Pseudo-static internal stability analysis of geosynthetic-reinforced earth slopes using horizontal slices method

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ABSTRACT
In this study, the well-established pseudo-static approach along with the horizontal slices method (HSM) is employed to investigate the seismic internal stability of geosynthetic-reinforced earth slopes. Previous simple HSM analyses were based on a primary assumption stating that the normal inter-slice forces are exerted on the mid-length of horizontal sections. However, this simplifying assumption could give rise to substantial errors in the calculation of design parameters, specifically in the case of high seismic excitations or low soil strength parameters. To address this deficiency, a balancing moment is considered as a new variable to account for the corresponding eccentricity. In the current HSM, two sets of unknown variables, including horizontal inter-slice forces and shear forces along failure surface, are determined using the well-known λ coefficient and the Mohr-Coulomb failure criterion. In this new technique, the traditional ‘5N-1’ type of HSM analysis is reduced to a robust and rigorous ‘3N’ one with the same predictive capability. The influence of various parameters, including soil characteristics, slope geometry and different earthquake coefficients are rigorously examined. Moreover, a number of useful graphs is provided to help engineers in the preliminary seismic design of geosynthetic-reinforced earth slopes.

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1. Introduction

Many geotechnical installations are constructed over the natural or man-made slopes and walls. Seismic stability of earth structures is one of the most important aspects of their design (Shahgholi et al. 2001, Kramer and Paulsen 2004, Nouri et al. 2006, 2008, Chandaluri et al. 2015, Khorasvizeh et al. 2016, Payan et al. 2016a, 2016b, 2016c, 2017a, 2017b, 2020, Senetakis and Payan 2018, Payan and Senetakis 2019, Payan and Jamshidi Chenari 2019, Farshidfar et al. 2020, Zamanian et al. 2020, 2021, Fathiopour et al. 2021b, He et al. 2021, Safaroost Siahmazgi et al. 2021). Reinforcing these soil structures is a relatively common practice for stability enhancement. In this regard, reinforced earth slopes and walls are used extensively due to their technical and economic advantages (Fathiopour et al. 2021a). Moreover, they have shown a quite well performance during the major recent earthquakes (Sandri 1997, Koseki et al. 2006). Most studies on the reinforced earth structures are restricted to the frictional type of backfill soils. Using the local cohesive soil for backfills, which requires less quality control and no special equipment, has many advantages and makes the projects quite affordable.

Two main analytical approaches employed by many researchers to evaluate the stability of earth slopes and walls are limit equilibrium method, LEM (Nouzari et al. 2021) and limit analysis, LA (Fathiopour et al. 2020, 2021a, 2021b). The pioneering studies of earthquake-induced pressure on retaining walls were performed by Okabe (1924), and Mononobe and Matsuo (1929). Their static analysis following Coulomb’s theory of earth pressure, known as Mononobe-Okabe method, has made the basis of pseudo-static approach (Kramer 1996). In this method, the inertia forces are simply simulated by the product of seismic coefficients and the weight of failure wedge.

The seismic stability analyses of slopes and retaining structures have been extensively studied throughout the literature (Chen et al. 2003, Chanda et al. 2017, Qin and Chian 2018a, 2018b, Li et al. 2020, Qin et al. 2020, Fathiopour et al. 2021b). In a study performed by Qin and Chian (2018a), a novel procedure was introduced to evaluate the seismic slope stability under the ultimate limit state using the pseudo-dynamic approach and discretisation technique. In a recent study, Chanda et al. (2018) presented the seismic stability analysis of homogeneous soil slope adopting the pseudo-dynamic
method and the well-established limit equilibrium approach.

The seismic stability of reinforced earth slopes and retaining walls have also been investigated in a great number of studies throughout the literature (Dong-ping et al. 2017, Rao et al. 2019, Deng et al. 2019, Farshidfar et al. 2020). Specifically, the pseudo-static method has been extended to the seismic analysis of reinforced soil structures in recent decades (Lo and Xu 1992, Ling et al. 1997, Ling and Leshchinsky 1998, Michalowski 1998, Ausilio et al., 2000, Shahgholi et al. 2001, Kramer and Paulsen 2004, Nouri et al. 2006 & 2008, Chandaluri et al. 2015, Khosravizadeh et al. 2016, Farshidfar et al. 2020). Ling et al. (1997) considered the seismic horizontal acceleration for the reinforced soil structures, through the limit equilibrium formulation, so as to calculate the required strength and length of geosynthetic layers. They introduced a permanent tolerable displacement for cases where the required reinforcement lengths are extremely high and/or in practical space limitations. Similar procedure was adopted by Ling and Leshchinsky (1998) to investigate the simultaneous effects of vertical and horizontal acceleration components. Using the kinematic theorem of limit analysis (LA), Michalowski (1998) evaluated the required reinforcement strength and length for the stability of earth structures against seismic loading. Ausilio et al. (2000) also exploited the LA method for the seismic stability analysis of reinforced soil slopes (RSS) to calculate the yield seismic acceleration and permanent displacement apart from the required reinforcement characteristics. Shahgholi et al. (2001), Nouri et al. (2006, 2008) and Farshidfar et al. (2020) used the so-called horizontal slices method (HSM), originally developed by Lo and Xu (1992), to perform the seismic stability analysis of the reinforced earth structures. Their HSM approach led to the computation of reinforcement design characteristics. Considering a surcharge on the general c-φ backfills, Chandaluri et al. (2015) also analyzed the seismic stability of reinforced walls through the limit equilibrium approach using horizontal slices method. The effects of other parameters, including wall inclination and horizontal seismic acceleration in addition to the surcharge and soil characteristics, were rigorously examined in their study. In a separate study, Khosravizadeh et al. (2016) analyzed the seismic stability of reinforced soil slopes using HSM based on a new procedure to determine the most critical arbitrary slip surface. This method of estimating the multilinear failure surface was advantageous due to its development and convergence towards the well-known circular, log spiral, or linear types of failure surfaces, depending on the specific slope conditions including soil type and geometry characteristics. More recently, Farshidfar et al. (2020) used HSM to evaluate the critical slip surface and calculate the reinforcement design parameters. They examined both the log-spiral and general (multilinear) types of slip surfaces and indicated that the general type renders greater values for the design characteristics.

In the present study, the seismic stability of reinforced soil slopes (RSS) has been analyzed using HSM adopting a pseudo-static earthquake loading accounting for both horizontal and vertical components. Considering a general c-φ backfill is a superior part of this study over the previous ones due to the very fact that it makes it possible to take best use of local materials. An inter-slice balancing moment introduced to account for the eccentricity of the vertical inter-slice forces, is another new aspect of this study. This bending moment simulates the non-uniform vertical stress distribution in horizontal sections due to the combined static (gravity) and pseudo-static earthquake loadings. A parametric survey is also conducted and the results of the reinforcement design are rigorously compared with the previous studies throughout the literature.

2. Method of analysis
2.1. Scope and aspects

Limit equilibrium using horizontal slices method (LEHSM) is one of the commonly used techniques for the seismic stability analysis of geosynthetic-reinforced soil structures. The horizontal slices method (HSM) is quite suitable for the stability analysis of geosynthetic-reinforced slopes, as each slice can contain a single reinforcement layer with its corresponding specific tension force acting at the center of the layer without any reinforcement-induced inter-slice forces. However, in the vertical slice method, each vertical slice contains several reinforcement layers with different tension forces acting as inter-slice forces, which in turn complicates the equilibrium analyses. The vertical slice method has been originally developed for the equilibrium analysis of earth (unreinforced) slopes, whereas the HSM has been mainly proposed to deal with the static and seismic analyses of reinforced slopes (Lo and Xu 1992, Shahgholi et al. 2001, Nouri et al. 2006, 2008, Khosravizadeh et al. 2016, Farshidfar et al. 2020). In this study, a new LEHSM technique is developed which not only considers all the forces and moment equilibrium equations in the seismic analysis of RSS; but also permits the non-uniform vertical stress distribution on the horizontal slices due to the simultaneous exertion of gravity and earthquake forces.
In the previous studies, different aspects of the seismic stability of reinforced earth structures, including stability modes, slip surface geometry, backfill soil type and seismic loading components have been investigated. The output items could be divided into two main categories of ‘reinforcement design parameters’ and ‘kinematics values’. The required strength and length of different layers are referred to as the reinforcement design parameters, whereas the yield acceleration and permanent displacement quantities are considered as the kinematic values. A brief comparison between some of the well-known and recent studies based on these scopes and aspects is presented in Table 1.

### 2.2. HSM assumptions and formulations

Many failure surfaces are assumed in the horizontal slices method and the critical one, corresponding to the maximum mobilized tension in the reinforcement layers, is acquired. Multilinear failure surfaces are considered in this study. The failure surfaces are deemed to pass through the base of the slope, i.e. a firm foundation is to be placed underneath. The failure wedge, limited to the slope borders and the failure surface, is divided into several horizontal slices with a rigid plastic behavior. Reinforcement layers are considered to be in the mid-height of horizontal slices. The seismic inertia force is simulated as a pseudo-static force acting at the center of gravity of each slice. In a pseudo-static analysis, the weight of the sliding mass, the shear and normal soil resistance forces along the sliding surface together with the constant horizontal and vertical forces caused by seismic loading are taken into account for the calculation of factor of safety. Although the pseudo-dynamic methods can better represent the dynamic behavior of slopes under seismic loading conditions (Safardoost Siahmazgi et al. 2021, Fathipour et al. 2021b), the pseudo-static methods are still widely used in practice due primarily to their simplicity and relatively fast solutions, especially for the preliminary design purposes (Ausilio et al., 2000, Shahgholi et al. 2001, Kramer and Paulsen 2004, Nouri et al. 2006, 2008, Chandaluri et al. 2015, Khosravizadeh et al. 2016, Farshidfar et al. 2020, Yazdandoust and Ghalandarzadeh 2020). In addition, apart from the resonance condition, the pseudo-static analysis always yields the lowest values of bearing capacity, thus rendering a more conservative design (Saha and Ghosh, 2015; Pain et al. 2016; Safardoost Siahmazgi et al. 2021).

Backfill soils in this study are considered to be of both frictional and cohesive-frictional types. The effects of facing elements and pore water pressure have been ignored. The tensile force within the reinforcement layers is linearly distributed along the slope height increasing with depth, i.e. $T_i = KyZD_i$ (Nouri et al. 2006), where $D_i = d_i$ for equally spaced reinforcement layers. In the analysis, the driving forces and moments are assumed to be equal to the resisting ones, thus yielding a factor of safety equal to one.

To analyze the stability of RSS, different formulations have been introduced in the literature, including simple ‘2N+1’ & ‘3N’, and rigorous ‘5N-1’ methods (Nouri et al. 2006). In the simple ones, either any of the force or moment equilibrium equations are ignored, or some other simplifying assumptions are incorporated into the calculations. In the rigorous formulation, however, not only are all the equilibrium equations satisfied but also

### Table 1. Comparison of some seismic stability analysis of reinforced soil structures.

| Research          | Stability/Failure mode | Analysis method | Critical slip surface | Backfill soil type | Acceleration components | Reinforcement | Kinematics |
|-------------------|------------------------|----------------|-----------------------|--------------------|-------------------------|---------------|------------|
| Ling et al. (1997)| * * *                 | LEM            | L & LS                | φ                  | $a_b$                   | T & L          | PD         |
| Ling and Leshchinsky (1998) | * * *                  | LEM            | L & LS                | φ                  | $a_b + a_o$             | T & L          | PD         |
| Michalowski (1998) | * * *                  | LA             | L & LS                | φ                  | $a_b$                   | T & L          |            |
| Ausilio et al. (2000) | * * *                  | LA             | L & LS                | φ                  | $a_b$                   | T & L          | YA & PD    |
| Shahgholi et al. (2001) | *                      | LEM            | ML                    | φ                  | $a_b$                   | T             |            |
| Nouri et al. (2006) | *                      | LEM            | LS                    | φ                  | $a_b$                   | T & L          |            |
| Nouri et al. (2008) | *                      | LEM            | LS & C                | φ                  | $a_b + a_o$             | T & L          |            |
| Chandaluri et al. (2015) | *                      | LEM            | ML                    | φ                  | $a_b + a_o$             | T & L          |            |
| Khosravizadeh et al. (2016) | *                      | LEM            | ML                    | φ                  | $a_b$                   | T & L          |            |
| Keshavarz et al. (2017) | *                      | LEM            | L & LS                | φ                  | $a_b$                   | T & L          |            |
| Farshidfar et al. (2020) | *                      | LEM            | LS & C                | φ                  | $a_b$                   | T & L          |            |
| Current study     | *                      | LEM            | G (ML)                | φ                  | $a_b$                   | T & L          |            |

(q*: surcharge loading is also considered in this study)

Stability modes: Int. = internal; Com. = compound; Ext. = external

Type of slip surface: L = linear; ML = multilinear; C = circular; LS = log spiral

Reinforcement parameters: T = total tension force; L = required length

Kinematics parameters: PD = permanent displacement; YA = yield acceleration
Table 2a. Simple ‘2N + 1’ method (from Nouri et al. 2006).

| Unknowns                                               | Number of unknowns | Equations                          | Number of equations |
|--------------------------------------------------------|--------------------|------------------------------------|---------------------|
| Normal force upon each slice (N)                       | N                  | \( \Sigma F_x = 0 \) (each slice)  | N                   |
| Shear force upon each slice (S)                        | N                  | \( \tau_r = \tau_r / F_S \) (each slice) | N                   |
| Total required force in reinforcement to maintain \( \tau_r \) | 1                  | \( \Sigma F_x = 0 \) (Shahgholi et al. 2001) (whole wedge) | 1                   |
| Sum                                                    | 2N + 1             | Sum                                | 2N + 1              |

Table 2b. Simple ‘3N’ method (from Nouri et al. 2006).

| Unknowns                                               | Number of unknowns | Equations                          | Number of equations |
|--------------------------------------------------------|--------------------|------------------------------------|---------------------|
| Normal force upon each slice (N)                       | N                  | \( \Sigma F_x = 0 \) (each slice)  | N                   |
| Shear force upon each slice (S)                        | N                  | \( \tau_r = \tau_r / F_S \) (each slice) | N                   |
| Horizontal inter-slice force \( (H_s) \)               | N-1                | \( \Sigma M_0 = 0 \) (each slice)  | N                   |
| Total required force in reinforcement to maintain \( \tau_r \) | 1                  | \( \Sigma M_0 = 0 \) (Fakher et al. 2002) (whole wedge) | 1                   |
| Sum                                                    | 3N                 | Sum                                | 3N                  |

Table 2c. Simple ‘5N - 1’ method (from Nouri et al. 2006).

| Unknowns                                               | Number of unknowns | Equations                          | Number of equations |
|--------------------------------------------------------|--------------------|------------------------------------|---------------------|
| Normal force upon each slice (N)                       | N                  | \( \Sigma F_x = 0 \) (each slice)  | N                   |
| Shear force upon each slice (S)                        | N                  | \( \Sigma F_x = 0 \) (each slice)  | N                   |
| Horizontal inter-slice force \( (H_s) \)               | N-1                | \( \Sigma M_0 = 0 \) (each slice)  | N                   |
| Vertical inter-slice force \( (V_s) \)                 | N-1                | \( \tau_r = \tau_r / F_S \) (each slice) | N                   |
| Location of vertical inter-slice force \( (x_s, y_s) \) | N-1                | \( H_s = M_s / (y_s V_s) \) (Morgenstern and Price assumption) | N-1                |
| Morgenstern and Price factor \( (\lambda) \)          | 1                  | \( \Sigma M_0 = 0 \) (each slice)  | N                   |
| Total required tension to maintain \( \tau_r \)        | 1                  | \( \Sigma M_0 = 0 \) (each slice)  | N                   |
| Sum                                                    | 5N-1               | Sum                                | 5N-1                |

due to two other subsidiary differences between the proposed ‘3N’ method with the previous ‘5N-1’ approach. Firstly, regarding the well-known assumption of Morgenstern and Price (1965), i.e. \( H_1 = \lambda f_r(y) V_1 \), the (N-1) unknowns

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The point of application of vertical inter-slice forces are considered as unknowns to account for the non-uniform vertical stress distribution in the horizontal sections. Different unknowns and equations for the simple and rigorous formulations are presented in Table 2 (a, b, c).

Table 3 shows the unknowns and equations used in the current study as a new rigorous ‘3N’ method. The free body diagram of applied forces on a typical slice in this new ‘3N’ method is shown in Figure 1. This approach is indeed equivalent to the rigorous ‘5N-1’ method due to considering quite similar conditions and assumptions. In addition to introducing the inter-slice bending moments as the new unknowns instead of the application points of vertical inter-slice forces, there are

![Figure 1. Free body diagram of a horizontal slice.](image-url)
of \(H_i\) could be eliminated by substituting them from the aforementioned relation as \(\lambda.V_i\), since \(f_i(y) = 1\) is a common accepted assumption. This simplification is due to the very fact that different distribution types of \(f_i(y)\) in height have a quite negligible effect on the obtained results (Lo and Xu 1992). Secondly, the \(N\) unknowns of \(S_i\) are computed from the Mohr-Coulomb failure envelope, i.e. \(S_i = \frac{1}{F_S}(c.b_i + N_i\tan\varphi)\), in terms of \(N_i\) unknowns and other known parameters. Therefore, the previous ‘3N-1’ method is effectively reduced to the new ‘3N’ one by subtracting \((N)+(N-1)\) unknowns and equations based on the two above mentioned subsidiary techniques. However, it should be noted that \(H_i = \lambda.V_i\) relationship introduces a nonlinear term (multiplication of two unknowns).

To avoid solving a set of simultaneous nonlinear equations, a preliminary \(\lambda\) value is assumed, and the other 3N-1 unknowns are determined by 3N-1 equations excluding the last slice horizontal force equilibrium equation. Then, the excluded equation is applied to verify the accuracy of the assumed value for \(\lambda\). This equation (2.1a in the following paragraphs) is formed as \(\Sigma F_X = \varepsilon\), which should converge to a quite small value of \(\varepsilon\) by an iterative process of \(\lambda\) variation. The proper \(\lambda\) between 0 and 1 is determined by the aforementioned iterative process as a constant value for all slices in the slope limited to slip surface (failure wedge). Then, the total tension force in the reinforcement layers is calculated based on the equation: \(\Sigma T_i = 0.5\gamma.H^2\). Finally, the maximum total tension in geosynthetic layers is sought among all the considered slip surfaces. This maximum tension force and its associated critical slip surface, which pertains to the required geosynthetic length, are the fundamentals of reinforcement design.

Three equilibrium equations for each slice, based on the free body diagram shown in Figure 1 and a unit factor of safety \((FS)\), are presented herein. Assuming a unit safety factor is a common practice in the well-established Mohr-Coulomb relationship as \(S_i = \left( N_i\tan\varphi_i + c_i.h_i \right)\). The mobilized soil characteristics are as follow:

\[
\varepsilon_f = \frac{c}{FS} \tag{1.1}
\]

\[
\varphi_f = \arctan \left( \frac{\tan\varphi}{FS} \right) \tag{1.2}
\]

and the factor of safety \((FS)\) is considered via the mobilized soil characteristics. In the following two force and one moment equilibrium equations, soil parameters \(c\) and \(\varphi\) could be considered as \(\varepsilon_f\) and \(\varphi_f\) to account for the desired safety factor. The equilibrium equation in the \(X\) direction is as follows:

\[
\Sigma F_X = 0 \rightarrow K\gamma(i - 0.5)d_i^2 + H_{i+1} - H_i - k_bW_i + (N_i\tan\varphi_i + c_i)\sin\theta_i - N_i\cos\theta_i = 0 \tag{2.1a}
\]

Reordering the above equation based on the independent unknowns of the \(i\)th slice, i.e. \(N_i, V_i, V_{i+1}, M_i, M_{i+1}\), and \(K\), and substituting \(H_i = \lambda.V_i\) yield:

\[
(tan\varphi_i\sin\theta_i - \cos\theta_i)N_i - \lambda.V_i + \lambda.V_{i+1} + 0 \times M_i + 0 \times M_{i+1} + \gamma(i - 0.5)d_i^2K = k_bW_i - c_i\sin\theta_i \tag{2.1b}
\]

The above equation for the last slice, considered as the verification of the \(\lambda\) value, is as follows:

\[
\Sigma F_X = \varepsilon = (tan\varphi_i\sin\theta_i - \cos\theta_i)N_i - \lambda.V_i + \lambda.V_{i+1} + 0 \times M_i + 0 \times M_{i+1} + \gamma(N - 0.5)d_i^2K = k_bW_i + cl_i\sin\theta_i \tag{2.1a*}
\]

The \(N\) index refers to the last slice with no beneath inter-slice force and moment. In terms of equilibrium equation in the \(Y\) direction, we have:

\[
\Sigma F_Y = 0 \rightarrow N_i\sin\theta_i + (N_i\tan\varphi_i + c_i)\cos\theta_i + V_{i+1} - V_i - (1 + k_v)W_i = 0 \tag{2.2}
\]

Reordering the former equation gives:

\[
(tan\varphi_i\cos\theta_i + \sin\theta_i)N_i - V_i - V_{i+1} + 0 \times (M_i + M_{i+1} + K) = (1 + k_v)W_i - c_i\cos\theta_i \tag{2.2a}
\]

Finally, the moment equilibrium equation is taken about the mid-length of the slip surface on the \(i\)th slice, as follows:

\[
\sum M_o = 0 \rightarrow 0 \times N_i - 0.5(LB_i + d_i\tan\theta_i)V_{i+1} + 0.5d_i(H_i + H_{i+1}) + 0.5(LT_i - d_i\tan\theta_i)V_i + M_{i+1} - M_i + k_bW_i(Y_Gi - Y_i - 0.5d_i) + (1 + k_v)W_i(X_i - X_Gi + 0.5d_i\tan\theta_i) = 0 \tag{2.3}
\]

where \(LT_i\) and \(LB_i\) are the slice lengths at the top and bottom of the \(i\)th slice, respectively. Their values for \(i = 1\) to \(i = N\) are given below:

\[
LT_i = LT_{i-1} + d_i\tan\beta - d_i\tan\theta_i \tag{2.4a}
\]
\[ LB_i = LT_{i+1} \] (2.4b)

and

\[ LB_N = 0 \] (2.4c)

Rearranging Eq. 2.3 gives:

\[
0 \times N_i + 0.5(LT_i - d_i \tan \theta_i + \lambda d_i)V_i \\
- 0.5(LB_i + d_i \tan \theta_i - \lambda d_i)V_{i+1} - M_i + M_{i+1} + 0 \times K \\
= -k_h W_i (Y_{Gi} - Y_i - 0.5d_i) \\
- (1 + k_v) W_i (X_i - X_{Gi} + 0.5d_i \tan \theta_i)
\] (2.3a)

Eqs. 2.1a, 2.2a, and 2.3a could be written for all N slices to generate a set of 3N simultaneous equations. The equation of moment equilibrium (Eq. 2.3a) for the last slice is taken about its lower corner to have a nonzero factor for \( K \) and to avoid a zero value on the diagonal elements of the coefficients matrix.

The pseudo-static horizontal and vertical components of the seismic force on each slice are considered as the constant fractions of the corresponding slice weight calculated as \( k_h W_i \) and \( k_v W_i \), respectively, where the slice weight is estimated as follows:

\[ W_i = 0.5d_i (LT_i + LB_i) \gamma \] (2.5)

2.3. Critical slip surface determination

The proposed method by Khosravizadeh et al. (2016) is implemented herein to attain the critical slip surface. The multilinear slip surface in this method consists of N straight lines for N slices, as shown in Figure 2. For any top length of slip surface, \( L_c \), which makes an angle of \( \alpha_{lc} \) with the horizontal line, an arbitrary angle of \( \alpha_s \), between \( \alpha_{lc} \) and 90°, is assumed. The angle between \( \alpha_s \) and \( \alpha_{lc} \) is then divided into N equal angles, and the intersections of these divided angle lines with the horizontal slices render the multilinear slip surface shown in Figure 2. This slip surface would converge to a curvy line by increasing the number of slices (N). Its shape varies from a straight line for \( \alpha_s \) being equal to \( \alpha_{lc} \) with minimum slip wedge area, to the curviest one for \( \alpha_s \) equal to 90° with a maximum slip wedge area. The critical slip surface for the assumed \( L_c \) is the multilinear line associated with the maximum mobilized tension in the reinforcement layers. Finally, the critical slip surface for the reinforced slope is sought among different values of \( L_c \), so that the greatest maximum mobilized tension force for different \( L_c \) values is the required strength, and its corresponding length, \( L_c \), is known as the critical required length, \( L_c \).

Different failure mechanisms for a reinforced earth slope are illustrated in Figure 3 (Javankhoshdel and Bathurst 2017) including external, internal and transition modes. In the external mode of failure, none of the reinforcement layers are intersected by the critical slip surface, whereas the failure surface passes through all the reinforcing elements in the internal failure mode. The transition mode, in turn, is defined as a state between the two aforementioned failure mechanisms. A proper design commonly consists of an internal mode of failure with adequate extension of reinforcements beyond the critical slip surface to account for the required pullout embedment. Accordingly, similar approach is followed in the current study.

Figure 2. Slip surface generation in the proposed HSM analysis.
3. Results & discussion

In this study, two general types of frictional (non-cohesive, \(c = 0\)) and cohesive – frictional (c-\(\varphi\)) soils are considered for backfills of earth slopes. For the corresponding soil types, the influence of different parameters, including horizontal and vertical earthquake coefficients (\(k_h = a_h/g\), and \(k_v = a_v/g\), respectively), slope angle (\(\beta\)) and soil strength characteristics (\(c\) and \(\varphi\)), on the stability of slopes is thoroughly examined. Apart from these parameters, slope height (\(H\)) and soil unit weight (\(\gamma\)) are considered as input values. Two main outputs are the non-dimensional reinforcement length (\(L_C/H\)), and non-dimensional total tension in geosynthetic layers (\(K = T/0.5yF^2\)). Critical slip surface, which dominates the required reinforcement lengths, is also presented for some of the typical cases considered. The values and variation ranges of input data of this study are presented in Table 4. It is worth noting that a wide range of soil strength parameters are considered in the analyses so as to cover all practical situations and desired factors of safety.

### 3.1. Frictional backfill (\(c = 0\))

#### 3.1.1. Failure slip surface and critical length (\(L_C\))

The critical slip surfaces for a wide range of soil friction angles (\(\varphi\)) and horizontal seismic coefficients (\(k_h\)) are investigated in this study. Typical results of the analyses for different slope angles (\(\beta = 45 \sim 90^\circ\)) and two horizontal seismic coefficients (\(k_h = 0.1\) and 0.2) are presented in Figure 4. It should be noted that the slope angles are chosen between 45° and 90° with a 15° interval in this paper, which are the commonly used values in previous research studies and geotechnical engineering practice. The vertical slopes with 90° angle are also used in many projects, especially for the trenches excavated in cohesive soils. However, the angles for the earth slopes are commonly restricted to 70° in practice, and the steeper slopes are generally categorized as walls.

The computed multilinear failure slip surfaces obtained in the course of this study are deemed to be quite accurate due to their automatic convergence towards the suitable relevant form. It could lead to a linear failure surface for walls and steep slopes, and to a log-spiral or circular slip surface for moderate slopes. The top length of the uppermost slice, known as the critical length (\(L_C\)), is an important factor that determines the required reinforcement length. As depicted in Figure 4, increasing \(k_h\) or decreasing \(\varphi\) leads to greater values of \(L_C\), whereas the variation of \(\beta\) imposes no specific trend in the design parameters.

#### 3.1.2. Effect of horizontal seismic coefficient (\(k_h\))

Figure 5 presents the design outputs for different values of \(\varphi\) (between 15° and 45° with increments of 2.5° for the accurate and sufficiently fine variations) based on various
Figure 4. Critical slip surface for different values of soil friction angle ($\gamma = 18$ kN/m$^3$, $k_v = 0$, $c = 0$).
Figure 5. Variation of $L_c/H$ & $K$ with $\varphi$ for different $k_h$ values ($y = 18$ kN/m$^3$, $k_v = 0$, $c = 0$).
$k_v$ values between 0 and 0.3. Variation of $L_C/H$ and $K$ with the friction angle for different values of horizontal pseudo-static coefficient ($k_h$) have been demonstrated for each slope angle (45, 60, 75, and 90°). It is obvious that increasing $k_h$ or decreasing $\varphi$, increases both the required reinforcement length ratio ($L_C/H$) and the total tension force in geosynthetic layers for all slope angles. For the values of $k_h$ greater than 0.2, the reinforcement length ratios for $\varphi \leq 20^\circ$ will be greater than 3, which is quite impractical. In this regard, the horizontal parts of the non-dimensional reinforcement tension ($K$) graphs mean that these situations are not acceptable and the backfill soil is too weak to tolerate the relatively massive seismic loads. In other words, these regions indicate that the internal stability is not the case for the failure mode, and the corresponding reinforced earth slope will be prone to either the transition or external mode of failure, which are not proper for design purposes.

3.1.3. Effect of vertical seismic coefficient ($k_v$)

The simultaneous effects of both horizontal and vertical seismic coefficients have been shown in Figure 6 herein and Figure S1 in the supplementary material, where $L_C/H$ and $K$ have been plotted against friction angle for different values of pseudo-static coefficient ratios ($k_v/k_h$). In each set of figures, $k_h$ is constant and $k_v$ varies in a way that the $k_v/k_h$ ratios are 0, +0.5, -0.5, +1, and -1. Similar to the previous case, again increasing $k_h$ or decreasing $\varphi$ values leads to the increase in both $L_C/H$ and $K$. It can be observed that the negative values of $k_v/k_h$ (downward $k_v$) cause a reduction in the length ratio and an increase in the reinforcement tension factor with respect to the case of $k_v = 0$, whereas the opposite trends are true for the positive values of $k_v/k_h$ (upward $k_v$). The corresponding variations could be observed to be more pronounced for higher absolute values of $k_h$ and $k_v/k_h$. The aforementioned trends are not valid for some regions of $\beta = 45^\circ$ at $k_h \geq 0.2$ and $\beta = 60^\circ$ at $k_h = 0.3$. For the latter cases, which include all graphs at $k_h = 0.3$ and a small portion of $0.2 \leq k_h < 0.3$ (only for $\beta = 45^\circ$ and $\varphi \geq 37.5^\circ$), both the $L_C/H$ and $K$ parameters increase for positive values of $k_v/k_h$ and vice versa.

It can also be generally concluded that, the vertical seismic component, $k_v$, has greater effect on steeper

![Figure 6](image-url). Variation of $L_C/H$ & $K$ with $\varphi$ for different $k_v/k_h$ values ($k_h=0.1, \gamma = 18 \text{ kN/m}^3, c = 0$).
slopes, i.e. higher $\beta$, and of weaker backfill soils, i.e. lower $\varphi$. For small $k_h$ values, say less than 0.15, $k_r$ has a low impact on K factor, and almost a quite negligible effect on $L_C/H$. For higher values of $k_h$, say greater than 0.15, the influence of $k_r$ is more notable on the design parameters of reinforcement, especially for $L_C/H$.

### 3.1.4. Effects of slope height ($H$) and soil unit weight ($\gamma$)

The height of slope and unit weight of soil have no significant effect on the non-dimensional design parameters of geosynthetic-reinforced slopes ($K$ and $L_C/H$) for the granular type of backfill soil ($c = 0$). It is obvious that increasing the values of $H$ and/or $\gamma$ will increase the total tension force of geosynthetic layers based on the well-known relation: $\Sigma T = 0.5K\gamma H^2$. Also, increasing the $H$ parameter will increase the critical length of slip surface ($L_C$) linearly, whereas the $\gamma$ parameter has no effect on it.

### 3.2. Cohesive – Frictional backfill ($c - \varphi$)

Exploiting local cohesive soils for reinforced earth slopes has many technical and economic advantages. It is shown that a small increase of cohesion for backfill soil would remarkably decrease the required length and strength of reinforcement.

#### 3.2.1. Failure slip surface and critical length ($L_C$)

The critical slip surfaces for a wide range of soil characteristics ($c$ and $\varphi$) and horizontal seismic coefficients ($k_h$) are examined in this study. Typical results of the analyses for different slope angles ($\beta = 45 \sim 90^\circ$) and backfill cohesion ($c = 0, 5, 10$, and $15$ kPa) at fairly high constant horizontal seismic coefficient of $k_h = 0.3$ are presented in Figure 7 and S2 in the supplementary material. The results indicate that similar to the effect of a decrease in the value of $k_h$, or an increase in $\varphi$, the required length of reinforcement declines for higher values of cohesion. This decline is more severe for moderate slopes and the lower angles of soil friction. In other words, the desired effect of cohesion enhancement is greater for lower values of $\beta$ and $\varphi$. General shapes of the critical slip surfaces are curved for moderate slopes or small friction angles; however, it tends to a linear surface for higher values of $\beta$ or $\varphi$. 

---

**Figure 6.** (Continued.)
Figure 7. Critical slip surface for different $\phi$ angles ($\beta = 45^\circ$ and $60^\circ$, $\gamma = 18$ kN/m$^3$, $k_i=0$).
Figure 8. Variation of $L_c/H$ & $K$ with $\varphi$ for different $k_h$ values ($\beta = 45^\circ$, $\gamma = 18$ kN/m$^3$, $k_r=0$).
3.2.2. Effect of horizontal seismic coefficient (k_h)

Figure 8 along with Figure S3 in the supplementary material presents the design outputs for different values of φ between 15° and 45°, based on various k_h values between 0 and 0.3. The backfill soil cohesion is considered to be 0, 5, 10, and 15 kPa for every set of graphs corresponding to each slope angle β (45, 60, 75, and 90°). It is obvious that increasing k_h, similar to the decrease of strength parameters φ or c, leads to the increase of both design parameters, L_c/H and K for all values of β. Regarding the design parameters for each slope angle, it is observed that the declines of L_c/H and K values corresponding to the enhancement of c are quite significant, especially for moderate slopes. No reinforcement is required for a β angle of 45° at k_h values of up to 0.3, provided that an average frictional soil (i.e. φ =30°) with a cohesive strength of 10 kPa is prepared as the backfill of slope. The similar limit values of k_h for β angles of 60° and 75° are 0.2 and 0.025, respectively, whereas a vertical slope (β =90°) with the same values of soil strength necessitates the design parameters of about 0.6 and 0.03, for L_c/H and K, respectively, to stay in static stability (k_h = 0).

3.2.3. Effect of vertical seismic coefficient (k_v)

In order to examine the simultaneous effects of horizontal and vertical seismic coefficients on the design parameters, similar values of soil cohesion, friction angle and horizontal seismic coefficient, as in the previous section, are considered; whereas the k_v/k_h ratio has been assumed to be 0.5 and 1 (Figures 9, Figure 10, S4 and S5). Both downward and upward components of k_v are considered and the greater values of L_c/H and K are selected between ±k_v for the sake of design requirements. It is obvious that increasing either k_v or k_v/k_h leads to the increase in both of the design parameters, L_c/H and K, for all values of β. Regarding the effect of cohesion on the ratio of reinforcement length, it is noticeable that enhancing the value of c brings the L_c/H ratios to some rational limits and eliminates the deleterious mechanisms of external or transition failure modes. For a fairly high seismic condition (k_h=0.3 and k_v/k_h=1); and a moderate frictional soil (φ =25°), the cohesion value of 5 kPa can limit the L_c/H ratios to about 1.35, 1.45, 1.55, and 1.60 for the slope angles of 45°, 60°, 75° and 90°, respectively. For similar friction angles, seismic coefficients and slope angles, the cohesion strength of 10 kPa confines the aforementioned ratios to the corresponding values of about 0.80, 0.95, 1.05, and 1.15, respectively. It should be noted that the relevant values of L_c/H ratios for similar slope conditions (i.e. the same φ, β and seismic coefficients) made of a non-cohesive (c =0) soil are all greater than 3, which coincides with an undesired mechanism other than the practical internal mode.

3.2.4. Effects of slope height (H) and soil unit weight (γ)

The height of slope and unit weight of soil have no significant effect on the non-dimensional design parameters of geosynthetic-reinforced slopes (K and L_c/H), provided that the non-dimensional cohesion term of c/γH is constant. Since the H and γ parameters of design graphs (Figures 8, Figure 9, Figure 10, S3, S4, S5) have specific values: H = 5m and γ = 18kN/m^3, the equivalent cohesion (c_eq) must be used for other H and γ parameters to be able to use the design graphs 

\[
\left( \frac{c}{\gamma H} \right)_{\text{graph}} = \left( \frac{c}{\gamma H} \right)_{\text{actual}} \text{, i.e. } c_{\text{eq}} = \left( \frac{90x}{7H} \right)_{\text{actual}}.
\]

4. Comparison with previous studies

4.1. Frictional backfill (c =0)

Most previous studies on the seismic stability of reinforced earth slopes have been restricted to the frictional type of backfill soils subjected solely to the horizontal component of seismic loading. Therefore, it is possible to compare and verify the results of current study with some of those throughout the literature. Some benchmark earlier studies based on the well-established LEM and LA approaches along with a more recent LEHSM study have been considered herein for comparison purposes. These previous works include the LEM study of Ling et al. (1997), the LA study of Michalowski (1998), the LA research of Ausilio et al. (2000), and the LEHSM study of Nouri et al. (2006). It should be noted that the experimental data are usually concerned with the stress-strain monitoring during the service load application until the failure state. However, the limit equilibrium approach used in the course of this study deals mainly with the determination of the factor of safety of the reinforced earth slope against failure. In other words, seismic stability analysis of reinforced slopes deals with the maximum developed tension in reinforcement layers and its required minimum length for design purposes with any specified factor of safety. Therefore, comparison of the proposed method of analysis with experimental data, which are mainly reported in service load conditions, is actually of no practical use.

Figure 11 shows the comparative results of L_c/H and K values for a typical earth slope subjected to the horizontal earthquake component of k_h=0.2 (k_v=0). As can be observed, the outputs are more scattered for L_c/H
Figure 9. Variation of $L_c/H$ & $K$ with $\varphi$ for different $k_h$ values ($\beta = 45^\circ$, $k_v/k_h = 0.5$).
Figure 10. Variation of $L_c/H$ & $K$ with $\varphi$ for different $k_h$ values ($\beta = 45^\circ$, $k_v/k_h = 1$).
Figure 11. Comparison of design outputs with previous results from literature ($k_h = 0.2, k_v = 0, c = 0$).
and lower slope angles. Based on these comparisons, it can be generally inferred that the computed K values are more precise than the estimated $L_c/H$ ones. Furthermore, the outputs could be observed to match better for relatively steep slopes; and for the vertical slope ($\beta = 90^\circ$), the K values are quite similar for all studies.

The evaluated K values of the current study lie among the other results and near to the upper bound (Michalowski 1998) outputs, whereas the estimated $L_c/H$ values are lower than the other results and near to the lower bound (Ausilio et al. 2000) outputs, except for the vertical slope.

A few studies on reinforced earth structures have included the vertical component of seismic loading. In this regard, the relevant results of the LEM analysis conducted by Ling and Leshchinsky (1998) and those of the LEHSM study accomplished by Nouri et al. (2008) have been compared with the outputs of the current study. Figures 12 and Figure 13 show the comparison between the computed $L_c/H$ and K values of these studies for the RSS with two different $\beta$ angles of 60° and 90°, respectively. In this comparison, two horizontal component values ($k_h = 0.1$ and 0.2) along with a constant vertical one ($k_v/k_h = 1$) are considered as the seismic loading. The maximum value of unity for the $k_v/k_h$ ratio has been selected herein so as to represent its utmost effect on the design outputs. Similar to the previous comparison for the case of pure $k_h$ application, the K outputs of the three compared studies match better than the $L_c/H$ ones and the results are less scattered for the higher $\beta$ value.

To better clarify, the outputs of the current study are very close to the previous ones for the K values, but have shown a little difference for the $L_c/H$ values with the results of other compared studies. The closest $L_c/H$ values of the other studies to those of the current one have a difference up to about 14% and 4% for $\beta$ angles of 60° and 90°, respectively.

4.2. Cohesive-frictional backfill ($c - \phi$)

Few studies have been presented in literature adopting cohesive backfills in reinforced earth structures, although most of them have not been concerned with the reinforcement design under seismic loading. The effect of cracks on reinforcement design for static loading (Abd and Utili 2017) and the distribution of active seismic earth pressure along the height of the retaining wall (Ghanbari and Ahmadiabadi 2010) are among the investigated parameters in these previous studies.

In a theoretical study, performed by Chandaluri et al. (2015) on the seismic stability of reinforced earth walls with cohesive backfills, the effects of various parameters, including surcharge loading (q), wall inclination angle ($\beta$), horizontal seismic coefficient ($k_h$) and soil cohesion ($c$), on the non-dimensional reinforcement tension (K) were thoroughly examined. K outputs for a vertical earth wall ($\beta = 90^\circ$) with $\phi = 25^\circ$, $q = 0$ (no surcharge) and $c = 0, 5, 10$, and 15 kPa under $k_h = 0.2$ are of few existing analytical data to be compared with the present work. The comparison of these data with the outputs of the current study, presented in Table 5, indicates a satisfactorily good agreement.

Another recent study on the stability of the RSS with cohesive backfills has been conducted by Abd and Utili (2017). Although this study is concerned with crack effects and static loading, its outputs for non-dimensional tension in the reinforcement layers (K), could be exploited to capture the effect of cohesion for intact (no cracks) slopes. A comparison of the results obtained in the course of this study with those presented by Abd and Utili (2017) for different slope angles ($\beta = 45, 60, 75$, and 90°), various c/\(\gamma H\) values between 0 and 0.1 ($c = 0, 1.8, 3.6, 5.4, 7.2$ and 9 kPa), $\phi = 20^\circ$ and $k_h = 0$ (static loading) is shown in Figure 14.

It can be obviously seen that the increase in backfill cohesion generally results in the reduction of the tension force in geosynthetic layers. The results match better for lower slope angles and also for higher values of cohesion; except for the vertical slope. It should be noted that for frictional soil ($c = 0$), the K outputs reported by Abd and Utili (2017) are quite similar to the upper bound results presented by Michalowski (1998). Therefore, their outputs for cohesive-frictional soil ($c-\phi$) could similarly be an upper bound for the results.

5. Design example

In order to illustrate how the developed flow charts above could be implemented for the design purpose, an example will be presented hereafter. A geosynthetic-reinforced earth slope, with the typical characteristics as listed in Table 6, is to be designed with the safety factors equal to 1, 1.25 and 1.5. It should be noted that the parameters in Table 6 are selected as typical values commonly encountered in various projects in the geotechnical engineering practice. It is intended to find the design parameters of reinforcements, including the required length and strength of geosynthetic layers, for different factors of safety. The detailed procedure, followed so as to
evaluate the design parameters for $FS = 1.25$, is elaborated as an instance and the results for all safety factors are then presented.

As explained earlier, an equivalent cohesion ($c_{eq}$) is determined to enable us to utilize the presented design graphs.

$$
\left( \frac{c_{eq}}{yH} \right)_\text{graph} = \left( \frac{c}{yH} \right)_\text{actual} \tag{3.1}
$$

$$
c_{eq} = \left( \frac{90c}{yH} \right)_\text{actual} = \frac{90(8)}{18(6)} = 6.67 \text{kN/m}^2 \tag{3.2}
$$

The mobilized $c$ and $\varphi$, based on the required factor of safety (1.25) are as follow:

$$
c_f = \frac{c_{eq}}{FS} = \frac{6.67}{1.25} = 5.33 \text{kPa} \tag{3.3}
$$

$$
\varphi_f = \arctan \left( \frac{\tan 28^\circ}{1.25} \right) = 23.04^\circ \tag{3.4}
$$

Based on the above strength parameters of backfill soil and the slope angle of 70°, design parameters for $c = 5$ and 10 kPa, $\varphi = 22.5^\circ$ and 25°, $\beta = 60^\circ$ and 75° and $k_h = \pm k_v = 0.25$ are extracted from the respective design charts, as shown in Tables 7 and 8. Interpolating between these values for $\beta = 70^\circ$ and $c_f = 5.33 \text{kPa}$ leads to the values presented in Table 9. Finally, interpolating between $\varphi = 22.5^\circ$ and 25° for $\varphi_f = 23.04^\circ$ gives:

$$
K = 0.399 - \frac{(0.399 - 0.345)(23.04 - 22.5)}{(25 - 22.5)} = 0.387 \tag{3.5}
$$

$$
\frac{L_C}{H} = 1.192 - \frac{(1.192 - 1.049)(23.04 - 22.5)}{(25 - 22.5)} = 1.16 \tag{3.6}
$$

Similar calculations for $FS = 1$ and 1.5 lead to the design parameters as presented in Table 10. Alternatively, the required design parameters of this example could be directly assessed using the developed program, as summarized in Table 11. The above direct results are more precise due to excluding any errors from either graph reading or interpolation process. Nevertheless, the graph outputs are quite satisfying for design purposes. The maximum difference between the two sets of
answers is about 1%, except for the $L_C/H$ parameter of $FS = 1.5$ which is about 5%. The latter significant difference is due to the very fact that some of its involved interpolations are performed on the $L_C/H$ values of up to 3, which are not acceptable for the desired internal mode of failure and/or practical limitations. Exploiting the former design outputs obtained from design graphs and adding approximately an extra 0.01FS to $L_C$ to account for the required embedment length against pullout with rounding it to the top 0.3 m multiple yield:

For $FS = 1.5 \rightarrow L_C = 1.015(1.68)(6) + 0.27 = 10.5m$  
\[(4.3)\]

Regarding the maximum tension in the last reinforcement layer ($i = 10$), the required reinforcement strengths for different factors of safety are as follows:

\[T_i = \gamma K (i - 0.5) d_i^2 = 18(10 - 0.5) \left(\frac{6}{10}\right)^2 K = 61.56K\]  
\[(5.1)\]

For $FS = 1 \rightarrow K = 0.248 \rightarrow T_{max}$
\[= 15.3kN/m (L_C = 5.4m)\]  
\[(5.2)\]

For $FS = 1.25 \rightarrow K = 0.387 \rightarrow T_{max}$
\[= 23.8kN/m (L_C = 7.2m)\]  
\[(5.3)\]

For $FS = 1.5 \rightarrow K = 0.503 \rightarrow T_{max}$
\[= 31.0kN/m (L_C = 10.5m)\]  
\[(5.4)\]

The finite element (FE) method is increasingly being implemented in the slope stability analyses. One of the most common methods to perform FE slope analysis is the shear strength reduction (SSR) approach. In the SSR
Figure 14. Comparison of obtained K values with the results of Abd and Utili (2017): φ = 20°, β = 45 − 90°, c = 0 − 9 kPa, and k_h = 0 (static loading).

Table 6. Characteristics of a typical slope for the design example.

| Characteristics                  | Value |
|----------------------------------|-------|
| H                                | 6 m   |
| β                                | 70°   |
| Number of slices                 | 10    |
| γ                                | 18 kN/m² |
| c                                | 8 kPa |
| φ                                | 28°   |
| k_h                              | 0.25  |
| k_u/k_h                          | ±1    |

Table 7. Design parameters for an slope with β = 60° subjected to k_h = ±k_u = 0.25.

| c (kPa) | 5 kPa | 10 kPa |
|---------|-------|--------|
| φ       | 22.5° | 25°    |
| K       | 0.35  | 0.285  |
| L_c/H   | 1.12  | 0.94   |

Table 8. Design parameters for an slope with β = 75° subjected to k_h = ±k_u = 0.25.

| c (kPa) | 5 kPa | 10 kPa |
|---------|-------|--------|
| φ       | 22.5° | 25°    |
| K       | 0.44  | 0.39   |
| L_c/H   | 1.26  | 1.13   |

Table 9. Design parameters for an slope with β = 70° subjected to k_h = ±k_u = 0.25.

| c (kPa) | 5 | 10 | c_r=5.33 kPa |
|---------|---|----|-------------|
| φ       | 22.5° | 25° | 22.5° | 25° |
| K       | 0.410 | 0.355 | 0.250 | 0.202 | 0.399 | 0.345 |
| L_c/H   | 1.213 | 1.067 | 0.897 | 0.803 | 1.192 | 1.049 |

Table 10. Design parameters for an slope with β = 70° subjected to k_h = ±k_u = 0.25 for FS = 1, 1.25 and 1.5 using the design charts.

| FS   | 1   | 1.25 | 1.5 |
|------|-----|------|-----|
| φ    | 28° | 23.04° | 19.52° |
| c_r (kPa) | 6.67 | 5.44 | 4.44 |
| K    | 0.248 | 0.387 | 0.503 |
| L_c/H | 0.85 | 1.16 | 1.68 |

Table 11. Design parameters for an slope with β = 70° subjected to k_h = ±k_u = 0.25 for FS = 1, 1.25 and 1.5 using the direct software output.

| FS   | 1   | 1.25 | 1.5 |
|------|-----|------|-----|
| K    | 0.245 | 0.382 | 0.502 |
| L_c/H | 0.84 | 1.15 | 1.60 |
method, the shear strength of the materials systematically reduces by a factor of safety until the obtained deformations from the FE analysis are unrealistically large or the solutions do not converge within a specified tolerance. The SSR technique for slope stability analysis involves systematic use of finite element analysis to evaluate a stress reduction factor (SRF) or the factor of safety value that brings a slope to the verge of failure state. The SSR method is commonly utilized to find the factor of safety for reinforced earth slopes.

The general procedure followed in the SSR method could be summarized as below:

1. The shear strength parameters of an earth slope are reduced by a certain factor (SRF), and the finite element stress analysis is conducted.
2. This process is then repeated for different values of strength reduction factor (SRF), until the deformations are large and the results of analyses do not converge (the model becomes basically unstable).
3. This determines the critical strength reduction factor (critical SRF), or factor of safety, of the slope.

In order to verify the accuracy of the aforementioned calculations and the proposed design charts in the course of this study, one of the cases presented above has been modelled and analyzed in the software RS2 (Rocscience Inc 2019). Indeed, it is attempted to utilize a different numerical approach, i.e. finite elements method, to provide comparative confidence about the results of the approximate limit equilibrium method presented here-with. In this finite element simulation, by adopting the well-established shear strength reduction (SSR) method, the stability of earth slopes could be assessed by finding the critical strength reduction factor (SRF). The SSR method in RS2 allows the user to automatically perform a finite element slope stability analysis, and compute a critical strength reduction factor for the geotechnical model. The critical strength reduction factor is equivalent to the ‘factor of safety (FS)’ of the slope.

As depicted in Figure 15, the stability of a typical slope with the properties as listed in Table 6 and also with the length of reinforcements (Lc) equal to 7.2 m and their tensile strength ( Tmax) equal to 23.8 kN/m has been analyzed with RS2. It should be further noted that all the common parameters were considered to be equal between the modellings of the RS2 and the current study. Few other parameters required for the finite element analysis were assumed to be as their default values in the software, i.e. Young’s modulus (E) = 20,000 kPa and Poisson’s ratio (υ) = 0.3. The width and height of the FE model have also been assumed to be 24 m and 17.5 m, respectively. The left and right boundaries are restricted to move horizontally while free to move vertically. The bottom boundary is, however, restricted from movement in both horizontal and vertical directions. As can be observed in Figure 15, the SRF value for the RS2 simulation has been obtained to be 1.24, thus confirming the accuracy of the design parameters extracted from the corresponding charts presented in the current study. It is worth mentioning that the displacement measurements are automatically incorporated in the SSR approach, as it lies within the convergence criterion itself. In other words, the convergence criterion satisfaction can’t be met without monitoring the displacement field across the model geometry.

6. Concluding remarks and future prospective

A new limit equilibrium formulation using horizontal slices method (HSM) is proposed for the internal stability analysis of geosynthetic-reinforced earth slopes. In this procedure, a new unknown as the balancing inter-slice moment is introduced to account for non-uniform vertical stress distribution due to the concurrent effect of static and seismic loadings. The pseudo-static approach with a new rigorous HSM, including ‘3N’ unknowns and equations instead of the traditional ‘5N–1’ method, is applied to precisely calculate the required reinforcement strength and to estimate its required length for reinforced slopes. Different parameters, including soil characteristics (c and ϕ), slope angle (β) and seismic loading coefficients (k_s and k_c/k_h), are studied and their effects on the reinforcement values are presented. A wide range of soil parameters are selected to cover all practical conditions for both frictional and cohesive-frictional backfills associated with the desired factor of safety.

It is shown that for all slope angles, an increase in k_s or k_c/k_h will raise the required reinforcement strength (K) and length (L_c/H). Similarly, reinforcement quantities have increased by a decrease in c or ϕ. The non-dimensional design parameters (K and L_c/H) have also been observed to increase for steeper slopes. The effect of vertical seismic component on the design outputs is quite considerable for high values of k_s or k_c/k_h, especially for lower soil friction angles and smaller soil cohesions. In fact, for high seismic loads and low soil strength parameters, the L_c/H values are out of the practical limits of 1.5 ~ 2. In such cases where the required reinforcement length ratio converges to the values greater than 3, specified by the horizontal part of K graphs, the governing failure mode is not the internal one. Therefore, these conditions are out of the
scope of the current study, and to include them requires the backfill soil strength to be improved.

The proposed design graphs for different conditions were verified for frictional backfill type, and were observed to be also reliable for cohesive-frictional backfills, provided that the mobilized soil strength parameters are safe for practical conditions. Therefore, the preliminary design is prepared by the presented graphs for a relatively wide range of soil types, slope angles and seismic loading conditions. For detailed design purposes, the transition and external modes of failure are to be considered to complete and develop the current study. Furthermore, the required pullout length of reinforcement is to be added to the estimated $L_C$, so as to find the precise design length of reinforcement at the internal mode of failure. A pseudo-dynamic approach will also be a proper extension to the current pseudo-static study to optimize the design outputs. Although the simple pseudo-static analysis is usually safe and slightly overestimates the reinforcement design parameters, the dynamic nature of seismic loading will be better simulated with the pseudo-dynamic method. It should be noted that the current approach is readily applicable to the pseudo-dynamic analysis for seismic loading.

**Nomenclature**

$a_h, a_v$  Horizontal and vertical seismic acceleration, respectively
c  Cohesion of backfill soil
$c_{eq}$  Equivalent cohesion of backfill soil for the constant parameter of $c/\gamma H$
$c_f$  Mobilized cohesion of backfill soil
$d_i$  Thickness of all slices (equally spaced reinforcement layers)
$D_i$  Tributary distance of layer i (distance between layers i and i+1)
$E$  Young’s modulus
$F_S$  Factor of safety
$g$  Gravity acceleration
$H$  Height of slope
$H_i, H_{i+1}$  Horizontal inter-slice forces at top and bottom of the ith slice, respectively
$i$  Number index of the slices
$k_h, k_v$  Horizontal and vertical seismic coefficient, respectively
$K$  Normalized total tension force in geosynthetic layers
$l_i$  Length of slip surface in the ith slice
$L_c$  Length of top chord of an arbitrary slip wedge
$L_{ic}$  Critical length of reinforcement
$M_i, M_{i+1}$  Balancing inter-slice moment at top and bottom of the ith slice, respectively
$N$  Number of slices
$N_i$  Normal force on the ith slice
$S_i$  Shear force on the ith slice
$T$  Total tension force in reinforcement layers per unit length of slope
$T_j$  Tension force of the ith slice’s reinforcement
$V_j, V_{i+1}$  Vertical inter-slice force at the top and bottom of the ith slice, respectively
$W_i$  Weight of the ith slice
$X_{Gi}$  X coordinate of the gravity center of the ith slice
$X_i$  X coordinate of the lower corner of the ith slice on the slip surface
$X_{Gi}, X_{Gi+1}$  X coordinate of exertion point of $V_i$ and $V_{i+1}$, relative to midpoint of ith slice edge
$Y_{Gi}$  Y coordinate of the gravity center of the ith slice
$Y_i$  Y coordinate of the lower corner of the ith slice on the slip surface
$Z_i$  Vertical distance of the ith slice reinforcement from the top of slope
$\alpha_i, \theta_i$  Horizontal and vertical angle of the ith slice edge, respectively
$\alpha_{LC}$  Angle of linear slip surface with horizontal line
$\alpha_S$  An arbitrary slip surface angle between $\alpha_{LC}$ and 90°
$\beta$  Slope angle
$\gamma$  Unit weight of backfill soil
$\epsilon$  Value of $\Sigma F_X$ of the last slice, as the verification equation
$\lambda$  Unknown coefficient in Morgenstern and Price method; constant for all slices
$\nu$  Poisson’s ratio
$\varphi$  Friction angle of backfill soil
$\varphi_f$  Mobilized friction angle of backfill soil

**Abbreviation**

| Abbreviation | Description |
|--------------|-------------|
| HSM          | Horizontal slices method |
| LA           | Limit analysis |
| LEM          | Limit equilibrium method |
| RSS          | Reinforced soil slopes |

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