Consensus Target Tracking in Switching Wireless Sensor Networks with Outliers

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The problem of consensus-based distributed tracking in wireless sensor networks (WSNs) with switching network topologies and outlier-corrupted sensor observations is considered. First, to attack the outlier-corrupted measurements, a robust Kalman filtering (RKF) scheme with weighted matrices on innovation sequences is introduced. The proposed RKF possesses high robustness against outliers while having similar computational burden as traditional Kalman filter. Then, each node estimates the network-wide agreement on target state using only communications between one-hop neighbors. In order to improve the convergent speed of the consensus filter in case of switching topologies, an adaptive weight update strategy is proposed. Note that the proposed algorithm relaxes the requirement of Gaussian noise statistics in contrast to the decentralized/distributed Kalman filters. Besides, unlike the existing consensus-based filters, we do not need to perform consensus filtering on the covariance matrices, which will reduce the computational and communicational burden abundantly. Finally, simulation examples are included to demonstrate the robustness of the proposed RKF and effectiveness of adaptive consensus approach.

1. Introduction

Wireless sensor networks (WSNs) have emerged as an ideal means for data gathering and target tracking in a variety of home [1, 2], industrial [3], transportation [4], and security [5] applications. Although target tracking system through a WSN can have several advantages, it is a challenging task since each sensor node typically has very limited power supply and communication bandwidth [6, 7]. Reducing the energy cost in communication and computation can significantly increase the node life span [8, 9]. This makes distributed estimation and fusion very popular in target tracking system by WSNs. Generally speaking, research in distributed estimation and fusion in WSNs can be categorized into two classes of networks: (a) fusion center- (FC-) based WSNs and (b) ad hoc WSNs. FC-based WSNs can perform distributed estimation but have limitations arising due to (i) the high transmission power required at each sensor to transmit its local information to the FC that is proportional to the covered geographic area and (ii) lack of robustness in case of FC failures. For example, the well-known FC-based strategy is decentralized Kalman filtering, which involves state estimation using a set of local Kalman filters that communicate with all other nodes [10, 11]. The information flow in the traditional decentralized Kalman filtering is all-to-all with communication complexity of $O(N^2)$ which is not scalable for WSNs [12]. As is known, the energy cost usually related to communication between sensor nodes and computation in each node is significant when using such an algorithm in sensor networks. On the contrary, these limitations are not encountered with ad hoc WSNs whereby each sensor communicates with its neighbors, and the estimation task can be performed in a totally distributed fashion [13, 14]. For an ad hoc WSN, one usually uses average consensus strategy to estimate the target state in a totally distributive way.

The average consensus has been proven to be an effective tool for performing network-wide distributed computation
task ranging from flocking to robot rendezvous [15, 16]. During the last decade, average consensus-based target tracking in WSNs has obtained extensive researching interest [17–21]. For example, Olfati-Saber introduces a distributed Kalman filtering (DKF) algorithm that uses dynamic consensus strategy [17, 18]. The DKF algorithm consists of a network of micro-Kalman filters each embedded with a high-gain high-pass consensus filter. The role of consensus filters is fusion of sensor and covariance data obtained at each node. Later on, the problem of estimating a simpler scenario with a scalar state of a dynamical system from distributed noisy measurements based on consensus strategies is considered in [19]; the focuses are on the interaction between the consensus matrix, the number of messages exchanged per sampling time, and the Kalman gain for scalar systems. In [20], the authors consider the distributed estimation problem for a continuous-time moving target under switching interconnection topologies. Using state-consensus strategy, a recursive distributed estimation algorithm is proposed. There are also some results on consensus-based target tracking for nonlinear dynamic (see, e.g., [14, 21]) and the reference therein). However, the distributed estimation and fusion problem through consensus strategy so far is mainly focused on Gaussian noises scenario in a static sensor network. It is a challenging task when measurements generated by a sensor node are corrupted by outliers (e.g., [22]). Unfortunately, the problem of outliers is of practical importance in a target tracking system using radar or infrared sensor [23, 24], communication applications where non-Gaussian (heavy-tailed) noise occurs, such as in underwater acoustics, satellite communications through the ionosphere [25], and others. More importantly, a node in the WSN may break in practical applications due to the strict energy constraint. This may make the observations from this node become outliers. In this case, the aforementioned consensus-based tracking approaches will degrade the fusion performance greatly. Therefore it is of practical interest to consider distributed estimation and fusion when outlier is presented, especially in case of switching topologies.

In this paper, we focus on consensus-based distributed tracking in a switching WSN when sensor observations are corrupted by outliers. First, to attack the outlier-corrupted measurements, a robust Kalman filtering (RKF) scheme with weighted matrices on innovation sequences is introduced. It has been proven that the proposed RKF possesses high robustness against outliers while having similar computational burden as traditional Kalman filter. Then, each node estimates the network-wide agreement on target state using only communications between one-hop neighbors. In order to improve the convergent speed of consensus approach in case of switching topologies, an adaptive weight update strategy is proposed. It should be noted that the proposed algorithm relaxes the requirement of Gaussian noise statistics compared with the decentralized/distributed Kalman filters (see, e.g., [11, 18, 26]). Besides, unlike the existing consensus-based filters, we do not need to perform consensus filter on the covariance matrices, which will reduce the computational and communicational burden abundantly. Simulation examples are included to demonstrate the robustness of the proposed RKF and effectiveness of adaptive consensus approach.

The rest of the paper is organized as follows. In Section 2, the problem of consensus tracking in a switching sensor network with outlier-corrupted measurements will be formulated. In Section 3, the outlier-resilient robust filter will be given according to $M$-estimation. The proposed robust filter is further distributed by adaptive consensus strategy in Section 4. In Section 5, we give several simulation examples, which are followed by concluding remarks in Section 6.

### 2. Problem Statement

Consider the target tracking problem in a network of $N$ sensors distributed randomly deployed on a field, each with measurements corrupted by outliers. As is well known the sensor network can be modeled by using algebraic graph theory [27]. A vertex of the graph corresponds to a node and edges of the graph capture the dependence of interconnections. Formally, a graph $G(t) = (V,E(t))$ consists of a set of vertices $V = \{v_1,v_2,\ldots,v_N\}$, indexed by nodes in the network, and a set of edges $E(t) = \{(v_i,v_j) \in V \times V\}$ represent the available communication link. Assume that only single-hop communications are allowed. The set of neighbors of node $i$ on graph $G$ is defined as $\mathcal{N}_i(t) = \{j: (i,j) \in E(t)\}$. The degree of vertex $i$ is defined as $d_i(t) = |\mathcal{N}_i(t)|$ and maximum degree is $d_{\text{max}}(t) = \max_i d_i(t)$. Let $\Delta(t)$ be the degree matrix, $\Delta(t) = \text{diag}(d_i(t))$. The adjacency matrix $J(t)$ is the integer matrix with rows and columns indexed by the vertices; that is, the $ij$-entry of $J(t)$ is equal to the number of edges from $i$ to $j$. Following [27], Laplacian matrix of a graph $G(t)$ is defined as $L(t) = \Delta(t) - J(t)$.

For the distributed estimation fusion problem, each node in the WSN is deployed to estimate the state of a dynamic target based on noise-corrupted measurements. We consider the discrete linear stochastic system

$$
x(t+1) = Fx(t) + Gw(t),
$$

$$
y_i(t) = H_i x(t) + v_i(t) + z_i(t), \quad i = 1, 2, 3, \ldots, N,
$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y_i(t) \in \mathbb{R}^m_i$ is the $ith$ measurement in the sampling period $iT$, $w(t) \in \mathbb{R}^m$ is the disturbance input or system white noise with zero mean and variance matrix $Q$, and $e_i(t) = [v_i(t) + z_i(t)] \in \mathbb{R}^{m_i}, i = 1, 2, 3, \ldots, N$, are the outlier-contaminated measuring noise vectors. The matrices $F$, $G$, and $H_i$ are known real constant matrices with appropriate dimensions.

**Assumption 1.** $w(t)$ and $v_i(t), \quad i = 1, 2, 3, \ldots, N$, are independent white noises with zero mean and

$$
E \left[ \begin{bmatrix} w(t) \\ v_i(t) \\ v_i^T(k) \\ v_i^T(k) \\ v_j^T(k) \end{bmatrix} \right] = \begin{bmatrix} Q(t) \\ 0 \\ 0 \\ R_i(t) \end{bmatrix} \delta_{ij},
$$

$\forall t, k, \text{ if } i \neq j,$
where $E$ is the expectation, the superscript $T$ denotes the transpose, and $\delta_{ik}$ is the Kronecker delta function.

**Remark 2.** The practical challenge which is of our interest also lies in the outlier measurement which means mismatch in measurement noise model. The modeling error in measurement noise model is related to the sensor failure, spikes, or jamming which is not Gaussian. In order to simulate this unmodeled measurement uncertainty, a non-Gaussian error term $z_i(t)$ is included. Commonly, the outlier-corrupted measuring noise can be represented by Turkey’s gross error model, a contaminated Gaussian model. That is, $e_i(t) = v_i(t) + z_i(t)$ are with non-Gaussian density function $f(e)$ described by

$$F_i = (1 - \alpha) G_i + \alpha \Delta G_i,$$

where $G_i$ is the zero-mean Gaussian density and $\Delta G_i$ is some unknown symmetric function representing the impulsive part of the noise density or outliers.

**Assumption 3.** The initial state $x(0)$ is independent of $\omega(t)$ and $e_i(t), i = 1, 2, 3, \ldots, N$, and

$$Ex(0) = x_0, \quad E[(x(0) - x_0)(x(0) - x_0)^T] = P_0.$$  

The problem of interest is to design a robust filter capable of performing state estimation and target tracking tasks with less communication and computational load. The robustness lies in two aspects: (i) the algorithm is robust in the sense that heavy-tailed errors or outliers do not affect the solution; (ii) the entire network works regardless of switching topologies as long as the network is still connected. To ease the analysis, we also assume that all sensors are synchronized and have the same measurement rate.

**Lemma 4.** Under Assumptions 1 and 3, for ith sensor subsystem of the system (1)-(2) without outlier (i.e., $z_i(t) = 0$), one has the local optimal Kalman filters [28]

$$\tilde{x}_i(t + 1 | t + 1) = \tilde{x}_i(t + 1 | t) + K_i(t + 1) e_i(t + 1),$$  

$$\tilde{x}_i(t + 1 | t) = F\tilde{x}_i(t | t),$$  

$$e_i(t + 1) = y_i(t + 1) - H_i\tilde{x}_i(t + 1 | t),$$  

$$K_i(t + 1) = P_i(t + 1 | t) H_i^T [H_i P_i(t + 1 | t) H_i^T + R_i]^{-1},$$  

$$P_i(t + 1 | t) = FP_i(t | t) F^T + GGQG^T,$$

$$P_i(t + 1 | t) = [I_n - K_i(t + 1 | t) H_i] P_i(t + 1 | t),$$  

$$\tilde{x}_i(0 | 0) = x_0, \quad P_i(0 | 0) = P_0,$$

where $P_i(t + 1 | t), P_i(t | t)$ are the one-step prediction and filtering error variance, respectively, and $K_i(t)$ is the filtering gain matrix. As described in Section 3, the innovation $e_i(t)$ plays an important role in robust Kalman filter recursive process.

### 3. Outlier-Resilient Robust Kalman Filtering

The conventional Kalman filter can be formulated as a solution to a particular weighted least squares problem [29]. Unfortunately, it is not robust because extreme outliers with arbitrarily large residuals can have an infinitely large influence on the resulting estimate. From (6) to (12), we can see that, in $(t + 1)$th sampling period, $\tilde{x}_i(t + 1 | t + 1)$ is corrected by the linear combination of $e_i(t + 1)$ [29]. Therefore, if the measurements $y_i(t + 1)$ are contaminated by outliers, $e_i(t + 1)$ will correct $\tilde{x}_i(t + 1 | t + 1)$ in a wrong way, which should make traditional Kalman filter degraded or even divergent. To handle this, the $M$-estimator, one of the most sophisticated approaches among the robust statistics approaches, is proposed [30]. $M$-estimators attempt to suppress the influence of outliers by replacing the square of the residuals with a less rapidly increasing loss function; that is,

$$J = \sum_{j=1}^{m} \rho(jy_i(t) - h_j x(t)) = \sum_{j=1}^{m} \rho(\epsilon_j(t)).$$

where $y_i(t), e_i(t), h_j$ stand for the $j$th row of $y_i(t)$, $e_i(t)$, and $H_j$, respectively (cf. (9)). $\rho(\cdot)$ is a scalar robust convex function that has to cut off the outliers. Particularly, if one chooses $\rho(\cdot)$ to be a quadratic function, the estimation reduces to the least squares estimator or Kalman filter solution (6) [31, 32].

Equating the first partial derivatives with respect to the state to be estimated $x(t)$ leads to the following relation:

$$\sum_{j=1}^{m} \psi(y_i(t) - h_j \tilde{x}(t)) h_j = \sum_{j=1}^{m} \psi(e_j(t)) h_{ij} = 0.$$  

The score function $\rho(\cdot)$ is usually nonnegative and symmetric, and $\psi(\cdot)$, the derivative of $\rho(\cdot)$, is often called the influence (score) function, since it describes the influence of the measurement errors on the solutions.

Now, (14) can be rewritten as

$$\sum_{j=1}^{m} \psi(e_j) h_{ij} = 0.$$  

Letting $d(e_{ij}) = \psi(e_j)/e_{ij}$, then (15) can be reformulated as the matrix form

$$H_i^T D_i(e) e_i = 0,$$

where $D_i(e) = \text{diag}(d(e_{i1}), d(e_{i2}), \ldots, d(e_{im}))$.

In the light of the above comparison and analysis of conventional Kalman filtering and $M$-estimator, the proposed RKF is given in Theorem 5 as follows.
Theorem 5. Under Assumptions 1 and 3, the $i$th sensor subsystem of the system (1)–(2), one has the RKF

$$
\hat{x}_i (t + 1 | t + 1) = \hat{x}_i (t + 1 | t) + K_i (t + 1) D_i (t) e_i (t + 1),
$$

(17)

$$
K_i (t + 1) = P_i (t + 1 | t) H_i^T
\times \left[ I_n - K_i (t + 1) D_i (t) H_i \right]^{-1},
$$

(18)

$$
P_i (t + 1 | t + 1) = [ I_n - K_i (t + 1) D_i (t) H_i ] P_i (t + 1 | t)
\times [ I_n - K_i (t + 1) D_i (t) H_i ]^T
+ K_i (t + 1) D_i (t) R_i D_i^T K_i^T (t + 1).
$$

(19)

Other recursive steps are just the same as (7), (8), (10), and (12) in Lemma 4.

Proof. The formula (17) can be derived from above directly, and the covariance of weighted innovation is

$$
E \left[ \left( D_i (t) e_i (t + 1) \right) \left( D_i (t) e_i (t + 1) \right)^T \right] = D_i (t) R_i D_i^T\ (t)
$$

(20)

from which we have the robust Kalman gain matrix as (18). Substituting (17) into the filtering error equation

$$
\hat{x}_i (t + 1 | t + 1) = x_i (t + 1) - \hat{x}_i (t + 1 | t + 1)
= \left[ I_n - K_i (t + 1) D_i (t) H_i \right] \hat{x}_i (t + 1 | t)
- K_i (t + 1) D_i (t) u_i (t),
$$

(21)

where $\hat{x}_i (t + 1 | t)$ is the one-step prediction residual, and after mathematical manipulation, the robust filter covariance can be computed as

$$
P_i (t + 1 | t + 1) = E \left[ \hat{x}_i (t + 1 | t + 1) \hat{x}_i^T (t + 1 | t + 1) \right]
= [ I_n - K_i (t + 1) D_i (t) H_i ] P_i (t + 1 | t)
\times [ I_n - K_i (t + 1) D_i (t) H_i ]^T
+ K_i (t + 1) D_i (t) R_i D_i^T K_i^T (t + 1).
$$

(22)

This completes the proof. □

Remark 6. $\rho(\cdot)$ is a robust $M$-estimate function for suppressing the outliers and is important for the estimation performance. Different $\rho(\cdot)$ will result in different $M$-estimate and fusion performance. Say, for a given density $f$, the choice $\rho(v) = -\log f(v)$ yields the maximum likelihood fuser. According to the above analysis, we propose a more general $M$-estimate function extended from Huber’s robust cost function:

$$
\rho \left( e_{ij} (t) \right) = \begin{cases} 
\frac{e_{ij}^2 (t)}{2}, & \text{for } |e_{ij} (t)| \leq a \\
$\frac{a|e_{ij} (t)| - a^2}{2}, & \text{for } a < |e_{ij} (t)| \leq b \\
abla - \frac{a^2}{2}, & \text{for } b < |e_{ij} (t)|,
\end{cases}
$$

(23)

where $a$ and $b$ have to be chosen to provide the desired efficiency at the Gaussian model while possessing robustness at the non-Gaussian model. Usually, they are chosen empirically [24]. For simplicity, we let $a = 3 \sqrt{R_{ij}^2(t)}$, $b = 5 \sqrt{R_{ij}^2(t)}$ in this paper, where $R_{ij}^2(t)$ stands for the $(j, j)$ entry of the covariance matrix $R(t)$.

It can be seen that $\rho(\cdot)$ is an even real-valued function and it is quadratic when $e_i (t)$ is smaller than $a$, which is just the same as the maximum likelihood (ML) cost function and keeps the efficiency of the $M$-estimate. For larger values of $e_i (t)$ in the interval $[a, b]$, the function is linear and increases more slowly than ML. For residuals greater than $b$, the function is equal to a constant.

Remark 7. Based on (23), the weighted matrix of innovations can be formulated as

$$
d \left( e_{ij} (t) \right) = \begin{cases} 
1, & \text{for } |e_{ij} (t)| \leq a \\
\frac{a}{|e_{ij} (t)|}, & \text{for } a < |e_{ij} (t)| \leq b \\
0, & \text{for } |e_{ij} (t)| > b.
\end{cases}
$$

(24)

The three different intervals of $D(\cdot)$ serve to deal with different kinds of residuals. In order to keep the accuracy and efficiency, when $|e_{ij} (t)| \leq a$, $D(\cdot)$ is set to be 1; when sampling from the moderate innovations, $D(\cdot)$ is decreased with the residuals; and while sampling from a heavy-tailed distribution or outliers, the weighted matrix is set to be zero.

Remark 8. Several other robust cost functions can be used in the robust statistics setting, such as Huber’s robust cost function, Andrews’ method, Vapnik’s loss function, or the biweight approach. Take Huber’s robust score function as an example:

$$
\rho \left( e_{ij} (t) \right) = \begin{cases} 
\frac{e_{ij}^2 (t)}{2}, & \text{for } |e_{ij} (t)| \leq a \\
\frac{a|e_{ij} (t)| - a^2}{2}, & \text{for } a < |e_{ij} (t)| \leq b; \\
\frac{a^2}{2}, & \text{for } b < |e_{ij} (t)|,
\end{cases}
$$

(25)

then the weighted matrix of innovations can be formulated as

$$
d \left( e_{ij} (t) \right) = \begin{cases} 
1, & \text{for } |e_{ij} (t)| \leq a \\
\frac{a}{|e_{ij} (t)|}, & \text{for } |e_{ij} (t)| > a.
\end{cases}
$$

(26)

It is obvious that Huber’s robust score function is a special case of our proposed function by letting $b = \infty$. However, in case of large outliers presented, the Huber’s cost function (25) based approaches will seriously degrade since the weighting entry $d \left( e_{ij} (t) \right) \neq 0$. 
4. Adaptive Consensus for Distributed Estimation Fusion

We now consider how to extend the aforementioned RKF to be applicable in distributed state estimation with only one-hop communications based on average consensus. For distributed robust Kalman filtering based on consensus, each node maintains a local filter according to the recursions in Theorem 5. However, in order to reach a network-wide agreement on target state estimate, we use an adaptive consensus on the local estimate in each time slot.

Assume the state of each node at the kth iteration is denoted by $\hat{x}_i^k(t + 1 | t + 1)$, initialized to be the local estimate of each node. The state is updated in the following form:

$$\hat{x}_i^k = w_i \hat{x}_i^{k-1} + \sum_{j \in N_i} w_{ij} \hat{x}_j^{k-1}, \quad k = 1, 2, \ldots, $$

(27)

where $w_{ij}$ denotes the weight on the state of node $j$ at node $i$ satisfying that $w_{ij} + \sum_{j \in N_i} w_{ij} = 1$. Notice that the time slot index is omitted in case without ambiguity.

**Remark 9.** The iteration (27) means that each node iteratively updates its state estimate based on a linear combination of its own and neighbor's state estimates. The iterative estimate update process (27) will compel the state estimate of each node to asymptotically reach a consensus value, provided that the weights $w_{ij}$ are appropriately selected. Two well-known weights that satisfy the above are the maximum-degree weight and the Metropolis weight [33].

In a practical application, it is hoped that the consensus algorithm converge to the steady state as fast as possible, especially for a switching network. To this end, we propose an adaptive weight modification approach to enhance the convergence speed of the consensus (27). Noticing that the weights satisfy $w_i + \sum_{j \in N_i} w_{ij} = 1$, we can rewrite (27) in the following form:

$$\hat{x}_i^k = \hat{x}_i^{k-1} + \sum_{j \in N_i} w_{ij} [\hat{x}_j^{k-1} - \hat{x}_i^{k-1}], \quad k = 1, 2, \ldots.$$

(28)

Now we propose an adaptive consensus scheme for our state estimate update. Let $\tilde{x}_i^k$ be the desired signal that is defined by

$$\tilde{x}_i^k = \frac{1}{d_i} \left( \hat{x}_i^k + \mathbf{1}_{d_i} \otimes \hat{x}_i^k \right),$$

(29)

where $\mathbf{1}_{d_i}$ is a $d_i \times 1$ vector of value ones, $\tilde{x}_i^k = [x_1^k, x_2^k, \ldots, x_{d_i}^k]^T$ is the vector of estimate from neighbors of the $i$th node, and $\otimes$ is the Kronecker product.

Remark 10. Note that we only perform consensus filtering on the state estimate in this paper, instead of simultaneous consensus on both information estimate and covariance as in [18, 21, 26] and the references therein. This is motivated by observation that enlarging the covariance matrices to an appropriate size can also guarantee the convergence of the filter [34, 35]. With the consensus iteration, the fused estimate converges to a network-wide agreement. This means the corresponding covariance matrices should be smaller than the one each node keeps. On the contrary, we do not perform consensus on the covariance matrices, which takes the effect of making the covariance matrices enlarged in another perspective. This makes the whole filtering expected to converge. The simulation results in Section 5 will demonstrate this expectation. However, our approach only needs communication burden of $n \times E_0$ for each filtering iteration per node (where $E_0$ is the total energy required to send and receive the message for entry in a state vector), instead of $(n^2 + n) \times E_0$.

5. Simulation Examples

The proposed consensus tracking approach is applied to tracking a target moving on noisy circular trajectories. Consider $N$ sensors randomly deployed in the region of interest, which is $50 \times 50$ m with the coordinates from $(-25, -25)$ to $(25, 25)$. Consider a target with dynamics [18, 21]

$$\dot{x} = F_c x + G_c \omega,$$

(33)

with

$$F_c = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

(34)

and $G_c = 25I_2$. We use the discrete-time model (1) of the target with parameters

$$F = I_2 + \delta F_c + \frac{\delta^2}{2} F_c^2 + \frac{\delta^3}{6} F_c^3, \quad G = \delta G_c.$$
The step size is $\delta = 0.015$. We use the root mean squared error (RMSE) as performance criterion; that is, 

$$\text{RMSE} = \left( \frac{\sum_{i=1}^{M} [\tilde{x}_i(t | t) - \hat{x}_i(t | t)]^2}{M} \right)^{1/2},$$  

(36)

where $M$ is the number of runs and $\tilde{x}_i(t | t)$ represents the estimation error of the fused estimate in the $j$th Monte Carlo run. The simulation is performed by 500 Monte Carlo runs, each with 200 time steps.

Two scenarios are considered to verify the robustness of the proposed RKF and the effectiveness of the adaptive consensus, respectively.

**Scenario 1 (one sensor case—robustness of the proposed RKF).**

To verify the robustness of proposed RKF under condition of outlier-corrupted measuring noises, we first consider just a single sensor which is used to track the target. The sensor makes noisy measurements of the target according to (2) with $H = [1, 1]$. We add outliers into $e_i(t), i = 1$ all over the simulation period. The outlier-corrupted noise $e_i(t)$ is with the contaminated Gaussian density function $\mathcal{F}_i = (1 - \alpha)N(0, \sigma_i^2) + \alpha N(0, k \sigma_i^2)$, where $\sigma_i$ is set as 5. For the RKF, we set $a = 3 \sigma_i$ and $b = 5 \sigma_i$ in (27).

The average RMSEs with different parameters over time steps after 500 Monte Carlo runs are shown in Table 1. From Table 1, we can see, in case of no outlier presented, the proposed RKF performs a little poorer than traditional Kalman filter (TKF). This is because RKF has deweighted the elements of matrix $D$ in case of larger innovations. However, when outliers present, the adaptive weight of matrix $D$ according to different innovations makes the proposed RKF more robust than TKF. For example, in case of $\alpha = 0.05$ and $k = 500$, the degradation of RKF and TKF is, respectively, $(1.48 - 1.05)/1.05 = 40.95\%$ and $(5.07 - 0.92)/0.92 = 451.09\%$. We highlight some results in Table 1 to show that, for some cases, the TKF does not converge while the proposed RKF degrades not so much. Moreover, the performance discrepancy of RKF with different $\alpha$ and $k$ is not very large (cf. the worst case is when $\alpha = 0.2$ and $k = 1000$). All the analyses above well demonstrate the robustness of proposed RKF.

Besides, we added some point outliers with $k = 100$ into the measuring noises when time step equals 20, 35, 50, 75, 150, and 175. The RMSEs on $x$-direction and $y$-direction are shown in Figure 1. It is obvious that even a point outlier can degrade the TKF very much while the proposed RKF performs smoothly. This is very important in case of tracking targets using radar or infrared sensor with glint noise.

**Scenario 2 (multiple sensors case—effectiveness of the adaptive consensus).**

Suppose there are $N = 50$ sensors randomly deployed in the ROI as shown in Figure 2, while other parameters are the same as in Scenario 1. The whole time periods are divided into 4 phases, each with different randomly deployed 50 sensors. In other words, the network topologies are switching with phases. For the 1st phase, the layout of the network is illustrated in Figure 2 while the other 3 topologies are omitted for the space reason. In the particular setup of Figure 2, there are 240 links with $d_{\text{max}} = 9$.

The nodes make noisy measurements of the position of the target either along the $x$-axis or along the $y$-axis; that is, in the observing model (2), $H_i = H_x = [1 \ 0]$ or $H_i = H_y = [0 \ 1]$ for $i = 1, 2, \ldots, 50$. Two cases are considered for the measuring noise: (i) no outlier is presented; (ii) outlier is presented. In the simulation, we also applied the centralized measurement fusion by traditional Kalman filter (CentrTKF) for the first case, the centralized measurement fusion by the proposed robust Kalman filter (CentrRKF) for the second case, consensus traditional KF (ConsenTKF), and the traditional KF without consensus (TKFwoConsen) to compare with our proposed consensus KF (ConsenRKF). Note that, in case of no outlier presented, the CentrTKF is the optimal estimation. On the other hand, the CentrRKF is expected to perform the best in the case of outlier-corrupted measurement.

Average RMSEs over sensors in the Gaussian noises case are shown in Figure 3. It is noted that the proposed Consen-RKF yields almost identical performance compared to the ConsenTKF. Both approaches based on adaptive consensus perform very close to the CentrTKF, which is the minimum mean squared error estimator in the case of Gaussian white noise. However, the TKFwoConsen ranks the worst among the 4 approaches, which show that the consensus algorithm makes the estimates of each sensor achieve an agreement over the whole network.

The average RMSEs over sensors in the outlier-corrupted noises case are shown in Figure 4. A quick look at Figure 4 reveals that the CentrRKF performs the best among the 4 approaches. However, it is worth noting that, for the centralized fusion approach such as CentrTKF and CentrRKF, a fusion center (FC) is needed. This makes it not robust against FC failure and imposes rather a high computational burden on the FC. More importantly, the communicational cost for the centralized fusion approaches is $O(N^2)$ since

| Parameters | RKF | TKF |
|------------|-----|-----|
| $\alpha = 0$ (no outliers) | $1.05$ | $0.92$ |
| $\alpha = 0.05$ | $1.29$ | $2.37$ |
| $k = 100$ | $1.48$ | $5.07$ |
| $k = 500$ | $1.61$ | $18.75$ |
| $k = 1000$ | $2.07$ | NaN |
| $\alpha = 0.1$ | $1.42$ | $4.32$ |
| $k = 100$ | $1.71$ | $9.88$ |
| $k = 500$ | $2.07$ | NaN |
| $k = 1000$ | $2.28$ | NaN |

The nodes make noisy measurements of the position of the target either along the $x$-axis or along the $y$-axis; that is, in the observing model (2), $H_i = H_x = [1 \ 0]$ or $H_i = H_y = [0 \ 1]$ for $i = 1, 2, \ldots, 50$. Two cases are considered for the measuring noise: (i) no outlier is presented; (ii) outlier is presented. In the simulation, we also applied the centralized measurement fusion by traditional Kalman filter (CentrTKF) for the first case, the centralized measurement fusion by the proposed robust Kalman filter (CentrRKF) for the second case, consensus traditional KF (ConsenTKF), and the traditional KF without consensus (TKFwoConsen) to compare with our proposed consensus KF (ConsenRKF). Note that, in case of no outlier presented, the CentrTKF is the optimal estimation. On the other hand, the CentrRKF is expected to perform the best in the case of outlier-corrupted measurement.
each node should send its measurement to the FC. This is undesirable for WSNs which suffer stringent energy and communication constraints. Besides, it is obvious that the proposed ConsenRKF obtains a considerable estimation error regardless of the outliers-corrupted noises, while the ConsenTKF degrades extensively. It is worth noting that, when some nodes in the network are out-of-battery, the communication links between these nodes and their one-hop neighbors will break. In this case, the topology of the network changed.

However, as far as the network is connected, the adaptive consensus proposed will converge.

Finally, in a peer-to-peer estimation architecture every node is supposed to know the estimate of the target state. Therefore, estimation error by itself is no longer the only measure of performance in consensus-based estimation in ad hoc sensor networks. The agreement of estimate of every node permits the query on any node in the network about the estimation. In Figure 5, the consecutive snapshots of estimates of all nodes are shown. The estimates appear as a cohesive set of particles that move around the position of the target. From Figure 5, we can see that the estimate over node reaches a consensus very quickly by our adaptive weights update (32).

6. Conclusion

The problem of consensus-based distributed tracking in wireless sensor networks (WSNs) with switching network topologies and outlier-corrupted sensor observations has been considered in this paper. To attack the outlier-corrupted measurements, a robust Kalman filtering scheme with weighted matrices on innovation sequences has been introduced. It has been proven that the proposed RKF possesses high robustness against outliers while having similar computational burden as traditional Kalman filter. To reach network-wide consensus, each node estimates the target state by an adaptive consensus strategy by using only communications between one-hop neighbors. Simulation examples have been included to demonstrate the robustness of the proposed RKF and effectiveness of adaptive consensus approach. Future work
Figure 3: Comparison of RMSE without outliers present.

Figure 4: Comparison of RMSE in case of outlier-corrupted measurements.
will be focused on nonlinear distributed estimation fusion by consensus strategy in switching ad hoc WSNs with non-Gaussian noises.

Conflict of Interests
The authors declare that there is no conflict of interests regarding the publication of this paper.

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