Research Article

Analytical Method for Fluid-Solid Coupling Vibration Analysis of Hydraulic Pipeline System with Hinged Ends

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The vibrations in hydraulic pipeline systems inevitably involve the interaction between the two-phase media of a solid and fluid. The fluid flowing in the pipeline generates pressure on the pipeline wall and thus causes vibration and deformation in the pipeline, which in turn changes the fluid flow condition, thereby cyclically affecting the deformation and motion of the solid and fluid and making the pipeline system vulnerable to vibration damage. Therefore, it is of great theoretical and practical value to investigate the vibration behavior and characteristics of hydraulic pipeline systems. In this study, the fluid flow-induced vibrations in a pipeline system are investigated based on Housner’s differential vibration equation of fluid pipelines. Through the application of relevant mathematical theories and methods, the derivation of the inherent characteristics and dynamic behavior of a hydraulic pipeline system with two hinged ends is simplified, and the corresponding equations for the natural frequencies and dynamic response of the system are obtained. The analytical method for analyzing the vibration behavior and characteristics of the system is presented, and the analytical results of the vibration analysis of the system are obtained through computer simulation, providing a theoretical and technical basis for the safe and reliable operation of the hydraulic pipeline system.

1. Introduction

Modern industry has placed increasingly high demands on project quality as well as product precision and reliability, and hence it has become an urgent task to study and solve various vibration problems in the mechanical industry. The vibratory quantities of a vibration system, such as displacement, velocity, and acceleration, are used to measure the vibration intensity of mechanical equipment. If the structure of a mechanical system is subjected to a large dynamic load, the vibratory quantities may exceed the allowable range of values, causing intense vibration in the mechanical equipment and noise. As a result, the mechanical equipment is affected in terms of working performance and service life and may undergo fault/failure in severe cases and even experience catastrophic accidents. As modern machinery and equipment systems are increasingly more complex, faster, and more accurate, the vibration hazards become more prominent. Thus, in the design and development of machinery and equipment, not only static effects but also dynamic effects should be considered. In the mechanical and hydraulic engineering field, the factors such as pressure and flow should be fluctuant, so the fluctuation in the hydraulic pipeline is inevitable. The vibration in the hydraulic pipeline system not only influences the normal operation of the system but also degrades the performance of the hydraulic system and even causes the pipeline or braced structures to break. The vibration damping is necessary to ensure the safe and reliable operation of the hydraulic system. There is coupled vibration of fluid and solid besides solid vibration and fluid vibration in the hydraulic system.

There are vibrations in solids and fluctuations in fluid as well as coupled vibration resulting from the interaction between two-phase media of a fluid and solid, which produces deformation and motion in the solid and fluid under the interaction of loads. Deformation occurs in the solid and motion occurs in the fluid under the interacting loads, thereby changing the distribution and magnitude of the fluid
fluctuations as well as the behavior and characteristics of the solid vibrations. It is this interaction that produces a variety of fluid-solid coupling phenomena under different conditions. The flow-solid coupling effect of a fluid-conveying pipeline system is essentially the coupling of the fluid pulsation in the pipeline and the solid vibrations of the pipe, which can change the natural frequencies and dynamic response properties of the pipeline system. The study of fluid-conveying pipeline systems can be traced back to as early as Lamb [1], who defined the concept of fluid-solid coupling in fluid-conveying pipeline systems, studied the effect of pipeline solid motion on the fluid pressure fluctuations, and investigated the effect of the fluid within the pipeline on the axial and radial vibrations of the pipe. Currently, research on the vibrations in fluid-conveying pipeline systems is mainly focused on vibration models, numerical methods, and experimental techniques for fluid-solid coupling of pipeline systems [2–17]. The solution methods mainly include analytical methods and numerical methods. Analytical methods are capable of computational analysis of the fluid-solid coupling vibrations of simple pipeline systems. The applicable numerical methods mainly include the finite element method, finite volume method, and smooth particle hydrodynamics method, which can provide approximate numerical solutions of fluid-solid coupling vibration in pipeline systems. Areas of application focus on vibration analysis of components and equipment with high safety standards (e.g., water-conveying pipelines, aircraft hydraulic pipelines, engine fuel pipelines, oil transmission pipelines, and steam pipelines in nuclear power plants). The famous monograph by Blevins [2] has exerted an important influence on the study of fluid-solid coupling mechanics, providing a powerful analytical tool for researchers in mechanical, aviation, construction, marine, and agricultural fields.

The vibrations in a hydraulic pipeline system can cause hidden dangers to the safe operation of the instrumentation and equipment, which not only affects the working quality of the hydraulic devices and mechanical equipment but also reduces the life of the components, as well as generates noise, pollutes the environment, and even damages the pipelines and accessories, causing major safety accidents. The vibrations in hydraulic pipeline systems are often derived from mechanical vibrations and fluid flow fluctuations. In fact, in a hydraulic system, factors such as pressure and flow cannot be completely free of fluctuations, and hence there are inevitable factors that cause vibrations in a hydraulic pipeline system. Thus, the key lies in the reduction and mitigation of the vibrations in hydraulic pipeline systems. For this reason, the behavior and characteristics of fluid-solid coupling vibrations in hydraulic pipeline systems need to be adequately studied to obtain effective guidance to prevent vibrations in hydraulic pipeline systems [18–20]. Ebrahimi et al. developed a nonlocal couple stress theory to investigate static stability and free vibration characteristics of functionally graded nanobeams [21]. Akgoz and Civalek carried out thermal and shear deformation effects on the vibrational response of non-homogeneous microbeams made of functionally graded materials [22]. Civalek et al. performed free vibration analyses of embedded carbon and silica carbide nanotubes lying on an elastic matrix based on Eringen’s nonlocal elasticity theory [23]. Due to the relative complexity and specificity of hydraulic systems, not much research has been conducted on the vibration behavior and characteristics of relevant hydraulic pipeline systems. However, the vibration analysis of hydraulic pipeline systems is of great significance for improving the performance, reducing the vibration noise, and improving the efficiency of hydraulic systems and devices.

In this paper, by applying the theory and methods of modern mathematics and mechanics and fully considering factors that cause the vibrations in fluid pipelines, a theoretical model and analytical method for the analysis of the vibration behavior and characteristics of a hydraulic pipeline system were proposed based on Housner’s differential equation regarding the vibrations in fluid pipelines. The vibration theory of a continuous system is applied to derive analytical equations for the natural frequencies and vibration response of a hydraulic pipeline system with two hinged ends. Thus, the vibrations in this system are analyzed and determined. The dynamic simulation analysis of this system is implemented using a computer program, providing solid theoretical and technical support for the safety and reliability of hydraulic pipeline systems. Note that the present study only investigated the behavior and characteristics of fluid-solid coupling vibrations in hydraulic pipeline systems with simple hinged ends, so it is still difficult to theoretically analyze the fluid-solid coupling vibrations in the fluid-conveying pipeline systems with complex support boundaries and complex flow fields. Nevertheless, through the application of the method proposed in this study along with approximation, the dynamic behavior and inherent characteristics of the fluid-solid coupling vibrations in hydraulic pipeline systems in engineering practice can be analyzed, which can effectively improve the vibration-resistant design of hydraulic pipeline systems in actual engineering projects.

2. Model for Fluid-Solid Coupling Vibrations in the Hydraulic Pipeline System

The vibrations in a hydraulic system are mainly derived from the fluctuations in the fluid flow pressure, which should therefore be minimized to control the vibrations in the hydraulic pipeline system. There are many reasons for the vibrations in a hydraulic system, such as the opening and closing of valves, unbalanced rotors, the starting and braking of moving parts, cavitation, and resonance in the hydraulic system. Therefore, the causes of vibration in hydraulic systems must be analyzed, and effective measures must be taken to reduce the hazards and losses from vibration. In fact, a vibratory hydraulic pipeline system is a continuous system with distributed physical parameters such as mass, elasticity, and damping, which requires a two-dimensional equation of time and coordinates to describe the vibration state of the hydraulic pipeline system. A hydraulic pipeline system with hinges at both ends is shown in Figure 1. Since the beginning of the 1950s, efforts have been made to study similar vibration and stability problems in fluid-conveying
pipeline systems. The Housner equation [3] is a representative outcome of such studies. It is the free vibration equation of a fluid-conveying pipeline system and gives the basic differential equation of the vibrations induced by the fluid flowing in the pipeline, that is,

\[ EI \frac{\partial^4 y}{\partial x^4} + \rho Av \frac{\partial^2 y}{\partial x^2} + 2\rho Av \frac{\partial y}{\partial x} \frac{\partial y}{\partial t} + \left( m_p + \rho A \right) \frac{\partial^2 y}{\partial t^2} = 0, \]

(1)

where \( E \) and \( L \) are the inertia modulus (Pa) and moment of inertia (m\(^4\)) of the pipeline, respectively; \( A \) is the cross-sectional area of the pipeline (m\(^2\)); \( m_p \) is the mass per unit length of the empty pipeline (kg/m); \( \rho \) is the fluid density (kg/m\(^3\)); and \( v \) is the fluid velocity (m/s). Equation (1) is usually referred to as the Housner equation, which has been used as basis by many subsequent studies on the vibration and stability of fluid-conveying pipeline systems.

One of the main factors causing the vibrations in the hydraulic pipeline system is excitation from the fluctuations in the fluid flow pressure, \( \Delta p \) (Pa). Based on the flow fluctuation model of the hydraulic pipeline system, the differential equation for the forced vibrations in the fluid pipeline is

\[ EI \frac{\partial^4 y}{\partial x^4} + \rho Av \frac{\partial^2 y}{\partial x^2} + 2\rho Av \frac{\partial y}{\partial x} \frac{\partial y}{\partial t} + \left( m_p + \rho A \right) \frac{\partial^2 y}{\partial t^2} = F_0 \sin \omega t, \]

(2)

where the amplitude of the forced vibrations is \( F_0 = \Delta pL \) and \( L \) (m) is the length of the pipeline.

In the study of the vibrations in hydraulic pipeline systems, it is assumed that the material is uniform, continuous, and isotropic and obeys Hooke’s law in the elastic range. Obviously, an actual hydraulic pipeline system is often very complex, and the corresponding partial differential equation of vibration can be solved analytically only in some relatively simple special cases.

### 3. Inherent Characteristics of the Hydraulic Pipeline System

The partial differential equation (1) of the free vibrations in the hydraulic pipeline system contains the fourth-order spatial derivative and the second-order time derivative. To solve this type of partial differential equation, two initial conditions and four boundary conditions are needed. In the following discussion, the boundary conditions of a simply supported beam with hinged ends, common in hydraulic pipeline systems, are considered. The displacement and bending moment at a hinged end are equal to zero, i.e.,

\[ y(0,t) = y(L,t) = 0, \]

\[ EI \frac{\partial^4 y(0,t)}{\partial x^4} = EI \frac{\partial^4 y(L,t)}{\partial x^4} = 0, \quad (x = 0 \text{ or } x = L). \]

(3)

Then, the solution to equation (1) is separated in space and time. Let

\[ y(x,t) = Y(x)F(t). \]

(4)

Substitution of equation (4) into equation (1) gives

\[ EI \frac{\partial^4 Y(x)}{\partial x^4} + \rho Av \frac{\partial^2 Y(x)}{\partial x^2} + 2\rho Av \frac{\partial Y(x)}{\partial x} \frac{\partial Y(x)}{\partial t} + \left( m_p + \rho A \right) \frac{\partial^2 Y(x)}{\partial t^2} = 0. \]

(5)

According to the natural frequencies and mode shapes of the bending vibrations in a homogeneous simply supported beam with a uniform cross section, the inherent frequency and mode shape equations of the simply supported beam without consideration of the fluid motion are, respectively, set as follows:

\[ \omega_r = r^2 \frac{\pi^2}{L^2} \sqrt{\frac{EI}{m_p + \rho A}}, \quad (r = 1, 2, \ldots), \]

(6)

\[ Y_r(x) = \sin \frac{r\pi}{L} x, \quad (r = 1, 2, \ldots). \]

(7)

The related time function can be set as

\[ F(t) = A \sin \omega t + B \cos \omega t = C \sin(\omega t + \phi), \]

(8)

where \( A \) and \( B \) or \( C \) and \( \phi \) are the integration constants, which are determined by the initial conditions of \( y(x,0) = y_0 \), \( y_t(x,0) = y_0 \).

In the presence of the mixed derivative term \( \frac{\partial^2 y}{\partial x \partial t} \), use of the existing expressions of the mode shape equation (7) and the time function (8) above for derivation inevitably produces symmetric and antisymmetric terms in the derived equation, making it difficult to solve. For this reason, it is more convenient and reasonable to use the complex form of \( e^{ix} \) than the trigonometric equations of \( \cos x \) and \( \sin x \) in the derivation. Hence, mode shape equation (7) and time function (8) take a complex form during the operation process; after the complex solution is obtained, it is projected on the real or imaginary axis to obtain the related equations expressed by trigonometric equations. Therefore, it is assumed that

\[ Y_r(x) = e^{i\pi \frac{r}{L} x}, \quad (r = 1, 2, \ldots), \]

(9)

\[ F_r(t) = C_r e^{i\omega t}, \]

(10)

where \( i = \sqrt{-1} \); here, complex vectors are used to express the mode shape equation and time function. The exponential expression \( e^{ix} \) and the trigonometric equations \( \cos x \) and \( \sin x \) are related by the expression \( e^{ix} = \cos x + i \sin x \), where \( \cos x \) is the projection of the complex vector \( e^{ix} \) on the real axis, i.e., the real part of \( e^{ix} \), and \( i \sin x \) is the imaginary part of \( e^{ix} \).

Substitution of equations (9) and (10) into equation (5) yields the following equation:

\[ EI \left( \frac{r\pi}{L} \right)^4 - \rho Av \left( \frac{r\pi}{L} \right)^2 - 2\rho Av \left( \frac{r\pi}{L} \right) \omega_r - \left( m_p + \rho A \right) \omega_r^2 = 0. \]

(11)
Let the critical flow velocities of different orders of pipeline instability be
\[ v_r^i = \frac{r\pi}{L} \sqrt{\frac{EI}{\rho A}} \quad (r = 1, 2, \ldots). \] (12)

Considering the inherent frequency (6) and the critical flow velocity (12), after mathematical derivation and rearrangement of terms, equation (11) can be expressed as
\[ 1 - \left( \frac{v}{v_r} \right)^2 - 2 \sqrt{\frac{\rho A}{m_p + \rho A}} \frac{v}{v_r} \omega_r - \left( \frac{\omega_r}{\omega} \right)^2 = 0. \] (13)

Equation (13) is solved to obtain the expression of the inherent frequency of the hydraulic pipeline system:
\[ \frac{\omega_r^2}{\omega_r^2} = - \frac{\rho A}{m_p + \rho A} \frac{v}{v_r} \left[ \left( \frac{\rho A}{m_p + \rho A} \frac{v}{v_r} \right)^2 + \left[ 1 - \left( \frac{v}{v_r} \right)^2 \right] \right]. \] (14)

In the range of \( (v/v_r) \leq 1 \), the imaginary root can be rejected in equation (14) to obtain
\[ \frac{\omega_r}{\omega_r^2} = - \frac{\rho A}{m_p + \rho A} \frac{v}{v_r} \left[ \left( \frac{\rho A}{m_p + \rho A} \frac{v}{v_r} \right)^2 + \left[ 1 - \left( \frac{v}{v_r} \right)^2 \right] \right], \] (15)
where the frequency \( \omega_r \) is called the fundamental frequency or fundamental harmonic. Equation (15) shows that if the flow velocity of fluid in the pipeline is equal to zero, \( \omega_r = \omega_r^2 \).

In addition, the mass ratio \( \rho A/(m_p + \rho A) \rightarrow 0 \) or is equal to zero. Then, the simplified result of equation (15) is
\[ \frac{\omega_r}{\omega_r^2} = \sqrt{1 - \left( \frac{v}{v_r} \right)^2}. \] (16)

On this basis, the variation patterns of the inherent characteristics of the hydraulic pipeline system, including the inherent frequency, critical flow rate, and inherent frequency ratio \( \omega_r/\omega_r^2 \), with the flow ratio \( v/v_r \) can be analyzed.

4. Dynamic Behavior of the Hydraulic Pipeline System

Similarly, substitution of equation (4) into equation (2) gives
\[ \left[ \frac{EI}{\rho A} \frac{\partial^4 Y(x)}{\partial x^4} + \rho \frac{Av^2}{\rho A} \frac{\partial^2 Y(x)}{\partial x^2} \right] F(t) + 2 \rho A v \frac{\partial Y(x)}{\partial x} \frac{\partial F(t)}{\partial t} \]
\[ + \left( m_p + \rho A \right) Y(x) \frac{\partial^2 F(t)}{\partial t^2} = F_0 \sin \omega t = \text{Im}(F_0 e^{i\omega t}), \] (17)
where \( \text{Im}(e^{i\omega t}) = \sin \omega t \) is the imaginary part of \( e^{i\omega t} \).

Substituting equations (9) and (10) into equation (17) and taking the modulus of the complex number with consideration of \( |e^{i\omega t}| = 1 \) yields the following equation:

\[ |C_r| = \left| \frac{F_0}{\frac{EI}{\rho A} \frac{\partial^4 Y(x)}{\partial x^4} + \rho \frac{Av^2}{\rho A} \frac{\partial^2 Y(x)}{\partial x^2} - 2 \rho A v \frac{\partial Y(x)}{\partial x} \frac{\partial F(t)}{\partial t} + \left( m_p + \rho A \right) Y(x) \frac{\partial^2 F(t)}{\partial t^2}}{2 \rho A v} \right| \] (18)

and rearrangement of terms, equation (18) can be expressed as
\[ |C_r| = \frac{\Delta p L}{\frac{EI}{\rho A} \frac{\partial^4 Y(x)}{\partial x^4} + \rho \frac{Av^2}{\rho A} \frac{\partial^2 Y(x)}{\partial x^2} - 2 \rho A v \frac{\partial Y(x)}{\partial x} \frac{\partial F(t)}{\partial t} + \left( m_p + \rho A \right) Y(x) \frac{\partial^2 F(t)}{\partial t^2}}{2 \rho A v} \] (19)
The relevant time function $F_r(t)$ can be obtained by solving equation (18) or (19). Hence, the response of the fluid-solid coupling vibrations in the hydraulic pipeline system is

$$y(x, t) = \sum_{r=1}^{\infty} Y_r(x)F_r(t) = \sum_{r=1}^{\infty} \sin \frac{r \pi}{L} x C_r \sin \omega t. \quad (20)$$

On this basis, the dynamic behavior of the hydraulic pipeline system can be analyzed.

### 5. Numerical Examples

In a hydraulic pipeline system, the fluid flows from an external container into the pipeline. The parameters of the system are as follows: $L = 1$ (m) (pipeline length), $d = 16$ (mm) (pipeline inner diameter), $\delta = 3$ (mm) (pipeline wall thickness), $\rho = 900$ (kg/m$^3$) (fluid density), $E = 2.06 \times 10^5$ (MPa) (elastic modulus of pipeline), $\gamma = 7.85 \times 10^3$ (kg/m$^3$) (density of empty pipeline), and $v = 2$ (m/s) (fluid velocity). Given that $K = 1.226 \times 10^3$ (MPa) (fluid bulk modulus), $\Delta p = 3.6$ (KPa) (fluid flow pressure fluctuations), and $\omega = 3$ (1/s) (fluid flow fluctuation frequency), the vibrations in the fluid pipeline system are analyzed herein.

Based on the known conditions and the given data, the first three orders of the inherent frequency $\omega_r$ and the inherent frequency without consideration of the fluid flow motion $\omega'_r$ of the hydraulic pipeline systems as well as the relative errors between the two can be calculated as shown in Table 1.

| Vibration order/inherent frequency | Inherent frequency $\omega_r$ (1/s) | Inherent frequency without consideration of fluid flow motion $\omega'_r$ (1/s) | Relative error $(\% \{\omega'_r - \omega_r\}/\omega_r) \times 100\%$ |
|-----------------------------------|-----------------------------------|-------------------------------------------------|---------------------------------|
| First order                       | $3.187148 \times 10^2$           | $3.196776 \times 10^2$                           | $0.301178$                      |
| Second order                      | $1.276795 \times 10^3$           | $1.278710 \times 10^3$                           | $0.149760$                      |
| Third order                       | $2.874232 \times 10^3$           | $2.877098 \times 10^3$                           | $0.099614$                      |

The fluid flow motion has little effect on the inherent frequency, and the relative error of the inherent frequency decreases as the harmonic order increases. Therefore, for the sake of convenience in practical engineering applications, the inherent frequency without consideration of fluid flow motion $\omega'_r$ can be used to replace the inherent frequency with consideration of fluid flow motion $\omega_r$.

Based on the known conditions and the given data, the variation in the inherent frequency ratio of the pipeline $\omega_r/\omega'_r$ with the fluid flow velocity ratio $v/v_r'$ is obtained, as shown in Figure 2.

Figure 1 and equation (15) show that as the fluid flow velocity increases, the inherent frequency decreases; if the fluid flow velocity of the pipeline system is in a critical state, $\omega_r = 0$.

Based on the known conditions and the given data, the variation in the first three orders of the relevant time function $F_r(t)$ of the pipeline system with time $t$ (s) is calculated, with the corresponding curves shown in Figure 3.

![Figure 2: The change curve of natural frequency ratio with liquid velocity ratio of the pipe.](image1)

**Figure 2: The change curve of natural frequency ratio with liquid velocity ratio of the pipe.**

Based on the known conditions and the given data, the variation in the first three orders of the amplitude $C_r$ of the relevant time function $F_r(t)$ with the fluid flow velocity ratio $v/v_r'$ is calculated, with the corresponding curves shown in Figure 4.

When $v = v_r'$, the vibration amplitude is very large, and the hydraulic pipeline system becomes unstable.

Based on the known conditions and the given data, the variation in the first three orders of the amplitude $C_r$ of the relevant time function $F_r(t)$ with the frequency ratio $\omega/\omega_r'$ of the pipeline is calculated, with the corresponding curves shown in Figure 5.

When $\omega = \omega_r'$, the vibration amplitude tends to infinity, and the hydraulic piping system resonates.
Based on the known conditions and the given data, the variation in the superposition of the first three orders of vibration response $y(x, t)$ of the hydraulic pipeline system with time $t$ (s) is calculated, with the corresponding curved surface shown in Figure 6.

With increasing pressure of the hydraulic pipeline system and the coupling effect of the hydraulic system vibration, the fluid flow fluctuations of the hydraulic pipeline system increase, and the inherent frequency of the hydraulic pipeline system is close to the pressure pulsation frequency. This can certainly induce intense vibrations and even resonance in the hydraulic pipeline system, which results in the occurrence of fluid-solid coupling vibrations in the hydraulic pipeline system, posing a challenge in the design of the hydraulic pipeline system. In engineering practice, vibration problems in hydraulic pipeline systems are usually solved by changing the positions of pipeline supports as well as by varying the fluid flow velocity, pipeline diameter, and pipeline direction. However, research on fluid-solid coupling vibrations in hydraulic pipeline systems offers theory and technology to explain and solve the vibration problems of hydraulic pipeline systems, based on which the dynamic behavior and characteristics of hydraulic pipeline systems can be fully studied.

To compare the accuracy, we present a comparison with reference [2], as shown in Figure 7.

Figure 7 indicates that the theoretical equations and numerical results proposed in this paper are consistent with famous work reference [2], and the derivation process in the paper is relatively simple and clear; the numerical results can present some qualitative conclusions.

6. Conclusion

The study of fluid-solid coupling vibrations in hydraulic pipeline systems can provide a theoretical basis for vibration suppression and design optimization of hydraulic pipeline...
systems with design requirements of high pressure, high velocity, and high reliability. The dynamic characteristics of the fluid pipeline system were studied. A theoretical model of the vibrations in a fluid pipeline system was established that can be used for practical calculation, the vibrations in the fluid pipeline system under the specified operating conditions were estimated or predicted, the mechanism by which the fluid pipeline system generates vibrations was revealed, and an analytical method for analyzing the fluid-solid coupling vibrations in a hydraulic pipeline system with two hinged ends was presented. However, the application of this method and the approximate treatment can analyze the dynamic behavior and inherent characteristics of the fluid-solid coupled vibrations in the actual hydraulic pipeline system, which can effectively improve the anti-vibration design of the actual fluid pipeline system.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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