New Physics effects on hadronic decay asymmetries of the Top quark.

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We study some New Physics effects on the hadronic decays of the Top quark, like $t \to b \bar{b} c$, and on a forward-backward-like CP even asymmetry $A_t$ constructed in such a way that it is zero in the SM. We find that an anomalous right-handed contribution of the effective $tbW$ vertex may induce an asymmetry $A_t$ of the order of 20%. A light $W'$ boson with pure right handed couplings $tdW'$ may induce an asymmetry $A_t$ of the same order of magnitude.

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I. INTRODUCTION

The effects of flavor changing neutral currents (FCNC) in processes involving the top quark have been studied even before the actual discovery of this quark \cite{1}. While these processes are highly suppressed in the Standard Model (SM), several FCNC top-quark decays may be enhanced by several orders of magnitude in scenarios beyond the SM and some of them falling within the reach of the LHC \cite{2}. FCNC top-quark processes may thus serve as a window to test new physics effects. Recently, the FCNC contributions to the decay $t \to b \bar{b} c$ have been used to identify deviations from the SM predictions\cite{3}. In particular, it was pointed that four observables, two CP-even and two CP-odd, that are formulated in such a way that they are zero in the SM, may produce measurable values at the LHC and thus may be used to identify new physics effects. In this report, we want to focus on a forward-backward-like CP even asymmetry proposed in Ref. \cite{3} and compute the predictions
from different New Physics scenarios. In particular, we are interested in 1) the contributions from an effective $t b W$ vertex, and in 2) the contributions induced by the new heavy gauge bosons introduced to explain the $A_{FB}$ asymmetry measured at the Tevatron\cite{10}. We find that a right-handed ($f_2^R$) $t b W$ coupling may induce a CP-even asymmetry $A_t$ of the order of 20\%. On the other hand, a new light $W'$ boson, with mass of order 180-300 GeV, and with pure right-handed $t d W'$ couplings may also contribute to $A_t$ with the same order of magnitude.

In any three body decay ($M \to m_1, m_2, m_3$) one can define three invariant masses $m^2_{ij} = (p_i + p_j)^2$, that satisfy the constraint $m^2_{12} + m^2_{23} + m^2_{13} = M^2 + m^2_1 + m^2_2 + m^2_3$. Only two are independent variables and we can define two asymmetries depending which $i, j$ pairs we choose. The asymmetry proposed in Ref. \cite{3} for the decay $t \to b \bar{b} c$ depends on $\rho^2 \equiv (p_b + p_c)^2$. However, the SM amplitude squared in the limit $m_b = 0$ depends on a function of $(p_b + p_c)^2$ divided by the $W$-propagator which depends on $(p_b + p_c)^2$. Therefore, we have chosen $(p_b + p_c)^2$ instead of $\rho^2$ to parametrize the asymmetries of interest.

\section{Asymmetry in the Hadronic Decay of Top}

Let us start by writing the tree level amplitude for the hadronic decay $t \to b W^+ \to b \bar{b} c$ of the top quark. For the sake of generality, we will use the notation $t \to b W^+ \to b \bar{d} u$ for the initial formula. Even though our main interest is in $t \to b W^+ \to b \bar{b} c$, other decay modes like $t \to d W^+ \to d \bar{b} c$, or $t \to s W^+ \to s \bar{s} c$ could be addressed in a similar manner. Besides $u, d$ and $s$, we consider $c$ to be massless. We allow the $b$ quark mass to be nonzero. As one would expect, effects from non-zero $m_b$ are very small and we usually set $m_b = 0$ in our calculations, but we have kept $m_b$ terms in the initial equations as they can be useful for future studies. For the propagator of the intermediate $W$ boson we have first taken the general expression given in Ref. \cite{4} and then we have simplified it by taking $m_c = 0$. The SM amplitude is

$$M(t \to bu \bar{d}) = \bar{u}_b \frac{ig}{\sqrt{2}} \gamma_\mu P_L u_t \bar{u}_u \frac{ig}{\sqrt{2}} V_{ud} \gamma_\nu P_L v_d \left( -g^{\mu \nu} + (1 + ir_W) \frac{q^\mu q^\nu}{m_W^2} \right) G_T^{-1},$$

with $r_W \equiv \Gamma_W/m_W$ the width-to-mass ratio of the $W$ boson. By momentum conservation $p_t = p_b + p_u + p_d$, and we define $q \equiv p_u + p_d = p_t - p_b$, $t \equiv p_b + p_u$ and $\rho \equiv p_b + p_d$. 
Let us now consider the particular case $t \to b\bar{b}c$ with $m_b$ non-zero. After summing (averaging) over final (initial) spins, and neglecting terms of order higher than $m_b^2$ the amplitude squared is given by

$$|\mathcal{M}_{SM}|^2(t \to b\bar{b}c) = \frac{3}{2} g_4 |V_{cb}|^2 |V_{tb}|^2 M_2(q^2,t^2)|G_f|^2,$$

$$M_2(q^2,t^2) = t^2(m_t^2 - q^2) + m_b^2 \left( t^2 - \frac{m_t^2}{m_W^2} \right) + \frac{1}{4} m_t^2 q^2 \left( m_t^2 - q^2 \right) \left( 1 + \frac{r_W^2}{4m_W^4} - m_t^2 \right).$$

Notice that $M_2^{SM}$ depends on $t^2$ in the numerator and on $q^2$ in the denominator. One can choose to apply the constraint $q^2 + t^2 + \rho^2 = m_t^2$ and change the $t^2$ dependence for a $\rho^2$ dependence as is done in Ref. [3]. As expected, our asymmetry based on $t^2$ is equivalent to the asymmetry based on $\rho^2$. As we will see, the definition of the asymmetry involves finding a particular integration limit that satisfies a cubic equation. We will find that it is easier to solve the equation based on $t^2$.

The total width for a three body decay can be written as:

$$\Gamma = \frac{1}{(2\pi)^3 32 m_t^3} \int dm_{12}^2 \int dm_{23}^2 |\mathcal{M}|^2.$$

Let us now assign $p_1 = p_b$, $p_2 = p_c$ and $p_3 = p_b$, then $m_{12}^2 = q^2$ and $m_{23} = t^2$. Furthermore, let us rewrite the integral in terms of dimensionless variables $x \equiv q^2/m_t^2$ and $y \equiv t^2/m_t^2$ ($\hat{m}_b = m_b/m_t$, $\hat{m}_W = m_W/m_t$):

$$\Gamma^{SM}(t \to b\bar{b}c) = \frac{m_t}{28\pi^3} \int_{\hat{m}_b^2}^{(1-\hat{m}_b)^2} dx \int_{y_{\min}}^{y_{\max}} dy \, \frac{3}{2} g_4 |V_{cb}|^2 |V_{tb}|^2 f_2^{SM}(x,y).$$

Where $f_2^{SM}$ is defined as

$$f_2^{SM}(x,y) = \frac{-y^2 + ay + \frac{1}{4} c_x}{(x - \hat{m}_W^2)^2 + x^2 \frac{r_W^2}{m_W^4}}, \quad a = 1 + 2\hat{m}_b^2 - \frac{\hat{m}_b^2}{\hat{m}_W^2}, \quad c_x = \hat{m}_b^2 \left( \frac{1 + \frac{r_W^2}{4m_W^4}}{m_W^4} - 4 \right),$$

and the integration limits are

$$y_{\max(\min)} = \hat{m}_b^2 + \frac{x - \hat{m}_b^2}{2x} (1 - x - \hat{m}_b^2) \pm \frac{x - \hat{m}_b^2}{2x},$$

$$\lambda = \sqrt{1 + x^2 + \hat{m}_W^4 - 2x - 2\hat{m}_W^4 - 2x\hat{m}_W^4}.$$

Our goal is to define an asymmetry that is zero in the SM but not necessarily so for other models. We will split the integral in $y$ into two equal parts, so that:

$$\int_{y_{\min}}^{t_x} dy \, (-y^2 + ay + \frac{1}{4} c_x) = \int_{t_x}^{y_{\min}} dy \, (-y^2 + ay + \frac{1}{4} c_x).$$
After solving the integral in the above equation we obtain a cubic equation for $t_x$

$$t_x^3 - \frac{3}{2}at_x^2 + \frac{3}{4}c_xt_x + \frac{b_x}{4} = 0,$$

where $b_x$ is defined by

$$b_x = \frac{3}{2}c_x(y_{max} + y_{min}) + 3a(y_{max}^2 + y_{min}^2) - 2(y_{max}^3 + y_{min}^3).$$

There are three solutions to this equation. The following one satisfies that $0 \leq t_x \leq 1$ for $0 \leq x \leq 1$:

$$t_x = \frac{1}{2}\Re e \left[ a - (1 + i\sqrt{3})z_x^{1/3} \right], \quad (5)$$

$$z_x = a^3 - b_x + \frac{3}{2}ac_x + i\sqrt{r_x}$$

$$r_x = (2a^3 - b_x)b_x + c_x \left( c_x^2 + 3ab_x + \frac{3}{4}a^2c_x \right).$$

Notice that in the limit $\hat{m}_b \to 0$, $a \to 1$, $c_x \to 0$, $y_{min} \to 0$, $y_{max} \to 1-x$, $b_x \to (1-x)^2(1+2x)$ and the expression for $z_x$ simplifies greatly: $z_x = 1 - b_x + i\sqrt{b_x(2 - b_x)}$. It is easy to see that in this limit we end up with a rather simple analytical expression for $t_x$ in Eq. (5). As mentioned before, choosing the variable $\rho^2$ instead of $t^2$ leads to a more complicated cubic equation. Notice that the authors in Ref. [3] decided to use a numerical method to obtain the solution in their study.

The forward-backward-like asymmetry is defined as

$$A_t = \frac{\int_{0}^{1}(1-\hat{m}_b)^2 \, dx \int_{t_x}^{max} dy |M|^2 - \int_{0}^{1}(1-\hat{m}_b)^2 \, dx \int_{y_{min}}^{t_x} dy |M|^2}{\int_{0}^{1}(1-\hat{m}_b)^2 \, dx \int_{t_x}^{max} dy |M|^2 + \int_{0}^{1}(1-\hat{m}_b)^2 \, dx \int_{y_{min}}^{t_x} dy |M|^2}. \quad (6)$$

Notice that if NP effects do not change significantly the value for $BR(t \to \bar{b}bc)$ in the SM, we can indeed make an approximation and use the SM value for the denominator in $A_t$.

The Asymmetry defined in [3] that is based on $\rho^2$ yields similar results. For instance, the coupling constant $X_{LR}^V$ (expected to be of order 1) associated to a four fermion vector operator yields $A_{\rho} = 0.0393|X_{LR}^V|^2$ (see Eq. (24) in [3]). If we consider this coupling and calculate its contribution to the $A_t$ asymmetry we obtain a value of $0.041|X_{LR}^V|^2$. 
III. EFFECTIVE \(tbW\) EFFECTS ON \(A_t\)

The first example of NP effects that we want to consider is based on the effective \(tbW\) vertex\[6\]:

\[
\mathcal{L}_{tbW} = \frac{g}{\sqrt{2}} W_{\mu}^+ \bar{b} \gamma^\mu (f_1^L P_L + f_1^R P_R) t \\
- \frac{g}{\sqrt{2} M_W} \partial_\nu W_{\mu}^- \bar{b} \sigma^{\mu\nu} (f_2^L P_L + f_2^R P_R) t + h.c.,
\]

(7)

In the SM (tree level) the coupling constants are \(f_1^L = V_{tb} \approx 1\) and \(f_1^R = f_2^R = f_2^L = 0\). The amplitude squared given by this vertex is (see Eq. [2])

\[
M_2(x, y) = -2 \tilde{m}_b f_1^L f_1^R x + f_1^L f_2^R \frac{2}{\tilde{m}_W} x(y - \tilde{m}_b^2) - f_1^L f_2^L \frac{2 \tilde{m}_b}{\tilde{m}_W} x(1 - y) \\
+ (f_2^R)^2 (x + y - \tilde{m}_b^2)(1 - x - y) + (f_2^R)^2 x(y - \tilde{m}_b^2)(x + y - \tilde{m}_b^2)/\tilde{m}_W^2 \\
+ (f_2^L)^2 x(1 - y)(1 - x - y)/\tilde{m}_W^2 - f_2^L f_2^R \frac{2 \tilde{m}_b^2}{\tilde{m}_W^2} x^2.
\]

The asymmetry in terms of the effective couplings is then given by

\[
A_t = 0.01 f_1^L f_1^R + 0.04 f_1^L f_2^R + 0.60 f_1^L f_2^L \\
- 0.44(f_1^R)^2 - 1.23(f_2^L)^2 + 0.78(f_2^R)^2 + 0.01 f_2^L f_2^R.
\]

(8)

In principle, we expect the coefficients to assume values somewhat (maybe much) less than one. To estimate how large can \(A_t\) become due to effects from the general \(tbW\) vertex we will consider the bounds presented in one recent study based on \(b \to s \gamma\) measurements\[7\]: \(|f_1^R| \leq 2.5 \times 10^{-3}\), \(|f_2^L| \leq 1.3 \times 10^{-3}\) and \(|f_2^R| \leq 0.57\). The potential contributions to the asymmetry are \(A_t \leq 3 \times 10^{-5}\), \(A_t \leq 5 \times 10^{-5}\) and \(A_t \leq 0.34\) respectively.

A. Asymmetry based on \(q^2\)

We can define an asymmetry in terms of the \(x = q^2/m_t^2\) variable. Let us simplify formulas by taking \(m_b = 0\) in this case, the asymmetry is defined as

\[
A_q = \frac{\int_0^1 dy \int_{1-y}^1 dx |\mathcal{M}|^2 - \int_0^1 dy \int_0^{q_y} dx |\mathcal{M}|^2}{\int_0^1 dy \int_{1-y}^1 dx |\mathcal{M}|^2 + \int_0^1 dy \int_0^{q_y} dx |\mathcal{M}|^2}
\]

(9)

The value of \(q_y\) is

\[
q_y = \frac{m_W^2 - \sqrt{(1-y - \tilde{m}_W^2)^2 + r_W^2 a_y^2}}{2 - (1-y)(1 + r_W^2)/\tilde{m}_W^2}.
\]
For some values of $y$, the denominator in $q_y$ approaches zero. In this region, we can expand $q_y$ in terms of a variable $\beta \ll 1$:

$$\beta = \frac{2 - (1 - y)(1 + r^2_W) / \hat{m}_W^2}{(1 + r^2_W)^2}$$

$$q_y = \frac{\hat{m}_W^2}{(1 + r^2_W)^2} \left( 1 - \frac{r^2_W}{2} (\beta + \beta^2 + \ldots) \right)$$

The asymmetry in terms of the effective $tbW$ couplings as well as the four fermion operator of Ref. [3] is given by

$$A_q = 0.31 f_1^L f_2^R + 0.01 (f_1^R)^2 + 0.10 (f_2^L)^2 + 0.13 (f_2^R)^2 + 0.01 |X^Y_{LR}|^2$$

Comparing with the coefficients in Eq. [3] we see that the sensitivity of $A_q$ to NP effects is much lower than the sensitivity of $A_t$. Thus, we focus our attention on the latter.

### IV. ASYMMETRY FROM MODELS ASSOCIATED WITH $A_{FB}$

There are a good number of models proposed in the literature that attempt to explain the FB asymmetry of $t\bar{t}$ production at the Tevatron [8]. In many cases the production process is modified by NP effects that are based on FCNC couplings [9].

Let us first refer to three examples presented in Ref. [10]. In particular, we want to consider the cases where there is a heavy boson involved with a mass of 2 TeV that can contribute to the $q\bar{q} \to t\bar{t}$ process via a t-channel diagram. One is a neutral vector $Z'$, another is a charged vector $W'$, and the third one is a neutral $SU(2)$ scalar $S$. It is possible that these new bosons contribute to the $t \to b\bar{c}$ decay mode that we are interested. For instance, the top quark could make a transition $t \to b W' \to b\bar{c}$ (i.e. just as the SM decay but with $W$ being replaced by $W'$). Unlike the SM $W$ boson, the $W'$ can couple to both left and right chiralities and the amplitude squared will have a different form than Eq. (2). Another case is a heavy neutral scalar $H'$ with strong flavor violating couplings that contributes via the process $t \to c H' \to c\bar{b}$ (or maybe $t \to c H' \to c\bar{s}$). Similarly the process induced by the scalar could also stem from a heavy vector boson $Z'$. For more details we refer to Ref. [10]. Table II shows the contributions to $A_t$ from each case.

Another possible FCNC (this one is not used to explain $A_{FB}$) that we would like to consider involves the gluon field via a dimension 5 $tcg$ tensor coupling [11]. This coupling could be generated at loop level and could induce a new single top production process...
model couplings $A_t$
\begin{tabular}{|c|c|}
\hline
$e \bar{q} \gamma^\mu (f_L P_L + f_R P_R) t W'_\mu$ & $f_L = 2$, $f_R = 20$ \ \ $5 \times 10^{-3}$ \\
\hline
$y_i j \bar{q}_i q_j H'$ & $y_{tc} = 15$, $y_{bb} = 1$ \ \ 0.05 \\
\hline
$ef_R \bar{c} \gamma^\mu t_R Z'_\mu$ & $f_R = 15$ \ \ $-0.09$ \\
\hline
$\sqrt{2} g_s \frac{n_{fa}}{\Lambda} \bar{b} L \sigma^{\mu\nu} t_R G_{\mu\nu}$ & $\frac{n_{fa}}{\Lambda} \leq 0.06$ \ \ $-0.07$ \\
\hline
\end{tabular}

TABLE I: Asymmetries predicted by FC interactions. The first three involve new heavy bosons with mass $m_V = 2$ TeV that can give rise to a $t \bar{t}$ FB asymmetry $A_{F B} = 0.2$\cite{10}. The last row is for an effective dimension 5 $tcg$ operator\cite{11} (limits from Tevatron measurements\cite{12}).

$g c \rightarrow t \rightarrow b W^+$ where the top quark is produced without any additional particles. Table II shows the contribution to $A_t$ from this coupling.

It is not the purpose of this work to make a comprehensive study of NP models and their contribution to the asymmetry $A_t$. For the results shown so far it has become apparent that there are several NP scenarios, that are of interest for the Top quark research program and that could also yield a non-zero $A_t$. From the results in Table II the heavy $W'$ has no significant contribution to $A_t$. However, a light $W'$ with pure right handed FV couplings $tdW'$\cite{13} could indeed give significant contributions. Table II shows the contributions to $A_t$ for this case. Notice that the decay mode is not $t \rightarrow b \bar{b} c$ as before but $t \rightarrow d \bar{b} c$. This mode has a negligible width in the SM ($\sim 10^{-8}$ GeV), so the mere observance of this decay would signal NP effects. For this decay the denominator in Eq. (6) is not the SM value but the value obtained from the same $W'$ contribution.

\begin{tabular}{|c|c|c|}
\hline
$m_{W'}$(GeV) & $g_R$ & $BR(t \rightarrow d \bar{b} c)$ & $A_t$ \\
\hline
180 & 1.4 & $3.7 \times 10^{-3}$ & 0.30 \\
\hline
200 & 1.5 & $2.3 \times 10^{-3}$ & 0.11 \\
\hline
300 & 2.0 & $1.0 \times 10^{-3}$ & $-0.13$ \\
\hline
\end{tabular}

TABLE II: Asymmetries predicted by three values of $m_{W'}$ and coupling constant $g_R$ in the case of a light $W'$\cite{13}. All cases are consistent with $A_{F B} = 0.2$.

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