Flavor-Changing Higgs Decays in Grand Unification with Minimal Flavor Violation

Seungwon Baek\textsuperscript{1,*} and Jusak Tandean\textsuperscript{2,3,†}

\textsuperscript{1}School of Physics, Korea Institute for Advanced Study, 85 Hoegiro Dongdaemun-gu, Seoul 02455, Korea
\textsuperscript{2}Physics Division, National Center for Theoretical Sciences, Hsinchu 300, Taiwan
\textsuperscript{3}Department of Physics and Center for Theoretical Sciences, National Taiwan University, Taipei 106, Taiwan

Abstract

We consider the flavor-changing decays of the Higgs boson in a grand unified theory framework which is based on the SU(5) gauge group and implements the principle of minimal flavor violation. This allows us to explore the possibility of connecting the tentative hint of the Higgs decay $h \rightarrow \mu \tau$ recently reported in the CMS experiment to potential new physics in the quark sector. We look at different simple scenarios with minimal flavor violation in this context and how they are subject to various empirical restrictions. In one specific case, the relative strengths of the flavor-changing leptonic Higgs couplings are determined mainly by the known quark mixing parameters and masses, and a branching fraction $B(h \rightarrow \mu \tau) \sim 1\%$ is achievable without the couplings being incompatible with the relevant constraints. Upcoming data on the Higgs leptonic decays and searches for the $\mu \rightarrow e\gamma$ decay with improved precision can offer further tests on this scenario.

\textsuperscript{*}Electronic address: swbaek@kias.re.kr
\textsuperscript{†}Electronic address: jtandeant@yahoo.com
I. INTRODUCTION

The ongoing measurements on the 125 GeV Higgs boson, \( h \), at the Large Hadron Collider (LHC) have begun to probe directly its Yukawa interactions with fermions \([1–7]\). In particular, for the branching fractions of the standard decay modes of \( h \), the ATLAS and CMS experiments have so far come up with

\[
\frac{\mathcal{B}(h \to b\bar{b})}{\mathcal{B}(h \to b\bar{b})_{\text{SM}}} = 0.70^{+0.29}_{-0.27} \quad [1], \quad \frac{\mathcal{B}(h \to \tau^+\tau^-)}{\mathcal{B}(h \to \tau^+\tau^-)_{\text{SM}}} = 1.12^{+0.24}_{-0.22} \quad [1],
\]

where the upper limits in the second line are at 95% confidence level (CL). Overall, these data are still in harmony with the expectations of the standard model (SM).

However, there are also intriguing potential hints of physics beyond the SM in the Higgs Yukawa couplings. Especially, based on 19.7 fb\(^{-1}\) of Run-I data, CMS \([4]\) has reported observing a slight excess of \( h \to \mu^+\mu^- \) events with a significance of 2.4\(\sigma\), which if interpreted as a signal implies

\[
\mathcal{B}(h \to \mu\tau) = \mathcal{B}(h \to \mu^-\tau^+) + \mathcal{B}(h \to \mu^+\tau^-) = (0.84^{+0.39}_{-0.37})\%,
\]

but as a statistical fluctuation translates into the bound

\[
\mathcal{B}(h \to \mu\tau) < 1.51\% \text{ at 95\% CL} \quad [4].
\]

Its ATLAS counterpart has a lower central value and bigger error, \( \mathcal{B}(h \to \mu\tau) = (0.53 \pm 0.51)\% \) corresponding to \( \mathcal{B}(h \to \mu\tau) < 1.43\% \text{ at 95\% CL} \quad [5] \). Naively averaging the preceding CMS and ATLAS signal numbers, one would get \( \mathcal{B}(h \to \mu\tau) = (0.73 \pm 0.31)\% \). More recently, upon analyzing their Run-II data sample corresponding to 2.3 fb\(^{-1}\), CMS has found no excess and given the bound \( \mathcal{B}(h \to \mu\tau) < 1.20\% \text{ at 95\% CL} \quad [8] \). This indicates that the analyzed integrated luminosity is not large enough to rule out the Run-I excess and further analysis with more data is necessary to exclude or confirm it. In contrast, although the observation of neutrino oscillation \([9]\) suggests lepton flavor violation, the SM contribution to lepton-flavor-violating Higgs decay via W-boson and neutrino loops, with the neutrinos assumed to have mass, is highly suppressed due to both their tiny masses and a Glashow-Iliopoulos-Maiani-like mechanism. Therefore, the \( h \to \mu\tau \) excess events would constitute early evidence of new physics in charged-lepton interactions if substantiated by future measurements. On the other hand, searches for the \( e\mu \) and \( e\tau \) channels to date have produced only the 95%-CL bounds \([6]\)

\[
\mathcal{B}(h \to e\mu) < 0.036\%, \quad \mathcal{B}(h \to e\tau) < 0.70% \quad (4)
\]

from CMS and \( \mathcal{B}(h \to e\tau) < 1.04\% \) from ATLAS \([5]\).

In light of its low statistics, it is too soon to draw firm conclusions about the tantalizing tentative indication of \( h \to \mu\tau \) in the present LHC data. Nevertheless, in anticipation of upcoming measurements with improving precision, it is timely to speculate on various aspects or implications of such a new-physics signal if it is discovered, as has been done in very recent literature \([10–14]\). In this paper, we assume that \( \mathcal{B}(h \to \mu\tau) \sim 1\% \) is realized in nature and
entertain the possibility that it arises from nonstandard effective Yukawa couplings which may have some linkage to flavor-changing quark interactions beyond the SM. For it is of interest to examine how the potential new physics responsible for $h \to \mu \tau$ may be subject to different constraints, including the current nonobservation of Higgs-quark couplings deviating from their SM expectations.

To handle the flavor-violation pattern systematically without getting into model details, we adopt the principle of so-called minimal flavor violation (MFV). Motivated by the fact that the SM has been successful in describing the existing data on flavor-changing neutral currents and $CP$ violation in the quark sector, the MFV hypothesis presupposes that Yukawa couplings are the only sources for the breaking of flavor and $CP$ symmetries. Unlike its straightforward application to quark processes, there is no unique way to formulate leptonic MFV. As flavor mixing among neutrinos has been empirically established, it is attractive to formulate leptonic MFV by incorporating new ingredients that can account for this fact. One could assume a minimal field content where only the SM lepton doublets and charged-lepton singlets transform nontrivially under the flavor group, with lepton number violation and neutrino masses coming from the dimension-five Weinberg operator. Less minimally, one could explicitly introduce right-handed neutrinos, or alternatively right-handed weak-SU(2)-triplet fermions, which transform nontrivially under an enlarged flavor group and play an essential role in the seesaw mechanism to endow light neutrinos with Majorana masses. One could also introduce instead a weak-SU(2)-triplet of unflavored scalars which participate in the seesaw mechanism. Here we consider the SM expanded with the addition of three heavy right-handed neutrinos as well as effective dimension-six operators conforming to the MFV criterion in both the quark and lepton sectors. To establish the link between the lepton and quark interactions beyond the SM, we consider the implementation of MFV in a grand unified theory (GUT) framework with SU(5) as the unifying gauge group. In this GUT scheme, there are mass relations between the SM charged leptons and down-type quarks, and so we will deal with only the Higgs couplings to these fermions.

In the next section, we first briefly review the application of the MFV principle in a non-GUT framework based on the SM somewhat enlarged with the inclusion of three right-handed neutrinos which participate in the usual seesaw mechanism to generate light neutrino masses. Subsequently, we introduce the effective dimension-six operators with MFV built-in that can give rise to nonstandard flavor violation in Higgs decays, specifically the purely fermionic channels $h \to f \bar{f} \ell^\prime$. Then we look at constraints on the resulting flavor-changing Higgs couplings to quarks and leptons, focusing on the former, as the leptonic case has been treated in detail in Ref. which shows that the CMS $h \to \mu \tau$ signal interpretation can be explained under the MFV assumption provided that the right-handed neutrinos couple to the Higgs in some nontrivial way. In Section III, we explore applying the MFV idea in the Georgi-Glashow SU(5)

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1 Other aspects or scenarios of leptonic MFV have been discussed in the literature.
2 A similar approach has been adopted in to study some lepton-flavor-violating processes that might occur as a consequence of the recently observed indications of anomalies in rare $b \to s$ decays.
3 A detailed analysis of the interplay between quark and lepton sectors in the framework of a supersymmetric SU(5) GUT model with right-handed neutrinos can be found in.
GUT \cite{28}, following the proposal of Ref. \cite{26}. As the flavor group is substantially smaller than in the non-GUT scheme, the number of possible effective operators of interest becomes much larger. Therefore, we will consider different scenarios involving one or more of the operators at a time, subject to various experimental constraints. We find that there are cases where the restrictions can be very severe if we demand $\mathcal{B}(h \to \mu\tau) \sim 1\%$. Nevertheless, we point out that there is an interesting scenario in which the flavor-changing leptonic Higgs couplings depend mostly on the known quark mixing parameters and masses and $\mathcal{B}(h \to \mu\tau)$ at the percent level can occur in the parameter space allowed by other empirical requirements. Our analysis serves to illustrate that different possibilities in the GUT MFV context have different implications for flavor-violating Higgs processes that may be testable in forthcoming experiments. We give our conclusions in Section IV. An appendix contains some extra information.

\section{Higgs Fermion Decays with MFV}

The renormalizable Lagrangian for fermion masses in the SM supplemented with three right-handed Majorana neutrinos is

$$
\mathcal{L}_m = -(Y_u)_{kl} \overline{Q}_{k,L} U_{l,R} \tilde{H} - (Y_d)_{kl} \overline{Q}_{k,L} D_{l,R} H - (Y_e)_{kl} \overline{L}_{k,L} \nu_{l,R} \tilde{H} - (Y_\nu)_{kl} \overline{\nu}_{k,L} E_{l,R} H
$$

where summation over the family indices $k, l = 1, 2, 3$ is implicit, $Y_{u,d,e}$ denote $3 \times 3$ matrices for the Yukawa couplings, $Q_{k,L}$ ($L_{k,L}$) is a left-handed quark (lepton) doublet, $U_{l,R}$ and $D_{l,R}$ ($\nu_{l,R}$ and $E_{l,R}$) represent right-handed up- and down-type quarks (neutrinos and charged leptons), respectively, $H$ stands for the Higgs doublet, $\tilde{H} = i\tau_2 H^*$ with $\tau_2$ being the second Pauli matrix, $M_\nu$ is a $3 \times 3$ matrix for the Majorana masses of $\nu_{l,R}$, and the superscript of $(\nu_{k,R})^c$ refers to charge conjugation. We select the eigenvalues of $M_\nu$ to be much greater than the elements of $\nu Y_\nu / \sqrt{2}$, so that the type-I seesaw mechanism becomes operational \cite{19}, leading to the light neutrinos’ mass matrix $m_\nu = -(v^2/2) Y_\nu M_\nu^{-1} Y_\nu^t = U_{PMNS} m_\nu U_{PMNS}^\dagger$, which also involves the Higgs vacuum expectation value $v \simeq 246 \text{ GeV}$, the Pontecorvo-Maki-Nakagawa-Sakata (PMNS \cite{31}) mixing matrix $U_{PMNS}$ for light neutrinos, and their eigenmasses $m_{1,2,3}$ in $\hat{m}_\nu = \text{diag}(m_1, m_2, m_3)$. This suggests that \cite{32}

$$
Y_\nu = \frac{i\sqrt{2}}{v} U_{PMNS} \hat{m}_\nu^{1/2} O M_\nu^{1/2},
$$

where $O$ is in general a complex orthogonal matrix, $OO^\dagger = \mathbb{1} \equiv \text{diag}(1, 1, 1)$.

Hereafter, we suppose that $\nu_{k,R}$ are degenerate in mass, and so $M_\nu = \mathcal{M} \mathbb{1}$. The MFV hypothesis \cite{16,17} then implies that $\mathcal{L}_m$ is formally invariant under the global flavor symmetry group $\mathcal{G}_f = G_q \times G_\ell$, where $G_q = \text{SU}(3)_Q \times \text{SU}(3)_U \times \text{SU}(3)_D$ and $G_\ell = \text{SU}(3)_L \times O(3)_{\nu} \times \text{SU}(3)_E$. This entails that the above fermions are in the fundamental representations of their respective flavor groups,

$$
Q_L \to V_Q Q_L, \quad U_R \to V_U U_R, \quad D_R \to V_D D_R,
$$

$$
L_L \to V_L L_L, \quad \nu_R \to \mathcal{O}_\nu \nu_R, \quad E_R \to V_E E_R,
$$

\begin{equation}
\label{eq:7}
\end{equation}
where $V_{Q,U,D,L,E} \in \text{SU}(3)_{Q,U,D,L,E}$ are special unitary matrices and $O_\nu \in O(3)_\nu$ is an orthogonal real matrix $[16,17,23]$. Moreover, the Yukawa couplings transform under $G_t$ in the spurion sense according to

$$Y_a \rightarrow V_Q Y_a V_U^\dagger, \quad Y_d \rightarrow V_Q Y_d V_D^\dagger, \quad Y_\nu \rightarrow V_L Y_\nu O_\nu^T, \quad Y_e \rightarrow V_L Y_e V_E^\dagger. \quad (8)$$

To construct effective Lagrangians beyond the SM with MFV built-in, one inserts products of the Yukawa matrices among the relevant fields to devise operators that are both $G_t$-invariant and singlet under the SM gauge group $[16,17]$. Of interest here are the combinations

$$A_q = Y_a Y_u^\dagger, \quad B_q = Y_d Y_d^\dagger, \quad A_\ell = Y_\nu Y_\nu^\dagger, \quad B_\ell = Y_e Y_e^\dagger. \quad (9)$$

Given that the largest eigenvalues of $A_q$ and $B_q$ are $y_t^2 = 2m_t^2/v^2 \sim 1$ and $y_b^2 = 2m_b^2/v^2 \sim 3 \times 10^{-4}$, respectively, at the mass scale $\mu \sim m_h^2/2$, for our purposes we can devise objects containing up to two powers of $A_q$ and drop contributions with $B_q$, as higher powers of $A_q$ can be connected to lower ones by means of the Cayley-Hamilton identity $[33]$. As for $A_\ell$, we assume that the right-handed neutrinos’ mass is big enough, $M \sim 6 \times 10^{14}$ GeV, to make the maximum eigenvalue of $A_\ell$ order 1, which fulfills the perturbativity condition $[25,33]$. Hence, as in the quark sector, we will keep terms up to order $A_q^2$ and ignore those with $B_\ell$, whose elements are at most $y_\tau^2 = 2m_\tau^2/v^2 \sim 10^{-4}$. Accordingly, the relevant spurion building blocks are

$$\Delta_q = \zeta_1 \mathbb{1} + \zeta_2 A_q + \zeta_4 A_q^2, \quad \Delta_\ell = \xi_1 \mathbb{1} + \xi_2 A_\ell + \xi_4 A_\ell^2, \quad (10)$$

where in our model-independent approach $\zeta_{1,2,4}$ and $\xi_{1,2,4}$ are free parameters expected to be at most of $O(1)$ and with negligible imaginary components $[25,33]$, so that one can make the approximations $\Delta_q^\dagger = \Delta_q$ and $\Delta_\ell^\dagger = \Delta_\ell$.

Thus, the desired $G_t$-invariant effective operators that are SM gauge singlet and pertinent to Higgs decays $h \rightarrow f\bar{f}$ into down-type fermions at tree level are given by $[17]$

$$\mathcal{L}_{\text{MFV}} = \frac{O_{RL}}{\Lambda^2} + \text{H.c.}, \quad O_{RL} = (D^a H)^\dagger \overline{D}_R Y_d^\dagger \Delta_q D_a Q_L + (D^a H)^\dagger \overline{E}_R Y_e^\dagger \Delta_\ell D_a L_L, \quad (11)$$

where the mass scale $\Lambda$ characterizes the underlying heavy new physics and the covariant derivative $D^a = \partial^a + (i g/2) \tau_a W^a + ig' Y^a B^a$ acts on $H, Q_L, L_L$ with hypercharges $Y' = 1/2, 1/6, -1/2$, respectively, and involves the usual $SU(2)_L \times U(1)_Y$ gauge fields $W^a$ and $B^a$, their coupling constants $g$ and $g'$, respectively, and Pauli matrices $\tau_a$, with $a = 1, 2, 3$ being summed over. There are other dimension-six MFV operators involving $H$ and fermions, particularly

$$i \left[ H^\dagger D_a H - (D_a H)^\dagger H \right] \overline{Q}_L \gamma^a \Delta_q Q_L, \quad g \overline{D}_R Y_d^\dagger \Delta_q \sigma_{\alpha \omega} H^\dagger Q_L B^{\alpha \omega}, \quad (12)$$

$$i \left[ (D_a H)^\dagger \tau_a H \right] \overline{Q}_L \gamma^a \Delta_{\alpha q3} \tau_a Q_L, \quad g \overline{D}_R Y_e^\dagger \Delta_q \sigma_{\alpha \omega} H^\dagger \tau_a Q_L W^{\alpha \omega}.$$  

In this study, we do not address $h$ couplings to up-type quarks for the following reason. As the operator $(D^a H)^\dagger \overline{U}_R Y_{u4}^\dagger \Delta_q D_a Q_L$ with $\Delta_q$ from $[16]$ conserves flavor, others with $B_q$, such as $(D^a H)^\dagger \overline{U}_R Y_{l4}^\dagger B_q D_a Q_L$, would be needed, but with only one Higgs doublet they are relatively suppressed by the smallness of the $B_q$ elements, which makes the present empirical bounds $[3,34]$ on $t \rightarrow uh, ch$ and $h \rightarrow uc$ not strong enough to offer meaningful constraints.
in the quark sector and

\begin{align}
E \left[ H^\dagger D \alpha H - (D \alpha H)^\dagger H \right] L_L^\gamma a \Delta l \gamma L_L; & \quad gF R Y \left[ D \alpha \sigma \omega \gamma H^l B \gamma L \right], \\
E \left[ H^\dagger \tau \alpha D \alpha H - (D \alpha H)^\dagger \tau \alpha H \right] L_L^\gamma a \Delta l \gamma \tau \alpha L_L; & \quad gF R Y \left[ D \alpha \sigma \omega \gamma H^\tau \gamma L L \right]
\end{align}

(13)

in the lepton sector, where \( \Delta qn \) and \( \Delta \ell n \) are the same in form as \( \Delta q \) and \( \Delta \ell \), respectively, except that they have their own coefficients \( \zeta_r \) and \( \xi_r \), but these operators do not induce \( h \to f f' \) at tree level. In the literature the operators \( H^l H D R Y \left[ \Delta q H^l Q \right] \) and \( H^l H E R Y \left[ \Delta q H^l L \right] \) are also often considered (e.g., \([10]\)) but they can be shown using the equations of motion for SM fields to be related to \( O_{RL} \) and the other operators above \([35]\).

It is worth remarking that there are relations among \( \Delta q \) and \( \Delta qn \) above (among their respective sets of coefficients \( \zeta \)) which are fixed within a given model, but such relations are generally different in a different model. As a consequence, stringent bounds on processes induced by one or more of the quark operators in Eqs. (11) and (12) may not necessarily apply to the others, depending on the underlying new-physics model. Similar statements can be made regarding \( \Delta l \), \( \Delta \ell n \), and the lepton operators in Eqs. (11) and (13). For these reasons, in our model-independent analysis on the contributions of \( O_{RL} \) to \( h \to f f' \) we will not deal with constraints on the operators in Eqs. (12) and (13). Our results would then implicitly pertain to scenarios in which such constraints do not significantly affect the predictions for \( h \to f f' \).

In view of \( O_{RL} \) in Eq. (11) which is invariant under the flavor symmetry \( G_f \), it is convenient to rotate the fields and work in the basis where \( Y_{d,e} \) are diagonal,

\[
Y_d = \text{diag}(y_d, y_s, y_b), \quad Y_e = \text{diag}(y_e, y_\mu, y_\tau), \quad y_f = \sqrt{2} m_f / v,
\]

(14)

and \( U_k, D_k, \tilde{v}_{k,L}, \nu_{k,R} \) and \( E_k \) refer to the mass eigenstates. Explicitly, \((U_1, U_2, U_3) = (u, c, t), \)

\( (D_1, D_2, D_3) = (d, s, b), \)

and \( (E_1, E_2, E_3) = (e, \mu, \tau) \). Accordingly,

\[
Q_{k,L} = \left( (V_{\text{CKM}}^\dagger)_{kl} U_{k,L} \right), \quad L_{k,L} = \left( (U_{\text{PMNS}})_{kl} \tilde{v}_{l,L} \right), \quad Y_u = V_{\text{CKM}}^\dagger \text{diag}(y_u, y_e, y_t),
\]

\[
A_q = V_{\text{CKM}}^\dagger \text{diag}(y_u^2, y_e^2, y_t^2) \quad \nu_{k,L}, \quad A_\ell = \frac{2 M}{v^2} U_{\text{PMNS}} \hat{m}_{\nu}^{1/2} O O^{1/2} \hat{m}_{\nu}^{1/2} U_{\text{PMNS}}^\dagger,
\]

\[
B_q = \text{diag}(y_d^2, y_s^2, y_b^2), \quad B_\ell = \text{diag}(y_e^2, y_\mu^2, y_\tau^2),
\]

(15)

where \( V_{\text{CKM}} \) is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix.

Now, we express the effective Lagrangian describing \( h \to f f' \) as

\[
\mathcal{L}_{hff'} = - \mathcal{F}(Y_{f'}_P L + Y_{f'}_P R) f' h,
\]

(16)

\[\text{This was explicitly done for the leptonic operators in \([14]\).}\]

\[\text{The high degree of model dependency in the relationships among the } \Delta s \text{ belonging to the different operators is well illustrated by the results of the papers in \([11, 13]\) which address } h \to \mu \tau \text{ in the contexts of various scenarios. Particularly, there are models \([12]\) in which } B(h \to \mu \tau) \sim 1\% \text{ is achievable from tree-level contributions without much hindrance from the strict experimental requirements on } \ell \to l' \gamma \text{ transitions, including lepton } g - 2, \text{ which arise from one-loop diagrams. In some other models \([13]\) all these processes only occur at the loop level and the limiting impact of the } \ell \to l' \gamma \text{ restrictions on } h \to \mu \tau \text{ is considerable. It follows that one cannot make definite predictions for } \ell \to l' \gamma \text{ in a model-independent way based on the input from } h \to \mu \tau.\]
where \( \mathcal{Y}_{ff',ff} \) are the Yukawa couplings, which are generally complex, and \( P_{L,R} = (1 \mp \gamma_5)/2 \) are chirality projection operators. This leads to the decay rate

\[
\Gamma_{h \to ff'} = \frac{m_h}{16\pi} \left( |\mathcal{Y}_{ff'}|^2 + |\mathcal{Y}_{ff'}'|^2 \right),
\]

where the fermion masses have been neglected compared to \( m_h \). Thus, from Eq. (11), which contributes to both flavor-conserving and -violating transitions, we find for \( h \to D_k \bar{D}_l, E_k^- E_l^+ \)

\[
\mathcal{Y}'_{D_k D_l} = \mathcal{Y}'_{D_k D_l}^{SM} - \frac{m_D m_h^2}{2 \Lambda^2 v} (\Delta_q)_{kl},
\]

\[
\mathcal{Y}'_{E_k E_l} = \delta_{kl} \mathcal{Y}'_{E_k E_l}^{SM} - \frac{m_E m_h^2}{2 \Lambda^2 v} (\Delta_l)_{kl},
\]

where we have included the SM contributions, which are separated from the \( \Delta_{q,l} \) terms and can be flavor violating only in the quark case due to loop effects, and \( \mathcal{Y}'_{ff} = m_f / v \) at tree level. Since approximately \( \Delta_{q,l} = \Delta_{q,l}^1 \), it follows that in our MFV scenario \( |\mathcal{Y}_{ff'}| \gg |\mathcal{Y}_{ff'}'| \) for \( ff' = ds, db, sb, e\mu, e\tau, \mu\tau \) and \( \mathcal{Y}_{ff} \) are real.

For \( \mathcal{Y}_{ds,db,bs} \), it is instructive to see how they compare to each other in the presence of \( \Delta_q \). In terms of the Wolfenstein parameters \( (\lambda, \rho, \eta) \), the matrices \( A_q \) and \( A_q^2 \) in \( \Delta_q \) are given by

\[
A_q \approx \begin{pmatrix}
\lambda^6 A^2 [(1 - \rho)^2 + \eta^2] & -\lambda^6 A^2 (1 - \rho + i\eta) & \lambda^3 A (1 - \rho + i\eta) \\
-\lambda^5 A^2 (1 - \rho - i\eta) & \lambda^4 A^2 & -\lambda^2 A \\
\lambda^3 A (1 - \rho - i\eta) & -\lambda^2 A & 1
\end{pmatrix} \approx A_q^2
\]

(20)

to the lowest nonzero order in \( \lambda \approx 0.23 \) for each component, as \( y_u^2 \ll y_c^2 \sim 1.4 \times 10^{-5} \sim 2\lambda^8 \) and \( y_t \sim 1 \) at the renormalization scale \( \mu \sim m_h/2 \). If the \( \Delta_q \) part of \( \mathcal{Y}_{D_k D_l} \) for \( k \neq l \) is dominant, we then arrive at the ratio

\[
|\mathcal{Y}_{ds}| : |\mathcal{Y}_{db}| : |\mathcal{Y}_{sb}| \zeta \lambda^3 A |1 - \rho + i\eta|m_s : \lambda|1 - \rho + i\eta|m_b : m_b = 0.00016 : 0.21 : 1,
\]

(21)

the numbers having been calculated with the central values of the Wolfenstein parameters from Ref. 30\(^7\) as well as \( m_s = 57 \) MeV and \( m_b = 3.0 \) GeV at \( \mu \sim m_h/2 \).

The SM coupling \( \mathcal{Y}_{D_k D_l}^{SM} \) with \( k \neq l \) arises from one-loop diagrams with the W boson and up-type quarks in the loops. Numerically, we employ the formulas available from Ref. 37\(^7\) to obtain \( \mathcal{Y}_{ds}^{SM} = (7.2 + 3.1i) \times 10^{-10} \), \( \mathcal{Y}_{db}^{SM} = -(9.2 + 3.8i) \times 10^{-7} \), \( \mathcal{Y}_{sb}^{SM} = (4.7 - 0.1i) \times 10^{-6} \), and relatively much smaller \( |\mathcal{Y}_{sd,db,bs}^{SM}| \). These SM predictions are, as expected, consistent with the ratio in Eq. (21), but still lie very well within the indirect bounds inferred from the data on \( K^-\bar{K}, B_d^-\bar{B}_d, \) and \( B_s^-\bar{B}_s \) oscillations, namely \( 34\)

\[
-5.9 \times 10^{-10} < \text{Re}(\mathcal{Y}_{ds, sd}^2) < 5.6 \times 10^{-10}, \quad |\text{Re}(\mathcal{Y}_{ds}^* \mathcal{Y}_{sd})| < 5.6 \times 10^{-11},
\]

\[
-2.9 \times 10^{-12} < \text{Im}(\mathcal{Y}_{ds, sd}^2) < 1.6 \times 10^{-12}, \quad -1.4 \times 10^{-13} < \text{Im}(\mathcal{Y}_{ds}^* \mathcal{Y}_{sd}) < 2.8 \times 10^{-13},
\]

\[
|\mathcal{Y}_{db, bd}|^2 < 2.3 \times 10^{-8}, \quad |\mathcal{Y}_{db}^* \mathcal{Y}_{bd}| < 3.3 \times 10^{-9},
\]

\[
|\mathcal{Y}_{sb, bs}|^2 < 1.8 \times 10^{-6}, \quad |\mathcal{Y}_{sb}^* \mathcal{Y}_{bs}| < 2.5 \times 10^{-7}.
\]

\(^7\) Explicitly, \( \lambda = 0.22543 \), \( A = 0.823 \), \( \rho \approx 0.1536 \), and \( \eta \approx 0.3632 \).
FIG. 1: Regions of $\zeta_1/\Lambda^2$ and $\zeta_2/\Lambda^2$ for $\zeta_4 = 0$ which fulfill the experimental constraints in Eqs. (22)-(23). The $\zeta_2/\Lambda^2$ range is determined by $|\mathcal{Y}_{ub}|^2 < 2.3 \times 10^{-8}$ from Eq. (22).

Hence there is ample room for new physics to saturate one or more of these limits. Before examining how the $\mathcal{L}_{\text{MFV}}$ contributions may do so, we need to take into account also the $h \to b\bar{b}$ measurement quoted in Eq. (1). Thus, based on the 90%-CL range of this number in view of its currently sizable error, we may impose

$$0.4 < |\mathcal{Y}_{bb}/\mathcal{Y}_{bb}^{\text{SM}}|^2 < 1.1,$$

where $\mathcal{Y}_{bb}^{\text{SM}} \simeq 0.0125$ from the central values of the SM Higgs total width $\Gamma_h^{\text{SM}} = 4.08 \text{ MeV}$ and $B(h \to b\bar{b})^{\text{SM}} = 0.575$ determined in Ref. [38] for $m_h = 125.1 \text{ GeV}$ [9]. Upon applying the preceding constraints to Eq. (18), we learn that $|\mathcal{Y}_{ub}|^2 < 2.3 \times 10^{-8}$ in Eq. (22) and the one in Eq. (23) are the most consequential and that the former can be saturated if at least both the $\zeta_1$ and $\zeta_2$, or $\zeta_4$, terms in $\Delta q$ are nonzero. We illustrate this in Fig. 1 for $\zeta_4 = 0$, where the $\zeta_2/\Lambda^2$ limits of the (blue) shaded areas are fixed by the just mentioned $|\mathcal{Y}_{ub}|$ bound and the $\zeta_1/\Lambda^2$ values in these areas ensure that Eq. (23) is satisfied. Interchanging the roles of $\zeta_2$ and $\zeta_4$ would lead to an almost identical plot. If $|\zeta_{1,2}| \sim 1$, these results imply a fairly weak lower-limit on the MFV scale $\Lambda$ of around 50 GeV.

For the leptonic Yukawa couplings, $\mathcal{Y}_{E_k E_l}$ in Eq. (19), the situation is different and not unique because the specific values and relative sizes of the elements of $A_{\ell}$ in $\Delta_{\ell}$ can vary greatly [14]. In our MFV scenario with the type-I seesaw, this depends on the choices of the right-handed neutrinos’ mass $\mathcal{M}$ and the orthogonal matrix $O$ as well as on whether the light neutrinos’ mass spectrum $(m_1, m_2, m_3)$ has a normal hierarchy (NH) or an inverted one (IH).

For instance, if $O$ is real, $A_{\ell} = (2\mathcal{M}/v^2)U^{\text{PMNS}}_{\ell m} \tilde{m}_\nu U^{\dagger}_{\text{PMNS}}$ from Eq. (15), and using the central values of neutrino mixing parameters from a recent fit to global neutrino data [39] we find in the NH case with $m_1 = 0$

$$A_{\ell} \simeq 10^{-15}\mathcal{M}/\text{GeV} \begin{pmatrix} 0.12 & 0.19 + 0.12i & 0.01 + 0.14i \\ 0.19 - 0.12i & 0.82 & 0.7 - 0.02i \\ 0.01 - 0.14i & 0.70 + 0.02i & 0.98 \end{pmatrix}. \quad (24)$$
Incorporating this and selecting $\xi_4 = 0$ in $\Delta_4$ to be employed in Eq. (19), we then arrive at $|\mathcal{Y}_{\mu\mu}| : |\mathcal{Y}_{e\tau}| : |\mathcal{Y}_{\mu\tau}| = |(A_{e})_{12}| m_{\mu} : |(A_{e})_{13}| m_{\tau} : |(A_{e})_{23}| m_{\tau} \simeq 0.019 : 0.19 : 1$. Interchanging the roles of $\xi_2$ and $\xi_4$ would modify the ratio to $0.013 : 0.21 : 1$. In the IH case with $m_3 = 0$, the corresponding numbers are roughly about the same. These results for the Yukawas in the real-$O$ case turn out to be incompatible with the following experimental constraints on the Yukawa couplings if we demand $B(h \rightarrow \mu\tau) \sim 1\%$ as CMS suggested, but with $O$ being complex instead it is possible to satisfy all of these requirements [14].

For the first set of constraints, the direct-search limits in Eqs. (3) and (4) translate into
\[ \sqrt{|\mathcal{Y}_{\mu\mu}|^2 + |\mathcal{Y}_{e\tau}|^2} < 5.43 \times 10^{-4}, \quad \sqrt{|\mathcal{Y}_{e\tau}|^2 + |\mathcal{Y}_{\mu\tau}|^2} < 2.41 \times 10^{-3}, \tag{25} \]
and $\sqrt{|\mathcal{Y}_{\mu\mu}|^2 + |\mathcal{Y}_{\mu\tau}|^2} < 3.6 \times 10^{-3}$ under the no-signal assumption, while Eq. (2) for the $h \rightarrow \mu\tau$ signal interpretation implies
\[ 2.0 \times 10^{-3} < \sqrt{|\mathcal{Y}_{\mu\mu}|^2 + |\mathcal{Y}_{\mu\tau}|^2} < 3.3 \times 10^{-3}. \tag{26} \]

Additionally, the latest experimental bound $B(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}$ at 90\% CL [40] on the loop-induced decay $\mu \rightarrow e\gamma$ can offer a complementary, albeit indirect, restraint [10, 34, 41] on different couplings simultaneously [14].

\[ \sqrt{(|\mathcal{Y}_{\mu\mu} + r_\mu| \mathcal{Y}_{e\tau} + 9.19 |\mathcal{Y}_{\mu\tau} \mathcal{Y}_{e\tau}|^2 + |(\mathcal{Y}_{\mu\mu} + r_\mu) \mathcal{Y}_{e\mu} + 9.19 |\mathcal{Y}_{e\mu} \mathcal{Y}_{\mu\tau}|^2 < 4.4 \times 10^{-7}, \tag{27} \]
with $r_\mu = 0.29$ [34]. This could be stricter especially on $\mathcal{Y}_{e\mu,\mu\tau}$ than its direct counterpart in Eq. (25) if destructive interference with other potential new physics effects is absent. Compared to Eqs. (25)-(27), the indirect limits [34] from the data on $\tau \rightarrow e\gamma, \mu\gamma$ and leptonic anomalous magnetic and electric dipole moments are not competitive for our MFV cases. Finally, the $h \rightarrow \mu^+\mu^-, \tau^+\tau^-$ measurements quoted in Eq. (11) are also relevant and may be translated into
\[ |\mathcal{Y}_{\mu\mu}/\mathcal{Y}_{\mu\tau}|^2 < 5, \quad 0.9 < |\mathcal{Y}_{\tau\tau}/\mathcal{Y}_{\tau\tau}^\text{SM}|^2 < 1.3, \tag{28} \]
where $\mathcal{Y}_{\mu\mu}^\text{SM} \simeq 4.24 \times 10^{-4}$ and $\mathcal{Y}_{\tau\tau}^\text{SM} \simeq 7.19 \times 10^{-3}$ from $B(h \rightarrow \mu^+\mu^-)_{\text{SM}} = 2.19 \times 10^{-4}$ and $B(h \rightarrow \tau^+\tau^-)_{\text{SM}} = 6.30\%$ supplied by Ref. [38].

As pointed out in Ref. [14], the aforementioned leptonic MFV scenario with the $O$ matrix in $A_\ell$ being real is unable to accommodate the preceding constraints, especially Eqs. (20) and (27), even with the $\xi_{1,2,4}$ terms in $\Delta_\ell$ contributing at the same time. Rather, it is necessary to adopt a less simple structure of $A_\ell$ with $O$ being complex, which can supply extra free parameters to achieve the desired results, one of them being $|\mathcal{Y}_{e\mu}/\mathcal{Y}_{\mu\tau}| \lesssim 10^{-3}$. This possibility was already explored in Ref. [14] and therefore will not be analyzed further here.

### III. HIGGS FERMIONIC DECAYS IN GUT WITH MFV

In the Georgi-Glashow grand unification based on the SU(5) gauge group [28] the conjugate of the right-handed down-type quark, $(D_{k,R})^c$, and the left-handed lepton doublet, $L_{k,L}$, appear

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For a review see, e.g., [42].
in the 5 representations $\psi_k$, whereas the left-handed quark doublets, $Q_{k,L}$, and the conjugates of the right-handed up-type quark and charged lepton, $(U_{k,R})^c$ and $(E_{k,R})^c$, belong to the 10 representations $\chi_k$. With three SU(5)-singlet right-handed neutrinos being included in the theory, the Lagrangian for fermion masses is \cite{26,29}

$$\mathcal{L}^\text{GUT}_m = (\lambda_5)_{kl} \psi_k^\dagger \chi_l H_5^c + (\lambda_{10})_{kl} \chi_k^\dagger \chi_l H_5^c + \frac{(\lambda_{14})_{kl}}{M_p} \psi_k^\dagger \Sigma_{24} \chi_l H_5^c + (\lambda_1)_{kl} u^{\dagger}_{k,R} \psi_l H_5 - \frac{(M_p)_{kl}}{2} \nu_{k,R} \nu_{l,R} + \text{h.c.}, \quad (29)$$

where SU(5) indices have been dropped, $H_5$ and $\Sigma_{24}$ are Higgs fields in the 5 and 24 of SU(5), and compared to the GUT scale the Planck scale $M_p \gg M^\text{GUT}_m$. Since $\mathcal{L}^\text{GUT}_m$ contains $\mathcal{L}_m$ for the SM plus 3 degenerate right-handed neutrinos, the Yukawa couplings in these Lagrangians satisfy the relations \cite{26,29}

$$Y_u^\dagger \propto \lambda_{10}, \quad Y_d^\dagger \propto \lambda_5 + \epsilon \lambda_5', \quad Y_e^* \propto \lambda_5 - \frac{3}{2} \epsilon \lambda_5', \quad Y_\nu^\dagger = \lambda_1,$$  \quad (30)

where $\epsilon = M^\text{GUT}_m / M_p \ll 1$. Evidently, in the absence of the dimension-five nonrenormalizable $\lambda_5'$ term in $\mathcal{L}^\text{GUT}_m$ the down-type Yukawas would be related by $Y_d \propto Y_e^\dagger$ which is inconsistent with the experimental masses \cite{29}. In this work, we do not include the corresponding term for the up-type quark sector, $(\lambda_{14})_{kl} \chi_k^\dagger \Sigma_{24} \chi_l H_5^c / M_p$ \cite{26}, which could significantly correct the up-quark mass, but does not lead to any quark-lepton mass relations.

The application of the MFV principle in this GUT context entails that under the global flavor symmetry group $G^\text{GUT} = \text{SU}(3)_5 \times \text{SU}(3)_{10} \times \text{O}(3)_1$ the fermion fields and Yukawa spurions in $\mathcal{L}^\text{GUT}_m$ transform as \cite{26}

$$\psi \rightarrow V_5 \psi, \quad \lambda_5^{(o)} \rightarrow V_5^{\dagger} \lambda_5^{(o)} V_5^\dagger, \quad \lambda_{10}^{(o)} \rightarrow V_5^{\dagger} \lambda_{10}^{(o)} V_5^\dagger, \quad \nu \rightarrow \mathcal{O}_1 \nu_R, \quad \lambda_1 \rightarrow \mathcal{O}_1 \lambda_1 V_5^\dagger,$$ \quad (31)

where we have assumed again that the right-handed neutrinos are degenerate, $V_{5,10} \in \text{SU}(3)^5_{5,10}$, and $\mathcal{O}_1 \in \text{O}(3)_1$. It follows that the flavor transformation properties of the fermions and Yukawa coupling matrices in $\mathcal{L}_m$ are

$$Q_L \rightarrow V_{10} Q_L, \quad U_R \rightarrow V_{10}^\dagger U_R, \quad D_R \rightarrow V_{10}^\dagger D_R,$$

$$L_L \rightarrow V_5 L_L, \quad E_R \rightarrow V_{10}^\dagger E_R,$$

$$Y_u \rightarrow V_{10} Y_u V_{10}^\dagger, \quad Y_d \rightarrow V_{10} Y_d V_{10}^\dagger, \quad Y_e \rightarrow V_5 Y_e V_{10}^\dagger,$$

$$Y_\nu \rightarrow V_5 Y_\nu \mathcal{O}_1.$$ \quad (32)

As in the non-GUT scheme treated in the previous section, one can then put together the spurion building blocks $\Delta_q = \zeta_1 \mathbb{I} + \zeta_2 A_q + \zeta_3 A_q^2$ and $\Delta_\ell = \xi_1 \mathbb{I} + \xi_2 A_\ell + \xi_3 A_\ell^2$, after dropping contributions involving products of down-type Yukawas, which have more suppressed elements.\footnote{Like before, we have assumed that the right-handed neutrinos’ mass $\mathcal{M} \sim 6 \times 10^{14}$ GeV, so that the biggest eigenvalue of $A_\ell$ is around one. Otherwise, if $\mathcal{M} \ll 10^{14}$ GeV, the flavor-violating impact of $\Delta_\ell$ would decrease accordingly.}
In analogy to the non-GUT scenario, the effective operators of interest constructed out of the spurious and SM fields need to be invariant under both $G^\text{GUT}_{\ell}$ and the SM gauge group. However, since $G^\text{GUT}_{\ell}$ is significantly smaller than $G_{\ell}$, in the GUT MFV framework there are many more ways to arrange flavor-symmetry-breaking objects for the operators \[26\]. It is straightforward to see that those pertaining to Higgs decays into down-type fermions at tree level are given by

$$
\mathcal{L}^\text{GUT}_{\text{MFV}} = \frac{1}{\Lambda^2} (D^a H)^\dagger D_R \left( Y^\dagger_d \Delta_{q_1} + Y^*_e \Delta_{q_2} + \Delta_{e_3}^T Y^\dagger_d + \Delta_{e_4}^T Y^*_e + \Delta_{e_7}^T Y^\dagger_{e_1} + \Delta_{e_8}^T Y^*_e \Delta_{q_2} \right) D_\alpha Q_L
$$

$$
+ \frac{1}{\Lambda^2} (D^a H)^\dagger E_R \left( Y^\dagger_e \Delta_{e_1} + Y^*_e \Delta_{e_2} + \Delta_{e_7}^T Y^\dagger_e + \Delta_{e_8}^T Y^*_e \Delta_{e_2} + \Delta_{e_4}^T Y^\dagger_{e_1} \right) D_\alpha L_L
$$

$$
+ \text{H.c.},
$$

where $\Delta^{(q)}_{q_4}$ and $\Delta^{(e)}_{e_r}$ are the same in form as $\Delta_q$ and $\Delta_e$, respectively, but have their own coefficients $\zeta^{(q)}_r$ and $\zeta^{(e)}_r$ ($r = 1, 2, 4$). We notice that, while the $\Delta_q$ and $\Delta_e$ terms in $\mathcal{L}^\text{GUT}_{\text{MFV}}$ already occur in the non-GUT case, Eq. \([\text{[11]}]\), the others are new here. In general, the different quark and lepton operators in Eq. \([33]\) may be unrelated to each other, depending on the specifics of the underlying model, and so it is possible that only one or some of the terms in $\mathcal{L}^\text{GUT}_{\text{MFV}}$ dominate the nonstandard contribution to $h \to f \bar{f}'$. Therefore, we will consider different possible scenarios below. As in the non-GUT framework of the last section, we will evaluate the contributions of $\mathcal{L}^\text{GUT}_{\text{MFV}}$ to Higgs decay model-independently and not deal with the constraints on the GUT-MFV counterparts of the operators in Eqs. \([12]\) and \([13]\), as the potential links among the $\Delta$s belonging to these various operators again depend on model details.

Working in the mass eigenstate basis, we derive from Eq. \([33]\)

$$
\mathcal{L}^\text{GUT}_{\text{MFV}} \supset \frac{\partial^a h}{\sqrt{2} \Lambda^2} D_R \left( Y^\dagger_d \Delta_{q_1} + G^\dagger Q^\dagger C \Delta_{q_2} + G^\dagger \Delta_{e_3}^T G Y^\dagger_d + G^\dagger \Delta_{e_4}^T Y^*_e C \right)
$$

$$
+ \frac{\partial^a h}{\sqrt{2} \Lambda^2} E_R \left( Y^\dagger_e \Delta_{e_1} + C^\dagger G^\dagger \Delta_{e_2} + C^\dagger \Delta_{e_7}^T Y^\dagger_e G^\dagger + C^\dagger \Delta_{e_8}^T C^\dagger Y^*_e \Delta_{e_2} \right)
$$

$$
+ \text{H.c.},
$$

where now the column matrices $D_{L,R}$ and $E_{L,R}$ contain mass eigenstates, $Y_{d,e}$ are diagonal and real as in Eq. \([14]\), the formulas for $A_{q,\ell}$ in $\Delta^{(q)}_{q_n,\ell_n}$, respectively, are those in Eq. \([15]\), and

$$
C = V^\dagger_{e_R} V_{d_L}, \quad G = V^\dagger_{e_L} V_{d_R},
$$

with $V_{d_{L,R}}$ and $V_{e_{L,R}}$ being the unitary matrices in the biunitary transformations that diagonalize $Y_d$ and $Y_e$, respectively. Since the elements of $V_{d_{L,R}}$ and $V_{e_{L,R}}$ are unknown, so are those of $C$ and $G$. Nevertheless, it has been pointed out in Ref. \([26]\) that the two matrices have hierarchical textures. As indicated in Appendix \([A]\), this implies that the limit $C = G = \mathbb{1}$ is one possibility that may be entertained for order-of-magnitude considerations \([26, 43]\). It corresponds to neglecting the subdominant $\lambda'_5$ contributions in Eq. \([30]\). Due to the lack of additional information about $C$ and $G$, in what follows we concentrate on this special scenario for simplicity, in which
case the Yukawa couplings from Eq. (34) are

\[ Y_{\ell h D_i} = \frac{m_h^2}{2\Lambda^2 v} \left[ (\Delta_{q1})_{kl} m_{D_i} + (\Delta_{q2})_{kl} m_{E_i} + m_{D_k} (\Delta_{q3})_{lk} + m_{E_k} (\Delta_{q4})_{lk} \right] 
- \frac{m_h^2}{2\Lambda^2 v} \left( \Delta_{q1}' M_d \Delta_{q3}' + \Delta_{q2}' M_e \Delta_{q4}' \right)_{kl}, \]

\[ Y_{E_k E_l} = \delta_{kl} Y_{E_k E_l}^\text{SM} - \frac{m_h^2}{2\Lambda^2 v} \left[ (\Delta_{q1})_{kl} m_{E_i} + (\Delta_{q2})_{kl} m_{D_i} + m_{D_k} (\Delta_{q3})_{lk} + m_{E_k} (\Delta_{q4})_{lk} \right] 
- \frac{m_h^2}{2\Lambda^2 v} \left( \Delta_{q2}' M_d \Delta_{q3}' + \Delta_{q1}' M_e \Delta_{q4}' \right)_{kl}, \] (36)

where \( M_d = Y_d v/\sqrt{2} = \text{diag}(m_d, m_s, m_b) \) and \( M_e = Y_e v/\sqrt{2} = \text{diag}(m_e, m_\mu, m_\tau) \).

To gain some insight into the potential impact of the new terms on these Yukawas, we can explore several different simple scenarios in which only one or more of the \( \Delta \)s are nonvanishing. If \( \Delta_{q1} \) and \( \Delta_{q2} \) are the only ones present and independent of each other, their effects are the same as those of \( \Delta_q \) and \( \Delta_\ell \), respectively, investigated in the previous section and Ref. [14].

In the rest of this section, we look at other possible cases.

In the first one, we assume that \( \Delta_{q1} \) is the only new source in Eq. (36). In view of the rough similarity between the \( \Delta_{q1} \) and \( \Delta_{q2} \) portions of \( Y_{E_k E_l} \), due to \( m_\mu/m_s \sim m_b/m_\tau \sim 2 \) at the renormalization scale \( \mu \sim m_h/2 \), we can infer that the situation in this case is not much different from its \( \Delta_\ell \) counterpart addressed briefly in the last section and treated more extensively in Ref. [14]. In other words, for the \( \Delta_\ell \) term alone to achieve \( B(h \rightarrow \mu\tau) \sim 1\% \) and meet the other requirements described earlier simultaneously, the \( O \) matrix occurring in \( A_\ell \), as defined in Eq. (15), must be complex in order to provide the extra free parameters needed to raise \( |Y_{\mu\tau}| \) and reduce \( |Y_{e\mu}| \) sufficiently. If \( \Delta_{\ell 1} \) is also nonvanishing and equals \( \Delta_{\ell 2} \), the picture is qualitatively unchanged. We have verified all this numerically.

Still another possibility with \( \Delta_{\ell n} \) is that all the \( \Delta_{qn} \) are absent and that \( Y_{D_k D_i} \) and \( Y_{E_k E_l} \) each have at least one \( \Delta_{q3} \). In this case, if, say, only \( \Delta_{q1,3} \) are present and \( \Delta_{q2} = \Delta_{q4} \), we find that it is not possible to reach the desired \( |Y_{\mu\tau}| > 0.002 \) and satisfy the constraints in the quark sector at the same time. The situation is not improved by keeping all the \( \Delta_{\ell n} \), while still taking them to be equal. However, if the \( \Delta_{\ell n} \) contributions to \( Y_{D_k D_i} \) are weakened by an overall factor of 2 or more, at least part of the requisite range of \( |Y_{\mu\tau}| \) can be attained.

An interesting case is where \( \Delta_{q3} \) is nonvanishing and all of the other \( \Delta \)s in Eq. (36) are absent. This implies that the flavor changes depend entirely on the known CKM parameters and quark masses. Furthermore, \( |Y_{\mu e, \tau e, \tau \mu}| \gg |Y_{e\mu, e\tau, \tau\mu}| \), respectively, as can be deduced from Eq. (36). It turns out that the lepton restrictions in Eqs. (25)-(28) can be satisfied together with only the \( \zeta_1 \) and \( \zeta_3 \), or \( \zeta_4 \), terms in \( \Delta_{q3} \) being present. We also find that the largest \( |Y_{\tau\mu}| \) that can be attained is \( \sim 0.0029 \). We illustrate this in Fig. 2 where the cyan and dark blue (orange and dark red) areas correspond to only \( \zeta_{1,2} \) (\( \zeta_{1,4} \)) being nonzero. The widths of the two (colored) bands in this graph are controlled by the \( Y_{\tau\tau} \) constraint, whereas the vertical and horizontal ranges are restrained by Eq. (26) as well as the \( Y_{\mu\mu} \) constraint and Eq. (27). To show some more details of this case, we collect in Table I a few sample values of the Yukawa couplings in the allowed parameter space. Evidently, the predictions on \( Y_{\mu\mu, \tau\tau} \) can deviate markedly from their SM values and, therefore, will likely be confronted with more precise measurements of \( h \rightarrow \mu^+\mu^-, \tau^+\tau^- \).
in the near future. As expected, the flavor-violating couplings obey the magnitude ratio $|Y_{\mu e}| : |Y_{\tau e}| : |Y_{\tau \mu}| \simeq |(A_q)_{12}|m_s : |(A_q)_{13}|m_b : |(A_q)_{23}|m_b \simeq 0.00017 : 0.21 : 1$, compatible with Eq. (21). Also listed in the table are the branching fractions of the decay $\mu \rightarrow e\gamma$ and $\mu \rightarrow e$ conversion in aluminum nuclei, computed with the formulas collected in Ref. [14] under the assumption that these transitions are induced by the Yukawas alone. The $\mu \rightarrow e\gamma$ numbers are below the current experimental bound $B(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}$ [40], but not by very much. Hence they will probably be checked by the planned MEG II experiment with sensitivity anticipated to reach a few times $10^{-14}$ after 3 years of data taking [24]. Complementarily, the $B(\mu Al \rightarrow e Al)$ results can be probed by the upcoming Mu2E and COMET searches, which utilize aluminum as the target material and are expected to have sensitivity levels under $10^{-16}$ after several years of running [44].

In contrast to the preceding paragraph, if $\Delta_{q4}$ instead of $\Delta_{q3}$ is nonvanishing and the other $\Delta$s remain absent, the desired size of $|Y_{\tau \mu}|$ becomes unattainable, as it can be at most $\sim 0.001$, even with $\zeta_{1,2,4}$ being nonzero. If both $\Delta_{q3,q4}$ are the only ones present and they are identical, we find $|Y_{\tau \mu}| \sim 0.0017$ to be the biggest achievable, somewhat below the lower limit in Eq. (21).

| $\mathcal{V}_{ee}$ | $\mathcal{V}_{\mu\mu}$ | $\mathcal{V}_{\tau\tau}$ | $\mathcal{V}_{\mu e}$ | $\mathcal{V}_{\tau e}$ | $\mathcal{V}_{\tau\mu}$ | $B(\mu \rightarrow e\gamma)$ | $B(\mu Al \rightarrow e Al)$ |
|---|---|---|---|---|---|---|---|
| $-31$ | $-2.1$ | $0.95$ | $-4.3 - 1.9i$ | $5.5 + 2.3i$ | $-2.8 + 0.05i$ | $4.0 \times 10^{-13}$ | $2.0 \times 10^{-15}$ |
| $-28$ | $-1.8$ | $1.1$ | $-4.0 - 1.7i$ | $5.1 + 2.1i$ | $-2.6 + 0.05i$ | $3.1 \times 10^{-13}$ | $1.6 \times 10^{-15}$ |
| $-24$ | $-1.4$ | $1.0$ | $-3.4 - 1.5i$ | $4.3 + 1.8i$ | $-2.2 + 0.04i$ | $1.7 \times 10^{-13}$ | $9.5 \times 10^{-16}$ |

TABLE I: Higgs-lepton Yukawa couplings if the $\Delta_{q3}$ term with $\zeta_4 = 0$ is the only new-physics contribution in Eq. (36), and the resulting branching fractions of the $\mu \rightarrow e\gamma$ decay and $\mu \rightarrow e$ conversion in aluminum nuclei.

FIG. 2: Regions of $\zeta_1/\Lambda^2$ and $\zeta_2/\Lambda^2$ for $\zeta_4 = 0$ (cyan and dark blue) which satisfy the experimental constraints in Eqs. (25) - (28) if the $\Delta_{q3}$ term is the only new-physics contribution in Eq. (36). For the orange and dark red regions, the roles of $\zeta_2$ and $\zeta_4$ are interchanged. The dark (blue and red) patches correspond to $|Y_{\tau \mu}| \simeq 0.0029$ and hence $B(h \rightarrow \mu\tau) \simeq 1\%$. 

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If instead $\Delta_{q1}$ and $\Delta_{q3}$ are the only ones nonvanishing and $\Delta_{q1} = \Delta_{q3}$, the quark sector constraints in Eqs. (22)-(23) do not permit $|Y_{r\mu}|$ to exceed 0.00072, which is almost 3 times less than the required minimum in Eq. (26). This implies that, alternatively, if the $\Delta_{q1}$ contribution to $Y_{D_LD_L}$ is decreased by an overall factor of 3 or more, at least part of the desired $|Y_{r\mu}|$ range can be reached and the other restrictions fulfilled.

Lastly, we look at the $\Delta'_{q2}M_d\Delta_{q3}^{\tau}$ and $\Delta'_{e1}M_e\Delta_{q4}^{\tau}$ parts in $Y_{E_kE_l}$. With $\Delta'_{q3} = \zeta'_11 + \zeta'_2A_q + \zeta'_4A_q^2$ and $\Delta'_{q2} = \xi'_11 + \xi'_2A_l + \xi'_4A_l^2$, using in particular $A_q$ from Eq. (20) and $A_l$ from Eq. (24), we see that $\Delta'_{q2}M_d\Delta_{q3}^{\tau}$ has two more free parameters, $\zeta'_2,4 (\xi'_2,4)$, compared to $\Delta'_{q2}M_d (\hat{M}_d\Delta_{q3}^{\tau})$. It turns out, however, that the presence of additional parameters does not necessarily translate into more freedom for the $\Delta'_{q2}M_d\Delta_{q3}^{\tau}$ contributions due to the following reason. With $\hat{M}_d$ being sandwiched between $\Delta'_{q2}$ and $\Delta_{q3}^{\tau}$, in general $Y_{ff'}$ for $f \neq f'$ can be comparable in size to $Y_{f'f}$ because they both have terms linear in $m_b$, as do $Y_{ee,\mu\mu}$, which is unlike the situation of the $Y_{E_kE_l}$ parts containing only one $\Delta$. We find that, once the two extra free parameters are fixed to suppress the $m_b$ effects on $\mu \to e\gamma$ as well as $h \to \mu^+\mu^-$, the predictions for the various $Y_{E_kE_l}$ are not very different qualitatively from those in the $\Delta'_{q2}$ ($\Delta_{q3}$) case examined earlier. Similarly, the implications of the contributions of $\Delta'_{e1}M_e\Delta_{q4}^{\tau}$ do not differ much from those of $\Delta_{e1}M_e$ or $\hat{M}_e\Delta_{q4}^{\tau}$ also discussed earlier.

The above simple scenarios have specific predictions for the flavor-conserving and -violating Yukawa couplings and hence are all potentially testable in upcoming measurements of $h \to f\bar{f}'$ and searches for flavor-violating charged-lepton transitions such as $\mu \to e\gamma$. If the predictions disagree with the collected data, more complicated cases could be proposed in order to probe further the GUT MFV framework that we have investigated.

IV. CONCLUSIONS

We have explored the flavor-changing decays of the Higgs boson into down-type fermions in the MFV framework based on the SM extended with the addition of right-handed neutrinos plus effective dimension-six operators and in its SU(5) GUT counterpart. As a consequence of the MFV hypothesis being applied in the latter framework, we are able to entertain the possibility that the recent tentative indication of $h \to \mu\tau$ in the LHC data has some connection with potential new physics in the quark sector. Here the link is realized specifically by leptonic (quark) bilinears involving quark (leptonic) Yukawa combinations that control the leptonic (quark) flavor changes. We discuss different simple scenarios in this context and how they are subject to various experimental requirements. In one particular case, the leptonic Higgs couplings are determined mainly by the known CKM parameters and quark masses, and interestingly their current values allow the couplings to yield $B(h \to \mu\tau) \sim 1\%$ without being in conflict with other constraints. Forthcoming measurements of the Higgs fermionic decays and searches for flavor-violating charged-lepton decays will expectedly provide extra significant tests on the GUT MFV scenarios studied here.
Acknowledgments

The work of J.T. was supported in part by the MOE Academic Excellence Program (Grant No. 102R891505) of Taiwan. He would like to thank S.B. and Pyungwon Ko for generous hospitality at the Korea Institute for Advanced Study during the course of this research. This work is supported in part by National Research Foundation of Korea (NRF) Research Grant NRF-2015R1A2A1A05001869 (S.B.).

Appendix A: C and G matrices

The unitary matrices C and G defined in Eq. (35) have unknown elements, but are expected to be hierarchical in structure [26]. Expressing each of them as an expansion in the Wolfenstein parameter $\lambda \simeq 0.23$, we have

$$C = \begin{pmatrix} C_{11} & C_{12} & \lambda^2(C_{11}C_1 + C_{12}C_2) \\ C_{21} & C_{22} & \lambda^2(C_{21}C_1 + C_{22}C_2) \\ -\lambda^2C_1^* - \lambda^2C_2^* & 1 \end{pmatrix},$$

$$G = \begin{pmatrix} G_{11} & G_{12} & \lambda^2(G_{11}G_1 + G_{12}G_2) \\ G_{21} & G_{22} & \lambda^2(G_{21}G_1 + G_{22}G_2) \\ -\lambda^2G_1^* - \lambda^2G_2^* & 1 \end{pmatrix} \tag{A1}$$

up to order $\lambda^2$, where $C_{ac}$, $C_a$, $G_{ac}$, and $G_a$ are parameters with magnitudes below 1 and we have used the approximation $y_\mu/y_\tau \sim \lambda^2$. For discussion purposes, it suffices to look at only two of the flavor-violating matrix combinations occurring in Eq. (33), namely $G^\dagger Y_e C_\eta A_q$ and $C^* A^\dagger Y_d G^\tau$, which are parts of $G^\dagger Y_e C_\Delta \varphi_2$ and $C^* \Delta^\tau \varphi_2 Y_d G^\tau$, respectively. Expanding their matrix elements in $\lambda$, we express these combinations as

$$G^\dagger Y_e C_\eta A_q = \begin{pmatrix} O(\lambda^5) y_\tau & O(\lambda^4) y_\tau & \lambda^2[C_{21}C_1 + C_{22}(c_2 - A)]G_{21}^* y_\mu - \lambda^2C_1 y_\tau \\ O(\lambda^5) y_\tau & O(\lambda^4) y_\tau & \lambda^2[C_{21}C_1 + C_{22}(c_2 - A)]G_{22}^* y_\mu - \lambda^2C_2 y_\tau \\ \lambda^3(1 - \rho - i\eta)y_\tau & -\lambda^2A y_\tau & y_\tau \end{pmatrix},$$

$$C^* A^\dagger Y_d G^\tau = \begin{pmatrix} O(\lambda^4) & O(\lambda^4) & \lambda^2[C_{11}C_1^* + C_{12}(c_2^* - A)] \\ O(\lambda^4) & O(\lambda^4) & \lambda^2[C_{21}C_1^* + C_{22}(c_2^* - A)] \\ \lambda^2(G_{11}G_1 + G_{12}G_2) & \lambda^2(G_{21}G_1 + G_{22}G_2) & 1 \end{pmatrix} y_b. \tag{A2}$$

where we have kept only $y_{\tau b}$ terms to the leading nonzero order in $\lambda$ and $y_\mu$ terms to order $\lambda^2$, made use of $y_e^2/y_t^2 \sim 2\lambda^8$ and $y_\tau/y_b \sim 2\lambda^3$, and set $y_t = 1$. Being unknown, one or more of $C_{ac}$ and $C_a$ may be small or vanishing, although the unitarity of $C$ implies

$$|C_{11}|^2 + |C_{12}|^2 = 1, \quad |C_{11}| = |C_{22}|, \quad |C_{12}| = |C_{21}|, \quad C_{11}C_{12}^* = -C_{21}C_{22}^*. \tag{A3}$$
valid to order $\lambda^2$. The same can be said of the elements of $G$. It follows that we may choose $C = G = 1$ as a possible limit for these matrices \cite{26,13}, in which case Eq. (A2) becomes

$$Y_e A_q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\lambda^2 A y_{\mu} \\ \lambda^3 A (1 - \rho - i \eta) y_{\tau} & -\lambda^2 A y_{\tau} & y_{\tau} \end{pmatrix} + O(\lambda^5),$$

$$A_q^T Y_d = \begin{pmatrix} 0 & 0 & \lambda^3 A (1 - \rho - i \eta) y_{b} \\ 0 & 0 & -\lambda^2 A y_{b} \\ 0 & 0 & y_{b} \end{pmatrix} + O(\lambda^5). \quad (A4)$$

Taking this limit corresponds to neglecting the nonleading $\epsilon \lambda_5^\prime$ terms in Eq. (30) which break the $Y_d = Y_e^T$ relation ($C, G \to 1$ if $\epsilon \to 0$) and simplifies the treatment of quantities that depend on $C$ and $G$. However, since not much is known about their elements, their presence precludes a precise evaluation of such quantities \cite{26}. The implication is that the results of our GUT MFV calculations involving the Yukawas with $C = G = 1$ from Eq. (36) should be understood as only order-of-magnitude estimates.

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