1 INTRODUCTION

Binary stars play a fundamental role in the evolution of globular clusters for at least two important reasons. First, the evolution of stars in binaries, whether in a cluster or in the galactic field, can be very different from the evolution of the same stars in isolation. In a dense environment like a globular cluster, this difference is exacerbated by dynamical encounters, which affect binaries much more than single stars. Second, binary stars crucially affect the dynamical evolution of globular clusters, providing (through inelastic collisions) the source of energy that supports them against gravothermal collapse (Goodman & Hut 1989; Gao et al. 1991; Freguca et al. 2003). In the “binary burning” phase, a cluster can remain in quasithermal equilibrium with nearly constant core density and velocity dispersion for many relaxation times, in a similar way to that in which a star can maintain itself in thermal equilibrium for many Kelvin-Helmholtz times by burning hydrogen in its core. The binary fraction (and the initial, primordial binary fraction in particular), is therefore one of the most important parameters that determine the evolution of globular clusters. However, most previous dynamical studies of globular clusters—even those including binaries—have neglected stellar evolution, which can significantly impact the properties and survival of binaries and hence the reservoir of energy they provide.

At present, there are very few direct measurements of binary fractions in clusters. However, even early observations showed that binary fractions in globular cluster cores are smaller than in the solar neighborhood (e.g., Cote et al. 1996). Recent Hubble Space Telescope (HST) observations have provided further constraints on the binary fractions in many globular clusters (Bellazzini et al. 2002b; Rubenstein & Bailyn 1997). The measured binary fractions in dense cluster cores are found to be very small. As an example, the upper limit on the core binary fraction of NGC 6397 is only 5-7% (Cool & Bolton 2002). On the other hand, in very sparse clusters, like NGC 288 (Bellazzini et al. 2002b), but also in some other “core-collapsed” clusters, like NGC 6752 (Rubenstein & Bailyn 1997), the upper limit for the binary fraction can be as high as ~30%.

For the initial binary fraction in globular clusters, there are of course no direct measurements. However, there are no observational or theoretical arguments suggesting that the formation of binaries and hierarchical multiples in dense stellar systems should be significantly different from other environments like open clusters, the Galactic field, or star-forming regions. Binary frequencies >50% are found in the solar neighborhood and in open clusters (Habwachs, Mayor, Udry, & Arenou 2003). T Tauri stars also have a very high binary fraction (Kohler, Leinert, & Zinnecker 2001). For the range of separations between 120 and 1800 AU, their binary fraction is comparable to that of main sequence stars in the...
solar neighborhood (Brandner et al. 1996), while at shorter periods it is higher (Mellon 2003). Furthermore, many stars are formed in systems of multiplicity 3 or higher: in the field their abundance is no less than 40% for inner periods $\lesssim 10$ days (Tokovinin 1997). All this suggests that, in dense stellar systems as well, most stars could be formed in binary and multiple configurations.

Most dynamical interactions in dense cluster cores tend to destroy binaries (the possible exception is tidal capture, which may form binaries, but turns out to play a negligible role; see § 5.2). Soft binaries (with orbital speeds lower than the cluster velocity dispersion) can be disrupted easily by any strong encounter with another passing star or binary. Even hard binaries can be destroyed in resonant binary–binary encounters, which typically eject two single stars and leave only one binary remaining (Mikkola 1983), or produce physical stellar collisions and mergers (Bacon et al. 1996; Fregeau et al. 2004).

In addition, many binary stellar evolution processes lead to disruptions (e.g., following a supernova explosion of one of the stars) or mergers (e.g., following a common envelope phase). These evolutionary destruction processes can also be enhanced by dynamics. For example, more common envelope systems form as a result of exchange interactions (Rasio, Pfahl, & Rappaport 2000), and the orbital shrinkage and the development of high eccentricities through hardening encounters may lead to the coalescence of binary components (Hills 1984; Hurley & Shara 2003).

It is therefore natural to ask whether the small binary fractions measured in old globular clusters today result from these many destruction processes, and what the initial binary fraction must have been to explain the current numbers. We address these questions in this paper by performing calculations that combine binary star evolution with a treatment of dynamical interactions in dense cluster cores. In § 3 we describe in detail the method we use, following a brief overview of the theoretical background in § 2. We test our simplified dynamical model by comparing it against full Monte Carlo $N$-body simulations in § 4.1. In § 4.2 we use semi-analytical estimates to predict the upper limit for the final binary fraction in dense clusters. In § 4.3 we estimate the lower limit for the final binary fraction and analyse which mechanisms of binary destruction are most efficient as a function of cluster age. In § 5 we present our numerical results for the evolution of the binary fraction in dense cluster cores, and we compare these results with observations. In particular, using our theoretically predicted period distribution, we re-examine observations of 47 Tuc and re-derive constraints on the core binary fraction. In the final discussion (§ 6), we point out how our results may be helpful in interpreting observations of core binary fractions in other clusters, and we discuss the required initial conditions for simulations of clusters with binaries, as well as which methods are best suited for these simulations.

2 BINARY POPULATION SYNTHESIS WITH DYNAMICS

There are several possible ways to approach the study of binary evolution in dense clusters. The traditional approach is to start from full $N$-body simulations to study the dynamics of the stellar system and introduce on top of this various simplified treatments of single and binary star evolution. This has been used for many years (for recent examples see Portegies Zwart et al. 2001; Shara & Hurley 2002, Hurley & Shara 2003). Unfortunately, even with the fastest special-purpose computers available today, this direct $N$-body approach remains extremely computationally, so that previous studies have been limited to small systems like open clusters and with limited coverage of parameter space. In addition, because binaries are particularly expensive to handle computationally (as they increase enormously the dynamic range of direct $N$-body simulations), these previous studies have also been performed with unrealistically small numbers of binaries. For example, the time required to perform just one direct $N$-body simulation of a cluster containing $2 \times 10^5$ stars with all stars formed initially in binaries would be at least a year on the GRAPE-6, with some dependence on the initial binary parameters.\footnote{J. Hurley, personal communication. The estimate is based on 5 days required to simulate an open cluster of 20000 stars with 2000 binaries on the GRAPE-6 in Shara & Hurley 2003.}

Alternatively, a binary population synthesis code (e.g., Hurley et al. 2002), normally used to evolve large numbers of stars and binaries without dynamical interactions, can be extended by introducing a simple treatment of dynamics. In this type of approach it is often assumed that all the relevant parameters of the cluster (e.g., central density and velocity dispersion) remain constant throughout each dynamical simulation, i.e., the dynamics is assumed to take place in a fixed background cluster. Many previous studies of dense stellar systems have been based on this type of approximation (see, e.g., Hut, McMillan, & Roman 1992; Di Stefano & Rappaport 1994; Sigurdsson & Phinney 1995; Portegies Zwart, Hut, McMillan, & Verbunt 1997; Davies 1995; Davies & Benz 1997; Davies 1997; Rasio, Pfahl, & Rappaport 2000; Smith & Bonnell 2001). This approach, sometimes called “encounter rate technique” (Benacquista 2002), is computationally much less expensive than direct $N$-body simulations and hence allows the systematic exploration of the vast parameter space of initial conditions for clusters and their primordial binary populations. In addition, the use of sufficiently large numbers of stars and binaries makes the simulations more realistic. Although obviously much less accurate in its description of the overall cluster dynamics, this method opens the possibility of studying “star cluster ecology” in considerably greater detail than has been possible with $N$-body simulations. In particular, it makes it possible to study in detail the rare but important evolutionary channels that may play a crucial role in the formation of some of the most interesting tracers of dynamical interactions in dense clusters, such as ultracompact X-ray binaries, millisecond pulsars, and cataclysmic variables (Ivanova & Rasio 2004).

Unfortunately, it is difficult to compare these two approaches, as each is based on a very different set of simplifying assumptions. There are no comprehensive studies of dense stellar systems including a self-consistent treatment of both dynamics and binary star evolution. In many recent $N$-body simulations for large clusters (using either Aarseth-type codes or Hénon’s Monte Carlo method; Aarseth 2001; Fregeau et al. 2003), binary stars are treated in the point mass limit and soft binaries are eliminated from the start. Binary destruction can then occur only through resonant 4-body interactions. However, $N$-body studies of open clusters that incorporate realistic treatments of binary stellar evolution have shown that stellar evolution affects the binaries significantly, and that, even in these low-density environments, the complex interplay between binary evolution and dynamics, even for soft binaries, can play an important role in the overall cluster evolution and in determining the properties of surviving binaries (Hurley & Shara 2003).

The second approach, “binary population synthesis with dynamics”, which we have adopted in this work, suffers from the lack
of self-consistent dynamical evolution of the cluster, which is assumed to remain in a constant state of thermal equilibrium for its entire evolution. This state, where the energy production through “binary burning” in the core is balanced by the outer energy flux into the cluster halo, does indeed provide nearly constant conditions throughout a typical globular cluster’s lifetime. The exception might be “core-collapsed” clusters, which may have run out of binaries and evolved to a much more centrally concentrated state. Typically, the density and “temperature” profiles of a cluster do not change much as long as there are enough binaries remaining to provide support against gravothermal contraction. Stellar interiors provide a useful analogy: as long as a star keeps enough hydrogen to burn in its core, it can remain in thermal equilibrium on the main sequence and avoid core contraction and envelope expansion. Just like main-sequence stars, globular clusters can maintain a nearly constant interior structure for many billion years. For the case of an isolated Plummer model with 10% initial hard binaries they found (see their Fig. 4) that the core radius of this cluster can remain nearly constant (to within a factor of 2) for many tens of half-mass relaxation times (i.e., more than a Hubble time for most clusters initially and tidal truncation, they found that, after about 40 \( t_{rh} \), when the cluster is about to disrupt in the Galactic tidal field, the core radius still has not varied by more than a factor of 2 over the entire evolution (see their Fig. 1); and over the first 10 \( t_{rh} \), the core radius changed by less than \( \sim 20\% \). The central velocity dispersion also does not vary much with time (see, e.g., Giersz & Spurzem 2003, Fig. 1). Similar results have been obtained from direct N-body simulations of open clusters, where the central density and velocity dispersion also remain nearly constant in models with significant numbers of binaries (Hurley & Shara 2003).

3 METHODS AND ASSUMPTIONS

3.1 Cluster Initial Conditions

Our initial conditions are described by the following parameters: total number of stars \( N \) (single or in a binary), initial mass function (IMF), binary fraction \( f_b \), distribution of binary parameters (period, \( P \), eccentricity, \( e \), and mass ratio, \( q = m_2/m_1 < 1 \)). We typically adopt standard choices used in previous population synthesis studies, which are based on available observations for stars in the field and in young star clusters (Sills et al. 2003). For most of the calculations reported here, we use the following “standard” initial conditions:

- We adopt the IMF of Kroupa (2002), which can be written as a broken power law \( dN \propto m^{-\alpha} dm \), where \( \alpha = 0.3 \) for \( m/M_\odot < 0.08 \), \( \alpha = 1.3 \) for \( 0.08 \leq m/M_\odot < 0.5 \), \( \alpha = 2.3 \) for \( m/M_\odot \geq 0.5 \). We assume that all stars are formed in a single burst of star formation at \( t = 0 \) in our simulations.
- We consider a wide range of stellar masses from 0.05 \( M_\odot \) to 100 \( M_\odot \).
- The binary mass ratio, \( q \), is assumed to be distributed uniformly in the range \( 0 < q < 1 \). This is in agreement with observations for \( q \gtrsim 0.2 \) (Woitas et al. 2001).
- The binary period, \( P \), is taken from a uniform distribution in \( \log_{10} P \) over the range \( P = 0.1-10^7 \) d.
- The binary eccentricity, \( e \), follows a thermal distribution with probability density \( p(e) = 2e \).
- We reject systems where binary components would overflow their Roche lobe at pericenter.

The initial average stellar mass is then \( \langle m \rangle \approx 0.5 M_\odot \), and, with the flat mass ratio distribution, the average binary mass is \( \langle m_b \rangle \approx 0.75 M_\odot \).

3.2 Stellar Evolution

We evolve all stars (single and binary) using the population synthesis code StarTrack (Belczynski et al. 2003), which has recently been updated significantly (Belczynski et al. 2005, in preparation). This is the only current population synthesis code that incorporates detailed treatments of all possible types of mass transfer (MT) episodes: stable MT (conservative or non-conservative), unstable MT (thermally or dynamically), and thermal time-scale MT. Also included are the effects of Eddington-limited mass accretion and transient behavior of accretion discs (based on the disc instability model). StarTrack allows us to follow the evolution of binaries with a large range of stellar masses, metallicities, and star formation histories (constant rate, sudden or exponential bursts, etc.). StarTrack also models in detail the loss of mass and angular momentum through stellar winds (dependent on metallicity) and gravitational radiation, asymmetric core collapse events with a realistic spectrum of compact object masses, and the effects of magnetic braking and tidal circularization on close binaries. In our simulations, we adopted the new prescription for magnetic braking given by Ivanova & Taam (2003). Compared to the older prescription (Verbunt & Zwaan 1981) closer binaries lose their angular momentum at a slower rate and hence survive as binaries longer. The evolution of single stars in StarTrack is based on the analytic

\footnote{The lower limit is chosen in order to provide a self-consistent mass-ratio distribution for binaries with primaries down to 0.15 \( M_\odot \).}
fits provided by Hurley et al. 2000, but includes a more realistic determination of compact object masses (Fryer & Kalogera 2001).

We treat the evolution of stellar collision and binary merger products following the general “rejuvenation” prescription of Hurley et al. 2003. It ensures that the merger product has the same total amount of hydrogen, helium, and carbon as the two parent stars together. For some stars, the assumptions made in the treatment of the merger depend on the types of stars and the type of merger (collision vs binary coalescence). For example, we assume that there is no accretion on to a neutron star during a physical collision, and that the other star, if it is unevolved, is destroyed completely (e.g., we do not consider the possible formation of a Thorne-Zytkow object). We treat as a “dynamical common envelope” (CE) event the outcome of a physical collision between a compact object and a red giant, applying a standard “alpha prescription” (where we adopt $\alpha_{CE} = 1$; see Iben & Livio 1993), but taking into account the initial positive energy. In particular, if this compact object is a neutron star, a compact binary containing a neutron star and a white dwarf is formed. We assume that, during the CE phase, the neutron star will accrete a significant amount of the envelope material and will become a millisecond pulsar (Bethe & Brown 1993). If the resulting mass of the neutron star exceeds the limit for a neutron star (taken in our simulations to be $2M_\odot$), we assume that a black hole is formed.

To evolve the cluster population of single stars and binaries in our code, we consider two basic time-scales. One is associated with the evolutionary changes in the stellar population, $\Delta t_{ev}$, and the other with the rate of encounters, $\Delta t_{coll}$ (see § 3.4). The evolution timestep $\Delta t_{ev}$ is computed so that no more than 2% of all stars change their properties (mass and radius) by more than 5%. The global timestep for the cluster evolution is taken to be $\Delta t = \min(\Delta t_{ev}, \Delta t_{dyne})$.

### 3.3 Dynamical Cluster Model

As we described in § 2, our model for the cluster dynamics is highly simplified. We adopt a simple two-zone, core-halo model for the cluster. We assume that the core number density, $n_c$, and one-dimensional velocity dispersion, $\sigma_{1D}$, as well as the half-mass relaxation time, $t_{rh}$, remain strictly constant throughout the evolution. While dense globular clusters of interest have $\sigma_{1D} \sim 10 \, \text{km s}^{-1}$, the core density can vary by several orders of magnitude. Here we set $n_c = 10^5 \, \text{pc}^{-3}$ for most calculations, representative of a fairly dense cluster like 47 Tuc. In general, $n_c$ is the main “knob” that we can turn to increase or decrease the importance of dynamics. Setting $n_c = 0$ corresponds to a traditional population synthesis simulation, where all binaries and single stars evolve in isolation after a single initial burst of star formation. To model a specific cluster, we match its core mass density today, $\rho_{c,\text{obs}}$, central velocity dispersion, and half-mass relaxation time.

The escape speed from the cluster core can be estimated from observations as $v_e \approx 2.5 \, \sigma_{3D}$ (Webbink 1983), where $\sigma_{3D}$ is the three-dimensional central velocity dispersion. Following an interaction or a supernova explosion, any object that has acquired a recoil speed exceeding $v_e$ is removed from the simulation. Acquiring a large recoil velocity in a dynamical encounter is a very efficient mechanism for ejecting low-mass objects from the cluster. We find generally that recoil to the halo does not play a significant role: the recoil velocity into the halo differs by $\sim 10\%$ from the escape velocity from the cluster, and affects only a small number of objects.

For computing interactions in the core, the velocities of all objects are assumed to be distributed according to a lowered Maxwellian (King 1963), with

$$f(v) = v^3/\sigma^2(m) \left[ \exp(-1.5v^2/\sigma^2(m)) - \exp(-1.5v_e^2/\sigma^2(m)) \right],$$

(1)

where $\sigma(m) = \langle m_c/m \rangle^{1/2} \sigma_{3D}$ (assuming energy equipartition in the core) and $v_e$ is the escape speed. Here $\langle m_c \rangle$ is the average mass of an object in the core. In addition, we use $\sigma$ to impose a cut-off for soft binaries entering the core: Any binary with maximum orbital speed $< 0.1 \sigma_{3D}$ is immediately broken into two single stars (Hills 1994).

In the presence of a broad mass spectrum, the cluster core is always dominated by the most massive objects in the cluster, which tend to concentrate there via mass segregation. As stars evolve, the composition of the core will therefore change significantly over time. Mass segregation in globular clusters was investigated recently in Fregeau et al. 2003, by considering both light and heavy tracers in two-component models. It was found that the characteristic mass-segregation time-scale is given by

$$t_{sc} \approx 10 \, C \langle m \rangle_h/m \, t_{rh}.$$

(2)

Here $C$ is a constant of order unity and $\langle m \rangle_h$ is the average mass of an object in the halo, and $m$ is the current mass of the object. This equation represents a diffusion process and can be applied to all stars, not just to those more massive than average. Indeed, even low-mass objects may (rarely) diffuse into the cluster core on a long time-scale, although on average they will tend to drift outwards. However, for very light objects with masses $\lsim 0.4 \langle m \rangle_h$ (which is typically $\sim 0.3 M_\odot$ at the beginning and $\sim 0.15 M_\odot$ after $\sim 10$ Gyr), this expression becomes less accurate, although it remains valid qualitatively. For a more recent discussion of mass segregation in the presence of a broad mass spectrum, see Girkan, Freitag, & Rasio 2004.

To model mass segregation in our simulations, we adopted the time-scale given by equation 2, but treated the process as stochastic: the probability for an object of mass $m$ to enter the core after a time $t_s$ is sampled from a Poisson distribution,

$$p(t_s) = (1/t_{sc}) \exp(-t_s/t_{sc}).$$

(3)

This treatment ensures that all stars heavier than $\sim 0.4 \langle m \rangle_h$ diffusing into the core will have the appropriate mass spectrum and that interactions will occur between objects drawn from the correct distribution.

Eq. 2 was derived for the restricted case of a two-component cluster – without a realistic IMF – therefore $C$ is unknown by a factor of a few. We find in simulations that the final core mass is nearly proportional to $1/C\langle m \rangle_h$. In order to obtain a better fit to observations for the core mass versus total cluster mass relation, we have fine-tuned eq. 2 using data for 47 Tuc, in particular the ratio of the core mass to the total mass of the cluster. For this cluster we adopted a core mass of $10^3 M_\odot$ and a total cluster mass of $10^6 M_\odot$ at present (Freire et al. 2001); we also take $t_{rh} = 10^7$ yr (Hills 1996) and an age of 11 Gyr (Gratton et al. 2003). While the core mass can be found directly from our simulations, the total cluster mass has an uncertainty due to the IMF cut-off at the low mass end in our standard cluster model. First, we found the total mass of a cluster model evolved to 11 Gyr and the initial number of very massive stars (defined as those producing a black hole at the end of their stellar evolution). We find that at 11 Gyr, the cluster has $145 M_\odot$ per black hole (or per heavy primordial star), when the IMF extends down to $0.01 M_\odot$. This allows us to normalize our model to the real cluster mass: with this ratio we have an estimate for the total cluster mass when we use a higher cut-off for the IMF.
(0.05 \ M_\odot). This now gives us the ratio of the core mass to the total cluster mass corresponding to our simulations. We find that the best fit for 47 Tuc gives \( C(m)_{\text{n}} = 3\ M_\odot \).

3.4 Treatment of Dynamical Interactions

All objects in our simulations are allowed to have dynamical interactions only after they have entered the cluster core. We use a simple Monte Carlo prescription to decide which pair of objects actually have an interaction during each timestep.

The cross section for an encounter between two objects, of masses \( m_i \) and \( m_j \), with relative velocity at infinity \( v_{iJ} \), is computed as

\[
S_{iJ} = \pi d_{\text{max}}^2 \left( 1 + v^2_{p}/v^2_{iJ} \right),
\]

where \( d_{\text{max}} \) is the maximum distance of closest approach that defines a significant encounter and \( v^2_{p} = 2G(m_i + m_j)/d_{\text{max}} \) is the velocity at pericenter. Here the index \( i \) (lowercase) reflects an individual object in the core, while \( J \) (uppercase) denotes a random representative object from the subclass of objects \( J \). In order to more accurately determine encounter rates, at each timestep binaries and single stars in the core are divided into 100 subclasses: 10 by size (radius for single stars or semimajor axis for binaries) and 10 by mass. Boundaries between mass subclasses are fixed approximately as 0.1 \times 2^n. Subclasses by size depend on the current sizes of single and binary populations (separately), with the step between subclasses taken as \( \delta \log_{10} R_{\text{min}} = 0.1(\log_{10} R_{\text{max}} - \log_{10} R_{\text{min}}) \). The encounter rate for a given object \( i \) and an object from subclass \( J \) is \( \Gamma_{iJ} = n_j S_{iJ} v_{iJ} \), where \( n_j \) is the number density of objects in subclass \( J \), and the cross section and relative velocity are defined for an average object in subclass \( J \).

The total interaction rate for a given object \( i \) is the sum of the interaction rates with all relevant subclasses, \( \Gamma_i = \sum_j n_j S_{iJ} v_{iJ} \). The corresponding interaction time is \( \tau_i = 1/\Gamma_i \). The actual time for an encounter \( t_i \) follows a Poisson distribution with mean \( \tau_i \). In practice, we generate a random number \( 0 < X < 1 \), and assume that the encounter happened if \( t_i = X/\tau_i < \Delta t \). The timestep is limited so that \( \Delta t_{\text{syn}} < 0.25 \ \text{min} \ \tau_i \). We keep track separately of the time-scales \( \tau_{iJ} \) for interactions with each subclass \( J \), and the corresponding \( t_{iJ} = X/\tau_{iJ} \) is generated from an independent random number. If an encounter happened, it is assumed to be with an object from the subclass with the smallest \( t_{iJ} \). The actual interacting object \( J \) from that subclass \( J \) is randomly selected from the list of non-interacted objects in that subclass.

In this paper we consider separately binary–binary, binary–single, and single–single interactions. The cross sections and rates are calculated using \( d_{\text{max}} = 5(R_i + (R_J)) \) for single–single, \( d_{\text{max}} = 3(b_i + (R_J)) \) for binary–single, and \( d_{\text{max}} = 3(b_i + (b_J)) \) for binary–binary.

For a total cluster mass of \( 1 \ M_\odot \) and \( \sigma_c = 10 \ \text{km/s} \). For a total cluster mass of \( 10^5 \ M_\odot \), these conditions imply \( t_{\text{sh}} = 8 \times 10^5 \ \text{yr} \) and \( n_c = 2000 \ \text{pc}^{-3} \). We evolved the cluster for 20 \( t_{\text{sh}} \).

In Fig. \ref{fig:4} we show our results for the core and halo binary fractions as a function of time, compared with the model of F03. The agreement with F03 is excellent: the core binary fraction rises very quickly to \( \sim 35\% \) and then remains close to this value for \( \sim 10t_{\text{sh}} \). In contrast, the halo binary fraction decreases more gradually from \( 20\% \) to \( 10\% \). Considering the differences between the two treatments, especially for binary interactions, this agreement is quite remarkable.

In the bottom panel of Fig. \ref{fig:4} we show the evolution of the core radius \( r_c \). The core radius evolves in the same way as in the dynamical simulation of F03. Overall, the core radius does not change much during the entire evolution and its value is consistent with the measured values for observed globular clusters with similar parameters (e.g., NGC 3201 or NGC 6254, in which the total cluster mass, the central number density, and the central velocity dispersion are similar to those in our model).

4 TEST CALCULATIONS AND SIMPLE ESTIMATES

4.1 Comparison with N-body Simulations

We have compared our simple dynamical model to recent results from fully self-consistent Monte Carlo simulations based on Hénon’s algorithm for solving the Fokker-Planck equation. Joshi, Rasio, & Portegies Zwart (2000); Joshi, Nave, & Rasio (2000). For our test we used the results obtained for an idealized model cluster containing 20% primordial hard binaries (binding energies in the range \( 1 < kT < 133 \ kT \), where \( kT \) is the average kinetic energy of an object in the cluster) for a King model with dimensionless central potential \( W_0 = 7 \) (Fregeau et al. 2003; hereafter F03). In this simulation all stars had equal masses, were treated as point masses (no physical collisions), no stellar evolution was taken into account, and all binary interactions were treated using simple recipes.

We used our code to perform a similar simulation: we considered a cluster consisting of equal-mass stars, and we turned off all stellar evolution and physical collisions. To fit the F03 model, we took the core mass as 8.3% of the total cluster mass (corresponding to a King model with \( W_0 = 7 \)), and we took an average star mass of \( 1 \ M_\odot \) and \( \sigma_c = 10 \ \text{km/s} \). For a total cluster mass of \( 10^5 \ M_\odot \), these conditions imply \( t_{\text{sh}} = 8 \times 10^5 \ \text{yr} \) and \( n_c = 2000 \ \text{pc}^{-3} \). We evolved the cluster for 20 \( t_{\text{sh}} \).

4.2 Semi-analytic Estimates

One can estimate the final binary fraction \( f_{b,i} \) in a dense environment by considering several mechanisms of binary destruction:

- all soft binaries are usually destroyed during a strong encounter.
- some fraction of hard binaries is destroyed through stellar evolution (mergers or disruptions after supernova explosions)
- when a hard binary has a strong encounter with another hard binary or a single star, it can exchange its less massive component

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gravitational dynamics that provides automatic calculation termination and classification of outcomes (for a detailed description see Fregeau et al. 2004). Fewbody numerically integrates the orbits of small-\(N\) systems, and performs collisions in the sticky-star approximation. Fewbody's ability to automatically classify and terminate calculations as soon as the outcome is unambiguous makes it well-suited for carrying out large sets of binary interactions, for which calculations must be as computationally efficient as possible.
for a more massive star, shrink its orbit, or be destroyed in a collisional merger.

It should be noted that in our simplified semi-analytical treatment we neglect the effect of mass segregation, which tends to increase the core binary fraction. (This issue is discussed in more detail in Section 5.2.)

Let us consider a dense environment with number density $10^5$ pc$^{-3}$, $\sigma_{1D} = 10$ km/s and with an average mass of 0.5 $M_\odot$. With our choices of initial parameters, and an average mass of 0.5 $M_\odot$, 40% of all primordial binaries initially are soft (this fraction would be 50% with respect to the average mass of 1 $M_\odot$). We introduce $\eta$ — the hardness of a binary system — as

$$\eta = \frac{G m_1 m_2}{a \sigma^2 \langle m \rangle},$$

where $a$ is the binary separation, $m_1$ and $m_2$ are the masses of the binary components, and $\langle m \rangle$ is the average mass of a single star. Binaries that have $\eta < 1$ are termed soft, and those with $\eta > 1$ are termed hard.

To find how many hard binaries will be destroyed by stellar evolution alone, we calculated the probability of binary destruction as a function of its initial total mass and orbital period, using the binary population synthesis code (see Fig. 1). This simulation was done with $1.25 \times 10^7$ binaries distributed initially flat in $\log P$ and $\log M_{tot}$ (in order to have better resolution for destruction rates for high mass binaries), where $P_d$ is the binary period in days and $M_{tot} = M_1 + M_2$ is the total binary mass in $M_\odot$, and was evolved for 14 Gyr.

The result is striking: most of the very hard binaries, with hardness ratios $\eta \gtrsim 100$, are destroyed by stellar evolution. The empty space near the bottom right corner of Fig. 2 reflects the absence of systems below the minimum period for binaries on the main sequence (the period at which stars come into contact). For binaries with total mass $\gtrsim 10 M_\odot$, destructions mainly occur during the first $10^5$ years of cluster evolution. Binaries with period $\gtrsim 10^3$ days are mainly destroyed through SN explosions. Binaries with period $\lesssim 10$ days are destroyed mainly via mergers at the MS stage. For periods $10^3$–$10^4$ days the destructions are associated with common envelope evolution and occur at later times. Destinations in binaries of smaller masses are not much different from more massive binaries at small periods, but are not destroyed through SN explosions at large periods. Also, the CE event in less massive binaries occurs when the donor is a less evolved giant (at the first red giant branch).

One may expect that destruction rates should vary smoothly; however, binary evolution involves many qualitatively different events. In particular, an interesting fluctuation in destruction rates can be seen at $\log P \sim 1.75$ and $\log M_{tot} \sim 0.95$, where local destruction rates are lower than for nearby binaries. For these and nearby binaries the destruction rates are about the same for mass ratio $q \lesssim 0.5$ but different for larger $q$. For binaries of masses close to and smaller than these, most destructions for $q \gtrsim 0.5$ occur during the CE event between a WD and a giant. This CE phase is the second interaction in the binary and follows the stable MT event with the other donor. When the total binary mass is smaller, the WD mass is $\lesssim 0.9 M_\odot$, and CE event leads to a merger. For binaries of higher total mass, the second interaction occurs between a He star or a WD and a star at the Hertzsprung gap. This MT is unstable and leads to a merger. This, therefore, provides for a small local decrease in destruction rates.

With our IMF and considering binaries of all masses, the fraction of hard binaries destroyed during their evolution is 20%.

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**Figure 1.** Evolution of the core and halo binary fractions (top) and the core radius (bottom) in our test model, compared with the F03 model (see text). In the top panel, the solid line shows the binary fraction in the core and the dashed line shows the binary fraction in the halo, both for the test model. The dotted line shows the binary fraction in the core in the F03 model and the dash-dotted line shows the binary fraction in the halo in F03 model. In the bottom panel the solid line shows the core radius in the test model and the dotted line shows the core radius in the F03 model.

**Figure 2.** Destruction of primordial binaries by stellar evolution, shown in the parameter space of total initial binary mass and initial binary period. Solid lines are lines of constant binary hardness and dashed lines are lines of constant collision time.
Among binaries where at least one star is more massive than 0.5 $M_\odot$, the destruction fraction is 50%, and for those with total initial mass above 1 $M_\odot$, this fraction is closer to 60%. The fraction of hard binaries that is not destroyed but is instead softened by evolution is very small, $\lesssim 1\%$. In §4.3 we will discuss in more detail how binary destruction rates change with time.

Therefore, even before any dynamical processes are considered for the hard binary population, we estimate that the final binary fraction cannot be higher than about 50%, and for binaries with at least one star more massive than 0.5 $M_\odot$ this upper limit becomes 30%. For relatively massive binaries, with total initial mass above 1 $M_\odot$, the upper limit for $f_{b,c}$ is only 24%. Overall, this estimate already shows that (i) the expected final binary fraction in a dense star cluster will be low and (ii) stellar evolution cannot be neglected when estimating binary fractions from dynamical models of dense star clusters.

Let us now consider the effects of dynamical interactions. The time-scale for a binary to undergo a strong encounter with another single star, the collision time, can be estimated as $\tau_{\text{coll}} = 1/\eta v_\infty$. Assuming that the strong encounter occurs when the distance of closest approach $d_{\text{max}} \leq k a$ with $k \approx 2$, we obtain

$$
\tau_{\text{coll}} = 3.4 \times 10^{13} \text{ yr} \, k^{-2} \rho_{\text{d}}^{-4/3} M_{\text{tot}}^{-2/3} n_5^{-1} v_{10}^{-1} \times \left(1 + 915 \frac{\langle M \rangle}{k n_2^{2/3} M_{\text{tot}}^{1/3} v_{10}} \right)^{-1} \tag{6}
$$

Here $\langle M \rangle$ is the mass of an average single star in $M_\odot$, $v_{10} = v_\infty/(10 \text{ km/s})$, where $v_\infty$ is the relative velocity at infinity and $n_5 = n/(10^5 \text{ pc}^{-3})$, where $n$ is the number density. Using eq. (5), eq. (6) can be rewritten in a more convenient form:

$$
\tau_{\text{coll}} = 1.7 \times 10^{10} \text{ yr} \, \eta^{-2} k^{-2} n_5^{-1} \frac{\langle M \rangle^2}{M_1^2 M_2^2} \left(1 + \eta^{-2} \frac{M_{\text{tot}} + \langle M \rangle}{M_1} \frac{\langle M \rangle}{M_2} \right)^{-1} \tag{7}
$$

It can be seen from eq. (7) that the collision time for a binary with $\eta = 1$ and $M_{\text{tot}} = 1 M_\odot$ is only $\sim 10^8$ yr. Overall, with our IMF, $\sim 50\%$ of all hard binaries have collision times shorter than $\sim 10$ Gyr (see also Fig. 3), and 15% have collision times as short as $\sim 1$ Gyr. In addition, most of the hard binaries that could have experienced an encounter are binaries with $\eta = 1-100$, and therefore they are from the population that is destroyed by stellar evolution to a lesser degree than binaries harder than $\eta = 100$. While this estimate considered only binary–single encounters, binary–binary encounters will further enhance the destruction rate. Moreover, when the binary fraction is higher than $\sim 25\%$, binary–binary encounters dominate over binary–single encounters.

Each binary–single encounter with a hard binary can result in a hardening of this binary, an exchange of a companion with a more massive single star, or binary destruction in a physical collision. The probability of a physical collision during an encounter increases strongly as the binary becomes harder (Fregeau et al. 2004). In addition, a very hard binary can be ejected from the core if its recoil velocity exceeds the escape speed from the cluster. Each of these three processes, directly or indirectly, leads to the depletion of binaries (immediate or delayed): acquiring a more massive companion, as well as orbital shrinkage or the eccentricity increase, makes systems more likely be destroyed through stellar evolution. As a conservative lower limit, we assume that half of the interacting hard binaries will be destroyed as a result of an encounter (immediately or later). This is clearly a lower limit, as scattering experiments show that, for hard binaries containing main-sequence stars, most encounters will lead to a physical collision (Fregeau et al. 2004).

Taking everything into account, we conclude that 64% of all binaries will be destroyed – only $k_n = 36\%$ of primordial binaries can survive to the present. The expected final binary fraction is therefore $f_b = N_b/(N_s + N_b) = k_b/(2 - k_n) = 22\%$ (for stars in the entire mass range), and it is 13% ($k_b = 23\%$) for binaries with at least one star more massive than 0.5 $M_\odot$, while for $M_{\text{tot}} \geq 1 M_\odot$ it is only 10% ($k_b = 18\%$). We note again that this upper limit for the final binary fraction does not take into account several other mechanisms leading to binary destruction, such as binary–binary encounters, which are more likely to cause binary destruction than binary–single encounters.

### 4.3 Encounters in Dense Environments

In order to verify our encounter rates, we considered the evolution of a binary population that is completely immersed into a dense environment with $n_e = 10^5 \text{ pc}^{-3}$ for its entire evolution. We adopted a velocity dispersion $\sigma_{1D} = 10 \text{ km/s}$ and evolved the system for 14 Gyr. This is an even simpler model of a star cluster, where all objects are effectively inside the “core” at all times and the effects of mass segregation and diffusion between a core and a halo are completely ignored. It allows us to study more clearly the various types of dynamical interactions as separate processes. We ran a sequence of simulations, with (i) only binary–single encounters, with no physical collisions allowed during an encounter; (ii) binary–single encounters with physical collisions; (iii) both binary–single and binary–binary encounters but still without physical collisions; (iv) binary–single and binary–binary encounters with physical collisions; and (v) all types of encounters allowed, including single–single collisions. In this last case we also eliminated from the sys-

![Figure 3](image-url) Binary destruction rates as a function of time for a field population (i.e., without dynamical interactions). Rates are given as numbers of events per binary per Gyr for mergers and disruptions (following a supernova explosion). Note the peak in evolutionary disruptions at $t \sim 10^7-10^8$ yr, which is due to supernovae.
tem any object that acquired a recoil speed exceeding the escape velocity \(v_r = 2.5 \sigma_{3D}\).

In § 4.2 we estimated that, with only binary–single encounters taken into account, the final binary fraction should not exceed 22%. This assumed that all objects in the core had a mass equal to the average mass 0.5 \(M_\odot\). With the complete IMF, a single star participating in the encounter can have mass higher than 0.5 \(M_\odot\). Also, a fraction of initially hard binaries can become soft during the evolution, e.g., because of mass loss. As a result, we obtain even lower remaining binary fractions: \(f_{b,c} = 16.5\%\) for case (i) and \(f_{b,c} = 16.4\%\) for case (ii). These numbers are in agreement with the upper limits we estimated in §4.

When binary–binary encounters are taken into account, the result is even more dramatic: we obtain \(f_{b,c} = 8.0\%\) and \(f_{b,c} = 7.9\%\) for cases (iii) and (iv), respectively. As expected, we see that binary–binary encounters further enhance the binary destruction rate.

When all dynamical processes are taken into account, in case (v), we obtain a final binary fraction \(f_{b,c} = 6.9\%\). While our semi-analytical estimate of the binary fraction provided an upper limit, the result obtained in this section is clearly a lower limit: in a real cluster, not all binaries will be exposed to the high interaction rates of the core at all times. However, since more massive objects are more likely to diffuse into the core, and since their destruction rate can be higher than for objects of average mass, this lower limit is only approximate, but we may expect it to be much closer to the real value than our previous, conservative upper limit.

Let us now consider in detail which mechanisms of binary destruction are most important. In Fig. 5 we show the rates of binary destruction (events per binary per Gyr) for a field population (with no interactions). Except for the interval between \(\sim 10^7\) and \(\sim 10^8\) years, when black holes and neutron stars are formed, the binary destructions are mainly coming from mergers. The power-law behavior observed for the overall destruction rate at late times is on the one hand imposed by the rate of orbital decay driven by magnetic braking, and on the other hand depends on the evolutionary time-scale for the companion to become a red giant.

In Fig. 5 we show the binary destruction rates again, but for the dense environment. During the first few Gyr the main destruction mechanism is the dynamical disruption of soft binaries. The rate of disruption through stellar evolution is smaller than in the field population: some massive binaries are destroyed by dynamical encounters before they evolve to a supernova explosion. However, the rate of evolutionary mergers is about the same as for the field for the first \(\sim 10^6\) years. This is consistent with our estimate for the collision time of hard binaries: at this time, hard binaries have not yet been hardened significantly by dynamical encounters. After \(\sim 10^6\) yr the rate of binary evolutionary mergers is increased compared to the field population, as the dynamical hardening of hard binaries has now started. The rates for binary destruction through mergers and physical collisions are very similar: in most cases, a binary is tight enough for an evolutionary merger, the most likely outcome of a dynamical interaction is a physical collision.

At about 5 Gyr, the rate of binary destruction through physical collisions starts dominating over dynamical disruptions: at this stage, almost no soft binaries are left, and the hard binaries are more likely to lead to physical collisions during an encounter. Consider a binary with \(\eta \sim 1\) at this stage. From equation (4) we can find that a collision time of 5 Gyr corresponds typically to binary components of 0.1 \(M_\odot\) and 0.05 \(M_\odot\) (assuming an average mass ratio of 0.5 and average field mass as 0.5 \(M_\odot\)). This binary is clearly at the lowest-mass end of our IMF, and therefore by this time almost all soft binaries in the simulation have been destroyed. On the other hand, consider some more massive binary, which has evolved through about half of its main-sequence lifetime, and has component masses of 0.8 \(M_\odot\) and 0.4 \(M_\odot\) (and still the same collision time of 5 Gyr). The hardness of such a binary is \(\eta \approx 100\), corresponding to a binary separation \(\approx 10 R_\odot\). For such a tight binary, the probability of a physical collision in an encounter is almost 100%. The total rate of binary destructions through physical collisions become comparable to that of dynamical disruptions, as can be seen from Fig. 5.

5 RESULTS

5.1 Overview of Cluster Models

Initial conditions for all our models are given in Table 1. The first group of models is used to cover the parameter space of initial conditions over fairly wide ranges. Our main reference model, Model 1, has core density \(n_c = 10^5\) pc\(^{-3}\), half-mass relaxation time \(t_{rh} = 10^9\) yr, initial binary fraction \(f_{b,0} = 1\), central velocity dispersion \(\sigma_{1D} = 10\) km s\(^{-1}\), and central escape speed \(v_e = 2.5\sigma_{3D} = 43\) km s\(^{-1}\). We then consider three models with different central densities (D3, D4, and D6), two with different half-mass relaxation time (T8 and T10) and one with an initial binary fraction decreased to 50% (B05). The initial total number of stars is \(N = 2.5 \times 10^5\) for all models, except for the “47 Tuc” model (see below), where we used \(N = 5 \times 10^5\). In all these models the cluster core was assumed to contain about 1% of the stars initially, and the metallicity was fixed at \(Z = 0.001\) (these two parameters have very little influence on our results).

We performed several simulations with parameters that attempt to match those of specific globular clusters in the Galaxy (the bottom part of Table 1). All these clusters are classified observationally as “non-core-collapsed”, meaning that they are well
Table 1. Initial Conditions for All Models

| Model | log \( n_c \) | log \( \rho^b_M \) | log \( t_{1/2} \) | \( f_{b,0} \) | \( \sigma_{1D} \) | \( v_v \) |
|-------|---------------|----------------|----------------|---------|----------------|---------|
| 1     | 5.0           | 9.0            | 1.0            | 10.0    | 43.0           |         |
| D3    | 3.0           | 9.0            | 1.0            | 10.0    | 43.0           |         |
| D4    | 4.0           | 9.0            | 1.0            | 10.0    | 43.0           |         |
| D6    | 6.0           | 9.0            | 1.0            | 10.0    | 43.0           |         |
| T8    | 5.0           | 8.0            | 1.0            | 10.0    | 43.0           |         |
| T10   | 5.0           | 10.0           | 1.0            | 10.0    | 43.0           |         |
| B05   | 5.0           | 9.0            | 0.5            | 10.0    | 43.0           |         |
| M12   | 3.8           | 3.5\(^a\)      | 9.02           | 1.0     | 4.5            | 19.6    |
| M4    | 4.4           | 4.1            | 8.82           | 1.0     | 4.2            | 20.3    |
| 47 Tuc | 5.3          | 5.1            | 9.48           | 1.0     | 11.5           | 56.8    |
| NGC 6388 | 5.9    | 5.7            | 9.08           | 1.0     | 18.9           | 78.2    |

Notations: \( n_c \) is the core number density in \( pc^{-3} \) (assumed fixed), \( \rho^b_M \) is the observed core mass density in \( M_\odot pc^{-3} \), \( t_{1/2} \) is the half-mass relaxation time in yr, \( f_{b,0} \) is the initial binary fraction, \( \sigma_{1D} \) is the 1-D velocity dispersion in \( km s^{-1} \), and \( v_v \) is the escape speed in \( km s^{-1} \).

\(^a\) \( t_{1/2} \) for specific clusters are taken from Harris (1996). \( \rho^b_M \) and \( \sigma_{1D} \) from Pryor & Meylan (1992), and \( v_v \) from Webbink (1985).

Table 2. Results for Reference Models

| Model | log \( \rho_M \) | \( f_{b,c} \) | \( f_{0.5} \) | \( f_{wd} \) |
|-------|----------------|-------------|-------------|-------------|
| 1     | 4.7            | 0.095       | 0.15        | 0.080       |
| D3    | 2.7            | 0.265       | 0.37        | 0.165       |
| D4    | 3.7            | 0.170       | 0.25        | 0.115       |
| D6    | 5.7            | 0.035       | 0.055       | 0.055       |
| T8    | 4.5            | 0.11        | 0.13        | 0.085       |
| T10   | 4.8            | 0.055       | 0.07        | 0.055       |
| B05   | 4.7            | 0.072       | 0.010       | 0.055       |

Here \( \rho_M \) is the core mass density in \( M_\odot pc^{-3} \), \( f_{b,c} \) is the binary fraction in the core, \( f_{0.5} \) is the binary fraction for non-degenerate stars more massive than \( 0.5 M_\odot \), and \( f_{wd} \) is the binary fraction among white dwarfs. Values for all quantities are given at 14 Gyr.

Model 1 is fitted by standard King models. These are precisely the kinds of clusters that, theoretically, we expect to be in the “binary burning” thermal equilibrium state. For this set, we tried to consider the maximum range of dynamical parameters, while concentrating on clusters at relatively small distances and/or very well studied observationally, so that current or future observations could test our predictions. For our most important model, 47 Tuc, we also considered additional models in which we varied the initial binary fraction or introduced a time-dependent core density (See § 5.5). For all our models of specific observed globular clusters, the central number density was chosen in order to provide (at the actual age of the cluster, \( \sim 11 - 13 \) Gyr) the best fit to the observed mass density \( \rho^b_M \) (Pryor & Meylan 1992). Metallicities for these models are taken from Harris (1996).

5.2 Main Reference Model

First we present our results for a typical dense cluster, represented by Model 1. With 100% binaries initially, the final core binary fraction (at 14 Gyr), \( f_{b,c} \), is only 9.5%. This is strikingly low, given that the cluster started with all stars in binaries, and that binaries should concentrate more into the core through mass segregation, but it is expected from our estimates in § 4. Decreasing the initial binary fraction, \( f_{b,0} \), to a more reasonable (but still large) 50% reduces \( f_{b,c} \) further to 7%, as shown in Model B05 (see Table 2). The dependence of \( f_{b,c} \) on \( f_{b,0} \) is not linear. This is mainly due to mass segregation: decreasing \( f_{b,0} \) also increases the ratio of mean binary mass to mean stellar mass in the cluster, thereby resulting in a higher concentration of binaries in the core.

The majority (about 75%) of destroyed binaries were disrupted by close dynamical encounters (or, rarely, following a supernova explosion). Note that some binaries that are initially hard eventually become soft after undergoing significant mass loss due to stellar evolution. About 20% of the destroyed binaries experienced mergers, typically after significant hardening through interactions. A few percent of the binaries lost were actually not destroyed but instead were ejected from the cluster as a result of recoil in strong encounters. Tidal capture did not play a significant role: the total number of tidal-capture binaries formed during the cluster lifetime is less than 1% of the final number of binaries in the core. The total core mass during most of the cluster evolution (the last \( \sim 10 t_{1/2} \)) did not vary by more than a factor of two.

While the final core binary fraction is extremely low, the overall cluster binary fraction, which takes into account all halo binaries, remains high. However, the halo binaries consist mainly of very low-mass systems: the average primary mass among binaries remaining outside the core at 14 Gyr is 0.2 \( M_\odot \), with the average companion mass around 0.1 \( M_\odot \). These binaries would be extremely faint and hard to detect observationally.

We have also examined different stellar subpopulations in the cluster: (i) the subpopulation of non-degenerate objects with mass (for a single star or for the primary in a binary) \( \geq 0.5 M_\odot \) and (ii) the subpopulation of all white dwarfs, single or in binaries. The binaries in group (i) may be easier to detect, e.g., from broadening of the main sequence in a colour-magnitude diagram.
5.3 Different Densities and Relaxation Times

Let us now compare results for different central densities, in Models 1, D3, D4, and D6. The evolution of \( f_{b,c} \) for these models is shown in Figure 6. As expected, the core binary fraction decreases as \( n_c \) increases. The dependence is steeper at high densities, where dynamical interaction rates play a more dominant role compared to stellar evolution.

With respect to the half-mass relaxation time (Models T8 and T10), we find that, surprisingly, the model with shorter relaxation time has a higher core binary fraction. There are two competing mechanisms that play a role here: mass segregation, which brings binaries into the core, and dynamical interactions, which destroy binaries in the core. A shorter segregation time increases the rate at which binaries enter the core but also allows less massive binaries to interact. Therefore, the average mass of a binary in the core and the effective mass density in the core is smaller. As a result, fewer binaries are destroyed.

5.4 Binary Period Distribution

Through dynamical interactions, we would expect that the initial period distribution of binaries should be depleted above the boundary between hard and soft binaries. Stellar evolution should deplete a fraction of hard binaries, especially at very short periods, and dynamical encounters should further deplete some of the wider hard binaries. Indeed, for lower-density clusters, we find that the distribution remains much flatter in \( \log P \) (see Fig. 7), with more and more of the wider hard binaries disappearing as the density increases.

This period distribution can be used to better extrapolate observed binary fractions, which are usually limited to rather narrow ranges of masses and periods. In particular, it is clear that the usual assumption of a flat distribution in \( \log P \) for hard binaries at present in a cluster core (e.g., \( \text{Albrow et al. 2001} \)) can be very misleading. This will be done in § 5.5 for our models of several real clusters.

5.5 Comparison with Observations

We performed several simulations with parameters that attempt to match those of specific globular clusters in the Galaxy (Tables 1 and 2). For these models, the total core masses we compute at present match well those of King models fitted to the observed cluster parameters, and the corresponding core radii of our models (from total core mass and density) are all \( \sim 0.5 \) pc, matching the
The middle plot shows the period distribution of binaries with at least one white-dwarf component (in the period range from 2 days to 3 years) is \( f_{b,c} \approx 15\% \). This is in good agreement with our overall predicted \( f_{b,c} = 11.5\% \), and also with our prediction for heavier main-sequence binaries, \( f_{0.5} = 17.5\% \).

In the 47 Tuc model (for which we increased the total number of stars initially to a more realistic \( N = 5 \times 10^5 \)), \( f_{b,c} \) is only 8% at an age of 11 Gyr and about 7% at 14 Gyr (we provide this value at different ages given the uncertainty in observationally determined values for the ages of 47 Tuc; see, e.g., Schiavon et al. 2002; Zoccali et al. 2001; also note that \( f_{b,c} \) does not change much over the last several Gyr). At first glance, this may seem to conflict with observations. In particular, Albrow et al. 2001 derive an overall binary fraction for the core of 47 Tuc of about 13%, from observations of eclipsing binaries with periods in the range \( P \approx 4–16 \) d. This estimate was based on an extrapolation assuming a period distribution flat in \( \log P \) from about 2.5 d to 50 yr (soft primordial binaries with \( P > 50 \) yr are assumed destroyed and short-period primordial binaries with \( P \leq 4 \) d are assumed to evolve toward much shorter periods through angular momentum loss by magnetic braking). In Figure 2 we show the final period distribution of core binaries in our simulation. Note that the period range of eclipsing binaries is near the peak of the distribution, while for longer periods the number of binaries drops rapidly. In particular, if we concentrate on the binaries consistent with the observed set, with primary masses \( M_1 > 0.25 M_\odot \) and \( q > 0.3 \), we find that the number of systems with periods in the range 16 d to 50 yr is about 6 times smaller than would be predicted by adopting a distribution flat in \( \log P \). For the whole period range from 2.5 d to 50 yr we have 2.2 times fewer binary systems compared to the flat distribution. If we take into account this depletion by wider binaries when modelling the number of observed eclipsing binaries in 47 Tuc, we are led to revise the observed core binary fraction from \( f_{b,c} \approx 6\% \) to \( 6\% \pm 2\)%, which is much closer to our theoretical prediction.

We performed three additional simulations for 47 Tuc, with \( f_{b,0} = 0.25, 0.5 \), and 0.75. The corresponding core binary fraction extrapolated from observations (corrected as above) does not vary much among these different models. Its maximum value is obtained for \( f_{b,0} = 0.5 \), which gives a core binary fraction of about 8%, and in this case the total number of binary systems is 1.6 times less than with an assumed flat distribution.

An alternative estimate of the binary fraction in the core of 47 Tuc is based on observations of BY Dra stars [Albrow et al. 2001]. Their estimated core binary fraction, which can be considered a lower limit, is approximately 0.8%, 18 times lower than the estimate based on eclipsing binaries. This estimate was based on 31 BY Dra binaries (observed in the period range 0.4–10 d) and 5 eclipsing binaries (period range 4–16 d). We analysed the core binary population in our model in order to identify BY Dra systems and eclipsing binaries, and considering the ratio of the two. We adopted the standard definition for a BY Dra binary: primary mass in the range \( 0.3–0.7 M_\odot \) (see, e.g., [Bopp & Fekel 1977]) and period in the range 0.4–10 d (as for the observed sample in 47 Tuc). For eclipsing binaries we adopted the observed period range 4–16 d with each star \( \geq 0.25 M_\odot \). The ratio between the number of observed values. In all cases the initial binary fraction is assumed to be 100%, so our results for final core binary fractions represent upper limits. As in all reference models, we predict low values for \( f_{b,c} \) in all globular clusters, with smaller values in higher-density cores.

We compare our derived binary fraction for M4 with the results from [Cote & Fischer 1995]. Using a Monte Carlo modelling of the binary population, they found that the best fit to the observed binaries (in the period range from 2 days to 3 years) is \( f_{b,c} \approx 15\% \). This is in good agreement with our overall predicted \( f_{b,c} = 11.5\% \), and also with our prediction for heavier main-sequence binaries, \( f_{0.5} = 17.5\% \).
BY Dra systems and the number of eclipsing binaries is found to be 5.9, 6.7, 7.2 and 3.5 for models with \( f_{b,0} = 1.0, 0.75, 0.5 \) and 0.25, respectively. Therefore a large initial binary fraction \( \gtrsim 75\% \) is most likely.

Of the quantities that we explicitly assume in our cluster model to be constant, the central (core) density is the one with the largest dynamic range in models that provide for dynamical evolution. Hence it is also the quantity that is most likely to significantly affect our final results. To test the sensitivity of our results to a changing central density, we have run our model of 47 Tuc with a central density assumed to increase by a factor of 10 from \( t = 0 \) to the present. Specifically, we still match the currently observed core density while ramping up the value either exponentially or linearly in time, starting with a value 10 times smaller than at present. This could represent qualitatively the gradual core contraction observed in some \( N \)-body models of star clusters with binaries (see Aarseth & Heggie 1993 and, for a steeper core contraction, see Giersz & Spurzem 2003). These models predict a present-day core binary fraction that is only slightly higher, by about 1–2%. To increase the binary fraction more significantly, the time-averaged core density in the cluster would have had to be many orders of magnitude lower than what is observed today. There is no reasonable dynamical history that would produce such an unusual result. In contrast, recent \( N \)-body simulations show that the presence of just 10% hard primordial binaries leads to core radius expansion and therefore the core density might be higher in the past (Wilkinson et al. 2003). Though we did not run another 47 Tuc model with a central density decreasing with time, we can predict that the present-day binary fraction would then be smaller by about 1–2%.

**6 CONCLUSIONS**

We have considered in detail processes of binary destruction (and formation) in dense stellar systems. In particular, we have shown that the present binary fraction in cluster cores should be relatively small (\( \lesssim 10\% \)). This is caused not only by dynamical encounters, but also by binary stellar evolution, which is the dominant mechanism for the destruction of hard binaries. We also find that the destruction rate due to stellar evolution is enhanced significantly by dynamical hardening of binaries.

We have shown that values of binary fractions for stars in different mass ranges and at different evolutionary stages can differ significantly. The fraction of dynamically formed binaries is higher when one considers stars at more advanced evolutionary stages. The binary period distribution evolves from flat in \( \log P \) for loose clusters toward a sharply peaked distribution in denser clusters, even if all clusters have identical velocity dispersions and therefore identical hard-soft binary boundaries. This implies that a flat period distribution should not be assumed when deriving overall binary fractions by extrapolation from the distribution of observed binaries in a narrow period range (e.g., eclipsing binaries).

We considered several models that attempted to match observed globular clusters. For those with good available data on binaries (M4 and 47 Tuc), we found our predicted binary fractions to be in good agreement with observations once we take into account the correct binary period distribution. The main conclusion we derive from these calculations and our semi-analytic estimates is that the currently observed binary fractions in cluster cores suggest very high initial (primordial) binary fractions—close to 100%.

In addition to their implications for the interpretation of observed binary fractions in cluster cores, our results have important consequences for the theoretical modeling of globular clusters using \( N \)-body simulations. Indeed, it is clear that “realistic” dynamical simulations of globular cluster evolution should include large populations of primordial binaries, with initial binary fractions in the range \( \sim 50\%–100\% \) (similar to what is usually assumed for the field; see, e.g., Duquennoy & Mayor 1991). This poses a particular challenge for direct \( N \)-body simulations, where the treatment of even relatively small numbers of binaries can add enormous computational costs. For this reason, current direct \( N \)-body simulations of star clusters with large initial binary fractions typically have \( N \) too small to be considered representative of globular clusters (see, e.g., Portegies Zwart et al. 2003; Wilkinson et al. 2003). Other methods, such as Monte Carlo simulations, do not suffer from the same limitations, and routinely simulate clusters with reasonably large \( N (\sim 10^5 – 10^6) \) and binary fractions (\( \sim 30\% \)), but have not yet included advanced treatments of binary star evolution (see, e.g., Fregeau et al. 2003 and references therein).

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The Evolution of Binary Fractions in Globular Clusters

Consider the rate of three-body binary formation (via the close approach of three single stars) in a dense cluster core. We denote by \( \Gamma(E_b) \) the rate per star of the formation of a binary with binding energy \( E_b \). First, we consider \( \Gamma(b \leq \sigma_{\text{max}}) \) – the rate per star at which three objects come together within a region of size \( \sigma_{\text{max}} \).

The probability that a third object will be in the vicinity of two other interacting objects is the product of the probability of the first two bodies meeting and the probability that during this time a third object will be in the same vicinity. The rate of two-body encounters for masses \( m_1 \) and \( m_2 \), with number density \( n_{b2} \) is

\[
\Gamma_2(b \leq \sigma_{\text{max}}) = \pi b_{\text{max}}^2 \left( 1 + \frac{v_p^2}{v_{12}^2} \right) n_{b2} v_{12}.
\]  

Here

\[
v_p = \frac{2G(m_1 + m_2)}{b_{\text{max}}}
\]

is the velocity at closest approach and

\[
< v_{12} >^2 \equiv (\sigma_1^2 + \sigma_2^2) = \sigma^2 < m > m_1 + m_2, \]

where \( \sigma \) is the three-dimensional velocity dispersion.

We define the minimum hardness for the binary to be formed as

\[
\eta_{\text{min}} = \frac{G m_1 m_2}{b_{\text{max}}^2 < m > \sigma_{\text{max}}^2}.
\]

Then

\[
\Gamma_2(b \leq \sigma_{\text{max}}) = \pi b_{\text{max}}^2 (1 + 2\eta) n_{b2} v_{12}.
\]

The second object spends in the vicinity of the first object a time \( \Delta t \approx \frac{v_{12}}{v_p} \). The probability that a third object will be within the same vicinity is then

\[
p_3 \approx n_3 (b_{\text{max}}^2 \Delta tv_3 + b_{\text{max}}^3) = n_3 b_{\text{max}}^3 \left( 1 + 2\frac{v_3}{v_p} \right).
\]

Here \( n_3 \) is the relative velocity of the third object with respect to the centre of mass of the first two. Effectively, it reflects velocity at which the population at the vicinity \( b \) increases. We do not consider here the population decrease, assuming that if the object was in the neighborhood, a three-body encounter has happened.

Finally, the result then takes the form given in [Binney & Tremaine 1987, §8], but with an additional mass- and hardness-dependent factor \( f \),

\[
\Gamma_3(\eta > \eta_{\text{min}}) \propto \frac{\eta^2 G^3 < m > ^5}{\sigma^9} f(m_1, m_2, m_3, \eta)
\]

\[\]
\[ f(m_1, m_2, m_3, \eta) = \pi \frac{n_1 n_2 n_3}{n_c^3} \frac{m_1^5}{m_c} \frac{m_2^5}{m_c} \eta^{-5} \left( \frac{1 + \frac{m_1 m_2}{m_c^2}}{2 \left( \frac{m_1 + m_2}{m_c} \right)^{-1}} \right)^{\frac{1}{2}}. \] (A8)

It is clear that the rate is highly dependent on the masses of the participating stars and decreases steeply with increasing hardness of the binary that is formed.

If all encounters resulted in the formation of a binary with hardness \( \eta \), then the rate of binary formation would be completely described by equation (A7).

Using the equation (A7), the time-scale for three-body binary formation can also be written as

\[ T_{3b} = \frac{2.08 \times 10^{16}}{\text{yr}} \left( \frac{10^5}{n_c} \right)^2 \left( \frac{M_\odot}{m_c} \right)^5 \left( \frac{\sigma}{10 \ \text{km/s}} \right)^9 \eta^5 (1 + 2\eta)^{-1} \left( 1 + \frac{\lambda}{\sigma} \eta^{-0.5} \frac{m_1 m_2}{(m_1 + m_2)^{1/2}} \right)^{-1}. \] (A9)

Even in the core of a large, very dense cluster, with density \( \sim 10^5 \text{pc}^{-3} \) and containing \( \sim 10^5 \) stars, no binaries with \( \eta > 1 \) will be formed from \( \sim 1 M_\odot \) stars in a Hubble time. It has also been shown that many three-body binary formation events will lead to physical collisions for stars as small as white dwarfs (Chernoff & Huang 1996). We therefore neglect all three-body interactions in our cluster simulations.