Are Supersymmetric Models with Large $\tan \beta$ Natural?

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Abstract

We point out that, contrary to general belief, generic supersymmetric models are not technically unnatural in the limit of very large values of the parameter $\tan \beta$ when radiative corrections are properly included. Rather, an upper limit on $\tan \beta$ only arises from the requirement that Yukawa couplings remain perturbative up to some high scale. We quantify the relation between this scale and the maximum value of $\tan \beta$. Whereas $\tan \beta$ is limited to lie below 50–70 in the mSUGRA model, models with a much lower scale of new physics (beyond supersymmetry) may have $\tan \beta \lesssim 150 - 200$. 
I. INTRODUCTION

Although the Standard Model (SM) is spectacularly successful in accommodating experimental data over a wide range of energies, it is widely believed to be an effective theory that is applicable below an energy scale $\mathcal{O}(1 - 10)$ TeV. New degrees of freedom (or at least evidence of structure via form factors) are expected to manifest themselves in experiments designed to probe energies above this scale. This belief stems from the instability of the parameters of the elementary scalar field sector to radiative corrections. This instability, in turn, may be interpreted as an extreme sensitivity of weak scale physics to parameters that describe physics at much higher energy scales. This is referred to as the fine-tuning problem of the SM. We stress that this is not a logical problem in that the SM provides an internally consistent predictive framework, but more a problem of what we expect of a fundamental theory (which the SM is not).

A conceptually distinct issue refers to the introduction of small dimensionless parameters, be they dimensionless couplings or small ratios of mass scales, into a theory. It has been proposed [1] that a dimensionless parameter $P$ may be much smaller than unity only if the replacement $P \to 0$ increases the symmetry of the theory. Theories that satisfy this requirement are technically referred to as natural. A small Yukawa coupling in grand unified theories (GUTs) is technically natural (because setting it to zero leads to a new chiral symmetry), but the introduction of the tiny ratio of the electroweak Higgs boson mass parameter to the grand unification scale is not. Likewise, within the framework of the simplest supersymmetric GUTs the choice $|\mu| \ll M_{GUT}$ is technically unnatural. This is the well-known "$\mu$ problem". Various dynamical mechanisms have been suggested to explain why $\mu$ is of the same size as the SUSY breaking scale [2].

While supersymmetry, by itself, does not address the naturalness question, it has received a lot of attention in the last two decades because it leads to an elegant solution to the fine-tuning problem\(^1\) of the first paragraph, provided that the SUSY breaking scale is comparable to the weak scale [3]. Supersymmetry thus preserves the hierarchy between the weak and GUT (or Planck) scales even in the presence of radiative corrections. But why this hierarchy exists at all requires an independent dynamical explanation.

The interpretation of the atmospheric neutrino data of the super-Kamiokande collaboration [4] as neutrino oscillations has led to a renewed interest in $SO(10)$ GUTs since neutrinos necessarily acquire masses within this framework. SUSY models based on $SO(10)$ require that the parameter $\tan \beta$ is large [5]. It has been argued, however, that models with large $\tan \beta$ are technically unnatural [6]. It is an evaluation of this claim that forms the subject of this note.

We begin by examining the part of the scalar potential relevant for spontaneous electroweak symmetry breaking (EWSB). At tree-level, this takes the form,

\[
V_{\text{tree}} = \left( m_{H_u}^2 + \mu^2 \right) |h_u^0|^2 + \left( m_{H_d}^2 + \mu^2 \right) |h_d^0|^2 + g_Z^2 \left( |h_u^0|^2 - |h_d^0|^2 \right)^2 - B \mu (h_u^0 h_d^0 + h.c.),
\]

\(^1\)What we call the fine-tuning problem has been referred to as the naturalness problem by some authors.
with
\[ g_2^2 = \frac{1}{8} \left( g^2 + g'^2 \right) = \frac{g^2}{8 \cos^2 \theta_W}. \] (2)

The minimization conditions can readily be derived from this potential and take the well
known form,
\[ \mu B = \sin \beta \cos \beta \left( m_{H_u}^2 + m_{H_d}^2 + 2\mu^2 \right), \] (3)
\[ \mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{(\tan^2 \beta - 1)} - \frac{1}{2} m_Z^2. \] (4)

It follows from (3) that if \( \tan \beta \gg 1 \), the parameter \( B\mu \) has a much smaller magnitude than
the other parameters in the scalar potential. Thus the model is technically unnatural unless
the limit \( B\mu \to 0 \) (equivalently, \( \tan \beta = \infty \)) increases the symmetry of the Lagrangian.
Indeed \( B \) can naturally be made small by an approximate R symmetry, while \( \mu \) could be
small because of an approximate Peccei–Quinn symmetry, which is taken to commute with
supersymmetry [7]. However, since either of these symmetries requires a chargino with a
mass below its experimental lower limit, an enlargement of the Higgs sector was proposed
to make the large \( \tan \beta \) scenario natural [6].

This simple argument is based on an analysis of the vacuum of the tree-level potential.
In the next section, we show that (unlike at tree level) if we take into account radiative
corrections to the potential, the value of \( B\mu \) does not vanish even if \( \tan \beta \to \infty \). If this
radiatively corrected value of \( B\mu \) (though loop suppressed) is not much smaller than other
soft SUSY breaking parameters (this could be because of hierarchies of \( \sim 10 \) in their values,
which is certainly allowed in a generic SUSY model), the \( \tan \beta \to \infty \) limit is not unnatural
in the sense of Ref. [1], as implied by the tree-level analysis. This is our main point. In
Section III, we exhibit technically natural scenarios with very large values of \( \tan \beta \). We note
that the upper bound on \( \tan \beta \) comes from the requirement that Yukawa couplings remain
perturbative up to a scale \( Q_{NP} \), and quantify the relation between the maximum value of
\( \tan \beta \) and this scale. We conclude in the last section with a discussion of our analysis.

II. ONE LOOP MINIMIZATION OF THE SCALAR POTENTIAL

Radiative corrections cause the ground state of a quantum theory to differ from the
ground state of the corresponding classical theory. These radiative corrections are automatically
included when the vacuum state is computed by minimizing the effective potential
\[ V = V_{\text{tree}} + V_{\text{rad}}, \]
of the quantum theory, where the the one loop radiative correction to the potential, renormalized at the scale $Q$, is given by

$$V_{\text{rad}} = \sum_k \frac{1}{64\pi^2} (-1)^{J_k} (2J_k + 1) c_k m_k^4 \left( \log \left( \frac{m_k^2}{Q^2} \right) - \frac{3}{2} \right).$$

(5)

Here the sum is taken over independent real boson or Majorana fermion fields in the loop (complex boson fields and Dirac fermions, therefore, contribute twice as much), $m_k^2$ are the field dependent squared masses of the particles in the loops, $J_k$ is their spin, and the factor $c_k = 3(1)$ for coloured (uncoloured) particles.

For the minimal supersymmetric Standard Model, the relevant part of $V_{\text{tree}}$ is given by (1). Gauge invariance dictates that both $V_{\text{tree}}$ and $V_{\text{rad}}$ are functions of the field combinations,

$$|h_u|^2, |h_d|^2$$

and ($h_u h_d + \text{h.c.}$), so that

$$\frac{\partial V_{\text{rad}}}{\partial |h_u|^2} \bigg|_{\text{min}} = \frac{\partial V_{\text{rad}}}{\partial |h_u|^2} \bigg|_{\text{min}} v_u + \frac{\partial V_{\text{rad}}}{\partial (h_u h_d + \text{h.c.})} \bigg|_{\text{min}} v_d,$$

$$\frac{\partial V_{\text{rad}}}{\partial |h_d|^2} \bigg|_{\text{min}} = \frac{\partial V_{\text{rad}}}{\partial |h_d|^2} \bigg|_{\text{min}} v_d + \frac{\partial V_{\text{rad}}}{\partial (h_u h_d + \text{h.c.})} \bigg|_{\text{min}} v_u.$$  (6)

It is then easy to see that the effect of including the one loop correction to the potential is equivalent to the replacements,

$$m_{H_u}^2 \rightarrow m_{H_u}^2 + \Sigma_{uu},$$

$$m_{H_d}^2 \rightarrow m_{H_d}^2 + \Sigma_{dd},$$

$$B\mu \rightarrow B\mu - \Sigma_{ud},$$

in the tree level minimization conditions (3) and (4), where,

$$\Sigma_{uu} = \frac{\partial V_{\text{rad}}}{\partial |h_u|^2} \bigg|_{\text{min}}, \quad \Sigma_{dd} = \frac{\partial V_{\text{rad}}}{\partial |h_d|^2} \bigg|_{\text{min}}, \quad \Sigma_{ud} = \frac{\partial V_{\text{rad}}}{\partial (h_u h_d + \text{h.c.})} \bigg|_{\text{min}}.$$  (7)

The radiatively corrected minimization conditions are thus given by,

$$\mu B = \sin \beta \cos \beta \left[ m_{H_u}^2 + m_{H_d}^2 + 2\mu^2 \right] + \sin \beta \cos \beta \left( \Sigma_{uu} + \Sigma_{dd} \right) + \Sigma_{ud},$$

$$\mu^2 = \frac{(m_{H_d}^2 + \Sigma_{dd}) - \tan^2 \beta (m_{H_u}^2 + \Sigma_{uu})}{(\tan^2 \beta - 1)} - \frac{1}{2} m_Z^2.$$  (8)

(9)

In the $\tan \beta \rightarrow \infty$ limit we see that

$$\mu B = \Sigma_{ud},$$  (10)

which, though suppressed by a loop factor, does not vanish. Indeed as long as $\Sigma_{ud}$ is sizeable, models with large $\tan \beta$ do not suffer from the naturalness problem. This is our main observation.
The dominant contribution to $\mathcal{V}_{rad}$ arises from the third generation Yukawa interactions. To illustrate solutions with very large values of $\tan \beta$, we will ignore electroweak gauge coupling corrections to the effective potential which are known to be small, and whose inclusion will not significantly affect our results. In this case, $\Sigma_{ud}$ that enters the determination of $B\mu$ is given by,

$$\Sigma_{ud} = \Sigma_{ud}(\tilde{t}) + \Sigma_{ud}(\tilde{b}) + \Sigma_{ud}(\tilde{\tau}),$$

with

$$\Sigma_{ud}(\tilde{f}) = \frac{c_f}{16\pi^2}(-\mu)f_f^2 A_f \left( f(m_{f_2}^2) - f(m_{f_1}^2) \right) \left( \frac{m_{f_2}^2 - m_{f_1}^2}{m_{f_2}^2 - m_{f_1}^2} \right). (11)$$

Here, $f = t, b, \tau$, $f_f$ is the Yukawa coupling of fermion $f$, $\tilde{f}_i$ are the sfermion mass eigenstates, the colour factor $c_f = 3(1)$ when $f$ is a quark (lepton), and the function $f(x)$ that appears in (11) is given by,

$$f(x) = x \left( \ln \frac{x}{Q^2} - 1 \right).$$

In a generic SUSY model, it is entirely possible that the weak scale $A$-parameters and $|\mu|$ are a few times larger than the soft SUSY breaking masses; in this case, the factor $\frac{\mu A_f}{m_f^2}$ would largely compensate for the loop suppression, and $\Sigma_{ud}$ (and hence, $B\mu$) would be comparable to other soft SUSY breaking masses. In any case, fine-tuning of parameters at the level of $\mathcal{O}(\frac{1}{\tan \beta})$, that is suggested by the tree-level analysis, is not needed.

III. VERY LARGE $\tan \beta$ AND THE SCALE OF NEW PHYSICS

Although we have argued that very large $\tan \beta$ solutions are not necessarily unnatural within the MSSM framework, it still remains to be shown that those solutions can be phenomenologically viable and theoretically interesting. By this, we mean that we look for large $\tan \beta$ solutions with third generation matter fermion masses given by their experimental values, and with the corresponding Yukawa couplings in the perturbative range.3

In a supersymmetric theory, the experimental values for the third generation fermion masses determine the corresponding Yukawa couplings at $Q = M_Z$, but only if we know the sparticle mass spectrum. This is because supersymmetric particle loops affect the fermion masses through threshold corrections [9]. Analytical expressions for the one loop susy threshold corrections were given in the literature [10]. To leave our approach as model-independent as possible, we parametrize our ignorance of the supersymmetric threshold corrections to third generation fermion masses through a set of coefficients, $\delta_t, \delta_b, \delta_{\tau}$, which appear in the relations between matter fermion masses and the corresponding Yukawa couplings:

3We ignore Yukawa couplings and fermion masses for the first two generations.
\[ f_t = \frac{m_t}{v \sin \beta (1 + \delta_{QCD} + \delta_t)}, \]
\[ f_b = \frac{m_b(m_Z)}{v \cos \beta (1 + \delta_b \tan \beta)}, \]
\[ f_\tau = \frac{m_\tau(m_Z)}{v \cos \beta (1 + \delta_\tau \tan \beta)}, \]

where \( v = \sqrt{v_u^2 + v_d^2} \). Here, \( m_t \) is the top quark pole mass while \( m_b(Q) \) and \( m_\tau(Q) \) are running bottom quark and tau lepton masses at the scale \( Q \) in the \( \overline{DR} \) scheme. The coefficient \( \delta_{QCD} \) is the usual QCD correction relating the pole and running top masses. This does not appear in the formula for \( f_b \) since for this we use the running mass \( m_b(M_Z) = 2.83 \) GeV as the experimental input [11]. The appearance of \( \tan \beta \) in the expressions for bottom and tau Yukawa couplings captures the fact that, for large \( \tan \beta \) for \( \delta_b \) the experimental input \([11]\). The appearance of \( \tan \beta \) is both model-dependent and a matter of judgement just how large a

Thus the largest values of \( \tan \beta \) in the range of 100-1000 GeV. Specifically, \( m_\tilde{g} \) reasonable values of the coefficients \( \delta_b, \delta_b \) and \( \delta_\tau \) as given in typical models with sparticles depend on the unknown sparticle spectrum. In the following, we implement these by adopting reasonable values of the coefficients \( \delta_b, \delta_b \) and \( \delta_\tau \) as given in typical models with sparticles in the range of 100-1000 GeV. Specifically, \( \delta_b \) is positive, and increases logarithmically with \( m_\tilde{g} \). Our results are insensitive to its precise value which we take to be 0.04, corresponding to \( m_\tilde{g} \) somewhat larger than 1 TeV. The threshold corrections for the down type fermions depend on \( \mu \), and so can have either sign. We take \( \delta_b = \pm 0.008 \) which gives a susy threshold of 40\% for \( \tan \beta = 50 \), typical of the mSUGRA framework with TeV scale parameters. For \( \delta_\tau \), we take \( \pm 0.0016 \) which correspond to \( \pm 8\% \) for \( \tan \beta = 50 \) as a typical value, and \( \pm 0.003 \) as a somewhat extreme case.

If all other things are the same, the bottom Yukawa coupling is clearly larger if \( \delta_b < 0 \). Thus the largest values of \( \tan \beta \) for which \( f_t \) remains in the perturbative range occur when \( \delta_b > 0 \) (assuming \( f_b \) is positive). Likewise, we would expect that the requirement that \( f_\tau \) lie in the perturbative range would allow larger values of \( \tan \beta \) when \( \delta_\tau > 0 \); however, if \( \delta_\tau \) is negative, for \( \tan \beta > \frac{1}{\delta_\tau}, f_\tau < 0 \), so that \( \frac{f_\tau}{f_b} \) decreases as \( \tan \beta \) increases beyond this value. It is easy to check that this branch of the \( \delta_\tau < 0 \) curve asymptotically approaches the \( \delta_\tau > 0 \) curve, as shown in Fig. 1 where we show the value of \( \alpha_f \equiv \frac{f_\tau}{f_b} \) evaluated at \( Q = M_Z \) versus \( \tan \beta \) for \( \delta_b = 0.008 \) and \( \delta_\tau = \pm 0.0016 \). Naively, one might conclude that “viable” solutions to the MSSM are possible for \( \tan \beta \) values up to 800.

This is, however, not the case since a Yukawa coupling close to the “perturbative limit” (aside from the fact that this might be phenomenologically unacceptable) would blow up at a scale \( Q_{NP} \) not far above \( M_Z \), and we would lose our main motivation for weak scale supersymmetry. Weak scale supersymmetry is well-motivated only if the scale \( Q_{NP} \) where any coupling becomes non-perturbative is sufficiently separated from \( M_Z \). We stress that the electroweak scale is destabilized even if \( Q_{NP} \) is much smaller than \( M_{Planck} \) or \( M_{GUT} \); it is both model-dependent and a matter of judgement just how large a \( Q_{NP} \) is acceptable in any extension of the SM without supersymmetry to stabilize the electroweak scale. Since a discussion of this would take us away from our main point, we show the value of \( Q_{NP} \) as a function of \( \tan \beta \) in Fig. 2. We have again fixed \( \delta_b = 0.008 \) and illustrate our result for \( \delta_\tau = -0.003 \) (dotted green curve, labelled a), \( \delta_\tau = -0.0016 \) (dashed-dotted pink line, labelled b), \( \delta_\tau = 0.0016 \) (dashed blue line, labelled c) and \( \delta_\tau = 0.003 \) (solid red line, labelled d). In all these cases, it is the coupling \( \alpha_\tau \) that exceeds unity at \( Q = Q_{NP} \), with the other
Yukawa couplings remaining perturbative. This figure, which updates previous work by Haber and Zwirner [12], includes two-loop Yukawa coupling RGEs and models the effect of SUSY threshold corrections. We see that for $Q_{NP}$ close to $M_{GUT}$, the maximum value of $\tan \beta$ is never much above what one obtains within the mSUGRA framework. We stress though that the value of this maximum is dictated by the measured fermion masses and not by the naturalness considerations. To emphasize this, we show the corresponding curve (dashed black line, labelled e) for $\delta_\tau = 0.0016$ but with tau lepton and bottom quark masses fixed at half their experimental values. In such a universe, it is easily possible to find natural models with $\tan \beta$ larger than 200, and couplings in the perturbative range all the way up to $Q = M_{Planck}$. Returning to the case of realistic masses, we see from the figure that (depending on the value of SUSY thresholds) models with $\tan \beta$ as large as 150 may be natural if the new physics scale is smaller than $\sim 10^7$ GeV, and we use SUSY to stabilize the electroweak scale relative to this intermediate scale.\footnote{We recognize that we would still have to be careful about the new physics at this scale in order}
FIG. 2. Relation between $\tan \beta$ and the scale of new physics for $\delta_b = +0.008 > 0$ and four values of $\delta_\tau$: $-0.003$ (dotted–green line, labelled a), $-0.0016$ (dash/dotted–pink line, labelled b) $+0.0016$ (dashed–blue line, labelled c) and $+0.003$ (solid–red line, labelled d). The dashed black line labelled e shows the value of $Q_{NP}$ for a fictitious case with both $m_\tau$ and $m_b$ set at one half their experimental values.

It is well known that SUSY phenomenology of large $\tan \beta$ models differs considerably [13] from that of models with low or moderate values of $\tan \beta$. What is less clear (because in the well-studied models, $\tan \beta \lesssim 50 - 70$) is whether the phenomenology is altered as $\tan \beta$ is changed from $\sim 50$ to $\gtrsim 100$. As we have already explained, $\tan \beta \geq 100$ can only be accommodated if $Q_{NP}$ is relatively low. This led us to examine the gauge-mediated SUSY breaking (GMSB) framework with a low messenger scale. Within the minimal version of this model, the radiative electroweak symmetry breaking mechanism breaks down, leading not to have to attribute the apparent unification of gauge couplings measured by LEP experiments to an accident.
TABLE I. Selected sparticle masses for two non–minimal GMSB scenario with $M_{mess} = 2\Lambda = 300 \text{ TeV}$, $n_5 = 2$, $\mu > 0$ and $\tan \beta = 50$ and $\tan \beta = 100$. We take $\delta m_{H_u}^2 = - (1000 \text{ GeV})^2$ and $\delta m_{H_d}^2 = 0$. For the corresponding minimal model, the upper limit on $\tan \beta$ is about 68. The entry in the last column is $10^9 \times B(B_s \to \mu^+\mu^-)$.

| $\tan \beta$ | $m_{\tilde{\tau}_1}$ | $m_A$ | $m_{\tilde{\chi}_1}$ | $m_{\tilde{b}_1}$ | $m_{\tilde{\nu}_R}$ | $m_{\tilde{\nu}_L}$ | $m_{Z_1}$ | $B(B_s \to \mu^+\mu^-)$ |
|-------------|----------------|--------|----------------|----------------|----------------|----------------|-------------|-----------------|
| 50          | 275            | 1245.4 | 2046.7         | 2102           | 385            | 2289          | 425.4       | 4.3             |
| 100         | 99.9           | 419.7  | 2038.4         | 1908           | 386            | 2286          | 425.5       | 33              |

To obtain larger values of $\tan \beta$, we introduced additional contributions $\delta m_{H_u,d}^2$ to the soft SUSY breaking Higgs boson masses, since these can facilitate radiative electroweak symmetry breaking. We attribute their origin to additional interactions needed to generate the $\mu$ and $B\mu$ parameters within this framework [14]. We have used ISAJET v7.64 [15] to evaluate the mass spectrum for two scenarios, with $\tan \beta = 50, 100$ whose parameters are listed in Table I, where selected sparticle masses are shown. In both scenarios, $\tilde{\tau}_1$ is the second lightest sparticle. The sfermions of the first two generations and the charginos and neutralinos have the same masses to within about a percent in the two cases. Third generation squark masses differ by a fraction of a percent for $t$–squarks to about 10% for $\tilde{b}_1$. The most striking difference is in the mass of the $A$ (and associated $H$ and $H^\pm$), and the mass of the lighter stau. The value of $m_{\tilde{\nu}_L}/m_{\tilde{\nu}_R}$ would suggest a GMSB scenario; the relative lightness of $\tilde{\tau}_1$ would point to the very large value of $\tan \beta$. Tevatron experiments may be able to probe the $\tan \beta = 100$ scenario in the Table via the decay $B_s \to \mu^+\mu^-$ whose branching fraction is usually very small in GMSB models [16]. We have checked that $B(b \to s\gamma)$ is similar in both cases: $3.44 \times 10^{-4}$ for $\tan \beta = 50$ (100), while $\Delta a_\mu = 11 (22) \times 10^{-10}$ scales with $\tan \beta$ as expected. The message of this illustrative example is that changing $\tan \beta$ from a large to a very large value has experimentally interesting implications.

IV. DISCUSSION

We have pointed out that the usual arguments [6] that suggest that supersymmetric extensions of the SM are unnatural for large values of the parameter $\tan \beta$ are inapplicable.

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5Within this framework, Higgs boson mass squared parameters are mainly driven down because squarks are heavy. If $\tan \beta$ is very large, the bottom and top Yukawa couplings are similar, so that $m_{H_u}^2$ and $m_{H_d}^2$ start from a common value and roughly evolve together. This then causes $m_A^2 \simeq m_{H_d}^2 - m_{H_u}^2 - M_Z^2$ (valid for large values of $\tan \beta$) to turn negative.

6Remember that $m_A$ always becomes small near the upper bound of $\tan \beta$, so by itself would not be indicative of the value $\tan \beta$. 
when 1-loop corrections to the effective potential are included: as can be seen from (10), the soft mass parameter $\mu B$ does not vanish as $\tan \beta \to \infty$, and the question of checking whether there is an increased symmetry when $\mu B \to 0$ becomes moot. In a generic SUSY model, it appears to us that there is no naturalness problem in the sense discussed in Ref. [1].

We stress though that there may be a different fine-tuning required in specific models if $\tan \beta$ is very large. For instance, in the mSUGRA framework, characterized by the soft breaking parameters, $m_0, m_{1/2}, A_0, (B\mu)_0$ and $\mu_0$ at the high scale, the parameter $B\mu$ at the weak scale will be of loop-suppressed magnitude only if the high scale parameters are all of comparable size, but related in a specific manner. From the perspective of a low energy theorist who does not have an understanding of the SUSY breaking mechanism, this appears to require an unexplained adjustment of the underlying parameters. But, perhaps, it is better to view this as a necessary property of the physics underlying supersymmetry breaking; i.e. the soft parameters that emerge from the theory of supersymmetry breaking must be related so that $B\mu$ is small at the weak scale. However, this issue seems to be separate from the naturalness question that we have focussed upon.

In summary, it appears to us that SUSY models are not unnatural in the technical sense of 't Hooft even if the parameter $\tan \beta$ is large. We have argued that the parameter $B\mu$ does not vanish in the limit $\tan \beta \to \infty$ when radiative corrections are included. Our considerations could be especially relevant in the context of low scale supersymmetric models as GMSB models [17] or supersymmetric extra dimensional models which are the subject of recent interest [18], and for which new physics (beyond minimal SUSY) intervenes at a relatively low scale.

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7 Of course, the necessary “fine-tuning” is much less severe than for the SM Higgs mass parameter because the underlying supersymmetry still precludes quadratic divergences in the corrections to $B\mu$. 
REFERENCES

[1] G. ’t Hooft, “Naturalness, Chiral Symmetry, And Spontaneous Chiral Symmetry Breaking,” Lecture given at Cargese Summer Inst. (Aug 26 - Sep 8, 1979), reprinted in Under the spell of the gauge principle, G.’t Hooft (published by World Scientific)

[2] J. E. Kim and H. P. Nilles, Phys. Lett. B138, 150 (1984) and *ibid* B263, 79 (1991); G. Giudice and A. Masiero, Phys. Lett. B206, 480 (1988); J. Casas and C. Muñoz, Phys. Lett. B306, 288 (1993); G. Dvali, G. Giudice and A. Pomarol, Nucl. Phys. B478, 31 (1996).

[3] E. Witten, Nucl. Phys. B188, 513 (1981); S. Dimopoulos and H. Georgi, Nucl. Phys. B193, 150 (1981); N. Sakai, Z. Phys. C11, 153 (1981); R. Kaul, Phys. Lett. B109, 19 (1982).

[4] Y. Fukuda *et al.* Phys. Rev. Lett. 82, 2644 (1999); S. Fukuda *et al.* *ibid* B85, 3999 (2000).

[5] H. Baer and J. Ferrandis, Phys. Rev. Lett. 87, 211803 (2001); H. Baer, M. Brhlik, M. A. Diaz, J. Ferrandis, P. Mercadante, P. Quintana and X. Tata, Phys. Rev. D63, 015007 (2001); H. Baer, M. A. Diaz, J. Ferrandis and X. Tata, Phys. Rev. D61, 111701 (2000); T. Blazek, R. Dermisek and S. Raby, Phys. Rev. D65, 115004 (2002); T. Blazek, R. Dermisek and S. Raby, Phys. Rev. Lett. 88, 111804 (2002).

[6] A. E. Nelson and L. Randall, Phys. Lett. B316, 516 (1993).

[7] L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D50, 7048 (1994); R. Rattazzi and U. Sarid, Phys. Rev. D53, 1553 (1996)

[8] C. F. Kolda, L. Roszkowski, J. D. Wells and G. L. Kane, Phys. Rev. D50, 3498 (1994); B. Ananthanarayan, K. S. Babu and Q. Shafi, Nucl. Phys. B428, 19 (1994); R. Rattazzi, U. Sarid and L. J. Hall, [arXiv:hep-ph/9405313]; M. Drees, Pramana 45, S85 (1995) [arXiv:hep-ph/9407254]; M. Schmaltz, Phys. Rev. D52, 1643 (1995); R. Rattazzi and U. Sarid, Ref. [7]; C. Csaki and L. Randall, Nucl. Phys. B466, 41 (1996) M. Carena, H. E. Haber and C. E. Wagner, Nucl. Phys. B472, 55 (1996); F. M. Borzumati, Y. Grossman, E. Nardi and Y. Nir, Phys. Lett. B384, 123 (1996); M. Dine, Y. Nir and Y. Shirman, Phys. Rev. D55, 1501 (1997); J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. Lett. 84, 2322 (2000).

[9] R. Hempfling, Phys. Rev. D49, 6168 (1994); 7048 (1994); M. Carena *et al.* Nucl. Phys. B426, 269 (1994); L. J. Hall, R. Rattazzi and U. Sarid, Ref. [7].

[10] D. M. Pierce, J. A. Bagger, K. T. Matchev and R. j. Zhang, Nucl. Phys. B491, 3 (1997)

[11] H. Baer, J. Ferrandis, K. Melnikov and X. Tata, Phys. Rev. D66, 074007 (2002).

[12] H. E. Haber, Properties of SUSY Particles, L. Cifarelli and V. A. Khoze, Editors (World Scientific, 1993).

[13] H. Baer, C. H. Chen, M. Drees, F. Paige and X. Tata, Phys. Rev. Lett. 79, 986 (1997); [80, 642 (1998) (E)], and Phys. Rev. D 58, 075008 (1998).

[14] S. Dimopoulos, S. Thomas and J. Wells, Nucl. Phys. B488, 39 (1998).

[15] F. Paige, S. Protopopescu, H. Baer and X. Tata, hep-ph/0001086 (2000).

[16] J. K. Mizukoshi, X. Tata and Y. Wang, Phys. Rev. D 66, 115003 (2002); S. Baek, P. Ko and W. Y. Song, Phys. Rev. Lett. 89, 271801 (2002).

[17] G. F. Giudice and R. Rattazzi, Phys. Rept. 322, 419 (1999)

[18] W. D. Goldberger, Y. Nomura and D. R. Smith, [arXiv:hep-ph/0209158]; K. w. Choi, H. D. Kim and Y. W. Kim, [arXiv:hep-ph/0202257]; D. Marti and A. Pomarol, Phys.
Rev. D 64, 105025 (2001); T. Gherghetta and A. Pomarol, Nucl. Phys. B 602, 3 (2001); A. Pomarol, Phys. Rev. Lett. 85, 4004 (2000)