Self-similar galaxy dynamics below the de Sitter scale of acceleration

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ABSTRACT
Radial accelerations \( \alpha \) in galaxy dynamics are now observed over an extended range in redshift that includes model calculations on galactic distributions of cold dark matter (CDM) in \( \Lambda \)CDM. In a compilation of data of the Spitzer Photometry and Accurate Rotation Curves (SPARC) catalogue, the recent sample of Genzel et al. (2017) and the McMaster Unbiased Galaxy Simulations 2, we report on effective self-similarity in the variable \( \zeta = a_N/a_{dS} \), given by the Newtonian acceleration \( a_N \) based on baryonic matter content over the de Sitter scale of acceleration \( a_{dS} = cH \), where \( c \) is the velocity of light and \( H \) is the Hubble parameter. SPARC, MUGS2 and theory satisfy \( a_N/\alpha \approx 2.1 \zeta^2 \) (\( \zeta << 1 \)). At \( \zeta = 1 \) in transition to Newtonian gravity (\( \zeta >> 1 \)), however, there is a 6σ gap between SPARC and MUGS2. This poses a novel challenge to CDM in \( \Lambda \)CDM against the apparent \( C^0 \) galaxy dynamics observed in SPARC. We attribute the latter to reduced inertia below the de Sitter scale of acceleration (\( \zeta < 1 \)), based on a causality constraint imposed by the cosmological horizon \( \mathcal{H} \).

Key words: galaxy dynamics: observations

1 INTRODUCTION
Advances in high resolution spectroscopy of galaxy rotation curves across a range of redshifts give a detailed view on radial accelerations over an extended range in radius \( r \) and redshift \( z \) up to about two (Famae & McGaugh 2012; Lelli et al. 2016; McGaugh et al. 2016; Genzel et al. 2017). In \( \Lambda \)CDM, these observations suggest a diminishing of cold dark matter content with \( z \), as observed accelerations \( \alpha \) increasingly match the Newtonian acceleration

\[
a_N = \frac{G M_b}{r^2}
\]

by baryonic matter content \( M_b \) within \( r \), where \( G \) is Newton’s constant. These results provide important benchmarks for galaxy models in \( \Lambda \)CDM from high resolution smoothed particle hydrodynamics simulations of galaxy formation. A recent comparison of the McMaster Unbiased Galaxy Simulations 2 (MUGS2) sample of galaxy models, for instance, suggests excellent agreement with the “missing mass” in galaxy rotation curves from the Spitzer Photometry and Accurate Rotation Curves (SPARC) (Keller & Wadsley 2017). Here, we revisit this claim focused on the transition regime of gravitational acceleration consistent with Newton’s theory based on baryonic matter and weak gravitation, marked by anomalous dynamics commonly attributed to dark matter or a modification of Newtonian gravitation (Famae & McGaugh 2012).

In a model-independent approach, redshift dependence in galaxy dynamics shows evolution with background cosmology described by the Hubble parameter \( H = H(z) \), carrying a de Sitter scale of acceleration

\[
a_{dS} = cH,
\]

where \( c \) is the velocity of light and \( H \) is the Hubble parameter (Fig. 1). For a galaxy such as the Milky Way, \( a_N = a_{dS} \) corresponds to a distance (van Putten 2016)

\[
r_t = \sqrt{R_g R_H} = 4.6 \text{kpc} M_11^{1/2},
\]

where \( R_g = c/H \) is the Hubble radius in a three-flat Friedmann-Robertson-Walker universe and \( R_g = GM/c^2 \) is the gravitational radius of a galaxy of mass \( M = M_11 \times 10^{11} M_\odot \). In quantum cosmology, \( a_{dS} \) represents the surface gravity of the cosmological horizon at Hubble radius \( R_H = c/H \) in de Sitter space (Gibbons & Hawking 1977). Based on dimensional analysis, this suggests evolution in galaxy dynamics in

\[
\zeta = \frac{a_N}{a_{dS}},
\]

where \( \zeta = 1 \) corresponds to a collision of Rindler and cosmological horizon (van Putten 2017b).

We here consider data on galaxy dynamics as a function of \( \zeta \), of galaxy rotation curves of observed galaxies and numerical galaxy models in \( \Lambda \)CDM side-by-side (§2). This compilation highlights self-similar behavior in galaxy dynamics in \( \zeta \) - by which data over different redshifts coalescence - and a transition across \( \zeta = 1 \) to weak gravitation (\( \zeta < 1 \))
from normal, Newtonian gravitation \((\zeta > 1)\), where observed and modeled galaxy dynamics differ. These observations are interpreted in §3. This study is restricted to late-time cosmology with redshifts \(z\) up to about two, over which range the Hubble parameter varies by a factor up to about three (Fig. 1). In §4, we give our conclusions and outlook on future observations.

2 SELF-SIMILAR GALAXY DYNAMICS

Galaxy rotation data considered here are taken from SPARC \(\text{[McGaugh et al. 2016; Lelli et al. 2016]}\), MUGS2 \(\text{(Keller & Wadsley 2017)}\) and Genzel et al. \(\text{(2017)}\). SPARC provides a sample of rotation curves observed from 175 nearby mostly late Hubble type galaxies observed by spectroscopy and photometry in HI/H\(_{\alpha}\), covering a broad range in luminosity \((10^{7−12} L_\odot)\), radii \((0.3−15 \text{ kpc})\), effective surface brightness \((5−5000 L_\odot \text{pc}^{-2})\) and rotation velocities \((20−300 \text{ km s}^{-1})\) consistent with a stellar mass-to-light ratio \(0.5M_\odot/L_\odot\).

MUGS2 provides a sample of 18 galaxy models with halo masses \(3.7 \times 10^{11}− 2.2 \times 10^{12} M_\odot\) and disk masses \(1.8 \times 10^{10}− 2.7 \times 10^{11} M_\odot\) in a ΛCDM cosmology from high resolution smoothed particle hydrodynamics simulations with radiative cooling, star-formation and feedback from supernovae \(\text{(Wadsley et al. 2004; Volker 2005; Shen et al. 2010; Keller et al. 2014)}\). Excluding galaxies that experience appreciable tidal interactions and limited to galaxies modeled by at least 100 star particles, it is extended to a total of 32 galaxies at \(z = 0\) \(\text{(Keller & Wadsley 2017)}\).

Genzel et al. \(\text{(2017)}\) provides a sample of six rotation curves of galaxies at intermediate redshifts \(z \in \{0.854, 1.5, 1.613, 2.196, 2.242, 2.383\}\) with respective baryonic masses \(M_{b,11} = \{1.7, 2.3, 1.0, 1.7, 1.7, 2.1\}\) and ΛCDM Hubble parameters \(H(z)/H_0 = \{1.599, 2.288, 2.425, 3.190, 3.253, 3.454\}\) featuring rotation velocities \(V_c = \{276, 310, 257, 301, 364, 299\}\) km/s at radii \(R_{25}/R_\odot = \{7.3, 7.4, 4.9, 5.5, 3.3, 6\}\) kpc. Following \(\text{Genzel et al. 2017}\), their \(\zeta\) values cluster about \(\zeta = 1\) \(\text{(van Putten 2017b)}\), \(\zeta = \{0.2942, 0.3100, 0.3162, 0.3378, 0.4034, 0.8521\}\).

Fig. 2 shows a compilation of MUGS2 rotation curve data plotted as a function of \(\zeta\). Over \(0 < \zeta < 2\), \(H(z)\) varies by a factor of about three, implying variations of order unity in dimensional quantities such as \(R_\odot\). For MUGS2, averaging of rotation curve data \(\langle \zeta, a_N/a_\perp \rangle_{z_0}\) over different redshifts leaves a dispersion much smaller than scatter in the data.

Fig. 3 shows a compilation of the three galaxy samples of SPARC \(\text{[Genzel et al. 2017; Keller & Wadsley 2017]}\) and MUGS2 combined. While there is excellent agreement between SPARC and MUGS2 in the weak gravity limit \(\zeta << 1\), there appears to be an appreciable gap about \(\zeta = 1\) in transition to the Newtonian limit \(\zeta >> 1\). As a function of \(\zeta\), the relatively high redshift data from \(\text{Genzel et al. 2017}\) agree within uncertainties with SPARC except for the outlier \(\zeta = 2006690\).

Plotted as a function of \(\zeta\), the aforementioned “missing mass” in galaxy rotation curves appears to be self-similar over an extended range of redshift, absorbed in normalization by \(a_{\text{DS}}\) giving a reduction in independent variables by one.

In the outskirts of galaxies, rotation curves satisfy

3 A 6σ GAP AT \(\zeta = 1\)

In transition from Newtonian gravity \(\text{(\(\zeta >> 1\)))}\) to weak gravity \((\zeta << 1)\), Fig. 3 shows an onset to the latter which is smooth in MUGS2 in contrast to what appears to be \(C^0\) galaxy dynamics - continuous with discontinuous derivatives - in SPARC \(\text{(van Putten 2017b)}\). (Uncertainties in the data do not resolve whether the transition is truly \(C^0\) or nearly so.) Smoothness in MUGS2 is expected and inherent to \(N\)-body simulations by diffusion due to small angle gravitational scattering and gas dynamics. The noticeable gap between MUGS2 and SPARC at \(\zeta = 1\) hereby might be characteristic for galaxy models in ΛCDM, not limited to MUGS2.

It is perhaps paradoxical, that \(\zeta\) is a similarity variable familiar from the theory of linear diffusion, yet the apparent \(C^0\) onset to weak gravity in SPARC runs counter to the
same. We attribute this result to a break in Newton’s second law - assuming a constant inertia at arbitrarily small accelerations - on a cosmological background with finite Hubble radius $R_H$, equivalently attributed to thermodynamic properties of the associated cosmological horizon $\mathcal{H}$.

According to the equivalence principle of general relativity, inertia can be identified with inertial mass-energy (van Putten 2017b):

$$ U = mc^2 $$

(8)
given by the gravitational binding energy in the gravitational field over the distance

$$ \xi = \frac{c^2}{\alpha} $$

(9)
to the Rindler horizon $h$ at a given acceleration $\alpha$. Here, $U$ obtains by integrating the inertial force $F = ma$ over a distance $\xi$.

According to quantum field theory, the vacuum seen by a Rindler observer is described by a finite temperature diffusion constant (reviewed by Son & Starinets 2007):

$$ D = \frac{\hbar c^3}{2\pi k_B T} $$

(10)

where $\hbar$ is Planck’s constant and $k_B$ is the Boltzmann constant. With $D = \xi C$, (10) is the thermodynamic interpretation of Rindler’s relation (9) at the Unruh temperature $T = T_U$ (Unruh 1976),

$$ k_B T_U = \frac{\hbar c}{2\pi c} $$

(11)

Identifying $T_U$ with the temperature of $h$, $U$ derives from the entanglement entropy $I_1 = 2\pi \Delta \varphi C$, where $\Delta \varphi C$ is the distance $\xi$ expressed in Compton phase (van Putten 2015), giving

$$ U = \int_0^\xi T_U dI_1 $$

(12)

In (12), $h$ is an apparent horizon surface. Apparent horizon surfaces are familiar concept in numerical relativity signaling black hole formation (Penrose 1965; Brevik 1988; Cook 2000; York 1998; Wald & Iyer 1999; Cook & Abrahams 1992; Thorne 2007). In a three-flat Friedmann-Robertson-Walker universe, the cosmological horizon $\mathcal{H}$ provides an apparent horizon in the background.
whose Hubble radius $R_H$ puts a bound on $\xi$. As $h$ formally drops beyond $H (\xi > R_H)$, $U$ in (9) drops below its Newtonian value $m = m_0$, since integration over the gravitational field is cut-off at $R_H$ by $H$ as a causal boundary on $D$ in (10). At a given $a_N$, the observed acceleration

\[ a = \left( \frac{m_0}{m} \right) a_N \]

experiences a $C^0$ transition across $\xi = 1$ (van Putten 2017a). The apparent $C^0$ galaxy dynamics in the SPARC data can hereby be attributed to causality imposed on inertial mass-energy by $H$, leading to a $6\sigma$ gap at $\xi = 1$ between it and MUGS2.

4 CONCLUSIONS

An effective self-similarity $\xi$ in galaxy dynamics enables a comprehensive confrontation between galaxy rotation curves from observations and simulations over an extended range of redshifts, here shown in Fig. 3 for SPARC, MUGS2 and Genzel et al. galaxies covering redshifts up to about two. In weak gravity ($\xi << 1$), SPARC, MUGS2 and theory agree. At $\xi = 1$, however, there is a $6\sigma$ gap between SPARC and MUGS2, where the first appears to show $C^0$ galaxy dynamics while the second gives a smooth transition between $\xi << 1$ and the Newtonian regime $\xi >> 1$. In Fig. 3, the latter is emphasized by a simple fitting function

\[ \frac{a_N}{a} = \left( \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{x} + \frac{1}{\sqrt{3}}} \right)^{-1} \]

with $x = 4\pi\xi$. This gap appears to have eluded the previous analysis of Keller & Wadsley (2017).

The low apparent dark matter content in Genzel et al. (2017) arises from clustering of $\xi$ close to the transition point $\xi = 1$, that agrees with SPARC but deviates from MUGS2. Conversely, there is no apparent low dark matter content in high redshift galaxies of MUGS2.

The SPARC-MUGS2 gap is expected to be generic for CDM galaxy models in $\Lambda$CDM, resulting from smoothness inherent to diffusion by small angle gravitational scattering. At $6\sigma$, this discrepancy appears to be fundamental to the nature of CDM, unless perhaps the mass of the putative dark matter particle is anomalously small. The apparent $C^0$ galaxy dynamics in SPARC, however, points to a departure of Newton’s second law as inertia drops at accelerations below $a_{\xi}$, when inertial mass-energy $U$ reduces to gravitational binding energy to the cosmological horizon $H$.

While a reduced inertia obviates the need for CDM in galaxies, a cosmological distribution of CDM is still required in light of the three-flat condition $\Omega_M + \Omega_{\Lambda} = 1$ on the dimensionless densities of dark matter ($\Omega_M$) and dark energy ($\Omega_{\Lambda}$). The Compton wave length of the putative dark matter particle, greater than the scale of galaxies, may reach the scale of galaxy clusters.

In light of the above, we anticipate that the apparent self-similarity and $C^0$ galaxy dynamics shown in Figs. 2-3 extends to elliptical galaxies, which may be obtained through future studies given the very large samples of elliptical galaxies available from, e.g., the Sloan Digital Sky Survey (Abolfati et al. 2018).

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