Complexity Factor for Static Cylindrical System in Energy-momentum Squared Gravity

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Abstract

This paper investigates some physical features that give rise to complexity within the self-gravitating static cylindrical structure coupled with anisotropic distribution in the energy-momentum squared gravity. To accomplish this, we formulate the modified field equations and explore the structure of the astronomical body. The C-energy and Tolman mass are also calculated to discuss the matter distribution. We then obtain some structure scalars via orthogonal splitting of the Riemann tensor. Since, the complexity of the considered structure is influenced by a variety of variables, including anisotropic pressure and inhomogeneous energy density, etc. thus, we adopt the factor $Y_{TF}$ as the complexity factor. Further, the complexity-free condition along with the Gokhroo-Mehra model and polytropic equation of state are taken to generate their corresponding solutions. We deduce that the inclusion of additional terms of this modified theory leads to a more complicated system.

Keywords: Self-gravitating system, Energy-momentum squared gravity; Structure scalars; Complexity factor.

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1 Introduction

Cosmic structures developed entirely under the effect of gravity result in the formation of self-gravitating systems (like stars, planets, galaxies and stellar clusters). Numerous astronomical observations [1] demonstrate that such large scale self-gravitating systems provide crucial details about the beginning and evolution of our universe. In order to comprehend the structure and formation of the cosmos, it is significant to investigate these compact structures. The physical characteristics of a complicated astronomical structure may substantially change as a result of a little disruption in the system. Therefore, it is necessary to develop a complexity factor that links fundamental physical factors. Furthermore, an adequate complexity factor must evaluate the impact of both external and internal perturbations on the evolution and stability of stellar configurations.

There is a large body of literature [2] in developing a suitable definition of complexity but a more appropriate conventional definition has not been achieved. Entropy and information have been considered in the earlier definitions of complexity but could not accurately estimate complexity of two standard models in physics: perfect crystal and ideal gas. Since particles in perfect crystal are distributed in a specific manner hence, minimum information is required to explain its symmetric distribution. For the ideal gas, atoms are arbitrarily arranged, therefore the maximum amount of data is required to determine any of its potential states. These two systems exhibit contradictory behavior but both give zero complexity. Since entropy and information cannot adequately describe the complexity, so some other parameters must be included in the definition of complexity.

The perception of complexity has been expanded by Lopez-Ruiz et al. [3] through analyzing the idea of disequilibrium, that estimates the discrepancy between the equiprobable arrangement and multiple probabilistic states of the system. In order to calculate the complexity of compact objects like white dwarfs and neutron stars, researchers have substituted the energy density of the system in place of probability distribution [4]. As particles are compactly organized in the core of dense celestial bodies, thus fewer radial pressure than tangential pressure is created, leading to pressure anisotropy in the system. Consequently, anisotropy plays a crucial role in evaluating the stability of self-gravitating structures. The concept of complexity defined by Lopez-Ruiz and his collaborators did not propose productive criteria to determine complexity because it focused solely on energy density while ignoring other
state parameters like anisotropic pressure.

In the framework of general relativity (GR), Herrera [5] recently introduced a new approach to estimate the complexity factor for a static sphere. In this technique, he assumed that the simplest system has isotropic pressure and homogenous energy density, i.e., the complexity of such a system is zero. He connected these state parameters in a single frame by splitting the Riemann tensor orthogonally and established a complexity factor. Herrera et al. [6] extended this concept by introducing minimal complexity condition for a dynamical fluid distribution evolving in a homologous pattern. Sharif and Butt [7] investigated complexity of a static cylindrical configuration in GR and concluded that the two parameters, i.e., inhomogeneous energy density and anisotropy in pressure make the system more complicated. The same authors [8] studied the complexity of the charged cylindrical structure and deduced that the electromagnetic field can create more complexity in a self-gravitating structure. A static axially symmetric distribution has also been examined by evaluating three complexity factors [9]. Herrera et al. [10] studied a quasi-homologous system with complexity-free criteria.

Cylindrical systems have been discussed at various scales to study their behavior and role of different physical characteristics [11]. Levi-Civita [12] developed vacuum solutions (having two independent components), which motivated researchers to investigate relativistic phenomena and the underlying mysteries of various celestial bodies. Einstein and Rosen [13] obtained solutions of the cylindrical gravitational waves. The presence of naked singularity producing strong gravitational waves at the end of collapse provided motivation to examine other physical properties of cylindrical structures. Herrera and Santos [14] determined the matching conditions for dynamical collapsing cylindrical structure. They demonstrated that the radial pressure exists at the surface of the cylinder and its temporal component depends on the collapsing matter. Sharif and Abbas [15] explored the dynamics of gravitational collapse of charged non-adiabatic cylinder and investigated the influence of charge as well as heat on its gravitational mass.

The evolution of the universe is effectively explained by GR through the Λ Cold Dark Matter model. However, there are certain issues with this model namely the coincidence problem and fine-tuning [16]. In order to address the issues related to cosmic expansion, many researchers introduced different extended theories such as $f(R)$, $f(R, T)$, where $R$ is the Ricci scalar and $T$ denotes trace of the energy-momentum tensor $T_{\delta\lambda}$, etc. The first generalization to GR was $f(R)$ theory, which is established by substituting the generic
function $f(R)$ in the Einstein-Hilbert action in place of $R$ \[17\]. Harko et al. \[18\] proposed an extension of $f(R)$ gravity, termed as $f(R,T)$ through the use of gravitational Lagrangian density in the form of $R$ and $T$. The cosmic accelerated expansion and the interaction between dark matter/dark energy are effectively described by the curvature-matter coupling scenarios in $f(R,T)$ gravity \[19\]. Haghani et al. \[20\] proposed $f(R,T,Q)$ theory, where $Q = R_\delta^\lambda T^\delta_T$, by defining a strong dependence of geometry and matter distribution. A thorough analysis of several standard problems and the most recent progress of modified theories in cosmology is provided in \[21\]. The complexity condition for both static and dynamic anisotropic matter configurations has been examined in $f(R)$ scenario \[22\]. Abbas and Ahmad \[23\] explored the complexity of several compact stars in $f(R,T)$ theory and deduced that the complexity is minimal near the surface. Several people \[24\] used Herrera’s concept of complexity in the framework of these extended theories.

Katirci and Kavuk \[25\] recently presented a new theory which generalized GR by describing a particular coupling between matter and gravity through a factor $T^\delta_T T^\delta_T$. This theory is known as the energy-momentum squared gravity (EMSG) or $f(R,T^2)$ gravity with $T^2 = T^\delta_T T^\delta_T$. The predictions of GR regarding singularities (like the big bang singularity) at higher energy levels are no longer applicable due to expected quantum fluctuations. In this context, EMSG is regarded as a valuable framework since it addresses the big bang singularity by supporting regular bounce having the least scale factor and finite maximum energy density in the beginning of the universe. The conservation law does not hold in this theory due to the coupling between matter and geometry, which implies the existence of some additional force. Consequently, the motion of test particles diverges from the standard geodesic trajectory. Numerous astrophysical and cosmological phenomena have been investigated in this theory.

Roshan and Shojai \[26\] determined an exact solution of the modified field equations and verified the probability of bounce at early time by discussing isotropic and homogeneous distribution. Using several cosmological models of this theory, Broad and Barrow \[27\] found a variety of exact solutions for an isotropic cosmos and analyzed their behavior for the accelerated expansion, early and late-time evolution, and the presence or absence of singularities. Many astrophysical systems, such as neutron stars have been explored by using particular model like $f(R,T^2) = R + \chi T^\delta_T T^\delta_T$, $\chi$ is the model parameter \[28\]. In order to discuss the current cosmic expansion, Bahamonde et
al. [29] investigated the dynamical characteristics of two distinct \( f(R, T^2) \) models. Sharif and Gul [30] examined this theory through the Noether symmetry approach and investigated some feasible cosmological models. They also investigated the dynamics associated with the cylindrical collapse in the presence of electromagnetic field and dissipative matter, reaching the conclusion that charge, dissipative matter and modified parameters decrease the collapse rate [31]. Recently, we have discussed the complexity of charged static sphere in \( f(R, T^2) \) scenario and concluded that the electromagnetic field reduces the complexity of a spherical system [32].

The purpose of this article is to establish the complexity factor for a static cylindrical distribution within \( f(R, T^2) \) framework. The paper is organized in the following manner. The modified field equations for anisotropic fluid configuration are derived in the next section. Section 3 addresses certain physical characteristics of matter distribution. The structure scalars are then developed in section 4. We construct complexity-free constraint in section 5 to generate solutions of the EMSG field equations corresponding to a particular form of energy density provided by Gokhroo-Mehra and polytropic equation of state. Lastly, we summarize the main outcomes in section 6.

2 The \( f(R, T^2) \) Field Equations

The general Einstein-Hilbert action of \( f(R, T^2) \) gravity is given by the following expression [25].

\[
I_{f(R, T^2)} = \int L_m \sqrt{-g} \, d^4x + \int \frac{f(R, T^2)}{2\kappa^2} \sqrt{-g} \, d^4x, \quad (1)
\]

where \( \kappa^2, L_m \) and \( g \) are the coupling constant, matter Lagrangian and determinant of the metric tensor \( g_{\delta \lambda} \), respectively. The energy-momentum tensor is related to the Lagrangian density as follows

\[
T_{\delta \lambda} = 2(-\frac{\partial L_m}{\partial g^{\delta \lambda}} + \frac{1}{2} g_{\delta \lambda} L_m).
\]

Varying Eq. (1) with respect to \( g_{\delta \lambda} \), we obtain the EMSG field equations as

\[
\Theta_{\delta \lambda} f_{T^2} + R_{\delta \lambda} f_R + g_{\delta \lambda} \square f_R - \nabla_\delta \nabla_\lambda f_R - \frac{1}{2} g_{\delta \lambda} f = \kappa^2 T_{\delta \lambda}, \quad (2)
\]
here, \( R_{\delta\lambda} \) represents the Ricci tensor. Also, \( f_R = \frac{\partial f}{\partial R}, \) \( f_{T^2} = \frac{\partial f}{\partial T^2}, \) \( \Box = \nabla^\delta \nabla_\delta, \) while \( \Theta_{\delta\lambda} \) is given as

\[
\Theta_{\delta\lambda} = \frac{\delta T^2}{\delta g^{\delta\lambda}} = \frac{\delta (T_{\delta\lambda} T^{\delta\lambda})}{\delta g^{\delta\lambda}} = -2(\mathcal{L}_m + \frac{1}{2}T)T_{\delta\lambda} + \mathcal{L}_m g_{\delta\lambda}T \\
- 4 \frac{\partial^2 \mathcal{L}_m}{\partial g^{\delta\lambda} \partial g^{\beta\alpha}} T_{\beta\alpha} + 2T^\beta_{\delta} T_{\lambda\beta}.
\]

(3)

The energy-momentum tensor describing anisotropic distribution of matter inside the cylinder is expressed as

\[
T^\delta_{\lambda} = \Pi^\delta_{\lambda} + \rho v^\delta v_\lambda - p h^\delta_{\lambda},
\]

(4)

where \( p, \Pi^\delta_{\lambda}, \rho \) and \( v^\delta \) denote the pressure, anisotropic tensor, energy density and four-velocity, respectively. These terminologies are described by the following expressions

\[
\Pi^\delta_{\lambda} = \frac{\Pi}{3} (h^\delta_{\lambda} + 3s^\delta s_{\lambda}), \quad \Pi = -p_\perp + p_r, \quad h^\delta_{\lambda} = -v^\delta v_\lambda + \delta^\delta_{\lambda}, \quad p = \frac{1}{3}(2p_\perp + p_r),
\]

where the tangential and radial pressures are \( p_\perp \) and \( p_r \), respectively. The four-vector and four-velocity are given as

\[
v^\delta = (0, \frac{1}{G}, 0, 0), \quad s^\delta = (\frac{1}{f}, 0, 0, 0),
\]

satisfying \( v_\delta v^\delta = 1, \ s_\delta s^\delta = -1, \ v_\delta s^\delta = 0. \) Different matter Lagrangians generate different field equations because such Lagrangian has no precise definition. It is noticed that \( \mathcal{L}_m = \rho \) and \(-p\) are the most extensively employed choices in the literature. In GR, these options are not problematic but in the non-minimal coupling, these choices lead to different outcomes \[33\]. Thus, for the sake of convenience, we take \( \kappa = 1 \) and \( \mathcal{L}_m = \rho \) which yields \[34\]

\[
\Theta_{\delta\lambda} = -2p(T_{\delta\lambda} - \frac{1}{2}g_{\delta\lambda}T) - TT_{\delta\lambda} + 2T^\beta_{\delta} T_{\lambda\beta},
\]

\[
\mathcal{G}_{\delta\lambda} = R_{\delta\lambda} - \frac{1}{2}R g_{\delta\lambda} = \frac{1}{k^2 f_R} (T_{\delta\lambda} + T^{(C)}_{\delta\lambda}) = T^{(D)}_{\delta\lambda},
\]

(5)

where \( \mathcal{G}_{\delta\lambda} \) is the Einstein tensor and the modified terms (or the correction terms) of \( f(R, T^2) \) theory are denoted by \( T^{(C)}_{\delta\lambda} \) and have the following form

\[
T^{(C)}_{\delta\lambda} = \frac{1}{f_R} \left( \Box f - g_{\delta\lambda} \Box f_R + g_{\delta\lambda} \left( \frac{f - R f_R}{2} \right) \right)
\]
\[
- \rho g_{\delta\lambda} f_{T^2} T + (T + 2\rho) f_{T^2} T_{\delta\lambda} - 2T^\beta_{\delta} T_{\beta\lambda} f_{T^2} \Bigg) .
\] (6)

To investigate the compact structure, we assume a static cylindrically symmetric spacetime confined by the hypersurface (\(\Sigma\)) as
\[
ds^2 = F^2(r) dt^2 - \left( G^2(r) dr^2 + H^2(r) d\theta^2 + \alpha^2 H^2(r) dz^2 \right). \tag{7}
\]
Here, \(F, G, H\) are functions of \(r\) and \(\alpha\) is the arbitrary constant. The metric representing the external geometry is given as \([15]\)
\[
ds^2 = -\frac{2\mathcal{M}}{\mathcal{R}} dr^2 + 2d\mathcal{R} d\nu - \mathcal{R}^2 (d\theta^2 + \alpha^2 dz^2), \tag{8}
\]
where \(\mathcal{M}\) and \(\nu\) indicate the total mass in the exterior region and the retarded time, respectively. The necessary and sufficient constraints for the smooth matching of two metrics (7) and (8) on the hypersurface are provided in \([15]\).

The spacetime (7) can be made analogous to general cylindrically symmetric distribution by specifying \(r\) in such a manner that coefficient of \(dz^2\) (i.e., \(H^2(r)\)) equals to \(r^2\). This conversion is referred to as the tangential gauge and it transforms the metric (7) into the following form
\[
ds^2 = F^2(r) dt^2 - \left( G^2(r) dr^2 + r^2 d\theta^2 + \alpha^2 r^2 dz^2 \right). \tag{9}
\]
Taking covariant divergence of Eq.(2), we have
\[
\nabla^\delta T_{\delta\lambda} = \frac{1}{\kappa^2} \nabla^\delta (\Theta_{\delta\lambda} f_{T^2}) - \frac{1}{2} g_{\delta\lambda} \nabla^\delta f, \tag{10}
\]
which indicates the non-conservation of energy-momentum tensor in \(f(R, T^2)\) gravity implying the existence of an unknown force which is responsible for the non-geodesic motion of particles in celestial bodies. The modified field equations associated with the spacetime (9) are given as
\[
- \frac{1}{r^2 G^2} \left( \frac{r}{r} - \frac{2G'}{G} \right) = \frac{1}{f_R} (\rho + \varphi + \varphi_{00}), \tag{11}
\]
\[
\frac{1}{r^2 G^2} \left( \frac{2r F'}{G} + 1 \right) = \frac{1}{f_R} \left\{ (\varphi_{11} - \varphi + p_r) + (2\rho p_r + \rho^2 \right.
- \left. 2\rho p_{\perp} - 2p_r p_{\perp} + p_r^2 \right) f_{T^2} \right\}, \tag{12}
\]
\[
\frac{1}{rG^2} \left( \frac{rF''}{F} - \frac{rF' G'}{FG} - \frac{G'}{G} + \frac{F'}{F} \right) = \frac{1}{f_R} \left\{ (\varphi_{22} - \varphi + p_\perp) + (\rho p_\perp + \rho^2
- \rho \rho_r - p_r p_\perp) f_T^3 \right\}, \tag{13}
\]

where
\[
\begin{align*}
\varphi &= \frac{1}{2} (f - R f_R), \\
\varphi_{00} &= \frac{1}{G^2} \left( \frac{G'}{G} - \frac{2}{r} \right) f_R - \frac{f''}{G^2}, \\
\varphi_{11} &= -\frac{f_R}{G^2} \left( \frac{F'}{F} + \frac{2}{r} \right), \\
\varphi_{22} &= -\frac{f''}{G^2} - \frac{1}{G^2} \left( \frac{F'}{F} - \frac{G'}{G} + \frac{1}{r} \right) f_R,
\end{align*}
\]

prime denotes derivative with respect to \( r \).

### 3 Physical Characteristics of Matter Distribution

The C-energy formula \([35]\) is used to determine the matter composition of the cylindrically symmetric structure. This is given as
\[
m(r) = l \mathcal{E} = l \left( \frac{1}{8} - \frac{1}{8 \ell^2} \nabla_\delta \hat{r} \nabla^\delta \hat{r} \right), \tag{14}
\]
where \( \hat{r} = \mathcal{P} l \), \( \mathcal{P}^2 = \psi_{(1)k} \psi_{(1)}^k \), \( \mathcal{P}^2 = \psi_{(2)k} \psi_{(2)}^k \). The quantities \( \mathcal{P} \) and \( l \) represent the circumference radius and specific length, respectively. Also, \( \mathcal{E} \) is the gravitational energy per specific length, \( \psi_{(1)} = \frac{\partial}{\partial \theta} \) and \( \psi_{(2)} = \frac{\partial}{\partial z} \). The inner mass of the considered distribution becomes
\[
m(r) = \frac{r \alpha}{2} \left( \frac{1}{4} - \frac{1}{G^2} \right) = \frac{\alpha}{2} \left( \frac{r}{4} + \int_0^r \tilde{r}^2 T_0^{0(0)} d\tilde{r} \right). \tag{15}
\]

Utilizing Eqs.\((11)-(13)\) with \((15)\), the mass function takes the form
\[
m = \left( T_0^{0(0)} - T_1^{1(0)} + T_2^{2(0)} \right) + \frac{\alpha r}{8} - \frac{1}{G^2} \left( \frac{\alpha r}{2} + \frac{1}{r^2} \right)
\]
\[
- \frac{1}{\mathcal{F}\mathcal{G}^2}\left(\mathcal{F}'' - \frac{\mathcal{F}'\mathcal{G}'}{\mathcal{G}} - \frac{\mathcal{F}'}{r} + \frac{\mathcal{F}\mathcal{G}'}{r\mathcal{G}} + \frac{\mathcal{F}}{r^2}\right).
\]

Equation (12) yields the value of \(\frac{\mathcal{F}'}{\mathcal{F}}\) as

\[
\frac{\mathcal{F}'}{\mathcal{F}} = \frac{m}{\alpha r^2} - \frac{r}{8} - \frac{r}{2} T_1^{1(D)}.
\]

The Riemann tensor determines the distortion of spacetime and is expressed in terms of the Ricci scalar, the Weyl (\(C_{\alpha\beta\gamma\delta}^{\mu\nu}\)) and Ricci tensors as

\[
R_{\alpha\beta\gamma\delta}^{\mu\nu} = \frac{1}{2} R_{\alpha\beta\gamma\delta} - \frac{1}{2} R_{\alpha\gamma\delta\epsilon} + \frac{1}{2} R_{\alpha\epsilon\delta\gamma} - \frac{1}{2} R_{\epsilon\gamma\delta\alpha} + \frac{1}{6} R (\delta_{\alpha\beta} \delta_{\gamma\delta} - \delta_{\alpha\gamma} \delta_{\beta\delta} - \delta_{\alpha\delta} \delta_{\beta\gamma}).
\]

The tidal force on an object is determined by the Weyl tensor, which is the trace-free part of the Riemann tensor. This can be separated into electric (\(E_{\mu\nu}\)) and magnetic (\(H_{\mu\nu}\)) components using the observer’s four-velocity as

\[
H_{\delta\lambda} = \frac{1}{2} \eta_{\delta\alpha\beta\epsilon} C_{\lambda\alpha\beta} v^\alpha v^\epsilon, \quad E_{\delta\lambda} = C_{\delta\gamma\lambda\sigma} v^\gamma v^\sigma,
\]

where \(g_{\alpha\beta} = g_{\alpha\beta} g_{\gamma\delta} - g_{\alpha\gamma} g_{\beta\delta}\), \(C_{\sigma\epsilon\mu\nu} = (g_{\sigma\epsilon \alpha\beta} g_{\mu\nu \pi\kappa} - \eta_{\sigma\epsilon \alpha\beta} \eta_{\mu\nu \pi\kappa}) v^\alpha v^\pi \mathbb{E}^{\beta\kappa}\), and \(\eta_{\delta\lambda\mu\nu}\) denotes the Levi-Civita tensor. Different purely electric spacetimes including all static ones are known in the literature [36]-[39]. Since the spacetime being examined is of static nature, therefore the magnetic component disappears. The electric part in terms of the projection tensor and unit four-vector is written as

\[
E_{\delta\lambda} = \frac{\mathcal{E}}{3} (3 s_{\delta\lambda} + h_{\delta\lambda}),
\]

where

\[
\mathcal{E} = \frac{1}{2 \mathcal{F}\mathcal{G}^2}\left(\mathcal{F}'' - \frac{\mathcal{F}'\mathcal{G}'}{\mathcal{G}} - \frac{\mathcal{F}'}{r} + \frac{\mathcal{F}\mathcal{G}'}{r\mathcal{G}} + \frac{\mathcal{F}}{r^2}\right),
\]

and its non-vanishing components are

\[
\mathbb{E}_{11} = \frac{2}{3} \mathcal{E} \mathcal{G}^2, \quad \mathbb{E}_{22} = \frac{-1}{3} \mathcal{E} r^2, \quad \mathbb{E}_{33} = \frac{-1}{3} \mathcal{E} \alpha^2 r^2,
\]

with \(\mathbb{E}_{\delta} = 0\) and \(\mathbb{E}_{\mu\lambda} v^\lambda = 0\).

We would like to highlight that the electric component of the Weyl tensor in Eq. (19) is defined through a single scalar function because of the
constraints created by the Weyl gauge, however, it is expressed in terms of
two scalar functions for the general cylindrical symmetric distribution. We
establish a connection between the mass function and the Weyl ten-
sor to investigate several properties of the cylindrical framework by us-
ing formulations of C-energy and Tolman mass [40]. Employing Eqs. (5), (15) and (20),
we obtain the following relation

$$\frac{m}{\alpha r^3} = \frac{1}{8r^2} - \frac{1}{6} \left( T_{0}^{0(D)} - T_{1}^{1(D)} + T_{2}^{2(D)} \right) + \frac{\mathcal{E}}{3}, \quad (21)$$

yielding

$$\mathcal{E} = \frac{1}{2r^3} \int_{0}^{r} \tilde{r}^3 \left( T_{0}^{0(D)} \right)' d\tilde{r} - \frac{1}{2} \left( T_{1}^{1(D)} - T_{2}^{2(D)} \right). \quad (22)$$

Using Eq. (22) in (21), we obtain

$$m(r) = \frac{1}{6} \int_{0}^{r} \tilde{r}^3 \left\{ \left( \frac{1}{f_R} \right)' \left( \rho + \varphi + \varphi_{00} \right) + \left( \frac{1}{f_R} \right) \left( \rho + \varphi + \varphi_{00} \right)' \right\} d\tilde{r} - \frac{\alpha}{6} r^3 T_{0}^{0(D)} + \frac{\alpha \alpha r}{8}. \quad (23)$$

This demonstrates the connection between energy density inhomogeneity and
mass function in modified theory. The effects of modified terms on the struc-
tural changes resulting from the inhomogeneity of the energy density can be
examined through the above-mentioned equation. When the inward-directed
force of gravitation is counterbalanced by the outward pressure of a celestial
body, then the system is said to be in equilibrium. In GR, the Tolman-
Opphenheimer-Volkoff (TOV) equation is the counterpart of the hydrostatic
equilibrium equation. For anisotropic matter distribution, we obtain the
TOV equation in $f(R, T^2)$ theory through Eq. (10) as

$$p_r' = \frac{1}{(p_r + 2p - 2p_r) f_{T^2} + 1} \left\{ 2 \left( p_r' \left( 2\rho - 7p_r + 2p_r \right) + \rho ' \left( 2p_r - \rho - 2p_r \right) \right) \right. + \left. \frac{f'}{f} \left( - \rho^2 + 2\rho p_r + 2\rho p_r + p_r^2 - 4p^2 + 2p_r p_r \right) - \frac{2}{r} \left( 3\rho p_r - 3\rho p_r + p_r^2 \right) \right. \right. \\
- \left. \alpha \rho p_r + 4p_r^2 \right\} f_{T^2} - \left\{ \frac{\alpha (p_r + p_r)}{r(\alpha r - 2m)} \left( \frac{r^3}{f_R} \left( 2\rho p_r + \rho^2 - 2\rho p_r - 2p_r p_r \right) + p_r^2 \right) f_{T^2} - \left( \varphi - p_r - \varphi_{11} \right) \right. + \left. \frac{m}{\alpha} \right\} + \frac{2}{r} (p_r - p_r) \right\}. \quad (24)
Tolman described another formula for the mass of a cylindrically symmetric distribution having radius $r$ within the boundary $\Sigma$ as [40]

$$m_{Tol} = \frac{\alpha}{2} \int_0^r r^2 F G \left( T_0^{0(D)} - T_1^{1(D)} - 2T_2^{2(D)} \right) \, d\tilde{r}. \quad (25)$$

Using the field equations, the above expression turns out to be

$$m_{Tol} = -\alpha F' F r^2. \quad (26)$$

Inserting the value of $F'$ from Eq.(17), the Tolman mass is rewritten as

$$m_{Tol} = \frac{FG}{8} \left( 4\alpha r^3 T_1^{1(D)} - 8m + \alpha r \right). \quad (27)$$

In a static gravitational field, a test particle’s gravitational acceleration is associated with the Tolman mass as

$$a = \frac{G^{-1} F'}{F} = -\frac{m_{Tol}}{\alpha F' r^2}.$$

This equation interprets the Tolman mass as the effective gravitational mass. Equation (26) can be written in a more suitable way after some simplifications as [41]

$$m_{Tol} = (m_{Tol})_\Sigma \left( \frac{r}{R} \right)^3 - \alpha r^3 \int_0^R \frac{FG}{\tilde{r}} \left( E - \frac{1}{2} (T_1^{1(D)} - T_2^{2(D)}) \right) \, d\tilde{r}, \quad (28)$$

where $R$ is the radius at the boundary. Using Eq.(22), the Tolman mass formula can be rewritten as

$$m_{Tol} = (m_{Tol})_\Sigma \left( \frac{r}{R} \right)^3 + \alpha r^3 \int_0^R \frac{FG}{\tilde{r}} \left( (T_1^{1(D)} - T_2^{2(D)}) - \frac{1}{2r^3} \int_0^r r^3 (T_0^{0(D)})' \, dr \right) \, d\tilde{r}. \quad (29)$$

This equation describes the Tolman mass with modified corrections for static cylindrical symmetric spacetime that could be very essential in identifying how the Weyl scalar $E$, inhomogeneity in the energy density and effective pressure anisotropy interact.
4 The Orthogonal Splitting of the Riemann Tensor

Bel [42] was the first to investigate the orthogonal splitting of the Riemann tensor, establishing its left, right, and double dual in accordance with standard fashion. All the information in the Riemann tensor is contained in these tensors. Employing Bel’s approach, Herrera [43] derived structures scalars which are a collection of tensors representing in terms of some scalar functions. There are various distinct features of such scalars. First of all, they are scalars, which make complex systems easier to deal with than tensors. Additionally, this single tool interacts with various system’s components and provides a wide range of information about the evolution of the structure including expansion, inhomogeneity, shear evolution, etc. Using his method, we take into account the following tensor quantities

\[ Y_{\delta\lambda} = R_{\delta\mu\lambda\nu} v^\mu v^\nu, \]  
(30)

\[ Z_{\delta\lambda} = * R_{\delta\mu\lambda\nu} v^\mu v^\nu = \frac{1}{2} \eta_{\delta\mu\gamma\varsigma} R_{\gamma\varsigma\lambda\nu} v^\mu v^\nu, \]  
(31)

\[ X_{\delta\lambda} = * R^*_{\delta\mu\lambda\nu} v^\mu v^\nu = \frac{1}{2} \eta_{\gamma\varsigma\delta\mu} R^*_{\gamma\varsigma\lambda\nu} v^\mu v^\nu. \]  
(32)

Here, * represents the dual tensor which is defined as \( R^*_{\delta\lambda\mu\nu} = \frac{1}{2} \eta_{\alpha\beta\delta\lambda} R^\alpha\beta_{\mu\nu}. \)

The Riemann tensor can be written by using the field equations in (18) as

\[ R^\delta_{\lambda\mu} = C^\delta_{\lambda\mu} + 2 T^{(D)[\delta_{\beta]}_{\alpha]}_{\mu]} + T^{(D)} \left( \frac{1}{3} \delta^\delta_{[\lambda\delta]} - \delta^\delta_{[\lambda\delta]} \right). \]  
(33)

Using the above expression, the Riemann tensor can be splitted as

\[ R^\delta_{\lambda\mu} = R^\delta_{(I)\lambda\mu} + R^\delta_{(II)\lambda\mu} + R^\delta_{(III)\lambda\mu}. \]

Here,

\[ R^\delta_{(I)\lambda\mu} = \frac{2}{f_R} \left[ (\rho + p_p) + \nabla^\delta \nabla_{[\lambda\delta]} + \left\{ (\varphi - p_p - \Box f_R) + (-\rho^2 + \rho p_r + \rho p_{\perp} + \rho p_{\perp}) f_T^2 \right\} \right. \]

\[ \times v^\delta v_{[\lambda\delta]} + \left\{ (\varphi - p_p - \Box f_R) + (-\rho^2 + \rho p_r - \rho p_{\perp} + p_r p_{\perp}) f_T^2 \right\} \]

\[ \left. \times \delta^\delta_{[\lambda\delta]} + \left\{ (p_r - p_{\perp}) + (\rho^2 + 3 \rho p_r - 3 \rho p_{\perp} - p_r p_{\perp}) f_T^2 \right\} s^\delta s_{[\lambda\delta]} \right], \]

\[ R^\delta_{(II)\lambda\mu} = \frac{1}{f_R} \left\{ p_r + 2 p_{\perp} - \varphi + 3 \Box f_R - (\rho^2 - 3 \rho^2 + 4 p_r p_{\perp}) f_T^2 \right\} \]
where $\epsilon_{\delta\mu\nu} u^\mu = 0$ and $\eta_{\delta\mu\nu} = u_\delta \epsilon_{\mu\nu}$. The structure scalars are a combination of state variables that are important in evaluating the complexity of stellar structure and particularly useful in examining physical characteristics of the system.

The Riemann tensor allows us to express $Y_{\delta\lambda}$, $X_{\delta\lambda}$ and $Z_{\delta\lambda}$ in terms of matter variables. Further, these tensors are the source of five structure scalars. It is mentioned here that instead of five, there are eight structure scalars that correspond to the general cylindrical symmetric case. Since the scalar associated with $Z_{\delta\lambda}$ does not contain state variables that are necessary to calculate the complexity, therefore, we only consider four scalars in this work. The tensors $Y_{\delta\lambda}$ and $X_{\delta\lambda}$ can be written in terms of their trace ($X_T = X_\delta^\delta$, $Y_T = Y_\delta^\delta$) and trace-free ($X_{TF}$, $Y_{TF}$) parts as

$$X_{\delta\lambda} = \frac{1}{3} h_{\delta\lambda} X_T + X_{TF} \left( \frac{h_{\delta\lambda}}{3} + s_\delta s_\lambda \right),$$

$$Y_{\delta\lambda} = \frac{1}{3} h_{\delta\lambda} Y_T + Y_{TF} \left( \frac{h_{\delta\lambda}}{3} + s_\delta s_\lambda \right).$$

In $f(R, T^2)$ gravity, the trace-free and trace components are given as

$$X_{TF} = \frac{1}{2f_R} \left[ \left( 3p_r + p_r^2 - p_\perp p_\perp - 3\rho p_{\perp} \right) f_{T^2} + \left( p_r - p_\perp \right) \right] - \mathcal{E},$$

$$Y_{TF} = \frac{1}{2f_R} \left[ \left( 3p_r + p_r^2 - p_\perp p_\perp - 3\rho p_{\perp} \right) f_{T^2} - \left( p_\perp - p_r \right) \right] + \mathcal{E},$$

$$X_T = -\frac{\varphi}{f_R} + \frac{3\Box f_R}{2f_R} - \frac{11}{2f_R} (p_r p_\perp + \frac{2}{11} \rho) f_{T^2},$$

$$Y_T = \frac{\varphi}{f_R} + \frac{3}{2f_R} (p + 3\Box f_R + \frac{1}{3} \rho) + \frac{1}{2f_R} \left( 3\rho^2 + p_r^2 - 6p_r p_\perp \right) f_{T^2},$$

Equation (34) shows that the scalar $X_{TF}$ determines energy density inhomogeneity in fluid distribution. The overall energy content of the system is evaluated via $X_T$ in the presence of correction terms, while $Y_T$ examines the influence of anisotropic stresses caused by inhomogeneous density. Equations
(39), and (35) can be used to interpret physical significance of the scalar $\mathcal{Y}_{TF}$ as

$$m_{Tol} = (m_{Tol})_\Sigma \left( \frac{r}{R} \right)^3 - \alpha r^3 \int_r^\infty \frac{FG}{\tilde{r}} \left( \mathcal{Y}_{TF} - \frac{1}{2f_R}(\varphi_{22} - \varphi_{11}) \right) d\tilde{r}. \quad (38)$$

Equations (35) and (38) demonstrate that $\mathcal{Y}_{TF}$ determines how inhomogeneous energy density, non-linear $f(R, T^2)$ terms and anisotropic pressure affect the Tolman mass. The local anisotropic pressure in the presence of modified corrections can be obtained by utilizing Eqs. (34) and (35) as

$$X_{TF} + \mathcal{Y}_{TF} = \frac{1}{f_R} \left[ (3p_r + p_r^2 - p_r p_\perp - 3\rho p_\perp) f_{T^2} - (p_\perp - p_r) \right]. \quad (39)$$

5 The Complexity Factor

Complexity in a celestial structure is developed by a variety of factors. The electromagnetic field, inhomogeneity, heat dissipation, viscosity and pressure anisotropy, etc. are the examples of such factors. In general, the only framework with zero complexity is the one that has isotropic pressure and homogeneous energy density. Anisotropic pressure, energy density inhomogeneity and dark source terms of EMSG are responsible for creating complexity in the system under consideration. The scalar $\mathcal{Y}_{TF}$ relates these factors and also evaluates their impacts on the Tolman mass. Therefore, $\mathcal{Y}_{TF}$ is an appropriate choice for the complexity factor of the current setup. Here, $\mathcal{Y}_{TF}$ in terms of state parameters is produced by substituting Eq.(22) in (35) as

$$\mathcal{Y}_{TF} = \frac{1}{2r^3} \int_0^r \tilde{r}^3 (T_0^{(D)}) \tilde{r} d\tilde{r} - \frac{1}{f_R} \left\{ (p_\perp - p_r) + \frac{1}{2}(\varphi_{11} - \varphi_{22}) \right. $$

$$- \left. (p_r^2 + 3\rho p_r - 3\rho p_\perp - p_r p_\perp) f_{T^2} \right\}. \quad (39)$$

The set of field equations in EMSG comprises of five unknown parameters ($p_r$, $\rho$, $p_\perp$, $F$, $G$), so we need additional conditions to get a solution. For this purpose, the vanishing complexity factor is used to establish one constraint which is obtained from Eq.(39) as

$$\Pi = \frac{1}{(p_r + 3\rho)f_{T^2}} + \frac{f_R}{2r^3} \int_0^r \tilde{r}^3$$

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\[ \times \left\{ \left( \frac{1}{f_R} \right) (\varphi + \rho + \varphi_{00})' + \left( \frac{1}{f_R} \right)' (\varphi + \rho + \varphi_{00}) \right\} d\tilde{r} \right] . \quad (40) \]

For homogenous and isotropic matter distribution in GR, the complexity factor disappears. On the other hand, in \( f(R, T^2) \) gravity, the complexity vanishes for isotropic and homogeneous distribution if the system satisfies the following condition
\[ \frac{r^3}{f_R} (\varphi_{11} - \varphi_{22}) - \int_0^r \tilde{r}^3 \left\{ \left( \frac{1}{f_R} \right) (\varphi + \rho + \varphi_{00})' + \left( \frac{1}{f_R} \right)' (\varphi + \rho + \varphi_{00}) \right\} d\tilde{r} = 0. \]

Now, we examine the zero complexity constraint for a particular \( f(R, T^2) \) model given as \( 26 \)
\[ f(R, T^2) = R + \chi T^2, \quad \text{(41)} \]
which leads Eq.(40) to
\[ \Pi = \frac{1}{r^3(2 + (6\rho + 2p_r)\chi)} \left[ \int_0^r \tilde{r}^3 \rho' d\tilde{r} + \frac{\chi}{2} \int_0^r \tilde{r}^3 \left( p_r^2 + 2p_\perp^2 + \rho^2 \right)' d\tilde{r} \right] . \quad \text{(42)} \]

We still need a constraint to solve the field equations even after applying the condition \( \mathcal{Y}_{TF} = 0 \). To achieve this goal, we employ the energy density of the Gokhroo-Mehra solution and the polytropic equation of state to construct the relevant solutions.

### 5.1 The Gokhroo-Mehra Solution

To evaluate the solutions to the field equations corresponding to anisotropic self-gravitating structure, Gokhroo and Mehra \( 44 \) took into the account a particular type of energy density. They developed a model which describes the behavior of neutron star as well as higher redshifts of several quasi-stellar configurations. We use this specific form of energy density for the current configuration to analyze how compact structures will behave when the condition of vanishing complexity is applied \( 44 \). Thus, the energy density is expressed as
\[ \rho = \rho_o \left( 1 - \frac{\mathcal{K}r^2}{R^2} \right) , \quad \text{(43)} \]
where $K \in (0, 1)$ and $\rho_o$ is a constant. For the assumed energy density, the mass function takes the form

$$m(r) = \frac{\alpha}{2} \left[ \frac{\rho_o r^3}{3} \left( 1 - \frac{3Kr^2}{5R^2} + \frac{\chi}{2} + \frac{3\chi K^2 r^4}{14R^4} - \frac{3\chi K^2 r^6}{5R^4} \right) + \frac{r}{4} \right] + \frac{\chi}{2} \int_0^r \tilde{r}^2 (2p^2_\perp + p^2_r) d\tilde{r},$$  

which gives the following form of the metric function $G$

$$\frac{1}{G^2} = \frac{\chi}{2} \left( \frac{6\chi K^4 r^4}{5R^2} - \frac{3\chi K^2 r^6}{7R^4} - \frac{1}{r^2} \int_0^r \tilde{r}^2 (2p^2_\perp + p^2_r) d\tilde{r} \right) + \frac{3\chi K^2 r^2}{5R^2} - r^2 \varsigma,$$

$\varsigma = \rho_o/3$. From Eqs. (12) and (13), it is evident that

$$\frac{1}{r^2G^2} \left( 1 + \frac{rF}{F} - \frac{r^2F''}{F} + \frac{r^2F'G'}{FG} + \frac{rG'}{G} \right) = \Pi \left\{ (p_r + 3\rho) \chi + 1 \right\}.  \tag{46}$$

In order to find the unknowns, we introduce new variables as

$$G^{-2} = g(r), \quad F^2(r) = e^{\int (f(r)-\frac{1}{2}) dr},$$

hence, Eq. (46) reduces to

$$g' + 2g \left[ f + \frac{f'}{f} + \frac{2}{r^2f} - \frac{3}{r} \right] = -\frac{2}{f} \left[ \Pi \left\{ 1 + \chi(p_r + 3\rho) \right\} \right].$$

Integration of the above expression provides the radial metric function as

$$G^2(r) = \frac{f^2(r) e^{\int \left( \frac{4}{f(r)^2} + 2f(r) \right) dr}}{r^3} \left[ -2 \int f(r) e^{\int \left( \frac{4}{f(r)^2} + 2f(r) \right) dr} \left( 1 + \Pi(r) \right) \left( 1 + \chi(p_r + 3\rho) \right) dr + C \right],$$

where $C$ represents an integration constant. Hence, the line element can be expressed in the form of $\Pi$ and $f(r)$ as follows

$$ds^2 = \frac{-f^2(r) e^{\int \left( \frac{4}{f(r)^2} + 2f(r) \right) dr} dr^2}{r^3} \left[ -2 \int f(r) e^{\int \left( \frac{4}{f(r)^2} + 2f(r) \right) dr} \left( 1 + \Pi(r) \right) \left( 1 + \chi(p_r + 3\rho) \right) dr + C \right] - r^2 d\theta^2 - r^2 \alpha^2 dz^2 + e^{\int (f(r)-\frac{1}{2}) dr} dt^2.  \tag{47}$$
5.2 The Polytropic Model with Complexity-free Condition

In analyzing the internal structure of self-gravitating systems, several physical parameters play significant role. However, some factors are more important than others in examining the structure. For the current scenario, it is helpful to use an equation of state which accurately describes the relationship of key factors. Anisotropic celestial structures have extensively been studied through the polytropic equation of state, which expresses how energy density and radial pressure are related to each other [45]. The polytropic equation of state has the following form

\[ p_r = \mathbb{K} \rho^\gamma = \mathbb{K} \rho^{\frac{1+\gamma}{\gamma}}, \]  

(48)

where \( \gamma, n \) and \( \mathbb{K} \) indicate the polytropic exponent, polytropic index and polytropic constant, respectively. To derive the dimensionless mass function and TOV equation, we apply the following variables

\[ \rho_o = \frac{p_{ro}}{\tau}, \quad \xi = r \mathcal{J}, \quad \mathcal{J}^2 = \frac{\rho_o}{(2n + 2)\tau}, \quad \Upsilon^n(\xi)\rho_o = \rho, \]

\[ \Omega(\xi) = \frac{2\mathcal{J}^3 m(r)}{\rho_o}, \]  

(49)

where the entities \( \Upsilon, \Omega, \tau \) and \( \xi \) are dimensionless. We obtain the dimensionless forms of Eq.(15) and (24) by inserting the above variables

\[ \frac{d\Omega}{d\xi} = \frac{\alpha \rho_o}{8(n+1)\tau} + \xi^2 \Upsilon^n \left\{ 1 + \frac{\chi}{2} \left( \rho_o \Upsilon^n + 3\tau^2 \rho_o \Upsilon^{n+2} + \frac{2\Pi^2}{\rho_o \Upsilon^n} - 4\tau \Upsilon \Pi \right) \right\}, \]

\[ \tau \xi^3 \Upsilon^{n+1} \left\{ 1 + \frac{\chi}{2} \left( \rho_o \Upsilon^n + 3\Upsilon^2 \rho_o \Upsilon^{n+2} + \frac{2\Pi^2}{\rho_o \Upsilon^n} - 4\tau \Upsilon \Pi \right) \right\} - \frac{\xi^3 \chi}{\alpha \rho_o} \left( \rho_o \Upsilon^n \right) \]

\[ \times \left( \frac{1}{2} \Upsilon^n + \frac{1}{2} \tau^2 \Upsilon - \tau^2 \Upsilon^{n+2} - \tau^2 \Upsilon^{n+1} \right) + 4\tau \rho_o \Pi \Upsilon^{n+1} \right\} \]  

\[ \times \left\{ \left( -\rho_o \Upsilon^n - 2\Pi - 7\tau^2 \Upsilon^{n+2} - \frac{8\Pi^2}{\rho_o \Upsilon^n} \right) \right\}^{-1} \left\{ \left( \xi^2 \frac{d\Upsilon}{d\xi} \right) (1 + (2\Pi - \tau \rho_o \Upsilon^{n+1} + 2\rho_o \Upsilon^n)) \right\} \]

\[ - \frac{2\chi \xi^2}{\tau(n+1)} \left\{ -\rho_o n \Upsilon^{n-1} \frac{d\Upsilon}{d\xi} + 2\tau \rho_o (n+1) \Upsilon^n \frac{d\Upsilon}{d\xi} - 2 \frac{d\Pi}{d\xi} \right\} - 2n \Upsilon^{-1} \Pi \frac{d\Upsilon}{d\xi} - 12\tau^2 \rho_o (n+1) \Upsilon^{n+1} \frac{d\Upsilon}{d\xi} + 12\tau \Upsilon \frac{d\Pi}{d\xi} + 14\tau (n+1) \Pi \frac{d\Upsilon}{d\xi} \]
\[ \frac{2 \xi}{\rho_o n} \left[ 1 + (\tau \Upsilon + 3) \rho_o \chi \Upsilon^n - \tau \chi \rho_o \Upsilon^{n+1} + \chi \Pi \right] \frac{d\Pi}{d\xi} = \left[ \frac{-2 \xi \Pi}{\rho_o n} \left( 3 \rho_o n \Upsilon^{n-1} + \tau \rho_o (n + 1) \Upsilon^n \right) \right] \frac{d\Upsilon}{d\xi} - \frac{6 \Pi}{\rho_o n} \left( 1 + (\tau \Upsilon + 3) \rho_o \Upsilon^n \chi \right). \] 

(52)

For some arbitrary values of \( \tau \) and \( n \), we obtain a unique solution for the cylindrical celestial structure with zero complexity. A physically acceptable model must have positive, finite and maximum state parameters (pressure and energy density) at its center \((r = 0)\). Also, they must have decreasing trend towards the boundary. Moreover, the mass function must be an increasing and positive function of the radial coordinate. We fix \( \chi = 3, n = 5, \alpha = 1 \) and \( \rho_o = 7 \) for graphical analysis. The behavior of the energy density, anisotropy and mass function are illustrated through Figures 1-3, respectively. Figure 1 shows that \( \Upsilon \) is maximum at the center and exhibits decreasing trend for smaller values of \( \tau \). However, it shows a divergence of the magnitudes.
as we approach to zero of the abscissa for larger values of $\tau$ (left plot). Moreover, anisotropy also shows diverging behavior near the center and then exhibits a rapid increase followed by a decreasing trend towards the boundary for $\tau = 1.3, 1.5, 1.7$ (Figure 2). However, this factor exhibits decreasing behavior throughout for $\tau = 0.01, 0.03, 0.05$ (right plot). Figure 3 indicates that the mass function varies directly with $\xi$ while it has an inverse relation with $\tau$. Since the left plots in Figures 1 and 2 show a divergence, thus our resulting model is no more valid for larger $\tau$. We conclude that this model has physically valid solution only for $\tau = 0.01, 0.03$ and 0.05.
6 Conclusions

The presence of complexity in a structure identifies the existence of pressure anisotropy and inhomogeneity in the energy density. A physically uniform system in all directions has no complexity. In this paper, we have analyzed the complexity of static cylindrical object within the EMSG scenario. In this regard, the modified field equations corresponding to anisotropic cylindrical distribution have been computed. The mass functions $m$ and $m_{Tol}$ are calculated by using the C-energy and Tolman formulations, respectively and a specific relationship between them has also been developed. We have then discussed how the Weyl tensor and matter variables are related to $m_{Tol}$ and $m$. There are several definitions of complexity based on various factors in the literature. Since the definition suggested by Herrera included all the factors which cause complexity and thus, considers as the most suitable definition. Using his method for the orthogonal splitting of the Riemann tensor, four structure scalars are obtained to formulate the complexity factor. The scalar $\mathcal{Y}_{TF}$ incorporates the impacts of all matter variables, including anisotropic pressure and inhomogeneous energy density along with additional terms of $f(R, T^2)$ gravity. This scalar also deals with the effects of inhomogeneous and anisotropic factors upon the Tolman mass and, therefore, taken as the complexity factor.

The vanishing complexity condition has been constructed by assigning $\mathcal{Y}_{TF} = 0$. If a self-gravitating system in GR has an isotropic and homogeneous configuration, then it is regarded as the complexity-free. However, this does not imply vanishing complexity in this theory, which reflects the influence of modified terms. Thus, we can conclude that EMSG corrections are accounted for enhancing the complexity of a cylindrical body. The complexity factor will be zero if

$$\frac{r^3}{f_R} (\varphi_{11} - \varphi_{22}) - \int_0^r r^3 \left\{ \frac{1}{f_R} (\varphi + \rho + \varphi_{00})' + \left( \frac{1}{f_R} \right)' (\varphi + \rho + \varphi_{00}) \right\} d\tilde{r} = 0.$$  

The complexity-free constraint for a particular model $f(R, T^2) = R + \chi T^2$ gives an additional constraint which helps in solving the field equations by lessening the degrees of freedom.

Finally, two distinct models have been discussed to compute the solutions of modified field equations. Utilizing the energy density of the stellar configuration proposed by Gokhroo-Mehra, we have explored characteristics
of compact objects and obtained the corresponding solution. For the second model, the polytropic equation of state has been employed to establish a set of dimensionless equations by incorporating certain new parameters. The current setup consists of dimensionless zero complexity condition, mass and TOV equation have calculated numerical solutions of this system and examined them graphically by varying the parameter $\tau$. Figure 1 demonstrates that the smaller values of $\tau$ provide maximum density at the center and decreasing towards boundary while larger values of $\tau$ yield divergence in its magnitude near the center. Furthermore, for $\tau = 1.3, 1.5$ and $1.7$, anisotropy also displays a diverging trend near the core. However, this parameter consistently shows decreasing behavior for $\tau = 0.01, 0.03$ and $0.05$. Figure 3 shows an inverse relationship between the mass function and $\tau$, while it varies directly with $\xi$. The graphs of $\Upsilon$ and $\Pi$ show a diverging behavior for larger $\tau$, resulting in an invalid model. That our results for smaller values of $\tau$ provide physically viable solution contrary to the charged \cite{32} and uncharged \cite{47} sphere in this theory. It can be observed from the graphical analysis that $f(R, T^2)$ theory produces more dense structure as compared to GR. All our results coincide with GR \cite{7} for $f(R, T^2) = R$.

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