Correlations between low energy leptonic CP violation and leptogenesis in the light of recent experiments.

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Abstract

Leptogenesis is the most favourable mechanism for generating the observed baryon asymmetry of the Universe (BAU) which implies CP violation in the high energy scale. The low energy leptonic CP violation is expected to be observed in the neutrino oscillations and $0\nu2\beta$ decay experiments. Generally it is not possible to connect both the CP violations. Here we revisit the issue of connecting the two in flavoured leptogenesis scenario within the Type I seesaw in the light of recent neutrino oscillation and Planck data. With the recent precise measurements of $\theta_{13}$ and BAU we are able to find new correlations between the low and high energy CP violating phases when leptogenesis occurs at temperature between $10^9$ to $10^{12}$ GeV and there is no contribution to CP violation from the heavy neutrino sector.

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I. INTRODUCTION

The recent Planck experiment [1] gives baryon density ($\Omega_b h^2$) to be $0.02205 \pm 0.00028$ in the 68% confidence level and the present value of baryon to entropy ratio is estimated to be $Y_B = (8.294 - 8.508) \times 10^{-11}$ from the relation $\eta = 3.81 \times 10^{-9}\Omega_b h^2$. This baryon number asymmetry can be generated by the process of thermal leptogenesis [2]. It is in this context that the Type I seesaw model becomes very interesting. It can explain the smallness of the neutrino mass as well as satisfy the Sakharov conditions necessary for successful baryogenesis. In addition to the Standard Model (SM) particles, three right handed (RH) heavy neutrinos which are singlet under the SM gauge group are added to generate a light neutrino mass in Type I seesaw model. These singlet heavy neutrinos are of Majorana type and therefore can decay into a particle and as well as to an anti-particle. Such lepton number violating decays with different decay rates for the particles and for the anti-particles can give rise to a net CP asymmetry of the particular flavour of the final state lepton. These CP asymmetries can survive only when the decays are out-of-equilibrium. Therefore a CP violating out-of-equilibrium decay of the heavy right-handed neutrinos can produces a lepton asymmetry, which is then converted into the baryon asymmetry by the $B + L$ violating sphaleron process [3]. Leptogenesis occurs at a high energy scale of temperature $T \simeq M_1$, where $M_1$ is the mass of the heavy RH neutrino.

In low energy neutrino physics the CP violating parameters are expressed in terms of three phases in the neutrino mixing matrix, the Dirac phase $\delta$, and two Majorana phases $\alpha_1$ and $\alpha_2$. The Dirac phase will be measured in long baseline experiments and the recent very precise measurement of sizeable neutrino reactor angle $\theta_{13}$ by [4, 5] has made it within the reach of these future experiments ([6] and references therein). The Majorana phases does not appear in the oscillation measurements but can be observed in the effective Majorana mass in neutrinoless double beta ($0\nu\beta\beta$) decay experiments. For a detailed discussion on $0\nu\beta\beta$ decays see Ref. [7].

CP violation in low energy sector may come from the phases appearing in the left handed and the right handed fields, whereas the phases responsible for leptogenesis are those appearing in the right handed fields. So observation of low energy CP violation does not necessarily also mean high energy CP violation. But there has been many efforts in the past to connect them in the flavoured leptogenesis scenario [8–10]. Thermal leptogenesis is basically studied in a 'single flavour' regime where all the lepton flavours are considered to be indistinguishable and all the charged lepton Yukawa coupling are out-of-equilibrium, which is true only for temperatures $T \geq 10^{12} GeV$. However, if we consider CP to be an exact symmetry of the right handed (RH) sector then the total CP asymmetry, which is sum of the asymmetry in each flavour goes to zero. Therefore leptogenesis occurring at temperature greater than $10^{12} GeV$ cannot generate a non-zero CP asymmetry for this class of models. But for temperatures within $10^9 \leq T \leq 10^{12} GeV$ the charged $\tau$ Yukawa couplings come into equilibrium and the lepton asymmetry can still survive even in the presence of CP invariant RH sector. It was shown in [9, 10] that in these class of models with the CP invariant RH neutrino sector it is possible to connect the low to the high scale CP violation when flavour effects are considered. In [9] the authors studied the correlation between the low CP violating phases for the observed baryon asymmetry for different light neutrino mass hierarchies around the central values of the neutrino oscillation parameters considering the value of $\sin \theta_{13}$ to be 0.01 and 0.15. In [10] the authors drew a correlation between low energy CP invariant term $J_{CP}$ and the baryon asymmetry of the universe. In the present
work we have scanned for the range of recent 1σ values of the oscillation parameters and carried the analysis for the three possible light neutrino mass spectrum and have observed a considerable change from the earlier results. This becomes interesting in the light of the recent measurements of precise value of $\theta_{13}$ and stringent limits on BAU by the Planck and WMAP 9. We would also like to mention here that we have considered the right handed heavy neutrinos to be hierarchical.

In next section we give the recent neutrino oscillation parameters. In section III we give a brief introduction to Type I seesaw and give the expressions for CP asymmetry for the different neutrino mass hierarchies and discuss each case explicitly in flavoured leptogenesis scenario. Finally in section IV we conclude with summary and discussion.

II. NEUTRINO OSCILLATION PARAMETERS

The best fit and 1σ values of the mass squared differences and the mixing angles given by the recent neutrino oscillation data are:

$$
(\Delta m^2_{21})_{bf} = 7.50 \times 10^{-5} eV^2, \quad 7.31 \times 10^{-5} \leq \Delta m^2_{21} \leq 7.68 \times 10^{-5} eV^2
$$

$$
(\Delta m^2_{31})_{bf} = 2.473 \times 10^{-3} eV^2 (NH), \quad 2.40 \times 10^{-3} \leq \Delta m^2_{31} \leq 2.54 \times 10^{-3} eV^2
$$

$$
(\Delta m^2_{23})_{bf} = 2.427 \times 10^{-3} eV^2 (IH), \quad 2.39 \times 10^{-3} \leq \Delta m^2_{23} \leq 2.49 \times 10^{-3} eV^2
$$

$$
(\sin^2 \theta_{12})_{bf} = 0.302, \quad 0.290 \leq \sin^2 \theta_{12} \leq 0.315
$$

$$
(\sin^2 \theta_{23})_{bf} = 0.413, \quad 0.388 \leq \sin^2 \theta_{23} \leq 0.450
$$

$$
(\sin^2 \theta_{13})_{bf} = 0.0227, \quad 0.020 \leq \sin^2 \theta_{13} \leq 0.025.
$$

The above data gives three possible hierarchies of neutrino mass spectrum:

- **Normal Hierarchy**: $m_1 \ll m_2 < m_3$.
- **Inverted Hierarchy**: $m_3 \ll m_1 < m_2$.
- **Quasi-degenerate**: $m_1 \simeq m_2 \simeq m_3 \simeq m$.

The $\Delta m^2_{21}$ denotes the solar mass squared difference $\Delta m^2_{SOL}$ while $\Delta m^2_{31}$ and $\Delta m^2_{23}$ denote the atmospheric mass squared differences $\Delta m^2_{atm}$ for normal and inverted hierarchy respectively. The leptonic mixing matrix is generally parametrised as:

$$
U_{PMNS} = \begin{pmatrix}
    c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
    -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & s_{13}s_{23} \\
    s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - c_{23}s_{13}s_{12}e^{i\delta} & c_{13}s_{23}
\end{pmatrix} \text{diag} \left(1, e^{i\alpha_1}, e^{i\alpha_2}\right),
$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$ respectively. Here $\delta$ is the Dirac CP phase and the Majorana phases are given by $\alpha_1$ and $\alpha_2$. Although the present oscillation data gives very precise values of the mass squared differences and mixing angles but there are still no bounds on the Dirac and Majorana phases from the experiments. Moreover we do not have information about the exact value of the absolute neutrino mass and also the correct neutrino mass pattern.
III. TYPE I SEESAW MODEL AND LEPTOGENESIS

In the Type I seesaw addition of three RH neutrinos to the Standard Model (SM), which are singlet under the group $SU(2) \times U(1)$, gives the $3 \times 3$ light neutrino mass matrix to be

$$m_\nu = m_D^T M_R^{-1} m_D,$$

(8)

where $m_\nu$ is the light neutrino mass matrix, $M_R$ is the right handed (RH) neutrino mass matrix and $m_D$ is the Dirac neutrino mass matrix. In the basis where the charged lepton Yukawa coupling are diagonal, the relevant terms in the lagrangian that gives the above seesaw formula after spontaneous symmetry breaking is:

$$L_m = -\frac{1}{2} m_L^\nu T C^\dagger L - m_D^\nu_R L - \frac{1}{2} M_R^\nu T C^\dagger R + h.c.$$

(9)

The first term in the above equation goes to zero as it is not invariant under $SU(2) \times U(1)$. Therefore, we have

$$L_m = -N_L^\nu T C^\dagger M N_L + h.c.,$$

(10)

where

$$N_L = \left( \begin{array}{c} \nu_L \\ \nu_R \end{array} \right) \text{ and } M = \left( \begin{array}{cc} 0 & m_D^T \\ M_D & M_R \end{array} \right).$$

(11)

For $M_R \gg m_D$ the above mass matrix can be block diagonalised and we get the effective light neutrino mass matrix to be

$$m_\nu \simeq m_D^T M_R^{-1} m_D = v^2 Y_\nu^T M_R^{-1} Y_\nu,$$

(12)

where $v$ is the vev and $Y_\nu$ is the neutrino Yukawa coupling matrix. Here we consider the RH Majorana mass matrices to be real and diagonal such that $M_R = M_R^{dia} = Dia(M_1, M_2, M_3)$. Under such an assumption the low energy phases in $m_\nu$ can appear only in the Yukawa couplings. The PMNS matrix diagonalising $m_\nu$ is given by

$$U^T m_\nu U = dia(m_1, m_2, m_3) = m^{dia}. $$

(13)

Using the Casas Ibarra parametrisation [12] the $Y_\nu$’s can be written as

$$Y_\nu = \frac{1}{v} \sqrt{M_R^{dia}} R \sqrt{m^{dia}} U^T.$$

For a particular lepton flavour $l$ where $l = e, \mu, \tau$,

$$Y_{i\ell} = \frac{1}{v} \sqrt{M_i} R_{ik} \sqrt{m^{dia}} U_{1k}.$$

(14)

The matrix $R$ is a orthogonal matrix which is in general complex but in our case it is a real matrix as CP is an exact symmetry of the RH sector. The self energy and vertex corrections of the decays : $N_1 \rightarrow l + \phi$ and $N_1 \rightarrow \bar{l} + \phi$ gives a CP asymmetry because of the difference in the decay rates of the two modes. Taking flavour effects [13] into account the lepton asymmetry for each flavour $l$ is given by

$$\epsilon_l = -\frac{3M_1}{16\pi v^2} \frac{Im \left( \sum_{\alpha\beta} m_\alpha^{1/2} m_\beta^{3/2} U_{1\alpha}^* U_{1\beta} R_{1\alpha} R_{1\beta} \right)}{\sum_{\alpha} m_\alpha |R_{1\alpha}|^2}$$

(15)
and the total CP asymmetry $\epsilon_1 = \sum l \epsilon_l$. Since $U$ is a unitary matrix and $R$ is real here, we get the total asymmetry $\epsilon_1$ to be zero when summed over all the lepton flavours. We should note here that for the CP asymmetry in each flavour to be non-zero it also requires $R$ to be non-diagonal. It shows that single flavour approximation models with exact CP symmetry in the RH neutrino sector gives vanishing lepton asymmetry. But we have already mentioned that single flavour approximation is true only for temperature greater than $10^{12}\text{GeV}$. If we go to temperatures $T \leq 10^{12}\text{GeV}$ the tau lepton Yukawa interactions come into equilibrium and if we go below $10^9\text{GeV}$, the $\mu$ Yukawa couplings also come into equilibrium. It is in this temperature region $10^9 \leq T \leq 10^{12}\text{GeV}$ where the tau leptons becomes distinguishable from the $e$ and $\mu$. Then the total CP asymmetry, which is the sum of CP asymmetry due to $\tau$ and $e + \mu$ is non zero. We prefer to work in the range $10^9 \leq T \leq 10^{12}\text{GeV}$ in our analysis, where only $\tau$ leptons are in equilibrium. The baryon asymmetry also depends on the wash out parameter, which for each flavour $l = e, \mu, \tau$ is given by

$$\tilde{m}_l \equiv \frac{Y_{l1}^* Y_{l1} v^2}{M_1} = \sum_i R_{1j}^2 m_j^2 U_{ij}^* U_{lj}, \quad j = 1, 2, 3$$

(16)

and

$$m_* \equiv 8\pi v^2 H|_{T=M_1} \simeq 1.1 \times 10^{-3}\text{eV},$$

(17)

where the $\tilde{m}_l$ and $m_*$ are related to the decay rate of the RH neutrino $N_1$ and the expansion rate of the universe respectively. Therefore the efficiency factor is

$$\eta(\tilde{m}_l) \simeq \left( \frac{\tilde{m}_l}{8.25 \times 10^{-3}\text{eV}} \right)^{-1} + \left( \frac{0.2 \times 10^{-3}}{\tilde{m}_l} \right)^{-1.16}.$$  

(18)

The final baryon asymmetry which is the sum of the asymmetries in each flavour, can be obtained after solving the Boltzmann equations taking flavour effects into account [13]:

$$Y_B \simeq -\frac{12}{37 g^*} \left( \epsilon_2 \eta \left( \frac{417}{589} \tilde{m}_2 \right) + \epsilon_\tau \eta \left( \frac{390}{589} \tilde{m}_\tau \right) \right).$$

(19)

As the lepton asymmetries in $e$ and $\mu$ are indistinguishable in this range, we can combine the two lepton asymmetries and the wash out parameters such that

$$\epsilon_2 = \epsilon_e + \epsilon_\mu = -\epsilon_\tau, \quad \tilde{m}_2 = \tilde{m}_e + \tilde{m}_\mu.$$  

(20)

Moreover, the above equation shows that it is sufficient to calculate the CP asymmetry of the $\tau$ in this particular limit. Throughout the analysis we have also neglected the scatterings by heavier RH neutrinos $N_{2,3}$.

**A. Leptogenesis in $\nu$ mass models with Normal hierarchy ($m_1 \ll m_2 \ll m_3$):**

In this section we consider leptogenesis in light neutrino mass models with Normal hierarchy mass pattern such that $m_2 \simeq \sqrt{\Delta m_{\odot}^2}$ and $m_3 \simeq \sqrt{\Delta m_{\text{atm}}^2}$. As $m_1 \ll m_{2,3}$ the CP
asymmetry term in eqn.(15) gives

\[ \epsilon_l = -\frac{3M_1}{16\pi v^2} \left( \frac{m_2^{1/2} m_3^{3/2} R_{12} R_{13} \text{Im}(U_{12}^* U_{13})}{m_2 R_{12}^2 + m_3 R_{13}^2} \right) \]

\[ + \frac{m_2^{1/2} m_3^{3/2} R_{12} R_{13} \text{Im}(U_{12}^* U_{13})}{m_2 R_{12}^2 + m_3 R_{13}^2} \]

\[ = -\frac{3M_1}{16\pi v^2} \left( \frac{\Delta m_2^2 \Delta m_{atm}^{1/4} (1 - \rho) R_{12} R_{13}}{\rho R_{12}^2 + R_{13}^2} \right) \text{Im}(U_{12}^* U_{13}), \text{ where } \rho = \sqrt{\frac{\Delta m_2^2}{\Delta m_{atm}}} \]

\[ = -\frac{3M_1}{16\pi v^2} \frac{\sqrt{\Delta m_{atm}^2} \sqrt{\rho (1 - \rho) R_{12} R_{13}}}{\rho R_{12}^2 + R_{13}^2} \text{Im}(U_{12}^* U_{13}). \] (21)

Therefore the CP asymmetry \( \epsilon_r \) is

\[ \epsilon_r = -\frac{3M_1}{16\pi v^2} \sqrt{\Delta m_{atm}^2} \sqrt{\rho (1 - \rho) R_{12} R_{13}} \times c_{13} c_{23} \left\{ \begin{array}{l} c_{12} s_{23} \sin \left( \frac{\alpha_1 - \alpha_2}{2} \right) \\ + c_{23} s_{13} \sin \left( \frac{\alpha_1 - \alpha_2}{2} + \delta \right) \end{array} \right\}. \] (22)

Further we get

\[ \bar{m}_2 = \sqrt{\Delta m_{atm}^2} \left\{ R_{13}^2 \left( s_{13}^2 + c_{13}^2 s_{23}^2 \right) + R_{12}^2 \rho \left[ c_{12}^2 c_{23}^2 + s_{12}^2 c_{13}^2 + s_{13}^2 s_{23}^2 - \frac{1}{2} \sin 2\theta_{12} \sin 2\theta_{23} \sin s_{13} \cos \delta \right] \right. \]

\[ + \left. R_{12} R_{13} \sqrt{\rho} \left[ c_{12} c_{13} \sin 2\theta_{23} \cos \left( \frac{\alpha_1 - \alpha_2}{2} \right) + s_{12} \sin 2\theta_{13} c_{23} \cos \left( \frac{\alpha_1 - \alpha_2}{2} + \delta \right) \right] \right\} \] (23)

and

\[ \bar{m}_r = \sqrt{\Delta m_{atm}^2} \left\{ R_{13}^2 c_{13}^2 c_{23}^2 + R_{12}^2 \rho \left[ c_{23}^2 s_{12}^2 s_{13}^2 + c_{12}^2 s_{23}^2 + \frac{1}{2} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta \right] \right. \]

\[ - \left. R_{12} R_{13} \sqrt{\rho} \left[ c_{12} c_{13} \sin 2\theta_{23} \cos \left( \frac{\alpha_1 - \alpha_2}{2} \right) + c_{23} s_{12} \sin 2\theta_{13} \cos \left( \frac{\alpha_1 - \alpha_2}{2} + \delta \right) \right] \right\}. \] (24)

Using eqn.(20) in eqn.(19) we get

\[ Y_B \simeq -\frac{12}{37} \frac{\epsilon_r}{g^*} \left( \eta \left( \frac{390}{589} \bar{m}_r \right) + \eta \left( \frac{417}{589} \bar{m}_2 \right) \right). \] (25)

1. **Values of \( R_{12} \) and \( R_{13} \) for strong wash-out**

The condition for strong wash-out is \( \bar{m}_r \gg m^* \). We use eqn.(17) and get the bound on the value of \( R_{12} \) and \( R_{13} \) to have a strong wash out

\[ R_{12} \gg \frac{1.1 \times 10^{-3} \text{eV}}{\sqrt{\Delta m_{atm}^2} \left[ c_{23}^2 s_{12}^2 s_{13}^2 + c_{12}^2 s_{23}^2 + \frac{1}{2} \sin 2\theta_{12} \sin 2\theta_{23} \sin s_{13} \cos \delta \right]}. \]

The dominating term in the denominator of the above equation is \( c_{23}^2 s_{12}^2 s_{13}^2 \). For the central value of the oscillation parameters, the term \( c_{23}^2 s_{12}^2 s_{13}^2 \) is of the order of \( \mathcal{O}(100) \) smaller than the dominating term due to the presence of \( s_{23}^2 \). The term containing \( \cos \delta \) can be
FIG. 1: The left panel and the right panel show the allowed low energy Dirac and Majorana phases for the observed 1σ range of BAU in normal hierarchical neutrino mass model in strong wash-out and weak wash-out regime respectively. The figures are for 1σ value of the oscillation parameters and $M_1 = 5.0 \times 10^{11} \text{GeV}$.

either positive or negative depending upon whether we are taking δ to be maximum(zero) or minimum(π). Thus we find the approximate lower bound for $R_{12}$ to be

$$R_{12} \gg 0.75 \quad \text{for} \; \delta = 0 \; \text{and} \; R_{12} \gg 0.59 \quad \text{for} \; \delta = \pi$$

(26)

for the strong wash-out regime and similarly for $R_{13}$ we get

$$R_{13}^2 \gg \frac{1.1 \times 10^{-3} eV}{\sqrt{\Delta m_{atm}^2 c_{12}^2 c_{23}^2}} \quad \Rightarrow \; R_{13} \gg 0.2.$$  

(27)

For the calculations in the strong wash-out regime we use $R_{12} = 0.81$, $R_{13} = 0.5$ and $M_1 = 5.0 \times 10^{11} \text{GeV}$. Now using eqn.(22), eqn.(23), eqn.(24) and the 1σ values of the neutrino oscillation parameters and putting these into the eqn.(25) we try to fix the allowed low energy phases for the observed 1σ range of $Y_B(8.294 \times 10^{-11} - 8.508 \times 10^{-11})$. The left panel of fig.1 shows the correlation between the effective Majorana phases $(\alpha_1 - \alpha_2)$ and Dirac CP phase δ which appears in the expression for CP asymmetry in the strong wash out regime in normal hierarchy. We can see from the fig.1 that for δ varying from 0 to 2π only certain regions of $(\alpha_1 - \alpha_2)$ are allowed for the observed range 1σ of baryon asymmetry. This is different from the earlier analysis [9] where the authors did the analysis for the best-fit value of the neutrino oscillation parameters. In the present work we scan over the 1σ range of the recent neutrino oscillation data and try to find the allowed values of Dirac and Majorana phase which can generate the required baryon asymmetry. The study becomes important with the recent measurement of the neutrino reactor angle $\theta_{13}$ and also with the updated measurement of the BAU.
2. Values of $R_{12}$ and $R_{13}$ for weak wash-out

The condition for weak wash-out ($\bar{m}_r \ll m^*$) requires the value of $R_{12} \ll 0.75$ and $R_{13} \ll 0.2$. So we can safely consider $R_{12} = 0.45$ and $R_{13} = 0.01$ for our analysis in the weak-wash out regime. The right panel of fig. 1 shows the correlation of the phases $(\alpha_1 - \alpha_2)$ and $\delta$ in the weak wash out regime for normal hierarchy mass model for the above values of $R_{12}$ and $R_{13}$. Here too we see that for all the values of $\delta$ i.e. from zero to $2\pi$, only certain values of $(\alpha_1 - \alpha_2)$ are allowed and most of the region are excluded. Fig. 2 depicts the relation

FIG. 2: The figure shows the allowed Majorana phases in strong(blue) and weak(cyan) wash out regime for neutrino mass model with normal hierarchy. The plots are for $1\sigma$ value of the $\nu$ oscillation parameters and $M_1 = 5.0 \times 10^{11} \text{GeV}$.

between Majorana phases in strong and weak wash out regime denoted by blue and cyan colours respectively. It is clear from the figure that some of the regions are totally excluded both in weak as well in strong wash out.

The left panel of fig. 3 shows the relation between $Y_B$ and the effective Majorana mass when the CP violating Dirac phase $\delta$ is set to zero. The range of the effective Majorana mass $|m_{ee}|$ in the allowed range of $Y_B$ is far below the experimental reach of GERDA [15] which is 10 meV. But we can relate the Jarlskog Invariant $J_{CP}$ to $Y_B$ as shown in the right panel of fig. 3 setting the Majorana phases to zero. Although this was studied by Pascoli et al. [10], we find a significant change in the allowed values of $J_{CP}$ with the present values of $Y_B$ and recent oscillation parameters. These allowed values now lie between -0.01 to -0.006 and 0.006 to 0.01 for the best-fit values of the neutrino oscillation parameters, whereas the earlier allowed values of $J_{CP}$ were lying between 0.02 to 0.03 and -0.03 to -0.02.

B. Leptogenesis in $\nu$ mass models with Inverted hierarchy ($m_2 > m_1 \gg m_3$):

In the inverted hierarchy model we consider the case where the contributions from the terms containing $m_3$ are negligible as compared to $m_1$ and $m_2$. Here, the lepton flavour CP
asymmetry expression under this approximation looks like

\[
\epsilon_l = -\frac{3 M_1}{16 \pi v^2} \left( \frac{m_1^{1/2} m_2^{3/2} R_{11} R_{12} \text{Im} (U^*_{11} U_{12})}{m_1 R_{11}^2 + m_2 R_{12}^2} + \frac{m_1^{1/2} m_2^{3/2} R_{112} R_{11} \text{Im} (U^*_{12} U_{11})}{m_1 R_{11}^2 + m_2 R_{12}^2} \right) \\
= -\frac{3 M_1}{16 \pi v^2} \frac{m_1^{1/2} m_2^{1/2}}{m_1 R_{11}^2 + m_2 R_{12}^2} \left[ m_2 \text{Im} (U^*_{11} U_{12}) + m_2 \text{Im} (U^*_{12} U_{11}) \right]
\]

We have \( m_2 \simeq m_1 \simeq \sqrt{\Delta m^2_{\text{atm}}} \) and from the relations \( m_2^2 - m_1^2 = \Delta m^2_\odot \) and \( m_2^2 - m_3^2 = \Delta m^2_{\text{atm}} \) we get \( m_2 - m_1 \simeq \frac{\Delta m^2_{\text{atm}}}{2 \Delta m^2_{\odot}} \). Substituting all these in the above expression we get

\[
\epsilon_l = -\frac{3 M_1}{32 \pi v^2} \frac{R_{11} R_{12}}{R_{11}^2 + R_{12}^2} \sqrt{\Delta m^2_\odot \rho \text{ Im} (U^*_{11} U_{12})},
\]

\[
\approx -\frac{3 M_1}{32 \pi v^2} \frac{R_{11} R_{12}}{R_{11}^2 + R_{12}^2} \sqrt{\Delta m^2_\odot \rho} \frac{1}{2} c_{12} s_{13} \left( c_{23}^2 s_{13}^2 - s_{23}^2 \right) \sin \frac{\alpha_1}{2} \\
+ \frac{1}{2} \sin \theta_{23} s_{13} \left[ c_{12}^2 \sin \left( \frac{\alpha_1}{2} - \delta \right) - s_{12}^2 \sin \left( \frac{\alpha_1}{2} + \delta \right) \right],
\]

\[
\approx -\frac{3 M_1}{32 \pi v^2} \frac{R_{11} R_{12}}{R_{11}^2 + R_{12}^2} \sqrt{\Delta m^2_\odot \rho} \frac{1}{2} c_{12} s_{13} \left( c_{23}^2 s_{13}^2 - s_{23}^2 \right) \sin \frac{\alpha_1}{2} \\
+ \frac{1}{2} \sin \theta_{23} s_{13} \left( \cos \delta \sin \frac{\alpha_1}{2} \cos \theta_{12} - \cos \frac{\alpha_1}{2} \sin \theta_{12} \right),
\] (28)
The wash-out factors $\tilde{m}_{2\tau}$ are

$$
\tilde{m}_{2} \simeq \sqrt{\Delta m_{\text{atm}}^2} \left\{ R_{12}^2 c_{12}^2 c_{23}^2 + R_{12}^2 c_{12}^2 s_{13}^2 + R_{12}^2 s_{12}^2 s_{13}^2 s_{23}^2 - \frac{1}{2} R_{12}^2 \sin 2\theta_{12} \sin 2\theta_{23}s_{13} \cos \delta + R_{11}^2 c_{23}^2 s_{12}^2 \\
+ R_{11}^2 c_{12}^2 c_{13}^2 + R_{11}^2 c_{12}^2 s_{13}^2 s_{23}^2 - \frac{1}{2} R_{11}^2 \sin 2\theta_{12} \sin 2\theta_{23}s_{13} \cos \delta + R_{11} R_{12} \sin 2\theta_{12} c_{12}^2 s_{23}^2 \cos \frac{\alpha_1}{2} \\
- R_{11} R_{12} \sin 2\theta_{23} c_{12}^2 s_{13} \cos \left( \frac{\alpha_1}{2} - \delta \right) + R_{11} R_{12} \sin 2\theta_{23} c_{12}^2 s_{13} \cos \left( \frac{\alpha_1}{2} - \delta \right) \right\}.
$$

(29)

The wash-out factors $\tilde{m}_{\tau}$ are

$$
\tilde{m}_{\tau} \simeq \sqrt{\Delta m_{\text{atm}}^2} \left\{ c_{23}^2 s_{12}^2 s_{13}^2 R_{12}^2 + c_{23}^2 s_{12}^2 s_{13}^2 R_{12}^2 + R_{12}^2 \sin 2\theta_{12} \sin 2\theta_{23} s_{13} c_{23} \cos \delta \\
+ R_{11}^2 \left( c_{23}^2 s_{12}^2 s_{13}^2 + s_{12}^2 s_{23}^2 - \frac{1}{2} \sin 2\theta_{12} \sin 2\theta_{23} s_{13} \cos \delta \right) \\
+ R_{11} R_{12} \sin 2\theta_{12} \left( c_{23}^2 s_{12}^2 - s_{23}^2 \right) \cos \frac{\alpha_1}{2} + R_{11} R_{12} \sin 2\theta_{23} c_{12}^2 s_{13} \cos \left( \frac{\alpha_1}{2} - \delta \right) \\
- R_{11} R_{12} \sin 2\theta_{23} c_{12}^2 s_{13} \cos \left( \frac{\alpha_1}{2} + \delta \right) \right\}.
$$

(30)

We have $\tilde{m}_{2} + \tilde{m}_{\tau} = \sqrt{\Delta m_{\text{atm}}^2}$. In this particular case of the heaviest RH neutrino $N_3$ decoupling, we have $R_{11}^2 + R_{12}^2 = 1$ [14]. Therefore we vary the values of $R_{11}^2$ and $R_{12}^2$ between 0 and 1. The left and the right panel of fig.4 shows the baryon asymmetry that can be generated for normal and inverted hierarchy respectively. The thick green colour curves are for $Y_B$ generated with only Majorana phases and the dashed red colour curves denote $Y_B$ only via Dirac CP phase. In the fig[4] we have put the figure for normal hierarchy for the sake of comparison with the inverted hierarchy. As we can see that the normal hierarchy model can generate the required $Y_B$ for the Majorana phase contribution but the inverted hierarchy model can generate a maximum $Y_B \sim 6.0 \times 10^{-12}$. This is far below the observed...
value and therefore it is not possible to generate the observed baryon asymmetry in the inverted hierarchy neutrino mass models with CP symmetric RH sector. This is because in this case the CP symmetry term is suppressed by the term $\sqrt{\Delta m^2_{\text{atm}}} \times \rho$. Our result for the inverted hierarchy agrees with the earlier works [9, 10].

C. Leptogenesis in Quasi-degenerate models ($m = m_1 \simeq m_2 \simeq m_3 \gg \Delta m^2_{\text{atm}}$):

For the quasi-degenerate models the CP asymmetry term is given by:

$$
\epsilon_\tau = \frac{3M_1}{16\pi v^2} \sum_i \frac{1}{m_i R_{i1}} \text{Im} \left\{ R_{11}^2 m_1 |U_{\tau 1}|^2 + R_{11} R_{12} m_1^{1/2} m_2^{3/2} U_{\tau 1}^* U_{\tau 2} + R_{11} R_{13} m_1^{1/2} m_3^{3/2} U_{\tau 1}^* U_{\tau 3} + R_{11} R_{12} m_1^{1/2} m_2^{3/2} U_{\tau 2}^* U_{\tau 1} + R_{22}^2 m_1 |U_{\tau 2}|^2 + R_{12} R_{13} m_2^{1/2} m_3^{3/2} U_{\tau 2}^* U_{\tau 3} + R_{11} R_{13} m_3^{1/2} m_1^{3/2} U_{\tau 3}^* U_{\tau 1} + R_{13} R_{12} m_3^{1/2} m_2^{3/2} U_{\tau 3}^* U_{\tau 2} + R_{33}^2 m_3 |U_{\tau 3}|^2 \right\}
$$

$$
= \frac{3M_1}{16\pi v^2} \sum_i \frac{1}{m_i R_{i1}} \left\{ R_{11} m_1 |U_{\tau 1}|^2 + R_{22} m_2 |U_{\tau 2}|^2 + R_{33} m_3 |U_{\tau 3}|^2 \right\}
$$

$$
+ R_{11} R_{12} m_1^{1/2} m_2^{1/2} (m_2 - m_1) \text{Im} \left( U_{\tau 1}^* U_{\tau 2} \right) + R_{11} R_{13} m_1^{1/2} m_3^{1/2} (m_3 - m_1) \text{Im} \left( U_{\tau 1}^* U_{\tau 3} \right) + R_{12} R_{13} m_2^{1/2} m_3^{1/2} (m_3 - m_2) \text{Im} \left( U_{\tau 2}^* U_{\tau 3} \right) \right\}
$$

$$
= \frac{3M_1}{16\pi v^2} \sum_i \frac{1}{R_{i1}} \left\{ R_{11} R_{12} (m_2 - m_1) \text{Im} \left( U_{\tau 1}^* U_{\tau 2} \right) + R_{11} R_{13} (m_3 - m_1) \text{Im} \left( U_{\tau 1}^* U_{\tau 3} \right) + R_{12} R_{13} (m_3 - m_2) \text{Im} \left( U_{\tau 2}^* U_{\tau 3} \right) \right\}.
$$

Ignoring the terms $m_2 - m_1 \simeq \sqrt{\Delta m^2_{\text{atm}}} / 2m_0$, $m_3 - m_1 \simeq \Delta m^2_{\text{atm}} / 2m_0$ and $m_3 - m_2 \simeq \Delta m^2_{\text{atm}} / 2m_0$ we get

$$
\epsilon_\tau \approx \frac{3M_1}{16\pi v^2} \frac{\Delta m^2_{\text{atm}}}{2m} \times \left\{ R_{11} R_{13} \text{Im} \left( U_{\tau 1}^* U_{\tau 3} \right) + R_{12} R_{13} \text{Im} \left( U_{\tau 2}^* U_{\tau 3} \right) \right\}
$$

$$
= \frac{3M_1}{32\pi v^2} \frac{\Delta m^2_{\text{atm}}}{m} \times \left\{ \frac{1}{2} R_{11} R_{13} \left( c_{13} \sin 2\theta_{23} s_{12} \sin \frac{\alpha_2}{2} - \sin 2\theta_{13} c_{23}^2 c_{12} \sin \left( \frac{\beta}{2} - \delta \right) \right) \right\}
$$

$$
= \frac{1}{2} R_{12} R_{13} \left( c_{13} \sin 2\theta_{23} s_{12} \sin \frac{\alpha_1 - \alpha_2}{2} + c_{23}^2 s_{12} \sin 2\theta_{12} \sin \left( \frac{\alpha_1 - \alpha_2}{2} + \delta \right) \right) \right\}
$$

$$
= \frac{3M_1}{64\pi v^2} \frac{\Delta m^2_{\text{atm}}}{m} \times \left\{ R_{11} R_{13} \left[ c_{13} \sin 2\theta_{23} s_{12} \sin \frac{\alpha_2}{2} - \sin 2\theta_{13} c_{23}^2 c_{12} \sin \left( \frac{\beta}{2} - \delta \right) \right] \right\}
$$

$$
R_{12} R_{13} \left[ c_{13} \sin 2\theta_{23} s_{12} \sin \frac{\alpha_1 - \alpha_2}{2} + c_{23}^2 s_{12} \sin 2\theta_{12} \sin \left( \frac{\alpha_1 - \alpha_2}{2} + \delta \right) \right] \right\}.
$$

The wash out factor (see eqn. 16) in the quasi-degenerate case becomes

$$
\bar{m}_e + \bar{m}_\mu + \bar{m}_\tau = m \left( R_{11}^2 + R_{12}^2 + R_{13}^2 \right)
$$

and

$$
\bar{m}_l = m |R_{11} U_{11}^* + R_{12} U_{12}^* + R_{13} U_{13}^*|^2.
$$

In the degenerate case the wash out factor is proportional to the absolute neutrino mass $m$. So we can see from eqn. 18 that for smaller values of the neutrino absolute mass we can
have large efficiency factors. Therefore for the strong wash out condition \( \tilde{m}_\tau \) is much smaller than the absolute mass \( m \), i.e., \( \tilde{m}_\tau \ll m \). For the best-fit values of the neutrino oscillation parameters we get

\[
\tilde{m}_\tau = m R_{13}^2 \times 0.1889
\]

(34)

for \( \alpha_1 = \alpha_2 = \pi \) and \( \delta = \pi/2 \) and also taking \( R_{11} = R_{12} = R_{13} \). Therefore we have

\[
R_{13}^2 \gg \frac{1.1 \times 10^{-3}}{m \times 0.1889}
\]

(35)

for strong wash-out. The Planck measurements \(^\text{[1]}\) gives sum of the neutrino mass bound to be \( \sum m_\nu < 0.23 \text{eV} \). For the smallest allowed value of absolute mass \( m = 0.07 \text{eV} \), we get \( R_{13}^2 \gg 0.13 \). In the strong wash out regime considering \( R_{13}^2 = 0.26 \) we get the maximum efficiency to be

\[
\eta_{\text{max}} \simeq \left( \frac{0.2 \times 10^{-3}}{390 \tilde{m}_\tau} \right)^{1.16}
\]

\[
= 0.0595.
\]

(36)

Using eqn.(31) one can obtain the CP asymmetry for the above values of CP phases \( (\alpha_1 = \alpha_2 = \pi \) and \( \delta = \pi/2 \) \) and taking \( R_{11} = R_{12} = R_{13} \). Using eqn.(19) and \( M_1 = 5.0 \times 10^{11} \text{GeV} \) we get the maximum value of the \( Y_B \) to be less than \( 2.58 \times 10^{-19} \) for lowest allowed value of the neutrino absolute mass \( m \). This is much smaller than the observed value of \( Y_B \).

Similarly for the weak wash-out condition we require \( \tilde{m}_\tau \ll 0.13 \). Considering \( R_{13}^2 = 0.06 \) the maximum efficiency is found to be 0.0638 and \( Y_B \) to be less than \( 2.76 \times 10^{-19} \). Thus we see that in order to achieve the range of observed BAU we need \( M_1 \gg 10^{12} \text{GeV} \) and a much lower value of the absolute neutrino mass. Therefore the Quasi-Degenerate neutrino mass model does not seem to be favourable choice if we have an exact CP symmetric and hierarchical heavy right handed neutrinos.

IV. SUMMARY AND CONCLUSION:

Low energy CP violating phases responsible for low energy leptonic CP violation may or may not be responsible for CP violation in high energy scale. Therefore it is in general not possible to connect both the CP violations. But if we consider the right handed sector to be CP symmetric then we have the advantage of generating a non-zero CP asymmetry from the low energy phases alone in flavoured leptogenesis scenario \(^\text{[3,4]}\). Under this particular assumption only normal hierarchical light neutrino mass model is able to generate the required baryon asymmetry of the universe with the present values of the neutrino oscillation data.

In this work we have analysed this particular scenario in details taking into account the recent neutrino oscillation data \(^\text{[11]}\) and Planck results \(^\text{[1]}\) and also considering the right handed heavy neutrinos to be hierarchical. We find that only certain combinations of effective Majorana and Dirac CP phases are allowed in normal hierarchy for the recent \( 1\sigma \) value of the baryon asymmetry \( Y_B \). These combinations are different for the strong and weak wash-out regimes. Our analysis differs from the previous works, as we have varied the neutrino masses and mixing parameters within \( 1\sigma \) range of recent oscillation data. We have also shown that in inverted hierarchy models the maximum BAU generated is about \( 6 \times 10^{-12} \).
which is much less than the observed value of $8.40 \times 10^{-11}$ and is in agreement with the earlier works. Therefore within this particular scenario the inverted hierarchy neutrino mass model is not a favourable one. In the quasi-degenerate case also it is not possible to generate the observed $Y_B$ in the temperature region we are considering i.e., $(10^9 \leq T \leq 10^{12} \text{GeV})$ and with an absolute neutrino mass $m$ greater than 0.07eV. Therefore, in future if the experiments measuring the sum of the neutrino masses $\sum m_\nu$ restricts it to the present value of the bound then the degenerate model will also not survive if CP is an exact symmetry of right handed sector and if leptogenesis takes place within the above mentioned temperature range. But this would not be true if the right handed heavy neutrinos are also quasi-degenerate.

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