Comparison of harmonic estimation methods for power quality assessment

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Abstract. Power quality is regulated by international accepted standards from IEEE and IEC. One of the parameters that is defined in the standards is the harmonic distortion. The grouping method is the standard-advised method to evaluate harmonics but other approaches can be used. In this paper, three different methods are compared considering memory requirements, relative time consumption and the uncertainty of the harmonic estimation with spectral leakage, noise and different harmonic phases.

1. Introduction
Power Quality (PQ) has been regulated through international standards from IEEE and IEC [1 - 5] due to the importance of power quality in ensuring the normal operation of electrical equipment [6-8]. Nowadays with the developments in power electronics and their nonlinear nature, the voltage and current waveforms have even higher harmonic interferences that need to be monitored. PQ measurements include harmonic amplitudes and the overall total harmonic distortion parameter [6]. Harmonic distortion is addressed in two IEC standards [2, 5] and accordingly, its analysis must be obtained from a 10 cycle segment for a 50 Hz nominal frequency and 12 cycle segment if the nominal frequency is 60 Hz. Both cases correspond to 200 ms waveform segments.

In this paper three different approaches for harmonic amplitude estimation are compared: (i) the standard-advised grouping method [2]; (ii) multiharmonic least-squares fitting [9, 10] and; (iii) a spectrum analysis with leakage compensation algorithm [11]. The comparison includes memory requirements, relative time consumption and error/uncertainty estimation.

2. Description of the methods
2.1. Grouping
The standard-advised method for harmonic amplitude estimation [2] is based in grouping frequency components to estimate each harmonic amplitude. Without spectral leakage, the harmonic amplitude is in one single DFT (Discrete Fourier Transform) frequency component. However, with leakage (caused by frequency changes), each harmonic voltage is spread over more frequency DFT bins. For a 5 Hz spectral resolution (which results from the 200 ms of the 10 cycles in 50 Hz systems or 12 cycles in 60 Hz systems) there are 9 frequency components between harmonics to be added at each group (11 for
60 Hz). The frequency component that is in the middle of two harmonic groups is divided/included in the two groups. The estimated power for each harmonic group, \( C_{h_k}^2 \), is, for the 50 Hz case

\[
C_{h_k}^2 = \frac{|c_{10kHz}|^2}{2} + \left( \sum_{i=1}^{I} |c_{10kHz+i}|^2 \right) + \frac{|c_{10kHz}|^2}{2}
\]

where \( h \) is the harmonic order, and \( c_k \) is the \( k \) DFT component.

2.2. Multiharmonic least-squares fitting algorithms

The multiharmonic method consists on fitting a multiharmonic signal model to the measured voltage/current signals. The least squares algorithm iteratively fits the acquired data to the model, adjusting the signal frequency until the signal frequency correction reaches a sufficiently low predefined threshold. Two different approaches of this method that lead to the same estimation results are used [9-10]. The base algorithm described in [9] is well-suited for computer based implementations where memory requirements are not important. The adaptation presented in [10] is better suited for memory limited devices (embedded measurement devices) but requires more time.

2.3. Spectral leakage-compensated method

In a spectrum without spectral leakage, each component is not influenced/affected by other frequency components. With leakage, each component can influence its neighbor components. Spectral analysis with leakage compensation [11] is a method that estimates the amplitude and phase of each harmonic. The contribution of this harmonic is then subtracted from the original signal spectrum. In each iteration, a frequency estimation of the highest harmonic amplitude is required and the process is repeated until the contribution of all harmonics are estimated and removed.

The full DFT contribution of each component with or without leakage is

\[
P_t = -\frac{j}{2} \sum_{k=1}^{N} A_h \left\{ \exp[j(a(\lambda_h - i) + \phi_h)] \frac{\sin[\pi(\lambda_h - i)/N]}{\sin[\pi(\lambda_h - i)/N]} - \exp[-j(a(\lambda_h + i) + \phi_h)] \frac{\sin[\pi(\lambda_h + i)/N]}{\sin[\pi(\lambda_h + i)/N]} \right\}
\]

where \( \lambda_h = f_h / \Delta f \) is the harmonic \( h \) DFT bin (non-integer when leakage occurs), \( f_h \) is the harmonic frequency, \( \Delta f \) is the DFT frequency resolution and \( \phi_h \) is the harmonic phase.

3. Performance comparison

The required memory for the grouping method is \( 3N / 2 \) where \( N \) is the number of acquired/processed samples from the 200 ms record. This value derives from the memory space for the signal DFT and its single sided spectrum. The memory required by the multiharmonic algorithm is \( 2NH + 7N / 2 + 4H^2 + 10H + 6 \) [10] where \( H \) is the number of considered harmonics to compute. In the multiharmonic algorithm, the \( 2NH \) component has a high influence in the memory requirement. In the efficient multiharmonic algorithm this memory dependence is reduced. With the efficient algorithm the memory requirement is \( 3N / 2 + 4H^2 + 10H + 6 \) [10] while in the spectral leakage-compensated method it is \( 9N / 2 + 2H + 44 \).

Figure 1 shows that, for a signal with 100 000 samples, the efficient multiharmonic and grouping methods are the ones that require less memory (almost identical). The method that is most dependent on the number of harmonics is the multiharmonic method [9]. In Figure 2, the number of harmonics is fixed and the results are shown as a function of the number of samples. Grouping is always the method that requires lower memory. With a higher number of samples, the memory required by the efficient multiharmonic method tends to the grouping required memory due to its similar dependence of \( N \).
Considering a nominal frequency \( f_0 \) of 50 Hz, oscillations up to ±5 Hz are possible in the electrical power grid frequency. In addition, there are multiple harmonics with different phases \( \phi_h \) and amplitudes \( A_h \), at multiples of \( f_0 \) and gaussian noise \( n \). To compare the relative time consumption of each method, and also their error/uncertainty a signal described by

\[
x(t) = \sum_{h} A_h \sin(2\pi f_0 t + \phi_h) + n(t)
\]  

with five harmonics \( A_1 = 1 \text{V}, A_2 = 0.5 \text{V}, A_3 = 0.25 \text{V}, A_4 = 0.1 \text{V}, A_5 = 0.05 \text{V} \), gaussian RMS noise of 0.01 V, and \( N = 200 \, 000 \) samples is used.

Figure 3 presents the expanded uncertainty intervals of 1 000 generated signals without spectral leakage (fundamental frequency of 50 Hz) with random phases. Figure 4 shows the results for the same situation but with 52.5 Hz fundamental frequency that results in a spectrum with leakage. Without spectral leakage the methods have similar uncertainty intervals. However, with leakage, the results obtained with grouping have higher uncertainties.

The multiharmonic algorithm has lower uncertainties than the grouping and leakage compensated methods. The grouping method is the fastest overall method and is used as a reference time. To compute five harmonics, the matrix based multiharmonic method has a normalized time of 10 while the leakage compensated method is 40 and the efficient multiharmonic method is 8000.

4. Conclusions

Grouping is the standard-advised method for harmonic estimation and with respect to memory requirements and time consumption it is the advisable option. However, when compared with the other tested methods, the uncertainties are higher in the presence of spectral leakage. The multiharmonic methods have lower uncertainties but the non-efficient algorithm requires more memory due to its number of samples and harmonics dependence and the efficient version has a higher computation time. The leakage-compensated method has lower uncertainty than the standard-advised grouping method, in particular when spectral leakage occurs.
5. References

[1] IEEE Std. 1159-2009 2009 IEEE recommended practice for monitoring electric power quality DOI: 10.1109/IEEESTD.2009.5154067.
[2] IEC 61000-4-7:2002+AMD1:2008 CSV 2008 Electromagnetic compatibility (EMC) - Part 4-7: Testing and measurement techniques - General guide on harmonics and interharmonics measurement and instrumentation, for power supply systems and equipment connected thereto ISBN: 978-2-88910-377-5.
[3] IEC 61000-4-11:2004+AMD1:2017 2017 Electromagnetic compatibility (EMC) - Part 4-11: Testing and measurement techniques - Voltage dips, short interruptions and voltage variations immunity tests ISBN: 978-2-8322-4402-9.
[4] IEC 61000-4-15:2010 RLV 2010 Electromagnetic compatibility (EMC) - Part 4-15: Testing and measurement techniques - Flickermeter - Functional and design specifications ISBN: 978-2-88912-076-5.
[5] IEC 61000-4-30:2015 RLV 2015 Electromagnetic compatibility (EMC) - Part 4-30: Testing and measurement techniques - Power quality measurement methods ISBN: 978-2-8322-2324-6.
[6] Arrillaga J, Watson N R and Chen S 2000 Power System Quality Assessment (John Wiley & Sons) ISBN: 978-0-471-98865-6
[7] Arrillaga J, Smith B C, Watson N R and Wood A R 1997 Power System Harmonic Analysis (John Wiley & Sons). ISBN: 978-0-471-97548-9.
[8] Fuchs E F and Masoum M A S 2008 Power Quality in Power Systems and Electrical Machines (Academic Press) ISBN: 978-0-12-369536-9.
[9] Ramos P M, Silva M F, Martins R C and Serra A C 2006 Simulation and experimental results of multiharmonic least-squares fitting algorithms applied to periodic signals IEEE Trans. Instrum. and Meas. 56 646-651 DOI: 10.1109/TIM.2006.864260
[10] Xavier P E, Janeiro F M and Ramos P M 2011 Efficient implementation of multiharmonic least-squares fitting algorithms. Proc. IMEKO TC4 Symposium (Natal: Brazil)
[11] Renders H, Schoukens J and Vilain G 1984 High-accuracy spectrum analysis of sampled discrete frequency signals by analytical leakage compensation IEEE Trans. Instrum. and Meas. 33 287-292 DOI: 10.1109/TIM.1984.4315226

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