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Analysis of boundary condition in modelling the low-velocity impact carbon-fiber reinforced polymer composite laminates

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Abstract. The effect of modelling strategy on boundary condition and impact energy of low-velocity impact laminate has been investigated. The paper deals with the finite element model (FEM) to study the structural responses of carbon fibre reinforced polymer (CFRP) composite laminate of CCF300/epoxy. In FE calculation, a proposed three-dimensional progressive damage model is used to determine the intralaminar damage, while cohesive contact formulation is employed to analyse the interlaminar damage. The failure model performances are validated and verified based on different boundary conditions and impact energies. Through experiment and simulation, it can be said that variation in boundary condition as well as impact energy, significantly change the force-time, displacement-time, energy-time, and load-displacement histories.

Keywords. Low-velocity impact (LVI); Carbon-fibre reinforced polymer; Finite element model; Intralaminar damage; Interlaminar damage.

1. Introduction
A low-velocity impact (LVI) is one of the loading scenarios which can cause significant reduction in stiffness and strength of the composite structures [1]. The detection of this type of failure is quite intricate because the damage cannot be easily observed from the surface of components by the naked eyes, thus exposing the structures into a great danger. For this reason, high number of researches focussing on low-velocity impact, and investigated in form of experiment [2, 3], simulations [4–6] and combination of both approaches [7–10].

During the LVI process in composite laminates, most experimental works used ASTM D7136 standard [11] for dimensioning and other related information. In most papers, the standard size of the laminates used is 150 mm x 100 mm with a cut-out rectangular support base of 125 mm x 75 mm. However, there also several literatures adopted different geometry of the laminates as well as non-standard support fixture. Liu and Liao [8] used a 100 mm x 100 mm composite plate together with top and bottom cut-out support fixtures. They tested plastic fibre-reinforced polymer matrix laminate. Wu et al. [12] carried out the LVI tests using fibre-metal laminates (FML), which the 100 mm x 100 mm plates are used for this purpose. Two steel supports at top and bottom which have the circular cut-out of diameter 76.2 mm are attached together to hold the specimens tightly. Non-standard specimen
dimension of 87.5 mm x 65 mm made of carbon-fibre reinforced polymer (CFRP) composite with 90 and 0 layers subjected to impact loading produced from hemispherical impactor was experimentally tested by Aymerich et al. [13] and later is used in work of Zhang and Zhang [14] as guideline in their numerical works.

To minimise the computational time, boundary conditions (i.e. clamping zone) and layup arrangement especially at the interface between layers are crucially emphasised in FE model. Full scale model including gripping areas are modelled explicitly as described in paper of Lou et al. [9], Tie et al. [17] and Riccio et al. [18]. All paper used cohesive elements to capture the onset and propagation of delamination. Other researchers such as Long et al. [19] simplified the clamping areas so that the calculation can be made faster and also acceptable results. They clamped the areas different from laminate and cut-out region both sides. Another similar approach performed by Liao and Liu [8], where the supports top and under the laminate are modelled physically to represent the clamping zone. The most simplified boundary condition to portray the clamped areas is realised by the development of the FE model only at the region/dimension of cut-out (i.e. 125 mm x 75 mm) [4, 7]. The simplification has shown significant effect on time consumption as well as quality of predicted results. Arrangement of cohesive interface between layers are most highly discussed in literatures. Long et al. [19] have built 5 models that can capture economically the delamination. The concept is based on experimental observation that conclude that delamination took place between the layers away from impacted surface. Layers which located nearby impacted zone are tied together without cohesive interfaces. Han et al. [7] established FE model using global-local approach to model the cohesive interfaces. Since the debonding normally occur when the interface interacts with two layers which have different in orientation, thus the layers are grouped together based on sequences and less cohesive interfaces were built. The more complex model for modelling the cohesion between layer were accomplished by inserting one layer of cohesive for each interface [4, 9, 16, 17, 20]. More delamination interfaces in the model contribute to higher accuracy and increase the computational effort [21].

For the completeness of the modelling strategy in LVI process, failure initiation and progression need to be predicted in proper way through the implementation of the progressive damage model for composite laminates, which include damage criteria and evolution law. In previous research works, three dimensional (3D) Hashin failure criteria [22] has been extensively used as initiation criteria to detect the failure especially for uni-directional (UD) composite laminates because of its credibility to isolate different mode of failures. Long et al. [19] and Tie et al. [23] performed the failure analysis of LVI using Hashin formulation and linear degradation scheme for progression law. Another researcher like Tie et al. [17] predicted the failure in laminates due to impact loading using Hashin criteria together with continuum damage mechanics (CDM) degradation law. It is found that those combination agree well with test results. In addition, combination between Hashin and Puck criteria [24] to predict failure due to fibre breakage and matrix or interfiber cracking are frequently observed in scholar articles [4, 10, 25]. To extent the capability of modern computer, LVI failure analysis also been numerically evaluated using multi-scale criteria (MMF) as described clearly in work of Han et al. [7] and Lou et al. [9].

Although the above researchers have analysed many aspects of modelling techniques in low velocity impact through experiment and numerical simulations, still many areas need to be improved. Thus, the research is inconclusive. In this article, the detailed clamping approaches (boundary condition) and failure prediction are described and discussed extensively, and specific simulation outputs are compared with test data. Four different BCs are used to study the effect of simplification of clamping areas together with Puck failure criteria to predict the intralaminar damage in the laminates for such type of loading. Prediction of delamination is achieved via implementation of cohesive contact formulation which also used bilinear traction function as embedded in Abaqus software. Validations of proposed model have been made by comparing the results with test data. It is hoped that the methodologies in this article can be a great design tool for more realistic composite parts and structures in the case of LVI.
2. Computational model
In this article, finite element method (FEM) model is used to simulate the impact damage based on the intralaminar damage analysis and interlaminar cohesive contact formulation.

2.1. FE modelling
The impact test is simulated using FE method in Abaqus/Explicit platform. The laminates are fabricated from CCF300/epoxy composite material. The laminate material properties are summarised in table 1, while test data is obtained from publication of Han et al. [7]. The laminate consists of [45/0/-45/90]₄s stacking sequences with a total thickness of 4 mm. To reduce the computational time, the global-local approach is employed where the layup arrangement is modified and rearranged according to [45/4/0/4/-45/4/90/8/-45/4/0/45]₄s stacking sequence. For this configuration, only six cohesive interfaces required. The layup modification can be viewed in figure 1, while the interfacial properties are shown in table 2. The intralaminar region is meshed using eight node linear brick reduced integration elements (C3D8R). To capture the damage pattern effectively, the region nearby impacted areas are modelled with finer mesh size compare with areas further away from impacted zone.

| Category | Properties |
|----------|------------|
| Elastic  | $E_1 = 123.91 \text{ GPa}, E_2 = E_3 = 9.72 \text{ GPa}, G_{12} = G_{13} = 4.53 \text{ GPa}, G_{23} = 2.56 \text{ GPa}, \nu_{12} = \nu_{13} = 0.288, \nu_{23} = 0.347$ |
| Strength | $X_T = 1762.3 \text{ MPa}, X_C = 1362.2 \text{ MPa}, Y_T = 71.1 \text{ MPa}, Y_C = 218.3 \text{ MPa}, S_{12} = S_{13} = S_{23} = 83.5 \text{ MPa}$ |
| Density  | $\rho = 1.5 \times 10^{-9} \text{ tonne/mm}^3$ |

Table 1. The Mechanical Properties of CCF300/epoxy.

| Properties |
|------------|
| $K_{nn} = K_{ss} = K_{tt} = 1 \times 10^5 \text{ MPa}$ |
| $t_{\sigma^n} = t_{\sigma^s} = t_{\sigma^t} = 80 \text{ MPa}$ |
| $G_{n^c} = 556 \text{ J/m}^2, G_{s^c} = G_{t^c} = 1497 \text{ J/m}^2$ |

Table 2. The Cohesive Properties of CCF300/epoxy.

In this article, four different models which emphasizing on boundary condition of gripping/clamping zones are analysed together with proposed progressive damage model. The complete description on the boundary condition and specific model developed is illustrated in figure 2. Further analysis is carried out based on two impact energies of 4.45 J/mm and 6.67 J/mm which produced an initial velocity of 2.577 m/s and 3.155 m/s, respectively. The contact between impactor and the laminate is defined using general contact, and “hard contact” is specified in normal direction. Finally, the contact force-time, displacement-time, kinetic energy-time, and load -displacement curves are extracted from the output files, and the quality of the results are compared with experimental data, as well as among the simulation data.
2.2. Intralaminar damage model

A user-defined material model for intralaminar failure is established for uni-directional composite laminate and has been implemented for Abaqus/Explicit using VUMAT subroutine. The identification and evaluation of damage progression has been evaluated using 3D Puck failure criteria [26, 27], which can distinguish fibre failure and inter-fibre failure in tension and compression. The background of this theory is extended from Hashin failure criteria [22].

2.2.1. Puck failure criteria. The analytical equations represent the fibre failure (FF) in composite laminate is written in the following form:

\[
FF = \begin{cases} 
\frac{1}{X_T} \left[ \sigma_1 - \left( v_{12} - v_{12f} m_{sf} \frac{E_{11}}{E_{11f}} \right) (\sigma_2 + \sigma_3) \right] & \text{for } \{ \ldots \ldots \} \geq 0 \\
\frac{1}{X_C} \left[ \sigma_1 - \left( v_{12} - v_{12f} m_{sf} \frac{E_{11}}{E_{11f}} \right) (\sigma_2 + \sigma_3) \right] & \text{for } \{ \ldots \ldots \} < 0 
\end{cases}
\]  

(1)

Where \( X_T \) and \( X_C \) are the tensile and compressive strengths of a UD layer in the longitudinal direction and \( v_{12} \) and \( v_{12f} \) are the Poisson’s ratio for UD lamina and fibre, respectively. The mean stress magnification factor, \( m_{sf} \) is assumed to be 1.3 for glass fibre and 1.1 for carbon fibre [28].

**Figure 1.** Local-global approach in defining the stacking sequences and meshing strategy for the LVI plate.
Figure 2. Description on boundary condition used in this paper for specific developed models.
For inter-fibre failure (IFF), also referred as matrix cracking assumes that fracture in the laminate is resulted by the stresses acting on the fracture plane (FP) \((\sigma_\tau, \tau_\nu \text{ and } \tau_\phi)\) inclined \(\theta_f\) with respect to material plane. The classical transformation equations are used to obtain the normal and shear stresses previously mentioned. The IFF function relies on the stresses acting on the fracture plane, and formulated as:

\[
IFF(\theta) = \begin{cases} 
\sqrt{\frac{1}{R_\perp - R_\perp^+} \sigma_n(\theta)}^2 + \frac{(\tau_{nt}(\theta))^2}{R_\perp} + \frac{(\tau_{nl}(\theta))^2}{R_\perp} + \frac{P_{\perp}^+}{R_\perp^+} \sigma_n(\theta) & \text{for } \sigma_n \geq 0 \\
\sqrt{\frac{\tau_{nl}(\theta)^2}{R_\perp} + \frac{(\tau_{nt}(\theta))^2}{R_\perp} + \frac{P_{\perp}^-}{R_\perp^-} \sigma_n(\theta) + \frac{P_{\perp}^+}{R_\perp^+} \sigma_n(\theta) & \text{for } \sigma_n < 0} 
\end{cases}
\]

The parameter \(\psi\) denotes the shear angle in action plane, \(R_\perp\) is failure resistance normal to fibres direction, and \(R_{\perp\perp}, R_{\perp\parallel}\) and \(R_{\parallel\parallel}\) are the fracture resistances of the action plane due to the shear stressing.

The Puck’s inclination parameters used here is shown in Table 3. The parameters shown before can be calculated using:

\[
\frac{P_{\perp\perp}^+}{R_{\perp\perp}^-} = P_{\perp\parallel}^+ \cos^2 \psi + P_{\perp\parallel}^- \sin^2 \psi; \quad \frac{P_{\perp\parallel}^-}{R_{\perp\parallel}^-} = \frac{P_{\perp\perp}^+}{R_{\perp\perp}^-} \cos^2 \psi + \frac{P_{\perp\perp}^-}{R_{\perp\perp}^-} \sin^2 \psi
\]

\[
\cos^2 \psi = \frac{\tau_{nt}^2(\theta)}{\tau_{nt}^2(\theta) + \tau_{nl}^2(\theta)}; \quad \sin^2 \psi = \frac{\tau_{nl}^2(\theta)}{\tau_{nt}^2(\theta) + \tau_{nl}^2(\theta)}
\]

\[
R_\perp = Y_T; \quad R_{\perp\parallel} = S_{2\theta}; \quad R_{\perp\parallel} = \frac{Y_C}{2(1 + P_{\perp\parallel})}
\]

Where \(Y_T\) and \(Y_C\) are the transverse tensile and compressive strength, respectively, while \(S_{2\theta}\) is the in-plane shear strength of composite.

| Type          | Inclination parameter (-) |
|---------------|----------------------------|
|               | \(P_{\perp\perp}^+\) | \(P_{\perp\parallel}^+\) | \(P_{\perp\perp}^-\) | \(P_{\perp\parallel}^-\) |
| Glass fibre   | 0.30                      | 0.25                      | 0.20-0.25               | 0.20-0.25                |
| Carbon fibre  | 0.35                      | 0.30                      | 0.25-0.30               | 0.25-0.30                |

2.2.2. Damage evolution law. The use of failure criteria alone is insufficient to represent the total failure behaviour of laminated composite structures. The proper degradation algorithm of stiffness matrix or elastic properties is crucially required to evaluate the progression of damage since damage is an accumulation of several series of failure in the laminate. The most prominent approach is to implement the ply-discount method to gradually decrease the stiffness of the structure.

To describe the elastic-brittle behaviour of fibre-reinforced composites, a constitutive model suited for composite material is used, where Lee et al. [29] was successfully performed their numerical model to identify the onset of failure as well as damage progression. A 3D-damaged stiffness matrix, \(C^d\) is written as:
\[ C^d = \begin{bmatrix}
\beta C_{11} & \kappa C_{12} & \kappa C_{13} & 0 & 0 & 0 \\
\kappa C_{21} & \kappa C_{22} & \kappa C_{23} & 0 & 0 & 0 \\
\kappa C_{31} & \kappa C_{32} & \kappa C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & \beta \omega G_{12} & 0 & 0 \\
0 & 0 & 0 & 0 & \beta \omega G_{13} & 0 \\
0 & 0 & 0 & 0 & 0 & \beta \omega G_{23}
\end{bmatrix} \quad (4) \]

Where \( C_0 \) is undamaged stiffness component, and \( G_{12}, G_{13} \) and \( G_{23} \) are the in-plane and out-of-plane shear modulus of composite material. The multiplication factors \( \beta, \kappa, \omega \) are defined as following:

\[
\beta = 1 - d_f \\
\kappa = (1 - d_f)(1 - d_m) \\
\omega = (1 - S_{mt}d_{mt})(1 - S_{mc}d_{mc})
\]

Where \( d_f \) and \( d_m \) are the global damage variables corresponding to fibre and inter-fibre failure, respectively. Individual damage variables based on failure mode are represented by \( d_{ft}, d_{fc}, d_{mt}, d_{mc} \) for fibre failure in tension and compression and inter-fibre failure in tension and compression, respectively. The relationship between global and local variables is defined as

\[
d_f = 1 - (1 - d_{ft})(1 - d_{fc}) \\
d_m = 1 - (1 - d_{mt})(1 - d_{mc})
\]

2.3. Cohesive model

Delamination is simulated by cohesive surface behaviour using cohesive contact interface. Based on the formulation, the fracture-separation law is employed to control the interaction between traction stress and separation displacement in the model as written in matrix form below:

\[
t = \begin{bmatrix}
t_n \\
t_s \\
t_t
\end{bmatrix} = \begin{bmatrix}
K_{nn} & 0 & 0 \\
0 & K_{ss} & 0 \\
0 & 0 & K_{tt}
\end{bmatrix} \begin{bmatrix}
\delta_n \\
\delta_s \\
\delta_t
\end{bmatrix}
\]

(5)

Where \( t_n, t_s, \) and \( t_t \) are the interface strength under the failure mode I, II, and III respectively. The damage initiation and progression based on delamination mode is summarized in table 4.

**Table 4. Failure criteria for delamination used in this article.**

| Approach            | Cohesive contact interface |
|---------------------|----------------------------|
| Damage initiation   | Quadratic nominal stress criterion |
|                     | \[
\left( \frac{t_n}{t_n^*} \right)^2 + \left( \frac{t_s}{t_s^*} \right)^2 + \left( \frac{t_t}{t_t^*} \right)^2 = 1
\]
| Damage evolution    | Power law fracture criterion |
|                     | \[
\frac{G_n}{G_i} + \frac{G_s}{G_i} + \frac{G_t}{G_i} = 1
\]

From table 4, \( t_i^* (i = n, s, t) \) is the interface strength parameters, \( G_i^* (i = n, s, t) \) is the critical fracture energy needed to cause damage in the normal and two shear directions and \( \alpha \) is the material parameter (i.e. \( \alpha = 1.45 \)). The onset of delamination can be identified whenever the quadratic function achieved
unity (=1). Once the damage criterion is satisfied, the cohesive stiffness degraded according to power law.

3. Results and discussion

3.1. Mesh sensitivity
To effectively analyzed the proposed damage model, the mesh dependency test is conducted for three different number of element (NoE) having 4032, 7168, and 11200 elements, respectively. The elements are established using structured mesh type in Abaqus/CAE. From figure 3, it is found that increasing the number of elements, decrease the contact force, while gaining the computational time. Due to computational cost saving, NoE of 7168 are satisfactorily predict the peak contact force, and no significant effect observed when increasing the element number to 11200.

![Figure 3](image)

Figure 3. Mesh sensitivity analysis based on peak force and calculation time.

3.2. Analysis of modelling boundary condition
This sub-section analyses the effect of boundary condition on LVI specimen in predicting the failure in composite laminate. The simulated impact force-time curves from 4 models are compared with experiment curve, and results in figure 4 revealed that the duration of impact force in virtual test curves are slightly shorter than real curve which is mainly due to philosophy of applied boundary condition. From all four models, model 1 is closely mimicked the test curve because of the boundary condition includes the rubber grips and cut-out support, however, overestimates the ultimate impact force. Model 3 and 4 are performed better in predicting the contact force, and the simulated results are agreed well with experiment, while model 2 performed least in both aspects.

Because of no reference for depth of indentation, kinetic energy and force-displacement curve from experiment, comparison is only made based on these four models. Model 1 used less time to bounce back the impactor and utilised low kinetic energy as compared to other models. The prediction of displacement and load-displacement curve show similar pattern where model 1 gave better results in producing lower indentation and energy absorption. Model 3 and 4 can be used as optimised tools to reduce the calculation time and at the same time produced the reasonable results.
Delamination or debonding is another area of interest in predicting the total failure due to impact loading. As can be seen in figure 5, the damage morphology indicates the area of delamination getting bigger towards the 90° layups, and slowly decreases approaching the last bottom layers. The existence of delamination boosts the process of damage accumulation.

3.3. Analysis of impact energy

The influence of the impact energy in determining the contact force and energy absorption is discussed graphically in figure 6. It can be noticed that higher impact energy resulted in increasing the contact force magnitude. Thus, energy absorption became greater while increasing the impact energy as clearly depicted in curve of load-displacement (bottom-right). With the comparison made between simulated force-time graph and experiment of impact energy of 4.45 J/mm, it should be said that the failure prediction is agreed well with test data. Contrary to the above achievement, an over-estimated result is produced under impact energy of 6.67 J/mm, which was around 35% discrepancy with experiment contact force curve. No clear explanation can be said, since the result from test obtained show less difference for both impact energies.

Compared with lower impact energy, the displacement is higher in 6.67 J/mm impact energy as the kinetic energy of 10 J deviated at the beginning of the LVI process and recorded around 5 J deviation at the end of the process. As a general conclusion, higher impact energy applied to the structure increase the amount of results in almost all aspects, proportionally.
Figure 5. Delamination morphology for the impact energy of 4.45 J/mm (model 3)

Figure 6. Simulated results of CCF300/epoxy LVI laminate under impact energy of 4.45 J/mm (2.577 m/s) and 6.67 J/mm (3.155 m/s)
4. Conclusion
In this paper, Puck damage criteria incorporated with simple degradation scheme and cohesive damage formulation was proposed to study the structural response of CCF300/epoxy composite laminate subjected to low-velocity impact loading. The damage model was used to study the variation of force, displacement and kinetic energy of the impactor. The cohesive theory is applied to the FE model to detect and trace the phenomena of delamination.

In general, the proposed damage model can predict the contact force and damage failure modes consistently with test result, while for higher impact energy showed larger discrepancy due to unclear structural response in the experiment. Besides, the full-scale FE model (model 1) performed excellent in capturing the structural response, while satisfactorily predict the peak force. Since test data for kinetic energy and displacement are not revealed to the public, no further comparison can be made regarding to the developed model. Increasing the impact energy rises the force response, as well as kinetic energy used and depth of indentation. Finally, the low-velocity impact results in the delamination in all sub-laminate, which varies in term of areas based on the location from impacted point.

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References
[1] Yang, L., Wu, Z., Gao, D., Liu, X.: “Microscopic damage mechanisms of fibre reinforced composite laminates subjected to low velocity impact.” Computational Materials Science, vol. 111, pp. 148–156, 2016.
[2] Tie, Y., Sapanathan, T., Rachik, M., Hou, Y., Zhou, X., Li, C.: “An insight into the low-velocity impact behavior of patch-repaired CFRP laminates using numerical and experimental approaches.” Composite Structures, vol. 190, pp. 179–188, 2018.
[3] Gliszczynski, A.: “Numerical and experimental investigations of the low velocity impact in GFRP plates.” Composites Part B: Engineering, vol. 138, pp. 181–193, 2018.
[4] Perillo, G., Vedvik, N.P., Echtermeyer, A.T.: “Numerical analyses of low velocity impacts on composite. Advanced modelling techniques.” In: Proceedings of the Simulia customer conference (2012).
[5] Zhang, J., Zhang, X.: “An efficient approach for predicting low-velocity impact force and damage in composite laminates.” Composite Structures, vol. 130, pp. 85–94, 2015.
[6] Maio, L., Monaco, E., Ricci, F., Leece, L.: “Simulation of low velocity impact on composite laminates with progressive failure analysis.” Composite Structures, vol. 103, pp. 75–85, 2013.
[7] Han, G., Guan, Z., Li, X., Du, S.: “Failure analysis of carbon fiber reinforced composite subjected to low velocity impact and compression after impact.” Journal of Reinforced Plastics and Composites, vol. 35, pp. 727–746, 2016.
[8] Liu, P.F., Liao, B.B., Jia, L.Y., Peng, X.Q.: “Finite element analysis of dynamic progressive failure of carbon fiber composite laminates under low velocity impact.” Composite Structures, vol. 149, pp. 408–422, 2016.
[9] Lou, X., Cai, H., Yu, P., Jiao, F., Han, X.: “Failure analysis of composite laminate under low-velocity impact based on micromechanics of failure,”, 2017.
[10] Singh, H., Namala, K.K., Mahajan, P.: “A damage evolution study of E-glass/epoxy composite under low velocity impact.” Composites Part B: Engineering, 2015.
[11] D7136, A.: “Standard Test Method for Measuring the Damage Resistance of a Fiber-Reinforced-Polymer Matrix Composites to a Drop-Weight Impact event.” Book of Standards, vol. 15, 2005.
[12] Yu, G.C., Wu, L.Z., Ma, L., Xiong, J.: “Low velocity impact of carbon fiber aluminum
laminates." *Composite Structures*, vol. 119, pp. 757–766, 2015.

[13] Aymerich, F., Dore, F., Priolo, P.: “Simulation of multiple delaminations in impacted cross-ply laminates using a finite element model based on cohesive interface elements.” *Composites Science and Technology*, vol. 69, pp. 1699–1709, 2009.

[14] Zhang, J., Zhang, X.: “Simulating low-velocity impact induced delamination in composites by a quasi-static load model with surface-based cohesive contact.” *Composite Structures*, vol. 125, pp. 51–57, 2015.

[15] Gliszczynski, A., Kubiak, T., Rozylo, P., Jakubczak, P., Bieniaś, J.: “The response of laminated composite plates and profiles under low-velocity impact load.” *Composite Structures*, vol. 207, pp. 1–12, 2019.

[16] Liu, H., Falzon, B.G., Tan, W.: “Experimental and numerical studies on the impact response of damage-tolerant hybrid unidirectional/woven carbon-fibre reinforced composite laminates.” *Composites Part B: Engineering*, vol. 136, pp. 101–118, 2018.

[17] Tie, Y., Hou, Y., Li, C., Zhou, X., Sapanathan, T., Rachik, M.: “An insight into the low-velocity impact behavior of patch-repaired CFRP laminates using numerical and experimental approaches.” *Composite Structures*, vol. 190, pp. 179–188, 2018.

[18] Riccio, A., De Luca, A., Di Felice, G., Caputo, F.: “Modelling the simulation of impact induced damage onset and evolution in composites.” *Composites Part B: Engineering*, vol. 66, pp. 340–347, 2014.

[19] Long, S., Yao, X., Zhang, X.: “Delamination prediction in composite laminates under low-velocity impact.” *Composite Structures*, vol. 132, pp. 290–298, 2015.

[20] Liao, B.B., Liu, P.F.: “Finite element analysis of dynamic progressive failure of plastic composite laminates under low velocity impact.” *Composite Structures*, vol. 159, pp. 567–578, 2017.

[21] Heimbs, S., Bergmann, T., Schueler, D., Tosso-Pentecôte, N.: “High velocity impact on preloaded composite plates.” *Composite Structures*, vol. 111, pp. 158–168, 2014.

[22] Hashin, Z.: “Fatigue Failure Criteria for Unidirectional Fiber Composites.” *Journal of Applied Mechanics*, vol. 48, pp. 846, 2009.

[23] Du, J., Tie, Y., Li, C., Zhou, X.: “Numerical and experimental study for damage characterization of composite laminates subjected to low-velocity impact.” *Materials Physics and Mechanics*, vol. 27, pp. 195–204, 2016.

[24] Puck, A., Schürmann, H.: “Failure analysis of FRP laminates by means of physically based phenomenological models.” In: Soden, M.J.H.S.K.D. (ed.) Failure Criteria in Fibre-Reinforced-Polymer Composites. pp. 832–876. Elsevier, Oxford (2004).

[25] Shi, Y., Pinna, C., Soutis, C.: “Modelling impact damage in composite laminates: A simulation of intra- and inter-laminar cracking.” *Composite Structures*, vol. 114, pp. 10–19, 2014.

[26] Puck, A.: “Failure Analysis of Frp Laminates By Means of Physically Based Phenomenological Models.” *Composites Science and Technology*, vol. 58, pp. 1045–1067, 1998.

[27] Puck, A., Deuschle, H.M.: “Progress in the Puck Failure Theory for Fibre Reinforced Composites: Analytical solutions for 3D-stress,”, 2002.

[28] Puck, A., Kopp, J., Knops, M.: “Errata to ‘Guidelines for the determination of the parameters in Puck’s action plane strength criterion’ [Composites Science and Technology 2001;62(3):371–8].” *Composites Science and Technology*, vol. 62, pp. 1275, 2002.

[29] Lee, C.S., Kim, J.H., Kim, S.K., Ryu, D.M., Lee, J.M.: “Initial and progressive failure analyses for composite laminates using Puck failure criterion and damage-coupled finite element method.” *Composite Structures*, vol. 121, pp. 406–419, 2015.