Analysis of the new Crystal Ball data on $K^- p \rightarrow \pi^0 \Lambda$ reaction with beam momenta of $514 \sim 750$ MeV/c

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The Crystal Ball Collaboration has recently reported the differential cross sections and $\Lambda$ polarization for the reaction $K^- p \rightarrow \pi^0 \Lambda$ using an incident $K^-$ beam with momenta between 514 and 750 MeV/c. We make a partial wave analysis for this process with an effective Lagrangian approach and study the properties of some $\Sigma$ resonances around this energy range. With the inclusion of the 4-star resonances $\Sigma(1189)$, $\Sigma^*(1385)$, $\Sigma(1670)\frac{1}{2}^-$, $\Sigma(1775)\frac{1}{2}^-$, as well as a $\Sigma(1635)\frac{1}{2}^+$, which is compatible with the 3-star $\Sigma(1660)\frac{3}{2}^-$ in PDG, our results can well reproduce the experimental data. The parameters on the $\Sigma$ resonances and related couplings are studied.

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I. INTRODUCTION

The $K^- p$ interactions at resonance region are important methods for the study of resonance spectroscopy and interactions, especially for hyperon with $S = -1$. Recently, the differential cross sections as well as the $\Lambda$ polarization for $K^- + p \rightarrow \pi^0 + \Lambda$ are measured with very high precision with the Crystal Ball spectrometer at the BNL Alternating Gradient Synchrotron [1], where neutron and photon final states from $\pi^0 \Lambda$ decays are well detected. The new data provides a good opportunity for studying $\Sigma$-hyperon resonances in the experimental energy range, which is between 514 and 750 MeV/c for incident momentum, corresponding to $\sqrt{s} = 1569 - 1676$ MeV for c.m. energy.

The $\Sigma$-hyperon resonances in the Particle Data Group (PDG) [2] are mainly known from the analysis of $\overline{K}N$ reactions in the 1970s, and large uncertainties may exist not only for the unestablished resonances with one or two stars, but also for the established ones with three or four stars because of the limited data and knowledge of background contributions. Moreover, there still may be some new resonances that have not been discovered. Past analyses of the reaction $\overline{K}N \rightarrow \pi \Lambda$ include the energy dependent partial wave analysis with c.m. energy between 1540 and 2215 MeV [3], and the energy independent analysis with c.m. energy between 1540 and 2150 MeV [4]. Both analyses considered the reaction amplitude parameterized as the sum of resonance terms of Breit-Wigner form and a background term of certain form. Different ways of background extraction may bring large uncertainty to results.

In this work, benefitted from the available new data of high precision, we make a partial wave analysis with an effective Lagrangian approach. We aim at an improvement in the knowledge of the $\Sigma$ resonances around the energy range concerned, as well as their interactions with some other hadrons.

This paper is organized as follows. In section II, the theoretical framework and amplitudes are presented for the reaction $\overline{K}N \rightarrow \pi \Lambda$. In section III, the analysis results are presented and compared with the experimental data, with some discussions. In section IV, we give the summary and conclusion of this work.

II. THEORETICAL FRAMEWORK

The effective Lagrangian method is an important theoretical approach in describing various processes at resonance region, and is widely used in partial wave analysis for the properties of resonances. For the reaction $K^- + p \rightarrow \pi^0 + \Lambda$, the Feynman diagrams are shown in Fig. 1, where the incoming momenta are $k$ and $p$ for kaon and proton, respectively, and the outgoing momenta are $q$ and $p'$ for $\pi^0$ and the $\Lambda$, respectively. The main contributions come from the t-channel $K^*$ meson exchange, the u-channel proton exchange, and the s-channel $\Sigma$ and its resonances exchanges. Note that in some previous analysis, the t-channel and u-channel contributions were treated differently, where they are treated as the backgound term with certain parametrization.

For the t-channel $K^*$ meson exchange, the effective Lagrangian for $K^* K \pi$ coupling is

$$L_{K^*K}\pi = ig_{K^*K}\pi K^*_\mu(\tau \partial^\mu K - \partial^\mu \pi \tau K),$$

(1)

where the isospin structure for $K^* K \pi$ is $\overline{K} \pi \pi \tau K$ with

$$\overline{K} = (K^* - K^0, \pi^-), \pi \tau = \begin{pmatrix} \pi^0 & \sqrt{2} \pi^+ \\ \sqrt{2} \pi^- & -\pi^0 \end{pmatrix}, K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix},$$

(2)
Thus we constrain $g_{K^*K\pi} = -3.23$.

The effective Lagrangian for $K^*N\Lambda$ coupling is

$$\mathcal{L}_{K^*N\Lambda} = -g_{K^*N\Lambda} \bar{\chi}(\gamma_\mu K^\mu - \frac{\kappa_{K^*N\Lambda}}{2M_N}\sigma_{\mu\nu}\partial_\nu K^\mu)N + \text{H.c.},$$

where $g_{K^*N\Lambda}$ and $\kappa_{K^*N\Lambda}$ are effective coupling constants and can only be estimated from model predictions or fit to some data. The popular potential model by Stoks and Rijken gave two sets of these coupling constants [2, 6]:

$$g_{K^*N\Lambda} = -4.26 \quad \kappa_{K^*N\Lambda} = 2.66 \quad \text{(NSC97a)},$$
$$g_{K^*N\Lambda} = -6.11 \quad \kappa_{K^*N\Lambda} = 2.43 \quad \text{(NSC97t)}.$$ (4)

Thus we constrain $g_{K^*N\Lambda}$ between $-4.26$ and $-6.11$, and $\kappa_{K^*N\Lambda}$ between $2.43$ and $2.66$ in our analysis. A recent prediction from light cone QCD sum rules (LCSR) gives a larger range for $g_{K^*N\Lambda} = -5.1 \pm 1.8$, while very different values for $\kappa_{K^*N\Lambda}$ [7]. Some other works for vector meson-baryon couplings also have large deviations on $\kappa_{VNB}$ [8–10]. For these uncertainties, we also try the parameters in larger range and give some discussions.

For the u-channel nucleon exchange, the effective Lagrangians are

$$\mathcal{L}_{\pi NN} = \frac{g_{\pi NN}}{2M_N} \bar{\chi} \gamma_\mu \gamma_5 \partial_\mu \pi \cdot N,$$

$$\mathcal{L}_{KNN} = \frac{g_{KNN}}{M_N + M_\Lambda} \bar{\chi} \gamma_\mu \gamma_5 \partial_\mu K + \text{H.c.},$$

where $g_{\pi NN} = 13.26$ and $g_{KNN} = -13.24$ are estimated from flavor SU(3) symmetry relations [11, 12].

For the s-channel $\Sigma$ and its resonances exchange, we consider effective couplings up to D-wave, which include intermediate states with $J^P = \frac{1}{2}^+$, $\frac{3}{2}^+$, and $\frac{5}{2}^+$. For $\Sigma(1189)$ and its resonance with $J^P = \frac{1}{2}^+$ contributions in s-channel, the effective Lagrangians are

$$\mathcal{L}_{K\Sigma} = \frac{g_{K\Sigma}}{M_N + M_\Sigma} \partial_\mu \bar{\chi} \Sigma \gamma_5 \gamma_5 \mu \gamma_5 N + \text{H.c.},$$

and

$$\mathcal{L}_{\Sigma\Lambda} = \frac{g_{\Sigma\Lambda}}{M_\Lambda + M_\Sigma} \bar{\chi} \gamma_\mu \gamma_5 \partial_\mu \Sigma + \text{H.c.}$$

Where the isospin structure for $KN\Sigma$ coupling is

$$\mathcal{K} = (K^-, K^0), \Sigma \cdot \tau = \left( \begin{array}{c} \Sigma^0 \\ \sqrt{2} \Sigma^+ \\ -\sqrt{2} \Sigma^- \end{array} \right), N = \left( \begin{array}{c} p \\ n \end{array} \right).$$

The coupling constants from SU(3) flavor symmetry relations predict $g_{KNN} = 3.58$ and $g_{\Sigma\Lambda} = 9.72$ for $\Sigma(1189)$. With consideration of possible SU(3) symmetry breaking effect, we multiply a tunable factor between $1/\sqrt{2}$ and $\sqrt{2}$ to the central value of $g_{KNN}\Sigma\Lambda$ in our analysis.

For intermediate $\Sigma$ state with $J^P = \frac{1}{2}^+$, the effective Lagrangians are

$$\mathcal{L}_{K\Sigma}(\frac{1}{2}^-) = -ig_{K\Sigma}(\frac{1}{2}^-) \bar{\chi} N + \text{H.c.},$$

and

$$\mathcal{L}_{\Lambda\Sigma}(\frac{1}{2}^-) = -ig_{\Lambda\Sigma}(\frac{1}{2}^-) \bar{\chi} N + \text{H.c.}$$

The product of the coupling constants $g_{K\Sigma}(\frac{1}{2}^-)g_{\Lambda\Sigma}(\frac{1}{2}^-)$ is set to be a free parameter in our analysis.

For intermediate $\Sigma^*$ state in s-channel with $J^P = \frac{3}{2}^+$, the effective Lagrangians are

$$\mathcal{L}_{KN\Sigma^*} = \frac{f_{KN\Sigma^*}}{m_K} \partial_\mu \bar{\chi} \Sigma^* \mu \gamma_5 \Lambda + \text{H.c.},$$

and

$$\mathcal{L}_{\Sigma^*\Lambda} = \frac{f_{\Sigma^*\Lambda}}{m_\Sigma} \partial_\mu \bar{\chi} \Sigma^* \mu + \text{H.c.},$$

For $\Sigma^*(1385)$, the coupling constant $f_{\Sigma^*\Lambda}$ can be calculated from the decay width $\Gamma_{\Sigma^*\Lambda} \approx 31$ MeV [2], and $f_{KN\Sigma^*} = -3.22$ can be estimated from flavor SU(3) symmetry relation [12]. With consideration of possible SU(3) symmetry breaking effect, we multiply a factor between $\sqrt{2}$ and $1/\sqrt{2}$ as a free parameter to the central value of $f_{KN\Sigma^*}$, and thus $f_{KN\Sigma^*}$ is constrained between $-2.9$ and $-5.8$ in our analysis.

For intermediate $\Sigma$ state in s-channel with $J^P = \frac{5}{2}^+$, the effective Lagrangians are

$$\mathcal{L}_{KN\Sigma}(\frac{5}{2}^-) = \frac{f_{KN\Sigma}(\frac{5}{2}^-)}{m_K} \partial_\mu \bar{\chi} \Sigma \gamma_5 N + \text{H.c.},$$

and

$$\mathcal{L}_{\Lambda\Sigma}(\frac{5}{2}^-) = \frac{f_{\Lambda\Sigma}(\frac{5}{2}^-)}{m_\Sigma} \partial_\mu \Lambda + \text{H.c.}$$

The $\Sigma(1670)D_{13}$ is a four-star resonance in PDG. The above coupling constants can be estimated from the decay width $\Gamma_{\Sigma}(1670)\rightarrow KN$ and $\Gamma_{\Sigma}(1670)\rightarrow \pi\Lambda$, which still have
the coupling constants 1775 MeV and 120 MeV, respectively, and the product of the averaged amplitudes in the propagators. The mass and width are fixed to be 50 GeV$^{-1}$, corresponding to $(\Gamma_{\pi\Lambda} \Gamma_{\Sigma N})^{1/2}/I_{\text{tot}} \sim -0.28$ in our analysis.

For each vertex of these channels, a form factor is attached to describe the off-shell properties of the amplitudes. For all the channels considered, we adopt the form factor $|\mathcal{M}|^2$ can be expressed as

$$|\mathcal{M}|^2 = \frac{1}{2} \sum_{r_1, r_2} |\mathcal{M}|^2$$

where $r_1$ and $r_2$ denote polarization of initial and final state, respectively; and $p$ and $p'$ denote the 4-momenta of proton and $\Lambda$ in the reaction. $\mathcal{A}$ is part of the total amplitude, which can be expressed as

$$\mathcal{M} = \pi \mathcal{A}(p') A \mathcal{U}_r(p) = \pi \mathcal{A}(p') (\sum_i A_i) u_{r_i}(p).$$

with

$$\pi g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2},$$

$$\pi = \gamma_\mu - \frac{p_\mu}{m^2}.$$
where $p_n'$ is the 3-momenta of the produced nucleon in the $\Lambda$ rest frame, and $\Gamma_\Lambda = \tau_\Lambda^{-1}$ is $\Lambda$ decay width; the amplitude $M'$ is expressed as

$$M' = \pi \pi_3 \langle p_n | G_F | m_n^2 (A - B \gamma_5) | p_n' \rangle (\sum_i A_i) | u_{r_1, r_3} \rangle (p),$$

and $|M'|^2 = \frac{1}{2} \sum_{r_1, r_3} M' M'^*.$

### III. RESULTS AND DISCUSSIONS

In this analysis, the t-channel $K^*$ exchange and the u-channel proton exchange amplitudes are fundamental ingredients, which are different from previous analyses using some polynomial parametrization for background contributions and are more physically based, although there are also some parameters to be fitted in reasonable ranges. The $\Sigma(1189)_{1/2}^+, \Sigma(1385)_{3/2}^+, \Sigma(1670)_{3/2}^-$ and $(1775)_{3/2}^-$ contributions are always included in our analysis, partly because these channels should contribute to the reaction by the knowledge of their existence as well established four-star resonances, partly because the present data favor the inclusion of them. Still some parameters in the above channels have uncertainties and are to be fitted in the analysis. The ranges of the parameters have been constrained from the PDG estimates or model predictions, which have been explained in section II. From the above 6 channels of 14 tunable parameters constrained in the allowed range, the best fit to the differential cross sections and the $\Lambda$ polarization gives a $\chi^2$ of about 763 for the total 248 data points. The results are shown by the (blue) dashed lines in Fig. 2 and Fig. 3 for the differential cross sections and the $\Lambda$ polarization, respectively. Although the fit looks already quite good qualitatively, from detailed comparison with the very precise data and the quite large $\chi^2$, some systematic deviations still exist.

For a better description of the data, we need to introduce some other $\Sigma$ resonances in s-channel. We try them in the analysis with their coupling constants, mass, and width as free parameters and check if they are favored by the present data.

Among the $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ $\Sigma$ resonances in s-channel, our best fit comes from the inclusion of a $J^P = \frac{1}{2}^+$ $\Sigma$ resonance with mass near 1635 MeV, and width around 121 MeV.

The solid lines in Fig. 2 and Fig. 3 shows this best fit compared with the experimental data of the differential cross sections and the $\Lambda$ polarization. The analysis includes 18 tunable parameters in the allowed range and the $\chi^2$ for this best fit is 223 for the total 248 data points. From the solid lines in Fig. 2 and Fig. 3, one can see that the experimental data can be much better described with the inclusion of the $\Sigma(1635)_{1/2}^+$ resonance, especially with the $\Lambda$ polarization.

In Table I, we show the central values and statistic uncertainties for 6 of the parameters on the s-channel $\Sigma$ resonances in this energy range. From Table I, the mass of $\Sigma(1670)$ is precisely around 1673 MeV, and its width is around 54 MeV, which are consistent with the PDG estimates. The mass of $\Sigma(1775)$ is much larger than the c.m. energies of this experiment, and thus the fits to the data are not sensitive to the parameters of $\Sigma(1775)$ within their PDG ranges. We fix the parameters of $\Sigma(1775)$ to their PDG central values. The $J^P = \frac{1}{2}^+ \Sigma(1635)$ from our analysis is well in accordance with the 3-star $\Sigma(1660)_{1/2}^+$ in PDG, giving a further support for the existence of the resonance. Our fit with the $\Sigma(1635)_{1/2}^+$ is stable with the parameters well constrained. By including this resonance, the $\chi^2$ drops from about 763 to 223 for the total 248 data points.

The propagators of the s-channel $\Sigma$ resonance exchanges are Breit-Wigner form in this analysis, where the mass and width of the resonance are constant parameters. We also check the energy dependent form of the Breit-Wigner propagator, i.e., to replace the constant $i\sigma_i \Sigma(1/2)$ in the propagator by the energy dependent form $i\sqrt{s} \Gamma(s)$. From the check on the $\Sigma(1670)_{1/2}^+$, we find that the central values of the mass and width of $\Sigma(1670)$ become larger by 2 MeV and 10 MeV, respectively, while the effects on other parameters are very small.

The other 12 free parameters of this analysis includes 5 coupling constants and 7 cutoff parameters in the form factors of the total 7 channels. In Table II we list the fitted results of the 5 free parameters on the couplings of the t-channel, u-channel and s-channel $\Sigma(1189)$ and $\Sigma(1385)$ contributions. Note that the first two parameters are couplings of the t-channel $K^*$ exchange, and their ranges are constrained by the potential model. We also check to widen the ranges of the two parameters, and find only small shifts of the other parameters. Thus the uncertainties on the parameters of the t-channel do not change the main results of the analysis.

The research for the possible new $\Sigma(1/2^-)$ near 1380 MeV has always been our concern, and previous work has shown some evidence of it. In this work, we also check whether this data set is compatible with the existence of the $\Sigma(1380)$. Without including the $\Sigma(1635)$, we try to include a $\Sigma(1/2^-)$, and constrain its mass above 1360 MeV. From our analysis, the best fit gives $\chi^2 = 385$ a minimum mass, a small coupling constant $g_{KN \Sigma(1/2^-)} g_{\Sigma(1/2^-)\Lambda} \sim -1.26$ and width around 315 MeV. This shows that the existence of the $\Sigma(1/2^-)$ near 1380 MeV with sizeable couplings is not ruled out by the present data, although there is no strong evidence of it. This result is understandable since 1380 MeV is much smaller than the energy range of the experiment.

With the same procedure, we also try $\Sigma^* (3/2^-)$ states in the analysis. The resulted $\chi^2$ are not significantly improved as the case of the $\Sigma(1635)_{1/2}^+$, which shows no con-
FIG. 2: The differential cross sections of the reaction $K^- + p \rightarrow \pi^0 + \Lambda$ compared with the experimental data [1], where $\theta$ denotes the angle of the outgoing $\pi^0$ with respect to beam direction in the c.m. frame. The dashed lines (blue) are the best results with inclusion of only well established (4-star) $\Sigma$ resonances in s-channel; the solid lines are best results of including an additional $\Sigma(1236)$ resonance in s-channel, with its mass near 1635 MeV and width around 121 MeV.

FIG. 3: Fits to the $\Lambda$ polarization as a function of $\cos \theta$ for the reaction $K^- + p \rightarrow \pi^0 + \Lambda$ compared with the experimental data [1], where $\theta$ denotes the angle of the outgoing $\pi^0$ with respect to beam direction in the c.m. frame. The dashed lines (blue) are results with inclusion of only well established (4-star) $\Sigma$ resonances in s-channel; the solid lines are results of including an additional $\Sigma(1236)$ resonance in s-channel with mass near 1635 MeV and width around 121 MeV.
TABLE I: Adjusted parameters for high mass Σ resonances, which includes the Σ(1635)± resonance. Statistic uncertainties and PDG estimates are also listed.

| Σ(1670)γ− | mass(MeV)(PDG estimate) | Γ_{tot}(MeV) (PDG estimate) | (Γ_{πΛ}Γ_{KΛ})/Γ_{tot} (PDG range) |
|------------|-------------------------|----------------------------|----------------------------------|
| Σ(1670)γ− | 1673.4^{+1.1}_{-0.8} (1665,1685) | 54 ± 5 (40,80) | 0.08^{+0.02}_{-0.015} (0.02, 0.17) |
| Σ(1635) or Σ(1660)γ± | 1635^{+1.1}_{-1.0} (1630,1690) | 121^{+12}_{-11} (40,200) | −0.064^{+0.012}_{-0.015} (0.024) |

TABLE II: Adjusted parameters with statistic uncertainties for the couplings in t-channel, u-channel and s-channel Σ(1189) and Σ∗(1385) exchange.

| g_{K^{−}NΛ} (model range) | g_{K^{−}NΛK^{−}NΛ} (model range) | g_{πNN}g_{K^{−}NΛ} (SU(3)) | g_{K^{−}NΛΣ^{−}Λ} (SU(3)) | f_{K^{−}NΛΣ^{−}Λ} (SU(3)) |
|---------------------------|----------------------------------|-----------------------------|---------------------------|-----------------------------|
| −6.11^{+0.15}_{−0.03} (−6.11, −4.26) [5] | −11.33^{+0.14}_{−0.12} (−16.3, −10.4) [5] | −177^{+2}_{−3} (−176) | 49.1^{+0}_{−0} (34.8) | −3.95^{+0.35}_{−0.28} (−4.1) |
vincing evidence of their existences in the energy range of the experiment.

Some uncertainty may still exist from the uncertainty in the coupling constants and cutoffs, while the main results of this analysis will not change.

As in previous analyses listed in PDG for the same reaction, here we have not imposed the unitarity by a multichannel description. It is clear that coupled channel analysis is most appropriate way to evaluate resonance properties. However, we found that other relevant channels are not well studied and there are no new precise data. In principle that is a general problem. That is why in PDG the listed analyses are also based on the single channel fit. Measurements with wider energy ranges and combined channel analysis in the future will be helpful to provide more information on properties and interactions of the Σ resonances.

IV. SUMMARY

The neutral particles production from \( K^- p \) interactions have been measured by the Crystal Ball Collaboration for incident momentum of \( K^- \) between 514 and 750 MeV/c. Using the high precision new data, we analyze the differential cross sections and the Λ polarization of the reaction \( K^- + p \rightarrow \pi^0 + \Lambda \) with the effective Lagrangian method. We include the contributions from t-channel \( K^* \) exchange, u-channel proton exchange, and four-star Σ resonances exchanges in s-channel, i.e., \( \Sigma(1189) \), \( \Sigma(1385) \), \( \Sigma(1670) \) and \( \Sigma(1775) \) in our analysis. We find that these 6 ingredients are still insufficient, with \( \chi^2 \sim 763 \) for the total 248 data points. We try to include some new ingredient in our analysis and the best result is to include a Σ resonance with \( J^P = \frac{1}{2}^+ \), mass near 1635 MeV and width around 121 MeV. The \( \chi^2 \) drops from 763 to 223 for the 248 data points. The properties of this resonance is well in accordance with the 3-star \( \Sigma(1660) \frac{1}{2}^+ \) in PDG, providing a further support for the existence of the resonance.

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