Fiducial Drell-Yan production at the LHC improved by transverse-momentum resummation at $N^4\text{LL}_p+N^3\text{LO}$

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Abstract

Drell-Yan production is one of the precision cornerstones of the LHC, serving as calibration for measurements such as the $W$-boson mass. Its extreme precision at the level of $1\%$ challenges theory predictions at the highest level. We present the first independent calculation of Drell-Yan production at order $\alpha_s^3$ in transverse-momentum ($q_T$) resummation improved perturbation theory. Our calculation reaches the state-of-the-art through inclusion of the recently published four loop rapidity anomalous dimension and three loop massive axial-vector contributions. We compare to the most recent data from CMS with fiducial and differential cross-section predictions and find excellent agreement at the percent level. Our resummed calculation including the matching to $Z+\text{jet}$ production at NNLO is publicly available in the upcoming CuTe-MCFM 10.3 release and allows for theory-data comparison at an unprecedented level.

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1. Introduction

Drell-Yan ($Z$-boson) production is among the most important standard candles of the high-energy LHC physics program due to its very precise measurement at the level of one percent \cite{1-4}. It is used for the extraction of the strong coupling \cite{5, 6}, fitting of parton distribution functions \cite{7, 8} that further constrain and determine Standard Model (SM) input parameters, and is also a crucial ingredient of the $W$-boson mass determination \cite{9-11}.

The current precision in QCD for Drell-Yan predictions is at the level of $\alpha_s^3$ both fully differentially \cite{12-15} and more inclusively \cite{16, 17}. Calculations at this order have been performed at fixed order ($N^3\text{LO}$) and including the effects of transverse momentum ($q_T$) resummation up to $N^3\text{LL}$ logarithmic accuracy. Currently all fully differential calculations at the level of $\alpha_s^3$ employ transverse momentum subtractions or transverse momentum resummation. They have been enabled by the recent availability of the three-loop beam-functions \cite{18-20}, complete three-loop hard function \cite{21-25} and the existence of a NNLO calculation of $Z+\text{jet}$ production \cite{26-30}. Beyond pure QCD corrections, the full set of two-loop mixed QCD$\times$EW corrections have been calculated very recently \cite{31-33}.

Traditionally there has been a focus on fixed-order calculations for total fiducial cross-sections, but now that relatively high perturbative orders have been reached, convergence issues of the perturbative series due to fiducial cuts have been identified \cite{34-36}. These issues trace back to a linear sensitivity of acceptance cuts to small transverse momenta, where fixed-order predictions are unreliable, leading to factorially divergent contributions \cite{35}. It has shifted the focus towards resummation-improved results even for total fiducial cross-sections, which can cure such problems without requiring any modification of analysis cuts.

All calculations matched to NNLO $Z+\text{jet}$ fixed-order at large $q_T$ have so far been based on the NNLOjet results
[27]. Different implementations of $q_T$ resummation and subtractions are built on top of this calculation. Results for a matching to the resummation in DYTurbo [37] have been presented in ref. [13] where only non-singlet and vector singlet contributions are included and truncation uncertainties are estimated by considering differences between successive orders. A matching to the RadISH resummation approach [14, 38] has been presented in refs. [12, 14], also neglecting axial singlet contributions. Axial singlet contributions in the $m_t \to \infty$ EFT have been included in the resummed calculation of ref. [39] but without the matching to $\alpha_s^3$ fixed-order. The NNLO-jet setup has subsequently been extended to calculate fiducial cross-sections also at fixed-order $N^3$LO, comparing the impact of power corrections through studying the difference between symmetric and product cuts [15] and comparing with 13 TeV ATLAS data [4]. The RadISH based calculations provide uncertainty estimates for differential and fiducial results for the first time.

Despite these studies, it is crucial to have an independent calculation of both the fixed-order components and the resummation implementation. While the NNLOjet calculation is tested by the correct approach of the triple singular limits through an implementation of (differential) $q_T$ subtractions, it is important to also probe the finite contributions. As well as acting as a cross-check, an additional calculation also provides an independent estimate of uncertainties.

In this paper we present both a publicly available calculation of $Z$-boson production as well as differential and fiducial cross-sections at the state-of-the-art level $N^4LL_p+N^3$LO. The “p” subscript denotes that we are $\alpha_s^3$ accurate in fixed-order and RG-improved perturbation theory up to missing effects from exact $N^3$LO PDFs that contribute both to fixed-order and logarithmic accuracy at $\alpha_s^3$. We include the four loop rapidity anomalous dimension [40, 41], pushing the accuracy to this level for the first time. We also include the massive three-loop axial singlet contributions [25] without the need for approximations. We compare at $\alpha_s^3$ accuracy with the CMS 13 TeV precision measurement. All parts, both resummation and fixed-order are publicly available in the next CuTe-MCFM release 10.3. Public codes are crucial to ensure reproducibility, allow the community to perform independent checks, to calculate predictions with different parameters, and provide the basis for future theoretical improvements as strongly advocated by our community [42].

In section 2 we provide technical details of our calculation before presenting results in section 3 and concluding in section 4 with an outlook.

2. Calculation

We consider QCD corrections to the process $q + \bar{q} \to Z/\gamma(\rightarrow l^- + l^+)$.

Our calculation in CuTe-MCFM [43, 44] matches resummation at the level of $N^3LL_p$ to $\alpha_s^3$ fixed-order $Z+\text{jet}$ production. Apart from missing $N^3$LO PDF effects we achieve full $\alpha_s^3$ fixed-order and transverse momentum renormalization-group-improved (RG-improved) logarithmic accuracy by counting $\log(q_T^2/Q^2) \sim 1/\alpha_s$.\footnote{While we are neglecting $N^3$LO PDFs for full $N^4LL+N^3$LO accuracy, it has been customary in the literature to refer to predictions as $N^3$LO despite the lack of these corrections.} We further calculate fixed-order $N^3$LO results based on $q_T$-subtractions. Our calculation involves many contributions at the fixed-order and at the resummation level, which we discuss separately below.

2.1. Resummation

The resummation is based on the SCET formalism derived in refs. [45–47] and originally implemented as CuTe-MCFM in ref. [43] to $N^3$LL. Large logarithms $\log(q_T^2/Q^2)$ are resummed through RG evolution of hard- and beam functions in a small-$q_T$ factorization theorem. Rapidity logarithms are directly exponentiated through the collinear-anomaly formalism.

At large $q_T$ the small-$q_T$ factorization theorem becomes invalid and one has to switch to fixed-order predictions. We switch using a transition function that smoothly interpolates between resummation and fixed-order without disturbing subleading power corrections, as detailed in ref. [43]. Within this procedure the overlap between fixed-order and resummation has to be subtracted by expanding the resummation to a fixed-order. This difference is referred to as matching corrections. For $Z$ boson production they quickly approach zero for $q_T \to 0$.
and remain at the few percent level up to $\sim 30$ GeV, see our dedicated discussion below.

Three loop transverse momentum dependent beam functions have been calculated in refs. [18–20] and implemented in ref. [48] in CuTe-MCFM. Together with the $\alpha_s^3$ hard function this enables resummation at the level of $\mathcal{N}^3LL'$. The resummation of linear power corrections [34] has been included in CuTe-MCFM since its initial implementation through a recoil prescription [49]. They are crucial to improve the resummation itself as well as the numerical stability by allowing a larger matching cutoff (the value of $q_T$ below which matching corrections are set to zero). It is also crucial for the stability of our fixed-order N$^3$LO results in the presence of symmetric lepton cuts, see below.

In this study we have upgraded the resummation to the logarithmic accuracy of $\mathcal{N}^4LL_p$ through the inclusion of the four loop rapidity anomalous dimension [40, 41]. While the five loop cusp anomalous dimension is also a necessary ingredient, it only enters through the hard function evolution and is numerically completely negligible. Already at a lower order the hard function evolution is precise at the level of one per-mille. We nevertheless include such effects in the hard function evolution by taking four loop collinear anomalous dimensions from ref. [50] and a five loop cusp estimate from ref. [51] that agrees with our own Padé approximant estimate. The five loop beta function is taken from ref. [52].

Transverse momentum Fourier conjugate logarithms $L_\perp \sim \log(x_T^2/\mu^2)$ appearing in the factorization theorem would traditionally be integrated over the full range of $x_T$. This requires the introduction of a prescription to avoid the Landau pole. Following the SCET resummation formalism of ref. [45, 46] this is not necessary as scales are always set in the perturbative regime. The formalism further employs an improved power counting $L_\perp \sim 1/\sqrt{\alpha_s}$ that is crucial to improve the resummation at small $q_T$ [46]. At $\mathcal{N}^4LL$ the $\alpha_s^3$ beamfunctions [18–20] are then not sufficient for improved $\alpha_s^3$ accuracy. Using the beamfunction RGEs we reconstructed the logarithmic beamfunction terms up to order $\alpha_s^6L_1^1$, $\alpha_s^4L_1^2$ and $\alpha_s^3L_2^2$. We performed the Mellin convolutions of beam function kernels and splitting functions up to three loops [53, 54] using the MT package [55].

The hard function entering the factorization formula consists of MS-renormalized virtual corrections. For Drell-Yan production one typically distinguishes between different classes of corrections based on the following decomposition. The Feynman rule vertex for the photon coupling to fermions is $-ieQ_f\gamma^\mu$, while the $Z$ coupling is $-ie\gamma^\mu(v_L^fP_L + v_R^fP_R)$. In terms of vector and axial-vector components this decomposes as

$$v_L^fP_L + v_R^fP_R = \left(\frac{1}{2}v_L^f + \frac{1}{2}v_R^f\right) - \gamma_5\left(\frac{1}{2}v_L^f - \frac{1}{2}v_R^f\right).$$

The first term constitutes the vector coupling and is dressed by a vector form-factor $F_V$ that encapsulates higher-order corrections. The second term constitutes the axial-vector coupling and is dressed by an axial-vector form-factor $F_A$. For a photon exchange $v_L = v_R = 1$ and $F_A = 0$. On the other hand, the coupling of $Z$ bosons to quarks involves both a vector ($F_V$) and an axial-vector ($F_A$) form factor. A common approximation is to include only non-singlet contributions, which leads to $F_A = F_V$.

The three-loop corrections to the vector part have been known for a while now [21–23], while the three-loop corrections to the axial singlet part have only been computed recently in purely massless QCD [24] and with full top-quark mass dependence [25]. In our calculation we include the complete three-loop corrections with full top-quark mass dependence. While these contributions are small, the top-quark mass dependence does not decouple in either the $m_t \to \infty$ limit or the low-energy limit, in contrast to the vector case.

2.2. $Z$+jet NNLO fixed order

Our fixed-order NNLO $Z$+jet calculation is based on ref. [28], employing 1-jettiness subtractions [26, 56, 57]. For 1-jettiness subtractions at NNLO a crucial new ingredient compared to 0-jettiness is the NNLO soft function which has been calculated in refs. [58, 59]. Top-quark loop corrections to $Z$+jet and $Z$+2 jet production have been known analytically for some time [60] and are included in our calculation. Two-loop jet axial singlet contributions in the $Z$+jet hard function are unknown so far and have been neglected in our calculation.

We have performed extensive cross-checks of all elements of the calculation. We find numerical agreement between all bare amplitude expressions and Recola [61], and have reproduced the non-singlet hard function that
was originally taken from the code PeTeR [62, 63] with an independent re-implementation from refs. [64–66]. We have thoroughly tested the implementation of the subtraction terms using the same methodology as in ref. [67]. Compared to the original implementation [28] we identified an inconsistency in a small number of subtraction terms and in the crossing of one-loop axial-vector helicity amplitudes. As a final check, we compared with fiducial results presented in ref. [68] for different partonic channels and find agreement.

2.3. Matching corrections and cutoff effects

Since our calculation is based on 1-jettiness slicing subtractions, unlike the local antenna subtractions used in the NNLOjet calculation [27], we have to pay attention to residual slicing cutoff effects. Jettiness slicing at the level of NNLO in association with one jet is widely believed to have reached its limits of applicability. But, as we demonstrate in this paper, optimized phase-space generation and the inclusion of linear power corrections together with an efficient parallelization for the use of modern HPC resources [69] allows us to compute results at the level of $N^3LL_p+N^3LO$ and $N^3LO$ with negligible systematic cutoff uncertainties.

Nevertheless, we had to choose the $q_T$ cutoff for the resummation matching corrections and the $q_T$ cutoff for the $q_T$ subtractions at $N^3LO$ low enough that residual cutoff effects can be neglected:

In fig. 1 we show the matching corrections of the $\alpha_s, \alpha_s^2$ and $\alpha_s^3$ coefficients relative to the naively matched result at $N^3LL_p$ for the CMS analysis in the results section. The naively matched result consists of matching corrections and resummed result without transition function. The size of the matching corrections on the one hand indicates where the transition function needs to switch between resummed and fixed-order calculations. In this case matching corrections become sizable around 50 GeV and the resummation quickly breaks down beyond 60 GeV. This motivates our choice to use a transition function as detailed in ref. [43] using $x_T^{\text{max}} = (q_T^{\text{max}}/M_Z)^2$ with $q_T^{\text{max}}$ in the range 40 to 60 GeV. The transition uncertainties are then comparable to uncertainties in the fixed-order and resummation region and we are therefore minimally sensitive to the precise range and shape of the transition.

| $q_T^2$ | $\alpha_s$ coeff. | $\alpha_s^2$ coeff. | $\alpha_s^3$ coeff. | sum |
|---------|---------------------|---------------------|---------------------|-----|
| 1       | 0.00                | 0.00                | 0.00                | 0.00 |
| 2       | 0.00                | 0.00                | 0.00                | 0.00 |
| 3       | 0.00                | 0.00                | 0.00                | 0.00 |
| 5       | 0.00                | 0.00                | 0.00                | 0.00 |
| 10      | 0.00                | 0.00                | 0.00                | 0.00 |
| 20      | 0.00                | 0.00                | 0.00                | 0.00 |
| 30      | 0.00                | 0.00                | 0.00                | 0.00 |
| 50      | 0.00                | 0.00                | 0.00                | 0.00 |

Figure 1.: Matching corrections of the $\alpha_s, \alpha_s^2$ and $\alpha_s^3$ coefficients relative to the naively matched result at $N^3LL$ (matching corrections + resummed result without transition function) for the CMS cuts as in the main document.

At $\alpha_s$ and $\alpha_s^2$ matching corrections can be safely neglected below 1 GeV, but the numerical implementation allows for smaller cutoffs if necessary. Figure 1 further justifies our neglect of matching corrections below 5 GeV at $\alpha_s^3$. The approach to zero of the matching corrections towards smaller $q_T$ shows that the large logarithms present in the fixed-order and expanded resummation calculations cancel. While the $\alpha_s^3$ matching corrections at 5 GeV are zero within numerical uncertainty, from the lower order results we see fluctuations at the level of one percent for smaller values of $q_T$. On the fiducial cross-section we therefore estimate an uncertainty due to missing matching corrections by multiplying the resummed result integrated up to 5 GeV with one percent. This is about 1 pb, our quoted numerical precision. Similarly the effect on the $Z$-boson $q_T$ distributions below 5 GeV is expected to be less than 1%. This is also the region with substantial resummation uncertainties from a variation of the low scale. The effect is therefore negligible. The size of the corrections is in line with the findings of previous studies [13, 15].

The $\alpha_s^3$ coefficient of fig. 1 has been obtained using a dynamic $\tau_1^{\text{cut}}$ value of $7.6 \cdot 10^{-5} \cdot \sqrt{(q_T^2)^2 + m_Z^2}$, which is about 0.007 GeV for small $q_T$. Our one-jettiness is defined by

$$\tau_i = \sum_{\text{partons}} \min_{k} \left\{ \frac{2r_i q_k}{Q_i} \right\},$$

where the sum over $i$ is over the two beam momenta and the jet axis determined by anti-$k_T R = 0.5$ clustering.
Table 1.: Fiducial cuts for $Z \to l^+l^-$ used in the ATLAS 13 TeV analysis [4].

| Lepton cuts | $q_T^l > 27$ GeV, $|\eta^l| < 2.5$ |
| Mass cuts   | $66.0$ GeV $< m_{l^+l^-} < 116.0$ GeV |

and $Q_i$ are chosen to be $2E_i$. We have checked the $\tau_1^{\text{cut}}$ dependence to determine that with the given $\tau_1^{\text{cut}}$ cutoff we can only reliably use a $q_T$ resummation matching cutoff of 5 GeV, as shown in fig. 1.

Smaller matching cutoffs would require smaller $\tau_1^{\text{cut}}$ values for the large $q_T$ logarithms to cancel between fixed-order NNLO $Z+\text{jet}$ calculation and fixed-order expansion of the resummation, increasing computational costs significantly: For a $q_T$ cutoff of 5 GeV the small size of the 1-jettiness parameter requires computing resources of about 6000 NERSC Perlmutter node hours for all fiducial and differential results presented in the following (we ran with 256 nodes for about one day). While a cutoff of 2 GeV to 3 GeV could likely be achieved with more resources (due to requiring a smaller jettiness parameter), the inclusion of subleading 1-jettiness power corrections, which have currently only been computed at a lower order [70], could be a more promising resource-saving approach.

2.4. $N^3$LO fixed order

For our $N^3$LO fixed-order results we have integrated the $q_T$ factorization theorem expanded to $\alpha_s^3$ over $q_T$ up to a slicing cutoff $q_T^{\text{cut}}$. This allows us to implement $q_T$ subtractions by combining this contribution with the fixed-order $Z+\text{jet}$ NNLO calculation, regulating IR divergences through the cutoff and extrapolating $q_T^{\text{cut}}$ to zero. We have checked that NNLO results obtained with this implementation agree with previous implementations of jettiness subtractions and $q_T$-subtractions in MCFM [71, 72].

As an additional cross-check of our calculation and validation of results in the literature, we compare with the fiducial cross-section results in ref. [15] for the most challenging case of symmetric lepton cuts. The authors employ cuts for the 13 TeV ATLAS analysis [4] as in table 1. We furthermore adopt their choice of PDF, NNPDF4.0 at NNLO with $\alpha_s(m_Z) = 0.118$ [73], and the $G\mu$ scheme with $m_Z = 91.1876$ GeV, $m_W = 80.379$ GeV, $\Gamma_Z = 2.4952$ GeV, $\Gamma_W = 2.085$ GeV, $G_F = 1.663787 \times 10^{-5}$ GeV$^{-2}$.

We can naturally fully reproduce their fixed-order results up to NNLO. At $N^3$LO we must estimate the slicing uncertainty due to our cutoff of $q_T^{\text{cut}} = 5$ GeV. With a 5 GeV slicing cutoff, and including linear power corrections, we obtain a value of $-22.6 \pm 1.4$ pb (numerical). Using a 10 GeV slicing cutoff instead we obtain a value of $-21.3 \pm 1.4$ pb (numerical). From this variation we therefore assign a further slicing uncertainty of about 1 pb.

This uncertainty estimate is also supported by an examination of the linear power corrections, which can be easily computed in our formalism. The knowledge of the size of the linear power corrections allows another estimate of slicing cutoff effects, since further corrections should be even more power suppressed. Figure 2 shows the size of the linear power corrections for our CMS cuts in the results section. For the $N^3$LO coefficient the linear power corrections are accidentally small around a slicing cutoff of 10 GeV and would be larger for a 1 GeV slicing cutoff. Around 0.5 GeV the linear power corrections at $N^3$LO are largest, about $-5$ pb.
Knowing that further slicing cutoff power corrections are suppressed by a factor of $q_T^{\text{cut}}/Q$, we expect further corrections of less than 1 pb with a cutoff of 10 GeV or less. This is in line with our cutoff variation discussed above. Ultimately in the presence of symmetric cuts a slicing uncertainty of about 1 pb is unavoidable for slicing calculations and can only be cured by a local subtraction scheme at fixed-order.

The final comparison with the NNLOjet result of ref. [15], for the N^3LO corrections is:

this work: $-22.6 \text{ pb} \pm 1.4 \text{ pb (num.)} \pm 1 \text{ pb (slicing)}$

Ref. [15]: $-18.7 \text{ pb} \pm 1.1 \text{ pb (num.)} \pm 0.9 \text{ pb (slicing)}$

The NNLOjet result is obtained with a slicing cutoff of $q_T^{\text{cut}} = 0.8 \text{ GeV}$ that is varied by a factor of two to obtain the slicing uncertainty. Our estimated slicing uncertainty is similar, which is only because the (dominant) linear power corrections are known. Cuts that eliminate the linear power corrections altogether [35] improve this situation [15].

The two results are in agreement within mutually large uncertainties of about 10% on the N^3LO coefficient. Fortunately this uncertainty on the N^3LO coefficient reduces to about three to four per mille for the full result and is currently insignificant compared to truncation uncertainties and the experimental precision (which is limited by a 2% luminosity uncertainty). But in the future local subtraction methods for N^3LO are clearly preferred.

3. Results

We present results at $\sqrt{s} = 13 \text{ TeV}$ using the NNPDF4.0 PDF set at NNLO with $\alpha_s(m_Z) = 0.118$ [73]. Electroweak input parameters are chosen in the $G_\mu$ scheme with $m_Z = 91.1876 \text{ GeV}$, $m_W = 80.355 \text{ GeV}$, $\Gamma_Z = 2.4952 \text{ GeV}$ and $G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$. We denote the matched resummation accuracy with $\alpha_s$ for N^3LL+NLO, $\alpha_s^2$ for N^3LL+NNLO and $\alpha_s^3$ for N^4LL+p^3LO.

Our fiducial selection cuts in table 2 are chosen to compare with the most recent $Z$-boson precision measurement by CMS in ref. [3]. The symmetric lepton cuts used in this analysis cause a poor perturbative convergence for fixed-order calculations and can also lead to numerical issues. However, the use of resummation resolves such issues [34–36].

In our calculation we distinguish between three scales for estimating uncertainties. We use a low (resummation) scale $\sim q_T$ (see ref. [43] for details) to which RGEs are evolved down from the hard scale chosen as $\sqrt{m_Z^2 + p_T^2_{Z,T}}$. The CuTe-MCFM resummation formalism [45–47] is originally derived using an analytic regulator to regulate rapidity divergences in the transverse position dependent PDFs (collinear anomaly formalism). This is opposed to using a rapidity regulator that introduces a rapidity scale [74]. We have re-introduced a scale estimating the effect of a different rapidity scale as suggested in ref. [75]. We vary hard and low scale by a factor of two, and rapidity scale by a factor of six, tuned on the truncation of the improved power counting, to obtain a robust estimate of truncation uncertainties. Most importantly our formalism allows for the variation of the low scale, which dominates uncertainties at small $q_T$. Last, in our uncertainty bands we include the effect of varying the transition function in the region of about 40 GeV to 60 GeV where matching corrections become significant, following the same procedure as in ref. [43] at a lower order.

While for Drell-Yan production our resummation formalism does not set the central low scale below $\sim 2 \text{ GeV}$ [43], a downwards variation would probe close towards the non-perturbative regime. We therefore set a minimum of 2 GeV and symmetrize the uncertainty bands since the variation becomes ineffective at small scales. Note that about 2% of the total fiducial cross-section comes from the region $q_T < 1 \text{ GeV}$ where one might expect additional non-perturbative effects of an unknown size.

The CMS collaboration [3] provides both differential results to compare with as well as a total fiducial cross-section measurement, that we discuss in turn below.

3.1. Differential results

In fig. 3 we present the $Z$ boson transverse momentum distribution predictions at order $\alpha_s$, $\alpha_s^2$ and $\alpha_s^3$ and compare it to the CMS 13 TeV measurement [3] with cuts as in table 2.

Overall there is an excellent agreement between theory
Table 2.: Fiducial cuts for $Z \rightarrow t^+t^-$ used in the CMS 13 TeV analysis [3].

| Lepton cuts | $q_T^l > 25 \text{GeV}, \mid y^l \mid < 2.4$ |
| Separation cuts | $76.2 \text{GeV} < m_t^{l^+l^-} < 106.2 \text{GeV}$, $\mid y^{l^+l^-} \mid < 2.4$ |

and data at the highest order. Going from $\alpha_s^2$ to $\alpha_s^3$ decreases uncertainties and improves agreement with data noticeably at both large and small $q_T$. In the first bin $0 \text{GeV} < q_T < 1 \text{GeV}$ we notice a relatively large difference to the data, but this is also where one would expect a non-negligible contribution from non-perturbative effects. We note that the impact of the corrections included in $N^4LL_\alpha$ is a noticeable shift in this distribution, compared to $N^3LL'$, as discussed further in appendix B.

For the $\Phi^*$ distribution shown in fig. 4 results are overall very similar. For the transverse momentum distribution we neglect matching corrections at $\alpha_s^3$ below $q_T < 5 \text{GeV}$. Here we correspondingly neglect them below $\Phi^* < 5 \text{GeV}/m_Z \sim 0.05$ and at lower orders below $\Phi^* < 1 \text{GeV}/m_Z \sim 0.01$, an overall per-mille level effect in that region.

Since our resummation implementation is fully differential in the electroweak final state we can naturally also present the transverse momentum distribution of the final state lepton, see fig. 5. This is plagued by a Jacobian peak at fixed-order and crucially requires resummation. The higher-order $\alpha_s^3$ corrections further stabilize the results with smaller uncertainties.

### 3.2. Total fiducial cross-section

In table 3 we present total fiducial cross sections. Uncertainties of the fixed-order NNLO ($\alpha_s^2$) result, obtained by taking the envelope of a variation of renormalization and factorization scales by a factor of two, are particularly small at the level of 0.5% and do not improve towards $N^3LO$ with large corrections. The resummation improved results are obtained by integrating over the matched $q_T$ spectrum shown in fig. 3. Uncertainties of the resummation improved predictions are obtained by taking the envelope of the variation of hard, low and rapidity scales in the fixed-order and resummation region. The matching uncertainty from the transition function variation is quoted separately. We estimate the effect of neglecting matching corrections at $\alpha_s^3$ below $q_T \leq 5 \text{GeV}$ to be less than 1 pb.

The resummation improved result at $\alpha_s$ has large uncertainties that stem from an insufficient order of the resummation ($N^3LL$), which still has substantial uncertainties in the Sudakov peak region (c.f. fig. 3). The results quickly stabilize, with less than a percent difference between the central $\alpha_s^2$ and $\alpha_s^3$ predictions. Nevertheless, the uncertainties we obtain are noticeably larger than the fixed-order uncertainties. We further observe that going from $N^3LL/\alpha_s^2$ to $N^4LL_p/\alpha_s^3$ does not reduce uncertainties as substantially as when going from $\alpha_s$ to $\alpha_s^2$. This is because the resummation uncertainties around the Sudakov peak region at small $q_T \sim 5 \text{GeV}$ do not improve dramatically.

While this behavior, of only moderately decreasing uncertainties going from $\alpha_s^2$ to $\alpha_s^3$, is consistent with the
Table 3.: Fiducial cross-sections in pb for the cuts in table 2 and input parameters as in the text. Uncertainties for the resummation-improved results include matching to fixed-order (mat.), neglected matching corrections (m.c.), and by scale variation (sc.). The fixed-order result at N^3LO has an additional slicing-cutoff uncertainty. For comparison, the final row shows the CMS measurement (for electron and muon channels combined) [3].

| Order $k$ | fixed-order $\alpha_s^k$ | res. improved $\alpha_s^k$ |
|-----------|-----------------|------------------|
| 0         | $694^{+85}_{-92}$ | —                |
| 1         | $732^{+19}_{-30}$ | $637 \pm 8_{\text{mat.}} \pm 70_{\text{sc.}}$ |
| 2         | $720^{+4}_{-3}$   | $707 \pm 3_{\text{mat.}} \pm 29_{\text{sc.}}$ |
| 3         | $700^{+4}_{-6} \pm 1_{\text{slicing}}$ | $702 \pm 1_{\text{mat.}} \pm 1_{\text{m.c.}} \pm 17_{\text{sc.}}$ |

$699 \pm 5$ (syst.) $\pm 17$ (lumi.) ($e, \mu$ combined) [3]

findings of ref. [15] using RadISH resummation, our uncertainties of the resummation improved fiducial cross-section are larger than the uncertainties presented there. Our $\alpha_s^3$ prediction has uncertainties of about 2.5%, while using RadISH for the resummation results in uncertainties of about 1%. Given that differentially in fig. 3 we see still some variation in the low $q_T$ region between the central $\alpha_s^2$ and $\alpha_s^3$ results, we are confident in our more conservative uncertainty estimate.

Indeed, theory uncertainties have become an important topic within recent years [76]. First, they cannot be interpreted statistically and second, perturbative predictions are limited to the level presented here for the foreseeable future. It is therefore important to study them with as much scrutiny as possible. An approach followed in ref. [13] has been to take half the difference between the two highest order results as an uncertainty. This would bring our uncertainties closer in line with the uncertainties presented in ref. [15], less than one percent.

4. Conclusions & Outlook.

Z-boson production is the most precisely measured process at the LHC and meanwhile solely limited in precision by the beam luminosity uncertainty. At the same time it is one of the most important standard candles and enters many precision prediction ingredients like PDFs and SM input parameters. It is crucial that theory predictions are available at the same level of precision to make best use of the available measurements.

In this paper we presented the first transverse-momentum ($q_T$) resummation improved calculation at the level of N^3LL_p+N^3LO, which broadly reduces theory uncertainties to the few percent level. Our results show excellent agreement with the 13 TeV CMS measurements within a few percent both at the differential level from $q_T^Z = 1$ GeV to $\sim 500$ GeV and for $\Phi^*$ over the whole spectrum, as well as for the total fiducial cross-section. As a consequence of the resummation (and inclusion of linear power corrections), our calculation can provide reliable predictions also for past experimental analyses that would induce factorially divergent contributions at fixed order due to cuts, e.g. symmetric lepton cuts [35].

All previous calculations of order N^3LL'+N^3LO rely on a single Z+jet NNLO calculation [27]. Further, uncertainties (via scale variation) for resummation improved results were only estimated by using the RadISH resummation framework [14, 38]. Due to the utmost importance of this process, it is crucial to provide an independent calculation using completely different methods to reliably estimate uncertainties. It allows future (experimental) studies to assess the validity of their input theory predictions through independent results. This becomes increasingly important with the advent of very precise collider measurements that might indicate tension with the SM [11]. The public availability of our calculation as part of the upcoming CuTe-MCFM release allows for a much larger audience to make use of this
state-of-the-art precision, to implement modification of
cuts and input parameters, and also to re-use parts and
to validate other calculations [42].

Previously it was found that fiducial cross-section un-
certainties at the level of $\alpha_s^3$ are similar to those at
$\alpha_s^2$, about 1% using RadISH resummation [15]. With
resummation, this uncertainty is dominated by the un-
certainties around the Sudakov peak at small $q_T$, i.e.
mostly within the pure resummation region. We find
more conservative uncertainties of about 2.5% using
CuTe-MCFM resummation.

Although the theoretical precision of the calculation
discussed in this paper is now at an impressive level,
there are two important aspects that require further
work. Statistical PDF uncertainties have reached the
level of one percent [73, 77] and systematic effects can
no longer be neglected. Since these uncertainties are
at the same level as perturbative truncation uncertain-
ties, a careful study of PDF effects at this order will be
an important future direction. Indeed, while finalizing
this manuscript, approximate N$^3$LO PDFs have been
introduced by the MSHT group [78]. They take into
account approximations for the four loop splitting func-
tions through known information on small and large $x$
and available Mellin moments. Such theory approxi-
mations of missing higher-order effects are included in
their Hessian procedure as nuisance parameters.\footnote{A
preliminary study of the potential impact of this PDF set on
the results shown in this paper is presented in appendix A.}

In addition, in order to better match with data at very
small $q_T$, it is possible to include a parametrization of
non-perturbative effects, see e.g. refs. [79, 80]. This
can then inform the modeling of the related process of
$W$-boson production and thus have implications for the
extraction of the $W$-boson mass. Extending $W$-boson
production in CuTe-MCFM to $\alpha_s^3$ accuracy will thus be
a valuable extension that allows for very precise $W/Z$
boson ratio predictions [39].

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A. Impact of N^{3}LO PDFs.

Here we give a first impression of the impact of the approximate N^{3}LO PDFs of Ref. [78] by comparing the PDF uncertainties of this set to our default set NNPDF40 NNLO [73] and to MSHT20 NNLO [77]. Figure 6 shows the purely resummed spectrum up to 40 GeV, where matching corrections of about 5% are neglected at 20 GeV (less than 2% below 10 GeV). We do not ex- pect that the matching corrections change the relative PDF results and uncertainties substantially. About two-thirds of the total fiducial cross-section originates from the integrated purely resummed spectrum up to 20 GeV. The results demonstrate that systematic differences be- tween PDF sets are still dominant, comparable to the effect of N^{3}LO corrections in the PDFs. Uncertainties for the MSHT20 aN^{3}LO PDF set are larger since it includes missing higher-order effects with the PDF uncertainties. Overall, combined statistical and systematic PDF un- certainties are comparable to the residual truncation uncertainties found in our paper.

B. Comparison of N^{3}LL' with N^{4}LL_{p}.

In fig. 7 we show the purely resummed q_{T} spectrum at order N^{3}LL, N^{3}LL’ and N^{4}LL normalized to N^{4}LL and using NNPDF40 NNLO PDFs in all cases. The N^{3}LL’ result does not include the four-loop rapidity anomalous dimension (the additional contribution from the estimated five-loop cusp anomalous dimension to the hard function evolution is completely negligible). This figure indicates that a substantial decrease in uncertainties comes from the N^{3}LL’ result, with little additional reduc- tion at N^{4}LL. However the N^{4}LL result shifts noticeably, and its central value is only marginally compatible with the N^{3}LL’ uncertainty estimate. It is therefore an im- portant step in the full α_{s}^{3} precision.

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