We are presenting here the new formulae for Bose-Einstein correlations (BEC) which contain effects of final state interactions (FSI) of both strong (in $s$-wave) and electromagnetic origin. We demonstrate the importance of FSI in BEC by analysing data for $e^+e^-$ annihilation and for heavy collisions. The inclusion of FSI results in the practical elimination (at least in $e^+e^-$ data) of the so-called degree of coherence parameter $\lambda$ (which becomes equal unity) and the long range parameter $\gamma$ (which is now equal zero).

1 Introduction

Bose-Einstein correlations (BEC) are among the most important current topics in high energy collisions (in particular in heavy-ion reactions). One of the interesting problems present in the BEC is the physical meaning of the degree of coherence parameter $\lambda$ and the long range correlation parameter $\gamma$ which are usually introduced by hand when analysing BEC data by means of the so-called standard formula:

$$N^{(\pm\pm)}/N^{BG} = c [1 + \lambda E^{2B}] (1 + \gamma Q).$$  \hspace{1cm} (1)

The function $E^{2B}$ appearing here is called the exchange function and $c$ is the so-called normalization parameter (also introduced by hand but we shall not discuss it here).

Our approach to this problem is based on the observation that BEC formula Eq.(1), which has been obtained by using the plane wave approximation (asymptotic states) for both observed particles, should be corrected for the effect of final state interactions (FSI) which can be of strong and electromagnetic type (Coulomb interactions). We have recently obtained several theoretical formulae for the BEC including effects of FSI of the strong type (in the $s$-wave,
isospin $I = 2$ channel, of the Coulomb interactions and of both types of FSI acting together.

The second order BEC without FSI is usually presented as the following convolution

$$N^{(\pm \pm)} / N^{BG} = \int_0^\infty \rho(x_1) \rho(x_2) |A_{12}|^2 d^3 x_1 d^3 x_2,$$

$$= \int_0^\infty \rho(R) \rho(r) |A_{12}|^2 d^3 R d^3 r$$  \hspace{1cm} (2)

of the single particle source density functions $\rho$ and the squared two particle amplitude

$$A_{12} = \frac{1}{\sqrt{2}} \{ \exp\{ (ip_1 (x_A - x_1) + ip_2 (x_B - x_2)) \}$$

$$+ \exp\{ (ip_1 (x_A - x_2) + ip_2 (x_B - x_1)) \} \}$$  \hspace{1cm} (3)

which is symmetrized accordingly, cf. Fig. 1.

Fig. 1 Identical boson exchange diagram.

Assuming now the following Gaussian distribution for the source function, $\rho(r) = e^{-r^2/2\beta^2}$, one obtains Eq. (1) with $E_{2B} = e^{-a^2 Q^2/2}$ (and with $c = 1$, $\lambda = 1$ and $\gamma = 0$).

In the next section we shall present how to correct this formula for the presence of strong interactions FSI which are seen in the phase shift analysis of the $\pi\pi$ and $K^0_s K^0_s$ correlations. The inclusion of Coulomb interactions, which are important when produced bosons are charged, demands however a profound change of the two particle amplitude $A_{12}$. This is discussed in Section 3. It is shown there also that, we can obtain information on the interaction range from data for unlike charged pairs $\pi^+ \pi^-$. Section 4 deals with the most general case when strong and Coulomb FSI coexist together. Our concluding remarks are presented in the last section.
2 Final state interaction in BEC for neutral particles

FSI of the strong type are limited (due to the short range of strong interactions) to a small number of partial waves (in practice to s-wave only) and it is therefore sufficient to use the following expression for the amplitude describing the system of two identical bosons in their rest frame:

\[ A_{12} = \frac{1}{\sqrt{2}} e^{i(p_1 + p_2) \cdot R} \left( \frac{e^{2i\delta} - 1}{ikr} \right) e^{ikr} + e^{-ikr} \cdot e^{i\delta} \cdot r + e^{i\delta} \cdot r \right], \quad (4) \]

Here \( R = \left( r_1 + r_2 \right)/2 \), \( k = (p_1 - p_2)/2 \), and \( r = (r_1 - r_2) \) and \( \delta \) denotes the phase shift describing the corresponding FSI. The data of phase shifts of \( \pi \pi \) and \( K_0^0 K_0^0 \), which were used in our analysis, are shown in Fig. 2.

After some algebra we obtain a following new formula of the BEC containing the strong FSI given by the respective phase shift function \( \delta \):

\[ N(2^-)/N^{BG} = 1 + \lambda \left\{ e^{-\beta^2 Q^2/2} \text{Re}[\text{erfc}(z)] + \frac{8 \sin^2 \delta}{\beta^2 Q^2} e^{-\beta^2 Q^2/2} \text{Re}[\text{erfc}(z)] \right. \]
\[ + \left. \frac{8 \sin \delta \cos \delta}{\beta^2 Q^2} e^{-\beta^2 Q^2/2} \text{Im}[\text{erfc}(z)] \right\}, \quad (5) \]

where \( z = -i\beta Q/\sqrt{2} \), and where \( \text{Re}[\text{erfc}(\sqrt{-i\beta Q/\sqrt{2}})] = 1 \) relation was used (the degree of coherence parameter \( \lambda \) was added by hand here). Notice that for \( \delta \to 0 \), i.e., when the FSI is switched off, we are recovering standard formula given by Eq. (1) with \( E = e^{-\beta^2 Q^2/2} \) (if we add by hand additional parameters \( c \) and \( \gamma \) in the form of long range correlation factor \( (1 + \gamma Q) \)). In Fig. 3 we
present our analysis of BEC data for $K_s^0K_s^0$ pair production using the above approach.

### Fig. 3. Analyses of the data of $K_s^0K_s^0$ pair.

#### 3 Final state interaction in BEC for charged particles - Coulomb interactions

In the case of Coulomb type of FSI the amplitude $A_{12}$ has to be described by the so called Coulomb wave functions:

$$A_{12} = \frac{1}{\sqrt{2}} [\Psi(k, r) + \Psi_S(k, r)], \quad (6)$$

$$\Psi(k, r) = \Gamma(1 + i\eta)e^{-\pi\eta/2}e^{ik\cdot r}\Phi(-i\eta; 1; ikr(1 - \cos \theta)), \quad (7)$$

$$\Psi_S(k, r) = \Gamma(1 + i\eta)e^{-\pi\eta/2}e^{-ik\cdot r}\Phi(-i\eta; 1; ikr(1 + \cos \theta)), \quad (8)$$

where $r = x_1 - x_2$, the parameter $\eta = m_{red}a/k$, the momentum transfer $Q = (p_1 - p_2) = 2k$ and $\Phi$ denotes the confluent hypergeometric function $\Phi$. The BEC formula now reads:

$$\frac{N(\pm) / N^{BG}}{G(2k)} = \sum_{n=0}^{\infty} \sum_{m=0}^{n+m} \frac{(-i)^n(i)^m}{n + m + 1} 2k^{n+m}I_R(n, m)A_nA_m^*$$

$$\times \left[ 1 + \frac{n!m!}{(n + m)!} \left( 1 + \frac{n}{i\eta} \right) \left( 1 - \frac{m}{i\eta} \right) \right]$$

$$= (1 + \Delta_{1C}) + (\Delta_{EC} + E_{2B}), \quad (7)$$

where $G(2k) = 2\pi\eta/(e^{2\pi\eta} - 1)$ denotes Gamow factor, the first and the second parentheses in Eq. (5) correspond to the first and the second terms in Eq. (7).
and (cf. ref. [3])

\[ I_R(n, m) = 4\pi \int dr \, r^{2+n+m} \rho(r), \quad A_n = \frac{\Gamma(i\eta + n)}{\Gamma(i\eta)} \frac{1}{(n!)^2}. \]

This formula, as it was shown in ref. [3], can be also used to analyse data on unlike sign \( \pi^+\pi^- \) pair production. This data cover the Coulomb interaction region at small momentum transfer \( Q \leq 40 \text{ MeV/c} \) in \( P + Ta \rightarrow \pi^+ + \pi^- + X \) at proton energy 70 GeV and have been so far analysed by using only Gamow factor correction.\[11\] However, when analysed by using instead our Coulomb formula with \( \eta = m_{\pi}a/2k \),

\[ N^{(+/-)} / N^{BG} = G(-\eta)(1 + \Delta_{1C}(-\eta)) \] (9)

they can provide us information on the interaction region in \( p + Ta \rightarrow \pi^+ + \pi^- + X \) collisions which turns out to be about 2 fm, cf. Fig. 4. It should be noticed that the interaction range cannot be estimated from the analysis performed only by using the Gamow factor.

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Fig. 4. Analysis of data for the \( \pi^+\pi^- \) pair production.

When one applies our formulae to single pion production in the external Coulomb field one finds an apparent quasi-scaling behavior in the pion production yield \( N^\pi(\beta, Z_{eff}) = 1 + \Delta_{1C} \); namely one observes that

\[ N^\pi(Q; \beta, Z_{eff}) \approx N^\pi(Q; \lambda \times \beta, \lambda \times Z_{eff}), \quad (1 \leq \lambda \leq 3). \] (10)

Because of this property, we have found that it is difficult to estimate the magnitude of nuclear fragments \( Z_{eff} \) produced in the central region in heavy-ion collisions from the yield ratio \( N^\pi^+ / N^\pi^- \) only.

5
4 Final state interaction including both strong and Coulomb interactions

To describe a pair of the identical bosons including both strong and Coulomb interactions, we have to symmetrize the total wave function in the following way:

\[ A_{12} = \frac{1}{\sqrt{2}} [\Psi_C(k, r) + \Psi_C^S(k, r) + \Phi_{st}(k, r) + \Phi_{st}^S(k, r)]. \] (11)

Here \( \Psi_C(k, r) \) denotes Coulomb wave function described above, superscript \( S \) denotes the symmetrization of the wave function and function \( \Phi_{st}(k, r) \) stands for the wave function induced by strong interactions. Assuming a source function \( \rho(r) \) we obtain the following expression for the BEC in this case:

\[
\frac{N^{(\pm \pm)}}{N^{BG}} = \frac{1}{G(2k)} \int \rho(r) d^3r |A_{12}|^2, \]

\[
I_C = (1 + \Delta_{1C}) + (E_{2B} + \Delta_{EC}), \]

\[
I_{Cst} = 2\Re\left[ \frac{2}{k}(2k)^i n \exp(-i(\eta_0 + \delta^{(2)}_0)) \sin \delta^{(2)}_0 \sum_{n=0}^\infty I_R(1+n)A(0) \right], \]

\[
I_{st} = \frac{2}{k^2} I_{R1}(0) \sin^2 \delta^{(2)}_0. \]

Explicit expressions for quantities present here are given in ref.[5]. It is also shown there that our method is equivalent to the numerical solution of the Schrödinger equation with strong and Coulomb potentials [3]. Our final formula (containing three additional parameters: \( c, \lambda \) and \( \gamma \), added by hand) is therefore given as:

\[
\frac{N^{(\pm \pm)}}{N^{BG}}(Q = 2k) = c \left( 1 + \Delta_{1C} + \Delta_{EC} + I_{Cst} + I_{st} \right) \left[ 1 + \frac{E_{2B}}{1 + \Delta_{1C} + \Delta_{EC} + I_{Cst} + I_{st}} \right] (1 + \gamma Q). \] (13)

It should be noted that the normalization \( c \) and an effective degree of coherence, i.e., the denominator of the ratio \( E_{2B}/(1 + \Delta_{1C} + \Delta_{EC} + I_{Cst} + I_{st}) \), are related to each other. For the sake of reference we shall also use in our analyses the standard formula as given be Eq. (1) (with \( E_{2B} = \exp(-\beta^2 Q^2/2) \)). We apply now our formulae to data for \( e^+ e^- \) annihilation [3]. Results of our analyses are shown in Fig. 5 and Table I. As seen in Table I, our (Eq.(13)) estimated values of the degree of coherence parameter \( \lambda \) are systematically larger (approaching
than those obtained by the standard formula (Eq.(1)). Similarly, the long range correlation parameter $\gamma$ approaches now (approximately) zero.

Fig. 5. Example of the analysis of data in $e^+e^-$ annihilation for TPC and OPAL collaborations.

|       | $\beta$ [fm] | $\lambda$ | $\gamma$ | $c$       | $\chi^2$/NDF |
|-------|--------------|-----------|----------|-----------|---------------|
| TPC   |              |           |          |           |               |
| ref.[14] | 0.92 ± 0.06* | 0.61 ± 0.05 | –        | –         | –             |
| Eq.(13)   | 0.74 ± 0.05  | 1.10 ± 0.04 | –0.00 ± 0.02 | 1.00 ± 0.02 | 44.2/35       |
| Eq.(1)    | 0.91 ± 0.06  | 0.61 ± 0.05 | 0.08 ± 0.03 | 0.88 ± 0.02 | 41.0/35       |
| OPAL     |              |           |          |           |               |
| ref.[15] | 1.12 ± 0.02* | 0.85 ± 0.03 | –        | –         | 336/73        |
| Eq.(13)   | 1.09 ± 0.04  | 1.04 ± 0.03 | 0.00 ± 0.00 | 0.99 ± 0.01 | 124.4/74      |
| Eq.(1)    | 1.34 ± 0.04  | 0.71 ± 0.04 | 0.04 ± 0.00 | 0.94 ± 0.00 | 118.7/74      |

Table 1: Estimated parameters

5 Concluding remarks

We have presented several analytic formulae for BEC including the Coulombic and strong FSI obtained by us recently. Combining the seamless fitting method and the CERN MINUIT program in Eq.(13) we have analysed different sets of BEC data showing respectively:

1. The role of strong FSI by using data on $K^0_sK^0_s$ phase shifts;

2. The ability to obtain by using our method the range of interaction from precise data on $\pi^+\pi^-$ production;
The possibility that $\lambda < 1$ and $\gamma \neq 0$ values of parameters encountered in the standard analyses of data could be a reflection of the combined action of strong and Coulombic FSI which were not taken properly into account there. In fact, also the values of the source size parameter reported by various collaborations (after using the relation: $\beta = \sqrt{2R}$) and obtained by the standard formula Eq.(1) are systematically larger than values estimated by our formula Eq.(13).

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Fig. 1
Fig. 2(a)
$K_S^0 K_S^0$

$\delta_0^{(1)}$ (deg)

$Q$ (GeV/c)

Fig. 2(b)
DELPHI [10]

4-parameters

\[ \text{eq. (5)} \quad \lambda = 0.54, \chi^2 = 24.1 \]

\[ \text{eq. (1)} \quad \lambda = 0.62, \chi^2 = 17.8 \]

3-parameters

\[ \text{eq. (5)} \quad \lambda = 0.9 \text{ (fixed)}, \chi^2 = 25.8 \]

Fig. 3
Fig. 4

\[ p + Ta \rightarrow \pi^+\pi^- + X \text{ at proton energy 70 GeV} \]

- \text{our calc. with } R=2.30 \text{ [fm]}
- \text{Gamow}
Fig. 5(a)

TPC

- eq. (13)
- eq. (1)

$N^{(\pm)}/N^{BG}$ vs $Q = 2k$ [GeV/c]
$N(N^\pm)/N^\text{BG}$

$Q = 2k$ [GeV/c]

Fig. 5(b)