Novel approach to single-screw compressors and expanders design*

Davide Ziviani1, Patrick J. Goeghegan2 and Eckhard A. Groll1

1Ray W. Herrick Laboratories, School of Mechanical Engineering, Purdue University, West Lafayette, 47907-2099, USA
2Oak Ridge National Laboratory (ORNL), Oak Ridge, TN, USA
E-mail: dziviani@purdue.edu; geogheganpj@ornl.gov; groll@purdue.edu

Abstract. Single-screw machines are currently employed as both compressors and expanders in vapor compression systems and organic Rankine cycles (ORCs), respectively. The working principle of single-screw machines is based on the simultaneous meshing of two starwheels with one central grooved rotor. The performance of the machine is heavily affected by the wear of the tooth meshing profile after several hours of running. In order to improve the wear resistance of the conventional straight-line profile, multi-column tooth flank designs have been introduced and investigated by several researchers. Cylindrical multi-column envelope profiles are able to distribute the local contact over a larger surface area reducing the wear of the tooth flank profile. Nevertheless, the manufacturability of such profiles is directly linked to the accuracy and limitations of CNC machines.

In this work, an attempt is made to overcome the aforementioned limitations. To push the boundaries of possible tooth and rotor profile designs, 3D printing is utilized as manufacturing technique. Such approach allows the investigation of more complex designs that improves the sealing lines during the meshing process.

A detailed geometry model based on polygon intersections is utilized to calculate the sealing lines, groove volume curves, and surface areas of the different designs. As a result of this study, a novel rotor design has been manufactured.

1. Introduction

The single-screw concept was first introduced by Bernard Zimmern in the early 1960s [1] and consisted of a single six helically grooved central rotor with a cylindrical periphery and a globoid core and two identical 11-teeth starwheels engaging with the rotor [2, 3]. After being rejected several times from French and American companies, a first prototype of single-screw compressor for compressed air applications was built in 1963 and it was licensed two years later. After that point, the development of the single-screw continued and its application was extended to refrigeration within a decade [4, 5, 6, 7].

*This manuscript has been authored by UT-Battelle, LLC, under contract DE-AC05-00OR22725 with the US Department of Energy (DOE). The US government retains and the publisher, by accepting the article for publication, acknowledges that the US government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this manuscript, or allow others to do so, for US government purposes. DOE will provide public access to these results of federally sponsored research in accordance with the DOE Public Access Plan (http://energy.gov/downloads/doe-public-access-plan).
Figure 1. Classification of single screw machines depending on the meshing conditions. Adapted from Zimmern B. [7].

The central position of the rotor in the single-screw potentially leads to different configurations depending on the shape of the grooved rotor and the position of the starwheels. In particular, four main configurations can be distinguished at the moment, as shown in Figure 1 [8]:

- **Plate-Cylindrical (PC):** planar main screw rotor and two cylindrical starwheels. The working process occurs only on the front face of the screw rotor. The starwheel shafts are oriented at opposing angles from the axial face of the main screw.

- **Cylindrical-Plate (CP):** cylindrical main screw rotor and two planar starwheels. The starwheel shafts are parallel to each other with their rotation axes perpendicular to the rotation axis of the main screw. The working process occurs within the volume created when the starwheels mesh with the main screw rotor.

- **Plate-Plate (PP):** planar screw rotor and planar starwheels. The grooves are located on the front and back faces of the main screw rotor. Two starwheels are employed for each side or the rotor and the rotation axis of their shafts are parallel to each other.

- **Cylindrical-Cylindrical (CC):** cylindrical starwheels and cylindrical screw rotor. The two starwheels are engaged on the top and bottom face of the main screw, respectively. As with the PC design, the starwheel shafts are oriented at opposing angles.

The most common single-screw configuration is the CP-type, which includes as primary components a screw rotor and two starwheels. One of the main reasons is the higher complexity of the manufacturing process of the other configurations that leads only to a limited increase in the efficiency and performance with respect to the CP-type.

The improvement of the single-screw design is directly related to advancements in the manufacturing techniques. Over the years, the accuracy of the Computer Numerically Controlled (CNC) machines has increased allowing for a better control of the machining process of the main rotor. Initially, specially designed cutting tools that mimicked the shape and motion of
the starwheel were employed to manufacture the main rotor grooves, resulting in long machining times and limited range of sizes [9]. The introduction of 5-axis CNC machine centers enabled advancements in the manufacturing process of the rotors. In fact, each groove became a ruled surface created by the milling tool while sweeping through the body of the main rotor [10, 11]. However, difficulties in the manufacturing process of the grooves still exist, especially with respect to the root of the grooves [12].

The machining accuracy of the meshing pair is an important aspect and according to Li et al. [13], the average maximum machining error is on the order of two times the design precision required of the single screw rotor. Furthermore, the indexing accuracy of a common CNC machine tool can reach 10 seconds, which means the maximum machining error is around 0.01 mm for a 200 mm diameter, while the design precision of the meshing pair in a single-screw is around 0.05 mm. The uncertainties related to the manufacturing process of the meshing pair and the type of the generating profile yield a potential rapid wear of the starwheel tooth flank surfaces meshing with the screw groove flanks. Ultimately, the wear leads to larger gaps which increase the leakages and decrease the volumetric efficiency.

The shape of the tooth flanks, the materials and the machining accuracy of the meshing pair represent the important aspects with respect to improving the wear resistance. Different geometric meshing conditions have been proposed to achieve a smoother contact during the meshing. The original profile proposed for the single-screw compressor meshing pair was substantially a straight-line envelope meshing pair (SEMP or LEMP) [14]. The contact area between the starwheel tooth flank and the screw rotor is a fixed straight line on the starwheel tooth flank, resulting in a low wear resistance because the contacts are concentrated on a line. The detailed model for LEMP was presented by Yang S.-C. [15]. In order to better distribute the contact area and reduce the progressive wearing, a column envelope meshing profile (CEMP) was proposed by Zimmern [3]. A more comprehensive mathematical model of CEMP with design purposes was presented by Wu et al. [16], but due to limitation in machining capabilities, the solution was not suitable for industrial applications. Successively, Wu and Feng [17, 18] improved the feasibility of the column profile by introducing a multi-column (or multi-straight line) approach as well. The proposed approach allows the contact line between the tooth flank and the groove flank to move back and forth in the whole area of the tooth flank, enhancing the lubrication between tooth and groove flanks and improving the service life. Basically, the meshing area of a multi-column envelope is not continuous but characterized by jumps between one cylindrical segment to another. Furthermore, different curved profiles of the gate-rotor teeth, e.g., elliptical, hyperbolic, or involute, were analyzed by Li et al. [13] in order to enhance the lubrication between tooth and groove flanks. It is possible to find an overview of the different meshing pair profiles in a recent work published by Wu et al. [19]. Conical teeth were also proposed by Yang and Liang [20].

Novel manufacturing techniques such as 3D printing would open the possibility of designing novel types of single-screw meshing configurations that are not cost effective with the current manufacturing processes, such as CC single screw compressor described by Heidrich and Yang [21, 22].

In this work, a non-conventional design of the starwheel tooth profile has been explored to improve the manufacturing control over the meshing pair and the sealing lines. In order to verify the feasibility of such profile, 3D printing has been employed to prototype the main rotor and the starwheel. Such methodology enables advancements in the design of single-screw compressors and expanders which are not suitable to be manufactured with conventional CNC machines.

2. Novel tooth design
As described by Ziviani et al. [23], the geometry model of single-screw machine is based on three groups of parameters denoted as independent variables (such as the number of grooves/teeth,
number of starwheels, diameters of rotor and starwheels), dependent variables (such as engaging angles, length of the rotor), and limit-set variables that enforce a realistic design space for the meshing pair. The selection of the generating profile uniquely defines the meshing pair. In this work, a single straight line envelope meshing pair (SSLEMP) is considered to have a direct comparison with the conventional tooth design, shown Figure 2(a). The proposed tooth profile consists of two straight lines and a circular arc, shown in Figure 2(b). Furthermore, new tooth design does not feature an upper and lower surfaces that contribute to the meshing (and sealing) process. In the conventional tooth profile, the cross section of the tooth tip is characterized by two design angles, $\alpha_1$ and $\alpha_2$, that merge to a single flank angle, $\alpha_{11}$, at the root of the tooth. Whereas, in the new design, the upper surface of the tooth is the meshing plane facilitating the alignment and mating between central rotor and starwheels. The tooth flanks are drafted accordingly to avoid interference with the groove flanks.

3. Kinematics of the meshing profile
From the gear kinematics theory [24], the meshing process of the screw rotor ($sr$) with two starwheels ($sw$) can be described as an envelope of two-parameter family of surfaces with a cylindrical-plate (CP) configuration. In particular, the generating surface is represented by the
starwheel, i.e. plate, (direct problem). The profile of the starwheel is obtained by applying the inverse envelope concept. A right-handed fixed, $O_{1f}(x_{1f}, y_{1f}, z_{1f})$, and a rotating, $O_{i}(x_{i}, y_{i}, z_{i})$, coordinate system have to be defined for both screw rotor $(i = 1)$ and starwheel $(i = 2)$, as shown in Figure 3(a).

The starwheel profile is expressed in general terms by:

$$R_{2j}(u, \theta) = [R_{2j,x}(u, \theta) R_{2j,y}(u, \theta) R_{2j,z}(u, \theta)]^T,$$

where $j = AB$, $BC$, $CD$. The corresponding family of curves on the main rotor, $R_{1j}(u, t, \theta)$, are obtained through a rotation matrix $M_{12}$ and ensuring the meshing condition at each angular position. The meshing equation imposes the sliding velocity between the two bodies in contact as zero, i.e. the cross product between the normal vector to the contact surface, $N_{j}$, and the relative velocity $V_{j}^{(12)}$ is zero. Mathematically, the system of equations that has to be solved

![Figure 3. (a) Fixed and rotating reference systems of the meshing pair; (b) representation of the new tooth design.](image)

Figure 3. (a) Fixed and rotating reference systems of the meshing pair; (b) representation of the new tooth design.
simultaneously is given by:

\[
\begin{align*}
\mathbf{N}_f^{(1)} \times \mathbf{v}_f^{(12)} &= \left( \frac{\partial \mathbf{R}_{1j}}{\partial u} \right) \times \mathbf{v}_f^{(12)} = 0 \\
\mathbf{R}_{1j} &= \mathbf{M}_{12} \mathbf{R}_{2j}
\end{align*}
\]

(2)

where \( j = AB, BC, CD \). In particular, the rotation matrix can be expanded as follows:

\[
\mathbf{R}_{1j}(u, \theta_1) = \mathbf{M}_{1,1j} \mathbf{M}_{1f,2j} \mathbf{M}_{2f,2} \mathbf{R}_{2j}(u, \theta_1).
\]

(3)

Usually, to perform coordinate transformations, mixed matrix operations are needed. Homogeneous coordinates can be adopted to include only multiplications [24].

By considering the reference system \( O_1 \) on the screw rotor as in Figure 3(a), the system of equations for meshing is given by:

\[
\begin{align*}
\mathbf{x}_1 &= -x_2 \sin \theta_{sw} \sin \theta_{sr} - y_2 \cos \theta_{sw} \sin \theta_{sr} + z_2 \cos \theta_{sw} \cos \theta_{sr} + d_{sr,sw} \sin \theta_{sr} \\
\mathbf{y}_1 &= -x_2 \sin \theta_{sw} \cos \theta_{sr} - y_2 \cos \theta_{sw} \cos \theta_{sr} - z_2 \sin \theta_{sr} + d_{sr,sw} \cos \theta_{sr} \\
\mathbf{z}_1 &= x_2 \cos \theta_{sw} - y_2 \sin \theta_{sw}
\end{align*}
\]

(4)

where \( d_{sr,sw} \) is the distance between rotation axes of the main rotor and a starwheel, the relationship between \( \theta_{sw} \) and \( \theta_{sr} \) is given by:

\[
\frac{i}{z_{sw}} = \frac{\theta_{sr}}{\theta_{sw}} = \frac{\omega_{sr}}{\omega_{sw}} = \frac{11}{6}.
\]

(5)

where \( \omega_{sr} \) and \( \omega_{sw} \) are the angular speed of main rotor and starwheel, respectively.

By introducing the parametric curves of the tooth profile, it is possible to obtain a family of meshing curves. To be noted is that the meshing equations refer the plain containing the straight-line profile, shown in Figure 2(b).

### Table 1. Geometry details of the designed single-screw machine.

| Parameter | Value | Description                        |
|-----------|-------|------------------------------------|
| D_{sr}    | 140 mm| Main rotor diameter                |
| D_{sw}    | 170 mm| Starwheel diameter                 |
| d_{sw, sr}| 119 mm| Distance main rotor and starwheel  |
| w_{tooth} | 25 mm | Tooth width                        |
| t_{tooth} | 15 mm | Tooth thickness                    |
| L_{rotor}| 134.9 mm | Rotor length suction side     |

4. Results and discussion

In order to prove the feasibility of the concept tooth profile, a rotor with a diameter of 140 mm has been chosen as reference case. The main dimensions of the meshing pair are listed in Table 1. The meshing equations have been implemented in SOLIDWORKS to generate the groove profiles starting from the new tooth design. Then, a 3D printer has been employed to manufacture the assembly in a plastic material, as shown in Figure 4(a) and in Figure 4(b). A second CAD model has been obtained from a conventional tooth profile by adopting the same rotor diameter and the meshing pair can be seen in Figure 5(a). Such conventional rotor profile
has been previously studied by Ziviani et al. [25] as an expander for organic Rankine cycle applications. For completeness, a view of the actual rotor is provided in Figure 5(b).

By looking at the two designs a number of observations can be made. In the conventional design, the manufacturing process is done by CNC machines and entails the control over the flank surfaces and the bottom surface of the groove. Inaccuracies of the tolerances during the manufacturing lead to leakage gaps as well as abnormal wear of the tooth profile. In the new design, the groove profile does not present discontinuities yielding to a higher control of the meshing process as well as the sealing lines, as shown in Figure 4. The continuity of the meshing helps establishing hydrodynamic lubrication conditions which can potentially reduce the tooth wear and leakage flows. Furthermore, due to the fact that the upper surface of the new tooth coincides with the meshing plane, the correct alignment of the starwheels can be facilitated. Whereas in the conventional tooth profile (see Figure 2(a)), upper and lower tooth surfaces need to be considered during the machining process to ensure the proper sliding of the tooth inside the mating groove.

Additional considerations can be made with respect to the usability of 3D printing. In fact, on the one hand, 3D printing enables new families of meshing profiles that optimize the single-screw geometry as well as allows to have higher uniformity of tolerances on the groove surfaces (both flanks and bottom). On the other hand, the current state-of-the-art of 3D printing does not allow for high-volume production required in industry. Moreover, the printed surfaces still require post treatment to ensure the correct surface roughness. Nevertheless, it is believed that such manufacturing technique will reduce current limitations in rotor profiles and research work in ongoing to evaluate different aspects of the manufacturing process.

Figure 4. (a) View of the 3D printed meshing pair; (b) View of the main rotor end face.

5. Conclusions
In this paper, a non-conventional tooth profile for single-screw machines has been developed. The new profile is believed to enable higher control over the sealing lines reducing leakage flows and wear of the starwheel teeth. The proposed design has been manufactured by using 3D
Figure 5. View of a conventional straight line meshing pair: (a) CAD model; (b) Example of single-screw expander rotor [23].

Figure 6. Close-up view of the tooth sealing line while engaged in the mating groove.

printing technique. Such methodology opens up more complex single-screw meshing profiles that could improve the performance of such machines over a wide range of rotor sizes.

As a next step, a metallic rotor and a PEEK (polyetheretherketone) starwheel will be manufactured and tested to validate the performance improvements and durability of the new profile.

Acknowledgments
The authors would like to thank Bob Gemmer, Technology Manager, Research and Development for the Department of Energy Advanced Manufacturing Office.
References

[1] Vrinat G 1984 *International Journal of Refrigeration* 7 107–114
[2] Zimmern B 1965 Worm rotary compressors with liquid joints
[3] Zimmern B 1976 Rotary injection worm and worm wheel with specific tooth shape
[4] Zimmern B and Patel G C 1972 *International Compressor Engineering Conference* paper 16
[5] Clark J R, Hodge J M and Hundy G F and Zimmern B 1976 *Institute of Refrigeration*
[6] van Male J 1978 *International Journal of Refrigeration* 1 242–248
[7] Zimmern B 1984 *Int. Compr. Eng. Conf. at Purdue*
[8] Masuda M, Ueno H, Inoue T, Hori K and Hessain M 2013 *Proceedings of 8th Int. Conf. on Compressors and their Systems, City University of London, London* pp 257–264
[9] Jensen D 1998 *Int. Compr. Eng. Conf. at Purdue* Paper 1306
[10] Zhou S, Li X, Zhang Y and Zhou D 2006 *Int. Compr. Eng. Conf. at Purdue* Paper 1815
[11] Wu W, Feng Q and Yu X 2009 *Journal of Mechanical Design* 131 1–5
[12] Wu W, Li J and Feng Q 2011 *Computer-Aided Design* 43 67–71
[13] Li J, Liu F, Feng Q and Wu W 2012 *International Compressor Engineering Conference at Purdue* Paper 1209
[14] Sun S, Wu W, Yu X and Feng Q 2010 *International Compressor Engineering Conference at Purdue* Paper 2024
[15] Yang S C 2002 *Proceedings of IMechE, Part C: J. of Mechanical Engineering Science* 216 343–351
[16] Wu W, Feng Q, Xu J and Feng X 2010 *International Compressor Engineering Conference at Purdue* Paper 1863
[17] Wu W and Feng Q 2009 *Journal of Zhejiang University* 10 31–36
[18] Wu W and Feng Q 2009 *Journal of Mechanical Design* 131 1–4
[19] Wu W, Hao X, He Z and Li J 2014 *Journal of Mechanical Design* 136 1–5
[20] Yang S C and Liang T L 2008 *Transactions of the CSME* 32 333–352
[21] Heidrich F L 1996 *International Compressor Engineering Conference at Purdue* pp 145–150
[22] Yang S C 2004 *IMechE Part C: J. Mechanical Engineering Science* 218 437–448
[23] Ziviani D, Braun J E, Groll E A and De Paepe M 2018 *International Journal of Refrigeration* 92 10–26
[24] Litvin F L 1994 *Gear Geometry and Applied Theory* (Prentice-Hall, New Jersey)
[25] Ziviani D, Sergei S, Lecompte S, Groll E A, Braun J E, Horton W T, van den Broek M and De Paepe M 2016 *Applied Energy* 181 155–170