Mechanics of non-Newtonian blood flow in an artery having multiple stenosis and electroosmotic effects

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Abstract
The electro-osmotically modulated hemodynamic across an artery with multiple stenosis is mathematically evaluated. The non-Newtonian behaviour of blood flow is tackled by utilizing Casson fluid model for this flow problem. The blood flow is confined in such arteries due to the presence of stenosis and this theoretical analysis provides the electro-osmotic effects for blood flow through such arteries. The mathematical equations that govern this flow problem are converted into their dimensionless form by using appropriate transformations and then exact mathematical computations are performed by utilizing Mathematica software. The range of the considered parameters is given as 0.03 < δ₁ < 0.12, 2 < m < 3.5, 0.03 < Q < 2, 0 < U_HS < 3, 2 < B_r < 2.9, 0.01 < S < 0.025. The graphical results involve combine study of symmetric and non-symmetric structure for multiple stenosis. Joule heating effects are also incorporated in energy equation together with viscous effects. Streamlines are plotted for electro-kinetic parameter m and flow rate Q. The trapping declines in size with incrementing m, for symmetric shape of stenosis. But the size of trapping increases for the non-symmetric case.

Keywords
Electro-osmosis, multiple stenosis, Casson fluid, Joule heating

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Introduction

The electro-osmotic phenomenon emerges when a channel under consideration is filled with an electrolyte solution and then by application of a high voltage, a charge is produced at inner surface of this tube when this electrolyte comes in contact with inner walls. Finally, the flow is developed due to this electric field.\textsuperscript{1} Electro-osmosis has immense uses in medical field and helps in treatment of diseases like cellular anomalies, sickle cells, and delivery of drugs by using diagnostic kits.\textsuperscript{2} The capillary electro-kinetic detailed study and various micro-chip methods are addressed.\textsuperscript{3} Wu and Papadopoulos\textsuperscript{4} had presented a mathematical model that compares the cylindrical and annulur electro-kinetic flows. The electro-kinetically produced flow between two parallel plates was mathematically studied by Yang et al.\textsuperscript{5} Zhao et al.\textsuperscript{6} had mathematically examined the two dimensional flow of a power law fluid by application of electroosmosis. The electro-kinetic flow of non-Newtonian fluids in small length tubes was first time interpreted by Tang et al.\textsuperscript{7} Liu et al.\textsuperscript{8} had interpreted the micro-slit channel flow using Jeffery fluid model by utilizing electro-kinetic mechanism. The Bingham plastic fluid’s electro-osmotic flow across a micro length channel was mathematically examined by Nadeem et al.\textsuperscript{9} Some of the recent research articles that interpret the electro-osmotic flow phenomenon are referred by Narla and Tripathi,\textsuperscript{10} Tripathi et al.,\textsuperscript{11} Akram et al.,\textsuperscript{12} and Saleem et al.\textsuperscript{13}

The blood arteries with stenosis result in restriction of hemodynamics across these diseased arteries. In some certain conditions, such arteries may also have more than one stenosis. The study of flow across such multiple stenosed arteries is also a topic of recent interest for researchers. The flow across such stenosed arteries was firstly reported by Ponalagusamy\textsuperscript{14} in his doctoral dissertation. This arterial study of mild stenosis is also covered for stenosis with various shapes.\textsuperscript{15} Varshney et al.\textsuperscript{16} had presented the mathematical study of a non-Newtonian fluid flow across a channel with multiple stenosis. Sreenadh et al.\textsuperscript{17} had studied the flow of blood across a multiple stenosed tube, treating blood as a Casson fluid. Nadeem and Ijaz\textsuperscript{18} had mathematically examined the blood flow across a multiple stenosed tube with variable fluid properties. The blood flow across such diseased multiple stenosed arteries, considering distinct models of non-Newtonian fluids is given.\textsuperscript{19–27}

The heat transfer study of blood flow across an artery with multiple stenosis was interpreted by Tashtoush and Magableh.\textsuperscript{28} The analysis of heat phenomenon for a mild stenotic tube, considering two phase blood flow model was conveyed by Ponalagusamy and Selvi.\textsuperscript{29} The heat transfer analysis of Williamson blood flow model for a stenotic tube was mathematically interpreted by Akbar et al.\textsuperscript{30} The heat transfer details combined with dissipation effects and Joule heating for an electro-kinetically developed flow was studied by Sadeghi and Saidi.\textsuperscript{31} Moreover, some further recent researches that evaluate blood flow as well as heat transfer are referred by Yan et al.,\textsuperscript{23} Li et al.,\textsuperscript{32} Ho et al.,\textsuperscript{33,34} and Chien et al.\textsuperscript{35}

The review of literature has shown that the electro-osmotic flow of blood across a multiple stenosed artery is not mathematically considered yet. We have analyzed the electro-kinetic flow of blood across an artery with multiple stenosis. The non-
Newtonian behavior of blood is incorporated by using Casson fluid model for this problem. In order to describe a thorough heat transfer mechanism, Joule heating effect is also incorporated together with viscous dissipation. Exact mathematical solutions are prevailed for governing flow equations. Further, these results are studied in detail with graphs.

**Mathematical model**

The electro-osmotically driven hemodynamic across an artery with multiple stenosis is studied. The non-Newtonian behaviour of blood is considered by utilizing Casson model for this flow problem.

The multiple stenosis wall geometry $\eta(z)$, with its dimensional mathematical expression (Figure 1). $^{36}$

$$\eta(z) = \begin{cases} \frac{R}{1 - K \{s_i^{n-1}(z - d_i) - (\bar{z} - d_i)^n\}}, & d_i \leq \bar{z} \leq d_i + s_i \\ R & \text{otherwise} \end{cases}. \quad (1)$$

The expression for value of $K$ is

$$K = \frac{\delta_i^* n_i}{R s_i^n} \frac{n_i/n - 1}{n - 1}, \quad (2)$$

The governing mathematical equations that manipulate the incompressible flow of Casson fluid are

\[
\begin{align*}
\frac{\partial \bar{u}}{\partial t} + \bar{u} \cdot \nabla \bar{u} + \nabla p &= 0, \\
\nabla \cdot \bar{u} &= 0, \\
\frac{\partial \bar{u}}{\partial t} + \bar{u} \cdot \nabla \bar{u} + \nabla p &= \frac{1}{\sigma} \frac{\partial \tau}{\partial \bar{u}}, \\
\nabla \cdot \tau &= \frac{1}{\sigma} \frac{\partial \tau}{\partial \bar{u}},
\end{align*}
\]
\[
\frac{\partial \bar{u}}{\partial r} + \frac{\bar{u}}{r} + \frac{\partial \bar{w}}{\partial z} = 0,
\]

\[
\rho \left( \frac{\partial \bar{u}}{\partial r} + \bar{w} \frac{\partial \bar{u}}{\partial z} \right) = -\frac{\partial \bar{p}}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} (r \bar{S}_{r\bar{r}}) + \frac{\partial}{\partial z} (\bar{S}_{\bar{r}z}),
\]

\[
\rho \left( \frac{\partial \bar{w}}{\partial r} + \bar{w} \frac{\partial \bar{w}}{\partial z} \right) = -\frac{\partial \bar{p}}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (r \bar{S}_{r\bar{z}}) + \frac{\partial}{\partial z} (\bar{S}_{\bar{z}z}) + \rho_c E_z,
\]

\[
\rho C_p \left( \frac{\partial \bar{T}}{\partial r} + \bar{w} \frac{\partial \bar{T}}{\partial z} \right) = \bar{S}_{r\bar{r}} \frac{\partial \bar{u}}{\partial r} + \bar{S}_{r\bar{z}} \frac{\partial \bar{w}}{\partial r} + \bar{S}_{\bar{z}r} \frac{\partial \bar{u}}{\partial z} + \bar{S}_{\bar{z}z} \frac{\partial \bar{w}}{\partial z} + k \left( \frac{\partial^2 \bar{T}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{T}}{\partial r} + \frac{\partial^2 \bar{T}}{\partial z^2} \right) + s^*,
\]

The Casson fluid’s extra stress tensor.\(^{37}\) The Casson fluid model is chosen to consider the non-Newtonian nature of blood.

\[
\bar{S}_{ij} = (\mu_B + \frac{p_v}{\sqrt{2 \pi c}}) \bar{e}_{ij},
\]

Where

\[
\bar{e}_{ij} = \frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i},
\]

The value of \(s^*\) given in equation (6) is \(s^* = \mu^2 \sigma\), where \(i_e = \frac{E}{\sigma}.\) An electrolyte mixture (\(Na^+ Cl^-\)) is uniformly considered and it’s electrical potential dispersion is mathematically expressed by Poisson-Boltzmann equation as

\[
\nabla^2 \bar{\Phi} = -\frac{\rho_e}{E},
\]

The value of \(\rho_e = ez^* (n^+ - n^-)\), the density of ionic energy, when “no EDL overlap” is considered, is given

\[
n^\pm = n_0 \text{Exp} \left( \frac{ez^* \bar{\Phi}}{K_BT^*} \right),
\]

Now substituting the value of \(\rho_e\) and \(n^\pm\) in equation (9), we get

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial \bar{\Phi}}{\partial r} \right) = \frac{2n_0 ez^* \text{Sinh} \left( \frac{ez^* \bar{\Phi}}{K_BT^*} \right)}{E},
\]

The Debye-Huckel approximation is utilized and we get \(\text{Sinh} \left( \frac{ez^* \Phi}{K_BT^*} \right) \approx ez^* \frac{\Phi}{K_BT^*}\). Also the non-dimensional variables \(\Phi = \bar{\Phi}/\zeta, r = \bar{r}/R\) are used in equation (11) and we get
The Exact solution of equation (12) is obtained with these conditions \( \frac{\partial \Phi}{\partial r} = 0, atr = 0, \) and \( \Phi = 1atr = \eta(z) \).

\[
\Phi = \frac{I_0(mr)}{I_0(m\eta)},
\]

(13)

The variables used in their dimensionless form are

\[
r = \frac{\bar{r}}{R}, z = \frac{\bar{z}}{s_l}, w = \frac{\bar{w}}{u_0}, u = \frac{\bar{u}}{u_0\delta_l^*}, p = \frac{\bar{p}}{u_0s_l\mu_f}, \theta = \frac{T - T_0}{T_0},
\]

(14)

The following assumptions are used in this study, in order to consider mild case of multiple stenosis

\[
\delta_l = \frac{\delta_l^*}{R} \ll 1, \frac{Rn}{s_l} \frac{1}{\sqrt{1 - 1}} \sim 1
\]

(15)

The dimensionless form of boundary conditions is

\[
\frac{\partial w}{\partial r} = 0 atr = 0, \frac{\partial w}{\partial r} = 0 atr = \eta,
\]

(19)

\[
\frac{\partial \theta}{\partial r} = 0 atr = 0, \frac{\partial \theta}{\partial r} = 0 atr = \eta,
\]

(20)

The dimensionless mathematical form of multiple stenosis wall is
\[ \eta(z) = \begin{cases} 1 - \delta_t \left(\frac{n}{n-1}\right)^{\eta - 1}[(z - h_t) - (z - h_t)^n], & h_t \leq z \leq h_t + 1, \\ 0 & \text{otherwise} \end{cases} \]  

(21)

**Exact solution**

The mathematical solution of axial velocity is

\[ w(r, z) = \frac{\beta \left[4U_{HS} + \frac{dp}{dz}(r^2 - \eta^2) - \frac{4U_{HS}I_1(m\eta)}{I_0(m\eta)}\right]}{4(1 + \beta)}, \]  

(22)

The volume rate of flow is evaluated by considering

\[ Q = 2\pi \int_0^\eta rwdr, \]  

(23)

Thus, the mathematical result for pressure gradient is

\[ \frac{dp}{dz} = \frac{8}{\eta^4} \left[-\frac{Q(1 + \beta)}{\pi\beta} + \frac{U_{HS}\eta^2 - 2U_{HS}\eta I_1(m\eta)}{mI_0(m\eta)}\right]. \]  

(24)

The shear stress at multiple stenosed wall is provided

\[ \tau_w = -\frac{\partial w}{\partial r} \bigg|_{r=\eta} = -\frac{\beta \left[2\frac{dp}{dz}\eta - \frac{4mU_{HS}I_1(m\eta)}{I_0(m\eta)}\right]}{4(1 + \beta)}, \]  

(25)

The exact temperature profile solution is

\[ \theta(r, z) = \frac{1}{64m^2(1 + \beta)(I_0(m\eta))^2} \]  

(26)

**Results and discussion**

The mathematical solutions acquired in above portion are explained in detail with graphical results. In Figure 2(a)–(d), the velocity graphs are represented for enlarging values of distinct physical parameters. It is observed in Figure 2(a) that there is enhance in velocity at the centre, as the value of \( \delta_t \) increases but it declines toward walls with multiple stenosis. The velocity gains magnitude due to narrowing of channel with incrementing \( \delta_t \) but at the same time velocity reduces toward walls with multiple stenosis. Figure 2(b) depicts that velocity declines with increasing \( m \). In Figure 2(c), it is seen that velocity increments with enhancing values of \( Q \). Figure 2(d) shows the decline in velocity for increasing \( U_{HS} \). Moreover, velocity
attains its highest value for zero $U_{HS}$ and gradually declines with enhancing value of $U_{HS}$. Thus, the speed of flow can mainly be governed by electric field that is axially applied. It is seen in these graphs of velocity that the increase in velocity is less for non-uniform shape as compared to uniform shape of multiple stenosis. The shear stresses at walls having multiple stenosis are plotted for various parameters and shown in Figure 3(a)–(d). The value of $\tau_w$ increments with increasing $\delta$, provided in Figure 3(a). Also, there is enhance in $\tau_w$ with increasing electro-kinetic parameter $m$, given in Figure 3(b). Figure 3(c) convey that $\tau_w$ gains magnitude, as the flow rate $Q$ enhances. In Figure 3(d), it is noted that $\tau_w$ increments for incrementing value of $U_{HS}$. The shear stress increases in all cases mainly due to “no slip” at walls. In Figure 4(a)–(f), the temperature graphs are displayed for varying values of involved parameters. Figure 4(a) shows enhance in temperature with incrementing $B_r$. Figure 4(b) depicts that there is enhance in temperature at the center but declines with walls having multiple stenosis, as the value of $\delta$ increases. There is decrease in temperature with incrementing $m$, displayed in Figure 4(c). The temperature gains magnitude for enhancing values of $Q$, given in Figure 4(d). Figure 4(e) depicts enhance in temperature with enhancing Joule heating $S$. There is

![Graphs showing velocity and shear stress](image)

**Figure 2.** (a) Velocity for $\delta$, (b) velocity for $m$, (c) velocity for $Q$, and (d) velocity for $U_{HS}$. 
decline in temperature, as the values of $U_{HS}$ increases, as shown in Figure 4(f). All these temperature graphs reveal that the increment in temperature for uniform shape of multiple stenosis is higher when compared with non-uniform shape. Further, both velocity as well as temperature declines with increasing axial electric field. Thus, the flow can be controlled in terms of both speed and temperature by application of electro-osmosis. Figure 5(a)–(d) show streamline graphs for increasing values of electro-osmosis parameter $m$, while both symmetric as well as non-symmetric shapes of multiple stenosis are considered. The trapping declines in size with incrementing $m$, for symmetric shape of stenosis. But there is enhance in size of trapped streamlines, for non-symmetric case. Further, the walls with multiple stenosis can clearly be seen in these streamlines. Also, the symmetric and non-symmetric shapes are clearly observed. The streamlines for flow rate $Q$ are also plotted and provided in Figure 6(a)–(d). When the multiple stenosis have symmetric shape, then there is enhance in size of trapping with incrementing $Q$. But when the shape is non-symmetric then the trapping declines in size for increasing $Q$. Moreover, when the multiple stenosis have symmetric shape then the trapping pattern is also symmetric in shape but it turns to be non-symmetric in shape, when the multiple stenosis have non-symmetric shape.
Conclusions

The electro-osmotically developed hemodynamics across an artery with multiple stenosis is examined. The non-Newtonian behaviour of blood is incorporated by using Casson fluid model. The important results are

- The speed of flow can mainly be governed by electric field that is axially applied.

Figure 4. (a) Temperature for $B_n$, (b) temperature for $d_l$, (c) temperature for $m$, (d) temperature for $Q$, (e) temperature for $S$, and (f) temperature for $U_{H5}$. 
The enhance in velocity is less for non-uniform shape as compared to uniform shape of multiple stenosis.

The medical advantages of electro-osmosis include treatment of cellular anomalies, sickle cells and delivery of drugs, etc.

The present analysis is limited due to theoretical approach and there is no experimental work performed during this research.

The flow can be controlled in terms of both speed and temperature by application of electro-osmosis.

The multiple stenosis have symmetric shape then the trapping is also symmetric in shape but it turns to be non-symmetric in shape, when the multiple stenosis have non-symmetric shape.

**Figure 5.** (a) Streamlines for \( m = 2, n = 2 \), (b) streamlines for \( m = 3, n = 2 \), (c) streamlines for \( m = 2, n = 6 \), and (d) streamlines for \( m = 3, n = 6 \).
Declaration of conflicting interests

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Figure 6. (a) Streamlines for \( Q = 1, n = 2 \), (b) streamlines for \( Q = 2, n = 2 \), (c) streamlines for \( Q = 1, n = 6 \), and (d) streamlines for \( Q = 2, n = 6 \).
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**Appendix**

**Notations**

\((\bar{r}, \bar{z})\) cylindrical coordinate system

\((\bar{u}, \bar{w})\) radial and axial velocity components

\(R\) non-stenotic radius of artery

\(d_l\) stenosis position \((l = 1, 2, 3)\)

\(s_l\) stenosis length \((l = 1, 2, 3)\)

\(B_r\) Brickman number

\(\lambda_d\) Debye-length

\(E_z\) axial electrical field

\(n_0\) ions concentration

\(U_{HS}\) Helmholtz-Smoluchowski velocity

\(z^*\) charge balance

\(i_e\) current density

\(n^+, n^-\) cation and anion densities

\(\zeta\) zeta potential

\(\mu_B\) plastic viscosity

\(n \geq 2\) multiple stenosis shape parameter

\(n = 2\) symmetric shape of multiple stenosis

\(n = 6\) non-symmetric shape of multiple stenosis

\(\delta^*_l\) maximum height of stenosis in dimensional form

\(\sigma\) electrical resistivity of fluid

\(\rho_e\) density of total ionic charge

\(T^*\) average temperature of electrolyte solution

\(e\) electronic charge

\(K_B\) Boltzmann Constant

\(S\) Joule heating parameter

\(m\) electro-osmotic parameter

\(E\) permittivity
\[ \phi \]  electro-kinetic potential function
\[ \beta \]  Casson parameter
\[ p_y \]  yield stress

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