Deterministic moore intuitionistic fuzzy sequential machine acceptors of intuitionistic fuzzy regular languages

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Abstract
Inspired by the Deterministic acceptors of fuzzy regular languages, a new approach is proposed as the Deterministic moore intuitionistic fuzzy sequential machine acceptors of intuitionistic fuzzy regular languages.

Keywords
Intuitionistic Fuzzy Sequential Machine.

AMS Subject Classification
03E72.

1. Introduction
Automata theory is firmly similar to formal Language theory. Fuzzy automata has been widely used in diverse applications, such as lexical analysis, learning systems, control system, etc [3]. The fuzzy language and fuzzy automata was introduced for dealing with uncertainty in a system in 1960 by santos [2]. Fuzzy automata can be classified into nondeterministic and deterministic fuzzy finite automata. Fuzzy automata depend on the membership value, which lies between 0 and 1 [3].

A simple technique for dealing such problems is grammatical inference, which is a method of extracting grammatical rules from the set example. Fuzzy system models based on fuzzy set theory which have been developed include a description of decision making in a fuzzy environment [1], and fuzzy grammars and languages [5]. According to Chomsky classification there can be four types of fuzzy grammars viz., Regular, Context-free, Context Sensitive and Unrestricted [5]. Fuzzy grammars were first discussed by Lee and Zadeh. There by many researches investigated various issue on fuzzy grammars and languages such as max-min, max-product etc [2].

Fuzzy automata, Grammars, and Languages are leading to greater understanding of nondeterministic algorithms. Inspired with the work of Lee and Zadeh, an algorithm is developed for automata which classifies the strings of a language with a regular fuzzy grammar whose derivations are governed by the "max(min)" rule.

Using the notion of intuitionistic fuzzy sets [8, 9] it is possible to obtain intuitionistic fuzzy language [6] by introducing the nonmembership value to the strings of fuzzy language. This is a natural generalization of a fuzzy language as it is characterized by two functions expressing the degree of belongingness and the degree of non-belongingness. An intuitionistic fuzzy language is called intuitionistic fuzzy regular language if its strings are regular having the finite membership and non-membership values between [0, 1] [7].

1. Introduction

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The paper is organized as follows. In section 2, we recall the definitions of fuzzy automata and fuzzy regular grammar. In section 3, we introduce the definition of intuitionistic fuzzy regular grammar and intuitionistic fuzzy Moore Sequential Machine, with examples. In section 4, we introduce the intuitionistic fuzzy regular grammar to intuitionistic fuzzy automata with theorem and examples. In section 5, Deterministic Moore intuitionistic fuzzy sequential machine acceptors of intuitionistic fuzzy regular languages with theorem and examples.

2. Preliminaries

Definition 2.1. Let a set 'E' be fixed. An intuitionistic fuzzy set 'A' in 'E' is an object having the form \( A = \{(x, \mu_a(x), \gamma_a(x)) | x \in E\} \) where, the functions \( \mu_a(x): E \rightarrow [0,1] \) and \( \gamma_a(x): E \rightarrow [0,1] \) define the degree of membership and the degree of nonmembership of the element \( x \in E \) to the set 'A', the subset of 'E' respectively, and for every \( x \in E \), \( 0 < \mu_a(x) + \gamma_a(x) \leq 1 \).

Definition 2.2. An intuitionistic fuzzy automata is a triple \( M = (Q, X, \delta) \)
where,
* \( Q \) is a set of states.
* \( X \) is an input alphabet.
* \( \delta : Q \times X \rightarrow Q \)
* \( q_0 \) is the starting state.
* \( f = (\mu, \gamma) \) is intuitionistic fuzzy subset of \( Q \).
* \( F = \{(q, (\mu, \gamma)) | 0 < \mu + \gamma \leq 1\} \).

Example 3.4. Let,
\[ Q = \{q_0, q_1, q_2, q_3\} \]
\[ q_0 = \{q_0\} \]
\[ X = \{x, y\} \]
\[ f = \{(0, 0.2), (0.2, 0.3), (0.5, 0.3)\} \]
\[ F = \{q_3(0.4, 0.3)\} \]
\[ \delta(q_0, x) = q_1 \]
\[ \delta(q_1, x) = q_1 \]
\[ \delta(q_0, y) = q_2 \]
\[ \delta(q_1, y) = q_3 \]
\[ \delta(q_2, x) = q_1 \]
\[ \delta(q_2, y) = q_3 \]

4. Intuitionistic fuzzy regular grammar to
Intuitionistic fuzzy automata

Theorem 4.1. If \( IG=(V, T, S, P) \) is an intuitionistic fuzzy regular grammar, then prove that there exist an intuitionistic fuzzy finite automata \( IM \) such that \( L(IM) = L(IG) \).

**Proof.** Let \( IG=(V, T, S, P) \) is an intuitionistic fuzzy regular grammar. Define Intuitionistic fuzzy automata \( IM = (Q, X, \delta, f, q_0) \)
where,
* \( Q \) is a set of states.
* \( X \) is a input alphabet.
* \( \delta : Q \times X \rightarrow Q \)
* \( q_0 \) is the starting state.
* \( f = (\mu, \gamma) \) is intuitionistic fuzzy subset of \( Q \).
* \( F = \{(q, (\mu, \gamma)) | 0 < \mu + \gamma \leq 1\} \).

Define \( IG = (V, T, S, P) \) is an intuitionistic fuzzy regular grammar, then prove that there exist an intuitionistic fuzzy finite automata \( IM \) such that \( L(IM) = L(IG) \).

\[ Q = V \cup \{q_f\}, \]
\[ q_f \in V, \]
\[ S = \{q_0\}, \]
\[ F = \{q_f\} \]
\[ \delta \]

3. Intuitionistic fuzzy regular grammar

Definition 3.1. Intuitionistic fuzzy regular grammar is a four tuple \( IG = (N, T, S, P) \). Where,
* \( N \) is a non empty finite set of non-terminals.
* \( T \) is a non empty finite set of terminals such that \( N \cap T = \emptyset \).
* \( S \in N, S \) is the starting symbol.
* \( P \) is a finite set of productions of the form \( \frac{(\mu', \gamma')}{(\mu, \gamma)} \rightarrow aB \) or \( \frac{(\mu', \gamma')}{(\mu, \gamma)} \rightarrow a \) for \( a, B \in N \), \( a \in T, 0 < \mu + \gamma \leq 1 \).

Example 3.2. \( IG = \{\{S, A, B\}, \{x, y\}, S, P\} \)
\[ P \Rightarrow S \]
\[ \rightarrow \]
\[ xA, A \rightarrow xA, S \rightarrow yB \]
\[ N \rightarrow y, B \rightarrow \]
\[ yA, B \rightarrow \]

3.1 Intuitionistic Fuzzy Sequential Moore Machine

Definition 3.3. Intuitionistic fuzzy sequential moore machine is a six tuple \( IM = (Q, X, \delta, f, q_0, F) \)
where,
* \( Q \) is a set of states.
* \( X \) is a input alphabet.
* \( \delta : Q \times X \rightarrow Q \)
* \( q_0 \) is the starting state.
* \( f = (\mu, \gamma) \) is intuitionistic fuzzy subset of \( Q \).
* \( F = \{(q, (\mu, \gamma)) | 0 < \mu + \gamma \leq 1\} \).

Example 4.2. Consider \( IG = (V, T, S, P) \) be a intuitionistic fuzzy regular grammar, where \( V = \{q_0, q_1\}, T = \{x, y\}, \)
\[ S = \{q_0\}, \]
\[ P = \{q_0 \rightarrow xq_0, q_0 \rightarrow xq_1, q_1 \rightarrow x, q_1 \rightarrow xq_0, \]

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Then \( L(IG) = (x^* y + x^+) \).

Construct \( IM = (Q, X, \delta, q_0, F) \) be a intuitionistic fuzzy finite automata, where \( q_0 = \{ S \}, F = \{ q_f \} \), \( Q = V \cup \{ q_f \} \) and 
\[
\delta(q_0, x, q_0) = 0.2, 0.3, \quad \delta(q_0, x, q_1) = 0.5, 0.3, \quad \delta(q_0, y, q_2) = 0.4, 0.3, \\
\delta(q_1, x, q_1) = 0.3, 0.2, \quad \delta(q_1, y, q_2) = 0.6, 0.3, \\
\delta(q_1, x, q_0) = 0.7, 0.2. \]
Then \( L(IM) = L(IG) \).

### 5. Deterministic Moore Intuitionistic Fuzzy Sequential Machine Acceptors of Intuitionistic Fuzzy Regular Languages

**Theorem 5.1.** If \( IG = (N, T, S, P) \) is an Intuitionistic fuzzy regular grammar generate a language \( L(IG) \), then there exist a Intuitionistic Regular Fuzzy Sequential machine (IM) that accepts the language \( L(IG) \). That is \( L(IM) = L(IG) \).

**Proof.** The proof is a five step algorithm for constructing the Deterministic Intuitionistic Fuzzy sequential machine Automata.

**Step 1:-**
Given the intuitionistic regular fuzzy grammar construct the corresponding Intuitionistic fuzzy automata.

**Step 2:-**
Obtain the set IM of possible membership and non membership grades of strings in the Intuitionistic language.

**Step 3:-**
For each \((\theta_1, \theta_2)\) in IM, obtain the Intuitionistic Regular expression \( F'(\theta_1, \theta_2) \) describing those strings \( x \) of \( X^* \) such that 
\[
\mu(x) \geq \theta_1, \quad \gamma(x) \leq \theta_2, \quad 0 < \theta_1 + \theta_2 \leq 1.
\]
\( F'(\theta_1, \theta_2) = \{ x : x \in X^*, \mu(x) \geq \theta_1 \text{ and } \gamma(x) \leq \theta_2, 0 < \theta_1 + \theta_2 \leq 1 \} \).

**Step 4:-**
Consider the finite set IM of possible membership and non membership grades \((\theta_1, \theta_2), (\theta_3, \theta_4)\) in IM such that \( \theta_1 > \theta_3, \theta_2 > \theta_4 \); then the intuitionistic fuzzy regular expression
\[
F(\theta_1, \theta_4) = F'(\theta_1, \theta_1) \cap F'(\theta_1, \theta_2)
\]
defines the set of strings,
\[
\{ x : x \in X^*, \mu(x) \geq \theta_1, \gamma(x) \leq \theta_4, \gamma(x) > \theta_2 \}.
\]

**Step 5:-**
Use the intuitionistic fuzzy regular expressions \( F(\theta_1, \theta_2), (\theta_1, \theta_2) \in IM \) to obtain the state transition diagram of the Deterministic Moore intuitionistic fuzzy sequential automata.

An immediate consequence of this theorem is the following corollary.

**Corollary 5.2.** Given a intuitionistic fuzzy regular grammar \( IRFG_1 \), there is an equivalent unambiguous \( IRFG_2 \) in which productions have the form \( A \overset{(\mu, \gamma)}{\rightarrow} xB \) or \( A \overset{(\mu, \gamma)}{\rightarrow} x \), with \( A, B \in V \), \( x \in T \), and \( 0 < \mu + \gamma \leq 1 \).
**Step 4:**

Let \( (\theta_1, \theta_2), (\theta_3, \theta_4) \)

\[ \theta_1 > \theta_2 \]

\[ \theta_2 < \theta_4 \]

\[ F(\theta_3, \theta_4) = F'(\theta_3, \theta_4) \cap \overline{F}(\theta_1, \theta_2) \]

\[ F(\theta_3, \theta_4) = \{ x : x \in X^*, \mu(x) \geq \theta_3, \mu(x) < \theta_4 \} \]

\[ \gamma(x) \leq \theta_4, \gamma(x) > \theta_2 \}

\( \theta_1 = 0.3 \quad \theta_2 = 0.6 \quad \theta_3 = 0.2 \quad \theta_4 = 0.7 \)

\( 0.3 > 0.2 \quad 0.6 < 0.7 \)

\[ F(0.2, 0.7) = F'(0.2, 0.7) \cap \overline{F}(0.3, 0.6) \]

\[ \{ (x+y)(yx)|\mu(x) \geq 0.2, \mu(x) < 0.3 \} \]

\[ \mu(x) < 0.4 \]

\[ \gamma(x) \leq 0.6, \gamma(x) > 0.4 \} \]

\[ \theta_1 = 0.4 \quad \theta_2 = 0.4 \quad \theta_3 = 0.3 \quad \theta_4 = 0.6 \]

\[ 0.4 > 0.3 \quad 0.4 < 0.6 \]

\[ F(0.3, 0.6) = F'(0.3, 0.6) \cap \overline{F}(0.4, 0.4) \]

\[ \{ (x+y)(x+y)(xy)|\mu(x) \geq 0.3, \mu(x) < 0.4 \} \]

\[ \gamma(x) < 0.7, \gamma(x) > 0.6 \} \]

\[ \theta_1 = 0.5 \quad \theta_2 = 0.2 \quad \theta_3 = 0.4 \quad \theta_4 = 0.5 \]

\[ 0.5 > 0.4 \quad 0.2 < 0.4 \]

\[ F(0.4, 0.4) = F'(0.4, 0.4) \cap \overline{F}(0.5, 0.2) \]

\[ \{ (y)|\mu(x) \geq 0.4, \mu(x) < 0.5 \} \]

\[ \gamma(x) < 0.4, \gamma(x) > 0.2 \} \]

\[ \theta_1 = 0.6 \quad \theta_2 = 0.1 \quad \theta_3 = 0.5 \quad \theta_4 = 0.2 \]

\[ 0.6 > 0.5 \quad 0.1 < 0.2 \]

\[ F(0.5, 0.2) = F'(0.5, 0.2) \cap \overline{F}(0.6, 0) \]

\[ \{ (x)|\mu(x) \geq 0.5, \mu(x) < 0.6 \} \]

\[ \gamma(x) < 0.2, \gamma(x) > 0.1 \} \]

\[ \theta_1 = 0.5 \quad \theta_2 = 0.2 \quad \theta_3 = 0.6 \quad \theta_4 = 0.1 \]

\[ 0.5 > 0.6 \quad 0.2 < 0.1 \]

which not satisfies the condition so, \( F(0.6, 0.1) = \phi \)

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**Step 5:** The deterministic classifier of strings in the intuitionistic fuzzy regular language IFRG is shown in Fig. 3. The productions of the equivalent grammar IRFG, found by the method of the corollary, are as follows:

\[ S \rightarrow xA \] $\xrightarrow{\mu_{\gamma}(xy)}$ $\rightarrow y$

\[ A \rightarrow xC \] $\xrightarrow{\mu_{\gamma}(xy)}$ $\rightarrow y$

\[ B \rightarrow xC \] $\xrightarrow{\mu_{\gamma}(xy)}$ $\rightarrow y$

\[ C \rightarrow xC \] $\xrightarrow{\mu_{\gamma}(xy)}$ $\rightarrow y$

\[ D \rightarrow xC \] $\xrightarrow{\mu_{\gamma}(xy)}$ $\rightarrow y$

\[ E \rightarrow xC \] $\xrightarrow{\mu_{\gamma}(xy)}$ $\rightarrow y$

\[ F \rightarrow xC \] $\xrightarrow{\mu_{\gamma}(xy)}$ $\rightarrow y$

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**Example 5.4.** Consider string \( xy \). Using IFRG, \( S^0 \xrightarrow{\mu_{\gamma}(xy)} xA \)

\[ S^0 \xrightarrow{\mu_{\gamma}(xy)} xB \]

\[ x \rightarrow xy \] so that \( \mu_{\gamma}(xy) = \max(\min(0.5, 0.5), \min(0.6, 0.4)), \min(\max(0.2, 0.2), \max(0.1, 0.4)) \]

\[ = (0.5, 0.2) \]

Using IRFG, \( S \rightarrow xA \)

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**6. Conclusion**

Classical automata theory can not deal with uncertainty to deal with system uncertainly, nondeterministic finite automata have been generalized into fuzzy automata. The use of regular expressions in describing strings generated by a regular fuzzy grammar using the “max(min)” as rule has been discussed.

In this paper, the Deterministic Moore intuitionistic fuzzy sequential machine acceptors of intuitionistic fuzzy regular languages.

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