About microlensing optical depth and rates for free-floating planets towards the Kepler's field of view

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Abstract. In this work we examine the possibility of observing microlensing events in the Kepler space observatory field of view, caused by brown dwarfs or free-floating planets. We calculate the optical depth towards the field of view of the Kepler satellite and the rate of these events based on latest results about mass distribution of astrophysical objects from brown dwarf down to Earth mass order. With the current data, the probability of such events is insignificant, due to the small number of stars observed by this instrument compared to other experiments devoted to the microlensing method. Nevertheless, this probability may increase significantly in the case of a higher presence of free-floating planets, whose number is poorly defined so far.

1. Gravitational microlensing
The gravitational microlensing effect happens when the gravitational field of an astrophysical object acts as a lens to magnify background source stars ([1]). A gravitational lens of mass \(M\) is characterized by its Einstein ring radius, the radius of the ring image created when the observer, the lens at distance \(D_l\) and the source at distance \(D_s\) are perfectly aligned, which is defined as:

\[
R_E(M, x) = \frac{4GM}{c^2D_s}x(1-x),
\]

where \(x = \frac{D_l}{D_s}\) is the normalized lens distance. The image separation for astrophysical bodies in galactic distances is too small to be resolved. In such microlensing events, the observable feature is the light magnification due to the lens-source relative motion with transverse velocity \(v_t\). The microlensing technique has already shown its great potentiality and has allowed the detection of thousands of events towards galactic bulge, several tens of events towards the LMC, SMC and M31 galaxy and 11 exoplanets until now.

The key parameter of a microlensing light curve is the Einstein radius crossing time given by:

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which can be considered as the characteristic event duration. Its value towards the Galactic centre is of the order of $t_E \approx \sqrt{\frac{M}{M_J}}$ days, $M_J = 9.5 \times 10^{-4} M_S$ being the Jupiter mass, $M_S$ the solar mass. Microlensing events with $t_E < 2$ days indicate, so, planetary-mass lenses. Thus, although $t_E$ depends on the velocity and distance of the lens object, in addition to its mass, the $t_E$ distribution can be used to probe statistically the mass function of the lens objects.

The microlensing optical depth is defined as the probability that at any time a random star is magnified more than a threshold amplification $A_{th} = 1.34$ by a certain population of lenses (i.e., thin and thick stellar disk, white and brown dwarfs or planetary mass populations). It is given by:

$$\tau_i = \frac{4\pi G D_L^2}{c^2} \int_0^1 \rho_i(x)(1-x)dx,$$

where $\rho_i(x) = m_i n_i(x)$ is the lens mass density of the i-th lens object galactic population ([2]).

The microlensing rate $\Gamma_i$, i.e. the number of events per unit time and per monitored star due to i-th lens object population is given by:

$$\Gamma_i = 2 \int R_E(x)n_i(x)\nu_L f(\tilde{v}_L - \tilde{v}_i)f(\tilde{v}_i)dx d\tilde{v}_L d\tilde{v}_i,$$

where $\tilde{v}_L, \tilde{v}_i, \tilde{v}_s$ are the lens, source and microlensing tube two-velocities in the plane transverse to the line of sight ([2]). The velocity distribution functions $f(\tilde{v}_L - \tilde{v}_i)$ and $f(\tilde{v}_i)$ are assumed to have a Maxwellian form with one-dimensional dispersion velocity $\sigma_i$ different for each lens and source population.

2. Free-floating planetary populations

In a recent work ([3]) is reported the discovery of a population of unbound Jupiter-mass objects, known as free-floating planets and whose existence has been questioned several times during this decade ([4,5]). After two years of gravitational microlensing surveys towards Galactic Bulge performed by MOA (Microlensing Experiments in Astrophysics) and OGLE (Optical Gravitational Lensing Experiment) groups, 10 microlensing events with $t_E < 2$ days are obtained out of 474 with $t_E$ from fractions of a day to more than a hundred days. The lenses for these 10 short duration events are inferred to be either unbound planets or bound with wide separations of more than ten astronomical units from their host stars, so that one cannot detect the host star in the light curves. For this reason, they are called free-floating planets.

By considering a broken power-law mass function $\frac{dN}{dM} = M^{-\alpha_i}$ with $\alpha_i = 2$ for $0.7 < M / M_S < 1, \alpha_2 = 1.3$ for $0.08 < M / M_S < 0.7$ (the initial stars with masses above the solar mass are assumed to have evolved into stellar remnants) and $\alpha_3$ as a fitting parameter for the brown dwarf regime $0.01 < M / M_S < 0.08$, the ten microlensing events with $t_E < 2$ lead the authors of [3] to a fourth population, that is a planetary population with masses $M / M_S < 0.01$ having a mass function $\frac{dN}{dM} = M^{-\alpha_4}$, with $\alpha_4 = 1.3^{+0.3}_{-0.4}$, much steeper than the brown dwarf slope $\alpha_3 = 0.49^{+0.24}_{-0.27}$. The abrupt change in the mass function favours the idea that the formation process of this planetary population is
different from that of stars and brown dwarfs. They may have formed in proto-planetary disks and subsequently scattered into unbound or very distant orbits.

The broken power-law mass function analysis indicates that the number of brown dwarfs would be \( \frac{0.73^{+0.22}_{-0.19}}{1.0} \) times the number \( N_e = 2 \times 10^{11} \) of the main sequence stars in the mass range \( 0.08 < M / M_\odot < 1 \). The model also predicted a larger, but poorly constrained number of planetary mass objects per star with \( \frac{N_{pl}}{N_e} = 5.5^{+1.6}_{-1.3} \), due to weak sensitivity of observations to very low-mass lenses. The lower mass limit for planets is taken \( 10^{-5} M_\odot \).

A consequence of the above mentioned work is a kind of correlation between variations of \( \alpha_3 \) and \( \alpha_{pl} \), which we parameterize in our analysis by an approximate linear relation:

\[
\alpha_{pl} = -\alpha_3 + 1.7. \tag{5}
\]

3. Free-floating planets towards Kepler’s field of view

In this work we are looking for the possibility to see microlensing events produced by brown dwarfs or free-floating planets and detected by Kepler mission. Although the principal goal of this mission is the detection of bound planets transiting their host stars, its capacity to analyze fine light fluctuations might lead to identification of eventual microlensing events. The Kepler Space observatory is placed at a heliocentric orbit, it is designed to survey main sequence stars at a portion of sky of 115 square degrees, with galactic coordinates centered at \( b = 13.5^\circ \), \( l = 76.32^\circ \) and lying from 150 to 1000 pc. The total number of stars in its field of view is about half million, 150 000 of them observed simultaneously every 30 minutes.

In this analysis we calculate the microlensing optical depth and the rate of microlensing events towards this field of view for two populations, brown dwarfs (BD) and free-floating planets (PL), based on relations (3) and (4) corrected with the mass distribution function \( f(M) \) for this case of lenses with a certain range of masses.

The data we need are spatial and velocity distributions for lenses and sources. For the two lens populations we assume the thin disk spatial distribution (see [6] for more details):

\[
\rho(M, R, z) = f(M) e^{-|z|/H} e^{-(R-R_0)/h},
\]

with \( H = 300 \text{pc} \), \( h = 3.5 \text{kpc} \), \( R_0 = 8.5 \text{kpc} \) and \( f(M) = \frac{dN}{dM} = M^{-\alpha} \) described above. For the velocity distribution we consider \( \sigma_{pl} = \sigma_{BD} = 50 \text{km/s} \ (\{7\}) \).

Concerning the sources, the observed stars lie in a depth of the order of 1 kpc, so we take into account their double exponential thin and thick luminous disk distribution:

\[
\rho_s(R, z) = \frac{\Sigma_0}{2H} e^{-|z|/H} e^{-(R-R_0)/h},
\]

with \( \Sigma_0 = 25 M_\odot \text{pc}^{-2} \), \( H = 0.3 \text{kpc} \), \( h = 3.5 \text{kpc} \) for the thin component and \( \Sigma_0 = 35 M_\odot \text{pc}^{-2} \), \( H = 1 \text{kpc} \), \( h = 3.5 \text{kpc} \) for the thick component.

Instead of the fixed lens mass, we make use of the broken power-law mass function \( f(M) = M^{-\alpha} \) with respectively \( \alpha_3 = 0.49^{+0.24}_{-0.22} \) and \( \alpha_{pl} = 1.3^{+0.3}_{-0.4} \), as concluded in [3]. The lower value of the mass range is taken of the order of \( 10^{-5} M_\odot \), corresponding to the lens mass which could produce a microlensing event with a characteristic duration 30 minutes, the lower value of the observation interval for the Kepler instrument. The higher limit of the free-floating mass range gets to the brown dwarf lower limit \( 0.01 M_\odot \).
Figure 1. The microlensing rate expected for brown dwarf and free-floating planetary disk models shown as a function of brown dwarfs mass function index $\alpha_3$. The dotted line corresponds to the minimum of the planetary number $N_{pl} / N_*$ = 1.2, the continuous to the most plausible value $N_{pl} / N_* = 5.5$ and the dashed line to the maximum value of the planetary number $N_{pl} / N_* = 23.6$.

Figure 2. Here we show the importance of the planetary population in microlensing rate (dashed line) compared to that of the brown dwarf population (continuous line). We obtain that the planetary population becomes dominant in such events for higher values of $\alpha_3$.

4. Model results
In figure 1 and 2 we present the results we obtained for the microlensing rate dependence on brown dwarf mass function index, this last one varying in the interval of values found in [3]: $0.1 < \alpha_3 < 0.8$. Owing to relation (5), free-floating planet mass function index varies also in the range $0.9 < \alpha_{pl} < 1.6$. For the brown dwarf number we keep a fixed value in the process of mass function calibration, the value $0.73 N_*$ mentioned above. On the contrary, the free-floating planet number varies in a wide range, $N_{pl} / N_* = 5.5^{+18.1}_{-4.3}$. We need thus to reflect the results for extreme limits of this number. We show in figure 1 three cases of free-floating planet abundance: two extreme values $N_{pl} = 1.2 N_*$, $N_{pl} = 23.6 N_*$, and the most probable one $N_{pl} = 5.5 N_*$.

In figure 1 we present the microlensing rate expected for both brown dwarf and free-floating planetary disk model. We obtain rate values increasing with the mass index, therefore we conclude that steeper mass functions for brown dwarfs (and softer for planets) give higher probabilities to get microlensing event signs in Kepler observations.

In figure 2 we show the weight of free-floating planets in the microlensing rate in comparison to that of brown dwarfs. Here we have chosen the most plausible value for the planetary abundance, $N_{pl} = 5.5 N_*$. We find that this rate increases with the mass index, whereas the rate for brown dwarfs...
Figure 3. The optical depth for brown dwarf and free floating planetary disk model shown as a function of brown dwarfs mass function index $\alpha_3$. The dotted line corresponds to the minimum of the planetary number $N_{pl} / N_\star = 1.2$, the continuous to the most plausible value $N_{pl} / N_\star = 5.5$ and the dashed one to the maximum value of the planetary number $N_{pl} / N_\star = 23.6$.

Figure 4. Here we show the importance of the planetary population in optical depth (continuous line) compared to that of the brown dwarfs population (dashed line). We obtain that the planetary population is at least one order less important.

decreases very softly. For values of the index above 0.7, the eventual microlensing events would be caused mainly by free-floating planet lenses.

In figure 3 and 4 we repeat for the optical depth the same calculations as above. The gap in the optical depth between the two considered populations is higher than in microlensing rate.

Once the microlensing rate for each lens population is calculated, it is straightforward to obtain the expected number of microlensing events by the simple multiplication of the rate with the number of observed stars. The most optimistic value we find is about 0.022 events per year, attained in the case of the upper limit for the number of free-floating planets.

5. Conclusions
With 150 000 stars observed by Kepler mission, it is obvious to obtain very small probabilities of detecting microlensing events during its time of observation. However, the present analysis is relevant since the detection of even a short-time microlensing event (for example by the forthcoming space-based telescopes like WFIRST and/or EUCLID) would require to review our work-hypotheses and look for:

1. A steeper mass function for brown dwarfs, associated to a softer one for free-floating planets, which leads to higher values of microlensing rate and optical depth.
2. A higher presence of free-floating planets than the limit found up to now.
3. Different models for their spatial and velocity distributions.
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