Entanglement versus quantum discord in two coupled double quantum dots

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Abstract. We study the dynamics of quantum correlations of two coupled double quantum dots containing two excess electrons. The dissipation is included through the contact with an oscillator bath. We solve the Redfield master equation in order to determine the dynamics of the quantum discord (QD) and the entanglement of formation (EoF). Based on our results, we find that the QD is more resistant to dissipation than the EoF for such a system. We observe that this characteristic is related to whether the oscillator bath is common to both qubits or not and to the form of the interaction Hamiltonian. Moreover, our results show that the QD might be finite, even for higher temperatures in the asymptotic limit.

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1. Introduction

Among the ‘unusual’ manifestations observed in the quantum world, entanglement is undoubtedly one of the most interesting. Entanglement corresponds to global states of two or more quantum systems that cannot be separated into direct product states of individual subsystems. This characteristic yields correlations between quantum systems that cannot be found in any classical system [1]. Moreover, entanglement is a very important ingredient of a quantum computer’s architecture [2] and quantum communication [1]. It is well known that some tasks executed by a quantum computer can be performed exponentially faster than by existing computers. These potential applications of a quantum computer have motivated a large number of experimentalists, and different approaches have been used to build entangled states in laboratories [3]. Despite the successful experimental achievements, there are many difficulties to be overcome before a functional quantum computer becomes a reality. One of the most trivial is the fact that the usual quantum system candidates for qubits (see below) are intrinsically open to their environments and therefore suffer their effects by losing coherence. Thus, pure quantum states become mixed states under the environment’s influence. Fortunately, it has been demonstrated that computers based on mixed states can also be used to solve certain tasks more efficiently than classical computers [4], although they are less powerful than the computation using pure states. The reason for such performance is attributed to correlations not presented in classical systems. These quantum correlations (QCs) can be identified through a quantity called quantum discord (QD) [5]. Recently, interest in this subject has increased greatly due to the possibility of achieving quantum computation without entangled states [6]. As a consequence, many different aspects of QD have been discussed, e.g. robustness to quantum sudden death [7, 8], relation to the speed-up in deterministic quantum computation with one qubit [9], the interplay between quantum phase transitions and QD [10], and the condition to obtain completely positive maps [11].

There are many different systems proposed as candidates for qubits. Among them, double quantum dots (DQDs) (see figure 1) are very interesting due to their easy integration with existing electronics and the advantage of scalability [12]. Moreover, arrays of quantum dots can be employed to perform logical operations, which can be used to implement a universal quantum computation based on quantum [13] and classical effects [14]. In this work, we investigate the QC dynamics of two DQDs, including the basic elements to simulate a realistic situation, e.g. finite temperatures, interaction between qubits and detuning. We also compare the effects of dissipation on the behavior of the QD with that of the entanglement of formation (EoF) [15]. Our results show the longer duration of QD against the EoF. To explain such a result, we analyze how the environment acts on the system by comparing the case where the bath is coupled to both DQDs, which is the more realistic case, to that where each DQD is coupled to its own bath. Based on this comparison, we verify that the common character of the bath might be very important to prolong the QD. In fact, this character of the environment brings different properties to the dissipative dynamics. Contrary to the case of independent environments, by which the coherences are completely lost, we can find regions of the state space where the coherences are preserved ([16] and references therein). We observe that this robustness can be transferred to the QD.

The present paper is organized as follows. In section 2, we describe the basic concepts of QD. In section 3, we present the Hamiltonian of the system and the master equation that accounts for the interaction between the system and the environment. In section 4, we discuss
the obtained results for the QCs of two coupled DQDs. Finally, we summarize our work in section 5.

2. Quantum discord

QD has been proposed as a quantity that captures the QCs between two subsystems [5]. To evaluate such QCs, one needs to subtract the classical correlations from the total correlation [5, 17]. The total correlation of bipartite quantum systems, A and B, is calculated by the quantum mutual information [18]:

$$I(A : B) = S(A) - S(A|B), \tag{1}$$

where $S(A|B) = S(AB) - S(B)$ and $S(X) = -\text{Tr}[X \log X]$ is the von Neumann entropy of the density matrix $X$. Here, we adopt the following definitions: $AB \equiv \rho_{AB}$, $A \equiv \rho_A = \text{Tr}_B(\rho_{AB})$ and $B \equiv \rho_B = \text{Tr}_A(\rho_{AB})$, where $\rho_{AB}$ is the bipartite density matrix.

To calculate the classical correlations we observe that the projective measurements on a subsystem remove all nonclassical correlations between the parts, i.e. after a measurement on a particular subsystem, all QCs are destroyed. Thus we define a quantity that evaluates the mutual information after a measurement on one of the subsystems

$$I(A : B)_{\{\Pi_j^B\}} = S(A) - S(A|\{\Pi_j^B\}), \tag{2}$$

where $S(A|\{\Pi_j^B\})$ is the system conditional entropy after the measurement and $\{\Pi_j^B\}$ defines a complete set of 1D projectors, and the different outcomes of this measurement are accounted for by $j$. To quantify the QCs, since $I(A : B)_{\{\Pi_j^B\}}$ depends on the projector basis $\{\Pi_j^B\}$, we take the maximum of $I(A : B)_{\{\Pi_j^B\}}$, taking into account all possible projectors. Thus we can define the following quantity:

$$J(A : B) = S(A) - \min_{\{\Pi_j^B\}} S(A|\{\Pi_j^B\}), \tag{3}$$

which gives a measure of the total classical correlations between two subsystems [17].

Therefore, the QD can be written

$$\delta(A : B) = I(A : B) - J(A : B). \tag{4}$$

It is straightforward to observe that, if all information can be obtained locally by $B$, this subsystem has QD equal to zero. This implies that a measurement on $B$ does not alter the state.
of $A$. Otherwise, if just part of the information can be obtained locally, $B$ is quantum correlated to $A$. The QD thus gives the total information that is not locally accessible and has been accepted as a measure of the QC. Besides, the QD is possibly finite even for separable quantum states and can be utilized as a new resource for quantum computation [4, 6]. Also, when only pure states are considered, QD and the EqF [15] are indistinguishable.

3. Theoretical model

3.1. Hamiltonian of the system

The system considered in this work is composed of two DQDs (see figure 1), where each has an excess electron localized in either the left $|L\rangle$ or the right dot $|R\rangle$. This system can be modeled by the following pseudospin Hamiltonian:

$$H_S = \Delta \left( \sigma_x^{(1)} + \sigma_x^{(2)} \right) + J \sigma_z^{(1)} \otimes \sigma_z^{(2)}.$$  \hspace{1cm} (5)

where the first term describes the tunneling coupling energy $\Delta$ between the qubits, and the last term takes into account the effects of the Coulomb interaction between the two electrons, which favors anti-parallel configurations $|L, R\rangle$ and $|R, L\rangle$ over the parallel ones $|L, L\rangle$ and $|R, R\rangle$. We adopt the convention $|L\rangle \equiv | \downarrow \rangle$ and $|R\rangle \equiv | \uparrow \rangle$. For simplicity, we set the energy offset of the single-qubit states as equal to zero.

3.2. Master equation

In order to analyze the two DQDs’ dynamics in an open quantum system, we suppose that both qubits are coupled to a bath of harmonic oscillators (phonons). The total Hamiltonian that computes the environment perturbation is given by

$$H = H_S + \left( \sigma_z^{(1)} + \sigma_z^{(2)} \right) \mathcal{L} + H_B,$$  \hspace{1cm} (6)

where $\mathcal{L} = B + B^\dagger$ with $B = \sum_k g_k a_k$, $H_B = \sum_k \omega_k a_k^\dagger a_k$, $\omega_k$ is the frequency of the $k$th normal mode of the bath and $\hbar = 1$. Here $a_k$ is the usual annihilation operator of this mode and $g_k$ is its complex coupling constant with the DQDs. It is important to note that the interaction Hamiltonian acts in a correlated way with both qubits, thereby representing a common bath. To numerically determine the evolution of the two DQDs’ reduced density matrix, we employ the Redfield master equation [20]

$$\frac{d\rho_I(t)}{dt} = -\int_0^t dt' \text{Tr}_B \left\{ \left[H_I(t'), \left[H_I(t'), \rho_B(t') \right] \right] \right\},$$  \hspace{1cm} (7)

where $H_I(t)$ is the interaction Hamiltonian in the interaction picture,

$$H_I(t) = U_S^\dagger(t) U_B^\dagger(t) \left\{ \left( \sigma_z^{(1)} + \sigma_z^{(2)} \right) \mathcal{L} \right\} U_B(t) U_S(t),$$  \hspace{1cm} (8)

where $U_B(t) = \exp(-i H_B t)$ and $U_S(t) = \exp(-i H_S t)$. Explicitly, the unitary evolution $U_S(t)$ is given by

$$U_S(t) = \frac{1}{2} \left[ \cos(\Omega t) + \cos(J t) \right] I - i \frac{\Delta}{\Omega} \left[ \sin(\Omega t) \right] \left( \sigma_x^{(1)} + \sigma_x^{(2)} \right) + \frac{1}{2} \left[ \cos(\Omega t) - \cos(J t) \right] \sigma_x^{(1)} \sigma_x^{(2)} - \frac{i}{2} \left[ \sin(J t) - \frac{J}{\Omega} \sin(\Omega t) \right] \sigma_y^{(1)} \sigma_y^{(2)} - \frac{i}{2} \left[ \sin(J t) + \frac{J}{\Omega} \sin(\Omega t) \right] \sigma_z^{(1)} \sigma_z^{(2)},$$  \hspace{1cm} (9)
where \( \Omega = \sqrt{J^2 + 4\Delta^2} \). Here, we suppose that the oscillator bath density matrix \( \rho_B \) is initially decoupled from the system,

\[
\rho_B = \frac{1}{Z} \exp(-\beta H_B),
\]  

(10)

where \( Z \) is the partition function \( Z = \text{Tr}_B[\exp(-\beta H_B)] \), \( \beta = 1/k_B T \), \( k_B \) is the Boltzmann constant and \( T \) is the absolute temperature of the environment. Defining \( U_B^\dagger(t)LU_B(t) \equiv \tilde{L}(t) \) and \( \Lambda(t) \equiv \sum_{s=0}^1 (\sigma_z^{(s)} + \sigma_x^{(s)})U_S(t) \), we can write the interaction Hamiltonian in the interaction picture as follows:

\[
H_I(t) = \Lambda(t)\tilde{L}(t).
\]  

(11)

Thus, substituting equation (10) and (11) into the master equation, equation (7), we obtain

\[
\frac{d\rho_I(t)}{dt} = \int_0^t dt' D(t, t') \left[ \Lambda(t), \rho_I(t') \Lambda(t') \right] + \int_0^t dt' D^*(t, t') \left[ \Lambda(t'), \rho_I(t), \Lambda(t) \right],
\]  

(12)

where \( D(t, t') = T_1(t - t') + T_2(t - t') \) with

\[
T_1(t - t') = \text{Tr}_B \left\{ \tilde{B}(t) \tilde{\rho}_B \tilde{B}^\dagger(t') \right\}
\]  

(13)

\[
= \sum_k |g_k|^2 n_k \exp[-i\omega_k(t - t')],
\]

and

\[
T_2(t - t') = \text{Tr}_B \left\{ \tilde{B}^\dagger(t) \tilde{\rho}_B \tilde{B}(t') \right\}
\]  

(14)

\[
= \sum_k |g_k|^2 (n_k + 1) \exp[i\omega_k(t - t')],
\]

with \( \tilde{B}(t) = U_B^\dagger(t)BU_B(t) \), \( \tilde{\rho}_B = U_B^\dagger(t)\rho_B U_B(t) \) and \( n_k \) is the average occupation number of mode \( k \):

\[
n_k = \frac{1}{\exp(\beta\omega_k) - 1}.
\]  

(15)

Defining, as usual, the spectral function

\[
J(\omega) = \sum_k |g_k|^2 \delta(\omega - \omega_k),
\]  

(16)

we can replace the summations above by integrals

\[
T_1(t) = \int_0^\infty d\omega J(\omega)n(\omega)\exp(-i\omega t)
\]  

(17)

and

\[
T_2(t) = \int_0^\infty d\omega J(\omega)\exp(i\omega t)[n(\omega) + 1].
\]  

(18)

If we assume an ohmic spectral density to the reservoir, \( J(\omega) = \eta \omega \exp(-\omega/\omega_c) \), where \( \omega_c \) is a cutoff frequency and \( \eta \) is the damping constant, we can explicitly evaluate the integrals above that yield:

\[
D(t, t') = \frac{\eta\omega_c^2}{|1 + i\omega_c(t - t')|^2} + \frac{2\eta}{\beta^2} \text{Re} \left\{ \Psi^{(1\text{st})}(1 + 1/(\beta\omega_c) - i(t - t')/\beta) \right\},
\]  

(19)

where \( \Psi^{(1\text{st})}(\cdot) \) is the first polygamma function.

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Figure 2. QC (QD and EoF) for the unitary evolution ruled by $H_S$. We consider different values of $J/\Delta$ and we fix the coupling constant $\eta = 0$. In (a)–(c) we have $J = \Delta/8$, $J = \Delta/4$ and $J = \Delta/2$, respectively. This is what we define as the weak-coupling regime. For the strong-coupling regime, we have (d)–(f) with $J = 4\Delta$, $J = 8\Delta$ and $J = 16\Delta$, respectively.

4. Quantum correlation dynamics of double quantum dots

To calculate the DQDs’ QC dynamics, we numerically solve equation (4) using the density matrix dynamics computed by equation (7) as input. In our simulations, we fixed $\Delta = \pi \hbar / 2\tau$ ($\Delta \approx 10 \mu \text{eV}$) and the cutoff frequency $\omega_c = 200 / \tau$ [21], where $\tau = 10^{-10} \text{s}$. These parameters correspond to typical experimental values in GaAs/AlGaAs lateral DQDs [19]. Our expressions are given as a function of the weight of the Coulomb interaction between the two electrons, $J$, and the damping constant $\eta$. First, we analyze the unitary dynamics in figure 2, supposing that the two DQDs are initially uncorrelated, $\rho_I(0) = \left| \uparrow \downarrow \right\rangle \left\langle \uparrow \downarrow \right|$, and assuming that the coupling with the environment is zero, i.e. $\eta = 0$. Since we assume an initial pure state, the EoF and the QC dynamics coincide. In figure 2, we observe that similar dynamics can be reached for different Coulomb coupling values $J$. Thus, one can find entanglement dynamics in the weak-coupling regime ($J \ll \Delta$) that correspond to similar dynamics in the strong-coupling regime ($J \gg \Delta$), and vice versa. To determine these equivalent entanglement dynamics, we examine the period that the initial state $\left| \uparrow \downarrow \right\rangle$ takes to reach a maximum entangled state $t_{\text{max}}$ for $J \ll \Delta$ and $J \gg \Delta$. For this purpose, we estimate these periods by analyzing the coherences of
In the weak-coupling regime $J \ll \Delta$, we obtain

$$t_{\text{max}} \approx \frac{1}{J_\prec} \left( \frac{\pi}{2} + n\pi \right),$$

where $n \in \mathbb{N}$ and $J_\prec$ denotes that $J \ll \Delta$. For the strong coupling regime $J \gg \Delta$ ($J_\succ$), the period during which the initial state reaches a maximum entangled state depends on $\Delta$ and is given by

$$t_{\text{max}} \approx \frac{J_\succ}{\Delta^2} \left( \frac{\pi}{4} + n\frac{\pi}{2} \right).$$

Based on equations (20) and (21), we find that the time to achieve a maximum entangled state is approximately the same in both regimes when $J_\prec J_\succ = 2\Delta^2$. This behavior can be noted by comparing figure 2(a) ($J_\prec = \Delta/8$) with figure 2(f) ($J_\succ = 16\Delta$), and by comparing figure 2(b) ($J_\prec = \Delta/4$) with figure 2(c) ($J_\succ = 8\Delta$).

In figure 3, assuming that $\rho_1(0) = |\uparrow\downarrow\rangle\langle\uparrow\downarrow|$, we present the numerical results for the QD and the EoF dynamics within the strong-coupling regime ($J = 4\Delta$), for different temperatures ($T = 0.1, 0.5$ and $2$ K) and for finite damping constant ($\eta = 1/600$). The EoF for two qubits can be expressed in terms of the concurrence [22], EoF $=-f(C)\log_2(C) - (1-f(C))\log_2(1-f(C))$, where $f(C) = (1 + \sqrt{1-C^2})/2$. On the other hand, the concurrence $C$ is defined as the maximum between zero and $\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4$, where $\lambda_i$, for $i = 1, 2, 3$ and 4, is the square root of the eigenvalues of the matrix $\rho\sigma_y \otimes \sigma_y \rho^* \sigma_y \otimes \sigma_y$, with $\lambda_1$ being the largest one among them, and $\rho^*$ is the complex conjugate of $\rho$. In figure 3, one notes that the QCs (QD and EoF) oscillate rapidly, which resembles the behavior observed when there is no coupling to the environment (see figure 2(d)). However, when the coupling to the bath is included, the QCs decay as

Figure 3. Two-qubit QCs (QD and EoF) under dissipative dynamics for $J = 4\Delta$ and $T = 0.1$ K in (a), $T = 0.5$ K in (b) and $T = 2$ K in (c). The blue (dotted) line is the QD dynamics and the red (solid) line is the EoF dynamics. Here we use $\eta = 1/600$. 

$$U_S(t)\rho(0)U_S^\dagger(t).$$ 

In the weak-coupling regime $J \ll \Delta$, we obtain 

$$t_{\text{max}} \approx \frac{1}{J_\prec} \left( \frac{\pi}{2} + n\pi \right).$$

where $n \in \mathbb{N}$ and $J_\prec$ denotes that $J \ll \Delta$. For the strong coupling regime $J \gg \Delta$ ($J_\succ$), the period during which the initial state reaches a maximum entangled state depends on $\Delta$ and is given by 

$$t_{\text{max}} \approx \frac{J_\succ}{\Delta^2} \left( \frac{\pi}{4} + n\frac{\pi}{2} \right).$$

Based on equations (20) and (21), we find that the time to achieve a maximum entangled state is approximately the same in both regimes when $J_\prec J_\succ = 2\Delta^2$. This behavior can be noted by comparing figure 2(a) ($J_\prec = \Delta/8$) with figure 2(f) ($J_\succ = 16\Delta$), and by comparing figure 2(b) ($J_\prec = \Delta/4$) with figure 2(c) ($J_\succ = 8\Delta$).

In figure 3, assuming that $\rho_1(0) = |\uparrow\downarrow\rangle\langle\uparrow\downarrow|$, we present the numerical results for the QD and the EoF dynamics within the strong-coupling regime ($J = 4\Delta$), for different temperatures ($T = 0.1, 0.5$ and $2$ K) and for finite damping constant ($\eta = 1/600$). The EoF for two qubits can be expressed in terms of the concurrence [22], EoF $=-f(C)\log_2(C) - (1-f(C))\log_2(1-f(C))$, where $f(C) = (1 + \sqrt{1-C^2})/2$. On the other hand, the concurrence $C$ is defined as the maximum between zero and $\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4$, where $\lambda_i$, for $i = 1, 2, 3$ and 4, is the square root of the eigenvalues of the matrix $\rho\sigma_y \otimes \sigma_y \rho^* \sigma_y \otimes \sigma_y$, with $\lambda_1$ being the largest one among them, and $\rho^*$ is the complex conjugate of $\rho$. In figure 3, one notes that the QCs (QD and EoF) oscillate rapidly, which resembles the behavior observed when there is no coupling to the environment (see figure 2(d)). However, when the coupling to the bath is included, the QCs decay as
Figure 4. Two-qubit QCs (QD and EoF) under dissipative dynamics for $J = 4\Delta$ and $T = 0.1$ K in (a), $T = 0.5$ K in (b) and $T = 2$ K in (c). The blue (dotted) line is the QD dynamics and the red (solid) line is the EoF dynamics. Here we use $\eta = 1/200$.

a function of time due to the loss of correlations between the qubits and the environment. Moreover, we verify that QD is more robust than EoF for higher temperatures and for higher values of the coupling to the environment $\eta$ (see figure 4), and remarkably reaches a constant value in the asymptotic limit when $t \to \infty$. In figures 5 and 6, the EoF and QD dynamics are analyzed for different temperatures and bath couplings $\eta$ within the weak regime $J = \Delta/4$. The QCs for $\eta = 1/600$ presented in figure 5 have an oscillation period that corresponds to the unitary evolution (figure 2(b)). However, when the coupling to the environment is more relevant $\eta = 1/200$, we observe a short oscillation period (see figure 6). Again, the QCs decay as a function of time due to the interaction with the environment in both figures 5 and 6. By comparing the results of the weak-coupling regime to those in the strong-coupling regime, we note that the QD is also more robust than EoF for higher temperatures and the QD tends to a finite value, in this case. This behavior is related to the convergence of the system when $t \to \infty$ to quantum subspaces defined by the eigenvectors of $\sigma_z^{(1)} + \sigma_z^{(2)}$, i.e. the system operator that is coupled to the environment [23]. In such a case, the resultant system density matrix can be written as

$$\rho(t \to \infty) \approx p_1 |\uparrow\uparrow\rangle \langle \uparrow\uparrow| + p_2 |\Psi_{\pm}\rangle \langle \Psi_{\pm}| + p_3 |\downarrow\downarrow\rangle \langle \downarrow\downarrow|,$$

(22)

where $|\Psi_{\pm}\rangle = \alpha|\uparrow\downarrow\rangle + \beta|\downarrow\uparrow\rangle$, and $p_1$, $p_2$ and $p_3$ are the weights of each eigenvector in the asymptotic limit. Because $\sigma_z^{(1)} + \sigma_z^{(2)}$ is degenerate, the density matrix coherence, whose modulus is given by $p_2 |\alpha\beta|$, is sustained. This fact induces a finite QC that is preserved by the two DQDs. On the other hand, the equilibrium density matrix describes a disentangled state since $p_2 |\alpha\beta| \leq \sqrt{p_1 p_3}$ [24]. To verify that this is the cause of the robustness of the two DQDs’ QD, we compare two distinct situations. In the first situation, each DQD is
Figure 5. Two-qubit QCs (QD and EoF) under dissipative dynamics for $J = \Delta/4$ and $T = 0.1$ K in (a), $T = 0.5$ K in (b) and $T = 2$ K in (c). The blue (dotted) line is the QD dynamics and the red (solid) line is the EoF dynamics. Here we use $\eta = 1/600$.

Figure 6. Two-qubit QCs (QD and EoF) under dissipative dynamics for $J = \Delta/4$ and $T = 0.1$ K in (a), $T = 0.5$ K in (b) and $T = 2$ K in (c). The blue (dotted) line is the QD dynamics and the red (solid) line is the EoF dynamics. Here we use $\eta = 1/200$. 
Figure 7. EoF (a) and QD (b) as a function of time, considering $J = 0$ for independent (dotted curve) and common (solid curve) baths with $T = 0.1$ K and $\eta = 1/600$. The insets show the results in the logarithmic scale.

coupled to its own heat bath, and these are completely uncorrelated. The second situation has a common environment, i.e. the same bath is coupled to both DQDs and the interaction Hamiltonian is the one already considered $(\sigma_z^{(1)} + \sigma_z^{(2)}) \mathcal{L}$. For independent environments, the interaction Hamiltonian in equation (6) is slightly different and is given by $\sigma_z^{(1)} \mathcal{L}^{(1)} + \sigma_z^{(2)} \mathcal{L}^{(2)}$. Therefore, the master equation’s correlation functions, equation (13) and (14), are written as $T_1(t - t') = \text{Tr}_B \left\{ \sum_{k,k'=1}^2 B(k) \rho_B B(k') \right\}$ and $T_2(t - t') = \text{Tr}_B \left\{ \left( \sum_{k,k'=1}^2 B(k) \right)^\dagger (t) \rho_B B(k') (t') \right\}$, respectively. In this case, because the qubit’s environments are totally uncorrelated, we have $T_1(t - t') = T_2(t - t') = 0$ if $k \neq k'$. We also suppose a simple case where the Coulomb coupling is null $J = 0$. Initially, the two DQDs are considered in a maximal quantum correlated state given by $\rho_I(0) = |\Phi \rangle \langle \Phi|$, where $|\Phi \rangle = (|\uparrow \downarrow \rangle + |\downarrow \uparrow \rangle)/\sqrt{2}$.

By examining figure 7, we observe that, for independent environments, the EoF and the QD exponentially tend to zero. On the other hand, for common environments, the QD has an exponential decay that tends to a finite value due to the specific form of the coupling between the system and the environment. The fact that the environment acts correlated with both DQDs makes a more robust QD. To investigate this robustness, in figure 8 we compare the EoF and QD dynamics, considering only the common bath case, and observe that QD is more resistant to the environment’s disturbance. Moreover, the inset of figure 8 shows only the exponential decay of the EoF and the QD, i.e. the EoF curve and the QD curve minus the long time saturation value. In this case, we note that the exponential behavior of QD decays faster than EoF.

Based on our results, we conclude that the QC dynamics of the two DQDs can be summarized as follows. The Coulomb interaction introduces the correlations between both DQDs, and the environment acts in a way to preserve these correlations for a longer time. Despite the fact that the entanglement decays exponentially to zero, the QD is sustained, depending on the way in which the DQDs are coupled to the environment.
5. Summary

We have numerically solved the Redfield master equation to study the QC dynamics of two DQDs at finite temperatures, including the interaction between the qubits and detuning. We have verified that dissipation in this system has an impact on the EoF that is stronger than that on the QD, thereby causing a shorter duration of the EoF. We have shown that the QD is not completely destroyed through the interaction with the environment and that it remains stable, even at finite temperatures. To explain these facts, we have explored the characteristics of the bath, i.e. whether both qubits are coupled simultaneously to a single bath or each qubit is coupled to its own bath. The results have shown that, even though the coupling between the two qubits is zero, the QCs survive for longer periods in the case of a common bath. This behavior results from the particular form of the interaction Hamiltonian, and there are systems for which the robustness of QD may not occur, so each different case must be analyzed carefully.

As a final comment, it is worth studying the same effects in the context of the dynamics of a couple of Brownian particles in a common bath, which has recently been analyzed in [25] and where it has been shown that the reservoir mediates an effective coupling between the particles. The alternative (and more general) coupling to the environment used in this problem could also be applied to the present model, aiming at a possible maintenance of the QCs between the qubits for a much longer time interval.

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