Surface Shape Stability Design of Mesh Reflector Antennas Considering Space Thermal Effects

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ABSTRACT
On-orbit periodic thermal loads degrade the reflector surface accuracy of AstroMesh antennas. To address this problem, a surface shape stability design method is proposed to passively pre-control the on-orbit thermal deformation of mesh reflectors, in which the structural parameters are properly designed to make the internal forces of the whole structure change coordinately and the surface shape of cable-mesh antennas insensitive to thermal loads. First, mathematical models of mechanical-thermal matching (MTM) are established for AstroMesh reflectors, in which an MTM model is developed for the cable net structure and the reflector membrane is equivalent to a cable net structure according to the force balance and thermal deformation coordination relationships. Then, based on the mathematical models, a surface shape stability design strategy is presented for AstroMesh antennas to properly design the cross-sectional dimensions of the cables. Finally, typical AstroMesh reflector structures are designed using the proposed method and simulation results show that the thermal deformations of the obtained AstroMesh reflectors are quite small under the whole temperature range.

INDEX TERMS
Mesh reflector antennas, thermal loads, surface shape stability, mechanical-thermal Matching (MTM).

I. INTRODUCTION

A. MOTIVATION
With the rapid development of high-resolution Earth observation satellites and telecommunications, high-accuracy spaceborne deployable antennas are in great demand [1]–[3]. AstroMesh reflectors have become one popular kind of spaceborne antennas with the advantages of large aperture, light mass and small-folded volume [4], [5] in the past few decades. As shown in Fig. 1, an AstroMesh reflector mainly consists of a deployable rim truss, two curved cable nets placed back-to-back across the truss, some vertical tension ties, and a RF mesh stretched across the front net.

There are mainly two evaluation criteria in the design of AstroMesh reflectors: surface accuracy and surface shape stability under space thermal loads. On one hand, the initial surface accuracy is determined by the configuration of the mesh reflector. Many researchers, such as Tibert [6], Shi et al. [7], Morterolle et al. [8], Yang et al. [9]–[11], have researched the configuration design methods of mesh reflectors and obtained satisfactory results. On the other hand, as on-orbit AstroMesh reflectors are affected by time-variant space thermal environment, the surface accuracy will be deteriorated due to thermal deformation [12], [13]. Therefore, how to effectively control the on-orbit thermal deformation of AstroMesh reflectors has become a hot topic in recent years [14]–[20].

B. RELATED LITERATURE
Two approaches have been investigated to control the thermal deformation of mesh reflectors: active control and passive
pre-control. In the aspect of active control, Tanaka [14], Tanaka and Natori [15], and Wang et al. [16], [17] proposed shape adjustment methods for active mesh reflectors based on active control strategies, which is satisfactory in theory. Nevertheless, in practice, the auxiliary equipment, such as the active actors and the voltage supply equipment, will not only greatly increase the weight and cost, but also reduce the reliability of the antenna system for on-orbit service. Therefore, the study on passive pre-control of the thermal deformation of on-orbit mesh reflectors is of great concern.

Unfortunately, researches on the pre-control of the on-orbit thermal deformation of mesh reflectors are very few [18]–[21]. Yang et al. [10], [11] proposed equal-tension form-finding methods to improve the shape stability of mesh reflectors, in which the surface shape stability was considered to be dependent on the tension uniformity of the cables. The viewpoint is not comprehensive, because the thermal deformation is also dependent on the structural material and geometry parameters. Shoji et al. [18] proposed a method to suppress the thermal deformation of a modular mesh reflector by adjusting the thermal expansion coefficients of the structural members and controlling the internal forces of the springs used to deploy the antenna. Ding [19] presented an optimization method to reduce the thermal deformation of AstroMesh reflectors, in which the structural sizes are optimized with the extreme temperature conditions being taken into consideration. To reduce the range of on-orbit thermal deformation of AstroMesh reflectors, Yang et al. [20] proposed a shape pre-adjustment optimization method, in which element lengths of the adjustable cables were properly designed considering space thermal loads. Nie et al. [21] provided a pretension optimization approach to minimized the maximal amplitude of the surface shape errors within the temperature range in space.

However, the above methods can only suppress the deformation of mesh reflectors under specific temperature conditions, and the surface errors of the reflector under extreme high or low temperature conditions are still very large, which cannot fully meet the requirement of on-orbit high-accuracy mesh reflectors.

C. CONTRIBUTIONS
To address the aforementioned problems, a surface shape stability design method is proposed to minimize the thermal deformation under the whole on-orbit temperature range. The contributions of this paper are summarized as follows.

1) A new design idea is proposed to control the on-orbit thermal deformation of tensioned cable-membrane structures. The cross-sectional areas of the cables are properly designed to make the internal forces of the whole tensioned structure change coordinately with thermal loads and the surface shape of the mesh reflectors insensitive to thermal loads.

2) Based on the thought of MTM design, mathematical models are established for tensioned cable-membrane structures. A MTM model is developed for tensioned cable net structures, and the reflector membrane structure is equivalent to a cable net structure according to the force balance and thermal deformation coordination relationships.

3) Based on the mathematical models of MTM design, a surface shape stability design strategy is presented for AstroMesh reflectors to properly design the cross-sectional dimensions of the cables.

D. PAPER ORGANIZATION
This paper is organized as follows. The mathematical models of MTM are established for AstroMesh reflectors in Section II. Section III proposes a shape stability design strategy for AstroMesh reflectors in detail. Typical reflector structures are designed using the proposed method in Section IV to validate the effectiveness of the method, and some brief conclusions are drawn in Section V.

II. MATHEMATICAL MODELS OF MTM
As shown in Fig. 1, the AstroMesh reflector is a cable-membrane-beam composite structure, of which the cable net and mesh can be taken as 2-node link elements and 3-node triangular membrane elements, respectively [22]. In general, the thermal expansion coefficient of the truss is obviously smaller than those of the cable and mesh. Thus, this work focuses on the MTM of cable-membrane structures. In this section, the thermal deformation problem of cable net structures is firstly discribed and the mechanism of thermal deformation is analyzed, and then a MTM model and a mechanical-thermal equivalent model are established for cable net and membrane structures, respectively.

A. THERMAL DEFORMATION PROBLEM OF CABLE NET STRUCTURES
For the AstroMesh reflector shown in Fig. 1, if there is no mesh stretched across the front cable net, the 2-layer cable net structure is tensioned in equilibrium state, and the equilibrium equation [6] can be expressed as

\[ BT_c = 0 \]  \hspace{1cm} (1)

where \( T_c \) is the pretension vector of the cables, and \( B \) is the coefficient matrix which is determined by the geometric topology and node positions of the cable net structure.

Considering the mesh reflector’s good transmittance and weak shielding of sunlight, it is assumed that the temperature of the whole AstroMesh structure is uniform [20], [21] in this work. Generally, when the temperature of the AstroMesh structure changes by \( \Delta t \), the cable tensions will change by \( \Delta T_c \) and the node positions will also change accordingly to reach a new equilibrium state. The new equilibrium equation can be written as

\[ (B + \Delta B)(T_c + \Delta T_c) = 0 \]  \hspace{1cm} (2)

where \( \Delta B \) is the variation of the coefficient matrix \( B \) due to temperature variation, which reflects the thermal deformation of the structure.
Generally, the thermal deformation problem of cable net structures can be understood as the following process: thermal strains and stresses are generated firstly due to the temperature change, and then the thermal deformation occurs under the action of thermal stresses. In general, the thermal strains and stresses caused by temperature change cannot be zero due to the inherent characteristics of thermal expansion and contraction. Nevertheless, it should be noted that, although the thermal strains and stresses caused by temperature change are inevitable, whether the thermal deformation will occur depends on whether the equivalent loads of the thermal stresses are a set of equilibrium forces.

For the cable net structures obtained by the existing methods, the equivalent load of the thermal stresses of the cables are always not a set of equilibrium forces. As a result, the thermal stresses of the cables inevitably lead to thermal deformation of the whole cable net structure, and the surface errors of the reflector under extreme high or low temperature conditions are still very large [20], [21].

Based on the above analysis, we try to properly design and tailor each cable individually to make the cross-sectional areas and tensions of the cables matched with each other and the equivalent loads of the thermal stresses caused by the temperature change be a set of equilibrium forces. Consequently, the obtained cross-sectional area vector of the cables, respectively, and is proportional to $1/\lambda_A$. Thus, if the pre-tension vector $T_c$ and the value of $\lambda_A$ are given, the desired cross-sectional area $A_c$ can be uniquely obtained to satisfy equation (3).

Here, in order to keep the material cost of the whole cable net structure unchanged, the total mass of the cable net structure is required to be constant, and the mass equation is used to determine the value of $\lambda_A$. The total mass of cable net structure is figured out as

$$m_c = \rho_c L_c^T A_c = \lambda_A \rho_c L_c^T T_c$$

where $\rho_c$ and $L_c$ are the mass density and element length vector of the cables, respectively.

According to (9), if the initial equilibrium state of the cable net structure is given, i.e., $L_c$ and $T_c$ are given, $m_c$ can be regarded as a unary function of $\lambda_A$. Consequently, by fixing the total mass of the cables, $\lambda_A$ can be uniquely determined as

$$\lambda_A = \frac{m_c}{\rho_c L_c^T T_c}$$

As a result, the desired cross-sectional area $A_c$ can be uniquely obtained according to (7) and (10) as

$$A_c = \frac{m_c}{\rho_c L_c^T T_c}$$

It can be seen from (11) that the obtained cross-sectional areas of the cables are independent of temperature variations. Therefore, the obtained cable net structures can adapt to different temperature conditions and satisfy the equilibrium condition (3) under the whole temperature range.

### B. MECHANICAL-THERMAL MATCHING MODEL FOR CABLE NET STRUCTURES

According to Section II-A, if the equivalent loads of the thermal stresses caused by the temperature change are a set of equilibrium forces, there will be no thermal deformation occurring on the cable net structure, i.e., $\Delta B = 0$. For this purpose, the tension variation $\Delta T_c$ should satisfy

$$B(\Delta T_c + T_c) = 0$$

Substituting (1) into (3), we can obtain that

$$B\Delta T_c = 0$$

Generally, equation (4) has multiple solutions. One special solution of (4) can be obtained from (1) and (4) as

$$\Delta T_c = \lambda_T T_c$$

where $\lambda_T$ is the variation coefficient of cable tensions, which is proportional to $\Delta t$.

According to Hooke’s law, $\Delta T_c$ can be calculated as

$$\Delta T_c = E_c \alpha_c \Delta T T_c$$

where $E_c$ and $\alpha_c$ are the Young’s modulus of elasticity and thermal expansion coefficient of the cables, respectively, and $A_c$ is the cross-sectional area vector of the cables.

The cross-sectional area vector of the cables can be written from (5) and (6) as

$$A_c = \lambda_A T_c$$

where constant $\lambda_A$ is the proportional coefficient between $A_c$ and $T_c$, and $\lambda_A = \lambda_T/(E_c \alpha_c \Delta T)$.

From (7), an arbitrary cable element $i$ should satisfy

$$\sigma_{ci} = \frac{T_{ci}}{A_{ci}} = \frac{1}{\lambda_A}$$

where $T_{ci}, A_{ci}$, and $\sigma_{ci}$ are the pre-tension, cross-sectional area and stress of cable element $i$, respectively.

Obviously, if all the cable elements satisfy (8), an equal-stress cable net structure will be obtained, and the stress constant $\sigma_c$ is exactly equal to $1/\lambda_A$. Therefore, the obtained cable net structure will be equivalent to a cable net structure based on the thought of MTM to consider the effect of the membrane. The equivalent process contains two aspects:

1) the tensions of the equivalent cable net structure are figured out according to the force balance relationship;
2) the cross-sectional areas of the equivalent cable net structure are determined according to the thermal deformation coordination relationship.

As shown in Fig. 2, an arbitrary triangular membrane element $k$ with a uniform stress distribution is equivalent to
three cable elements along the sides of a triangle. The tensions of the three equivalent cables can be calculated as [23]

$$T_{mk}^p = \frac{\sigma_m d_m L_{mk}^p}{2 \tan \beta_{mk}^p} \quad (p = 1, 2, 3)$$  \hspace{1cm} (12)

where $\sigma_m$ and $d_m$ are the stress and thickness of membrane element $k$, respectively, and $L_{mk}^p$ and $\beta_{mk}^p$ are the length and corresponding inner angle of side $p$, respectively, and $T_{mk}^p$ is the tension of the equivalent cable element along side $p$. The meanings of the parameters are illustrated in Fig. 2.

Accordingly, the tensions of the obtained three equivalent cables can be expressed in matrix form as

$$T_{mc} = \begin{bmatrix} T_{mk1}^1, T_{mk2}^2, T_{mk3}^3 \end{bmatrix}^T$$  \hspace{1cm} (13)

If the temperature of the reflector changes by $\Delta t$, the stress variation of the membrane element $k$ can be calculated as

$$\Delta \sigma_m = \frac{E_m \alpha_m \Delta t}{1 - \mu_m}$$  \hspace{1cm} (14)

where $E_m$, $\alpha_m$, and $\mu_m$ are Young’s modulus of elasticity, thermal expansion coefficient and Poisson’s ratio of the membranes, respectively.

Then, the tension variations of the three equivalent cables can be written from (12) and (14) as

$$\Delta T_{mk}^p = \frac{E_m \alpha_m d_m L_{mk}^p \Delta t}{2 (1 - \mu_m) \tan \beta_{mk}^p} \quad (p = 1, 2, 3)$$  \hspace{1cm} (15)

The tension variations of the cables are also related to the cable stiffness as

$$\Delta T_{mk}^p = E_m \alpha_m A_{mk}^p \Delta t \quad (p = 1, 2, 3)$$  \hspace{1cm} (16)

The cross-sectional areas of the equivalent cables from membrane element $k$ can be expressed through (15) and (16) as

$$A_{mk}^p = \frac{E_m d_m}{E_c \alpha_c} \frac{d_m L_{mk}^p}{2 (1 - \mu_m) \tan \beta_{mk}^p} \quad (p = 1, 2, 3)$$  \hspace{1cm} (17)

It follows from (17) that the cross-sectional areas of the equivalent cables are independent of temperature variations. Thus, the obtained cables can adapt to different temperature conditions and satisfy the equilibrium condition (3) under the whole temperature range.

Similarly, the cross-sectional areas of the equivalent cables from membrane $k$ can be expressed in matrix form as

$$A_{mc}^k = \begin{bmatrix} A_{mk1}^1, A_{mk2}^2, A_{mk3}^3 \end{bmatrix}^T$$  \hspace{1cm} (18)

Based on the above deduction, arbitrary triangular membrane element $k$ can be equivalent to three cable elements, and the equivalent tensions and cross-sectional areas of the cables can be obtained from (13) and (18). Furthermore, for the AstroMesh reflector, there is a common boundary between any two triangular membrane elements. In order to obtain the equivalent cable net structure of the whole membrane structure, the equivalent tension and cross-sectional area at the common boundary should be superimposed. With the finite element assembly operation, the tensions and cross-sectional areas of the equivalent cable net structure can be written in matrix form as

$$T_{mc} = \sum_k T_{mc}^k$$  \hspace{1cm} (19)

$$A_{mc} = \sum_k A_{mc}^k$$  \hspace{1cm} (20)

### III. SURFACE SHAPE STABILITY DESIGN STRATEGY

According to Section II, if a cable net structure meets the MTM model, the tensions of the cables will change coordinate and there will be no thermal deformation under uniform thermal loads. Moreover, the reflector membrane structure can be equivalent to a cable net structure. Therefore, the cross-sectional areas of the cables of AstroMesh reflectors can be designed to realize that the internal tensions of the whole cable-membrane structure change coordinate and there is no thermal deformation under uniform thermal loads.

The overall design flow of the surface shape stability design is illustrated in Fig. 3, and the detailed design processes can be implemented through the following process:

1) The initial AstroMesh reflector structure is decomposed into a 2-layer cable net structure and a reflector membrane structure, denoted by Cable-I and Membrane-I, respectively, as shown in Fig. 4. The known parameters contain: the geometry configuration and material parameters of the AstroMesh reflector, the pretension $T_c$ and initial cross-sectional area $A_0$. 

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**FIGURE 2.** Equivalence of a membrane element to cable elements. (a) Stress distribution of triangular membrane element $k$. (b) Obtained equivalent cables.
TABLE 1. Pretension distribution of the AstroMesh cable net structure.

| Front net (N) | Rear net (N) | Vertical cables (Tension ties) (N) |
|---------------|--------------|----------------------------------|
| Internal cables | Boundary cables | Internal cables | Boundary cables | Tension ties |
| 25.000 | 25.000 – 39.750 | 98.012 – 99.853 | 157.000 | 1.531 – 2.853 |

TABLE 2. Prestress distribution of the AstroMesh cable net structure.

| Front net (Mpa) | Rear net (Mpa) | Vertical cables (Tension ties) (Mpa) |
|-----------------|----------------|-------------------------------------|
| Internal cables | Boundary cables | Internal cables | Boundary cables | Tension ties |
| 5.000           | 5.000 – 7.950 | 19.602 – 19.971 | 19.971 – 31.400 | 0.306 – 0.571 |

Then, based on the MTM model in Section II-B, the cross-sectional area of Cable-III can be calculated as

\[ A_{eq} = \frac{m_c}{\rho_c(T_c)^{1/3}} \]  

4) Consequently, the cross-sectional areas of Cable-I in the actual AstroMesh structure can be obtained as

\[ A_c = A_{eq} - A_{mc} \]  

IV. SIMULATION RESULTS AND DISCUSSIONS

A. ASTROMESH CABLE NET STRUCTURE

The AstroMesh reflector, as shown in Fig. 5, which was studied in [10] with parameters of \( D_a = 12 \) m, \( F/D_p = 0.45 \), and \( d = 8.3 \) m, is employed to demonstrate the effectiveness of the proposed method. The rim truss is realized with an equilateral 30-bay ring. The Young’s modulus of elasticity of the cables is 20 GPa. In order to illustrate the effectiveness of the method under a wide range of materials, the thermal expansion coefficients of the cables are set to be \( \alpha_c = 2 \times 10^{-6} \) C\(^{-1} \) and \( \alpha_c = -2 \times 10^{-6} \) C\(^{-1} \), denoted by Case-I and Case-II, respectively.

For the initial cable net structure shown in Fig. 5, the cross-sectional area of all the cables is \( A_0 = 5 \) mm\(^2 \). The pretension distribution of the front net cables is the same as that in [10]. In order to allow the driving force of the truss to deploy the structure and avoid the cables being slack under temperature loads, the tension values of the front net cables are reduced to 1/4 of those in [10], and the corresponding tensions of the rear net and vertical cables can be determined through the method in [22]. The pretension distribution of the whole cable net structure is listed in Table 1.

To ensure the reliability of the analysis model, the finite element model of the AstroMesh cable net structure is established through commercial software ANSYS [20], [24]. With the pretensions listed in Table 1 applied to the cables, the nodal displacements and cable stress distribution of the cable net structure under room temperature 20°C are shown in Fig. 6, which is obtained by nonlinear static analysis of software ANSYS. The maximum displacement of the structure is \( 0.513 \times 10^{-5} \) mm and the stresses of the cables are listed in Table 2. By comparing the data listed in
Table 1 and Table 2, we can get that the values of the cable stresses are just one fifth of the values of the pretensions, which is because the stress and cross-sectional area of arbitrary cable element $i$ satisfies $\sigma_{ci} = T_{ci}/A_{ci}$ and the cross-sectional areas of all the cables are equal to $5 \text{ mm}^2$. According to the results of the nodal displacements and cable stresses, it can be conducted that the cable net structure is in an ideal equilibrium state without temperature loads, and the established finite element analysis model is reliable and credible.

Here, it should be pointed out that, all the cable-membrane models are established through software ANSYS and all the thermal deformation analyses of the models are implemented by the nonlinear static analysis of ANSYS in this work, which are also feasible and reliable.

On-orbit mesh reflectors are periodically exposed to sunlight and the Earth’s shadow, depending on their orbit. This causes a periodic change in reflector temperature. In this study, it is assumed that the temperature of the whole AstroMesh structure is uniform, and the temperatures of on-orbit antenna structures range from $-200^\circ \text{C}$ to $100^\circ \text{C}$ [20], [21].

First, based on the finite element model in Fig. 6, the thermal deformation analyses of the cable net structure under the temperature range $-200^\circ \text{C} \sim 100^\circ \text{C}$ are implemented by nonlinear static analysis of ANSYS. In order to illustrate the shape stability of the initial reflector under thermal loads, the reflector surface $RMS$ errors of the cable net under the whole temperature range are calculated, as shown in Fig. 7 (a). In this work, the $RMS$ (Root Mean Square) error is the $RMS$ value of surface nodal deviations between the deformed mesh reflector and the initial one, which can be calculated as

$$RMS = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \delta_i^2}$$

where $n$ is the number of the reflector nodes, and $\delta_i$ is the displacement of node $i$ due to temperature change.

For the initial cable net structure, the maximum $RMS$ errors induced by thermal deformation under the whole temperature range are $8.492 \text{ mm}$ and $9.103 \text{ mm}$ for Case-I ($\alpha_c = 2 \times 10^{-6} \text{ C}^{-1}$) and Case-II ($\alpha_c = -2 \times 10^{-6} \text{ C}^{-1}$), respectively, which can seriously deteriorate the surface accuracy of the AstroMesh reflector.

Then, keeping the total mass of the cables constant, the cable net structure is designed based on the MTM model presented in Section II-B to control the thermal deformation under the whole temperature range. Fig. 7 (b) shows the thermal deformation results of the obtained cable
net structure. The thermal RMS errors of the reflector over the whole temperature range are approximately equal to zero for both Case-I and Case-II. We can get that the shape of the obtained cable net structure through the proposed method is insensitive to uniform thermal loads.

Fig. 8 shows the tensions of all the cables under different temperatures, which is drawn through the function \( Mesh() \) of software MATLAB. It can be seen that tensions of the cables decrease linearly with the temperature changing from \(-200^\circ C\) to \(100^\circ C\) for Case-I, while it is the opposite for Case-II. Moreover, the tensions of all the cables are positive, so there is no slack cable under the whole temperature range. In addition, there is a large gap in the tension value of some cable elements with adjacent numbers, so there will be some vertical lines occurring in Fig. 8. Fig. 9 shows the relative tension variations of the cables under different temperatures. Here, the relative tension variation of each cable refers to the ratio of the actual tension under a certain temperature to the initial tension under \(20^\circ C\). We can get that all the relative tension variations of the cables are equal under each specific temperature. Although the actual values of cable tensions change with temperature, the distribution of cable tensions remains unchanged and the whole cable net structure is still in equilibrium. In other words, the cable tensions change coordinately under uniform thermal loads. That is why the thermal RMS errors of the obtained reflector over the whole temperature range are approximately equal to zero, as shown in Fig. 7.

Accordingly, the cross-sectional areas of all the cables are shown in Fig. 10, and the corresponding data results are listed in Table 3. According to Table 1 and Table 3, the obtained cross-sectional area is proportional to the pretension of the cable net structure, as described in equation (7). Moreover, the spring stiffness \((AE/L)\) results of the cables are listed in Table 4. We can get that the obtained spring stiffness \((AE/L)\) of the tension ties is much smaller than those of the cable net elements, i.e., the tension ties are softer than the cable net elements, which is beneficial for the assembly precision in practical engineering.

B. ASTROMESH CABLE-MEMBRANE STRUCTURE

To further demonstrate the effectiveness of the method, a reflector membrane structure is added to the original 2-layer
TABLE 3. Cross-sectional area results of the cables of the AstroMesh cable net structure.

|         | Front net (mm²) | Rear net (mm²) | Vertical cables (Tension ties) (mm²) |
|---------|-----------------|----------------|-------------------------------------|
|         | Internal cables | Boundary cables | Internal cables | Boundary cables |                     |
|         | 2.214           | 2.214 – 3.520   | 8.680 – 8.843 | 8.843 – 13.904 | 0.136 – 0.253 |

TABLE 4. Spring stiffness (AE/L) results of the cables of the AstroMesh cable net structure.

|         | Front net (N/mm) | Rear net (N/mm) | Vertical cables (Tension ties) (N/mm) |
|---------|-----------------|-----------------|--------------------------------------|
|         | Internal cables | Boundary cables | Internal cables | Boundary cables |                     |
|         | 33.750 – 90.935 | 35.574 – 109.234 | 135.001 – 363.741 | 142.296 – 436.937 | 2.589 – 8.498 |

**FIGURE 8.** Cable tensions under different temperatures. (a) Case-I. (b) Case-II.

**FIGURE 9.** Relative variations of cable tensions under different temperatures. (a) Case-I. (b) Case-II.

cable net structure of Section IV-A, as shown in Fig. 5 and Fig. 11. The geometry parameters, material parameters, and the pretension distribution of the cable net, are the same with those in Section IV-A. In addition, the thickness, Young’s modulus of elasticity, thermal expansion coefficient and Poisson’s ratio of the membrane structure are 0.1 mm, 20 MPa, $1 \times 10^{-5}$ C$^{-1}$ and 0.3, respectively. In the initial equilibrium state of the cable-membrane structure, the stresses of all the membrane elements are equal and set to be 0.05 MPa to avoid the slackness of the membrane elements.
First, the cable-membrane structure is designed using the design strategy presented in Section III. Fig. 12 shows the thermal deformation results of the obtained AstroMesh reflector. The RMS errors of the reflector under the whole temperature range are approximately equal to zero for both Case-I and Case-II. In other words, the shape of the obtained AstroMesh reflector is stable and insensitive to uniform thermal loads, which is exactly what we need to achieve the on-orbit high-accuracy mesh reflectors.

The actual values and relative variations of the cables are close with those shown in Fig. 7 and Fig. 8. The membrane stresses under different temperatures are shown in Fig. 13. It can be seen that the stress of each membrane decreases linearly with the temperature changing from −200°C to 100°C for both Case-I and Case-II and there is no slack membrane element under the whole temperature range. Moreover, all the stresses of the membranes are equal under each specific temperature. Thus, the internal forces of the obtained cable-membrane structure also change coordinately, which just illustrates the rationality of the thermal deformation results shown in Fig. 12.

Accordingly, the cross-sectional area results of the cables for Case-I and Case-II are shown in Fig. 14, and the corresponding data results are listed in Table 5. Obviously, the obtained cross-sectional area results of Case-I and Case-II
TABLE 5. Cross-sectional area results of the cables of the AstroMesh cable-membrane structure.

| Items       | Front net (mm²) | Rear net (mm²) | Vertical cables (mm²) |
|-------------|-----------------|----------------|-----------------------|
|             | Internal cables | Boundary cables | Internal cables | Boundary cables | (Tension ties) |
| Case-I      | 1.947           | 1.947 ~ 3.366  | 8.304 ~ 9.452        | 8.436 ~ 13.821 | 0.137 ~ 0.257 |
| Case-II     | 2.038           | 2.038 ~ 3.550  | 7.995 ~ 9.101        | 8.122 ~ 13.308 | 0.132 ~ 0.248 |

TABLE 6. Cross-sectional area results of the cables of the AstroMesh cable-membrane structure after merging variables.

| Items       | Front net (mm²) | Rear net (mm²) | Vertical cables (mm²) |
|-------------|-----------------|----------------|-----------------------|
|             | Internal cables | Boundary cables | Internal cables | Boundary cables | (Tension ties) |
| Case-I      | 2.012           | 2.799          | 9.037                | 11.232           | 0.171          |
| Case-II     | 2.443           | 2.765          | 8.701                | 10.815           | 0.165          |

are different with each other. The reason for this phenomenon is that the cross-sectional areas $A_{mc}$ of the equivalent cables from the membrane structure are positive for Case-I and negative for Case-II according to (17). Consequently, with the total mass of all the cables unchanged, the cross-sectional areas of the front net cables of Case-II are larger than those of Case-I according to (23), while the cross-sectional areas of the rear net and vertical cables of Case-II are smaller than those of Case-I.

Second, the cross-sectional area variables of the cables should be merged to improve the feasibility of manufacture and assembly. All the cables of the AstroMesh reflector are divided into five parts: 1) the internal cables of the front net; 2) the boundary cables of the front net; 3) the internal cables of the rear net; 4) the boundary cables of the front net; 5) the vertical cables (the tension ties), which is coincident with the classifications in Table 5. Here, the corresponding cross-sectional areas in Table 5 are also divided into five parts, denoted as $A_{cJ} (J = 1 \sim 5)$. Then, keeping the total mass of the cables constant for each parts, the weighted average value of the cross-sectional area $A_{cJ}$ can be calculated as

$$A_{cJ} = \frac{(A_{cJ})^T \|L_c^{J}\|}{\|L_c^{J}\|_1}$$

(25)

where $L_c^{J} (J = 1 \sim 5)$ is element length vector of the cables, and $\|L_c^{J}\|_1$ is the 1-norm of $L_c^{J}$, which is the sum of the absolute values of all elements of $L_c^{J}$.

The cross-sectional area results of the cables through (25) are listed in Table 6. Fig. 15 shows the thermal deformation results of the obtained AstroMesh reflector. The maximum $RMS$ error induced by thermal deformation under the whole temperature range is just 0.063 mm and 0.052 mm for Case-I and Case-II, which shows that the proposed method is still effective with the merging of the cross-sectional areas of the cables.

Then, the manufacturing accuracy of the cables should also be taken into consideration. In Table 5 and Table 6, the accuracy of the cross-sectional areas is retained to a prohibitive value of 0.001 mm². Thus, we try to reduce the manufacturing difficulty and keep the manufacturing accuracy of the cables to 0.1 mm². The cross-sectional area data in Table 6 are rounded with the accuracy of 0.1, and the corresponding results are listed in Table 7. Then, the thermal deformation analysis of the obtained AstroMesh reflector is also implemented, as shown in Fig. 16. The maximum $RMS$ error induced by thermal deformation under the whole temperature range is just 0.055 mm and 0.155 mm, respectively, which is still quite satisfactory compared with the millimeter-scale thermal deformation of the initial model.

In summary, the proposed method is still effective when further considering the feasibility of assembly and the manufacturing accuracy of the cables in practical engineering.

C. UMBRELLA-LIKE CABLE-MEMBRANE STRUCTURE

To further demonstrate the versatility of the method, an umbrella-like cable-membrane structure which is similar with that in [24], is also analyzed and designed using the proposed method. The umbrella-like structure, as shown in Fig. 17, is composed by frame, cable net and membrane.
The cable-membrane is supported by the frame structure, and the membrane is supported by the cable net structure. Here, it is assumed that the frame structure is fixed. Thus, the cable-membrane structure is in equilibrium state under the cable tensions and membrane stresses. The materials of the cables and membrane are the same with those in the Case-II of Section IV-B. The initial pretension distribution of the cables of the umbrella-like structure is listed in Table 8, and the uniform stress of the membranes is 0.05 MPa. In this example, it is assumed that the temperatures of umbrella-like structure also range from $-200^\circ$C to $100^\circ$C.

First, to ensure the reliability of the analysis model, the finite element model of the umbrella-like cable-membrane structure is established by the commercial software ANSYS. Correspondingly, the pretensions and the prestress listed in Table 8 are applied to the cables and membrane, and the initial cross-sectional area of all the cables is $A_0 = 5 \text{ mm}^2$. By nonlinear static analysis, the nodal displacements and membrane stress distribution of the umbrella-like structure under room temperature $20^\circ$C are obtained and shown in Fig. 18. It can be seen that the maximum displacement of the structure is $0.830 \times 10^{-7}$ mm and the membrane stress is exactly 0.05 MPa. That is to say, the structure is in an ideal equilibrium state without temperature loads, and the established finite element model is reliable and credible.

Then, based on the finite element model in Fig. 18, the thermal deformation analyses of the umbrella-like cable-membrane structure under the temperature range $-200^\circ$C $\sim$ $100^\circ$C are implemented by nonlinear static analysis. The maximum nodal displacements of the structure under the whole temperature range are calculated and shown in Fig. 19. It can be seen that the worst temperature condition is the extreme high or low temperature. We can get that the maximum nodal displacements under $100^\circ$C and $-200^\circ$C are $0.442 \text{ mm}$ and $2.127 \text{ mm}$, respectively, which are relatively large compared with that under $20^\circ$C.

To control the thermal deformation of the umbrella-like structure under the whole temperature range, the umbrella-like cable-membrane structure is designed using the proposed method. The thermal deformation analyses of the umbrella-like structure under the whole temperature range are also implemented by nonlinear static analysis of ANSYS. The red line in Fig. 19 shows the thermal deformation results of the obtained AstroMesh reflector under different temperatures. It can be seen that the maximum nodal displacements of the umbrella-like structure under the whole temperature range are approximately equal to zero. It can be
V. CONCLUSION

In this study, mathematical models of MTM design is established for tensioned cable-membrane structures and a surface shape stability design strategy is proposed to properly design the cross-sectional dimensions of the cables to make the surface shape of AstroMesh reflectors insensitive to the on-orbit thermal loads. Numerical simulations verified the effectiveness of the proposed method. Several conclusions can be drawn:

1) The proposed surface shape stability design method is effective for both cable net and cable-membrane structures, in which the cable net structure can be regarded as a special case of cable-membrane structures.

2) For the obtained AstroMesh structures, the internal forces of the structure change coordinately under thermal loads and the thermal deformation under the whole temperature range is quite small, so the reflector surface shape is insensitive to thermal loads, which is of great significance for the realization of on-orbit high-accuracy mesh reflectors.

3) No matter the thermal expansion coefficient of the cables is positive or negative, the surface shape stability design of AstroMesh reflectors can both be implemented. Due to the total mass of the cables being fixed, the obtained distribution of the cross-sectional areas changes with the thermal expansion coefficient of the cables.

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