Identification of Linear Time-Variant Systems Based on Ensemble Empirical Mode Decomposition and Hilbert Transform

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Abstract. This paper proposes an identification method based on ensemble empirical mode decomposition (EEMD) and Hilbert transform. The main procedures are: First, the system responses of an \( n \) degrees of freedom (DOFs) system are decomposed into intrinsic mode functions (IMFs) and residues by EEMD. The obtained ensemble mean IMFs and residues are then analysed by the Hilbert Transform to form the corresponding analytical signals. Second, the analytical signals of the \( n \) IMFs and the residue of each system response of each DOF are added to form a new analytical response. Third, the system equation of motion is rewritten into complex matrix equations for parameter identification. Numerical simulations are carried out on multi-DOFs systems with varying system parameters to demonstrate good robustness, effectiveness and accuracy of the proposed method, which attempts to offer a new means to study identification for linear time variant systems.

1. Introduction
In virtue of the prolonged impact of environmental degradation and excessive load in operation of high-rise and super high-rise buildings, the structural materials perish over time and structural damages usually occur in local areas, which could unavoidably result in performance degradation, or even structural failures after continuous accumulation, possibly causing disasters with huge casualty and property losses. Hence, it is quite significant to implement structural health monitoring (SHM) of the building structures, so that structural damages could be detected in time, structural safety could be secured, and early warnings about probable dangers and unwanted anomalies of structures could be issued [2].

As the core of SHM, damage identification technology refers to the methods and technologies of identifying the presence, location, severity and type of the damage. Among the various damage identification methods, signal processing techniques are indispensable means for SHM. This paper
proposes an adaptive signal processing technique based on Hilbert-Huang transform (HHT), and linear time-variant multi-degrees-of freedom (MDOF) systems with system parameters having smooth, abrupt and periodical variations are investigated by numerical simulations for demonstration of the proposed method.

2. Hilbert-Huang Transform

2.1. Empirical Mode Decomposition
Huang et al. first proposed Hilbert-Huang transform (HHT) which combines the adaptive empirical mode decomposition (EMD) with Hilbert transform (HT) for nonlinear and non-stationary signal analysis, in which EMD method was used to decompose general signals into intrinsic mode functions (IMFs) that are supposed to satisfy two conditions: first, the number of extrema and the number of zero crossings of an IMF must either be equal or differ at most by one; and second, the mean value of the local maxima defined envelope and the local minima defined envelope at any point is zero [1]. By application of EMD, a system response of an n-DOF system is given by

\[ z(t) = \sum_{j=1}^{n} z_j(t) + r(t) \]  

in which \( z(t), j=1, 2, \ldots, n \) denote the IMFs of the system response, \( r(t) \) denotes the residue. The obtained IMFs are then analyzed by HT.

EMD has proven to be a universal local and adaptive time-frequency data analysis method in a wide variety of fields for signal extraction of nonlinear and non-stationary data even with the existence of noise [3]. Even though it is so powerful, there still exist some unresolved difficulties in application of EMD. The most significant drawback appears to be the frequent occurrence of mode mixing caused by signal intermittency that is defined when an IMF consists of signals with different frequency components, or a signal having a similar frequency component appears in diverse IMFs. Mode mixing can be avoided by using the intermittence test [4]. However, the intermittence test also has limitations: first, since it is based on a pre-assumed scale, the EMD applying the intermittence test becomes no longer totally adaptive; second, if the timescales in the signal are not clearly separable and definable, the intermittence test usually does not work very well.

2.2. Ensemble Empirical Mode Decomposition
For avoidance of the mode mixing without using the intermittence test, a Noise-assisted data analysis (NADA) method-Ensemble Empirical Mode Decomposition (EEMD) [5] has been developed by Wu and Huang, of which the basic principle is: add different white noise series to the signal to be decomposed, since the white noise series would distribute uniformly in the whole time-frequency space, the components of the signal with different scales will be automatically projected on proper reference scales set by the white noise series. By application of EEMD, the ensemble mean of IMFs can be obtained

\[ z_k(t) = \lim_{l \to \infty} \sum_{i=1}^{l} \frac{z_{ki}(t)}{l} \]  

in which \( z_{k}(t) \) are the \( k \)th IMF obtained by decomposing the signal added with the \( i \)th white noise serie, \( i=1, 2, \ldots, l \) is the index of white noise series, and \( l \) is the ensemble number.

Since the white noise series generated in different trials have no correlation with each other, they cancel each other out in the ensemble means of enough trials, only the IMF components of the noiseless signal is capable to persist in the final ensemble means. The finally obtained ensemble means of IMFs of the noise-added signal are treated as the true answers. By utilizing the statistical characteristics of white noise, mode mixing is avoided by EEMD in all cases automatically. Wu and Huang also an EEMD post-processing algorithm to guarantee that the obtained IMF components satisfy the two conditions of IMFs.

2.3. Hilbert Transform
After the process of EEMD and post-processing procedures, a signal is decomposed into \( n \) IMFs suitable for HT. For an IMF \( z_k(t) \), the HT of \( z_k(t) \) is given by
\( \tilde{z}_k(t) = \text{HT}[z_k(t)] = P \int_{-\infty}^{\infty} z_k(\tau) / (t-\tau) \, d\tau / \pi \) \hfill (3)

where \( P \) denotes the Cauchy principal value. The complex conjugate pair \( \tilde{z}_k(t) \) and \( z_k(t) \) can form an analytical signal \( Z_k(t) \) by

\[ Z_k(t) = z_k(t) + i\tilde{z}_k(t) = A_k(t) \exp[i\psi_k(t)] \] \hfill (4)

in which

\[ z_k(t) = A_k(t) \cos[\psi_k(t)] \] \hfill (5)

\[ A_k(t) = \sqrt{z_k^2(t) + \tilde{z}_k^2(t)} \] \hfill (6)

\[ \psi_k(t) = \arctan[\tilde{z}_k(t) / z_k(t)] \] \hfill (7)

where \( A_k(t) \) denotes the instantaneous amplitude, \( \psi_k(t) \) denotes the instantaneous phase angle, and non-subscript \( i = (-1)^{1/2} \). The instantaneous frequency \( \omega_k(t) \) is defined by

\[ \omega_k(t) = \dot{\psi}_k(t) = [z_k(t) \tilde{z}_k(t) - \tilde{z}_k(t) z_k(t)] / A_k^2(t) = \text{Im}[\dot{Z}_k(t) / Z_k(t)] \] \hfill (8)

The time-derivative of \( A_k(t) \) can be calculated by

\[ \dot{A}_k(t) = [z_k(t) \ddot{z}_k(t) + \tilde{z}_k(t) \dot{\tilde{z}}_k(t)] / A_k(t) = A_k(t) \text{Re}[\dot{Z}_k(t) / Z_k(t)] \] \hfill (9)

Equations (3) to (9) show the instantaneous dynamic properties of the ensemble mean IMF \( z_k(t) \) at any time \( t \).

### 3. EEMD and HT based identification

In this section, an EEMD and HT based method is proposed for damage identification of linear time-variant multi-degrees-of-freedom (MDOF) systems.

The equation of motion of forced vibration of a time-variant MDOF system is given by

\[ M(t)\ddot{z}(t) + D(t)\dot{z}(t) + K(t)z(t) = u(t) \] \hfill (10)

in which \( M(t) = [m_{ij}] \), \( D(t) = [d_{ij}] \), \( K(t) = [k_{ij}] \) represent \( n \times n \) mass, damping and stiffness matrices, \( u(t) \) represents excitation vector.

The damage identification method can be carried out as follows:

1. For each system response of (10), given the standard deviation of added white noise \( w \) and the ensemble number \( l \), EEMD and its post-processing method are applied to decompose the system response, obtaining \( l \) ensemble means of IMFs;

2. By application of Bedrosian’s theorem and HT with the assumption that the system matrices do not vary quickly over time, we have

\[ H[M(t)\ddot{z}(t)] = M(t)\ddot{z}(t) \]
\[ H[D(t)\dot{z}(t)] = D(t)\dot{z}(t) \]
\[ H[K(t)z(t)] = K(t)z(t) \] \hfill (11)

Then, HT is applied for both sides of (10) with the help of (11), and (10) can be represented by analytic signals as

\[ M(t)\ddot{Z}(t) + D(t)\dot{Z}(t) + K(t)Z(t) = U(t) \] \hfill (12)

in which the analytic signal of the displacement vector is given by \( Z(t) = [Z_1(t), \ldots, Z_n(t), \ldots, Z_m(t)]^T \) with the \( i \)th element calculated by \( Z_i(t) = \sum_{i=1}^{m} Z_{0i}(t) + R_i(t) \), and \( U(t) \) denotes the analytic signal of the excitation vector.

3. With the help of the system responses and the above obtained analytic signals, (12) can be simplified and rewritten as follows

\[ \text{Re}[P(t)\beta(t) + Q(t)\gamma(t)] = \text{Re}[U(t) - M(t)O(t)] \] \hfill (13)

\[ \text{Im}[P(t)\beta(t) + Q(t)\gamma(t)] = \text{Im}[U(t) - M(t)O(t)] \]

where
\[ \beta(t) = [k_{11}, \ldots, k_{in}, \ldots, k_{nn}, \ldots, k_{nm}] \]  
\[ \gamma(t) = [d_{11}, \ldots, d_{in}, \ldots, d_{nn}, \ldots, d_{nm}] \]  
\[ P(t) = \begin{bmatrix} (Z(t))^T & 0 \\ 0 & (Z(t))^T \end{bmatrix} \]  
\[ Q(t) = dP(t) / dt \]  
\[ O(t) = d^2Z(t) / dt \]

4. Solve Equation (13) for each time instant \( t \) of the required time period, the identified values of unknown parameters are obtained.

4. Numerical simulations

The method is adopted in damage identification of linear time-variant 2-DOF systems with system parameters having smooth, abrupt and periodical variations for demonstration of the method. The systems at time \( t=0s \) are regarded as undamaged systems, whereas those having smoothly, abruptly and periodically varying system parameters are regarded as systems with different damage modes. The time step between adjacent data points in the simulations is specified as 1/2048s with time period specified as 2s. Mass parameters are assumed as \( m_1 = m_2 = 1kg \). Each of the simulated systems is excited at mass 1 by excitation \( u_1(t) = 200\cos(30\pi t)N \), and is excited at mass 2 by excitation \( u_2(t) = 200\cos(15\pi t)N \).

Simulations on systems with different time-variant stiffness parameters are conducted as follows:

1. System with smoothly varying parameters: stiffness parameter \( K_1 = 5000-100t \) N/m, damping parameter \( E_1 = 0.8+0.05t \) Ns/m; other parameters are given as: \( K_2 = 4000 \) N/m, \( E_2 = 1 \) Ns/m for any \( t \).

2. System with an abruptly varying stiffness parameter: \( K_1 = 5000 \) N/m when \( t < 1s \); \( K_1 = 4900 \) N/m when \( t \geq 1s \); other parameters are given as constants: \( K_2 = 4000 \) N/m, \( E_1 = 0.8 \) Ns/m, \( E_2 = 1 \) Ns/m for any \( t \).

3. System with periodically varying stiffness parameters: \( K_1 = 5000+50\sin(\pi t) \) N/m, \( K_2 = 4000+50\sin(\pi t) \) N/m; damping parameters are given as constants: \( E_1 = 0.8 \) Ns/m, \( E_2 = 1 \) Ns/m for any \( t \).

The proposed EEMD method and HT based method is applied to the simulations of the above time-variant systems under multi-point harmonic excitations.

The identified results of the simulations are presented in Figs. 1-3.

![Fig. 1. Parameters of a 2-DOF linear smoothly varying system: (a) True value and identified values of stiffness parameter \( K_1 \); (b) True value and identified values of damping parameter \( E_1 \).](image-url)
For simulations of 2-DOF linear smoothly varying and periodically varying systems, the stiffness parameters obtained by applying the proposed damage identification method almost overlap with their corresponding true values, while the identified damping parameter of the smoothly varying system has small fluctuations around its true value. For simulation of a 2-DOF linear abruptly varying system, the identified stiffness parameter $K_1$ has large identification errors around $t = 1s$, which is caused by the sudden change of the system stiffness parameter at $t = 1s$, implying the bad capability of the proposed method to identify the abrupt variation of the system parameters due to the assumption of (11). At other time instants, the identified values of $K_1$ are close to their true values.

The proposed EEMD and HT based method is then adopted to the simulations of the above time-varying systems under multi-point excitations with consideration of different signal-to-noise ratio values, the mean absolute percentage error (MAPE) values of the identified results is shown in Table 1.

Table 1: MAPE values of the identified results of system parameters of linear time-varying systems for different signal-to-noise ratio values

| SNR | Smoothly varying system | Abruptly varying system | Periodically varying system |
|-----|-------------------------|-------------------------|---------------------------|
|     | $K_1$ (%) | $E_1$ (%) | $K_1$ (%) | $K_2$ (%) | $K_1$ (%) | $K_2$ (%) |
| $+\infty$ | 0.010 | 1.254 | 0.021 | 3.351E-04 | 3.50E-04 |
| 100 | 0.288 | 18.488 | 0.159 | 0.153 | 0.028 |
| 90 | 0.320 | 20.507 | 0.175 | 0.170 | 0.031 |
| 80 | 0.360 | 23.035 | 0.196 | 0.192 | 0.035 |
| 50 | 0.575 | 36.725 | 0.309 | 0.307 | 0.056 |
Table 1 shows that the identified results obtained by applying the proposed identification method under different SNR values are close to the true values of the corresponding structural parameters. For 2DOF forced vibration systems with linear smooth, sudden and periodic changes, identified results of stiffness parameters obtained by the proposed method have very small MAPE values. Even in the case of SNR=50, the MAPE values of the identified stiffness parameters still do not exceed 0.6%, which indicates that the proposed identification method has high identification accuracy for stiffness parameters. In the case of a large signal-to-noise ratio, good identified result of the damping parameter $E_1$ is obtained with small MAPE value (1.254%). As the value of signal-to-noise ratio continues to decrease, the MAPE value of the identified damping parameter $E_1$ keeps increasing, indicating that the damping parameter is very sensitive to noise.

The above results demonstrate that the proposed EEMD and HT based method is capable of identifying 2DOF linear time-varying forced vibration systems with high accuracy, effectiveness and robustness.

5. Conclusion
This paper proposes an EEMD and HT based method for identification of multi-DOF linear time-variant systems under multi-point excitations. 2-DOF linear time-variant systems having system parameters with smooth, abrupt and periodical variations are investigated by numerical simulations with consideration of noise perturbations in system responses. Structural parameters of the above systems are well identified, their MAPE values under different signal-to-noise ratio values are collected and compared, which demonstrate that the proposed identification method has good accuracy, robustness and effectiveness for 2-DOF linear time-variant systems.

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