Preliminary studies of the effect of thinning techniques over muon production profiles

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Abstract.
In the context of air shower simulations, thinning techniques are employed to reduce computational time and storage requirements. These techniques are tailored to preserve locally mean quantities during shower development, such as the average number of particles in a given atmosphere layer, and to not induce systematic shifts in shower observables, such as the depth of shower maximum. In this work we investigate thinning effects on the determination of the depth in which the shower has a maximum muon production ($X_{\mu_{\text{sim}} \text{max}}$). We show preliminary results in which the thinning factor and maximum thinning weight might influence the determination of $X_{\mu_{\text{sim}} \text{max}}$.

1. Introduction
Typical quantities of interest regarding the study of ultra-high energetic cosmic rays (UHECR) are their energy spectra, mass composition and distribution of arrival directions. Particle interaction properties are also studied in specific analysis. Regarding the mass composition and interaction properties studies, the most commonly used observable is the depth of shower maximum, $X_{\text{max}}$, defined as the atmospheric slant depth where the number of electromagnetic particles in the shower is maximum. The underlying theory that relates $X_{\text{max}}$ to the primary particle composition and particle interaction properties was developed by Heitler in the 1940s [1, 2].

The Pierre Auger Collaboration makes direct measurements of $X_{\text{max}}$ by using fluorescence detector (FD). The Collaboration analyzed eight years of data comprising more than two million events before cuts. The $X_{\text{max}}$ evolution with energy shows a break at $10^{18.27} \text{ eV}$ [3] and if air-shower simulations are used to interpret the data, it is possible to conclude the mass composition is getting heavier above this energy [4]. The Telescope Array Collaboration currently cannot discriminate between a proton dominated composition or a changing composition, due to low statistics limitations [5]. Moreover, the FD analysis suffers from a small duty cycle ($\sim 10 - 15\%$), since the telescopes can only be operated during clear moonless nights. Thus, one would greatly benefit from a second, independent observable for composition, exploiting Pierre Auger’s surface detector (SD) stations which have a nearly 100% duty cycle.

Muons have been used as a important observable related to the primary particle mass and particle interaction properties [6, 7]. Muons are mostly generated by pion and kaon decay. The majority of pions and kaons are generated by the hadronic component of the shower, and since
primary particles that differ in mass have different hadronic properties, it is natural to expect the muon content of the shower to be sensitive to the primary mass composition and to the hadronic interaction properties. Indeed, one would expect that, regarding muon production (the number of muons created at a given slant depth), heavy primaries should yield a shallower profile with a greater amplitude, since heavy primary particles generate muon rich showers and one can think of a primary particle of atomic mass \( A \) and energy \( E \) as \( A \) independent nuclei each with energy \( E/A \) (a reasonable approximation since the nuclear binding energies, in the MeV scale, are much smaller than the typical primary cosmic ray energy \( > 10^{12} \text{ MeV} \)).

In order to interpret the measurements of the observables, one needs to perform Monte Carlo simulations and compare the results to the measurements. Extensive air showers (EASs) are phenomena consisting of a large numbers of particles produced by cosmic rays that interacts with Earth’s atmosphere. Since the total number of particles in an extensive air shower can be very large, a statistical technique - the thinning technique - is implemented in simulation software. However, by employing this technique there is, in principle, a risk of inducing some bias in the simulation results. So far, such biases were either not found or found to be much smaller than systematics due to experimental features [8, 9, 10, 11].

In this paper we present a preliminary study of the effect of the thinning on the depth of maximum muon production \( X_{\mu_{\text{sim}}}^{\text{max}} \). The study is based only in the shower simulations, no detector simulation is taken into account. The Pierre Auger Collaboration published results based on maximum muon production depth [6], however the analysis done in this paper are very different and much more sophisticated than the ones presented here. We will discuss the differences in section 3.1.

2. Thinning Algorithm

The thinning technique consists in following just a subset of particles in an EAS simulation: all the particles above a defined energy threshold \( \epsilon E_p \) (where \( \epsilon \) is the thinning factor and \( E_p \) is the primary particle energy), and for particles below the threshold the technique adopts different policies depending on the interaction that gave them birth. If all particles in the interaction have an energy below threshold, then the algorithm will follow a single particle with probability \( \epsilon/E_{\text{sum}} \), where \( \epsilon \) is its energy and \( E_{\text{sum}} \) is the sum of all particle energies. To this particle, the weight \( E_{\text{sum}}/\epsilon \ast w \) is assigned to statistically preserve shower quantities (\( w \) is the weight of the mother particle; unthinned particles have unit weight). If, however, some of the particles have energies greater than the thinning threshold, one will follow all those particles above threshold, and follow a sub-threshold particle of energy \( \epsilon \) with probability \( \epsilon/E_t \) (where \( E_t \) is the sum of all particle energies below the thinning threshold). If a particle below the thinning threshold is followed, a weight \( E_t/\epsilon \ast w \) is assigned to it.

The above technique is know as the Hillas thinning [8]. Alternative thinning techniques, such as the statistical thinning described in [12] are also employed, depending on what observables one is interested in analyzing with the simulation software. Additionally, statistical fluctuations are considerably reduced by setting a maximum particle weight: a particle with weight \( w > w_{\text{max}} \) is no longer subject to thinning. The thinning threshold and maximum weight sometimes differ for hadronic and electromagnetic particles, since the former are less numerous than the latter. Finally, a radial thinning technique can be employed, which consists in defining a radial scale \( r_{\text{max}} \), and then retaining all particles with radial position (with respect to the shower axis) \( r > r_{\text{max}} \) and otherwise selecting the particle with probability \( r/r_{\text{max}} \) and multiplying its weight by \( r_{\text{max}}/r \).

Thinning techniques are known to preserve mean shower quantities [8], to not induce bias in observables such as the depth of shower maximum [9, 11] or the risetime [10]. In other cases, it has been shown that the spurious fluctuations introduced by the thinning in the risetime are smaller than statistical fluctuations due to detection and reconstruction [10]. The thinning
3. Muon production profile at its maximum

We studied the muon production profiles and its maximum using Monte Carlo simulations of air showers. The simulation software of choice was CORSIKA 7.5000 [13]; we used QGSJetII-04 [14] as the high energy interaction model and UrQMD 1.3cr [15] as the low energy model. The primary particle was fixed to $1 \times 10^9$ GeV with zenith angle of 65°. The geomagnetic field was chosen so as to be the same as in Malargue, in Mendoza, Argentina (the site of the Pierre Auger Observatory).

In order to investigate thinning effects on muon production profiles we simulated showers with two thinning factors ($10^{-6}$ and $10^{-5}$) and three maximum weight for the hadronic and electromagnetic thinning $w_{\text{max}} = 10^3$, $10^4$ and $10^5$. In our study, we consider all muons produced in the shower development. The number of muons in the shower are sampled in steps of 5 g/cm$^2$ along the slant depth. We used the longitudinal tables produced in the output of CORSIKA and subtracted the number of muons from consecutive steps. But this procedure we evaluate the “production rate” of muons, because in each step muons are created and decay and we can not separate the two effects in our estimation.

Figure 1 shows an example of the development of the muon profile. The simulated muon profile was fitted by a second order polynomial as shown by the red line in the figure. We used

![Figure 1. Example of the evolution of the number of muons as a function of depth. All muons produced in the shower development were taken into account. Shower simulated with CORSIKA and QGSJetII-04 and UrQMD 1.3cr hadronic interaction models at 65° zenith angle. Primary particle was a carbon nucleus with energy $1 \times 10^9$ GeV. Red line shows a fit of a second order polynomial to the points used to calculate the maximum of the profile $X_{\mu \text{-sim}}$. The thinning factor was set to $10^{-6}$ and $w_{\text{max}} = 10^4$. The technique greatly improves computational speed and reduces required storage space. Without it, it would not be possible to simulate large samples of UHECR showers.](image-url)
Figure 2. Simulated $X_{\mu \text{max}}$ distribution for proton and iron nuclei as primary particles. All muons produced in the shower development were taken into account. Showers simulated with CORSIKA and QGSJETII-04 and UrQMD 1.3cr hadronic interaction models at $65^\circ$ zenith angle. Primary particles had energy $1 \times 10^9$ GeV. The thinning factors were set to $10^{-6}$ and $w_{\text{max}} = 10^4$.

The fitted function to calculate $X_{\mu \text{max}}^{\mu - \text{sim}}$. Figure 2 shows the calculated $X_{\mu \text{max}}^{\mu - \text{sim}}$ for all proton and iron nuclei primaries simulated. As expected, iron primaries generate narrow profiles with a shallow maximum as compared to proton primaries.

3.1. Differences from the Pierre Auger Collaboration analysis

The Pierre Auger Collaboration has elaborated a sophisticated data analysis to extract from the measured air showers the relevant information from the muon profile. References [6, 16] summarizes the data analysis procedure. The data analysis starts with the arrival time of the particles in the SD stations. A detailed selection procedure was applied in order to minimize systematic and statistic uncertainties, maximize the signal-to-noise ratio and deconvolve detector limitations. Only stations far from the shower axis ($r > 1700$ m) are used and only events with zenith angle within the interval $[55^\circ, 65^\circ]$ and energy between $\log(E/eV) = 19.3 - 20$ are considered.

The arrival time of particles is converted into a muon production depth using a model described in references [17, 18, 19]. Note that by this procedure, only muons arriving on ground contributes to the measured muon profile, therefore low energy muons which might decay in the shower development are not taken into account. A Gaisser-Hillas function is fitted to the longitudinal muon production depth profile and $X_{\mu \text{max}}^{\mu}$ is determined. Finally, in an internal publication [20], the Pierre Auger Collaboration has evaluated the effect of thinning factors in the muon production depth analysis and it has shown to be negligible when thinning levels below $10^{-6}$ are used.

The summary of the data analysis of the Pierre Auger Collaboration presented above clarifies
the difference between the preliminaries studies presented here and the detailed analysis done by the Pierre Auger Collaboration. We call the main quantity studied here \(X_{\mu}^{\text{max}}\) to make it explicit that we take all muons produced in the simulation (sim) irrespectively of their distances from the shower core. Our study also considers all muons produced in the shower independently if they would hit ground or not. Moreover we made our study for one fixed energy \(1 \times 10^9\) GeV and zenith angle 65°.

Finally, we would like to make it clear that the quantity calculated here \(X_{\mu}^{\text{max}}\) can not be directly compared to the quantity used by the Pierre Auger Collaboration in its publications and therefore the conclusion can not be extrapolated.

![Figure 3. \(X_{\mu}^{\text{max}}\) for different thinning factors and weight limits. All muons produced in the shower development were taken into account. Shower simulated with CORSIKA and QGSJETII–04 UrQMD 1.3cr hadronic interaction models at 65° zenith angle. Primary particle was a proton with energy \(1 \times 10^9\) GeV. Points were slightly displaced horizontally so that the error bars would not overlap.](image)

4. Results

Figure 3 displays the preliminary results obtained. Each point in the figure corresponds to the \(X_{\mu}^{\text{max}}\) from 1000 proton simulated showers, for a given thinning factor and weight limit. Figure 3 suggests that the thinning factor and the maximum weight limit might have an effect on the \(X_{\mu}^{\text{max}}\) values. The effect is not trivial and the error bars are too large for a definitive conclusion. Currently, we are investigating this matter by writing a toy Monte Carlo code, which implements the same interaction and decay procedure as CORSIKA and running other thinning factors and weight limits.
5. Conclusion
The muon content of an EAS provides valuable information, which can be used to study the primary particle and to discriminate between competing interaction models. For this reason it is crucial that all possible errors be kept in check. In this work we analysed the effect of the thinning factor and maximum weight on the $X_{\mu_{\text{max}}-\text{sim}}$ parameters. It has been shown that $X_{\mu_{\text{max}}-\text{sim}}$ can be affect by different values of thinning factor and maximum weight. Further investigations are under way.

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References
[1] Heitler W 1947 The Quantum Theory of Radiation (Oxford University Press)
[2] Matthews J 2005 Astroparticle Physics 22 387–97
[3] The Pierre Auger Collaboration 2014 Phys. Rev. D 90(12) 122005–30
[4] The Pierre Auger Collaboration 2014 Phys. Rev. D 90(12) 122006–18
[5] Barcikowski, E and Bellido, J and Belz, J and Egorov, Y and Knurenko S and Souza, V et al 2013 EPJ Web Conf. 53 1–14
[6] Aab A and et al 2014 Phys. Rev. D 90 012012–27
[7] Unger M 2016 Proc. of Int. Symp. for Ultra-High Energy Cosmic Rays 2014 010020 1–7
[8] Hillas A M 1997 Nuclear Physics B B 29–42
[9] Gorbunov D S, Ruhtsov G I and Troitsky S V 2007 Phys. Rev. D 76 043004–11 (Preprint 0703546)
[10] Brulju R, Schmidt F, Ilee J and Knapp J 2009 Proc. of the 31st Int. Cosmic Ray Conf. 2 1–4
[11] Toderu C J, Souza V and Bellido J A 2013 Astroparticle Physics 47 18–30
[12] Kobal M 2001 Astroparticle Physics 15 259–73
[13] Heck, D et al 1988 CORSIKA: A Monte Carlo Code to Simulate Extensive Air Showers FZKA-6019 (FZKA)
[14] Ostapchenko S 2011 Phys. Rev. D 83 1010–869
[15] Bleicher, M et al 1999 J.Phys.G: Nucl.Part.Phys. 25 1859–96
[16] Laura Collica for the Pierre Auger Collaboration 2016 Eur. Phys. J. Plus 131
[17] Cazon L, Vázquez R and Zas E 2005 Astroparticle Physics 23 393–409 ISSN 0927-6505
[18] Cazon L, Vázquez R Watson A and Zas E 2004 Astroparticle Physics 21 71–86 ISSN 0927-6505
[19] Cazon L, Conceição R, Pimenta M and Santos E 2012 Astroparticle Physics 36 211–223 ISSN 09276505
[20] Bueno A, García-Gámez D and Bueno L M GAF-Notes - Pierre Auger Internal Notes 2015-34