Quantum Pumping in the Magnetic Field: Role of Discrete Symmetries

I. L. Aleiner 1, B. L. Altshuler 2, and A. Kamenev 3

1 Department of Physics and Astronomy, SUNY at Stony Brook, Stony Brook, NY 11794
2 Physics Department, Princeton University, NJ 08544
and NEC Research Institute, 4 Independence Way, Princeton, NJ 08540
3 Department of Physics, Technion, Haifa 32000, Israel.

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We consider an effect of the discrete spatial symmetries and magnetic field on the adiabatic charge pumping in mesoscopic systems. In general case, there is no symmetry of the pumped charge with respect to the inversion of magnetic field \( Q(B) \neq Q(-B) \). We find that the reflection symmetries give rise to relations \( Q(B) = Q(-B) \) or \( Q(B) = -Q(-B) \) depending on the orientation of the reflection axis. In presence of the center of inversion, \( Q(B) \equiv 0 \). Additional symmetries may arise in the case of bilinear pumping.

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The phenomenon of adiabatic charge pumping has attracted considerable theoretical and experimental interest during the last decade. It occurs when the Hamiltonian of the system is changed periodically with time: under certain condition a finite charge may be transmitted through the system during each period of the oscillation. Such a charge transfer takes place even if no dc voltage is applied. The idea is originally due to Thouless, who showed that in some one-dimensional systems the transmitted charge is quantized in the adiabatic limit.

Since the adiabatic pumping is a phase coherent mesoscopic effect it may be strongly sensitive to an external magnetic field, \( B \). The qualitatively important aspect is the presence (or absence) of any symmetry relations upon reversing the sign of \( B \). It was suggested theoretically that the transmitted charge, \( Q \), is invariant upon such field reversal

\[
Q(B) = Q(-B) . \tag{1}
\]

This relation, if true, would be an analog of the famous Onsager symmetry for the two–terminal conductance: \( G(B) = G(-B) \). The subsequent experiment on mesoscopic quantum dots appeared to be in a good agreement with Eq. (1). It soon became clear, however, that there is no real theoretical justification for the symmetry relation like Eq. (1). Indeed, unlike the conductance which is determined only by the moduli of the transmission eigenvalues, the pumped charge involves eigenfunctions as well. The latter, in general, do not obey any symmetry relation and so does not transmitted charge. As a result, the experimental confirmation of Eq. (1) in Ref. 2 appears to be a puzzle.

![Schematic representation of the possible reflection symmetries: (a) left–right (LR); (b) up–down (UD); (c) inversion (I).](image)

The purpose of this work is two–fold. First, we intend to demonstrate explicitly the role of the dissipation in lifting the symmetry with respect to the magnetic field inversion. Second, we consider how discrete spatial symmetries which the quantum dot may have manifest themselves in the magnetic symmetries of the transmitted charge.

It has been known for some time that the discrete symmetries may change level statistics as well as influence quantum correction to the transport coefficients. It is therefore natural to explore their effect on the adiabatic pumping and in particular its magneto–dependence. A quantum dot with two leads may posses three distinct types of spatial symmetries which, following Ref. 2, we call LR, UD, and I, see Fig. 1. If these spatial symmetries are kept intact in the pumping cycle, they give rise to definite magnetic symmetries of the transmitted charge. Curiously the corresponding magnetic symmetry are qualitatively distinct. For the UD symmetry we find

\[
Q_{UD}(B) = Q_{UD}(-B) , \tag{2}
\]

in agreement with an earlier speculation and experimental results. On the other hand, dots with LR symmetry obey a qualitatively different relation...
\[ Q_{LR}(B) = -Q_{LR}(-B). \]

When both of the symmetries are present, one finds that the only possibility consistent with both Eqs. (2) and (3) is

\[ Q(B) = 0, \]

The same relation holds under the weaker condition that the dot has the center of inversion (I), see Fig. 1(c). Finally, dots without any spatial symmetry, in general, do not have any definite relation between \( Q(B) \) and \( Q(-B) \). Let us present a formal proof of the advertised results.

In these notations, the Landauer formula for the two–terminal conductance is

\[ G = \frac{e^2}{2\pi h} \text{Tr} \{ \hat{S}^{\dagger} \hat{S} \hat{\sigma}_z \} = \frac{e^2}{2\pi h} \text{Tr} \{ \hat{S}^{\dagger} \hat{S} \hat{\sigma}_z \} \],

where \( \hat{S} \) is a matrix of the form Eq. (5) with diagonal reflection and transmission blocks: \( \hat{S} = \begin{pmatrix} \hat{S}^{\dagger} \hat{S} \end{pmatrix} \).

In terms of the decomposition, Eq. (5), this symmetry implies that

\[ \hat{S} = \hat{U} \hat{S} \hat{V}^{\dagger}. \]

The Pauli matrices \( \hat{\sigma}_i \) are defined as

\[ \hat{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \hat{\sigma}_z = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \]

where the \( 2 \times 2 \) structure represents the space of left–right leads (the same as in Eq. (1) and \( \hat{I} \) is the unit matrix. In these notations, the Landauer formula for the two–terminal conductance can be written as

\[ G = \frac{e^2}{2\pi h} \text{Tr} \{ \hat{S}^{\dagger} \hat{S} \hat{\sigma}_z \} = \frac{e^2}{2\pi h} \text{Tr} \{ \hat{\sigma}_z \} \cdot \text{Tr} \{ \hat{S}^{\dagger} \hat{S} \hat{\sigma}_z \}. \]

The \( \hat{S} \)–matrix must be invariant upon simultaneous transposition and magnetic field inversion [14]:

\[ \hat{S}(-B) = \hat{S}^{T}(B). \]

In terms of the decomposition, Eq. (5), this symmetry implies that

\[ \hat{U}(-B) = \hat{V}^{\dagger}(B), \quad \hat{V}(-B) = \hat{U}^{\dagger}(B), \quad \hat{S}(-B) = \hat{S}(B). \]

The last equality combined with Eq. (1) immediately yields the Onsager relation for the conductance, \( G(-B) = G(B) \). Applying these relation to the adiabatic pumping, one finds that the quantized component of the transmitted charge, \( Q_1 \), is a symmetric function of the magnetic field, \( Q_1(-B) = Q_1(B) \). On the other hand, the dissipative component of the pumped charge, \( Q_2 \), in general, does not exhibit any definite symmetry upon field reversal. Therefore, the experimental finding [4] of the magnetic field symmetric pumping in open quantum dot requires some additional understanding. In what follows we show that spatial symmetries of the dot indeed can enforce magnetic symmetries.
Consider a two-fold spatial symmetry operation, \( \hat{O} \), such that \( \hat{O}^2 = 1 \). Upon this symmetry the \( \hat{S} \)-matrix transforms according to
\[
\hat{S} \rightarrow \hat{O} \hat{S} \hat{O}^{-1} = \hat{O} \hat{S} \hat{O}.
\]
(15)
The system conductance, as well as shot–noise and higher order current correlators, must remain invariant under such transformation. This means that \( \text{Tr}\{\hat{S}^{k} \hat{\sigma}_{t} \hat{S}_{r} \hat{\sigma}_{t}^{k}\} \), is invariant under transformation (15), for any positive integer \( k \). Such invariance is possible only if one of the two conditions is fulfilled: either
\[
\hat{O} \hat{\sigma}_{z} = \hat{\sigma}_{z} \hat{O} \tag{16a}
\]
(consequently \( \hat{\sigma}_{t/r} \rightarrow \hat{\sigma}_{t/r} \) meaning that each lead is transformed to itself upon the symmetry transformation), or
\[
\hat{O} \hat{\sigma}_{z} = -\hat{\sigma}_{z} \hat{O} \tag{16b}
\]
(the leads are interchanged by the symmetry, \( \hat{\sigma}_{t/r} \rightarrow \hat{\sigma}_{r/t} \)).

An example of the symmetry (16a) is reflection relative to the axis connecting the two leads, see Fig. 1(b), i.e. UD symmetry. All the channels may be classified by the parity of their wavefunctions with respect to inversion around the reference axis. The corresponding \( \hat{S} \)-matrix at \( B = 0 \) acquires a block-diagonal structure in the space of even–odd channels. In other words,
\[
\hat{S}(B = 0) = \hat{\Sigma}_{z} \hat{S}(B = 0) \hat{\Sigma}_{z}^{\dagger},
\]
where \( \hat{\Sigma}_{z} \) is a diagonal \( 2n \times 2n \) matrix \( [\hat{\Sigma}_{z}]_{ik} = p_{i} \delta_{ik} \), and \( p_{i} = 1 (-1) \) for the even (odd) \( i \) th channel. If the magnetic field \( B \) is applied to the system, one can write the Hamiltonian in the Landau gauge as
\[
(p_{x} - B y)^{2} + p_{y}^{2} + V_{UD}(x, y),
\]
where the \( x \)-axis is the symmetry axis, \( V_{UD}(x, y) = V_{UD}(x, -y) \). It is easy to see that the Hamiltonian is invariant under simultaneous inversion of the magnetic field and reflection with respect to the \( x \)-axis. As a result
\[
\hat{S}(-B) = \hat{\Sigma}_{z} \hat{S}(B) \hat{\Sigma}_{z}^{\dagger},
\]
(17)
and therefore \( \hat{V}(B) = \hat{\Sigma}_{z} \hat{V}(-B), \hat{U}(B) = \hat{U}(-B) \hat{\Sigma}_{z} \). Substituting these relations into Eq. (16), one finds that the dissipative component of pumped charge is magnetic field symmetric, \( Q_{2}(B) = Q_{2}(-B) \). Since the quantized component \( Q_{1} \) is always symmetric, one concludes that for the UD reflection symmetry the transmited charge obeys Eq. (16).

An example of the symmetry of type (16b) is the reflection symmetry depicted on Fig. 1(a). We call it LR symmetry. In this case \( \hat{O} = \hat{\sigma}_{x} \), where \( \hat{\sigma}_{x} \) is defined in Eq. 3. In the appropriate gauge the Hamiltonian takes the form
\[
p_{x}^{2} + (p_{y} + B x)^{2} + V_{LR}(x, y)\text{ with } V_{LR}(x, y) = V_{LR}(-x, y).
\]
The system is invariant under inversion of the magnetic field direction and reflection with respect to the \( y \)-axis simultaneously. As a result one obtains
\[
\hat{S}(-B) = \hat{\sigma}_{x} \hat{S}(B) \hat{\sigma}_{x}.
\]
(18)

Employing Eq. (14) and the fact that \( \hat{\sigma}_{x} \hat{\sigma}_{x} \hat{\sigma}_{x} = -\hat{\sigma}_{x} \), one finds \( Q(-B) = -Q(B) \), as was announced in Eq. (4). Since the quantized component is always symmetric, it must be absent for the LR symmetry. The remaining pure dissipative component, \( Q_{2} \), is antisymmetric in this case. In the absence of the field, \( Q(B = 0) = 0 \), which is obviously true in case of the LR symmetry, because the two directions of the current flow are equivalent.

If the dot exhibits both UD and LR symmetries (four-fold symmetry in the terminology of Ref. (2)) which are preserved in the pumping process, then the pumped charge vanishes, \( Q(B) = 0 \). Indeed this is the only possibility to satisfy both Eqs. (2) and (3). The same is true if the dot has a center of inversion, Fig. 1(c). In this case the Hamiltonian is invariant upon changing direction of both \( x \) and \( y \) axis, but keeping \( B \) intact. As a result,
\[
\hat{\sigma}_{x} \hat{\Sigma}_{z} \hat{\Sigma}_{z} \hat{\sigma}_{x} = \hat{S}(B).
\]
Employing Eq. (14), one obtains \( Q = -Q = 0 \).

Due to the presence of the dissipative contribution \( Q_{2} \), see Eq. (2), the transmitted charge fluctuates from cycle to cycle; the resulting random process can be characterized by the distribution function \( P(Q) \). Using the formalism of Ref. (3), one may show that not only the average charge, but the entire distribution function, \( P(Q) \), exhibits magnetic symmetries if the dot is spatially symmetric. Namely, for the case of the UD symmetry one easily obtains \( P(Q, B) = P(Q, -B) \), for the LR symmetry \( P(Q, B) = P(-Q, -B) \), and for the inversion symmetry \( P(Q, B) = P(-Q, B) \) and for four-fold symmetry \( P(Q, B) = P(-Q, -B) = P(Q, -B) \).

So far we considered the pumping of arbitrary strength however preserving the initial symmetry of the dot. In the case of so-called bilinear response (4) one may make some conclusions even for perturbations violating the symmetry. Consider the time dependent potential of the form
\[
V(x, y, t) = V(x, y) + X_{1}(t)V_{1}(x, y) + X_{2}(t)V_{2}(x, y)
\]
(19)
the last two terms describe potential profiles created by the pumps. In the lowest non-vanishing order in \( X_{1,2} \), Eq. (10) takes the form
\[
Q_{\text{d}} = \frac{e}{2\pi} A_{X} \text{ Im } \text{Tr} \left\{ \frac{\partial \hat{S}}{\partial X_{1}} \frac{\partial \hat{S}^{\dagger}}{\partial X_{2}} \hat{\sigma}_{z} \right\}_{X_{1,2}=0},
\]
(20)
where \( A_{X} \) is the area on the \( (X_{1}, X_{2}) \) plane enclosed by the contour \( [X_{1}(t), X_{2}(t)] \).
Let the unperturbed system be UD-symmetric, i.e. $V(x, y) = V(x, -y)$. We wish to consider the perturbations within irreducible representation of the discrete symmetry group, i.e. either symmetric or antisymmetric: $V_i(x, y) = p_i V_i(x, -y)$, where $p_i = \pm 1$. We have already considered the case of $p_1 = p_2 = 1$, i.e. not changing the symmetry of the dot, see Eq. (2). The question we are going to address now is what happens if at least one of the factors $p_i$ is negative. Similar problem can be posed for the LR symmetric dot: $V(x, y) = V(-x, y)$ and $V_i(x, y) = p_i V_i(-x, y)$.

Analogously to Eqs. (17) and (18), one obtains

$$\frac{\partial \hat{S}(-B)}{X_i} = p_i \hat{\sigma}_x \frac{\partial \hat{S}(B)}{X_i} \hat{\sigma}_x, \quad (LR);$$

$$\frac{\partial \hat{S}(-B)}{X_i} = p_i \hat{\Sigma}_{eo} \frac{\partial \hat{S}(B)}{X_i} \hat{\Sigma}_{eo}, \quad (UD).$$

Thus, we find instead of Eqs. (2) and (3)

$$Q_{UD}^{bl}(B) = p_1 p_2 Q_{UD}^{bl}(-B), \quad (21a)$$
$$Q_{LR}^{bl}(B) = -p_1 p_2 Q_{LR}^{bl}(-B). \quad (21b)$$

To avoid a confusion, notice that Eqs. (21) are not valid beyond the bilinear response approximation if at least one of $p_i = -1$.

To conclude, we have demonstrated that a general adiabatic pump does not obey any symmetry with respect to magnetic field reversal. Such symmetry, however, may be recovered as a result of the discrete spatial symmetry of the pump. Moreover, different spatial symmetries lead to a qualitatively distinct behavior in the magnetic field. The average transmitted charge (and its higher moments) is a symmetric or antisymmetric (or identically zero) function of the field depending on the type of spatial symmetry of the dot. Our findings may possibly clarify the origin of the experimentally observed magnetic symmetry [7], as well as motivate further experiments.

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