Pre-service teacher’s pedagogical content knowledge: What teachers know about students’ errors?

W Setyaningrum, A Mahmudi and Murdanu
Department of Mathematics Education, Universitas Negeri Yogyakarta, Indonesia

E-mail : wahyu_setyaningrum@uny.ac.id

Abstract. Content knowledge is one of the important aspect for the improvement of teaching and learning, attention to its development and study has been conducted frequently. This study was intended to explore the pedagogical content knowledge of mathematics pre-service teachers in Indonesia. This descriptive qualitative study involving twenty five pre-service teachers enrolled in mathematics education department in one university in Yogyakarta. The collected data of the pedagogical content knowledge were examined by using descriptive analysis. This study revealed that many pre-service mathematics teachers could identify students’ errors. Nevertheless, they struggle to provide alternate ways to minimize the errors. The possible reasons behind these findings are discussed.

1. Introduction
Some experts across decades argue that being an effective mathematics teachers not only requires strong content knowledge (CK) but also a strong foundation in pedagogical content knowledge (PCK) [1]–[3]. However, there are disagreement among the experts on the definitions of PCK. A broader definition of PCK is knowledge about classroom teaching [3]. From the cognitive point of view, PCK related to “knowledge needed for teaching a specific subject” [4]. In more specific, PCK is “knowledge in action” [5].

Shulman [6] proposed two component of PCK: (1) teachers’ knowledge of common conceptions, misconceptions, and difficulties held by students regarding particular subject matter, and (2) teachers’ knowledge on various instructional strategies and representations that could help students learning particular subject matter. The first category can be divided into two (a) teachers’ knowledge of common (mis)conceptions and difficulties held by students, and (b) teachers’ knowledge of possible sources of these (mis)conceptions and difficulties [7]–[8]. Referring to these ideas, PCK in this study defines as a teachers’ attributes on the subject matter, planning, and teaching certain topics in certain ways to certain students. Teachers, therefore, need to know the nature of the subject matter they are taught and the characteristics of students including common misconception, common difficulties and source of misconception and difficulties.

PCK can be prepared and improved during teachers training program. Pre-service teachers are not only learn what material should be taught but also what method should be employed. In order to be able to select appropriate method, pre-service teachers should know which material are difficult for students and what kind of errors students often make. Research on PCK of pre-service mathematics teacher have been conducted in many countries such as Kılıç [9] who investigated six knowledge of
pre-service mathematics teachers on students’ knowledge. The study found that pre-service teachers had difficulty in both identifying the source of students’ misconceptions, and errors and generating effective ways other than teaching the rules or procedures to eliminate such misconceptions. This study, therefore, aims to investigate the knowledge of pre-service mathematics teachers in identifying students’ errors, its possible sources and solution to elude such errors.

2. Methods
This study is a descriptive qualitative study particularly a case study of pre-service mathematics teachers on their knowledge of the students. The students’ knowledge refers to familiarity with the students’ mathematical thinking, especially with common mistakes they present this including ability to identify common errors, explain the errors and acknowledge difficult topic students usually experiences [9].

The participants are third-year-students of mathematics education department at one university in Yogyakarta. Considering that the third-year-students have received all courses related to content mathematics and pedagogy, therefore they might have enough knowledge to the content as well as knowledge on how to teach the content.

The data were gathered through written test and interview. Twenty-five pre-service teachers of were given two open ended questions or cases of students works, they were asked to identify students error in solving mathematics problems. In addition, they were asked to offer solution to minimize such students’ errors. The pre-service students were then interviewed to give more explanation on their written work. The validity of their explanations were counted as the indicators of their knowledge of subject matter and knowledge of pedagogy.

The data were then analysed descriptively. The answers of the participants are coded and categorised in regards to the type of knowledge demonstrated in the questions. The interviews data are transcribed and coded in term of the type of knowledge.

3. Results and Discussions
Students were asked to response two cases of students’ errors. The first case related to the issue of determining based and height of the parallelogram and its area. The later is problem related to the concept of quadrilateral. The responses of the pre-service teachers are explained below.

![Figure 1. Example of students’ work](image-url)
In the first case, pre-service teachers were asked to identify the errors or misconception students have when solving a parallelogram which the length of the two parallel sides are given. Figure 1 shows that pre-service mathematics teacher actually could identify the error made by student which assumed the base of the triangle is 13 instead of 12. But unfortunately, 19 out of 25 pre-service mathematics teachers could not offer the teaching method to eliminate such misconception. Among the 6 pre-service teachers who propose teaching methods, the common method they offer focusing on how to teach the area of parallelogram. The believed that teaching how to get the formula of the area of parallelogram would help students to deal with this such misconception. They did not point out the issues of the height and base of parallelogram. In fact, this the common misconception hold by students.

The data above implies that the pre-service students did not recognize one of the common misconception held by students. The common students’ misconception related to parallelogram is parallelogram has only one height and one base. The students usually assume that the base of the parallelogram is the horizontal side \( b \) and the height is the vertical side \( h \) in Figure 2. They determine the base and the height of parallelogram based on its position instead of its concepts. Therefore when the position of the parallelogram changes, the students confuse to determine the height and base. It assumes that students have a mental image of parallelogram in way that it commonly presented in textbook which is in general position and shape (Figure 2 (i)).

![Figure 2. Parallelogram in several position](image)

The second case was aimed to identify whether pre-service teachers master the concepts of square, rectangle, kite, trapezium, and parallelogram. The pre-service students were given eleven geometrical objects (Figure 3) and asked to check whether the example answer was correct.
Figure 3. Case 2: Types of quadrilateral

Among the 25 pre-service student, 19 the pre-service teachers could recognise the errors given in the second case. One of the responses of the pre-service students can be seen in Figure 4. The pre-service teacher could explain the concept of each geometrical shape and able to classify the shapes correctly.

However, among 19 pre-service mathematics teachers who could recognize the error and give a revision towards that error, there is only two pre-service mathematics teachers who could identify possible reason of the errorness. One of the pre-service teacher assume that the mental image hold by students as the main issues in forming their misconception (Figure 4). The students were not accustomed to various kinds of positions of the shapes. The students more focused on the common position of the objects rather than understanding the concept.

Another pre-service teacher point the importance of mastering definition or concepts if various kinds of quadrilaterals and their characteristics (Figure 5). One of two pre-service mathematics teachers who could give a suggestion related to the best alternative learning trajectory that can be used to facilitate students to construct their knowledge about the correct concept of quadrilateral. In fact, one method that can be used in learning this material is meaningful learning, with a focus both on relational understanding and instrumental understanding. The students should have opportunities to develop and gain their knowledge. Students are exposed to various kinds of quadrilaterals in various orientation. In addition, the students are urge to explore the characteristics of each kind of quadrilaterals (instrumental understanding) and understand the relationship among many kinds of quadrilaterals (relational understanding).
Figure 5. Student’s Response to Case 2

Data from case 1 and 2 suggested that the pre-service teachers could understand the material or concepts they are taught and recognize students' mistakes as well as identify several possible sources of student errors. The possible sources they are pointed out, however, more focus on the technical issues rather than on the pedagogical aspect. The pre-service teachers believed that figural mental imagination is one of possible sources of misconception. This finding is line with the previous finding of Zilkova [10] in which students often failed to recognize a geometrical object when it presented in non-standard orientation or uncommon position. The findings of this study imply that some pre-service teachers have good knowledge on students and profound understanding of fundamental mathematics yet little on pedagogy knowledge.

Conclusion
The ability of the pre-service teachers in identifying students' errors on quadrilateral might be sufficient yet the ability to recognize sources of errors still needs to be improved. Giving more exposure of student misconceptions during teacher training program, might improve pre-service teachers skills in recognizing sources of students’ errors. In conclusion, the picture emerging from this study was of pre-service teachers demonstrating low knowledge of pedagogy, there is a need for programs that focus on developing pedagogy skills in addition to content capabilities.
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