Reaction and break-up cross sections of $^{11}$Li at 0.8 and 0.28 GeV/u.

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Abstract

In this paper we calculate reaction and breakup cross sections for the two-neutron halo nucleus of $^{11}$Li using the optical limit of Glauber theory. Calculations are presented and compared to experimental data at 0.8 and 0.28 GeV/u on a series of targets. The $^{11}$Li nucleus is described as a three-body system, a core plus two neutrons, with a phenomenological neutron-core potential and a density dependent neutron-neutron interaction of zero range. Three different wave functions are constructed which have different $(2s_{1/2})^2$ and $(1p_{1/2})^2$ two-neutron components but correspond to the same binding energy close to the experimental value. We show that the agreement with all the experimental observables is achieved only if the $^{11}$Li wave function contains about 30% of $(2s_{1/2})^2$ configuration.

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1 Introduction

In two-neutron halo nuclei the reaction and two-neutron break-up cross sections are much larger than in normal nuclei because of their very low two-neutron separation energy and large radius. These two observables are thus very useful tools to investigate properties of halo nuclei. Numerous works, experimental and theoretical, have been devoted to reactions with a $^{11}\text{Li}$ projectile. For long time there was some ambiguity about the structure of $^{11}\text{Li}$ due to the lack of information about the unbound nucleus of $^{10}\text{Li}$. In precursor experiments a $\frac{1}{2}$ neutron resonance at $0.8 \text{ MeV}$ was assumed to be the ground state of $^{10}\text{Li}$ and first theoretical works assumed that the ground state of $^{11}\text{Li}$ was mainly formed of two neutrons in a $\frac{1}{2}$ resonance. Now it is well accepted that the ground state is nearly bound with an energy of 0.1-0.2 MeV and that it is a $l=0$ state while the $\frac{1}{2}$ resonance is an excited state at an energy ranging from 0.35 to 0.6 MeV, implying that the correlated two halo-neutron wave function in $^{11}\text{Li}$ will involve both $(2s\frac{1}{2})^2$ and $(1p\frac{1}{2})^2$ two-neutron components. However for a given neutron-neutron interaction, calculations in Faddeev or pairing model show that the measured two-neutron separation energy in $^{11}\text{Li}$ can well be reproduced with different positions of $s\frac{1}{2}$ and $p\frac{1}{2}$ neutron states therefore with different $(2s\frac{1}{2})^2$ and $(1p\frac{1}{2})^2$ components in the wave function. Then one needs other information to discriminate between the different scenarios. Different mixtures in the wave function correspond to different radii and comparison of calculated and measured radii gives a first indication. However the determination of the experimental radius depends on the reaction model and on the assumed $^{11}\text{Li}$ density while the calculated radius seems to depend on the structure microscopic model. This will be discussed in the present work. Other information comes from the presence of a low energy dipole mode in $^{11}\text{Li}$ and the measured B(E1) which has been shown to favor a wave function with 30-40% of $(2s\frac{1}{2})^2$ configuration. Reaction and break-up cross sections are expected to be sensitive to both, radius and configuration mixing, and then to give further constraints on the structure of $^{11}\text{Li}$.

In the present work we study reaction and two-neutron removal cross sections in the optical Glauber approximation describing the $^{11}\text{Li}$ nucleus in a two-neutron model. The interaction is assumed to be density dependent with zero range. Two of the three parameters of the interaction are taken
from the work of Schuck et al [14] while the third one, the strength of the density independent term, is fitted on the two-neutron separation energy in $^{14}\text{C}$ which has the same number of neutrons as $^{11}\text{Li}$. Then varying the energies of the $s_{1/2}$ and $p_{1/2}$ states in $^{10}\text{Li}$ we look for a two-neutron binding energy in $^{11}\text{Li}$ close to the experimental value. This can be obtained for different couples of neutron states but leads to different mixtures of $(2s_{1/2})^2$ and $(1p_{1/2})^2$ configurations in the wave function, thus to different $^{11}\text{Li}$ radii. We have chosen three such situations, calculated the corresponding cross sections for $^{11}\text{Li} + ^{12}\text{C}$ reactions at 0.8 GeV/u and show that we are able to discriminate between these different $^{11}\text{Li}$ wave functions. Using the wave function which reproduces at best the measured cross sections for this system we consider other targets and make the calculation of cross sections at 0.8 and 0.28 GeV/u incident energies. Numerous theoretical works have reported on calculations of cross sections but most of them assumed two independent halo-neutrons in a $(1p_{1/2})$ state. In the present paper we shall compare our results mainly with the three most recent works by Al-Khalili et al. [15], Bertsch et al. [16] and Garrido et al. [17] who use correlated wave functions.

This paper is organized as follows: in Section II we introduce the optical limit of Glauber reaction model and give the detailed expressions of the cross sections; Section III deals with the choice of the numerical parameters entering the calculations; Section IV presents the results for reaction and breakup cross sections at the incident energies of 0.8 and 0.28 GeV/u on the targets $^{12}\text{C}$, $^{27}\text{Al}$, $^{63}\text{Cu}$ and $^{208}\text{Pb}$; finally in Section V we give our conclusions.

2 Model of reaction

In a Glauber eikonal model [18] the reaction cross section for nucleus-nucleus collisions is:

$$\sigma_R = \int d^2b \ (1 - P(b))$$

(1)

where $b$ is the impact parameter of the projectile relative to the target and $P(b)$ the probability that the projectile passes through the target without interacting. $P(b)$ is related to the Glauber phase by:

$$P(b) = \exp (-2Im\chi(b))$$

(2)
In the optical limit [19] with a t-matrix approach for the projectile-target effective interaction, the phase $\chi$ is simply related to the nucleon-nucleon profile function $\gamma_{NN}$ by:

$$\chi(b) \simeq \chi_0(b) = i \int \int d\mathbf{r}_i \, d\mathbf{r}_j \, \rho_p(\mathbf{r}_i) \, \gamma_{NN}(|\mathbf{s}_i - \mathbf{s}_j - b|) \, \rho_t(\mathbf{r}_j) \quad (3)$$

$\rho_p$ and $\rho_t$ are respectively the projectile and target densities and $\mathbf{s}$ is the projection of the 3-dimensional coordinate $\mathbf{r}$ on the plane perpendicular to the $z$-axis. The profile function is defined in terms of the two-dimensional Fourier transform of the nucleon-nucleon scattering amplitude $f_{NN}(q)$ by:

$$\gamma_{NN}(\mathbf{s}) = \frac{1}{2\pi i k_{NN}} \int \exp(-i\mathbf{q} \cdot \mathbf{s}) \, f_{NN}(\mathbf{q}) \, d\mathbf{q} \quad (4)$$

while the scattering amplitude can be parametrised as:

$$f_{NN}(\mathbf{q}) = \frac{k_{NN}}{4\pi} \sigma_{NN}(i + \alpha_{NN}(q)) \exp(-q^2 r_0^2 / 4) \quad (5)$$

where $\sigma_{NN}$ is the average nucleon-nucleon cross section and $r_0$ has been defined to be the range of the profile function.

For spherically symmetric target and projectile the equations (2) to (5) lead to a probability $P(b)$:

$$P(b) = \exp\left(-\frac{\sigma_{NN}}{2} \int_0^{\infty} q \, d\mathbf{q} \, \rho_p(q) \, \rho_t(q) \, e^{-q^2 r_0^2 / 4} \, J_0(qb)\right) \quad (6)$$

with $\rho_{p(t)}(q)$ the 3-dimensional Fourier transform of the projectile(target) ground state density defined below in eq.(7).

Our projectile of $^{11}$Li will be described as a core plus two valence neutrons in a two-neutron pairing model where the ground state wave function is assumed have the form:

$$\Psi_0 = \Phi_0(1,2) \cdot \Phi_c. \quad (7)$$

Here $\Phi_c$ is the core wave function and $\Phi_0$ the correlated two-neutron wave function. The coordinates $1,2$ include spin as well as spatial ($\mathbf{x}_i$) coordinates of the valence neutrons. The coordinates $\mathbf{x}_i$ are defined relative to the core center of mass.
In eq.(6) the density \( \rho_p \) is expressed in terms of coordinates \( r_i \) relative to the center of mass of the projectile as shown in Fig.1. By definition the density is:

\[
\rho_p(r) = \langle \Psi_0(1, \ldots, A) | \sum_{i=1}^{A} \delta(r - r_i) | \Psi_0(1, \ldots, A) \rangle \tag{8}
\]

Using the relations between \( r_i \) and \( x_i \) coordinates we may write the 3-dimensional Fourier transform of the \(^{11}\text{Li} \) density with respect to \( q \equiv (q_x, q_y, 0) \) as:

\[
\rho_p(q) = \int dr e^{iqr} \langle \Psi_0 | \sum_{i=1}^{A} \delta(r - r_i) | \Psi_0 \rangle \tag{9}
\]

\[
= \sum_{i=1}^{A} \langle \Psi_0 | \exp(iq \cdot (x_1 + x_2/A)) | \Psi_0 \rangle \tag{10}
\]

Inserting eq.(7) in eq.(9) we get:

\[
\rho_p(q) = \tilde{\rho}_c(q) + \tilde{\rho}_h(q) \tag{11}
\]

\[
\tilde{\rho}_c(q) = \tilde{\rho}_c(q) \int dx_1 dx_2 \rho(x_1, x_2) \exp(-iq \cdot (x_1 + x_2/A)) \tag{12}
\]

\[
\tilde{\rho}_h(q) = 2 \int dx_1 dx_2 (1 - \tilde{\rho}_c(q) \exp(-iq \cdot (x_1 + x_2/A))) \tag{13}
\]

where \( \rho_c(q) \) is the Fourier transform of the core density expressed in its own center of mass system and \( \rho(x_1, x_2) \), the two-neutron density normalized to 1, is defined as:

\[
\rho(x_1, x_2) = \sum_{\text{spins}} |\Phi_0(1, 2)|^2 \tag{14}
\]

Eq.(11) inserted in eq.(3) gives:

\[
P(b) = \tilde{P}_c(b) P_h(b) \tag{15}
\]

where \( \tilde{P}_c \) and \( P_h \) are given by eq.(6) with \( \rho_p \) replaced by \( \tilde{\rho}_c \) and \( \tilde{\rho}_h \) respectively.

Inserting eq.(15) in eq.(4) we may write \( \sigma_R \) as:

\[
\sigma_R(^{11}\text{Li}) = 2\pi \int b \, db \, (1 - \tilde{P}_c \, P_h) \tag{16}
\]

\[
= 2\pi \int b \, db \, ((1 - \tilde{P}_c) + (\tilde{P}_c(1 - P_h))) \tag{17}
\]

\[
= \tilde{\sigma}_R(^{9}\text{Li}) + \sigma_{-2n} \tag{18}
\]

where $\tilde{\sigma}_R(^9\text{Li})$ is the core contribution to $\sigma_R(^{11}\text{Li})$ but is different from the $^9\text{Li}$-core reaction cross section due to the core recoil. Indeed $\tilde{\rho}_c$ of eq. (12) can be written in a more explicit way by introducing $r_{12}$, the distance between the two halo-neutrons and $r_{cm}$, the distance between the $^{11}\text{Li}$ and $^9\text{Li}$ centers of mass related to $x_1$ and $x_2$ by:

$$r_{cm} = \frac{x_1 + x_2}{A} \quad r_{12} = x_1 - x_2$$

(19)

To simplify the equations we ignore spin variables and write, following eq. (14):

$$\rho(x_1, x_2) = |\Phi_0(x_1, x_2)|^2 \equiv |\Phi(r_{cm}, r_{12})|^2$$

(20)

Then eq. (12) can be transformed into:

$$\tilde{\rho}_c(q) = \rho_c(q) \rho_{cm}(q)$$

(21)

where $\rho_{cm}(q)$ is the Fourier transform of the density distribution of the core center of mass motion relative to the $^{11}\text{Li}$ center of mass given by:

$$\rho_{cm}(r_{cm}) = \left(\frac{A}{2}\right)^2 \int dr_{12} |\Phi(r_{cm}, r_{12})|^2$$

(22)

Eq. (17) shows that $\tilde{P}_c$ can be interpreted as the probability that the core, inside the projectile, has no interaction with the target while $\tilde{P}_h$ concerns the two halo-neutrons.

The second term of eq. (18), which we are calling $\sigma_{2n}$ represents dominantly valence particle effects. It is the contribution of the two neutrons to the reaction cross section and gives the two-halo-neutron removal cross section, including break-up, neutron transfer to the target and inelastic processes.

Note that if $\frac{x_1 + x_2}{A}$ is a small quantity, namely if the core and projectile centers of mass are close, the exponential $exp(-iq \cdot \frac{x_1 + x_2}{A})$ in eqs. (12) and (13) can be replaced by 1 and we can write:

$$\tilde{\rho}_c(q) \approx \rho(q)$$

(23)

$$\tilde{\rho}_h(q) \approx 2 \int dx_1 dx_2 \rho(x_1, x_2) exp(-iq \cdot x_1) \quad (24)$$

$$= 2 \rho_n(q)$$

(25)
where $\rho_n(q)$ is the Fourier transform of the one-neutron average density given by:

$$\rho_n(x) = \int d\mathbf{x}' \rho(\mathbf{x}, \mathbf{x}') \quad (26)$$

In this limit which is valid for normal (or heavy) nuclei we get:

$$\sigma_R = \sigma_R(^9\text{Li}) + \sigma_R(2n) \quad (27)$$

then the reaction cross section is the sum of two independent cross sections for the core and the two valence-neutrons. This however does not hold for $^{11}\text{Li}$ where the two halo-neutrons are far away from the core.

Before ending this section we discuss the validity of the reaction model presented above. Equation (3) is based on the leading order term in the cumulant expansion of the multiple scattering series [21] for the eikonal operator: $\int d^3r \rho_{2n} \exp(-\sigma_{NN} \int \rho_T dz') \approx \exp(-\sigma_{NN} \int \rho_{2n} \rho_T d^3r dz')$. According to Yabana et al. [20] this is not very well justified in the intermediate energy region and for extended halo nucleon wave functions. However its use can be justified in the cases studied in this paper in view of the fact that we work in the high energy regime and that the separation between valence and core particles cannot be done exactly working in the center-of-mass of a halo projectile nucleus as we do in the present work. Furthermore, as we shall see in the following our numerical results are quite close to those of Bertsch et al. [16] who do not use the cumulant expansion but on the other hand use a purely imaginary nucleon-nucleon scattering amplitude as we do here. A second drawback of the optical limit giving eq.(3) is the fact that only the imaginary part of the nucleon-nucleon scattering amplitude enters the calculation. The eikonal phase shift eq.(3) is thus equivalent to a phase shift due only to an imaginary nucleus-nucleus optical potential. At high energy where $\alpha_{NN}$ of eq.(5) is small this is a good approximation. However at 0.28 GeV per nucleon $\alpha_{NN}$ is small but might not be negligible. Ray [22] gives $\alpha_{PN} = 0.16$ at 325MeV. Thus Bertsch et al. [16] neglected it. On the other hand Garrido et al. [17] used a phenomenological optical potential with both real and imaginary parts for the neutron target scattering. At this point an important remark is in order. As it has been discussed in [23, 24] and references therein, the neutron target interaction optical potential varies from light to heavy targets and increasing the incident energy reflecting the change in the reaction mechanism. In particular the relative amount of real
and imaginary parts can be rather different from one target to the other leading to different amount of neutron elastic and inelastic scattering. This in turn is reflected in rather different amounts of the so called diffraction and absorptive breakup from a halo projectile. Around 300MeV experimental data [25] show that the total n+^{12}C cross section is largely dominated by the reaction cross section. The same is not true for a Pb target, for example, where, at the same energy, reaction and elastic free neutron cross sections are of comparable magnitude. We conclude that our calculated cross sections at the lower energy might underestimate the measured cross sections because the so called diffraction component of the breakup cross section, corresponding to the neutron elastic rescattering on the target [23] is calculated only with the imaginary part of the nucleon-nucleon amplitude.

3 Inputs of the calculations

We apply the reaction model of the previous section to ^{11}\text{Li} on ^{12}\text{C}, ^{27}\text{Al}, ^{63}\text{Cu} and ^{208}\text{Pb} for incident energies of 0.8 and 0.28 GeV/u. To perform the calculations we need the nucleon-nucleon parameters, average cross section and range of the profile function, target densities and the ^{11}\text{Li} wave function.

3.1 Nucleon-nucleon parameters

The nucleon-nucleon cross section \( \sigma_{NN} \) of eq.(5) averaged over the neutron-neutron \( \sigma_{nn} \), proton-proton \( \sigma_{pp} \) and neutron-proton \( \sigma_{np} \) pairs is taken as defined by Charagi and Gupta [26] and calculated for each target from their parametrisation of \( \sigma_{pp} (= \sigma_{nn}) \) and \( \sigma_{np} \) at 0.8 and 0.28 GeV. As they depend only very slightly on the targets, we use a unique value \( \sigma_{NN} = 4.1 \text{ and } 3.1 \text{ fm}^2 \) for incident energies of 0.8 and 0.28 GeV/u respectively.

The average range of the nucleon-nucleon profile function, eq.(4), is extrapolated from the tabulation given by Ray [22] and found to be 0.64 and 1.41 fm at 0.8 and 0.28 GeV respectively. The value of 0.64 fm is close to the values used at high energy by Charagi and Gupta [23] and Cziz and Maximon [13]. In most of the calculations for a ^{11}\text{Li} projectile a zero range is assumed but we shall see that this choice has some effect on the determination of the ^{9}\text{Li} radius.
3.2 Target densities

For a $^{12}$C target we use a Gaussian density with a range fitted to reproduce the radius of 2.32 fm. We have checked that an harmonic oscillator density gives the same cross sections. For the heaviest targets we take a Fermi density with parameters determined from electron scattering \[27\] and neglect differences between neutrons and protons. For $^{208}$Pb we have also used the theoretical density of Brack et al.\[28\] with different parameters for neutron and proton densities but the difference between the results on cross sections are very small compared to the uncertainties due the reaction model and to the structure model used to describe $^{11}$Li.

3.3 $^{11}$Li wave function

We construct several wave functions following ref.\[10\]. We replace continuum states by discrete states calculated in a radial box of radius 20 fm and take all neutron states up to an energy of 8 MeV. Instead of a Woods-Saxon neutron-core potential with a strength fitted separately on $1p_{1/2}$ and $2s_{1/2}$ neutron energies, as usually done, we take an usual Woods-Saxon potential with fixed parameters and correct it for the two low energy $2s_{1/2}$ and $1p_{1/2}$ resonances in $^{10}$Li by a surface term due to neutron-core vibrations coupling and fitted to each resonance \[10, 29\]. These two choices of neutron-core potential are not equivalent since the surface term in the second choice modifies the radius of the potential without changing its strength. Our modified average neutron-core potential has been shown to give simultaneous good description of the two mirror nuclei $^{11}$Be and $^{11}$N \[30\].

The neutron-neutron effective interaction is chosen of simple form:

\[
V_{nn}(1, 2) = - \left( V_0 - V_\rho \left( \frac{\rho_c(r_1)}{\rho_0} \right)^p \right) \delta(r_1 - r_2) \tag{28}
\]

where $\rho_c(r)$ and $\rho_0$ are the core and nuclear matter densities respectively. In our first papers on $^{11}$Li structure we have taken $p=1.2$ and a constraint $V_0 \simeq V_\rho$. The strengths $V_0$ and $V_\rho$ were then fitted to $^{14}$C and $^{12}$Be two-neutron separation energies. However the determination of the three parameters of eq.\(28\) is not unique and to restrict the number of parameters we have taken the two parameters $p$ and $\alpha = V_\rho/V_0$ determined by Garrido et al. \[14\] in order to reproduce the nuclear matter gap calculated with the Gogny effective...
pairing interaction. To get agreement in all domain of $k_F$ they have to assume $p=0.47$ and $\alpha=0.45$, then very different parameters compared to what is usually employed \[10, 11, 31\]. With a similar adjustment, close parameters have been found by Bertsch and Esbensen \[12\]. Taking these two parameters we have determined the third one, $V_0$, in order to reproduce closely the two-neutron separation energy in $^{14}\text{C}$. This gives $V_0=890 \text{ MeV.fm}^3$ ($V_0=440 \text{ MeV.fm}^3$).

To construct different wave functions we keep the effective pairing interaction fixed and vary the $1p_{1/2}$ and $2s_{1/2}$ neutron energies simultaneously in order to get the same two-neutron separation energy $S(2n)=0.36 \text{ MeV}$. This value corresponds to the highest value compatible with measurements but we have checked that fixing $S(2n)=0.32 \text{ MeV}$ as in ref.\[1\] or $0.295 \text{ MeV}$ as in ref.\[16\] would not change our conclusions. This way of deriving different wave functions is similar to Thompson and Zukhov \[9\] but different from Esbensen et al. \[16\] who fix the $1p_{1/2}$ energy and vary both the $2s_{1/2}$ energy and the strength of the density dependent term of the effective interaction. We present in Table I three such typical wave functions, called $F_1$, $F_2$ and $F_3$, with the corresponding $1p_{1/2}$ and $2s_{1/2}$ neutron energies, the rms-radius of $^{11}\text{Li}$ assuming the $^9\text{Li}$ radius to be 2.32 fm, the percentages of $(2s_{1/2})^2$ and $(1p_{1/2})^2$ configurations and the mean distance between the core and two halo neutrons centers of mass. This last quantity will tell us about the validity of the optical Glauber approximation. We see that for $2s_{1/2}$ and $1p_{1/2}$ energies close to the most recent experimental neutron energies the $F_2$ wave function has a large(27%) component of $(2s_{1/2})^2$ two-neutron state with a $^{11}\text{Li}$ radius compatible with the value determined from reaction cross sections by Tanihata et al. \[33\] but smaller than the value given in a more recent analysis of Al-Khalili et al. \[15\]. This question will be discussed in more detail in section IV.

4 Results and discussion

From the equations of section II and the inputs of the previous section we calculate the reaction and two-neutron removal cross sections for a $^{11}\text{Li}$ projectile on different targets at two incident energies of 0.8 and 0.28 GeV/u. We first study $^{11}\text{Li}$ on $^{12}\text{C}$ at 0.8 GeV/u. Indeed for a $^{12}\text{C}$ target the Coulomb contribution to cross sections is negligible and we can compare directly the
calculated cross sections of eqs. [15] with measurements. On the other hand at such a high energy the validity of the optical Glauber approximation is better. At 0.8 GeV/u it has been evaluated in a simple model by Tostevin et al. [35] and shown to overestimate the reaction cross section by 1 to 5 % for the domain of distances between the core and two-neutron centers of mass implied by our $^{11}$Li wave functions of Table I. It is well beyond what can be expected from nuclear structure models. We calculate $\sigma_R$ and $\sigma_{-2n}$ for our three wave functions and keep the one which reproduces at best the measurements. Then with this 'best' wave function we report on our results for the other targets at the two incident energies. At 0.28 GeV/u our results represent only a part of the breakup cross section since, as discussed in section II, the diffractive contribution to the cross sections is calculated only via an imaginary potential.

Before going to $^{11}$Li we look at the reaction $^{9}$Li + $^{12}$C at 0.8 GeV/u and investigate the sensitivity of the extracted rms radius of $^{9}$Li to the range of the nucleon-nucleon profile function.

4.1 $^{9}$Li + $^{12}$C at 0.8 GeV/u and $^{9}$Li radius

We take the $^{9}$Li core density as a Gaussian and fit the radius to reproduce the measured reaction cross section $\sigma_R = 796 \pm 6$ mb [33]. With $r_0$, the range of the n-n profile function, equal to zero one finds $< r^2 >^{1/2} = 2.32$ fm for $^{9}$Li as already given by Tanihata et al. [33]. Taking $r_0 = 0.64$ fm we have to assume a radius of 2.18 fm to recover the experimental cross section of 796 mb. This new value is significantly smaller and in fact in better agreement with the variation as $A^{1/3}$ compared to the $^{12}$C radius for example. It shows that what we call the experimental radius is in fact model dependent.

In the rest of the paper we use for definiteness $r_0 = 0.64$ fm and a radius of 2.18 fm for $^{9}$Li.

4.2 $^{11}$Li + $^{12}$C at 0.8 GeV/u

Our cross sections are summarized in Table II for the wave functions $F_1$, $F_2$ and $F_3$ of Table I. We give the three quantities, $\tilde{\sigma}_R (^{9}$Li ), $\sigma_R (^{11}$Li ) and $\sigma_{-2n} (^{11}$Li ). We see that $\tilde{\sigma}_R (^{9}$Li ), the core contribution to the $^{11}$Li reaction

\footnote{We have checked that a harmonic oscillator density leads to the same results}
cross section of eq. (18), is always larger that the reaction cross section for a $^9\text{Li}$ projectile implying that $\sigma_{-2n}$ is always smaller than $\Delta\sigma_R$, the difference between $^{11}\text{Li}$ and $^9\text{Li}$ reaction cross sections, in agreement with experimental observation when available. Comparing our results with the measured values \[38, 37\] for the three quantities $\sigma_R$, $\Delta\sigma_R$ and $\sigma_{-2n}$ simultaneously we see that an overall good agreement between theory and experiment is achieved for the $F_2$ wave function only. This wave function has 27% of $(2s_{1/2})^2$ and 52% of $(1p_{1/2})^2$ configurations respectively. The $F_1$ wave function which has 39% of $(2s_{1/2})^2$ and the $F_3$ wave function which has instead 80% of $(1p_{1/2})^2$ configuration give too large and too small cross sections. 

At this point it is instructive to compare our wave function $F_2$ to wave functions derived in other works and shown to give the best results for reaction or two-neutron removal cross sections. We again point out that our approximation of working with an $S$ matrix for the whole $^{11}\text{Li}$ instead of a product $S_nS_{c1}S_{c2}$ implies, following ref. [15], for $F_2$ which corresponds to $<r_{c2n}^2>^{1/2}=4.5$ fm an overestimate of few % which leaves our results within the experimental error bars. In a Faddeev model, Thompson and Zukhov [9] get two wave functions, $P_2$ and $P_3$, which lead to agreement with respectively the minimum and maximum of the measured reaction cross sections for $^{11}\text{Li} + ^{12}\text{C}$ at 0.8 GeV/u \[15, 35\]. For breakup at the same energy Bertsch et al. [16] et al obtain also for their two-neutron wave function $S_{23}$ of ref. [11] slightly too small cross section; considering other observables they conclude that the best mixture of $(2s_{1/2})^2$ state should be around 30-40%. 

The properties of all these wave functions are summarized in Table III. Following the above papers we give the neutron-core s-state scattering length rather than the $2s_{1/2}$ energy. We note that the percentage of $(2s_{1/2})^2$ component is similar in the four wave functions. The wave functions $P_2$, $P_3$ and $F_2$ are constructed in different structure models but with similar neutron energies, a low $2s_{1/2}$ state and a $1p_{1/2}$ resonance at 0.25-0.35 MeV lower than given by measurement. The wave function $S_{23}$ however corresponds to higher $2s_{1/2}$ and $1p_{1/2}$ states. We see from the table that for similar neutron energies and mixtures the radius calculated in Faddeev model is larger than the radii given by pairing model. Such a discrepancy could have several origins. Concerning pairing model, one can invoke the discretization of continuum used in ref. \[10, 14\] and in this work. Indeed the discrete, positive energy states are calculated in a box of finite radius and the final $^{11}\text{Li}$ radius could depend on the radius of the box. This approximation is currently used in
normal heavier nuclei where continuum has little effect on ground state properties. In $^{11}$Li where all neutron states are unbound this approximation may be suspected. An indication of the effect of the radial cut-off could be found by comparing the radii of $F_2$ and $S_{23}$ since they were calculated in a box of 20 and 40 fm respectively. Indeed $F_2$ has a smaller radius but the radius depends also on the single neutron energies which are different in the two works and perhaps on the energy cut-off which is higher for $S_{23}$. An example of a possible disease coming from continuum discretization can be found in $^6$He where, using experimental neutron resonance energies in $^5$He and the same pairing force as in $^{11}$Li, a pairing model with continuum discretization gives too large binding due to strong couplings of the resonances with higher discretized continuum states. A treatment of continuum avoiding discretization seems to give weaker couplings with however a larger radius [36]. These two points have to be confirmed but suggest a sensitivity of the radius of two-neutron halo nuclei to the nuclear model and, inside the model, to the treatment of the nonresonant continuum neutron states.

As a conclusion of this subsection our analysis shows a great sensitivity of reaction and two-neutron removal cross sections to the percentage of $(2s_{1/2})^2$ two-neutron configuration and agrees with other analysis in predicting a percentage of about 30% . It is difficult to make a precise prediction on $^{11}$Li radius which is strongly model dependent. Besides the uncertainty discussed above, there is another uncertainty coming from the $^9$Li radius. For example a radius of 3.4 fm calculated assuming a radius of 2.32 fm for $^9$Li becomes 3.3 fm if we adopt the value of 2.18 fm deduced above. Note also that the removal cross section is more sensitive to the details of the $^{11}$Li wave function and is a more stringent constraint than the reaction cross section.

4.3 $^{11}$Li +$^{27}$Al , $^{63}$Cu and $^{208}$Pb at 0.8 GeV/u

We present now our cross sections for different targets using the results of the previous subsection that the best overall agreement is obtained for our wave function $F_2$ which is used from now on in all our calculations. The cross sections are presented in Table IV (upper part) and compared with available measurements [37]. In the table the indices N and C mean nuclear and Coulomb cross sections respectively, the sum of the two being compared with measurements. For the heavier targets Coulomb contributions cannot be neglected. We have calculated $\sigma_{C_{2n}}$ following our previous paper on low
energy dipole mode and Coulomb break-up [13] which assumes that the two-halo neutrons are ejected simultaneously. The low energy dipole state and corresponding B(E1) are calculated with the same neutron energies and effective n-n interaction than used to construct the ground state wave function $F_2$. We see that $\sigma_{-2n} = \sigma_{-2n}^N + \sigma_{-2n}^C$ is in reasonable agreement with measurements. For $^{63}$Cu and $^{208}$Pb targets the total cross section is close to measurement even though $\sigma_{-2n}^C$ and $\sigma_{-2n}^N$ differ from the values extracted by Kobayashi et al [37] using general properties of the nuclear contribution. For $^{27}$Al our $\sigma^N_R$ is already close to the largest value indicated by experiment, suggesting that the Coulomb contribution should be very small, smaller than the evaluation given in ref.[30]. This is true for $\sigma_{-2n}^C$ which is less than 5% of $\sigma_{-2n}^N$. It is also the case for $^9$Li + $^{27}$Al where our calculated $\sigma_R$ of 1.22 b is only slightly larger than the measured one of 1.135±7 b.

For all targets including $^{12}$C, our cross sections are close to the higher experimental values or even slightly larger but one has to remember that using the optical limit of Glauber theory leads to an overestimate of few %. On the other hand we have to keep in mind that nuclear models introduce also some uncertainty on the final results.

4.4 $^{11}$Li +$^{12}$C, $^{27}$Al and $^{208}$Pb at 0.28 GeV/u

At this relatively low incident energy our optical Glauber approximation which involves only the imaginary part of the nucleon-nucleon scattering amplitude does not seem to be able to describe the whole two-neutron contribution to the reaction cross section. However our calculated $\sigma_{-2n}^N$ should describe well what is called the absorption or inelastic or stripping contribution ($\sigma_{-2n}^S$). We give in Table IV (bottom) our results and the measured cross sections when available [5, 6, 38]. Blank et al [39] have measured $\sigma_R$ ($^9$C+$^{12}$C ) at 280 MeV/u to be 812±34 mb. If we assume that $^9$C and $^9$Li have similar density our value of 842 mb agrees well with this result.

Our two-neutron removal cross sections are always smaller than the measured ones as expected. Bertsch et al. [16] obtain also for their two-neutron wave function $S_{23}$ of ref.[11] a too small two-neutron removal cross section. However our value $\sigma_{-2n}$ ($^{11}$Li +$^{12}$C )=0.2 b is in good agreement with the stripping contribution extracted from the measured $\sigma_{-2n}$ by Zinser et al. [8] and with the value obtained by Garrido et al. [17] with a Faddeev $^{11}$Li wave function built from low energy 2s$_{1/2}$ and 1p$_{1/2}$ neutron states in $^{10}$Li .
It is worth noticing, as it has already been remarked in [16], that the experimental two neutron removal cross section at 280 MeV/u is larger than the cross section measured at 800 MeV/u. One-step breakup models of the Glauber type discussed here [16, 34] and based on the nucleon-nucleon cross section or based on the use of a phenomenological nucleon-target optical potential [17, 20, 23] would predict an opposite trend.

Finally we notice that Zinser et al [6], with an invariant-mass analysis of $^9\text{Li} + n$ system after break-up of $^{11}\text{Li}$ on the $^{12}\text{C}$ target, have extracted energies of s and p states in $^{10}\text{Li}$ and the corresponding relative intensity of $(2s_{1/2})^2$ and $(1p_{1/2})^2$ components in $^{11}\text{Li}$. Our wave function $F_2$ qualitatively agrees with their results even though our $1p_{1/2}$ resonance energy is lower. In their analysis they see a higher broad peak at about 1.6 MeV assigned as a p-resonance. In our discrete basis simulating continuum we have a $21p_{1/2}$ state at 1.43 MeV which gives rise to $(1p_{1/2}, 2p_{1/2})$ and $(2p_{1/2})^2$ components in $^{11}\text{Li}$ with probabilities of 10 and 1.5% respectively and which could, perhaps, be identified with this experimental peak. However we have not checked that this discrete state corresponds to a resonance.

5 Conclusion

In this paper reaction and two-neutron removal cross sections have been calculated for a $^{11}\text{Li}$ projectile at two incident energies of 0.28 and 0.8 GeV per nucleon and several targets, using the optical approach to Glauber model where the $^9\text{Li}$-core recoil has been taken into account. We have worked with a finite range nucleon-nucleon profile function and observed that the “measured” $^9\text{Li}$ radius fitted to reproduce the $^9\text{Li} + ^{12}\text{C}$ reaction cross section at 0.8 GeV/u is quite sensitive to this parameter. With a range $r_0 = 0.64$ fm taken from nucleon-nucleon systematic we get $<r^2>^{1/2} = 2.18$ fm instead of 2.32 fm, the commonly accepted value obtained with $r_0 = 0$. This new value is in better agreement with a variation as $A^{1/3}$ when compared for example to the $^{12}\text{C}$ radius of 2.32 fm determined from electron scattering.

The $^{11}\text{Li}$ wave function has been calculated in a two-neutron pairing model with a zero range and density dependent pairing interaction suggested by a fit of the nuclear matter gap calculated with a Gogny finite range effective interaction. We have found that for $^{12}\text{C}$, $^{27}\text{Al}$, $^{63}\text{Cu}$ and $^{208}\text{Pb}$ targets and an incident energy of 0.8 GeV/u, reaction and two-neutron removal cross sections.
sections are simultaneously well reproduced for a $^{11}$Li wave function having about 30% of $(2s)^2$ configuration. At an incident energy of 280 MeV/u however, where what is called diffraction contribution has been measured to be 30% of the total removal cross section, we get too low cross sections because we use the optical approach to Glauber model which involves only the absorptive part of the nucleus-nucleus potential. However our results compare well with the calculated or measured absorption contribution.

Our finding of about 30% of $(2s)^2$ component in the $^{11}$Li wave function agrees with the conclusion of other works on $^{11}$Li+$^{12}$C reaction cross section at 0.8 GeV/u or removal cross section at 0.28 GeV/u. However our theoretical radius is smaller than that deduced by Al-Khalili et al using three-body Faddeev model but close to the “measured” radius deduced by Tanihata from reaction cross section assuming a Gaussian or harmonic oscillator $^{11}$Li density. This reveals a strong dependence of the radius on the structure model which comes, one may guess, from the large distance part of the wave function. In our pairing model continuum neutron states are approximated by discrete states calculated in a radial box, thus with a radial cut-off of the wave function. If the wave function has large enough components on two-neutron configurations involving continuum non resonant neutron states, the rms radius will be possibly underestimated in our model as well as in an empirical analysis of data using Gaussian or harmonic oscillator density. In the case of our ‘best’ wave function $F_2$ such configurations contribute to about 12%, thus may introduce indeed a sensitivity of the radius to the radial cut-off.

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Table 1: Binding energies, r.m.s. radii and weights of the \((2s)^2\) and \((1p_{1/2})^2\) components in the wave function obtained with different single neutron energies. The r.m.s. radius of \(^9\text{Li}\) is taken as 2.32 fm.

| wave funct. | \(\epsilon(1p_{1/2})\) (MeV) | \(\epsilon(2s)\) (MeV) | \(S(2n)\) (MeV) | <\(r^2\)>\(^{1/2}\) (fm) | \((2s)^2\) | \((1p_{1/2})^2\) |
|-------------|----------------|----------------|----------------|----------------|---------|---------|
| \(F_1\)     | 0.41           | 0.14           | 0.36           | 3.30           | 49      | 36      |
| \(F_2\)     | 0.35           | 1.20           | 0.36           | 3.11           | 27      | 54      |
| \(F_3\)     | 0.28           | 0.56           | 0.36           | 2.84           | 1       | 80      |

Table 2: Reaction and two-neutron removal cross sections defined in the text for \(^{11}\text{Li}^{+}\)^{12}\text{C} at 0.8 GeV/n calculated with the three wave functions of Table I. The experimental values are taken from references [33] and [37]. The cross sections are expressed in barns.

| wave f. | \(\tilde{\sigma}_R(^9\text{Li})\) | \(\sigma_R(^{11}\text{Li})\) | \(\sigma_R^{exp}(^{11}\text{Li})\) | \(\Delta\sigma_R\) | \(\Delta\sigma_R^{exp}\) | \(\sigma_{-2n}\) | \(\sigma_{-2n}^{exp}\) |
|----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \(F_1\)  | 0.839          | 1.127          | 1.060±0.010    | 0.331          | 0.264±0.016    | 0.288          | 0.220±0.010    |
| \(F_2\)  | 0.832          | 1.072          |                | 0.276          |                | 0.241          |                |
| \(F_3\)  | 0.818          | 1.00           |                | 0.204          |                | 0.183          |                |

Table 3: \(s\)-wave scattering length, \(1p_{1/2}\) single neutron energy, r.m.s. radius of \(^{11}\text{Li}\) and weights of \((s)^2\) and \((p_{1/2})^2\) components for wave functions \(P_2\), \(P_3\) of ref.[9], \(S_{23}\) of ref.[11] and \(F_2\) of this work. The * means that 54% is the percentage of \((1p_{1/2})^2\) state only, the sum over all \((p_{1/2})^2\) states is 65%.

| wave f. | \(a_0\) (fm) | \(\epsilon(1p_{1/2})\) (MeV) | <\(r^2\)>\(^{1/2}\) (fm) | \((2s)^2\) (%) | \((1p_{1/2})^2\) (%) |
|----------|---------------|----------------|----------------|-------------|----------------|
| \(P_2\)  | -18           | 0.25           | 3.39           | 31          | 64            |
| \(P_3\)  | -27           | 0.3            | 3.64           | 45          | 51            |
| \(S_{23}\)| -5.6          | 0.54           | 3.22           | 23.1        | 61            |
| \(F_2\)  | -11.7         | 0.35           | 3.11           | 27          | 54\(^*\)      |
Table 4: Cross sections defined in the text at two incident energies and for several targets expressed in barns. Experimental values are taken from references [6] for \( a \) and [38] for \( b \).

| Energy   | Target | \( \sigma^N_R(7\text{Li}) \) | \( \sigma^N_R(9\text{Li}) \) | \( \sigma^N_R(11\text{Li}) \) | \( \sigma^{2n}_R \) | \( \sigma^{-2n}_R \) | \( \sigma^{2n}_R \) | \( \sigma^{-2n}_R \) |
|----------|--------|-------------------------------|-------------------------------|-------------------------------|----------------|----------------|----------------|----------------|
| 0.8 GeV/n | \( ^{12}\text{C} \) | 0.796                         | 0.832                         | 1.07                          | 1.06±0.01      | 0.240          | -              | 0.240          | 0.220±0.010    |
|          | \( ^{27}\text{Al} \) | 1.22                          | 1.26                          | 1.62                          | 1.56±0.04      | 0.361          | 0.017          | 0.378          |
|          | \( ^{63}\text{Cu} \) | 1.83                          | 1.88                          | 2.41                          | 2.55±0.22      | 0.527          | 0.080          | 0.607          | 0.52±0.04      |
|          | \( ^{208}\text{Pb} \) | 3.11                          | 3.19                          | 4.03                          | 5.38±0.64      | 0.840          | 0.580          | 1.42           | 1.31±0.1       |
| 0.28 GeV/n | \( ^{12}\text{C} \) | 0.842                         | 0.870                         | 1.07                          | -              | 0.200          | -              | 0.200          | 0.28±0.03\(^a\) |
|          | \( ^{27}\text{Al} \) | 1.27                          | 1.31                          | 1.61                          | -              | 0.300          | 0.025          | 0.325          | 0.47±0.08\(^b\) |
|          | \( ^{208}\text{Pb} \) | 3.26                          | 3.33                          | 4.0                           | -              | 0.679          | 0.823          | 1.502          | 2±0.5\(^a\)    |