A linear distribution of orbits in compact planetary systems?

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ABSTRACT
We report a linear ordering of orbits in a sample of multiple extrasolar planetary systems with super-Earth planets. We selected 20 cases, mostly discovered by the Kepler mission, hosting at least four planets within ~0.5 au. The semimajor axis $a_n$ of an nth planet in each system of this sample obeys $a(n) = a_1 + (n-1)\Delta a$, where $a_1$ is the semimajor axis of the innermost orbit and $\Delta a$ is a spacing between subsequent planets, which are specific for a particular system. For instance, the Kepler-33 system hosting five super-Earth planets exhibits the relative deviations between the observed and linearly predicted semimajor axes of only a few per cent. At least half of systems in the sample fulfill the linear law with a similar accuracy. We explain the linear distribution of semimajor axes as a natural implication of multiple chains of mean-motion resonances between subsequent planets, which emerge due to planet-disc interactions and convergent migration at early stages of their evolution.

Key words: planets and satellites: general.

1 INTRODUCTION
The Kepler photometric mission (Borucki et al. 2010) brought many discoveries of multiple low-mass planetary systems. There are several known systems with four or more super-Earths and/or Neptune/Uranus mass planets. In particular, there are the Kepler-11 system with six planets (Lissauer et al. 2011), five-planet systems Kepler-33 (Lissauer et al. 2012), Kepler-20 (Gautier et al. 2012) and Kepler-32 (Fabrycky et al. 2012). There are also a few systems with five candidate planets: KOI-435 (Ofir & Dreizler 2012), KOI-500, KOI-505 (Borucki et al. 2011) and several more four-planet systems. Configurations of this type were first discovered with Doppler spectroscopy, e.g. Gliese 876 (Rivera et al. 2010), Gliese 581 (Forveille et al. 2011), HD 10180 (Lovis et al. 2011) and HD 40307 (Tuomi et al. 2013). In the Gliese 876 system, however, two of the companions are Jovian planets, similarly to the Kepler-94 system. All studied systems, with a few discussed furthermore, host at least four planets with orbital semimajor axes $\lesssim 0.5$ au.

These discoveries raise a question on mechanisms leading to such compact ordering of the planetary systems, and simultaneously providing their long-term stability. Our recent study of the Kepler-11 system (Migaszewski, Słonina & Goździewski 2012) revealed that this system of six super-Earths is chaotic, and its marginal, long-term dynamical (Lagrangian) stability is most likely possible due to particular multiple mean-motion resonances (MMRs) between the planets. In the sample quoted above, we may pick up multiple configurations even more compact, and bounded to the distance as small as 0.08 au, e.g. Kepler’s KOI-500 with five planets. The orbital architecture of planetary systems of this class recalls the hypothesis of the packed planetary systems (PPS; Barnes & Raymond 2004), though originally formulated for configurations with Jovian companions. In the Jovian mass range, the orbital stability of multiple systems is statistically preserved, if planets in subsequent pairs with semimajor axes $a_1$, $a_2$ and masses $m_1$, $m_2$ are separated by more than $K \sim 4, 5$ mutual Hill radii $R_{H,M}$, where $R_{H,M} = (1/2)(a_1 + a_2)(m_1 + m_2)/(m_1)^{1/3}$ and $m_1$ is the mass of the parent star (Chatterjee et al. 2008). However, $R_{H,M}$ in the above systems is of the order of $10^{-3}$ au, hence their typical separation is at least one order of magnitude larger, $K \sim 10$, for the outermost pairs of planets, while for the innermost planets $K \sim 30–50$. A study of systems with 1 Earth-mass planets orbiting a Sun-like star conclude that the stability is maintained for $K$ roughly larger than 10–13 (Smith & Lithwick 2009). This seems in accord with the analysis of the Kepler-11 (Migaszewski et al. 2012) and similar systems which reveals, that likely they evolved into a particular architectures helping to maintain the stability. Indeed, due to small eccentricities, these systems unlikely suffered planet–planet scattering, often quoted in the literature to explain the observed eccentricity distribution in the sample of multiple extrasolar systems (e.g. Raymond et al. 2009). In the fight of the PPS hypothesis, the multiple, compact systems with super-Earths will be classified as packed multiple-planet systems, from hereafter. In this Letter, we report a detection of a linear ordering of the planets with their number (index) and argue that such particular architecture might stem from the planetary migration.

2 A PROTOTYPE CASE: THE KEPLER-33 SYSTEM
The Kepler-33 system hosts five planets. For the parent star mass $m_0 = (1.29 \pm 0.12)$ $M_\odot$ (see caption to Table 1 for references to all

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Table 1. The results of analysis of a sample of 20 systems. References: 1 – Lissauer et al. (2012), 2 – Ofir & Dreizler (2012), 3 – Borucki et al. (2011), 4 – Fabrycky et al. (2012), 5 – Tuomi et al. (2013), 6 – Hirano et al. (2012), 7 – Weiss et al. (2013), 8 – Lissauer et al. (2011), 9 – Forveille et al. (2011), 10 – Gautier et al. (2012), 11 – Rivera et al. (2010), 12 – Lovis et al. (2011).

| Star          | $m_0(M_\odot)$ | $N$ Ref. | $\Delta a$ (au) | $a_1$ (au) | $\delta$ (per cent) | $f_{2,3}$ (per cent) | $f_3$ (per cent) | Sequence |
|---------------|----------------|----------|-----------------|------------|----------------------|----------------------|-----------------|----------|
| Kepler-33     | 1.29 ± 0.12    | 5 1      | 0.0466 ± 0.0012 | 0.024 ± 0.004 | 6.1                 | 5.8                  | 1.8            | 2 – 3 – 4 – 5 – 6 |
| KOI-435       | 0.9            | 5 2      | 0.0507 ± 0.0012 | 0.0419 ± 0.0033 | 6.9                  | 8.0                  | 2.5            | 1 – 2 – 3 – 4 – 6 |
| KOI-1955      | 1.0            | 4 2      | 0.0497 ± 0.0015 | 0.025 ± 0.004 | 6.0                 | 14.4                 | 6.6            | 1 – 3 – 4 – 5   |
| KOI-719       | 0.68           | 4 3      | 0.0291 ± 0.0004 | 0.0155 ± 0.0017 | 4.3                 | 8.0                  | 3.6            | 2 – 3 – 6 – 8   |
| KOI-408       | 1.05           | 4 3      | 0.0306 ± 0.00035 | 0.0157 ± 0.0013 | 3.0                 | 3.8                  | 1.7            | 2 – 3 – 4 – 7   |
| KOI-671       | 0.96           | 4 3      | 0.0242 ± 0.00082 | 0.001 ± 0.003    | 5.7                 | 11.3                 | 5.2            | 3 – 4 – 5 – 6   |
| Kepler-32     | 0.58 ± 0.05    | 5 4      | 0.0195 ± 0.00036 | 0.0132 ± 0.0012 | 6.5                 | 6.7                  | 2.1            | 1 – 2 – 3 – 4 – 7 |
| KOI-500       | 0.66           | 5 3      | 0.0146 ± 0.0006 | 0.004 ± 0.002    | 10.2                | 23.2                 | 7.9            | 2 – 3 – 4 – 5 – 6 |
| HD 40307      | 0.77 ± 0.05    | 6 5      | 0.02795 ± 0.00016 | 0.0229 ± 0.0015  | 7.2                 | 4.1                  | 0.9            | 2 – 3 – 5 – 7 – 9 – 22 |
| KOI-730       | 1.07           | 4 3      | 0.01400 ± 0.00044 | 0.0071 ± 0.0033  | 8.3                 | 26.8                 | 12.8           | 6 – 7 – 9 – 11   |
| KOI-94        | 1.25 ± 0.40    | 4 6.7    | 0.0637 ± 0.0027 | 0.0437 ± 0.0062  | 8.8                 | 29.0                 | 13.8           | 1 – 2 – 3 – 5   |
| Glines 81     | 0.31 ± 0.02    | 5 9      | 0.01469 ± 0.0013 | 0.013 ± 0.001    | 7.3                 | 9.7                  | 3.1            | 2 – 3 – 5 – 10 – 15 |
| KOI-510       | 1.03           | 4 3      | 0.0243 ± 0.0005  | 0.0184 ± 0.0022  | 7.4                 | 21.4                 | 10.1           | 2 – 3 – 5 – 9   |
| KOI-505       | 1.01           | 5 3      | 0.01181 ± 0.0006  | 0.008 ± 0.001    | 9.8                  | 20.3                 | 6.9            | 4 – 6 – 7 – 10 – 33 |
| Kepler-31     | 1.21 ± 0.17    | 4 4      | 0.0521 ± 0.0013  | 0.047 ± 0.006   | 8.0                 | 24.8                 | 11.8           | 2 – 3 – 5 – 8   |
| Kepler-11     | 0.95 ± 0.10    | 6 8      | 0.01427 ± 0.00011 | 0.0077 ± 0.0019  | 13.4                | 33.9                 | 8.8            | 7 – 8 – 11 – 14 – 18 – 33 |
| Kepler-20     | 0.912 ± 0.035  | 5 10     | 0.0149 ± 0.0001  | 0.0027 ± 0.0011  | 7.9                 | 11.8                 | 3.8            | 4 – 5 – 7 – 10 – 24 |
| KOI-623       | 1.21           | 4 3      | 0.0353 ± 0.0027  | 0.024 ± 0.008    | 11.8                | 47.3                 | 23.8           | 2 – 3 – 4 – 5 – 15 |
| Gliese 876    | 0.33 ± 0.03    | 4 11     | 0.1030 ± 0.0063  | 0.021 ± 0.012    | 9.5                  | 33.5                 | 16.1           | 1 – 2 – 3 – 4   |
| HD 10180      | 1.06 ± 0.05    | 5 12     | 0.0717 ± 0.0003  | 0.0597 ± 0.0027  | 4.8                 | 3.0                  | 1.0            | 1 – 2 – 4 – 7 – 20 |

Figure 1. The $(n, a_n)$-diagrams of the best-fitting linear solutions computed for chosen planetary systems. See the text for details.

discussed systems) and the reported orbital periods, we computed the semimajor axes of the planets, $a_n$, where $n = 2, 3, 4, 5, 6$. The plot of $a_n$ against $n$ (the top left-hand panel of Fig. 1) reveals a clear linear correlation $a(n) = 0.024 + 0.047 (n - 1)$ (shown as a green line). We start the sequence of indices from 2 rather than from 1 to have $a_1 \in (0, \Delta a)$. We want this condition to be fulfilled in all studied examples. The uncertainties of the best-fitting parameters $a_1$, $\Delta a$ (accompanied by other quantities introduced below) are given in
the first row of Table 1. All of \( a_n \) are very close to the line on the
\((n, a_n)\)-graph. We found a few other systems exhibiting a similar
dependence of the semimajor axes on the planet index. To express
deviations between observed and predicted semimajor axis (O–C)
of a planet in a given system, we introduce \( \Delta a \equiv [a_n - a(n)]/a_n \) and
\( \bar{\Delta} a \equiv [a_n - a(n)]/\Delta a \), which are the (O–C) scaled by \( a_n \) and \( \Delta a \)
respectively. The top left-hand panel of Fig. 1 is labelled by \( \Delta a \) and
\( \bar{\Delta} a \) expressed in percentages, close to each red filled-circle marking
a particular \( a_n \). Values of \( \Delta a \) are given above the linear graph, and
\( \bar{\Delta} a \) below it. To measure the ‘goodness of fit’ of the linear model
for a whole \( N \)-planet system, we define \( \delta \equiv (\frac{1}{N} \sum_{i=1}^{N} \bar{\Delta} a_{i(i)})^{1/2} \times 100 \) per cent, where \( n(i) \) is an index given to \( i \)th planet. Therefore, \( \delta \)
is equivalent to the common rms scaled by \( \Delta a \). When the indices
\( n(i) \) for subsequent planets of the Kepler-33 are 2, 3, 4, 5, 6, the
resulting \( \delta \approx 6.1 \) per cent. We did not find any better parameters
and planets numbering. However, in other cases, as shown below,
non-unique solutions may appear for different \( \Delta a \).

To find the best-fitting combination of \( a_1, \Delta a \) and a sequence
of indices \( n(i) \), for each studied system, we perform a simple
optimization. We fix a point in the \((a_1, \Delta a)\)-plane, where \( a_1 \in [0, \Delta a] \)
and look for a set of \( n(i) \) providing \( \min \delta \). The results for the
Kepler-33 system are illustrated in the top left-hand panel of Fig. 2
in the form of one-dimensional scan over \( \Delta a \). Red colour is for
solutions for which minimal difference between subsequent indices
equals 1. For instance, a solution of a given \( \Delta a \) corresponding to
a sequence \( 1 - 2 - 4 - 6 - 7 \) would be plotted in red, because
differences between indices \( n(2) = 2 \) and \( n(1) = 1 \) as well as \( n(5) = 7 \)
and \( n(4) = 6 \) equal 1. On the other hand, \( \Delta a \) corresponding to a
sequence \( 1 - 3 - 5 - 8 - 11 \) would be plotted in black (minimal
difference between indices equals 2). It is obvious that when \( \Delta a \)
is much smaller than the distance between planets forming the closest
pair in a system, one can obtain very low values of \( \delta \). To avoid such
artificial solutions, we limit our analysis to solutions from the red
part of scans.

2.1 Testing the linear ordering for known packed systems

The sample consists of 20 systems (including Kepler-33). The results
are gathered in Table 1. Its columns display the name of the
star, its mass, the number of planets, the reference, \( \Delta a, a_1, \delta, f_2/3, \)
\( f_1 \) (False Alarm Probabilities, FAPs, defined below) and a sequence
of \( n(i) \). A few planetary systems have more than one record. Fig. 1
shows the \((n, a_n)\)-diagrams for nine chosen systems. For a reference,
one-dimensional scans of \( \delta(\Delta a) \) are presented in Fig. 2. The choice
of systems to be shown in Figs 1 and 2 was made on basis of a few
criteria, as low \( \delta \) and FAP, and as few gaps as possible. Because
this is a multiple-criteria choice it has to be, to some degree, arbitrary.
KOI-435 is a system with five planetary candidates in orbits
of \( a \lesssim 0.4 \) au and the sixth, much more distant object, for which
only one transit was observed. Here, we take into account only five
inner candidates. Fig. 2 reveals that the linear model corresponds to
the minimum of \( \delta \) around \( \Delta a \approx 0.05 \) au, which is close to the value
for the Kepler-33 system. The quality of this model is very good,
\( \delta \approx 7 \) per cent. Indices of the planets are 1, 2, 3, 4, 6, hence there is
a gap between planet 4 and planet 6. We did not find any better nor
alternative solution. It is not yet possible to say if such a gap should
be filled by yet undetected planet. The question is if such gaps are
frequent outcomes of physical processes leading to discussed archi-
tecture. If they are rare one might expect a planet with \( n = 5 \) in the
KOI-435, otherwise we cannot make any predictions. Fig. 1 shows
\((n, a_n)\)-diagram for this sequence. This system seems very similar
to the Kepler-33 system. A difference of \( a_1 \) means that the orbits
of planets in KOI-435 are slightly shifted, when compared to the
Kepler-33 orbits.

![Figure 2. Goodness of the linear fit \( \delta \) as a function of \( \Delta a \). Each panel is for one system.](https://academic.oup.com/mnrasl/article-abstract/436/1/L25/989782/989782)
We estimate, that the remaining seven systems shown in Fig. 2 obey the linear law similarly well. The most interesting example here is KOI-500 with five planets, which form a sequence $2 \sim 3 \sim 4 \sim 5 \sim 6$ (the same as Kepler-33). All planets reside within the distance of 0.08 AU from the parent star.

The Kepler-31 system (not shown in Fig. 2) of four candidate planets, exhibits non-unique solutions ($\Delta a \approx 0.052$ AU and $\Delta a \approx 0.081$ AU). For both of them $\delta \approx 8$ per cent. Indices of these models are 2, 3, 5, 8 and 2, 3, 4, 6, respectively. The next system, Kepler-11 has six planets. Two of its inner orbits are separated by only $\sim 0.015$ AU. Other orbits are separated by $\sim 0.05$ AU except of the last one, which is relatively distant. There are two possible solutions: $\Delta a \approx 0.0154$ AU ($\delta \gtrsim 13$ per cent) and $\Delta a \approx 0.052$ AU ($\delta \approx 12$ per cent). For the second case, the indices are 2, 2, 3, 4, 5, 9, i.e., two innermost planets have the same number 2. The parameters are almost the same as for KOI-435 and Kepler-31 (the first solution). The remaining members of the group of systems not shown in Fig. 2 exhibit relatively large $\delta$ or the best-fitting models have many ‘gaps’. Moreover, in some cases, more than one model is possible (see Table 1).

Having in mind systems with many gaps and/or large values of $\delta$, one might ask whether the linear ordering might be just a matter of blind coincidence, like the widely criticized Titius–Bode (TB) rule. To check the linear rule on statistical grounds, we applied the Monte Carlo approach of Lynch (2003). He expressed the TB model in the logarithmic scale, which can be directly used in our case. We then analyse a random sample of $10^4$ synthetic orbits of $a_i = a_i + [(n - 1) + k n] \Delta a$, where $n \in [-0.5, 0.5]$ is chosen randomly, while $k > 0$ is a scaling parameter. We optimize each synthetic system and compute the percentage of systems for which $\delta < \delta_0$, where $\delta_0$ is for the observed system. The resulting FAPs for $k = 2/3$ and $k = 1$ are displayed as $f_{2/3}$ and $f_1$ in Table 1, respectively. We conclude that the random occurrence of the linear ordering is unlikely ($f_{2/3} \lesssim 10$ per cent, $f_1 \lesssim 5$ per cent) for approximately half of the sample. Nevertheless, these results are not definite, as the FAPs might depend on the sampling strategy (Lynch 2003).

We stress here that the linear ordering of orbits is not expected to be a universal rule which all systems would obey. We found that some of them are ordered according to this rule while some other systems are built differently. Our next step is to explain this.

### 3 IS THE LINEAR RULE REFLECTING MMRs?

Since the linear spacing of orbits cannot be pure coincidence for all systems, there should be a physical mechanism leading to this particular ordering of them. Searching for possible explanations of this phenomenon, we found that it may appear naturally due to the inward, convergent migration of the planets interacting with the remnant protoplanetary disc. The migration of two planets in a gaseous disc has been studied in many papers (e.g. Papaloizou & Terquem 2006; Szuszkiewicz & Podlew ska-Gaca 2012). It is known that the migration usually leads to trapping orbits into the MMRs. It is reasonable to foresee that systems with more planets might be trapped into chains of MMRs, see Conclusions. We ask now if there is any combination of MMRs between subsequent pairs of planets resulting in the linear spacing of the orbits.

There are no MMRs leading to the exact linear spacing of the orbits ($\delta = 0$). However, we can pick up easily many different linear model possessing $\delta \sim 1$ per cent. We examined synthetic planetary systems of five and six planets involved in multiple MMRs. We searched for such combinations of MMRs which lead to the linear distribution of semimajor axes with no ‘gaps’. We found many models with $\delta < 4$ per cent. Let us quote some interesting examples. For a five-planet system, subsequent MMRs 7 : 3, 5 : 3, 2 and 4 : 3 correspond to a sequence $2 \sim 3 \sim 4 \sim 5 \sim 6$ and $\delta \approx 2.4$ per cent. Actually, this is very similar to the Kepler-33 system. Its planets are close to the same resonances.

A proximity of a particular pair of planets $i$ and $i + 1$ to a given MMR $q:p$, i.e., $P_{i+1}/P_i \approx q/p$ (where $q, p$ are relatively prime natural numbers), can be expressed through $\epsilon_{i+1,i} \approx (qP_i/pP_{i+1} - 1) \times 100$ per cent. For Kepler-33, one finds $\epsilon_{5,4} \approx 0.4$ per cent, $\epsilon_{6,5} \approx 0.8$ per cent, $\epsilon_{7,6} \approx 0.3$ per cent, $\epsilon_{8,7} \approx 0.3$ per cent, where the subsequent planets are called as $b$, $c$, $d$, and $f$, respectively. Periods ratios of the first two pairs of planets are almost exactly equal to rational numbers 7/3 and 5/3. For two more distant pairs, deviations from 3/2 and 4/3 are slightly larger, still as small as $\sim 3$ per cent.

If, in accord with the linear law, there existed one more innermost planet, it would be involved in 7 : 1 MMR with planet b. In such a case, the six-planet sequence would correspond to the MMRs chain of 7 : 1, 3 : 2, 4 : 3 and $\delta \approx 2.2$ per cent. Yet other MMRs between planets 1 and 6 are possible (6 : 1, 8 : 1, 9 : 1, 11 : 2), leading to $\delta < 3$ per cent. One more example of six planets involved in low-order MMRs is: 5 : 1, 2 : 1, 5 : 3, 3 : 2, 4 : 3 with $\delta \approx 3.4$ per cent (the first MMR could be also 6 : 1, 9 : 2); 7 : 1, 5 : 2, 5 : 3, 3 : 2, 4 : 3 ($\delta \approx 3.4$ per cent); 9 : 2, 9 : 4, 5 : 3, 3 : 2, 4 : 3 ($\delta \approx 3.6$ per cent); 6 : 1, 7 : 3, 4 : 2, 3 : 2, 4 : 3 ($\delta \approx 3.6$ per cent). There are many other solutions with higher order resonances and/or larger $\delta$. The most frequent MMRs in such sequences are 3 : 2, 4 : 3, 5 : 2, 3 : 2, 2 : 1 and 7 : 3. Considering the 4 : 3 MMR, Rein et al. (2012) argue that it is difficult to construct this resonance on the grounds of the common planet formation scenario. However, Rein et al. (2012) studied two-planet systems and their results might be not necessarily extrapolated for systems with more planets. Indeed, a recent paper by Coissou, Raymond & Pierens (2013) suggests quite opposite that forming the low-order MMRs, and the 4 : 3 MMR in particular, might be quite a natural and common outcome of a joint migration of planetary systems with low-mass members.

Kepler-33 is not the only system whose planets are close to MMRs. In Table 2, we gathered other systems with at least two MMRs with $|\epsilon| < 2$ per cent. For most systems from the studied

| System   | Res. ($|\epsilon| < 2$ per cent) | Res. ($|\epsilon| > 2$ per cent) | Res. ($|\epsilon| > 6$ per cent) |
|----------|----------------------------------|----------------------------------|----------------------------------|
| Kepler-33| 7b : 3c (+0.4)                    | 5c : 3d (+0.8)                    |                                  |
| KOI-435  | 5b : 2c (−0.9)                    | 8d : 5e (−0.5)                    |                                  |
| KOI-1955 | 7c : 4d (+1.2)                    | 3d : 2e (−0.3)                    |                                  |
| KOI-671  | 7b : 4c (−0.9)                    | 3c : 2d (+0.6)                    |                                  |
| Kepler-32| 3d : 2e (+1.2)                    | 13e : 5f (−0.1)                   |                                  |
| KOI-500  | 3c : 2d (−0.8)                    | 3d : 2e (−1.3)                    | 4e : 3f (−1.3)                    |
| HD-40307 | 9b : 4e (+0.9)                    | 5d : 3e (−1.7)                    | 3f : 2f (−0.3)                    |
| KOI-730  | 4b : 3c (−0.06)                   | 3c : 2d (−0.1)                    | 4d : 3e (−0.006)                 |
| KOI-94   | 14b : 5c (+0.6)                   | 12d : 5e (−1.3)                   |                                  |
| Kepler-11| 5b : 4c (−1.1)                    | 7c : 4d (+0.5)                    | 7d : 5e (−0.8)                    |
| Kepler-20| 5b : 3c (+1.0)                    | 9c : 5d (+1.1)                    | 9d : 5e (−0.2)                    |
| KOI-510  | 11b : 5c (+1.3)                   | 9c : 4d (−1.8)                    | 12d : 5e (−0.4)                   |
| KOI-623  | 3c : 2d (−1.0)                    | 8d : 5e (−0.5)                    |                                  |
| KOI-505  | 4c : 3d (−1.1)                    | 5d : 3e (+1.1)                    |                                  |
| Gliese 876| 2c : 1d (−1.6)                   | 2d : 1e (−1.7)                    |                                  |
| HD 10180 | 5c : 3d (−0.6)                    | 5d : 3e (−1.6)                    | 3c : 1f (−1.4)                    |
| KOI-730  | 4b : 3c (−0.06)                   | 3c : 2d (−0.1)                    |                                  |

Note: Values of $|\epsilon|$ for each system are shown in per cent.

Table 2. List of MMRs ($|\epsilon| < 2$ per cent). See the text for an explanation.
sample, there are two or even more resonant pairs. The KOI-730 system is a good example here, as all three pairs of planets exhibit almost exact period commensurabilities (Fabrycky et al. 2011): planets b and c are close to 4:3 MMR (\(\epsilon \approx -0.06\) per cent), planets c and d lie in a vicinity of the 3:2 MMR (\(\epsilon \approx -0.1\) per cent), and planets d and e are trapped in the 4:3 MMR (\(\epsilon \approx -0.006\) per cent). Furthermore, planets b and d, as well as planets c and f are very close to 2:1 MMR, and planets b and e are close to 4:1 MMR. This is an amazing example of a multiple, chain structure of MMRs. Still, it is not the only known system with all planets trapped into multiple MMRs. The Kepler-20 systems exhibit the following chain of MMRs, 5:3, 9:5, 9:5, 4:1. This implies that planets b and d are close to 3:1 MMR.

We do not attempt to study here whether a given system is involved in an exact MMR or only evolves close to this MMR. The migration does not necessarily result in trapping super-Earth into exact MMRs. Indeed, there are several mechanisms proposed in the literature to explain systematic and significant deviations of orbits in multiple Kepler systems from the MMRs (e.g. Lithwick & Wu 2012; Rein 2012; Petrovich, Malhotra & Tremaine 2013).

An inward migration of already formed planets is not the only scenario of an early history of a planetary system. A migration of small `pebbles’ may take place before they form a planet (Boley & Ford 2013; Chatterjee & Tan 2013) or both migration and formation may occur simultaneously. Although this is a very complex issue, because different mechanisms have to be taken into account, trapping planets into MMRs seems to be a natural outcome of a dissipative evolution of a young planetary system.

4 CONCLUSIONS AND DISCUSSIONS

Although in the sample of 20 PPS there are stunning examples of the linear architecture, not all studied systems could be satisfactorily described by the proposed rule. One possible explanation is that there exist additional planets in these systems, not yet detected (due to unfavourable orbit orientation or too small radii) or the systems are trapped into such chains of MMRs, which do not necessarily imply the linear architecture. It is also possible that the migration was stopped before pairwise MMRs were attained by the orbits, for instance due to relatively early disc depletion.

Some of the systems exhibit multiple-resonant structure, which, as we found here, might explain the linear spacing law. This means an occurrence of a chain of two-body MMRs. Remarkably, some of combinations of MMRs imply indexing of the planets without gaps. Nevertheless, there are many other combinations which may lead to sequences including `gaps’.

It is widely believed that a convergent migration of relatively small planets within protoplanetary disc or due to tidal interaction with the outer disc leads to trapping the planets into MMRs. Still, the underlying astrophysics is very complex (Paardekooper, Rein & Kley 2013; Quillen, Bodman & Moore 2013). We performed preliminary numerical studies of a simple model of planet-disc interaction (Moore, Hasan & Quillen 2013). We found that multiple-resonance capture is very likely. Recently, Moore et al. (2013) showed that, for appropriately chosen initial orbits and rates of migration, it is possible to simulate appearance of the chain of MMRs in KOI-730. This result is encouraging for the explanation of the linear spacing as the final outcome of relatively `quiet’ and slow migration of the whole, interacting systems towards the observed state. Obviously final chain of MMRs as well as \(\Delta\alpha\) depend on initial orbits as well as on disc properties. Longer migration at a given rate can result in smaller \(\Delta\alpha\). Our finding might be also confirmed by the fact that in most Kepler systems planets are not captured into exact MMRs – but are found close to them (e.g. Jenkins et al. 2013). Detailed studies of migration of multiple-planet systems are necessary to tell which final states, determined by the observed architectures, are likely. We postpone this problem to future papers.

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