CONSTRAINT ON THE PARAMETERS OF THE INVERSE COMPTON SCATTERING MODEL FOR RADIO PULSARS

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ABSTRACT

The inverse Compton scattering (ICS) model can explain various pulse profile shapes and the diversity of the pulse profile evolution based on the mechanism where the radio emission is generated through ICS between secondary relativistic particles and radio waves from polar gap avalanches. In this paper, we study the parameter space of the ICS model for 15 pulsars that share the common pulse profile evolution phenomenon, where the pulse profiles are narrower at higher observing frequencies. Two key parameters, the initial Lorentz factor and the energy loss factor of secondary particles, are constrained using the least-squares fitting method, where we fit the theoretical curve of the “beam-frequency mapping” of the ICS model to the observed pulse widths at multiple frequencies. The uncertainty of the inclination and viewing angles are taken into account in the fitting process. It is found that the initial Lorentz factor is larger than 4000, and the energy loss factor is between 20 and 560. The Lorentz factor is consistent with the prediction of the inner vacuum gap model. Such high-energy loss factors suggest significant energy loss for secondary particles at altitudes of a few tens to hundreds of kilometers.

Key words: methods: data analysis -- pulsars: general -- radiation mechanisms: non-thermal

Online-only material: color figure

1. INTRODUCTION

Since the discovery of pulsars, a wealth of observational data have been accumulated for radio pulsars and the morphological characteristics of pulsar profiles have been widely investigated. The core–double-cone model (Rankin 1983a, 1983b), a widely used empirical model, attributes the various kinds of morphologies to geometrical origins. Assuming that the emission beam consists of a hollow core component, a nesting inner cone, and an outer cone, the observed single, double, triple, quadruple, and five-component pulse profiles can be explained as geometrical effects that the line of sight sweeps across the emission beam from different locations. Despite the success of the empirical model, physical mechanisms of radio emission still remain open. Among the models of coherent curvature radiation (Ruderman & Sutherland 1975, hereafter RS75; Gil & Snakowski 1990), plasma instabilities (Asseo et al. 1990; Luo & Melrose 1995; Weatherall 1998; Gedalin et al. 2002; Melrose & Luo 2004), and inverse Compton scattering (hereafter ICS; Qiao & Lin 1998; Qiao et al. 2001), the ICS model manifests itself in simplicity to generate the frequency-dependent beam structures and flexibility to reproduce various kinds of frequency dependent on the profile shape and the pulse width. Aside from pulse morphologies, the high brightness temperature and polarization properties of pulsars can also be explained by the ICS model (Qiao & Lin 1998; Zhang et al. 1999; Xu et al. 2000).

In the ICS model, radio emission is generated by the ICS process between secondary relativistic electrons/positrons and initial low-frequency waves (with frequency $\nu_0 \sim 10^5$–$10^6$ Hz), which are produced by avalanches of the inner vacuum gap. The relation between the altitude and the Lorentz factor of secondary particles determines the beam pattern. For example, if the Lorentz factor of secondary particles decreases as they flow out along open field lines, the emission at a given frequency would emerge at three different altitudes, which correspond to the hollow core and the inner and outer cone components. If the Lorentz factor remains constant, it would form a beam pattern of only one hollow core and one cone (the inner cone). The ICS model predicts that, in the outer cone, the emission with a higher frequency is generated at a relatively lower altitude, while the radiation in the inner cone follows the opposite relation. Observationally, the anti-correlation between the pulse width and the frequency is found in many pulsars with conal components, although a small number of pulsars show constant or increasing pulse widths at higher frequencies. The ICS model agrees with these observations and attributes these two types of relations to the outer and inner conal components, respectively (Qiao et al. 2001).

In order to constrain the parameter space of the ICS model, some authors have already compared the predictions of the ICS model with the observational data. There are three basic parameters for the ICS model: the initial Lorentz factor $\gamma_0$, the frequency of initial radio wave $\nu_0$, and the energy loss factor $\xi$ for the secondary particles. Lee et al. (2009, hereafter LCW09) found that the ratio between different radiation altitudes is insensitive to inclination angles or for radio pulsars with a large linear polarization position-angle (P.A.) swing rate. Assuming that the Lorentz factor decreases with altitude, the authors constrained the parameter space of $\gamma_0$ and the energy loss factor $\xi$ for five radio pulsars by fitting the ratios of the profile width at multiple frequencies. Four out of five pulsars show clear decreasing pulse width with frequency, while the other one shows nearly constant pulse width.

In this paper, we extend the above reverse-engineering test to a larger sample of pulsars and check the parameter space of the ICS model, where 15 pulsars with anti-correlations between the pulse width and the frequency are selected to check the radiation behavior of the conal beam in the ICS model. We calculate the “beam-frequency mapping” of the ICS model in Section 2. The geometrical method for calculating the beam...
width from the pulse width is presented in Section 3. Details on the data reduction and related results are given in Section 4. Conclusions and discussions are summarized in Section 5.

2. BEAM-FREQUENCY MAPPING IN THE ICS MODEL

In the ICS model, low-frequency waves $\nu_0$ are excited by the periodic breakdown of the inner vacuum gap. The waves are then inversely Compton scattered by the relativistic secondary particles produced in the pair cascade process in the gap. The scattered waves have the frequency

$$\nu = 2\gamma^2 \nu_0 (1 - \nu \cos \theta_i/c),$$

(1)

where $\nu$ is the velocity of the particle, $\theta_i$ is the incident angle between the particle motion direction and the incoming photons, and $\nu_0$ is assumed to be $10^6$ Hz. There are a few physical considerations for the value of $\nu_0$. RS75 pointed out that the growth and the fluctuation timescale of the inner vacuum gap might be (30–40)$\tau_0$, where $\tau_0$ is the time needed for the conversion of a gamma-ray photon to a pair in the gap, which has the order $\sim$(gap height)/$c$. The fluctuation timescale is estimated to be about 10 $\mu$s, but it is not yet conclusive even with the recent simulation (Timokhin 2010). Observationally, durations of a few microseconds were indeed observed in some pulsars (e.g., Bartel & Sieber 1978; Lange et al. 1998).

For the geometrical configuration in Figure 1, when the radiation altitudes are far from the pulsar surface, we have (Qiao et al. 2001)

$$\cos \theta_i = \frac{2 \cos \theta - (R/r)(1 - 3 \cos^2 \theta)}{\sqrt{(1 + 3 \cos^2 \theta)(1 - 2(R/r) \cos \theta + (R/r)^2)}}.$$  

(2)

where $r$ is the distance between the scattering point and the center of the pulsar, $R$ is the pulsar radius, and $\theta$ is the polar angle between the radiation location and the magnetic axis. For a dipole magnetic field, we have

$$r = R_\phi \sin^2 \theta,$$

(3)

where $R_\phi$ is the maximal radius of a given magnetic field line. The angular beam width $\theta_\mu$ of the radiation beam coming from the place with polar angle $\theta$ is

$$\tan \theta_\mu = \frac{3 \sin 2\theta}{1 + 3 \cos 2\theta}.$$  

(4)

where $\theta_\mu$ is the angle between the direction of the magnetic field at point Q and the magnetic axis (Figure 1). From the radiation altitude $r$ and Lorentz factor $\gamma$, one can determine the outgoing radio wave frequency $\nu$ and the angular beam width $\theta_\mu$ using the above equations.

The energy of the secondary particles decreases as they flow along the field lines; it is usually assumed that the Lorentz factor follows

$$\gamma = \gamma_0 [1 - \xi (r - R)/R_e],$$  

(5)

where $\gamma_0$ is the initial Lorentz factor at the top of the gap and $\xi$ is the energy loss factor. From Equations (1) and (5), one can see that $\nu_0$ and $\gamma_0$ are degenerate, i.e., different groups of $\nu_0$ and $\gamma_0$ can give identical results in the ICS model. Thus, in the subsequent test for the ICS model, we fix the value of $\nu_0$ without losing generality.

Using the above equations, the relation between the outgoing frequency $\nu$ and the beam width $\theta_\mu$, i.e., the function $\nu = v(\nu_0, \gamma_0, \xi, \theta_\mu)$, can be calculated. We refer to this $v$–$\theta_\mu$ relation as the “beam-frequency mapping.” A characteristic beam-frequency mapping with the dropping–rising–dropping pattern is given in Figure 2. Compared with the empirical core–double-cone framework for pulsar radiation, the leftmost branch corresponds to the core component, the middle rising part corresponds to the inner cone and the rightmost part corresponds to the outer cone. In this paper, we fit such beam-frequency mapping to the observational data to infer the parameters of the ICS model; the fitting method is given in the following section.

3. METHOD

In our method, we first determine $\theta_\mu$ from the pulse width using the conventional geometrical model (Gil 1984; Lyne & Manchester 1988) and then fit the beam-frequency mapping to...
the $\theta_i$ data at multiple frequencies to infer the parameters. Here the pulse width $\phi$ is measured using the Gaussian decomposition method (Wu et al. 1992; Kramer 1994; Kramer et al. 1994; LCW09).

3.1. Measurement of the Pulse Width

The Gaussian decomposition method is widely used to measure the pulse width of a pulse profile, where one models the profile with multiple Gaussian components and extracts the pulse width using curve fitting techniques, i.e., one fits the pulse profile with the following form (Wu et al. 1992; Kramer 1994; Kramer et al. 1994):

$$I = \sum_{i=1}^{n} I_i \exp \left[ -\frac{(\phi - \phi_i)^2}{2\sigma_i^2} \right]. \quad (6)$$

Here we denote the intensity of the pulse profile $I(\phi)$ as a function of the longitudinal phase $\phi$, which is modeled with $n$ Gaussians, $I_i$ is the amplitude of the $i$th Gaussian, and $\phi_i$ and $\sigma_i$ are the central phase and standard deviation of a Gaussian component, respectively.

We use 10% width throughout this paper, which is defined as the full pulse width $2\Delta\phi$ between the leading phase of the leftmost component and the trailing phase of the rightmost component down to the 10% level of their maximal intensities.

The number of Gaussians, the amplitude, the peak phase, and the typical width of each Gaussian are free parameters. Following LCW09, we use the Levenberg–Marquardt method to perform the least-squares fitting, which is accepted only when (1) the nonparametric Kolmogorov–Smirnov test is passed when comparing the distributions of residues in the on-pulse and off-pulse regions, and (2) the difference between the rms levels of two residues is close to zero, i.e., $\eta = |\sigma_{on} - \sigma_{off}| / \sigma_{eff} \simeq 0$.

The number of Gaussians is determined by minimizing $\eta$. We run a small Monte Carlo simulation to determine the final fitting parameters, where the fitting is repeated several times (about 10) with randomly generated initial values, and the final parameters are the averaged values from each individual accepted fitting. We list the measured final pulse width in Table 1.

3.2. Determining Angular Sizes of Radiation Beams

From the pulse width, we calculate the angular size of the radiation beam following the conventional geometrical model, which assumes that (1) the magnetic field in the magnetosphere is dipolar and (2) the radiation direction is parallel to the local magnetic field (Figure 3). When the line of sight locates in the $\Omega-\mu$ plane, the rate of linear polarization P.A. swing reaches the maximum $\kappa$ (Radhakrishnan & Cooke 1969), where

$$\kappa = \frac{\sin \alpha}{\sin \beta}. \quad (7)$$

Here, $\alpha$ is the inclination angle and the impact angle $\beta$ is defined as the angle between the line of sight and the magnetic axis.

Given the inclination angle $\alpha$ and $\kappa$, we can determine $\beta$. We can further substitute $\beta$, $\Delta\phi$, and $\alpha$ into the following geometrical relation to calculate the beam width $\theta_\mu$ (Gil 1984; Lyne & Manchester 1988):

$$\sin^2 \left( \frac{\theta_\mu}{2} \right) = \sin^2 \left( \frac{\Delta\phi}{2} \right) \sin \alpha \sin(\alpha + \beta) + \sin^2 \left( \frac{\beta}{2} \right). \quad (8)$$

Figure 3. Geometry of the pulsar radiation beam. The line of sight sweeps the beam from A to B and the radiation is observed at C (Lyne & Manchester 1988).

3.3. Measurement for the ICS Model Parameter

The ICS model parameters are derived via fitting the $\theta_\mu$ data at multiple frequencies with the beam-frequency mapping. On the one hand, given $N$ frequency bands at central frequencies of $\nu_i$, we can measure $N$ beam widths $\theta_{\mu,i}$ from the observation, where $i = 1, \ldots, N$. On the other hand, with Equations (1)–(4), we can calculate the predicted beam width $\theta_{\mu,ics}$ of the ICS model at those frequencies as a function of parameters $\gamma_0$ and $\xi$. Thus, we can fit the ICS model to the data by minimizing the following standard $\chi^2$,

$$\chi^2 = \sum_{i=1}^{N} \left( \frac{\theta_{\mu,i} - \theta_{\mu,ics}}{\delta\theta_\mu} \right)^2, \quad (9)$$

where $\delta\theta_\mu$ is the error of $\theta_\mu$. In our fitting, the third ICS model parameter, the background wave frequency $\nu_0$, is fixed as $10^6$ Hz. Such fixation is due to parameter degeneracy, i.e., the effect of $\nu_0$ is completely absorbed into the parameter $\gamma_0$ as indicated in Equation (1).

4. DATA REDUCTION AND RESULTS

The pulse profile data are from the European Pulsar Network Database, where 15 pulsars are selected according to two criteria: (1) a high pulse profile quality at more than five frequencies ranging over one order of magnitude and (2) the absolute values of the maximum slope rate of linear polarization P.A. $\kappa$ is much larger than 1. The second requirement, i.e., $\kappa \gg 1$, is to make sure that the ratio between angular sizes of radiation beams at different frequencies is insensitive to the inclination angle (Lee et al. 2009).

As explained in the previous section, given the inclination angle $\alpha$, the maximal P.A. swing slope $\kappa$, and the pulse width, we can calculate the angular sizes $\theta_\mu$ of the radiation beams at multiple frequencies. With the beam-frequency mapping, we fit for the ICS model parameters.

One caveat is that it is difficult to measure the inclination angle accurately. Two types of methods have been used to estimate $\alpha$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{beam.png}
\caption{Geometry of the pulsar radiation beam. The line of sight sweeps the beam from A to B and the radiation is observed at C (Lyne & Manchester 1988).}
\end{figure}
Table 1
Pulsar Parameters and Their 10% Pulse Widths as Functions of Observing Frequencies

| Name          | $P$ (s) | $B$ ($10^{12}$ G) | $\kappa$ | $\nu$ (Hz) | $2\Delta\phi$ ($^\circ$) | $\eta$ | Reference |
|---------------|---------|-------------------|----------|------------|--------------------------|-------|-----------|
| B0301+19      | 1.388   | 1.36$^{-17}$      | 0.408    | 20.2 ± 3.8 | 0.206                    | GL98  |
|               |         |                   | 0.61     | 19.6 ± 1.1 | 0.078                    | GL98  |
|               |         |                   | 0.925    | 18.9 ± 0.7 | 0.189                    | GL98  |
|               |         |                   | 1.408    | 17.7 ± 0.7 | 0.133                    | GL98  |
|               |         |                   | 1.642    | 16.3 ± 0.5 | 0.154                    | GL98  |
|               |         |                   | 4.85     | 15.8 ± 0.4 | 0.035                    | HX97  |
| B0525+21      | 3.745   | 12.4 31           | 0.408    | 21.7 ± 1.9 | 0.031                    | GL98  |
|               |         |                   | 0.61     | 22.1 ± 0.7 | 0.303                    | GL98  |
|               |         |                   | 0.925    | 21.7 ± 2.2 | 0.347                    | GL98  |
|               |         |                   | 1.642    | 20.9 ± 0.7 | 0.458                    | GL98  |
|               |         |                   | 4.85     | 17.7 ± 0.6 | 1.607                    | HX97  |
| B0540+23      | 0.246   | 1.97 9            | 0.91     | 43.2 ± 2.8 | 0.068                    | GL98  |
|               |         |                   | 1.408    | 41.9 ± 2.0 | 0.235                    | GL98  |
|               |         |                   | 1.642    | 43.7 ± 2.9 | 0.044                    | GL98  |
|               |         |                   | 4.85     | 38.8 ± 1.4 | 0.069                    | HX97  |
|               |         |                   | 10.45    | 34.0 ± 4.0 | 0.094                    | HX97  |
| B0823+26      | 0.531   | 0.96 4.5          | 0.408    | 21.1 ± 4.3 | 0.067                    | GL98  |
|               |         |                   | 0.61     | 21.2 ± 2.4 | 0.369                    | GL98  |
|               |         |                   | 0.925    | 18.9 ± 2.5 | 0.397                    | GL98  |
|               |         |                   | 1.404    | 12.8 ± 0.7 | 0.839                    | GL98  |
|               |         |                   | 4.85     | 12.7 ± 1.2 | 0.727                    | HX97  |
|               |         |                   | 10.55    | 10.9 ± 0.3 | 0.803                    | SGG95 |
| B0919+06      | 0.431   | 2.45 7            | 0.408    | 27.9 ± 1.6 | 0.290                    | GL98  |
|               |         |                   | 0.61     | 22.8 ± 1.5 | 0.411                    | GL98  |
|               |         |                   | 0.925    | 17.1 ± 0.6 | 0.115                    | GL98  |
|               |         |                   | 1.408    | 18.8 ± 1.2 | 0.070                    | GL98  |
|               |         |                   | 1.642    | 15.6 ± 2.4 | 0.087                    | GL98  |
|               |         |                   | 4.85     | 10.4 ± 1.1 | 0.233                    | HX97  |
|               |         |                   | 10.55    | 7.6 ± 0.3  | 0.149                    | SGG95 |
| B1039−19      | 1.386   | 1.16 18           | 0.4      | 24.8 ± 1.3 | 0.229                    | ALN94 |
|               |         |                   | 0.61     | 20.3 ± 0.4 | 0.394                    | GL98  |
|               |         |                   | 0.8      | 22.4 ± 1.0 | 0.139                    | ALN94 |
|               |         |                   | 0.925    | 19.5 ± 1.0 | 0.230                    | GL98  |
|               |         |                   | 1.408    | 17.6 ± 0.5 | 0.147                    | GL98  |
|               |         |                   | 1.642    | 18.1 ± 0.8 | 0.326                    | GL98  |
|               |         |                   | 4.85     | 17.6 ± 1.2 | 0.032                    | KKW97 |
| B1133+16      | 1.188   | 2.13 12           | 0.102    | 22.9 ± 1.3 | 1.633                    | KL99  |
|               |         |                   | 0.408    | 14.7 ± 0.6 | 0.095                    | GL98  |
|               |         |                   | 0.925    | 13.4 ± 0.7 | 0.077                    | GL98  |
|               |         |                   | 1.408    | 14.0 ± 1.5 | 0.959                    | GL98  |
|               |         |                   | 1.71     | 12.3 ± 0.8 | 0.614                    | HX97  |
|               |         |                   | 2.25     | 11.1 ± 0.4 | 0.645                    | KX96  |
|               |         |                   | 4.85     | 11.2 ± 0.6 | 2.485                    | HX97  |
|               |         |                   | 10.45    | 9.1 ± 0.8  | 0.577                    | HX97  |
| B1702−19      | 0.299   | 1.13 14           | 0.61     | 20.3 ± 0.7 | 0.388                    | GL98  |
|               |         |                   | 0.925    | 18.2 ± 0.9 | 0.121                    | GL98  |
|               |         |                   | 1.408    | 17.4 ± 0.4 | 0.437                    | GL98  |
|               |         |                   | 1.642    | 17.2 ± 0.4 | 0.407                    | GL98  |
|               |         |                   | 4.85     | 16.7 ± 0.4 | 0.091                    | KKW97 |
| B1706−16      | 0.653   | 2.05 9            | 0.408    | 14.8 ± 1.0 | 0.272                    | GL98  |
|               |         |                   | 0.61     | 14.2 ± 0.6 | 0.251                    | GL98  |
|               |         |                   | 0.925    | 14.6 ± 0.8 | 0.300                    | GL98  |
|               |         |                   | 1.408    | 14.5 ± 0.5 | 0.065                    | GL98  |
|               |         |                   | 4.75     | 10.8 ± 0.5 | 0.103                    | SGG95 |
| B2020+28      | 0.343   | 0.82 15           | 0.41     | 18.5 ± 0.5 | 0.522                    | GL98  |
|               |         |                   | 0.925    | 18.5 ± 0.6 | 2.365                    | GL98  |
|               |         |                   | 1.408    | 19.7 ± 0.4 | 0.549                    | GL98  |
|               |         |                   | 1.642    | 19.4 ± 0.6 | 1.893                    | GL98  |
|               |         |                   | 4.85     | 18.6 ± 0.8 | 1.780                    | HX97  |
|               |         |                   | 10.45    | 14.9 ± 0.7 | 0.275                    | HX97  |
Table 1 (Continued)

| Name          | $P$ (s) | $B$ ($\times 10^{12}$ G) | $\kappa$ | $\nu$ (Hz) | $2\Delta \phi$ (°) | $\eta$ | Reference |
|---------------|---------|--------------------------|----------|------------|---------------------|-------|-----------|
| B2021+51      | 0.529   | 1.29                     | 4        | 0.41       | 25.6 ± 1.4          | 0.489 | GL98      |
|               |         |                          |          | 0.925      | 22.3 ± 0.5          | 0.788 | GL98      |
|               |         |                          |          | 1.408      | 22.9 ± 0.8          | 1.567 | GL98      |
|               |         |                          |          | 4.75       | 17.8 ± 0.3          | 0.177 | SGG95     |
|               |         |                          |          | 10.45      | 13.0 ± 0.4          | 0.045 | HX97      |
|               |         |                          |          | 14.6       | 12.7 ± 0.6          | 0.027 | KXJ96     |
| B2045−16      | 1.962   | 4.69                     | −30      | 0.606      | 17.8 ± 0.2          | 2.131 | GL98      |
|               |         |                          |          | 0.925      | 17.9 ± 0.8          | 0.080 | GL98      |
|               |         |                          |          | 1.408      | 16.8 ± 0.3          | 1.746 | GL98      |
|               |         |                          |          | 1.642      | 16.4 ± 0.4          | 1.788 | GL98      |
|               |         |                          |          | 4.85       | 15.7 ± 0.5          | 0.203 | HX97      |
| B2154+40      | 1.525   | 2.32                     | 8        | 0.408      | 32.1 ± 2.1          | 0.410 | GL98      |
|               |         |                          |          | 0.61       | 31.4 ± 0.7          | 0.294 | GL98      |
|               |         |                          |          | 0.925      | 28.8 ± 1.4          | 0.105 | GL98      |
|               |         |                          |          | 1.408      | 29.9 ± 2.0          | 0.108 | GL98      |
|               |         |                          |          | 1.642      | 27.4 ± 1.2          | 0.191 | GL98      |
|               |         |                          |          | 4.85       | 26.3 ± 1.2          | 0.178 | HX97      |
| B2310+42      | 0.349   | 0.20                     | 4        | 0.408      | 19.4 ± 1.2          | 0.049 | GL98      |
|               |         |                          |          | 0.61       | 18.4 ± 0.9          | 1.417 | GL98      |
|               |         |                          |          | 0.925      | 19.6 ± 1.1          | 0.194 | GL98      |
|               |         |                          |          | 1.408      | 17.8 ± 0.7          | 1.581 | GL98      |
|               |         |                          |          | 1.615      | 17.6 ± 0.5          | 1.419 | SGG95     |
|               |         |                          |          | 4.85       | 15.7 ± 0.3          | 0.221 | HX97      |
|               |         |                          |          | 10.55      | 15.2 ± 0.8          | 0.127 | SGG95     |
| B2319+60      | 2.256   | 4.03                     | −8       | 0.925      | 24.7 ± 0.8          | 0.079 | GL98      |
|               |         |                          |          | 1.408      | 23.6 ± 1.0          | 0.379 | GL98      |
|               |         |                          |          | 1.642      | 22.6 ± 0.3          | 0.188 | GL98      |
|               |         |                          |          | 4.85       | 19.8 ± 0.6          | 0.146 | HX97      |
|               |         |                          |          | 10.55      | 20.6 ± 1.4          | 0.554 | SGG95     |

References. Pulsar parameters of the periods and the surface magnetic field intensities are taken from the ATNF pulsar catalog (Manchester et al. 2005). The maximal slopes of P.A. curves $\kappa$ are from Lyne & Manchester (1988). The original references for the profile data are given in the last column of the table as the following abbreviations: ANT94: Arzoumanian et al. (1994); GL98: Gould & Lyne (1998); HX97: von Hoensbroech & Xilouris (1997); KKW97: Kijak et al. (1997); KL99: Kuz’min & Losovskii (1999); KXJ96: Kramer et al. (1996); SGG95: Seiradakis et al. (1995).

The first type of method is fitting the polarization P.A. data with the conventional (Radhakrishnan & Cooke 1969; Lyne & Manchester 1988; Everett & Weisberg 2001) or modified versions (Blaskiewicz et al. 1991) of the rotation vector model. The second type of method is based on some statistical relations between the pulse period and the opening angle of the emission beam (Rankin 1990; Kijak & Gil 1997). Due to the limitation of each method and data quality, in many cases, different authors get inconsistent values of the inclination angle. The two major methods are used here to reduce the effect of uncertainty of the inclination angle. First, we select pulsars with a larger $\kappa$. As shown by LCW09, although the absolute beam width relies on the inclination angle, the ratio between the beam widths at multiple frequencies is insensitive to the inclination angle. Such an invariant ratio is already able to determine the ICS model parameters. Second, we perform the measurement of $\theta_\mu$ and the fitting with the ICS model for all the possible $\alpha$ values whose range is presented in Table 2. By enumerating all the possible $\alpha$, we ensure that the invalid parameter space is really not viable.

We show the allowed regions in the $\gamma_0$–$\xi$ parameter space in Figure 4, where the contours are at the 2$\sigma$ level. We also show the results using the method in LCW09. The results immediately

Table 2

| Name          | $\alpha$ (°) | Range of $\xi$ | Range of $\gamma_0$ | References for Possible $\alpha$ |
|---------------|--------------|----------------|--------------------|----------------------------------|
| B0301+19      | 5–90         | 50–420         | 5000–20000         | 1, 2, 3, 5, 10                   |
| B0525+21      | 14–67        | 50–560         | 5500–20000         | 1, 4, 5, 6, 8, 9, 10             |
| B0540+23      | 17–90        | 10–40000       | 16000–16000        | 1, 2, 4, 8                       |
| B0823+26      | 60–90        | 26–50          | 5800–10000         | 1, 2, 5, 6, 9                    |
| B0919+06      | 58–90        | 34–80          | 4200–7600          | 1, 8                            |
| B1039−19      | 23–31        | 175–410        | 11000–20000        | 5, 10                           |
| B1133+16      | 33–60        | 80–425         | 6500–20000         | 1, 5, 6, 8, 9                    |
| B1702−19      | 75           | 64–110         | 10000–20000        | 5                               |
| B1706−16      | 40–70        | 70–290         | 6000–20000         | 5, 6                            |
| B2020+28      | 60–71        | 60–130         | 9200–20000         | 5, 6                            |
| B2021+51      | 16–30        | 68–170         | 11500–20000        | 9                               |
|               |              |                |                   |                                  |
| B2045−16      | 24–90        | 27–375         | 8000–20000         | 2, 5                            |
| B2154+40      | 15           | 210–430        | 12000–20000        | 5                               |
| B2310+42      | 24–56        | 25–175         | 4000–15000         | 5, 6, 7                         |
| B2319+60      | 12–35        | 85–245         | 10800–20000        | 5, 8                            |

References. (1) Blaskiewicz et al. 1991; (2) von Hoensbroech & Xilouris 1997; (3) Narayan & Vivekanand 1982; (4) Gould 1994; (5) Kuz’min & Wu 1992; (6) Lyne & Manchester 1988; (7) Wang & Wu 2003; (8) Kijak & Gil 1997; (9) Rankin 1993; (10) Everett & Weisberg 2001.
Figure 4. Joint parameter space of the initial Lorentz factor $\gamma_0$ and the energy loss factor $\xi$. The gray areas represent the allowed parameter space derived at the 95% confidence level by fitting the index of the $\theta_{\mu}-\nu$ relation, which is insensitive to inclination angles (LCW09). The solid-line and dotted-line contours represent the 2σ confidence regions of $\gamma_0$ and $\xi$ constrained by fitting the absolute values of $\theta_{\mu}$ for different inclination angles, where the symbol “+” stands for the best-fit $\gamma_0$ and $\xi$. For each pulsar, some inclination angles within the possible range given in the literature are selected as examples and their values are shown within the contour regions. For a particular inclination angle, the joint parameter space is the overlapping region of the gray area and the contour corresponding to the inclination angle. See Table 2 for details of the joint parameter space.

(A color version of this figure is available in the online journal.)
Figure 4. (Continued)
show that fitting the absolute values of $\theta_0$ can tell us how the allowed parameter space depends on the inclination angle. However, it is not sensitive enough to constrain $\gamma_0$, because the outer cone branches of beam-frequency curves with the same $\xi$ but different $\gamma_0$ are compressed very much along the $\theta_0$ dimension, as presented in Figure 2. The constrained parameter ranges are also listed in Table 2. The general features are summarized as follows.

1. Combining all the samples, the allowed parameter spaces are $\gamma_0 > 4000$ and $20 < \xi < 560$. Since the Lorentz factor of secondary particles is likely well below $10^5$ according to the inner vacuum gap model, we set the cutoff of $\gamma_0$ to be 20,000 in calculation. No clear correlation is found between the constrained parameter values and observational quantities, e.g., the pulse width, the surface magnetic field, the pulsar age, and the spin-down energy loss rate.

2. It shows a general trend wherein both the initial Lorentz factor $\gamma_0$ and the energy loss factor $\xi$ increase as $\alpha$ decreases. This is because $\theta_0$ and the emission altitude $r$ become smaller when $\alpha$ decreases; it requires faster energy loss so that the Lorentz factor decreases to proper values to produce the radio emission at the observed frequencies.

5. CONCLUSIONS AND DISCUSSIONS

We collected a sample of 15 pulsars that have the anti-correlation phenomenon between the pulse width and the observing frequency. Their pulse widths are measured from multi-frequency profiles by using the Gaussian decomposition techniques. Beam widths are calculated with the classical beam geometry model for possible inclination angles. They are then fitted with the beam-frequency relation of the ICS model to constrain two free parameters, i.e., the initial Lorentz factor $\gamma_0$ and the energy loss factor $\xi$. The fitting is performed for a group of possible $\alpha$. It shows a trend wherein $\gamma_0$ and $\xi$ could be larger for smaller inclination angles. The allowed parameter spaces are $\gamma_0 > 4000$ and $20 < \xi < 560$. The range of the initial Lorentz factor is generally consistent with the prediction of the inner vacuum gap model (Timokhin 2010). The constrained values of $\xi$ suggest that the secondary particles need to lose a large fraction of their initial energy.

In our calculations, we assume $v_0 = 10^8$ Hz without losing generality due to the parameter degeneracy, which also agrees with the physical consideration of the inner vacuum gap model. If we take $v_0 = 10^7$ Hz, according to Equation (1), the resulting Lorentz factors would be about three times higher than the present results.

Our results indicate a bit higher Lorentz factor $\gamma_0 > 4000$ than the traditional picture of the inner vacuum gap model ($\sim 1000$ in RS75), but this agrees with the requirement to produce the radio luminosity of pulsars in the ICS model. Noting that the primary particles usually have a Lorentz factor of $10^5$--$10^7$ due to radiation reaction (e.g., RS75), the present results imply that the multiplicity of secondary particles is about a few hundred. With such a multiplicity, only a small fraction of secondary particles that are coherent (Qiao & Lin 1998), or even incoherent radiation in some cases (Zhang et al. 1999), is sufficient to account for the observed radio luminosity because the ICS emission of a single particle is much more efficient than the curvature radiation. Harding & Muslimov (2011) found that a slightly asymmetric distortion can significantly increase the accelerating electric field on one side of the polar cap and, combined with a smaller field line radius of curvature, would lead to larger pair multiplicity. This increase of the primary accelerating electric field may also be the origins of high Lorentz factors of the secondary particles.

The result that $20 < \xi < 560$ indicates an efficient energy loss for the secondaries. It has been suggested that the resonant ICS between secondaries and thermal photons from the pulsar surface can cause efficient energy loss at a certain surface temperature, but it is only effective within about one stellar radius above the polar cap surface (Zhang et al. 1997; Lyubarskii & Petrova 2000). The other emission mechanisms are much less efficient (Sturner 1995) for secondaries and can hence be neglected. Therefore, the mechanism for such efficient energy loss is still unclear. LCW09 revealed that the decay of the secondary Lorentz factor varies significantly from pulsar to pulsar. For some pulsars (e.g., PSR B2016+28), whose profile width nearly remains constant at different frequencies, a small value of $\xi$ is thus inferred. Such diversity of energy loss behaviors may be due to the interplay between the residual electrical field and the radiation reaction at higher altitudes, whose detailed studies are still in demand.

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REFERENCES

Arzoumanian, Z., Nice, D. J., Taylor, J. H., & Thorsett, S. E. 1994, ApJ, 422, 671
Asoeo, E., Pelletier, G., & Sol, H. 1990, MNRAS, 247, 529
Bartel, N., & Sieber, W. 1978, A&A, 70, 307
Blaskiewicz, M., Cordes, J. M., & Wasserman, I. 1991, ApJ, 370, 643
Everett, J. E., & Weisberg, J. M. 2001, ApJ, 553, 341
Gedalin, M., Gruman, E., & Melrose, D. B. 2002, MNRAS, 337, 422
Gil, J. 1984, A&A, 131, 67
Gil, J. A., & Snadowski, J. K. 1990, A&A, 234, 237
Gould, D. M. 1994, PhD thesis, Univ. Manchester
Gould, D. M., & Lyne, A. G. 1998, MNRAS, 301, 235
 Harding, A. K., & Muslimov, A. G. 2011, ApJ, 726, L10
 Kijak, J., & Gil, J. 1997, MNRAS, 288, 631
 Kijak, J., Kramer, M., Wielebinski, R., & Jessner, A. 1997, A&A, 318, 63
 Kramer, M. 1994, A&A, 107, 527
 Kramer, M., Wielebinski, R., & Jessner, A., Gil, J. A., & Seiradakis, J. H. 1994, A&A, 107, 515
 Kramer, M., Xilouris, K. M., Jessner, A., et al. 1996, A&A, 322, 846
 Kuz’min, A. D., & Lofofsky, B. Ya. 1999, Astron. Rep., 43, 288
 Kuz’min, A. D., & Wu, X. J. 1992, Ap&SS, 190, 209
 Lange, Ch., Kramer, M., Wielebinski, R., & Jessner, A. 1998, A&A, 332, 111
 Lee, K. J., Cui, X. H., Wang, H. G., Qiao, G. J., & Xu, R. X. 2009, ApJ, 703, 507
 Luo, Q., & Melrose, D. B. 1995, MNRAS, 276, 372
 Lyne, A. G., & Manchester, R. N. 1988, MNRAS, 234, 477
 Lyubarsky, Y. E., & Petrova, S. A. 2000, A&A, 355, 406
 Manchester, R. N., Hobbs, G. B., Teoh, A., & Hobbs, M. 2005, AJ, 129, 693
 Melrose, D. B., & Luo, Q. 2004, Phys. Rev. E, 70, 6404
 Narayan, R., & Vivekanand, M. 1988, MNRAS, 234, 671
 Qiao, G. J., & Lin, W. P. 1998, A&A, 333, 406
 Qiao, G. J., Lin, W. P. 1998, A&A, 333, 406
 Qiao, G. J., Liu, J. F., Zhang, B., & Han, J. L. 2001, A&A, 377, 964
 Qiao, G. J., & Lin, W. P. 1998, A&A, 333, 406
 Radhakrishnan, V., & Cooke, D. J. 1969, Astrophys. Lett., 3, 225
 Rankin, J. M. 1983a, ApJ, 274, 333
 Rankin, J. M. 1983b, ApJ, 274, 359
 Rankin, J. M. 1990, ApJ, 352, 247
 Rankin, J. M. 1993, ApJS, 85, 145
Ruderman, M. A., & Sutherland, P. G. 1975, ApJ, 196, 51
Seiradakis, J. H., Gil, J. A., Graham, D. A., et al. 1995, A&AS, 111, 205
Sturner, S. J. 1995, ApJ, 446, 292
Timokhin, A. N. 2010, MNRAS, 408, 2092
von Hoensbroech, A. V., & Xilouris, K. M. 1997, A&A, 324, 981
Wang, H. X., & Wu, X. J. 2003, Chin. J. Astron. Astrophys., 3, 469
Weatherall, J. C. 1998, ApJ, 506, 341
Wu, X. J., Zhang, C. S., Shi, Y. L., & Xu, W. 1992, in Proc. Supernovae and Their Remnants, ed. Q. Li, E. Ma, & Z. Li, 113
Xu, R. X., Liu, J. F., Han, J. L., & Qiao, G. J. 2000, ApJ, 535, 354
Zhang, B., Hong, B. H., & Qiao, G. J. 1999, ApJ, 513, L111
Zhang, B., Qiao, G. J., & Han, J. L. 1997, ApJ, 491, 891