KKMChh: matching CEEX photonic ISR to a QED-corrected parton shower

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KKMChh adapts the CEEX (Coherent Exclusive Exponentiation) formalism of the Monte Carlo Program KKMC for Z boson production and decay to hadron scattering. Amplitude-level soft photon exponentiation of initial and final state radiation, together with initial-final interference, is matched to a perturbative calculation to second order next-to-leading logarithm, and electroweak corrections to the hard process are included via DIZET. The first release of KKMChh included complete initial state radiation calculated with current quark masses. This version assumes idealized pure-QCD PDFs with negligible QED contamination. Traditional PDFs neglect QED evolution but are not necessarily free of QED influence in the data. QED-corrected PDFs provide a firmer starting point for precision QED work. We describe a new procedure for matching KKMChh’s initial state radiation to a QED-corrected PDF, and compare this to earlier approaches.

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1. Introduction

KKMChh [1, 2] adapts Coherent Exclusive Exponentiation (CEEX) [3] to hadronic collisions producing lepton pairs with multi-photon radiation, \( pp \rightarrow Z/\gamma^* \rightarrow ll+ny \). CEEX is an amplitude-level implementation of YFS exponentiation [4]. KKMChh includes hard photon residuals through order \( a^2 L \), where \( L \) is an appropriate “big logarithm.” A separate DIZET6.45 library [5] tabulates electroweak form factors before a MC run. KKMC [6] was originally designed for \( ee \) collisions. The was recently reprogrammed in C++ and released as KKMCee5.00.2 [7]. Here, we focus on the effect of ISR on the quark joint parton luminosity and describe “Negative ISR,” a recent addition to KKMChh to allow it to be used with parton distribution functions (PDFs) incorporating QED corrections.

2. Joint Parton Luminosity

We will examine the effect of ISR radiation on the colliding quarks by focusing on the joint parton luminosity function for a quark \( q \). Consider a collision between quarks with momentum fractions \( x_q, x_\bar{q} \) in a proton collision with CM energy \( \sqrt{s} \). Neglecting quark masses relative to the CM energy, the squared CM energy of the quark collision is \( \hat{s} \) with \( \hat{s} = x_q x_\bar{q} \). Without QED corrections, the cross section for \( pp \rightarrow Z/\gamma^* \rightarrow f\bar{f} \) would be proportional to the joint parton luminosity \( L_{q\bar{q}} \) defined in terms of PDFs \( f_q, f_\bar{q} \) by

\[
L_{q\bar{q}}(\hat{s}) = \int dx_q dx_\bar{q} \delta(\hat{s} - x_q x_\bar{q}) f_q(x_q, \hat{s}) f_\bar{q}(x_\bar{q}, \hat{s}).
\]  

(1)

Exponentiation of photonic ISR in a soft-photon approximation leads to an ISR radiation factor of the form [4, 6]

\[
\rho_{\text{ISR}}^{(0)}(v, \gamma) = F_{\text{YFS}}(\gamma) \gamma v^{\gamma - 1},
\]  

(2)

where \( v \) is the soft photon radiation fraction, leaving the quarks with squared CM energy \( xs = (1 - v)\hat{s} \), and

\[
F_{\text{YFS}}(\gamma) = \frac{e^{-CE\gamma}}{\Gamma(1 + \gamma)}
\]  

(3)

is the YFS form factor, \( CE \approx 0.57722 \) is Euler’s constant, and

\[
\gamma \equiv \gamma_{\text{ISR}}(\hat{s}, Q_q, m_q) = \frac{2q}{\pi} Q_q^2 \left[ \ln \left( \frac{\hat{s}}{m_q^2} \right) - 1 \right].
\]  

(4)

We use current quark masses \( m_d = 4.7 \text{ MeV}, m_u = 2.2 \text{ MeV}, m_s = 93 \text{ MeV}, m_c = 1.2 \text{ GeV}, \) and \( m_b = 4.7 \text{ GeV} \). [8] The superscript \( (0) \) indicates that this ISR radiation factor includes no fixed-order corrections, and is an approximation to the full \( \rho_{\text{ISR}}^{(2)} \) in KKMChh, which has fixed-order corrections through \( a^2 L \), where \( L = \ln(\hat{s}/m_q^2) \).

The joint quark luminosity function after photonic ISR is then

\[
L_{q\bar{q}}^{\text{QED}}(xs) = \int_0^{1-x} dv L_{q\bar{q}} \left( \frac{xs}{1-v} \right) \rho_{\text{ISR}}^{(0)}(v, \gamma_{\text{ISR}} \left( \frac{xs}{1-v}, Q_q, m_q \right)).
\]  

(5)

2
Fig. 1 shows the ratio of eq. 5 to eq. 1 for each quark, using NNPDF3.1 NLO PDFs. Over the narrower range on the left, the ratio varies slowly, and at $M_{q\bar{q}} = M_Z$, QED ISR reduces the joint luminosity distribution by $-1.2\%$ for the up quark and $-0.30\%$ for the down quark. Over the range $10-1000$ GeV, the up quark distribution varies from $+0.11\%$ to $-4.2\%$, while the down quark distribution varies less, from $+0.30\%$ to $-1.0\%$.

Figure 1: These graphs show the ratios of joint quark luminosity distributions $L_{q\bar{q}}^\text{QED} / L_{q\bar{q}}$ with and without photonic ISR for each quark in two ranges: $60 - 150$ GeV on the left and $10 - 1000$ GeV on the right. The graphs were produced using ManeParse [10] with NNPDF3.1 NLO PDFs [9].

3. PDF Matching via Negative Initial State Radiation

In its original form, KKMChh was intended to be used with PDF sets having a negligible photonic contribution, adding complete ab initio QED ISR to idealized PFDs representing pure QCD. Even without QED evolution, QED can influence a PDF via the data. It is conceptually cleaner to start with a PDF set that incorporates QED corrections consistently. To match KKMChh’s exponentiated ISR to a QED-corrected PDF set without double-counting, we introduced a feature called “Negative ISR,” or NISR, which backs out QED from the PDF using ISR radiator factors in reverse, starting from a scale $q_0$ which, for QED-corrected PDFs, we take to be the hard process scale $s_x$.

In short, the idea is to combine each PDF function with inverse “half-radiator” factors

$$
\rho_{\text{ISR}}^{(2)} \left( u_q, -\frac{1}{2} \gamma_{\text{ISR}}(\hat{x}s) \right), \quad \rho_{\text{ISR}}^{(2)} \left( u_{\bar{q}}, -\frac{1}{2} \gamma_{\text{ISR}}(\hat{x}s) \right)
$$

and define modified quark momentum fractions $x'_q = x_q (1 - u_q)$, $x'_{\bar{q}} = x_{\bar{q}} (1 - u_{\bar{q}})$ before ISR, so that when the PDFs are convoluted with these inverse half-radiators and the forward KKMChh radiator $\rho_{\text{ISR}}^{(2)} (v, \gamma_{\text{ISR}}(\hat{x}s))$, the original joint luminosity distribution is recovered, and ISR photons are generated with a modified momentum fraction $v'$ satisfying the constraint

$$
1 - v' = (1 - v)(1 - u_q)(1 - u_{\bar{q}}).
$$

When we calculated the inclusive quark-level Drell-Yan cross section for muons with $60 \text{ GeV} < M_{\mu\mu} < 150$ GeV using NNPDF3.1 NLO alone and then turned on KKMChh ISR with NISR, we
found that the original PDF-only result is recovered to within 0.3% for charge $\frac{2}{3}e$ quarks and 0.08% for charge $-\frac{1}{3}e$ quarks. Both of these differences are due to an $O(\alpha)$ non-logarithmic correction

$$
\delta_Q = Q^2 \alpha \pi \left( -\frac{1}{2} + \frac{\pi^2}{3} \right).
$$

This term is not included in NISR and precisely accounts for the difference seen. We also found that the quark-level cross sections with ISR and NISR are unchanged if the quark masses are all set to a high value of 500 MeV, demonstrating the quark mass independence of this procedure.

Figs. 2 and 3 show the dimuon invariant mass and transverse momentum of the hardest photon for approximately 200 million KKMChh events including at least one photon, with a requirement that each muon have transverse momentum $p_T \mu > 25$ GeV and pseudorapidity $|\eta_\mu| < 2.5$. All results except (3) (green) use NNPDF3.1-LuxQED NLO PDFs [12], while (3) uses standard NNPDF3.1 NLO PDFs [9]. The black histogram (0) on the left side of each figure includes FSR only with quarks generated using a LuxQED PDF set. The blue histogram (1) adds ISR to quarks generated using the LuxQED version without NISR, the red histogram (2) does the same with NISR, and the green histogram (3) adds ISR to quarks generated using the ordinary PDFs.

The center graph in each figure shows the fractional ISR contribution for cases (1), (2) and (3) in blue, red, and green, respectively. The red plot in the graph on the right of each figure is the difference between the fractional ISR contributions with and without NISR. We see that NISR makes a change between 0.5% and 1.0% in each distribution. The green plot shows the difference between the fractional ISR contribution calculated with an ordinary PDF set without NISR and the fractional ISR contribution calculated with a LuxQED PDF set including NISR. For NNPDF3.1, these two methods agree within statistical errors.

**Figure 2:** The invariant mass $M_{\mu\mu}$ distribution of the final muon pair for events with at least one photon and each muon having $p_T \mu > 25$ GeV, $\eta_\mu < 2.5$ calculated with (0) FSR only (black), (1) FSR + ISR (blue), and (2) FSR + ISR with NISR (red) for NNPDF3.1-LuxQED NLO PDFs. For comparison, (3) shows FSR + ISR with ordinary NNPDF3.1 NLO PDFs (green). The center graph shows ISR on/off ratios (1)/(0) (blue), (2)/(0) (red) and (3)/(0) (green). The right-hand graph shows the fractional differences ((1) - (2))/(0) in red and ((2) - (3))/(0) in green.
The agreement shown by the green plots on the right of each figure can be traced to a close agreement between the ratio \( \frac{L_{q\bar{q}}^{\text{QED}}}{L_{q\bar{q}}} \) of joint quark luminosity distributions with ISR on/off in Fig. 1 and the ratio of joint quark luminosity distributions calculated for NNPDF with and without LuxQED at scales near the Z mass.

Earlier phenomenological studies with KKMChh [2] followed this approach out of necessity. The fact that NISR with the NNPDF LuxQED PDF set gives compatible results confirms the validity of that approach for NNPDF sets, which is also the one followed in the recent calculations for the ATLAS single-photon events presented at this conference [13].

We expect to publish more details on the effects of exponentiated ISR on quark distributions and the implementation of NISR in KKMChh shortly. [14] We also envisage updating the photos and tauola libraries used by KKMChh and KKMCee; see Ref. [15] and references therein.

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