How strange a non-strange heavy baryon?

Ariel R. Zhitnitsky

Physics Department, University of British Columbia, 6224 Agricultural Road, Vancouver, BC V6T 1Z1, Canada

and

Budker Inst. of Nuclear Physics, Novosibirsk, 630090, Russia

Abstract

We give some general arguments in favor of the large magnitude of matrix elements of an operator associated with nonvalence quarks in heavy hadrons. We estimate matrix element \( \frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | \bar{s}s | \Lambda_b \rangle \simeq 1 \div 2 \) for \( \Lambda_b \) baryon whose valence content is \( b, u, d \) quarks. This magnitude corresponds to a noticeable contribution of the strange quark into the heavy baryon mass \( \frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | m_s \bar{s}s | \Lambda_b \rangle \simeq 200 \div 300 MeV \). The arguments are based on the QCD sum rules and low energy theorems. The physical picture behind of the phenomenon is somewhat similar to the one associated with the large strange content of the nucleon where matrix element \( \langle p | \bar{s}s | p \rangle \simeq 1 \) by no means is small. We discuss some possible applications of the result.

1 e-mail address: arz@physics.ubc.ca
1 Introduction and Motivation.

Nowadays it is almost accepted that a nonvalence component in a hadron could be very high, much higher than naively one could expect from the naive perturbative estimations. Experimentally, such a phenomenon was observed in a number of places. Let me mention only few of them.

First of all it is anomalies in charm hadroproduction. As is known, the cross section for the production of $J/\psi'$s at high transverse momentum at the Tevatron is a factor $\sim 30$ above the standard perturbative QCD predictions. The production cross sections for other heavy quarkonium states also show similar anomalies[1].

The second example of the same kind is the charm structure function of the proton measured by EMC collaboration [2] is some 30 times larger at $x_{Bj} = 0.47$, $Q^2 = 75\, GeV^2$ than that predicted on the standard calculation of photon-gluon fusion $\gamma^{ast} g \rightarrow c\bar{c}$.

Next example is the matrix element $\langle N|\bar{s}s|N\rangle$ which does not vanish, as naively one could expect, but rather, has the same order of magnitude as valence matrix element $\langle N|\bar{d}d|N\rangle$.

One can present many examples of such a kind, where “intrinsic” non-valence component plays an important role. This is not the place to analyze all these unexpected deviations from the standard perturbative predictions. The only point we would like to make here is the following. Few examples mentioned above ( for more examples see recent review [3]) unambiguously suggest that a non-valence component in a hadron in general is not small. In QCD-terms it means that the corresponding matrix element has non-perturbative origin and has no $\alpha_s$ suppression which is naively expected from perturbative analysis ( we use the term “intrinsic component” to describe this non-perturbative contribution in order to distinguish from the “extrinsic component” which is always present and is nothing but a perturbative amplitude of the gluon splitting $g \rightarrow Q\bar{Q}$ with non-valence quark flavor $Q$).

The phenomenon we are going to discuss here is somewhat similar to those effects mentioned above. We shall argue that a non-valence component in a heavy-light quark system could be very large. However, before to present our argumentation of why, let say, the matrix element $\langle \Lambda_b|\bar{s}s|\Lambda_b\rangle$ is not suppressed (i.e. has the same order of magnitude as valence matrix element $\langle \Lambda_b|\bar{u}u|\Lambda_b\rangle$), we would like to get some QCD-based explanation of the similar effects we mentioned earlier.

Before to do so, let me remind that for a long time it was widely believed that the admixture of the pairs of non-valence quarks in hadrons is small. The main justification of this picture was the constituent quark model where
there is no room, let say, for a strange quark in the nucleon, (see, however, the recent paper [4] on this subject). It has been known for a while that this picture is not quite true: In scalar and pseudoscalar channels one can expect a noticeable deviation from this naive prediction. This is because, these channels are very unique in a sense that they are tightly connected to the QCD-vacuum fluctuations with $0^+, 0^-$ singlet quantum numbers. Manifestation of the uniqueness can be seen, in particular, in the existence of the axial anomaly ($0^-$ channel) and the trace anomaly ($0^+$ channel).

Well-known example where this uniqueness shows up is a large magnitude of the strange content of the nucleon. In formal terms one can show that the matrix element $\langle N|\bar{s}s|N \rangle$ has the same order of magnitude as valence matrix element $\langle N|\bar{d}d|N \rangle$. We shall give a QCD-based explanation of why a naively expected suppression is not present there. After that, using an intuition gained from this analysis, we turn into our main subject: non-valence matrix elements in heavy hadrons.

We should note from the very beginning of this letter that the ideology and methods (unitarity, dispersion relations, duality, low-energy theorems) we use are motivated by QCD sum rules. However we do not use the QCD sum rules in the common sense. Instead, we reduce one complicated problem (the calculation of non-valence nucleon matrix elements) to another one (the behavior of some vacuum correlation functions at low momentum transfer). One could think that such a reducing of one problem to another one (may be even more complicated) does not improve our understanding of the phenomenon. However, this is not quite true: The analysis of the vacuum correlation functions with vacuum quantum numbers, certainly, is a very difficult problem. However some nonperturbative information based on the low energy theorems is available for such a correlation function. Besides that, one and the same vacuum correlation functions enters into the different physical characteristics. So, we could extract the unknown correlation function, let say, from $\langle N|\bar{s}s|N \rangle$ and use this information in evaluation of the matrix element we are interested in: $\langle \Lambda_b|\bar{s}s|\Lambda_b \rangle$. Such an approach gives a chance to estimate some interesting quantities.

## 2 Strangeness in the nucleon.

Let us start from the standard arguments (see e.g. the text book [5]) showing a large magnitude of of $\langle N|\bar{s}s|N \rangle$. Arguments are based on the results of the fit to the data on $\pi N$ scattering and they lead to the following estimates
the so-called $\sigma$ term [3]:

$$\frac{m_u + m_d}{2} \langle p | \bar{u} u + \bar{d} d | p \rangle = (45 \text{MeV}).$$

(1)

(Here and in what follows we omit kinematical structure like $\bar{p} p$ in expressions for matrix elements.). Taking the values of quark masses to be $m_u = 5.1 \pm 0.9 \text{MeV}$, $m_d = 9.3 \pm 1.4 \text{MeV}$, $m_s = 175 \pm 25 \text{MeV}$ [4], from (1) we have

$$\langle p | \bar{u} u + \bar{d} d | p \rangle \simeq 6.2,$$

(2)

where we literally use the center points for all parameters in the numerical estimations. Further, assuming octet-type $SU(3)$ breaking to be responsible for the mass splitting in the baryon octet, we find

$$\langle p | \bar{u} u | p \rangle \simeq 3.5, \; \langle p | \bar{d} d | p \rangle \simeq 2.8, \; \langle p | \bar{s} s | p \rangle \simeq 1.4.$$

(3)

We should mention that the accuracy of these equations is not very high. For example, the error in the value of the $\sigma$ term already leads to an error of order of one in each matrix element discussed above. Besides that, chiral perturbation corrections also give noticeable contribution into matrix elements [3], see [5]. However, the analysis of possible errors in eq. (3) is not the goal of this paper. Rather, we wanted to demonstrate that these very simple calculations explicitly show that the strange matrix element is not small. Recent lattice calculations [8] also support the large magnitude for the strange matrix element.

We would like to interpret the relations (3) as a combination of two very different (in sense of their origin) contributions to the nucleon matrix element:

$$\langle p | \bar{q} q | p \rangle \equiv \langle p | \bar{q} q | p \rangle_0 + \langle p | \bar{q} q | p \rangle_1,$$

(4)

where index 0 labels a (sea) vacuum contribution and index 1 a valence contribution for a quark $q$. In what follows we assume that the vacuum contribution which is related to the sea quarks is the same for all light quarks $u, d, s$. Thus, the nonzero magnitude for the strange matrix elements comes exclusively from the vacuum fluctuations. At the same time, the matrix elements related to the valence contributions are equal to

$$\langle p | \bar{u} u | p \rangle_1 \simeq 2.1, \; \langle p | \bar{d} d | p \rangle_1 \simeq 1.4.$$

(5)

These values are in remarkable agreement with the numbers 2 and 1, which one could expect from the naive picture of non-relativistic constituent quark model. In spite of the very rough estimations presented above, we believe we convinced a reader that: a) a magnitude of the nucleon matrix element
for $\bar{s}s$ is not small; b) the large value for this matrix element is due to the nontrivial QCD vacuum structure where vacuum expectation values of $u, d, s$ quarks are developed and they have the same order in magnitude: $\langle 0|\bar{d}d|0\rangle \sim \langle 0|\bar{u}u|0\rangle \sim \langle 0|\bar{s}s|0\rangle$.

Once we realized that the phenomenon under discussion is related to the nontrivial vacuum structure, it is clear that the best way to understand such a phenomenon is to use some method where QCD vacuum fluctuations and hadronic properties are strongly interrelated. We believe, that the most powerful analytical nonperturbative method which exhibits these features is the QCD sum rules approach \cite{9},\cite{10}.

2.1 Strangeness in the nucleon and QCD vacuum structure.

To calculate $\langle N|\bar{s}s|N\rangle$ using the QCD -sum rules approach, we consider the following vacuum correlation function \cite{11}:

$$T(q^2) = \int e^{iqx}dxdy\langle 0|T\{\eta(x), \bar{s}s(y), \bar{\eta}(0)\}|0\rangle$$

at $-q^2 \to \infty$. Here $\eta$ is an arbitrary current with nucleon quantum numbers. In particular, this current may be chosen in the standard form $\eta = \epsilon^{abc}\gamma_\mu d^a(u^b C\gamma_\mu u^c)$. For the future convenience we consider the unit matrix kinematical structure in (6).

Let us note that due to the absence of the $s$ -quark field in the nucleon current $\eta$, any substantial contribution to $T(q^2)$ is connected only with non-perturbative, so-called induced vacuum condensates. Such a contribution arises from the region, when some distances are large. Thus, this contribution can not be directly calculated in perturbative theory, but rather should be coded (parameterized) in terms of a bilocal operator $K$ \cite{11}:

$$\langle p|\bar{s}s|p\rangle \simeq \frac{-m}{\langle 0|\bar{u}u|0\rangle}K,$$  \hspace{1cm} (7)

$$K = i \int dy\langle 0|T\{\bar{s}s(y), \bar{u}u(0)\}|0\rangle$$  \hspace{1cm} (8)

where $m$ is the nucleon mass. For the different applications of this approach where the bilocal operators play an essential role, see refs. \cite{12},\cite{13},\cite{14}.

The main assumptions which have been made in the derivation of this relation are the following. First, we made the standard assumption about local duality for the nucleon. The second assumption is that the typical
scales (or what is the same, duality intervals) in the limit $-q^2 \to \infty$ in the three-point sum rules (7) and corresponding two-point sum rules

$$ P(q^2) = \int e^{iqx} dx \langle 0| T\{\eta(x), \eta(0)\}|0\rangle, \quad (9) $$

are not much different in magnitude from each other. In different words we assumed that a nucleon saturates both correlation functions with approximately equal duality intervals $\sim S_0$. In this case the dependence on residues $\langle 0|\eta|N\rangle$ is canceled out in the ratio of those correlation functions and we are left with the matrix element $\langle p|\bar{s}s|p\rangle$ we are interested in.

One can estimate the value of $K$ by expressing this in terms of some vacuum condensates [11]:

$$ K \simeq \frac{18}{b} \frac{\langle \bar{q}q \rangle^2}{\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \rangle} \simeq 0.04 GeV^2 \quad (10) $$

where $b = \frac{11}{3} N_c - \frac{2}{3} N_f = 9$ and we use the standard values for the condensates [3], [10]:

$$ \langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \rangle \simeq 1.2 \cdot 10^{-2} GeV^4, \quad \langle \bar{q}q \rangle \simeq -(250 MeV)^3. $$

The estimation (11) might be too naive, however, if we literally adopt this estimate for $K$, formula (7) gives the following expression for the nucleon expectation value for $\bar{s}s$

$$ \langle p|\bar{s}s|p\rangle \simeq -m \cdot \frac{18}{b} \frac{\langle \bar{q}q \rangle}{\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \rangle} \simeq 2.4, \quad (11) $$

which is not far away from ”experimental result” (3). Having in mind a large uncertainties in those equations, we interpret an approach which leads to the final formula (11) as a very reasonable method for estimation of non-valence matrix elements.

It is very important that our following formulas for the non-valence content in heavy quark system (next section) will be expressed in terms of the same correlator $K$. Therefore, we could use formula (7) in order to extract the corresponding value for $K$ from experimental data instead of using our estimation (11). In this case $K$ is given by

$$ K \simeq -\frac{1}{m} \langle p|\bar{s}s|p\rangle \langle 0|\bar{u}u|0\rangle \sim 0.025 GeV^2. \quad (12) $$

Let us stress: we are not pretending to have made a reliable calculation of the matrix element $\langle p|\bar{s}s|p\rangle$ here. Rather, we wanted to emphasize on the qualitative picture which demonstrates the close relation between non-valence matrix elements and QCD vacuum structure. This is the lesson number one. More lessons to be learned will follow.
3 Zweig rule violation in the vacuum channels. Lessons.

The result (3,11) means that $s$ quark contribution into the nucleon mass is not small. Indeed, by definition

$$m = \langle N | \sum_q m_q \bar{q}q | N \rangle - \frac{b}{8} \langle N | \frac{\alpha_s}{\pi} G_{\mu\nu}^2 | N \rangle,$$

where sum is over all light quarks $u,d,s$. Adopting the values for $\langle p|\bar{s}s|p\rangle \simeq 1.4$ and $m_s \simeq 175MeV$ [7], one can conclude that a noticeable part of the nucleon mass (about $200 \div 300$ MeV) is due to the strange quark. We have mentioned this, well known result, in order to emphasize that the same phenomenon takes place (as we argue in the next section) in heavy quark system. Namely, we shall see that $s$ quark contribution to $\bar{\Lambda} \equiv m_{HQ} - m_Q|_{m_Q \rightarrow \infty}$ for heavy hadron $HQ$ is not small. This result is in a variance with the standard Zweig rule expectation predicting that any non-valence matrix element is suppressed in comparison with a similar in structure, but valence one.

The method presented above gives a very simple QCD-based physical explanation of why the Zweig rule in the scalar and pseudoscalar channels is badly broken and at the same time, in the vector channel the Zweig rule works well. In fact, we reformulated the original problem of the calculating of a non-valence matrix element in terms of some vacuum nondiagonal correlation function $\sim \langle 0|T\{\bar{s}\gamma_{\mu}s(x), \bar{u}\gamma_{\nu}u(0)\}|0\rangle$ with a Lorenz structure $\Gamma$.

In particular, the matrix element $\langle N|\bar{s}\gamma_{\mu}s|N\rangle$ is reduced to the analyses of the nondiagonal correlation function $\int dx \langle 0|T\{\bar{s}\gamma_{\mu}s(x), \bar{u}\gamma_{\nu}u(0)\}|0\rangle$, which is expected to be very small in comparison with the diagonal one $\int dx \langle 0|T\{\bar{u}\gamma_{\mu}u(x), \bar{u}\gamma_{\nu}u(0)\}|0\rangle$. Therefore, the corresponding matrix element as well as the coupling constant $g_{\phi NN}$ are also small. In terms of QCD such a smallness corresponds to the numerical suppression (of order $10^{-2} - 10^{-3}$) of the nondiagonal correlation function in comparison with the diagonal one, see QCD-estimation in [9].

In the scalar and pseudoscalar channels the diagonal and non-diagonal correlators have the same order of magnitude; therefore, no suppression occurs. This is the cornerstone of the paper and is the fundamental explanation of the phenomenon we are discussing here. Specifically, magnitude of correlator $K$ is not changing much if we replace $s$ quark to $u$ quark in formula (8).

Of course, it is in contradiction with large $N_c$ (number of colors) counting rule where a non-diagonal correlator should be suppressed. The fact that the naive counting of powers of $N_c$ fails in channels with total spin 0 is well-known: quantities small in the limit $N_c \rightarrow \infty$ turn out to be large and vice
versa. This is manifestation of the phenomenon discovered in ref. [15]: not all hadrons in the real world are equal to each other.

Because the issue of the violation of $N_c$ counting rule (Zweig rule) is so important and because all our results are based on this violation, we believe it is appropriate to give more examples (explanations) where naive $N_c$ counting fails. We hope that arguments presented below convince a reader that effect we are talking about is not extraordinary one, but rather is a very common phenomenon if we have dealt with $0^\pm$ vacuum channels.

We follow [15] and introduce the following ratio

$$r = \frac{\langle 0| \frac{\alpha_s}{8\pi} G_{\mu\nu}^2 |2\text{gluons}\rangle^2}{\langle 0| \frac{\alpha_s}{8\pi} G_{\mu\nu}^2 |\pi\pi + KK + \eta\eta\rangle^2}. \tag{14}$$

This ratio is very convenient since all normalization factors due to phase volume cancel out. Besides that, it is not difficult to find that this ratio is proportional to $N_c^2$ because of the suppression of an amplitude creating a pair of mesons in comparison with a creation of gluon pair. This ratio can be explicitly calculated from low energy theorem and is given by [15]:

$$r = \frac{N_c^2 - 1}{16 \ln^2(M_{QCD})}, \tag{15}$$

where $M \sim 1GeV$ is invariant mass of the pair of mesons, small enough for low energy expansion to be valid, large enough to have a small coupling constant $\alpha_s(M) < 1$. In accordance with general rules $r \sim N_c^2 \rightarrow \infty$ in the large $N_c$ limit. However, for the real world with $N_c = 3$ the ratio is small rather than large, $r \sim 0.1 \ll 1$. This is a clear indication of the anomalously strong coupling in $0^\pm$ vacuum channel.

The same phenomenon is responsible for large $\eta'$ mass. Indeed, as was noted by Witten [16], unlike all normal mesons whose masses are $N_c$ independent, the $\eta'$ mass vanishes in the large $N_c$ limit. However, in the real world this rule is badly broken, where $m_{\eta'}^2 / m_{\rho}^2 \simeq 1.8$. The reason is the same as before – the anomalously strong coupling in $0^-$ vacuum channel.

Our next argument is as follows. It has been known for a while that in some special cases[2] the QCD sum rules do not work. In particular, in the pseudoscalar quark channel the standard QCD sum rule for the correlation function[3]

$$T(q^2) = \int e^{iqx} dx \langle 0| T\{J^\dagger(x), J(0)\} |0\rangle, \quad J = \bar{u}i\gamma_5d \tag{16}$$

Note, that [14] is very similar to our correlator $K(8)$.

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2 The guess that it happens exactly in the vacuum $0^\pm$ channels is correct.
3 Note, that [14] is very similar to our correlator $K(8)$. 7
can not reproduce the residue $\langle 0|J|\pi \rangle \simeq f_\pi 2 GeV$ which is known exactly from PCAC, and which has much bigger scale than QCD sum rule can provide. So called direct instanton contributions play a decisive role in these channels [17].

Another manifestation of the same phenomenon is somewhat different behavior of the four-quark condensates $\langle \bar{q}\Gamma qq\Gamma q \rangle$ with different Lorenz and color structure ($\Gamma$ in this formula denotes a combination any of the $\gamma$ and $\lambda_2^a$ or $1$ matrices from the color and flavour groups). The so-called factorization hypothesis (which is justified in the large $N_c$ limit) works perfectly well for the vector and axial-vector cases ($\Gamma = \gamma_\mu$, $\gamma_\mu \gamma_5$) [17], but does not work in general, see Shifman’s comment paper on this subject in the book [10]. In particular, the vacuum condensate $\langle \bar{u}\sigma_{\mu\nu}\lambda^a ud\sigma_{\mu\nu}\lambda^a d \rangle$ is not small in spite of the fact that the factorized value is exactly zero [4]. Such a behavior of condensates has a qualitative explanation based on the properties of the fermion zero modes within the instanton approach, see e.g. review [19].

The same instanton picture gives also a qualitative explanation of the enhancements mentioned above in the $0^\pm$ channels. In fact, the forementioned Zweig rule violation is related to the fermion zero modes, which always accompany an instanton. As is known, those zero modes are very selective in a sense of the quantum numbers they carry on: they do contribute, let say, to the scalar correlator and they do not contribute to the vector one.

One could estimate the correlator $K(8)$ using the instanton ideas of ref. [19]. The numerical result obtained in this way will be very close to our estimation (10). However, in spite of the very attractive and simple picture of the QCD vacuum structure advocated in ref. [19], it is very difficult to estimate an error of such a calculation. Therefore, we prefer to use a less direct, but more solid approach based on the low-energy theorems. In this case the enhancement of the vacuum channels (at least qualitatively) can be easily understood.

Instead of the analysis of the original correlator (8) which enters into our formulae, we introduce the following correlation function containing a heavy quark $Q$:

$$K^{(Q)} = i \int dy \langle 0|T\{\bar{Q}Q(y), \bar{u}u(0)\}|0\rangle. \tag{17}$$

In the limit when a quark $Q$ is very heavy, the correlator $K^{(Q)}$ can be calculated exactly! Indeed, in this limit, one can use the standard operator product expansion $\sim (1/m_Q)^n$ in order to express the quark operator $\bar{Q}Q$ in

\footnote{Actually, it has the same order of magnitude as the condensate $\langle \bar{d}\sigma_{\mu\nu}\lambda^a ud\sigma_{\mu\nu}\lambda^a d \rangle$ and both condensates are much bigger than the factorization hypothesis predicts [18].}
terms of the gluon operators\cite{10}:

\[ \bar{Q}Q = -\frac{\alpha_s}{12m_Q\pi}G^2_{\mu\nu} + \frac{G^3_{\mu\nu}}{m_Q^3} + \ldots \quad (18) \]

The correlation function \( K^{(Q)} \) then takes the form

\[ K^{(Q)} = -i\frac{1}{12m_Q} \int dy \langle 0 | T \{ \frac{\alpha_s}{\pi}G^2_{\mu\nu}(y), \bar{u}u(0) \} | 0 \rangle + O(1/m_Q^3) \quad (19) \]

where we mainly interested in the leading term \( \sim 1/m_Q \). Fortunately, the obtained correlation function is known exactly \cite{15}:

\[ i \int dy \langle 0 | T \{ \frac{\alpha_s}{\pi}G^2_{\mu\nu}(y), \bar{u}u(0) \} | 0 \rangle = \frac{8d}{b} \langle \bar{u}u \rangle, \quad (20) \]

where \( d = 3 \) is the dimension of the operator \( \bar{u}u \) and \( b = \frac{41N_c}{3} - \frac{2N_f}{3} = 9 \).

Finally, we get the following expression for the correlation function we are interested in:

\[ K^{(Q)} = i \int dy \langle 0 | T \{ \bar{Q}Q(y), \bar{u}u(0) \} | 0 \rangle = -\frac{2}{9} \frac{\langle \bar{u}u \rangle}{m_Q} + O(1/m_Q^3). \quad (21) \]

This is exact formula for large \( m_Q \).

Few remarks are in order. First, formula (21) for \( K^{(Q)} \) shows the correct sign “ + ” which is expected for the light \( s \)-quark (12,10). This formula also demonstrates the correct \( N_c \) dependence at large \( N_c \): The correlator (21) is of order of one\footnote{Let us remind that \( \langle \bar{u}u \rangle \sim N_c, \quad b \sim N_c \). Therefore, the combination on the right hand side of eq. (20) is of order \( \frac{1}{b} \langle \bar{u}u \rangle \sim 1 \).} rather than \( N_c \) expected for a diagonal quark correlation function. One could estimate the next \( 1/m_Q^n, n > 1 \) corrections in the eq. (21) with the result that the series blows up when \( m_Q \leq 300 \div 400 MeV \). Of course, this result was expected from the very beginning: one can not take the limit \( m_Q \to m_s \) in the expansion like (21).

However, from the general consideration we expect that the correlator \( K^{(Q)} \) is a monotonic function of \( m_Q \) in the extended region of \( m_Q \) (except the region of the extremely small \( m_Q \leq 30 MeV \) where the chiral perturbation theory predicts somewhat different behavior\cite{13}).

The main goal of the present analysis of the correlator \( K^{(Q)} \) is not a numerical estimation (which is strongly model dependent magnitude in the interesting region of \( m_Q \simeq m_s \simeq 150 MeV \)). Rather, we want to give a qualitative explanation of the enhancement in the vacuum channels by analysing this correlator. On the qualitative level, one could expect from the perturbative analysis that the correlation function (17) should be suppressed by a
factor of $\alpha_s^2$. Indeed, an annihilation of the quark $Q$ into two gluons and a creation of a pair with a different flavour $u$ is suppressed in perturbative calculation as $\alpha_s^2$. Our formula (21) shows that this naive estimation is wrong: no any suppression occurs in the exact formula.

It is clear why our intuition, based on the perturbative calculations is failed: transition, we are talking about, is the large distance phenomenon. Therefore, the perturbative analysis can not be applied to such an amplitude. This statement can be easily understood from the analysis of exact low-energy theorem (20), where a similar factor $\alpha_s^2$ has disappeared from the right hand side of the equation.

Interpretation of the disappearing of this factor $\alpha_s^2$ is very simple: At large distances the most important configurations which are responsible for the transition like (20) have an enhancement like $G_{\mu\nu} \sim \frac{1}{g}$. Therefore, semiclassical configurations with $G_{\mu\nu} \sim \frac{1}{g}$ saturate the corresponding low-energy theorems; they clearly can not be seen in perturbative analysis. This remark closes our qualitative analysis of the correlation function $K$ (8). As we discussed earlier, one can not use formula (21) for the quantitative calculations for $m_s \simeq 175MeV$. However, if we literally adopt this formula for $K$ with the assumption about its monotonic behavior formulated above, we get

$$K^{(Q)} = i \int dy \langle 0 | T \{ \bar{Q}Q(y), \bar{u}u(0) \} | 0 \rangle \simeq -\frac{2}{9} \frac{\langle \bar{u}u \rangle}{m_Q} \to 0.02 GeV^2 \hspace{1cm} (22)$$

at $m_Q \simeq m_s \simeq 0.175 GeV$,

which is very close to the “experimental” value (12).

Let us stress: we are not pretending to have made a reliable calculation of the correlation function $K$ here. Rather, we wanted to emphasize on the enhancement mechanism of the vacuum channels which could be understood from the analysis of the low-energy theorems. This analysis also shows that the corresponding enhancement is due to some semiclassical configurations in the functional integral with $G_{\mu\nu} \sim \frac{1}{g}$.

Finally, it is fair to say, that the limit of large $N_c$ nicely explains a lot of empirical regularities. The Zweig rule is particular example of this kind. However, in vacuum channels this naive counting rule does not work. Therefore, we should not be surprised if we find some strong deviation from the naive picture in $0^\pm$ vacuum channels. Relatively large magnitude for the correlation function $K$ (8) (which is fundamentally important parameter for our estimations), is another manifestation of the same kind.
4 Heavy hadrons

In this section we shall apply the ideas described above for the calculation of the non-valence matrix element \( \langle \Lambda_b | \bar{s}s | \Lambda_b \rangle \). It should be considered as an explicit demonstration of the general idea (formulated in the introduction) that a non-valence component \( \bar{s}s \) in a heavy quark system \( \Lambda_b \sim bud \) could be large and comparable with valence matrix element like \( \langle \Lambda_b | \bar{u}u | \Lambda_b \rangle \). We notice, that a similar conclusion was obtained previously in the toy model of two-dimensional \( QCD_2(N) \) [20].

We start from the definition of the fundamental parameter \( \bar{\Lambda} \) [21] of HQET (heavy quark effective theory), see e.g. nice review paper [22]:

\[
\bar{\Lambda} \equiv m_{H_Q} - m_Q |_{m_Q \to \infty}
\] (23)

All hadronic characteristics in HQET should be expressed in terms of \( \bar{\Lambda} \) which is defined as the following matrix element:

\[
\bar{\Lambda} = \frac{1}{2m_{H_Q}} \langle H_Q | \sum_q m_q \bar{q}q + \frac{\beta(\alpha_s)}{4\alpha_s} G_{\mu\nu}^2 | H_Q \rangle.
\] (24)

Numerically \( \bar{\Lambda} \sim 500 MeV \) [23].

Now we can use the same technique (we have been using in the previous section) to estimate the strange quark contribution into the mass of a heavy hadron:

\[
\bar{\Lambda}(s) = \frac{1}{2m_{H_Q}} \langle H_Q | m_s \bar{s}s | H_Q \rangle.
\] (25)

Lessons we learned from the similar calculations teach us that this matrix element might be large enough.

Technically, to calculate \( \langle \Lambda_b | \bar{s}s | \Lambda_b \rangle \) we use the same approach we described in Section 2; namely we consider the following vacuum correlation function:

\[
T(q^2) = \int e^{i q x} dxdy \langle 0 | T \{ \eta(x), \bar{s}s(y), \eta(0) \} | 0 \rangle
\] (26)

Here \( \eta = e^{\alpha_\gamma}(u^T \gamma_5 b_\beta)_{\alpha}\gamma_\beta \) is the current with \( \Lambda_b \) quantum numbers. It is much more convenient in the case of heavy quark, to use heavy quark expansion within QCD sum rules, as it was suggested for the first time in

\footnote{To be more precise, two baryons: \( \Lambda_b(I = 0) \) as well as \( \Sigma_b(I = 1) \) contribute to this correlation function. However, for qualitative analysis we assume that their matrix elements are similar. Therefore, in order to simplify things, we do not separate those states.}
In this case, instead of external parameter $q_\mu$, one should introduce parameter $E$ in the following way: $q_\mu = (m_Q + E, 0, 0, 0)$. Similarly, the resonance energy is defined as $m_{HQ} = m_Q + E_r$ etc. Therefore, all low energy parameters do not depend on $m_Q$ and they scale like $\Lambda$ at large $m_Q$.

Let us note that, similar to the nucleon case, due to the absence of the $s$-quark field in the current $\eta$, the most important contribution to $T(q^2)$ comes from the induced vacuum condensates $K$. We should consider, along with the analysis of the correlator (26), the following two-point correlation function

$$ P(q^2) = \int e^{i q x} \langle 0 | T \{ \eta(x), \bar{\eta}(0) \} | 0 \rangle. $$

As before, we assume that $\Lambda_b$ baryon saturates both correlation functions (24, 27) with approximately equal duality intervals $\sim S_0$. In this case the dependence on residues $\langle 0 | \eta | \Lambda_b \rangle$ is canceled out in the ratio of those correlation functions and we are left with the matrix element $\langle \Lambda_b | \bar{s}s | \Lambda_b \rangle$ we are interested in.

This is the standard first step of any calculation of such a kind: Instead of direct calculation of a matrix element, we reduce the problem to the computation of some correlation function. As the next step, we use the duality and dispersion relations to relate a physical matrix element to the QCD-based formula for the corresponding correlation function. This is essentially the basic idea of the QCD sum rules.

With this remark in mind, the calculations very similar to (6,7, 8) bring us to the following formula

$$ \frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | \bar{s}s | \Lambda_b \rangle \simeq \frac{3}{4} S_0 + E_r \frac{K}{\langle \bar{q}q \rangle} \simeq 1 \div 2, $$

where $S_0$ and $E_r$ are duality interval and binding energy for the lowest state with given quantum numbers. This formula is direct analog of the expression (7) we derived previously for the nucleon. In the course of calculation we have made the same assumptions we made before, see previous section. Therefore, we believe we have the same accuracy as before which we estimate on the level of 50%. The only difference with formula (7) is a replacement of nucleon mass $m \simeq 1GeV$ by a combination of two parameters $S_0$ and $E_r$. Similar to the nucleon case, we use the local duality arguments (so-called, finite energy sum rules) to estimate the matrix element (25). Besides that, we use the standard technical trick [25] which suggests to use the combination $(E - E_r)T(E)$ in sum rules (24) rather than $T(E)$ itself. This trick allows to exponentially suppress an unknown contribution from the nondiagonal transitions which include higher resonances.

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*We use $S_0 \sim E_r \sim (0.5 \div 0.7)GeV$ and $K \simeq 0.025GeV^2$ [12] for numerical estimations.
\( E_r \) which have the same order of magnitude as nucleon mass. In a sense, those parameters are trivial kinematical factors which always have a hadronic \( \sim 1 GeV \) scale.

There is a non-trivial factor in our formula which is very important for us and deserves an additional explanation. The fact is: the nonperturbative correlation function \( K \) which enters into the expression (28) is the same correlator we have been using for the calculation \( \langle p|\bar{s}s|p \rangle \) (7). This factor is not small as naively one could expect. It sets the scale of the phenomenon.

Moral: If we accept the large value for \( \langle p|\bar{s}s|p \rangle \) we should also accept the large value for

\[
\frac{1}{2m_{HQ}}\langle H_Q|m_s\bar{s}s|H_Q \rangle \sim (200 \div 300) MeV, \tag{29}
\]

as a consequence of absence of any suppression for nondiagonal correlator \( K \).

Let us repeat: we are not pretending to have made a reliable calculation of the matrix element \( \frac{1}{2m_{HQ}}\langle H_Q|m_s\bar{s}s|H_Q \rangle \) here. Rather, we wanted to emphasize on the qualitative picture which demonstrates a close relation between the matrix element (28) and corresponding nucleon matrix element (7). Both those matrix elements are related to each other and relatively large because of the strong fluctuations in vacuum \( 0^\pm \) channels. We can not calculate the nontrivial part (correlator \( K \)) from the first principles. However, the analysis of different low-energy theorems supports our expectation that its magnitude is large. For numerical estimations, we can extract a relevant information from one problem in order to use this info somewhere else.

5 Conclusion

We have argued that matrix element (29) could be numerically large. The arguments are very similar to the case of strange matrix element over nucleon and based on the fundamental property of nonperturbative QCD that there is no suppression for flavor changing amplitudes in the vacuum channels \( 0^\pm \) (the Zweig rule in these channels is badly broken). Few consequences of the result (29) are in order:

1. The value of \( \Lambda \) continues to be controversial, because the QCD sum rules indicate that \( \Lambda \sim 0.5 GeV \) which does not contradict to the lower bound stemming from Voloshin’s sum rules \( \Lambda \sim 0.2 – 0.3 GeV \), see [22] for more details. The possible interpretation is: lattice definition of \( \Lambda \) does not correspond to the continuum theory because the \( s \)-quark contribution (29) was not accounted properly. It would be very interesting to calculate
matrix element \((29)\) in somewhat independent way; for example, in chiral perturbation theory or on the lattice (similar to the nucleon calculation of ref.\([3]\)).

2. Scalar and pseudoscalar light mesons (\(\eta, f_0\)) strongly interact with \(\Lambda_b\); \(\phi\) meson does not interact with \(\Lambda_b\).

3. We expect a similar situation for all heavy hadrons. Therefore, for inclusive production of strangeness heavy hadrons we expect some excess of strangeness in comparison with naive calculation. However, we do not know how to estimate this effect in appropriate way.

We conclude with few general remarks:

4. A variation of the strange quark mass may considerably change some vacuum and hadronic characteristics. Therefore, the standard lattice calculations of those characteristics using a quenched approximation is questionable simply because such a calculation clearly not accounting the fluctuations of the strange (non-valence)quark. At the same time, the QCD sum rules approach clearly includes those contributions implicitly. Indeed, all relevant vacuum condensates (like \(\langle \bar{u}u \rangle\)) which appear in the QCD sum rules approach for the non-strange hadrons do depend on \(s\) quark.

In fact, the correlator \(K\) enters to expression \((3)\), as well as it determines the variation of the condensate \(\langle \bar{u}u \rangle\) with \(s\) quark mass:

\[
\frac{d}{dm_s} \langle \bar{u}u \rangle = -i \int dy \langle 0 | T \{ \bar{s} s(y), \bar{u} u \} | 0 \rangle = -K \simeq -0.025 GeV^2. \tag{30}
\]

To understand how large this number is and in order to make some rough estimations, we assume that this behavior can be extrapolated from physical value \(m_s \simeq 175 MeV\) till \(m_s = 0\). In this case we estimate that

\[
| \frac{\langle \bar{u}u \rangle_{m_s=175} - \langle \bar{u}u \rangle_{m_s=0}}{\langle \bar{u}u \rangle_{m_s=175}} | \simeq 0.3. \tag{31}
\]

Such a decrease of \(| \langle \bar{u}u \rangle |\) by a 30% as \(m_s\) varies from \(m_s \simeq 175 MeV\) to \(m_s = 0\) is a very important consequence of QCD. Therefore, QCD sum rules approach implicitly accounts an existence of strange quark in the theory.

5. An analysis of the low-energy theorems (similar to \((21)\)) might be useful tool for the future investigations on the Zweig rule violations in different channels.

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