Modeling and Analysis of Sorting Traffic and Buffer Capacities for Disassembly Systems with Reverse Blocking

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Abstract: For environmentally conscious manufacturing, end-of-life (EOL) assembly products should be disassembled for reuse and recycling. In this remanufacturing, the disassembly systems essentially consist of disassembly and sorting processes, and the product sorting process involves reverse blocking which impacts and reduces the total productivity of the remanufacturing. This paper models and analyzes the sorting process in the disassembly system with reverse blocking in consideration of traffic and buffer capacities by a queueing theory. First, the traffic and buffer capacities in the sorting stations are modeled on a sorting queueing system, and the evaluation functions of this system are set as the reverse blocking probability and the system throughput. Next, a transition diagram of the system is drawn, and the stationary state equations of the system are generally formulated with the traffic and buffer capacities. Finally, the system performance is discussed in terms of the buffer capacities, the system balancing among arrival, sorting and disassembly abilities, and a buffer allocation plan.

Key Words: Remanufacturing, Reuse and Recycling, Queueing Analysis, Environmentally Conscious Manufacturing.

1. Introduction

As the world population and economic development increase, demand for products/services from natural resources is also increasing. Additionally, waste from End-of-Life (EOL) products has become an increasingly serious issue, such that material circulation by reuse and recycling are required all over the world [1]. To promote material circulation for assembly products, a closed-loop supply chain [2] is necessary in environmentally conscious manufacturing [3] [4]. The closed-loop supply chain for the assembly products includes disassembly [5] for reuse and recycling by remanufacturing [6], where the collected EOL products are disassembled into parts and/or materials. To realize the closed-loop supply chain economically, higher productivity is essential for disassembly systems [5].

Unlike assembly systems, the disassembly systems have not only disassembly but also sorting processes. One of the reasons is that the collected EOL products have different parts and materials depending on their product types; therefore, they are sorted according to the product/material type before the disassembly process. The importance of the sorting process has been heeded for not only disassembly systems, but also closed-loop supply chains [2] [7].

In the disassembly systems, there are two types of sorting systems [8]: product sorting [9] [10] and material sorting [11]. In the product sorting system, the reverse blocking phenomenon [9] [10] is known, which reduces the productivity of the disassembly system and impacts the total productivity of the remanufacturing because the sorting process takes place during the first process of the disassembly system. According to [10], the reverse blocking phenomenon is explained as follows:

In an actual product sorting disassembly system [12] [13], the used and collected products from users arrive at a sorting station where they are serviced, and then the product types are identified and sorted into each type and sent to the appropriate succeeding stations. The sequences and mixed ratio for input products of multiple types are unknown at the sorting station because they consist of used products unlike finished products in assembly systems. When the same type of products intensively arrives at a particular succeeding station, the limited buffer capacity may become full at the succeeding station. Then, the preceding sorting station is blocked if the next product of the same type is serviced at the sorting station and sent to the same succeeding station. Additionally, the other succeeding stations starve due to no arriving product and stop during the blocking, even though they are available, and this decreases the throughput of the sorting process.

The fundamental performances of such a system have already been demonstrated by a queueing analysis [10] including a buffer allocation problem (BAP) [14] [15] [16]. The BAP is concerned with the allocation of a certain amount of buffer among buffer locations to achieve some specific objective [16], and is treated for the forward sup-
ply chains in [17] [18] and for the reverse ones in [19] [20] [21] [22]. However, there has been no paper for the disassembly systems with the reverse blocking except [10]. In there [10], a part of the model and conditions are simplified by ignoring the arrival/service (traffic) and the buffer capacities in the sorting station in order to overcome the complexities for the equations and the explosions of the number of states.

This paper models and analyzes the sorting process in the disassembly system with reverse blocking in consideration of the traffic and buffer capacities by a queuing theory [10]. In Section 2, the traffic and buffer capacities in the sorting stations are modeled on a sorting queuing system [10], and the evaluation functions of this system are set as the reverse blocking probability and the system throughput. In Section 3, a transition diagram of the system is drawn, and the stationary state equations of the system are generally formulated with the traffic and buffer capacities. Finally, Section 4 discusses the system performance in terms of the effects of the buffer capacities at a succeeding disassembly station with the sorting traffic and buffers. Additionally, the system balancing among arrival, sorting and disassembly abilities and a buffer allocation plan among the sorting and disassembly stations are considered in Section 5. Finally, Section 6 concludes this study and proposes future works.

2. Modeling of Disassembly Systems with Reverse Blocking in Consideration of Traffic and Buffer Capacities

2.1 Explanation of the disassembly systems with reverse blocking

This study develops the sorting process with reverse blocking in a disassembly system [10] in consideration of sorting traffic (system arrival/sorting service) and buffer capacities as shown in Figure 1. The sorting process consists of one sorting station (station 0) and the K succeeding disassembly stations. It is assumed that the collected products from users first arrive at the sorting station according to Poisson Arrival with the mean arrival rate λ. The in-process inventories at the sorting station have infinite or finite buffer capacities. There are K types of arrival products at the system, and the mixed ratio for product i is assumed and set as qi (i = 1, 2, · · · , K, i = 1 and 0 ≤ qi ≤ 1). If an arriving product is identified as product i after being serviced at the sorting station, this product is sorted into product i and sent to the succeeding station i. Then the routing probability from the sorting station to station i becomes qi. Also, it is assumed that the sorting and each succeeding disassembly station i independently follow the exponential service with the mean service rate μi (i = 0, 1, · · · , K). The finite buffer capacity at each succeeding station is set as Bi at station i (i = 1, 2, · · · , K).

Unlike Yamada et al. (2009) [10], this study focuses not only on the sorting but also on each succeeding disassembly station for the number of in-process inventories to calculate system throughput and reverse blocking probability. Each state is represented as state (l0, l1, · · · , lK), where the number of in-process inventories is li at the sorting and each succeeding disassembly station i (0 ≤ li ≤ Bi + 1, li = 0, 1, · · · , Bi + 1, Bi + 1 + 2, i = 1, 2, · · · , K). The reverse blocking phenomenon [9] [10] occurs in the queuing model, and is described as follows: When the buffer capacity at the succeeding station i is full (i.e., the number of in-process inventories, li = Bi + 1), a new product at the sorting station has just been processed and then identified as a type i product. However, the type i product cannot enter succeeding disassembly station i because the buffer capacity at station i is already full. Therefore, the product stays at the preceding sorting station, and blocks and stops the sorting station until the service at station i is completed and the buffer capacity becomes available. We represent this reverse blocking state as li = Bi + 2 (i = 1, 2, · · · , K). This means that one shared buffer space among the succeeding stations is assumed and set at the preceding sorting station during the blocking. Let PI(l0, l1, · · · , lK) be the stationary probability for a state (l0, l1, · · · , lK) [10].

A summary of the notation used in this paper is given below:

\[ i \]: station number (i = 0, 1, 2, · · · , K) where station 0 means the sorting station
\[ K \]: number of succeeding stations
\[ λ \]: mean arrival rate at system
\[ μ_i \]: mean service rate at station i (i = 0, 1, 2, · · · , K)
\[ B_i \]: buffer capacity at station i (i = 0, 1, 2, · · · , K)
\[ q_i \]: routing probability to station i (mixed ratio for product i)
\[ ρ_i \]: utilization rate at station i (i = 0, 1, 2, · · · , K)
\[ l_i \]: number of in-process inventories at station i (buffer + service) (i = 0, 1, 2, · · · , K)
\[ BL_i \]: reverse blocking probability at station i (i = 1, 2, · · · , K)
\[ BL \]: total reverse blocking probability at system
\[ TH_i \]: throughput at station i (i = 0, 1, 2, · · · , K)
\[ TH \]: throughput at system

2.2 Evaluation functions for the disassembly systems with reverse blocking

The system performances of the disassembly systems with the reverse blocking are measured and evaluated by the system throughput and the total reverse blocking probability, respectively [10]. The throughput / total reverse blocking probability at the system is the sum of the throughput / reverse blocking probability at each disassembly station as follows:

Reverse blocking probability at station i (i = 1, 2, · · · , K), BLi:

\[ BL_i = \sum_{l_j \in C_i} P(l_0, l_1, \cdots, l_j, \cdots, l_K) \]
In the case of \( K \) disassembly stations, the blocking probability at station \( j \), \( B_j \), is written as follows:

\[
B_j = \left\{ \left( \sum_{l=0}^{B_j+1} \sum_{l_0, l_1, \ldots, l_i} P(l_0, l_1, \ldots, l_i, l_j) \right) \right\} f_i(x_0, x_i)
\]

where \( l_i = B_i + 2 \) and \( l_j = B_j + 2 \), then,

\[
P(l_0, l_1, \ldots, l_i, l_j, l_K) = 0.
\]

Fig. 1 Disassembly queueing model with reverse blocking with sorting traffic/buffer capacities

\[
\sum_{i=1}^{K} \sum_{l_i=0}^{B_i+1} \sum_{l_0=0}^{B_0+1} P(l_0, l_1, \ldots, l_i, l_K) = 1, i = 1, 2, \ldots, K, (3)
\]

where \( \alpha_i \) takes value 0 if \( l_i = 0 \) and 1 if \( l_i \geq 1 \), and \( \beta \) takes value 0 if \( l_0 = B_0 + 1 \) or there exists \( l_i \) such that \( l_i = B_i + 2, l_0 = B_0, \) and 1 otherwise. 

\[
I(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } x \neq 0 \end{cases}
\]

The assumptions of the model used in this paper are given below:

- The system is stationary.
- The travel time of each product is zero.
- There is no failure at each station.
- The dispatching rule at all stations is First Come First Served (FCFS).
- \( \rho_0 = \lambda/\mu_0 < 1, \rho_i = \mu_0 q_i/\mu_i < 1 (i = 1, 2, \ldots, K). \)

3. Stationary State Equations for Reverse Blocking with Traffic and Buffer Capacities in the Sorting Station

3.1 Generalized formulation with traffic and buffer capacities in the sorting station

In the case of \( K \) stations with the buffer capacity \( B_i \) at station \( i \), the stationary state equations for the sorting process are generally formulated such that the sum of output rates from a state on the left side equals the sum of input rates to the state on the right side at each equation, and is written as follows:

\[
\lambda \beta + \sum_{i=1}^{K} \mu_i \alpha_i + I(x) \sum_{i=1}^{K} \mu_0 q_i P(l_0, l_1, \ldots, l_i, l_K)
\]

\[
= \left( \sum_{i=1}^{K} \sum_{l_i=0}^{B_i+1} \sum_{l_0=0}^{B_0+1} P(l_0, l_1, \ldots, l_i, l_K) \right) f_i(x_0, x_i)
\]

\[
-(K-1) \sum_{x_0 \in l_0-1} \sum_{x_i \in l_i} P(x_0, l_1, \ldots, x_i, l_K) f_i(x_0, x_i)
\]

3.2 Transition diagram for reverse blocking with traffic and buffer capacities in the sorting station

Figure 2 shows a transition diagram of the states and transitions in the case of two disassembly stations with finite sorting buffer \( B_0 = 1 \). The buffer capacities at disassembly station 1 are set as \( B_1 = 0 \) in the left diagram and \( B_2 = 1 \) in the right diagram. The states with the red circles in the right diagram show the additional states.
from the cases of the buffer capacity at disassembly station 1, $B_1 = 0$ to $B_1 = 1$. In both the right and bottom edges of Figure 2 in the right diagram, the states $(l_0, 3, l_2)$ for $l_0 = 0, 1$ and $(l_0, 2, l_2)$ for $l_0 = 0, 1, 2$ and $l_2 = 0, 1, 2$ indicate the reverse blocking. The stationary state equations in the cases of the buffer capacities at disassembly station 1, $B_1 = 1$ are written as follows:

$$\lambda P(0, 0, 0) = \mu_1 P(0, 1, 0) + \mu_2 P(0, 0, 1)$$

$$\lambda + \mu_1) P(0, 1, 0) = \mu_1 P(0, 2, 0) + \mu_2 P(0, 1, 1) + \mu_0 q_1 P(1, 0, 0)$$

$$\lambda + \mu_1) P(0, 2, 0) = \mu_1 P(0, 3, 0) + \mu_2 P(0, 2, 1) + \mu_0 q_1 P(1, 1, 0)$$

$$\lambda + \mu_2) P(0, 0, 1) = \mu_1 P(0, 1, 1) + \mu_2 P(0, 0, 2) + \mu_0 q_2 P(1, 0, 0)$$

$$\lambda + \mu_1 + \mu_2) P(0, 1, 1) = \mu_1 P(0, 2, 1) + \mu_2 P(0, 1, 2) + \mu_0 q_1 P(1, 0, 1) + \mu_0 q_2 P(1, 1, 0)$$

$$\lambda + \mu_1 + \mu_2) P(0, 2, 1) = \mu_1 P(0, 3, 1) + \mu_2 P(0, 2, 2) + \mu_0 q_1 P(1, 1, 1) + \mu_0 q_2 P(1, 2, 1)$$

$$\lambda + \mu_1 + \mu_2) P(0, 3, 0) = \mu_2 P(0, 3, 1) + \mu_0 q_1 P(2, 0, 0)$$

$$\lambda + \mu_1 + \mu_2) P(0, 3, 1) = \mu_0 q_1 P(2, 1, 0)$$

$$\lambda + \mu_2) P(0, 0, 2) = \mu_1 P(0, 1, 2) + \mu_0 q_2 P(1, 0, 1)$$

$$\lambda + \mu_1 + \mu_2) P(0, 2, 2) = \mu_0 q_2 P(1, 1, 1)$$

$$\lambda + \mu_1 + \mu_2) P(0, 0, 1) = \mu_0 q_2 P(1, 2, 1)$$

$$\lambda + \mu_1 + \mu_0 q_1 + \mu_0 q_2) P(1, 0, 0) = \lambda P(0, 0, 0)$$

$$\mu_1 P(1, 2, 0) + \mu_2 P(1, 1, 1) + \mu_0 q_1 P(2, 0, 0)$$

$$\lambda + \mu_1 + \mu_0 q_1 + \mu_0 q_2) P(2, 0, 0) = \lambda P(0, 0, 0)$$

$$\mu_1 P(1, 3, 0) + \mu_2 P(1, 2, 1) + \mu_0 q_1 P(2, 0, 0)$$

$$\lambda + \mu_1 + \mu_0 q_1 + \mu_0 q_2) P(2, 0, 0) = \lambda P(0, 0, 0)$$

$$\mu_1 P(1, 1, 1) + \mu_2 P(1, 0, 2) + \mu_0 q_1 P(2, 0, 0)$$

$$\lambda + \mu_1 + \mu_2 + \mu_0 q_1 + \mu_0 q_2) P(1, 1, 1) = \lambda P(0, 1, 1)$$

$$\mu_1 P(1, 2, 1) + \mu_2 P(1, 1, 2) + \mu_0 q_1 P(2, 0, 1) + \mu_0 q_2 P(2, 1, 0)$$

$$\lambda + \mu_1 + \mu_2 + \mu_0 q_1 + \mu_0 q_2) P(1, 1, 1) = \lambda P(0, 1, 1)$$

$$\mu_1 P(1, 2, 1) + \mu_2 P(1, 1, 2) + \mu_0 q_1 P(2, 0, 1) + \mu_0 q_2 P(2, 1, 0)$$

$$\lambda + \mu_1 + \mu_2 + \mu_0 q_1 + \mu_0 q_2) P(1, 1, 1) = \lambda P(0, 1, 1)$$

$$\mu_1 P(1, 2, 1) + \mu_2 P(1, 1, 2) + \mu_0 q_1 P(2, 0, 1) + \mu_0 q_2 P(2, 1, 0)$$

$$\lambda + \mu_1 + \mu_2 + \mu_0 q_1 + \mu_0 q_2) P(1, 1, 1) = \lambda P(0, 1, 1)$$

$$\mu_1 P(1, 2, 1) + \mu_2 P(1, 1, 2) + \mu_0 q_1 P(2, 0, 1) + \mu_0 q_2 P(2, 1, 0)$$

$$\lambda + \mu_1 + \mu_2 + \mu_0 q_1 + \mu_0 q_2) P(1, 1, 1) = \lambda P(0, 1, 1)$$

$$\mu_1 P(1, 2, 1) + \mu_2 P(1, 1, 2) + \mu_0 q_1 P(2, 0, 1) + \mu_0 q_2 P(2, 1, 0)$$

$$\lambda + \mu_1 + \mu_2 + \mu_0 q_1 + \mu_0 q_2) P(1, 1, 1) = \lambda P(0, 1, 1)$$

$$\mu_1 P(1, 2, 1) + \mu_2 P(1, 1, 2) + \mu_0 q_1 P(2, 0, 1) + \mu_0 q_2 P(2, 1, 0)$$

4. System Performance by Sorting Traffic and Buffers

4.1 Effect of disassembly buffer capacities and service rates

In order to compare the system behaviors in the case without the sorting traffic and buffer capacities [10], the system performances in the case of the sorting traffic and buffer capacities proposed in this paper are numerically experiment and evaluated by the blocking probabilities and throughputs. The numerical experiments are conducted using numerical analysis software, Maple 14, to solve the difficult stationary state equations as shown in Section 3.

In the case without the sorting traffic and buffers [10], it was found that the total reverse blocking probability should be decreased in order to increase the throughput at the system. One of the reasons is that there was an inverse proportion between the reverse blocking probability and the throughput at the system for the effects of the disassembly buffer capacities and service rates.

To validate that phenomenon in this case with the sorting traffic and buffers proposed in this paper, Section 4 discusses the system performances as follows: Section 4.1 Effect of the disassembly buffer capacities and service rates, Section 4.2 Effect of sorting traffic and buffers by arrival and service rates, and Section 4.3 Effect of mixed ratio.

Figure 3 shows the behavior of the blocking probability $BL$ at the system for the disassembly buffer capacity $B_1$. The blocking probability $BL$ at the system decreases monotonously as disassembly buffer capacity $B_1$ at station 1 increases. It is noted that blocking probability $BL_2$ at another disassembly station 2 increases as the disassembly
buffer capacity $B_1$ at station 1 increases. One of the reasons is that the arrival units to disassembly station 2 from sorting station 0 increase by the decreases of the blocking probability $BL_1$ at station 1 with the additional buffers. However, the total blocking probability at the system $BL$ is decreased.

On the other hand, Figure 4 shows the behavior of the throughput $TH$ at the system for the disassembly buffer capacity $B_1$. The throughput $TH$ at the system increases slightly but monotonously as buffer capacity $B_1$ at the disassembly stations increases. Like the case without the sorting traffic and buffers [10], it is seen that there is an inverse proportion between the reverse blocking probability and the throughput at the system in terms of the effects of the different buffer capacities at a succeeding disassembly station. The similar behaviors between the total blocking probability and the throughput at the system are observed in terms of the effects of the disassembly service rate at a succeeding disassembly station as shown in Figures 5 and 6.

In addition, the improvement effects for the total blocking probability $BL$ and the throughput $TH$ at the system are compared in the experiments by the disassembly buffer capacity $B_1$ at station 1 (Figures 3 and 4) and the service rate $\mu_1$ and $\mu_2$ (Figures 5 and 6). Essentially, the blocking probability $BL$ at the system in Figures 3 and 5 decreases monotonously as buffer capacity $B_1$ or service rate $\mu_1$ and $\mu_2$ at the disassembly stations increase. Unlike the decreases of the blocking probability $BL$ at the system in Figures 3 and 5, the throughput $TH$ at the system increases slightly in Figure 4 but drastically in Figure 6 as buffer capacity $B_1$ or service rate $\mu_1$ and $\mu_2$ at the disassembly stations increase. It seems that service rate $\mu_1$ and $\mu_2$ at the disassembly stations may be the bottleneck of the system by comparing it to the effect of buffer capacity $B_1$ at disassembly station 1 in the experiments.

### 4.2 Effect of sorting traffic and buffers by arrival and service rates

In the case without sorting traffic and buffers [10], the buffer capacity and the service rate at the sorting station are ignored to avoid the complex stationary state equations due to the explosions of state spaces. Section 4.2 discusses the effects of the sorting traffic and buffers by the arrival rate $\lambda$ and by the sorting service rate $\mu_0$ with different buffer allocation plans, such that $(B_0, B_1, B_2) = (1, B_1, 0)$ where $B_1 = 0, 1, 2$ drawn by the three graphs.

Similar to Section 4.1), the paired figures are respectively prepared for the behaviors of the blocking probability $BL$ at the system and the throughput $TH$ at the system by the arrival rate $\lambda$ at the system as shown in Figures 7 and 8, and by the sorting service rate $\mu_0$ as shown in Figures 9 and 10.

Essentially, the blocking probability $BL$ at the system in Figures 7 and 9 increases monotonously as arrival rate $\lambda$ at the system or the service rate $\mu_0$ at sorting station 0 increases at any number of buffer capacities at station 1. It is found that the blocking probability $BL$ for buffer capacity $B_1 = 0$ at the disassembly station 1 is the highest at buffer capacity $B_1 = 0, 1, 2$. However, there are few differences for the blocking probability $BL$ in the case with buffers between $B_1 = 1$ and $B_1 = 2$ at disassembly station.
Fig. 7 Behavior of blocking probability $BL$ at the system for arrival rate $\lambda$ ($\lambda = 0.5, q_1 = q_2 = 0.5, \mu_0 = \mu_1 = \mu_2 = 1$)

Fig. 8 Behavior of throughput $TH$ at the system for arrival rate $\lambda$ ($\lambda = 0.5, q_1 = q_2 = 0.5, \mu_0 = \mu_1 = \mu_2 = 1$)

Fig. 9 Behavior of blocking probability $BL$ at the system for sorting service rate $\mu_0$ ($\lambda = 0.5, q_1 = q_2 = 0.5, \mu_1 = \mu_2 = 1$)

Fig. 10 Behavior of throughput $TH$ at the system for sorting service rate $\mu_0$ ($\lambda = 0.5, q_1 = q_2 = 0.5, \mu_1 = \mu_2 = 1$)

On the other hand, the throughput $TH$ increases monotonously as the arrival rate $\lambda$ at the system or the service rate $\mu_0$ at sorting station 0 increases; this was observed in an inverse proportion between the reverse blocking probability and the throughput in Section 4.1. Effect of the disassembly buffer capacities and service rates. Additionally, there are few differences for the throughput $TH$ among three types of buffer allocations plans in disassembly station 1. Therefore, there is a possibility that the arrival rate $\lambda$ at the system or the service rate $\mu_0$ at sorting station 0 may be the bottleneck of the system in this case. Thus, further discussion for a system balancing and buffer allocation is presented in Section 5.

4.3 Effect of mixed ratio

This section 4.3 discusses an effect of a different mixed ratio $q_i$ for types of products sorted into a succeeding disassembly station $i$. The mixed ration also means a routing probability to each station.

Fig. 11 Behavior of blocking probability $BL$ at the system for mixed ratio $q_1$ and $q_2$ ($\lambda = 0.5, B_0 = 1, B_2 = 0, \mu_0 = \mu_1 = \mu_2 = 1$)

Fig. 12 Behavior of throughput $TH$ at the system for mixed ratio $q_1$ and $q_2$ ($\lambda = 0.5, B_0 = 1, B_2 = 0, \mu_0 = \mu_1 = \mu_2 = 1$)

Figure 11 shows the behavior of the blocking probability $BL$ at the system for mixed ratio $q_1$ and $q_2$. The sum of $q_1$ and $q_2$ equals to 1 so that the increasing value of $q_1$ means the decreasing value of $q_2$. As shown in Figure 11, the blocking probability $BL$ decreases as the buffer capacities $B_1$ at disassembly station 1. At any mixed ratio for $q_1$ and $q_2$, the blocking probability $BL$ in the case with the buffer capacities $B_1 = 2$ as disassembly station 1 is always lower than one in the case with the buffer capacity at disassembly station 1 at $B_1 = 0$ and 1. As the routing probability $q_1$ to station 1 increases, the number of the arriving products at disassembly station 1 increases so that the blocking probability $BL$ essentially increases.
However, in the case with the buffer capacities $B_1 = 2$, the disassembly station 2 has the two buffer capacities such as $B_1 = 2$ so that the blocking probability $BL$ can be decreased.

Also, it is observed that the throughput $TH$ in Figure 12 has an inverse proportion for the blocking probability $BL$ in Figure 11.

5. System Balancing and Buffer Allocation  
5.1 System balancing among arrival, sorting and disassembly abilities  
In Section 5.1, the system balancing among arrival, sorting and disassembly capacities is discussed. Figures 13 and 14 show the behavior of throughput $TH$ at the system and at each station in the case of arrival rate $\lambda = 0.1$ and $0.8$. In the case of arrival rate $\lambda = 0.1$ as shown in Figure 13, the sorting throughput $TH_0$ is almost 0.1 in spite of the sorting service capacity $\mu_0 = 0.9$. Additionally, the sorting throughput $TH_0$ is slightly lower than the arrival rate $\lambda = 0.1$ due to the small reverse blocking rate. However, there are few differences between the sorting throughput $TH_0$ and the system throughput $TH$. Therefore, it is considered that the arrival rate $\lambda$ of the system reflects the system throughput $TH$ and becomes a bottleneck of the system when the arrival rate is lowered as $\lambda = 0.1$.

![Fig. 13 Behavior of throughput $TH$ at the system and each station: Case of arrival rate $\lambda = 0.1$ ($\lambda = 0.1, q_1 = q_2 = 0.5, \mu_0 = 0.9, \mu_1 = \mu_2 = 0.5, B_0 = 1, B_1 = B_2 = 0$)](image1)

![Fig. 14 Behavior of throughput $TH$ at the system and each station: Case of arrival rate $\lambda = 0.8$ ($\lambda = 0.8, q_1 = q_2 = 0.5, \mu_0 = 0.9, \mu_1 = \mu_2 = 0.5, B_0 = 1, B_1 = B_2 = 0$)](image2)

On the other hand, it is seen that the sorting throughput $TH_0$ is lower than the arrival rate $\lambda = 0.9$ by about 50% in case of arrival rate $\lambda = 0.8$ shown in Figure 14. It is considered that the throughput $TH_0$ at the sorting station is obstructed with the lower sorting service capacity $\mu_0$ and the large reverse blocking rate because there is no waiting space in service at disassembly stations 1 and 2 such as $B_1 = B_2 = 0$. Therefore, the system throughput $TH$ is lower than the arrival rate $\lambda = 0.9$ and is almost the same as the sorting throughput $TH_0$. It is considered that the sorting service capacity $\mu_0$ becomes a bottleneck of the system when the arrival rate $\lambda$ is higher such as $\lambda = 0.8$.

As developed in the above discussion, the proposed modeling and analysis with the sorting traffic and buffer capacities in this study enable us to consider the system balancing among the arrival, sorting and disassembly capacities, while the previous study [10] had limited system balancing among the disassembly capacities.

5.2 Buffer allocation among sorting and disassembly stations  
As pointed out in Cruz et al. (2010) [15], there is a crucial trade-off between the overall amount of buffer space and its resulting throughputs. However, the additional buffer space basically brings higher costs; thus it is necessary to decrease the total amount of buffer capacities with a buffer allocation plan [15] [16] [19]. Section 5.2 discusses the buffer allocation plan among the sorting and the disassembly stations.

![Fig. 15 Behavior of blocking probability $BL$ at system: sorting vs. disassembly buffers $B$: ($\lambda = 0.5, q_1 = q_2 = 0.5, \mu_0 = \mu_1 = \mu_2 = 1$)](image3)

![Fig. 16 Behavior of throughput $TH$ at system: sorting vs. disassembly buffers $B$: ($\lambda = 0.5, q_1 = q_2 = 0.5, \mu_0 = \mu_1 = \mu_2 = 1$)](image4)

Figures 15 and 16 show the behavior of blocking probability $BL$ and throughput $TH$ at the system in the cases of a different number of sorting and blocking probability $BL$ are similar to the ones of the throughput $TH$. Also,
it is seen that the throughputs $TH$ with the buffers at sorting station 0 for the buffer allocation $(B_0, B_1, B_2) = (1, 0, 0), (2, 0, 0)$ and $(1,1,0)$ are higher than the ones with the buffers at disassembly station 1 for $(0,1,0)$ and $(0,2,0)$. Therefore, it is considered that the better buffer allocation plan can be selected among the sorting and disassembly stations.

On the other hand, it is also observed that there are differences of throughputs $TH_0$ at the sorting station 0 and $TH$ at the system when the number of sorting buffers $B_0$ becomes larger. Therefore, it is considered that the additional buffer at the sorting station is more effective than the one at the disassembly station in the experiments.

6. Conclusions

This paper modeled and analyzed the sorting station in the disassembly system with the reverse blocking in consideration of the traffic and buffer capacities by the queueing analysis. First, the conditions and capacities with the traffic and sorting capacities were modeled on the sorting queueing system. Next, the stationary state equations of the system were generally formulated by drawing the transition diagram. Finally, the system performance and the system balancing / buffer allocation plan were numerically evaluated in terms of the total blocking probability and the throughput at the system. The main conclusions drawn are as follows:

- By using the proposed queueing modeling and analysis, the disassembly system with the reverse blocking [10] was further developed in consideration of the sorting traffic and buffer capacities.

- The total reverse blocking probability should be decreased in order to increase the throughput at the system for the disassembly buffer capacities and service rates as well as for the case without the sorting traffic and buffers [10].

- An appropriate system balancing arrival, sorting and disassembly capacities could be discussed quantitatively based on the behaviors of the total reverse blocking and the throughputs among the stations in consideration of the sorting traffic and buffers.

- Additionally, better buffer allocation plans could be selected among the sorting and disassembly stations.

- There was a case in which the throughputs with the buffers at sorting station 0 were higher than the ones with the buffers at the disassembly station when the total number of buffers was limited in the experiments.

Future works should look for optimal disassembly design parameter sets, develop an approximation for a larger disassembly system with the reverse blocking to overcome the states explosions, model a material sorting system [8] [11] with this queueing analysis, and so on.

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