ON GLOBAL BOUNDEDNESS OF THE CHEN SYSTEM

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Abstract. This paper deals with the open problem of the global boundedness of the Chen system based on Lyapunov stability theory, which was proposed by Qin and Chen (2007). The innovation of the paper is that this paper not only proves the Chen system is global bounded for a certain range of the parameters according to stability theory of dynamical systems but also gives a family of mathematical expressions of global exponential attractive sets for the Chen system with respect to the parameters of this system. Furthermore, the exponential rate of the trajectories is also obtained.

1. Introduction. The well-known systems of Lorenz, Rossler, Chua’s circuit, Chen, Lü, Glukhovsky-Dolzhansky, and other chaotic dynamical systems have served as important chaotic models in the development of chaos theory [1-2, 4-11, 13, 15].

As a dual system to the classical Lorenz system, the Chen system has been seriously studied in recent years (see [2-3,7,14] and some references therein). The
Chen system is described by [3]:

\[
\begin{align*}
\frac{dx}{dt} &= a(y - x), \\
\frac{dy}{dt} &= (c - a)x - xz + cy, \\
\frac{dz}{dt} &= xy - bz.
\end{align*}
\]  

(1)

where \(a > 0, b > 0, \) and \(c > 0\) are constant parameters.

Despite the fact that many qualitative and quantitative results on the Chen system have been obtained, there is a fundamental question that has not been completely answered so far: is there a global trapping region for the Chen system? Global exponential attractive set is an important concept in dynamical systems.

If one can show that a chaotic or a hyperchaotic system has a global attractive set, then one knows that the system cannot possess equilibrium points, periodic or quasiperiodic solutions, or other chaotic or hyperchaotic attractors outside the globally attractive set. This is very important for engineering applications, since it is very difficult to predict the existence of hidden attractors and they can lead to crashes [1,13]. Therefore, how to get the global attractive sets of a chaotic dynamical system is particularly significant both for theoretical research and practical applications.

It is well known that there exist global exponential attractive sets for the Lorenz system for all positive parameters [12, 22]. Global attractive sets of the Lorenz system have been firstly investigated by Leonov et al. in a series of articles [6, 11]. One can show that there is a bounded ellipsoid in \(R^3\), which all orbits of the Lorenz system with exponential rate will eventually enter.

In searching for a global bounded region, one generally would like to choose a Lyapunov-like function, as simple as possible, and apply the Lyapunov stability criteria. The former dynamical systems that we are searching for a global bounded region have a common characteristic: the elements of main diagonal of matrix \(A\) are all negative, where matrix \(A\) is the Jacobian matrix \(\frac{df}{dx}\) of a continuous dynamical system defined by \(\dot{x} = f(x), x \in R^3\), evaluated at the origin \((0, 0, 0)\). And the origin \((0, 0, 0)\) is an equilibrium point of the system. However, there are positive numbers in the elements of main diagonal of matrix \(B\), where matrix \(B\) is the Jacobian matrix of the Chen system for \(a > 0, b > 0, c > 0\) evaluated at the origin \((0, 0, 0)\). And the origin \((0, 0, 0)\) is an equilibrium point of the Chen system. So, the known results on the localization and global exponential attractive sets of the Lorenz system cannot (or can in some cases) be directly applied to the Chen system. In conclusion, it is a very difficult task to study the global attractive sets of the Chen system.

Motivated by the above discussion, we will investigate the global attractive sets of the Chen system (1). The rest of the paper is organized as follows. Section 2 gives the main results of this paper. The conclusions are drawn in Sect.3.

2. Main results. Consider the system

\[
\frac{dX}{dt} = f(X)
\]

(2)

where \(X = (x_1, x_2, \ldots, x_n) \in R^n, f : R^n \rightarrow R^n, t_0 \geq 0\) is the initial time, \(X_0 \in R^n\) is a initial value and \(X(t, t_0, X_0)\) is the solution to the system (2) satisfying \(X(t_0, t_0, X_0) = X_0\) which for simplicity is denoted as \(X(t)\). Assume \(\Omega \subset R^n\) is a compact set. Define the distance between the solution \(X(t, t_0, X_0)\) and the set \(\Omega\) as \(\rho(X(t, t_0, X_0), \Omega) = \inf_{Y \in \Omega} \|X(t, t_0, X_0) - Y\|\).
We will give the following definition and introduce Lemma 1 which will be used in Theorem 1.

**Definition 1.** ([12, 16]). If there exists a positive definite and radially unbounded generalized Lyapunov-like function \( V(X) \), and positive numbers \( L > 0, \alpha > 0 \) for system (2), such that the following inequality

\[
[V(X(t)) - L] \leq [V(X(t_0)) - L] e^{-\alpha(t-t_0)},
\]

is valid for \( V(X(t)) > L (t \geq t_0) \), then the set \( \Omega =\{X|V(X) \leq L\} \) is called a globally exponential attractive set of system (2).

**Lemma 1.** When \( 2a > b > 0 \), the following inequality holds for the Chen system (1)

\[
\lim_{t \to +\infty} [2az - x^2] \geq 0.
\]

**Proof.** Let us define

\[
V(x,z) = x^2 - 2az.
\]

Then, its derivative along the orbits of system (1) is

\[
\frac{dV(x,z)}{dt} = 2x \frac{dx}{dt} - 2a \frac{dz}{dt} = 2ax(y-x) - 2a(xy-bz) = -2ax^2 + 2abz.
\]

And,

\[
\frac{dV}{dt} + bV = (b-2a)x^2.
\]

When \( 2a > b > 0 \), we have

\[
\frac{dV}{dt} + bV = (b-2a)x^2 \leq 0.
\]

For any initial value \( V(t_0) = V_0 \), we have

\[
V(t) \leq V_0 e^{-b(t-t_0)} \rightarrow 0 (t \to +\infty).
\]

Thus,

\[
\lim_{t \to +\infty} V(t) = \lim_{t \to +\infty} [x^2 - 2az] \leq 0.
\]

This completes the proof. \( \Box \)

**Remark 1.** The method to prove the inequality in Lemma 1 is using the method in [11]. As early as in 1987, G. A. Leonov et al. have proposed the method to prove this kind of inequalities for the Lorenz system in the excellent paper [11].

In the following, we will study the global exponential attractive sets of Chen system (1). In order to simplify the calculation process, let us take model (1) with the following reversible linear transform: \( x = x, y_1 = y - \eta x, z = z \). Then the model (1) takes the form as:

\[
\begin{align*}
\frac{dx}{dt} &= -a_1 x + a y_1, \\
\frac{dy_1}{dt} &= -c_1 y_1 + c_2 x - x z, \\
\frac{dz}{dt} &= -b_0 + x y_1 + \eta x^2,
\end{align*}
\]

where

\[
\eta = \frac{b + 2c}{4a}, a_1 = a (1 - \eta), c_1 = a \eta - c,
\]

\[
c_2 = c - a + c \eta + a \eta (1 - \eta) = \frac{1}{4a} \left[ ab + 2ac - \frac{b^2}{4} + 4a(c - a) + c^2 \right].
\]
Let us define the function

\[ V_\lambda (X) = \frac{1}{2} \left[ \lambda x^2 + y_1^2 + (z - \tau_\lambda)^2 \right], \]

where \( \lambda > 0 \). This completes the proof. □

According to Lemma 1 and Lemma 2, we can get the global exponential attractive sets of the Chen system (1). The global exponential attractive sets of the Chen system (1) is described by the following Theorem 1.

**Theorem 1.** For \( 2a > b > 2c > 0 \), let us denote

\[ \eta = \frac{b + 2c}{4a}, \varepsilon = \frac{b - 2c}{4}, c_1 = a\eta - c, a_1 = a(1 - \eta), \]

\[ c_2 = c - a + c\eta + a\eta(1 - \eta) = \frac{1}{4a} \left[ ab + 2ac - \frac{b^2}{4} + 4a(c - a) + c^2 \right]. \]

Then we can get the conclusions that

\[ 1 > \eta > 0, 0 < \varepsilon < \min \{ a_1, b - 2a\eta \}, \varepsilon = c_1. \]

**Proof.** When \( 2a > b > 2c > 0 \), we can get

\[ 1 > \eta = \frac{b + 2c}{4a}, \varepsilon = \frac{b - 2c}{4}, \]

\[ a_1 = a(1 - \eta) - \frac{b - 2c}{4} = a \left( 1 - \frac{b + 2c}{4a} \right) - \frac{b - 2c}{4} = \frac{2a - b}{2} > 0, \]

\[ b - 2a\eta - \varepsilon = \frac{b - 2c}{4} > 0, \]

\[ c_1 - \varepsilon = a\eta - c - \frac{b - 2c}{4} = 0. \]

By the definition,

\[ \Omega_\lambda = \left\{ (x, y, z) | \lambda x^2 + y_1^2 + (z - \tau_\lambda)^2 \leq 2L_\lambda, \forall \lambda > \max \left\{ -\frac{c_2}{a}, 0 \right\} \right\}, \]

is the global exponential attractive set of system (3). Therefore,

\[ \Phi_\lambda = \left\{ (x, y, z) | \lambda x^2 + \left( y - \frac{b + 2c}{4a} x \right)^2 + (z - \tau_\lambda)^2 \leq 2L_\lambda, \forall \lambda > \max \left\{ -\frac{c_2}{a}, 0 \right\} \right\}, \]

is the global exponential attractive set of the Chen system (1).
Let \( f'(z) = -2(b - 2a\eta)z + 2\varepsilon(z - \tau_{\lambda}) + b\tau_{\lambda} = 0. \) (4)

Then we can get the solution of the above equation (4)

\[
z = z_0 = \frac{2\varepsilon - b}{-2(b - 2a\eta - \varepsilon)}\tau_{\lambda} = \frac{b + 2c}{b - 2c}\tau_{\lambda}.
\]

And

\[
f''(z_0) = -2(b - 2a\eta - \varepsilon).
\]

According to Lemma 2, we can get \( 0 < \varepsilon < \min\{a_1, b - 2a\eta\} \). So we can obtain

\[
f''(z_0) = -2(b - 2a\eta - \varepsilon) < 0.
\]

Therefore, we have

\[
\max_{z \in \mathbb{R}} f(z) = f(z_0) = \frac{(b^2 + 4c^2)}{2(b - 2c)\lambda^2} > 0.
\]

Define the following generalized positively definite and radially unbounded Lyapunov-like function

\[
V_{\lambda}(X) = V_{\lambda}(x, y_1, z) = \frac{1}{2}\left[\lambda x^2 + y_1^2 + (z - \tau_{\lambda})^2\right], \quad \forall \lambda > \max\left\{\frac{-c_2}{a}, 0\right\}.
\]

When \( V_{\lambda}(X) > \lambda \), \( V_{\lambda}(X_0) > \lambda \), computing the derivative of \( V_{\lambda}(X) \) along the trajectory of system (3), we have

\[
\left. \frac{dv_{\lambda}(x, y_1, z)}{dt} \right|_{(3)} = \lambda x \frac{dx}{dt} + y_1 \frac{dy_1}{dt} + (z - \tau_{\lambda}) \frac{dz}{dt},
\]

\[
= \lambda x (-a_1 x + a y_1) + y_1 (-c_1 y_1 + c_2 x - x z) + (z - \tau_{\lambda}) (bz + xy_1 + \eta x^2),
\]

\[
= -\left(\lambda a_1 + \eta \tau_{\lambda}\right)x^2 - c_1 y_1^2 - b z^2 + \eta x^2 z + b \tau_{\lambda} z,
\]

\[
\leq -\left(\lambda a_1 + \eta \tau_{\lambda}\right)x^2 - c_1 y_1^2 - b z^2 + 2a_2 \eta z^2 + b \tau_{\lambda} z.
\]

According to Lemma 1 and Lemma 2, we have

\[
\lim_{t \to \infty} [x^2 - 2az] \leq 0, \eta > 0, \varepsilon< b - 2a\eta, \varepsilon< a_1, \varepsilon = c_1 > 0.
\]

Since \( \lambda > \max\left\{\frac{-c_2}{a}, 0\right\} \), so we have

\[
\tau_{\lambda} = a\lambda + c_2 > 0.
\]

Furthermore, we have

\[
\lambda a_1 + \tau_{\lambda} \eta - \varepsilon \lambda = \lambda (a_1 - \varepsilon) + \tau_{\lambda} \eta > 0.
\]

Since \( \lim_{t \to \infty} [x^2 - 2az] \leq 0 \), so there exists a positive constant \( T_0 > 0 \), when \( t > T_0 \), we have

\[
\left. \frac{dv_{\lambda}(x, y_1, z)}{dt} \right|_{(3)} = \lambda x \frac{dx}{dt} + y_1 \frac{dy_1}{dt} + (z - \tau_{\lambda}) \frac{dz}{dt},
\]

\[
= \lambda x (-a_1 x + a y_1) + y_1 (-c_1 y_1 + c_2 x - x z) + (z - \tau_{\lambda}) (bz + xy_1 + \eta x^2),
\]

\[
= -\left(\lambda a_1 + \eta \tau_{\lambda}\right)x^2 - c_1 y_1^2 - b z^2 + \eta x^2 z + b \tau_{\lambda} z,
\]

\[
\leq -\left(\lambda a_1 + \eta \tau_{\lambda}\right)x^2 - c_1 y_1^2 - b z^2 + 2a_2 \eta z^2 + b \tau_{\lambda} z.
\]

\[
\leq -\varepsilon \left[\lambda x^2 + y_1^2 + (z - \tau_{\lambda})^2\right] - (c_1 - \varepsilon) y_1^2 - (\lambda a_1 + \tau_{\lambda} \eta - \varepsilon \lambda) x^2
\]

\[
- (b - 2a \eta) z^2 + \varepsilon (z - \tau_{\lambda})^2 + b \tau_{\lambda} z,
\]

\[
-\varepsilon \left[\lambda x^2 + y_1^2 + (z - \tau_{\lambda})^2\right] - (\lambda a_1 + \tau_{\lambda} \eta - \varepsilon \lambda) x^2
\]

\[
- (b - 2a \eta) z^2 + \varepsilon (z - \tau_{\lambda})^2 + b \tau_{\lambda} z,
\]

\[
\leq -\varepsilon \left[\lambda x^2 + y_1^2 + (z - \tau_{\lambda})^2\right] - (b - 2a \eta) z^2 + \varepsilon (z - \tau_{\lambda})^2 + b \tau_{\lambda} z,
\]
holds, then \( \max_f(z) \) is the global exponential attractive set of the Chen system (1).

\( \text{ii)} \) Let us take \( \lambda \) unknown, leaving an important and yet nontrivial problem for future research.

This completes the proof. \( \square \)

**Remark 2.** i) In Theorem 1, we have shown the global exponential attractive sets of the Chen system but only for some cases with \( 2a > b > 2c > 0 \). The global exponential attractive sets of the Chen system for other parameters ranges are still unknown, leaving an important and yet nontrivial problem for future research.

ii) Let us take \( \lambda = \frac{2|c_2|}{a} \), then we can get

\[
\phi_1 = \left\{ (x, y, z) | \lambda x^2 + \left( y - \frac{b + 2c}{4a} x \right)^2 + (z - \tau)^2 \leq \frac{2 (b^2 + 4c^2)}{(b - 2c)^2} \delta^2 \right\}
\]

is the global exponential attractive set of the Chen system (1), where

\[
\delta = 2|c_2| + c_2, c_2 = c-a+ac+a \eta (1-\eta), \eta = \frac{b+2c}{4a}.
\]

iii) Suppose \( c_2 > 0 \), and let \( \beta = \left( \frac{b+2c}{4a} \right)^2 \). Then we have \( \beta = \left( \frac{b+2c}{4a} \right)^2 > \max \left\{ -\frac{\pi}{2}, 0 \right\} \). In fact, simple calculations show that if \( \frac{b+2c}{4a} > 2a > b > 2c > 0 \) holds, then \( c_2 > 0 \) will hold.
In this case, let us take $\lambda = \left( \frac{b + 2c}{4a} \right)^2$, then we can get
\[
\Omega_2 = \left\{ (x, y, z) \left| \left( \frac{b + 2c}{4a} x \right)^2 + \left( y - \frac{b + 2c}{4a} x \right)^2 + (z - \tau)^2 \leq \frac{2 \left( b^2 + 4c^2 \right)}{(b - 2c)^2} \tau^2 \right. \right\},
\]
is the global exponential attractive set of the Chen system (1), where
\[
\tau = \frac{(b + 2c)^2}{16a} + c_2, c_2 = c - a + c\eta + a\eta(1 - \eta), \eta = \frac{b + 2c}{4a}.
\]
For the reason that $(y - 2\eta x)^2 \geq 0$.

Therefore, we have
\[
y^2 \leq (y - 2\eta x)^2 + y^2 = 2y^2 - 4\eta xy + 4\eta^2 x^2
\]
\[
= 2\eta^2 x^2 + 2\eta^2 x^2 - 4\eta xy + 2y^2 = 2(\eta x)^2 + 2(y - \eta x)^2.
\]
Namely,
\[
y^2 \leq 2 \left( \frac{b + 2c}{4a} x \right)^2 + 2 \left( y - \frac{b + 2c}{4a} x \right)^2.
\]
Combining (8)-(9), we have the estimate
\[
\lim_{t \to +\infty} \left[ \frac{y^2}{2} + (z - \tau)^2 \right] \leq \frac{2 \left( b^2 + 4c^2 \right)}{(b - 2c)^2} \tau^2,
\]
where
\[
\tau = \frac{(b + 2c)^2}{16a} + c_2, c_2 = c - a + c\eta + a\eta(1 - \eta), \eta = \frac{b + 2c}{4a}.
\]

iii) Currently it is being actively discussed the question of the equivalence of various Lorenz-like systems and the possibility of universal consideration of their behavior in view of the possibility of reduction of such systems to the same form with the help of various transformations [14]. Although in some case the Chen system (1) can be transformed to the famous Lorenz system by reversible linear transform [14], the known results on the localization and global exponential attractive sets in the Lorenz system cannot (or can in some cases) be directly applied to the Chen system (1). It is mainly because that there is positive number in the elements of main diagonal of matrix $A$, where matrix $A$ is the Jacobian matrix of the Chen system (1) for $a > 0, b > 0, c > 0$ evaluated at the origin. And the origin is an equilibrium point of Chen system (1). However, there are all negative numbers in the elements of main diagonal of matrix $C$, where matrix $C$ is the Jacobian matrix of the Lorenz system for its positive parameters evaluated at the origin. And the origin is an equilibrium point of the Lorenz system. So, the method of constructing Lyapunov-like functions applied to the former dynamical systems (see, e.g. Lorenz system [22], the Lorenz-like systems [17-20,23], the low-order atmospheric circulation system [21] etc), is not applicable to the Chen system (1) for $a > 0, b > 0, c > 0$ and the known results on the localization and global exponential attractive sets in the Lorenz system cannot (or cannot in some cases) be directly applied to the Chen system. To conclude, it is a very difficult work to get the global exponential attractive sets of the Chen system for $a > 0, b > 0, c > 0$.
3. Conclusions. This paper deals with the global exponential attractive sets of the Chen system. Using Lyapunov stability theory and many proper inequalities, global exponential attractive sets of the Chen system for a certain range of the parameters are obtained.

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