RSS Positioning Algorithm Based on Maximum Likelihood Recursive Estimation and CKF

Xiao Ning¹, ⁰, Shi Shuai², *

¹Bowen College of Management, Guilin University of Technology, Guilin, China
²The 34th Research Institute of CETC, Guilin, China
⁰icyxiao@163.com, *2008shishuai@163.com

Abstract—In order to improve the positioning accuracy of indoor and dense obstacles, a positioning algorithm based on maximum likelihood recursive estimation and cubature Kalman filter is proposed for the positioning technology based on the received signal strength. The algorithm consists of two steps: initial position estimation and mobile location. Firstly, according to the principle of triangulation, the possible target region of mobile terminal is determined and the region is divided into smaller possible target region step by step. Then, based on the hybrid cooperation of the received signal strength and time of arrival, the nonlinear CKF filter is used to realize the mobile location. Finally, MATLAB is used to simulate the algorithm, and the simulation results show that the proposed method has better positioning performance even in the shadow area.

1. INTRODUCTION

Nowadays, hybrid cooperative technology plays an important role in the field of wireless positioning and can be used in indoor, underground parking lot and other harsh environments without enough GPS signal. Some literatures also put forward different cooperative positioning technologies. In [1], GNSS cooperative positioning is used to achieve high-precision positioning with the help of cooperative nodes. In [2], mobile terminals cooperate with each other to improve accuracy of positioning, enhance the continuity and coverage of location services, and solve the uncertainty of local geometric positioning through spatial diversity and information redundancy. Centralized weighted least squares (WLS) and improved multidimensional scaling analysis (MDS) are commonly used to process location data of mobile terminals [3][4]. The distributed positioning algorithm mentioned in [5] and [6] can also provide better position accuracy in case of deviation measurement. Most of these algorithms are based on the triangulation technology of different measurement values, focusing on the intersection or cross region formed by three circles.

This paper focuses on positioning based on RSS in extreme cases. It is necessary to know the initial position of mobile terminal when using Kalman filter for mobile positioning. However, there is no intersection or cross area sometimes in the process of triangle positioning, that is, there is a blind area for positioning, so it is impossible to use the traditional algorithm to obtain a more accurate initial position. In order to solve this problem, this paper constructs the possible target region of mobile terminal, and gradually divides it into smaller target regions for maximum likelihood (ML) recursive iteration to estimate the initial location of the mobile terminal. Then, the cubature Kalman filter (CKF) is used to locate the target by cooperating with TOA measurements.
2. SYSTEM MODEL

In this paper, it is assumed that the average receiving power varies exponentially with distance in the case of shadow and multipath effects in a large fading propagation channel\(^5\). The path-loss model is generally:

\[
P = P_0 + X_{\alpha_d} = P_0 + 10\alpha \log\left(\frac{d}{d_0}\right) + X_{\alpha_d}
\]

\[
P_0 = 10\log\left(\frac{4\pi d_0}{\lambda}\right)
\]

In (1) and (2), \(d\) is the distance between transceivers, \(d_0\) is the reference distance (usually 1m), \(P_0\) is the strength of received signal when the distance is \(d_0\), \(\alpha\) is the path loss index, \(P_\alpha\) is the mean of received power, \(X\) is the Gaussian random variable with the mean value of 0 and the standard deviation of \(\sigma_\alpha\). The change of power of received signal \(P\) is caused by shadow effect. The obstacles between the transceiver make the signal arrive at the receiver through absorption, reflection, scattering and diffraction, which weaken the power received.

As shown in Fig.1, the three circles with the radius of the distance estimated from the mobile terminal to the base station as the center do not intersect due to the deviation of RSS measurements. In this case, the traditional positioning algorithm LS or WLS does not necessarily converge to an optimal solution. Therefore, this paper adopts a geometric solution based on maximum likelihood recursive estimation.

3. MAXIMUM LIKELIHOOD RECURSIVE ESTIMATION

Once the unlocated mobile terminal (UMT) obtains enough RSS measurements (at least three RSS measurements are needed in the 2D positioning algorithm), the distance \(\hat{d}_k\),

\[
\hat{d}_{k,LS} = e^{\mu_{\hat{d}_k} S^2}
\]

\[
S = \frac{\sigma_{\alpha_d} \ln(10)}{10\alpha}
\]

\[
M = \frac{(L - L_0) \ln(10)}{10\alpha} + \ln(d_0)
\]
In (4) and (5), assuming that the location of the base stations serviced is known, the optimal solution of the location of the mobile terminal is calculated by using (3), (4) and (5) and the geometric positioning algorithm. Theoretically, the optimal solution should be located in the intersection or cross area formed by the circles with the base station as the center and \( \hat{d}_k \) as the radius. However, this cross region does not exist sometimes due to the complexity of the propagation path in practical applications, so the traditional positioning algorithm cannot provide the optimal solution. Therefore, this paper adopts recursive geometry algorithm based on maximum likelihood, which is divided into the following four steps:

1) **Build target region:** When the mobile terminal is not in the triangle region composed of three base stations A, B and C shown in Fig.2. Three new nodes P, M and N are obtained by constructing three parallelograms. The triangle region PMN enclosed by these three new nodes will be the target region to be studied.

2) **Subdivide target region:** As shown in Fig.3. The area PMN is divided into three small triangle regions, which are respectively represented by R1, R2 and R3. Each triangle region is composed of two nodes and the center of gravity of the original region.

3) **Select the best region:** Take enough samples for R1, R2 and R3 regions, and calculate ML estimation of sample \( i \) for each region according to (6).

\[
\hat{r}_{ML,i} = \min \left( \sum_{k=1}^{N} \frac{(r_{i,j} - \hat{d}_k)^2}{\sigma_k^2} \right)
\]

(6)

In (6), \( \hat{d}_k \) is calculated according to (2), \( r_{i,j} \) is the distance from point \( i \) to the service base station \( k \) and \( \sigma_k^2 \) is the corresponding distance estimation variance. The region of the sample with the minimum value \( \hat{r}_{ML,i} \) will be considered as the region where UMT is most likely to be located, and return to step 2 to start the next recursive iteration.

4) **Convergence interruption:** Once the new target region is small enough relative to the original region, the algorithm is interrupted to obtain the initial position of UMT estimated, so as to ensure the convergence and error of the algorithm proposed in this paper is within the allowable range.

4. **HYBRID CUBATURE KALMAN FOR MOBILE POSITIONING**

When UMT moves along a given path, there is a strong correlation between adjacent positions. Due to the nonlinear relationship between the received signal strength and the propagation distance, the target tracking scheme based on the nonlinear cubature Kalman filter (CKF) is adopted in this paper.

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Figure 2 Extend region  
Figure 3 Subdivide region

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CKF uses the numerical calculation method of cubature principle to calculate the mean value and covariance of nonlinear random variables with high accuracy [9][10].

In order to improve the performance of positioning, the measurements adopt the combined model of RSS and TOA in this paper [10]. The TOA measurement model is defined as:

$$h_t(s) = \sqrt{(x-x_{BS})^2 + (y-y_{BS})^2} + \delta_t + \omega_t$$  \hspace{1cm} (7)

Where, \((x, y)\) is the coordinate of UMT, \((x_{BS}, y_{BS})\) is the coordinate of \(BS_i\), \(\delta_t\) is measuring error caused by the non-line-of-sight propagation, and \(\omega_t\) is system measuring error. RSS measurement model is

$$h_t(s) = P_0 + 10\alpha \log d + X_{\omega_t}$$  \hspace{1cm} (8)

Where, \(P_0\) is the given reference of power, \(d\) is the distance from UMT to \(BS_i\), \(\alpha\) is the path loss exponent, and \(X_{\omega_t}\) is the measuring error caused by shadow effect.

The CKF filtering process consists of two steps: time update and measurement update [9]. When UMT moves at a constant speed, the system state variable is:

$$X = [x \quad y \quad \dot{x} \quad \dot{y}]$$  \hspace{1cm} (9)

\((\dot{x}, \dot{y})\) is the velocity of UMT along X-axis and Y-axis. According to the third-order cubature principle, the dimension of state variable is 4, and the total number of cubature points is 8.

$$\xi_{t,k} = \hat{X}_k \pm S_k \sqrt{\frac{n_{CP}}{2}} [I_{n_{a_0}},n_{a_0}] \quad i \in \{1, ..., n_{CP}\}$$  \hspace{1cm} (10)

\(n_D\) is the state dimension, \(n_{CP}\) is the number of cubature points, \(\xi_{t,k}\) is the cubature point \(i\) at time \(k\), \([a]\) is the cubature vector \(i\) of matrix \(A\), \(S_k\) is the chernsky decomposition of the state covariance matrix \(P_k\), that is \(P_k = S_k S_k^T\).

The time updating process of each accumulation point is as follows:

$$\hat{\xi}_{t,k}^- (1) = \xi_{t,k-1}^- (1) + \xi_{t,k-1}^- (3) \cdot T$$

$$\hat{\xi}_{t,k}^- (2) = \xi_{t,k-1}^- (2) + \xi_{t,k-1}^- (4) \cdot T$$  \hspace{1cm} (11) (12)

Where, the superscript “-” represents time propagation, \((j)\) represents state variable \(j\), and \(T\) represents measuring interval.

The time updating processes of state vector \(X\) and covariance matrix \(P\) are as follows:

$$\hat{X}_t = \frac{1}{n_{CP}} \sum_{i=1}^{n_{CP}} \xi_{t,k}^-$$  \hspace{1cm} (13)

$$P_t = \frac{1}{n_{CP}} \sum_{i=1}^{n_{CP}} (\xi_{t,k}^- - \hat{X}_t)(\xi_{t,k}^- - \hat{X}_t)^T + Q$$  \hspace{1cm} (14)

\(Q\) is the covariance of system noise. The measurement updating process is as follows:

$$\hat{X}_t = \hat{X}_t^+ + K_t (Z_t - \hat{Z}_t)$$  \hspace{1cm} (15)

$$P_t = P_t^+ + K_t P_t K_t^T$$  \hspace{1cm} (16)

$$P_{a,t} = \frac{1}{n_{CP}} \sum_{i=1}^{n_{CP}} (\xi_{t,k}^- - \hat{X}_t)(h(\xi_{t,k}^-) - h(\hat{X}_t))^T$$  \hspace{1cm} (17)

$$P_{\omega,t} = \frac{1}{n_{CP}} \sum_{i=1}^{n_{CP}} (h(\hat{\xi}_{t,k}^-) - \hat{Z}_t)(h(\hat{\xi}_{t,k}^-) - \hat{Z}_t)^T + R$$  \hspace{1cm} (18)

\(R\) is the covariance of measuring noise. When the measurement updating is completed, the cubature point is updated with (10), and then the process of (11) - (18) is repeated.
5. Simulation

Assuming that the mobile terminal moves in a square room of 100m², the coordinates of the three service base stations participating in the location are BS₁(0,100), BS₂(100,100), BS₃(50,0), and the unit is m.

5.1 ML Performance

When UMT moves randomly in the room, 20000 positions and the corresponding distance to each base station are obtained, and then the mean square error (MSE) of positioning is calculated in three different simulation situations.

1) Case 1: Change the estimated variance $\sigma^2$ of distance of one base station, the other two base stations remain unchanged. $\sigma^2 \in [0, 5]$.

2) Case 2: Change the estimated variance $\sigma^2$ of distance of two base stations, the third base station unchanged. $\sigma^2 \in [0, 5]$.

3) Case 3: Change the estimated variance $\sigma^2$ of distance of all base stations, $\sigma^2 \in [0, 5]$.

It can be seen from Fig.4 that the algorithm has good performance in estimating the initial position of UMT. When one of the estimated distances contains error, the resulting MSE is about 2.9 m². At this time, two circles with accurate radius can intersect and have higher weight in the maximum likelihood estimation. When two and three estimated distances contain errors at the same time, the MSE almost increases linearly. When the estimated variance $\sigma^2$ of three distances is 5m², the MSE of UMT position estimated by this algorithm is about 18.5m².

5.2 CKF Performance Analysis

Fig.5 shows the MSE changing curve estimated by CKF when the measuring interval is 200ms and the variance of RSS measuring error changes. Combining three RSS measurements and three TOA measurements for collaborative positioning, the MSE generated is much smaller than only using RSS measurements for positioning.

For example, the MSE of positioning using only RSS measurements is about 40 m² when the variance of measuring error is 16m². However, the MSE of collaborative positioning using RSS and TOA measurements is about 11 m².

As shown in Fig.6, when UMT moves along a given route, the moving trajectory estimated by CKF with two kinds of measurements is very close to the actual route, and the target tracking can be basically achieved.

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![Figure 4 Performance of ML](image-url)
6. CONCLUSION
A target positioning algorithm based on maximum likelihood recursive estimation and CKF is proposed in this paper. In the case that the traditional algorithm cannot get the optimal location, the algorithm estimates the initial location of UMT through the formation and segmentation of the target region, the maximum likelihood iteration and triangulation method, and then the nonlinear CKF combines the RSS and TOA measurements to achieve the location of mobile terminal. The simulation results show that the performance of CKF filter based on RSS and TOA is better than that based on only one measurement.

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