MULTIPLIERS ON A HILBERT SPACE OF FUNCTIONS ON $\mathbb{R}$

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Abstract. For a Hilbert space $H \subset L^1_{\text{loc}}(\mathbb{R})$ of functions on $\mathbb{R}$ we obtain a representation theorem for the multipliers $M$ commuting with the shift operator $S$. This generalizes the classical result for multipliers in $L^2(\mathbb{R})$ as well as our previous result for multipliers in weighted space $L^2_\omega(\mathbb{R})$. Moreover, we obtain a description of the spectrum of $S$.

Key words: multipliers, spectrum

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1. Introduction

Let $H \subset L^1_{\text{loc}}(\mathbb{R})$ be a Hilbert space of functions on $\mathbb{R}$ with values in $\mathbb{C}$. Denote by $\|\cdot\|$ (resp. $<\cdot,\cdot>$) the norm (resp. the scalar product) on $H$. Let $C_c(\mathbb{R})$ be the set of continuous functions on $\mathbb{R}$ with compact support. For a compact $K$ of $\mathbb{R}$ denote by $C_K(\mathbb{R})$ the subset of functions of $C_c(\mathbb{R})$ with support in $K$ and denote by $\hat{f}$ or by $\mathcal{F}(f)$ the usual Fourier transform of $f \in L^2(\mathbb{R})$. Let $S_x$ be the operator of translation by $x$ defined on $H$ by

$$(S_x f)(t) = f(t-x), \ a.e. \ t \in \mathbb{R}.$$ 

Let $S$ (resp. $S^{-1}$) be the translation by 1 (resp. -1). Introduce the set

$$\Omega = \left\{ z \in \mathbb{C}, -\ln \rho(S^{-1}) \leq \text{Im} z \leq \ln \rho(S) \right\},$$

where $\rho(A)$ is the spectral radius of $A$ and let $I$ be the interval $[-\ln \rho(S^{-1}), \ln \rho(S)]$. Assuming the identity map $i : H \rightarrow L^1_{\text{loc}}(\mathbb{R})$ continuous, it follows from the closed graph theorem that if $S_x(H) \subset H$, for $x \in \mathbb{R}$, then the operator $S_x$ is bounded from $H$ into $H$. In this paper we suppose that $H$ satisfies the following conditions:

(H1) $C_c(\mathbb{R}) \subset H \subset L^1_{\text{loc}}(\mathbb{R})$, with continuous inclusions, and $C_c(\mathbb{R})$ is dense in $H$.

(H2) For every $x \in \mathbb{R}$, $S_x(H) \subset H$ and $\sup_{x \in K} \|S_x\| < +\infty$, for every compact set $K \subset \mathbb{R}$.
(H3) For every $\alpha \in \mathbb{R}$ let $T_\alpha$ be the operator defined by
\[ T_\alpha : H \ni f(x) \mapsto f(x)e^{i\alpha x}, \ x \in \mathbb{R}. \]
We have $T_\alpha(H) \subset H$ and, moreover, $\sup_{\alpha \in \mathbb{R}} \| T_\alpha \| < +\infty$.

(H4) There exists $C > 0$ and $a \geq 0$ such that
\[ \| S_x \| \leq C e^{a|x|}, \forall x \in \mathbb{R}. \]
Set $||| f ||| = \sup_{\alpha \in \mathbb{R}} \| T_\alpha f \|$, for $f \in H$. The norm $||| . |||$ is equivalent to the norm of $H$ and without loss of generality, we can consider below that $T_\alpha$ is an isometry on $H$ for every $\alpha \in \mathbb{R}$. Obviously, the condition (H3) holds for a very large class of Hilbert spaces. We give some examples of Hilbert spaces satisfying our hypothesis.

**Example 1.** A weight $\omega$ on $\mathbb{R}$ is a non negative function on $\mathbb{R}$ such that
\[ \sup_{x \in \mathbb{R}} \frac{\omega(x + y)}{\omega(x)} < +\infty, \forall y \in \mathbb{R}. \]
Denote by $L^2_\omega(\mathbb{R})$ the space of measurable functions on $\mathbb{R}$ such that
\[ \int_{\mathbb{R}} |f(x)|^2 \omega(x)^2 dx < +\infty. \]
The space $L^2_\omega(\mathbb{R})$ equipped with the norm
\[ \| f \| = \left( \int_{\mathbb{R}} |f(x)|^2 \omega(x)^2 dx \right)^{\frac{1}{2}} \]
is a Hilbert space satisfying our conditions (H1)-(H3). Moreover, we have the estimate
\[ \| S_t \| \leq Ce^{a|t|}, \forall t \in \mathbb{R}, \quad (1.1) \]
where $C > 0$ and $m \geq 0$ are constants. This follows from the fact that $\omega$ is equivalent to the special weight $\omega_0$ constructed in [1]. The details of the construction of $\omega_0$ are given in [6], [1]. Below after Theorem 2 we give some examples of weights.

**Definition 1.** A bounded operator $M$ on $H$ is called a multiplier if
\[ MS_x = S_x M, \forall x \in \mathbb{R}. \]
Denote by $\mathcal{M}$ the algebra of the multipliers. Our aim is to obtain a representation theorem for multipliers on $H$ and to characterize the spectrum of $S$. These two problems are closely related. In [6] we have obtained a representation theorem for multipliers on $L^2_\omega(\mathbb{R})$. Here we generalize our result for multipliers on a Hilbert space and shift operators satisfying the conditions $(H1) - (H4)$. Our proof is shorter than that in [6]. The main improvement is based on an application of the link between the spectrum $\sigma(S_t)$ of a
element of the group \((S_t)_{t \in \mathbb{R}}\) and the spectrum \(\sigma(A)\) of the generator \(A\) of this group. In general, in the setup we deal with the spectral mapping theorem
\[
\sigma(S_t) \setminus \{0\} = e^{\alpha(tA)}
\]
is not true. To establish the crucial estimate in Theorem 4 we use the general results (see [3] and [5]) for the characterization of the spectrum of \(S_t\) by the behavior of the resolvent of \(A\). This idea has been used in [8] for \(L^2_\omega(\mathbb{R})\) but one point in our argument needs a more precise proof and in this paper we do this in the general case.

Denote by \((f)\) the function
\[
\mathbb{R} \ni x \longrightarrow f(x)e^{\alpha x}.
\]
We prove the following

**Theorem 1.** For every \(M \in \mathcal{M}\), and for every \(a \in I = [-\ln \rho(S^{-1}), \ln \rho(S)]\), we have
1) \((Mf)_a \in L^2(\mathbb{R})\), \(\forall f \in C_c(\mathbb{R})\).
2) There exists \(\mu(a) \in L^\infty(\mathbb{R})\) such that
\[
\int_{\mathbb{R}} (Mf)(x)e^{\alpha x}e^{-itx}dx = \mu(a)(t) \int_{\mathbb{R}} f(x)e^{\alpha x}e^{-itx}dx, \ a.e.
\]
i.e.
\[
\hat{(Mf)}_a = \hat{\mu(a)(f)}_a.
\]
3) If \(I \neq \emptyset\) then the function \(\mu(z) = \mu_{(\im z)}(\Re z)\) is holomorphic on \(\hat{\Omega}\).

**Definition 2.** Given \(M \in \mathcal{M}\), if \(\hat{\Omega} \neq \emptyset\), we call symbol of \(M\) the function \(\mu\) defined by
\[
\mu(z) = \mu_{(\im z)}(\Re z), \ \forall z \in \hat{\Omega}.
\]
Moreover, if \(a = -\ln \rho(S^{-1})\) or \(a = \ln \rho(S)\), the symbol \(\mu\) is defined for \(z = x + ia\) by the same formula for almost all \(x \in \mathbb{R}\).

Denote by \(\sigma(A)\) the spectrum of the operator \(A\). From Theorem 1 we deduce the following interesting spectral result.

**Theorem 2.** We have
\[
\sigma(S) = \left\{ z \in \mathbb{C} : \frac{1}{\rho(S^{-1})} \leq |z| \leq \rho(S) \right\}.
\]

To prove this characterization of the spectrum of \(S\) we exploit the existence of a symbol of every multiplier. Notice that in general \(S\) is not a normal operator and there are no spectral calculus which could characterize the spectrum of \(S\). On the other hand,
Theorem 2 has been used in [9] to obtain spectral mapping theorems for a class of multipliers. Now we give some examples of weights.

Example 2. The function \( \omega(x) = e^x \) is a weight. For the associated weighted space \( L^2_\omega(\mathbb{R}) \) we obtain \( \sigma(S) = \{ z \in \mathbb{C}, |z| = e \} \).

Example 3. The functions of the form \( \omega(x) = 1 + |x|^\alpha \), for \( \alpha \in \mathbb{R} \) are weights and we get \( \sigma(S) = \{ z \in \mathbb{C}, |z| = 1 \} \).

Example 4. Let \( \omega(x) = e^{a|x|^b} \) with \( a > 0 \) and \( 0 < b < 1 \). Then in \( L^2_\omega(\mathbb{R}) \) we have
\[
\sigma(S) = \{ z \in \mathbb{C}, e^{-a} \leq |z| \leq e^a \}.
\]

Example 5. Functions like \( e^{\mid x \mid \ln(2+|x|)} \), \( e^{\mid x \mid (1+|x|^2)^n} \), for \( n > 0 \) also are weights.

The weights in the Examples 4 and 5 are used to illustrate Beurling algebra theory (cf. [10]).

2. Proof of Theorem 1

For \( \phi \in C_c(\mathbb{R}) \) denote by \( M_\phi \) the operator of convolution by \( \phi \) on \( H \). We have
\[
(M_\phi f)(x) = \int_{\mathbb{R}} f(x-y)\phi(y)dy, \quad \forall f \in H.
\]
It is clear that \( M_\phi \) is a multiplier on \( H \) for every \( \phi \in C_c(\mathbb{R}) \).

In [7] we proved the following

Theorem 3. For every \( M \in \mathcal{M} \), there exists a sequence \( (\phi_n)_{n\in\mathbb{N}} \subset C_c(\mathbb{R}) \) such that:

i) \( M = \lim_{n \to \infty} M_{\phi_n} \) with respect to the strong operator topology.

ii) We have \( \|M_{\phi_n}\| \leq C\|M\| \), where \( C \) is a constant independent of \( M \) and \( n \).

The main difficulty to establish Theorem 1 is the proof of an estimate for \( \hat{\phi_n}(z) \) for \( z \in \Omega \) by the norm of \( M_{\phi_n} \).

Theorem 4. For every \( \phi \in C_c(\mathbb{R}) \) and every \( \alpha \in \Omega \) we have
\[
\left| \int_{\mathbb{R}} \phi(x)e^{-iax}dx \right| \leq \|M_\phi\|.
\]
Theorem 1 is deduced from Theorem 3 and Theorem 4 following exactly the same arguments as in Section 3 of [6] and Section 3 of [7]. The function \( \mu(a) \) introduced in Theorem 1 is obtained as the limit of \( \left( \hat{\phi}_n(a) \right)_{n \in \mathbb{N}} \) with respect to the weak topology of \( L^2(\mathbb{R}) \). The reader could consult [6] and [7] for more details.

Here we give a proof of Theorem 4 by using the link between the spectrum of \( S \) and the spectrum of the generator \( A \) of the group \( (S_t)_{t \in \mathbb{R}} \).

**Proof of Theorem 4.**

Let \( \lambda \in \mathbb{C} \) be such that \( e^\lambda \in \sigma(S) \). First we show that there exists a sequence \( (n_k)_{k \in \mathbb{N}} \) of integers and a sequence \( (f_{n_k})_{k \in \mathbb{N}} \) of functions of \( H \) such that

\[
\| (e^{tA} - e^{(\lambda+2\pi in_k)t}) f_{n_k} \| \longrightarrow 0, \quad n_k \rightarrow \infty, \quad \| f_{n_k} \| = 1, \quad \forall k \in \mathbb{N}.
\]

Let \( A \) be the generator of the group \( (S_t)_{t \in \mathbb{R}} \). We have to deal with two cases:

(i) \( \lambda \in \sigma(A) \),

(ii) \( \lambda \notin \sigma(A) \).

In the case (i) we have \( \lambda \in \sigma_p(A) \cup \sigma_c(A) \cup \sigma_r(A) \), where \( \sigma_p(A) \) is the point spectrum, \( \sigma_c(A) \) is the continuous spectrum and \( \sigma_r(A) \) is the residual spectrum of \( A \). If we have

\[
\lambda \in \sigma_p(A) \cup \sigma_c(A),
\]

it is easy to see that there exists a sequence \( (f_m)_{m \in \mathbb{N}} \subset H \) such that

\[
\| (A - \lambda) f_m \| \longrightarrow 0, \quad \| f_m \| = 1, \quad \forall m \in \mathbb{N}.
\]

Then the equality

\[
(e^{At} - e^{\lambda t}) f_m = \left( \int_0^t e^{\lambda(t-s)} e^{As} ds \right) (A - \lambda) f_m,
\]

yields

\[
\| (e^{At} - e^{\lambda t}) f_m \| \longrightarrow 0, \quad \forall t \in \mathbb{R}
\]

and we obtain (2.1). If \( \lambda \notin \sigma_p(A) \cup \sigma_c(A) \), we have \( \lambda \in \sigma_r(A) \) and

\[
\text{Ran}(A - \lambda I) \neq H,
\]

where \( \text{Ran}(A - \lambda I) \) denotes the range of the operator \( A - \lambda I \). Therefore there exists \( h \in D(A^*) \), \( \| h \| = 1 \), such that

\[
< f, (A^* - \lambda) h > = 0, \quad \forall f \in D(A).
\]

This implies \( (A^* - \lambda) h = 0 \) and we take \( f = h \). Then

\[
< (e^{At} - e^{\lambda t}) f, f > = < f, (e^{A^*t} - e^{\lambda t}) f >
\]
\[
\langle f, \left( \int_0^t e^{\lambda(t-s)}e^{A^*s}ds \right)(A^* - \lambda)f \rangle = 0.
\]

In this case we set \( n_k = k \) and
\[
f_k = f, \quad \forall k \in \mathbb{N}
\]
and we get again (2.1).

The case (ii) is more difficult since if \( \lambda \notin \sigma(A) \), we have \( e^\lambda \notin \sigma(e^A) \).

Taking into account the results about the spectrum of a semi-group in Hilbert space [5] satisfying the condition \((H4)\) (see also [3] for the contraction semi-groups), we deduce that there exists a sequence of integers \( n_k, \) such that \( |n_k| \to \infty \) and
\[
\| (A - (\lambda + 2\pi in_k)I)^{-1} \| \geq k, \quad \forall k \in \mathbb{N}.
\]

Let \( (g_{n_k})_{k \in \mathbb{N}} \) be a sequence such that
\[
\|g_{n_k}\| = 1, \quad \left\| \left( (A - (\lambda + 2\pi in_k)I)^{-1} \right) g_{n_k} \right\| \geq k/2, \quad \forall k \in \mathbb{N}.
\]

We define
\[
f_{n_k} = \frac{\left( (A - (\lambda + 2\pi in_k)I)^{-1} \right) g_{n_k}}{\left\| \left( (A - (\lambda + 2\pi in_k)I)^{-1} \right) g_{n_k} \right\|}
\]
Then we obtain
\[
\left( e^{tA} - e^{(\lambda + 2\pi in_k)t} \right) f_{n_k} = \int_0^t e^{(\lambda + 2\pi in_k)(t-s)}e^{sA}ds \left( A - (\lambda + 2\pi in_k) \right) f_{n_k}
\]
and for every \( t \) we deduce
\[
\lim_{k \to +\infty} \left\| \left( e^{tA} - e^{(\lambda + 2\pi in_k)t} \right) f_{n_k} \right\| = 0.
\]
Thus is established (2.1) for every \( \lambda \) such that \( e^\lambda \in \sigma(S) \).

Now consider
\[
\hat{\phi}(-i\lambda) = \langle \int_{\mathbb{R}} \phi(t) e^{(\lambda + 2\pi in_k)t} f_{n_k}, e^{2\pi in_k t} f_{n_k} \rangle dt + \langle \int_{\mathbb{R}} \phi(t) e^{tA} f_{n_k}, e^{2\pi in_k t} f_{n_k} \rangle dt
\]
\[
= J_{n_k} + \langle \int_{\mathbb{R}} \phi(t) e^{tA} f_{n_k}, e^{2\pi in_k t} f_{n_k} \rangle dt,
\]
where \( J_{n_k} \to 0 \) as \( n_k \to \infty \). On the other hand, we have
\[
I_{n_k} = \langle \int_{\mathbb{R}} \phi(t) e^{tA} f_{n_k}, e^{2\pi in_k t} f_{n_k} \rangle dt = \langle \int_{\mathbb{R}} \phi(t) e^{-2\pi in_k t} f_{n_k}(\cdot - t) dt, f_{n_k} \rangle
\]
\[
= \langle \int_{\mathbb{R}} \phi(\cdot - y) e^{-2\pi in_k (-y)} f_{n_k}(y) dy, f_{n_k} \rangle
\]
\[
= \langle M_{\phi}(f_{n_k} e^{2\pi in_k \cdot}), e^{2\pi in_k \cdot} f_{n_k} \rangle
\]
and \( |I_{n_k}| \leq \|M_\phi\| \). Consequently, we deduce that
\[
|\hat{\phi}(-i\lambda)| \leq \|M_\phi\|.
\]

Next a similar argument yields
\[
|\hat{\phi}(-i\lambda - a)| \leq \|M_\phi\|, \quad \forall a \in \mathbb{R}.
\] (2.2)

In fact, if for \( t \in \mathbb{R} \) there exists a sequence \( (h_n)_{n \in \mathbb{N}} \subset H \) such that \( (e^{tA} - e^{\lambda t})h_n \to 0 \) as \( n \to \infty \) with \( \|h_n\| = 1 \), we consider
\[
\langle \int_\mathbb{R} (\phi(t)(e^{\lambda t} - e^{tA}))h_n, e^{-iat}h_n \rangle dt = \hat{\phi}(-i\lambda - a) - \langle \int_\mathbb{R} \phi(t)e^{iat}e^{tA}h_n dt, h_n \rangle.
\]
The term on the left goes to 0 as \( n \to \infty \), so it is sufficient to show that the second term on the right is bounded by \( \|M_\phi\| \). We have
\[
\left( \int_\mathbb{R} \phi(t)e^{iat}e^{tA}h_n dt \right)(x) = \int_\mathbb{R} \phi(t)e^{iat}h_n(x-t)dt
\]
\[
= \int_\mathbb{R} \phi(x-y)e^{ia(x-y)}h_n(y)dy = e^{iax}[M_\phi(e^{-ai}.h_n)](x), \text{a.e.}
\]
and we obtain
\[
|\hat{\phi}(-i\lambda - a)| \leq \|M_\phi\|.
\]

Next consider the second case when we have a sequence \( (f_{n_k})_{k \in \mathbb{N}} \) with the properties above. Multiplying by \( e^{i(2\pi n_k - a)t}f_{n_k} \), we obtain
\[
\hat{\phi}(-i\lambda - a) = \langle \int_\mathbb{R} \phi(t)e^{tA}f_{n_k}, e^{i(2\pi n_k - a)t}f_{n_k} \rangle dt + I_{n_k},
\]
where \( I_{n_k} \to 0 \) as \( n_k \to \infty \). To examine the integral on the right, we apply the same argument as above, using the fact that \( (2\pi n_k - a) \in \mathbb{R} \). This completes the proof of (2.2).

The property (2.2) implies that if for some \( \lambda_0 \in \mathbb{C} \) we have
\[
|\hat{\phi}(\lambda_0)| \leq \|M_\phi\|,
\]
then
\[
|\hat{\phi}(\lambda)| \leq \|M_\phi\|, \quad \forall \lambda \in \mathbb{C}, \text{ s.t. } \text{Im } \lambda = \text{Im } \lambda_0.
\]

There exists \( \alpha_0 \in \sigma(S) \) such that \( |\alpha_0| = \rho(S) \). Then we obtain that
\[
|\hat{\phi}(\alpha)| \leq \|M_\phi\|,
\]
for every \( z \) such that \( \text{Im } z = \ln \rho(S) \). In the same way there exists \( \eta \in \sigma(S^{-1}) \) such that \( |\eta| = \rho(S^{-1}) \) and \( \alpha_1 = \frac{1}{\eta} \in \sigma(S) \). Then applying the above argument to \( \alpha_1 \), we get
\[
|\hat{\phi}(\alpha)| \leq \|M_\phi\|,
\]
for every $z$ such that $\text{Im} \, z = -\ln \rho(S^{-1})$. Since $\phi \in C_c(\mathbb{R})$ we have

$$|\hat{\phi}(z)| \leq C\|\phi\|_{\infty}e^{k|\text{Im} \, z|} \leq K\|\phi\|_{\infty}, \quad \forall z \in \Omega,$$

where $C > 0$, $k > 0$ and $K > 0$ are constants. An application of the Phragmen-Lindelöf theorem for the holomorphic function $\hat{\phi}(z)$ yields

$$|\hat{\phi}(\alpha)| \leq \|M_{\phi}\|$$

for all $\alpha \in \Omega$. □

Now we pass to the proof of Theorem 2. It is based on Theorem 1 combined with the arguments in [9] to cover our more general case. For the convenience of the reader we give the details.

**Proof of Theorem 2.** Let $\alpha \in \mathbb{C}$ be such that $e^{\alpha} \notin \sigma(S)$. Then it is clear that $T = (S - e^{\alpha}I)^{-1}$ is a multiplier. Let $a \in [-\ln \rho(S^{-1}), \ln \rho(S)]$. Then there exists $\nu_{(a)} \in L^\infty(\mathbb{R})$ such that

$$(Tf)_a = \nu_{(a)}(f)_a, \quad \forall f \in C_c(\mathbb{R}), \text{ a.e.}$$

For $g \in C_c(\mathbb{R})$, the function $(S - e^{\alpha}I)g$ is also in $C_c(\mathbb{R})$. Replacing $f$ by $(S - e^{\alpha}I)g$, for $g \in C_c(\mathbb{R})$ we get

$$\widehat{(g)_a}(x) = \nu_{(a)}(x)\mathcal{F}\left([S - e^{\alpha}I]g\right)_a(x), \quad \forall g \in C_c(\mathbb{R}), \text{ a.e.}$$

and

$$\widehat{(g)_a}(x) = \nu_{(a)}(x)\widehat{g}_a(x)[e^{a-ix} - e^{\alpha}], \quad \forall g \in C_c(\mathbb{R}), \text{ a.e.}$$

Choosing a suitable $g \in C_c(\mathbb{R})$, we have

$$\nu_{(a)}(x)(e^{a-ix} - e^{\alpha}) = 1, \text{ a.e.}$$

On the other hand, $\nu_{(a)} \in L^\infty(\mathbb{R})$. Thus we obtain that $\text{Re} \alpha \neq a$ and we conclude that

$$e^{a + ib} \in \sigma(S), \quad \forall b \in \mathbb{R}.$$ 

Since $S$ is invertible, it is obvious that

$$\sigma(S) \subset \{z \in \mathbb{C}, \frac{1}{\rho(S^{-1})} \leq |z| \leq \rho(S)\},$$

Consequently, we obtain

$$\sigma(S) = \{z \in \mathbb{C}, \frac{1}{\rho(S^{-1})} \leq |z| \leq \rho(S)\}$$

and this completes the proof. □
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