On exit times of Lévy-driven
Ornstein–Uhlenbeck processes

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Abstract

We prove two martingale identities which involve exit times of Lévy-driven Ornstein–Uhlenbeck processes. Using these identities we find an explicit formula for the Laplace transform of the exit time under the assumption that positive jumps of the Lévy process are exponentially distributed.

Keywords: exit times, Ornstein–Uhlenbeck process, martingale identities

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1 Introduction

Let $X_t, t \geq 0$, be an Ornstein–Uhlenbeck (O-U) process driven by a Lévy process $L_t$, i.e. $X_t$ is a solution of the equation

$$X_t = x - \beta \int_0^t X_s ds + L_t, \quad t \geq 0.$$ 

We assume that the parameter $\beta$ is positive and the initial value $X_0 = x$ is non-random.

In the special case when $L_t$ is a compound Poisson process, the process $X_t$ is also known in applications as a “shot-noise” process or a “storage process” with a linear release function.

One of the most important for the models of that sort problems is to determine or to approximate the distribution of the first passage time

$$\tau_b = \inf\{t > 0 : X_t \geq b\}$$

of a given level $b > x$. The problem was discussed for the Gaussian O-U processes by Darling & Siegert (1953). Explicit representations for the Laplace
transform $E(e^{-\mu \tau_b})$ were found in the papers of Hadjiev (1983) and Novikov (1990, 2003) in the case when $L_t$ has no positive jumps (the so-called spectrally negative case). Moreover, the papers of Novikov and Ergashev (1993) and Novikov (2003) provide some bounds and asymptotic approximations for the distribution of $\tau_b$. In particular, it was proved in Novikov (2003), Theorem 2, that the distribution of $\tau_b$ is exponentially bounded under the condition that $L_t$ has a diffusion part or positive jumps. The papers Perry et al. (2001) and Borovkov and Novikov (2001) contain some general results on integral equations for the distributions of $\tau_b$ and $X_{\tau_b}$.

It seems that the first results for Lévy-driven O-U processes with exponentially distributed jumps were obtained by Tsurui and Osaki (1976) for the case when the parameter $1/\beta$ is an integer. For the case of arbitrary $\beta > 0$ and exponentially distributed positive jumps, explicit formulas for the Laplace transform $E(e^{-\mu \tau_b})$ and the expectation of $\tau_b$ can be found in the paper of Novikov et al. (2005) who solved the corresponding integro-differential equation. Recently, Jacobsen and Jensen (2006) found the joint Laplace transform $E(e^{-\mu \tau_b + wX_{\tau_b}})$ in the form of a linear combination of contour integrals under the assumption that a distribution of the positive jumps is a mixture of exponential ones.

In what follows, we always assume that the following condition holds:

$$E \log(1 + |L_t|) < \infty$$ (1)

(this is a sufficient and necessary condition for convergence of $X_t$ in distribution to a proper limit, see e.g. Wolfe (1982)).

In Section 2 we prove two martingale identities (see Theorems 1 and 2 below) which involve both the first passage time $\tau_b$ and $X_{\tau_b}$. These identities enable one to obtain explicit bounds for the distribution of $\tau_b$ (e.g. an explicit lower bound for $E\tau_b$) just by neglecting the overshoot $\chi_b = X_{\tau_b} - b$.

In Section 3 we use Theorems 1 and 2 for deriving explicit representations of the Laplace transform and the mean of $\tau_b$ under the assumption that the positive jumps of $L_t$ are exponentially distributed but without any restrictions on the distribution of the negative jumps of $L_t$. We also prove the Exponential Limit Theorem for $\tau_b$ as $b \to \infty$.

2 Martingale identities

In what follows we always assume that $L_t$ has a non-zero component with positive jumps (or, equivalently, $\Pi(0, \infty) > 0$ where $\Pi(dx)$ is the Lévy-Khinchin measure associated with $L_t$). This assumption implies that $L_t$ has the following representation:

$$L_t = Q_t + R_t$$ (2)
with the compound Poisson process

\[ R_t = \sum_{k=1}^{N_t} \xi_k, \quad (3) \]

where \( \xi_k \) are the jumps of \( L_t \) which are greater than some positive number \( \delta \).

\( P\{\xi_k > \delta\} > 0 \), \( N_t \) is a Poisson process with rate \( E(N_1) = \lambda > 0 \). We also assume that the component \( Q_t \) can only contain a diffusion part and jumps less than or equal to \( \delta \) and therefore \( Q_t \) and \( R_t \) are independent.

Set

\[ K = \sup\{u \geq 0 : Ee^{uL_1} < \infty\}. \]

We shall further assume that \( K > 0 \) and set

\[ \varphi(u) = \int_0^u \frac{\log(Ee^{vL_1})}{v} dv, \quad 0 \leq u < K. \quad (4) \]

Since \( Q_t \) and \( R_t \) are independent, we have

\[ \varphi(u) = \Delta(u) + W(u), \quad (5) \]

where we put

\[ \Delta(u) = \int_0^u \frac{\log(Ee^{vQ_1})}{v} dv, \quad (6) \]

\[ W(u) = \int_0^u \frac{\log(Ee^{vR_1})}{v} dv = \int_0^u \frac{Ee^{v\xi_1} - 1}{v} dv. \]

Under the assumption (1), the integrals in (4) and (6) converge (see some details of the proof for this fact in Wolfe (1982) or Novikov (2003)) and so \( \varphi(u), \Delta(u) \) and \( W(u) \) are finite continuous functions. Besides, for all \( u \in [0, \infty) \) the following lower bound holds

\[ \Delta(u) \geq -c - Cu \quad (7) \]

(see Novikov (2003)).

Using the inequality \( e^x > 1 + x + x^2/2, \quad x > 0 \), we obtain also

\[ W(u) \geq \frac{\lambda}{\beta} (u\delta + u^2\delta^2/4) P\{\xi_k > \delta\} > 0. \quad (8) \]

Set

\[ \varphi(K) = \lim_{u \uparrow K} \varphi(u). \]

If \( K = \infty \), then \( \varphi(K) = \infty \) due to (8). If \( 0 < K < \infty \), then the value \( \varphi(K) \) could be finite or infinite as illustrated by the following example where the

\[ ^{1}c \text{ and } C \text{ are some positive constants} \]
Compound Poisson process $L_t = \sum_{k=1}^{N_t} \xi_k$, has the jumps $\xi_k$ with the Gamma distribution, i.e.

$$P(\xi_k \in dx) = \frac{x^{\rho-1}e^{-x}}{\Gamma(\rho)}dx, \ x > 0, \rho > 0.$$  

Then $K = 1$ and by direct calculation

$$\varphi(u) = \frac{\lambda}{\beta} \int_0^u \frac{1-(1-v)\rho}{(1-v)\rho}dv, \ u < 1,$$

so that

$$\varphi(1) < \infty \quad \text{for } \rho < 1$$

and

$$\varphi(1) = \infty \quad \text{for } \rho \geq 1.$$  

Note that when $\rho = 1$ (this is the case of exponentially distributed jumps with mean one) we have the explicit formula

$$\varphi(u) = -\frac{\lambda}{\beta} \log(1-u), \ u < 1. \quad \text{(9)}$$

Set

$$G(z, \mu) = \int_0^K e^{uz-\varphi(u)\mu}u^{\mu-1}du, \ \mu > 0. \quad \text{(10)}$$

This function is, obviously, finite when $K < \infty$. For the case $K = \infty$ the finiteness of $G(z, \mu)$ is implied by (7) and (8).

**Theorem 1.** Let condition (11) hold, $0 < K \leq \infty$ and $\varphi(K) = \infty$. Then

$$E(e^{-\mu\beta t}G(X_{\tau_b}, \mu)) = G(x, \mu), \ \mu > 0. \quad \text{(11)}$$

**Proof.** First consider the case

$K = \infty,$

in which it was shown by Novikov (2003) that the process $e^{-\mu\beta t}G(X_t, \mu)$ is a martingale with respect to the natural filtration $\mathcal{F}_t = \sigma\{L_s, s \leq t\}$.

Applying the optional stopping theorem, we have for any $t > 0$

$$E[I\{\tau_b \leq t\}e^{-\mu\beta t}G(X_{\tau_b}, \mu)] + E[I\{\tau_b > t\}e^{-\mu\beta t}G(X_t, \mu)] = G(x, \mu). \quad (12)$$

Since $X_t \leq b$ on the event $\{\tau_b > t\}$ and $G(x, \mu)$ is a nondecreasing function of $x$, we obtain

$$E[I\{\tau_b > t\}e^{-\mu\beta t}G(X_t, \mu)] \leq e^{-\mu\beta t}P\{\tau_b > t\}G(b, \mu) \to 0$$

as $t \to \infty$. The first term on the LHS of (12) clearly converges monotonically to $E(e^{-\mu\beta \tau_b}G(X_{\tau_b}, \mu))$ as $t \to \infty$ because $\tau_b$ is finite with probability one (in fact, it is even exponentially bounded). So (11) holds when $K = \infty$. 


To prove (11) in the case $0 < K < \infty$, we shall truncate positive jumps of $L_t$ by a positive constant $A$ and then justify a passage to the limit as $A \to \infty$. Set

$$L_t^A = Q_t + R_t^A$$

with

$$R_t^A = \sum_{k=1}^{N_t} \min(\xi_k, A).$$

Let $X_t^A$ be an O-U process driven by $L_t^A$,

$$\tau_b^A = \inf\{t \geq 0 : X_t^A \geq b\}$$

and

$$\varphi_A(u) = \frac{1}{\beta} \int_0^u \frac{\log(E e^{vL_t^A})}{v} dv = \Delta(u) + W(u, A),$$

where we put

$$W(u, A) = \frac{\lambda}{\beta} \int_0^u \frac{(E e^{v\min(\xi, A)} - 1)}{v} dv.$$

It is obvious from the Lévy-Khintchin formula that the right distribution tail of $L_t^A$ decays faster than any exponential function, so that the respective value $K = K(A) = \infty$. Hence, identity (11) does hold for the process $X_t^A$:

$$E \left[ e^{-\mu \tau_b^A} \int_0^\infty \exp\{uX_t^A - \varphi_A(u)\} u^{\mu-1} du \right] = \int_0^\infty e^{ux - \varphi_A(u)} u^{\mu-1} du. \quad (14)$$

Further we note that as $A \to \infty$

$$\int_0^\infty e^{ux - \varphi_A(u)} u^{\mu-1} du \to \int_0^K e^{ux - \varphi(u)} u^{\mu-1} du \quad (15)$$

which gives the RHS of (11). To see this, we note that, as $A \to \infty$, for any $u < K$

$$W(u, A) \to W(u), \quad \varphi_A(u) \to \varphi(u),$$

and for $u \geq K$

$$W(u, A) \to \infty.$$

and obviously, the last two relations imply (15). Next, on the LHS of (14) we write

$$\int_0^\infty = \int_0^K + \int_K^\infty$$

and consider convergence of the corresponding two terms separately. Note that

$$X_{\tau_b^A} \leq b + \delta + \min(\xi_{N_{\tau_b^A}}, A). \quad (16)$$
Obviously, \( \tau_b^A \) could only decrease as \( A \) increases. Choose now a positive constant \( A_0 \) such that \( P\{\xi_1 < A_0\} > 0 \) and so \( \tau_b^{A_0} \) is exponentially bounded. Then we have for all \( A > A_0 \)

\[
e^{uX^A_{\tau_b}} \leq e^{u(b+\delta)} \sum_{k=1}^{N_{\tau_b}^{A_0}} e^{u \min(\xi_k, A)}
\]

(17)

where by Wald’s identity

\[
E\left( \sum_{k=1}^{N_{\tau_b}^{A_0}} e^{u \min(\xi_k, A)} \right) = E(N_{\tau_b}^{A_0})E(e^{u \min(\xi_1, A)})
\]

(18)

and

\[
E(N_{\tau_b}^{A_0}) = \lambda E(\tau_b^{A_0}) < \infty.
\]

Collecting together the above bounds we obtain for the \( \int_K \) part of the LHS of (14) the following bound:

\[
E \left[ e^{-u\beta \tau_b^A} \int_K^{\infty} \exp\{uX^A_{\tau_b} - \varphi_A(u)\} u^{\mu-1} du \right] \leq C \int_K^{\infty} E(e^{u \min(\xi_1, A)}) e^{u(b+\delta)-\varphi_A(u)} u^{\mu-1} du,
\]

(19)

where \( C = \lambda E(\tau_b^{A_0}) \). To show that the last integral converges to zero as \( A \to \infty \), we note that (13) implies

\[
E(e^{u \min(\xi_1, A)}) = \frac{\beta u}{\lambda} \frac{\partial W(u, A)}{\partial u} + 1.
\]

This means that

\[
\int_K^{\infty} E(e^{u \min(\xi_1, A)}) e^{u(b+\delta)-\varphi_A(u)} u^{\mu-1} du
\]

\[
= \frac{\beta}{\lambda} \int_K^{\infty} \frac{\partial W(u, A)}{\partial u} e^{u(b+\delta)-W(u, A)-\Delta(u)} u^{\mu} du + \int_K^{\infty} e^{u(b+\delta)-W(u, A)-\Delta(u)} u^{\mu-1} du.
\]

(19)

The last integral tends to zero as \( A \to \infty \) due to the fact that \( W(u, A) \to \infty \) for \( u \geq K \).

Integrating by parts the first integral on the RHS of (19), we obtain:

\[
\int_K^{\infty} \frac{\partial W(u, A)}{\partial u} e^{u(b+\delta)-W(u, A)-\Delta(u)} u^{\mu} du = - \int_K^{\infty} e^{u(b+\delta)-\Delta(u)} u^{\mu} d(e^{-W(u, A)})
\]
\[ e^{K(b+\delta)-\Delta(K)}K^\mu e^{-W(K,A)} + \int_0^K e^{-W(u,A)} d(e^{u(b+\delta)-\Delta(u)} u^\mu). \]

Now it should be clear that the last two terms converge to zero as \( A \to \infty \) due to the fact that \( W(u,A) \to \infty \) for \( u \geq K \). So, we have proved the convergence of the \( \int_K^\infty \) part of the LHS of (14) to zero.

To study the part with \( \int_0^K \) on the LHS of (14), note that the random variable \( X^A_{\tau^b} \) coincides with \( X_{\tau^b} \) on the set \( \{ \max_k \xi_k < A \} \) (because no jumps are truncated up to the time \( \tau^b \leq \tau^b_A \)). Obviously, as \( A \to \infty \)
\[ P\{ \max_k \xi_k < A \} \to 1, \tag{20} \]
and hence
\[ E \left[ \begin{array}{c} I \{ \max_k \xi_k < A \} e^{-\mu \beta \tau^b} \int_0^K e^{uX_{\tau^b} - \varphi_A(u)} u^{\mu-1} du \\ \end{array} \right] \]
\[ \to E \left[ e^{-\mu \beta \tau^b} \int_0^K e^{uX_{\tau^b} - \varphi_A(u)} u^{\mu-1} du \right]. \]

To complete the proof, we need to check only that
\[ \lim_{A \to \infty} E \left[ I \{ \max_k (\xi_k) \geq A \} \int_0^K e^{uX^A_{\tau^b} - \varphi_A(u)} u^{\mu-1} du \right] = 0. \tag{21} \]

To see this, note that in view of (17), we have for all \( A \geq A_0 \)
\[ \int_0^K e^{uX^A_{\tau^b} - \varphi_A(u)} u^{\mu-1} du \leq \sum_{k=1}^{N_A} \int_0^K e^{u\min(\xi_k,A)} e^{u(b+\delta)-\Delta(u)} e^{-W(u,A)} u^{\mu-1} du \leq C \eta_A, \]
where we put
\[ \eta_A = \sum_{k=1}^{N_A} \int_0^K e^{u\min(\xi_k,A)} e^{-W(u,A)} u^{\mu-1} du, \quad C = e^{K(b+\delta)-\min_{u \leq K} \Delta(u)}. \]

Due to this bound and (20), for the validity of (21) it is sufficient to show that \( \{ \eta_A, A \geq A_0 \} \) is a family of uniformly integrable random variables or, equivalently, that
\[ \lim_{A \to \infty} E(\eta_A) = E( \lim_{A \to \infty} \eta_A ) < \infty. \]
In view of (18) the latter property is equivalent to

$$\lim_{A \to \infty} E\left( \int_0^K e^{u \min(\xi_k, A)} e^{-W(u, A)} u^{\mu-1} du \right) = E\left( \int_0^K e^{u \xi_k} e^{-W(u)} u^{\mu-1} du \right) < \infty.$$ 

which one can easily verify (e.g. using monotonicity of the functions \(\min(\xi_k, A)\) and \(W(u, A)\)).

This completes the proof.

**Theorem 2.** Let condition (1) hold, \(0 < K < \infty\) and \(\varphi(K) = \infty\). Then

$$\beta E(\tau_b) = E \int_0^K (e^{uX_{\tau_b}} - e^{ux}) e^{-\varphi(u)} u^{\mu-1} du.$$ 

**Proof.** We will derive this identity from Theorem 1 by passing to the limit as \(\mu \to 0\). To justify this procedure we observe that (11) can be written in the following form:

$$E(e^{-\mu \beta \tau_b} \int_0^K (e^{uX_{\tau_b}} - e^{ux}) e^{-\varphi(u)} u^{\mu-1} du) = (1 - E e^{-\mu \beta \tau_b}) \int_0^K e^{ux \varphi(u)} u^{\mu-1} du, \mu > 0.$$ 

Here the LHS converges to \(E \int_0^K (e^{uX_{\tau_b}} - e^{ux}) e^{-\varphi(u)} u^{\mu-1} du\) as \(\mu \to 0\). One can easily see (e.g. using integration by parts) that

$$\int_0^K e^{ux \varphi(u)} u^{\mu-1} du \sim \frac{1}{\mu}. \quad (22)$$

This implies that

$$\lim_{\mu \to 0} (1 - E e^{-\mu \beta \tau_b}) \int_0^K e^{ux \varphi(u)} u^{\mu-1} du = \beta E(\tau_b)$$

which completes the proof of Theorem 2.

**Remark.** The assertion of Theorem 2 was proved for the case \(K = \infty\) in Novikov (2003) under an additional assumption that

$$E(L_1^-)^\delta < \infty \text{ for some } \delta > 0.$$ 

### 3 Exponentially distributed positive jumps

In this section we use the same notation as in Section 2 and assume that the process \(Q_t\) in the decomposition (2) does not contain positive jumps while
the process $R_t$ is a compound Poisson process with exponentially distributed positive jumps, $E(\xi_k) = K$, $0 < K < \infty$; $N_t$ is a Poisson process with rate $E(N_t) = \lambda > 0$. Note that under these assumptions
\[ e^{-\varphi(u)} = (1 - u/K)^{\lambda/\beta} e^{-\Delta(u)}. \]

**Theorem 3.** For any $\mu > 0$,
\[ E(e^{-\mu \beta \tau_b}) = \frac{\int_0^K (1 - u/K)^{\lambda/\beta} u^{\mu-1} e^{ux-\Delta(u)} du}{\int_0^K (1 - u/K)^{\lambda/\beta-1} u^{\mu-1} e^{ub-\Delta(u)} du}, \] (23)
\[ E(\tau_b) = \frac{1}{\beta} \int_0^K (e^{ub} - e^{ux}(1 - u/K))(1 - u/K)^{\lambda/\beta-1} e^{-\Delta(u)} u^{-1} du. \] (24)

Besides, as $b \to \infty$,
\[ E(\tau_b) = C e^{Kb}(Kb)^{-\lambda/\beta}(1 + o(1)), \quad C = \frac{\Gamma(\lambda/\beta)}{\beta K e^{-\Delta(K)}} \] (25)

and the Exponential Limit Theorem holds:
\[ P\left\{ \frac{\tau_b}{E(\tau_b)} > x \right\} \to e^{-x}, \quad x > 0. \] (26)

**Proof.** Formulas (23) and (24) are direct consequences of Theorem 1, Theorem 2 and the following two well-known facts (which hold due to the memory-less property of the exponential distribution, see a similar statement in Borovkov (1976) for the case $\beta = 0$):

1) the overshoot $\chi_b = X_{\tau_b} - b$ has the density
\[ p_{\chi_b}(x) = \frac{1}{K} e^{-x/K}, \quad x > 0; \]

2) $\chi_b$ and $\tau_b$ are independent.

Relation (24) implies, using the change of variables $(1 - u/K)b = w$,
\[ \frac{d}{db} E(\tau_b) = \frac{1}{\beta} \int_0^K e^{ub}(1 - u/K)^{\lambda/\beta-1} e^{-\Delta(u)} du \]
\[ = \frac{1}{\beta} e^{Kb} b^{-\lambda/\beta} \int_0^b e^{-Kw} w^{\lambda/\beta-1} e^{-\Delta(K(1-w/b))} dw. \]
Since the function $\Delta(K(1 - w/b))$ is continuous and bounded in $w \in [0, K]$, the last integral converges as $b \to \infty$ to

$$e^{-\Delta(K)} \int_0^\infty e^{-Kw} w^{\lambda/\beta - 1} dw = e^{-\Delta(K)} \Gamma(\lambda/\beta) K^{-\lambda/\beta}.$$ 

Hence

$$\frac{d}{db} E(\tau_b) \sim \frac{\Gamma(\lambda/\beta)}{\beta K} e^{-\Delta(K)} e^{Kb} (bK)^{\lambda/\beta - \lambda/\beta}, \quad b \to \infty.$$ 

By well-known facts of theory of asymptotic expansions (see e.g. Olver (1997)) it implies

$$E(\tau_b) \sim \frac{\Gamma(\lambda/\beta)}{\beta} e^{-\Delta(K)} e^{Kb} (bK)^{\lambda/\beta}$$

and therefore we have proved (25).

To derive (26), we write the denominator in (23) as follows:

$$K \int_0^K (e^{ub} - e^{ux}(1 - u/K))(1 - u/K)^{\lambda/\beta - 1} u^{\mu-1} e^{-\Delta(u)} du + \int_0^K (1 - u/K)^{\lambda/\beta} u^{\mu-1} e^{ux - \Delta(u)} du.$$ 

Set

$$\mu = z/(\beta E(\tau_b))$$

with a fixed $z > 0$ and $E(\tau_b)$ defined in (24). Clearly, $\mu \to 0$ as $b \to \infty$. Due to (22), the second term in (27) (which is also the nominator in (23)) can now be written as

$$K \int_0^K (1 - u/K)^{\lambda/\beta} u^{\mu-1} e^{ux - \Delta(u)} du = \frac{\beta E(\tau_b)}{z} (1 + o(1)).$$ 

Using (24), the first term in (27) can be written as

$$K \int_0^K (e^{ub} - e^{ux}(1 - u/K))(1 - u/K)^{\lambda/\beta - 1} u^{\mu-1} e^{-\Delta(u)} du = \beta E(\tau_b) + \delta(b),$$ 

where

$$\delta(b) = K \int_0^K (e^{ub} - e^{ux}(1 - u/K))(1 - u/K)^{\lambda/\beta - 1} u^{\mu-1} (u^\mu - 1) e^{-\Delta(u)} du.$$ 

Note that for $u > 0$

$$|u^\mu - 1| = |e^{\mu \log u} - 1| \leq \mu \max(u^\mu, 1) |\log u|.$$
This implies

\[ |\delta(b)| \leq \mu \max(K^\mu, 1) \int_0^K (e^{ab} - e^{ux}(1-u/K))(1-u/K)^{\lambda/\beta - 1}e^{-\Delta(u)u^{-1}}\log u|du. \]

Applying the same change of variables as above, one can easily show that the last integral is \(O(e^{K^\beta b^{-\lambda/\beta}})\) as \(b \to \infty\).

Taking into account (25), due to the setting for \(\mu\) we get

\[ \delta(b) = O(1). \]

Now making the substitution in (23) relations (27), (28), (29) with the last result, we obtain

\[ E(e^{-z\tau_b/E(\tau_b)}) = \frac{\beta E(\tau_b)}{z\beta E(\tau_b) + O(1)} \to \frac{1}{z + 1}. \]

Since the function \(\frac{1}{1+z}\) is the Laplace transform of the exponential distribution with mean 1, this completes the proof.

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References

[1] Borovkov, A. (1973) *Stochastic processes in queueing theory*, Springer-Verlag, New York.

[2] Borovkov, K., and Novikov, A. (2001) On a piece-wise deterministic Markov process model. *Statist. Probab. Lett.* 53, no. 4, 421–428.

[3] Darling, D. A., and Siegert, A. J. F. (1953) The first passage problem for a continuous Markov process. *Ann. Math. Statistics* 24, 624–639.

[4] Hadjiev, D. (1983) The first passage problem for generalized Ornstein-Uhlenbeck processes with nonpositive jumps. In: *Séminaire de probabilités*, XIX, 1983/84, 80–90, Lecture Notes in Math., 1123, Springer, Berlin.

[5] Jacobsen, M. and Jensen, A. (2006) Exit times for a class of piecewise exponential Markov processes with two-sided jumps. Dept. of Applied Mathematics and Statistics, University of Copenhagen. Preprint No 5.

[6] Kella, O. and Stadje, W. (2001) On hitting times for compound Poisson dams with exponential jumps and linear release rate. *J. Appl. Prob.* 38, no. 3, 781–786.
[7] Novikov, A. A. (1990) On the first exit time of an autoregressive process beyond a level and an application to the ‘change-point’ problem. *Theory Probab. Appl.* 35, no. 2, 269–279.

[8] Novikov, A. A. and Èrgashev, B. A. (1993). Limit theorems for the time of crossing a level by an autoregressive process. In: *Trudy Mat. Inst. Steklova.* 202, 209–233. [In Russian. English translation in: *Proc. Steklov Math. Inst.* 1994, 4 (202), 169–186.]

[9] Novikov, A.A. (2003) Martingales and first-exit times for the Ornstein–Uhlenbeck process with jumps. *Theory Probab. Appl.* 48, 340–358.

[10] Novikov, A.A., Melchers, R.E., Shinjikashvili, E. and Kordzakhia, N. (2005) First passage time of filtered Poisson process with exponential shape function. *Probabilistic Engineering Mechanics,* 20, no. 1, 33-44.

[11] Olver, F.W.J. (1997) *Asymptotics and special functions.* 2nd ed. AK Peters, Wellesley, Mass.

[12] Perry, D., Stadje, W. and Zacks, S. (2001) First-exit times for Poisson shot noise. *Stoch. Models,* 17, no. 1, 25–37.

[13] Tsurui, A. and Osaki, Sh. (1976) On a first-passage problem for a cumulative process with exponential decay. *Stochastic Processes Appl.* 4, no. 1, 79–88.

[14] Wolfe, S. (1982) On a continuous analogue of the stochastic differential equation $X_n = \rho X_{n-1} + B_n$. *Stoch. Proc. Appl.,* 12, 301-312