A generalized model for rate-independent ferromagnetic hysteresis phenomena

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Abstract. In this paper a new approach is presented, how to calculate the inductance and the core losses of chokes considering DC premagnetization. The formulation is based on the well known Hodgdon model. Generalized material functions are applied that can be easily adapted to measured data. To obtain these data, an experimental set-up was developed according to the standards. It is shown that the measurement of symmetrical $B$-$H$ loops is sufficient to calibrate the model. By means of the adapted dynamical model also unsymmetrical hysteresis loops can be simulated with high accuracy in steady state. The influence of DC premagnetization on the inductance and the core losses is investigated under practical conditions using commercial ferrite ring core specimens. The results are verified by measurements and illustrated in various figures and tables.

1. Introduction
Considering climate protection requirements and rapidly rising energy prices, energy saving technology has become essential to realize the ecological and economical objectives. Improving the power efficiency is now a permanent challenge of research and development in power electronics. Here, the focus is put especially on small to medium power electronic systems (up to some kW), due to their widespread applications and high cost pressure. A typical commercial 300W power supply is exemplified in figure 1.

![Image of a 300W PC power supply](image1.jpg)

**Figure 1.** Commercial 300W PC power supply (total losses ~60W, losses caused by inductive components ~20W).
The total efficiency of such a component is usually ~ 80%, thus leading to a total power loss of ~60W. About one third (~20W) of the total power loss is generated by the inductive components. The example depicts that chokes and transformers are one of the main contributors to losses in switch mode power supplies (SMPS). This problem will become much more critical with increasing switching frequencies due to new generation power switches (e.g. silicon carbide technology [1]). Consequently, further optimization has to be based on the passives and particularly on the inductive components.

The aim of the development process is to find an optimum component design related to the desired inductance value(s). Therefore simulations of the assembly of core and coil have to be as accurate as possible. Due to the strong nonlinear behavior of the ferrite material usually preferred in that power range, the calculation of the inductance(s) and the core losses can lead to considerable problems, particularly under DC premagnetization. Conventional approximate formulas based on previously measured data or information supplied by the manufacturer are often inadequate for practical applications. To close this gap, the new approach of this paper is to calculate the inductance(s) and the core losses by means of a time domain simulation of the surrounded loop(s) in the B-H plane. Therefore the material behavior has to be known in detail, including its dependencies on the influencing quantities. Of course, the formulation of the hysteresis phenomena has to be in agreement with physics.

A measurement test set-up was assembled to obtain the power loss and to derive the material properties. Figure 2 shows the equivalent circuit according to the standards IEC 60404-6 (2003).

![Measurement test set-up diagram](image)

**Figure 2.** Measurement test set-up according to the standards IEC 60404-6 (2003).

Three independent windings are realized on the ring core specimens. The primary winding conducts the magnetizing high frequency (HF) current \( i_p(t) \) that is related to the \( H \)-field by Oersted’s law. The time derivation of the magnetic flux density can be determined via the secondary voltage \( u_s(t) \) evaluating Faraday’s law of electromagnetic induction. A third winding conducts the DC current \( I_{DC} \) that is required for a certain DC premagnetization. Of course, \( I_{DC} \) could be also applied to the primary winding, but in the configuration presented here, a current probe is used to measure the primary
current. The measurement accuracy of these components is very sensitive to DC bias and the sensor has to be prevented from running into saturation. If the current probe is replaced by a (low inductive) shunt resistor, a third winding is optional. During the measurement process, the data of at least one period of \( i_p(t) \) and \( u_s(t) \) is being sampled at the two channels of a digital oscilloscope, transmitted to a PC and post processed using Matlab\(^\text{TM} \) language. Finally, the desired values of the mean power loss per volume \( \overline{p} \) and the mean stored magnetic energy per volume \( \overline{w} \) are calculated via numerical integration following the formulas denoted in figure 2. In a previous work \[2\], the measurement accuracy of this experimental set-up has been determined by a detailed error analysis. It fulfills the requirements up to the MHz-range.

2. Transient hysteresis modeling based on Hodgdon’s constitutive relations

2.1. Model formulation

Sophisticated dynamical models are required for a realistic simulation of the nonlinear behavior of inductive components. Arbitrary transitions from one state \((B_1, H_1)\) in the \(B-H\) plane to another state \((B_{i+1}, H_{i+1})\) have to be described. Therefore a replication of surrounded symmetrical hysteresis loops as given e.g. in \[3\], \[4\] and \[5\] is not sufficient. Adequate models have been published by the authors L. O. Chua, M. L. Hodgdon, D. C. Jiles & D. L. Atherton and F. Preisach, their advantages and disadvantages have been investigated in \[6\], \[7\], \[8\] and \[9\]. The approach of this paper is based on the previous work of B. D. Coleman and M. L. Hodgdon published in \[10\], \[11\], \[12\], \[13\] and \[14\].

According to the authors, the time derivation of the magnetic flux density can be expressed by the following differential equation:

\[
\frac{\partial B}{\partial t} = \alpha \frac{\partial H}{\partial t} [f(H) - B] + \frac{\partial H}{\partial t} g(H) . \tag{1}
\]

In (1), the material functions \( f(H) \) and \( g(H) \) as well as the positive constant \( \alpha \) remain as the degrees of freedom and are used for model calibration. If static (rate independent) hysteresis is assumed, the time derivations can be reduced, thus leading to an expression for the differential permeability:

\[
\frac{\partial B}{\partial H} = \alpha s \left( \frac{\partial H}{\partial t} \right) [f(H) - B] + g(H) . \tag{2}
\]

Equation (2) contains the signum-function defined by

\[
s(\eta) = \begin{cases} 1 & \eta > 0 \\ 0 & \eta = 0 \\ -1 & \eta < 0 \end{cases} \tag{3}
\]

An alternate formulation of (2) is obtained by the interchange of \( B \) and \( H \) in (2):

\[
\frac{\partial H}{\partial B} = \alpha \bar{s} \left( \frac{\partial B}{\partial t} \right) [\bar{f}(B) - H] + \bar{g}(B) . \tag{4}
\]

The following symmetrical conditions have to be kept by the material functions in (2) and (4):

\[
f(H) = -f(-H), \quad f(0) = 0 \quad \text{and} \quad g(H) = g(-H) \quad \text{resp.}
\]

\[
\bar{f}(B) = -\bar{f}(-B), \quad \bar{f}(0) = 0 \quad \text{and} \quad \bar{g}(B) = \bar{g}(-B) . \tag{5}
\]

To give a better understanding of the physical meaning of the equations described above, a small signal analysis of (2) is shown below in detail.
2.2. Small signal analysis

To begin with, the material functions \( f(H) \) and \( g(H) \) are replaced by its first order Taylor approximations close to the origin \( H = 0 \):

\[
f(H) = f_1 H \quad \text{and} \quad g(H) = g_0. \quad (6, 7)
\]

Due to formality reasons, the following abbreviations are introduced to denote the different branches of the hysteresis loop:

\[
B(H) = \begin{cases} B_1(H) & \text{if} \ H > 0, \\ B_2(H) & \text{if} \ H < 0, \end{cases}
\]

Following the notation (8), two different partial differential equations are obtained from (2):

\[
\frac{\partial B_{1,4}}{\partial H} + \alpha B_{1,4} = \alpha f_1 H + g_0 \quad \text{and} \quad \frac{\partial B_{2,3}}{\partial H} - \alpha B_{2,3} = -\alpha f_1 H + g_0. \quad (9, 10)
\]

The equations (9) and (10) can be solved analytically. Superpositions of the homogeneous and particular solutions lead to

\[
B_i(H) = c_i e^{-\alpha t} + f_1 H - \frac{f_1}{\alpha} + \frac{g_0}{\alpha},
\]

\[
B_1(H) = c_1 e^{\alpha t} + f_1 H + \frac{f_1}{\alpha} - \frac{g_0}{\alpha}, \quad \text{and}
\]

\[
B_2(H) = c_2 e^{\alpha t} + f_1 H + \frac{f_1}{\alpha} + \frac{g_0}{\alpha}.
\]

with the unknown coefficients \( c_1, c_2, c_3 \) and \( c_4 \). If the magnetic field strength \( H \) oscillates between \( -\hat{H} \) and \( +\hat{H} \) for a long time, the solutions have to fulfill the steady state conditions

\[
-B_1(0) = B_2(0) = B_3(0) = B_4(0) := B_r.
\]

depending on the remanent flux density \( B_r \). With the coefficients specified by

\[
c_1 = -c_2 = -c_3 = c_4 = -B_r + \frac{f_1}{\alpha} - \frac{g_0}{\alpha},
\]

and the remanent flux density

\[
\frac{B_r}{B_r} = \left(1 - \frac{2e^{-\alpha t} - 2e^{-\alpha t}}{1 - e^{-\alpha t}}\right)\left(\frac{f_1}{\alpha} - \frac{g_0}{\alpha}\right),
\]

(11) now can be described by an entire formula:

\[
B(H) = s\left(\frac{\partial H}{\partial t}\right)\left[-B_r + \frac{f_1}{\alpha} - \frac{g_0}{\alpha}\right]e^{\alpha\left(\frac{\partial t}{\alpha}\right)} + \frac{f_1}{\alpha} + \frac{g_0}{\alpha} + f_1 H. \quad (11)
\]
With aid of the exponential approximation for small arguments $\eta$

$$e^\eta = 1 + \eta + \frac{1}{2} \eta^2,$$ \hfill (16)

(15) can be further reduced:

$$B(H) = s \left( \frac{\partial H}{\partial t} \right) \left[ \frac{\alpha^2}{2} \left( -B_r + \frac{f_l}{\alpha} - \frac{g_0}{\alpha} \right) H^2 - B_r \right] + (\alpha B_r + g_o) H.$$ \hfill (17)

The hysteresis loop for small signal excitations observed by Lord Rayleigh [15] has the form

$$B_{res}(H) = \mu_o \left[ s \left( \frac{\partial H}{\partial t} \right) \nu \left( H^2 - \hat{H}^2 \right) + \left( \mu_i + \nu \hat{H} \right) \right]$$ \hfill (18)

with the hysteresis coefficient $\nu$, the initial permeability $\mu_i$ and the amplitude permeability $\mu_a$ according to

$$\mu_i = \frac{1}{\mu_0} \frac{\partial B}{\partial H} \bigg|_{H \to 0}$$ \hfill (19, 20)

and

$$\mu_a = \mu_i + \nu \hat{H}.$$ \hfill (21, 22, 23)

By means of a comparison of (17) and (18), these material parameters can be calculated in turn:

$$\nu = \frac{2\alpha}{\mu_0} \frac{f_l - g_0}{2 + \alpha^2 \hat{H}^2}, \quad \mu_a = \frac{\alpha \nu \hat{H}^2}{2} + \frac{g_0}{\mu_0}$$ \hfill (24)

and

$$\mu_i = \mu_a - \nu \hat{H}.$$ \hfill (25)

By (21), (22), (23) and (24), it is demonstrated that the empirical relation (2) shows the expected physical behavior (18) for symmetrical small signal excitations. The surrounded loops can be directly associated with common material parameters.

### 2.3. Large signal analysis

Now (2) is solved also for large signal excitations up to the saturation level. Considering (5), the material functions can be expressed by the Fourier series

$$f(H) = \sum_{n=1}^{\infty} F_n \sin(p_n H) \text{ with } p_n = \frac{n\pi}{H_{max}}$$ \hfill (26)

and

$$g(H) = G_0 + \sum_{n=1}^{\infty} G_n \cos(p_n H).$$ \hfill (27)

Therein $H_{max}$ denotes an arbitrary positive value that should never be reached during the simulation. Analogously (9) and (10), two different partial differential equations are obtained from (2):

$$\frac{\partial B_{1,4}}{\partial H} + \alpha B_{1,4} = G_0 + \sum_{n=1}^{\infty} \left[ \alpha F_n \sin(p_n H) + G_n \cos(p_n H) \right]$$ \hfill (28)

and

$$\frac{\partial B_{2,3}}{\partial H} - \alpha B_{2,3} = G_0 + \sum_{n=1}^{\infty} \left[ -\alpha F_n \sin(p_n H) + G_n \cos(p_n H) \right].$$ \hfill (29)

As an advantage of the harmonic approaches (24) and (25), an analytical solution can be found for (26) and (27) as well.
If the homogeneous and particular parts are superimposed, the solutions for all of the different branches of the hysteresis loop are given by

\begin{align*}
B_1(H) &= c_1 e^{-\alpha t} + B_0 + \sum_{n=1}^{\infty} \left[ A_n \sin(p_n H) + B_n \cos(p_n H) \right], \\
B_2(H) &= c_2 e^{\alpha t} - B_0 + \sum_{n=1}^{\infty} \left[ A_n \sin(p_n H) - B_n \cos(p_n H) \right], \\
B_3(H) &= c_3 e^{\alpha t} - B_0 + \sum_{n=1}^{\infty} \left[ A_n \sin(p_n H) - B_n \cos(p_n H) \right], \\
B_4(H) &= c_4 e^{-\alpha t} + B_0 + \sum_{n=1}^{\infty} \left[ A_n \sin(p_n H) + B_n \cos(p_n H) \right].
\end{align*}

(28)

The set of equations (28) encloses the unknown coefficients $c_1$, $c_2$, $c_3$ and $c_4$ and the abbreviations

\begin{align*}
A_n &= \frac{\alpha^2 F_n + p_n G_n}{p_n^2 + \alpha^2}, & B_0 &= \frac{G_0}{\alpha}, & B_n &= \frac{-\alpha p_n F_n + \alpha G_n}{p_n^2 + \alpha^2}.
\end{align*}

(29, 30, 31)

If a symmetrical excitation with the amplitude $\hat{H}$ of the magnetic field strength is assumed, the conditions

\begin{align*}
-B_1(0) &= B_2(0) = B_3(0) = -B_4(0) =: B_r
\end{align*}

(32)

are fulfilled in steady state by the relations

\begin{align*}
c_1 = -c_2 = -c_3 = c_4 = -B_r - B_0 - \sum_{n=1}^{\infty} B_n.
\end{align*}

(33)

With $\hat{H}$ as a parameter, the remanent flux density is given by

\begin{align*}
B_r &= -B_0 - \sum_{n=1}^{\infty} B_n + \frac{2 e^{\alpha t} - 2 e^{-\alpha t}}{1 - e^{-2\alpha t}} \left[ B_0 + \sum_{n=1}^{\infty} B_n \cos(p_n \hat{H}) \right].
\end{align*}

(34)

Thus the different solutions (28) for each of the branches (8) can be condensed into one entire expression:

\begin{align*}
B_{\text{Hody}}(H) &= \sum \left[ \frac{\partial H}{\partial t} \left( -B_r - B_0 - \sum_{n=1}^{\infty} B_n \right) e^{-\alpha t} \left( \frac{\partial H}{\partial t} \right)^H + B_0 + \sum_{n=1}^{\infty} B_n \cos(p_n H) \right] + \sum_{n=1}^{\infty} A_n \sin(p_n H).
\end{align*}

(35)

The final equation (35) offers the opportunity to determine the material functions (24) and (25) directly by means of numerical curve fitting (e.g. by least squares method) to a vector of measured symmetrical subloops of different excitation levels. These data can be easily obtained from the measurement test set-up depicted in figure 2. In contrast to [12], [13] and [14], where distinctive analytical functions are suggested for $f(H)$ and $g(H)$, the presented model is not restricted. Of course, the stability and closure criteria given in [13] also have to be fulfilled by the generalized material functions (24) and (25). This requires a numerical check after model calibration up to the saturation value of the magnetic field strength $H_s$ that defines the saturation loop. Finally, all simulation has to be limited by $|H| \leq H_s$ to guarantee stability and closed loops in steady state.
2.4. Adaptation of the model to measured data

Figure 3 shows a set of symmetrical subloops of different excitation levels. The data is obtained from the measurement test set-up shown in figure 2 using commercial R16 ring core specimens. For \( f = 10\text{kHz} \), rate-independent hysteresis can still be assumed. To be able to investigate the influence of the temperature, it is possible to measure the samples inside a heating cabinet. According to the manufacturer’s data sheet, the investigated MMG\textsuperscript{TM} F49 material is adequate for power applications. These materials usually show lower core losses at high temperatures. As expected, thinner loops are obtained at \( T_e = 100^\circ\text{C} \) (lower picture) in comparison to room temperature \( T_e = 25^\circ\text{C} \) (upper picture).

![Figure 3. Measured symmetrical B-H loops used for model calibration (MMG\textsuperscript{TM} F49 ferrite [16]).](image)
To adapt the model to the measured subloops depicted in figure 3, an optimum value for the positive constant $\alpha$ in (2) has to be found. Therefore the differential permeability is determined in the remanent point $(B = B_r, H = 0)$ of the saturation loop. We obtain

$$\left. \frac{\partial B}{\partial H} \right|_{(B_r, H=0)} = \alpha B_r + g(0) = \alpha B_r + \mu_i \mu_i \rightarrow \alpha = B_r \left( \frac{\partial B}{\partial H} \right)_{(B_r, H=0)} - \mu_i \mu_i. \quad (36)$$

Now the unknown coefficients $F_n$, $G_0$, and $G_n$ of the generalized material functions (24) and (25) can be determined by means of numerical curve fitting of (35). To reduce the number of necessary iterations, it is recommended to use adequate starting parameters. These are directly given by the small signal analysis described above. A Fourier expansion of (6) and (7) leads to

$$f(H) = f_r H = \sum_{n=1}^{m} F_n \sin(p_n H) \quad \text{with} \quad p_n = \frac{n\pi}{H_{max}} \quad \text{and} \quad (37)$$

$$g(H) = g_0 = G_0 + \sum_{n=1}^{m} G_n \cos(p_n H) \quad \text{with} \quad (38)$$

$$F_n = \frac{2f_r}{p_n}(-1)^{n+1}, \quad G_0 = g_0 \quad \text{and} \quad G_n = 0. \quad (39, 40, 41)$$

Consequently the starting parameters (39), (40) and (41) only depend on $f_r$ and $g_0$ that can be derived from the data sheet values of $\nu$, $\mu_i$ and $\mu$ by solving the nonlinear system of equations (21), (22) and (23). Finally, the starting parameters have to be varied until the least squares condition

$$\left| B_{\text{Meas}}^2(H_{\text{Meas}}) - B_{\text{Hyst}}^2(H_{\text{Meas}}) \right| = \min! \quad (42)$$

is fulfilled. In praxis, the calculation can easily be done with aid of the Matlab™ function ‘lsqcurvefit.m’. Adequate values for $H_{\text{max}}$ in (38) are

$$H_s \leq H_{\text{max}} \leq 10H_s. \quad (43)$$

A comparison of the measured and the simulated results (dotted lines) in figure 3 shows the good agreement between the measured data and the calibrated model. Due to the simultaneous fitting of all subloops, the mean errors are equally distributed over the single loops, no runaways can be detected. For the example evaluation, the Fourier series (24) and (25) is truncated after 10 addends. A time domain resolution of 512 sampling points per period is sufficient for an adequate representation of the subloops. Consequently, the amount of calculation time for the fitting process is only some seconds on a standard office PC.

3. Results

3.1. Simulations based on the adapted model

During the investigation it was found that the alternate representation (4) is more adequate for simulations up to the saturation level. Therefore the alternate positive constant $\bar{\alpha}$ has to be determined instead of $\alpha$. It is given by the inverse differential permeability in the coercive point $(B = 0, H = H_{cb})$ of the saturation loop:

$$\left. \frac{\partial H}{\partial B} \right|_{(B=0, H=H_{cb})} = -\bar{\alpha} H_{cb} + \bar{g}(0) = -\bar{\alpha} H_{cb} + \left( \mu_i \mu_i \right)^{-1} \rightarrow \bar{\alpha} = H_{cb}^{-1} \left( \mu_i \mu_i \right)^{-1} - \left( \mu_i \mu_i \right)^{-1} \left. \frac{\partial H}{\partial B} \right|_{(B=0, H=H_{cb})} \quad (44)$$
By the interchange of $B$ and $H$ in (24) and (25), the Eigenvalues of the Fourier expansions have to be replaced by

$$\tilde{\lambda}_n = \frac{n\pi}{B_{\text{max}}}.$$  \hspace{1cm} (45)

Finally, the derivations of the alternate model (4) are substituted by finite differences

$$\left. \frac{\partial H}{\partial B} \right|_{(B-H,H-H_0)} = \frac{H_{i+1} - H_i}{B_{i+1} - B_i} = \tilde{\alpha} s \left( \frac{B_{i+1} - B_i}{t_{i+1} - t_i} \right) \left[ \tilde{f}(B_i) - H_i \right] + \tilde{g}(B_i).$$  \hspace{1cm} (46)

and expression (46) is solved for the desired value $H_{i+1}$ of the magnetic field strength in the future point:

$$H_{i+1} = H_i + \frac{\tilde{\alpha} s (B_{i+1} - B_i) \left[ \tilde{f}(B_i) - H_i \right] + \tilde{g}(B_i)}{\tilde{h}(B_i, H_i)} \cdot \frac{B_{i+1} - B_i}{\Delta t}.$$  \hspace{1cm} (47)

**Annotations:** In practical simulations, the magnetic devices are usually excited by voltage- and not by current sources. In case of voltage sources, the value $B_{i+1}$ of the magnetic flux density in the future point is identified, in case of current sources the value $H_{i+1}$ of the magnetic field strength is fixed. Therefore the alternate notation (47) is more adequate in praxis than those based on the original representation (2). Moreover no inversion of the “modification” $\tilde{h}(B_i, B_{i+1}, H_i)$ is necessary during evaluation. Another inherent problem is given by the determination of the DC premagnetization $B_{\text{DC}}$. Due to the transformer measurement principle depicted by figure 2, the measured average value of $B(t)$ over each period has to be zero. Of course, a Hall sensor could be used to measure $B_{\text{DC}}$, but as a disadvantage, the ring core specimens have to equipped with an air gap, thus leading to considerable problems due to the parasitic coupling of the stray field. To avoid these problems, the ‘commutation curve’ is utilized here as a representation of the nonlinear relation between $B_{\text{DC}}$ and $H_{\text{DC}}$. The curve shown in figure 4 is given by the closure points of the ascendant and descendant branches of a set of symmetrical subloops of different excitation levels. It is usually very close to the initial magnetization curve. Figure 5 shows the run of the normalized alternate material functions $\tilde{f}(B)$ and $\tilde{g}(B)$.

**Figure 4.** Measured commutation curves.  
**Figure 5.** Material functions $\tilde{f}$ and $\tilde{g}$. 
The numerical values of the parameters $\bar{\alpha}$ and $\bar{g}(0)$ are given in table 1. In figure 6 a comparison between time domain simulations and measured results is presented in steady state.

**Table 1.** Numerical values of the material parameters $\bar{\alpha}$ and $\bar{g}(0)$.

| Material: F49 | $f = 10\text{kHz}$ | $T_c = 25^\circ\text{C}$ | $T_c = 100^\circ\text{C}$ |
|---------------|-------------------|--------------------------|---------------------------|
| $\bar{\alpha}$ | $[\text{Ws}/\text{m}^2]^{-1}$ | 12.511 | 13.112 |
| $\bar{g}(0)$ | $[\text{Vs}/\text{Am}]^{-1}$ | 260.185 | 329.612 |

**Figure 6.** Measured and calculated magnetic field strength $H$ in time domain (steady state).
To give an impression of the achieved accuracy, the mean power loss per volume and per cycle

\[ \bar{p}_m = \frac{1}{T} \int_0^T p_m(t) \, dt \quad \text{with} \quad p_m(t) = H(t) \frac{\partial B(t)}{\partial t} \]  

is evaluated numerically as a function of the maximum flux density \( \dot{B} \) and compared to measurement points in figure 7. The results show that a very good replication of the hysteresis phenomena is obtained from the calibrated model up to the saturation level. This is more or less a direct consequence of the generalized material functions (24) and (25). Furthermore simulations of \( \bar{p}_m \) and the average stored magnetic energy per volume and per cycle

\[ \bar{w}_m = \frac{1}{T} \int_0^T w_m(t) \, dt \quad \text{with} \quad w_m(t) = \frac{1}{2} B(t) \cdot H(t) \]  

that is associated with the inductance of the component, are conducted based on the calibrated model.

In table 2 a comparison between calculated and measured points of \( \bar{p}_m \) and \( \bar{w}_m \) is listed.

**Table 2.** Simulation of \( \bar{p}_m \) and \( \bar{w}_m \) in time domain considering different values of the applied DC premagnetization \( B_{dc} \) (parameters: \( \dot{B} = 100\text{mT}, \ f = 20\text{kHz} \)).

| \( B_{dc} \) [mT] | \( \bar{p}_m \) [mW/cm\(^3\)] | \( T_e = 25^\circ \text{C} \) | \( T_e = 100^\circ \text{C} \) |
|------------------|---------------------|---------------------|---------------------|
| 0 mT             | 66.1899             | 66.1899             | 52.7062             | 40.8264             |
| 200 mT           | 67.8200             | 69.1616             | 55.5901             | 46.2143             |
| 300 mT           | 75.2274             | 72.1820             | 56.0040             | 48.4272             |
| 350 mT           | 80.1502             | 73.5691             | 55.3041             | 61.3847             |

| \( B_{dc} \) [mT] | \( \bar{w}_m \) [Ws/m\(^2\)] | \( T_e = 25^\circ \text{C} \) | \( T_e = 100^\circ \text{C} \) |
|------------------|---------------------|---------------------|---------------------|
| 0 mT             | 0.4363              | 0.4998              | 0.6775              | 0.7187              |
| 200 mT           | 1.2452              | 1.1623              | 1.1528              | 1.1853              |
| 300 mT           | 1.9330              | 1.7065              | 1.9705              | 2.1506              |
| 350 mT           | 2.8466              | 2.4852              | 3.4908              | 3.7068              |
Therein the AC excitation level defined by $\hat{B} = 0.5 (B_{\text{max}} - B_{\text{min}})$ is fixed at $\hat{B} = 100 \text{mT}$. All results are obtained for a frequency of $f = 20 \text{kHz}$ (the model was adapted to symmetrical loops measured at $f = 10 \text{kHz}$). It is found that the description of the physical phenomena by the model is precise enough to calculate results under DC premagnetization with high accuracy, even if none of these information is used for model adaptation. Figure 8 shows the surrounded loops with the DC premagnetization $B_{\text{dc}}$ as a parameter. The DC offset of the magnetic field strength $H_{\text{dc}}$ is automatically given by (47), if the desired DC premagnetization is applied to $B(t)$ in time domain. Hence no use of the commutation curves shown in figure 4 is required.

**Figure 8.** Simulated $B$-$H$ loops for different values of the applied DC premagnetization $B_{\text{dc}}$. 

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Parameters: $\hat{B} = 100 \text{mT}, \ f = 20 \text{kHz}, \ T_c = 25^\circ\text{C}$
4. Conclusion

A generalized approach is presented based on the well known Hodgdon dynamical model. It is shown how the model is calibrated using a set of measured symmetrical subloops – data that can be easily acquired by means of the shown experimental set-up. It is found that the physical behavior of the model is precise enough to simulate hysteresis loops also under DC premagnetization, even if none of these information is used for the adaptation of the model. The conditions are discussed how to obtain stability and closed loops in steady state. Of course, all of the verified results presented by the paper refer to the steady state in case of periodical excitation. No fast transient processes are investigated. This is due to the fact that fast transients usually are not of interest in the calculation of the choke inductance and the core losses. But regarding to the surveys of dynamical models to be found in literature, it can be supposed that Hodgdon’s constitutive relations are not adequate for a precise modeling of these effects. The well known approaches of D. C. Jiles & D. L. Atherton as well as F. Preisach seem to be more promising here.

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