Establishment of Altitude and Velocity Measurement
Arithmetic Model for a Lander

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Abstract. For successful landing of large lander, active control of lander and reliable landing are essential. The data of altitude and velocity between the cabin of the lander and the ground should be obtained accurately. In the stage of landing power descending, each wave beam direction of radar is designed reasonably according to landing control system, so that a radar waveform for altitude and velocity measurement is proposed. Based on the analysis of the waveform, an arithmetic model for measurement of altitude and velocity is established. Simulation and experimental results manifested that this model could meet the accuracy requirements.

Introduction
Recently, lots of research efforts have been made to study how to control the landing process of large lander. To obtain the altitude and velocity of lander is necessary, because which can indicate the lander landing on the interested location. Numbers of techniques have been proposed for the measurement of the altitude and velocity between the lander and ground [1-3]. For pinpoint safe landing and providing effective controlling data, some kinds of radar systems are adopted [4,5].

For providing effective controlling data, a radar single system is employed. It is step frequency pulse system, which can survey accurate altitude and velocity. A certain of waveform is applied to the survey system. With their excellent properties, the waveform can obtain wider bandwidth by carrier frequency hops between pulses in coherent pulse train. So high range resolution can be acquired by using IFFT. Due to the instantaneous bandwidth is smaller, the high range resolution can be obtained in the condition of lower sample rate. Furthermore the pulse accumulating improves the signal - noise ratio, which can satisfy with the system. The same echo sampling values are extracted from the multi-frame echo signal for FFT, the velocity is obtained, which is along the direction of wave beam.

Design of Radar Beams for Surveying Altitude and Velocity
According to the demand, the landing control system needs radar to obtain the vertical altitude and relative velocity vector, which are from the cabin of lander to the ground in the ground inertia coordinate system. This system adopts four pairwise orthogonal antenna waveforms. They measure respectively the altitude and velocity along the wave beam.

Radar working state in the stage of landing power descending is shown in Figure 1. In the light of the cabin attitude getting from other sensors on the lander, the vertical altitude and velocity vector between the cabin and the ground are calculated. Figure 1 shows that radar can guarantee at least 3 antennas (wave beams) to detect the echo of the ground during the whole landing process. Specific wave beam selection can be obtained according to the cabin pitch angle.

In general, wave beams $\odot$, $\odot$ and $\odot$ are selected, then the directions of 3 wave beam vectors is defined as basis 1 ($J_1$). The included angles of 3 coordinate axis vectors are known in the Cartesian coordinate system of cabin, which is defined as basis 2 ($J_2$). Then $J_1$ is transformed into $J_2$ in the coordinate system. The vertical altitude and velocity can be obtained by computation.
Building the Landing State Arithmetic Model

According to Figure 1, suppose the included angles between wave beam $\odot$ and Xd, Yd, Zd are respectively $\alpha_1, \beta_1, \gamma_1$; the included angles between wave beam $\odot$ and Xd, Yd, Zd are respectively $\alpha_2, \beta_2, \gamma_2$; the included angles between wave beam $\odot$ and Xd, Yd, Zd are respectively $\alpha_3, \beta_3, \gamma_3$. The transition vectors of J₁ to J₂ can be expressed as Eq. (1):

$$
\begin{align*}
\vec{e}_1 &= b_1 \cos \alpha_1 \hat{x} + b_2 \cos \beta_1 \hat{y} + b_3 \cos \gamma_1 \hat{z} \\
\vec{e}_2 &= b_1 \cos \alpha_2 \hat{x} + b_2 \cos \beta_2 \hat{y} + b_3 \cos \gamma_2 \hat{z} \\
\vec{e}_3 &= b_1 \cos \alpha_3 \hat{x} + b_2 \cos \beta_3 \hat{y} + b_3 \cos \gamma_3 \hat{z}
\end{align*}
$$

where $J_1 \in (b_1, b_2, b_3)$, $J_2 \in (\vec{e}_1, \vec{e}_2, \vec{e}_3)$.

The transition matrix C of J₁ to J₂ is defined as Eq. (2):

$$
C = \begin{bmatrix}
\cos \alpha_1 & \cos \beta_1 & \cos \gamma_1 \\
\cos \alpha_2 & \cos \beta_2 & \cos \gamma_2 \\
\cos \alpha_3 & \cos \beta_3 & \cos \gamma_3
\end{bmatrix}
$$

Supposing the three Euler angles of the cabin coordinate system relative to the ground inertial coordinate system are $\Phi$, $\Omega$, and $\Psi$ respectively, so the transition GIVENS matrix is shown in Eq.(3):

$$
A = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \Psi & \sin \Psi \\
0 & -\sin \Psi & \cos \Psi
\end{bmatrix}
$$

Supposing the range surveyed by three antennas are $R'_i (i=1,2,3)$, so $H$ is expressed as $(R'_1, R'_2, R'_3)$ under $J_1 \in (b_1, b_2, b_3)$. The coordination of vertical range $H$ is expressed as $(R'_1, R'_2, R'_3)$ under the $J_2 \in (\vec{e}_1, \vec{e}_2, \vec{e}_3)$. Then Eq.(4) can be obtained through basis transformation:

$$
(R'_1, R'_2, R'_3)^T = C^{-1} (R'_1, R'_2, R'_3)^T .
$$

Suppose that $H$ is expressed as $(R'_1, R'_2, R'_3)$ in the inertial coordinate system, according to GIVENS transformation, Eq. (5) is acquired:
\[(R_1^*, R_2^*, R_3^*)^T = A(R_1^*, R_2^*, R_3^*)^T = AC^{-1}(R_1^*, R_2^*, R_3^*)^T. \quad (5)\]

The altitude range between cabin and ground is expressed as \( H = |R_3^*|/3 \).

The solution of velocity vector \( \vec{v} \) is similar with vertical range \( H \). Suppose that the velocity surveyed by three effective antennas wave beams is \( \vec{v}_i (i = 1, 2, 3) \), \( \vec{v} \) can be expressed as \((\vec{v}_1^*, \vec{v}_2^*, \vec{v}_3^*)\) under \( J_1 \in (b_1, b_2, b_3) \). Assume \( \vec{v} \) is \((\vec{v}_1^*, \vec{v}_2^*, \vec{v}_3^*)\) under \( J_2 \in (e_1, e_2, e_3) \), then Eq.(6) can be obtained through basis transformation:

\[
(\vec{v}_1^*, \vec{v}_2^*, \vec{v}_3^*)^T = C^{-1}(\vec{v}_1^*, \vec{v}_2^*, \vec{v}_3^*)^T. \quad (6)
\]

Then the cabin coordinate system is converted into the ground inertial coordinate system. \( \vec{v} \) is expressed as \((\vec{v}_1^*, \vec{v}_2^*, \vec{v}_3^*)\) under the ground inertial coordinate system. According to GIVENS transformation, the Eq. (7) can be obtained:

\[
(\vec{v}_1^*, \vec{v}_2^*, \vec{v}_3^*)^T = A(\vec{v}_1^*, \vec{v}_2^*, \vec{v}_3^*)^T = AC^{-1}(\vec{v}_1^*, \vec{v}_2^*, \vec{v}_3^*)^T. \quad (7)
\]

The velocity vector of cabin is \( \vec{v} = (\vec{v}_1^*, \vec{v}_2^*, \vec{v}_3^*) \).

A single system pulse radar is adopted to survey velocity and altitude of the cabin. The design and analysis are below.

The Theoretical Analysis of Surveying Velocity and Altitude

Theoretical Analysis of Surveying the Altitude

The radar adopts step frequency pulse compression system, which has two functions for measurements of pulse compression and pulse Doppler velocity. For stepped frequency pulse, suppose pulse repeated time is \( T_r \), transmitting pulse width is \( \tau \), the initial frequency of carrier frequency is \( f_0 \), stepped frequency size is \( \Delta f \), frequency stepped number is \( N \), sample frequency is \( f_s \), \( T_s = 1/f_s \), velocity of light is \( c \). So transmitting pulse is expressed as Eq. (8):

\[
x(t) = \sum_{i=0}^{N-1} \text{rect}\left(\frac{t - iT_r - \tau/2}{\tau}\right) \exp\left[-j2\pi\left(f_0 + i\Delta f\right)t\right] \quad (8)
\]

where \( \text{rect}(t/\tau) = \begin{cases} 1, & -\tau/2 < t < \tau/2 \\ 0, & \text{others} \end{cases} \).

The target echo is located in range \( R \) \( (R < \frac{cT_r}{2}) \). The filtered echo signal mixed with local oscillator is expressed as Eq. (9):

\[
y(t) = \sum_{i=0}^{N-1} \text{rect}\left(\frac{t - iT_r - \tau/2 - 2R/c}{\tau}\right) \exp\left[-j2\pi\left(f_0 + i\Delta f\right)2R/c\right] \\
= \sum_{i=0}^{N-1} \text{rect}\left(\frac{t - iT_r - \tau/2 - 2R/c}{\tau}\right) \exp[-j2\pi f_0 2R/c] \exp[-j2\pi i\Delta f 2R/c] \quad (9)
\]

where \( i = 1, 2, \ldots, N \).

In the light of Eq. (9), the echo of a single frame emitting signal is in the form of a pulse train. The echo is mixed with local oscillator and filtered, whose amplitude is modulated by frequency \( f = 2R\Delta f/c \), pulse number is \( N \), modulated signal frequency \( f \) is corresponding to range \( R \). Therefore, the measurement of the target range \( R \) is converted to the measurement of the modulation signal frequency \( f \).
Theoretical Analysis of Surveying the Velocity

Stepped frequency system uses the principle of pulse Doppler velocity measurement [6]. The sampling echo signals with the same frequency pulse are selected among multiple frame echo signals for FFT. The target velocity is obtained along the wave beam direction. To every transmitting pulse, the echo signal frequency is superimposed Doppler frequency $f_d$ due to relative motion. Supposing the signal of transmitting carrier frequency $f_i$ is selected among m frame signals, stepped frequency pulse repeated period is $T_r$, stepped number is $N$. Then the echo signal pulse repeated period is $rNT_r$ whose carrier frequency is $f_i$. Therefore the velocity resolution $\Delta \nu$ is Eq. (10):

$$\Delta \nu = \frac{\Delta f_i \cdot c}{2f_i} = \frac{c}{2f_i mNT_r}.$$  \hspace{1cm} (10)

Because the pulse repeated frequency of the same carrier frequency pulse is smaller, and the resolution of FFT is higher, the higher velocity resolution can be obtained.

Simulation and Test Analysis

According to empirical formula the radar waveform is designed. Under the conditions of close loop and low velocity, using the cable delay to simulate the echo, the point target spectrum is shown in Figure 2.

![Figure 2. Frequency spectrum of point target.](image)

Figure 2 demonstrates that the subdivided point target spectrum width is still narrow and good signal-to-noise ratio is obtained. Therefore, the measurement results can be given accurately. The measured results of range obtained by multiple measurements are shown in Tab.1. The accuracy of the point target surveying is in the range of 0.04m.

| Real range  | 9m  | 9.375m | 9.525m | 9.75m | 10.5m |
|-------------|-----|--------|--------|-------|-------|
| Simulated range | 9m  | 9.37m  | 9.563m | 9.75m | 10.5m |
| Maximum error  | 0   | 0.005  | 0.04   | 0     | 0     |

The pulse Doppler system velocity is simulated. It is supposed there is a frequency difference between the initial frequencies of transmitting signal DDS and local oscillator signal. The surveying frequency accuracy and stability performance of the radar system are tested. Through repeated measurements the maximum error value is selected, as shown in Tab.2, the velocity maximum error of point target measurement is 0.17m/s.

| Real velocity | 1m/s | 1.4m/s | 0.8m/s |
|---------------|------|--------|--------|
| Simulated velocity | 1m/s | 1.57m/s | 0.97m/s |
| Maximum error  | 0    | 0.17   | 0.17   |
Assuming that the system demands surveying velocity range is 0~2200m/s, the Doppler frequency of different carrier frequency is generated by simulation. The result of difference between computing velocity and real velocity is shown in Figure 3. Figure 3 indicates that velocity changes in the range of 0~2200m/s, the difference between computing result and real velocity is smaller than 0.1m/s.

Conclusions

Based on the reasonable design of the wave beam direction of radar during landing power descending, a radar waveform is adopted, which can survey altitude and velocity. The simulation confirms that the system is satisfied with accuracy demand. For point target the surveying altitude accuracy is the range of 0.04m and the surveying velocity maximum error is 0.17m/s. Therefore, this system can provide landing control system with accurate altitude and velocity data.

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