Scattering polarization of hydrogen lines from electric-induced atomic alignment

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Abstract

We consider a gas of hydrogen atoms illuminated by a broadband unpolarized radiation with zero anisotropy. In the absence of external fields, atomic J-levels are thus isotropically populated. While this condition persists in the presence of a magnetic field, we show instead that electric fields can induce the alignment of these levels. We also show that this electric alignment cannot occur for a two-term model of hydrogen (e.g. if only the Lyα transition is excited), or if the level populations are distributed according to Boltzmann’s law.

1. Introduction

Atomic polarization is the property that describes the condition of population imbalances and quantum coherence between atomic levels [1, 2]. Population imbalances in a J-level occur when its Zeeman sublevels, characterized by the azimuthal quantum number \( M = -J, \ldots, J \), are differently populated. Quantum coherence (or interference) occurs instead when definite phase relations exist between wavefunctions representing two distinct Zeeman sublevels.

An atomic J-level is isotropically (or naturally) populated if its Zeeman sublevels have identical populations and the quantum coherences among them are completely relaxed.

The study of atomic polarization has played a fundamental role in the quantum mechanical understanding of the photon–atom interaction [1, 3–7], and in devising techniques for controlling and manipulating atoms through the radiation (e.g. laser cooling [8, 9]). Because atomic polarization is affected by physical conditions of the atomic gas (e.g. illumination condition, external field, gas density and collisions), and manifests itself in the polarization of the light emitted by the atoms, its study is fundamental also for the diagnostics of plasmas, both in the laboratory and in space [10–12].

In order to represent atomic polarization, it is convenient adopting the density-matrix formalism. The atomic density matrix is simply proportional to the identity matrix of rank \( 2J + 1 \). Hence, for an isotropically populated J-level, the density matrix is simply proportional to the identity matrix of rank \( 2J + 1 \).

A J-level is further said to be oriented if \( \rho_J(-M', -M) \neq \rho_J(M, M) \), and aligned if sublevels corresponding to different values of \( |M| \) have different populations. In general, an atomic level (and hence the atomic system) can be both oriented and aligned. The concept of the orientation and alignment of an atomic level will be clarified further in the next section.

Typically, there are two mechanisms considered in the literature, by which atomic polarization can be created: (1) anisotropic and/or polarized excitation by radiation or particles, which selectively populates the different magnetic sublevels in the transition; and (2) in the presence of external fields (electric and/or magnetic), a transition is excited by radiation that has spectral structure across the frequency range of level splitting [1, 13–19]. In this paper we consider instead a different mechanism which is the alignment of hydrogen levels in the presence of an electric field, when the atoms are illuminated with a broadband, unpolarized radiation with zero anisotropy (\( \int \frac{d\Omega}{4\pi} (3 \cos^2 \vartheta - 1) I(\Omega) \equiv 0 \), where \( I(\Omega) \) is the radiation intensity, and \( \Omega \equiv (\vartheta, \varphi) \) the propagation vector). This phenomenon of electric alignment has important implications for the diagnostics of electric and magnetic fields.
fields in hydrogen plasmas, because it affects the scattering polarization that is produced by anisotropic irradiation and modified by the presence of external fields. Interestingly, there is no magnetic counterpart of this mechanism. However, if a magnetic field is present simultaneously with the electric field, electric alignment can be converted into atomic orientation via the alignment-to-orientation (A-O) conversion mechanism [20], resulting in broadband circular polarization (BCP) of the scattered radiation.

It is important to remark that electric alignment does not violate any conservation or symmetry principle. In particular, this cannot occur if the atom is illuminated by Planckian radiation, or if the electric fields are isotropically distributed (e.g., Holtsmark-type fields [21]) and no magnetic fields are present.

2. The physics of electric alignment

We study the mechanism by which an electric field can induce atomic alignment in an ensemble of hydrogen atoms that are initially in an isotropically populated state because of the particular illumination conditions. This process bears resemblance with the phenomenon of polarized emission of Lyα radiation by electric quenching of hydrogen atoms prepared in the metastable state 2S1/2 [22, 23], since both phenomena are determined by the electric mixing of atomic states of different parities. However, the case studied in this paper has a much broader impact, because the electric-induced polarized emission of hydrogen lines (not restricted to Lyα) is attained in a stationary regime of radiative excitation of the hydrogen atoms (rather than as a transient phenomenon like in the electric quenching of metastable hydrogen atoms), under conditions that are very common in hydrogen plasmas, both in the laboratory and in astrophysical objects. In addition, from our analysis we derive a condition necessary for generating electric alignment, which applies to any hydrogen line, and this to our knowledge has not been clarified in any previous work.

Hydrogen atoms in an isotropically populated state are completely described by the lowest multipole order (K = 0) of the set of irreducible tensor components of the atomic density matrix (also known as statistical tensors),

\[ n^K_{Q} (LJ, L'J') = \sum_{MM'} (-1)^{M-M'} \sqrt{2K+1} \alpha^{J' -M' -Q} K_{J'MM'} \]

where \( n \) is the principal quantum number of the level of interest, and \( L, S, J \) are the orbital, spin, and total angular momentum quantum numbers, respectively. In fact, the quantity \( N_{L}(J) = \sqrt{2J+1} n^J_{O}(LJ, LJ) \) represents the total population of the level \( J \) in the atomic term \( nL \). A direct evaluation of equation (1) shows that our definition of alignment involves that the \( K = 1 \) multipole of the statistical tensor is not vanishing, whereas atomic alignment is a non-zero \( K = 2 \) multipole. Sometimes one speaks more generally of alignment (alignment) of an atomic system when at least one of the odd (even) multipoles of the statistical tensor is not vanishing [24].

The statistical equilibrium (SE) of hydrogen atoms subject to external electric and magnetic fields is governed by the following evolution equation for the atomic density operator, \( \rho \):

\[ \dot{\rho} = (i\hbar)^{-1} [H_0 + H_E + H_B, \rho] - (\Gamma \rho + \rho \Gamma) + \mathcal{T} \rho. \]  

A derivation of equation (2) within the formalism of the statistical tensors is given in [25]. In equation (2), \( H_0 \) is the field-free atomic Hamiltonian, \( H_E = -e_0 \mathbf{r} \cdot \mathbf{E} \) and \( H_B = \mu_0 \mathbf{B} \cdot (J + S) \) are the usual electric and magnetic Hamiltonians, and finally \( \Gamma \) and \( \mathcal{T} \) are two radiation operators that are responsible, respectively, for radiation damping and population transfer. In this work, we neglect the role of collisions. In the absence of external fields, and in the presence of a broadband unpolarized radiation with zero anisotropy, equation (2) reduces to the well-known rate equations for the atomic populations. Assuming broadband illumination of the atoms implies that the pumping radiation has no spectral structure across the frequency range of any given hydrogen line. It is well known [26] that under this hypothesis the scattering process can be described as the succession of the two incoherent mechanisms of radiation absorption and re-emission, even in the absence of collisions. Such flat-spectrum approximation is indeed verified for most astrophysical problems—as well as in many applications to laboratory plasmas, except in the case of laser excitation—and this comes as a fundamental assumption in deriving results for this work.

By studying explicit expressions of the magnetic and electric contributions to equation (2) (see equations (17b) and (17c) in [25]) one can see how an external field is able in transforming atomic populations, \( n^S_{Q}(LJ, LJ) \), into quantum coherences, under the form of non-diagonal orientation components, \( n^S_{O}(LJ, L'J') \). In the case of a magnetic field, these orientation components can only be generated between \( J \) levels belonging to the same \( nL \) term (because the magnetic Hamiltonian is diagonal with respect to \( L \)). On the other hand, since by the assumption the incident radiation is spectrally flat across the frequency range of any \( nL-n'L' \) transition, the population of these levels always satisfy the condition \( N_{L}(J) = N_{L}(J') = (2J+1)(2J'+1) \), characteristic of thermodynamic equilibrium (TE). In such case, one can show that the total magnetic contribution to atomic orientation vanishes. Hence, within the flat-spectrum approximation, a magnetic field cannot, by itself, generate polarization in an atomic system that is initially in an isotropically populated state. This is a well-known fact, which is the basis of the application of Hanle effect to the magnetic diagnostics of plasmas [11, 12].

In the presence of an electric field instead, the electric Hamiltonian mixes levels with \( \Delta L = 1 \) and it is found that population imbalances (i.e. out of TE) between such interfering levels can effectively be transformed into atomic polarization. This process is inhibited only under strict TE conditions for
where we indicated with $n_{\alpha L}/n_{\alpha L'} = (2L + 1)/(2L' + 1)$. However, in the absence of collisions, this can only happen if the illumination is Planckian ($I(\Omega) = B_\gamma$; in such case the TE distribution of level populations applies to the entire atomic model). Otherwise, we must expect a contribution from electric alignment to the polarized radiation scattered by a gas of hydrogen atoms, under a wide range of physical conditions that are commonly found in laboratory and astrophysical plasmas.

In order to quantify this mechanism, we consider a simplified model of the hydrogen atom, consisting of the first two Bohr levels, plus the 3P term (see figure 1). For simplicity, we neglect the fine structure of hydrogen (both spin–orbit interaction and Lamb shift), and assume that the simplified model of the hydrogen atom, consisting of the Bohr levels up to $n = 4$ with fine structure. After imposing the stationary condition $\dot{n} = 0$, equation (2) yields the following linear system

\begin{equation}
(R_{12} + R_{13})N_{1S} - R_{21}N_{2P} - R_{31}N_{3P} = 0, \tag{3a}
\end{equation}

\begin{equation}
R_{23}N_{2S} - R_{12}N_{1P} - 6\omega_{E}c_{2S,2P} = 0, \tag{3b}
\end{equation}

\begin{equation}
R_{12}N_{1S} - R_{31}N_{2P} - 6\omega_{E}c_{2S,2P} = 0, \tag{3c}
\end{equation}

\begin{equation}
R_{13}N_{1S} + R_{23}N_{2S} - (R_{31} + R_{32})N_{3P} = 0, \tag{3d}
\end{equation}

\begin{equation}
R_{31}\alpha_{2P} - 2\sqrt{6}\omega_{E}c_{2S,2P} = 0, \tag{3e}
\end{equation}

\begin{equation}
3\omega_{E}N_{2S} - \omega_{E}N_{2P} + \sqrt{6}\omega_{E}\alpha_{2P} + \frac{1}{2}(R_{21} + R_{21})c_{2S,2P} = 0, \tag{3f}
\end{equation}

where we indicated with $R_{\alpha\beta}$ the radiative rate for the transition from Bohr’s level $n$ to $n'$, and with $\omega_{E} = \omega_{0}E/h$ the angular frequency associated with the electric field strength. When determining completely the solution of this linear system, we must impose the conservation of the total atomic population, $N_{1S} + N_{2S} + N_{2P} + N_{3P} = 1$. Solving the linear system algebraically, we find

\begin{equation}
\alpha_{2P} = \frac{2\sqrt{\epsilon}}{R_{21}}c_{2S,2P}\omega_{E}, \tag{4a}
\end{equation}

\begin{equation}
c_{2S,2P} = \frac{2R_{21}\omega_{E}}{R_{21}(R_{21} + R_{23}) + 24\omega_{E}^{2}}(N_{2P} - 3N_{2S}). \tag{4b}
\end{equation}

These relations show that the alignment of the 2P term is a direct consequence of the quantum coherence between the 2S and 2P terms due to electric mixing of the same terms, as expected. However, under TE conditions, electric alignment cannot be generated because $N_{2P}/N_{2S} = 3$. In the limit of very strong fields, the quantum coherence between the 2S and 2P terms vanishes, whereas the atomic alignment of the 2P term reaches the asymptotic value of

\begin{equation}
\alpha_{2P}(\omega_{E} \rightarrow \infty) = \frac{1}{3}(N_{2P} - 3N_{2S}), \tag{5}
\end{equation}

which again is zero under TE conditions.

For comparison, we calculated numerically the alignment of the 2P term for a model of hydrogen that includes all Bohr’s levels up to $n = 4$ with fine structure (the dimension of the SE system is $1416 \times 1416$ in this case). This quantity depends on the alignment of the fine-structured levels according to the formula

\begin{equation}
n\rho_{Q}^{K}(L, L') = \sum_{J, J'}(-1)^{K + L + J + S}\sqrt{(2J + 1)(2J' + 1)}
\end{equation}

\begin{equation}
\times \begin{vmatrix}
L & L' & K \\
J & J' & S
\end{vmatrix} \rho_{Q}^{K}(LJ, L'J'). \tag{6}
\end{equation}

which gives in our case

\begin{equation}
\alpha_{2P} \equiv \rho_{Q}^{2}(1, 1) = \frac{1}{\sqrt{2}}\left[\rho_{Q}^{2}(1^{\frac{1}{2}}, 1^{\frac{1}{2}})
- \rho_{Q}^{2}(1^{\frac{1}{2}}, 1^{\frac{3}{2}}) + \rho_{Q}^{2}(1^{\frac{3}{2}}, 1^{\frac{3}{2}}) \right] \tag{7}
\end{equation}

(for simplicity of notation, we suppressed the configuration superscripts). In figure 2, we show the alignment of the 2P term normalized by the quantity $(N_{2P} - 3N_{2S})$, as a function of
In fact, one can show that the quantity $N_{\ell\ell'}$ below the minimal set of levels highlighted in figure 1. the 2S and 2P terms vanish identically restricting the atomic alignment of the 2P term and the coherence between the fine structure in the model of equations (3f) of hydrogen. The disagreement between the two cases for the electric alignment of the Lyα radiation is found to be completely radiatively atomic alignment of hydrogen levels, as expected, because the coherences $\alpha_3^J(J, L', \ell')$ are zero in this case. In contrast, the alignment reaches asymptotically finite values for very large electric fields, in agreement with equation (5). In the case of isotropic electric fields (thick solid curve), the BLP generally does not vanish, because different field directions in the isotropic distribution are not equivalent when the spherical symmetry is already broken by the presence of the magnetic field. However, it vanishes for very strong electric fields, as expected for symmetry reasons.

Figure 4 shows the BCP of Lyα observed along the quantization axis, for the same distributions of the electric and magnetic fields as those of figure 3. We see that the presence of the magnetic field realizes a conversion of the electric alignment of the hydrogen levels into atomic orientation via the A-O mechanism [20]. This orientation is responsible for the appearance of BCP in the scattered radiation. In contrast, the electric alignment generated in the presence of only electric fields cannot be further converted into BCP-producing atomic orientation, so all curves of figure 4 would collapse to zero in the absence of the magnetic field.

3. Examples

We now consider some examples of broadband polarization of the scattered radiation of hydrogen lines resulting from electric alignment. All results in this section are calculated assuming the same illumination conditions and hydrogen model (complete up to $n = 4$ with fine structure) adopted for figure 2.

Figure 3 shows an example of the broadband linear polarization (BLP) of the Lyα, scattered radiation, due to the electric field strength. The illumination conditions are such that $I(\Omega) = (1/2)B_T=20000\text{K}$ with zero anisotropy. The solid line shows the alignment calculated with equation (7), whereas the dashed line shows the alignment computed according to equations (4a) and (4b). We see that equation (5) gives the correct strong-field limit also in the case of a realistic model of hydrogen. The disagreement between the two cases for intermediate field strengths is mainly due to the omission of the fine structure in the model of equations (3a)–(3f).

From equations (4a) and (4b), we can also conclude that both the alignment of the 2P term and the coherence between the 2S and 2P terms vanish identically restricting the atomic model below the minimal set of levels highlighted in figure 1. In fact, one can show that the quantity $(N_{2\ell} - 3N_{2S})$ contains a factor $R_{12}R_{21}R_{31} - 3R_{11}R_{22}R_{31}$, which is zero if we eliminate the 3P term. It follows that electric alignment cannot be produced in a two-term model of hydrogen, for example, when the atoms are illuminated only by Lyα radiation.\footnote{On the other hand, it is sufficient to pump the model of equations (3a)–(3f) just with Lyα radiation for inducing electric alignment in the 2P term. In this case, we reproduce the results for the electric quenching of metastable hydrogen atoms considered previously in the literature [22, 23]. Specifically, for very large field strengths the Lyα radiation is found to be completely polarized along the applied electric field ($P_{\text{max}} = -100\%$), whereas for vanishing field strengths (taking into account the fine structure of hydrogen) we find $P_{\text{max}} \approx E_{32.88\%}$ in agreement with the results of [23] for the case of ‘diabatic’ quenching considered.}

4. Conclusions

In this paper we demonstrated that it is possible to generate atomic alignment of hydrogen levels radiatively through a mechanism that does not require anisotropic illumination, polarization or spectrally modulated radiation. The alignment is instead generated by an electric field that mixes atomic levels having imbalanced populations (out of TE).

In practice, such electric alignment always contribute alongside other competing polarizing mechanisms (for example, anisotropic excitation), and the relative importance...
of this effect depends on the physical conditions of the plasma and on the particular hydrogen line. A recent investigation of the polarization effects of an isotropic distribution of electric fields in magnetized plasmas, where all these competing mechanisms are taken into account, is presented in [27] for the case of Ly\textsuperscript{α} and H\textalpha lines.

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