Heavy–light charm mesons spectroscopy and decay widths

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We present the mass formula for heavy–light charm meson at one loop, using heavy quark effective theory. Formulating an effective Lagrangian, the masses of the ground state heavy mesons have been studied in the heavy quark limit, including leading corrections from finite heavy quark masses and nonzero light quark masses, using a constrained fit for the eight equations with 11 parameters including three coupling constants \( g, h, \) and \( g' \). Masses determined using this approach are fitted to the experimentally known decay widths to estimate the strong coupling constants, showing a better match with available theoretical and experimental data.

Subject Index
B31, B36, B37, B60, D32

1. Introduction

A heavy hadronic system contains heavy quarks with spin quantum number \( (S_Q) \) and light degrees of freedom, where light degrees of freedom include light quarks and gluons interacting through quark–antiquark pairs. The light degrees of freedom should have the quantum number of the light quark, \( S_l \), in order to have total conserved quantum number \( J \) where \( J = S_Q + S_l \). Defining \( J = j(j+1) \) and \( S_Q^2 = s_Q(s_Q+1) \) and \( S_l^2 = s_l(s_l+1) \), the total spin \( j \) is obtained by combining the heavy quark spin \( \frac{1}{2} \) with the spin of the light degrees of freedom. The ground state \( l = 0 \) heavy mesonic system forms a degenerate doublet with \( J^0 \oplus 1 \) and negative parity denoted as \( D \) and \( D^* \) for the charm meson. The first excited states are \( 0^+ \) and \( 1^+ \) with \( D_0 \) and \( D_1 \). The lowest-lying excited states \( J^P = 0^+, 1^+ \) heavy mesons are the members of the \( j^P = \frac{1}{2}^+ \) doublet. There is also an excited doublet of heavy mesons with \( j^P = \frac{3}{2}^+ \), whose members are \( J^P = 1^+ \) and \( 2^+ \). The members of various doublets are represented by \( (P, P^*) \) for \( l = 0 \), \( (P_0^*, P_1^{'}) \) for \( J^P = (0^+, 1^+) \), while those with \( J^P = (1^+, 2^+) \) for \( l = 1 \) correspond to \( (P_1, P_2^{'}) \).

Study of the heavy meson spectrum is a recent motivation for researchers. There are several resonances like \( D_J^* (3000) \) [1], \( D_sJ^* (2860), D_{sJ} (3040) \) [2] which still need confirmation regarding their \( J^P \) states. Heavy quark effective theory (HQET) is one of the effective tools for studying details of recent resonances observed at different experiments. The theory can be applied to estimate the various signatures like masses, decay widths, and their quantum numbers of meson states. Various models like quark models [3–5], potential models, and lattice studies have previously been used to calculate meson masses, but the calculated charm masses were found to be of higher values as compared with experiments [6–12]. The approximate symmetries of QCD for heavy quarks can be incorporated to get information about the heavy–light system of mesons like charm and bottom mesons. In the
infinite heavy quark mass limit, a heavy–light system $Qq$ can be classified into doublets depending upon their quantum numbers.

Heavy meson spectroscopy can be analyzed by electron collider experiments, but observations of such a wide range of resonances lead to different puzzles. One of the most important pieces of evidence of $c\bar{s}$ broad states was provided by the CLEO Collaboration [7], which observed a state of mass 2460 MeV and width 290 MeV. The BaBar Collaboration observed another narrow peak in the $D^+_s\pi^0$ invariant mass distribution, corresponding to a state of mass 2.317 GeV [8]. Both of these states were confirmed by the Focus and Belle Collaborations [9]. Recent experimental evidence includes $D_{sJ}(2860)$ observed by the BaBar Collaboration [10] with mass $2856.6 \pm 1.5 \pm 50$ MeV, and the Belle Collaboration measured another peak in $B^+$ decays with mass $M(D_{sJ}(2715)) = 2715 \pm 11^{+11}_{-14}$ MeV [8]. Moreover, in the DK mass distribution, the BaBar Collaboration noticed a broad structure with mass $M = 2688 \pm 4 \pm 3$ MeV [11,12] and width $\Gamma = 112 \pm 7 \pm 36$ MeV, likely the same resonance $D_{sJ}(2700)$ found by Belle. The most recent state in the strange sector was announced in 2009 as $D_{sJ}(3040)$ with mass $M = 3044 \pm 8(stat) +30^{-5}_{-5}$ (sys)MeV/$c^2$ [13]. This state was observed in $D^*K$ channel mode not DK bound state. Many of the excited charm mesonic states in the non-strange sector have recently been analyzed [14] and more data with more accuracy is needed. Even the experimental searches include $0^-$ and $1^-$ states in the non-strange sector but the excited states like $2^+$ and $3^+$ non-strange mesons have not yet been confirmed accurately.

In this paper, Sect. 2 presents a heavy quark effective theory for writing an effective Lagrangian with leading order corrections from finite heavy quark masses and nonzero light quark masses. In Sect. 3, we formulate the mass formulae for ground states and low-lying excited states charm meson in terms of 11 unknown parameters and their decay widths in terms of strong coupling constants. In Sect. 4, we present the results for charm masses, decays, and coupling constants, the constrained fits for the respective parameters, and further conclusions.

2. Theoretical framework: heavy quark effective theory

The properties of hadrons with a heavy quark coupled with light degrees of freedom can also be explained on the basis of symmetry occurring in the heavy quark limit which takes a particular simpler form in the limit $m_Q \rightarrow \infty$. Two theories related to two symmetries, chiral symmetry for light quarks u, d, s and heavy quark spin and flavor symmetry for heavy quarks c and b, can be exploited to explain a system with one heavy quark and another light one. Heavy–light mesons can be studied by implementing both the chiral symmetry and heavy quark symmetry in the form of an effective Lagrangian [15–17]. The effective Lagrangian here describes the interplay between the chiral symmetry and heavy quark symmetry in the form of low-energy gradients with heavy and light fields as operators.

For a heavy–light system, heavy quark doublets are represented by effective fields and the octet of light pseudo-Goldstone bosons are grouped in a single field. The chiral Lagrangian for heavy mesons incorporating the heavy quark and vector symmetry can be written by including a kinematic term and all the possible interactions with the Goldstone bosons, which involve the symmetry-breaking and -conserving terms. The expressions for effective fields describing the various doublets in the heavy quark limit are described as [18]:

$$H_a = \frac{1+\gamma}{2} \left( P^*_{a} \gamma_{\mu} - P_{a} \gamma_{5} \right) \tag{1}$$
$$S_a = \frac{1+\gamma}{2} \left( P^*_{a} \gamma_{\mu} \gamma_{5} - P_{a} \gamma_{5} \right) \tag{2}$$

Here, $a = u, d, s$ is the SU(3) index and various operators annihilate mesons of four-velocity $v$. 

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Equations (1) and (2) represent the ground-state negative parity doublet \((0^-, 1^-)\) and the low-lying excited state positive parity doublet \((0^+, 1^+)\), respectively. The interactions among heavy and light mesons are obtained by expanding the field \(\xi = e^{iH/t}\) and \(\sum = \xi^2\), where \(\xi\) is the pion decay constant. The pion octet is introduced here by the vector and axial combinations \(V^\mu = \frac{1}{2}(\xi \partial^\mu \xi^* + \xi^* \partial^\mu \xi)\) and \(A^\mu = \frac{1}{2}(\xi \partial^\mu \xi^* - \xi^* \partial^\mu \xi)\) as

\[
\begin{bmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
- \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
\pi^+ \\
\pi^- \\
K^- \\
K^0
\end{bmatrix}
\]

These fields are involved in forming an effective Lagrangian.

Now the Lagrangian that describes the dynamics of mesons with the heavy–light combination is

\[
L = L_{\text{kinetic}} + L_{\text{axial}} + L_{\text{ct}}.
\]  

The kinetic part of the Lagrangian is

\[
L_{\text{kinetic}} = -\text{Tr}\left[ H^T (ivD_{\text{ba}} - \delta_H \delta_{\text{ab}}) H_b \right] + \text{Tr}\left[ \bar{S}^\mu (ivD_{\text{ba}} - \delta_S \delta_{\text{ab}}) S_b \right],
\]

where \(\delta_H\) and \(\delta_S\) are the residual masses of the H and S fields, where the residual masses are defined to be the difference between the real mass and an arbitrarily chosen reference mass of \(O(m_q)\). The axial coupling of the field to the pseudo-Goldstone bosons is defined by the Lagrangian

\[
L_{\text{axial}} = g \text{Tr}\left[ \bar{H}_a H_b \ A_{\text{ba} \gamma 5} \right] + g' \text{Tr}\left[ \bar{S}_a S_b \ A_{\text{ba} \gamma 5} \right] + h \text{Tr}\left[ \bar{H}_a S_b \ A_{\text{ba} \gamma 5} + h.c.\right],
\]

where \(g\) and \(g'\) are coupling constants in the ground-state and excited-state doublets respectively, and \(h\) is the coupling between mesons belonging to different doublets. The mass counter-term Lagrangian is

\[
L_{\text{ct}} = \text{Tr}\left[ (a_H \bar{H}_a H_b - a_S \bar{S}_a S_b) \left( \xi m_q \xi^* + \xi^* m_q \xi \right)_{ab} \right]
+ \text{Tr}\left[ (\sigma_H \bar{H}_a H_a - \sigma_S \bar{S}_a S_a) \left( \xi m_q \xi^* + \xi^* m_q \xi \right)_{bb} \right],
\]

where \(m_q = \text{diag}(m_u, m_d, m_s)\), \(\xi^2 = \exp(2i\phi/f)\), with \(\phi\) being usual matrix of pseudo-Goldstone bosons and \(f \approx 130\,\text{MeV}\). In terms of heavy quark symmetry-conserving and symmetry-violating terms, the above Lagrangian can be written as

\[
L_{\text{ct}} = -\frac{\Delta_H}{8} \text{Tr}\left[ \bar{H}_a \sigma^{\mu \nu} H_a \sigma_{\mu \nu} \right] + \frac{\Delta_S}{8} \text{Tr}\left[ \bar{S}_a \sigma^{\mu \nu} S_a \sigma_{\mu \nu} \right] + a_H \text{Tr}\left[ \bar{H}_a H_b \right] m^\xi_{ba} - a_S \text{Tr}\left[ \bar{S}_a S_b \right] m^\xi_{ba}
+ a_H \text{Tr}\left[ \bar{H}_a H_a \right] m^\xi_{bb} - a_S \text{Tr}\left[ \bar{S}_a S_a \right] m^\xi_{bb}
- \frac{\Delta_H^{(a)}}{8} \text{Tr}\left[ \bar{H}_a \sigma^{\mu \nu} H_b \sigma_{\mu \nu} \right] m^\xi_{ba}
+ \frac{\Delta_S^{(a)}}{8} \text{Tr}\left[ \bar{S}_a \sigma^{\mu \nu} S_b \sigma_{\mu \nu} \right] m^\xi_{ba}
\]

where the \(\Delta_H, \Delta_S\) terms in the above equation are symmetry-(spin-)violating operators giving rise to hyperfine splitting, and the \(a_H, a_S, \sigma_H, \) and \(\sigma_S\) terms preserve spin symmetry while the other operators violate heavy quark spin symmetry.
3. Mass formula and decay width for charm mesons

In the framework of heavy hadron chiral perturbation theory chiral corrections and corrections due to chiral and heavy quark symmetry are encountered, and at one-loop level the residual masses can be given by the generalized formula

\[
m^0_{R_s} = \delta_R + \frac{n_j}{4} \left( \Delta_R + \Delta_R^{m} + \Delta_R^{(a)} \right) + \sigma_R^{m} + a_R m_a + \frac{g^2}{f^2} c^{R_s} K_1(\eta, m) + \frac{h^2}{f^2} c^{R_s} K_2(\eta, m),
\]

where R is an index that labels the ground state (H) and excited state (S), each of the ground and excited states having members corresponding to \( J = 0, 1 \), where \( n_j = n_0 = 3 \) and \( n_1 = 1 \). These coefficients come from the \( S_Q, S_V \) operator and give \(-3/4\) for pseudoscalar mesons and \(1/4\) for vector mesons. Here, \( m_a = u, d, s \) and \( m = m_u + m_d + m_s \). Here, \( g, g', \) and \( h \) are the coupling constants corresponding to \( H-H \), \( S-S \), and \( H-S \) interactions, respectively. Thus in total we obtain 12 equations representing the residual masses at one-loop level for the low-lying doublet combining with three light quarks respectively, but assuming isospin symmetry the equations reduce to eight constants corresponding to \( H-H, S-S, S-H, T-H \) state interactions, respectively. The coupling constants depend on the radial and \( c^{R_s} \) are the coefficients that signify the emission of pion, kaon, and eta Goldstone bosons from specific charm states. The index \( (a) \) labels the light flavor and runs over \( u, d, \) and \( s \), and \( K_1 \) and \( K_2 \) are the chiral loop functions defined as

\[
K_1(\eta, M) = \frac{1}{16\pi^2} \left[ (-2\eta^3 + 3M^2\eta) \ln \left( \frac{M^2}{\mu^2} \right) + 2\eta \left( \eta^2 - M^2 \right) F \left( \frac{\eta}{M} \right) + 4\eta^3 - 5\eta M^2 \right]
\]

\[
K_2(\eta, M) = \frac{1}{16\pi^2} \left[ (-2\eta^3 + M^2\eta) \ln \left( \frac{M^2}{\mu^2} \right) + 2\eta^3 F \left( \frac{\eta}{M} \right) + 4\eta^3 - \eta M^2 \right],
\]

where

\[
F(x) = \begin{cases} 
2 \sqrt{1-x^2} \left[ \frac{\pi}{2} - \tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right) \right] & |x| < 1 \\
-2 \sqrt{x^2-1} \ln \left( x + \sqrt{x^2-1} \right) & |x| > 1
\end{cases}
\]

The function \( K_1(\eta, M) \) appears whenever the virtual heavy meson inside the loop is in the same doublet as the external heavy meson, while \( K_2(\eta, M) \) appears when the virtual heavy meson is from the opposite parity doublet.

Here, \( \eta \) mass difference of heavy–light charm mesons in the initial and final state and \( M \) is the mass of the pseudo-scalar boson which is emitted during the process. \( \mu \) is the energy scale of 1 GeV.

Two-body strong decays of heavy–light mesons to pseudo-scalar mesons can be derived from the heavy meson chiral Lagrangian \( L_0, L_{HH}, L_{SH}, L_{TH}, \) etc., which can be written as

\[
L_0 = i \text{Tr} \left[ \bar{H}_a v. D_{ab} H_b + i \text{Tr} \left[ \bar{S}_a v. D_{ab} S_b \right] + i \text{Tr} \left[ T_{ab}^{\mu} v. D_{ab} T_{\mu b} \right] \right]
\]

\[
L_{HH} = g \text{Tr} \left[ \bar{H}_a H_b \gamma_\mu \gamma_5 A_{ba}^{\mu} \right],
\]

\[
L_{SH} = g' \text{Tr} \left[ \bar{H}_a S_b \gamma_\mu \gamma_5 A_{ba}^{\mu} \right] + \text{h.c.},
\]

\[
L_{TH} = h \text{Tr} \left[ \bar{S}_a S_b \gamma_\mu \gamma_5 A_{ba}^{\mu} \right] + \text{h.c.}
\]

(13)

Here, \( D_\mu \) is the covariant derivative and \( A_\mu, V_\mu \) have their usual meanings, \( \Lambda \) is the chiral symmetry breaking to be taken as \( \sim 1 \) GeV, and \( g, g', h, \) and \( g_{TH} \) are the hadronic coupling constants between the \( H-H, S-S, S-H, T-H \) state interactions, respectively. The coupling constants depend on the radial...
quantum number of the heavy mesons and are represented by $g^-$. From these chiral Lagrangian terms, we obtain the decay width for the strong interaction to final states $D^{(*)}\pi$, $D^{(*)}K$, $D^{(*)}\eta$ as

$$
\Gamma = \frac{1}{2J + 1} \sum p_f \frac{P_f}{8\pi M_i^2} |A|^2 \quad \text{where} \quad p_f = \sqrt{\left(M_i^2 - (M_f + m_p)^2\right) \left(M_i^2 - (M_f - m_p)^2\right) \over 2M_i} 
$$

(14)

and $A$ is the scattering amplitude, $i$ and $f$ denote the initial and final particles, $J$ is the total angular momentum of the initial heavy meson, $P$ denotes the light pseudo-scalar meson, the summation $\sum$ is of all the polarisation vectors, and $P_f$ is the momentum exchanged between the initial and final state. The expressions for the decay widths for various doublets are as follows, where $M$ refers to the emitted pseudo-scalar meson ($\pi$, $\eta$, $K$) fields:

$(0^-, 1^-)$ to $(0^-, 1^-) + M$

$$
\Gamma (1^- \to 0^-) = c_p \frac{g_{HH}^2 M_f p_f^2}{6\pi f_f^2 M_i} \quad (15)
$$

$$
\Gamma (1^- \to 0^-) = c_p \frac{g_{HH}^2 M_f p_f^2}{6\pi f_f^2 M_i} \quad (16)
$$

$$
\Gamma (1^- \to 1^-) = c_p \frac{g_{HH}^2 M_f p_f^2}{3\pi f_f^2 M_i} \quad (17)
$$

$$
\Gamma (0^- \to 1^-) = c_p \frac{g_{HH}^2 M_f p_f^2}{2\pi f_f^2 M_i} \quad (18)
$$

$(0^+, 1^+) \to (0^-, 1^-) + M$

$$
\Gamma (1^+ \to 1^-) = c_p \frac{g_{HH}^2 M_f \left(p_f^2 + m_p^2\right)}{2\pi f_f^2 M_i} \quad (19)
$$

$$
\Gamma (0^+ \to 0^-) = c_p \frac{g_{HH}^2 M_f \left(p_f^2 + m_p^2\right)}{2\pi f_f^2 M_i} \quad (20)
$$

The coefficients $C_p$ are different for different pseudo-scalar mesons,

$$
C_{\pi^\pm} = C_{K^\pm} = C_{K^0} = C_{\bar{K}^0} = 1, \quad C_{\pi^0} = \frac{1}{2}, \quad C_{\eta} = \frac{1}{6}.
$$

## 4. Results and conclusion

Within the heavy hadron chiral perturbation theory framework, the charm meson masses, decay widths, and couplings are analyzed using a mass formula up to one-loop chiral corrections [20]. With higher-loop corrections of order $O(Q^3)$ the mass formula in Eq. (9) consist of eight equations having the 11 parameters $\delta_s + \sigma_s \bar{m}$, $\delta_H - \sigma_H \bar{m}$, $a_H$, $a_S$, $\Delta_H^{(a)}$, $\Delta_S^{(a)}$, $g$, $g'$, $h$, $\Delta_H + \Delta_H^{(a)} \bar{m}$, $\Delta_S + \Delta_S^{(a)} \bar{m}$.

We will use the experimentally measured residual masses given below to fit our 11 parameters. Two of the parameters, the axial coupling for the ground state doublet of charmed mesons, and $h$ which dominates the strong coupling between even parity and the ground state charmed meson, have a range of $0–1$. These constraints are justified from the literature [15]. The experimentally measured
residual masses in reference to the non-strange spin averaged mass \( (m_{H_i} + 3/4m_{H^*_i}) \) are:

\[
\begin{align*}
&m_{H_1} = -106.1 \text{ MeV} & m_{H_2} = -4.75 \text{ MeV} & m_{H_1}^* = 35.4 \text{ MeV} & m_{H_2}^* = 139.1 \text{ MeV} \\
&m_{S_1} = 335.0 \text{ MeV} & m_{S_2} = 344.4 \text{ MeV} & m_{S_1}^* = 465.0 \text{ MeV} & m_{S_2}^* = 486.3 \text{ MeV}
\end{align*}
\]

In the tree-level mass formula where \( g, g', \) and \( h = 0 \), the residual masses can be reproduced with the many sets of parameters. There are large uncertainties in the parameters involved in the Lagrangian of the low-lying excited states, i.e. even parity mesons. Below is one such set of eight parameters reproducing the above residual masses at tree level:

\[
\begin{align*}
&\delta_S + \sigma_S m - \delta_H - \sigma_H m = 432 \pm 26 \text{ MeV}, \\
&\Delta_H + \Delta_H^{(S)} m = 146 \pm 1 \text{ MeV}, \\
&\Delta_S + \Delta_S^{(S)} m = 129 \pm 50 \text{ MeV}, \\
&a_H = 1.14 \pm 0.06, \\
&a_S = 0.21 \pm 0.29, \\
&\Delta_H^{(a)} = -0.03 \pm 0.01, \text{ and} \\
&\Delta_S^{(a)} = 0.14 \pm 0.55
\end{align*}
\]

Different predictions from relativistic and non-relativistic quark models, and decays of \( D^* \) in the one-loop calculation, restrict the values of coupling constants \( g \) and \( h \) to lie between 0 and 1, but \( g' \in [-1, 1] \) [21]. Using the literature value, we vary these values over the range 0–1. We used \( f = 120 \text{ MeV} \) extracted in Refs. [15,20] using the one-loop formulae for pion and kaon decay constants. We set \( m_u = m_d = 4 \text{ MeV} \) and \( m_s = 90 \text{ MeV} \). Using Mathematica 7.0 [22] as a programming language to fit the values, a number of sets can be obtained. The sets can be reduced by putting justified constraints on the unknown parameters. We cannot make strong conclusions on the sets, as some of the parameters cannot be significantly determined, and thus the uncertainties on the parameters are very large. Using some of the constraints from literature on spin-breaking and flavour-breaking terms, one set of value couplings that satisfies our residual experimental masses, i.e. Eq. (21), is given below. One of the best-fitting set of parameters to these residual masses is

\[
\begin{align*}
&g = 0.1, \quad g' = 0.1, \quad h = 0.07, \quad \delta_H = 4, \quad \delta_S = 431, \quad \Delta_H = 144, \quad \Delta_S = 126 \\
&a_H = 1.1, \quad a_S = 0.21, \quad \Delta_H^{(a)} = -0.04, \quad \Delta_S^{(a)} = 0.14
\end{align*}
\]

Based on the above parameters, central masses of the charmed meson for one-loop corrections have been calculated and presented in Table 1 column 5. Here, real masses and decay widths have been compared with the experimental data taken from Particle Data Group (PDG) [6]. \( D^0 \) mass has also been measured at Belle [23] and is equal to 2308 ± 36; similarly, other masses are also available. We present our results, which match well with the PDG data [6].

Here the importance of the results can be determined from the calculated data. The data is obtained for the higher loop correction into the mass formulae (9). Spin symmetry violating parameters \( \Delta_H \) and \( \Delta_S \) for the ground and the excited states resulting from \( (1/m_Q) \) corrections in the effective
Table 1. Ground state and excited state even- and odd-parity charm mesons with their residual and real masses and decay widths.

| S. no | State | Mesonic state | J^P | Calculated residual mass (MeV) | Calculated real mass (MeV) | Experimental real mass (MeV) | Decay width (Γ) (MeV) |
|-------|-------|---------------|-----|--------------------------------|----------------------------|-----------------------------|------------------------|
| 1.    | m_{H_1} | D^0_{1 0}, 0^- | 0^- | -105.97 | 1867.04 | 1869.61 | |
| 2.    | m_{H_0} | D^+_{2 0} | 0^- | -5.27028 | 1967.74 | 1968.30 | |
| 3.    | m_{S_1} | D_0^0 | 0^- | 329.722 | 2314.08 | 2318 | 267 |
| 4.    | m_{S_1} | D^+_{2 0} | 0^- | 341.077 | 2314.55 | 2317.7 | <3.8 |
| 5.    | m_{H_1} | D^0_{4 0}, 1^- | 1^- | 44.4608 | 2017.24 | 2010.26 | 0.0834 |
| 6.    | m_{H_1} | D^+_1 | 1^- | 136.485 | 2109.49 | 2112 | <1.9 |
| 7.    | m_{S_1} | D^0 | 1^- | 460.509 | 2433.52 | 2421.4 | 27.4 |
| 8.    | m_{S_1} | D^+_1 | 1^- | 483.281 | 2456.29 | 2459.5 | <3.5 |

Table 2. Experimental and calculated values for spin splitting and mass splitting of meson doublets.

| State J^P | Spin splitting | Mass splitting |
|-----------|----------------|----------------|
|           | Value from experiment (MeV) | Calculated value (MeV) | State | Value from experiment (MeV) | Calculated value (MeV) |
| D^*_s - D_s (1^- - 0^-) | 143.8 | 141.4 | D^*_s - D^*_u (1^- - 1^-) | 105.4 | 94.04 |
| D^*_s - D^*_u (1^- - 0^-) | 141.9 | 138.7 | D_s - D_u (0^- + 0^-) | 98.8 | 96.6 |
| D^*_u - D_u (1^- - 0^-) | 142.1 | 143.9 | D^*_s - D^*_u (1^- - 1^-) | 21.3 | 20.3 |
| D^*_u - D_u (1^- - 0^-) | 130 | 126.5 | D_s - D_u (0^- + 0^-) | 9.4 | 8.1 |

Table 3. Decay mode of S field strange charm meson.

| Decay mode | Width used (MeV) [25] | Coupling constant h calculated |
|------------|------------------------|-------------------------------|
| D^*_s → D_s Π (0^- → 0^-) | 260 | 0.7 |
| D_s → D^*_s Π (0^- → 0^-) | 160 | 0.5 |

Table 4. Decay width for non-strange P(L = 1) wave meson states.

| Decay mode | Width (MeV): calculated; h = 0.7 | Width (MeV): calculated; h = 0.5 | Width (MeV) |
|------------|----------------------------------|----------------------------------|-------------|
| D^* → DΠ | 236.76 | 120.79 | 267 [6] |
| D → D^*Π | 101.73 | 51.905 | 84 [26] |

Lagrangian break the heavy quark spin symmetry. This results in the breaking of mass degeneracy between the members of the same doublets. These (1/ m_Q) corrections will introduce hyperfine splittings of order 140 MeV. Column 2 of Table 2 shows the hyperfine splittings for our calculated masses and in the next column these splittings have been compared with the experimental values. This comparison shows that our calculated values are within the 2% deviation from the experimental values.

The SU(3) symmetry breaking arises when m_u = m_d ≠ m_s in the chiral Lagrangian for one-loop mass formulae. This flavor breaking between the strange and non-strange charm mesons, whose other
quantum numbers are identical, is expected to be of the order of $\sim 100 \text{MeV}$, corresponding to the mass of the strange quark. The SU(3) flavour-violating terms enter in the form of $a_H$ and $\Delta^{(a)}_H$. Due to the 11 parameter fit, the constraints on the SU(3) flavour-violating terms show unusual patterns but are within a 2%–13% deviation from the experimental data.

The decays channel of the radially excited strange charm meson state [4] are used to justify our calculated masses using the effective theory. Here we calculate the strong coupling constant and verify it with other measure values in Ref. [24].

\[ D^*_0(2700) \rightarrow (1^- \rightarrow 0^-) \ D^* \ K; \ (1^- \rightarrow 1^-) \ D^* \ K; \ (1^- \rightarrow 0^-) \ D_S^{\eta}; \ (1^- \rightarrow 1^-) \ D_S^{*\eta}; \ D^*K_0^* \]

(this mode is excluded in our calculation as it gives a negative momentum for the light meson).

Using the total decay width (125 MeV experimentally) we get $g^\sim = 0.30$ which can be compared with $g^\sim = 0.28$ [24]. A decay width of strange 0$^+$ and 1$^+$ is used from [3,4] and the coupling constant is determined and matched with $h = -0.6$ [3,4].

To check the validity of our approach we took the two different values of calculated strong coupling, i.e. $h = 0.7$ and $h = 0.5$ in Table 3, and found the decay width for non-strange P (L = 1) wave meson states in Table 4. The results of the decay width are compared and shown to match well with the experimental values.

The abovementioned calculations can be summarized as, in this work, experimental data of ground state charmed meson masses is used to constrain 11 parameters in the effective Lagrangian to compute the masses, including higher-loop corrections. The coupling constants and the mass parameters in the effective Lagrangian reproduce QCD in specific limits and represent important input parameters for the description of hadron properties like heavy–light charm meson masses and mass splitting. The results are obtained and matched with the available experimental and theoretical data. Also, to check the accuracy of the observed residual masses, these calculated masses have been used in decay width formulas and the coupling constant ($h$) and decay width of the excited state calculated, and they match with the available theoretical and experimental information.

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