Using the Bees Algorithm to solve combinatorial optimisation problems for TSPLIB

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Abstract. The Bees Algorithm (BA) is a metaheuristic algorithm to find good solutions to optimisation problems in reasonable computing times. This paper is the first to report on the use of the BA to solve 9 combinatorial optimisation problems (COP) with more than 100 cities from TSPLIB rigorously to test the performance of the algorithm. The work employed a basic version of the BA for COP and TSPLIB datasets involving between 100 and 200 cities. The results obtained show that deviations from the best-found tour lengths for the datasets with 100 cities and 200 cities were approximately 2.5% and 7.5%. The reason for this jump in deviations was that the number of iterations was kept constant for all experiments while the solution space increased factorially with the number of cities. This research can be replicated and modified through Google Colab.

Keywords: bees algorithm, combinatorial optimisation problem, metaheuristics algorithm, travelling salesman problem.

1. Introduction

One of the most intensive fields studied in mathematics, engineering, and computer science is the optimisation which involves finding the best solution or a feasible solution from candidates in the solution space. An important class of practical optimisation problems is that of combinatorial problems. Many COPs are NP-hard and thus challenging to solve because the number of combinations exponentially increases with the size of the problem [1]. The need to find acceptable near-optimal solutions in reasonable computing times has motivated researchers to develop heuristic and metaheuristic algorithms. According to Ouaraab, an advantage of metaheuristic algorithms is that they are simple and flexible [1]. Nature-inspired metaheuristic algorithms including those based on the behaviour of social insects such as honey bees have attracted the attention of many researchers [2].

As stated by Lones, there are 15 nature-inspired metaheuristic algorithms that have each received more than 1,000 citations [3]. One of those algorithms is the Bees Algorithm (BA) proposed by Pham et al. in 2005 [4], [5]. The BA is inspired by the natural honey bees foraging behaviour and can be used to solve continuous and combinatorial optimisation problems. A comparative study of the BA, Particle Swarm Optimisation (PSO), and Evolutionary Algorithm (EA) show that the BA outperforms PSO and EA [6]. The BA also outperforms the deterministic simplex method, genetic algorithm, ant colony system, and stochastic annealing optimisation procedure [5]. The BA has been very popular since its development and it has many applications in multi-discipline area from work scheduling,
image processing, pattern recognition, manufacturing, arc routing, PCB manufacturing, matrices structure, data partitioning, data transmission in a network, power distribution networks, and robot navigation [7], [8].

The BA has already found more than 400 different applications covering a variety of continuous or combinatorial problems, such as pattern recognition, scheduling, engineering design, neural network training, manufacturing cell formation, control system tuning, dynamic control, data clustering, mechanical design and supply chain design [7], [8], [9], [10]. There have been at least 56 endeavours to develop new versions of the BA. However, only 4 out of those 56 attempts have focused on combinatorial optimisation although COP formed approximately 33% of the total number of applications of the algorithm and the problem of combinatorial optimisation arises in many fields of engineering and other scientific disciplines that use computational approaches, such as artificial intelligence, operations research, electronic commerce, and bioinformatics. The authors believe that the reason for the lack of activity on combinatorial versions of the BA is that there are no available results on the performance of the BA on standard data-sets (test functions) that can be used for comparison in developing the algorithm.

Although there are many publications addressing real-world applications in the combinatorial domain such as machine scheduling [11], [12], cellular manufacturing [13], Printed Circuit Board (PCB) manufacturing [14], [15], [16], vehicle routing [17], it is important to use standard data sets when assessing the performance of an algorithm. The results for one or more standard data sets could be employed to compare different algorithms.

The basic combinatorial version developed by Koc [18] was adopted in this work as the basic version of the BA for combinatorial optimisation (“the Combinatorial BA”). This study extends Koc’s previous work that used simple problems up to 51 cities. The performance of many algorithms is satisfactory for simple problems. However, the problem becomes challenging when the number of cities increases. The Google Colab link for this work is provided in the appendix to give a clear starting point to develop the Combinatorial BA.

One of the most common problems in combinatorial optimisation is the travelling salesman problem (TSP). The problem has many variants such as PCB manufacturing, machine scheduling, cellular manufacturing, arc routing, vehicle routing, job shop scheduling, data routing, machine sequencing, computer wiring, DNA sequencing, etc. TSP is a NP-hard optimisation problem because the complexity of the computation increases exponentially with the number of cities [1]. The TSP was initially introduced in 1930 and is one of the most studied COP as it can be used as a benchmark for evaluating the performance of many combinatorial optimisation algorithms [19].

The most widely used dataset library for COP is TSPLIB, which was published by Reinelt in 1991 [20]. The library itself has been cited more than 2,400 times, according to Google Scholar (October 2019). It is a very popular library and has been used mostly by researchers to evaluate optimisation algorithms such as the GA, PSO and ACO. However, no work has been done to measure the performance of the BA on TSPLIB problems involving more than 100 cities. This paper will present the results of testing the basic version of the BA on TSPLIB datasets with 100 to 200 cities. A reason for choosing those datasets was that most metaheuristic algorithms, including the BA, give good results for datasets with fewer than 100 cities.

The basic combinatorial version of the BA adopted in this research employed a permutation random number generator as the global search operator. The basic local search operator was a combination of swap, insertion, and reversion. This local search operator was chosen as the algorithm performed poorly when swap, insertion or reversion alone was used.

1.1. Bees Algorithm
Pham et al. introduced the BA as a nature-inspired metaheuristic optimisation method inspired by the food foraging behaviour of the swarm honey bees [4], [5]. The BA classified as one of the swarm intelligence algorithms. Figure 1 illustrates the flow chart of The BA.
The BA for the continuous domain has become more advanced and better structured compared to the version for the combinatorial domain. Work on the algorithm can be classified into four major sections: parameter tuning and setting, initialisation, local search (exploitation), and global search (exploration). Most of the developments primarily concentrate on the local search procedure of the BA [7]. All of the previous work on the BA in the continuous domain had different characteristics from the combinatorial version, such as the solution space is real rather than discrete. Therefore, the BA needs to be modified to perform exploitation and exploration in the combinatorial domain without changing the principal procedure of the algorithm.

1.2. Bees Algorithm in Combinatorial Optimisation Problem
The first COP solved by BA was reported in 2007 for finding the solution of the machine scheduling [11]. In the same year, BA also had solved PCB assembly sequence as an early TSP based COP [14]. After those publication in 2007, other important works began to emerge in the combinatorial BA algorithm such as timetabling [21], [22], [23], [24], PCB assembly planning [15], [16], circuit designing [25], scheduling [12], [26], [27], [28], [29], cellular manufacturing [13], vehicle routing [17], disassembly sequence planning [30], [31], and path planning [32], [33].

Combinatorial and continuous problems have completely different search principles. The task of searching as the main feature in the combinatorial problem is distinctive from the continuous problem where there is no real distance value [18]. The combinatorial domain has main characteristics [34]: the search space is discrete; the constraints are finite; the solution has ordered sequence and a cost function which related to the combination. Since the BA was originally developed to solve continuous domains, it is important to change the searching section in global and local search with a discrete search operator.

In essence, the BA for COP is almost identical to the algorithm for continuous one. The main difference between continuous and combinatorial version is lying on their (global and local search) operators. The combinatorial version, of course, has to use a discrete random generator replacing the real number generator on the continuous version.

1.3. Travelling Salesman Problem
TSP can be classified as symmetrical or asymmetrical TSP. In symmetrical TSPs, the distances between the two cities do not rely on the trajectory direction. For example, if the distance between two

![Figure 1. Flowchart of Bees Algorithm (basic version) [5]](image-url)
cities i and j when travelling from i to j is denoted by $d_{i,j}$ and if $d_{i,j} = d_{j,i}$ then the TSP is symmetrical and vice versa.

TSP’s graph represented the Graph $G = (V,E)$, where $V$ is a set of vertices representing the cities, and all the connecting lines between the cities is $E$. Every edge indicates a possible route between two connected vertices or cities. The variable $d_{i,j}$ is associated with an edge $(i,j)$ and represents the Euclidean distance from vertex $(x_i,y_i)$ to $(x_j,y_j)$ as Equation (1). Before executing the BA, these distances of all edges have calculated and store as a distance matrix.

$$d_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$  

(1)

The TSP objective is to find the minimal total tour length of the final closed Hamilton cycle (only visiting once of all cities) tour as defined in Equation (2).

$$\text{tour length} = \sum_{i=1}^{n} d_{i,i+1} + d_{n,1}$$  

(2)

Koc, Otri and Zeybek were three of the researchers developing the BA for solving TSPs taken from the TSPLIB dataset up to 100 cities [18],[34],[35].

2. Methods

The first step before running the BA is setting up the parameters. There are 5 parameters that have to be initially set up as number of scout bees ($n$), elite bees ($n_{ep}$), best bees ($n_{sp}$), elite sites ($e$), and best sites ($m$). All the parameters dependently set to the $n$. The parameter $n_{sp}$ and $m$ are 50% of $n$. The $e$ is 40% of $m$. The $n_{ep}$ has a double number of $n_{sp}$. The $n$ set equal to 40 bees (see Table 1).

After setting up the parameters, random initialisation (see Figure 1) will generate $n$ initial solutions. The best $e$ out of $n$ initial solutions will be the elite sites which best seeds to be improved. The $m$ sites excluding the $e$ sites are sufficiently good seeds as sites to be exploited as well.

Table 1. Parameter value of the running experiments

| Parameter             | Value |
|-----------------------|-------|
| Number of runs        | 10    |
| Number of iterations  | 3,000 |
| Number of scout bees ($n$) | 40 |
| Number of elite bees ($n_{ep}$) | 40 |
| Number of best bees ($n_{sp}$) | 20 |
| Number of elite sites ($e$) | 16 |
| Number of best sites ($m$) | 20 |

The elite and best sites are the places needed to be exploited by worker bees ($n_{ep}$ and $n_{sp}$) using neighbourhood or local search (see Figure 2 to 5). The remaining $(n - e - m)$ bees as insufficiently seeds will explore the other solution spaces using the global search operator. All worker bees will continuously renew the patch a better position for every iteration and abandoned the worn-out one. The position that a worker bees had is representing a solution (complete tour). The best solution is generated by sorting all the best positions from all sites. The code is written in Python and could be retrieved in Google Colab.
3. Result and Discussion

Nine benchmark problems from TSPLIB [20] were used in this work to evaluate the performance of the combinatorial BA with the basic local search operator. The experimental results are summarised in Table 2. The first column of Table 2 gives the TSPLIB dataset names and the tour length for the best-known solution (BKS). The second column lists the tour length for the best solution of 10 runs, each of 3,000 iterations (Best). The third column gives the average tour length of the solutions found by the ten runs (Avg). The fourth column gives the deviation of the Avg from BKS (Davg). The last column
lists the deviation of the Best from BKS ($D_{best}$). $D_{avg}$ and $D_{best}$ as the accuracy indicators are defined in Equation (3) and (4).

$$D_{avg} = \frac{(\text{Avg} - \text{BKS}) \cdot (\text{BKS})^{-1} \cdot 100}{}\%$$ (3)

$$D_{best} = \frac{(\text{Best} - \text{BKS}) \cdot (\text{BKS})^{-1} \cdot 100}{}\%$$ (4)

**Table 2.** Results for ten runs each of 3000 iterations for 100-200 cities data-sets from TSPLIB using the combinatorial BA

| Dataset names (BKS) | Best   | Avg    | $D_{avg}$ | $D_{best}$ |
|---------------------|--------|--------|-----------|------------|
| KroA100 (21,282)    | 21,469 | 21,712.4 | 2.02%     | 0.88%      |
| KroB100 (22,141)    | 22,618 | 22,879.0 | 3.33%     | 2.15%      |
| KroC100 (20,749)    | 20,969 | 21,339.8 | 2.85%     | 1.06%      |
| KroD100 (21,389)    | 21,392 | 21,896.7 | 2.37%     | 0.01%      |
| KroE100 (22,068)    | 22,266 | 22,496.1 | 1.94%     | 0.90%      |
| KroA150 (26,524)    | 27,527 | 27,871.2 | 5.08%     | 3.78%      |
| KroB150 (26,130)    | 27,095 | 27,458.5 | 5.08%     | 3.69%      |
| KroA200 (29,368)    | 31,550 | 32,214.0 | 9.69%     | 7.43%      |
| KroB200 (29,437)    | 32,024 | 32,519.7 | 10.47%    | 8.79%      |

The overall result shows that the $D_{avg}$ starts from 1.94% up to 10.47% and for the $D_{best}$ from 0.01% up to 8.79%. The results of 100 cities’ datasets show that the $D_{avg}$ have range between 1.94% and 3.33% while the $D_{best}$ range between 0.01% and 2.15%. For the 150-200 cities results, the highest $D_{avg}$ is 10.47% and the lowest is 5.08%, while the $D_{best}$ has a range from 3.69% to 8.79%. The performance for 150-200 cities’ shows that the deviation is around three times larger than performance of 100 cities’ results due to the solution spaces were factorially increasing while the number of iteration remains the same. The authors believe that the accuracy performance of the combinatorial BA will increase if the number of iteration of the algorithm were adjusted according to the problem complexity (number of cities). The poor convergence has been found as other problem in this work. ACO’s previous work [36] in 1999 had proved that ACO convergence performance would increase when ACO utilising a constructive heuristic (nearest neighbour) on the candidate solution generator. Later, It provokes the other researchers to be more intense to develop the constructive heuristic [37],[35],[30] as initialisation or global search operator for the algorithm development.

Overall, the basic combinatorial BA has the ability to provide good solutions with average error around 2.5% for 100 cities datasets and 7.5% for 150-200 cities datasets. Figure 6, Figure 7, and Figure 8 show the best-found tour length results for every TSPLIB’s datasets in this research.

![Figure 6](image-url)  
**Figure 6.** Best solution of (a) KroA100, (b) KroB100, (c) KroC100
4. Conclusion
This paper has shown that the BA can find near optimal solutions to the COP. The algorithm uses the permutation random number generator as global search operator and a combination of swap, insertion and reversion as the basic local search operator. Experiments using TSPLIB datasets with 100 to 200 cities show that the deviation increases around three times from the 100 cities dataset to the 200 cities dataset. The reason for this surge was that the number of iterations was kept constant for all experiments while the solution space increased factorially with the number of cities. This poor convergence for larger problems is a weakness common to many swarm intelligence or nature-inspired algorithms. The convergence problem is a challenge to address in developing these nature-inspired algorithms including the BA.

For further research, the other constructive and improved heuristic algorithms potentially hybridised to the BA either as a global or local search operator to improve the performance of the algorithm for solving the COP.

Appendix A. Google Colab link
Google Colab link for this research: https://bit.ly/2MaJTyG or http://bit.ly/2LR7rlO or https://bit.ly/2YYXQVo

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