Classification of $osp(2|2)$ Lie super-bialgebras

Cezary Juszczak
Institute for Theoretical Physics, University of Wroclaw
pl. M. Borna 9, Wroclaw, Poland

Abstract
The co-Lie structures compatible with the $osp(2|2)$ Lie super-algebra structure are investigated and found to be all of coboundary type. The corresponding classical $r$-matrices are classified into several disjoint families. The $osp(1|2) \oplus u(1)$ Lie super-bialgebras are also classified.

Introduction
The need of classification of Lie bialgebras [1] comes from their close relation with $q$-deformations of universal enveloping algebras in the Drinfeld sense. To each such deformation there corresponds a Lie bialgebra which may be recovered from the first order of the deformation of the coproduct.

It has also been shown [4] that each Lie bialgebra admits quantization. So the classification of Lie bialgebras can be seen as the first step in classification of quantum algebras.

Along these lines several efforts (see e.g. [6, 7, 8, 9] to list only a few) have been undertaken in order to classify those Hopf algebras which can be of importance in physics.

The $Osp(2|2)$ super-group is a subgroup of two-dimensional $N = 2$ super-conformal symmetry which plays an important role in string theory. In [10] the correlation functions of $N = 2$ super-conformal field theory were found by using the $Osp(2|2)$ symmetry group. Lattice models based on $U_q(osp(2|2))$ symmetry were constructed in [11], where also new solutions to the graded Yang-Baxter equation were found.
A few examples of quantum deformations of $osp(2|2)$ [12, 13, 14, 15] were given so far and it became evident that their classification would be of much value.

In this paper we perform a complete classification of Lie super-bialgebras $osp(2|2)$ based on the brut-force computer approach combined with careful identification of equivalent structures. We also classify of the $u(1) \oplus osp(1|2)$ Lie super-bialgebras. $u(1) \oplus osp(1|2)$ is the simplest central extension of the $osp(1|2)$ subalgebra and is similar to $osp(2|2)$ in the fact that it containes $gl(2)$ and $osp(1|2)$ as subalgebras. In both cases all the obtained structures are coboundary, allowing for a brief exposition of the results in the form of list of classical $r$-matrices.

1 Lie super-bialgebra $osp(2|2)$

The $osp(2|2)$ Lie superalgebra $G = G_0 \oplus G_1$ is spanned by the generators $(g_1, \ldots, g_8) = (H, X_+, X_-, B, V_+, V_-, W_+, W_-)$, where $H, X_\pm, B$ span the subspace $G_0$ of grade 0, and $V_\pm, W_\pm$ span the subspace $G_1$ of grade 1. We refer to the elements of $G_0$ and $G_1$ as bosons and fermions respectively. The generators fulfill the following relations:

\[
\begin{align*}
[H, X_\pm] &= \pm X_\pm, & [X_+, X_-] &= -2H, \\
[H, B] &= 0, & [X_\pm, B] &= 0, \\
[H, V_\pm] &= \pm \frac{1}{2} V_\pm, & [H, W_\pm] &= \pm \frac{1}{2} W_\pm, \\
[B, V_\pm] &= \frac{1}{2} V_\pm, & [B, W_\pm] &= -\frac{1}{2} W_\pm, \\
[X_\pm, V_\mp] &= 0, & [X_\pm, W_\mp] &= 0, \\
[X_\pm, V_\pm] &= \mp V_\pm, & [X_\pm, W_\pm] &= \mp W_\pm, \\
\{V_\pm, V_\pm\} &= \{V_\pm, V_\pm\} = \{W_\pm, W_\pm\} = \{W_\pm, W_\pm\} = 0, \\
\{V_+, W_-\} &= H - B, & \{W_+, V_-\} &= H + B, \\
\{V_\pm, W_\pm\} &= X_\pm.
\end{align*}
\]

From the last three relations it is evident that the superalgebra is generated by its fermionic sector $G_1$.

For the approach taken in the present paper it is most convenient to use the definition of Lie super-bialgebra in terms of the structure constants. Thus we define
Definition 1  Lie super-bialgebra \([3]\) is a vector space \(G\) with two linear mappings:

\[
[\cdot, \cdot] : G \otimes G \ni g_i \otimes g_j \mapsto [g_i, g_j] = c_{ij}^k g_k \in G , \tag{2}
\]

\[
\delta : G \ni g_i \mapsto \delta(g_i) = f_{ij}^k g_j \otimes g_k \in G \otimes G . \tag{3}
\]

\([\cdot, \cdot]\) is a Lie bracket on \(G\) which means that its structure constants \(c_{ij}^k\) satisfy the relations:

\[
c_{ij}^k = 0 \text{ if grade}(i) + \text{grade}(j) \not\equiv \text{grade}(k) \pmod{2} \tag{4}
\]

\[
c_{ij}^k = -z(i, j)c_{ji}^k \tag{5}
\]

\[
c_{ij}^k c_{kl}^m z(i, l) + c_{jl}^k c_{ki}^m z(j, i) + c_{li}^k c_{kj}^m z(l, j) = 0 \tag{6}
\]

where

\[
z(i, j) \equiv (-1)^{\text{grade}(g_i) \cdot \text{grade}(g_j)} \tag{7}
\]

\(\delta^*\) defines Lie bracket on the dual space \(G^*\) so its structure constants \(f_{ij}^k\) fulfill similar relations:

\[
f_{kj}^i = 0 \text{ if grade}(i) + \text{grade}(j) \not\equiv \text{grade}(k) \pmod{2} \tag{8}
\]

\[
f_{kj}^i = -z(i, j)f_{jk}^i \tag{9}
\]

\[
f_{ij}^k f_{jk}^l z(k, m) + f_{ij}^l f_{jk}^m z(l, k) + f_{ij}^m f_{kl}^j z(m, l) = 0 . \tag{10}
\]

Moreover, the two mappings need to be compatible:

\[
c_{ij}^k f_{kl}^m = f_{ik}^l c_{jk}^m + c_{kj}^l f_{ki}^m z(m, j) + c_{jm}^l f_{ij}^m + f_{ij}^k c_{ik}^m z(i, l) \tag{11}
\]

If there exists an element \(r = r_{ij}^k g_i \otimes g_j \in G \otimes G\) such that :

\[
\delta(g_i) = [r, g_i \otimes 1 + 1 \otimes g_i] \tag{12}
\]

or, in terms of the structure constants,

\[
f_{ij}^k = r_{jm}^i c_{mi}^k - c_{im}^j r_{mk}^i \tag{13}
\]

then the \(G\) is called coboundary Lie super-bialgebra. It is easy to see that the graded antisymmetric part of \(r\) defined by \(\hat{r}_{ij} = (r_{ij} - z(i, j)r_{ji})/2\) yields the same \(f_{ij}^k\) so we will assume that \(r \in G \wedge G\).

\[
r_{ij} = -r_{ji} z(i, j) . \tag{14}
\]
Similarly, it can be shown that projection of $r$ on the $G_0 \wedge G_1$ subspace of $G \wedge G$ cannot influence $f_{ij}^k$ without violating the condition (8). After subtracting it from $r$ we obtain even $r$-matrix $r \in G_0 \wedge G_0 \oplus G_1 \wedge G_1$, i.e.

$$r^{ij} = 0 \text{ if grade}(g_i) \neq \text{grade}(g_j).$$

We start with the Lie bracket $[\cdot, \cdot]$ given by (1) and look for all the co-brackets $\delta$ which are compatible with it.

The commutation relations (1) fix the structure constants $c_{ijk}$. Then our task is to find all the $f_{ijk}$ that fulfill (8), (9), (10), and (11). To this end we use a computer and a symbolic algebra program REDUCE. We use (8), (9) just to reduce the number of unknowns, then we solve the set of linear equations (11) coming from the cocycle condition. At this point we are able to obtain a 16-parameter family of solutions and by solving the relations (13) are able to find the corresponding classical $r$-matrix.

This leads to the conclusion that all the solutions are coboundary. It is well known [1] that the Lie bialgebras of simple Lie algebras are all of coboundary type. Since the $osp(2|2)$ is a simple Lie superalgebra the fact that all its bialgebras are coboundary can probably be justified from the cohomological point of view.

We substitute the results into the quadratic equations (10) representing the co-Jacobi identity. Solving them yields 22 solutions, each parametrized by up to 6 complex numbers. Substituting these solutions into the generic $r$-matrix we obtain 22 families of classical $r$-matrices.

We consider two coalgebra structures $\delta$ (and their corresponding $r$-matrices) equivalent if they differ only by a linear transformation of the generators which preserves the algebra commutation relations (1).

In the next Section we prove that these transformations form a group $GL(2) \oplus \mathbb{Z}_2$. In Section 3 we use this 4-parameter symmetry to obtain families of nonequivalent $r$-matrices parametrized by at most 2 complex numbers.

### 2 Automorphisms of the algebra

We consider two coalgebra structures $\delta$ equivalent if they differ only by a change of basis. As a change of the basis we allow only such linear transformations of the generators that: (a) parity is preserved, and (b) algebra structure constants $c_{ij}^k$ are unaffected. (i.e. automorphisms of the Lie super-algebra).
Since the algebra is generated by the fermions, every such transformation is generated by a transformation within the fermionic sector $G_1$ which in turn can be identified as a nonsingular $4 \times 4$ matrix $A_F$:

$$
\begin{pmatrix}
\tilde{V}_+ \\
\tilde{V}_- \\
\tilde{W}_+ \\
\tilde{W}_-
\end{pmatrix} = A_F \cdot 
\begin{pmatrix}
V_+ \\
V_- \\
W_+ \\
W_-
\end{pmatrix}
$$

(16)

such that $\tilde{V}_+, \tilde{V}_-, \tilde{W}_+, \tilde{W}_-$ fulfill the relations (1).

**Statement 1** The matrix $A_F$ must be either block diagonal or block anti-diagonal i.e. it is of the form

$$
A_1 = \begin{pmatrix}
A_{VV} & 0 \\
0 & A_{WW}
\end{pmatrix} \text{ or } A_2 = \begin{pmatrix}
0 & A_{VW} \\
A_{WV} & 0
\end{pmatrix}.
$$

(17)

**Proof:**

Let us assume the following general expression for $\tilde{V}_+$

$$\tilde{V}_+ = aV_+ + bV_- + cW_+ + dW_-.$$  
(18)

Then

$$
0 = \{\tilde{V}_+, \tilde{V}_+\}/2
= \{aV_+ + bV_- + cW_+ + dW_-, aV_+ + bV_- + cW_+ + dW_-\}/2
= acX_+ + (ad + bc)H + (-ad + bc)B + bdX_-.
$$

(19)

The condition $0 = ac = ad + bc = -ad + bc = bd$ has two solutions ($a = b = 0$ or $c = d = 0$) which shows that $\tilde{V}_+$ is either a combination of $V$'s or a combination of $W$'s. Similar reasoning is valid for $\tilde{V}_-, \tilde{W}_+, \text{ and } \tilde{W}_-$.

Now we show that both $\tilde{V}_+$ and $\tilde{V}_-$ belong to the same sector ($V$ or $W$).

Indeed, assumption to the contrary, i.e. that $\tilde{V}_+ = aV_+ + bV_-, \tilde{V}_- = cW_+ + dW_-$ would imply:

$$
0 = \{\tilde{V}_+, \tilde{V}_-\}
= \{aV_+ + bV_-, cW_+ + dW_-\}
= acX_+ + (ad + bc)H + (-ad + bc)B + bdX_-.
$$

(20)

Then $0 = ac = ad + bc = -ad + bc = bd$ with the only two solutions being ($a = b = 0$ or $c = d = 0$) would mean that either $\tilde{V}_+ = 0$ or $\tilde{V}_- = 0$. This would contradict our assumption that $A_F$ is nonsingular.
Now when we have proved that $\tilde{V}_+, \tilde{V}_-$ belong to the same sector we see that $\tilde{W}_+$ and $\tilde{W}_-$ must belong to the other one for the $A_F$ to be nonsingular. The conclusion is that the matrix $A_F$ is block diagonal or block antidiagonal. $\square$

**Comment:** Every block antidiagonal matrix $A_2$ can be written in the form:

$$A_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \cdot A_1$$

where $A_1$ is block diagonal.

**Statement 2** The diagonal blocks $A_{VV}$ and $A_{WW}$ of $A_1$ are proportional to each other:

$$A_{VV} = k \cdot A_{WW}$$  \hspace{1cm} (21)

where $k = \det A_{VV}$.

**Proof:**

Assume that

$$A_{VV} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad A_{WW} = \begin{pmatrix} x & y \\ z & t \end{pmatrix},$$  \hspace{1cm} (22)

or equivalently:

$$\tilde{V}_+ = aV_+ + bV_- \, , \quad \tilde{W}_+ = xW_+ + yW_- \, ,$$  \hspace{1cm} (23)

$$\tilde{V}_- = cV_+ + dV_- \, , \quad \tilde{W}_- = zW_+ + tW_- \, .$$

Then

$$\tilde{X}_+ = \{\tilde{V}_+, \tilde{W}_+\} = axX_+ + (bx + ay)H + (bx - ay)B + byX_- \, ,$$  \hspace{1cm} (24)

$$\tilde{X}_- = \{\tilde{V}_-, \tilde{W}_-\} = czX_+ + (dz + ct)H + (dz - ct)B + dtX_- \, .$$  \hspace{1cm} (25)

Inserting (23)-(25) into

$$[\tilde{X}_+, \tilde{V}_-] = -\tilde{V}_+, \quad [\tilde{X}_-, \tilde{V}_+] = \tilde{V}_-,$$  \hspace{1cm} (26)

we obtain

$$(ad - bc)(-xV_+ - yV_-) = -aV_+ - bV_-,$$  \hspace{1cm} (ad - bc)( zV_+ + tV_-) = cV_+ + dV_-.$$  \hspace{1cm} (27)$$
from which follows
\[
\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (ad - bc) \begin{pmatrix} x & y \\ z & t \end{pmatrix}.
\]
(28)

The remaining relations (1) do not lead to further constraints on the numbers \(a, b, c, d\). Altogether, we have just shown that

**Statement 3** The matrix \(A_F\) has the following general form

\[
A_F = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}^m \begin{pmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & a/k & b/k \\ 0 & 0 & c/k & d/k \end{pmatrix}, \quad m = 0, 1
\]
(29)

where \(a, b, c, d\) are arbitrary complex numbers such that

\[
k \equiv \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} \neq 0.
\]
(30)

The action of the above symmetry on bosons is defined by the matrix \(A_B\)

\[
\begin{pmatrix} \tilde{H} \\ \tilde{X}_+ \\ \tilde{X}_- \\ \tilde{B} \end{pmatrix} = A_B \cdot \begin{pmatrix} H \\ X_+ \\ X_- \\ B \end{pmatrix}
\]
(31)

where

\[
A_B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & (-1)^m \end{pmatrix} k^{-1} \begin{pmatrix} ac + bc & ac & bd & 0 \\ 2ab & a^2 & b^2 & 0 \\ 2cd & c^2 & d^2 & 0 \\ 0 & 0 & 0 & k \end{pmatrix}
\]
(32)

We define block diagonal matrix \(A = \text{diag}(A_B, A_F)\) such that

\[
\tilde{g} = A \cdot \bar{g}
\]
(33)

where \(\tilde{g}^T = (g_1, \ldots, g_8) = (H, X_+, X_-, B, V_+, V_-, W_+, W_-)\). Since all the obtained coalgebra structures are coboundary, which means that the co-Lie bracket \(\delta\) is defined in terms of classical \(r\)-matrix:

\[
\delta(x) = [r, x \otimes 1 + 1 \otimes x],
\]
(34)
it is useful to know the action of the symmetry $A$ on classical $r$-matrices.

$$r = r^{ij} g_i \otimes g_j. \quad (35)$$

From

$$r = r^{ij} g_i \otimes g_j = \tilde{r}^{ij} \tilde{g}_i \otimes \tilde{g}_j = \tilde{r}^{kl} A^i_k g_i \otimes A^j_l g_j = \tilde{r}^{kl} A^i_k A^j_l g_i \otimes g_j, \quad (36)$$

we see that

$$r = A^T \tilde{r} A \quad \text{and} \quad \tilde{r} = (A^{-1})^T r (A^{-1}). \quad (37)$$

Because we require the $r$-matrix to be even (35) is equivalent to

$$r = \sum_{i,j=1}^4 r^{ij} g_i \otimes g_j + \sum_{i,j=5}^8 r^{ij} g_i \otimes g_j = r_B + r_F, \quad (38)$$

where we define

$$(r_B)_{ij} = r^{ij}, \quad (r_F)_{ij} = r^{i+4\, j+4}, \quad \text{for } i, j = 1, 2, 3, 4. \quad (39)$$

Then it follows from (37) that:

$$\tilde{r}_F = (A_F^{-1})^T r_F (A_F^{-1}), \quad \tilde{r}_B = (A_B^{-1})^T r_B (A_B^{-1}). \quad (40)$$

We will use this symmetry to identify which classes of $r$-matrices differ only by a change of basis. Furthermore, in most cases we will be able to eliminate some parameters from $r$ by an appropriate choice of parameters $a$, $b$, $c$, $d$ of $A_F$.

$$r_F = \begin{pmatrix} r_{VV} & r_{VW} \\ r_{WV} & r_{WW} \end{pmatrix}$$

where $r_{VV} = r_{VV}^T$, $r_{VW} = r_{VW}^T$ and $r_{WW} = r_{WW}^T$.

The action of the symmetry $S$ consists in exchanging $r_{VV}$ with $r_{WW}$ and $r_{WV}$ with $r_{VW}$ or, in other words, exchanging $V$ with $W$.

The action of symmetry $A$ on $r_{WW}$ and $r_{VV}$ is the following $\tilde{r}_{VV} = A^T r_{VV} A$, $\tilde{r}_{WW} = A^T r_{WW} A (\det A)^{-2}$. Since they are symmetric we may use the Sylvester theorem to prove that for example $r_{VV}$ can be made equal diag$(1,1)$, diag$(1,0)$ or 0 depending on its rank. Once one of these forms is achieved we may use the remaining symmetry to simplify $r_{VV}$ and then the remaining to simplify $r_{WW}$.
3 Details of the equivalence considerations

Our strategy of bringing the solutions to the ‘canonical’ form by using some change of basis can be summarized in the following steps:

1. If $r_{VV}$ has lower rank than $r_{WW}$ we apply the symmetry $S$ which makes them interchange. Now rank $r_{VV} \geq$ rank $r_{WW}$. It also turns out that rank $r_{WW} < 2$ after this step.

2. We can diagonalize $r_{WW}$ so that we obtain

$$r_{WW} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ or } r_{WW} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

depending on the initial rank of $r_{WW}$.

3. Now we try to simplify $r_{VV}$ while preserving the form of $r_{WW}$. Depending on whether $r_{WW}$ vanishes or not we have either the full $GL(2)$ symmetry at our disposal or just $A$ of the form

$$A = \begin{pmatrix} 1 & b \\ 0 & d \end{pmatrix}$$

(41)

In general the matrix $r_{VV}$ does not have to be symmetric nor antisymmetric. However, it almost always is. Three general possibilities occur in this case:

(a) When $r_{WW}$ is antisymmetric it is invariant with respect to $A$. We just proceed to simplify $r_{VV}$.

(b) When $r_{WW}$ is symmetric we make sure it is antidiagonal i.e. of the form $\begin{pmatrix} 0 & z \\ z & 0 \end{pmatrix}$ or diagonal $\begin{pmatrix} x & 0 \\ 0 & 0 \end{pmatrix}$.

(c) In other cases we easily obtain $r_{VV} = \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix}$

4. We use the remaining symmetry to simplify the part $r_{VV}$. Here what we can achieve depends on the previous steps and on the rank of $r_{VV}$.

5. We can use the WV scaling $(A_{VV} = \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix})$ to scale $r_{VV}$ with respect to $r_{WW}$. 

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6. If we still have any freedom we try to make simpler the bosonic part. The main rule is to get rid of $X_-$ if possible.

The list of computer generated solutions for the classical $r$-matrix consists of 22 entries, many of them equivalent. We list them below in the form $r_B, r_F$. The numbering of the cases has been changed from the original computer output in order to group equivalent cases.

The $r$-matrices which are equivalent are discussed together. The parameters $a, b, c, d,$ and $m$ of $A$ are always given for the symmetry (29) (32) which makes the $r$-matrices identical. Here ‘identical’ means the same up to renaming of some arbitrary constants.

It is well known that computer produced results are the generic ones. For example if we solve the equation $xy = 1$ with respect to $x$ we obtain the generic solution $x = \frac{1}{y}$ which does not make sense when $y = 0$. In this case the equation has no solutions. If however we start with equation $xy = z$ we obtain $x = \frac{z}{y}$. This makes no sense either when $y = 0$ but when $z = y = 0$ there are in reality continuum of solutions. They can be recovered from the generic one by taking the limit $z = \lambda y \mapsto 0$ where $\lambda = \text{const}$.

It is therefore very important to perform the analysis of what happens when some parameters of our set of solutions tend to 0. Such singular limits were investigated when needed and they are listed as footnotes. We did our best to find out all the special cases which might have been missing from the computer generated list of results.

We use the strategy outlined above to discuss each group and come up with a list of nonequivalent $r$-matrices.

**A Cases 1,2,3,4,5,6**

Case 1

$$r_1 = \begin{pmatrix}
0 & \frac{J}{2} & \frac{-(2K_L)}{J} & K + L \\
\frac{-(J)}{2} & 0 & \frac{-(K+L)}{2} & \frac{2K_L}{J} \\
\frac{(2K-L)}{J} & \frac{(K+L)}{2} & 0 & \frac{2K_L}{J} \\
-(K+L) & \frac{-(J)}{2} & \frac{-(2K-L)}{J} & 0
\end{pmatrix},$$

$$\begin{pmatrix}
\frac{(U-J)}{2L} & U & 0 & \frac{(K-L)}{2} \\
U & \frac{(2U-L)}{J} & \frac{-(K+L)}{2} & 0 \\
0 & \frac{-(K+L)}{2} & 0 & 0 \\
\frac{(K-L)}{2} & 0 & 0 & 0
\end{pmatrix}.$$
Case 2

\[
\begin{bmatrix}
0 & \frac{(-J)}{2} & \frac{(2 \cdot M \cdot K)}{M + K} \\
\frac{J}{2} & 0 & \frac{J}{2} \\
-\frac{(2 \cdot M \cdot K)}{J} & \frac{-(M + K)}{2} & 0 \\
\end{bmatrix}
\]

This is equivalent to case 1 by means of the \( S \) symmetry and then renaming some parameters.

Case 3

\[
\begin{bmatrix}
0 & 0 & \frac{-N}{2} \\
\frac{(-N)}{2} & \frac{B}{2} & 0 \\
-\frac{L}{2} & 0 & \frac{N}{2} \\
\end{bmatrix}
\]

Case 4

\[
\begin{bmatrix}
0 & 0 & \frac{(-N)}{2} \\
\frac{B}{2} & \frac{L}{2} & 0 \\
0 & \frac{-L}{2} & 0 \\
\end{bmatrix}
\]

Case 5

\[
\begin{bmatrix}
0 & 0 & \frac{-K}{2} \\
\frac{J}{2} & \frac{2}{2} & \frac{K}{2} \\
0 & \frac{(-K)}{2} & 0 \\
\end{bmatrix}
\]

Case 6

\[
\begin{bmatrix}
0 & \frac{J}{2} & \frac{-K}{2} \\
\frac{(-J)}{2} & \frac{K}{2} & 0 \\
0 & \frac{-J}{2} & 0 \\
\end{bmatrix}
\]

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All the above cases become identical with the case 6 after bringing them to the 'canonical' form. We take $r_6$ for further consideration. We have two distinct cases
(a) $K \neq 0$,
(b) $K = 0$.
In case (a) we apply the symmetry
\[
\begin{pmatrix}
a & b \\
c & d \\
\end{pmatrix} = \begin{pmatrix}
\frac{1}{t} & 0 \\
\frac{2K}{t} & t
\end{pmatrix}
\]
with $t = \sqrt{F}$ if $F \neq 0$ and $t = 1$ if $F = 0$. We obtain
\[
\tilde{r}_{a1} = \begin{pmatrix}
0 & 0 & 0 & K \\
0 & 0 & -K & 0 \\
0 & K/2 & 0 & 0 \\
-K & 0 & 0 & 0
\end{pmatrix},
\begin{pmatrix}
\alpha & 0 & 0 & -K/2 \\
0 & 0 & K/2 & 0 \\
0 & K/2 & 0 & 0 \\
-K/2 & 0 & 0 & 0
\end{pmatrix},
\]
where $\alpha = 0$ if $F = 0$ and $\alpha = 1$ otherwise. In standard notation we have
\[
\tilde{r}_{a1} = x(-2H \wedge B + X_+ \wedge X_+ + V_+ \wedge W_- - V_- \wedge W_+) + \alpha \frac{1}{2} V_+ \wedge V_+
\]
where $x = -K/2$. In case (b) we can use the scalings $(+\cdot)$ and $(W V)$ to obtain a nonstandard $r$-matrix
\[
\tilde{r}_{a2} = \alpha (H - B) \wedge X_+ + \beta V_+ \wedge V_+,
\]
where $\alpha, \beta = 0, 1$.

**B Case 7**

Case 7

\[
r_7 = \begin{pmatrix}
0 & Y & \frac{(L - 2K - L)}{Y} & 0 \\
-Y & 0 & -(K + L) & 0 \\
\frac{(L - 2K + 2L)}{Y} & K + L & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix},
\begin{pmatrix}
0 & 0 & 0 & K \\
0 & 0 & -K & 0 \\
0 & -K & 0 & 0 \\
K & 0 & 0 & 0
\end{pmatrix}
\]
Here if $L \neq 0$ we can make it vanish by the following symmetry. \(^1\)
\[
\begin{pmatrix}
a & b \\
c & d \\
\end{pmatrix} = \begin{pmatrix}
1 & -\frac{K + L}{Y} \\
0 & 1
\end{pmatrix}
\]
\(^1\)One may ask what happens when both $Y$ and $L$ tend to 0. Then the proposed symmetry becomes singular and thus unapplicable. We should notice that after performing
So we can assume that $L = 0$:

$$r_7 = \begin{pmatrix} 0 & Y & 0 & 0 \\ -Y & 0 & -K & 0 \\ 0 & K & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & K \\ 0 & 0 & -K & 0 \\ 0 & -K & 0 & 0 \\ K & 0 & 0 & 0 \end{pmatrix}$$

Now if $K \neq 0$ then we can make $Y = 0$ by using:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & \frac{(-2 \cdot d \cdot K)}{Y} \\ \frac{Y}{2 \cdot d \cdot K} & d \end{pmatrix}.$$ 

And we obtain a standard $r$-matrix

$$r_{b1} = x (X_+ \wedge X_- - V_+ \wedge W_- + V_- \wedge W_+). \quad (44)$$

where $x = -K$. Otherwise $K = 0$ and we use

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1/\sqrt{Y} & 0 \\ 0 & \sqrt{Y} \end{pmatrix}.$$ 

to obtain non standard $r$-matrix

$$r_{b2} = H \wedge X_+. \quad (45)$$

**C Case 8**

Case 8

$$r_8 = \begin{pmatrix} 0 & (J \cdot L) & (M \cdot (K^2 - L^2)) & M \\ \frac{(-J \cdot L)}{M} & 0 & -L & \frac{J}{2} \\ \frac{(M \cdot (-K^2 + L^2))}{(J \cdot L)} & L & 0 & \frac{(-J \cdot L)}{2} \\ -M & \frac{(-J \cdot L)}{2} & (M^2 \cdot (K^2 - L^2)) & (2 \cdot J \cdot L^2) \end{pmatrix}$$

that limit we would obtain

$$r_B = \begin{pmatrix} 0 & 0 & z & 0 \\ 0 & 0 & -K & 0 \\ z & K & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

which differs from the case when $L = 0$ with just the role of $X_+$ and $X_-$ interchanged and can be obtained as well by the symmetry

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$
In this case it is possible \(^2\) to perform the following symmetry:

\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix}
= \begin{pmatrix}
1 & -\frac{(L+K)M}{\frac{1}{J-L}} \\
0 & 1
\end{pmatrix}
\]

\(^2\)We assume that denominators of the entries should be different from 0. Let us however look at the limits when both numerator and denominator tends to 0. Please note that if \(K = 0\) then \(L\) cancels from all the denominators. We then obtain the purely bosonic \(r\)-matrix

\[
\begin{pmatrix}
R & N \\
0 & S
\end{pmatrix}
\]

Now, if \(L = 0\) we obtain

\[
\begin{pmatrix}
0 & 0 & 0 & M \\
0 & 0 & 0 & \frac{M}{2} \\
-\frac{M}{2} & \frac{M}{2} & 0 & 0
\end{pmatrix}
\]

which is always equivalent to

\[
r_8 = B \wedge X_+.
\]

If however \(L \neq 0\) then we may consider the limit \(J = \lambda M \to 0\) and obtain

\[
\begin{pmatrix}
0 & \frac{\lambda}{L} \cdot L & -\frac{L}{\lambda} & 0 \\
-\frac{\lambda}{L} \cdot L & 0 & -\frac{L}{\lambda} & 0 \\
\frac{L}{\lambda} & L & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

which is equivalent to

\[
r_8 = H \wedge X_+.
\]

When \(J \neq 0 \neq M\) we obtain (symmetry)

\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix}
= \begin{pmatrix}
1 & -\frac{dM}{d-M} \\
0 & 1
\end{pmatrix}
\]

with \(d = \sqrt{-\frac{J}{2}}\)

\[
r_{c0} = xH \wedge X_+ + B \wedge X_+.
\]

The situation is more complicated when \(K \neq 0\). Then we have just two possibilities of singular limits: (a) \(L \to 0, M \to 0, \frac{M}{J} = \text{const.} = M'\) (b) \(J \to 0, M \to 0, \frac{M}{J} = \text{const.} = J'.\) in case (a) we obtain:

\[
\begin{pmatrix}
0 & \frac{M}{J} & \frac{M'K^2}{J} \\
0 & 0 & \frac{M'K^2}{2J} \\
0 & \frac{M'K^2}{2J} & 0
\end{pmatrix}
\]

which is equivalent to (46).
and also rescale the parameter $M$ as $M' = M/L$ then the effect is the same as setting $L = -K$; we come up with the bosonic part:

$$r_B = \begin{pmatrix}
0 & \frac{-(J \cdot K)}{M'} & 0 & M' \\
\frac{(J \cdot K)}{M'} & 0 & K & \frac{J}{2} \\
0 & -K & 0 & 0 \\
-M' & \frac{-(J)}{2} & 0 & 0
\end{pmatrix}$$

and the $r_F$ unchanged. Then the symmetry

$$\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} = \begin{pmatrix}
1 & 0 \\
\frac{-J}{2M'} & 1
\end{pmatrix}$$

gives us effect of setting $J = 0$. We obtain a 2-parameter family

$$r_{c1} = yH \land B + x(X_+ \land X_- - V_+ \land W_- + V_- \land W_+)$$

where $x = K$ and $y = M'$.

**D Cases 9,10,11**

Case 9

$$r_9 = \begin{pmatrix}
0 & 0 & -2 \cdot Z & 0 \\
0 & 0 & M & 0 \\
2 \cdot Z & -M & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} \cdot \begin{pmatrix}
\frac{-(M \cdot U)}{Z} & U & 0 & 0 \\
\frac{(-U \cdot Z)}{M} & M & -Z \\
0 & \frac{M}{M} & 0 & 0 \\
0 & -Z & 0 & \frac{-(M \cdot Z)}{U}
\end{pmatrix}$$

Since $Z$, $M$ and $U$ cannot vanish \(^3\) we can perform the symmetry transformation

$$\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} = \begin{pmatrix}
\frac{-Z}{\sqrt{Z}} & \sqrt{\frac{Z}{U}} \\
-\sqrt{\frac{Z}{U}} & 0
\end{pmatrix}$$

In case (b) we obtain:

$$r_8 = \begin{pmatrix}
0 & J' \cdot L & \frac{(K^2 - L^2)}{(J' \cdot L)} & 0 \\
\frac{-J' \cdot L}{(J' \cdot L)} & 0 & -L & 0 \\
L & 0 & 0 & 0 \\
0 & 0 & 0 & K
\end{pmatrix} \cdot \begin{pmatrix}
0 & 0 & 0 & K \\
0 & 0 & -K & 0 \\
0 & -K & 0 & 0 \\
K & 0 & 0 & 0
\end{pmatrix}$$

which is equivalent to (46) with $y = 0$.

\(^3\)they cannot vanish separately but any two of them can tend to 0 at the same time. If $Z$ and $M$ go to 0 we obtain a case which is equivalent to $V_+ \land V_-$. The remaining cases are equivalent to the one described below.
we obtain one parameter family
\[
\tilde{r}_9 = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & M & 0 \\
0 & M & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad \begin{pmatrix}
0 & 0 & 0 & -M \\
0 & -M & 0 & 0 \\
0 & 0 & -M & 0 \\
-M & 0 & 0 & 0
\end{pmatrix},
\]
which can be written as
\[
r_{d1} = x(X_+ \wedge X_- + V_+ \wedge W_- + \frac{1}{2}V_- \wedge V_+ + \frac{1}{2}W_+ \wedge W_+), \quad (47)
\]
where \(x = -M\). The following two cases are equivalent to the above. Case 10
\[
r_{10} = \begin{pmatrix}
0 & 0 & -2 \cdot Z & 0 \\
0 & 0 & K & 0 \\
2 \cdot Z & -K & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad \begin{pmatrix}
0 & 0 & 0 & K \\
0 & 0 & 0 & -Z \\
0 & 0 & \frac{(-K \cdot Z)}{C} & C \\
K & -Z & \frac{(-K \cdot C)}{Z} & \frac{(-Z \cdot C)}{R}
\end{pmatrix}
\]
This is equivalent to the Case 9 after application of \(S\): Case 11
\[
r_{11} = \begin{pmatrix}
0 & -2 \cdot X & 0 & 0 \\
2 \cdot X & 0 & M & 0 \\
0 & -M & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad \begin{pmatrix}
\frac{(X \cdot M)}{C} & 0 & X & 0 \\
0 & 0 & \frac{M}{C} & 0 \\
X & M & \frac{(X \cdot C)}{M} & C \\
0 & 0 & \frac{(M \cdot C)}{X} & X
\end{pmatrix}
\]

E  Cases 12, 13
Case 12
\[
r_{12} = \begin{pmatrix}
0 & -X & -Z & -S \\
X & 0 & \frac{S}{2} & -X \\
Z & \frac{(-S)}{2} & 0 & Z \\
S & X & -Z & 0
\end{pmatrix}, \quad \begin{pmatrix}
0 & 0 & X & \frac{S}{2} \\
0 & 0 & \frac{S}{2} & -Z \\
X & \frac{S}{2} & \frac{P}{C} & C \\
\frac{S}{2} & -Z & C & T
\end{pmatrix}
\]
Case 13
\[
r_{13} = \begin{pmatrix}
0 & -X & -Z & K \\
X & 0 & \frac{K}{2} & X \\
Z & \frac{(-K)}{2} & 0 & -Z \\
-K & -X & Z & 0
\end{pmatrix}, \quad \begin{pmatrix}
F & U & X & \frac{K}{2} \\
U & B & \frac{K}{2} & -Z \\
X & \frac{K}{2} & 0 & 0 \\
\frac{K}{2} & -Z & 0 & 0
\end{pmatrix}
\]
The case 12 is equivalent to case 13 upon the symmetry $S$ so we just focus on the case 13. We act in line with our general strategy. Since $r_{WW} = 0$ we look at $r_{VW}$. There are three possibilities in this case
(a) $\text{rank}(r_{VW}) = 2$,
(b) $\text{rank}(r_{VW}) = 1$,
(c) $r_{VW} = 0$, \quad $(X = Z = K = 0)$.

In case (a) we can obtain a new matrix with the same structure but $X' = Z' = 0$ and $K' = \sqrt{K^2 - 4 \cdot X \cdot Z} \neq 0$. The symmetries preserving this form of $r_{VW}$ are generated by $(+-)$ and $(WV)$ scalings and $(+-)$ swapping. By using this operations we can bring $r_{VV}$ to one of the following forms:

$$
\begin{pmatrix}
F' & U' \\
U' & B'
\end{pmatrix} = \begin{pmatrix} y & 1 \\ 1 & 1 \end{pmatrix} \text{or} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \text{or} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{or} \begin{pmatrix} \alpha & 0 \\ 0 & 0 \end{pmatrix}
$$

This gives the following $r$-matrices

$$
r_{e0} = x(2H \wedge B + X_+ \wedge X_- + V_+ \wedge W_+ + V_- \wedge W_-), \quad (48)
$$

$$
r_{e1} = r_{e0} + y(V_+ \wedge V_+) + (V_+ \wedge V_-) + \frac{1}{2}(V_- \wedge V_-), \quad (49)
$$

$$
r_{e2} = r_{e0} + \frac{1}{2}(V_+ \wedge V_+) + (V_+ \wedge V_-), \quad (50)
$$

$$
r_{e3} = r_{e0} + \frac{1}{2}(V_+ \wedge V_+) + \frac{1}{2}(V_- \wedge V_-), \quad (51)
$$

$$
r_{e4} = r_{e0} + \frac{1}{2}(V_+ \wedge V_+), \quad (52)
$$

In case (b) we can obtain the same structure with $X' = 1$ and $K' = Z' = 0$. i.e.

$$
r_{VW} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.
$$

Symmetries preserving this form of $r_{VW}$ consists of $(WV)$ scaling combined with

$$
\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ c & 1 \end{pmatrix}
$$

If $c9' \neq 0$ we can make $U' = 0$, and then scale $B'$ to become 1. If $B' = 0$ but $U' \neq 0$ we use $c$ to make $F'$ vanish. If $B' = U' = 0$ we just scale $F'$ to 1 if is is not zero. Thus the following possibilities for $r_{VV}$ emerge

$$
\begin{pmatrix}
F' & U' \\
U' & B'
\end{pmatrix} = \begin{pmatrix} y & 0 \\ 0 & 1 \end{pmatrix} \text{or} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{or} \begin{pmatrix} \alpha & 0 \\ 0 & 0 \end{pmatrix}.
$$
So we have

\[ r_{e5} = -(H + B) \wedge X_+ + V_+ \wedge W_+ \]  
(53)

\[ r_{e6} = r_{e5} + \frac{1}{2}(V_+ \wedge V_-) \] 
(54)

\[ r_{e7} = r_{e5} + (V_+ \wedge V_-), \] 
(55)

\[ r_{e8} = r_{e5} + \frac{1}{2}(V_+ \wedge V_+). \] 
(56)

In case (c) we have all the symmetry and we can obtain depending on the rank of \( r_{VV} \)

\[ \begin{pmatrix} F' & U' \\ U' & B \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{or} \begin{pmatrix} \alpha & 0 \\ 0 & 0 \end{pmatrix} \] 
(57)

\[ r_{e9} = (V_+ \wedge V_-). \] 

\[ r_{e10} = \frac{1}{2}(V_+ \wedge V_+). \] 
(58)

F Case 14

Case 14

\[ r_{14} = \begin{pmatrix} 0 & -X & -Z & 0 \\ X & 0 & 0 & \frac{1}{2} \\ Z & 0 & 0 & \frac{(-Z \cdot J)}{(2 \cdot X)} \\ 0 & \frac{(-J)}{2} & \frac{(Z \cdot J)}{(2 \cdot X)} & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & X & 0 \\ 0 & 0 & 0 & -Z \\ X & 0 & 0 & 0 \\ 0 & -Z & 0 & 0 \end{pmatrix} \]

When \( Z = 0 \) then \( X \) may also vanish. In such a case after a simple (\(+\)\(-\)) rescaling we obtain just

\[ r_{f0} = B \wedge X_+ \]  
(59)

If \( Z = 0 \) but \( X \neq 0 \) we just take

\[ \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1/\sqrt{X} & 0 \\ 0 & \sqrt{X} \end{pmatrix} \]

and obtain

\[ \tilde{r}_{14} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{(-J)}{2} & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \]

\(^4\)if \( X \) and \( Z \) tend to 0 in such a way that \( \frac{Z}{X} \) remains finite we are left with a linear combination of \( B \wedge X_+ \) and \( B \wedge X_- \) which is equivalent either to \( B \wedge X_+ \) or \( yH \wedge B \).
which can be written as
\[ r_{f1} = (-H \land X_+ + V_+ \land W_+) + x(B \land X_+) , \] (60)
where \( x = -\frac{i}{2} \). If, however, \( Z \neq 0 \) than we take
\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} = \begin{pmatrix} 1/\sqrt{Z} & \frac{-1}{2}\sqrt{X} \\ 1/\sqrt{Z} & \frac{1}{2}\sqrt{X} \end{pmatrix}
\]
to obtain
\[
\tilde{r}_{10} = \begin{pmatrix}
0 & 0 & 0 & y \\
0 & 0 & x & 0 \\
0 & -x & 0 & 0 \\
y & 0 & 0 & 0 \\
\end{pmatrix},
\]
\begin{align*}
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} & = \begin{pmatrix} 1/\sqrt{Z} & \frac{-1}{2}\sqrt{X} \\ 1/\sqrt{Z} & \frac{1}{2}\sqrt{X} \end{pmatrix} \\
\tilde{r}_{10} & = \begin{pmatrix}
0 & 0 & 0 & x \\
0 & 0 & x & 0 \\
0 & x & 0 & 0 \\
x & 0 & 0 & 0 \\
\end{pmatrix},
\end{align*}

\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} = \begin{pmatrix} 1/\sqrt{Z} & \frac{-1}{2}\sqrt{X} \\ 1/\sqrt{Z} & \frac{1}{2}\sqrt{X} \end{pmatrix}
\]
to obtain
\[
\tilde{r}_{10} = \begin{pmatrix}
0 & 0 & 0 & y \\
0 & 0 & x & 0 \\
0 & -x & 0 & 0 \\
y & 0 & 0 & 0 \\
\end{pmatrix},
\]
\begin{align*}
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} & = \begin{pmatrix} 1/\sqrt{Z} & \frac{-1}{2}\sqrt{X} \\ 1/\sqrt{Z} & \frac{1}{2}\sqrt{X} \end{pmatrix} \\
\tilde{r}_{10} & = \begin{pmatrix}
0 & 0 & 0 & x \\
0 & 0 & x & 0 \\
0 & x & 0 & 0 \\
x & 0 & 0 & 0 \\
\end{pmatrix},
\end{align*}

or
\[ r_{f2} = x(X_- \land X_+ + V_+ \land W_+ - V_- \land W+) + y(H \land B) , \] (61)
where \( x = -\sqrt{X \cdot Z} \) and \( y = -J\sqrt{Z \cdot X} \).

**G  Cases 15,16**

Case 15
\[
r_{15} = \begin{pmatrix}
0 & 0 & \frac{(-N)}{2} & K \\
0 & 0 & \frac{K}{2} & 0 \\
\frac{N}{2} & \frac{(-K)}{2} & 0 & \frac{(-N)}{2} \\
-K & 0 & \frac{N}{2} & 0 \\
\end{pmatrix},
\]
\[
\begin{pmatrix}
0 & 0 & 0 & \frac{K}{2} \\
0 & 0 & \frac{(-K)}{2} & 0 \\
0 & \frac{(-K)}{2} & \frac{N}{C} & 0 \\
\frac{K}{2} & 0 & 0 & \frac{(-N-C)}{2} \\
\end{pmatrix}
\]
Here if \( C = 0 \) and \( K \neq 0 \) then we take
\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} = \begin{pmatrix} 1/\sqrt{Z} & \frac{-1}{2}\sqrt{X} \\ 1/\sqrt{Z} & \frac{1}{2}\sqrt{X} \end{pmatrix}
\]
to obtain
\[
\tilde{r}_{15} = \begin{pmatrix}
0 & 0 & 0 & K \\
0 & 0 & \frac{K}{2} & 0 \\
0 & \frac{(-K)}{2} & 0 & 0 \\
-K & 0 & 0 & 0 \\
\end{pmatrix},
\]
\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} = \begin{pmatrix} 1/\sqrt{Z} & \frac{-1}{2}\sqrt{X} \\ 1/\sqrt{Z} & \frac{1}{2}\sqrt{X} \end{pmatrix}
\]
to obtain
\[
\tilde{r}_{15} = \begin{pmatrix}
0 & 0 & 0 & K \\
0 & 0 & \frac{K}{2} & 0 \\
0 & \frac{(-K)}{2} & 0 & 0 \\
-K & 0 & 0 & 0 \\
\end{pmatrix},
\]
\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} = \begin{pmatrix} 1/\sqrt{Z} & \frac{-1}{2}\sqrt{X} \\ 1/\sqrt{Z} & \frac{1}{2}\sqrt{X} \end{pmatrix}
\]
to obtain
\[
\tilde{r}_{15} = \begin{pmatrix}
0 & 0 & 0 & K \\
0 & 0 & \frac{K}{2} & 0 \\
0 & \frac{(-K)}{2} & 0 & 0 \\
-K & 0 & 0 & 0 \\
\end{pmatrix},
\]
This is just
\[ r_{g1} = x(2H \land B + X_+ \land X_+ + V_+ - V_- \land W_+). \] (62)
with \( x = \frac{4K}{2} \). If \( C = 0 \) then we can also put \( K = 0 \) and obtain after rescaling

\[
 r_{g1.5} = (H - B) \land X_+ .
\]  

\( (63) \)

If \( C \neq 0 \) we take \(^5\)

\[
\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \frac{Z}{2bK} & \sqrt{\frac{-N \cdot C}{2bK}} \\ 0 & \frac{Z}{2bK} \end{pmatrix}
\]

and after performing additional \( S \) transformation obtain

\[
\tilde{r}_{12} = \begin{pmatrix} 0 & 0 & 0 & K \\ 0 & 0 & \frac{K}{2} & 0 \\ -K & 0 & 0 & 0 \\ -K & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & \frac{K}{2} \\ 0 & 0 & \frac{(-K)}{2} & 0 \\ 0 & \frac{(-K)}{2} & 0 & 0 \\ \frac{K}{2} & 0 & 0 & 0 \end{pmatrix},
\]

which differs from (62) only by \( \frac{1}{2} V_+ \land V_+ \)

\[
 r_{g1} = r_{g1} + \frac{1}{2} V_+ \land V_+ .
\]  

\( (64) \)

Case 16

\[
 r_{16} = \begin{pmatrix} 0 & 0 & \frac{N}{2} & K \\ 0 & 0 & \frac{(-K)}{2} & 0 \\ \frac{(-N)}{2} & \frac{K}{2} & 0 & \frac{(-N)}{2} \\ -K & 0 & \frac{N}{2} & 0 \end{pmatrix}, \begin{pmatrix} \frac{(-2U \cdot K)}{N} & U & 0 & \frac{K}{2} \\ \frac{(-U \cdot N)}{2} & \frac{(-K)}{2} & 0 & 0 \\ \frac{(2-U \cdot K)}{2} & 0 & 0 & 0 \\ \frac{K}{2} & 0 & 0 & 0 \end{pmatrix},
\]

This case is equivalent to the Case 15 upon the symmetry \( S \).

\(^5\)if \( N \) and \( K \) tend to 0 we are left with a purely fermionic \( r \)-matrix which is easily shown to be equivalent with \( V_+ \land V_+ \).
H Cases 17, 18

The matrices $r_{18}$ and $r_{17}$ differ only by the names of parameters

$$r_{17} = \begin{pmatrix}
0 & 0 & \frac{(N(-M+K))}{(2(M+K))} & M + K \\
\frac{(N(M-K))}{(2(M+K))} & \frac{(-M+K)}{2} & 0 & 0 \\
-(M + K) & 0 & \frac{N}{2} & 0 \\
0 & 0 & \frac{(-M+K)}{2} & 0
\end{pmatrix},$$

$$r_{18} = \begin{pmatrix}
0 & 0 & \frac{(L-N)}{2(2K + L)} & 2 \cdot K + L \\
0 & 0 & \frac{(-L)}{2} & 0 \\
\frac{(-L-N)}{2(2K + L)} & \frac{L}{2} & 0 & \frac{(-N)}{2} \\
-2 \cdot K - L & 0 & \frac{N}{2} & 0
\end{pmatrix},$$

so just look into the $r_{18}$. The symmetry given by \(^6\)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & \frac{N}{2(2K + L)} \\ 0 & 0 \end{pmatrix}$$

yields

$$\tilde{r}_{18} = \begin{pmatrix}
0 & 0 & \frac{(-L)}{2} & 0 \\
0 & \frac{L}{2} & 0 & 0 \\
-2 \cdot K - L & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix},$$

which can be written as

$$r_{h1} = x(X_+ \wedge X_- + V_+ \wedge W_- - V_- \wedge W_+) + y(H \wedge B), \quad (65)$$

with $x = \frac{-L}{2}$ and $y = 2 \cdot K + L$.

\(^6\)If $K$ and $L$ both tend to 0 we obtain any linear combination of $H \wedge X_-$ and $B \wedge X_-$.
I Cases 19,20

Case 19

\[
\tilde{r}_{19} = \begin{pmatrix}
0 & \frac{\sqrt{J}G}{\sqrt{N}} & \frac{\sqrt{J}G \cdot N}{(\sqrt{N} \cdot J)} & 0 \\
\frac{\sqrt{-J}G}{\sqrt{N}} & 0 & 0 & \frac{J}{2} \\
\frac{\sqrt{-J}G \cdot N}{(\sqrt{N} \cdot J)} & 0 & 0 & \frac{(-N)}{2} \\
0 & \frac{(-J)}{2} & \frac{N}{2} & 0
\end{pmatrix}
\begin{pmatrix}
0 & 0 & 0 & G \\
0 & 0 & -G & 0 \\
0 & -G & 0 & 0 \\
G & 0 & 0 & 0
\end{pmatrix}
\]

From the above we see that both \(N\) and \(J\) are non zero \(^7\). Upon the symmetry \((a\ b\ c\ d) = \begin{pmatrix}
\frac{1}{2} & \frac{\sqrt{N}}{2} \\
-\frac{1}{2} \sqrt{\frac{J}{N}} & 1
\end{pmatrix}\)

we obtain

\[
\tilde{r}_{19} = \begin{pmatrix}
0 & 0 & 0 & \sqrt{N} \cdot J \\
0 & 0 & -G & 0 \\
0 & G & 0 & 0 \\
-\sqrt{N} \cdot J & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
0 & 0 & 0 & G \\
0 & 0 & -G & 0 \\
0 & -G & 0 & 0 \\
G & 0 & 0 & 0
\end{pmatrix}
\]

This can be written as

\[
\tilde{r}_{11} = x(X_+ \wedge X_- - V_+ \wedge W_+ + V_- \wedge W_-) + y(H \wedge B) \quad (66)
\]

with \(x = -G\) and \(y = \sqrt{N \cdot J}\).

If \(G = 0\) we have

\[
r_{19} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{J}{2} \\
0 & 0 & 0 & \frac{(-N)}{2} \\
0 & \frac{(-J)}{2} & \frac{N}{2} & 0
\end{pmatrix}
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

then it is possible that just one of \(N\) and \(J\) vanishes. After putting \(N = 0\) and rescaling we obtain

\[
r_{i2} = B \wedge X_+ . \quad (67)
\]

Case when \(G = 0\) and \(J \neq 0 \neq N\) is just \(r_{11}\) with \(x = 0\).

\(^7\)However, we can look at the limit when both \(N\) and \(J\) tend to 0, leaving the ratio \(\frac{N}{J}\) finite. The same symmetry is applicable this case, leading to the \(r\)-matrix \(r_{11}\) with \(y = 0\).
Case 20 is equivalent to $r_{19}$ the only difference being the sign of $\sqrt{J}$ and $G$ renamed $G$.

$$r_{20} = \begin{pmatrix}
0 & (-\sqrt{J}G) & (-\sqrt{J}G:J) & 0 \\
(\sqrt{J}G) & 0 & 0 & \frac{J}{2} \\
(\sqrt{J}G:J) & 0 & 0 & (-\frac{N}{2}) \\
(\sqrt{N}J) & 0 & (-\frac{N}{2}) & 0
\end{pmatrix}, \begin{pmatrix}
0 & 0 & 0 & G \\
0 & 0 & -G & 0 \\
0 & -G & 0 & 0 \\
G & 0 & 0 & 0
\end{pmatrix}$$

**J Case 21**

Case 21

$$r_{21} = \begin{pmatrix}
0 & -2 \cdot X & (2U-C) & X \\
2 \cdot X & 0 & (U-C+K^2) & K \\
X & 0 & (U-C) & X \\
0 & 0 & 0 & 0
\end{pmatrix}, \begin{pmatrix}
\frac{(X-U)}{K} & U & X & K \\
\frac{(U-K)}{X} & U & X & K \\
\frac{X}{U} & (U-C) & K & C \\
\frac{(U-C^2)}{K} & C
\end{pmatrix}$$

If $C = 0$ we have\(^8\)

$$r_{21} = \begin{pmatrix}
0 & -2 \cdot X & 0 & 0 \\
2 \cdot X & 0 & K & 0 \\
0 & -K & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \begin{pmatrix}
\frac{(X-U)}{K} & U & X & K \\
\frac{(U-K)}{X} & U & X & K \\
X & 0 & (X-K) & 0 \\
K & 0 & 0 & 0
\end{pmatrix}$$

then after symmetry with

$$\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} = \begin{pmatrix}
\sqrt{\frac{U}{X}} & 0 \\
-\sqrt{\frac{U \cdot X}{K}} & \sqrt{\frac{X}{U}}
\end{pmatrix}$$

we obtain

$$r_{21} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & K & 0 \\
0 & -K & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \begin{pmatrix}
0 & 0 & 0 & K \\
0 & K & 0 & 0 \\
0 & 0 & K & 0 \\
K & 0 & 0 & 0
\end{pmatrix}$$

which is just

$$r_{j1} = x(X_+ \land X_- + V_+ \land W_- + \frac{1}{2} V_+ \land V_- + \frac{1}{2} W_+ \land W_+), \quad (68)$$

\(^8\)The singular limits were investigated but gave rise to no new $r$-matrices. We therefore assume that all denominators are different from 0.
with \( x = K \). When \( C \neq 0 \) we still have two possibilities. If \( K^2 - U \cdot C \neq 0 \) we can obtain a similar matrix (with \( K \) replaced with \( \frac{K^2 - U \cdot C}{K} \)). To do it we take

\[
\begin{pmatrix}
  a & b \\
  c & d
\end{pmatrix} = \begin{pmatrix}
  -c \cdot \frac{K}{X} & \frac{U \cdot C}{X - (K^2 - U \cdot C)} \\
  \frac{X - U \cdot C}{K^2 - U \cdot C} & \frac{K}{X - (K^2 - U \cdot C)}
\end{pmatrix}.
\]

If \( K^2 - U \cdot C = 0 \) we then let \( U = \frac{K^2}{C} \) and the symmetry

\[
\begin{pmatrix}
  a & b \\
  c & d
\end{pmatrix} = \begin{pmatrix}
  -\frac{c}{\sqrt{X}} & \frac{K}{\sqrt{X}} \\
  0 & -\frac{c}{\sqrt{X}}
\end{pmatrix}
\]

yields

\[
r_{21} = \begin{pmatrix}
0 & -2 & 0 & 0 \\
2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad \begin{pmatrix}
\frac{K}{c} & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & \frac{c}{K} & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

When we take \( k = \frac{K}{C} \) and it becomes

\[
r_{21} = \begin{pmatrix}
0 & -2 & 0 & 0 \\
2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad \begin{pmatrix}
1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix},
\]

which can be written as

\[
r_{j2} = -2H \wedge X_+ + \frac{1}{2}(V_++W_+) \wedge (V_++W_+). \tag{69}
\]

**K Case 22**

Case 22

\[
r_{22} = \begin{pmatrix}
0 & -X & -Z & \frac{K - S}{(K + S)} \\
X & 0 & \frac{(X - (K - S))}{2} & \frac{(K - S)}{(K + S)} \\
Z & \frac{(-X - S)}{2} & 0 & \frac{(X - (K - S))}{2} \\
-K + S & \frac{(Z - (K - S))}{2} & \frac{(Z - (K - S))}{(K + S)} & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & 0 & \frac{X}{(K + S)} & \frac{(K + S)}{2} \\
0 & 0 & \frac{(K + S)}{2} & -Z \\
\frac{X}{(K + S)} & \frac{(K + S)}{2} & 0 & 0 \\
\frac{(K + S)}{2} & -Z & 0 & 0
\end{pmatrix}
\]

Let us denote \( K = K' \) \( S = K - S' \). Then we have two possibilities;
(a) \( X \cdot Z + K^2 = 0 \),
(b) \( X \cdot Z + K^2 \neq 0 \).

In the latter case both \( X \) and \( Z \) can be made equal zero. If \( Z = 0 \) we just use the matrix
\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} = \begin{pmatrix} 0 & -1 \\ \frac{X}{K+S} \end{pmatrix}
\]
and when \( Z \neq 0 \) we use
\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} = \begin{pmatrix} \frac{Z}{2\sqrt{X \cdot Z + K^2}} & \frac{1}{2} \\ \frac{K}{2\sqrt{X \cdot Z + K^2}} - \frac{1}{2} \end{pmatrix} \frac{1}{Z}
\]
in either case we obtain
\[
\begin{pmatrix}
r_{22} = \begin{pmatrix} 0 & 0 & 0 & \frac{(K+S)}{2} \\ 0 & 0 & \frac{(K+S)}{2} & 0 \\ -K + S & 0 & 0 & 0 \\ \frac{(K+S)}{2} & 0 & 0 & 0
\end{pmatrix}
\end{pmatrix}
\]
This is just
\[
r_{k1} = x(X_+ \wedge X_- + V_+ \wedge W_- + V_- \wedge W_+) + y(H \wedge B) \tag{70}
\]
with \( x = \frac{K+S}{2} \) and \( y = K - S \). In the case (a), however, we use the symmetry
\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} = \begin{pmatrix} 0 & \sqrt{-Z} \\ \frac{\sqrt{-Z}}{Z} \frac{1}{K+S} \end{pmatrix}
\]
and obtain
\[
r_{22} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & \frac{K+S}{K+S} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{K+S}{K+S} & 0 & 0
\end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0
\end{pmatrix}
\]
or
\[
r_{k2} = -(H \wedge X_+) + (V_+ \wedge W_+) + x(B \wedge X_+) \tag{71}
\]
where \( x = \frac{K-S}{K+S} \).
4 Summary and discussion

Below we give the list nonequivalent $r$-matrices for the $osp(2|2)$. The lowercase latin letters $x$, $y$ denote arbitrary complex numbers, whereas $\alpha$, $\beta$ can only take value 0 or 1.

\begin{align*}
    r_{b2} &= H \wedge X_+ , \\
    r_{e0} &= xH \wedge X_+ + B \wedge X_+ , \\
    r_{a2} &= \alpha(H - B) \wedge X_+ + \beta(V_+ \wedge V_+ ) , \\
    r_{b1} \subset r_{c1} &= r_{e1} \sim r_{h1} &= x(X_+ \wedge X_- + V_+ \wedge W_- - V_- \wedge W_+) + y(H \wedge B) , \\
    r_{f2} &= r_{k1} &= x(X_+ \wedge X_- + V_+ \wedge W_- + V_- \wedge W_+) + y(H \wedge B) , \\
    r_{d1} = r_{j1} &= x(X_+ \wedge X_- + V_+ \wedge W_- + \frac{1}{2} V_- \wedge V_- + \frac{1}{2} W_+ \wedge W_+) , \\
    r_{j2} &= -2(H \wedge X_+) + \frac{1}{2}(V_+ + W_+) \wedge (V_+ + W_+) , \\
    r_g &= x(2H \wedge B + X_+ \wedge X_- + V_+ \wedge W_- - V_- \wedge W_+) + \alpha \frac{1}{2}(V_+ \wedge V_+ ) , \\
    r_{a1} &= x(-2H \wedge B + X_+ \wedge X_- + V_+ \wedge W_- - V_- \wedge W_+) + \alpha \frac{1}{2}(V_+ \wedge V_+ ) , \\
    r_{e0} &= x(2H \wedge B + X_+ \wedge X_- + V_+ \wedge W_- + V_- \wedge W_+) , \\
    r_{e1} &= r_{e0} + y(V_+ \wedge V_+) + (V_+ \wedge V_-) + \frac{1}{2}(V_- \wedge V_-) , \\
    r_{e2} &= r_{e0} + \frac{1}{2}(V_+ \wedge V_+) + (V_+ \wedge V_-) , \\
    r_{e3} &= r_{e0} + \frac{1}{2}(V_+ \wedge V_+) + \frac{1}{2}(V_- \wedge V_-) , \\
    r_{e4} &= r_{e0} + \frac{1}{2}(V_+ \wedge V_+) , \\
    r_{f1} = r_{k2} &= ((xB - H) \wedge X_+) + (V_+ \wedge W_+) , \\
    r_{e5} &= -(B + H) \wedge X_+ + (V_+ \wedge W_+) , \\
    r_{e6} &= r_{e5} + \frac{1}{2}(V_+ \wedge V_+) + \frac{1}{2}(V_- \wedge V_-) , \\
    r_{e7} &= r_{e5} + (V_+ \wedge V_-) , \\
    r_{e8} &= r_{e5} + \frac{1}{2}(V_+ \wedge V_+) , \\
    r_{e9} &= V_+ \wedge V_- , \\
    r_{e10} &= \frac{1}{2}(V_+ \wedge V_+ ) .
\end{align*}

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All the “generic” solutions were found by the computer. We carefully analysed the singular limits to obtain some more solutions. On this basis we claim that every co-Lie structure on the \( \text{osp}(2|2) \) algebra must be generated by an \( r \)-matrix which is equivalent to some member of the list (72)-(92). Due to the equivalence \( \text{osp}(2|2) \sim \text{sl}(1|2) \) (see e.g [2]) this classification is also valid for the \( \text{sl}(1|2) \) super-algebra.

The \( r \)-matrices (72), (73), (74), (78), (86), (87), (88), (89), (90), (91), (92) satisfy CYBE. The remaining ones satisfy CYBE only if the parameter \( x \) is equal to 0.

If the result of Etingof [5] can be generalized to the case of Lie superbialgebras, then \( r \)-matrices satisfying CYBE can be easily quantized.

In view of the sequence of inclusions:

\[
\text{sl}(2) \subset \text{gl}(2) \subset \text{osp}(1|2) \subset \text{osp}(2|2)
\]

it is relevant to look at the classification of each subalgebra in this chain. It is obvious that any nonstandard \( r \)-matrix (i.e. satisfying CYBE) of a subalgebra is also \( r \)-matrix for the whole algebra. We make sure that nonstandard \( r \)-matrices of \( \text{sl}(2) \), \( \text{gl}(2) \) and \( \text{osp}(1|2) \) which are known in the literature are also present in our classification.

(a) for \( \text{sl}(2) \) all the \( r \)-matrices satisfying CYBE are equivalent to \( H \wedge X_+ \) (72).

(b) The classification of \( \text{gl}(2) \) Lie bialgebras was first obtained by Balles
teros et al. in [9], where also the corresponding Hopf algebras were described. From their nonstandard \( r \)-matrices it is possible to pick up just two nonequivalent \( r_1 = H \wedge X_+ \) and \( r_2 = H \wedge B \) which coincide with (72) and (73).

(c) The subalgebra \( \text{osp}(1|2) \) is generated by \( H, X_+, X_- \), \( V'_+ = (V_+ + W_+)/2 \) and \( V'_- = (V_- + W_-)/2 \). The classification of super-Lie bialgebras was obtained in [18]. There were two nonstandard \( r \)-matrices: \( r_1 = H \wedge X_+ \) and \( r_2 = H \wedge X_+ - V_+ \wedge V_+ \). They correspond to (72) and (78) from our list.

The classification of \( \text{osp}(2|2) \) Lie superbialgebras was not known before, however, several examples have been investigated.
Deguchi et al. [14] constructed the deformation of the universal enveloping algebra $U_q(osp(2|2))$ and obtained its universal $R$-matrix. After identification of the generators $J_\pm = \pm X_\pm, V_\pm = V_\pm/\sqrt{2}, \nabla_\pm = W_\pm/\sqrt{2}, H = H, T = 2B$ we notice that the antisymmetric part of the first order term (in $\ln q$) of the $R$-matrix takes the form $X_+ \wedge X_- + V_+ \wedge W_- + V_- \wedge W_+$ which is a special case of our classical $r$-matrix (76). We also check that it generates the antisymmetric part of the first order term of their coproduct.

The universal $R$-matrix given by Aizawa [15] was obtained by a twisting element belonging to the $gl(2)$ subalgebra which had the following form (we use the following identification of generators used in [15]: $H_1 = 2H, Z = 2B, X_\pm = \pm X_\pm, v_\pm = V_\pm, \varphi_\pm = \pm W_\pm$ in order to give the original expression in our basis):

$$F = \exp\left(\frac{g}{h}\sigma \otimes B\right) \exp(-H \otimes \sigma)$$  \hfill (94)

where

$$\sigma \equiv -\ln(1 - 2hX_+)$$

The universal $R$-matrix takes the form:

$$R = \exp\left(\frac{g}{h}B \otimes \sigma\right) \exp(-\sigma \otimes H) \exp(H \otimes \sigma) \exp\left(-\frac{g}{h}\sigma \otimes B\right)$$

and in the classical limit $h \rightarrow 0, g \rightarrow 0$ it gives rise to the $r$-matrix (73).

Another two parameter deformation was investigated by Arnaudon et al. [16]. After the identification of the generators ($E_1^+ = V_-, E_1^- = W_-, E_2^+ = W_-, E_2^- = V_+ \cup V_\cup W_+ = X_+, H_1 = H + B, H_2 = H - B$) we were able to check that the super antisymmetric part of the first order term of their coproduct is generated by the classical $r$-matrix (76).

5 Classification of $osp(1|2) \oplus u(1)$ super Lie bialgebras

The $osp(1|2) \oplus u(1)$ Lie superalgebra has the same subalgebra structure as $osp(2|2)$.

$$sl(2) \subset \frac{gl(2)}{osp(1|2)} \subset osp(1|2) \oplus u(1)$$  \hfill (95)

However, it has only 6 generators so it is relatively easy to classify using the same technique. The $osp(1|2)$ algebra of is spanned by the generators $H,$
$X_+, X_-, Q_+ \equiv \frac{1}{2}(V_+ + W_+)$ and $Q_- \equiv \frac{1}{2}(V_- + W_-)$, whose commutation relations follow from (1). Supplementing them with a central generator $Z$ gives $osp(1|2) \oplus u(1)$.

All the Lie super-bialgebras $osp(1|2) \oplus u(1)$ are coboundary and their corresponding $r$-matrices are equivalent to one of the following

\begin{align*}
r_1 &= H \wedge X_+, \\
r_2 &= Z \wedge X_+, \\
r_3 &= H \wedge X_+ + Z \wedge X_+, \\
r_4 &= H \wedge X_+ - Q_+ \wedge Q_+, \\
r_5 &= H \wedge X_+ - Q_+ \wedge Q_+ + Z \wedge X_+, \\
r_6 &= x(X_+ \wedge X_- + 2Q_+ \wedge Q_-), \\
r_7 &= x(X_+ \wedge X_- + 2Q_+ \wedge Q_-) + H \wedge Z. \tag{102}
\end{align*}

$r$-matrices (96)-(100) satisfy CYBE whereas (101) and (102) don’t if $x \neq 0$.

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