Interferometric detection of gravitational waves arising from extended theories of gravity

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Abstract

Even if Einstein’s General Relativity achieved a great success and overcame lots of experimental tests, it also showed some shortcomings and flaws which today advise theorists to ask if it is the definitive theory of gravity. In this letter Proceeding we show that, by assuming that advanced projects on the detection of Gravitational Waves (GWs) will improve their sensitivity, allowing to perform a GWs astronomy, accurate angular and frequency dependent response functions of interferometers for GWs arising from various Theories of Gravity, i.e. General Relativity and Extended Theories of Gravity, could aim in discriminating among various theories.

Recently, the data analysis of interferometric GWs detectors has been started (for the current status of GWs interferometers see [1]) and the scientific community aims at a first direct detection of GWs in next years.

Detectors for GWs will be important for a better knowledge of the Universe and also to confirm or ruling out the physical consistency of General Relativity or of any other theory of gravitation [2, 3, 4, 5, 6, 7]. This is because, in the context of Extended Theories of Gravity, some differences between General Relativity and the others theories can be pointed out starting by the linearized theory of gravity [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. In this picture, detectors for GWs are in principle sensitive also to a hypothetical scalar component of gravitational radiation, that appears in extended theories of gravity like scalar-tensor gravity [4, 8, 9, 10, 12], bi-metric theory [5], high order theories [2, 3, 6, 7], Brans-Dicke theory [13] and string theory [14].
Motivations of extending gravity arise from the fact that even Einstein’s General Relativity achieved a great success (see for example the opinion of Landau who says that General Relativity is, together with Quantum Field Theory, the best scientific theory of all) and overcame lots of experimental tests, it also showed some shortcomings and flaws which today advise theorists to ask if it is the definitive theory of gravity. Differently from other field theories like the electromagnetic theory, General Relativity is very difficult to be quantized. This fact rules out the possibility of treating gravitation like other quantum theories and precludes the unification of gravity with other interactions. At the present time, it is not possible to realize a consistent Quantum Gravity Theory which leads to the unification of gravitation with the other forces.

On the other hand, one can define Extended Theories of Gravity those semi-classical theories where the Lagrangian is modified, in respect to the standard Einstein-Hilbert gravitational Lagrangian, adding high-order terms in the curvature invariants (terms like $R^2$, $R^{\alpha\beta}R_{\alpha\beta}$, $R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$, $R\Box R$, $R\Box^2 R$) or terms with scalar fields non minimally coupled to geometry (terms like $\phi^2 R$). In general, one has to emphasize that terms like those are present in all the approaches to perform the unification between gravity and other interactions. More, from a cosmological point of view, such modifies of General Relativity generate inflationary frameworks which are very important as they solve lots of problems of the Standard Universe Model. Note that we are not telling that General Relativity is wrong. It is well known that, even in the context of Extended Theories, General Relativity remains the most important part of the structure. We are only trying to understand if weak modifies to such a structure could be needed to solve some theoretical and observable problems. In this picture, we also recall that even Einstein told that General Relativity could not be definitive. In fact, during his famous research on the Unified Field Theory, he tried to realize a theory that he called “Generalized Theory of Gravitation”, and he said that mathematical difficulties precluded him to obtain the final equations.

In the general context of cosmological evidences, there are also other considerations, which suggest an extension of General Relativity. As a matter of fact, the accelerated expansion of the Universe, which is today observed, shows that cosmological dynamic is dominated by the so called Dark Energy, which gives a large negative pressure. This is the standard picture, in which such new ingredient is considered as a source of the right side of the field equations. It should be some form of non-clustered and non-zero vacuum energy which, together with the clustered Dark Matter, drives the global dynamics. This is the so called “concordance model” ($\Lambda$CDM) which gives, in agreement with the CMBR, LSS and SNeIa data, a good tapestry of the today observed Universe, but presents several shortcomings as the well known “coincidence” and “cosmological constant” problems. An alternative approach is changing the left side of the field equations, seeing if observed cosmic dynamics can be achieved extending General Relativity. In this different context, it is not required to find out candidates for Dark Energy and Dark Matter, that,
till now, have not been found, but only the “observed” ingredients, which are curvature and baryon matter, have to be taken into account. Considering this point of view, one can think that gravity is different at various scales and a room for alternative theories is present. In principle, the most popular Dark Energy and Dark Matter models can be achieved considering $f(R)$ theories of gravity, where $R$ is the Ricci curvature scalar, and/or Scalar-Tensor Gravity [17, 18, 22, 23].

In this letter Proceeding we show that, by assuming that advanced projects on the detection of GWs will improve their sensitivity allowing to perform a GWs astronomy, accurate angular and frequency dependent response functions of interferometers for gravitational waves arising from various Theories of Gravity, i.e. General Relativity and Extended Theories of Gravity, could aim in discriminating among various theories [2, 3, 4, 5, 6, 7, 10, 11, 12].

The line element for GWs arising from standard General Relativity is given by [1, 15, 24, 25] (note that in this letter Proceeding we work with $G = 1$, $c = 1$ and $\hbar = 1$)

$$ds^2 = dt^2 - dx^2 - (1 + h_+)dx^2 - (1 - h_+)dy^2 - 2h_x dxdy,$$

where $h_+(t + z)$ and $h_×(t + z)$ are the weak perturbations due to the + and the × polarizations which are expressed in terms of synchronous coordinates in the Transverse Traceless (TT) gauge. Using the “bouncing photon” analysis, in [24, 25] it has been shown that the total frequency and angular dependent response function (i.e. the detector pattern) of an interferometer to the + polarization is:

$$\tilde{H}^+(\omega) = \Omega^+_{\omega}(\omega) + \Omega^+_{\omega}(\omega)$$

$$= \frac{(\cos^2 \theta \cos^2 \phi - \sin^2 \phi)}{2L} \hat{H}_{u}(\omega, \theta, \phi) - \frac{(\cos^2 \theta \sin^2 \phi - \cos^2 \phi)}{2L} \hat{H}_{v}(\omega, \theta, \phi),$$

that, in the low frequencies limit ($\omega \to 0$) gives the well known low frequency response function of [26, 27] for the + polarization:

$$\tilde{H}^+(\omega) = \frac{1}{2}(1 + \cos^2 \theta) \cos 2\phi + O(\omega).$$

The same analysis works for the × polarization (see [24, 25] for details). One obtains that the total frequency and angular dependent response function of an interferometer to the × polarization is:

$$\tilde{H}^×(\omega) = \frac{-\cos \theta \cos \phi \sin \phi}{L} [\hat{H}_{u}(\omega, \theta, \phi) + \hat{H}_{v}(\omega, \theta, \phi)],$$

that, in the low frequencies limit ($\omega \to 0$), gives the low frequency response function of [26, 27] for the × polarization:

$$\tilde{H}^×(\omega) = -\cos \theta \hat{H}_{v}(\omega, \theta, \phi) \sin 2\phi + O(\omega).$$
In these equations \( u \) and \( v \) are the directions of the interferometer arms in respect to the propagating GW and we have defined:

\[
\tilde{H}_u(\omega, \theta, \phi) \equiv -\frac{1 + \exp(2i\omega L) - \sin \theta \cos \phi(1 + \exp(2i\omega L) - 2 \exp i\omega L(1 - \sin \theta \cos \phi))}{2i\omega(1 + \sin^2 \theta \cos^2 \phi)}
\]

(6)

and

\[
\tilde{H}_v(\omega, \theta, \phi) \equiv -\frac{1 + \exp(2i\omega L) - \sin \theta \sin \phi(1 + \exp(2i\omega L) - 2 \exp i\omega L(1 - \sin \theta \sin \phi))}{2i\omega(1 + \sin^2 \theta \sin^2 \phi)}.
\]

(7)

The case of massless Scalar-Tensor Gravity has been discussed in \([4, 12]\). In this case, the line-element in the TT gauge can be extended with one more polarization, labelled with \( \Phi(t + z) \), i.e.

\[
ds^2 = dt^2 - dz^2 - (1 + h_+ + \Phi)dx^2 - (1 - h_+ + \Phi)dy^2 - 2h_x dxdy.
\]

(8)

Now, the total frequency and angular dependent response function of an interferometer to this “scalar” polarization is \([4, 12]\):

\[
\tilde{H}^\Phi(\omega) = \frac{\sin \theta}{2i\omega L}\{\cos \phi[1 + \exp(2i\omega L) - 2 \exp i\omega L(1 + \sin \theta \cos \phi)] + \\
- \sin \phi[1 + \exp(2i\omega L) - 2 \exp i\omega L(1 + \sin \theta \sin \phi)]\},
\]

(9)

that, in the low frequencies limit (\( \omega \to 0 \)), gives the low frequency response function of \([9, 14]\) for the \( \Phi \) polarization:

\[
\tilde{H}^\Phi(\omega) = -\sin^2 \theta \cos 2\phi + O(\omega).
\]

(10)

In \([2, 3, 4, 7]\) it has also been shown that, in the framework of GWs, the cases of massive Scalar-Tensor Gravity and \( f(R) \) theories are totally equivalent (this is not surprising as it is well known that there is a more general conformal equivalence between Scalar-Tensor Gravity and \( f(R) \) theories, even if there is a large debate on the possibility that such a conformal equivalence should be a physical equivalence too \([17, 18, 21]\)). In such cases, because of the presence of a small mass, a longitudinal component is present in the third polarization, thus it is impossible to extend the TT gauge to the third mode \([2, 3, 4, 6, 7]\). But, by using gauge transformations, one can put the line-element due to such a third scalar mode in a conformally flat form \([2, 3, 4, 6, 7]\):

\[
ds^2 = [1 + \Phi(t, z)](-dt^2 + dz^2 + dx^2 + dy^2).
\]

(11)
If the interferometer arm is parallel to the propagating GW the longitudinal response function associated to such a massive mode is \[ \Upsilon_l(\omega) = \frac{1}{m^2 \omega^2 L} \left( 1 + \exp[2i\omega L] \right) m^2 \omega^2 L \left( m^2 - 2\omega^2 \right) + 
\]
\[-i \exp[2i\omega L] \omega^2 \sqrt{-m^2 + \omega^2} (4\omega^2 + m^2 (-1 - iL\omega)) + 
\]
\[+ \omega^2 \sqrt{-m^2 + \omega^2} (-4i\omega^2 + m^2 (i + \omega L)) + 
\]
\[+ \exp[iL(\omega + \sqrt{-m^2 + \omega^2})] \left( m^6 L + m^4 \omega^2 L + 8i\omega^4 \sqrt{-m^2 + \omega^2} + 
\]
\[+ m^2 (-2L\omega^4 - 2i\omega^2 \sqrt{-m^2 + \omega^2}) \right) + 2 \exp[i\omega L] \omega^4 (-3m^2 + 4\omega^2) \sin[\omega L] \right) \],
\[ (12) \]

where \( m \) is the small mass associated to the GW.

Thus, by assuming that advanced projects on the detection of GWs will improve their sensitivity allowing to perform a GWs astronomy (this is due because signals from GWs are quite weak) [1], one will only have to look the interferometer response functions to understand which is the correct theory of gravity. If only the two response functions (2) and (4) will be present we will conclude that General Relativity is the definitive theory of gravitation. If the response function (9) will be present too, we will conclude that massless Scalar-Tensor Gravity is the correct theory of gravitation. Finally, if the third response function will be given by Eq. (12), we will learn that the correct theory of gravity will be Scalar-Tensor Gravity which is equivalent to \( f(R) \) theories. In any case, such response functions will help in discriminating between gravity theories. This is because General Relativity is the only gravity theory which admits only the two response functions [2] and [4]. Such response functions correspond to the two “canonical” polarizations \( h_+ \) and \( h_\times \). Thus, if a third polarization will be present, a third response function will be detected by GWs interferometers and this fact will rule out General Relativity like the definitive theory of gravity confirming Einstein’s intuition that a modify is needed [28].

**Conclusion remarks**

In this review letter Proceeding we have shown that, by assuming that advanced projects on the detection of GWs will improve their sensitivity, allowing to perform a GWs astronomy, accurate angular and frequency dependent response functions of interferometers for gravitational waves arising from various Theories of Gravity, i.e. General Relativity and Extended Theories of Gravity, will be an important help in discriminating among various gravity theories.

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