Shot noise and tunnel magnetoresistance in multilevel quantum dots: Effects of cotunneling

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Spin-dependent transport through a multilevel quantum dot weakly coupled to ferromagnetic leads is analyzed theoretically by means of the real-time diagrammatic technique. Both the sequential and cotunneling processes are taken into account, which makes the results on tunnel magnetoresistance (TMR) and shot noise applicable in the whole range of relevant bias and gate voltages. Suppression of the TMR due to inelastic cotunneling and super-Poissonian shot noise have been found in some of the Coulomb blockade regions. Furthermore, in the Coulomb blockade regime there is an additional contribution to the noise due to bunching of cotunneling processes involving the spin-majority electrons. On the other hand, in the sequential tunneling regime TMR oscillates with the bias voltage, while the current noise is generally sub-Poissonian.

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Introduction: Transport properties of quantum dots coupled to ferromagnetic leads are currently a subject of extensive experimental and theoretical studies.\textsuperscript{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15} This interest is stimulated by expected applications in spintronics and quantum computing.\textsuperscript{16} When a quantum dot is coupled to two ferromagnetic leads, its transport properties depend on the magnetic configuration of the system. This is the so-called tunnel magnetoresistance (TMR) effect, which is characterized by the ratio \(TMR = (I_P - I_{AP})/I_{AP}\), where \(I_P\) (\(I_{AP}\)) is the current flowing through the system in the parallel (antiparallel) configuration.\textsuperscript{16,18} When the coupling between the dot and leads is strong, the Kondo physics emerges for one energy level participate in transport, leading to more Kondo dots.\textsuperscript{18} In turn, single-electron charging in the Coulomb blockade regime can be super-Poissonian, while the current noise is generally sub-Poissonian.

In real systems, however, usually more than one energy level participate in transport, leading to more complex and interesting transport characteristics.\textsuperscript{15,12} It has been shown recently that the Fano factor in the Coulomb blockade regime calculated in the first order approximation is larger than unity.\textsuperscript{19} Moreover, the TMR was found then to be independent of the gate voltage. In this paper we extend the theoretical studies by including the cotunneling processes, and show that the shot noise in the Coulomb blockade regime can be super-Poissonian, although the Fano factor is significantly reduced by the cotunneling processes. Apart from this, we show that the TMR in the blockade regime is considerably modified by cotunneling processes, and can be either enhanced or reduced in comparison to that in the first order approximation, depending on the transport regime. Our considerations are based on the real-time diagrammatic technique,\textsuperscript{20,21} which, after taking into account both the first and second-order contributions, allows us to analyze transport in the full weak coupling regime, i.e. in the cotunneling, cotunneling-assisted sequential, and sequential tunneling regimes. Furthermore, to calculate the shot noise in the cotunneling regime, we include the non-Markovian effects,\textsuperscript{21} which were neglected in previous considerations.\textsuperscript{22}

Model: We consider a two-level quantum dot weakly coupled to external ferromagnetic leads whose magnetic moments are either parallel or antiparallel. The Hamiltonian of the system reads, \(H = H_L + H_R + H_D + H_T\). The first two terms describe noninteracting itinerant electrons in the leads, \(H_L = \sum_{k\sigma} \varepsilon_{rk\sigma} c^\dagger_{rk\sigma} c_{rk\sigma}\) for the left (\(r = L\)) and right (\(r = R\)) leads, where \(c^\dagger_{rk\sigma}\) (\(c_{rk\sigma}\)) creates (annihilates) an electron with the wave vector \(k\) and spin \(\sigma\) in the lead \(r\), and \(\varepsilon_{rk\sigma}\) is the corresponding dispersion relation. The quantum dot is described by

\[
\hat{H}_D = \sum_j \varepsilon_j n_j + U \sum_j n_{j\uparrow} n_{j\downarrow} + U' \sum_{\sigma\sigma'} n_{1\sigma} n_{2\sigma'},
\]

where \(n_{j\sigma} = d^\dagger_{j\sigma} d_{j\sigma}\) is the creation (annihilation) operator of an electron with spin \(\sigma\) in the \(j\)th level \((j = 1, 2)\), \(\varepsilon_j\) is the corresponding single-particle energy, and \(U\) (\(U'\)) is the on-level (inter-level) Coulomb repulsion parameter. The tunnel Hamiltonian, \(\hat{H}_T\), takes the form: \(\hat{H}_T = \sum_{r=L,R} \sum_{k\sigma} (t_{rj} c^\dagger_{rk\sigma} d_{j\sigma} + t_{rj}^* d^\dagger_{j\sigma} c_{rk\sigma})\), where \(t_{rj}\) is the relevant tunneling matrix element. Coupling of the \(j\)th level to the spin-majority (spin-minority) electron band of the lead \(r\) is described by \(\Gamma_{rj}^{+(-)} = 2\pi |t_{rj}|^2 p_r^{+(-)} = \Gamma_{rj}(1 \pm p_r)\), where \(\Gamma_{rj} = (\Gamma_{rj}^+ + \Gamma_{rj}^-)/2\), while \(p_r^{+(-)}\) and \(p_r\) are the spin-dependent density of states and spin polarization in the lead \(r\), respectively.

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In the following we assume $\Gamma_{ij} = \Gamma/2$ and $p_L = p_R = p$.

**Method:** In order to calculate the spin-polarized transport through a two-level quantum dot in the sequential and cotunneling regimes, we employ the real-time diagrammatic technique, which consists in a systematic expansion of the quantum dot (reduced) density matrix and the current operator with respect to the dot-lead coupling strength $\Gamma$. The current operator $I$ is defined as $I = (I_R - I_L)/2$, with $I_L = -\imath \langle c_L \hat{c}_R \rangle \sum_{\kappa} \sum_{\sigma} \langle \tau_j \rangle_{\kappa \sigma} d_{\kappa \sigma} - t_{\kappa \sigma} d_{\kappa \sigma}^\dagger$ being the current flowing from the dot to the lead $r$. Time evolution of the reduced density matrix can be visualized as a sequence of irreducible self-energy blocks, $W_{\chi \chi'}$, on the Keldysh contour. The matrix elements $W_{\chi \chi'}$ describe transitions between the many-body states $|\chi\rangle$ and $|\chi'\rangle$ of the two-level dot.\(^{20}\) The full propagation of the dot density matrix is given by the Dyson equation, which is further transformed into a general kinetic equation for the elements of the reduced density matrix.\(^{21}\) With the aid of the matrix notation introduced in Ref.\(^{21}\), all the quantities of interest can be defined in terms of the following self-energy matrices: $W$, $W^I$, $W^{II}$, $\partial W$, and $\delta W^I$. The matrix $W^{I(II)}$ is the self-energy matrix with one internal vertex (two internal vertices) resulting from the expansion of the tunneling Hamiltonian replaced by the current operator, while $\partial W$ and $\delta W^I$ are partial derivatives of $W$ and $W^I$ with respect to the convergence factor of the Laplace transform.\(^{22}\) Using the above matrices, the stationary occupation probabilities can be found from, $(\tilde{W}p^{st})_{\chi} = \Gamma \delta_{\chi \chi_0}$, where $p^{st}$ is the vector containing probabilities and the matrix $W$ is given by $W$ with one arbitrary row $\chi_0$ replaced by $(\Gamma, \ldots, \Gamma)$ due to the normalization, $\sum_{\chi} p^{st}_{\chi} = 1$. The current flowing through the system can then be found from

$$I = \frac{e}{2 \hbar} \text{Tr} \{W^I p^{st}\}. \quad (2)$$

Finally, the zero-frequency current noise, $S = 2 \int_0^\infty dt \langle (I(t)I(0) + I(0)I(t)) - 2\langle I(t)^2\rangle \rangle$, is given by\(^{21}\)

$$S = \frac{e^2}{\hbar} \text{Tr} \left\{ \left[ W^{I} + W^{I} \left( PW^{I} + p^{st} \otimes e^T \partial W^{I} \right) \right] p^{st} \right\}, \quad (3)$$

where the object $P$ is calculated from: $\tilde{W}P = \tilde{I}(p^{st} \otimes e^T - 1 - \partial W^{st} \otimes e^T)$, with $\tilde{I}$ being the unit vector with row $\chi_0$ set to zero, and $e^T = (1, \ldots, 1)$\(^{21}\).

To calculate the transport properties order by order in tunneling processes, we expand the self-energy matrices, $W^{I(II)} = W^{I(II)}(1) + W^{I(II)}(2) + \ldots$, the dot occupations, $p^{st} = p^{st}(1) + p^{st}(2) + \ldots$, and, $P = P(-1) + P(0) + \ldots$, respectively. The self-energies can be calculated using the corresponding diagrammatic rules.\(^{8,21}\) The first order of expansion corresponds to the sequential tunneling, whereas the second one to cotunneling. Thus, by taking into account all the first and second-order contributions, we are able to resolve transport properties in the full range of the bias and gate voltages.

**Results on TMR:** The TMR as a function of the bias voltage and position of the dot levels is shown in Figs.\(^{1}(b)\). To facilitate the identification of different transport regimes, we present in Fig.\(^{1}(a)\) the density plot of the differential conductance $G$ in the parallel configuration (differential conductance in the antiparallel configuration is qualitatively similar). Since position of the dot levels can be shifted with a gate voltage, Figs.\(^{1}(a)\) and (b) can be viewed as a bias and gate voltage dependence of TMR and $G$, respectively. By sweeping the gate voltage in the linear response regime, the number of electrons in the dot can be changed successively. More precisely, this happens when: $\varepsilon \equiv \varepsilon_1 = 0$, $\varepsilon = -(\delta \varepsilon + U')$, $\varepsilon = -(U + U')$, and $\varepsilon = -(\delta \varepsilon + U + 2U')$, where $\delta \varepsilon = \varepsilon_2 - \varepsilon_1$ is the level spacing. Thus, for $\varepsilon \gtrsim 0$, $[-(\delta \varepsilon + U + 2U') \gtrsim \varepsilon]$, the dot is empty (fully occupied). When $0 \gtrsim \varepsilon \gtrsim -(\delta \varepsilon + U')$, there is a single electron (three electrons) in the dot. On the other hand, for $-(\delta \varepsilon + U + U') \gtrsim \varepsilon \gtrsim -(U + U')$, the dot is occupied by two electrons, one on each orbital level.

In the case of empty (fully occupied) dot in the Coulomb blockade regime the current flows only due to elastic non-spin-flip cotunneling processes. Such processes are fully coherent and do not affect the charge and spin state of the dot. As a result, the system behaves as a single ferromagnetic tunnel junction, yielding the TMR given by the Julliere formula.\(^{8,16}\) TMR is decreased. This is due to the spin-flip cotunneling, decreasing the difference between conductance in the parallel and antiparallel configurations and thus reducing the TMR. On the other hand, in the regime when each dot level is singly occupied (doubly occupied dot), the amount of spin-flip cotunneling is increased and the suppression of TMR is even more pronounced. However, when the bias voltage is increased, the central minimum in TMR [Fig.\(^{1}(c)\)] transforms into a local maximum due to a nonequilibrium spin accumulation in the dot, see Fig.\(^{1}(d)\) and (e).

The bias voltage dependence of the TMR for several values of the level position is displayed in Fig.\(^{1}(f)-(h)\). When the dot is empty in equilibrium ($\varepsilon = 2 \text{ meV}$), the TMR in the cotunneling regime is given by the Julliere value. However, once the bias voltage reaches the threshold voltage ($V \approx 3 \text{ mV}$), the sequential processes are allowed and TMR drops, see Fig.\(^{1}(f)\). If the ground state is singly occupied ($\varepsilon = -1.75 \text{ meV}$), a nonequilibrium spin accumulation in doublet states ($p_{\uparrow \downarrow}^{(1)} \neq p_{\downarrow \uparrow}^{(1)}$) is built up with increasing bias voltage.\(^{9}\) This leads to an enhanced TMR, which again starts to drop around the threshold for sequential tunneling, see Fig.\(^{1}(g)\). Finally, in Fig.\(^{1}(h)\) we show the bias dependence of TMR for the case when the dot is doubly occupied in equi-
librium ($\varepsilon = -5.25$ meV). Now, the spin accumulation in triplet states ($p_{1\uparrow\downarrow}^{\text{st}} \neq p_{1\downarrow\uparrow}^{\text{st}}$) gives rise to an increase in TMR. We also note that in the transport regime where the sequential tunneling is allowed, more and more charge states become active in transport with increasing the bias voltage. This gives rise to step-like characteristics and the oscillatory-like behavior of the TMR, see Fig. 1(f)-(h).

In Fig. 1(c)-(h) we also showed the TMR calculated in the first order (sequential) approximation (dashed lines). It is evident that the role of cotunneling processes is particularly pronounced in the blockade regions, where the cotunneling processes dominate over the sequential ones, and lead to a significant enhancement (or reduction) of TMR. Outside the blockade regions, TMR is determined mainly by sequential transport, so the contribution from cotunneling processes is rather minor.

**Results on shot noise:** Upon calculating the current $I$ and the zero-frequency current noise $S$, one can determine the Fano factor $F$, $F = S/(2e|I|)$. The Fano factor describes the deviation of $S$ from the Poissonian shot noise given by $S_p = 2e|I|$. The bias and gate voltage dependence of the Fano factor in the parallel and antiparallel magnetic configurations is shown in Fig. 2(a) and (b), respectively. When $|eV| \lesssim k_B T$, $S$ is dominated by thermal Nyquist-Johnson noise, otherwise the noise due to the discrete nature of charge (shot noise) dominates. In the limit of $V \to 0$, the current tends to zero whereas the current noise is dominated by thermal noise. This leads to a divergency of the Fano factor in the linear response regime. Therefore, in the density plots of $F$ we mark the low bias voltage regime with a black line.

In the cotunneling regime, where the dot is empty (or fully occupied), the shot noise is Poissonian, Fig. 2(c). This is because the current flows then only due to elastic non-spin-flip second-order processes. However, in the Coulomb blockade regions, where inelastic cotunneling processes also contribute, we find a pronounced super-Poissonian shot noise, see Fig. 2(d) and (e). This increased shot noise is related to bunching of electrons carried by different types of cotunneling processes. Furthermore, in the case of magnetic leads there is an additional contribution to the noise coming from the difference between the spin-up and spin-down channels which leads to bunching of fast cotunneling processes involving the majority electrons. This is more pronounced in the parallel configuration where the difference between the two channels is approximately equal to $(1 + p)^2/(1 - p)^2$, while for the antiparallel configuration the two channels are comparable, see Fig. 2(a) and (b). The variation of the Fano factor with the bias voltage is displayed in Fig. 2(f)-(h). When the dot is empty [Fig. 2(f)], the Fano factor is Poissonian and starts to drop at the onset of sequential tunneling. With increasing bias voltage, the noise becomes super-Poissonian in some range of the transport voltage. In the case of singly and doubly occupied quantum dot, the bias voltage dependencies of $F$ are qualitatively similar. In the Coulomb blockade regime the current noise becomes super-Poissonian (up to $F \approx 2.5$ in the parallel configuration), indicating the existence and role of inelastic and spin-flip cotunneling processes. For example, in the case of doubly occupied dot, Fig. 2(h), the maximum Fano factor is found for voltages where inelastic cotunneling between the two orbital levels is allowed, $|eV| \approx 2\delta\varepsilon$. On the other hand, in the sequential tunneling regime the Fano factor becomes suppressed and is generally sub-Poissonian due to the Coulomb correlations in sequential transport. For comparison, in Fig. 2(c)-(h) we show
the results calculated within the sequential tunneling approximation (dashed lines), which clearly show the role of cotunneling processes in shot noise, particularly in the blockade regions. We also note that the occurrence of super-Poissonian shot noise in the Coulomb blockade regime has also been reported experimentally in quantum dots coupled to nonmagnetic leads.\textsuperscript{25,26,27}

In conclusion, we have discussed the spin-polarized transport through a two-level quantum dot coupled to ferromagnetic leads in the sequential and cotunneling regimes. We have analyzed the dependence of the TMR and Fano factor on the bias and gate voltages. In the Coulomb blockade regime we have found a suppression of TMR for singly and doubly occupied quantum dot. Furthermore, in these transport regimes the inelastic cotunneling processes lead to super-Poissonian shot noise, irrespective of magnetic configuration of the system. On the other hand, in the sequential tunneling regime, the current noise is generally sub-Poissonian, while TMR exhibits oscillatory-like dependence on the bias voltage. We also notice that generally the sequential tunneling approximation in the Coulomb blockade regime underestimates the TMR and overestimates the Fano factor.

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{(Color online) The Fano factor $F$ as a function of the bias voltage and level position for the parallel (a) and antiparallel (b) magnetic configurations. Parts (c)-(h) show $F$ in the parallel configuration as a function of the level position [(c)-(e)] and the bias voltage [(f)-(h)]. The dashed lines present the first-order contributions. The parameters are the same as in Fig. [1].}
\end{figure}
63, 125315 (2001).
I. Weymann, Europhys. Lett. 76, 1200 (2006).
24 Ya. M. Blanter and M. Büttiker, Phys. Rep. 336, 1 (2000).
25 S. Gustavsson et al., Phys. Rev. B 74, 195305 (2006).

26 E. Onac et al., Phys. Rev. Lett. 96, 026803 (2006).
27 Yiming Zhang et al., Phys. Rev. Lett. 99, 036603 (2007).