THE NONRELATIVISTIC EVOLUTION OF GRBS 980703 AND 970508: BEAMING-INDEPENDENT CALORIMETRY

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Received 2004 April 19; accepted 2004 May 25

ABSTRACT

We use the Sedov-Taylor self-similar solution to model the radio emission from the γ-ray bursts GRB 980703 and GRB 970508 when the blast wave has decelerated to nonrelativistic velocities. This approach allows us to infer the energy of the GRBs independent of jet collimation. We find that for GRB 980703 the kinetic energy at the time of the transition to nonrelativistic evolution, $t_{\text{NR}} \approx 40$ days, is $E_{\text{ST}} \approx (1 - 6) \times 10^{51}$ ergs. For GRB 970508 we find $E_{\text{ST}} \approx 3 \times 10^{51}$ ergs at $t_{\text{NR}} \approx 100$ days, nearly an order of magnitude higher than the energy formerly derived by Frail, Waxman, and Kulkarni. This is due primarily to revised cosmological parameters and partly to the maximum likelihood fit we use here. Taking into account radiative losses prior to $t_{\text{NR}}$, the inferred energies agree well with those derived from the early, relativistic evolution of the afterglow. Thus, the analysis presented here provides a robust, geometry-independent confirmation that the energy scale of cosmological GRBs is about $5 \times 10^{51}$ ergs and also shows that the central engine in these two bursts did not produce a significant amount of energy in mildly relativistic ejecta at late times. Furthermore, a comparison to the prompt energy release reveals a wide dispersion in the γ-ray efficiency, strengthening our growing understanding that $E_\gamma$ is a not a reliable proxy for the total energy.

Subject headings: gamma rays: bursts — radiation mechanisms: nonthermal — shock waves

1. INTRODUCTION

The two fundamental quantities in explosive phenomena are the kinetic energy, $E_K$, and the mass of the explosion ejecta, $M_{ej}$, or equivalently the expansion velocity, $\beta \equiv v/c$, or Lorentz factor, $\Gamma = (1 - \beta^2)^{-1/2}$. Together, these gross parameters determine the appearance and evolution of the resulting explosion. Gamma-ray bursts (GRBs) are distinguished by a highly relativistic initial velocity, $\Gamma_0 \approx 100$, as inferred from their nonthermal prompt emission (Goodman 1986; Paczynski 1986). For the range of γ-ray isotropic equivalent energies observed in GRBs, $E_{\gamma,\text{iso}} \approx 10^{51} - 10^{54}$ ergs (Bloom et al. 2001), this indicates $M_{ej} \approx 10^{-2} - 10^{-3} M_\odot$, compared to several $M_\odot$ in supernovae (SNe).

The true energy release of GRBs depends sensitively on the geometry of the explosion. For a collimated outflow (“jet”) with a half-opening angle $\theta_j$, it is $E = f_b E_{\text{iso}}$, where $f_b \equiv 1 - \cos \theta_j$ is the beaming fraction; the true ejecta mass is also a factor of $f_b$. Over the past several years there has been growing evidence for such collimated outflows, coming mainly from achromatic breaks in the afterglow light curves (e.g., Kulkarni et al. 1999; Stanek et al. 1999). The epoch at which the break occurs, $t_j$, corresponds to the time at which the ejecta bulk Lorentz factor decreases below $\theta_j^{-1}$ (Rhoads 1999; Sari et al. 1999).

In this context, several studies have shown that the beaming-corrected energies of most GRBs, in both the prompt γ-rays and afterglow phase, are of the order of $10^{51}$ ergs (Frail et al. 2001; Panaitescu & Kumar 2002; Berger et al. 2003a; Bloom et al. 2003; Yost et al. 2003). The various analyses are sensitive to the energy contained in ejecta with different velocities, $\Gamma \gg 100$ in the γ-rays, $\Gamma \approx 10$ in the early X-rays, and $\Gamma \approx 1 \text{ few}$ in the broadband afterglow. However, none are capable of tracing the existence and energy of nonrelativistic ejecta.

Frail et al. (2000) overcame this problem in the case of GRB 970508 by modeling the afterglow radio emission in the nonrelativistic phase, thus inferring $E_K \approx 5 \times 10^{50}$ ergs. This analysis has two significant advantages. First and foremost, it is independent of jet collimation, since the blast wave approaches spherical symmetry on the same timescale that it becomes nonrelativistic (Livio & Waxman 2000). Second, this analysis relies on the simple and well-understood Sedov-Taylor dynamics of spherical blast waves, as opposed to the hydrodynamics of spreading relativistic jets. In addition, the peak of the synchrotron spectrum on the relevant timescale lies in the radio band, where the afterglow is observable for several hundred days.

Two recent developments make similar analyses crucial. We now recognize that some GRBs are dominated by mildly relativistic ejecta (Berger et al. 2003c). For example, for GRB 030329 the kinetic energy inferred from the afterglow emission, $E_K (\Gamma \approx \text{few}) \approx 5 \times 10^{50}$ ergs (Berger et al. 2003c), was an order of magnitude higher than the γ-ray energy release (Price et al. 2003). Similarly, for GRB 980425, $E_K \approx 8 \times 10^{47}$ ergs (Galama et al. 1998; Pian et al. 2000) was about 1% of the relativistic kinetic energy of the associated SN 1998bw, $E_K \approx 10^{50}$ ergs (Kulkarni et al. 1998; Li & Chevalier 1999). This begs the following question: Is there even more energy emerging from the engine, either at the time of the burst or later on, at nonrelativistic velocities?

Second, there is a growing interest in “unification models” for GRBs, X-ray flashes (XRFs), and core-collapse SNe of...
Type Ib/c, relying primarily on energetics arguments. For example, Lamb et al. (2004) argue that GRBs and XRFs share an energy scale of \(10^{49}\) ergs, and that all Type Ib/c SNe give rise to GRBs or XRFs. Both conclusions result from significantly smaller values of \(\theta_j\) compared to those inferred in the past, such that the energy scale, \(\propto \theta_j^2\), is lower by a factor of \(~100\) and the true GRB rate, \(\propto \theta_j^{-2}\), matches locally the Type Ib/c SN rate. Given the important ramifications of the GRB energy scale for progenitor scenarios, we would like to independently address the following question: Is the energy scale of cosmic explosions \(10^{49}\) ergs, implicating all Type Ib/c SNe? 

The answer will also provide an independent confirmation of the jet paradigm by comparison to the isotropic equivalent energies. This is crucial, since other explanations for the light curve breaks have been suggested, including changes in the density of the circumburst medium, a transition to a nonrelativistic evolution on the timescale of a few days (due to a high circumburst density), and changes in the energy spectrum of the radiating electrons (Dai & Lu 2001; Panaitescu 2001; Wei & Lu 2002).

Here we address the possibility of significant contribution from nonrelativistic ejecta and robustly determine the energy scale of GRBs independent of geometrical assumptions, using Very Large Array\(^1\) radio observations of the afterglows of GRBs 970508 and 980703 in the nonrelativistic phase. We generally follow the treatment of Frail et al. (2000), but unlike these authors we carry out a full least-squares fit to the data.

2. THE NONRELATIVISTIC BLAST WAVE

AND FIREBALL CALORIMETRY

The dynamical evolution of an ultrarelativistic blast wave expanding in a uniform medium (ISM) is described in terms of its Lorentz factor, \(\Gamma = (17E_{\text{iso}}/8\pi nm_p c^2 \rho^3)^{1/2}\), where \(r\) is the radius of the blast wave and \(n\) is the number density of the circumburst medium (Blandford & McKee 1976). This, along with the relation for the observer time, which for the line of sight to the center of the blast wave is \(t = r/8c^2\) (e.g., Sari 1997), determines the evolution of the radius and Lorentz factor. For a spherical blast wave the expansion will eventually become nonrelativistic on a timescale\(^2\) of \(t_{\text{NR}} \approx 65(E_{\text{iso}, 52}/n_0)^{1/3}\) days, determined by the condition that the mass swept up by the blast wave, \(M_{\text{sw}} \approx E_{\text{iso}}/c^2\).

An initially collimated outflow becomes nonrelativistic at \(t_{\text{NR}} \approx 40(E_{\text{iso}, 52}/n_0)^{1/4} t_j^{1/4}\) days (Livio & Waxman 2000). Moreover, as the jet expands sideways (at \(t \approx t_j\)) the outflow approaches spherical symmetry on a timescale \(t_j \approx 150(E_{\text{iso}, 52}/n_0)^{1/4} t_j^{1/4}\) days similar to \(t_{\text{NR}}\). Thus, regardless of the initial geometry of the outflow, the nonrelativistic expansion is well approximated as a spherical outflow. We note that this discussion can be generalized to a range of radial density profiles. Here, in addition to the ISM model, we focus on a density profile, \(\rho \propto Ar^{-2}\) (hereafter “Wind model”), appropriate for mass loss with a constant rate, \(\dot{M}_w\), and speed, \(v_w\) (Chevalier & Li 2000).

Following the transition to nonrelativistic expansion, the dynamical evolution of the blast wave is described by the Sedov-Taylor self-similar solution (Sedov 1946; von Neumann 1947; Taylor 1950). In this case the radius of the shock is given by \(r \propto (E_{\text{ST}}/A)^{1/3}\), with \(\rho \propto Ar^{-3}\). Thus, in the ISM case \(r \propto (E_{\text{ST}}/nm_p)^{1/3}\), while in the Wind case \(r \propto (E_{\text{ST}}/A)^{1/3}\).

The constant of proportionality, \(\xi(\gamma)\), depends on the adiabatic index of the gas, \(\gamma\), and is equal to 1.05 in the ISM case and 0.65 in the Wind case for \(\gamma = 13/9\). The latter is appropriate for pressure equilibrium between relativistic electrons and nonrelativistic protons\(^3\) (Frail et al. 2000). The circumburst material shocked by the blast wave is confined downstream to a thin shell of width \(r/\eta_i\), with \(\eta_i \approx 10\).

To calculate the synchrotron emission emerging from this shock-heated material, we make the usual assumptions. First, the relativistic electrons are assumed to obey a power-law distribution, \(N(\gamma) \propto \gamma^{-\gamma \pm 2}\) for \(\gamma \geq \gamma_m\). Second, the energy densities in the magnetic field and electrons are assumed to be a nonvarying fraction \((e_B \text{ and } e_e)\), respectively, of the shock energy density. Coupled with the synchrotron emissivity and taking into account self-absorption, the flux received by an observer at frequency \(\nu\) and time \(t\) is given by (e.g., Frail et al. 2000)

\[
F_{\nu} = F_0(t/t_0)^{-\gamma}(1 + z)^{\nu/\gamma}(1 - e^{-\gamma})(f_0(\nu/m_0) f_{\gamma}^{-1}(\nu/m_0)),
\]

and the function

\[
f_{\gamma}(x) = \int_0^x F(y)^{(p-1)/2} dy.
\]

Here \(\nu_m = \nu_0(t/t_0)^{-\gamma}/(1 + z)\) is the synchrotron peak frequency corresponding to electrons with \(\gamma = \gamma_m\). \(F(y)\) is given in, e.g., Rybicki & Lightman (1979), and the temporal indices \(\alpha_F, \alpha_e, \text{ and } \alpha_m\) are determined by the density profile of the circumburst medium. In the ISM case \(\alpha_F = 11/10, \alpha_e = -1\), and \(\alpha_m = -7/3\) (Waxman 2004a). Equations (1)–(3) include the appropriate redshift transformations to the rest frame of the burst.

Based on the thermal scalings, the synchrotron flux in the optically thin regime \((\nu \gg \nu_m, \nu_a)\) evolves as \(F_\nu \propto t^{(21-15\gamma)/10}\) (ISM) or \(F_\nu \propto t^{6-\gamma/6}\) (Wind); here the synchrotron self-absorption frequency, \(\nu_a\), is defined by the condition \(\tau_\nu(\nu_a) = 1\). Thus, for \(\nu \gg \nu_m, \nu_a\) the transition to nonrelativistic expansion is manifested as a steepening of the light curves at \(t_{\text{NR}}\) if the outflow is spherical (Sari et al. 1998; Chevalier & Li 2000) or a flattening if the outflow was initially collimated (Sari et al. 1999). Below we use this behavior to estimate \(t_{\text{NR}}\) for GRBs 980703 and 970508.

In \(\frac{3}{5}\) and 4 we use the temporal decay indices and equations (1)–(3) to carry out a least-squares fit to the data at \(t > t_{\text{NR}}\) with the free parameters \(F_0, \tau_0, \nu_0, \nu_a, \text{ and } p\). These parameters are, in turn, used to calculate the physical parameters of interest, namely \(r, n_e, \gamma_m, \text{ and } B; n_e \approx (\eta/3)\) is the shocked electron density (Frail et al. 2000). Since only three spectral

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1 The VLA is operated by the National Radio Astronomy Observatory, a facility of the National Science Foundation operated under cooperative agreement by Associated Universities, Inc.
2 Here and throughout the paper we use the notation \(g = 10^g\).
3 The relative pressure between the protons and relativistic electrons depends on the fraction of energy in relativistic electrons, \(e_e\). If this fraction is low, the pressure may be dominated by the nonrelativistic protons, in which case \(\gamma = 5/3\). As we show below, \(e_e\) for both GRBs 980703 and 970508 is in the range \(\sim 0.1–0.5\), and thus \(\gamma = 13/9\) is applicable.
parameters are available, this leaves the radius unconstrained, and thus,

\[ B = 11.7(p + 2)^{-2}F_{0-52}^{-2}(r_{17}/d_{28})^4 \text{ G}, \]  

\[ \gamma_m = 6.7(p + 2)F_{0-52}^{-1/2}(r_{17}/d_{28})^{-2}, \]  

\[ n_e = 3.6 \times 10^{10} c_\eta F_{0-52}^{3} n_{m}^{-1/9} \eta_{0.9}^{-1/9} (r_{17}/d_{28})^{-6} \text{ cm}^{-3}, \]  

\[ c_\eta = (1.67 \times 10^{3})^{-p}(5.4 \times 10^{5})^{(1-p)/2}(p + 2)^2/(p - 1). \]

In the Wind model, the density is appropriate at \( r_{\text{ST}} \equiv r(r_{\text{NR}}) \), i.e., \( \rho(r) = nm_p(r/r_{\text{NR}})^{-2} \).

To determine the radius of the blast wave, a further constraint is needed. We note that the energy contained in the electrons and magnetic field cannot exceed the thermal energy of the Sedov-Taylor blast wave, which accounts for about half of the total energy (Frail et al. 2000). The energy in the electrons is given by \( E_e = [(p - 1)/(p - 2)]n_e\gamma_m m_e c^2 V \), while the energy in the magnetic field is \( E_B = B^2 V/8\pi \); here \( V = 4\pi r^3/\eta \) is the volume of the synchrotron emitting shell. Thus, using equations (4)–(7) and the condition \( E_e + E_B \leq E_{\text{ST}}/2 \), we can constrain the range of allowed values of \( r \). In the ISM model \( E_{\text{ST}} = nm_p(r/1.05)^3(1/z)^2/(1+z)^2 \), while in the Wind model \( E_{\text{ST}} = A(r/0.65)^3(1+z)^2 \).

With a constraint on the radius we can also ensure self-consistency by calculating the velocity of the blast wave when it enters the Sedov-Taylor phase, \( v_{\text{ST}} = 2r(1+z)/3L_{\text{NR}} \) (ISM) or \( v_{\text{ST}} = 2r(1+z)/3L_{\text{NR}} \) (Wind). We expect that \( v \sim c \). Finally, the isotropic equivalent mass of the ejecta is given by \( M_{\text{ej}} = 4\pi nm_p r_{\text{ST}}^3 \) (ISM) or \( 4\pi A r_{\text{ST}} \) (Wind). The actual ejecta mass is reduced by a factor \( f_0 \) relative to this value.

3. GRB 980703

In Figure 1 we plot the radio light curves of GRB 980703. The data are taken from Berger et al. (2001) and Frail et al. (2003). Two gross changes in the light curve’s evolution are evident: a flattening at \( t \approx 40 \text{ days} \) at 4.8 and 8.5 GHz and a transition to a constant flux density at late times. The latter is due to radio emission from the host galaxy of GRB 980703 with flux densities at 1.4, 4.8, and 8.5 GHz of 65, 50, and 40 \( \mu \text{Jy} \), respectively (Berger et al. 2001). The flattening at \( t \approx 40 \text{ days} \) marks the transition to nonrelativistic evolution following a period of sideways expansion of the initially collimated outflow (Fig. 1). A similar value of \( L_{\text{NR}} \approx 30–50 \text{ days} \) has been inferred by Frail et al. (2003) from tracking the evolution of the blast wave Lorentz factor in the relativistic phase. Therefore, we use \( t_{\text{NR}} \approx 40 \text{ days} \) here.

We follow the method outlined in § 2 using both the ISM and Wind cases. The results of both fits, shown in Figure 1, are overall indistinguishable. In the following discussion we quote the results of the ISM model. The best-fit parameters (\( \chi^2_{\text{min}} = 123 \) for 45 degrees of freedom) are: \( F_{0-52} \approx 2.7, \gamma_m \approx 80, \gamma_m \approx 4.6, \) and \( p \approx 2.8 \). The relatively large value of \( \chi^2_{\text{min}} \) is primarily due to fluctuations induced by interstellar scintillation, particularly at 4.8 GHz.

Using \( d_{28} = d_{28}/(1+z)^{1/2} \), \( z = 0.966, H_0 = 71 \text{ km} \) \( s^{-1} \text{ Mpc}^{-1}, \Omega_m = 0.27, \) and \( \Omega_{\Lambda} = 0.73 \) and equations (4)–(7), we find \( B \approx 1.8 \times 10^{-2}r_{17}^{-2} \) G, \( \gamma_m \approx 300 r_{17}^{-2} \), and \( n_e \approx 4.9 \times 10^3 r_{17}^{-2} \) cm\(^{-3} \). From these parameters we calculate...
$E_e \approx 3.4 \times 10^{51} r_{17}^{-6}$ ergs, $E_B \approx 1.7 \times 10^{46} r_{17}^{11}$ ergs, and $E_{ST} \approx 6.2 \times 10^{51} r_{17}^{-2}$ ergs. These results are plotted in Figures 2 and 3.

The range of blast wave radii allowed by the constraint $E_e + E_B \leq E_{ST}/2$ is $r_{17} \approx 1.05 - 2.5$, resulting in a range of values for the Sedov-Taylor energy, $E_{ST} \approx (1 - 6) \times 10^{55}$ ergs (Table 1). Given the strong dependence on radius, the ratio of energy in the electrons to that in the magnetic fields ranges from $\epsilon_e/\epsilon_B \approx 0.03$ to $(9 \times 10^4)$, while the specific values range from $\epsilon_e \approx 0.01$ to 0.45 and $\epsilon_B \approx (5 \times 10^{-6})$ to 0.4. The circumburst density is in the range $n \approx 8$ to $(3.5 \times 10^3)$ cm$^{-3}$, while the blast wave velocity is $\beta_{ST} \approx 0.8 - 1.9$.

Finally, the isotropic equivalent mass of the ejecta ranges from $(1$ to $40) \times 10^{-4} M_\odot$.

A comparison to the values derived by Frail et al. (2003) using modeling of the afterglow emission in the relativistic phase is useful. These authors find $n \approx 30$ cm$^{-3}$, $\epsilon_e \approx 0.27$, and $\epsilon_B \approx 2 \times 10^{-3}$. Using the same density in our model (Fig. 3), as required by the ISM density profile, gives a radius

![Graph](image)

**TABLE 1**

| Parameter | GRB 98703 | GRB 970508 |
|-----------|-----------|------------|
| $r$ (10$^{17}$ cm) | 1.05 - 2.5 | 3.7 - 5.9 |
| $\beta$ (G) | 0.02 - 0.7 | 0.04 - 0.25 |
| $n$ (cm$^{-3}$) | 8 - 270 | 65 - 165 |
| $\epsilon_e$ | 8 - 3.5 x $10^5$ | 0.4 - 0.1 |
| $\epsilon_B$ | 0.01 - 0.45 | 0.07 - 0.5 |
| $\epsilon_{\gamma}$ | $5 \times 10^{-6}$ - 0.4 | $1 \times 10^{-3}$ - 0.45 |
| $M_{ej,iso}$ (10$^{-4} M_\odot$) | 1 - 40 | 2 - 18 |
| $E_{ST}$ (10$^{50}$ ergs) | 9 - 56 | 15 - 38 |
| $E_K(t_{dec})$ (10$^{51}$ ergs) | 4 | 30 |

**NOTE.**—Physical parameters of GRBs 980703 and 970508 derived from the nonrelativistic evolution of their blast waves. The range of allowed radii, and hence physical parameters, is determined by the condition $(E_e + E_B) \leq E_{ST}/2$. The last entry in the table, $E_K(t_{dec})$, is the total kinetic energy at the deceleration time, $t_{dec} \approx 90$ s, including synchrotron radiative losses ($\epsilon_5$).
distribution at energy. This can be avoided by assuming a break in the electron energy data required to constrain either $E_t$ or $E_B$ (Fig. 4), using $t_{\text{NR}}$. The best-fit parameters in the ISM model are shown (black lines); the wind model can be ruled out, since it requires $p < 2$.

$r_{17} \approx 1.75$ and hence $\epsilon_e \approx 0.06$ and $\epsilon_B \approx 4 \times 10^{-3}$, in rough agreement; the energy is $E_{\text{ST}} \approx 2 \times 10^{51}$ ergs.

If we assume, alternatively, that the energy in relativistic electrons and the magnetic field are in equipartition, we find $r_{17} \approx 2.05$. In this case, $E_{\text{ST}} \approx 1.5 \times 10^{51}$ ergs, $n \approx 10$ cm$^{-3}$, $B \approx 0.3$ G, and $\epsilon_e = \epsilon_B = 0.03$.

4. GRB 970508

The nonrelativistic evolution of GRB 970508 was studied by Frail et al. (2000). These authors provide a rough model for the radio emission beyond $t_{\text{NR}} \approx 100$ days and argue that the constraint $E_e + E_B < E_{\text{ST}}/2$ requires the electron and magnetic field energy to be in equipartition, $\epsilon_e = \epsilon_B \approx 0.25$, with $E_{\text{ST}} \approx 4.4 \times 10^{50}$ ergs. Here we perform a full least-squares fit (Fig. 4), using $t_{\text{NR}} = 100$ days, and find somewhat different results. We use $t_{\text{NR}} = 100$ days, noting that for GRB 970508 the outflow appears to be weakly collimated (Yost et al. 2003), and hence the transition is manifested as a mild steepening of the light curves (see § 2).

The best-fit parameters in the ISM model$^4$ ($\chi^2_{\text{min}} = 164$ for 58 degrees of freedom) are: $F_{0, -52} \approx 38$, $\tau_{0.32} \approx 3.1 \times 10^{-3}$, $\nu_{0.9} \approx 3$, and $p \approx 2.17$. The large value of $\chi^2_{\text{min}}$ is primarily due to interstellar scintillation.

$^4$ We do not consider the Wind case, since in this model the observed decay rates at 4.8 and 8.5 GHz, $F_e \propto t^{-1.2}$, require $p \approx 1.7$ and hence an infinite energy. This can be avoided by assuming a break in the electron energy distribution at $\gamma_0 > \gamma_\ast$, with a power law index $q > 2$, but we do not have the data required to constrain either $\gamma_0$ or $q$.

In comparison, Frail et al. (2000) use $F_{0, -52} \approx 41$, $\tau_{0.32} \approx 5.3 \times 10^{-3}$, $\nu_{0.9} \approx 9.5$, and they set $p = 2.2$; a solution with $\nu_{0.9} \approx 4.2$ is also advocated, but it is not used to derive the physical parameters of the blast wave. The formal $\chi^2$ values for these solutions are 225 and 254, respectively, somewhat worse than the solution found here.

As a result, we find that solutions away from equipartition are allowed. Adopting the cosmological parameters used by Frail et al. (2000), $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_m = 1$, and $\Omega_\Lambda = 0$, we find $E_{\text{ST}} \approx (6-11) \times 10^{50}$ ergs, a factor of about 20%–100% higher than the values inferred by these authors.

Using the currently favored cosmology (§ 3), we find instead that the distance to the burst is higher by about 30%, $d_{28} = 1.21$ compared to 0.94 (Frail et al. 2000). The change in distance has a significant effect on the derived parameters, since $E_e \propto d^6$, $E_B \propto d^3$, and $E_{\text{ST}} \propto n \propto d^6$. Thus, we find that the constraint on $E_e + E_B$ indicates $r_{17} \approx 3.7-5.9$ and therefore, $B \approx 0.04-0.25$ G, $\gamma \approx 65-165$, and $n \approx 0.4-10$ cm$^{-3}$. The Sedov-Taylor energy is $E_{\text{ST}} \approx (1.5-3.8) \times 10^{51}$ ergs, while $\epsilon_e \approx 0.07-0.5$ and $\epsilon_B \approx 0.001-0.45$ (Figs. 5 and 6 and Table 1). Assuming equipartition, we find $r_{17} = 3.3$, $E_{\text{ST}} = 1.8 \times 10^{51}$ ergs, and $\epsilon_e = \epsilon_B = 0.11$. The derived energy is about a factor of 4 higher than the previous estimate (Frail et al. 2000).

A comparison of our best-fit model with the flux of the afterglow in the optical $R$ band at $t = 110$ days, $F_{\nu, R} \approx 0.3$ Jy (Garcia et al. 1998), indicates a break in the spectrum. If we interpret this break as due to the synchrotron cooling frequency, above which the spectrum is given by $F_{\nu} \propto \nu^{-q/2}$, we find $\nu_c \approx 6 \times 10^{13}$ Hz. Since $\nu_c = 1.9 \times 10^{19} B^{-3}(t/110 \text{ days})^{-2}$ Hz we infer $B \approx 0.073$ G and hence $r_{17} = 4.3$, $E_{\text{ST}} = 2.8 \times 10^{51}$ ergs, $\epsilon_e = 0.25$, and $\epsilon_B = 8 \times 10^{-3}$. These values are in rough agreement with those inferred from modeling of the

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**Fig. 4.** Radio light curves of the afterglow of GRB 970508 at 1.4, 4.8, and 8.5 GHz. Only data at $t \geq t_{\text{NR}} = 100$ days (black circles) are used in the fit. The best-fit light curves for the ISM model are shown (black lines); the wind model can be ruled out, since it requires $p < 2$.

**Fig. 5.** Energies associated with the afterglow of GRB 970508 in the nonrelativistic Sedov-Taylor phase as a function of the (unconstrained) blast wave radius. The thin curve is the sum of the energy in relativistic electrons ($E_e \propto r^{-4}$) and in the magnetic fields ($E_B \propto r^{3}$). Also plotted are the Sedov-Taylor energy ($E_{\text{ST}} \propto r^{-4}$) and the thermal component, $E_{\text{ST}}/2$. The shading corresponds to an uncertainty of 30% in the value of the synchrotron frequency $\nu_0$ at $t = t_{\text{NR}}$. The value of $E_{\text{ST}}/2$ provides an additional constraint, $E_e + E_B < E_{\text{ST}}/2$, which limits the range of allowed radii in the solution (boxed region). Finally, the arrow marks the most likely solution using the value of the cooling frequency as estimated from a combination of the radio and optical data (§ 4). This additional parameter breaks the radius degeneracy, indicating $r \approx 4.2 \times 10^{17}$ cm and $E_{\text{ST}} \approx 8 \times 10^{51}$ ergs.
relativistic phase (Panaitescu & Kumar 2002; Yost et al. 2003), although our value of \( \epsilon_B \) is somewhat lower.

5. RADIATIVE CORRECTIONS

The energies derived in § 3 and § 4 are, in fact, lower limits on the initial kinetic energy of the blast wave due to synchrotron radiative losses. These play a role primarily in the fast-cooling regime \( (\nu_c < \nu_m) \), which dominates in the early stages of the afterglow evolution (e.g., Sari et al. 1998).

Yost et al. (2003) estimate the time at which fast-cooling ends: \( t_{cm} \approx 0.1 \) and 1.4 days after the burst for GRBs 970508 and 980703, respectively. Using these values, and our best estimate of \( \epsilon_e \approx 0.06 \) (GRB 980703) and \( \epsilon_e \approx 0.25 \) (GRB 970508), we calculate the radiative corrections, \( E \propto t^m \), going back from \( t_{NR} \) to about 90 s after the burst. Here \( m \approx -17/12 \epsilon_c \), with \( \epsilon = \epsilon_e/(1 + 1.05 \epsilon_e) \) for \( t < t_{cm} \), and it is quenched by a factor \( (\nu_m/\nu_c)(p^{-2}/2)^{1/2} < 1 \) at later times. Thus, at low values of \( \epsilon_c \) the radiative losses are negligible. The cutoff at 90 s corresponds to the approximate deceleration time of the ejecta, \( t_{dec} \approx 90(E_{52}/n_0 \Gamma_{2}^2)^{1/3} \) s.

We find that approximately 50% and 90% of the energy was radiated away before \( t_{NR} \) for GRB 980703 and GRB 970508, respectively. Thus, the initial kinetic energies are estimated to be \( 4 \times 10^{51} \) and \( 3 \times 10^{52} \) ergs, respectively. The corrections from \( t_{NR} \) back to \( t_{cm} \), 10% for GRB 980703 and 70% for GRB 970508, indicate \( E_K \approx 2 \times 10^{51} \) and \( 9 \times 10^{51} \) ergs, respectively. Both estimates of the energy are in excellent agreement with those inferred from the relativistic evolution of the fireball at \( t_{cm} \) (Yost et al. 2003), \( E_K \approx 3 \times 10^{51} \) ergs (GRB 980703) and \( E_K \approx 1.2 \times 10^{52} \) ergs (GRB 970508).

6. DISCUSSION AND CONCLUSIONS

Analysis of the synchrotron emission from a GRB blast wave in the nonrelativistic phase has the advantage that it is independent of geometry and is described by the well-understood Sedov-Taylor self-similar solution. Using this approach to model the late-time radio emission from GRB 980703 \( (t > 40 \text{ days}) \) and GRB 970508 \( (t > 100 \text{ days}) \), we infer kinetic energies in the range \( (1-6) \times 10^{51} \) and \( (1.5-4) \times 10^{51} \) ergs, respectively. Including the effect of radiative losses...
starting at $t_{\text{dec}} \sim 90$ s, we find that the initial kinetic energies were about $4 \times 10^{51}$ and $3 \times 10^{52}$ ergs, respectively.

The inferred kinetic energies confirm, independent of any assumptions about the existence or opening angles of jets, that the energy scale of GRBs is $\sim 5 \times 10^{51}$ ergs. Therefore, we unambiguously rule out the recent claim of Lamb et al. (2004) that the energy scale of GRBs is of the order of $10^{49}$ ergs. Since the claimed low energies were based on the apparent correlation between $E_{\gamma, \text{iso}}$ and the energy at which the prompt emission spectrum peaks, $E_{\text{peak}}$ (Amati et al. 2002), we conclude that this relation, and the prompt emission in general, does not provide a reliable measure of the total energy. As a corollary, we rule out the narrow jet opening angles used by Lamb et al. (2004), $\theta \sim 0.1^\circ$ and thus confirm that the true GRB rate is significantly lower than the rate of Type Ib/c SNe (Berger et al. 2003b).

Finally, the overall agreement between the energies derived here and those inferred from modeling of the relativistic phase of the afterglow indicates that the central engine in GRBs 980703 and 970508 did not produce a significant amount of energy in mildly relativistic ejecta ($\Gamma \beta \gtrsim 2$) at late times, $t \sim t_N$. However, a comparison to the beaming-corrected $\gamma$-ray energies (Bloom et al. 2003), $E_{\gamma} \approx 1.1 \times 10^{51}$ ergs (GRB 980703) and $E_{\gamma} \sim 10^{51}$ ergs (GRB 970508), reveals that the efficiency of the blast wave in producing $\gamma$-rays, $\epsilon_\gamma$, varies considerably: $\sim 20\%$ for GRB 980703 but only $\sim 3\%$ for GRB 970508. The wide dispersion in $\epsilon_\gamma$ strengthens the conclusion that $E_\gamma$ is not a reliable tracer of the total energy (Berger et al. 2003c).

The low value of $\epsilon_\gamma$ for GRB 970508 may indicate an injection of energy from mildly relativistic ejecta at early times. Both the optical and X-ray light curves of this burst exhibited a sharp increase in flux approximately 1 day after the burst, by a factor of about 4 and $\gtrsim 2$, respectively (Piro et al. 1998; Sokolov et al. 1998). The flux in these bands depends on energy as $F_\nu \propto E^{(p+3)/4}$ and $F_\nu \propto E^{(p+2)/4}$, respectively (Sari et al. 1998). Thus, if we interpret the flux increase as due to the injection of energy from ejecta with $\Gamma \sim 5-10$ (Panaitescu et al. 1998), we find an energy increase of about a factor of 3. The analysis performed here provides an estimate of the total energy following the injection, and thus $\epsilon_\gamma$ appears to be low. The actual value of $\epsilon_\gamma$ is thus $\sim 10\%$.

Although GRBs 980703 and 970508 are currently the only bursts with sufficient radio data to warrant the full Sedov-Taylor analysis, flattening of radio light curves at late times have been noted in several other cases, most notably GRBs 980329, 991208, 000301C, 000418, and 000926 (Frail et al. 2004). Interpreting the flattening as a transition to nonrelativistic expansion and using the expression for the flux at 8.5 GHz at the time of the transition, $F_\nu(t_N) \approx 50(1 + 2)^{1/2} \epsilon_{B,\text{iso}} E_{3}^{1/2} d_{28}^{3/2} \mu$Jy (Livio & Waxman 2000), we find the rough results $n_0^{1/3} E_3 \approx 6$ (GRB 980329), $\approx 4$ (GRB 991208), $\approx 25$ (GRB 000301C), $\approx 6$ (GRB 000418), and $\approx 22$ (GRB 000926). Thus, for typical densities, $\sim 1-10$ cm$^{-3}$ (Panaitescu & Kumar 2002; Yost et al. 2003), the inferred kinetic energies are again of the order of $10^{51}-10^{52}$ ergs.

This leads to the following conclusions. First, the energy scale of cosmological bursts is about $5 \times 10^{51}$ ergs, at least 3 orders of magnitude higher than the kinetic energies in fast ejecta determined for local Type Ib/c SNe from radio observations (Berger et al. 2002, 2003b) and an order of magnitude higher relative to the nearby ($d \approx 40$ Mpc) GRB 980425 associated with SN 1998bw (Kulkarni et al. 1998; Li & Chevalier 1999; Waxman 2004b) and GRB 031203 ($z = 0.105$; Prochaska et al. 2004; Soderberg et al. 2004). Second, as already noted in the case of GRB 030329 (Berger et al. 2003c), there is a wide dispersion in the fraction of energy in ultrarelativistic ejecta, such that the $\gamma$-rays are a poor proxy for the total energy produced by the engine.

Thus, radio calorimetry is uniquely suited for addressing the relation between various cosmic explosions. So far, such studies reveal a common energy scale in relativistic ejecta of about 5 foe ($\text{foe} \approx 10^{53}$ ergs) for cosmological GRBs (Berger et al. 2003c), about 0.1 foe for the low-redshift bursts (GRBs 980425 and 031203), and $\lesssim 10^{-3}$ foe in fast ejecta for Type Ib/c SNe. The open question now is whether we are beginning to trace a continuum in the energetics of cosmic explosions, or whether the various classes truly represent distinct physical mechanisms with different energy scales. Fortunately, the best example to date of an object possibly bridging the various populations, GRB 030329, still shines brightly in the radio a year after the burst.

We thank Eli Waxman, Sarah Yost, and Re’em Sari for valuable discussions, and the referee, Roger Chevalier, for useful comments. We acknowledge NSF and NASA grants for support.

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