The Perturbation Spectrum of Black Holes in 
\( N = 8 \) Supergravity

Finn Larsen

Department of Physics and Astronomy

University of Pennsylvania

Philadelphia, PA 19104

e-mail: larsen@cvetic.hep.upenn.edu

Abstract

The near horizon geometry of four-dimensional black holes in the dilute gas regime is \( \text{AdS}_3 \times S^2 \), and the global symmetry group is \( SU(2) \times USp(6) \). This is exploited to calculate their perturbation spectrum using group theoretical methods. The result is interpreted in terms of three extreme M5-branes, orthogonally intersecting over a common string. We also consider \( N = 8 \) black holes in five dimensions, and compute the spectrum by explicit decoupling of the equations of motion, extending recent work on \( N = 4 \) black holes. This result is interpreted in terms of \( D1 \)- and \( D5 \)-branes that are wrapped on a small four-torus. The spectra are compared with string theory.
1 Introduction

It has been proposed that, in certain limits, the near horizon geometry of brane configurations contain all the structure needed to construct the underlying quantum theory [1]. In particular, the spectrum of classical perturbations in the near horizon region is equivalent to that of the quantum operators [2], and the quantum correlation functions are similarly encoded in the geometry [3, 2]. These relations have been extensively exploited to study conformally invariant gauge theories in four and six dimensions. Another interesting system is the bound state of $D1$-branes and $D5$ branes, because of its relation to black holes in five dimensions [4], and plausibly to second-quantized string theory [5]. In the case of $D1 - D5$ the corresponding conformal field theory (CFT) is two dimensional, and so the techniques for its study are well-developed; moreover, the spectrum of perturbations in the near horizon geometry has been computed completely for $D1 - D5$ wrapped on a small $K3$ [6]. The correspondance with the CFT has been strikingly confirmed in some instances [7, 8]; but the map may nevertheless be imperfect [9].

The purpose of the present paper is to present the spectrum of the black holes in $N = 8$ supergravity, with emphasis on the four dimensional case [10, 11, 12]. Their spectrum is classified in general, using the global duality symmetries. In the dilute gas regime the global symmetries are enhanced to $SU(2) \times USp(6)$, and the near horizon geometry is of the form $AdS_3 \times S^2$. This structure is exploited to give a derivation of all the conformal weights and the associated global quantum numbers, using group theoretical methods. The result provides the starting point for a more detailed study of the conformal field theory that describes the internal structure of black holes in four dimensions [13, 14, 15]. This theory is not well-understood from fundamental string theory so the spectrum of perturbations gives new results, such as the complete list of chiral primary fields.

The spectrum of black hole perturbations can be applied to the study of the dynamics underlying the emission and absorption of Hawking radiation. In the simplest case, the minimally coupled scalar field, there is a quantitative microscopic model that gives both the amplitude [16], and the energy dependence of the Hawking radiation [17], in perfect agreement with semiclassical calculations of greybody factors.
For more complicated processes there are interesting results, including [18, 19, 20, 21, 22, 23, 24, 25], indicating that most or all processes can be interpreted as interactions in an effective string theory. The spectrum of black hole perturbations, presented in this paper, gives the quantum numbers of the black hole constituents, whose collisions give rise to the Hawking radiation. However, the detailed picture of the black hole dynamics remains incomplete: on the semi-classical side, it is not in general clear how the near-horizon wave functions match onto the free wave functions in the asymptotic Minkowski space. Microscopically, the couplings between operators in the conformal field theory and the fields in the bulk of spacetime are not yet understood in detail. And, perhaps most seriously, the present discussion appears to apply only for black holes in the “dilute gas” regime. Despite these caveats, the results presented here provide progress towards a theory of black hole dynamics.

In the above, the discussion has been framed in terms of the properties of black holes in four dimensions. However, in the dilute gas regime the spacetime is effectively that of a black string in five dimensions. This string can be interpreted as the intersection manifold of three intersecting $M5$-branes. The interpretation as a regular black hole in four dimensions requires that the effective string wraps a compact dimension, and momentum must be added that flows along the string, in one or both directions. However, in the near horizon region, the black hole spacetime and that of three extreme $M5$-branes, uncompactified along their common string, differ only in their global properties. Thus, since the present paper studies local properties of spacetime, the two perspectives are equally valid. Further discussion of the relation between the two interpretations was given, in the context of rotating black holes in five dimensions (or rotating black strings in six dimensions), in [26].

The AdS/CFT correspondence has been exploited extensively to study black holes in five dimensions [7, 8, 27] (see also [28, 29, 30]). Most of this work has been in the context of the black holes in $N = 4$ supergravity, interpreted microscopically as $D1 - D5$ wrapped on a small $K3$ manifold that is transverse to the $D1$, and within the $D5$. In sec. 3 we discuss how these results are extended to black holes in $N = 8$ supergravity, or $D1 - D5$ wrapped on a small transverse four-torus $T^4$. The perturbation spectrum is computed by explicit decoupling of the linearized wave equations satisfied by the bosonic perturbations, as an illustration of those methods.
This paper is organized as follows. In section 2 we consider black holes in four dimensions. First, in sec. 2.1, we analyze the global symmetries, considering in turn general black holes, extreme black holes, and dilute gas black holes. Then, in sec. 2.2, the superconformal multiplets of perturbations around the dilute gas black holes are calculated, using indirect arguments; finally, in sec. 2.3, the complete list of chiral primaries is presented. Section 3 concerns five-dimensional black holes, extending results previously derived for \( N = 4 \) black holes to the case of \( N = 8 \) black holes. In this section the spectrum is computed explicitly, by decoupling of the equations of motion. In the concluding section 4, we compare the spectrum of perturbations with that of the underlying conformal field theory. The five dimensional case is presented in some detail, and the four dimensional black holes are commented on.

2 \( N = 8 \) Black Holes in Four Dimensions.

2.1 Global Symmetries

We are interested in the perturbation spectrum of the background specified by 3 orthogonally intersecting \( M_5 \)-branes. It is assumed that the configuration is compactified along all dimensions within the \( M_5 \)-branes, leaving 4 noncompact dimensions. In practice, additional dimensions can be decompactified, as long as the background has been averaged over the “internal” directions; the global specification of spacetime will not enter our considerations. The configuration can be interpreted in terms of black holes in four dimensions and we begin the discussion by considering the most general black holes in four dimensions. This generality will give some additional properties that are not strictly needed here, but they are of interest in their own right.

General considerations: The duality group of \( N = 8 \) supergravity in four dimensions is \( E_{7(7)} \). Any choice of vacuum configuration breaks this global symmetry spontaneously to its maximal compact subgroup \( SU(8) \), due to the specification of the scalars at infinity. The supersymmetry generators \( Q^A_\alpha \) transform in the fundamental of \( SU(8) \), and thus the particle spectrum in the four dimensional Minkowski vacuum is 70 scalars \( P^{ABCD} \), 56 Weyl fermions \( \psi^{ABC}_\alpha \), 28 vectors \( F^{AB}_{\mu\nu} \), 8 gravitini \( \psi^A_{\alpha\mu} \), and the graviton \( G_{\mu\nu} \). In each case the \( SU(8) \) indices \( A = 1, \ldots, 8 \) are fully
antisymmetrized.

The global symmetry is in general broken further in a nontrivial background. This is most conveniently analyzed in terms of the central charge matrix $Z_{AB}$, an antisymmetric tensor that transforms in the $28$ of the $SU(8)$. It can be represented up to an $SU(8)$ transformation as an antisymmetric $8 \times 8$ matrix with the skew-eigenvalues (see [32, 33] and references therein):

$$Z_{12} = Q_1 + Q_2 + Q_3 + Q_4,$$

$$Z_{34} = Q_1 + Q_2 - Q_3 - Q_4,$$

$$Z_{56} = Q_1 - Q_2 + Q_3 - Q_4,$$

$$Z_{78} = Q_1 - Q_2 - Q_3 + Q_4 .$$

In this formula the boldfaced symbols denote the physical (dressed) charges. These are the charges that appear in the spacetime solution; they are related to the quantized microscopic charges by multiplication with moduli and dimensionful constants. The central charge matrix generically breaks the duality group $SU(8) \rightarrow SU(2)^4$. This organizes the scalar fields $P^{ABCD}$, transforming in the $70$ of $SU(8)$, into 1 ($2, 2, 2, 2$), 2 ($2, 2, 1, 1$) (+ $2 \times 5$ permutations) and 6 ($1, 1, 1, 1$). Similar results are easily derived for the fields with higher spin.

The symmetry breaking pattern determines the wave equations satisfied by the individual fields, via the coset construction (see e.g. [34]), and this gives the explicit connection to much work on greybody factors. We will not need these details, but let us note that the symmetry breaking pattern for the scalars show that there are exactly 16 minimally coupled scalars, 48 intermediate scalars (in 6 different varieties), and 6 fixed scalars. Furthermore, the spacetime parities are such that, for each type of scalar, there are equally many proper scalars and pseudo-scalars.

**Extreme black holes:** For *extreme* black holes the dynamics determines the moduli in terms of the microscopic charges so that, in the near horizon region, the physical charges are identical [35]:

$$Q_1 = Q_2 = Q_3 = Q_4 .$$

(5)
Then three of the skew-eigenvalues of the central charge matrix vanish and the global symmetry is broken $SU(8) \rightarrow SU(2) \times SU(6)$. The vacuum multiplets decompose as:

$$
SU(8) \quad SU(2) \times SU(6) \\
70 \quad (2, 20) \oplus 2(1, 15) \quad P^{ABCD} \\
56 \quad (1, 20) \oplus (2, 15) \oplus (1, 6) \quad \psi^{ABC}_\alpha \\
28 \quad (1, 15) \oplus (2, 6) \oplus (1, 1) \quad F^{AB}_{\mu \nu} \\
8 \quad (2, 1) \oplus (1, 6) \quad \psi^A_{\alpha \mu} \\
1 \quad (1, 1) \quad G_{\mu \nu}
$$

In the extreme case the background preserves $N = 1$ SUSY which, in the horizon region, is enhanced to $N = 2$ SUSY. Therefore the fields eq. 6 can be represented as multiplets of $N = 2$ SUSY. The supercharges transform according to the $(2, 1)$ of the $SU(2) \times SU(6)$, so the various components of the supermultiplet transform differently under $SU(2)$; however, a single $SU(6)$ quantum number can be assigned to the complete $N = 2$ multiplet. We can therefore organize table 6 as:

$$
\begin{array}{ccc}
\text{multiplet} & \text{content} & SU(6) \\
\text{Hyper} & 2S + F & 20 \\
\text{Vector} & 2S + 2F + V & 15 \\
\text{Gravitino} & Gi + 2V + F & 6 \\
\text{Graviton} & G + 2Gi + V & 1 \\
\end{array}
$$

The graviton, the gravitino, the vector, the Weyl fermion, and the scalar are denoted $G, Gi, V, F$ and $S$, respectively. In four dimensions each field has two degrees of freedom, except the scalar that has only one. With these degeneracies, the numbers of bosons and fermions in each multiplet agree.

Since supersymmetry is broken to $N = 2$, the gravitini in the 6 are massive; in particular, the Weyl fermions in the gravitino multiplet are in fact absorbed by the gravitini to form the physical components with a longitudinal vector index.

In the extreme limit, the intermediate scalars of the general classification combine with the minimal and the fixed scalars, forming larger multiplets under the enhanced symmetry group. In the near horizon region of an extreme black hole the 40 scalars
in the hyper-multiplets are all minimally coupled, because $(2, 20)$ contains $(2, 2, 2, 2)$ in its decomposition under $SU(2)^4$. Similarly, the 30 scalars in the vector-multiplets are all fixed scalars, because $(1, 15)$ contains $(1, 1, 1, 1)$ in its decomposition under $SU(2)^4$. These results agree with those of [30].

**Near extreme black holes:** Next, consider near extreme black holes in the “dilute gas” regime. Since the supersymmetry is not restored in the horizon region the “fixed” scalars do not take on precisely their fixed point values; however some of them do, so that:

\[ Q_1 \neq Q_2 = Q_3 = Q_4. \]  

(8)

This equation can be verified explicitly in specific examples; more generally it can be taken as a duality invariant definition of the dilute gas regime. Eq. (8) implies that three of the eigenvalues of the central charge matrix are identical, without being zero. Therefore the global symmetry is broken as $SU(8) \rightarrow SU(2) \times USp(6)$.

The $SU(6)$ multiplets that decompose nontrivially under $USp(6)$ are:

\[ 15 \rightarrow 14 \oplus 1, \]  

(9)

\[ 20 \rightarrow 14' \oplus 6, \]  

(10)

where the $14$ is the antisymmetric 2-tensor of $USp(6)$, and the $14'$ is the antisymmetric 3-tensor. In both cases traces are removed, making it possible that the two representations have the same dimension without being equivalent. It follows by comparing eqs. (9) and (10) with table 7 that the breaking from $SU(6)$ to $USp(6)$ only affects the hyper-multiplet and the vector-multiplet, dividing each of them into two smaller multiplets. In the dilute gas regime one of the internal dimensions effectively decompactifies; so the local Lorentz group is enhanced from $SO(3, 1)$ to $SO(4, 1)$. As result, the $D = 4$ graviton multiplet combines with the “little” vector-multiplet following from eq. (9), in the 1 of $USp(6)$, and form a single $D = 5$ graviton multiplet. Similarly, the $D = 4$ massive gravitino combines with the “little” hyper-multiplet following from eq. (10), in the 6 of $USp(6)$, and form a $D = 5$ massive gravitino multiplet.

In summary, the structure of the supermultiplets for perturbations of black holes in the dilute gas regime, is:
• The $N = 2$ graviton multiplet has 1 graviton, 2 gravitini (in the $\mathbf{2}$ of the global $SU(2)$), and 1 vector. They have 5, $2 \times 4$, and 3 degrees of freedom, respectively. The multiplet is 1 under the global $USp(6)$.

• The massive gravitino multiplet has 1 massive gravitino and 2 vectors (in the $\mathbf{2}$ of the global $SU(2)$); they have 6 and $2 \times 3$ degrees of freedom, respectively. The multiplet is 6 under the global $USp(6)$.

• The vector-multiplet has 1 vector, 2 fermions (in the $\mathbf{2}$ of the global $SU(2)$), and 1 scalar. They have 3, $2 \times 2$, and 1 degrees of freedom, respectively. The multiplet is 14 under the global $USp(6)$. The scalars are the fixed scalars.

• The hyper-multiplet has 1 fermion and 2 scalars (in the $\mathbf{2}$ of the $SU(2)$). They have $1 \times 2$ and $2 \times 1$, degrees of freedom, respectively. The multiplet is $14'$ under the global $USp(6)$. The scalars are the minimally coupled scalars.

The classification according to global symmetries is complete as it stands, but the background in general breaks Lorentz invariance. Thus the wave function of the various components of the $N = 2$, $D = 5$ multiplets are not in general related in any simple way.

Duality invariance is manifest in the discussion no specific higher dimensional interpretation of the configuration has been assumed. However, some configurations behave simpler under five dimensional boost invariance, leaving the organization into $D = 5$ multiplets more useful. In the remainder of this work we consider the three orthogonally intersecting $M5$-branes.

The multiplet structure given above can be recovered directly from the five-dimensional perspective, by considering the near-horizon geometry of an extreme black hole; the computation is similar to the extreme case in four dimensions, except that the compact duality group in five dimensions is $USp(8)$.

The extreme black holes and the dilute gas black holes are characterized by eq. $\S$ and eq. $\S$, respectively. The boundary conditions on the scalars at infinity can be cho-

\footnote{In particular, the classification persists for the most general black holes, with five independent charges $[37]$. In fact, the near horizon geometry of these black holes is identical to the one considered here, when the dilute gas condition (eq. $\S$) is satisfied.
sen so that these equations are maintained throughout spacetime. With this choice of moduli the global symmetries, respectively $SU(6)$ and $USp(6)$, can be applied away from the near-horizon region. Thus these symmetries classify the full greybody factors, rather than just the near-horizon wave functions.

In the case of three orthogonally intersecting $M5$-branes, it is possible to include momentum running along the line of intersection (in both directions, breaking supersymmetry). The geometry of this configuration is $\text{BTZ} \times S^2 \times T^6$ with the effective BTZ mass and angular momentum dependent on the momentum and the energy above extremality, as given in [39]. However, the BTZ black hole is locally $AdS_3$ and so the local geometry remains $AdS_3 \times S^2 \times T^6$ after this apparent generalization, leaving the spectrum of perturbations unaffected. Thus, for local properties, the possibility of momentum can be ignored without loss of generality. In particular, the spectrum is organized in supermultiplets, even when supersymmetry is broken.

### 2.2 Superconformal multiplets

In this section we find the perturbation spectrum of the near horizon region of three intersecting $M5$-branes, as classified under the $(4,0)$ superconformal symmetry of the underlying CFT in two dimensions. This is accomplished by an indirect strategy: the properties satisfied by the supermultiplets from general principles determines them uniquely. The precise steps are discussed in the following.

**Global symmetries:** In section 2.1 the perturbations of black holes in the dilute gas regime were classified under the global $SU(2) \times USp(6)$ symmetry. This symmetry is preserved by the near horizon geometry; so there can be no mixing between the hyper-multiplet, the vector-multiplet, the massive gravitino multiplet, and the $N = 2$ graviton multiplet. The field content of these multiplets was written out in the end of the previous section.

\[2\] In the extreme case such black holes are referred to as double extreme black holes (see e.g. [38]).
Superconformal symmetry: The near horizon geometry is $AdS_3 \times S^2$ [40]. This background can be expressed as the group manifold [360]:

$$AdS_3 \times S^2 \simeq \frac{SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R}{SL(2, \mathbb{R})_{\text{diag}}} \times \frac{SU(2)}{U(1)}.$$  \hfill (11)

In this form the bosonic symmetries are manifest. The $SO(2, 2) \simeq SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$ is the conformal group, with quantum numbers $(\tilde{h}, \bar{h})$; and the $SL(2, \mathbb{R})_R \times SU(2)$ symmetry is the bosonic subgroup of the supergroup $SU(2|1, 1)_R$. The background preserves the supersymmetry, and so its perturbations form representations of $SU(2|1, 1)_R$ [41]:

$$\begin{array}{ccc}
\bar{h} & \bar{j} & SU(2) \\
\begin{array}{c}
k + \frac{1}{2} \\
k + 1 \\
k + \frac{3}{2}
\end{array} & \\
\begin{array}{c}
k + \frac{1}{2} \\
k \\
k - \frac{1}{2}
\end{array} & 1
\end{array}$$  \hfill (12)

where $k$ can be integer or half-integer. These multiplets are perhaps more familiar as the short representations of $N = 4$ supersymmetry in two dimensions. The numbers of fermions and bosons at each level coincide and equals $2(2k + 1)$. Note that the $SL(2, \mathbb{R})_L$ is not a subgroup of any supergroup, so every element of the superconformal multiplet has identical eigenvalue $h$ of the $SL(2, \mathbb{R})_L$ generator $L_0$.

Light cone helicities: It is simplest to analyze the physical spectrum in the light-cone frame, where all modes are physical. The local $SO(4, 1)$ Lorentz group [3] has little group $SO(3)$ for massless fields; so the possible light-cone helicities for fields with spin $s$ are:

$$\lambda = -s, -s + 1, \cdots, s.$$  \hfill (13)

The light-cone can be chosen in the $t-x^{11}$ plane of the $AdS_3$, where the 11th dimension is along the intersection of the $M5$-branes, and then the helicity is also given by $\lambda = h - \bar{h}$. Thus, the transformation properties under the $D = 5$ Lorentz group give the possible values of the $AdS_3$ spin $s_0 = h - \bar{h}$, and their degeneracies.

---

3Global properties of the groups are disregarded in the present work, e.g., $SO(3) \simeq SU(2)$.

4We emphasize that this is not an isometry of the manifold, but it is realized locally in the tangent space.
The leading eigenvalues: Given a superconformal multiplet eq. 12 with parameter $k$, all the higher level multiplets $k + 1, k + 2, \cdots$ correspond to the expansion in partial waves. However, the smallest value of $k$ must be determined from separate considerations. The first step is to expand the fields in their $D = 4$ components and exploit that the angular momentum of a $D = 4$ field with spin $s$ satisfies $\tilde{j} \geq s$ so that the tower of states is precisely $\tilde{j} = s, s + 1, \cdots$. This observation gives the complete set of angular momenta for the fields in a given representation of $USp(6)$. However, the relation between these towers and specific spacetime fields is not clear, because different fields with the same $\tilde{j}$ can mix.

We will resolve the remaining ambiguity by requiring that the $USp(6)$ multiplets can be reassembled into $SU(6)$ multiplets. This is a definite requirement for extreme configurations, including the state with three intersecting $M5$-branes and no momentum along the common string.

Altogether these observations give a practical way to find all the superconformal multiplets: consider fields in a given $USp(6)$ representation and use their $D = 5$ Lorentz-properties to find the possible helicities and their degeneracies; now simply choose superconformal multiplets in the unique way that exhausts all the values of the helicity. This can be done systematically, noting that each element in the superconformal multiplet has the same value of $h$, so the highest $SU(2)$ component, given in the first row of eq. 12, is the entry with the minimal $AdS_3$ spin $s_0 = h - \tilde{h}$. After the form of the multiplets has been found, the leading eigenvalues are determined by expansion in $D = 4$ components, as described above.

In the following we make this procedure explicit by considering each of the $N = 2$, $D = 5$ multiplets in turn.

---

5 From the algebraic point of view the partial waves are generated by the $SU(2)$ level 1 currents of the $N = 4$ superconformal algebra.

6 There is an ordering ambiguity in the problem: we approach the near horizon limit first; and then take the momentum along the string to zero. Additionally, the leading eigenvalues of the supermultiplets could be interchanged in the dilute gas limit, where the symmetry of the central charge matrix is only $USp(6)$.
Hyper-multiplet: There are only scalars and spin-$\frac{1}{2}$ fermions, and all components fit in a single superconformal multiplet:

\[
(h, \tilde{h}) \quad \tilde{j} \quad SU(2) \times USp(6)
\]

\[
\begin{align*}
(l + 1, l + \frac{1}{2}) & \quad l + \frac{1}{2} & \quad (1, 14') \\
(l + 1, l + 1) & \quad l & \quad (2, 14') \\
(l + 1, l + \frac{3}{2}) & \quad l - \frac{1}{2} & \quad (1, 14')
\end{align*}
\]

The required partial wave numbers for scalars and fermions in four dimensions determine that \(l = 0, 1, \cdots\). We use the convention that multiplets with \(\tilde{j} < 0\) are absent. In other words, the \(l = 0\) entry is a particularly short multiplet, where the vacuum is acted on nontrivially by only one of the four supercharges.

The entry in the centre row are the minimally coupled scalars whose conformal weights have previously been identified by explicit calculation [19, 42, 22]; the results agree. By supersymmetry, the fermion with \(l = 0\) should share the property with minimally coupled scalars that, for each value of the helicity, the absorption cross-section \(\sigma_{\text{abs}}(\omega \to 0) = A_4\), where \(A_4\) is the area of the black hole; in contrast, \(\sigma_{\text{abs}}(\omega \to 0) = 0\) for fermions that satisfy the Weyl equation [43, 25]. It would be interesting to verify this prediction by explicit calculation.

Vector multiplets: The helicities of the \(D = 5\) vectors are \(h - \tilde{h} = \pm 1, 0\); and the scalar has \(h - \tilde{h} = 0\). This determines the supermultiplets as:

\[
(h, \tilde{h}) \quad \tilde{j} \quad SU(2) \times USp(6)
\]

\[
\begin{align*}
(l + 1, l + 1) & \quad l + 1 & \quad (1, 14) \\
(l + 1, l + \frac{3}{2}) & \quad l + \frac{1}{2} & \quad (2, 14) \\
(l + 1, l + 2) & \quad l & \quad (1, 14)
\end{align*}
\]

and

\[
(h, \tilde{h}) \quad \tilde{j} \quad SU(2) \times USp(6)
\]

\[
\begin{align*}
(l + 2, l + 1) & \quad l + 1 & \quad (1, 14) \\
(l + 2, l + \frac{3}{2}) & \quad l + \frac{1}{2} & \quad (2, 14) \\
(l + 2, l + 2) & \quad l & \quad (1, 14)
\end{align*}
\]

The helicities of the fermions work out too, as they should. From the \(D = 4\) perspective there are two scalars, so there must be exactly two towers with the angular momenta \(\tilde{j} = 0, 1, \cdots\); this requires that both multiplets have \(l = 0, 1, \cdots\).
The $AdS_3$ scalars, with spin $s_0 = h - \bar{h} = 0$, are in general mixtures of the $D = 5$ vectors and the $D = 5$ fixed scalars. However, there are precisely two states with $j = 0$, and the one with $s_0 = -1$ must be part of the $D = 5$ vector; so the state with $s_0 = j = 0$ can include no vector contribution. Thus it must be the fixed scalar in the S-wave, with no mixing. The conformal weights $(h, \bar{h}) = (2, 2)$ of this state agree with those that have previously been identified by direct calculation [18, 20].

**Massive gravitino multiplets:** In this case the $D = 5$ vector transforms in the $2$ of the global $SU(2)$; so its helicities determines the three supermultiplets:

\[
\begin{array}{ccc}
(h, \bar{h}) & \tilde{j} & SU(2) \times USp(6) \\
(l, l + \frac{1}{2}) & l + \frac{1}{2} & (1, 6) \\
(l, l + 1) & l & (2, 6) \\
(l, l + \frac{3}{2}) & l - \frac{1}{2} & (1, 6) \\
(l + 1, l + \frac{1}{2}) & l + \frac{1}{2} & (1, 6) \\
(l + 1, l + 1) & l & (2, 6) \\
(l + 1, l + \frac{3}{2}) & l - \frac{1}{2} & (1, 6) \\
(l + 2, l + \frac{1}{2}) & l + \frac{1}{2} & (1, 6) \\
(l + 2, l + 1) & l & (2, 6) \\
(l + 2, l + \frac{3}{2}) & l - \frac{1}{2} & (1, 6) \\
\end{array}
\]

Again, the fermion helicities serve as a check.

The $D = 5$ vector decomposes into a $D = 4$ vector, and a $D = 4$ scalar. Since $\tilde{j} = l$ for the bosons in these tables it follows that exactly two of the tables have $l = 1, 2, \cdots$, and one has $l = 0, 1, \cdots$.

The middle table is identical to the result for the hyper-multiplet eq. (14), except for the $USp(6)$ representation. Therefore the condition that the states can be assembled into $SU(6)$ multiplets forces that the middle table has range $l = 0, 1, \cdots$, just as the hyper-multiplet. Accordingly, the other two representations have $l = 1, 2, \cdots$.

**Graviton multiplet:** Finally, the $D = 5$ graviton has light cone helicities $s_0 = h - \bar{h} = \pm 2, \pm 1, 0$; so, together with the $D = 5$ vector helicities $s_0 = h - \bar{h} = \pm 1, 0$,
the supermultiplets are determined as:

\[
\begin{array}{ccc}
(h, \bar{h}) & \bar{j} & SU(2) \times USp(6) \\
(l, l + 1) & l + 1 & (1, 1) \\
(l, l + \frac{3}{2}) & l + \frac{1}{2} & (2, 1) \\
(l, l + 2) & l & (1, 1) \\
(l + 1, l + 1) & l + 1 & (1, 1) \\
(l + 1, l + \frac{3}{2}) & l + \frac{1}{2} & (2, 1) \\
(l + 1, l + 2) & l & (1, 1) \\
(l + 2, l + 1) & l + 1 & (1, 1) \\
(l + 2, l + \frac{3}{2}) & l + \frac{1}{2} & (2, 1) \\
(l + 2, l + 2) & l & (1, 1) \\
(l + 3, l + 1) & l + 1 & (1, 1) \\
(l + 3, l + \frac{3}{2}) & l + \frac{1}{2} & (2, 1) \\
(l + 3, l + 2) & l & (1, 1)
\end{array}
\]

From the $D = 4$ perspective the boson fields are 2 scalars, 2 vectors, and 1 graviton. The vectors and the graviton each have two degrees of freedom and the scalar has one. Thus there must be two towers with $SU(2)$ quantum numbers $\bar{j} = 0, 1, \cdots$, four with $\bar{j} = 1, 2, \cdots$ and two with $\bar{j} = 2, 3, \cdots$. Comparing with the tables we find that two supermultiplets have indices $l = 0, 1, \cdots$, and two have $l = 1, 2, \cdots$.

Two graviton multiplets must combine with the vector multiplets eqs. 15-16 and form larger multiplets, with the global group $SU(6)$. This condition determines that it is the middle two multiplets that have $l = 0, 1, \cdots$; the first and the last multiplet has $l = 1, 2, \cdots$.

The wave functions of particles with spin in the background of dilute gas black holes have previously been considered in [23, 24, 25]. It was found that the leading Hawking emission of particles with spin $s$ at low frequency is controlled by the conformal weights $(1, 1 + s)$ [25]. This result was based on “minimal” couplings, i.e. only the coupling to gravity was taken into account. In the present work, supergravity
specifies additional background fields. Nevertheless, the tables do have entries with the leading quantum numbers \((1, 1+s)\). This indicates the interpretation of Hawking emission of particles with spin remains qualitatively correct, even though a complete analysis of the greybody factors for the \(N = 8\) black holes has not yet been carried out.

### 2.3 The Chiral Primary Fields

The chiral primaries are useful because they generate the complete supermultiplet. They are defined as the states that satisfy \(\tilde{h} = \tilde{j}\). More precisely, the representation \(\tilde{j}\) of the \(SU(2)\) rotation group has elements with projections on some axis \(\tilde{j}_3 = -\tilde{j}, \ldots, \tilde{j}\). The states with \(\tilde{h} = \tilde{j}_3\) are the chiral primaries; those with \(\tilde{h} = -\tilde{j}_3\) are anti-chiral primaries. We will informally refer to all these fields as chiral primaries.

The first entry of each table in the previous section corresponds to a chiral primary. In the extreme limit the fields form representations of \(SU(6)\), and we collect them as:

\[
\begin{array}{cccc}
\hline
h & \tilde{h} = \tilde{j}_3 & SU(6) & l \\
\hline
l+1 & l + \frac{1}{2} & 20 & 0, 1, \ldots \\
l+1 & l + 1 & 15 & 0, 1, \ldots \\
l+2 & l + 1 & 15 & 0, 1, \ldots \\
l & l + \frac{1}{2} & 6 & 1, 2, \ldots \\
l+2 & l + \frac{1}{2} & 6 & 1, 2, \ldots \\
l & l + 1 & 1 & 1, 2, \ldots \\
l+3 & l + 1 & 1 & 1, 2, \ldots \\
\hline
\end{array}
\]

(19)

All the chiral primaries are singlets under the global \(SU(2)\).

The \(AdS/CFT\) correspondance predicts that this table gives the complete list of chiral primaries in the effective string theory describing three orthogonally intersecting \(M5\)-branes. In supergravity, it gives all the conformal weights underlying the greybody factors of extreme black holes. The result for the dilute gas regime is found by decomposing the \(SU(6)\) representation into \(USp(6)\), according to eqs. [9]-[10].

The physical significance of the precise ranges of \(l\) is not clear. It is known that the “missing” states with \(l = 0\) generally correspond to modes that are pure gauge. However, it is possible such modes induce boundary states at AdS-infinity, and so
they may nevertheless play a role in the AdS/CFT correspondance. The discussion in the following paragraphs disregard the range of $l$.

In the dilute gas regime, all the chiral primaries at a given level can be generated from the $14'$ state, by acting with two operators of the form $Q^A_{h,\bar{h}}$, where $A$ is a $USp(6)$ vector index, $\bar{h} = \frac{1}{2}$, and $h$ takes the two values $h = 0$ and $h = 1$, respectively. This suggests that these operators, together with their complex conjugates, are symmetries of the chiral algebra. The $Q^A_{h,\bar{h}}$ have half-integer spin; so this global symmetry is itself a supersymmetry. Global symmetries are generally robust; thus it is reasonable to expect that the $Q^A_{h,\bar{h}}$ persist in the full interacting string theory.

The $Q^A_{h,\bar{h}}$ admit a spacetime interpretation in terms of the broken supersymmetry. Namely, the infinitesimal generators of the broken supersymmetries act nontrivially on the background creating fermionic “perturbations”. Repeated actions form a closed algebra, by the underlying supersymmetry. The restriction of this algebra of broken supersymmetry to the chiral operators gives the $Q^A_{h,\bar{h}}$. These operators must have definite transformation properties under the preserved symmetries. They are in the 6 of $USp(6)$, by restriction of the original 8 of $SU(8)$. Generators of supersymmetry are spin-$\frac{1}{2}$ so $\bar{h} = \frac{1}{2}$, to preserve the chiral condition. Finally, there are two $D = 5$ helicities, so $h - \bar{h} = \pm \frac{1}{2}$ determines the two values of the conformal weight $h$ as $h = 0$ and $h = 1$. It would be interesting to investigate the properties of the $Q^A_{h,\bar{h}}$ in more detail.

3 $N = 8$ Black Holes in Five Dimensions

The perturbation spectrum of five-dimensional black holes and their associated six-dimensional black strings has been considered recently by several workers \cite{7, 8, 6}. In particular, the linearized equations of motion have been decoupled completely by explicit calculation, in the case of $N = 4$ supergravity \cite{7, 6}. The purpose of the present section is to extend this result to $N = 8$ supergravity. In other words, we consider the background $AdS_3 \times S^3 \times M$, with the small manifold $M = T^4$, rather than $M = K3$.

The matter content of $N = 8$ supergravity differs from the matter content of $N = 4$ supergravity by having $n = 5$ anti-selfdual tensor supermultiplets, instead of $n = 21$. 
There are also additional fields, namely 4 gravitini, 16 vector fields and 20 fermions, with counting from the $D = 6$ perspective. In the linearized approximation, there are no couplings between the original $N = 4$ and the additional $N = 8$ perturbations\footnote{It is consistent to take the 16 vector fields vanishing in the background, so they can appear only in quadratic order in the variations with respect to any of the $N = 4$ fields. Alternatively, the decoupling follows from global symmetries.}, so the results obtained previously for $N = 4$ remain valid, except that now $n = 5$. However, the additional gravitino multiplet requires further considerations.

The conformal weights of the additional multiplet can be worked out using group theory, as in the previous section. A new feature is that, for massless fields, the little group $SO(4) \simeq SU(2) \times SU(2)$ of the Lorentz group $SO(5,1)$ is not simple; so there are two independent helicities $\lambda$ and $\bar{\lambda}$. The $AdS_3$ spin is given in terms of the helicities as $s_0 = h - \bar{h} = \lambda + \bar{\lambda}$. The result obtained from group theory agrees with the one given below, in eq. 44. However, in the present section we follow the explicit calculation of \cite{6}, and decouple the linearized equations of motion explicitly for the bosons.

### 3.1 Decoupling of the Equations of Motion

The linearized equations of motion for the vector fields in $D = 6$ SUGRA is:

$$\nabla^I F_{IJ}^\alpha - \frac{1}{6} \epsilon_J^{KLMNO} (\Gamma_A)_{\alpha}^{\beta} F_{KL}^{\beta} H_{MNO}^A = 0 \ .$$

These wave equations follow from duality, with the numerical coefficient determined by explicit dimensional reduction from $D = 11$ of selected components. The duality group $SO(5,5)$ has spinor indices $\alpha, \beta = 1, \cdots, 16$ and vector index $A = 1, \cdots, 10$. The $SO(5,1)$ Lorentz group has vector indices $I, J, \cdots = 0, \cdots, 5$ that decompose into the $AdS_3$ indices $\mu, \nu, \cdots = 0, 1, 2$ and the $S^3$ indices $a, b, \cdots = 3, 4, 5$.

The only nonvanishing matter field in the black hole background is a selfdual component of the antisymmetric tensor, decomposed as:

$$H_{abc}^5 = \epsilon_{abc} \ ; \ H_{\mu\nu\rho}^5 = \epsilon_{\mu\nu\rho} \ ,$$

in units where the cosmological constant is $\Lambda = -l^2 = -1$.\footnote{It is consistent to take the 16 vector fields vanishing in the background, so they can appear only in quadratic order in the variations with respect to any of the $N = 4$ fields. Alternatively, the decoupling follows from global symmetries.}
The vacuum breaks the duality group \( SO(5, 5) \rightarrow SO(5) \times SO(5) \), and the black hole background breaks this global symmetry further as \( SO(5) \times SO(5) \rightarrow SO(4) \times SO(5) \). The field strength transforms as the spinor \( 16 \) under \( SO(5, 5) \), and thus as the bispinor \( (4, 4) \) in the \( SO(5) \times SO(5) \) symmetric vacuum. In the \( SO(4) \times SO(5) \) symmetric background created by the black hole this representation is further broken to \( (2_+, 4) \oplus (2_-, 4) \). The Gamma-matrices \( (\Gamma_A)_a^\beta \) of \( SO(5, 5) \) can be decomposed according to this symmetry breaking pattern. The two representations are distinguished by the eigenvalue \( P = \pm 1 \) of:

\[
(\Gamma_{11})_\alpha^\beta F_{KL}^\beta = P F_{KL}^\alpha .
\]

For definiteness we concentrate on \( P = 1 \) and so the representation \( (2_+, 4) \). The result for the alternative projection \( P = -1 \) will be recovered in due course. Thus the equation of motion becomes:

\[
\nabla^I F_{IJ} - \frac{1}{6} \epsilon^{KLMNO} F_{KL} H_{MNO}^5 = 0 ,
\]

or, in view of eq. 21:

\[
\nabla^I F_{Ia} - \epsilon_a^{bc} F_{bc} \epsilon_a = 0 ,
\]

\[
\nabla^I F_{I\mu} - \epsilon_\mu^{\nu\rho} F_{\nu\rho} \epsilon_a = 0 ,
\]

The general expansion in spherical harmonics on \( S^3 \) is:

\[
A_\mu(x, y) = \sum_l A^{(l, 0)}_\mu(x) Y^{(l, 0)}(y) ,
\]

\[
A_a(x, y) = \sum_l [ A^{(l, \pm 1)}_a(x) Y^{(l, \pm 1)}(y) + A^{(l, 0)}(x) \partial_a Y^{(l, 0)}(y) ] ,
\]

where the coordinates on \( AdS_3 \) and \( S^3 \) are denoted \( x \) and \( y \), respectively. The spherical harmonics satisfy \( ^8 \):

\[
\nabla^2 Y^{(l, 0)} = [1 - (l + 1)^2] Y^{(l, 0)} ,
\]

\[
\nabla^2 Y^{(l, \pm 1)} = [2 - (l + 1)^2] Y^{(l, \pm 1)} ,
\]

\[
\nabla_a Y^{(l, \pm 1)} = 0 ,
\]

\[
\epsilon_a^{bc} \partial_b Y^{(l, \pm 1)} = \pm (l + 1) Y^{(l, \pm 1)} .
\]

---

\(^8\)We use conventions where \( \epsilon^{\mu\nu\rho\sigma} = -\epsilon^{\sigma\mu\nu\rho} = \epsilon^{\nu\rho\sigma\mu} \).
It is straightforward to show that a gauge transformation can be chosen so that:

$$\partial^a A_\mu(x, y) = 0 .$$  \hspace{1cm} (32)

This condition defines the Lorentz-DeDonder gauge. In this gauge $A^{(l,0)} = 0$ so that the last term in eq. 27 vanishes. The gauge condition eq. 32 allows further gauge transformations of the form:

$$\delta A_{\mu}^{(0,0)}(x) = \partial_{\mu} \Lambda(x) .$$  \hspace{1cm} (33)

These are generated by the 0-mode $Y^{(0,0)}$ on the sphere.

The equations of motion eq. 24 for the AdS scalars become:

$$\sum_l [\nabla^\mu \partial_\mu - (l + 1)^2 \mp 2(l + 1)] A^{(l,\pm 1)} Y_a^{(l,\pm 1)} - \sum_l \nabla^\mu A^{(l,0)}_\mu \partial_a Y^{(l,0)} = 0 ,$$  \hspace{1cm} (34)

in the Lorentz-DeDonder gauge. Orthogonality relations of the spherical harmonics gives:

$$\nabla^\mu A^{(l,0)}_\mu (x) = 0 ,$$  \hspace{1cm} (35)

so that the longitudinal modes decouple, as they should. The remaining equations are those of minimally coupled scalars in the AdS$_3$, with effective masses:

$$m^2 = (l + 1)^2 \pm 2(l + 1) .$$  \hspace{1cm} (36)

Then the conformal weights given through:

$$m^2 = 4h(h - 1) ,$$  \hspace{1cm} (37)

become:

$$h = \frac{l + 2 \pm 1}{2} .$$  \hspace{1cm} (38)

The $SU(2)$ quantum numbers of the fields are related to the indices $(l_1, l_2)$ of the spherical harmonics through $(j, \bar{j}) = (l_{1+2}, l_{2+2})$. Moreover, the fields are AdS$_3$ scalars so $h = \bar{h}$. Thus we arrive at the following table:

| $(h, \bar{h})$ | $(j, \bar{j})$ | $SO(4) \times SO(5)$ | $l$ |
|---------------|---------------|----------------|---|
| $\left(\frac{l+3}{2}, \frac{l+3}{2}\right)$ | $\left(\frac{l+1}{2}, \frac{l-1}{2}\right)$ | $(2_+, 4)$ | $1, 2, \cdots$ |
| $\left(\frac{l+1}{2}, \frac{l+1}{2}\right)$ | $\left(\frac{l-1}{2}, \frac{l+1}{2}\right)$ | $(2_+, 4)$ | $1, 2, \cdots$ |
Next we consider the $AdS_3$ vectors. Their equation of motion is eq. 25. In the Lorentz-DeDonder gauge eq. 32 we immediately find independent equations for each partial wave:

$$\nabla^n \partial^\nu A^{(l,0)}_{\mu} - \epsilon_{\mu}^{\nu \rho} \partial^\nu A^{(l,0)}_{\rho} = l(l+2)A^{(l,0)}_{\mu}. \quad (40)$$

It is simplest to analyze this equation by taking advantage of the $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$ symmetry. From this point of view it is clear that vector fields in $AdS_3$ satisfy $h - \bar{h} = \pm 1$ and we can view eq. 40 as an equation for the energy $E_0 = h + \bar{h}$. The $SL(2, \mathbb{R})$ representation theory is analogous to that of $SU(2)$. We need the equation analogous to eq. 31, namely:

$$\epsilon_{\mu}^{\nu \rho} \partial^\nu A^{(l,0)}_{\rho} = \mp(h + \bar{h} - 1)A^{(l,0)}_{\mu}. \quad (41)$$

We find:

$$(h + \bar{h} - 1)^2 \pm 2(h + \bar{h} - 1) = l(l+2), \quad (42)$$

so $h + \bar{h} = l + 1$ and $h + \bar{h} = l + 3$ for $h - \bar{h} = \pm 1$, respectively. We thus arrive at the table:

$$\begin{array}{cccc}
(h, \bar{h}) & (j, \bar{j}) & SO(4) \times SO(5) & l \\
(\frac{l+2}{2}, \frac{l}{2}) & (\frac{l}{2}, \frac{l}{2}) & (2_+, 4) & 1, 2, \cdots \\
(\frac{l+2}{2}, \frac{l+4}{2}) & (\frac{l}{2}, \frac{l}{2}) & (2_+, 4) & 0, 1, 2, \cdots \\
\end{array} \quad (43)$$

Note that the functions $Y^{(l,0)}$ are the standard scalar spherical harmonics with indices $l = 0, 1, \cdots$. However, according to eq. 11 the corresponding mode is constant when $h + \bar{h} = 1$, and thus this mode is not propagating. This absence of the $l = 0$ mode is related to the residual gauge transformation eq. 33.

At this point we should recall that we have omitted half the story, due to the choice of $P = +1$ after eq. 22. Accordingly, tables 39 and 43 summarizing the results contain only entries with $SO(4)$ quantum numbers $2_+$. The fields with $P = -1$ and thus the global quantum numbers $(2_-, 4)$, are the complex conjugates of those we have considered. They correspond to the entries of table 39 with $j \leftrightarrow \bar{j}$, and the entries of table 43 with $h \leftrightarrow \bar{h}$.

### 3.2 Superconformal Multiplets

We shift the index $l$ of the second entry in both tables (43 and 39) so that $l = 0, 1, \cdots$; and, recalling the convention that multiplets with $SU(2)$ indices $j = -1$ or $\bar{j} = -1$
vanish, we allow \( l = 0 \) for the first entry of eq. \[39\]. Then all the modes in the supermultiplet can be assembled into the table:

\[
\begin{array}{ccc}
(h, \bar{h}) & (j, \bar{j}) & SO(4) \times SO(5) \\
\left(\frac{l+2}{2}, \frac{l+2}{2}\right) & \left(\frac{l+2}{2}, \frac{l}{2}\right) & (2_-, 4) \\
\left(\frac{l+3}{2}, \frac{l+3}{2}\right) & \left(\frac{l+1}{2}, \frac{l-1}{2}\right) & (2_+, 4) \\
\left(\frac{l+2}{2}, \frac{l+1}{2}\right) & \left(\frac{l+2}{2}, \frac{l+1}{2}\right) & (1, 4) \\
\left(\frac{l+2}{2}, \frac{l+3}{2}\right) & \left(\frac{l+2}{2}, \frac{l-1}{2}\right) & (1, 4) \\
\left(\frac{l+3}{2}, \frac{l+2}{2}\right) & \left(\frac{l+1}{2}, \frac{l}{2}\right) & (4, 4) \\
\left(\frac{l+4}{2}, \frac{l+3}{2}\right) & \left(\frac{l+4}{2}, \frac{l-1}{2}\right) & (1, 4) \\
\left(\frac{l+3}{2}, \frac{l+1}{2}\right) & \left(\frac{l+1}{2}, \frac{l+1}{2}\right) & (2_+, 4) \\
\left(\frac{l+1}{2}, \frac{l+2}{2}\right) & \left(\frac{l}{2}, \frac{l+3}{2}\right) & (2_-, 4) \\
\left(\frac{l+1}{2}, \frac{l+1}{2}\right) & \left(\frac{l}{2}, \frac{l+1}{2}\right) & (1, 4) \\
\end{array}
\]

(44)

plus its complex conjugate. The fermionic entries were inferred by supersymmetry, discussed below. At each level \( l \) there are \( 8(l+1)(l+2) \) bosons and \( 8(l+1)(l+2) \) fermions.

The underlying symmetry structure of the near horizon geometry of black holes in five dimensions is the factorized supergroup \( SU(2|1, 1)_L \times SU(2|1, 1)_R \). In particular, the two-dimensional supersymmetry is \((4,4)\). It follows that the states can be organized into supermultiplets under left and right moving generators independently. Indeed, the table above is the tensor product of two \( N=4 \) multiplets of the form given in eq. \[12\], with parameters \( k_L = (l+1)/2 \) and \( k_R = l/2 \); and the complex conjugate multiplet similarly derives from \( k_L = l/2 \) and \( k_R = (l+1)/2 \). The chiralities of the \( D=6 \) vectors, and the quantum numbers of the fermions were in fact determined precisely by demanding this structure.

The chiral primary fields with respect to any \((2,2)\) subalgebra of the \((4,4)\) supersymmetry satisfy \((h, \bar{h}) = (j, \bar{j})\) \[9\]. In the table above, the chiral primary fields are the entries with \((h, \bar{h}) = (\frac{l+2}{2}, \frac{l+1}{2})\). Similar tables, for the fields present in the \( N=4 \) case, are given in \[9\]. This allows the assembly of a complete list of chiral primaries.

\[9\] More precisely the \((j, \bar{j})\) representation of the \( SU(2) \times SU(2) \) has components with \( j_3 = -j, \cdots, j \) and \( \bar{j}_3 = -\bar{j}, \cdots, \bar{j} \). The chiral fields are those components that satisfy \((h, \bar{h}) = (\pm j_3, \pm \bar{j}_3)\), with the four possible signs corresponding to elements in the \((c, c)-, (c, a)-, (a, c)-\) and \((a, a)-\) rings.
in the $N = 8$ black hole background. It is:

$$
(h, \bar{h}) \quad SO(4) \times SO(5)
$$

$$
\left( \frac{l+3}{2}, \frac{l+1}{2} \right) + \text{c.c.} \quad (1, 1)
$$

$$
\left( \frac{l+1}{2}, \frac{l+1}{2} \right) \quad (1, 5)
$$

$$
\left( \frac{l+2}{2}, \frac{l+2}{2} \right) \quad (1, 1)
$$

$$
\left( \frac{l+2}{2}, \frac{l+1}{2} \right) + \text{c.c.} \quad (1, 4)
$$

(45)

where $l = 0, 1, \cdots$. This representation of the complete perturbation spectrum is convenient for comparison with string theory.

4 Microscopic Interpretation

According to Maldacena’s conjecture [1] the spectrum of black hole perturbations is identical to that of the underlying string theory. For five dimensional black holes the string theory result is given in the work of Strominger and Vafa [4]. This sets the stage for a detailed comparison, elaborating the one given in [7, 8].

Consider the string theory spectrum of $n_1$ $D1$-branes and $n_5$ $D5$-branes, wrapped on a small Calabi-Yau manifold $M$ with two complex dimensions, i.e. $M = T^4$ or $M = K3$. The spectrum is given by a superconformal $\sigma$-model on the target space [4]:

$$
\mathcal{C} = M^k / \Sigma_k
$$

(46)

where the level $k = n_1 n_5$. The RR states of the $\sigma$-model are in one-to-one correspondence with the elements of the cohomology of the target space. It is a property of the symmetric orbifold construction that the cohomology of $\mathcal{C}$ can be constructed as a Fock space over the cohomology of $M$ [4, 3]. In particular, the level $n$ $(p, q)$-form of $H^*(\mathcal{C})$, denoted $(p, q)_n$, is the permutation invariant product of $n$ $(p, q)$-forms of $H^*(M)$. More general elements in $H^*(\mathcal{C})$ are generated by combining these elementary ones, as in the construction of a Fock space.

We are interested in the chiral operators, more precisely the $(c, c)$ chiral ring. These are in the NS-NS sector, related to the RR-sector by spectral flow. Their spectrum is [7]:

$$
(h, \bar{h}) = (j_3, \bar{j}_3) = \frac{1}{2} \sum_i (p_i - 1 + n_i, q_i - 1 + n_i)
$$

(47)
The chiral operators are organized in a Fock-space whose building blocks are the “single particle operators”, just as the RR-states. The conformal weights of the single particle chiral primaries are given by the individual terms in the expression above, with levels \( n = 1, 2, \cdots \), except that \( n = 2, 3, \cdots \) for \( p = q = 0 \) because \((0,0)\) corresponds to the standard \( NS - NS \) vacuum. Thus, their degeneracies are:

\[
\begin{align*}
(h, \bar{h}) & \quad \text{degeneracy} \\
\left( \frac{l+2}{2}, \frac{l}{2} \right) + \text{c.c.} & \quad h_{2,0} + \text{c.c.} \\
\left( \frac{l+1}{2}, \frac{l+1}{2} \right) & \quad h_{0,0} + h_{1,1} \\
\left( \frac{l+2}{2}, \frac{l+2}{2} \right) & \quad h_{2,2} \\
\left( \frac{l+1}{2}, \frac{l}{2} \right) + \text{c.c.} & \quad h_{1,0} + \text{c.c.} \\
\left( \frac{l+2}{2}, \frac{l+1}{2} \right) + \text{c.c.} & \quad h_{2,1} + \text{c.c.}
\end{align*}
\]

where \( l = 0, 1, \cdots \).

The spectrum of chiral operators in string theory should be compared with that of black hole perturbations. In the linearized approximation, the perturbations can be superimposed, and therefore they form a Fock space, just as in string theory. Thus, it is sufficient to compare the single particle operators on the two sides. Moreover, it is only the chiral primaries that are needed, since these operators generate the complete supermultiplet.

For \( T^4 \) the “scalar” Betti-numbers are \( h_{2,2} = 1 \) and \( h_{0,0} + h_{1,1} = 5 \). This gives perfect agreement between string theory (table 48) and supergravity (table 45), as in [7] (except that there \( M = K3 \)). The remaining Betti-numbers are \( h_{2,0} = 1 \) and \( h_{2,1} = h_{1,0} = 2 \); and their complex conjugates. Here the degeneracies of string theory and supergravity agree again, but there is a minor discrepancy in the conformal weights: the first element of the \((2,0)\) and the \((1,0)\) string towers are absent in the supergravity description. The states that are missing are not propagating on the supergravity side because they are pure gauge modes; so it is quite proper that they are not included in the table. However, they may nevertheless induce physical degrees of freedom on the boundary at infinity; if these modes are included the agreement is restored. It would be interesting to work out this possibility in more detail.

There is also an important structural difference between string theory and supergravity: in string theory the levels take a finite range \( n = 1, \cdots, k \), but in the standard Fock space description they do not. This is the “stringy exclusion princi-
ple" [4]. Moreover, in multiparticle states, the total occupation number (weighted with respect to the level) of all varieties of excitations is similarly bounded by $k$. Thus the stringy exclusion principle also applies to particles that are not identical.

Let us conclude with a few comments on the conformal field theory underlying the four dimensional black holes. This theory is the effective two dimensional theory of three orthogonally intersecting $M5$-branes, wrapped on a small torus with their line of intersection kept large. Little is known about this theory, at least in comparison with the $D1 - D5$ bound state. In particular, there is no complete list of chiral primaries derived from string theory that can be compared with the perturbation spectrum of the near horizon geometry, displayed in table 13 [9]. The result given in the table can be viewed as a prediction for string theory.

In supergravity the spectrum of perturbations forms a Fock space, at the linearized level. Thus, it is inherent in the construction of the CFT via $AdS/CFT$ correspondence that there is a large set of multi-particle chiral primaries, organized in a Fock space over the single particle chiral primaries, as in the $D1 - D5$ case. This suggests that, in analogy with the $D1 - D5$ system, we seek a $\sigma$-model on a symmetric orbifold with the structure given in eq. 46. In the case of intersecting $M5$ branes we expect $k = n_1 n_2 n_3$, where $n_i$ is the respective number of $M5$-branes. It is not a priori clear what the manifold $M$ is; but it should be readily determined from the spectrum of chiral primaries [45].

Acknowledgments: I would like to thank M. Cvetič, S. Gubser, H. Verlinde and S. Mathur, for discussions; and R. Leigh for collaboration in the initial stages of this project. This work is supported in part by DOE grant DOE-FG02-95ER40893.

References

[1] J. Maldacena. The large N limit of superconformal field theories and supergravity. [hep-th/9711200]

[2] E. Witten. Anti-de Sitter space and holography. [hep-th/9802150]

[3] S.S. Gubser, I.R. Klebanov, and A.M. Polyakov. Gauge theory correlators from noncritical string theory. [hep-th/9802109]
[4] A. Strominger and C. Vafa. Microscopic origin of the Bekenstein-Hawking entropy. *Phys. Lett. B*, 379:99–104, 1996. [hep-th/9601029](https://arxiv.org/abs/hep-th/9601029).

[5] R. Dijkgraaf, G. Moore, E. Verlinde, and H. Verlinde. Elliptic genera of symmetric products and second quantized strings. *Commun. Math. Phys.*, 185:197–209, 1997. [hep-th/9608096](https://arxiv.org/abs/hep-th/9608096).

[6] S. Deger, A. Kaya, E. Sezgin, and P. Sundell. Spectrum of $D = 6$, $N=4$ supergravity on $AdS_3 \times S^3$. [hep-th/9804166](https://arxiv.org/abs/hep-th/9804166).

[7] J. Maldacena and A. Strominger. $AdS_3$ black holes and a stringy exclusion principle. [hep-th/9804083](https://arxiv.org/abs/hep-th/9804083).

[8] E. Martinec. Matrix models of AdS gravity. [hep-th/9804111](https://arxiv.org/abs/hep-th/9804111).

[9] C. Vafa. Puzzles at large N. [hep-th/9804172](https://arxiv.org/abs/hep-th/9804172).

[10] M. Cvetič and D. Youm. Dyonic BPS-saturated black holes of heterotic string theory on a six–torus. *Phys. Rev. D*, 53:584, 1996. [hep-th/9507090](https://arxiv.org/abs/hep-th/9507090).

[11] M. Cvetič and D. Youm. BPS saturated and nonextreme states in abelian Kaluza-Klein theory and effective $N=4$ supersymmetric string vacua. In *STRINGS 95: Future Perspectives in String Theory*, pages 131–147. Los Angeles, CA, 1995. [hep-th/9508058](https://arxiv.org/abs/hep-th/9508058).

[12] G. T. Horowitz, D. A. Lowe, and J. M. Maldacena. Nonextremal black hole microstates and U-duality. *Phys. Rev. Lett.*, 77:430–433, 1996. [hep-th/9603195](https://arxiv.org/abs/hep-th/9603195).

[13] R. Dijkgraaf, E. Verlinde, and H. Verlinde. Counting dyons in $N=4$ string theory. *Nucl. Phys. B*, 484:543–561, 1997. [hep-th/9607026](https://arxiv.org/abs/hep-th/9607026).

[14] J. Maldacena, A. Strominger, and E. Witten. Black hole entropy in M theory. *JHEP*, 12:002, 1997. [hep-th/9711053](https://arxiv.org/abs/hep-th/9711053).

[15] C. Vafa. Black holes and Calabi-Yau threefolds. [hep-th/9711067](https://arxiv.org/abs/hep-th/9711067).

[16] S. Das and S. Mathur. Comparing decay rates for black holes and D-branes. *Nucl. Phys. B*, 478:561–576, 1996. [hep-th/9606185](https://arxiv.org/abs/hep-th/9606185).
[17] J. Maldacena and A. Strominger. Black hole greybody factors and D-brane spectroscopy. *Phys. Rev. D*, 55:861–870, 1996. hep-th/9609026.

[18] C. G. Callan, S. S. Gubser, I. R. Klebanov, and A. A. Tseytlin. Absorption of fixed scalars and the D-brane approach to black holes. *Nucl. Phys. B*, 489:65–94, 1997. hep-th/9610172.

[19] J. Maldacena and A. Strominger. Universal low-energy dynamics for rotating black holes. *Phys. Rev. D*, 56:4975–4983, 1997. hep-th/9702013.

[20] M. Krasnitz and I. Klebanov. Testing effective string models of black holes with fixed scalars. *Phys.Rev.D*, 56:2173–217, 1997. hep-th/9703216.

[21] I. Klebanov, A. Rajaraman, and A. Tseytlin. Intermediate scalars and the effective string model of black holes. *Nucl.Phys.B*, 503:157–176, 1997. hep-th/9704112.

[22] S. Mathur. Absorption of angular momentum by black holes and D-branes. *Nucl. Phys. B*, 514:204–226, 1998. hep-th/9704156.

[23] S. Gubser. Absorption of photons and fermions by black holes in four-dimensions. *Phys.Rev.D*, 56:7854–7868, 1997. hep-th/9706100.

[24] K. Hosomichi. Fermion emission from five-dimensional black holes. hep-th/9711072.

[25] M. Cvetić and F. Larsen. Greybody factors for black holes in four dimensions: Particles with spin. *Phys. Rev. D*, 57:6297–6310, 1998. hep-th/9712118.

[26] M. Cvetić and F. Larsen. Near horizon geometry of rotating black holes in five dimensions. hep-th/9805097.

[27] J. DeBoer. Talk at the ITP, santa barbara. 5.7.98.

[28] A. Strominger. Black hole entropy from near horizon microstates. *JHEP*, 02:009, 1998. hep-th/9712251.

[29] E. Teo. Black hole absorption cross-sections and the Anti-de Sitter conformal field theory correspondence. hep-th/9805014.
[30] H. Lee, N. Kim, and Y. Myung. Probing the BTZ black hole with test fields. hep-th/9803227.

[31] E. Cremmer and B. Julia. The SO(8) supergravity. *Nucl. Phys. B*, 159:141–212, 1979.

[32] R. Kallosh and B. Kol. $E_7$ symmetric area of the black hole horizon. *Phys. Rev. D*, 53:5344–5348, 1996. hep-th/9602014.

[33] C. Hull and M. Cvetić. Black holes and U-duality. *Nucl. Phys. B*, 480:296–316, 1996. hep-th/9606193.

[34] P. Fre. Lectures on special Kähler geometry and electric-magnetic duality rotations. *Nucl. Phys. B. (Proc. Suppl)*, 45:59–114, 1996. hep-th/9512043.

[35] S. Ferrara and R. Kallosh. Universality of supersymmetric attractors. *Phys. Rev. D*, 54:1525–1534, 1996. hep-th/9603090.

[36] L. Andrianopoli, R. D’Auria, S. Ferrara, P. Fre, and M. Trigiante. $E_{7,7}$ duality, BPS black hole evolution and fixed scalars. *Nucl. Phys. B*, 509:463–518, 1998. hep-th/9707087.

[37] M. Cvetić and A. Tseytlin. Solitonic strings and BPS saturated dyonic black holes. *Phys. Rev. D*, 53:5619–5633, 1996. hep-th/9512031. Erratum-ibid. 55:3907, 1997.

[38] K. Behrndt, R. Kallosh, J. Rahmfeld, M. Schmakova, and W. Wong. STU black holes and string triality. *Phys. Rev. D*, 54:6293–6301, 1996. hep-th/9608059.

[39] V. Balasubramanian and F. Larsen. Near horizon geometry and black holes in four dimensions. hep-th/9802198.

[40] H. Boonstra, B. Peeters, and K. Skenderis. Duality and asymptotic geometries. *Phys. Lett. B*, 411:59–67, 1997. hep-th/9706192.

[41] M. Gunaydin, G. Sierra, and P.K. Townsend. The unitary supermultiplets of $D = 3$ Anti-de Sitter and $D = 2$ conformal superalgebras. *Nucl. Phys. B*, 274:429, 1986.
[42] S. Gubser. Can the effective string see higher partial waves? *Phys. Rev. D*, 56:4984–4993, 1997. [hep-th/9704195](http://arxiv.org/abs/hep-th/9704195).

[43] S. Das, G. Gibbons, and S. Mathur. Universality of low-energy absorption cross-sections for black holes. *Phys. Rev. Lett.*, 78:417–419, 1997. [hep-th/9609052](http://arxiv.org/abs/hep-th/9609052).

[44] C. Vafa and E. Witten. A strong coupling test of S-duality. *Nucl. Phys.*, 431:3–77, 1994. [hep-th/9408074](http://arxiv.org/abs/hep-th/9408074).

[45] F. Larsen and R. Leigh. To appear.