Parton distributions: Functional complexity and Lorentz parametrization

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ABSTRACT: In the paper we focus on the study of the functional complexity of the Lorentz parametrizing functions in connection with the time-reversal transformations. We argue that the interactions encoded in the corresponding correlators of non-local quark(-gluon) operators generate additional sources of functional complexity for the parametrizing functions which are not discussed in the literature. We also revisit the Lorentz parametrization of different correlators given by the hadron matrix elements of the non-local operators. The evidences for the new parametrizing function existence have been presented.
1 Introduction

The production in nucleon-nucleon collisions in the Drell-Yan (DY) processes and the (semi-inclusive) deep inelastic scattering (SIDIS/DIS) processes provide still much information on the composite structure of hadrons. From the theoretical viewpoint, these processes give additional possibilities to study the new sorts of parton distribution functions which accumulate information on the transverse motion of quarks inside hadrons. In fact, every of parton distribution functions is actually the Lorentz parametrizing function related to the given correlator. In this connection, it cannot be overestimated that the different properties of parametrizing functions which originate from the fundamental (discrete) symmetries play the very important role in investigations.

In the most general case, the functions which depend on the parton momentum are given by the hadron-hadron matrix element of non-local quark operators projected on the given $\Gamma$-combination. Before factorization, it reads

$$
\Phi^{[\Gamma]}_{(\pm)}(k) = \int (d^4z) e^{+ikz} \langle P,S|\bar{\psi}(0)\Gamma[0;z]_A^{(\pm)} \psi(z)|P,S\rangle^H, \tag{1.1}
$$

where $[0;z]_A^{(\pm)}$ stands for the future- and past-pointed Wilson line (WL) \(^1\). As usual, the $\Gamma$-combination corresponds to the Dirac $\gamma$-matrices which form the basis given by $\{1, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}\}$. The decomposition of $\Phi^{[\Gamma]}_{(\pm)}(k)$ based on the Lorentz covariance gives a number of parametrizing functions which, in its turn, can be associated with the given distribution functions (see, for example, [1–6]). It is important to emphasize that the distribution functions as functions of the parton

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\(^1\)Throughout the paper, we use the standard notations for the plus and minus light-cone directions.
momentum fractions appear only after the factorization procedure has been applied. In the simplest \( k_{\perp}\)-independent case, we have

\[
\Phi^\Gamma_{(\pm)}(x) = \int (d^4k) \delta(x - k^+/P^+) \Phi^\Gamma_{(\pm)}(k), \quad k = (k^+, k^- \mathbf{\hat{k}}_{\perp}).
\] (1.2)

The necessity to study the \( k_{\perp}\)-dependence leads to the \( k_{\perp}\)-unintegrated functions \( \Phi^\Gamma_{(\pm)}(x, k_{\perp}) \) and it requires, in a sense, the corresponding modification of factorization.

In Eqn. (1.1), \( H \) indicates the Heisenberg representation (H-representation). In other words, we have to assume that all states and operators of Eqn. (1.1), roughly speaking, are “dressed” ones due to the interactions. This is the well-known fact which is, however, forgotten or hidden in the literature quite frequently. Meanwhile, the presence of interactions in the correlators provides not only the evolution of given distribution functions, but it ensures the important fundamental properties of them.

In the present paper, we revisit the properties of distribution functions which stem from the time-reversal transformations together with the hermitian conjugations. Namely, we argue that the interactions encoded in the bound in- and/or out-states and in H-representation of quark(-gluon) operators forming the corresponding correlators result in additional sources of the functional complexity for the parametrizing functions. Then, based on the time-reversal and hermitian transforms, we demonstrate that the \( k_{\perp}\)-dependent parametrizing functions do not possess the certain symmetry properties under \( k_{\perp} \to -k_{\perp} \). This our finding is not stemmed from the past-pointed and future-pointed Wilson lines discussed in [7]. At the same time, the usual time-reversal properties of functions can be restored but after the \( k_{\perp}\)-integration of the corresponding functions.

In addition, we show that the careful taking into account of interactions in the correlators leads to a new type of parametrizing functions which are associated with the inner quark structure defined by the quark spin. These functions can contribute to the corresponding single quark spin asymmetry inside the unpolarized hadron or to the alignment \( k_{\perp}\)-dependent functions.

2 The time-reversal transforms of parameterizing functions

In this section, we dwell on the comprehensive analysis of the fundamental properties which have been imposed on the transverse momentum dependent distribution functions. Namely, we focus on the time-reversal transforms of the correlators with and without the taking into account of interactions.

2.1 The role of interactions in the correlators

At the beginning, we remind the role of H-representation (and the related interaction representation (I-representation)) in the definition of parametrizing functions. As an example, let us consider the most typical (forward) Compton scattering (CS) amplitude which takes the form of

\[
\mathcal{M}_{\mu\nu} = \langle P | a_{\nu}^- (q) \mathcal{S}[\psi, \bar{\psi}, A] a_{\mu}^+ (q) | P \rangle,
\] (2.1)

where \( \mathcal{S} \)-matrix is given by

\[
\mathcal{S}[\psi, \bar{\psi}, A] = T \exp \left\{ i \int (d^4z) [\mathcal{L}_{QCD}(z) + \mathcal{L}_{QED}(z)] \right\}.
\]
In Eqn. (2.1), in contrast to the photon Fock states the hadron states cannot be expressed through the relevant operators of creation and annihilation, 2.

Making use of the commutation relations of creation (or annihilation) operators with S-matrix (see, for example, Eqn. (38.20b) of [9]),

\[ [a_\mu (q), S[\bar{\psi}, \psi, A]] = \int (d^4 \xi) e^{\pm iq \xi} \frac{\delta S[\bar{\psi}, \psi, A]}{\delta A^\mu (\xi)} \]  

with (2.2)

\[ \frac{\delta S[\bar{\psi}, \psi, A]}{\delta A^\mu (\xi)} = T \{ \int (d^4 z) \frac{\delta Z_{QED}(z)}{\delta A^\mu (\xi)} S[\bar{\psi}, \psi, A] \}, \]  

the CS-amplitude can be rewritten as

\[ \mathcal{A}_{\mu \nu} = \int (d^4 \xi_1)(d^4 \xi_2) e^{-iq(\xi_1-\xi_2)} \langle P | \frac{\delta^2 S[\bar{\psi}, \psi, A]}{\delta A^\mu (\xi_1) \delta A^\nu (\xi_2)} | P \rangle \]  

\[ \Rightarrow \int (d^4 z) e^{-iqz} \langle P | T \{ [\bar{\psi}(0) \gamma_\mu \psi(0)] [\bar{\psi}(z) \gamma_\nu \psi(z)] S[\bar{\psi}, \psi, A] \} | P \rangle. \]  

(2.4)

Using Wick’s theorem and calculating the only quark operator contraction in Eqn. (2.4), we can readily obtain the simplest “hand-bag” diagram contribution to the CS-amplitude. Within the momentum representation, it reads

\[ \mathcal{A}_{\mu \nu}^{\text{hand-bag}} = \int (d^4 k) \text{tr} \left[ E_{\mu \nu}(k) \Phi(k) \right], \]  

where

\[ E_{\mu \nu}(k) = \gamma_\mu S(k+q) \gamma_\nu + \gamma_\nu S(k-q) \gamma_\mu, \]  

\[ \Phi(k) = \int (d^4 z) e^{ikz} \langle P | T \bar{\psi}(0) \psi(z) S[\bar{\psi}, \psi, A] | P \rangle_c. \]  

(2.7)

Here, the subscript c denotes the connected diagram contributions which we only consider.

It is worth to notice that in Eqn. (2.7) the nonperturbative correlator has been written in the interaction representation. It is more compact to use, however, the Heisenberg representation of this correlator, i.e.

\[ \Phi(k) = \int (d^4 z) e^{ikz} \langle P | T \bar{\psi}(0) \psi(z) | P \rangle^H. \]  

(2.8)

We stress that the neglecting H in Eqn. (2.8) may result in the wrong impression about the absence of interaction in the correlator.

In the relevant correlators of Eqns. (2.7) and (2.8), the interactions which are described by the S-matrix generate, first, the Wilson lines ensuring the gauge invariance of non-local operators and, second, the contributions which are not being exponentiated. The latter can be associated with the evolutions of the corresponding functions or/and with the tensor structure of Lorentz parametrization, see section 3. Since the Wilson lines can be eliminated by the contour gauge use [10–12], in what follows we mainly focus on the non-exponentiated contributions of S-matrix.

2The creation and annihilation hadron operators can be introduced with the help of the effective Lagrangian describing the transition of partons onto hadrons. This is the so-called effective quark-hadron Lagrangian of interaction [8].
The CS-amplitude of Eqn. (2.5) is not factorized yet. We omit the full description of the factorization procedure which is presented in the literature in detail. We would like to notice that even after the applied factorization procedure the CS-amplitude contains the corresponding correlator (forming the soft part of amplitude) where $S$-matrix has to be included either in $I$-representation or in $H$-representation.

2.2 On in- and out-states in the correlators

In this subsection, we remind the classical definitions of in- and out-states which appear in any correlators, see for example Eqns. (2.7) and (2.8). The properties of these states play an important role for our study.

As a example, we consider the physical (scalar hadron) states which are being described by the in- and out-fields. The in- and out-fields are nothing but the asymptotically free fields. From the viewpoint of the standard classical scattering theory, they can be treated as the asymptotes to the given trajectory. In other words, in order to find the correspondence between the initial and final asymptotic states, we have to know the potential of interaction and to solve the corresponding equations of motion (see [13, 14] for further details). Indeed, let us consider the following matrix element

$$\langle p_1, \ldots, p_n; \text{out} | q_1, \ldots, q_m; \text{in} \rangle = \langle 0 | a_{\text{out}}^+(p_1) \ldots a_{\text{out}}^-(p_n) | q_1, \ldots, q_m; \text{in} \rangle, \quad (2.9)$$

where the states have been constructed from the scalar hadron fields ($\varphi$-fields) and $\langle 0 \rangle$ is a mathematical vacuum state.

Using the relation [13, 14]

$$a_{\text{out}}^\pm (p_i) = S^\dagger [\varphi] \otimes a_{\text{in}}^\pm (p_i) \otimes S[\varphi] \quad (2.10)$$

with the unitary $S$-matrix describing the interaction of scalar fields $\varphi$, Eqn. (2.9) takes the form of

$$\langle 0 | a_{\text{out}}^-(p_1) \ldots a_{\text{out}}^-(p_n) | q_1, \ldots, q_m; \text{in} \rangle =$$

$$\langle 0 | a_{\text{in}}^-(p_1) \ldots a_{\text{in}}^-(p_n) S[\varphi] | q_1, \ldots, q_m; \text{in} \rangle = \langle p_1, \ldots, p_n; \text{in} | S[\varphi] | q_1, \ldots, q_m; \text{in} \rangle. \quad (2.11)$$

Hence, we derive the well-known relation between out- and in-states which correspond to the final Fock state given by

$$\langle p_1, \ldots, p_n; \text{out} \rangle = \langle p_1, \ldots, p_n; \text{in} | S[\varphi] \rangle. \quad (2.12)$$

It is important to notice that one-particle states meet the trivial condition

$$\langle 0 | a_{\text{out}}^- (p) = \langle 0 | a_{\text{in}}^- (p). \quad (2.13)$$

Indeed, the simple algebra gives us the following line of reasoning

$$\langle 0 | a_{\text{out}}^- (p) = \langle 0 | a_{\text{in}}^- (p) S[\varphi] = \langle 0 | [a_{\text{in}}^- (p), S[\varphi]] + \langle 0 | S[\varphi] a_{\text{in}}^- (p) = \langle 0 | a_{\text{in}}^- (p). \quad (2.14)$$

In this line of reasoning, we take into account that [9]

$$\langle 0 | \left[ a_{\text{in}}^- (p), S[\varphi] \right] = \int (d^4 \xi) e^{ip \xi} \langle 0 | \frac{\delta S[\varphi]}{\delta \varphi (\xi)} \rangle \sim \delta^{(4)} (p). \quad (2.15)$$

[9]
owing to the fact that the variation derivative of $\mathbb{S}[\varphi]$ gives only the tadpole contributions multiplying by the delta-function with the momentum conservation as an argument. And, if the hadron momentum $p$ is nonzero, the commutator term in Eqn. (2.14) does not contribute.

Within the center-mass system, Eqn. (2.12) can be simplified for the case of two-particle states with the help of partial wave expansion. It reads [13]

$$\langle p_1, p_2; \text{out} | = \langle p_1, p_2; \text{in} | \mathbb{S}[\varphi] = \langle p_1, p_2; \text{in} | e^{i \delta_{p_1 p_2}(W)},$$

(2.16)

where $\delta_{p_1 p_2}(W)$ implies the scattering phase as a function of energy $W$. Eqn. (2.16) shows that even in the most simple case the $\mathbb{S}$-matrix indeed generates the complexity.

To conclude this subsection, we discuss the most general form of correlators given by

$$\langle P; \text{out} | \hat{\mathcal{O}} (\bar{\psi}, \psi, A) | P; \text{in} \rangle,$$

(2.17)

where the quark-gluon operator $\hat{\mathcal{O}} (\bar{\psi}, \psi, A)$ is defined as $\hat{\mathcal{O}} (\bar{\psi}, \psi, A) \mathbb{S}[\bar{\psi}, \psi, A]$ (see Eqn. (2.7)). Here, the hadron states of $\langle P; \text{out} |$ and $| P; \text{in} \rangle$ are the one-particle states unless the composite parton (quark-gluon) structure of hadron has been incorporated. In the case of bound hadron states, the annihilation operator of hadron should be replaced by the unknown, from the point of view of QCD, function $A^-$ of annihilation operators of partons, i.e. we have

$$a^-_{\text{out}}(P) \Rightarrow A^-_{\text{out}}(a^-_{\text{out}}(k_1), \ldots, a^-_{\text{out}}(k_n)|P = k_1 + \ldots + k_n),$$

(2.18)

and the similar expression can be written for the in-state. The function $A^-$ should meet the condition as

$$\langle 0 | A^-_{\text{out}}(a^-_{\text{out}}(k_1), \ldots, a^-_{\text{out}}(k_n)) = \langle 0 | A^-_{\text{in}}(a^-_{\text{in}}(k_1), \ldots, a^-_{\text{in}}(k_n))$$

(2.19)

because in the limit where the hadron is described as a point-like particle one should get the condition of Eqn. (2.13). Moreover, Eqn. (2.19) resembles formally the condition known from the theory of group representations. Indeed, the necessary condition to construct the representation of a group is $F'(x') \equiv U_g F(O_g x) = F(x)$ or $F(O_g x) = U^{-1}_g F(x)$ where $U_g$ is an operator that acts on the representation space and $O_g$ is an operator defined on the given group.

Hence, for the quark-gluon partons, we have the following relation

$$\mathcal{S}[\bar{\psi}, \psi, A] \otimes A^-_{\text{in}}(\mathcal{S}[\bar{\psi}, \psi, A] \otimes a^-_{\text{in}}(k_1) \otimes \mathbb{S}[\bar{\psi}, \psi, A], \ldots) \otimes \mathcal{S}[\bar{\psi}, \psi, A] =$$

$$A^-_{\text{in}}(a^-_{\text{in}}(k_1), \ldots, a^-_{\text{in}}(k_n)),$$

(2.20)

or

$$\langle 0 | A^-_{\text{in}}(a^-_{\text{out}}(k_1), \ldots, a^-_{\text{out}}(k_n)|P = k_1 + \ldots + k_n) =$$

$$\langle 0 | \mathcal{S}[\bar{\psi}, \psi, A] \otimes A^-_{\text{in}}(a^-_{\text{in}}(k_1), \ldots, a^-_{\text{in}}(k_n)|P = k_1 + \ldots + k_n) \otimes \mathcal{S}[\bar{\psi}, \psi, A],$$

(2.21)

where $\mathcal{S}[\bar{\psi}, \psi, A]$ describes effectively the quark-gluon interactions inside the hadron. In other words, the relation given by Eqn. (2.21) can be treated as a definition of the unknown $\mathcal{S}$-operator which reflects the fact that the bound state is being always considered as the dressed (by the quark-gluon interaction) states. That is, we adhere the conception according to which the physical bound
states have been treated as the states which cannot be described by the free Lagrangian. The physical statement on that the bound states are always dressed is equivalent to the mathematical requirement on the nullification of the hadron renormalization constant, \( Z_h = 0 \). This is the so-called compositeness condition, see [8] and Appendix A for the clarifying details. Notice that for our further study the explicit form of \( \mathcal{S} \)-operator is not required, however the presence of this operator supports additionally the statement on the complexity of the given correlator.

With these, Eqn. (2.17) can be presented as

\[
\langle 0| A_{\mu}^C (a_{\text{out}}^C (k_1), ..., a_{\text{out}}^C (k_n)) | P = k_1 + ... + k_n \rangle \mathcal{O} (\psi, \psi, A) | P; \text{in} \rangle = \\
\langle 0| A_{\mu}^C (a_{\text{in}}^C (k_1), ..., a_{\text{in}}^C (k_n)) | P = k_1 + ... + k_n \rangle \mathcal{S} [\psi, \psi, A] \mathcal{O} (\psi, \psi, A) | P; \text{in} \rangle = \\
\langle P; \text{in} | \mathcal{O} [\bar{\psi}, \psi, A] \mathcal{O} (\psi, \psi, A) | P; \text{in} \rangle. \tag{2.22}
\]

The principle conclusion from this consideration is that the trivial relation between one-hadron in- and out-state given by Eqn. (2.13) has been non-trivially modified if the quark-gluon interactions are included in the given hadron.

### 2.3 The time-reversal properties without interactions in correlators

We are going over to the discussion of properties which are originated from the time-reversal transformations. The time-reversal transforms imply that

\[
\langle \Phi_2 | U^T \mathcal{O} (0, z) | \Phi_1 \rangle = \left[ \langle \Phi_2 | \mathcal{O} (0, z) | \Phi_1 \rangle \right]^T,
\]  

where \( \mathcal{O} \) is an arbitrary nonlocal operator with the closed Dirac indices, the operator \( U_T \) is acting on the Fock states. In Eqn. (2.23), the correlator does not contain any interactions, \( \mathcal{S} [\bar{\psi}, \psi, A] = 1 \).

For the sake of definiteness, we suppose that

\[
\mathcal{O} (0, z) = \bar{\psi} (0) \gamma^+ \psi (z), \quad \langle \Phi_1 \rangle = | P, S \rangle, \quad \langle \Phi_2 \rangle = | P, S \rangle.
\]  

As mentioned, the Wilson lines in \( \mathcal{O} (0, z) \) have been eliminated by the contour gauge of axial type [10–12] and, in the context of our study, they are not considered as the effect of the interaction presence.

Hence, Eqn. (2.23) takes the form of

\[
\langle P, \bar{S} | \bar{\psi} (0) \gamma^+ \psi (z) | P, \bar{S} \rangle = \left[ \langle P, S | \bar{\psi} (0) \gamma^+ \psi (z) | P, S \rangle \right]^T,
\]  

where \(^3\)

\[
\tilde{z} = (-z_0, \tilde{z}), \quad \tilde{A} = (A_0, -\tilde{A}) \quad \text{for} \quad A = (P, S).
\]  

Concentrating on the l.h.s. of Eqn. (2.25) and extracting only \((b^+ b^-)\)-combination of quark creation operators, the Fourier transforms of quark operators give the following representation

\[
\langle P, \bar{S} | \bar{\psi} (0) \gamma^+ \psi (z) | P, \bar{S} \rangle = \int (d^4 k) e^{-i\tilde{z}k} \Phi \bar{\psi} (k, P, \bar{S}) \frac{L_{\text{par}}}{2\pi} \\
\int (d^4 k) e^{-i\tilde{z}k} \mathcal{F} \left\{ \mathcal{F} f_1 (\tilde{k}^-, \tilde{k}^+) + e^{-i\tilde{k}^+ \tilde{A}_1 (n)} (\tilde{k}^-, \tilde{k}^+) + .... \right\}, \tag{2.27}
\]

\(^3\)More precisely, if we use the standard definition of covariant and contravariant vectors, we have \( \tilde{z}^\mu = (-z_0, \tilde{z}) = -z_\mu \) and \( \tilde{A}^\mu = (A_0, -\tilde{A}) = A_\mu \). However, this is irrelevant for the subject of our discussion
where the normalized function \( f_{1T}^{(n)} = f_{1T} / m_N \) has been introduced and the Lorentz parametrization (L. par.) has been applied. Also, in this representation we implement the integration over \( dk^+ \).

Then, taking into account that
\[
\tilde{k}^\pm = k^\mp, \quad \tilde{P}^\pm = P^\mp, \quad \tilde{z}^\pm = -z^\pm,
\]
\[
\tilde{k}_\perp = -\tilde{k}_\perp, \quad \tilde{P}_\perp = -\tilde{P}_\perp, \quad \tilde{z}_\perp = \tilde{z}_\perp
\]
(2.28)

Eqn. (2.27) can be rewritten as
\[
(P, S) |\bar{\psi}(0)\gamma^+ \psi(z)|P, S\rangle \xrightarrow{\text{L. par.}} \int (dk^+) (d^2\tilde{k}_\perp) e^{ik^+z^+ + \tilde{k}_\perp \tilde{z}_\perp} \left\{ P^+ f_1(k^+, k_\perp) - e^{+\tilde{P}_\perp S} f_{1T}^{(n)}(k^+, k_\perp) + \ldots \right\} = \int (dk^+) (d^2\tilde{k}_\perp) e^{ik^+z^+ - \tilde{k}_\perp \tilde{z}_\perp} \left\{ P^+ f_1(k^+, -k_\perp) - e^{+\tilde{P}_\perp S} f_{1T}^{(n)}(k^+, -k_\perp) + \ldots \right\}, \quad (2.29)
\]
where in the second term we use that
\[
e^{-\tilde{P}_\perp S} P^+ e^{-i\tilde{k}_\perp S} = P^+ e^{-i\tilde{P}_\perp S} = e^{i\tilde{P}_\perp S} = e^{i\tilde{P}_\perp S}.
\]

Let us now consider the r.h.s. of Eqn. (2.25) where one can see the following subtlety. Namely, the Lorentz parametrization (or decomposition) can be performed both (a) before the Hermitian conjugation and (b) after this conjugation:

- the first way (a) leads to
\[
\int (dk^+) (d^2\tilde{k}_\perp) e^{ik^+z^+ + \tilde{k}_\perp \tilde{z}_\perp} \left\{ P^+ f_1(k^+, k_\perp) + e^{+\tilde{P}_\perp S} f_{1T}^{(n)}(k^+, k_\perp) + \ldots \right\}^\dagger = \int (dk^+) (d^2\tilde{k}_\perp) e^{ik^+z^+ - \tilde{k}_\perp \tilde{z}_\perp} \left\{ P^+ f_1(k^+, k_\perp) + e^{+\tilde{P}_\perp S} f_{1T}^{(n)}(k^+, k_\perp) + \ldots \right\}; \quad (2.31)
\]

- while the second way (b) gives
\[
\int (dk^+) (d^2\tilde{k}_\perp) e^{ik^+z^+ - \tilde{k}_\perp \tilde{z}_\perp} \left\{ P^+ f_1(k^+, k_\perp) + e^{+\tilde{P}_\perp S} f_{1T}^{(n)}(k^+, k_\perp) + \ldots \right\}.
\]
(2.32)

Comparing Eqns. (2.31) and (2.32), we derive the following properties
\[
[f_1(k^+, k_\perp)]^* = f_1(k^+, k_\perp), \quad [f_{1T}^{(n)}(k^+, k_\perp)]^* = f_{1T}^{(n)}(k^+, k_\perp)
\]
(2.33)
provided (here, we do not specify the in- and out-states because the hadron states have been considered as one-particle states, see Eqn. (2.13))
\[
[P, S]^\dagger = (P, S).
\]
(2.34)

These properties together with Eqns. (2.25) and (2.29) result in
\[
f_1(k^+, -k_\perp) = [f_1(k^+, k_\perp)]^* = f_1(k^+, k_\perp) \quad \rightarrow \quad \text{T-even}
\]
\[
-f_{1T}^{(n)}(k^+, -k_\perp) = [f_{1T}^{(n)}(k^+, k_\perp)]^* = f_{1T}^{(n)}(k^+, k_\perp) \quad \rightarrow \quad \text{T-odd}.
\]
(2.35)

The properties of Eqn. (2.35) are well-known in the literature for the transverse momentum dependent functions.
2.4 The influence of interactions on the time-reversal properties

In the preceding subsection the interactions in correlators have been excluded. Now, we analyse the influence of interactions in the relevant correlators on the properties of parametrizing functions with respect to \( k_\perp \to -k_\perp \).

First, we dwell on the Hermitian conjugation which appears in the r.h.s. of Eqn. (2.25). Taking into account \( S \)-matrix, we have the following

\[
\left[ \langle P,S|\bar{\psi}(0)\gamma^+\psi(z)S[\bar{\psi},\psi,A]|P,S \rangle \right]^\dagger.
\]  

(2.36)

For the sake of illustration, it is convenient to expand \( S \)-matrix, say, up to the second order of interaction, see Fig. 1, i.e.

\[
\left[ \langle P,S|\bar{\psi}(0)\gamma^+\psi(z)S^{(2)}[\bar{\psi},\psi,A]|P,S \rangle \right]^\dagger.
\]

We stress that the order of expansion does not play any role for our final conclusions.

Having used the results of subsection 2.2, we have the following

\[
\langle P,S \rangle \equiv \langle P,S;out \rangle = \langle P,S;in|\mathcal{S}[\bar{\psi},\psi,A],
\]

\[
\langle P,S \rangle \equiv |P,S;in \rangle = \mathcal{S}^\dagger[\bar{\psi},\psi,A]|P,S;out \rangle.
\]  

(2.37)

In contrast to Eqn. (2.34), the Hermitian conjugation of Eqn. (2.37) shows that (see Fig. 1)

\[
\left[ \langle P,S;in \rangle \right]^\dagger = \langle P,S;out \rangle \mathcal{S}[\bar{\psi},\psi,A] \neq \langle P,S;out \rangle \quad \text{etc.}
\]  

(2.38)

owing to the included interactions of quarks and gluons inside the hadron. And as such, the correlator of Eqn. (2.36) can be presented in the form of

\[
\left[ \langle P,S;out \rangle \bar{\psi}(0)\gamma^+\psi(z)S[\bar{\psi},\psi,A],\mathcal{S}^\dagger[\bar{\psi},\psi,A]|P,S;out \rangle \right]^\dagger = 
\]

\[
\langle P,S;out \rangle \mathcal{S}[\bar{\psi},\psi,A]S^\dagger[\bar{\psi},\psi,A] \psi(z)\gamma^+\psi(0)|P,S;out \rangle.
\]  

(2.39)

(2.40)

From these, it is clear that the functions parametrizing the given correlator must be complex functions thanks for the presence of the product \( S[\bar{\psi},\psi,A],\mathcal{S}^\dagger[\bar{\psi},\psi,A] \). The difference between the parametrizing functions of Eqns. (2.39) and (2.40) appears only in the imaginary parts, see below.

Thus, the Lorentz parametrization is sensitive to the correlator which we deal with. Namely, the Hermitian conjugation is not a “commutative” operation with the Lorentz parametrization if the interactions are presented. Hence, the parametrizing functions in Eqns. (2.31) and (2.32) are not identical ones and the properties defined in Eqn. (2.33) are modified by

\[
[f_1^{(a)}(k^+,k_\perp)]^* = f_1^{(b)}(k^+,k_\perp), \quad [f_{1T}^{(n)(a)}(k^+,k_\perp)]^* = f_{1T}^{(n)(b)}(k^+,k_\perp).
\]  

(2.41)

or

\[
\Re f_1^{(a)}(k^+,k_\perp) = \Re f_1^{(b)}(k^+,k_\perp), \quad -\Im f_1^{(a)}(k^+,k_\perp) = \Im f_1^{(b)}(k^+,k_\perp);
\]

\[
\Re f_{1T}^{(n)(a)}(k^+,k_\perp) = \Re f_{1T}^{(n)(b)}(k^+,k_\perp), \quad -\Im f_{1T}^{(n)(a)}(k^+,k_\perp) = \Im f_{1T}^{(n)(b)}(k^+,k_\perp).
\]  

(2.42)
So, the time-reversal transformations given by
\[
\langle \tilde{P}, \tilde{S}; in | \bar{\psi}(0) \gamma^+ \psi(z) \tilde{S}[\bar{\psi}, \psi, A] | P, S; out \rangle + \text{inter.} = \langle P, S; out | \bar{\psi}(0) \gamma^+ \psi(z) \tilde{S}[\bar{\psi}, \psi, A] | P, S; in \rangle \tag{2.43}
\]
together with Eqn. (2.31) result in the following properties for the corresponding parametrizing functions \(^4\):
\[
\{ f_1(k^+, -k\perp), f_{1T}^{(n)}(k^+, -k\perp) \} \in \mathbb{C} \tag{2.44}
\]
which demonstrate that there are no the definite properties under the replacement \(k\perp \to -k\perp\) in contract to the standard consideration. Again, the usual time-reversal properties are restored by the \(k\perp\)-integrations.

The functional complexity of parametrizing functions we have discovered can be manifested in the DY-like processes with the essential contributions from the gluon poles \([16]\).

### 3 The Lorentz parametrization of correlators including the interactions

In the preceding section, it has been shown that due to the different sources of interactions in the correlators, \(i.e.\) in the hadron matrix element of quark-gluon operators, all of the Lorentz parametrizing functions become the complex functions. Now, we study the influence of interactions on the Lorentz parametrization in order to find new possible parametrizing functions. For this aim, the \(in\)- and \(out\)-states can be left without the specification explained in the preceding section because we now focus on the tensor structure of relevant correlators.

We begin with the vector (the plus light-cone projection) correlator written in the interaction representation. It reads \(^5\)
\[
\Phi^{(\gamma^+)}(k) = \int (d^4z) e^{ikz} \langle P, S; out | \bar{\psi}(0) \gamma^+ \psi(z) \tilde{S}[\bar{\psi}, \psi, A] | P, S; in \rangle. \tag{3.1}
\]
Here, the \(\tilde{S}\)-matrix generates the explicit and implicit loop integrations (modulo the Wilson lines which are irrelevant within the contour gauge, see below). The explicit loop integrations, see the right panel of Fig. 2, are responsible for the forming of evolution integral kernels. While the
Figure 2. The types of loop integrations in the corresponding correlators: the left panel corresponds to the demonstration of the implicit loop integrations defined the Lorentz structure; the right panel – to the explicit loop integrations contributing to the evolution integration kernels.

implicit loop integrations, see the left panel of Fig. 2 as a particular example, are determining the Lorentz structures of the correlators.

It is now a time to make the important comments on the Wilson lines. In Eqn. (3.1), the Wilson lines, which ensure the gauge invariance of non-local operators, are not shown because they can be eliminated by the corresponding contour gauge [10–12]. This point requires the additional explanations. Following to [11, 12], all the gluon radiation contributions that appears in the corresponding quark-gluon correlators of the Drell-Yan hadron tensor can be separated out in the three classes 6:

• (i) the longitudinal $A^+$ and transverse $A^\perp_i$ ($i = 1, 2$) gluons from the standard diagram (see the left panel of Fig. 3) which are being exponentiated in the corresponding Wilson lines;

• (ii) the longitudinal $A^-$ and transverse $A^\perp_i$ gluons from the non-standard diagram (see the right panel of Fig. 3) which are being exponentiated in the other Wilson lines;

• (iii) the transverse $A^\perp_i$ gluons from both the standard and non-standard diagrams which cannot be exponentiated in the corresponding Wilson lines, but they construct the higher twist quark-gluon correlators.

Notice that, in contrast to the Wilson lines with longitudinal gluons, the nullification of the exponential functional in the relevant Wilson lines,

$$W\left(A^\perp_i P(x^0_\perp \; x^\perp)\right) = \mathbb{P}\exp\left\{ig \int_{x^0_\perp}^{x^0_\perp} d\omega_i A^\perp_i (x^0_\perp, x^\perp, \omega^\perp)\right\},$$

is related to the nullification of the full integral (but not the integrand) containing $A^\perp_i$, see [10]. In other words, in this case the trivialization of the Wilson lines does not lead to the absence of $A^\perp_i$. Also, in the context of the contour gauge use, it is important to stress that the given contour gauge

\footnotetext[4]{Here, the specification of (a) and (b) ways is irrelevant for the complex functions.}

\footnotetext[5]{The symbol of time-ordering is omitted.}

\footnotetext[6]{The standard and non-standard diagrams have been defined in [11, 12]}
Figure 3. The standard diagram of DY-process, see the left panel, and the non-standard diagram of DY-process, see the right panel.

Figure 4. The function $\Phi^\gamma(k)$ at the forth order of strong coupling constant.

is not equivalent to the local axial gauge in the same manner as the given vector is not equivalent to its projection on the non-trivial direction [10]. By definition, the trivial direction needed for the relevant projection coincides with the giving vector. Therefore, in discussing the interaction influence, the Wilson lines have been excluded from our consideration. This our “assumption” does not effect on our principal conclusions.

Among the standard parametrizing functions which are associated with the vector correlator (see, for example, [3]), we introduce the functions that are accompanying the Lorentz tensor with the quark spin (covariant) vector provided the interaction has been included in the correlator [15]. In particular, we have argued that the new type of functions is not excluded in the parametrization:

$$\Phi^\gamma(k) = i e^{+ - V^\perp(x)} f_1^{(V)}(x; k^2_\perp) + ....$$  \hspace{1cm} (3.3)

where $s_\perp$ stands for the quark spin axial-vector and $V_\perp = \{P_\perp; k_\perp\}$.

In [15], in order to prove the existence of this type of functions the second order of the S-matrix decomposition over the strong coupling constant and the Fierz transformations applied for the relevant four-fermion combination have been used. Here, we give an alternative proof working with the fourth order of decomposition over the strong coupling constant and without the mentioned four-fermion Fierz transforms.
Let us rewrite Eqn. (3.1) in the form of expansion as, see Fig. 4,

\[ \Phi^{\gamma^1}(k) = \int (d^4 z) e^{+i k z} \langle P, S | \psi(0) \gamma^+ \psi(z) \Sigma^{(4)}[\psi, \psi, A] | P, S \rangle = \int (d^4 k_1)(d^4 \ell) \text{tr}[\gamma^\nu S(k) \gamma^\mu S(k - \ell) \gamma^\nu S(k)] D_{\mu\nu}(\ell) D_{\nu\nu}(\ell) \mathcal{F}^{\nu\mu}(k_1, \ell), \]

where

\[ \mathcal{F}^{\nu\mu}(k_1, \ell) = \int (d^4 \xi) e^{-ik_1 \xi} \langle P | \bar{\psi}(\xi) \gamma^\nu S(k_1 - \ell) \gamma^\mu \psi(0) | P \rangle. \]

Focusing on the axial-vector projection of Fierz decomposition of two fermions, the function \( \mathcal{F}^{\nu\mu}(k_1, \ell) \) takes the form of

\[ -4 \mathcal{F}^{\nu\mu}_{(A)}(k_1, \ell) = \text{tr}[\gamma^\nu S(k_1 - \ell) \gamma^\mu \gamma_5 \Phi^{[\gamma_5]}(k_1) \]

with

\[ \Phi^{[\gamma_5]}(k_1) = \int (d^4 \xi) e^{-ik_1 \xi} \langle P | \bar{\psi}(\xi) \gamma_5 \psi(0) | P \rangle. \]

The (sub)structure function \( \Phi^{[\gamma_5]}(k_1) \) in Eqn. (3.7) can be presented in the form of \( \mathcal{M} \)-amplitude written in the momentum representation:

\[ \mathcal{M}^{(4)}(P_f - P_i) \mathcal{M}(k_1) = \mathcal{M}^{(4)}(0) \Phi^{[\gamma_5]}(k_1) = \langle P | b^+(k_1) b^-(k_1) | P \rangle \left[ \bar{u}(k_1) \gamma_5 u(k_1) \right], \]

where the quark axial-vector combination gives the quark spin, i.e.

\[ \left[ \bar{u}(k_1) \gamma_5 u(k_1) \right] \sim s_\alpha. \]

Hence, the interactions which have been included in the corresponding correlator give the evidences for the existence of a new type of parametrizing functions in Eqn. (3.3). These functions can be treated as the corresponding single quark spin asymmetry inside the unpolarized hadron or as the alignment \( k_\perp \)-dependent functions introduced in [15].

To conclude the discussion presented in this subsection, we emphasize that the Lorentz tensor \( i e^{+ - V_{\perp, s_\perp}} \) of Eqn. (3.3) contains implicitly the information on the frame system where the factorization procedure has been implemented. The separation on the longitudinal and transverse components of different Lorentz vectors implies already the certain fixed frame. Indeed, the corresponding Lorentz invariant would take the form of \( i e^{pV_s} \) with the vectors \( p \) and \( n \) being the dominant and sub-sub-dominant directions which can be actually chosen in an arbitrary way but in the connection with the kinematics of a given process. As argued in [16], for the Drell-Yan-like processes the most appropriated frame of factorization is given by the Collins-Soper (CS) system where the factorization procedure takes the archetypal form. In the CS-frame, the proposed new type of parametrizing functions can be firmly singled out. However, the fact of the new function existence might be not easily seen in the other frames. Indeed, there is no doubt that any Lorentz invariant (say, an arbitrary scalar products) is independent on the chosen frame by construction, but the representation of the given scalar product by components depends certainly on the frame.
4 Conclusions

In the presented paper, we have demonstrated that the parametrizing functions which correspond to the certain correlator are actually complex functions owing to the discovered role of interactions in the correlator. As a result of the time-reversal transforms, the $k_\perp$-dependent parametrizing functions do not possess the definite properties regarding the replacement $k_\perp \rightarrow -k_\perp$. However, the usual time-reversal properties of functions can be restored after the integration over $k_\perp$.

Also, we have presented an additional evidences for discovering of a new type of parametrizing functions which are associated with the inner quark structure defined by the quark spin.

As a practical application of our findings, we suggest that the manifestation of the discussed functional complexity can be observed in the Drell-Yan-like (DY-like) processes with the essential role of the gluon pole contributions [16].

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A The compositeness condition for bound states

In the hadron physics, the compositeness condition used in the description of bound states is a key instrument for the practical calculations in the low and intermediate energy regions. All details on the compositeness condition and its practical application can be found in the seminal works [8, 17].

In this Appendix, for the convenience of readers, we present the main substantial points which should clarify the features of the compositeness condition for the high energy and QFT community where it is not widely known.

Let us consider two approaches/models with the interaction Lagrangians defined as (here we use the symbolical forms where the corresponding coordinate dependences and the tensor structures are neglected)

\[
\mathcal{L}_\psi^I = \lambda_0 (\bar{\psi} \psi)^2, \quad \mathcal{L}_Y^I = g_0 M (\bar{\psi} \psi), \tag{A.1}
\]

where the coupling constants and all fields are assumed to be bare (or unphysical) variables; $M$ and $\psi$ stand for the boson (hadron) and quark states.

Notice that the interaction Lagrangian $\mathcal{L}_\psi^I$ resembles formally the Nambu–Jona-Lasinio (NJL) model, while the interaction Lagrangian $\mathcal{L}_Y^I$ of the Yukawa-type takes the form of the NJL-model after the bosonization procedure [18]. As well-known, the approach with $\mathcal{L}_\psi^I$ in the $D = 4$ space refers to the unrenormalizable class of theories. In what follows we do not dwell on the questions of renormalization of this approach, instead we express all quantities appearing within this approach via the renormalized quantities that are derived in the approach with $\mathcal{L}_Y^I$.

The compositeness condition ensures that the description of the physical processes within both two approaches with $\mathcal{L}_\psi^I$ and $\mathcal{L}_Y^I$ coincides completely. We stress that the hadron states are
considered as the bound states of quarks in these approaches. In order to demonstrate that, based on $\mathcal{L}_I^Y$, we first calculate the two-point Green function of bosons within the chain approximation (which is fully enough for our discussion). It reads

$$G_Y^{(2)}(s) \equiv D_M(s) = \frac{1}{m_0^2 - s - g_0^2 \Sigma(s)}, \quad (A.2)$$

where the mass operator $\Sigma$ is determined by the quark 1-loop integration which is divergent as a logarithm. The physical boson with mass $m^2$ corresponds to the pole of this Green function which is a function of $s$:

$$m_0^2 - s - g_0^2 \Sigma(s) \bigg|_{s=m^2} = 0. \quad (A.3)$$

Hence, using the Taylor expansion of $\Sigma$ around $s = m^2$, we have

$$D_M(s) = \frac{1}{m^2 - s} \cdot \frac{1}{1 + g_0^2 \Sigma(m^2) + g_0^2 \Sigma(m^2, s)},$$

$$\Sigma'(m^2) = \frac{\partial \Sigma(s)}{\partial s} \bigg|_{s=m^2}, \quad \Sigma(m^2, s) = \frac{\Sigma(m^2, s)}{s - m^2} \quad (A.4)$$

where $\Sigma(m^2, s)$ implies the finite residual term of the Taylor expansion. Eqn. (A.4) can be rewritten as

$$D_M(s) = \frac{Z_M}{m^2 - s} \cdot \frac{1}{1 + g_R^2 \Sigma(m^2, s)}, \quad (A.5)$$

where

$$Z_M = \frac{1}{1 + g_0^2 \Sigma'(m^2)} \quad \text{and} \quad g_R^2 = \frac{g_0^2}{1 + g_0^2 \Sigma'(m^2)} \equiv Z_M g_0^2. \quad (A.6)$$

With the help of these equations, one can express the renormalization constant through the renormalized coupling constant as

$$Z_M = 1 - g_R^2 \Sigma'(m^2). \quad (A.7)$$

Then, we calculate the four-point Green function with the same interaction Lagrangian $\mathcal{L}_I^Y$. We obtain that

$$G_Y^{(4)}(s) \equiv \Gamma(s) = -\frac{g_0^2}{m_0^2 - s - g_0^2 \Sigma(s)} = -\frac{g_R^2}{m^2 - s} \cdot \frac{1}{1 + g_R^2 \Sigma(m^2, s)}. \quad (A.8)$$

On the other hand, we can calculate the four-point Green function within the approach with $\mathcal{L}_I^\psi$. In the chain approximation, we derive that

$$G_\psi^{(4)}(s) = \frac{\lambda_0}{1 + \lambda_0 \Sigma(s)}, \quad (A.9)$$

where the function $\Sigma(s)$ is the same function as in Eqn. (A.2). Again, this Green function considered as the scattering amplitude has a pole which corresponds to the physical boson with $m^2$ provided

$$1 + \lambda_0 \Sigma(s) \bigg|_{s=m^2} = 0 \quad \Rightarrow \quad \lambda_0 = -\frac{1}{\Sigma(m^2)}. \quad (A.10)$$
Inserting the second equation of Eqn. (A.10) into Eqn. (A.9), we get that
\[ G_\psi^{(4)}(s) = \frac{1}{\Sigma(s) - \Sigma(m^2)} = \frac{1}{\Sigma'(m^2)(s - m^2) + \Sigma(m^2,s)}. \]  
(A.11)

Having multiplied and divided Eqn. (A.11) by \( g_R^2 \), we obtain
\[ G_\psi^{(4)}(s) = - \frac{g_R^2}{m^2 - s} \cdot \frac{1}{g_R^2 \Sigma'(m^2)} \cdot \frac{1}{1 + \Sigma(m^2,s)/\Sigma'(m^2)}. \]  
(A.12)

In addition, we note that
\[ g_R^2 \Sigma'(m^2) = 1 - Z_M, \quad \frac{\Sigma(m^2,s)}{\Sigma'(m^2)} = \frac{g_R^2 \Sigma(m^2,s)}{1 - Z_M}. \]  
(A.13)

Thus, we derive that the four-point Green function within the approach with \( \mathcal{L}_\psi^l \) reads
\[ G_\psi^{(4)}(s) = - \frac{g_R^2}{m^2 - s} \cdot \frac{1}{1 - Z_M} \cdot \frac{1}{1 + g_R^2 \Sigma(m^2,s)/(1 - Z_M)}. \]  
(A.14)

According to the compositeness condition, the four-point Green functions (as the scattering amplitudes) of Eqns. (A.8) and (A.14) should describe the same physics, i.e.
\[ G_\psi^{(4)}(s) = G_\psi^{(4)}(s). \]  
(A.15)

This is possible if and only if we deal with the condition
\[ Z_M = 0. \]  
(A.16)

In other words, the compositeness condition states that the physical hadron state is always dressed by the quark-gluon interaction: \( M(x) = Z_M^{1/2} M_R(x) = 0 \) due to Eqn. (A.16).

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