Comparison second order versus zero order boundary element method for tomography imaging

T Rymarczyk and J Sikora

1 Research & Development Centre Netrix SA, Lublin, Poland
2 University of Economics and Innovation in Lublin, Poland

e-mail: tomasz.rymarczyk@netrix.com.pl

Abstract. In this article a new version of the algorithm for Electrical Impedance Tomography is presented. By describing the problem with differential equations brought to integral equations the inverse problem for tomography has been defined. This algorithm can be used for many types of nonclassical tomography. This approach is particularly useful where it is not possible to formulate accurately the boundary conditions at the outer boundary of the region. The influence of the order of BEM effect on precision of imaging was shown.

1. Introduction

The motivation for writing this work is the need of building a relatively universal algorithm [1,2,3-17] which could be able to provide a descent images for several modalities like EIT (Electrical Impedance Tomography) [2,18,19], ECT (Electrical Capacitance Tomography) [20], DOT (Diffuse Optical Tomography) [21], UTT (Ultra Sound Transmission Tomography) [22-26], RT (Radio Tomography [27] and the others [1,28]. All above modalities could be called as nonclassical ones. Such universal algorithm could open the way for the hybrid tomograph [27], combining the possibility of EIT and ECT for example. The main problem in hybridization is designing compact electrodes system suitable for both EIT and ECT tomography types [29].

If the frequency of excitation is different from zero (ω≠0), then material coefficient must be considered as a complex value:

\[ \gamma(x, \omega) = \sigma(x) + i \omega \varepsilon(x), \quad i = \sqrt{-1} \] (1)

where \( x \) is a position vector in a 2D space depending on coordinates \( x, y \), and \( i = \sqrt{-1} \) denotes imaginary unit and \( \omega \) is the pulsation (angular frequency).

But if the frequency is low than the real part is much higher than the imaginary one (\( \sigma(x) \gg \omega \varepsilon(x) \) ) then we are dealing with resistive tomography. But when the imaginary part of complex conductivity is much larger than its real part (resistivity), then the electrical impedance tomography acts as the true impedance tomography. In practice three kinds of nonconventional tomography are used: impedance tomography which refers also to resistance tomography when the imaginary part is negligible, capacitance tomography and induction, or eddy current, tomography, called magnetic tomography by some authors [30].

All three kinds of tomography are included in the group of impedance tomography, just having different characters: resistive, capacitive, or inductive.
The algorithm based on the PDE (Partial Differential Equations) solution (the forward problem) by BEM (Boundary Element Method) is presented in this paper. Immediately arise the question why BEM not the FEM (Finite Element Method)? First, the mathematical similarities of different kinds of nonconventional tomography predispose towards BEM rather than FEM because in BEM it is enough to change the Green function [31]. Secondly the boundary conditions are much easier to impose, particularly for UTT and RT.

The main goal of this paper is to present the basic assumptions for such an effective imaging algorithm.

The accuracy improvements for the forward problem solution are expected in increasing the order of the boundary elements. So, the results for the forward and inverse problems when the quadratic (second order approximation) and constant (zero order approximation) where compared. It is well known that the higher approximation should give more accurate results (see for example figure 2 or figure 3).

![Figure 1](image)

**Figure 1.** Boundary elements and nodes numbering (a) for the first subregion consisting of two edges $\Gamma_1^{(1)}$ and $\Gamma_2^{(1)}$ (b) for the second, internal subregion.

The number of elements, dimensions, material coefficients and all other parameters in both cases of the numerical experiment would be the same. The same number of elements means that the number of nodes provided calculated state functions for the second order BEM would be doubled in comparison to the zero order BEM.
Figure 2. BEM solution consisted of the electric potential and its normal derivative for the first subregion only (a) second order element (b) zero order element.

However, if the quadratic elements provide more exact solution for the forward problem is it also the truth for the inverse problem? To answer such a question several numerical experiments were carried out and presented in this paper.
2. Forward problem solution

Electrical properties such as conductivity $\sigma$ [S/m] and electrical permittivity $\varepsilon$ [F/m] define the behaviour of a material under the influence of an external electric field. For example, conductive materials have high resistivity $\sigma$ for direct (DC) and high complex conductivity $\gamma$ for alternate (AC) current. Dielectric materials have a high permittivity with comparison to conductivity which is close to zero. Let us consider a conducting material case for which conductivity become:

$$\gamma(x, \omega) = \sigma(x)$$

Let us consider the example of circular region with one circular shaped object inside (see figure 1). The second region temporarily was in the middle of the area for a better view of the numbering system. The internal boundary, separated two subregions, called an interface has the opposite direction of numbering regards the external boundary of the area. Due to such numbering the normal outside direction of the normal derivative of the electric potential would be preserved. A forward problem solution involves the Laplace equation solution for the EIT:

$$\nabla \cdot (r \nabla \varphi) = 0$$

with Dirichlet boundary conditions imposed in two voltage electrodes and Neumann boundary conditions on the rest of the boundary nodes. The Laplace’s equation in the integral form [3,16] is solved by Boundary Element Method (see results in figure 2, 3 and 4).
3. Interface conditions

Basically, the BEM can solve the homogeneous areas [21,31]. However very often we must find out the solution inside the regions which are spatially homogeneous (see for example figure 1). Then each region is considered separately, and the solution is “sewed” by the interface conditions (see equation (4)) on the common boundary between subregions (interface). In tomography task very often, there are more than two subregions, so the interface conditions must be considered on every interface. Such approach is general and describes 2D and 3D problems. Let introduce the superscripts \((i)\) denoting the \(i\)–th sub-region.

The electric potential and the normal derivative of potential (which is proportional to the electric current and for simplicity denoted as \(\Phi\)) along the nodes on the interface must therefore satisfy the following conditions:

\[
\phi_{i}^{(i-1)} = \phi_{i}^{(i)}
\]

\[
\gamma^{(i-1)} \frac{\partial \phi^{(i-1)}}{\partial n} \bigg|_{\Gamma_{i}} = -\gamma^{(i)} \frac{\partial \phi^{(i)}}{\partial n} \bigg|_{\Gamma_{i}}
\]

where: \(\Gamma_{i}\) – interface boundary, \(\gamma^{(i-1)}\) and \(\gamma^{(i)}\) material coefficient of the subregions separated by the interface.

4. Boundary conditions

When the boundary conditions are imposed on the whole external boundary, the number of unknowns is reduced from \(2N\) to \(N\) (\(N\)-number of nodes). The following cases could be distinguish:

1. Dirichlet boundary conditions,
2. Neumann boundary conditions,
3. Mixed boundary conditions when some nodes belong to case 1 and the rest of the nodes are case 2.

The general idea of rearranging system of equations for all of those cases will be described below.

Ad 1. Dirichlet boundary conditions

For Dirichlet boundary conditions vector \(\Phi = \Phi_{D}\) is specified (known) but the vector \(\partial \Phi / \partial n\) for those nodes is unknown. The matrix form of the BEM governing equation is

\[
A \Phi = B \partial \Phi / \partial n + q
\]

which can be rearranged to the following form:

\[
B \partial \Phi / \partial n = A \Phi_{D} - q = C_{D}
\]

Now the unknown values are on the left side of the equation but known values formulate the right-hand vector \(C_{D}\).

Ad 2. Neumann boundary conditions

In this case the vector \(\partial \Phi / \partial n = (\partial \Phi / \partial n)_{N}\) is known and vector \(\Phi\) remain unknown:

\[
A \Phi = B (\partial \Phi / \partial n)_{N} + q = C_{N}
\]

Ad 3 Mixed boundary conditions

When on some part of the boundary the Dirichlet boundary conditions are imposed and on the rest of the boundary the Neumann boundary conditions are imposed, then the rearranging process is more
complicated. Let assume that first $m_1$ rows of vector $\Phi$ are the Dirichlet boundary conditions and the rest equal to $m - m_1$ are of the Neumann type. Then:

$$
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
\Phi_D \\
\Phi
\end{bmatrix}
=
\begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \Phi_D}{\partial n} \\
\Phi
\end{bmatrix}
+ [q_1]
$$

where $\Phi_D$ - potential in $m_1$ nodes with Dirichlet boundary conditions, coefficient matrix was split for four submatrices according to boundary conditions.

After some mathematics final system of equations has the following form:

$$
\begin{bmatrix}
A_{11} - B_{12} & -B_{11} \\
A_{22} - B_{22} & -B_{21}
\end{bmatrix}
\begin{bmatrix}
\Phi \\
\frac{\partial \Phi_D}{\partial n}
\end{bmatrix}
=
\begin{bmatrix}
-A_{11} \Phi_D & q_1 \\
-A_{21} \Phi_D & q_2
\end{bmatrix}
$$

The whole solution vector composed of the electric potential and its normal derivative (for simplicity denoted as $\partial \Phi$) is presented in figure 2. The whole area under consideration is heterogenous.

The electrical potential distributions on the $\Gamma^{(1)}_1$ – external boundary and $\Gamma^{(1)}_2$ – internal boundary of the first subregion, are shown in figure 2 for second and zero order elements. Potential distribution inside of the first sub-region, are shown in figure 3.

Concluding the forward part of the analysis the differences of the solution between the second and the zero order are clearly visible but not too big. Particularly inside of the region what can be seen in figure 4 where the equipotential lines are shown.

![Figure 4](image-url)

**Figure 4.** Equipotential lines of electric potential for the first region only (a) second order element (b) zero order element.

In the rest of the paper the algorithm suitable for impedance/resistance tomography will be used, therefore to simplify the text we can introduce a material factor $k(x, y)$, which in this case, will represent conductivity $\gamma(x, y)$ but in other cases it will represent the permittivity $\varepsilon(x)$ or permeability $\mu(x)$.

5. Imaging algorithm

Tomography imaging has been brought to an inverse problem of optimal dimensioning and optimal location [32]. This problem belongs to the family of optimal shape design tasks. It could be solved by gradient free optimization method see for example [26,27,33].

This approach consists of the assumption that inside of the imaging area exists some number of trial objects. Imaging of the region rely on internal objects to be illustrated with respect to their location and
their dimensions. As a matter of fact, it is a parametrization of the image. Some simplifying assumptions were introduced:

1) inside the region under consideration only single object exists,
2) measurements were noisy and generated by computer. That is why we call them as a synthetic measurement.

For optimal dimensioning and optimal location, the BEM has big advantage over FEM as remeshing is not necessary during the iteration process because only boundaries are discretized.

One more advantage for UTT or High Frequency EIT which could represent the ECT where it is not possible to impose the boundary conditions precisely. In such cases the artificial boundary or artificial/approximate boundary conditions should be introduced. Implementation of such procedure is much easier for BEM than for FEM.

Above mentioned advantages were crucial when decision about the solution method of partial differential equation (PDE) was made.

6. Inverse problem for Electric Impedance Tomography

There is one big difference between tomographic and non-tomographic inverse problems. For tomographic problems existence of several projection angles make that input data are much more complicated in comparison to single projection angle for the standard inverse problems (see for example figure 7).

Let us consider a circular cross section of the region and single internal obstacle, for simplification also circular shape as it is shown in figure 5. The environment is heterogeneous, it was assumed that conductivity ratio of the background to the conductivity of internal object is equal to 1/100. External boundary was divided by 32 boundary elements with constant state function (interpolation with zero order polynomial). For the sake of attention, the potential value was placed in the middle of the element denoted by the bigger circles (see figure 5).

![Figure 5. Heterogeneous region under consideration and nodes distribution along the external boundary.](image)

The number of elements – 32 is twice as much as the number of electrodes/projection angles - 16. Now the forward tomography problem could be solved by BEM for two subregions with spatially homogeneous materials what could be a certain difficulty for BEM (see for example equation (9)).
The crucial point of the Inverse Problems is the Sensitivity Analysis, which is particularly difficult for BEM. So, to avoid the Sensitivity Analysis which for BEM is mathematically complicated and time consuming, the gradient free optimization offered by MATLAB [33] was selected.

7. Definition of the objective function
In order to match the signal calculated in each iteration step to the measured signal, the following objective function has been defined:

\[ \Phi = \sum_{j=1}^{j=p} \Phi_j = \frac{1}{2} \sum_{j=1}^{j=p} (f_j - v_{0j})^T (f_j - v_{0j}) = \frac{1}{2} (F - V_0)^T (F - V_0) \]  

(10)

where: \( \Phi \) – global objective function calculated for all \( p = 16 \) positions of the voltage source (so-called projection angles), \( j=1,2,...,p \), \( \Phi_j \) – objective function for the \( j \)-th position of the voltage source, \( f_j \) – vector of electrodes voltages obtained from calculations in the current iterative step for the assumed distribution of internal objects for the \( j \)-th position of the voltage source (projection angles), \( v_{0j} \) – vector of measured voltages for \( j \)-th position of the voltage source. The matrices \( F \) and \( V_0 \) are equal respectively: \( F = [f_1, f_2, ..., f_p]^T \) and \( V_0 = [v_{01}, v_{02}, ..., v_{0p}]^T \).

This objective function will be subject to minimization with a certain constrains.

8. Definition of inequality constrains
The imagining problem was turned into the optimization problem what in fact means parametrization of an image. The following parameters could sufficiently describe proposed image: radius of internal object \( r_1 \), and the position vector of the centre of circular internal object \( r_2 \), (see figure 5). During the optimization process internal object should not cross the external boundary of the region. Mathematically could be express as follows:

\[ \begin{bmatrix} -1 & 0 & \vdots & 0 \\ 0 & -1 & \vdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} < \begin{bmatrix} -0.1R_0 \\ 0 \\ 0.9R_0 \end{bmatrix} \]  

(11)

where: \( r_1 \) – radius of the internal object, \( r_2 \) – position vector, \( R_0 \) – radius of the region to be imaged (see figure 5).

The third parameter - an angle remains without of any constrains.

9. Adjacent measurement protocol
Numerical experiments were done for the adjacent protocol regarding projection angle and adjacent electrodes measurements as it is shown in figure 6. It is a common knowledge that adjacent protocol is much better idea when the internal object to be imaged is placed close to the external boundary.

The general idea of adjacent protocol for the source and measurements is presented in figure 6a for the first projection angle and in figure 6b for the last projection angle.
Figure 6. Adjacent protocol of measurements (a) first (b) the last projection angle; Illustration for 16 electrodes system.

Measured noisy signal composed of 16 projection angles is presented in figure 7 for both boundary elements second order (figure 7(a)) and zero order (figure 7(b)).
Figure 7. “Measuring” signal for all projection angles in case of adjacent electrodes with the presence of 20% noise (a) for second order (b) for 0-order.

Based on the adjacent protocol the optimization was carried out and the results of imaging process is presented in figure 8. The green colour means the final position and dimensions of the internal object. Blue dashed line denotes its real position and dimension.

The optimization is started from the yellow point (figure 8) and ended as the green object. The exact dimensions and position of the internal obstacle is denoted by blue dashed line (figure 8).
As the real position is known, the relative error of the optimization is possible to calculate. Result of calculations are presented in Table 1.

| noise | 2-nd order | 0 order |
|-------|------------|---------|
| 0%    | 3.76%      | 3.25%   |
| 5%    | 6.27%      | 3.75%   |
| 10%   | 8.64%      | 4.59%   |
| 15%   | 11.05%     | 5.26%   |
| 20%   | 13.47%     | 5.95%   |

Surprisingly second order BEM with comparison to zero order BEM provide much worse results. After experience in the forward solution where behaviour of higher order gives better results the worse results for inverse problem are rather unexpected. It means that the more sophisticated 2-nd order BEM is much more sensitive on the error which for the real measurement is always present.

The error changing with respect to the noise rate is shown in figure 9. Relation is linear.

![Figure 8](image.png)  
**Figure 8.** Result of image for: (a) 2-nd and (b) 0 order boundary elements for the noisy data of different noise rate.

![Figure 9](image.png)  
**Figure 9.** Error comparison for 2-nd and zero order element as a function of the noise rate.
History of gradient free optimization process is presented in figure 10. The optimization process is similar for both cases. However for the constant element optimization process is slightly longer but the value of the objective function goes deeper to the local minimum. It is worth to notice the long period of the optimization where the objective function almost do not change its value. Only after more than one hundred objective function evaluations the optimization process for both cases has really started (see figure 10).

![Figure 10. Optimization process for (a) 2-nd order (b) 0 order BEM.](image)

10. Conclusions
In this paper a novel approach to Electrical Impedance Tomography was presented. This new approach based on the solution of Partial Derivative Equations reformulated to the Integral Equations solved by BEM can be relatively easy adopted to other tomography modalities. This approach poses several important advantages over direct PDE solution by Finite Element Method. Those advantages were listed in previous paragraphs. But some other could be also added as:

1. converting the imaging problem to the inverse task demands the solution of PDE by the BEM the remeshing during the optimization process is not demanded contrary to the FEM solution,
2. proposed method strongly depends on the configuration of the object or objects inside of the region and the starting position and dimensions. Such parameters should be carefully selected depending on the experience of researcher and the case under consideration,
3. based on numerical experiments it is obvious that the lower order BEM is more robust and suitable for the inverse problem’s solution,
4. lower order BEM normally are much faster than higher order due to the simpler algorithm of integration demanding less numerical operations.
The authors would like to state that all figures were drawn with the aid of MATLAB [33].

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