Variability of GRB light curve: Shock wave model modification

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Abstract. The main topic of our investigation is a mysterious phenomenon of gamma ray bursts. If someone analyses the observation of this events, one can see that the most interesting data are among the gamma phase, during the first couple of seconds. In this stage central engine generate a high intensity and high energy radiation, observed in the form of pulses in the light curve. If we could understand the main physical processes in this phase, we could put some constraints for the model which try to explain the core of this phenomena, deeply hidden from observation by high optical thickness of surrounding material.

Observation of the GRB light curve can be easily done with the help of modern satellites, so that the data of this kind are vastly dispersed. We try to analyse them and to explain in details physical process which create the light curve pulses. Our research is based on the broadly accepted scenario of gamma phase in GRBs, which predicts that GRB core generate highly relativistic mater in some amount of time. This mater form shock waves of different velocities, due to highly differential motions. The shock waves can interact with each others, and in that moment radiation significantly increases, creating the observed pulses in the GRB light curve. We develop a phenomenological model based on the model developed by Huang and coauthors, to explain evolution of a shock wave expanding from some distance. In it, we implemented the ability to simulate collision between incoming shock and density barrier. We also propose that the density barrier is created by the material ejected from the core and spread around by the shocks. As a result of simulation we can get synthesized pulses, and by comparing them with the observational data we can acquire values of basic parameters used in our model.

1. Introduction
Investigation of the Gamma Ray Bursts (hereafter GRB) phenomena in the last decade has been pointed to high energy astrophysics involving relativistic jets and shock waves. Crucial research problem left unclear is functioning and nature of the GRB core, particularly mechanism for generation of relativistically moving material. We have try in our investigation to put some additional constraints on the GRB engine in the form of values of basic parameters of created shock waves. We have analyzed the shapes of pulses observed in the light curve of this event and simulate them numerically. Then in the process of comparisons between observer and synthesized pulses, we extract expected basic parameters of involved shock waves.

The main scenario presently accepted for this phenomena says that in some kind of explosion probably similar to supernova [1], (collapsar models) huge amount of material is ejected in the surrounding space. There is also another explanation of cause for this ejection, mostly based on the collision scenario, for example between two rotating neutron stars or black holes [2].
This highly accelerated material spread around center of explosion either isotropically (high total energy) or collimated into jets of matter (lower total energy). Differential motion of this relativistic matter create shock waves in its front. They present highly compressed and fast moving energetic, parts of ejecta which are created by the GRB core in some number. They have different velocities and by that ability for mutual collision and merging. In the moment of interaction the amount of radiation is highly increased creating the observed pulses in the light curve (internal shock wave model - [4]). Internal engine of GRBs is active a while, after it’s shut down, marking the ending of the first phase of GRB event, known as a gamma phase. The duration of the first phase is usually around a couple of tens of seconds. The ejected material in form of shock waves continues to move away from the center of explosion merging in to one more massive but slower shock wave, activating the second phase of GRB called the afterglow (external shock wave model - [4]).

This one is characterized with the lower energies and smooth continuous light curves, lasting for even years with transition from the X-ray band toward the lower energy radio bands.

Our goal is to examine the shock waves in the first phase of the event, simulating the expansion and collision of them with a suitable mathematical model. Additional comparison of theoretical and observational light curve, can give us values of the basic parameters used. In the next chapter §2 we will present model developed by us [5], [6] and give some fundamental constraints and assumptions. In the chapter §3 we will discuss process of extract and fitting of the observational data, as well as present and discuss obtained parameters.

2. Employed model and assumptions

Let us first describe the physical scenario of these processes. As in [7], we will consider an ultra relativistic flow of matter as an array of well defined collimated shells with random energy and mass, the so called ‘small shocks’, because of its lower mass compared with the mass of the afterglow shock wave. They are created by differential motion of ejected material, which can produce shocks of different energies and Lorentz factors. As a result, collision can occur between two expanding waves of different velocities. This can produce low intensity pulses in the light curve.

In the process of interaction the resulting shock wave is created by merging of initial two, so the mass of newly created shock wave is its sum. On the contrary, the Lorentz factor is highly decreased, which can produce effect of accumulation of material at the certain distance, thus creating a density perturbation. The newly created mass ejection rush into such density perturbation merging with it and moving it further from the center of explosion producing the pulses of higher intensity and duration. Depending on the intensity and width of a such created density barrier we can simulate the wider-tinner or higher-lower pulses.

Also, considering the energy density of shock wave and barrier, we can get more symmetrical shape light curve pulses in the case of energetic shock wave and low energy barrier, and high asymmetrical pulses in an opposite situation. This is the main mechanism we use to explain high temporal variability of the observed γ-ray light curve. Concerning the scenario mentioned above, we see that the selection of the shock parameters has random nature in a given interval of values.

The central engine continuously creates new small mass shocks that cause the flow of shock waves with different initial parameters. With time, they are able to accelerate particles of the Inter-Stellar Medium (ISM) surrounding the GRB and to accumulate in another, massive shock wave which continues to spread with lower velocity. Then, the second phase of the GRB event starts, with the creation of the afterglow.

For proper description of evolution of relativistically moving matter we have use a phenomenological system of first order differential equations which evolve three important variables, Lorentz factor, mass and distance of shock wave [6]. First two of them are rewritten
from the paper of [9], and third is derived by us, to incorporate sharp jump of basic variables in the moment of collision. Then we have:

$$\frac{dR}{dt} = c \sqrt{\Gamma^2 - 1} \left[ \Gamma + \sqrt{\Gamma^2 - 1} \right], \quad (1)$$

$$\frac{d\Gamma}{dm_s} = -\frac{\Gamma^2 - 1}{M_{ej} + 2(1 - \varepsilon)\Gamma m_s + \varepsilon m_s}, \quad (2)$$

$$\frac{dm_s}{dt} = 2\pi n_0 \Gamma p (1 - \cos \theta) R^2 \left( 3\Gamma \frac{dR}{dt} - 2R \frac{d\Gamma}{dt} \right), \quad (3)$$

where the parameter \(\varepsilon\) takes values from 0 for the adiabatic expansion, to 1 which describes a fully radiative case, and \(M_{ej}\) is the mass of a primary ejected material. Eqs. (1) - (3) are derived for an observer reference frame, and they have to be solved simultaneously, together with the density equation. Initial values of parameters and variables are highly dependent on the properties of the shocks.

For a specific density of surrounding media we use equation developed by [8], which we modified to include a density distribution to describe the barrier of accumulated ejected material. The barrier is assumed to have Gaussian profile of density, which is completely satisfied in the first order approximation. The final form of equation is:

$$n = n_0 \left( \frac{R_0}{R} \right)^s \left( 4\Gamma + 3 \right) \left( 1 + a \cdot \exp \left[ -\left( \frac{R - R_c}{b} \right)^2 \right] \right), \quad (4)$$

where \(R_c\) is the distance of the shock encounter, width at half maximum \((b)\) represents the width of the slower shock, and the intensity of the Gaussian \((a)\) represents the slower shock number density.

In principle, one can expect that the primary shock can collide with an ISM which has certain density distribution. In such highly relativistic physical system the relative motion of charged particles of ISM can generate intensive magnetic field in the reference frame of moving fluid. The geometry of magnetic field is considering to be mostly turbulent. We calculated it in a similar way as in [9], by assuming that the energy of the magnetic field is a certain fraction, \(\xi_B\), of the total energy of the shock wave. The parameter \(\xi_B\) determines this magnetic energy part, and in the first approximation we will consider it constant during the interaction. In the comoving reference frame the expression for the magnetic field is taken as:

$$B' = \sqrt{8\pi \xi_b n_0 \Gamma m_pc^2 (4\Gamma + 3) \left( \frac{R_0}{R} \right)^s \left( 1 + a \cdot \exp \left[ -\left( \frac{R - R_c}{b} \right)^2 \right] \right)}, \quad (5)$$

The emission mechanism of shock waves is mainly based on synchrotron radiation, but for higher energy bands additional flux can be gained by the Inverse Compton (IC) radiation [4]. In the first approximation we will neglect flux gained by the IC effect and preserve our calculation on the synchrotron source. To calculate the intensity of the radiation by particles in the shock wave we will use the formulae given by [11], then the total emitted flux can be calculated as e.g. in [10]. Also, we should note that the shock waves contain relativistic electrons and baryons which contribute to the synchronistic radiation, but taking into account the difference in velocities of these constituents, one can neglect the contribution of baryons to the total emitted flux. Then in the comoving reference frame the total flux is:

$$P'_{\nu} = A \cdot \int_0^{\theta_m} \tan \theta d\theta \int_{\gamma_{emin}}^{\gamma_{emax}} \gamma^{-p+1}_e \left( \frac{\nu'}{\nu_c} \right) F(\nu'/\nu_c) d\gamma_e \quad (6)$$
where $A$ is:

$$A = \frac{\sqrt{3}e^3 B' m_s C_1}{m_e c^2 m_p \ln(\cos(\theta_m))}$$

(7)

and $F(x) = \int_{x}^{\infty} K_{5/3}(x) dx$ where $K_{5/3}$ is the Bessel function of the second order. Here, $\nu'_c$ is the critical frequency of the radiation expressed by $\nu'_c = 3\gamma^2 e^2 B'/4\pi m_e c$.

In equation 6 we implemented the effects of curved surface of an emitting shock wave. Also, in the case of small shocks the cooling time is longer then dynamical time of expansion of shell, so the effect of electron cooling can be neglected.

With presented model we are able to simulate the collision of shock waves in the first phase of GRB, which produce the pulses in the light curve (see [6]). In the figures below we present evolution of the main magnitudes describing the shock wave in the case where there is a collision with slow density barrier.

**Figure 1.** Evolution of Lorentz factor (left) and shock wave mass $m_s \times 10^{22}$ [gr.] (right) given in the logarithmic scale. We use three different values of $\Gamma_0 = 120$ - full line, $\Gamma_0 = 100$ - dashed line and $\Gamma_0 = 80$ - dash-doted line.

**Figure 2.** Evolution of distance from center of explosion $R \times 10^{14}$ [cm] (left) and vector of magnetic field $B \times 10^6$ Gauss, given in the logarithmic scale. We use three different values of $\Gamma_0 = 120$ - full line, $\Gamma_0 = 100$ - dashed line and $\Gamma_0 = 80$ - dash-doted line.
We can easily see sharp discontinuity in the moment of interaction, which causes decreasing of Lorentz factor for an order of magnitude, as well as rapid increase of shock wave mass. Evolution of distance show us what we initially propose that after a collision, resulting shock wave accumulate at some distance, which is not influenced by initial values of the Lorentz factor and ejected mass. Magnetic field in that moment sharply increases gaining the additional radiation power (see Eq. 7). That is expected if we know that equation for synchrotron radiation power is directly proportional to the intensity of the magnetic field.

3. Results and discussion
We select a sample of 30 recorded GRB events from a large BATSE database (3rd channel, $E = 100 - 300$ keV, for the light curve) which are suitable for light curve pulse extraction. The main criteria we used is that: (i) the light curves are well isolated with clear peak maximum (process of pulse creation is stochastic in nature and result in a wide variety of pulse shapes and combinations, e.g. it is often observed that two pulses are superposed on another); (ii) we avoid small pulses, because of their low temporal resolution we could not achieve enough accuracy in a fitting process; (iii) the pulses are with different shapes as much as possible in order to include long and short duration pulses, as well as pulses of different intensity and symmetries in the shape.

During the fitting attempts we have done broad analysis of how parameters influence the shape of the light curve pulse. The one particularly interesting case is with the parameter $\theta_m$, the opening angle of the shock wave cone. In the Figure 3 (left) we can see that for the bigger value of $\theta_m$, resulting pulse is more wider and slow decay feature is much more expressed then in the case when the angle $\theta_m$ has lower values. In that case the generated pulse is more tinner and symmetrical in shape. That is in agreement with employed physical processes during shock expansion. Indeed, if we demand wider opening angle, we lower the energy density of the shock wave and the produced pulse is softer and more broadly disperse. On the contrary, if we narrow the jet, than the shock wave is more energetic and we have symmetrical but intensive light curve pulses.

Figure 3. Shape of the light curve pulses (left) and its spectral curves (right) given in relative units compared to highest pulse. Three lines are appropriate to different values of the opening angle of ejected material $\theta_m = 0.09$ rad - full line, $\theta_m = 0.07$ rad - dashed line and $\theta_m = 0.05$ rad - dash-doted line.

Also, other parameters have substantial influence on the light curve pulse, see Figures 3. For example, if we demand higher values for the initial Lorentz factor, we expect to have shorter and more intensive pulses. Similar, with the increase of shock wave mass, pulses also increase.
Figure 4. Shape of the light curve pulses for different values of $\Gamma_0$ (left) and $M_{ej}$ (right) given in relative units compared to highest pulse. Three lines on the left and right Figures are appropriate to following set of parameter: $\Gamma_0 = 80, M_{ej} = 9 \times 10^{-11} \times M_{\odot}$ - full line; $\Gamma_0 = 70, M_{ej} = 6 \times 10^{-11} \times M_{\odot}$ - dashed line and $\Gamma_0 = 60, M_{ej} = 3 \times 10^{-11} \times M_{\odot}$ - dash-doted line.

Table 1. Mean and most probable values taken from the large sample of fitted GRBs.

| parameter | $n_0$ [cm$^{-3}$] | $\Gamma_0$ | $M_{ej}$ x 10$^{-11}$[M$_{\odot}$] | $\theta_m$ [rad] | $R_c$ x 10$^{14}$[cm] | $\Delta R$ x 10$^{15}$[cm] | $n_b$ x 10$^5$[cm$^{-3}$] |
|-----------|------------------|------------|----------------------------------|-----------------|---------------------|------------------------|-------------------|
| Mean      | 22.5             | 12.4       | 3.9                              | 0.067           | 1.3                 | 5.9                    | 10.3              |
| Most probable | 16.              | 27.        | 0.1                              | 0.05            | 1.0                 | 1.6                    | 0.1               |

in its height. This behavior is connected with increase of initial energy of shock wave, when the interaction with the stationar barrier become quicker and more intense. In the opposite situation the produced pulses are smaller and less symmetrical in shape following the well known FRED (fast rise slow decay) law. In the case of density of surrounding media we have almost direct proportionality. That is well understanding if we know that the radiation power of the synchrotron emission is connected with the number of radiating particles. So, if we have denser barrier then the light curve pulse is more intensive and also broader.

Based on presented results we can make overall conclusion about parameters influence on the shape of the light curve pulses. Namely, in the interaction of the shock wave with the barrier, two cases can be achieved considering the parameters setup. First, the case when the shock wave is more energetic and compact compared with the barrier. In this case produced pulses are symmetrical, relatively shorter and intensive. Otherwise, when the incoming shock wave is broader and with lower energy then the barrier, pulses are more asymmetrical and broader in shape.

In the Figures 3 the results of fitting the light curve for particular GRBs are presented as well as theirs spectral curves. We select from our sample two opposite cases, first GRB911104 with the broad pulse duration and GRB000513 presenting the narrow pulses. With this we try to emphasize differences of shapes in the GRB pulses, and ability of our model to fit both mentioned types. With the similar results from others GRBs in the our sample we can group the data and statistically handle them to acquire most probable values and intervals. Example of basic parameters are presented in the Table 1 below.

At the end we try to synthesized overall l.c. that contains multiple pulses of different shapes and intensity. For this reason we propose that all pulses are created in the same manner, and that observed differences originate from the stochastic nature of basic parameters. In this case
Figure 5. Left Figure presents the best fit (solid line) of pulse form GRB911104 (dashed line), with the abscissa axis designated as emitted flux $F_\nu$ and time $t$ on the ordinate. On the right side we place the spectral fit for all four channels of BATSE instrument, where the values of the flux are put in the relative units (r.u.).

Figure 6. Same as in Fig. 3 but for GRB000513.

we account just a clear single pulses, which mean that all of them must have monotonic rise and decay. Pulses consisted from combination of two or more are avoided in this simulation. Another thing to consider is the time interval between pulse generation, which is tightly correlated with the dynamics of internal GRB engine. We have to notice that on this level of research we can take it as totally random in some range which is acquired mostly from the observations.

Using this assumptions we can synthesize light curve by generating one collision for every separate pulse, and by superposition of them we get final overall l.c. For any particular interaction a new set of basic parameters are selected in the range specified with the use of Table 1. In Figure 7 the result of our simulation is presented.

4. Conclusion
In this paper we have develop a phenomenological model to simulate the evolution of shock waves expanding from the center of GRB, and colliding mutually producing the observed light curve pulses. As a result we acquire a range of basic parameters which describe the shock waves. We present mean and most probable values of parameters in the Table 1 in order to get clearer picture of physical processes which are behind the GRB event, and we reach following conclusions:
Figure 7. Synthetic light curve produced by simultaneous application of our model.

(i) The derived model is able to synthesize pulses similar to one observed in the l.c. of GRBs. The main advantage of this model is its simplicity, compared to the models which are based on the strict magneto-hydrodynamical approach. Nevertheless, using our simplified model we preserve accuracy and clear picture of physical processes employed in the central engine of GRBs.

(ii) The obtained values for internal shock parameters are extracted for different shapes of pulses in the GRB light curve. They are in a good agreement with estimations given by other authors earlier (see [4]). As we see in Table 1 there is some most probable values which show us that the physical processes responsible for GRB creation are similar, and that observed variation of pulse shapes are generated by stochastic variation in values of basic parameters.

(iii) Extracted parameters can present fundamental requirements for construction of broad models which describe GRB progenitor.

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