Impact of Right-handed Strange-beauty Squark on $b \leftrightarrow s$ Transitions

Wei-Shu Hou and Makiko Nagashima
Department of Physics, National Taiwan University, Taipei, Taiwan 106, R.O.C.

As the hint for CP violating new physics in $B \to \phi K_S$ has weakened, we reconsider the possibility of near maximal mixing between $\tilde{s}_R-\tilde{b}_R$ squarks. Such a right-handed strange-beauty squark $\tilde{s}_b$ can be realized by combining supersymmetry with an approximate Abelian flavor symmetry, and comes with a unique new CP violating phase from right-handed quark mixing. Naturally heavy strange-beauty squark and gluino, of order 0.5 to 1 TeV, are easily accommodated by recent time-dependent CP violation measurements in $B_d \to \phi K^0$ and $\tau^+K^0$. Because of near maximal mixing, even with such heavy masses, the $\tilde{s}_b$ and $\tilde{g}$ can still strongly impact on $B_s$ mass difference and generate CP violation in the mixing, which can still be probed at Tevatron Run II. But if the scenario is realized, the LHC will provide definitive information on the new CP phase, and possibly discover the $\tilde{s}_b$ squark. Time-dependent CP violation in $B_d \to K^{*0}\gamma$ can be probed at the future $B$ factory upgrades. Other $b \to s$ decays influenced by large right-handed dynamics are also discussed.

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I. INTRODUCTION

The existence of flavor, and the observed patterns associated with it, are not understood. CP violation (CPV) also seems to be closely linked to the flavor problem. In the left-handed quark sector, which can be probed by the weak interaction, information is summarized in the CKM matrix elements exhibit a hierarchical pattern $|V_{us}| \approx 0.22$, $|V_{cb}| \sim \lambda^2$, $|V_{td}| \sim \lambda^3 - \lambda^4$, which echo the hierarchy in quark masses. However, since the weak interaction is purely left-handed, we have no information on right-handed flavor physics.

It was observed some time ago that [1,2], if there is an approximate Abelian flavor symmetry (AFS) in nature, then the order of magnitude of the elements of $M_u$ and $M_d$ mass matrices, in powers of $\lambda$, can be inferred from our current knowledge of quark masses and mixings. It turns out that the largest mixing effect, of order 1, would be between the right-handed $s_R$ and $b_R$ quarks. Such flavor mixing, though hidden in the Standard Model (SM), would be brought forward if supersymmetry (SUSY) is also realized. Near maximal mixing between the $\tilde{s}_R$ and $\tilde{b}_R$ squarks would give rise to two flavor-mixed “strange-beauty” squarks $\tilde{s}_{b1,2}$, with a single associate new CPV phase defined as $\sigma$ [2,3]. The strong interaction would now contain flavor-changing $\tilde{s}_{b1,2}-\{\tilde{s}_R,\tilde{b}_R\}$-$\tilde{g}$ couplings, and should affect $b \to s$ transitions.

Sure enough, some “anomalies” have been uncovered recently in CPV in $b \to s$ transitions, notably in $B \to \phi K_S$, which has illustrated the frontier nature of such studies in the past few years. As the B factories mature, it is exciting that the LHC will turn on in 2007, making CPV studies involving the $B_s$ system accessible. Furthermore, one can directly search for the flavor-mixed $\tilde{s}_{b1,2}$ squarks and probe a broad range of parameter space.

Time-dependent CPV (TCPV) in $B_d \to J/\psi K_S$ decay ($S_{\psi K_S}$) was established in 2001. Because the CKM factor $V_{ub}^*V_{cb}$ for the dominantly tree level $b \to c\bar{c}s$ transition is expected to be almost real, $S_{\psi K_S}$ is identical to $\sin 2\phi_1/\beta$, the CP phase in $B_d$ mixing (i.e. $\phi_1/\beta \equiv \arg(V_{ub}^*)$) to very good approximation. The current world average is $\sin 2\phi_1 = 0.685 \pm 0.032$ [4] ($0.73 \pm 0.04$ for PDG2005 [5]). TCPV in $B_d \to \phi K_S$ decay, $S_{\phi K_S}$, is of interest because, in SM one expects $S_{\phi K_S} \cong S_{\psi K_S}$ to the percent level. This is because the loop-induced $b \to s s\bar{s}$ transition that underlies $B_d \to \phi K_S$ is controlled by the CKM factor $V_{ub}^*V_{tb}$, which again is very close to being real in SM. This makes $S_{\phi K_S}$ an excellent probe of CP violating new physics (NP).

Interestingly, for the two consecutive years of 2002 and 2003, the combined result of BaBar and Belle strongly contradicted the SM prediction of $S_{\phi K} \cong \sin 2\phi_1/\beta$. Even the sign was inconsistent. This so-called sign anomaly stimulated many theoretical studies which showed that, to account for the $\phi K$ sign anomaly, one in general would need large $s-b$ flavor mixing, new CPV phases, and possibly right-handed dynamics (see, for example, [6–10]). This seemed to be an ideal situation for the strange-beauty squark. Indeed, our previous work [10] was stimulated by the 2003 result of $S_{\phi K} = -0.14 \pm 0.33$ [4]. We showed that a rather light $\tilde{s}_b$ and not too heavy gluino $\tilde{g}$, such as $[m_{\tilde{s}_b},m_{\tilde{g}}] \simeq [200,500]$ GeV, together with a large new CP phase $\sigma \sim 70^\circ$, could give $S_{\phi K} < 0$. However, for SUSY scale at TeV, $\tilde{s}_b = 200$ GeV would require fine-tuning to $O(10^{-2})$ in the squark mass matrix [3].

Since 2004, however, the experimental discrepancy has weakened considerably, and we need not adhere to our previous conclusion. Fine tuning of the $\tilde{s}_b$ mass is no longer necessary, and one could reconsider the model in a more natural setting. The Belle updated result of $S_{\phi K} = 0.44 \pm 0.27 \pm 0.05$ [11], based on 386 million $B\bar{B}$ events, is in agreement with BaBar result of $0.50 \pm 0.25^{+0.07}_{-0.04}$ based on 227 million $B\bar{B}$ events [12]. The combined result of $S_{\phi K} = 0.47 \pm 0.19$ [4] is $2.5 \sigma$ away from the sign anomaly.
of $S_{bK} < 0$. But there is still a hint of deviation between $S_{bK}$ and $\sin 2\phi_1/\beta$, which could be due to $s_b$. On the other hand, several other modes also show some discrepancy. The $B \rightarrow \pi^0 K_S$ decay involves both $b \rightarrow s$ penguin and $b \rightarrow u$ tree contributions. The latter is, however, suppressed by $V_{ub}^* V_{tb}/V_{tb}^* V_{tb} \lesssim \mathcal{O}(V_{ub}^2)$, so $S_{\pi^0 K_S} \approx \sin 2\phi_1/\beta$ is also expected in the SM. From same number of $B \bar{B}$ events with $S_{bK}$, BaBar finds $S_{\pi^0 K_S} = 0.35^{+0.30}_{-0.33} \pm 0.04$ [13], and Belle finds $0.22 \pm 0.47 \pm 0.08$ [11]. The combined result of $0.31 \pm 0.26$ [4] is different from $\sin 2\phi_1/\beta$ with more than 1.4 $\sigma$ significance, and is in the same direction as $S_{bK}$.

Similarly, $S_{\eta^0 K_S} \approx \sin 2\phi_1/\beta$ is also predicted in the SM. Yet this is again in some conflict with the combined result of $S_{\eta^0 K} = 0.50 \pm 0.09$ [4]. However, the results of BaBar [14] and Belle [11] are at some odds with each other, so the error of 0.09 probably should be rescaled to 0.13. Note that, thanks to the large decay rate, the measurement of $S_{\eta^0 K_S}$ has better accuracy compared with $S_{\pi^0 K_S}$. However, the large rate of $B \rightarrow \eta^0 K_S$ is not well understood, and is likely generated not by new physics. In contrast to $B \rightarrow \eta^0 K_S$, the $B \rightarrow \pi^0 K_S$ mode is simpler and more transparent. Hence we focus on the two modes of $B \rightarrow \phi K_S$ and $\pi^0 K_S$ in this work.

The main purpose of this work is to update the picture of right-handed strange-beauty squarks. We identify new preferred regions for $m_{\tilde{s}b_1}$ and $m_{\tilde{g}}$ from recent data, and revisit the implications for $B_s$ mixing and the associated $CP$ violating phase. We believe this update would be useful for LHC experiments, which would start in 2007. We follow the observations outlined in Ref. [15], which was written after 2004 data revealed drastic softening of $B \rightarrow \phi K_S$ results. With smaller deviations, it is now more customary to use the difference

$$\Delta S_f = S_f - \sin 2\phi_1/\beta,$$

which measures the deviation from SM expectations. We put up useful benchmarks to pin down our model parameters. Since the error in the $\pi^0 K_S$ data is still large, let us take the central value of $\phi K_S$ data as a criterion to be more conservative. We find three scenarios:

- **Scenario 1:** $\Delta S_{\phi K_S} \approx -0.22$,
- **Scenario 2:** $\Delta S_{\pi^0 K_S} \approx -0.22$,
- **Scenario 3:** $\Delta S_{\phi K_S, \pi^0 K_S} \approx -0.22$,

which could provide hint for NP. In addition, there is a fourth possibility,

- **Scenario 4:** $\Delta S_{\phi K_S, \pi^0 K_S} \approx 0$,

which, evidently, does not discriminate between NP and the SM.

In this work, we pay attention to the issue of naturalness, that is, that $s_b$ and $\tilde{g}$ are comparable to SUSY scale. We shall see that $m_{\tilde{s}b_1} \gtrsim 500$ GeV and $m_{\tilde{g}} \gtrsim 700$ GeV can be accounted for by the recent $S_{\phi K_S}$ data. These mass regions are well within the *discovery ranges* at LHC [16]. Measurements of $B_s$ oscillations and the associated $CP$ phase will provide further information on $m_{\tilde{s}b_1}$, $m_{\tilde{g}}$ and $\sigma$. In fact, these measurements may discover NP, even if further B factory results confirm Scenario 4.

This paper is organized as follows: Sec. II concentrates on the formalism for $CP$ violation observables. In Sec. III we give the SM expectations. In Sec. IV we briefly recapitulate our model of near maximal $s_R \leftrightarrow b_R$ mixing. Sec. V and VI gives our NP results. Discussion and conclusion are given in Sec. VII. We refer the hadronic parameters accompanied by the chromo-dipole effects to Appendix A.

## II. Time-Dependent $CP$ Violation

In this section we present the formalism for TCPV. For neutral B meson decays into $CP$ eigenstate $f_{CP}$, the CPV are studied by means of the time evolution $CP$ asymmetry [17],

$$a_{CP}(t) = \frac{A_{f_{CP}} \cos \Delta m_{B_d} t + S_{f_{CP}} \sin \Delta m_{B_d} t}{\cosh \Delta \Gamma t + A_{\Delta \Gamma} \sinh \Delta \Gamma t}. \quad (2)$$

The coefficients $A_{f_{CP}}$, $S_{f_{CP}}$ and $A_{\Delta \Gamma}$ are described in terms of decay amplitudes $M(B_q \rightarrow f_{CP})$ and $M(\bar{B}_q \rightarrow f_{CP})$ as

$$A_{f_{CP}} = \frac{|\lambda_{CP}|^2 - 1}{|\lambda_{CP}|^2 + 1}, \quad S_{f_{CP}} = \frac{2\text{Re}\lambda_{CP}}{|\lambda_{CP}|^2 + 1}, \quad A_{\Delta \Gamma} = \frac{2\text{Re}\lambda_{CP}}{|\lambda_{CP}|^2 + 1}, \quad (3)$$

with $\lambda_{CP} = e^{-i2\phi_{B_d}} M_{f_{CP}}^\dagger$, where $\Phi_{B_d}$ is the $CP$ phase in $B_d$ mixing. The state $f_{CP}$ satisfies $\bar{CP}(f_{CP}) = \xi f_{CP}$. For the $B_d$ system, due to $\Delta \Gamma_{d}/\Gamma_{d} \approx 0$, one finds the simpler form which is given by

$$a_{CP}(t) = A_{f_{CP}} \cos \Delta m_{B_d} t + S_{f_{CP}} \sin \Delta m_{B_d} t. \quad (4)$$

The $CP$ phase $\Phi_{B_d}$ corresponds to $\phi_1/\beta$.

With Wolfenstein parameterization, $\sin 2\phi_1$ can be expressed in terms of $CP$ violating parameters $\bar{\rho} \propto \sin 3\phi_3/\gamma$ and $\bar{\eta} \propto \sin \phi_3/\gamma$ as [18]

$$\sin 2\phi_1 = \frac{2\bar{\eta} (1 - \bar{\rho})}{(1 - \bar{\rho}^2)^{1/2} + \bar{\eta}^2}, \quad (5)$$

where $\phi_3/\gamma \equiv \text{arg} |V_{ub}^*|$. As we stated, the value of $\sin 2\phi_1$ is basically determined by $S_{\phi K_S}$. Although there is a small discrepancy between unitarity fit of $\sin 2\phi_1$ and direct experimental result of $S_{\phi K_S}$, the approximation $\sin 2\phi_1 \approx S_{\phi K_S}$ remains reasonable. Therefore, in order to evade uncertainties brought from $J/\psi K_S$ decay amplitude, we use Eq. (5) for calculation of $\sin 2\phi_1$. In what follows, we take $|V_{ub}/V_{cb}| = 0.09$ and $\lambda \equiv V_{us} = 0.22$. We then find $\sin 2\phi_1 \approx 0.73$ for $\phi_3 = 60^o$, and $\sin 2\phi_1 \approx 0.69$ for $\phi_3 = 47^o$. 

Impact of CP violating NP on TCPV in $b \to s$ transitions can be understood as follows. Let us take $B \to \phi K$ arising from purely $b \to s\bar{s}s$ as an example. We write its amplitude as $\mathcal{M} = a e^{i\delta_a} + b e^{i\delta_b} e^{i\Phi_{\text{new}}}$. The first term denotes the SM contribution which contains the strong phase $\delta_a$ but approximately no weak phase because of $\text{Im}[V_{ts}^* V_{tb}] \approx 0$. The second term, instead, corresponds to the NP contribution with strong phase $\delta_b$, accompanied by a new CP phase $\Phi_{\text{new}}$. One finds the simple expression for $\Delta S_{\phi K_S}$ up to $\mathcal{O}(b/a)$ [7, 19]

$$\Delta S_{\phi K_S} = -\frac{2 b}{a} \cos \delta \cos 2\phi_1 \sin \Phi_{\text{new}} / (1 + 2b/a \cos \delta \cos \Phi_{\text{new}}),$$

(6)

where $\delta = \delta_b - \delta_a$. $\Delta S_{\phi K_S}$ basically follows sin $\Phi_{\text{new}}$ around zero. We see from Eq. (6) that, unless $\Phi_{\text{new}} \neq 0$, $\Delta S_{\phi K_S}$ vanishes for any additional contributions that carry only strong phases. Therefore, either SM or NP without CPV mechanism, i.e. $b = 0$ or $\sin \Phi_{\text{new}} = 0$, would give $\Delta S_{\phi K_S} = 0$. However, we note that Eq. (6) could be diluted by the relative strong phase $\delta$. For $\delta = 90^\circ$ as a typical case, any NP effect is washed away in $\Delta S_I$ (one would then in general get large $\Delta A_f$, which is not the case).

To calculate decay amplitudes we need to evaluate hadronic matrix elements. It is known that the naive factorization (NF) framework [20], which involves small strong phase, has difficulties in explaining current experimental measurements on decay rates and on direct CP violation. Recent developments of factorization frameworks, QCDF [21] and PQCD (see, for example, [22, 23]), have shown the importance of annihilation processes which can generate sizable strong phases (but not at $90^\circ$ level). Such annihilation effects could manifest differently in SM and NP, and in different modes. Our interest is the genuine effects on $\Delta S_{\phi K_S}$ from NP. Because of the small strong phases, the analysis within NF framework would be rather transparent. In the following we use NF in our calculation and assume absence of final state interactions. We will illustrate the possible dilution from presence of additional strong phases by introducing a heuristic term.

### III. STANDARD MODEL EXPECTATIONS

This section is devoted to SM calculations. Besides following Ref. [20], we will take into account the so-called chromo-dipole effects from $b \to s g$ [24]. For hadronic decays from $b \to s$ transition, the effective Hamiltonian is given by

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{ts}^* V_{tb} C_{1,2} O_{1,2}^{nu} + V_{ts}^* V_{tb} C_{1,2} O_{1,2}^c \right\} - V_{ts}^* V_{tb} \left\{ \sum_{i=3}^{10} \left( C_i O_i + C'_i O'_i \right) + C_G O_S^G + C'^G O_{S'}^G \right\} ,$$

(7)

where $C_i$ and $C'_i$ are the color octets and singlets of the $b \to s$ transition, and $O_{1,2}^{nu}$ and $O_{1,2}^c$ are the nonetiuplet and octet singlet operators, respectively.

| TABLE I: Numerical values of $a_i$ for $b \to s$ ($\bar{b} \to \bar{s}$) transition |
|-----------------|-----------------|-----------------|
| $a_1$ | 1.046 |
| $a_2$ | 0.024 |
| $a_3$ | 72.24 |
| $a_4$ | -401.8 + 32.29i |
| $a_5$ | -27.57 |
| $a_6$ | -454.7 + 32.29i |
| $a_7$ | -1.447 - 0.043i |
| $a_8$ | 3.076 - 0.014i |
| $a_9$ | -93.02 - 0.043i |
| $a_{10}$ | 0.129 - 0.014i |

One has [20] current-current, strong penguin and electroweak penguin operators, $O_{1,2}^{nu}, O_3^{nu}$ and $O_2^{nu}$. In addition, the chromo-dipole operator $O_8^G$ is defined as

$$O_8^G = \frac{\alpha_s}{2\pi} (T_{ij} A^A_{ij} m_b q^2 / 4 \pi) \bar{s}_i \sigma_{\mu\nu} (1 + \gamma_5) q^\nu b_j \left[ \bar{q}_k \gamma_\mu q_l \right] ,$$

(8)

where $T_{ij}^A (A = 1, \ldots, 8)$ is a generator of SU(3) with color indices $ij$, and $q$ denotes the momentum carried by the virtual gluon. The operators $O_i^{nu}$ arise from right-handed dynamics, which are represented by exchanging $L \leftrightarrow R$ everywhere. Since the weak interaction probes only left-handed dynamics, the short-distance coefficient $C_i^{nu}$ in SM is suppressed by a factor of $m_u/m_b$. In what follows, we will neglect the SM contributions in the primed coefficients.

Let us start from $B \to \phi K_S$. The decay $B^0 \to \phi K^0$ does not occur at tree level. This decay had been studied within QCDF [21, 26] and PQCD framework [27], giving decay rate in agreement with the experimental result of $B \simeq (7 - 9) \times 10^{-6}$ [4].

With NF, the amplitude for $B^0 \to \phi K^0$ is given by,

$$\mathcal{M}(\phi K^0) / e^+ \cdot P_B = -\kappa_{\phi K} \left\{ a_3 + a_4 + a_5 \right\}$$

$$\left\{ \frac{1}{2} (a_7 + a_9 + a_{10}) + C_{S^G} \alpha_s m_b^2 q^2 / 4 \pi S_{\phi K} \right\} ,$$

(9)

where $\kappa_{\phi K} = \sqrt{2} G_F f_\phi m_\phi F_{BK} V_{ts}^* V_{tb}$, and $e^+ \cdot P_B$ is the scalar product of the $\phi$ meson polarization vector with $B$ meson momentum. The definition of the parameters $a_i$ can be found in Ref. [20]. Table I enumerates the numerical values of $a_i$ which we use for calculation. We take $C_{S^G}^G (m_b) = -0.15$ that would be derived from $b \to s g$. The hadronic parameter $S_{\phi K} / q^2$ is taken for $q^2 = m_b^2 / 3$, and $S_{\phi K} \simeq -1.32$ from the evaluation in NF, which can be found in the Appendix.
Taking $f_{\phi} = 237$ MeV and $F_{BK}(0) = 0.35$, we find $B_{SM}(\phi K^0) \simeq 2.4 \times 10^{-6}$. Without the chromo-dipole effects, i.e., $\mathcal{S}_{K \phi} \rightarrow 0$, one has $B_{SM}(\phi K^0) \simeq 5.3 \times 10^{-6}$. Due to $\mathcal{S}_{K \phi} \sim \mathcal{O}(1)$, the chromo-dipole contribution is substantial, and gives large reduction for the rate. But even for $\mathcal{S}_{K \phi} \rightarrow 0$, the decay rate obtained by NF calculation is far below the experimental data.

For the the decay rate, NF seems deficient. However, the SM result for $\mathcal{S}_{K \phi}$ is independent of the factorization framework. As we noted, any additional effects do not change Eq. (6) unless an extra CPV phase enters. Taking account of $\text{Im}[V_{ts}^* V_{tb}]$, we find $\Delta S_{\phi K^0} \simeq 0.02$, for $\phi_3/\gamma = 60^\circ$, which is in good agreement with Refs. [28, 29].

We turn to $B \rightarrow \pi^0 K^0$. For the decay rate, the QCDF result [21] and the PQCD result [22] are comparable to the current data $B \sim (11 - 13) \times 10^{-6}$ [5]. In the NF framework, the $B^0 \rightarrow \pi^0 K^0$ amplitude is

$$M(\pi^0 K^0) = \kappa_{\pi K} (\mathcal{B}) \left[ V_{ts}^* V_{tb} \frac{a_2}{\sqrt{2}} (a_9 - a_7) \right] + \kappa_{\pi K} (\mathcal{B}) \left[ \frac{a_4}{2} + \left( \frac{m_{K^*}^2}{m_{s} + m_{d}} \right) \right] - \frac{1}{4} \frac{\alpha_1}{C_8} \cdot \tilde{\mathcal{S}}_{K^0},$$

where $\kappa_{\pi K} (\mathcal{B}) = -\frac{iG_F}{2} f_\pi (m_B^2 - m_{K^*}^2) F_{BK} V_{ts}^* V_{tb}$ for the $B \rightarrow K$ transition, while $\kappa_{\pi K} (\mathcal{B}) = -\frac{iG_F}{2} f_K (m_B^2 - m_{K^*}^2) F_{BK} V_{ts}^* V_{tb}$ for the $B \rightarrow \pi$ transition. The chromo-dipole contribution appears only in the latter, and we shall use $\tilde{\mathcal{S}}_{K^0} \sim -1.55$. Taking $f_\pi = 132$ MeV and $m_s = 110$ MeV, Eq. (10) leads to $B_{SM}(\pi^0 K^0) \simeq 2.8 \times 10^{-6}$. Just as $\phi K$, $\mathcal{S}_{K^0} \sim \mathcal{O}(1)$ reduces the rate substantially. But even $B_{SM}(\pi^0 K^0) \simeq 4.0 \times 10^{-6}$ for $\tilde{\mathcal{S}}_{K^0} \rightarrow 0$ remains problematic.

As $\Delta S_{\phi K^0}$, we are not hampered by using NF. Eq. (10) is more complicated than Eq. (9). Furthermore, $\pi^0 K$ would be smeared with the tree contributions carrying $\text{Im}[V_{sb}]$ although $V_{sb}$ is highly suppressed. However, Eq. (6) permits. The amplitude in Eq. (10) gives $\Delta S_{\phi K^0} \simeq 0.03$, for $\phi_3/\gamma = 60^\circ$, again in agreement with Refs. [28, 29].

In similar way, we evaluate TCPV observables in $B \rightarrow \eta' K_S, \omega K_S, \rho K_S$ modes. The amplitudes of these modes are rather complicated. But the trend of our results are consistent with the results in Refs. [28, 29]. For $\phi_3/\gamma = 60^\circ$, we find $\Delta S_{\eta' K_S} \simeq 0.02$, $\Delta S_{\omega K_S} \simeq 0.08$, and $\Delta S_{\rho K_S} \simeq -0.01$, respectively. We also calculate TCPV observables in $\pi^0 K^*$ and $\eta' K^*$. We assume these final states to be the CP even state, via $K^* \rightarrow \pi^0 K_S$. As anticipated, we find $S_{\pi^0 K^*} \simeq -0.76$ and $S_{\eta' K^*} \simeq -0.75$, respectively, with minus sign coming from $\xi$. The relevant chromodiopole hadronic parameters $\mathcal{S}_{PP}$, $\mathcal{S}_{PV}$, and $\mathcal{S}_{VV}$ can be found in Appendix A.

IV. STRANGE-BEAUTY $\tilde{s}_R \tilde{b}_R$ SQUARKS

In this section, we briefly summarize our model without going into details. The left-handed flavor mixing is well understood in terms of the CKM matrix elements. The pattern of flavor mixing in left- and right-handed dynamics could be different. But within the SM we are unable to probe right-handed quark mixing.

Our model is one of the possibilities to address CP violation in the right-handed sector. If there is an underlying approximate Abelian flavor symmetry (AFS), the right-handed $s$ and $b$ quarks can have near maximal mixing [1, 2]. The effective AFS implies down-type quark mass matrix to be of the form [3],

$$\frac{M_d^b}{m_b} \sim \begin{bmatrix} m_s/m_b & m_s/m_b \\ 1 & 1 \end{bmatrix},$$

where we quote 2-3 sector only. The ratio $m_s/m_b$ is approximately $\mathcal{O}(\lambda^2)$. The near maximal right-handed mixing may be the largest off-diagonal element. However, its effect is hidden from our view in the SM for absence of right-handed dynamics. The combination of AFS and SUSY brings forth right-handed dynamics involving squarks, as well as realizing a near maximal $\tilde{s}_R \tilde{b}_R$ squark mixing [2, 3].

As pointed out in [2, 3], applying four texture zeros is needed to be safe from kaon low energy constraints. Decoupling $d$ flavor is implied by lack of NP indication in $B_d$ mixing. From this backdrop, we focus on 2-3 generation subsystem. With decoupling of $d$ flavor, the down squark mass matrix is reduced from $6 \times 6$ to $4 \times 4$, which is split up into

$$(\tilde{M}_d^2)_{(sb)} = \begin{bmatrix} (\tilde{M}_d^2)_{LL} & (\tilde{M}_d^2)_{LR} \\ (\tilde{M}_d^2)_{RL} & (\tilde{M}_d^2)_{RR} \end{bmatrix},$$

where each submatrix would in principle be complex, and the Hermitian nature implies $(\tilde{M}_d^2)_{(sb)}^{\dagger} = (\tilde{M}_d^2)_{(sb)}$. With squark mass scale $\tilde{m}$, one finds $(\tilde{M}_d^2)_{LL} \sim \tilde{m}^2 V_{CKM}^{(23)}$, and $(\tilde{M}_d^2)_{(sb)} \sim \tilde{m} M_d^{(sb)}$, while near democratic structure in $(\tilde{M}_d^2)_{(sb)} \sim \tilde{m}^2$ [3, 30].

We now parameterize $(\tilde{M}_d^{(sb)})_{RR}$ as

$$(\tilde{M}_d^2)_{(sb)} = \begin{bmatrix} \tilde{m}^2_{22} & \tilde{m} V_{23} e^{-i\sigma} \\ \tilde{m} V_{23} e^{i\sigma} & \tilde{m}^2_{33} \end{bmatrix},$$

with a unique new CP phase $\sigma$, which cannot be rotated away since phase freedom is already used in quark sector. Transforming

$$
\begin{bmatrix} \tilde{s}_b^1 \\ \tilde{s}_b^2 \end{bmatrix} = R \begin{bmatrix} \tilde{s}_R \\ \tilde{b}_R \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta e^{-i\sigma} \\ \sin \theta & \cos \theta e^{-i\sigma} \end{bmatrix} \begin{bmatrix} \tilde{s}_R \\ \tilde{b}_R \end{bmatrix},
$$

(14)
one obtains the mass eigenstates $\tilde{s}_{1,2}$, with the corresponding eigenvalues \( \tilde{m}_{1,2}^2 = (\tilde{m}_{22}^2 + \tilde{m}_{33}^2 \pm \sqrt{(\tilde{m}_{22}^2 - \tilde{m}_{33}^2)^2 + 4\tilde{m}_{23}^2})/2 \). In general, with $\tilde{m}_{22,33} \approx \tilde{m}_{23}^2$, strange-beauty squark $\tilde{s}_1$ could be made lighter as a consequence of level splitting.

We assume that SUSY is at TeV scale, and the AFS scale is not too far above the SUSY scale. To make $\tilde{s}_1$ as light as 200 GeV for TeV scale SUSY, one needs fine-tuning of $\tilde{m}_{23}^2/\tilde{m}_i^2 \approx 1$ to $O(10^{-2})$ level \([3]\). In fact, the scenario of our previous work \([10]\) used $\tilde{m}_{22}^2 = \tilde{m}_{33}^2 = \tilde{m}^2$ and $1 - \tilde{m}_{23}^2/\tilde{m}_i^2 = 0.04 (0.01)$ for $\tilde{m} = 1\, (2)$ TeV. The $\phi K_S$ sign anomaly in 2003 seemed to demand such fine-tuning for light $\tilde{s}_1$. Furthermore, we also demanded $m_{\tilde{g}}$ to be not heavier than 600 GeV. With the weakening of the current $S_{\phi K_S}$ data, the previous somewhat extreme model parameters can be relaxed. In the following, we continue to pursue near maximal $\tilde{s}_R$-$\tilde{b}_R$ mixing for convenience, i.e. $\theta = 45^\circ$, but make the tuning $1 - \tilde{m}_{23}^2/\tilde{m}_i^2$ less strict.

Let us not give the explicit expressions \([30]\) of gluino-quark-squark and squark-squark-gluino interactions in mass eigenbasis. Instead, we present the right-handed chromo-dipole contributions as an example. The relevant coefficient is described as

$$C_8^{\text{G}} = -\frac{\sqrt{2}a_{\text{G}}}{G_F m_t} \left\{ \frac{1}{6} F_1(\tilde{m}_1, m_\tilde{g}, \tilde{m}_i) + \frac{3}{2} F_2(\tilde{m}_1, m_\tilde{g}, \tilde{m}_i) \right\},$$

where

$$F_1(\tilde{m}_1, m_\tilde{g}, \tilde{m}_i) = -c_{\theta} g_{\theta} e^{-i\alpha} \left[ \frac{f(\tilde{x}_1)}{\tilde{m}_1^2} - \frac{f(\tilde{x}_2)}{\tilde{m}_2^2} \right] + i \tilde{m}_3 \left\{ \frac{c_{\theta} (a_1 - s_{\theta} e^{-i\alpha})}{\tilde{m}_1^2 - \tilde{m}_2^2} \left( \frac{g(\tilde{x}_1)}{\tilde{m}_1^2} - \frac{g(\tilde{x}_2)}{\tilde{m}_2^2} \right) + \frac{s_{\theta} (a_1 + c_{\theta} e^{-i\alpha})}{\tilde{m}_1^2 - \tilde{m}_2^2} \left( \frac{g(\tilde{x}_1)}{\tilde{m}_1^2} - \frac{g(\tilde{x}_2)}{\tilde{m}_2^2} \right) \right\},$$

(15)

with $\tilde{x}_0 = m_{\tilde{g}}^2/\tilde{m}_i^2$, $\tilde{x}_1 = m_{\tilde{g}}^2/\tilde{m}_1^2$, and $\tilde{x}_2 = m_{\tilde{g}}^2/\tilde{m}_2^2$, and $F_2(\tilde{m}_1, m_\tilde{g}, \tilde{m}_i)$ is obtained by the replacement $f(\tilde{x}_1) \rightarrow h(\tilde{x}_1)$ and $g(\tilde{x}_1) \rightarrow j(\tilde{x}_1)$. The expressions of $f(\tilde{x}_i)$, $g(\tilde{x}_i)$, $h(\tilde{x}_i)$, and $j(\tilde{x}_i)$ in Eq. (15) are given by

$$f(\tilde{x}_i) = \frac{1}{12(\tilde{x}_i - 1)^3} \left( 1 - 6\tilde{x}_i + 3\tilde{x}_i^2 + 2\tilde{x}_i^3 - 6\tilde{x}_i^2 \ln \tilde{x}_i \right),$$

$$g(\tilde{x}_i) = \frac{1}{2(\tilde{x}_i - 1)^3} \left( 1 - 3\tilde{x}_i^2 + 2\tilde{x}_i \ln \tilde{x}_i \right),$$

$$h(\tilde{x}_i) = \frac{1}{12(\tilde{x}_i - 1)^3} \left( 2 + 3\tilde{x}_i - 6\tilde{x}_i^2 + 3\tilde{x}_i^3 + 6\tilde{x}_i \ln \tilde{x}_i \right),$$

$$j(\tilde{x}_i) = \frac{1}{2(\tilde{x}_i - 1)^3} \left( -3 + 4\tilde{x}_i - \tilde{x}_i^2 - \ln \tilde{x}_i \right).$$

(16)

The coefficients at $m_{\tilde{b}}$ scale can be calculated by means of the leading order renormalization equations \([30]\),

$$C_8^{\text{G}}(\mu) = \left( \frac{\alpha_s(m_t)}{\alpha_s(\mu)} \right) \frac{4\pi}{\tilde{m}_i^2} \left( \frac{\alpha_s(\sqrt{M_{\text{SUSY}}})}{\alpha_s(m_t)} \right) C_8^{\text{G}}(\sqrt{M_{\text{SUSY}}}),$$

(17)

where we assume $M_{\text{SUSY}} = \sqrt{m_{\tilde{m}_i} m_{\tilde{b}}}$.

Right-handed strong penguins $a_{3,6}$ are also taken into account in our calculation, where $a_{3,4}$ and $a_{6,6}$ mainly enter $RR$ and $RL$ squark mixings, respectively. The size of $a_{3,6}$ is smaller than the size of $C_8^{\text{G}}$ by at least two orders of magnitude.

Before moving to the numerical study in the next session, one remark should be made. In general, the new effect on left-handed dynamics are not a sign of SM contribution in left-handed dynamics, and neglect the left-handed SUSY effects. It is important to note that, in this specific framework, a light $\tilde{s}_1$ can well survive the constraint of $b \rightarrow s\gamma$ decay rate \([3]\). As shown in Refs. \([3, 10]\), for $m_{\tilde{g}} > 800$ GeV, neither the $\tilde{s}_1$ mass nor the $\sigma$ phase are constrained. Furthermore, the impact of $\tilde{s}_1$ on the direct CPV in $b \rightarrow s\gamma$ \([31]\), $A_{b \rightarrow s\gamma}$, is rather small. This is based on the fact that there is no right-handed tree level weak interaction, and no operator mixing between right- and left-handed dynamics. We found that, even though $S_{\phi K_S} < 0$ had persisted, the modulation from the SM prediction for $A_{b \rightarrow s\gamma}$ (around +0.6%) is as small as ±0.15%.

V. IMPACT OF $\tilde{s}_R$ ON $\Delta S_f$

We now introduce the SUSY effects into $B_d \rightarrow \phi K^0$ and $\pi^0 K^0$. As stated in previous sections, for simplicity we neglect the right-handed SM contributions to $b \rightarrow s$ transition, as well as the left-handed SUSY contributions. Consequently, in our calculation, the left-handed coefficients $C_i$ are identical to the SM calculation, while the right-handed coefficients $C_i'$ arise purely from NP.

The decay amplitude of $B^0 \rightarrow \phi K^0$ in Eq. (9) is modified as

$$M(\phi K^0) = \frac{\mathcal{M}(\phi K^0)}{c_s \cdot P_B} \propto \left\{ \cdots + (C_8^{\text{G}} + C_8) \frac{\alpha_s}{4\pi} \frac{m_{\tilde{m}_i}^2}{q^2} S_{\phi K} \right\},$$

(18)

where $\cdots$ are the terms shown in Eq. (9), modified by $a_i \rightarrow a_i + a_i'$. Because of $S_{\phi K} \sim O(1)$ and $a_{3,6}/C_8^{\text{G}} < O(10^{-2})$, the chromo-dipole SUSY effect is the dominant NP effect in Eq. (18).

Fig. 1(a) illustrates our result for $\Delta S_{\phi K_S}$ vs new CP phase $\sigma$. We fix $\tilde{m}_i$ to be 1.5 TeV for illustration. Since
NF contains small strong phases, our results reflect genuine NP contributions. Here we set $\Delta S_{\phi K_S} \simeq -0.2$ as a target for extracting the preferred regions for $m_{\tilde{b}_1}$ and $m_{\tilde{g}}$. We illustrate with four examples spanning some range for $m_{\tilde{b}_1}$ and $m_{\tilde{g}}$, but with values remaining natural. We see that, for $\sigma \approx 90^\circ$, the range $m_{\tilde{b}_1} \simeq 500$–800 GeV with $m_{\tilde{g}} \approx 700$–900 GeV can be easily accommodated by the current $\Delta S_{\phi K_S}$ data. These mass regions are well within the discovery range at LHC [16], which would be commissioned in 2007. On the other hand, one has $\Delta S_{\phi K_S} \gg \Delta S_{\phi K_S}^{SM}$, $> 0$ for $\sigma \approx 270^\circ$, and $\Delta S_{\phi K_S} \simeq \Delta S_{\phi K_S}^{SM}$ at $\sigma \approx 0^\circ$/$180^\circ$.

Note that $m_{\tilde{b}} \ll m_{\tilde{g}} \ll m$, or $m_{\tilde{g}} \ll m_{\tilde{b}} \ll m$ as well as $m_{\tilde{b}} \sim m_{\tilde{g}} \ll m$ with small $\sigma$ phase [10] can still be accommodated and need not be thrown away. However, these ranges are not of interests in this work as we emphasize naturalness for $m_{\tilde{b}_1}$ on the SUSY scale. In the following, we take $[m_{\tilde{b}_1}, m_{\tilde{g}}] \simeq [500, 900]$ GeV as our standard value with $m$ at 1.5 TeV.

The decay amplitude of $B^0 \to \pi^0 \bar{K}^0$ is modified as

$$\mathcal{M}(\pi^0 \bar{K}^0) \propto \left\{ \cdots + \left( C_S^G - C_S^{G^c} \right) \frac{\alpha_S m_b^2}{4\pi} \tilde{S}_{\pi K} \right\},$$

where $\cdots$ are the terms shown in Eq. (10) with replacement $a_i \to a_i - a_i^*$. Contrary to Eq. (18), the primed terms change sign, which would cause anticorrelation effect between $S_{\phi K_S}$ and $S_{\pi K_S}$ [10]. Fig. 1(b) illustrates our result of $\Delta S_{\pi^0 K_S}$ which shows the opposite trend to $\Delta S_{\phi K_S}$. Like $\phi K$ amplitude, the dominant SUSY effect would come from the chromo-dipole term. However, the new effect on $\Delta S_{\pi^0 K_S}$ is milder since the chiral operator $O_6$ enhances the left-handed, i.e. the SM, contributions. Specifically, our standard value with $\tilde{S}_{\pi K} = 900$ GeV and $750$ GeV with $m_{\tilde{b}_1}$ fixed at $500$ GeV. Steeper solid (dashed) curve corresponds to $m_{\tilde{b}_1} (m_{\tilde{g}}) = 750$ GeV.

Because of the anticorrelation between $\Delta S_{\phi K_S}$ and $\Delta S_{\pi^0 K_S}$, one has $\Delta S_{\pi^0 K_S} > 0$ for $\Delta S_{\phi K_S} < 0$, and vice versa for $m_{\tilde{b}}$ effect. We see that Scenario 1 prefers the $\sigma$ range (a), which would demand $\Delta S_{\pi^0 K_S}$ to be positive, the same as the SM prediction, and likely larger. If the present $\Delta S_{\phi K_S}$ holds, but $\Delta S_{\pi^0 K_S}$ changes sign in the future, the range (a) with $m_{\tilde{g}} \lesssim 900$ GeV might be the solution. Scenario 2, instead, would require the further experimental results to raise up $\Delta S_{\phi K_S}$, but confirm the present sign of $\Delta S_{\pi^0 K_S}$. This scenario, if realized, would prefer range (b). Scenario 3 is consistent with the current experimental results of $\Delta S_{\phi K_S, \pi^0 K_S} < 0$. However, our model, as well as any SUSY model with underlying large right-handed dynamics, cannot accommodate this scenario because of the implied anticorrelations. If the present deviations in $\Delta S_{\phi K_S, \pi^0 K_S}$ persist, it would evidently be a hint for NP, but would be a different picture of NP than the current one (a study within a certain specific model will be described in Ref. [32]) studied here. The ranges (c) and (d) can fit Scenario 4 which implies that $\Delta S_{\phi K_S}$ and $\Delta S_{\pi^0 K_S}$ will converge to $\sin 2\phi_1/\beta$ in the future. In this scenario, we cannot distinguish between NP and SM since the effect from NP might overlap with the effect from SM. However, in the next section, we shall see that measurements of $B_S$ mixing and the associated CPV phase may discover NP, even if Scenario 4 is realized.

From straightforward calculations, we have also evaluated our SUSY model effects on $\Delta S_f$ for $f = \eta' K_S$, $\omega K_S$, $\rho^0 K_S$, $\pi^0 K^*$ and $\eta' K^*$. So far, $S_{\eta' K_S}$ and $S_{\omega K_S}$ have been measured. For $\omega K_S$, the BaBar result [34] and Belle result [11] are not in good agreement, as is the case for $S_{\eta' K_S}$. Recently, BaBar measured $S_{\pi^0 K_S}$ [35] which is, in principle, equivalent to $S_{\eta' K_S}$, where $K^* \to \pi^0 K_S$, however the error is very large.

We confirm that the effect on $\Delta S_{\eta' K_S}$ anticorrelates
\[ \Delta S_{\phi K_S} \]

Our result of \( \Delta S_{\phi K_S} \) is rather similar to \( \Delta S_{s^0 K_S} \). On the other hand, \( \Delta S_{\rho K_S} \) and \( \Delta S_{s^0 K_S} \) show the opposite trend with \( \Delta S_{\phi K_S} \). The hadronic parameters \( \tilde{S}_f \) in \( \eta'/K_S, \omega K_S \) and \( \rho^0 K_S \) are more involved than \( \tilde{S}_{\phi K_S} \) or \( \tilde{S}_{s^0 K_S} \). In particular, our results for \( \omega K_S \) and \( \rho^0 K_S \) are quite sensitive to form factors. This is because the hadronic parameters in these modes arise from \( B \to V \) transitions, unlike \( \phi K_S \), and depend strongly on the form factors (see Eq. (A3)). Within NF, our SUSY effects on \( \Delta S_{\phi K_S}, \Delta S_{\omega K_S} \) and \( \Delta S_{\rho K_S} \) could be measurable. However, the large rates for these modes are not well understood. Our SUSY effects do not help to reach the observed rates. The additional hadronic effects needed to enhance the rates could likely be rather large and smear the impact of our SUSY effects very much. This is why we refrain from going into the details about these decays. For our results on TCPV in \( \pi^0 K^* \) and \( \eta' K^* \), we just mention that we found sizable NP effects in \( \pi^0 K^* \) with variation around the SM result, while invisibly small for \( \eta' K^* \).

\[ \Delta S_{K^*} \]

Due to the softening of TCPV data on \( \Delta S_{\phi K_S} \) and \( \Delta S_{s^0 K_S} \), we find that the preferred regions for \( m_{sb_1} \) and \( m_\tilde{g} \) are now heavier hence more natural. However, the parameter \( S_f / q^2 \) related to the hadronic matrix element of chromo-dipole operator is highly uncertain. Furthermore, annihilation phases are likely present in these hadronic \( B \) decays. Such hadronic uncertainties would prevent the determination of the SUSY model parameters, even when \( \Delta S_{\phi K_S} \) and \( \Delta S_{s^0 K_S} \) become precisely measured. It is of interest to explore processes that are largely free of hadronic uncertainties.

As pointed out in Ref. [36], a polarization in \( \Lambda_b \to \Lambda_7 \) [37] would be a clean probe of the right-handed NP. However, the impact from our updated heavier \( m_{sb_1} \) and \( m_\tilde{g} \) would give rise to effects no larger than 10\%. Besides, this mode seems very hard to measure. In this sense, \( B_s \) mixing and TCPV in \( B_d \to K^{*\gamma} \) would be better probes for our SUSY effects [3, 10]. We therefore update these measurable to the current preferred parameter space for our model.

\[ B_s \text{ system} \]

\( B_s \) mixing can be studied by flavor specific decays, such as \( B_s \to D_s^+ \pi^- \). The present bound of the mass difference is \( \Delta m_{B_s} > 14.5 \text{ ps}^{-1} \) with 95\% confidence level [5]. The SM prediction of \( \Delta m_{B_s} \), on the other hand, is around 20 ps\(^{-1}\). Uncovering a \( \Delta m_{B_s} \) value considerably larger than this would be a hint for NP. The currently running Tevatron can cover part of the SM range. But at the LHC, besides the collider detectors ATLAS and CMS, the dedicated LHCb experiment claims to cover

\[ \Delta m_{B_s} \text{ up to 70 ps}^{-1}, \quad \text{and should have no problem in fully exploring } \Delta m_{B_s}^{\text{SM}}. \]

In our previous work, we studied the implication of \( \phi K_S \) sign anomaly for \( \Delta m_{B_s} \) [10]. The negative \( S_{\phi K_S} \) would imply that \( B_s \) probably oscillates faster than 70 ps\(^{-1}\), which would be challenging even for LHCb. The situation is now relaxed. It is useful to revisit the impact of our SUSY effect on \( \Delta m_{B_s} \).

The error on theoretical calculations are contained in the hadronic factor \( f_{B_s} \sqrt{B_{B_s}} \), which is being studied vigorously by lattice approaches. There is no strong phase, therefore \( \Delta m_{B_s} \) should eventually supply much cleaner information about new CPV phase.

Fig. 2(a) shows our prediction of \( \Delta m_{B_s} \). We plot the present bound as horizontal solid straight line. Let us focus on our standard value \( \{m_{sb_1}, m_\tilde{g}\} \simeq \{500, 900\} \text{ GeV} \). For the \( \sigma \) range (a) and (b), \( \Delta m_{B_s} \) lie in the measurable 20-70 ps\(^{-1}\) range. The curious Scenario 4 prefers the \( \sigma \) ranges (c) and (d). From \( \Delta S_f \) studies, one cannot distinguish these two possibilities, therefore cannot tell whether one has NP or not. Interestingly, \( \Delta m_{B_s} \) could disclose the remarkable difference between the ranges (c) and (d). For the range (c), \( \Delta m_{B_s} \) is similar to the SM prediction. For the range (d), instead, one has \( \Delta m_{B_s} \approx \text{70-80 ps}^{-1} \).

Assuming \( \Delta m_{B_s} \) can be observed, the associated CPV phase \( \sin 2\phi_{B_s} \) can provide further information on \( \sigma \). \( \sin 2\phi_{B_s} \) can be measured for example via \( B_s \to J/\psi \phi \) decay. Similar to the \( B_d \) case, this mode is dominantly a \( b \to c\bar{c} s \) transition, hence probes the weak phase as-

\[ \text{FIG. 2: (a) } \Delta m_{B_s} \text{ and (b) } \sin 2\phi_{B_s} \text{ vs } \sigma \text{ for } \tilde{m} = 1.5 \text{ TeV. Notation is the same as Fig. 1. The horizontal line in (a) is the current } \Delta m_{B_s} \text{ bound.} \]
associated with $B_s$ mixing. The SM asserts $\sin 2\Phi_{B_s} \approx -0.04$. More than ±0.1 deviation from the SM prediction, if measured, would indicate the presence of new CPV phase. As shown in Fig. 2(b), the various heavier $m_{\tilde{b}_1}$ and $m_{\tilde{g}}$ values can still make spectacular impact on $\sin 2\Phi_{B_s}$. Note that Scenario 1 implies $\sin 2\Phi_{B_s} \sim +1$, while Scenario 2 implies $\sin 2\Phi_{B_s} \sim -1$. Measurement of $\sin 2\Phi_{B_s}$ and $\Delta m_{B_s}$ might provide confirmation of Scenario 1, 2 or 4.

Besides $\sin 2\Phi_{B_s}$ measurement, one can study TCPV in nonleptonic $B_s$ decays as well. For $b \to s$ transition, the promising decays are $B_s \to K^+ K^-$ [38–40] and $\phi \phi$, while measuring $K^0\bar{K}^0$ might be hopeless. Especially, $B_s \to \phi \phi$ will be a help to understand the polarization anomaly in $B_d \to \phi K^*$. The SM predicts $S_{B_s \to K^+ K^-} \approx 0.5$ reflecting its decay phase from $V_{ud}^* V_{ub}$ accompanied by $a_1$ in Table I, while $S_{B_s \to \phi \phi} \approx 0$ due to the absence of the decay phase just as in $B_d \to \phi K_S$. We find that $S_{B_s \to K^+ K^-}$ and $S_{B_s \to \phi \phi}$ (longitudinal polarization state) show the opposite trend with $\sin 2\Phi_{B_s}$. Finding $S_f$ in $B_s$ decays would be a further crosscheck of hint for NP from corresponding $B_d$ decays.

B. TCPV in $B_d \to K^*\gamma$

TCPV in $B \to K^*\gamma$ is also a good place to look for SUSY effect in our model [10].

The photon radiated from $b \to s\gamma$ can, in principle, pick up the chirality information of the electromagnetic operators arising from the left- and right-handed dynamics. Therefore, the SM contribution dominantly generates the left-handed polarized photon, while our SUSY effect dominantly generates the right-handed photon. Although $S_{K^*\gamma}$ is studied without directly measuring the photon handedness, because of the requirement that the final state should be a $CP$ eigenstate, both components need to be present for $S_{K^*\gamma}$ to be nonvanishing. Summing over the two separate rates for $K^*\gamma_L$ and $K^*\gamma_R$, the $CP$ violating interference can occur. Following Refs. [36, 41], one has

$$S_{K^*\gamma} = \frac{2\text{Im}[e^{-i2\phi_1} C_2^L C_2^R]}{|C_7^L|^2 + |C_7^R|^2},$$

where we follow the convention of Eq. (3) which is opposite to that used in Ref. [41]. In the SM, $C_2^L / C_2^R \approx m_s/m_b$, hence $S_{K^*\gamma} \approx -0.04$ is predicted.

Fig. 3 illustrates our prediction with our updated $m_{\tilde{b}_1}$ and $m_{\tilde{g}}$, which shows a measurable effect [42]. We see that Scenario 1 gives small $S_{K^*\gamma}$ with negative sign, while sizable $S_{K^*\gamma}$ with positive sign will be realized in Scenario 2. Measurement of $S_{K^*\gamma}$ can also help discriminate between the $\sigma$ range (c) and (d). So far, errors in the experimental results at BaBar [43] and at Belle [44] are too large to observe a real hint for NP.

We make one cautionary remark. $S_{K^*\gamma}$ has been expected to be largely free from hadronic uncertainties. A recent study of TCPV in $B \to X\gamma$ [45] in Soft Collinear Effective Theory claims that the SM prediction of $S_{K^*\gamma}$ could easily be of order 0.1 (the sign cannot be fixed) with large uncertainties from unknown hadronic matrix elements. If such sizable uncertainties are indeed present in SM, $S_{K^*\gamma}$ may not be as good a probe of NP as $\sin 2\Phi_{B_s}$. Even so, however, improved measurements in the future will test our result for $S_{K^*\gamma}$ if $\sigma \in (180^\circ, 300^\circ)$, i.e. the ranges (b) and (d).

VII. DISCUSSION AND CONCLUSION

As stated previously, a more up to date application of the factorization framework could dilute the NP effects on $\Delta S_f$ [32]. Let us try to estimate the probable dilution effect on $\Delta S_{\phi K_S}$. To address this, we pay attention to the decay rate, and introduce a heuristic term for annihilation effect, by referring to QCDF framework [21]. Without the chromo-dipole term, multiplying $a_1 + a_1'$ in the SM by $(1 + 0.4 e^{30^\circ})$ enhances the decay rate from $B \approx 5.3 \times 10^{-6}$ to $9.8 \times 10^{-6}$. The direct CPV around $+0.02$ is not in disagreement with data. Adding the chromo-dipole term (but not scaled by the annihilation factor), our standard values for NP parameters give $B \approx 7.6 \times 10^{-6}$ for $\sigma \sim 90^\circ$, which remains within the experimental error. In this case, NP effect on TCPV gets diluted by the annihilation effect, and $\Delta S_{\phi K_S}$ becomes 35% weaker than shown in Fig. 1(a). Because of this dilution from non-$CP$ related physics, and because the associated strong phase is not very large, the effect on direct CPV is small and still consistent with data. From a similar study, we find that the dilution for $\Delta S_{\phi K_S}$ could be 42%. We therefore caution that NP effect in TCPV observables might get diluted by such hadronic effects.

As our model still adheres to large right-handed dynamics, we should comment on the constraint from EDM of $^{199}$Hg. As claimed in Ref. [46], within NP CPV with underlying SUSY, the chromoelectric dipole moment (CEDM) of $s$ quark ($d_s^C$) would strongly correlate with $S_{\phi K_S}$.

In our model, $d_s^C$ receives contributions from $RR$
squark mixing as well as $RL$ squark mixing, where the latter is enhanced by $m_{\tilde{b}}/m_s$ and accompanied with $\tilde{s}_L$-$\tilde{b}_L$ mixing parameter ($\delta_{LL}$)$_{sb}$. Even if we set ($\delta_{LL}$)$_{sb} = 0$, $d_s^C$ for our standard value turns out to be three times larger than the present bound [46]. However, there could be cancellations between effect from $RR$ mixing and from $RL$ mixing. As a particular case, taking ($\delta_{LL}$)$_{sb} \approx -0.01$ would allow our updated $m_{\tilde{s}b_1}$ and $\tilde{m}_g$ values. The left-handed squark mixing in our model should therefore be studied further.

Even if studies of CPV can determine $m_{\tilde{s}b_1}$, $\tilde{m}_g$ and $\sigma$, direct observation of $\tilde{s}b_1$ is much more exciting. A 200 GeV $\tilde{s}b_1$ as suggested by us when $S_{\phi K_S} < 0$ could be discovered at the Tevatron. However, with the weakening of the current $S_{\phi K_S}$ data, $m_{\tilde{s}b_1} \gtrsim 500$ GeV would be the more plausible range. Observation of such $\tilde{s}b_1$ is hopeless at the Tevatron, but very relevant at the LHC.

Assuming that a bino $\tilde{\chi}^0_1$ is lighter than $\tilde{s}b_1$ [3], the most interesting decay mode is $\tilde{s}b_1 \rightarrow b/s\tilde{\chi}^0_1$. Since $\tilde{s}b_1$ carries both $s$ and $b$ flavors, both the $\tilde{s}b_1 \rightarrow b\tilde{\chi}^0_1$ and $\tilde{s}b_1 \rightarrow s\tilde{\chi}^0_1$ decays, depending on the mixing angle $\sin \theta$ of Eq. (14), could be comparable in rate. If maximal $\tilde{s}_R$-$\tilde{b}_R$ mixing is realized, the standard $b$ squark search bound, based on $b$-tagging, would be weakened. We stress that this possibility should be kept in mind for direct search. It would be exciting to discover a flavor-mixed squark, as SUSY and flavor would become clearly linked to each other. The production of light $\tilde{s}b_1$ at hadron colliders have been studied by paying attention to lower mass $m_{\tilde{s}b_1}$ and $m_{\tilde{g}}$ as hinted by $S_{\phi K_S} < 0$ [47], which is aimed more at the Tevatron. But the era of LHC is approaching. With the heavier $m_{\tilde{s}b_1}$ and $m_{\tilde{g}}$ range now implied by the softened $\Delta S_f$, the studies for collider search should be updated.

Let us conclude. The hint for New Physics in $B_d \rightarrow \phi K_S$ has weakened. The mild hint no longer calls for rather light $m_{\tilde{s}b_1}$ and $m_{\tilde{g}}$. Comparing with recent experimental data, we extract the new mass range of $m_{\tilde{s}b_1} \simeq 500 - 800$ GeV and $m_{\tilde{g}} \simeq 700 - 900$ GeV, which is much more natural on the SUSY scale of $\tilde{m} \sim 1 - 2$ TeV. $B_s$ mixing, the associated $CP$ violation $\sin 2\Phi_B$, and time-dependent $CP$ violation in $B_d \rightarrow K^*\gamma$ would be better probes of our flavor/SUSY model effects, which are less affected by hadronic uncertainties. The impact on $\Delta m_{B_s}$ and $\sin 2\Phi_{B_s}$ is very relevant to LHCb. By the time of LHC turn-on, we may know better about the hints for NP from $\Delta S_f$. But $B_s$ studies and direct search for the $\tilde{s}b_1$ squark at the LHC would open a new era.

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\section*{APPENDIX A: HADRONIC PARAMETER $\tilde{S}_f$}

We compute the hadronic parameter $\tilde{S}_f$ of chromo-dipole operator contribution to $B \rightarrow f$ decay by following Ref. [24]. The expressions depend on $f$ being $PP$, $VP$ or $VV$ final state, where $P$ is a pseudoscalar, and $V$ a vector meson.

1. $B \rightarrow PP$ decays

With $B \rightarrow P_2$ transition, one gets

\[ \tilde{S}_{P_1 P_2} = - \frac{8}{9} \left[ \left( \frac{2m_1^2}{m_s + m_q} \right) \left( 1 - \frac{m_s}{m_b} \right) + \frac{(P_B \cdot p_1)}{m_b (m_b - m_q)} - \frac{m_1^2 (2m_1^2 - (P_B \cdot p_1))}{m_b (m_b - m_q)^2} \right] \cdot \frac{m_1}{2m_b (m_b - m_q)} \cdot \left( \frac{m_1}{m_b} \right)^2. \]

2. $B \rightarrow VP$ decays

For $B \rightarrow P$ transition, one has

\[ \tilde{S}_{V_1 P_2} = - \frac{8}{9} \left[ 1 + \frac{m_s}{m_b} + \frac{m_1^2 (m_b - m_q)}{2m_b (m_b - m_q)} - \frac{f_1 f_P}{f_1} \cdot \frac{m_1}{2m_b} - \frac{(P_B \cdot p_1)}{4m_b (m_b - m_q)} \right], \]

where we assume $f_1^T = f_1$ in our calculation.
For $B \to V$ transition, one has

$$
\tilde{S}_{V_1V_2} = -\frac{8}{9} \left[ \left(1 - \frac{2m_B^2}{m_s + m_q}(m_b + m_q) \right) \left(1 + \frac{m_s}{m_b} \right) + \frac{(P_B \cdot p_1)}{m_b(m_b + m_q)} \right. \\
- \frac{m_B^2(m_B + m_2)}{2m_b(m_s + m_q)m_2 A_1} + \frac{m_B^2(2m_B^2 - (P_B \cdot p_1))}{2m_b(m_s + m_q)m_2(m_B + m_2) A_2} \\
\left. + \frac{3(P_B \cdot p_1) - 2m_B^2 - m_B^2}{2m_0m_2} A_2 \right] + \frac{m_0^2 - (P_B \cdot p_1)g_+ + \frac{m_B^2m_B^2 - (P_B \cdot p_1)g_-}{m_0m_Bm_Bm_2} A_0 \right],
$$

(A3)

with

$$
g_+ = \frac{1}{2} \left[ \frac{m_B + m_2}{m_B} A_1 + \frac{m_B^2 - m_2^2 + m_B^2}{m_B(m_B + m_2)} \right], \\
g_- = \frac{1}{2} \left[ \frac{m_B + m_2}{m_B} A_1 - \frac{3m_B^2 + m_2^2 - m_B^2}{m_B(m_B + m_2)} \right], \\
h = \frac{m_B V}{m_B + m_2} - \frac{m_B A_2}{2(m_B + m_2)} - \frac{m_Bm_2 A_0}{2m_1^2} + \frac{m_B(m_B + m_2) A_1}{2m_1^2} - \frac{m_B(m_B - m_2) A_2}{2m_1^2},
$$

(A4)

where $A_{0,1,2}$ and $V$ are the $B \to V$ form factors [20, 48, 49].

3. $B \to VV$ decays

For longitudinally polarized state with $B \to V_2$ transition, one has

$$
\tilde{S}_{V_1V_2} = -\frac{8}{9} \left[ \left(\frac{x(m_B + m_2)}{2} - \frac{m_Bm_2 A_2}{m_B + m_2} A_1(x^2 - 1) \right) \left(1 - \frac{m_s}{m_b} \right) + \frac{1}{2m_B A_1} \left(m_Bm_2(x^2 - 1) - m_B^2x \right) \\
- \frac{m_Bp_c x g_-}{2m_b A_1} - \frac{m_B^2 x^2 + h}{2m_Bm_B A_1} \left(\sqrt{x^2 - 1} + x \right) + \frac{m_Bm_2 A_0}{m_B(m_B + m_q) A_1(x^2 - 1)} \right] ,
$$

(A5)

where $x = \frac{m_Bp_c}{m_Bm_2}^2$, and $p_c$ is the c.m. momentum of the final state particle.

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