Weakly localised bosons

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Abstract

In this paper, we argue that, in 2-d, the weak localisation of bosons, which occurs on the insulating side of the superconductor-insulator transition, is characterised by $\rho \sim \ln(1/T)$, as compared to $\sigma \sim \ln T$ for fermions. Such an unconventional behaviour is tied to the diffusion pole of the delocalised (uncondensed) vortices.

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Superconductivity in low-dimensional systems has been of considerable interest over the past few decades. This has received a strong boost with the discovery of high-$T_c$ superconductivity. High-$T_c$ superconductors are layered materials and exhibit a superconductor-insulator transition at low doping. In an attempt to understand the two-dimensional aspect of this new phenomenon, an appreciable amount of effort has been focussed on the low-temperature behaviour of thin films made out of the low-$T_c$ superconductors[1]. In these experiments, one observes that the film undergoes a transition from a superconductor to an insulator at $T \to 0$, as one tunes the thickness, disorder or the magnetic field[1]. There are two distinctively different schools of thought, for explaining this set of observations. One school [2] believes that the insulating behaviour occurs due to the localisation of charged bosons, viz. Cooper pairs. Whereas, the other school[3] believes that during the said transition, Cooper pairs break up into fermions which naturally localise in 2-d and cause insulating behaviour. Each school is supported by a corresponding theory. Cooper pair localisation is supported by Fisher’s theory[4], whereas the fermion localisation picture is interpreted in terms of Fukuyama’s theory [5]. In the latter scenario, Cooper pairing is hampered as a result of suppression of density of states due to enhanced coulomb repulsion. Although Fisher’s theory seemed to work in many cases[6], there has been unflinching evidence[3,7] that Fukuyama’s theory is right for certain systems. Thus each mechanism probably has an appropriate class of materials for which it is relevant.

So far, there has been no straightforward way to find which theory applies in a given system. In this paper, we suggest a simple way to resolve this debate. In particular, we argue that, in 2-d, weak localisation of bosons implies $\rho \sim ln(1/T)$, whereas for fermion localization one expects $\sigma \sim lnT$ [8]. The closest experimental observation of this non-fermionic effect seems to be the experiment by Gerber on granular Pb films[9]. A similar behaviour has been reported in Ref.[6]. It is interesting to note that such a $\rho \sim ln(1/T)$ dependence has been observed in the high-$T_c$ materials[10] as well, where bosonic models have been suggested to be relevant[11]. In what follows, most of our discussion will be restricted to two dimensional systems at finite temperatures, the focus being exclusively on
Why not $\sigma \sim \ln T$ for bosons? $\sigma \sim \ln T$ behaviour is profuse in 2-d electronic systems. Many of these logarithmic tendencies are associated with the critical properties of electrons, viz. the lower critical dimension of the electronic system is two. By contrast, in a bosonic system, the lower critical dimension is one[4]. There is a further difference between the two kinds of charge carriers. Let us consider, for example, a system of non-interacting electrons. Here $\sigma \sim \ln T$ follows from one-parameter scaling[8]. It has been argued that [12] the scaled conductance $g = G/(e^2/h)$ for such systems depends on a single parameter $\Delta E/\delta W$, where $\Delta E =$ bandwidth and $\delta W = (N_0 L^d)^{-1}$ is the scatter in the energy levels in a system of size $L$ in $d$-dimensions. $N_0$ is the density of states at the Fermi level. Let us repeat the argument[12] here for the sake of completeness. In this picture, one considers constructing a sample of size $(2L)^d$, in $d$ dimensions, out of $2^d$ samples of size $L^d$. A given eigenstate of the larger sample is a linear combination of the eigenstates of the smaller samples. What really matters here is the amount of admixture among the (old) eigenstates, which in turn is controlled by the overlap integral and energy denominator. The measure of the energy denominator is $\delta W$. To obtain an estimate for the overlap, one considers repeating a given $L^d$ sample in one direction, subject to appropriate boundary conditions. Each eigenstate broadens to form a band, and the bandwidth $\Delta E$ is an estimate of the overlap integral. If the wave function is localised, $\Delta E$, and hence, $\Delta E/\delta W$, is exponentially small. If the wavefunction is delocalised, $\Delta E/\delta W$ is large and $\Delta E$ is sensitive to boundary conditions. Thus, the scaled conductance $g$ is a simple function of a single parameter $\Delta E/\delta W$. But this is not enough for a bosonic system. The phase of the wavefunction is an important parameter here. A pretty argument due to Anderson[13] suggests that, for the Bose case, there are strong phase relationships among the (smaller) samples, discussed above, which would invalidate one-parameter scaling and also the logarithmic behaviour of the conductivity along with it. More precisely, in the localised phase, the so called Bose-glass(BG) phase[14], there are big patches of locally superfluid regions which strongly modify the scattering of the charge bosons in the following way. When a charge boson scatters off
an impurity, it correlates with (or, talks to) all the charge bosons within a distance of the order of the localisation length $\xi_{\text{loc}}$. As a result, the boson scatters as a blob, i.e. the scattering is inherently multiparticle/collective in nature, as compared to a single particle scattering typical of a non-interacting particle. (Plus, this is also the physical reason why we find a tenfold enhancement of resistance over the fermionic counterpart.) That is why it is much more convenient to think in terms of the collective excitations, viz. vortices, which are dual to the charge (boson) degrees of freedom[15]. Translated into the vortex picture, big superconducting patches convert into regions where vortices are localised. In between these large regions, there are narrow channels where the vortices are delocalised, and only these make a significant contribution to the vortex conductivity[16]. These channels are superfluid below a certain temperature $T_{\lambda V}$ (please see later), above which they lose phase coherence[17]. Further, we note that[18] the charge resistivity $\rho_c$ is related to the vortex conductivity $\sigma_v$ by

$$\rho_c = \left(\frac{h}{4e^2}\right)\sigma_v$$ (1)

In the rest of the paper, we are going to exploit this duality relation between charges and vortices extensively and evaluate the resistance as seen by the charges in terms of the vortex mobility.

In order to calculate the vortex conductivity, or equivalently the charge resistivity, we consider a model of interacting (charged) bosons in a random potential[14]. Before we write down this model explicitly, let us discuss the physical picture[Fig.1] generated by this model. Consider a system of bosons (e.g. Cooper pairs in a granular superconductor) interacting with a short-range repulsive potential $V$ in the presence of a finite disorder $\Delta$. Let $J$ be the zero point energy of the bosons. When the ratio $(J/V)$ is large, the bosons condense into a superfluid (SF) state. As the ratio $(J/V)$ is decreased (or, alternatively, $(\Delta/V)$ is increased), the bosons tend to localise and superfluidity decreases. At a critical value of the kinetic energy $(J/V)_c$, (global) superfluidity is completely destroyed and the bosons make a transition to the Bose-glass phase, where the bosons are essentially localised by disorder.
On the insulating side, very close to the phase boundary, the bosons are weakly localised and there are finite regions (of the order of the localisation length $\xi_{loc}$) where the bosons form a local condensate. As one moves further from the phase boundary by reducing $(J/V)$, one gradually enters the strongly localised regime. Here the overlap between the nearest neighbours is really small and Mott variable range hopping is expected to occur[18]. In this paper, we address the issue of conductivity of Cooper pairs in the Bose glass phase very close to the BG-SF phase boundary. For the sake of completeness, we note that when the average number of bosons per site is an integer, there is an additional Bose insulating(BI) phase. In this phase, there are exactly n bosons per site (where n is an integer) and the phase is gapped.

One can consider this entire phase diagram in the (dual) vortex picture as well. In the superfluid region, the vortices form closed loops in zero field. At the superfluid-Bose glass transition, the loops blow up as a result of the quantum zero point motion at $T = 0$ and thermal motion at finite $T$, and the vortex-antivortex pairs break up[19]. Since these vortices and antivortices are bosons, they form a boson condensate at $T = 0$[15]. However, for $T > T_\lambda$, the lambda transition of the superfluid vortex condensate[20], this is destroyed, because the disordering of the charges(viz. Cooper pairs for this case)[18], which represent the topological excitation over the vortex degrees of freedom, will destroy any possible phase coherence among the vortices. We further note that the existence of a vortex condensate would simply mean, from (1) that, $\rho_c = \infty$ at a finite temperature. This signature of a “quantum insulator” is rather counterintuitive, so far as the resistive behaviour of classical insulators is concerned. However, it can be made clearer in the dual “cooper lattice” phase where the system is essentially a charge density wave in which the pairs are strongly correlated. The superinsulator behaviour then corresponds to the response of a sliding charge density wave in the absence of pinning. This behaviour would probably be suppressed experimentally by finite size and nonlinear effects. In the rest of the paper, we shall work in the regime $T > T_\lambda$ which we assume may be quite small in the materials such as those studied by Gerber.
Although the above phase diagram is simplest to model on a lattice, in the rest of this paper we shall consider the bosons in a continuum[21,15,22]. The main purpose of this exercise is to demonstrate that under the duality transformation, vortices see potential disorder (as seen by the charges) as a random magnetic field (RMF). This feature is particularly explicit in a continuum description. Hence we consider the following model Hamiltonian ($\hbar = 1$)

\[
H = \int d^2x \left[ \frac{1}{2m} | i \vec{\nabla} \psi |^2 + \frac{1}{2} \int d^2y \delta \rho(x) V(x - y) \delta \rho(y) - \mu(x) \rho(\vec{x}, \tau) \right],
\]

where $\delta \rho = \rho - \bar{\rho}$, $\bar{\rho}$ being the (neutralising) background charge density with partition function $Z = \int \mathcal{D}\psi^* \mathcal{D}\psi \exp(-S)$, and $S = \int d\tau d^2x \mathcal{L} = \int d\tau [\int d^2x \psi^* \frac{\partial \psi}{\partial \tau} + H]$. Here $V(x)$ is a short-range potential $V(x) = V\delta(x)$.

Equivalence with the above phase diagram is established by setting $\bar{\rho}/m = J$. Now, invoke a duality transformation[23] $\psi = \sqrt{\rho} e^{i\theta} \phi_v$, where $\theta$ = non-topological part of the phase and $\phi_v$ = topological part of the wave-function, with $\phi^*_v \phi_v = 1$. Also, $\mu = \mu_0 + \delta \mu_i(\vec{x})$, where $\delta \mu_i(\vec{x})$ represents the impurity potential which is Gaussian distributed with variance $\Delta$. (We have not written down the chemical potential $\mu_0$ explicitly in what follows for convenience, and have taken $\mu(\vec{x})$ as $\delta \mu_i(\vec{x})$.) Further, define a gauge field $M_i(\vec{x})$ ($i = 1, 2$) by the relation $\epsilon_{ij} \partial_i M_j = -\mu(\vec{x})/V = B(\vec{x})$. Here $\vec{M}(\vec{x})$ is independent of $\tau$. As a result, one is led to the following Lagrangian –

\[
\mathcal{L} = \frac{m}{2\bar{\rho}} | \vec{J} |^2 + \frac{V}{2} (\delta \rho)^2 + 2\pi ij_\mu a_\mu + 2\pi j_k M_k,
\]

($\mu = 0, 1, 2, k = 1, 2$), with

\[
< B(\vec{x}) B(\vec{x}^\prime) > = (\Delta/V)^2 \delta(\vec{x} - \vec{x}^\prime).
\]

Here $J_\mu = (\delta \rho, \vec{J}) = \epsilon_{\mu\rho} \partial_\rho a_\rho$ represents the charge (boson) density and current, whereas $j_\mu^\nu = (1/2\pi i) \epsilon_{\mu\rho} \partial_\rho (\phi^*_v \partial_\mu \phi_v)$ represents the vortex density and current[24]. Integrating out the charge degrees of freedom, and transforming over to the real time, we are led to the following model Hamiltonian

\[
H_v = \sum_{i\alpha} \frac{1}{2m_\alpha} (p_{i\alpha} - 2\pi q_i M_{i\alpha})^2 + \frac{1}{2} 4\pi^2 \frac{\bar{\rho}}{m} \sum_{i \neq j} q_i q_j \ln(| x_i - x_j | / \xi_0) + \text{consts}
\]
Here \( q_i = \pm 1, (\alpha = 1, 2) \) represents the charge on a vortex and an antivortex respectively, \( x_i \) and \( p_i \) represent the positions and momenta of vortices and summation is over all vortices and antivortices. Here \( m_v \) refers to the vortex mass. Details of the notation and missing steps can be found in [23]. Thus, we find that vortices move in a random effective magnetic field under a duality transformation. A few comments are in order. In going from eqn (3) to (4), we have included a vortex mass term. This kinetic energy term is essential for the quantum melting of the vortices and is guaranteed by the underlying electronic degrees of freedom[25]. Also, we have neglected a current-current interaction term among vortices resulting from the short-range interaction among charges. This is valid when \( n_v/V m_v \ll 1/4\pi^2(n_v = \text{vortex density}) \) and at low temperatures. The physical content of this condition is as follows. The mass term for vortices generates a long-range interaction among the charges[26]. The above inequality simply states that this long-range component is much weaker than the short-range component. Thus, the BG phase discussed above is perturbed to as little extent as possible.

As we noted earlier, in the BG phase, the vortex-antivortex pairs break up. Since they are delocalised, they screen each other, and as a result, interact via short-range interactions. Thus, we are led to evaluate the conductivity of a vortex liquid as it diffuses in the presence of a random magnetic field at finite temperature. We shall do this using perturbation theory. As is well known in the context of the fermionic problem[27,28], the diffusion pole, viz. \( \Gamma(q, \omega_n) = (1/2\tau^2)/(|\omega_n | + D q^2) \), for \( \epsilon_m(\epsilon_m + \omega_n) < 0, \) (where \( \omega_n = 2\pi n T, n = \text{integer}, \) are Matsubara frequencies and \( D \) is the diffusion constant) generates singular corrections to the conductivity(at finite temperature). In the absence of a condensate, this is true of the Bose case as well, with \( m = m_v, D = \frac{1}{2} v_b^2 \tau_{tr}, \) where \( \frac{1}{2} m_v v_b^2 = \mu_v \equiv E_0 = \text{chemical potential of vortices, } \tau_{tr} = \text{transport time in a RMF= } m_v \xi_0^2/\pi^2(\Delta/V)^2, \) and \( \xi_0 = \text{pair size, defines the microscopic scale of the Bose problem. } \tau = \text{the elastic scattering time in the RMF, and we will take } \tau = \tau_{tr}[28]. \) Now, for superconductors, \( \mu_v = d(\Phi_0/4\pi \lambda)^2 \kappa \), where \( \kappa \) is the Ginzburg-Landau parameter and \( d \) is the film thickness. One can rewrite this as \( \mu_v = \alpha_0 \rho/m \equiv \alpha_0 J, \) with \( \alpha_0 = \frac{1}{4} ln \kappa. \) Using the usual value for the Josephson coupling energy \( J = (R_Q/2R_n)\Delta_0, \) where \( \Delta_0 \) is the superconducting gap, \( R_Q = h/4e^2 \simeq 6.45 K \Omega, \) and
$R_n$ is the normal resistance of the sample, we obtain the condition for validity of perturbation theory as

$$E_b \tau_{tr} = \frac{2}{\pi} \alpha_0 \frac{k_F d}{(\Delta/V)^2} \frac{E_F R_Q}{R_n} \gg 1$$

(5)

Because magnetic disorder is time-reversal symmetry breaking, the cooperon mode is suppressed[28]. The diagrams which contribute are the Altshuler-Aronov type diagrams, shown in fig.2. We merely quote the result here and refer the reader to Refs. [27,28] for more details. We obtain

$$\delta \sigma_v = -A_0 \ln(T \tau_{tr})$$

(6)

an extra minus sign coming from the replacement of anticommutators by commutators for bosons. Physically, this makes sense because as we go down in temperature the vortices being bosons tend to bose-condense. Here $A_0 = (2/\pi)(1 + F/2)$, a number of order unity, with $F = \frac{1}{2}\sqrt{1 + (\alpha_0/2\pi^2)}$. Eqn.(6) is an enhancement over a background part $R_{v0} |_{RMF} = (h/4e^2)(n \tau_{tr}/m_v) \sim (h/4e^2)(n_v \xi_0^2/(\Delta/V)^2)$. The constant part of the conductivity will receive contributions from the Bardeen-Stephen processes, $R_{BS} = 2\pi R_n (n_v \xi_0^2)[29]$.

Putting all these contributions together, we obtain,

$$\rho_c = R_b + R_0 \ln(T_{0B}/T),$$

(7)

where $R_b = [2\pi R_n + (R_Q/(\Delta/V)^2)](n_v \xi_0^2)$, and $R_0 = A_0 R_Q$. Also, in the BG phase, we expect $(\Delta/V)^2$ to be a number of order unity. Here $T_{0B} = 1/\tau_{tr}$. Eqn.(7) is the main result of the paper.

Comparison with experiment. The logarithmic behaviour for bosons, as predicted by eqns.(6) and (7), is much steeper than the analogous fermionic behaviour (with interactions included), viz. $\sigma_F = \sigma_0 - (2 - 2\ln2)(e^2/\pi h) \ln(T_{0F}/T)$ [12], with $\sigma_0 = (e^2/h)(k_F l)$. For fairly large values of $k_F l$, this implies

$$\rho_F \simeq \rho_0 + R_F l \ln(T_{0F}/T),$$

(8)
where $1/T_{OF} = \tau_F$ = electronic transport time, and $R_F = (2 - 2ln2)(\rho_0/\pi k_F l)$. Since we do not have all the necessary material parameters available for Gerber’s experiment[9], we make some estimates based on Mo-Ge samples discussed in Ref.[6]. In this case, $T_{\lambda V} \sim 10mK[20]$, much lower than the temperature range of measurements. This justifies our neglect of the “quantum insulator” phase in this discussion. Taking $k_F l \sim 5[30]$, we have $R_F \simeq 0.1 - 1K\Omega$. The scale of $R_0$ is set by $R_Q = (h/4e^2)$ and we have $R_0 \sim 5k\Omega$. Thus, we expect a change in slope of a $R$ vs $\ln T$ curve by a factor of about 10 when bosonic conduction sets in. There is one more difference between Bose and Fermi resistivities. In the case of the former, $T_{0B} \simeq 0.1 - 10K$, whereas $T_{0F} \simeq 10^2 - 10^3K$. Thus, the fermionic mechanism is a high temperature phenomenon, whereas the bosonic behaviour is a predominantly low temperature phenomenon. For low-$T_c$ materials, $T_{0B} \sim T_p$, the pair formation temperature, and the $\ln T$ dependence may be limited by $T_p$ and set in as soon as the pairs are formed. These features are clearly observable in Gerber’s experiment[9]. Also, in this experiment, at a very high magnetic field which kills quasi-reentrance (characteristic of pair formation), this low temperature logarithmic behaviour disappears as well. We consider this to be a telltale evidence of bosonic transport. However, it must be mentioned that in this experiment[9], the scale of the resistivity is unusually high, lying between megaohms and gigaohms. For a typical resistivity of fig.2 in Ref.[9], we get $R_Q/R_n \sim 10^{-4}$. This means, from eqn(5), that $E_b\tau_{tr} \sim O(1)$. So, higher order corrections in $(1/E_b\tau_{tr})$ are particularly important in this case and probably a certain class of diagrams needs to be resummed. We also take this opportunity to comment that Wolf’s observations[31], which originate on the high temperature side, are most likely not due to bosonic transport, but have some fermionic mechanism attached to it.

Our focus in this paper has been only on short range interactions among the charge bosons, although in the superconducting films like Mo-Ge[6], which are of current interest, long-range interactions dominate. It is quite possible that inclusion of such effects will renormalise the coefficients only, rather than affecting the temperature dependence strongly. This needs further investigation.
All these issues prevent us from making good contact with the existing experiments and we only hope to have conveyed to the reader the essence of bosonic transport, and how it is distinguished from its fermionic analog. We believe, however, that this behaviour should be seen in other systems where bosonic transport dominates.

**What this is not?** We would like to comment here, before closing, that this model is not the same as the traditional Bose Hubbard model with simple on-site repulsion. The latter does not generate a vortex mass term[32] which is particularly important for vortex mobility calculations discussed above. Thus, any simulation which tries to track this resistive behaviour must include an appropriate vortex mass term along with the usual Bose Hubbard model terms.

To summarise, we have argued that in the regime where Cooper pairs are weakly localised, the vortices are in a liquid phase. Thermal diffusion of this quantum liquid in the presence of (pseudo-magnetic) disorder leads to logarithmic temperature dependence of the (charge) resistivity. This kind of temperature dependence should be observable in a bosonic system, so long as the temperature is higher than that at which the vortices themselves would form a bosonic condensate.
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This simplifies a transport calculation of the weakly localised phase in the vortex picture.

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FIGURES

FIG. 1. A schematic phase diagram for the Bose localisation problem. The hatched region shows the region of interest.

FIG. 2. Altshuler-Aronov diagrams
Fig. 2  (BLR685)