Chapter 3

THEORY OF DISK ACCRETION ONTO SUPERMASSIVE BLACK HOLES

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Abstract

Accretion onto supermassive black holes produces both the dramatic phenomena associated with active galactic nuclei and the underwhelming displays seen in the Galactic Center and most other nearby galaxies. I review selected aspects of the current theoretical understanding of black hole accretion, emphasizing the role of magnetohydrodynamic turbulence and gravitational instabilities in driving the actual accretion and the importance of the efficacy of cooling in determining the structure and observational appearance of the accretion flow. Ongoing investigations into the dynamics of the plunging region, the origin of variability in the accretion process, and the evolution of warped, twisted, or eccentric disks are summarized.

3.1 Introduction

Most galactic nuclei are now believed to harbor supermassive black holes. These black holes are all accreting gas—at a minimum from the interstellar medium proximate to the event horizon, and in some cases from dense disks of gas in orbit around the hole. The observational signatures of this accretion, however, differ dramatically from galaxy to galaxy. The black hole in the Galactic Center, for example, has an X-ray luminosity in quiescence of only \( L_X \approx 2 \times 10^{33} \) ergs s\(^{-1}\), reaching \( L_X \sim 10^{35} \) ergs s\(^{-1}\) during flaring states (Baganoff et al. 2001, 2003; Goldwurm et al. 2003). Many nearby elliptical galaxies also show nuclear emission that is much weaker than might be expected on the basis

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of simple estimates of the black hole accretion rate (Fabian & Rees 1995; Di Matteo et al. 2000; Loewenstein et al. 2001). In stark contrast to these feeble displays, accretion onto black holes in quasars powers the most luminous steady sources in the universe. At a minimum, a theory of accretion needs to account for this dichotomy, and to explain at least some of the many secondary phenomena (jets, outflows, variability, etc.) associated with Active Galactic Nuclei (AGN). More ambitiously, one might hope to understand the role of accretion in actually forming supermassive black holes. Black hole formation is a difficult theoretical problem which, although currently untroubled by direct observations, is receiving increasing attention.

In this chapter, I discuss selected aspects of the theory of accretion onto supermassive black holes, with an emphasis on the physical processes that drive accretion and determine the qualitative properties of the flow. In §3.2, I outline the mechanisms which can lead to angular momentum transport within an accretion flow, thereby allowing rotationally supported gas to flow inward and liberate gravitational potential energy. Close to the black hole, turbulence driven by magnetohydrodynamic instabilities is probably the dominant mechanism for transport. Further out, other processes, such as gravitational instabilities, are likely to assume that role. Angular momentum transport, however, is only part of the story. Although central to our understanding of accretion, knowledge of its origin does not suffice to explain why the Galactic Center looks nothing like a powerful AGN. For that we need to consider the distinction between geometrically thin accretion disks, in which the gas can radiate efficiently and cool to sub-virial temperatures, and hot thick disks, which are radiatively inefficient and vulnerable to rapid mass loss. These questions are addressed in §3.3. Subsequent sections examine the status of several open questions in the study of black hole accretion, including the dynamics of gas executing its final plunge into the hole, the origin of variability, and the evolution of disks that are warped or eccentric.

### 3.2 Angular Momentum Transport

In almost all circumstances, gas in the nuclear regions of galaxies has far too much angular momentum to be swallowed directly by the black hole. Understanding the mechanisms that can lead to angular momentum transport in rotating flows is therefore the central problem in the theory of accretion onto supermassive black holes. At scales of $10 - 100$ pc or larger, this is a problem in galactic dynamics. Following galactic mergers, for example, gravitational torques from bars and other
transient structures can efficiently funnel gas into the nuclear regions (Shlosman, Frank, & Begelman 1989; Barnes & Hernquist 1991; Hernquist & Mihos 1995). Once within the black hole’s sphere of influence, however, which for a galaxy of central velocity dispersion $\sigma$ and black hole mass $M_\bullet$ extends out to

$$r_\bullet = \frac{GM_\bullet}{\sigma^2},$$

(3.1)

these galactic mechanisms become less efficient. Within the sphere of influence, which reaches $\approx 10$ pc for a black hole of mass $M_\bullet = 10^8 M_\odot$, the black hole's gravity overwhelms that of the host galaxy and dominates the potential, unless a comparably large mass of gas has managed to accumulate at such small radii. Timescale arguments suggest that gas at $r_\bullet$ probably cannot trickle down through an accretion disk all the way to the vicinity of the black hole (e.g., Shlosman, Begelman, & Frank 1990). Disks probably do form, however, at smaller radii of order $0.1$ pc, where maser emission in the nucleus of NCG 4258 has an unmistakably disk-like geometry (Miyoshi et al. 1995). At such radii, the specific angular momentum of a Keplerian disk exceeds that needed for direct capture by the black hole by a factor of order $10^2$. If that gas is to accrete, 99% of the angular momentum must either be redistributed within the disk—angular momentum transport—or lost entirely from the system. We will consider the known mechanisms which can accomplish this feat.

In the inner disk, turbulence driven by magnetic instabilities is a potent source of angular momentum transport. Although magnetohydrodynamic (MHD) disk turbulence has long been implicated in disk angular momentum transport and is mentioned prominently by Shakura & Sunyaev (1973), current confidence in this conclusion rests on two more recent developments. First, Balbus & Hawley (1991) demonstrated that the introduction of a weak magnetic field renders accretion flows linearly unstable to a powerful local instability. Subsequent numerical simulations (Hawley, Gammie, & Balbus 1995; Brandenburg et al. 1995; Matsumoto & Tajima 1995; Stone et al. 1996) showed that the instability rapidly develops into sustained turbulence, which transports angular momentum outward at a rate that is consistent with observational constraints derived from studies of accretion in mass transfer binaries (e.g., Pringle, Verbunt, & Wade 1986; Cannizzo 1993; Osaki 1996; Hameury et al. 1998). For a recent review of the role of MHD turbulence in accretion disks, see Balbus (2003).

In the absence of magnetic fields, a differentially rotating disk with angular velocity $\Omega(r)$ is linearly stable to axisymmetric perturbations
according to the Rayleigh criterion if

\[ \frac{d}{dr} \left( r^2 \Omega \right) > 0, \]  

i.e., if the specific angular momentum \( l(r) \) of the flow is an increasing function of radius. For a geometrically thin disk orbiting a point mass \( M \), the angular velocity is Keplerian,

\[ \Omega = \sqrt{\frac{GM}{r^3}}, \]

the specific angular momentum \( l(r) \propto \sqrt{r} \), and the disk is hydrodynamically stable. Numerical simulations support this conclusion (Balbus, Hawley, & Stone 1996). Although real disks—especially geometrically thick flows with significant pressure support—can have angular velocity profiles that differ from the simple Keplerian form, they too are invariably stable by the Rayleigh criterion.

Matters are drastically different if the disk contains a magnetic field. Analytic studies have shown that a weak magnetic field destabilizes astrophysical disks, provided that

\[ \frac{d\Omega^2}{d \ln r} < 0, \]

a condition which is almost always satisfied in real disks. This magnetorotational instability (MRI) exists regardless of the initial magnetic field configuration (Velikhov 1959; Chandrasekhar 1960; Balbus & Hawley 1991, 1992; Terquem & Papaloizou 1995; Ogilvie & Pringle 1996; Curry & Pudritz 1996), though the growth rates vary depending upon the magnetic field geometry, being fastest for vertical fields.

Proving the existence of the MRI in the general case requires a moderately involved calculation, which can be found in the comprehensive review by Balbus & Hawley (1998). Analyses of much simpler systems, however, reveal most of the important physics. Here I follow closely the treatment of Balbus & Hawley (2000). Consider a fluid element orbiting in a disk with a central gravitational potential \( \Phi(r) \). We will assume that pressure forces are negligible, as they will be, provided that perturbations to the disk velocity field are highly subsonic. In cylindrical polar coordinates \((r, z, \phi)\), the equations of motion then read

\[ \ddot{r} - r \dot{\phi}^2 = -\frac{\partial \Phi}{\partial r} + f_r, \]

\[ r \ddot{\phi} + 2 \dot{r} \dot{\phi} = f_\phi, \]  

(3.5)
where the dots denote derivatives with respect to time, and \( f_r \) and \( f_\phi \) are forces that we will specify shortly. We now concentrate our attention on a small patch of the disk at radius \( r_0 \) that is corotating with the overall orbital motion at angular velocity \( \Omega \). We define a local Cartesian coordinate system \((x, y)\) via

\[
\begin{align*}
  r &= r_0 + x, \\
  \phi &= \Omega t + \frac{y}{r_0},
\end{align*}
\]

and substitute these expressions into equation (3.5) above. Discarding quadratic terms, the result is

\[
\begin{align*}
  \ddot{x} - 2\Omega \dot{y} &= -x \frac{d\Omega^2}{d \ln r} + f_x, \\
  \ddot{y} + 2\Omega \dot{x} &= f_y.
\end{align*}
\]

These equations describe the epicyclic motion of pressureless fluid perturbed from an initially circular orbit.

If the disk contains a weak vertical magnetic field, perturbations to the fluid in the plane of the disk will be opposed by magnetic tension forces generated by the bending of the field lines. Considering in-plane perturbations varying with height \( z \) and time \( t \) as \( e^{i(\omega t - kz)} \), the magnetic tension force is \( f = -(kv_A)^2 s \), where \( s \) is the displacement vector and \( v_A = \sqrt{B^2/4\pi \rho} \) is the Alfvén speed. Using this expression for \( f_x \) and \( f_y \), and assuming the \( e^{i\omega t} \) time dependence, equation (3.8) becomes

\[
\begin{align*}
  -\omega^2 x - 2i\omega \dot{y} &= -x \frac{d\Omega^2}{d \ln r} - (kv_A)^2 x, \\
  -\omega^2 y + 2i\omega \dot{x} &= -(kv_A)^2 y.
\end{align*}
\]

Combining these equations yields a dispersion relation that is a quadratic in \( \omega^2 \),

\[
\omega^4 - \omega^2 \left[ \frac{d\Omega^2}{d \ln r} + 4\Omega^2 + 2(kv_A)^2 \right] + (kv_A)^2 \left[ (kv_A)^2 + \frac{d\Omega^2}{d \ln r} \right] = 0.
\]

The system is unstable if \( \omega^2 < 0 \), which occurs when the third term in the dispersion relation is negative. Instability therefore occurs if

\[
(kv_A)^2 + \frac{d\Omega^2}{d \ln r} < 0.
\]
For a sufficiently weak field \((v_A \rightarrow 0)\), or for long enough wavelength perturbations \((k \rightarrow 0)\), we then obtain the aforementioned criterion for a disk to be unstable to the MRI, namely

\[
\frac{d\Omega^2}{d\ln r} < 0.
\] (3.13)

In a real disk, the longest wavelength perturbation in the vertical direction will be of order the scale height \(h\) of the disk. There will therefore be instability, provided that the magnetic field is weaker than some threshold value \(B_{\text{max}}\).

The physical origin for this instability is fairly straightforward. Adopting the same configuration that we have just analyzed, consider the effect of perturbing a weak vertical magnetic field threading an otherwise uniform disk. If the field remains frozen into the plasma, then field lines which connect adjacent annuli in the disk will be sheared by the differential rotation of the disk into a trailing spiral pattern. Provided that the field is weak enough, magnetic tension is inadequate to snap the field lines back to the vertical. What tension there is, however, acts to reduce the angular momentum of the inner fluid element and boost that of the outer fluid element, providing angular momentum transport in the outward sense that is required to drive accretion.

The pervasive nature of the MRI, taken together with its rapid linear growth rate—\(\omega_{\text{max}}\) can be as large as \(3\Omega/4\), meaning that growth of magnetic field energy occurs on a dynamical timescale—implies that well-ionized astrophysical disks will be unstable. Analytic studies are unable, however, to investigate the most interesting consequence of the instabilities, namely, what happens when the MRI reaches a saturated state and turbulence has developed throughout the flow? Quantities we would like to determine in this state include the \(r\phi\) component of the stress tensor,

\[
W_{r\phi} = \langle v_r dv_\phi - v_{Ar} v_{A\phi} \rangle,
\] (3.14)

which includes both fluid (Reynolds) and magnetic (Maxwell) stresses. In this expression, \(dv_\phi = v_\phi - r\Omega\) is the fluctuation in the azimuthal velocity, \(v_{Ar}\) and \(v_{A\phi}\) are Alfvén velocities in the radial and azimuthal directions, respectively, and the angle brackets denote a density weighted average over height. If \(W_{r\phi}\) can be measured, in practice from a numerical MHD simulation, then it can be used to estimate the \(\alpha\) parameter introduced by Shakura & Sunyaev (1973). They suggested the scaling

\[
W_{r\phi} = \alpha c_s^2,
\] (3.15)
where $c_s$ is the sound speed and $\alpha$ is a dimensionless constant or parameter that measures the efficiency of angular momentum transport in disks.

Numerical simulations of the non-linear development of the MRI have now been performed in both local and global geometries. Local simulations (Hawley, Gammie, & Balbus 1995, 1996; Brandenburg et al. 1995; Matsumoto & Tajima 1995; Stone et al. 1996; Miller & Stone 2000) follow the evolution of the MRI in a small patch of disk, with periodic boundary conditions in $\phi$ and (in a modified form) $r$. Global simulations (Armitage 1998; Hawley 2000) have the advantage of being able to study large scale magnetic fields and are essential for the investigation of geometrically thick ($h \sim r$) flows (e.g., Hawley, Balbus, & Stone 2001). All such simulations have known limitations. The physical separation between the largest relevant scale (that of the whole disk, or the wavelength of the most important MRI modes) and the dissipative scale (either viscous or resistive) is almost always too large to resolve numerically. Moreover, many calculations rely on purely numerical effects at the grid scale to provide dissipation. Discussions of the effect of these limitations can be found, for example, in Balbus & Hawley (1998) or Schekochihin et al. (2004). Nevertheless, it is encouraging that a variety of simulations, using different numerical techniques, agree in several critical aspects. The MRI rapidly leads—on a timescale of just a few orbital periods—to a state of MHD disk turbulence in which angular momentum is transported outward. This is sustained, despite the presence of dissipation (often at unphysically large scales) within the numerical codes. The magnetic field energy remains smaller than the thermal energy in the disk. Finally, the Maxwell stress dominates by a large margin over the Reynolds stress, though the latter also provides some outward transport. Values of $\alpha$ between $10^{-2}$ and $10^{-1}$ are representative of the bottom line of most simulations.

The maintenance of a disk magnetic field, despite the presence of dissipation, implies that turbulence driven by the MRI sustains a disk dynamo, just as fluid motions in stars and planets generate their own self-sustaining magnetic fields (Parker 1955; Proctor & Gilbert 1994; Glatzmaier & Roberts 1995). The accretion disk case, however, is interestingly different from these better-observed systems. In a disk (at least according to current belief) there would be no turbulent motions in the absence of a magnetic field, which rather must generate the turbulent velocity field needed for its own amplification. This bootstrapping quality makes it harder to apply knowledge gleaned from the long history of work on solar and planetary dynamos to the disk case.
How do the values of $\alpha$ obtained from numerical simulations compare with those inferred from observations? The best constraints come from modeling the episodic accretion events seen in the dwarf novae subclass of cataclysmic variables. In the widely accepted thermal disk instability model for dwarf nova outbursts (Meyer & Meyer-Hofmeister 1981; Mineshige & Osaki 1983; Faulkner, Lin, & Papaloizou 1983), the eruptive behavior results from the inability of the disk to reach a steady-state at radii where hydrogen is partially ionized. Under these conditions, there is no thermally stable vertical structure for a range of accretion rates, resulting in a limit cycle in which the disk alternates between a cool mass-accumulating phase and a hot draining phase. The timescales of the outbursts are directly related to the values of $\alpha$ in the outburst and quiescent states. Different authors (e.g., Pringle et al. 1986; Cannizzo 1993; Osaki 1996; Hameury et al. 1998) derive typical values of $\alpha$ in the outburst state of $\sim 0.1$, with quiescent values a factor of a few lower. This is in reasonably encouraging agreement with the values predicted from MHD simulations. Moreover, as I will discuss later in the context of AGN disks, the difference in $\alpha$ between the outburst and quiescent states could be caused by a suppression of the MRI when the disk is cool and the magnetic field is imperfectly coupled to the gas (Gammie & Menou 1998).

Of course, in many circumstances of interest, it is either inconvenient or unfeasible to determine the structure and evolution of disks via three-dimensional MHD simulations. In particular, the evolution of AGN disks across the full range of radii spanned by real disks cannot be simulated directly. This raises the following questions: under what circumstances are vertically-integrated one-dimensional disk models, or two-dimensional simulations using a parameterized viscosity, good approximations to the real disk physics?

For geometrically thin disks ($h/r \ll 1$), the standard time-dependent one-dimensional treatment divides the problem into two parts (e.g., Frank, King, & Raine 2002). First, one solves for the vertical structure of an isolated annulus of the disk using methods and equations analogous to those of stellar structure. The vertical structure calculation yields the functional dependence of the kinematic viscosity $\nu$ on the surface density $\Sigma$, angular velocity $\Omega$, and $\alpha$. Armed with this knowledge, we can then solve a one-dimensional diffusion equation for the time-dependence of the disk,

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r} \frac{\partial}{\partial r} \left[ r^{1/2} \frac{\partial}{\partial r} \left( \nu r^{1/2} \Sigma r^{1/2} \right) \right],$$

subject to appropriate boundary conditions at the inner and outer edges of the disk. If thermal fronts are present (or develop) within the disk,
this equation for $\Sigma$ needs to be supplemented with an explicit equation for the time-dependence of the central temperature $T_c$ (e.g., Cannizzo 1993).

Provided that we restrict our attention to the axisymmetric evolution of geometrically thin disks, one-dimensional disk models constructed in this manner probably provide a reasonable approximation to the evolution of MRI-active disks. At the most basic level, the MRI is a local instability, and, as such, is in principle amenable to an $\alpha$-type description (Balbus & Papaloizou 1999). For the vertical structure part of the calculation, the main question is whether the distribution of the stress with height follows the simple scalings (stress proportional to pressure) normally employed as a generalization of the $\alpha$ ansatz (Eq. 3.15). In fact, numerical simulations of the MRI suggest that the stress may be roughly constant within the body of the disk (up to $\approx 2h$), with a significant fraction (perhaps a quarter) of the magnetic energy escaping buoyantly from the disk to form a hot, strongly magnetized corona (Miller & Stone 2000). These simulation results differ from the simplest assumptions used in modeling vertical structure, but they do not invalidate the basic approach. Indeed, it ought to be possible to compute vertical structure models—and vital observational properties of the disk, such as the local spectrum (e.g., Hubeny & Hubeny 1997; Hubeny et al. 2001)—using the computational results as a guide to where in the disk dissipation of energy occurs. This has not yet been done.

The second fundamental assumption of one-dimensional models is the diffusive evolution of the surface density predicted by Equation 3.16. This, too, is broadly compatible with what is known about the MRI. Figures 3.1 and 3.2 show the predicted evolution of a ring of gas in a Keplarian potential, computed directly from a global three-dimensional MHD simulation using methods described by Armitage (1998). Although large scale non-axisymmetric structure is evident in Figure 3.1, the diffusive evolution of the surface density that is predicted by the one-dimensional theory is nonetheless reproduced.

The preceding discussion has emphasized that one-dimensional disk models based on the $\alpha$ formalism can provide a reasonable description of the evolution of thin disks, especially if insights from numerical simulations of the MRI are used to guide the detailed modeling. Simulations, however, also reveal circumstances under which $\alpha$ models fail to describe disks driven by magnetically induced turbulence. Papaloizou & Nelson (2003) and Winters, Balbus, & Hawley (2003a) have shown that averaging the magnetically induced stress over tens or even hundreds of orbital times is required to yield stable estimates of the average stress. In the context of a one-dimensional $\alpha$ disk model, this implies that $\alpha$
Figure 3.1. Evolution of a narrow ring of gas under the influence of angular momentum transport, computed using a three-dimensional global MHD simulation (Armitage, unpublished). Panels show the surface density $\Sigma(r, \phi)$ at different times $t$, where $t$ is measured in units of the orbital period at the initial center of the ring. The simulation is isothermal, with $c_s/v_\phi = 0.2$ at $r = r_0$, the initial center of the ring. The MRI was triggered by threading an initially stable and axisymmetric ring of gas with a weak vertical magnetic field. A cylindrical disk approximation was used (i.e., no vertical stratification), and the evolution was computed using the ZEUS code (Stone & Norman 1992) with $200 \times 250 \times 40$ grid points in $r$, $\phi$, and $z$, respectively. As in the one-dimensional viscous disk solution to this problem, given by Lynden-Bell & Pringle (1974), the ring spreads out, with the mass flowing inward, while a fraction of the mass at large radii absorbs the angular momentum. In addition, however, there is obvious non-axisymmetric structure in the form of trailing spiral waves and relatively long-lived fluctuations in the magnitude of the derived stress.
cannot be regarded as a constant over shorter timescales. More generally, any attempt to extend $\alpha$ models to treat two-dimensional flows (for example, to study eccentric disks or disks strongly perturbed by binary companions) is dangerous and not to be recommended. Such generalizations frequently rest on the assumption that all the components of the stress tensor in a turbulent MHD disk flow behave in the same way as a Navier-Stokes viscosity. There is scant reason to believe that this is true, and indeed the limited numerical evidence collected to date suggests that the multidimensional evolution of magnetically driven disks differs significantly from that of their viscous counterparts (Nelson & Papaloizou 2003; Winters, Balbus, & Hawley 2003b).

So far, we have assumed that the disk orbits in the point mass potential of the black hole, ignoring the self-gravity of the disk itself. This approximation is generally valid in the inner regions of the disk and will obviously fail if the disk mass becomes comparable to the mass of the black hole (perhaps during formation of the hole via accretion). Even if the disk mass is relatively modest, however, self-gravity can still become
important at large enough radii. To demonstrate this, we make use of the $\alpha$-prescription and write the kinematic viscosity $\nu$ in the form

$$\nu = \alpha \frac{c_s^2}{\Omega}. \quad (3.17)$$

For a steady disk, away from the boundaries (e.g., Pringle 1981),

$$\nu \Sigma = \frac{M}{3\pi}, \quad (3.18)$$

which, together with Equation 3.17, specifies $\Sigma(r)$ in terms of the accretion rate, sound speed, and $\alpha$ parameter. The stability of a nearly Keplerian gas disk against axisymmetric perturbations is measured by the Toomre (1964) $Q$ parameter,

$$Q = \frac{c_s \Omega}{\pi G \Sigma}, \quad (3.19)$$

with instabilities developing as $Q \to 1$ from above. Making use of Equations 3.17 and 3.18, we find that for a steady disk,

$$Q \propto \frac{\alpha c_s^3}{M}. \quad (3.20)$$

Since the temperature and sound speed in the disk are decreasing functions of radius (at least if the heating is provided by accretion and/or irradiation from the central regions of the disk), the outer parts of a steady-state disk will inevitably reach the $Q \approx 1$ threshold for self-gravity to become important, provided that the disk is large enough. Indeed, models of AGN disks suggest that self-gravity typically sets in at $r \approx 10^{-2}$ pc, which is only $\sim 10^3$ Schwarzschild radii for a $10^8 M_\odot$ black hole (Clarke 1988; Kumar 1999; Goodman 2003). Interestingly, this radius is one to two orders of magnitude smaller than the radii from which maser emission has been observed in NGC4258 (Miyoshi et al. 1995) and NGC1068 (Greenhill & Gwinn 1997). This raises the immediate question: is the masing disk observed in these galaxies a roughly homogenous structure, or is it rather composed of discrete clumps (Shlosman et al. 1990; Kumar 1999; Kartje, Königl, & Elitzur 1999)?

Once $Q \approx 1$ has been attained, self-gravity triggers the development of spiral structure in the disk. Gravitational torques then act to transport angular momentum outward, in a manner superficially resembling the evolution of an ordinary viscous disk described by an $\alpha$ model (Laughlin & Bodenheimer 1994). Such a correspondence rests on shaky theoretical foundations (Balbus & Papaloizou 1999), because the non-local nature
of gravitational angular momentum transport immediately violates the local property of $\alpha$ models (Eq. 3.15). That said, examining the local properties of self-gravitating disks proves to be useful in delineating the circumstances under which a self-gravitating disk will fragment, rather than stably transport angular momentum (Lodato & Rice 2004). If, at a particular radius, the disk can cool via radiative losses on a timescale $t_{\text{cool}}$, then Gammie (2001) showed that the boundary between stable transport and fragmentation lies at $t_{\text{cool}} = t_{\text{crit}} \simeq 3 \Omega^{-1}$. For $t_{\text{cool}} > t_{\text{crit}}$, a stable state can be reached in which $Q \sim 1$ and angular momentum is transported with an effective $\alpha$ of

$$\alpha = \left[ \left(\frac{9}{4}\right) \gamma (\gamma - 1) \Omega t_{\text{cool}} \right]^{-1},$$

where $\gamma$ is a two-dimensional adiabatic index (Gammie 2001). As $t_{\text{cool}}$ decreases, turbulence in the disk (measured by the effective $\alpha$) has to become increasingly violent in order to balance the radiative losses, and the overdensities of spiral structures increase. Eventually, for $t_{\text{cool}} < t_{\text{crit}}$, the disk fragments into bound objects. Both local (Gammie 2001; Johnson & Gammie 2003) and global (Rice et al. 2003; Fig. 3.3) simulations broadly support these analytic results.
Application of these ideas to the outer regions of accretion disks in AGN is complicated by the dominant role that irradiation plays in setting the thermal conditions in the outer disk. In particular, fluctuations in the disk temperature (and, thus, $Q$) will occur as the illumination from the central source varies, with the disk being most vulnerable to fragmentation when the source is in its lowest state. Furthermore, there is no reason to expect that the disk at large radii is able to attain the steady-state described by Equation 3.18. The most plausible scenario is that the inner non-self-gravitating disk is surrounded by a region in which self-gravity transports angular momentum in the stable, non-fragmenting regime. Beyond that, at radii of order a pc or less, any disk formed from inflowing or outflowing gas is unstable to fragmentation (Goodman 2003). If correct, this suggests that AGN need to be resupplied with gas of low specific angular momentum (compared to that of a circular orbit at the sphere of influence) in order to prevent fragmentation occurring in the outer disk. More speculatively, if fragmentation does occur, the edge of the accretion disk in AGN could be a birth place for massive stars that would ultimately form stellar or intermediate mass black holes. If this process had occurred in the Galactic Center in the recent ($\sim$10 Myr) past, it could leave a distinctive signature in the form of disk-like kinematics for some subset of massive stars (Levin & Beloborodov 2003). Moreover, if some fraction of the stars ended up embedded in the inner disk, they would migrate inward on at most a viscous timescale. Gravitational waves from the final inspiral of the stellar remnants with the supermassive black hole are potentially detectable with the Laser Interferometer Space Antenna (LISA; Levin 2004; Goodman & Tan 2004).

In broad outline then, the MRI is likely to provide the main mechanism of angular momentum transport in the inner disk, while self-gravity is potentially important at relatively large radii. A more subtle question is whether there is a region of the disk in which both processes are operative, or, conversely, a zone in which neither drives efficient angular momentum transport. This depends upon whether the MRI is able to operate in the cool outer regions of the disk, where the gas has a low ionization fraction and the flux-freezing assumption of ideal MHD breaks down. For AGN, a quantitative discussion of the effect of the low temperature on angular momentum transport processes has been given by Menou & Quataert (2001). For a disk with sound speed $c_s$ and resistivity $\eta$, the magnetic Reynolds number is

$$\text{Re}_M = \frac{v_A h}{\eta}.$$
If Re_M is too low, then diffusion of magnetic fields due to the finite conductivity can suppress the MRI (e.g., Matsumoto & Tajima 1995; Gammie 1996; Wardle 1999; Sano & Stone 2002). The critical value of Re_M (which may also need to take account of the Hall effect) is probably of order unity or somewhat larger. Identical reasoning suggests that ambipolar diffusion can also shut off the MRI and prevent MHD disk turbulence from developing (Menou & Quataert 2001).

Applying these ideas to AGN disks, Menou & Quataert (2001) found that the ionization state of the outer disk—and hence the likelihood of the MRI being operative—was highly uncertain, depending upon (among other things) whether the outer disk was exposed to irradiation from the central source. If the MRI does shut off at radii where Q >> 1, then an intermediate zone in the disk could be both magnetically and gravitationally inactive. Such a “dead zone” would act to starve the inner disk of resupply from larger radii, until enough matter built up to initiate self-gravity or magnetorotational activity. Complicated and poorly understood time-dependent accretion would probably result (e.g., Gammie 1999b; Armitage, Livio, & Pringle 2001). Alternatively, if the regions of MHD turbulence and gravitational instability overlap, then there could be non-trivial interactions between the two angular momentum transport mechanisms. Planet formation simulations (Winters et al. 2003b) show that external gravitational torques can modify the properties of MHD disk turbulence, and the first simulations of self-gravitating magnetized disks (Fromang et al. 2004) likewise show that magnetic and gravitational torques are not simply additive. In the calculations of Fromang et al. (2004), the presence of MHD turbulence in a self-gravitating disk significantly reduced the rate of gravitational transport of angular momentum.

Finally, it remains possible that accretion is not always driven by angular momentum redistribution within the disk at all, but rather by angular momentum loss in an outflow. Outflows and jets are certainly present in some AGN, and if magnetically driven (Blandford & Payne 1982), they can potentially remove all of the angular momentum needed to allow accretion, while ejecting only a modest fraction of the mass. Models in which outflows are entirely responsible for driving accretion are not popular, in part because of skepticism that the required magnetically launched outflows are present across a sufficiently broad range of disk radii. In particular, although there is near-universal agreement that magnetic fields are implicated in the formation of relativistic jets, the slower outflows that originate further away from the black hole may well be partially or entirely accelerated by radiation pressure acting on resonance lines (Vitello & Shlosman 1988; Murray et al. 1995; Proga
Moreover, the stability of accretion arising from magnetic outflow torques is rather doubtful (Lubow, Papaloizou, & Pringle 1994; Cao & Spruit 2002; but see also Konigl & Wardle 1996 and Campbell 2001). Nonetheless, the possibility that angular momentum loss via MHD outflows dominates over internal stresses in restricted regions of AGN disks remains entirely viable. It constitutes a significant source of uncertainty in our understanding of the structure of the disk and when considering possible mechanisms for large-scale variability.

### 3.3 The Effect of the Accretion Rate

The accretion rate onto a black hole is most usefully measured as a fraction of the accretion rate at which radiation pressure becomes important for the dynamics of the flow. For a spherically symmetric flow, radiation pressure acting on free electrons balances gravity at the Eddington luminosity,

\[
L_{\text{Edd}} = \frac{4\pi G m_p c}{\sigma_T} M ,
\]

where \(\sigma_T\) is the Thomson scattering cross-section. If the luminosity is generated by accretion with radiative efficiency \(\epsilon\), then \(L = \epsilon\dot{M}c^2\) and the above expression defines an Eddington accretion rate. Taking \(\epsilon = 0.1\),

\[
\dot{M}_{\text{Edd}} = 2.2 \left( \frac{M}{10^8 \, M_\odot} \right) \, M_\odot \, \text{yr}^{-1} .
\]

The accretion rate can then be expressed in dimensionless form as a fraction \(\dot{m} \equiv \dot{M}/\dot{M}_{\text{Edd}}\) of the Eddington rate. Current theoretical understanding identifies \(\dot{m}\) as the most important parameter controlling the structure and observational appearance of black hole accretion flows. We will consider in turn the three cases: \(\dot{m} \ll 1\), \(\dot{m} \sim 1\), and \(\dot{m} \gg 1\). These cases, by definition, differ in the importance of radiation pressure for the dynamics of the flow. Equally important, the ability of the flow to cool also varies systematically with \(\dot{m}\).

For \(\dot{m} \ll 1\), the critical question is whether the inflowing gas can radiate the gravitational potential energy liberated by accretion. Most theoretical work suggests that the vast majority of the energy dissipated by MHD turbulence in a disk goes initially into heating the ions rather than the electrons (Quataert 1998; Blackman 1999; Quataert & Gruzinov 1999; Medvedev 2000), at least as long as the magnetic field remains sub-equipartition. Sub-equipartition fields are the normal outcome of the non-linear phase of the MRI in MHD simulations of accretion flows in the fluid limit (for a discussion of how the MRI itself is altered if
the plasma is collisionless, see Quataert, Dorland, & Hammett 2002). Radiative losses from bremsstrahlung radiation, synchrotron radiation, and Comptonization of soft photons, on the other hand, primarily take energy from the electrons. In this situation, where the ions are heated simultaneously with the electrons being cooled, the extent of coupling between the two fluids obviously determines the state of the plasma. If the dominant coupling process is ordinary Coulomb collisions, then the timescale for thermal equilibrium to be established between the ions and the electrons is (Spitzer 1962; Ichimaru 1977)

$$\tau_{ei} = \frac{3 (kT_e)^{3/2} m_p}{8 ne^4 \sqrt{2\pi m_e} \ln \Lambda}.$$  (3.25)

Here, $T_e$ is the electron temperature, $k$ is the Boltzmann constant, $m_p$ is the proton (ion) mass, $n$ is the number density of ions, and $m_e$ is the electron mass. The Coulomb logarithm has a value of $\ln \Lambda \sim 20$. When the accretion rate is low, $n$ is small, and this, coupled with the high temperature near the black hole (of the ions generally, and the electrons, as well, if we initially postulate a one-temperature plasma), implies a large value for $\tau_{ei}$. Once $\tau_{ei}$ becomes comparable or larger than the inflow time, electrons and ions can no longer maintain equilibrium via Coulomb collisions. Instead, a two-temperature plasma develops in which the ions stay close to the virial temperature while the electrons are substantially cooler. All current models for low $\dot{m}$ accretion flows onto black holes are based on the two-temperature idea (e.g., Ichimaru 1977; Rees et al. 1982; Narayan & Yi 1994; Abramowicz et al. 1995; Blandford & Begelman 1999; and the numerous variants on these models), and would not be viable otherwise. In particular, any collective mechanism that coupled the ions to the electrons substantially more efficiently than Coulomb collisions would undercut the foundations of two-temperature plasma accretion models. Although no such mechanism is known, the well known complexities of laboratory plasmas suggest that the possible existence of more efficient coupling cannot be entirely ruled out (Begelman & Chiueh 1988; Bisnovatyi-Kogan & Lovelace 1997; Quataert 1998; Quataert & Gruzinov 1999; Medvedev 2000).

Although from a purely theoretical perspective the question of whether low $\dot{m}$ accretion flows form a two-temperature plasma remains open, observations—particularly those that indicate a remarkably low luminosity for the black hole at the Galactic Center (Narayan et al. 1998)—are hard to reconcile with almost any imaginable single temperature disk model (see below). This has prompted most theorists to accept that accretion flows with $\dot{M} \ll \dot{M}_{\text{Edd}}$ develop a two-temperature structure, though the exact threshold below which a hot two-temperature flow sup-
plants a cool thin disk is less certain. Narayan & Yi (1995b) suggest that the critical accretion rate is around

$$\dot{m}_{\text{crit}} \simeq \alpha^2; \quad (3.26)$$

that is, one to two orders of magnitude below the Eddington limit.

Once a two-temperature flow has developed, three model-independent conclusions follow. First, since the ion temperature $T_i$ remains comparable to the virial temperature, the flow can be \textit{geometrically thick}, with $h/r \sim 1$. Second, since most of the accretion energy remains locked up in the ions, the flow is \textit{underluminous}, with an accretion efficiency much smaller than the $\epsilon \simeq 0.1$ values realized for thin disks. Finally, since the gas is both hot and rapidly rotating, it is at most marginally bound to the black hole (formally, the Bernoulli constant, which is conserved for adiabatic flows, is positive). As a consequence, it is certainly plausible, and possibly inevitable, that a substantial fraction of the putative fuel for the black hole actually ends up outflowing from the galactic nucleus (Narayan & Yi 1995a; Blandford & Begelman 1999, 2004). Outflows further reduce the already low radiative efficiency (now defined as $\epsilon = L/c^2 \dot{M}$, where the inflow rate is measured at large radius), because most of the gas then fails to sample the deep potential well close to the black hole. Figure 3.4 shows semi-quantitatively (not quantitatively, because the simulation depicted was two-dimensional and non-magnetic) what the resulting flow looks like.
Apart from such generalities, no consensus has yet been reached as to the detailed structure of non-radiative accretion flows (also described as advection dominated flows, since most of the liberated gravitational potential energy is advected with the fluid rather than being radiated). The problem lies in the fact that the structure of non-radiative accretion flows is expected to depend sensitively on the nature of energy and angular momentum transport. The numerical simulations needed to directly attack this issue are extraordinarily difficult. For an illustration of the difficulties, suppose that the black hole is accreting from the hot interstellar medium of an elliptical galaxy with $T_{\text{ISM}} \sim 10^6$ K and sound speed $c_s$ a few hundred kilometers per second. The ratio of the Bondi radius,

$$R_B = \frac{GM}{c_s^2},$$

(3.27)
to the gravitational radius of the hole—both of which must be resolved if we want to get the boundary conditions strictly correct—is then $R_B/R_g \sim 10^6$, vastly exceeding the dynamical range accessible to any present three-dimensional simulation. Two-dimensional simulations can do better but have their own limitations due to both the antidynamo theorem in axisymmetry (Cowling 1934) and the fact that the MRI has different properties in axisymmetry, as compared to three dimensions (Goodman & Xu 1994; Hawley et al. 1995). Recent examples of simulations include work by Stone & Pringle (2001), Machida, Matsumoto, & Mineshige (2001), Hawley & Balbus (2002), Proga & Begelman (2003), Pen, Matzner, & Wong (2003), and Igumenshchev, Narayan, & Abramowicz (2003). There is substantial agreement that outflows of various kinds occur generically as a side effect of non-radiative accretion, but opinions still differ on the steady-state radial structure of the flow, and on how to interpret the numerical results in terms of simpler analytic models.

Accretion rates above $\dot{m}_{\text{crit}}$ (Eq. 3.26) yield flows that are dense enough to provide good coupling between ions and electrons. Provided that the accretion rate is not extremely high, i.e., $\dot{m} \sim 1$, the flow is then radiatively efficient and cools to form a geometrically thin ($h/r \ll 1$) disk. The disk extends from an outer radius, which is probably set by the considerations of gravitational stability described in the previous section, down to the marginally stable circular orbit at $r = r_{\text{ms}}$. Interior to the marginally stable circular orbit, which lies at $r_{\text{ms}} = 6 \frac{GM}{c^2}$ for a Schwarzschild black hole, circular particle orbits are unstable—any inward radial perturbation suffices to place the particle onto a plunging trajectory into the black hole. In the simple—but possibly incorrect—case where there is zero torque at $r_{\text{ms}}$ (non-zero torque is discussed in §3.4), the radiative efficiency is determined by the binding energy $E_{\text{ms}}$
Figure 3.5. (Left panel) Location of the marginally stable circular orbit as a function of dimensionless spin parameter $a$ for prograde (solid line) and retrograde (dashed line) orbits. (Right panel) Efficiency of energy extraction if gas spirals slowly into $r_{\text{ms}}$ before plunging silently into the black hole. Plotted is $1 - E_{\text{ms}}$, where $E_{\text{ms}}$ is the binding energy of the marginally stable circular orbit for prograde and retrograde motion.

of gas at that radius. Plots of these quantities are shown as a function of the dimensionless spin parameter ($a \equiv Jc/GM^2$ for a hole with an angular momentum $J$) in Figure 3.5.

The basic theory of thin disks is well known (for reviews, see, e.g., Pringle 1981; Frank et al. 2002). The main open issues (at least for flat, circular disks) concern the role of magnetic fields and radiation. The structure of magnetic fields near $r_{\text{ms}}$ is of particular interest, since it determines not only the boundary condition at the disk’s inner edge (Krolik 1999), but also the strength of magnetic fields threading the black hole itself. If the black hole is spinning, magnetic fields admit the possibility of tapping some of the rotational energy via the Blandford-Znajek (1977) process. Spin energy extraction provides one way to power jets (e.g., Blandford 2000), and is frequently suggested—mostly on the basis of empirical arguments—as the source of the radio-loud/radio-quiet dichotomy in AGN (e.g., Wilson & Colbert 1995). As a counter to such claims, Ghosh & Abramowicz (1997) and Livio, Ogilvie, & Pringle (1999) have argued that since dynamo generated disk fields are expected to be weak (sub-equipartition), and dragging in ordered external fields is difficult (Lubow et al. 1994), the efficiency of the Blandford-Znajek process is likely to be negligibly small for thin disks. These arguments are plausible, but I doubt that they represent the last word on the subject, not least because of observational indications for spin energy extraction
in some AGN (Wilms et al. 2001; Reynolds et al. 2004, though note that even if spin energy is being extracted, this need not be the classical Blandford-Znajek process at work).

As the accretion rate increases, thin disks are predicted to remain the preferred mode of accretion below some upper limit set by the effects of radiation pressure. For a spherical flow, that upper limit lies at $\dot{m} = 1$. In disks, however, matters need not be so simple. Once radiation pressure dominates gas pressure in the inner disk, the disk is prone to several instabilities, even in the absence of magnetic fields (Shakura & Sunyaev 1976; Agol et al. 2001). When magnetic fields are included, it is found that the flow develops photon bubbles (Arons 1992; Gammie 1998; Blaes & Socrates 2001, 2003)—low density radiation filled cavities that coexist with dense gas pressure dominated regions. In principle, radiation can escape from such a highly inhomogenous structure at rates above the Eddington limiting luminosity, without being accompanied by catastrophic mass loss (Begelman 2001; Begelman 2002; see also Shaviv 1998). This could permit the existence of stable, thin disks, with an emergent luminosity as large as $L = 10 - 100 \, L_{\text{Edd}}$ (Begelman 2002). Detailed calculations of the outcome of these instabilities in magnetized disks, and their implications for the structure of radiation pressure dominated flows even below the Eddington limit, are now in progress (Turner, Stone, & Sano 2002; Turner et al. 2003; Turner 2004).

The final regime to consider is that of overfed accretion, with $\dot{m} \gg 1$. By definition, radiation pressure is central to the dynamics of such flows, though it need not act to shut off accretion directly. At sufficiently high accretion rates, the opacity can be large enough that photons in the inner regions are advected inward faster than they can random walk away from the black hole. Eventually, they are dragged across the event horizon along with the gas and are not radiated from the system. For a spherical accretion flow, it is straightforward to show that advection wins over diffusion inside a trapping radius given by

$$r_{\text{trap}} = \frac{\dot{M} c^2 r_s}{L_{\text{Edd}} 2},$$

where $r_s = 2GM/c^2$ is the Schwarzschild radius (Begelman 1979). The same behavior occurs for flows with non-zero angular momentum (Begelman & Meier 1982). The emergent luminosity from photon-trapped flows remains around $L_{\text{Edd}}$ (e.g., Blondin 1986; Ohsuga et al. 2002), so as $\dot{m}$ increases, the radiative efficiency decreases inversely with the accretion rate.

Clearly, the similarities between the $\dot{m} \ll 1$ and the $\dot{m} \gg 1$ regimes are striking. In both cases, the radiative efficiency is low, for $\dot{m} \ll 1$ be-
cause of inefficient ion-electron coupling, and for $\dot{m} \gg 1$ because the high opacity prevents photons from escaping the fluid. In the simplest analysis, overfed accretion will then also result in a hot, geometrically thick disk, with the main difference from the underfed case being the lower value of $\gamma$ appropriate for a radiation dominated gas. Strong outflows could then be inevitable, and these might even reduce the accretion rate onto the hole back down to levels comparable to the Eddington limit. Once again, simulations of radiation dominated disks will be needed to test such speculations.

In addition to providing an explanation for the dichotomy in the observed properties of supermassive black holes in AGN and quiescent galactic nuclei, the existence of these different modes of accretion also has implications for the growth of black holes. Accretion rates well below the Eddington limiting value probably provide the black hole with a negligible amount of mass, despite the fact that this mode lasts much longer than more active phases (Haehnelt, Natarajan, & Rees 1998). More surprisingly, attempts to flood the black hole with mass at rates vastly in excess of the Eddington value may also fail if, as we have speculated, such high $\dot{m}$ flows are as vulnerable to outflows as their low $\dot{m}$ counterparts. The entire accretional growth of black holes may be restricted to occur via radiatively efficient accretion at rates that lie within an order of magnitude or so of the Eddington limit (probably extending above as well as below the nominal “limit”). For the most luminous AGN, such a “what you see is what you get” model allows for a consistent match (within substantial uncertainties) between observations of the luminous output of AGN and the local density of supermassive black holes (Barger et al. 2001; Merritt & Ferrarese 2001; Yu & Tremaine 2002; following the basic approach of Soltan 1982). It may also apply to the earlier growth phase of supermassive black holes.

### 3.4 Dynamics of the Plunging Region

The inner boundary of a radiatively efficient accretion disk around a black hole lies close to the marginally stable circular orbit. At $r > r_{\text{ms}}$, the force balance for a fluid element in a thin disk is predominantly between gravity and centrifugal force, with angular momentum transport driving a slow, subsonic inflow. Interior to $r_{\text{ms}}$, circular orbits are unstable. In the standard model for black hole accretion disks (Novikov & Thorne 1973; Page & Thorne 1974), once gas reaches this plunging region, it flows into the black hole along geodesics, maintaining a constant energy and angular momentum. In this simple model, gas in the plunging region can have observational effects, mainly by reprocessing
Figure 3.6. Local flux from the disk (i.e., the emission per unit area in the rest frame of the orbiting gas, ignoring returning radiation) for disk models with different inner boundary conditions. Lowest curve shows the profile of the emission for a standard Novikov-Thorne disk (Page & Thorne 1974) around a Schwarzschild black hole, assuming zero torque at $r_{\text{ms}}$. $F(r)$ peaks well outside the marginally stable circular orbit. The three upper curves show the effect of adding increasing amounts of torque at $r_{\text{ms}}$ (Agol & Krolik 2000), with the labels showing the resulting increase $\Delta \epsilon$ in the radiative efficiency as a fraction of the original efficiency $\epsilon$. No emission is assumed to originate from the plunging region.

or reflecting radiation generated elsewhere in the system (Cunningham 1976; Reynolds & Begelman 1997a). The plunging region does not, however, alter the structure of the disk or the radiative efficiency of the flow. The inner boundary condition for the disk is simply that there should be zero torque at $r = r_{\text{ms}}$.

Early studies of black hole accretion assumed (often explicitly) that magnetic fields in the plunging region remained weak. Provided that this is so, the inflowing gas rapidly attains supersonic (and super-Alfvenic) radial velocities and becomes causally disconnected from the disk outside the marginally stable orbit. This justifies the zero-torque boundary condition at $r_{\text{ms}}$. If magnetic fields become strong, for example as a consequence of the shearing of frozen-in fields during the final inspiral, the argument fails. Energy and angular momentum can then be exported
from the plunging region to the disk, modifying its structure and emission (Krolik 1999; Gammie 1999a). The same basic process can occur in either Schwarzschild or Kerr geometry.

The consequences of a non-zero torque at the marginally stable orbit have been discussed in detail by Agol & Krolik (2000). The most basic is that energy that would otherwise be lost down the hole can be transmitted from the plunging region to the disk, and subsequently radiated. As a result, the radiative efficiency of thin disk accretion could be larger than normally assumed (Fig. 3.5). In particular, the standard value of $\epsilon = 0.06$ for the yield of accretion onto Schwarzschild holes may more usefully be regarded as a lower limit. This implies that—although the qualitative trend for more rapidly spinning black holes to be more luminous at a given accretion rate than Schwarzschild holes is very reasonable—there are significant theoretical uncertainties that need to be borne in mind when attempting to determine the average spin of black holes from comparison of black hole masses and integrated AGN output. For example, the relatively high inferred radiative efficiency ($\epsilon > 0.1$) derived by Elvis, Risaliti, & Zamorani (2002) and Yu & Lu (2004) need not necessarily imply a rapid average spin for the supermassive black hole population. Perhaps more interestingly, given that measuring $\epsilon$ to a factor of about two precision is obviously a challenging task, a non-zero torque at the disk inner edge makes a large change to the radial distribution of the disk emissivity. This is plotted in Figure 3.6, using expressions given in Agol & Krolik (2000). Unlike a “standard” disk, in which the local flux peaks well outside the last stable orbit, a torqued disk has a steeply rising emissivity profile right down to the marginally stable orbit (at large radius, the contribution to the emissivity due to the torque at $r_{\text{ms}}$ scales with radius as $F(r) \propto r^{-7/2}$, which is steeper than the dissipation profile for an untorqued disk). This difference in the dissipation with radius is potentially observable. It is worth noting that similarly steep dissipation profiles are predicted in some models in which magnetic fields act to extract energy from a Kerr black hole (Takahashi et al. 1990; Li 2002).

Numerical simulations have generally been supportive of the basic concept outlined by Krolik (1999). Pseudo-Newtonian calculations, which use a modified potential to mimic some of the most important relativistic effects (Paczynski & Wiita 1980), have confirmed that there can be significant non-zero torques at $r_{\text{ms}}$ (Hawley 2000; Reynolds & Armitage 2001; Hawley & Krolik 2001, 2002). As shown in Figure 3.7, this leads to a declining specific angular momentum profile that extends seamlessly through the last stable orbit to smaller radii. The same phenomenon has now been seen in General Relativistic MHD simulations of black hole ac-
Figure 3.7. (Left panel) Magnetic torque at the marginally stable orbit as a fraction of the torque in the disk at a radius of 2 $r_{ms}$, as derived from numerical simulations with relatively high (solid line) and relatively low (dashed line) sound speed (Armitage & Reynolds 2003). (Right panel) Specific angular momentum $l(r)$ at several different times for a simulated flow in which the disk magnetic field strength was relatively large (Reynolds & Armitage 2001). The inner disk is quite variable, but typically $l(r)$ continues to decline interior to $r_{ms}$.

3.5 Variability

The emission from AGN is variable (Mushotzky, Done, & Pounds 1993; Ulrich, Maraschi, & Urry 1997), as is the much weaker emission from the Galactic Center (Baganoff et al. 2001; Zhao, Bower, & Goss 2001; Genzel et al. 2003; Ghez et al. 2004). Although in some observationally important cases the correlations between variability in different wavebands can be understood through simple light travel time arguments (Blandford & McKee 1982), the basic origin of most of this variability is not well understood. Here, I discuss what insights into the problem recent MHD calculations of disk accretion provide.

The first question we might ask is, are there large amplitude changes in the luminosities of AGN akin to those seen in most other disk accreting systems? We have already discussed the outbursts of dwarf novae,
which are attributed to a global thermal instability of the accretion disk. Accretion onto some neutron stars and (stellar mass) black holes in low-mass X-ray binaries is similarly prone to thermal instability (Tanaka & Shibazaki 1996) and can be modeled using a minimally modified version of the theory developed for dwarf novae (Dubus, Hameury, & Lasota 2001). The same ideas have also been pressed into service to explain the outbursts of protostellar disks seen in FU Orionis objects (Hartmann & Kenyon 1996; Bell & Lin 1994), though the identification of these events with thermal disk instabilities is less compelling.

If thermal instabilities act on a global scale in AGN, the timescales for the cycling between outburst and quiescence would be long—probably $\sim 10^4$ yr or longer (Lin & Shields 1986). Transitions between “on” and “off” states might not occur on easily observable timescales, though they could leave traces in the form of intermittent activity from radio-loud AGN (Reynolds & Begelman 1997b; Owsianik, Conway, & Polatidis 1998). If such intermittent accretion is commonplace, there ought to exist a population of relatively quiescent galaxies in which the black hole is surrounded not by a radiatively inefficient flow, but rather by a temporarily inactive thin disk. Detailed models of this type have been developed by Siemiginowska, Czerny, & Kostyunin (1996) and by Siemiginowska & Elvis (1997).

Disks in AGN extend out to radii that encompass the zone in which hydrogen is partially ionized, so the basic ingredient necessary for thermal instabilities to occur is present (Burderi, King, & Szuszkiewicz 1998). What is less clear is whether the efficiency of angular momentum transport (i.e., the value of $\alpha$) varies between the ionized and neutral state in the AGN environment. A substantial difference between $\alpha_{\text{hot}}$ and $\alpha_{\text{cold}}$ is needed if the whole disk is to flip between outburst and quiescent states. In dwarf novae, the required change probably occurs because the hot and cold states lie on opposite sides of the critical degree of ionization required for sustained MHD turbulence (Eq. 3.22; Gammie & Menou 1998). It has been argued (Menou & Quataert 2001) that this is no longer the case in AGN disks, leading to thermal instabilities that produce local flickering but not global outbursts. If so, any observational evidence for large scale outbursts in AGN may require a different explanation. Numerous possibilities for time dependent behavior suggest themselves in the outer disk, where there is an interplay of gravitational and MHD disk instabilities, though none have been worked out in any great detail.

At frequencies between $\sim 10^{-4}$ Hz and $\sim 10^{-8}$ Hz (i.e., timescales of tens of minutes to around a year), X-ray observations provide good constraints on the properties of AGN variability (e.g., Edelson & Nan-
Figure 3.8. View of a simulated disk as seen by a distant observer at an inclination angle of 55° (Armitage & Reynolds 2003). The patchy emission is boosted on the approaching (left hand) side of the disk as a consequence of beaming. In constructing this image, the local disk emission has been assumed to scale with the vertically integrated magnetic stress. Raytracing through the Schwarzschild metric is then used to account for the relevant relativistic effects, including beaming, gravitational redshift, and light bending.

dra 1999; Uttley, McHardy, & Papadakis 2002; Vaughan & Fabian 2003; Markowitz et al. 2003; and references therein). The power spectral density $G(f)$ derived from these observations is often consistent with a power law broken at a frequency $f_{\text{break}}$,

$$
G(f) \propto f^{-\alpha_{\text{low}}} \quad f < f_{\text{break}},
$$

$$
G(f) \propto f^{-\alpha_{\text{high}}} \quad f > f_{\text{break}}.
$$

(3.29)

Representative values for the two slopes are $\alpha_{\text{low}} \simeq 1$ and $\alpha_{\text{high}} \geq 2$ (Vaughan & Fabian 2003; Markowitz et al. 2003).

Although a detailed comparison is beyond the capabilities of current numerical simulations, there appear to be good prospects for relating noise power spectra of this form to theoretical models of MHD disk turbulence. Simulations of magnetically active disks show that the flow displays variability across a wide range of timescales, with physical quantities such as the magnetic stress and mass accretion rate showing temporal power spectra that are described by power laws (Kawaguchi et al.
The patchy and rapidly fluctuating pattern of stress across the surface of the inner disk, which gives rise to this variability, is shown in Figure 3.8. If we assume that the local disk emission traces the magnetic stress, then the power spectra predicted from the simulations have the form shown in Figure 3.9. Regardless of the inclination of the disk to the line of sight (which affects the importance of relativistic effects such as beaming), the temporal power spectrum at frequencies comparable to those of the inner disk is around $G(f) \propto f^{-2}$. Individual annuli in the simulated disk, moreover, produce power spectra that break—at around the orbital frequency—to substantially steeper slopes of around $f^{-3.5}$ (Armitage & Reynolds 2003). These results support the common assumption that the break frequency in observed systems scales with the orbital frequency at the marginally stable orbit, and hence with the mass of the black hole (e.g., Nowak & Chiang 2000; Lee et al. 2000). Although there are significant uncertainties in many AGN black hole masses, the observations of Markowitz et al. (2003) suggest a correlation between black hole mass and break frequency. We note, however, that work to date has not been able to recover the observed values of $\alpha_{low}$ and $\alpha_{high}$—the theoretical power spectra are steeper than observed.

### 3.6 Warped, Twisted, and Eccentric Disks

Flat disks, in which the gas follows close to circular orbits, are special cases of the more general situation in which the disk may be eccentric and/or warped out of a single plane. A common assumption is that an initially warped or eccentric disk will relax to a flat circular one “on a viscous timescale”. This is at best a gross simplification of the evolution of warped or eccentric disks, and may even be qualitatively wrong. As one might expect, the evolution of warped or eccentric disks is rather complex, and in this section I attempt only to provide a framework for understanding some of the key results. For more details, the interested reader is well advised to consult the recent and exhaustive studies by Ogilvie (1999, 2000, 2001), which include references to the earlier literature on the subject.

Observations provide clear motivation for considering the evolution of warped disks in AGN. The masing disks in NGC4258 (Miyoshi et al. 1995), NGC1068 (Greenhill & Gwinn 1997), and the Circinus galaxy (Greenhill et al. 2003) all appear to be warped, albeit to different degrees. This raises two questions. First, are the warps merely decaying features that reflect the complex angular momentum distribution of the infalling gas that formed the disk, or are they self-excited by some pro-
Figure 3.9. Temporal power spectra $G(f)$ derived from a numerical simulation of accretion onto Schwarzschild black holes (Armitage & Reynolds 2003). The frequency $f$ has been rescaled such that the orbital frequency at the radius of marginal stability corresponds to $f = 1$. Shown here are the power spectra for $i = 1^\circ$, $i = 30^\circ$ (offset by $10^2$), $i = 50^\circ$ (offset by $10^3$), and $i = 80^\circ$ (offset by $10^4$). MHD turbulence in the disk generates variability across a wide range of timescales, with quantities such as the predicted emission or mass accretion rate showing steep, approximately power law spectra.

cess intrinsic to the disk or AGN itself? Second, if the warps are in fact decaying, how rapidly does that process occur compared to the timescale on which the gas would be accreted?

Although easily formulated, both of these questions have proved to be difficult to answer. The basic physics revolves around the fact that in a warped disk, neighboring annuli are inclined relative to one another, and as a consequence, there is shear in the vertical direction as well as in the radial direction. Dissipation of the energy associated with the out-of-plane motions is governed by the $(r, z)$ component of the stress, and this acts to flatten out the disk as, simultaneously, the $(r, \phi)$ stress drives accretion. The crucial point is that these stresses can be generated by entirely different physical processes (for a concrete example, see Gammie, Goodman, & Ogilvie 2000) and need not act on the same timescale, even roughly (Pringle 1992). The detailed physics of stresses within the disk will then determine whether the disk flattens before a significant fraction
of the gas accretes, or whether instead the warp is advected inward along with the gas.

To go further, we need to elaborate on the mechanisms that can allow annuli to exchange angular momentum. If the disk is thin enough, specifically, if

\[ \frac{h}{r} < \alpha , \]  

(3.30)

then the evolution of the tilt as a function of radius, like the evolution of the surface density, is diffusive (Papaloizou & Pringle 1983). If, conversely,

\[ \frac{h}{r} > \alpha , \]  

(3.31)

then the warp is communicated through the disk by waves (Papaloizou & Lin 1995). Assuming that \( \alpha \sim 0.1 \) in geometrically thin AGN disks, then most parts of the disk (except, perhaps, the innermost regions) are likely to fall into the diffusive regime, which we will consider exclusively from now on. We will also ignore the complications of self-gravity (Papaloizou, Terquem, & Lin 1998), even though, as already noted, the masing disks cited as motivation are probably self-gravitating.

In the diffusive limit, the evolution of the disk can be described to a first approximation using the simple theory described by Pringle (1992). In this theory, the stresses within the disk are reduced to two viscosities; \( \nu_1 \), which describes the usual kinematic viscosity leading to inflow, and \( \nu_2 \), which is related to the stresses which act to flatten out any warp. The timescales for accretion \( t_\nu \) and for decay of the warp \( t_{\text{warp}} \) are related to these viscosities in the usual way:

\[ t_\nu = \frac{r^2}{\nu_1} , \]  

\[ t_{\text{warp}} = \frac{r^2}{\nu_2} . \]  

(3.32)

The relation between these two viscosities can be calculated either analytically (Ogilvie 1999) or via numerical simulations (Torkelsson et al. 2000). Both approaches suggest that

\[ \frac{\nu_2}{\nu_1} \sim \frac{1}{2\alpha^2} \gg 1 . \]  

(3.33)

This is a striking result. It implies that warps are likely to decay rapidly and will require strong forcing if they are to persist for long periods. Torques from the reprocessing of radiation from the central source by
the disk provide one well studied forcing term, which could act to excite warps even in initially planar disks (Pringle 1996; Maloney, Begelman, & Pringle 1996). The large ratio of \( \nu_2/\nu_1 \) implied by the above equation, however, severely limits the circumstances in which radiation driven warping can overcome the disk’s intrinsic tendency to flatten out. Torques from a disk wind are potentially more promising (Schandl & Meyer 1994; Lai 2003), though harder to evaluate in detail.

Further complications ensue if the black hole is rotating, even modestly. Orbits whose angular momentum is inclined to the spin axis of a rotating hole will precess due to the Lense-Thirring effect (“frame dragging”). This forced precession is only significant at small radii, where it will affect the inclination of the disk. Again, the simplest case to analyze is the one in which the inner disk is in the diffusive regime of warp evolution. Differential precession, acting on an initially flat but misaligned disk, will rapidly induce a sharp twist in the disk close to the marginally stable orbit. This will be flattened out by viscosity, with the consequence that the disk near a spinning hole will be driven into the equatorial plane of the hole, regardless of the inclination at large radii (Bardeen & Petterson 1975). Over longer timescales, the torque between the disk and the hole will also act to realign the system until the angular momentum vector of the hole, and of the disk at all radii, are coincident. Since normally \( J_{\text{disk}} \gg J_\bullet \), this balancing act will usually result in a large change in the spin axis of the black hole until it roughly matches the angular momentum vector of gas in the outer disk. How quickly this process occurs depends, again, largely on \( \nu_2/\nu_1 \), with large values of this ratio leading to rapid alignment (Natarajan & Pringle 1998).

In some circumstances, the inner accretion disk around a rotating black hole may be thick enough that warps display wave-like evolution. In this case, the inner disk may be able to remain misaligned with respect to the spin axis of the black hole, even in the presence of dissipation (Demianski & Ivanov 1997; Ivanov & Illarionov 1997; Lubow, Ogilvie, & Pringle 2002). This could have important consequences for the direction of jets launched from the inner regions of the disk.

Unlike in the case of warped disks, there is little observational evidence to suggest that AGN disks—or any disk that is not strongly forced, for example by a binary companion—are significantly eccentric, though some sort of asymmetry is deduced in as many as 60% of AGN with double-peaked Balmer lines (Strateva et al. 2003). Theoretical studies, however, raise the interesting possibility that there could be circumstances in which isolated accretion disks spontaneously develop eccentricity. In particular, a two-dimensional disk model in which the viscosity follows a Navier-Stokes form, with shear viscosity \( \mu \) and bulk
viscosity $\mu_b$, is unstable to the growth of eccentricity if

$$\frac{d \ln \mu}{d \ln \Sigma} > 1 + \frac{1}{3} \left( \frac{\mu_b}{\mu} - \frac{2}{3} \right);$$  \hspace{1cm} (3.34)

that is, if the vertically integrated viscosity is increasing too rapidly with surface density (Ogilvie 2001). If the bulk viscosity is small, as is often assumed in accretion disk modeling, then this result implies that almost all disks ought to be unstable to the growth of eccentricity (Lyubarskij, Postnov, & Prokhorov 1994; Ogilvie 2001).

Earlier, of course, we specifically warned of the dangers of assuming that angular momentum transport in an MHD flow can be described via an effective Navier-Stokes viscosity. Indeed, Ogilvie (2001) shows that if angular momentum transport follows an alternative analytic prescription, which is motivated by the phenomenology of MHD turbulence, then the wholesale instability of disks to eccentricity can be avoided. The important point is that the behavior of eccentric disks— which at a minimum will exist in galactic nuclei following the tidal disruption of stars (Rees 1988), or when binary black holes are present—is closely related to subtle aspects of turbulent transport within the disk. Predictions as to their evolution cannot yet be made with confidence.

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