Bayesian Pool-based Active Learning With Abstention Feedbacks

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Abstract

We study pool-based active learning with abstention feedbacks, where a labeler can abstain from labeling a queried example with some unknown abstention rate. This is an important problem with many useful applications. We take a Bayesian approach to the problem and develop two new greedy algorithms that learn both the classification problem and the unknown abstention rate at the same time. These are achieved by simply incorporating the estimated abstention rate into the greedy criteria. We prove that both of our algorithms have near-optimality guarantees: they respectively achieve a \((1 - \frac{1}{e})\) constant factor approximation of the optimal expected or worst-case value of a useful utility function. Our experiments show the algorithms perform well in various practical scenarios.

1 Introduction

We consider the problem of active learning with abstention feedbacks. This problem is one of the several attempts to deal with imperfect labelers in active learning who may give incorrect or noisy labels to queried examples (Zhang and Chaudhuri, 2015; Yan et al., 2015) or in our case, give abstention feedbacks to queries (Fang et al., 2012; Ramirez-Loaiza et al., 2014; Yan et al., 2015). Abstention feedbacks indicate that the labeler abstains from providing labels to some queried examples, and thus the active learning algorithm will not receive labels for these examples in this case.

Learning with abstention feedbacks is important in many real-life scenarios. Below we discuss some examples where this problem is useful. In these examples, although the reasons for the abstention may be different, from the learner’s view they are the same: the learner will receive no labels for some queries and the true labels for the others.

Crowdsourcing: In crowdsourcing, we have many labelers, each of whom only has expertise in some certain area and therefore can only provide labels for a subset of the input domain. These labelers were also called labelers with a knowledge blind spot (Fang et al., 2012). In this case, active learning is a good approach to quickly narrow down the expertise domain of a labeler and focus on querying examples in this region to learn a good model. By adapting active learning algorithms to each labeler, we can also gather representative subsets of labeled data from the labelers and combine them into a final training set.

Learning with Corrupted Labels: In this problem, the abstention feedbacks do not come from the labeler but occur due to corruptions in the labels received by the learner. The corruptions
could be caused by bad communication channels that distorts the labels or could even be caused by attackers attempting to corrupt the labels (Zhao et al., 2017). The setting in our paper can deal with the case when the corrupted labels are completely lost, i.e. they cannot be recovered and are not converted to incorrect ones.

In this paper, we consider active learning with abstention feedbacks in the pool-based and fixed budget setting, where a finite pool of unlabeled examples is given in advance and we need to sequentially select \( N \) examples from the pool to query their labels. Our setting assumes that abstention feedbacks count towards the budget \( N \), so we need to be careful when selecting the queried examples. In addition, we make an assumption that there is an unknown \textit{abstention rate} \( r^*(x) \) according to which the labeler decides whether or not to label an example \( x \).

Our work takes a Bayesian approach to the problem and learns both the classification model and the abstention rate at the same time. We contribute to the understandings of this problem both algorithmically and theoretically. Algorithmically, we develop two novel greedy algorithms for the considered problem. The algorithms use two different greedy criteria to select queried examples that can give information for both the classification model and the abstention rate. Theoretically, we prove that our proposed algorithms have theoretical guarantees for a useful utility of the selected examples in comparison to the optimal active learning algorithms. To the best of our knowledge, these are the first theoretical results for active learning with abstention feedbacks in the Bayesian pool-based setting.

The first greedy algorithm that we propose in this paper aims to maximize the expected version space reduction utility (Golovin and Krause, 2011) of the joint deterministic space deduced from the spaces of possible classification models and abstention rates. Version space reduction was shown to be a useful utility for active learning (Golovin and Krause, 2011; Cuong et al., 2013, 2014), and we apply it here in our algorithm. In essence, the proposed algorithm is similar to the maximum Gibbs error algorithm (Cuong et al., 2013) except that we incorporate the terms controlling the estimated abstention rate into the greedy criterion. By using previous theoretical results for adaptive submodularity (Golovin and Krause, 2011), we are able to prove that our algorithm has an \textit{average-case near-optimality} guarantee: the average utility value of its selected examples is always within a \((1 - \frac{1}{e}) \) constant factor of the optimal average utility value.

In contrast to the first algorithm, the second algorithm that we propose aims to maximize the worst-case version space reduction utility above. This algorithm resembles the least confidence active learning algorithm (Lewis and Gale, 1994) with the main difference that we also incorporate the estimated abstention rate into the greedy criterion. From previous theoretical results for pointwise submodularity (Cuong et al., 2014), we can prove that the proposed algorithm has a \textit{worst-case near-optimality} guarantee: the worst-case utility value of its selected examples is always within a \((1 - \frac{1}{e}) \) constant factor of the optimal worst-case utility value.

We conduct experiments to evaluate our proposed algorithms on various binary classification tasks under three different realistic abstention scenarios. The experiments show that our algorithms are useful compared to the passive learning and normal active learning baselines with various abstention rates under these scenarios.

### 2 Bayesian Pool-based Active Learning With Abstention Feedbacks

In pool-based active learning, we are given a finite set (or a pool) \( \mathcal{X} \) of unlabeled examples and a budget \( N \). We need to select \( N \) examples from \( \mathcal{X} \) and query a human labeler for their labels in order to learn a good classifier. Let \( \mathcal{Y} \triangleq \{1, 2, \ldots, \ell\} \) be a finite set of all possible labels and \( \mathcal{H} \) be a (possibly infinite) set of probabilistic hypotheses, where each hypothesis \( h \in \mathcal{H} \) is a random function from \( \mathcal{X} \) to \( \mathcal{Y} \). Formally, for any \( x \in \mathcal{X} \), \( h(x) \) is a categorical distribution with probability mass function (pmf) \( \mathbb{P}[h(x) = y] \) for all \( y \in \mathcal{Y} \).
Given any $S = \{x_1, x_2, \ldots, x_n\} \subseteq \mathcal{X}$ and $h \in \mathcal{H}$, we define $h(S) \triangleq \{h(x_1), h(x_2), \ldots, h(x_n)\}$. Throughout this paper, we assume $h(x_i)$ and $h(x_j)$ are independent for any fixed $h$ and $i \neq j$. Thus, $h(S)$ is also a categorical distribution with pmf $P[h(S) = y] = \prod_{i=1}^{n} P[h(x_i) = y_i]$ for all $y = \{y_1, y_2, \ldots, y_n\} \in \mathcal{Y}^{|S|}$. We call $y$ a labeling of $S$ as it contains the labels of the examples in $S$ accordingly.

We take the Bayesian approach and assume a prior distribution $p_0[h]$ on $\mathcal{H}$. If we observe some labels, we can use Bayes’ rule to obtain a posterior distribution. For any $S \subseteq \mathcal{X}$ and any distribution $p[h]$ on $\mathcal{H}$, let $Y$ be the random variable for the labeling of $S$ w.r.t. the distribution $p$. We note that $Y$ takes values in $\mathcal{Y}^{|S|}$ with pmf $P[Y = y; S] = \int p[h] \ P[h(S) = y]dh$ for all $y \in \mathcal{Y}^{|S|}$. As a special case, if $S$ is a singleton $\{x\}$ and $Y$ is the random variable for the label of $x$, we write $P[Y = y; x]$ for $y \in \mathcal{Y}$ to denote the pmf of $Y$.

Any pool-based active learning algorithm is a policy for choosing training examples from $\mathcal{X}$. A policy is a mapping from a set of labeled examples to the next unlabeled example to be queried. We assume at the beginning, a fixed labeling $y^*$ of the whole pool $\mathcal{X}$ is drawn randomly from the prior $p_0$ and is hidden from the learner. The labeling $y^*$ can be generated by first drawing a ground truth hypothesis $h^* \sim p_0$ and then drawing a label $y \sim P[h^*(x) = y]$ for each $x \in \mathcal{X}$. During the active learning process, the learner sequentially selects unlabeled training examples from $\mathcal{X}$ and asks the labeler for their labels, which would be returned to the learner according to $y^*$. The learner will iteratively select one unlabeled example at a time, and the decision to select any example depends on the labels of previously selected examples. This process can be represented by a policy tree whose nodes are unlabeled examples to be selected and edges from a node are its possible labels.

In a normal active learning process, the labeler always gives a label for a queried example. For the problem of active learning with abstention feedbacks that we consider here, the labeler can abstain from labeling a queried example. In other words, the labeler may return “no label” to a queried example. We note that this abstention decision may come from an attacker instead of the labeler in the security example above, but from the view of the learner, these two cases are the same: the learner will receive no labels for some queries and the true labels for the other queries. Our setting is general and can cover both cases. We also assume the abstention decision is permanent; that is, the labeler always makes the same decision if an example is queried many times.

In this setting, an active learning algorithm is also a policy for choosing examples to query, and which examples to query depend on the labels and abstention feedbacks of previously selected examples. Illustrations of policy trees for normal pool-based active learning and pool-based active learning with abstention feedbacks are given in Figure 1. In this paper, we shall assume there is an unknown function $r^*: \mathcal{X} \rightarrow [0, 1]$ such that $r^*(x)$ is the probability that the labeler abstains from labeling $x$. The function $r^*$ is called the abstention rate.
3 The Average-case Algorithm

In this section, we describe our first algorithm for Bayesian pool-based active learning with abstention feedbacks. We shall also prove the average-case near-optimality guarantee for the algorithm. Hence, we call this algorithm the average-case algorithm.

3.1 Algorithm Description

Since the abstention rate \( r^* \) is unknown, we take a Bayesian approach and consider a set of possible functions \( \mathcal{R} = \{r_1, r_2, \ldots, r_{|\mathcal{R}|}\} \) from \( \mathcal{X} \) to \([0, 1]\). In general, \( \mathcal{R} \) can be infinite, but we assume it to be finite here for notational simplicity. We also assume a prior \( p_0[r] \) on \( \mathcal{R} \). Note that we have slightly abused the notation \( p_0 \) for both priors on \( \mathcal{H} \) and \( \mathcal{R} \). In this case, \( p_0 \) can be thought of as a joint distribution on \( \mathcal{H} \times \mathcal{R} \) where the two elements are independent, i.e., \( p_0[h \wedge r] = p_0[h] p_0[r] \) for \( h \in \mathcal{H} \) and \( r \in \mathcal{R} \).

Algorithm 1 describes our average-case algorithm in details. In this algorithm, we sequentially select \( N \) examples for query in \( N \) iterations. At each iteration \( i \), we first estimate the abstention rate using an estimator \( \tilde{r}(x) \) based on the current posterior \( p_i[r] \). Then we select the example \( x^* \) to query using the greedy criterion:

\[
x^* = \arg\max_{x \in \mathcal{X}} \left\{ 1 - \tilde{r}(x)^2 - (1 - \tilde{r}(x))^2 \sum_{y \in \mathcal{Y}} p_i[Y = y; x]^2 \right\}.
\]

Intuitively, this criterion maximizes the expected one-step utility increment, with the utility function being defined in Eq. (2) in Section 3.2.

In the algorithm, we assume the labeler will give the label \( 0 \not\in \mathcal{Y} \) if he abstains from labeling an example; otherwise, he will return a label in \( \mathcal{Y} \). If we receive a label \( y^* \in \mathcal{Y} \) for \( x^* \), we update both posteriors \( p_{i+1}[h] \) and \( p_{i+1}[r] \) using Bayes’ rule. On the other hand, if we do not receive a label for \( x^* \) (i.e., \( y^* = 0 \not\in \mathcal{Y} \)), we update only the posterior \( p_{i+1}[r] \). After \( N \) iterations, we return the final posteriors \( p_N[h] \) and \( p_N[r] \).

Eq. (1) resembles the maximum Gibbs error criterion (Cuong et al., 2013) which selects \( x^* = \arg\max_y \{1 - \sum_y p_i[Y = y; x]^2\} \), except that we incorporate the terms \( \tilde{r}(x)^2 \) and \( (1 - \tilde{r}(x))^2 \) into the criterion. If we fix the distribution \( p_i[Y = y; x] \), the criterion value in Eq. (1) achieves its maximum when \( \tilde{r}(x) = \sum_y p_i[Y = y; x]^2 / (1 + \sum_y p_i[Y = y; x]^2) \) and achieves its minimum when \( \tilde{r}(x) = 1 \). Figure 2 (left) illustrates this criterion value as a function of \( \tilde{r}(x) \). Thus, given \( \tilde{r}(x) \) reasonably approximates \( r^*(x) \), Algorithm 1 would give more preference to the examples with \( r^*(x) \approx \sum_y p_i[Y = y; x]^2 / (1 + \sum_y p_i[Y = y; x]^2) \) and less preference to the examples with \( r^*(x) \approx 1 \).

3.2 Average-case Near-optimality Guarantee

We now prove the average-case near-optimality guarantee for Algorithm 1. In the context of this paper, near-optimality means the algorithm can achieve a constant factor approximation to the optimal algorithm w.r.t. some objective function.

To define an objective function that is useful for active learning with abstention feedbacks, we first induce a deterministic hypothesis space equivalent to the original probabilistic hypothesis space \( \mathcal{H} \). In particular, consider the hypothesis space \( \mathcal{F} \triangleq \{f : \mathcal{X} \to \mathcal{Y}\} \) consisting of all deterministic functions from \( \mathcal{X} \) to \( \mathcal{Y} \). We induce a new prior \( q_0 \) on \( \mathcal{F} \) from the original prior \( p_0 \) such that \( q_0[f] \triangleq p_0[Y = f(\mathcal{X}); \mathcal{X}] \). For any \( S \subseteq \mathcal{X} \) and \( y \in \mathcal{Y}^{|S|} \), we can define \( q_0[Y = y; S] \) in the way described in Section 2 with the hypothesis space \( \mathcal{F} \) and distribution \( q_0 \).

Also consider the space \( \mathcal{K} \triangleq \{k : \mathcal{X} \to \{0, 1\}\} \) consisting of all deterministic functions from \( \mathcal{X} \) to \( \{0, 1\} \). In essence, \( k(x) = 1 \) means the labeler abstains from labeling \( x \) while \( k(x) = 0 \) means the labeler gives a label for \( x \). We will call each \( k \in \mathcal{K} \) an abstention pattern. The prior
Algorithm 1 Average-case algorithm for Bayesian pool-based active learning with abstention feedbacks

**input:** Priors $p_0[h]$ and $p_0[r]$, budget $N$

**output:** Final posteriors $p_N[h]$ and $p_N[r]$ after $N$ queries

for $i = 0$ to $N - 1$ do
  Let $\bar{r}(x) \triangleq \mathbb{E}_{r \sim p_i[r]}[r(x)]$ for $x \in \mathcal{X}$
  Select $x^* = \arg \max_{x \in \mathcal{X}} \left\{ 1 - \bar{r}(x)^2 - (1 - \bar{r}(x))^2 \sum_{y \in \mathcal{Y}} p_i[Y = y; x|^2 \right\}$
  $y^* \leftarrow$ Query-label($x^*$)
  if $y^* \in \mathcal{Y}$ then
    Update $p_{i+1}[h] \propto p_i[h] \mathbb{P}[h(x^*) = y^*]$, and $p_{i+1}[r] \propto p_i[r] (1 - r(x^*))$
  else
    Update $p_{i+1}[r] \propto p_i[r] r(x^*)$
  end if
end for
return $p_N[h], p_N[r]$

Algorithm 2 Worst-case algorithm for Bayesian pool-based active learning with abstention feedbacks

**input:** Priors $p_0[h]$ and $p_0[r]$, budget $N$

**output:** Final posteriors $p_N[h]$ and $p_N[r]$ after $N$ queries

for $i = 0$ to $N - 1$ do
  Let $\bar{r}(x) \triangleq \mathbb{E}_{r \sim p_i[r]}[r(x)]$, for $x \in \mathcal{X}$
  Select $x^* = \arg \min_{x \in \mathcal{X}} \left\{ \max_{y \in \mathcal{Y}} \{ \bar{r}(x), (1 - \bar{r}(x)) \} \mathbb{P}[h(x^*) = y^*] \right\}$
  $y^* \leftarrow$ Query-label($x^*$)
  if $y^* \in \mathcal{Y}$ then
    Update $p_{i+1}[h] \propto p_i[h] \mathbb{P}[h(x^*) = y^*]$, and $p_{i+1}[r] \propto p_i[r] (1 - r(x^*))$
  else
    Update $p_{i+1}[r] \propto p_i[r] r(x^*)$
  end if
end for
return $p_N[h], p_N[r]$
Figure 2: Graphs showing the greedy criterion values in Algorithm 1 (left) and Algorithm 2 (right) as a function of \( \tilde{r}(x) \). The graphs are plotted with the fixed distribution \( p_i[Y = 1; x] = p_i[Y = 2; x] = 0.5 \). The red and yellow points indicate the maximum and minimum points in the graphs respectively.

\( p_0[r] \) induces a probability distribution \( q_0[k] \) on \( \mathcal{K} \) where:

\[
q_0[k] \triangleq \sum_{r \in \mathcal{R}} p_0[r] \, P_r[k], \quad \text{and} \\
P_r[k] \triangleq \prod_{x \in \mathcal{X}} (1 - r(x))^{1-k(x)} r(x)^{k(x)}
\]

is the probability (w.r.t. the rate \( r \)) that the labeler gives or abstains from giving labels to the whole pool \( \mathcal{X} \) according to the abstention pattern \( k \). For any \( S \subseteq \mathcal{X} \) and \( z \in \{0,1\}^{|S|} \), we can define \( q_0[Z = z; S] \) as in Section 2 with the hypothesis space \( \mathcal{K} \) and distribution \( q_0 \), where \( Z \) is the random variable for the abstention pattern of \( S \). Note that the induced prior \( q_0 \) can also be thought of as a joint prior on \( \mathcal{F} \times \mathcal{K} \) where the two elements are independent, i.e., \( q_0[f \land k] = q_0[f] \, q_0[k] \).

For \( S \subseteq \mathcal{X} \), \( f \in \mathcal{F} \), and \( k \in \mathcal{K} \), we consider the utility function:

\[
g(S, (f, k)) \triangleq 1 - q_0[Y = f(S) \land Z = k(S); S],
\]

where \( q_0[Y = f(S) \land Z = k(S); S] \) is the joint probability (w.r.t. \( q_0 \)) that the labeling of \( S \) is \( f(S) \) and the abstention pattern of \( S \) is \( k(S) \). This is a useful utility function for active learning because it is the version space reduction utility w.r.t. the joint prior \( q_0 \) on the joint space \( \mathcal{F} \times \mathcal{K} \) (Golovin and Krause, 2011).

With this utility, our objective function is defined as:

\[
G_{\text{avg}}(\pi) \triangleq \mathbb{E}_{f,k \sim q_0}[g(x^\pi_{f,k}, (f,k))],
\]

where \( x_{f,k}^\pi \) is the set of examples selected by the policy \( \pi \) given that the true labeling is \( f \) and the true abstention pattern is \( k \). This objective function is the average of the above utility w.r.t. the joint prior \( q_0 \). The following theorem proves the average-case near-optimality guarantee for Algorithm 1.

**Theorem 1.** For any budget \( N \geq 1 \), let \( \pi \) be the policy selecting \( N \) examples using Algorithm 1 and let \( \pi^*_\text{avg} \) be the optimal policy w.r.t. \( G_{\text{avg}} \) that selects \( N \) examples. We have: \( G_{\text{avg}}(\pi) > (1 - 1/e)G_{\text{avg}}(\pi^*_\text{avg}) \).

**Proof.** To prove this theorem, we first apply Theorem 5.2 in (Golovin and Krause, 2011). This requires us to prove that the utility function \( g(S, (f, k)) \) is adaptive monotone and adaptive submodular w.r.t. the joint prior distribution \( q_0 \). Note that \( g(S, (f, k)) \) is the version space reduction function w.r.t. to the joint prior \( q_0 \) on the joint space \( \mathcal{F} \times \mathcal{K} \). From the results
in Section 9 of (Golovin and Krause, 2011), version space reduction functions are adaptive monotone and adaptive submodular w.r.t. the corresponding prior. Thus, the utility function $g(S, (f, k))$ is adaptive monotone and adaptive submodular w.r.t. the joint prior $q_0$.

With the above properties of $g$, applying Theorem 5.2 in (Golovin and Krause, 2011), we have $G_{\text{avg}}(\pi_{\text{greedy}}) > (1 - 1/e)G_{\text{avg}}(\pi^*_{\text{avg}})$, where $\pi_{\text{greedy}}$ is the greedy algorithm that selects the examples maximizing the expected utility gain at each step. From the proof of Theorem 4 of (Cuong et al., 2013), this greedy algorithm is equivalent to the maximum Gibbs error algorithm that selects the examples according to the criterion:

$$x^* = \arg\max_{x \in X} \left\{ 1 - p_i[Z = 1; x]^2 - \sum_{y \in \mathcal{Y}} p_i[Y = y \land Z = 0; x]^2 \right\},$$

(4)

where $p_i$ is the current posterior distribution, $Y$ is the random variable for the label of $x$, and $Z$ is the random variable for the abstention pattern of $x$. To understand this equation, we can think of the considered problem as a classification problem with labels $(y, z = 0)$ or $(z = 1)$, where $(y, z = 0)$ indicates an example is labeled the label $y$ and $(z = 1)$ indicates an example is not labeled.

Since $y$ and $z$ are independent, Eq. (4) is equivalent to:

$$x^* = \arg\max_{x \in X} \left\{ 1 - p_i[Z = 1; x]^2 - \sum_{y \in \mathcal{Y}} (p_i[Z = 0; x] p_i[Y = y; x])^2 \right\}$$

$$= \arg\max_{x \in X} \left\{ 1 - p_i[Z = 1; x]^2 - p_i[Z = 0; x]^2 \sum_{y \in \mathcal{Y}} p_i[Y = y; x]^2 \right\}.$$

We also have $p_i[Z = 1; x] = \sum_{r \in \mathcal{R}} p_i[r] r(x) = \mathbb{E}_{r \sim p_i[r]}[r(x)] = \tilde{r}(x)$. Similarly, $p_i[Z = 0; x] = 1 - \tilde{r}(x)$. Hence, the previous equation is equivalent to:

$$x^* = \arg\max_{x \in X} \left\{ 1 - \tilde{r}(x)^2 - (1 - \tilde{r}(x))^2 \sum_{y \in \mathcal{Y}} p_i[Y = y; x]^2 \right\},$$

which is Eq. (1). Therefore, Algorithm 1 is equivalent to $\pi_{\text{greedy}}$ and Theorem 1 holds.

4 The Worst-case Algorithm

We now describe our second algorithm for Bayesian pool-based active learning with abstention feedbacks. We shall also prove its worst-case near-optimality guarantee. Hence, we call this algorithm the worst-case algorithm.

4.1 Algorithm Description

The worst-case algorithm (see Algorithm 2) is essentially similar to the previous average-case algorithm, except that we replace the greedy criterion in Eq. (1) by the following greedy criterion:

$$x^* = \arg\min_{x \in X} \left\{ \max_{y \in \mathcal{Y}} \{ \tilde{r}(x), (1 - \tilde{r}(x)) \} \max p_i[Y = y; x] \right\}.$$  

(5)

Intuitively, this criterion maximizes the worst-case one-step utility increment, with the version space reduction utility in Eq. (2).

The criterion (5) resembles the least confidence criterion (Lewis and Gale, 1994), which selects $x^* = \arg\min_x \{ \max_y p_i[Y = y; x] \}$, except that we also incorporate the terms $\tilde{r}(x)$ and $1 - \tilde{r}(x)$ into the criterion. If we fix the distribution $p_i[Y = y; x]$, the criterion value in Eq. (5) achieves its maximum when $\tilde{r}(x) = 1$ and achieves its minimum when $\tilde{r}(x) = \max_y p_i[Y = y; x]/(1 + \max_y p_i[Y = y; x])$. Figure 2 (right) illustrates this criterion value as a function of $\tilde{r}(x)$. Thus, given $\tilde{r}(x)$ reasonably approximates $r^*(x)$, Algorithm 2 would give more preference to the examples with $r^*(x) \approx \max_y p_i[Y = y; x]/(1 + \max_y p_i[Y = y; x])$ and less preference to the examples with $r^*(x) \approx 1$. 

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4.2 Worst-case Near-optimality Guarantee

We now prove the worst-case near-optimality guarantee for Algorithm 2. For this guarantee, we still make use of the version space reduction utility function $g(S,(f,k))$ defined in Eq. (2). Using this utility, we define the following worst-case objective function for Algorithm 2:

$$G_{\text{worst}}(\pi) \triangleq \min_{(f,k) \in F \times K} [g(x_{f,k}, (f,k))].$$  \hfill (6)

The following theorem proves the worst-case near-optimality guarantee for this algorithm.

**Theorem 2.** For any budget $N \geq 1$, let $\pi$ be the policy selecting $N$ examples using Algorithm 2 and let $\pi_{\text{worst}}^*$ be the optimal policy w.r.t. $G_{\text{worst}}$ that selects $N$ examples. We have: $G_{\text{worst}}(\pi) > (1 - 1/e)G_{\text{worst}}(\pi_{\text{worst}}^*)$.

**Proof.** To prove this theorem, we first apply Theorem 3 in (Cuong et al., 2014). This requires us to prove that the utility $g(S,(f,k))$ is pointwise monotone and pointwise submodular. Note that $g(S,(f,k))$ is the version space reduction function w.r.t. to the joint prior $q_0$ on the joint space $F \times K$. From the proof of Theorem 5 in (Cuong et al., 2014), version space reduction functions are both pointwise monotone and pointwise submodular. Thus, $g(S,(f,k))$ is pointwise monotone and pointwise submodular.

With the above properties of $g$, applying Theorem 3 in (Cuong et al., 2014), we have $G_{\text{worst}}(\pi'_{\text{greedy}}) > (1 - 1/e)G_{\text{worst}}(\pi_{\text{worst}}^*)$, where $\pi'_{\text{greedy}}$ is the greedy algorithm that selects the examples maximizing the worst-case utility gain at each step. From the proof of Theorem 5 of (Cuong et al., 2014), this greedy algorithm is equivalent to the least confidence algorithm that selects the examples according to the criterion:

$$x^* = \arg \min_{x \in \mathcal{X}} \left\{ \max_{y \in \mathcal{Y}} p_i[Z = 1; x], \max_{y \in \mathcal{Y}} p_i[Y = y \land Z = 0; x] \right\},$$ \hfill (7)

where $p_i$ is the current posterior distribution, $Y$ is the random variable for the label of $x$, and $Z$ is the random variable for the abstention pattern of $x$. Similar to the proof of Theorem 1 above, to understand this equation, we can think of the considered problem as a classification problem with labels $(y,z) = (0,0)$ or $(z = 1)$, where $(y,z = 0)$ indicates an example is labeled the label $y$ and $(z = 1)$ indicates an example is not labeled.

Since $y$ and $z$ are independent, Eq. (7) is equivalent to:

$$x^* = \arg \min_{x \in \mathcal{X}} \left\{ \max_{y \in \mathcal{Y}} p_i[Z = 1; x], \max_{y \in \mathcal{Y}} p_i[Y = y; x, p_i[Z = 0; x]] \right\}$$

$$= \arg \min_{x \in \mathcal{X}} \left\{ \max_{y \in \mathcal{Y}} p_i[Z = 1; x], p_i[Z = 0; x] \max_{y \in \mathcal{Y}} p_i[Y = y; x] \right\}.$$ 

From the proof of Theorem 1, we have $p_i[Z = 1; x] = \tilde{r}(x)$ and $p_i[Z = 0; x] = 1 - \tilde{r}(x)$. Hence, the previous equation is equivalent to:

$$x^* = \arg \min_{x \in \mathcal{X}} \left\{ \max \{ \tilde{r}(x), (1 - \tilde{r}(x)) \max_{y \in \mathcal{Y}} p_i[Y = y; x] \} \right\},$$

which is Eq. (5). Therefore, Algorithm 2 is equivalent to $\pi'_{\text{greedy}}$ and Theorem 2 holds. \hfill $\square$

5 Experiments

In this section, we experimentally evaluate the proposed algorithms. In particular, we compare four algorithms:

- **PL**: the passive learning baseline with randomly selected examples,
• ALg: the active learning baseline using the maximum Gibbs error criterion (Cuong et al., 2013),
• ALa: the average-case algorithm (Algorithm 1), and
• ALw: the worst-case algorithm (Algorithm 2).

For binary classification, ALg is equivalent to other well-known active learning algorithms such as the least confidence (Lewis and Gale, 1994) and maximum entropy (Settles, 2010) algorithms. The PL and ALg baselines do not learn the abstention probability of the examples, i.e., they ignore whether an example would be labeled or not when making a decision. In contrast, the proposed algorithms ALa and ALw take into account the estimated abstention probability \( \hat{r}(x) \) when making decisions.

To show the potential of our algorithms further, we also consider two variants of ALa and ALw that are assumed to know a good estimate of the training examples’ abstention rates \( r^*(x) \). In particular, for these versions of ALa and ALw (shown as dashed lines in Figures 3 and 4), we train a logistic regression model using the actual abstention pattern on the whole training set to predict the abstention probability for each example. We keep this classifier fixed throughout the experiments and use it to estimate \( \hat{r}(x) \) in these versions of ALa and ALw.

In the algorithms, we use Bayesian logistic regression models for both \( H \) and \( R \). That is, each hypothesis \( h \in H \) and each candidate abstention rate function \( r \in R \) is a logistic regression model. We put an independent Gaussian prior \( \mathcal{N}(0, \sigma^2) \) on each parameter of the logistic regression models (for both \( H \) and \( R \)). In this case, the posteriors are proportional to the regularized likelihood of the observed data with \( \ell_2 \) penalty. To avoid sampling from the posteriors, we use the maximum a posteriori (MAP) hypotheses to estimate the probabilities in our algorithms. Finding the MAP hypotheses is equivalent to maximizing the regularized log-likelihood of the observed data.

Following previous works in active learning (Settles and Craven, 2008; Cuong et al., 2013, 2014), we evaluate the algorithms using the area under the accuracy curve (AUAC) scores. For each task in our experiments, we compute the scores on a separate test set during the first 300 queries and then normalize these scores so that their values are between 0 and 100. The final scores are obtained by averaging 10 runs of the algorithms using different random seeds.

We shall consider three scenarios: (1) the labeler abstains from labeling examples unrelated to the target classification task, (2) the labeler abstains from labeling easy examples, and (3) the labeler abstains from labeling hard examples.

5.1 Abstention on Data Unrelated to Target Task

We consider the binary text classification task between two recreational topics: \texttt{rec.motorcycles} and \texttt{rec.sport.baseball} from the 20 Newsgroups data (Joachims, 1996). In the pool of unlabeled data, we allow examples from other classes (e.g., in the \texttt{computer} category) that are not related to the two target classes. The labeler always abstains from labeling these redundant examples while always giving labels for examples from the target classes. Thus, the abstention is on examples unrelated to the target task, and this satisfies the independence assumption in Section 3.1. In the experiment, we fix the pool size to be 1322 and vary the abstention percentage (%) of the labeler by changing the ratio of the redundant examples.

Figure 3 shows the results for various abstention percentages. From the figure, our algorithms ALa and ALw are consistently better than the baselines for abstention percentages above 40%. When a good estimate of \( r^* \) is available, our algorithms perform better than all the other algorithms for abstention percentages above 30%. This shows the advantage of modeling the labeler’s abstention pattern in this setting, especially for medium to high abstention percentages.
5.2 Abstention on Easy Examples

In this scenario, we test with the labeler who abstains from labeling easy data, which are far from the true decision boundary. This setting may seem counter-intuitive, but it is in fact not unrealistic. For example in the learning with corrupted labels setting discussed in Section 1, easy examples may be considered less important than hard examples and thus were less protected than hard ones. In this case, an attacker may attempt to corrupt the labels of those easy examples to bring down the performance of the learned classifier. Furthermore, under a heavy attack, we may expect a high abstention percentage. As another example, in medical diagnosis, lung cancer screening is only recommended for the high-risk group (heavy smoking, 55-74 years old, etc.) (Roberts et al., 2013), so labels (cancer or no cancer) for the low-risk group (easy data) are often unavailable.

We simulate the abstention pattern for this scenario by first learning a logistic regression model with regularizer $\sigma^2 = 0.5$ on the whole training data set and then measuring the distance between the model’s prediction probability to 0.5 for each example. The labeler would always abstain from labeling the subset of the training data (with size depending on the abstention percentage) that have the largest such distances while he would always give labels for the other examples. Figure 4 (first row) shows the results for this setting on 4 binary text classification data sets from the 20 Newsgroups data (from left to right): comp.sys.mac.hardware/comp.windows.x, rec.motorcycles/rec.sport.baseball, sci.crypt/sci.electronics, and sci.space/soc.religion.christian.

From the results, ALa and ALw work very well when the abstention percentage is above 50%. This shows that it is useful to learn and take into account the abstention probabilities when the abstention percentage is high (e.g., under heavy attacks), and our algorithms provide a good way to exploit this information. When the abstention percentage is small, the advantages of ALa and ALw diminish. This is expected because in this scenario, learning the abstention pattern is more expensive than simply ignoring it. However, when a good estimate of $r^*$ is available, ALa and ALw perform better than all the other algorithms for most abstention percentages.

5.3 Abstention on Hard Examples

In this scenario, we test with the labeler who abstains from labeling hard data, which are near to the true decision boundary. This setting is common when the labeler wants to maximize the number of labels giving to the learner (e.g., in crowdsourcing where he is paid for each label provided). The abstention pattern in this experiment is generated similarly to the previous scenario, except that the labeler abstains from labeling the examples having the smallest distances above instead of those with the largest distances.

Figure 4 (second row) shows the results for this scenario on the same 4 data sets above. These results suggest that this is a more difficult setting for active learning. From the figure,
ALa and ALw are only better than the baselines when the abstention percentage is from 20-40%. For other abstention percentages, ALa, ALw, and ALg do not provide much advantage compared to PL. However, when a good estimate of \( r^* \) is available, ALa and ALw perform very well and are better than all the other algorithms.

**Summary:** The results above have shown that the proposed algorithms are useful for pool-based active learning with abstention feedbacks when the abstention percentage is within an appropriate range that depends on the problem. The algorithms are especially useful when a good estimate of the abstention rate \( r^* \) is available. In practice, this estimate can be pre-computed from previous interactions between the learning systems and the labeler (e.g., using previous labeling preferences of the labeler), and then inputted into our algorithms as the priors \( p_0[r] \). During the execution of our algorithms, this estimate will be gradually improved and can be reused in future interactions with the labeler.

6 Related Work

The theoretical guarantees considered in this paper have been studied for normal Bayesian pool-based active learning where the labeler always gives labels to queried examples (Golovin and Krause, 2011; Chen and Krause, 2013; Cuong et al., 2013, 2014; Chen et al., 2015; Cuong and Xu, 2016; Cuong et al., 2016). The theory for the average case was originally developed in (Golovin and Krause, 2011) with adaptive submodular utilities, while that of the worst case was developed in (Cuong et al., 2014) for pointwise submodular utilities. In both cases, \((1-\frac{1}{e})\)-factor approximation guarantees were proven for the corresponding greedy algorithms.

The problem of active learning with abstention feedbacks was previously investigated in (Fang et al., 2012; Ramirez-Loaiza et al., 2014; Yan et al., 2015). Fang et al. (2012) considered a setting similar to ours where the labeler may have knowledge blind spots and would be incapable of labeling examples in such blind spots. On the other hand, Ramirez-Loaiza et al. (2014) studied a situation where the learner may interrupt the labeler rather than waiting for his response, thus allowing the possibility of receiving “I don’t know” labels. In both papers, greedy algorithms were used to create a balance between maximizing the received information and minimizing the abstention probability; however, no theoretical guarantees about their performance were obtained. Our work in this paper, although similar to theirs in spirit, provides theoretical guarantees for the proposed algorithms.
A theoretical work on active learning with abstention feedbacks is (Yan et al., 2015), where both noisy labels and abstention feedbacks are considered. However, they only examined a simple one-dimensional classification problem and made a low-noise assumption on both the labeling noise and the abstention rate. Furthermore, the labeler’s abstention in their model is non-persistent and that allows the learner to repeatedly query an example until a label is received. Under this framework, the paper derived an algorithm with a near-optimal asymptotic convergence rate for estimating model parameters. In contrast, our work in this paper investigates the persistent label scenario, which is much less understood and more difficult to resolve (Chen et al., 2015, 2017), and we focus on near-optimal query strategies with a finite budget. Thus, our results are not directly comparable to those in (Yan et al., 2015).

Besides abstention feedbacks, there were other works on active learning with unreliable labelers. For examples, many authors considered labelers that give incorrect or corrupted labels from various types of noise models (Donmez and Carbonell, 2008; Golovin et al., 2010; Naghshvar et al., 2012; Cuong et al., 2016; Chen et al., 2017). Ni and Ling (2012) considered a setting where the labeler can return both labels and confidences, while in (Malago et al., 2014; Zhang and Chaudhuri, 2015), multiple labelers with different fidelity are available and the learner is given the option of obtaining labels from either weak or strong labelers. Our work also relates to other works on active learning and adaptive sampling in crowdsourcing such as (Yan et al., 2011; Zhao et al., 2011; Mozafari et al., 2014; Manino et al., 2016; Singla et al., 2016).

7 Conclusion

We proposed two new greedy algorithms for Bayesian pool-based active learning with abstention feedbacks. This setting is useful in many real-world scenarios, including learning from multiple labelers and under corrupted labels. We proved that the algorithms have theoretical guarantees in the average and worst cases and also showed experimentally that they are useful for classification, especially when a good estimate of the abstention rate is available. Our results suggest that keeping track and learning the abstention patterns of labelers are important for active learning with abstention feedbacks.

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References

Yuxin Chen and Andreas Krause. Near-optimal batch mode active learning and adaptive submodular optimization. In ICML, 2013.

Yuxin Chen, S. Hamed Hassani, Amin Karbasi, and Andreas Krause. Sequential information maximization: When is greedy near-optimal? In COLT, 2015.

Yuxin Chen, S. Hamed Hassani, and Andreas Krause. Near-optimal Bayesian active learning with correlated and noisy tests. In AISTATS, 2017.

Nguyen Viet Cuong and Huan Xu. Adaptive maximization of pointwise submodular functions with budget constraint. In NIPS, 2016.

Nguyen Viet Cuong, Wee Sun Lee, Nan Ye, Kian Ming A. Chai, and Hai Leong Chieu. Active learning for probabilistic hypotheses using the maximum Gibbs error criterion. In NIPS, 2013.
Nguyen Viet Cuong, Wee Sun Lee, and Nan Ye. Near-optimal adaptive pool-based active learning with general loss. In *UAI*, 2014.

Nguyen Viet Cuong, Nan Ye, and Wee Sun Lee. Robustness of Bayesian pool-based active learning against prior misspecification. In *AAAI*, 2016.

Pinar Donmez and Jaime G. Carbonell. Proactive learning: Cost-sensitive active learning with multiple imperfect oracles. In *CIKM*, 2008.

Meng Fang, Xingquan Zhu, and Chengqi Zhang. Active learning from oracle with knowledge blind spot. In *AAAI*, 2012.

Daniel Golovin and Andreas Krause. Adaptive submodularity: Theory and applications in active learning and stochastic optimization. *JAIR*, 2011.

Daniel Golovin, Andreas Krause, and Debajyoti Ray. Near-optimal Bayesian active learning with noisy observations. In *NIPS*, 2010.

Thorsten Joachims. A probabilistic analysis of the Rocchio algorithm with TFIDF for text categorization. Technical report, DTIC Document, 1996.

David D. Lewis and William A. Gale. A sequential algorithm for training text classifiers. In *SIGIR*, 1994.

Luigi Malago, Nicolo Cesa-Bianchi, and J Renders. Online active learning with strong and weak annotators. In *NIPS Workshop on Learning from the Wisdom of Crowds*, 2014.

Edoardo Manino, Long Tran-Thanh, and Nicholas R. Jennings. Efficiency of active learning for the allocation of workers on crowdsourced classification tasks. In *NIPS CrowdML Workshop*, 2016.

Barzan Mozafari, Purna Sarkar, Michael Franklin, Michael Jordan, and Samuel Madden. Scaling up crowd-sourcing to very large datasets: a case for active learning. *PVLDB*, 2014.

Mohammad Naghshvar, Tara Javidi, and Kamalika Chaudhuri. Noisy Bayesian active learning. In *Allerton*, 2012.

Eileen Ni and Charles Ling. Active Learning with c-Certainty. *Advances in Knowledge Discovery and Data Mining*, 2012.

Maria Eugenia Ramirez-Loaiza, Aron Culotta, and Mustafa Bilgic. Anytime active learning. In *AAAI*, 2014.

Heidi Roberts, Cindy Walker-Dilks, Khalil Sivjee, Yee Ung, Kazuhiro Yasufuku, Amanda Hey, Nancy Lewis, and Lung Cancer Screening Guideline Development Group. Screening high-risk populations for lung cancer: guideline recommendations. *Journal of Thoracic Oncology*, 2013.

Burr Settles. Active learning literature survey. Computer Sciences Technical Report 1648, University of Wisconsin–Madison, 2010.

Burr Settles and Mark Craven. An analysis of active learning strategies for sequence labeling tasks. In *EMNLP*, 2008.

Adish Singla, Sebastian Tchiatschek, and Andreas Krause. Noisy submodular maximization via adaptive sampling with applications to crowdsourced image collection summarization. In *AAAI*, 2016.
Songbai Yan, Kamalika Chaudhuri, and Tara Javidi. Active learning from noisy and abstention feedback. In *Allerton*, 2015.

Yan Yan, Glenn M. Fung, Rómer Rosales, and Jennifer G. Dy. Active learning from crowds. In *ICML*, 2011.

Chicheng Zhang and Kamalika Chaudhuri. Active learning from weak and strong labelers. In *NIPS*, 2015.

Liyue Zhao, Gita Sukthankar, and Rahul Sukthankar. Incremental relabeling for active learning with noisy crowdsourced annotations. In *PASSAT and SocialCom*, 2011.

Mengchen Zhao, Bo An, Wei Gao, and Teng Zhang. Efficient label contamination attacks against black-box learning models. In *IJCAI*, 2017.