Polarization degeneracy of TE and TM eigenmodes for dielectric metasurface in the microwave

Syuzanna Asadulina, Andrey Bogdanov, Stanislav Glybovski, and Oleh Yermakov
School of Physics and Engineering, ITMO University, St. Petersburg, Russia
E-mail: o.yermakov@metalab.ifmo.ru

Abstract. We analyze the TE-TM polarization degeneracy of the guided modes of a dielectric metasurface in the microwave frequency range. We find the optimum metasurface design and investigate the dependence of the degeneracy degree on the propagation direction. Finally, we simulate the possible microwave experiment demonstrating the extraction of isofrequency contours for this metasurface. The results obtained could be useful for flat photonic devices with the function of polarization control.

1. Introduction
Metasurfaces allow to engineer efficiently the properties of electromagnetic plane waves (reflection, refraction, diffraction, polarization transformation, etc.) [1, 2]. Besides, metasurfaces pave way towards controlling the properties of strongly localized light [2, 3, 4]. Nevertheless, the polarization control of surface and guided waves still remains relevant and unresolved fundamental problem requiring the polarization degree of freedom for localized light.

The spectrum of electromagnetic plane waves in any isotropic medium is twice degenerate for all frequencies in any direction, because the dispersions of TE- and TM-polarized eigenmodes are absolutely the same. The operational principle of any bulk classical polarizer is based on the removal of the polarization degeneracy by using an anisotropic slab. However, there is no polarization degree of freedom for planar photonic devices with in-plane electromagnetic wave propagation, since the TE and TM localized modes are not degenerate. It significantly limits the functionality of flat optics and planar photonics devices. In particular, the polarizer of guided waves cannot be implemented without the broadband TE-TM polarization degeneracy.

In this work, we theoretically study the dielectric metasurface based on high-index cylinders that supports the broadband degeneracy of TE and TM guided modes in the microwave frequency range. We analyze numerically the degree of degeneracy and its dependence on the propagation direction for guided waves.

2. Design optimization
We perform a numerical optimization of a metasurface design representing a square periodic array of ceramic cylinders (Fig. 1) with a fixed diameter and permittivity equal to 5.2 mm and 40, respectively. The broadband degeneracy of the TE and TM polarized modes has been achieved with the cylinders period and height equal to 10 mm and 4.85 mm, respectively.
Figure 1: (a) The geometry of the metasurface with the lattice constant $a$ surrounded by air. (b) Top and (c) side views of the unit cell of a metasurface representing a single cylinder with a diameter $d$ and a height $h$.

The dispersion dependencies of guided modes of the dielectric metasurface demonstrate the TE-TM polarization degeneracy in the frequency range up to 8.2 GHz (Fig. 2), which was calculated in CST Microwave Studio using the Eigenmode Solver package. We introduce the degree of degeneracy value defined as the difference between the wave vectors of TE and TM guided modes at the same frequency ($\delta k$) or vice versa ($\delta f$). By increasing the period of the structure with fixed cylinders, the degree of degeneracy increases, but the eigenmodes become less localized, i.e. their dispersions become closer to the light line, and vice versa. For instance, the localization can be characterized by the in-plane wavevectors which are about $k_1 = 1.75k_0$, $k_2 = 1.41k_0$ and $k_3 = 1.32k_0$, where $k_0 = 2\pi f_0/c$, for the periods $a_1 = 9$ mm, $a_2 = 10$ mm.

Figure 2: The dispersions of TE (blue line) and TM (red dashed line) guided modes along $x$-axis direction for different periods: (a,d) $a = 9$ mm, (b,e) $a = 10$ mm, (c,f) $a = 11$ mm. Black line corresponds to the light line. (d-f) Dependence of the degeneracy degree defined as the difference between the frequencies of TM and TE modes on the wavevector in the dimensionless units for corresponding lattice constants.
and $a_3 = 11$ mm, respectively, at the frequency $f_0 = 8$ GHz (Fig. 2). Hence, we have chosen a metasurface design giving the relatively high localization of guided waves ($k = 1.41k_0$) saving the degree of degeneracy not exceeding $\delta f_{\text{max}} = 0.002 c/a$ (Fig. 2), or in other terms $\delta k_{\text{max}} = 0.004 \pi/a$ in the frequency range up to 8.1 GHz along $\Gamma X$ direction.

3. Dependence on propagation direction

In addition, we analyze the sustainability of eigenmodes polarization degeneracy in different propagation directions. The degeneracy is removed at high frequencies in $\Gamma M$ propagation direction (at angle 45° with respect to $x$- or $y$-axis), see Fig. 3. Isofrequency contours of TE and TM guided modes explicitly show that the degeneracy is achieved for all propagation directions in the frequency range up to 7.85 GHz (see the insert in Fig. 3). In the vicinity of 8 GHz one can notice the perceptible difference between TE and TM modes, especially at the propagation angle 45°. At slightly higher frequencies, only TM mode exists which completely breaks the polarization degeneracy. So, we conclude that the polarization degeneracy for the chosen metasurface design is fulfilled in the frequency range up to 7.85 GHz in all propagation directions.

![Figure 3: The dispersion of TE (blue line) and TM (red dashed line) guided modes along the propagation direction at 45° with respect to $x$-axis. Insert shows the isofrequency contours within the first Brillouin zone at the frequencies 7.5, 7.85, 8 and 8.2 GHz. The light cone at 7 GHz is shown by yellow circle.](image)

We have numerically simulated the guided TE and TM modes at the dielectric metasurface under consideration in CST Microwave Studio using the Transient package. The guided waves were excited by the effective electric and magnetic point dipoles representing as coaxial probe and loop, respectively. Figure 4 shows the distribution of the normal component of electric field for guided waves excited by vertical probe-like (electric) dipole. For frequencies 7.7 GHz and 8 Ghz (Figs. 4a-4b), one can observe the conventional dipole-like field distribution corresponding to the elliptic isofrequency contours (Figs. 4d-4e). The field distribution at 8.2 GHz is shown in Fig. 4c, it corresponds to the arc-like isofrequency contour of TM mode (Fig. 3). After that we have reconstructed the isofrequency contours applying the 2D Fourier transform operation to normal components of electric and magnetic fields (see color maps in Figs. 4d-4f) following the same procedure as in Refs. [5, 6]. The extracted isofrequency contours are in good agreement with the ones calculated directly in Eigenmode solver. This retrieval technique can be used in the experimental verification of the broadband TE-TM polarization degeneracy of eigenmodes at all-dielectric metasurface.
Figure 4: (a-c) The spatial distribution of the normal component of electric field for guided waves excited by a vertical point-like electric dipole (coaxial probe) located 1 mm above the surface at the frequencies: (a,d) 7.7 GHz, (b,e) 8 GHz and (c,f) 8.2 GHz. The size of the simulated region is $250 \times 250$ mm$^2$. (d-f) The isofrequency contours retrieved from the field distributions (color maps) and calculated numerically (dashed lines). The green circles correspond to the light cones.

4. Conclusions
We have proposed the optimum design of a metasurface consisting of dielectric high-index cylinders to achieve the polarization degeneracy of TE and TM guided modes in a wide spectral range. We have investigated the stability of degeneracy degree for this metasurface under different propagation directions by calculating the isofrequency contours. Finally, we have numerically simulated the potential microwave experiment and retrieved the isofrequency contours from the field distribution. This concept can be transferred to the optical and infrared ranges using other material platforms and opens new opportunities for the polarization control of guided waves such as metasurface-based waveguide polarizer [7].

Acknowledgements
This work was supported by the Russian Foundation for Basic Research (20-02-00636) and the Grant of the President of the Russian Federation (MK-2224.2020.2). O.Y. and A.B. acknowledge the support from the Foundation for the Advancement of Theoretical Physics and Mathematics "BASIS".

References
[1] Yu N, Capasso F 2014 Nat. Mater. 13(2) 139
[2] Glybovski S B, Tretyakov S A, Belov P A, Kivshar Y S and Simovski C R 2016 Phys. Rep. 634 1
[3] Kildishev A V, Boltasseva A, Shalaev V M 2013 Science 339 6125
[4] Chen H T, Taylor A J, Yu N 2016 Rep. Prog. Phys. 79 076401
[5] Yang Y, Jing L, Shen L, Wang Z, Zheng B, Wang H, Li E, Shen N-H, Koschny T, Soukoulis C M and Chen H 2017 NPG Asia Mater. 9(8) e428
[6] Yermakov O Y, Hurshkainen A A, Dobrykh D A, Kapitanova P V, Iorsh I V, Glybovski S B and Bogdanov A A 2018 Phys. Rev. B 98(19) 195404
[7] Yermakov O Y, Bogdanov A A, Lavrinenko A V 2019 IEEE J. Sel. Top. Quant. Electron. 25(3) 1