Charmonium suppression in the presence of dissipative forces in a strongly coupled quark-gluon plasma

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We study the survival of charmonium states in a strongly-coupled quark-gluon plasma in the presence of dissipative forces. We consider first-order dissipative corrections to the plasma equation of motion in the Bjorken boost-invariant expansion with a strongly-coupled equation of state for QGP and study the survival of these states with the dissociation temperatures obtained by correcting the full Cornell potential not its Coulomb part alone with a dielectric function encoding the effects of deconfined medium. We further explore the sensitivity of prompt and sequential suppression of these states to the shear viscosity-to-entropy density ratio, \( \eta/s \) from perturbative QCD and AdS/CFT predictions. Our results show excellent agreement with the recent experimental results at RHIC.

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I. INTRODUCTION

One of the amazing discoveries of experimental measurements at RHIC is the surprising amount of collective flow exhibited by the outgoing hadrons which is evidenced in both the single-particle transverse momentum distribution\textsuperscript{1} and the asymmetric azimuthal distribution around the beam axis\textsuperscript{2}, quantified by the parameters, \( v_1 \) and \( v_2 \), respectively. The parameter, \( v_2 \), in particular, was expected to be much smaller at RHIC than it was at the lower energies of the SPS\textsuperscript{3}; in fact, it is about twice as large. Theoretical calculations cannot predict such higher values of \( v_1 \) and \( v_2 \) unless the partonic cross-sections are artificially enhanced by more than an order of magnitude over perturbative QCD predictions\textsuperscript{4}. Thus quark-gluon matter created in these collisions is strongly interacting (SIQGP), unlike the type of weakly interacting quark-gluon plasma (wQGP) expected to occur at very high temperatures on the basis of asymptotic freedom\textsuperscript{5}. On the other hand, lattice QCD calculations yield an equation of state that differs from an ideal gas only by about 10% once the temperature exceeds 1.5\( T_c \)\textsuperscript{6}. Furthermore, perfect fluid dynamics with zero shear and bulk viscosities reproduces \( v_2 \) well up to transverse momenta of order 1.5 GeV/c; however, it cannot be zero. Indeed, the calculations within AdS/CFT suggest that \( \eta \geq s/(4\pi) \).

Charmonium (\( J/\psi \)) suppression has long been proposed as a signature of QGP formation\textsuperscript{7} and has indeed been seen at SPS\textsuperscript{8} and RHIC experiments\textsuperscript{9}. The heavy quark pair leading to the \( J/\psi \) mesons is produced in nucleus-nucleus collisions on a very short time-scale \( \sim 1/2m_c \), where \( m_c \) is the mass of the charm quark. The pair develops into the physical resonance over a formation time \( \tau \psi \) and traverses the plasma and (later) the hadronic matter before leaving the interacting system to decay (into a dilepton) to be detected. This long ‘trek’ inside the interacting system is fairly ‘hazardous’ for the \( J/\psi \). Even before the resonance is formed it may be absorbed by the nucleons streaming past it\textsuperscript{4}. By the time the resonance is formed, the screening of the colour forces in the plasma may be sufficient to inhibit a binding of the \( c\bar{c} \)\textsuperscript{7}. The resonance(s) could also be dissociated either by an energetic gluon\textsuperscript{10} or by a comoving hadron. A complete and deep understanding of the various stages of absorption is still missing and is under intense investigations\textsuperscript{12}. In order to disentangle these different effects, we must know the properties of quarkonium in medium and determine their dissociation temperatures.

The physics of a given quarkonium state is encoded in its spectral function\textsuperscript{13, 14}. So following how the spectral function changes with temperature can give us a theoretical insight to the temperature-dependence of quarkonium properties. There are mainly two lines of theoretical studies to determine quarkonium spectral functions: the first one is the potential models\textsuperscript{15,16,17,18} which have been widely used to study quarkonium, but their applicability at finite temperature is still under scrutiny and the second one is the lattice QCD\textsuperscript{19} which provides the most straightforward way to determine spectral functions, but the results suffer from discretization effects and statistical errors, and thus are still inconclusive. These two approaches show poor matching between their predictions because of the uncertainties coming from a variety of sources. None of the approaches give a complete framework to study the properties of quarkonia states at finite temperature.

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However, some degree of qualitative agreement had been achieved for the S-wave correlators. The finding was somehow ambiguous for the P-wave correlators and the temperature dependence of the potential model was even qualitatively different from the lattice one. Refinement in the computations of the spectral functions have recently been done by incorporating the zero modes both in the S- and P-channels \cite{20,21}. It was shown that, these contributions cure most of the previously observed discrepancies with lattice calculations. This supports the use of potential models at finite temperature as an important tool to complement lattice studies.

It is now pertinent to ask how a quarkonium behaves in strongly interacting QGP unlike in wQGP. The propagation of charm quark in SIQGP is different from wQGP because in SIQGP there will be a rapid equilibration due to the multiple momenta from nuclear matter to QGP is not a phase transition, rather a crossover \cite{23}. It is then reasonable to assume that the string-tension does not vanish abruptly at the deconfined point, so we may expect presence of non-perturbative effects such as non-zero string tension in the deconfined phase and one should study its effects on heavy quark potential even above $T_c$. This is indeed compatible with the strongly interacting nature of QGP where large number of non-perturbative effects are expected. This issue, usually overlooked in the literature where only a screened Coulomb potential was assumed above $T_c$ and the linear/string term was assumed zero, was certainly worth of investigation. This was exactly we had done recently \cite{24} where a medium-modified heavy quark potential was derived by correcting the full Cornell potential not its Coulomb part alone with a dielectric function encoding the effects of the deconfined medium. We found that this approach led to a long-range Coulomb potential with an (reduced) effective charge in addition to the usual Debye-screened form. With such an effective potential, we investigated the effects of perturbative and non-perturbative contributions to the Debye masses on the dissociation of quarkonium states. Finally we determined the binding energies and dissociation temperatures of the ground and the first excited states of charmonium and bottomonium spectra \cite{24,25}.

Now let us consider a central collision in a nucleus nucleus collision, which results in the formation of quark gluon plasma at initial time $\tau_0$. Let us concentrate on the region of energy density greater than a characteristic energy density, say, screening energy density $\epsilon_s$ which encloses the plasma which is dense enough to cause the melting of a particular state of quarkonium. We assume the plasma to cool, according to Bjorken’s boost invariant (longitudinal) hydrodynamics. If the plasma has cooled to a energy density less than $\epsilon_s$, the $c\bar{c}$ pair would escape and quarkonium would be formed. If however, the energy density is still larger than $\epsilon_s$, the resonance will not form and we shall have a quarkonium suppression. It is easy to see that the $p_T$ dependence of the survival probability will depend on how rapidly the plasma cools. If the initial energy density is sufficiently high, the plasma will take longer to cool and only the pairs with very high $p_T$ will escape. If however the plasma cools rapidly, then even pairs with moderate $p_T$ will escape. There are many attempts \cite{26,28,29} where the above picture of plasma expansion dynamics had been employed to study charmonium suppression, but in their works following points are missing: a) the effects of dissipative forces was not included in hydrodynamic expansion. The presence of dissipative term in the energy-momentum tensor \cite{34} causes the plasma to expand slowly and the system will take longer to reach $\epsilon_s$ (screening time $\tau_s$) and only the pairs with very high $p_T$ will escape resulting overall more suppression. b) The EoS employed in their works was either ideal or bag model EoS which was then directly used to calculate two crucial factors in quantifying the suppression in relativistic heavy ion collisions: the energy density $\epsilon_s$ corresponding to the dissociation temperature ($T_D$) and the time elapsed by the system ($\tau_s$) to reach $\epsilon_s$ by the expansion (cooling) of the system. As we now know the matter formed at RHIC is far from its ideal limit even at $T \geq T_c$ so the EoS employed in their works was not reliable to capture the properties of strongly-interacting matter. This is evident from the substantial difference in the numerical values of the crucial quantities calculated in the ideal and strongly-interacting equation of state (in the Tables II-V).

The main motivation of this article is to remedy the above shortcomings in multifold respects: i) First we use an appropriate equation of state (EoS) which should reproduce the lattice results verifying the strongly-interacting nature of QGP. This also explains the non-ideal behaviour of QGP through the speed of sound which will play an important role in the expansion dynamics. ii) Then we study hydrodynamic boost-invariant Bjorken expansion in $(1+1)$ dimension with the EoS discussed in i) as an input. In addition we explore the effects of dissipative terms on the hydrodynamic expansion by considering the shear viscosity $\eta$ up to first-order in the stress-tensor. Basically we consider the ratio of the shear viscosity-to-entropy density ($\eta/s$) as $1/4\pi$ and 0.3 which was predicted from the AdS/CFT correspondence \cite{31} and perturbative QCD \cite{28}, respectively. We also consider ideal hydrodynamics with $\eta/s = 0$ for the sake of comparison. iii) The most important point is to know the properties of quarkonium in the medium. For that we need to know the
correct form of medium-modified potential to determine the dissociation temperatures of charmonium states in a static, thermal medium. For this purpose we follow our recent work where medium modified potential \cite{21, 25} was obtained by correcting the full form of the Cornell potential through a dielectric function embodying the medium effects. iv) Finally we study the survival of charmonium states with all the refinements discussed above. The extent of suppression will be decided by a competition among the dissociation scale, the transverse momentum of J/ψ’s and the rate of expansion, making it sensitive to such details as the speed of sound \cite{26, 28} through the EoS. Thus a study of the survival of J/ψ is poised to provide a wealth of information about the dissociation mechanism of c ¯c states in a hot QCD medium, the evolution of the plasma and its properties.

The paper is organized as follows. In Sec.II.A, we briefly describe the strongly-interacting equation of state developed by Bannur \cite{33, 34} and determine the pressure, energy density and the speed of sound etc. In Sec.II.B, we then use the aforesaid equation of state to study boost-invariant (1+1) dimensional longitudinal expansion in the presence of viscous forces. In Sec.III.A, we study the dissociation phenomenon of J/ψ by studying the in-medium modifications to the heavy quark potential and its suppression in a longitudinally expanding QGP in Sec.III.B. Results are presented in Sec.IV with a discussion on important aspects of our approach. Finally in Sec.V, we present the conclusions and future prospects of the present work.

II. STRONGLY-INTERACTING QGP AND LONGITUDINAL EXPANSION OF QGP

In this section, we first discuss the equation of state for SIQGP. We shall then discuss hydrodynamical expansion in the presence of viscous forces with the SIQGP EoS as an input.

A. Equation of state

The equation of state for the quark matter produced in RHIC is a very important observable and the properties of the matter are sensitive to it. The expansion of QGP is quite sensitive to the equation of state (EoS) through the speed of sound. There have been many attempts to explain what is called strongly coupled plasma \cite{35}, widely studied in QED plasma where the plasma parameter, $\Gamma$, defined as the ratio of the average potential energy to the average kinetic energy of the particles, is of the order of 1 or larger. An extensive study of strong-coupled plasma in QED with proper modifications to include colour degrees of freedom and the strong running coupling constant gives an expression for the energy density as a function of the plasma parameter, $\Gamma$:

$$\epsilon = (2.7 + u_{ex}(\Gamma))nT,$$

where the first term corresponds to the ideal EoS and the second term, $u_{ex}(\Gamma)$, gives the non-ideal (or excess) contribution to EoS as

$$u_{ex}(\Gamma) = -\frac{\sqrt{3}}{2}\Gamma^{3/2}$$

So, the scaled energy density (in terms of Stefan-Boltzmann limit) is given by

$$\epsilon(\Gamma) \equiv \frac{\epsilon}{\epsilon_{SB}} = 1 - \frac{1}{2.7}\frac{\sqrt{3}}{2}\Gamma^{3/2}$$

where $\epsilon_{SB} \equiv 3a_fT^4$ with degrees of freedom $a_f \equiv (16 + 21n_f/2)\pi^2/90$ and using the relation, $\epsilon = T\frac{\partial p}{\partial T} - p$, we get the pressure

$$\frac{p}{T^4} = \left(\frac{p_0}{T_0^4} + 3a_fT^4\int_0^{T_0} d\tau \tau^2e(\Gamma(\tau))\right)/T^3$$

where $p_0$ is the pressure at some reference temperature $T_0$. If we calculate $p(T)/T^4$ versus $T$ for pure gauge, 2-flavor and 3-flavor QGP, a surprisingly good fit with the lattice data was found \cite{35, 36}.

Another important observable, the speed of sound which appears in the equation of motion for the expansion through the relation,

$$c_s^2 = \frac{\partial p}{\partial \epsilon}.$$

It is found that strongly-interacting matter created at RHIC has a signature of non-zero interaction measure, $\Delta = \epsilon - 3p$ which measures the deviation of the EoS from its ideal counterpart. This quantity has been extensively studied in lattice QCD \cite{39} which shows that $\Delta \geq 0$ and asymptotically approaches the ideal value (zero). This implies that $c_s^2 \leq 1/3$ at $T = T_c$ or even much above $T_c$. This is exactly found in Bannur’s model \cite{33} where $c_s^2$ is very much less than its ideal limit around $T_c$ and increases towards its ideal value. In view of the excellent agreement with lattice simulations the above phenomenological EoS is a right choice for the strongly-interacting matter possibly formed at RHIC to calculate the thermodynamical quantities viz. screening energy density ($\epsilon_s$), the speed of sound etc. and also to study the hydrodynamical expansion of plasma.
B. Longitudinal expansion in the presence of dissipative forces

The energy momentum tensor of the plasma in the absence of dissipative forces is written as:
\[ T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + g^{\mu\nu}p. \]  
(6)

If the effects of viscosity are neglected, conservation of energy-momentum tensor leads to
\[ \partial_\mu T^{\mu\nu} = 0. \]  
(7)

We now consider the Bjorken boost invariant longitudinal flow in (1+1) dimension where the equation (7) simplifies to
\[ \partial_\tau \epsilon = -\frac{\epsilon + p}{\tau}. \]  
(8)

The effect of the speed of sound \( c_s^2 \) is seen immediately through the EoS: \( p = c_s^2 \epsilon \) as
\[ \epsilon(\tau)^{1+c_s^2} = \epsilon(\tau_i)^{1+c_s^2} = \text{const.} \]  
(9)

With the condition \( \epsilon \sim T^4 \) and taking the speed of sound \( c_s^2 = 1/3 \), the above equation translates to the cooling law:
\[ T^3 \tau = T_i^3 \tau_i, \]  
(10)

where \( T_i \) is the initial temperature and \( \epsilon(\tau_i) \) is the initial energy density.

Now, we study the correction to the equation of motion (7) in the presence of dissipative forces in the form of shear viscosity, \( \eta \) which is itself a function of temperature. The hydrodynamic evolution preserves the boost-invariance, which in ultrarelativistic heavy ion collisions is expected to be realized near mid-rapidity [40]. In the presence of viscous forces the energy-momentum tensor is written as,
\[ T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + g^{\mu\nu}p + \pi^{\mu\nu}, \]  
(11)

where \( \pi^{\mu\nu} \) is the stress-energy tensor. Dissipative corrections to the first-order in the gradient expansion are recovered by setting the relaxation time to zero. This leads to:
\[ \pi^{\mu\nu} = \eta(\nabla^\mu u^\nu), \]  
(12)

where \( \eta \) is the co-efficient of the shear viscosity and \( \langle \nabla^\mu u^\nu \rangle \) is the symmetrized velocity gradient.

In (1+1) dimensional Bjorken expansion in the first-order dissipative hydrodynamics, only one component \( \pi^{\tau\tau} \) of the viscous stress tensor is non-zero. In this case the equation of motion reads,
\[ \partial_\tau \epsilon = -\frac{\epsilon + p}{\tau} + \frac{4\eta}{3\tau^2}. \]  
(13)

The solution of the above equation in the case of constant value of \( \eta/s \) is known analytically [41, 42] and is given by,
\[ T(\tau) = T_i \left( \frac{\tau_i}{\tau} \right)^{1/3} \left[ 1 + \frac{2\eta}{3s\tau_i T_i} \left( 1 - \left( \frac{\tau_i}{\tau} \right)^{2/3} \right) \right]. \]  
(14)

The first term in the RHS is the same as in the case of zeroth-order (non-viscous) hydrodynamics and the second term is the correction arising from constant \( \eta/s \) which causes the system to expand slowly compared to the perfect fluid \( \eta = 0 \) (10). It is important to note that temperature will be maximum after a span of time
\[ \tau_{\text{max}} = \tau_i \left( \frac{1}{3} + \frac{s}{\eta} \frac{\tau_i}{T_i} \right)^{-3/2}. \]  
(15)

For times \( \tau > \tau_{\text{max}} \), the temperature decreases with time, as expected for matter undergoing expansion. For early times \( \tau < \tau_{\text{max}} \), however, the solution shows an unphysical reheating [43] which will not be discussed here.

We shall now study the solution of Eq. (13) for the strongly-interacting equation of state discussed in Sec.II.A where \( c_s^2 \) is always less than 1/3 and is a function of temperature. The solution of Eq. (13) is obtained as,
\[ \epsilon(\tau)^{1+c_s^2} + \frac{4a}{\tilde{\tau}^2} \tau^{(1+c_s^2)} = \epsilon(\tau_i)^{1+c_s^2} + \frac{4a}{3\tilde{\tau}_i^2}, \]  
(16)

where \( a = \left( \frac{2}{3} \right) T_i^3 \tau_i \) and \( \tilde{\tau}^2 \) and \( \tilde{\tau}_i^2 \) are given by \( (1 - c_s^2)\tau^2 \) and \( (1 - c_s^2)\tau_i^2 \), respectively. The first term in both LHS and RHS accounts for the contributions coming from the zeroth-order expansion and the second term is the first-order viscous corrections (In other words, Eq. (16) reduces to Eq. (13) for \( \eta = 0 \).)

In our work we consider three values of the shear viscosity-to-entropy density ratio to see the effects of nonzero values of the shear viscosity on the expansion. The first one is from perturbative QCD [52] calculations where \( \eta/s = 0.3 \) near \( T \sim T_c - 2T_c \). The second one is from AdS/CFT studies [31] where \( \eta/s = 1/4\pi \sim 0.08 \). Finally we consider \( \eta/s = 0 \) (for the ideal fluid) for the sake of comparison. We shall employ Eq. (16) to study the charmonium suppression in a strongly interacting QCD medium formed in a ultra-relativistic heavy-ion collisions in the next section.

III. Suppression of J/ψ in a Longitudinally Expanding Plasma

Let us now turn our attention to the most important quantity, viz., the J/ψ survival probability. Recall that, to study the fate of charmonium in SIQGP,
A. Dissociation of charmonium in SIQGP

The large distance property of the heavy quark interaction is important for our understanding of the bulk properties of the QCD plasma phase, e.g. the screening property of the quark gluon plasma [44, 45], the equation of state [46, 47] and the order parameter (Polyakov loop) [48, 49, 50, 51]. In all of these studies deviations from perturbative calculations and the ideal gas behavior are expected and were indeed found at temperatures which are moderately larger than the deconfinement temperature. This calls for quantitative non-perturbative calculations. Recent findings [24] indicate the phase transition in full QCD appears as a crossover rather than a ‘true’ phase transition with the related singularities in thermodynamic observables. In light of this finding, one can not simply ignore the effects of string tension between the quark-antiquark pairs beyond T_c. This is indeed a very important effect which needs to be incorporated while setting up the criterion for the dissociation of quarkonia in QGP.

In our recent works [24, 25], this issue has successfully been addressed for the dissociation of quarkonium in QGP. In these works, an appropriate form of the medium modified inter-quark potential have been obtained. The authors have considered the Cornell form of the potential: $V(r) = -\frac{\alpha}{r} + \sigma r$ and correct its Fourier transform (FT) $\tilde{V}(k)$ as

$$\tilde{V}(k) = \frac{V(k)}{\epsilon(k)}, \quad (17)$$

where $\epsilon(k)$ is the dielectric permittivity given in terms of the static limit of the longitudinal part of the gluon self-energy [52]:

$$\epsilon(k) = \left(1 + \frac{\Pi_{L}(0, k, T)}{k^2}\right) \equiv \left(1 + \frac{m_D^2}{k^2}\right), \quad (18)$$

and $V(k)$ is the Fourier transform (FT) of the Cornell potential:

$$V(k) = -\sqrt{\frac{2}{\pi}} \frac{\alpha}{k^2} \left(\frac{4\sigma}{\sqrt{2}\pi k^4}\right), \quad (19)$$

Substituting Eqs. (18) and (19) into (17) and then evaluating its inverse FT one obtains the $r$-dependence of the medium modified potential [24, 25],

$$V(r) = \frac{2\sigma}{m_D^2} \exp\left(-\frac{m_D r}{\sigma}\right) - \frac{2\sigma}{m_D^2} + \frac{2\sigma}{m_D} \alpha m_D, \quad (20)$$

where the constant terms are introduced to yield the correct limit of $V(r, T)$ as $T \to 0$. Such terms could arise naturally from the basic computations of real time static potential in hot QCD [55] and from the real and imaginary time correlators in a thermal QCD medium [54]. The medium modified form of the potential thus obtained has an additional long range Coulomb term with an (reduced) effective charge in addition to the conventional Yukawa term. A new picture of quarkonium suppression has emerged out of these studies where both the range and the charge of the potential get screened due to in-medium modifications. The binding energies and dissociation temperatures for various quarkonium states have been determined by solving the Schrödinger equation numerically with the potential [20]. We considered three possible forms of the Debye masses ($m_D$) in the potential: the leading-order term in QCD coupling ($m_D^{LO}$) which is a perturbative result [22], the non-perturbative corrections to the leading-order ($m_D^{NP}$) [55] and the lattice parametrized form ($m_D^{L}$) [50] to examine the effects of perturbative and nonperturbative contributions on the dissociation mechanism. The upper limit of the dissociation temperature of a particular state is obtained when its binding energy is of the order of temperature of the system: $E_{\text{bin}} \approx T$ so that the state becomes feebly bound and thermal fluctuations can excite it to the continuum. The perturbative result for the Debye mass in the potential gives rise higher values (in Table II) of the dissociation temperatures compared to the spectral function studies [54, 55] calculated in a potential model but smaller than the results from the analysis of lattice (temporal) correlators [58] (in Table IV). On the other hand, when we use the lattice parametrized form (by fitting the lattice free-energy for color-singlet states) for the Debye mass, the dissociation temperatures (given in Table III) exactly matches (discussed in in details in Ref. [24]) with the results of the potential model calculations [50, 55] but much smaller than the lattice studies [58] (Table IV). Finally the non-perturbative corrections of $\mathcal{O}(g^2 T)$ and $\mathcal{O}(g^3 T)$ to the perturbative result [55] in Debye mass give unrealistically smaller values which will not be considered in the present work.

In addition, we also take the advantage to demonstrate the flavor dependence of the dissociation process. Hence we determined the dissociation tempera-
tures for the 2-flavor QGP (Table V) with the Debye mass only in the leading-order term in strong coupling. The dissociation temperatures are found to be higher than the 3-flavor case (Table II). However the detailed study of the flavor dependence was given in our earlier works \[21, 22\]. In the present analysis, we have used the four sets of the dissociation temperatures (Tables II-V) to study the centrality dependence of survival probability of charmonium as a function of number of participants \(N_{\text{part}}\) in an expanding (strongly interacting) QCD medium.

### B. Survival probability

We now have all the ingredients to write down the survival probability and we closely follow Chu and Matsui for this. In the work of Chu and Matsui \[23\], the \(p_T\) dependence of the survival probability of charmonium was studied by choosing the speed of sound \(c_s^2 = 1/3\) (ideal EoS) and the extreme value \(c_s^2 = 0\). This work was further generalized by invoking the various parameters for Au-Au collisions at RHIC in \[23\] to include the effects of realistic EoS in an adhoc manner by simply choosing a lower value of \(c_s^2 = 1/5\). Instead of taking arbitrary values of \(c_s^2\) we tabulated the values of \(c_s^2\) in Tables II-V corresponding to the dissociation temperatures calculated from Bannur model \[22\] where the values of \(c_s^2\) match perfectly with the lattice results. Moreover, in the light of recent experimental finding from RHIC, one can not ignore the viscous effects while studying charmonium suppression. Here, we address these issues. Note that in our analysis, we follow the idea of Chu and Matsui \[26\], but with the more appropriate criteria for the charmonium dissociation and the dissipative hydrodynamic expansion of the plasma.

Let us take a simple parametrization for the initial energy-density profile on any transverse plane:

\[
\epsilon(\tau_i, r) = \epsilon_i A_T(r); A_T(r) = \left(1 - \frac{r^2}{R_T^2}\right)^\beta \theta(R_T - r) \tag{21}
\]

where \(r\) is the transverse co-ordinate and \(R_T\) is the transverse radius of the nucleus. One can define an average energy density \(<\epsilon_i>\) as

\[
\pi R_T^2 <\epsilon_i> = \int 2\pi r dr \epsilon(r) \tag{22}
\]

so that

\[
\epsilon_i = (1 + \beta) <\epsilon_i>. \tag{23}
\]

We have taken \(\beta = 1\), which will reflect the proportionality of the deposited energy to the nuclear thickness. The average energy-density is obtained from the Bjorken formula:

\[
<\epsilon_i> = \frac{1}{\pi R_T^2 \tau_i} \frac{dE_T}{dy} \tag{24}
\]

where \(E_T\) is the transverse energy deposited in the collision. The appropriate characterization of kinematic quantities in Au+Au collisions is presented in Table I.

The time \(\tau_s\) when the energy density drops to the screening energy density \(\epsilon_s\) is estimated from Eq. (10) as

\[
\tau_s(r) = \tau_i \left[\frac{\epsilon_i(r) - \frac{4a}{3 F^2}}{\epsilon_s - \frac{4a}{3 F^2}}\right]^{1/1+c_s^2} \tag{25}
\]

where \(\epsilon_i(r) = \epsilon(\tau_i, r)\) and \(\tau_s^2 = (1-c_s^2) \tau_i^2\). The critical radius \(r_s\), is seen to mark the boundary of the region where the quarkonium formation is suppressed, can be obtained by equating the duration of screening \(\tau_s(r)\) to the formation time \(t_F = \gamma \tau_F\) for the quarkonium in the plasma frame and is given by:

\[
r_s = R_T(1-A)^{\frac{1}{2}} \theta(1-A), \tag{26}
\]

where \(A\) is given by

\[
A = \left[\frac{\epsilon_s}{\epsilon_i} \left(\frac{t_F}{\tau_i}\right)^{1+c_s^2} + \frac{1}{\epsilon_i} \left(\frac{t_F}{\tau_i}\right)^{1+c_s^2} \left(\frac{4a}{3 F^2} \epsilon_i^{3/2} \right)^{1/3}\right] \tag{27}
\]

| Nuclei   | \(\sqrt{s_{NN}}\) | \(N_{\text{part}}\) | \(<\epsilon>\) | \(R_T\) |
|----------|------------------|-----------------|-------------|--------|
| Au+Au 200 | 212.9           | 103.0           | 5.86        | 3.45   |
|          | 220.0           | 100.0           | 7.92        | 3.61   |
|          | 240.0           | 90.0            | 10.14       | 3.79   |
|          | 250.0           | 80.0            | 12.76       | 3.96   |
|          | 260.0           | 70.0            | 15.69       | 4.16   |
|          | 270.0           | 60.0            | 18.58       | 4.37   |
|          | 280.0           | 50.0            | 21.36       | 4.61   |
|          | 290.0           | 40.0            | 24.38       | 4.85   |
|          | 300.0           | 30.0            | 27.36       | 5.12   |
|          | 310.0           | 20.0            | 30.52       | 5.38   |
|          | 320.0           | 10.0            | 34.17       | 5.64   |
|          | 330.0           | 10.0            | 37.39       | 5.97   |
|          | 340.0           | 10.0            | 41.08       | 6.31   |
|          | 350.0           | 10.0            | 45.09       | 6.68   |
with $\tilde{r}_F = (1 - c_s^2) r_F$. The quark-pair will escape the screening region (and form quarkonium) if its position $\mathbf{r}$ and transverse momentum $p_T$ are such that

$$|\mathbf{r} + \tau_F p_T / M| \geq r_s,$$  

(28)

Thus, if $\phi$ is the angle between the vectors $\mathbf{r}$ and $p_T$, then

$$\cos \phi \geq \left( (r_s^2 - r^2) M - \tau_F^2 p_T^2 / M \right) / [2 r \tau_F p_T],$$  

(29)

which leads to a range of values of $\phi$ when the quarkonium would escape. Now we can write for $\phi$ which leads to a range of values of $\phi$

Thus, if $J/\psi$ the production of $J/\psi$ mesons in hadronic reactions occurs in-part through production of higher excited $c\bar{c}$ states and their decay into quarkonia ground state. Since the lifetime of different sub-threshold quarkonium states is much larger than the typical life-time of the medium which may be produced in nucleus-nucleus collisions so their decay occurs almost completely outside the produced medium. This means that the produced medium can be probed not only by the ground state quarkonium but also by different excited quarkonium states. Since, different quarkonium states have different sizes (binding energies), one expects that higher excited states will dissolve at smaller temperature as compared to the smaller and more tightly bound ground states. These facts may lead to a sequential suppression pattern in $J/\psi$ yield in nucleus-nucleus collision as the function of the energy density or number of participants in the collision. The $J/\psi$ yield could show a significant suppression even if the energy density of the system is not enough to melt directly produced $J/\psi$ but is sufficient to melt the higher resonance states because they are loosely bound compared to the ground state $J/\psi$.

In nucleus-nucleus collisions, it is known that about 60% of the observed $J/\psi$ originate directly in hard collisions while 30% of them come from the decay of $\chi_c$ and 10% from the decay of $\psi'$. Hence, the $p_T$-integrated inclusive survival probability of $J/\psi$ in the QGP becomes

$$\langle S^{inc} \rangle = 0.6 \langle S^{dir} \rangle_{\psi} + 0.3 \langle S^{dir} \rangle_{\chi_c} + 0.1 \langle S^{dir} \rangle_{\psi'},$$  

(36)

The hierarchy of dissociation temperatures in lattice correlator studies leads to sequential suppression pattern with an early suppression of $\psi'$ and $\chi_c$ decay products and much later one for the direct $J/\psi$ production. However, with our recent results (Table II-III) employing medium modification to the full Cornell potential and also results from potential model studies on dissociation temperatures, all three species will show essentially almost the same suppression pattern, i.e., the concept of sequential melting will not have any dramatic effect which will be seen in the next section.

### IV. RESULTS AND DISCUSSIONS

There are three time-scales involved in the screening scenario of $J/\psi$ suppression in an expanding plasma. First one is the screening time, $\tau_s$ as the time available for the hot and dense system during which $J/\psi$’s are suppressed. Second one is the formation time of $J/\psi$ in the plasma frame ($t_F = \tau_F$) which depends on the transverse momentum by which the $c\bar{c}$ pairs was produced. Third one is the cooling rate which depends on the speed of sound, $c_s$ through the equation of state. The screening time not only depends upon the screening energy
TABLE II: Formation time (fm), dissociation temperature $T_D$ (in units of $T_c=197$ MeV for a 3-flavor QGP) with the Debye mass in the leading-order [24, 25], the speed of sound $c^2_s$ and the screening density $\epsilon_s$ (GeV/fm$^3$) calculated in SIQGP and ideal EoS for $J/\psi$, $\psi'$, $\chi_c$ states $[24, 25]$, respectively.

| State    | $\tau_F$ | $T_D$ | $c^2_s$(SIQGP) | $c^2_s$(Id) | $\epsilon_s$ (SIQGP) | $\epsilon_s$(Id) |
|----------|----------|-------|----------------|-------------|---------------------|------------------|
| $J/\psi$ | 0.89     | 1.61  | 0.26           | 1/3         | 17.65              | 21.77            |
| $\psi'$  | 1.50     | 1.16  | 0.24           | 1/3         | 04.51              | 06.53            |
| $\chi_c$ | 2.00     | 1.25  | 0.24           | 1/3         | 06.15              | 08.47            |

TABLE III: Same as Table II but the dissociation temperature is calculated with the Debye mass in the lattice parametrized form $[24, 25]$.

| State    | $\tau_F$ | $T_D$ | $c^2_s$(SIQGP) | $c^2_s$(Id) | $\epsilon_s$ (SIQGP) | $\epsilon_s$(Id) |
|----------|----------|-------|----------------|-------------|---------------------|------------------|
| $J/\psi$ | 0.89     | 1.18  | 0.24           | 1/3         | 4.83                | 7.81             |
| $\psi'$  | 1.50     | 0.85  | 0.18           | 1/3         | 1.21                | 2.89             |
| $\chi_c$ | 2.00     | 0.90  | 0.19           | 1/3         | 1.54                | 3.36             |

TABLE IV: Same as Table II but with different values of dissociation temperature(s) obtained from lattice studies $[58]$ in units of $T_c=175$ MeV.

| State    | $\tau_F$ | $T_D$ | $c^2_s$(SIQGP) | $c^2_s$(Id) | $\epsilon_s$ (SIQGP) | $\epsilon_s$(Id) |
|----------|----------|-------|----------------|-------------|---------------------|------------------|
| $J/\psi$ | 0.89     | 2.10  | 0.27           | 1/3         | 32.85              | 69.83            |
| $\psi'$  | 1.50     | 1.12  | 0.21           | 1/3         | 02.36              | 05.80            |
| $\chi_c$ | 2.00     | 1.16  | 0.22           | 1/3         | 02.74              | 06.53            |

TABLE V: Dissociation temperature(s) $T_D$ (in units of $T_c=203$ MeV for a 2-flavor QGP) with the Debye mass in the leading-order $[24, 25]$ of strong coupling.

| State    | $\tau_F$ | $T_D$ | $c^2_s$(SIQGP) | $c^2_s$(Id) | $\epsilon_s$ (SIQGP) | $\epsilon_s$(Id) |
|----------|----------|-------|----------------|-------------|---------------------|------------------|
| $J/\psi$ | 0.89     | 1.69  | 0.238          | 1/3         | 17.76              | 23.14            |
| $\psi'$  | 1.50     | 1.21  | 0.172          | 1/3         | 04.07              | 06.70            |
| $\chi_c$ | 2.00     | 1.31  | 0.192          | 1/3         | 05.77              | 08.79            |

$\eta/s$ ratio: if the ratio is larger then the cooling will be slower, so the system will take longer to reach $\epsilon_s$ resulting the higher value of screening time and hence more suppression compared to $\eta/s = 0$. With this physical understanding we analyze $\langle S(p_T)\rangle$ as a function of the number of participants $N_{\text{Part}}$ in an expanding QGP.

In our analysis, we have employed the dissociation temperatures for $J/\psi$, $\chi_c$ and $\psi'$ recently computed by us $[24, 25]$ in the Table(s) II-III for 3-flavor and V for 2 flavor QGP, respectively, where we employed an appropriate form of medium modified potential which includes both non-zero string tension effects beyond $T_c$ and the usual screened coulomb term. To compare our predictions with the lattice correlator studies, we employ the dissociation temperatures (in Table IV) computed from the first principle of (lattice) QCD $[58]$ where a simple screened Coulomb form was assumed for the medium-dependent interquark potential. The corresponding values for $\epsilon_s$ and $c^2_s$ calculated in the strongly-interacting and ideal EoS are also listed in the respective Tables. We shall employ these next to study $\langle S(p_T)\rangle$.

We have shown the variation of $p_T$-integrated survival probability (in the range allowed by invariant $p_T$ spectrum of $J/\psi$ by the Phenix experiment $[1]$) with $N_{\text{Part}}$ at mid-rapidity in Figs.1-8. The experimental data (the nuclear-modification factor $R_{AA}$) are shown by the squares with error bars whereas circles and diamonds represent with $\langle S^{\text{incl}}(p_T)\rangle$ without $\langle S^{\text{dir}}(p_T)\rangle$ sequential melting. We have employed three values of $\eta/s$ viz. 0, 0.08, and 0.3 in each figure to see the effects of dissipative terms in the expansion of the plasma. In Fig. 1, we find that the survival probability of directly produced $J/\psi$ is slightly higher than $\langle S^{\text{incl}}(p_T)\rangle$ and is closer to the experimental results $[1]$. For the lower value of $\eta/s$ our predictions are closer to the experimental ones. As the ratio $\eta/s$ is increased, the expansion of the system becomes slower leading to the higher values for the screening time and hence lower values of $\langle S(p_T)\rangle$ i.e., $J/\psi$’s will be suppressed more. However, for all the three values of $\eta/s$ ratio, $\langle S(p_T)\rangle$ for both the directly produced and sequential $J/\psi$ sit below the experimental numbers for $N_{\text{Part}} \gtrsim 100$.

In Fig.2, we used the same values of dissociation...
FIG. 1: The variation of $p_T$ integrated survival probability (in the range allowed by invariant $p_T$ spectrum of $J/\psi$ by the Phenix experiment) versus number of participants at mid-rapidity. The experimental data (the nuclear-modification factor $R_{AA}$) are shown by the squares with error bars whereas circles and diamonds represent with $(\langle S^{incl} \rangle)$ without $(\langle S^{dir} \rangle)$ sequential melting using the values of $T_D$’s and related parameters from Table II using SIQGP equation of state.

FIG. 2: Same as Fig. 1 but the related parameters from the ideal EoS.
temperatures as in Table II but the thermodynamic variables \( \epsilon_s, c_s^2 \) etc. have been calculated in the ideal EoS to see the sensitivity of the EoS to the plasma dynamics which, in turn, indirectly affect suppression in relativistic collision. In this case too, the \( \eta/s \) dependence of \( \langle S(p_T) \rangle \) pattern remains the same and \( \langle S(p_T) \rangle \) is always higher than the corresponding values (with SIQGP) in Fig.1. The matching is almost perfect for \( \eta/s=0 \). This can be understood in terms of the sensitivity of \( \langle S(p_T) \rangle \) on the speed of sound, \( c_s^2 \) for a fixed difference in \( \epsilon_f-\epsilon_s \) because cooling of the system with ideal EoS is much faster compared to the strongly-interacting EoS so the system will spend less time in the screening region results in less suppression.

Another interesting observation, which is common to both Fig.1 and 2 is that as the ratio \( \eta/s \) is increased from 0 to 0.3, \( \langle S(p_T) \rangle \) for both directly and sequential \( J/\psi \)'s become smaller. To be more accurate, \( \langle S(p_T) \rangle \) for the sequential decays will be affected much causing the suppression stronger and is being separated from the \( \langle S(p_T) \rangle \) of the directly pro-
duced $J/\psi$. As the ratio $\eta/s$ is increased from zero, cooling becomes slower so that $\chi_c$ and $\psi'$ show more suppression due to their smaller value of $\epsilon_s$ (smaller $T_D$) compared to $J/\psi$ making the difference between $\epsilon_i$ and $\epsilon_s$ larger. This leads to more suppression for $\psi'$.

In Figs. 3 and 4, we used the dissociation temperatures employing the lattice parametrized form of the Debye mass (in Table III) which is the same as the values obtained from potential models but the expansion of the system is taken into account through SIQGP and ideal EoS, respectively. We again vary $\eta/s$ between 0 and 0.3. Interestingly, in both the cases (Fig.3 and Fig.4) the suppression is large compared to the previous cases (Fig.1 and Fig.2) and is much smaller than the experimental results. Since the dissociation temperatures are much smaller than the values in Table II so that the difference $(\epsilon_i - \epsilon_s)$ becomes large. Therefore, the system will take longer time to reach $\epsilon_s$, as a result the system have enough time to kill $J/\psi$'s. The sensitivity to the EoS and the ratio $\eta/s$ is the same as in Figs.1 and 2. As expected, there is more suppression in SIQGP as compared to ideal EoS.

In Figs. 5 and 6, we use the dissociation temperatures from the lattice correlator studies (in Table IV) and the expansion of the system are taken into account through SIQGP and ideal EoS, respectively. Interestingly, in both Figs. the suppression is much smaller compared to the previous cases. The hierarchy in the dissociation temperatures in Table IV led the sequential suppression to play an important role which causes more suppression for the sequentially produced $J/\psi$'s which is closer to the Phenix results compared to very less suppression for the directly produced $J/\psi$'s. On the contrary, in the earlier sets on the dissociation temperatures the concept of sequential melting has not any dramatic effect on $\langle S(p_T) \rangle$. In this set of dissociation temperatures, the agreement between theory and experiment is better with the SIQGP EoS (Fig.5) compared to ideal EoS (Fig.6) because cooling is slower in the former EoS and results in more suppression for the sequential melting. In addition to the effect of EoS, the effect of the dissipative forces in terms of $\eta/s$ becomes prominent because the matching (with the experiment) is good for higher values of $\eta/s$. This is due to the fact that the larger value of viscous force makes the expansion slower which results in more suppression.

To examine the flavor dependence we have plotted the $p_T$-integrated survival probability for the 2-flavor QGP with the dissociation temperatures in Table V in Figs.7, 8. The suppression pattern remains the same in comparison to Figs.1 and 2. It seems surprising because of the following reason: As the number of flavors in the system decrease, $\epsilon_s$ decrease rapidly so that the difference $(\epsilon_i - \epsilon_s)$ becomes larger compared to 3-flavor system. As a result the system will get more time to kill $J/\psi$'s resulting more suppression. But it did not happen in the Figs. 7 and 8 because we take into account the flavor dependence of the dissociation temperature where the dissociation temperature for the 2-flavor system is much higher than 3-flavor system which compensates the decrease in the screening energy density due to the decrease in the degrees of freedom. This balance makes the suppression pattern same in Fig.1 and Fig.7 (or Fig.2 and Fig.8).

Let us now compare Fig.1 with Fig.5 and Fig.2 with Fig.6. It is clear from Figs.5 and 6 that $\langle S(p_T) \rangle$ for both the directly and the sequentially produced
$J/\psi$ are quite high with the higher values of $T_D$'s obtained from the lattice correlator studies (in Table IV) compared to the values obtained from our model (in Table II). Since the dissociation temperature for the $J/\psi$ is high so that $\epsilon_s$ is much larger than $\epsilon_i$, up to the number of participants 200 resulting no suppression at all. However, $T_D$'s are not so high for $\chi_c$ and $\psi'$ so that the sequential probability $\langle S^{\text{incl}} \rangle$ has been suppressed that makes them closer to the experimental results for the higher values of $\eta/s$. However, $\langle S^{\text{incl}} \rangle$ gets 60% contribution from the directly produced $J/\psi$, it has a dominating contribution on the behavior of sequential $S(p_T)$. This is indeed reflected in Fig.5 and Fig.6. To be conclusive, the dissociation temperatures exploited in the present work (in Table II) shows better agreement with the experimental results from RHIC as compared to the spectral function technique calculated either in the potential model [56] or in the lattice (temporal) correlator [58].

V. CONCLUSIONS AND FUTURE SCOPE

In conclusion, we have studied the charmonium suppression in a longitudinally expanding QGP in the presence of dissipative forces. We find that presence of dissipative terms in the fluid equation of motion slower the expansion rate and eventually lead to the enhanced suppression of $J/\psi$. In other words, the presence of viscosity enhances the screening time for $J/\psi$ in the SIQGP medium and hence the survival probability gets decreased compared to that without the viscous forces. These conclusions are true for both the directly and sequentially produced $J/\psi$.

In this work, we have exploited a recent understanding of dissociation of quarkonia in the QGP medium which rely on the fact that the transition from the hadronic matter to QGP is a crossover not a phase transition in the true sense. We have employed the results of [24, 25] on dissociation temperatures of various charmonium states. We have employed the SIQGP equation of state to estimate the screening energy density and the speed of sound to study the $J/\psi$ yield. To compare our results with those obtained by employing the simple screening picture of quarkonia commonly considered in the literature, we employ dissociation temperatures estimated in lattice correlator studies [58]. We find that the results on $J/\psi$ survival probability agree with the Phenix Au-Au data [1] with the set of dissociation temperatures (Table II) obtained with the perturbative result of the Debye mass. We had shown in our earlier work [24] that the inclusion of non-perturbative contributions to the Debye mass lower the dissociation temperatures substantially which looks unfeasible to compare to the spectral analysis of lattice temporal correlator of mesonic current [58] which finally makes the $J/\psi$ survival probability too small to compare with the experimental results. This does not immediately imply that the non-perturbative effects should be ignored. It is rather interesting to investigate the disagreement between the non perturbative result obtained with a dimensional-reduction strategy and the Debye mass arising from the Polyakov-loop correlators. Only future investigations may throw more light on this issue.
FIG. 7: Same as Fig.1 but using the values of $T_D$’s and related parameters in SIQGP EoS from Table V for 2-flavor QGP.

This leaves an open problem of the agreement between these two kind of approaches. This could be partially due to the arbitrariness in the criteria/definition of the dissociation temperature. To examine this point we estimated both the upper and lower bound on the dissociation temperatures. Thus, this study provides us a handle to decipher the extent up to which non-perturbative effects should be incorporated into the Debye mass.

Finally, our study does provide a systematic way to analyze the fate of charmonium in QGP medium. We have included two important aspects of the QGP at RHIC which are based on recent experiment findings: (i) strongly-interacting picture of QGP and (ii) non-vanishing string tension (between $q\bar{q}$) contributions beyond the deconfinement point. The first one we have incorporated by considering the phenomenological equation of state and the speed of sound determined from it and the second one is by considering a right criterion for the dissociation of quarkonia in QGP. Our attempt is perhaps the first one to understand $J/\psi$ suppression systematically in SIQGP. It would be of interest to extend the present study by incorporating the higher order contributions coming from the viscous forces including contributions of the bulk viscosity. These issues will be taken up separately in the near future.

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