Hipsters on Networks: How a Small Group of Individuals Can Lead to an Anti-Establishment Majority

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The spread of opinions, memes, diseases, and “alternative facts” in a population depends both on the details of the spreading process and on the structure of the social and communication networks on which they spread. One feature that can change spreading dynamics substantially is heterogeneous behavior among different types of individuals in a social network. In this paper, we explore how anti-establishment nodes (e.g., hipsters) influence spreading dynamics of two competing products.

We consider a model in which spreading follows a deterministic rule for updating node states (which describe which product has been adopted) in which an adjustable fraction \( p_{\text{Hip}} \) of the nodes in a network are hipsters, who always choose to adopt the product that they believe is the less popular of the two. The remaining nodes are conformists, who choose which product to adopt by considering only which products their immediate neighbors have adopted. We simulate our model on both synthetic and real networks, and we show that the hipsters have a major effect on the final fraction of people who adopt each product: even when only one of the two products exists at the beginning of the simulations, a very small fraction of hipsters in a network can still cause the other product to eventually become more popular. Our simulations also demonstrate that a time delay \( \tau \) in the knowledge of the product distribution in a population, as compared to immediate knowledge of product adoption among nearest neighbors, has a large effect on the final distribution of product adoptions. Using a local-tree approximation, we derive an analytical estimate of the spreading of products and obtain good agreement if a sufficiently small fraction of the population consists of hipsters. In all networks, we find that either of the two products can become the more popular one at steady state, depending on the fraction of hipsters in the network and on the delay of the knowledge of the product distribution in the total population. Our simple model and analysis may help shed light on the road to success for anti-establishment choices in elections, as such success — and qualitative differences in final outcomes between competing products, political candidates, and so on — can arise rather generically from a small number of anti-establishment individuals and ordinary processes of social influence on normal individuals.

I. INTRODUCTION

The study of spreading phenomena on networks has received considerable attention in many disciplines, including sociology, economics, physics, biology, computer science, and others\[1\][17]. In analogy with the spread of infectious diseases in populations of susceptible individuals, the spread of social phenomena (such as opinions, actions, memes, information, misinformation, and alternative facts) is often viewed as a contagion process that spreads through a network’s nodes, which are connected to each other via one or more types of edges. The nodes can represent entities such as people or institutions\[18\][20] (or other things), and the edges can represent physical proximity, communication channels, sociological interactions (e.g., different types of relationships), or something else. An important goal of many studies of the spread of social contagions is identifying criteria that determine when the phenomenon that is spreading reaches a large fraction of a population or subpopulation \[1\].

Scholars have used various approaches for studying contagions on networks. These include game theory\[21\], statistical physics\[9\], agent-based models\[22\], and systems of coupled differential equations or stochastic processes\[1, 23\][28]. The temporal dynamics and peak size of an outbreak are influenced both by the specific model of a contagion and by the structure of the network on which it spreads\[11\][19]\[26\][28]\[35\]. One of the focal ideas is to examine when a disease or idea, initially carried by a local fraction of nodes (so-called “infected” nodes, “active” nodes, or “adopters”), can become widespread in a network. When a large fraction of a population or subpopulation becomes infected (or adopts an idea), one says that a cascade has occurred, and cascading phenomena have been studied in a wide variety of systems, ranging from financial networks\[20\] to social media like Twitter\[30\]. Cascading phenomena are also often studied using

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Various types of percolation models [1]. Cascading behavior can be either beneficial or harmful. For example, a failing financial institution can cause a cascade of failures of numerous other financial institutions, a tweet can result in a cascade of tweets that promotes the opinion of the original tweeter (perhaps influenced by the actions of ‘bot’ accounts [16]), and widely-spread alternative facts can influence the opinions of a large population of voters [37].

Models for cascading behavior on networks can have either stochastic or deterministic state-update rules, and the update rules in most models only consider nodes that are adjacent to the node under consideration. For example, in the compartmental model known as the susceptible–infected (SI) model, when examining a susceptible node that may update to become infected, one considers all of its incident edges that are also incident to an infected node, and each of these edges gives an independent adoption rate of \( \alpha \) to infect the node. Nodes that become infected stay infected forever. Infection cascades can also occur in more complicated compartmental models, which can include recovery, a return to susceptibility, or additional node states. There are also many models with deterministic update rules, such as numerous threshold models for social contagions. The simplest example of such a model is the Watts threshold model (WTM) [4, 5, 8], a type of bootstrap percolation [38], in which each node is assigned a threshold from some distribution. When considering a node for updating, if the fraction of its neighbors that are adopters is at least as large as its threshold, it becomes an adopter itself. There are numerous variants and generalizations of the WTM, including ones with adoption thresholds based on number of adopted neighbors [11, 39], ones with multiple adoption stages [29], ones with “synergy” based on other nearby nodes that are adopters [25], and ones with timers in addition to adoption thresholds [40].

Efforts to develop mathematical models for product and innovation spreading date at least as far back as the 1960s. Rogers [41] described qualitatively (as sigmoidal-shape) how the number of adopters should look as a function of time. Bass [42] developed a mathematical model for the adoption of innovations that was inspired by models for biological contagions. Bass’s model results in sigmoidal-shaped adoption curves, and it has been generalized in various ways [33, 46].

More recent studies have considered models in which agents of different types can have significant effects on the final distributions of products or innovations in a population. For example, [17] showed that if some nodes are allowed to regret adopting an innovation, while other (“contrarians”) nodes resist adopting innovations, then temporal cycles of adoption can occur. Dodd et al. [18] found rich behavior (including chaotic dynamics) in a social contagion model that incorporates an aversion to complete conformity.

Reference [49] examined a population in which nodes can either adopt a product or become “luddites”, who oppose the spread of innovation, and found that luddites greatly limit adoption if the adoption rate is high but not if it is low. Reference [50] examined what they call the “majority illusion”, in which an opinion or product is overrepresented in some part of a network (such as in the “echo chambers” of online social networks) and thereby potentially accelerate global adoption cascades, depending on network structure. They also suggested that the majority illusion is much stronger in networks (such as Twitter and political blogs) in which low-degree nodes tend to link to high-degree. Gambaro and Crokidakis [61] illustrated that contrarian agents can be a source of disorder in opinion dynamics, and Ferrara and collaborators have reported that individual social-media accounts controlled by bots can exert a considerable influence on political elections and social cascades [16, 52, 53].

In the present paper, we examine the influence of anti-establishment nodes, such as hipsters, on spreading processes in a social network. Individuals specifically preferring something other than the established standard in society have manifested in several ways over the last decade. They include members of anti-establishment movements in Western Europe and the United States of America, who have hugely impacted the geopolitical landscape, to the curious style of hipsters in cities throughout the world. In some cases, such as the 2016 “Brexit” vote [54] and the 2016 American presidential election [55], anti-establishment opinions appear to have spread to so many people that they exerted a major influence on political outcomes. In our paper, we ask the following questions: (1) How does a large fraction of a population decide to choose something different from the established standard? (2) How can a small fraction of individuals spread their anti-establishment opinions to a majority (or at least very large minority) of the rest of a population? (3) Can we capture these ideas using a simple mathematical model of a spreading process on a network?

To explore the above phenomena, we examine the spread of two competing products in a network of heterogeneous agents who act differently when introduced to a product. Specifically, we investigate how the spread of the two products is influenced if some fraction of nodes choose to adopt the product that is less popular in the total population, rather than the more-popular product among their neighboring nodes. We use the term hipsters for these anti-conformist nodes, but in some contexts (such as political ones) they can be viewed as anti-establishment agents.

Previously, a statistical-physics approach was used to examine how anti-conformists (i.e., hipsters) who make decisions against the majority, thereby attempting to stand out from the crowd, may all end up “looking the same” [56] (wearing the same clothes, buying the same products, having the same opinions, and so on). This study found that the dynamics of a population was influenced greatly by delays in the knowledge of hipsters and by how large a fraction of the population are hipsters.
II. A THRESHOLD MODEL WITH HIPSTERS

A popular type of model for spreading processes on networks are threshold models of social influence\cite{1, 5}. Let’s set up a simple example of a threshold model. Consider a network with $N$ nodes, and suppose that each node $i$ is independently assigned a threshold $\phi_i$ from a distribution $f(\phi)$. We also suppose that a node can be in one of two states: active or inactive. An active node has adopted the product (or meme, opinion, product, etc.) that is spreading through a population, and an inactive node has not adopted the product. (We will use the term “product” from now on.) Once a node becomes active, it stays that way forever. The threshold of a node determines how difficult it is to activate that node, so a node’s threshold value can be construed as its stubbornness level. Node $i$ becomes active if a peer pressure, which in the WTM is equal to the fraction of active nodes among $i$’s neighbors, is greater than or equal to its threshold $\phi_i$.

We seek to develop a model for competing products that spread in a population that includes hipsters. Therefore, in our model, each node $i$ is assigned a value $H_i \in \{0, 1\}$; such that $H_i = 0$ indicates that node $i$ is a conformist and $H_i = 1$ indicates that node $i$ is a hipster. We update nodes synchronously. At each discrete time $t \geq 1$, we assume that conformists know the distribution of products among their immediate neighbors at the previous time step $t’ = t - 1$, whereas hipsters know the distribution of products in the total population at an earlier time step $t_r = t - \tau$ (where $\tau \in \mathbb{N}$). The first updating step is at $t = 1$. If $t - \tau \leq -1$, we let $t_r = 0$.

A node chooses to adopt a specific product in two steps. First, the node must become active, as determined by whether sufficiently many of its neighbors are active. If the fraction of neighbors that are active at time $t - 1$ is at least as large as the node’s threshold, it becomes active at time $t$. If node $i$ becomes active, it immediately adopts one of two possible products. If $H_i = 0$, node $i$ is a conformist and thus adopts the product that most of its active neighbors have adopted at time step $t - 1$. However, if $H_i = 1$, node $i$ is a hipster and thus adopts the product which is the less popular in the total population at time $t_r = t - \tau$. For both $H_i$ values, a tie results in the node choosing one of the two products with equal probability. Each node can adopt only a single product, and once it has adopted a product, it never switches to the other product or becomes inactive. To keep track of the product distribution, we associate a variable $S_i$ to each node $i$. If $S_i = 0$, node $i$ is inactive; if $S_i = \lambda \geq 1$, node $i$ has adopted product $\lambda$. We summarize the model and the decision process in Fig. 1 and its caption.

It is possible in principle to generalize our model to consider arbitrarily many products spreading on a network, we consider only the case of two products for simplicity. At $t = 0$, we activate a single node with product 1, and we introduce product 2 when the first hipster chooses to adopt a product.

III. SIMULATION OF OUR MODEL ON A FACEBOOK NETWORK

We start by simulating our model on the Northwestern25 network from the Facebook100 data set\cite{58}. This network consists of the friendship relationships on Facebook at Northwestern University on one day in autumn 2005. Its largest connected component has 10537 nodes, and it has a mean degree of $\langle k \rangle \approx 92$ and a maximum degree of $k_{\text{max}} = 2105$. We assign a threshold of
adopted each of the products at time $t$ for our simulations in which the hipster fraction is 0.04. For these parameters, the curves are indistinguishable for the two values of the delay time $\tau$. A much larger fraction of nodes adopts product 1 than product 2. In Fig. 2(b), we show the corresponding curves for simulations in which the hipster fraction is 0.30. The results for different delay times $\tau$ are now clearly distinguishable. For $\tau = 1$, the fraction of nodes that have adopted the two products are approximately equal; for $\tau = 4$, however, the fraction of nodes that have adopted product 2 is much larger than the fraction of nodes that have adopted product 1.

IV. ANALYSIS

We approximate the temporal spreading of products on a network using a pair approximation (as in [26, 28, 33, 39]), which relies on the hypothesis that the network is locally tree-like [60]. Let $\rho^{(\phi,k)}_\lambda(t)$ denote the density of nodes with threshold $\phi$ and degree $k$ that have adopted product $\lambda \in \{1, 2\}$ at time step $t$. We write the recursion relation

$$
\rho^{(\phi,k)}_\lambda(n+1) = \rho^{(\phi,k)}_\lambda(n) + (1 - \rho^{(\phi,k)}_\lambda(n)) \sum_{k'=1}^k F(k,k',\phi)\
\times B^n_k \left( \sum_{\beta=1}^m \tilde{q}^{(\phi,k)}_{\beta}(n) \right) \left[ (1 - p\text{Hip}) \sum_{k''>k'} B^n_{k''} \left( \frac{\rho^{(\phi,k)}_\lambda(n)}{\sum_{\beta=1}^m \tilde{q}^{(\phi,k)}_{\beta}(n)} \right) 
+ p\text{Hip} \prod_{\beta \neq \lambda} \Theta \left( \sum_{k,\phi} \rho^{(\phi,k)}_{\lambda}(n+1-\tau) - \sum_{k,\phi} \rho^{(\phi,k)}_{\beta}(n+1-\tau) \right) \right],
$$

(1)

where $\tilde{q}^{(\phi,k)}_{\beta}(n)$ is the probability that a uniformly random neighbor of an inactive node with threshold $\phi$ and degree $k$ is active and has adopted product $\beta \in \{1, 2\}$; the “response function” $F(k,k',\phi) = 1$ if $k'/k \geq \phi$ and $F(k,k',\phi) = 0$ otherwise; $\Theta(x)$ is the step function (so it equals 1 for $x > 0$ and 0 otherwise); and

$$
B^n_k(p) = \binom{k}{l} p^l (1-p)^{k-l}
$$

(2)

is the binomial function for probability $p$. We can write $\tilde{q}^{(\phi,k)}_k(n)$ as a function of $\tilde{q}^{(\phi,k')}_k(n)$, the probability that, for a given inactive node, a neighbor with degree $k'$ and threshold $\phi'$ is active at time step $n$. We can then write

$$
\tilde{q}^{(\phi,k)}_{\lambda}(n) = \frac{\sum_{k',\phi'} P((k,\phi),(k',\phi')) \tilde{q}^{(\phi',k')}_k(n)}{\sum_{k',\phi'} P((k,\phi),(k',\phi'))},
$$

(3)

where $P((k,\phi),(k',\phi'))$ is the probability that a node with degree $k$ and threshold $\phi$ is adjacent to a node with degree $k'$ and threshold $\phi'$. Given an active node, the

FIG. 1. Illustration of the types of nodes. A node is a hipster with probability $p_{\text{Hip}}$, and it is a conformist with probability $1 - p_{\text{Hip}}$. If at least a fraction $\phi$, of the neighbors of node $i$ are active at discrete time step $t-1$, the node activates and adopts a product at time step $t$ (for $t \geq 1$). We then need to consider which products have been adopted by node $i$’s neighbors and the relative popularity of different products in the whole network. If node $i$ is a conformist, it adopts the product that most of its active neighbors have adopted at time $t-1$. However, if node $i$ is a hipster, it adopts the product that is less popular among the active nodes in the network at time step $t-\tau$ (where $\tau \in \mathbb{N}$). For both node types, a tie results in a node choosing one of the two products with equal probability.

$\phi = 1/33$ to each node. In addition to this threshold, we independently assign each node a value $H_i \in \{0,1\}$ with some uniform probability $p_{\text{Hip}}$. Hence, different simulations of our model with a specified hipster probability do not in general have the same number of hipster nodes.

We examine our model with two different time delays and two different values for the fraction of hipsters in the network. We consider $\tau = \{1, 4\}$ and $p_{\text{Hip}} = \{0.04, 0.30\}$, and we conduct simulations for the four combinations of these parameter choices.

For each parameter pair, we choose a single node uniformly at random and suppose that it has adopted product 1 at time $t = 0$. This node acts as a seed for the spreading process on the network. We introduce another node, labeled with 2, when the first hipster node is activated. Thus, product 2 will never be adopted by any node if the network has no hipsters. We stop the simulation after the dynamics reaches a steady state, in which no further adoptions occur. At each time step, we track the fraction of the network nodes that have adopted each of the two products. We conduct the simulation 200 times (each with a seed chosen uniformly at random, and determining new hipster nodes for each simulation), and we average the fraction of nodes that have adopted each product at time step $t$ over these 200 simulations. We show the results of these simulations in Fig. 2.

In Fig. 2(a), we plot the fraction of nodes that have
model on a Facebook network for delay values $	au$. FIG. 2. Example of the behavior of the hipster threshold (a) a hipster fraction of $p_{Hip}$ on the same network (the both panels, each data point is a mean over 200 simulations product 2 is ultimately adopted more than product 1. For $\tau = 4$. For $\tau = 1$, the final fractions that have adopted products 1 and 2 are approximately equal. However, for $\tau = 4$, product 2 is ultimately adopted more than product 1. For both panels, each data point is a mean over 200 simulations on the same network (the Northwestern25 network of the Facebook100 data set[58], where we choose both seed nodes and hipster nodes uniformly at random for each of the simulations.

To focus on situations in which many nodes adopt (either of the products), we only consider realizations in which at least some minimal fraction of nodes eventually adopt a product. We take this minimal threshold to be 0.10. (Another way to examine situations with a lot of spreading is through “cluster seeding” [30], in which one considers initial conditions in which a uniformly random seed of the spreading process. There is a risk that a single active neighbor is known as a “vulnerable” node which at least some minimal fraction of nodes eventually will not observe a cascade of adoptions in which many of the products), we only consider realizations in which many nodes adopt (either of the products).)

As with our simulations on the Facebook network in Section III, we select a single node uniformly at random to have adopted product 1 at time $t = 0$. This node is the seed of the spreading process. There is a risk that the chosen seed node is located in a neighborhood of very few vulnerable nodes. (A node that can be activated by a single active neighbor is known as a “vulnerable” node[58].) If such a seed is chosen, few nodes will be activated a single active neighbor is known as a “vulnerable” node which at least some minimal fraction of nodes eventually will not observe a cascade of adoptions in which many of the products), we only consider realizations in which many nodes adopt (either of the products).)

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larger than the mean fraction of adopters in discarded realizations, and it is much smaller than the fraction of adopter nodes in realizations that we keep. Thus, this choice of threshold ensures a clear separation between realizations with cascades of adoption and without cascades of adoption.

A. 5-regular configuration-model networks

We first consider configuration-model networks in which every node has degree 5. As described in [62], we match stubs (i.e., ends of edges) uniformly at random. We suppose that each node has a of threshold \( \phi_i = 0.19 \) with probability \( p_0 = 0.8 \) and a threshold of \( \phi_i = 0.8 \) with probability \( 1 - p_0 = 0.2 \). Hence, on average, 80\% of the nodes are vulnerable, and 20\% can only be activated if 4 or more of their nearest neighbors are activated.

We examine our hipster threshold model on the networks for time delays \( \tau \in \{1, 2, 3, 4, 5, 6\} \) and hipster probabilities of \( p_{\text{hip}} \in [0, 1] \) (in increments of 0.01). For each parameter pair \( (\tau, p_{\text{hip}}) \), we examine the model on 200 different networks. We independently draw the specific sets of networks for different parameter values, so in general they are not the same networks. For each realization, we stop the simulations after the distribution of product adoptions reaches a steady state, and we track the adoption fractions of the two products. From these values, we calculate the mean fraction of nodes that adopt each product over the 200 realizations and the corresponding standard deviations of the means. We plot these values in Fig. 3.

For all hipster delay times \( \tau \), the steady-state fraction \( \rho_{2,\text{tot}}(t \to \infty) \) of nodes that have adopted product 2 increases rapidly for small \( p_{\text{hip}} \). For \( \tau = 1 \) [see Fig. 3(a)], the hipsters have access to information without any delays, and their behavior leads to a balancing of the distributions of products 1 and 2. If a sufficiently large fraction of the nodes are hipsters, the mean final fraction of nodes that have adopted one product is almost indistinguishable from the other. This occurs for \( p_{\text{hip}} \approx 0.09 \).

In all examined cases, \( \rho_{2,\text{tot}}(t \to \infty) \) equals \( \rho_{1,\text{tot}}(t \to \infty) \) (i.e., the final fraction of nodes that have adopted product 1) for \( p_{\text{hip}} \approx 0.09 \). For \( \tau \geq 2 \) [see Fig. 3(b)–(f)] and \( p_{\text{hip}} \approx 0.09 \), there exists an interval of \( p_{\text{hip}} \) values in which \( \rho_{2,\text{tot}}(t \to \infty) > \rho_{1,\text{tot}}(t \to \infty) \). This interval is larger for larger values of \( \tau \), and the peak of \( \rho_{2,\text{tot}}(t \to \infty) \) in this interval grows with \( \tau \), taking a value above 0.8 for \( \tau = 6 \) [see Fig. 3(f)]. In other words, the fraction of hipsters must be larger than about 0.09 for product 2 to become adopted by a larger fraction of the population than product 1 at steady state.

For \( \tau = 2 \) [see Fig. 3(b)], we observe another (and wider) \( p_{\text{hip}} \) interval (specifically, at about \( 0.35, 0.69 \)) in which product 2 beats product 1. For \( p_{\text{hip}} \geq 0.69 \), product 1 dominates. Hence, for \( \tau = 2 \), product 2 dominates in two \( p_{\text{hip}} \) intervals, and product 1 dominates in three \( p_{\text{hip}} \) intervals. However, \( \tau = 3 \) [see Fig. 3(c)] results in two intervals of dominance for each product.

B. 3-regular configuration-model networks

We now examine our hipster threshold model on 3-regular configuration-model networks. Suppose that a fraction \( p_0 = 0.8 \) of the nodes have a threshold of \( \phi = 0.3 \) and that the remaining fraction \( 1 - p_0 = 0.2 \) of the nodes have a threshold of \( \phi = 0.65 \). We perform simulations as in the 5-regular configuration-model networks (see Section VA) and show our results in Fig. 4.

Our results on 3-regular configuration-model networks differ from those on 5-regular configuration-model networks in several ways. One interesting result is that the fractions that adopt products 1 and 2 are very similar for \( \rho_{2,\text{tot}}(t \to \infty) \) first becomes larger than \( \rho_{1,\text{tot}}(t \to \infty) \) at about \( p_{\text{hip}} \approx 0.05 \), which is about half of the hipster probability that we observed for the analogous result for 5-regular configuration-model networks. Additionally, the height of this first peak is lower when simulating our model on 3-regular networks than on the 5-regular networks. For large \( p_{\text{hip}} \), the most popular product at steady state is the same for \( \tau = 3 \) [see Fig. 4(c)–(d)] as for the same delay times on the 5-regular networks, while it is opposite to that on the 5-regular networks for \( \tau = 5 \) [see Fig. 4(e)–(f)].

Our analytical approximation and numerical simulations match well for \( p_{\text{hip}} \approx 0.05 \). After this, our analytical approximation again shows abrupt jumps that we do not observe in computations. Our analytical approximation also does not predict the fraction of nodes that adopt each product for \( p_{\text{hip}} = 1.0 \) as well as it did on 5-regular networks. This may be because 3-regular configuration-model networks have a higher edge density than 5-regular configuration-model networks, so the former depart further from satisfying the local tree hypothesis on which our analytical approximation relies. The mean fraction of nodes that were activated in the discarded realizations was 0.0002, which is much less than the threshold of 0.10.
C. Erdős–Rényi networks

We now examine our hipster threshold model on Erdős–Rényi (ER) networks. Specifically, we examine $G(N, p)$ graphs, in which one specifies the total number $N$ of nodes, and each pair of nodes is linked independently with uniform probability $p$. We choose the expected mean degree of the networks to be $z = 5$ (so the probability of an edge between any two nodes is $p = z/N$) to match the mean degree of the 5-regular networks that we examined in Section V A.

We assign the same threshold $\phi_1 = \phi^* = 0.2$ to each node. With this threshold, all nodes with degree $k \leq 5$ are vulnerable. We again consider $p_{\text{Hip}} \in [0, 1]$ (in increments of 0.01) and $\tau \in \{1, 2, 3, 4, 5, 6\}$. For each parameter pair $(\tau, p_{\text{Hip}})$, we simulate the dynamics on 200 different networks, stop the simulations after reaching a steady state, track the final fractions of nodes that have adopted each of the products, and calculate the corresponding mean and standard deviation of the mean from these data. As in prior simulations, we use a different set of 200 networks for each parameter value. We plot our results in Fig. 5.

As with our simulations on 5-regular and 3-regular configuration-model networks, the absence of time delay (i.e., $\tau = 1$) in the information possessed by hipsters results in $p_{2,\text{tot}}(t \rightarrow \infty)$ being almost indistinguishable from $p_{1,\text{tot}}(t \rightarrow \infty)$ [see Fig. 5(a)]. For all examined values of $\tau$, we observe that $p_{2,\text{tot}}(t \rightarrow \infty)$ again increases rapidly for small values of $p_{\text{Hip}}$. For $p_{\text{Hip}} \approx 0.07$, we observe that $p_{1,\text{tot}}(t \rightarrow \infty)$ and $p_{2,\text{tot}}(t \rightarrow \infty)$ have similar steady-state fractions, though one can also observe rather interesting dynamics. For large values of $p_{\text{Hip}}$, the same product becomes the more-popular one at steady state as with the 3-regular networks for the delay times $\tau = \{4, 5, 6\}$ [see Fig. 5(d)–(f)], but product 1 is the more-popular one for $\tau = 3$ [see Fig. 5(c)]. The mean fraction of nodes that adopt a product in the discarded realizations is 0.0072. This is higher than what we observed for 3-regular and 5-regular configuration-model networks, but it is still much less than the threshold of 0.10.

Our analytical approximation and numerical simulations once again match well for small values of $p_{\text{Hip}}$ (specifically, for $p_{\text{Hip}} \lesssim 0.07$). For larger values of $p_{\text{Hip}}$ values, our analytical approximation has jumps in the fraction that adopts each product, which we again do not observe in the simulations. One possibility, which we suggested in our discussion of 3-regular configuration model networks in Section VB is whether our analytical approximation is running into problems because we are considering networks that are not locally tree-like (though similar approximations are known to be effective when using many networks that are not locally tree-like [23]). Additionally, note that the mean local clustering coefficient for our ER networks with $z = 5$ is 0.00058±0.00018, so our networks have very 3-cycles. If we ignore which product is adopted and pretend that the two products are the same, we recover the usual WTM model, and we have employed an analytical approximation that is known to work in that situation [63]. Our own recent work has demonstrated that this type of analytical approximation is also effective for a WTM augmented with “synergistic” social influence from nodes other than nearest neighbors [25], so the incorporation of different nodes (rather than the lack of a locally tree-like network structure) appears to be the likely cause of the breakdown of the approximation, especially given that the approximation becomes worse as we increase the hipster probability $p_{\text{Hip}}$.

D. The Northwestern25 Facebook network

In Section V B we showed simulations of our hipster threshold model model on the Northwestern25 network from the Facebook100 data set for two choices of the $(p_{\text{Hip}}, \tau)$ parameter pair. We now examine the model on the Northwestern25 network more systematically (with more initial conditions and for a wider range of parameter values). Suppose that each node has a threshold of $\phi^* = 1/33$. In each of our simulations, we use a single node, chosen uniformly at random, as a seed at $t = 0$ and consider $\tau \in \{1, 2, 3, 4, 5, 6\}$ and $p_{\text{Hip}} \in [0, 1.0]$ (in increments of 0.01).

In Fig. 6 we show the mean fraction of nodes that have adopted product 1 and 2 at steady state. For each choice of parameters, we choose a set of 200 initial conditions, and we calculate means over these simulations. The most striking difference between these plots compared to those of the model on synthetic networks in previous sections is that now it takes more hipsters to observe equal fractions of nodes adopting the two products at steady state. In the Northwestern25 network, the fractions that have adopted the two products become equal when roughly one fifth of the nodes are hipsters. We also observe that the height of the first peak of $p_{2,\text{tot}}(t \rightarrow \infty)$ increases with $\tau$, as was also the case in the synthetic networks that we examined. The mean fraction of nodes that adopt some product in the discarded realizations is 0.0001, which again is much less than the threshold of 0.10.

VI. CONCLUSIONS

It is important to study what makes information, opinions, diseases, memes, products, misinformation, alternative facts, and other things that originate in a small subpopulation spread to a large fraction of nodes in a network. This type of scenario can arise in the adoption of products and spreading of memes, and it can also occur in anti-establishment behavior, which can significantly impact the geopolitical landscape.

We developed a threshold model to examine the impact of anti-conformists (i.e., hipsters) on the spreading of two competing products (one of which, labeled “2”, was not adopted by any node at the beginning of our simulations). We examined our hipster threshold model
on various types of networks, and we considered different fractions of the hipster nodes and different amounts of time delay in the global information possessed by hipsters. In the absence of a time delay, we found that hipsters tend to balance the adoption of the two competing products. For all other delay values and all network types, we observed that the fraction of nodes that adopt product 2 (i.e., the product that would not be adopted in the absence of hipsters) grows rapidly with the fraction of hipsters. For all networks, we needed a very low fraction of hipsters for product 2 to become comparably popular, or even more popular, than product 1 at steady state. In our simulations on a variety of synthetic networks, we found that product 2 needs fewer than 10% of the nodes to be hipsters to become as widespread as product 1 (the lone product that spreads at time \( t = 0 \)). For the Northwestern25 Facebook network, roughly 20% of the nodes need be hipsters for product 2 to become as widespread as product 1 at steady state. In a recent study \[28]\, in which we generalized threshold models to incorporate synergistic effects from nodes other than nearest neighbors, we also found that the spreading dynamics on Facebook networks exhibited rather nontrivial behavior.

Our hipster threshold model behaved in a variety of fascinating ways on different types of networks. For example, when there is no delay in global information (i.e., \( \tau \geq 2 \)) and the hipster probability \( p_{\text{hip}} \) is large, we observed nontrivial dynamics in the number of intervals of hipster probabilities for which a given product is more popular at steady state. The quality of the match between our analytical approximation and numerical simulations also depended on both network model and hipster probability. Specifically, our analytical approximation generally does well for small values of \( p_{\text{hip}} \), and it correctly produces a fast increase in product 2 adopters with increasing values of \( p_{\text{hip}} \); it does reasonably well for large values of \( p_{\text{hip}} \) for 5-regular configuration-model networks (except for abrupt jumps that are not present in the simulations), it had more problems for 3-regular configuration-model networks (although it yielded the correct result for the more-popular product at steady state for \( p_{\text{hip}} \approx 1 \) in all but one instance), and it was not good for Erdős–Rényi networks (where it was incorrect about which product is more popular at steady state for \( p_{\text{hip}} \approx 1 \) in roughly half of the cases).

If the global adoption information available to hipsters is delayed, we found that the steady-state fraction of nodes that adopt a product varies nonmonotonically with the fraction of hipsters. For some delay values, this steady-state fraction peaks for two values of the fraction of hipsters; for other delay values, however, there is only a single peak. This behavior also depends on the network type on which spreading occurs. If the global adoption information available to hipsters is not delayed, we found that the final fraction of nodes that have adopted product 2 first increases rapidly with \( p_{\text{hip}} \) and then stabilizes in such a way that approximately half of the nodes end up adopting each product.

In summary, we find that even if only one of two products is adopted when the spreading begins, very small fractions of anti-establishment nodes can make a competing product become adopted by a majority of a population. Our simple model and numerical experiments may help shed light on the road to success for anti-establishment choices in elections, as such success (and qualitative differences in final outcomes between competing products, political candidates, and so on) can arise rather generically from a small number of anti-establishment individuals and ordinary processes of social influence on normal individuals. In our model, the hipsters always choose to adopt the product that is less popular at time step \( t - \tau \). If all hipsters regard product 1 as the established choice at all time steps — regardless of the actual distribution of adopted products — the steady-state adoption fractions of product 2 naturally become even larger, and the anti-establishment choice (which is product 2, in our example) becomes even more successful than what we observed in our simulations. This more extreme situation may be relevant in elections in which the conception of who is part of the establishment may not change during weeks of campaigning and polls predicting which candidate will win an election and take office.

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| Network                      | Number of discarded realizations | Mean   | Standard deviation of the mean |
|------------------------------|----------------------------------|--------|-------------------------------|
| 5-regular configuration model| 36                               | 0.0001 | 0.0024                        |
| 3-regular configuration model| 1843                             | 0.0002 | 0.0001                        |
| ER                           | 52214                            | 0.0072 | 0.0000                        |
| Northwestern25               | 35171                            | 0.0001 | 0.0001                        |

TABLE I. Summary statistics of the discarded realizations of our hipster threshold model on each network family (or individual network, for Northwestern25). We show the mean fraction of nodes that are activated at steady state for discarded realizations and the standard deviation of this mean. For all networks, the mean fraction of active nodes is much smaller than the threshold fraction 0.10, below which we discarded realizations.
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Adoption fraction at steady state

The nodes have a threshold of $\phi = 0.19$ with probability $p_0 = 0.8$ and threshold of $\phi = 0.8$ with probability $1 - p_0 = 0.2$. For each simulation, we activate a single node, chosen uniformly at random, with product 1 at time $t = 0$. We stop simulations when products are no longer spreading on the network. We plot the mean fraction of nodes that have adopted products 1 and 2 in the 200 realizations and the corresponding standard deviations of the means. (For each $(\tau, p_{\text{Hip}})$ parameter pair, we independently construct 200 networks, and we also independently determine the initial condition for each network.)

For all values of $\tau$, the fraction of nodes that have adopted product 2 at steady state increases rapidly with $p_{\text{Hip}}$ for small $p_{\text{Hip}}$, reaching 0.5 at $p_{\text{Hip}} \approx 0.09$. For $\tau = 1$, which we show in panel (a), hipsters have information about the product distribution in the network without any delay, and the fraction of nodes that have adopted products 1 and 2 are almost indistinguishable for $p_{\text{Hip}} \approx 0.09$. For larger values of $\tau$, which we show in panels (b)–(f), the fraction of nodes that have adopted each product varies for $p_{\text{Hip}} \approx 0.09$. The fraction of nodes that have adopted product 2 is larger than the fraction that have adopted product 1 at steady state for an interval of $p_{\text{Hip}}$ values around $p_{\text{Hip}} \approx 0.10$. For $\tau \in \{2, 3\}$ [see panels (b) and (c)], we also observe that more nodes adopt product 2 for large values of $p_{\text{Hip}}$. The height of the peak in the fraction of product-2 adopters around $p_{\text{Hip}} \approx 0.80$ increases with $\tau$, reaching a value above 0.80 for $\tau = 6$ [see panel (f)]. We also plot the analytically-estimated product-adopting fractions given by Eq. [1]. Our analytical approximation matches the behavior well for small values of $p_{\text{Hip}}$ and large values of $p_{\text{Hip}}$. Between these extremes, however, our analytical approximation yields discontinuities in the steady-state adoption fractions of products that do not arise in our numerical computations.
Adoption fraction at steady state

FIG. 4. Distribution of products at steady state in 10,000-node 3-regular configuration-model network. The different panels give results of simulations of our hipster threshold model with different delay times $\tau$ for the hipster nodes. For each value of $\tau$, we consider hipster probabilities $p_{\text{hip}} \in [0, 1]$ in increments of 0.01. For each $(p_{\text{hip}}, \tau)$ parameter pair, we simulate the model on 200 different networks created using a configuration model (in which we connect stubs uniformly at random). The model on 200 different networks created using a configuration model (in which we connect stubs uniformly at random). The nodes have a threshold of $\phi = 0.3$ with probability $p_0 = 0.8$ and threshold of $\phi = 0.65$ with probability $1 - p_0 = 0.2$. For each simulation, we activate a single node, chosen uniformly at random, with product 1 at time $t = 0$. We stop our simulations when the products are no longer spreading on the network. We plot the mean fraction of nodes that have adopted products 1 and 2 in the 200 realizations and the corresponding standard deviations of the means. (For each $(\tau, p_{\text{hip}})$ parameter pair, we independently construct 200 networks, and we also independently determine the initial condition for each network.) For all values of $\tau$, the fraction of nodes that have adopted product 2 at steady state increases rapidly with $p_{\text{hip}}$ for small $p_{\text{hip}}$, reaching 0.5 at $p_{\text{hip}} \approx 0.05$. For $\tau = 1$, which we show in panel (a), hipsters have information about the product distribution in the network without any delay, and the fractions of nodes that have adopted products 1 and 2 at steady state are almost indistinguishable for $p_{\text{hip}} \approx 0.05$. For $\tau = 2$, which we show in (b), the fractions of nodes that have adopted the two products are similar (though one can see some interesting dynamics) until $p_{\text{hip}} \approx 0.93$, above which product 2 is the more-popular product. For larger values of $\tau$ [see panels (c)–(f)], the fraction of nodes that have adopted each product varies for $p_{\text{hip}} \lesssim 0.05$. The height of the peak, which occurs at $p_{\text{hip}} \approx 0.06$, of the node fraction that has adopted product 2 at steady state does not take place at $p_{\text{hip}}$ values near 0.06 but instead occurs for much larger values of $p_{\text{hip}}$. We also plot the analytically-estimated fractions of product adoption given by Eq. (1). Our analytical approximation gives a good match to our computations well for small values of $p_{\text{hip}}$. For $p_{\text{hip}} \lesssim 0.05$, however, our analytical approximation does not do well. Our analytical solution includes discontinuities in the steady-state adoption fractions of the products, but these do not arise in our numerical simulations.
FIG. 5. Distribution of products at steady state in 10,000-node ER networks with $z = 5$. The different panels give results of simulations of our hipster threshold model with different delay times $\tau$ for the hipster nodes. For each value of $\tau$, we consider hipster probabilities $p_{\text{Hip}} \in [0, 1]$ in increments of 0.01. For each $(\tau, p_{\text{Hip}})$ parameter pair, we simulate the model on 200 different networks and initial conditions. Each node has a threshold of $\phi = 0.2$. For each simulation, we activate a single node, chosen uniformly at random, with product 1 at time $t = 0$. We stop simulations when products are no longer spreading on the network. We plot the mean fraction of nodes that have adopted products 1 and 2 in the 200 realizations and the corresponding standard deviations of the means. (For each $(\tau, p_{\text{Hip}})$ parameter pair, we independently construct 200 networks, and we also independently determine the initial condition for each network.) For all values of $\tau$, the fraction of nodes that have adopted product 2 increases rapidly with $p_{\text{Hip}}$ for small $p_{\text{Hip}}$, reaching 0.5 at $p_{\text{Hip}} \approx 0.07$. For $\tau = 1$, which we show in panel (a), hipsters have information about the product distribution in the network without any delay, and the fractions of nodes that have adopted products 1 and 2 at steady state are almost indistinguishable for $p_{\text{Hip}} \gtrapprox 0.07$. For larger values of $\tau$ [see panels (b)–(f)], the fraction of nodes that have adopted product 2 is largest for a small $p_{\text{Hip}}$ interval around $p_{\text{Hip}} \approx 0.10$. For $\tau \geq 4$ [see panels (d)–(f)], an additional, large $p_{\text{Hip}}$ interval results in a majority of nodes adopting product 2. We also plot the analytically-estimated fractions of product adoption given by Eq. (1). The analytical curves and numerical simulations match well for small values of $p_{\text{Hip}}$. For larger hipster probabilities, however, our analytical approximation is not accurate. For $\tau = 5$ [see panel (e)], it predicts incorrectly that product 1 is the more-popular product at steady state for large values of $p_{\text{Hip}}$, and our analytical results include discontinuities in the steady-state adoption fractions of products that are not present in our numerical simulations.
FIG. 6. Distribution of products at steady state in the Northwestern25 network from the Facebook100 data set. The different panels give results of simulations of our hipster threshold model with different delay times \( \tau \) for the hipster nodes. For each value of \( \tau \), we consider hipster probabilities \( p_{\text{Hip}} \in [0, 1] \) in increments of 0.01. For each \((\tau, p_{\text{Hip}})\) parameter pair, we simulate the hipster threshold model on the Northwestern25 network with 200 choices for the seed node, chosen uniformly at random, which adopts product 1 at \( t = 0 \). We use a different set of 200 nodes for different parameter values. Each node has a threshold of \( \phi = 1/33 \). We plot the mean fractions of nodes that have adopted products 1 and 2 at steady state in the 200 realizations and the corresponding standard deviations of the means. For all values of \( \tau \), the fraction of nodes that have adopted product 2 increases rapidly with \( p_{\text{Hip}} \) for small \( p_{\text{Hip}} \), reaching \( 0.5 \) at \( p_{\text{Hip}} \approx 0.2 \) for \( \tau \geq 3 \) [see panels (c)–(f)] and for larger values of \( p_{\text{Hip}} \) for \( \tau \leq 2 \) [see panels (a), (b)]. For \( \tau = 1 \), which we show in panel (a), hipsters have information about the product distribution in the network without any delay, and the fraction of nodes that have adopted product 1 and 2 are very similar for \( p_{\text{Hip}} \geq 0.3 \). For larger values of \( \tau \) [see panels (b)–(f)], the fraction of nodes that have adopted each product varies nonmonotonically for \( p_{\text{Hip}} \geq 0.2 \). For \( \tau \geq 3 \) [see panels (c)–(f)], the fraction of nodes that have adopted product 2 is largest for a small \( p_{\text{Hip}} \) interval around \( p_{\text{Hip}} \approx 0.3 \). This is the single peak in the adoption of product 2 in these simulations. For \( \tau = 2 \) [see panel (b)], product 2 is the more-popular product for high values of \( p_{\text{Hip}} \). For \( \tau = 2 \) [see panels (b)–(f)], product 2 is the more-popular product for \( p_{\text{Hip}} \geq 0.20 \). The maximum fraction of nodes that adopt product 2 for these values of \( p_{\text{Hip}} \) occurs for \( p_{\text{Hip}} \approx 0.30 \) for small \( \tau \), and both the hipster probability that produces the peak fraction and (especially) the peak fraction increase with \( \tau \). For \( \tau = 6 \) [see panel (f)], the maximum fraction of nodes that adopt product 2 is about 0.90.