Large Negative Poisson’s Ratio and Anisotropic Mechanics in New Penta-PBN Monolayer

Shambhu Bhandari Sharma,* Issam A. Qattan,* Santosh KC, and Ahmad M. Alsaad

ABSTRACT: The scarce negative Poisson’s ratio (NPR) in a two-dimensional (2D) material is an exceptional auxetic property that offers an opportunity to develop nanoscale futuristic multifunctional devices and has been drawing extensive research interest. Inspired by the buckled pentagonal iso-structures that often expose NPR, we employ state-of-the-art first-principles density functional theory calculations and analyses to predict a new 2D metallic ternary auxetic penta-phosphorus boron nitride (p-PBN) with a high value of NPR. The new p-PBN is stable structurally, mechanically, and dynamically and sustainable at room temperature, with experimental feasibility. The short and strong quasi sp$^3$-hybridized B–N bond and unique bond variation and geometrical reconstruction with an applied strain allow p-PBN to inherit a high value of NPR (−0.236) along the (010) direction, the highest among any other ternary penta iso-structures reported to date. Despite having a small elastic modulus, the highly asymmetric Young’s modulus and Poisson’s ratio along the (100) and (010) directions indicate large anisotropic mechanics, which are crucial for potential applications in nanomechanics and nanoauxetics.

INTRODUCTION

In recent years, a new class of two-dimensional (2D) Cairo-pentagonal lattice monolayers and their one-dimensional (1D) nanoribbon are drawing considerable attention due to their extraordinary chemical and physical properties$^{1,2}$ identified by both experiments$^{3−6}$ and theories. Unlike ultra-thin planar hexagonal graphene,$^{7}$ the low-symmetric, trilayered, and buckled geometry of penta monolayers allow them more degrees of freedom to possess novel mechanical, piezoelectric, electronic, thermal, and optical properties.$^{8−13}$ In addition, the inheritance of the robust band topology and visible light-harvesting direct-band gap semiconductor to metallic electronic behavior of penta monolayers make them highly desirable in cutting-edge technological devices such as optomechanical sensors, lithium-ion batteries, and solar cells.$^{11,15,16}$

Particularly, the finding of a very rare mechanical behavior, the negative Poisson’s ratio (NPR), from the first penta monolayer, monoelemental penta-graphene,$^{17}$ to a binary,$^{18−20}$ up to a ternary$^{21}$ penta monolayer shows tremendous possibilities of NPR, which is attributed to the unique buckled/puckered geometry and deformation mechanism dominated by the inter-atomic interactions.

Unlike the positive Poisson’s ratio material, NPR materials shrink (expand) laterally when compressed (stretched) axially and transform into a dome-sshape, while the out-of-plane bending moment is applied.$^{22}$ Most of these materials manifest NPR values ranging from −1 to 0. However, some materials such as AlP exhibited a large NPR value of −1.779.22 The higher the NPR value, the more lateral to longitudinal expansion and contraction the material undergoes. High NPR materials have a higher indentation, impact, and crack-propagation resistance with superior sound absorption performance.$^{23,24}$ These excellent properties allow NPR materials to have broad applications in the automotive, aerospace, marine, and other industrial fields.$^{25}$ More specifically, they hold great promise in advanced applications such as sensors, biomedicine, fasteners, bullet-proof shields, sports equipment, tissue engineering, and an untold number of unrevealed applications.$^{26−31}$

Even though the Gibson theory predicted the auxetic behavior of 2D materials earlier,$^{32}$ the experimental and theoretical findings of intrinsic NPR in black phosphorus (−0.027),$^{33,34}$ in 2014 inspired the exploration of the different classes of low-dimensional materials. Since then, NPR is reported in several 2D materials$^{18}$ including borophene (−0.053),$^{35}$ BP$_1$ (−0.037),$^{36}$ TiN,$^{37}$ Be$_2$C (−0.566),$^{38}$ 2D Be$_3$C$_2$ (−0.041),$^{39}$ penta-B$_2$N$_4$ (−0.02) and penta-B N$_2$ (−0.19),$^{40}$ penta-graphene (−0.07),$^{41}$ and penta-PdSe$_2$ (−0.022),$^{19}$ most of which have puckered and/or buckled
configurations. One of many, pentagonal monolayers are reported to be highly favorable structures for NPR due to their unique geometry and inter-atomic interaction. Besides Raman spectroscopy, the current experimental efforts to synthesize 2D auxetic materials are still premature, challenging, and lagging behind the theoretical studies. Nevertheless, the synthesis of such 2D auxetic materials is fascinating, highly desirable, and expected to prosper with the advancement of technologies in the near future.

Understanding the exceptional properties of NPR materials as well as their diverse applicabilities in non-planar structures, pentagonal monolayer research is not only limited to device replication, which preserves the periodic boundary conditions. Moving beyond monoelemental penta-graphene to the binary up to the new ternary pentagonal monolayer (4P, 2B, and 2N atoms in 1:1:1 proportion) in trilayered, three virtual layers in a monolayer, which are formed by the P, B, and N atoms, buckled form. Here, the designed structure is fully relaxed to achieve the minimum energy, atomic forces, and stresses. The fully relaxed structure maintains the pentagonal symmetry belonging to P-21 symmetry (space group no. 4).

The monolayer has a centered rectangular 2D lattice with lattice parameters $a = 4.31$ Å and $b = 4.28$ Å. The lattice parameters of p-BN are equivalent to the recently predicted ternary pentagonal p-SiCN. Interestingly, the supercell consists of four distinct irregular pentagonal Cairo tiles replication, which preserves the periodic boundary conditions.

In the primitive cell, the two P atoms have slightly different bond distances from the N atom. Therefore, they are labeled as P1 and P2 for better visualization. The importance of this choice is realized later when explaining the mechanical properties. The stable configuration of p-BN has N–P1, P1–B, B–N, and B–P2 bond lengths of 1.79, 1.89, 1.44, and 1.89 Å, respectively.

Remarkably, the quasi sp$^3$-hybridized B–N bond is even shorter in p-BN than that of the hexagonal sp$^3$-hybridized bond in h-BN (1.45 Å). This uniquely shorter bond length plays an important role in the strength, stability, and advanced mechanical responses of the monolayer. Similarly, the thickness (buckling height) of the monolayer ($h$), calculated by measuring the difference between the vertical extremities of atoms, is 1.41 Å, which is higher than that of penta-graphene (1.20 Å), penta-SiCN (1.24 Å), penta-BCN (1.34 Å), and penta-CN (1.52 Å). The measured bond angles of P–N–B ($\alpha$), N–B–P ($\beta$), and B–P–B ($\gamma$) are 111.8, 118.6, and 98.2 degrees, respectively.

The Mulliken charge density analysis is performed to corroborate the bonding mechanism and inter-atomic charge distribution in p-BN. The iso-surface plot (Figure 1b) demonstrates that most of the charge is accumulated around the N atom and shared significantly between the B and N atoms compared to that of the N and P atoms. Furthermore, the nature of the chemical bond is analyzed by visualizing the 2D valence charge density contour plot (Figure 1c). Here, the highest charge density is represented by blue > green > orange > red, in that descending order. The presence of a high electronic charge (green) between B and N and the overlapping of concentric lines with the considerable

RESULTS AND DISCUSSION

Structural Properties. The geometry of p-BN comprises three layers lattice planes, top/bottom with 3-coordinated boron/nitrogen ($B_3/N_3$) and middle layer with 4-coordinated phosphorus ($P_4$) following the mother penta-graphene isostructure (Figure 1a). The primitive unit cell consists of 2P, 2B, and 2N atoms (in 1:1:1 proportion) in trilayered, three virtual layers in a monolayer, which are formed by the P, B, and N atoms, buckled form. Here, the designed structure is fully relaxed to achieve the minimum energy, atomic forces, and stresses. The fully relaxed structure maintains the pentagonal symmetry belonging to P-21 symmetry (space group no. 4).

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Figure 1. (Color online) (a) fully optimized p-BN monolayer (4 x 4 x 1 supercell). The blue, magenta, and gray balls represent the P, B, and N atoms, respectively, (b) charge density iso-surface (0.05 e/Å$^3$), (c) charge density contour plots with different viewpoints including a magnified view of charge distribution between atoms (bottom), (d) phonon dispersion of the optimized p-BN, and (e) AIMD simulation at $T = 300$ K for the energy fluctuation as a function of time. The inset shows the top view of the final p-BN structure.
conjoining of their electronic wavefunctions confirm the covalent bonding similar to that of h-BN.\textsuperscript{53,44} In contrast, the smaller overlap of wavefunctions (dumbbell-shaped contour lines) with small charge sharing between P and N indicates partly ionic and covalent P–N bonding. The deformed conjoining contour lines with some charge sharing (faded-green) demonstrate a polar covalent bond in P–B. Chemically, the strength of bonding in p-PBN follows the order B–N > P–N > P–B, which is also tested against the applied strains while explaining the mechanical results. The cohesive energy ($E_c$) is calculated to examine the average interatomic binding and structural stability of p-PBN using $E_c = (E_{PBN} - 2E_P - 2E_B - 2E_N)/6$, where $E_{PBN}$, $E_P$, $E_B$, and $E_N$ represent the energy of p-PBN, isolated P, B, and N atoms, respectively. The high negative $E_c$ of $-7.42$ eV/atom, comparable to that of h-BN ($-8.83$ eV/atom),\textsuperscript{45,46} justifies it as one of the most stable configurations. Furthermore, the formation energy ($E_f$) is calculated using $E_f = (E_{PBN} - 2E_{bulk} - 2E_{P} - 2E_{B} - 2E_{N})/6$, where $E_{PBN}$, $E_{P}$, $E_{B}$, and $E_{N}$ represent the energy of P–B, and N atoms at their bulk phase, respectively. The computed $E_f$ (1.16 eV/atom) indicates the feasibility of endothermic experimental synthesis of p-PBN. We have also performed formation energy calculations using the energy of atoms from the most stable possible binary (h-BN) and elementary (phosphorene) phases. The calculated $E_f$ of p-PBN using this approach is 1.10 eV/atom, which is equivalent to the results obtained using the bulk-phase approach. However, no binary or other ternary phases were included in the assessment of the thermodynamic stability. In addition, the lattice vibration and corresponding dynamical stability are verified with the presence of real phonon dispersion for the optimized p-PBN (Figure 1d). For the 6 atomic unit cell, 3 acoustic and 15 optical branches with high vibrational frequencies are observed, which are comparable to that of penta-graphene\textsuperscript{17} and graphene,\textsuperscript{16,47} indicating the relative dynamical stability. To verify the thermal response of p-PBN, the ab initio molecular dynamics (AIMD) simulation is performed at room temperature ($T = 300$ K), with a time step of 1 fs up to 4000 fs (Figure 1e). At this temperature, the structure remains intact without suffering any significant distortion, bond breaking, or melting, indicating robust thermal stability.

**Mechanical Properties and Anisotropy.** In low-dimensional materials, mechanical stability, strength, and anisotropy are the crucial factors for machinability, material synthesis, and diverse functionality. Keeping that in mind, we investigate the 2D Young’s/shear modulus and Poisson’s ratio along the x-direction (100) and y-direction (010) directions at different orientations of the lattice. In addition, to identify the anisotropy, important elastic anisotropic indices are evaluated. For this, the relevant second-order elastic constants ($C_{ij}$) are computed using the strain-energy method.\textsuperscript{10} In the standard Voigt notation,\textsuperscript{17,48} the strain energy per unit area as a function of strain $U_s(\varepsilon) = E_s(\varepsilon)/area$ for a 2D sheet is defined as

$$U_s(\varepsilon) = \frac{1}{2} C_11^{xy} \varepsilon_{xx}^2 + \frac{1}{2} C_{22}^{xy} \varepsilon_{yy}^2 + C_{12}^{xy} \varepsilon_{xy} \varepsilon_{yx} + 2C_{66}^{xy} \varepsilon_{xy}^2$$

where $C_{11}$, $C_{22}$, $C_{12}$, and $C_{66}$ are second-order elastic tensors, and their values are extracted by fitting the energy curves associated with the uniaxial ($\varepsilon_{x,x}$) and biaxial ($\varepsilon_{xy}$) strains.

The Li’s elastic anisotropy measurement methods\textsuperscript{49} are used to analyze the mechanical anisotropy. The needed anisotropy indices such as the universal $A^SU$, Ranganathan $A^{Ranganathan}$, and Kube $A^{Kube}$\textsuperscript{51} are expressed as

$$A^{SU} = \left( \frac{1}{4} (C_{11} + C_{22} + 2C_{12})(S_{11} + S_{22} + 2S_{12}) - 1 \right)^{1/2} + \frac{1}{4} (C_{11} + C_{22} + 2C_{12} - 2C_{66})$$

$$A^{Ranganathan} = \frac{K^V}{K^R} + \frac{2G^V}{G^R} - 3 \geq 0$$

and

$$A^{Kube} = \left( \frac{\ln K^V}{K^R} \right)^2 + \frac{\ln G^V}{G^R}$$

where $K^V$ and $K^R$ and $G^V$ and $G^R$ are area and shear moduli for Voigt and Reuss parameters, respectively, and expressed by the following relations\textsuperscript{59}

$$K^V = \frac{C_{11} + C_{22} + 2C_{12}}{4}$$

$$G^V = \frac{C_{11} + C_{22} - 2C_{12} + 4C_{66}}{8}$$

$$K^R = \frac{1}{S_{11} + S_{22} + 2S_{12}}$$

$$G^R = \frac{2}{S_{11} + S_{22} - 2S_{12} + S_{66}}$$

where $S_{ij}$ are the elements of the compliance matrix, which is the reciprocal of the $C_{ij}$ matrix.\textsuperscript{50}

For graphical visualization of the anisotropy, the Poisson’s ratio $\nu(\theta)$, Young’s $Y(\theta)$, and shear $G(\theta)$ moduli as functions of the direction angle ($0^\circ \leq \theta \leq 360^\circ$) are also calculated using\textsuperscript{62}

$$\nu(\theta) = \frac{C_{12}\cos^4(\theta) - B \cos^2(\theta)\sin^2(\theta) + C_{12}\cos^4(\theta)}{C_{22}\cos^4(\theta) + A \cos^2(\theta)\sin^2(\theta) + C_{11}\cos^4(\theta)}$$

$$Y(\theta) = \frac{C_{11}C_{22} - C_{12}^2}{C_{22}\cos^4(\theta) + A \cos^2(\theta)\sin^2(\theta) + C_{11}\cos^4(\theta)}$$

and

$$1\frac{1}{G(\theta)} = [S_{11} + S_{22} - S_{12}]\cos^2(\theta)\sin^2(\theta) + \frac{1}{4} S_{66}\cos^4(\theta) + \sin^4(\theta) - 2\sin^2(\theta)\cos^2(\theta)$$

where $A = (C_{11}C_{22} - C_{12}^2)/C_{66} - 2C_{12}$ and $B = C_{11} + C_{22} - (C_{11}C_{22} - C_{12}^2)/C_{66}$.

We start by applying uniaxial (longitudinal $\varepsilon_x$ and lateral $\varepsilon_y$) and equi-biaxial ($\varepsilon_{xy}$) strains. The parabolic strain energy $E_s$ profile (Figure 2) demonstrates that the strain-free state ($\varepsilon = 0$) is energetically favorable within $\pm 6\%$ range of applied strains. The wideness of these curves suggests that the impact of the strain follows the trend $E_x < E_y < E_{xy}$. Following the above formalism, the computed values of $C_{11}$, $C_{22}$, $C_{12}$, and $C_{66}$ are found to be 86.87, 136.83, $-20.51$, and 53.69 N/m, respectively (Table 1). The mechanical stability is...
approved by obeying the Born-Huang criteria,\(^5\)\(^5\) \(C_{11}C_{22} - C_{12}^2 > 0\), and \(C_{00} > 0\). This ensures the positive-definiteness of \(E_s(\varepsilon)\) followed by lattice distortion. The two most important mechanical parameters: 2D Young’s modulus \((Y)\) and Poisson’s ratio \((\nu)\) are investigated along the \((100)\) and \((010)\) directions. The 2D Young’s modulus for the \((100)\) and \((010)\) directions, \(Y_a = (C_{11}C_{22} - C_{12}C_{21})/C_{22}\) and \(Y_b = (C_{11}C_{22} - C_{12}C_{21})/C_{11}\), are 83.79 and 131.99 N/m, respectively, and \(Y_a\) is equivalent to that of \(p\)-SiCN.\(^7\) The significantly lower value of \(Y_a\) not only indicates the relatively softer plane but also demonstrates a large mechanical anisotropy (LMA), which is unique from other penta monolayers.\(^7\),\(^17\)

On the other hand, the Poisson’s ratio in the corresponding directions \((\nu_a = C_{12}/C_{22}\) and \(\nu_b = C_{12}/C_{11}\)) are \(-0.150\) and \(-0.236\), respectively, which also supports LMA. In addition, the absolute value of the shear modulus is \(G_{xy} = 53.69\) N/m, which is relatively smaller compared to that of penta-BCN.\(^10\)

To further corroborate, the anisotropy indices are calculated to get insights into the mechanical anisotropy (Table 2). The close to zero value of the anisotropy indices denotes the isotropic elastic response, while an increased value from that reference point indicates the degree of anisotropy. The high values of \(A_{50}^{0}\), \(A_{K}\), and \(A_{hub}\) indicate mechanical anisotropy, which is visualized by the polar plots (Figure 3). The orientation-dependent polar plots \((0° ≤ \theta ≤ 360°)\) for \(Y(\theta)\) and \(\nu(\theta)\) show the deformed circles, justifying the presence of LMA (Figure 3a,b). However, the orientation dependency of \(G(\theta)\) shows a quasi-anisotropic mechanical response as demonstrated by the relatively small distortion from a perfect circle as shown in Figure 3c.

The maximum value of \(Y_{a/b}\) and \(\nu_{a/b}\) for the monolayer is located along the \((010)\) \((90°)\), while that of \(G_{a/b}\) is located along \((110)\) \((45°)\). On the other hand, the minimum values of \(Y_{a/b}\), \(\nu_{a/b}\), and \(G_{a/b}\) are at \((100)\) \((0°)\), \((110)\) \((45°)\), and \((010)\) \((90°)\), respectively.

Even though the negative value of \(C_{12}\) leaves the early signature of NPR, it is crucial to verify the presence of NPR. Following refs \(^17\) and \(^55\), a longitudinal strain along the \(x\)-axis \((\varepsilon_x = 1, 3, \) and \(5\%)\) is applied, and the corresponding strain energy \((E_s)\) is observed as a function of the lateral strain along the \(y\)-axis \((\varepsilon_y)\). The increase in \(E_s\) and the shift of the equilibrium lattice constant with increasing \(\varepsilon_x\) confirm the presence of NPR (Figure 4). It is important to mention here that, while the theory of continuum mechanics does not preclude the possibility of inception of NPR in a stable 2D material, it is uncommon to find such rare NPR material,\(^17\) and our first-principles prediction of NPR in \(p\)-PBN is scarce. This exceptional NPR in \(p\)-PBN is attributed to the buckled and asymmetric geometrical structure, particularly the shorter and quasi sp\(^3\)-hybridized B–N bond and the rigidity against the strain (discussed below). This exciting finding makes the monolayer a superior candidate material for multiple applications such as a tension-activatable substrate, a nano-auxetic, and a deformable variable stiffness device.

Further, high strains are loaded in a \(4 \times 4 \times 1\) supercell of \(p\)-PBN along both the uniaxial \((\varepsilon_{x/y})\) and biaxial \((\varepsilon_{x/y})\) directions to identify the ultimate bond breaking, corresponding geometrical reconstruction, and elastic–plastic region. The tensile strain is loaded with full atomic relaxation until \(p\)-PBN completely breaks and is deformed. The variations of stress, strain-energy, bond length, and thickness (buckling height) as functions of the different modes of strains applied are shown in Figures 5 and 6. With increasing \(\varepsilon_{x/y}\) the stress increases monotonically, reaching the maximum value, the so-called ultimate stress \((\sigma_u)\) of 3.40 N/m at a corresponding strain \(\varepsilon_c = 6\%\). The stress then starts to drop with a higher strain \((8 \leq \varepsilon_x \leq 20\%)\), indicating a small elastic response of \(p\)-PBN. On the other hand, the strain energy grows exponentially until saturation at \(\varepsilon_s = 16\%.\) The variation of bond length and thickness \((h)\) against \(\varepsilon_s\) describes the corresponding geometrical reconstruction. In the range \(0 \leq \varepsilon_s \leq 6\%\), only the \(P1–B\) bond increases contrary to other bond lengths, including the \(B–P2\) bond. This converse variation of longitudinal bond \(P1–B\) and lateral bond \(B–P2\) supports the existence of NPR within 6% of strain. The small variation of the short covalent \(B–N\) bond against \(\varepsilon_s\) proves the strongest quasi sp\(^3\)-hybridized character. Furthermore, the regaining of bond length of \(P1–B\) and \(B–P2\) and the small bond variation in the range \(16\% ≤ \varepsilon_s\)

\[\text{Table 1. Elastic Constants } C_{ij}^{x,y,z} \ (\text{N/m}), \ 2D \text{ Young’s Modulus } Y_{a/b} \ (\text{N/m}), \text{ and Poisson’s Ratio } \nu_{a/b} \text{ from This Work and for Other Ternary Penta Monolayers}\]

| materials | \(C_{11}\) | \(C_{22}\) | \(C_{12}\) | \(C_{66}\) | \(Y_a\) | \(Y_b\) | \(\nu_a\) | \(\nu_b\) |
|-----------|----------|----------|----------|----------|--------|--------|--------|--------|
| \(p\)-PBN | 86.87    | 136.83   | -20.51   | 53.70    | 83.79  | 131.99 | -0.150 | -0.236 |
| \(p\)-SiCN| 132.15   | 133.59   | -17.44   | 74.80    | 129.88 | 131.29 | -0.131 | -0.132 |
| \(p\)-BNSi| 114.46   | 112.21   | 12.76    | 48.97    | 113    | 109    | 0.11   | 0.11   |
| \(p\)-BCN | 210.15   | 170.77   | 4.27     | 102.93   | 210.05 | 170.66 | 0.020  | 0.025  |
| \(p\)-CNP | 214.45   | 176.10   | 4.38     | 103.54   | 214.36 | 175.99 | 0.020  | 0.025  |
| \(p\)-CNP | 223.56   | 189.16   | 4.90     | 104.80   | 223.45 | 189.03 | 0.022  | 0.026  |
| \(p\)-CNP | 173.32   | 183.57   | 4.52     | 99.01    | 172    | 190    |        |        |

\[\text{Figure 2. (Color online) the strain energy } E_s (\text{eV}) \text{ as a function of different modes of strains } \varepsilon.\]
≤ 20% demonstrate the unique geometrical reconstruction and justify the stress drop and strain energy saturation. A similar stress and strain energy profile is observed when εy is applied. However, the Uy value is higher (5.79 N/m) as compared to Ux. On the other hand, all bonds seem to increase in length (except for P1–B). The continuation of contrasting bond length variation of P1–B and B–P2 within the elastic range approves the existence of NPR. A similar bond variation was detected in p-SiCN, which demonstrates the high value of NPR. In addition, when the equi-biaxial strain (εxy) is applied, a similar stress and strain energy profile with an inappreciable bond lengths variation is detected until εxy reaches the ultimate value. The corresponding ultimate stress (Uxy) is measured to be 6.56 N/m, which is relatively higher than Ux and Uy, indicating higher strength against εxy. Beyond Uxy notable bond length variations are observed. For the different strains applied, a clear drop in the monolayer thickness (h) is observed, with a drastic reduction clearly seen when the strain εxy is applied. At εxy = 20%, the thickness rockets to 1.95 Å, showing a complete deformation of p-PBN. Noting that, the ultimate stress and strain of p-PBN are relatively smaller as compared to other penta ternary monolayers. The deformed structures at higher loading of strains are shown in Figure 7. It suggests that p-PBN is in the best operable phase within 6% of stretching.

Electronic Properties. To get a deep insight into the electronic behavior of p-PBN, the electronic band structure and partial density of states (PDOSs) are computed for both spin-polarized and non-polarized configurations. The symmetry in both the spin-up and spin-down channels with zero magnetic moments confirms the non-magnetic ground state of the monolayer. The high-symmetry points along the Γ−X−M−Y−Γ direction of the first Brillouin zone are chosen for band calculations (Figure 8). Interestingly, the valence band maximum (VBM) and conduction band minimum (CBM) are entwined near the Fermi level (FL) between the X−M path, demonstrating the metallic behavior of p-PBN. This metallic feature is also confirmed when the hybrid functional HSE06 is applied. However, the Uc value is higher (5.79 N/m) as compared to Ux. On the other hand, all bonds seem to increase in length (except for P1–B). The continuation of contrasting bond length variation of P1–B and B–P2 within the elastic range approves the existence of NPR. A similar bond variation was detected in p-SiCN, which demonstrates the high value of NPR. In addition, when the equi-biaxial strain (εxy) is applied, a similar stress and strain energy profile with an inappreciable bond lengths variation is detected until εxy reaches the ultimate value. The corresponding ultimate stress (Uxy) is measured to be 6.56 N/m, which is relatively higher than Ux and Uy, indicating higher strength against εxy. Beyond Uxy notable bond length variations are observed. For the different strains applied, a clear drop in the monolayer thickness (h) is observed, with a drastic reduction clearly seen when the strain εxy is applied. At εxy = 20%, the thickness rockets to 1.95 Å, showing a complete deformation of p-PBN. Noting that, the ultimate stress and strain of p-PBN are relatively smaller as compared to other penta ternary monolayers. The deformed structures at higher loading of strains are shown in Figure 7. It suggests that p-PBN is in the best operable phase within 6% of stretching.

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employed. See Figure S1 in the online Supporting Information. In addition, the PDOS plot, as obtained by both Perdew–Burke–Ernzerhof (PBE) and HSE06 functionals, confirms that the B-2p, N-2p, and P-3p with B-2s electronic orbitals (sp³-
hybridized) contribute in descending order in the vicinity of the FL. Furthermore, both the uniaxial and biaxial strains are loaded to examine the metallicity. Nevertheless, only minor shifting of the VBM and CBM from the FL is observed, suggesting the rigid metallicity of $p$-PBN. This remarkable electronic response was also detected in the auxetic $p$-SiCN monolayer, but it is different from other penta iso-structures. This rigid metallicity is highly desirable for the fabrications of electrode materials in high-performance lithium-ion batteries and high-quality electronic devices.

**CONCLUSIONS**

In summary, using first-principles density functional theory (DFT) calculations, a new pentagonal monolayer $p$-PBN is predicted, which is structurally, chemically, mechanically, dynamically, thermodynamically stable, and experimentally feasible. The new $p$-PBN exhibits metallic behavior, which is preserved even under high strains. Remarkably, the short and strong quasi $sp^3$-hybridized B–N bond and unique bond variation and geometrical reconstruction with applied strain allow $p$-PBN to possess a high NPR value, the highest among all the ternary penta iso-structures. The large difference between the (100) and (010) directions' Young's modulus and Poisson's ratio values demonstrate the large anisotropic mechanics of the monolayer. Nevertheless, because of the unique geometrical reconstruction against the strain, the monolayer is sustainable and operable up to 6% of stretching. Our findings not only introduce a new ternary monolayer but also pave the way for future possibilities of experimental fabrication and characterization of modern nanoscale electro-mechanical devices based on auxetic nanomaterials and nanostructures.

**COMPUTATIONAL METHODS**

In this work, the DFT implemented in the Spanish Initiative for Electronic Simulations with Thousands of Atoms (SIESTA)\(^{36,37}\) is used for electronic structure calculations.
The semi-local forms of norm-conserving pseudopotentials\(^5\) are utilized to determine the interaction between ions and electrons. The generalized gradient approximation of PBE\(^9\) within the double zeta plus polarization basis sets, is used to determine the exchange-correlation potential. A 20 × 20 × 1 k-points within the Monkhorst pack scheme,\(^6\) and 350 Rydberg cutoff energy are set to meet the convergence criteria. The atomic force of 0.002 eV/Å and self-consistent field of 10\(^−6\) eV are set for tight convergence within the conjugate-gradient scheme. In addition, a vacuum-gap separation of 25 Å is used along the z-axis of the monolayer to avoid unnecessary interlayer interactions. The chemical stability and strength are checked by analyzing the cohesive \(E_c\) and formation \(E_f\) energies. The frozen phonon method, implemented in the Vibra utility of SIESTA, is used for the phonon dispersion calculation and dynamical stability test. A large supercell of 6 × 6 × 1 is converged to obtain an accurate force matrix. Furthermore, to account for the thermal stability, the Viena Ab-initio Simulation Package\(^6\) is utilized to perform the AIMD simulations with a 6 × 6 × 1 supercell used for convergence.

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**ASSOCIATED CONTENT**

Supporting Information

The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acsomega.2c03567.

Lattice parameters and coordinates of the optimized geometry for p-PBN in the POSCAR format and the PDOSs confirming the metallic behavior of p-PBN as obtained using the HSE06 functional (PDF).

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Notes

The authors declare no competing financial interest.

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