Muons anomalous magnetic moment from effective supersymmetry

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Abstract

We present a detailed analysis on the possible maximal value of the muon \((g - 2)_\mu \equiv 2a_\mu\) within the context of effective SUSY models with \(R\) parity conservation. First of all, the mixing among the second and the third family sleptons can contribute at one loop level to the \(a^{\text{SUSY}}_\mu\) and \(\tau \rightarrow \mu \gamma\) simultaneously. One finds that the \(a^{\text{SUSY}}_\mu\) can be as large as \((10 - 20) \times 10^{-10}\) for any \(\tan \beta\), imposing the upper limit on the \(\tau \rightarrow \mu \gamma\) branching ratio. Furthermore, the two-loop Barr-Zee type contributions to \(a^{\text{SUSY}}_\mu\) can be significant for large \(\tan \beta\), if a stop is light and \(\mu \) and \(A_t\) are large enough \((\sim O(1) \text{ TeV})\). In this case, it is possible to have \(a^{\text{SUSY}}_\mu\) upto \(O(10) \times 10^{-10}\) without conflicting with \(\tau \rightarrow l \gamma\). We conclude that the possible maximal value for \(a^{\text{SUSY}}_\mu\) is about \(\sim 20 \times 10^{-10}\) for any \(\tan \beta\). Therefore the BNL experiment on the muon \(a_\mu\) can exclude the effective SUSY models only if the measured deviation is larger than \(\sim 30 \times 10^{-10}\).
The anomalous magnetic dipole moment (MDM) of a muon, \( a_\mu \equiv (g_\mu - 2)/2 \), is one of the best measured quantities. Recently, the Brookhaven E821 collaboration announced a new data on anomalous magnetic moment \( a_\mu \) [1]:

\[
a_\mu^{\text{exp}} = (11659202 \pm 14 \pm 6) \times 10^{-10}.
\]

(1)

On the other hand, the SM prediction for this quantity has been calculated through five loops in QED and two loops in the electroweak interactions [2]. Using the corrected light–light scattering contribution to the \( a_\mu \) through pion exchanges [3], the difference between the data and the SM prediction is

\[
\delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (26 \pm 16) \times 10^{-10},
\]

(2)

which is only 1.6\( \sigma \) deviation. Therefore, the present data does not indicate any indirect evidence of new physics at electroweak scale. However, since the ultimate goal of the BNL experiment is to reduce the experimental error down to \( \sim 4 \times 10^{-10} \), the \( \delta a_\mu \) may provide a useful constraint on various new physics scenarios just around the electroweak scale.

The most promising new physics beyond the SM is the minimal supersymmetric standard model (MSSM) and its various extensions, and the muon \( g - 2 \) was one of the basic observables one considered in various SUSY models [4]. After the BNL data was announced in the year of 2001, there appeared a lot of works on the muon \( g - 2 \) in the context of SUSY models within the general MSSM (even with \( R \) parity violation), minimal SUGRA, gauge mediation, anomaly mediation and gaugino mediation scenarios [5]. The conclusions of these works can be summarized as follows in a model independent manner: it is rather easy to accommodate \( \delta a_\mu \sim (10 - 70) \times 10^{-10} \) in SUSY models, if \( \mu \tan \beta \) is relatively large and SUSY particles are not too heavy. Also the sign of the \( a_\mu^{\text{SUSY}} \) is correlated with the sign of the \( \mu \) parameter.

This general conclusion seems to allude that the so-called effective (or decoupling) SUSY models [6], which is an attractive way to solve the SUSY flavor and CP problems, have serious troubles if it eventually turns out \( \delta a_\mu > 10^{-10} \), since the 1st/2nd generation sfermions have to be very heavy (\( \sim O(20) \) TeV) and almost degenerate for squark sector. One way to evade this conclusion within the effective SUSY models is simply to invoke \( R \) parity violations in order to explain the muon \( g - 2 \) within the effective SUSY models [6]. However, the mixing between the staus and the smuons were ignored in Ref. [7], which is not a valid assumption in generic effective SUSY models. This mixing arises from mismatches between lepton and slepton mass matrices in the flavor space. The effects of such a mixing among the down squarks and its effects on \( B \) physics were discussed in Ref. [8] sometime ago. Our present work is an analogy of these works within the lepton sector (see also Ref. [9]). The flavor mixing between the staus and the smuons that contribute to the \( a_\mu^{\text{SUSY}} \) can also enhance the decay \( \tau \to \mu \gamma \), for which there exists a new strong bound from BELLE [10]:

\[
B(\tau \to \mu \gamma) < 1.0 \times 10^{-6}.
\]

Thus one has to consider the \( a_\mu^{\text{SUSY}} \) and \( \tau \to \mu \gamma \) simultaneously.

In this letter, we present a detailed analysis on the muon \( g - 2 \) in the effective SUSY models with \( R \) parity conservation, especially the possible maximal value of \( a_\mu^{\text{SUSY}} \) in view of the expected new BNL data. Lacking definite effective SUSY models, we will basically preform a numerical analysis in a model independent way, imposing the constraint from the unobserved decay \( \tau \to \mu \gamma \). This constraint turns out to be especially strong in the large
tan β region. For relatively small tan β (up to \( \lesssim 10 \)), the slepton mixing allows \( a_\mu^{\text{SUSY}} \) to be as large as \( \sim 20 \times 10^{-10} \) without having too large \( \tau \to \mu \gamma \), if there are large mixing between the staus and smuons in both chirality sectors (namely, \( \tilde{\mu}_L - \tilde{\tau}_L \) and \( \tilde{\mu}_R - \tilde{\tau}_R \) mixings). For larger tan β > 30, the constraint from \( \tau \to \mu \gamma \) becomes very strong. Still the \( a_\mu^{\text{SUSY}} \) can be as large as \( 9 \times 10^{-10} \) at one loop level. Furthermore, the Barr–Zee type two loop contribution can enhance the \( a_\mu^{\text{SUSY}} \) up to \((10 - 20) \times 10^{-10} \), if \( A_t \) and \( \mu \) are of size \( \sim O(1) \) TeV and tan β is large. In short, it is not impossible to have \( a_\mu^{\text{SUSY}} \) as large as \( \sim 20 \times 10^{-10} \) regardless of tan β in effective SUSY models. Therefore the BNL experiment on the muon \((g - 2)_\mu \) can exclude the effective SUSY models without any ambiguities only if \( \delta a_\mu \geq 30 \times 10^{-10} \) within the errors.

Let us first define the \( l_i \to l_j \gamma \) form factors \( L_{ji} \) and \( R_{ji} \) as follows:

\[
\mathcal{L}_{\text{eff}}(l_i \to l_j \gamma) = e \frac{m_{l_i}}{2} \tilde{\tau}_j \sigma^\mu_\nu F^\mu_\nu (L_{ji} P_L + R_{ji} P_R) l_i. \tag{3}
\]

Then, the muon \((g - 2)\) or \( a_\mu \) is related with \( L(R)_{22} \) by

\[
a_\mu = \frac{1}{2} \left( g_\mu - 2 \right) = m_{\mu}^2 (L_{22} + R_{22}), \tag{4}
\]

whereas the decay rate for \( l_i \to l_j \neq i + \gamma \) is given by

\[
\frac{Br(l_i \to l_{j\neq i} + \gamma)}{Br(l_i \to l_{j\neq i} + \nu_{\bar{\nu}_j})} = \frac{48 \pi^3 \alpha}{G_F^2} \left( |L_{ji}|^2 + |R_{ji}|^2 \right) \tag{5}
\]

We will calculate \( L, R \)'s relevant to \( a_\mu^{\text{SUSY}} \) and \( \tau \to \mu \gamma \) in the framework of effective SUSY models. Our notations and conventions follow those of Ref. [11].

The slepton mass matrix in the super-CKM basis is given by

\[
M_i^2 = \begin{pmatrix}
V_L^E M_L^2 V_L^{E\dagger} + m_i^2 + \frac{\cos 2\beta}{2} (M_Z^2 - 2M_W^2) 1 & -m_i \mu \tan \beta 1 + A_i^\ast \\
-m_i (\mu^* \tan \beta 1 + A_i) & V_R^E M_E^2 V_R^{E\dagger} + m_i^2 - \sin 2\beta M_Z^2 \sin^2 \theta_W 1
\end{pmatrix} \tag{6}
\]

This matrix is taken to be of the following form (neglecting the trilinear couplings for charged leptons for the time being):

\[
\begin{pmatrix}
\tilde{m}_{LL11}^2 & \tilde{m}_{LL22}^2 & \tilde{m}_{LL23}^2 & \tilde{m}_{LL33}^2 \\
-m_\epsilon \mu \tan \beta & -m_\mu \mu \tan \beta & -m_\tau \mu \tan \beta
\end{pmatrix}
\begin{pmatrix}
\tilde{m}_{RR11}^2 & \tilde{m}_{RR22}^2 & \tilde{m}_{RR23}^2 & \tilde{m}_{RR33}^2 \\
-m_\epsilon \mu \tan \beta & -m_\mu \mu \tan \beta & -m_\tau \mu \tan \beta
\end{pmatrix} \tag{7}
\]

Since we are looking at a \( CP \)-conserving effect, all these mass parameters are assumed to be real. The origin of this kind of mixing may be the form of \( M_{LL,E}^2 \), the soft mass matrices in the flavor basis, or \( V_L^{E,R} \), the lepton mixing matrices. We can diagonalize the 2-3 submatrix of the \( LL \) sector into a mixing angle \( \theta_L \) and two mass eigenvalues \( \tilde{M}_L^2, \tilde{m}_L^2 \) in the limit of no \( LR \) mixing:

\[
\begin{pmatrix}
\tilde{m}_{LL22}^2 & \tilde{m}_{LL23}^2 \\
\tilde{m}_{LL32}^2 & \tilde{m}_{LL33}^2
\end{pmatrix} = \begin{pmatrix}
\cos \theta_L & \sin \theta_L \\
-\sin \theta_L & \cos \theta_L
\end{pmatrix} \begin{pmatrix}
\tilde{M}_L^2 \\
\tilde{m}_L^2
\end{pmatrix} \begin{pmatrix}
\cos \theta_L & -\sin \theta_L \\
\sin \theta_L & \cos \theta_L
\end{pmatrix}, \tag{8}
\]
and likewise for the $RR$ sector. The sneutrino mass matrix with the neutrino masses neglected is

$$M_{\tilde{\nu}}^2 = V_{L}^{\nu} M_{L}^{2} V_{L}^{\nu}\dagger + \frac{\cos 2\beta}{2} M_{W}^{2} \mathbb{1},$$

(9)

and the lightest sneutrino mass is

$$m_{\tilde{\nu}_3}^2 = \tilde{m}_{L}^2 + \cos 2\beta M_{W}^{2},$$

(10)

when we also ignore the lepton masses. If $V_{L}^{\nu}$ is different from $V_{L}^{E}$, $M_{\tilde{\nu}}^2$ is diagonalized by a different unitary matrix than the $LL$ sector of $M_{\tilde{\nu}}^2$. However, this misalignment is compensated by the MNS matrix at the chargino-lepton-sneutrino vertex, and the chargino amplitudes can be expressed in terms of the slepton mixing angles, $\theta_{L}$ and $\theta_{R}$, if we ignore neutrino and lepton masses in $M_{\tilde{\nu}}^2$ and $M_{\nu}^2$.

The question about the sizes of $\tilde{m}_{A_{A33}}^2$ and $\tilde{m}_{A_{A23}}^2$ (with $A = L, R$) is highly model dependent one, depending on the details of the underlying model and may be closely related with understanding flavor structures in the MSSM. Note that the SUSY flavor problem is stated in the super-CKM basis as follows: the sfermion mass matrices should be flavor diagonal in this basis and/or the sfermion masses should be almost degenerate. Most effective SUSY models in the literature have hierarchical sfermion mass structures (which are almost diagonal with small mixing angles among different generations) in the flavor basis, namely $M_{L}^2$ and $M_{E}^2$. However, it’d not be impossible to construct a model of large flavor mixings in the second and third generation sfermions, especially considering the large mixings in the neutrino sector. In the super-CKM basis, the slepton mass matrices $M_{\tilde{\nu}}^2$ and $M_{\nu}^2$ are multiplied by $V_{L}^{E}$, $V_{R}^{E}$ and $V_{L}^{\nu}$ with $V_{MNS} \equiv V_{L}^{E} V_{L}^{\nu}\dagger$. Because of the large mixings among three light neutrinos, the resulting slepton mass matrices can have large and comparable elements. (A similar argument may be true for the right-handed slepton sector as well.) This is a source of the large mixings among the sleptons, which can enhance the $a_{\mu}^{SUSY}$ in the effective SUSY models.

The heavier mass eigenstates decoupling, it is straightforward to show that the $a_{\mu}^{SUSY} = a_{\mu}^{C} + a_{\mu}^{N}$ is given by

$$a_{\mu}^{C} = \frac{2}{(4\pi)^2} \frac{m_{\mu}^2}{m_{\tilde{\nu}_3}^2} \left[ \sum_{j} g_{2}^{2} |Z_{ij}^+|^2 f_{1}(x_{j}) - \frac{m_{C_{j}}}{v \cos \beta} g_{2} Z_{2j}^+ Z_{ij} f_{2}(x_{j}) \right] \sin^2 \theta_{L},$$

$$a_{\mu}^{N} = \frac{2 m_{\mu}^2}{(4\pi)^2} \left[ \frac{1}{\sqrt{2}} \left( g_{1} Z_{N}^{1j} + g_{2} Z_{N}^{2j} \right) \frac{m_{N_{j}}}{v \cos \beta} Z_{N}^{3j} f_{4}(x_{jL}) - \frac{1}{2} |g_{1} Z_{N}^{1j} + g_{2} Z_{N}^{2j}|^2 f_{3}(x_{jL}) \right] \frac{\sin^2 \theta_{L}}{m_{L}^2}$$

$$- \left( \sqrt{2} g_{1} \frac{m_{N_{j}}}{v \cos \beta} Z_{N}^{1j} Z_{N}^{3j} f_{1}(x_{jR}) + 2 g_{2} |Z_{N}^{1j}|^2 f_{3}(x_{jR}) \right) \frac{\sin^2 \theta_{R}}{m_{R}^2}$$

$$- g_{1} Z_{N}^{1j} g_{2} Z_{N}^{2j} m_{N_{j}} \tan \beta \frac{m_{N_{j}}}{m_{L}^2} \left( \frac{f_{4}(x_{jL})}{m_{L}^2} - \frac{f_{4}(x_{jR})}{m_{R}^2} \right) \frac{m_{\tau} \sin 2\theta_{L} \sin 2\theta_{R}}{4} \right],$$

(11)

where $x_{j} \equiv m_{C_{j}}^2 / m_{\tilde{\nu}_3}^2$, $x_{jL(R)} \equiv m_{N_{j}}^2 / \tilde{m}_{L(R)}^2$, and $v^2 = 2 m_{Z}^2 / (g_{1}^2 + g_{2}^2)$. The loop functions are
defined as follows:

\[ f_1(x) = \frac{1}{12(x - 1)^4}(2 + 3x - 6x^2 + x^3 + 6x \log x), \]
\[ f_2(x) = \frac{1}{2(x - 1)^3}(3 - 4x + x^2 + 2 \log x), \]
\[ f_3(x) = \frac{1}{12(x - 1)^4}(1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x), \]
\[ f_4(x) = \frac{1}{2(x - 1)^3}(-1 + x^2 - 2x \log x). \]

In the limit of no slepton flavor mixing (\( \theta_L = \theta_R = \pi/2 \)), we have checked that our results reduce to the previous results in the MSSM. Let us note that the neutralino-stau loop contribution to the \( a_\mu \) can be enhanced by \( m_\tau / m_\mu \) if both \( \tilde{\mu}_L - \tilde{\tau}_L \) and \( \tilde{\mu}_R - \tilde{\tau}_R \) mixing are (near) maximal. On the other hand, if the mixing is significant only in one chirality sector (namely, if \( \theta_L = 0 \) or \( \theta_R = 0 \)), there is no such an enhancement factor, and the resulting \( a_\mu^{SUSY} \) will be less than the case \( \theta_L = \theta_R = \pi/4 \). This was also noticed in Ref. \([9]\).

One can also calculate the amplitude for the decay \( \tau \to \mu \gamma \). The coefficients relevant to this process read as

\[
L_{23}^L = \frac{1}{(4\pi)^2} \frac{1}{m_{\tilde{\mu}_3}^2} \sum_j \left[ \frac{g_2^2}{\nu \cos \beta} Z_{1j}^1 Z_{1j}^2 f_1(x_j) - \frac{m_{\tilde{\mu}_j}}{g_2 Z_{2j}^1} Z_{1j}^1 f_2(x_j) \right] \frac{m_\mu \sin 2\theta_L}{2},
\]

\[
L_{23}^N = \frac{1}{(4\pi)^2} \sum_j \left[ \left( \frac{1}{\sqrt{2}} (g_1 Z_{1j}^1 + g_2 Z_{2j}^2) v \cos \beta \frac{m_{N_j}}{g_2 Z_{N}^1} f_4(x_j L) - \frac{1}{2} |g_1 Z_{1j}^1 + g_2 Z_{2j}^2| |f_3(x_j L)| \right) \frac{m_\mu \sin 2\theta_L}{2 m_R^2} \right. \\
\left. \left. - \left( \sqrt{2} g_1 \frac{m_{N_j}}{\nu \cos \beta} Z_{1j}^1 Z_{N}^1 f_4(x_j R) + 2 g_1^2 \sin 2\theta_L \sin 2\theta_R \right) \cos^2 \theta_L \sin 2\theta_R \right] \\
- \left( g_1 Z_{1j}^1 (g_1 Z_{1j}^1 + g_2 Z_{2j}^2) \frac{m_{N_j} \mu \tan \beta}{m_L^2 - m_R^2} \left( \frac{f_4(x_j L)}{m_L^2} - \frac{f_4(x_j R)}{m_R^2} \right) \cos^2 \theta_L \sin 2\theta_R \right] \frac{1}{2},
\]

\[
R_{23}^L = \frac{1}{(4\pi)^2} \frac{1}{m_{\tilde{\mu}_3}^2} \sum_j \left[ \frac{g_2^2}{\nu \cos \beta} Z_{1j}^1 Z_{1j}^2 f_1(x_j) - \frac{m_{\tilde{\mu}_j}}{g_2 Z_{2j}^1} Z_{1j}^1 f_2(x_j) \right] \frac{\sin 2\theta_L}{2},
\]

\[
R_{23}^N = \frac{1}{(4\pi)^2} \sum_j \left[ \left( \frac{1}{\sqrt{2}} (g_1 Z_{1j}^1 + g_2 Z_{2j}^2) v \cos \beta \frac{m_{N_j}}{g_2 Z_{N}^1} f_4(x_j L) - \frac{1}{2} |g_1 Z_{1j}^1 + g_2 Z_{2j}^2| |f_3(x_j L)| \right) \frac{m_\mu \sin 2\theta_L}{2 m_R^2} \right. \\
\left. \left. - \left( \sqrt{2} g_1 \frac{m_{N_j}}{\nu \cos \beta} Z_{1j}^1 Z_{N}^1 f_4(x_j R) + 2 g_1^2 \sin 2\theta_L \sin 2\theta_R \right) \cos^2 \theta_L \sin 2\theta_R \right] \\
- \left( g_1 Z_{1j}^1 (g_1 Z_{1j}^1 + g_2 Z_{2j}^2) \frac{m_{N_j} \mu \tan \beta}{m_L^2 - m_R^2} \left( \frac{f_4(x_j L)}{m_L^2} - \frac{f_4(x_j R)}{m_R^2} \right) \cos^2 \theta_L \sin 2\theta_R \right] \frac{1}{2}. \]
In order that our numerical analysis be as model independent as possible, we fixed $\tilde{m}_{LL22} = \tilde{m}_{RR22} = 10\, 0 \, \text{TeV}$, and scanned the following parameter range:

$$2 \leq \tan \beta \leq 50, \quad 0.2\, \text{TeV} \leq \mu, \quad M_2 \leq 1\, \text{TeV}, \quad (0.1\, \text{TeV})^2 \leq \tilde{m}_{LL33}^2, \quad \tilde{m}_{RR33}^2 \leq (10\, \text{TeV})^2, \quad -(10\, \text{TeV})^2 \leq \tilde{m}_{LL23}^2, \quad \tilde{m}_{RR23}^2 \leq +(10\, \text{TeV})^2. \quad (20)$$

Note that the effective SUSY models do not necessarily imply that the slepton mass parameters $\tilde{m}_{LL33}^2$ and/or $\tilde{m}_{RR33}^2$ should be (electroweak scale)$^2$. Since slepton Yukawa couplings are small, their effects on the one loop corrected Higgs mass are negligible. Therefore staus and tau sneutrinos need not be light in the effective SUSY models. However the resulting $a_{\mu}^{\text{SUSY}}$ will be very small for very heavy staus and tau sneutrinos. Then we selected parameter sets yielding positive slepton (mass)$^2$ and satisfying the direct search bounds: $m_{\tilde{\tau}} \geq 85\, \text{GeV}$, $m_{\tilde{\chi}} > 44.7\, \text{GeV}$ and $m_{\tilde{\chi}^+} > 103.5\, \text{GeV}$ \cite{12}. We used the GUT relation $M_1/M_2 = 5\alpha_1/3\alpha_2$ to fix $M_1$ for a given $M_2$. The trilinear couplings for the charged leptons are set to zero. For large $\tan \beta$, the trilinear couplings are almost irrelevant. For small and moderate $\tan \beta$, it changes the $LR$ mixing parameters, and we have checked the constrained maximal $a_{\mu}^{\text{SUSY}}$ can change upto $\pm 2 \times 10^{-10}$ when we varied the $A_i$’s from $-1\, \text{TeV}$ to $+1\, \text{TeV}$.

In Fig. 1, we show the possible maximal value of $a_{\mu}^{\text{SUSY}}$ (at one loop level) as a function of $\tan \beta$ with and without the $\tau \rightarrow \mu \gamma$ constraint in solid and dotted curves, respectively. If $\tan \beta$ is not too large, the $\tau \rightarrow \mu \gamma$ constraint does not overkill the $a_{\mu}^{\text{SUSY}}$. For large $\tan \beta$, the one loop contribution to $a_{\mu}^{\text{SUSY}}$ can be much larger, but is strongly constrained by $\tau \rightarrow \mu \gamma$. Still the resulting $a_{\mu}^{\text{SUSY}}$ can be as large as $9 \times 10^{-10}$. This point is also illustrated by Figs. 2 (a) and (b), where we show the region plots for $\tan \beta = 3$ (a) and $\tan \beta = 30$ (b). In Fig. 2 (a), $a_{\mu}^{\text{SUSY}}$ can reach $O(20 \times 10^{-10})$ for $\tan \beta = 3$, still satisfying the $\tau \rightarrow \mu \gamma$ constraint. For $\tan \beta = 30$, we have $a_{\mu}^{\text{SUSY}} \lesssim 10 \times 10^{-10}$ [Fig. 2 (b)]. This behavior can be easily understood, since $a_{\mu}^{\text{SUSY}} \propto \tan \beta$ whereas $B(\tau \rightarrow \mu \gamma) \propto \tan^2 \beta$. Therefore the constraint becomes much more severe when $\tan \beta$ is large, in which the one loop $a_{\mu}^{\text{SUSY}}$ is essentially smaller than $10 \times 10^{-10}$. The possible maximal value for $a_{\mu}^{\text{SUSY}}$ will decrease as the upper limit on $B(\tau \rightarrow \mu \gamma)$ gets improved.

In the effective SUSY models, the two-loop contributions to the EDM’s and MDM’s through a third (s)fermion loop could be substantial for large $\tan \beta$ \cite{13, 14}. Since previous discussion implies that the one loop contribution to $a_{\mu}^{\text{SUSY}}$ cannot be larger than $\sim 10 \times 10^{-10}$ for large $\tan \beta$ in the effective SUSY models, it is important to estimate these two loop contributions which may dominate in the large $\tan \beta$ region. The basic formulae for these contributions have been derived both for the neutral and the charged Higgs exchanges with (s)top and/or (s)bottom loops:

$$a_{\mu}^{2\text{-loop}} = -\frac{\alpha}{2\pi} \left( \frac{G_F m_{\mu}^2}{4\sqrt{2}\pi^2} \right) \lambda_5^S \sum \tilde{N}_f Q_f^2 \frac{\lambda_f}{m_S} \mathcal{F}(m_f^2/m_S^2) \quad (21)$$

where $\tilde{N}_f$, $Q_f$ and $m_f$ are the number of colors, the electric charge and the mass of the internal sfermion in the loop, and $m_S$ (with $S = h^0$ or $H^0$) is the mass of the exchanged scalar Higgs $h^0$ or $H^0$. $\lambda_{\mu}^{(h^0,H^0)} = (-\sin \alpha, \cos \alpha)/\cos \beta$, where $\alpha$ is the mixing angle of neutral CP-even Higgs bosons. The explicit form of the loop function $\mathcal{F}(z)$ can be found in
Note that the expression in the parenthesis,
\[ \frac{G_F m^2_a}{4\sqrt{2\pi^2}} = 23.3 \times 10^{-10} \]
is the size of the SM electroweak corrections to the muon \((g - 2)_\mu\), and thus the above two-loop Barr-Zee type contributions to the muon \((g - 2)_\mu\) can be substantial for large \(\tan \beta\), and the large positive \(\mu\) or the large negative \(A_f\). The larger the trilinear coupling \(A_t\) is, the larger \((g - 2)_\mu\) one can afford.

In Fig. 1, we also show the two-loop Barr-Zee type contribution to \(a^\text{SUSY}_\mu\) for three different \(\mu = 0.5, 1\) and 2 TeV’s (the long dashed, the dot-dashed and the short dashed curves, respectively). We have assumed the maximal mixing angle for neutral Higgs bosons, and set \(m_S = 100\) GeV (just above the current lower limit on the CP-even heavier neutral Higgs boson \(H\)) in order to maximize the desired effect. There is a clear evidence that this two-loop effects becomes important as \(\tan \beta\) grows. Adding the two-loop Barr-Zee type contribution to the one loop effects, the possible maximal value for \(a^\text{SUSY}_\mu\) can easily extend to \((20 - 30) \times 10^{-10}\) even for large \(\tan \beta\). Therefore it’d not be possible to completely rule out the effective SUSY models from the BNL experiment on the muon MDM, unless the deviation between the SM prediction and the data is larger than, say, \(\sim 30 \times 10^{-10}\).

We also plot the dependence of the possible maximal value of \(a^\text{SUSY}_\mu\) on the SUSY breaking parameter \(\tilde{m}_{LL33} = \tilde{m}_{RR33} = \tilde{m}_{33}\) in Fig. 3 for \(\tan \beta = 3, 10\) and 40, respectively. The lower (the upper) curves are with (without) \(\tau \to \mu \gamma\) constraint. A larger value of \(a^\text{SUSY}_\mu\) is possible, if \(\tilde{m}_{33}\) becomes larger. The reason lies in that in this case one needs a large mixings \(\tilde{m}^2_{LL33}\) and \(\tilde{m}^2_{RR23}\) in order to have light stops at the electroweak scale if \(\tilde{m}_{33}\) becomes large. (Note that we had fixed \(\tilde{m}_{LL22} = \tilde{m}_{RR22} = 10\) TeV and we need light stops around a few hundred GeV’s in order to have a significant effect on the muon \((g - 2)\).) Therefore the \(a^\text{SUSY}_\mu\) in the effective SUSY models can be \(\sim 20 \times 10^{-10}\) at one loop level, if \(\tan \beta\) is not too large and the slepton mass parameters involving the 3rd generations are also very large (upto \(O(\text{few} - 10)\) TeV) so that one can have light slepton spectra and large mixings. On the other hand, if one naively apply the idea of light staus directly to the mass parameters \(\tilde{m}^2_{LL33}\) and \(\tilde{m}^2_{RR33}\) (and necessarily with small flavor mixings \(\tilde{m}^2_{LL23}\) and \(\tilde{m}^2_{RR23}\) in order to have light but non-tachyonic stops), the resulting \(a^\text{SUSY}_\mu\) cannot be large : \(a^\text{SUSY}_\mu \lesssim 3 \times 10^{-10}\) if \(\tilde{m}_{LL33} = \tilde{m}_{RR33} < O(1)\) TeV, for example (see Fig. 3).

Note that the maximum of \(\tan \beta = 40\) curve in Fig. 3 is lower than the tan \(\beta = 40\) point of Fig. 1 about \(10 \times 10^{-10}\). This is because \(\tilde{m}_{LL33}\) and \(\tilde{m}_{RR33}\) were assumed to be equal in Fig. 3, but not in Fig. 1. It turns out that in small \(\tan \beta\) case, \(a^\text{SUSY}_\mu\) gets maximized when \(\tilde{m}^2_L \simeq \tilde{m}^2_R\) and \(\theta_L \simeq \theta_R \simeq \pi/4\), while in large \(\tan \beta\) case, \(\tilde{m}^2_L/\tilde{m}^2_R \simeq 60\) and \(\theta_L \simeq 0.18, \theta_R \simeq \pi/4\). Let us note another point here. The result that \(a^\text{SUSY}_\mu\) reaches \(9 \times 10^{-10}\) when \(\tan \beta = 40\), was obtained from Eqs. (11). If we treat the \(LR\) mixing by fully diagonalizing the \(4 \times 4\) mass matrix, this maximal number gets reduced to \(6 \times 10^{-10}\).

In conclusion, we considered the muon \((g - 2)_\mu\) within the effective SUSY models. In this case, the smuon and the sneutrino loop contributions to the muon \((g - 2)_\mu\) is negligible. However, the staus can contribute to the muon \((g - 2)_\mu\) through the flavor mixing in the slepton sector. Including the current constraint from \(\tau \to \mu \gamma\), we find that \(a^\text{SUSY}_\mu\) in the effective SUSY model can be as large as \(\sim 20 \times 10^{-10}\) in a reasonable region of parameter space. This bound is fairly model independent with the effective SUSY models, and will become smaller once the upper bounds on \(\tau \to \mu \gamma\) is improved. Our study shows that the \(a^\text{SUSY}_\mu\) can be as large as \(\sim 20 \times 10^{-10}\) in the effective SUSY models for all \(\tan \beta\) if there
is a large mixing between the second and third generation sfermions. For large tan $\beta$, the constraint from $\tau \rightarrow \mu \gamma$ is very strong but $a^{\text{SUSY}}_\mu$ can be as large as $9 \times 10^{-10}$. Also it can receive additional contributions from two-loop Barr-Zee type contributions of the similar size. Overall, the possible maximal value for $a^{\text{SUSY}}_\mu$ is about $20 \times 10^{-10}$ so that the BNL experiment on the muon $(g-2)_\mu$ can exclude the effective SUSY models only if the measured deviation is larger than $\sim 30 \times 10^{-10}$.

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FIG. 1: The possible maximal value of $a_{\mu}^{\text{SUSY}}$ at one loop order in the effective SUSY models as a function of $\tan \beta$, with and without the $\tau \to \mu \gamma$ constraint (the solid and the dotted curves, respectively). The lower three curves represent the two-loop Barr-Zee type contributions to $a_{\mu}^{\text{SUSY}}$ for $m_S = 100$ GeV and the maximal mixing angle for neutral Higgs bosons.

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FIG. 2: Regions on $a_{\mu}^{SUSY} - Br(\tau \rightarrow \mu\gamma)$ plane swept as the parameters are varied within Range (20) with $\tan \beta$ fixed at 3 and 30. The vertical dashed line shows upper bound on the branching ratio of 90% confidence level.

FIG. 3: The possible maximal value of $a_{\mu}^{SUSY}$ as a function of $\tilde{m}_{33} = \tilde{m}_{LL33} = \tilde{m}_{RR33}$, with and without the $\tau \rightarrow \mu\gamma$ constraint (the lower and the upper curves, respectively).