On the impact of unsprung weight of mobile vehicles

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Abstract: The article analyzes the impact of the value of unsprung weight on the main operating characteristics of mobile vehicles and on the load incurred by the elements of suspension, wheels and tires. It is concluded that the reduction in the unsprung weight has a positive impact on drawing and dynamic as well as fuel economy features of mobile vehicles, their stability and maneuverability, on the load incurred by wheels, tires and roads. At the same time, it has an insignificant influence on the performance of springing or dissipative elements of the suspensions as well as on the smoothness of the ride.

1. Introduction
It is well known that the total weight of the mobile vehicle is divided into sprung and unsprung ones. The former includes the weight of all the components influencing the springing elements of the suspension. If the weight of the elements does not impact the springing elements of the suspension, it is considered unsprung. The latter is typically several times smaller than the former one.

As it is widely known that the unsprung weight affects the smoothness of the ride, stability and maneuverability of the vehicle as well as drawing, speed and fuel economy characteristics of the mobile vehicle, and the load of the elements of suspension, wheels and tires, there is a trend to lowering this weight. The case with stability, maneuverability, drawing and braking dynamics is rather clear as the bigger the weight, including the unsprung one, the more difficult it is to accelerate, decelerate, or change the direction of the vehicle, relation between the smoothness of the ride and the load of the elements is not that transparent.

2. Formulating the task
There is an ongoing debate on the optimal ratio between the sprung and unsprung weights. For example, the analogy between colliding balls is often referred to: the smaller the weight of the ball that is hit, the shorter the distance it will go [2]. In other words, the lower the value of the unsprung weight in relation to the sprung one, the shorter the distance and the lower the acceleration of the latter affected by the oscillations of the latter arising as a result of the kinematic impact from the road surface. This is definitely justified. However, on the other hand, the higher the value of the unsprung weight, the less it will react to external factors (with similar tire characteristics). Therefore, the impact on its sprung weight will be lower.

This discourse leads us to further question the increase in the load of springing and dissipative elements of the suspension with the rise of the unsprung weight.

3. Solution
Let us carry out a numeric investigation of the task to identify amplitude-frequency characteristics (AFC) of the movements, speeds and accelerations of sprung and unsprung weights [1], which will
allow us to determine the load on tires, springing and dissipative elements of the suspension and the smoothness of the ride. The solution will be linear due to the simplification of the assumption on the relation between the oscillations of front and rear sprung parts. The respective calculation model is featured in figure 1. The mathematical model will be as follows:

\[
\begin{align*}
M\ddot{y} + c_p (y - \xi) + k_p (\dot{y} - \dot{\xi}) &= 0; \\
m\ddot{\xi} - c_p (y - \xi) - k_p (\dot{y} - \dot{\xi}) + c_{sh} (\xi + q(t)) + k_{sh} (\dot{\xi} + \dot{q}(t)) &= 0,
\end{align*}
\]

where \( M, m \) are sprung and unsprung weights of the mobile vehicle; \( y, \xi \) are the vertical movements of sprung and unsprung weights; \( c_p, k_p \) are reduced rigidity factor of the springing element and viscous drag coefficient for the dissipative element; \( c_{sh} \) is the tire normal stiffness coefficient; \( q(t) \) is the kinematic impact from the unevenness of the road surface.

We assume that \( q(t) \) is a harmonic function with the cyclical frequency of \( \nu \).

As the differential equations in the system are linear, we can apply the Laplace transformations to them and present them as a system of algebraic equations with complex variables \( \tilde{y}(s), \tilde{\xi}(s), \tilde{q}(s) \), where \( s = a + bi \). With zero initial conditions, it will be stated in the following way:

\[
\begin{align*}
\left\{ \begin{align*}
(Ms^2 + k_p s^2 + c_p)\tilde{y}(s) + (-k_p s - c_p)\tilde{\xi}(s) &= 0; \\
-k_p s\tilde{q}(s) + (ms^2 + (k_p + k_{sh})s + c_p + c_{sh})\tilde{\xi}(s) &= (-k_{sh}s - c_{sh})\tilde{q}(s)
\end{align*} \right.
\]

As the kinematic impact is described using a harmonic function with the rotational frequency \( \nu \), the constrained oscillations will also have the frequency equalling \( \nu \). This is why we replace the complex variable \( s \) with its imaginary component \( vi \). As a result, we get a system of equations related to the unknown frequency characteristics \( \tilde{y}(vi), \tilde{\xi}(vi) \) of the corresponding functions \( y(t), \xi(t) \):

\[
\begin{align*}
\left\{ \begin{align*}
(-M\nu^2 + k_p vi + c_p)\tilde{y}(vi) + (-k_p vi - c_p)\tilde{\xi}(vi) &= 0; \\
-k_p vi\tilde{q}(vi) + (m\nu^2 + (k_p + k_{sh})vi + c_p + c_{sh})\tilde{\xi}(vi) &= (-k_{sh} vi - c_{sh}).
\end{align*} \right.
\]

For the sake of convenience, we use a single external impact amplitude in this system, i.e. \( \tilde{q}(vi) = 1 \).

Let us take the following values by way of example: \( M = 6000 \text{kilos}; \ c_p = 870 \text{kN m}^{-1}; \)
\( c_{sh} = 3236 \text{kN m}^{-1}; \ k_p = 12390 \text{Ns m}^{-1}; \ k_{sh} = 2544 \text{Ns m}^{-1}. \)

Figure 2 features the graphs of amplitude and frequency characteristics of the movements of the vehicle body (unbroken line) and of the wheels (broken lines). In this graph and in the following ones (Figures 2–5, 7–9) the red color refers to the unsprung weight of 1200 kilos, the blue color stands for that of 800 kilos, the green color is for 400 kilos. As it is seen in Figure 2, the change in the value of the unsprung weight does not provoke any substantial changes as all the three curves are practically

![Figure 1. Calculation model.](image-url)
identical. However, this is not the case with the AFCs of the wheel movement: the reduction in the value of the unsprung weight, the amplitude of the wheel movement decreases, whereas the extreme point corresponding to the area of high-frequency resonance shifts into the area of higher frequencies. Concurrently, dispersion, and correspondingly, mean square deviations (MSD) of the shift increase. If we compare the results for 1200 kilos and 400 kilos, the MSD will increase by approximately 8%.

Figure 3 shows the graphs for AFCs of the body accelerations. The reduction in the unsprung weight has a little effect on the acceleration values in the area of low-frequency resonance, however, it reduces their amplitude in the area of high-frequency one, shifting the extreme point into the area of higher frequencies. MSD of accelerations with the reduction of the unsprung mass from 1200 kilos to 400 kilos has vice versa increased (instead of decreasing!) by approximately 65%.

Figure 4 illustrates AFCs of the speeds of the sprung (unbroken lines) and unsprung (broken lines) weights that allow to analyze the load on dissipative elements of the suspension. We observe the same pattern of shifting the extreme point to the area of higher frequencies and a significant increase in the mean square accelerations of the unsprung weights with the reduction in its value.

It should be mentioned that the reduction in the unsprung weight will have practically no effect on amplitude frequency characteristics of movements and speeds of sprung and unsprung weights. However, it will have a considerable negative effect on the smoothness of the ride, which is common knowledge. Figure 5 illustrates AFCs of the vehicle body acceleration with $M = 4000$ kilos.
Figure 5. Sprung weight of 4000 kilos acceleration AFCs.

As the real micro-profile of the bearing surface, on which the vehicle moves, has uneven spots of various lengths, the final conclusion on the impact of the value of the unsprung weight will depend, other factors being equal, on the micro-profile characteristics and the motion speed. To exemplify this, Figure 6,a shows the graphs of spectral densities of the impact of the y-coordinates of the micro-profile of the asphalt road at the speed of vehicle movement at 80 km h\(^{-1}\) (red), 60 km h\(^{-1}\) (blue), 40 km h\(^{-1}\) (green). Figure 6,b gives a similar graph for a paved road with the speed of 30 km h\(^{-1}\) (red), 20 km h\(^{-1}\) (blue), 10 km h\(^{-1}\) (green). It is clear that with the real speeds of movement at the frequencies above 3.5 – 4 Hz the graphs for each micro-profile are practically identical. It follows from Figures 2 – 5, the significant difference in AFCs begins with these frequencies specifically.

Figure 6. Spectral densities of impact of y-coordinates of the road surface micro-profile.
Figures 7 and 8 features the graphs of spectral densities of (respectively) speeds (the indicators used are the same as in figure 2); figure 9 shows the vehicle body acceleration at the movement of the mobile vehicle on the asphalt road at the speed of 60 km h⁻¹, figure 10 makes more vivid the mean square deviations of dynamic loads on tires (red), on springing elements of the suspension (blue), on dissipative elements of the suspension (green). Figure 11 shows the MSD of the vehicle body acceleration. As it is seen from these figures, the change in the value of the unsprung weight has an insignificant effect on the load on springing and dissipative elements of the suspension, on the smoothness of the ride, and only tires incur a considerable increase in static and dynamic forces. Thus with the increase in the unsprung weight from 400 kilos to 1200 kilos (by 200), the static load rose by 12.5%, and the MSD of dynamic forces by 17.5% approximately. A similar increase in the force is observed in the area of the tire contract with the road surface.

**Figure 7.** Spectral densities of the movement of sprung and unsprung weights.

**Figure 8.** Spectral densities of speeds of sprung and unsprung weights.

4. Conclusions
Thus the results of the calculations conducted allow to conclude that the reduction of unsprung weights has a positive impact on the drawing, dynamic and fuel economy characteristics of mobile
vehicles, their stability and maneuverability, the load on the wheels, tires and roads. It also has an insignificant effect on the operating conditions of springing and dissipative elements of the suspensions as well as on the smoothness of the ride. This contradicts the generally accepted opinion stated at the beginning of the article.

The following stage of the research will involve a more precise solution of the problem in the non-linear mode.

References
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[2] https://www.kolesa.ru/article/chto-takoe-nepodressorennaya-massa-i-na-chto-ona-vliyayet

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