Study on Vibration Characteristics of Cables Excited by Lateral Displacement of Ends

Wen-gang YANG and Zhe ZHENG*
North China Electric Power University, Baoding, China 071003
*Corresponding author

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Abstract. Considering the axial vibration, the partial differential nonlinear vibration equations of the cable excited by the lateral displacement of the end is established. The Galerkin method is used to perform the modal truncation and transform the equations into the ordinary differential nonlinear equations. The typical examples are selected and the vibration characteristics are studied by numerical simulation. The conclusions of the study are as follows: when the frequency ratio is close to 2, the resonance phenomenon occurs. When the frequency ratio is greater than 3, the nonlinear characteristic is obvious. The amplitude of the in-plane vibration of the cable increases with the increase of the excitation amplitude, and small displacement excitation at the end causes large amplitude vibrations in the cable surface. The initial disturbance outside the plane is introduced. When the excitation frequency is close to 2.1 times the natural frequency of the in-plane vibration, the resonance phenomenon occurs and the amplitude is large.

Introduction

The end of the single span wire is not fixed and will be excited by the lateral displacement given by the insulator string. Therefore, the wire is simplified into a cable structure. It is engineering to study the vibration characteristics of the cable under the lateral displacement of the end.

In recent years, domestic scholars have studied the vibration characteristics of cable structures and achieved many results. In the literature [1], the two-dimensional vibration equation of stay cable is established and its resonance phenomenon is studied. In the literature [2], the two-dimensional vibration differential equation of stay cable is established, and the nonlinear coupled vibration characteristics of the cable under the end excitation are studied. The variation of the coupled vibration characteristics of the cable under different damping ratios and excitation amplitudes is discussed. In the literature [3], the vibration of the iron tower under the action of wind force is simplified to induce the displacement of the wire by the end displacement, and the bow-shaped vibration characteristics of the out-of-plane are studied.

Foreign scholar Irvine [4] first conducted a comprehensive and in-depth study on the cable, summed up a series of analytical theories. Later on, Benedettini [5] studied the coupled in-plane and out-of-plane finite free oscillations by a two degree-of-freedom model. M.EI-Attar [6] analyzed the effects of cable sag and phase difference between the input support excitations on the response. J.W. Larsen [7] studied the two states of the small sag cable, namely vibration and a whirling state. G. Ricciardi and F. Saitta[8] established a continuous vibration analysis model of cable considering bending stiffness and sag, and obtained dimensionless frequency and mode equations. Augelo Luongo [9] studied the analytical and numerical methods of nonlinear vibration of internal resonance cables. In summary, domestic and foreign scholars have done a lot of research on the mechanical properties of the cable, and established different models for different problems. The research methods have reference significance.

In this paper, considering the axial vibration, the three-dimensional vibration model of the cable under the lateral displacement excitation is established. The three-dimensional partial differential nonlinear equations are obtained, and the modal truncation is performed by the Galerkin method,
which transformed the equations into the ordinary differential nonlinear equations. The vibration characteristics are studied by numerical simulation.

**Analysis Model**

**Basic Assumption**

Fig. 1 shows the analytical model of the cable under the lateral displacement of the end. One end of the cable is fixed, and the other end is excited by a simple harmonic displacement. The coordinate system is established with the O point as the origin. The $u$, $v$ and $w$ are the dynamic displacement in the directions of $x$, $y$ and $z$, and the harmonic excitation of the end is: $U = U_0 \sin(\omega t)$.

![Figure 1. Analysis model of cable.](image)

The theoretical analysis of the cable is based on the following assumptions:

1. The cable has no stiffness, and its bending, torsion and shear stiffness are ignored. The stiffness of the horizontal cable has little effect on the shape of the curve, and cable can only withstand tensile forces.

2. The horizontal cable is a small vertical cable, and its gravity is evenly distributed along the chord direction.

**Establishment of Vibration Equation**

Using Newton's law to establish in-plane and out-of-plane three-dimensional vibration differential equations are as follows [4]:

\[
\begin{align*}
\frac{\partial}{\partial s} \left[ (T + \tau) \left( \frac{dx}{ds} + \frac{\partial u}{\partial s} \right) \right] &= m \frac{dx}{ds} \frac{\partial^2 u}{\partial t^2} + C_u \frac{\partial u}{\partial t} \\
\frac{\partial}{\partial s} \left[ (T + \tau) \left( \frac{dy}{ds} + \frac{\partial v}{\partial s} \right) \right] &= m \frac{dy}{ds} \frac{\partial^2 v}{\partial t^2} - m \frac{dx}{ds} g + C_v \frac{\partial v}{\partial t} \\
\frac{\partial}{\partial s} \left[ (T + \tau) \frac{\partial w}{\partial s} \right] &= m \frac{dx}{ds} \frac{\partial^2 w}{\partial t^2} + C_w \frac{\partial w}{\partial t}.
\end{align*}
\]  

(1)

Where $T$ is the axial tensile force of the cable, $\tau$ is the axial tensile force increment of the cable, $m$ is the linear density of the cable, and $C_u, C_v, C_w$ are viscous damping in the directions of $x$, $y$ and $z$.

The horizontal component of $\tau$ is expressed as:

\[ h = \tau \frac{dx}{ds}. \]  

(2)

The following expressions are obtained by static balance:
\[
\begin{align*}
\frac{d}{ds} \left( T \frac{dx}{ds} \right) &= 0 \\
\frac{d}{ds} \left( T \frac{dy}{ds} \right) &= -mg \frac{dx}{ds}
\end{align*}
\]  \quad (3)

Bringing Eq. (2) and Eq. (3) into Eq. (1), Eq. (1) can be reduced as:

\[
\begin{align*}
(H + h) \frac{\partial^2 u}{\partial x^2} &= m \frac{\partial^2 u}{\partial t^2} + C_u \frac{\partial u}{\partial t} \\
(H + h) \frac{\partial^2 v}{\partial x^2} + h \frac{d^2 y}{dx^2} &= m \frac{\partial^2 v}{\partial t^2} + C_v \frac{\partial v}{\partial t} \\
(H + h) \frac{\partial^2 w}{\partial x^2} &= m \frac{\partial^2 w}{\partial t^2} + C_w \frac{\partial w}{\partial t}
\end{align*}
\]  \quad (4)

Where \( H \) is the horizontal component of the initial axial force and \( h \) is the horizontal component of the axial dynamic pull. The expression of \( h \) [10] is:

\[
h = \frac{EA}{L_0} \left[ U + \int_0^l \frac{dy}{dx} dv + \frac{1}{2} \int_0^l \left( \frac{\partial u}{\partial x} \right)^2 \, dx + \frac{1}{2} \int_0^l \left( \frac{\partial v}{\partial x} \right)^2 \, dx + \frac{1}{2} \int_0^l \left( \frac{\partial w}{\partial x} \right)^2 \, dx \right],
\]  \quad (5)

\[
L_0 = \int_0^l \frac{ds}{dx} \, dx = \int_0^l \left[ 1 + \left( \frac{dy}{dx} \right)^2 + \left( \frac{dz}{dx} \right)^2 \right]^\frac{3}{2} \, dx.
\]  \quad (6)

**Galerkin Dispersion**

In order to perform nonlinear dynamic analysis on differential equations, the Galerkin method is needed to perform modal truncation. Since the first-order mode is the main component in the vibration of the cable, the first-order modal truncation is used here [11]. The function used for truncation is as shown in Eq. (7).

\[
\begin{align*}
u(x,t) &= \alpha(x) \beta(t) + U \frac{x}{l} \\
v(x,t) &= \phi(x) P(t) \\
w(x,t) &= \phi(x) Q(t)
\end{align*}
\]  \quad (7)

Where \( \alpha(x) \) is the axial first-order mode function, \( \phi(x) \) is the first-order mode function in the in-plane direction, and \( \phi(x) \) is the out-of-plane first-order mode function. \( U \) is the lateral displacement excitation, and \( l \) is the span.

After integral and simplification, Eq. (4) can be reduced as:

\[
\begin{align*}
a_1 \beta'' + a_2 \beta' + a_3 \beta + a_4 \beta P + a_5 \beta P^2 + a_6 \beta Q^2 + a_7 \beta U + a_8 \beta U^2 + a_9 \beta^2 U &= 0 \\
b_1 \beta'' + b_2 \beta' + b_3 \beta + b_4 \beta P + b_5 \beta P^2 + b_6 \beta Q^2 + b_7 \beta U + b_8 \beta U^2 + b_9 \beta &= 0 \\
+ b_{10} U^2 + b_{11} P U + b_{12} P U^2 + b_{13} P \beta U + b_{14} \beta U^2 + b_{15} \beta^2 &= 0 \\
c_1 \beta'' + c_2 \beta' + c_3 \beta + c_4 \beta P + c_5 \beta Q + c_6 \beta Q^2 + c_7 \beta U + c_8 \beta U^2 + c_9 \beta U &= 0
\end{align*}
\]  \quad (8)

The vibration frequency of the axial vibration is \( \omega_a = \sqrt{\frac{a_1}{a_4}} \), the vibration frequency of the in-plane
vibration is $\omega_v = \sqrt{\frac{b_1}{b_3}}$, and the vibration frequency of the out-of-plane vibration is $\omega_w = \sqrt{\frac{c_3}{c_1}}$.

Case Analysis

In the single span, the typical cable parameters are selected for simulation study. The cable parameters are shown in Table 1.

| parameters | $L$(m) | $m$(kg/m) | $E$(GPa) | $A$(mm$^2$) | $T$(kN) |
|------------|--------|-----------|-----------|-------------|---------|
| values     | 400    | 0.6507    | 66        | 215.48      | 17.15   |

Using the data in the table, the first-order natural frequency of the cable is calculated as:

$$
\begin{align*}
\omega_u &= 0.2027\text{Hz} \\
\omega_v &= 0.3214\text{Hz} \\
\omega_w &= 0.2027\text{Hz}
\end{align*}
$$

The initial parameters are selected: the lateral displacement excitation amplitude is 0.2m, and the frequency is equal to the natural frequency of the in-plane vibration; in the initial state, the in-plane and out-of-plane displacement and velocity are both 0; the damping effect is not considered. Using Matlab for numerical solution, the time-history curve of the midpoint of the cable is plotted as shown in Fig. 2.

![Figure 2. Time history curve of vibration amplitude in cable midpoint.](image)

As can be seen from the Fig. 2:

(1) Under the given initial parameters, the in-plane vibration of the cable is caused, and the amplitude of the vibration is much larger than the amplitude of the end excitation. So the micro-displacement excitation at the end will cause large in-plane vibration.

(2) Under the action of the end displacement excitation, the corresponding vibration is generated in the axial direction.

(3) There is almost no vibration outside the surface. Because there is no displacement or velocity outside the initial state, the motion of the cable only occurs in the plane.

In the experiment of Lintao Gao [7], the out-of-plane vibration of the cable under different initial parameters was studied. The following conclusions are consistent with the conclusions of this paper: When the frequency ratio of the excitation frequency to the natural frequency of the out-of-plane vibration is close to 2, it is found that the out-of-plane vibration produces a resonance phenomenon.
This verifies the correctness of the conclusions of this paper.

**Vibration Characteristics**

Change the initial parameters and study the influence of each parameter on the vibration characteristics of the cable. Since the end excitation mainly causes vibration in the cable surface, the following is mainly for the in-plane vibration development.

**Influence of Excitation Frequency**

The amplitude of the end excitation is taken as 0.2m, the frequency of the excitation is changed, and the influence of the excitation frequency on the vibration of the cable is studied. Here, the ratio of the end excitation frequency to the in-plane natural frequency is called the frequency ratio. The frequency ratio is changed from 0.1 to 3.5, and the maximum amplitude of the midpoint of the cable at the corresponding frequency is extracted. The amplitude-frequency curve is plotted as shown in Fig. 3.

![Amplitude-frequency curve of cable.](image)

As can be seen from the Fig. 3:

1. The amplitude of the vibration in the midpoint of the cable increases with the increase of the excitation frequency, but the amplitude of the change is not large.
2. When the frequency ratio is close to 2, resonance remarkably occurs. The accurately ratio is 2.091 instead of 2, which proves that the vibration of the cable is non-linear and the resonance frequency shifts to the right.
3. When the frequency ratio is greater than 3, the amplitude of the cable is significantly increased. The vibration becomes more disordered and the nonlinearity is more pronounced.

**Influence of Excitation Amplitude**

Keep the frequency of the end excitation constant, take the obvious resonance frequency of 2.091, adjust the amplitude of the excitation, and plot the in-plane vibration displacement time history of the amplitudes of 0.1m, 0.2m and 0.3m as shown in Fig. 4.

![Time history curve of vibration displacement of cables under different excitation amplitudes.](image)

As can be seen from the above figure that the in-plane vibration of the cable increases with the
increase of the excitation amplitude. And the vibration amplitude is more than 10 times the excitation amplitude, indicating that a small excitation amplitude will cause large vibration in the cable surface.

Influence of Damping

In order to study the influence of damping on the in-plane vibration, the damping coefficient $C_v$ is introduced. The amplitude and frequency of the end excitation are kept constant. The vibration displacement time history curves with damping values of 0.01, 0.05, 0.1, 0.15 and 0.5 are respectively plotted as shown in Fig. 5.

![Figure 5. Time history curve of vibration displacement at the midpoint of cable under different damping.](image)

It can be seen from the above figure that the amplitude of the in-plane vibration is attenuated after the damping coefficient is introduced. As the damping coefficient increases, the rate of in-plane amplitude attenuation increases, but the amplitude does not change much during steady state.

Conclusions

In this paper, considering the influence of axial vibration, the three-dimensional vibration model of the cable under the lateral displacement of the end is established, and the differential equations of vibration are obtained. Considering the coupled vibration, the first-order truncation is performed by Galerkin method, and the simulation is solved by Runge-Kutta method. After studying the vibration characteristics of the cable, the following conclusions are found:

(1) When the ratio of the excitation frequency to the natural frequency of the in-plane vibration is close to 2, the resonance phenomenon obviously occurs; when the frequency ratio is greater than 3, the vibration of the cable becomes disordered and the nonlinear characteristic is obvious. This is consistent with the experimental comparison.

(2) The amplitude of the in-plane vibration of the cable increases with the increase of the excitation amplitude, and the amplitude of the vibration is more than 10 times of the excitation amplitude, indicating that the slight end excitation causes a large vibration in the cable surface.

(3) After the damping coefficient is introduced, as the damping coefficient increases, the amplitude of the in-plane amplitude attenuation increases, but the amplitude does not change much during steady-state vibration.
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