Edge-state Fabry-Perot interferometer as a high sensitivity charge detector

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Abstract

We present a scheme for high sensitivity charge detection in the integer quantum Hall regime using two point contacts in a series. The setup is an electronic analog of an optical Fabry-Perot interferometer. We show that for small transmission through the point contacts the sensitivity of the interferometer is very high due to multiple reflections at the point contacts. The sensitivity can be further enhanced twice by using electrons in spin entangled state. We show that for point contacts having different reflection probabilities, the interferometer can be tuned for the quantum limited measurement.

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Measurement of the charge-state of a mesoscopic system has generated a lot of interest in recent years \[1,2,3\], mainly due to the applications of charge qubits in solid-state realization of quantum information processing \[4\]. Mesoscopic devices such as quantum point contact (QPC) \[5\] and single electron transistor (SET) \[6\] have been widely used as the charge detectors. These detectors do not perform instantaneous measurement, but the measurement is performed as a sequence of continuous weak measurements \[7\]. The merits of these detectors can be understood from the two points of view: (1) efficiency and (2) sensitivity. The former is related to the back-action noise produced by the detector and the latter is related to the precision. The quantum mechanical complementarity establishes a trade-off between acquisition of information about the state of the system and the back-action dephasing. A detector is called 100% efficient (quantum-limited) if the dephasing occurred in the measured system is only due to the acquisition of information by the detector. Performing more sensitive measurements have often led to reveal new physics \[8\]. A high sensitivity charge detector working in the quantum limit can have wider applications in quantum metrology \[9\]. The improvements in measurements can be accomplished either through new designs of measurement devices or by developing methods that rely on properties like correlations \[10\] and entanglement \[11,12\].

In this Letter, we present an interferometry model of a high sensitivity charge detector in the integer quantum Hall regime \[13\]. For fractional quantum Hall states, a similar arrangement has been proposed for measuring fractional charge and non-Abelian statistics \[14\]. Our model is an electronic analog of Fabry-Perot interferometer \[15\]. We show that the charge sensitivity of our model is higher than a two-path interferometer due to multiple reflections of electrons at QPCs. We report the possibility of tuning the interferometer for quantum limited measurement for \( R_a < R_b \), where \( R_a \) (\( R_b \)) is reflection probability of quantum point contact QPC\(_a\) (QPC\(_b\)) (cf.Fig. \[\])\). We note that, two-path interferometer with edge channel (Mach-Zehnder interferometer) has been realized \[13\]. Further, the possibility of quantum limited detection of charge using Mach-Zehnder interferometer has also been proposed \[16\].

In Fig. \[\] we show a schematic setup, constructed using electrical gates on a Hall bar, for measurement of charge. Our detector consists of two QPCs, QPC\(_a\) and QPC\(_b\), arranged in a series. The input electrons are injected from the source terminals \( \alpha \) and \( \gamma \). The outgoing electrons are collected at the drain terminals \( \beta \) and \( \delta \). In the quantum Hall regime, QPCs
FIG. 1: Schematic arrangement for measurement of charge qubit. Two spatially separated point contacts form the Fabry-Perot interferometer. The qubit is capacitively attached in one arm of the interferometer.

act as the beam splitters for the incoming electrons. The point contact QPC$_a$ splits the incoming edge-state current from source $\alpha$ into two parts with one reflected back to the drain $\beta$ and the other transmitted to the second point contact QPC$_b$. The edge-state beam on reaching at QPC$_b$ is further split into two parts, one transmitted to the drain $\delta$ and other part reached at QPC$_a$, where it is again partially transmitted to drain $\beta$ and partially reflected back to QPC$_b$ and so on. Thus our detector is analogous to optical Fabry-Perot interferometry. A charge-qubit is capacitively attached to the lower arm of the interferometer between the two QPCs. The qubit, having two charge states $|0\rangle$ and $|1\rangle$, could be a double-quantum-dot or a two path interferometer. There is no electron transfer from the qubit to the interferometer. Due to Coulomb interaction the charge on the qubit deflects edge-state in the lower arm without changing transmission through QPCs, which modifies the phase of the edge-state-current via the Aharonov-Bohm effect.

The information of the measured state of the qubit is reflected in the electrons collected at drain reservoirs. We follow scattering matrix analysis for input-output probability amplitudes. The scattering matrix in terms of Fermi operators at $m$-th terminal $c_m$, $m = \alpha, \beta, \gamma, \delta$ is written as follows:

$$
\begin{pmatrix}
    c_\beta \\
    c_\delta
\end{pmatrix} =
\begin{bmatrix}
    \bar{r}_i & \bar{t}_i' \\
    \bar{t}_i & \bar{r}_i'
\end{bmatrix}
\begin{pmatrix}
    c_\alpha \\
    c_\gamma
\end{pmatrix},
$$

(1)

$$
\bar{r}_i = r_a + \frac{t_a t'_b r_b e^{i(\phi+\theta)}}{1 - r_a' r_b e^{i(\phi+\theta)}}, \bar{t}_i = \frac{t_a t_b e^{i\phi}}{1 - r_a' r_b e^{i(\phi+\theta)}},
$$

(2)

$$
\bar{r}_i' = r'_b + \frac{t_b' t'_a r_a e^{i(\phi+\theta)}}{1 - r_a' r_b e^{i(\phi+\theta)}}, \bar{t}_i' = \frac{t_a' t_b' e^{i\phi}}{1 - r_a' r_b e^{i(\phi+\theta)}},
$$

(3)

where $\phi$ is the Aharonov-Bohm phase acquired by the electron along one complete loop.
between QPCs and $\theta_i$ is the phase produced by the qubit. The phase $\theta_i$ has two values corresponding to different charge states of the qubit $|i\rangle$, $i = 0, 1$. Typical value of the phase difference $\Delta \theta = \theta_1 - \theta_0$ generated by the Coulomb interaction is about $\Delta \theta = 0.03$ [3].

Effectively, charge state of the qubit modifies the amplitude as well as the phase of the transmission through the detector. All other phases in scattering are included in the transmission amplitudes $t_n$ ($t'_n$) from the left (right) and the reflection amplitudes $r_n$ ($r'_n$) on the left (right) for QPC$_n$, $n = a, b$.

First, we consider electrons are injected only from the source terminal $\alpha$ and collected at the drain terminal $\delta$. The transmission probability $\bar{T}_i (= |\bar{t}_i|^2)$ of the interferometer is given by

$$\bar{T}_i(\Phi_i) = \frac{T_aT_b}{1 + R_aR_b - 2\sqrt{R_aR_b}\cos \Phi_i},$$

where $\Phi_i = \theta_i + \phi + \arg(r'_a r'_b)$ and $T_n = |t_n|^2 = 1 - R_n$. Sensitivity of the transmission probability $T$ to variation in phase $\Phi_i$ makes it possible to measure the charge state of the qubit. The transmission probability has Lorentzian-like resonances when $\Phi_i$ is multiples of $2\pi$. The half width at half maximum of the resonance is $\Gamma_w \approx (1 - \sqrt{R_aR_b})/(R_aR_b)^{1/4}$. The resonances are narrower for larger values of $R_a$ and $R_b$, which provides larger change in current for small variations in phase $\Phi_i$. The phase sensitivity of the interferometer is determined by the phase fluctuations due to intrinsic shot noise. In the linear regime, the average source-drain current is $\langle I_i \rangle = (e^2V/h)\bar{T}_i$ and the shot noise is given by $S_i = (2e^3V/h)\bar{T}_i(1 - \bar{T}_i)$, where $V$ is source-drain voltage. For time interval $t$, the average number of electrons transmitted is $\langle N_i \rangle = \langle I_i \rangle t/e$ and the fluctuation of number of electron is $\langle (\Delta N_i)^2 \rangle = S_i t/(2e^2)$. Therefore, the rms phase fluctuation [11] for the interferometer is given by

$$\Delta \Phi_i \equiv \sqrt{\langle (\Delta N_i)^2 \rangle / \langle N_i \rangle} = \sqrt{\frac{\hbar}{eVt}} \sqrt{T_i(1 - T_i)} |\partial T_i/\partial \Phi_i|.$$  

From Eq. (4) and (5) one can calculate the sensitivity of Fabry-Perot interferometer. We compare the sensitivity of Fabry-Perot interferometer with a two-path (Mach-Zehnder) interferometer for which transmission probability is cosine function of the form $\bar{T}_i(\Phi_i) = R_aR_b + T_aT_b + 2\sqrt{R_aR_bT_aT_b}\cos \Phi_i$ [13]. Near the resonance, for $R_a \approx R_b$, the ratio of $\Delta \Phi_i$ for Fabry-Perot interferometer to Mach-Zehnder interferometer is approximately $T_a^{3/2}$. Clearly, Fabry-Perot interferometer can be used as a very high precision charge detector for small transmission probabilities $T_a, T_b$. 

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In real devices, this high precision would be limited by the finite source-drain bias voltage, because the phase $\Phi_i$ acquires an additional energy dependent fluctuating part $\Gamma_d$. Considering drift velocity $v_d$ as constant along the edges, we can write energy dependence of phase as $\Phi_i(\epsilon) = \Phi_i(E_F) + \epsilon/E_c$, $E_c = \hbar v_d/L$, where $L$ is the length of one complete loop between the QPCs, $E_F$ is Fermi energy and $\epsilon$ is small energy difference for electrons from Fermi level. The averaging of the energy dependent fluctuations gives average transmission probability and average shot noise, respectively, as $\langle \bar{T}_i \rangle = (eV)^{-1} \int_{-eV/2}^{eV/2} \bar{T}_i(\Phi_i(\epsilon))d\epsilon$, $\langle S_i \rangle = 2e^3/h \int_{-eV/2}^{eV/2} \bar{T}_i(\Phi_i(\epsilon))(1 - \bar{T}_i(\Phi_i(\epsilon)))d\epsilon$. At small bias $(eV/E_c \ll \Gamma_w)$, we find that $\Delta \Phi_i$ is changed by the factor $[1 + (eV/E_c)^2(\Gamma_w^2 - \Phi_i^2)/2(\Gamma_w^2 + \Phi_i^2)]^2$ (for $-\pi < \Phi_i < \pi$).

In order to understand the measurement process and the back action of the detector, we consider evolution of the state of the combined system of detector and qubit. When an electron is injected from source $\alpha$ and the initial state of the qubit is $a_0|0\rangle + a_1|1\rangle$, the state of the combined qubit-detector system evolves as

$$|\psi\rangle = (a_0|0\rangle + a_1|1\rangle)e^{iF}|F\rangle \rightarrow a_0|0\rangle|\xi_0\rangle + a_1|1\rangle|\xi_1\rangle,$$

where $|F\rangle$ denotes Fermi sea of all the electrodes and $|\xi_i\rangle = (\tilde{r}_i c_i^\dagger + \tilde{t}_i c_i \dagger)|F\rangle$ for $i = 0,1$ are detector states. The final state of the qubit is given by the reduced density matrix $\rho = Tr_{\text{det}}|\psi\rangle\langle\psi|$, obtained after tracing over the detector states. The dephasing of qubit can be expressed in terms of off-diagonal elements of density matrix $\rho$ as $|\rho_{01}(t)| = |\rho_{01}(0)|\exp(-\Gamma_d t)$, where $\Gamma_d$, detector back action induced dephasing rate, is given by $\Gamma_d = -h^{-1} \int \text{d}\epsilon \log |\tilde{r}_0 \tilde{r}_1^* + \tilde{t}_0 \tilde{t}_1^*|$. In the linear regime, for weak measurement ($|\tilde{r}_0 \tilde{r}_1^* + \tilde{t}_0 \tilde{t}_1^*| \sim 1$), the dephasing rate $\Gamma_d$ can be expanded in terms of the change in the transmission probability, $\Delta T = |\tilde{r}_0|^2 - |\tilde{t}_1|^2$, and the change in the relative scattering phase $\Delta \zeta = arg(\tilde{t}_1/\tilde{r}_1) - arg(\tilde{t}_0/\tilde{r}_0)$ as follows,

$$\Gamma_d = \Gamma_T + \Gamma_\zeta,$$
$$\Gamma_T = eV (\Delta T)^2 / 8h T(1 - T), \quad \Gamma_\zeta = eV / 2h T(1 - T)(\Delta \zeta)^2,$$

where $T = (|\tilde{t}_1|^2 + |\tilde{t}_0|^2)/2$. The information of the state of qubit is reflected in the change of source-drain current. Therefore only the information of the qubit in the part of dephasing related to the change in current $\Gamma_T$ is utilized by the detector. One can find that the measurement rate of the detector $\Gamma_m$ is equal to $\Gamma_T$. However, the information lost in the part of dephasing $\Gamma_\zeta$ goes undetected. For a quantum limited detector it is necessary that
FIG. 2: (Color online) The renormalized measurement rate $\Gamma_m/\Gamma_0$ (blue line) and dephasing rate $\Gamma_d/\Gamma_0$ (red line) for $\Gamma_0 = eV/h$, $\Delta \theta = 0.05$, and (a) for symmetric interferometer ($R_a = R_b$=0.5), (b) for $R_a > R_b$ i.e ($R_a = 0.7, R_b = 0.5$), (c) for $R_a < R_b$ ($R_a = 0.5, R_b = 0.7$). The interferometer operates in quantum limit for $\Phi_0 = \pm \cos^{-1} \frac{\sqrt{R_a}}{\sqrt{R_b}}$, shown as black asterisks. (d) Same as (c) for small finite bias ($eV/E_c = 0.5$). Note that $\Gamma_w \approx 0.53$ for $R_a = 0.5$ and $R_b = 0.7$.

the unutilized information in phases should be eliminated, i.e. $\Delta \zeta = 0$. In a single QPC detector that obeys mirror reflection symmetry and time reflection symmetry the relative phase between transmission and reflection amplitude remains constant and change in relative phase $\Delta \zeta = 0$ [7, 18, 19].

From Eqs. (2) and (3) change in relative phases between transmission and reflection amplitude for Fabry-Perot interferometer is given by

$$\Delta \zeta = \arg \left\{ e^{i\Delta \theta} \frac{\sqrt{R_a} - \sqrt{R_b}e^{i\Phi_0}}{\sqrt{R_a} - \sqrt{R_b}e^{i(\Phi_0 + \Delta \theta)}} \right\}. \quad (8)$$

For $R_a = R_b$, from Eq. (8), we get $\Delta \zeta = \Delta \theta/2 + \pi$, for $0 > \Phi_0 > -\Delta \theta/2$, and $\Delta \zeta = \Delta \theta/2$, otherwise.

In the case when both QPCs in Fabry-Perot interferometer have same reflection probabilities ($R_a = R_b$), $\Delta \zeta$ always remains nonzero. Therefore there is always some information loss in the phases which goes undetected and detector cannot perform quantum limited measurement. Note that this behavior is different from the detection with resonant transmission at zero magnetic field [20], where the quantum-limited detection is possible only for symmetric double QPCs. In Fig. 2(a) we show measurement rate $\Gamma_m$ and dephasing rate $\Gamma_d$ calculated
from Eq. (7) for \( R_a = R_b \). We find that dephasing rate of the qubit is always higher than the measurement rate. In this case some information is always lost in scattering phases, which means quantum limited measurement is not possible. For higher values of \( R_a \) and \( R_b \), detector has higher sensitivity and the measurement is nearly quantum limited except at resonance. At resonance relative scattering phase \( \Delta \zeta \) faces an abrupt change by \( \pi \) which results maximum loss of information. Further because of the sensitivity of the detector is minimum at resonance, the measurement rate faces dip. For smaller values of \( R_a \) and \( R_b \) sensitivity of detector is smaller and more information is lost in scattering phases. From Eq. (8), change in relative scattering phases for \( R_a \neq R_b \) is given by

\[
\Delta \zeta = \Delta \theta/2 + \tan^{-1}\left[\frac{(R_a - R_b) \sin(\Delta \theta/2)}{(R_a + R_b) \cos(\Delta \theta/2) - 2\sqrt{R_aR_b} \cos(\Phi_0 + \Delta \theta/2)}\right]. \tag{9}
\]

In this case, we find the condition for quantum limited measurement \( \Delta \zeta = 0 \) simplifies to \( R_a/R_b = \cos^2(\Phi_0 + \Delta \theta/2)/\cos^2(\Delta \theta/2) \). For small value of \( \Delta \theta \), \( \cos^2(\Phi_0 + \Delta \theta/2)/\cos^2(\Delta \theta/2) \) is always less than unity except at resonance where quantum limited measurement is not possible. This clearly shows that in Fabry-Perot interferometer quantum limited measurement can only be possible if \( R_a < R_b \), and the value of \( \Phi \) for quantum limited measurement is given by \( \Phi_0 \approx \pm \cos^{-1} \sqrt{R_a/R_b} \). In Fig. 2(b)-(d), we show dephasing rate and measurement rate of qubit for Fabry-Perot interferometer having QPCs with different reflection probabilities \( (R_a \neq R_b) \). For \( R_a > R_b \), shown in Fig. 2(b), dephasing rate is always larger than the measurement rate. This shows that the detector has poor efficiency for such construction. On the other hand, in Fig. 2(c) for \( R_a < R_b \), there exist two points where the measurement rate is equal to the dephasing rate at \( \Phi_0 \approx \pm \cos^{-1} \sqrt{R_a/R_b} \). These points are symmetrically placed on both sides of resonance. For finite bias we average over the energy of the injected electrons. We find that at small bias \( eV/E_c = 0.5 \lesssim \Gamma_w \) (see Fig. 2(d)), our results are not modified much. The measurement rate is reduced very much at large biasing, \( eV/E_c \gg \Gamma_w \), and the quantum limited operation of the detector is not possible. Similarly, we also found that (not shown here) thermal broadening at high temperature \( kT/E_c \gg \Gamma_w \) reduces the sensitivity and the efficiency.

If we also include effect of environment on the qubit, the coupling to the environment relaxes the state of the qubit to its lower energy state. The condition when environment can produce dephasing and the measurement of relaxation rate has been discussed in detail in Ref. [21]. Coupling of the qubit with environment can reduce the efficiency of the detector...
only when environment also produces dephasing.

Our findings are unique because of the following facts. For a single QPC as a quantum limited charge detector, satisfaction of time reversal symmetry and mirror-reflection symmetry is essential \[7, 18, 19\]. Technically construction of such QPC may not be trivial, and the information loss is usually large for generic QPC. The dephasing rate is reported about 30 times larger than the measurement rate \[3, 22, 23\]. Here we report that in Fabry-Perot interferometer quantum limited measurement is possible only if the first QPC has smaller reflection than the second QPC, i.e. \( R_a < R_b \). Further, this Fabry-Perot construction provides much higher precision than a two-path (Mach-Zehnder) interferometer does.

Next, we briefly discuss improvement in sensitivity using quantum entanglement. For our purpose we consider spin entangled singlet pairs injected through identically biased input terminals \( \alpha \) and \( \gamma \). The state of injected electrons can be expressed as \( |\psi_{in}\rangle = \frac{1}{\sqrt{2}} (c_\alpha \uparrow c_\gamma \downarrow - c_\alpha \downarrow c_\gamma \uparrow) |F\rangle \), where \( \uparrow \) and \( \downarrow \) represent up and down spin of an electron. Methods for production and transport of spin entangled electron in solid-state structures have been discussed in Ref. \[24\]. For this input state electrons show bunching behavior and the current shot noise in the interferometer is enhanced \[25\]. Electron bunching, in turn, leads to improvement in sensitivity. For each up or down spin Fermi operators in this state scattering matrix is given by Eq. (1). The final state of the two electrons at drains \( \beta \) and \( \delta \) is given by (for the qubit charge \( i \))

\[
|\psi_i^f\rangle = \sqrt{2} \left[ \tilde{r}_i \tilde{t}_i^\dagger c_{\beta i}^\dagger c_{\delta i}^\dagger + \tilde{t}_i \tilde{r}_i^\dagger c_{\delta i}^\dagger c_{\beta i}^\dagger \right] + \frac{1}{2} (\tilde{t}_i \tilde{t}_i^\dagger + \tilde{r}_i \tilde{r}_i^\dagger) (c_{\beta i}^\dagger c_{\delta i}^\dagger + c_{\delta i}^\dagger c_{\beta i}^\dagger) |F\rangle,
\]

From this state one finds that the dephasing rate of the qubit as \[12\]

\[
\Gamma_d^s = \frac{eV}{h} \frac{(\Delta T)^2}{T(1-T)} + 4 \frac{eV}{h} T(1-T)(\Delta \zeta)^2
\]

The dephasing rate \( \Gamma_d^s \) is enhanced by a factor of eight compared to the case of injecting independent electrons at a single input (Eq. \[7\]). Taking into account biasing two inputs with spin degeneracy in Eq. \[11\], the charge sensitivity (per electron) of the singlet state is enhanced by a factor of two \[26\]. The average current at the output \( \beta \) or \( \delta \) is independent of the phase change \( \Delta \phi \). In order to detect the phase shift \( \Delta \phi \), it is necessary to measure shot noise or cross correlation at the output leads.
In conclusion, we have discussed high sensitivity quantum limited charge detection using electronic Fabry-Perot interferometer with edge states. We note that in the realization of electronic Mach-Zehnder interferometer significance of electron-electron interactions at nonlinear bias \cite{27} and temperature dependence on dephasing \cite{17} have been reported. Such studies in our scheme may also have experimental relevance.

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\[\text{(References)}\]

[1] S. A. Gurvitz, Phys. Rev. B 56, 15215 (1997); A. N. Korotkov, \textit{ibid.} 60, 5737 (1999); T. Gilad and S. A. Gurvitz, Phys. Rev. Lett. 97, 116806 (2006); A. A. Clerk, \textit{ibid.} 96, 056801 (2006); S. D. Barrett and T. M. Stace, \textit{ibid.} 96, 017405 (2006).

[2] E. Buks \textit{et al.}, Nature 391, 871 (1998); D. Sprinzak \textit{et al.}, Phys. Rev. Lett. 84, 5820 (2000).

[3] D.-I. Chang \textit{et al.}, Nature Physics 4, 205 (2008).

[4] D. Loss and D. P. DiVincenzo, Phys. Rev. A 57, 120 (1998); D. Vion \textit{et al.}, Science 296, 886 (2002); S. M. Clark, \textit{et al.}, Phys. Rev. Lett. 99, 040501 (2007).

[5] M. Field \textit{et al.}, Phys. Rev. Lett. 70, 1311 (1993); D. Sprinzak \textit{et al.}, \textit{ibid.} 88, 176805 (2002); E. Onac \textit{et al.}, \textit{ibid.} 96, 176601 (2006).

[6] M. H. Devoret and R. J. Schoelkopf, Nature 406, 1039 (2000); W. Lu, \textit{et al.}, Nature 423, 422 (2003).

[7] A. N. Korotkov and D. V. Averin, Phys. Rev. B 64, 165310 (2001).

[8] V. Giovannetti \textit{et al.}, Science 306, 1330 (2004).

[9] M. W. Keller, \textit{et al.}, Science 285, 1706 (1999).

[10] A. N. Jordan and M. Büttiker, Phys. Rev. Lett. 95, 220401 (2005).

[11] B. Yurke, Phys. Rev. Lett. 56, 1515 (1986).

[12] Y. Lee, G. L. Khym, and K. Kang, J. Phys.: Condens. Matter 20, 395212 (2008).

[13] Y. Ji, \textit{et al.}, Nature(London) 422, 415 (2003); I. Neder, \textit{et al.}, Nature(London) 448, 333 (2007).

[14] C. de C. Chamon \textit{et al.}, Phys. Rev. B 55, 2331 (1997); B. Rosenow \textit{et al.}, Phys. Rev. Lett. 100, 226803 (2008).

[15] B. J. van Wees \textit{et al.}, Phys. Rev. Lett. 62, 2523 (1989); F. E. Camino \textit{et al.}, Phys. Rev. B
76, 155305 (2007); E. V. Deviatov and A. Lorke, *ibid.* 77, 161302(R) (2008).

[16] D. V. Averin and E. V. Sukhorukov, Phys. Rev. Lett. 95, 126803 (2005).

[17] V. S.-W. Chung *et al.*, Phys. Rev. B 72, 125320 (2005).

[18] S. Pilgram and M. Büttiker, Phys. Rev. Lett. 89, 200401 (2002).

[19] A. A. Clerk *et al.*, Phys. Rev. B 67, 165324 (2003).

[20] G. L. Khym, Y. Lee, and K. Kang, J. Phys. Soc. Jpn. 75, 063707 (2006); Y. Lee, G. L. Khym, and K. Kang, J. Kor. Phys. Soc. 51, 2004 (2007).

[21] A. N. Korotkov, Phys. Rev. B 63, 085312 (2001); S. A. Gurvitz *et al.*, Phys. Rev. Lett. 91, 066801 (2003).

[22] K. Kang, Phys. Rev. Lett. 95, 206808 (2005); K. Kang and G. L. Khym, New. J. Phys. 9, 121 (2007).

[23] M. Avinun-Kalish *et al.*, Phys. Rev. Lett. 92, 156801 (2004).

[24] D. S. Saraga and D. Loss, Phys. Rev. Lett. 90, 166803 (2003); G. Burkard, J. Phys.: Condens. Matter 19, 233202 (2007).

[25] G. Burkard *et al.*, Phys. Rev. B 61, R16303 (2000).

[26] This is valid at low bias where the wave-packet size is larger than the distance between the two QPCs.

[27] E. V. Sukhorukov and V. V. Cheianov, Phys. Rev. Lett. 99, 156801 (2007).