Macrodynamics of users’ behavior in Information Retrieval

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Abstract
We present a method to geometrize massive data sets from search engines query logs. For this purpose, a macrodynamic-like quantitative model of the Information Retrieval (IR) process is developed, whose paradigm is inspired by basic constructions of Einstein’s general relativity theory in which all IR objects are uniformly placed in a common Room. The Room has a structure similar to Einsteinian spacetime, namely that of a smooth manifold. Documents and queries are treated as matter objects and sources of material fields. Relevance, the central notion of IR, becomes a dynamical issue controlled by both gravitation (or, more precisely, as the motion in a curved spacetime) and forces originating from the interactions of matter fields. The spatio-temporal description ascribes dynamics to any document or query, thus providing a uniform description for documents of both initially static and dynamical nature. Within the IR context, the techniques presented are based on two ideas. The first is the placement of all objects participating in IR into a common continuous space. The second idea is the ‘objectivization’ of the IR process; instead of expressing users’ wishes, we consider the overall IR as an objective physical process, representing the IR process in terms of motion in a given external-fields configuration. Various semantic environments are treated as various IR universes.

Introduction
The goal of this paper is to provide a framework in which to compare and introduce new Information Retrieval methods, rather than to propose a particular retrieval strategy. In order to enhance the capabilities of search engines, we need to know

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how well the engines satisfy the user requests. We try to answer this question by trying to understand the user or user group behavior.

New insights can be gained by mining search patterns, or as complementary approach, by visualizing the click streams in an intelligent way so that an expert can make sense of the structures he detects in the visualizations (visual data mining). Especially in the case of large data sets, a method of geometrization from search engines query logs is very much in demand. To manage huge data corpora, a proper theory for its description is required. Once the data are represented in the database, two different types of queries can be started, resulting in very different query processing stages. The interpretability of the returned results is different as well. In Data Retrieval, only exact matches to a query are considered, whereas in Information Retrieval, documents with a certain probability of relevance to the query are searched. Information Retrieval queries are, technically speaking, $k$-nearest-neighbor queries with similarities adopted for the specific information need.

In order to build an Information Retrieval theory, structured data are to be represented in the data retrieval context in some way. The data are treated as discrete by nature, but this does not imply that they have to be put in a discrete environment (consider, for example, an appropriate analogy in the theory of solid states or liquids: everyone knows that they are composed of discrete molecules). However, continuous geometrical methods proved high efficiency and predictive power. This, in turn, is the result of a crucial simplification of the model by disregarding its micro-details.

The vector model was the first considerable step in this direction. It introduced vector spaces (which are spanned on terms or their generalizations) and treated documents and queries uniformly as vectors in the same space. These spaces are still discrete.

Theoretical physics and, more generally, the physical world remain a source of inspiration for computer scientists [8]. The first really continuous model was suggested by C. J. Keith van Rijsbergen who introduced Hilbert spaces for this purpose (as in quantum mechanics). By nature, quantum mechanics is a genuine combination of continuous and discrete. In van Rijsbergen’s model, the relevance becomes the angle or the distance between appropriate continuous vectors. This is an effective illustration of the idea of a quantum-like description [6]: it has nothing to do with its roots in quantum physics, nonetheless, it efficiently uses its mathematical language and results.

We, however, should strike a new path. Two major fundamental theories in modern physics exist which are mutually exclusive to a great extent: Quantum Mechanics and General Relativity. The former mostly deals with the microworld, whereas the latter deals with cosmic distances. Our everyday intuition is in between and called classical physics; both theories admit the so-called classical limit.
Vector models are based on a quantum-mechanical, linear vector space paradigm which plays the rôle of a Room to store data and pose queries.

Our basic idea is the following. We consider a smooth continuous manifold \( B \), and call it Information Retrieval space. Note that \( B \) is neither a document space, nor a query space; instead, it has a more fundamental and unstructured nature. It may be thought of as the set of all transmitted bits. The elements of \( B \) are all the same; they have no structure. This is a complete analogy to the points of spacetime, or the configuration space in physical theories.

For the time being, let us return to van Rijsbergen’s geometrical model. The documents in his approach are vectors, but, if we look at the model in more detail, we see that if we multiply a vector by a number, we do not get a new document. As in quantum mechanics, only unit vectors are of physical (operationalistic) meaning. These vectors, in turn, form a smooth manifold — the unit sphere in the appropriate vector space. Therefore, even in van Rijsbergen’s approach, curved spaces are already implicitly used as document spaces. In quantum mechanics, quantum dynamics have been successfully described as classical Lagrangian mechanics on unit vectors [1]. It should be mentioned that all this has no direct relevance to our approach, but shows that what we suggest is a natural development of standard, conventional approaches.

To be more specific, we treat \( B \) as an analogon to physical spacetime. We place both documents and queries into \( B \), providing them with both temporal and spatial dimensions. As a consequence, the idea that a document may change in time is automatically incorporated in the theory. The second consequence is that the static documents, and those generated on-the-fly, are described as entities of exactly the same nature, differing only in ‘shape’ in our IR spacetime \( B \).

1 Information retrieval as dynamics

In this section we develop one of the idea highlighted in the beginning, namely the objectification of the IR process. Information Retrieval is commonly treated as an analogy to data search: there is a user with a (more or less) definite goal wishing to gain this or that knowledge from the retrieved information.

We suggest an alternative approach: When we are speaking about a huge community of users, we no longer treat their behavior as intelligent. This contrasts with the viewpoint of ‘intelligent crowd behavior’, but the community of users in our setting is a large collection of autonomous individuals rather than a crowd, and we dwell on their average behavior. This gives us the right to shift the focus of our research from the task of finding a good way to satisfy users’ requests to the task of analyzing typical user behavior. From this perspective, a typical user of a search
environment is not more intelligent than an elementary particle or a molecule, and we may apply the good old principle of least action, which stems from the work of Fermat and Euler. They proved its efficiency by providing simple and strongly predictive models. The power of the least action principle is that we do not have to make any difference between users and resources—we are free to include anything we like in a uniform way to describe the dynamics.

**IR environment.** This notion is informal; we need it to link the mathematical model with practical situations. Within a mathematical model, the IR environment is specified by the IR space $\mathcal{B}$, a collection of effective fields on it, and the Lagrangian (which is a concrete expression for the action). As soon as all this is specified, the IR process itself is represented by trajectories in $\mathcal{B}$ which show the behavior of users.

The standard IR paradigm treats the IR process as a search. That is, the initial condition is posing a query, then, according to this or that formula, the indexed documents are ranked. Subsequently, the results are delivered to the user according to the ranking. But, typically, the user never makes a single query and the process is usually progressive. After parsing the results and considering their relevance, the user poses further queries, repeating the process iteratively.

Our suggestion is to get above these particularities. We replace the notion of relevance feedback by that of least action\(^1\). This can be drawn as follows

![Figure 1. A point on IR surface together with users' intention vector.](image)

and interpreted as geodesic motion. The dynamics replace the notion of relevance, and the displacement of a user from point $t_1$ to point $t_2$ is what replaces relevance feedback making it, in a sense, a relevance feedforward.

**Users.** We should not treat users literally as persons. In our setting, a user is just an entity which pursues a particular goal. This means that a single physical

\(^1\)A similar approach appears in the ostensive model of information retrieval \([2]\). Within this model, there is an implicit unobservable entity—state of knowledge, or awareness of a user—and the behavior of the user is interpreted as the change of the user’s knowledge. The user acquires knowledge after performing a certain action.
A person can represent a number of simultaneously acting users and, conversely, there may be a group of people whose retrieval behavior looks like that of a single user from the outside. Later we shall examine the problem of detecting users and user clickstreams in more detail.

2 An outline of differential geometry.

The aim of this section is to present the basic geometric ingredients for our model and introducing the notation. We start by presenting the basic notions of a smooth manifold, followed by the Riemannian and pseudo-Riemannian metrics and geodesics.

A smooth manifold $A$ is an analogon to a curved surface with the difference that it is considered *per se*, not merged into any outer space. Metric in differential geometry has a double meaning, a global and a local one. The global metric is a distance function ascribed to any pair of points and satisfying the triangle inequality. Locally, a metric is a nondegenerate quadratic form $g(u, v)$ which defines the scalar product for any pair of tangent vectors. If the quadratic form $g$ is positive-definite, the appropriate metric is called *Riemannian*. However, the metric is called *pseudo-Riemannian* when squared lengths of vectors maybe both positive and negative (and zero as well). The latter is the mathematical ground for special relativity as spacetime is assumed to define such a metric; that is, time is treated as complex-valued distance.

A geodesic line is an analogon to a straight line on a plane. This is the locally shortest curve, shortest with respect to the defined metric on the manifold $A$. For instance, circumstances are geodesics on a sphere. The explicit formula for geodesics is as follows: Given a metric $g$, fix a coordinate system, then $g$ takes matrix form $g = g_{ik}$, each matrix entry is a function defined on the manifold $A$. Combine their derivatives, introducing the coefficients:

$$
\Gamma^j_{kl} = \frac{\partial g_{lk}}{\partial x^j} + \frac{\partial g_{jk}}{\partial x^l} - \frac{\partial g_{kl}}{\partial x^j}.
$$

(1)

When the coefficients are calculated, the equation of geodesic motion $x(t)$ along the manifold $A$ is the following second-order differential equation:

$$
\ddot{x}^j = \Gamma^j_{kl} \dot{x}^k \dot{x}^l.
$$

(2)

Here, the dot above indicates the derivative over a parameter counting the points of the trajectory $\dot{x}^j = \partial x^j / \partial t$, and the summation over repeated indices is assumed. This means the expression $\Gamma^j_{kl} \dot{x}^k \dot{x}^l$ is in fact $\sum_{k,l} \Gamma^j_{kl} \dot{x}^k \dot{x}^l$. 
In a local sense the metric is connected to the global one as follows. Given a curve \( x(t) \), its length is given by the integral
\[
\int_0^T g(\dot{x}(t), \dot{x}(t)) \, dt = \int_0^T g_{jk}(t) \dot{x}^j(t) \dot{x}^k(t) \, dt
\] (3)
where (i) the summation is carried over repeated indices and (ii) \( g_{ik} = g_{ik}(a) \) are functions, defined at each point \( a \in A \).

**Dynamics.** As stated above, we replace the study of users’ needs with the study of users’ behaviors in a way analogous to the study of a deterministic physical processes. For that, we introduce the notion of action as a function which evaluates every curve (the basic example of action is the length of the curve). Given an action, we then use the well-known fundamental physical principle of least action: Among all possible trajectories it happens that (only) those yielding the minimum to the action are realized. In our approach, all the variety of evaluating relevance is assumed to be hidden in the calculation of the action.

**Describing manifolds.** How can we generally describe infinite, continuous objects? This immediately brings us to the question of how we can describe a function which, in turn, has commonly accepted answers. We treat certain sets of functions as elementary and construct new functions from them using elementary operations. What is elementary? This is a matter of the particular setup of the problem to be defined individually.

In our case we are going to deal with regular geometrical objects and simply treat smooth manifolds as surfaces in Euclidean space, defined by appropriate smooth functions. In particular, when we reconstruct smooth surfaces from discrete data, we use standard approximations such as the mean square method with respect to Euclidean distance.

### 3 Building IR spaces

We begin by drawing an analogy between IR spaces and differential geometry, in the context of smooth manifolds. When we just say ‘given a manifold’ this still means nothing unless we specify it. We have already presented a method of building IR spaces by representing them as graphs of smooth functions. Another way to represent IR spaces is to specify a manifold by describing the set of all smooth functions on it. (These sets are different. For instance, any such function on a circle attains its maximal value, which is no longer the case for a straight line.) An algebra is a linear space with an extra operation of multiplication. One
can easily observe that, given a space, the set of all functions on it is closed under pointwise addition and multiplication. That is, the set of functions is a linear space equipped with an extra operation of multiplication, such spaces are called algebras.

### 3.1 Dimensionality reduction

A dimension can be defined as one of a number of parameters needed to describe an object. This may sound abstract, but there are parallels with our everyday experience. A cake recipe, for example, may be defined by the amount of the various ingredients in grams. If one writes down the amounts of flour, sugar, butter, eggs, and baking powder in the form (200, 100, 80, 20, and 3), then this representation contains the most important information. So, there is essentially nothing complicated with five dimensions from a common sense point-of-view (one may even use this example to explain the vector space model for IR).

Mathematical methods of dimensionality reduction can be used for feature transformation. Feature selection, for example, focuses on uncovering subsets of variables predictive of a prespecified target variable. In our context, dimensionality reduction comes into consideration when we want to control the number of parameters for the results of visualization.

The dimension is one of the main properties of linear spaces; it may finite or infinite. In the case of an algebra of functions on a manifold, the dimension is infinite. What does that mean? Suppose we would like to specify a straight line. We might consider the linear space of polynomials, treated not as functions, but defined formally, as, say, formal series. The dimension of this space is obviously infinite as nobody limits the degree of polynomials. In the meantime we know that the space, on which these polynomials are defined, is just a straight line, a one-dimensional object! And it is completely a matter of our choice which of the descriptions of the straight line we prefer: either functional and infinite-dimensional, or geometrical and one-dimensional.

After that, we can present to our basic suggestion. By analogy with algebras we see that we may define the IR space in terms—thus making it huge-dimensional—or, rather, observe some ‘massive regularities’ and define the IR space geometrically, as an abstract manifold B. The terms will then become functions on B, exactly as in differential geometry.

Dimensionality reduction is one of the key features of our approach. This is reason why we do not treat terms as basic objects: the appropriate vector space would have an immense dimension. What we suggest is a kind of holographic approach. Its closest analogy in image processing is the JPEG format. If we draw an analogy with image processing, terms will be a counterpart to pixels, vector
models are then similar to the BMP format; we parameterize the search space by holistic patterns.

3.2 IR Space from discrete skeleton

In this section we will dwell on the first basic principle of our techniques: merging everything—users, queries and data—into a single space.

Return to equation 2. It is of second order, that is, in order to specify its particular solution, we must specify the initial conditions which are the initial position \( x(0) \) and the initial ‘intention’ \( \dot{x}(0) \). A typical user clickstream will be represented as a line, whereby the points of the line \( x(t) \) are associated with the state of knowledge the user has gained from interpreting the retrieved information until that point.

Next, let us specify what are we going to visualize. The object of our inquiry is the IR semantic environment, which consists of a typical community of users with specific needs, using certain information or knowledge retrieval techniques. In fact, this requirement is not crucial, we may take a random collection of users, and even carry out its visualization, but the point is that this visualization will remain a thing in itself. If, conversely, we determine some common features of the team of users, we may afterwards vary the parameters of the problem and the obtained visualization may give us an immediate tracking of the results. However, at present, we may not put \textit{a priori} restrictions on the contents of the environment.

We study the behavior of an IR environment by analyzing the logs of user querying activities. Let us first produce the ‘flesh’ of IR space. Its elementary constituent, a point, is a click: a query/HTTP request together with a body response (HTML page accessed by a result link – this way we do not take into account broken links).

**Step 1. Extract the clickstreams.** A clickstream is a progressive, ‘continuous’ sequence of user queries and responses which have a definite start and end. The end of a clickstream is marked by a breach in the continuity of the requests. What does ‘continuous’ mean? To specify it we need a distance function between points. This distance is beyond our exploration in this paper, but we may use one from, say, vector space model. The result of Step 1 is a collection of clickstreams, an ordered sequences of points:
Step 2. Creating the discrete pre-space B. At this step, our input is a collection of clickstreams. Their points are ordered and we know the distances between them. We assume that we use certain distance between points of the threads, and therefore, any particular relevance formula can be applied. There is a well-defined distance between the neighbor points of each thread. That means, beside order, clickstreams acquire metric:

Now we make a layered structure. We start with points with label 0 (this will be a starting layer), and, using the same distance function, place them as points on a metric space. Then we pass to label 1, and form the same discrete metric space, and so on. As a result, we have a sequence of layers labeled 0, 1, . . . , forming altogether a discrete metric space:
Step 3. Geometrization and dimensionality reduction. At this step we continue binding the threads and complete the skeleton with the ‘bones’ linking nearest neighbors, now irrespective of the thread, to which they belong.

we choose, or set up by force, the dimension $n + 1$ of our IR space. (Since it has a spatiotemporal structure, we reserve one dimension for the temporal parameter and $n$ for ‘spatial’.) Once $n$ is chosen, we project each layer on an $n$-dimensional space. Technically, this can be done as follows: when the dimension $n$ is fixed, we form cells of $n + 1$ neighboring points for each point and then treat each such collection of points as a simplex (simplex is a generalization of a triangle, pyramid and so on). So, we form a foliation, labeled 1, 2, . . . , together with threads, which we retained from Step 1. Finally, we treat the resulting space as IR space.
4 Possible applications

Here we give a brief overview of the potential benefits of the techniques we introduce.

**IR spaces as comparison tools.** Now, how can our geometrical picture serve as a comparison tool between different IR environments? Suppose we have a kind of contest. There are, say, two search environments and two similar teams of users with the same tasks and wishes. After some time we may represent the results of the contest for each environment as a geometrical picture, i.e., a manifold and a collection of users’ trajectories on it. Since the teams of users are similar, we may put them into a correspondence and thus establish a mapping between the two manifolds.

This comparison can also be viewed from a physical perspective. Suppose we have carried out such an experiment with a team of observers. Then, we change the circumstances and a new set of relevant documents emerges and is indexed. As a consequence, the behavior of users will also have changed. This result has a direct physical analogy: Suppose we have a cloud of test particles and we record their trajectories. Then, a massive body emerges in the neighborhood. As a result, the trajectories will be biased.

In general, this representation is not a way to judge which IR environment is better; rather, it is a way to put them together and visually compare them, thus making it an instance of visual data mining in the sense of visual pattern recognition or...
the like.

**Geometrodynamics and relevance feedback.** Our visualization method can also be used as a relevance feedback tool. We may use it to modify the relevance-distance function. The idea behind is the following: Suppose we look at a typical picture of users’ behavior and discover that there are sharp peaks on the IR surface. What does that mean? Users typically make big jumps:

![Figure 7. High peaks on IR surface.](image)

From this we may infer that our distance is not adequately calculated and this will force us to correct the ranking formula.

Here, our goal is to make the surface more smooth and less lumpy, according to the requirement to make the ranking function more consistent with the users requests and their evaluations of the results the retrieval. It works as depicted in Figures 7, 8: the smoother is the surface, the better the IR is organized and the simpler it is for users to achieve their goals.
5 Related works.

_If I have seen further it is only by standing on the shoulders of Giants._ - I. Newton.

In this section we summarize and compare some previous works and point out their relevance to our model.

Keith van Rijsbergen [7] has already been mentioned in the introduction. Iain Campbell [2] developed an ostensive model of IR. Duality issues were studied by R. Rousseau and Leo Egghe [3].

In this paper the duality relation between documents and queries, and between indexing and retrieval are studied. This is an important step towards merging all of these objects in a unique space. Recently, a gravitation-based model (GBM) of IR was proposed, where relevance is treated as Newtonian gravitation between a query and a document. This provides not only a holistic view, but also a mathematical background to deduce particular ranking formulas. In particular, the famous Okapi BM25 formula

\[
\text{score}(D, Q) = \sum_{i=1}^{n} \text{IDF}(q_i) \cdot \frac{f(q_i, D) \cdot (k_1 + 1)}{f(q_i, D) + k_1 \cdot \left(1 - b + b \cdot \frac{|D|}{\text{avgdl}}\right)}
\]

is naturally derived within their approach. GBM represents documents as cylindrical objects and considers only attraction between documents and point-like queries, according to the Newtonian gravity law. These are, however, advantages over GBM and argumentations in favor of our model:

- Right from the beginning one sees that the Newtonian formula for gravity is too rigid and cannot properly capture the subtleties of relevance.

- In order to adjust the function \( f \) in [4] properly, the authors of GBM suggest replacing the Newtonian quadratic law with a different one, varying the power of the distance. This immediately destroys the beauty, simplicity, and self-consistence of the Newtonian world.
• Our suggestion is different. Instead of modifying the law of gravity, we modify the geometry of the space, but leave the laws intact—exactly as it is done in Einstein’s General Relativity.

An introduction to the possible applications, the topics of interpreting public search queries can be found in [9]; [4] provide a review of web searching studies, and [5] address the difficulties when processing web search clickstreams.

Conclusion

We presented a method to geometrize massive data sets from search engines query logs. For this purpose, a cosmological-like quantitative model of the Information Retrieval (IR) process has been developed, where documents and queries are treated as matter objects and sources of material fields.

One of the peculiarities of our approach is that we practically do not use and do not consider terms as basic entities. We do that deliberately in order to simplify the construction in some sense.

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