Smart Hybrid Beamforming and Pilot Assignment for 6G Cell-Free Massive MIMO

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Abstract—We investigate Cell-Free massive MIMO networks, where each access point (AP) is equipped with a hybrid transceiver, reducing the complexity and cost compared to a fully digital transceiver. Asymptotic approximations for the spectral efficiency are derived for uplink and downlink. Capitalizing on digital transceiver. Asymptotic approximations for the spectral efficiency are derived for uplink and downlink. Capitalizing on these expressions, a max-min problem is formulated enabling us to optimize the (i) analog beamformer at the APs and (ii) pilot assignment. Simulations show that the optimization of these variables substantially increases the minimum user throughput.

Index Terms—Cell-Free, MIMO, MMSE, RZF; hybrid beamforming, large-scale, optimization, SINR

I. INTRODUCTION

A prospective candidate considered for beyond-5G wireless networks is the cell-free massive MIMO (CF-mMIMO) topology, where every user (UE) potentially connects to every access point (AP), and takes the principles of cell cooperation to the limit; see [1]–[5] and the references therein.

In parallel, forthcoming technologies will be operating at higher frequencies (i.e. mmWave or THz bands), and therefore the transceivers complexity experiences a key trade-off: data rate vs power consumption. Additionally, CF networks will cover larger areas compared to cellular systems, and therefore the severity of the path loss requires the APs to be equipped with large arrays to compensate the attenuation, demanding even more power if fully digital structures are used.

A possible solution that has attracted a lot of attention is a hybrid transceiver [6], [7], composed by two stages: (a) the analog part, in which the antennas are connected to a few RF chains by means of phase shifters, and (b) the digital part. While the former stage dramatically reduces the AP complexity and power consumption, the performance decreases as well. Consequently, properly designing the analog beamformer might be a mean to reduce the performance gap with respect to fully digital transceivers. To the best of our knowledge, there are two main works dealing with the construction of the analog beamformer as a function of slow fading channel parameters [8], [9], which is also investigated in this paper and shown to outperform the previous references.

Once the analog part is designed, we investigate the uplink (UL) and downlink (DL) of two digital benchmarks: (i) the regularized zero forcing (RZF) precoding. Asymptotic approximations of the signal-to-interference-and-noise-ratio (SINR) are derived based on [10], and shown to be tight for finite-dimension systems under the previous decoding/precoding. For a given hybrid structure, and capitalizing on the asymptotic approximations, another relevant problem is studied in this paper: pilot assignment, for which a greedy algorithm based on the asymptotic expressions is provided.

Finally, two novel bounds on the gap between hybrid and fully digital transceivers are derived.

II. SYSTEM MODEL

Consider a CF massive MIMO system composed by $M$ APs, each equipped with $N$ antennas and $L(\leq N)$ RF chains serving $K$ single antenna users (UEs). We assume each AP is connected to a central processing unit (CPU) through high capacity fronthaul links. Denote by $h_{m,k} \in \mathbb{C}^{N \times 1}$ the channel between AP $m$ and UE $k$.

$$h_{m,k} \sim \mathcal{N}(0, R_{m,k}),$$

(1)

with $R_{m,k}$ being the spatial correlation matrix. Each AP performs hybrid beamforming with the aim of reducing the number of RF chains at the transceivers, and therefore their cost and complexity. Particularly, each AP contains an analog matrix $W_m \in \mathbb{C}^{N \times L}$ such that $\left( |W_m| \right)_{n,l} = \frac{1}{\sqrt{N}}$, emulating phase shifters and whose entries will be designed later. As a consequence, the effective channel between AP $m$ and UE $k$ is represented by $g_{m,k} \in \mathbb{C}^{L \times 1}$

$$g_{m,k} = W^*_m h_{m,k}.$$  

(2)

Hence, $g_{m,k} \sim \mathcal{N}(0, R^{(g)}_{m,k})$ with $R^{(g)}_{m,k} = W^*_m R_{m,k} W_m$.

A. Channel Estimation Process

A portion of the total number of resource units, the latter denoted by $\tau_e$, is used for channel estimation. During $\tau_e(\leq \tau_c)$ channel uses, UE $k$ is assigned a pilot $\phi_k \in \mathbb{C}^{\tau \times 1}$ with $||\phi_k||^2 = \tau$ and the pilot matrix is denoted by $\Phi = (\phi_1, \ldots, \phi_K) \in \mathbb{C}^{\tau \times K}$. Upon pilot transmission at a certain power $p(t)$, the observations at the $m$th AP are

$$Y_m = \sqrt{p(t)} (g_{m,1}, \ldots, g_{m,K}) \Phi^T + W^*_m Z_m,$$

(3)

with $Z_m \sim \mathcal{N}(0, \sigma^2 I_N)$ for $\sigma^2$ being the noise power. Standard MMSE estimation leads to the next estimates $[11]$

$$\hat{g}_{m,k} = \sqrt{p(t)} R^{(g)}_{m,k} (\Phi \otimes I_L)^* \Psi^{-1} \text{vec}(Y_m),$$

(4)

with

$$\Psi = p(t) (\Phi \otimes I_L) R^{(g)}_{m,k} (\Phi \otimes I_L)^* + \sigma^2 I_T \otimes W^*_m W_m,$$

(5)
for $R_{m,k}^{(g)} = \text{diag}\{R_{m,k}^{(g)}_{kk} \}$ for $k = 1, \ldots, K$. It can be verified that $g_{m,k} = \hat{g}_{m,k} + \tilde{g}_{m,k}$ with $\tilde{g}_{m,k}$ denoting the error, uncorrelated with the estimate. More concretely, $g_{m,k} \sim \mathcal{N}_\mathbb{C}(0, \Gamma_{m,k}^{(g)})$ with $\Gamma_{m,k}^{(g)}$ defined by
\[
\Gamma_{m,k}^{(g)} = E[\hat{g}_{m,k}^* \hat{g}_{m,k}] = R_{m,k}^{(g)} (\Phi_k \otimes I_L)^* \Psi_m^{-1} (\Phi_k \otimes I_L) R_{m,k}^{(g)}. \tag{6}
\]
and the channel error following $\hat{g}_{m,k} \sim \mathcal{N}_\mathbb{C}(0, \Gamma_{m,k}^{(g)})$ with $\Gamma_{m,k}^{(g)} = R_{m,k}^{(g)} - \Gamma_{m,k}^{(g)}$.

### B. Scalable Cell-Free

Although CF networks allow users to establish connectivity to multiple APs, scalability must be taken into account. Therefore only a subset of APs jointly serve a particular user. Hence, we define by $\mathcal{F}_k$ the subset of APs involved in the decoding of the $k$th UE and by $\mathcal{U}_m$ the subset of UEs treated as signal by AP $m$. Thus, the binary matrix $M = (m_1, \ldots, m_K) \in \mathbb{Z}_2^{K \times K}$ whose entries are
\[
(M)_{m,k} = \begin{cases} 1 & \text{if } k \in \mathcal{U}_m \\ 0 & \text{otherwise} \end{cases}, \tag{8}
\]
accounts for scalability. Provided that each AP observes an $L$-dimensional signal after the hybrid beamforming stage, the expanded version of $M$ is $\hat{M} = M \otimes I_L$ with $I_L$ an $L$-dimensional vector of ones. The complementary matrix $\hat{M} = I - M$ accounts for the disregarded UEs per AP.

### C. Uplink & Downlink Data Transmission

After data transmission, the signal collected by the $M$ APs is $y = (y_1, \ldots, y_M)^T \in \mathbb{C}^{ML \times 1}$ with $y_m \in \mathbb{C}^{L \times 1}$
\[
y = (M^{(s)} \circ G)x + (M^{(i)} \circ G)x + W^*n, \tag{9}
\]
with $\circ$ denoting the Hadamard product, $G \in \mathbb{C}^{ML \times K}$ being the effective channel matrix whose entries are $(G)_{m,k} = g_{m,k} \in \mathbb{C}^{L \times 1}$. Vector $x = (\sqrt{p_{s1}}, \ldots, \sqrt{p_{sK}})^T$ for given UE transmit powers and symbols, denoted by $p_{sk}$ and $s_k$, respectively. Finally, $W = \text{diag}\{W_m\}$ for $m = 1, \ldots, M$ and $n = (n_1, \ldots, n_M)^T$ where $n_m \sim \mathcal{N}(0, \sigma^2 I_N)$.

In the downlink, the APs jointly decode the users data. More particularly, the precoder intended for UE $k$ is denoted by $v_k \in \mathbb{C}^{ML \times 1}$ and after data transmission, the signal collected at UE $k$ is
\[
y_k = \sum_{i=1}^K g_{k,i}^* v_i \sqrt{p_i s_i} + n_k, \tag{10}
\]
where $n_k \sim \mathcal{N}(0, \sigma^2)$.

## III. Spectral Efficiency Analysis

### A. Uplink MMSE Reception

Provided that for UE $k$ only $|\mathcal{F}_k|$ APs are relevant, taking the rows of $y$ in (9) associated to $\mathcal{F}_k$ produces the following reduced signal model
\[
y_k = M_k^{(s)} \circ \hat{G}_k x + (M_k^{(s)} \circ \hat{G}_k + M_k^{(i)} \circ G_k) x + W_k^* n, \tag{11}
\]
where matrices in (11) are $|\mathcal{F}_k|L \times N$-dimensional, i.e. the reduced version of the original matrices associated to $\mathcal{F}_k$. Note that $z_k \sim \mathcal{N}_\mathbb{C}(0, \Sigma_k)$ with $\Sigma_k$ being a block diagonal matrix $\Sigma_k = \text{diag}\{\Sigma_{k,m} \in \mathbb{C}^{L \times L} \text{ for } m \in \mathcal{F}_k\}$ where the diagonal terms are
\[
\Sigma_{k,m} = \sum_{i \in \mathcal{U}_m} C_{m,i}^{(g)} p_i + \sum_{i : i \neq k} R_{m,i}^{(g)} p_i + \sigma^2 W_m^* W_m. \tag{12}
\]
In the uplink, the combiner maximizing the SINR is the MMSE, achieving a maximum value of
\[
\text{SINR}_k = \hat{g}_k^* \left( \sum_{i \neq k} (m_{k,i}^{(s)} \circ \hat{g}_i)(m_{k,i}^{(s)} \circ \hat{g}_i)^* p_i + \Sigma_k \right)^{-1} \hat{g}_k. \tag{13}
\]
where $\hat{g}_k$ and $\hat{g}_i$ are the $k$th and $i$th columns of $\hat{G}_k$, respectively, and a similar definition applies to $m_{k,i}^{(s)}$. As a consequence, after accounting for the pilot overhead $\frac{\tau}{L}$, the ergodic spectral efficiency that the $k$th UE can achieve is
\[
\text{SE}_k = \left( 1 - \frac{\tau}{\tau_c} \right) \text{E}\{\log_2(1 + \text{SINR}_k)\}. \tag{14}
\]

### B. Downlink RZF Precoding

Various precoding strategies can be used to encode the users data. However, RZF provides an outstanding performance as studied in the literature. More particularly, the subset RZF precoding, denoted by $V = (v_1, \ldots, v_K)$, follows
\[
V = (v_1, \ldots, v_K) = [(M^{(s)} \circ \hat{G})(M^{(s)} \circ \hat{G})^* + \rho I_{ML}]^{-1} (M^{(s)} \circ \hat{G}) \Lambda, \tag{15}
\]
with $\rho$ being the regularization parameter and $\Lambda = \text{diag}(\lambda_1, \ldots, \lambda_K)$. Different formulations can be used for $\lambda_k$, such as to ensure (i) $E[||W v_k||^2] \leq 1$ or (ii) $||W v_k||^2 \leq 1$. In our case, since perfect CSI is not available, we use the former formulation. Once User $k$ receives $y_k$, as defined in Eq. (10), the following spectral efficiency can be achieved:
\[
\text{SE}_k = \left( 1 - \frac{\tau}{\tau_c} \right) \log_2(1 + \text{SINR}_k), \tag{17}
\]
with
\[
\text{SINR}_k = \frac{\text{E}\{||g_k^* v_k||^2||p_k \}}{\sum_{i \neq k} \text{E}\{||g_i^* v_k||^2||p_i + \text{var}(g_k^* v_k)\} + \sigma^2}. \tag{18}
\]

## IV. Asymptotic Analysis

To evaluate the previous SINR expressions, we consider the asymptotic regime, $MNK \to \infty$ with finite $MN/K$ and investigate the convergence of the spectral efficiency expressions to deterministic limits. Provided that the subsets account for the non-zero entries in the random matrices, it is required that they grow with the network as well, i.e., $|\mathcal{F}_k|N, |\mathcal{U}_m| \to \infty \forall k,m$. The premises for this convergence need the involved matrices to satisfy two technical conditions: (a) the inverse of the resolvent matrix in (13) and (15) to
exist, ensured by $\Sigma_k$ and $\rho I_{ML}$, respectively, and that (b) $\Gamma_k^{(g)} = \text{diag}\{m_mk \cdot \Gamma_{m,m}^{(g)} \} m = 1, \ldots, M$ has uniformly bounded spectral norm, for $m_mk$ being the $(m,k)$ element of (8). Under these conditions, the following approximations can be made.

**Theorem 1.** For $|F_k[N, |U_m|] \rightarrow \infty \forall k, m$ and UL MMSE combining, $\text{SINR}_k \approx \overline{\text{SINR}}_k$ with $\overline{\text{SINR}}_k$ given in (19).

\[
\text{SINR}_k = \frac{p_k}{|F_k[N]|} \sum_{m \in F_k} \text{tr}\left[\Gamma_{m,k}^{(g)} T_{m,k}\right],
\]

where

\[
T_{m,k} = \left(\frac{1}{|F_k[N]|} \sum_{i=1}^{K} \frac{m_{m,i} \cdot \Gamma_{m,i}^{(g)} p_i}{1 + e_i} + \frac{1}{|F_k[N]|} \Sigma_{m,k}\right)^{-1}.
\]

The coefficients $e_i$ are obtained iteratively, $e_i = \lim_{n \rightarrow \infty} e_i^{(n)}$, given $e_i^{(0)} = |F_k[N]$ and the recursion in (21).

\[
e_i^{(n)} = p_i \text{tr}\left[\Gamma_i^{(g)} \left(\sum_{j=1}^{K} \frac{\Gamma_j^{(g)} p_j}{1 + e_j^{(n-1)}} + \Sigma_i\right)^{-1}\right].
\]

**Proof.** The proof can be found in [12, App. C].

Moreover, matrix

\[
T' = \left(\frac{\rho}{MN}, \Gamma_k^{(g)}\right) = TT_i^{(g)} + T + \frac{1}{M} \sum_{k=1}^{K} \frac{\Gamma_k^{(g)} e_k'}{(1 + e_k')^2},
\]

and coefficients $e' = (e'_1, \ldots, e'_K)$ are calculated as

\[
e'(\frac{\rho}{MN}) = (I_K - J)^{-1} v(\frac{\rho}{MN}),
\]

with $J \in \mathbb{C}^{K \times K}$ and $v(\frac{\rho}{MN}) \in \mathbb{C}^{K \times 1}$ defined as

\[
(J)_{k,l} = \frac{1}{MN} \text{tr}\left[\Gamma_{k,l}^{(g)} T_{T_i}^{(g)}\right],
\]

\[
\left(v(\frac{\rho}{MN})\right)_{k,l} = \frac{1}{MN} \text{tr}\left[\Gamma_{k,l}^{(g)} T_{T_i}^{(g)}T\right].
\]

**Proof.** The proof can be found in [12, App. D].

From the continuous mapping theorem [13], the following holds: $\text{SE}_k \approx \left(1 - \frac{1}{2e}\right) \log_2(1 + \text{SINR}_k)$ with the corresponding $\text{SINR}_k$ provided above for $MN$ and $K \rightarrow \infty$.

**V. Spectral Efficiency Optimization**

Note that the asymptotic SE approximations derived in the previous section only depend on large scale parameters. Therefore, we can formulate different asymptotic optimization problems. However, with the aim of increasing fairness in the network, we consider the following max-min problem:

\[
\max_{W, \Phi} \min_k \overline{\text{SINR}}_k.
\]

\[\text{s.t. } \left(|W_m|\right)_{n,l} = \frac{1}{\sqrt{N}},
\]

where the optimization variables are two: (i) analog beamforming matrix $W$ and (ii) pilot matrix $\Phi$, studied separately.

**A. Analog Beamformer Design**

The design of $W = \text{diag}\{W_m\}$ for $m = 1, \ldots, M$ is challenging given the complexity of the SINR. Therefore, directly solving (32) poses a major challenge. However, under perfect CSI, some algebraic properties on $W$ can be extracted and therefore used for its design. Concretely, we first disregard the unit-modulus constraint and after SVD decomposition $W$ factorizes as $W = U Q$ with semi-unitary $U$, i.e. $U^*U = I_{ML}$.

**Proposition 1.** Under perfect CSI UL-MMSE reception, any nonsingular $Q$ provides maximum SINR.

**Proof.** The proof can be found in [12, App. E].

According to [14], $\sum_{k \neq k} |g_k^* v_i|^2 p_i + \sigma^2 \approx \sum_{k \neq k} |g_k^* v_k|^2 p_i + \sigma^2$. Under the condition that the previous approximation is tight, the following proposition, which is similar to the result obtained in [9] for another metric, can be obtained.

**Proposition 2.** Under perfect CSI DL-RZF precoding, the SINR is maximum when $Q$ is semi-unitary: $QQ^* = I_{ML}$.
Proof. The proof can be found in [12, App. F].

In order to full-fill both propositions, for UL and DL, $Q$ can be set to $Q = I_ML$ and therefore $W = U$ meaning that the analog matrix should have orthogonal columns. The idea behind having orthogonal columns is that interference is reduced. To the best of our knowledge, there are two ways of smartly creating $W$ explained in [8] and [9], respectively. While the latter is based on perfectly known channels, the former fails to capture the complete spectrum of the channel covariance matrices. In this work, we propose a method that takes into account all possible eigenvectors/eigenvalues of all $R_{m,k}$ with the aim of maximizing the minimum average UE power signal, which is shown to maximize that takes into account all possible eigenvectors/eigenvalues covariance matrices. In this work, we propose a method to improve the design of the analog matrix exploiting the following proposition.

**Proposition 3.** Assume that instead of using $W_m^{(p)}$ as the analog matrix, $W_m^{(p)}A_m$ is the new analog beamformer with $A_m \in \mathbb{C}^{LxL}$ nonsingular. The product between $W_m^{(p)}A_m$ provides the same optimality as $W_m^{(p)}$ and therefore $W_m^{(p)}A_m$ is an optimal unconstrained analog matrix.

Proof. The proof can be found in [12, App. G].

Using the previous proposition, the initial unconstrained beamformer $W_m^{(p)}$ can be replaced by $W_m^{(p)}A_m$ without a performance degradation as long as $A_m$ is nonsingular. As a consequence, we can formulate the following optimization problem:

$$
\min_{W_m, A_m} \|W_m - W_m^{(p)}A_m\|_F
$$

s.t. $\|W_m\|_{\infty,i} = 1/\sqrt{N}$

Thanks to the degrees of freedom added by $A_m$, the constrained analog beamformer $\hat{W}_m$, can be made closer to the unconstrained one $W_m^{(p)}$. By alternating minimization, we split the previous problem into two sub-problems: (i) find the optimal $A_m$ for fixed $\hat{W}_m$ and (ii) find the optimal $\hat{W}_m$ for fixed $A_m$. The solution to the previous subproblems is

$$
A_m = W_m^{(p)}\hat{W}_m^*,
$$

$$
\hat{W}_m = \frac{1}{\sqrt{Nn}}\exp(\mathbf{g}_m)^T(A_m)^T.
$$

An iterative process based on the block coordinate descend method follows until convergence is reached [15]. Therefore, a constrained analog matrix will be obtained and thus from Eq. (36) we can create $F_m$ that goes into the baseband (or digital) part. As a consequence, the equivalent channel between AP $m$ and UE $k$ has an extra component:

$$
g_{m,k} = F_m^*\hat{W}_m^*h_{m,k}.
$$

B. Pilot Assignment Optimization

The optimal solution to (32) with respect to $\Phi$ requires an exhaustive search over the set of possible pilot sequences. However, based on the correlation between effective channels: $
\Delta_{k,i} = \text{tr}(\Gamma_k^T \Gamma_i^T)$ for $k \neq i$, an initial pilot assignment can be made, denoted by $\Phi^{(0)}$. Particularly, a set of users is assigned the same pilot if their normalized cross-correlation,
The proof can be found in [12, App. H].

Algorithm 1 Greedy pilot assignment

Require: Set of available pilots, \( \mathcal{S} = \{s_1, \ldots, s_{|\mathcal{S}|}\} \) and initial pilot assignment \( \Phi^{(0)} \) at iteration \( j = 0 \).

Define the cost function \( \mu^{(0)} = \min_k \text{SINR}_k(\Phi^{(0)}) \).

while \( |\mu^{(j+1)} - \mu^{(j)}| > \epsilon \) do

For each UE \( u = 1, \ldots, K \) solve

\[
\phi_u^{(j+1)} = \arg \max_{\phi \in \mathcal{S}} \min_k \text{SINR}_k(\phi) \quad (41)
\]

Update cost function \( \mu^{(j+1)} = \min_k \text{SINR}_k(\phi^{(j+1)}) \).

end while

VI. \( M \to \infty \) REGIME

Finally, we focus on the case where \( M \to \infty \) under perfect CSI. For simplicity, assume \( M^{(1)} = 1 \) and recall that a full digital structure is the one providing the best performance in terms of SE, attained when \( L = N \) and \( W_m = I_N \). Then, the following can be derived.

Proposition 4. Define the gap as the difference in SINR between full digital and hybrid. Then, there exist lower and upper bounds for the gap, denoted by \( \delta_{\text{LB}} \) and \( \delta_{\text{UB}} \), given by

\[
\delta_{\text{LB}} = \frac{p_k}{\sigma^2} \sum_{m=1}^{M} \sum_{n=L+1}^{N} \lambda_{m,k}^{(n)}.
\]

\[
\delta_{\text{UB}} = \frac{p_k}{\sigma^2} \sum_{m=1}^{M} \left( \sum_{n=1}^{N} (\lambda_{m,k}^{(n)} - \lambda_{m,k}^{(N-L+n)}) + \sum_{n=L+1}^{M} \lambda_{m,k}^{(n)} \right) \quad (43)
\]

Proof. The proof can be found in [12, App. H].

\[\Box\]

Note that if the channel matrices are rank-deficient, i.e. \( \text{rank}(R_{m,k}) \leq L \), the gap can be as small as zero and therefore a hybrid structure would achieve the same performance as digital.

VII. SIMULATION RESULTS

For the purpose of performance evaluation, we consider a \( 200 \times 200 \) m\(^2\) wrapped around universe. To generate the channel model, we assume that the APs are deployed in urban environments at around 10 m, matching with the 3GPP Urban Microcell model in [16, Table B.1.2.1-1] at an operating frequency of 2 GHz. The shadowing terms given an AP to different UEs present a certain correlation, given by the model in [16, Table B.1.2.2.1-1]. The number of total channel uses is \( \tau_c = 200 \). Unless otherwise specified, in order to take into account the effects of pilot contamination \( \tau = 8 \) orthogonal pilots and \( K = 16 \) UEs (i.e. reuse factor of two). Additionally, each AP has \( N = 32 \) antennas. The UE transmit power is set to \( 200 \) mW, \( \sigma^2 = -96 \) dBm and \( \rho = 10^{-4} \). Moreover, to account for scalability, the \([m,k]\) entry of \( M_{[m,k]} \) is 1 if \( d_{m,k} \leq R_{\text{max}} \), which ensures connectivity to multiple APs per GU for \( d_{m,k} \) the Euclidean distance between AP \( m \) and UE \( k \). Finally, \( \epsilon = 0.001 \) to ensure enough iterations until convergence is reached.

The applicability of Theorems 1 and 2 to finite-dimensional systems is first verified in Figs. 1 and 2, where the approximations are denoted by RMT in the legend. For different network setups, corresponding to \( (M = 4, N = 32, L = 16) \) and \( (M = 12, N = 32, L = 8) \), the approximations obtained in Th. 1 and 2 respectively are indeed accurate for \( K = 16 \) and \( \tau = 8 \) orthogonal pilots.

In Fig. 3, we compare the UL pilot assignment obtained by Alg. 1 (Greedy) and a random assignment (RA) for different values of \( N \) and \( L \). For \( N = L = 16 \) we assume a digital structure while for \( N = 32 \) and \( L = 8 \) the analog matrices
analog beamformer and pilot assignment. The solution to

\( N \) 

is measured and is of about 60% and 90% for UL and DL, respectively. Additionally, the improvement in terms of minimum SE is highly dependant on the eigenvalues of the channel correlation matrices.

Finally, theoretical bounds for the gap between full digital and hybrid structures are presented, showing that such a gap is highly dependant on the eigenvalues of the channel correlation matrices.

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