Fast universal logical gates on topologically encoded qubits at arbitrarily large code distances

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A fundamental question in the theory of quantum computation is to understand the ultimate space-time resource costs for performing a universal set of logical quantum gates to arbitrary precision. To date, all proposed schemes for implementing a universal logical gate set, such as magic state distillation or code switching, require a substantial space-time overhead, including a time overhead that necessarily diverges in the limit of vanishing logical error rate. Here we demonstrate that non-Abelian anyons in Turaev-Viro quantum error correcting codes can be moved over a distance of order the code distance by a constant depth local quantum circuit followed by a permutation of qubits. When applied to the Fibonacci surface code, our results demonstrate that a universal logical gate set can be implemented on encoded qubits through a constant depth unitary quantum circuit, and without increasing the asymptotic scaling of the space overhead. The resulting space-time overhead is optimal for topological codes with local syndromes. Our result reformulates the notion of anyon braiding as an effectively instantaneous process, rather than as an adiabatic, slow process.

The possibility of a universal quantum computer, which can perform accurate, arbitrary quantum computations, rests on the possibility of quantum error correction and quantum fault-tolerance. The threshold theorems prove that arbitrarily accurate logical qubits and logical quantum gates can be achieved with noisy physical qubits, as long as the error rate associated with the physical qubits is below an appropriate error threshold. Specifically, the logical error rate \( p_L \propto (p/p_{th})^{d/2} \), where \( p_{th} \) is the error threshold, \( p \) is the error rate of the physical qubits, and \( d \) is the code distance [1,2].

There are two known routes for realizing quantum error correcting codes with arbitrarily low logical error rates: code concatenation and topological quantum error-correcting codes (QECC) [1,4]. These two schemes are actually intimately related, as concatenated codes use successive layers of small QECCs, such as Shor’s code, Steane code, or the Read-Muller code, which are actually topological QECCs defined on small lattices. In topological QECCs, repeated local syndrome measurements ensure that the many-body quantum wave function of the physical qubits is projected into a topologically ordered quantum state of matter [1,3].

In topological QECCs, logical qubits can be encoded through the spatial topology of the system, or through various types of defects, such as non-Abelian anyons, extrinsic twist defects, holes, or various types of boundary defects [3-7]. The code distance can be made arbitrarily large by considering arbitrarily large lattices and arbitrarily large spacings between the defects of the topological state that are used to encode logical qubits. In contrast to schemes based on code concatenation, topological QECCs can have arbitrarily large code distances while maintaining locality.

In order for a QECC to be useful for fault-tolerant quantum computation, it must be possible to perform fault-tolerant logical gates on the encoded logical qubits. Each QECC admits its own set of fault-tolerant logical operations. For example, braiding of non-abelian anyons, holes, or twist defects in topological QECCs implement certain logical gates in the code space. In particular, braiding of Fibonacci anyons in certain non-Abelian topological QECCs [4,5,8] can form a universal logical gate set [9,11].

Proposed methods to realize a universal fault-tolerant gate set involve magic state distillation [12] or code switching [13]. However, such methods necessarily carry a large space-time overhead of \( O(d^3) \). Either they require time overhead that is linear in \( d \), or they can partially trade space for time and use at least \( \log d \) time overhead (including classical computation costs), at the cost of an increased factor of \( d \) in space overhead [2,12,22]. It is thus an open question whether this space-time overhead cost to perform universal quantum computation is fundamentally necessary.

This space-time overhead is potentially prohibitive—it has been estimated that in surface code, for \( p \sim 10^{-3} \) and \( p_L \sim 10^{-15} \), more than \( 10^4 \) physical qubits will be needed per logical qubit [1,3]. Thus, to implement near-term quantum algorithms such as quantum chemistry simulations using a scheme with an \( O(d) \) time overhead, \( \sim 10^6 \) physical qubits will be needed [23]. A major experimental challenge is hence the vast number of physical qubits needed in the device, and it is therefore crucial to reduce the time overhead without further increasing the space overhead.

In this paper, we show that braiding of non-abelian anyons, and hence universal logical gates on encoded qubits, can be performed through a constant depth circuit acting on the physical qubits. The circuit depth is independent of the separation between the anyons, and thus independent of \( d \). In particular, the braiding circuit is composed of a local quantum circuit, \( \mathcal{U} \), which implements a local geometry deformation, and a permutation of qubits, \( \mathcal{P}_\tau \), separated by distance of \( O(d) \). The permutation can be implemented by moving qubits, i.e., \( j \rightarrow \sigma(j) \), or a two-step process using long-range SWAPs where the quantum states are swapped from the data register \( \{j\} \) to a temporary register \( \{j\}' \) and then back to the target sites in the data register \( \{\sigma(j)\} \). Our result can be generalized to arbitrary braids and Dehn twists, which generate the mapping class group of genus \( g \) surfaces with \( n \) punctures [24,25].

Turaev-viro codes

We present our results in terms of Turaev-Viro codes [5,8], which can capture all non-chiral topological states in 2D, and include the surface code and Kitaev quantum double models as special cases. For the application to universal gate sets we...
triangulation $\Lambda$

trivial graph $\hat{\Lambda}$

Figure 1. Definition of the Levin-Wen Hamiltonian and Turaev-Viro codes on a triangulated manifold (light grey lines indicate the triangulation $\Lambda$) and the corresponding trivalent graph $\hat{\Lambda}$ (blue lines). The thin red lines represent the string nets. The thick blue and red lines illustrate the plaquette and vertex projectors respectively.

are interested in the doubled Fibonacci state, which can be realized by a specific type of Turaev-Viro code.

The Turaev-Viro code associates to a closed surface $\Sigma$ a finite-dimensional code space $H_\Sigma$. We use $\Lambda$ to denote a triangulation of $\Sigma$ and $\hat{\Lambda}$ to denote the dual cellulation associated with $\Sigma$. In particular, $\Lambda$ defines a trivalent graph, such as the honeycomb lattice shown in Fig. 1. Each edge of $\Lambda$ (equivalently, of $\hat{\Lambda}$) is associated with an $N$-state qudit. If the qudit on a particular edge is in the state $|a\rangle$, we say that there is a string of type $a$ passing through that edge. The wave functions in the code space can be viewed as superpositions of closed string-net configurations consistent with certain string branching rules [8].

In the Turaev-Viro code, the states in the code space are exact ground states of a commuting projector Hamiltonian $H_\Lambda$, as illustrated on the right side of Fig. 1, where the vertices and plaquettes of $\hat{\Lambda}$ are labeled with the vertices and plaquettes of $\Lambda$. The branching rules [8] define a controlled-unitary operation; the external $a, b, c, d$ legs are the control qudits that determine the resulting unitary $F_{\Lambda}^{abc}$, whose matrix elements are $[F_{\Lambda}^{abc}]_{ef}$.

In the Fibonacci code, the only non-trivial $F$-matrix is:

$$\left(F_{111}^{111}\right) = \begin{pmatrix} \phi^{-1} & \phi^{-1/2} \\ \phi^{-1/2} & -\phi^{-1} \end{pmatrix}. \tag{5}$$

All other $F$-symbols are either 1 or 0, depending on whether they are consistent with the branching rules [Eq. (4) and Eq. (6) introduced below]. A quantum circuit implementing the $F$-operations in the Fibonacci surface code was presented in Ref. 26 and is shown in Fig. 2(a). The circuit inside the dashed box, consisting of a 5-qubit Toffoli gate sandwiched by two single-qubit rotations, implements the $F$-matrix in Eq. (5). Here, $R_y(\pm\theta) = e^{\pm i\theta/2}$ are single-qubit rotations about the y-axis with angle $\theta = \tan^{-1}(\phi^{-1/2})$. All the other maps are taken care of by the rest of the quantum circuit.

The Fibonacci code can be implemented by repeated measurements of the vertex and plaquette operators $Q_v$ and $B_p$, which can be performed with the aid of an ancilla qubit and a local single and two-qubit quantum circuit [26]. Ongoing progress has been made on syndrome extraction, decoding and error correction [27, 29]. In particular, the decoder for a phenomenological Fibonacci code has been simulated numerically, yielding an error threshold $p_{th} = 0.125$ [29].

Local geometry deformation

The wave functions in the code space on two different triangulations (dual trivalent graphs) $\Lambda$ ($\hat{\Lambda}$) and $\Lambda'$ ($\hat{\Lambda}'$) that differ locally can be related by moves known as 2-2 Pachner moves (also called F-moves) and 1-3 Pachner moves, with the following relations represented on the trivalent graph:

$$\Psi_\Lambda \left( \begin{array}{ccc} b & c \\ a & d \end{array} \right) = \sum_f F_{abc}^{def} \Psi_{\Lambda'} \left( \begin{array}{ccc} b & c \\ a & d \end{array} \right) \tag{6}$$

where the tensor coefficients are

$$F_{abc}^{def} = \frac{\phi^2}{\sqrt{\phi}} \left( \begin{array}{ccc} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{array} \right).$$

The plaquette operator is composed of $F$-symbols, $F_{abc}^{def}$. Together with the branching rules, the $F$-symbols define the topological order of the state, and therefore the code. The $F$ symbols also define a controlled-unitary operation; the external $a, b, c, d$ legs are the control qudits that determine the resulting unitary $F_{\Lambda}^{abc}$, whose matrix elements are $[F_{\Lambda}^{abc}]_{ef}$.
\[ F_{a|b|c} = \begin{bmatrix} a & b & c \\ 0 & 0 & d \\ e & f & 0 \end{bmatrix} \]

\[ \Psi_{\hat{\Lambda}} \left( \begin{bmatrix} b \\ d \\ c \\ a \end{bmatrix} \right) = [F_{f|d|e}] \frac{d_{f}d_{e}}{d_{c}} \Psi_{\hat{\Lambda}} \left( \begin{bmatrix} b \\ d \\ c \\ a \end{bmatrix} \right). \]

(7)

The local geometry deformation of the triangulation and dual trivalent graph corresponding to the two types of Pachner moves are illustrated in Fig. 2(b, c). Note that the F-move preserves the number of qubits (and also their locations) and is obviously a unitary transformation which can be implemented by the conditional circuit in Fig. 2(a). On the other hand, the 1-3 Pachner move adds (entangles) three additional ancilla qubits to the code space, as shown in Fig. 2(c) (from left to right). The reverse process (from right to left) removes (disentangles) three qubits from the code space. Therefore, the 1-3 Pachner move can be related to entanglement renormalization performing either fine-graining or coarse-graining of the lattice, which has been studied in the context of MERA (multi-scale entanglement renormalization ansatz) of string-nets [30, 31].

Taking into account the additional qubits, the 1-3 Pachner move can also be implemented by a sequence of unitary gates as illustrated in Fig. 2(d). We first consider three extra qubits, each initialized to the \(|0\rangle\) state. Next, we apply a CNOT (indicated by the purple arrow), which takes \(|b|0\rangle \rightarrow |b|b\rangle\). This is equivalent to an isometry in the MERA language. At the same time, we apply modular \(S\): \(|0\rangle \rightarrow \sum_{i} \alpha_{i} |i\rangle\) to the top-most qubit, which effectively builds a ‘tadpole diagram’ connected to the original graph through the edge with the remaining ancilla in the \(|0\rangle\) state. Note that the original edge labeled by \(b\) is split into two edges with the same label \(b\), where we have implicitly used the concept of a smooth string net (see Refs. [30] for details). Next, we apply two successive F-moves and hence end up with the desired trivalent graph with a triangular plaquette replacing the original vertex in the center. Since the process is unitary, it is also reversible.

Moving anyons in constant time
An intuitive way to understand the moving protocol is through the picture of local entanglement renormalization. The essence of entanglement renormalization and the MERA circuit can be understood as a global coarse-graining (fine-graining) process that ‘merges’ (‘splits’) several qubits together, effectively removing (adding) qubits in the code, as illustrated in Fig. 3(a). In the context of topological order, one can think of this process as squeezing (stretching) the manifold which supports the topological states. Now one can consider anyons or defects as punctures (yellow circles) in the manifold as illustrated in the lower panel. In order to separate two adjacent punctures to distance \(d\), one needs a MERA circuit with depth (layers) \(\log_{2}(d)\), where each step stretches the manifold by a factor of 2. When the two punctures are already separated by a distance \(d\), one can perform one layer of the local entanglement renormalization circuit (with constant depth) locally to stretch (fine-grain) the region between...
of arbitrary length moves) and local SW APs, we can effectively split a single row triangulation. By a finite sequence of F-moves (2-2 Pachner Pachner moves, which increase the number of vertices of the length. By utilizing ancilla qubits, we can implement the 1-3 Pachner move.

Now we investigate fault tolerance of the braiding circuit. Discussion of fault tolerance and experimental platforms

Figure 4. Gadgets for local geometry deformation in Turaev-Viro codes. The solid (dashed) purple lines represent added (removed) edges during the 1-3 Pachner moves. The pink line represent the switched edges in F-moves. The yellow arrow indicates the equivalence between two triangulations by locally shifting the positions of the edges, which can be physically implemented by local SWAPs.

the two punctures to increase the distance to 2d, which effectively adds qubits into the system, as illustrated in Fig. 4(b). Now the manifold is effectively enlarged due to the addition of qubits. In order to preserve the shape of the manifold away from the region of the punctures, one can also perform one layer of the entanglement renormalization circuit locally to squeeze (coarse-grain) the region on the left and right sides of the punctures, as shown in Fig. 4(c). Thus one effectively ends up with the same overall shape of the manifold, with the two punctures being separated by a factor of 2, i.e., \( d \rightarrow 2d \). Note that according to the left panel of Fig. 4(c), in order to map the qubit lattice exactly to the original shape, one performs SWAPs (green arrows indicated in the bottom layer) with largest distance of \( O(d) \). The long-range SWAP ensures that the actual location of each puncture is moved by a distance \( d/2 \).

**Instantaneous braiding circuit**

For the implementation of the braiding circuit, we need to introduce two elementary gadgets, as illustrated in Fig. 4. In Fig. 4(a), we consider triangulation of a single row of arbitrary length. By utilizing ancilla qubits, we can implement the 1-3 Pachner moves, which increase the number of vertices of the triangulation. By a finite sequence of F-moves (2-2 Pachner moves) and local SWAPs, we can effectively split a single row of arbitrary length \( L \) into two rows, with a constant (independent of \( L \)) number of steps (i.e., a constant depth local unitary circuit).

In Fig. 4(b), we illustrate how two rows can be converted into a single row by a finite number of steps. Note that in both of these protocols, the qubits on the outer boundary of the rows shown are completely unaffected, acting as control qubits for the unitary operations. This then allows the transformations to be applied to a large number of rows in parallel.

Using the above gadgets, we can now demonstrate our braiding circuit on a triangulated planar surface \( \Lambda \), shown in Fig. 5. In the first step, in the region between anyon I (red) and III (pink), we split rows of varying lengths in two rows, while combining rows in the region above the anyon, in a manner illustrated in Fig. 5(b). We create a lattice \( \Lambda' \) with a shearing pattern on the left and right sides of anyon I (red); the regions above anyon I being coarse grained (effectively squeezed) while the region below it is fine grained (effectively stretched).

Now via long-range permutation of qubits (indicated by green arrows) \( \mathcal{P}_\sigma \), where \( \mathcal{P}_\sigma \) is the unitary representation of the permutation \( \sigma \), we reach the configuration in Fig. 5(c) which is isomorphic to the configuration in Fig. 5(b), with anyon I (red) being moved up in space. To recover the original triangulation \( \Lambda \), we apply another step of retriangulation in the strip on the right of the (red) anyon (indicated by the pink thick lines), and hence map back to the original lattice in Fig. 5(d).

The above protocol, which uses a constant-depth local quantum circuit and long-range qubit permutations, effectively moves one anyon vertically by a distance of the order of the separation between the nearest anyon II, which is on the order of the code distance \( d \). The (vertical) separation between anyon I (red) and III (pink) is also increased by a factor of 2, which concretely demonstrates the local entanglement renormalization idea in Fig. 5(b). To complete a braiding cycle, we apply another 5 shots of a similar procedure, which then effectively braid anyons I and II around each other as illustrated in Fig. 5(e, f). Here, we show the qubits (black dots) and trivalent graph (light blue lines) explicitly for concreteness.

To summarize, a single braiding operation can be performed in a constant number of steps, independent of the system size and code distance. Note that this is in contrast with the previous computation schemes of the Turaev-Viro code presented in Ref. 5 where braiding or Dehn twists are implemented by sequential F-moves with circuit depth of \( O(d) \). In this case we have demonstrated a 6-step procedure:

\[
B_{1,II} = \prod_{i=1}^{6} \mathcal{U}_i \mathcal{P}_{\sigma,i} \mathcal{U}_{i+1}. \tag{8}
\]

Note that each step is composed of a constant-depth local quantum circuit \( \mathcal{U}_i \) corresponding to a retriangulation of the manifold, a permutation of qubits \( \mathcal{P}_{\sigma,i} \) over a distance \( O(d) \), and another local circuit \( \mathcal{U}_{i+1} \) in order to retriangulate the manifold back to the original triangulation.

**Discussion of fault tolerance and experimental platforms**

Now we investigate fault tolerance of the braiding circuit. We first consider the constant depth local quantum circuit
LU. These are also referred to as locality-preserving unitaries \cite{32,33}, and are protected logical gates since the ‘light cone’ \cite{34} bounds the propagation of the pre-existing errors.

The permutation $\mathcal{P}_\sigma$ used in our scheme belongs to a specific class that we can refer to as a connectivity-preserving isomorphism (CPI). While the CPI can permute qubits over long distances, it preserves the local connectivity of the underlying lattice structure of the codes (and Hamiltonian). More concretely, for a pair of neighboring vertices $v_1$ and $v_2$ in the original lattice (triangulation) and the permuted vertices $\sigma(v_1), \sigma(v_2)$, the edge $e[v_1, v_2]$ connecting the original vertices $v_1$ and $v_2$ is exactly transformed to the new edge connecting the new vertices, i.e., $\mathcal{P}_\sigma : e[v_1, v_2] \mapsto e[\sigma(v_1), \sigma(v_2)]$, which has length of $O(1)$ and hence remains local.

To analyze fault tolerance of a CPI, we first consider propagation of pre-existing errors under a perfect (error-free) CPI. Let us consider an error string (the two end points of the string corresponding to anyons) with a length $l$ much smaller than the code distance, $l \ll d$, and which has support on sites $\{j_1, j_2, \ldots, j_n\}$, as illustrated in Fig. 5(g). Our CPI permutation $\mathcal{P}_\sigma$ maps the string onto the new sites $\{\sigma(j_1), \sigma(j_2), \ldots, \sigma(j_n)\}$, which is a deformed error string with a length of the same order as before, i.e., $O(l) \ll d$. One can compare the two configurations in Fig. 5(g), and see that the error string A gets squeezed, B and C gets deformed, and D gets stretched. Therefore, although CPI does not preserve the location of errors, it only changes the length of the error string by a constant factor (independent of code distance), so that it does not introduce logical errors.

Now we further consider additional errors that can be generated during the process of the qubit permutation. Just as with any discussion of fault-tolerance, we have to assume a particular reasonable noise model. In our case, the permutations will yield fault-tolerant operations if errors in any site-to-site permutation process, i.e. $j \mapsto \sigma(j)$, are independent. We consider two schemes and their experimental platforms for implementing the permutations:

(1) Moving qubits. For certain experimental systems, one can directly move the qubits to desired positions. For example, high-fidelity fast shuttling of individual ion qubits has been realized experimentally \cite{35,36} and proposed for a scalable quantum computation architecture \cite{37,38}. The individual shuttling processes have independent noise, e.g., ion heating \cite{36} (as the dominant error source).

(2) Long-range SWAP. For experimental systems with long-range connectivity, one can implement the permutation $\mathcal{P}_\sigma$ using long-range SWAP operations, possible for the following platforms: I. Long-range connectivity in ion traps mediated by motional (phonon) modes of ions \cite{39}. II. Modular architecture of 3D superconducting cavities \cite{40–42} with photonic qubits, using reconfigurable long-range connectivity between cavity nodes, routed by microwave circulators and superconducting cables \cite{40}. One scheme is through direct quantum state transfer between remote cavity nodes in a network, equivalent to a long-range SWAP \cite{41}. The noise is uncorrelated if different cables are used for individual SWAP processes. An alternative is through remote entanglement generation and teleportation \cite{40,42}, which also has uncorre-
lated noise for individual teleportation channels. III. Circuit QED with cavity buses. Here, long-range interaction between superconducting qubits or semiconductor spin qubits can be mediated by cavity array serving as quantum buses \[43\]-[45].

We have shown that a single logical gate implemented by our constant depth circuit is fault-tolerant since it does not spread errors by more than a \(O(1)\) factor. A fully fault-tolerant computation will require syndrome measurements, decoding algorithms, and error recovery operations, a full analysis of which requires more detailed studies. Note that our protocols do not introduce plaquette or vertex operators whose measurement outcomes are unknown (in contrast to, for example, lattice surgery methods for surface code). Thus, while not studied explicitly here, we expect that the space-time paths of the errors can be fault-tolerantly decoded using a constant number of syndrome measurements per logical gate, with the similar approach in Ref. \[46\].

Our scheme demonstrates, for the first time, at a fundamental level the significant advantage of long-range connectivity in quantum architectures for implementing fault-tolerant quantum computation. In addition, our study essentially provides a vision to bridge ideas from quantum communication, such as robust quantum state transfer and teleportation, and ideas from fault-tolerant quantum computation.

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