Lattice with a Twist: Helical Waveguides for Ultracold Matter

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Abstract

We investigate the waveguiding properties of the optical interference pattern of two counter-propagating Laguerre-Gaussian beams. The number, helicity, radius, pitch, depth and frequencies of transverse confinement of the waveguides are simply related to the beam parameters. Quantitative connections to the familiar Gaussian optical lattice are made and an application to quantum transport is suggested.

Key words: Optical lattices, waveguides, ultracold atoms and molecules

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1. Introduction

Optical lattices arising from the interference of laser beams have proved to be a versatile tool for probing the physics of periodic systems [1]. When loaded with cold atoms or molecules they can be used to investigate paradigms ranging from single particle Bloch physics [1] to coherent [2] and strongly correlated many-body systems [3]. Compared to their counterparts in the solid-state, optical lattices can be modulated easily in time as well as space, possess no phonons and are nearly free from defects.

The most easily available laser modes are Gaussian (G), whose phase structure is essentially that of plane waves. Their interference therefore gives rise to optical lattices with discrete symmetries, which have been explored quite intensely [1,2,3]. Cold atoms loaded into these lattices can be used to simulate other physical systems which share the same discrete symmetries, such as the Hubbard model on a honeycomb lattice [4], or quasi-crystals [5]. However there are also interesting physical systems which possess continuous symmetries, such as a Bose-Einstein condensate (BEC) in a toroidal potential expected to display a persistent current [6,7] or a particle in a helical waveguide expected to be bound geometrically [8].

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In order to simulate these scenarios with cold atoms optical lattices with continuous symmetries are required. Such lattices can be made by interfering laser modes with azimuthal phase structure such as Laguerre-Gaussian (LG) beams which have recently become available ([9] and references therein).

Some work has been done regarding continuous optical lattices (see below); however they are yet to be explored in as much detail as discrete lattices. In this Communication we examine the structure of the optical lattice arising from the interference of two LG beams. While it is generally known that this produces a helical potential for ultracold matter, the waveguiding properties of this potential have hitherto been unexplored. Here we provide an elementary demonstration of the fact that the lattice can indeed work as a waveguide. Further we show that the confinement in both transverse directions in the continuous helical lattice can be related to that in the more familiar one-dimensional discrete optical lattice via simple geometrical factors. Lastly, we present a compact expression for the aspect ratio of the waveguide. This enables us to prove that the waveguide is always anisotropic. It also provides a convenient but realistic limit to spectral geometers analysing the potential for its ability to support geometrically bound states [8].

2. Interference of Laguerre-Gaussian beams

We consider the intensity pattern arising from the interference of two identical linearly polarized LG beams with charge $l$ and a single radial node at the origin, counter-propagating along the cylindrical $z-$axis

$$I = 4I_l(\rho) \cos^2(kz - l\phi) \cos^2(\omega t),$$  

where $I_l(\rho)$ is the intensity due to either beam, $k$ the wavevector and $\omega$ the optical frequency. In writing Eq. (1) we have considered displacements much smaller than the Rayleigh range, i.e. $z/z_R \ll 1$, where $z_R = \pi \omega_0^2/\lambda$, $\omega_0$ being the beam waist and $\lambda$ the wavelength of light.

3. The helical dipole potential

We now examine what happens when a cold atom is brought into a region illuminated by light with the intensity distribution (1). Without loss of generality we consider an isolated ground-excited transition for the atom. We let the frequency $\omega$ of the LG beams be red-detuned ($\Delta < 0$) by $\Delta \gg \Gamma$ from the atomic transition, where $\Gamma$ is the linewidth of the excited state. In this case the cold atom experiences a nondissipative attractive potential $|21|

$$U = U_l(\rho) \cos^2(kz - l\phi),$$

where $U_l(\rho) = \frac{\hbar k^2}{2\Delta} \left[ \frac{l(\rho)}{I_{Sat}} \right] < 0$, and $I_{Sat}$ is the saturation intensity for the transition. The non-resonant potential $U$ attracts atoms to the most intense part of its intensity distribution. $U$ can trap sufficiently cold molecules as well.
3.1. The geometry of the potential

The minima of the potential \ref{eq:potential} lie on 2\(l\) intertwined helices of radius \(R_0 = \omega_0 \sqrt{l/2}\),

\[
R = \omega_0 \sqrt{l/2},
\]

and pitch

\[
P = l\lambda \left[1 - \left(\frac{3l + 2}{4}\right) \theta^2\right]^{-1}.
\]

The pitch of each helix is practically \(\sim l\lambda\); the correction to it has been calculated by including terms linear in \(z/z_R\) and has been defined in terms of the asymptotic half-angle of divergence of either LG beam \cite{22},

\[
\theta = \frac{\lambda}{\pi \omega_0},
\]

as shown in Fig.1 which displays the potential for the case \(l = 1\). For regular laser beams \(\theta\) is quite small, about \(10^{-3}\). We note that \(\theta = 1\) corresponds to the diffraction limit \cite{22}.

The helices are displaced \(\lambda/2\) parallel to each other along the \(z\)-axis, and rotated \(\pi/l\) radians away from each other on any plane where \(z\) is constant. For high enough laser power because of the red detuning \((\Delta < 0)\) a cold atom moves along a particular minimum of the potential Eq.\ref{eq:potential} and effectively sees a potential that is helical in symmetry.

3.2. Atomic motion along the waveguide

Here we establish the fact that \(U\) can act as a waveguide. Without loss of generality we restrict ourselves to a single strand of the ensemble. For an atom in such a helix an orthogonal co-moving coordinate system is supplied by the Frenet frame consisting of the unit vectors \((\hat{\tau}, \hat{\nu}, \hat{\beta})\), which are the tangent, normal and binormal to the helix respectively at a point \((\rho, \phi, z)\) (Fig.1) \cite{25}. The double-helical lattice, arising from the optical interference of two Laguerre-Gaussian \(l = 1\) beams. Two right-handed helical waveguides drawn using solid red and dotted blue curves are formed, each with radius proportional to the beam radius \(\omega_0\) and with pitch equal to the optical wavelength \(\lambda\). They are translated by \(\lambda/2\) and rotated by \(\pi\) with respect to each other. The half-angle of divergence of either beam \(\theta\), and the pitch angle or chirality of each helical strand \(\alpha_R\) are depicted in the figure. Also shown are the orthogonal unit vectors of the right-handed Frenet frame local to the helix, consisting of the tangent \((\hat{\tau})\), normal \((\hat{\nu})\) and binormal \((\hat{\beta})\) at a point.

\[
\hat{\tau} = \sin \alpha \hat{z} + \cos \alpha \hat{\phi},
\]

\[
\hat{\nu} = \cos \alpha \hat{z} - \sin \alpha \hat{\phi},
\]

\[
\hat{\beta} = \hat{\rho},
\]

where the transformation from the cylindrical unit vectors involves the pitch angle \(\alpha\) of a helix of radius \(\rho\) defined by

\[
\tan \alpha = \frac{P}{2\pi \rho}, \quad 0 \leq \alpha < \pi/2.
\]

If the laser detuning \((\Delta)\) from the atomic transition is large enough, photon scattering due to spontaneous emission is negligible and the potential Eq.\ref{eq:potential} is essentially conservative. This implies that we can calculate the force due to the potential simply as

\[
F = -\nabla U.
\]

Expressing Eq.\ref{eq:potential} in the Frenet frame Eq.\ref{eq:force} we find that the component of force
along the $\hat{\tau}$ direction vanishes identically. However there are constraining forces in the directions $\hat{\nu}$ and $\hat{\beta}$. The potential $U$ therefore acts as a waveguide that confines the atom in the locally transverse directions while allowing free motion along the local longitudinal direction. It is worth noting that this is true everywhere in the potential, and not only at its minimum.

3.3. Atomic motion transverse to the waveguide

We now turn to a detailed consideration of the atomic motion in the transverse directions $\hat{\nu}$ and $\hat{\beta}$. From Eq. (7) we find that at the minimum ($\rho = R$) of the potential $U$ the pitch angle, or the chirality of the helix (Fig. 1), is $\alpha_R$ where

$$\tan \alpha_R = \frac{P}{2\pi R} = \theta \sqrt{\frac{1}{2}}.$$  \hspace{1cm} (9)

To investigate the nature of the potential seen in the transverse direction by the atom, we use Eq. (9) and the curvilinear coordinates local to the Frenet frame

$$\nu = (kz - l\phi)/k,$$

$$\beta = \rho - R.$$ \hspace{1cm} (10)

Using these quantities we Taylor expand Eq. (4) near its minimum (where $\nu \sim 0$, $\beta \sim 0$, and $\alpha \sim \alpha_R$) and obtain

$$U \sim U_{00}(R) + \frac{1}{2}m\omega_{\nu}^2\nu^2 + \frac{1}{2}m\omega_{\beta}^2\beta^2.$$  \hspace{1cm} (11)

Thus to lowest order in $\nu$ and $\beta$ the transverse confinement is simple harmonic. Let us consider the trapping frequencies $\omega_{\nu}$ and $\omega_{\beta}$. The trapping frequency in the $\hat{\nu}$ direction is given by

$$\omega_{\nu} = \frac{1}{\hbar} \left[4E_R U_{00}(R) \sec \alpha_R\right]^{1/2},$$  \hspace{1cm} (12)

where we have used the single photon recoil energy

$$E_R = \frac{\hbar^2k^2}{2m}$$ \hspace{1cm} (13)

to write $\omega_{\nu}$ in a transparent form. In Eq. (13) $m$ is the atomic mass. Importantly, Eq. (12) allows us to relate the helical lattice to the more familiar one-dimensional optical lattice if we recall that $l = 0$ corresponds to the interference of G beams. If we put $l = 0$ in Eq. (9), the tangent vanishes. This implies $\sec \alpha_R = 1$, and Eq. (12) reduces to

$$\omega^{1d} = \frac{1}{\hbar} \left[4E_R U_{00}(R)\right]^{1/2},$$  \hspace{1cm} (14)

which is the trapping frequency for an atom in the well of a 1-d lattice constructed from G beams. In this case the $\hat{\nu}$ direction points along the $z$–axis. Now for LG beams, $l \neq 0$. Nevertheless since $\theta \sim 10^{-3}$ is usually a small quantity, the tangent in Eq. (9) is still close to zero and hence $\sec \alpha_R$ is not much greater than 1. Therefore we may not expect the trapping frequency in the normal direction for a helical lattice to be very different than that for the one-dimensional G lattice for comparable laser parameters ($U_{00} \sim U_{10}$).

The trapping frequency in the $\hat{\beta}$ direction can be written as

$$\omega_{\beta} = \theta \left[4E_R U_{00}(R)\right]^{1/2} \sim \theta \omega^{1d}.$$ \hspace{1cm} (15)

Eq. (15) shows that for $l \neq 0$ and comparable parameters ($U_{00} \sim U_{10}$) the confinement along the binormal in the helical waveguide is typically weaker than in the corresponding one-dimensional G lattice by a factor of $\theta \sim 10^{-3}$.

Relating the continuous helical LG lattice to the discrete G lattice allows us to transfer some of the well-established physical intuition from the latter to the former. For example the sub-wavelength confinement of atoms, characteristic of deep discrete optical lattices [21], can typically be obtained only along the $\hat{\nu}$ direction in a helical lattice.

The ratio of the two transverse frequencies Eqs. (12) and (15), i.e. the aspect ratio $A$ of the waveguide, is given by
\[ A = \frac{\omega_\nu}{\omega_\beta} = \frac{1}{\theta} \left( 1 + \frac{l \theta^2}{2} \right)^{1/4}. \]  

Eq. (16) enables us to examine the cross section of the helical waveguide quite generally. For example perfect isotropy \((A = 1)\) requires \(\theta = 1\) and \(l = 0\). The first condition corresponds to the diffraction limit and the second to the absence of any helical structure. This implies that perfect isotropy for the waveguide is not possible even in principle. Staying in the diffraction limit and allowing for just a single pair of helices \((l = 1)\) introduces a 10\% anisotropy: \(A = 1.1\). For typical experimental parameters however \(\omega_\nu \sim 10\) MHz, \(\omega_\beta \sim 10\) KHz and \(A \sim 10^3\), i.e. the anisotropy is quite large. An important implication of this anisotropy is the ability of the waveguide to serve as a three, two or one-dimensional structure for microscopic objects. For sufficiently cold atoms or molecules \((T < \hbar \omega_\nu/k_B \sim 10 \mu K)\) the motion along the \(\hat{\nu}\) direction can be frozen out and the waveguide is rendered essentially two dimensional. Similarly, for even lower temperatures \((T < \hbar \omega_\beta/k_B \sim 10nK)\) the atomic motion along the \(\hat{\beta}\) direction can be frozen out and the waveguide made one-dimensional. Here \(k_B\) is Boltzmann’s constant.

4. Applications

The helical potential (2) allows access to a less-explored symmetry, and may reveal interesting physics such as negative group velocities \([23]\) and Berry’s phase \([24]\) for particles moving under its influence. Below we outline a possible application to quantum transport.

The availability of a waveguide that bends and twists naturally suggests a problem that has been considered (as with much of optical lattice physics) initially in the condensed matter literature. This is the phenomenon of geometrically bound states relevant to electron transport through nanoscopic wires in quantum heterostructures \([26]\) and references therein). The basic idea can be understood by considering the transmission of a quantum particle through a straight waveguide of uniform cross-section. A low energy cut-off for the transmission exists because the transverse wavelength of the particle cannot be larger than the width of the waveguide. Now imagine a bulge in the waveguide, which locally allows for a longer wavelength, i.e. an energy lower than the cut-off. In an infinitely long waveguide this would correspond to the localization of the particle at the bulge, or a bound state. In a waveguide of finite extent, this shows up as an exponentially decaying resonance: the particle eventually leaks out of the waveguide. The same effect can be achieved even in a waveguide of constant width, if the waveguide is bent \([27]\). In fact it has been shown quite generally that curvature in a waveguide is equivalent to the presence of an attractive potential \([28]\). Classically analogous ‘trapped’ electromagnetic modes have experimentally been observed in bent microwave waveguides \([29]\).

Motivated by our work, Exner et al. have examined a curved helical potential for the presence of bound states and suggested an answer in the affirmative \([8]\). Their analysis implies that for large aspect ratios \((A \gg 1)\) and small pitch angles \((\alpha R \sim 0)\) appropriate to the optical potential \(U\) described here, particles can be bound by the waveguide if the radius goes through a local maximum. It would be interesting to test this prediction as in practice the pitch angle of \(U\) can be adjusted using \(\theta\) and \(l\) while local extrema in the radius can easily be obtained using (de)focussing lenses. One experimental signature of the bound state would be a minimum in the transmission through the waveguide at the binding energy \([26]\).
5. Conclusion

We have investigated the waveguiding properties of the optical lattice arising from the interference of two identical counter-propagating Laguerre-Gaussian beams. We have provided a quantitative connection to the more familiar Gaussian lattices. We have also provided a basic description of atomic motion in the waveguide.

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