Gauge theory on $Z_2 \times Z_2 \times Z_2$ Discrete Group and a Spontaneous $CP$ Violation Toy Model

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Abstract

In the spirit of Non-commutative differential calculus on discrete group, we construct a toy model of spontaneous $CP$ violation (SCPV). Our model is different from the well-known Weinberg-Branco model although it involves three Higgs doublets and preserve neutral flavor current conservation (NFC) after using the $Z_2 \times Z_2 \times Z_2$ discrete symmetry and imposing some constraints on Yukawa couplings.

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1 Introduction

Ever since the discovery of \textit{CP} violation, many mechanisms have been put forward to explain it. Among them, Kobayashi-Maskawa(K-M) model \cite{10} is the most famous one and has been accepted as the standard model. However, because all experimental data indicate that the origin of \textit{CP} violation is like to be of superweak type, K-M model is not a natural one for its milliweak character despite it behave like a superweak model.

Among other mechanisms, the Weinberg-Branco model \cite{7} of SCPV has attracted much attention for its theoretical value. There are three Higgs doublets in their model, and in order to achieve so-called ”natural flavor conservation” a discrete symmetry is imposed to eliminate the flavor changing neutral current. The dominant \textit{CP} violation mechanisms is mediated by the charged Higgs bosons. As a result the model is milliweak in character and predicts large \textit{CP} violation in many experiments other than the neutral kaon system. Therefore it has been given a very tight constraints from the experiments. Very rescently, Y-L Wu propose a $SU(2)_L \times U(1)_Y$ gauge theory with two Higgs doublets which has both the SCPV and the NFC \cite{8, 9}. He pointed that the Glashow-Weinberg criterion for NFC is to be only sufficient but not necessary. After imposing some constraints on Yukawa coupling matrix, his model allow each Higgs doublet couple to all the fermions so that the lagrangian does not possess any additional discrete symmetries.

None of the existing \textit{CP} violation mechanisms is good enough to explain all experimental data, \textit{CP} violation remains a mystery. We think that this fact is related to our lack of knowledge on the origin of Higgs particle. Fortunately with the help of non-commutative geometry some improvements have been made to understand the origin of Higgs particle\cite{1 \sim 5}. The Higgs field can be viewed as the gauge field with respect to the discrete $Z_2$ group\cite{4, 5}. Due to the new discrete symmetry, the potential can be calculated within less freedom. This allow us to understand spontaneous violation better.

In this paper we construct a toy model to explore the possible relation between the SCPV and discrete symmetry in the spirit of non-commutative geometry. In our
model, we take the discrete group to be $Z_2 \times Z_2 \times Z_2$, and following the formalism in [5], we get the lagrangian that admits SCPV. After using the result of Y-L Wu, we can preserve NFC in our model but allow each Higgs doublet couple to all flavor of fermions.

This paper is scheduled as follows: In section 2, we briefly give out the result of gauge theory on discrete group $Z_2 \times Z_2 \times Z_2$. In section 3, we construct our toy model. At last section, we end with some conclusions and remarks.

# 2 The Gauge theory on the Discrete Group $Z_2 \times Z_2 \times Z_2$

## 2.1 Differential Calculus on Discrete Groups $G$

Let $G$ be a discrete group of size $N_G$, its elements are $\{e, g_1, g_2, \ldots, g_{N_G-1}\}$, and $\mathcal{A}$ the algebra of the all complex valued functions on $G$. The basis $\partial_i$, ($i = 1, \ldots, N_G-1$) of the left invariant vector space $\mathcal{F}$ on $\mathcal{A}$ are defined as

$$\partial_i f = f - R_i f, \quad \forall f \in \mathcal{A},$$

(2.1)

where

$$(R_i f)(g) = f(g \cdot g_i).$$

(2.2)

Obviously $\partial_i$ is nothing but the difference operator acting on $\mathcal{A}$, and satisfies

$$\partial_i \partial_j = \sum_k C^k_{ij} \partial_k, \quad C^k_{ij} = \delta^k_i + \delta^k_j - \delta^k_{i,j}$$

(2.3)

where $i, j, \ldots, (i \cdot j)$ denote $g_i, g_j, \ldots, (g_i \cdot g_j)$ respectively. Let $\Omega^1$ be the dual space of $\mathcal{F}$, whose basis $\chi^i$ are one-forms, satisfy

$$\chi^i(\partial_j) = \delta^i_j.$$  

(2.4)

The exterior derivative on $\mathcal{A}$ is given by

$$df = \sum_{i=1}^{N_G-1} \partial_i f \chi^i.$$  

(2.5)
The nilpotency of $d$ and the graded Leibniz rule

\begin{align}
(i) \quad & d^2 = 0, \\
(ii) \quad & d(fg) = df \cdot g + (-1)^{\deg f} f \cdot dg, \quad \forall f, g \in \Omega^*,
\end{align}

(2.6)

could be obtained provided that $\chi^i$ satisfy the following two conditions\[4]\n
\begin{align}
\chi^i f &= (R_i f) \chi^i, \quad \forall f \in \mathcal{A}, \\
\quad & d\chi^i = -\sum_{j,k} C^i_{jk} \chi^j \otimes \chi^k.
\end{align}

(2.7)

The involution operator $^*$ on the differential algebra is well defined if it agrees with the complex conjugation on $\mathcal{A}$, takes the assumption that $(\chi^g)^* = -\chi^{g^{-1}}$, and (graded) commutes with $d$, i.e. $d(\omega^*) = (-1)^{\deg \omega}(d\omega)^*$. The Haar integral, which remains invariant under group action, is introduced as a complex valued linear functional on $\mathcal{A}$ as,

\begin{align}
\int_G f = \frac{1}{N_G} \sum_{g \in G} f(g).
\end{align}

(2.8)

2.2 Gauge Theory on Discrete Group $G$

Let us now construct the generalized gauge theory on finite groups using the above differential calculus. Like the usual gauge theory, the $d + \phi$ is gauge covariant which requires the transformation of gauge field one form $\phi$ as,

\begin{align}
\phi \rightarrow H\phi H^{-1} + HdH^{-1}.
\end{align}

(2.9)

If we write $\phi = \sum_g \phi_g \chi^g$, the coefficients $\phi_g$ transform as

\begin{align}
\phi_g \rightarrow H\phi_g (R_g H^{-1}) + H\partial_g H^{-1}.
\end{align}

(2.10)

It is convenient to introduce a new field $\Phi_g = 1 - \phi_g$, then (2.10) is equivalent to

\begin{align}
\Phi_g \rightarrow H\Phi_g (R_g H^{-1}).
\end{align}

(2.11)
The extended anti-hermitian condition $\phi^* = -\phi$ results in the following relations on its coefficients $\phi_g$ as well as $\Phi_g$,

$$\phi_g^\dagger = R_g(\phi_g^{-1}), \quad \Phi_g^\dagger = R_g(\Phi_g^{-1}) \quad (2.12)$$

which will be very useful in following discussions.

It can be easily shown that the curvature two form $F = d\phi + \phi \otimes \phi$ is gauge covariant and can be written in terms of its coefficients

$$F = \sum_{g,h} F_{gh} \chi^g \otimes \chi^h \quad (2.13)$$

$$F_{gh} = \Phi_g R_g(\Phi_h) - \Phi_{h-g}. \quad (2.14)$$

In order to construct the Lagrangian of this gauge theory on discrete groups we need to introduce a metric on the forms. Let us first define the metric $\eta$ as a bilinear form on the bimodule $\Omega^1$ valued in the algebra $\mathcal{A}$,

$$\eta : \Omega^1 \otimes \Omega^1 \to \mathcal{A} \quad (2.15)$$

$$<\chi^g, \chi^h> = \eta^{gh}. \quad (2.16)$$

The gauge invariance requires that $\eta^{gh} \sim \delta^{gh^{-1}}$. However, the metric on the two forms becomes more complicated, if only the condition of gauge invariance is required.

$$<\chi^g \otimes \chi^h, \chi^p \otimes \chi^q> = \alpha \eta^{gh} \eta^{pq} + \beta \eta^{gp} \eta^{hq} + \gamma \eta^{qp} \eta^{hq}, \quad (2.17)$$

where the term proportional to $\gamma$ is only appeared when $G$ is commutative. Then the most general Yang-Mills action is given,

$$\mathcal{L} = -\int_G <F, \overline{F}> \quad (2.18)$$

where we have used the involution relations

$$\overline{F} = (\chi^g)^* \otimes (\chi^p)^* F^\dagger_{pq}, \quad (\chi^g)^* = -\chi^{g^{-1}}. \quad (2.19)$$
2.3 Gauge Theory on $Z_2 \times Z_2 \times Z_2$

We apply the general discussion in section 2.2 to the case of $G = Z_2 \times Z_2 \times Z_2$. There are 8 elements in $G$, which could be denoted as $\{g_0, g_1, g_2, g_3, g_4, g_5, g_6, g_7\}$, or written in terms of the elements $\{e, r\}$ in $Z_2$,

\[
\begin{align*}
g_0 &= \{e, e, e\}, \quad g_1 = \{r, e, e\} \\
g_2 &= \{e, r, e\}, \quad g_3 = \{e, e, r\} \\
g_4 &= \{r, r, e\}, \quad g_5 = \{r, e, r\} \\
g_6 &= \{e, r, r\}, \quad g_7 = \{r, r, r\}
\end{align*}
\]

The multiplication rule of $Z_2 \times Z_2 \times Z_2$ is given in the following table,

|     | $g_0$ | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ |
|-----|------|------|------|------|------|------|------|------|
| $g_0$ | $g_0$ | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ |
| $g_1$ | $g_1$ | $g_0$ | $g_4$ | $g_5$ | $g_2$ | $g_3$ | $g_7$ | $g_6$ |
| $g_2$ | $g_2$ | $g_4$ | $g_0$ | $g_6$ | $g_1$ | $g_7$ | $g_3$ | $g_5$ |
| $g_3$ | $g_3$ | $g_5$ | $g_6$ | $g_0$ | $g_7$ | $g_1$ | $g_2$ | $g_4$ |
| $g_4$ | $g_4$ | $g_2$ | $g_1$ | $g_7$ | $g_0$ | $g_6$ | $g_5$ | $g_3$ |
| $g_5$ | $g_5$ | $g_3$ | $g_7$ | $g_1$ | $g_6$ | $g_0$ | $g_4$ | $g_2$ |
| $g_6$ | $g_6$ | $g_7$ | $g_3$ | $g_2$ | $g_5$ | $g_4$ | $g_0$ | $g_1$ |
| $g_7$ | $g_7$ | $g_6$ | $g_5$ | $g_4$ | $g_3$ | $g_2$ | $g_1$ | $g_0$ |

There are seven independent one-forms in the space of $\Omega^1$ which we denote by $\chi^q$ with subscripts $g_1, g_2, g_3, g_4, g_5, g_6, g_7$. The components of the connection, corresponding to the components of the Higgs, is denoted by two entries, one for the entry of the space of $\Omega_1$ and the other for the elements of group. According to the general formalism in section 2.2, we can write the connection one forms as,

\[
A = \sum_{i=1}^{7} \phi_{g_i} \chi^{g_i},
\]
and $\Phi_{g_i} = 1 - \phi_{g_i}$. The curvatures of $A$ are easily obtained,

$$F_{g_i g_j} = \Phi_{g_i} R_{g_i} \Phi_{g_j} - \Phi_{g_i g_j}, \quad \text{if} \quad g_i \neq g_j$$

$$F_{g_i g_j} = \Phi_{g_i} R_{g_i} \Phi_{g_j} - 1, \quad \text{if} \quad g_i = g_j. \quad (2.22)$$

In order to get the potential for three Higgs doublets, we take the following assumption,

$$\Phi_{g_1}(h) = \begin{pmatrix} 0 & \Phi_1(x) \\ \Phi_1^\dagger(x) & 0 \end{pmatrix},$$

$$\Phi_{g_2}(h) = \begin{pmatrix} 0 & \Phi_2(x) \\ \Phi_2^\dagger(x) & 0 \end{pmatrix}, \quad (2.23)$$

$$\Phi_{g_3}(h) = \begin{pmatrix} 0 & \Phi_3(x) \\ \Phi_3^\dagger(x) & 0 \end{pmatrix},$$

$$\Phi_{g_4}(h) = \Phi_{g_5}(h) = \Phi_{g_6}(h) = \Phi_{g_7}(h) = 0$$

where $h$ is any element of $G$. Then the only nontrivial curvatures are,

$$F_{g_1 g_1} = \Phi_{g_1} \cdot R_{g_1} \Phi_{g_1} - 1, \quad F_{g_2 g_2} = \Phi_{g_2} \cdot R_{g_2} \Phi_{g_2} - 1,$$

$$F_{g_3 g_3} = \Phi_{g_3} \cdot R_{g_3} \Phi_{g_3} - 1, \quad F_{g_4 g_2} = \Phi_{g_4} \cdot R_{g_2} \Phi_{g_2},$$

$$F_{g_5 g_3} = \Phi_{g_5} \cdot R_{g_3} \Phi_{g_3}, \quad F_{g_6 g_1} = \Phi_{g_6} \cdot R_{g_1} \Phi_{g_1},$$

$$F_{g_7 g_2} = \Phi_{g_7} \cdot R_{g_2} \Phi_{g_2}. \quad (2.24)$$

If one use the most simple metric form on $\Omega^1$, i.e. $<\chi^g, \chi^h> = E_g \delta^{gh}$, where $E_{g_1}, E_{g_2}$ and $E_{g_3}$ are real numbers and $\{E_{g_4}, E_{g_5}, E_{g_6}, E_{g_7}\}$ are zero, such that one will get the
perfect three Higgs potential,

\[ V(\Phi_1, \Phi_2, \Phi_3) = \alpha Tr[ E_1(\Phi_1 \Phi_1^\dagger - 1) + E_2(\Phi_2 \Phi_2^\dagger - 1) + E_3(\Phi_3 \Phi_3^\dagger - 1)]^2 \]

\[ + \beta [E_1^2 Tr(\Phi_1 \Phi_1^\dagger - 1)^2 + E_2^2 Tr(\Phi_2 \Phi_2^\dagger - 1)^2 \]

\[ + E_3^2 Tr(\Phi_3 \Phi_3^\dagger - 1)^2 + 2E_1E_2 Tr(\Phi_1 \Phi_2^\dagger \Phi_2 \Phi_1^\dagger) \]

\[ + 2E_1E_3 Tr(\Phi_1 \Phi_2^\dagger \Phi_3 \Phi_3^\dagger + \Phi_2 \Phi_1^\dagger \Phi_2 \Phi_3^\dagger) \]

\[ + \gamma [E_1^2 E_2 Tr(\Phi_1 \Phi_1^\dagger - 1)^2 + E_2^2 E_3 Tr(\Phi_2 \Phi_2^\dagger - 1)^2 + E_3^2 E_1 Tr(\Phi_3 \Phi_3^\dagger - 1)^2 \]

\[ + E_1E_2 Tr(\Phi_1 \Phi_2^\dagger \Phi_1 \Phi_1^\dagger + \Phi_2 \Phi_1^\dagger \Phi_2 \Phi_1^\dagger) \]

\[ + E_1E_3 Tr(\Phi_1 \Phi_2^\dagger \Phi_3 \Phi_3^\dagger + \Phi_2 \Phi_1^\dagger \Phi_2 \Phi_3^\dagger) \]

\[ + E_3E_2 Tr(\Phi_3 \Phi_2^\dagger \Phi_3 \Phi_3^\dagger + \Phi_2 \Phi_3^\dagger \Phi_2 \Phi_3^\dagger)] \]

(2.25)

After taking the suitable value of \( \{E_{g_1}, E_{g_2}, E_{g_3}, \alpha, \beta \text{ and } \gamma \} \) one will get the three Higgs potential with nontrivial vacuum expectation value.

### 3 A Toy Model with Spontaneous CP violation

In order to get a model with spontaneous CP violation, the \( SU(2)_L \times U(1)_Y \) electroweak gauge fields, quark-lepton spinors and three Higgs doublets should be arranged as the fields over \( M^4 \times Z_2 \times Z_2 \times Z_2 \) according to the three \( Z_2 \) symmetry in those fields.

Here for the simplicity we take the three \( Z_2 \) group as the same \( Z_2 \) symmetry, for example, the \( (CPT)^2 \) discrete symmetry in fermions. Thus the fermions are arranged as,

\[ \psi(x, g_0) = \psi(x, g_4) = \psi(x, g_5) = \psi(x, g_6) = \begin{pmatrix} L \\ R \end{pmatrix} \]

\[ \psi(x, g_1) = \psi(x, g_2) = \psi(x, g_3) = \psi(x, g_7) = - \begin{pmatrix} L \\ R \end{pmatrix} \]

(3.1)
and the gauge fields as,

\[ A_\mu(x,h) = \begin{pmatrix} L_\mu & 0 \\ 0 & R_\mu \end{pmatrix}, \text{ for } h \in G \]  

(3.2)

where \( L, R, L_\mu, R_\mu \) are

\[ L = \begin{pmatrix} U_i \\ D_i \\ N_i \\ E_i \end{pmatrix}_L, \quad R = \begin{pmatrix} U_i \\ D_i \\ 0 \\ E_i \end{pmatrix}_R \quad i = 1, 2, 3 \]  

(3.3)

here

\[ U_i = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad D_i = \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad N_i = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \quad E_i = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}. \]  

(3.4)

and

\[ L_\mu = -\frac{ig}{2} f_2 \otimes \tau_i W^i_\mu \otimes I_3 - ig B_\mu \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes I_2 \otimes I_3; \right. \]

\[ R_\mu = -ig_1 B_\mu \left( \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \otimes I_3. \right. \]  

(3.5)

Furthermore, the Higgs fields \( \Phi_n, \ n = 1, 2, 3 \) are,

\[ \Phi_n(x) = \begin{pmatrix} \phi^0_n^+ \\ \phi^0_n^- \end{pmatrix} \otimes I_3^G \cdot \begin{pmatrix} \Gamma^U_n \\ \Gamma^D_n \end{pmatrix}. \]  

(3.6)

where \( \Gamma^U_n, \Gamma^D_n, \Gamma^L_n \) are the coefficient of Yukawa couplings.

After some algebraic calculations one gets the Lagrangian for that model. The Yang-Mills terms for \( SU(2)_L \times U(1)_Y \) gauge fields, the kinetic terms for fermions and its coupling to gauge fields remain the same as in \([5]\), i.e.,

\[ L_{YM}(x) = -\frac{1}{4N_L} 3g^2 W^i_\mu W^{i\mu} - \frac{1}{4N_Y} \frac{19g'^2}{3} B_\mu B^{\mu}. \]  

(3.7)
and

$$L_{\text{Fermion}}(x) = \overline{L} i \gamma^\mu (\partial \mu + L_\mu) L + \overline{R} i \gamma^\mu (\partial \mu + R_\mu) R \quad (3.8)$$

We are going to give other terms, which are not the same as in standard model. First, the Yukawa coupling terms are,

$$L_{\text{Yukawa}} = \sum_{n=1}^{3} \left( \begin{array}{cc} \bar{\psi}_L & \psi_R \end{array} \right) \left( \begin{array}{c} \Phi_n^\dagger \\ \psi_L \end{array} \right) \left( \begin{array}{c} \psi_R \\ \Phi_n \end{array} \right)$$

$$L_{\text{Yukawa}} = \sum_{n=1}^{3} \bar{\psi}_L \Phi_l \psi_R + \bar{\psi}_R \Phi_l^\dagger \psi_L \quad (3.9)$$

In general the Yukawa coupling in (3.9) can not preserve the neutral flavor current conservation (NFC) provided the matrices $\Gamma^F_n$, ($F = U, D, L$) can not be diagonalized simultaneously by a biunitary or biothogonal transformation. So Weinberg and Glashow introduced a discrete symmetry in Lagrangian to ensure the NFC. But there is another way to overcome this difficulty by imposing some constraints on Yukawa coupling matrix so that the matrix $\Gamma$ can be diagonalized simultaneously\cite{8, 9}. In other words we require that the real matrices $\Gamma^a_F$ be written into the following structure

$$\Gamma^F_n = \sum_{\alpha=1}^{3} g^F_{\alpha} O^F_n \Omega^\alpha (O^F_R)^T, \quad n = 1, 2, 3. \quad (3.10)$$

with $\Omega^\alpha, \alpha = 1, 2, 3$ the set of diagonalized projection matrices $\Omega^\alpha_{ij} = \delta_{i\alpha} \delta_{j\alpha}$, $O^F_{L,R}$ are the arbitrary orthogonal matrices and independent of the Higgs doublet label $n$. This is the crucial point for ensuring the NFC.

For the kinetic terms of Higgs fields we get

$$L_H(x) = \frac{1}{N} \sum_{n=1}^{3} 2 E_n \sigma_n (D_\mu \pi_n)^\dagger D^\mu \pi_n \quad (3.11)$$

where we introduce an usually notation of doublet scalar field $\pi_n$,

$$\pi_n = \left( \begin{array}{c} \phi_n^+ \\ \phi_n^0 \end{array} \right), \quad (3.12)$$
\begin{equation}
D_\mu \pi_n = \left( \partial_\mu - \frac{ig}{2} \tau_i W^i_\mu - \frac{ig'}{2} B_\mu \right) \pi_n,
\end{equation}
and the notations \( \sigma_n \)
\begin{equation}
\sigma_n = Tr \left( \begin{array}{ccc}
\Gamma^U_n \Gamma^{U\dagger}_n \\
\Gamma^D_n \Gamma^{D\dagger}_n \\
\Gamma^L_n \Gamma^{L\dagger}_n
\end{array} \right),
\end{equation}
If the \( \Gamma^F_n, (F = U, D, L) \) have been chosen, we should take the values of \( E_1, E_2, E_3 \) such that,
\begin{equation}
\frac{2}{N} E_n \sigma_n = 1
\end{equation}
for all \( n = 1, 2, 3 \) in order to get the normalized kinetic terms for the three Higgs fields.

Finally, from (2.25) and above assignments, we get the potential of three Higgs fields \( \pi_n \),
\begin{equation}
V(\pi_1, \pi_2, \pi_3) = \sum_{n=1}^{3} \left[ a_{nn} Tr(\pi_n \pi_n^\dagger)^2 - a_n Tr(\pi_n \pi_n^\dagger) \right] \\
+ \sum_{n<m} \left\{ a_{mn} Tr(\pi_n \pi_m^\dagger)(\pi_m \pi_n^\dagger) + b_{mn} Tr(\pi_n \pi_m^\dagger)(\pi_m \pi_n^\dagger) \\
+ c_{mn} Tr(\pi_n \pi_m^\dagger \pi_n \pi_m^\dagger) + h.c. \right\}
\end{equation}
where
\begin{align*}
a_{nn} &= 2(\alpha + \beta + \gamma) E_n^2 p_{nn} \\
a_n &= 4(\alpha + \beta + \gamma) E_n^2 \sigma_n + 4\alpha \sum_{m \neq n} E_m E_n \sigma_n \\
a_{mn} &= 4\alpha E_m E_n p_{mn} \\
b_{mn} &= 4\beta E_m E_n q_{mn} \\
c_{mn} &= 2\gamma E_m E_n s_{mn}
\end{align*}
and
\begin{align*}
p_{mn} &= Tr(\Gamma^U_m \Gamma^{U\dagger}_n)(\Gamma^U_n \Gamma^{U\dagger}_m) + (\Gamma^D_m \Gamma^{D\dagger}_n)(\Gamma^D_n \Gamma^{D\dagger}_m) + (\Gamma^L_m \Gamma^{L\dagger}_n)(\Gamma^L_n \Gamma^{L\dagger}_m) \\
q_{mn} &= Tr(\Gamma^U_m \Gamma^{U\dagger}_n)(\Gamma^U_m \Gamma^{U\dagger}_n) + (\Gamma^D_m \Gamma^{D\dagger}_n)(\Gamma^D_m \Gamma^{D\dagger}_n) + (\Gamma^L_m \Gamma^{L\dagger}_n)(\Gamma^L_m \Gamma^{L\dagger}_n) \\
s_{mn} &= Tr(\Gamma^U_m \Gamma^{U\dagger}_n)(\Gamma^U_m \Gamma^{U\dagger}_n) + (\Gamma^D_m \Gamma^{D\dagger}_n)(\Gamma^D_m \Gamma^{D\dagger}_n) + (\Gamma^L_m \Gamma^{L\dagger}_n)(\Gamma^L_m \Gamma^{L\dagger}_n)
\end{align*}
We find that the terms propotional to \( c_{mn} \) possess CP nonconservation. Furthermore we read that the potential is invariant under some discrete symmetry transformation.
4 Remark and Conclusion

In this paper, we construct a three-Higgs toy model of SCPV with the help of the differential calculus on discrete group in the spirit of non-commutative geometry. In our model the NFC is ensured by imposing some constraints on the Yucawa coupling matrices (just as in [8]) and we allow each Higgs doublet couple to all the fermions. By a straightforward calculation we obtain a potential which has a $CP$ violation term and is invariant under a discrete symmetry transformation. However we should address that the NFC is not necessary in our discussions. If we abandon the NFC, i.e. abandon the so called simultaneously diagnoized condition of Yukawa coupling matrix, we can obtain the Weinberg-Hall model [11].

Furthermore the Yucawa coupling will be carefully choosed so that the following two physical conditions satify:
1. if $CP$ violate, the values of $c_{mn}$ should satisfy the triangular condition [7].
2. the masses of three Higgs bosons cannot differ too much, therefore the expectation values of Higgs doublets must be at the same level.

Then we may construct a physical $CP$ violation model. This question will be investigate more.

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