Electromagnetic form factors of $\Sigma^+$ and $\Sigma^-$ in the vector meson dominance model

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Based on the recent measurements of the $e^+e^- \rightarrow \Sigma^+\Sigma^-$ and $e^+e^- \rightarrow \Sigma^-\Sigma^+$ processes by BESIII collaboration, the electromagnetic form factors of the hyperon $\Sigma^+$ and $\Sigma^-$ in the timelike region are investigated by using vector meson dominance model, where the contributions from the $\rho$, $\omega$, and $\phi$ mesons are taken into account. The model parameters are determined from the BESIII experimental data on the timelike effective form factors $|G_{\text{eff}}|$ of $\Sigma^+$ and $\Sigma^-$ baryons for center-of-mass energy from 2.3864 to 3.02 GeV. It is found that we can provide quantitative descriptions of available data as few as one adjustable model parameter. We then progress to an analysis of the electromagnetic form factors in the spacelike region and evaluate the spacelike form factors of the hyperons $\Sigma^+$ and $\Sigma^-$. The obtained electromagnetic form factors of the $\Sigma^+$ and $\Sigma^-$ baryons are comparable with other model calculations.

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I. INTRODUCTION

The study of the structure of hyperons which contain strange quark through their electromagnetic form factors (EMFFs) is crucial for deep understanding the nonperturbative quantum chromodynamics (QCD) effects that quarks are bounded in baryons [1-5]. As discussed in Refs. [6-9], the EMFFs are crucial experimental observables of baryons which are intimately related to their internal structure and dynamics. In the last few decades, there are great progress in the study of baryon EMFFs, especially for the nucleon both in timelike [9-20] and spacelike regions [21-31]. For example, in Ref. [32] the nucleon form factors and the $N-\Delta(1232)$ transitions were theoretically investigated in a framework of constituent quark model, where the effect of $\pi$ meson cloud is also considered. While for the case of hyperons, since they cannot be targets, the hyperon EMFFs in the spacelike region are hardly to be measured by experiments. In the timelike region, the BESIII collaboration investigated the $\Sigma$ hyperon EMFFs from the reaction $e^+e^- \rightarrow \Sigma^+\Sigma^-$ and $e^+e^- \rightarrow \Sigma^-\Sigma^+$ [4,33]. From the determined high precision Born cross sections of these two above reactions for center-of-mass energy from 2.3864 to 3.02 GeV, the effective form factors $|G_{\text{eff}}|$ of $\Sigma^+$ and $\Sigma^-$ and also the ratios of $\Sigma^+$ electric and magnetic form factors $|G_E/G_M|$, are obtained [4].

Very recently, the EMFFs of hyperons are investigated in the timelike region through $e^+e^- \rightarrow Y\bar{Y}$ ($Y$ is the hyperon) reaction [5], where the $Y\bar{Y}$ final state interactions are taken into account, and the EMFFs in the timelike region are calculated for the $\Lambda$, $\Sigma$, and $\Xi$ baryons based on one-photon approximation for the elementary reaction mechanism. In Ref. [2], the timelike EMFFs of hyperons are well reproduced within a relativistic quark model. In addition, the $\Sigma$ hyperon EMFFs have been calculated within Lattice QCD [34], light cone sum rule (LCSR) [35], and chiral perturbative theory (ChPT) [36]. In addition to these, the vector meson dominance (VMD) model is a very successful approach to study the nucleon electromagnetic form factors both in spacelike and timelike regions [37-59]. Within a modified VMD model, in Ref. [1] the EMFFs of $\Lambda$ hyperon were studied in the timelike region from $e^+e^- \rightarrow \Lambda\bar{\Lambda}$ reaction, where the contributions from $\phi$ and $\omega$ mesons, and their excited states are included. It was found that the VMD model can simultaneously describe the effective form factor and also the electromagnetic form factor ratio of the $\Lambda$ hyperon.

In this work, based on the recently measurements of the $e^+e^- \rightarrow \Sigma^+\Sigma^-$ and $e^+e^- \rightarrow \Sigma^-\Sigma^+$ reactions, we aim to determine the parameters of VMD model by fitting them to the experimental data of $|G_{\text{eff}}|$ of $\Sigma^+$ and $\Sigma^-$. We have included the contributions from the $\rho$, $\omega$, and $\phi$ mesons. Then the ratios $|G_E/G_M|$ of $\Sigma^+$ are estimated with the model parameters, which are comparable with other theoretical calculations.

This article is organized as follows: formalism of $\Sigma$ hyperon form factors in VMD model are shown in the following section. In Sec. III, we introduce effective form factors and the method that analytically continue the expressions of the form factors from the timelike region to the spacelike region. Numerical results of the timelike form factors of $\Sigma^+$ and $\Sigma^-$ hyperon and the ratios $|G_E/G_M|$ of $\Sigma^+$ and $\Sigma^-$ are presented. In Sec. IV, followed by a short summary in the last section.

II. THEORETICAL FORMALISM

We will study the EMFFs of $\Sigma^+$ and $\Sigma^-$ within the VMD model. As in Ref.[11], we first introduce the electromagnetic current of $\Sigma$ hyperon with spin-1/2 in terms of the Dirac form factor $F_1(Q^2)$ and Pauli form factors $F_2(Q^2)$ as

$$J^\mu = \gamma^\mu F_1(Q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2m_\Sigma} F_2(Q^2), \quad (1)$$
where $F_1$ and $F_2$ are functions of the squared momentum transfer $Q^2 = -q^2$. In the VMD model, the Dirac and Pauli form factors are parametrized into two parts. One is the intrinsic three-quark structure described by the form factor $g(Q^2)$, and the other one is the meson cloud, which is used to describe the interaction between the bare baryon and the photon through the intermediate isovector $\rho$ meson and isoscalar $\omega$ and $\phi$ mesons [37]. Following Refs. [37, 39], $F_1$ and $F_2$ can be decomposed as $F_i = F_i^S + F_i^V$, with $F_i^S$ and $F_i^V$ the isoscalar and isovector components of the form factors, respectively. The Dirac and Pauli form factors of $\Sigma^+$ and $\Sigma^-$ are easily obtained without considering the total decay widths of the vector mesons, as follows

\begin{align}
F_{1 \Sigma^+}^S (Q^2) &= \frac{1}{2} g_1 (Q^2) \left[ (1 - \beta_\omega - \beta_\phi) + \beta_\omega \frac{m_\omega^2}{m_\omega^2 + Q^2} + \beta_\phi \frac{m_\phi^2}{m_\phi^2 + Q^2} \right], \\
F_{1 \Sigma^+}^V (Q^2) &= \frac{1}{2} g_1 (Q^2) \left[ (1 - \beta_\omega - \beta_\phi) + \beta_\rho \frac{m_\rho^2}{m_\rho^2 + Q^2} \right], \\
F_{2 \Sigma^+}^S (Q^2) &= \frac{1}{2} g_2 (Q^2) \left[ (2\mu_{\Sigma^+} - 2 - \alpha_\phi - \alpha_\rho) \frac{m_\omega^2}{m_\omega^2 + Q^2} + \alpha_\phi \frac{m_\phi^2}{m_\phi^2 + Q^2} \right], \\
F_{2 \Sigma^+}^V (Q^2) &= \frac{1}{2} g_2 (Q^2) \left[ \alpha_\rho \frac{m_\rho^2}{m_\rho^2 + Q^2} \right], \\
F_{1 \Sigma^-}^S (Q^2) &= \frac{1}{2} g_2 (Q^2) \left[ (-1 - \beta_\omega - \beta_\phi) + \beta_\omega \frac{m_\omega^2}{m_\omega^2 + Q^2} + \beta_\phi \frac{m_\phi^2}{m_\phi^2 + Q^2} \right], \\
F_{1 \Sigma^-}^V (Q^2) &= \frac{1}{2} g_2 (Q^2) \left[ (-1 - \beta_\rho) + \beta_\rho \frac{m_\rho^2}{m_\rho^2 + Q^2} \right], \\
F_{2 \Sigma^-}^S (Q^2) &= \frac{1}{2} g_2 (Q^2) \left[ (2\mu_{\Sigma^-} + 2 - \alpha_\phi - \alpha_\rho) \frac{m_\omega^2}{m_\omega^2 + Q^2} + \alpha_\phi \frac{m_\phi^2}{m_\phi^2 + Q^2} \right], \\
F_{2 \Sigma^-}^V (Q^2) &= \frac{1}{2} g_2 (Q^2) \left[ \alpha_\rho \frac{m_\rho^2}{m_\rho^2 + Q^2} \right],
\end{align}

where we take the intrinsic form factor $g(Q^2)$ as a dipole form

\begin{align}
g_1(Q^2) &= (1 + \gamma_1 Q^2)^{-2}, \\
g_2(Q^2) &= (1 + \gamma_2 Q^2)^{-2}.
\end{align}

This form is consistent with $g = 1$ at $Q^2 = 0$, and the free model parameters $\gamma_1$ and $\gamma_2$ will be determined by fitting them to the experimental data.

On the other hand, the observed electric and magnetic form factors $G_E$ and $G_M$ can be expressed in terms of Dirac and Pauli form factors $F_1$ and $F_2$ by,

\begin{align}
G_E(Q^2) &= F_1 - \tau F_2 = F_1^S + F_1^V - \tau (F_2^S + F_2^V), \\
G_M(Q^2) &= F_1 + F_2 = F_1^S + F_1^V + F_2^S + F_2^V,
\end{align}

where $\tau = \frac{Q^2}{M^2}$ in this work. At $Q^2 = 0$, $G_M(Q^2)$ defines the value of the magnetic moment of the $\Sigma$ hyperon, $\mu_{\Sigma} = G_M(0)$, in natural unit, i.e., $\mu_{\Sigma} = e/(2M_{\Sigma})$. For easily compare magnetic moments of different particles masses it is usual to express magnetic moments in terms of $\mu_{\Sigma}^z = e/(2M_N)$, the nucleon magneton [40, 41]. In this work we take $\mu_{\Sigma^+} = 3.112$ and $\mu_{\Sigma^-} = -1.479$ with natural unit [40]. In addition, we take $m_\rho = 0.775$, $m_\omega = 0.782$, and $m_\phi = 1.019$ GeV [40].

In Eqs. (2)-(9), $\beta_\rho$, $\beta_\omega$, $\beta_\phi$, $\alpha_\phi$, and $\alpha_\rho$ represent the product of a vector-meson-photon coupling constant and a $V\Sigma\Sigma$ coupling constant. For the $V\Sigma\Sigma$ coupling constants, we obtain them through SU(3) flavor symmetry as in Ref. [42].

\begin{align}
g_{\Sigma \Sigma \omega} &= g_{BBV} 2\alpha_{BBV}, \\
g_{\Sigma \Sigma \phi} &= -g_{BBV} \sqrt{2} (2\alpha_{BBV} - 1), \\
g_{\Sigma \Sigma \rho} &= g_{BBV} 2\alpha_{BBV},
\end{align}

where $g_{BBV} = g_{NN \rho} = 3.20$ and $\alpha_{BBV} = 1.15$ as used in Refs. [42, 43]. Then we can easily get $g_{\Sigma \Sigma \omega} = 7.36$, $g_{\Sigma \Sigma \phi} = 5.88$, $g_{\Sigma \Sigma \rho} = 7.36$. While the tensor couplings, they are given by

\begin{align}
f_{\Sigma \Sigma \phi} &= f_{NN \rho} \frac{1}{\sqrt{2}}, \\
f_{\Sigma \Sigma \rho} &= f_{NN \rho} \frac{1}{2},
\end{align}

where $f_{NN \rho} = g_{NN \rho} \kappa_{\rho}$ with $\kappa_{\rho} = 6.142, 43$. Therefore we can get $f_{\Sigma \Sigma \phi} = 13.80$ and $f_{\Sigma \Sigma \rho} = 9.76$. 
In addition, we calculate $V\gamma$ coupling constants following Refs. [44–46].

$$\mathcal{L}_{V\gamma} = \sum_{V} \frac{eM_{V}^{2}}{f_{V}} V_{\mu} A^{\mu},$$

$$\frac{e}{f_{V}} = \left[\frac{3V_{e+e+\rightarrow \gamma\gamma}}{2\alpha_{e} |\vec{p}_{e}|}\right]^{1/2}.$$  

(18)

where $\alpha_{e} = e^{2}/(4\pi)\pi/137$ is the fine-structure constant and $\vec{p}_{e}$ is the three momentum of electron in the rest frame of the vector meson. $\Gamma_{e+e+\rightarrow \gamma\gamma}$ is the partial decay width of the vector meson. $\Gamma_{V+e+e+\rightarrow \gamma\gamma}$ is the partial decay width of the vector meson decaying into $e^{+}e^{-}$ pair. Then, with the experimental values we get $1/f_{\rho} = 0.200$, $1/f_{\omega} = 0.059$, and $1/f_{\phi} = 0.075$.

Finally we obtain $\beta_{\rho}, \beta_{\omega}, \beta_{\phi}$ and $\alpha_{\phi}$ through

$$\beta_{V} = g_{\Sigma \Sigma V} \frac{1}{f_{V}}, \quad \alpha_{V} = f_{\Sigma \Sigma V} \frac{1}{f_{V}},$$

(19)

which are summarized in Table I.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $\beta_{\rho}$ | 0.736 | $\alpha_{\phi}$ | 0.976 |
| $\beta_{\omega}$ | -0.441 | $\alpha_{\phi}$ | 1.035 |
| $\beta_{\phi}$ | 0.434 | | |

TABLE I: Parameters used in this work.

Next we explain how the large total width of $\rho$ meson contributions are implemented. 1 For this purpose, one needs to replace [37]

$$\frac{m_{\rho}^{2}}{m_{\rho}^{2} + Q^{2}} \rightarrow \frac{m_{\rho}^{2} + 8\Gamma_{\rho} m_{\rho}/1\pi}{m_{\rho}^{2} + Q^{2} + (4m_{\rho}^{2} + Q^{2})\Gamma_{\rho} \alpha(Q^{2})/m_{\rho}},$$

(20)

where we take $\Gamma_{\rho} = 149.1$, and a average value of $m_{\rho} = 138.04$ MeV [40]. The function $\alpha(Q^{2})$ is given by

$$\alpha(Q^{2}) = \frac{2}{\pi} \left(\frac{4m_{\rho}^{2} + Q^{2}}{Q^{2}}\right)^{1/2} \ln \left(\frac{\sqrt{4m_{\rho}^{2} + Q^{2}} + \sqrt{Q^{2}}}{2m_{\rho}}\right).$$

(21)

Timelike formfactors can be obtained from the spacelike form factors by an appropriate analytic continuation. Within the above ingredients, $g(Q^{2})$ has the form of an analytical continuation form,

$$g(Q^{2}) = (1 - \gamma Q^{2})^{-2},$$

(22)

where $Q^{2} = -q^{2} = q^{2}e^{i\pi}$. We want to mention that $\gamma$ has positive value, hence $g(q^{2})$ will be divergent at $q^{2} = 1/\gamma$. To evade this problem, one can restrict $\gamma > 1/(4m_{\Sigma}^{2}).$

III. NUMERICAL RESULTS AND DISCUSSION

In the timelike region, the EMFFs of hyperon $\Sigma^{+}$ and $\Sigma^{-}$ are experimental studied via electron-position annihilation processes. Under the one-photon exchange approximation, the total cross section of $e^{+}e^{-} \rightarrow Y Y$, with $Y$ the $\Sigma^{+}$ or $\Sigma^{-}$, can be expressed in terms of the electric and magnetic form factors $G_{E}$ and $G_{M}$ as [47]

$$\sigma = \frac{4\pi^{2}}{3q^{2}} C_{Y} \left(\left|G_{M}(q^{2})\right|^{2} + \frac{2M_{\Sigma}^{2}}{q^{2}} \left|G_{E}(q^{2})\right|^{2}\right),$$

(23)

where $\beta = \sqrt{1 - 4M_{\Sigma}^{2}/q^{2}}$ is a phase-space factor. $q^{2} = s$ is the invariant mass square of the $e^{+}e^{-}$ system. In addition, the Coulomb correction factor $C_{Y}$ is given by [47]

$$C(y) = \frac{y}{1 - ey},$$

(24)

with $y = \frac{\alpha_{e}}{q} 2M_{\Sigma}$. In general, one can easily obtain the effective form factor $G_{\text{eff}}(q^{2})$ from the total cross section of $e^{+}e^{-}$ annihilation process. The effective form factor $G_{\text{eff}}(q^{2})$ is defined as

$$|G_{\text{eff}}(q^{2})| = \sqrt{\frac{2\pi\left|G_{M}(q^{2})\right|^{2} + \left|G_{E}(q^{2})\right|^{2}}{1 + 2\pi}}.$$  

(25)

We then perform a $\chi^{2}$ fit to the experimental data of the effective form factor $|G_{\text{eff}}|$ of $\Sigma^{+}$ and $\Sigma^{-}$ taken from [3]. In the fitting we have only one free parameter: $\gamma_{1}$ for $\Sigma^{+}$, and $\gamma_{2}$ for $\Sigma^{-}$. The fitted parameters are $\gamma_{1} = 0.46 \pm 0.01$ GeV$^{-2}$ and $\gamma_{2} = 1.18 \pm 0.13$ GeV$^{-2}$, with $\chi^{2}/dof = 2.0$ and 1.1, respectively. The corresponding best-fitting results for the the effective form factor $|G_{\text{eff}}|$ of $\Sigma^{+}$ and $\Sigma^{-}$ in the energy range 2.3864 GeV $< \sqrt{s} < 2.9884$ GeV are shown in Fig. 2 with the solid curves. We also show the theoretical band obtained from the above uncertainties of the fitted parameters. The numerical results show that we can give a good description for the experimental data.

From the fitted results of the effective form factors, we can easily obtain the values of $|G_{\Sigma^{+}}^{\text{eff}}|/|G_{\Sigma^{-}}^{\text{eff}}|$, which are shown in Fig. 3. One can see that the ratio is about three in the energy region of $2.4 < \sqrt{s} < 3.0$ GeV. The value of 3 is just the ratio of the incoherent sum of the squared charges of the $\Sigma^{+}$ and $\Sigma^{-}$ valence quarks.

Through the vector meson dominant model, with the fitted parameter $\gamma$, one can also easily calculate the ratio of $|G_{E}|$ and $|G_{M}|$. This ratio is determined to be one at the mass threshold of a pair of baryon and anti-baryon due to the kinematical restriction. We show our theoretical calculations in Fig. 4 where the results are obtained with $\gamma_{1} = 0.46$ and $\gamma_{2} = 1.18$. It is found that, with the reaction energy $\sqrt{s}$ increasing, the ratio of $|G_{E}|$ and $|G_{M}|$ for $\Sigma^{+}$ is slowly decreased, while for the case of $\Sigma^{-}$, it is almost flat. Our results here cannot explain well the experimental data that the ratio is larger than one within uncertainties close to threshold. This may indicated that there should be also other contributions

1 We will not consider the effects from the widths of $\omega$ and $\phi$, since they are so narrow.
around that energy region. For example, the electromagnetic form factors should be significantly influenced by the interaction in the final $\Sigma\Sigma$ system \[5\]. However, since the experimental and empirical information about the $\Sigma\Sigma$ final state interaction is so limited, we leave those contributions to further study when more precise data are available.

Next, we pay attention to the EMFFs in the spacelike region, which can be straightforwardly calculated with the parameter $\gamma$ determined by the experimental measurements in the timelike region. Since the parametrization forms shown in Eqs. \[6\]-\[9\] are valid in the low $Q^2$ regime, we calculate $G_E$ and $G_M$ below $Q^2 = 3$ GeV$^2$, and compare our numerical results with other calculations.

The numerical results for the $G_M$ and $G_E$ obtained with $\gamma_1 = 0.46$ and $\gamma_2 = 1.18$ are shown in Fig.\[4\] and \[5\] respectively. In Fig.\[4\] predictions from light cone sum rules \[35\] and lattice QCD calculations \[34\] are also shown for comparing. Our results for the magnetic form factor of $\Sigma^+$ and $\Sigma^-$ are somewhat quantitatively different from other theories. Our results for the magnetic form factor of $\Sigma^-$ are more consistent with other calculations. While for the case of $\Sigma^-$ electric form factor in Fig.\[5\] our results are disagreement with the ChPT and LCSR calculations. However, our results for the $\Sigma^+$ are closer to the lattice QCD in Ref. \[34\] and ChPT in the very low $Q^2$ region. It is expected that these theoretical calculations can be tested by future experiments on the EMFFs of hyperons $\Sigma^+$ and $\Sigma^-$ and thus will provide new insight into the complex internal structure of the baryons.

To compare our estimations with other calculations, we convert the unit of our results into nucleon magneton.
IV. SUMMARY

In this work, we have investigated the electromagnetic form factors of the hyperon $\Sigma^+$ and $\Sigma^-$ within the vector meson dominance model. The contributions from the $\rho$, $\omega$, and $\phi$ mesons are taken into account. The model parameters, $\gamma_1$ and $\gamma_2$, are determined with the BESIII experimental data on the timelike effective form factors $|G_{\text{eff}}|$ of $\Sigma^+$ and $\Sigma^-$. It is found that the experimental data can be well reproduced with only one model parameter. Then, we analytically continue the electromagnetic form factors to spacelike region and evaluate the spacelike form factors of $\Sigma^+$ and $\Sigma^-$. The obtained electromagnetic form factors of the $\Sigma^+$ and $\Sigma^-$ and their ratio are qualitatively comparable with other model calculations, but slightly different quantitatively.

Finally, we would like to stress that, the estimations of $\Sigma$ form factors in this work, the $\Lambda$ form factors in Ref. [1] and proton form factors in Refs. [37–39] indicate that the vector meson dominance model is valid to study the electromagnetic form factors of the baryons, accurate data for the $e^+e^- \rightarrow$ baryon + anti-baryon reaction can be used to improve our knowledge of baryon form factors, which are at present poorly known.

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