Phase coherent transport in hybrid superconducting structures: the case of d-wave superconductors

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We examine the effect of d-wave symmetry on zero bias anomalies in normal-superconducting tunnel junctions and phase-periodic conductances in Andreev interferometers. In the presence of d-wave pairing, zero-bias anomalies are suppressed compared with the s-wave case. For Andreev interferometers with aligned islands, the phase-periodic conductance is insensistive to the nature of the pairing, whereas for non-aligned islands, the nature of zero-phase extremum is reversed.

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I. INTRODUCTION

During the past few years, studies of the subgap conductance of normal-insulator-superconductor (N-I-S) structures have revealed new and unexpected behaviour [1]. Whereas conventional tunneling theory suggests that the subgap conductance must vanish, experiments reveal that when the normal region becomes phase-coherent, there exists a zero-voltage peak in the differential conductance, which can be comparable with the normal state value. The interplay between Andreev scattering [2] at an N-S interface and disorder-induced scattering in the normal region has been recognised as the main physical origin of this zero bias anomaly (ZBA) [3]- [11].

Andreev scattering [2] involves the simultaneous tunneling of two electrons of opposite momenta through the N-I-S structure. In the case of a low-conductance tunnel junction ($G_{\text{tun}} \ll 1$), this two-particle process occurs with a probability $G_{\text{tun}}^2$. As a consequence, the conductance in the superconducting state is expected to be much smaller than that in the normal state. However, disorder-induced scattering on the normal side may give rise to particle-particle correlations, which effectively, increase the probability for two particles with opposite momenta to tunnel into the superconductor. This increase manifests itself as a low total-momentum singularity in the particle-particle scattering channel (the so-called Cooperon) [2], which is also the relevant scattering channel for the occurrence of s-wave pairing in BCS theory [3]. In the latter case, the low total-momentum instability signals the formation of a bound state of two electrons with opposite momenta, while in the former it leads to enhanced backscattering in disordered systems. In both cases, the singularity in the particle-particle Cooperon channel is characterized by the s-wave symmetry of the relative two-particle wavefunction and is ultimately responsible for the occurrence of the ZBA [9]. In contrast, for a clean normal region, Andreev scattering is almost suppressed [14] and conventional tunneling theory applies.

The aim of this paper is to demonstrate that ZBAs and related phenomena are sensitive to the symmetry of the superconducting order parameter and therefore can be used to distinguish between s-wave and d-wave pairing in high-$T_c$ superconductors. In what follows, we extend a numerical multiple scattering approach [15] to the case of non-local anisotropic superconductors. In the case of local s-wave pairing, this approach has been shown to be equivalent to other techniques and has been used to study the cross-over between different transport regimes [16]. To start with, in section II , we present an analytic treatment of a ballistic N-I-S structure, which generalizes the theory of Ref. [14] to the case of a tight-binding lattice and of a non-local superconducting order parameter. In section III, we show that, in the presence of disorder, the subgap conductance of a normal-insulating- d-wave junction is suppressed compared with the corresponding normal-insulating-s-wave structure. In particular, in the regime of small tunnel junction conductance $G_{\text{tun}} \ll 1$, we predict that the d-wave junction shows only a weak zero bias anomaly. Similarly, the zero energy dip of a N-I-S structure, in the high tunnel junction conductance regime $G_{\text{tun}} \approx 1$, is considerably smaller for the d-wave case.

Having examined the case of a single superconducting contact, in section IV, we extend the analysis to a phase-coherent structure in contact with two superconductors. The electrical conductance of such structures is known to be a periodic function of the difference between the order parameters phases of the two superconductors, which in turn is an externally controllable quantity. Such Andreev interferometers have recently been the subject of intensive theoretical [17]- [24] and experimental studies [23]- [25]. In what follows, we predict that certain features of the
phase-periodic conductance, such as the nature of the zero-phase extremum, are sensitive to the symmetry of the order parameter, and, therefore, interferometers of this kind provide a further probe into the nature of the pairing.

II. ANDREEV REFLECTION AT A N-S INTERFACE

To begin with, in this section, we examine a normal-superconducting interface (N-S) in the presence of a non-local pairing potential, described by the Hamiltonian

$$H = \sum_{i,\sigma} \epsilon_i c_{i,\sigma}^\dagger c_{i,\sigma} + \sum_{i,\delta,\sigma} \left[ \gamma c_{i,\sigma}^\dagger c_{i+\delta,\sigma} + h.c. \right] + \sum_{i,\delta} \left[ (c_{i,\delta}^\dagger c_{i+\delta,\uparrow} - c_{i,\delta} c_{i+\delta,\uparrow}^\dagger) \Delta_{i,\delta} + h.c. \right].$$

(1)

Here, the index $i$ runs over a two-dimensional tight-binding lattice, with unit lattice constant, $\delta$ sums over nearest neighbours, $\delta = \hat{x}, \hat{y}$, and the pairing potential, $\Delta_{i,\delta}$, is defined on the bond from site $i$ to site $i + \delta$. The operator $c_{i,\sigma}^\dagger$ ($c_{i,\sigma}$) creates (destroys) an electron at site $i$, with site energy $\epsilon_i$, and $\gamma$ is the hopping matrix element between nearest-neighbours sites. All energies will be measured in units of $\gamma$, which will be set to unity throughout the paper. The above Hamiltonian is diagonalised by solving the Bogoliubov-de Gennes equation:

$$E \psi_i = \epsilon_i \psi_i + \sum_{\delta} \gamma (\psi_{i+\delta} + \psi_{i-\delta}) + \sum_{\delta} \left( \Delta_{i,\delta} \psi_{i+\delta} + \Delta_{i-\delta,\delta} \psi_{i-\delta} \right),$$

$$E \phi_i = -\epsilon_i \phi_i - \sum_{\delta} \gamma (\phi_{i+\delta} + \phi_{i-\delta}) + \sum_{\delta} \left( \Delta_{i,\delta}^* \psi_{i+\delta} + \Delta_{i-\delta,\delta}^* \psi_{i-\delta} \right),$$

(2)

where $\psi_i (\phi_i)$ indicates the particle (hole) wavefunction. In general, the Hamiltonian of eq.(1) is a mean-field approximation to a more complex Hamiltonian containing electron-electron interactions and all parameters should be determined self-consistently. However, in many cases of experimental interest, the qualitative form of parameters such as $\Delta_{i,\delta}$ is known and for the purpose of highlighting generic transport properties, self-consistency is not required.

The Bogoliubov-de Gennes equation may be solved by means of a transfer matrix method or a recursive Green function technique. These methods work in any dimension and constitute the only exact approach when translational invariance is absent. As a prelude to such a calculation, in this section, we begin by considering a system with translational invariance in the direction perpendicular to the current flow. Our motivation for doing so is two-fold. On the one hand, one obtains new results for Andreev scattering in the presence of anisotropic pairing. On the other, the analysis provides a controllable limit, which can be used to test the numerical machinery used in more complicated situations.

In the presence of translational invariance in the direction transverse to the current, the problem reduces to one of many independent one-dimensional channels, each characterised by one or more discrete quantum numbers. In what follows, we consider the case of a 2-dimensional system with a N-S interface, whose normal vector points in the $x$ direction, although by redefining the parameters $\bar{\mu}$, $\bar{\Delta}$ introduced below, all results are trivially generalized to 3-dimensions. The number of independent channels is determined by the width $M$ of the system, and each channel has a discrete wavevector $k_y$, along the $y$ direction. Choosing $\epsilon_i = -\mu$ yields an energy dispersion relation in the normal region of the form

$$\epsilon_k = -2\gamma (\cos(k_x) + \cos(k_y)) - \mu$$

(3)

where the uniform site energy $\mu$ determines the filling of the tight-binding band. Writing

$$\bar{\mu} = \mu + 2\gamma \cos(k_y),$$

(4)

shows that the dispersion relation reduces to that of a one-dimensional system with a channel-dependent chemical potential $\bar{\mu}$.

In the superconducting region, the dispersion relation is replaced by

$$E^2 = (\epsilon_k)^2 + |\Delta_k|^2,$$

(5)

where $\epsilon_k = -2\gamma (\cos(k_x) + \cos(k_y)) - \mu$, and the momentum dependent gap function, $\Delta_k$, is given by

$$\Delta_k = 2(\Delta_x \cos(k_x) + \Delta_y \cos(k_y)),$$

(6)

where for d-wave symmetry, $\Delta_x = -\Delta_y$. 

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In writing the above equation, we have assumed a uniform pairing potential for all sites $i > 0$. The transverse component, $k_y$ is conserved through the interface, $k_y = k_y$, and for a fixed value of $k_y$, the gap function can be written

$$\Delta_k = 2\Delta_x \cos(k_x) + \Delta,$$

where $\Delta = 2\Delta_x \cos(k_y)$, which demonstrates that the pairing potential along the $y$ direction yields a local contribution to the pairing potential for a particular channel. Hence for $i < 0$, the Bogoliubov - de Gennes equation reduces to

$$E\psi_i = -\gamma(\psi_{i-1} + \psi_{i+1}) - \bar{\mu}\psi_i$$

$$E\phi_i = \gamma(\phi_{i-1} + \phi_{i+1}) + \bar{\mu}\phi_i$$

while for $i > 0$,

$$E\psi_i = -\gamma(\psi_{i-1} + \psi_{i+1}) - \bar{\mu}\psi_i + \Delta_x(\phi_{i-1} + \phi_{i+1})$$

$$E\phi_i = \gamma(\phi_{i-1} + \phi_{i+1}) + \bar{\mu}\phi_i + \Delta_x(\psi_{i-1} + \psi_{i+1}).$$

Since the value of the transverse component of the momentum enters only through the effective chemical potential $\bar{\mu}$ and the local contribution to the gap $\Delta$, it is convenient to drop the $x$ suffix in labelling the various longitudinal momenta and write $k$ and $q$, ($\bar{k}$ and $\bar{q}$), for the longitudinal momenta of particles and holes in the normal (superconducting) region. With this notation and writing $(\psi^R, \phi^R)$ and $(\psi^L, \phi^L)$ for the solutions when $i > 0$ and $i < 0$, respectively, one obtains for $i < 0$

$$\psi_i^L = e^{ikR_i} + r_a e^{-ikR_i},$$

$$\phi_i^L = r_a e^{iqR_i},$$

and for $i > 0$,

$$\psi_i^R = t_a e^{ikR_i} + t_\bar{a} e^{-iqR_i},$$

$$\phi_i^R = t_\bar{a} e^{iqR_i} + t_a e^{-ikR_i}.$$

The coherence factors $u$ and $v$ identify a particle-like excitation of energy $E$ and momentum $\bar{k}$, while $\bar{u}$ and $\bar{v}$ correspond to an hole-like excitation at the same energy and momentum $\bar{q}$. To compute these quantities one notes that

$$E^2 = (-2\gamma \cos(\bar{p}) - \bar{\mu})^2 + |\Delta + 2\Delta_x \cos(\bar{p})|^2$$

where $\bar{p}$ may be $\bar{k}$ or $\bar{q}$, which yields

$$\cos(\bar{p}) = -\frac{1}{2} (\bar{\mu} + A \pm B) / \gamma$$

with

$$A = \Delta_x \frac{\gamma \Delta - \Delta_x \bar{\mu}}{\gamma^2 + \Delta_x^2},$$

$$B = \gamma \sqrt{\frac{E^2}{\gamma^2 + \Delta_x^2} - \left(\frac{\gamma \Delta - \Delta_x \bar{\mu}}{\gamma^2 + \Delta_x^2}\right)^2}.$$

For the coherence factors one has to distinguish the case of real $B$, which corresponds to quasi-particle transmission through the interface, from the case of imaginary $B$, which corresponds to no quasi-particle transmission. For $\bar{B}$ real, one obtains

$$u^2(v^2) = \frac{1}{2}(1 + (-)(A + B)/E)$$

$$\bar{u}^2(\bar{v}^2) = \frac{1}{2}(1 + (-)(A - B)/E)$$

with $u/v = \text{sign}(\Delta_k/(E - \epsilon_k))$, and $\bar{u}/\bar{v} = \text{sign}(\Delta_{\bar{q}}/(E - \epsilon_{\bar{q}})).$

For $\bar{B}$ imaginary, we write $\bar{B} \equiv i\bar{B}$, to yield

$$\frac{u}{v} = \Delta_k/(E - A - i\bar{B})$$

$$\bar{u}/\bar{v} = \Delta_{\bar{q}}/(E - A + i\bar{B}) = (u/v)^*.$$
To solve for the scattering coefficients $r_\alpha$, $r_o$, $t_o$, and $t_a$, one needs matching conditions at the interface. These are obtained by evaluating equations (8) and (9) at $i = 0$ and at $i = -1$ and can be written in the form

$$
\psi^L_0 = \psi^R_0
$$

and

$$
\psi^L_{-1} = \tilde{\psi}^R_{-1}
$$

where (see appendix) $\tilde{\phi}^R_{-1}$, $\tilde{\psi}^R_{-1}$ are obtained by acting on $\phi^R_i$, $\psi^R_i$ with the appropriate transfer matrix at the interface. Equation (16) yields

$$
1 + r_o = t_o u + t_o \bar{u}
$$

$$
r_o = t_o v + t_o \bar{v},
$$

whereas the matching conditions eq.(17) yield (see appendix)

$$
e^{-ik} + r_a e^{ik} = t_a u e^{-ik} (1 - (v/u)(\Delta_x/\gamma)) + t_a \bar{u} e^{ik} (1 - (\bar{v}/\bar{u})(\Delta_x/\gamma))
$$

$$
r_a e^{-iq} = t_a v e^{-ik} (1 + (u/v)(\Delta_x/\gamma)) + t_a \bar{v} e^{iq} (1 + (\bar{u}/\bar{v})(\Delta_x/\gamma)).
$$

The eqs.(18-19) yield all the scattering coefficients associated with a particle incident from the left on a clean N-S interface. A direct analytic evaluation is rather messy, but it is trivial to solve these numerically. Explicit results will be presented in section III. In the presence of a tunnel barrier modelled by a delta function potential at the interface, $\epsilon_i$ is replaced by $\epsilon_i = -\mu + U \delta_{i,0}$ and the matching conditions eq.(19) by

$$
e^{-ik} + r_a e^{ik} = t_a u e^{-ik} (1 - (v/u)(\Delta_x/\gamma)) + t_a \bar{u} e^{ik} (1 - (\bar{v}/\bar{u})(\Delta_x/\gamma)) + (1 + r_o) U/\gamma
$$

$$
r_a e^{-iq} = t_a v e^{-ik} (1 + (u/v)(\Delta_x/\gamma)) + t_a \bar{v} e^{iq} (1 + (\bar{u}/\bar{v})(\Delta_x/\gamma)) + r_a U/\gamma.
$$

Various limiting forms of the above expressions are discussed in appendix. Here we merely note that with the convention adopted in eqs.(10-11), if $v_k$ ($v_q$) and $v_{-k}$ ($v_{-q}$) are group velocities for particles (holes) in the normal and superconducting regions respectively, then the following unitarity condition is satisfied, $R_a + R_0 + T_a + T_0 = 1$, where $R_a = (v_q/v_k)|r_a|^2$, $R_0 = |r_0|^2$, $T_a = (v_q/v_k)|t_a|^2$ and $T_0 = (v_q/v_k)|t_0|^2$.

### III. RESULTS FOR N-I-S STRUCTURES

In this section we present explicit results for the above scattering coefficients and for the electrical conductance

$$
G = \left(\frac{2e^2}{h}\right) \sum_{k} (1 - R_0 + R_a)
$$

where the sum is over all channels.

For an N-S interface with no potential barrier, fig.(a) shows the behaviour of the differential conductance as a function of energy, along with various scattering coefficients. In a conventional superconductor under sub-gap conditions, only Andreev reflection contributes to the current flow, because sub-gap quasi-particle transmission is forbidden. In a gapless superconductor, the situation is more complicated, because each channel has its own effective gap, so that normal quasi-particle transmission occurs even at low energies. To illustrate this point, fig.(a) shows the behaviour of transmission and reflection probabilities for a system width $M = 10$. The number of open channels $N$ depends on the position of the chemical potential $\mu$ within the band. Here we have used $\mu = -0.2\gamma$ so that $N = 9$. For free-end boundary conditions in the transverse direction, the allowed values of $k_y$ are $\pi n/(M + 1)$, with $n = 1, 2, ..., M$. For this choice, all channels have non-vanishing gap and at zero energy, normal transmission vanishes. By increasing the energy, one eventually crosses the effective gap of a particular channel, at which point transmitting channels appear in the superconducting region and Andreev scattering is suppressed. Since Andreev scattering contributes a factor of 2 (in units of $2e^2/h$) to the electrical conductance, while transmission processes only contribute a factor of unity, the conductance decreases at such energies. In fig.(b), we show results for a system width $M = 1000$. At zero energy, it is interesting to note that the conductance per channel is insensitive to the width of the system and agrees

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perfectly with the $M = 10$ value. At finite energy the results differ slightly, because in the case of a large number of channels, channels open continuously with increasing energy, to yield the smooth behaviour shown in fig.6(b).

We now consider the case of an N-I-S structure, in which a tunnel junction is present at the interface. Fig.3(a-b) shows results for the electrical conductance $G$ and for various scattering coefficients in the presence of a barrier height $U = 2.0\gamma$ for a system width $M = 10$. The oscillating behavior is a finite size effect which can be understood by observing that a peak occurs when the energy becomes greater than the effective gap $\Delta$ of a particular channel. The oscillation arises because for $E < \Delta$ the contribution from such a channel is proportional to the square of the barrier transmission coefficient $\Gamma$ (i.e., $\approx G_{\text{tun}}$), whereas for $E > \Delta$ it is proportional to the first power of $\Gamma$ (i.e., $\approx G_{\text{tun}}$).

In fig.3(c-d) we show the conductance for a width $M = 1000$. Again the conductance per channel, at zero energy, is found to be insensitive to the number of open channels, and oscillations at intermediate energies are no-longer present.

Having examined a clean metal in contact with a superconductor, we now introduce disorder to the normal metal, to produce a diffusive conductor in contact with a d-wave superconductor. In this case the conductance is obtained using the transfer matrix method outlined in reference [22] and computer resources impose restrictions on the system width $M$.

The N-I-S structure of fig.3 consists of a diffusive metallic region placed in series with a tunnel junction, which in turn is adjacent to a superconductor. In the absence of disorder, the numerical code agrees exactly with the analytical results of figures 1 and 2. In what follows, the simulated structure is a two-dimensional tight-binding lattice of width $M = 10$ sites. The disordered region is of length $L_{\text{dif}}$ sites, the tunnel junction is $L_{\text{tun}}$ sites long and the superconductor has a length $L_{\text{sup}}$. The conductance of the entire structure is denoted by $G$ and the average over an ensemble of disorder realizations is $< G >$. The physical variables in the following calculation are the averaged conductance $< G_{\text{dif}} >$ of the diffusive region and the conductance of the tunnel junction $G_{\text{tun}}$. To identify a suitable choice of parameters, we considered first a normal diffusive portion of length $L_{\text{dif}}$ and width $M$, connected to crystalline, normal leads. The conductance of a diffusive material is inversely proportional to its length and therefore a plot of $< G_{\text{dif}} >$ as a function of $L_{\text{dif}}$ will exhibit a plateau in the diffusive regime, with a mean free path given by $l = < G_{\text{dif}} > L_{\text{dif}}/(2e^2/h)$. A diffusive system must satisfy $l \ll L_{\text{dif}}$ and $l \ll M$. Furthermore, if weak localization corrections are to be neglected, we require $N! \gg L_{\text{dif}}$. In the calculations which follow, having in mind also the necessity of minimizing the CPU time, we have made the following choice of parameters: $L_{\text{dif}} = 30$ sites, disorder width $W = 1$, system width $M = 10$, $\mu = 0$ so that the number of open channels is $N = 10$. This yields a mean free path $l = 4.2$ and an average conductance $< G_{\text{dif}} > = 1.6$ (in units of $2e^2/h$). The superconductor has a length $L_{\text{sup}} = 100$ with order parameter $\Delta_x = 0.1\gamma$. One characteristic energy scale is the maximum gap, which for $\Delta_x = 0.1\gamma$ is equal to $0.4\gamma$. The tunnel junction is $L_{\text{tun}} = 1$ site long and the potential on the line of sites defining the junction is $-\mu + \epsilon_b$ with $\epsilon_b$ taking the value $7\gamma$ and $2\gamma$, which yields $G_{\text{tun}} = 0.4(2e^2/h)$ and $G_{\text{tun}} = 3(2e^2/h)$ in the low and high tunnel junction regimes, respectively.

For the case of low tunnel junction conductance, fig.4 shows the sub-gap conductance as a function of the energy, which in the linear response regime, corresponds to experimentally measured I-V characteristic. The strong peak, present in the s-wave case, is strongly suppressed in the d-wave case. Fig.4 shows the corresponding behaviour in the high tunnel junction conductance regime. In this case, the zero energy behaviour is characterized by a dip in the conductance. As before, the d-wave conductance and the zero bias feature is suppressed.

IV. RESULTS FOR D-WAVE ANDREEV INTERFEROMETERS.

We now consider the effect of d-wave symmetry on the properties of Andreev interferometers. Two different interferometer geometries are analyzed below and shown as inserts in figures 6 and 7. In each example, the system consists of two superconducting regions with order parameter phases $\phi_1$ and $\phi_2$, separated by a normal region $N$, with a quasi-particle current flowing vertically. In figure 6, the S-N-S structure is placed in contact with a tunnel junction and the N and S regions are clean. In figure 7, there is no tunnel junction, but the whole S-N-S structure is disordered.

The physical parameters used to obtain figure 6 are as follows: $\mu = 0$ and the tunnel barrier site-energy $\epsilon_b = 2\gamma$, which corresponds to an average conductance per channel of $G_{\text{tun}}/N = 0.176(2e^2/h)$. As shown in figure 6, the electrical conductance in the presence of s-wave pairing shows a large amplitude of oscillation and a zero-phase minimum, which are characteristic of a ballistic structure, as discussed in Refs. [14, 23]. In contrast, for a d-wave interferometer, the nature of the zero phase extremum depends on the relative orientation of the two superconducting islands. In one case (solid line in figure 6) the x axes with a positive value of the order parameter are parallel in both islands, while in the other case (dashed line in figure 6) the x and y axes are exchanged in going from one island to the other. For the case of parallel orientation, the d-wave interferometer is almost identical to the s-wave case, exhibiting
a large amplitude of oscillation and a zero-phase minimum. When the islands are orientated perpendicular to one another, the minimum becomes a maximum and the entire curve is shifted by \( \pi \). This arises because an electron Andreev reflected at an N-S interface, acquires the phase of the bonds in the longitudinal direction (with respect to the incoming direction of the electron). When the two islands are orientated parallel, they have the same value for the longitudinal bonds, so that there is no effective phase difference between the two islands, as in the s-wave case. When one of the two superconducting islands is rotated by \( \pi/2 \), the longitudinal bonds have a phase difference of \( \pi \), which adds to the external phase difference. This effect is similar in origin to that predicted \[33\] and later verified \[34\] in a corner SQUID experiment on the high-\( T_c \) superconducting compound YBCO. The relevance of the orientation of the crystal axis with respect to the N-S interface has also recently put forward \[35\] as a possible explanation of zero bias anomalies.

For figure 7 we choose parameters corresponding to a diffusive region. The structure has a length \( L = 50 \) with each of the three regions having a width \( M' = 15 \), so that the entire structure has a width \( M = 45 \). We used \( \mu = 0 \), disorder width \( W = 0.5\gamma \), which yields, in the normal state, a mean free path \( l = 12 \). For an s-wave interferometer (shown as an inset), the zero phase extremum switches to a maximum and the value at \( \phi = \pi \) becomes a minimum. For aligned islands (solid line), the d-wave interferometer shows the same qualitative behaviour as in the the s-wave case. However, rotating one of the islands by \( \pi/2 \) again shifts the curve by \( \pi \) and changes the zero-phase extremum from a maximum to a minimum.

V. DISCUSSION

Zero bias anomalies and phase-periodic conductances in Andreev interferometers are paradigms of phase-coherent transport in hybrid N-S structures. In this paper we have examined the sensitivity of these phenomena to the symmetry of the superconducting order parameter. We find that ZBAs are suppressed, reflecting the fact that disorder induced scattering and Cooper pairing no longer occur in the same particle-particle scattering channel. We also find that for aligned islands, Andreev interferometers are relatively insensitive to the nature of the pairing. Nevertheless, such devices could be used to reveal the presence of d-wave pairing, since the positions of conductance extrema are sensitive to the relative orientation of the two order parameters.

For simplicity, we have avoided the necessity of a fully self-consistent calculation, by restricting the analysis to a particular crystal orientation. Analyses \[30\] of the proximity effect in ballistic systems reveal that a self-consistent theory can yield a drastic suppression of the superconducting order parameter in the vicinity of the N-I-S interface, but only for certain crystal orientations of the d-wave superconducting order parameter with respect to the interface \[36\]. In this paper, our main focus has been the interplay of Andreev scattering at the interface and disorder-induced scattering in the normal region, and therefore we have not considered these particular crystal orientations. For this reason we expect that our main conclusions will not be qualitatively changed by a fully self-consistent calculation, but for the future it would be of interest to explore transport in the presence of alternative crystal orientations.

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APPENDIX A: TECHNICAL DETAILS OF THE ANALYSIS OF SECTION II

To obtain equation (19) one notes that the left-hand sides of eq. (17) are

\[
\psi^L_1 = -((E + \bar{\mu})/\gamma)\psi^L_2 - \psi^L_3 = e^{-ik} + r_0 e^{i\phi} \\
\phi^L_1 = ((E - \bar{\mu})/\gamma)\phi^L_2 - \phi^L_3 = r_a e^{-iq},
\]

whereas the right-hand sides are
the expressions for the momenta \( k \)

at the interface, and the momenta \( \bar{k} \) considering the relevant limit of zero quasiparticle energy. In this case there is no quasiparticle transmission through the interface, because the resulting equations look rather cumbersome. We will instead content ourselves by rewriting equation (20) in the form

\[
\gamma \bar{\psi}_1^R = - (E + \bar{\mu}) \psi_0^R - \gamma \psi_1^R + \bar{\Delta} \phi_0^R + \Delta_x \phi_1^R,
\]

\[
\gamma \bar{\phi}_1^R = (E - \bar{\mu}) \phi_0^R - \gamma \phi_1^R - \Delta_x \psi_0^R.
\]

Notice that \( \bar{\psi}_1^R \) and \( \bar{\phi}_1^R \) are decoupled because there is no superconducting bond between \( i = -1 \) and \( i = 0 \).

Inserting eq.(11) into the above expressions and using the eigenvalue equations, yields

\[
\bar{\psi}_1^R = t_o u e^{-ik}(1 - (v/u)(\Delta_x/\gamma)) + t_o \bar{u} e^{i\tilde{\phi}}(1 - (\tilde{v}/\bar{u})(\Delta_x/\gamma))
\]

\[
\bar{\phi}_1^R = t_o v e^{-ik}(1 + (u/v)(\Delta_x/\gamma)) + t_o \bar{v} e^{i\tilde{\phi}}(1 + (\bar{u}/\bar{v})(\Delta_x/\gamma))
\]

Finally one obtains the last two matching conditions in the form (19).

The equations obtained in section II are very general and yield a variety of useful results. To illustrate this we first rewrite equation (21) in the form

\[
t_o u e^{ik} e^{i(\Delta_x/\gamma) - (U/\gamma)} + t_o \bar{u} e^{i\tilde{\phi}} e^{i(\Delta_x/\gamma) - (U/\gamma)} = 1
\]

\[
t_o v e^{-ik} e^{i(\Delta_x/\gamma) - (U/\gamma)} + t_o \bar{v} e^{i\tilde{\phi}} e^{i(\Delta_x/\gamma) - (U/\gamma)} = 0.
\]

(A1)

Once the above pair of equations is solved for \( t_o \) and \( t_a \), eq.(18) yields \( r_o \) and \( r_a \). In the case of no quasi-particle transmission, the only physical quantities are \( r_o \) and \( r_a \), which, from the form of eq.(A1), depend only upon the ratios \( u/v \) and \( \tilde{u}/\tilde{v} \).

Results corresponding to the case of a local order parameter \( \Delta_a \) can be obtained from the above eqs.(A1) by simply setting \( \Delta_x = 0 \), and \( \Delta = \Delta_a \). By considering for simplicity an ideal interface \( (U = 0) \), one obtains

\[
t_o u e^{ik} e^{-i(\Delta_x/\gamma) - (U/\gamma)} + t_o \bar{u} e^{i\tilde{\phi}} e^{-i(\Delta_x/\gamma) - (U/\gamma)} = 1
\]

\[
t_o v e^{-ik} e^{-i(\Delta_x/\gamma) - (U/\gamma)} + t_o \bar{v} e^{i\tilde{\phi}} e^{-i(\Delta_x/\gamma) - (U/\gamma)} = 0.
\]

(A2)

which gives

\[
t_o = \bar{v}(e^{i\tilde{\phi}} - e^{-i\tilde{\phi}})(e^{ik} - e^{-ik})/d
\]

(A3)

\[
t_a = v(e^{-ik} - e^{-ik})(e^{ik} - e^{-ik})/d
\]

(A4)

and, by using eq.(18),

\[
\frac{r_o = \frac{\bar{v}u(e^{i\tilde{\phi}} - e^{-i\tilde{\phi}})(e^{-ik} - e^{-ik}) + \bar{u}v(e^{-i\tilde{\phi}} - e^{i\tilde{\phi}})(e^{-ik} - e^{-ik})}{d}}{d}
\]

(A5)

\[
\frac{r_a = \bar{v}v(e^{i\tilde{\phi}} - e^{-i\tilde{\phi}})(e^{ik} - e^{-ik})/d}{d}
\]

(A6)

where

\[
d = \frac{\bar{v}u(e^{ik} - e^{-ik})(e^{i\tilde{\phi}} - e^{-i\tilde{\phi}}) - \bar{u}v(e^{-ik} - e^{-ik})(e^{-ik} - e^{-i\tilde{\phi}})}{d}
\]

Eqs.(A3,A6) solve the problem of determining the various scattering coefficients. As a last step one has to substitute the expressions for the momenta \( k, q, \bar{k}, \) and \( \tilde{q} \) in terms of the energy \( E \) as given by eq.(13). We will not do this substitution here, because the resulting equations look rather cumbersome. We will instead content ourselves by considering the relevant limit of zero quasi-particle energy. In this case there is no quasi-particle transmission through the interface, and the momenta \( \tilde{k} \) and \( \bar{q} \) are complex. By setting \( k(q) = p + (-)i\bar{l} \) and observing that \( q = k \) and \( \bar{\mu} = -2\cos(k) = -2\cos(p) \cosh(l) \), \( \Delta_a = 2\sin(p) \sinh(l) \) yields for the Andreev reflection scattering probability

\[
R_a = (v_q/v_{\bar{q}})|r_a|^2 = \frac{2(1 - (\mu^2/4\gamma^2))}{1 - (\mu^2/4\gamma^2) + (\Delta_a/4\gamma^2)^2 + \sqrt{(1 + (\mu^2 + \Delta_a/4\gamma^2))^2 - \mu^2}}.
\]

(A7)
It is clear from the above equation that in the limit \( \Delta_o \to 0 \), the Andreev scattering probability \( R_a \to 1 \), which amounts to the so-called Andreev’s approximation largely used in a number of theoretical treatments. To close this appendix, it is useful to make connection with the continuum limit, which is most conveniently obtained from eqs. (A3-A6). To make this connection, we restore the lattice step \( a \) in all expressions involving the momenta, i.e., \( k \to ka \) and take the limit \( a \to 0 \), \( \gamma \to \infty \), such that \( \frac{\gamma a^2}{\hbar^2} = \frac{\gamma a^2}{2} \). This leads to

\[
 r_o = \frac{[u\bar{v}(q + \bar{q})(k - \bar{k}) - \bar{u}v(q - \bar{k})(\bar{q} + k)]}{d} \tag{A8}
\]

\[
 r_a = \frac{2\bar{v}k(\bar{q} + \bar{k})}{d} \tag{A9}
\]

\[
 t_o = \frac{2\bar{v}k(q + \bar{q})}{d} \tag{A10}
\]

\[
 t_a = \frac{-2\bar{v}k(q - \bar{k})}{d} \tag{A11}
\]

where

\[
 d = u\bar{v}(k + \bar{k})(q + \bar{q}) - \bar{u}v(q - \bar{k})(k - \bar{q}).
\]

In the limit \( (\Delta_o/\mu \ll 1) \), Andreev’s approximation of ignoring both Andreev transmission and normal reflection is valid and one obtains \[14\]

\[
 R_a = 1, \quad E < \Delta_o \tag{A12}
\]

and

\[
 R_a = \frac{E - \sqrt{E^2 - \Delta_o^2}}{E + \sqrt{E^2 - \Delta_o^2}}, \quad E > \Delta_o \tag{A13}
\]

In contrast, at zero energy, without invoking Andreev’s approximation, one obtains the exact result

\[
 R_a = \frac{2\mu}{\mu + \sqrt{\mu^2 + \Delta_o^2}} \tag{A14}
\]

with \( R_o = 1 - R_a \). The above equation clearly shows how Andreev’s approximation breaks-down in the large \( \Delta_o \) regime.

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We notice, however, that self-consistency in the presence of d-wave pairing in hybrid superconducting structures has been addressed explicitly by Ch. Bruder, Phys. Rev. B 41, 4017 (1990), and by Y. S. Barash, A. V. Galaktionov, and A. D. Zaikin, Phys. Rev. B 52, 665 (1995). See also the discussion in section V.

According to the analysis by Y. S. Barash et al. (see Ref. [30]) when one of the crystal axes of the d-wave superconducting structure coincides with the equilibrium value far from the interface itself.

Figure 1. Electrical conductance (thick solid line), Andreev reflection (solid line), normal transmission (dashed line). Normal reflection and Andreev transmission are negligible on this scale. Parameters used are: (a) width \( M = 10 \), number of open channels \( N = 9 \); (b) width \( M = 1000 \), number of open channels \( N = 857 \); in all cases \( \mu = -0.2\gamma \), \( \Delta_{s} = 0.01\gamma \), barrier height \( U = 0 \).

Figure 2. Electrical conductance (thick solid line), Andreev reflection (solid line), normal transmission (dashed line), figures (a) and (c); normal reflection (solid line), Andreev transmission (dashed line), figures (b) and (d). Parameters used: (a) and (b) width \( M = 10 \), number of open channels \( N = 9 \); (c) and (d) width \( M = 1000 \), number of open channels \( N = 857 \); in all cases \( \mu = -0.2\gamma \), \( \Delta_{s} = 0.01\gamma \), barrier height \( U = 2\gamma \). Values are normalised to the normal state conductance in order to compare with the case of no potential barrier at the interface.

Figure 3. Schematic picture of an N-I-S structure, considered in the text. Parameters used are: width \( M = 10 \), length of the diffusive region \( L_{\text{diff}} = 30 \), length of the tunnel junction \( L_{\text{run}} = 1 \), disorder width \( W = 1 \), \( \mu = 0 \), number of open channels \( N = 10 \).

Figure 4. Total conductance as function of energy in the low tunnel junction conductance regime. s-wave: dotted line; d-wave: solid line. \( G_{\text{run}} = 0.4 \) in units of \( 2e^{2}/h \). The conductance is normalised to the normal state conductance \( G_{0} \). The two insets (left d-wave and right s-wave) show the conductance behaviour on a more extended energy range. The value of the superconducting order parameter in the s-wave case is \( \Delta_{s} = 0.1\gamma \).

Figure 5. Total conductance as function of the energy in high tunnel junction conductance regime. s-wave: dotted line; d-wave: solid line. \( G_{\text{run}} = 3 \) in units of \( 2e^{2}/h \). The conductance is normalised to the normal state conductance \( G_{0} \). The two insets (left d-wave and right s-wave) show the conductance behaviour on a more extended energy range. The value of the superconducting order parameter in the s-wave case is \( \Delta_{s} = 0.1\gamma \).

Figure 6. The figure shows results for a system with an insulating barrier placed at one end of the interferometer (See the structure displayed in the top right corner). The solid line (dashed line) indicates results for the conductance with aligned (not aligned) superconducting islands, for a d-wave interferometer. The inset shows the corresponding result for an s-wave interferometer.
FIG. 7. The figure shows the conductance for a system without a barrier but when the interferometer is dirty (See the structure displayed in the top right corner). The solid line (dashed line) indicates results for the conductance with aligned (not aligned) superconducting islands, for a d-wave interferometer. The inset shows the corresponding result for an s-wave interferometer.