Bi-level planning model of conventional missile multi-strike strike maneuvers

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Abstract. In order to optimize the maneuvering route of missile launchers in multi-wave fire strikes of conventional missiles, a bi-level planning model for maneuvers with two-strike fire strikes and a multi-tiered maneuvering route model for multi-strike fire strikes are constructed; The dissertation summarizes the basic algorithms for solving such models, and specifically designs a branch and bound algorithm for solving such problems.

1. Preface
With the rapid development of military science and technology, multi-wave operations of conventional missiles have become possible. In the large-scale and multi-wave missile operations, the missile unit will undergo launch from the standby position to the launch position. After the launch is complete, it will be transferred and loaded from the launch position to the converted position, and then will be maneuvered to the launch position for launch. In the context of large missiles, optimizing the missile launcher's travel route and reasonably calculating the travel route, travel time, launch time, and order of transfer of each launcher during each wave are of great significance to the missile force's efficient combat.

2. Model establishment [2]

2.1. Basic ideas
The research idea of the multi-level planning method is to divide the multi-stage planning problem into the upper and lower levels of the bi-level planning, and adjust the results through the feedback mechanism between the upper and lower levels. The specific research ideas are as follows:
The upper model of a conventional missile multi-wave-strike bi-level planning model describes the first wave of maneuvers, that is, the launcher launches from the standby position to the launch position; the lower model describes the second wave of the maneuvering path, that is to say the launching maneuvering process of the launching device from the launching position to the transfer position to another launching position. The upper layer optimizes the first wave of the optimal path, the lower layer selects the path within the constraints, and performs feedback adjustment on the upper layer. After the upper layer’s path adjustment changes, the lower layer is affected and the lower layer adjusts again. Through multiple feedback adjustments, the balance between the upper and lower layers is obtained, and a feasible solution to achieve the best wave strike is found.

2.2. Objective Function of Bi-level Programming Model

In order to achieve the purpose of destroying the enemy’s own operations, the shorter the exposure time of the launcher, the better. The exposure time of the launching device during the entire combat consists of three parts: the maneuvering time of the launching device, the waiting time of the launching device at the node and the launching time of the launching device at the launching position.

(1) Motor time

\[ T_{1i} = \sum_{i=1}^{\text{motor}} \sum_{u=1}^{\text{motor}} \sum_{t=1}^{\text{motor}} D_{iu} S_{iv} \frac{z_{iu}}{V} \]  \hspace{1cm} (1)

\[ T_{2i} = \sum_{i=1}^{\text{motor}} \sum_{u=1}^{\text{motor}} \sum_{t=1}^{\text{motor}} S_{iu v} \frac{z_{iu}}{V} \]  \hspace{1cm} (2)
Among them, $T1_1$ Indicates the first wave of maneuvers, $T2_2$ Indicates the second wave of maneuvers; When $D_v$ is 1, it means $i$ launcher is in the $v$ standby position, When $D_v$ is 0, it means $i$ launcher is not in the $v$ standby position; $z_u$ is 1 indicates that the $i$ emitting device is in the $u$ emitting position, $z_u$ is 0 indicates that the $i$ emitting device is not in the $u$ emitting position; $S_{vu}$ indicates the distance between $v$’s standby position and $u$’s firing position; $V$ Indicates the launcher’s maneuvering speed; $S_{zu'}$ means that after the first wave is completed, $i$ launcher reaches the $u'$ launcher's distance at the $u$ launch position via the $z$ transfer position.

(2) Waiting time for waiting at the node

In the completion of the first wave launch mission, the total vehicle waiting time for all launch devices $T1_2$ is:

$$T1_2 = \sum_{i=1}^{n} \sum_{k=1}^{m} (t_{ik_i} - t_{ik_2})$$

(3)

In the completion of the second wave launch mission, the total vehicle waiting time for all launch devices $T2_2$ is:

$$T2_2 = \sum_{i=1}^{n} \sum_{k=1}^{m} (t_{ik'_i} - t_{ik'_2})$$

(4)

Among them, $t_{ik_i}$ and $t_{ik'_i}$ indicate the time at which the $i$ emitting device arrives at node $k$ and $k'$ respectively. $t_{ik_2}$ and $t_{ik'_2}$ denote the time at which the $i$ emitting device leaves node $k$ and $k'$ respectively.

(3) Launch point waiting time

The waiting time $T1_3$ for all transmitting devices at the launching point after completing the first wave launch mission is:

$$T1_3 = \sum_{i=1}^{n} \sum_{u=1}^{u'} (t_{iu_i} - t_{iu_2})$$

(5)

The waiting time $T2_3$ for all transmitting devices at the launching point after completing the second wave launch mission is:

$$T2_3 = \sum_{i=1}^{n} \sum_{u'=1}^{u'} (t_{iu_i} - t_{iu'_2})$$

(6)

Among them, $t_{iu_i}$, $t_{iu'_i}$ indicate the time at which the $i$ emitting device arrives at node $u$ and $u'$ respectively. $t_{iu_2}$, $t_{iu'_2}$ denote the time at which the $i$ emitting device leaves node $u$ and $u'$ respectively.

(4) Objective function

According to formulae (1), (3), (5), the objective function of the second layer model is established:
According to formulae (2), (4), (6), the objective function of the second layer model is established:

$$\min T1 = T1_1 + T1_2 + T1_3$$

$$= \sum_{i=1}^{n} \sum_{u=1}^{m} D_{uv} z_{iu} \frac{z_{iu}}{V} + \sum_{i=1}^{n} \sum_{k=1}^{m} (t_{ik} - t_{ik_2}) + \sum_{i=1}^{n} \sum_{u=1}^{m} (t_{iu} - t_{iu_2})$$

(7)

According to formulae (2), (4), (6), the objective function of the second layer model is established:

$$\min T2 = T2_1 + T2_2 + T2_3$$

$$\sum_{i=1}^{n} \sum_{u=1}^{m} S_{iu} z_{iu} \frac{z_{iu}}{V} + \sum_{i=1}^{n} \sum_{k=1}^{m} (t_{ik} - t_{ik_2}) + \sum_{i=1}^{n} \sum_{u=1}^{m} (t_{iu} - t_{iu_2})$$

(8)

2.3. Bi-level planning model constraints

Due to the condition of the firing position, each launch point can only accommodate one launcher at a time when each mission is performed:

$$\sum_{i=1}^{u} z_{iu} = 1 \quad u = 1, 2\ldots$$

(9)

At the same time, each launcher can only launch at one launch position:

$$\sum_{u=1}^{u} z_{iu} = 1 \quad i = 1, 2\ldots$$

(10)

Different earth projections of different missiles on the same wave cannot intersect, and the coordinates of the intersection between missile aiming direction and critical point [6] $J(x_j', y_j', z_j')$:

$$\begin{align*}
  x_j' &= \frac{-B + \sqrt{B^2 - 4AC}}{2A} \\
  y_j' &= \frac{mp'_{y}}{lp'}(x_j' - x_{og}) + y_{og}' \\
  z_j' &= \frac{np'}{mp'}(y_j' - y_{og}) + z_{og}'
\end{align*}$$

(11)

Among them, $A = 1 + \frac{mp'^2}{lp'^2}$, $B = 2z_{og}' \frac{np'^2}{lp'^2} - 2 \frac{np'^2}{lp'^2}$, $(lp, mp, np)$ is the directional vectors for missiles, The missile base coordinates are $(x_{og}, y_{og}, z_{og})$, $C = \frac{np'^2}{lp'^2} z_{og}^2 - 2 \frac{np'}{lp'} x_{og}' + z_{og}'^2 - D^2$, $D$ is the missile launch cylinder radius, if $\sqrt{B^2 - 4AC} < 0$, there is no intersection, that is, firepower does not cross. In a two-wave launch mission, once the firing position $u$ is used, it must not be used again:

$$\sum_{m=1}^{m} z_{um} = 1 \quad u = 1, 2\ldots$$

(12)
Among them, \( z_{um} \) indicates that \( u \) firing position is used in \( m \) waves. In the event of a road conflict, if the waiting time exceeds the time that the transmitting device passes through this section of the road, the road is abandoned, and the waiting time of the transmitting device is not more than half of the total time of passing the road.

\[
Tm_{2ij} \leq \frac{l_{ij}}{2V}
\]

**2.4. Construction of a Bi-level Planning Model**

(U) \( \min TL = T1_1 + T1_1 + T1_1 + \sum_{i=1}^{n} \sum_{u=1}^{m} D_{iu} \sum_{i=1}^{m} (t_{ik} - t_{ik}) + \sum_{i=1}^{m} (t_{iu} - t_{iu}) \)

\[
\sum_{j=1}^{m} x_{ij} = 1 \quad i = 1,2,...
\]

\[
\sum_{j=1}^{m} z_{iu} = 1 \quad u = 1,2,...
\]

s.t.: \( \sum_{u=1}^{m} z_{iu} = 1 \quad i = 1,2,... \)

\[
\sqrt{B^2 - 4AC} < 0
\]

\[
Tm_{2ij} \leq \frac{l_{ij}}{2V}
\]

(L) \( \min T2 = T2_1 + T2_1 + T2_1 + \sum_{i=1}^{n} \sum_{u=1}^{m} S_{iu} \sum_{i=1}^{m} (t_{ik} - t_{ik}) + \sum_{i=1}^{m} (t_{iu} - t_{iu}) \)

s.t.: \( \sum_{u=1}^{m} z_{iu} = 1 \quad u = 1,2,... \)

For the multi-wave stratified multi-level planning model, its ideas and model expressions are similar to those of the bi-level planning model, and will not be repeated here.

**3. Model algorithm design**

The bi-level programming algorithm [4] mainly discusses six methods: pole search method, branch and bound method, complementary rotation method, descent direction method, penalty function method and intelligent optimization algorithm. This paper proposes a branch-and-bound algorithm suitable for this model to solve the optimal maneuvering scheme. The algorithm comprehensively applies the breadth-first search and depth-first search methods, starting with breadth-first search. In the breadth-first search, one or more infringement complementary slack conditions select two or more sub-problems to establish and solve. By introducing Lagrange multipliers \( \mu \) and \( \lambda \), the original bi-level programming model is transformed into a single-level planning model. In each instance, all complementary relaxation conditions satisfy \( \mu_i = 0 \) and \( G_i = 0 \) various combinations of constraints. When the number of live nodes becomes excessive, it switches to depth-first search. In the depth-first search, a solution that violates constraints is selected, and two sub problems are established, the first corresponding to \( \mu_i = 0 \) and the second corresponding to \( G_i = 0 \). In both cases, the constraints are satisfied. Get the solution of one of the two sub-problems and repeat this process until all the constraints are satisfied. When the algorithm terminates, all the problems are solved or known as suboptimal or infeasible solutions. Let
Let $T_U$ be the upper constraint variable, $T_L$ the lower variable, $G(T_U, T_L)$ be the inequality constraint condition vector, and $H(T_U, T_L)$ be the equality constraint condition vector. Reduce the original planning model to:

$$
\min T1 = \sum_{i=1}^{n} \sum_{u=1}^{m} D_{i,u} S_{iu} z_{iu} + \sum_{i=1}^{n} \sum_{k=1}^{m} (t_{ik} - t_{ik}) + \sum_{i=1}^{n} \sum_{u=1}^{m} (t_{iu} - t_{iu})
$$

$$
\nabla T_L \sum_{i=1}^{n} T2(T_U, T_L) + \mu \nabla T_L G(T_U, T_L) + \lambda \nabla T_L H(T_U, T_L)
$$

$$
s.t.: \begin{align*}
H(T_U, T_L) \\
\mu \geq 0 \\
\mu G(T_U, T_L)
\end{align*}
$$

Branch and delimitation algorithm process shown in Figure 2:

4. examples
For a given road network with 2 standby positions, 5 relocated positions, 60 firing positions, and 62 road nodes, the path plan for the two wave strike problems is shown in the specific road network shown in the figure. Each wave uses 24 launchers. Each node represents a standby position (D), a retransmission position (Z), a firing position (F), and a road node (J), a blue road represents a one-way street, and a red road represents a two-way street.
The maneuver scheme obtained through the solution at the beginning is shown in Table 1:

| D  | F(1) | Z  | F(2) | D  | F(1) | Z  | F(2) |
|----|------|----|------|----|------|----|------|
| D1 | F14  | Z1 | F17  | D2 | F48  | Z1 | F49  |
|    | F15  |    | F18  |    | F49  |    | F50  |
|    | F21  |    | F19  |    | F50  |    | F51  |
|    | F20  | Z1 | F23  |    | F51  | Z1 | F52  |
|    | F12  | Z2 | F24  |    | F52  | Z2 | F53  |
|    | F11  |    | F25  |    | F53  |    | F54  |
|    | F41  |    | F37  |    | F54  |    | F55  |
|    | F43  |    | F38  |    | F55  |    | F56  |

The total maneuvering distance is 5017.231km. The total maneuver time is 154.2h.

5. Conclusion
In this paper, the multi-wave fire attack problem of conventional missiles is analyzed. A two-level planning model of multi-wave fire attack and a multi-level planning model of multi-wave fire attack are established. The bi-level programming algorithm is studied. The branch-and-bound algorithm is selected to investigate the problem and the basic steps of calculation are given. The bi-level programming model can complete the multi-wave maneuver launch in better time on the basis of satisfying the constraint conditions. It is of great practical significance to rationalize the allocation of firepower, to better deploy the positions and to relocate positions, and to improve the combat effectiveness of missile units.
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