Helical multiferroics for electric field controlled quantum information processing

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Magnetoelectric coupling in helical multiferroics allows to steer spin order with electric fields. Here we show theoretically that in a helical multiferroic chain quantum information processing as well as quantum phases are highly sensitive to electric \(E\) field. Applying \(E\)-field, the quantum state transfer fidelity can be increased and made directionally dependent. We also show that \(E\) field transforms the spin-density-wave/nematic or multipoles of frustrated ferromagnetic spin–\(\frac{1}{2}\) chain in chiral phase with a strong magnetoelectric coupling. We find sharp reorganization of the entanglement spectrum as well as a large enhancement of fidelity susceptibility at Ising quantum phase transition from nematic to chiral states driven by electric field. These findings point to a new tool for quantum information with low power consumption.

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effect of electric field on the ground state properties have been addressed yet. The present study is a contribution to fill these gaps.

We note that the electric field coupling term resembles a Dzyaloshinskii-Moriya (DM) anisotropy, with a coupling constant $d = g_{ME}E$. Experiments indicate the presence of a small DM anisotropy in MF cuprates made of frustrated spin chains [57, 58]; previous theories considered it negligible, however. Here we show that even a tiny DM anisotropy modifies considerably the spin 1/2 chain characteristics. In particular, nematic spin-density-wave (SDW) state of magnon as well as multipolar phases transform into a chiral Luttinger liquid with non-zero spin current in the ground state.

First we focus analytically on a minimal system of four spins for different strengths of magnetic and electric (driving) fields for establishing an efficient protocol to field-control the entanglement. We also inspect quantum state transfer fidelity (QSTF) through MF chain and its E-field dependence.

For strong B-fields, i.e. $B$ is larger than $|J_1|$, 1, and $d = g_{ME}E$, the ground state is fully polarized, namely $|F\rangle = |↑↑↑↑\rangle$. The corresponding energy is $E_F = J_1 + 1 - 2B$. The pair entanglement between any two arbitrary spins and the chirality vanish. Decreasing the magnetic field so that $B_0 < B < d + J_1 + 2$, where $B_0 = \sqrt{(J_1 - d)^2 + 8d^2/2 + (J_1 - 2d)/2}$, the ground state is $|\psi_1\rangle = \frac{1}{\sqrt{2}} |\uparrow\downarrow\uparrow\uparrow\rangle + \frac{1}{\sqrt{2}} |\uparrow\uparrow\downarrow\uparrow\rangle$ with the corresponding energy $E_1 = -1 - B - d$. The chirality jumps to $k = \langle\psi_1|\kappa|\psi_1\rangle = 1$.

We observe a finite entanglement, as quantified by the pair concurrence between spins on and m sites [60] $C_{nm} = \max(0, \sqrt{R_1 - \sqrt{R_2 - \sqrt{R_3 - \sqrt{R_4}}}})$, where $R_n$ are the eigenvalues of the matrix $R = \rho_m^R \mathbf{O}_{2m} \rho_m^R$, and $ρ_R m$ is the reduced density matrix of the four spins system obtained from the density matrix $\hat{ρ}$ after tracing over two spins. One can contrast the amount of the entanglement stored in the pair correlations, quantified by the so-called two-tangle $\tau_2$, with the multi-spin entanglement of the whole spin chain, encapsulated in the one-tangle, $\tau_1 = 4\text{det} \rho_1$ [60] ($\rho_1$ is the single spin reduced density matrix). Two-tangle is calculated as $\tau_2 = \sum_m C_{nm}$. For the state $|\psi_1\rangle$, we find the ratio $\tau = \frac{\tau_1}{\tau_2} = 1$, thus half of the entanglement generated by decreasing the magnetic field (or increasing the electric field) in $|\psi_1\rangle$ is stored in the collective multi-spin correlations and half in the pair correlations. It is instructive to study the effect of $E$ and $B$ fields on quantum-transfer fidelity, QSTF, [62] between different states,

$$F(E, B, t) = \frac{|f_{j,s}(E, B, t)| \cos \gamma}{3} + \frac{|f_{j,s}(E, B, t)|^2}{6} + \frac{1}{2},$$

$$\gamma = \text{arg}\{f_{j,s}(E, B, t)\},$$

$$(2)$$

$|f_{j,s}(E, B, t)| = (j) \exp(-i\hat{H}t)|s\rangle$ is the transition amplitude between the states $|j\rangle$ and $|s\rangle$.

Time dependencies of QSTF obtained analytically between the initial state $|1\rangle = |\uparrow\uparrow\uparrow\uparrow\rangle$ and final states $|2\rangle = |\uparrow\downarrow\uparrow\uparrow\rangle$ and $|3\rangle = |\uparrow\uparrow\downarrow\uparrow\rangle$ are depicted in the Fig.1. The results evidence that $E$-field increases QSTF, particularly from $|1\rangle$ to $|3\rangle$. By inspecting (2) we infer that the oscillating behavior of $F$ in Fig.1, is related to the interference effect between different quantum states $\mathcal{E}_n(E)/\hbar$. Note that electric field $E$ enters in the energy levels through the DM coupling leading to a shift of state energies and the transition strength. For the explicit expression of Fidelity see supporting materials.

For confirmation we performed numerical calculations for systems with a large number of spins (not shown) and observed similar behavior of QSTF on $E$. We note that $E$-field breaks the parity symmetry of the MF spin chain. Hence, when $E$-field is present, clockwise and anticlockwise QSTF between the states $|j\rangle \rightarrow |s\rangle$ and $|s\rangle \rightarrow |j\rangle$ differ considerably (cf. Fig.1, which might be used for information transfer control via magnetic chirality [61]). Further decreasing the magnetic field below $B_0$, the ground state becomes

$$|\psi_2\rangle = \beta(|\uparrow\downarrow\uparrow\downarrow\rangle - i|\downarrow\uparrow\downarrow\uparrow\rangle - |\uparrow\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle + i|\downarrow\uparrow\uparrow\downarrow\rangle + |\uparrow\uparrow\downarrow\uparrow\rangle),$$

$$\lambda = \left(\frac{J_1}{4} - 1 + \sqrt{1 + J_1^2/16 - J_1^2/2 + d^2/2}\right)/2$$

and $\beta = 1/\sqrt{4 + 2\lambda^2}$. In this case for chirality we have $k = \langle\psi_2|\kappa|\psi_2\rangle = 8\lambda\beta^2$. and we plot its electric field dependence in Fig. 2 (a). The ratio between one-tangle $\tau_1$ and two-tangle $\tau_2$ in the ground state $|\psi_2\rangle$ reads $\tau = \frac{\tau_1}{\tau_2} = (\frac{2 - \lambda^2}{2\lambda})^2 < 1$, for $0 < d < \frac{2 - J_1^2/2}{2}$. Therefore, in this case the entanglement generated by the electric field is stored basically in many spin correlations rather then in two spin correlations.

Response sensitivity with changing the driving field amplitude is quantified by the fidelity susceptibility (FS) [63]. FS with respect to magnetic field vanishes as the magnetization is conserved in our model. FS with E-field changes is finite. E.g., for $|\psi_2\rangle$ state we obtain: $\chi_{FS}^E = (\alpha\beta/d)^2$ and depict it in Fig. 2 (b). As we see even small amplitude of the electric field
leads to the substantial reduction of the FS. Physical reason of the observed effect is transition to the chiral phase. We will study FS for long chains later, especially its behavior near the nematic to chiral quantum phase transition (QPT).

Hence depending on the driving fields, quantum information characteristics such as many particle entanglement and QSTF differ considerably. For macroscopic number of sites driving fields lead to different quantum phases and QPTs in frustrated FM chain. For MF chain we can expect thus a similar behavior that can possibly be controlled by $E$ field. Hence, we study below $E$-field steered quantum phases and their transitions in a macroscopic MF chain. We focus on the thermodynamic limit. Before addressing the many-body physics it is instructive to start with the two-magnon problem: For $d = 0$ and weak $J_1 < 0$ a bound state of two magnons forms below the scattering continuum. The bound state branch has a minimum for the total momentum $K = \pi$, for antiferromagnetic $J_2$ disfavors two-magnons occupying sites of the same parity.

We solved analytically the two-magnon problem for $d \neq 0$ (for $L \to \infty$). The solution of two-magnon problem [67] is shown in Fig. 3. One can clearly see that with including $d \neq 0$, the bound state minimum of the two-magnon state shifts from $K = \pi$ to $K = \pi - K_0$, where $K_0 \sim d$. The binding energy decreases as well gradually and after the critical value of $d > d_c(J_1)$ (e.g. for $J_1 = -1$, $d_c \approx 0.183$) the two-magnon scattering state minimum becomes energetically lower. Hence, bound states disappear from the ground state.

When the density of magnons is increased with decreasing the magnetic field we expect that the two-magnon bound states quasi-condense in the minimum of the two-magnon dispersion at $K = \pi - K_0$. Hence, the ground state will enter the nematic-chiral state for an arbitrary small $d \neq 0$. However, when $d > d_c$, the nematicity (magnon pair quasi-condensate) disappears via QPT, and the low energy behavior is dominated by a single-particle picture with $(S_i^z S_j^z)$ quasi-long-range ordered as shown in right panel of Fig. 4. Hence, we anticipate an $E$-field driven phase transition from the 'molecular' (2-magnon bound state) quasi-condensate to the 'atomic' (single-particle) quasi-condensate. This expectation is fully confirmed by the effective field theory description within bosonization techniques [67] where the competition between ferromagnetic $J_1$ (that binds magnons and produces nematic order) and electric field (promoting chirality) is resolved via an Ising QPT with changing $d$.

We have checked our analytical results with large scale numerical calculations using the density matrix renormalization group (DMRG) method [66] on chains up to $L = 240$ sites.

FIG. 2: a) Electric field dependence of chirality for the following values of the parameters $-J_1 = J_2 = 1$, $B = 1/4$. We see that electric field generates chirality. Qualitatively similar dependence holds even in thermodynamic limit. Electric field control of the magnetic chirality in the ferroaxial MF system RbFe(MoO$_4$)$_2$ was addressed in Ref. [62]. b) Electric field fidelity susceptibility. As we see, due to the transition to the chiral phase, even a weak electric field leads to a substantial reduction of the FS.
behavior of the correlation functions in nematic \(d < d_c\) and chiral \(d > d_c\) phases. In Fig. 5 we depict the phase diagram as a function of driving fields \(E\) and \(B\) at \(J_1 = -1\) (a) and \(J_1 = -3\) (b). To witness the transition from the nematic to the chiral state induced by \(E\) we studied the behavior of the entanglement spectrum (Fig. 6 (a)) and DM FS (Fig. 6 (b)). In the chiral phase of a \(J_1 - J_2\) chain and for \(d = 0\) the complete entanglement spectrum is doubly degenerate due to the spontaneously broken parity symmetry, however in the presence of \(d\) the degeneracy is lifted. Linear in \(L\) scaling of the peak of DM FS relative to the overall background shown in inset of Fig. 6 b) confirms the Ising nature of QPT.

We have studied as well the effect of DM anisotropy on multipolar phases of the \(J_1 - J_2\) chain for \(-4 < J_1 < -2.7\) involving bound states with more than 2 magnons. The minum of the multi-body bound state dispersion which is at \(K = \pi\) for \(d = 0\) (in both phases T and Q) shifts from \(\pi\) for \(d \neq 0\). In fact, 1\% \(\sim 2\%\) DM anisotropy in \(J_1\) is sufficient to remove the three-body and the four-body multipolar phases from the ground state phase diagram below the saturation magnetization. Instead, in the presence of a tiny \(d \neq 0\) the ground state magnetization experiences a macroscopic jump to the fully saturated value when increasing the magnetic field as depicted in Fig. 7. Note, for \(d = 0\) the metamagnetic region is squeezed in the close right-side vicinity of \(J_1 = -4\) point. In the presence of DM anisotropy the metamagnetic jump is observed in much broader region, starting at \(J_1 \simeq -2\) and extending even in the region \(J_1 < -4\) [67].

**In summary**, based on the spin current model for a helical multiferroic spin-\(\frac{1}{2}\) chain in external \(B\) and \(E\)-fields we find that both quantum information processing as well as ground state phases are extremely sensitive to an electric field that affects the magnetoelectric coupling. \(E\)-field increases strongly the quantum state transfer fidelity and makes it directional dependent (transfer in clockwise direction differs from that in anticlockwise direction). A tiny magnetoelectric coupling is sufficient to change the spin-density-wave/nematic or multipolar phases in favor of the chiral phase. We analyzed QPT induced by ME coupling and find in particular a sharp change of the entanglement spectrum and a large enhancement of the fidelity susceptibility at Ising QPT from nematic to chiral states. Our findings serve as the basis for \(E\) field controlled quantum information processing in helical multiferroics.

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DETAILS OF BOSONIZATION

Here we provide details of effective field theory description, bosonization applied to microscopic Hamiltonian

\[ \hat{H} = J_1 \sum_{i=1}^{2} \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_{i=1}^{2} \mathbf{S}_i \cdot \mathbf{S}_{i+2} - B \sum_{i=1}^{2} \mathbf{S}_i^z - d \sum_{i=1}^{2} (\mathbf{S}_i \cdot \mathbf{S}_{i+1})^z. \]  

(3)

To develop bosonization description it is convenient to consider the limit of strong frustration \( J_2 \gg |J_1| \) and weak DM anisotropy \( d \ll J_2 \). In this case the system may be viewed as two antiferromagnetic spin-\( \frac{1}{2} \) chains weakly coupled by the zigzag interchain coupling \( J_1 \) with DM anisotropy \( d \).

Low-energy properties of a single spin-\( \frac{1}{2} \) chain in a uniform magnetic field is described by the standard Gaussian theory known also as the Tomonaga-Luttinger liquid:

\[ \mathcal{H} = \frac{v}{2} \int dx \left\{ \frac{1}{K}(\partial_x \phi)^2 + K(\partial_x \theta)^2 \right\}. \]  

(4)

Here \( \phi \) is a real scalar bosonic field and is its dual field, \( \partial_x \phi = v \partial_x \theta \), with the commutation relations \( [\phi(x), \theta(y)] = i \Theta(x-y) \), where \( \Theta(x) \) is the Heaviside function. \( K \) is Luttinger liquid parameter and \( v \) is spin-wave velocity.

The exact functional dependences \( v(J_2, B) \) and \( K(J_2, B) \) for isolated chains are known (see [3] and references therein) from the numerical solution of the Bethe ansatz integral equations [4]. In particular, \( K \) monotonously with the magnetic field, whereas \( v \) decreases: \( K(B=0) = \frac{1}{2} \), \( v(B=0) = J_2 \pi/2 \) and \( K \rightarrow 1 \), \( v \rightarrow 0 \) for \( B \rightarrow B_{sat} \), where saturation value \( B_{sat} = 2J_2 \).

Long wave-length fluctuations of spin-1/2 chain are captured by the following representation of the lattice spin operators [2]:

\[ S_n^z \rightarrow \frac{1}{\sqrt{\pi}} \partial_x \phi + \frac{a}{\pi} \sin \left\{ 2k_F x + \sqrt{4\pi} \phi \right\} + M \]  

(5)

\[ S_n^- \rightarrow (1)^n e^{-i\theta} \sqrt{\pi} \left\{ c + b \sin \left\{ 2k_F x + \sqrt{4\pi} \theta \right\} \right\}, \]

Here \( M(B) \) is the ground state magnetization per spin which determines the Fermi wave vector \( k_F = (\frac{1}{2} - M) \pi \) and \( a, b \), and \( c \) are non-universal numerical constants.

For \( J_1 = d = 0 \), two decoupled chains are described by two copies of Gaussian models of the form (4) with pair of dual bosonic fields \([\phi_1, \theta_1]\) and \([\phi_2, \theta_2]\). Treating interchain couplings \( J_1 \) and DM anisotropy \( d \) perturbatively and introducing the symmetric and antisymmetric combinations of the fields describing the individual chains, \( \phi_{\pm} = (\phi_1 \pm \phi_2)/\sqrt{2K} \) and \( \theta_{\pm} = (\theta_1 \pm \theta_2)/\sqrt{K/2} \), the effective Hamiltonian density describing low-energy properties of (3) takes the following form:

\[ \mathcal{H}_{\text{eff}} = \mathcal{H}_0^+ + \mathcal{H}_0^- + \mathcal{H}_{\text{int}}, \]

\[ \mathcal{H}_0^\pm = \frac{v}{2} \left\{ (\partial_x \theta_{\pm})^2 + (\partial_x \phi_{\pm})^2 \right\}, \]

\[ \mathcal{H}_{\text{int}} = g_1 \cos \left( k_F + \sqrt{8\pi K} - \theta_{-} \right) - (g_2 \partial_x \theta_{+} + g_3) \sin \left( \sqrt{2\pi K} - \theta_{-} \right). \]  

(6)

The Fermi velocities \( v_{\pm} \propto J_2 \) and coupling constants are \( g_1 \propto J_1 \cos k_F [5] \), \( g_2 \propto J_1 \) and \( g_3 \propto d \), with proportionality coefficients involving short-distance cut-off. The Luttinger Liquid parameter of antisymmetric sector is given by

\[ K_- = K(h) \left\{ 1 + J_1 K(B)/\sqrt{2\pi v(B)} \right\}. \]  

(7)

The inter-sector coupling in Eq. (6) contains a term with coupling constant \( g_2 \) that represents an infrared limit of the product of \( z \)-components of in-chain and inter-chain vector chiralities [6].

\[ (\kappa_{2i-1,2i+1}^z + \kappa_{2i,2i+2}^z) \kappa_{2i,2i+1}^z \rightarrow \partial_x \theta_+ \sin \sqrt{2\pi K^-} \theta_-, \]  

(8)

where \( \kappa_{i,j}^z \equiv (\mathbf{S}_i \cdot \mathbf{S}_j)^z \).

The Hamiltonian (6) provides with the effective field theory describing the low-energy behavior of a strongly frustrated spin-\( \frac{1}{2} \) zigzag chain with DM anisotropy for nonzero magnetization \( M \). For small values of magnetization the Luttinger liquid parameter \( K_- \approx 1/2 \), and the inter-sector \( g_2 \) term has a higher scaling dimension than the strongly relevant \( g_1 \) and \( g_3 \) terms in the antisymmetric sector. In this case the system is in a phase with relevant competing couplings in antisymmetric sector. In contrast to that, at \( B = 0 \) all terms generated by the \( J_1 \) zigzag coupling are marginal and only DM coupling \( g_3 \) is a relevant perturbation.

The competition between \( \cos \sqrt{8\pi K^-} \theta_+ \) (nematicity) and \( \cos \sqrt{2\pi K^-} \theta_- \) (chirality) terms is resolved with an Ising phase transition in the antisymmetric sector with changing \( d/J_1 \) [7].

EFFECT OF DM ANISOTROPY IN FERROMAGNETIC REGION \( J_1 < -4J_2 \)

We now discuss the effect of DM interaction on ferromagnetic region \( J_1 < -4J_2 \). For \( d = 0 \), due to SU(2) symmetry the magnon gas behaves as non-interacting bosons. Deep inside ferromagnetic region DM interaction introduces repulsion (repulsion increases monotonously with increasing \( d \)) between magnons and below the fully polarized state chiral Luttinger liquid phase is realized for any \( d \neq 0 \) [8]. However, in close left-side vicinity of \( J_1 = -4 \) (hence \( J_1 < -4 \)) non-monotonous effect of DM on the effective interaction between magnons is observed. First, for small values \( d \rightarrow 0 \) DM anisotropy introduces repulsion between magnons, however with increasing \( d \) repulsion transforms into attraction and with further increasing \( d \) interaction between magnons becomes repulsive once again as shown in Fig. 1. Effective coupling constant of the magnon gas we extracted from the following relation [9, 10],

\[ g = \frac{2\hbar^2}{ma_{1D}} \]  

(9)
where \( m \) is mass of magnon and \( a_{1D} \) is one-dimensional scattering length, which we calculated analytically from the low energy scattering phase shift \( \delta(k) \),

\[
a_{1D} = \lim_{k \to 0} \frac{\delta(k)}{k}, \tag{10}
\]

where \( k \) is a relative momentum of scattering magnons.

For attractive regime \( g < 0 \), \( a_{1D} > 0 \), scattering length extracted from scattering problem coincides with the correlation length of the bound state of magnons. We depict in Fig. 1 scattering length from which one can observe due to Eq. (9) that effective interaction changes sign twice via resonance-like behavior when changing \( d \). For the values of \( d \) which correspond to the positive scattering length (and hence \( g < 0 \)), the external magnetic field induces a metamagnetic transition (macroscopic jump of the magnetization) from chiral Luttinger liquid to the fully polarized state (resulting in first order phase transition). For the parameters corresponding to negative scattering length (and hence \( g > 0 \)) magnetization will change smoothly all the way from \( M = 0 \) till \( M = 1/2 \), in particular leading to usual commensurate-incommensurate phase transition from chiral Luttinger liquid to fully polarized state when increasing the magnetic field strength.

\[ f_{1,2} = \frac{1}{4}(\exp[-i\varphi_2 t] - \exp[-i\varphi_4 t]) \]
\[ -\frac{i}{4}(\exp[-i\varphi_2 t] - \exp[-i\varphi_4 t]), \]
\[ f_{2,1} = -\frac{1}{4}(\exp[-i\varphi_1 t] - \exp[-i\varphi_3 t]) \]
\[ -\frac{i}{4}(\exp[-i\varphi_1 t] - \exp[-i\varphi_3 t]), \tag{11} \]
\[ f_{1,3} = f_{3,1} = -\frac{1}{4}(\exp[-i\varphi_2 t] + \exp[-i\varphi_3 t] \]
\[ -\exp[-i\varphi_4 t] - \exp[-i\varphi_5 t]), \]
\[ \varphi_2 = -J_2 - B - d, \varphi_3 = -J_2 - B + d, \]
\[ \varphi_4 = -J_1 + J_2 - B, \varphi_5 = J_1 + J_2 - B. \tag{12} \]

**FIDELITY**

Transition amplitudes and energy levels entering in the expression for fidelity Eq. (2), used for plotting Fig. 1:

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