Electric field correlation measurements on the electromagnetic vacuum state

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Quantum mechanics ascribes to the ground state of the electromagnetic radi-
diation\textsuperscript{1} zero-point electric field fluctuations that permeate empty space at all
frequencies. No energy can be extracted from the ground state of a system
and, therefore, these fluctuations cannot be measured directly with an inten-
sity detector. The experimental proof of their existence came thus from more
indirect evidence, such as the Lamb shift,\textsuperscript{2,3} the Casimir force between close
conductors\textsuperscript{4,5} or spontaneous emission.\textsuperscript{6} A direct method to determine the
spectral characteristics of vacuum field fluctuations has been missing so far. In
this work, we perform a direct measurement of the field correlation on these
fluctuations in the terahertz frequency range using electro-optic detection\textsuperscript{9} in a
non-linear crystal placed in a cryogenic environment. We investigate their tem-
poral and spatial coherence, which, at zero time delay and spatial distance, has
a peak value of $6.2 \times 10^{-2} \text{ V}^2/\text{m}^2$, corresponding to a fluctuating vacuum field\textsuperscript{10,11}
of $0.25 \text{ V/m}$. With this measurement, we determine the spectral composition of
the ground state of electromagnetic radiation which lies within the bandwidth
of electro-optic detection.

The spectral properties of the ground state of a quantum system intimately determine
its behavior. An optical cavity, for example, shapes the spectral density of states of vacuum
fluctuations and spontaneous emission is enhanced at its resonance frequency.\textsuperscript{12} Moreover,
systems in which polaritons are created by the ultra-strong coupling of matter excitations to
light\textsuperscript{13–15} are predicted to have a ground state which contains virtual photons.\textsuperscript{16,17} A method
to measure the spectral properties of the electromagnetic ground state in-situ would provide
a direct experimental test of these properties predicted theoretically.

The electric field correlator $G^{(1)}(\tau) = \langle E^*(t)E(t+\tau) \rangle$ defines the coherence properties
of light and yields its power spectrum after a Fourier transformation. Typically, $G^{(1)}(\tau)$ is
retrieved by measuring the intensity of the electric field interfering with a delayed version
of itself. The real part of the field correlation function $\Re(G^{(1)}(\tau))$ is readily retrieved by
varying the time delay $\tau$ between the two arms of an interferometer.

Whereas the measured response of an interferometer on the vacuum state is zero due to energy conservation, a different situation arises if, instead, electro-optic detection is used to measure directly its electric field at two space-time points. In electro-optic detection, an ultrashort near-infrared (NIR) pulse enables the measurement of the instantaneous electric field of multimode free-running terahertz (THz) electro-magnetic waves by probing the local field-induced birefringence of a crystal with $\chi^{(2)}$ second order non-linear susceptibility. Using this technique, we have recently measured the first and second order correlation of classical fields with sub-cycle temporal resolution. Quantum mechanically, electro-optic detection corresponds to the measurement of an operator that depends linearly on the multi-mode vacuum field:

$$\hat{S}_{eo}(t) = \sqrt{C} \sum_{\Omega} \sqrt{\frac{\hbar \Omega}{2 \epsilon_0 \epsilon_r V}} (\hat{a}(\Omega)R(\Omega)e^{-i\Omega t} - h.c.)$$  \hspace{1cm} (1)

$$\sqrt{C} = -ir_{41}n^3L_{cryst}\omega I_p/c$$ where $c$ is the light velocity, $\omega = 2 \pi \times 375$ THz the frequency of the probe of intensity $I_p$, $L_{cryst}$ the crystal length, $n$ the refractive index, $r_{41}$ the electro-optic coefficient. $E_{vac} = \sqrt{\frac{\hbar \Omega}{2 \epsilon_0 \epsilon_r V}}$ is the vacuum field amplitude of a THz mode at frequency $\Omega$ confined to the volume $V$ of dielectric constant $\epsilon_r$. The sum represents the superposition of all frequency components contained in the detection bandwidth $R(\Omega)$ that describes the frequency dependent efficiency of electro-optic detection due to phase matching.

As a result, the electro-optic field correlation operator is defined as the anti-commutator of the electro-optic operators at $t$ and $t+\tau$, $\hat{G}_{eo}^{(1)}(\tau) = -\frac{1}{2C}\{\hat{S}_{eo}(t+\tau), \hat{S}_{eo}(t)\}$. Its expectation value for a thermal state with a mean photon occupation number per mode $\langle \hat{n}(\Omega) \rangle$ is the electro-optic field correlation function $G_{eo}^{(1)}(\tau)$

$$G_{eo}^{(1)}(\tau) = \Re F^{-1}\{\sum_{\Omega} \frac{\hbar \Omega}{2 \epsilon_0 \epsilon_r V} (1 + 2\langle \hat{n}(\Omega) \rangle) |R(\Omega)|^2\}.$$  \hspace{1cm} (2)

$\Re$ represents the real part of the inverse Fourier transformation $F^{-1}$. For a vanishing THz photon population, the input state is the vacuum state $|0\rangle$, described by $\langle \hat{n}(\Omega) \rangle = 0$. The
electro-optic correlation function yields in this case \( G_{eo}^{(1)}(\tau) = \Re \mathcal{F}^{-1} \{ \sum_{\Omega} \frac{\Omega}{2\pi\epsilon_0\epsilon_V} |R(\Omega)|^2 \} \), precisely the power spectrum of the vacuum field within the bandwidth of detection \( R(\Omega) \).

In this work, we show experimentally that the electro-optic field correlation measurement on a vacuum state is non-zero and analyse its dependency on \( \tau \) to determine its power spectrum. We implemented the two-point correlation measurement using a pair of mode-matched 80 fs pulses of waist \( w_0 = 125 \, \mu m \) that sample the multi-mode THz vacuum field in two space-time volumes, characterised by a temporal delay \( \tau \) and a lateral spatial separation \( \delta x \) as shown in figure 1 A and in the zoom 1 B. By changing these two parameters, we investigate the spatial and temporal dependence of the field correlation function. The chosen probe polarisation maximises the electro-optic effect but prohibits all unwanted coherent \( \chi^{(2)} \) effects (supplementary information). The crystal is inserted in a cryostat that enables a control of its thermal environment with temperatures between 4 K and 300 K. \( \delta x \) is changed by steering mirrors outside the cryostat and \( \tau \) by a mechanical delay stage. Two polarisation measurements are performed separately on the two probe pulses at the repetition rate of the laser.

To describe the infinity of electro-magnetic modes of arbitrary k-vector and frequency \( \Omega \) present in the detection crystal, we use a modal decomposition in an arbitrary quantizing box, taken here as the non-linear crystal itself (supplementary information). The contribution of each THz mode to the total electro-optic correlation function \( G_{sim}^{(1)}(\tau) \) depends on its photon population, which is strongly temperature dependent as shown in figure 1 C, and on its phase matching with the probe pulse, described by the coherence length shown in figure 1 D. In the temperature range around 4 K, only the multi-mode vacuum field is detected.

The measured electro-optic field correlation \( G_{eo}^{(1)}(\tau) \) is shown together with the associated power spectrum in figure 2 for two distinct temperatures, 300 K and 4 K. They are compared to the predicted results \( G_{sim}^{(1)}(\tau) \).

In figure 2 A and C we show results obtained when the system is at \( T = 300 \) K. In this condition, the black-body radiation from the environment dominates the vacuum field,
Figure 1: A. Schematic description of the set-up used to measure the field correlation of vacuum fluctuations and thermal fields using electro-optic sampling. The detection crystal (ZnTe, 110-cut) is placed inside a closed-cycle cryostat and is thermally anchored to the 4 K plate. It is shielded from the radiation of the environment by a shield cooled to 4 K and the built-in one at 45 K. The probe pulse pairs originate from a Mai Tai laser with 80 fs pulse duration and 80 MHz repetition rate. A mechanical delay stage imposes a delay of duration $\tau$ on one of the probe pulses by means of an optical path. B. The vacuum field fluctuations couple from the environment into the detection crystal, where their electric field $\hat{E}(t)$ and $\hat{E}(t + \tau)$ is measured with a pair of probe pulses displaced by a distance $\delta x$. C. Mean photon occupation number per mode for black-body radiation at different temperatures in the frequency range of detection, shown in units of coherence length in D. QWP = quarter wave plate, WP = Wollaston prism, BD = balanced detector, BSS = beam stabilisation system, ZnTe = zinc telluride.
Figure 2: A.-B. Electro-optic field correlation measurements $G_{eo}^{(1)}(\tau)$ are compared to the predicted correlation $G_{sim}^{(1)}(\tau)$ for two temperatures, 300 K and 4 K, respectively. Faded lines denote raw measurements and thick lines the curves filtered by a low-pass Fourier filter of maximal frequency of 3 THz, corresponding to an upper bound for the detection frequency of electro-optic detection in a 3 mm thick crystal. C.-D. The power spectra of the detected fields are retrieved by computing the real part of the Fourier transform of the raw (unfiltered) $G_{eo}^{(1)}(\tau)$. E. Spatial coherence of the probed vacuum and thermal fields. The timetraces from which these values were extracted are shown in the supplementary material. F. Detection of field correlation employing spontaneous parametric down conversion: $E_{THz}^{(2)}$ represents the back-action and $E_{NIR}^{(2)}$ the measurement.
with a photon occupation number of for example \( \langle \hat{n}(\Omega = 1 \, THz) \rangle = 5 \). The field coherence is preserved for a duration of 250 fs. The real part of the power spectrum reveals a large contribution of low frequency components, which exhibit a large photon population. The contribution of high frequency components is reduced by THz absorption in the detection crystal. The peak-peak signal is \( G_{eo,pp}^{(1)} = 0.98 V^2/m^2 \), and the standard deviation of the signals is \( \sigma = 0.134 V^2/m^2 \).

In figure 2 B and D, we show results when the system is at 4 K base temperature (optional 4 K aperture in use). In this condition, the trace of the field correlation changes dramatically and the thermal contribution is suppressed. The power spectrum of vacuum fluctuations contains frequency components in the frequency band around 0.75 THz and around 2 THz. This is well reproduced by our model prediction, and matches very well the coherence properties in figure 1 D. The peak-peak signal is \( G_{eo,pp}^{(1)} = 0.084 V^2/m^2 \), and the standard deviation of the signals is \( \sigma = 0.018 V^2/m^2 \). To achieve this sensitivity, the integration time has been increased as discussed in the supplementary material. We demonstrate the pure electro-magnetic origin of our measurements by removing the optional aperture and thereby changing solely the properties of the detected light by unblocking thermal radiation from the 45 K plate. The field correlation function changes dramatically and the results are shown in the supplementary information.

Finally, we investigate the spatial field correlation of the probed vacuum and thermal fields by displacing the two probe beams in the crystal. It is directly related to the numerical aperture of efficient detection and the frequency components in the detection bandwidth. In figure 2 E, we report the peak-peak magnitude of \( G_{eo}^{(1)}(\tau) \) and \( G_{sim}^{(1)}(\tau) \) as a function of the probe spacing \( \delta x \) and find that the spatial coherence is maintained over multiple wavelengths of the probed radiation. This result suggests that the detection is limited to paraxial waves. We attribute the difference between our model and the experimental results to an uncertainty in the phase matching properties of the two interacting waves.

We now provide a quantum mechanical interpretation of the field correlations measure-
ments. We first note that, unlike a coherent state, the vacuum state has no well-defined amplitude and phase. As such, two putative electric field measurements that would have no back-action on the state would be uncorrelated, seemingly in contradiction with our results. However, the electric field measurement cannot be performed on the vacuum state without measurement-induced back-action, as it is not an eigenstate. Conceptually, this is equivalent to a position measurement on a mechanical oscillator cooled to its ground state. By the employed $\chi^{(2)}$ nonlinearity, both the measurement and its back-action can be described by the emission of entangled photon pairs by spontaneous parametric down conversion. As shown schematically in figure 2F and in more details in the supplementary material, the rotation of the polarization experienced by the near-infrared pulse originally polarized along the z direction can be interpreted as arising from the generation of a new field $E_{NIR}^{(2)}$ in the x direction mediated by the two vacuum fields, $E_{THz}^{vac}$ and $E_{NIR}^{vac}$. Also, a new THz field $E_{THz}^{(2)}$ is generated along the x axis and represents the measurement induced back-action. The superposition of this idler field, containing in our case an average photon number much less than unity, to the vacuum state creates the measurable correlation shown in figure 2B.

In our measurements, we employ linear (field) detectors, which, in contrast to those used in superconducting circuits measure the electric field of a broadband wave with sub-cycle temporal resolution instead of the two quadratures of a narrowband one. We achieve a noise equivalent field squared of $1.8 \cdot 10^{-2} \, V^2/m^2$, which suffices to detect vacuum field fluctuations as well as fractions of thermal photons. As compared to the results of Riek et al., the technique presented here enables the access to quantum signals much below the single shot noise because it is background-free and, in addition, provides spectral information. These characteristics may be essential to detect the emission of pure quantum light from nonadiabatic modulation of a quantum system in the ultra-strong coupling regime. While the present measurements, performed in an inorganic zinc telluride crystal, required long integration times to extract the signal, much larger signal over noise could be achieved using organics-based THz detectors with extremely large electro-optic coefficients. By embedding
the non-linear material in a resonator, the presence of a superradiant phase transition in a coupled light-matter system as well as the influence of the vacuum field on the charge transport could be investigated. In addition, by changing the detection of the near-infrared pulse from an ellipsometric to a projective polarization measurement, electro-optic sampling may provide a path for the generation of heralded single photons in the terahertz frequency range.

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Methods

Measurement settings

Full details on the complex measurement conditions (electro-optic detection scheme, noise analysis, fast acquisition) are given in the supplementary information. Also, we provide there more detailed measurements that sustain the claims of the main paper.

Analytical model for electro-optic correlation measurements.

An analytical model was developed to predict the experimental results starting from energy conservation and electro-optic detection at an arbitrary angle. The full derivation is given in the supplementary information.