Research Article

Marine Propeller Optimization Based on a Novel Parametric Model

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This paper presents a novel parametric model of marine propellers based on Non-Uniform Rational B-Splines. It involves eight parameters and five categories of spanwise parameter distributions, which are utilized for determining hydrofoil and blade shapes. 20 different hydrofoils and 5 types of well-known marine propellers are employed to detect the accuracy of the proposed parametric model. Furthermore, a propeller optimization problem was addressed with the aid of the parametric model. In the propeller optimization problem, a common AU-series propeller is treated as the baseline propeller. The proposed parametric model is used for the representation and deformation of the propeller geometric model. A hydrodynamic performance evaluation model is developed based on gene expression programming. Also, non-dominated sorting genetic algorithm II is used in the applications of the propeller optimization problem. The results demonstrate that the accuracy of the proposed parametric model satisfies the engineering requirements well, and a propeller with higher efficiency than the baseline propeller can be derived by settling the propeller optimization problem.

1. Introduction

Marine propeller is a crucial propulsion device. Its hydrodynamic performance is forcefully linked to the characteristics and the shape of its blades. With the development of marine shipping and the concept of Energy Efficiency Design Index (EEDI), stringent environmental regulations are applied by the international and national organizations to reduce the exhaust emissions from ships [1]. Several researchers began to do a series of research on the ship propulsion system to reduce ship emissions and make it adapt to this standard. The propeller as the main equipment of the ship propulsion system will be widely concerned. In this context, a method is needed to quickly and automatically generate propeller geometry models in the process of propeller optimization design. Therefore, it is necessary to find a suitable parametric expression of propellers. Some researchers had demonstrated the need for parametric models [2–5]. However, most of the parametric models are bound with the calculation software to a certain extent and most of the parameters in the parameter model are some physical meanings, such as the diameter (D) of the propeller, the pitch ratio (P/D), and so on. There are a lot of commercial software programs and open-source academic software programs that can help us generate the geometric model of propellers, including PropCAD and PropElements from HydroComp Inc [6], OpenProp [2, 7], and JavaProp [8]. In all of the above cases, we are once again dealing with propeller models tightly linked to the computational procedure employed. Modeling the geometry of a propeller is different from modeling other industrial objects. The blade constitutes the so-called functional free form surface of significant complexity, and its design requires specialized procedures both concerning its representation in a Computer-Aided Design (CAD) package as well as its analysis using Computer-Aided Engineering (CAE) tools [9, 10]. Transforming a Computer-Aided Design (CAD) model into an appropriate Computer-Aided Engineering (CAE) model is a time-consuming, labor-intensive, and costly process. Therefore, in order to generate smooth and suitable example from the parametric model directly.
Arapakopoulos et al. provided two parametric models based on NURBS and T-splines [11]. These geometric parametric models can quickly and automatically produce valid geometric representations of marine propellers. Simultaneously, these two models can provide model instances suitable for engineering analysis following the isogeometric analysis paradigm. Pérez-Arribas and Pérez-Fernández generated a B-Spline surface representation of propeller blades while securing a reduced number of employed control points [12]. This general approach is implemented by the presented models and the benchmark models. Herath et al. presented a layout optimization algorithm for composite marine propellers using Non-Uniform Rational B-Splines (NURBS)-based FEM coupled with real-coded genetic algorithm (GA) [13]. In addition, a well-responsive, smooth, and direct-use propeller parameterized model is also very beneficial to the research of propeller optimization.

In the past, the study of propellers was usually based on experimental tests in the absence of a powerful software system. With the development of the times and the continuous improvement of computer computing power, several researchers had tried to apply optimization algorithms to propeller design. Dai et al. optimized the chord length and blade thickness distribution to minimize the propeller mass under the constraint of constant propeller efficiency [14]. Mishima and Kinnas optimized the camber, pitch, and chord distribution under the constraint of a constant mean torque or constant mean power subject to a given maximum allowed cavity area or a maximum cavity volume velocity in non-uniform flow. They also provided an option for optimum skew [15]. Their method was further improved by Griffin and Kinnas [16]. Kawakita and Hoshino optimized the pressure distribution in non-uniform flow [17]. Jang et al. optimized the pitch distribution to maximize the propeller efficiency [18]. Takekoshi used the two-dimensional wing theory and vortex lattice method to realize the optimal design of the propeller [19]. The vortex lattice method is used to evaluate the performance and the time-dependent pressure distribution on the blade surface in a non-uniform flow. The propeller efficiency was increased by 1.2% under the constraints of constant thrust and a prescribed margin for surface cavitation. Zeng and Kuiper developed an optimization technique for the effective blade section by using a genetic algorithm [20]. Taheri and Mazaheri developed a propeller design method based on a vortex lattice algorithm and optimized the shape and efficiency of two propellers using gradient-based and non-gradient-based optimization algorithms [21]. Gaggero et al. proposed the design and the analysis of the performance of an improved tip loaded propeller geometry [22]. Gaggero et al. solved the design problem of high-speed ship propeller by using a multi-objective numerical optimization method. By combining fast and reliable boundary element method (BEM), viscous flow solver based on RANSE approximation, parameterized 3D description of blades and genetic algorithm, the new propeller shape is designed to improve propulsion efficiency, reduce cavitation expansion, and increase cavitation start speed while maximizing ship speed [23]. Gaggero proposed a simulation-based design optimization (SBDO) tool for the design of rim-driven thrusters. The optimization framework consists of a parametric description of the rim blade geometry and a multi-objective optimization algorithm which makes use of the results from high-fidelity RANS calculations to drive the choice towards optimal blade shapes [24]. Vesting et al. improved the commonly used group-based algorithms (NSGA-II and PSO) to be applied to marine propeller design [25]. In addition, Vesting et al. also discussed several response surface methods to replace the time-consuming propeller performance evaluation tool. By combining the response surface method to fill the design space and the calculation in the local search method, a practical application method for the minimum calculation workload was proposed [26].

In addition, many scholars had also researched the parameters affecting the propeller. Ghassemi et al. numerically discussed the effect of tip rake angle on the open water characteristics and sound pressure level around the marine propeller [27]. Mahmoudi et al. used the computational fluid dynamics (CFD) data of propeller thrust, torque, and cavitation volume under different influence parameters, like pitch ratio (Pr), rake angle (RA) and skew angle (SA), advance velocity ratio (J), and cavitation number (ζ), as inputs-outputs of GEP models to treat gene expression programming for forecasting the hydrodynamic performance and cavitation volume of the marine propeller [28]. Shora et al. also used the above parameters as the input-output of the neural network (ANN) model to predict the hydrodynamic performance of the propeller [29]. The most notable thing is that Mirjalili and Zheng transformed the propeller optimization design into a 20-parameter optimization problem; ant lion optimizer and diffusion algorithm were used in their research [30, 31]. This means that the number of parameters in the propeller optimization problem will no longer be a limitation.

The remainder of this paper is organized as follows. Section 2 provides a brief review of the NURBS theory and develops a novel parametric model. The accuracy of the parametric model is tested and discussed in this section. In Section 3, the mathematical model of propeller optimization is proposed. In Section 4, a gene expression programming model for the propeller hydrodynamic prediction tool is constructed, and the results of the propeller optimization and accuracy test of the GEP model are discussed. Finally, the conclusions and future research directions are given in Section 5.

2. Parametric Model Based on NURBS

2.1. NURBS Parametric Representations. NURBS have been already extensively and successfully used in CAD industry. At the same time, they are also widely used and have good
performance in the field of isogeometric analysis. Due to the complex shape of propellers, conventional Lagrange can only approximate the geometry. We use the NURBS function to replace the Lagrange for getting an analysis-suitable model. Non-Uniform Rational B-Splines were proposed by Piegl and Tiller [32].

A \( m^{th} \) degree NURBS curve is then defined as

\[
P(K) = \frac{\sum_{i=0}^{n} N_{i,m}(K) R_i}{\sum_{i=0}^{n} N_{i,m}(K)}
\]

where \( p_i \) are the control points, \( R_i \) are the associated weights, and \( K \) is the knot vector.

The \( j^{th} \) B-Spline basis function is defined as

\[
N_{i,j} = \begin{cases} 
1, & (K_i \leq K \leq K_{i+1}), \\
0, & \text{(else)},
\end{cases}
\]

\[
N_{i,m} = \frac{(K - K_i) N_{i,m-1}(K)}{K_{i+m} - K_i} + \frac{(K_{i+m+1} - K) N_{i+1,m-1}(K)}{K_{i+m+1} - K_{i+1}}, \quad m \geq 1.
\]

The function of a NURBS surface is defined as

\[
S(u, v) = \sum_{i=0}^{n} \sum_{j=0}^{m} p_{ij} w_i N_i^{k_1}(u) N_j^{k_2}(v)
\]

where \( u \) and \( v \) represent the direction, \( k_1, k_2 \) represent the surface’s order in \( u \) and \( v \) directions, \( p_{ij} \) and \( w_i \) represent the control points and their corresponding weights, and \( N_i^{k_1}(u), N_j^{k_2}(v) \) represent the B-Spline basis function in \( u \) and \( v \) directions, respectively.

2.2. The Geometry and Parameters of the Propeller. A general-purpose propeller consists of a hub and blades, as shown in Figure 1. Generally, the following definitions are used to describe a marine propeller, the number of blades, the blade area ratio, the pitch ratio, the propeller type (AU and B-series propellers), diameter, etc.

The number of blades is used to describe the number of blades of a propeller. The blade area ratio refers to the ratio of the area of the extension profile of each blade of the propeller to the area of the propeller disk. Pitch refers to the distance of advance due to the rotation of the propeller, and pitch ratio is the ratio of pitch to diameter. The blade surfaces of a propeller are commonly constructed through a single hydrofoil, appropriately transformed in the span direction, or via multiple, differently shaped hydrofoil profiles for different areas of the blade.

The parameters of the propeller can be divided into three categories, one type is used to control the shape of the hydrofoil, the other is used to control the deformation of the hydrofoil in the span direction of the propeller, and the last type is used to control the overall shape of the propeller, such as the number of blades. We define the first category as hydrofoil parameters, the second category as radial parameters, and the third category as general parameters. A hydrofoil generates propeller blades as shown in Figure 2.

2.3. Definition of Parametric Model for Hydrofoils. Usually, we describe the shape of the propeller’s hydrofoil by giving its corresponding propeller type; another way is to give its model such as NACA0012. As shown in Figure 3, there are two cases in which the blade profile shape is determined according to the given propeller type.

The above definition method has certain limitations. Some unusual hydrofoil shapes may be overlooked when designing the propeller. In our work, the propeller blade sections are defined with the aid of an additional parametric model. The parametric model generates a closed cubic B-Spline curve via a set of 8 parameters. All parameters use hydrofoil’s chord length for normalization to enhance the robustness of the model, and their definitions are included in Table 1.

The assumed coordinate system’s origin is at hydrofoil’s leading-edge point and the longitudinal axis coincides with the chord line with the positive direction towards the trailing edge. Therefore, the ordinates of hydrofoil’s upper side will always be non-negative numbers, while the lower-side values can be either negative or mixed, depending on hydrofoil’s camber. A parametric model instance with chord length equal to one is depicted in Figure 4. We assume that the final B-Spline curve starts at the leading-edge point and traverses the hydrofoil in a clockwise direction, and then we will use the 7 control points as ordered in Figure 4 and the corresponding knot vector to describe the hydrofoil. Finally, every hydrofoil instance produced by this model is a closed cubic B-Spline curve with 7 control points, the first and last one being coincident.

By interactively modifying parameter values, this parameter model can be used to approximate any desired hydrofoil shape. Such an example is depicted in Figure 5 where a set of NACA0012 points are approximated by the hydrofoil parametric model using the design vector \( v = [0.0941, 0.4266, 0.4479, 0.0473, 0.0941, 0.4266, 0.4479, 0.0473] \).
Figure 2: Hydrofoils along the blade.

Figure 3: Examples of specifying blade profile shape according to propeller type. (a) AU-series propeller. (b) B-series propeller.

Table 1: Parameters’ definition.

| Nr. | Name                             | Symbol      | Dimensionless | In the equations (9) |
|-----|----------------------------------|-------------|---------------|-----------------------|
| 0   | Chord length                     | L           | Free          | 1                     |
| 1   | Upper-side front edge shift length | u_in_shift | (0, L) → [0, 1] | P1                    |
| 2   | Upper-side front edge in angle   | u_in_angle  | [0, π] → [0, 1] | P2                    |
| 3   | Upper-side trailing edge shift length | u_out_shift | (0, L) → [0, 1] | P3                    |
| 4   | Upper-side trailing edge out angle | u_out_angle | [0, π/2] → [0, 1] | P4                    |
| 5   | Lower-side front edge shift length | l_in_shift | (0, L) → [0, 1] | P5                    |
| 6   | Lower-side front edge in angle   | l_in_angle  | [−π, π/2] → [0, 1] | P6                    |
| 7   | Lower-side trailing edge shift length | l_out_shift | (0, L) → [0, 1] | P7                    |
| 8   | Lower-side trailing edge out angle | l_out_angle | [−π, π/2] → [0, 1] | P8                    |

Figure 4: Hydrofoil parametric model.

Figure 5: Approximation of NACA0012 unit chord-length profile with the hydrofoil parametric model.
2.4. Construction of Propeller. Before building a propeller model, we need to define the hydrofoil parameters, radial parameters, and general parameters mentioned above.

We can use the parametric model mentioned above to generate the 2D hydrofoil shape. Then, the next step in the process is regarding the provision of general propeller parameters (see Table 2).

After the general parameters are specified, the radial parameters need to be defined. The radial parameters of the propeller presented collectively in Figure 6 which affect the blade’s section in the spanwise direction are effectively modifying the shape of the template profile along the blade. Specifically, the chord length and thickness distributions determine the size of each section, while the skew, rake, and pitch distributions define the corresponding rotations and/or translations. Typical distributions of these parameters are illustrated in Figure 6. Figure 7 depicts a series of hydrofoil sections twisted and positioned along the blade’s spanwise direction. In Figure 6, what we see are the mathematical functions based on real data (the unit is meters). But our purpose is to achieve a propeller model based entirely on NURBS, so we parameterize these distributions with simple Bézier curves with four control points. An example case of such distribution curve is depicted in Figure 8. We realize the parameterization of these distributions with the help of Grasshopper [33]. Figure 9 shows an example of all relevant distributions after parameterization. On the basis of these theories, we can initially generate the blade shape of the propeller.

For the compilation of the complete blade, additional processing is required at the hub and tip area. We use interpolation to refine these two parts locally. As for hub of the propeller, we divide the propeller hub into three parts: the front, middle, and rear parts. In order to make the hub part more close to the actual shape, the three parts are divided into different sections, each section is also expressed by B-Spline, and a more flexible hub geometry is obtained by rotating the two-dimensional curve. Refer to Figure 10 for details. The middle part is divided into 7 sections, and the front and rear positions are divided into 2 sections. The resulting surface is illustrated in Figure 11.

2.5. Validation of the Parametric Model. To address the accuracy of the proposed parametric model, the two-dimensional hydrofoil and three-dimensional propeller models are evaluated. The shape of propeller is closely related to the shape of each radius section. The first thing we would like to assess is the approximation level achieved by the parametric model when compared to the prototype of blade sections (hydrofoils). The validation is depicted through point set deviation. Specifically, we choose 20 hydrofoil shapes such as NACA 2421, NACA 4418, NACA 0006, etc. as samples and implement distance error (including mean distance, median distance, and standard deviation) statistics on them. The information is depicted in a graphical form in Figure 12. These hydrofoil data were downloaded from https://mselig.ae.illinois.edu/ads/coordDatabase.html.

In Figure 12, we can easily get that the model-generated hydrofoil has an average deviation of approximately 0.0015 mm and we can hardly find any surface point deviating more than 0.002 mm. In our analysis, the basic chord length we defined is 0.1 mm, so this parameter model can approximate the hydrofoil airfoil within the error range of 1.5%–2%.

In sequel, we demonstrate the use of the parametric models in approximating the original propeller model (including B-Screw Series, AU Series, Gawn, Kaplan, and SK). Our comparison is limited to the propeller blade which is the most complex and decisive geometrical element for the propeller’s performance. The diameter (D) and number of blades are set to 0.25 m and 4. As for the number of sections, 12 are employed for the generation of AU, B series, Gawn, and SK and 10 for Kaplan. When modeling with this parametric model, the coordinate system used in the model is different from the global space coordinate system. When the chord line of the profile does not coincide with the x-axis of the global coordinate, a simple rotation operation will be involved.

The first thing we would like to assess is the approximation level achieved by the parametric models when compared to the prototype surface blades of the five types of propellers. To this end, we generated points from the prototype surface blades of the five types of propellers and compared them against the corresponding sections belonging to surface blades of models. In Figure 13, we visualize the deviation about some propellers using Point set deviation. Specifically, a detailed comparison of deviations about Kaplan propeller in all generated sections is included in Table 3. We present the standard deviations achieved by parametric model when compared to prototype blade. From these results, it can be seen that the parametric model of propeller proposed by us can reach the approximate level required by engineering design.

2.6. Propeller Optimization Based on Parametric Model. The essence of the propeller optimization problem is usually to find a propeller with better hydrodynamic performance for a defined operating condition, using a certain propeller as a reference (called the baseline propeller in this paper). To determine the geometry of the new propeller, the parameters used to describe the propeller need to be specified. The propeller parametric model proposed in Section 2, as a form of propeller representation, can express and reconstruct the shape of the propeller with flexibility. The purpose of the propeller optimization problem is to improve the hydrodynamic performance of the propeller by modifying the propeller geometry, which includes the open water efficiency of the propeller, the cavitation performance of the propeller, and the vibration noise of the propeller. Additionally, a series of constraints such as strength constraints are usually specified when propeller optimization is implemented. The mathematical model of the propeller optimization problem is expressed as follows:

\[
\max F(x) = \text{Hydrodynamic (} J, x),
\]

\[
st. \text{Constraint},
\]
where \( x \) represent the parameters affecting propeller geometry, \( J \) is the advance coefficient, \( \text{Hydrodynamic}(x) \) is the hydrodynamic function (open water efficiency, the cavitation performance, and vibration noise) of the propeller, and Constraint represents a series of constraints.

### 3. Results and Discussion

For the propeller optimization, the AU4-40 was chosen as the baseline propeller, while the propeller efficiency was chosen as the function \( F(x) = \text{Hydrodynamic}(J, x) \) and
Figure 8: A distribution about hydrofoil transformation along the propeller’s radius.

Figure 9: Generating distributions of radial parameters with Grasshopper.

Figure 10: Hub of propeller and the details.
Figure 11: Model of a marine propeller.

Figure 12: Graphical representation of different hydrofoils’ approximation level.

Figure 13: Continued.
solved using an approximate model. In this part of the results, the approximate model used (gene expression programming) and its construction process are first described. Subsequently, the optimization results are analysed.

4. Gene Expression Programming and Construction Process

GEP is a new evolutionary artificial intelligence technique [34, 35]; GEP combines the advantages of genetic algorithm (GA) [36] and genetic programming (GP). In the form of expression, GEP inherits the characteristics of GA’s fixed length linear coding, which is simple and fast; in the gene expression (language expression), GEP inherits the characteristics of GP’s flexible tree structure. However, the key difference between the three algorithms resides in the nature of the individuals: in GA, the individuals are chromosomes (a fixed length linear string), GP’s individuals are parse trees (a non-linear solid with different sizes and shapes), and in GEP, the individuals are encoded as linear strings of fixed length that are expressed as non-linear entities of different sizes and shapes (expression trees (ETs)). An example of the expression tree is illustrated in Figure 14.

In GEP, chromosomes are usually composed of multiple genes, and then each gene is connected by a connecting symbol (such as “+”, “−”, “×”, “÷”) or other elementary functions such as sin (x) and cos (x)) and terminal symbol set (e.g., {a, b, c, −2}), while the tail can only come from the latter. Function set, terminal set, fitness function, control parameters, and termination condition are the major components of a GEP model. More information about GEP can be found in [34, 35]. The flowchart of GEP is shown in Figure 15.

Like most machine learning methods, the premise of gene expression programming is to have the datasets for training. The datasets for the GEP model are calculated using the Reynolds-averaged Navier–Stokes (RANS) method. In the CFD solver, the computational domains were discretized and solved using a finite volume method. The second-order upwind convection scheme was used for the momentum equations. The overall solution procedure was based on a semi-implicit method for pressure-linked equations type algorithm. The shear stress transport k-ω turbulence model was used to predict the effects of turbulence. Mesh generation was performed using the built-in automated meshing tool of STAR-CCM+. Trimmed hexahedral meshes were used for the high-quality grid for the complex domains. Local refinements were made for finer grids in the critical regions. These areas can be areas such as the root, middle and slightly of the propeller such as blade edges and areas where the tip and hub. The prism layer meshes were used for near-wall refinement, and the thickness of the first layer cell on the
The surface was chosen such that the $y^+$ value is always higher than 30. The boundary conditions of the simulations were selected to represent the propeller that is completely submerged. The computational domain consists of a stationary region and a rotating region. Velocity inlet boundary condition was applied for the inlet free stream boundary condition, and a pressure outlet was chosen for the outlet boundary condition. The inlet and outlet were placed at 5D and 10D distance from the propeller to avoid any reflections downstream of the propeller and to ensure uniform incoming flow upstream of the propeller. The radius of the rotating region is 1.2D, and the radius of the static region is 5D, as shown in Figure 16. In addition, the RANS simulations are steady. In each simulation sample, 5000 iterations will be carried out under each advance coefficient as the convergence criterion.

In the dataset, the same dimensionless blade profile shape is used to construct the propeller geometric model, which is very similar to DTMB4119 and other propellers. The blade profile shape of DTMB4119 is in the form of NACA66 (MOD) $+$ $a = 0.8$. At the same time, this type of propeller is very common on the trunk line of the Yangtze River in China. Specifically, the sample is obtained by adjusting the eight parameters of the two-dimensional hydrofoil on the premise of keeping the other parameters of the propeller unchanged, as shown in Figure 17. The specific values of other parameters are given in detail in Table 4.

In order to demonstrate and ensure the capability of the CFD solver, mesh uncertainty estimations are carried out with the AU4-40 propeller at $J = 0.8$. The details of grid size information are listed in Table 5.

The results of the mesh uncertainties are shown in Figure 18(a). In addition, Figure 18(b) provides a graph that illustrates the comparison between experimental data [37] and numerical results (based on the first grid environment) for the open water characteristics of the AU-series conventional propeller. Through the comparison, we can know that the dataset obtained by CFD simulation method is effective. The method of the Grid Convergence Index is adopted for discretization error estimation [38]. It can be seen from Table 6 that the better quality of mesh is effective in the CFD simulation.

For data mining purposes, the collected dataset size is dependent on the complexity and degree of non-linearity of the studied problem. The dataset used in this work contains 1170 items. The details of the dataset are presented in Table 7.

In order to build a GEP model that avoids overfitting, the dataset is randomly categorized into two classes of training and testing, containing 70% and 30% of the total data, respectively. In this paper, GEP is used to train the propeller efficiency model and thrust coefficient model for subsequent optimization work. The input parameters are the eight parameters about hydrofoil and advance coefficient $J$ as input variables of the GEP model. The details of input-output variables of propeller efficiency model and thrust coefficient model are presented in Table 8.

The values of GEP parameters (like gene transposition rate, gene recombination rate, and so on) have important influence on the fitness of the output model. We will correlate the setting of these parameters in detail in Table 9. In the present paper, geppy (Gao 2019) is used to implement GEP models.

GEP model development consisted of five major following steps:

1. Selecting the fitness function: in the present study, the RMSE is employed as fitness function.
2. Set the terminal and function sets to create the chromosomes: in this work, the function set includes $\{+, -, \times, \div, \sin(x), \cos(x), \tan(x)\}$.
**Figure 16:** Domain and boundary conditions of CFD.

**Figure 17:** Partial propeller shape for CFD calculation.

**Table 4:** The parameters of sample propeller.

| Diameter ($D$) (m) | Blades | Blade area ratio | DRL         | Pitch (m) | Number of sections |
|-------------------|--------|------------------|-------------|-----------|-------------------|
| 0.25              | 4      | 0.4              | Right hand | 0.25      | 10                |
The architecture of the chromosomes including head size and the number of genes per chromosomes are selected.

Choosing the linking function.

Selecting genetic operators including mutation, inversion, transposition, and recombination.

5. GEP Model Result

In this section, four statistical error criterion measures are used to evaluate the performance of the model. The root mean square error (RMSE), mean absolute error (MAE), mean squared error (MSE), and also the coefficient of determination ($R^2$) are calculated as follows:
0.5, 0.7, and 0.9, respectively, the optimization problem with 3 optimization objectives is carried out. In addition, other parameters were kept consistent with the baseline propeller AU4-40 in the selection of optimization variables. To investigate the significance of the proposed hydrofoil parametric model in propeller optimization applications, the blade section shape of the propeller is considered as the optimization variable. As for the constraints, we hope that the thrust of the propeller after the improvement of the baseline propeller should be consistent with the thrust of the baseline propeller and meet the requirements of China Classification Society (CCS) for propeller strength, and thus the whole propeller optimization mathematical model is summarized as equation (9). The whole optimization process is depicted in a graphical form in Figure 21. 

Suppose: \( X = [P1, P2, P3, P4, P5, P6, P7, P8] \).

Maximize : \( f_1(x) = \eta_0(0.2, X), f_2(x) = \eta_0(0.4, X), f_3(x) = \eta_0(0.6, X), \)

\[ \text{s.t. } K_1^0 = K_1^0, K_2^0 = K_2^0, \ldots, K_n^0 = K_n^0, \]

where \( X \) represent the parameters of the parametric model, \( \eta_0 \) is the efficiency, \( K_t^0 \) is the thrust coefficient of the optimized propeller, and \( K_t^0 \) is the thrust coefficient of the original propeller. Thickness in regulation meets the “Regulations for Classification and Construction of Sea-Going Steel Ships” issued by China Classification Society. It should be noted that the objectives under each working condition are equally important.

We use the non-dominated sorting genetic algorithm to solve the optimization problem. The initial population number is 500, and the evolution algebra is 2000 generations. Non-dominated sorting genetic algorithm is a classic
Table 10: The accuracy assessments for the dataset.

| Item                     | Data       | RMSE | MAE  | MSE  | $R^2$ |
|--------------------------|------------|------|------|------|-------|
| Efficiency model         | Total data | 0.031| 0.022| 0.00097| 0.94  |
|                          | Training data | 0.028| 0.021| 0.00081| 0.94  |
|                          | Test data   | 0.033| 0.023| 0.00107| 0.93  |
| Thrust coefficient model | Total data | 0.006| 0.005| $3.7E-05$| 0.98  |
|                          | Training data | 0.006| 0.005| $3.7E-05$| 0.98  |
|                          | Test data   | 0.007| 0.006| $3.9E-05$| 0.97  |

Figure 19: Comparison between the CFD result and GEP result of efficiency model. (a) Training data. (b) Test data.

Figure 20: Continued.
and effective optimization algorithm, which can effectively avoid falling into local optimal solutions. At the end of iteration, the results are summarized in a compact manner, as shown in Figure 22, from which we can see that the Pareto front is relatively clear.

In order to verify the validity of the optimization results, the TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) [39] method is utilized to select an optimized blade profile in the Pareto front. Also, the open water efficiency of the propeller with the optimized blade profile is compared with AU4-40 propeller. It is worth mentioning here that the pitch, area ratio, rake, and skew of the propeller with optimized blade profile and the AU4-40 propeller are consistent for ensuring the validity of comparison. When using the TOPSIS method, the weight coefficient of each indicator needs to be specified. The indicators in here are the advance coefficient $j = [0.2, 0.4, 0.6]$ and the weight coefficient $w = [0.2, 0.3, 0.5]$. The open water efficiency is obtained by open water simulation based on the Reynolds-averaged Navier–Stokes (RANS) method.

The geometric parameters of the optimized blade profile are given in Table 11 and visualized in Figure 23. The pressure distribution of this propeller under the different advance speed condition is shown in Figures 24 and 25. In Figure 26, the performance comparison between the optimized propeller and the AU4-40 propeller is given. The $D$ value in Figure 26 is the difference between the efficiency of the optimized propeller and the AU4-40 propeller. It can be seen that the value of $K_T$ has hardly changed, and the performance of the propeller with optimized blade profile is
Figure 22: Pareto front for the optimization.

Table 11: The parameters of the optimized hydrofoil.

| Item          | Value  |
|---------------|--------|
| u_in_shift    | 0.302  |
| u_in_angle    | 0.168  |
| u_out_shift   | 0.155  |
| u_out_angle   | 0.124  |
| l_in_shift    | 0.179  |
| l_in_angle    | -0.308 |
| l_out_shift   | 0.400  |
| l_out_angle   | 0.053  |

Figure 23: The comparison between optimized hydrofoil and the baseline hydrofoil (x-axis coincides with the chord direction).

Figure 24: The pressure distribution of optimized propeller suction side.
better than that of the AU4-40 propeller. The open water efficiency increases by 1.52% when \( J = 0.2 \), 1.9% when \( J = 0.4 \), and 2.16% when \( J = 0.6 \). This means that the proposed parametric model and GEP model are effective for propeller optimization.

In this propeller optimization case, we choose one type of AU4-40 propeller with a pitch ratio of 1 as the baseline propeller. The purpose is to improve the open water efficiency by modifying the propeller blade section shape while keeping the same area ratio and pitch ratio. If a new type of propeller with better open water efficiency is found with one disc ratio and one pitch ratio, it means that new propeller open water mapping information can be provided by performing open water experiments with different areas and pitch ratios.

7. Conclusions

In this work, we have presented a geometric parametric model. In the whole process of propeller parameterization, the template hydrofoil profile is constructed with fewer parameters, and then the template hydrofoil profile is copied and transformed along the spanwise direction according to a series of distribution rules defined by radial parameters. These distribution rules describe the scaling, rotation, and translation of the template hydrofoil which correspond to the distribution of chord length, thickness, pitch, rake angle, and skew. Through the parametric expression of several existing marine propellers (AU Series, Wageningen B-Screw Series, Gawn, Kaplan, and SK), its accuracy is evaluated by the level of approximation that can be achieved when parameterized. It has been demonstrated that the model can quickly and automatically produce valid geometric representations of marine propellers, mainly blade surfaces, based on a small set of geometrically and physically meaningful parameters. However, for some special hydrofoil shape, the proposed parameter model may have certain limitations. This will also be our future work. Moreover, with the help of the proposed parametric model, we solve a propeller optimization problem. In the propeller optimization problem,
the GEP model is used to construct an approximate model for the hydrodynamic calculation of the propeller. By analysing the constructed GEP model, the model can better provide an approximate model similar to the response surface model. Also, for the optimization results, the final obtained propeller shows good performance in terms of efficiency compared to the base propeller. This means that the parametric model proposed in this paper can play a certain role in solving the propeller optimization problem.

For future work, the consideration of cavitation will be added into the propeller optimization based on the GEP model. In addition, the proposed approach can be developed and improved to solve marine propeller design problems.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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