The Galactic One-Way Shapiro Delay to PSR B1937+21

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The time delay experienced by a light ray as it passes through a changing gravitational potential by a non-zero mass distribution along the line of sight is usually referred to as Shapiro delay. Shapiro delay has been extensively measured in the Solar system and in binary pulsars, enabling stringent tests of general relativity as well as measurement of neutron star masses. However, Shapiro delay is ubiquitous and experienced by all astrophysical messengers on their way from the source to the Earth. We calculate the “one-way” static Shapiro delay for the first discovered millisecond pulsar PSR B1937+21, by including the contributions from both the dark matter and baryonic matter between this pulsar and the Earth. We find a value of approximately 5 days (of which 4.74 days is from the dark matter and 0.22 days from the baryonic matter). We also calculate the modulation of Shapiro delay from the motion of a single dark matter halo, and also evaluate the cumulative effects of the motion of matter distribution on the change in pulsar’s period and its derivative. The time-dependent effects are too small to be detected with the current timing noise observed for this pulsar. Finally, we would like to emphasize that although the one-way Shapiro delay is mostly of academic interest for electromagnetic astronomy, its ubiquity should not be forgotten in the era of multi-messenger astronomy.

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I. INTRODUCTION

In 1964, I. Shapiro [1] calculated and then measured [2] the round-trip time delay of a radar signal to the inner planets of our solar system, which is caused by the gravitational field of the Sun. This delay is known as “Shapiro delay” and has been measured precisely in the solar system over the last five decades, allowing very stringent tests of general relativity (GR) and in particular the PPN $\gamma$ parameter [3]. The current best solar system constraints come from the Cassini mission, which agree with the GR prediction to within $10^{-5}$ [4]. The calculation of Shapiro delay has also been generalized for a time-varying gravitational field [5–7] and experimentally confirmed to agree with the predictions of GR [7,8]. It has also been calculated for some alternate theories of gravity in anticipation of future solar system based satellite experiments [9–13]. Besides its use as a test of GR, Shapiro delay has been routinely used as an astrophysical probe to measure the masses of pulsars in binary systems [11–13], allowing us to constrain the neutron star equation of state. Shapiro delay has been proposed as the possible cause of the low frequency noise in timing residuals of pulsars in globular clusters [14]. This delay is also one of the contributing factors for the observed time delays between multiply lensed images from quasars [15]. These time delays have been measured and used to constrain Hubble constant and other cosmological parameters [16].

In this paper, we would like to focus on another facet of Shapiro delay, which is the total delay experienced by any astrophysical messenger from cosmic rays to gravitational waves on its way to the Earth from the source, due to the gravitational potential of all the intervening mass distribution along the line of sight. We refer to this as the “one-way” Shapiro delay. Although this was first calculated in 1988 [17,18] for the gravitational potential of our galaxy, it is rarely mentioned in astrophysical literature. This is because one can never measure the absolute value of this delay and only its time-dependence has observational consequences, in case the source is a steady-state emitter. However, one can measure the difference in the static component of the Shapiro delay between photons and neutrinos/gravitational waves in the case of simultaneous detection of multiple cosmic messengers, and when we know the relative departure time at the source. In this situation, although the absolute static component of the delay can never be measured, we can use one astrophysical messenger as a clock to time the other one and thereby test the equivalence principle for the non-electromagnetic messenger.

However, we would like to enumerate some examples of various astrophysical measurements, which have already been done or planned in the future, for which one-way Shapiro delay plays a central role in the final results which are derived from these observations. As a strawman, we then focus on the Shapiro delay calculation for one astrophysical source, which is the millisecond pulsar PSR B1937+21. We calculate the total Shapiro delay from both the dark matter and baryonic matter assuming static spherically symmetric geometry. We then estimate the time-dependent corrections on the pulsar period and its derivative due to the velocity of the matter distribution along the line of sight and then discuss the prospects for detection.

The outline of this paper is as follows. In Sect. 1
we provide a brief history of one-way Shapiro delay calculations in literature, including measurements from SN1987A and implications for fundamental physics from these, as well as point out other astrophysical examples where one-way Shapiro delay is relevant. We then set up our formalism for calculating the static Shapiro delay from PSR B1937+21 in Sect. II, including the mass models used for dark matter (Sect. II.A) and baryonic matter (Sect. II.C), and then calculate the total delay assuming a static potential. In Sect. II.D we calculate the corrections due to the velocity of the matter distribution. We then conclude in Sect. IV.

II. ONE-WAY SHAPIRO DELAY

The first paper which explicitly mentions the existence of one-way Shapiro delay from our galaxy (to the best of our knowledge) is by Backer and Hellings [19] (see their Eqn. 4.3), which was written in the context of pulsar timing observations. However, they argued that since the galactic gravitational potential is essentially static over a ten year period, there is no need to model for it. Following the detection of neutrinos from SN1987A [20][21], it was pointed out that the neutrinos from SN1987A also experience this one-way Shapiro delay due to the gravitational potential of the intervening matter along the line of sight. Two independent groups in back to back papers calculated the delay by modeling the gravitational field of the Milky Way as a point mass. The value for the one-way delay ranged from one to six months for different models of the galactic gravitational potential [17][18].

This delay calculation was also generalized for a non-zero neutrino mass [22] and the difference due to the neutrino mass was shown to be negligible for the neutrino energies detected from SN1987A. The near-simultaneous arrival of photons and neutrinos from this core-collapse supernova confirmed that Shapiro delay for neutrinos is same as that for photons to within 0.2-0.5% [17][18]. To date, this is the only direct observational evidence we have that neutrinos are affected by gravity and obey the weak equivalence principle. These observations also constrained the difference in relative couplings of gravitational interactions of matter and anti-matter to within $10^{-5} - 10^{-6}$ [23]. We should point out that the calculation of one-way Shapiro delay for sources in our galaxy and local neighborhood is decoupled from measurement of distances, and if the gravitational potential along the line of sight changes, then so would the arrival time.

The one-way Shapiro delay is also a very important factor in searches for gravitational waves from sources with electromagnetic counterparts, which should be expected with advanced LIGO commencing operations in Sept. 2015. In case gravitational waves arrive at the same time as photons, it would be the first direct evidence that gravitational waves gravitate, or that “gravity begets gravity”. We could also use measurements of relative Shapiro delay between gravitational waves and photons/neutrinos to confirm or rule out modified theories of gravity designed to explain the dark matter conundrum [24][25].

However, besides the above examples the galactic one-way Shapiro delay due to the gravitational potential of our galaxy is hardly ever discussed in literature. This could be because for all practical purposes, this delay is only of academic interest, since it is much smaller than the vacuum light travel time. Other possible reasons are that one can never know when a photon left an astrophysical source and the gravitational potential of our galaxy changes very slowly. The measurement of one-way Shapiro delay has practical relevance only in case of two independent messengers seen from an astrophysical source, and for which we know the delay between them at the source. To date, the only such example Mother Nature has provided us, is the simultaneous detection of neutrinos and photons from SN1987A. However with the recent detection of high energy astrophysical neutrinos from IceCube [27], and the expected detection of gravitational waves now that advanced LIGO has started taking data, we hope that the ubiquity of one-way Shapiro delay is not forgotten.

We now point out the relevance of one-way Shapiro delay for some more astrophysical measurements. In anticipation of expected gravitational wave observations from sources with electromagnetic and neutrino counterparts, there have been many proposed tests of various fundamental physics parameters such as vacuum speed of gravitational waves, mass of the graviton, mass of the neutrino. [28][29]. A key assumption for all such measurements (even though it's rarely stated) is that the one-way Shapiro delay is the same for gravitational waves and neutrinos/photons. However, we don’t know as of now whether gravitational waves follow the same geodesics (as photons) from the intervening matter distribution, because they have not been directly detected yet. In some alternate gravity theories which violate the strong equivalence principle, they do not [25]. In purely electromagnetic astronomy, the equality of one-way Shapiro delay along different lines of sight is also a key assumption in the observations of light echoes from distant supernovae [31].

Therefore, even though the general lore is that one-way Shapiro delay is only of academic interest, the above examples show some applications of Shapiro delay observations in both astrophysical and fundamental physics measurements, and especially for multi-messenger astronomy.

III. SHAPIRO DELAY FOR PSR B1937+21

We now turn to the delay calculation for the pulsar PSR B1937+21. This is the first ever discovered millisecond pulsar, which has a rotational period of 1.55 milliseconds [32], dispersion measure of about 71 pc cm$^{-3}$, and timing residuals of about 2 $\mu$s from over three decades of observations [33].
The total light travel time from this pulsar includes the geometric propagation delay due to distance and proper motion of the pulsar, the one-way Shapiro delay from all intervening masses along the line of sight \(19, 33\), as well as two additional frequency-dependent delay terms from propagation in the interstellar medium due to dispersion and birefringence \(35\). However, these frequency dependent terms are small compared to the light travel time. We also note that the Shapiro delay calculation from the inner solar system planets is included in the TEMPO2 software, which is routinely used in the analysis of pulsar data \(34\). We now calculate the total one-way Shapiro delay for this pulsar from the static gravitational potential of the dark matter, and also the modulations of density profiles or mass functions.

\[ M(r) \equiv 4\pi \int_0^r r'^2 dr' \rho(r') . \]  

The linearized solution of Eqn. 2 is

\[ \Delta A(r) = \frac{8\pi G}{c^2 r} \int_0^r dr' r'^2 \rho(r') = \frac{2G}{c^2} \frac{M(r)}{r} , \]  

\[ \Delta B(r) = -\int_r^\infty dr' \frac{\Delta A(r')}{r'} . \]  

A. Formalism for Calculating the one-way Shapiro Delay

We discuss the details of the Shapiro delay calculation experienced by the radio wave on its way from the pulsar to the Earth. Previously, we have calculated this using only the dark matter contributions \(25\) for a few selected astrophysical sources (SN1987A, GRB070201, and Sco-X1), and also as a function of distance in our galaxy \(26\). In this paper, we do this calculation for PSR B1937+21 and also as a function of distance in our galaxy \(26\). We briefly review the formalism for calculating the static Shapiro delay, which follows the same method and notations as in \(25, 26\) and the reader can refer to these papers for more details.

To calculate the delay, we shall assume a static spherically symmetric geometry, which is a good approximation for dark matter models of the Milky Way. The Schwarzschild line element for this geometry is given by

\[ ds^2 = -B(r) c^2 dt^2 + A(r) dr^2 + r^2 d\Omega^2 . \]  

We can calculate \(A(r)\) and \(B(r)\) as a function of the mass distribution by solving Einstein’s equations. For pressure-less dust, they take the form:

\[ \frac{B'}{rB} - \left( \frac{A - 1}{r^2} \right) = 0 . \]  

We assume that the deviations from flat geometry are small and therefore \(A(r)\) and \(B(r)\) can be written as

\[ A(r) \equiv 1 + \Delta A(r) , \quad B(r) \equiv 1 + \Delta B(r) . \]  

Linearizing eqns. \(23\) and solving these two equations give the following expressions for \(\Delta A(r)\) and \(\Delta B(r)\) in terms of density profiles or mass functions.

\[ M(r) \equiv 4\pi \int_0^r r'^2 dr' \rho(r') . \]  

The linearized solution of Eqn. 2 is

\[ \Delta A(r) = \frac{8\pi G}{c^2 r} \int_0^r dr' r'^2 \rho(r') = \frac{2G}{c^2} \frac{M(r)}{r} , \]  

\[ \Delta B(r) = -\int_r^\infty dr' \frac{\Delta A(r')}{r'} . \]  

B. Shapiro Delay from Dark Matter

Over the last decade there has been a lot of effort to model the dark matter halo of the Milky Way, using different observational tracers \(35\). For this paper, we shall use the Milky Way dark matter profile from Klypin et al \(37\). Currently, the uncertainty in dark matter mass for our Milky Way Halo is about 30% \(36\). However,
this uncertainty should be reduced using results from the GAIA satellite.

We briefly review Klypin et al’s posited dark matter profile and its associated parameters \[44\]. They assumed a Navarro-Frenk-White (NFW) \[38\] profile given by:

\[
\rho_{\text{halo}}(r) = \frac{\rho_s}{x(1 + x)^2}, \quad x = r/r_s
\]

\[10\]

\[
M_{\text{halo}}(r) = 4\pi \rho_s r_s^3 f(x)
\]

\[11\]

\[
f(x) = \ln(1 + x) - \frac{x}{1 + x},
\]

\[12\]

\[
C = r_{\text{vir}}/r_s,
\]

\[13\]

where \(C\) and \(M_{\text{vir}}\) are the halo concentration and virial mass, and \(r_{\text{vir}}\) is the virial radius. Details about the other terms can be found in \[37\].

Following this paper, we assume \(r_{\text{vir}} = 258\) kpc, \(M_{\text{vir}} = 1.02 M_\odot\) and \(C = 12\). Using these values, we can calculate \(\rho_s \simeq 0.186\) GeV/cm\(^3\).

Therefore, using this value of \(\rho_s\) and assuming an NFW profile (given in Eqn. \[10\]), we can calculate \(\Delta A(r)\) and \(\Delta B(r)\) at any distance \((r)\) using Eqns. \[8\] and \[7\] respectively. The dark matter induced Shapiro delay can then be evaluated using Eqn. \[3\].

We now do these numerical integrations for PSR B1937+21. For the distance to the pulsar and its position on the sky, we use the tabulated values from Table I in Nicastro et al. \[39\]. From this paper, we also note that the lower limit on the distance of the pulsar is \(\simeq 3.6\) kpc with an upper limit of 21 kpc. This pulsar is located at Right Ascension of 19hr 39mt 38.5 sec and declination of 21°34′59″. For the calculations in this paper, we shall assume that the pulsar is at a distance of 3.6 kpc. The total dark mass between the Earth and the pulsar for the above parameters is \(1.8 \times 10^{10} M_\odot\).

Given this spatial location and distance, the calculated dark matter induced Shapiro delay is shown below as a function of distance and celestial coordinates near PSR B1937+21 in Figs. 1 and 2 respectively. Therefore for this dark matter profile, we find that the dark matter induced Shapiro delay is approximately 4.74 days.

### C. Shapiro Delay from Baryonic Matter

To calculate the Shapiro delay from baryonic matter, we assume that the total baryonic matter is given by the sum of bulge and disk components, and assume spherical symmetry for mass distributions of both the bulge and the disk. We use the mass models from Xue et al \[40\], which assumed a Hernquist profile \[41\] for the galactic bulge with total mass given by \(M_{\text{bulge}} = 1.5 \times 10^{10} M_\odot\), and the Miyamoto & Nagai profile \[42, 43\] for the disk, with total mass given by \(M_{\text{disk}} = 5 \times 10^{10} M_\odot\). Therefore the total mass is equal to \(6.5 \times 10^{10} M_\odot\). We should however point out that estimates for the total disk and bulge mass of the Milky Way differ a lot in literature.

For example, using measurements of the stellar luminosity function mass of the bulge and disk were estimated to be, \(M_{\text{bulge}} = 1.3 \times 10^{10} M_\odot\) and \(M_{\text{disk}} = (4.9 - 6.7) \times 10^{10} M_\odot\) \[44\]. Therefore, in this model the total baryonic mass they obtain is \((7.1 \pm 0.9) \times 10^{10} M_\odot\), of which \((4.9 \pm 0.4) \times 10^{10} M_\odot\) lies within the solar circle. Some other estimates of bulge mass are about twice this value. From the DENIS near-infrared large scale survey, the mass of bulge is assumed to be \((2.4 \pm 0.6) \times 10^{10} M_\odot\) \[45\]. However, since we are interested in an order of magnitude estimate we use the bulge and disk mass models from Xue et al \[40\]. Using this value for the total baryonic mass, we can calculate \(\Delta A(r)\) and \(\Delta B(r)\) directly and hence the Shapiro delay in the same way as for the dark matter. After doing this, we get a value of 0.13 days for the bulge and 0.086 days for the disk. The total Shapiro delay from baryonic matter along the line of sight is approximately 0.22 days and is negligible compared to the dark matter.
contributions.

Therefore, the total galactic Shapiro delay to PSR B1937+21 by summing both the baryonic and dark matter contributions is about five days. Since the purpose of this calculation is only for pedagogy, we do not calculate any systematic errors for this delay.

D. Velocity Dependent Corrections to Shapiro Delay

Since only the modulations to the static Shapiro delay are potentially detectable, we do a feasibility study of the detection prospects by estimating its time dependence. We first calculate the corrections to the static Shapiro delay estimated in Sect. II A by the motion of matter along the line of sight between the Earth and the pulsar. For an elaborate calculation, we would need to linearize the Kerr metric and use the number density of dark matter haloes from theories of structure formation, along with their phase space velocity distribution to calculate the velocity dependent corrections to the static Shapiro delay. Here, we use simplified assumptions to calculate order of magnitude effects of the Shapiro delay induced modulations. We first do the calculation for a single gravitating dark matter mass close to the line of sight between the Earth and the pulsar. For the orbiting matter along the line of sight, only those dark matter haloes or baryonic clumps which have small impact parameters will leave an observational imprint on the timing properties of the pulsar signal. We first estimate the time variation by a single dark matter halo transit having the characteristic bell-shaped signature of Shapiro delay with an amplitude of \( \sim 0.05 \, \text{year} \) between the Earth and PSR B1937+21. The expected number of transits from all dark matter haloes with such impact parameters can be calculated using the gravitational lensing optical depth and is \( \sim 0.05 \, \text{year} \) between the Earth and PSR B1937+21. The expected number of transits from baryonic clumps is even smaller. Therefore, we need to monitor the pulsar for about 25 years to experience at least one such dark matter halo transit having the characteristic bell-shaped signature of Shapiro delay with an amplitude of about 5 ns. However, in practice this is much smaller than the timing residuals seen for this pulsar which is \( O(\mu \text{sec}) \) and the dominant sources of timing noise, such as from pulsar spin-down are usually fitted for in the timing analysis. So the first pre-requisite for detection is that the timing residuals be reduced by at least three orders of magnitude. The full list of all sources of timing noise in millisecond pulsars and ways to mitigate or accurately model these terms to reduce the residuals to \( O(\text{nsec}) \) during the Square Kilometer array (SKA) era are discussed in \[33\] .

We now calculate the cumulative effects of all the dark and baryonic matter on the change in pulsar’s period and its derivative at a given instant. We assume that the pulsar’s true period is \( P_0 \) and the observed period due to Shapiro-delay induced modulation is \( P \), then by taking the derivatives of Eqn. [17] we obtain the following expressions for the fractional change in the period and its derivative [48].

\[
\Delta T_{vel} = -\frac{2GM}{c^3} \ln(1 + (v/d)^2 \cdot (t - T_0)^2). \tag{17}
\]

where \( \Delta T_{vel} \) is the approximate velocity-dependent Shapiro delay correction which must be added to the static part given by Eqn. [8]. We note that the velocity dependent corrections are opposite in sign to the static component of the Shapiro delay. Since the proper motion of PSR 1937+21 is small, we assume that \( |v| \) is dominated by the Galactic rotation velocity at the position of the Sun or the Local Standard of Rest velocity, and has a value of \( \sim 220 \, \text{km/sec} \) in the direction of Galactic Center. For the orbiting matter along the line of sight, only those dark matter haloes or baryonic clumps which have small impact parameters will leave an observational imprint on the timing properties of the pulsar signal. We first estimate the time variation by a single dark matter halo transit having the characteristic bell-shaped signature of Shapiro delay with an amplitude of about 5 ns. However, in practice this is much smaller than the timing residuals seen for this pulsar which is \( O(\mu \text{sec}) \) and the dominant sources of timing noise, such as from pulsar spin-down are usually fitted for in the timing analysis. So the first pre-requisite for detection is that the timing residuals be reduced by at least three orders of magnitude. The full list of all sources of timing noise in millisecond pulsars and ways to mitigate or accurately model these terms to reduce the residuals to \( O(\text{nsec}) \) during the Square Kilometer array (SKA) era are discussed in [33] .

We now calculate the cumulative effects of all the dark and baryonic matter on the change in pulsar’s period and its derivative at a given instant. We assume that the pulsar’s true period is \( P_0 \) and the observed period due to Shapiro-delay induced modulation is \( P \), then by taking the derivatives of Eqn. [17] we obtain the following expressions for the fractional change in the period and its derivative [48].
\[ \Delta = -\frac{4GM}{c^3} \frac{v}{d} \cos(\theta), \]  
\[ \dot{\Delta} = -\frac{4GM}{c^3} \left( \frac{v}{d} \right)^2 [1 - 2 \cos^2(\theta)]. \]  

where \( \Delta = \frac{\tau - \tau_0}{\tau_0} \) (after neglecting \( P_0 \Delta \) term), and \( \cos(\theta) = R \cdot x \).

If we assume that the dark matter haloes are randomly aligned between the Earth and the pulsar [13], then \( \langle \cos(\theta) \rangle = 0 \), and \( \langle \cos^2(\theta) \rangle = 0.5 \), and the total effect from summing the terms in Eqn. [18] will average out to zero. We therefore compute the variance of the above quantities, which have non-zero values and can be written as [13]:

\[ \langle \Delta^2 \rangle = 8 \left( \frac{GM}{c^3} \right)^2 \pi \Sigma v^2 \ln N, \]  
\[ \langle \dot{\Delta}^2 \rangle = 8 \left( \frac{GM}{c^3} \right)^2 \left( \pi \Sigma v^2 \right)^2. \]  

where \( \Sigma \) is the surface density of all dark or baryonic matter which contributes to the Shapiro delay, and \( N \) is the total number of distinct gravitating objects which contribute to Shapiro delay. In the above equations, the averaging is done over shorter time-scales, over many multiples of the pulsar period. For the dark matter between the Earth and the pulsar, assuming \( M = 100M_\odot \), \( \Sigma = 4 \times 10^5 \text{pc}^{-2} \), and \( N = 6 \times 10^{13} \) we obtain \( \langle \Delta^2 \rangle \sim 1.7 \times 10^{-37} \) and \( \langle \dot{\Delta}^2 \rangle \sim 8 \times 10^{-50} \text{sec}^{-2} \). For baryons, assuming local stellar density in our galaxy [40], we get \( M = 1M_\odot \), \( \Sigma = 310 \text{ pc}^{-2} \), and \( N = 1.2 \times 10^{11} \), from which \( \langle \Delta^2 \rangle \sim 10^{-41} \) and \( \langle \dot{\Delta}^2 \rangle \sim 2.24 \times 10^{-64} \text{sec}^{-2} \). Therefore for PSR B1937+21, we obtain \( |P - \dot{P}_0| \sim 4.1 \times 10^{-22} \text{ sec} \) and \( |P - \dot{P}_0| \sim 3.6 \times 10^{-28} \). As we can see, these are too small to be observed with current precision.

Therefore, we conclude that it is not possible to detect any observational signatures of time-dependent modulation of galactic Shapiro delay on the observed period of the pulsar with the current technology. However, optimistically once SKA comes online and the timing residuals are reduced to nanoseconds, one would need to monitor this pulsar over a baseline of 20-25 years to see at least one dark matter halo transiting near the line of sight between the Earth and the pulsar. Moreover to detect such signatures, one would need to use non-standard data analysis methods, where the time-dependent component of galactic Shapiro delay is kept as a free parameter [50].

The cumulative effects from dark and baryonic matter on the variance of the pulsar period and its derivative are much smaller and cannot be detected. Alternately, another strategy to look for time-dependent Shapiro delay would be to look at the residual power spectrum over a long period of time to distinguish it from other sources of noise and signals from a gravitational wave background [49], or to look at the auto-correlation function of the timing residuals [13].

IV. CONCLUSIONS

In this paper, we have stressed that the total time required for any astrophysical messenger to reach the Earth from a given source, includes the Shapiro delay contribution from the total gravitational potential of our Milky Way galaxy. We provide some examples of current and future astrophysical measurements, where one-way Shapiro delay plays a pivotal role. We then calculate the static one-way Shapiro delay for PSR B1937+21 (which is the first ever discovered millisecond pulsar), by summing up the contributions from both the dark matter as well as baryonic matter. For the dark matter mass distribution, we have used the NFW profile parameters from [37] and obtain a delay of about 4.74 days. For the baryonic mass, we have used the bulge and disk mass from [40] and get a value of approximately 0.22 days. Therefore, the total additional delay experienced by the radio signal is about five days. We then estimated the velocity dependent modulation of Shapiro delay on the pulsar period by one dark matter halo with mass of 100 \( M_\odot \) orbiting near the line of sight between the Earth and the pulsar, to be about five nanoseconds over a five year time-scale. However, realistically one expects only one such transit in 20 years from all the matter distribution along the line of sight. We also calculate the variance of the change in pulsar’s period and its derivative, due to the cumulative effects of all the matter distribution over shorter time scales, and find that the absolute value of the difference in the pulsar period is \( O(10^{-22}) \) sec. Therefore, the time-dependent modulation to the static one-way Shapiro delay for this pulsar from the matter distribution along the line of sight is currently unobservable. In order to detect observational signatures, one would need to reduce the timing residuals to \( O(ns) \), monitor this pulsar for over a 25 year time-scale, and use sophisticated signal processing techniques with non-standard assumptions. In principle however, once the uncertainty in distance and dark matter mass distribution is reduced, the static Shapiro delay calculation can be incorporated in TEMPO2 or other pulsar data analysis packages.

Therefore, the absolute value of the one-way Shapiro delay can never be measured and its time-variation for this millisecond pulsar is too small to be detected within the next decade. However, one should not forget that one-way Shapiro delay is ubiquitous and experienced by all photons, neutrinos, gravitational waves, and cosmic rays from the gravitational potential of our Galaxy along the line of sight, as they reach the Earth from distant sources in the Universe. It will play a pivotal role in multi-messenger astronomy.

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