Neutrino energy loss rates due to $^{66-71}$Ni in stellar matter

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Abstract. Rates for (anti-)neutrino energy loss on nickel isotopes, due to interactions involving weak decays ($\beta^\pm$-decay and lepton captures) are regarded as having fundamental importance during late evolutionary stages of massive stars. These rates substantially affect the leptonic ratio ($Y_e$) of stellar interior. For the densities less than $10^{11}$ g/cm$^3$, weak processes produce (anti-)neutrinos which cause reduction in the stellar core’s entropy. In this paper, rates for neutrino and anti-neutrino energy loss on nickel neutron-rich isotopes ($^{66-71}$Ni) have been presented. Rates for energy loss have been determined by applying the deformed $pn$-QRPA model. The ranges for temperature and density, have been used to determine the rates, are from 0.01 to 30 (10$^9$ K) and $10^1$ to $10^{11}$ (g/cm$^3$), respectively. Our computed rates for energy loss, at higher temperature regions, are enhanced in comparison with previously reported rates of Pruet and Fuller (PF).

1. Introduction

Stars are considered to be nucleosynthesis laboratories in the universe, which give out almost all the elements in the interstellar medium via the supernova explosions. Hydrodynamically stable massive stars (> 8 M$_\odot$) run through several burning phases and develop an iron core. At this stage the inward self-gravity force is balanced by the electron degeneracy pressure [1,2,3]. Further burning of silicon in shell exceeds the mass of core than the Chandrasekhar mass (~1.5 M$_\odot$), there the degeneracy pressure fails to overcome enormous gravity force and core is derived towards collapse forcibly [4, 5]. The gravitational core-collapse results in star’s death through gigantic burst known as supernova (e.g. type II) explosions.

For the better understanding of the core-collapse supernova dynamics, the nuclear processes mediated by weak-interactions have considerable importance. The weak-processes; viz $\beta^-$-decays and lepton (electron and positron) captures cause reduction in the $Y_e$ (fraction of lepton to baryon) and core entropy in the presupernova phase of massive stars [4, 6]. These processes are also responsible for an abundant production of (anti-)neutrinos up to $\sim 10^{11}$ g/cm$^3$ staller densities [7,8]. Energy budget of stars are decided by the (anti-)neutrinos which escape from stars and take away entropy and energy, resulting in cooling of the core. Therefore, it is valuable to calculate the rates for (anti-)neutrino energy loss for the Fe-peak nuclei. Throughout in this paper $\nu$EnL ($\bar{\nu}$EnL) will be used for neutrino (anti-neutrino) energy loss.

The weak decay rates and the associated rates for ($\bar{\nu}$)$\nu$EnL are ruled by the Gamow-Teller (GT) distributions strength [4]. In the presupernova and core-collapse phases, the GT transitions for Fe-peak nuclei are of central importance [9]. To precisely determine the GT transitions requires a reliable and microscopic theory. In this work, we are using the proton-neutron quasi-particle random phase approximation (pn-QRPA) [10] model for the calculations of ($\bar{\nu}$)$\nu$EnL rates of nickel neutron-rich isotopes.
In the late evolutionary stages of massive stars, isotopes of nickel are produced abundantly and play a crucial role to change Y_e. As per studies of Ref. [11,12], the electron captures (EC) and decay rates of Ni isotopes have significant importance in astrophysical environment. Ref. [13] first reported the GT strength and EC rates of 56Ni by using the pn-QRPA model. Later, the results were improved [14] and rates were calculated for heavier nickel isotopes (mass ranging 57-65). Ref. [15,16] recently calculated the weak-interaction rates and GT transition properties on Ni isotopes. In this paper our focus is to calculate the rates for (ν̅ν)νEnL of nickel neutron-rich isotopes 66-71Ni in both leptons capture and lepton emission directions. The deformed pn-QRPA model has been used to compute the rates and our results have been compared with the earlier results of Pruet and Fuller (PF) [17].

In the next section, pn-QRPA has been described briefly. The detailed formalism can be found in Ref. [10]. Section 3 comprises discussion on our calculated rates and comparison of our results with other model calculations. Conclusion of our present work is given in Section 4.

2. Formalism

The Hamiltonian for the pn-QRPA model is

$$H_{QRPA} = H^{pp} + H^{pair} + H_{GT}^{pp} + H_{GT}^{ph}$$

Nilsson model [18] was used to calculate the single-particle energies and wave functions. Pairing interactions $H^{pair}$ in nuclei were handled by using the BCS approximation. The residual interactions for proton-neutron pairs were included by $H_{GT}^{pp}$ (particle-particle) and $H_{GT}^{ph}$ (particle-hole) GT forces, respectively.

Among other necessary parameters for pn-QRPA model are the Nilsson potential parameters [18], the Nilsson oscillator constant $\Lambda = 41A^{-1/3}$ MeV, the pairing gaps $\Delta_n = \Delta_p = 12/\sqrt{A}$, the nuclear quadrupole deformation parameters [19], and the reaction $Q$-values [20].

For a transition from the $k^{th}$ parent to the $m^{th}$ daughter state, the $(\nu\bar{\nu})\nu$EnL rate of weak interactions is given by

$$\lambda_{km} = \langle \frac{\ln 2}{D} \rangle f_{km}(T, \rho, E_f) [B(F)_{km} + (g_A/g_V)^2 B(GT)_{km}], \quad N \equiv \nu, \bar{\nu}$$

In equation (2) $B(F)_{km}$ and $B(GT)_{km}$ are reduced Fermi and GT transition probabilities, respectively. For current calculations, $D=6143s$ (taken from [21]) and the value of ratio $g_A/g_V$ is $-1.2694$ (taken from [22]). The $f_{km}$ is the phase space integrals (natural units: h = c = $m_e = 1$), for electron and positron (upper and lower sign, respectively) emission

$$f_{km} = \int_{w_i}^{w_m} w(w^2 - 1)(w_m - w)^2 F(\pm Z, w)(1 - G_m) dW$$

for electron (upper sign) and positron (lower sign) capture, the integral is

$$f_{km} = \int_{w_i}^{w_m} w(w^2 - 1)(w_m - w)^2 F(\pm Z, w) G_m dW$$

In equations (3) and (4) $w$ is total energy (kinetic plus rest) of electron or positron, $w_i$ is the total threshold energy (kinetic plus rest) for electron or positron capture, and $w_m$ is the total energy of $\beta$-decay. The $G_m$ are the Fermi-Dirac electron (positron) distribution functions and $F(Z, w)$ are Fermi functions.

The total $(\nu\bar{\nu})\nu$EnL rates are given by

$$\lambda^N = \sum_{km} P_k \lambda^N_{km}, \quad N \equiv \nu, \bar{\nu}$$

Here $P_k$ is occupation probability of parent nucleus excited states which obeys the normal Boltzmann distribution and $\lambda^N_{km} = \lambda^{pc(\nu)}_{km} + \lambda^{pc(\bar{\nu})}_{km}$. 
3. Results and discussion
In this unit, our estimated ($\bar{\nu}$)uEnL rates on $^{66-71}$Ni are presented. Tables 1 and 2 comprise the results of the deformed pn-QRPA computed ($\bar{\nu}$)uEnL rates for various densities region at different values of stellar temperature. The selected values for density $\rho$ (g/cm$^3$) are given in the first column of the tables and the second column displays the chosen values of temperature $T$ (10$^9$ K). For each isotope, two columns represent the rates for uEnL ($\lambda_{0}^{uu+pe}$) and $\bar{\nu}$EnL ($\lambda_{0}^{pp+ee}$), where ec (pe) and ee (pe) stands for electron (positron) capture and electron (positron) emission, respectively. The total ($\bar{\nu}$)uEnL rates take contributions from positron capture and electron emission (electron capture and positron emission).

Table 1. The pn-QRPA calculated rates due to $^{66-68}$Ni at certain densities and stellar temperatures. the terms $\lambda_{0}^{uu+pe}$ and $\lambda_{0}^{pp+ee}$ are total (anti-)neutrino rates respectively in units of Mev/s.

| $\rho$ (g/cm$^3$) | $T$ ($\times10^9$ K) | $^{66}$Ni | $^{67}$Ni | $^{68}$Ni |
|------------------|------------------|-----------|-----------|-----------|
| $1.00\times10^2$ | 1.00             | 8.93E-57  | 9.62E-07  | 3.97E-04  |
|                  | 5.00             | 7.29E-11  | 1.07E-02  | 1.00E-09  |
|                  | 10.00            | 8.89E-04  | 1.42E+00  | 1.36E-03  |
|                  | 30.00            | 5.27E+02  | 3.32E+03  | 1.16E+03  |
| $1.00\times10^5$ | 1.00             | 1.03E-54  | 3.67E-07  | 4.56E-07  |
|                  | 5.00             | 7.45E-11  | 1.05E-02  | 1.03E-09  |
|                  | 10.00            | 8.91E-04  | 1.42E+00  | 1.37E-03  |
|                  | 30.00            | 5.28E+02  | 3.32E+03  | 1.16E+03  |
| $1.00\times10^8$ | 1.00             | 1.53E-44  | 1.68E-16  | 7.18E-37  |
|                  | 5.00             | 1.17E-08  | 4.55E-04  | 1.63E-07  |
|                  | 10.00            | 5.02E-03  | 3.89E-01  | 7.71E-03  |
|                  | 30.00            | 5.73E+02  | 3.07E+03  | 1.26E+03  |
| $1.00\times10^{11}$ | 1.00          | 5.71E+03  | 1.00E-100 | 1.02E+05  |
|                  | 5.00             | 8.45E+03  | 1.99E-24  | 1.52E+05  |
|                  | 10.00            | 1.62E+05  | 1.25E-11  | 3.56E+05  |
|                  | 30.00            | 1.41E+06  | 4.59E-01  | 3.26E+06  |

Table 2. Same as table 1 but for $^{69-71}$Ni.

| $\rho$ (g/cm$^3$) | $T$ ($\times10^9$ K) | $^{69}$Ni | $^{70}$Ni | $^{71}$Ni |
|------------------|------------------|-----------|-----------|-----------|
| $1.00\times10^2$ | 1.00             | 6.75E-54  | 1.30E-01  | 1.49E-69  |
|                  | 5.00             | 3.97E-11  | 9.10E-02  | 9.73E-13  |
|                  | 10.00            | 5.13E-04  | 1.37E+00  | 1.66E-04  |
|                  | 30.00            | 7.21E+02  | 1.69E+03  | 5.47E+02  |
| $1.00\times10^5$ | 1.00             | 7.74E-52  | 1.30E-01  | 1.70E-67  |
|                  | 5.00             | 4.06E-11  | 9.08E-02  | 9.95E-13  |
|                  | 10.00            | 5.14E-04  | 1.37E+00  | 1.67E-04  |
|                  | 30.00            | 7.21E+02  | 1.69E+03  | 5.48E+02  |
| $1.00\times10^8$ | 1.00             | 1.22E-41  | 5.96E-02  | 2.68E-57  |
|                  | 5.00             | 6.44E-09  | 4.42E-02  | 1.58E-10  |
|                  | 10.00            | 2.90E-03  | 7.55E-01  | 9.40E-04  |
|                  | 30.00            | 7.82E+02  | 1.57E+03  | 5.94E+02  |
| $1.00\times10^{11}$ | 1.00         | 2.37E+04  | 2.01E-96  | 4.51E+03  |
|                  | 5.00             | 2.56E+04  | 3.84E-21  | 5.05E+03  |
|                  | 10.00            | 1.24E+05  | 1.39E-10  | 4.71E+04  |
|                  | 30.00            | 1.99E+06  | 2.43E-01  | 1.53E+06  |
Tables 1 and 2 show that when temperature \((\times 10^9 \text{ K})\) changes from 1 to 5, the values of \(\nu\text{EnL}\) rates increase abruptly, in the domain of low- and medium-density. Above \(T = 5 \times 10^9 \text{ K}\), a small increase occurs in the values of \(\nu\text{EnL}\) rates with the rise of core density. In general, \(\bar{\nu}\text{EnL}\) rates also increase as temperature increases at each density. For \(\rho = 1.00 \times 10^{11} \text{ g/cm}^3\), the values of \(\bar{\nu}\text{EnL}\) rates are smaller in magnitude as compared to those in the regions of low- and medium-density, particularly at low temperature.

Next, we present the comparison of our estimated \(\nu\text{EnL}\) and \(\bar{\nu}\text{EnL}\) rates with the PF [17] rates for three selected Ni-isotopes (\(^{66}, {68}, {71}\)Ni for space consideration). Figure 1 illustrates the ratio of \((\bar{\nu})\nu\text{EnL}\) rates calculated by our model to the PF rates \((R_{\nu(\bar{\nu})})\). These ratios are plotted against stellar temperature values for four particular densities \((10^2, 10^5, 10^8, 10^{11} \text{ g/cm}^3)\).

![Figure 1](image-url)

**Figure 1.** Ratios of calculation of \((\bar{\nu})\nu\text{EnL}\) rates due to \(^{66}, {68}, {71}\)Ni at selected temperatures and densities in stellar matter. \(R_{\nu} (\text{QRPA/PF})\) represents the ratios of our calculated \(\nu\text{EnL}\) rates to those of PF rates and \(R_{\bar{\nu}} (\text{QRPA/PF})\) for \(\bar{\nu}\text{EnL}\) rates. \(\rho\) is the baryon density in units of \(\text{g/cm}^3\), \(Y_e\) is the leptonic ratio. \(T_9\) \((T \times 10^9 \text{ K})\) represents the temperature in stellar matter.

The first and second row of the figure 1 displays the ratios of neutrino \((R_\nu)\) and anti-neutrino \((R_{\bar{\nu}})\) rates, respectively. The graphical trend of \(R_\nu\) shows that in the domain of low- and medium-densities, our computed neutrino \(\nu\text{EnL}\) rates are larger than those of PF rates. It can be noted from figure 1 that in case of anti-neutrino, at low temperature and for low- and medium-densities, the rates calculated by two models are in reasonable agreement. A large difference in the results of both pn-QRPA calculated rates and PF rates occur, at high density and low temperature (where values of rates are in itself very small). At high temperatures, both of the \(\nu\text{EnL}\) and \(\bar{\nu}\text{EnL}\) rates by using the deformed pn-QRPA are largely enhanced in comparison with PF rates.
4. Conclusion
We have determined the $\nu_{\text{EnL}}$ and $\bar{\nu}_{\text{EnL}}$ rates for Ni neutron-rich isotopes, over wide-ranging densities and temperatures. We also presented the comparison of our determined rates with the earlier Pruet and Fuller work. At higher stellar core temperatures (around $30 \times 10^9$ K) pn-QRPA rates are bigger by up to an order of magnitude than PF. We hope that our calculated $(\bar{\nu})_{\text{EnL}}$ rates may help in an improved comprehension of core-collapse simulations.

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