A Secure Distributed Authentication scheme based on CRT-VSS and Trusted Computing in MANET

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Abstract—With the rapid development of MANET, secure and practical authentication is becoming increasingly important. The existing works perform the research from two aspects, i.e., (a) secure key division and distributed storage, (b) secure distributed authentication. But there still exist several unsolved problems. Specifically, it may suffer from cheating problems and fault authentication attack, which can result in authentication failure and DoS attack towards authentication service. Besides, most existing schemes are not with satisfactory efficiency due to exponential arithmetic based on Shamir’s scheme. In this paper, we explore the property of verifiable secret sharing (VSS) schemes with Chinese Remainder Theorem (CRT), then propose a secret key distributed storage scheme based on CRT-VSS and trusted computing for MANET. Specifically, we utilize trusted computing technology to solve two existing cheating problems in secret sharing area before. After that, we do the analysis of homomorphism property with CRT-VSS and design the corresponding shares-product sharing scheme with better concision. On such basis, a secure distributed Elliptic Curve-Digital Signature Standard (ECC-DSS) authentication scheme based on CRT-VSS scheme and trusted computing is proposed. Furthermore, as an important property of authentication scheme, we discuss the refreshing property of CRT-VSS and do thorough comparisons with Shamir’s scheme. Finally, we provide formal guarantees towards our schemes proposed in this paper.

Keywords—secret sharing, trusted computing, Chinese Remainder Theorem, distributed authentication, CSP security model

I. INTRODUCTION

Mobile ad hoc network (MANET) is a new kind of wireless mobile network, where nodes are dispersed and easy to be compromised. So it isn’t reasonable to assume such a trust center in MANET. This makes the central management of secret keys and traditional authentication scheme with single CA weak and dangerous. In order to solve such problem and build dependable schemes, most of the existing work contributes to the design of secure distributed authentication scheme in such two steps as follows,

- divide the secret key in some way (e.g., secret sharing scheme) and distribute them to different nodes.
- design the detailed secure and dependable authentication process, whose essence is the reconstruction of original secret key and generation of authentication signature.

Specifically, in the first step, the private key of CA will be divided into $n$ different nodes and each node has one. As for the second step, distributed authentication, many schemes are proposed to provide the service of a distributed CA. And there have been several corresponding schemes, e.g. the multi-hop authentication scheme with RSA function in [22], ElGamal shape function scheme in [15] and threshold DSS (Digital Signature Standard) scheme in [8].

However, there still exist several problems. The threshold secret sharing schemes all have two categories of cheating problems against secret distributor and participants respectively. And existing authentication solutions can’t eliminate the possibility of cheating (just detect it with some computation overhead). Most of existing distributed authentication schemes, e.g., [8], [22], lack the validation of the secret shares and trusted property of the nodes, which will result in fault attack problem [17]. Furthermore, the misbehavior of authentication nodes may lead to DoS towards the authentication service. What’s worse, most of existing works based on Shamir’s scheme [21] suffer from a relative high complexity of exponent arithmetic in secret reconstruction phase.

Due to the reasons above, we aim to contribute as below:

- First, we propose a new scheme based on CRT-VSS (Verifiable Secret Sharing Scheme Based on Chinese Remainder Theorem) scheme instead of Shamir’s scheme, tackling security problems with better efficiency. Specifically, we utilize the trusted computing technology to solve two cheating problems above.
- Then we explore $(\oplus, \oplus)$, $(\otimes, \otimes)$ homomorphism property for CRT-VSS scheme and design the secure shares-product sharing scheme with CRT scheme. It owns better concision and equal security property compared with Shamir’s scheme.
- On such basis, a trusted ECC-DSS distributed authentication scheme is proposed to eliminate the possibility of the DoS and fault attack problems before. Furthermore, we discuss the refreshing property of CRT-VSS scheme and do thorough comparisons with Shamir’s scheme as an important property of authentication scheme towards mobile adversaries [24] tolerance.
- Finally, we model our schemes and ensure the security with formal guarantees.
The rest of this paper is organized as follows. In Section II we introduce related works in this paper. In Section III, we design and analyze the framework of authentication between two nodes (as initiator and responder) and prepare it as a module which will be used in detailed scheme below. Section IV shows the secret key distributed storage scheme based on CRT-VSS and trusted computing. In section V we explore and do some analysis for the homomorphism property of the CRT-VSS scheme. Then we propose our trusted ECC-DSS distributed authentication scheme in section VI. In Section VII, we present security analysis on the schemes in this paper. Finally, Section IX concludes the paper.

II. RELATED WORK

A. Threshold secret sharing scheme

In traditional $(t,n)$ threshold secret sharing schemes, the secret distributor $D$ divides the secret $S$ into $n$ shares and deliver them to $n$ participants securely. The recovery of secret $S$ can only be accomplished by a coalition not less than $t$ participants. The most popular secret sharing scheme is Shamir’s scheme in \[21\]. The reconstruction of secret can be realized with complexity $O(t\log^2 t)$ \[21\]. The first secret sharing scheme based on the Chinese Remainder Theorem (CRT) is proposed in \[1\] with reconstruction complexity $O(t)$. There exist two kinds of cheating problems in secret sharing area. (1) cheating from distributor. Distributor delivers the false secret shares to corresponding authentication nodes, braking the base of authentication. (2) cheating among participants. During the reconstruction phase, malicious participant will provide false/fault shares, leading to unfair, inconsistent results, etc.

The verifiable secret scheme development based on zero-knowledge proof evidence has both improvements on the original Shamir’s and CRT scheme (e.g. \[13\], \[18\]), which can only be used to detect such cheating problems but without elimination. The detection scheme in \[9\] can achieve the goal in some specified condition, but inefficient and restricted to specified cheater number condition. Though work in \[14\] uses the range proof techniques to prove the right range of the generated parameters, it can’t provide further assurance or evidence about the right property of the node.

B. Homomorphism property and distributed authentication

In order to tolerate mobile adversaries, the key management service proposed in \[24\] employs share refreshing as an important scheme which relying on the $(\oplus, \otimes)$ homomorphism property. It is indicated in \[3\] that the traditional Shamir’s threshold scheme only has the $(\oplus, \oplus)$-homomorphism property, without the $(\oplus, \otimes)$ or $(\otimes, \otimes)$- property. While it’s only mentioned in \[4\], \[13\] that the extended Asmuth-Bloom scheme has homomorphism proprieties but without any proof about it. The discussion about homomorphism property of CRT-VSS scheme is rare.

In the Distributed CA scheme in \[24\], the private key of CA is divided into several shares and distributed to several chosen entities, and each has one and only one part. Distributed authentication is the result of the Distributed CA scheme in the distributed environment of MANET. The scheme proposed in \[22\] realize the authentication with the process of the multi-hop signing. The encrypted functions mechanism \[19\] is used to strengthen the privacy and security of the authentication process, similar related work can be seen in \[7\]. In \[8\], a meaningful robust DSS signature schemes are proposed under different security requirements. But in all the schemes proposed above, the choice of nodes coalition for authentication is random and lacks validation, including the secret shares and the trusted property of the nodes. This may lead to DoS and fault attack problem \[17\]. Besides, RSA/Elgamal encrypt/decrypt and signature mechanism utilized by all the schemes above has a relative limited efficiency in MANET according to analysis in \[7\].

C. Trusted computing

Trusted Computing (TC) is a new technology developed and promoted by the Trusted Computing Group (TCG) for enhancing the security of computers and networks. The root of trust is a tamper resistant hardware engine, called TPM. TPM is assumed robust against both hardware and software attacks from either the underlying host or external sources. The aim of TC is to allow someone else to verify that only authorized code runs on a system. There are at least 16 PCRs in a TPM and a PCR is a 160-bit shielded storage location to hold an unlimited number of measurements in the way like this: $PCR_{new} = SHA-1(PCR_{old} || \text{measurement})$. A simple way to accomplish the measurement mentioned here is to do a digest of the configuration of the platform to ensure the integrity of a Platform. Direct anonymous attestation (DAA) protocol \[5\] was adopted by TCG as the method for remote authentication of the hardware module TPM, while preserving the privacy of the user of the platform that contains the module. Furthermore, the Property-Based Attestation (PBA) scheme proposed in \[6\] present a new privacy-preserving approach to attest the right configuration of host. Now the application of TC technology appears in various areas, e.g., inference Control \[23\], Ad hoc routing protocol security \[12\], digital rights management, etc.

III. FRAMEWORK OF AUTHENTICATION PROTOCOL BASED ON TRUSTED COMPUTING BETWEEN TWO NODES

In the two important steps of a secure authentication scheme, i.e. secure secret division/distribution and secure distributed authentication, the right behavior of the nodes is so important that it should be ensured in some reliable authentication way. Otherwise the malicious action of nodes may lead to cheating problems and DoS/fault attack. In order to solve this problem, we will utilize the trusted computing technology, including the property of TPM, the secure and authentication property of trusted computing protocols, e.g. DAA \[5\] and PBA \[6\] protocol. Because the authentication between two nodes will be referred and used repeatedly in the detailed schemes below, here we will first illustrate the framework of the authentication protocol between two nodes, one initiator and responder, in the MANET. Then the other part of this paper can refer it as a module easily. Without
loss of generality, suppose the authentication process happens between node $P_i$ and $P_j$, node $P_i$ request for authentication from node $P_j$. The authentication process between them is as follows in Figure 1

1. $P_i \rightarrow P_j$: $id_i, id_j$, INIT

2. $P_j \rightarrow P_i$: $id_j, id_i, n_1$

3. $P_i \rightarrow P_j$: $id_i, id_j$, im=$SHA-1(id_i||n_1||PCR)$,
   $\mathcal{D}_S = DAASign(im), PS_i = PBASign(im)$,
   $SHA-1(id_j||K_{INFO}||im||\mathcal{D}_S||PS_i, K_{INFO}||)$

4. $P_j \rightarrow P_i$: $id_j, id_i, S_{sk_{AIK}}, (K_{INFO}||id_j), \{id_j||n_2\}_{k_{ij}}$

5. $P_i \rightarrow P_j$: $id_i, id_j, \{id_i||n_2\}_{k_{ij}}$

Fig. 1. $P_i$ request for authentication from node $P_j$

In the framework, each node maintains three lists:
- Neighbor list $N$. records the IDs of the neighbor nodes.
- Trust list $T$. records the authentication result of different neighbors with different symbols, e.g. 1 represents trusted, 0 for no authentication before and -1 for authentication failure, initially all 0.
- Key list $K$. stores the established session with trusted neighbor nodes, initially empty.

Now we will do some explanation to the details of the framework proposed above as follows.

1. $P_i \rightarrow P_j$: $id_i, id_j$, INIT

Node $P_i$ sends the request for authentication towards node $P_j$ with its ID $id_i$ and destination ID $id_j$.

2. $P_j \rightarrow P_i$: $id_j, id_i, n_1$

After receiving the request from node $P_i$, $P_j$ first check the ID $id_i$ in neighbor list $N$, if there is no match then drop the request without any reply, else node $P_j$ reply with its ID $id_j$ and a random nonce $n_1$ as the challenge.

3. $P_i \rightarrow P_j$: $id_i, id_j$, im=$SHA-1(id_i||n_1||PCR)$,
   $\mathcal{D}_S = DAASign(im), PS_i = PBASign(im)$,
   $SHA-1(id_j||K_{INFO}||im||\mathcal{D}_S||PS_i, K_{INFO}||)$

After receiving the $n_1$ from $P_j$, node $P_i$ will do the following work.

- Compute the measurement of integrity towards the platform by $im=SHA-1(id_i||n_1||PCR)$. Here we note that the measurement of the integrity consists of the content of PCR. Due to the TPM will record the measurement of the platform from time to time by the function and update of PCRs. Here we use the command TPM Quote to get the content of the specified PCR. The PCR can be updated by the command TPM Extend.
- Generate the DAA signature to attest to $P_j$ its perfect TPM. Here the the integrity measurement $im$ computed above will be the parameter $\mathcal{D}_S = DAASign(im)$.
- Generate the PBA signature to furthermore attest to $P_j$ that its configuration and property is in a good set but without revealing the details. It is assumed that before running the PBA protocol, node $P_i$ and $P_j$ have already agreed on a set of configuration values denoted $CS = \{cs_1, \ldots, cs_n\}$. \[\]

The generation denoted as $PS_i = PBASign(CS, im)$. Note that $CS$ is constructed and planted inside the nodes initially and will be updated consistently if necessary according to the technology of trust computing.

- If there is no key established with node $P_j$, $K_{[id_j]}$ is empty, node $P_i$ will generate its public part parameters according to Diffie-Hellman key exchange scheme, denoted as $K_{INFO} = (g, p, g^y \mod p)$ and provide the integrity assurance by $SHA-1(id_j||K_{INFO}||im||\mathcal{D}_S||PS_i)$, otherwise the digest assurance is $SHA-1(id_i||im||\mathcal{D}_S||PS_i)$

4. $P_j \rightarrow P_i$: $id_j, id_i, S_{sk_{AIK}}, (K_{INFO}||id_j), \{id_j||n_2\}_{k_{ij}}$

After receiving the signature attest message from $P_i$, node $P_j$ will do the following work.

- Check the integrity of the message by digest assurance above. If check fails, then drop the message and abort.
- Validate the DAA and PBA signature by $DAAV erify(\mathcal{D}_S), PBAV erify(CS, im)$. If any verification fails, abort the protocol and mark the trusted symbol by $T[id_i] = -1$, otherwise $T[id_i] = 1$.
- Like above, if there is no key established before, then generate its public part parameters as $K_{INFO} = (g, p, g^y \mod p)$. Furthermore sign the $K_{INFO}$ structure with its ID under the private key of AIK inside the TPM to attest the right origin, in order to avoid the possibility of man-in-the-middle attack. Then the session key between the nodes can be generated according to DH scheme, denoted as $k_{ij} = (g^y)^{xp} \mod p$. Then node $P_j$ stores the session key $k_{ij}$ in the key list as $K[id_i] = k_{ij}$ and generate the encryption part $\{id_j||n_2\}_{k_{ij}}$ with its ID $id_j$ and random nonce $n_2$ inside.

5. $P_i \rightarrow P_j$: $id_i, id_j, \{id_i||n_2\}_{k_{ij}}$

- $P_i$ first validate the signature with the public AIK of $P_j$ by $S_{pk_{AIK}}(S_{sk_{AIK}}, (K_{INFO}||id_j)) = K_{INFO}||id_j$ If it fails, abort and without any reply.
- Similarly, if there is no key established before, $P_i$ first generate the session key $k_{ij}$ with the $K_{INFO}$ and its own part $K_{INFO}$ according to DH scheme denoted as $k_{ij} = (g^y)^{x} \mod p$. Then stores the key $k_{ij}$ in its key list.
- Then node $P_i$ decrypt the encrypted message $\{id_j||n_2\}_{k_{ij}}$ and get the nonce $n_2$.
- At last node $P_i$ generates the encryption part $\{id_i||n_2\}_{k_{ij}}$ with its ID $id_i$ and nonce $n_2$ above together to inform neighbor node $P_j$ that the key $k_{ij}$ has been well established between them.

Some additional explanation about the details of the scheme above is provided as follows.

1) The Neighbor list is constructed by routing protocols, e.g., OLSR, AODV. Besides, it and can consist of

\[\]

\[\]

\[\]

\[\]
the nodes not neihbored stringently, e.g., the two-hop neighbors in OLSR, in order to contain enough neighbors (not less than n).

2) The update and change of Neightbor list N will result in the update of the trust list T and key list K. Once neighbor node moves out of the region, the item will be removed out from the neighbor list and that will lead to corresponding item abolishment in the trust list and key list respectively.

3) In order to enhance the security, the trust list T and key list K should be re-initialized periodically. So the authentication process should be executed periodically when necessary.

In the other parts of the paper, we will refer this authentication process as module denoted as auth(id_i, id_D) indicating node P_i running authentication protocol with node P_j and requesting for the authentication from the later. The security analysis about the protocol will be left in section VII.

IV. SECRET KEY DISTRIBUTED STORAGE SCHEME BASED ON CRT-VSS AND TRUSTED COMPUTING

Our scheme has two phases. One is the distributing phase in which distributor divides the secret S into n shares (denoted as \{S_i\}_{i \in \{1,...,n\}}) and distributes them to a set of n nodes as participants (denoted as set P). The other is reconstruction phase, any coalition of t nodes (denoted as C) can reconstruct the secret S together. The details can be described as follows.

A. Distribution phase

To share a secret S among a set of n nodes with verifiable shares, the distributor does the following:

1) Setup work. Distributor D setup and set the related parameters, including the big prime number m_0 (m_0 > S), the positive sequence m = \{m_1,m_2,...,m_n\} meets the requirements below:
   • the sequence m = \{m_1,m_2,...,m_n\} is increasing;
   • (m_i, m_j) = 1, (i \neq j);
   • (m_i, m_0) = 1, (i \in \{1,2,...,n\};
   • N = \prod_{i=1}^t m_i > m_0^2 \prod_{i=1}^{t-1} m_{n-i+1};
   • ensure the p_i = 2m_i + 1 is still prime.

After that distributor need to choose a random positive integer A satisfied the condition 0 < A < M, and y = S + Am_0 < M, here the M = \prod_{i=1}^t m_i. Then y is the real shape of the secret S to be divided later.

2) The distributor D choose n trusted neighbors nodes \{P_i\}_{i \in \{1,...,n\}} (denoted as set P) to be qualified participants to distribute the shares later.
   • D \xrightarrow{B} \{N_D\}: id_D, AUTH, S_{sk_AIKD}(id_D). \{N_D\} is the neighbor set of D. The distributor D broadcasts (denoted as \xrightarrow{B} notation) a AUTH message to inform its neighbors the authentication election event. The message consists of the ID of D, authentication symbol AUTH and the signature of the id_D by the private key of AIK inside the TPM of distributor D to attest to P_i \in \{N_D\} the identity of D.
   • P_i \rightarrow D: id_i, id_D, auth(id_i, id_D).

After receiving the AUTH message, P_i first check the signature with the public AIK of D by S_{pk_AIKD}(S_{sk_AIKD}(id_i)) = id_i. If it holds, then P_i begins to execute the authentication process as auth(id_i, id_D), else P_i drop the AUTH message without any reply.

   • After the authentication process, Distributor D choose n qualified nodes from the trust list T as participants.

3) Reverse, the distributor D executes the authentication process to attest to the selected participants \{P_i\}_{i \in \{1,...,n\}}:
   • D \rightarrow \{P_i\}(P_i \in P): auth(id_D, id_i).
   • P_i \rightarrow D: id_i, id_D, AGREE.

If P_i pass the authentication process from D and agree to undertake the responsibility as a participant, P_i reply with a AGREE message to inform the distributor D, else deny and refuse the request without reply.

If D doesn’t receive the AGREE message from P_i, D have to remove the id of the P_i from the participant set P. This step won’t finish until the participant set has n participants, equally receiving n AGREE messages totally.

4) The distributor D generates and distributes shares and validation information securely.
   • D \xrightarrow{B} \{P_i\}: id_D, PUBLIC, S_{sk_AIKD}(id_D||p_i||g_i||m_i). Then distributor generates the validation information by z_i = g_i^{S_i} (mod m_i) and makes the values p_i, g_i, z_i public by the broadcast PUBLIC message. In order to ensure the correctness of the origin of the public information, the signature with private AIK sk_{AIKD} is included in the message.

After receiving the PUBLIC message, the participants can verify the correctness of the origin of public information by the signature. With the correct validation information, participant P_i can find whether his share is valid or not by checking z_i = g_i^{S_i} (mod m_i).

The work and schemes proposed before all assume that D is honest. But once the malicious distributor D distributes the shares for some y > M, then y will be greater than M_0 for all coalition C of size t. Hence, according to the analysis in [13], C may not compute the correct y value or recover correct secret S with inconsistent results. But here in the scheme, with the authentication help of the trusted computing technology, the first cheating problem exists in this phase before can be eliminated.

B. Reconstruction phase

As can be seen in the notations Table I, without loss of generality, suppose C = \{P_{i_k}\}_{k \in \{1,...,t\}}. M_C = \prod_{i \in C} m_i, M_{C \setminus \{i_k\}} = \prod_{j \in C, j \neq i_k} m_j, M_{C_{ik}} \equiv M_{C \setminus \{i_k\}}(\text{mod } m_i), M_{C_{ik}}^{-1} \equiv 1(\text{mod } m_i), the exchange of shares and
reconstruction happens between the participants in the coalition \( C \). \( P_{ik}, P_{ij} \in C \). The details are as follows.

1) If \( T[i_k] = 1 \) and \( T[i_k] \) not empty, then \( P_{ik} \) is trusted and goto the next step.
2) If \( T[i_k] = -1 \), then the authentication from \( P_{ik} \) before failed, \( P_{ij} \) abort and the coalition should be rebuilt.
3) If \( T[i_k] = 0 \), no authentication before. \( P_{ij} \) asks for new auth(id_{ij}, id_{ik}, \{S_i\}_{k\in\{i_k\}}).

The other participants can decrypt the message with the session key \( k_{ij,k} \), then validate the shares from other nodes, e.g. The share \( S_{ij} \) can be verified by the other participants in \( C \) by the equation:

\[
z_{ij} \equiv S_{ij} (\text{mod } p_{ij}).
\]

3) Similarly, \( P_{ij} \) can receive the encrypted shares from the other \((t-1)\) participants, after the decryption and validation success, \( P_{ij} \) can get the equation set established as below:

\[
S_{ij} \equiv S (\text{mod } m_{ij}), \quad (i = 1, 2, \ldots, t)
\]

According to the Chinese remainder theorem, the only solution can be easily computed as

\[
y \equiv \sum_{k=1}^{t} S_{ik} \cdot m_{C,ik} \cdot M_{C \setminus \{ik\}} \quad (\text{mod } M_{C})
\]

So \( S = y (\text{mod } m_{0}) \), the reconstruction can be accomplished and all secret \( S \in Z_{m_{0}} \) can be reconstructed. The work and schemes proposed before all assume that the exchange of the encrypted shares are all correct and without malicious forgery. But this can’t be ensured when the participants are corrupted and act as a malicious attacker. But in the scheme here, the authentication process ensures that the second cheating problem exists in this phase can be eliminated too.

In other parts of the paper, we will use a concision form \( S_{CRT} \rightarrow \{S_{i}\}_{i \in \{1, \ldots, n\}} \) to indicate secret \( S \)'s division into \( n \) shares \( \{S_{i}\}_{i \in \{1, \ldots, n\}} \) according to CRT-VSS scheme with prime modulus parameters \( \{m_{i}\}_{i \in \{1, \ldots, n\}} \). Similarly, \( S_{CRT} \) represents secret reconstructed by coalition \( I = \{i_{k}\}_{k \in \{1, \ldots, t\}} \), \( |I| = t \) as Equation (1)

V. HOMOMORPHISM PROPERTY FOR CRT-VSS SCHEME, BASIS AND PRELIMINARIES

In this section, we will explore the homomorphic property of CRT-VSS scheme and do some preliminaries for the trusted ECC-DSS distributed authentication scheme later. It’s only mentioned in \( [4, 8] \) that the extended Asmuth-Bloom scheme has homomorphic proprieties but without any proof about it and such discussion about the homomorphic property of CRT-VSS scheme is rare.

**Lemma V.1.** For any coalition \( |I| = t \), the notation \( CRT_{I} \) donates the reconstruction scheme, then all CRT-VSS scheme has \( \langle \oplus, \otimes \rangle \) and \( \langle \otimes, \otimes \rangle \)-homomorphism property, whenever \( S_{1} = CRT_{I}(\{S_{1}'\}_{k \in \{1, \ldots, t\}}) \), \( S_{2} = CRT_{I}(\{S_{2}'\}_{k \in \{1, \ldots, t\}}) \), then

\[
\langle \oplus, \otimes \rangle. S_{1} \oplus S_{2} \equiv CRT_{I}(\{S_{1}' \oplus S_{2}'\}_{k \in \{1, \ldots, t\}})
\]

\[
\langle \otimes, \otimes \rangle. S_{1} \otimes S_{2} \equiv CRT_{I}(\{S_{1}' \otimes S_{2}'\}_{k \in \{1, \ldots, t\}})
\]

**Proof:** The proof is simple and easy similar to the proof on Shamir’s scheme in \( [4] \) and we neglect it here.

Among the technical difficulties for designing distributed authentication schemes, the secure shares-product sharing scheme combing shares of two secrets, e.g. \( a \) and \( b \), into shares of the product of \( a \cdot b \), e.g. \( \{a_{i}b_{i}\}_{i \in \{1, \ldots, n\}} \) is outstanding and important. Though related work and discussion can be seen in \( [3, 8] \), but they are all designed with Shamir’s polynomial VSS scheme. Here we will first do some description about them, then design the corresponding scheme for CRT-VSS scheme.

**Lemma V.2.** The scheme combing shares of two secrets, e.g. \( a, b \in Z_{n} \), into shares of the product of \( a \cdot b \) with Shamir’s polynomial VSS scheme is as follows.

1) **Setup work.** With \((t, n)\) Shamir’s VSS scheme, shares \( \{a_{i}\}_{i \in \{1, \ldots, n\}} \) and \( \{b_{i}\}_{i \in \{1, \ldots, n\}} \) are generated by random polynomial denoted as \( f_{a} \) and \( f_{b} \) with secret \( a \) and \( b \) respectively.

\[
e.g. a \xrightarrow{f_{a}} \{a_{i}\}_{i \in \{1, \ldots, n\}}, \quad b \xrightarrow{f_{b}} \{b_{i}\}_{i \in \{1, \ldots, n\}}.
\]

2) **Refreshing shares.** \( \{c_{i}\}_{i \in \{1, \ldots, n\}} \) are generated by random polynomial \( f_{c} \) with secret 0. e.g. 0 \( \xrightarrow{f_{c}} \{c_{i}\}_{i \in \{1, \ldots, n\}} \).

3) **After participant node \( P_{i} \) exchange \( v_{i} = a_{i}b_{i} + c_{i} \), each Participant node \( P_{i} \) locally computes \( ab = F_{1}(\{v_{i}\})_{i \in \{1, \ldots, t\}} \), the scheme is \((t-1)\) eavesdropping adversary tolerant secure under the \((t, n)\) threshold condition.

The details about the correctness and security analysis can be seen in \( [3, 8] \). The scheme consists of two main steps, degree reduction step ensure the correctness and randomization step strengthen the security. Since 0 \( \xrightarrow{f_{c}} \{c_{i}\}_{i \in \{1, \ldots, n\}} \), the reconstruction is without retortion. Furthermore, in order to maintain the \((t-1)\) eavesdropping adversary tolerant secure property(against the decompose attack towards Shamir’s polynomial), the additional permutation term \( c_{i} \) is necessary to keep the Shamir’s polynomial secret and irreducible. However, the weakness of the Shamir’s scheme doesn’t exist in CRT scheme and there is no need to keep such permutation term. Thus we can design the corresponding scheme with better concision and equal security.

**Theorem V.1.** The scheme combing shares of two secrets, e.g. \( a \) and \( b \), into shares of the product of \( a \cdot b \) with CRT scheme is \((t-1)\) eavesdropping adversary tolerant as follows.

1) **Setup work.** CRT scheme parameters \( \{m_{i}\}_{i \in \{1, \ldots, n\}} \).

\[
M = \prod_{1 \leq i \leq n} m_{i}, \quad a, b \in Z_{M}, \quad ab \in Z_{M}.
\]

With CRT
scheme, e.g. \( a \xrightarrow{CRT} \{a_i\}_{i \in \{1,\ldots,n\}}, \) \( b \xrightarrow{CRT} \{b_i\}_{i \in \{1,\ldots,n\}} \).

2) After participant node \( P_i \) exchange \( v_i = a_i b_i \mod n \), each Participant node \( P_i \) locally computes \( ab = \text{CRT}_t\{v_i\}_{i \in \{1,\ldots,t\}} \).

the scheme is \((t - 1)\) eavesdropping adversary tolerant secure under the \((t, n)\) threshold condition.

Proof: We will prove the correctness and security respectively.

1) Correctness. According to Lemma \([\text{V.1}]\)

\[
\text{CRT}_t(\{v_i\}_{i \in \{1,\ldots,t\}}) = \text{CRT}_t(\{a_i b_i\}_{i \in \{1,\ldots,t\}}) = \text{CRT}_t(\{a_i\}_{i \in \{1,\ldots,t\}} \cdot \text{CRT}_t(\{b_i\}_{i \in \{1,\ldots,t\}})
\]

So the reconstruction of \( ab \) maintains well.

2) Security. On one hand, the reconstruction of \( ab \) is \((t - 1)\) tolerant, on the other hand, though the scheme is designed without such refreshing permutation term \( c_i \), while the weakness of decomposing for Shamir’s scheme doesn’t exist in CRT scheme. So the adversary can’t get more information like the decomposing attack with Shamir’s scheme. Hence, the scheme is \((t - 1)\) eavesdropping adversary tolerant secure.

According to Theorem \([\text{V.1}]\), the secure shares-product sharing scheme based on CRT scheme has better concision(without the refreshing permutation term \( c_i \)) compared with Shamir’s scheme in Lemma \([\text{V.2}]\) From Lemma \([\text{V.1}]\) and Theorem \([\text{V.1}]\)

VI. A DISTRIBUTED ECC-DSS AUTHENTICATION SCHEME BASED ON CRT-VSS AND TRUSTED COMPUTING

Among the schemes about the distributed authentication protocol, the robust Digital Signature Standard(DSS) signature scheme proposed in [8] is significant, it can achieve security in different conditions. While the scheme is built on the basis of Shamir’s scheme and the complexity of exponent arithmetic is high. Besides, the biggest problem is that the validation of the secret shares in the node and the trusted property may lead to DoS and fault attack security problem. The distributed ECC-DSS authentication scheme based on CRT-VSS and trusted computing proposed later will solve those problems together. The whole protocol consists of three phases, including trusted choice phase, signature generation phase and signature verification phase.

A. trusted choice phase

Suppose the request node \( A \) apply for a signature for distributed authentication with message \( M \). Here we don’t focus on a password-protected mechanism [2] with secret sharing in some e-commercial scenarios(e.g., [10]) and don’t contain it explicitly. The details are as follows.

1) Compute the digest of the message \( M \) with some Hash function, e.g. \( m=\text{SHA}-1(M) \).

2) A choose \( n \) trusted authentication nodes \( P_i(1 \leq i \leq n) \) to ask for distributed authentication. The process is similar to that in distribution phase in CRT-VSS scheme in section [IV]

- A \( \xrightarrow{\mathbb{B}} \{P_i\}_{i \in \mathbb{P}} \): \( id_D, \text{REQUEST} \).
- \( P_i \rightarrow A \): \( id_i, id_A, \text{auth}(id_i, id_A) \).

After the authentication process, request node \( A \) choose \( t \) qualified nodes as a coalition \( C \) to finish the distributed authentication later.

3) Request node \( A \) broadcast the digest \( m \) to the nodes in coalition \( C \) as \( A \xrightarrow{\mathbb{B}} \{P_i\}_{i \in \mathbb{C}} \): \( id_A, m \).

B. DSS signature generation phase

This phase will do the generation work of the DSS signature. The scheme will be founded on the basis of Elliptic curve and the DSS signature[8]. Here are some notations and preliminaries before the phase. Suppose point \( G \) is one point on the Elliptic curve \( E_p(a, b) \in \mathbb{Z}_q \) with the order \( q, f \) is a big prime. The secret private key \( S \) has been divided into \( S_i \)'s and shared by node \( P_i \) respectively. On the Elliptic curve, the public key denoted as \( Q = SG \). Some necessary constrains: \( 0 \leq S_i \leq q, Mc \leq M \leq n \). The notation \( \text{CRT}_t(\{\lambda_1\}_{i \in \{1,\ldots<n\}}) \) represents that number \( \lambda \) can be divided into shares \( \{\lambda_i\}_{i \in \{1,\ldots<n\}} \) with our CRT scheme proposed in section [III] and notation \( \lambda = \text{CRT}_t(\{\lambda_i\}_{i \in \{1,\ldots,t\}} \) represents the reconstruction of \( \lambda \) with \( t \) shares \( \{\lambda_i\}_{i \in \{1,\ldots,t\}} \). The details of the protocol are as follows.

1) Setup work. Choose random values \( k, a \in \mathbb{Z}_f \), \( k \xrightarrow{\text{CRT}} \{k_i\}_{i \in \{1,\ldots,n\}}, a \xrightarrow{\text{CRT}} \{a_i\}_{i \in \{1,\ldots,n\}} \).

2) Participant node \( P_i \) broadcast \( v_i = k_i a_i \mod n \) and \( w_i = a_i G \mod n \). After collecting the \( v_i \)'s and \( w_i \)'s, each Participant node \( P_i \) locally computes

- \( ka = \text{CRT}_t(\{v_i\}_{i \in \{1,\ldots,t\}} \mod f) \)
- \( aG = \text{CRT}_t(\{w_i\}_{i \in \{1,\ldots,t\}} \mod f) \)
- \( \text{Compute } r = k^{-1}G = (ka)^{-1}(aG) \mod f \)

3) Each Participant node \( P_i \) broadcasts \( sig_i = k_i (m+rS_i) \). After collecting, each Participant node \( P_i \) computes global signature \( sig = \text{CRT}_t(\{sig_i\}_{i \in \{1,\ldots,t\}} \mod f) \)

4) Output the \((r, s)\) as the signature for \( m \).

C. signature verification phase

1) Check the public parameters and limitations, do the digest as \( m=\text{SHA}-1(M) \);

2) Do some preliminaries. \( w = s^{-1}, u_1 = mw \mod n, u_2 = rw \mod f) \);

3) Use the public key \( Q = SG \mod f \) to do the verification as \( X = u_1 G + u_2 Q \). If \( X = 0 \), then declare it as a fault signature, or else suppose \( X = (x, y) \), compute \( v = x \mod f \), if \( v = r \) holds, then accept the signature, or else refuse.

Here we will do some simple analysis about the correctness of the protocol and its security analysis will be done later in section [VIII] separately. The proof of the correctness will have two aspect on the \( s \) and \( r \) respectively.

1) Correctness of \( r \). According to the protocol above,
r = (ka)^{-1}(aG)(mod f)
= (CRT_1(v_i))_{i \in \{1,\ldots,t\}} \cdot CRT_1\{w_i\}_{i \in \{1,\ldots,t\}}(mod f)
= (CRT_1(k,v_i))_{i \in \{1,\ldots,t\}} \cdot CRT_1\{aG\}_{i \in \{1,\ldots,t\}}(mod f)
≡ (ka)^{-1} \cdot (aG)(mod f) ≡ k^{-1}G(mod f).

2) Correctness of sig, combining the equation r = k^{-1}G which has been proved in (1),
sig = CRT_1\{sig_i\}_{i \in \{1,\ldots,t\}}(mod f)
= CRT_1\{k(m + rS_i)\}_{i \in \{1,\ldots,t\}}(mod f)
≡ km + rCRT_1\{k\}_{i \in \{1,\ldots,t\}}CRT_1\{S_i\}_{i \in \{1,\ldots,t\}}(mod f)
≡ km + rkS(mod f).

It is the right shape of the DSS signature s.

VII. SOME DISCUSSION ABOUT SHARES REFRESHING SCHEME

A. refreshing discussion on Shamir’s scheme

In the scheme proposed in [24] and other literature, the traditional refreshing scheme of the shares use the \((\oplus, \oplus)\)-homomorphism property for Shamir’s scheme [21]. Each participant nodes randomly generate refresh shares and divide them into subshares by randomly chosen t-degree random Shamir’s polynomial with sharing of 0. After participants finish exchanging the subshares, they refresh the shares with subshares by addition operation. The literature before all designs the share refreshing scheme with the Shamir’s threshold scheme in [24,22]. They all set the refreshing shares 0, then generate the refreshing subshares. Here we will extend such refreshing schemes based on \((\oplus, \oplus)\)-homomorphism property for Shamir’s scheme.

Theorem VII.1. The refreshing scheme based on \((\oplus, \oplus)\)-homomorphism property for Shamir’s scheme [21] is correct as follows. For each participant P_i, Refreshing shares(RS) constraints and subshares generation:

\[
\begin{cases}
\sum_{i=1}^{n} w_i = 0 & (a) \\
w_i \xrightarrow{\cdot} \{w_{ij}\}_{j \in \{1,\ldots,n\}} & (b)
\end{cases}
\]

Refreshing operation:

\[S'_j = S_j + \sum_{1 \leq i \leq n} w_{ij} (\mod m_i)
\]

Proof: The proof is easy and similar to [21], thus we neglect it due to the space limitation here. □

Obviously, the correctness of refreshing scheme above is built on the assumption that all the participants are correct and honest. As mentioned before that the lack of the trusted assurance of the generation of the refreshing subshares will result to the retortion problems of the secret S.

B. refreshing discussion on CRT-VSS scheme

Now we discuss the refreshing scheme on CRT-VSS scheme, combining the \((\oplus, \oplus)\) and \((\times, \otimes)\) property in Lemma [6], we can easily get two trivial distributed refreshing schemes in the similar way as Shamir’s scheme in Theorem VII.1 which can furthermore deduce a general mixed scheme. Here We skip the illustration of such two trivial schemes and derive the mixed scheme below.

Proposition VII.1. The \((\oplus, \oplus)\) and \((\times, \otimes)\)-mixed scheme based on CRT-VSS scheme is correct as follows. RS constraints and subshares generation:

\[
\begin{cases}
\prod_{i=1}^{k} w_i \equiv 1, \sum_{i=k+1}^{n} w_i = 0 (\mod M) & (a) \\
\{w_i\}_{i \in \{1,\ldots,k\}} \neq 1 & (mod M) \quad (b) \\
\{w_i\}_{i \in \{k+1,\ldots,n\}} = 0 & (mod M) \quad (c)
\end{cases}
\]

Refreshing operation:

\[S'_j = S_j \times \prod_{1 \leq i \leq k} w_{ij} + \sum_{i=k+1}^{n} w_{ij} (\mod m_i)
\]

We conduct modulus arithmetic operation,

\[S'_j \mod m_i = S_j \times \prod_{1 \leq i \leq k} w_{ij} + \sum_{k+1 \leq i \leq n} w_{ij} (\mod m_i)
\]

\[\equiv S_j \times (\prod_{1 \leq i \leq k} w_{ij} mod m_i) + (\sum_{k+1 \leq i \leq n} w_{ij} mod m_i)
\]

\[\equiv S_j \times 1 + 0 \equiv S_j
\]

Obviously, the refreshing results has little function under the modulus m_i operation. So the three trivial refreshing schemes aren’t correct refreshing schemes for CRT-VSS scheme.

Unluckily, besides the failure with trivial schemes above, we can have the furthermore upsetting conclusion as follows.

Theorem VII.2. Without changing the prime parameters set \(\{m_i\}_{i \in \{1,\ldots,n\}}\), the refreshing scheme without the retortion problems can not avoid the refreshing failure under the modulus m_i operation, formally denoted as

\[
\begin{cases}
S = CRT_1\{S'_i\}_{i \in \{1,\ldots,n\}} & (a) \\
S = CRT_1\{S'_i\}_{i \in \{1,\ldots,n\}} & (b)
\end{cases}
\]

\[\Rightarrow S_i \equiv S'_i (mod m_i) \quad (1 \leq i \leq n).
\]

Proof: The proof can be finished in two different ways. Here we just provide one due to the constraints of limitation. According to the equation (a),(b), suppose the reconstruction result before the modulus operation

\[
\begin{cases}
A = \sum_{k=1}^{t} S_i M_{C,i\setminus\{i\}} \\
A' = \sum_{k=1}^{t} S'_i M_{C,i\setminus\{i\}}
\end{cases}
\]

and

\[
\begin{cases}
S = S_i (mod m_i) \\
S' = S'_i (mod m_i)
\end{cases}
\]

If the reconstruction without retortion holds, then
TABLE I
COMPARISON BETWEEN SHAMIR’S SCHEME AND CRT SCHEME

| Comparison Items                  | Shamir’s | CRT          |
|----------------------------------|----------|--------------|
| Computation complexity           | O(t)log^t | O(t)         |
| Reconstruction parameter related | No       | Yes.         |
| Fit distributed refreshing       | Yes      | No.          |
| Fit TTP refreshing               | No       | Yes.         |
| Homomorphism property            | (⊕, ⊕)  | (⊕, ⊕)  |
| Secure shares-product concision   | Not good | Good         |

\[ A \equiv A’ (mod \; M) \Rightarrow \exists k \in \mathbb{Z}, A = kM + A’ \]

hence, we can get
\[ A(mod \; m_i) = kM + A’ (mod \; m_i) \]
\[ \equiv (kM + \mod \; m_i) + A’ \mod \; m_i \]
\[ \equiv 0 + S’(mod \; m_i) \equiv S’ . \]

So such refreshing scheme results to little function by modulus operation with \( \{m_i\}_{i \in \{1, \ldots, n\}} \).

According to the Theorem VII.2, a correct refreshing scheme for CRT-VSS scheme need to change the prime parameters set \( \{m_i\}_{i \in \{1, \ldots, n\}} \), the essence of the necessity is the newly computation of a new share \( S_i’ \) modulus new \( m_i’ \)’s. And this will bring to some requirements as follows:

- The right computation of new share \( S_i’ \) modulus new \( m_i’ \)’s; The computation should be right and dependable, this can be done by a Trusted Third Party(TTP).
- The right and secure delivery of the new shares; this can be easily done by secure channel or by cryptography mechanisms.
- The right update of the shares and remove of the old ones. This can be assured by the normal process of the protocol, so it can be ensured by the trusted computing techniques.

C. comparison between Shamir’s and CRT scheme

From the analysis in Section VII-A and VII-B we note that there exist some differences between CRT and Shamir’s schemes. Thus we summarize these comparison results in Table I.

VIII. SECURITY ANALYSIS

In this section, we will prove the security of simple authentication framework between two nodes with CSP [11] model and rank function [20]. Then extend the secure assurance to multi-run and arbitrary size authentication case. After that we model our CRT-VSS scheme, ECC-DSS authentication scheme based on trusted computing and prove the security of the schemes.

The TPM can provide the authentication property towards the remote party in a secure and private way. The security of our protocol is built on the assumptions including: 1)TPM can not be compromised by the intruder maintaining a right property. By now, there is no successful way to compromise the TPM without any damage of the right property of TPM. Whatever happens, the TPM can record and reveal the property of software/hardware within the node. 2) The security of DAA and PBA protocol. It is mentioned in [5] and [6] that the two protocol was secure under different assumptions. Besides, the cryptographic primitives, including encryption \( \{\cdot\} \) and signature \( S_k(\cdot) \) we employed are secure. We will formally model the protocol and security property based on communicating sequential processes(CSP) [11], and use the rank function [20] to do some analysis and proof furthermore.

A. The preliminaries

1) The network: The network considered below will consist of several different parties: Distributor denoted as \( D \), Applicant for distributed authentication signature denoted as \( A \), Users acted as the participants with the secret shares denoted as set \( USER \) and the intruder denoted as \( ENEMY \). The model of the network is standard, based on the Dolev-Yao model. The \( ENEMY \) is in full control of the network, and all communication passes through it. The attacker may choose to block or redirect messages, send messages of its choice, and imitate as other users. The capabilities of the attacker are bounded by a finite set of deductions, by which it can generate new messages from the set of messages it already knows. Let “generates” relation \( X \vdash m \) denotes that new message \( m \) is deduced from a set \( X \) of messages. Here if \( m,n \) are messages, \( k \) is a key and \( k^{-1} \) is the inverse of \( k \), then the smallest \( \vdash \) relation satisfies. \( \{m,n\} \vdash m.n/m/n \), \( \{m,n\} \vdash \{m\}_k/S_k(m) \), \( \{\{m\}_k\}_k^{-1} \vdash m \), etc.

2) Authentication: Authentication can be captured in terms of events whose occurrence guarantees the prior occurrence of other messages. The key property is concerned with precedence between events. Conditions can be expressed as traces specifications on \( NET \), requiring that no event from a set \( T \) has occurred unless another event from a set \( R \) has previously occurred, which can be illustrated as: \( NET \; sat \; R \; precedes \; T \). The rank function approach proposed in [20] assigns a rank 0 to the messages that should remain secret during the protocol and a rank 1 to the messages that should be public and might be get held by the enemy. The central rank theorem from [20] gives four conditions the rank function must satisfy in order to maintain the rank with the rank function.

B. Security analysis for simple authentication framework between two nodes

1) Simplification for protocol: The protocol framework illustrated in Figure 1 can be simplified as Figure 2

Fig. 2. \( P_i \) request authentication to node \( P_j \)
2) Protocol modeling: single run and node I as the initiator and node J as the responder.

Node I can be modeled as follows.

\[ USER(I) = \square_{i}^{\text{trans.i.j.id, id}_j.\text{INIT}} \]
\[ \rightarrow \text{recv.i.id, id}_1, n_1 \]
\[ \rightarrow \text{trans.i.j.id, id}_j, \text{im, DS}_i, \text{PS}_i, \text{K_INFO}_J \]
\[ \rightarrow \text{recv.i.id, id}_i, \text{S}_{pk_AIK_J}((\text{K_INFO}_J)[id_j]), \{id_j\}_{n_2}, k_{ij} \rightarrow \text{STOP} \]

Node J can be modeled in a similar way as follows.

\[ USER(J) = \text{recv.j.id, id}_j.\text{INIT} \]
\[ \rightarrow \text{trans.j.id, id}_1, n_1 \]
\[ \rightarrow \text{recv.j.id, id}_j, \text{im, DS}_i, \text{PS}_i, \text{K_INFO}_J \]
\[ \rightarrow \text{validate}&\text{Signal.Trust.i.n}, n_1, n_2 \]
\[ \rightarrow \text{trans.j.id, id}_j, \text{S}_{pk_AIK_J}((\text{K_INFO}_J)[id_j]), \{id_j\}_{n_2}, k_{ij} \rightarrow \text{STOP} \]

The rank function for the authentication framework between the two nodes are as in Figure 3.

\[ \text{ENEMY}(X) = \text{trans.i.j.id, id}_j, \text{im}, \text{DS}_i, \text{PS}_i, \text{K_INFO}_J \]
\[ \text{recv.i.id, id}_1, n_1 \]
\[ \text{recv.j.id, id}_j, \text{im}, \text{DS}_i, \text{PS}_i, \text{K_INFO}_J \]
\[ \text{STOP} \]

The rank function for the authentication framework between node P_i and P_j is as follows.

- User and host: \( \rho(id_i) = 1, \rho(\text{INIT}) = 1 \)
- Nonces: \( \rho(n_1) = 1, \rho(n_2) = 0 \)
- Integrity metrics: \( \rho(\text{im}) = 1 \)
- TPM signatures: \( \rho(\text{DS}_i) = 0, \rho(\text{PS}_i) = 0 \)
- K_INFO: \( \rho(g) = 1, \rho(g^r) = 0, \rho(g^s) = 0 \)
- Encryption: \( \rho(\{m\}_{k_{ij}}) = \rho(k_{ij}) \)
- AIK Signature: \( \rho(S_{pk_AIK_J}((\text{K_INFO}_J)[id_j])) = 0 \)
- Concatenations: \( \rho(n_1, n_2) = \min\{\rho(n_1), \rho(n_2)\} \)
- Events: \( \rho(\text{trans.i.j.id, id}_j, \text{im}, \text{DS}_i, \text{PS}_i, \text{K_INFO}_J) \)
- Signals: \( \rho(\text{Signal.Trust.i.n}) = 0 \)

Checking the four conditions above is easy and just routine. We neglect the details due to the limitation of paper. With the rank function in the figure above, we can see that:

- The messages in the initial set X known to the ENEMY process are of rank 1.
- Any message generated by ENEMY from X under the deductions rules has a rank pub whenever messages in X have a rank 1.
- \( \rho(\text{Signal.Trust.i.n}) \) in R has a rank 0.
- When R = \{trans.i.j.id, id_j, im, DS_i, PS_i, K_INFO_J\} is blocked, user can never give out a message of rank 0 unless it has previously received one.

So according to the central theorem of rank function, the rank function above can maintain the rank well. Furthermore, the authentication property of the simple authentication framework can be proved secure well.

C. Security analysis for multi-run and arbitrary size authentication framework

In this subsection we show that if there is an attack up on the authentication framework of arbitrary size with multi-run given in the subsection before, then there is an attack up on the small framework described above with a single initiator A and a single responder B. Similar work with the classical Needham-Schroeder Public-Key Protocol can be seen in [16]. The process of proof can be carried out by two aspects from the attacks upon responder and initiator respectively.

1) Attacks upon the responder: Consider a run \( \alpha \) where the intruder imitates the initiator A to attack the responder B as in Figure 4.

```
Message \( \alpha.1 \) \( I(A) \rightarrow B: A, B, \text{INIT} \)
Message \( \alpha.2 \) \( B \rightarrow I(A): B, A, n_1 \)
Message \( \alpha.3 \) \( I(A) \rightarrow B: A, B, \text{im}_A, DSA_A, PSA_A, g^x \)
Message \( \alpha.4 \) \( B \rightarrow I(A): B, A, g^\gamma, \{B\}_{n_2} k_{AB} \)
Message \( \alpha.5 \) \( I(A) \rightarrow B: A, B, \{A\}_{n_2} k_{AB} \)
```

Fig. 4. Intruder imitates initiator A to attack responder B

In order to attack upon the responder B successfully, the intruder needs to produce message 1, 3 and 5. Message 1 can be produced easily. As to the message 3, the intruder can produce it by replay the signature \( DSA_A \) and \( PSA_A \) by eavesdropping the message with the signatures inside. While the get of the message 5 can be rather difficult since the session key \( k_{AB} \) only established between the real A and B party. So the intruder cannot decrypt the message \( \alpha.4 \), so he learns neither \( n_2 \) nor \( \{A\}_{n_2} k_{AB} \) from message \( \alpha.4 \), denoted as: \( \neg \) (Intruder learns \( n_2 \) or \( \{A\}_{n_2} k_{AB} \) from \( \alpha.4 \)).

According to the analysis delivered in [16], the way to solve this problem for intruder is to replay the encrypted part of message \( \alpha.4 \) in message 4 in another run called \( \beta \), and learns the \( n_2 \) or \( \{A\}_{n_2} k_{AB} \) from message \( \beta.5 \). Note that the initiator of run \( \beta \) must be A. Hence the form of the run \( \beta \) should be as follows in Figure 5. Now we see that the intruder learns

```
Message \( \beta.1 \) \( A \rightarrow I(B): A, B, \text{INIT} \)
Message \( \beta.2 \) \( I(B) \rightarrow A: B, A, n_1 \)
Message \( \beta.3 \) \( A \rightarrow I(B): A, B, \text{im}_A, DSA_A, PSA_A, g^x \)
Message \( \beta.4 \) \( I(B) \rightarrow A: B, A, g^\gamma, \{B\}_{n_2} k_{AB} \)
Message \( \beta.5 \) \( A \rightarrow I(B): A, B, \{A\}_{n_2} k_{AB} \)
```

Fig. 5. The form of auxiliary run \( \beta \)

the component \( \text{im}_A, DSA_A, PSA_A, g^x \) from Message \( \beta.3 \) and replays this in \( \alpha.3 \), learns the component \( \{A\}_{n_2} k_{AB} \) from Message \( \beta.5 \) and replays this in \( \alpha.5 \). So only these two runs are necessary for the intruder to learn all the knowledge it uses in the attack upon the responder. Thus if the intruder can
imitate the initiator A to attack the responder B then such an attack would have been found by considering the small system above.

2) Attacks upon the initiator: Similar analysis can be done and we neglect it here.

Combining these two aspects, we can get the conclusion that our scheme with multi-run and arbitrary size can also be secure under authentication property.

D. Security analysis for the complete protocols in this paper

1) Distributing phase for CRT-VSS scheme: The model of process $T$ characterize trusted election before the distributing process, $D$ dealing with the distributing procedure. Process $USER$ represents the behavior of node during the first phase in the CRT-VSS scheme.

$$
T_i (i) = \text{trans} . D . i . id_D , \text{AUTH} , S_{sk_{AIK_D} (id_D)} \\
\rightarrow \text{recv} . D . i . id , \text{auth}(id_i , id_D) \\
\rightarrow \text{validate} & \text{signal} . \text{Trust} . \text{..} \\
\rightarrow \text{STOP}
$$

$$
T_R (i) = \text{recv} . D . i . id_D , \text{AUTH} \\
\rightarrow \text{trans} . D . i . id_D , \text{auth}(id_i , id_D) \\
\rightarrow \text{recv} . D . i . id_D , \text{AGREE} \\
\rightarrow \text{STOP}
$$

$$
D (i) = \square_i \in \text{trans} . D . i . id_D , id_i , S_{sk_{AIK_D} (id_D)} , \text{enc}_{id_D} (S_i) \\
\rightarrow \text{STOP} \quad \square_i \in \text{trans} . D . i . id_D , id_i , S_{sk_{AIK_D} (id_D)} ||| id_i ||| \text{PUBLIC} \\
\rightarrow \text{STOP}
$$

$$
USER_R (i) = \text{recv} . i . id_D , id_i , \text{AUTH} , S_{sk_{AIK_D} (id_D)} \\
\rightarrow \text{validate} (S_{sk_{AIK_D} (id_D)}) \\
\rightarrow \text{trans} . i . D . id_D , \text{AGREE} \\
\rightarrow \text{STOP}
$$

$$
USER_I (i) = \text{trans} . i . D . id_D , id_i , \text{AUTH} \\
\rightarrow \text{trans} . i . D . id_D , \text{auth}(id_i , id_D) \\
\rightarrow \text{validate} (\text{auth}(id_i , id_D)) & \text{signal} . \text{Trust} . . . \\
\rightarrow \text{trans} . i . D . id_D , \text{AGREE} \\
\rightarrow \text{STOP}
$$

The process $T_R (i)$ and $USER_I (i)$ are similar to $T_I (i)$ and $USER_R (i)$. The entire network is thus modeled as follows.

$$
D = \text{trans} . i . D . id_D , \text{AUTH} \\
T (i) = ((T_i (i) ||| T_R (i)), T = \square_i \in \text{filter} (T (i)), \\
USER (i) = (((USER_I (i)) ||| USER_R (i)), \\
USER = \square_i \in \text{USER} (i), \\
\text{DIS} = \text{USER} (i) ||| \text{TRAN}, D \\
\text{NET} = \text{DIS} \quad ||\quad \text{ENEMY}
$$

The objective of the distribution phase scheme is $R = \{\text{trans} . i . D . id_D , \text{auth}(id_i , id_D) \& \text{trans} . D . i . id_D , \text{auth}(id_D , id_i)\}$ precedes $T = \{\text{trans} . i . D . id_D , \text{AGREE}\}$.

2) Reconstruction phase for CRT-VSS scheme: In this phase, the participant exchange the secret shares with each other. Each node send and receive and shares at the same time as initiator and responder respectively.

$$
USER_R (J) = \square_i \in \text{recv} . i . j . id_j , id_i , \text{AUTH} \\
\rightarrow \text{trans} . i . j . id_j , \text{auth}(id_i , id_j) \\
\rightarrow \text{recv} . i . j . id_j , \{S_i\}_{k_{ij}} \\
\rightarrow \text{STOP}
$$

$$
USER_I (J) = \square_i \in \text{trans} . i . j . id_i , id_j , \text{AUTH} \\
\rightarrow \text{recv} . i . j . id_i , \text{auth}(id_j , id_i) \\
\rightarrow \text{validate}(\text{auth}(id_j , id_i)) & \text{signal} . \text{Trust} . . . \\
\rightarrow \text{trans} . i . j . id_i , \{S_i\}_{k_{ij}} \\
\rightarrow \text{STOP}
$$

The entire network for the reconstruction scheme is thus modeled as follows.

$$
USER = \square_i \in \text{USER} (i), \\
\text{NET} = USER \text{ ||| ENEMY}
$$

Similarly, the objective of the reconstruction scheme is as follows: $R = \{\text{trans} . i . j . \text{auth}(id_i , id_j)\}$ precedes $T = \{\text{trans} . i . j . \{S_i\}_{k_{ij}}\}$. Here we neglect the model details like above due to the limitation of paper.

3) Distributed ECC-DSS authentication scheme based on trusted computing: In the scheme, the node $A$ will request for the authentication and signature from the participant nodes. Each node will send and receive shares at the same time as initiator and responder respectively. The model can be described as follows.

$$
\text{REQA} (I) = \square_i \in \text{trans} . a . i . id_A , id_i , \text{REQU} \\
\rightarrow \text{trans} . A . i . id_i , \text{AGREE} \\
\rightarrow \text{trans} . A . i . id_i , \text{AUTH} \\
\rightarrow \text{trans} . i . A . id_A , \text{AGREE} \\
\rightarrow \text{trans} . A . i . m , \text{AGREE} \\
\rightarrow \text{STOP}
$$

$$
\text{USER}_A (I) = \square_i \in \text{trans} . a . i . id_A , id_i , \text{REQU} \\
\rightarrow \text{trans} . A . i . id_i , \text{AGREE} \\
\rightarrow \text{trans} . A . i . id_i , \text{AUTH} \\
\rightarrow \text{trans} . i . A . id_A , \text{AGREE} \\
\rightarrow \text{trans} . i . A . m , \text{AGREE} \\
\rightarrow \text{STOP}
$$

$$
\text{USER}_R (J) = \square_i \in \text{recv} . i . j . id_j , id_j , k_{ij} \\
\square_i \in \text{trans} . i . j . id_j , id_j , a_{ij} \\
\rightarrow \text{trans} . i . j . id_j , \text{AGREE} \\
\rightarrow \text{trans} . i . A . id_A , \text{AGREE} \\
\rightarrow \text{trans} . A . i . m , \text{AGREE} \\
\rightarrow \text{STOP}
$$

$$
\text{USER}_R (J) = \square_i \in \text{recv} . i . j . id_j , id_j , k_{ij} \\
\square_i \in \text{trans} . i . j . id_j , id_j , a_{ij} \\
\rightarrow \text{trans} . i . j . id_j , \text{AGREE} \\
\rightarrow \text{trans} . i . A . id_A , \text{AGREE} \\
\rightarrow \text{trans} . A . i . m , \text{AGREE} \\
\rightarrow \text{STOP}
$$

The process $USER_I (J)$ and $USER_R (J)$ is similar to $\text{REQA} (I)$ and $\text{USER}_I (J)$. So the entire network is thus modeled as follows.

$$
\text{USER} (I) = \square_i \in \text{USER} (I), \text{USER}_R = \square_i \in \text{USER} (R), \\
\text{NET} = \text{((REQA ||| USER) ||| ENEMY}}
$$
The objective of the scheme is: \( R = \{\text{trans}.A.i.d_1, \text{id}_A, \text{auth}(\text{id}_1, \text{id}_A)\} \) precedes \( T = \{\text{trans}.A.i.m\} \).

4) Overall analysis: From the CSP model illustrated above about the schemes above, we can see that: that the authentication objective essentially can be reduced to the authentication objective within the authentication framework between the single initiator and responder. (i) All the authentication objectives in the schemes has the essence that \( R \) precedes \( T \), here \( R \) can be summarized as the multi-run execution of the authentication framework \( \text{auth}(\cdot) \). In detail, in the reconstruction phase and distributed ECC-DSS scheme \( R = \{\text{trans}.U.U, \text{auth}(\cdot)\} \) represents the maintain unidirectional authentication property. While the bi-directional authentication property holds in distribution phase \( R \). And the \( T \) represents the corresponding service and action in different schemes. (ii) The security of multi-run execution of the authentication framework can be reduced to the single small authentication framework. The detailed proof can be seen in section VIII-C Take distribution phase scheme as example, the multi-run with arbitrary size authentication component, i.e \( \text{USER}(i), \text{USER}, T \), can be reduced to simple small framework instance. The \( \text{USER} \) component appears in the complete model of reconstruction and ECC-DSS model.

According to analysis from two aspects above, we can conclude that the security proof towards the authentication property for the small authentication framework can hold the authentication property of complicate multi-run and arbitrary case. So the authentication property of schemes can be maintained well above.

IX. CONCLUSION

In this paper, we proposed a secret key distributed storage scheme based on CRT-VSS and trusted computing, solving the two category cheating problems in the VSS area before. Then we explore and do overall analysis of the \((\oplus, \odot), (\odot, \odot)\) homomorphic property with the CRT-VSS scheme and design the corresponding secure shares-product sharing scheme base on CRT scheme. At last, on the foundation of the robust DSS scheme, our proposed ECC-DSS distributed authentication scheme based on trusted computing eliminates the possibility of the malicious attack, e.g. DoS attack and fault attack, during the process of signature generation. The security analysis proves that our schemes can maintain relative security assurance under certain conditions.

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