An Approach to Analyze Duality of Current-type DC-DC Circuit and Voltage-type DC-DC circuit based on Gyrator

Zhenhong Zheng1*, Li Ran1
1School of electrical engineering, Chongqing University, Chongqing, 400044, China
*zhengzhh_stw@cqu.edu.cn

Abstract. This paper proposes a method that can derive the current-type (C-T) DC-DC circuit topology from the voltage-type (V-T) DC-DC circuit topology by the duality of two-ports gyrator. In addition, the mathematical relationships between C-T DC-DC circuit and V-T DC-DC circuit parameters are established using the gyration resistance and the duty cycle of power switching device. Hence, C-T DC-DC parameters of the passive components, and the expressions of the mathematical relationships between the input and output, including the ripples of inductance current and capacitance voltage are obtained by analysing and calculating the V-T DC-DC circuit. Finally, the analysis results are verified by simulation in SIMULINK.

1. Introduction
Classical DC-DC converters based on inductive volt-second balance buck, boost and buck-boost are usually referred to as V-T DC-DC converters. The analysis and calculation of V-T DC-DC have been discussed in detail in [1] and [2]. Even if researches on C-T DC-DC circuit started later than V-T DC-DC circuit, the converter based on current source has been studied and applied in bi-directional current source converter [3], inverter [4], photovoltaic [5] and light-emitting diode (LED) [6] and [7] also discussed a method for obtaining a C-T DC-DC circuit from a V-T DC-DC circuit based on the duality principle. However, there are few papers reports the mathematical relationship between C-T DC-DC circuit and V-T DC-DC circuit. In this paper, the method of C-T DC-DC circuit deriving from the V-T DC-DC circuit through the gyrator is described, and mathematical relationship of the parameters between the V-T DC-DC circuit and the C-T DC-DC circuit in duality is presented.

2. The Proposed Approach
The gyrator is a two-port component in the network theory. The component symbol is shown in the Fig. 1, and the circuit parameters relationships of the ideal gyrator are expressed as

\[
\begin{bmatrix}
    \mu_1 \\
    i_1
\end{bmatrix} = \begin{bmatrix}
    0 & r \\
    \frac{1}{r} & 0
\end{bmatrix} \begin{bmatrix}
    \mu_2 \\
    -i_2
\end{bmatrix}
\]

(1)

where the constant \( r \) is the gyration resistance. Obviously, Gyration transformation is not suitable for nonlinear DC-DC converter circuit. However, in the continuous conduction mode, the circuit, of which the switch is in the on state or off state, can be approximately regarded as a linear circuit. Taking the buck converter as an example, the circuit diagram of the buck converter is shown in Fig. 2. In continuous conduction mode, the buck circuit can be equivalent to blocking state circuit and conduction state circuit according to the state of controllable switch \( S \), as shown in the Fig. 3 and Fig.
4. According to the Fig. 5, the switch conduction state circuit is divided into three parts, which are the input circuit, the conversion circuit and the output circuit, as shown in Fig. 6 to 8 respectively.

![Figure 1. Gyrator.](image1)

![Figure 2. Buck circuit diagram.](image2)

![Figure 3. Conduction state circuit.](image3)

![Figure 4. Blocking state circuit.](image4)

The electrical characteristics of the ports of the divided circuits can be equivalent to the ports characteristics of the primary side of the gyrators, thereby obtaining circuits of the secondary side without the gyrators, as shown in Fig. 6 to 8. According to the ports voltage-current relationship of the gyrator, the following relation exists

\[
Z_1(s) = \frac{U_1(s)}{I_1(s)} = \frac{-rI_2(s)}{U_2(s)} = r^2 \frac{I_2(s)}{U_2(s)} = r^2 \frac{1}{Z_2(s)}
\]

(2)

where \(Z_1(s)\) is the port equivalent impedance of the primary side of the gyrator, and \(Z_2(s)\) is the port external impedance of the secondary side of the gyrator.

![Figure 5. Circuit structure division.](image5)

![Figure 6. Input circuit equivalent transformation.](image6)

![Figure 7. Transform circuit equivalent transformation.](image7)

![Figure 8. Output circuit equivalent transformation.](image8)
Figure 9. Secondary side equivalent circuit of gyrator of conduction state circuit

Figure 10. Secondary side equivalent circuit of gyrator of blocking state circuit

Figure 11. Secondary side equivalent circuit of gyrator

According to (1) and (2), the equivalent switch conduction state circuit of the secondary side of the gyrator is obtained by the three-part circuits of the secondary sides based on Kirchhoff Voltage Law (KVL) and Kirchhoff Current Law (KCL), as shown in Fig. 9. The equivalent switch blocking state circuit of the secondary side of the gyrator can be obtained by repeating the above operations, as shown in Fig. 10. To obtain the dual switch circuit topology, firstly, the switch conduction state circuit and the switch blocking state circuit on the secondary side of the gyrator should be combined. Then the common branches of the two circuits are reserved, and the non-public branches are replaced by switches. Next, the switch device type is determined by the current. Finally, the C-T DC-DC converter topology dual to the V-T DC-DC converter topology is obtained by simplifying the number of switch devices, as shown in Fig. 11.

Table 1. The Circuit of Analysis

| V-T Buck Parameters | V-T Buck Equations | C-T Buck Equations | C-T Buck Parameters |
|---------------------|--------------------|--------------------|--------------------|
| $P_{0\text{-V-T}}$ (W) | $\frac{D^2V_s^2}{R}$ | $\frac{D^3f^2(D)V_s^2}{r}$ | $P_{0\text{-C-T}}$ (W) |
| $L_{\text{V-T}}$ (H) | $(1 - D)DV_s$ | $\frac{[1 - Df(D)]Df(D)V_s}{r}$ | $C$ (F) |
| $C_{\text{V-T}}$ (F) | $\frac{(1 - D)DT^2V_s}{8LU}$ | $\frac{[1 - Df(D)]Df(D)T^2V_s}{r}$ | $L$ (H) |
| Gain ($V_o/V_s$) | $D$ | $Df(D)$ | Gain ($I_o/I_s$) |
| ripple of $V_C$ (V) | $\Delta U$ | $\Delta U / r$ | ripple of $V_C$ (V) |
| ripple of $I_L$ (A) | $\Delta i$ | $r\Delta i$ | ripple of $I_L$ (A) |
| $R$ (Ohm) | $R$ | $R/r^2$ | $G$ (S) |

3. Circuit Analysis and Simulation Results

It should be noted that the switching devices control states between the V-T buck and the C-T buck are completely opposite. $D$ represents the duty cycle of V-T buck, $D'$ represents the equivalent duty cycle of C-T buck, $D$ and $D'$ satisfy the following equation

$$f(D) = \frac{D'}{D} = \frac{1 - D}{D}$$ (3)
To describe the effect of the difference in switch states to their corresponding circuits parameters, \( f(D) \) is defined as switch coefficient. Switching coefficient and gyration resistance are the key parameters for the analysis of unified V-T and C-T DC-DC circuits. And the details of circuit analysis are shown in Table 1, and the simulation results based on Simulink model are shown in Table 2. The data of second column in Table 2 is the design parameters of the V-T DC-DC in the continuous conduction mode, and the third column is the simulation results using the duty cycle, input current, capacitance, inductance and load values calculated in the second column. The fifth column is the design parameter of the C-T DC-DC calculated by the equation of Table 1, and the sixth column is the simulation results using the duty cycle, input current, capacitance, inductance and load values calculated in the fifth column. Fig. 12 and Fig. 13 are the simulation circuits, and simulation waveforms are shown in Fig. 13 and Fig. 14 respectively. The gyration resistance is 10. It is found that the simulation results are in good agreement with the analysis and calculation results of dual transformation based on gyrator.

### Table 2. The Simulation Results

| V-T Buck parameters | Design | Simulation | C-T Buck parameters | Calculation | Simulation |
|---------------------|--------|------------|---------------------|-------------|------------|
| Input voltage (V)   | 30     | 30         | Input current (A)   | 3           | 3          |
| Duty                | 0.8    | 0.8        | Duty                | 0.2         | 0.2        |
| Frequency (/kHz)    | 40     | 40         | Frequency (/kHz)    | 40          | 40         |
| Inductor (\( L_{V-T} /mH \)) | 0.15  | 0.15       | Capacitor (\( C_{C-T} /\mu F \)) | 1.5 | 1.5 |
| Capacitor (\( C_{V-T} /\mu F \)) | 12.5  | 12.5       | Inductor (\( L_{C-T} /mH \)) | 1.25 | 1.25 |
| Resistance (\( R/Ohm \)) | 6     | 6          | Conductance (\( G/S \)) | 0.06 | 0.06 |
| Output voltage (V)  | 24     | 23.5       | Output current (A)  | 0.6         | 0.595      |
| \( I_{L_{ripple,pp}} \) (A) | 0.8    | 0.8        | \( V_{C_{ripple,pp}} \) (V) | 8 | 8 |
| \( V_{C_{ripple,pp}} \) (V) | 0.2    | 0.13       | \( I_{L_{ripple,pp}} \) (A) | 0.02 | 0.014 |
| \( P_{0V-T} \) (W) | 96     | 92.5       | \( P_{0C-T} \) (W) | 6 | 5.95 |

Figure 12. Simulation circuit of V-T buck.

4. CONCLUSIONS

Gyration resistance determines the parameters of capacitance and inductance between V-T and C-T circuits, gyration resistance and switch coefficient together determine the output power, capacitance voltage ripple and inductance current ripple. It also means that when gyration resistance and switch...
After the coefficient are determined, the analysis of V-T circuit and C-T circuit with dual relationship is consistent.

![Simulation circuit of C-T buck.](image)

**Figure 13. Simulation circuit of C-T buck.**

![Simulation result of V-T buck.](image)

**Figure 14. Simulation result of V-T buck.**

![Simulation result of C-T buck.](image)

**Figure 15. Simulation result of C-T buck.**
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