario 9), i.e. $b_i = 0$ and $B_i \sim \text{Gl}(c, d)$, with $c = 3$ and $d = 2$; hierarchical prior with two stages on the mean and variance (Scenario 10), i.e. $b_i \sim \text{N}(a, b)$ and $B_i \sim \text{Gl}(c, d)$, with $a = 0$ and $b = 25$. The choices described lead to diffuse priors on the ideal points, since if $c = 3$ and $d = 2$, then the coefficient of variation of the Inverse Gamma distribution is 1 (see also the discussion in this regard provided in Section 4). The hyperparameters of the discrimination and approval parameters are $a_0 = 0$ and $A_0 = 25$.

The information criteria (Scenario 4, DIC = 8563.98, WAIC = 8775.22; Scenario 9, DIC = 8555.55, WAIC = 8774.05; Scenario 10, DIC = 8553.48, WAIC = 8771.59) show that Scenario 10 presents a better predictive performance. However, note once again that the corresponding improvement is not substantial. Therefore, we can dispense such a prior hierarchy in order to preserve parsimony. Under all three scenarios, the groups are recovered on a similar scale, revealing the same pattern as in Figure 2(b).

6. Senate of the Republic of Colombia 2010–2014

The Sixth Congress of the Republic of Colombia 2010–2014 is characterized mainly by two aspects: (i) The leadership of the National Unity, a coalition of political parties formed to support the first government of Juan Manuel Santos Calderón, and (ii) a substantial number of motions proposed for deliberation. Several parties take part in this particular Senate. Partido Social de unidad Nacional (PU), Conservador Colombiano (CC), Liberal Colombiano (LC) and Cambio Radical (CR) are those collectivities that make up the government coalition, whereas Partido Polo Alternativo Democrático (PDA) is the only one in open opposition. Furthermore, Partido Integración Nacional (PIN), Partido Alianza Verde (PAV), and MIRA are independent political groups. Finally, Alianza Social Indígena (ASI) along with Autoridades Indígenas de Colombia (AICO) are political parties representing minorities. The extensive legislative activity, the partisan configuration, and the importance of constitutional reforms and bills in the context of the peace process (e.g. [34]) make this four-year period very interesting to characterizing the legislative electoral behavior of the deputies as well as identifying partisan and coalition patterns from their individual preferences.

In order to evaluate the political preferences underlying the deputies voting behavior, we consider the plenary votes available for 2010–2014. The choice of plenary votes obeys the intention of providing inferences based on the observed voting behavior of deputies when they act on the same voting lists. We also support this decision considering that plenary votes are the most relevant activity of parliament for different groups. In particular, they are votes that capture the attention of political groups, media, and voters, because of the impact of their results on state policy [3].

We were able to obtain the voting records through the Congreso Visible website at https://congresovisible.uniandes.edu.co/. The dataset provides information on bills, legislators, and voting decisions. We exclude legislators who participate in less than 95% of the votes. They correspond to 8% of the total number of deputies (102 deputies and 8
permanent replacements). Their low participation comes as a result of different causes: Death, resignation, disciplinary sanctions, or permanent replacement. Eliminating such deputies does not imply eliminating any political group.

6.1. Ideal points patterns

We fit a Bayesian spatial voting model to these data as discussed in Section 4. Figure 4 shows the ideal points estimates of the Senate deputies that take part in the analysis. Such estimates present a particular behavior according to the political group the deputies belong to. For example, members of the opposition reveal a location on the left side of the political spectrum. This position is opposite to that of the ruling coalition members, whose estimated ideal points, for the most part, are greater than zero. Minorities reveal a center-left location, between $-0.5$ and $0$, whereas independent parties are more dispersed across the political space. The opposition and the coalition are the political groups with the least within-variability (the coefficient of variation of the ideal points of their members is 29% and 42%, respectively). Minorities and independents are the most heterogeneous groups (the coefficient of variation of the ideal points of their members is 62% and 167%, respectively).

Deputies with ideal points located towards the extreme ends of the political spectrum and with high percentages of abstention and non-attendance are those who reveal wider credibility intervals, thus, greater uncertainty in the estimation of their positions. This is the case of Senators Alexander López Maya of the PDA, Jorge Hernando Pedraza Gutiérrez of the CC and Héctor Julio Alfonso López of the PIN.

The credibility intervals with the smallest amplitude are recorded for legislators whose ideal points lie on $[-0.5, 0.5]$, which reflects less uncertainty in the corresponding positions of centrist parliamentarians. On the other hand, the legislator Jorge Eliecer Guevara of the PDA, who changed parties during his tenure (he ends his four-year term in the PAV), reveals a position consistent with the political group he started his legislative work. Also, 83 out of 89 ideal points are significantly different from zero (i.e. the corresponding credibility interval does not contain zero). Legislators German Bernardo Carlosama López of AICO, Feliz Jose Varela Ibañez of PAV, Camilo Armando Sanchez Ortega of LC, Juan Francisco Lozano Ramirez, Maritza Martinez Aristizabal, and Claudia Janneth Wilches Sarmiento of PU have ideal points indistinguishable from zero.

Senators with a higher probability of being at the extreme ends of the political spectrum belong to either the opposition or the coalition. From the PDA legislators, those with a high probability of having an ideal point lower than $-1$ are Alexander López Maya (99%), Camilo Romero Galeano (98%), and Mauricio Ernesto Ospina Gomez (75%). From the coalition, those who have a high probability of having an ideal point greater than 1 are Jaime Alonso Zuluaga Aristizabal (93%) and Jorge Eduardo Gechem Turbay (92%) of the PU, Bernabe Celis Carrillo of CR (90%), and Gabriel Ignacio Zapata Correa of the CC (89%). Legislators from the LC have a probability of 76% that being at the upper end of the political spectrum. Some members of the minorities and independent parties present a high probability of being at the center of the spectrum. In particular, the deputies with a high probability of having an ideal point between $-0.2$ and $0.2$ (i.e. having locations close to zero) are Germán Bernardo Carlosama López (95%) of AICO and Feliz Jose Valera Ibañez (92%) of PAV. All PIN members have a probability at least of 98% of having an ideal point.
higher than 0.2. The latter indicates that legislators affiliated with the PIN tend to cluster with the coalition since they are distant from the opposition and centrist legislators.

Figure 5 shows the ideal points behavior by political party. We see similar patterns in members that make up the coalition (CC, LC, CR, and PU) and PIN. The latter does not declare himself for or against the government, but its political career has shown affinity with parties such as the PU [11]. Therefore, we consider just natural that PIN ideal points lie

Figure 4. Senate ideal points estimates under the election of presumed opposing legislators and at different distances from the center as anchors. Points represent the posterior mean, and horizontal lines 95% symmetric credibility interval based on percentiles.
Figure 5. Classification of the Senate's ideal points estimates by political party. Points represent the posterior mean, and horizontal lines 95% credibility interval based on percentiles.

in the same direction as those associated with the government parties. On the other hand, the PDA, the only opposition party, reveals ideal points at the opposite end of the political spectrum. The parties with fewer seats in the Senate (AICO, ASI, MIRA, and PAV) show ideal points around zero. Minorities show center-left tendencies (ideal points between $-0.5$ and 0), and both MIRA and PAV exhibit senators with center-left and center-right locations (ideal points between 0 and 0.5).
Table 2. Ideal point estimates mean and coefficient of variation (CV) by political party.

| Party | PU | CC | LC | CR | PIN | PDA | MIRA | PAV | ASI | AICO |
|-------|----|----|----|----|-----|-----|------|-----|-----|------|
| Mean  | 0.77 | 0.76 | 0.70 | 0.68 | 0.67 | -1.03 | -0.29 | 0.10 | -0.24 | -0.09 |
| CV (%) | 47 | 27 | 46 | 61 | 39 | 29 | 150 | 311 | NA | NA |

Note: The ASI and AICO parties only have one seat, so their point estimate corresponds to the party average, and the CV is not defined.

Location patterns in Figure 5 demonstrate that the latent trait underlying the 2010–2014 Senate roll call is non-ideological. For instance, ideologically antagonistic parties such as CC and LC lie on the same side of the political spectrum. Furthermore, we firmly believe that it is also not convenient proposing a non-ideological feature of opposition-government (suggested by other authors in Latin American scenarios, [31]) since some parties openly declare themselves alien to either side. Consequently, we postulate a non-ideological dimension that divides the political spectrum into opposition-non-opposition, revealing a conflict centered on power control. Such a finding is entirely consistent with other research that has studied the political space from a broader perspective (e.g. [21]).

Unlike other studies, we do not slice the political spectrum down the middle. It is common in the case of balanced parliaments to take zero as the cut-off point to make sense of the nature of the dimension, typically ideological [10]). Previous studies in the Colombian context also favor this position (e.g. [9]). However, in this Senate, we firmly believe that zero is not an appropriate reference point for interpreting the nature of the latent trait since the non-opposition parties (coalition, minority, and independent) are on one side of the space delimited by zero.

Finally, Figure 5 shows that the coalition parties, the PIN, and the opposition, are the ones that reveal less internal variability in their ideal points. Table 2 shows the mean and coefficient of variation of the ideal points estimated by political party. Our findings show that the PDA and CC are the parties whose members reflect more homogeneous voting decisions than the other political groups. On the contrary, MIRA and PAV show a high dispersion around their mean. The latter suggests that these two parties are highly volatile in their legislative voting decisions.

6.2. Dimension of the political space via discrimination parameters

Regarding the discrimination parameters estimates, we see that 251 of the 417 differ significantly from zero. This implies that, under the proposed unidimensional model, 60.2% of the motions can discriminate among legislators across the political spectrum. We are able to determine that 81.7% of the motions that discriminate against a dimension are those about governmental initiatives, a finding that is consistent with the proposed latent trait (opposition-non-opposition).

Figure 6 contrasts the percentage of votes in favor of the voting list $j$ and the respective point estimate of the discrimination parameter $\alpha_j$. When comparing the behavior of the $\alpha_j$ that are distinguishable from zero (Figure 6(a)) with those that do not segregate for the proposed dimension (Figure 6(b)), we notice that the voting lists with 0% and 100% (unanimous votes) do not distinguish between legislators. Therefore, they are not relevant to recovering information about the deputies’ political preferences. Furthermore, 89 out of
166 non-segregating motions correspond to votes of this type. Meanwhile, the remaining 77 motions possibly contain information about an additional latent trait. Voting lists that discriminate among deputies having a percentage of votes in favor less than 45% reveal negative discrimination parameters (Figure 6(a)), which implies that legislators with ideal points lower than zero increase their probability of positive voting, while legislators with ideal points higher than zero such a probability decreases. We observe an inverse pattern for the voting lists with a percentage of votes in favor greater than 70% (Figure 6(a)). Voting lists with a percentage of positive votes between 40% and 70% are related to points estimates for $\alpha_j$ scattered around zero.

Finally, we obtain the posterior predictive distribution for a set of test statistics (see [15] for details). All subsequent predictive $p$-values provide reasonable evidence supporting a good model-fit since they are not extreme (less than 0.05 or greater than 0.95). Furthermore, all distributions (not shown here) evidence that the observed test statistics values fall within the corresponding 95% symmetric credibility intervals. Such predictive tests give conclusive evidence that the proposed one-dimensional model reveals a reasonable fit to the roll-call data for the Senate of the Republic of Colombia 2010–2014.

7. Discussion

Our simulation study provides strong evidence about the conditions under which the results obtained after fitting a one-dimensional Bayesian ideal point model to nominal voting data of an unbalanced parliament are invariant. In particular, we find that the choice of anchor legislators can be arbitrary. Also, we see that there are no substantial differences between logit and probit link functions working in such an univariate context. However, it
is up to future studies to analyze the sensitivity of these binding functions when working in higher dimensions and even under non-parametric conditions [33].

The sensitivity analysis to the prior distributions of the ideal points indicates that the results are consistent with hyperparameters with and without extra hierarchies. In future studies, it is of interest to inquire about the sensitivity of the model when other parametric or non-parametric families are considered for the model hyperparameters, including those associated with the discrimination and approval parameters. On the other hand, removing missing data does not generate erroneous conclusions regarding the inference of the model parameters. However, a higher rate of missing data generates a greater uncertainty in estimation. The 2010–2014 Senate abstentions and non-attendance rate is quite high (approximately 40%), which prompts further studies that inquire about the mechanism that generates the lost data (e.g. [45]).

This paper is the first in its class to apply the Bayesian ideal point estimator in the Colombian landscape. Our results constitute a substantial contribution regarding the dimension of political space as well as the identification of pivot legislators. The proposed methodology allows us to identify those legislators who have a higher probability of lying at the extremes or in the center of the political spectrum, but it does not determine the order of location of parliamentarians (e.g. [10]). Legislative analysis at the individual level allows us to personalize patterns of electoral behavior and include legislators from political parties with a low number of seats in parliament, such as AICO, ASI, PAV, and MIRA, which are usually excluded in quantitative research that only extrapolates at group levels.

Employing the estimated ideal points patterns, we identify a non-ideological latent trait (opposition-non-opposition) underlying the voting behavior of Senate deputies. However, the possibility of extending the one-dimensional standard Bayesian ideal point estimator to evaluate other latent factors in the electoral behavior of parliamentarians is open for future investigations. It is worth noting that the estimator in its canonical version operates under the assumption of sincere voting (i.e. the legislator only votes according to his/hers ideal point). Therefore, it does not allow to measure ideological, partisan, or coalition effects in deputy voting (e.g. [51]).

The one-dimensional model exhibits a reasonable fit to the nominal voting data of the Senate of the Republic of Colombia 2010–2014. However, we identified some voting lists that do not discriminate among legislators on the continuum of recovered policies. This finding raises the possibility of evaluating a model in higher dimensions. It also leaves an open possibility of evaluating the static and collective character of the political space dimension in the Colombian case (e.g. [32]). Finally, during the revision process, one of the referees suggested a way of handling ‘multiple policy dimensions’ as in Wiesehomeier and Benoit [53], which we find very appealing. Even though some issues need to be resolved before considering such an approach for roll-call data directly, we encourage readers to pursue this possibility if necessary. We do not follow this path in this case because our simulations and our goodness-of-fit assessment using tests statistics prevent us from doing so. We might study this possibility elsewhere.

**Disclosure statement**

No potential conflict of interest was reported by the author(s).
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Appendices

Appendix 1. MCMC algorithm

In the same spirit of Albert and Chib [1], we rely on the fact that any logit or probit model can be expressed as a latent linear regression model. In the case of a probit model, we have that:

\[ y_{ij}^* = \mu_j + \alpha_j \beta_i + \epsilon_{ij}, \quad \epsilon_{ij} \sim N(0, 1). \]

The corresponding expression in the logit case is similar to the previous one, except that the \( \epsilon_{ij} \) follow a standard Logistic distribution instead of a standard Normal distribution. Thus, the parameter space is increased by including auxiliary variables \( y_{ij}^* \), which makes it easier to sample the \( \mu_j, \alpha_j, \) and \( \beta_i \).

The steps of the algorithm are detailed below (\( b \) indexes iterations).

1. \( y_{ij}^{* (b)} \) is sampled from the full conditional distribution \( p(y_{ij}^* \mid y_{ij}, \mu_j, \alpha_j, \beta_i) \) as follows:
   (a) Evaluate \( \rho_{ij}^{(b-1)} = \mu_j^{(b-1)} + \alpha_j^{(b-1)} \beta_i \).
   (b) Sample \( y_{ij}^{* (b)} \) from a Truncated Normal distribution depending on the observed value \( y_{ij} \) as follows:

\[
p(y_{ij}^* \mid \mu_j^{(b-1)}, \alpha_j^{(b-1)}, \beta_i^{(b-1)}) = \begin{cases} 
N(0, \infty) (y_{ij}^* \mid \rho_{ij}^{(b-1)}, 1) & \text{if } y_{ij} = 1, \\
N(-\infty, 0) (y_{ij}^* \mid \rho_{ij}^{(b-1)}, 1) & \text{if } y_{ij} = 0.
\end{cases}
\]

2. \( \mu_j^{(b)} \) and \( \alpha_j^{(b)} \) are sampled from the full conditional distribution \( p(\mu_j, \alpha_j \mid B, y_{ij}^{* (b)}) \) as follows:
   (a) Compute \( c_j \) and \( C \), with

\[
c_j = (B^T B + A_0^{-1})^{-1} B^T y_{ij}^{* (b)} + A_0^{-1} a_0 \quad \text{and} \quad C = (B^T B + A_0^{-1})^{-1}
\]

where \( B^* \) is an array of \( n \times (d + 1) \) whose \( r \)th row is \( \beta_i^* = (1, \beta_i^{(b-1)}) \), and \( y_{ij}^{* (b)} \) is a vector of \( n \times 1 \) storing samples of the latent dependent variables for the \( j \)th proposal.
   (b) Sample \( (\mu_j^{(b)}, \alpha_j^{(b)}) \sim N(c_j, C) \).

3. Note the latent linear regression model is rewritten as \( \omega_{ij} = y_{ij}^* - \mu_j = \alpha_j \beta_i + \epsilon_{ij} \). If \( \omega_i = (\omega_{i1}, \ldots, \omega_{im}) \) is taken as the vector of observations associated with legislator \( i \), and \( A = [\alpha_1, \ldots, \alpha_m]^T \) as the corresponding design matrix, the previous equations can be expressed
as \( w_i = A \beta_i \). Thus, \( \beta_i^{(b)} \) is sampled from the full conditional distribution \( p(\beta_i \mid \mu_j, \alpha_j, y_{ij}^*) \) as follows:

(a) Compute \( h_i \) and \( H_i \), with

\[
  h_i = [A^T A + B_i^{-1}]^{-1} [A^T w_i + B_i^{-1} b_i] \quad \text{and} \quad H_i = [A^T A + B_i^{-1}]^{-1}.
\]

(b) Sample \( \beta_i^{(b)} \sim N(h_i, H_i) \).

**Appendix 2. Notation**

Matrices and vectors with entries consisting of subscripted variables are denoted by a boldfaced version of the letter for that variable. For example, \( x = (x_1, \ldots, x_n) \) denotes an \( n \times 1 \) column vector with entries \( x_1, \ldots, x_n \). We use 0 and 1 to denote the column vector with all entries equal to 0 and 1, respectively, and \( I \) to denote the identity matrix. A subindex in this context refers to the corresponding dimension; for instance, \( I_n \) denotes the \( n \times n \) identity matrix. The transpose of a vector \( x \) is denoted by \( x^T \); analogously for matrices. Moreover, if \( X \) is a square matrix, we use \( \text{tr}(X) \) to denote its trace and \( X^{-1} \) to denote its inverse. The norm of \( x \), given by \( \sqrt{x^T x} \), is denoted by \( \|x\| \).