Indirect identification of damage functions from damage records

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In order to assess future damage caused by natural disasters, it is desirable to estimate the damage caused by single events. So called damage functions provide – for a natural disaster of certain magnitude – a specific damage value. However, in general, the functional form of such damage functions is unknown. We study the distributions of recorded flood damages on extended scales and deduce which damage functions lead to such distributions when the floods obey Generalized Extreme Value statistics and follow Generalized Pareto distributions. Based on the finding of broad damage distributions we investigate two possible functional forms to characterize the data. In the case of Gumbel distributed extreme events, (i) a power-law distribution density with an exponent close to 2 (Zipf’s law) implies an exponential damage function; (ii) stretched exponential distribution densities imply power-law damage functions. In the case of Weibull (Fréchet) distributed extreme events we find correspondingly steeper (less steep) damage functions.

I. INTRODUCTION

Natural disasters, such as floods or storms, represent extreme events [1–4] with severe consequences including numerous killed and affected people as well as huge economic damage. Independent of the problem on how to project the occurrence of future extreme events, one is interested in which damage can typically be expected from an extreme event of certain magnitude. One approach to tackle this question is to separate the statistics of extreme events from the damage caused by them. The former can be obtained from measurements, such as water level records, but the latter needs to be estimated by some empirical studies.

Accordingly, so called damage functions provide a monetary value as a function of the magnitude of an event, such as the maximum flood level. We distinguish damage functions on a microscopic scale from those on a macroscopic scale [5–7]. The former describes the typical costs of damages to single assets, such as residential buildings, and the latter describes damage costs at larger areas, such as an entire city. This macroscopic (aggregated) damage function represents a composition of information on asset values, their location, and their vulnerability. We assume that in this case the damages are well characterized by the function, i.e. noise is reduced due to spatial aggregation.

In general, the functional form of damage functions is unknown. By definition, they are monotonic increasing with the magnitude of the natural hazard and eventually exhibit saturation. For floods, on the microscopic scale a considerable set of recorded direct monetary damage values of inundated buildings has been related to the corresponding water depth leading to an exponential dependence for private housing [8, Fig. 5]. Furthermore, a set of different microscopic damage functions, models, and the related damages have been compared and evaluated [9], among them linear, quadratic, and square-root damage functions (see also [10]). In [11] a storm surge damage function has been estimated for the city of Copenhagen. In what follows we refer to macroscopic damage functions, i.e. the direct damage to an urban agglomeration as a function of the maximum flood height.

Since from single sites there are usually too few damage records to obtain a damage function, we tackle the problem on a larger scale and study which functional form a damage function must follow so that the distribution of extreme events transforms to the distribution of observed damages. Relating these two distributions we obtain macroscopic damage functions. Therefore, we analyze flood data assembled by CRED [12] and find broad damage distributions. In order to characterize them, we elaborate two functional forms. We show that for the Gumbel case, Zipf’s law, i.e. a power-law distribution density with an exponent $\alpha \approx 2$, implies an exponential damage function. Stretched exponential distribution densities imply power-law damage functions.

As it is known analytically, maximum values of samples of fixed size (such as "block maxima" of time series) follow distributions which converge for sufficiently large samples towards Generalized Extreme Value (GEV) distributions, if the values are independent and identically
distributed. Thus, we assume that changes in the statistics are small compared to the implied damages. GEV distributions are commonly fitted to maxima in order to estimate annualities and future occurrences \[13\].

Further, if the data obeys GEV characteristics, it is possible to use the Generalized Pareto (GP) distributions as an approximation of the distribution function of the level \( s \) above a sufficiently high threshold \( s_T \) \[14\]. Therefore, the upper tail of the distribution function of the (damage causing) events is described by one of the three GP distributions

\[
P^{\text{GP}}_{(s)} = \begin{cases} 
1 - \left( 1 + \frac{s - s_T}{\gamma} \right)^{-\frac{1}{\xi}} & \text{for } \xi \neq 0 \\
1 - e^{-\left( s - s_T \right) / \gamma} & \text{for } \xi = 0.
\end{cases}
\]

They are defined on \( s \in [s_T, \infty) \) and have a scale parameter, \( \gamma \in \mathbb{R}^+ \), as well as a shape parameter, \( \xi \in \mathbb{R} \) (for \( \xi < 0 \): \( s \in (s_T, s_T - \frac{1}{\xi}) \)). According to the GEV-shape, one distinguishes three cases: (i) the Gumbel-shape (\( \xi = 0 \)), (ii) the heavy-tailed Fréchet distribution (\( \xi > 0 \)), and (iii) the bounded-tailed reversed Weibull distribution (\( \xi < 0 \)). For a more detailed presentation of extreme value assessment and applications we refer to \[14\,17\].

II. DAMAGE RECORDS

We consider the EM-DAT database \[12\] collected by the Centre for Research on Epidemiology of Disasters (CRED) in version v12.07 as created on Oct-28-2009. The information listed for each event entry consists of: start, end, country, location, type of disaster, sub-type, name, number of people killed, number of people affected, an estimated damage, and an ID. For the years 1950-2008 we extract the information on floods, which include general floods, flash floods, as well as storm surges respectively coastal floods, and obtain 3469 entries worldwide, while for 1225 entries an estimated damage in units of Million US-Dollars is available.

In Figure II(a) we show the estimated probability densities, \( \tilde{p}(D) \), for all flood damage values of the database. However, since one may argue that the result could be biased by regional differences, in Fig. II(b) we show \( \tilde{p}(D) \) for only those floods that occurred in the USA. Thus, we can weaken influences due to different economic power of different countries. In order to reduce possible trends in the data II, we also exclude floods before 1980 and plot the distribution density for Europe in Fig. II(c). In any case, we observe broad distributions with damages reaching the order of 10 Billion US-Dollars. We would also like to note that we obtain similar distributions for the number of killed or affected people as well as for other natural disasters.

In the simplest approach, the tail of the probability densities can be described with a functional form involving one parameter, namely a power-law according to

\[
\tilde{p}(D) \sim D^{-\alpha},
\]

where we find \( \alpha \approx 2 \) [Fig. II(a-c)]. Such a size distribution is also known as Zipf’s law (a special case of the Pareto distribution) and is found in many different fields, such as word usage, city sizes, firm sizes, wealth, intensity of solar flares, etc. For an overview we refer to \[19\,21\] and references therein. Minor deviations from Eq. (2) for floods with small damage could be due to the fact that small damages are more likely to be missing in the database.

III. RELATING EXTREME EVENTS AND DAMAGES

The damage costs of a large flood magnitude event depend on a variety of damage influencing factors, such as orography, flow velocity, contamination, preparedness,
according to since it is integrable – namely a stretched exponential we also elaborate a two parameter fit – which we choose. Therefore, we write the probability as an integral over the density and substitute the magnitude with the damage \((s \to D = D(s)):\)

\[
\int_{s_1}^{s_2} p(s)ds = \int_{D(s_1)}^{D(s_2)} \frac{ds}{dD}D = \int_{D(s_1)}^{D(s_2)} \tilde{p}(D)dD.
\]

(3)

Here, \(p(s)\) and \(\tilde{p}(D)\) are the probability density of the extreme event and the damage, respectively. Furthermore, the density transformation \(\frac{\tilde{p}(D)}{p(s)} = \frac{ds}{dD}\) was used. Next, choosing \(s_1 = s_T\) (the threshold for which GP distributions are applicable) and \(s_2 = s\), we obtain the equation

\[
P(s) = \int_{D(s_T)}^{D(s)} AD^{-\alpha}dD = 1 - D^{\alpha - 1}_s D(s)_T^{1 - \alpha},
\]

(4)

which holds for any (reasonable) damage distribution density \(\tilde{p}(D)\).

Using Eq. (2), i.e. the probability density \(\tilde{p}(D) = AD^{-\alpha}\) as indicated in Fig. 1(a-c), we write

\[
P(s) = \int_{D(s_T)}^{D(s)} AD^{-\alpha}dD = 1 - D^{\alpha - 1}_s D(s)_T^{1 - \alpha},
\]

(5)

where \(A = (\alpha - 1)D^{\alpha - 1}_s\) such that \(\tilde{p}(D)\) is normalized in \(D \in [D(s_T), D(s_\to \infty)]\). Further we suppose that \(D(s_\to \infty) \to \infty\). Solving the equation for \(D(s)\) we get

\[
D(s) = D(s_T) \left(1 - P(s)\right)^{\frac{1}{\alpha - 1}}.
\]

(6)

Finally, we insert the GP distribution for the Gumbel case, i.e. Eq. (1) with \(\xi = 0\), and obtain that the damage increases exponentially,

\[
D(s) = D(s_T)e^{\frac{s - s_T}{\lambda}},
\]

(7)

which holds for \(s \in [s_T, \infty)\). Accordingly, the exponential damage function transforms the Gumbel distribution, approximated by an exponential one, into a Pareto distribution.

However, the exponential damage function is based on damages following power-law distributions [Fig. 1(a-c)]. Since a one parameter description might not be sufficient, we also elaborate a two parameter fit – which we choose since it is integrable – namely a stretched exponential according to

\[
\tilde{p}(D) \sim \frac{a}{b} D^{a - 1} e^{-\frac{D}{b}},
\]

(8)

where \(a \) and \(b\) are the parameters \((a, b > 0)\). Equation (8) is also known as Weibull distribution, see e.g. [24, 25] and references therein. For the same data as before the fitted curves are shown in Fig. 1(d-f) providing values for the exponent \(a\) roughly between 1/3 and 1/2. The stretched exponential (Weibull) distribution, Eq. (8), is sometimes used as penultimate approximation for pre-asymptotic behavior of extreme value distributions [26, 27]. This suggests, that the usage of the stretched exponential distribution could be justified by an insufficiently high damage threshold.

Now, the integral relating the extreme events \(s\) and damages \(D\), Eq. (4), is over \(\tilde{p}(D)\) from Eq. (8), instead of Eq. (2):

\[
P(s) = \int_{D(s_T)}^{D(s)} B\frac{a}{b} D^{a - 1} e^{-\frac{D}{b}}dD = 1 - B e^{-\frac{D(s)}{b}},
\]

(9)

where \(B = e^{-\frac{D(s_T)}{b}}\) such that \(\tilde{p}(D)\) is normalized in \(D \in [D(s_T), D(s_\to \infty)]\). Solving for \(D(s)\) we find

\[
D(s) = \left[D(s_T) - b \ln(1 - P(s))\right]^{\frac{1}{\alpha}}.
\]

(10)

Finally, we insert the GP distribution for the Gumbel case, i.e. Eq. (1) with \(\xi = 0\), and obtain

\[
D(s) = \left(D(s_T)^{\alpha} + \frac{b}{\alpha} (s - s_T)\right)^{\frac{1}{\alpha}}.
\]

(11)

Accordingly, the power-law damage function transforms the Gumbel distribution, approximated by an exponential one, into a stretched exponential distribution. For \(a\) between 1/3 and 1/2 we obtain the asymptotic power-law relation \(D(s) \sim s^3\) or \(D(s) \sim s^2\), respectively.

IV. SUMMARY AND DISCUSSION

In summary, we characterize distributions of recorded flood damages, argue that they are caused by extreme events, and employ density transformation to deduce damage functions relating both. For Gumbel distributed extreme events \((\xi = 0)\) we find asymptotically

\[
D(s) \sim \begin{cases} 
\left(e^{\gamma(\alpha - 1)}\right) & \text{for } \tilde{p}(D) \sim D^{-\alpha} \text{ with } \alpha > 1 \\
\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha}} \left(s - s_T\right) & \text{for } \tilde{p}(D) \sim \frac{a}{b} D^{a - 1} e^{-\frac{D}{b}} \text{ with } a > 0.
\end{cases}
\]

(12)

The involved GP parameters are in an ultimate sense and in practice penultimate approximations might be necessary. In particular, it needs to be considered that if the underlying geophysical variable has only an approximate exponential distribution [i.e. \(\xi = 0\) in Eq. (1)], then the form of the obtained damage functions would not be unique. An analogous argument applies to the cases
The functional forms of Eq. (12) are illustrated in Fig. 2 which also includes the cases $\xi \neq 0$. For power-law distributed damages and Weibull distributed extreme events ($\xi < 0$) in average the damage increases faster than exponentially with the magnitude. For Fréchet distributed extreme events ($\xi > 0$) the opposite is the case. Intuitively, since in the Weibull case the extreme events have a less heavy tail, a steeper damage function is needed to result in the same damage distribution as the Gumbel case [Fig. 2(a)]. In the Fréchet case, which has a faster tail than the Gumbel distribution, a less steep damage function is sufficient to result in the same damage distribution as the Gumbel case. Similar arguments hold for stretched exponential damages [Fig. 2(b)].

We would like to remark that, allowing also negative values of $a$ in Eq. (9), it represents the Fréchet instead of the Weibull distribution. Moreover, with a scale parameter $a = -1$, in the limit $s \to \infty$ the Fréchet distribution is equivalent to Zipf’s law, Eq. (2).

In particular the role of aggregation on both, the extremes and the damages functions requires further investigations. While floods involve an integration of precipitation over a water basin, damages involve an integration of flood impacts over affected assets. Thus, it would be interesting to understand, to which extend the tail becomes heavier at each stage, from precipitation extremes to flood magnitude and then to damages.

FIG. 2. Illustration of the obtained damage functions. (a) In the case of power-law distributed damages for $\xi = 0$ we obtain an exponential damage function [Eq. (7), solid line]. For $\xi \neq 0$ we insert Eq. (1) in Eq. (6) and find faster (dotted) or slower (dashed) than exponential damage functions, Weibull ($\xi < 0$) or Fréchet ($\xi > 0$) case, respectively. (b) In the case of stretched exponential distributed damages for $\xi = 0$ we obtain a power-law damage function with exponent $1/a$ [Eq. (11), solid line]. For $\xi \neq 0$ we insert Eq. (1) in Eq. (10) and find that the damage function increases faster (dotted) or slower (dashed), Weibull ($\xi < 0$) or Fréchet ($\xi > 0$) case, respectively, than the corresponding power-law. To generate the curves, we choose: Fréchet, $\xi = 0.1$ and Weibull, $\xi = -0.1$ as well as $\alpha = 2$ in (a) and $a = 0.4$ in (b).

Since the obtained damage functions represent an average for large temporal and spatial scale, neglecting any further local differences, they have limited predictive power. However, the results could provide qualitative insight because exponential increasing damage is much more catastrophic than polynomial. Thus, it would be of interest to systematically analyze how the results depend on the considered spatial scale.

Our approach could help to address the questions raised by decision makers or insurers, which costs certain regions, countries, or even the globalized economy are facing from coastal floods [29–31]. The assessment of damage costs is fundamental in the context of adaptation to climate change [32]. Finally, we would like to note that the presented approach can be applied to any type of natural disasters as long as the concept of damage functions can be used (see e.g. [33, 34]).

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