Constraints on flavour-dependent long-range forces from atmospheric neutrino observations at Super-Kamiokande

Anjan S. Joshipura\textsuperscript{a} and Subhendra Mohanty \textsuperscript{a,b}
\textsuperscript{a} Physical Research Laboratory, Ahmedabad 380009, India.
\textsuperscript{b} International Center for Theoretical Physics, Trieste, Italy.
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In the minimal standard model it is possible to gauge any one of the following global symmetries in an anomaly free way: (i) $L_e - L_\mu$, (ii) $L_e - L_\tau$ or (iii) $L_\mu - L_\tau$. If the gauge boson corresponding to (i) or (ii) is (nearly) massless then it will show up as a long range composition dependent fifth force between macroscopic objects. Such a force will also influence neutrino oscillations due to its flavour-dependence. We show that the latter effect is quite significant in spite of very strong constraints on the relevant gauge couplings from the fifth force experiments. In particular, the $L_e - L_{\mu,\tau}$ potential of the electrons in the Sun and the earth is shown to suppress the atmospheric neutrino $\nu_\mu \to \nu_\tau$ oscillations which have been observed at Super-Kamiokande. The Super-K data of oscillation of multi-GeV atmospheric neutrinos can be used to put an upper bound on coupling $\alpha_{e\mu} < 6.4 \times 10^{-52}$ and $\alpha_{e\mu} < 5.5 \times 10^{-52}$ at 90% CL when the range of the force is the earth-sun distance. This is an improvement by two orders on the earlier fifth force bounds in this range.

The standard model is invariant under four global symmetries corresponding to the baryon number and lepton numbers $L_\alpha$ of the three families $(\alpha = e, \mu, \tau)$ of leptons. None of these symmetry by themselves can be gauge symmetries but there exists three combinations each of which can be googed in an anomaly free way along with the standard $SU(2) \times U(1)$ group. These correspond \[1\text{] to (i) $L_e - L_\mu$, (ii) $L_e - L_\tau$ or (iii) $L_\mu - L_\tau$. Recent experimental indications of neutrino oscillations lead us to conclude that none of these three symmetries can be an exact symmetry of nature since exact conservation of the corresponding charges prevent mixing of different neutrino species and hence oscillations among them contrary to strong indications from the solar, KamLand and atmospheric neutrino experiments. Hence these symmetries must be broken in nature. The phenomenological consequences of relatively heavy gauge bosons corresponding to these symmetries have been discussed in \[1,2\]. Here we concentrate on an alternative possibility corresponding to very light gauge bosons with typical masses corresponding to a range greater than or equal to the earth-sun distance. Such a scenario is strongly constrained by the fifth force experiments but it still has interesting consequences in neutrino physics which we discuss.

For very light masses, the exchange of an $L_e - L_{\mu,\tau}$ gauge boson between electrons will give rise to a composition dependent long range force between macroscopic bodies. A variety of experiments have been performed to look for such equivalence principle violating long range forces \[3,4,5\]. A $(L_e - L_{\mu,\tau})$ gauge boson exchange between electrons will give rise to a potential $V(r) = \frac{\alpha_{e\mu}^2}{\lambda} e^{-\lambda m_\mu}$ where $m_\mu$, the gauge boson mass determines the range $\lambda$, of the force $\lambda = 1/m_\mu$. For composition dependent forces in the earth-sun distance range the most stringent bounds come from lunar laser ranging experiments \[4,5\] and from earth based torsion balance experiments where a search is made of a composition dependent force on torsion balance which would be in phase with the diurnal rotation of the earth \[6\]. The earth and the moon have different $Z$ composition and in the presence of a solar distance $L_e - L_{\mu,\tau}$ potential caused by the electrons in the sun, they will have different acceleration towards the sun. From the differential acceleration of the earth-moon system towards the sun and torsion balance experiments one can put an upper bound

\[
\sin^22\theta_{23} = 1.
\]

The solid line is a fit with $\alpha_{e\mu} = 0$, $\Delta m_{23}^2 = 3.9 \times 10^{-3}$, $\sin^22\theta_{23} = 1$. The dashed curve is for $\alpha_{e\mu} = 5.5 \times 10^{-52}$ with the same values for $\Delta m_{23}^2$ and $\sin^22\theta_{23}$.
FIG. 2: $L_e - L_\tau$ gauge symmetry: Allowed values of $\Delta m_{23}^2$ and $\sin^2\theta_{23}$ at 90C.L.% with $\alpha_{e\tau} = 0$ (solid curve), $\alpha_{e\tau} = 4 \times 10^{-52}$ (dashed curve) and $\alpha_{e\tau} = 5 \times 10^{-52}$ (dotted curve). For $\alpha_{e\tau} = 6.4 \times 10^{-52}$ there is no allowed parameter space of $\Delta m_{23}^2$ and $\sin^2\theta_{23}$ which is consistent with the Super-K atmospheric neutrino data.

FIG. 3: $L_e - L_\mu$ gauge symmetry: Allowed values of $\Delta m_{23}^2$ and $\sin^2\theta_{23}$ at 90C.L.% with $\alpha_{e\mu} = 0$ (solid curve), $\alpha_{e\mu} = 4 \times 10^{-52}$ (dashed curve) and $\alpha_{e\mu} = 5 \times 10^{-52}$ (dotted curve). For $\alpha_{e\mu} = 5.5 \times 10^{-52}$ there is no allowed parameter space of $\Delta m_{23}^2$ and $\sin^2\theta_{23}$ which is consistent with the Super-K atmospheric neutrino data.

of $\alpha(e,\mu,\tau) < 3.3 \times 10^{-50}$ for a $Z$ dependent force with the range $\lambda \simeq 10^{13} cm$.

The basic observation of this paper is that in spite of very stringent constraints on $\alpha(e,\mu,\tau)$, the $L_e - L_{\mu,\tau}$ forces can significantly influence the neutrino oscillations. This comes about due to the long range nature and the flavour dependence of the potential generated by the gauge bosons of the $L_e - L_{\mu,\tau}$ symmetry through the $\nu_{\mu,\tau} - e$ elastic scattering. For example, the electrons inside the Sun generate a potential $V_{e,\mu,\tau}$ at the earth surface which is given by

$$V_{e,\mu,\tau} = \alpha_{e,\mu,\tau} \frac{N_e}{R_{es}} \frac{Y_e M_\odot}{m_n} \frac{1}{A.U.} = 3.3 \cdot 10^{-11} eV \left( \frac{\alpha_{e,\mu,\tau}}{3.3 \cdot 10^{-50}} \right)$$

where $\alpha_{e,\mu,\tau} \equiv g_{e,\mu,\tau}^2/4\pi$ and $g_{e,\mu,\tau}$ is the gauge coupling of the $L_e - L_{\mu,\tau}$ symmetry. The electron fraction in the sun $Y_e \sim (2/3)$, the solar mass $M_\odot = 1.12 \times 10^{57}$ GeV, the earth sun distance $A.U. = 1.5 \times 10^{13} cm = 7.6 \times 10^{26}$ GeV$^{-1}$ and the nucleon mass $m_n \simeq 0.939$GeV lead to the numerical value quoted above. The corresponding potential due to electrons in the earth of an earth-radius range force is about 20 times smaller. This means that the bounds on the $\alpha_{e\mu,\tau}$ established for solar-distance forces reduce by a factor 20 for earth distance forces i.e $\alpha_{e\mu} < 1.1 \times 10^{-50}$ and
\(\alpha_{\text{ee}} < 1.2 \times 10^{-50}\) for \(\lambda \sim 6400\text{km}\). The improvement in the bound on earth-radius range fifth force is an improvement on the existing bounds \[3\] by more than five orders of magnitude.

The potential given in eq.1 is comparable or greater than the \(\Delta m^2_E\) probed in various neutrino experiments, e.g. \(\Delta m^2_E \sim (10^{-12} - 10^{-14})\text{eV}^2\) in case of the multi-GeV atmospheric neutrinos. Thus the \(V_{\mu\tau}\) can lead to observable changes in the oscillations of the terrestrial, solar and atmospheric neutrinos. At the very least, these experiments can be used to put more stringent constraints on \(\alpha_{e\mu,\tau}\) than the existing fifth force experiments. We illustrate this through a study of the atmospheric neutrinos.

The observations of atmospheric neutrinos at Super-Kamiokande have shown that the \(\nu_e\) and \(\bar{\nu}_e\) produced by cosmic rays in the atmosphere are largely unaffected, whereas the \(\nu_\mu\) and \(\bar{\nu}_\mu\) get converted partially to \(\nu_\tau\) and \(\bar{\nu}_\tau\) respectively \[7\]. The neutrino parameters from the Super-K data are \(\Delta m^2_{23} = 2.8 \times 10^{-3}\text{eV}^2\) and \(\sin^2\theta_{23} = 1\) for vacuum oscillations. These parameters would get affected in the presence of the potential \[1\]. For a small value of \(\alpha_{(e,\mu,\tau)}(< 10^{-52})\) the oscillation probability observed in Super-K can be reproduced by shifting \(\Delta m^2\) and \(\sin^2\theta_{23}\). With increase in \(\alpha_{e\mu,\tau}\) the allowed parameter space becomes smaller and finally at some \(\alpha_{e\mu,\tau}\) no values of the parameters \(\Delta_{23}, \sin^2\theta_{13}\) can fit the Super-K multi-GeV muon neutrino event data \[8\]. This way we obtain a 90\% CL upper bound of \(\alpha_{e\mu} < 5.5 \times 10^{-52}\) and \(\alpha_{e\tau} < 6.4 \times 10^{-52}\) on the couplings of the possible \(L_e - L_\mu\) and \(L_e - L_\tau\) gauge forces respectively.

\(L_e - L_\tau\) gauge symmetry : The \(\nu_\mu - \nu_\tau\) oscillations are governed by the evolution equation

\[
\frac{d}{dt} \begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2_{23}}{2E} \cos\theta_{23} & \frac{\Delta m^2_{23}}{4E} \sin\theta_{23} \\ \frac{\Delta m^2_{23}}{4E} \sin\theta_{23} & -\frac{\Delta m^2_{23}}{2E} \cos\theta_{23} - V_{\nu_e} \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix}
\]

(2)

Where the potential \(V_{\nu_e}\) is due to exchange of \(L_e - L_\tau\) gauge boson. For anti-neutrinos the potential in \[2\] will appear with a negative sign. The survival probability of the atmospheric muon neutrinos can be written as

\[
P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{23} L}{4E}\]

(3)

where the effective mixing angle \(\tilde{\theta}_{23}\) and mass squared difference \(\Delta\tilde{m}^2_{23}\) are given in terms of the corresponding vacuum quantities appearing in the Hamiltonian \[2\] by the relations

\[
\Delta\tilde{m}^2_{23} = \Delta m^2_{23} \left[ (\xi_{\nu_e} - \cos 2\theta_{23})^2 + \sin^2 2\theta_{23}\right]^{1/2}
\]

(4)

and

\[
\sin^2 2\tilde{\theta}_{23} = \frac{\sin^2 2\theta_{23}}{[\xi_{\nu_e} - \cos 2\theta_{23}]^2 + \sin^2 2\theta_{23}}
\]

(5)

where the strength of the potential is characterized by the parameter

\[
\xi_{\nu_e} = \frac{2V_{\nu_e} E}{\Delta m^2}
\]

(6)

with \(V_{\nu_e}\) given as in eq.1. The \(\bar{\nu}_\mu\) survival probability is obtained from the \(\nu_\mu\) survival probability by replacing \(\xi \rightarrow -\xi\) in the expressions \[4\] and \[5\].

Note that there is a possibility of the resonant enhancement of the atmospheric neutrino mixing angle due to the MSW effect generated by \(V_{\nu_e}\). This depends on the sign of \(\xi_{\nu_e}\) and could occur either for neutrino or anti neutrino. Since the atmospheric flux contains comparable fractions of both, this effect would get washed out and one still needs large mixing angle to explain the atmospheric data as our detailed analysis presented below shows.

In Super-Kamiokande the neutrino flavor is identified by the charged current interaction \(\nu_l + N \rightarrow N_l + l + X\) \((l = e, \mu)\). The outgoing muon or electron is identified by its characteristic Cerenkov cone. We use multi-GeV and partially contained mu-like events data for 3 years of operation \[8\]. The ratio of the observed mu-like events to the corresponding Monte Carlo data \[8\] in the multi-GeV range \((E_\nu \sim (1 - 100)\text{GeV})\) is shown as a function of the zenith-angle in Fig 1. The cosine of the zenith angle, \(\cos \theta_z\) is related to the neutrino flight path-length \(L\) in \[8\] as

\[
L = (\kappa + h)^2 - \kappa \sin^2 \theta_z^{1/2} - \kappa \cos \theta_z
\]

(7)

where \(\kappa = 6374\text{km}\) is the mean radius of the earth and \(h \simeq 15\text{km}\) is the average height in the atmosphere where the neutrinos are produced.
The multi-GeV data are presented as bins in energy and cosine of the zenith angle. We average over the energy bins in our analysis. The theoretical prediction for the mu-like events/Monte Carlo can then be written as

\[
\frac{\langle \mu - \text{like} \rangle}{MC} (\cos \theta_z) = \frac{\int dE_\nu (P_{\mu\mu} \Phi_\mu + rP_{\tilde{\mu}\tilde{\mu}} \Phi_{\tilde{\mu}})}{\int dE_\nu (\Phi_\mu + r \Phi_{\tilde{\mu}})}
\]

where \( P_{\mu\mu} \) and \( P_{\tilde{\mu}\tilde{\mu}} \) are the survival probabilities and \( \Phi_\mu (E_\nu, \cos \theta_z) \) and \( \Phi_{\tilde{\mu}} (E_\nu, \cos \theta_z) \) are fluxes of the atmospheric \( \nu_\mu \) and \( \nu_{\tilde{\mu}} \) respectively. We use the Fluka-3D flux given in \([8]\) in our analysis. In \([8]\) \( r \) is the ratio of the cross sections \( \sigma_{\nu_eN}/\sigma_{\nu_N} \) which is \( \sim 0.5 \). In writing eq.\([8]\), we have neglected small energy dependent of the relevant charged current cross section given in \([10]\).

We calculate the chi-square for the 10 zenith angle bins with \( \cos \theta_z = (-0.9 - 0.9) \) as a function of the parameters \( \Delta m^2_{23}, \sin^2 2\theta_{23} \) for different values of \( \alpha_{e\tau} \). For \( \alpha_{e\tau} = 0 \), we find that the minimum chi-square is 4.82 and corresponds to the best fit values \( \Delta m^2_{23} = 3.9 \times 10^{-3} eV^2, \sin^2 2\theta_{23} = 1 \). In Fig 2, we show the 90\% C.L allowed region when the long-range force is taken to be zero (solid line). We increase \( \alpha_{e\tau} \) in small steps and observe that the \( \chi^2_{\text{min}} \) increases from the \( \alpha = 0 \) value, and the allowed parameter space of \( \Delta m^2_{23}, \sin^2 2\theta_{23} \) shrinks as shown in Fig 2. We find that when \( \alpha_{e\tau} = 6.4 \times 10^{-52} \) there is no allowed parameter space which is consistent with the Super-K observations of muon-neutrino events in the multi-GeV energy range. From this we derive the upper bound on \( \alpha_{e\tau} < 6.4 \times 10^{-52} \) at 90\% C.L.

\( \nu_\tau \rightarrow \nu_\tau \) gauge symmetry

The \( \nu_\mu \rightarrow \nu_\tau \) oscillations are governed by the following evolution equation when the long range potential arise from the exchange of the \( L_e - L_\mu \) gauge bosons.

\[
\begin{align*}
\frac{d}{dt} (\nu_\mu) &= \left( \frac{\Delta m^2_{23} \cos 2\theta_{23} - V_{\mu\tau}}{4E_\nu} \right) (\nu_\mu) + \left( \frac{\Delta m^2_{23} \sin 2\theta_{23}}{4E_\nu} \right) (\nu_\tau) \\
\frac{d}{dt} (\nu_\tau) &= \left( \frac{\Delta m^2_{23} \cos 2\theta_{23}}{4E_\nu} \right) (\nu_\mu) + \left( \frac{\Delta m^2_{23} \sin 2\theta_{23}}{4E_\nu} \right) (\nu_\tau)
\end{align*}
\]

The expression for the \( \nu_\mu \) and \( \nu_\tau \) survival probabilities are identical to \([8][8][8][8]\) with \( V_{e\tau} \) replaced with \( -V_{e\mu} \). Therefore, the survival probabilities of \( \nu_\mu \) and \( \nu_\tau \) in case of the \( L_e - L_\tau \) and the \( L_e - L_\mu \) symmetry satisfy the following relations:

\[
P_{\mu\mu}(V_{e\tau}) = P_{\tilde{\mu}\tilde{\mu}}(-V_{e\tau}) = P_{\mu\mu}(-V_{e\mu}) = P_{\tilde{\mu}\tilde{\mu}}(V_{e\mu})
\]

Using the same procedure as discussed above we find that in case of the \( L_e - L_\tau \) symmetry, the upper bound on the coupling constant is \( \alpha_{e\mu} < 5 \times 10^{-52} \) (Fig 3).

We have concentrated here on the \( \nu_\tau - \nu_\tau \) oscillations and \( \mu \)-like events at Super-K. In general, the \( \nu_\tau - \nu_\mu \) oscillations would also get affected by the presence of the additional potentials considered here. When the \( \nu_e - \nu_\mu \) oscillations are governed by the solar scale, the survival probability of the atmospheric electron neutrinos is nearly one. Addition of potential tend to only suppress the \( \nu_e - \nu_\mu \) oscillations and one would not get any bound from the study of the electron-like events at Super-K. There would exist some limited ranges of parameters where these oscillations would be resonantly enhanced due to the contribution from \( V_{e\mu,\tau} \). Such parameter space would any way be ruled out from the non-observations of the atmospheric electron neutrinos.

While we concentrated on the atmospheric neutrinos, \( \Delta m^2_{12}/2E \) in case of the solar neutrinos is not significantly larger than the value of the potential in eq.\([11]\) and the parameter \( \xi \) in eq.\([10]\) can become comparable to \( \cos 2\theta_{\text{sol}} \) for the upper bound on \( \alpha_{e\mu,\tau} \) found here. This would effect the effective solar mixing angle both inside the Sun and at the earth. Thus the \( L_{e,\mu,\tau} \) would be expected to produce observable effects on the solar neutrino oscillations also.

Let us now give a possible example of the theoretical generation of the oscillation parameters in the presence of the \( L_e - L_{\mu,\tau} \) symmetry. We choose a specific case of the \( L_e - L_\tau \) symmetry. Without specifying underlying mechanism, we assume that neutrino masses are generated by effective five dimensional operators constructed from the standard model fields and an additional Higgs doublet \( \phi' \) having the \( L_e - L_\tau \) charge -1. The gauge invariance of the model implies the following structure:

\[
-L_m = \frac{1}{2} \left[ \phi^T T_2 \tau^a \phi \right] M \left[ \frac{1}{2} \left[ \beta_{13} (l^T C \tau^a l_\tau + \beta_{23} l^T \nu^a l_\mu) + \phi^T T_2 \tau^a \phi' \right] \right] + \frac{1}{2} \left[ \beta_{12} (l^T C \tau^a l_\mu) + \phi^T T_2 \tau^a \phi' \right],
\]

where \( M \) is a high scale associated with the physics generating the above operators, e.g. scale of the right handed neutrinos in the seesaw model. We have suppressed Lorentz indices in writing above equation. \( C \) refers to the usual charge conjugate matrix, \( l \) to the leptonic doublet and \( \tau_2, \tau_a (a = 1, 2, 3) \) act in the \( SU(2) \) space.
Eq. (11) generates the following neutrino mass matrix

$$ M_\nu = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & 0 \\ m_{13} & 0 & 0 \end{pmatrix}. \tag{12} $$

The $m_{ij}$ are proportional to $\beta_{ij}$ and can be read off from eq. (11). In the exact symmetry limit corresponding to $< \phi' > = 0$ the above mass matrix describes a Dirac neutrino with mass $m_{13}$ and a Majorana neutrino with mass $m_{22}$. These remain unmixed and there are no neutrino oscillations. A non-zero $< \phi' >$ leads to the required mixing and mass splitting. If parameters $\beta$ and relevant vacuum expectation values (vevs) in eq. (11) are chosen to give a hierarchy $m_{11} \lesssim m_{22} \ll m_{13}$ then the above mass matrix displays an approximate $L_e - L_\mu - L_\tau$ symmetry. This symmetry is known [11] to lead to the successful explanation of the atmospheric neutrino problem. The presence of $m_{11}, m_{22}$ breaks this symmetry and generates the splitting required to explain the solar neutrino oscillations.

It is interesting to note that although we need very light gauge boson $Z_\tau$ with typical mass $M_{Z_\tau} \lesssim 4.8 \times 10^{-22}$ MeV corresponding to the radius of the earth, we do not need to fine tune the symmetry breaking vev $< \phi' >$ to such an extent. This follows since $M_{Z_\tau} \sim g_\tau < \phi' >$ and since $g_\tau \sim 9.9 \times 10^{-26}$, a value of $< \phi' >$ in few GeV range would still keep $M_{Z_\tau}$ very light. Likewise, the $Z - Z_\tau$ mixing $g_{Z-Z_\tau} \sim \frac{g \times \phi'}{< \phi'^2 >}$ also remains very small and does not lead to observable effects such as shift in the $Z$ mass.

We have neglected possible long range Debye-screening of the long range forces due to the solar plasma in our analysis above. This is justified since the screening length for the force is given by $r_D = (4\pi\alpha N_e/T)^{-1/2}$. In the center of the sun the electron density is $N_e \approx 10^{26} cm^{-3}$ and the temperature is $T = 1.5 \times 10^7 K$. Even for $\alpha$ as large as $10^{-49}$ the Debye screening length turns out to be $r_D \approx 10^{16} cm \approx 10^3 AU$. So in the range of $\alpha$ which we discuss there will be no screening at earth-sun distance. The electron density in the sun drops off faster than the temperature so the average $r_D$ of the sun is larger than that in the core.

We conclude that the atmospheric neutrino observations at Super-Kamiokande enable us to put bounds on long range equivalence principle violating forces which are two orders of magnitude more stringent than the corresponding bounds from the classic fifth force experiments.

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