A Closed-Form Localization Algorithm in Scan-Based Sonar

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Abstract. Multi-station TDOA positioning is a more accurate positioning method, which can locate the acoustic emitter by processing the arrival time of signals collected by three or more measuring stations. This paper presents a closed-form localization algorithm using the angle-of-arrival and difference time of scan time measurements from the scan-based sonar (SBS). The basic principle of multi station TDOA passive location is proposed. Location scheme based on the combination of different platforms is used. The basic principle of TDOA passive location is discussed, as well as the realization scheme of the main problems such as the synchronization of observation station and acoustic target, the selection of ground observation station, error correction and so on.

1. Introduction
Multi-station TDOA positioning is a more accurate positioning method, which can locate the emitter by processing the arrival time of signals collected by three or more measuring stations[1-3]. This paper discusses the basic principle of multi station TDOA passive location, and proposes a passive location scheme based on the combination of land and underwater platforms. This paper mainly discusses the basic principle of TDOA bistatic passive location, as well as the realization scheme of the main problems such as the synchronization of observation station and target, the selection of ground observation station, error correction and so on. In the application of underwater target detection, early warning and long-distance sudden covert attack have become one of the most concerned attack means. Whether the armament system has the ability of long-range detection, positioning and precision strike has become an important factor to determine the outcome of the war. At present, the main ways to solve the problem of underwater target detection are sonar, geomagnetic detection, infrared detection, gravity field detection and so on. Because all kinds of detection methods have their limitations, in order to realize the target detection, positioning and tracking in modern high-tech war, we must use a variety of detection means, multi-channel, multi-level, all-round detection and monitoring of the target.

Using sonar detection system to locate and track underwater targets can give full play to the advantages of good concealment of sonar, and improve the survivability and effective working ability of the system in complex environment. So far, the research on multi station sonar positioning and tracking is relatively few, and the technology is not very mature. It will have important theoretical and practical significance to carry out this research for improving the ability of early warning detection, long-range positioning and strike. Therefore, this paper studies the problem of three-dimensional passive location and tracking of underwater moving targets. Using the residual vector variance matrix as the weighted matrix, the weighted least square (WLS) method is extended for AOA and SA positioning systems.
2. Measurements model[4]

This section describes the AOA and DTOST measurement models used for the proposed localization algorithm. Figure 1 shows the geometry of SBSs.

![Geometry of SBSs](image)

The AOA measurement model is as follows:

$$\hat{\alpha}_k = \alpha_k + n_{\alpha_k}$$  \hspace{1cm} (1)

$$E(n_{\alpha_k}n_{\alpha_m}) = \begin{cases} \sigma_{\alpha}^2 & k = m \\ 0 & k \neq m \end{cases}$$  \hspace{1cm} (2)

where $\alpha_k$ is the AOA with zero mean white Gaussian noise.

The AOA at the $k$th sensor can be obtained from the SBS location $x$:

$$\alpha_k = f_{\alpha}(x) = \tan^{-1}(y - y_k)(x - x_k)^{-1}.$$  \hspace{1cm} (3)

The estimated SA, obtained by DTOST model, can be expressed as:

$$\hat{\beta}_{k1} = \beta_{k1} + n_{\beta_{k1}}.$$  \hspace{1cm} (4)

$$\beta_{k1} = \cos^{-1}\left[\frac{r_1^2 + r_k^2 - d_{k1}^2}{2r_1r_k}\right] = \omega(t_k - t_1).$$  \hspace{1cm} (5)

where $\omega$ is the transducer scanning speed.

The measurement error of the arrival time for sensor $\hat{t}_k$, which is modeled as the white Gaussian noise with zero mean. These errors are assumed to be uncorrelated:

$$E(\hat{t}_k\hat{t}_m) = \sigma_t^2\delta(k - m).$$  \hspace{1cm} (6)

3. Localization algorithm

Passive location can be divided into triangle location, cross location and TDOA location according to the information used in location. Multi station TDOA location, also known as hyperbolic location, is a more accurate location method. It locates the emitter by processing the arrival time of signals collected by three or more measuring stations. In the two-dimensional plane, a pair of hyperbolas focusing on the two stations is determined by the time difference between the two stations. If two pairs of hyperbolas can be formed by using three stations to generate the intersection point, and then the false points can be eliminated by using the direction finding information, the position of the radiation source can be determined. In the three-dimensional space, the time difference of the emitter signal arriving at the two measuring stations specifies the hyperboloid with the two stations as the focus. To determine any emitter in the three-dimensional space, at least four stations need to form three unilateral hyperboloids to generate the intersection point to determine the location of the emitter. Using TDOA location method, when there is measurement noise, there will be location error. In order to reduce the
influence of positioning error, maximum likelihood estimation, least square estimation and other methods can be used to minimize the positioning error; direct calculation method, spherical intersection method, spherical complement method, plane intersection method can also be used for positioning [5-7].

The noisy measurements of the AOA and SA can be expressed by the following vector-matrix equations:

\[
\begin{bmatrix}
\hat{\alpha}_1, \cdots, \hat{\alpha}_k
\end{bmatrix}^T = \begin{bmatrix}
\alpha_1, \cdots, \alpha_k
\end{bmatrix}^T + \mathbf{n}_\alpha
\]

(7)

\[
\begin{bmatrix}
\hat{\beta}_1, \cdots, \hat{\beta}_k
\end{bmatrix}^T = \begin{bmatrix}
\beta_1, \cdots, \beta_k
\end{bmatrix}^T + \mathbf{n}_\beta
\]

(8)

\[
E[\mathbf{n}_\alpha^T \mathbf{n}_\alpha] = \sigma_\alpha^2 \mathbf{I}, \quad E[\mathbf{n}_\beta^T \mathbf{n}_\beta] = \sigma_\beta^2 \mathbf{I}.
\]

(9)

The linear relationship between AOA and SA can be expressed as:

\[
\beta = \mathbf{H}\alpha = [-\mathbf{I}_{K-1}, \mathbf{I}_{K-1}].
\]

(10)

The corresponding BLUE can be obtained as:

\[
\alpha_\beta = \left(\mathbf{Q}_\alpha^{-1} + \mathbf{H}^T \mathbf{Q}_\beta \mathbf{H}\right)^{-1} \left(\mathbf{Q}_\alpha^{-1} \hat{\alpha} + \mathbf{H}^T \mathbf{Q}_\beta^{-1} \hat{\beta}\right).
\]

(12)

The following linear approximation:

\[
\begin{bmatrix}
\sin(\hat{\alpha}_{B,1}) \\
\vdots \\
\sin(\hat{\alpha}_{B,K}) \\
\cos(\hat{\alpha}_{B,1}) \\
\vdots \\
\cos(\hat{\alpha}_{B,K})
\end{bmatrix}
\begin{bmatrix}
x_1 \sin(\hat{\alpha}_{B,1}) - y_1 \cos(\hat{\alpha}_{B,1}) \\
\vdots \\
x_K \sin(\hat{\alpha}_{B,K}) - y_K \cos(\hat{\alpha}_{B,K})
\end{bmatrix}
+ \text{diag}(c_1, \cdots, c_K) \mathbf{n}_{B,K}
\]

(13)

where,

\[
c_k = (x-x_k) \cos(\hat{\alpha}_{B,k}) + (y-y_k) \sin(\hat{\alpha}_{B,k}).
\]

(14)

In order to determine the position of the emitter in three-dimensional space, three pairs of hyperboloids formed by three baselines are needed in multi station TDOA location. This requires four stations to measure the time difference of arrival of the target signal synchronously, and the intercepting station needs to solve the problem of distinguishing different radiation sources and matching the same source multi station measurements. Therefore, the system also has higher requirements for signal processing and multi-target correlation processing.

\[
\hat{x}_{LS} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}.
\]

(15)

Procedure of is summarized as follows:

- Step1: given sensor locations \(x_1, \cdots, x_k\);
- Step2: given noisy measurements for AOA and SA \(\hat{\alpha}_k, \hat{\beta}_k\);
- Step3: given measurements errors variance \(\sigma_\alpha^2, \sigma_\beta^2\);
- Step4: construct the vectors and matrix;
- Step5: compute (12) ~ (15) to get the SBS location.
4. Simulation results
The location accuracy of TDOA passive location system is affected by the following factors: the measurement accuracy of time difference of arrival, the influence of multi-path and non line of sight propagation in signal propagation, the geometric structure of measurement base station distribution, etc. The geometric structure of base station distribution is usually measured by geometric accuracy factor, which is one of the key factors affecting the positioning accuracy. Monte Carlo simulation is used to evaluate the effectiveness. Assume that SBS is located at (50, 50). The unit of distance is meter. Sensors are pre distributed at (0,0), (20,0), (40,0), (60,0), (80,0) and (100,0).

Group I: AOA measurement noise is white Gaussian noise, the average value is zero, and the standard deviation is 0.1 ° to 5 °. The standard deviation of SA measurement noise is fixed at 1 degree. Group II: SA measurement noise was white Gaussian noise, the mean value was zero, and the standard deviation was 0.1 ° to 5 °. Suppose that the standard deviation of AOA measurement noise is 1 degree.

Figure 2 plots the root-mean-square errors (RMSEs) for the proposed localization algorithm.

(a)
5. Conclusions
Multi-base-station TDOA passive location has high accuracy. This paper proposes a multi base station passive location scheme based on underwater platform of time difference of arrival, which solves the problem of underwater target location and tracking to a certain extent. The system is an assumption, and its feasibility is still in the research stage. Simulation results show that the presented approach is efficient due to its closed-form and is sufficient so that the corresponding CRLB can be attained.

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