MAXIMIZING RELIABILITY OF THE CAPACITY VECTOR FOR MULTI-SOURCE MULTI-SINK STOCHASTIC-FLOW NETWORKS SUBJECT TO AN ASSIGNMENT BUDGET

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Abstract. Many real-world networks such as freight, power and long distance transportation networks are represented as multi-source multi-sink stochastic flow network. The objective is to transmit flow successfully between the source and the sink nodes. The reliability of the capacity vector of the assigned components is used an indicator to find the best flow strategy on the network. The Components Assignment Problem (CAP) deals with searching the optimal components to a given network subject to one or more constraints. The CAP in multi-source multi-sink stochastic flow networks with multiple commodities has not yet been discussed, so our paper investigates this scenario to maximize the reliability of the capacity vector subject to an assignment budget. The mathematical formulation of the problem is defined, and a proposed solution based on genetic algorithms is developed consisting of two steps. The first searches the set of components with the minimum cost and the second searches the flow vector of this set of components with maximum reliability. We apply the solution approach to three commonly used examples from the literature with two sets of available components to demonstrate its strong performance.

1. Introduction. Many real-life systems such as computer systems, telecommunication systems, urban traffic systems and logistics systems as are treated as multi-source multi-sink flow networks. As an example of a transportation network, assume that two factories wish to ship their products to two locations. Determining how to ship the products with maximum reliability and minimum budget can be treated as multi-source multi-sink flow problem under budget constraint. Locations correspond to nodes in the graph and roads connecting the locations correspond to arcs. The number of trucks and their size correspond to component capacities, and the shipping prices correspond to component costs.

The Components Assignment Problem (CAP) in a single-source single-sink stochastic flow network (SFN) was found by [16], they studied the problem of searching the optimal components to be assigned to the network for maximizing reliability. In [20], the system reliability of an SFN is optimized by searching the optimal resource to be assigned by using an approach based on a genetic algorithm (GA). In the case of the two resource types, transmission line and transmission facility,
existing, Lin and Yeh [21] stated the problem as a double-resource assignment problem and proposed an optimization approach solution. In [19], they developed an approach based on the GA to solve the CAP with optimal reliability under the assignment budget constraint. The Network Reliability-based Transmission Line Assignment problem under the Budget constraint (NRTLAB), studied in [17], was solved through their developed optimization algorithm that integrates the GA, Minimal cuts (MCs), and the Recursive Sum of Disjoint of Products (RSDP) [25], called the MCRSDP-GA.

Considering the computer network arcs and nodes, Lin and Yeh [15] addressed the problem as a double-component assignment with maximal system reliability. An approach integrating the Non-dominated Sorting Genetic Algorithm II (NSGA-II) and the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) to solve CAP was proposed by [18]. The problem of searching the minimum capacity assignment of each edge in the capacitated flow network was addressed by Chen [2] who defined a robust design problem and an exact algorithm for a proposed solution. The CAP was formulated as a multi-objective minimization problem and solved by a proposed approach based on the NSGA-II by [13], and then solved by a hybrid ant-tabu (HAT) algorithm to maximize the reliability of an SFN by [22].

In the case of each link (vertex) having several capacities or states, Chen [3] proposed an algorithm to solve the optimal double-resource assignment problem. Lin and Yeh [14], introduced an optimization algorithm based on a GA to solve the two-class (transmission lines and facilities) allocation problem subject to a budget. In the case of considering the total lead-time of the assigned components, Hassan [8] studied the CAP subject to total-lead time constraint and solved it as a multi-objective optimization problem in [7]. In the case of each component having an assignment cost and lead-time, Aissou et al. [1] proposed an approach based on a random weighted genetic algorithm (RWGA) to maximize the system’s reliability subject to these constraints.

In multi-source multi-sink SFN, the problem of allocating various resources at the source nodes to maximize reliability is studied in [12] who developed an algorithm to solve it. In the case of considering predetermined transmission cost constraints, the resource allocation problem was studied and solved by [10]. In an unreliable multi-source multi-sink flow network, when the resource demand or the characteristic of the flow network changes, Haish and Lin in [11] proposed updating schemes instead of recomputing schemes to take advantage of existing minimal path vectors and corresponding flow patterns to reduce effort as well as to incorporate the efforts made in obtaining the existing resource allocation. The flow assignment problem in the multi-source multi-sink SFN was studied by [23] and [21], while in [9] studied the flow assignment problem subject to the system’s reliability maximization and solved with a proposed GA. Liu et al. [24] studied the flow assignment problem subject to the transmission cost by maximizing the reliability of the capacity vector.

The CAP in single-source single-sink stochastic flow networks was studied and solved using both single- and multi-object GAs. Also, the flow assignment problem in multi-source multi-sink stochastic flow networks with multiple commodities was studied in the literature. However, the CAP in multi-source multi-sink SFNs was not discussed in the previous literature, so the current study investigates the CAP for this network type under an assignment budget constraint to maximize the reliability of the capacity vector where each component has a unit cost. The challenge
is comprised of two issues. The first is to search the minimum cost components assigned to a given network. The second is to search the flow vector that corresponds to the assigned components to calculate the reliability of the capacity vector. The best flow vector is characterized by the maximum reliability of its corresponding capacity vector.

This paper is organized as follows. Section 2 presents notations and assumptions. Section 3 investigates the mathematical formulation of the problem. Section 4 describes the components of the proposed approach. Section 5 provides the pseudo-code of the proposed approach. Section 6 demonstrates illustrative examples, and Section 7 offers our conclusions.

2. Notations and assumptions.

2.1. Notations. A \{a_e | 1 \leq e \leq n\}, the set of arcs.
\(N\) The set of nodes.
\(M = \{M_1, M_2, \ldots, M_n\}\), where \(M_e\) is the maximum capacity of each arc \(a_e\) and \(M_e\) is an integer.
\(S = \{s_1, \ldots, s_q\}\): the set of source nodes.
\(T = \{t_1, \ldots, t\}\): the set of sink nodes.
\(G(A, N, M, S, T)\) is a multi-source multi-sink stochastic-flow network.
\(MP\) Minimal path.
\(MP_{i,j,k}\) The \(k^{th}\) MP from \(s_i\) to \(t_j\).
\(B = \{b_1, b_2, b_3, \ldots, b_n\}\), the set of assigned components.
\(F = (f_{i,1,1,1}, f_{i,1,1,2}, \ldots, f_{i,j,k_1,1}, \ldots, f_{\sigma,\theta,\sigma,\theta}m)\), where \(f_{i,j,k,w}\) represents the flow quantity of resource \(w\) on \(MP_{i,j,k}\).
\(d_{w,j}\) The demand for resource \(w\) at sink node \(t_j\).
\(r_{w,i}\) The maximum quantity of resource \(w\) that source node \(s_i\) can supply.
\(X_F(B)\) The capacity vector related the flow vector \(F\) under assigned components \(B, X_F(B) = (x_{F_1}, x_{F_2}, \ldots, x_{F_n})\), where \(x_{F_e} = \sum_{j=1}^{\sigma} \sum_{\theta=1}^{\theta} \sum_{k=1}^{k_{i,j}} \sum_{w=1}^{m} f_{i,j,k,w} \{a_e \in MP_{i,j,k}\}\).
\(R(X_F(B))\) The reliability of the capacity vector \(X_F(B)\). The probability that transmission is successful from source nodes to sink nodes.
\(R_{min}\) is the specified lower bound for \(R(X_F(B))\).
\(G\) Maximum number of generations.
\(P\) Population size.
\(\lambda\) Probability of crossover.
\(\mu\) Probability of mutation.

2.2. Assumptions.

1. No component is assigned to any node.
2. The flow must satisfy the flow-conservation law.
3. Each component can be assigned to at most one arc.
4. The capacities of resources are statistically independent.
5. The state capacities of each arc \(a_e\) is an integer-valued random variable, which takes the values \(0 < 1 < 2 < \ldots < M_e\) according to a given distribution.
6. The capacities of the arcs are statistically independent.
7. The flow along a path does not exceed its maximum capacity.

3. The problem formulation. The problem is separated into two models as shown in the following subsections.
3.1. **Searching the best components.** Let $\mathcal{B} = \{b_1, b_2, b_3, \ldots, b_n\}$ be the set of components assigned to the set of arcs $A$ and $b_i$ is a candidate component selected from the set of available components such that $b_i \neq b_e$ for $i \neq e$. Then, the problem is formulated as:

$$\text{Minimize} Z(\mathcal{B})$$

s.t.

$$R(X_F(\mathcal{B})) \geq R_{\text{min}}$$

where $Z(\mathcal{B}) = \sum_{i=1}^{n} c_{b_i}$ is the total cost of the assigned components and $c_{b_i}$ is cost of the assigned component $b_i$. The reliability of the capacity vector $R(X_F(\mathcal{B}))$ is greater than or equal to a specified amount $R_{\text{min}}$.

3.2. **Searching the best flow vector.** The flow vector $F$ of the assigned components is essential to find the capacity vector $X_F(\mathcal{B})$ and calculate $R(X_F(\mathcal{B}))$, as discussed in [23] and [9].

Find the best $R(X_F(\mathcal{B}))$

s.t.

$$\sum_{i=1}^{\sigma} \sum_{k=1}^{k_{i,j}} f_{i,j,k,w} = d_{w,j}, \ w = 1, \ldots, m; j = 1, \ldots, \theta$$

(3)

$$\sum_{j=1}^{\theta} \sum_{k=1}^{k_{i,j}} f_{i,j,k,w} = r_{w,i}, \ w = 1, \ldots, m; i = 1, \ldots, \sigma$$

(4)

$$x_{F_e} \leq M_e, e = 1, 2, 3, \ldots, n$$

(5)

$$f_{i,j,k,w} \leq C \ i = 1, \ldots, \sigma, j = 1, \ldots, \theta$$

$$w = 1, \ldots, m \ k = 1, \ldots, k_{i,j}$$

(6)

where $C = \text{Min} \{L_k | k = 1, 2, \ldots, np\}$ and $L_k$ is the maximum capacity of the path $MP_{i,j,k}$ given by $L_k = \text{min} \{M_e | e \in MP_{i,j,k}\}$.

4. **The proposed approach.** Our proposed approach consists of two GAs. The first is an outer GA described below to search the best components, and the second is from [23] and [9] to search the best $F$. The following subsections illustrate the proposed approach and explains how the function of the two GAs.

4.1. **The outer GA.** The components of our proposed GA, including chromosomal representation, the crossover and mutation operations, and the fitness function are described in the following subsections.

4.1.1. **Chromosomal representation.** The chromosome $\mathcal{B} = \{b_1, b_2, b_3, \ldots, b_n\}$ with length $n$, where $n$ is the number of arcs (components) for the network, represents the set of candidate components. The component $b_1$ is assigned to arc $a_1$, the component $b_2$ is assigned to arc $a_2$, . . . , and the component $b_l$ is assigned to arc $a_l$. Note that $b_i = z \in \{1, 2, \ldots, p\}$, where $p$ is the number of available components.
4.1.2. Crossover and mutation. Our proposed algorithm uses both the modified uniform crossover approach and the swap mutation to avoid duplicate genes in the chromosome [8]. Figures 1 and 2 summarize the crossover and mutation processes.

![Modified uniform crossover](image1)

**Figure 1.** Modified uniform crossover.

![Mutation operation](image2)

**Figure 2.** Mutation operation

4.1.3. Evaluation. Evaluation is an essential step in the searching process. Once the set of components $B$ is generated, then the corresponding total assignment cost $Z(B)$ and the reliability of the capacity vector $R(X_F(B))$ calculated. The total cost $Z(B)$ is calculated as shown in section 3. While the reliability of the capacity vector $R(X_F(B))$ is calculated by:

$$R(X_F(B)) = \prod_{e=1}^n \Pr(x_e \geq x_{Fe})$$

(7)

Where,

$$x_{Fe} = \sum_{i=1}^{\sigma} \sum_{j=1}^{\theta} \sum_{k=1}^{m} \sum_{w=1}^{m} \{f_{i,j,k,w} | a_e \in MP_{i,j,k}\}$$

(8)

The flow vector $F = (f_{1,1,1,1}, f_{1,1,1,2}, \ldots, f_{i,j,k_{i,j},1}, \ldots, f_{i,j,k_{i,j},m}, \ldots, f_{\sigma,\theta,k_{\sigma,\theta},m})$, where $f_{i,j,k,w}$ represents the flow quantity of resource $w$ on $MP_{i,j,k}$ is generated by using the inner GA described in Section 4.2.

4.1.4. Fitness function. The objective function $Z_{obj}(B)$ is equal to the total assignment cost $Z(B)$ only if the reliability of the capacity vector $R(X_F(B))$ is greater than or equal to the specified amount $R_{min}$. Otherwise, $Z_{obj}(B)$ is penalized [4]-[5].

$$Z_{obj}(B) = Z(B) + \delta[c_{max}(R(X_F(B) - R_{min}))^2]$$

(9)

where $c_{max}$ is the maximum cost of the assigned components.

$$\delta = \begin{cases} 
0, & \text{if } R(X_F(B)) \geq R_{min} \\
1, & \text{otherwise } x \geq 0 
\end{cases}$$

(10)

The fitness function is

$$E(B) = Z_{max}(B) - Z_{obj}(B)$$

(11)

where $Z_{max}(B)$ is the maximum value of Equation 11 for the current population.
4.1.5. *Selection process.* The algorithm uses the Roulette Wheel selection mechanism to select new parents [6], and the selection of a chromosome is based on its fitness value [16]. This process is summarized as:

1. Compute the sum of the fitness values of all chromosomes \( S \) in the current population.
2. Generate a random number \( r \) belonging to \([0, S]\).
3. Compute the sum of the fitness values \( H \) from the first chromosome to the current one.
4. If \( H \geq r \), then return the corresponding index of the current chromosome. Otherwise, repeat Steps i through iv.

4.1.6. *Keeping the best parents.* Through the generation process, we combine the current and previous generations and select the best parents for the next generation.

4.2. *The inner GA.* The following subsections describe how to deduce \( F \) that satisfies the conditions 2, 3, 4, and 6 as shown in section 3.

4.2.1. *Representation, crossover and mutation.* The chromosome \( F \) is represented by a string of length \( m \times np \) where \( m \) and \( np \) are the number of resources and paths, respectively.

\[
F = (f_{1,1}, f_{1,1}, f_{i,j,1}, f_{i,j,1}, \ldots, f_{i,j,1}, f_{i,j,1}, \ldots, f_{\sigma,\theta,\sigma,\theta,}, m)
\]

The one cut point crossover is used in the inner GA to generate two new offspring, and a simple mutation process is used to mutate the offspring. The mutated bit is selected randomly to change its value, and the crossover and mutation processes are described in Figures 3 and 4.

![Figure 3. Crossover operation.](image1)

![Figure 4. Mutation operation.](image2)
4.2.2. **Objective function.** The probability of the capacity vector \( R(F) \) of the generated \( F \) is feasible if it satisfies all constraints. Otherwise, it is set to zero. Hence, the fitness function has the form

\[
fit(F) = \begin{cases} 
R(F), & \text{if } F \text{ satisfies all conditions} \\
0, & \text{otherwise}
\end{cases}
\] (12)

5. **The whole algorithm.** The following steps describe how the outer GA searches the set of components with minimum cost based on the results of the inner GA after discovering the best reliability of the capacity vector that corresponds to this set of components.

**Start**

- Input the information of the available components, network arcs, and paths.
- Set the GA parameters.
- Generate initial population.
- Evaluate the initial population following Section 4.1.3 and calculate \( E(B) \) for each chromosome.

while \( g \leq G \), do

while \( p \leq P \), do

- Use the Roulette Wheel to select two parents.
- Apply crossover and mutation according to \( \lambda \) and \( \mu \), respectively.

\[ p = p + 1; \]

End do.

Evaluate the current population.

Save the best solution found according to \( E(B) \).

\[ g := g + 1; \]

Save the best parents to the next generation as described in Section 4.1.6.

End do.

**End**

6. **Illustrative examples.** The proposed approach is programmed using MATLAB 2013a and all information of the studied problems stored as input files. It is applied to three network examples as described in Sections 6.1, 6.2, and 6.3. The valid values of the GA parameters were \( G=100, P = 10, \lambda=90, \) and \( \mu=0.10 \). Ten runs performed for each problem with \( R_{min} \) equal to 0.30, 0.40, and 0.40, respectively.

6.1. **Network with three sources and two sinks.** The network in Figure 5 includes two sources and three sink nodes. The available components with their capacities, the corresponding probabilities, and the costs are listed in Table 1, [10].

The MPs are \( MP_{1,1,1} = \{a_1, a_7\}, MP_{1,1,2} = \{a_2, a_9\}, MP_{1,2,1} = \{a_1, a_8\}, MP_{2,1,1} = \{a_3, a_9\}, MP_{2,2,1} = \{a_4, a_{10}\}, MP_{3,1,1} = \{a_5, a_9\}, \) and \( MP_{3,2,1} = \{a_6, a_{10}\} \). Here, we let \( R = (r_{1,1}, r_{1,2}, r_{1,3}, r_{2,1}, r_{2,2}, r_{2,3}, r_{3,1}, r_{3,2}, r_{3,3}) = (5, 2, 3, 5, 3, 2, 2, 3) \) and \( D = (d_{1,1}, d_{1,2}, d_{2,1}, d_{2,2}, d_{3,1}, d_{3,2}) = (3, 1, 2, 2, 1, 3) \).

The initial population is listed in Table 2, and Figure 6 shows the minimum cost found at each generation. The best solution discovered by the proposed approach occurred at generation three. The assigned components of this best solution are \((9 7 4 11 1 12 6 3 10 5)\), and the corresponding cost and reliability are 19 and 0.363370,
Figure 5. Network with three source and two sink nodes.

Table 1. Component information for the network in Figure 5.

| p | Capacity | Cost |
|---|----------|------|
|   | 0        | 1    | 2    | 3    | 4    |
| 1 | 0.02     | 0.04 | 0.14 | 0.80 | 0.00 | 1    |
| 2 | 0.04     | 0.06 | 0.10 | 0.15 | 0.65 | 3    |
| 3 | 0.02     | 0.03 | 0.05 | 0.90 | 0.00 | 4    |
| 4 | 0.05     | 0.08 | 0.87 | 0.00 | 0.00 | 2    |
| 5 | 0.01     | 0.04 | 0.10 | 0.85 | 0.00 | 3    |
| 6 | 0.02     | 0.05 | 0.15 | 0.78 | 0.00 | 2    |
| 7 | 0.05     | 0.10 | 0.85 | 0.00 | 0.00 | 1    |
| 8 | 0.04     | 0.06 | 0.15 | 0.75 | 0.00 | 4    |
| 9 | 0.03     | 0.05 | 0.12 | 0.80 | 0.00 | 1    |
| 10| 0.01     | 0.04 | 0.05 | 0.15 | 0.75 | 3    |
| 11| 0.03     | 0.05 | 0.07 | 0.85 | 0.00 | 1    |
| 12| 0.01     | 0.02 | 0.07 | 0.90 | 0.00 | 1    |

respectively. The capacity vector of the assigned components, \( M = (3, 2, 2, 3, 3, 3, 3, 3, 3) \).

6.2. Network with two sources and two sinks. We next applied our approach to the network in Figure 7, which has two source and two sink nodes. The available components are listed in Table 3 [20] and [21], and the MPs are \( MP_{1,1,1} = \{a_1, a_5\}, MP_{1,1,2} = \{a_1, a_6, a_9\}, MP_{1,1,3} = \{a_2, a_7, a_9\}, MP_{1,2,1} = \{a_1, a_6, a_{14}\}, MP_{1,2,2} = \{a_2, a_7, a_{14}\}, MP_{2,1,1} = \{a_3, a_7, a_{14}\}, MP_{2,1,2} = \{a_4, a_8, a_{13}, a_9\}, MP_{2,2,1} = \{a_3, a_7, a_{11}\}, MP_{2,2,2} = \{a_4, a_6, a_{13}, a_{14}\}, MP_{2,2,3} = \{a_4, a_6, a_{10}\}, \) and \( MP_{2,2,4} = \{a_4, a_{11}, a_{12}\} \). Here, \( R = (r_{1,1}, r_{1,2}, r_{2,1}, r_{2,2}) = (15, 17, 10, 13) \), \( D = (d_{1,1}, d_{1,2}, d_{2,1}, d_{2,2}) = (9, 10, 5, 8) \), and the costs of the available components are (41, 52, 51, 32, 61, 52, 31, 42, 21, 62, 51, 52, 41, 22, 31, 22, 11, 32, 51, 51).
Table 2. The initial population for the network example in Figure 5.

| No. | The components \((B)\) | The Flow vector \((F)\) | \(Z_{obj}(B)\) | \(R(XF(B))\) |
|-----|------------------|------------------|--------------|--------------|
| 1   | 2 10 3 12 7 1 8 11 6 9 | 1 0 1 1 0 0 0 1 1 0 1 | 21.0067 | 0.259087 |
| 2   | 1 7 4 11 6 8 9 10 2 3 | 1 0 1 1 0 0 0 1 1 0 1 | 22.0005 | 0.288896 |
| 3   | 6 12 8 11 11 10 9 3 2 4 | 1 0 1 1 0 0 0 1 1 1 1 0 1 0 | 22.0164 | 0.236832 |
| 4   | 12 6 9 7 11 1 3 8 10 2 | 1 0 1 1 0 0 0 1 1 1 1 0 1 0 | 21.0002 | 0.293038 |
| 5   | 8 6 4 9 7 11 2 6 10 2 | 1 0 1 1 0 0 0 1 1 1 1 0 1 0 | 21.0038 | 0.269935 |
| 6   | 10 7 12 9 1 5 6 3 2 4 | 1 0 1 1 0 0 0 1 1 1 1 1 0 1 0 | 21.0000 | 0.292652 |
| 7   | 1 9 6 10 5 3 11 4 2 1 1 2 | 1 0 1 1 0 0 0 1 1 1 1 0 1 0 | 21.0000 | 0.512310 |
| 8   | 2 1 9 8 12 4 3 6 10 1 1 | 1 0 1 1 0 0 0 1 1 1 1 0 1 0 | 22.0012 | 0.282989 |
| 9   | 9 3 12 5 4 1 6 10 2 7 | 1 0 1 1 0 0 0 1 1 1 1 0 1 0 | 21.0002 | 0.293636 |
| 10  | 2 8 4 1 7 5 11 9 10 3 | 1 0 1 1 0 0 0 1 1 1 1 0 1 0 | 23.0000 | 0.314006 |

Figure 6. The minimum cost found at each generation.

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Figure 7. Two-source two-sink computer network.

malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

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Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.
Table 3: The component information for the network of Figure 7.

| P  | Capacity                  |
|----|---------------------------|
| 0  | 0.001 0.001 0.003 0.004 0.005 0.005 0.006 0.007 0.010 0.015 |
| 1  | 0.001 0.003 0.003 0.004 0.005 0.007 0.007 0.008 0.009 0.010 |
| 2  | 0.002 0.002 0.003 0.006 0.007 0.007 0.010 0.012 0.015 0.017 |
| 3  | 0.001 0.001 0.002 0.003 0.005 0.008 0.010 0.011 0.012 0.015 |
| 4  | 0.001 0.001 0.002 0.009 0.012 0.020 0.040 0.050 0.060 0.080 |
| 5  | 0.001 0.002 0.002 0.005 0.010 0.012 0.015 0.017 0.020 0.025 |
| 6  | 0.001 0.001 0.002 0.005 0.008 0.010 0.012 0.015 0.015 0.017 |
| 7  | 0.001 0.002 0.005 0.005 0.007 0.008 0.010 0.012 0.015 0.015 |
| 8  | 0.001 0.001 0.002 0.005 0.005 0.007 0.008 0.010 0.012 0.015 |
| 9  | 0.001 0.001 0.002 0.002 0.003 0.004 0.005 0.008 0.009 0.010 |
| 10 | 0.002 0.003 0.005 0.006 0.007 0.009 0.012 0.015 0.015 0.017 |

Continue on the next page
Table 3: The component information for the network of Figure 7 (cont.).

|      |      |      |      |      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.019 | 0.020 | 0.023 | 0.025 | 0.026 | 0.740 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

The behavior of the minimum cost found at each generation is shown in Figure 8 with the corresponding cost and reliability of the best solution being 499 and 0.608402, respectively, and found at generation seven. The assigned components of the best solution are (7 14 5 18 20 19 12 8 17 16 7 13 9 15). The capacity vector of the assigned components, $M = (31 22 61 32 51 51 52 42 11 22 31 41 21 31)$.

**Figure 8.** The minimum cost found at each generation.
6.3. Network with two source and three sink nodes. The algorithm was again applied to another network given in Figure 9. The information of each component is given in Table 3. Here, $R = (r_{1,1}, r_{1,2}, r_{2,1}, r_{2,2}) = (10,19,14,19)$, $D = (d_{1,1}, d_{1,2}, d_{1,3}, d_{2,1}, d_{2,2}, d_{1,3}) = (7, 8, 6, 8, 7, 8)$, and the costs of the available components are $(41, 52, 51, 32, 61, 52, 31, 42, 21, 62, 51, 52, 41, 22, 31, 22, 11, 32, 51, 51)$. The MPs are $MP_{1,1,1} = \{a_1, a_7\}$, $MP_{1,1,2} = \{a_2, a_5, a_7\}$, $MP_{1,2,1} = \{a_1, a_8\}$, $MP_{1,2,2} = \{a_2, a_9\}$, $MP_{1,2,3} = \{a_2, a_5, a_6\}$, $MP_{1,3,1} = \{a_2, a_{10}\}$, $MP_{2,1,1} = \{a_3, a_5, a_7\}$, $MP_{2,1,2} = \{a_4, a_5, a_7\}$, $MP_{2,2,1} = \{a_3, a_9\}$, $MP_{2,2,2} = \{a_4, a_6, a_9\}$, $MP_{2,3,1} = \{a_3, a_{10}\}$, $MP_{2,3,2} = \{a_4, a_6, a_{10}\}$, and $MP_{2,3,3} = \{a_4, a_{11}\}$.

![Figure 9. Network with two source and three sink nodes.](image)

The minimum cost calculated at each generation is shown in Figure 10 with the best cost of 367 found at generation three and a reliability value equal to 0.458603. The assigned components of the best solution are $(17, 4, 14, 18, 9, 13, 11, 2)$. The capacity vector of the assigned components, $M = (11, 32, 22, 41, 32, 42, 21, 41, 51, 52)$.

7. Conclusion and future work. This research studied the CAP in multi-source multi-sink stochastic flow networks with multiple commodities. In addition, an approach based on genetic algorithms is presented as a solution. The presented approach consists of two steps. The first, called the outer GA, searches the best components that could be assigned to the network with minimum cost. The second, called the inner GA, finds the flow vector with maximum reliability of the capacity vector for the assigned components. The obtained solutions are characterized by minimum cost and maximum reliability of the capacity vector. Based on the value of $R_{\min}$ supported by the decision maker, the proposed approach searches the best solution to the given problem. From the managerial point of view, the decision maker determines the transmission budget according to the best $R(X_F(B))$ value found by the proposed algorithm. By solving the designated problem, the data or goods can be reliably transmitted through the network.
For future work, we will study the CAP in multi-source multi-sink stochastic flow networks with multiple commodities subject to system reliability constraints.

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