UX Ori Stars Eclipses by Large-Scale Disc Humps

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ABSTRACT

The eclipses of UX Ori type stars by large-scale disc humps are studied in detail. The influence of the hump extension on the eclipse depth and parameters of the linear polarization is modelled and compared with a compact cloud eclipse model. Eclipses were investigated both for the flared disc and for the disc with a puffing-up in the dust sublimation zone. It is shown that the large-scale hump eclipse may be significantly deeper than a compact cloud eclipse and show a greater linear polarization degree compared to it. It is also demonstrated that the hump extension together with the disc puffing-up can strongly affect the degree of polarization and colour index of the star during the eclipse. The position angle of the linear polarization may also change markedly during and after an eclipse by a large scale hump for the model with a puffed-up inner rim. Also, in this model, the maximum degree of the linear polarization can be achieved not at the brightness minimum, but closer to the end of the eclipse.

Key words: radiative transfer – stars: variables: T Tauri, Herbig Ae/Be – circumstellar matter — polarization

1 INTRODUCTION

Herbig Ae/Be stars are intermediate mass pre-main sequence stars (Herbig 1960). Among them, variable UX Ori stars are distinguished into a separate subclass. UX Ori stars demonstrate photopolarimetric variability with deep sporadic fades up to 2–3 magnitudes and duration from several days to months. Such fades are usually accompanied by a rise in the linear polarization degree up to 5–8% (Grinin et al. 1991). Selective absorption leads to star colour changes during the eclipse. When the star fades it usually turns red first and then turns blue in the deep minimum. Sometimes the position angle of the linear polarization also may change. It may be explained by the increased contribution of the interstellar polarization with a different position angle compared to intrinsic polarization.

Such photopolarimetric changes are explained by the increasing contribution of the polarized disc radiation to the system total light, while the star is eclipsed by gas-dust clumps in the disc atmosphere. Grinin (1988) proposed the eclipse model in which dust clouds obscuring the star are presumed to be sufficiently compact, and they do not affect significantly the circumstellar disc scattered light. Consequently, the polarization parameters of the scattered disc radiation do not change during an eclipse. This model explains the limitation on the eclipses depth, polarization degree increase and the changes in the colour indices during the eclipses.

Based on the high linear polarization observed at deep brightness minima, it was suggested that the photometric activity of UX Ori type stars is caused by the small inclination of circumstellar discs to the line of sight. The interferometric observations of these stars in the near-infrared wavelengths generally support this hypothesis (see Kreplin et al. 2016, and references there). Exceptions are T Tauri stars demonstrated the extinction events (Ansdell et al. 2020). Due to their low luminosity, the circumstellar dust can survive in their nearest neighborhood and can penetrate the magnitosphere of the star (Nagel & Bouvier 2020), which makes extinction events less sensitive to the inclination of their discs to the line of sight.

In various papers, the eclipses were studied within this model. In general, a good agreement with observations was obtained. In the very first studies, the scattering medium had an elliptical geometry (e.g. Voshchinnikov 1989), later the flared disc model was used (Whitney & Hartmann 1992; Natta & Whitney 2000).

Natta et al. (2001) showed the flared disc model to be unable to explain the near-infrared excesses of Herbig Ae/Be stars. A new model with a disc puffed up in the inner region (in the dust evaporation zone) was introduced. Such a model is recognized as the most in line with reality now. Nevertheless, various possible mechanisms of the puffing-up are considered now (for a review, see Dullemond & Monnier 2010; Turner et al. 2014).

Originally the heating of the inner disc region by stellar radiation was considered as a cause of the disc puffing-up. Later Vinković & Jurkić (2007) shown that radiational heating is not enough to explain infrared excesses and noticed that disc wind may raise dust from the disc surface and lead to disc thickening. Tambovtseva & Grinin (2008) demonstrated that the dust particles survive in the disc wind, despite the gas component high temperature. Thereby the dust could make a significant contribution to the circumstellar extinction, particularly in T Tauri stars. Bans & Königl (2012) used the centrifugally driven winds (Safier 1993a,b) to model infrared excess of the stars. They showed that the dusty disc wind may have a significant contribution to the infrared excess.

In one of our later papers, we considered the conservative compact dust cloud model of the eclipses for the star with a puffed-up disc (Shulman & Grinin 2019a). We also used Safier (1993a,b) wind models and obtained new results that did not fit into the disc model without the puffed-up inner boundary:

(i) The position angle changing by 90° when changing the wave-
length. Some young stars show such changes when switching between optical and near-infrared spectral ranges (Pereyra et al. 2009).

(ii) With a certain combination of the disc wind parameters and the radiation wavelength the polarization degree may not change or change in a narrow range during an eclipse. Rostopchina-Shakhovskaja et al. (2012) observed such eclipses of the UX Ori star WW Vul.

(iii) The disc puffing-up may lead to unusual colour changes during the eclipse. There may not be the star reddening during the fading at all. The colour indices immediately begin to decrease. In our model, it is easier to get this behaviour in the blue region of the optical spectrum rather than in the red one. Such changes of the UX Ori colour indices were observed from the IUE satellite (Grady et al. 1995).

These results show that the dust component of the disc wind can be very important for the intrinsic polarization of young stars. Blandford & Payne (1982) showed that the wind density is strongly tied with the accretion rate. Therefore the intrinsic polarization of young stars must also depend on the accretion rate. As a result, the accretion rate fluctuations may lead to polarization variability. Note that the large scatter of polarization parameters at the same brightness was observed in some of the UX Ori stars (e.g., Rostopchina et al. 2000).

The disc puffing-up can explain some unusual polarization parameters changes during the eclipses. Nonetheless strong polarization parameters changes may also occur shortly after the eclipse. Significant changes in the linear polarization position angle soon after the deep photometric minimum have been observed in UX Ori (Grinin 1994) and WW Vul (Grinin et al. 1988). These eclipses have lasted for a few months. This is comparable in time to the Keplerian rotation period of the UX Ori type star disc on the dust evaporation radius.

In our paper (Shulman & Grinin 2019b) we calculated such eclipses for the first time for a flared disc model and a disc with the puffing-up in the inner region. We used as a screen a simple 1-D model of the disc perturbation. We received that in the model with a puffed-up disc, the position angle changes may reach 60°, which is comparable to the observational results.

In the present paper, we consider a more realistic elongated in azimuth disc perturbation and study other features of the eclipses by disc perturbations with different sizes. Thus this paper may be viewed as a continuation and a generalization of our previous studies. Its results can be used for the modelling of the very deep and long-lasting minima of UX Ori stars similar to that observed recently by Belan and Shakhovskoy (in preparation, see also The 2nd international Workshop ‘UX Ori type stars and related topics’ materials).

We will leave the nature of large-scale disc perturbations outside the scope of this work. We just note that different physical process may give large disc perturbations. Among them, there are disc atmosphere vortices (see, e.g., Surville & Barge 2015; Barge et al. 2017), charged dust grains rising above the disc due to magnetorotational turbulence (Turner et al. 2014), and accretion and disc wind instability and asymmetry.

2 METHOD OF CALCULATIONS

A lot of different numerical methods may be used to model scattered light intensities and polarization. Some of the methods are semi-analytical, consider only single scatterings or have high computational complexity in three-dimensional cases. The Monte-Carlo method is rather simple in the implementation, does not require analytical integration, does not have a limitation of the number of considered scatterings and works fine in complex three-dimensional studies. As a result, the Monte-Carlo method is the most popular choice for radiation transfer simulations.

The Monte-Carlo method is based on calculating the photon packets propagating in random directions. A photon packet propagates in a random direction for a random optical depth. After that, it is scattered in another random direction. The procedure may be repeated for a desired number of scatterings.

One of us (Shulman 2018) proposed to calculate photon packets propagating into uniformly distributed directions instead of random ones and scatter them after propagating several precalculated optical depth in every direction. It was shown, that such a technique leads to the same results but significantly faster computationally. It can be shown that from the mathematical point of view it is the integral calculation in terms of the Riemann sums. A three-dimensional space partition is a combination of a two-dimensional partition of the unit sphere for propagation direction and a one-dimensional partition by optical depth along each direction. Within the scope of our task, we considered the star to be a point source of radiation.

The disc mass density distribution, which is required for fast optical depth calculation, was approximated with an unstructured tetrahedron grid, based on Delaunay’s triangulation. To obtain such a grid gmsh program was used (Geuzaine & Remacle 2009). This kind of grids allows elements that vary greatly in size. As a result, we can use small elements in the central region of the disc, where high-density gradients are, and large ones at the disc periphery with low-density gradients. Thus, this way we can more closely model more significant and complex areas, while large simple areas can be modelled relatively quickly. Moreover, tetrahedron grid-based continuous function interpolation is also continuous, which is also well for numerical simulation.

In this paper, we limited ourselves to taking into account the contribution of the first four scattering. In the appendix A, we consider the contribution of multiple scattering using the model from the section 4.1 as an example.

3 DISC MODEL

We consider a model of a single early spectral type A star with a flared circumstellar disc. The star radius is $R_\ast = 2.9R_\odot$, and mass is $M_\ast = 2M_\odot$. A disc wind is used to produce a puffing-up in the inner region of the disc, in the dust evaporation zone. We study the eclipses of the star with a large-scale disc perturbation.

3.1 Disc

Following previous studies (e.g., Shulman & Grinin 2019a,b; Teixeira et al. 2009; Robitaille 2011) we use the following flared disc form:

$$\rho(x, y, z) = \begin{cases} \rho_0 \left(\frac{R_0}{r}\right)^\alpha \exp \left[ -\frac{1}{2} \left(\frac{r}{h(r)}\right)^2 \right], & R_{in} \leq r \leq R_{out}, \\ 0, & \text{otherwise} \end{cases}$$

where $r = \sqrt{x^2 + y^2}$ is the radius in the disc plane,

$$h(r) = h_0 \left(\frac{r}{R_0}\right)^\beta$$

is the scale height. Other values are disc model parameters.

In this paper we use the same values as in Natta & Whitney (2000) and Shulman & Grinin (2019a): $\alpha = 2.79$ is the radial density exponent, $\beta = 1.29$ is the flaring power, $R_{in} = 4R_\ast$ and $R_{out} = 100AU$.
are the inner and outer disc radii respectively. The disc thickness at radius $R_0 = R_*$ is taken to be $h_0 = 0.008 R_*$. The density normalization constant $\rho_0$ is obtained from the disc mass, which is $0.1 M_\odot$.

### 3.2 Disc hump

We use a disc perturbation model based on two Gaussian functions determining the hump shape by distance from the star and azimuth. This shape resembles the vortex structures (cyclones and anticyclones) predicted by the gas-dynamic models of protoplanetary discs (see, e.g., Godon & Livio 2000; Wolf & Klahr 2002). Similar geometry of the hump was previously used in O’Sullivan et al. (2005) for AA Tau photopolarimetry modelling. In our previous work (Shulman & Grinin 2019b) we used a simpler hump model, determining its shape only by distance from the hump centre.

Below the hump is described in the polar coordinate $(r = \sqrt{x^2 + y^2}$, $\phi = \arctan(y/x))$ system with the equation:

$$h_{\text{hump}}(\phi, r) = h_{\text{hump}_0} \exp \left[ \frac{1}{2} \left( \frac{\phi - \phi_0}{\Delta \phi} \right)^2 \right] \exp \left[ \frac{1}{2} \left( \frac{r - r_0}{\Delta r} \right)^2 \right].$$

(3)

where $h_{\text{hump}_0}$ is a relative hump height in the units of $h$, $\phi_0$ and $r_0$ are polar coordinates of the hump centre. $\phi_0$ and the longitude of the observer $\phi_{\text{obs}}$ together determine the relative position of the hump and the observer. As a result, we can always assume $\phi_0 = 0$ and change only $\phi_{\text{obs}}$, $\Delta \phi$ and $\Delta r$ characterize the hump extension along azimuth and radius.

For the height of the disc with a hump we use the following equation

$$h(\phi, r) = h_0 \left( \frac{r}{R_0} \right)^\beta \left( 1 + h_{\text{hump}}(\phi, r) \right).$$

(4)

instead of (2). Thus, the hump increases the disc height and may lead to star eclipses.

Fig. 1 shows the effective height and the image of the disc with a hump observed from the pole. By the effective height, we mean the height at which the optical thickness of the disc, when approaching its plane, reaches 1. The hump parameters are: $h_{\text{hump}_0} = 3$, $\Delta \phi = 0.6$ and $\Delta r = 0.3$ AU.

### Table 1. Parameters of the disc wind models.

| Model | $\kappa_w$ | $\lambda_w$ | $\xi_0$ |
|-------|-----------|-------------|--------|
| C     | 0.01      | 75.43       | 1.73   |
| E     | 0.10      | 25.63       | 3.73   |
| G     | 0.01      | 189.34      | 3.73   |

In the current paper we continue to use the disc wind models from Safier (1993a,b), as in our previous two.

In these wind models, three parameters define the radially self-similar wind solution. The first parameter is $\kappa_w$, the normalized mass-to-magnetic flux ratio. It describes the wind mass loading. The second parameter is $\lambda_w$, the normalized total specific angular momentum. $\lambda_w$ characterizes the efficiency of the angular momentum transport of the wind. The third one, $\xi_0 = \tan \theta_0$, measures the initial inclination of the magnetic field lines. Where $\theta_0$ is the angle between the poloidal field component and the disc normal at the disc surface. These parameters for three wind models used in our paper are listed in table 1.

The model C disc wind rises from the disc at a large angle to the disc plane compared to models E and G. As a result, the disc puffing-up with model C wind is more noticeable. The mass loading in model E is higher than in model G, so the wind model G provides the less significant puffing-up among the three considered models.

The disc wind density is described by the expression

$$\rho_{\text{wind}}(r, z) = \rho_{\text{wind}_0} \left( \frac{r}{r_0} \right)^{-3/2} \eta(z/r).$$

(5)

Here, the function $\eta(z/r)$ is derived from the solution to the gas-
dynamic equations. Safier (1993b) provides approximations for this function for several disc wind models, including models from table 1. \( \rho_0 \) is the wind density in g cm\(^{-3} \) on the disc surface at distance 1 AU from the star. It can be expressed through the following parameters: the mass outflow rate in solar masses per year \( M_{\text{out}} \), the stellar mass in solar masses \( M_* \), the inner and outer radii of the wind formation region \( r_{\text{min}} \) and \( r_{\text{max}} \), the ratio of the vertical speed to the Keplerian speed at the disc surface \( \psi_0 \), and the dimensionless height from which the wind begins \( h_0 \):

\[
\rho_{\text{wind}} = 1.064 \times 10^{-15} \frac{M_{\text{out}}}{10^{-7} M_\odot \text{yr}^{-1}} \left( \frac{M_*}{0.5 M_\odot} \right)^{-0.5} \times \\
\ln \left( \frac{r_{\text{max}}}{r_{\text{min}}} \right) \psi_0 \left( 1 - h_{\text{wind}} \zeta_0^2 \right).
\]

(6)

The approximation for \( \psi_0 \) is also available in Safier (1993b). The density is not very sensitive to \( r_{\text{min}} \) and \( r_{\text{max}} \) radii, so we can use \( r_{\text{max}}/r_{\text{min}} \sim 20 \) (Bans & Königl 2012). Cauley & Johns-Krull (2015) estimate the accretion rate in Herbig Ae/Be stars from 10\(^{-6} \) to 10\(^{-5} \) \( M_\odot \) yr\(^{-1} \). It is usually estimated that the mass outflow rate is one order less than the accretion rate. Bans & Königl (2012) accept the mass outflow rate as 1–5\% of the accretion rate. Finally, \( h_{\text{wind}} \) is defined from the disc geometry.

As a result, the mass outflow rate and the disc wind model are two parameters defining the disc wind for the considered star with the flared disc.

When we consider the disc with a hump and the disc wind together, we apply the hump to the disc and use \( \max[\rho(x, y, z), \rho_{\text{wind}}(r, z)] \) as the resulting density of the puffed-up disc with the hump.

### 3.4 Dust properties

For the dust scattering computation, we used the Henyey-Greenstein phase function with the approximations for the polarization functions (White 1979). The absorption coefficient \( \kappa \), the single-scattering albedo \( \omega \), and the peak linear polarization \( p_l \) were taken from Natta & Whitney (2000).

The phase function asymmetry parameter \( g \) was taken from Kim et al. (1994). We neglect circular polarization and assume the peak circular polarization equal to 0. Following White (1979) the skew factor was taken equal to 1. For simplicity, the dust properties were assumed to be constant for the entire model. Table 2 presents the dust properties for five photometric bands.

### 4 RESULTS FOR DEEPEST ECLIPSE POINTS

In this section, we consider the eclipses by large scale humps and compare them with the eclipses by a compact dust cloud. For each hump, we consider the deepest point of the eclipse, when the hump centre is between the star and the observer (\( \phi_{\text{obs}} = 0 \)). We start from the several eclipse examples for the disc without a disc wind, demonstrate multiple scatterings contribution, and study the disc wind influence on the eclipses. After that, we examine colour indices for different eclipses and briefly discuss the eclipses of the star with a different disc model.

To eclipse a star with a disc hump, the observer must look along the edge of the disc. Therefore, a significantly smaller range of values is available for the inclination angle \( i \) between the observer and the disc axis. If this angle is too large, we will see the system from the edge, and the disc will cover the star. On the contrary, if \( i \) is too small, the disc hump will not be able to intersect the line of sight. For the disc model from the section 3.1, we take the angle between the direction to the observer and the disc axis equal to 65\°. This makes it possible to obtain eclipses by a disc hump with not too high relative height.

#### 4.1 Eclipses with large scale humps

In our work, the disc hump is described with four parameters. Showing the dependence on all parameters at once leads to an over-complication of the figure. Therefore, we will consider these dependencies separately. The relative hump height \( (h_{\text{rel}}) \) and the hump extension along azimuth \( (\Delta \phi) \) determine which parts of the disc will be eclipsed. Together with the hump extension along the radius \( (\Delta r) \) the relative hump height also defines the optical depth along the line of sight. The polar radius of the hump centre \( r_{0} \) affects whether the side regions farther from the star are eclipsed.

Here we start from dependency on the relative hump height and the hump extension along the azimuth. An increase in the relative hump height leads to deeper eclipses, while the change in hump extension along the azimuth gives the maximum variation in polarization depending on magnitude changes. Fig. 2 demonstrates points corresponding to maximum magnitude changes \( (\Delta V) \) and degrees of polarization \( (\rho) \) in the minima of the eclipses with various humps. For all humps on this figure \( r_{0} = 0.5 \text{ AU} \) and \( \Delta r = 0.3 \text{ AU} \). \( \Delta \phi \) was changed from 0.03 to 0.6. \( h_{\text{rel}} \) was varied from 0 to 5.

It should be noted that the eclipse depth varies greatly with small
changes in the hump relative height. At first, the hump height is not enough for any noticeable star eclipse. Then, with small changes in the hump height, the star quickly weakens. Finally, the star is completely eclipsed, and, therefore, a further increase in the hump height leads only to a weakening of the brightness of some areas of the disc. As a result, the depth of the eclipse increases slightly. Thus, most of the results in Fig. 2 were obtained with hump heights from 2.3 to 2.8.

Additionally, we show three line types in Fig. 2: the thick line is the graph of the compact cloud eclipse, thin lines connect points with equal \( h_{hump} p_0 \) and \( \Delta \phi \). We call these lines isoisheights and isoextensions respectively.

In general, isoextensions go along the compact cloud eclipse track (see Fig. 2 for the illustration). With higher humps, we obtain deeper minima with higher polarization degrees. Deeper minima are achieved since the inner areas of the disc are also eclipsed together with a star. The large changes in the polarization degree are associated with different orientations of the scattered light polarization. Behind the star, the scattered disc radiation is polarized along the disc plane while the lateral disc parts polarization is perpendicular to the disc plane. A geometrically thin disc is polarized perpendicular to its plane, the disc humps eclipse the region behind the star stronger than the lateral areas. Hence, the total polarization degree of the system increases.

Isoheights go across the compact cloud track. With narrow humps, we obtain results close to the compact cloud eclipse. With a wide hump, we get deeper eclipses since the star fading is the same but large areas of the disc are also eclipsed. The polarization degree changes in a difficult way depending on the disc and hump geometry. In the case of a low hump, the polarization decreases due to lateral disc areas eclipse. When the disc height is large enough for significant magnitude changes, with an increase in the hump extension, the polarization first increases and then decreases. Usually, at the same fading level the polarization in the model with a large scale hump is lower than in the conservative model, but sometimes it may be slightly higher.

We should pay attention to two important results: the maximum eclipse depth and the polarization degree in the minimum are significantly greater compared to the compact cloud model. We achieve \( \Delta V \) about 3.77\(^{m} \) instead of 2.66\(^{m} \) and polarization degree about 14\% instead of 8.5\%. Finally, we can see the change in the nature of the dependence of \( p \) on \( \Delta V \): near the minimum, the polarization degree almost stops growing and may start to decrease. This effect is explained by the total star eclipse, due to which the contribution of the disc to the system radiation can not increase anymore. As a result, all changes occur only because the hump unevenly eclipses different parts of the disc with different linear polarization orientations.

The polar radius of the war centre \( (r_0) \) affects the eclipse in two ways. Firstly, with larger \( r_0 \) and the same hump extension along the azimuth, the hump can eclipse wider areas of the disc. So the eclipse depth will increase. The eclipsed lateral disc regions are polarized orthogonal to the disc plane. Therefore, when considering a thin disc, the polarization degree will slightly decrease.

Secondly, since we are considering a flared disc, increasing the distance to the hump centre results in a larger effective hump height at a fixed relative height of \( h_0 \). At first, the same \( h_0 \) is not enough to eclipse the star, but with larger \( r_0 \) the star can be completely eclipsed. In Fig. 3 the magnitude changes and polarization degree dependencies on the hump centre radius are shown for four hump models. All other hump parameters are fixed for each model.

Different \( r_0 \) leads to small changes in the linear polarization degree corresponding to the fixed eclipse depth compared to the azimuthal extension.

The hump radial extension \( \Delta r \) has even less impact on the eclipse. The dependence on the radial extent turns out to be simpler than the dependence on the \( r_0 \). In Fig. 4 we present the magnitude changes and polarization degree dependencies on the radial extension for four hump models. A hump with a large radial extension stronger eclipses the star and the disc, which leads to deeper minima with a large polarization degree. Gradually, the radial extension becomes large enough so its further increase leads only to a slight increase in the eclipse depth and small changes in the polarization degree.

We conclude this section by noting that the dependencies on the hump centre radius and the radial extension turned out to be less interesting than the dependence on the azimuthal extension. Also, they are not so convenient for increasing the eclipse depth as the dependence on the relative hump height. Consequently, in the following sections, we will use Fig. 2 analogues with isoisheights and azimuthal
Figure 5. The same as in Fig. 2 for the disc with the disc wind. C disc wind model is used. The mass outflow rates are signed in the units of $10^{-9} M_\odot$ year$^{-1}$.

isoextensions to discuss the large-scale disc hump influence on the star eclipses.

4.2 Disc wind influence

The numerical modelling showed that the disc wind influence the eclipses with large scale humps is very similar to the case of the compact cloud eclipse Shulman & Grinin (2019a). In this section, we will present a brief demonstration of the wind influence on the eclipses. It generalizes our previous results to a more complex model of the eclipsing screen.

In Fig. 5 we show eclipses obtained with C wind model and different mass outflow rates. The observer position and the parameters of all humps are the same as in Fig. 2. The considered mass outflow rates are $0, 2 \cdot 10^{-9}, 5 \cdot 10^{-9}, 10^{-8}$, and $1.5 \cdot 10^{-8} M_\odot$ year$^{-1}$. The results with 0 mass outflow rate are the same as in Fig. 2.

With an increase in the mass outflow rate, the eclipses become less deep, as the starlight is partly absorbed in the wind. Linear polarization degree changes are more complicated: when the wind density is low, the wind increases the polarization degree on the same level fadeings; with higher wind density the polarization degree decreases to zero; finally, with a further increase in the mass outflow rate, the degree of polarization begins to grow. The nature of this behaviour is the fact that the wind thickens the disc, therefore the orientation of the linear polarization changes. For a thin disc with a light wind, it is perpendicular to the disc plane. Otherwise, for a thick disc with a dense wind, the linear polarization is parallel to the disc plane. Some isoextensions on the plot reach zero polarization. On such isoextensions there are eclipses with different linear polarization orientations. When the disc is in the borderline state between thick and thin ones, large scale hump, eclipsing various disc areas, may have a major influence on the linear polarization orientation.

Fig. 6 shows in detail the changes in the eclipse parameters with a change in the mass outflow rate and, therefore, the wind density. This figure is very similar to the graphs from our paper (Shulman & Grinin 2019a), only we replaced the graphs for compact cloud eclipses by isoextensions, as the closest to them in behaviour and meaning. Only isoextensions for the minimum and maximum mass outflow rate are shown in the figure. Intermediate ones run along with them to the constant hump shape line. We can conclude that, in general, the disc wind has the same effect on eclipse by a large-scale disc hump as in the conservative model: it makes possible an eclipse without an increase in the linear polarization degree and the position angle changing by 90° when changing the wavelength. Additionally, it increases the possible extension of the linear polarization degree at a fixed fading.

When the disc wind density increases we can see one more interesting result: the scatter of polarization parameters at one brightness level strongly increases for deep eclipses. This is because the star is almost completely hidden from the observer by the disc wind and...
Figure 8. Changes in colour indices during eclipses for discs with different wind densities. The mass outflow rate in units of $10^{-9} M_\odot$ year$^{-1}$ is signed on the graphs. Thick lines show colour indices for the compact cloud model, dots show colour indices for large scale humps, and thin lines are for isoheights and isoextensions.

As a result, polarization does not increase due to an increase in the contribution of the scattered by the disc radiation. On the contrary, the most important thing turns out to be which parts of the disc are eclipsed. So, the humps with different extensions give us eclipses with very different polarization degrees. In this model, the mass outflow rate when the polarization orientation changes depends not only on the wavelength, disc model, and the observer position, as in the compact cloud case but also on the hump shape.

As was mentioned in our paper (Shulman & Grinin 2019a), the eclipse depth and other parameters are closely connected with the wind model. In Fig. 7 we again show the results for the same observer position and the parameters of the humps. But this time the mass outflow rate is always $10^{-9} M_\odot$ year$^{-1}$, and the wind models are C, G, and E.

One can see that the wind model choice may give a difference similar to changing mass outflow rate several times. So, it is rather tricky to estimate the mass outflow rate based on the eclipse shape. At the same time, we should conclude that the wind may strongly influence the eclipse and lead to new phenomena. The disc wind is only one of possible reasons for the disc puffing-up in the dust sublimation zone. The conclusions above should be generally true for any puffing-up in this zone regardless of its physical nature.

4.3 Colour indices

Above we considered the eclipses in the V band. The influence of large-scale disc hump on the eclipse parameters in different spectral bands is approximately the same. In all bands, we obtain deeper minima with higher polarization degrees. The large-scale hump eclipses reach almost $4^m$, while the depth of the compact cloud eclipse is about $2.6^m$. The linear polarization degree differs from band to band. The compact cloud eclipse model gives us maximal linear polarization degree equal to 7% in the U band and 13% in the I band. The current model increases maximal polarization degree to 12% and 18% in U and I bands respectively.

In the conservative model, we obtain colour changes due to selective absorption. The colour indices increase during the star fading and then decrease near the photometric minimum. This is called ‘blueing effect’. A large hump also eclipses the inner regions of the disc. As a result, we obtain larger colour indices on the same fading level. Moreover, in the deep minimum, the behaviour changes, when the star is fully eclipsed and colours change due to the inner disc regions eclipse.

In Fig. 8 colour indices for the results from Fig. 5 are presented. Only the data with $M_{\text{out}} = 1.5 \cdot 10^{-8} M_\odot$ year$^{-1}$ is omitted as ΔV was too low. The compact cloud model colour indices are shown with the thick lines. It is interesting, that the eclipse of the inner regions of the disc turns out to be more significant for colour indices than for the polarization degree. On the figures with the polarization degree versus the magnitude change the results for a hump with a small azimuth extension almost coincided with the result of the conservative model. On the contrary, in Fig. 8 there are noticeable gaps between compact cloud model results and the results of the model with the large-scale hump.

We have already obtained earlier in the compact cloud eclipse model that the disc wind instability increases the possible variation of the colour indices at the same fading level. Now we can see that the ability to change the size and shape of the absorbent screen also provides such an effect. Together, they allow explaining very strong scatter in the colour indices during different eclipses of the same star.

4.4 Different disc models and observer positions

The above results are presented for only one disc model and one observer position. Of course, the choice of these parameters also affects the result.

We have already mentioned above that the choice of the angle between the disc axis and the direction towards the observer is strongly limited by the fact that the star should be clearly visible before an eclipse, and the deep minimum must be achieved at not too large disc height. A variation of the observer’s position in this range changes the eclipse depth and the maximum degree of polarization. Nonetheless, the hump shape and disc wind influence the eclipse in the same manner. Also, there are some minor changes in secondary parameters of eclipses, such as the mass outflow rate, at which the linear polarization orientation rotates. These changes are of no fundamental importance and can be omitted in the current paper.

The parameters of the disc also affect the characteristics of the
It is worth noting that in our model the hump height $h$ is considered relative to the flared disc without any puffing-up. Accordingly, when the puffing-up is significant, the actual hump height is smaller than $h$. In the previous sections, the results of our models were weakly sensitive to the maximum considered $h$ value. For significant deviations of the positional angle, which we will study in this section, large disc perturbations are needed. This is the reason to increase the maximum considered value $h$ from 5 to 6 in this section.

The possibility of varying several parameters at once allows us to consider many models of eclipses. Nevertheless, we restrict ourselves to a relatively small number of them, allowing us to get an idea of the possible effects. Below, we will omit the change in $\Delta r$, which mainly affects the optical thickness of the hump, and $r_0$ since it complicates the result comparison and adds the need to choose a different $h$ value due to the flared disc shape. However, larger $r_0$ allows changing wide areas of the disc and enhance some obtained effects.

Fig. 9 shows the dependence of the radiation parameters (magnitude changes, the polarization degree and the position angle) depending on the observer longitude $\phi_{\text{obs}}$ for a disc without puffing-up in the dust sublimation zone. There is nothing unusual about these eclipses: the maximum degree of linear polarization is achieved at the minimum brightness. With the increasing brightness of the star, the degree of polarization decreases. The linear polarization positional angle varies within five degrees in both directions.

The observed changes in the position angle after passing through the minimum distinguish the presented eclipses from the compact gas-dust cloud model, but we can hardly consider these changes significant. The reason for these changes is that the disc hump violates the disc symmetry. The large-scale hump influences the polarized scattered radiation of different disc areas: the hump can both additionally eclipse some areas of the disc and increase the scattered radiation. Therefore, its effect on the position angle can be different and depends on the geometric properties of the disc, the hump and the observer position.

The observed behaviour weakly depends on the parameters of the hump: the depth of the eclipse, the maximum linear polarization degree, the duration of the eclipse phases can change. Minor variations in the position angle can also present, but the shape of the eclipse is about the same.

From the results in Fig. 9, it is difficult to assess how the degree of polarization depends on the fading level. We have shown this dependence in Fig. 10 for the five humps with different $\Delta \phi$. For comparison, the conservative model eclipse is also presented in it. The
lines of eclipses by moving large-scale disc humps go along the compact cloud eclipse line and isosextensions in Fig. 2. For each $\Delta \phi$ the resulting graph differs from the corresponding isosextension, because the hump has a different shape and eclipses the disc differently.

A largely displaced disc hump weakens the polarized scattered disc radiation more strongly than a small hump with the centre exactly on the line of sight. As a result, the considered eclipses have a lower polarization degree than isospreads on the same fading level, and differ even more from the conservative model eclipses. Thus, the shape of the eclipse from sec. 4.1 (by a hump with the constant position and an increasing height) and the shape of the eclipse from the current section (by moving around the star hump with a constant shape) in general turn out to be the same. However, in the second case, it is possible to achieve even greater parameters scatter on the diagram presenting the linear polarization versus the change in magnitude.

For the disc with a puffing-up in the dust sublimation zone, the eclipses begin to change noticeably. Therefore, we considered several models with a constant hump shape and an increase in the mass outflow rate. As follows from the results presented above, with increasing puffing-up, the depth of eclipses and the polarization degree at the minimum decrease. But the differences don’t stop there.

In Fig. 11 we present results for the constant hump shape and an increasing mass outflow rate, which defines the disc puffing-up. With an increase in the mass outflow rate, we can pay attention to the following features on the eclipses: the depth of the eclipse and the polarization degree in the minima decrease; from a certain mass outflow rate, the maximum degree of linear polarization is achieved not at the minimum, but after it, when $\phi_{\text{obs}} \sim 25^\circ$; the amplitude of the position angle deviations increases.

The maximum of the linear polarization degree is shifted from the brightness minimum because the stellar radiation and scattered disc radiation are strongly absorbed by the extended disc hump and the puffing-up. In such a situation the degree of polarization depends not only on the intensity ratio of unpolarized stellar radiation to polarized disc radiation (as in the conservative model), but also on the polarization of the scattered radiation. The disc with a large-scale hump becomes highly asymmetric. As a result, the scattered light polarization can change noticeably, depending on the geometric arrangement of the observer and the perturbation. In our case, the Stokes parameter Q weakly changes when the hump eclipses the star. On the contrary, the Stokes parameter U is zero for the symmetrical geometry (when the hump centre is between the star and the observer) and grows rapidly with increasing $\phi_{\text{obs}}$. Consequently, the disc scattered light polarization increases noticeably. There may also be a slight increase in linear polarization degree when the hump and the observer are on opposite sides of the star.

The mass outflow rate and the disc wind model determine the disc puffing-up in the dust sublimation zone. The greater the mass outflow rate, the thicker the disc becomes. A thicker disc is polarized perpendicularly to its plane weakly than a thin one. Hence large-scale disc humps lead to stronger changes in the linear polarization positional angle. This dependence on the mass outflow rate is retained until the disc becomes thick enough that its scattered light is polarized along the disc plane. Thereafter, a further increase in the mass outflow rate will cause the amplitude of the position angle changes to decrease. The maximum angle changes are possible for an almost unpolarized disc. In Fig. 11 the considered model gives deflections of the position angle up to $60^\circ$ for a thin disc and up to $30^\circ$ for a thick disc. Significant position angle deviations are observed both during the eclipse and after it. In the presented results, the deviations are mainly in one direction.

In order to present the eclipses in more detail in Fig. 12 we show eclipses for all hump models from Fig. 9 for a disc with the model C
disc wind. The mass outflow rate is $10^{-8} M_\odot \text{ year}^{-1}$. With a small hump height, a large deviation of the positional angle is obtained during an eclipse. And with a more significant hump, the essential change in the positional angle can persist for some time after the end of the eclipse. Moreover, in the brightness minimum, there may be no change in the position angle at all or its change by 90°. The last option is observed when the hump has a large azimuthal spread. This is explained by the fact that the extended large-scale disc hump, located between the star and the observer, increases the effective disc puffing-up for the observer. Therefore, a thin disc with a slight puffing-up and a large hump can behave like a thick disc with a strong puffing-up. When the hump shifts around the star, the observer sees a thin disc again.

In our previous paper (Shulman \& Grinin 2019b), round hump did not give us 90° positional angle change during the eclipse. Herewith the maximum deviation of the positional angle was achieved after the eclipse, and not during it, as in the above examples. We have found that even simple models of the disc hump lead to new features of the positional angle behaviour during the eclipse and after it. The positional angle dependence on the observer longitude is rather sensitive to the hump model, model parameters and the puffing-up. Nevertheless, we can confidently conclude that for a star surrounded by the disc with a puffing-up, there might be strong changes in the positional angle. These changes are possible for stars with thin and thick discs. During an eclipse, the changes can reach 90°, and, after the eclipse, they are up to 60°. A star surrounded by a disc without puffing-up can hardly show noticeable changes in the position angle during the considered eclipses.

The disc puffing-up optical depth also depends on the optical properties of the dust and, as a consequence, on the wavelength. In our models, the disc with a puffing-up turns out to be thinner in the I band than in the V band and especially in the U band, as a result, the position angle changes for a thin disc in the I band are less than in the V band, and in the V band they are less than in the U band.

A more complex form of the disc hump opens up additional possibilities for changing the positional angle of linear polarization. In particular, the hump may be asymmetrical and has forward and backward parts with different spreads.

Depending on the optical properties of the dust, the thickening in the central regions of the disc has a different effect on the shape of the eclipse. With the higher dust absorption coefficient and the same wind model, we get a more noticeable puffing-up. As a result, the eclipse will be less deep, and the degree of polarization will be lower. It is possible that in one spectral band the disc will be thick, while in the other it will be thin. Then the position angle in different spectral bands will be different. Fig. 13 demonstrates such behavior for three spectral bands. Consequently, the deviations of the positional angle during the eclipse will be different: in one band there will be deviations up to 60° from the orientation perpendicular to the disc plane (V strip in Fig. 13), and in the other — up to 30° from the position along the disc plane (U strip in Fig. 13).

**Figure 12.** The same as in Fig. 9 for the same hump models and a disc with the disc wind. The wind model C with the mass outflow rate $10^{-8} M_\odot \text{ year}^{-1}$ is used.

**Figure 13.** The same as in Fig. 9 for one scattering geometry model in U, V, and I spectral bands. The hump parameters are $\Delta r = 0.3 \text{ AU}$, $r_0 = 0.5 \text{ AU}$, $h = 6$ and $\Delta \phi = 0.4$. The wind model C with the mass outflow rate $10^{-8} M_\odot \text{ year}^{-1}$ is used.
6 CONCLUSIONS

We studied the eclipses of a star by a large-scale disc hump. We considered both a model of a geometrically thin flared disc and a model of a disc with a puffing-up in the dust sublimation zone. The disc wind (Safler 1993a,b) was used to create the disc puffing-up. It is rather convenient the mechanism of disc thickening in the inner regions, but it is worth remembering that several other physical processes are leading to disc thickening (see, e.g., Turner et al. 2014). We compared the results obtained in the large-scale hump model with the results for the conservative compact gas-dust cloud model (Natta & Whitney 2000; Shulman & Grinin 2019a).

We found that in both cases the disc puffing-up at a certain ratio of the model parameters leads to the following new physical phenomena:

(i) Change in the orientation of the polarization plane by 90 degrees when changing the radiation wavelength.
(ii) No increase in the polarization degree during an eclipse
(iii) Unusual colour changes during the eclipse without the star reddening during the fading at all.
(iv) An increase in the parameters scatter (degree of polarization and colour indices) corresponding to the same fading level.

Regardless of the disc puffing-up in the dust sublimation zone, the model of an eclipse by a large-scale disc hump has important differences from the compact gas-dust cloud eclipse model.

Firstly, in the large-scale hump model we obtain a significant restriction on the observer position: the angle between the direction towards the observer and the disc plane cannot be large so that the line of sight passes through the disc hump.

Secondly, the absorbing extended screen eclipses not only the star but also the central regions of the disc. As a result, we get deeper minima with a higher polarization degree. The minimum depth can increase by about 1.5 times. The increase in polarization degree also depends on the wavelength and disc puffing-up. In our work for the disc without puffing-up, it is about 1.7 times in U band and about 1.4 times in I band. With a significant disc puffing-up, the possible increase in polarization turns out to be less significant. Instead, the polarization plane orientation may depend on the hump shape: during an eclipse by a compact cloud or a hump with a small extension, the disc is polarized perpendicular to its plane, and, during an eclipse by an extended hump, the disc is polarized along its plane.

Belan and Shakovskoy (in preparation) have recently observed a very deep and extended (about a year) UX Ori minimum, during which very high (about 10%) linear polarization was observed. The calculations presented above show that the considered models with an extended hump can be applicable to describe such observations.

Thirdly, large-scale humps with different shape additionally increase the parameters scatter (polarization degree and colour indices) on the same fading level. This is observed when considering values at the minimum brightness. When studying the large-scale hump motion around the star, the effect is further enhanced. Usually, an increase in the length of the disturbance leads to a lower degree of polarization at the same fading level, but there may be exceptions for narrow humps. Close to a deep minimum, the degree of polarization can vary by 1–2%, depending on the hump azimuthal extension. In general, an increase in the hump azimuthal extension leads to higher modulus values of colour indices.

Also, when the large-scale hump moves around the star, new effects arise which are not possible for an eclipse by a compact gas-dust cloud. In a model with a significant disc puffing-up, the maximum of the polarization degree can be achieved not at the minimum brightness. It may be slightly shifted relative to it. In this case, at the minimum brightness, the degree of polarization can be less than a per cent, while the maximum polarization degree will be about 3%.

The behaviour of the linear polarization positional angle turns out to be very complex. For a disc without thickening in the dust sublimation zone, small (on the order of several degrees) changes in the position angle in both directions are possible. As the puffing-up increases, the position angle deviations become more noticeable, but at the same time, they are mainly directed in the same direction. In some models, during an eclipse, the position angle can change up to 90°. In others, the position angle at the brightness minimum does not change at all, but its strong changes are observed after the minimum.

In both cases, these changes may present for some time after an eclipse, when the star brightness returned to pre-eclipse level. These deviations can be up to 60° for a thin disc and up to 30° for a thick one. The effect depends on the optical properties of the dust. It is possible that in one spectral band the disc with a puffing-up is thin while in the other band the disc is already thick. As a result in the first band, we have a strong change in the positional angle while in the second band there are smaller changes in the opposite direction.

Thus the above conclusions of the models can be summarized as an extension of the conservative model of a compact gas-dust cloud eclipse of a star with a thin disc (Grinin 1988). Computations for a disc with a puffed-up inner rim demonstrate that both models (the compact cloud eclipse and the large-scale hump eclipse) explain several observational phenomena that do not fit the predictions of the thin disc model. The large-scale hump model not only generalizes the predictions of the compact cloud model, allowing one to obtain deeper eclipses and a greater scatter of parameters, but also leads to new effects associated with changes in the position angle of linear polarization.

In this paper, the puffing-up is simulated by a disc wind. The wind density depends on the mass outflow rate and may change for various reasons. In particular, the resulting eclipses may vary greatly with a change in the mass outflow rate. Usually, the mass outflow rate is assumed to be equal to a certain fraction of the accretion rate. For example, Bans & Königl (2012) estimate the mass outflow rate as 1–5% of the accretion rate. At the same time, the accretion rate of young stars is unstable. Its variability usually does not exceed 0.5 dex for Herbig Ae/Be stars (Mendigutia et al. 2011; Pogodin et al. 2012).

But a change in the wind density by 0.5 dex has a very strong effect on the eclipse shape. Also, the shape of the hump can be quite complex and change over time. A simultaneous change in the shape of the disc hump and the mass outflow rate can lead to complex changes in the stellar magnitude, the polarization degree, and its positional angle in time. This can bring us closer to understanding the nature of the unusual deep and lasting minima of UX Ori and WW Vul (Grinin et al. 1988; Grinin 1994). At the moment we can separately obtain the observed changes in the magnitude or the positional angle of linear polarization, but we cannot reproduce all the features of the eclipse at the same time.

A detailed study of such eclipses of UX Ori type stars can give us more information in the future about the inner regions of the protoplanetary disc and the processes occurring in them. It was previously known that large-scale structures in the disc can be seen from the disc pole when it is possible to obtain its resolved image. Our models show that under certain conditions, large-scale disc structures have important observational manifestations with a line of sight close to the disc plane. Since it is reasonable to expect large-scale disc structures to be independent of the disc orientation relative to an observer, this gives us an additional way to study such structures as shadows on the disc images. As noted Stolker et al. (2017), the dense in time
observations of such moving shadows can be considered as a face-on version of the studies of the UXOR phenomenon.

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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

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Figure A1. The comparison of the results obtained taking into account only single ($ΔV_1$ and $p_1$), first two ($ΔV_2$ and $p_2$) and first four ($ΔV_4$ and $p_4$) scatterings for the disc model from Fig. 2 with $Δr = 0.3$ AU and $r_0 = 0.5$ AU. The angle between the direction to the observer and the disc axis is 65°.

APPENDIX A: MULTIPLE SCATTERINGS CONTRIBUTION

It is very important to take multiple scatterings into account when we study the dust radiative transfer. Here we consider eclipses from the section 4.1. Here we use the models from Fig. 2 with $Δr = 0.3$ AU and $r_0 = 0.5$ AU. In Fig. A1 we show the difference between the results obtained taking into account four first scatterings and the results obtained taking into account only single or double scattering. For this purpose in the current subsection we use the following notation for the main eclipse parameters: $ΔV_1$, $ΔV_2$, and $ΔV_4$ are maximal changes in the magnitude in V band when we take into account only single, first two, and first four scatterings, respectively; $p_1$, $p_2$, and $p_4$ are the corresponding maximal polarization degrees. Again, we show the results for compact cloud eclipses with thick lines, and the deepest points of the large-scale humps with dots. Thin lines connect points with equal $ΔV/V$ (isoheights) and $Δφ$ (isoextentions). Isoextentions go along the compact cloud eclipse track, and isoheights go across it.

Maximal changes in the magnitude have simple behaviour: $ΔV_1$ is always greater than $ΔV_2$ and $ΔV_2$ is always greater than $ΔV_4$. When we take into account more scatterings, the disc becomes brighter, and the star input into the system light decreases. As a result, the same star fading leads to a smaller magnitude change. When the hump becomes more extended the eclipsed section of the disc grows, multiply scattered light from this region is also absorbed by the hump, so the disc brightening is reduced, and the magnitude differences become smaller than in the compact cloud case.
The polarization has more difficult behaviour. The main contribution to polarization is from the single scatterings. After multiple scatterings, the light is only slightly polarized. Hence, the total light of the system has three components: nonpolarized direct starlight, strongly polarized single scattered light, and slightly polarized multiply scattered light. As a result, when the star is weakly eclipsed, slightly polarized light increases the total polarization degree. In this situation $p_1$ and $p_2$ are less than $p_4$. Conversely, when the star is heavily eclipsed and makes a small contribution to the total system light, slightly polarized multiple scattered light decreases the total polarization degree. So, $p_1$ and $p_2$ become greater than $p_4$.

The difference between taking into account the first two or first four scatterings for our model is always less than 0.15$^m$ and 1%. The difference between the first three and four scatterings is even less (not more than 0.035$^m$ and 0.3%) and we do not show it in Fig. A1 so as not to overload it.

The difference between the first four and five scatterings is not more than 0.02$^m$ and 0.15%. The contribution of subsequent scattering to the total radiation of the system decreases rapidly. Therefore, in this paper, we limited the computations to modelling only the first four scatterings.

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