Microwave Diagnostics of Ultracold Neutral Plasma

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Abstract

The microwave method is suggested to diagnose the ultracold neutral plasma. Based on the calculations of the dipole radiation, we derived the microwave scattering cross section of the ultracold neutral plasma. The significant results indicate that we can diagnose the total electron number and recombination rate by this method.

Key words: ultracold neutral plasma, microwave diagnostics

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1 INTRODUCTION

The ultracold neutral plasma (UNP) is first generated by photoionizing a ultracold gas[1], in which the typical electron and ion temperature are around $1 \sim 1000K$ and 1K respectively. UNP extends greatly the boundaries of classical neutral plasma physics and has been widely studied in recent years[2][3]. The diagnostics methods of UNP are mainly developed from some well-defined technique of optic probes, such as laser induced fluorescence imaging[4],

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optical absorption imaging\textsuperscript{5} and even recombination fluorescence\textsuperscript{6}. The optic probes can excite the fluorescence or be absorbed in the case of ions and Rydberg atoms. However, free electrons in UNP can not respond to the laser beam except the Thomason scattering which is too weak to be measured in the current techniques. Furthermore, most traditional plasma diagnostics are not accessible due the size-limited UNP. In this paper, we suggest a new method of using microwave radiation for the study of UNP. Using this method, we can measure the amount of electrons $N_e$ and the recombination rate of plasma, which is extremely important in the research of UNP.

2 DISCUSSION

Normally, when the microwave wavelength $\lambda$ is much smaller than the size plasma size $L$, microwave-based diagnostics such as interferometry and reflectometry have found very broad application in classical plasma diagnostics. However, the UNP size limited by the beamwidth of the cooling laser is usually very small around the range of $mm$. In the small plasma objects situation $\lambda/L \gg 1$, those microwave diagnostics based on propagation, absorption or reflection fail to work. Shneider and Miles develop the theory in the case of small plasma objects by measuring the radiation scattered by the effective oscillating plasma dipole\textsuperscript{7} [8]. In their work \textsuperscript{7}, the plasma is static uniform plasma column, and the microwave frequency used is smaller than plasma frequency $\omega < \omega_{pe}$. However, UNP is an expanding spherical plasma cloud with radial inhomogeneity, the density profile of UNP decay radically quickly in the few of $mm$. There is still no any discussion about microwave diagnostics on this special plasma so far. In this paper we investigate the dipole radiation
of UNP in an incident microwave as the physical scheme shown in Fig[1].

From linearized electron motion equation, we can easily derive the high frequency conductivity $\sigma$ and dielectric constant $\epsilon$ of plasma[9]:

$$\sigma = \frac{i \omega_{pe}^2}{4\pi \omega}$$  \hspace{1cm} (1)

$$\epsilon = 1 + \frac{i 4\pi \sigma}{\omega} = 1 - \frac{\omega_{pe}^2}{\omega^2}$$ \hspace{1cm} (2)

where $\omega_{pe} = \sqrt{n_e e^2/m_e \epsilon_0}$ is the plasma frequency, $m_e$ is the electron mass and $n_e$ is electron density.

We calculate the dipole of an uniform dielectric ball with the radius $r$ and dielectric constant epsilon $\epsilon$ in the first step. The dielectric ball responds to the incident electric field $\vec{E}$ and the electric dipole is induced. we can get that outside the sphere the potential is equivalen to that of the applied field plus the field of a point electric dipole $\vec{p}_{uniform}$, so the equivalent dipole of the dielectric ball is

$$\vec{p}_{uniform} = \epsilon - 1 + 2 4\pi \epsilon_0 r^3 \vec{E}$$ \hspace{1cm} (3)

Next, we consider a thin spherical shell with spherical shell of radius $r$, thickness $dr$ and dielectric constant $\epsilon(r)$, From Eq[3], the corresponding dipole of the shell is

$$d\vec{p}(r) = \frac{\epsilon(r) - 1}{\epsilon(r) + 2} \frac{12\pi \epsilon_0 r^2 dr \vec{E}}{12\pi \epsilon_0 r^2 dr \vec{E}}$$ \hspace{1cm} (4)

So the integral on the Eq[4] along the radius yields the equivalent dipole of the UNP ball
\[ \vec{p} = \int d\vec{p}(r) = \int \frac{-\omega_{pe}(r)^2}{3 - \frac{\omega_{pe}(r)^2}{\omega^2}} 12\pi\epsilon_0 r^2 dr \vec{E} \]  

(5)

Because the size of UNP is much less than the wavelength of microwave, we can neglect the field variance across the UNP ball. We set \( \vec{E} = E_0 e_r \exp i\omega t \), the equivalent dipole of the UNP ball will vibrate in the same frequency \( \omega \) of the incident electric field. The oscillation mainly comes from the oscillation of \( \vec{E} \), so we get total dipole radiation power in space

\[ P = \frac{|\vec{p}|^2}{12\pi\epsilon_0 c^3} = \frac{12\pi\epsilon_0 E_0^2}{c^3} \left[ \int \frac{-\omega_{pe}(r)^2}{3 - \frac{\omega_{pe}(r)^2}{\omega^2}} r^2 dr \right]^2 \]  

(6)

and the effective scattering cross section

\[ \sigma(\omega) = \frac{P}{0.5\epsilon_0 c E_0^2} = \frac{24\pi}{c^4} \left[ \int \frac{-\omega_{pe}(r)^2}{3 - \frac{\omega_{pe}(r)^2}{\omega^2}} r^2 dr \right]^2 \]  

(7)

\( \omega_{pe}(r) \) can be written as \( \omega_{pe0} f_e(r) \), where \( \omega_{pe0} \) is plasma frequency at the center of UNP and \( f_e(r) \) is the radial density profile.

In Eq(5) and Eq(7), the dipole radiation power and the effective scattering cross section tend to a constant when the microwave frequency is much greater than the plasma frequency. But it is worthy to note the corresponding constant reflects the crucial properties of plasma. The UNP has a typical guassian-like radial density profile \( f_e(r) = \exp(-r^2/2r_0^2) \). In the UNP situation, Fig2 and Fig3 illuminate the constant tendency when the ratio of microwave and plasma frequency is greater than 5 in three different density cases. In the two figures, the dipole radiation power \( P(\omega) \) and the effective scattering cross section \( \sigma(\omega) \) are scaled vertically by \( \omega_{pe0}^4 E_0^2 \) and \( \omega_{pe0}^4 \) respectively. The overlapping horizontal lines at large frequency ratio indicate the radiation power and cross section never change while \( \omega \) increases at large \( \omega \). The classical \( \omega^4 \)
dependence of dipole radiation is not satisfied here. Specially, when the microwave frequency is large enough it shows the frequency independence as we explained above. The underlying reason is the frequency dependence of the equivalent dipole instead of the assumption of constant dipole in the classical description.

For large $\omega/\omega_{pe}$, we can ignore the term of $\omega_{pe}^2(r)/\omega^2$ in Eq.6 and Eq.7 and get the dipole radiation power

$$P = \frac{4\pi\varepsilon_0 E_0^2}{3c^3} \left[ \int \omega_{pe}^2(r)r^2dr \right]^2 = \frac{4\pi e^4 E_0^2}{3\varepsilon_0 m_e^2 c^3} \left[ \int n_e(r)r^2dr \right]^2 = \frac{e^4 N_e^2 E_0^2}{12\pi\varepsilon_0 m_e^2 c^3} \quad (8)$$

the effective scattering cross section

$$\sigma = \frac{8\pi}{3c^4} \left[ \int \omega_{pe}^2(r)r^2dr \right]^2 = \frac{8\pi e^4}{3c^4 m_e^2 \varepsilon_0^2} \left[ \int n_e(r)r^2dr \right]^2 = \frac{e^4 N_e^2}{6\pi c^4 m_e^2 \varepsilon_0^2} \quad (9)$$

and the radiation electric field $E_r$ at distance $R$ and direction $\phi$

$$E_r(R, \phi) = \frac{\sin \phi}{2R} \sqrt{\frac{3P}{\pi \varepsilon_0 c}} = \frac{e^2 N_e}{4\pi \varepsilon_0 c^2 R m_e} E_0 \sin \phi \quad (10)$$

where $N_e = \int n_e(r) 4\pi r^2dr$ is the total number of electron.

Eqns 8-10 demonstrate the formula of UNP dipole radiation is free from the the frequency $\omega$ or plasma density profile $f_e(r)$, but only dependent of total electron number $N_e$. Though we still don’t break the resolution limitation on the microwave diagnostics technology, it is significant to give a clue of the space integral on density i.e. $N_e$. Moreover, the recombination rate can be calculated based the $N_e$ measurement.

The ultracold neutral plasma is produced in very low electron temperature. In this range of temperature, the three body recombination(TBR) dominates over
the electron and ion recombination. Through the TBR, one ion recombines with two electrons into a Rydberg atom and an leftover electron with the extra energy. The classical TBR theory predicts the recombination rate per ion is $K_{TBR} \approx 3.8 \times 10^{-9} T_e^{-9/2} n_e^2 \text{s}^{-1}$, where $T_e$ is given in $K$ and the density $n_e$ in $cm^{-3}$, so the TBR rate varies with temperature as $T_e^{-9/2}$ and is very fast at ultracold temperatures\cite{2}. The TBR effect is very important in UNP, because it is the main heating mechanism for the electron at low $T_e$ \cite{10}. It has attracted wide interest and much controversy. Microwave diagnostics of UNP may offer a new way to detect the recombination rate of UNP.

As one example of the application, we consider an expanding UNP with $r_0 = 2mm$ and initial center density $n_{e0} = 5 \times 10^9 cm^{-3}$. These two parameters are determined by the cool laser and are typical in the experiment. Though in general case the plasma cloud expands with characteristic expansion time $\tau = \sqrt{m_i r_0^2(0)/k_b[T_e(0) + T_i(0)]}$ after creation\cite{11}, the total electron number $N_e$ would not been changed during the expansion if there was no any recombination. So when we calculate the total electron number $N_e(t)$, we need consider only TBR without expansion. We use a simple formulations for TBR $K_{TBR} \approx 3.8 \times 10^{-9} T_e^{-9/2} n_e^2 \text{s}^{-1}$ (There is some other discussion about TBR \cite{6}, but the details of TBR mechanism are out of this paper’s scope). So we can get $N_e(t) = N_e(0) \prod (1 - K_{TBR} \Delta t)$, and $T_e(t) = T_e(0) \prod (1 + K_{TBR} \Delta t)$. Fig4 shows the typical time evolution of $N_e(t)$ at three distinct initial electron temperatures. Clearly $N_e$ falls too sharply in the initial short time when initial electron temperature $T_e(0) = 1K$, it is hard to get enough temporal resolution. So in the next figure, we only plot curves in $T_e(0) = 10K$ and $20K$ condition.

Finally, we give a numerical case. The initial center density $n_{e0} = 5 \times 10^9 cm^{-3}$,
so the plasma frequency at UNP center \( f_{pe0} \approx 0.6 \text{GHz} \). If incident microwave frequency \( f \) is greater than \( 5f_{pe0} = 3 \text{GHz} \), we can directly use Eqns.8-10 to estimate the equivalent dipole of the UNP cloud unrelated to \( f \). Fig.5 shows the results of effective cross section, radiation power and radiation electric filed when the incident microwave is \( I_0 = 10W/m^2 \), the radiation electric filed \( E_r \) in the fig is calculated at \( \phi = \pi/3 \) and \( r = 0.2m \).

3 SUMMARY

Our calculations indicate that the dipole radiations of UNP do not depend on specific density profile \( n_e(r) \) and incident frequency \( \omega \) when \( \omega \gg \omega_{pe0} \), but on the total electron number \( N_e \). We suggest that the microwave radiation from UNP may offer a new way to get the information of recombination in UNP, or other expanding inhomogeneous plasma in similar.

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**Figure captions**

Fig.1 The physical scheme of the dipole radiation of UNP in an incident microwave field.

Fig.2 Dipole radiation power to $\omega_{pe}^{4}E_{0}^{2}$ ratios at different density $n_{e0} = 5 \times 10^{8} cm^{-3}, 5 \times 10^{9} cm^{-3}, 5 \times 10^{10} cm^{-3}$.

Fig.3 Effective scattering cross section to $\omega_{pe}^{4}$ ratios at different density $n_{e0} = 5 \times 10^{8} cm^{-3}, 5 \times 10^{9} cm^{-3}, 5 \times 10^{10} cm^{-3}$.

Fig.4 The evolution total electron number $N_{e}$ at different initial $T_{e}$. $N_{e}$ decrease more rapidly with lower $T_{e}$.

Fig.5 Effective cross section, radiation power and radiation electric filed when the intensity of the incident microwave equals $10W/m^{2}$.
Fig. 1. The physical scheme of the dipole radiation of UNP in an incident microwave field

![Physical scheme of dipole radiation](image)

\[ \frac{P}{\omega} = \frac{\omega^4}{\omega_{pe0}^2} \times 10^{8} \text{ cm}^{-3}, 5 \times 10^{9} \text{ cm}^{-3}, 5 \times 10^{10} \text{ cm}^{-3} \]

Fig. 2. Dipole radiation power to $\omega_{pe0}^4 E_0^2$ ratios at different density $n_{e0} = 5 \times 10^8 \text{ cm}^{-3}, 5 \times 10^9 \text{ cm}^{-3}, 5 \times 10^{10} \text{ cm}^{-3}$

![Dipole radiation power graph](image)

Fig. 3. Effective scattering cross section to $\omega_{pe0}^4$ ratios at different density $n_{e0} = 5 \times 10^8 \text{ cm}^{-3}, 5 \times 10^9 \text{ cm}^{-3}, 5 \times 10^{10} \text{ cm}^{-3}$

![Effective scattering cross section graph](image)
Fig. 4. The evolution total electron number $N_e$ at different initial $T_e$. $N_e$ decrease more rapidly with lower $T_e$.

Fig. 5. Effective cross section, radiation power and radiation electric filed when the intensity of the incident microwave equals $10W/m^2$. 