We discuss the systematic uncertainties in the recovery of dark energy properties from the use of baryon acoustic oscillations as a standard ruler. We demonstrate that while unknown relativistic components in the universe prior to recombination would alter the sound speed, the inferences for dark energy from low-redshift surveys are unchanged so long as the microwave background anisotropies can measure the redshift of matter-radiation equality, which they can do to sufficient accuracy. The mismeasurement of the radiation and matter densities themselves (as opposed to their ratio) would manifest as an incorrect prediction for the Hubble constant at low redshift. In addition, these anomalies do produce subtle but detectable features in the microwave anisotropies.

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I. INTRODUCTION

In standard cosmology, the acoustic oscillations imprinted in the matter power spectrum at recombination have a length scale that can be accurately calculated based on measurements of the CMB anisotropy power spectrum [1, 2, 3, 4, 5]. It should then be possible to measure this "standard ruler" scale at low redshifts, for example in large galaxy redshift surveys, and thereby constrain the matter and energy content of the universe [6, 7, 8, 9, 10]. However, if the CMB measurements were misled by some new physics, e.g. a new undetected relativistic particle, then the misinterpretation could potentially spread to the low-redshift application and bias the inferences.

Here, we show that the interpretation of the low-redshift acoustic oscillations are robust if the CMB correctly tells us the baryon-to-photon ratio and the epoch of matter-radiation equality. These quantities are robustly measured in the CMB. The actual densities of matter and radiation drop out of the calculation; only their ratio matters. The result is that even if the physical matter density $\rho_m \propto \omega_m \equiv \Omega_m h^2$ is misinterpreted from the CMB due to undetected relativistic components, the inferences for dark energy from the combined CMB and low-redshift survey data sets are unchanged. Knowledge of actual densities, e.g. $\omega_m$, translates into improved constraints on the Hubble constant, $H_0 = 100 h \text{km s}^{-1} \text{Mpc}^{-1}$.

II. THE PHYSICAL SCALE

The acoustic peak method depends upon measuring the sound horizon, which is the comoving distance that a sound wave can travel between the end of inflation and the epoch of recombination [11]. Nearly all of this distance is accumulated just prior to the epoch of recombination at $z \sim 1100$. The sound horizon integral depends only on the Hubble parameter $H(z)$ and the sound speed $c_s$ in the baryon-photon plasma. If we assume dark energy is sub-dominant at $z \sim 10^3$, then

$$H(z) \simeq H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_r (1+z)^4} = \sqrt{\Omega_m H_0^2 (1+z)^3 \left(1 + \frac{1+z}{1+z_{eq}}\right)},$$

where $z_{eq} = \Omega_m/\Omega_r$ is epoch of matter-radiation equality. The sound speed depends only on the baryon-to-photon ratio and is $c_s = c/\sqrt{3(1+R)}$ with $R \equiv 3 \rho_b/4 \rho_r \propto \omega_b (1+z)^{-1}$. These two produce the sound horizon

$$s = \int_0^{z_{rec}} c_s (1+z) \, dt = \int_{z_{rec}}^{\infty} c_s \, dz = \frac{1}{\Omega_m H_0^2} \frac{2c}{\sqrt{3} \Omega_r R_{eq}} \ln \frac{1 + R_{rec} + \sqrt{R_{rec} + R_{eq}}}{1 + \sqrt{R_{eq}}},$$

where ‘rec’ and ‘eq’ refer to recombination and equality respectively. One sees that the aside from a prefactor of $\omega_m^{-1/2}$, the sound horizon depends only on the baryon-to-photon ratio and the redshift of equality.

The epoch of recombination, being controlled by atomic physics, is very insensitive to the cosmology. For reasonable changes in the early universe and our current uncertainties of the theory of recombination [12], any shift in $z_{rec}$ is...
negligible. The baryon-to-photon ratio is also exquisitely well measured in the CMB power spectrum by both the ratios of the odd and even acoustic peaks and by the Silk damping tail \[^{14,17}\]. The former effect depends only on the gravitational inertia of the baryons driven by the potentials near the epoch of recombination. Thus the modulation gives us a precise measurement of the baryon-to-photon ratio \(p_b/p_\gamma\), which with our precise knowledge of \(T^\text{cmb}\), fixes \(\rho_b \propto \omega_b \equiv \Omega_b h^2\). Moreover, for the established value of \(R \approx 0.6\) near \(z \approx 10^3\), the effect on the sound horizon is already small. It seems very likely that the CMB will determine the baryon-to-photon ratio to sufficient accuracy for this portion of the sound horizon computation \[^{12}\].

Information about matter-radiation equality is encoded in the amplitudes of the peaks through the manner in which the potentials evolve as they cross the horizon: the potential envelope \[^{14,12}\]. Measurements of the potential envelope thus robustly constrain equality. Normally, one interprets this constraint as the matter density \(\omega_m\), on the assumption that the photons and standard neutrino background are the full radiation density. However, one could imagine other relativistic components, and in this case, measuring the redshift of equality does not imply the matter density \(\omega_m\) (we continue to assume that the extra components are “undetected” in the CMB and return to this point in the next section). As we can see from Eq. (2), the dependence of \(\omega_m\) on \(\rho_m\) is relatively small since \(\Omega_m \approx \sqrt{1+z}\), thus even a crude determination suffices to fix \(\rho_m\) up to an overall factor of \(\omega_m^{-1/2}\), i.e., \(\omega_m^{1/2}/\rho_m\) is very well measured. The sound horizon decreases by only 5% if \(\omega_m\) is lowered by 50%.

Understanding the acoustic oscillations at \(z \approx 1100\) allows us to translate knowledge of the sound horizon into knowledge of wavelength of the baryonic features in the mass power spectrum up to the same normalization uncertainty. We then wish to consider the measurement of this scale at lower redshift, such as could be accomplished in large galaxy surveys. Measuring the scale along and across the line of sight, as a redshift or angular scale, constrains low-redshift distances in equations (3) and (4) would still have prefactors of \(\Omega_{cdm}/(1+z)^3\). Another way of saying this is that the standard ruler measurements of this ruler do not measure \(\Omega_{cdm}\) rather than \(\Omega_m\) itself, as this is the quantity that isolates the cosmological densities at low redshift.

Parameter estimation for acoustic oscillations based on standard models \[^{7,8,9,10}\] will be unchanged by the presence of undetectable new radiation provided that the redshift of equality is measured correctly. Only the Hubble constant would be incorrect. In the context of CMB parameter estimation, it would be more useful to report \(\rho_m\) rather than \(\rho_m\) itself, as this is the quantity that isolates the cosmological densities at low redshift.

Massive neutrinos should be counted as radiation at \(z > 10^3\) but would be counted as matter at low redshift. That does create a small error in the dark energy inferences. One would be computing \(\omega_m^{1/2}/\rho_m\) from the CMB, but the low-redshift distances in equations (3) and (4) would still have prefactors of \(\rho_m^{1/2} = (\Omega_{cdm} + \rho_{\text{wDM}})^{-1/2}\). As the observations constrain \(\rho_m^{1/2} = (\Omega_{cdm} + \rho_{\text{wDM}})^{-1/2}\), many regions of parameter space would be excluded. As the matter neutrino fraction inferred for non-degenerate neutrino species and the atmospheric neutrino mass splitting \[^{17}\] this is a 0.2% correction; however, it could be a few percent correction at the upper limit of the allowed masses for degenerate neutrino species \[^{13}\]. Fortunately, such neutrino masses should be detectable in upcoming data due to their suppression of the late-time matter power spectrum \[^{10}\].

Because the integral for the sound horizon extends to early times (essentially to the time when the cosmic perturbations were established), alterations to the Hubble parameter (eq. (3)) could alter the sound horizon even at fixed redshift of equality. For example, there might exist a non-relativistic particle that decays into relativistic unseen particles sometime prior to recombination \[^{20,21,22,23}\]. However, such decays create alterations to the gravitational potentials that would be detectable in the CMB if the horizon scale at that epoch is visible in the primary anisotropies (i.e., wavenumbers \(k \lesssim 0.2h\) Mpc\(^{-1}\)). This makes decays at \(10^3 \lesssim z \lesssim 10^5\) difficult to hide. Furthermore, such decays alter the transfer function and would affect the amplitude of the late-time matter power spectrum. Precision measurement of the matter power spectrum out to \(k \sim 1h\) Mpc\(^{-1}\) from the Lyman-alpha forest could push the limits
on particle decays to earlier times. In the standard theory, the first 1% of the sound horizon integral is contributed by \( z = 200,000 \). It therefore seems very unlikely that a decaying particle could significantly affect the sound horizon (relative to the standard theory at fixed redshift of equality) and escape detection in the CMB or large-scale structure.

As a corollary, it is interesting to note that when one measures the acoustic oscillation scale at \( z_{\text{rec}} \) with the CMB and at some low redshift \( z_g \) in a galaxy survey, one can construct an observable (the difference of the suitably scaled angular wavenumbers of the acoustic peaks) that isolates the cosmology between \( z_g \) and \( z_{\text{rec}} \), i.e.,

\[ \ell_{z_{\text{rec}}} - \ell_{z_g} \propto \int_{z_g}^{z_{\text{rec}}} \frac{dz}{\omega_m^{-1/2} H(z)} \]  

This integral is trivial when the universe is completely matter-dominated. Hence, if we can perform a galaxy survey at some suitably high redshift, e.g., \( z_g = 4 \), where the dark energy is supposed to be negligible, we could search for dynamical shenanigans at \( 4 < z < 10^3 \). A photometric-redshift survey over large amount of sky would be an economical route to this goal, as one can acquire very large survey volumes [10].

\section*{III. WHITHER THE ACTUAL DENSITY?}

We have argued that only the ratios of the radiation and matter densities enter the acoustic oscillation standard ruler method, but of course it is interesting to measure the actual values of the densities as they might reveal new relativistic components or unexpected evolution [28].

The CMB is sensitive to the non-photon radiation density primarily through its effect on equality and the evolution of the potentials. This encodes a sensitivity to both the amount of radiation and its quadrupole moment. In general any particle that free streams rather than behaves as a fluid will have a local quadrupole that will affect the evolution of the potentials and thus the anisotropy in the CMB. This breaks the degeneracy in the CMB between changes in \( \omega_m \) and \( \omega_{\text{rad}} \) [10] and allows us to constrain a “conventional” neutrino at the \( N_\nu \simeq 0.1 - 0.2 \) level [12]. At the other extreme, extra radiation which is of the perfect fluid form leads to a increase in the small-scale power which is easily discerned from a change in \( \omega_m \) (see Fig. 1). Only a radiation component whose higher moments track closely that of the traditional mix of photons and neutrinos could be confused with a change in \( \omega_m \).

Altering the radiation and matter density while holding the baryon (and photon) densities fixed would alter the baryon fraction \( \Omega_b/\Omega_m \) and would produce significant offsets in the late-time matter power spectrum (e.g. [2, 3, 25]). Higher baryon fractions suppress power on small scales compared to large, with the transition occurring near the

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1}
\caption{The angular power spectra for a fiducial ΛCDM model (solid) and three variants with a lower \( z_{eq} \). The dotted line shows a model with \( \omega_m \) reduced by 10%, the long dashed line \( N_\nu \) increased from 3.04 to 3.78 (which raises \( \rho_r \) by 10%) and the short dashed line 10% extra \( \rho_r \) in the form of a fluid. The upper panel shows the spectra while the lower panel shows the ratio to the ΛCDM curve. All spectra have been normalized (arbitrarily) at \( \ell = 200 \).}
\end{figure}
sound horizon at $k \approx 0.05 h \text{ Mpc}^{-1}$. The acoustic oscillations shortward of this scale would also be increased. The small-scale amplitude (e.g., $\sigma_8$) would be reduced. These changes are observable in galaxy surveys, weak lensing surveys, and cluster abundance measurements. Of course one can also directly measure the Hubble constant \[26\].

In short, although the acoustic peak method depends only on the redshift of equality, other cosmological measurements both at $z = 1000$ and at $z \sim 0$ are well poised to measure the actual densities and thereby constrain the presence of unknown relativistic species.

IV. CONCLUSIONS

We have shown that the standard ruler defined by the acoustic oscillations prior to recombination can deal gracefully with uncertainties in the matter density $\omega_m$ provided that the redshift of matter-radiation equality is well measured. The anisotropies of the CMB have very good leverage on this quantity, and so the acoustic peak method of probing dark energy is robust. In addition, it is likely that the CMB, perhaps in combination with other probes, will be able to constrain the actual matter and radiation densities.

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[27] The same thing occurs in non-flat cosmologies. For example, in an open cosmology, we have $d_A = R_c \sinh(r/R_c)$, where $r$ is equation 4. But we can write $R_a = c \sqrt{\Omega_m H_0^2} / \sqrt{\Omega_k H_0^2}$, where $\Omega_k$ is the usual curvature term. This substitution shows that $\omega_m^{1/2} d_A$ depends only on the bare $\Omega$.
[28] We assume implicitly that any extra radiation is introduced between 1 second and 10,000 years, since radiation present prior to 1 second would modify big bang nucleosynthesis.