Determining Weights in Multi-Criteria Decision Making Based on Negation of Probability Distribution under Uncertain Environment

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Abstract: Multi-criteria decision making (MCDM) refers to the decision making in the limited or infinite set of conflicting schemes. At present, the general method is to obtain the weight coefficients of each scheme based on different criteria through the expert questionnaire survey, and then use the Dempster–Shafer Evidence Theory (D-S theory) to model all schemes into a complete identification framework to generate the corresponding basic probability assignment (BPA). The scheme with the highest belief value is then chosen. In the above process, using different methods to determine the weight coefficient will have different effects on the final selection of alternatives. To reduce the uncertainty caused by subjectively determining the weight coefficients of different criteria and further improve the level of multi-criteria decision-making, this paper combines negation of probability distribution with evidence theory and proposes a weights-determining method in MCDM based on negation of probability distribution. Through the quantitative evaluation of the fuzzy degree of the criterion, the uncertainty caused by human subjective factors is reduced, and the subjective error is corrected to a certain extent.

Keywords: Multi-criteria decision making; negation of probability distribution; Dempster–Shafer Evidence Theory

1. Introduction

Multi-criteria decision making (MCDM) refers to the decision making in the limited or infinite set of conflicting schemes. According to whether the alternatives are limited or infinite, MCDM can be divided into multi-attribute decision making (limited) and multi-objective decision making (infinite). In essence, multi-attribute decision-making is mainly used to study the evaluation and optimization of known schemes, and multi-objective decision-making is mainly used to study the planning and design of unknown schemes. This paper mainly discusses the multi-attribute decision-making problem with multiple conflicting attributes. According to the performance of each scheme under different criteria, it carries out a comprehensive evaluation, and then selects the best scheme from the limited schemes. Most of the project decision-making problems in our life can be modeled as multi-criteria decision making problems [1–5]. Because of its universality and importance, the exploration of such problems has shown progress. Many math tools are presented to deal with multi-criteria decision-making, such as fuzzy sets [6–11], which offer a framework to address uncertainty and vagueness; soft sets [12–14]; evidence theory [15–19], which enables any union of classes to be addressed and expresses both
uncertainty and imprecision; Z numbers [20], which can not only express uncertainty, imprecision and incompleteness of information but can also represent the reliability of information; D numbers [21,22], which are more capable of expressing and handling both uncertainty and imprecision; network modeling [23–25]; etc. [26].

In the process of solving multi-criteria decision making problems, the determination of criteria weight is very important. It is used to reflect the relative importance of a certain criterion. If a certain criterion is relatively important, it is given a relatively large weight; otherwise, it is given a relatively small weight. There are three methods to determine the weight: subjective method, objective method, and comprehensive method. At present, the general solution is the comprehensive method. First, the alternatives are subjectively evaluated by experts based on various criteria. Then, the linguistic evaluation is converted to the corresponding quantitative evaluation; a comprehensive measure of subjective evaluation and objective reflection is calculated through certain rules; and finally the alternatives are evaluated, ranked and optimized [27,28]. On this basis, Fei et al. developed a MCDM method where the evaluation information is expressed and handled by Dempster–Shafer theory [29].

In the evaluation of a scheme, the evaluation under each criterion affects the final result to some extent [30–32]. Due to the relative importance difference between different criteria, the final evaluation results are obtained by evaluating the importance degree of each criterion and weighting the evaluation based on each criterion, respectively. The relatively important criterion has a larger weight, and the corresponding evaluation of the criterion has a greater impact on the final results. On the contrary, the relatively unimportant criterion has a smaller weight, and the corresponding evaluation of the criterion has a smaller impact on the final result. Thus, we need to assign an appropriate weight to each influencing factor carefully to reduce the impact of subjective factors on the result. To solve this problem, we carefully studied some methods to determine the combination weight, such as the method for obtaining ordered weighted average OWA weights proposed by Xu, which can relieve the influence of unfair arguments on the decision results [33]; the method to obtain a weights vector [34] proposed by Lamata et al.; the PPMIS (Project and Portfolio Management Information Systems) selection/evaluation approach that combines TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) with intuitionistic fuzzy group decision making presented by Gerogiannis et al. [35]; and two reliability-based combination rules are used to synthesize multi-source information [36].

Furthermore, to improve the level of decision-making in uncertain environment, researchers have previously carried out long-term and in-depth studies. Srivastava et al. defined a new entropy function for determination of uncertainty associated with the negation of a probability distribution and the events associated with it [37]. Srivastava et al. also developed uncertainty measures that measure the uncertainty associated with the negation of a probability distribution [38]. Luo and Deng presented a new definition of negation of BBA [39], which is used to measure fuzziness for D-S theory. Xie and Xiao proposed negation method of BPA based on the maximum uncertainty [40]. The negation of probability distribution provides a new view to represent the uncertainty information, which leads to the application of negation of probability distribution in related fields and in-depth exploration [41]. In addition, the negation model has the maximum entropy allocation, which attracts studies on uncertainty measures that can be applied in the negation process.

Recently, Xiao proposed a method of evidence fuzzy multi-criteria decision-making based on belief entropy, named as EFMCDM [42]. The combination of D-S evidence theory and belief entropy should be used to solve the problem of multi-criteria decision making. First, the experts evaluate the linguistic rating of all alternatives and the importance of each criterion based on each criterion. Then, the fuzzy number is used to transform the language level evaluation into quantitative evaluation of numbers, and then the weighted decision matrix is obtained through a series of calculations. Then, the D-S evidence theory is used to model the whole scheme into a complete identification framework, and the BPA of each criterion is generated by using the belief entropy. After fusing multiple pieces of evidence, the best scheme is selected according to the belief value of the alternatives. In the process of decision-making, EFMCDM considers the influence of subjective and objective weight, and builds
a quantitative model of uncertainty, which successfully reduces the uncertainty brought by human
subjective cognition and improves the decision-making level.

However, EFMCDM does not consider the fuzziness of evaluation criteria. To further improve
the decision-making level, this paper proposes a weights-determining method in MCDM based on
negation of probability distribution [43] under uncertain environment. In the process of constructing
the weighted decision matrix, it can be found that the column elements in the final weighted decision
matrix satisfy the probability distribution. The probability distribution of each criterion has its

2. Preliminaries

2.1. Dempster–Shafer Evidence Theory

How to handle the problem of uncertainty has received much attention in recent years [44,45].
Thus far, various methods have been exploited, including grey prediction model [46], network
model [47,48], risk-based model [49–52], evidential reasoning [53,54], entropy-based [55–57], etc.
These methods were used in many applications, e.g. failure analysis [58,59], risk assessments [60],
reliability evaluation [61,62], classification [63,64], and decision-making [65–68]. As one of the most
effective methods, D-S evidence theory [69,70] is a kind of imprecise reasoning theory, which satisfies
the axiomatic system weaker than probability theory. The main knowledge we need to master includes:
the basic concepts of D-S evidence theory (identification framework, proposition, and basic probability
distribution), and combination rules.

Theorem 1. The identification framework \( \Theta \) is defined as an exhaustive set of all possible values that are
mutually exclusive to variables [69,70].

\[
\Theta = \{A_1, A_2, \ldots, A_n\}
\]
Theorem 2. The power set \(2^{\Theta}\) is defined as the set of all combinations of possible cases in the identification framework, that is, the set of all subsets of the identification framework \(\Theta\). Proposition \(A\) is a subset of the identification framework \(\Theta\), that is, the elements of the power set \(2^{\Theta}\) \([69,70]\).

\[
2^{\Theta} = \{\emptyset, \{A_1\}, \cdots, \{A_n\}, \{A_1, A_2\}, \cdots, \{A_1, A_2, \cdots, A_i\}, \cdots, \Theta\} \tag{2}
\]

Theorem 3. BPA is the basic probability distribution function and \(m\) meets the following two conditions: \(m(\emptyset) = 0\) and \(\sum_{A \subseteq \Theta} m(A) = 1\). \(m(A)\) represents the precise trust degree to proposition \(A\), which means that you do not know how to allocate this number when \(A = \Omega\) \([69,70]\).

Since BPA is effective to model the uncertain information, it has been well studied, e.g. for divergence measure \([71,72]\), distance measure \([73]\), correlation coefficient \([74,75]\), entropy measure \([76–78]\), dependent evidence \([79]\), etc. \([80,81]\).

Theorem 4. The combination rule is to calculate the orthogonal sum of different probability distribution functions \([69,70]\).

\[
m(A) = m_1 \oplus m_2 = \begin{cases} 0, & A = \emptyset \\ \frac{\sum_{A_1 \cap A_2 = A} m_1(A_1)m_2(A_2)}{1 - \sum_{A_1 \cap A_2 = \emptyset} m_1(A_1)m_2(A_2)}, & A \neq \emptyset \end{cases} \tag{3}
\]

2.2. Negation of Probability Distribution

The negation of probability distribution \([43]\) generally does not lead to the unique probability distribution, but generates a set of probability distributions that is regarded as consistent with the negation idea. Specially, based on the maximum entropy distribution of the weight related to each focus element, there is a unique negative implementation.

Theorem 5. Probability distribution. If the reference system is set \(X = \{x_1, x_2, \cdots, x_n\}\) and \(P = \{p_1, p_2, \cdots, p_n\}\) is the probability distribution of \(X\), the following requirements should be met \([43]\):

\[
\begin{cases} \sum p_i = 1 \\ p_i \in [0, 1] \end{cases} \tag{4}
\]

Theorem 6. Negation of probability distribution. \(\overline{P} = \{\overline{p_1}, \overline{p_2}, \cdots, \overline{p_n}\}\) is the negation of probability distribution \(P\) \([43]\),

\[
\overline{p_i} = \frac{1 - p_i}{n - 1} \tag{5}
\]

Because \(\sum_{i=1}^n \overline{p_i} = \frac{1}{n-1} \sum_{i=1}^n (1 - p_i) = 1\) and \(\overline{p_i} \in [0, 1]\), \(\overline{P}\) is also a probability distribution.

3. Proposed Method

In this section, we introduce a multi-criteria decision-making method based on negation of probability distribution. The specific flow chart is shown in Figure 1. By combining the D-S evidence theory with the negation of probability distribution, the fuzzy degree of the criterion is taken as the reference standard to calculate the discount coefficient of the criterion, which corrects the errors brought by the experts’ subjective evaluation of each criterion and improves the decision making level.
Problem statement: Let a set of \( m \) possible exclusive alternatives \( \Theta = \{ A_1, A_2, \cdots, A_m \} \) be the frame of discernment, \( C = \{ C_1, C_2, \cdots, C_n \} \) be the set of \( n \) decision criteria, and \( DM = \{ DM_1, DM_2, \cdots, DM_t \} \) be the set of \( t \) decision makers. A set of fuzzy ratings for \( A_i (i = 1, 2, \cdots, m) \) with respect to the criteria \( C_j (j = 1, 2, \cdots, n) \) given by the decision maker \( DM_k (k = 1, 2, \cdots, t) \) is denoted as \( \tilde{x}_{ijk} \), \( i = 1, 2, \cdots, m; j = 1, 2, \cdots, n; k = 1, 2, \cdots, t \), where \( \tilde{x}_{ijk} = (x_{ijk1}, x_{ijk2}, x_{ijk3}, x_{ijk4}) \), and a set of fuzzy importance weights for \( C_j (j = 1, 2, \cdots, n) \) given by the decision maker \( DM_k (k = 1, 2, \cdots, t) \) is denoted as \( \tilde{w}_{jk} = (w_{jk1}, w_{jk2}, w_{jk3}, w_{jk4}) \). Consider this fuzzy MCDM problem: The best candidate should be selected from \( \Theta \).

Step 1: According to Xiao’s idea [42], obtain the normalized defuzzified weight decision matrix \( D_{\text{ef}}(\tilde{D}) \).

Step 1-1: Multiply fuzzy coefficient \( x_{ijk} \) and fuzzy importance weight \( w_{jk} \) to get weighted fuzzy coefficient \( \tilde{x}_{ijk}^w = \tilde{x}_{ijk} \times \tilde{w}_{jk} \), where the superscript \( w \) is used to represent the weight of importance of different criteria. Due to different decision makers having different judgments on the importance degree of different criteria, by scalar multiplication, the weight factors \( (\tilde{x}_{ijk}) \) of the same criteria of the same decision-maker and the importance factors \( (\tilde{w}_{jk}) \) of the criteria are multiplied correspondingly, and the weighted weight factors \( (\tilde{x}_{ijk}^w) \) of a decision-maker for a scheme under a certain criterion are obtained. Then, the weighted fuzzy coefficient matrix \( \tilde{D}_k (k = 1, 2, \ldots, t) \) is obtained:

\[
\tilde{D}_k = \begin{bmatrix}
\tilde{x}_{11k}^w & \tilde{x}_{1jk}^w & \cdots & \tilde{x}_{1nk}^w \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{x}_{11k}^w & \tilde{x}_{1jk}^w & \cdots & \tilde{x}_{1nk}^w \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{x}_{m1k}^w & \tilde{x}_{mjk}^w & \cdots & \tilde{x}_{mnk}^w
\end{bmatrix}
\]  

(6)

Step 1-2: The aggregated fuzzy value \( \tilde{x}_{ijk}^w \) of weighted fuzzy value and the aggregated decision matrix \( \tilde{D} \) are obtained by considering the evaluation of \( t \) decision-makers. Then, the aggregated fuzzy values are normalized and defuzzified, respectively. Then, the defuzzified weight decision matrix.
\( \text{Def} (\bar{D}) \) is obtained. After another normalization, obtain \( \text{Def} \left( \bar{x}_{ij}^{w} \right) = \frac{\text{Def} (\bar{x}_{ij}^{w})}{\sum_{m=1}^{m} \text{Def} (\bar{x}_{ij}^{w})}, j = 1, 2, \ldots, n. \)

Then, the normalized defuzzified weight decision matrix is obtained:

\[
\text{Def} (\bar{D}) = \begin{bmatrix}
\text{Def} \left( \bar{x}_{11}^{w} \right) & \cdots & \text{Def} \left( \bar{x}_{1j}^{w} \right) & \cdots & \text{Def} \left( \bar{x}_{1n}^{w} \right) \\
\vdots & \ddots & \vdots & \cdots & \vdots \\
\text{Def} \left( \bar{x}_{j1}^{w} \right) & \cdots & \text{Def} \left( \bar{x}_{jj}^{w} \right) & \cdots & \text{Def} \left( \bar{x}_{jn}^{w} \right) \\
\vdots & \ddots & \vdots & \cdots & \vdots \\
\text{Def} \left( \bar{x}_{m1}^{w} \right) & \cdots & \text{Def} \left( \bar{x}_{mj}^{w} \right) & \cdots & \text{Def} \left( \bar{x}_{mn}^{w} \right)
\end{bmatrix}
\]  

(7)

Step 2: Calculate the discount factor for each criterion.

Step 2-1: Since the column elements of the above matrix are obtained by normalization, \( \text{Def} (\bar{x}_{ij}^{w}) = \frac{\text{Def} (\bar{x}_{ij}^{w})}{\sum_{m=1}^{m} \text{Def} (\bar{x}_{ij}^{w})}, j = 1, 2, \ldots, n, \) each column conforms to the characteristics of probability distribution. Through the negation formula,

\[
N\text{Def} \left( \bar{x}_{ij}^{w} \right) = \frac{1 - \text{Def} \left( \bar{x}_{ij}^{w} \right)}{m - 1}
\]  

(8)

the negation weighted decision matrix \( N\text{Def} (\bar{D}) \) can be obtained.

\[
N\text{Def} (\bar{D}) = \begin{bmatrix}
N\text{Def} \left( \bar{x}_{11}^{w} \right) & \cdots & N\text{Def} \left( \bar{x}_{1j}^{w} \right) & \cdots & N\text{Def} \left( \bar{x}_{1n}^{w} \right) \\
\vdots & \ddots & \vdots & \cdots & \vdots \\
N\text{Def} \left( \bar{x}_{j1}^{w} \right) & \cdots & N\text{Def} \left( \bar{x}_{jj}^{w} \right) & \cdots & N\text{Def} \left( \bar{x}_{jn}^{w} \right) \\
\vdots & \ddots & \vdots & \cdots & \vdots \\
N\text{Def} \left( \bar{x}_{m1}^{w} \right) & \cdots & N\text{Def} \left( \bar{x}_{mj}^{w} \right) & \cdots & N\text{Def} \left( \bar{x}_{mn}^{w} \right)
\end{bmatrix}
\]  

(9)

Step 2-2: For each criterion \( C_j \), the Euclidean distance \( d_j \) between the probability distribution and the negation is calculated.

\[
d_j = \sqrt{\sum_{i=1}^{m} \left[ \text{Def} \left( \bar{x}_{ij}^{w} \right) - N\text{Def} \left( \bar{x}_{ij}^{w} \right) \right]^2}, j = 1, 2, \ldots, n
\]  

(10)

Step 2-3: Normalize to get the discount coefficient \( U_{C_j} \) of each criterion.

\[
U_{C_j} = \frac{d_j}{\sum_{j=1}^{n} d_j}, j = 1, 2, \ldots, n
\]  

(11)

Step 3: Generate BPAs and combine them.

Step 3-1: According to the following formula, BPA corresponding to each proposition \( A_i \) is generated based on each criterion \( C_j \).

\[
\begin{align*}
    m_{C_j} (\phi) &= 0 \\
    m_{C_j} (A_i) &= \text{Def} \left( \bar{x}_{ij}^{w} \right) \times U_{C_j}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \\
    m_{C_j} (\Theta) &= 1 - \sum_{i=1}^{m} m_{C_j} (A_i)
\end{align*}
\]  

(12)

Step 3-2: Through \((n-1)\) times orthogonal sum operation, BPAs of different criteria are combined with each other.

\[
m_C = \left( \cdot \left( m_{C_1} \oplus m_{C_2} \right) \lor \ldots \lor m_{C_n} \right)
\]  

(13)
Step 3-3: According to the value of \( m_C \), the scheme \( A_a \) with the largest \( m_C \) is the best scheme, and the best scheme may not be unique.

\[
\alpha = \arg \max_{1 \leq i \leq m} \{ m_C \}
\]  

4. Examples

In this section, the feasibility of the proposed method is verified by experiments.

4.1. Problem Statement

Problem statement: Consider the decision-making problem of supplier selection from Xiao [42] associated with a committee of three decision makers established by a company who give their anticipation and definition of the evaluation criteria for the final four candidate suppliers after a preliminary screening, where \( \Theta = \{ A_1, A_2, A_3, A_4 \} \) is the frame of discernment consisting of four alternatives, \( DM = \{ DM_1, DM_2, DM_3 \} \) is the set of decision makers, and \( C = \{ C_1, C_2, C_3, C_4, C_5 \} \) is the set of criteria, which is considered to be the set of evidence. The criteria are defined as follows: C1 is the quality of products, C2 is the effort to establish cooperation, C3 is the technical level of each supplier, C4 is the delay on delivery of each supplier, and C5 is the price/cost.

A set of fuzzy ratings \( x_{ijk}^j, i = 1, 2, 3; j = 1, 2, 3, 4, 5; k = 1, 2, 3 \) for \( A_i (i = 1, 2, 3, 4) \) with respect to the criteria \( C_j (j = 1, 2, 3, 4, 5) \) given by the decision maker \( DM_k (k = 1, 2, 3) \) is shown in Table 1, and a set of fuzzy importance weights \( w_{jk}^k, j = 1, 2, 3, 4, 5; k = 1, 2, 3 \) for \( C_j (j = 1, 2, 3, 4, 5) \) given by the decision maker \( DM_k (k = 1, 2, 3) \) is shown in Table 2.

**Table 1.** A set of fuzzy ratings \( \tilde{x}_{jk} \).

| \( D1 \) | \( C1 \) | \( C2 \) | \( C3 \) | \( C4 \) | \( C5 \) |
|---|---|---|---|---|---|
| \( A_1 \) | (0.5, 0.6, 0.7, 0.8) | (0.4, 0.5, 0.5, 0.6) | (0.5, 0.6, 0.7, 0.8) | (0.7, 0.8, 0.8, 0.9) | (0.8, 0.9, 1.0, 1.0) |
| \( A_2 \) | (0.5, 0.6, 0.7, 0.8) | (0.5, 0.6, 0.7, 0.8) | (0.7, 0.8, 0.8, 0.9) | (0.7, 0.8, 0.8, 0.9) | (0.5, 0.6, 0.7, 0.8) |
| \( A_3 \) | (0.7, 0.8, 0.8, 0.9) | (0.5, 0.6, 0.7, 0.8) | (0.4, 0.5, 0.5, 0.6) | (0.5, 0.6, 0.7, 0.8) | (0.8, 0.9, 1.0, 1.0) |
| \( A_4 \) | (0.8, 0.9, 1.0, 1.0) | (0.7, 0.8, 0.8, 0.9) | (0.7, 0.8, 0.8, 0.9) | (0.5, 0.6, 0.7, 0.8) | (0.7, 0.8, 0.8, 0.9) |

**Table 2.** A set of fuzzy importance weights \( \tilde{w}_{jk} \).

| \( W_{jk} \) | \( C1 \) | \( C2 \) | \( C3 \) | \( C4 \) | \( C5 \) |
|---|---|---|---|---|---|
| \( D1 \) | (0.2, 0.3, 0.4, 0.5) | (0.4, 0.5, 0.5, 0.6) | (0.7, 0.8, 0.8, 0.9) | (0.8, 0.9, 1.0, 1.0) | (0.7, 0.8, 0.8, 0.9) |
| \( D2 \) | (0.2, 0.3, 0.4, 0.5) | (0.2, 0.3, 0.4, 0.5) | (0.8, 0.9, 1.0, 1.0) | (0.8, 0.9, 1.0, 1.0) | (0.5, 0.6, 0.7, 0.8) |
| \( D3 \) | (0.4, 0.5, 0.5, 0.6) | (0.4, 0.5, 0.5, 0.6) | (0.8, 0.9, 1.0, 1.0) | (0.8, 0.9, 1.0, 1.0) | (0.5, 0.6, 0.7, 0.8) |

4.2. Implementation Based on the Proposed Method

Step 1: Get the normalized defuzzified weight decision matrix.

Step 1-1: Generate the fuzzy values of the weighted supplier ratings \( \tilde{x}_{ijk}^w \). Then, the weighted fuzzy coefficient matrix \( \tilde{D}_k \) is obtained, as shown in Table 3.
Step 1-2: By integrating evaluations from different decision makers, the aggregated fuzzy value of weighted fuzzy value \( \tilde{x}_{ij}^{w} \) can be obtained, as shown in Table 4. Through the normalization and the defuzzification, the defuzzified weight decision matrix \( \text{Def}(\tilde{D}) \) can be obtained, as shown in Table 5. Then, through another normalization, the normalized defuzzified weight decision matrix \( \text{Def}(\tilde{D}) \) can be obtained, as shown in Table 6.

### Table 4. The normalized aggregated decision matrix \( \tilde{D} \).

| \( \tilde{x}_{ij}^{w} \) | \( c_1 \) | \( c_2 \) | \( c_3 \) | \( c_4 \) | \( c_5 \) |
|-----------------|--------|--------|--------|--------|--------|
| \( A_1 \) | (0.10, 0.24, 0.32, 0.48) | (0.10, 0.29, 0.34, 0.60) | (0.35, 0.64, 0.72, 0.90) | (0.40, 0.66, 0.77, 0.90) | (0.25, 0.54, 0.66, 0.90) |
| \( A_2 \) | (0.10, 0.22, 0.30, 0.48) | (0.10, 0.29, 0.34, 0.54) | (0.40, 0.63, 0.71, 0.90) | (0.40, 0.66, 0.77, 0.90) | (0.25, 0.44, 0.54, 0.72) |
| \( A_3 \) | (0.14, 0.30, 0.37, 0.54) | (0.14, 0.28, 0.34, 0.48) | (0.28, 0.46, 0.53, 0.80) | (0.40, 0.54, 0.70, 0.80) | (0.35, 0.58, 0.69, 0.90) |
| \( A_4 \) | (0.14, 0.30, 0.37, 0.54) | (0.10, 0.29, 0.34, 0.54) | (0.49, 0.75, 0.88, 1.00) | (0.40, 0.66, 0.77, 0.90) | (0.25, 0.49, 0.56, 0.81) |

### Table 5. The defuzzified weight decision matrix \( \text{Def}(\tilde{D}) \).

| \( \text{Def}(\tilde{x}_{ij}^{w}) \) | \( c_1 \) | \( c_2 \) | \( c_3 \) | \( c_4 \) | \( c_5 \) |
|-----------------|--------|--------|--------|--------|--------|
| \( A_1 \) | 0.53  | 3.38  | 0.65  | 1.69  | 2.34  |
| \( A_2 \) | 0.52  | 3.19  | 0.66  | 1.69  | 1.95  |
| \( A_3 \) | 0.63  | 3.10  | 0.52  | 1.52  | 2.51  |
| \( A_4 \) | 0.63  | 3.19  | 0.77  | 1.64  | 2.12  |

### Table 6. The normalized defuzzified weight decision matrix \( \text{Def}(\tilde{D}) \).

| \( \text{Def}(\tilde{x}_{ij}^{w}) \) | \( c_1 \) | \( c_2 \) | \( c_3 \) | \( c_4 \) | \( c_5 \) |
|-----------------|--------|--------|--------|--------|--------|
| \( A_1 \) | 0.2296 | 0.2629 | 0.2481 | 0.2581 | 0.2626 |
| \( A_2 \) | 0.2243 | 0.2481 | 0.2531 | 0.2581 | 0.2181 |
| \( A_3 \) | 0.2730 | 0.2408 | 0.2015 | 0.2327 | 0.2818 |
| \( A_4 \) | 0.2730 | 0.2481 | 0.2973 | 0.2511 | 0.2375 |

Step 2: Calculate the discount factor of each criterion.
Step 2-1: Through the negation formula, the negation weighted decision matrix \( \text{NDef}(\tilde{D}) \) can be obtained, as shown in Table 7.
Table 7. The negation weighted decision matrix $N\bar{D}$.

|       | $C_1$   | $C_2$   | $C_3$  | $C_4$  | $C_5$  |
|-------|---------|---------|--------|--------|--------|
| $A_1$ | 0.2568  | 0.2457  | 0.2506 | 0.2473 | 0.2458 |
| $A_2$ | 0.2586  | 0.2506  | 0.2490 | 0.2473 | 0.2606 |
| $A_3$ | 0.2423  | 0.2531  | 0.2662 | 0.2558 | 0.2394 |
| $A_4$ | 0.2423  | 0.2506  | 0.2342 | 0.2496 | 0.2542 |

Step 2-2: The Euclidean distance $d_j$ of each criterion is calculated by the difference between the weighted decision matrix and its negation matrix, as shown in Tables 8 and 9.

Table 8. The difference between the weighted decision matrix and its negation matrix.

|       | $C_1$   | $C_2$   | $C_3$   | $C_4$   | $C_5$   |
|-------|---------|---------|---------|---------|---------|
| $A_1$ | -0.0272 | 0.0172  | -0.0025 | 0.0108  | 0.0168  |
| $A_2$ | -0.0343 | -0.0025 | 0.0041  | 0.0108  | -0.0425 |
| $A_3$ | 0.0307  | -0.0123 | -0.0647 | 0.0231  | 0.0424  |
| $A_4$ | 0.0307  | -0.0025 | 0.0631  | 0.0015  | -0.0167 |

Table 9. The Euclidean distance $d_j$.

|       | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|-------|-------|-------|-------|-------|-------|
|        | 0.0617| 0.0214| 0.0905| 0.0277| 0.0645|

Step 2-3: Normalize to get the discount coefficient $U_{C_j}$ of each criterion, as shown in Table 10.

Table 10. The discount coefficient $U_{C_j}$.

|       | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|-------|-------|-------|-------|-------|-------|
|        | 0.2319| 0.0806| 0.3404| 0.1043| 0.2427|

Step 3: Generate BPAs and combine them.

Step 3-1: According to the following formula, BPA corresponding to each proposition $A_i$ is generated based on each criterion $C_j$, as shown in Table 11.

Table 11. The BPAs of the propositions with respect to different criteria.

|       | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|-------|-------|-------|-------|-------|-------|
| $m(A_1)$ | 0.0532| 0.0212| 0.0845| 0.0269| 0.0637|
| $m(A_2)$ | 0.0520| 0.0200| 0.0862| 0.0269| 0.0529|
| $m(A_3)$ | 0.0633| 0.0194| 0.0686| 0.0243| 0.0684|
| $m(A_4)$ | 0.0633| 0.0200| 0.1012| 0.0262| 0.0577|
| $m(\tilde{0})$ | 0.7681| 0.9194| 0.6596| 0.8957| 0.7573|

Step 3-2: Combine BPAs of different criteria $C_j$ each other, as shown in Table 12.

Table 12. The combined BPAs of the propositions.

|       | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|-------|-------|-------|-------|-------|
|        | 0.1502| 0.1428| 0.1456| 0.1641|

Step 3-3: Sort the $m_C$ of different schemes $A_i$ in descending order, and select the best scheme $A_4$. 


4.3. Comparison and Discussion

We compare the proposed method with the related methods, i.e., EFMCDM [42], where the experimental results are shown in Table 13.

| Decision Method       | Ranking Order | Optimal Choice |
|-----------------------|---------------|----------------|
| The proposed method   | $A_4 > A_1 > A_3 > A_2$ | $A_4$          |
| EFMCDM method         | $A_4 > A_1 > A_3 > A_2$ | $A_4$          |

The method proposed in this paper obtains the order of $A_4 > A_1 > A_3 > A_2$ and the best choice $A_4$. The result is the same as EFMCDM method, and it can make a reasonable choice, which shows the feasibility and correctness of this method.

In addition, we compare the belief value of the uncertainty of the criteria obtained by the proposed method after evidence fusion with the mean belief value of the uncertainty of the criteria. The results are shown in Table 14.

| The Proposed Method | Mean |
|--------------------|------|
| $m(\theta)$        | 0.3973 0.8000 |

According to the data in Table 14, after evidence fusion, the belief value of the uncertainty decreased from 0.8000 to 0.3973. While reducing the complexity of the algorithm, to some extent, it still retains the advantages of EFMCDM method, that is, it can quantitatively describe and effectively reduce the uncertainty caused by subjective cognition.

4.4. Sensitivity Analysis

To study the robustness and stability of the proposed method, it is necessary to analyze the sensitivity of the subjective weight of the criteria. Since the importance evaluation of the criteria is determined by the decision-maker subjectively, there must be human errors caused by the subjective evaluation. Therefore, six groups of importance weights of the criteria are randomly selected, as shown in Table 15. The belief values corresponding to the alternative schemes are calculated, and the best scheme is selected.

| Sets | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|------|-------|-------|-------|-------|-------|
| Set 1 | (0.8, 0.9, 1.0, 1.0) | (0.5, 0.6, 0.7, 0.8) | (0.4, 0.5, 0.5, 0.6) | (0.2, 0.3, 0.4, 0.5) | (0.1, 0.2, 0.2, 0.3) |
| Set 2 | (0.1, 0.2, 0.2, 0.3) | (0.8, 0.9, 1.0, 1.0) | (0.5, 0.6, 0.7, 0.8) | (0.4, 0.5, 0.5, 0.6) | (0.2, 0.3, 0.4, 0.5) |
| Set 3 | (0.2, 0.3, 0.4, 0.5) | (0.1, 0.2, 0.2, 0.3) | (0.8, 0.9, 1.0, 1.0) | (0.5, 0.6, 0.7, 0.8) | (0.4, 0.5, 0.5, 0.6) |
| Set 4 | (0.4, 0.5, 0.5, 0.6) | (0.2, 0.3, 0.4, 0.5) | (0.1, 0.2, 0.2, 0.3) | (0.8, 0.9, 1.0, 1.0) | (0.5, 0.6, 0.7, 0.8) |
| Set 5 | (0.5, 0.6, 0.7, 0.8) | (0.4, 0.5, 0.5, 0.6) | (0.2, 0.3, 0.4, 0.5) | (0.1, 0.2, 0.2, 0.3) | (0.8, 0.9, 1.0, 1.0) |
| Set 6 | (0.7, 0.8, 0.8, 0.9) | (0.1, 0.2, 0.2, 0.3) | (0.8, 0.9, 1.0, 1.0) | (0.1, 0.2, 0.2, 0.3) | (0.7, 0.8, 0.8, 0.9) |

According to six groups of importance weights of the criteria, the belief values of each alternative are calculated, as shown in Table 16 and Figure 2.
Table 16. Sensitivity analysis of the subjective weights of the criteria.

| \( m(C) \) | Set 1 | Set 2 | Set 3 | Set 4 | Set 5 | Set 6 |
|------------|-------|-------|-------|-------|-------|-------|
| A1         | 0.1504| 0.1521| 0.1506| 0.1497| 0.1503| 0.1496|
| A2         | 0.1429| 0.1436| 0.1427| 0.1413| 0.1420| 0.1420|
| A3         | 0.1485| 0.1482| 0.1467| 0.1504| 0.1499| 0.1475|
| A4         | 0.1695| 0.1696| 0.1682| 0.1654| 0.1665| 0.1681|

Figure 2. Sensitivity analysis of the subjective weights of the criteria.

Except that the alternative ranking of Set 4 is \( A_4 > A_3 > A_1 > A_2 \), the alternative ranking calculated by other groups is \( A_4 > A_1 > A_3 > A_2 \), indicating that the change of belief value of alternative is basically stable against the variation in importance weights of the criteria. The criteria ranking corresponding to Set 4 is \( C_4 > C_3 > C_1 > C_2 > C_3 \) according to the relative importance, which results in the lowest evaluation weighting coefficient based on \( C_3 \) criterion and affects the final \( A_1 \) and \( A_3 \) ranking results, which shows that the method is sensitive to the variation in subjective weight. Because the proposed method is affected by some subjective weight changes, it can result in human errors in the decision results. Therefore, the proposed method is suitable for a decision-making environment with less human error.

In addition, as shown in Table 17, no matter how the relative importance of the criteria changes, the best scheme is always \( A_4 \), which confirms the effectiveness of the method, and shows that the proposed method is effective for the correction of subjective errors.

Table 17. Ranking order under the variation of criteria weights.

| Sets  | 1     | 2     | 3     | 4     |
|-------|-------|-------|-------|-------|
| Set 1 | \( A_4 \) | \( A_1 \) | \( A_3 \) | \( A_2 \) |
| Set 2 | \( A_4 \) | \( A_1 \) | \( A_3 \) | \( A_2 \) |
| Set 3 | \( A_4 \) | \( A_1 \) | \( A_3 \) | \( A_2 \) |
| Set 4 | \( A_4 \) | \( A_1 \) | \( A_3 \) | \( A_2 \) |
| Set 5 | \( A_4 \) | \( A_1 \) | \( A_3 \) | \( A_2 \) |
| Set 6 | \( A_4 \) | \( A_1 \) | \( A_3 \) | \( A_2 \) |
5. Conclusions

This paper presents a weights-determining method in MCDM based on negation of probability distribution under uncertain environment. The main contribution of this paper is the combination of probability distribution negation and evidence fusion, which provides a new way to solve MCDM problems. While inheriting the advantage of EFMCDM in quantitative description of the uncertainty, through quantitative evaluation of the fuzzy degree of the criteria, to a certain extent, the error of experts’ subjective evaluation of each criterion is corrected to improve the decision-making level.

In the future work, the proposed method presents a new improvement scheme for the multi-attribute decision-making problem, which can be widely used in all aspects of life, e.g., supplier selection. In addition, on the premise of ensuring the correctness of the best scheme, we intend to further eliminate the influence of human subjective error on the order of alternatives.

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Abbreviations

The following abbreviations are used in this manuscript:

- MCDM: Multi-criteria Decision Making
- BPA: Basic Probability Assignment
- D-S theory: Dempster–Shafer Evidence Theory
- OWA: Ordered Weighted Average
- BBA: Basic Belief Assignment
- EFMCDM: Evidence Fuzzy Multi-criteria Decision Making
- PPMIS: Project and Portfolio Management Information Systems
- TOPSIS: Technique for Order Preference by Similarity to Ideal Solution

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