1. Introduction

According to the current research, harmful substances inside different premises (social, industrial, housing) are having increasingly adverse effects on the human health and wellbeing. In its current state, the performance of the maintenance staff is down six or more percent with more errors made in the process. The losses accounted for by the reduction in the productivity due to low quality of the indoor air might be higher than the costs of maintaining optimal and acceptable microclimate parameters in the operating area of maintained premises and thus this aspect is central in designing systems of maintaining social and industrial buildings and structures. A mere increase in the multiplicity of air exchange does not necessarily cause the quality of premises to improve. This is due to infiltration of the indoor air from the area of the aerodynamic shadow of a building as well as possible dead air regions occurring due to turbulence of air flows inside premises of complex configuration.

In order to provide the required microclimate parameters in such premises it is increasingly common to use displacement ventilation. It was proved that displacement ventilation has a number of advantages compared to mixed-mode ventilation. The major principle of this method of air distribution is even supply and removal of air at a low velocity. The efficiency of these systems increases as air flows are distributed more evenly.

The main problem in designing displacement ventilation is a complex configuration of premises or enclosures between them. In this case it is necessary to accurately calculate current lines of air flows, their directions and velocities. Presently in order to do this mathematical modeling is applied. Mathematical modeling of velocity, concentration and temperature...
Conformal Mapping in Mathematical Modelling of Air Flows in Premises

fields is the most promising method of studying microclimate parameters influencing health and safety and technological requirements for the indoor air of premises. This allows one to obtain quick and accurate solution of multidimensional tasks as well as to identify the effects of variable thermal physical characteristics, boundary conditions and duration of processes on them.

Mathematical modeling is seen as promising in developing methods of calculating and designing air conditioning and ventilation systems in mathematical physics in handling problems regarding low speed aerodynamics. The major application of conformal mappings is calculation of a flat harmonic vector field which is defined as a field whose vector is parallel to some plane $Y$. Its value and direction are identical at all points of any straight line perpendicular to plane $Y$.

The main task facing the theory of conformal mapping is designing functions imaging one area onto another. As there is no general algorithm to address this, the development of the theory of conformal mapping has several directions. Firstly, general conditions for conformal mapping and its unity are specified. Secondly, different specific classes of areas that can be mapped using a combination of simplest functions are identified. Thirdly, based on general properties of analytical functions different properties of conformal mappings are studied depending on a type of mapped areas. Fourthly, approximate methods of conformal mapping are being developed.

This problem of calculating velocity fields of air in premises using the theory of functions of a complex variable (conformal mapping) takes several stages: Designing current lines of the air, determining numerical values of the air velocity. Let us look at each of them more closely.

2. Designing Current Lines

Velocity fields of air flows using conformal mapping are designed in a two-dimensional setup. Therefore the premises in question are rectangular with the sides parallel to the abscissa axis and ordinate axis. Fields of velocity of air flows in this case are designed with a transition to the complete system of the Navier-Stokes equations (which describes motion of air flows in premises) to a more simple system of the Cauchy–Riemann equations. The medium and its vortex-free flow should be non-condensable. According to in this case the components of the velocity vector of moving air flows $\omega_x, \omega_y$ constitute a gradient of some potential function $\varphi(x, y)$:

$$\omega_x = \frac{\partial \varphi}{\partial x} \quad \omega_y = \frac{\partial \varphi}{\partial y}, \quad (1)$$
A path of the moving air or a current line is lines of a level of another function $\psi(x, y)$ adjoining to the function $\varphi(x, y)$\textsuperscript{3}. A complex potential of a flow, function (2) are an analytical (holomorphic) function in some initial figure (premise):

$$f(z) = \varphi(z) + i\psi(z); z = x + iy,$$  \hspace{1cm} (2)

For the derivative $f'(z)$ of a complex potential there is the following equality:

$$f'(z) = \omega_x - i\omega_y,$$  \hspace{1cm} (3)

On condition that the vector (3) never turns to zero in the original figure, the reflection performed with the function (2) is conformal in this figure. Therefore at the initial stage it is crucial that the initial boundary conditions are determined correctly by specifying the size, shape, enclosures, air inlet devices.

In order to design current lines in the initial premise it can be assumed that there is air moving inside the figure and so using the symmetry of the premise it can be argued that current lines propagate symmetrically in relation to the enclosure and only either of the halves of the figure can thus be considered and the principle of symmetry can be multiply employed repeating all the transformations of the previous stage. The general principle of applying conformal mapping relies on the properties of the first-order elliptic integral and a reverse function called an elliptic sine\textsuperscript{3–5}. It should be noted that an elliptic sine cannot be applied for an arbitrary-sized rectangle, it should be stretched (compressed) so that it could be of an appropriate size. Figure 1 shows the main schemes of air distribution in premises and approximate development of air flows depending on the position of air inlet and exhaust devices.

**Figure 1.** Principal schemes of the development of air flows for displacement ventilation.

In order to design current lines and subsequently determine the velocities of air flows in premises shown in Figure 1, it is necessary to design conformal mapping of the initial figure as "a more simple one" which is an elementary rectangle where inlet and outlet openings of a ventilation system should be places so that they suggested a "natural" configuration of the current lines.

In this case of restrained figures they are mapped onto a horizontal rectangle with the inlet and outlet openings being the right and left sides of the rectangle respectively. The transformation of the initial figures into elementary ones followed by design of current lines is detailed in\textsuperscript{3}. The main stages of these transformations are basically as follows: Transformation of a real-size rectangle into a rectangle for which there is a solution in this specific parameter so that before designing conformal mapping it is necessary to expand the initial figure; transformation of a rectangle into a half-plane under the effect of an elliptic sine; transformation of a half-plane into a half-plane under the effect of a linear-fractional mapping (4):

$$w = g(\zeta) = \frac{A\zeta + B}{\zeta + D},$$  \hspace{1cm} (4)

Transformation of a half-plane into a rectangle under the effect of an elliptic integral with a parameter $\alpha$ (5) is as follows

$$F_\alpha(z) = F(z, \alpha) = \int_0^z \frac{dt}{\sqrt{(1-t^2)(1-\alpha^2t^2)}} (0 < \alpha < 1).$$  \hspace{1cm} (5)

A general mapping is designed in the reverse order as a superposition of simple transformations. In the case when premises are more complexly configured, calculations become more difficult as a new transformation is added, e.g., in Figure 2 it is necessary to repeat the principle of symmetry or to force a division of the premise into more simple areas.

**Figure 2.** Principal schemes of the development of air flows for symmetrical division of premises.

For visual purposes the final result of the first stage of the calculations of the premise presented in Figure 1a can be shown using the Maple software (Figure 3).
3. Determining Numerical Values of the Velocity of an Air Flow

In order to determine the numerical value of the air at a certain point of the premises it is traditionally assumed that a derivative of the complex potential is a complex adjoint vector to the velocity vector. I.e. if mapping of the premise onto a half-plane is known, a tangent to the current lines determines the air flow velocity and is calculated using simple differentiation. Therefore for a known complex potential of a flow velocity mappings are given by the formula:

$$ \frac{dv}{dz} = \omega_x + i\omega_y, $$  \hspace{1cm} (6)

Where $\omega_x$ is a velocity mapping onto the axis $OX$; $\omega_y$ is a velocity mapping onto the axis $OY$; $v$ is the complex potential of a flow.

A scalar air velocity is given by the Equation:

$$ \omega = |\omega_x + i\omega_y| = \sqrt{\omega_x^2 + \omega_y^2}, $$  \hspace{1cm} (7)

As the previously described mapping is used in designing the current lines of the air in the premises, the air velocity is the product of the derivatives of each transformation step:

$$ \omega = A_1 \cdot A_2 \cdot \ldots \cdot A_n, $$  \hspace{1cm} (8)

Where $A_1, A_2, A_n$ are the derivatives of the functions at each transformation step.

The methodology of a mathematical and any other modeling relies on studies of the characteristics of different objects using investigations of similar structures. It should be noted that a mathematical model is an idealized real object it is only expected to clearly represent the most relevant features of a particular phenomenon without much emphasis on what seems to be of less significance. As a result it is not capable of a comprehensive description of all the typical features of an object making it thus important to compare the results of numerical and experimental studies obtained by means of mathematical or physical modeling respectively.

It should be noted that since as the air velocity is on the rise, turbulence of a flow increases as well, a mathematical model based on the theory of conformal mapping is not always capable of a relevant description of ongoing changes and merely deals with the Reynolds number range.

4. Conclusions

The described method of calculating the velocity fields of air flows based on the theory of conformal mapping allows one to determine the velocities at any point of particular premises regardless of its geometric characteristics and ways of supplying and removing the air. However the previously mentioned restriction of the Reynolds number for which the calculations hold true and thus the air velocities, it is necessary to identify an acceptable velocity range where the theory of conformal mapping can be applied to design velocity fields of air flows of particular premises. At the initial stage the condition of vortex-free flow of a medium can only be made use of for current lines and its scalar velocities to be more likely to be accurate.
5. References

1. Chuikin S. Ventilyaciya i e’kologicheskayabezopasnost’ zh-ilyx i obshhestvennyxpomeshhenij [Ventilation and environmental safety of living and public premises]. ‘Ekologiya i promyshlennost’ Rossi[Ecology and Industry of Russia]. 2015; 19(2):424.Doi no: 10.18412/1816-0395-2015-2-42-44.

2. Sklyarov KA, Cheremisin AV, Pavlyukov SP. Two-dimensional stationary movement of air flow in premises with partitions. Scientific herald of the Voronezh State University of Architecture and Civil Engineering.Construction and architecture. 2009; (2):69–76.

3. Chuykin SV, Loboda AV. Determination of velocity fields of air streams in ventilated rooms with conformal mappings. Scientific herald of the Voronezh State University of Architecture and Civil Engineering.Construction and Architecture. 2013; (3):39–51.

4. Mel’kumov VN, Loboda AV, Chuykin SV. Mathematical modelling of air streams in large spaces. Scientific herald of the Voronezh State University of Architecture and Civil Engineering.Construction and Architecture. 2015; (1):15–24.

5. Mel’kumov VN, Kolodyazhniy SA, Chuykin SV. Modeling air flows in premises using conformal mapping. Middle-East Journal of Scientific Research. 2014; 22(1):78–81.

6. Wang B, Zhao B, Chen, C. A simplified methodology for the prediction of mean air velocity and particle concentration in isolation rooms with downward ventilation systems. Building and Environment. 2010; 45(8):1847–53.

7. Zuo W, Hu J, Chen Q. Improvements in ffd modeling by using different numerical schemes. Numerical Heat Transfer, Part B: Fundamentals. 2010; 58(1):1–16.

8. Zuo W, Chen Q. Fast and informative flow simulations in a building by using fast fluid dynamics model on graphics processing unit. Building and Environment. 2010; 45(3):747–57.

9. Loboda AV, Kuznetsov SN. The use of the method of conformal mappings to determine velocity fields of air flows in ventilation problems. Scientific herald of the Voronezh State University of Architecture and Civil Engineering. Construction and Architecture. 2011; (4):18–26.

10. Li R, Sekhar SC, Melikov AK. Thermal comfort and iaq assessment of under-floor air distribution system integrated with personalized ventilation in hot and humid climate. Building and Environment. 2010; 45(9):1906–13.

11. Chen Q, Lee K, Mazumdar S, Poussou S, Wang L, Wang M, Zhang Z. Ventilation performance prediction for buildings: Model assessment. Building and Environment. 2010; 45(2):295–303.

12. Sushil Kumar TVK, Chandrasekhar J, Shriram K, Vasudevan, Balachandran A. An analysis of effect of wing walls on natural ventilation.Indian Journal of Science and Technology. 2015 Aug; 8(17):1–4.Doi no:10.17485/ijst/2015/v8i17/68171.

13. Visagavel K, Srinivasan PSS. Experimental investigation on solar air heater assisted natural ventilation in single-sided ventilated room. Indian Journal of Science and Technology. 2010 Jul; 3(7):802–6.Doi no: 10.17485/ijst/2010/v3i7/29819.

14. Nedhal A, Al-Tamimi. Toward sustainable building design: improving thermal performance by applying natural ventilation in hot–humid climate. Indian Journal of Science and Technology. 2015 Oct; 8(28):1–8.Doi no: 10.17485/ijst/2015/v8i28/83620.

15. Taemthong W, Sirikulyanon W. Improving indoor air quality in university buildings in Thailand. Indian Journal of Science and Technology. 2015 Nov; 8(29):1–8.Doi no: 10.17485/ijst/2015/v8i29/83640.

16. Sushil Kumar TVK, Chandrasekar J, Moorthy SK, Sakhthi kala A, ArvindBharath SR. Optimization of building enve lop e to reduce air conditioning. Indian Journal of Science and Technology. 2016 January; 9(4):1–4.Doi no: 10.17485/ ijst/2016/v9i4/79072.

17. Prakash D, Ravikumar P. Study of thermal comfort in a room with windows at adjacent walls along with additional vents. Indian Journal of Science and Technology. 2013 Jan; 6(6):4659–69.Doi no: 10.17485/ijst/2013/v6i6/33918.

18. Ryu JH, Hong WH, Lee JA, Na WJ. Optimal operation of ERV system installed in the apartment houses in South Korea.Indian Journal of Science and Technology. 2015 Sep; 8(24):1–9.Doi no: 10.17485/ijst/2015/v8i24/80037.