THE GLUON CHAIN MODEL REVISITED

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Abstract. I describe how the gluon chain model of QCD string formation meets a number of criteria which are required of any theory of the confining force, including: the correct center dependence and (at large-N) Casimir scaling of the string tension, the logarithmic broadening of the QCD flux tube, and the existence of a Lüscher term in the static quark potential.

1. Introduction

The gluon chain model [1-4] is a picture of the formation and composition of the QCD string in terms of the perturbative excitations of the theory. In this talk I would like to discuss some recent work [5] in this area, which was carried out in collaboration with Charles Thorn.

Normally gluon (particle) excitations are useful for describing high-energy scattering processes, while non-perturbative effects, such as confinement and chiral symmetry breaking, are usually ascribed to some special class of field configurations with particular topological properties. However, it is not excluded that particle and field descriptions of various phenomena in QCD can overlap in some cases. An example is the color screening of higher representation Wilson loops. In this case one is easily convinced, just by thinking about string breaking due to gluon pair production, that the asymptotic string tension of a higher-representation loop can depend only on the transformation properties of the representation with respect to the center subgroup (i.e. on the “N-ality” of the representation). For an adjoint string in particular, one ends up with two gluelump states, each consisting

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of a gluon bound to a heavy source. On the other hand, from the “field” point of view, the area law falloff of the Wilson loop is due to large scale vacuum fluctuations of some kind. The N-ality dependence of the string tension tells us that these large-scale field fluctuations must induce fluctuations in Wilson loop holonomies only among the center elements, rather than coset elements, of the SU(N) gauge group. This simple fact implies (very strongly, in my opinion) the existence of a center vortex mechanism of some kind.

In principle we then have two complementary ways of understanding the N-ality dependence of the QCD string tension: (i) in terms of color-screening by constituent gluons; and (ii) in terms of the vortex confinement mechanism. But if we can picture string breaking in terms of particle excitations, is it also possible to describe string formation in terms of constituent gluons? As it happens, large-$N_c$ considerations lead to just such a description. Consider a very high-order planar Feynman diagram contributing to a Wilson loop expectation value, as shown in Fig. 1. A time-slice of the diagram reveals a sequence of gluons, interacting only with their nearest neighbors in the diagram. If we take this picture seriously, it suggests that the QCD string might be regarded, in some gauge, as a “chain” of constituent gluons, with each gluon held in place by its attraction to its two nearest neighbors in the chain.

![Figure 1](image)

*Figure 1.* The gluon chain as a time slice of a planar diagram (shown here in double-line notation). A solid (open) hemisphere indicates a quark (antiquark) color index.

The linear potential in this “gluon chain” model comes about in the following way: As the heavy quarks separate, we expect that at some point the interaction energy increases rapidly due to the running coupling. Eventually, it becomes energetically favorable to insert a gluon between the quarks, to reduce the color charge separation. As the quarks continue to separate, the process repeats, as shown in Fig. 2, and we end up with a chain of gluons. The average gluon separation $R$ along the axis joining the quarks is fixed, regardless of the quark separation $L$, and the total energy of the chain is the energy per gluon times the number $N$ of
gluons in the chain, i.e.

$$E_{\text{chain}} \approx NE_{\text{gluon}} = \frac{E_{\text{gluon}}}{R} L = \sigma L$$

(1)

where $E_{\text{gluon}}$ is the (kinetic+interaction) energy per gluon, and $\sigma = E_{\text{gluon}}/R$ is the string tension. In this picture, the linear growth in the number of gluons in the chain is the origin of the linear potential.

**I.**

**II.**

**III.**

Figure 2. Constituent gluons (small circles) appear as quarks separate, to keep the average color charge separation below some maximum value. Dotted lines indicate nearest-neighbor interactions.

Every theory of confinement aims at explaining the linearity of the static quark potential at large distances. However, it is by now well understood that linearity is only one of several conditions that a theory of the confining force must satisfy. The requirements also include:

1. **Casimir Scaling.** At intermediate distance scales, the string tension should be proportional to the quadratic Casimir of the quark color group representation.
2. **Center Dependence.** Asymptotically, the string tension should depend only on the N-ality, not the Casimir, of the quark color group representation.
3. **String Behavior:**
   - **Roughening.** There is a logarithmic broadening of the flux tube with quark separation $L$.
   - **Lüscher term.** There is a universal $-c/L$ term in the static quark potential at large distances.

Taken together, this is a challenging set of conditions. The abelian projection theory, for example, has difficulties with both Casimir scaling [10] and center dependence [11]. Other proposals, which seek to derive confinement from a $1/k^4$
behavior of the gluon propagator in some gauge, will run into trouble when con-
fronted with roughening and the Lüscher term. So does the gluon chain model,
which is yet another idea about confinement, do any better in meeting these con-
ditions?

Having already discussed the linearity of the static potential, let us consider
Casimir scaling. In any SU($N_c$) representation $r$, the group character can be ex-
pressed as a product

$$\chi_r[g] \propto \left(\chi_F[g]\right)^n \left(\chi_F^*[g]\right)^{\bar{n}} + \text{sub-leading terms.}$$

where $F$ denotes the fundamental representation. By factorization at large-$N_c$, a
Wilson loop in representation $r$ has a string tension

$$\sigma_r = M_r \sigma_F$$

at $N_c \to \infty$, where $M_r = n + \bar{n}$. In this limit, the quadratic Casimir is $C_r = M_r N_c / 2$.
Exact Casimir scaling is therefore a property of the planar limit. The gluon chain
model, which is motivated by large-$N_c$ considerations, inherits this property. In
the gluon-chain model, there are $M_r$ chains terminating at each heavy source,
and these are non-interacting at $N_c \to \infty$. The total energy of the system is then
proportional to the number of chains times the length of each chain, and in this
way Casimir scaling

$$\sigma_r \propto C_r \quad (N_c \to \infty)$$

is obtained at large $N_c$. The situation for heavy adjoint-representation quarks,
which have two chains between them, is shown in Fig. 3(I).

The appropriate center dependence of the string tension is due to color screen-
ing. Screening is accomplished by a $(1/N_c^2$ suppressed) contact interaction be-
 tween constituent gluons in different chains, leading to string-breaking processes
such as those shown in Fig. 3.

As for string behavior, the gluon chain is clearly some sort of discretized
string, so string-like properties are not entirely unexpected. I will return to this
topic in section 4, below.

2. The Force Renormalization Scheme

The growth of the running coupling with color charge separation is an essential
ingredient of gluon chain picture, but the relationship of the running coupling to
the interaction potential, which depends on the choice of renormalization scheme,
is an important consideration. Suppose one defines the running coupling in terms of
the static potential via

$$V_{qq}(R) = -\left(1 - \frac{1}{N_c^2}\right) \frac{N_c g_s^Y(R)}{2R}.$$


III. 

II. 

I. 

Figure 3. Adjoint string-breaking in the gluon chain model. Two gluons in separate chains (I) scatter by a contact interaction, resulting in the re-arrangement of color indices indicated in II. This corresponds to chains starting and ending on the same heavy source. The chains then contract down to smaller “gluelumps” (III).

![Graph](image)

Figure 4. A running coupling in the V-scheme which is monotonically increasing takes the potential in the negative-V direction, away from the actual potential.

This is the V-renormalization scheme. But if $\alpha_V^s$ grows monotonically, the resulting behavior of $V(R)$ is the opposite of what is required for confinement, behaving
roughly like the curve labeled “running coupling” in the sketch shown in Fig. (4). In order that the potential becomes positive at some point, as it does in the actual potential found in numerical simulations, it is obviously necessary that \( \alpha_s^\prime(R) \) becomes smaller and eventually changes sign as \( R \) increases. This is asking a lot of perturbation theory, which tends in the opposite direction. A better renormalization scheme, advocated by Grunberg [7], Sommer [8], and also by Thorn, is to define the running coupling in terms of the force

\[
|F(R)| = \left(1 - \frac{1}{N_c^2}\right) \frac{N_c \alpha_s^F(R)}{2R^2}
\]

and derive the potential at intermediate distances from integration

\[
V_{qq}(R) = V_{qq}(R_A) + \int_{R_A}^R dR \left| F(R) \right|
\]

In this “F-scheme” the running coupling doesn’t have to change sign, and in fact the three loop term in the beta function

\[
\beta(g) = -g^3 \left( \frac{11}{(4\pi)^2} + \frac{102}{(4\pi)^4} g^2 + \frac{1.65}{(4\pi)^3} g^4 + \ldots \right)
\]

is substantially smaller (by a factor of 2.6) in the F-scheme than the V-scheme. The three-loop potential has been computed in the F-scheme by Necco and Sommer in ref. [6]. I have extracted Fig. 5 below from their article; this figure displays the perturbative potential compared to the corresponding Monte Carlo data. The parameter \( r_0 \approx 0.5 \) fm is the Sommer scale. Notice that the F-scheme gives a remarkably good fit to the numerical data almost up to the Landau pole, while the V-scheme results in a very poor fit to the data in this interval.

According to a rigorous theorem [9], the static potential must be concave downwards; i.e. the force cannot increase with distance. This implies that the the perturbative 3-loop result must certainly break down at quark separation \( L \approx 0.6r_0 \approx 0.3 \) fm in the F-scheme. In the gluon-chain model, the rising force is avoided by inserting constituent gluons between the quarks. From the breakdown of perturbation theory at \( L = 0.6r_0 \), we can estimate that the inter-gluon separation along the line joining the quarks is \( R = 0.3r_0 \).

3. A Variational Approach

Let \( \Psi_0[A] \) be the QCD vacuum wavefunctional. An excited state can be written as

\[
\Psi_{ex}[A] = Q[A]\Psi_0[A].
\]

e.g. for a glueball

\[
\Psi_G[A] = \sum_{N=1}^{N_{max}} \Psi_G^{(N)}[A]
\]
Figure 5. The static potential in the F and V-renormalization schemes, compared to numerical data. This figure is taken from Necco and Sommer, ref. [6].

where

\[
\Psi_{G}^{(N)}[A] = \left\{ \int d\vec{x}_1 d\vec{x}_2 \ldots d\vec{x}_N f_{\mu_1 \mu_2 \ldots \mu_N} (\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_N) \right. \\
\left. \text{Tr} A_{\mu_1}(\vec{x}_1) A_{\mu_2}(\vec{x}_2) \ldots A_{\mu_N}(\vec{x}_N) \right\} \Psi_0[A] \tag{11}
\]

is the N constituent gluon state. The energy is

\[
E = \frac{\langle \Psi_{ex} | H | \Psi_{ex} \rangle}{\langle \Psi_{ex} | \Psi_{ex} \rangle} - \langle H \rangle \tag{12}
\]

Defining

\[
Q_t = Q[A(\vec{x}, t)] \tag{13}
\]

it is easy to show that

\[
E = -\frac{1}{2} \lim_{T \to 0} \frac{d}{dT} \log \langle Q_T^T Q_{-T} \rangle \tag{14}
\]
where \( ⟨...⟩ \) represents the Euclidean VEV.

The idea is to choose \( Q \) as a function of a few parameters, and then determine the optimal parameters by minimizing the value of \( E \) computed perturbatively [3]. For the gluon chain, we propose to use

\[
Q_{\text{chain}}[A] = q^{a_1}(0) \left\{ \int d\vec{x}_1 d\vec{x}_2 ... d\vec{x}_N \, \psi_{\mu_1 \mu_2 ... \mu_N}(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N) \right. \\
\left. A_{\mu_1}^{a_1} A_{\mu_2}^{a_2} ... A_{\mu_N}^{a_N}(\vec{x}_N) \right\} \hat{q}^{a_{N+1}}(L)
\]

Before trying this approach in the full-blown field theory, it is interesting to apply these ideas to a simple quantum-mechanical model with most of the same qualitative features. Let us take the multi-gluon Hamiltonian to be

\[
H = \sum_{n=1}^{N-1} p_n^2 + \sum_{n=2}^{N-1} V(\vec{x}_n - \vec{x}_{n-1}) + V_{qg}(\vec{x}_1) + V_{qg}(L - \vec{x}_{N-1})
\]

The **Product Ansatz** for the (N-1)-gluon chain ground state is

\[
Ψ(\vec{x}_1, \vec{x}_2, ..., \vec{x}_{N-1}) = A \prod_{i=1}^{N} \psi(u_i)
\]

where

\[
u_i = \vec{x}_i - \vec{x}_{i-1}
\]

and

\[
\psi(u) = e^{-u^2/2r^2}
\]

with the constraint

\[
\vec{x}_0 \equiv 0 , \quad \vec{x}_N = L
\]

After some work, one finds that the string tension (= energy/unit length) is

\[
\frac{E}{L} = \frac{1}{rR} \sqrt{\frac{8}{\pi}} + \frac{1}{R} \langle V(u) \rangle
\]

In particular, taking for \( V \) the instantaneous Coulomb potential \( V(u) = -C_F \alpha_s / |u| \), we have

\[
\frac{E}{L} = \frac{1}{rR} \sqrt{\frac{8}{\pi}} - \frac{C_F \alpha_s}{R^2} \text{erf} \left( \frac{R}{r} \right)
\]

where both \( r \) and \( N \) (no. of gluons), or equivalently \( r \) and \( R = L/N \), are taken as variational parameters.

Our second trial wavefunction, the **String Ansatz**, is adapted from discretized light-cone string theory [12]:

\[
a_m^{a_i} \Psi(\vec{x}_1, \vec{x}_2, ..., \vec{x}_{N-1}) = 0 \quad \left\{ \begin{array}{l} m = 1, 2, ..., N-1 \\ i = 1, 2, 3 \end{array} \right.
\]
with

\[ \vec{x}_k = \frac{L}{N} k + \sqrt{\frac{2}{NT_0}} \sum_{m=1}^{N-1} \frac{1}{\sqrt{2\omega_m}} (\vec{a}_m + \vec{a}_m^\dagger) \sin \left( \frac{m\pi}{N} k \right) \]

\[ \vec{p}_k = -i \sqrt{\frac{2T_0}{N}} \sum_{m=1}^{N-1} \sqrt{\frac{\omega_m}{2}} (\vec{a}_m - \vec{a}_m^\dagger) \sin \left( \frac{m\pi}{N} k \right) \]  

(24)

where \( \omega_m = 2\sin \frac{m\pi}{2N} \). Define \( r \equiv \frac{2}{\sqrt{\pi T_0}} \). For a Coulomb potential, we then find

\[ \frac{\mathcal{E}}{L} = \frac{1}{rR} \left[ \frac{8}{\pi^{3/2}} + \langle V(u) \rangle \right]. \]

(25)

Comparison with the product ansatz shows that the string ansatz achieves a slightly lower energy.

To estimate the string tension, we begin from

\[ \frac{\mathcal{E}}{L} = \frac{1}{R} \left[ \frac{8}{\pi^{3/2}} + \langle V(u) \rangle \right]. \]

(26)

and make the approximation that \( \langle V(u) \rangle = V(s) \), where

\[ \frac{1}{s} \equiv \langle \frac{1}{|u|} \rangle = \frac{1}{R} \operatorname{erf} \left( \frac{R}{r} \right) \]

(27)

Minimizing wrt \( R \) and \( r \) results in two conditions

\[ \frac{\mathcal{E}}{L} = \frac{\partial s}{\partial R} F(s) \]

\[ \frac{8}{\pi^{3/2}} \frac{1}{r^2} = \frac{\partial s}{\partial r} F(s) \]

(28)

which together imply

\[ \sigma = \frac{\partial s}{\partial R} \left( \frac{\partial s}{\partial r} \right)^{-1} \frac{8}{\pi^{3/2}} \frac{1}{r^2} \]

(29)

Given \( F(s) \equiv dV/ds \), we could determine \( r, R \) and \( \sigma \). Even without \( F(s) \), we can still make a “ballpark” estimate of \( \sigma \). First, the static force increases beyond \( L = 0.6r_0 \), so a constituent gluon must appear between the quarks at that point, to stabilize the force. This suggests that \( R \approx 0.3r_0 \). Secondly, the running coupling \( \alpha(s) \) must be \( \mathcal{O}(1) \), if the growth in potential energy over-compensates the falloff in kinetic. This argues for \( s \approx 0.6r_0 \) and \( r \approx 0.63r_0 \). With these guesses, one finds \( \sigma = 1.26/r_0^2 \), which is to be compared with the experimental value \( \sigma_{\exp} = (430 \text{ Mev})^2 = 1.18/r_0^2 \). We conclude that the value of the string tension \textit{may} work out in a genuine field-theory/variational calculation, but of course this remains to be seen.
4. String Behavior

Using the string ansatz wavefunction, one finds for the transverse coordinate of the $k$-th gluon in the chain

\[
(0|x_{2\perp}k|0) = \frac{D-2}{2\pi T_0} \sum_{m=1}^{N-1} \frac{\sin^2(m\pi k/N)}{\sin(m\pi/2N)} \\
\sim \frac{D-2}{2\pi T_0} \ln N \\
= \frac{r^2(D-2)}{8} \ln \frac{L}{R} \tag{30}
\]

which demonstrates the expected logarithmic broadening of the flux tube with quark separation. Note that there is no need to insert a high-frequency cutoff; this is taken care of by the discreteness of the gluon chain.

The total energy of the gluon chain in the string ansatz can also be computed, and the result minimized wrt the variational parameters. The result is

\[
\mathcal{E} = \frac{L}{\pi^{3/2}} \int d^3u \, e^{-\frac{1}{2} \vec{R} \cdot \nabla \bar{V} \left( \vec{R} + 2\vec{u} \sqrt{\Delta_1} \right)} \\
- \frac{\pi}{24L} \left( \frac{4R}{r\sqrt{\pi}} \right) - \left[ 1 + \frac{\pi}{2} \right] \frac{4}{r\pi^{3/2}} \tag{31}
\]

(see ref. [5] for details). The first term on the rhs is the linear potential, the second is a Lüscher-like term. For the bosonic string, the factor multiplying $\pi/24L$ in the Lüscher term would be $D-2=2$. With our estimate above of $R \approx r/2$ this coefficient is estimated to be $2/\sqrt{\pi} \approx 1.13$. At this stage, however, numerical values should not be taken too seriously.

5. Numerical Studies

It is possible to investigate the constituent gluon composition of the QCD string by numerical methods, and in fact a study along these lines was carried out many years ago (second reference of ref. [2]). The idea is the following: Observe that the ground state of the QCD flux tube, in a physical gauge, can be written as

\[
\Psi_{\text{string}}[A_\mu(x,t=0)] = \\
\lim_{T \to \infty} \left(W(R,2T)Z\right)^{-1/2} \int DA_\mu(x,t<0) \delta[F(A)]\Delta[A] \\
\times P\exp[i \oint_{C_-} dx^\mu A_\mu] \exp \left[ - \int_0^{\infty} dt \, L[A] \right] \tag{32}
\]
where $C_-$ is the $R \times T$ open contour shown in Fig. 6. The $R \times 2T$ Wilson loop factor $W(R, 2T)$ ensures that $\Psi_{\text{string}}$ is normalized to unity.

If the gluon chain model is correct, then $\Psi_{\text{string}}$ can also be expressed as a sum of $n$-gluon states

$$\Psi_{\text{string}}[A] = \sum_n c_n \Psi_n[A]$$

$$\Psi_n[A] = N_n Q_n \Psi_0[A]$$

(33)

where $Q_n$ is an $n$-gluon operator, and

$$N_n = \left( \frac{1}{2} \text{Tr}[Q_n^\dagger Q_n] \right)^{-1/2}$$

(34)

is a normalization constant. Assuming the $\Psi_n$ states are orthogonal, the overlap of an $n$-gluon state with the true string state is given, in the $T \to \infty$ limit, by

$$c_n = \frac{\left\langle \text{Tr}[Q_n^\dagger \exp[i \oint_{C_-} dx A_\mu]] \right\rangle} {\sqrt{\left\langle \text{Tr}[Q_n^\dagger Q_n] \right\rangle W(R, 2T)}}$$

(35)

The second article of ref. [2] used the following trial operators on the lattice (with $A_\mu = (U_\mu - U_\mu^\dagger)/2i$):

$$Q_0^{ab} = \delta^{ab}$$

$$Q_1^{ab} = \sum_{x_1 < x < x_2} \overline{A}_x^{ab}(x)$$

$$Q_2^{ab} = \sum_{x_1 < x < x_2} \sum_{x' < x_2} \left[ \overline{A}_x^{ac}(x) \overline{A}_{x'}^{cb}(x') - \frac{1}{2} \text{Tr}[\overline{A}_x(x) \overline{A}_{x'}(x')] \right]$$

(36)

where

$$\overline{A}_x^{ab}(x) = \sum_{y,z} A_x^{ab}(x,y,z) e^{-\delta y z}$$

(37)
is a “smeared” vector potential, \( r_\perp \) is the transverse distance from the flux tube axis, and \( \delta \) is a variational parameter. The corresponding zero, one, and two-gluon overlaps \((c_0, c_1, c_2)\) were then computed from eq. (35) via lattice Monte Carlo. It was found that for small \( R \), the zero-gluon overlap \( c_0 \) is the largest of the three overlaps. As \( R \) increases, the \( c_0 \) overlap falls and the \( c_1 \) term becomes dominant, and at the largest \( R \) separations that were used in the simulations we found the two-gluon overlap \( c_2 \) becoming the largest of the three. The sum of the three overlaps accounts for about 90% of the norm of \( \Psi_{\text{string}} \), which means that for the given range of \( R \) the sum of zero, one and two gluon states is a fairly good approximation to the true flux tube state. For details, please see the cited reference.

All of this is in good qualitative agreement with the gluon chain picture, but the old results can surely be much improved. It may be useful to repeat the investigation of ref. [2] with modern computers and better noise-reduction techniques.

6. Conclusions

The gluon-chain model offers a simple and concise explanation of many features of the confining force — Casimir scaling at large-\( N_c \), center dependence, roughening, and the Lüscher term — which are problematic in many other approaches. The model is essentially a “particle” picture of string formation, and I regard it as complementary to the “field” explanation of confinement in terms of center vortices. The next step will be to apply the field theory/variational approach outlined above to obtain quantitative estimates for the string tension and, perhaps, for the masses of the low-lying glueballs.

References

1. C. B. Thorn, Phys. Rev. D19 (1979) 639; D20 (1979) 1435; D20 (1979) 1934.
2. J. Greensite, Nucl. Phys. B249 (1985) 263; B315 (1989) 663.
3. J. Greensite and M. Halpern, Nucl. Phys. B271 (1986) 379.
4. C. B. Thorn, Phys. Rev. D51 (1995) 647.
5. J. Greensite and C. B. Thorn, J. High Energy Phys. 02 (2002) 014; hep-ph/0112326.
6. S. Necco and R. Sommer, Phys. Lett. B523 (2001) 135; hep-ph/0109093.
7. G. Grunberg, Phys. Rev. D40 (1989) 680.
8. R. Sommer, Nucl. Phys. B411 (1994) 839.
9. C. Bachas, Phys. Rev. D33 (1986) 2723.
10. L. Del Debbio, M. Faber, J. Greensite, and S. Olejnik, Nucl. Phys. Proc. Suppl. 53 (1997) 141, hep-lat/9607053.
11. J. Ambjörn, J. Giedt, and J. Greensite, J. High Energy Phys. 02 (2000) 033, hep-lat/9907021.
12. C. B. Thorn, Nucl. Phys. B263 (1986) 493.