Multiple-Re trapping Process in High-$T_c$ Intrinsic Josephson Junctions

Myung-Ho Bae$^{1,*}$, M. Sahu$^1$, Hu-Jong Lee$^2$, and A. Bezryadin$^{1,†}$

$^1$Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801-3080, USA and
$^2$Department of Physics, Pohang University of Science and Technology, Pohang 790-784, Republic of Korea

(Dated: July 24, 2008)

We report measurements of switching-current distribution (SWCD) from a phase-diffusion branch to a quasiparticle-tunneling branch as a function of temperature in a cuprate-based intrinsic Josephson junction. Contrary to the thermal-activation model, the width of the SWCD increases with decreasing temperature, down to 1.5 K. Based on the multiple-retrapping model, we quantitatively demonstrate that the quality factor of the junction in the phase-diffusion regime determines the observed temperature dependence of the SWCD.

PACS numbers: 74.72.Hs, 74.50.+r, 74.40.+k, 74.45.+c

The escape of a system trapped in a metastable state governs the reaction rate in various dynamical systems, where the escape is made by a noise-assisted hopping [1]. In the case of a Josephson junction (JJ), a thermal noise induces an escape of a phase particle representing the system from a local minimum of the potential well. In an underdamped JJ with hysteresis in the voltage-current (V-I) characteristics a single escaping event induces a switching from a zero-voltage phase-trapped state to a finite-voltage phase-rolling state [2]. In an overdamped JJ without hysteresis, however, the energy of an escaped phase particle is dissipated during its motion so that the particle is retrapped in another local minimum of the potential. The phase particle repeats this thermally activated escape and retrapping process, i.e., the multiple-retrapping process. It results in a phase-diffusion branch (PDB); a resistive branch with a small but finite voltage for a bias current below a switching current, $I_{SW}$ [3].

A hysteretic JJ can also evolve into this multiple-retrapping state with increasing temperature ($T$) when the energy fed by a bias current, near a mean switching current that is suppressed by an increment in $T$, gets comparable to the dissipated energy. Recently, this phase-retrapping phenomenon in the hysteretic JJ has been intensively studied in association with the phase-diffusion model. The values of $Z_J$, turn out to be an order of the measurement-line impedance $Z_L$, which indicates that the phase dynamics of our JJJs is governed by the environmental dissipation. Our measurements of the $T$ dependence of the switching rate $\Gamma_S$ and the corresponding SWCD in a single JJ are in good agreement with those estimated by the multiple-retrapping model. This study quantitatively clarifies how the impedance- and temperature-related dissipations, and the corresponding $Q$ determine the switching dynamics in a hysteretic JJ with phase-diffusion characteristics.

A stack with the lateral size of 2.5×2.9 μm$^2$ in Bi-2212 single crystal was defined by using focused-ion-beam (FIB) process [9]. High-intensity FIB irradiation is known to degrade the peripheral region by the scattered secondary ion beam [10]. In the milling process, we used a relatively high ion-beam current of 3 nA, corresponding to $\sim 200$ pA/μm$^2$. This high ion-beam current much reduced the interlayer tunneling critical current density down to $\sim 8$ A/cm$^2$ in $N=12$ junctions out of total $\sim 100$ junctions in a stack, which were estimated from the number of QTBS in V-I curves of the stack. Four-terminal transport measurements were carried out in a pumped He$^4$ dewar with the base temperature of 1.45 K. Room-temperature ε-filters were employed and measurement lines were embedded in silver paste at cryogenic temperatures to suppress high-frequency noises propagating along the leads. The measurements were made by using battery-operated low-noise amplifiers (PAR-113). The ramping speed of the bias current and the threshold volt-
a frequency-dependent effective junction impedance $Z_{\text{eff}}$. FIG. 1: (color online) $V$-$I$ characteristics at various temperatures of the Bi-2212 IJJs stack. The upward and downward arrows indicate the switching and the return currents, respectively, at $T=15.1$ K. Black solid curves are the best fits to the phase diffusion model at three different temperatures of 15.1 K, 18 K, and 23 K, with the effective junction impedance $Z_{J}=125, 125$, and 128 $\Omega$ as the best-fit parameters, respectively. Inset: the $R$-$v$-$T$ curve, where $T_c$ is indicated by an arrow.

The inset of Fig. 1 shows the tunneling $R$ vs $T$ curves with the superconducting transition at $T_c=90$ K, which is indicated by an arrow. The $c$-axis tunneling nature in the Bi-2212 stacked junctions is evident by the increasing resistance with decreasing $T$ above $T_c$. At $T<T_c$, the junction resistance does not vanish completely, even down to $T=10$ K, which is attributed to the phase diffusion in the junctions. Fig. 1 shows the $V$-$I$ curves at various $T$ below $T_c$. The nonlinear curve at $T=50$ K begins to show a hysteresis below $T\sim30$ K. For a bias below a $I_{SW}$ in each curve, a pronounced non-zero resistive branch appears below $T=23$ K. Here, the finite resistance is caused by the phase diffusion in the condition of $k_BT\sim2E_J$. The phase-diffusion model predicts the current-voltage relation of $I(V)=\frac{eI_{SW}k_BT}{\hbar}\frac{Z_{J}V}{v^2+2\pi Z_{J}k_BT/\hbar}$ with a frequency-dependent effective junction impedance $Z_{J}$. Here, the phase diffusion is assumed to take place in all twelve junctions with FIB-suppressed critical currents, so that the voltage bias per junction becomes $v=V/N$. We use this relation to fit the PDB as shown by black curves in Fig. 1 with $Z_{J}$ as the best-fit parameter. $Z_{J}$ turns out to be 125, 125, and 128 $\Omega$ for $T=15.1, 18$, and 23 K, respectively. These values of $Z_{J}$ are comparable to the measurement line impedance $Z_L$ (50$\sim$100 $\Omega$), even with a much higher quasiparticle tunneling resistance of $R_{qp}\sim35$ k$\Omega$ as seen in Figure 1. This implies that the dissipation in the PDB is dominated by the high-frequency (of an order of the Josephson plasma frequency, $\omega_p$) dissipation through the measurement lines [6, 7]. Since the energy of the escaped phase particle is dissipated through the environment in the phase-diffusive regime, the switching also becomes sensitive to this dissipation process.

Now, we turn to the switching event from the PDB to the QTB. FIG. 2(a) shows the SWCD (scattered symbols) at various temperatures. The standard deviation ($\sigma$) and the mean switching currents ($I_{SW}$) are shown as a function of $T$ in the inset of Fig. 2(a). The $T$ dependence of $\sigma$ contradicts to that of a conventional underdamped JJ, where $\sigma$ increases with $T$ in a thermal-activation regime [2]. FIG. 2(b) shows $I_S$ (scattered points) vs $I$ calculated from the SWCD of Fig. 2(a) following the Fulton and Dunkleberger analysis [2]. With lowering $T$, $G_S(I)$ shows a pronounced change from an almost linear to a down-turn nonlinear bias-current dependence in a semi-logarithmic plot. To explain this behavior, we adopted the multiple-retrapping model [5, 8]. In the phase-diffusive regime, the successive retrapping processes suppress the switching rate, $\Gamma_S$. The switching to the high-voltage branch (i.e. the QTB) occurs only when the phase particle is not retrapped after escaping from a local energy minimum. A phase particle escaped from a potential minimum has a probability, $P_{RT}$, to be retrapped in the next potential minimum. The switching rate $\Gamma_S$, including $P_{RT}$, is expressed by [8]

$$\Gamma_S = \Gamma_{TA}(1 - P_{RT}) \frac{\ln(1 - P_{RT})}{P_{RT}}. \quad (1)$$

Here, $\Gamma_{TA}=\frac{\hbar}{2\pi} \exp(-\Delta U_{\phi}/k_B T)$ is the thermally activated escape rate, $\omega_p=\omega_{p0}(1-\gamma^2)^{1/4}$, $\omega_{p0}=2eI/k_B T$ is the escape energy barrier, and $\gamma=I/I_c$ is a normalized bias current. The retrapping probability can be obtained by an integration [8] of the retrapping rate $\Gamma_{RT}=\frac{\gamma\omega_p}{4\pi} \exp(-\Delta U_{\phi}/k_B T)$, where $\Delta U_{\phi}=E_J(1-\gamma^2)/2$ and $\gamma=I/I_c$ is the noise-free return current from a QTB to a PDB. FIG. 3(a) shows the experimental switching distribution (red dots) at $T=1.5$ K with the corresponding fit (green curve) obtained by using Eq. (1) with the best-fit parameters of $Z_J=61.9$ $\Omega$, $I_c=1.26$ $\mu$A, and $I_{0}=63$ $n$A. The junction capacitance, 330 $fF$, was estimated from the typical value of 45 $fF/\mu m^2$ for Bi-2212 IJJs [17]. The corresponding switching rate (red dots) and the fit (green curve) are shown in Fig. 3(b), with the same parameters. An excellent agreement is obtained in both fittings.

These results are analyzed in terms of $Z_{J}$- and $T$-dependence of $P_{RT}$. The inset of Fig. 3(a) shows the calculated $P_{RT}$-$vs$-$I$ curves for various $Z_{J}$ at $T=1.5$ K, with the values of $I_c$ and $I_{0}$ obtained from the best fits of Figure 3. The retrapping-probability curve shifts to
higher currents as $Z_J$ decreases, due to the presence of a $Z_J$ dependence of $\Delta U_{RT}$ in the exponential factor of $\Gamma_{RT}$. The current positions of almost vanishing $P_{RT}$, indicated by downward arrows, approximately correspond to the maximum current allowing the retrapping. We denote this current as $I_{PD}$. Physically this is the same current as the one denoted $I_m$ in Ref. [6]. The system can hardly be retrapped at a current higher than $I_{PD}$, because in this case the energy fed to the system by the bias current gets larger than the dissipated energy. By equating the energy fed and the energy dissipated, similar to McCumber and Stewart analysis [4], one obtains the relation

$$I_{PD} = 4I_c/\pi Q_{PD}, \tag{2}$$

where $Q_{PD}$ is the phase-diffusion quality factor at $\omega \approx \omega_p$. In fact, the noise-free retrapping current can be written in a form similar to Eq. (2), namely as $I_{\alpha} = 4I_c/\pi Q(\omega=0)$. The thick black curves in Figs. 3(a) and 3(b) show SWCD and the corresponding $\Gamma_{TA}(I)$, respectively. Other solid curves in Fig. 3(b) are $\Gamma_S(I)$ in Eq. (1) for varying $Z_J$ under the multiple retrapping processes. $\Gamma_S(I)$ with each $Z_J$ in Fig. 3(b) starts to drop quickly from $\Gamma_{TA}(I)$ at $I=I_{PD}$, which are denoted by arrows in the figure. We define $I_{PD}$ as the current value corresponding to $P_{RT}=0.01$, where $\Gamma_S(I_{PD})$ is nearly the same as the $\Gamma_{TA}(I_{PD})$ as shown in Fig. 3(b) [8]. The impedance of 106 $\Omega$ gives the same SWCD as thermally activated SWCD without retrapping because $\Gamma_S(I)$ in $\Delta\Gamma$ well overlaps with $\Gamma_{TA}(I)$ although the impedance is of an order of $Z_J$. When $\Gamma_S(I_{PD})$ crosses over the bottom line of $\Delta\Gamma$ while $Z_J$ keeps decreasing, the high-frequency dissipation affects the SCDW: the observable window of $\Delta\Gamma$ (between the two dashed lines) for a fixed $I_b$ shifts to the steeper section with decreasing $Z_J$, resulting in the decreasing width of the SWCD in Figure 3(a) at a constant $T$. This shift itself is caused by a reduction of $Q_{PD}$ with decreasing $Z_J$ at a constant $T$.

Since $I_{PD}$ is also affected by a change in $T$ as followings, the shape of $\Gamma_S(I)$ does not simply follow the variation of $Z_J$ as $T$ changes. The inset of Fig. 3(b)
illustrates $P_{RT}$ vs $I$ at various temperatures. Here, $Z_J$ is fixed at 61.9 Ω and other parameters, except for $T$, are set to be the same as for the inset of Fig. 3(a). The current position of the zero-temperature curve, indicated by an upward arrow in the inset of Fig. 3(b), corresponds to the fluctuation-free return current $I_0$. The value of $I_{PD}$ shown by arrows increases with increasing $T$. Thus, in effect, the retrapping probability curve shifts to higher currents as $I_{PD}$ increases with increasing $I_{PD}$ at $4.2$ K. This behavior leads to the conclusion that the retrapping rate, $\Gamma_{RT}$, is insensitive to $T$ variations at sufficiently high temperatures, and it is only a function of $I_{PD}$ and the window of $\Delta T$. The slope of the calculated $\Gamma_S(I)$ in the window of $\Delta T$ at the temperature of 0 K equals to $\frac{\exp[-\beta V]}{\exp[-\beta V']}$, where the estimated noise-free $I_0$ matches with the value used in $\Gamma_S$ fitting in Table I. The junction impedance $Z_J$ estimated from the return currents is significantly larger than $Z_J(\omega \sim \omega_p)$ for the observed switching events. It indicates that the retrapping phenomena from QTB to PDB are mainly determined by a zero-frequency damping with $Q(0)=11.4$ with $I_c=572$ nA and $I_0=63.8$ nA at $T=4.2$ K as shown in Table I.

Table I: Fitting parameters for the switching events for selected temperatures

| $T$(K) | $I_c$(µA) | $I_0$(nA) | $Z_J$(Ω) | $I_{PD}$(nA) | $P_{PD}$ |
|--------|-----------|-----------|-----------|-------------|----------|
| 1.5    | 1.263     | 63.00     | 61.9      | 678         | 2.37     |
| 2.4    | 1.209     | 65.10     | 77.6      | 683         | 2.25     |
| 3.0    | 1.006     | 63.44     | 86.0      | 685         | 1.87     |
| 3.6    | 0.735     | 65.22     | 94.7      | 682         | 1.37     |
| 4.2    | 0.572     | 63.79     | 101.5     | 572         | 1.27     |

In summary, we clearly show that the multiple retrapping processes in underdamped IJJs, with much suppressed critical current and high tunneling resistance, govern the switching from the PDB to the QTB. The predicted SWCD and $\Gamma_S$ in the multiple-retrapping model are in good agreement with the observed broadening of the distribution of switching currents with decreasing temperature. We also demonstrate that the change of the shapes of the observed SWCD and the $\Gamma_S$ in various temperatures can be understood by impedance and temperature dependence of $P_{PD}$, taking the retrapping probability into account. As the macroscopic quantum tunneling has been observed recently in IJJs of Bi-2212 single crystals [19], this study provides in a quantitative manner the role of the dissipation in quantum devices based on cuprate-based JJs.

This work was supported by DOE Grants No. DEFG02-07ER46453. We acknowledge the access to the fabrication facilities at the Frederick Seitz Materials Research Laboratory. This work was also partially supported by POSTECH Core Research Program, Acceleration Research Center (No. R17-2008-007-01001-0), and the Korea Research Foundation Grants No. KRF-2006-352-C00020.

*mbh@uiuc.edu, †bezryadi@uiuc.edu

References

[1] P. Hänggi, P. Talkner, and M. Borkovec, Rev. Mod. Phys. 62, 251 (1990).
[2] T. A. Fulton and L. N. Dunkleberger, Phys. Rev. B 9, 4760 (1974).
[3] M. Tinkham, Introduction to Superconductivity, 2nd ed. (McGraw-Hill, New York, 1996).
[4] J. M. Kivioja et al., Phys. Rev. Lett. 94, 247002 (2005).
[5] V. M. Krasnov et al., Phys. Rev. Lett. 95, 157002 (2005).
[6] M. Martinis and R. L. Kautz, Phys. Rev. Lett. 63, 1507 (1989); R. L. Kautz and J. M. Martinis, Phys. Rev. B 42, 9903 (1990).
[7] A. T. Johnson, C. J. Lobb, and M. Tinkham, Phys. Rev. Lett. 65, 1263 (1990).
[8] J. Männik et al., Phys. Rev. B 71, R20509 (2005).
[9] D. Vion et al., Phys. Rev. Lett. 77, 3435 (1996).
[10] S. J. Kim et al., Appl. Phys. Lett. 74, 1156 (1999).
[11] M.-H. Bae et al., Phys. Rev. B 77, 144501 (2008).
[12] In our measurement setup, the voltage spacing between the maximum voltage value of the PDB and the next jumping voltage is ∼150 µV so that the value of our $V_{th}$ of 110µV is reasonable to obtain $I_{SW}$ values.
[13] G.-L. Ingold, H. Grabert, and U. Eberhardt, Phys. Rev. B 50, 395 (1994).
[14] A. Franz et al., Phys. Rev. B 69, 014506 (2004).
[15] A. Garg, Phys. Rev. B 51, 15592 (1995). The retrapping current distribution with increasing bias current is given by the Kurkişari-Fulton-Dunkelberger formula, $P(I) = \frac{I_{RT}}{I_{PD}} \exp\left[-\int_0^I \frac{\Gamma_{RT}(I')}{{I'}^2} dI'\right]$ and the retrapping probability at $I$ is obtained by $P_{RT}(I) = \frac{I_{RT}}{I_{PD}} \exp\left[-\int_0^I \frac{\Gamma_{RT}(I')}{{I'}^2} dI'\right]$.
\[
\int_{I_c}^{I_c'} P(I') dI' / \int_{0}^{I_c} P(I') dI'.
\]

[16] E. Ben-Jacob et al., Phys. Rev. A 26, 2805 (1982).

[17] A. Irie, Y. Hirai, and G. Oya, Appl. Phys. Lett. 72, 2159 (1998).

[18] The noise-free critical current, \(I_c\), is strongly suppressed with \(T\), resulting in a reduction of \(\omega_p\). Thus, the frequency related to a phase diffusion dynamics gets smaller, leading to a larger value of \(Z_J\) with \(T\) as shown in Table I [6].

[19] K. Inomata et. al., Phys. Rev. Lett. 95, 107005 (2005); X. Y. Jin et. al., Phys. Rev. Lett. 96, 177003 (2006); S.-X. Li et. al., Phys. Rev. Lett. 99, 037002 (2007).