Analysis of nonlinear stratified convective flow of Powell-Eyring fluid: Application of modern diffusion

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Abstract
The stratification phenomena have great importance in fishery management, insufficiency of dissolved oxygen in the lower parts of lakes, rivers and ponds, and phytoplankton populations. Thus the present article examines vital role of stratification phenomena in Powell-Eyring fluid flow due to inclined sheet which is stretched in a linear way. Collaboration of Cattaneo-Christov heat and mass flux model instead of Fourier Law of heat conduction is also accounted. Interpretation of heat transport is carried out with heat generation/absorption. Thermal stratification supports heat transport. Chemical reaction and solutal stratification also helped out mass transport. Non-linear governing equations with partial derivatives are converted into ordinary differential equation with the help of similarity transformations. Homotopic method is applied to solve arising dimensionless governing equations. Pertinent parameters and their physical behavior are displayed graphically. Drag force coefficient is also examined graphically. In culmination, substantial parameters of radiation and heat generation/absorption raised the temperature field while thermal relaxation time and solutal relaxation time parameters lower the temperature and concentration fields, respectively.

Keywords
Powell-Eyring, inclined sheet, dual stratification (nonlinear), linear stretching, Cattaneo-Christov heat and mass flux, thermal radiation, heat generation/absorption, chemical reaction

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Introduction
There are very useful applications of heat transfer phenomena like heat conduction in tissues and drugs, cooling in nuclear reactor etc. In the past emphasis has been given on the use of Fourier Law of heat conduction for heat exchange and Fick’s law for mass diffusion although the anomaly occurred in these laws is ignored. Law of heat conduction yields parabolic equation which means that any incipient transition is felt instantly all around the whole object. To overcome this bug Cattaneo introduced a thermal relaxation time factor in which propagated thermal waves are made to transfer heat at low speed. Later on Christov gave more improvement to this law. Farooq et al.1 have considered squeezed flow in a porous medium to analyze Cattaneo-Christov model. Dogonchi and Ganji2 considered nanofluid MHD flow to analyze Cattaneo-Christov heat flux

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between parallel plates. Hayat et al. used heat and mass diffusivity theory of Cattaneo-Christov model in 3D nanofluid flow. Hashim and Khan have discussed the flow of Carreau fluid over a slandering sheet by considering the Cattaneo-Christov model. Hayat et al. explained stretching flow in the presence of stratification and Cattaneo-Christov heat flux. Ijaz and Ayub discussed the modified model of fluxes in stratified flow of Walter-B fluid with activation energy. Shah et al. described the modified heat flux features in stagnation flow deformed by Riga sheet considering mixed convection. Khan et al. explored the stagnation flow of Carreau liquid under the Cattaneo-Christov theory.

In engineering and industrial applications non-Newtonian fluid possesses practical and fundamental importance. Non-Newtonian fluids play significant role in nuclear slurries, plasma, paper coating, polymers, mercury, etc., these fluids have non-linear relationship between shear stress and strain rate. Food products, inks, glues, soaps, shampoos, paints, etc., are other few examples of non-Newtonian fluids. In 1944, Powell and Eyring two scientists introduced a non-Newtonian fluid model which is derived from kinetic theory of gases instead of empirical formula and one of the advantages of Powell-Eyring fluid model is, it acts like a viscous fluid at high shear rate. Hayat et al. have applied non-Fourier heat flux theory on Powell-Eyring fluid flow. Hayat and Nadeem considered exponentially stretching sheet over Powell-Eyring fluid flow. Rehman et al. considered inclined stretching cylinder for the analysis of Powell-Eyring fluid flow with heat generation/absorption. Upadhay and Raju utilized heat and mass fluxes conditions in the flow of Powell-Eyring fluid. Hayat et al. used variable thermal conductivity in Powell-Eyring fluid flow. Jayachandra Babu et al. have considered porous medium to analyze Eyring-Powell-nanofluid flow induced by a cone. Ogunsaye and Sibanda investigated Eyring-Powell fluid flow deformed by a catalytic surface due to para-boloid revolution. Seyedi et al. discussed the Darcian flow of Powell-Eyring nanoliquid with entropy generation. Waqas et al. depicted the chemically reactive flow of hydromagneto Powell-Eyring fluid caused by stretching phenomenon. Abegunrin et al. explored the radiative and magnetic dipole features in Powell-Eyring nanoliquid with activation energy.

Convection, conduction and radiation are three sources of the heat transference. The collective fundamental of mixed convection and thermal radiation has great importance in science and technology especially in medical science. Radiations are used in the treatment of cancer, to kill bacteria and viruses, in microwave oven, to sterilize food stuffs and electromagnetic radiation in cell phone is some of the applications of radiation. Sulochana et al. analyzed fluid flow due to rotating cone in the presence of radiation. Ahmad et al. exposed radiation and chemical reaction properties in Sutterby fluid flow. Hayat et al. addressed Carreau fluid to check the thermal radiation influence in the presence of chemical reaction. Soomro et al. analyzed radiation effects on nanofluid flow with heat absorption/generation. Akbar et al. elaborated the effects of radiation on nanofluid flow with stagnation point. Qayyum et al. have given analysis of thermal radiation in third grade fluid flow with Newtonian conditions. Bhatti et al. explained the irreversibility in radiative nanofluid flow caused by stretching surface. Ramzan et al. disclosed the radiation phenomenon in MHD stratified CNT nanofluid flow. Khan et al. discussed the radiative flow of non-Newtonian nanofluid with magnetic dipole effect.

Five novel aspects are there in our present research. Firstly Powell-Eyring fluid flow is applied by inclined stretching sheet. Secondly, we analyzed heat and mass transfer via inclined stretching cone. Thirdly, we analyzed heat and mass transfer with Hamiltonian approach. Fourthly, we considered heat transfer with nonlinear stratification over stretchable inclined surface with the combination of Cattaneo-Christov model. Presence of heat absorption/generation and first order chemical reaction can also be visualized here. Hence to fill this breach is our main theme.

Mathematical modeling

We consider Powell-Eyring fluid (steady and incompressible) flow persuaded by linearly stretchable sheet which makes an angle $\beta_1$ with the horizontal axis (see Figure 1(a)). Cattaneo-Christov model is implemented to elaborate heat and mass transport. Nonlinear stratification is also implemented to analyze the features of allied mass and heat flux. Heat absorption/generation and thermal radiation assisted heat transfer phenomena. Mass transfer mechanism is evaluated in the light of constructive chemical reactions. After the implementation of boundary layer approximations we’ve,
with the boundary conditions

\[ u = u_0(x) = cx, \quad T_u(x) = T_0 + d_1x^2, \]
\[ C_w(x) = C_0 + e_1x^2, \quad v = 0, \quad at \quad y = 0, \]
\[ u \rightarrow 0, \quad T_u(x) = T_0 + d_2x^2, \quad C_w(x) = C_0 + e_2x^2, \]
\[ T \rightarrow T_0(x), \quad C \rightarrow C_0(x) \hspace{0.5cm} as \hspace{0.5cm} y \rightarrow \infty, \]

In above expressions \( u_c, \ u_w, \ T_u, \ \rho, \ g, \ T_0, \ C_0, \ \beta_1, \ C_w, \ \beta_c, \ K_1, \ D, \ C_p, (C_1, \ \beta), \ h_f, \ h_r, \ \beta_1, \ T_0, \ C_0, \ \tau_0, \ \tau_1, \ k^*, \ \sigma^*, \)
\( v, \ k, \ (d_1, \ d_2, \ e_1, \ e_2) \) represents free stream velocity, stretching velocity, stretching temperature of fluid, density, gravitational acceleration, variable ambient fluid temperature, variable ambient concentration, coefficient of thermal expansion, stretching concentration of heated fluid, mass expansion coefficient, chemical reaction coefficient, diffusion species coefficient, fluid specific heat, material parameters, heat transfer coefficient, mass transfer coefficient, angle of inclination, reference temperature, thermal relaxation time coefficient, solutal relaxation time coefficient, absorption coefficient, Stephen Boltzman constant, fluid kinematic viscosity, thermal conductivity, dimensional constants.

Implementing the transformations

\[ \xi = y \sqrt{\frac{c}{v}}, \quad \eta_1 = \frac{c}{v}, \quad \eta_2 = \frac{c}{v}, \quad \phi = -\sqrt{\frac{c}{v}}, \]
\[ \theta(\xi) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\xi) = \frac{C - C_\infty}{C_w - C_\infty}, \]

Equation (1) becomes zero identically, whereas equations (2)–(4) are as follows:

\[ (1 + \psi)f''' + f'' - f' \psi' = 0, \]
\[ \theta''(1 + R) - 2f' S_1 \Pr - 2f'' \theta \Pr + \psi f' \Pr + \delta \theta \Pr - \Pr \beta_s \]
\[ [4f^2 S_1 + 4f^2 \theta - 3ff'' \theta'] \]
\[ [-2ff'' S_1 - 2ff'' S_2 - 2ff'' \theta + f^2 \theta''] \]
\[ - \delta \beta_s \Pr[2f' \theta - \theta f'] - \beta_s R(2f'' \theta' - \theta' f) = 0, \]
\[ \phi'' - S_c(2S f^2 + 2f' \phi - f \phi') + S c k \phi - L \phi \]
\[ 4f^2 S_2 + 4f^2 \phi - 3ff' \phi' \]
\[ [-2ff'' S_2 - 2ff'' S_2 = -2ff'' \phi + f^2 \phi''] \]
\[ - L k \phi(2f'' \phi' - \phi f') = 0, \]

The corresponding dimensionless boundary conditions

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]
\[ \frac{\partial T}{\partial x} + \frac{\partial v}{\partial y} \leq 0, \]
\[ \frac{\partial C}{\partial x} + \frac{\partial v}{\partial y} = 0, \]
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]
\[ f'(0) = 1, f(0) = 0, \theta(0) = 1 - S_1, \phi(0) = 1 - S_2, \]
\[ f''(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0, \]
\[ (10) \]
where \( R \) is radiation parameter \( Sc \) represents Schmidt number, \( \delta \) is heat generation/absorption parameter, \( Pr \) indicates Prandtl number, \( \varepsilon, \delta \) material parameters, \( kr \) chemical reaction parameter, \( \lambda \) thermal buoyancy parameter, \( \beta_s \) thermal relaxation time parameter, \( S_2 \) represent stratified parameter (solutal), \( S_1 \) indicates thermal stratified parameter, solutal relaxation time parameter \( L \), and angle of inclination \( \beta_1 \). Parameters can be represented as:
\[
Pr = \frac{v}{\alpha}, \quad \varepsilon = \frac{1}{\mu_0 C_1}, \quad \delta = \frac{x^2 c^3}{2 x C^2}, \quad kr = \frac{K_1}{c},
\]
\[
Sc = \frac{v}{D}, \quad \lambda = g \beta_r (T_m - T_0) x^2, \quad \beta_s = c \tau_o,
\]
\[
L = c \tau_i, \quad R = \frac{16 u^*}{3 \kappa^* T^3}, \quad \delta_1 = \frac{Q_o}{\rho C_p c}
\]
Surface drag force is as follows:
\[
C_f = \frac{\tau_w}{\rho u_w^2}
\]
where wall shear stress is,
\[
\tau_w = \tau_{xy} = \left( \mu + \frac{1}{\beta C_1} \right) \left( \frac{\partial u}{\partial y} \right) - \frac{1}{6 \beta C_1} \left( \frac{\partial u}{\partial y} \right)^3
\]
\[
(13) \]
above equation after using equations (6), (13), and (12) reduced to
\[
C_f Re_{w}^{1/2} = (1 + \varepsilon) f''(0) - \frac{\varepsilon}{3} \delta f''(0)
\]
\[
(14) \]
Reynolds number is represented by \( Re_w = \frac{u_w^*}{v} \).

**Homotopic solutions**

To find out the solutions of governing equations we have adopted homotopy analysis method introduced by Liao.\(^{32,33}\) It has many advantages over other methods. Firstly it provides liberty to select initial guesses and linear operators. Secondly non-linear equations (weak or strong) frequently can be solved by this method. Thirdly it is independent of small or large parameters. The initial guesses are:
\[
\begin{align*}
f_0(\xi) &= 1 - \exp(-\xi), \\
\theta_0(\xi) &= (1 - S_1) \exp(-\xi), \\
\phi_0(\xi) &= (1 - S_2) \exp(-\xi).
\end{align*}
\]
\[
(15, 16, 17) \]
Supporting linear operators are:
\[
\begin{align*}
\mathbf{L}_f(f) &= \frac{d^3 f}{d \xi^3} - \frac{d f}{d \xi}, \\
\mathbf{L}_0(\theta) &= \frac{d^2 \theta}{d \xi^2} - \theta, \\
\mathbf{L}_\phi(\phi) &= \frac{d^2 \phi}{d \xi^2} - \phi,
\end{align*}
\]
\[
(18) \]
which satisfy the specified properties
\[
\begin{align*}
\mathbf{L}_f(B_1 + B_2 \exp(\xi) + B_3 \exp(-\xi)) &= 0, \\
\mathbf{L}_0(B_4 \exp(\xi) + B_5 \exp(-\xi)) &= 0, \\
\mathbf{L}_\phi(B_6 \exp(\xi) + B_7 \exp(-\xi)) &= 0,
\end{align*}
\]
\[
(19, 20, 21) \]
where \( B_i \) \((i = 1 - 7)\) are the optional constants.

**Zeroth-order problems**

\[
\begin{align*}
(1 - q) \mathbf{L}_f \left[ f(\xi; q) - f_0(\xi) \right] &= q h_1 \mathbf{N}_f \left[ f(\xi; q), \theta(\xi; q), \phi(\xi; q) \right], \\
(1 - q) \mathbf{L}_0 \left[ \theta(\xi; q) - \theta_0(\xi) \right] &= q h_0 \mathbf{N}_0 \left[ \theta(\xi; q), \theta(\xi; q) \right], \\
(1 - q) \mathbf{L}_\phi \left[ \phi(\xi; q) - \phi_0(\xi) \right] &= q h_0 \mathbf{N}_\phi \left[ \phi(\xi; q), \phi(\xi; q) \right],
\end{align*}
\]
\[
(22, 23, 24) \]
\[
\begin{align*}
\tilde{f}'(0; q) &= 1, \quad \tilde{f}(0; q) = 0, \quad \tilde{\theta}(0; q) = 0, \\
\tilde{\phi}'(0; q) &= 0, \quad \tilde{\phi}(0; q) = 0, \\
\tilde{\theta}(\infty; q) &= 0, \quad \tilde{\phi}(\infty; q) = 0,
\end{align*}
\]
\[
(25) \]
\[
\begin{align*}
\mathbf{N}_f \left[ f(\xi; q), \theta(\xi; q), \phi(\xi; q) \right] &= (1 + \varepsilon) \frac{\partial f(\xi; q)}{\partial \xi} + \tilde{f}(\xi; q) \frac{\partial^2 f(\xi; q)}{\partial \xi^2} \\
&\quad + \left( \frac{\partial \tilde{f}(\xi; q)}{\partial \xi} \right)^2 - \varepsilon \Delta \left( \frac{\partial \tilde{f}(\xi; q)}{\partial \xi} \right)^2 \frac{\partial^2 \tilde{f}(\xi; q)}{\partial \xi^2} \\
&\quad + \lambda \tilde{\theta}(\xi; q) \sin \beta_1 + \lambda_1 \tilde{\phi}(\xi; q) \sin \beta_1,
\end{align*}
\]
\[
(26) \]
\[ N_0 \left[ \tilde{f}(\xi, q), \tilde{\theta}(\xi, q) \right] = \left[ 1 + R \right] \frac{\partial^2 \tilde{f}(\xi, q)}{\partial \xi^2} - 2S_1 \text{Pr} \frac{\partial \tilde{f}(\xi, q)}{\partial \xi} - 2 \tilde{\theta}(\xi, q) \text{Pr} \frac{\partial \tilde{f}(\xi, q)}{\partial \xi} + \text{Pr} \tilde{f}(\xi, q) \frac{\partial^2 \tilde{\theta}(\xi, q)}{\partial \xi^2} + \delta \text{Pr} \tilde{\theta}(\xi, q) - \text{Pr} \beta_s \left[ 4 \left( \frac{\partial \tilde{f}(\xi, q)}{\partial \xi} \right)^2 S_1 + \frac{\partial \tilde{f}(\xi, q)}{\partial \xi} \frac{\partial \tilde{\theta}(\xi, q)}{\partial \xi} \right] - 2 \tilde{\theta}(\xi, q) \frac{\partial \tilde{f}(\xi, q)}{\partial \xi} - 3 \tilde{f}(\xi, q) \frac{\partial \tilde{\theta}(\xi, q)}{\partial \xi} \right] \]

\[ + \beta_s \left[ 2 \frac{\partial \tilde{f}(\xi, q)}{\partial \xi} \frac{\partial \tilde{\theta}(\xi, q)}{\partial \xi} - \frac{\partial \tilde{\theta}(\xi, q)}{\partial \xi} \tilde{f}(\xi, q) \right] , \]  

\[ N_0 \left[ \tilde{f}(\xi, q), \tilde{\phi}(\xi, q) \right] = \frac{\partial^2 \tilde{\phi}(\xi, q)}{\partial \xi^2} - \text{Sc} \left[ 2S_2 \frac{\partial \tilde{\phi}(\xi, q)}{\partial \xi} + \frac{\partial \tilde{f}(\xi, q)}{\partial \xi} \tilde{\phi}(\xi, q) - \frac{\partial \tilde{\phi}(\xi, q)}{\partial \xi} \tilde{f}(\xi, q) \right] + \text{ScKr} \tilde{\phi}(\xi, q) - \text{LSc} \left[ 2 \frac{\partial \tilde{\phi}(\xi, q)}{\partial \xi} - \frac{\partial \tilde{f}(\xi, q)}{\partial \xi} \tilde{\phi}(\xi, q) - \tilde{f}(\xi, q) \frac{\partial \tilde{\phi}(\xi, q)}{\partial \xi} \right] , \]  

\[ \text{Auxiliary parameters} \ h_f, \ h_n, \ h_o \ \text{hold non-zero values while} \ q \ \text{which is called embedding parameter belongs to} \ [0, 1]. \]

\[ m \text{-th order problems} \]

\[ L_0 \left[ \tilde{f}_m(\xi) - \chi_m f_{m-1}(\xi) \right] = h_f R_{m}^f(\xi), \]  

\[ R_{m}^a(\xi) = (1 + R) \theta_m - 2S_1 \text{Pr} f_{m-1}^a - 2 \text{Pr} \sum_{k=0}^{m-1} \left( f_{m-1-k}^a \theta_k \right) + \delta \text{Pr} \theta_{m-1} \]

\[ + \text{Pr} \beta_s \left[ 4 \left( f_{m-1-k}^a \right)^2 S_1 + \frac{\partial f_{m-1-k}^a}{\partial \xi} \frac{\partial \theta_k}{\partial \xi} \right] - 2 \text{Pr} \sum_{k=0}^{m-1} \left( f_{m-1-k}^a \theta_k \right) - 3 \left( f_{m-1-k}^a \right) \frac{\partial \theta_k}{\partial \xi} \right] \]

\[ + \beta_s \text{Pr} \text{Pr} \left[ 2 \left( f_{m-1-k}^a \theta_k \right) - \sum_{k=0}^{m-1} \left( \theta_k \right) \right] - \text{R} \beta_s \left[ 2 \sum_{k=0}^{m-1} \left( \theta_k f_{m-1-k}^a \right) - \sum_{k=0}^{m-1} \left( \theta_k f_{m-1-k}^a \right) \right] , \]

\[ R_{m}^p(\xi) = \varphi_{m-1}^p - \text{Sc} \left[ 2S_2 f_{m-1}^p + \frac{\partial f_{m-1-k}^p}{\partial \xi} - \sum_{k=0}^{m-1} \varphi_{m-1-k}^p \right] + \text{ScKr} \varphi_{m-1}^p \]

\[ - \text{LSc} \left[ 4S_2 \left( f_{m-1-k}^p \right)^2 + \frac{\partial f_{m-1-k}^p}{\partial \xi} \frac{\partial \varphi_{m-1-k}^p}{\partial \xi} \right] - 2 \text{LSc} \sum_{k=0}^{m-1} \left( f_{m-1-k}^p \varphi_{m-1-k}^p \right) - 3 \left( f_{m-1-k}^p \right) \frac{\partial \varphi_{m-1-k}^p}{\partial \xi} \right] \]

\[ - \text{LKrSc} \left[ 2 \left( f_{m-1-k}^p \varphi_{m-1-k}^p \right) - \sum_{k=0}^{m-1} \varphi_{m-1-k}^p \right] , \]

\[ \text{For} \ q = 0 \ \text{and} \ q = 1, \ \text{one can write} \]

\[ f(0) = 0, \ \theta(0) = 0, \ \varphi(0) = 0, \]  

\[ f(\infty) = 0, \ \theta(\infty) = 0, \ \varphi(\infty) = 0, \]  

\[ \chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1 \end{cases} \]
\[ \varphi(\xi; 0) = \varphi_0(\xi), \quad \varphi(\xi; 1) = \varphi(\xi), \tag{39} \]

and \( q \) varies from zero to one solution begins from initial approximations \( f_0(\xi), \theta_0(\xi) \) and \( \varphi_0(\xi) \) to final solutions. By using \( q = 1 \) and Taylor’s series we have

\[ f(\xi) = f_0(\xi) + \sum_{m=1}^{\infty} f_m(\xi), \tag{40} \]
\[ \theta(\xi) = \theta_0(\xi) + \sum_{m=1}^{\infty} \theta_m(\xi), \tag{41} \]
\[ \varphi(\xi) = \varphi_0(\xi) + \sum_{m=1}^{\infty} \varphi_m(\xi), \tag{42} \]

The appropriate solutions \( f_m, \theta_m, \) and \( \varphi_m \) of equations (42)–(44) corresponding to \( (f_m', \theta_m' \text{ and } \varphi_m') \) are

\[ f_m(\xi) = f_m'(\xi) + A_1 + A_3 e^\xi + A_3 e^{-\xi}, \tag{43} \]
\[ \theta_m(\xi) = \theta_m'(\xi) + A_4 e^\xi + A_3 e^{-\xi}, \tag{44} \]
\[ \varphi_m(\xi) = \varphi_m'(\xi) + A_6 e^\xi + A_7 e^{-\xi}, \tag{45} \]

**Convergence analysis**

To find the convergent series solution we’ve adopted HAM. It is clear from the Figure (1), the ranges provided by auxiliary parameters \( (h_f, h_u, h_\theta) \) are \(-1.8 \leq h_f \leq -0.1, -1.5 \leq h_u \leq -0.1, \) and \(-1.8 \leq h_\theta \leq -0.7, \) where the \( h \)–curves give horizontal plot, indicated as convergence region for series solution.

**Discussion**

Function of velocity, concentration, and temperature distributions are elaborated this section. Figure 2 demonstrates the reaction of thermal buoyancy parameter \( \lambda \) on \( f'(\xi) \). With the growing behavior of thermal buoyancy parameter \( \lambda \) velocity grows. Physically, with the rising values of \( \lambda \), gravitational forces enhance the buoyancy forces which in turn reduce the viscous effects and consequently, more deformation occurs in fluid motion. Hence velocity increases. Figure 3 indicates the behavior of material fluid parameter \( \varepsilon \) on \( f'(\xi) \). It is noted that fluid parameter raises the fluid velocity. In fact, as fluid parameter relates inversely to the viscosity so that higher fluid parameter decays the viscous forces and as a result, fluid deforms rapidly. Thus, velocity field grows. Figure 4 indicates the response of velocity field due to \( \beta_1 \) (angle of inclination). It describes that due to increase in \( \beta_1 \) buoyancy forces also increase with the effect of gravitational force which shows elevation in velocity field. Figure 5 illustrates the features of (Pr) Prandtl number on temperature field. Decrement in temperature field occurs for higher Prandtl number. Physically it justifies that dominant Prandtl number results in low thermal diffusivity which is responsible for low temperature distribution. Figure 6 exhibits the increasing trend of radiation
parameter $R$ on $\theta(\xi)$. Temperature distribution raises with dominant radiation parameter $R$. Physically higher radiation parameter $R$ results in low absorption coefficient which helps in enhancing the temperature field. Figure 7 shows impact of heat generation/absorption parameter $\delta_1$ on temperature enclosure $\theta(\xi)$. It has been found that dominant $\delta_1$ rises the fluid temperature. Physically, higher $\delta_1$ generates extra heat in the fluid system which resultantly intensifies the temperature field. Thermal boundary layer grows when heat generation parameter rises. Figure 8 depicts the variation of thermal relaxation time parameter $\beta_s$ on temperature profile $\theta(\xi)$. It is noticed that thermal layer thickness and temperature decreases with the increment of $\beta_s$. Physically, with thermal relaxation time increments, material particles need more time in transferring heat to their neighboring particles and as a result, temperature field decays. Figure 9 exhibits the decreasing profile of temperature distribution for higher stratification parameters $S_1$. Higher stratification parameter causes in decrement of temperature field. Physically, growing stratification parameter decays the difference in temperature between surface fluid and ambient fluid. Hence low temperature is detected. Figure 10 reflects the impact of chemical reaction parameter $kr$ on...
temperature field. Concentration field enhances for dominant values of destructive chemical reaction. For dominant destructive chemical reaction parameter, more heat is generated and consequently chemical reaction is destructive and not completed. Hence concentration field decays. Analysis of $L$ (solutal relaxation time parameter) on $f(x)$ is displayed in Figure 11. It depicts concentration and thickness of related layer minimizes with the increment of $L$. It sees that increment in $L$, material particles take extra time in mass transferring from sheet to the colder region. Hence, concentration field decays. Figure 12 examines $S_2$ (solutal stratified parameter) on concentration field. Leading $S_2$ is liable for lower concentration. Physically the concentration difference between surface and outside the boundary layer reduces for larger $S_2$. Hence declined concentration field appears. Analysis of Schmidt number is illustrated in Figure 13 for concentration field. Schmidt number ($Sc$) depicts the inverse relation with diffusion co-efficient. As expected, higher $Sc$ decays the mass diffusivity and therefore, concentration field reduces. In Figure 14 represents behavior of $e$ (fluid material parameter) and $\lambda_1$ (solutal) buoyancy parameter on $Re^{1/2}/Cf$. It is noticed that coefficient of drag force decreases with increment in $e$ and $\lambda_1$. In Figure 15 streamlines are sketched for flow behavior of the considered fluid.

Summary

In current investigation phenomena of radiation and quadratic stratification with heat generation/absorption in Powell-Eyring fluid flow deformed by inclined sheet is described. With Cattaneo-Christov heat and mass flux conditions whole problem is analyzed and declared. The salient features are:

- Angle of inclination executes gain in velocity field because of higher rate of transfer of heat.
- Radiation parameter concludes raise in temperature field due to higher heat flux on surface.
Heat generation/absorption results increased temperature profile because of heat generation. Thermal relaxation time parameter results decrement in temperature field. Increment of solutal relaxation time parameter declares decrement of concentration field. Thermal and solutal stratified parameters accordingly lessen the temperature and concentration fields. Chemical reaction parameter lowers the concentration field.

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**Figure 14.** Analysis of $\lambda_1$ and $\varepsilon$ on $C_f$.

**Figure 15.** Streams lines.

- Heat generation/absorption results increased temperature profile because of heat generation.
- Thermal relaxation time parameter results decrement in temperature field.
- Increment of solutal relaxation time parameter declares decrement of concentration field.
- Thermal and solutal stratified parameters accordingly lessen the temperature and concentration fields.
- Chemical reaction parameter lowers the concentration field.

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### Appendix I

**Notation**

- $u, v$: velocity components $ms^{-1}$
- $\rho$: fluid density $kgm^{-3}$
- $\beta_T$: thermal expansion $K^{-1}$
- $T_a$: ambient temperature $K^{-1}$
- $T_w$: wall temperature $K^{-1}$
- $T_f$: fluid temperature $K^{-1}$
- $T_0$: reference temperature $K^{-1}$
- $g$: gravitational acceleration $ms^{-2}$
- $\beta_1$: angle of inclination *radial*
- $Q_0$: heat generation coefficient $Wm^{-2}K^{-1}$
- $C_p$: specific heat capacity $kJ^{-1}s^{-2}$
- $k^s$: Stephen Boltzmann constant $Wm^{-2}K^{-4}$
- $k$: mean absorption coefficient $m^{-1}$
- $\tau_0$: thermal conductivity $Wm^{-1}K^{-1}$
- $\tau_1$: thermal relaxation time $s$
- $\tau_s$: solutal relaxation time $s$
- $K_1$: chemical reaction constant $s^{-1}$
- $v$: kinematic viscosity $m^2s^{-1}$
- $D$: diffusion species coefficient $m^2s^{-1}$
- $c$: dimensional constants $m$
- $d_1, d_2, e_1, e_2$: dimensional constants $m$