Theoretical investigation of weight-dependent optical spike timing dependent plasticity based on VCSOA

Tao Tian, Min Ni, Zhengmao Wu, Guangqiong Xia, Xiaodong Lin, Tao Deng*

School of Physical Science and Technology, Southwest University, Chongqing 400715, China
*dengt@swu.edu.cn

Abstract. We propose an implement scheme of the weight-dependent optical spike timing dependent plasticity (STDP) based on the vertical-cavity semiconductor optical amplifiers (VCSOAs) subject to double optical spike injections, and numerically investigate the effects of the internal and external parameters of VCSOAs on the optical STDP curve by the well-known Fabry-Pérot approach. Additionally, the optical weight-dependent STDP is realized by introducing the various bias current according to the feedback signal. The simulation results show that, the width and height of the optical STDP curve window can be effectively controlled by adjusting the relevant parameters of VCSOAs. Moreover, the optical weight-dependent STDP is analogous to the biological STDP synapses and can be used to balance the stability and competition among synapses. These results can offer great potential for constructing a large-scale energy efficient optical spiking neural networks (SNNs).

1. Introduction

The human brain possesses the powerful information processing capability and can perform the intricate tasks with the low energy consumption [1]. Therefore, the neuromorphic engineering has become a research hot. Especially, the previous works have demonstrated that the photonic spiking neural network (SNN) possesses the ultrafast information processing capability with a speed up to several orders of magnitude faster than their biological or electronic counterparts [2,3]. As one of the important information function units in the photonic SNN, the synapses are responsible to the connection and communication between various neurons and its spiking dynamics have been considerably concerned.

The synaptic learning is one of the key process to realize the neuron network. The spike timing dependent plasticity (STDP), strongly associated with the learning and memory in the brain [4,5], has been applied in the photonic SNN to adjust the synapse weight [6]. In 1998, Bi and Poo experimentally measured the STDP learning function in the first time [5]. Subsequently, some photonic implement methods of the STDP synapse have been successively proposed. Fok et al. experimentally demonstrated the first optical STDP circuit by adopting a semiconductor optical amplifier (SOA) and an electro-absorption modulator (EAM) and the STDP curve can be adjusted through varying the EAM bias and SOA current [3]. Toole experimentally realized the photonic STDP based on the cooperative effects of cross gain modulation and nonlinear polarization rotation within an SOA and proposed a STDP-based photonic approach for measuring the angle of arrival of a microwave signal [7]. Ren et al. implemented the weight-dependent optical STDP through modifying the SOA current according to the feedback signal and realized the reward-based reinforcement
learning [6]. Consequently, the VCSOA-based STDP has highlighted the huge potential in the future energy-efficient photonic information processing system.

In the biological neural system, the boundary mechanisms are particularly important for the STDP synapses to prevent synaptic weight from exceeding a certain range, which can be adjusted by a third factor [8,9]. Generally, the additive or multiplicative STDP learning rules include the boundary mechanism and are used to adjust the synapse weight [10]. As for optical STDP scheme, previous works mainly focused on the SOA or VCSOA synapses. Ren et al. adjusted the bias current of SOA according to the local or global feedback signals to realize the reward-based reinforcement learning [6]. Compared with the first optical STDP without boundary mechanism proposed by Fok et al., this proposed STDP scheme can be regarded as an intermediate configuration between additive STDP with hard bounds and multiplicative STDP with soft bounds, which can better balance stability and competition among synapses. Obviously, after taking into account the advantages of the optical STDP scheme with boundary mechanism and VCSOA, exploring the VCSOA-based STDP learning rule with boundary mechanism becomes significant for the practical application of VCSOA-based synapse in future ultrafast photonic neural networks.

In this paper, we demonstrated an optical STDP with weight-dependent learning window by utilizing two VCSOAs and numerically investigated the VCSOA-based optical STDP characteristics with weight-dependent learning rule. The adjusting mechanism of VCSOA-based optical STDP and the effects of the internal and external parameters on the STDP curve are given. After introducing the present weight as the feedback signal, the bias current of VCSOA can be adjusted in an adaptive way and the weight-dependent STDP with the boundary mechanism can be realized. During the weight training, the synaptic connection weight can tend to the maximum or minimum values at slower rate, which is analogous with the biological prototype and can balance the stability and competition among synapses.

2. Theoretical model

![Diagram](image)

**Figure 1.** (a) A simplified model of the photonic SNN with an optical STDP synapse. W: variable weight regulating device; VCSOA: vertical cavity semiconductor optical amplifier; CCU: current control unit; T: variable delay line. (b) Schematic diagram of the optical weight-dependent STDP. $\lambda_{\text{pre}}$, $\lambda_{\text{post}}$: pre- and postsynaptic spikes, C: coupler, PD: photodetector.

Figure 1(a) shows a simplified model of the photonic SNN, which includes two photonic neurons and a VCSOA-based STDP synapse. All of the external stimuli are injected into the presynaptic neuron (neuron 1) and a spike is excited until the integrated strength exceeds the exciting threshold of the neuron, and then the output spike is transmitted to postsynaptic neuron (neuron 2) after a certain time.
delay. The synaptic weight in the SNN model is trained through this proposed weight-dependent photonic STDP module. Fig.1(b) further shows the photonic implement model of the VCSOA-based photonic STDP with weight-dependent learning capability. Two spike injections respectively denote the pre- and postsynaptic spikes. A variable optical delay line (T) is utilized to precisely control the relative time delay. Both pre- and postsynaptic spikes are respectively injected into the promotion and depression modules via a coupler, where the spikes with larger power are used as trigger spikes of VCSOAs while the relative smaller spikes are the probe spikes. Through recording the output power of probe spike with different time delays, the synapse weight variation can be determined and the photonic STDP characteristics of VCSOA can be obtained. The updated weight is feedback to the implemented. For convenience, we define $P_{ts}$ and $P_{ps}$ as the power of the trigger and probe spikes, respectively.

The F-P approach was first proposed by Adams and has been extensively used to analyze the output characteristics of VCSOA [11]. Considering the introduction of double optical spike injections, the modified rate equation of carrier density $N$ is described as follows [11]:

$$\frac{dN}{dt} = -\frac{N}{\tau_D} - (AN + BN^2 + CN^3) - \frac{\Gamma \xi a(N - N_0)}{n_0} (B R_s + S + S_p)$$  \hspace{1cm} (1)

where $J$ is the bias current of VCSOAs, $\eta$ is the internal quantum efficiency, $e$ is the electron charge, $a$ is the linear material gain coefficient, $\Gamma$ is the longitudinal confinement factor, $\beta$ is the linear recombination coefficient, $B$ is the bimolecular recombination coefficient, $N_0$ is the transparency carrier density, $n_c$ is the cavity refractive index, $\beta$ is the spontaneous emission factor, $V = \pi r^2 L_c$ is the volume of the cavity, and $L_c$ is the effective cavity length, $r$ is the circular active region radius, $\xi = 1 + \sin(2\pi n_c L_{MQW}/\lambda_p) \times \lambda_p / 2 \pi n_c L_{MQW}$ denotes the gain enhancement factor and $L_{MQW}$ is the single quantum well length [12]. $S_t$ is the averaged spontaneous photon density, $S_n$ and $S_p$ respectively denote the average stimulated photon density associated with the trigger spikes and probe spikes, and can be expressed as follows [11,12]:

$$S_t = \left(\frac{e^{\delta_L} - 1}{2} - 1 \right) \left(1 - e^{\delta_L} - 1 \right) \frac{P_{ts} \lambda_p}{\hbar c V g}$$  \hspace{1cm} (2)

$$S_n = \frac{\left(1 - e^{\delta_L} - 1 \right) \frac{P_{ts} \lambda_p}{\hbar c V g}}{\left(1 - e^{\delta_L} - 1 \right) \frac{P_{ts} \lambda_p}{\hbar c V g}}$$  \hspace{1cm} (3)

$$S_p = \frac{\left(1 - e^{\delta_L} - 1 \right) \frac{P_{ps} \lambda_p}{\hbar c V g}}{\left(1 - e^{\delta_L} - 1 \right) \frac{P_{ps} \lambda_p}{\hbar c V g}}$$  \hspace{1cm} (4)

where $e^{\delta_L}$ represents the single-pass gain, $\gamma = \Gamma \xi a(N - N_0) - a_0$ and $a_0$ is the fixed internal loss, $\lambda_p$ is the wavelength of resonant mode, $R_t$ is the top DBR reflectivity, $R_b$ is the bottom DBR reflectivity, The single-pass phase change can be expressed as [11,12]:

$$\phi_{n,p} = \phi_{n,p,0} - \beta_t \xi L_a (N - N_e) / 2$$  \hspace{1cm} (5)

The carrier density $N_t$ is corresponding to the case without the optical spike injection into VCSOAs. The items $\phi_{n,p,0} = 2 \pi n_c L_c / (1 / \lambda_{n,p} - 1 / \lambda_p)$ denote initial phase detuning, $\Gamma$ denotes the linewidth enhancement factor, $\lambda_{n,p}$ are corresponding to the wavelengths of the trigger and probe spikes, $\lambda_p$ is the resonant wavelength of VCSOA. $\Delta \lambda_{n,p} = \lambda_{n,p} - \lambda_p$ denote the initial wavelength detuning between two injection spikes and the resonant wavelength of VCSOA. In our study, the relative time difference $\Delta t = t_{\text{post}} - t_{\text{pre}}$ can be controlled by delay line, where $t_{\text{pre}}$ and $t_{\text{post}}$ are the timing of the pre- and postsynaptic spikes respectively. $c$ is the velocity of light and $h$ is the Planck constant.

When the VCSOA operates in the reflection mode, the gain of VCSOA can be defined as [12]:

$$G_{n,p} = \frac{\left(\frac{P_{ts} \lambda_p}{\hbar c V g} \right) \left(1 - e^{\delta_L} - 1 \right) \frac{P_{ps} \lambda_p}{\hbar c V g}}{\left(1 - e^{\delta_L} - 1 \right) \frac{P_{ps} \lambda_p}{\hbar c V g}}$$  \hspace{1cm} (6)

The corresponding peak output power of two spike signals can be obtained by $P_{\text{ops}} = P_{\text{ops}} G_{R_s,R_p}$. 

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3. Results and discussions

Figure 2. Formation of promotion window (a), Formation of depression window (b). The blue and red pulses correspond to the pre- and postsynaptic spikes respectively. The upper sections denote the injected spikes and the lower sections denote the response outputs of the probe signals.

The rate Eqs. (1-6) are numerically solved by the fourth-order Runge-Kutta method. During the calculations, the used data are adopted as following [13]: \( \lambda_p = 1549.75 \text{nm} \), \( R_t = 0.9947 \), \( R_b = 0.9995 \), \( n_c = 3.2 \), \( L_c = 4.5 \lambda_p / n_c \), \( r = 5 \times 10^{-9} \text{m} \), \( L_{MQW} = 7 \times 10^{-9} \text{m} \), \( \Gamma_1 = 0.0108 \), \( \Gamma = 1 \), \( a = 2.8 \times 10^{-20} \text{m}^2 \), \( b = 2.2 \), \( A = 1 \times 10^8 / \text{s} \), \( B = 10^{-16} \text{m}^3 / \text{s} \), \( C = 5 \times 10^{-42} \text{m}^6 / \text{s} \), \( N_0 = 2 \times 10^{24} / \text{m}^3 \), \( \eta = 1 \), \( \alpha_l = 800 \text{m}^{-1} \), \( \beta = 10^{-5} \). Firstly, we explore the gain-recovery dynamics of VCSOA, which determine the VCSOA-based STDP synapse characteristics, the presynaptic spike (trigger or probe spike) and postsynaptic spike (probe or trigger spike) are injected into VCSOA and the responsive output power of the probe spike from VCSOA is recorded to mapping the STDP curve. The delay time of one spike signal can be adjusted through the variable delay line while the arrival time of another spike is fixed. Fig. 2 shows the mechanism of photonic STDP in VCSOA. For the promotion window, when the probe spike (postsynaptic spike) is injected into VCSOA before the trigger spike (presynaptic spike), it is amplified by the VCSOA without carrier depletion and the output power \( P_{const} \) remains constant. The corresponding outputs are depicted in the left section of Fig. 2(a). Inversely, when the probe spike arrives after the trigger spike, which can experience the carrier depletion due to the stimulated recombination caused by the trigger spike and the output power \( P_{ops} \) is smaller than normal value as shown in the right section of Fig. 2(a), and the corresponding magnitude of the weight update is denoted as \( \Delta P(\Delta t, I) = P_{const} - P_{ops}(\Delta t, I) \). For the depression window, as shown in Fig. 2(b), the presynaptic spike is used as the probe spike while the postsynaptic spike is used as the trigger spike, and then similar results can be obtained and \( \Delta P(\Delta t, I) = P_{ops}(\Delta t, I) - P_{const} \) represents the magnitude of the weight update in the depression module. Consequently, the weight variation \( \Delta w \) in the promotion and depression sections is calculated as follows [14]:

\[
\Delta w(\Delta t, I) = \begin{cases} 
\frac{(P_{const} - P_{ops}(\Delta t, I))}{P_{const}} & \text{if } \Delta t > 0 \\
\frac{(P_{ops}(\Delta t, I) - P_{const})}{P_{const}} & \text{else}
\end{cases}
\]

(7)

Where \( \Delta t \) denotes the time difference between the trigger spike and probe spike. For the case \( \Delta t = 0 \), the synaptic weight can be regarded as ideal. Consequently, the weight variation is set as \( \Delta w = 0 \).
For exploring the effect of the system parameters on the optical STDP curve, we investigated the variations of the height and width of the optical STDP curve window with some typical internal and external parameters of VCSOA in detail. Fig. 3 shows the effects of the bias current of VCSOA on the STDP curve window. From this diagram, it can be observed that, a larger $I$ can increase the height of the optical STDP window at a certain extent.

Figure 4 shows the optical STDP curves variations with the different fixed internal losses under three cases of cavity volumes. From these diagrams, one can see that, for relative large cavity volumes, a larger $a_l$ leads to larger height of the optical STDP window but the window width remains almost unchanged. For the same $a_l$, the smaller cavity volume leads to larger height of the optical STDP window, as shown in Fig. 4(a) and 4(b). For the relative small cavity volume, the height and width of the optical STDP window has little variation under a certain range of fixed internal losses, as shown in Fig. 4(c).

Figure 5 shows the effects of the linear material gain coefficients on the optical STDP curves for different cavity volumes. For relative large cavity volume, with the increase of the linear material gain coefficient, the width and height of the optical STDP window are slightly increased. For the same linear material gain coefficient, the height of the optical STDP window become larger with the decrease of the cavity volume. As the cavity volume is decreased to $V_3$, the relative small linear material gain coefficients have little effects on the the optical STDP window. When the linear material gain coefficient is decreased to a certain extent, the width and height of the the optical STDP window will be obviously decreased. Above-mentioned results demonstrate the adjustable optical STDP curve can be obtained by adjusting these system parameters, which is considerably valuable for the realization of large-scale photonic spike neural networks.
Figure 5. The influence of linear material gain coefficients on the optical STDP curve for the different volumes of the cavity $V_1$ (a), $V_2$ (b), and $V_3$ (c), other parameters is the same as in Fig. 4.

For the biological neurons, the synaptic learning with boundary mechanism is necessary for preventing the synaptic weight to exceed the limitation. Correspondingly, the weight-dependent synaptic learning characteristics with boundary mechanism for the optical synapse also need to be further explored. In this work, the bias current of VCSOAs is determined by the present weight $w$ integrating the weight variation $\Delta w$ and controlled by the CCU through introducing the positive and negative feedback for the promotion and the depression modules respectively. The two weight-dependent bias currents are respectively set as $I_p(w) = 0.2\text{mA} + \epsilon(1 - w)\text{mA}$ and $I_d(w) = 0.2\text{mA} + w\text{mA}$, where $w$ is standardized within the interval $[0,1]$ to avoid the excessive bias current, $\epsilon$ denotes the range factor and the value is $0.65\text{mA}$. During synaptic learning, the evolution of the synaptic weight $w$ from $i$ learning cycle to $i + 1$ learning cycle is calculated as [13]:

$$w(i + 1) = w(i) + w_l \times \Delta w(\Delta t),$$

where $w_l$ is the learning efficiency and is set as $0.001$, $w(i + 1)$ and $w(i)$ are the synaptic weight at $i$ and $i + 1$ learning cycles, respectively. In our simulation, the period of the repeated spikes is set as $10\text{ns}$, which is much larger than the recovery time of carrier density of the VCSOAs after spike injection.

Figure 6(a) shows the evolution of the synaptic weight $w$ for $\Delta t = 50\text{ps}$. For the case without feedback signal, denoted by the red solid line, the weight $w$ linearly increases with the increase of training time until the maximum bound is arrived, which is analogous to the additive STDP with a hard bound. For the case with feedback signal, denoted by the blue dotted line, the synaptic weight $w$ firstly exhibits a linear increase trend, and then slowly increase to a maximum value. Finally, the $w$ will remain at this level. This is analogous to the multiplicative STDP with a soft bound. As a result, the optical weight-dependent STDP can be used to balance competition and stability among synapses and can be treated as intermediate structure between the additive and multiplicative STDP [13]. Fig. 6(b) depict the evolution of synaptic weight $w$ for $\Delta t = 1050\text{ps}$. Compared with the cases of $\Delta t = 50\text{ps}$, the weight variation becomes slower. Consequently, the weight promotion and depression processes will need longer time.

Figure 6. Evolution of weight value with training time for $\Delta t = 50\text{ps}$ (a), $1050\text{ps}$ (b), where $\Delta \lambda_{ps} = 0$, $\Delta \lambda = 0.03\text{nm}$ and $P_{ts} = 30\mu\text{w}$, $P_{ps} = 5\mu\text{w}$. 
4. Conclusion
In summary, we implement an optical STDP scheme with weight-dependent learning window based on VCSOAs. Firstly, the effects of the internal and external parameters of VCSOA on the optical STDP are verified in detail. Then, the present synaptic weight is used as feedback signal to control the variation of the bias current of VCSOA and the boundary mechanism is considered. Compared with the optical STDP without weight-dependent learning rule, the optical weight-dependent STDP can be used as an intermediate configuration between additive and multiplicative STDP, which can efficiently balance the stability and competition among synapses, and then can be used in the future high-speed neuromorphic computing and intelligent recognition.

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