Critical Fluctuations in Beam-Plasma Systems and Solar Type III Radio Bursts

G. Thejappa\(^1\), R. J. MacDowall\(^2\)

\(^1\) Department of Astronomy, University of Maryland, College Park, MD 20742, USA
\(^2\) NASA/Goddard Space Flight Center, Greenbelt MD 20771, USA

Abstract. It is shown that the Langmuir waves are excited similar to critical fluctuations during phase transitions when the negative absorption due to electron beam traveling radially outward in the solar atmosphere is balanced by the positive absorption due to collisions in the corona and due to scattering on electron density inhomogeneities in the interplanetary medium. The effective temperature of the Langmuir fluctuations range from \(10^{11}\) to \(10^{13}\) K, explaining the majority of the type III bursts. The Rayleigh scattering and direct coupling due to density gradient as well as due to density inhomogeneities are discussed in the context of fundamental radiation and the combination scattering for second harmonic. The number density of electrons in type III beams is estimated and compared with observations. It is also shown that the stabilization of type III beams is achieved automatically since the instability does not develop in the case of critical fluctuations.

1. Introduction

For four decades the solar type III radio burst phenomenon has attracted the attention of both solar radio astronomers and space plasma physicists because of the intriguing complexity revealed by observations and theory. The direct detection of electron beams [17, 18] and Langmuir waves [10, 11] in association with type III bursts together with tracking of the type III burst sources in the interplanetary medium [4, 5] confirmed that electrons and Langmuir waves are associated with type III bursts [7, 8]. However the dynamics of type III electron beams and their interaction with the ambient plasma through the excited Langmuir waves is an unsolved problem. In order that the electron beams don’t loose their energy by resonantly interacting with Langmuir waves, various nonlinear processes such as the induced scattering of Langmuir waves off the background ions in weak turbulence regime [14] and oscillating two-stream instability in strong turbulence regime [22] were invoked. However the observed electron density inhomogeneities [2, 3] do not allow Langmuir waves to grow to very high intensities as pointed out by several authors [20]. As far as the conversion of Langmuir wave energy into electromagnetic energy is concerned, there is no consensus either for the fundamental...
or for the second harmonic emission.

In the present paper we investigate the phenomenon of critical fluctuations near the boundary of the instability which is analogous to the anomalously growing fluctuations near the phase transition points called critical points [12, 23]. We show that the problems concerning the beam stabilization [25] and the estimation of the number density of beam electrons are automatically solved if type III bursts are assumed as manifestations of critical fluctuations. We also examine different physical processes involved in conversion of Langmuir into electromagnetic waves at the fundamental and second harmonic plasma frequencies.

2. Critical Fluctuations in the Beam-Plasma Systems

The effective temperature $T_{\text{eff}}$ of Langmuir waves excited by an electron beam propagating radially outward in the solar atmosphere is determined in the linear approximation by the transfer equation:

$$\frac{dT_{\text{eff}}}{dl} = \alpha - \mu T_{\text{eff}}, \quad (1)$$

where $\alpha$ and $\mu$ are the emission and absorption coefficients respectively in a layer of thickness $dl$. The emission coefficient $\alpha$ is connected to the emissivity $\alpha_\omega$ as:

$$\alpha = (2\pi)^3 \alpha_\omega \kappa k^2, \quad (2)$$

where $\kappa$ is the Boltzmann constant and $k$ is the wave number. The emissivity $\alpha_\omega$ is given by [1]:

$$\alpha_\omega = \frac{\omega_{\text{pe}}^2 \omega_{\text{ph}}^2 m_e N_b}{6(2\pi)^{5/2} \Delta v_b v_{T_e} N_e} \exp\left(-\frac{(v_{\text{ph}} - v_b \cos\theta)^2}{2\Delta v_b^2}\right), \quad (3)$$

where $m_e$ is the electron mass; $\omega_{\text{pe}}, N_e$ and $v_{T_e}$ are the plasma frequency, density and thermal velocity of electrons in the ambient plasma; $N_b$, $v_b$ and $\Delta v_b$ are the density, velocity and velocity spread of the electron beam; $\omega$ and $v_{\text{ph}}$ are the frequency and the phase velocity of the excited Langmuir waves. The absorption coefficient $\mu$ is determined by the absorption due to collisions ($\mu_c$), effective absorption due to scattering on electron density inhomogeneities ($\mu_{\text{sca}}$), Landau damping by the ambient electrons ($\mu_L$) and negative absorption due to electron beam ($\mu_b$):

$$\mu = \mu_c + \mu_{\text{sca}} + \mu_L + \mu_b. \quad (4)$$

The absorption due to collisions is given by:

$$\mu_c = \frac{2v_{\text{ph}} v_e}{3v_{T_e}^2}, \quad (5)$$

where $\nu_e$ is the effective electron-ion collision frequency determined by the temperature ($T_e$) and density ($N_e$) as:

$$\nu_e = \frac{5.5 N_e}{T_e^{3/2}} \ln\left(220 \frac{T_e}{N_e^{1/3}}\right). \quad (6)$$
The effective absorption coefficient due to scattering by density inhomogeneities ($\mu_{sca}$) is given by:

$$\mu_{sca} = \frac{2v_{ph}}{3v_T^2_e} \nu_{sca},$$

(7)

where the effective damping $\nu_{sca}$ is given by the diffusion coefficient $D_\theta$:

$$\nu_{sca} = D_\theta (\Delta \theta)^2,$$

(8)

where ($\Delta \theta$) is the mean angle of deflection given approximately as:

$$(\Delta \theta)^2 \simeq \left(\frac{\Delta v_b}{v_b}\right)^2$$

(9)

and $D_\theta$ is given by [21, 20]:

$$D_\theta = \frac{\pi \omega}{12} \frac{\omega pe}{k^3} \frac{q}{\bar{q}^2} \frac{\delta n^2}{n^2}.$$  

(10)

Here $\lambda_D$ is the Debye length $\bar{q}$ is the typical wave number associated with the fluctuations and $\langle \delta n^2 / n^2 \rangle$ is the relative level of density fluctuations. The absorption due to Landau damping by the ambient electrons is:

$$\mu_L = \sqrt{\frac{\pi}{18}} \frac{\omega}{k^2 v_T^2_e} \exp(-\frac{\omega^2}{2v_T^2_e k^2}),$$

(11)

In the corona, the absorption due to Landau damping can also be important due to smoothly varying nature of the density distribution as shown by [28, 27, 29]. However in the interplanetary medium it is negligibly small. In the present study we neglect it both in the corona as well as in the interplanetary medium. And also the relative level of density fluctuations increases with the radial distance in the solar atmosphere and hence we consider that in the corona the main contribution to the absorption is due to collisional damping whereas the absorption due to scattering of Langmuir waves by the electron density inhomogeneities plays the dominant role in the interplanetary medium.

The absorption due to beam ($\mu_b$) is given by:

$$\mu_b = \sqrt{\frac{\pi}{18}} \frac{\omega}{v_T^2_e} \frac{\omega pe}{\Delta v_b^2} \frac{N_b}{N_e} \left(v_{ph} - v_b \cos \theta\right) \exp\left(-\frac{(v_{ph} - v_b \cos \theta)^2}{2\Delta v_b^2}\right).$$

(12)

The value $\mu_b$ is positive for $v_{ph} > v_b \cos \theta$, whereas it is negative in the region where $v_{ph} < v_b \cos \theta$. Therefore for $(v_{ph} - v_b \cos \theta) = -\Delta v_b$, and for marginally stable condition, where the absorption $\mu$ tends to zero, we can obtain the ratio $\frac{N_b}{N_e}$ in the corona by equating $\mu_b$ to $\mu_c$, and in the interplanetary medium by equating $\mu_{sca}$ with $\mu_b$, which can be written as:

$$\frac{N_b}{N_e} = \sqrt{\frac{13.2}{\pi}} \frac{\omega \Delta v_b^2}{\omega pe v_T^2} \nu,$$

(13)

where $\nu = \nu_c$ in the corona and $\nu = \nu_{sca}$ in the interplanetary medium. The solution of equation (1) in the marginally stable condition i.e., when the negative absorption due to
electron beam is approximately balanced by the damping due to collisions in the corona or to scattering on electron density fluctuations in the interplanetary medium, i.e., optically thin case $\mu L \leq 1$ can be written as:

$$T_{eff} = \alpha L.$$  \hspace{1cm} (14)

Here $L$ is the thickness of the layer in which the Langmuir waves exist which is determined by:

$$L = \frac{N_e v_{Ti}}{2|\nabla N_e| v_{ph}},$$  \hspace{1cm} (15)

where $v_{Ti}$ is the ion thermal velocity. In the corona approximately for 100 MHz plasma frequency layer the thickness $L$ can be approximated as $L \approx 10^9$ cm whereas in the interplanetary medium for plasma frequency layer of 13 kHz it can be approximated as $L \approx 4.5 \times 10^{10}$ cm.

From equation (13) for $\Delta v_b \approx v_b \approx 10^{10}$ cm, $\omega \approx \omega_{pe}$ and $\nu = \nu_c$, we obtain $\frac{N_b}{N_e} \approx 2.63 \frac{v_c}{\omega_{pe}}$. In the interplanetary medium, since it is difficult to measure the effective damping due to scattering on density fluctuations, we evaluate it by assuming that the values for the beam density measured by [18] simultaneously with type III bursts as the critical density. Therefore for the values reported by [18], where $\frac{N_b}{N_e} \approx 3.5 \times 10^{-5}$, $v_b \approx 3.5 \times 10^9$ cm/s, we obtain $\nu_{sca} \approx 0.15$, the effective damping rate ($\nu_{sca}$) is calculated as $\nu_{sca} \approx 0.6 \times 10^{-3} \omega_{pe}$, i.e., for $f_{pe} \approx 13$ kHz, we obtain $\nu_{sca} \approx 48.4$ s$^{-1}$. Therefore we obtain the ratio of critical beam density to electron density of ambient plasma ($\frac{N_b}{N_e}$)$_{cri} = 5.23 \times 10^{-8}$ for $f_{pe} \approx 100$ MHz and $v_c \approx 12.5$ s$^{-1}$. The effective temperature of Langmuir waves emitted spontaneously by such beams as critical fluctuations can be obtained by using equations (2), (3) and (14). We obtain $T_{eff} \approx 10^{12}$ K for $(v_{ph} - v_b \cos \theta) \approx -\Delta v_b$, $v_{ph} \approx v_b \approx \Delta v_b \approx c/3$, $v_{Te} = 3.89 \times 10^8$ cm/s, $f \approx f_{pe} \approx 100$ MHz and $L \approx 10^9$ cm. In the case of interplanetary type III bursts, for $\frac{N_b}{N_e} \approx 3.5 \times 10^{-5}$; $v_{ph} \approx v_b \cos \theta \approx -\Delta v_b$; $v_{ph} \approx 3.5 \times 10^9$ cm/s; $\Delta v_b \approx 0.15$; $v_{Te} \approx 1.74 \times 10^8$ cm/s; $f \approx f_{pe} \approx 13$ kHz and $L \approx 4.5 \times 10^{10}$ cm, we obtain $T_{eff} \approx 2.1 \times 10^{12}$ K.

[13] estimated the maximum value of the energy density of Langmuir waves due to critical fluctuations when the damping approaches zero as $\sqrt{g}$ by including the damping due to nonlinear interactions, where $g$ is the plasma parameter defined as $g = \frac{\Lambda}{N_e \lambda_D}$. Thus approximated effective temperature is much higher than the effective temperature obtained by assuming that critical fluctuations are manifestations of spontaneous emission.

3. Conversion of Langmuir Fluctuations into Electromagnetic Waves

Various nonlinear mechanisms were proposed for conversion of Langmuir waves into electromagnetic waves at the fundamental plasma frequency $f_{pe}$. These are (1) spontaneous and induced scattering of Langmuir waves by thermal ions [7, 24, 19, 30, 31], (2) scattering of Langmuir waves by low frequency waves, such as ion-acoustic and whistler waves [26, 19], (3) strong turbulence processes [9, 16] and (4) gradient coupling of Langmuir waves with electromagnetic waves [6, 34, 35, 15]. In the present case, the energy density of critical Langmuir fluctuations is much less than the threshold
energy densities required for the induced scattering on thermal ions, decay of Langmuir waves into electromagnetic and low frequency waves and strong turbulence. Therefore only spontaneous scattering of Langmuir waves on thermal ions and direct coupling of Langmuir waves with electromagnetic waves may be important for conversion process at fundamental plasma frequency.

The brightness temperature of electromagnetic radiation $T_B$ emitted by Langmuir waves of effective temperature $T_{eff}$ is given by [32]:

$$T_B \simeq \frac{c^2 \Omega_L}{3v_T^2 \Omega_T} T_L Q \exp (-\tau), \quad (16)$$

where $\Omega_L$ and $\Omega_T$ are the solid angles of Langmuir and electromagnetic waves respectively, $\tau$ is the optical depth from the source to the observer and $Q$ is the efficiency of conversion from Langmuir to electromagnetic waves. The efficiency of conversion due to Rayleigh scattering is [7]:

$$Q = \frac{\omega_{pe}^2}{4\pi c^2} \left( \frac{3c}{2\omega L_n} \right)^2,$$

where $L_n$ is the scale length of the density variation which is $\simeq 1.7 \times 10^{10}$ cm and $1.5 \times 10^{13}$ cm respectively making $Q$ as $5.7 \times 10^{-7}$ and $6.24 \times 10^{-7}$ at 100 MHz and 13 kHz respectively. Therefore we believe that the direct coupling due to the gradient in the electron density in the ambient plasma may be the appropriate conversion mechanism for generation of electromagnetic waves from the Langmuir waves. By using the equation (16) for the parameters assumed in the present paper for 100 MHz and 13 kHz, we obtain $1.72 \times 10^8$ K and $7.3 \times 10^5$ K respectively for the effective temperatures of the Langmuir critical fluctuations.

As far as the electromagnetic emission at $2\omega_{pe}$ is concerned, if the generation mechanism is assumed to be the merging of beam excited Langmuir waves with back-scattered secondary Langmuir spectrum i.e., the modified idea of [7], the absorption due to decay of electromagnetic wave into two plasma waves is the most dominant absorption process. The optical depth due to such a decay is [33]:

$$\mu L = \frac{2c^2 \omega_{pe}^2 \kappa T_{eff} L}{15 \sqrt{3} m_e^2 c^3 v_T^2 v_{ph}}.$$ 

For the parameters assumed for 100 MHz and 13 kHz $\mu L \ll 1$ and the brightness temperature $T_B$ is given by:

$$T_B \simeq \alpha_1 L, \quad (18)$$

where the spontaneous scattering coefficient in this case is given by [33]:

$$\alpha_1 \simeq \frac{2}{15 \sqrt{3} m_e^2 c^3 v_T^2 v_{ph} L}.$$ 

Therefore for $T_{eff} \simeq 10^{12} K$ we obtain $T_B \simeq 3 \times 10^{10}$ K and $3.1 \times 10^5$ K at 100 MHz and 13 kHz respectively.
4. Discussion and Conclusions

Since the velocity of the electron beams responsible for type III bursts is known directly by measuring the drift rate of type III bursts in the dynamic spectrum, the ratio of beam to ambient electron density is an unknown quantity in type III phenomena. The critical beam density derived in the present paper where the phenomenon of critical fluctuations starts playing the dominant role in generation of Langmuir waves may give the required beam densities. Higher densities where the instability is playing the role in excitation of Langmuir fluctuations may be necessary to explain the brightest type III bursts. In the case of the interplanetary medium, the exact knowledge of electron density fluctuations is necessary to estimate the effective damping due to scattering of Langmuir waves on fluctuations. The problem of beam stabilization is not important in the case of critical fluctuations since the effective growth is almost negligible.

As far as conversion mechanism for the fundamental emission is concerned, the direct coupling between Langmuir and electromagnetic waves due density gradient in the ambient electron distribution appears to be the most favorable mechanism both in the corona as well as in interplanetary medium. The efficiency of conversion due to Rayleigh scattering is comparable to that due to direct coupling in the corona whereas it is much smaller at lower frequencies. But the efficiency of conversion at 100 MHz due to Rayleigh scattering is comparable to direct coupling. Therefore we believe that direct coupling may be the dominant process in converting the Langmuir waves into electromagnetic waves both in the corona as well as in the interplanetary medium.

In the case of second harmonic emission, the combination scattering may account for the emission at higher frequencies whereas it is too inefficient at lower frequencies thus requiring to invoke other mechanisms. Therefore we conclude that: (1) the phenomenon of critical fluctuations in the beam-plasma system may be very important for type III bursts, (2) the ratio $N_b/N_e$ in weak type III bursts can be determined by the collisional damping in the corona, and by damping due to scattering on density fluctuations in the interplanetary medium, (3) since the instability does not develop in the case of critical fluctuations, the question of beam stabilization does not arise in the present model, (4) the direct coupling between Langmuir and electromagnetic waves appears to play a dominant role in converting the Langmuir energy into electromagnetic energy at fundamental plasma frequency and (5) even though the combination scattering is capable of explaining the emission at twice the plasma frequency, it is too inefficient to explain the type III bursts at lower frequencies.

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5. References

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