Fault detection and isolation of malicious nodes in MIMO Multi-hop Control Networks

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Abstract—A MIMO Multi-hop Control Network (MCN) consists of a MIMO LTI system where the communication between sensors, actuators and computational units is supported by a (wireless) multi-hop communication network, and data flow is performed using scheduling and routing of sensing and actuation data. We provide necessary and sufficient conditions on the plant dynamics and on the communication protocol configuration such that the Fault Detection and Isolation (FDI) problem of failures and malicious attacks to communication nodes can be solved.

I. INTRODUCTION

Wireless networked control systems are spatially distributed control systems where the communication between sensors, actuators, and computational units is supported by a wireless multi-hop communication network. The use of wireless Multi-hop Control Networks (MCNs) in industrial automation results in flexible architectures and generally reduces installation, debugging, diagnostic and maintenance costs with respect to wired networks. Although MCNs offer many advantages, co-design of the network configuration and of the control algorithm for a MCN requires addressing the joint dynamics of the plant and of the communication protocol.

Recently, a huge effort has been made in scientific research on Networked Control Systems (NCSs), see e.g. [2], [25], [11], [24], [10], [8], [14] and references therein for a general overview. In general, the literature on NCSs addresses non–idealities (such as quantization errors, packets dropouts, variable sampling and delay and communication constraints) as aggregated network performance variables, losing irreversibly the dynamics introduced by scheduling and routing communication protocols. What is needed for modeling and analyzing control protocols on MCNs is an integrated framework for analyzing/co-designing network topology, scheduling, routing and control. In [1], a simulative environment of computer nodes and communication networks interacting with the continuous-time dynamics of the real world is presented. To the best of our knowledge, the first formal model of a Multi-hop Control Network has been presented in [18], [19], where a mathematical framework has been proposed that allows modeling the MAC layer (communication scheduling) and the Network layer (routing) of recently developed wireless industrial control protocols, such as WirelessHART and ISA-100. In [5] we extended the formalism proposed in [19] by defining a MCN \( M \), that consists of a continuous-time SISO LTI plant \( P \) interconnected to a controller \( C \) via two (wireless) multi-hop communication networks \( G_{R} \) (the controllability network) and \( G_{O} \) (the observability network), as illustrated in Figure 1, and by modeling redundancy in data communication - i.e. sending actuation/sensing data via multiple paths and then merging these components according to a weight function. This approach, which can be interpreted as a form of network coding at the level of the application layer of the ISO/OSI protocol stack, is called multi-path routing (or flooding, in the communication scientific community) and aims at enabling the detection and isolation of node failures and malicious intrusions, which cannot be done using single-path routing or strategies that use timestamps to discard redundant packets. It is well known that redundancy can also render the system fault-tolerant with respect to node failures and mitigate the effect of packet losses. We remark that, as illustrated in [16], the implementation of multi-path routing in a Wireless HART device only requires a minor change which retains backward compatibility with standard devices.

Paper contribution: Because of wireless networking, a MCN can be subject to failures and/or malicious attacks. In [5] we addressed and solved the problem of designing a set of controllers and the communication protocol parameters of a SISO MCN, so that it is possible to detect and isolate the faulty nodes of the controllability and observability networks and apply an appropriate controller to stabilize the system, as depicted in Figure 1. In [21] and in this paper we extend such investigation to MIMO MCNs: in particular, while in [21] we extended the MCN formalism to MIMO LTI plants and developed a procedure to guarantee the existence of a stabilizing controller \( C_i \) for any node failure, in this paper we provide conditions on network topology, scheduling and routing that enable detection and isolation of node failures.

The extension with respect to the results in [5] is not trivial: indeed in the MIMO case the geometric approach exploited in [5] does not easily provide a relation between the conditions that enable FDI of node failures and the network topology, scheduling and routing. To overcome this issue we exploit formalism and FDI methodologies of structured systems [9]. This methodology leverages on the classical observer-based results in [12], where the system with failures is required to be left-invertible: therefore our results imply that almost any failure signal can be detected and isolated. In order to detect and isolate any non-zero failure signal we should require input-observability. Indeed a carefully chosen
Related work: As can be inferred from the recent survey [7], fault tolerant control and fault diagnosis is one of the main issues addressed in the research on NCSs. However, most of the existing literature does not consider the effect of the communication protocol introduced by a Multi-hop Control Network. In [4], a procedure to minimize the number and cost of additional sensors, required to solve the FDI problem for structured systems, is presented. In [20] the design of an intrusion detection system is presented for a Wireless Control Network: our work differs from such results in the following four aspects. (1) In [13] the wireless network is an autonomous system where the network itself acts as a decentralized controller, while in our model the wireless network transfers sensing and actuation data between a plant and a centralized controller, namely it acts as a relay network. This modeling choice is motivated by the fact that WirelessHART and ISA-100 are designed for control loops where a centralized controller exploits a wireless network to relay sensing and actuation data, which is often a forced choice in industrial environments. (2) As a consequence of the previous issue, in [13] FDI is performed only exploiting the output signal from a subset of communication nodes, while we can exploit the input and output signals of the centralized controller. (3) In our model we take into account the effect of the scheduling ordering of the node transmissions in the sensing and actuation data relay, which provides a more accurate modeling of the effect of scheduling on the closed loop dynamics. Indeed, the conditions we derive in Section V show that, in order to guarantee FDI of faulty nodes, the link scheduling ordering is irrelevant: this is an interesting result because, as widely discussed in [6], [5], [22], it strongly reduces the scheduling period, avoids the necessity of on-the-fly scheduling re-definition, and always guarantees the existence of an admissible scheduling when multiple loops exploit the same communication network. (4) In [20] FDI is performed by on-the-fly testing the rank of a number of matrices which is a combinatorial function of the number of communication links, while our method only requires to apply a logic operator to a number of Luenberger observers which is at most equal to the number of communication nodes. To the best of our knowledge, our work is pioneering in addressing FDI for a MCN that implements standardized communication protocols. An extended version of this paper can be found on ArXiv.

Notation: We will denote by $\mathbb{N}$ and $\mathbb{R}$ respectively the sets of natural and real numbers. Given $n \in \mathbb{N}$, we denote by $n$ the set $n = \{1, 2, \ldots, n\}$. We denote by $0_{n \times m}$ the matrix of zeros with $n$ rows and $m$ columns and by $I_n$ the identity matrix of dimension $n$. Given a finite set $A$ and a subset $B \subseteq A$, we define $|A|$ and $|B|$ their cardinality, $A \setminus B$ the difference set and $2^A$ the power set. We denote by $\text{diag}(F_1(z), \ldots, F_n(z))$ the $n \times n$ diagonal transfer function matrix whose diagonal consists of the scalar transfer functions $F_1(z), \ldots, F_n(z)$. Given a directed graph $(\mathcal{V}, \mathcal{E})$ we define path an alternating sequence of vertices and edges. A path is defined simple if no vertices are repeated. We define a set of paths vertex disjoint if each two of them consist of disjoint sets of vertices. We call a set of $r$ disjoint and simple paths from a set $\mathcal{V}_1 \subseteq \mathcal{V}$ to a set $\mathcal{V}_2 \subseteq \mathcal{V}$ an $r$-linking from $\mathcal{V}_1$ to $\mathcal{V}_2$. We denote by $\{\}$ the disjoint union operator among directed graphs. For a formal definition of further graph properties and notations (e.g. weakly connected graph, weakly connected component, bridge nodes etc.) the reader is referred to [26].

II. MCN MODEL

We propose a mathematical framework for modeling wireless multi-hop communication networks that implement time-triggered protocols such as WirelessHART and ISA-100. In these standards the access to the shared communication channel is specified as follows: time is divided into slots of fixed duration $\Delta$ and groups of $T$ time slots are called frames of duration $T = \Pi \Delta$ (see Figure 2). For each frame, a communication scheduling allows each node to transmit data only in a specified time slot. The scheduling is periodic with period $T$, i.e. it is repeated in all frames.

\[
\begin{array}{c|c|c|c}
\text{Cycle } n-1 & \Delta & \text{Cycle } n & \text{Cycle } n+1 \\
1 & 2 & 3 & 4 & 5 & 1 & 2 & 3 & 4 & 5 \\
\hline
\end{array}
\]

Fig. 2. Time-slotted structure of frames.

Definition 1: A MIMO Multi-hop Control Network is a tuple $\mathcal{M} = (\mathcal{P}, G, W, \eta, \Delta)$ where:
- $\mathcal{P}$ is a continuous-time MIMO LTI system, with $n$, $m$ and $\ell$ respectively the dimensions of the internal state, input and output spaces.

\( G = (G_R, G_O) \). \( G_R = (V_R, E_R) \) is a directed graph, where the vertices correspond to the communication nodes of the network and an edge from \( v \) to \( v' \) means that node \( v' \) can receive messages transmitted by node \( v \) through the wireless communication link \((v, v')\). We denote by \( v_{u,c} \) the special node of \( V_R \) that corresponds to the controller and by \( v_{u,i} \in V_R, i \in m \), the special nodes that correspond to the actuators of the input components. \( G_O = (V_O, E_O) \) is defined similarly to \( G_R \). We denote by \( v_{y,c} \) the special node of \( V_O \) that corresponds to the controller and by \( v_{y,i} \in V_O, i \in l \), the special nodes that correspond to the sensors of the outputs \( y_i \), \( i \in l \).

\[
W = (W_R, W_O), \quad W_R = \{W_{R,i}\}_{i \in m}, \quad W_{R,i} : \mathbb{R} \rightarrow \mathbb{R} \text{ is a weight function for the } i\text{-th input component that associates to each link a real constant.} \quad W_O = \{W_{O,i}\}_{i \in l} \text{ is defined similarly to } W_R.
\]

\[\eta = (\eta_R, \eta_O), \quad \eta_R = \{\eta_{R,i}\}_{i \in m}, \quad \text{where } \eta_{R,i} : \mathbb{N} \rightarrow \mathbb{R}^E_R \text{ is the controllability scheduling function for the } i\text{-th input component that associates to each time slot of each frame a set of edges of the controllability radio connectivity graph } G_R. \text{ Since in this paper we only consider a periodic scheduling, that is repeated in all frames, we define the controllability scheduling functions by } \eta_{R,i} : \{1, \ldots, \Pi\} \rightarrow \mathbb{R}^E_R. \]

The integer constant \( \Pi \) is the period of the controllability scheduling. The semantics of \( \eta_{R,i} \) is that \( (v, v') \in \eta_{R,i}(h) \) if at time slot \( h \) of each frame the data associated to the \( i\)-th input component and contained in node \( v \) is transmitted to the node \( v' \), multiplied by the weight \( W_{R,i}(v, v') \). For any \( \eta_{R,i} \), we assume that each link can be scheduled only one time for each frame\(^1\). \( \eta_O = \{\eta_{O,i}\}_{i \in l} \) is defined similarly to \( \eta_{R,i} \)^2

\( \Delta \) is the time slot duration. As a consequence, \( T = \Pi \Delta \) is the frame duration.

The main difference with respect to the MCN definition given in [15] for SISO systems is that here the network needs to relay \( m \) actuation signals and \( l \) sensor signals: we assume that nodes routes data of each of these signals separately.

Designing a scheduling function induces a communication scheduling (namely the time slot when each node is allowed to transmit) and a multi-path routing (namely the set of paths that convey data from the input to the output of the connectivity graph) of the communication protocol. Since the scheduling function is periodic, the induced communication scheduling is periodic and the induced multi-path routing is static.

**Definition 2:** Given \( G_R \) and \( \eta_R \), we define \( G_R(\eta_{R,i}(h)) \) the sub-graph of \( G_R \) induced by keeping the edges scheduled in the time slot \( h \). We define \( G_R(\eta_{R,i}) = \bigcup_{h=1}^{\Pi} G_R(\eta_{R,i}(h)) \) the sub-graph of \( G_R \) induced by keeping the union of edges scheduled during the whole frame. \( G_R(\eta_{R,i}) \) consists of a set of simple (routing) paths starting from \( v_{u,c} \) and terminating in \( v_{u,i} \). As a consequence \( G_R(\eta_{R,i}) \) is a directed, weakly connected and acyclic graph.

\[G_R(\eta_{R,i}) \text{ is directed, weakly connected and acyclic graph. The above definition can be given similarly for } G_O \text{ and } \eta_O,\]

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**Fig. 3.** MCN interconnected system.

**Nominal MCN:** The dynamics of a MCN \( \mathcal{M} \) can be modeled by the interconnection of blocks as in Figure 3. The block \( \mathcal{P} \) is characterized by the discrete-time transfer function matrix \( \mathcal{P}(z) \) obtained by discretizing the system \( \mathcal{P} \) with sampling time \( T = \Pi \Delta \). The block \( \mathcal{R} \) models the dynamics introduced by the flow of the actuation data of all components of the control input \( u \) via the communication network represented by \( G_R \). In order to define the dynamics of \( \mathcal{R} \), we need to define the semantics of the data flow through the network induced by the scheduling and the weight functions. We assume that each communication node, when scheduled to transmit by \( \eta_{R,i} \), computes a linear combination of the data associated to the input component \( u_i \) and received from the incoming links according to the weight function \( W_{R,i} \). This linear combination is then transmitted via the outgoing scheduled links. As in [21] the input/output behavior of \( u_i(kT) \) with respect to \( u_i(kT) \) can be formalized, for any \( i \in m \), by the following transfer function:

\[
\mathcal{R}_i(z) = \frac{\tilde{U}_i(z)}{U_i(z)} = \sum_{d=1}^{D_{R,i}} \frac{\gamma_{R,i}(d)}{z^d},
\]

where \( D_{R,i} \in \mathbb{N} \) is the maximum delay introduced by the (routing) paths of \( G_R(\eta_{R,i}) \) and \( \forall d \in D_{R,i}, \gamma_{R,i}(d) \in \mathbb{R}, \gamma_{R,i}(D_{R,i}) \neq 0 \). The reader is referred to [5], [22] for the formal definition of the coefficients \( \gamma_{R,i}(d), d \in D_{R,i} \), which depend on the weight function \( W_{R,i} \).

The block \( \mathcal{R} \) is characterized by the transfer function matrix \( \mathcal{R}(z) = \text{diag}(\mathcal{R}_1(z), \ldots, \mathcal{R}_m(z)) \). The same holds for the block \( \mathcal{O} \). The dynamics of a MIMO MCN \( \mathcal{M} \) can be modeled by the cascade of the transfer function matrices \( \mathcal{R}(z), \mathcal{P}(z) \) and \( \mathcal{O}(z) \), thus \( \mathcal{M}(z) = \mathcal{O}(z) \mathcal{P}(z) \mathcal{R}(z) \).

**Faulty MCN:** We assume that a failure or a malicious attack associated to a communication node \( v \in V_R \) can be modeled by a set of arbitrary signals \( f_{v,i}(k) \), for any \( i \) such that there exist \( u' \in V_R \), \( h \in \Pi \) with \( (v, v') \in \eta_{R,i}(h) \), each summed to the \( i\)-th input component routed via node \( v \). This general framework, as illustrated in [5], models several classes of failures (e.g. a node stops sending data or sends fake data) and malicious attacks (e.g. an arbitrary signal is injected, which overrides/sums to the original data).

Following the same reasoning as in the definition of \( \mathcal{R}_i(z) \), we can define the transfer function from \( f_{v,i}(k) \) to \( u_i(k) \) as follows:

\[
\mathcal{F}_{v,i}(z) = \frac{U_i(z)}{F_{v,i}(z)} = \sum_{d=1}^{D_{v,i}} \frac{\gamma_{v,i}(d)}{z^d},
\]

where \( D_{v,i} \in \mathbb{N} \) is the maximum delay introduced by the (routing) paths from \( v \) to the actuator node \( v_{u,i} \) and \( \forall d \in \mathbb{N}, \gamma_{v,i}(d) \in \mathbb{R}, \gamma_{v,i}(D_{v,i}) \neq 0 \).
structured system can also be represented by a directed graph $D$ of $\lambda$. If the corresponding entry of the original matrix is zero), or a free parameter (if the corresponding entry the original matrix is nonzero). For instance, consider a system $\eta(i)$ we derive, for each component $i \in m$, a free parameter $\lambda$, relating the corresponding variables in the equations. The graph representation of the example above is given by $V_{S\lambda} = \{(x_1, x_1), (x_2, x_1), (u, x_2), (x_1, y_1)\}$.

The following proposition formalizes the graph structured representation of the block $R$ when a failure signals is applied to communication nodes.

**Proposition 1:** Given $G_{\lambda, \eta}$ and a set of faulty nodes $V \subseteq R$, we define the graph structured representation $(V_{S\lambda}, E_{\lambda})$ of the block $R$ as follows:

$$V_{S\lambda} = \{u_1, \ldots, u_m\} \cup \{\bar{u}_1, \ldots, \bar{u}_m\} \cup \bigcup_{v \in V} \{(x, i) \in \lambda, (f, i) \in \lambda\},$$

$$\forall i \in m, \forall d \in D_{R\lambda}, (u, x, i, d) \in E_{\lambda} \iff \gamma_{\lambda, i}(d) \neq 0,$$

$$\forall i \in m, \forall d \in D_{R\lambda}, \forall v \in V, (f, i, x, i, d) \in E_{\lambda} \iff \gamma_{\lambda, i}(d) \neq 0,$$

$$\forall i \in m, \forall d \in D_{R\lambda}, (x, i, d, i, d) \in E_{\lambda} \iff d_1 = d_2 + 1,$$

$$\forall i \in m, (x, i, \bar{u}_i) \in E_{\lambda}.$$

**Proof:** By applying the principle of superposition to (1) and (2) we derive, for each component $i \in m$ of the input signal, a state space representation

$$x_i(t) = A_i x(t) + B_i u_i(t) + \sum_{v \in V} F_i f_v, i(t),$$

$$\bar{u}_i(t) = C_i x(t),$$

of $U_i(z) = \mathcal{R}_i(z) U(z) + \sum_{v \in V} F_{v,i}(z) E_v(i, z)$. In particular, we define $x = [x_{i,1}, \ldots, x_{i,D_{R\lambda}}]^T$ and:

$$A_i = \begin{bmatrix} 0 & I_{D_{R\lambda} - 1} \\ 0 & (x \times (D_{R\lambda} - 1)) \end{bmatrix},$$

$$B_i = \begin{bmatrix} \gamma_{\lambda, i}(1) & \cdots & \gamma_{\lambda, i}(D_{R\lambda}) \end{bmatrix}^T,$$

$$F_i = \begin{bmatrix} \gamma_{\lambda, i}(1) & \cdots & \gamma_{\lambda, i}(D_{R\lambda}) \end{bmatrix}^T,$$

$$C_i = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}.$$

The result follows by definition of graph representation of a structured system.

**Figure 4** provides an example of the graph representation of $R_{\lambda}$ when a failure in the node $v$ occurs. The $\gamma$'s listing some edges of $E_{R_{\lambda}}$ just indicate that such edges are present if and only if the corresponding $\gamma$'s are not equal to 0. Note that $(V_{R_{\lambda}, E_{R_{\lambda}}})$ is composed by $m$ weakly connected components, each associated to the data flow of the $i$-th input component. Also note that each node $x_{i, d}$ is a variable associated to the $i$-th input component that will be delivered with a delay $d$ to the actuator node $v_{u, i}$. Finally, note that the sets of input and output nodes are respectively $U = \{u_1, \ldots, u_m\}$ and $\bar{U} = \{\bar{u}_1, \ldots, \bar{u}_m\}$.

The same holds for defining the structured graph representation $(V_{O\lambda}, E_{O\lambda})$ of block $O$, where the sets of input and output nodes are respectively $\bar{U} = \{\bar{u}_1, \ldots, \bar{u}_m\}$ and $\bar{Y} = \{\bar{y}_1, \ldots, \bar{y}_m\}$.

The model of a MCN $M$ is the cascade of the blocks $R$, $P$ and $O$. As a consequence, its structured graph representation $(V_{M\lambda}, E_{M\lambda})$ is given by the union of the structured graph representations of $R_{\lambda}$, $P_{\lambda}$ and $O_{\lambda}$, and the set of nodes $\bar{U}$ and $\bar{Y}$ represent the interconnection nodes among such graphs. It is easy to see that all nodes in $\bar{U}$ and $\bar{Y}$ are (weak) bridges of $(V_{M\lambda}, E_{M\lambda})$ since the removal of one of them increases the number of weakly connected components.

**IV. FDI ON STRUCTURED SYSTEMS**

**Assumption 1:** We assume in this section that $f_v(i) = f_v(k)$ for all $i$ such that there exist $v' \in V_R$, $h \in \Pi$ with $(v, v') \in \eta_{\lambda}(h)$.

In other words, when a failure on a node $v$ occurs, it equally affects all the input components routed via $v$. This assumption is satisfied when $f_v(i)$ models a node failure, or when the malicious attack is not able to access separately the input components. We will discuss the case when this assumption does not hold in Section VI.

Let a structured system $S_{\lambda}$ be given in the form:

$$x(k + 1) = A x(k) + B u(k) + E_d d(k) + F_1 f_1(k),$$

$$y(k) = C x(k) + D u(k) + E_2 d(k) + F_2 f_2(k),$$

where $d(k)$ is a vector of disturbance signals and $f(k) = [f_1(k), \ldots, f_r(k)]^T$ is a vector of $r$ failure signals. In this paper we will not consider the disturbance (i.e. $E_1 = E_2 = 0$) and leave such generalization to further work. In [3] necessary and sufficient conditions have been derived on the graph representation of $S_{\lambda}$ that (generically) guarantee the existence of a bank of Luenberger observers, which takes
as inputs $u(k), y(k)$, generates as output the residual signals vector $\hat{f}(k) = [\hat{f}_1(k), \ldots, \hat{f}_r(k)]^T$ and is characterized by a transfer function

$$
\begin{bmatrix}
F_1(z) \\
\vdots \\
F_r(z)
\end{bmatrix} =
\begin{bmatrix}
T_{11}(z) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & T_{rr}(z)
\end{bmatrix}
\begin{bmatrix}
F_1(z) \\
\vdots \\
F_r(z)
\end{bmatrix},
$$

where $\forall i \in S$, $T_{ii}(z) \neq 0$. Characterizing the existence of such bank of observers is called the bank of observer-based diagonal FDI problem. It is well known that the control input effects can be taken into account in the observer structure, therefore we will consider without loss of generality $B = D = 0$. The theorem below characterizes the bank of observer-based diagonal FDI problem when there are no disturbances, and is a particular case of Theorem 3 in [3].

**Theorem 2 (from Theorem 3 in [3]):** The bank of observer-based diagonal FDI problem is generically solvable for a system $S$ if and only if (i) $S$ is structurally observable and (ii) $k = r$, where $k$ is the maximum number of fault-output vertex disjoint paths in the graph representation of $S_A$.

V. FDI OF NODE FAILURES ON MCNS

Given a MCN $\mathcal{M}$ subject to failures on communication nodes, if the bank of observer-based diagonal FDI problem is generically solvable for node failures in $\mathcal{M}_\lambda$, then the residual signals can be used to detect and identify possibly simultaneous occurrence of node failures. In [21] we proved that, given a MIMO MCN $\mathcal{M}$ and if the plant $P$ is controllable and observable, it is always possible to design a weight function $W$ such that $\mathcal{M}$ is controllable and observable. As a consequence, we assume in this paper that $\mathcal{M}_\lambda$ always satisfies Condition (i) of Theorem 2.

In order to design the network configuration of a MCN $\mathcal{M}$ to enable FDI of node failures, as the main result of this paper we state a formal relation between the network topology $G$ and scheduling/routing $\eta$ of $\mathcal{M}$, and solvability conditions of the bank of observer-based diagonal FDI problem for node failures. With this aim we first propose an algorithm to construct a graph $(\mathcal{V}, \mathcal{E})$, which essentially consists of the disjoint union of the controllability and observability graphs associated to each schedule, and of the structured graph representation of the plant. Then we will state a formal relation between solvability conditions of the bank of observer-based diagonal FDI problem for $\mathcal{M}_\lambda$ and $(\mathcal{V}, \mathcal{E})$.

**Definition 3:** Given a MCN $\mathcal{M}$, we define a graph

$$(\mathcal{V}, \mathcal{E}) = (\bigcup_{i=1}^{m} G_{\mathcal{R}}(\eta_{\mathcal{R}_i})) \cup (V_{\mathcal{P}_1} \cup E_{\mathcal{P}_1}) \cup (\bigcup_{i=1}^{r} G_{\mathcal{O}}(\eta_{\mathcal{O}_i})), \tag{4}$$

with the addition of the following edges:

$$\forall i \in m, \ (v_{u,i}, \bar{u}_i) \in \mathcal{E}, \tag{5}$$

$$\forall i \in r, \ (\tilde{y}_i, v_{y,i}) \in \mathcal{E}. \tag{6}$$

The disjoint union operator induces a map $\Gamma_{\mathcal{R}}$ that associates to each communication node $v \in V_{\mathcal{R}}$ a set of $m$ corresponding vertices $\{v_1, \ldots, v_m\} \subset \mathcal{V}$, one for each graph $G_{\mathcal{R}}(\eta_{\mathcal{R}_i}), i \in m$. A map $\Gamma_{\mathcal{O}}$ can be defined similarly.

**Example 1:** Consider a MCN $\mathcal{M}$ where the plant is given by

$$A = [1, 2; 0, 3], \ B = C = I_2, \ G_{\mathcal{R}} = \{v_{y,1}, v_{y,2}, v_3, v_{y,c}\}, \ G_{\mathcal{O}} = \{v_{u,c}, v_{y,1}, v_{y,2}, v_{y,c}\}, \ \eta_{\mathcal{R}} = \{(v_{y,1}, v_{y,c})\}, \ \eta_{\mathcal{O}} = \{(v_{y,1}, v_{y,2}, v_{y,c})\}, \ \eta_{\mathcal{R}_2} = \{(v_{y,c}, v_{y,1}, v_{y,2})\}, \ \eta_{\mathcal{O}_2} = \{(v_{y,c}, v_{y,1}, v_{y,2}, v_{y,c})\}. \tag{7}$$

The corresponding graph $(\mathcal{V}, \mathcal{E})$ constructed as in Definition 3 is depicted in Figure 5. It is easy to see that $G_{\mathcal{R}}(\eta_{\mathcal{R}_1}) \bigcup G_{\mathcal{R}}(\eta_{\mathcal{R}_2})$ do not split in the disjoint union, since each of them only belongs respectively to $G_{\mathcal{R}}(\eta_{\mathcal{O}_1})$, $G_{\mathcal{R}}(\eta_{\mathcal{O}_2})$. 

![Graph (V,E) of Example 1](image)
where \( \{k_1, \ldots, k_p\} \subseteq m \). By (10), (4) and for the symmetric weakly connected components structure of \( R_{\lambda_i} \) and \( G_{R}(\eta_{R_i}), i \in m \), it follows that in \((V, E)\)

\[
\exists \text{ a } p\text{-linking from } \{v_{R_1, k_1}, \ldots, v_{R_p, k_p}\} \text{ to } \{v_{u, k_1}, \ldots, v_{u, k_p}\}, \quad (13)
\]

where \( \forall \vartheta \in p, v_{R_\vartheta, k_\vartheta} \in R_{(v_{R_\vartheta})} \). By (5) it follows that

\[
\exists \text{ a } p\text{-linking from } \{v_{u, k_1}, \ldots, v_{u, k_p}\} \text{ to } \hat{U}_R. \quad (14)
\]

By (11) and since (4) the structured graph representation \((V_{R_\lambda}, E_{R_\lambda})\) of the plant belongs both to \((V, E)\) and \((V_{M_{\lambda_i}}, E_{M_{\lambda_i}})\), it follows that

\[
\exists \text{ a } p\text{-linking from } \hat{U}_R \text{ to } \hat{Y}_R. \quad (15)
\]

By (6) it follows that

\[
\exists \text{ a } p\text{-linking from } \hat{Y}_R \text{ to } \{v_{y, c, i_1}, \ldots, v_{y, c, i_p}\}. \quad (16)
\]

By (12), (4) and for the symmetric weakly connected components structure of \( O_{\lambda_i} \) and \( G_O(\eta_{O_i}), i \in \ell \), it follows that

\[
\exists \text{ a } p\text{-linking from } \{v_{y, c, i_1}, \ldots, v_{y, c, i_p}\} \text{ to } \{v_{y, c, i_1}, \ldots, v_{y, c, i_p}\}. \quad (17)
\]

By (13), (14), (15), (16) and (17) it follows that

\[
\exists \text{ a } q\text{-linking from } \{v_{O_1, j_1}, \ldots, v_{O_q, j_q}\} \text{ to } \{v_{y, c, i_1}, \ldots, v_{y, c, i_p}\}, \quad (18)
\]

where \( \forall \vartheta \in q, v_{O_{\vartheta, j_{\vartheta}}} \in O_{(v_{O_{\vartheta}})} \). By (9), (18) and (19) it follows that there exists an \( p + q \)-linking in \((V, E)\) from the \( p + q \) dimensional set \( \{v_{R_1, k_1}, \ldots, v_{R_p, k_p}, v_{O_1, j_1}, \ldots, v_{O_q, j_q}\} \) to the \( p + q \) dimensional set \( \{v_{y, c, i_1}, \ldots, v_{y, c, i_p}, v_{y, c, j_1}, \ldots, v_{y, c, j_q}\} \). Since \( p + q = r \), this completes the proof.

\[\left(\Leftarrow\right)\] Consider a set of \( r \) nodes given by the union of \( \{v_{R_1, \ldots, v_{R_p}}\} \subseteq V_R \) and \( \{v_{O_1, \ldots, v_{O_q}}\} \subseteq V_O \). By (10), (4) and for the symmetric weakly connected components structure of \( R_{\lambda_i} \) and \( G_{R}(\eta_{R_i}), i \in m \), it follows that in \((V, E)\)

\[
\exists \text{ a } p\text{-linking from } \hat{U}_R \text{ to a set } \hat{Y}_R. \quad (23)
\]

By (23), (4) and for the symmetric weakly connected components structure of \( R_{\lambda_i} \) and \( G_{R}(\eta_{R_i}), i \in m \), it follows that in \((V_{M_{\lambda_i}}, E_{M_{\lambda_i}})\)

\[
\exists \text{ a } p\text{-linking from } \{v_{R_1, \ldots, v_{R_p}}\} \text{ to } \hat{U}_R. \quad (24)
\]

By (24) and (25) it follows that

\[
\exists \text{ a } p\text{-linking from } \hat{Y}_R \text{ to } \hat{Y}_R. \quad (26)
\]

Reasoning as above for (26), by (24) and (25) it follows that

\[
\exists \text{ a } p\text{-linking from } \hat{U}_R \text{ to } \hat{Y}_R, \quad (27)
\]

By (26), (27) and (28) it follows that

\[
\exists \text{ a } p\text{-linking from } \{v_{R_1, \ldots, v_{R_p}}\} \text{ to } \hat{U}_R. \quad (29)
\]

By (21), (4) and for the symmetric weakly connected components structure of \( O_{\lambda_i} \) and \( G_O(\eta_{O_i}), i \in \ell \), it follows that

\[
\exists \text{ a } q\text{-linking from } \{v_{O_1, j_1}, \ldots, v_{O_q, j_q}\} \text{ to } \hat{Y}_R. \quad (30)
\]

By (22), (29) and (30) it follows that there exists an \( p + q \)-linking in \((V_{M_{\lambda_i}}, E_{M_{\lambda_i}})\) from the \( p + q \) dimensional set \( \{v_{R_1, \ldots, v_{R_p}}, v_{O_1, \ldots, v_{O_q}}\} \) to the \( p + q \) dimensional set \( \{v_{y, c, 1}, \ldots, v_{y, c, r}\} \). Since \( p + q = r \), this completes the proof.

The following corollary provides necessary and sufficient conditions on network topology, scheduling and routing for FDI of node failures over a MIMO MCN.

**Corollary 5**: Given a MCN \( \mathcal{M} \), the associated graph \((V, E)\) constructed as in Definition 3 and a set \( \{v_1, \ldots, v_r\} \subseteq V_R \cup V_O \) of faulty nodes, then the bank of observer-based diagonal FDI problem is generically solvable for node failures in \( \mathcal{M} \) if and only if there exists an \( r\)-linking from a set \( \{v_i \in V \ (v_i) \subseteq V_R \cup V_O\} \) to \( \{v_{y, c, 1}, \ldots, v_{y, c, r}\} \) in \((V, E)\).

**Proof**: Straightforward by Lemma 3 and Theorem 4.

**Example 2**: Consider the MCN as in Example 1. It is easy to see that the conditions of Corollary 5 are satisfied for any 2-dimensional set of communication nodes, except for \( (v_2, v_3) \). As a consequence, the FDI problem cannot be solved for 2 simultaneous failures. However, by adding to any time slot of the scheduling function \( \eta_{R_i} \), the transmission of links \( (v_{u_c}, v_2) \) and \( (v_2, v_{u_1}) \), the conditions are satisfied and we can detect and isolate up to 2 simultaneous failures. An alternative solution is adding to the scheduling function \( \eta_{O_i} \), the transmission of links \( (v_{y, c, 1}) \) and \( (v_4, v_{y, c}) \). It is interesting that, in order to guarantee FDI of faulty nodes, the link scheduling order is irrelevant.

Example 2 shows that our results, for small graphs, can be used to enable FDI by manual designing the graph topology, scheduling and routing over the graph \((V, E)\). For more complex networks graph theory algorithms can...
be exploited to automate the network design process, by searching for disjoint paths for any tentative set of faulty nodes in $2|V_R|+|V_O|$, which is characterized by an exponential complexity. In the following we provide a sufficient condition that is easier to be verified.

Lemma 6: [26] Let a graph $(V, E)$ have connectivity $r$, and let $V_1, V_2$ be subsets of $V$ each of size at least $r$, then there exists an $r$-linking from $V_1$ to $V_2$ (and vice versa).

Proposition 7: Given a MCN $\mathcal{M}$ and a positive integer $r \geq \min\{m, \ell\}$, let:

$$\forall v \in V_R, |\Gamma_R(v)| \geq r,$$

(31)

$$\forall v \in V_O, |\Gamma_O(v)| \geq r,$$

(32)

$$\forall v \in V_R, |\Gamma_R(v)| \geq r,$$

(33)

Then for any $r$-dimensional set of faulty nodes the bank of observer-based diagonal FDI problem is generically solvable for $\mathcal{M}_\lambda$.  

Proof: Consider a set of $r$ nodes given by the union of $\{v_{R_1}, \ldots, v_{R_p}\} \subseteq V_R$ and $\{v_{O_1}, \ldots, v_{O_p}\} \subseteq V_O$, with $p + q = r$. Let $(V, E)$ be the graph constructed as in Definition 3 from $\mathcal{M}$, and let $F = \mathcal{F}_R \cup \mathcal{F}_O$, where $\mathcal{F}_R = \{v_{R_1}, \ldots, v_{R_p}\} \subseteq \mathcal{V}$ and $\mathcal{F}_O = \{v_{O_1}, \ldots, v_{O_p}\} \subseteq \mathcal{V}$, where $\{v_{R_1}, \ldots, v_{R_p}\} \subseteq \mathcal{V}$ and $\{v_{O_1}, \ldots, v_{O_p}\} \subseteq \mathcal{V}$. By (31) it follows that there exists a $p$-linking from $\mathcal{F}_R$ to a set $\{v_{u_1}, \ldots, v_{u_{k_p}}\}$, where $\{k_1, \ldots, k_p\} \subseteq \mathcal{O}$. By (5) and since $r \leq m$ it follows that there exists a $p$-linking from $\{v_{u_1}, \ldots, v_{u_{k_p}}\}$ to $\{\tilde{u}_1, \ldots, \tilde{u}_p\}$. By (32) it follows that there exists a $p$-linking from $\{\tilde{u}_1, \ldots, \tilde{u}_p\}$ to $\{\tilde{y}_1, \ldots, \tilde{y}_p\}$. By (6) and since $r \leq \ell$ it follows that there exists a $p$-linking from $\{\tilde{y}_1, \ldots, \tilde{y}_p\}$ to a set $\{v_{y_1}, \ldots, v_{y_p}\}$. By (33) it follows that there exists an $r$-linking from $\{v_{y_1}, \ldots, v_{y_p}\}$ to a set $\{v_{y_1}, \ldots, v_{y_p}\} \cup \mathcal{F}_O$. For all the above properties it follows that there exists an $r$-linking from $\mathcal{F}_R$ to $\{v_{y_1}, \ldots, v_{y_p}\}$ by Corollary 5, and since $p + q = r$, this completes the proof.

Conditions (31) and (33) can be verified in linear time with respect to $|V_R| + |V_O|$, but they are conservative. Indeed, it is easy to see that, for the solutions proposed in Example 2 where the FDI problem is solvable for up to 2 simultaneous failures, they are not satisfied. Condition (32) can be verified by searching for disjoint paths on a graph characterized by cardinality $n + m + \ell$, and it is easy to show that it is a necessary condition (i.e. by assuming that all $r$ node failures occur in the controllability network).

VI. REMOVING ASSUMPTION 1

If we do not impose Assumption 1 a failure on a node $v \in V_R$ affects all the input components routed via $v$ with possibly different signals $\{f_{e,i}(k)\}_{i \in \phi(v)}$, where

$$\phi(v) = \{i \in m : (\exists v' \in V_R, \exists h \in \Pi : (v, v') \in \eta_R(h))\}$$

represents the set of input components routed via $v$. In this case, since we aim at isolating node failures, we are just interested in detecting whether at least one of the signals $\{f_{e,i}(k)\}_{i \in \phi(v)}$ is active. As a consequence the conditions for solvability of the bank of observer-based diagonal FDI problem can be defined as follows.

Lemma 8: Given a MCN $\mathcal{M}$ and a set $\bar{V} = \{v_1, \ldots, v_r\} \subseteq V_R \cup V_O$ of faulty nodes, define $\bar{F} = \bigcup_{\forall v \in \bar{V}} \{f_{e,i}(k)\}$ the set of all failure signals. The bank of observer-based diagonal FDI problem is generically solvable for node failures in $\mathcal{M}$ if and only if, for any $\bar{v} \in \bar{V}$ and any $i \in \phi(\bar{v})$, there exists an $r$-linking from $\mathcal{F} \setminus \bigcup_{i \in \phi(v)} \{f_{e,i}(k)\}$ to $\bar{Y}$.

Proof: By applying the same reasoning as in the proof of Theorem 3 in [3] it can be stated that, given $\bar{v} \in \bar{V}$ and $i \in \phi(\bar{v})$, an $r$-linking exists from $\mathcal{F} \setminus \bigcup_{i \in \phi(v)} \{f_{e,i}(k)\}$ to $\bar{Y}$. This completes the proof.

Proposition 9: Given a MCN $\mathcal{M}$ and 2 faulty nodes $v_1, v_2 \in V_R$, the bank of observer-based diagonal FDI problem is generically solvable for node failures in $\mathcal{M}$ if and only if $\phi(v_1) \cap \phi(v_2) = \emptyset$.  

Proof: Assume that $\phi(v_1) \cap \phi(v_2) = \{k\} \subseteq m$, and let $\bar{v} = v_1, \bar{i} = k$. In the graph $(V_{\mathcal{M}_k}, E_{\mathcal{M}_k})$ there does not exist a 2-linking from $\{f_{e_1, k}, f_{e_2, k}\}$ to $\bar{Y}$ since they only have outgoing links to the weakly connected node $R_{\lambda, k}$ of the structured graph representation of the block $\mathcal{R}$ (see Figure 4). As a consequence, the maximal linking from $\mathcal{F} \setminus \bigcup_{i \in \phi(v_1)} \{f_{e,i}(k)\}$ to $\bar{Y}$ in $(V_{\mathcal{M}_k}, E_{\mathcal{M}_k})$ is $k = 1$. Since $r = 2$ the results follows.

The above proposition can be proven similarly for the observability graph and shows that, if Assumption 1 is not raised, the bank of observer-based diagonal FDI problem is generically solvable for node failures in $\mathcal{M}$ only for trivial network topologies, namely when all graphs $G_{\mathcal{R}}(\eta_R), i \in m$ and $G_{\mathcal{O}}(\eta_O), i \in \ell$ consist of a single communication node (namely, they are not multi-hop networks). This can be easily seen by considering that, for any 2 nodes $v_1, v_2$ belonging to a given $G_{\mathcal{R}}(\eta_R), k \in \phi(v_1) \cap \phi(v_2)$. This negative result is intuitive: if a malicious attack is able to access separately the input components and to inject unrelated signals to each of them, it is much more difficult to exploit redundancy to perform FDI. To overcome this difficulty an interesting venue for future work is providing milder conditions that enable to detect and isolate failures of clusters of nodes, instead of isolating singleton node failures. In particular, it would be interesting to compute for a given MCN the minimal node clustering that enables FDI.
REFERENCES

[1] M. Andersson, D. Henriksson, A. Cervin, and K. Arzen. Simulation of Wireless Networked Control Systems. In Proceedings of the 44th IEEE Conference on Decision and Control and European Control Conference. CDC-ECC 2005, pages 476 – 481, December 2005.

[2] K. Astöm and B. Wittenmark. Computer-Controlled Systems: Theory and Design. Prentice Hall, 1997.

[3] C. Commault, J.-M. Dion, O. Sename, and R. Motieyian. Observer-based fault detection and isolation for structured systems. IEEE Transactions on Automatic Control, 47(12):2074–2079, December 2002.

[4] C. Commault and J.-M. Dion. Sensor Location for Diagnosis in Linear Systems: A Structural Analysis. IEEE Transactions on Automatic Control, 52(2):155–169, February 2007.

[5] A. D’Innocenzo, M.D. Di Benedetto, and E. Serra. Fault tolerant control of multi-hop control networks. IEEE Transactions on Automatic Control, 58(6):1377–1389, June 2013.

[6] A. D’Innocenzo, G. Weiss, R. Alur, A.J. Isaksson, K.H. Johansson, and G.J. Pappas. Scalable Scheduling Algorithms for Wireless Networked Control Systems. In Proceedings of the 5th IEEE Conference on Automation Science and Engineering (CASE), pages 409–414, 2009.

[7] R.A. Gupta and M.-Y. Chow. Networked Control System: Overview and Research Trends. IEEE Transactions on Industrial Electronics, 57(7):2527 –2535, July 2010.

[8] V. Gupta, A.F. Dana, J.P. Hespanha, R.M. Murray, and B. Hassibi. Data transmission over networks for estimation and control. IEEE Transactions on Automatic Control, 54(8):1807–1819, 2009.

[9] J.-M. Dion, C. Commault, and J. van der Woude. Generic Properties and Control of Linear Structured Systems: a Survey. Automatica, 39(7):1125 –1144, July 2003.

[10] J.P. Hespanha, P. Naghshtabrizi, and Y. Xu. A Survey of Recent Results in Networked Control Systems. Proceedings of the IEEE, 95(1):138–162, January 2007.

[11] K.-E. Årzen, A. Bicchi, S. Hailes, K. H. Johansson, and J. Lygeros. On the design and control of wireless network embedded systems. In Proceedings of the 2006 IEEE Conference on Computer Aided Control Systems Design, Munich, Germany, pages 440–445, October 2006.

[12] M.-A. Massoumnia, G.C. Verghese, and A.S. Willsky. Failure Detection and Identification. IEEE Transactions on Automatic Control, 34(3):316 –321, March 1989.

[13] M. Pajic, S. Sundaram, G.J. Pappas, and R. Mangharam. Topological conditions for wireless control networks. In Proceedings of the 50th IEEE Conference on Decision and Control and European Control Conference, Orlando, FL, pages 2353–2360, December 2011.

[14] M.C.F. Donkers, W.P.M.H. Heemels, N. van de Wouw, and L.Hetel. Stability Analysis of Networked Control Systems: A Structural Analysis. Automatica, 56(7):1495–1508, July 2011.

[15] M. Tabbara, D. Nešić, and A.R. Teel. Stability of Wireless and Wireline Networked Control Systems. IEEE Transactions on Automatic Control, 52(7):1615–1630, September 2007.

[16] W. Zhang, M.S. Branicky, and S.M. Phillips. Stability of Networked Control Systems. IEEE Control Systems Magazine, 21(1):84–99, February 2001.

[17] D. B. West. Introduction to Graph Theory. Prentice-Hall Inc., Upper Saddle River, New Jersey, 2001.