Robust $L_2 - L_\infty$ filtering for Markovian jump neutral systems with distributed delays

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ABSTRACT

Over the past decades, a great deal of attention has been devoted to parameter-invariant neutral systems. However, in practical applications, the jump of parameters frequently occurs to neutral systems on account of sudden environmental changes, which can be described by Markovian jump systems. This paper mainly deals with the problem of robust $L_2 - L_\infty$ filtering for uncertain Markovian jump neutral systems with distributed delays. The uncertain parameters are assumed to be time varying and norm bounded. By utilizing the Newton–Leibniz formula and integral inequality technique, the delay-dependent sufficient conditions for the existence of the filter are derived such that the filtering error system is stochastically stable with an $L_2 - L_\infty$ performance constraint. Based on the obtained criteria and by employing congruent transformation, the robust $L_2 - L_\infty$ filter is designed in terms of linear matrix inequalities. Finally, a numerical example is provided to illustrate the effectiveness of the proposed method.

1. Introduction

It is well known that time delays exist universally in nature, the existence of them does have a great impact on the dynamic performance of the system which cannot be ignored. Many works about time-delay systems have been done (Gao & Li, 2011). Neutral systems as an important branch of time-delay systems, which depend not only on the state but also the derivatives of the state, have drawn growing interest due to their extensive applications in various areas. For example, a four-dimensional linear neutral system can be obtained from the voltage and current fluctuations of a lossless transmission line. In addition, a two-stage dissolved groove in a chemical process and a turbo jet aircraft engine are generally modelled by neutral systems. So far, fruitful results of neutral systems have been reported (Parlakçı, 2015; Zhang & Yu, 2010). Nevertheless, just like numerous physical systems such as networked control systems and manufacturing systems, neutral systems also have variable structures and parameters subject to abrupt changes. This phenomenon can be adequately depicted by a special kind of stochastic hybrid systems, i.e. Markovian jump systems, which include a series of linear subsystems and a switching law governed by the Markovian stochastic process. In Qiu, Wei, and Karimi (2015) and Wei, Qiu, Karimi, and Wang (2013), a new input–output approach to the stability analysis for Markovian jump systems with time-varying delays and deficient transition descriptions has been proposed. When Markovian jump parameters emerge in neutral systems, the delay-dependent stability problem for neutral Markovian jump systems with generally unknown transition rates has been investigated in Kao, Wang, Xie, and Karimi (2015). By means of the linear matrix inequality (LMI) approach, the mode-dependent $L_2 - L_\infty$ fuzzy controller for nonlinear Markovian jump systems with neutral delays has been obtained in He and Liu (2013). Therefore, it is of great significance to discuss Markovian jump neutral systems.

Deserve to be mentioned, all the above results are employed for systems with either discrete delays or neutral delays. Distributed delays will appear in such circumstances when the number of delay summation terms in a system equation is increased and the differences between parameter values in neighbouring are decreased (An, Wen, Gan, & Li, 2011). Distributed delay systems, which consider uneven delays, are widely used in the fields of aerospace, communication, chemistry and so on. From an objective point of view, distributed delays can reveal the nature of things change more profoundly and more accurately. For instance, in the networked control systems, the signal is transmitted in a large quantity of parallel paths, in which there are different sizes of nodes,
so the signal transmission is delay distribution in a certain time. Besides, in the process of distillation, the thermal integration method is used to save energy in every distillation column, so that the distillation column has the characteristic of distributed delays. Hence, considerable attention has been focused on the analysis and synthesis problems of distributed delay systems. In Karimi (2012), a sliding-mode approach for exponential $H_{\infty}$ synchronization problem of a class of master–slave time-delay systems with both discrete and distributed time delays, norm-bounded nonlinear uncertainties and Markovian switching parameters has been studied. He, Song, and Liu (2014) have investigated the unbiased switching parameters with Markovian switching parameters and mixed discrete, neutral, and distributed delays has been established in Karimi (2011). In summary, the research on Markovian jump neutral systems with distributed delays has important theoretical value and engineering significance.

In recent years, $L_2 - L_{\infty}$ filtering ($L_2 - I_{\infty}$ for discrete-time systems) problem has attracted increasing interest and has made great progress. In Li and Zhong (2014), generalised nonlinear $L_2 - I_{\infty}$ filtering for discrete-time Markovian jump descriptor systems with partially unknown transition probabilities has been presented. The issue on gain-scheduled robust $L_2 - L_{\infty}$ filtering for neutral systems with jumping and time-varying parameters has been addressed in Yin, Liu, and Shi (2012). Based on a novel version of mode-dependent Lyapunov function, the problem of exponential $L_2 - L_{\infty}$ filter for distributed delay systems with Markovian jump parameters has been proposed in Zhang and Li (2013). To the best of our knowledge, little effort has been made to uncertain Markovian jump neutral systems with distributed delay, not to mention the robust $L_2 - L_{\infty}$ filtering problem which is still open and needs to be solved.

It is noted that the works in Yin et al. (2012) and Zhang and Li (2013) have investigated only the Markovian jump systems with neutral delays and Markovian jump systems with distributed delays, respectively, owing to some technical difficulties. However, many actual systems encountered time delays may contain discrete delays, neutral delays and distributed delays at the same time. The existence of mixed delays in Markovian jump systems may make the filter design much more complicated and harder to handle, especially for the case where the mixed delays are different from each other. Furthermore, the Lyapunov function constructed in Yin et al. (2012) does not take neutral delays into consideration and there is no sudden jump of the filter parameters, which is more conservative. In addition, methods introduced in Yin et al. (2012) and Zhang and Li (2013) are not the most appropriate to establish sufficient conditions for the stochastic stability, which should not be extended to solve the robust $L_2 - L_{\infty}$ filtering problem for uncertain Markovian jump neutral systems with distributed delays, it restricts their application scope.

In this article, the robust $L_2 - L_{\infty}$ filtering problem for uncertain Markovian jump neutral systems with distributed delays is investigated. Compared with the Lyapunov function of conventional neutral systems, the mode-dependent Lyapunov function is constructed for Markovian jump neutral systems. Subsequently, the sufficient conditions for the existence of the filter are presented to guarantee the stochastic stability of the filtering error system with a prescribed $L_2 - L_{\infty}$ performance level. And the mode-dependent filter can be gained by solving the convex optimization problem of LMI. At last, a numerical simulation verifies the feasibility of the proposed approach. The main contributions of this paper are threefold. First, the robust $L_2 - L_{\infty}$ filtering for uncertain Markovian jump neutral systems with distributed delays is considered for the first time, which comprises and generalizes the previous results. Second, a general version of the delay-dependent Lyapunov function is selected to account for mixed delays and the sufficient conditions are obtained based on the integral inequality technique, which could reduce the conservatism. Third, the considered system is more common than those in the literature, the mixed delays in both states and outputs can describe a real dynamic process comprehensively.

2. Problem formulation

Given a complete probability space $(\Omega, F, P)$, where $\Omega$, $F$ and $P$ represent the sample space, the algebra of events and the probability measure defined on $F$, respectively. Considering the following uncertain Markovian jump neutral system with distributed delays over the space $(\Omega, F, P)$:

$$
\dot{x}(t) = E_1(r(t))\dot{x}(t - \tau) + A(r(t), t)x(t) + A_1(r(t), t)x(t - h) + A_2(r(t), t) \int_{t-h}^{t} \dot{x}(s) \, ds + B_1(r(t), t)w(t),
$$

$$
y(t) = E_2(r(t))\dot{x}(t - \tau) + C(r(t), t)x(t) + C_1(r(t), t)x(t - h) + C_2(r(t), t) \int_{t-h}^{t} \dot{x}(s) \, ds + B_2(r(t), t)w(t),
$$

where $r(t)$ is a Markov chain taking values in a finite set $\{1, 2, \ldots, N\}$, $x(t)$ is the state vector, $\dot{x}(t)$ is the derivative of $x(t)$, $E_1(r(t))$ and $E_2(r(t))$ are state matrices, $A(r(t), t)$ and $A_1(r(t), t)$ are delay matrices, $A_2(r(t), t)$ and $B_1(r(t), t)$ are matrices related to the neutral and distributed delays, respectively, $C(r(t), t)$ and $C_1(r(t), t)$ are output matrices, $C_2(r(t), t)$ and $B_2(r(t), t)$ are matrices related to the neutral and distributed delays, respectively, $\dot{x}(t)$ is the derivative of $\dot{x}(t)$, and $w(t)$ is a stochastic process.
\[ z(t) = L(r(t))x(t) + L_1(r(t))x(t - h), \]
\[ x(t) = \phi(t), \quad \forall t \in [-d, 0], \quad (1) \]

where \( x(t) \in \mathbb{R}^n \) is the state vector; \( y(t) \in \mathbb{R}^q \) is the measured output; \( z(t) \in \mathbb{R}^m \) is the signal to be estimated; \( w(t) \in \mathbb{R}^p \) is the disturbance input which belongs to \( L_2[0, \infty); \) \( \tau > 0, h > 0 \) and \( \eta > 0 \) are constant delays, \( d = \max\{\tau, h, \eta\}; \) \( \phi(t) \) is a vector-valued initial continuous function defined on the segment \([-d, 0]; \) \( r(t), t \geq 0 \) is a continuous-time Markovian process taking values in a finite set \( S = \{1, 2, \ldots, N\}, \) with the transition rate matrix \( \Pi = \{\pi_{ij}\}, (i, j) \in S \) given by

\[
\Pr[r(t + \Delta t) = j \mid r(t) = i] = \begin{cases} 
\pi_{ij}\Delta t + o(\Delta t), & i \neq j, \\
1 + \pi_{ij}\Delta t + o(\Delta t), & i = j,
\end{cases} \quad (2)
\]

where \( \Delta t > 0, \) and \( \lim_{\Delta t \to 0} o(\Delta t)/\Delta t = 0. \) \( \pi_{ij} \geq 0, \forall i \neq j \) is the transition rate from mode \( i \) at time \( t \) to mode \( j \) at time \( t + \Delta t \) such that \( \sum_{j=1}^{N} \pi_{ij} = -\pi_{ii}. \)

In system (1), \( E_1(r(t)), E_2(r(t)), L(r(t)) \) and \( L_1(r(t)) \) are known mode-dependent real constant matrices, while \( A(r(t), t), A_1(r(t), t), A_2(r(t), t), C(r(t), t), C_1(r(t), t), C_2(r(t), t), B_1(r(t), t), B_2(r(t), t) \) and \( B_2(r(t), t) \) are mode-dependent time-varying matrices, which are assumed to satisfy

\[
\begin{align*}
A(r(t), t) &= \tilde{A}(r(t)) + \Delta \tilde{A}(r(t), t), \\
C(r(t), t) &= \tilde{C}(r(t)) + \Delta \tilde{C}(r(t), t), \\
A_1(r(t), t) &= \tilde{A}_1(r(t)) + \Delta \tilde{A}_1(r(t), t), \\
C_1(r(t), t) &= \tilde{C}_1(r(t)) + \Delta \tilde{C}_1(r(t), t), \\
A_2(r(t), t) &= \tilde{A}_2(r(t)) + \Delta \tilde{A}_2(r(t), t), \\
C_2(r(t), t) &= \tilde{C}_2(r(t)) + \Delta \tilde{C}_2(r(t), t), \\
B_1(r(t), t) &= \tilde{B}_1(r(t)) + \Delta \tilde{B}_1(r(t), t), \\
B_2(r(t), t) &= \tilde{B}_2(r(t)) + \Delta \tilde{B}_2(r(t), t),
\end{align*} \quad (3)
\]

where \( \tilde{A}(r(t)), \tilde{A}_1(r(t)), \tilde{A}_2(r(t)), \tilde{C}(r(t)), \tilde{C}_1(r(t)), \tilde{C}_2(r(t)), \tilde{B}_1(r(t)), \tilde{B}_2(r(t)) \) and \( \tilde{B}_2(r(t)) \) are known mode-dependent real constant matrices, while \( \Delta \tilde{A}(r(t), t), \Delta \tilde{A}_1(r(t), t), \Delta \tilde{A}_2(r(t), t), \Delta \tilde{C}(r(t), t), \Delta \tilde{C}_1(r(t), t), \Delta \tilde{C}_2(r(t), t), \Delta \tilde{B}_1(r(t), t) \) and \( \Delta \tilde{B}_2(r(t), t) \) are unknown mode-dependent time-varying uncertain matrices taking the form

\[
\begin{bmatrix}
\Delta \tilde{A}(r(t), t) & \Delta \tilde{A}_1(r(t), t) & \Delta \tilde{A}_2(r(t), t) & \Delta \tilde{B}_1(r(t), t) \\
\Delta \tilde{C}(r(t), t) & \Delta \tilde{C}_1(r(t), t) & \Delta \tilde{C}_2(r(t), t) & \Delta \tilde{B}_2(r(t), t)
\end{bmatrix} =
\begin{bmatrix}
M_1(r(t)) \\
M_2(r(t))
\end{bmatrix}
\times F(r(t), t)[N_1(r(t)) N_2(r(t)) N_3(r(t)) N_4(r(t))], \quad (4)
\]

where \( M_1(r(t)), M_2(r(t)), N_1(r(t)), N_2(r(t)), N_3(r(t)) \) and \( N_4(r(t)) \) are known mode-dependent real constant matrices, \( F(r(t), t) \) is unknown mode-dependent time-varying matrices with Lebesgue norm measurable elements satisfying \( F^T(r(t), t)F(r(t), t) \leq I. \)

To simplify the description, when \( r(t) = i \in S, \) each system matrix, for example, \( \tilde{A}(r(t)) \) is abbreviated as \( \tilde{A}_i \) and so on.

**Assumption 2.1:** In order to ensure that the robustness of the results with respect to small changes of neutral delays, in this paper, we assume \( \|E_1i\| < 1. \)

For system (1), the mode-dependent \( L_2 - L_{\infty} \) filter is constructed as follows:

\[
\begin{align*}
\dot{x}(t) &= A_0x(t) + B_0y(t), \\
x(t) &= C_0x(t), \quad (5) \\
x(t) &= 0,
\end{align*}
\]

where \( x(t) \in \mathbb{R}^n \) is the filter state; \( z(t) \in \mathbb{R}^m \) is the output of the filter; \( A_0, B_0 \) and \( C_0 \) are filter parameters to be determined. Augmenting system (1) and the filter (5), the filtering error system can be obtained

\[
\dot{\xi}(t) = \dot{\xi}(t) - \dot{\xi}(t) + \tilde{A}_1(t)\dot{\xi}(t) + \tilde{A}_1(t)\dot{\xi}(t) - h) \\
+ \tilde{A}_2(t)\int_{t-h}^t \dot{\xi}(s) ds + \tilde{B}_2(t)w(t), \quad (6)
\]

\[
\xi(t) = [x^T(t) \quad x^T(t)]^T, \quad e(t) = z(t) - z(t), \\
L_\xi = [L_{\xi} - C_{\xi}], \quad H = [H_{\xi} 0], \\
\tilde{A}_1(t) = \tilde{A}_1 + \Delta \tilde{A}_1(t), \quad \tilde{A}_1(t) = \tilde{A}_1 + \Delta \tilde{A}_1(t), \\
\tilde{A}_2(t) = \tilde{A}_2 + \Delta \tilde{A}_2(t), \quad \tilde{B}_1(t) = \tilde{B}_1 + \Delta \tilde{B}_1(t), \\
\tilde{A}_i = \begin{bmatrix} \tilde{A}_i & 0 \\
B_{\xi}C \end{bmatrix}, \quad \Delta \tilde{A}_i(t) = \begin{bmatrix} \Delta \tilde{A}_i(t) & 0 \\
B_{\xi}\Delta \tilde{C}_i(t) & 0 \end{bmatrix}, \\
\tilde{A}_i = \begin{bmatrix} \tilde{A}_i & 0 \\
B_{\xi}C \end{bmatrix}, \quad \Delta \tilde{A}_1(t) = \begin{bmatrix} \Delta \tilde{A}_1(t) & 0 \\
B_{\xi}\Delta \tilde{C}_i(t) & 0 \end{bmatrix}, \\
\tilde{A}_2(t) = \begin{bmatrix} \tilde{A}_2 & 0 \\
B_{\xi}C \end{bmatrix}, \quad \Delta \tilde{A}_2(t) = \begin{bmatrix} \Delta \tilde{A}_2(t) & 0 \\
B_{\xi}\Delta \tilde{C}_2(t) & 0 \end{bmatrix}, \\
\tilde{B}_i = \begin{bmatrix} \tilde{B}_i & 0 \\
B_{\xi}B_{\xi} \end{bmatrix}, \\
\Delta \tilde{B}_1(t) = \begin{bmatrix} \Delta \tilde{B}_1(t) & 0 \\
B_{\xi}\Delta \tilde{B}_2(t) & 0 \end{bmatrix}, \quad \tilde{E}_i = \begin{bmatrix} E_{1i} \\
B_{\xi}E_{2i} \end{bmatrix}. 
\]
Throughout this paper, some definitions are given below, which will be utilized in the proof.

**Definition 2.1 (He et al., 2014):** The filtering error system (6) is said to be stochastically stable if, when \( w(t) = 0 \), for any initial state \( \xi(0) = \xi_0 \) and mode \( r_0 \in S \), such that the following holds

\[
\lim_{t \to \infty} E \left\{ \int_0^t \left( \| e(s) \|^2 \right) \, ds \mid \xi_0, r_0 \right\} < \infty,
\]

where \( E(\cdot) \) is the mathematical expectation.

**Definition 2.2 (Yin et al., 2012):** Given a scalar \( \gamma > 0 \), the filtering error system (6) is said to be stochastically stable with an \( L_2 - L_\infty \) performance constraint \( \gamma \), if it is stochastically stable and under zero-initial conditions for any nonzero \( w(t) \in L_2(0, \infty) \), satisfying

\[
E(\| e(t) \|^2_{\infty}) \leq \gamma^2 \| w(t) \|^2_2,
\]

where \( E(\| e(t) \|^2_{\infty}) = E(\sup_t (e^T(t)e(t))) \), \( \| w(t) \|^2_2 = \int_0^\infty \langle w^T(t)w(t) \rangle \, dt \).

**Definition 2.3 (He and Liu, 2011):** Let \( V(\xi(t), r(t) = i, t > 0) \) be the stochastic Lyapunov function of the filtering error system (6), the weak infinitesimal operator is defined as follows:

\[
\mathbf{L}V(\xi(t), i) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \mathbb{E} [V(\xi(t + \Delta t), r(t + \Delta t), t + \Delta t) | \xi(t) = \xi, r(t) = i] - V(\xi(t), i, t)]
\]

\[
= \frac{\partial}{\partial t} V(\xi(t), i, t) + \frac{\partial}{\partial x} V(\xi(t), i, t) \dot{x}(t, i)
\]

\[
+ \sum_{j=1}^N \pi_{ij} \mathbb{E}(\xi(t), j, t).
\]

The main intent of this paper is to design the filter (5) for system (1), guaranteeing that the filtering error system (6) is stochastically stable with a prescribed \( L_2 - L_\infty \) performance constraint \( \gamma \).

**Remark 2.1:** System (1) is composed of continuous states \( x(t) \) and discrete jump modes \( r(t) \), succeeded in modelling many important practical systems that are subject to random changes caused by sudden environment changes, failures occurred in components and so on. For such a system, a mode-dependent filter is more proper for selection and can be acquired by denoting \( A_{fi}, B_{fi}, C_{fi} \) rather than \( A_r, B_r, C_r \), which brings in some computational complexity but exhibits less conservative.

### 3. Robust \( L_2 - L_\infty \) filter design

In this section, the sufficient conditions for stochastic stability and \( L_2 - L_\infty \) performance analysis of the filtering error system (6) are derived firstly. Based on the obtained criteria and by taking the advantage of convex optimization method, the robust \( L_2 - L_\infty \) filter is designed to minimize the influence of the unknown disturbances.

**Theorem 3.1:** For any mode \( i \in S \), given scalars \( \tau > 0, h > 0, \eta > 0 \) and \( \gamma > 0 \), the filtering error system (6) is stochastically stable with an \( L_2 - L_\infty \) performance index \( \gamma \), if there exists positive-definite symmetric matrices \( P_i, Q_i, Q_2, Q_3, Q_4, Q_5, Q_6 \) such that the following LMIs hold

\[
\begin{bmatrix}
\Theta_{1i} \\
\Phi_{1i} \\
\Phi_{2i}
\end{bmatrix} < 0,
\]

where

\[
W = Q_1 + Q_3 + \eta^2 Q_6 - \frac{1}{\tau} Q_5 - \frac{1}{h} Q_2,
\]

\[
\Phi_{11i} = P_i A_{2i}(t) \bar{B}_i(t) (\bar{A}_i(t) H^T K)
\]

\[
= 0, \quad \Phi_{12i} = P_i A_{1i}(t) (\bar{A}_i(t) H^T K)
\]

\[
= 0, \quad \Phi_{22i} = -Q_4
\]

\[
\Phi_{33i} = -Q_3 - \frac{1}{\tau} Q_5, \quad K = h Q_2 + Q_4 + \tau Q_5.
\]
Proof: For any mode $i \in S$, consider the Lyapunov function candidate for filtering error system (6) as

$$V(\xi(t), i) = V_1(\xi(t), i) + V_2(\xi(t), i) + V_3(\xi(t), i) + V_4(\xi(t), i),$$

where

$$V_1(\xi(t), i) = \xi^T(t)P_i\xi(t),$$

$$V_2(\xi(t), i) = \int_{t-h}^t \xi^T(s)H_1^TQ_1H_1\xi(s)\,ds + \int_{t-h}^t \int_{t-h}^s \xi^T(s)H_2^TQ_2H_2\xi(s)\,ds\,d\theta,$$

$$V_3(\xi(t), i) = \int_{t-\tau}^t \xi^T(s)H_3^TQ_3H_3\xi(s)\,ds + \int_{t-\tau}^t \int_{t-\tau}^s \xi^T(s)H_3^TQ_3H_3\xi(s)\,ds\,d\theta,$$

$$V_4(\xi(t), i) = \int_{t-h}^t \left[ \int_{t-h}^s H_4\xi(s)\,ds \right]^T Q_6 \left[ \int_{t-h}^s H_4\xi(s)\,ds \right] d\theta + \int_{t-h}^t \int_{t-h}^s (s - t + \theta)\xi^T(s)H_5^TQ_6H_6\xi(s)\,ds\,d\theta,$$

Recalling Definition 2.3, we can obtain the time derivative of $V(\xi(t), i)$ as follows:

$$\dot{E}V_1(\xi(t), i) = \xi^T(t) \left[ P_i\tilde{A}_i(t) + \tilde{A}_i^T(t)P_i + \sum_{j=1}^N \beta_jP_j \right] \xi(t) + 2(\xi(t))P_1\tilde{A}_1(t)\tilde{H}_1\xi(t) + 2\xi^T(t)P_1\tilde{A}_1(t)\tilde{H}_2\xi(t - h) + 2\xi^T(t)P_2\tilde{A}_2(t)H\int_{t-h}^t \xi(s)\,ds + 2\xi^T(t)P_3\tilde{B}_3(t)w(t),$$

$$\dot{E}V_2(\xi(t), i) = \xi^T(t)H_1^TH_1^TQ_1H_1\xi(t) - \xi^T(t + h)H_1^TH_1^TQ_1H_1\xi(t - h) + \xi^T(t)H_1^TH_1^TQ_1H_1\xi(t) - \int_{t-h}^t \xi^T(s)H_1^TH_2^TQ_2H_2\xi(s)\,ds,$$

$$\dot{E}V_3(\xi(t), i) = \xi^T(t)H_3^TH_3^TQ_3H_3\xi(t) - \xi^T(t + \tau)H_3^TH_3^TQ_3H_3\xi(t - \tau) + \xi^T(t + \tau)H_3^TH_3^TQ_3H_3\xi(t - \tau) - \int_{t-\tau}^t \xi^T(s)H_3^TH_3^TQ_3H_3\xi(s)\,ds,$$

$$\dot{E}V_4(\xi(t), i) = 2\xi^T(t)H_5^TH_5^TQ_6 \int_{t-h}^t H_4\xi(s)\,ds d\theta + \frac{\eta^2}{2} \xi^T(t)H_5^TH_5^TQ_6\xi(t) - \int_{t-h}^t (s - t + \theta)\xi^T(s)H_5^TH_5^TQ_6\xi(s)\,ds + \frac{\eta^2}{2} \xi^T(t)H_5^TH_5^TQ_6\xi(t) - \int_{t-h}^t (s - \tau + \theta)\xi^T(s)H_5^TH_5^TQ_6\xi(s)\,ds.$$

By Lemma 2.3 in He and Liu (2011), it can be verified that

$$- \int_{t-h}^t \xi^T(s)H_5^TH_5^TQ_6\xi(s)\,ds \leq -\frac{1}{\tau} \int_{t-h}^t \xi^T(s)\,ds H_5^TH_5^TQ_6\int_{t-h}^t \xi(s)\,ds,$$

$$- \int_{t-\tau}^t \xi^T(s)H_5^TH_5^TQ_6\xi(s)\,ds \leq -\frac{1}{\tau} \int_{t-\tau}^t \xi^T(s)\,ds H_5^TH_5^TQ_6\int_{t-h}^t \xi(s)\,ds.$$

Using the Newton–Leibniz formula, when $w(t) = 0$, we can obtain

$$\dot{E}V(\xi(t), i) \leq \psi_1^T(1)\Phi_1(1)\psi_1(t),$$

where

$$\psi_1(t) = \left[ \xi^T(t) \left( H_5^T(t - h) \right)^T \left( H_5^T(t - \tau) \right)^T \right] \left( \int_{t-h}^t H_5\xi(s)\,ds \right)^T,$$

$$\Phi_1(1) = \left[ \begin{array}{c} \xi^T(t) \left( H_5^T(t - h) \right)^T \left( H_5^T(t - \tau) \right)^T \\ \left( H_5^T(t - \tau) \right)^T \left( \int_{t-h}^t H_5\xi(s)\,ds \right)^T \end{array} \right].$$
Thus, when \( \Phi_{1i}(t) \lessgtr 0 \) holds. Then for any nonzero \( \Phi_{1i}(t) \lessgtr 0 \), we obtain \( EV(\xi(t), i) \leq -\lambda_0 \psi_1(t) \psi_1(t) \leq -\lambda_0 \xi(t) \xi(t) \). (9)

where \( \lambda_0 = \min(\lambda_{\text{min}}(-\Phi_{1i}(t)), i \in S) > 0 \). Integrating both sides of Equation (9) from 0 to \( t > 0 \) and taking the expectation gives

\[
E[V(\xi(t), i)] = E[V(\xi_0, r_0)] - \lambda_0 \int_0^t \xi(s) \xi(s) \, ds.
\]

Since \( E[V(\xi(t), i)] > 0 \), we can receive

\[
E\left[\int_0^t \|\xi(s)\|^2 \, ds\right] \leq \lambda_0^{-1} E[V(\xi_0, r_0)].
\]

Taking the limit as \( t \to \infty \), we have \( E[\int_0^\infty \|\xi(t)\|^2 \, dt] < \infty \). Hence, the filtering error system (6) is stochastically stable by Definition 2.1.

Next, we will establish the \( L_2 - L_\infty \) performance for filtering error system (6) under zero-initial conditions. Consider the following index:

\[
J = E[V(\xi(t), i)] - \int_0^{T_1} w^T(t)w(t) \, dt.
\]

Then for any nonzero \( w(t) \in L_2(0, \infty) \) and \( T_1 > 0 \), we can get

\[
J = E\left\{\int_0^{T_1} (EV(\xi(t), i) - w^T(t)w(t)) \, dt\right\}
\]

\[
= E\left\{\int_0^{T_1} \psi_2^T(t) \Phi_2(t) \psi_2(t) \, dt\right\},
\]

where

\[
\psi_2(t) = \begin{bmatrix} \xi^T(t) & (H\xi(t) - h)^T & (H\xi(t) - \tau)^T & (H\xi(t) - \tau)^T \left(\int_{t-\eta}^t H\xi(s) \, ds \right)^T & w^T(t) \end{bmatrix}^T,
\]

Applying the Schur complement formula to Equation (8) gives \( \Phi_{2i}(t) \lessgtr 0 \) which implies \( J < 0 \), then we have

\[
E[\xi^T(t)P_\xi(t)] \leq E[V(\xi(t), i)] \leq \int_0^{T_1} w^T(t)w(t) \, dt.
\]

On the other hand, condition (7) shows that

\[
E[e^T(t)e(t)] = E[(\tilde{\xi}e(t) + L_1H\xi(t - h))^T(\tilde{\xi}e(t) + L_1H\xi(t - h))] < \gamma^2 E[\xi^T(t)P_\xi(t)]
\]

\[
\leq \gamma^2 \int_0^{T_1} w^T(t)w(t) \, dt
\]

\[
\leq \gamma^2 \int_0^{\infty} w^T(t)w(t) \, dt.
\]

Therefore, \( E[\|\xi(t)\|^2_{L_\infty}] \leq \gamma^2 \|w(t)\|^2_2 \). From Definition 2.2, the filtering error system (6) is stochastically stable and satisfies \( L_2 - L_\infty \) performance \( \gamma \), which concludes the proof.

\section*{Remark 3.1:}
It is worth pointing out that the reasonable construction of Lyapunov function is directly related to the conservatism of the results. In this paper, the delay-dependent conditions are derived by exploiting an appropriate type of Lyapunov function, which makes full use of the information of the bounds and derivatives of time delays without using the differential operator \( De(t) \), so that the relevant results are more feasible and achievable. And the Lyapunov function has matrix \( P \), depended on the jump mode \( r(t) \), which is less conservative than the results discussed by using the conventional matrix \( P \).

\section*{Remark 3.2:}
Compared with existing literature, the system we considered is more comprehensive and has more...
applications in reality since discrete delays, neutral delays and distributed delays are simultaneously taken into account. Further, the mixed delays, which are not equal to each other, exist in both states and outputs. It is worth to note that Parlakçı (2015), An et al. (2011), Yin et al. (2012), and Zhang and Li (2013) are included here as special cases.

Based on Theorem 3.1, the design problem of robust $L_2 - L_\infty$ filter can be established in the following theorem.

**Theorem 3.2:** For any mode $i \in \Sigma$, given scalars $\tau > 0, h > 0, \eta > 0$ and $\gamma > 0$, the filtering error system (6) is stochastically stable with an $L_2 - L_\infty$ disturbance attenuation $\gamma$, if there exist scalars $\epsilon_i > 0$, positive-definite symmetric matrices $X_i, Y_i, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6$ and matrices $R_i, U_i, V_i$ such that the following LMIs hold

$$
\Theta_{2i} = \begin{bmatrix}
-\frac{1}{2}\gamma^2 I & 0 & L_i - R_i & -R_i & 0 & 0 \\
* & -\frac{1}{2}\gamma^2 I & 0 & 0 & L_{1i} & 0 \\
* & * & Y_i - X_i & 0 & 0 & 0 \\
* & * & * & -Y_i & 0 & 0 \\
* & * & * & * & Y_i - X_i & 0 \\
\end{bmatrix} < 0, (10)
$$

$$
\Phi_{2i} = \begin{bmatrix}
\Phi_{11i} & \Phi_{12i} & \Phi_{13i} & \frac{1}{\tau} Q_5 & Y_1 M_{1i} \\
* & \Phi_{22i} & \Phi_{23i} & 0 & \Phi_{25i} \\
* & * & \Phi_{33i} & 0 & 0 \\
* & * & * & \Phi_{44i} & 0 \\
* & * & * & * & -Q_4 \\
* & * & * & * & * \\
\end{bmatrix} < 0, (11)
$$

where

$$
\begin{align*}
\Phi_{11i} &= Y_i A_i + A_i^T Y_i + W + \sum_{j=1}^N \pi_{ij} Y_j + \epsilon_i N_{1i}^T N_{1i}, \\
\Phi_{12i} &= A_i^T Y_i - A_i^T X_i + \tilde{C}_i^T \bar{V}_i + U_i^T, \\
\Phi_{13i} &= Y_i A_i + \frac{1}{\tau} Q_2 + \epsilon_i N_{1i}^T N_{2i}, \\
\Phi_{16i} &= Y_i A_{2i} + \epsilon_i N_{1i}^T N_{3i}, \\
\Phi_{17i} &= Y_i B_{1i} + \epsilon_i N_{1i}^T N_{4i}, \\
\Phi_{22i} &= U_i + U_i^T + \sum_{j=1}^N \pi_{ij} (X_j - Y_j), \\
\Phi_{23i} &= (Y_j - X_i) A_{1i} + V_i \tilde{C}_{1i}, \\
\Phi_{25i} &= (Y_j - X_i) E_{1i} + V_i E_{2i}, \\
\Phi_{26i} &= (Y_j - X_i) A_{2i} + V_i \tilde{C}_{2i}, \\
\Phi_{27i} &= (Y_j - X_i) B_{1i} + V_i \tilde{B}_{2i}, \\
\Phi_{29i} &= (Y_j - X_i) M_{1i} + V_i M_{2i}, \\
\Phi_{33i} &= -Q_1 - \frac{1}{\tau} Q_2 + \epsilon_i N_{2i}^T N_{2i}, \\
\Phi_{44i} &= \Phi_{33i}, \\
\Phi_{66i} &= -Q_6 + \epsilon_i N_{3i}^T N_{3i}, \\
\Phi_{77i} &= -I + \epsilon_i N_{4i}^T N_{4i}.
\end{align*}
$$

Moreover, the mode-dependent filter matrices are given as follows:

$$
\begin{bmatrix}
A_{fi} & B_{fi} \\
C_{fi} & 0
\end{bmatrix} = \begin{bmatrix}
(X_i - Y_i)^{-1} & 0 \\
0 & I
\end{bmatrix} \begin{bmatrix}
U_i & V_i
\end{bmatrix}. (12)
$$

**Proof:** By Lemma 2 in An et al. (2011), and recalling Schur complement, Equation (8) is equivalently written as

$$
\tilde{\Phi}_{1i} = \begin{bmatrix}
\Phi_{11i} & \Phi_{12i} & \frac{1}{\tau} H_i^T Q_5 & P_i \tilde{E}_i \\
* & \Phi_{22i} & 0 & 0 \\
* & * & \Phi_{33i} & 0 \\
* & * & * & -Q_4 \\
* & * & * & * \\
* & * & * & * \\
\end{bmatrix} < 0, (13)
$$
where

\[ \Phi_{1i} = P_i A_i + A_i^T P_i + \sum_{j=1}^{N} \pi_j P_j + H_i^T WH + \varepsilon_i \bar{N}_i \bar{N}_i^T, \]

\[ \Phi_{2i} = P_i A_i + \frac{1}{h} H_i^T Q_2 + \varepsilon_i \bar{N}_i \bar{N}_2, \]

\[ \Phi_{3i} = P_i A_i + \varepsilon_i \bar{N}_i \bar{N}_3, \quad \Phi_{16i} = P_i \tilde{A}_i + \varepsilon_i \bar{N}_i \bar{N}_4, \]

\[ \Phi_{22i} = -Q_1 - \frac{1}{h} H_i^T Q_2 + \varepsilon_i \bar{N}_2 \bar{N}_2, \]

\[ \tilde{\Phi}_{3i} = \Phi_{33i} = \tilde{\Phi}_{66}, \quad \Phi_{66i} = \tilde{\Phi}_{77i}, \]

\[ \tilde{M}_1 = [\tilde{M}_1^T (B_i M_2)^T]^T, \quad \bar{N}_i = [N_i 0]. \]

For any mode \( i \in S \), define

\[ P_i = \begin{bmatrix} X_i & Y_i - X_i \\ Y_i - X_i & X_i - Y_i \end{bmatrix}. \]

It is easy to see that \( X_i - (Y_i - X_i)(X_i - Y_i)^{-1}(Y_i - X_i) = Y_i > 0 \), which together with Equation (10) implies \( P_i > 0 \).

Introduce

\[ \Upsilon_i = \begin{bmatrix} Y_i^{-1} & 0 \\ Y_i^{-1} & I \end{bmatrix}. \]

Performing congruence transformations to Equation (13) by \( \theta_{1i} = \text{diag}(\Upsilon_i^T, l, l, l, l, l, l) \) and to Equation (7) by \( \nu_{1i} = \text{diag}(l, l, \Upsilon_i^T, Y_i^T) \) yield Equations (14) and (15), respectively.

\[ \Phi_i = \theta_{1i} \Phi_1 \theta_{1i}^T < 0, \quad (14) \]

\[ \Theta_i = \nu_{1i} \Theta_1 \nu_{1i}^T < 0. \quad (15) \]

Using \( \theta_{2i} = \text{diag}(Y_i, l, l, l, l, l, l, l, l, l, l) \), \( \nu_{2i} = \text{diag}(l, l, Y_i, l, l, l) \) and their transposes to pre- and post-multiplying the LMIs in Equations (14) and (15) respectively, and setting

\[ U_i = (X_i - Y_i) A_i, \quad V_i = (X_i - Y_i) B_i, \quad R_i = C_i \quad (16) \]

then we obtain Equations (11) and (10).

Therefore, from Equation (16), we can conclude that the filter parameters can be constructed by Equation (12). This completes the proof.

Remark 3.3: Note that Theorem 3.2 proposes sufficient conditions for robust \( L_2 - L_\infty \) filtering design such that the filtering error system (6) is stochastically stable with a prescribed performance. It is possible to minimize the noise-attention level \( \gamma^2 \) according to Theorem 3.2, and the robust \( L_2 - L_\infty \) filter can be readily found by solving the following convex optimization problem:

\[ \begin{aligned}
\min_{\tau, h, \eta, \epsilon_i, X_i, Y_i, U_i, V_i, Q_i, T_i, \eta, \tilde{\eta}, \gamma} & \quad \rho \\
\text{subject to} & \quad (15) \text{ and } (16) \text{ with } \rho = \gamma^2.
\end{aligned} \]

Remark 3.4: It is worth mentioning that Lemma 2 in An et al. (2011) is successfully used to tackle the norm-bounded parameter uncertainties in the proof of Theorem 3.2, which is very important for deriving LMIs solutions in general. According to LMIs (Equations (10) and (11)) which are obtained by using the integral inequality technique and congruent transformation method, we can find that the variable numbers are fewer than Theorem 2 in Chen et al. (2015). Consequently, the filter design method provides a more simple form.

Remark 3.5: It is easy to see that Theorem 3 in Chen et al. (2015) provides an easy technique using the block diagonal matrix \( P_i = \text{diag}(X_i, Y_i) \) during the process of designing the filter. Due to the restriction of 0 on the vice diagonal, the conservatism is larger. However, this constraint is relaxed in this paper by defining \( P_i = \begin{bmatrix} X_i & Y_i - X_i \\ Y_i - X_i & X_i - Y_i \end{bmatrix} \). Thus, the results in Theorem 3.2 are more general than that in Chen et al. (2015).

It is worth pointing out that our main results can be easily extended to time-varying delays. When considering time-varying delays \( \tau(t), h(t), \eta(t) \) which satisfy \( 0 \leq \tau(t) < \tau, \ h(t) \leq h_d, \ \eta(t) \leq \eta_d < 1, \) and \( d = \max \{ \tau(t), h(t), \eta(t) \}, \) we can get the next corollary following the similar lines as in the proof of Theorems 3.1 and 3.2, so it is omitted here.

Corollary 3.1: For any mode \( i \in S, \) given scalars \( \tau, h, \eta, \tau_d, h_d, \eta_d \) and \( \gamma > 0, \) the filtering error system (6) is stochastically stable with an \( L_2 - L_\infty \) disturbance attenuation \( \gamma, \) if there exist scalars \( \epsilon_i > 0, \) positive-definite symmetric matrices \( X_i, Y_i, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, R_i, U_i, V_i \) such that the following LMIs hold

\[ \Lambda_i = \begin{bmatrix}
-\frac{1}{2} \gamma^2 I & 0 & L_i - R_i & -R_i & 0 & 0 \\
* & -\frac{1}{2} \gamma^2 I & 0 & 0 & L_i & 0 \\
* & * & -Y_i & 0 & 0 & 0 \\
* & * & * & Y_i - X_i & 0 & 0 \\
* & * & * & * & -Y_i & 0 \\
* & * & * & * & * & Y_i - X_i 
\end{bmatrix} < 0, \quad (18) \]
where

\[
\begin{bmatrix}
\Gamma_{11i} & \Gamma_{12i} & \Gamma_{13i} & \frac{1}{\tau}Q_5 & Y_iE_{1i} \\
* & \Gamma_{22i} & \Gamma_{23i} & 0 & \Gamma_{25i} \\
* & * & \Gamma_{33i} & 0 & 0 \\
* & * & * & \Gamma_{44i} & 0 \\
* & * & * & * & \Gamma_{55i} \\
\end{bmatrix} + \begin{bmatrix}
\epsilon_i N_{2j1} N_{3j1} & \epsilon_i N_{2j2} N_{4j1} & \epsilon_i N_{2j3} N_{5j1} & \epsilon_i N_{2j4} N_{6j1} & \epsilon_i N_{2j5} N_{7j1} & \epsilon_i N_{2j6} N_{8j1} & \epsilon_i N_{2j7} N_{9j1} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \xi_i T_i K & 0 & 0 & 0 & 0 \\
0 & 0 & \bar{\lambda}_i T_i K & 0 & 0 & 0 & 0 \\
0 & 0 & \tilde{\lambda}_i T_i K & 0 & 0 & 0 & 0 \\
\end{bmatrix} < 0,
\]

**Remark 3.6:** Noted that the case of constant time delays provides feasible solutions irrespective of the size of delays. However, the Lyapunov function \( V(\xi(t), \bar{i}) \) of time-varying delays, which is set as \( V(\xi(t), \bar{i}) = \xi^T(t)P_i \xi(t) + \int_{t-h(t)}^{t} \xi^T(s)H_1^{\tau} Q_1 H_1(s) \) for the case of constant time delays, which is set as \( V(\xi(t), \bar{i}) = \xi^T(t)P_i \xi(t) + \int_{t-h(t)}^{t} \xi^T(s)H_1^{\tau} Q_1 H_1(s) \) for the case of time-varying delays, which is set as \( V(\xi(t), \bar{i}) = \xi^T(t)P_i \xi(t) + \int_{t-h(t)}^{t} \xi^T(s)H_1^{\tau} Q_1 H_1(s) \). \]

**Remark 3.7:** As is known to all, the Riccati equation is widely used in modern control theory. However, there are a large number of parameters and positive-definite symmetric matrices need to be adjusted in advance, which brings great inconvenience to the practical application. Our main results are formulated in terms of LMIs, which makes up the deficiency of the Riccati equation. The LMI Toolbox implements interior-point LMI solvers, which are significantly faster than classical convex optimization algorithms.

**Remark 3.8:** A series of random jumps of parameters, all of which are mutually independent but obey the Markovian process, are introduced in the considered system. The jump parameters exist in the obtained LMIs, which occurs in a random way and satisfies the transition rate. Meanwhile, conditions (7) and (8) are nonlinear matrix inequalities for the unknown matrix variables, which makes us unable to use the standard LMI steps to solve the parameters of the filter. Therefore, the matrix transform method is used to realize the decoupling between the system matrices and the Lyapunov matrix, and the filtering problem is transformed into a convex optimization problem based on LMIs, then the conservatism of the filter design is reduced. However, this method creates new matrix variables. To solve these questions, we require heavier computational burdens and take more time.

### 4. Illustrative Example

This section provides a numerical example to demonstrate the validity of the proposed method. Consider system (1) with two modes, the corresponding parameters are as follows. Mode 1:

\[
\tilde{A}_1 = \begin{bmatrix}
-1 & 0 \\
-0.6 & -2.2 \\
\end{bmatrix}, \quad \tilde{A}_{11} = \begin{bmatrix}
-0.8 & -0.1 \\
-0.8 & 0.4 \\
\end{bmatrix},
\]

then the \( L_2 - L_\infty \) filter can be designed in Equation (16).
\[ \tilde{A}_{21} = \begin{bmatrix} 0.5 & -0.4 \\ 0.2 & 0.4 \end{bmatrix}, \quad \tilde{B}_{11} = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}, \]

\[ E_{11} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad \tilde{C}_1 = \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.6 \end{bmatrix}, \]

\[ \gamma \]

\[ \tilde{C}_{11} = \begin{bmatrix} -0.8 & -0.8 \\ 0.1 & 0.2 \end{bmatrix}, \quad \tilde{C}_{21} = \begin{bmatrix} -0.2 & 0.6 \\ 0.8 & -0.5 \end{bmatrix}, \]

\[ \tilde{B}_{21} = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, \quad \tilde{E}_1 = \begin{bmatrix} -0.2 & 0.3 \\ 0.2 & -0.4 \end{bmatrix}, \]

\[ L_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & -0.7 \end{bmatrix}, \quad L_{11} = \begin{bmatrix} 0.5 & 0 \\ 0 & -0.5 \end{bmatrix}, \]

\[ M_{11} = \begin{bmatrix} -0.2 \\ 0.2 \end{bmatrix}, \quad M_{21} = \begin{bmatrix} 0.2 \\ -0.4 \end{bmatrix}, \quad N_{11} = \begin{bmatrix} -0.2 & 0.4 \end{bmatrix}, \]

\[ N_{21} = \begin{bmatrix} 0.3 & -0.4 \end{bmatrix}, \quad N_{31} = \begin{bmatrix} 0.5 & -0.4 \end{bmatrix}, \quad N_{41} = \begin{bmatrix} 0 & 0 \end{bmatrix}. \]

**Mode 2:**

\[ \tilde{A}_2 = \begin{bmatrix} -3 & 0.6 \\ -0.8 & -2.5 \end{bmatrix}, \quad \tilde{A}_{12} = \begin{bmatrix} -0.5 & 0.1 \\ -0.3 & 0.4 \end{bmatrix}, \]

\[ \tilde{A}_{22} = \begin{bmatrix} 0.3 & -0.4 \\ -0.1 & 0.6 \end{bmatrix}, \quad \tilde{B}_{12} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \]

\[ E_{12} = \begin{bmatrix} 0.2 & -0.3 \\ -0.1 & 0.4 \end{bmatrix}, \quad \tilde{C}_2 = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \]

\[ \gamma \]

\[ \tilde{C}_{12} = \begin{bmatrix} -0.7 & -0.6 \\ 0.3 & 0.2 \end{bmatrix}, \quad \tilde{C}_{22} = \begin{bmatrix} -0.3 & 0.5 \\ 0.4 & -0.3 \end{bmatrix}, \]

\[ \tilde{B}_{22} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad \tilde{E}_2 = \begin{bmatrix} -0.3 & 0.4 \\ 0.1 & -0.5 \end{bmatrix}, \]

\[ L_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0 & -0.5 \end{bmatrix}, \quad L_{12} = \begin{bmatrix} 0.4 & 0 \\ 0 & -0.4 \end{bmatrix}, \]

\[ M_{12} = \begin{bmatrix} -0.1 \\ 0.2 \end{bmatrix}, \quad M_{22} = \begin{bmatrix} 0.2 \\ -0.3 \end{bmatrix}, \quad N_{12} = \begin{bmatrix} -0.2 & 0.4 \end{bmatrix}, \]

\[ N_{22} = \begin{bmatrix} 0.2 & -0.4 \end{bmatrix}, \quad N_{32} = \begin{bmatrix} 0.4 & -0.4 \end{bmatrix}, \quad N_{42} = \begin{bmatrix} 0 & 0 \end{bmatrix}. \]

The simulation results are given in Figures 1–4, where the initial conditions are assumed as \( x(0) = [1 1]^T, \) \( x_f(0) = [0 0]^T, \) respectively, and the external disturbance input is given as \( w(t) = e^{-0.5t}. \) Figure 1 plots out the jumping modes. Figure 2 displays the system state \( x_1(t) \) and its estimation \( x_{f1}(t). \) Figure 3 displays the system state \( x_2(t) \) and its estimation \( x_{f2}(t). \) Figure 4 shows the filtering error \( e(t). \) It is obvious that the obtained filter can track the real states smoothly and approach zero quickly as time goes, guaranteeing that the filtering error system is stochastically stable with the specified requirements.

In order to further illustrate the design method proposed in this paper is less conservative, we will make

**Figure 1.** Jumping modes.

**Figure 2.** The system state \( x_1(t) \) and its estimation.
It can be verified from the case of constant time delays in the table that for given $h = \tau = 1$, $\eta = 1.5$, the mode-independent and block diagonal matrix methods are not satisfied for this system, which implies that the methods fail to conclude whether or not there exist $L_2 - L_\infty$ filters. However, applying our results to this case, we have obtained $L_2 - L_\infty$ filter that guarantees the stochastic stability of the filtering error system, which shows the merits of our method.

The table shows that the distributed delays have a direct influence on the $L_2 - L_\infty$ performance index. That is, the smaller distributed delay is, the better $L_2 - L_\infty$ performance index will be achieved. It is clear from the table that the $\gamma^*$ obtained by our method tends to be the minimum, which shows less conservative results and better performance than other two methods. Meanwhile, the case of constant time delays provides a larger $\gamma^*$ than the case of time-varying delays, especially in situations where delays are big, which indicates that the conservatism increases when the size of delays is abandoned.

5. Conclusion

As we all know, previous results, for the most part, are under the assumption that the systems investigated are subject to time-invariant parameters. But in real uncertain Markovian jump neutral systems, it may always be the case that the jumping parameters are time varying. The issue on robust $L_2 - L_\infty$ filtering for such systems with distributed delays has been discussed in this paper. The time delays assumed to arise in both states and outputs are generalized by combining discrete delays, neutral delays and distributed delays. By choosing the mode-dependent Lyapunov function and using matrix transformation method, the less conservative filter is formulated in terms of LMIs, which ensures that the filtering error system is stochastically stable and satisfies a prescribed $L_2 - L_\infty$ performance level for all admissible uncertainties. A numerical example has demonstrated that the designed filter can estimate original system states and suppress the interference effectively, which indicates the validity of the proposed results.

| $\eta$  | 0.6  | 0.8  | 1    | 1.3  | 1.5    |
|---------|------|------|------|------|--------|
| Constant time delays | Mode-dependent | 0.2045 | 0.2216 | 0.2500 | 0.3831 | 0.5560 |
|          | Mode-independent | 0.2272 | 0.2522 | 0.3208 | ~      | ~      |
|          | Block diagonal method | 0.2820 | 0.3056 | 0.3480 | 0.5327 | ~      |
| Time-varying delays | Mode-dependent | 0.1857 | 0.1949 | 0.2066 | 0.2341 | 0.2683 |
|          | Mode-independent | 0.2025 | 0.2140 | 0.2298 | 0.2921 | 0.4314 |
|          | Block diagonal method | 0.2569 | 0.2685 | 0.2837 | 0.3211 | 0.3701 |
Disclosure statement

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