Summary: For finite semidistributive lattices the map $\kappa$ gives a bijection between the sets of completely join-irreducible elements and completely meet-irreducible elements.

Here we study the $\kappa$-map in the context of torsion classes. It is well-known that the lattice of torsion classes for an artin algebra is semidistributive, but in general it is far from finite. We show the $\kappa$-map is well-defined on the set of completely join-irreducible elements, even when the lattice of torsion classes is infinite. We then extend $\kappa$ to a map on torsion classes which have canonical join representations given by the special torsion classes associated to the minimal extending modules introduced by the first and third authors and A. Carroll in [Algebr. Comb. 2, No. 5, 879–901 (2019; Zbl 1428.05314)].

For hereditary algebras, we show that the extended $\kappa$-map on torsion classes is essentially the same as Ringel’s $\varepsilon$-map on wide subcategories. Also in the hereditary case, we relate the square of $\kappa$ to the Auslander-Reiten translation.

**MSC:**

- 16G10 Representations of associative Artinian rings
- 16S90 Torsion theories; radicals on module categories (associative algebraic aspects)
- 06A07 Combinatorics of partially ordered sets
- 05E10 Combinatorial aspects of representation theory

**Keywords:** lattice; torsion class; kappa map; Auslander-Reiten translation; minimal extending module; wide subcategory

**Full Text:** DOI arXiv

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