Foundation design taking into account non-linear soil behavior in karst-hazardous areas

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Abstract. The article solves the problem of stress and strain state of the subgrade waned by cylindrical neckline, modeling the karst pothole area. The solution was obtained taking into account nonlinear behavior of the soil subgrade based on using the A.I.Bonkin bilinear subjection. In the article are presented the visual results of calculations as the isofields of stresses and strains. The obtained solution allowed us to offer for consideration an updated analytical model of karst-resisting foundations of buildings and constructions. In the proposed analytical model was adopted a variable coefficient of subgrade reaction, varying according to the bilinear law.

1. Introduction
The stress and strain state of the soil subgrade waned by karst pothole is advisable to predict using analytical models describing the changing of the coefficient of subgrade reaction [1,2,3]. On the fig. 1 two analytical models are presented. On the first one (Fig. 1 (a)), the coefficient of subgrade reaction \(K\) changes with stepwise from zero value to the calculation value beyond its borders. On the second one (Fig. 1 (b)), enters the decreasing coefficient \(K_1'\) which is not regulated by the distance from the pothole area and its quantitative value. This article is devoted to the definition of this coefficient for using it in the calculations of karst-resisting foundation of buildings and constructions.

Figure 1. Existing analytical models

The Authors of the article are already considered [1] the problem of the stress and strain state of a uniformly loaded layer of soil with limited thickness, waned by cylindrical neckline (Fig. 2), the diameter of which is regulated by regulatory documents [2].
Fig. 2. Analytical model of the problem

As an output of solving the problem in the linear formulation, expressions for stresses and strains in a cylindrical coordinate system are obtained (Eq. 1):

\[
\begin{align*}
\sigma_z &= p_0; \quad \sigma_r = \frac{\nu p_0}{(1 - \nu)} \left[1 - \left(\frac{b}{r}\right)^2\right]; \quad \sigma_\varphi = \frac{\nu p_0}{(1 - \nu)} \left[1 + \left(\frac{b}{r}\right)^2\right] \\
\varepsilon &= \frac{\beta p_0 (H - z)}{E}; \quad u = -\frac{v p_0 b^2}{2rG(1 - \nu)}.
\end{align*}
\]

(1)

It is important to note that the formula for vertical deformations \( w \) does not depend on the distance from the cylindrical area to the pothole; therefore, the effect of waningof the subgrade in the zone of contact with karst pothole is not realized. The calculation model, describing this behavior of the foundation in approximate way is shown on the fig. 1 (a).

This work proposed to give a quantitative estimate of the waning of the subgrade in the zone of contact with a karst pothole and to develop a refined analytical model for calculating karst-resisting foundations, the assigned problem will solve in a nonlinear formulation.

2. Pre-conditions for using nonlinear calculations

Instead of the usual deformation characteristics of the soil - \( E \) and \( \nu \), there are two alternative deformation characteristics in the formula: the Bulk Modulus Elasticity-\( K \) and the shear modulus- \( G \).

Then, Hooke’s law takes the following form:

\[
\varepsilon_i = \frac{\sigma_i - \sigma_m}{2G} + \frac{\sigma_m}{K}, \quad (i = r, \varphi, z); \quad \sigma_m = \frac{\sigma_z + \sigma_\varphi + \sigma_r}{3}.
\]

The separation of the deformation characteristics of the soil into shear and volumetric is very important for the nonlinear theory of soil strain, because the soil differently resists to volumetric deformations and to strains of shape changings.

Thus, as the average stresses \( \sigma_m \) increase, the volume strain \( \varepsilon_v \) increases too, but asymptotically it tends to a certain value, that is, the module of the volume strain \( K (\sigma_m) \) is a function of the average
stresses $\sigma_{m}$ and increases with their growth [4, 5, 6]. With strains of shape changings, the opposite is true: with increasing tangential stresses $\tau$, the angular strain increases hyperbolically and tends to infinity with a certain limiting intensity of shearing stresses $\tau^{*}$, so the shear modulus $G$ is a function of shear stresses $\tau$ and decreases with their growth [5, 6, 7, 8].

It is important to note, that shear modulus- $G$ depends on the average stresses $\sigma_{m}$, because the limited value of intensity of shearing stresses $\tau^{*}$ also depends on them (fig. 3).

There are a lot of theories for describing nonlinear deformability of soils. In the article authors apply the A.I.Bonkin bilinear subjection [4, 5, 6] (Eq. 3):

$$G_{pl} = G_{0} \frac{\tau_{i}^{*} - \tau_{i}}{\tau_{i}^{*}}; \quad \tau_{i}^{*} = \sigma_{m} \cdot tg \varphi + c. \tag{3}$$

In addition to the deformation characteristics of the soil, this dependence also allows to introduce the stress parameters: the angle of internal friction $\varphi$ and the coefficient of adhesion $c$. Thus, having a “standard” set of mechanical characteristics of the soil ($E$, $\nu$, $\varphi$ and $c$), you can get a solution to a nonlinear problem, approximate to the actual behavior of the subgrade.

**3. Nonlinear settlement calculation**

To obtain the convenient for analytical work expression for $\tau_{i}^{*}$, we substituted the obtained expressions of the components of the tensor of tensions $\sigma_{z}$, $\sigma_{r}$ and $\sigma_{\varphi}$ and carried out the simple transformations (Eq. 4):

$$\tau_{i}^{*} = \frac{p_{0}(1 + \nu)}{3(1 - \nu)} \cdot tg \varphi + c. \tag{4}$$

As a result of the analysis of the expression for the components of tension, taking into account the equality of tension, we obtain the following expression for the intensity of the tension (Eq. 5):
\[ \tau_i = \frac{p_0}{\sqrt{3(1 - \nu)}} \sqrt{3\nu^2 \left( \frac{b}{r} \right)^4 + (1 - 2\nu)^2}. \] (5)

We obtain the expression for vertical deformations \( w \) by substituting the obtained expressions for \( \tau_i^* \) and \( \tau_i \) in (Eq. 3) and carrying out the simple transformations. On the base of this expression we build the isofields of vertical displacements (Fig.4).

Figure 4. Isofields of vertical strains \( w \), taking into account the nonlinear behavior of subgrade

This pattern of the isofields of vertical displacements of the array is fundamentally different from the solution obtained in the linear formulation — there is a significant increase of displacements in the pothole area, which decays at some distance from the pothole.

Comparing the obtained results with the data of the linear calculation in the article [1] it is unequivocally traced that the settlement of the bottom, obtained with due accounting for the nonlinearity of the soil deformation, is greater than the settlements obtained by the linear theory. It is important to note that as the load \( p_0 \) increases, this difference increases as the intensity of the tangential stresses \( \tau_i \) with a tendency toward \( \tau_i^* \). The value of the shear modulus \( G \), in turn, tends to zero \( G \to 0 \), thus increasing the first term (shape changing) in the expression for the deformation.

On the basis of the obtained data was built the graph of varying of the coefficient of subgrade reaction \( C_z \) in the karst pothole area under the base slab (Fig.5).
The obtained expressions and graphs for the variable coefficient of subgrade reaction $C_z$ are non-linear in nature and are difficult to use in engineering practice.

4. Formation of the analytical of the subgrade.

In the article, for the simplified description of the dependence of the variable coefficient of subgrade reaction $C_z$ from the radial distance to the axis of the pothole $r$, the bilinear dependence is accepted, which is divided into two sections at the radial distance $r = R$.

- when $r \geq R$, the coefficient of subgrade reaction $C_z$ takes on a value without taking into account the effect of waning of the subgrade by the cylindrical neckline $C_z^{\text{max}}$;
- when $r \leq R$, the coefficient of subgrade reaction $C_z$ increases with the increasing of radial distance $r$ be linear dependence, it increases from $C_z^{\text{min}}(b)$ to $C_z^{\text{max}}(R)$.

A graphic representation to the description of the bilinear dependence is given below on Fig.6.
However, in engineering calculations it is more convenient to use not two stiffness coefficients $C_{z,0}$ and $C_{z,red}^{(b)}$, but the ratio of them - the attenuation coefficient $k_{red}$ (Eq. 6):

$$k_{red} = \frac{C_{z,red}^{(b)}}{C_{z,0}}$$

(6)

The radius of the waning area $R$ is taken from the assumption that in this point the value of coefficient of subgrade reaction $C_z$, taking into account the wane, $C_{z,red}^{(R)}$ differs by no more than 5% from coefficient of subgrade reaction without taking into account the wane in $C_{z,0}(R)$. In this case, 2 (two) main parameters of this dependence are determined unambiguously, and it is much more convenient to use them in engineering practice than the complex non-linear dependencies. Formulas for determining these parameters are given below (Eq. 7):

$$k_{red} = \left(\frac{(1+v)}{3(1-v)} \cdot tg\varphi + \frac{c}{p_0} \cdot \frac{\sqrt{7v^2-4v+1}}{\sqrt{3(1-v)}}\right) \cdot \left(\frac{(1+v)}{1-v} \cdot tg\varphi + \frac{c}{p_0} - \frac{(1-2v)}{\sqrt{3(1-v)}}\right)$$

$$\cdot \left(\frac{(1+v)}{1-v} \cdot tg\varphi + 3 \cdot \frac{c}{p_0} - \frac{\sqrt{7v^2-4v+1}}{\sqrt{3(1-v)}}\right)$$

(7)

$$R = \frac{b}{\sqrt{4 \left[0.05 \cdot \frac{(1+v)}{\sqrt{3}} \cdot tg\varphi + 0.05 \frac{c\sqrt{3(1-v)}}{p_0} + 0.95 (1-2v)^2 \right] - (1-2v)^2}}$$
5. Conclusion
The obtained parameters for the wane of the foundation subgrade ($k_{red}$ and $R$) have a wide range of changes depending on the mechanical characteristics of the soil subgrade and structural elements of the underground part of buildings and structures. Even with the presence of "weak" soils and in the case of load transfer of a conditionally low load on the subgrade, the attenuation coefficient $k_{red}$ is in the range from 0.3 to 0.7, and the zone of waning of the subgrade $R$ expands beyond the limits of the karst pothole by a value from $b$ to $2b$.

References

[1] Shebunyaev A.N., Yudina I.M. Analysis of stress and strain state of the subgrade in the karst pothole area. Inzhenerniy vestnik Dona, (Rus), 2019, no1. URL: ivdon.ru/ru/magazine/archive/n1y2019/5637. (Rus)

[2] Ilichev V.A., Mangushev R.A. and others. Spravochnik geotehnika. Osnovaniya, fundamenty i podzemnie sooruzheniya [Handbook of geotechnical engineering. Subgrades, foundations and underground structures]. Moscow, Izd ASB, 2016, 1040 p. (Rus)

[3] Gotman A.L., Gotman N,Z., Kayumov M.Z. Method of calculation of the foundations of buried structures in the karst areas. Zhilischnoe stroitelstvo, Moscow, 2011, no. 9, p. 13-15. (Rus)

[4] Ter-Martirosyan Z.G, Mehanika gruntov [Soil mechanics], Izd. ASB, 2009, 488 p. (Rus)

[5] Nadai A. Plastichnost I razrushenie tverdih tel [Plasticity and fracture of solids], 1969, 863 p. (Rus)

[6] Botkin A.I. About the strength of loose and brittle materials. Izd. VNII Gidrotehniki, 1940, no. 26, p.205-236. (Rus)

[7] Karl Terzaghi, Ralfh B. Peck, Cholamreza Mesri. Soil Mechanics in Engineering Practice, Third Edition, 1995, 549 p.

[8] James K. Mitihell Fundamentals of soil behavior. 1993, 437 p.