$D_6$ Family Symmetry and Cold Dark Matter at LHC

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Abstract

We consider a non-supersymmetric extension of the standard model with a family symmetry based on $D_6 \times Z_2 \times Z_2$, where one of $Z_2$’s is exactly conserved. This $Z_2$ forbids the tree-level neutrino masses and simultaneously ensures the stability of cold dark matter candidates. From the assumption that cold dark matter is fermionic we can single out the $D_6$ singlet right-handed neutrino as the best cold dark matter candidate. We find that an inert charged Higgs with a mass between 300 and 750 GeV decays mostly into an electron (or a positron) with a large missing energy, where the missing energy is carried away by the cold dark matter candidate. This will be a clean signal at LHC.

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I. INTRODUCTION

It is now clear that the standard model (SM) has to be extended at least in two ways: Neutrino masses \[1\] and cold dark matter (CDM) \[2\] have to be accommodated. Since neutrinos are a part of dark matter \[2\], the nature of cold dark matter and neutrinos may be somehow related. A natural possibility to connect apparently unrelated physics is a symmetry. By introducing an unbroken discrete symmetry, for instance, we can make a weakly interacting neutral particle stable so that it can become a CDM candidate, while making this discrete symmetry responsible for the smallness of neutrino masses. Proposals along this line of thought have been suggested in refs. \[3, 4, 5, 6, 7\]. The basic idea of refs. \[3, 4, 5\] is to introduce an unbroken $Z_2$ symmetry to forbid tree-level neutrino masses and to assign an odd $Z_2$ parity to CDM candidates. For this mechanism to work, one introduces an additional $SU(2)_L$ doublet Higgs, which does not acquire VEV \[8\], along with right-handed neutrinos. We will adopt this idea in this paper. However, the introduction of an additional Higgs doublet and the right-handed neutrinos into the SM introduces additional ambiguities in the Yukawa sector. Because of these ambiguities, it would be difficult to make quantitative tests of this idea, except may be the existence of the additional Higgs particles which could be found at LHC.

A natural guidance to constrain the Yukawa sector is a flavor symmetry \[1\]. In this paper we would like to consider a nonabelian discrete symmetry $D_6$, which is one of the dihedral groups $D_N$. The smallest dihedral group is $D_3$ which is isomorphic to the smallest nonabelian group $S_3$. $D_4$ has been used as a flavor symmetry in refs. \[12, 13, 14\], while $D_5$ and $D_7$ have been considered in refs. \[15, 16\], respectively. We will consider a non-supersymmetric extension of the SM, which possesses a flavor symmetry based on $D_6 \times \hat{Z}_2 \times Z_2$, where $Z_2$ is exactly conserved and $D_6 \times \hat{Z}_2$ is spontaneously broken by the VEV of the $SU(2)_L$ doublet Higgs fields. The unbroken $Z_2$ ensures the stability of the CDM candidate and at the same time forbids the tree-level neutrino masses, while $\hat{Z}_2$ is responsible for the suppression of FCNCs in the quark sector. The $D_6$ assignment is so chosen that the leptonic sector of the model is made predictive as possible without having contradictions with experimental observations in this sector: There are eight independent parameters to describe six lepton masses, and three mixing angles and three CP violating phases of the neutrino mixing matrix $V_{\text{MNS}}$. This will be discussed in Sect. III, where we will calculate the radiative neutrino mass matrix \[17\].

The $\mu \rightarrow e\gamma$ amplitude in our model does not vanish \[18\]. We will investigate it in Sect. IV, making it possible to single out the best CDM candidate, once we assume that CDM is fermionic. The fermionic CDM candidate is the $D_6$ singlet right-handed neutrino. Its mass and the masses of the additional Higgs fields are constrained by $\mu \rightarrow e\gamma$ and the observed dark matter relic density. In Sect. V we will plot these masses from various aspects. It will

\[1\] Recent flavor models are reviewed, for example, in \[9, 10\] and \[11\]
turn out, among other things, that the $D_6$ singlet inert charged Higgs with a mass between 300 and 750 GeV decays mostly into an electron (or a positron) with a large missing energy, where the missing energy is carried away by the CDM candidate. They are within the accessible range of LHC [19]. Sect. V is devoted to summarizing our findings.

II. THE MODEL

A. $D_6$ group theory

The dihedral groups, $D_N$ ($N = 3, 4, \ldots$), are nonabelian finite subgroups of $SO(3)$, where all the irreps. of $D_N$ are real, and there exist only two- and one-dimensional irreps [20, 23]. The irreps of $D_6$ are $2, 2', 1, 1'', 1''', 1''''$, and the group multiplication rules are given as follows [20]:

\[
\begin{align*}
1' \times 1' &= 1'' \times 1'' = 1''' \times 1''' = 1, \\
1' \times 1'' &= 1'', \\
1' \times 1''' &= 1'''.
\end{align*}
\]  

(1)

The Clebsch-Gordan coefficients for multiplying the irreps are [23]

\[
\begin{align*}
2 \times 2 &= 1' + 1 + 2' \\
(a_1) \times (b_1) &= (a_1 b_1 + a_2 b_2) \quad (a_1 b_2 - a_2 b_1) \quad (-a_1 b_1 + a_2 b_2) \\
2' \times 2' &= 1 + 1' + 2' \\
&= \begin{pmatrix} x_1 y_2 - x_2 y_1 \\ x_1 y_2 + x_2 y_1 \end{pmatrix} \begin{pmatrix} x_1 y_1 - x_2 y_2 \\ x_1 y_1 + x_2 y_2 \end{pmatrix},
\end{align*}
\]

\[
\begin{align*}
2 \times 2' &= 1'' + 1'' + 2 \\
(a_1) \times (b_2) &= (a_1 b_2 + a_2 b_1) \quad (a_1 b_2 - a_2 b_1) \\
&= \begin{pmatrix} x_1 a_2 + x_2 a_1 \\ x_1 a_1 - x_2 a_2 \end{pmatrix} \begin{pmatrix} x_1 a_1 + x_2 a_2 \\ x_1 a_2 - x_2 a_1 \end{pmatrix}.
\end{align*}
\]

In what follows we will use these multiplication rules to construct a flavor model.

B. The Lepton Yukawa interaction

The Yukawa sector of the SM contains a large number of independent parameters. We would like to reduce this number to as few as possible by a symmetry argument and then to

\[2^{\text{The "covering group" of } D_N \text{ is } Q_{2N} [24], \text{ which contains complex irreps. } Q_4 \text{ and } Q_6 \text{ have been considered in refs. [22] and [20, 23], respectively.}}\]
understand the flavor structure in terms of the symmetry. In the following discussions we will concentrate on the leptonic sector of a non-supersymmetric model.

As we will see below, it is possible to construct a model with a family symmetry based on $D_6 \times \hat{Z}_2 \times Z_2$, in which (1) the neutrino mass matrix $M_\nu$ contains only four real parameters, (2) the maximal mixing of atmospheric neutrinos follows from the family symmetry, and (3) the absolute value of the $e - 3$ element of the neutrino mixing matrix $V_{MNS}$ can be expressed in terms of the charge lepton masses. The leptons $L, e^c, n$ and $SU(2)_L$ Higgs doublets $\phi, \eta$ belong to irreducible representations of $D_6$, and we give the $D_6 \times \hat{Z}_2 \times Z_2$ assignment in Table I and II. $\hat{Z}_2 \times Z_2$ is an abelian factor which we impose on the model, where $Z_2$ shall remain unbroken after the spontaneous symmetry breaking of $SU(2)_L \times U(1)_Y$. We use the two component notation for the Weyl spinors in an obvious notation, where $e^c$'s are the charge-conjugate states of the right-handed electron family. Under $Z_2$ (which plays the role of $R$ parity in the MSSM), only the right-handed neutrinos $n_S, n_I$ and the extra Higgs $\eta_S, \eta_I$ are odd. The quarks are assumed to belong to 1 of $D_6$ with $(+, +)$ of $\hat{Z}_2 \times Z_2$ so that the quark sector is basically the same as the SM, where the $D_6$ singlet Higgs $\phi_S$ with $(+, +)$ of $\hat{Z}_2 \times Z_2$ plays the role of the SM Higgs in this sector. No other Higgs can couple to the quark sector at the tree-level. In this way we can avoid tree-level FCNCs in the quark sector. So, $\hat{Z}_2$ is introduced to forbid tree-level couplings of the $D_6$ singlet Higgs $\phi_S$ with the leptons and simultaneously to forbid tree-level couplings of $\phi_I, \eta_I$ and $\eta_S$ with the quarks.

| $SU(2)_L \times U(1)_Y$ | $L_S$ | $n_S$ | $e^c_S$ | $L_I$ | $n_I$ | $e^c_I$ |
|-------------------------|-------|-------|---------|-------|-------|---------|
| $D_6$                   | 1     | 1'    | 1''     | 2'    | 2'    | 2'      |
| $\hat{Z}_2$            | +     | +     | -       | +     | -     | +       |
| $Z_2$                   | +     | -     | +       | +     | -     | +       |

**TABLE I:** The $D_6 \times \hat{Z}_2 \times Z_2$ assignment for the leptons. The subscript $S$ indicates a $D_6$ singlet, and the subscript $I$ running from 1 to 2 stands for a $D_6$ doublet. $L$'s denote the $SU(2)_L$-doublet leptons, while $e^c$ and $n$ are the $SU(2)_L$-singlet leptons.

| $SU(2)_L \times U(1)_Y$ | $\phi_S$ | $\phi_I$ | $\eta_S$ | $\eta_I$ |
|-------------------------|-----------|-----------|-----------|-----------|
| $D_6$                   | 1         | 2'        | 1''       | 2'        |
| $\hat{Z}_2$            | +         | -         | +         | +         |
| $Z_2$                   | +         | +         | -         | -         |

**TABLE II:** The $D_6 \times \hat{Z}_2 \times Z_2$ assignment for the $SU(2)_L$ Higgs doublets.
The most general renormalizable $D_6 \times \hat{Z}_2 \times Z_2$ invariant Yukawa interactions in the leptonic sector can be described by

$$\mathcal{L}_Y = \sum_{a,b,d=1,2,S} \left[ Y_{ab}^d (L_a i \sigma_2 \phi_d) e_b^c + Y_{ab}^{vd} (\eta_d^I L_a) n_b \right] - \sum_{I=1,2} \frac{1}{2} M_I n_I n_I - \frac{1}{2} M_S n_S n_S + h.c.$$  \hspace{1cm} (2)

The Yukawa matrices $Y$’s are given by

$$Y^{e1} = \begin{pmatrix} -y_2 & 0 & y_5 \\ 0 & y_2 & 0 \\ y_4 & 0 & 0 \end{pmatrix}, \quad Y^{e2} = \begin{pmatrix} 0 & y_2 & 0 \\ y_2 & 0 & y_5 \\ 0 & y_4 & 0 \end{pmatrix}, \quad Y^{cS} = 0,$$ \hspace{1cm} (3)

$$Y^{\nu 1} = \begin{pmatrix} -h_2 & 0 & 0 \\ 0 & h_2 & 0 \\ h_4 & 0 & 0 \end{pmatrix}, \quad Y^{\nu 2} = \begin{pmatrix} 0 & h_2 & 0 \\ h_2 & 0 & 0 \\ 0 & h_4 & 0 \end{pmatrix},$$

$$Y^{\nu S} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & h_3 \\ 0 & 0 & 0 \end{pmatrix}.$$ \hspace{1cm} (4)

C. The scalar potential and the $\eta$ masses

The most general renormalizable Higgs potential, invariant under $D_6 \times \hat{Z}_2 \times Z_2$, is given by \(^3\)

$$V(\phi, \eta) = V_1[\phi; \mu_1^1, \mu_2^1, \lambda_1^1, \ldots, \lambda_7^1] + V_1[\eta; \mu_1^\eta, \mu_2^\eta, \lambda_1^\eta, \ldots, \lambda_7^\eta] + V_2[\phi, \eta], \hspace{1cm} (5)$$

where

$$V_1[\phi; \mu_1^1, \mu_2^1, \lambda_1^1, \ldots, \lambda_7^1]$$

$$= -\mu_1^1 (\phi_1^\dagger \phi_1) - \mu_2^1 (\phi_2^\dagger \phi_2) + V_3[(\phi_1^\dagger \phi_1), (\phi_2^\dagger \phi_2); \lambda_1, \lambda_2, \lambda_3] + \lambda_4^1 (\phi_1^\dagger \phi_1) (\phi_1^\dagger \phi_1) + \lambda_5^1 (\phi_2^\dagger \phi_2) (\phi_2^\dagger \phi_2) + \lambda_6^2 (\phi_1^\dagger \phi_2)^2 + h.c.] + \lambda_7^1 (\phi_1^\dagger \phi_1)^2,$$ \hspace{1cm} (6)

$$V_2[\phi, \eta]$$

$$= V_3[(\phi_1^\dagger \phi_1), (\eta_1^\dagger \eta_1); \kappa_1, \kappa_2, \kappa_3] + \lambda_4 (\phi_1^\dagger \phi_1)(\eta_1^\dagger \eta_1) + \lambda_5 (\phi_2^\dagger \phi_2)(\eta_2^\dagger \eta_2) + \lambda_6 (\phi_1^\dagger \phi_2)(\eta_1^\dagger \eta_2) + \lambda_7^2 (\phi_1^\dagger \phi_1)^2 + h.c.] + \lambda_8 (\phi_2^\dagger \phi_2)^2,$$ \hspace{1cm} (7)

\(^3\) See also for instance \cite{24}.
and \( I \) runs from 1 to 2. Here \( V_3 \) is defined as

\[
V_3[(A^\dagger B), (C^\dagger D); \kappa_1, \kappa_2, \kappa_3] = \kappa_1 (A^\dagger B_I) (C^\dagger D_J) + \kappa_2 (A^\dagger I (i\sigma_2)_I J_B) (C^\dagger I (i\sigma_2)_I J D_L) + \kappa_3 [ (A^\dagger I (\sigma_1)_I J_B) (C^\dagger I (\sigma_1)_I J D_L) + (A^\dagger I (\sigma_3)_I J_B) (C^\dagger I (\sigma_3)_I J D_L) ],
\]

(8)

where \( A, B, C \) and \( D \) are \( SU(2)_L \) doubles and belong to \( 2' \) of \( D_6 \), and \( (A^\dagger B) \) is an \( SU(2)_L \) invariant product. As we can see from (7) and (8), an exact lepton number \( U(1)_L' \) invariance emerges in the absence of \( \kappa_{13} \sim \kappa_{18} \), where the right-handed neutrinos \( n_I \) and \( n_S \) are neutral under \( U(1)_L' \) in contrast to the conventional seesaw models \[25\]. This \( U(1)_L' \) forbids the neutrino masses, so that the smallness of the neutrino masses has a natural meaning. Therefore, the radiative neutrino masses will be proportional to these Higgs couplings.

We assume that \((\mu_1^\eta)^2, (\mu_2^\eta)^2 < 0 \) so that

\[
< \eta_S > = < \eta_I > = 0
\]

(9)

corresponds to a local minimum of the scalar potential \[5\]. This is the essence of refs. \[4, 5\] to connect the neutrino masses and the nature of CDM, because \( Z_2 \) remains unbroken. We further observe that the scalar potential \[5\] has an accidental symmetry \( S_2 \):

\[
\phi_1, \eta_1 \leftrightarrow \phi_2, \eta_2,
\]

(10)

while the \( D_6 \) singlets Higgs fields \( \phi_S \) and \( \eta_S \) remain unchanged. This symmetry ensures that if \((\mu_1^\phi)^2, (\mu_2^\phi)^2 > 0 \),

\[
< \phi_1 >= < \phi_2 > = \left( \begin{array}{c} \frac{v_D}{\sqrt{2}} \\ 0 \end{array} \right) \quad \text{and} \quad < \phi_S > = \left( \begin{array}{c} \frac{v_S}{\sqrt{2}} \\ 0 \end{array} \right)
\]

(11)

can correspond to a local minimum of the potential, suggesting an appropriate field redefinition \(^4\)

\[
\phi_\pm = \frac{1}{\sqrt{2}} (\phi_1 \pm \phi_2), \quad \eta_\pm = \frac{1}{\sqrt{2}} (\eta_1 \pm \eta_2).
\]

(12)

A consequence of (9) is that \( \eta' \)’s do not mix with \( \phi' \)’s in the mass matrix. Furthermore, because of the absence of the \( \phi_S \phi_I \eta_S \eta_I \) type couplings, \( \eta_\pm \) and \( \eta_S \) do not mix with each other. Keeping these observations in mind, we can write down the mass terms for \( \eta' \)’s as

\[
L_{M_\eta} = - \sum_{a=\pm, -S} \left[ m_a^2 \eta_+^{(a)} \eta_-^{(a)} + \frac{1}{2} (m_a^R)^2 \eta_+^{(0)} \eta_-^{(0)} + \frac{1}{2} (m_a^L)^2 \chi_+^{(0)} \chi_-^{(0)}
\]

\[
+ (m_a^{RI})^2 \eta_+^{(0)} \chi_-^{(0)}, \right]
\]

(13)

\(^4\) The tree-level W boson mass constraint is \((v_D^2 + v_S^2) = v^2 \simeq (246 \text{ GeV})^2\).
where \( \eta^{(-)} (= \eta^{(+)\ast}) \), \( \eta^{(0)} \) and \( \chi^{(0)} \) are \( SU(2)_L \) components of \( \eta \), i.e.,

\[
\eta = \left( \frac{(\eta^{(0)} + i\chi^{(0)})/\sqrt{2}}{\eta^{(-)}} \right).
\]

(14)

Since the neutrino masses will be proportional to the \( U(1)_L \) violating Higgs couplings \( \kappa_{13} \sim \kappa_{18} \), we may assume that they are small. In the absence of these Higgs couplings, there will be no mixing of the scalar and pseudo scalar components of the neutral \( \eta \)'s, i.e., \( m_a^{RI} = 0 \), and \( m_a^R = m_a^I \). In general, even though it is small, there exists mixing:

\[
\eta_a^{(0)} = \cos \gamma_a \hat{\eta}_a + \sin \gamma_a \hat{\chi}_a, \quad \chi_a^{(0)} = -\sin \gamma_a \hat{\eta}_a + \cos \gamma_a \hat{\chi}_a, \quad (a = \pm, S),
\]

(15)

where \( \hat{\eta}_a \) and \( \hat{\chi}_a \) are mass eigenstates. We denote their masses by \( m_a^\eta \) and \( m_a^\chi \), respectively. The difference \( (m_a^\eta)^2 - (m_a^\chi)^2 \) will be proportional to the \( U(1)_L \) violating couplings, and we find that

\[
\Delta m_a^2 = (m_a^\eta)^2 - (m_a^\chi)^2
\]

\[
= [(m_a^{R})^2 - (m_a^{I})^2] \cos 2\gamma_a - 2(m_a^{RI})^2 \sin 2\gamma_a,
\]

(16)

where

\[
(m_a^{R})^2 - (m_a^{I})^2 = \begin{cases} 
2[(\text{Re} \nu_D)^2 - (\text{Im} \nu_D)^2](\text{Re} \kappa_{13} + \text{Re} \kappa_{15}) + 2[(\text{Re} \nu_S)^2 - (\text{Im} \nu_S)^2]\text{Re} \kappa_{16} \\
+4\text{Re} \nu_D \text{Im} \nu_D (\text{Im} \kappa_{13} + \text{Im} \kappa_{15}) + 4\text{Re} \nu_S \text{Im} \nu_S \text{Im} \kappa_{16} \\
\end{cases}
\]

and

\[
(m_a^{RI})^2 = \begin{cases} 
2[(\text{Re} \nu_D)^2 - (\text{Im} \nu_D)^2]\text{Re} \kappa_{17} + 4\text{Re} \nu_D \text{Im} \nu_D \text{Im} \kappa_{17} \\
+2[(\text{Re} \nu_S)^2 - (\text{Im} \nu_S)^2]\text{Re} \kappa_{18} + 4\text{Re} \nu_S \text{Im} \nu_S \text{Im} \kappa_{18} \\
2\text{Re} \nu_D \text{Im} \nu_D (\text{Re} \kappa_{13} + \text{Re} \kappa_{15}) + 2\text{Re} \nu_S \text{Im} \nu_S \text{Re} \kappa_{16} \\
-[(\text{Re} \nu_D)^2 - (\text{Im} \nu_D)^2](\text{Im} \kappa_{13} + \text{Im} \kappa_{15}) - [(\text{Re} \nu_S)^2 - (\text{Im} \nu_S)^2]\text{Im} \kappa_{16} \\
2\text{Re} \nu_D \text{Im} \nu_D (\text{Re} \kappa_{14} + \text{Re} \kappa_{15}) + 2\text{Re} \nu_S \text{Im} \nu_S \text{Re} \kappa_{16} \\
-[(\text{Re} \nu_D)^2 - (\text{Im} \nu_D)^2](\text{Im} \kappa_{14} + \text{Im} \kappa_{15}) - [(\text{Re} \nu_S)^2 - (\text{Im} \nu_S)^2]\text{Im} \kappa_{18} \\
\end{cases}
\]

for \( a = +, -, S \). As we will see below, the absence of the mixing among \( \eta_\pm \) and \( \eta_S \) is responsible for the reason that the neutrino masses and mixing of the present model have the same structure as that of the \( S_3 \) model of ref. [27].
III. LEPTON MASSES AND MIXING

A. CP phases

Let us first figure out the structure of CP phases. To this end, we introduce phases explicitly as follows:

\[ y_a \rightarrow e^{ip_a} y_a \ (a = 2, 4, 5) \]  

(17)

for the Yukawa couplings, where we assume that the possible phases coming from the VEVs of \( \phi' \)'s are absorbed into the Yukawa couplings. The \( y' \)'s on the right-hand side are supposed to be real and \(-\pi/2 \leq p' \leq \pi/2\), and similarly for the fields

\[ L_I \rightarrow e^{ip_L} L_I, \quad L_S \rightarrow e^{ip_L} L_S, \quad e^I \rightarrow e^{ip_e} e^I, \quad e^S \rightarrow e^{ip_e} e^S, \]  

\[ n_I \rightarrow e^{ip_n} n_I, \quad n_S \rightarrow e^{ip_n} n_S. \]  

(18)

The phases of the right-handed neutrinos are used to absorb the phases of their Majorana masses \( M_1 \) and \( M_S \). The phases of \( y_2, y_4 \) and \( y_5 \) can be rotated away if

\[ 0 = p_L + p_{y_2} + p_e, \quad 0 = p_L + p_{y_4} + p_e, \quad 0 = p_L + p_{y_5} + p_{e_S} \]  

(19)

are satisfied. So, only one free phase is left, which we assume to be \( p_L \). We will use this freedom to make certain entries of the one-loop neutrino mass matrix real. After that no further phase rotation which does not change physics is possible.

B. Charged fermion masses

The charged lepton masses are generated from the \( S_2 \) invariant VEVs, and the mass matrix becomes

\[ M_e = \begin{pmatrix} -m_2 & m_2 & m_5 \\ m_2 & m_2 & m_5 \\ m_4 & m_4 & 0 \end{pmatrix}, \]  

(20)

where

\[ m_2 = |v_D y_2|/2, \quad m_4 = |v_D y_4|/2, \quad m_5 = |v_D y_5|/2. \]  

(21)

All the mass parameters appearing in (20) can be assumed to be real. Diagonalization of the mass matrices is straightforward. The mass eigen values are approximately given by

\[ m^2_e = \frac{(m_4 m_5)^2}{(m_2)^2 + (m_5)^2} + O((m_4)^4), \]  

(22)

\[ m^2_\mu = 2(m_2)^2 + (m_4)^2 + O((m_4)^4), \]  

(23)

\[ m^2_\tau = 2\left[ (m_2)^2 + (m_5)^2 \right] + \frac{(m_4 m_2)^2}{(m_2)^2 + (m_5)^2} + O((m_4)^4). \]  

(24)
Concrete values are given as $m_4/m_5 \simeq 0.00041$ and $m_2/m_5 \simeq 0.0596$ and $m_5 \simeq 1254$ MeV to obtain $m_e = 0.51$ MeV, $m_\mu = 105.7$ MeV and $m_\tau = 1777$ MeV. The diagonalizing unitary matrices (i.e., $U_{eL}^T M_e U_{eR}$) assume a simple form in the $m_e \to 0$ limit, which is equivalent to the $m_4 \to 0$ limit. We find that $U_{eL}$ can be approximately written as

$$U_{eL} = \begin{pmatrix} \epsilon_e (1 - \epsilon_\mu^2) & - (1/\sqrt{2})(1 - \epsilon_e^2 + 2 \epsilon_e^2 \epsilon_\mu^2) & 1/\sqrt{2} \\ - \epsilon_e (1 + \epsilon_\mu^2) & (1/\sqrt{2})(1 - \epsilon_e^2 - 2 \epsilon_e^2 \epsilon_\mu^2) & 1/\sqrt{2} \\ 1 - \epsilon_e^2 & \sqrt{2} \epsilon_e & \sqrt{2} \epsilon_e \epsilon_\mu \\ \end{pmatrix}, \tag{25}$$

where $\epsilon_\mu = m_\mu/m_\tau$ and $\epsilon_e = m_e/(\sqrt{2}m_\mu)$. In the limit $m_e = 0$, the unitary matrix $U_{eL}$ becomes

$$\begin{pmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & 0 & 0 \end{pmatrix}, \tag{26}$$

which is the origin of a maximal mixing of the atmospheric neutrinos.

C. Radiative neutrino masses

The neutrino mass matrix is generated from the one-loop diagram fig. 1 and is given by

$$(M_\nu)_{ij} = \sum_{a=\pm,S} \sum_{k=1,2,S} (Y^{\nu a})^*_{ik} (Y^{\nu a})^*_{jk} \Gamma^a(M_k), \tag{27}$$
where

\[
\begin{align*}
Y^{\nu+} &= \frac{1}{\sqrt{2}} (Y^{\nu 1} + Y^{\nu 2}) = \frac{1}{\sqrt{2}} \begin{pmatrix}
-h_2 & h_2 & 0 \\
0 & h_2 & 0 \\
0 & h_4 & 0
\end{pmatrix}, \\
Y^{\nu-} &= \frac{1}{\sqrt{2}} (Y^{\nu 1} - Y^{\nu 2}) = \frac{1}{\sqrt{2}} \begin{pmatrix}
-h_2 & -h_2 & 0 \\
0 & h_2 & 0 \\
0 & -h_4 & 0
\end{pmatrix},
\end{align*}
\]

(28)

(29)

\[
\Gamma^a(M_k) = \frac{M_k}{8\pi^2} \exp(-i2\gamma_a) \left[ \frac{(m^\eta_a/M_k)^2 \ln(m^\eta_a/M_k)^2}{1 - (m^\eta_a/M_k)^2} - \frac{(m^\chi_a/M_k)^2 \ln(m^\chi_a/M_k)^2}{1 - (m^\chi_a/M_k)^2} \right]
\]

(30)

\[
\simeq \frac{M_k}{8\pi^2} \exp(-i2\gamma_a) \Delta m_a \frac{1 - (m^\eta_a/M_k)^2 + \ln(m^\eta_a/M_k)^2}{(1 - (m^\eta_a/M_k)^2)^2}.
\]

(31)

The Yukawa matrices, \(m^\eta_a, \gamma_a\) and \(\Delta m_a^2\) are defined in [3], [4], [13], [15] and [16], respectively. (Recall that \(M_1 = M_2\) because of the \(D_6\) symmetry.)

Using the explicit form of the Yukawa matrices we obtain

\[
\mathcal{M}_\nu = \begin{pmatrix}
G^+(M_1)h_2^2 & 0 & 0 \\
0 & G^+(M_1)h_2^2 & G^-(M_1)h_2h_4 \\
0 & G^-(M_1)h_2h_4 & G^+(M_1)h_4^2 + \Gamma^S(M_S)h_3^2
\end{pmatrix},
\]

(32)

where

\[
G^\pm(M_1) = \Gamma^+(M_1) \pm \Gamma^-(M_1).
\]

(33)

At this stage we recall that the phase \(p_L\) was left undetermined (see [19]), and so we use it to make the \((1,1)\) and \((2,2)\) entries of \(\mathcal{M}_\nu\) real. Then \((2,3)\) and \((3,2)\) entries of \(\mathcal{M}_\nu\) can be made real by multiplying a diagonal phase matrix

\[
P = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \exp(ip)
\end{pmatrix}
\]

(34)

with \(\mathcal{M}_\nu\) from left and right, and we can rewrite the neutrino mass matrix as

\[
P\mathcal{M}_\nu P = \mathcal{M}_\nu = \begin{pmatrix}
2(\rho_2)^2 & 0 & 0 \\
0 & 2(\rho_2)^2 & 2\rho_2\rho_4 \\
0 & 2\rho_2\rho_4 & (\rho_4)^2 + (\rho_3)^2 \exp i2\varphi_3
\end{pmatrix},
\]

(35)

where \(p\) is an independent parameter and enters into the definition of the neutrino mixing matrix \(V_{MNS}\), and the \(\rho\)'s in \((35)\) are real numbers. One can convince oneself that \(\mathcal{M}_\nu\) can be diagonalized as \([11, 27]\)

\[
U^\nu_T \mathcal{M}_\nu U_\nu = \begin{pmatrix}
m_{\nu_1}e^{i\phi_1-\iota\phi_\nu} & 0 & 0 \\
0 & m_{\nu_2}e^{i\phi_2+i\phi_\nu} & 0 \\
0 & 0 & m_{\nu_3}
\end{pmatrix},
\]

(36)
where

\[
U_\nu = \begin{pmatrix}
0 & 0 & 1 \\
-s_{12} & c_{12}e^{i\phi} & 0 \\
c_{12}e^{-i\phi} & s_{12} & 0
\end{pmatrix}, \tag{37}
\]

\[
m_{\nu_3} \sin \phi_\nu = m_{\nu_2} \sin \phi_2 = m_{\nu_1} \sin \phi_1, \quad 2\varphi_3 = \phi_1 + \phi_2 \tag{38}
\]

\[
m_{\nu_3} = 2\rho_2^2, \quad \frac{m_{\nu_1}m_{\nu_2}}{m_{\nu_3}} = \rho_3^2, \tag{39}
\]

\[
\tan \phi_\nu = \frac{\rho_3^2 \sin 2\varphi_3}{2(\rho_2^2 + \rho_4^2) + \rho_3^2 \cos 2\varphi_3}, \tag{40}
\]

and \(c_{12} = \cos \theta_{12}\) and \(s_{12} = \sin \theta_{12}\). We also find that

\[
\tan^2 \theta_{12} = \frac{(m_{\nu_2}^2 - m_{\nu_3}^2 \sin^2 \phi_\nu)^{1/2} - m_{\nu_1} \cos \phi_\nu}{(m_{\nu_2}^2 - m_{\nu_3}^2 \sin^2 \phi_\nu)^{1/2} + m_{\nu_1} \cos \phi_\nu}, \tag{41}
\]

from which we obtain

\[
\frac{m_{\nu_2}^2}{\Delta m_{23}^2} = \frac{(1 + 2t_{12}^2 + t_{12}^4 - r t_{12}^4)^2}{4t_{12}^2(1 + t_{12}^4 - r t_{12}^4) \cos^2 \phi_\nu} - \tan^2 \phi_\nu \tag{42}
\]

\[
\simeq \frac{1}{\sin^2 2\theta_{12} \cos^2 \phi_\nu} - \tan^2 \phi_\nu \text{ for } |r| << 1, \tag{43}
\]

where \(t_{12} = \tan \theta_{12}, r = \Delta m_{21}^2/\Delta m_{23}^2\). It can also be shown that only an inverted mass spectrum

\[
m_{\nu_3} < m_{\nu_1}, m_{\nu_2} \tag{44}
\]

is consistent with the experimental constraint \(|\Delta m_{21}^2| < |\Delta m_{23}^2|\) in the present model. Note that eq. (39) is satisfied for

\[
2\varphi_3 = \phi_1 + \phi_2 \sim \pm \pi \tag{45}
\]

and not for \(\phi_1 \sim \phi_2\). That is, if \(2\varphi_3 \sim (+)(-) \pi\), then \(\cos \phi_1 < (>) 0\) and \(\cos \phi_2 > (<) 0\).

Now the product \(U_{eL}^\dagger PU_\nu\) defines the neutrino mixing matrix \(V_{\text{MNS}}\), where \(P\) is defined in (34). We find

\[
|(V_{\text{MNS}})_{13}| = \frac{m_e}{\sqrt{2}m_\mu} + O(m_e m_\mu/m_\mu^2) \simeq 3.4 \times 10^{-3}, \tag{46}
\]

\[
|(V_{\text{MNS}})_{23}| = \frac{1}{\sqrt{2}} + O((m_e/m_\mu)^2), \quad |(V_{\text{MNS}})_{33}| = \frac{1}{\sqrt{2}}. \tag{47}
\]

So the mixing of the atmospheric neutrinos are very close to the maximal form \(^5\). A reparametrization independent indicator for CP violation in neutrino oscillations is the Jackob determinant, which is given by

\[
|J| = |\text{Im} \left[(V_{\text{MNS}})^*_{22}(V_{\text{MNS}})_{33}(V_{\text{MNS}}^*_{13})(V_{\text{MNS}}^*_{23})\right]| = \frac{m_e}{\sqrt{2}m_\mu} \sin 2(\theta_{12}) \sin(p - \phi_\nu) + O(m_e m_\mu/m_\mu^2) < 3.5 \times 10^{-4} \tag{48}
\]

\(^5\) Unfortunately, this value of \(|(V_{\text{MNS}})_{13}|\) is too small to be measured [28].
in the present model.

The effective Majorana mass \( < m_{ee} > \) in neutrinoless double \( \beta \) decay is given by

\[
< m_{ee} > = | \sum_{i=1}^{3} m_{\nu_i} V_{ei}^2 | \simeq | m_{\nu_1} e_{12}^2 + m_{\nu_2} s_{12}^2 \exp i2\alpha |,
\]

where

\[
\sin 2\alpha = \sin(\phi_1 - \phi_2) = \pm \frac{m_{\nu_3} \sin \phi_\nu}{m_{\nu_1} m_{\nu_2}} \left( \sqrt{m_{\nu_2}^2 - m_{\nu_3}^2 \sin^2 \phi_\nu} + \sqrt{m_{\nu_1}^2 - m_{\nu_3}^2 \sin^2 \phi_\nu} \right)
\]

for \( 2\phi_2 \sim \pm \pi \), and \( \phi_1, \phi_2 \) and \( \phi_\nu \) are defined in \[39\]. In fig. 2 we plot \( < m_{ee} > \) as a function of \( \sin \phi_\nu \) for \( \sin^2 \theta_{12} = 0.3, \Delta m_{21}^2 = 6.9 \times 10^{-5} \text{ eV}^2 \) and \( \Delta m_{23}^2 = 1.4, 2.3, 3.0 \times 10^{-3} \text{ eV}^2 \) \[29\]. As we can see from fig. 2, the effective Majorana mass stays at about its minimal value \( < m_{ee} >_{\text{min}} \) for a wide range of \( \sin \phi_\nu \). Since \( < m_{ee} >_{\text{min}} \) is approximately equal to \( \sqrt{\Delta m_{23}^2} / \sin 2\theta_{12} = (0.034 - 0.069) \text{ eV} \), it is consistent with recent experiments \[2, 30\].

![FIG. 2: The effective Majorana mass \( < m_{ee} > \) as a function of \( \sin \phi_\nu \) with \( \sin^2 \theta_{12} = 0.3 \) and \( \Delta m_{21}^2 = 6.9 \times 10^{-5} \text{ eV}^2 \). The dashed, solid and dot-dashed lines stand for \( \Delta m_{23}^2 = 1.4, 2.3 \) and \( 3.0 \times 10^{-3} \text{ eV}^2 \), respectively. The \( \Delta m_{21}^2 \) dependence is very small.](image)

IV. \( \mu \rightarrow e\gamma \) CONSTRAINT

The \( Z_2 \) even neutral components of the Higgs fields \( \phi_I \) have tree-level FCNC couplings in the lepton sector as we can see from \[3\]. Since the Yukawa couplings \( y_2, y_4 \) and \( y_5 \) are
related to the charged lepton masses (see [21]), they are very small. As a consequence, the
tree-level FCNCs are well suppressed in the present model, as it is shown in ref. [27]. The
one-loop $\mu \to e\gamma$ amplitude mediated by $\phi_I$ is negligibly small, too, because of the same
reason. In the following discussions, therefore, we consider $\mu \to e\gamma$ mediated only by the $\eta$
exchange.

To this end, we express the Yukawa couplings in terms of the mass eigenstates. We use
the approximate unitary matrix (25) for the charged leptons, and we find the Yukawa terms
relevant for $\mu \to e\gamma$ are given by

$$L_{Y\nu} = -Y^+_{ij} e_L i n_j n_{\eta^+} + Y^-_{ij} e_L i n_j n_{\eta^-} - Y^S_{ij} e_L i n_j n_{\eta^S} + h.c.,$$

where

$$Y^\pm = U^T_{eL} (Y^{\nu 1} \pm Y^{\nu 2}) / \sqrt{2}, \quad Y^S = U^T_{eL} Y^{\nu 3},$$

and

$$Y^+ \approx \begin{pmatrix} (h_4 - 2\epsilon_e h_2) / \sqrt{2} & h_4 / \sqrt{2} & 0 \\ h_2 + \epsilon_e h_4 & \epsilon_e h_4 & 0 \\ 0 & h_2 & 0 \end{pmatrix},$$

$$Y^- \approx \begin{pmatrix} h_4 / \sqrt{2} & (h_4 - 2\epsilon_e h_2) / \sqrt{2} & 0 \\ \epsilon_e h_4 & h_2 - \epsilon_e h_4 & 0 \\ -h_2 & 0 & 0 \end{pmatrix},$$

$$Y^S \approx \begin{pmatrix} 0 & 0 & h_3 \\ 0 & 0 & \sqrt{2} \epsilon_e h_3 \\ 0 & 0 & 0 \end{pmatrix}, \quad (\sqrt{2}\epsilon_e = m_e / m_\mu \approx 0.0048).$$

The inert Higgs fields $\eta^\pm, \eta_S$ and the Yukawa matrices $Y^{\nu}$'s are defined in (12) and (4),
respectively.

Using the Yukawa interaction term (51), we then compute the branching fraction of
$\mu \to e\gamma$ from fig. 3 and find

$$B(\mu \to e\gamma) = \frac{3\alpha}{64\pi G_F} X^4 \approx |X^2 900 \text{ GeV}^2|^2,$$
where
\[ X^2 \approx \frac{h_4 h_2}{\sqrt{2}} \left[ \frac{F_2(M_1^2/m_+^2)}{m_+^2} - \frac{F_2(M_1^2/m_-^2)}{m_-^2} \right] + h_3^2 \frac{m_e}{m_\mu} \frac{F_2(M_5^2/m_S^2)}{m_S^2}, \] (57)
and
\[ F_2(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1 - x)^4}. \] (58)

\((m_\pm\) and \(m_S\) are the masses of \(\eta_\pm\) and \(\eta_S\), respectively, which are defined in (12).\) We recall that \(M_1 = M_2\) because of the \(D_6\) symmetry. As we see the third term of (57) contains a suppression factor \(m_e/m_\mu\). There is no suppression factor like \(m_e/m_\mu\) for the first two terms in (57), but there are two cancellation mechanisms, where these two mechanisms can be combined. The first one takes place if \(M_1 >> m_+, m_-\), for which the leading terms in the parenthesis cancels, and one finds
\[ \frac{F_2(M_1^2/m_+^2)}{m_+^2} - \frac{F_2(M_1^2/m_-^2)}{m_-^2} = -\frac{1}{M_1^2} \left[ \frac{11}{6} (m_+^2 - m_-^2) - m_-^2 \ln(M_1^2/m_-^2) + m_+^2 \ln(M_1^2/m_+^2) + O(1/M_1^2) \right]. \] (59)
The second one is obvious because of the relative sign in (59): it cancels exactly if \(m_+ = m_-\).

The second possibility appears naturally for relatively heavy \(\eta_\pm\), because \(m_+^2 - m_-^2 \sim O(\lambda)v^2\) as we can see from the Higgs potential (5). To get an idea on the size of the suppression we compute the branching fraction for \(M_1 = 2\) TeV, \(m_+ = 700\) GeV and \(m_- = 750\) GeV, and find
\[ B(\mu \to e\gamma) \approx 1.6 \times 10^{-12}, \] (60)
where we have neglected the contribution coming from the third term in (57) and used \(|h_4 h_2| = 1\). This should be compared with \(B(\mu \to e\gamma) \approx 1.1 \times 10^{-9}\), which one would obtain in the absence of the second term in the parenthesis. As we see from (58) the function \(F_2(x)\) can vary between \(1/12(x = 1)\) and \(1/6(x = 0)\) for \(x \leq 1\). From this we can roughly estimate the lower bound of \(m_S\) for the case that \(n_S\) is a CDM candidate, i.e., \(M_S < m_S\). We find that \(m_S > O(300)\) GeV for \(F_2 \simeq 1/12\) and \(h_3 \simeq 0.93\) should be satisfied to satisfy \(B(\mu \to e\gamma) < 1.2 \times 10^{-11}\).

Now, suppose that CDM is fermionic. From Table I we see that only \(D_6\) doublet and singlet right-handed neutrinos, i.e. \(n_I\) and \(n_S\), come into question. If \(n_I\) are CDM, then \(M_1 < m_+\) should be satisfied. (Otherwise \(n_I\) would decay into \(\eta_\pm\).) As we can find from (57), we have to impose a fine tuning of the form
\[ (m_+^2 - m_-^2)/m_+^2 \lesssim (m_+/500\text{GeV})^2 \times 10^{-2} \] (61)
to sufficiently suppress \(\mu \to e\gamma\) in this case. This means that, for instance, if \(m_+ = 500\) GeV, then \(m_-\) should be within \(500 \pm 5\) GeV. Therefore, it is more natural to assume that
$M_1 > m_\pm$ to suppress $\mu \rightarrow e\gamma$. Then only the $D_6$ singlet right-handed neutrino $n_S$ remains as a fermionic CDM candidate. As we can see from the Yukawa matrices (53), (54) and (55), only $\eta_S$ couples to $n_S$ with $e_L$ and with $\mu_L$, where the coupling with $\mu_L$ is suppressed by $m_e/m_\mu \simeq 0.005$. Therefore, there would be a clean signal if the charged component of $\eta_S$ were produced at LHC. In the next section we will investigate whether or not the $\mu \rightarrow e\gamma$ constraint above is consistent with the observed dark matter relic density $\Omega d h^2 \simeq 0.12$ assuming that $n_S$ is CDM.

V. COLD DARK MATTER

Here we would like to investigate whether or nor $n_S$ can be a good CDM candidate. For simplicity we assume that $m_S \simeq m_S^R \simeq m_S^I >> m_S^{RI}$ (which are defined in (13)), that is, the breaking of $U(1)'_L$ by the VEV (11) is small (which is needed to obtain small neutrino masses). To obtain a thermally averaged cross section for the annihilation of two $n_S$’s, we compute the relativistic cross section $\sigma$ from fig. 4 and expand it in powers of the relative velocity $v$ of incoming $n_S$ [31]. Note that the $\eta_\pm$ exchange diagrams are suppressed, which one sees from the Yukawa matrices (53) and (54). Lepton number violating diagrams being proportional to $\Delta m^2_\alpha$ in (16) are also very small. So they are annihilated mostly into an $e^+ - e^-$ pair and a $\nu_\tau - \bar{\nu}_\tau$ pair.

Using the result of ref. [31], we find in the limit of the vanishing lepton masses

$$\sigma v = a + bv^2 + \cdots, \quad a = 0, \quad b = \frac{|h_3|^4 r^2 (1 - 2r + 2r^2)}{24 \pi M_S^2},$$

$$r = M_S^2/(m_S^2 + M_S^2).$$

Then we can compute the relic density of $n_S$ from [32]

$$\Omega d h^2 = \frac{Y_{\infty} s_0 M_S}{\rho_c/H^2} \quad \text{with} \quad Y_{\infty}^{-1} = 0.264 g^1/2 M_p M_S (3b/x_f^2),$$

FIG. 4: Annihilation diagram of $n_S$. 


where \( Y_\infty \) is the asymptotic value of the ratio \( n_{n_s}/s \), \( s_0 = 2970/\text{cm}^3 \) is the entropy density at present, \( \rho_c = 3H^2/8\pi G = 1.05 \times 10^{-5}h^2 \) GeV/cm\(^3\) is the critical density, \( h \) is the dimensionless Hubble parameter, \( M_{pl} = 1.22 \times 10^{19} \) GeV is the Planck energy, and \( g_* \) is the number of the effectively massless degrees of freedom at the freeze-out temperature. Further, \( x_f \) is the ratio \( M_S/T \) at the freeze-out temperature and is given by \( \left[32\right] \)

\[
x_f = \ln \frac{M_{pl}(6b/x_f)c(2+c)M_S}{(g_*x_f)^{1/2}}. \tag{65}
\]

Using \( g_*^{1/2} = 10 \) and \( c = 1/2 \) \( \left[32\right] \) we obtain

\[
\frac{M_S}{\text{GeV}} = 5.86 \times 10^{-8}x_f^{-1/2}(\exp x_f) \left[ \frac{\Omega_d h^2}{0.12} \right], \tag{66}
\]

\[
\frac{b}{\text{GeV}^2} = 2.44 \times 10^{-11}x_f^2 \left[ \frac{0.12}{\Omega_d h^2} \right], \tag{67}
\]

where \( b \) is given in \( \left[62\right] \).

FIG. 5: \( M_S \) versus \( m_S \) for \( |h_3| = 1.3 \) and \( \Omega_d h^2 = 0.13 \) (dot-dashed), 0.12 (solid) and 0.11 (dashed). \( M_S \) and \( m_S \) denote the masses of the CDM \( n_S \) and the \( D_6 \) singlet neutral Higgs, respectively.

Since \( b/|h_3|^4 \) is a function of \( M_S \) and \( m_S \), eqs. \( \left[66\right] \) and \( \left[67\right] \) give \( M_S \) and \( m_S \) in unit of GeV for a given set of \( |h_3|^2, x_f \) and \( \Omega_d h^2 \). In fig. \( \left[5\right] \) \( M_S \) against \( m_S \) is plotted for \( \Omega_d h^2 = 0.13 \) (dot-dashed), 0.12 (solid) and 0.11 (dashed) \( \left[2\right] \), where \( |h_3| \) is fixed at 1.3. We see from the figure that the dark matter mass \( M_S \) and the \( D_6 \) singlet Higgs mass \( m_S \) should be closely related with each other to obtain an observed relic density of dark matter. We plot
in fig. 6, $M_S$ against $m_S$ for $y = 0.3, 0.5, 0.7, 1.0$. As we can see also from the figure, the dark matter constraint requires that the dark matter mass $M_S$ increases as the Higgs mass $m_S$ increases and reaches at its maximal value $m_S$ at a certain value of $m_S$. We see that $m_S$ can not exceed $\sim 750$ GeV for the perturbative regime $|h_3| \leq 1.5$. Therefore, it is by no means trivial to satisfy the $\mu \to e\gamma$ constraint. In fig. 7 we present the allowed region in the $m_S - M_S$ plane, in which $\Omega_d h^2 = 0.12$ and $B(\mu \to e\gamma) < 1.2 \times 10^{-11}$ are satisfied, where we assume $|h_3| < 1.5$ and only the last term in $X^2$ of (57) contributes to $\mu \to e\gamma$. If we allow larger $|h_3|$, then the region expands to larger $m_S$ and $M_S$, and for $|h_3| \lesssim 0.8$ there is no allowed region. As we can also see from fig. 7 the mass of the CDM and the mass of the inert Higgs should be larger than about 230 and 300 GeV, respectively. If we restrict ourselves to a perturbative regime, they should be lighter than about 750 GeV.

VI. CONCLUSION

We have assumed that two important issues, the origin of neutrino masses and the nature of CDM, are related, and considered a non-supersymmetric extension of the standard model with a family symmetry based on $D_6 \times \hat{Z}_2 \times Z_2$. The gross structure of the Yukawa couplings is fixed by $D_6$, while $\hat{Z}_2$ is responsible to suppress FCNCs in the quark sector, e.g. in the mixing of the neutral meson systems. They are spontaneously broken together with $SU(2)_L \times U(1)_Y$. The remaining $Z_2$ is unbroken; it forbids the tree-level neutrino masses and simultaneously
FIG. 7: The region in the $m_S - M_S$ plane in which $\Omega \chi^2 = 0.12, B(\mu \to e\gamma) < 1.2 \times 10^{-11}$ and $|h_3| < 1.5$ are satisfied.

ensures the stability of CDM candidates. From the assumption that CDM is fermionic we can single out the $D_6$ singlet right-handed neutrino as the best CDM candidate. We find that the $D_6$ singlet inert charged Higgs with a mass between 300 and 750 GeV has a tree-level coupling to the electron, the muon and the tau with the relative strength $(1, m_e/m_\mu, 0)$, respectively. The lower value results from the $\mu \to e\gamma$ constraint, while we obtain the upper value from the assumption that the Yukawa sector remains within a framework of perturbation theory. (This assumption is reflected on the condition $M_S < m_S$.) From this observation we have concluded that the charged Higgs decays mostly into an electron (or a positron) with a large missing energy, where the missing energy is carried away by the CDM candidate. This will be a clean signal at LHC.

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[1] Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Lett. B433, 9 (1998); SNO Collaboration, Q. R. Ahmad et al., Phys. Rev. Lett. 89, 011301 (2002); K2K Collaboration, M. H. Ahn et al., Phys. Rev. Lett. 93, 051801 (2004); KamLAND Collaboration, T. Araki et al., Phys. Rev. Lett. 94, 081801 (2005).
[2] WMAP Collaboration, D. N. Spergel et al., Astrophys. J. Suppl. 148, 175 (2003); SDSS Collaboration, M. Tegmark et al., Phys. Rev. D69, 10350 (2004).
[3] L. M. Krauss, S. Nasri and M. Trodden, Phys. Rev. D67, 085002 (2003).
[4] E. Ma, Phys. Rev. D73, 077301 (2006); Mod. Phys. Lett. A2, 1777 (2006); “Supersymmetric Model of Radiative Seesaw Majorana Neutrino Masses”, hep-ph/0607142.
[5] J. Kubo, E. Ma and D. Suematsu, “Cold Dark Matter, Radiative Neutrino Mass, $\mu \rightarrow e\gamma$, and Neutrinoless Double Beta Decay”, hep-ph/0604114, to appear in Phys. Lett. B.
[6] T. Hambye, K Kannike, E. Ma and M. Raidal, “Emanations of Dark Matter: Muon Anomalous Magnetic Moment, Radiative Neutrino Mass, and Novel Leptogenesis at the TeV Scale”, hep-ph/0609228.
[7] J. Kubo and D. Suematsu, “Neutrino masses and CDM in a non-supersymmetric model”, hep-ph/0610006.
[8] R. Barbieri, L.J. Hall and V.S. Rychkov, Phys. Rev. D74, 015007 (2006).
[9] G. Altarelli and F. Feruglio, New J. Phys. 6, 106 (2004).
[10] R.N. Mohapatra and A.Y. Smirnov, “Neutrino Mass and New Physics”, hep-ph/0603118
C.H. Albright and M-C. Chen, “Model Predictions for Neutrino Oscillation Parameters”, hep-ph/0608137.
[11] A. Mondragón, “Models of flavour with discrete symmetries”, hep-ph/0609243.
[12] G. Seidl, “Deconstruction and bilarge neutrino mixing”, hep-ph/0301044
K.R.S. Balaji, M. Lindner and G. Seidl, Phys. Rev. Lett. 91, 161803 (2003).
[13] W. Grimus and L. Lavoura, Phys. Lett. B572, 189 (2003); W. Grimus, A.S. Joshipura, S. Kaneko, L. Lavoura and M. Tanimoto, JHEP 0407, 078 (2004).
[14] C. Hagedorn and W. Rodejohann, JHEP 0507, 034 (2005).
[15] C. Hagedorn, M. Lindner and F. Plentinger, Phys. Rev. D74, 025007 (2006); E. Ma, Memorial issue dedicated to Dubravko Tadic, hep-ph/0409288.
[16] S-L. Chen and E. Ma, Phys. Lett. B620, 151 (2005).
[17] A. Zee, Phys. Lett. B93, 339 (1980) [Erratum-ibid. B95, 641 (1980)]; Nucl. Phys. B264, 99 (1986); K.S. Babu, Phys. Lett. B203, 132 (1988); E. Ma, Phys. Rev. Lett. 81, 1171 (1998); D. Aristizabal Sierra and M. Hirsch, “Experimental tests for the Babu-Zee two-loop model of Majorana neutrino masses”, hep-ph/0609307 and references therein.
[18] E. Ma and M. Raidal, Phys. Rev. Lett. 87, 011802 (2001); Erratum: ibid. 87, 159901 (2001).
[19] Higgs Working Group Collaboration, K.A. Assamagan et al., “The Higgs working group:
Summary report 2003”, hep-ph/0406152; D. Zeppenfeld, “Higgs Bosons at the LHC”, Talk given at SUSY06, UC Irvine, 12-17 June 2006.

[20] P.H. Frampton and T.W. Kephart, Int. J. Mod. Phys. A10, 4689 (1995); Phys. Rev. D64, 086007 (2001).

[21] J. Kubo, Phys. Lett. B622, 303 (2005).

[22] M. Frigerio, S. Kaneko, E. Ma and M. Tanimoto, Phys. Rev. D71, 011901 (2005).

[23] K.S. Babu and J. Kubo, Phys. Rev. D71, 056006 (2005); E. Itou, Y. Kajiyama and J. Kubo, Nucl. Phys. B743, 74 (2006).

[24] J. Kubo, H. Okada and F. Sakamaki, Phys. Rev. D70, 036007 (2004).

[25] M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, eds. P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, 1979), p. 315; T. Yanagida, in Proc. of the Workshop on the Unified Theory and the Baryon Number in the Universe, eds. O. Sawada and A. Sugamoto, KEK Report No. 79-18 (Tsukuba, Japan, 1979), p. 95; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[26] T. Kobayashi, J. Kubo and H. Terao, Phys. Lett. B568, 83 (2003).

[27] J. Kubo, A. Mondragón, M. Mondragón and E. Rodríguez-Jáuregui, Prog. Theor. Phys. 109, 795 (2003); Erratum-ibid. 114, 287 (2005); J. Kubo, Phys. Lett. B578, 156 (2004); Erratum-ibid. B619, 387 (2005).

[28] H. Minakata, H. Sugiyama, O. Yasuda, K. Inoue and F. Suekane, Phys. Rev. D68, 033017 (2003); Erratum-ibid. D70, 059901 (2004); O. Yasuda, “Measurement of $\theta_{13}$ by reactor experiments”, hep-ph/0309333; H. Minakata, Nucl. Phys. B (Proc. Suppl.) 137, 74 (2004); P. Huber, M. Lindner, M. Rolice, T. Schwetz, and W. Winter, Phys. Rev. D70, 073014 (2004); H. Sugiyama, O. Yasuda, F. Suekane and G.A. Horton-Smith, Phys. Rev. D73, 053008 (2006); T. Kajita, H. Minakata, S. Nakayama and H. Nunokawa, “Resolving Eight-Fold Neutrino Parameter Degeneracy by Two Identical Detectors with Different Baselines”, hep-ph/0609286.

[29] M. Maltoni, T. Schwetz, M.A. Törnalo and J.W.F. Valle, New J. Phys. 6, 122 (2004).

[30] H.V. Klapdor-Kleingrothaus et al., Eur. Phys. J. A12, 147 (2001); C.E. Aalseth et al., Phys. Atm. Nucl. 63, 1268 (2000); H.V. Klapdor-Kleingrothaus, A. Dietz, H.L. Harney and I.V. Krivosheina, Mod. Phys. Lett. A 16, 2409 (2001).

[31] K. Griest, Phys. Rev. D38, 2357 (1988).

[32] K. Griest, M. Kamionkowski and M.S. Turner, Phys. Rev. D41, 3565 (1990).