Higher order electro-magneto-elastic free vibration analysis of piezomagnetic nano panel

Guoping Wang¹, Huadong Hao² and Mohammad Arefi³,*

¹Department of Mechanical Engineering, Xi’an Jiaotong University City College, Xi’an 710018, Shaanxi, China; ²Zhoushan Institute of Calibration and Testing for Quality and Technology Supervision, Zhoushan 316013, Zhejiang, China and ³Department of Solid Mechanics, Faculty of Mechanical Engineering, University of Kashan, Kashan 87317-51167, Iran

*Corresponding author. E-mail: arefi63@gmail.com

Abstract

This paper investigates electro-magneto-elastic free vibration responses of piezomagnetic cylindrical nano panel subjected to electro-magneto-mechanical loads based on third-order theory. Third-order shell theory is used for description of the displacement field. The zero transverse shear strains are obtained using the third-order displacement field. Hamilton’s principle is employed to obtain the governing equations of motion. The nano panel is subjected to a coupling of magnetic and electric loads, including a linear function along with the thickness direction and a 2D function along with the axial and circumferential directions. To account the effect of nanoscale in governing equations, the Eringen nonlocal elasticity theory is used. The numerical results are obtained to investigate the impact of significant parameters such as axial and circumferential mode numbers, the nanoscale parameter, applied electromagnetic potentials, and length-to-radius ratio. It is concluded that an increase in initial electric potential and a decrease in magnetic potential lead to an increase in natural frequencies of the nano panel.

Keywords: higher order shear deformation theory; nonlocal elasticity; piezomagnetic nano panel; applied electromagnetic potentials; span angle

1. Introduction

Free vibration analysis of structures is one of the essential requirements for designing structures and mechanical elements. The calculation of natural frequencies of systems and structures guides the designers to reach an acceptable interval for actuation frequency. This act can avoid resonance and leads to an acceptable design. The calculation of natural frequency based on various theories in various scales leads to different results that encourage researchers to investigate new theories and new size-dependent theories. In this work, a higher order shear deformation theory is used for free vibration analysis of a piezomagnetic nano panel. Shear deformation theories have been proposed for the 2D analysis of plates and shells instead of a three-dimensional (3D) one. More accurate results can be obtained by selecting new theories with new shape functions for better estimation of transverse shear strains along the thickness direction. To account the effect of very small sizes of structures, especially in nanoscale and microscale, some size-dependent theories have been proposed by many scientists. To capture the impact of nanoscale sizes, Eringen’s nonlocal elasticity theory has been used in multiple works. The literature review is presented in this section with considering related works on the cylindrical panel, nonlocal elasticity, and piezomagnetic materials.

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Chung (1981) presented a general analytical method for the dynamic investigation of circular cylindrical shells with classical boundary conditions based on Sanders’ shell theory. The numerical results were confirmed with the finite element approach (FEA). The effect of a new higher order shear deformation theory on the dynamic analysis of circular cylindrical shells was investigated by Bhimaraddi (1984). Plane elasticity theory was used by Tutuncu and Ozturk (2001) for bending analysis of spherical and cylindrical pressure vessels made from radially variable material properties. The effect of variation of material properties on the dynamic behavior of laminated composite shear deformable cylindrical panels was studied by Singh et al. (2002) based on the perturbation approach. Effect of some important parameters such as core thickness-to-face-sheet thickness ratios, various boundary conditions, variable material properties was studied on the free vibration responses. Ding et al. (2002) employed a 3D model for vibration responses of a circular cylindrical panel made from transversely isotropic piezoelectric with various boundary conditions using the variable separation method. It has been deduced that the natural frequencies for the piezoelectric panel are more than the nonpiezoelectric one.

There are some practical (Berlincourt et al., 1964; Ballato, 2001; Liu et al., 2005; Hasayana et al., 2012) and theoretical (Fan et al., 2015; Yan, 2018; Fan, 2020) models for the application of piezoelectric/piezomagnetic materials in nanoscale and microscale in various situations. These materials are used as energy harvester and some other applications as sensor, actuator, and transducers. Yang and Shen (2003) investigated the effect of an arbitrary combination of static and dynamic forces as well as thermal environment on the dynamic instability of functionally graded (FG) cylindrical panels with different boundary conditions based on Reddy’s higher order shear deformation theory. Temperature dependence, and functionality, was accounted for all material properties along the thickness direction. The numerical results were obtained using the one-dimensional differential quadrature method (DQM) as well as Galerkin and Bolotin methods. Zhao et al. (2004) studied the dynamic analysis of cylindrical panels with two simply supported (SS) boundary conditions with the development of Love theory and classical thin shell theory using the mesh-free method. The effect of uniform heat generation was studied on the time-dependent thermoelastic analysis of an FG hollow cylinder by Ootao and Tanigawa (2006). Free vibration analysis of a cylindrical panel made of homogeneous and isotropic materials with SS boundary conditions was studied by Sharma and Pathania (2003) based on 3D piezoelasticity equations and using modified Bessel function solution with complex arguments. The effect of various electric potential conditions was studied on the responses.

The effect of a dynamic load was studied on the transient responses of a thick-walled cylindrical shell made from FG materials by Shakeri et al. (2006). The governing equations of motion were reduced to simpler form using Newmark methods and finally were solved using the Galerkin finite element. They used a computational method named as division method where the cylinder has been divided into some annular cylinders. The continuity of displacements was applied to the obtained displacements. Thermoelastic vibration and buckling analyses of FG piezoelectric (FPG) cylindrical shells were studied based on Hamilton’s principle and quadratic variation of electric potential as well as first-order shear deformation theory (FSDT). Rahmani et al. (2010) developed a new higher order model for the dynamic analysis of composite sandwich cylindrical shell including a soft core. The influence of the geometrical parameters of the shell was studied on the responses. Asgari and Akhlaghi (2011) studied the effect of bi-directional functionality on the dynamic responses of a short-length FG cylindrical shell. Eshmatov and Khodjaev (2007) studied vibration and stability analyses of a viscoelastic cylindrical panel with a concentrated mass. The kinematic relations were developed based on the Kirchhoff-Love hypothesis considering the nonlinear geometric form of strain-displacement relations. The formulation of the problem using the proposed theories has led to an integro-differential equation. The effect of radially temperature gradient on the elastoplastic results of a cylindrical panel was studied based on generalized plane strain by Arslan and Haskul (2015). Elastic limits of the cylindrical panels were computed based on Tresca and von Mises criteria. Shojaeae et al. (2017) studied the free vibration analysis of skewed cylindrical panel reinforced by FG carbon nanotubes based on the first-order shear deformation shell theory and Hamilton’s principle. The effect of various electric potential parameters such as radius to length and side length ratios as well as various characteristics of carbon nanotubes was studied on the free vibration responses.

Yas and Sobhani Aragh (2011) investigated 3D free vibration analysis of FG fiber orientation cylindrical panel based on Hamilton’s principle and DQM. They studied the effect of symmetric and asymmetric fiber orientation profiles and a random variation of fiber orientation on the dynamic responses. Laplace transform and Laplace inverse methods were employed for the dynamic analysis of SS FGP cylindrical panel subjected to time-dependent blast pulses based on Hamilton’s principle and the FSDT by Bodaghi and Shakeri (2012) with various electrical boundary conditions. The effect of von Karman nonlinearity as well as Donnel shell theory was studied on the nonlinear dynamic response of eccentrically stiffened FG cylindrical panels with geometric imperfections by Bich et al. (2012). Zuo et al. (2015, 2017) studied the transport of intensity phase retrieval and computational imaging for partially coherent fields. Wang et al. (2020) developed a new regularization method for dynamic load identification. Zhang et al. (2020a, b, c) studied transport-of-intensity equation and resolution analysis in a lens-free on-chip digital holographic microscope. Hu et al. (2020) provided a review on the microscopic fringe projection profilometry in micro-electro-mechanical systems. Huo et al. (2021) studied the determination of interface modes in 2D acoustic systems. Some works (see e.g. Yang et al., 2015; Sun et al., 2015; Zhang et al., 2019, 2020; Gao & Lu, 2020; Wang et al., 2020a; Yan et al., 2020; Gao et al., 2021a, b, c, d) on the application of electromagnetic loads in engineering and industrial application were developed by various researchers. There are some works (see e.g. Zhu et al., 2020; Chen et al., 2021; Wang et al., 2021a, b) on applying nanostructures in new situations. The Budiansky–Roth method was used for finding nonlinear dynamic critical buckling loads. Free vibration analysis single-walled carbon nanotubes reinforced cylindrical panel was studied by Yas et al. (2013) based on the 3D theory of elasticity. Effective material properties of the composite reinforced cylindrical panel were computed based on the extended rule of mixture. The generalized differential quadrature method (GDQM) was used to solve variable coefficients differential equations of motion to investigate the effect of type and amount of reinforcement.

Influences of centrifugal and Coriolis forces were studied on the dynamic results of the rotating FG cylindrical shells in the thermal environment based on the FSDT by Malekzadeh and Heydarpour (2012) based on DQM. They investigated the effect of main parameters such as rotational speed, in-homogeneity, and temperature dependence of material on the responses. FSDT was used by Arefi and...
Rahimi (2014) to study thermoelastic analysis of an FG cylindrical shell and an FGP shell. Alibeigloo and Liew (2014) developed an analytical method for 3D free vibration analysis of cylindrical sandwich panels, including an FG core based on the Fourier series. Based on the analytical method, the natural frequencies were obtained in terms of in-homogeneous index, span angle, and geometric parameters. 3D thermoelastic analysis of an SS sandwich panel made from FGM integrated with two face-sheets was studied by Alibeigloo (2014b). The power-law function was used to describe material properties variation along the thickness direction except for the Poisson’s ratio that was assumed constant. The trueness and accuracy of the presented method were justified using comparison with some related works. The effect of a higher order shear deformation theory was studied on the electroelastic results of FG plates and shells by Mohammadimehr et al. (2016) and Zur et al. (2020), respectively. Arefi et al. (2012) investigated exact solution of an FGP cylindrical shell subjected to thermal, mechanical, and electrical loads. The effect of nonuniform pressure and short length was studied on the elastic analysis of FG cylindrical shell by Khoshgoftar et al. (2013); Shen et al. (2015) and Shen & Wang (2014) developed relations of the curvilinear coordinate system for nonlinear vibration analysis of the doubly curved panels and cylindrical shell subjected to thermal loads, respectively. The effect of various distributions and amounts of the carbon nanotube reinforcement was studied on the 3D dynamic responses of the reinforced composite cylindrical shell by Alibeigloo (2014a). The solution of the governing equations was performed by the Fourier series expansion and state-space technique. Dynamic responses of the reinforced composite plate were studied by Mirzaei and Kiani (2016) based on FSDT and Donnell shell theory. Effective material properties of carbon nanotubes reinforced material were computed based on refined rule of mixtures. The solution method for finding the system’s eigenvalue was developed based on the Ritz method including the Chebyshev polynomials. Uysal and Güven (2015) studied buckling analysis of sandwich plate made from an epoxy reinforced with two various types of graphite. The sandwich plate was subjected to a combination of external loads such as in-plane compression and shear force. Image processing program was used for the estimation of the effective material properties. The effect of the type and volume of graphite powders was studied on the buckling characteristics based on the FEA. Nonlinear bending analysis of reinforced composite cylindrical panel was studied by Shen and Xiang (2014) based on higher order shear deformation theory and accounting von Kármán-type kinematic nonlinearity. After the derivation of the governing equations based on virtual work principle and performing the solution procedure based on perturbation method, the obtained results were presented to investigate the effect of main parameters such as kind and amount of carbon nanotubes as reinforcement on the nonlinear bending results. Kiani et al. (2018) developed the Chebyshev–Ritz formulation for free vibration analysis of carbon nanotube reinforced composite cylindrical panel based on FSDT. Effective material properties of the reinforced cylindrical shells were computed based on the rule of mixture for various types of reinforcements in terms of some different efficiency parameters and volume fraction. Wang and Li (2015) studied nonlinear thermal buckling and post-buckling analysis of double-walled carbon nanotubes subjected to thermal loads based on a nonlocal continuum model. It was concluded that large mode numbers have a significant influence on the nonlinear post-buckling characteristics. Furthermore, it was deduced that maximum deflection becomes very important on the nonlinear post-buckling characteristics for short-length nanotubes. Wang and Li (2016) studied dynamic instability analysis of double-walled carbon nanotubes subjected to axial harmonic excitation based on nonlocal continuum theory. The governing equations were derived using Mathieu form by the Galerkin’s theory. The instability conditions were defined using Bolotin’s method. They expressed that accounting the van der Waals force leads to significant improvement of stability condition of double-walled nanotubes. Wang (2017) studied the nonlinear internal resonance of double-walled nanobeams subjected to external parametric load based on nonlocal continuum theory. It was concluded that the gap between stable and unstable regions might be reduced by accounting the van der Waals force that leads to increase in excitation amplitude. Samaniego et al. (2020) developed a machine learning method for solving partial differential equations using some specific functions with desirable properties. They summarize some incompleteness of some solution methods such as finite element method, mesh-free method, and isogeometric analysis. Anitescu et al. (2019) developed an artificial neural network and an adaptive collocation strategy to solve the partial differential equations such as Poisson and Helmholtz equations. The method was contained an initial stage with a coarse grid of training points with addition of more points to further stages. Rabczuk et al. (2019) developed the application of a novel nonlocal operator to the solution of partial differential equations and eigenvalue analysis. They presented advantages of the proposed method for solution of differential electromagnetic vector wave equations based on electric fields through the conversion of governing equations into nonlocal integral form. Developed a sensitivity analysis to identify the influence of main input parameters on the energy conversion factor of a structure made of flexoelectric material. The governing equations were derived using isogeometric analysis by applying higher order continuity. Vu-Bac et al. (2016) studied the influence of uncertainties of input parameters on the noise and industrial data based on probability density function. Guo et al. (2019) studied bending analysis of a thin plate based on a deep collocation method. The proposed method included a loss function to minimize differential governing equations and boundary/initial boundary conditions at the collocation points.

The author provided a literature review on the various works related to shear deformation analysis of cylindrical panels subjected to various loadings such as thermal, mechanical, electrical, and magnetic loads. Furthermore, some works on the nanoscale problems were reviewed comprehensively. The author’s knowledge indicated that there is no published work on the magneto-electro-elastic free vibration analysis of piezomagnetic nano panel subjected to applied electromagnetic potentials. To increase the accuracy of the modeling and corresponding numerical results, a third-order shear deformation theory is employed to ensure vanishing transverse shear strains at the top and bottom. Furthermore, accounting size dependence based on nonlocal elasticity theory in conjunction with higher order shear deformation theory is the main novelty of this paper. This paper uses nonlocal elasticity theory and higher order shear deformation theory for magneto-electro-elastic free vibration response analysis of piezomagnetic nano panel.
2. Higher Order Formulation of Piezoelectric Nano Panel

The electro-magneto-elastic governing equations of a piezoelectric nano panel (Figure 1) are derived in this section based on third-order shear deformation theory, nonlocal elasticity, and the principle of virtual work. The nano panel is assumed to be constrained with four SS edges. Furthermore, the nano panel is subjected to a combination of electromagnetic loads along the thickness direction. Shear strains are assumed at the top and the bottom of shell because of proposed higher order shear deformation theory.

The displacement field based on third-order shear deformation theory is expressed as follows:

\[
\begin{align*}
\mathbf{u}(x, \theta, z, t) &= u_0(x, \theta, t) + z\chi_1^x(x, \theta, t) + z^2\chi_2^x(x, \theta, t) + z^3\chi_3^x(x, \theta, t), \\
v(x, \theta, z, t) &= \left(1 + \frac{z}{R}\right)v_0(x, \theta, t) + z\chi_1^\theta(x, \theta, t) + z^2\chi_2^\theta(x, \theta, t) + z^3\chi_3^\theta(x, \theta, t), \\
w(x, \theta, z, t) &= w_0(x, \theta, t),
\end{align*}
\]

(1)

where \(u, v, \) and \(w\) are displacement components along the axial, circumferential, and radial directions, respectively, and \(\chi_1^x, \chi_2^x, \) and \(\chi_3^x\) are rotation components. Furthermore, \(u_0, v_0, \) and \(w_0\) are displacement components of middle surface. The rotation components \(\chi_1^x, \chi_2^x, \) and \(\chi_3^x\) will be chosen to satisfy zero shear strains \(\gamma_{xz} = \frac{1}{R} \frac{\partial w}{\partial \theta} + \frac{1}{R} \frac{\partial u}{\partial z} - \frac{v}{R}, \gamma_{zx} = \frac{1}{R} \frac{\partial v}{\partial x} + \frac{1}{R} \frac{\partial u}{\partial z}\) at top and bottom of shell thickness. Finally, the displacement field is defined as (Shen et al., 2018a, b) follows:

\[
\begin{align*}
u &= u_0 + z\chi_3^x + kz^2 \left(\chi_1^x + \frac{\partial w}{\partial x}\right), \\
v &= \left(1 + \frac{z}{R}\right)v_0 + z\chi_3^\theta + kz^3 \left(\chi_1^\theta + \frac{1}{R} \frac{\partial w}{\partial \theta}\right), \\
w &= w_0.
\end{align*}
\]

(2)
The kinetic energy of cylindrical nano panel is defined based on third-order shear deformation theory as follows:

\[
\delta T = \int_V \left( I_1 \dot{u}_0 \delta u_0 + I_2 \dot{\chi}_1 \delta u_1 + I_3 \dot{\chi}_2 \delta u_2 + I_4 \frac{\partial \dot{u}_0}{\partial x} \delta u_0 + I_5 \frac{\partial \dot{u}_1}{\partial x} \delta u_1 + I_6 \frac{\partial \dot{u}_2}{\partial x} \delta u_2 + I_7 \frac{\partial \dot{u}_0}{\partial y} \delta u_0 + I_8 \frac{\partial \dot{u}_1}{\partial y} \delta u_1 + I_9 \frac{\partial \dot{u}_2}{\partial y} \delta u_2 \right) \mathrm{d}V
\]

in which the integration constants \((i_0, i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9)\) are defined as follows:

\[
(i_0, i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9) = \int_{-h/2}^{h/2} \rho (R + z) \left( \frac{1}{R} \frac{\partial}{\partial y} \left( 1 + z^2, k^2 x^2, k^2 x^2, k^2 z^2, k^2 \frac{\partial w}{\partial y}, -k^2 \frac{\partial w}{\partial y}, k^2 \frac{\partial w}{\partial y}, -k^2 \frac{\partial w}{\partial y}, \frac{\partial w}{\partial y}, \frac{\partial w}{\partial y} \right) \right) \mathrm{d}z .
\]

Rearranging after integration by part leads to:

\[
\delta T = \int_V \int_x \left( \left[ I_1 \dot{u}_0 + I_2 \dot{\chi}_1 + I_3 \dot{\chi}_2 + I_4 \frac{\partial \dot{u}_0}{\partial x} \right] \delta u_0 - \left( I_1 + I_3 \right) \dot{u}_0 - \left( I_1 + I_3 \right) \dot{\chi}_1 - \right. \]

\[
\left. \left( I_1 + I_3 \right) \dot{\chi}_2 + \left( I_4 \right) \frac{\partial \dot{u}_0}{\partial x} \right) \delta u_0 - \left( I_0 + I_1 \right) \dot{v}_0 \frac{\partial \dot{u}_0}{\partial y} + \left( I_0 + I_1 \right) \dot{\chi}_1 \frac{\partial \dot{u}_0}{\partial y} + \left( I_0 + I_1 \right) \dot{\chi}_2 \frac{\partial \dot{u}_0}{\partial y} \right) \delta u_0 \right) \mathrm{d}x \mathrm{d}y .
\]

Based on assumed displacement field, the strain components are obtained as follows:

\[
\varepsilon_x = 0 ,
\]

\[
\varepsilon_y = \frac{u_0}{R} + \frac{1}{R} \frac{\partial u_0}{\partial y} + z \left( 1 + k^2 \chi_1 \right) \frac{\partial \chi_1}{\partial y} ,
\]

\[
\varepsilon_z = \frac{\dot{u}_0}{R} + z \left( 1 + k^2 \chi_2 \right) \frac{\partial \chi_2}{\partial y} + k^2 \frac{\partial^2 u_0}{\partial y^2} ,
\]

\[
g_{yt} = \left( 1 + 3 k^2 \right) \left( \chi_2 + \frac{1}{\partial y} \frac{\partial u_0}{\partial y} \right) ,
\]

\[
g_{zt} = \left( 1 + 3 k^2 \right) \left( \chi_1 + \frac{1}{\partial y} \frac{\partial u_0}{\partial y} \right) ,
\]

\[
y_{yt} = \frac{\dot{u}_0}{R} + \frac{1}{R} \frac{\partial u_0}{\partial y} + z \left( 1 + k^2 \chi_1 \right) \frac{\partial \chi_1}{\partial y} + z \left( 1 + k^2 \chi_2 \right) \frac{\partial \chi_2}{\partial y} + 2 k^2 \frac{\partial^2 u_0}{\partial y^2} \right) \frac{\partial \dot{u}_0}{\partial y} \right) \delta u_0 \mathrm{d}x \mathrm{d}y .
\]

Hamilton’s principle is used to derive governing equations of motion. Using the strain components defined in equation (6), the variation of strain energy is calculated as:

\[
\delta U = \int_V \int_x \left( N_x \frac{\partial \delta u_0}{\partial x} + M_x \frac{\partial \delta \chi_1}{\partial x} + S_x \frac{\partial^2 \delta u_0}{\partial x^2} \right) + \left[ N_y \delta u_0 + N_z \delta \chi_1 + P_x \frac{\partial \delta \chi_1}{\partial y} + P_y \frac{\partial \delta \chi_2}{\partial y} + N_x \delta \chi_1 + N_z \frac{\partial \delta u_0}{\partial x} \right] \right) \mathrm{d}x \mathrm{d}y .
\]

in which the resultant components are defined in Appendix A.

Arranging the variables after integration by part on equation (7) yields variation of strain energy as follows:

\[
\delta U = \int_V \int_x \left( \frac{\partial N_x}{\partial x} \frac{\partial \delta u_0}{\partial x} - \frac{\partial N_x}{\partial x} \delta u_0 + \delta u_0 \frac{\partial \delta u_0}{\partial x} \right) \mathrm{d}V + \left[ \frac{\partial N_y}{\partial y} \frac{\partial \delta u_0}{\partial y} - \frac{\partial N_y}{\partial y} \delta u_0 + \delta u_0 \frac{\partial \delta u_0}{\partial y} \right] \mathrm{d}V + \left[ \frac{\partial N_z}{\partial z} \frac{\partial \delta u_0}{\partial z} - \frac{\partial N_z}{\partial z} \delta u_0 + \delta u_0 \frac{\partial \delta u_0}{\partial z} \right] \mathrm{d}V .
\]

Variation of external work is composed of effect of in-plane mechanical, electrical, and magnetic loads and Pasternak’s foundation. Based on the aforementioned comment, we will have (Zhang et al., 2021)
\[ \delta W = \int \int \left[ (N_0 + N_{\text{me}} + N_{\text{ed}}) \frac{\partial^2 w}{\partial x^2} + (N_0 + N_{\text{me}} + N_{\text{ed}}) \frac{1}{R^2} \frac{\partial^2 w}{\partial \theta^2} - F_f + P_l \right] \delta w \, d\theta, d\phi. \]  

where \( F_f \) is the reaction of Pasternak's foundation expressed as \( F_f = K_1 w_0 - K_2 V^2 w_0 \). Furthermore, \( N_0, N_{\text{me}}, \) and \( N_{\text{ed}} \) are pre-mechanical, electrical, and magnetic loads, respectively.

The constitutive relations based on Eringen nonlocal magneto-electro-elastic theory are obtained as follows (Farajpour et al., 2016; Chen et al., 2020; Zhang et al., 2020):

\[
(1 - \xi^2) \begin{bmatrix}
\sigma_x & \sigma_\theta & \tau_{x\theta} \\
\sigma_\theta & \tau_{x\theta} & \tau_{x\phi} \\
\tau_{x\theta} & \tau_{x\phi} & \tau_{xz}
\end{bmatrix} = \begin{bmatrix}
C_{xxx} & C_{x\phi\theta} & 0 & 0 & 0 & 0 \\
C_{x\phi\theta} & C_{\theta\phi\phi} & 0 & 0 & 0 & 0 \\
0 & 0 & C_{\phi\phi\phi} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{\phi\phi\phi} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{\phi\phi\phi} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{\phi\phi\phi}
\end{bmatrix} \begin{bmatrix}
\epsilon_x \\
\epsilon_\theta \\
\gamma_{x\phi} \\
\gamma_{x\phi} \\
\gamma_{x\phi} \\
\gamma_{x\phi}
\end{bmatrix} - \begin{bmatrix}
\epsilon_0 \\
\epsilon_0 \\
\gamma_{x\phi} \\
\gamma_{x\phi} \\
\gamma_{x\phi} \\
\gamma_{x\phi}
\end{bmatrix} \begin{bmatrix}
\eta_{x\phi} \\
\eta_{x\phi} \\
\eta_{x\phi} \\
\eta_{x\phi} \\
\eta_{x\phi} \\
\eta_{x\phi}
\end{bmatrix} \begin{bmatrix}
E_x \\
E_\theta \\
\mu_{x\phi} \\
\mu_{x\phi} \\
\mu_{x\phi} \\
\mu_{x\phi}
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
\mu_{x\phi} \\
\mu_{x\phi} \\
\mu_{x\phi} \\
\mu_{x\phi}
\end{bmatrix} \begin{bmatrix}
d_{x} \\
d_{x} \\
d_{x} \\
d_{x} \\
d_{x} \\
d_{x}
\end{bmatrix} \begin{bmatrix}
H_x \\
H_x \\
H_x \\
H_x \\
H_x \\
H_x
\end{bmatrix}. \tag{10}
\]

where \( C_{ijkl} \) are stiffness coefficients, \( \varepsilon_{ij} \) are piezoelectric coefficients, and \( q_{ij} \) are piezomagnetic coefficients. In addition, \( E_x, E_\theta, \) and \( E_z \) represent electric field components, and \( H_x, H_\theta, \) and \( H_z \) are magnetic field components.

The electric displacement relations are developed as (Farajpour et al., 2016)

\[
(1 - \xi^2) \begin{bmatrix}
D_x \\
D_\theta \\
D_z
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\epsilon_x \\
\epsilon_\theta \\
\gamma_{x\phi}
\end{bmatrix} + \begin{bmatrix}
\eta_{x\phi} \\
\eta_{x\phi} \\
\eta_{x\phi}
\end{bmatrix} \begin{bmatrix}
E_x \\
E_\theta \\
\mu_{x\phi}
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
\mu_{x\phi}
\end{bmatrix} \begin{bmatrix}
d_{x} \\
d_{x} \\
d_{x}
\end{bmatrix} \begin{bmatrix}
H_x \\
H_x \\
H_x
\end{bmatrix}. \tag{11}
\]

in which \( \eta_{ij} \) are dielectric coefficients and \( d_{ij} \) are magnetoelectric coefficients.

The magnetic induction relations are developed as (Farajpour et al., 2016)

\[
(1 - \xi^2) \begin{bmatrix}
B_x \\
B_\theta \\
B_z
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\epsilon_x \\
\epsilon_\theta \\
\gamma_{x\phi}
\end{bmatrix} + \begin{bmatrix}
\eta_{x\phi} \\
\eta_{x\phi} \\
\eta_{x\phi}
\end{bmatrix} \begin{bmatrix}
E_x \\
E_\theta \\
\mu_{x\phi}
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
\mu_{x\phi}
\end{bmatrix} \begin{bmatrix}
d_{x} \\
d_{x} \\
d_{x}
\end{bmatrix} \begin{bmatrix}
H_x \\
H_x \\
H_x
\end{bmatrix}. \tag{12}
\]

where \( q_{ij} \) are piezomagnetic coefficients and \( \mu_{ij} \) are magnetic coefficients.

The electric and magnetic fields are obtained using electric and magnetic potentials as

\[
\psi = \frac{2z}{h} \psi_0 - \psi(x, \theta, t) \cos \frac{\pi z}{h} = \begin{cases}
E_x = \frac{\delta \psi}{\delta x} \cos \frac{\pi z}{h} \\
E_\theta = \frac{\delta \psi}{\delta \theta} \cos \frac{\pi z}{h} \\
E_z = -\frac{\delta \psi}{\delta t} \cos \frac{\pi z}{h}
\end{cases}, \tag{13}
\]

\[
\phi = \frac{2z}{h} \phi_0 - \phi(x, \theta, t) \cos \frac{\pi z}{h} = \begin{cases}
H_x = \frac{\delta \phi}{\delta x} \cos \frac{\pi z}{h} \\
H_\theta = \frac{\delta \phi}{\delta \theta} \cos \frac{\pi z}{h} \\
H_z = -\frac{\delta \phi}{\delta t} \cos \frac{\pi z}{h}
\end{cases}.
\]

One can find components of stress, electric displacement, and magnetic induction in detailed form in Appendix B.

The substitution of variations of strain energy, kinetic energy, and energy due to external works into Hamilton's principle gives final governing equations as follows:

\[
\delta \Psi_0 = -\frac{\partial N_0}{\partial x} - \frac{\partial P_\theta}{\partial \theta} = - \left[ I_0 \dddot{w}_0 + I_1 \dddot{w}_0 + I_3 \dddot{w}_0 + I_1 \dddot{w} \right],
\]

\[
\delta \chi^2 = N_\text{xx} - \frac{\partial M_\text{xx}}{\partial x} = - \left[ (I_1 + I_3) \dddot{w}_0 + (I_2 + I_4 + I_6) \dddot{w}_0 + (I_4 + I_6) \dddot{w} \right],
\]

\[
\delta \chi^2 = \frac{\partial N_{xx}}{\partial x} - \frac{\partial P_{xx}}{\partial \theta} = - \left[ J_0 \dddot{v}_0 + J_2 \dddot{v}_0 + J_2 \dddot{v} + J_3 \dddot{v} \right],
\]

\[
\delta \chi^2 = N_{xx} - \frac{\partial M_{xx}}{\partial x} = - \left[ (I_1 + I_3) \dddot{v}_0 + (I_2 + I_4 + I_6) \dddot{v}_0 + (I_4 + I_6) \dddot{v} \right],
\]

\[
\delta \psi = \frac{\partial \delta \psi}{\partial x} + \frac{\partial \delta \psi}{\partial \theta} + \delta D_2 = 0
\]

\[
\delta \phi = \frac{\partial \delta \phi}{\partial x} + \frac{\partial \delta \phi}{\partial \theta} + \delta B_2 = 0
\]  

in which the resultant components are defined in Appendix C.
The substitution of resultant components from Appendix C into governing equations yields final governing equations as follows:

$$
\delta u_0 : A_0 \frac{\partial^2 u_0}{\partial x^2} + A_6 \frac{\partial^2 u_0}{\partial y^2} + A_2 \frac{\partial^2 x_1}{\partial x^2} + (A_6 + A_8) \frac{\partial^2 u_0}{\partial x\partial y} + (A_6 + A_9) \frac{\partial^2 \chi_1}{\partial x^2} \\
+ A_3 \frac{\partial^2 u_0}{\partial y^2} + A_4 \frac{\partial u_0}{\partial x} + A_7 \frac{\partial \psi}{\partial x} + A_8 \frac{\partial \phi}{\partial x} \\
= I_0 \delta u_0 + (I_1 + I_3) \frac{\partial^2 \chi_1}{\partial x^2} + A_1 \frac{\partial \psi}{\partial y} - (N_{m_4} - N_{m_1}) \frac{\partial \phi}{\partial y}
$$

$$
\delta x_1 : A_0 \frac{\partial^2 u_0}{\partial x^2} + A_6 \frac{\partial^2 u_0}{\partial y^2} + A_2 \frac{\partial^2 x_1}{\partial x^2} + (A_6 + A_8) \frac{\partial^2 u_0}{\partial x\partial y} + (A_6 + A_9) \frac{\partial^2 \chi_1}{\partial x^2} \\
+ A_3 \frac{\partial^2 u_0}{\partial y^2} + A_4 \frac{\partial u_0}{\partial x} + A_7 \frac{\partial \psi}{\partial x} + A_8 \frac{\partial \phi}{\partial x} \\
= I_0 \frac{\partial \psi}{\partial y} + (I_1 + I_3) \frac{\partial \chi_1}{\partial x} + A_1 \frac{\partial \psi}{\partial y} - (N_{m_2} - N_{m_1}) \frac{\partial \phi}{\partial y}
$$

$$
\delta v_0 : (A_{45} + A_{35}) \frac{\partial^2 u_0}{\partial x\partial y} + (A_{56} + A_{35}) \frac{\partial^2 x_1}{\partial x\partial y} + A_{56} \frac{\partial^2 \chi_1}{\partial x^2} + A_{45} \frac{\partial^2 \psi}{\partial x^2} + A_{45} \frac{\partial^2 \phi}{\partial x^2} \\
+ (A_{46} + A_{45}) \frac{\partial^2 u_0}{\partial x^2} + A_{46} \frac{\partial^2 \psi}{\partial x^2} + A_{46} \frac{\partial^2 \phi}{\partial x^2} \\
= \frac{\partial \psi}{\partial y} + (I_1 + I_3) \frac{\partial \chi_1}{\partial x} + A_1 \frac{\partial \psi}{\partial y} - (N_{m_2} - N_{m_1}) \frac{\partial \phi}{\partial y}
$$

$$
\delta x_\psi : (A_{55} + A_{57}) \frac{\partial^2 \chi_1}{\partial x^2} + (A_{58} + A_{57}) \frac{\partial^2 x_1}{\partial x^2} + A_{58} \frac{\partial^2 \chi_1}{\partial x^2} + A_{58} \frac{\partial^2 \psi}{\partial x^2} + A_{58} \frac{\partial^2 \phi}{\partial x^2} \\
+ (A_{49} + A_{48}) \frac{\partial^2 u_0}{\partial x^2} + (A_{49} - A_{48}) \frac{\partial^2 \psi}{\partial x^2} + (A_{49} - A_{48}) \frac{\partial^2 \phi}{\partial x^2} \\
= (I_1 + I_3) \frac{\partial \psi}{\partial y} + (I_1 + I_3) \frac{\partial \phi}{\partial y}
$$

$$
\delta u_9 : \frac{\partial^2 u_0}{\partial x^2} - 2A_0 \frac{\partial^2 u_0}{\partial y^2} A_0 \frac{\partial^2 \chi_1}{\partial x^2} - 2A_0 \frac{\partial^2 \chi_1}{\partial x^2} + (A_{49} - A_{48}) \frac{\partial^2 \chi_1}{\partial x^2} + (2A_{49} + A_{48}) \frac{\partial^2 \psi}{\partial x^2} + 2A_{49} \frac{\partial^2 \psi}{\partial x^2} \\
- (2A_{49} + A_{48}) \frac{\partial^2 \chi_1}{\partial x^2} + (A_{57} - A_{58}) \frac{\partial^2 \phi}{\partial x^2} - 2A_0 \frac{\partial^2 \psi}{\partial x^2} + 2A_9 \frac{\partial^2 \psi}{\partial x^2} + (A_{49} - A_{48}) \frac{\partial^2 \chi_1}{\partial x^2} + (2A_{49} + A_{48}) \frac{\partial^2 \psi}{\partial x^2} + 2A_{49} \frac{\partial^2 \psi}{\partial x^2} \\
- (A_{23} + A_{49}) \frac{\partial^2 \psi}{\partial x^2} - A_{23} \frac{\partial^2 \phi}{\partial x^2} - A_{23} \frac{\partial^2 \psi}{\partial x^2} - A_{23} \frac{\partial^2 \phi}{\partial x^2} - A_{23} \frac{\partial^2 \phi}{\partial x^2} - A_{23} \frac{\partial^2 \phi}{\partial x^2} \\
+ \lambda \left[ \left( \frac{N_{m_3} + N_{m_1} \frac{\partial^2 w}{\partial x^2} + (N_{m_3} + N_{m_1} \frac{\partial^2 \psi}{\partial x^2} - F_1 + P_1 \right) \right] \\
\delta \psi : A_{60} \frac{\partial \psi}{\partial x} + (A_{60} + A_{65}) \frac{\partial \chi_1}{\partial x} + (A_{65} + A_{60}) \frac{\partial \psi}{\partial x} + (A_{60} + A_{65}) \frac{\partial \chi_1}{\partial x} + (A_{60} + A_{65}) \frac{\partial \psi}{\partial x} + A_{65} \frac{\partial \phi}{\partial x} \\
+ (A_{60} + A_{65}) \frac{\partial \chi_1}{\partial x} + (A_{60} + A_{65}) \frac{\partial \psi}{\partial x} + (A_{60} + A_{65}) \frac{\partial \chi_1}{\partial x} + (A_{60} + A_{65}) \frac{\partial \psi}{\partial x} + A_{65} \frac{\partial \phi}{\partial x} \\
\delta \phi : + \lambda \left[ \left( \frac{N_{m_3} + N_{m_1} \frac{\partial^2 w}{\partial x^2} + (N_{m_3} + N_{m_1} \frac{\partial^2 \psi}{\partial x^2} - F_1 + P_1 \right) \right] \\
$$

3. Solution, Numerical Results, and Discussion

Numerical results are expressed in this section to investigate the effect of significant parameters such as axial and circumferential mode numbers, applied electromagnetic potentials, nanoscale size, and length to mean radius ratio on the free vibration responses. The displacements are assumed as double series of trigonometric functions that satisfy boundary
and consequently a decrease in natural frequency. Stiffness and consequently a decrease in natural frequency.

The natural frequencies of cylindrical are derived using the solution of the characteristic equation as follows:

\[ \omega^2 = \frac{X}{M} \]

where \( \omega \) is the natural frequency.

The natural frequencies of piezomagnetic nano panel are presented in this section. These results are presented in terms of side length ratio based on the present results with other available results in literature.

The natural frequencies are obtained using the determinant of the characteristic equation as follows:

\[ \det [K - \omega^2 M] = 0. \]

The natural frequencies are obtained using the determinant of the characteristic equation as follows: \( \det [K - \omega^2 M] = 0 \).

Before the presentation of the complete numerical results, a comparative study is presented for the verification of formulation and corresponding numerical results. Listed in Table 1 is the comparison of dimensionless natural frequencies of the cylindrical panels in terms of side length ratio based on the present results with other available results in literature.

Natural frequencies of cylindrical piezomagnetic nano panel are presented in this section. These results are presented in terms of main input parameters such as axial and circumferential mode numbers, initial electromagnetic potentials, span angle, and different geometric parameters.

The material properties of piezomagnetic materials are extracted from reference (Buchanan et al., 2003).

\[
\begin{align*}
C_{xx} & = 218 \text{ GPa}, & C_{yy} & = 218 \text{ GPa}, & C_{zz} & = 215 \text{ GPa}, & C_{xy} & = 120 \text{ GPa}, & C_{xz} & = 120 \text{ GPa}, \\
C_{tx} & = 120 \text{ GPa}, & C_{ty} & = 50 \text{ GPa}, & C_{tz} & = 50 \text{ GPa}, & C_{e} & = 49 \text{ GPa}, \\
e_{e} & = -2.5C/\text{Vm}, & e_{t} & = -2.5C/\text{Vm}, & q_{e} & = 265N/\text{Am}, & q_{t} & = 265N/\text{Am}, \\
\eta_{e} & = 0.4C/\text{Vm}, & \eta_{t} & = 0.4C/\text{Vm}, & \eta_{e} & = 5.8N/\text{Am}, \\
d_{e} & = 0.0074Ns/\text{VC}, & d_{t} & = 0.0074Ns/\text{VC}, & d_{e} & = 2.82Ns/\text{VC}, \\
\mu_{e} & = -200 \times 10^{-6} \text{Ns}^2/\text{C}^2, & \mu_{t} & = -200 \times 10^{-6} \text{Ns}^2/\text{C}^2, & \mu_{e} & = 95 \times 10^{-6} \text{Ns}^2/\text{C}^2.
\end{align*}
\]

Variation of natural frequencies of the piezomagnetic nano panel is presented in terms of applied electric potential \( \psi_0 \) and nonlocal parameter \( \xi \) in Fig. 2. An increase in applied electric potential \( \psi_0 \) yields a stiffer nano panel that lead to a significant increase in natural frequency of piezomagnetic nano panel. Furthermore, a decrease in natural frequencies with the increase in the nonlocal parameter is observed. It is concluded that an increase in natural frequency of piezomagnetic nano panel. Furthermore, a decrease in natural frequencies with the increase in the nonlocal parameter is observed. It is concluded that an increase in nonlocal parameter leads to a significant decrease in stiffness and consequently a decrease in natural frequency.

Figure 3 shows the variation of the natural frequencies of cylindrical nano panel in terms of applied magnetic potential \( \Phi_0 \) and nonlocal parameter \( \xi \). Unlike electric potential, the increase in applied magnetic potential yields a softer nano panel that leads to a decrease in natural frequencies. It is concluded that effect of magnetic potential is more than electric potential, numerically.

Shown in Fig. 4 is the variation of natural frequencies of the piezomagnetic nano panels in terms of the span angle \( \Theta \) and nonlocal parameter \( \xi \). Investigating the effect of span angle on the natural frequencies of piezomagnetic nano panel indicates that an increase in span angle leads to a decrease in natural frequency. It is concluded that an increase in span angle leads to a decrease in panel stiffness and consequently a decrease in natural frequency.

Figure 5 shows the variation of natural frequencies of cylindrical nano panel in terms of nonlocal parameter \( \xi \) for various length-to-mean radius ratios (L/R). It is concluded that an increase in length-to-mean radius ratio (L/R) leads to an increase in natural frequencies. One can conclude that the stiffness of cylindrical panel is decreased with an increase in length-to-mean radius ratio (L/R) and consequently a decrease in natural frequency.

| \( b/a \) | \( a/h = 10 \) | Present | \( a/h = 100 \) |
|---|---|---|---|
| 0.5 | 1.31631 | 1.31742 | 1.30597 | 0.16001 | 0.16066 | 0.15751 |
| 1 | 0.55010 | 0.55049 | 0.54523 | 0.07301 | 0.07368 | 0.07234 |
| 1.5 | 0.39963 | 0.39987 | 0.39501 | 0.04901 | 0.04912 | 0.04845 |
| 2 | 0.34600 | 0.34600 | 0.34122 | 0.03921 | 0.03925 | 0.03879 |

Comparison of dimensionless natural frequencies of cylindrical panel with various references.
Figure 2: Effect of applied electric potential $\Psi_0$ and nonlocal parameter $\xi$ on the variation of natural frequencies of piezomagnetic nano panel.

Figure 3: Effect of applied magnetic potential $\Phi_0$ and nonlocal parameter $\xi$ on the variation of natural frequencies of piezomagnetic nano panel.
Figure 4: Effect of span angle $\Theta$ and nonlocal parameter $\xi$ on the variation of natural frequencies of piezomagnetic nano panel.

Figure 5: Effect of length-to-mean radius ratio ($L/R$) and nonlocal parameter $\xi$ on the variation of natural frequencies of piezomagnetic nano panel.
4. Conclusions

Magneto-electro-elastic free vibration analysis of a piezomagnetic nano panel was studied in this paper based on nonlocal piezomagnetoelasticity relations and higher order shear deformation theory. For more accurate modeling of the shell and satisfying zero transverse shear strains at top and bottom of thickness, third-order shear deformation theory was used. The effect of nanoscales in the geometry is accounted in the formulation based on Eringen nonlocal elasticity theory. The piezomagnetic nano panel was subjected to initial electromagnetic potentials. After the derivation of the governing equations of motion, the numerical results for an SS piezomagnetic nano panel were presented. The effect of significant parameters of the problem such as axial and circumferential mode numbers, span angle, applied electromagnetic potentials, nonlocal parameter, and length-to-mean radius ratio was investigated on the electro-magneto-elastic free vibration responses. The main results of this paper are summarized as follows:

The natural frequencies were presented in terms of initial electromagnetic potentials. It is deduced that the natural frequencies are increased with increase of applied electric potential and decrease of applied magnetic potential.

The effect of various geometric parameters was studied in the free vibration responses. It is concluded that increase in length-to-mean radius ratio (L/R) and decrease in length-to-thickness ratio (L/h), mean radius-to-thickness ratio (R/h), and span angle lead to increase of natural frequencies of nano panel.

Investigating the effect of axial and circumferential mode numbers on the natural frequency of piezomagnetic nano panel indicates that the natural frequencies are increased with increase of mode numbers in which the circumferential mode number has more effect than axial one.
Figure 7: Effect of mean radius-to-thickness ratio \((R/h)\) and nonlocal parameter \(\xi\) on the variation of natural frequencies of piezomagnetic nano panel.

Figure 8: Effect of circumferential mode number \(n\) and nonlocal parameter \(\xi\) on the variation of natural frequencies of piezomagnetic nano panel.
Figure 9: Effect of axial mode number $n$ and nonlocal parameter $\xi$ on the variation of natural frequencies of piezomagnetic nano panel.

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Conflict of interest statement
None declared.

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Appendix 1

\[
\{N_x, M_x, S_x\} = \int_{-h/2}^{h/2} (R + z) \{1, z(1 + k z^2), k z^2\} \, dz,
\]
\[
\{N_y, P_y\} = \int_{-h/2}^{h/2} s_0 (R + z) \left\{ \frac{1}{R} \frac{z}{r} (1 + k z^2) \right\} \, dz.,
\]
\[
\{N_{x2}, P_{x2}\} = \int_{-h/2}^{h/2} r_{x2} (R + z) (1 + 3 k z^2) \left\{ \frac{1}{r} \frac{1}{r} \right\} \, dz.
\]
\[
\{N_{x3}, P_{x3}, M_{x3}, S_{x3}, Q_{x3}\} = \int_{-h/2}^{h/2} r_{x3} (R + z) \left\{ \frac{1}{r} \frac{1}{r} z (1 + k z^2), \frac{z}{r} (1 + k z^2), \frac{k z^2}{r} \right\} \, dz.
\]
\[
\{D_x, D_y, D_z\} = \int_{-1}^{2} \left\{ D_x (R + z) \cos \frac{\pi z}{h}, D_y \cos \frac{\pi z}{h}, D_z \cos \frac{\pi z}{h} (R + z) \sin \frac{\pi z}{h} \right\} \, dz
\]
\[
\{E_x, E_y, E_z\} = \int_{-1}^{2} \left\{ E_x (R + z) \cos \frac{\pi z}{h}, E_y \cos \frac{\pi z}{h}, E_z \cos \frac{\pi z}{h} (R + z) \sin \frac{\pi z}{h} \right\} \, dz
\]

Appendix 2

\[
(1 - \xi^2 \nu^2) \sigma_x = C_{x0xx} \left( \frac{\partial u}{\partial x} + z (1 + k z^2) \frac{\partial j_x}{\partial x} + k z^2 \frac{\partial^2 u}{\partial x^2} \right) + C_{x0xy} \left( \frac{u_0}{R} + \frac{1}{R} \frac{\partial u_0}{\partial \theta} + \frac{z}{r} (1 + k z^2) \frac{\partial j_x}{\partial \theta} \right)
\]
\[
- \epsilon_{x0z} \left( \frac{2}{h} \Psi_0 - \frac{\pi}{h} \psi \sin \frac{\pi z}{h} \right) - q_{xz} \left( \frac{2}{h} \Phi_0 - \frac{\pi}{h} \phi \sin \frac{\pi z}{h} \right)
\]
\[
(1 - \xi^2 \nu^2) \sigma_y = C_{y0xx} \left( \frac{\partial u}{\partial y} + z (1 + k z^2) \frac{\partial j_y}{\partial y} + k z^2 \frac{\partial^2 u}{\partial y^2} \right) + C_{y0xy} \left( \frac{u_0}{R} + \frac{1}{R} \frac{\partial u_0}{\partial \theta} + \frac{z}{r} (1 + k z^2) \frac{\partial j_y}{\partial \theta} \right)
\]
\[
- \epsilon_{y0z} \left( \frac{2}{h} \Psi_0 - \frac{\pi}{h} \psi \sin \frac{\pi z}{h} \right) - q_{yz} \left( \frac{2}{h} \Phi_0 - \frac{\pi}{h} \phi \sin \frac{\pi z}{h} \right)
\]
\[
(1 - \xi^2 \nu^2) \tau_{x0} = C_{x0xz} \left( \frac{\partial v}{\partial x} + z (1 + k z^2) \frac{\partial j_x}{\partial x} + \frac{z}{r} (1 + k z^2) \frac{\partial j_x}{\partial \theta} \right) - \epsilon_{x0z} \left( \frac{2}{h} \Phi_0 - \frac{\pi}{h} \phi \sin \frac{\pi z}{h} \right) - q_{xz} \left( \frac{2}{h} \Phi_0 - \frac{\pi}{h} \phi \sin \frac{\pi z}{h} \right)
\]
\[
(1 - \xi^2 \nu^2) \tau_{y0} = C_{y0xz} \left( \frac{\partial v}{\partial y} + z (1 + k z^2) \frac{\partial j_y}{\partial y} + \frac{z}{r} (1 + k z^2) \frac{\partial j_y}{\partial \theta} \right) - \epsilon_{y0z} \left( \frac{2}{h} \Phi_0 - \frac{\pi}{h} \phi \sin \frac{\pi z}{h} \right) - q_{yz} \left( \frac{2}{h} \Phi_0 - \frac{\pi}{h} \phi \sin \frac{\pi z}{h} \right)
\]
\[
(1 - \xi^2 \nu^2) D_x = \epsilon_{x0z} \left( \frac{\partial j_x}{\partial x} + \frac{\partial u}{\partial x} \right) - \epsilon_{x0z} \left( \frac{\partial j_x}{\partial \theta} + \frac{\partial u}{\partial \theta} \right) + \eta_{x0z} \left( \frac{\partial j_x}{\partial \theta} + \frac{\partial u}{\partial \theta} \right)
\]
\[
(1 - \xi^2 \nu^2) D_y = \epsilon_{y0z} \left( \frac{\partial j_y}{\partial y} + \frac{\partial u}{\partial y} \right) - \epsilon_{y0z} \left( \frac{\partial j_y}{\partial \theta} + \frac{\partial u}{\partial \theta} \right) + \eta_{y0z} \left( \frac{\partial j_y}{\partial \theta} + \frac{\partial u}{\partial \theta} \right)
\]
Appendix 3

\( (1 - \xi^2 \nu^2) N_x = A_1 \frac{\partial u_0}{\partial x} + A_6 \frac{\partial x_1^0}{\partial x} + A_8 \frac{\partial^2 w}{\partial x^2} + A_{14} w_0 + A_{12} \frac{\partial v_0}{\partial y} + A_{11} \frac{\partial x_1^0}{\partial y} + N_{y_0} + A_3 \Psi + N_{z_0} + A_8 \Phi. \)

\( (1 - \xi^2 \nu^2) M_x = A_2 \frac{\partial u_0}{\partial x} + A_{10} \frac{\partial x_1^0}{\partial x} + A_{11} \frac{\partial^2 w}{\partial x^2} + A_{12} w_0 + A_{13} \frac{\partial v_0}{\partial y} + A_{14} \frac{\partial x_1^0}{\partial y} + M_{y_0} + A_{15} \Psi + M_{z_0} + A_{16} \Phi. \)

\( (1 - \xi^2 \nu^2) S_x = A_{17} \frac{\partial u_0}{\partial x} + A_{18} \frac{\partial x_1^0}{\partial x} + A_{19} \frac{\partial^2 w}{\partial x^2} + A_{20} w_0 + A_{21} \frac{\partial v_0}{\partial y} + A_{22} \frac{\partial x_1^0}{\partial y} + S_{y_0} + A_{16} \Psi + S_{z_0} + A_{24} \Phi. \)

\( (1 - \xi^2 \nu^2) \Phi = A_{25} \frac{\partial u_0}{\partial x} + A_{26} \frac{\partial x_1^0}{\partial x} + A_{27} \frac{\partial^2 w}{\partial x^2} + A_{28} w_0 + A_{29} \frac{\partial v_0}{\partial y} + A_{30} \frac{\partial x_1^0}{\partial y} + \Phi_{y_0} + A_{31} \Psi + \Phi_{z_0} + A_{32} \Phi. \)

\( (1 - \xi^2 \nu^2) \Psi = A_{33} \frac{\partial u_0}{\partial x} + A_{34} \frac{\partial x_1^0}{\partial x} + A_{35} \frac{\partial^2 w}{\partial x^2} + A_{36} w_0 + A_{37} \frac{\partial v_0}{\partial y} + A_{38} \frac{\partial x_1^0}{\partial y} + \Psi_{y_0} + A_{39} \Psi + \Psi_{z_0} + A_{34} \Phi. \)

Appendix 4

\( \{A_1, A_5, A_3\} = \int \left( R + z \right) C_{xxxx} \left[ 1, z \left( 1 + kz^2 \right), kz^2 \right] \text{dz}. \)

\( \{A_4, A_5, A_3\} = \int \frac{1}{R} \left( R + z \right) C_{xxxx} \left[ \frac{1}{R} z, \frac{1}{R}, \frac{1}{R} \left( 1 + kz^2 \right) \right] \text{dz}. \)

\( \{A_4, A_5, A_1\} = \int \left( R + z \right) \frac{\pi}{R} \sin \frac{\pi z}{R} \left( \rho_{xxx}, q_{xxx} \right) \text{dz}. \)

\( \{N_0, N_0\} = \int \frac{1}{R} \left( R + z \right) \left( \rho_{xxx} \varphi_0, \varphi_{xxx} \Phi_0 \right) \text{dz}. \)

\( \{A_7, A_5, A_1\} = \int \left( R + z \right) z \left( 1 + k z^2 \right) C_{xxxx} \left[ 1, z \left( 1 + k z^2 \right), kz^2 \right] \text{dz}. \)
\begin{align*}
\{A_{12}, A_{13}, A_{14}\} &= \int_{-\frac{3}{2}}^{\frac{1}{2}} (R + z) z (1 + k z^2) C_{xxyy} \left\{ 1 + \frac{1}{R} \frac{z}{r} \left( 1 + k z^2 \right) \right\} dz, \\
\{A_{15}, A_{16}\} &= \int_{-\frac{3}{2}}^{\frac{1}{2}} (R + z) \frac{\pi}{R} \sin \frac{\pi z}{R} (1 + k z^2) \left[ e_{xy}, q_{xzy} \right] dz, \\
\{M_{30}, M_{31}\} &= \int_{-\frac{3}{2}}^{\frac{1}{2}} \frac{1}{R} (R + z) z (1 + k z^2) \left[ e_{xy}, \Psi_0, q_{xzy} \Phi_0 \right] dz, \\
\{A_{17}, A_{18}, A_{19}\} &= \int_{-\frac{3}{2}}^{\frac{1}{2}} (R + z) k z^3 C_{xxyy} \left\{ 1, z (1 + k z^2) \right\} dz, \\
\{A_{20}, A_{21}, A_{22}\} &= \int_{-\frac{3}{2}}^{\frac{1}{2}} (R + z) k z^3 C_{xxyy} \left\{ 1 + \frac{1}{R} \frac{z}{r} \left( 1 + k z^2 \right) \right\} dz, \\
\{A_{23}, A_{24}\} &= \int_{-\frac{3}{2}}^{\frac{1}{2}} (R + z) \frac{\pi}{R} \sin \frac{\pi z}{R} k z^4 \left[ e_{xy}, q_{xzy} \right] dz, \\
\{S_{30}, S_{31}\} &= \int_{-\frac{3}{2}}^{\frac{1}{2}} \frac{1}{R} (R + z) k z^3 \left[ e_{xy}, \Psi_0, q_{xzy} \Phi_0 \right] dz, \\
\{A_{30}, A_{35}, A_{37}\} &= \int_{-\frac{3}{2}}^{\frac{1}{2}} (R + z) C_{xxyy} \left\{ 1, z (1 + k z^2) \right\} dz, \\
\{A_{38}, A_{39}, A_{40}\} &= \int_{-\frac{3}{2}}^{\frac{1}{2}} (R + z) C_{xxyy} \left\{ 1 + \frac{1}{R} \frac{z}{r} \left( 1 + k z^2 \right) \right\} dz, \\
\{A_{31}, A_{32}\} &= \int_{-\frac{3}{2}}^{\frac{1}{2}} (R + z) \frac{\pi}{R} \sin \frac{\pi z}{R} \left[ e_{xy}, q_{xzy} \right] dz, \quad \{N_{6, \Phi_1}, N_{6, \Phi_2}\} = \int_{-\frac{3}{2}}^{\frac{1}{2}} \frac{1}{R} (R + z) \left[ e_{xy}, \Psi_0, q_{xzy} \Phi_0 \right] dz, \\
\{A_{37}, A_{38}, A_{39}\} &= \int_{-\frac{3}{2}}^{\frac{1}{2}} (R + z) C_{xxyy} \left\{ 1 + 3 k z^2 \right\} \left\{ 1, \frac{1}{R} \right\} dz, \quad \{A_{35}, A_{36}\} = \int_{-\frac{3}{2}}^{\frac{1}{2}} \cos \frac{\pi z}{R} \left( 1 + 3 k z^2 \right) \left[ e_{xy}, q_{xzy} \right] dz, \\
\{A_{32}, A_{33}\} &= \int_{-\frac{3}{2}}^{\frac{1}{2}} (R + z) C_{xxyy} \left\{ 1 + 3 k z^2 \right\} \left\{ 1, \frac{1}{R} \right\} dz, \\
\{A_{37}, A_{38}\} &= \int_{-\frac{3}{2}}^{\frac{1}{2}} (R + z) C_{xxyy} \left( 1 + 3 k z^2 \right) \left\{ 1, \frac{1}{R} \right\} dz, \\
\{A_{39}, A_{40}\} &= \int_{-\frac{3}{2}}^{\frac{1}{2}} \cos \frac{\pi z}{R} \left( 1 + 3 k z^2 \right) \left[ e_{xy}, q_{xzy} \right] dz, \\
\{A_{41}\} &= \int_{-\frac{3}{2}}^{\frac{1}{2}} (R + z) C_{xxyy} \left( 1 + 3 k z^2 \right) dz, \quad \{A_{42}, A_{43}\} = \int_{-\frac{3}{2}}^{\frac{1}{2}} (R + z) \cos \frac{\pi z}{R} \left( 1 + 3 k z^2 \right) \left[ e_{xy}, q_{xzy} \right] dz.
\end{align*}
\[ \{A_{44}, A_{45}, A_{46}, A_{47}, A_{48}\} = \frac{3}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} (R + z) C_{x\omega z} \left\{ 1, \frac{1}{R}, z(1 + k z^2), \frac{z}{R} (1 + k z^2), \frac{2}{R} \right\} dz. \]

\[ \{A_{49}, A_{50}, A_{51}, A_{52}, A_{53}\} = \frac{3}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} (R + z) C_{x\omega z} \left\{ 1, \frac{1}{R}, z(1 + k z^2), \frac{z}{R} (1 + k z^2), \frac{2}{R} \right\} dz. \]

\[ \{A_{64}, A_{65}, A_{66}, A_{67}, A_{68}\} = \frac{3}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} (R + z) C_{x\omega z} z(1 + k z^2) \left\{ 1, \frac{1}{R}, z(1 + k z^2), \frac{z}{R} (1 + k z^2), \frac{2}{R} \right\} dz. \]

\[ \{A_{69}, A_{70}, A_{71}, A_{72}, A_{73}\} = \frac{3}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} (R + z) C_{x\omega z} z(1 + k z^2) \left\{ 1, \frac{1}{R}, z(1 + k z^2), \frac{z}{R} (1 + k z^2), \frac{2}{R} \right\} dz. \]

\[ \{A_{84}, A_{85}, A_{86}, A_{87}, A_{88}\} = \frac{3}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} (R + z) C_{x\omega z} z(1 + k z^2) \left\{ 1, \frac{1}{R}, z(1 + k z^2), \frac{z}{R} (1 + k z^2), \frac{2}{R} \right\} dz. \]