Peak Effect and the Transition from Elastic to Plastic Depinning

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We demonstrate for the first time an observation of the peak effect in simulations of magnetic vortices in a superconductor. The shear modulus $c_{66}$ of the vortex lattice is tuned by adding a fictitious attractive short range potential to the usual long-range repulsion between vortices. The peak effect is found to be most pronounced in low densities of pinning centers, and is always associated with a transition from elastic to plastic depinning. The simulations suggest in some situations that over a range of values of $c_{66}$ the production of lattice defects by a driving force enhances the pinning of the lattice.

One of the most important aspects of superconducting materials is their behavior in magnetic fields. In Type II superconductors, magnetic fields induce the formation of quantized vortices through which magnetic flux may penetrate the system. The application of a current in the presence of vortices generates an effective force that causes them to flow through the superconductor, thereby dissipating energy and spoiling perfect conductivity. Much work, both theoretical and experimental, has been devoted over the years to finding and understanding mechanisms that pin the vortices, so that the superconductor will remain dissipationless even in the presence of the field. In the presence of pinning centers, the superconductor displays a critical current $j_c$ above which dissipation sets in; physically this current is proportional to a critical force $F_p$ at which the vortices become depinned and can move through the system.

The competition between intervortex interactions and pinning by disorder results in a surprising and long studied phenomenon known as the “peak effect”. As the critical field or critical temperature at which a sample loses its superconducting properties is approached, in many situations one observes an enhancement of the superconductivity just before it is completely suppressed. Thus, $j_c$ exhibits a peak as a function of field or temperature, just before it vanishes. The earliest understanding of this phenomenon, the collective pinning theory, involves the softening of the elastic moduli of the vortex lattice as the superconducting order is suppressed, so that the vortices may settle more deeply into the pinning potential and thus become more difficult to dislodge. One difficulty with collective pinning is that tearing of the lattice is ignored in estimating $j_c$: i.e., the lattice depins elastically, not plastically. In recent years this assumption has been increasingly questioned in the peak effect regime, particularly for high-purity superconductors (e.g., NbSe\textsubscript{2}) and systems with strong pinning centers. Although there is accumulating experimental evidence of plastic motion in the peak effect regime, its precise effect on the size of $j_c$ is not known, and there is disagreement as to whether $j_c$ is enhanced or suppressed by the onset of plastic motion.

In principle much of this debate could be settled by direct imaging of the vortices near the depinning critical current. However, in the peak effect regime such experiments are exceedingly difficult because the order parameter is suppressed near the critical temperature or critical magnetic field. Numerical simulations thus offer a unique window through which one may view the qualitative behavior of the vortices. In this work, we present results demonstrating for the first time (albeit, in a two-dimensional geometry) the peak effect in a simulated vortex system, show conclusively that it is associated with a crossover from elastic to plastic motion, and find that under different circumstances lattice tearing may enhance or suppress $j_c$.

In recent years, the peak effect has been associated with the proximity of the vortex lattice to a melting transition. Direct simulations of depinning in this situation pose enormous practical problems because near a critical point one inevitably has large thermal fluctuations. To circumvent this problem, we take note that in nearly every theoretical approach to the peak effect, it is not actually melting itself but rather the softening of $c_{66}$ and other elastic moduli near the melting transition that is responsible for the effect. We thus consider a system of vortices in which the interaction may be varied so that the elastic properties of the system may be tuned directly, without the introduction of critical fluctuations. To simulate a large number of vortices ($N_V = 1600$) we confine ourselves to two-dimensional systems, so that our simulations are most directly applicable to very thin films or superconductors consisting of effectively decoupled layers. In contrast to previous simulations, we focus on pinning centers that are dilute compared to the vortex density, which is most appropriate for systems with strong pinning centers.

The precise form of the intervortex interaction we use is
where $\vec{R}$ are the position vectors of vortices, $e^2$ is the strength of the logarithmic interaction, and $A_v$ and $\xi_v$ are \textit{the} strength and the range of a short range attractive interaction. For large enough values of $A_v$ the interaction may in principle be attractive over a range of vortex separations; however, in all the simulations we report here, $A_v$ is small enough that the net interaction is repulsive at all distances. A uniform background is assumed to cancel out the diverging energy due to the logarithmic interaction. Because of the long-range potential, the bulk modulus is formally divergent, while the shear modulus $c_{66}$ may be shown to have the form
\begin{equation}
\begin{aligned}
c_{66} &= n_v \left\{ \frac{e^2}{8} - \frac{A_v}{2} \sum_{\vec{R}} \left[ \frac{1}{\xi_v^2} \right] R^4 \right\} - \left( \frac{R}{\xi_v^2} \right)^2 e^{-R^2/\xi_v^2} 
\end{aligned}
\end{equation}
where $n_v = 2/(\sqrt{3}a_0^2)$ is the density of vortices. We take $\xi_v = 0.5a_0$ so that the shear modulus has values $c_{66} = (n_v/8)(e^2 - 1.768A_v)$, and tune $c_{66}$ by varying $A_v$. The vortices interact with the pinning centers through the potential:
\begin{equation}
\begin{aligned}
H_{\text{pin}} &= -A_p \sum_{\vec{R}, \vec{r}} e^{-|\vec{R} - \vec{r}|^2/\xi_p^2} 
\end{aligned}
\end{equation}
where $A_p$ is \textit{the} strength of the pinning centers and $\vec{r}$ are \textit{the} positions of the pinning centers.

Several different choices of $\xi_p$, $A_p$, and $N_p$, the number of pinning centers. In our simulations, we find that the peak effect is most pronounced when the density of pinning centers is small compared to the number of vortices, so we focus our attention on simulations with $N_p = 50$ and 100. For increasing values of $c_{66}$, we expect the number of pinned vortices to decrease, as in collective pinning. This general trend is confirmed in the inset of Fig. 1, which illustrates the fraction of occupied pinning sites $P_{\text{occupied}}$, defined as the fraction of sites for which a vortex may be found within $\xi_p/\sqrt{2}$ of the pinning site center. When the lattice depins elastically (i.e., when tearing and defect formation may be ignored), one expects the threshold depinning force to be proportional to the density of occupied pinning sites and the maximum pinning force that a site may exert, $f_{\text{max}} = (A_p/\xi_p)\sqrt{2}e^{-1/2}$. In the main part of Fig. 1, $f_{\text{max}}$ has been scaled out of $F_p$, and two curves proportional to $P_{\text{occupied}}$ are plotted for the $N_p = 50, 100$ data with the proportionality constants chosen to match the threshold force for the largest values of $c_{66}$, where the depinning is most elastic. As may be seen, the pinning force matches the expectations for elastic depinning reasonably well down to $c_{66}/A_p n_v \approx 2.0$. For $N_p = 100$, $F_p$ decreases monotonically to zero as $c_{66} \to 0$, whereas for $N_p = 50$ there is a clear tendency for $F_p$ to overshoot the elastic depinning estimate before dropping to zero. This non-monotonic behavior of $F_p$ vs. $c_{66}$ is the peak effect, and one of the surprising results of this study is that it is more pronounced for lower densities of pinning centers. This latter result is in qualitative agreement with experiment, for which samples that are more weakly disordered exhibit stronger peak effects.

The deviations from the elastic depinning estimate occur because for small values of $c_{66}$, the lattice easily deforms and tears, allowing motion without forcing out all the vortices trapped in pinning sites. Fig. 2 illustrates the trajectories of the vortices for large, intermediate, and small values of $c_{66}$. For the largest values, the vortex lattice largely retains its order as it depins. As the maximum of the peak in $F_p$ is approached, a crossover from elastic to more plastic motion is observed, in which dynamically changing channels form where vortex motion takes place. The width of these channels decrease with decreasing $c_{66}$. This behavior is reminiscent of plastic motion observed in superconductors with ordered arrays of pinning centers \cite{7} and in simulations of disordered Wigner crystals \cite{23}. As the falling edge of the peak effect is entered, a new qualitative behavior emerges in which the active channels of motion of the vortices are no longer dynamic, and the motion becomes very much like river flow \cite{24}.

Scenarios in which the peak effect is associated with a crossover from elastic to plastic motion have been advocated by several groups in the last few years \cite{23,22,4}. The present simulations strongly support this viewpoint, although the precise evolution of the flows with $c_{66}$ dif-
fers in some important aspects from what previously has been supposed. In particular the onset of plastic motion in the peak effect regime has been thought to be associated with either a monotonically increasing \( F_p \) or decreasing \( F_p \) with decreasing \( c_{66} \). Our simulations demonstrate that in a sense both scenarios are true. Plastic flow, when it first sets in with decreasing \( c_{66} \), is associated with an increasing critical current. This is particularly true for very low densities of pinning centers, for which \( F_p \) is enhanced by tearing \[9,11\]. For low enough values of \( c_{66} \), however, river flow motion sets in and \( F_p \) becomes proportional to \( c_{66} \). The latter behavior is quite sensible once one recognizes from the simulations that the motion of vortices for the smallest values of \( c_{66} \) correspond to river flow through channels that do not change dynamically. In this case the depinning force comes about due to interactions of the “rivers” (moving vortices) with the “river banks” (stationary vortices), whose ability to hold the rivers in place decreases with decreasing shear modulus.

A useful way of characterizing the depinning force for the smallest values of \( c_{66} \) (the “static river” limit) is to assume that if few of the vortices trapped in pinning sites are pulled free by the depinning driving force, then the only relevant length scale in the system at the depinning transition is \( d \propto (n_v/n_p)^{1/2} \), the average distance between pinning sites. In particular this implies a characteristic displacement scale for the lattice in the presence of the driving force \( u_{max} \equiv r_c d \), with \( r_c \) a unit-less constant, above which defects are produced so that the lattice becomes depinned. The work done by the driving force must provide the elastic deformation energy just before defects and depinning set in, so that \( F_p n_v r_c d = c_{66} n_v d^2 \). \( F_p n_v \propto c_{66} (n_v/n_p)^{1/2} \). Fig. 3 illustrates this scaling relation, and one may clearly see a collapse of the data onto a single straight line for \( c_{66}/A_p n_v < 1.0 \). The collapse of the data indicate that \( d \) is indeed the only relevant length scale in the static river flow regime. We note finally that in our limit of dilute pinning centers, river flow motion is possible for an unmelted vortex system \((c_{66} > 0)\), in contrast to what has been speculated for systems with dense pinning centers \[16\].

In summary we have reported the first simulations of the peak effect in a vortex lattice. We observe a peak in the depinning force near the smallest values of \( c_{66} \), demonstrate with particle trajectories that this peak is associated with a crossover from elastic to plastic motion, and find that the peak is most pronounced for low pinning center densities.

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Note added: After the submission of this work, a publication \[26\] appeared reporting experiments on Nb using neutron scattering to measure correlation lengths of a vortex lattice. It was found that in the peak effect regime, the correlation length corresponding to shear displacements decreases monotonically to a minimum through the rising edge of the peak effect. This observation corroborates our finding that the softening of \( c_{66} \) may be the controlling parameter in the peak effect.

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FIG. 1. Depinning force $F_p$ as a function of shear modulus $c_{66}$ for $N_V$ vortices and $N_p$ pinning centers of strength $A_p$ and range $\xi_p$. The maximum possible force $f_{\max}$ and the number of pinning sites per vortex $N_p/N_V$ are scaled out so that data should collapse onto a single curve if the lattice depins elastically. Solid and dotted lines illustrate the expected behavior for elastic depinning. Inset: Fraction of occupied pinning sites $P_{\text{occupied}}$ for the groundstate configurations found by simulated annealing.

FIG. 2. Trajectory plots for depinned vortices at different values of $c_{66}$, illustrating the evolution from elastic to plastic motion. Crosses represent the locations of pinning centers. $A_p = 0.03 e^2$, $N_V = 1600$, and $N_p = 50$, and (a) $c_{66}/(A_p n_v) = 4.17$; (b) $c_{66}/(A_p n_v) = 1.96$; (c) $c_{66}/(A_p n_v) = 0.48$.

FIG. 3. Depinning force $F_p$ times inter-pinning-center spacing as a function of shear modulus $c_{66}$. The symbols represent parameters as used in Fig. 1. The figure clearly shows that in the region where $c_{66}$ is small $F_p$ is proportional to $c_{66}$, and that the average distance between pinning centers is the only relevant length scale for “static river flow” depinning (see text).
