Implementing Genuine Multi-Qubit Entanglement of Two-Level-System Inside a Superconducting Phase Qubit

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(Dated: April 8, 2009)

The interaction between a superconducting phase qubit and the two-level systems locating inside the Josephson tunnel barrier is shown to be described by the XY model, which is naturally used to implement the iSWAP gate. With this gate, we propose a scheme to efficiently generate genuine multi-qubit entangled states of such two-level systems, including multipartite W state and cluster states. In particularly, we show that, with the help of the phase qubit, the entanglement witness can be used to efficiently detect the produced genuine multi-qubit entangled states. Furthermore, we analyze that the proposed approach for generating multi-qubit entangled states can be used in a wide class of candidates for quantum computation.

PACS numbers: 03.67.Mn, 42.50.Dv, 85.25.Cp

I. INTRODUCTION

Generation of entangled states of an increasing number of qubits has been an important goal and benchmark in the field of quantum information. Multi-qubit entangled states serve as the essential physical resources for measurement-based quantum computing and quantum error-correcting codes. Some of them, such as W states, GHZ states and cluster states, have been investigated both theoretically and experimentally; however, the experimental preparation of multi-particle entanglement has been proved to be extremely challenging. To date, the entangled states up to eight atoms or six photonic qubits have been experimentally reported. As for solid-state systems, due to the difficulty to decouple the qubits with the environments, only the two-qubit entanglement of the superconducting qubits has been demonstrated in experiments. Therefore, generation of up to ten qubits entangled states of the candidates for solid-state quantum computation will be a next significant and very challenging step towards quantum information processing.

Among the solid-state systems, superconducting circuit is one of the most promising candidates served as hardware implementation of quantum computer. But the short coherence time limits both of the qubit state manipulation and information storage. The loss of quantum coherence in the most of solid state qubits is mainly due to the unwanted coupling of the qubits with the environments. In particular, the coherence time would be decay quickly with increasing number of qubits because each qubit usually has a control and a measurement circuit. Besides fulfilling the manipulations and measurements required for the necessary information processing, all of circuits can also disturb the qubits and lead to decoherence. Therefore, a possible way to experimentally prepare more than two-qubit entangled states of solid state systems, which have not yet been demonstrated, may need to suppress the decoherence from the environments, such as to reduce the number of control and measurement lines.

In this paper, we propose a distinct scheme to prepare genuine multipartite entangled states for several to ten qubits, while the decoherence from the control and measurement lines may be minimized by using single control and measurement set-up. The system we have in mind is several to ten of two-level systems (TLSs) locating inside a superconducting phase qubit, e.g., a current-biased Josephson junction (CBJJ). Recent experiments have shown that some of TLSs locate inside the Josephson tunnel barrier, while the parameters for such TLSs can be detected through spectroscopic measurements. The lifetime of the TLS is much longer than the decoherence time of the phase qubit, thus TLS can be used as high quality quantum memory. Furthermore, macroscopic quantum jump has been experimentally demonstrated for a hybrid model consisting of a phase qubit and a TLS inside the Josephson tunnel barrier. In particular, it has been proposed that the TLS itself can be used as qubits for quantum computation, and typical operations required for the information processing, such as the state initialization, universal logical gate operations and readout, have been experimentally demonstrated.

Motivated by the progress, in this paper, we show that the interaction between the superconducting phase qubit and the two-level systems may be described by the XY model, which is naturally used to implement the iSWAP gate. With this gate, we can effectively generate various genuine multi-qubit entangled states for TLSs, including the W state and multipartite cluster states. Moreover, the states of TLSs can be manipulated by controlling the interaction between them and CBJJ. Such CBJJ-TLS coupling system offers a natural candidate to realize quantum information processing through combining the advantages of microscopic and macroscopic scale systems. In particularly, we found that such entangled states may be efficiently detected by measuring the entanglement witness with the help of the phase qubit. Finally, the proposed approach to produce genuine multi-qubit entanglement may be applied to various systems for quantum computing, including the trapped ions and C60 etc.

The paper is organized as follows. In sec. II, we briefly...
introduce the superconducting phase qubit and TLSs locating inside the Josephson tunnel barrier, and then we show that such TLSs can serve as qubits for information processing. Furthermore, we demonstrate that an iSWAP gate between the phase qubit and each of TLSs can be achieved. In Sec. III, a scheme to implement genuine multi-qubit entanglement of such TLSs is proposed. In Sec. IV, the detection of the prepared multi-qubit entangled states based on the entanglement witness is studied. In Sec. V, we show that the proposed approach can be used to generate genuine multi-qubit entangled states in a wide class of quantum systems, and the paper ends with a brief discussion.

II. TWO-LEVEL-SYSTEMS INSIDE A SUPERCONDUCTING PHASE QUBIT

The system we consider is a hybrid consisting of a standard superconducting phase qubit and several to ten TLSs inside the Josephson tunnel barriers, as shown in Fig. 1. The superconducting phase qubit is a CBJJ, and recent experiments have shown that some of TLSs are located inside the Josephson tunnel barriers. Furthermore, such TLSs can be considered as qubits for the information processing, whereas the Josephson phase qubit itself is a ‘register’ qubit capable of general logic operations between TLSs qubits. The Hamiltonian of the phase qubit as shown in Fig. 1a reads

\[ H_p = \frac{1}{2C} \dot{Q}^2 - \frac{I_0 \Phi_0}{2\pi} \cos \delta - \frac{I \Phi_0}{2\pi} \delta, \]

where \( I_0 \) is the critical current of the Josephson junction, \( I \) is the bias current, \( C \) is the junction capacitance, \( \Phi_0 = h/2e \) is the flux quantum, \( \dot{Q} \) and \( \delta \) are the charge and gauge-dependent phase difference across the junction, which obeys the convectional quantizing commutation relation \([\delta, \dot{Q}] = 2\pi i\). For large area junctions, the Josephson coupling energy \( E_J = I_0 \Phi_0/2\pi \) is much larger than the single charging energy \( E_C = e^2/2C \). The phase is a well defined macroscopic variable and quantum behavior can be observed when the bias current is slightly smaller than the critical current. In this regime, the two lowest energy levels, \(|0\rangle \) and \(|1\rangle \), are usually employed as two quantum states to form a so-called phase qubit. Truncating the full Hilbert space of the junction to the qubit subspace, the phase qubit Hamiltonian can be written as

\[ H_P = -\frac{1}{2} \omega_{10} \sigma_z, \]

where \( \omega_{10} \) is the frequency difference between \(|0\rangle \) and \(|1\rangle \). The states of the qubit can be fully controlled with the bias current in the form of

\[ I(t) = I_{dc} + I_1 \cos \omega_{10} t + I_{\mu wc} \sin \omega_{10} t, \]

where the classical bias current is parameterized by \( I_{\mu wc} \), \( I_{dc} \), and \( I_{\mu wc} \).

The transition frequency \( \omega_{10} \) can be measured using spectroscopy and is a continuous function of bias current. It was found that some TLSs may locate inside the Josephson tunnel barrier (see Fig. 1a). A TLS is understood to be an atom, or a small group of atoms, that tunnels between two lattice configurations. A TLS can induce an energy splitting in the curve of the function \( \omega_{10}(I) \), as shown in Fig. 1(c). The splittings \( \Delta_j \in [20, 100] \) MHz were observed and they are separated by \( \delta f \approx 200 \) MHz on average in the spectroscopy. Therefore, one can characterize the positions and sizes of TLSs in the CBJJ energy spectrum through spectroscopic measurements. When the register qubit is detuned from the TLS by \( \delta f \), the effective coupling strength is described by \( \Delta_j^2/\delta f \). Therefore, with a splitting magnitude of several tens of MHz, it is reasonable to assume that only a single TLS satisfies the near-resonance condition while the other TLSs are far off-resonance. Furthermore, as the number of TLS mainly depends on the tunnel junction area, the design with small area and external low-loss capacitor (keeping the critical current constant) will greatly improve its performance. It is now possible to obtain a tunnel junction within about ten useful TLSs in the CBJJ spectroscopy ranging over \( \sim 2 \) GHz with the improved design.

As shown in Fig. 1(a), each TLS can be modeled as a charged particle that can tunnel between two nearby different positions with different wave functions \(|R\rangle \) and \(|L\rangle \) (correspond to critical currents \( I^R \) and \( I^L \)) within the tunnel barrier. The interaction Hamiltonian between the resonators and the critical-current is

\[ H_{int} = -\frac{I^R \Phi_0}{2\pi} \cos \delta \otimes |R\rangle \langle R| - \frac{I^L \Phi_0}{2\pi} \cos \delta \otimes |L\rangle \langle L|. \]
Assume a symmetric potential with energy separated by $\hbar \omega_i^j$ for the $i$th TLS, then the ground and excited states are $|g\rangle \simeq (|R\rangle + |L\rangle)/\sqrt{2}$ and $|e\rangle \simeq (|R\rangle - |L\rangle)/\sqrt{2}$. In practical experiments, the junction is biased near its critical current, $\delta \to \delta + \pi/2$ with $\delta \ll 1$, thus $\cos \delta \to \delta$. In the basis \{|0g\rangle, |1g\rangle, |0e\rangle, |1e\rangle\}, as shown in Fig.1(b), denoting $\delta_{ij} = \langle i|\delta|j\rangle$, usually $\delta_{ii} \approx 0$ and $\delta_{01} = \delta_{10} = 2\pi/\sqrt{2\hbar \omega_{10} C}$. Then the Hamiltonian of CBJJ-TLS system becomes

$$H_{\text{total}} = -\frac{\hbar \omega_{10}}{2} \sigma_z - \sum_j \left[ \frac{\hbar \omega_j^2}{2} \sigma_j^2_z + S_j \sigma_x^j \sigma_y^j \right], \quad (3)$$

where the effective coupling strength between the CBJJ and the $j$th TLS is $S_j = \frac{(I_R^j - I_L^j)}{2\sqrt{\hbar \omega_{10} C}}$. The Pauli matrices $\sigma_j^x, \sigma_j^y$ operate on the $j$th TLS states. The CBJJ and the $j$th TLS are tuned into resonance when the coupling term satisfy $|\hbar \omega_{10} - \hbar \omega_j^2| < S_j$. The parameters $\omega_j^2$ and $S_j$ of the $j$th TLS are usually known, but they can be independently determined by the energy splitting on the spectroscopy, as shown in Fig.1.

In the interaction picture and under the rotating-wave approximation, the effective interaction between the $j$th TLS and the phase qubit is given by

$$\hat{H}_{\text{int}}^j = -\frac{S_j}{2} (\sigma_x \sigma_j^x + \sigma_y \sigma_j^y). \quad (4)$$

So the effective CBJJ-TLS interaction is described as the XY model. Assume that the interaction between CBJJ and the $j$th TLS is tuned to the resonance with time $t$, one obtain the evolution operator given by

$$U_j(t) = \exp \left[ \frac{i t S_j}{2} (\sigma_x \sigma_j^x + \sigma_y \sigma_j^y) \right] \quad (5)$$

between them. This operator results in an oscillation between $|1g\rangle$ and $|0e\rangle$ at a frequency $S_j$, i.e.,

$$|0g\rangle \to |0g\rangle, |1e\rangle \to |1e\rangle, |1g\rangle \to \cos(S_j t)|1g\rangle - i \sin(S_j t) |0e\rangle, |0e\rangle \to \cos(S_j t) |0e\rangle - i \sin(S_j t) |1g\rangle. \quad (6)$$

Note that the iSWAP gate between CBJJ and TLS can be obtained when $t_j = \pi/(2S_j) \equiv \tau_j$. In the paper, the TLS states, $|g\rangle$ and $|e\rangle$, represent our logic qubit. The experiments have shown that the lifetime of such qubits is sufficiently long to carry out precise gate operations between the phase qubit and TLSs. In addition, we assume that the states of different TLSs are well separated from each other in frequency. Therefore, by adjusting the bias currents, the phase qubit and a TLS can be tuned into and out of resonance (turning on and off their coupling), i.e., it allows the independent manipulation of each TLS.

III. IMPLEMENTING THE $W$ AND CLUSTER STATES OF TWO-LEVEL-SYSTEMS

We now turn to demonstrate that the genuine entanglement of several to ten TLSs could be generated through the unitary operator described in Eq.(6) by controlling the interaction time between CBJJ and TLSs.

We assume that there are $N$ TLSs locating inside the Josephson tunnel barrier, while the coupling constant $S_j (j = 1, 2, \cdots, N)$ have been measured by using the spectroscopy. We first show that the $N$-qubit $W$ state can be achieved simply by two steps.

The first step is to initialize each TLS qubit in any general state $|\psi\rangle_j = \alpha_j|g\rangle_j + \beta_j|e\rangle_j$ to the ground state $|g\rangle_j$ by performing an iSWAP operation between this TLS and the phase qubit. We have demonstrated in the previous section that such isSWAP gate can be realized by switching on the interaction between the $j$th TLS and the phase qubit with the fixed time $\tau_j \equiv \pi/2S_j$. The ground state of the $j$th TLS is initialized by the following transformation

$$[i\text{SWAP}_{(j, P)}]|\psi\rangle_j \otimes |0\rangle \rightarrow |g\rangle_j \otimes (\alpha_j|0\rangle - i \beta_j|1\rangle). \quad (7)$$

with $[i\text{SWAP}_{(j, P)}]$ denoting the iSWAP operation between the $j$th TLS and the phase qubit, provided that the initial state of the phase qubit is $|0\rangle$. The ground state $\otimes_{j=1}|g\rangle_j$ for all TLSs is then realized after an iSWAP gate is performed between each TLS and the phase qubit, hybrid an operation to initialize the phase qubit to the state $|0\rangle$ between two iSWAP gates.

In the second step, the phase qubit is first tuned far off-resonance with all TLSs and excited on the $|1\rangle$ state. Then the phase qubit is adiabatically tuned into resonance with one of TLSs (e.g., the $j$th TLS), effectively turning on the coupling between this TLS and the phase qubit with the time $t_j$. Sequentially, the register qubit is interacted with every TLSs only once with appropriate time $t_j$, and then an operator $U_j = e^{-i\hat{H}_{\text{int}}^j t_j/\hbar}$ between the phase qubit and the $j$th TLS is performed. In this case (as shown in the Fig.2(a)), the final state of the TLSs and the phase qubit becomes

$$\prod_{l=1}^{N} U_l|1\rangle = \prod_{l=1}^{N} \prod_{j=1}^{N} U_l|\cos(S_1 t_1)|g\rangle_1 - i \sin(S_1 t_1)|0\rangle|e\rangle_1 \prod_{j=2}^{N} |g\rangle_j \nonumber$$

$$- \prod_{l=3}^{N} U_l|\cos(S_1 t_1)|g\rangle_1 (\cos(S_2 t_2)|1\rangle|g\rangle_2 \nonumber$$

$$- i \sin(S_2 t_2)|0\rangle|e\rangle_2 - i \sin(S_1 t_1)|e\rangle_1|g\rangle_2 |0\rangle \prod_{j=3}^{N} |g\rangle_j \nonumber$$

$$: \nonumber$$

$$\rightarrow -i|0\rangle \sum_{l=1}^{N} \prod_{j=1}^{N} \cos(S_1 t_j) \sin(S_1 t_j) \prod_{k \neq l}^{N} |g\rangle_k|e\rangle_l. \quad (8)$$

If the interaction time between the $j$th TLS and the phase qubit is chosen specifically as $t_j = \frac{\pi}{2S_j} \arcsin(\frac{1}{\sqrt{N+1}})$, i.e.,

$$\prod_{j=1}^{N-1} \cos(S_1 t_j) \sin(S_1 t_j) = \frac{1}{\sqrt{N}}, \quad \text{then the final state of Eq.(8) indeed becomes the standard } W\text{-state of the } N\text{ TLSs described by } |W\rangle^N = \frac{1}{\sqrt{N}} \sum_{l=1}^{N} \prod_{k \neq l}^{N} |g\rangle_k|e\rangle_l.$$
By using the above process, two important entangled states may be realized. Firstly, the Bell state of two arbitrary TLSs \( j \) and \( k \) can be achieved as

\[
U_j U_k |1\rangle_j |g\rangle_k \rightarrow \frac{-i}{\sqrt{2}} (|g\rangle_j |e\rangle_k + |e\rangle_j |g\rangle_k) |0\rangle,
\]

(9)

when the interaction time are chosen as \( t_j = \tau_j/2 \) and \( t_k = \tau_k \). Secondly, the W state of arbitrary three TLSs, such as \( j \), \( k \) and \( l \)-th TLS, may also be obtained through

\[
U_j U_k U_l |1\rangle_j |g\rangle_k |g\rangle_l \rightarrow \frac{-i}{\sqrt{3}} (|g\rangle_j |g\rangle_k |e\rangle_l + |g\rangle_j |e\rangle_k |g\rangle_l + |e\rangle_j |g\rangle_k |g\rangle_l) |0\rangle,
\]

(10)

by choosing the interaction time \( t_j = \tau_j/3 \), \( t_k = \tau_k/2 \), and \( t_l = \tau_l/2 \), respectively.

Furthermore, the cluster state \( \ket{C_N} \equiv \frac{1}{2^{N/2}} \bigotimes_{j=1}^{N} (|g\rangle_j \sigma_z^{j+1} + |e\rangle_j) \) of \( N \) TLSs may be implemented by a similar process. The process is also two steps. (i) The first step is to initialize the \( N \)-th TLS to the ground state \( |g\rangle_N \) and all the other TLSs to the states \( \ket{+}_j = \frac{1}{\sqrt{2}} (|g\rangle_j + |e\rangle_j) \), while initialize the phase qubit to \( \ket{+}_p = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \). (ii) Sequentially perform the iSWAP operations between CBJJ and each of TLSs. Note that, to cancel the single-qubit phase factor, a \( Z_P^2 \) (which denotes a \( \pi/2 \) rotation around the \( z \) axis) pulse needs to be applied to the phase qubit both before and after each iSWAP operation. After that, all \( N \) TLSs may be connected to make a large cluster chain (as shown in the Fig.2(b)), i.e.,

\[
[i\text{swap}]_{(N,P)} \prod_{j=1}^{N-1} \{ \bigotimes_j (Z^2_P[i\text{swap}]_{(j,P)}[Z^2_P]) \}
\]

\[
N-1
\}
\]

\[
\frac{1}{2^{N/2}} \bigotimes_{j=1}^{N} (|g\rangle_j \sigma_z^{j+1} + |e\rangle_j) |0\rangle
\]

\[
= \frac{1}{\sqrt{3}} \bigotimes_{j=1}^{N} (|g\rangle_j |g\rangle_k |e\rangle_l + |g\rangle_j |e\rangle_k |g\rangle_l + |e\rangle_j |g\rangle_k |g\rangle_l) |0\rangle
\]

\[
= \ket{C_N} |0\rangle.
\]

(11)

Moreover, The chain cluster states can be connected to produce higher dimensions cluster states by repeating iSWAPs.\(^39\)

**IV. DETECTION OF THE GENUINE MULTI-QUBIT ENTANGLEMENT**

To ensure the obtained state is the desired multi-qubit entangled state, one must detect the state of TLSs. Because the TLSs cannot be directly measured, one must previously perform iSWAP operation to transform the state of the \( j \)-th TLS to the phase qubit (up to a correctable Z rotation), then the information can be read out through measuring the phase qubit with quantum state tomography (QST)\(^24\). Currently the measurement of the phase qubit with high-fidelity \( \langle F = 0.96 \rangle \) can be completed with a short time (less than 5 ns). The readout technique is achieved by applying a short bias current pulse \( \delta I(t) \) that adiabatically reduces the well depth \( \Delta U/\hbar \omega_p \), so that the first excited state lies very near the top of the well when the current pulse is at its maximum value. In this way, one can read out the states of TLSs one by one.

For detecting two-qubit entangled states, one can use tomographic state analysis\(^12,13,16\) to reconstruct the density matrix with near-unity detection efficiency. This is achieved by single qubit rotations and subsequent projective measurements. For the two-qubit system, a convenient set of operators is given by 16 operators \( \sigma_\alpha^{(1)} \otimes \sigma_\beta^{(2)} \) (\( \alpha, \beta = 0, x, y, z \)), where \( \sigma_\alpha^{(j)} \) denote Pauli matrices of qubit \( j \). The reconstruction of the density matrix \( \rho \) is accomplished by measuring the expectation values \( \langle \sigma_\alpha^{(1)} \otimes \sigma_\beta^{(2)} \rangle_\rho \). It has been shown that only the expectation of \( \sigma_z \) in the phase qubit can be measured; however, the other direction measurements can be achieved by applying a transformation that maps the detected eigenvector onto the eigenvector of \( \sigma_z \) before detecting. To obtain all 16 expectation values, nine different settings have to be used.

A disadvantage of the tomography is that the operators required to detect the entanglement are growing exponentially with the number of qubits. However, if one knows about some priori information about the generated entangled state, one can use entanglement witness operator to distinguish and characterize the \( N \)-particle entangled state\(^33,34,35\). The W state and cluster states proposed in the paper are actually two typical types of genuine multipartite entangled states (i.e., the state whose reduced density operator of any subsystem has rank larger than 1). The entanglement witness \( W \) is an operator such that for every product state

\[
\text{Tr}(\rho W) \geq 0 \quad (\rho \in S_+)
\]

with \( S_+ \) denoting the set of separable states. From the definition of the operator \( W \), it is clear that the witness has a positive or zero expectation value for all separable states, and thus a negative expectation value signals the presence of genuine multipartite entanglement. In order to measure the witness \( W \)
of the generating entanglement of the TLSs proposed here, we should decompose the witness operator into a sum of locally measurable operators. By appropriately constructing $W$, the required measurement settings are much less than the requirement of the tomography.

As for the $N$-qubit W state denoted as $|W_N\rangle$, one of the universal methods to construct the entanglement witness $W_{W_N}$ is given by

$$W_{W_N} = \frac{N-1}{N} I - |W_N\rangle\langle W_N|$$

with $I$ denoting the identity operator. Especially, it has been proven that the optimal decomposition of the witness $W_{W_3}$ is given by

$$W_{W_3} = \frac{2}{3} I - |W_3\rangle\langle W_3|$$

$$= \frac{1}{24}[17 \cdot I_{\otimes3} + 3 \cdot \sigma_3^{\otimes3} + 3 \cdot (\sigma_z I I + I \sigma_z I + I I \sigma_z)$$

$$+ 5 \cdot (\sigma_z \sigma_z I + I \sigma_z \sigma_z + I \sigma_z \sigma_z)$$

$$- (I + \sigma_z + \sigma_y)^{\otimes3} - (I + \sigma_z - \sigma_y)^{\otimes3}$$

$$- (I + \sigma_z + \sigma_y)^{\otimes3} - (I + \sigma_z - \sigma_y)^{\otimes3}].$$

This decomposition requires five measurement settings, namely $\sigma_3^{\otimes3}$ and $((\sigma_z + \sigma_y)/\sqrt{2})^{\otimes3}$, $\eta = x, y$. Since three qubits entangled states have not been realized in solid state systems, the above optimal decomposition is very useful to detect a purely genuine entangled state in near future experiments. In addition, a universal method to construct the witness $W_{W_N}$ which requires $N^2 - N + 1$ measurement settings is developed in Ref.\textsuperscript{33}.

We now turn to address a very efficient method to construct the entanglement witness $W_{C_N}$ of the $N$-qubit cluster state $|C_N\rangle$. By using the stabilizing operators $S_j^{(C_N)}$ of the cluster state (i.e., $S_j^{(C_N)}|C_N\rangle = |C_N\rangle$) defined as

$$S_1^{(C_N)} = \sigma_z^{(1)} \sigma_z^{(2)},$$

$$S_j^{(C_N)} = \sigma_z^{(j-1)} \sigma_z^{(j)} \sigma_z^{(j+1)} \ (j = 2, 3, \ldots, N-1),$$

$$S_N^{(C_N)} = \sigma_z^{(N-1)} \sigma_z^{(N)},$$

one can construct the entanglement witness $W_{C_N}$, which detects genuine $N$-qubit entanglement around the $N$-qubit cluster state, given by

$$W_{C_N} = 3 I - 2 \left[ \prod_{\text{even } k} \frac{S_k^{(C_N)}}{2} + \prod_{\text{odd } k} \frac{S_k^{(C_N)}}{2} \right].$$

A remarkable feature of this entanglement witness is that only two local measurement settings, i.e.,

$$\sigma_z^{(1)} \otimes \sigma_z^{(2)} \otimes \sigma_z^{(j-1)} \otimes \sigma_z^{(j)} \ldots \sigma_z^{(N-1)} \otimes \sigma_z^{(N)},$$

$$\sigma_z^{(1)} \otimes \sigma_z^{(2)} \otimes \sigma_z^{(j-1)} \otimes \sigma_z^{(j)} \ldots \sigma_z^{(N-1)} \otimes \sigma_z^{(N)},$$

are needed independent of the number of qubits. Comparing with quantum state tomography that the number of measurement settings increase exponentially with the number of qubits, the required settings for the entanglement witness increase at most polynomial with the number of qubits, and specially for the cluster state, only two local measurement settings are needed for any number of qubits.

V. GENERALIZATION AND CONCLUSION

Actually the proposed approach to produce genuine multi-qubit entanglement may be applied to a wide class of the candidates for quantum computation, examples including trapped-ion quantum computation and quantum computation based on $C_60$ etc.. For concreteness, we generalize this method to achieve multi-qubit entanglement of trapped ions proposed in Ref.\textsuperscript{30}. Trapped atomic ions remain one of the most attractive candidates for the realization of a quantum computer, owing to their long-lived internal qubit coherence. The central challenge now is to scale up the number of trapped ion qubits. However, scaling the ion trap to interesting numbers of ions poses significant difficulties. In Ref.\textsuperscript{30}, the authors proposed an interesting scheme to scale the ion qubits. As shown in Fig.3, ion qubits locate in a two-dimensional array of micro-traps, where the distance between the ions in the plane can be very large, since no direct interaction between them is required. A different ion named the head ion can move above the plane of the ion array. By switching on a laser propagating in the perpendicular direction to the plane, one can perform the two-qubit gate between the target ion and the head ion. In particular, if the two-qubit iSWAP gate described in the present paper can be performed between the head ion and all ion qubits in the array, one can produce a large number of the cluster state or $W$ state. IF the cluster state of the ion qubits in this two-dimensional array can be achieved, an one-way quantum computation may be implemented, since single...
qubit gate for each ion qubits and the measurement operators can be easily realized for trapped ions.

Furthermore, if each trapped ions in Fig.3 is replaced by a qubit consisting of $C_0^{12}$, the genuine multi-qubit entanglement for such qubits can also be achieved by using the scheme we proposed here. Noted that it is difficult to realize the strong couplings between all nearest neighbor qubits. So the approach proposed here is promising.

In conclusion, we have presented an efficient scheme to realize and detect the multi-qubit entangled states of TLSs locating inside the Josephson phase qubit, and the proposed method can be able to applied in a wide class of candidates for quantum computation.

VI. ACKNOWLEDGEMENTS

This work was supported by the NSFC (No. 10674049), the State Key Program for Basic Research of China (Nos. 2006CB921801 and 2007CB925204), and the RGC of Hong Kong (Nos. HKU7051/06P and HKU7049/07P).