Think Global, Act Local: The Influence of Environment Age and Host Mass on Type Ia Supernova Light Curves

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Abstract

The reliability of Type Ia Supernovae (SNe Ia) may be limited by the imprint of their galactic origins. To investigate the connection between supernovae and their host characteristics, we developed an improved method to estimate the stellar population age of the host as well as the local environment around the site of the supernova. We use a Bayesian method to estimate the star formation history and mass weighted age of a supernova’s environment by matching observed spectral energy distributions to a synthesized stellar population. Applying this age estimator to both the photometrically and spectroscopically classified Sloan Digital Sky Survey II supernovae (N = 103), we find a 0.114 ± 0.039 mag “step” in the average Hubble residual at a stellar age of ~8 Gyr; it is nearly twice the size of the currently popular mass step. We then apply a principal component analysis on the SALT2 parameters, host stellar mass, and local environment age. We find that a new parameter, PC_1, consisting of a linear combination of stretch, host stellar mass, and local age, shows a very significant (4.7σ) correlation with Hubble residuals. There is a much broader range of PC_1 values found in the Hubble flow sample when compared with the Cepheid calibration galaxies. These samples have mildly statistically different average PC_1 values, at ~2.5σ, resulting in at most a 1.3% reduction in the evaluation of H_0. Despite accounting for the highly significant trend in SN Ia Hubble residuals, there remains a 9% discrepancy between the most recent precision estimates of H_0 using SN Ia and the CMB.

Key words: distance scale – galaxies: distances and redshifts – galaxies: general – galaxies: photometry – galaxies: stellar content – supernovae: general

Supporting material: figure sets, machine-readable tables

1. Introduction

For decades, astronomers have been developing methods to better understand the variation in peak luminosities of Type Ia supernovae (SN Ia) and improve their use as precision distance indicators. Phillips (1993) identified a relationship between peak magnitude and the rate of fading in the light curves of SN Ia. A connection between supernova color and peak luminosity was also shown to improve distance estimates of SN Ia (Riess et al. 1996; Phillips et al. 1999; Tripp & Branch 1999). SN Ia quickly became useful cosmological probes, measuring the expansion rate of the universe (Hamuy et al. 1995; Riess et al. 1995), the density of matter (Garnavich et al. 1998; Perlmutter et al. 1998), and the accelerated expansion of the universe (Riess et al. 1998; Perlmutter et al. 1999).

The principle energy source that powers a SN Ia light curve derives from the decay chain of 56Ni that is synthesized in the runaway nuclear fusion at the start of the explosion (Arnett 1982). The radioactive nickel yield appears to vary by a factor of eight over the extreme range of typical SN Ia luminosities. The mass of radioactive elements helps regulate the rate of recombination in iron group elements, and this results in the famous “Phillips relation” between the light curve decline rate and luminosity (Kasen & Woosley 2007).

The reason some SN Ia synthesize nearly a solar mass of radioactive nickel while others are powered by a tenth of a solar mass remains uncertain. Models suggest that the speed of the fusion front moving through the white dwarf has a major influence on the radioactive yield. The transition between deflagration (subsonic fusion) and detonation (supersonic fusion) may vary from supernova to supernova and this could explain the diversity in their luminosities. The variation in nickel yield appears to be influenced by host properties as Hamuy et al. (1996, 2000) noted that low-luminosity SN Ia tend to occur in passive galaxies like large ellipticals. This observation has been confirmed and expanded in several subsequent studies (Gallagher et al. 2005; Sullivan et al. 2006), which clearly demonstrate that host galaxies provide an important clue to the progenitors and explosion mechanisms of SN Ia.

Metal abundance and population age are global properties of galaxies that could conceivably have an impact on a supernova’s 56Ni yield. Host properties that correlate with age or metallicity, such as galaxy mass, could also influence the character of a supernova explosion. For example, Timmes et al. (2003) proposed that the metal abundance during the main-sequence stage could affect the neutron fraction in the resulting white dwarf stars. They predicted that high metal abundance populations will generate low radioactive yields and thus faint supernovae. Applying the galactic mass–metallicity relationship to this finding, SN Ia from large galaxies would then be systematically fainter just as was seen in Hamuy et al. (2000). Bravo et al. (2010) derived a similar luminosity–metallicity relationship, but 3D calculations by Röpke & Hillebrandt (2004) suggest the effect is much smaller than originally envisioned. An observational test of this hypothesis by Gallagher et al. (2008) looked at SN Ia in elliptical galaxies and found trends with both age and metallicity, although this was disputed by Howell et al. (2009).

A sensitive test of the environmental effects on SN Ia is to study the scatter in a SN Ia Hubble diagram after light-curve shape and color corrections have been applied. This type of analysis is also important for constraining systematic errors in
cosmological measurements. Hubble residuals are the difference between the luminosity distance and the expected distance from the best-fit cosmology and are most powerful in the “Hubble flow” where peculiar velocities are small compared with the expansion velocity. Woosley et al. (2007) and Kasen et al. (2009) showed that explosion asymmetries, metallicity variations, kinetic energy variations, and other explosion parameters can produce a dispersion in the Phillips relation for a fixed radioactive yield. These relationships present the possibility of using Hubble residuals to probe supernova physics. Research over the past several years indicates that some host-galaxy properties correlate with Hubble residuals (e.g., Lampeitl et al. 2010; Sullivan et al. 2010; Childress et al. 2013).

Surprisingly, the strongest correlation between Hubble residuals and galaxy properties appears to be with the host stellar mass. The effect is called the “mass step,” because at \( \sim 10^{10} M_\odot \) there appears to be a jump in the average Hubble residual. This led to extensive work on understanding the physical properties such as population age and metallicity that could underlie the mass correlation (e.g., Gupta et al. 2011; Hayden et al. 2013; Moreno-Raya et al. 2016a, 2016b). Childress et al. (2014) has proposed that the mass correlation is really an age effect amplified by galaxy “downsizing.” This research has been fruitful but not definitive. Ongoing analyses of SN Ia data sets continue to debate the significance of these effects (e.g., Graur et al. 2017; Jones et al. 2018, for LOSS and Foundation, respectively).

Rigault et al. (2013, 2018) looked at star formation and specific star formation rates, respectively, at the sites of SN Ia explosions by measuring \( H_\odot \) emission strength using spatially resolved spectroscopy. They found a very significant step in corrected SN Ia peak luminosity as a function of the specific star formation rate at the location of the explosion. This research identified a set of SN Ia with a 0.2 mag offset in Hubble residual that are generally found in regions of lower star formation rate. There is still not a consensus on the impact of local star formation rates on Hubble residual, especially when these trends are measured using UV observations (Jones et al. 2015; Rigault et al. 2015).

A very recent study (Jones et al. 2018) on a large low-redshift data set (the Foundation sample, Foley et al. 2017) compared Hubble residual with host-galaxy stellar mass, local environment mass (the region within a radius of 1.5 kpc of the SN Ia), host-galaxy rest frame \( u - g \) color, and local environment rest frame \( u - g \) color. The rest frame \( u - g \) color is a simple way to estimate recent star formation and, therefore, a crude age estimator. They found that the local environment contained no new information compared to the global parameters.

In addition, there appears to be a tension between the \( H_0 \) estimated from the cosmic microwave background observations (Planck Collaboration 2016; 2018) and estimates based on SN Ia. These precision measurements now disagree by 3.8\( \sigma \) (Riess et al. 2016, 2018) due to either new physics or a systematic bias in one of the measurements. The SN Ia-host environment is of particular importance to the precision measurements of the Hubble constant (\( H_0 \)). The Cepheid variables used to calibrate the SN Ia peak absolute magnitude are massive stars, so are observed only in star-forming galaxies. In contrast, supernovae discovered in unbiased searches are found in all types of galaxies. A mass step correction between the Cepheid calibrated hosts and the Hubble flow galaxies is currently applied to the data (at \( \pm 0.03 \) mag) and provides a relatively minor tweak to the value of \( H_0 \) derived from supernovae. So, at present, host environment is not a major contributor to the uncertainty budgets of cosmological measurements.

Here, we scrutinize the relation between Hubble residual and the age of the stellar population derived from host-galaxy colors. We develop a technique to translate multi-band galaxy photometry into an estimate of the star formation history and finally an average age for the stellar population. In principle, colors should be a better indicator of SN Ia progenitor age than \( H_\odot \). This is because after a single burst of star formation, \( H_\odot \) emitting \( H\alpha \) regions are dissipating just as SN Ia are beginning to explode. We apply our technique to both the global photometry of SN Ia-host galaxies and to the specific populations at the sites of the explosions. We then compare the local and global ages with SN Ia characteristics and other host properties to determine the parameters that have the largest impact on the measured Hubble residual. Finally, we investigate how our estimated ages may influence the current SN Ia measurements of \( H_0 \) and its error budget.

2. Data

For our primary analysis, we use SN Ia that were discovered with the SDSS-II Supernova survey (Sako et al. 2008). We selected both spectroscopic and photometric classified SN Ia that passed cosmology cuts as described and released by Campbell et al. (2013). This cosmological data set, including their SALT2 (Guy et al. 2007, 2010) calibration parameters, are available online. Hereafter we will refer to Campbell et al. (2013) as C13. For our analysis, a few additional cuts were applied. For quality, we limited our analysis to objects whose photometric uncertainty is less than 1.5 mag. These cuts are dominated by low quality \( u \)-band magnitudes. In the end, the resulting \( g \)- and \( i \)-band maximum uncertainties are less than 0.3 mag, and less than 0.16 mag for \( r \)-band. In addition, we removed objects that had a Hubble residual greater than 0.7 mag. Looking at Figure 16 of C13, these objects are likely core collapse supernovae that passed the color–magnitude cut.

This data set does not use the most recent standardization tools, such as BEAMS with Bias Corrections applied to the Pantheon data set (Kessler & Scolnic 2017; Scolnic et al. 2018). Restricting ourselves to SDSS photometry and low-redshift events avoids several of the biases mitigated in the Pantheon analysis while still providing a large, uniform sample.

The software developed for the data aggregation and analysis described in this paper is available at https://github.com/benjaminrose/nc-age.

2.1. Local Environment Photometry

The photometry of the environment around the site of the supernovae are taken from the “Scene Modeling” method described in Holtzman et al. (2008). The method incorporates all the images taken during the survey to build a photometric model at the location of the transient. The resulting photometry at the site is a convolution of the galaxy on the scale of the typical seeing. By applying this fixed angular aperture, we always get the most compact local environment possible.

\footnote{\url{http://www.icg.port.ac.uk/stable/campbelh/SDSS_Photometric_SN_e_In TERFACE}}
Finally, in order to keep the environment truly "local," a redshift cutoff was imposed, \( z < 0.2 \). With the average seeing for SDSS being 1.0 arcsec, the maximum extent of a galaxy’s local environment was 3 kpc in radius. At higher redshifts, the angular resolution encompasses a majority of typical galaxy, and there is little difference between local and global properties. The typical size of the projected aperture defining the environment at the supernova location was 1.5 kpc.

Since core collapse supernovae are less luminous than SN Ia, their contamination percentage increases in the low-redshift portion of any data set. Anticipating this higher percentage of core collapse supernova (CC) interlopers, we added further Hubble residual cut to minimize the CC contamination. In addition, the statistics used in this work are robust against the \( \geq 3.9\% \) CC contamination estimated in C13. A detailed study of CC contamination affecting SN Ia standardization with host-galaxy properties should be done because the ratio of SN Ia and CC is highly dependent on host properties.

This results in a final data set of 103 SN Ia. A partial list of the final data set used in the local environment analysis is visible in Table 1.

### 2.2. Global Photometry

In addition to the photometry of the local environment, we also study the correlation between the supernova characteristics and the host properties as a whole (hereafter the global properties). From the global host properties, we can compare our results directly to the work presented in Gupta et al. (2011), hereafter G11, to check if our age estimator is more sensitive to younger stellar populations. Second, we can contrast the local and global properties of hosts to check if there is more information contained in the local environment rather than the more easily studied global properties.

The final analysis of G11 included 206 SN Ia and hosts. We looked at the 76 objects that passed our redshift and other quality cuts to use in our method validation. A sample of the data set used for this validation is visible in Table 2.

For each of the 103 hosts where we had local environment photometry, we gathered the SDSS-DR12 model magnitudes for the estimate of the global properties. A sample of these data set can be seen in Table 3.

### 2.3. Photometry of Nearby Galaxies

The SDSS model magnitudes are unreliable for galaxies with a large angular extent. For the nearby galaxies with distances calibrated with Cepheid variable stars, we used aperture photometry to obtain both the local and global magnitudes. Images of the large galaxies were downloaded from the SDSS-DR12 and individual apertures were designed to capture 90% of the combined light in all the filters. After masking out

### Table 1

Local Environment Data for Campbell et al. (2013) SN Ia

| SDSS ID | Redshift | Uncert. \( \times 10^{-5} \) | \( u \) (mag) | \( g \) (mag) | \( r \) (mag) | \( i \) (mag) | \( z \) (mag) | \( \sigma_u \) (mag) | \( \sigma_g \) (mag) | \( \sigma_r \) (mag) | \( \sigma_i \) (mag) | \( \sigma_z \) (mag) | HR uncert. (mag) |
|---------|----------|--------------------------|-------------|-------------|-------------|-------------|-------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 762     | 0.19138  | 2.4                      | 24.65       | 23.82       | 22.95       | 22.61       | 22.18       | 0.79          | 0.09          | 0.04          | 0.04          | 0.13          | 0.15          | 0.08          |
| 1032    | 0.12975  | 3.3                      | 24.92       | 24.74       | 23.73       | 23.32       | 22.87       | 0.61          | 0.16          | 0.09          | 0.11          | 0.26          | -0.15         | 0.12          |
| 1371    | 0.11934  | 2.6                      | 23.22       | 21.52       | 20.42       | 20.00       | 19.62       | 0.16          | 0.01          | 0.00          | 0.006         | 0.01          | -0.13         | 0.06          |
| 1794    | 0.14191  | 6.3                      | 23.76       | 23.77       | 23.09       | 22.89       | 22.81       | 0.43          | 0.11          | 0.08          | 0.06          | 0.24          | 0.27          | 0.08          |
| 2372    | 0.18046  | 2.2                      | 24.81       | 22.85       | 21.84       | 21.40       | 21.02       | 0.87          | 0.03          | 0.01          | 0.01          | 0.03          | -0.12         | 0.06          |

(This table is available in its entirety in machine-readable form.)

### Table 2

Data used for Validation with Gupta et al. (2011) Ages

| SDSS ID | Redshift | \( u \) (mag) | \( g \) (mag) | \( r \) (mag) | \( i \) (mag) | \( z \) (mag) | \( \sigma_u \) (mag) | \( \sigma_g \) (mag) | \( \sigma_r \) (mag) | \( \sigma_i \) (mag) | \( \sigma_z \) (mag) |
|---------|----------|-------------|-------------|-------------|-------------|-------------|----------------|----------------|----------------|----------------|----------------|
| 1166    | 0.3820   | 22.54       | 21.83       | 20.04       | 19.38       | 19.03       | 0.48            | 0.11           | 0.04           | 0.03           | 0.08           |
| 1580    | 0.1830   | 24.99       | 20.41       | 19.21       | 18.72       | 18.28       | 1.35            | 0.03           | 0.02           | 0.02           | 0.04           |
| 2165    | 0.2880   | 22.82       | 23.35       | 22.04       | 22.22       | 21.28       | 0.38            | 0.24           | 0.12           | 0.20           | 0.36           |
| 2422    | 0.2650   | 23.64       | 22.86       | 22.00       | 21.95       | 22.01       | 0.90            | 0.23           | 0.17           | 0.24           | 0.76           |
| 2789    | 0.2905   | 22.01       | 20.92       | 19.42       | 18.84       | 18.39       | 0.35            | 0.05           | 0.02           | 0.02           | 0.06           |

(This table is available in its entirety in machine-readable form.)

### Table 3

Global Host Data for Campbell et al. (2013) SN Ia

| SDSS ID | \( u \) (mag) | \( g \) (mag) | \( r \) (mag) | \( i \) (mag) | \( z \) (mag) | \( \sigma_u \) (mag) | \( \sigma_g \) (mag) | \( \sigma_r \) (mag) | \( \sigma_i \) (mag) | \( \sigma_z \) (mag) |
|---------|-------------|-------------|-------------|-------------|-------------|----------------|----------------|----------------|----------------|----------------|
| 762     | 20.34       | 18.50       | 17.46       | 17.01       | 16.70       | 0.13           | 0.01           | 0.01           | 0.01           | 0.02           |
| 1032    | 21.49       | 19.40       | 18.30       | 17.83       | 17.47       | 0.19           | 0.01           | 0.01           | 0.01           | 0.02           |
| 1371    | 20.60       | 18.62       | 17.55       | 17.10       | 16.68       | 0.08           | 0.01           | 0.01           | 0.01           | 0.01           |
| 1794    | 22.37       | 20.76       | 20.37       | 20.10       | 20.12       | 0.45           | 0.05           | 0.06           | 0.08           | 0.29           |
| 2372    | 21.79       | 20.50       | 19.53       | 19.02       | 18.59       | 0.24           | 0.03           | 0.02           | 0.02           | 0.05           |

(This table is available in its entirety in machine-readable form.)
stars projected on the galaxy, the aperture was then applied to the image of each filter. The magnitude was then calibrated using nearby stars. The photometry for these galaxies can be seen in Table 4.

2.4. Supernova Properties

We use the C13 supernova sample to provide light-curve properties and Hubble residual information. We use the Malmquist bias corrected distances derived from the best-fit cosmology ($H_0 = 73.8 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$, $\Omega_M = 0.24$, and $\Omega_{\Lambda} = 0.76$). When needed, we use these values for our assumed cosmology. For the maximum redshift in our sample, the Malmquist bias correction is $\sim 0.01$ mag. However, C13 noted that the stretch correction coefficient, $\alpha$ they found for their full sample was significantly larger than typical and larger than the $\alpha$ derived from their spectroscopically classified sub-sample. After our cuts, we found a significant correlation between the supernova stretch parameters, $x_t$, and the C13 Hubble residuals, which would likely result in spurious correlations with our host-galaxy analysis. We corrected the C13 Hubble residuals using their spectroscopically derived $\alpha$ value of 0.16 and no longer detected a significant correlation between $x_t$ and our sample’s Hubble residuals. The resulting Hubble residuals can also be found in Table 1.

For the nearby SN Ia used in the Cepheid calibration of SN Ia peak luminosity, we obtained light curves from the SNANA database and fit them using SALT2.4 implemented from the sncosmo.3 The model also corrected for Milky Way dust extinction from the dust maps of Schlegel et al. (1998) and Schlafly & Finkbeiner (2011) via sfedmap.4

3. Stellar Population Model

A direct estimate of the age of the stellar population requires a robust model for the observed population. Flexible Stellar Population Synthesis (FSPS; Conroy et al. 2009; Conroy & Gunn 2010) takes a star formation history and outputs either a spectrum or a redshifted spectral energy distribution (SED) of the resulting stellar population. The version of FSPS we used (commit ae31b26f from 2016 November) uses the MIST isochrones (Choi et al. 2016; Dotter 2016) and the MILES spectral libraries (Falcón-Barroso et al. 2011).

3.1. FSPS Settings

Many of the FSPS parameters were set at their default values, but a number of key settings were adjusted to produce the desired model space. To control the metallicity of our stellar population, we set $z_{\text{continuous}} = 2$. This setting convolves the SSPs (simple stellar populations) with a metallicity distribution function. The metallicity distribution is defined as

$$ (Z e^{-Z})^{p_{\text{metals}}} $$

with

$$ Z \equiv \frac{z}{z_0 \log(z/z_0)} $$

where $z$ is the metallicity in linear units and $z_0 = 0.019$. This metallicity distribution is governed by two more FSPS parameters: $p_{\text{metals}}$ and $\log z_{\odot}$ (i.e., $\log(z/z_\odot)$). We left $p_{\text{metals}}$ at its default value of 2, and during the fitting process $\log z_{\odot}$ was allowed to vary but was marginalized over when the age probability distribution was determined.

The next set of parameters govern the treatment of dust. We use the default power-law dust model as explained in Conroy et al. (2009), based off of Charlot & Fall (2000). The attenuation curve of a star, as a function of stellar age, is defined as

$$ \tau(t) = \begin{cases} \tau_1(\lambda/5500 \, \text{Å})^{-0.7} & t \leq 10^7 \, \text{yr} \\ \tau_2(\lambda/5500 \, \text{Å})^{-0.7} & t > 10^7 \, \text{yr} \end{cases} $$

where $\tau_1$ and $\tau_2$ are the attenuation around a young stellar population and in the ISM, respectively. See Charlot & Fall (2000), Figure 1, for a visual representation. In FSPS these two parameters are controlled via the dust1 and dust2 variables.

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5. \url{http://snana.uchicago.edu}

3. \url{sncosmo.readthedocs.io/}

4. \url{https://github.com/kbarbary/sfedmap}
For this analysis, dust1 was set to two times dust2, and dust2 was allowed to vary to match the observations. Conroy et al. (2009) claims good values of dust1 and dust2 are 1.0 and 0.3, respectively, while Charlot & Fall (2000) prefers values of dust1 and dust2 of 1.0 and 0.5, respectively. The allowed range for dust2 in this analysis is explained in Section 4.2 and is consistent with these recommendations.

A few of the host SEDs show an unusual $r$-band feature (e.g., SN4019). Adding nebular emission (setting add_neb_emission = True and cloudy_dust = True) adjusts the $r$-band magnitude for a young stellar population and allows the model to match this observed feature. This characteristic is shown to be achievable in the self-consistency validation test number 3, as explained in Section 5.1.

Finally, FSPS outputs the luminosity of 1 $M_\odot$, so an extra constant, $\delta$, is used to scale the output SED of FSPS to match the observed SEDs.

### 3.2. Star Formation History

FSPS has many inputs for describing the star formation history of a galaxy. The $sfh$ parameter allows the user to select the functional form of star formation history. $G11$ used the simple $\tau$-model: the star formation rate is proportional to $e^{-t/\tau}$, with $t$ being the time since the start of star formation and $\tau$ is a free parameter. G11 fit both $\tau$ and the length of star formation history. This is the simplest model, which is important when fitting a small number of free parameters. However, such a simple prescription makes it difficult to create a model with both old stars and recent star formation.

Simha et al. (2014) investigated the ability of several star formation history models to match simulated galaxies. This research looked at the simple $\tau$-model, a linear-exponent model, and a four-parameter $\tau$-model. A visual comparison is presented in Figures 3–5 of Simha et al. According to the calculations in Simha et al., the simple $\tau$-model can overestimate the age by $\sim 1$–2 Gyr, particularly for younger populations. Since we expect some supernovae to explode in young ($\leq 2$ Gyr) stellar populations, we decided to use a four-parameter $\tau$-model.

The main feature of the four-parameter $\tau$-model (4$\tau$-model) is that it separates the properties of early and late time star formation. This model can describe a wide range of star formation histories: an early burst, a history dominated by recent star formation, or both an early burst and recent star formation. The 4$\tau$-model is a piecewise combination of a linear-exponent star formation history, then a linearly rising or falling star formation. This model is used by FSPS when $sfh = 5$. Mathematically the 4$\tau$-model can be written as

$$
\Psi'(t) \propto \begin{cases} 
(t - t_0) e^{-(t-t_0)/\tau} & t_0 \leq t \leq t_i \\
\Psi(t_i) + m_{sd} \mathcal{A}(t - t_i) & t_i < t \leq \mathcal{A}(z), \\
0 & \text{else}
\end{cases}
$$

where $t_0$ is when the star formation started, $\tau$ is the e-folding parameter, $t_i$ is the star formation transition time, $m_{sd}$ is the slope of the late time star formation, $\mathcal{A}$ is the ramp function, and $\mathcal{A}(z)$ is the redshift dependent time when the observed light was emitted. Note that the equation above allows negative values of $\Psi$, which is nonphysical. So we add an extra

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Figure 1. Example of several four-parameter $\tau$-model star formation histories. These star formation histories are normalized to produce the same total stellar mass. The old bursts have a $\tau = 0.5$ and “flat” histories have a $\tau = 7.0$. The two examples of young bursts show how the number of young stars produced after $t = t_i$ depends heavily on the amount of previously formed stars, even for the same $m_{sd}$.

### 3.3. Calculating Ages

For any set of star formation parameters, we determine the average population age. The mass weighted average age is

$$
\langle A \rangle_{\text{mass}} = \mathcal{A}(z) - t_0 - \frac{\int_0^{\mathcal{A}(z)} (t - t_0) \Psi'(t) dt}{\int_0^{\mathcal{A}(z)} \Psi'(t) dt},
$$

where all of the variables are the same as described in Equation (4), so $t - t_0$ is simply the length of star formation. In the integral, a variable substitution of $t - t_0 \rightarrow t$ transforms the time zero-point from the beginning of the universe to the start of star formation. We would also need to transform $\Psi'(t) \rightarrow \Psi'_s(t)$ so that $\Psi'_s$ assumes $t = 0$ is the start of star formation. This makes the equation for the mass weighted average age to be

$$
\langle A \rangle_{\text{mass}} = \mathcal{A}(z) - t_0 - \frac{\int_0^{\mathcal{A}(z) - t_0} t \Psi'_s(t) dt}{\int_0^{\mathcal{A}(z) - t_0} \Psi'_s(t) dt},
$$

or the equation used in G11. In this paper, the age of a stellar population will refer to the mass weighted average described here.

The model using the star formation parameters $t_0 = 8.0$, $\tau = 0.1$, $t_i = 12$, and $m_{sd} = 20$ produces a population with an average age of 437 Myr. This demonstrates that our SFH prescription can generate a dominant young population when SN Ia are expected to start exploding. A small $\tau$ is needed to keep the number of old stars from building up over the cosmic time and dominating over the recent linear star formation.
4. Determining the Most Probable SFH

Using Bayesian statistics and a Markov chain Monte Carlo (MCMC) sampling method, we determine the probability of each free parameter described in Section 3: \( \log(z/z_\odot) \), \( \tau_2 \), \( \tau \), \( t_0 \), \( t_1 \), \( m_{sf} \), and \( \delta \). Data modeling and parameter estimation are often done with Bayesian statistics because they calculate the probability of the model parameters, given the observed data by using Bayes’ theorem,

\[
P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)}, \tag{6}
\]

where \( \theta \) is a given model’s parameters and \( D \) represents the observed data. Each probability is

- \( P(\theta|D) \): The posterior, which is the probability of the model parameters given the data
- \( P(D|\theta) \): The standard likelihood function, \( \mathcal{L}(D|\theta) \)
- \( P(\theta) \): The model prior, which describes what we know about the model before considering the data \( D \), such as model parameter limits
- \( P(D) \): The model evidence, which in practice amounts to a normalization term

For MCMC sampling, only relative probabilities are needed, so \( P(D) \) is generally ignored and Equation (6) becomes a proportionality, not an equality. It is common to use flat priors, \( P(\theta) \propto 1 \). In this case Bayes’ theorem simplifies to a standard Frequentist likelihood estimation, \( P(\theta|D) \propto \mathcal{L}(D|\theta) \). More generally, some prior information is used and Bayes’ theorem becomes

\[
P(\theta|D) \propto P(\theta) \times \mathcal{L}(D|\theta). \tag{7}
\]

To find the maximum posterior probability of the model parameters, we need only to know the priors, \( P(\theta) \), and the likelihood of the data, \( \mathcal{L}(D|\theta) \). A more complete description of Bayesian statistics and MCMC sampling is available in VanderPlas (2014) and Hogg & Foreman-Mackey (2017).

4.1. Likelihood

This method uses a standard log-likelihood function for data with Gaussian uncertainties. Summing over each filter, we compare the observed apparent magnitude (\( m_i \)) with the resulting apparent magnitude from FSPS (\( m_{FSPS,i} \)) plus a scaling factor (\( \delta \)) to account for FSPS’s 1 M\(_\odot\) output. Mathematically this is written as

\[
\ln(\mathcal{L}) \propto \sum_i \frac{(m_i - (m_{FSPS,i} + \delta))^2}{\sigma_i^2} + \ln(2\pi\sigma_i^2), \tag{8}
\]

with \( \sigma_i \) as the uncertainty in each \( m_i \) measurement.

4.2. Priors

For five of the variables, we use bounded flat tops:

- \( 2.5 < t_1 < 6 \) (Gyr)
- \( 0.1 < \tau < 10 \)
- \( -1.520838 < \phi < 1.520838 \)
- \( 0.5 < t_0 < t_1 - 2 \)
- \( -45.0 < \delta < -5.0 \). \tag{9}

Since a flat distribution in a slope parameter preferentially searches the high values space,\(^7\) the MCMC was performed over \( \phi \), the angle the ramp function makes with respect to the \( x \)-axis; therefore, \( m_\phi = \arctan(\phi) \). The prior bound above, \(-1.520838 < \phi < 1.520838\), corresponds to \(-20 < m_\phi < 20\).

In addition, \( \log(z/z_\odot) \) uses a Gaussian distribution with \( \mu = -0.5 \) dex and \( \sigma = 0.5 \) dex limited to the range of \(-2.5 < \log(z/z_\odot) < 0.5 \). This is a common assumption as seen in Belczynski et al. (2016). Our model reaches a lower metallicity than the grid search used in G11.

For the ISM dust parameter, \( \tau_2 \), we assume a Gaussian prior on the top bounds. The Gaussian distribution is defined by \( \mu = 0.3 \) and \( \sigma = 0.2 \), but only values between 0 and 0.8 are accepted. This allows for some variability but keeps the values near the 0.3 and 0.5 as recommended by Conroy et al. (2009) and Charlot & Fall (2000), respectively.

5. Validation

Following the statistical method described in Section 4, we derive probability distributions for the model parameters defined in Section 3. Using Equation (5) at each step in the MCMC chain, we build a probability distribution function for the age marginalized over metallicity and host-galaxy dust.

5.1. Self-consistency

The first validation of this newly developed age estimator was to verify that it was self-consistent (i.e., it could correctly estimate the star formation parameters from an SED generated by FSPS).

Eight different models were used in this test. The initial metallicity, dust, and star formation parameters can be seen in Table 5. These values were put into FSPS, at a redshift of

\(^7\) A mathematical description is available in VanderPlas (2014, Appendix A); he also has a nice graphical example on his website (http://jakevdp.github.io/blog/2014/06/14/frequentism-and-bayesianism-4-bayesian-in-python/#The-Prior).
Table 6
SEDs for the Self-consistency Test

| ID | g (mag) | r (mag) | i (mag) | z (mag) | \( <A > \) (Gyr) |
|----|---------|---------|---------|---------|-----------------|
| 1  | 20.36   | 18.76   | 17.99   | 17.67   | 17.39 ± 0.8      |
| 2  | 20.31   | 18.74   | 17.98   | 17.66   | 17.39 ± 0.7      |
| 3  | 16.15   | 15.43   | 15.40   | 16.19   | 15.21 ± 1.4     |
| 4  | 17.65   | 16.74   | 16.49   | 16.26   | 16.16 ± 0.4     |
| 5  | 19.69   | 18.29   | 17.70   | 17.45   | 17.29 ± 0.7     |
| 6  | 17.66   | 16.58   | 16.25   | 16.01   | 15.86 ± 0.7     |
| 7  | 17.62   | 16.80   | 16.57   | 16.34   | 16.26 ± 0.4     |
| 8  | 19.72   | 18.37   | 17.88   | 17.68   | 17.56 ± 0.4     |

Note. SEDs were scaled with a \( \delta = -25 \) mag.

\( z = 0.05 \), to generate observed SEDs. The resulting SEDs, Table 6, were then analyzed with our Bayesian estimator. This set of models produced old populations (~10.5 Gyr) and very young populations (~0.5 Gyr). They explored the effect of metallicity (Model 5) and dust (Model 6). Model 2 also looked at a “mixed” population with an old burst of star formation and a strong increase of star formation to the present epoch, a stellar population that is not possible with a simpler star formation history.

This method can model a young stellar population, (like Model 3), but is unable to recover a starburst or mixed populations (Models 8 and 2, respectively) based on SED fitting. The “old and young burst” of Model 2 is not particularly blue, and as such, we identify an age of 4.4 Gyr. We do better with Model 3, where we estimate the correct age of 1.4 Gyr with a small uncertainty.

Figure 2 shows the FSPS output SEDs of the initial input star formation parameters and the best-fit parameters from our analysis. The SEDs are fit very well across all the models used in this self-consistency test. In addition to fitting the SEDs, our analysis was able to approximate the underlying parameters. An example corner plot of the posterior probabilities is displayed in Figure 3, and the full set of figures is available online.

5.2. Comparison to Previous Work

The final validation was to recalculate the global host ages originally presented in G11. A direct comparison between ages from G11 and the results from our analysis can be seen in Figure 4. Most points fit along the one-to-one line with a scatter of around 2 Gyr, implying that our analysis is consistent with the previous work. However, there are six hosts that our method estimates to be \(< 2 \) Gyr, whereas none of the ages in G11 were that young. At \(< 4 \) Gyr, our method systematically estimates a younger age. This is because our star formation history model allows for more recent star formation than the simple \( \tau \) models permit. In their discussion on this topic, Simha et al. (2014) claimed that the star formation model used in G11 can overestimate young populations by \(~ 2 \) Gyr. This overestimation can be seen in Figure 4.

6. Results

Our analysis generates a probability distribution for the age of a stellar population. Probability distributions can be summarized by a median and 68% confidence intervals, especially if the distribution is Gaussian. For non-normal distributions, particularly ones with long tails or multiple modes, the distribution can still be summarized by a median age, but its interpretation is not as clear. To accurately represent the estimated age posterior probabilities shown in Figure 5, 6, and 8, we plotted the results of 100 random samples from both the age and Hubble residual distributions for each SN Ia. This technique results in a probability density plot of finding a SN Ia at a given age and Hubble residual.

We test for correlations between parameters with the Spearman’s rank-order correlation coefficient. This is the non-parametric version of the more common Pearson’s correlation coefficient. There are several differences between Pearson’s and Spearman’s correlations. The most important difference for our study is that the Spearman’s correlation has a high absolute value for any monotonic relationship rather than just linear relationships. This means a linear, exponential, or a single step function would all rank highly with the Spearman’s correlation. Because several previous host-galaxy studies have seen steps in Hubble residuals (or more generally sigmoid functions), it is reasonable to use a statistical measurement that is sensitive to these nonlinear correlations. See Wall & Jenkins (2012, Section 4.2.3) for more information on the Spearman’s correlation.

A large absolute rank-order correlation refers to a tighter association of points, indicated by a small scatter around the
correlation. The significance of a Spearman’s correlation can be described by a standard \( p \)-value, or the probability under the null hypothesis of obtaining a result equal to or more extreme than what was observed. Given a sample size and Spearman’s correlation, a \( p \)-value can be calculated. If we let \( 3\sigma \) be our significance limit, then for our main sample of 103 objects, the Spearman’s critical correlation value would be \( \pm 0.30 \). That is, there is a 0.2% chance of seeing a Spearman’s correlation value of \( >0.3 \) or \( < -0.3 \) from our data set, assuming no underlying correlation.

6.1. Global Environments

First, we compare the Hubble residuals with ages derived from the global photometry of the hosts using C13 (Table 3) sample. The comparison is presented in Figure 5. This data set
has a low 2.1σ correlation, as defined by the Spearman’s value of −0.23. In addition, the distribution in Hubble residual-age space appears to show a distinctive “step” between 7 and 8 Gyr.

6.2. Local Environments

Finally, we compare the Hubble residuals versus average local environment age for the data set derived from C13 (see Table 1). The data are presented the same way as Figure 5. The Spearman’s coefficient (−0.21) is only slightly different than the one seen in Figure 5 but is insignificant only have a 1.8σ significance. The data also have a stronger “step” at ∼8 Gyr to the global analysis. There seems to be significantly fewer SN Ia with a Hubble residual at ≥0.0 mag with an age ≥8 Gyr.
contain any additional information not already present in the global age. Jones et al. (2018) found a marginally significant difference between their global and local analyses.

Because a fraction of our sample is photometrically classified, there may be some CC contamination that would be found preferentially in the upper left of Figure 5 and 6. This contamination might contribute to the observed correlations, but at a level that is small compared with the 2.1σ and 1.8σ trends.

7. Analysis and Discussion

7.1. Comparison between Local and Global Ages

The global age estimated for spiral galaxies is an average of several stellar populations. The prompt component of SN Ia are expected to be found in the youngest regions of a galaxy. So we may see that the population age at the supernova location would tend to be younger than the global age of the galaxy. In Figure 7 we show the difference between the estimated global and local ages. For massive galaxies with local ages less than 4 Gyr, there is a tendency for the supernova to be located in a younger than average spot in the host. The effect is less apparent for low-mass galaxies, but that is likely because the size of the region measured for the local environment is a large fraction of the size of a small galaxy.

Between local ages of 4 and 8 Gyr the difference between global and local age estimates show a large scatter with no apparent trend. Beyond 8 Gyr the scatter between the local and global ages is reduced, probably because the stellar population in ellipticals is fairly uniform. In these older hosts the global age estimates tend to be 1–2 Gyr younger than the local ages. The reason for this difference is not clear, but it may be due to activity at the center of ellipticals contaminating the stellar colors.

Although we see no statistical difference between the use of local and global ages when comparing with Hubble residuals, our results suggest that using local photometry to characterize the supernova environment has some benefit over the global average. For example, the SED measured local to the supernova can indicate a population that is a factor of two younger than the global average in large star-forming galaxies. When feasible, the measurement of the local environment, particularly of younger populations, provides a more accurate representation of the progenitor age than simple averaging the light from the host.

7.2. Investigating the Age Step

The Hubble residuals in the C13 sample have a monotonically decreasing trend (at 2.1σ) that appears to be a break or step at an age of ~8 Gyr. Figure 8 plots the same data as Figure 6, but this time splits the data into two age bins: ≤8 Gyr and >8 Gyr. Both age ranges show very small Spearman’s correlations (−0.07 and −0.09, respectively) that are consistent with a flat distribution. The correlation decreasing in significance when the data set is split in two implies that the monotonic function seen in Figure 6 is really a step-like function with a transition at ~8 Gyr. This function may have a transition width making it more like a continuous sigmoid function with a transition faster than our age resolution. In both the local and global age measurements, we find a significant age step in Hubble residuals.

7.3. Age as the Cause of the Mass Step

Childress et al. (2014) argue for a link between host stellar mass and the delay time between stellar formation and SN Ia explosion, with results summarized in their Figure 4 where the SN Ia progenitor age distribution is divided into host-galaxy stellar mass bins. They find, with reasonable assumptions of star formation histories and SN Ia delay times, that there is a natural division in host mass and age between prompt and “tardy” SN Ia. Prompt SN Ia occur in lower mass galaxies with ages ≤6 Gyr, while tardy events continue in high mass galaxies with ages ≥6 Gyr. Projecting their model onto the host mass axis results in an overlapping distribution of prompt/tardy explosions with a transition near 10^{10.5} M_{⊙}. This transition in progenitor age may correspond to the mass step observed at
$10^{10} M_\odot$. Projection of the Childress et al. (2014) model onto the age axis provides a clean separation between the prompt and tardy SN Ia with a dearth of events between 4 and 8 Gyr. Our age estimates are not consistent with the deficit of supernovae exploding in that range, but we do see a shift in the SN Ia light curve properties around a stellar age of about 8 Gyr.

We have estimated the host-galaxy masses in our sample to test if our measurements agree with the prediction of Figure 4 in Childress et al. (2014). We use the kcorrect code (v4_3)\(^8\) described in Blanton & Roweis (2007). This code utilizes spectral fitting templates based on the stellar population synthesis models of Bruzual & Charlot (2003), which are calculated using the Chabrier (2003) initial mass function. We input the SDSS model magnitudes and the pipeline redshifts for each galaxy (presented in Table 3) to calculate the $k$-corrections and stellar mass-to-light ratios. The stellar masses are output in units of $M_\odot h^{-1}$, and we convert them to units of $M_\odot$ using the C13 cosmology described in Section 2.4. The uncertainties on the stellar masses are approximately $\pm 0.3$ dex. The resulting estimated stellar masses are reported in Table 7.

Figure 9 shows the distribution of the local SN Ia age versus stellar mass for our sample. For our sample, the distribution of hosts in age-stellar mass space shows similarities and differences with that predicted in Childress et al. (2014). The observations do show young hosts extending to low stellar masses reproducing the backward "L" shape seen in the Childress et al. (2014) simulation. This indication of "cosmic downsizing" is not as pronounced in our data, as we are probably still overestimating the ages of the extremely young populations. Most notably missing is that the predicted bimodal age feature is not present. An island of young galaxies is expected between 0.5–1.0 Gyr and that is not seen in Figure 9. Our age distribution is relatively flat from 2 to 5 Gyr and then declines out to 11 Gyr. This does not match the predicted distribution in Figure 4 of Childress et al. (2014), and there is no clear peak of old hosts. The fact that our sample of supernovae extends to a redshift of 0.2 may contribute to the lack of a clear peak in old age. Childress et al. does predict that the old-age peak decreases in height and age, as the data are taken at higher redshifts simply due to the finite age of the universe.

### 7.4. Principal Component Analysis

Light curve shape and color of SN Ia have been shown to be strongly correlated with peak luminosity. Other modest trends with population age, host mass, and gas metallicity have been reported, and here we have identified a rather significant jump in Hubble residuals with local and global stellar age. Because these environmental observables are highly correlated with each other, it is interesting to ask if there is a linear combination of observables that have a clear and significant correlation with SN Ia Hubble residual. As an initial test, we performed a principal component analysis (PCA) on our data set.

PCA is a linear algebra tool that transforms the basis of a matrix of data to orthogonal axes where the new axes are maximally aligned and sorted with the intrinsic scatter of the data.\(^9\) Therefore the first principal component contains the most amount of information and the last principal component contains the least. Thus, it is possible to reduce the dimensions of a problem by retaining only the principal components that comprise most of the variance (or relative information) in the original data set. In addition, the first principal component will identify a linear combination of the original variables that accounts for most of the variance in the data. Interpretation of PCA is difficult because the results are sensitive to noise,

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\(^8\) Available through http://kcorrect.org or http://github.com/blanton144/kcorrect.

\(^9\) For more information, see Ivezić et al. (2014, Section 7.3), Wall & Jenkins (2012, Section 4.5), or visit https://towardsdatascience.com/pca-using-python-scikit-learn-e65388938e60.
this is a strong correlation. The unexpected PC1 of ± values. PC1 is not simply a stretch value, red colors being a negative value and blue colors being positive uncorrelated variables. The color of each data point represents its SALT2 significance. It is extremely unlikely to measure a correlation like this from uncorrelated variables. The color of each data point represents its SALT2 significance. We applied PCA on the parameters of SALT2 stretch value, red colors being a negative value and blue colors being positive values. PC1 is not simply an \( x_1 \) effect because the red points are distributed across a significant range of PC1 and two blue points \((x_1 \approx 1.5)\) have an unexpected PC1 of \( \sim -1 \). The best-fit linear regression has a slope of \( 0.051 \pm 0.011 \text{ mag} \) and an intercept of \( -0.012 \pm 0.016 \text{ mag} \).

The complete figure set (8 images) is available.

**Table 9**

PCA Coefficients Using Local and Global Ages

| \( x_1 \) | \( c \) | \( \log(M/M_\odot) \) | Age\(^a\) | \% Variance | \( x_1 \) | \( c \) | \( \log(M/M_\odot) \) | age\(^b\) | \% Variance |
|---|---|---|---|---|---|---|---|---|---|---|
| PC1 | 0.557 | -0.103 | -0.535 | -0.627 | 44 | 0.465 | -0.134 | -0.596 | -0.641 | 47 |
| PC2 | -0.159 | 0.956 | -0.213 | -0.118 | 25 | -0.206 | 0.937 | -0.263 | -0.101 | 25 |
| PC3 | -0.651 | -0.258 | -0.710 | 0.071 | 18 | -0.846 | -0.315 | -0.385 | -0.190 | 19 |
| PC4 | 0.490 | 0.086 | -0.404 | 0.767 | 11 | -0.158 | 0.070 | 0.654 | -0.737 | 8 |

**Note.** All observables are normalized via Equation (10).

\(^{a}\) The median age of the local environment posterior.

\(^{b}\) The median global age posterior.

**Figure 10.** Trend between Hubble residual and principal component 1 (using local age) is clearly visible. With a Spearman’s correlation coefficient of 0.44, this is a strong correlation. The \( p \)-value of 3.86 \( \times 10^{-7} \) corresponds to a 4.7\( \sigma \) significance. Hubble residuals are not included as a PCA parameter. Only after the PCA did we search for correlations between the resulting principal components and the Hubble residuals.

Before running PCA, we normalized all parameters by removing the mean and scaling to unit variance resulting in normalized input parameters \((x_1', c', m', \text{ and } a')\). An example, for \( x_1 \), is defined as

\[
x_1' = \frac{(x_1 - \mu_{x_1})}{\sigma_{x_1}}.
\]

The means and standard deviations used in the normalization process can be seen in Table 8.

From these four observables, PCA yields four principal components (PCs). Table 9 shows the linear combination of the observable variables that make up each principal component, as well as the explained variance. Substituting global age for local age resulted in some minor differences between the PCA coefficients and variance, but the overall conclusions are very similar between the two analyses. PC2 accounts for a quarter of the variance and is dominated by the SALT2 color parameter. The other two PCA components contribute only a small portion of the variance.

As a first analysis, it is important to see if there are any correlations between Hubble residual and these SN Ia-host-galaxy principal components. Looking at the PCA done with local age, PC3 and PC4 versus Hubble residual have extremely low Spearman’s coefficients of 0.21 and \(-0.15\), respectively. PC2, dominated by the SALT2 color term, when compared with Hubble residual is also a scatter plot with a low Spearman’s correlation of \(-0.06\). Surprisingly there is an increase in the scatter at the high PC2 (high color) domain. Using the PCA done with the global age, PC2, PC3, and PC4 have similar Spearman’s correlation coefficients. Figures showing the relationship between the Hubble residual and each principal component (four per analysis method, eight total figures) are available as a figure set in the online journal.

**Figure 10** shows the relationship between Hubble residuals and PC1 using the estimated local age. The correlation when PCA is applied using the global host age can be seen in Figure 11(a). PC1 with either local or global ages shows a very strong correlations with Hubble residuals. The first principal component, using local age estimates, is defined as

\[
PC_1 = 0.56x_1' - 0.10c' - 0.54m' - 0.63a',
\]
or it can be approximated by ignoring the color term because it barely contributes to PC1.

Uncertainties for these coefficients can be obtained via bootstrap resampling (Wall & Jenkins 2012, Section 6.6). Here we run a PCA on 100,000 data sets that were created randomly (with replacement) from our original data set using local ages. Since principal components are invariant to being multiplied by \(-1\), a constraint was made that the bootstrap eigenvector needed to have a positive dot product with the original eigenvector. If this was not true, the bootstrap eigenvector’s direction was reversed. The resulting distribution of coefficients for PC1 can be seen in Figure 12 and a statistical summary can be seen in Table 10. As expected, the color coefficient is consistent with zero. Interestingly, the coefficient for local age is more constrained than for stellar mass.

In PC1, the SALT2 color coefficient is very small, implying supernova color is not a strong contributor to this Hubble residual correlation. The mass, age, and stretch parameters in our PCA analysis are similar in amplitude and likely are of similar importance in any further standardization of SN Ia distances. The SALT2 stretch parameter, \( x_1 \), has a surprising large contribution to PC1, given that the SN Ia have already been corrected for light-curve shape. Figure 10 suggests a correlation between stretch and Hubble residual, but this
correlation was removed at the start of the analysis. Instead, PC1 shows that the value of $x_1$ is related to mass and age. Attempting to correct for the stretch-luminosity relationship requires including host properties, as all three have an influence on Hubble residuals. Our results suggest that the $\alpha$ parameter derived from the SALT2 fit is not ideal, because the effects of host mass and population age are not distributed uniformly with stretch.

The Hubble residual-PC1 correlation in Figure 10 has a Spearman’s correlation of 0.44. This trend is highly significant, with a p-value of $3.86 \times 10^{-6}$ corresponding to a 4.7σ significance. The trend in Hubble residual versus age has a significance of only 2.1σ (Figure 5), while including mass and stretch greatly increases the significance. This strong correlation suggests that host properties influence the luminosities of SN Ia beyond the currently applied corrections.

This data set did not have any significant correlations between Hubble residual and $x_1$ or $c$. But when $x_1$ is combined with stellar mass and age, there is a significant correlation with Hubble residual.

When using global ages, the underlying trend has a Spearman’s correlation of 0.40 or a 4.0σ significance. The Spearman’s correlation with PC1 from local ages is slightly more significant than for global ages, suggesting that measurements near the event provide some improvement in correlating supernova with environment.

The best-fit linear regression of this trend is

$$HR = 0.051 \text{ mag } \times \text{PC1} - 0.012 \text{ mag},$$

with uncertainties in the slope and intercept as $\pm 0.011$ mag and $\pm 0.016$ mag, respectively. This trend reduces the $1\sigma$ scatter in Hubble residual from 0.17 to 0.15 mag. The best-fit linear regression using global ages values is $HR = 0.045 \times \text{PC1} - 0.012$, with uncertainties in the slope and intercept as $\pm 0.011$ mag and $\pm 0.016$ mag, respectively. This reduces the $1\sigma$ scatter in the Hubble residual to 0.16 mag. As a note, the PCA does not maximize this correlation or minimize the $\chi^2$ parameter in a linear regression. For example, reversing the sign of the $x_1$ coefficient nearly doubles the slope of the correlation with Hubble residual as seen in Figure 11(b) and significantly reduces the $\chi^2$ parameter of a linear fit. Understanding the best use of these coefficients will be part of future work already in preparation.

Since the age and mass coefficients make up nearly two-thirds of the correlation amplitude, we see that a change of $\pm 2\sigma$ in the normalized mass and age values results in a 0.24 mag shift in SN Ia brightness. Cosmic downsizing suggests these two parameters are correlated since young galaxies tend to be small while typical old hosts are massive in the current epoch. Low-mass, young galaxies tend to be metal deficient, while old, massive galaxies can be metal-rich. Thus, this combination of age and mass may indicate progenitor metallicity influences the peak luminosities of SN Ia.

The correlation between Hubble residual and PC1 is very significant, and it is unlikely that PCA could generate such a strong correlation from a random distribution. To test the probability that this correlation is caused by chance, we applied a bootstrap style method for hypothesis testing. We took our PCA input matrix ($x_1$, $c$, host mass, local environment age) and shuffled the order along each parameter, creating 103 “new” SN Ia. There was no cross shuffling, so a stretch always stays a stretch; it just corresponds to a different SN Ia. We applied PCA on each shuffled sample and tested for any correlations with Hubble residual. After 32,000 runs, a 4σ test, the maximum Spearman’s correlation coefficient between Hubble residual and any principal component was 0.43, while the measured Spearman’s test of the non-shuffled values was 0.44. The distribution of the bootstrapped Spearman’s correlation coefficients is shown in Figure 13. This bootstrap style analysis shows that the false-positive Spearman’s correlation follows a Gaussian distribution with $\sigma \approx 0.1$. This is the expected distribution for Spearman’s correlation coefficients for a data set of $N \approx 100$. It is exceedingly unlikely that the correlation between Hubble residual and PC1 found in Figure 10 would appear at random. As a result, this luminosity-stretch-mass-age relationship is the most significant systematic seen between calibrated SN Ia and a host-galaxy environment.

### 7.5. Correcting for the Hubble Residual-PC1 Correlation

Since our PC1 strongly correlates with Hubble residuals, it is reasonable to consider a modification to the Tripp formula that is used to correct SN Ia peak luminosities. The new correction coefficients would be the multiplication of the PCA coefficients and the slope of the Hubble residual-PC1 trend. Except for the SALT color, the individual components making up PC1 have significant weights and thus are included in this modified equation. The stretch parameter in PC1 can be grouped with the...
original Tripp coefficient, leaving a new term with just host properties. Since PC1 has a positive correlation with Hubble residual, the coefficient should come in with a negative sign. From these results, and following the example of others (e.g., Moreno-Raya et al. 2016b), we propose a change in the distance modulus corrections performed by SALT2 by modifying the equation to include PC1. This new equation would be

$$\mu = m_B - M_B + (\alpha - \alpha')x_1 - \beta c + \gamma m' + \gamma' a',$$  \hspace{1cm} (13)

where $\alpha' \approx \gamma \approx 0.03$ mag. This uses both the approximate form of PC1 and encompasses the very slight renormalization of $x_1$ into $x_1'$. More research is needed to accurately determine $\alpha'$ and $\gamma$.

This modified distance correction formula now includes a host-galaxy stellar mass correction and a host age term. Childress et al. (2014) attempted to explain the mass step as an age dependency, and we see that both appear to have an impact on SN Ia luminosities. Our result is bolstered by Jones et al. (2018), who showed that stellar population color effects are still present after stellar mass corrections. Stellar mass plus population age is similar to the Mannucci relationship (Mannucci et al. 2010) that connects galaxy mass and star formation rate with gas phase oxygen abundance. Our combined age plus mass parameters work in the same sense as the Mannucci relation and may be a proxy for metallicity. Both Hayden et al. (2013) and Moreno-Raya et al. (2016b) have indicated that metallicity is the underlying galactic variable influencing Hubble residuals, and our results support this suggestion.

Kasen et al. (2009) shows that varying the metallicity of SN Ia progenitors will shift the slope of the Phillips relationship. To investigate this, we assume our SN Ia have a $M_{\text{peak}} = -19.0$. We then add in the Phillips relationship (with $\alpha = 0.16$) and our Hubble residual-PC1 relationship. The result is seen in

![Figure 13. Distribution of 32,000 absolute value Spearman’s correlation coefficient for the relationship between Hubble residual and principal components of randomized SN Ia $x_1$, $c$, host stellar mass, and local environment age data sets. This distribution was generated from a bootstrap style approach that accounts for any signal PCA might produce when no underline correlation exists. With 32,000 iterations and a maximum Spearman’s correlation of 0.43, the correlation between Hubble residual and PC1 (0.44, red vertical line Figure 10) is at least 4x significance. This bootstrap style analysis shows that a false-positive Spearman’s correlation, including any PCA effects, appears to follow a Gaussian distribution with a $\sigma = 0.1$ (black dashed line). This is the expected distribution for Spearman’s correlation coefficients for a data set of $N \approx 100$.](image1)

![Figure 14. Phillips relation reconstructed from the PC1 parameters assuming all SN Ia in our sample have a peak $M_{\text{peak}} = -19.0$. For the reconstruction, no color correction was applied and we set the slope to the effective stretch coefficient of $\alpha = 0.16$. The color of the points represents the sum of the mass and age parameters. Blue points are young, low-mass galaxies in our sample, and red points are high mass, old hosts. The arrows approximate the predictions of Figure 4 in Kasen et al. (2009) and represent how an increase of a factor of 10 in metallicity would impact the Phillips relationship. For luminous events, metallicity mainly reduces the stretch at a fixed absolute magnitude. For low-luminosity supernovae, the effect of metallicity moves the points parallel to the stretch correction correlation, resulting in little impact on distance estimates.](image2)
Figure 14. As fast decliners are preferentially in larger and older hosts, we see the host-galaxy’s effect on the Phillips relationship is stretch dependent. Put another way, the slope of the Phillips relationship, $\alpha$, is dependent on the host-galaxy properties of the sample. These results show that these parameters are interrelated and that there is likely a multi-dimensional relationship between peak absolute magnitude, decline rate, and progenitor metallicity. Research into this multi-dimensional relationship will be a part of a future work already in preparation.

8. The Effect on $H_0$

The most precise SN Ia measurement of the local value of $H_0$ was presented in Riess et al. (2016, 2018). Riess et al. rebuilt the distance ladder connecting geometric distances, Cepheid distances, and finally SN Ia in the Hubble flow. These works take into account the host-galaxy mass step, but this correction has a minor impact (0.7%, Riess et al. 2016) on the resulting $H_0$. The work of Rigault et al. (2015), has shown that host-galaxy correlations can have a large effect on the value of $H_0$. Our trend in PC1, which includes stellar age, that has the potential for a significant impact on $H_0$, as Cepheid hosts tend to have a significantly younger stellar population than the average Hubble flow galaxy. Here we put a constraint on the influence this Hubble residual-PC1 trend, seen in Figure 10, may have on the measurement of $H_0$ presented in Riess et al. (2016).

To test if the correlation in the PC1 parameter could affect $H_0$ tension, we estimated the local age, global age, and stellar mass of the SN Ia hosts that have distances calibrated using Cepheid variables. We then compare the average PC1 parameter found for the Cepheid sample with the average for a Hubble flow sample. As a representative Hubble flow data set, we use our analysis of the C13 galaxies. This assumption makes our estimated shift an upper limit, as the work in C13 does not include more recent corrections of small biases (e.g., Betoule et al. 2014; Kessler & Scolnic 2017) and the mass step that was performed in Riess et al. (2016). The difference in the average PC1 values for the two sets of hosts, multiplied by the slope in Equation (12), provides an estimate of the shift in peak absolute magnitude, $\Delta M$ between the calibration sample and the Hubble flow. The fractional error in distance, and therefore $H_0$, due to the differences in age, mass, and stretch between the two samples is then

$$10^{\Delta M/5} - 1.$$  

SDSS imaging was available for 14 of the 19 SN Ia hosts presented in Table 1 of Riess et al. (2016). The others were outside of the SDSS footprint and without SDSS-$ug$ photometry, and therefore we were not able to apply our age estimator in a consistent way. For this calibration sample, we followed the same procedures to get the local environment age, global age,

| SN Ia    | Host        | $x_1$ | $\sigma_{x_1}$ | $\epsilon$ | $\sigma_{\epsilon}$ | Citation | Local Age | $\sigma_{\epsilon}$ | Global Age | $\sigma_{\epsilon}$ | $\log(M/M_\odot)$ | $PC1_{\text{local}}$ | $PC1_{\text{global}}$ |
|----------|-------------|-------|----------------|-------------|-----------------------|----------|------------|-------------------|-------------|-------------------|-------------------|----------------|---------------------|
| 1981B    | NGC 4536    | −0.32 | 0.14           | 0.030       | 0.010                | J07      | 6.5        | 2.4               | 3.0          | 2.4               | 10.2              | −0.49          | 0.61                |
| 1990N    | NGC 4639    | 0.63  | 0.04           | 0.014       | 0.004                | J07      | 6.5        | 1.4               | 4.8          | 3.0               | 10.2              | 0.06           | 0.55                |
| 1994ac   | NGC 3370    | 0.32  | 0.10           | −0.065      | 0.033                | J07      | 5.4        | 1.3               | 5.0          | 1.9               | 10.0              | 0.46           | 0.62                |
| 1995al   | NGC 3021    | 0.71  | 0.08           | 0.051       | 0.006                | J07      | 6.4        | 1.7               | 6.0          | 1.5               | 10.2              | 0.06           | 0.16                |
| 1998aq   | NGC 3982    | −0.40 | 0.07           | −0.086      | 0.007                | J07      | 5.2        | 1.4               | 6.1          | 1.4               | 10.0              | 0.10           | 0.01                |
| 2002fk   | NGC 1309    | 0.22  | 0.04           | −0.101      | 0.003                | S12a     | 6.6        | 2.6               | 5.2          | 1.4               | 10.1              | −0.06          | 0.44                |
| 2003du   | UGC 9391    | 0.30  | 0.04           | −0.100      | 0.004                | J07      | 4.6        | 3.1               | 4.4          | 1.4               | 9.0               | 1.50           | 1.73                |
| 2007af   | NGC 5584    | −0.45 | 0.02           | 0.053       | 0.004                | H12      | 6.2        | 1.6               | 6.2          | 2.0               | 9.8               | −0.26          | −0.13               |
| 2009ig   | NGC 1015    | 1.76  | 0.15           | −0.058      | 0.013                | H12      | 8.8        | 4.4               | 7.9          | 2.1               | 10.3              | 0.01           | 0.20                |
| 2011by   | NGC 3972    | 0.02  | 0.13           | 0.012       | 0.014                | B14a     | 4.5        | 3.0               | 4.1          | 2.6               | 9.8               | 0.60           | 0.78                |
| 2011fe   | M101        | −0.21 | 0.07           | −0.066      | 0.021                | P13b     | 3.1        | 1.0               | 4.9          | 1.2               | 9.9               | 0.88           | 0.46                |
| 2012cg   | NGC 4424    | 0.45  | 0.04           | 0.080       | 0.020                | V18b     | 3.5        | 2.3               | 2.8          | 2.5               | 10.0              | 0.88           | 1.07                |
| 2012ht   | NGC 3447    | −1.25 | 0.05           | −0.080      | 0.030                | V18b     | 4.0        | 1.3               | 4.4          | 1.4               | 9.2               | 0.63           | 0.79                |
| 2013dy   | NGC 7250    | 0.70  | 0.04           | 0.089       | 0.025                | V18b     | 4.6        | 1.2               | 4.2          | 1.4               | 9.2               | 1.33           | 1.51                |

Notes. Light curve parameters were estimated from cited light curves using sncosmo (Barbary et al. 2016). Citation key: J07-Jha et al. (2007), S12-Silverman et al. (2012), H12-Hicken et al. (2012), B14-Brown et al. (2014), P13-Pereira et al. (2013), V18-Vinkó et al. (2018).

a Light curve data supplied by the Open Supernova Catalog (Guillochon et al. 2016).

b Citation is for the SALT2 parameters.
A KS-test concludes that these samples are different at a 1.9\% significance. A Mann–Whitney U test says that the calibration sample has a higher mean at a 1.2\% significance. Using the relationship between Hubble residual and PC1, this difference would translate to a 0.021 mag shift in peak luminosity, or a 1.0\% effect on $H_0$.

and host stellar mass. These values, along with the full list of SALT2 parameters, can be found in Table 11. A visual comparison of these parameter distributions can be seen in Figure 15.

The individual parameters of PC1 have only mild differences between the SN Ia calibration sample and the Hubble flow hosts. The most extreme fast decliners ($x_1 < -2$) are only in the Hubble flow sample because they are preferentially found in passive galaxies. The mass distributions are significantly different, with a low-mass tail in the calibration sample. The Hubble flow sample has a wider distribution of local ages than the calibration set, and the same is true for the global age. What is interesting about PC1 is that all these slight differences work in the same direction. The lower stretch, higher mass, and older ages in the Hubble flow combine for a significantly lower average PC1 when compared with the larger stretch, lower mass, younger hosts from the calibration sample.

For the calibration sample, we calculated a PC1 from both the local and global age PCA methodology, PC1,local and PC1,global, respectively. These values can also be seen in Table 11. A comparison of these two PC1 distributions is shown in Figures 16 and 17.

Using the local age, the calibration sample in Figure 16 has a mean $PC_{1,\text{local}}$ of $0.41 \pm 0.15$ (2.7\%), while the Hubble flow PC1 mean is defined as zero. To account for the sample sizes, we performed the Kolmogorov–Smirnov test (KS-test), resulting in a 5.5\% chance (1.9\%) that these samples are drawn from a common distribution, and a Mann–Whitney U test indicates that the calibration sample has a higher mean at a 1.2\% level. Using the best fit of the correlation between Hubble residual and PC1, this corresponds to a difference of 0.021 mag.

Rerunning this analysis with the global age PCA normalization and methodology, there is a $0.63 \pm 0.14$ (4.6\%) shift in the PC1,global means. The resulting KS-test says there is a 0.8\% chance (2.7\%) that these two samples are drawn from a common distribution. A Mann–Whitney U test indicates that the calibration sample has a higher mean at a 2.3\% significance. Applying this shift to the trend seen in Figure 11(a), there is a shift in peak luminosity of 0.028 mag.

These shifts in peak luminosity produce at most a 1.0\% or a 1.3\% effect on $H_0$, respectively. This is about twice the size of the already accounted for mass step, but is less than the current 1\% uncertainty in $H_0$ (2.3\%). A large SN Ia systematic effect was found, but it had a minimal effect on $H_0$. For this effect to relieve the full 3.8\% tension (Riess et al. 2018), these two samples would need to have a PC1 shift of $\sim 3.5$, about six times larger than currently seen. The SN Ia systematic found in this paper is extremely unlikely to fully resolve the $H_0$ tension.

9. Conclusion

Host-galaxy properties have an effect on the absolute magnitude of SN Ia. Host-galaxy stellar mass, age, and metallicity have all been shown to be a secondary correction to the Phillips relation. Using a Bayesian method to estimate the age, we were able to look at how Hubble residuals of SN Ia correlate with the mass weighted average age for both the local environment and the galaxy as a whole. This method is better at correctly estimating younger populations than previous methods. A 2.1\% significant correlation between Hubble residual and age was seen. This correlation may be an age step of $\sim 3.5$, about six times larger than currently seen. The SN Ia systematic found in this paper is extremely unlikely to fully resolve the $H_0$ tension.
the general trends of old galaxies with high stellar masses and a tail of low-mass young galaxies are seen in the data.

Using PCA on the two SALT2 parameters, host stellar mass, and local environment age, we see a very significant correlation (0.44) between Hubble residual and the first principal component \( PC_1 = 0.56x_1 - 0.11c - 0.54m - 0.63a' \). This trend was fit with a slope of 0.051 \( \pm 0.011 \) mag. The mixture of parameters making up \( PC_1 \) suggests that to understand the luminosity variations in SN Ia and to properly correct for them requires simultaneous knowledge of their host and supernova properties. This data set lacked any significant correlations between Hubble residual and \( x_1 \) or \( c \), but the combination of \( PC_1 \) does have a significant correlation with Hubble residual. As a result of this significant trend, \( PC_1 \) should be used as part of an updated light-curve fitter.

The dominant components of \( PC_1 \) are stretch, mass, and age. Using the Mannucci relationship, \( PC_1 \) may be implying that \( \alpha \) has a metallicity dependence. A theoretical case for this was already made by Kasen et al. (2009), and the observational trends found in this our work are able to reproduce this predicted effect.

A correlation of this magnitude could have major effects on the precision measurement of \( H_0 \). Looking at the difference in the calibration sample and a proxy Hubble flow sample, we see that these data sets have a meaningful difference in \( PC_1 \). Using the PCA methodology and normalization from the global age analysis, there is a shift in mean \( PC_1,global \) of 0.63 \( \pm 0.14 \) (4.6\( \sigma \)). In addition, a KS-test shows that they are drawn from different underlying populations at the 2.7\( \sigma \) significance level, and a Mann–Whitney U test says the calibration sample has a higher mean at a 2.3\( \sigma \) significance. Similar differences in the mean were seen using the local age. This difference between these two samples would correspond to a shift in SN Ia peak absolute magnitude of 0.028 mag, or at most a 1.3% shift in \( H_0 \). This analysis only places an upper limit on this effect, because several minor bias corrections were not applied. With at most a \( \sim 0.5 \sigma \) effect on \( H_0 \), this is extremely unlikely to relieve the full 3.8\( \sigma \) tension between the most recent measurements of the CMB from the Planck collaboration.

A major systematic in SN Ia was discovered, but it had only a small effect on \( H_0 \). This correction should be further investigated and applied to SN Ia used in cosmological studies. Moreover, it appears that even a large SN Ia systematic cannot fully relieve the tension between the local and CMB measurements of \( H_0 \).

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