Fixing Two-Nucleon Weak-Axial Coupling $L_{1,A}$ From $\mu^-d$ Capture

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We calculate the muon capture rate on the deuteron to next-to-next-to-leading order in the pionless effective field theory. The result can be used to constrain the two-nucleon isovector axial coupling $L_{1,A}$ to $\pm 2$ fm$^3$ if the muon capture rate is measured to 2% level. From this, one can determine the neutrino-deuteron break up reactions and the $pp$-fusion cross section in the sun to a same level of accuracy.

The strong evidence of neutrino oscillations observed at the Sudbury Neutrino Observatory (SNO) is based on detecting the $^8$B solar neutrino flux through the following three reactions:

$$\nu_e + d \rightarrow p + p + e^- \quad \text{(CC)},$$
$$\nu_x + d \rightarrow p + n + \nu_x \quad \text{(NC)},$$
$$\nu_x + e^- \rightarrow \nu_x + e^- \quad \text{(ES)}.$$  (1)

The charged current (CC) reaction involves only the electron neutrinos, while the neutral current (NC) reaction and elastic scattering (ES) involve all the active neutrinos ($x = e, \mu, \tau$). The $\nu_e$ and $\nu_x$ fluxes are found to be significantly different. Further detailed measurements of the fluxes could sharpen the constraints to neutrino oscillation parameters and provide precision tests to the standard solar model. However, while the ES cross section is known to high accuracy, the CC and NC cross sections have hadronic uncertainties. As shown by Butler, Chen and Kong, the dominant uncertainties in low energy CC and NC cross sections come from the coupling of a two-body isovector axial current, $L_{1,A}$, in pionless effective field theory (EFT($\pi$)). The potential model results of Refs. and can be reproduced by different choices of $L_{1,A}$, indicating that the $\sim 5\%$ difference between the models comes from the different assumptions about the short-distance nuclear physics. There are other interesting weak reactions involving the same two-body current, for example, the $pp$ and $pep$ fusion processes ($pp \rightarrow d e^- \nu_e, pep \rightarrow d \nu_e$) which power the sun and. It is one of the great current interests to measure these neutrino fluxes to further test the standard solar model.

Recently much effort has been going into determining the effective two-body axial current interaction. Butler, Chen, and Vogel attempted to fix $L_{1,A}$ from reactor antineutrino-deuteron breakup reactions, and they found $L_{1,A} = 3.6 \pm 5.5$ fm$^3$. Chen, Heeger, and Robertson obtained $L_{1,A} = 4.0 \pm 6.3$ fm$^3$ by using SNO’s CC and NC data, calibrated by the ES events of SNO and Super-Kamiokande(SK). Schiavilla et al.’s idea of using the tritium $\beta$ decay rate to control the strength of the two-body current was adopted by Park et al. in their hybrid EFT calculation, and the pp fusion rate was predicted with a small error. When compared with the EFT($\pi$) calculation, their result yields $L_{1,A} = 4.2 \pm 2.5$ fm$^3$.

In this paper we aim to make a high-precision determination of $L_{1,A}$ from the $\mu^-d$ capture process

$$\mu^- + d \rightarrow \nu_\mu + n + n,$$  (2)

by calculating the rate to next-to-next-to leading order (N$^2$LO) in EFT($\pi$). The $\mu^-d$ capture rate has been measured previously by different groups with rather different results $\Gamma^{\exp} = 470 \pm 29$ s$^{-1}$ and $\Gamma^{\exp} = 409 \pm 40$ s$^{-1}$. A measurement of this rate with 1% precision is under investigation at PSI. An earlier potential model calculation gave $\Gamma = 397 \sim 400$ s$^{-1}$. More recently, the hybrid approach mentioned above gave $\Gamma = 386 \pm 5$ s$^{-1}$.

A concern in applying EFT($\pi$) to the $\mu^-d$ capture is that the energy transfer into the hadronic system might be too large to apply EFT($\pi$). However, as shown in Ref. and also in this calculation, the contribution to the total rate from high-energy neutrons is small, and it is possible to impose a neutron energy cut to isolate the low-energy ($\leq 10 \sim 20$ MeV) neutron events without significantly increasing the statistical errors.

Effective field theory is useful when low and high energy scales in the problem are widely separated. For low-energy processes, short-distance physics can be taken into account by local operators in an effective lagrangian involving only low-energy degrees of freedom. For $\nu(\bar{\nu}) - d$ scattering with neutrino energy below 20 MeV and $\mu^-d$ capture with small final-state neutron energy, the pion and other meson exchanges are not dynamical degrees of freedom, and their physics can be captured by contact interactions involving nucleons and the external currents. To make predictions with controlled precision, calculations are done with the perturbative expansion parameter $Q \equiv (1/a_{nn}^{(1S_0)}, \gamma, p)/\Lambda$, which is the ratio of light to heavy scales. The light scales include the inverse $S$-wave neutron-neutron scattering length $1/a_{nn}^{(1S_0)} = -10.6$ MeV in the $^1S_0$ channel, the deuteron binding momentum $\gamma = 45.7$ MeV in the $^3S_1$ channel, and typical nucleon momentum $p$ in the system. The heavy scale $\Lambda$ is...
set by the pion mass $m_\pi$. This EFT($\phi$) (see e.g. [17]) and its dibaryon version [18, 19, 20] have been applied successfully to many processes involving the deuteron, including electro-magnetic processes such as Compton scattering $\gamma d \rightarrow \gamma d$ [21, 22], $np \rightarrow d\gamma$ relevant to the big-bang nucleosynthesis [23, 24], weak processes such as $\nu d$ reactions for SNO physics [4], the solar $pp$ fusion process [6, 7], and parity violating observables [14, 15]. For reviews on three-body systems, see [26].

The effective Lagrangian for the CC weak interaction is given by $\mathcal{L}^{CC} = -\frac{G_F}{\sqrt{2}} F_{\nu d} \bar{u}_d J^{\mu} \nu / \sqrt{2} + \text{h.c.}$, where $G_F = 1.166 \times 10^{-5} \text{GeV}^{-2}$ is the Fermi coupling constant and $\omega = 1.024$ takes into account the inner electroweak radiative correction [27]. $F^{\mu} = \tau^\mu\gamma^\mu(1 - \gamma_5)\mu$ is the leptonic current. The quark current $J^{\mu}_q = \bar{V}_{\sigma}^\mu - A^{\mu}_q = (V^{\mu}_q - A^{\mu}_q) - i(V^{\mu}_q - A^{\mu}_q)$ contains both vector and axial-vector interactions, where the superscripts 1 and 2 are the isospin indices. At the scale relevant to nuclear physics, the quark current need be matched to a hadronic current which in general contains one-nucleon, two-nucleon, etc., operators.

Up to the order of our interest, the one-nucleon isovector vector and axial vector currents are

$$V^{0,a}_{(1)} = \frac{N^a}{2} \left(1 + \frac{1}{6}(\tau^2_{c})_{c=1} \vec{\nabla} \right) N c$$

$$V^{k,a}_{(1)} = \frac{i}{2M_N} N^a \vec{\nabla} \cdot \vec{\tau}^{a}_{c} N - \frac{\mu^{(1)}}{M_N} \epsilon_{kij} N^a \nabla_i \nabla_j \vec{\tau}^{a}_{c}$$

$$A^{0,a}_{(1)} = \frac{ig_s}{2M_N} N^a \vec{\nabla} \cdot \vec{\tau}^{a}_{c} N$$

$$A^{k,a}_{(1)} = g_A N^a \vec{\nabla} \cdot \vec{\tau}^{a}_{c} N$$

where $\vec{\nabla} = \vec{\nabla} + \vec{\nabla}_d$, $\vec{\nabla}_d = \vec{\nabla}_0 + \vec{\nabla}_d$, and $\vec{\nabla} = \vec{\nabla} - \vec{\nabla}$. The superscript $a$ is the isospin index, $\mu^{(1)} = (\mu_p - \mu_n) / 2 = 2.353$, is the isovector magnetic moment. Isovector Dirac charge radius $(\langle \tau^2 \rangle_{c=1} - \langle \tau^2 \rangle_{c} = 0.873 \text{ fm}^2$, and the isovector axial-charge radius $(\langle \tau^2 \rangle_{A=1} \approx 0.45 \text{ fm}^2$. We have neglected terms of order $p^2 / M_N^2$ or even $\mu^{(1)} p^2 / M_N^2$.

The pseudoscalar form factor is, to a good approximation, dominated by the pion-pole $G_p(q^2) = \frac{4M_p^2}{}$ whose $q^2$ dependence will not be expanded because the momentum transfer $|q|$ is of order muon mass $M_{\mu}$ with low energy final state neutrons. The $G_p$ contribution to the axial current is counted of order $Q$.

The lowest dimensional two-nucleon isovector currents, in the dibaryon version of EFT($\phi$), relevant to the $\mu^-d$ capture process are

$$V^{k,a}_{(2)} = \frac{L^{db}_{1}}{M_N \sqrt{r^{(3S_1)}_{1} r^{(1S_0)}_{nn}}} \epsilon_{kij} t_i \nabla_j s_a + \text{h.c.},$$

$$A^{k,a}_{(2)} = \frac{L^{db}_{1A}}{M_N \sqrt{r^{(3S_1)}_{1} r^{(1S_0)}_{nn}}} (t_i^{A} s_a + \text{h.c.})$$

where $t_i$ and $s_a$ are dibaryon fields for the two-nucleon $3S_1$ and $1S_0$ states, respectively. The second term in $A^{k,a}_{(2)}$ is induced by the $G_p$ term in the one-nucleon current. $r^{(3S_1)}_{1} = 1.764 \text{ fm}$ and $r^{(1S_0)}_{nn} = 2.8 \text{ fm}$ are the effective ranges in triplet and two-neutron singlet channels, respectively. $L^{db}_{1}$ and $L^{db}_{1A}$ are coupling constants in dibaryon formalism. The vector current is NLO and its coupling $L^{db}_{1} = -4.08 \text{ fm}$ has been determined by the rate of $n + p \rightarrow d + \gamma$ near threshold. The axial current is NLO, and its coupling $L^{db}_{1A}$ is proportional to the renormalization-scale-$\mu$-independent $L_{1A}$ in Ref. [3] as $L^{db}_{1A} = \frac{M_{\mu}}{2 \pi} L_{1A}$, through which $L^{db}_{1A}$ is related to the $\mu$-dependent $L_{1A}(\mu)$ in Ref. [3]. The numerical relation between $L^{db}_{1A}$ and $L_{1A}(\mu)$ is $L^{db}_{1A} = L_{1A} = -13.8 + 0.28 L_{1A}(M_{\pi})$, where $L_{1A}(M_{\pi})$ is in units of $\text{fm}^3$ and has a natural size $\sim 6 \text{ fm}^3$.

The $\mu^-d$ atom has a ground state with a hyperfine structure, corresponding to the total angular momentum $F = 3/2$ and $F = 1/2$. The $\mu^-d$ capture process is known to take place almost uniquely from the doublet $F = 1/2$ state. The differential capture rate for muon and deuteron in their specific polarization states can be written in terms of leptonic tensor $\mu^{\nu\sigma}$ and hadronic tensor $W_{\mu\nu}$ as

$$d\Gamma(S, \hat{\epsilon}) \frac{dE d\Omega}{dE d\Omega} = \frac{\omega_i G^{2}_F |V_{ud}|^2 |\psi(0)|^2}{32\pi^2 M_{\mu}(1 + \frac{M_{d}}{M_{\mu}})^3} \mu^{\nu\sigma}(S) W_{\mu\nu}(\hat{\epsilon}),$$

where $|\psi(0)|^2 = \frac{1}{\pi} \left(\alpha_{em} \frac{M_{\mu} M_{d}}{M_{\mu} + M_{d}}\right)^3$ is the 1S-state wave-function-at-origin-squared, $E$ is the outgoing neutrino energy, and $M_d$ is the mass of the deuteron.

The capture rate depends on the polarization vector of the muon $S_{\mu}$ and the deuteron polarization vector $\hat{\epsilon}$. The leptonic tensor is given by $\mu^{\nu\sigma}(S) = 4(k^\mu k^\nu + k^\mu k^\nu - k^\mu g^{\nu\sigma} + i e \mu^{\nu\sigma} k p k_d) - 4 M_{\mu}(S^\nu k^\mu + k^\mu S^\nu - k^\cdot S g^{\mu\nu} + i e \mu^{\nu\sigma} S^\nu k_d)$, where $k = (M_{\mu}, 0)$ and $k' = (E, \vec{k}^{'})$, with $E = |k'|$, are the four-momenta of initial muon and final $\nu_{\mu}$, respectively. The hadronic tensor is

$$W_{\mu\nu}(\hat{\epsilon}) = \frac{1}{\pi} \text{Im} \left[ \int d^4x e^{i\xi x} T(d|J_{\mu}^+(x)J_{\nu}^-(0)|d) \right],$$

where $q_{\mu} = k_{\mu} - k_{\mu}'$, and $|d\rangle \equiv |d(P, \hat{\epsilon})\rangle$ is the deuteron state with momentum $P = (M_d, \vec{0})$ and polarization $\hat{\epsilon}$. 


The diagrams contributing to $W_{\mu\nu}(\hat{\xi})$ up to N²LO are shown in Fig. 1. A straightforward calculation finds

\[ \frac{d\Gamma}{dE} = \frac{G_F^2 |V_{ud}|^2 E^2 |\psi(0)|^2}{2\pi(1 + \frac{M_N}{M_\pi})} \left( 1 - \frac{1}{1 - \varepsilon \gamma r(\xi_{1f})} \right) \left( F_1 - F_2 \right) \]

\[
\left( 3G_V^2 - 2(G_V - G_A)^2 - \frac{4g_\pi (1 - g_A) M_\mu E}{3(M_N^2 - q^2)} \right) + \varepsilon \kappa(1) \frac{8(1 - g_A) E}{3M_N} + \left( F_1 - F_2 \right) \frac{2}{3} F_{1a} (9G_A^2 - \frac{6g_\pi^2 M_\mu E}{M_N^2 - q^2} + \frac{2}{3} g_A^2 E^4 )
\]

\[
+ (10F_1 - F_2 + 8F_{4b}) \varepsilon \kappa(1)^2 E^2 \frac{8}{3M_N^2 - q^2} + \frac{2}{3} \varepsilon \kappa(1)^3 E^3 \frac{8g_\pi^2}{3M_N(M_N^2 - q^2)} + F_5 \varepsilon^2 E \left( \frac{14}{3} g_A + 16 \right)
\]

In the energy region where the EFT is valid, it is possible to define and measure the integrated capture rate up to a threshold $p/m_\pi$ to estimate the accuracy of the expansion. For example, it is well-known that in the nucleon-nucleon scattering, the effective theory without pion works rather well at the nucleon momentum on the order of 100 MeV, a value close to the pion mass. This can also be seen in the present calculation because the NLO result at large neutron momentum does not completely modify the leading-order result, whereas the naive power counting would indicate otherwise. Fortunately, for muon capture, the most of the events occur when the neutron momentum is about half of the muon mass, that is, about 50 MeV.

In Fig. 2 we show the differential rate $d\Gamma/dE_{nn}$ in terms of the relative motion energy $E_{nn} = 2(\sqrt{M_N^2 + p^2} - M_N)$ of two final-state neutrons in the region where the EFT(?) calculation is most reliable. It is clear from the figure that the differential rate in the energy region $E_{nn} \geq 10$ MeV is very small, and is negligible for $E_{nn} \geq 15$ MeV. By comparing the results of LO, NLO, and N²LO, we find good convergence of the expansion.

In the case that a neutron energy cut can be imposed on experimental data [14], it is possible to define and measure the integrated capture rate up to a threshold.
energy $E_{nn}^{th}$

$$\Gamma(E_{nn}^{th}) \equiv - \int_0^{E_{nn}^{th}} \frac{d\Gamma}{dE_{nn}} dE_{nn}. \quad (9)$$

The result up to 30 MeV is shown in Fig. 3. In the whole energy region, the NLO contribution is less than 10% of the LO contribution, while the N$^2$LO contribution is less than 1%. This small size of N$^2$LO contribution is accidental and does not happen for unpolarized and $F = 3/2$ rates. For example, the unpolarized rate has the expansion (for $L_{1,A} = 6$ fm$^3$)

$$\Gamma_{unpol} = \Gamma_{unpol}^{LO} (1 + 5.3\% + 4.9\%). \quad (10)$$

where the NLO correction is abnormally small, and NNLO is of the normal size. A similar expansion is obtained for $L_{1,A} = -6$ fm$^3$:

$$\Gamma_{unpol} = \Gamma_{unpol}^{LO} (1 + 10.9\% + 5.2\%) \quad (11)$$

which shows a nice convergence pattern. Based on this trend, we assign a 2-3% correction at NNNLO, corresponding to an error in $L_{1,A} \sim 2$ fm$^3$. This is consistent with the naive estimation of 3% if the small expansion parameter is 1/3. A calculation shows the N$^3$LO final state P-wave re-scattering contributes only $\sim 1\%$. Furthermore, the result is insensitive to the uncertainty in $a_{nn}^{(iS_0)}$. Choosing $L_{1,A} = 5.6$ fm$^3$, the energy dependence of our result matches the previous hybrid calculation very well [10].

To extract $L_{1,A}$ from experimental data, it is useful to provide the dependence of the rate on $L_{1,A}$

$$\Gamma(E_{nn}^{th}) = a(E_{nn}^{th}) + b(E_{nn}^{th}) L_{1,A}, \quad (12)$$

where $L_{1,A}$ is in unit of fm$^3$. The energy-cut dependent functions $a(E_{nn}^{th})$ and $b(E_{nn}^{th})$ for a set of $E_{nn}^{th}$s are listed in the Table 1, from which we observe that, for the whole range of $E_{nn}^{th}$, the size of $b(E_{nn}^{th})$ is about 1.3 $\sim$ 1.5% of the size of $a(E_{nn}^{th})$. This shows how an error in capture rate is translated into an uncertainty of $L_{1,A}$.

Table 1: Coefficients functions $a(E_{nn}^{th})$ and $b(E_{nn}^{th})$ for specific values of two-neutron relative energy $E_{nn}^{th}$ from the EFT(#) calculation.

| $E_{nn}^{th}$ (MeV) | 5.0 | 10.0 | 15.0 | 20.0 |
|---------------------|-----|-----|-----|-----|
| $a(E_{nn}^{th})$ (s$^{-1}$) | 3.3 | 4.2 | 4.7 | 4.9 |
| $b(E_{nn}^{th})$ (s$^{-1}$fm$^{-3}$) | 239.2 | 308.0 | 332.0 | 342.3 |

In summary, we calculated the $\mu^-$d capture rate using EFT(#) to N$^2$LO. The major goal is to fix the two-nucleon isovector axial coupling constant $L_{1,A}$ from future precision experimental data. An experimental result on the integrated rate up to some neutron energy $E_{nn}^{th}$ with a 2% error should be able to, through comparison with our calculation with theoretical error 2-3%, fix the $L_{1,A}$ with error $\pm 2.0$ fm$^3$. This in turn allows us to determine the neutrino deuteron breakup cross section and the pp fusion rate in the sum to 2-3%.

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[1] Q.R. Ahmad et al., Phys. Rev. Lett. 87, 071301 (2001); 89, 011301 (2002); 89, 011302 (2002).
[2] J.N. Bahcall, A.M. Serenelli, and S. Basu, Astrophys. J. 621, L85 (2005).
[3] M.N. Butler and J.W. Chen, Nucl. Phys. A675, 575 (2000); M.N. Butler, J.W. Chen, and X. Kong, Phys. Rev. C63, 035501 (2001).
[4] S. Ying, W.C. Haxton and E.M. Henley, Phys. Rev. C45, 1982 (1992); Phys. Rev. D40, 3211 (1989).
[5] S. Nakamura, T. Sato, V. Gudkov and K. Kubodera, Phys. Rev. C63, 034617 (2001).
[6] X. Kong and F. Ravndal, Nucl. Phys. A656, 421 (1999); A665, 137 (2000); Phys. Lett. B470, 1 (1999); Phys. Rev. C64, 044002 (2001).
[7] M. Butler and J.W. Chen, Phys. Lett. B520, 87 (2001).
[8] M. Butler, J.W. Chen, and P. Vogel, Phys. Lett. B549, 26 (2002).
[9] J.W. Chen, K.M. Heeger, and R.G.H. Robertson, Phys. Rev. C67, 025801 (2003).
[10] R. Schiavilla et al., Phys. Rev. C58, 1263 (1998).
[11] T.-S. Park et al., nucl-th/0106025, nucl-th/0107012.
[12] G. Bardin et al., Nucl. Phys. A453, 591 (1986).
[13] M. Cargelli et al., in: M. Morita, H. Ejiri, H. Ohtsubo, T. Sato (Eds.), Proceedings of the XXIII Yamada Conference on Nuclear Weak Processes and Nuclear Structure, Osaka, 1989, World Scientific, Singapore, P. 115 (1989).
[14] P. Kammel et al., nucl-ex/0202011, nucl-ex/0304019.
[15] N. Tatara, Y. Kohyama, K. Kubodera, Phys. Rev. C42, 1694 (1990).
[16] S. Ando, T.S. Park, K. Kubodera and F. Myhrer, Phys. Lett. B533, 25 (2002).
[17] J.W. Chen, G. Rupak, and M.J. Savage, Nucl. Phys. A653, 386 (1999).
[18] D. B. Kaplan, Nucl. Phys. B494, 471 (1997).
[19] S. R. Beane and M. J. Savage, Nucl. Phys. A694, 511, (2001).
[20] S. Ando and C. H. Hyun, Phys. Rev. C72, 014008 (2005).
[21] H.W. Griesshammer and G. Rupak, Phys. Lett. B529, 57 (2002).
[22] J.W. Chen, X. Ji, and Y. Li, Phys. Lett. B620, 33 (2005);
[23] S. Ando, T.S. Park, K. Kubodera and F. Myhrer, Phys. Lett. B533, 25 (2002).
[24] J.W. Chen and M.J. Savage, Phys. Rev. C71, 044321 (2005).
[25] G. Rupak, Nucl. Phys. A678, 405 (2000).
[26] M.J. Savage, Nucl. Phys. A695, 365 (2001).
[27] A. Sirlin, Rev. Mod. Phys. 50, 573 (1978).