Stock portfolio selection based on investors’ risk preference

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Abstract. Stock portfolio investment gives investors the opportunity to engage in diversification among stocks which helps to achieve higher financial risk-adjusted return. A properly diversified stock portfolio should include stocks that have different economic variable such as interest rate, exchange rate and share price. It helps investors to achieve higher risk-adjusted return, nevertheless there are investors who love risk known as Risk Premium investors and Risk Averse investors who prefer lower risk. The risk value of stocks acts as an indicator in selecting stocks to be included in a portfolio. Risk value of stock portfolios is used in selecting stock portfolio that follows investors’ risk preference. This study forecast share price by using Geometric Brownian Motion and hence use the Variance Covariance to calculate Value at Risk of each stock and stock portfolio in future investment. The stock portfolios in this study are based on five sectors which are industrial product, consumer, construction, technology, and trading and services. Thus, there are five stock portfolios that are produced for each type; Risk Averse and Risk Premium investors. Using the Value at risk of each stock portfolio, it is found that Stock portfolio from the Industrial Product Trading and Service sectors is the most suitable for Risk Averse and Risk Premium investors respectively. This study may serve as a guide for investors in creating and choosing stock portfolios for future investment based on their risk preference and also forecast its Value at Risk.

1. Introduction

Stock or equity security is one of the financial tradable assets. The stock is traded or invested to gain profit and is believed to be the best financial instrument in promising a higher possible return. Stocks can be invested in a portfolio form. A stock portfolio consists of a combination of stocks of different companies, location or sector. Investing in a stock portfolio helps to achieve risk-adjusted return instead of investing in a single stock. The risk is adjusted when loss caused by a poor performance stock is counter back with a profit gain from a good stock performance.

Risk is chance of having negative outcome and unexpected loss. It cannot be eliminated but can be minimized the chance of loss or risk in an investment indicates the possible return. The higher the risk, the higher the possible return and the lower the risk, the lower the possible return. Risk Premium is an investor who is willing to take high risks to achieve above average return; while an investor who is the opposite to a Risk Premium investor is called Risk Averse. The availability of risk value is required to assist the process in selecting the right stock that should be included in a portfolio based on investors’ risk preference. There are various different portfolios that can be set up. Each portfolio will have stocks...
of different companies. Thus, investors also had to evaluate risk value for each portfolio, so that the best stock portfolio for Risk Averse or Risk Premium investors can be determined [6]. It is not a simple task to evaluate risk value for single stock and multiple stock portfolio. Hence, a systematic method such as a mathematical model is required to assist investors in determining risk value before entering stock investment.

The main objective in this study is to proposed a mathematical method in determining stock portfolios that follow investors risk preference by evaluating Value at Risk of multiple stock portfolio using Variance Covariance method.

2. Materials and methods

The collected data are share prices of Shariah-compliant securities (stocks) which are listed on Bursa Malaysia starting from 1st October 2017 until 30th April 2018; which is the latest 7-months period when the study is conducted [4]. The stocks selected are the Top Ten Gainers ranking stocks on 26th April 2018 for each five sectors; which are industrial-product, consumer, construction, technology, and trading and service [8].

Each sector would build two portfolios that consist five stocks for each Risk Averse and Risk Premium [5]. Thus, the total stock number of companies that are involved in this study are 25 in which five portfolios are formed from each sector.

In determining the time interval, this study has compared three distinct time interval which are one-week (5 day), two-week (10 day) and one-month (20 day). It is found that the two-week time interval has the most number of stocks with MAPE value below 10%. Thus, two-week forecast share price is used in evaluating Value at Risk of each single stock and multiple stock portfolio.

The methodology undertaken comprises of three parts. First, the forecast share price is determined by using Geometric Brownian Motion [11],[1],[9],[2]. Then, future VaR is obtained for each stock to build a stock portfolio [3]. Lastly, future VaR is calculated for each stock portfolio in order to determine the best portfolio to invest [7]. Numerical implementations are explained in three parts.

2.1 Part 1: Forecasting share price by using Geometric Brownian Motion

The first six-month actual share prices is used as historical data in the process of computing the next one-month forecast share price by using Geometric Brownian Motion [11]. Hence, the seventh month actual share price is used in measuring the accuracy of forecast share price by using Mean Absolute Percentage Error [9],[10].

Rate of return, drift and volatility is determined before estimating the future shares price by using Geometric Brownian Motion.

Rate of return, \( R_t \) is calculated in equation (2.1)

\[
R_t = \ln \frac{S_t}{S_{t-1}} \tag{2.1}
\]

where, \( S_t \) is share price at time \( t \) and \( S_{t-1} \) is share price at time \( t-1 \). Rate of return indicates the percentage growth of shares at time \( t \).

Growth rate of shares or drift rate, \( \mu \) is calculated as shown in equation (2.2)

\[
\mu = \frac{1}{M \delta t} \sum_{t=1}^{M} R_t \tag{2.2}
\]

where, \( M \) is total number of returns in a sample and \( \delta t \) is the timestep.

Volatility, \( \sigma \) is calculated in this study by using Log Volatility as shown in equation (2.3)

\[
\sigma = \left[ \frac{1}{(M - 1) \delta t} \sum_{t=1}^{M} (log S_t - S_{t-1})^2 \right]^{\frac{1}{2}} \tag{2.3}
\]
In this study, total number of returns in sample, $M$ for six months is 123 days due to trading day from 1st October 2017 until 30th March 2018 excluding public holiday and weekends and the timestep $\delta t$ is 1/123.

Share price is forecasted by using stochastic model as shown in equation (2.4)

$$S(t) = S(0)e^{(\frac{1}{2}\sigma^2)t + \sigma(X(t) - X(0))}$$

(2.4)

where $S(t)$ is forecast share price at time $t$, $S(0)$ is actual share price at $t = 0$ and $X(t) - X(0)$ is random value. A two-week (10-day) time interval is applied to produce $n = 10$ of highly accurate forecast share price [9].

MAPE is calculated to ensure that the forecast share price is valid and reliable by using (2.5)

$$\varepsilon = \frac{\sum |Y_t - F_t|}{10} \times 100$$

(2.5)

MAPE indicate the model accuracy by using its range as shown in table 2.1. The model accuracy increases as the MAPE value decreases.

**Table 2.1. MAPE Judgement Range**

| MAPE range     | Level of accuracy |
|----------------|-------------------|
| Below than 10% | Highly accurate   |
| 11%-20%        | Good forecast     |
| 21%-50%        | Reasonable forecast|
| 51% and above  | Inaccurate forecast|

**Table 2.2. MAPE and VaR values for each stock.**

| Sector          | Stock | MAPE value | VaR value $\alpha = 95\%$ | VaR value $\alpha = 99\%$ |
|-----------------|-------|------------|-----------------------------|-----------------------------|
| Construction    | CT 1  | 2.632495   | 0.00644173                  | 0.00912416                  |
|                 | CT 2  | 3.237548   | 0.0472796                   | 0.0669675                   |
|                 | CT 3  | 3.574447   | 0.0232269                   | 0.0328989                   |
|                 | CT 4  | 21.59978   | 0.0391735                   | 0.0554858                   |
|                 | CT 5  | 9.497645   | 0.0114713                   | 0.0162489                   |
|                 | CT 6  | 2.54013    | 2.6749183                   | 3.7887901                   |
|                 | CT 7  | 2.872458   | 0.060674                    | 0.0142799                   |
|                 | CT 8  | 5.773059   | 0.0859395                   | 0.0202263                   |
|                 | CT 9  | 9.356149   | 0.0456278                   | 0.0346228                   |
|                 | CT10  | 2.369117   | 0.0523321                   | 0.0373411                   |
| Consumer        | CS 1  | 1.053082   | 0.021900                    | 0.031020                    |
|                 | CS 2  | 4.609359   | 0.066903                    | 0.094762                    |
|                 | CS 3  | 5.750369   | 0.048036                    | 0.068039                    |
|                 | CS 4  | 2.706814   | 0.010964                    | 0.015530                    |
|                 | CS 5  | 3.370079   | 0.004602                    | 0.006518                    |
|                 | CS 6  | 3.840424   | 0.001691                    | 0.002395                    |
|                 | CS 7  | 6.227788   | 0.016779                    | 0.023766                    |
|                 | CS 8  | 3.652712   | 0.384833                    | 0.545083                    |
|                 | CS 9  | 2.563078   | 0.037649                    | 0.053326                    |
|                 | CS10  | 2.018872   | 0.053222                    | 0.075384                    |
| Industrial Product | IP 1  | 2.9262     | 0.25789648                  | 0.36528803                  |
The MAPE value and VaR for each stock is shown in Table 2.2. From the MAPE value, 90% of the forecast stock prices are highly accurate, 8% are good, and 2% are reasonable forecast. For each sector, the companies with top five VaR will be included in the Risk premium and the remaining companies will be included in the Risk averse portfolio.

2.2 Evaluate VaR for single/each Stock

The purpose of evaluating VaR for every stock in each sector is to select stocks that have the lowest or the highest VaR value to form a portfolio. Portfolio with stocks that have lowest VaR is created for Risk Averse, while portfolio with stocks that have highest VaR is created for Risk Premium investors. VaR for single stock would be computed by using Variance Covariance method. Forecast data of share prices is the main input. Variance Covariance method have two main components which are standard normal deviation corresponding to specific confidence level, \( \alpha \) and standard deviation, \( \sigma \). This study uses confidence levels, at 95% and 99%. Both of the confidence levels are commonly used in solving real life financial institution problem (Aslinda, 2018).

Rate of return is computed by using forecast price and calculated by using equation (2.6):

\[
\tilde{R}_t = \ln \frac{F_t}{F_{t-1}} \tag{2.6}
\]

where, \( \mu \) is rate of return of forecast share price, \( F_t \) is forecast share price at time \( t \) and \( F_{t-1} \) is forecast share price at time \( t-1 \). Drift, \( \mu \) is calculate by using equation (2.7)
\[ \mu = \frac{1}{n \delta t} \sum_{t=1}^{n} \bar{R}_t \]  

(2.7)

where, \( \mu \) is drift, \( n = 10 \) is total number of returns in a sample of a two-week (10-day) time interval; thus, the timestep \( \delta t = \frac{1}{10} \).

Standard deviation, \( \sigma \) is calculated using equation (2.8)

Finally, VaR for single asset (stock), \( \nu \) can be calculated using equation (2.9)

\[ \sigma = \left( \frac{1}{n-1} \sum_{t=1}^{n} (\bar{R}_t - \mu)^2 \right)^{\frac{1}{2}} \]  

(2.8)

\[ \nu = \frac{1}{(n-1)} \sum_{t=1}^{n} F_t \alpha \sigma \]  

(2.9)

where, \( \sigma \) is the standard normal deviation corresponding to specific confidence level \( \alpha \).

2.3 Part 3: Evaluate VaR for Multiple Stocks Portfolio by using Matrix Approach

Calculation process of Multiple Asset Portfolio by using matrix multiplication approach is divided into three steps which are; determine standard normal distribution corresponding to confidence level, estimate standard deviation and obtain VaR value for a portfolio.

2.3.1 Step 1: Determine standard normal distribution corresponding to confidence level, \( \alpha \). The confidence level use in evaluating VaR for Multiple Stocks Portfolio is same as confidence level use in evaluating VaR for Single Stocks. At 95% and 99% confidence levels, standard normal deviations are 1.65 and 2.33 respectively.

2.3.2 Step 2: Estimate standard deviation in a portfolio, \( \sigma_p \). Standard deviation for Multiple Asset Portfolio deals with weightage assumption, standard deviation for single asset, covariance and correlation.

The weightage matrix is a column matrix \( W = [ w_i ]_{1 \times m} \) where \( w_i \) is weightage of asset \( i \) \( (i = 1, 2, \ldots, m) \) and \( m \) is total number of assets in a portfolio. This study assumes each company has equal weightage; which means each company holds 20% proposition or weightage in a portfolio. Thus, weightage matrix, \( W \) of each portfolio in this study is a column matrix \( W = [ w_i ]_{1 \times 5} \) where each \( w_i = 0.2 \) \( (i = 1, 2, \ldots, 5) \).

Standard deviation measures dispersion of rate of return in sample from its average value. Standard deviation for each single asset, \( \sigma_i \) is calculated by using equation (3.8). Standard deviation for single asset is combined to construct volatility matrix, \( \Sigma = [ \sigma_{ij} ]_{m \times m} \) \( (i = 1, 2, \ldots, m) \). Since this study considers a portfolio is built up by five assets, \( m=5 \); thus \( \Sigma = [ \sigma_{ij} ]_{5 \times 5} \) as shown in (2.10).

\[
\Sigma = \begin{bmatrix}
\sigma_1 & 0 & 0 & 0 & 0 \\
0 & \sigma_2 & 0 & 0 & 0 \\
0 & 0 & \sigma_3 & 0 & 0 \\
0 & 0 & 0 & \sigma_4 & 0 \\
0 & 0 & 0 & 0 & \sigma_5 \\
\end{bmatrix} 
\]  

(2.10)

Covariance explains movement of return for each two assets in a portfolio and is obtained by using equation (2.11).

\[ \sigma_{x,y} = \frac{\sum_{t=1}^{n}(x_{t} - \bar{x})(y_{t} - \bar{y})}{n - 1} \]  

(2.11)
where $\sigma_{x,y}$ is covariance between asset $x$ and asset $y$, $x_t$ is rate of return asset $x$ at time $t$, $y_t$ is rate of return asset $y$ at time $t$, $\bar{x}$ is average rate of return of asset $x$ and $\bar{y}$ is average rate of return of asset $y$; $n$ is total number of return sample.

Return deviation is the numerator in covariance fraction in equation (2.11). The strength of covariance is determined by its correlation value, $\rho_{x,y}$ which is obtained using equation (2.12).

$$
\rho_{x,y} = \frac{\text{Cov}(x,y)}{\sigma_x \sigma_y} = \frac{\sigma_{x,y}}{\sigma_x \sigma_y}
$$

(2.12)

where, $\sigma_x$ is standard deviation of asset $x$ and $\sigma_y$ is standard deviation of asset $y$. Both $\sigma_x$ and $\sigma_y$ is computed by using equation (2.8).

The range of correlation is between -1 and +1. If correlation value is -1, the two assets are said to move perfectly in opposite direction. If correlation value is +1, the two assets are said to move perfectly in same direction. Thus, correlation matrix, $C = [\rho_{i,j}]_{m \times m}$ ($i, j = 1, 2, \ldots, m$). Since this study considers a portfolio is built up by five assets, $m=5$; thus

$$
C = \begin{bmatrix}
1 & \rho_{1,2} & \rho_{1,3} & \rho_{1,4} & \rho_{1,5} \\
\rho_{2,1} & 1 & \rho_{2,3} & \rho_{2,4} & \rho_{2,5} \\
\rho_{3,1} & \rho_{3,2} & 1 & \rho_{3,4} & \rho_{3,5} \\
\rho_{4,1} & \rho_{4,2} & \rho_{4,3} & 1 & \rho_{4,5} \\
\rho_{5,1} & \rho_{5,2} & \rho_{5,3} & \rho_{5,4} & 1
\end{bmatrix}
$$

(2.13)

Covariance matrix, $\Phi = [\phi_{ij}]_{m \times m}$ ($i,j = 1,2,\ldots,m$) is obtained by applying cross equation (2.12) which would give covariance equation as in (2.14).

$$
\phi_{x,y} = \rho_{x,y} \sigma_x \sigma_y = \sigma_x \rho_{x,y} \sigma_y
$$

(2.14)

We know that $x$ and $y$ are two assets that are correlated to each other. However, there are $m=10$ assets that are possible to be asset $x$ and asset $y$ in a portfolio. Thus, possible asset would be $i = 1, 2, \ldots, m$. Hence, according to cross equation (2.14), covariance also can be obtained by multiplying volatility matrix, $\nu$ and correlation matrix, $C$ and volatility matrix, $V$ as in equation (2.15).

$$
\phi_{x,y} = \sigma_x \rho_{x,y} \sigma_y = VCV
$$

(2.15)

$$
\phi_{x,y} = VCV = \begin{bmatrix}
\sigma_1^2 & \sigma_1 \rho_{1,2} \sigma_2 & \sigma_1 \rho_{1,3} \sigma_3 & \sigma_1 \rho_{1,4} \sigma_4 & \sigma_1 \rho_{1,5} \sigma_5 \\
\sigma_2 \rho_{2,1} \sigma_1 & \sigma_2^2 & \sigma_2 \rho_{2,3} \sigma_3 & \sigma_2 \rho_{2,4} \sigma_4 & \sigma_2 \rho_{2,5} \sigma_5 \\
\sigma_3 \rho_{3,1} \sigma_1 & \sigma_3 \rho_{3,2} \sigma_2 & \sigma_3^2 & \sigma_3 \rho_{3,4} \sigma_4 & \sigma_3 \rho_{3,5} \sigma_5 \\
\sigma_4 \rho_{4,1} \sigma_1 & \sigma_4 \rho_{4,2} \sigma_2 & \sigma_4 \rho_{4,3} \sigma_3 & \sigma_4^2 & \sigma_4 \rho_{4,5} \sigma_5 \\
\sigma_5 \rho_{5,1} \sigma_1 & \sigma_5 \rho_{5,2} \sigma_2 & \sigma_5 \rho_{5,3} \sigma_3 & \sigma_5 \rho_{5,4} \sigma_4 & \sigma_5^2
\end{bmatrix}
$$

Standard deviation in portfolio is product square root of variance, $\sigma_p^2$ as shown in equation (2.16)

$$
\sigma_p^2 = W^T \Phi W = W^T VCVW
$$

(2.16)

where $W^T$ weightage transpose matrix and $W$ is weightage matrix. Thus, in case of five number of stocks in a portfolio, variance value for a portfolio is computed by using equation (2.16).
\[ \sigma_p^2 = \sigma_{1,1}^2 + 2w_1w_2\sigma_{1,2} + 2w_1w_3\sigma_{1,3} + 2w_1w_4\sigma_{1,4} + 2w_1w_5\sigma_{1,5} + \sigma_{2,2}^2 + 2w_2w_3\sigma_{2,3} + 2w_2w_4\sigma_{2,4} + 2w_2w_5\sigma_{2,5} + \sigma_{3,3}^2 + 2w_3w_4\sigma_{3,4} + 2w_3w_5\sigma_{3,5} + 2w_3w_5\sigma_{3,5} + 2w_4w_5\sigma_{4,5} + \sigma_{5,5}^2 \]

Thus, standard deviation of a portfolio, \( \sigma_p \), is obtained by square root of the variance in portfolio, \( \sigma_p^2 \), value as equation (2.17).

\[ \sigma_p = \left( \sigma_p^2 \right)^{1/2} \]  

(2.17)

2.3.3 Step 3: Compute VaR value. Finally, VaR of Multiple Asset Portfolio, \( V \), is computed using the equation (2.18). (Noor Baizura, 2016).

\[ V = \alpha \times \sigma_p \]

(2.18)

The values of the computed VaR for each sector is shown in Table 3.3.

3. Result and Discussion

This study involves five sectors which are consumer, construction, trading and service, technology and industrial product. Each sector forms two portfolios which are for Risk Premium and Risk Averse. Stock portfolio for Risk Averse is combination of stocks that have the lowest VaR values and the vice versa for stock portfolio for Risk Premium.

Tables 3.1 and 3.2 summarize stock combination in portfolio formed for Risk Averse and Risk Premium respectively.

| Sectors | Construction | Consumer | Industrial Product | Technology | Trading and Service |
|---------|--------------|----------|--------------------|------------|---------------------|
| Portfolio | 1 | 2 | 3 | 4 | 5 |
| Stocks | CT 1 | CS 1 | IP 2 | TH 1 | TS 1 |
| | CT 3 | CS 4 | IP 4 | TH 4 | TS 3 |
| | CT 6 | CS 5 | IP 5 | TH 8 | TS 4 |
| | CT 9 | CS 6 | IP 6 | TH 9 | TS 5 |
| | CT 10 | CS 7 | IP 8 | TH 10 | TS 8 |

| Sectors | Construction | Consumer | Industrial Product | Technology | Trading and Service |
|---------|--------------|----------|--------------------|------------|---------------------|
| Portfolio | 1 | 2 | 3 | 4 | 5 |
| Stocks | CT 2 | CS 2 | IP 1 | TH 2 | TS 2 |
| | CT 4 | CS 3 | IP 3 | TH 3 | TS 6 |
| | CT 5 | CS 8 | IP 7 | TH 5 | TS 7 |
| | CT 7 | CS 9 | IP 9 | TH 6 | TS 9 |
| | CT 8 | CS 10 | IP 10 | TH 7 | TS 10 |

Table 3.1. List of Stocks in Portfolios for Risk Averse

Table 3.2. List of Stocks in Portfolios for Risk Premium

Table 3.3 presents VaR values which are obtained from equation 2.18 for five portfolios that are suggested for Risk Averse and Risk Premium at 95% and 99% confidence level.

| Sector | VaR values | Rank |
|--------|------------|------|
| Construction | 0.014044 0.019863 | R4 |
| Risk Averse | Risk Premium |
| VaR values | VaR values |
| \( \alpha = 95\% \) | \( \alpha = 99\% \) | \( \alpha = 95\% \) | \( \alpha = 99\% \) | Rank |
| 0.016937 0.023954 R4 | |
It shows that for Risk Averse, the lowest VaR portfolio value for Risk Averse is Portfolio 3 followed by Portfolio 4, 2, 1 and 5. Thus, Portfolio 3 from sector Industrial Product is the most suggested stock portfolio to Risk Averse investors. As for Risk Premium, the highest VaR is Portfolio 5 followed by Portfolio 4, 2, 1 and 3. Hence, Portfolio 5 from sector Trading and Service is the most recommended stock portfolio for Risk Premium investors.

4. Conclusion
This study gives significant to investor who are planning to enter stock investment. The existing Variance Covariance method is only able to calculate VaR for the next day. This study comes out with Variance Covariance method that calculates VaR for the next two-week. Thus, investor have ample time to think before confirms their decision to start the investment. Furthermore, this study may serve as a guide for investors in creating and choosing stock portfolios based on their risk preference for future investment and also forecast its Value at Risk.

This study can be improved in many aspects for future study. Firstly, this study only limit to five industry sectors due to time constraint. Thus, next study can be expanded to the rest of all the industry sector. Next, the forecasting method used in this study may replace by another method than GBM such as ARIMA or by using fuzzy approach. The main reason this study used GBM because forecasting share price for short term. Thus, future study may forecast share price for a longer term so that the study can guide investors to know future VaR for one or two years ahead of time. In addition, instead of computing VaR value by using Variance Covariance, further study may use other VaR methods such as Monte Carlo Simulation or Historical Simulation.

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