Direct $CP$ violation for $\bar{B}_s^0 \to K^0\pi^+\pi^-$ decay in QCD factorization

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Abstract

In the framework of QCD factorization, based on the first order of isospin violation, we study direct $CP$ violation in the decay of $\bar{B}_s^0 \to K^0\rho^0(\omega) \to K^0\pi^+\pi^-$ including the effect of $\rho - \omega$ mixing. We find that the $CP$ violating asymmetry is large via $\rho - \omega$ mixing mechanism when the invariant mass of the $\pi^+\pi^-$ pair is in the vicinity of the $\omega$ resonance. For the decay of $\bar{B}_s^0 \to K^0\rho^0(\omega) \to K^0\pi^+\pi^-$, the maximum $CP$ violating asymmetries can reach about 46%. We also discuss the possibility to observe the predicted $CP$ violating asymmetries at the LHC.

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I. INTRODUCTION

$CP$ violating asymmetry is one of the most important areas in the decays of bottom hadrons. In the standard model (SM), a non-zero complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix is responsible for $CP$ violating phenomena. In recent years $CP$ violation in several $B$ decays such as $B^0 \rightarrow J/\psi K^0_S$ and $B^0 \rightarrow K^+ \pi^-$ has indeed been found in experiments [1, 2]. Due to its much higher statistics, the Large Hadron Collider (LHC) will provide a new opportunity to search for more $CP$ violation signals.

Direct $CP$ violating asymmetries in $b$-hadron decays occur through the interference of at least two amplitudes with the weak phase difference $\phi$ and the strong phase difference $\delta$. The weak phase difference is determined by the CKM matrix while the strong phase is usually difficult to control. In order to have a large $CP$ violating asymmetries signal, we have to apply some phenomenological mechanism to obtain a large $\delta$. It has been shown that the charge symmetry violating mixing between $\rho^0$ and $\omega$ can be used to obtain a large strong phase difference which is required for large $CP$ violating asymmetries. Furthermore, it has been shown that the measurement of the $CP$ violating asymmetries can be used to remove the mod($\pi$) ambiguity in the determination of the $CP$ violating phase angle $\alpha$ [3–7].

Naive factorization approximation has been shown to be the leading order result in the framework of QCD factorization when the radiative QCD corrections of order $O(\alpha_s(m_b))$ ($m_b$ is the $b$-quark mass) and the $O(1/m_b)$ corrections in the heavy quark effective theory are neglected [8]. In naive factorization scheme, the hadronic matrix elements of four-quark operators are assumed to be saturated by vacuum intermediate states. Since the bottom hadrons are very heavy, their hadronic decays are energetic. Hence the quark pair generated by one current in the weak Hamiltonian moves very fast away from the weak interaction point. Therefore, by the time this quark pair hadronizes into a meson, it is already far away from other quarks and is unlikely to interact with the remaining quarks. This quark pair is factorized out and generates a meson [9, 10]. This approximation can only estimate the $CP$ violation order neglecting QCD correction. Furthermore, as pointed out in previous studies [3–7], in order to take into account the nonfactorizable contributions, an effective parameter, $N_c$, is introduced. The deviation of the value of $N_c$ from the color number, 3, measures the nonfactorizable effects in the naive factorization scheme. Obviously, $N_c$ should depend on the hadronization dynamics of different decay channels. In this scheme, $CP$ violation depends strongly on $N_c$ values, which makes the results uncertainties.

In the heavy quark limit, QCD factorization [8] includes nonfactorization strong interaction correction, and the decay amplitudes can be calculated at leading power in $\Lambda_{QCD}/m_b$ and at next-to-leading order in $\alpha_s$, which can be expressed in terms of form factors and meson light-cone distribution amplitudes. One can take into account the nonfactorizable and chirally enhanced hard-scattering spectator and annihilation contributions which appear at order $O(\alpha_s(m_b))$ and $O(1/m_b)$, respectively. In this work we adopt the QCD factorization scheme including order-$\alpha_s$ correction to compute $CP$ violating asymmetry of the decay $\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$ via the $\rho - \omega$ mixing mechanism. As will be shown later, the $CP$ violating asymmetries in this decay channel could be large and may be observed in the LHC experiments.

The remainder of this paper is organized as follows. In Sec. II, we present the form
of the effective Hamiltonian and the general form of QCD factorization. In Sec. III, we give the formalism for $CP$ violating asymmetries in $B_s^0 \rightarrow K^0\pi^+\pi^-$ decay. In Sec. IV, we calculate the branching ratio for decay process of $B_s^0 \rightarrow K^0\rho^0(\omega)$ via $\rho - \omega$ mixing. We briefly discuss the input parameters in Sec. V. The numerical results are given in Sec. VI. In Sec. VII we discuss the possibility to observe the predicted $CP$ violating asymmetries at the LHC. Summary and conclusions are included in Sec. VIII.

II. THE EFFECTIVE HAMILTONIAN

With the operator product expansion [11], the effective Hamiltonian in bottom hadron decays is

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \sum_{q=d,s} V_{pb} V_{pq}^* (c_1 O_1^p + c_2 O_2^p) + \sum_{i=3}^{10} c_i O_i + c_{7\gamma} O_{7\gamma} + c_{8g} O_{8g} + H.c.,$$

where $c_i \ (i = 1, \ldots, 10, 7\gamma, 8g)$ are the Wilson coefficients, $V_{pb}, V_{pq}$ are the CKM matrix elements. The operators $O_i$ have the following form:

$$
\begin{align*}
O_1^p &= \bar{p}\gamma_\mu(1 - \gamma_5)b\bar{q}\gamma^\mu(1 - \gamma_5)p, \\
O_2^p &= \bar{p}\gamma_\mu(1 - \gamma_5)b\bar{q}\gamma^\mu(1 - \gamma_5)p, \\
O_3 &= \bar{q}\gamma_\mu(1 - \gamma_5)b\sum_{q'} q'\gamma^\mu(1 - \gamma_5)q', \\
O_4 &= \bar{q}\gamma_\mu(1 - \gamma_5)b\sum_{q'} q'\gamma^\mu(1 - \gamma_5)q', \\
O_5 &= \bar{q}\gamma_\mu(1 - \gamma_5)b\sum_{q'} q'\gamma^\mu(1 + \gamma_5)q', \\
O_6 &= \bar{q}\gamma_\mu(1 - \gamma_5)b\sum_{q'} q'\gamma^\mu(1 + \gamma_5)q', \\
O_7 &= \frac{3}{2} \bar{q}\gamma_\mu(1 - \gamma_5)b\sum_{q'} e_{q'} q'\gamma^\mu(1 + \gamma_5)q', \\
O_8 &= \frac{3}{2} \bar{q}\gamma_\mu(1 - \gamma_5)b\sum_{q'} e_{q'} q'\gamma^\mu(1 + \gamma_5)q', \\
O_9 &= \frac{3}{2} \bar{q}\gamma_\mu(1 - \gamma_5)b\sum_{q'} e_{q'} q'\gamma^\mu(1 - \gamma_5)q', \\
O_{10} &= \frac{3}{2} \bar{q}\gamma_\mu(1 - \gamma_5)b\sum_{q'} e_{q'} q'\gamma^\mu(1 - \gamma_5)q', \\
O_{7\gamma} &= \frac{e}{8\pi^2} m_b\bar{s}\sigma_{\mu\nu}(1 + \gamma_5)F^{\mu\nu}b, \\
O_{8g} &= \frac{e}{8\pi^2} m_b\bar{s}\sigma_{\mu\nu}(1 + \gamma_5)G^{\mu\nu}b,
\end{align*}
$$

where $\alpha$ and $\beta$ are color indices, $O_1^p$ and $O_2^p$ are the tree operators, $O_3 - O_8$ are QCD penguin operators which are isosinglets, $O_7 - O_{10}$ arise from electroweak penguin operators which have both isospin 0 and 1 components. $O_{7\gamma}$ and $O_{8g}$ are the electromagnetic and chromomagnetic dipole operators. $e_{q'}$ are the electric charges of the quarks and $q' = u, d, s, c, b$ is implied.

The Wilson coefficients can be calculated at a high scale $M_W$ and then evolved to scale $m_b$ using renormalization group equation. In QCD factorization, We consider weak decay $B_s \rightarrow M_1 M_2$ ($M_1, M_2$ refer to $K^0$ and $\rho^0$ mesons, respectively) in the heavy-quark limit. Up to power corrections of order $\Lambda_{\text{QCD}}/m_b$, the transition matrix element of an operator
\[ \langle M_1 M_2 | O_i | \bar{B} \rangle = \sum_j \mathcal{F}_{j}^{B \rightarrow M_1}(m_2^2) \int_0^1 du T_{ij}(u) \Phi_{M_2}(u) + (M_1 \leftrightarrow M_2) + \int_0^1 d\xi du dv T_{ij}(\xi, u, v) \Phi_B(\xi) \Phi_{M_1}(v) \Phi_{M_2}(u) \]  
if \( M_1 \) and \( M_2 \) are both light,  
\[ (3) \]
Here \( \mathcal{F}_{j}^{B \rightarrow M_{1,2}}(m_{2,1}^2) \) denotes a \( B \rightarrow M_{1,2} \) form factor, and \( \Phi_X(u) \) is the light-cone distribution amplitude for the quark-antiquark Fock state of meson \( X \). \( T_{ij}(u) \) and \( T_{ij}^{II}(\xi, u, v) \) are hard-scattering functions, which are perturbatively calculable. The hard-scattering kernels and light-cone distribution amplitudes (LCDA) depend on a factorization scale and scheme, which is suppressed in the notation of \( (3) \). Finally, \( m_{1,2} \) denote the light meson masses.

We match the effective weak Hamiltonian onto a transition operator, the matrix element is given by \((\lambda_p^{(D)} = V_{pb} V_{pD}^* \) with \( D = d \) or \( s)\)
\[ \langle M'_1 M'_2 | \mathcal{H}_\text{eff} | \bar{B} \rangle = \sum_{p=u,c} \lambda_p^{(D)} \langle M'_1 M'_2 | \mathcal{T}_A^p + \mathcal{T}_B^p | \bar{B} \rangle. \]  
\[ (4) \]
Using the unitarity relation
\[ \lambda_u^{(D)} + \lambda_c^{(D)} + \lambda_t^{(D)} = 0 \]  
\[ (5) \]
we can get
\[ \sum_{p=u,c} \lambda_p^{(D)} \mathcal{T}_A^p = \sum_{p=u,c} \lambda_p^{(D)} \left[ \delta_{pu} \alpha_1(M_1 M_2) A([\bar{q}_s u][\bar{u}D]) + \delta_{pu} \alpha_2(M_1 M_2) A([\bar{q}_s D][\bar{u}u]) \right] \]
\[ + \lambda_u^{(D)} \left[ (\alpha_4^u(M_1 M_2) - \alpha_4^c(M_1 M_2)) \sum_q A([\bar{q}_s q][\bar{q}D]) + (\alpha_4^u_{\text{EW}}(M_1 M_2) - \alpha_4^c_{\text{EW}}(M_1 M_2)) \sum_q \frac{3}{2} e_q A([\bar{q}_s q][\bar{q}D]) \right] \]
\[ - \lambda_t^{(D)} \left[ \alpha_5^c(M_1 M_2) \sum_q A([\bar{q}_s D][\bar{q}q]) + \alpha_5^c(M_1 M_2) \sum_q A([\bar{q}_s q][\bar{q}D]) + \alpha_5^c_{\text{EW}}(M_1 M_2) \sum_q \frac{3}{2} e_q A([\bar{q}_s D][\bar{q}q]) \right] \]
\[ + \alpha_4^c_{\text{EW}}(M_1 M_2) \sum_q \frac{3}{2} e_q A([\bar{q}_s q][\bar{q}D]) \]
where the sums extend over \( q = u, d, s \), and \( \bar{q}_s \) denotes the spectator antiquark. The operators \( A([\bar{q}_M_1 q_{M_1}][\bar{q}_{M_2} q_{M_2}]) \) also contain an implicit sum over \( q_s = u, d, s \) to cover all possible \( B \)-meson initial states.

Next we need change the annihilation part
\[
\sum_{p=u,c} \lambda_p^{(D)} T^p_B = \sum_{p=u,c} \lambda_p^{(D)}
\]
\[
\times \left[ \delta_{pu} b_1(M_1 M_2) \sum_{q'} B([\bar{u}q'][\bar{q}'u][Db]) 
+ \delta_{pu} b_2(M_1 M_2) \sum_{q'} B([\bar{u}q'][\bar{q}'D][\bar{u}b]) \right] 
- \lambda_i^{(D)} \left[ b_3(M_1 M_2) \sum_{q,q'} B([\bar{q}q'][\bar{q}'q][\bar{u}b]) 
+ b_4(M_1 M_2) \sum_{q,q'} B([\bar{q}q'][\bar{q}'q][\bar{D}b]) 
+ b_{3,EW}(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q}q'][\bar{q}'q][\bar{D}b]) 
+ b_{4,EW}(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q}q'][\bar{q}'q][\bar{D}b]) \right]
\]

(7)

where \( b_i \), \( b_{i,EW} \) and \( B \) are given by following. The coefficients of the flavor operators \( \alpha_i^p \) can be expressed in terms of the coefficients \( a_i^p \) defined in [8] as follows:

\[
\alpha_1(M_1 M_2) = a_1(M_1 M_2),
\]

\[
\alpha_2(M_1 M_2) = a_2(M_1 M_2),
\]

\[
\alpha_3^p(M_1 M_2) = \begin{cases} 
  a_3^p(M_1 M_2) + a_5^p(M_1 M_2) ; \\
  \text{if } M_1M_2 = PV,
\end{cases}
\]

\[
\alpha_4^p(M_1 M_2) = \begin{cases} 
  a_4^p(M_1 M_2) + r_{M_2}^M a_6^p(M_1 M_2) ; \\
  \text{if } M_1M_2 = PV,
\end{cases}
\]

\[
\alpha_{3,EW}^p(M_1 M_2) = \begin{cases} 
  a_3^{p,EW}(M_1 M_2) + a_5^{p,EW}(M_1 M_2) ; \\
  \text{if } M_1M_2 = PV,
\end{cases}
\]

\[
\alpha_{4,EW}^p(M_1 M_2) = \begin{cases} 
  a_4^{p,EW}(M_1 M_2) + r_{M_2}^M a_6^{p,EW}(M_1 M_2) ; \\
  \text{if } M_1M_2 = PV,
\end{cases}
\]

(8)

For pseudoscalar (P) meson \( M_1 \), the ratios \( r_{M_1}^M \) are defined as

\[
r_{M_1}^M(\mu) = \frac{2m_{M_1}^2}{m_b(\mu)(m_q + m_s)(\mu)},
\]

(9)

All quark masses are running masses defined in the \( \overline{\text{MS}} \) scheme, and \( m_q \) denotes the average of the up and down quark masses. For vector (V) meson \( M_2 \) we have

\[
r_{M_2}^V(\mu) = \frac{2m_V}{m_b(\mu) f_V^+ (\mu)} f_V^-, 
\]

(10)
where the scale-dependent transverse decay constant $f_{\perp V}$ is defined as
\[
\langle V(p, \varepsilon^*) | \bar{q} \sigma_{\mu \nu} q' | 0 \rangle = f_{\perp V}^{\perp}(p_\mu \varepsilon_{\nu}^{*} - p_\nu \varepsilon_{\mu}^{*}) .
\] (11)

Note that all the terms proportional to $r^{M_2}$ are formally suppressed by one power of $\Lambda_{\text{QCD}}/m_b$ in the heavy-quark limit.

The general form of the coefficients $a_i^p$ at next-to-leading order in $\alpha_s$ is
\[
a_i^p(M_1M_2) = \left( C_i + \frac{C_{i+1}}{N_c} \right) N_i(M_2) \\
+ \frac{C_{i+1}}{N_c} C_F \alpha_s \left[ V_i(M_2) + \frac{4\pi^2}{N_c} H_i(M_1M_2) \right] \\
+ P_i^p(M_2) ,
\] (12)

where $N_c$ is the number of colors, the upper (lower) signs apply when $i$ is odd (even). It is understood that the superscript ‘$p$’ is to be omitted for $i = 1, 2$. The quantities $V_i(M_2)$ account for one-loop vertex corrections, $H_i(M_1M_2)$ for hard spectator interactions, and $P_i^p(M_1M_2)$ for penguin contractions. The $N_i(M_2)$ and $C_F$ are given by
\[
N_i(M_2) = \begin{cases} 
0; & i = 6, 8 \text{ and } M_2 = V, \\
1; & \text{all other cases.} 
\end{cases}
\] (13)
\[
C_F = \frac{N_c^2 - 1}{2N_c} .
\] (14)

The vertex corrections are given by
\[
V_i(M_2) = \begin{cases} 
\int_0^1 dx \Phi_{M_2}(x) \left[ 12 \ln \frac{m_b}{\mu} - 18 + g(x) \right] & (i = 1, 4, 9, 10), \\
\int_0^1 dx \Phi_{M_2}(x) \left[ -12 \ln \frac{m_b}{\mu} + 6 - g(1-x) \right] & (i = 5, 7), \\
\int_0^1 dx \Phi_{m_2}(x) \left[ -6 + h(x) \right] & (i = 6, 8),
\end{cases}
\] (15)

with
\[
g(x) = 3 \left( \frac{1 - 2x}{1 - x} \ln x - i \pi \right) + \left[ 2 \text{Li}_2(x) - \ln^2 x \\
+ \frac{2 \ln x}{1 - x} - (3 + 2i \pi) \ln x - (x \leftrightarrow 1 - x) \right] ,
\] (16)
\[
h(x) = 2 \text{Li}_2(x) - \ln^2 x - (1 + 2i \pi) \ln x - (x \leftrightarrow 1 - x).
\] (17)

The constants $-18, 6, -6$ are scheme dependent and correspond to using the NDR scheme for $\gamma_5$. The light-cone distribution amplitude (LCDA) $\Phi_{M_2}$ is the leading-twist amplitude
of $M_2$, whereas $\Phi_{m_2}$ is the twist-3 amplitude. LCDA for pseudoscalar and vector mesons of twist-2 are

\begin{align}
\Phi_P(x, \mu) &= 6x(1 - x) \left[ 1 + \sum_{n=1}^{\infty} a_n^P(\mu) C_n^{3/2} (2x - 1) \right], \\
\Phi^V_\parallel(x, \mu) &= 6x(1 - x) \left[ 1 + \sum_{n=1}^{\infty} a_n^V(\mu) C_n^{3/2} (2x - 1) \right], \\
\Phi^V_\perp(x, \mu) &= 6x(1 - x) \left[ 1 + \sum_{n=1}^{\infty} a_n^{V, \perp}(\mu) C_n^{3/2} (2x - 1) \right],
\end{align}

(18)

and twist-3 ones

\begin{align}
\Phi_p(x) &= 1, \quad \Phi_\sigma(x) = 6x(1 - x), \\
\Phi_v(x, \mu) &= 3 \left[ 2x - 1 + \sum_{n=1}^{\infty} a_n^{\perp, V}(\mu) P_{n+1} (2x - 1) \right],
\end{align}

(19)

where $C_n(x)$ and $P_n(x)$ are the Gegenbauer and Legendre polynomials, respectively. $a_n(\mu)$ are Gegenbauer moments that depend on the scale $\mu$. $\Phi^V_\parallel(x, \mu)$ and $\Phi^V_\perp(x, \mu)$ are the transverse and longitudinal quark distributions of the polarized mesons.

At order $\alpha_s$ a correction from penguin contractions is present only for $i = 4, 6$. For $i = 4$ we obtain

\begin{align}
P^p_4(M_2) &= \frac{C_F \alpha_s}{4\pi N_c} \left\{ C_1 \left[ \frac{4}{3} \ln \frac{m_b}{\mu} + \frac{2}{3} - G_{M_2}(s_p) \right] \\
&\quad + C_3 \left[ \frac{8}{3} \ln \frac{m_b}{\mu} + \frac{4}{3} - G_{M_2}(0) - G_{M_2}(1) \right] \\
&\quad + (C_4 + C_6) \left[ \frac{4n_f}{3} \ln \frac{m_b}{\mu} - (n_f - 2) G_{M_2}(0) \\
&\quad - G_{M_2}(s_c) - G_{M_2}(1) \right] \\
&\quad - 2C_{8g}^{\text{eff}} \int_0^1 \frac{dx}{1-x} \Phi_{M_2}(x) \right\}, 
\end{align}

(20)

where $n_f = 5$ is the number of light quark flavors, and $s_u = 0$, $s_c = (m_c/m_b)^2$ are mass ratios involved in the evaluation of the penguin diagrams. The function $G_{M_2}(s)$ is given
by
\begin{align*}
G_{M_2}(s) &= \int_0^1 dx \, G(s - i\epsilon, 1 - x) \Phi_{M_2}(x), \\
G(s, x) &= -4 \int_0^1 du \, u(1 - u) \ln[s - u(1 - u)x] \\
&= \frac{2(12s + 5x - 3x \ln s)}{9x} \\
&- \frac{4\sqrt{4s - x}(2s + x)}{3x^{3/2}} \arctan \sqrt{\frac{x}{4s - x}}.
\end{align*}
(21)

For $i = 6$, if $M_2$ is a vector meson, the result for the penguin contribution is
\begin{align*}
P^6_p(M_2) &= -\frac{C_F \alpha_s}{4\pi N_c} \left\{ C_1 \hat{G}_{M_2}(s_p) + C_3 \left[ \hat{G}_{M_2}(0) + \hat{G}_{M_2}(1) \right] \\
&+ (C_4 + C_6) \left[ (n_f - 2) \hat{G}_{M_2}(0) + \hat{G}_{M_2}(s_c) \\
&+ \hat{G}_{M_2}(1) \right] \right\}. \quad (23)
\end{align*}

In analogy with (21), the function $\hat{G}_{M_2}(s)$ is defined as
\begin{align*}
\hat{G}_{M_2}(s) &= \int_0^1 dx \, G(s - i\epsilon, 1 - x) \Phi_{M_2}(x). \quad (24)
\end{align*}

Electromagnetic corrections are present for $i = 8, 10$ and correspond to the penguin diagrams. For $i = 10$ we obtain
\begin{align*}
P^p_{10}(M_2) &= \frac{\alpha}{9\pi N_c} \left\{ (C_1 + N_c C_2) \left[ \frac{4}{3} \ln \frac{m_b}{\mu} \\
&- \frac{2}{3} - G_{M_2}(s_p) \right] - 3C^{\text{eff}}_{\tau \gamma} \int_0^1 dx \, \frac{1 - x}{1 - x} \Phi_{M_2}(x) \right\}. \quad (25)
\end{align*}

For $i = 8$
\begin{align*}
P^p_8(M_2) &= -\frac{\alpha}{9\pi N_c} (C_1 + N_c C_2) \hat{G}_{M_2}(s_p), \quad (26)
\end{align*}
if $M_2$ is a vector meson.

The correction from hard gluon exchange between $M_2$ and the spectator quark is given by
\begin{align*}
H_i(M_1 M_2) &= \frac{B_{M_1 M_2}}{A_{M_1 M_2}} \frac{m_B}{\lambda_B} \int_0^1 dx \int_0^1 dy \left[ \frac{\Phi_{M_2}(x) \Phi_{M_1}(y)}{xy} \\
&+ r^M_{M_1} \frac{\Phi_{M_2}(x) \Phi_{M_1}(y)}{xy} \right], \quad (27)
\end{align*}
for $i = 1$–$4, 9, 10$.

$$H_i(M_1 M_2) = -\frac{B_{M_1 M_2}}{A_{M_1 M_2}} \frac{m_B}{\lambda_B} \int_0^1 dx \int_0^1 dy \left[ \frac{\Phi_{M_2}(x) \Phi_{M_1}(y)}{x y} \right. \right. $$

$$+ \left[ r_X^{M_1} \frac{\Phi_{M_2}(x) \Phi_{M_1}(y)}{x y} \right],$$

(28)

for $i = 5, 7$, and $H_i(M_1 M_2) = 0$ for $i = 6, 8$.

where $\lambda_B$ is defined by

$$\int_0^1 \frac{d\xi}{\xi} \Phi_B(\xi) \equiv \frac{m_B}{\lambda_B}$$

(29)

with $\Phi_B(\xi)$ is one of the two light-cone distribution amplitudes of the $B$ meson.

If $M_1 = P$, $M_2 = V$, $f$ refers to decay constant of relevant meson, $A_{M_1 M_2}$ and $B_{M_1 M_2}$ are given by

$$A_{M_1 M_2} = i \frac{G_F}{\sqrt{2}} (-2) m_{M_1} \epsilon^*_{M_1} \cdot p_B F_{0}^{B \rightarrow M_1}(0) f_{M_2},$$

(30)

$$B_{M_1 M_2} = -\frac{G_F}{\sqrt{2}} f_{B_i} f_{M_1} f_{M_2}.$$  

(31)

where $m_{M_1}$ and $\epsilon_{M_1}$ are the mass and polarization vector of the vector meson. $F_{0}^{B \rightarrow M_1}$ is the form factor for $B \rightarrow M_1$ transition.

We recall that the term involving $r_X^{M_1}$ is suppressed by a factor of $\Lambda_{\text{QCD}}/m_b$ in heavy-quark power counting. Since the twist-3 distribution amplitude $\Phi_{m_1}(y)$ does not vanish at $y = 1$, the power-suppressed term is divergent. We extract this divergence by defining a parameter $X_{H_i}^{M_1}$ through

$$\int_0^1 \frac{dy}{y} \Phi_{m_1}(y) = \Phi_{m_1}(1) \int_0^1 \frac{dy}{y}$$

$$+ \int_0^1 \frac{dy}{y} \left[ \Phi_{m_1}(y) - \Phi_{m_1}(1) \right]$$

$$\equiv \Phi_{m_1}(1) X_{H_i}^{M_1} + \int_0^1 \frac{dy}{[y]} \Phi_{m_1}(y).$$

(32)

The remaining integral is finite (it vanishes for pseudoscalar mesons since $\Phi_p(y) = 1$), but $X_{H_i}^{M_1}$ is an unknown parameter representing a soft-gluon interaction with the spectator quark. Since $X_{H_i}^{M_1}$ varies within a certain range (specified later) and $X_{H_i}^{M_1} \sim \ln(m_b/\Lambda_{\text{QCD}})$, we treat the resulting variation of the coefficients $\alpha_i^p$ as an uncertainty. We also assume that $X_{H_i}^{M_1}$ is universal, i.e., that it does not depend on $M_1$ and on the index $i$ of $H_i(M_1 M_2)$. For the convolution integrals, one can find the results in Ref. [8].

For the annihilation contribution, one can get [8]:

$$b_3^p = \frac{C_F}{N_c^2} \left[ C_3 A_1^i + C_5 (A_3^i + A_2^i) + N_c C_6 A_3^i \right].$$

(33)

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The weak annihilation kernels exhibit endpoint divergences, which we treat in the same manner as the power corrections to the hard spectator scattering. The divergent subtractions are interpreted as
\[ \int_0^1 \frac{dy}{y} \to X_{M1}, \quad \int_0^1 \frac{\ln y}{y} \to -\frac{1}{2} (X_{M1})^2, \]
and similarly for \( M_2 \) with \( y \to \bar{x} \). The treatment of weak annihilation is model-dependent in the QCD factorization approach. We treat \( X_{M1} \) as an unknown complex number of order \( \ln(m_b/\Lambda_{QCD}) \) and make the simplifying assumption that this number is independent of the identity of the meson \( M_1 \) and the weak decay vertex. Here,
\[ A_i^1 \approx -A_i^2 \approx 6\pi\alpha_s \left[ 3\left(X_A - 4 + \frac{\pi^2}{3}\right) \right. \]
\[ \left. + r_{\chi}^{M1} r_{\chi}^{M2} (X_A^2 - 2X_A) \right], \]
\[ A_i^3 \approx 6\pi\alpha_s \left[ -3r_{\chi}^{M2} \left(X_A^2 - 2X_A - \frac{\pi^2}{3}\right) \right. \]
\[ \left. + 4 \right] + r_{\chi}^{M1} \left(X_A^2 - 2X_A + \frac{\pi^2}{3}\right), \]
\[ A_i^f \approx -6\pi\alpha_s \left[ 3r_{\chi}^{M2}(2X_A - 1)(2 - X_A) \right. \]
\[ \left. - r_{\chi}^{M1} (2X_A^2 - X_A) \right], \]
and \( A_1^f = A_2^f = 0 \). Here, \( M_1 \) is \( K^0 \) meson and \( M_2 \) is \( \rho^0 \) meson.

III. \( CP \) VIOLATION IN \( \bar{B}_s^0 \to K^0\pi^+\pi^- \) DECAY

A. Formalism

In the vector meson dominance model [12], the photon propagator is dressed by coupling to vector mesons. Based on the same mechanism, \( \rho - \omega \) mixing was proposed [13]. The formalism for \( CP \) violation in the decay of a bottom hadron, \( B_s \), will be reviewed in the following. The amplitude for \( B_s \to K^0\pi^+\pi^- \), \( A \), can be written as
\[ A = \langle \pi^+\pi^- K^0|H^T|\bar{B}_s \rangle + \langle \pi^+\pi^- K^0|H^P|\bar{B}_s \rangle, \]
where \( H^T \) and \( H^P \) are the Hamiltonians for the tree and penguin operators, respectively. We define the relative magnitude and phases between these two contributions as follows:
\[ A = \langle \pi^+\pi^- K^0|H^T|\bar{B}_s \rangle [1 + re^{i\delta} e^{i\phi}], \]
where $\delta$ and $\phi$ are strong and weak phase differences, respectively. The weak phase difference $\phi$ arises from the appropriate combination of the CKM matrix elements: $\phi = \arg[(V_{tb}V_{ts}^*)/(V_{ub}V_{us}^*)]$. The parameter $r$ is the absolute value of the ratio of tree and penguin amplitudes,

$$r = \left| \frac{\langle \pi^+\pi^- K^0 | H^T | B_s \rangle}{\langle \pi^+\pi^- K^0 | H^T | B_s \rangle} \right|. \quad (41)$$

The amplitude for $B_s \to \bar{K}^0\pi^+\pi^-$ is

$$\bar{A} = \langle \pi^+\pi^- \bar{K}^0 | H^T | B_s \rangle + \langle \pi^+\pi^- \bar{K}^0 | H^P | B_s \rangle. \quad (42)$$

Then, the CP violating asymmetry, $a$, can be written as

$$a = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{-2r \sin \delta \sin \phi}{1 + 2r \cos \delta \cos \phi + r^2}. \quad (43)$$

We can see explicitly from Eq. (40) that both weak and strong phase differences are needed to produce $CP$ violation. $\rho - \omega$ mixing has the dual advantages that the strong phase difference is large and well known [3, 4]. In this scenario one has

$$\langle \pi^+\pi^- K^0 | H^T | B_s \rangle = \frac{g_\rho}{s_\rho s_\omega} \bar{\Pi}_{\rho\omega}(t_\omega + t_\omega^a) + \frac{g_\rho}{s_\rho} (t_\rho + t_\rho^a), \quad (44)$$

$$\langle \pi^+\pi^- K^0 | H^P | B_s \rangle = \frac{g_\rho}{s_\rho s_\omega} \bar{\Pi}_{\rho\omega}(p_\omega + p_\omega^a) + \frac{g_\rho}{s_\rho} (p_\rho + p_\rho^a), \quad (45)$$

where $t_V (V = \rho$ or $\omega$) is the tree amplitude and $p_V$ is the penguin amplitude for producing a vector meson, $V$. $t_\rho^a (V = \rho$ or $\omega$) is the tree annihilation amplitude and $p_\rho^a$ is the penguin annihilation amplitude. $g_\rho$ is the coupling for $\rho^0 \to \pi^+\pi^-$. $\bar{\Pi}_{\rho\omega}$ is the effective $\rho - \omega$ mixing amplitude, and $s_V$ is from the inverse propagator of the vector meson $V$,

$$s_V = s - m_V^2 + i m_V \Gamma_V, \quad (46)$$

with $\sqrt{s}$ being the invariant mass of the $\pi^+\pi^-$ pair.

The direct $\omega \to \pi^+\pi^-$ is effectively absorbed into $\bar{\Pi}_{\rho\omega}$, leading to the explicit $s$ dependence of $\bar{\Pi}_{\rho\omega}$ [13]. Making the expansion $\bar{\Pi}_{\rho\omega}(s) = \bar{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_\omega^2) \bar{\Pi}_{\rho\omega}'(m_\omega^2)$, the $\rho - \omega$ mixing parameters were determined in the fit of Gardner and O’Connell [15]:

$\text{Re} \bar{\Pi}_{\rho\omega}(m_\omega^2) = -3500 \pm 300 \text{ MeV}^2$, $\text{Im} \bar{\Pi}_{\rho\omega}(m_\omega^2) = -300 \pm 300 \text{ MeV}^2$, and $\bar{\Pi}_{\rho\omega}'(m_\omega^2) = 0.03 \pm 0.04$. In practice, the effect of the derivative term is negligible. From Eqs. (40)(41)(44)(45)(46) one has

$$r e^{i\delta} e^{i\phi} = \frac{\bar{\Pi}_{\rho\omega}(p_\omega + p_\omega^a) + s_\omega (p_\rho + p_\rho^a)}{\bar{\Pi}_{\rho\omega}(t_\omega + t_\omega^a) + s_\omega (t_\rho + t_\rho^a)}. \quad (47)$$
Defining

\[ \frac{p_\omega + p_\omega'}{t_\rho + t_\rho'} = r'e^{i(\delta_q + \phi)}, \quad \frac{t_\omega + t_\omega'}{t_\rho + t_\rho'} = \alpha e^{i\delta_\alpha}, \quad \frac{p_\rho + p_\rho'}{p_\omega + p_\omega'} = \beta e^{i\delta_\beta}, \]

(48)

where \( \delta_\alpha, \delta_\beta, \) and \( \delta_q \) are strong phases, one finds the following expression from Eq. (47):

\[ re^{i\delta} = r'e^{i\delta_q} \frac{\tilde{\Pi}_{\rho\omega} + \beta e^{i\delta_\beta} s_\omega}{s_\omega + \Pi_{\rho\omega} \alpha e^{i\delta_\alpha}}. \]

(49)

\( \alpha e^{i\delta_\alpha}, \beta e^{i\delta_\beta}, \) and \( re^{i\delta} \) will be calculated in the QCD factorization approach later. With Eq. (49), we can obtain \( r \sin \delta \) and \( r \cos \delta \). In order to get the \( CP \) violating asymmetry, \( a \), in Eq. (43), \( \sin \phi \) and \( \cos \phi \) are needed. \( \phi \) is determined by the CKM matrix elements.

In the Wolfenstein parametrization [16], one has

\[ \sin \phi = \frac{\eta}{\sqrt{[\rho(1-\rho)-\eta^2]^2 + \eta^2}}, \]

\[ \cos \phi = \frac{\rho(1-\rho)-\eta^2}{\sqrt{[\rho(1-\rho)-\eta^2]^2 + \eta^2}}. \]

(50)

(51)

B. \( CP \) violation via \( \rho - \omega \) mixing

In the following we will study the \( CP \) violating asymmetries in the following decay: \( \bar{B}_s^0 \to K^0\rho^0(\omega) \to K^0\pi^+\pi^- \). With the Eq. (4)(6)(7)(8), we can calculate the decay amplitudes in QCD factorization scheme. The expressions for the \( \bar{B}_s^0 \to K^0\rho^0(\omega) \) amplitudes are given by

\[ \sqrt{2}A_{\bar{B}_s^0 \to K^0\rho^0} = A_{K^0\rho}(\delta_{\rho\alpha_2} - \alpha_4^p + \frac{3}{2}\alpha_{3,EW}^p) \]

\[ \quad + \frac{1}{2}\alpha_{4,EW}^p - \beta_3^p + \frac{1}{2}\beta_{3,EW}^p), \]

(52)

\[ \sqrt{2}A_{\bar{B}_s^0 \to K^0\omega} = A_{K^0\omega}(\delta_{\rho\alpha_2} + 2\alpha_3^p + \alpha_4^p + \frac{1}{2}\alpha_{3,EW}^p) \]

\[ \quad - \frac{1}{2}\alpha_{4,EW}^p + \beta_3^p - \frac{1}{2}\beta_{3,EW}^p), \]

(53)

where

\[ A_{K^0\rho} = (-2)i \frac{G_F}{\sqrt{2}} m_{\rho\rho} e_{\rho}^* \cdot p_B F_{0B_3 \to K^0}(0) f_{\rho}, \]

(54)

\[ A_{K^0\omega} = (-2)i \frac{G_F}{\sqrt{2}} m_{\omega\omega} e_{\omega}^* \cdot p_B F_{0B_3 \to K^0}(0) f_{\omega}. \]

(55)
Here $F_0$ denote $B_s \rightarrow K^0$ meson form factor. $m_{\rho}, m_{\omega}$ are the mass of $\rho^0$ and $\omega$ mesons. $\varepsilon^*_\rho, \varepsilon^*_\omega$ correspond to polarizing vectors. $f$ refers to the decay constant. Then we can get

$$\sqrt{2}A_{B_s \rightarrow K^0, \rho} = A_{K^0, \rho} \left[ \delta_{\rho} a_2, K^0, \rho - a_{4, K^0, \rho}^\rho - \gamma_\chi^\rho a_{6, K^0, \rho}^\rho 
+ \frac{3}{2} \left( a_{9, K^0, \rho}^\rho + a_{7, K^0, \rho}^\rho \right) + \frac{1}{2} (a_{10, K^0, \rho}^\rho 
+ \gamma_\chi^\rho a_{8, K^0, \rho}^\rho) - \beta_3 + \frac{1}{2} \beta_3, EW \right],$$

(56)

$$\sqrt{2}A_{B_s \rightarrow K^0, \omega} = A_{K^0, \omega} \left[ \delta_{\omega} a_2, K^0, \omega + 2 (a_{3, K^0, \omega}^\omega + a_{5, K^0, \omega}^\omega) 
+ a_{4, K^0, \omega}^\omega + \gamma_\omega^\omega a_{6, K^0, \omega}^\omega + \frac{1}{2} (a_{7, K^0, \omega}^\omega + a_{9, K^0, \omega}^\omega) 
- \frac{1}{2} (a_{10, K^0, \omega}^\omega + \gamma_\omega^\omega a_{8, K^0, \omega}^\omega) + \beta_3 - \frac{1}{2} \beta_3, EW \right].$$

(57)

where the form of the coefficients $a_i^\rho$ at next-to-leading order in $a_s$ is given by Eq.(12), which $M_1$ is $K^0$ meson and $M_2$ is $\rho^0$ meson. $\beta_i$ is the weak annihilation contribution in QCD factorization. $\gamma_\chi$ is chirally-enhanced terms which we have denoted above.

From Eq. (6)(7)(48), one can get

$$\alpha e^{i \delta_\alpha} = \frac{t_\omega + t_\rho^a}{t_\rho + t_\rho^a} = \frac{Q_1}{Q_2}$$

(58)

$$Q_1 = A_{K^0, \omega} \left\{ \delta_{\rho} a_2, K^0, \omega + 2 (a_{3, K^0, \omega}^u - a_{3, K^0, \omega}^c) 
+ a_{5, K^0, \omega}^u - a_{5, K^0, \omega}^c + (a_{4, K^0, \omega}^u - a_{4, K^0, \omega}^c) 
+ \gamma_\omega (a_{6, K^0, \omega}^u - a_{6, K^0, \omega}^c) + \frac{1}{2} (a_{7, K^0, \omega}^u - a_{7, K^0, \omega}^c) 
+ a_{9, K^0, \omega}^u - a_{9, K^0, \omega}^c - \frac{1}{2} (a_{10, K^0, \omega}^u - a_{10, K^0, \omega}^c) 
+ \gamma_\omega (a_{8, K^0, \omega}^u - a_{8, K^0, \omega}^c) \right\}$$

(59)
\[ Q_2 = A_{K^0\rho} \delta_{p\mu} a_{2,K^0\rho} - (a_{4,K^0\rho}^u - a_{4,K^0\rho}^c) \]
\[ - \gamma_x^{K^0} (a_{6,K^0\rho}^u - a_{6,K^0\rho}^c) + \frac{3}{2} (a_{9,K^0\rho}^u - a_{9,K^0\rho}^c) \]
\[ + a_{7,K^0\rho}^u - a_{7,K^0\rho}^c + \frac{1}{2} \left[ a_{10,K^0\rho}^u - a_{10,K^0\rho}^c \right] \]
\[ + \gamma_x^{K^0} (a_{8,K^0\rho}^u - a_{8,K^0\rho}^c) \] \quad (60)

In a similar way, with the aid of the Fierz identities, we can evaluate the penguin operator contributions \( p_{\rho} \) and \( p_{\omega} \). From Eq. (48) we have
\[ \beta e^{i\delta_\beta} = \frac{p_\rho + p_\rho^a}{p_\omega + p_\omega^a} = \frac{Q_3}{Q_4} \] \quad (61)

where
\[ Q_3 = A_{K^0\rho} \left[ - a_{4,K^0\rho}^c - \gamma_x^{K^0} a_{6,K^0\rho}^c \right. \]
\[ + \frac{3}{2} (a_{9,K^0\rho}^c + a_{7,K^0\rho}^c) + \frac{1}{2} (a_{10,K^0\rho}^c) \]
\[ + \gamma_x^{K^0} a_{8,K^0\rho}^c - \beta_3 + \frac{1}{2} \beta_3, EW \] \quad (62)

\[ Q_4 = A_{K^0\omega} \left[ 2(a_{3,K^0\omega}^c + a_{5,K^0\omega}^c) \right. \]
\[ + a_{4,K^0\omega}^c + \gamma_\omega^{K^0} a_{6,K^0\omega}^c + \frac{1}{2} (a_{7,K^0\omega}^c + a_{9,K^0\omega}^c) \]
\[ - \frac{1}{2} (a_{10,K^0\omega}^c + \gamma_\omega^{K^0} a_{8,K^0\omega}^c) + \beta_3 - \frac{1}{2} \beta_3, EW \] \quad (63)

and
\[ r' e^{i(\delta_\phi + \phi)} = \frac{p_\omega + p_\omega^a}{t_\rho + t_\rho^a} = \frac{Q_4}{Q_2} \] \quad (64)
\[ r' e^{i\delta_\phi} = \frac{Q_4}{Q_2} \left| \frac{V_{tb} V_{td}^*}{V_{ub} V_{ud}^*} \right| \] \quad (65)

where
\[ \left| \frac{V_{tb} V_{td}^*}{V_{ub} V_{ud}^*} \right| = \frac{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}}{(1 - \lambda^2/2)(\rho^2 + \eta^2)}. \] \quad (66)
It can be seen that \( r' \) and \( \delta_q \) depend on both the Wilson coefficients and the CKM matrix elements, as shown in Eqs. (65). Substituting Eqs. (58) (61) (65) into Eq. (49), we can obtain \( r, \sin \delta, \) and \( \cos \delta \). Then, in combination with Eqs. (50) and (51) the \( CP \) violating asymmetries can be obtained.

**IV. BRANCHING RATIO OF \( B_s^0 \rightarrow K^0 \rho^0(\omega) \)**

The matrix element for \( B_s \rightarrow P \) and \( B_s \rightarrow V \) (where \( P \) and \( V \) denote pseudoscalar and vector mesons, respectively) can be decomposed as follows \[17\]:

\[
\langle P|J_\mu|B_s\rangle = \left(p_{B_s} + p_P - \frac{m_{B_s}^2 - m_P^2}{k^2}k\right)\epsilon_\mu F_1(k^2)
+ \frac{m_{B_s}^2 - m_P^2}{k^2}k_\mu F_0(k^2),
\]

\[
\langle V|J_\mu|B_s\rangle = \frac{2}{m_{B_s} + m_V}\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu*\rho\sigma}p_P^\nu p_V^\sigma V(k^2)
+ i\left\{\epsilon_\mu(m_{B_s} + m_V)A_1(k^2) - \frac{\epsilon^* \cdot k}{m_{B_s} + m_V}
\times (p_{B_s} + p_V)_\mu A_2(k^2) - \frac{\epsilon^* \cdot k}{k^2}2m_V \cdot k_\mu A_3(k^2)\right\}
+ \frac{i\epsilon^* \cdot k}{k^2}2m_V \cdot k_\mu A_0(k^2),
\]

where \( J_\mu \) is the weak current \( (J_\mu = \bar{q}\gamma^\mu(1 - \gamma_5)b \) with \( q = u,d,s)\), \( p_{B_s}(m_{B_s}), p_P(m_P), p_V(m_V) \) are the momenta (masses) of \( B_s, P, V \), respectively, \( k = p_{B_s} - p_P(p_V) \) for \( B_s \rightarrow P(V) \) transition and \( \epsilon_\mu \) is the polarization vector of \( V \). \( F_i \) \((i = 0,1)\) and \( A_i \) \((i = 0,1,2,3)\) in Eq. (67) are the weak form factors which satisfy \( F_1(0) = F_0(0), A_3(0) = A_0(0), \) and \( A_3(k^2) = [(m_B + m_V)/2m_V]A_1(k^2) - [(m_B - m_V)/2m_V]A_2(k^2) \).

With the factorizable decay amplitudes in Eq. (56)(57) we can calculate the decay rate for \( B_s \) to a pseudoscalar meson \( (P) \) and a vector meson \( (V) \) transition by using the following expression \[18\]:

\[
\Gamma(B_s \rightarrow PV) = \frac{p_c}{8\pi m_V^2}|A(B_s \rightarrow PV)/(\epsilon \cdot p_{B_s})|^2,
\]

where

\[
p_c = \sqrt{[m_{B_s}^2 - (m_P + m_V)^2][m_{B_s}^2 - (m_P - m_V)^2]} / 2m_{B_s}
\]

is the c.m. momentum of the product particle and \( A(B_s \rightarrow PV) \) is the decay amplitude.

In the QCD factorization approach. Here \( V_u^{T,P} \) are the CKM factors,

\[
V_u^T = |V_{ub}V_{uq}^*|, \text{ for } i = 1,2,
\]

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\begin{align*}
V_u^P &= |V_{ub}V_{cq}^*|, \text{ for } i = 3, \ldots, 10. \quad (70)
\end{align*}

In our case we take into account the \( \rho - \omega \) mixing contribution when we calculate the branching ratios since we are working to the first order of isospin violation. We can explicitly express the branching ratio for the process \( \bar{B}_s \to K^0\rho(\omega) \) as the following:

\begin{align*}
BR(\bar{B}_s \to K^0\rho(\omega)) &= \frac{G_F^2p_1^3}{16\pi m_\rho^2\Gamma_{Bs}} \left[ |V_T^T(a_1, a_2) - V^P_T(a_3, \ldots, a_{10})| \right] \\
+ &\left[ V^T_T(a_1, a_2) - V^P_T(a_3, \ldots, a_{10}) \right] \\
\times &\frac{\tilde{\Pi}_\rho^\perp}{(s_\rho - m_\rho^2 + im_\rho\Gamma_{Bs})^2},
\end{align*}

where \( \Gamma_{Bs} \) is the total decay width of \( B_s \).

V. INPUT PARAMETERS

In the numerical calculations, we have several parameters, i.e. \( N_c \) and the CKM matrix elements in the Wolfenstein parametrization. For the CKM matrix elements, which should be determined from experiments, we use the results of Ref. [2]:

\begin{align*}
\bar{\rho} &= 0.132^{+0.022}_{-0.014}, \quad \bar{\eta} = 0.341 \pm 0.013, \\
\lambda &= 0.2253 \pm 0.0007, \quad A = 0.808^{+0.022}_{-0.015}. \quad (72)
\end{align*}

In QCD factorization scheme, since power corrections have been considered, \( N_c \) is only color parameter, hence we use \( N_c = 3 \). In naïve factorization \( N_c \) includes the nonfactorizable effects which are model and process dependent and cannot be theoretically evaluated accurately and can be determined by experiment.

The running quark masses is taken by the scale \( \mu \) in \( B_s \) decay. One has

\begin{align*}
m_b(m_b) &= 4.2GeV, \quad m_c(m_b) = 0.91GeV, \\
m_u(m_b) &= m_d(m_b) = 0, \quad m_s(2.1GeV) = 0.095GeV.
\end{align*}

\begin{align*}
(73)
\end{align*}

The values of the scale dependent quantities \( f_+(\mu) \) and \( a_{1,2}^+(\mu) \) are given for \( \mu = 1GeV \). The value of Gegenbauer moments are taken from [19].

\begin{align*}
a_1^\rho &= 0, \quad a_2^\rho = 0.15 \pm 0.07 \\
a_1^\omega &= 0, \quad a_2^\omega = 0.15 \pm 0.07 \\
a_1^\rho &= 0, \quad a_2^\rho = 0.14 \pm 0.06 \\
a_1^\omega &= 0, \quad a_2^\omega = 0.14 \pm 0.06 \\
a_1^K &= 0.06 \pm 0.03, \quad a_2^K = 0.25 \pm 0.15 \\
f_\rho &= 216 \pm 3MeV, \quad f_\rho^+(\mu) = 165 \pm 9MeV, \\
f_\omega &= 187 \pm 5MeV, \quad f_\omega^+(\mu) = 151 \pm 9MeV, \quad (74)
\end{align*}
For $B_s$ meson, we use the value\[^{[2]}\]:

$$\tau = 1.47\text{ps}, \ m_{B_s} = 5.366\text{GeV}$$  \hspace{1cm} (75)

The Wilson coefficients $c_i$ can be found in \[^{[8]}\]. As discussed in detail in \[^{[8]}\], there are large theoretical uncertainties related to the modeling of power corrections corresponding to weak annihilation effects and the chirally-enhanced power corrections to hard spectator scattering. So we parameterize these effects in terms of the divergent integrals $X_H$ (hard spectator scattering) and $X_A$ (weak annihilation). We also model these quantities by using the parameterization\[^{[8]}\]

$$X_A = \left(1 + \varphi_A e^{i\phi_A}\right) \ln \frac{m_B}{\Lambda_h}; \quad \varphi_A \leq 1 \quad \Lambda_h = 0.5\text{GeV},$$  \hspace{1cm} (76)

and similarly for $X_H$. Here $\varphi_A$ is an arbitrary strong-interaction phase, which may be caused by soft rescattering. The fitted $\varphi_A$ and $\phi_A$ are taken from \[^{[20]}\]. For $B_s \to PV$ decay, $\rho_{PV} \approx 0.87$, $\phi_{PV} \approx -30^\circ$. For the estimate of theoretical uncertainties, we shall assign an error of $\pm 0.1$ to $\rho_A$ and $\pm 20^\circ$ to $\phi_A$\[^{[20]}\].

The form factors associated with the weak transitions depend on the inner structure of the hadrons and are hence model dependent. Here we will consider the form factors obtained in several phenomenological models. For $B_s$ decay form factors, we will use the results (form factors are referred to the ones at $q^2 = 0$):

1. Model 1 \[^{[8]}\]

$$F_0^{B_s \to K} = 0.31 \pm 0.05,$$

2. Model 2 (in the pQCD approach)\[^{[21]}\]

$$F_0^{B_s \to K} = 0.24^{+0.05+0.00}_{-0.04-0.01},$$

3. Model 3 (form factors obtained by QCD sum rules)\[^{[22]}\]

$$F_0^{B_s \to K} = 0.30^{+0.04}_{-0.03},$$

4. Model 4 (light-cone sum rule calculation based on heavy quark effective theory)\[^{[23]}\]

$$F_0^{B_s \to K} = 0.296 \pm 0.018,$$

5. Model 5 (A light cone quark model in conjunction with soft collinear effective theory)\[^{[24]}\]

$$F_0^{B_s \to K} = 0.290,$$
FIG. 1: Plot of $a$ as a function of $\sqrt{s}$ corresponding to central parameter values of CKM matrix elements for $\bar{B}_s^0 \to K^0 \rho^0(\omega) \to K^0 \pi^+ \pi^-$.  

6). Model 6 (lattice QCD calculation) \[^{[25]}\]

$$F_{0}^{\bar{B}_s \to K} = 0.23 \pm 0.05 \pm 0.04.$$  

In above Models, the $k^2$ dependence of the form factors has the following form under the nearest pole dominance assumption:

$$h(k^2) = \frac{h(0)}{1 - \frac{k^2}{m_h^2}}, \quad (77)$$

where $h$ could be $F_0$, and $m_h$ is the pole mass.  

It is noted that since the value of $k^2$ (which is actually the square of the mass of the factorized light meson) is much smaller than the square of the pole mass which is of order $m^2$, only the values of the form factors at $k^2 = 0$ are most relevant and hence how the form factors depend on $k^2$ has little effects (less than 2%). From the above values we see that the form factor $B_s \to K$ at $q^2 = 0$ ranges from 0.23 to 0.31.

VI. NUMERICAL RESULTS AND DISCUSSIONS

A. $CP$ violation via $\rho - \omega$ mixing in $\bar{B}_s^0 \to K^0 \pi^+ \pi^-$

In the numerical calculations, we find the $CP$ violating asymmetry, $a$, is large when the invariant mass of $\pi^+ \pi^-$ is in the vicinity of the $\omega$ resonance within QCD factorization scheme.  

In the respective error ranges, when $\sqrt{s} = 0.782 \ GeV$, we get maximum $CP$ violating asymmetry

$$a = (45.9^{+16.2+27.5}_{-15.7-29.1}) \times 10^{-2} \quad (78)$$
FIG. 2: Plot of $\sin \delta$ as a function of $\sqrt{s}$ corresponding to central parameter values of CKM matrix elements with $\rho - \omega$ mixing for $\bar{B}_s^0 \to K^0 \rho^0(\omega) \to K^0 \pi^+ \pi^-$. 

FIG. 3: Plot of $r$ as a function of $\sqrt{s}$ corresponding to central parameter values of CKM matrix elements with $\rho - \omega$ mixing for $\bar{B}_s^0 \to K^0 \rho^0(\omega) \to K^0 \pi^+ \pi^-$. 

In QCD factorization, the theoretical errors are large which follows to the uncertainties of results. Generally, power corrections beyond the heavy quark limit give the major theoretical uncertainties. This implies the necessity of introducing $1/m_b$ power corrections. Unfortunately, there are many possible $1/m_b$ power suppressed effects and they are generally nonperturbative in nature and hence not calculable by the perturbative method. There are more uncertainties in this scheme. The first error refers to the variation of the CKM parameters. The second error comes from form factors and decay constants. The third error corresponds to the Gegenbauer moments. The last error is the wave function of the $B_s$ meson characterized by the parameter $\lambda_B$, the power corrections due to weak annihilation and hard spectator interactions described by the parameters $\rho_{A,H}, \phi_{A,H}$, respectively. Using the central values of above parameters, we first calculate the numerical results of $CP$ violation and branching ratio, and then add errors according to standard deviation. In Fig.1, We give the central value of $CP$ violating asymmetry as a function of $\sqrt{s}$. From the figure one can see the $CP$ asymmetry parameter is dependent on $\sqrt{s}$ and changes rapidly due to $\rho - \omega$ mixing when the invariant mass of $\pi^+ \pi^-$ is in the vicinity of
the $\omega$ resonance. The CP violating asymmetry vary from around $-37\%$ to around $45\%$.

From Eq. (43), one can see that the CP violating asymmetry parameter depends on both $\sin \delta$ and $r$. The plots of $\sin \delta$ and $r$ as a function of $\sqrt{s}$ are shown in Fig. 2 and Fig. 3. It can be seen that when $\rho - \omega$ mixing is taken into account $\sin \delta$ and $r$ change sharply when the invariant mass of $\pi^+\pi^-$ is around 0.782 GeV. From the Fig. 2, one can find $\rho - \omega$ mixing make the $\sin \delta$ value oscillate from $-0.56$ to $0.44$ which cannot reach the value $-1$. This result is not in agreement with conclusion from naive factorization which can measure the CP violating parameter to remove the $\text{mod}(\pi)$ phase uncertainty in the determination of the CKM angle $\alpha$ arising from the conventional determination through $\sin^2 \alpha$ [7].

We have shown that $\rho - \omega$ mixing does enhance the direct CP violating asymmetries and provide a mechanism for large CP violation in QCD factorization scheme. On the other hand, it is important to see whether it is possible to observe these large CP violating asymmetries in experiments. This depends on the branching ratio for $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega)$. We will study this problem in the next section.

B. Branching ratios via $\rho - \omega$ mixing in $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega)$

Including $\rho - \omega$ mixing, we calculate the values of branching ratios for $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega)$. Base on the reasonable parameters range, we obtain the branching ratio of $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega)$ is $(9.8^{+2.6+3.4}_{-1.2-0.7}) \times 10^{-7}$ which is consistent with the result [20]. In other words, although we calculate the branching ratio due to $\rho - \omega$ mixing in QCD factorization scheme, we find the contribution of $\rho - \omega$ mixing for branching ratio is small and can be neglected. $\rho - \omega$ mixing mechanism presents new phase differences and produce extremely small effect for branching ratio of $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega)$.

VII. DISCUSSIONS ON POSSIBILITY TO OBSERVE CP VIOLATING ASYMMETRIES AT THE LHC

The LHC is a proton-proton collider currently have started at CERN. With the designed center-of-mass energy 14 TeV and luminosity $L = 10^{34} \text{cm}^{-2}\text{s}^{-1}$, the LHC gives access to high energy frontier at TeV scale and an opportunity to further improve the consistency test for the CKM matrix. The production rates for heavy quark flavours will be large at the LHC, and the $b\bar{b}$ production cross section will be of the order 0.5 mb, providing as many as $0.5 \times 10^{12}$ bottom events per year [26]. In particular, the LHCb detector is designed to exploit large number of $b$-hadrons produced at the LHC in order to make precise studies on CP asymmetries and on rare decays in $b$-hadron systems. The other two experiments, ATLAS and CMS, are optimized for discovering new physics and will complete most of their $B$ physics program within the first few years [26, 27]. Obviously, the LHC has a great advantage over the current experiments on $b$-hadrons [28].

In the present work, we have predicted possible large CP violating asymmetries in decay channel of $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega) \rightarrow K^0\pi^+\pi^-$ via the $\rho - \omega$ mixing. At the LHC, the $b$-hadrons are produced from the $pp$ collisions. The possible asymmetry between the numbers of the $b$-hadrons, $H_b$, and those of their antiparticles, $\bar{H}_b$, has been studied in the Lund string fragmentation model and the intrinsic heavy quark model [20, 29, 30]. It has
been shown that this asymmetry can only reach values of a few percent. In our following discussions, we will ignore this small asymmetry and give the numbers of $H_b \bar{H}_b$ pairs needed for observing the $CP$ violating asymmetries we have predicted. These numbers depend on both the magnitudes of the $CP$ violating asymmetries and the branching ratios of heavy hadron decays which are model dependent. For one (three) standard deviation signature, the number of $H_b \bar{H}_b$ pairs we need is \[31-33\]

$$N_{H_b \bar{H}_b} \sim \frac{1}{BRa^2} \left(1 - a^2\right) \left(\frac{9}{BRa^2} \left(1 - a^2\right)\right),$$  \hspace{1cm} (79)$$

where $BR$ is the branching ratio for $H_b \to f \rho^0$.

For central value of $CP$ asymmetry in Eq. \[78\], we present the numbers of $B_s \bar{B}_s$ pairs for observing the large $CP$ violating asymmetries at LHC. For the channel $\bar{B}_s^0 \to K^0 \rho^0(\omega) \to K^0 \pi^+ \pi^-$, the numbers of $B_s \bar{B}_s$ pairs are $3.8 \times 10^6$ ($3.4 \times 10^7$) for $1\sigma$ ($3\sigma$) signature. At the LHC the average $B_s \bar{B}_s$ production is about 10% out of $10^{12}$ $b \bar{b}$ events \[26\]. From Fig.1, one can see $CP$ violating asymmetries vary sharply at small energy range, and reach peak value at $\sqrt{s} = 0.782$ GeV. Hence, it is very possible to observe the large $CP$ violating asymmetries in small energy range of $\rho^0 \sim \omega$ resonance at the peak values of $CP$ violating asymmetries from the LHC experiment. For the experiments, it is possible to reconstruction $\pi^+$, $\pi^-$ and $K^0$ mesons when the invariant masses of $\pi^+ \pi^-$ pairs are in the vicinity of the $\omega$ resonance. Therefore, it is very possible to observe the large $CP$ violating asymmetries in $\bar{B}_s^0 \to K^0 \rho^0(\omega) \to K^0 \pi^+ \pi^-$ at the LHC.

### VIII. SUMMARY AND CONCLUSIONS

In this paper, we have studied $CP$ violation in $\bar{B}_s^0 \to K^0 \rho^0(\omega) \to K^0 \pi^+ \pi^-$. It has been found that, by including $\rho - \omega$ mixing, the $CP$ violating asymmetries can be large when the invariant masses of $\pi^+ \pi^-$ pairs are in the vicinity of the $\omega$ resonance. For the decay $\bar{B}_s^0 \to K^0 \rho^0(\omega) \to K^0 \pi^+ \pi^-$, the maximum $CP$ violation can reach 46%. Furthermore, taking $\rho - \omega$ mixing into account, we have calculated the branching ratio of the decay. We have also presented the numbers of $B_s \bar{B}_s$ pairs required for observing the predicted $CP$ violation in experiments at the LHC. We have found the channel is the likely channel in which the large $CP$ violating asymmetries may be observed at LHC. We expect that our predictions will provide a useful guidance for future investigations and experiments.

In our calculations there are some uncertainties. We have worked in the QCD factorization which is expected to be a reliable approach in the heavy-quark limit. In the QCD factorization scheme, $\alpha_s(m_b)$ and some $1/m_b$ (annihilation) corrections are included. In this framework, there is cancellation of the scale and renormalization scheme dependence between the Wilson coefficients and the hadronic matrix elements. However, the QCD factorization scheme suffers from endpoint singularities which are not well controlled. The $CP$ violating asymmetry depends on the unknown parameters which are associated with such endpoint singularities. The CKM matrix elements also lead to some uncertainties in the $CP$ violating asymmetry. Uncertainties also come from the weak form factors associated with the hadronic matrix elements. This lead to uncertain $CP$ violating asymmetries in the QCD factorization scheme. This needs further detailed investigations.
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