Efficient Refreshing Protocol for Leakage-Resilient Storage Based on the Inner-Product Extractor

Marcin Andrychowicz
University of Warsaw

Abstract. A recent trend in cryptography is to protect data and computation against various side-channel attacks. Dziembowski and Faust (TCC 2012) have proposed a general way to protect arbitrary circuits against any continual leakage assuming that: (i) the memory is divided into the parts, which leaks independently (ii) the leakage in each observation is bounded (iii) the circuit has an access to a leak-free component, which samples random orthogonal vectors. The pivotal element of their construction is a protocol for refreshing the so-called Leakage-Resilient Storage (LRS).

In this note, we present a more efficient and simpler protocol for refreshing LRS under the same assumptions. Our solution needs $O(n)$ operations to fully refresh the secret (in comparison to $\Omega(n^2)$ for a protocol of Dziembowski and Faust), where $n$ is a security parameter that describes the maximal amount of leakage in each invocation of the refreshing procedure.

1 Introduction

A leakage-resilient cryptography has been intensively studied in the recent years (cf. for instance [MR04, DP08, FKPR10, DHLAW10, BKKV10, LLW11, DF11, DF12, GR12]). This note is based on a work by Dziembowski and Faust [DF12]. It follows the assumptions, construction and notation from the mentioned work. We briefly review the settings, for the complete description of the model we refer the reader to [DF12].

We first start with the definition of the Leakage-Resilient Storage (LRS) [DDV10], which is a randomized encoding scheme $(\text{Enc}: \mathcal{M} \rightarrow \mathcal{L} \times \mathcal{R}, \text{Dec}: \mathcal{L} \times \mathcal{R} \rightarrow \mathcal{M})$, resilient to leakage in the following sense. Let $m \in \mathcal{M}$ be a message, and let $(l, r) := \text{Enc}(m)$. Then, an adversary that learns some partial information $f(l)$ about $l$ and (independently) $g(r)$ about $r$ should gain no information about the encoded message $m$. The idea is to keep $l$ and $r$ on the different memory parts, which leak independently. We will model that setting assuming that they are kept by different parties, which can perform computation and exchange messages.

* This work was supported by the WELCOME/2010-4/2 grant founded within the framework of the EU Innovative Economy (National Cohesion Strategy) Operational Programme.
More precisely (citing verbatim [DF12]), for some \(c, \ell, \lambda \in \mathbb{N}\) let \(M_1, \ldots, M_\ell \in \{0, 1\}^c\) denote the contents of the memory parts, then we define a \(\lambda\)-leakage game played between an adaptive adversary \(A\), called a \(\lambda\)-limited leakage adversary, and a leakage oracle \(\Omega(M_1, \ldots, M_\ell)\) as follows. For some \(m \in \mathbb{N}\), the adversary \(A\) can adaptively issue a sequence \(\{(x_i, f_i)\}_{i=1}^m\) of requests to the oracle \(\Omega(M_1, \ldots, M_\ell)\), where \(x_i \in \{1, \ldots, \ell\}\) and \(f_i : \{0, 1\}^c \rightarrow \{0, 1\}^\lambda_i\) with \(\lambda_i \leq \lambda\).

To each such a query the oracle replies with \(f_i(M_{x_i})\) and we say that in this case the adversary \(A\) retrieved the value \(f_i(M_{x_i})\) from \(M_{x_i}\). The only restriction is that in total the adversary does not retrieve more than \(\lambda\) bits from each memory part. In the following, let \((A \rightleftharpoons (M_1, \ldots, M_\ell))\) be the output of \(A\) at the end of this game.

An LRS \(\Phi\) is said to be \((\lambda, \epsilon)\)-secure, if for any \(S, S' \in \mathcal{M}\) and any \(\lambda\)-limited adversary \(A\), we have \(\Delta(A \rightleftharpoons (L, R); A \rightleftharpoons (L', R')) \leq \epsilon\), where \((L, R) \leftarrow \text{Enc}(S)\) and \((L', R') \leftarrow \text{Enc}(S')\), for any two secrets \(S, S' \in \mathcal{M}\).

A variant of LRS \(\Phi^w\) introduced in [DF11] is based of the inner-product extractor. A secret \(S \in \mathbb{F}\) (where \(\mathbb{F}\) is an arbitrary finite field) is encoded using two random vectors \(L, R \in \mathbb{F}^n\), such that \(S = \langle L, R \rangle\). In this note we only allow the encodings such that \(L, R \in (\mathbb{F} \setminus \{0\})^n\). Moreover, we will assume that \(F\) is fairly large in comparison to \(n\), that is \(|\mathbb{F}| \geq 4n\). Dziembowski and Faust [DF12] showed the following lemma.

**Lemma 1.** Suppose \(|\mathbb{F}| = \Omega(n)\). Then, LRS \(\Phi^w\) is \((0.49 \cdot \log_2 |\mathbb{F}|^n - 1, \text{negl}(n))\)-secure, for some negligible function \(\text{negl}\).

Dziembowski and Faust [DF12] have proposed a compiler, which transforms arbitrary circuits over \(\mathbb{F}\) into functionally equivalent circuits secure against any continual leakage assuming that:

1. the memory is divided into the parts, which leak independently,
2. the leakage from each memory part is bounded,
3. the circuit has an access to a leak-free component, which samples random orthogonal vectors.

A pivotal point in the construction is the \(\text{Refresh}^w\) protocol, which refreshes the encoding of the secret. It is run by two parties \(P_L\) holding \(L\) and \(P_R\) holding \(R\). At the end of the protocol \(P_L\) outputs \(L'\) and \(P_R\) outputs \(R'\) such that \(\langle L, R \rangle = \langle L', R' \rangle\) but except of this \((L', R')\) is uniform and independent of \((L, R)\).

The only fact about \(\text{Refresh}^w\), which is used in the security proof presented in [DF12] is the existence of the reconstructor procedure (an idea introduced earlier in [FRR+10]). Informally, the reconstructor is a protocol that for inputs \((L, L')\) held by \(P_L\) and \((R, R')\) held by \(P_R\) (where \(L, L', R, R' \in (\mathbb{F} \setminus \{0\})^n\) such that \(\langle L, R \rangle = \langle L', R' \rangle\) allows the parties to reconstruct the views that they would have in the \(\text{Refresh}^w(L, R)\) protocol, assuming that \((L', R')\) is an output of \(\text{Refresh}^w(L, R)\).

The \(\text{Refresh}^w\) protocol presented in [DF12] performs \(O(n^2)\) operations. It is there used in a "generalized multiplication" protocol as a sub-routine, what leads at the end to \(O(n^3)\) blow-up of the circuit’s size while securing it against leakages. The protocol presented in this note needs \(O(n)\) operations to refresh the secret, what leads to \(O(n^2)\) blow-up of the circuit’s size.
2 Leakage-Resilient Refreshing of LRS

Similarly as in [DF12] we assume that the players have access to a leak-free component that samples uniformly random pairs of orthogonal vectors. Technically, we will assume that we have an oracle $O'$ that samples a uniformly random vector $(A, \tilde{A}, B, \tilde{B}) \in (\mathbb{F}^n)^4$, subject to the constraint that the following three conditions hold:

1. $\langle A, B \rangle + \langle \tilde{A}, \tilde{B} \rangle = 0$,
2. $A_i \neq 0$ for $1 \leq i \leq n$,
3. $\tilde{B}_i \neq 0$ for $1 \leq i \leq n$.

Note that although our oracle is slightly different from the oracle $O$ used in [DF12], it may be easily „simulated” by the players having access to $O$.

The refreshing scheme is presented in Figure 1. The general idea behind the protocol is similar to one, which appeared in [DF12]. Denote $\alpha := \langle A, B \rangle = -\langle \tilde{A}, \tilde{B} \rangle$. The Steps 2 and 3 are needed to refresh the share of $P_R$. This is done by generating, with the “help” of $(A, B)$ (coming from $O'$) a vector $X$ such that

$$\langle L, X \rangle = \alpha. \quad (1)$$

The key difference between our approach and the protocol from [DF12] is a new and more efficient way of generating such $X$. Eq. (1) comes from a summation: $\langle L, X \rangle = \sum_{i=1}^n L_iX_i = \sum_{i=1}^n L_iV_iB_i = \sum_{i=1}^n L_iL_i^{-1}A_iB_i = \langle A, B \rangle = \alpha$. Then, vector $X$ is added to the share of $P_R$ by setting (in Step 3) $R' := R + X$. Hence we get $\langle L, R' \rangle = \langle L, R \rangle + \langle L, X \rangle = \langle L, R \rangle + \alpha$. Symmetrically, in Steps 5 and 6 the players refresh the share of $P_L$, by first generating $\tilde{X}$ such that $\langle \tilde{X}, R' \rangle = -\alpha$, and then setting $L' = L + \tilde{X}$. By similar reasoning as before, we get $\langle L', R' \rangle = \langle L, R' \rangle - \alpha$, which, in turn is equal to $\langle L, R \rangle$. Hence, $\langle L, R \rangle = \langle L', R' \rangle$.

3 Reconstructor for Refresh$_F^n$

We now show a reconstructor for the Refresh$_F^n$ protocol. Informally, the reconstructor is a protocol that for inputs $(L, L')$ held by $P_L$ and $(R, R')$ held by $P_R$ (where $L, L', R, R' \in (\mathbb{F} \setminus \{0\})^n$) such that $\langle L, R \rangle = \langle L', R' \rangle$ allows the parties to reconstruct the views that they would have in the Refresh$_F^n(L, R)$ protocol, assuming that $(L', R')$ is an output of Refresh$_F^n(L, R)$. The key feature of this reconstructor is that it does not require any interaction between the players. The only “common randomness” that the players need can be sampled offline before the protocol starts. These properties are used in a security proof presented in [DF12].

We now formalize what it means that Reconstruct$\text{Refresh}_F^n$ is a reconstructor for Refresh$_F^n$. This is done by considering two experiments depicted on Fig. 3. The next lemma shows that these experiments produce the same distributions.
Protocol \((L', R') \leftarrow \text{Refresh}_n^o((L, R))\):

**Input** \((L, R)\): \(L \in (\mathbb{F} \setminus \{0\})^n\) is given to \(P_L\) and \(R \in (\mathbb{F} \setminus \{0\})^n\) is given to \(P_R\).

1. Let \(((A, \tilde{A}), (B, \tilde{B})) \leftarrow \mathcal{O}'\) and give \((A, \tilde{A})\) to \(P_L\) and \((B, \tilde{B})\) to \(P_R\).

**Refreshing the share of \(P_R\):**

2. The player \(P_L\) computes a vector \(V\) such that \(V_i := L_i^{-1} \cdot A_i\) for \(1 \leq i \leq n\) and sends \(V\) to \(P_R\).
3. The player \(P_R\) computes a vector \(X\) such that \(X_i := V_i \cdot B_i\) for \(1 \leq i \leq n\) and sets \(R' := R + X\).
4. If there exists \(i\) such that \(R'_i = 0\), then the protocol is restarted from the very beginning with the new vectors sampled from \(\mathcal{O}'\).

**Refreshing the share of \(P_L\):**

5. The player \(P_R\) computes a vector \(\tilde{V}\) such that \(\tilde{V}_i := R'_i^{-1} \cdot \tilde{B}_i\) for \(1 \leq i \leq n\) and sends \(\tilde{V}\) to \(P_L\).
6. The player \(P_L\) computes a vector \(\tilde{X}\) such that \(\tilde{X}_i := \tilde{V}_i \cdot \tilde{A}_i\) for \(1 \leq i \leq n\) and sets \(L' := L + \tilde{X}\).
7. If there exists \(i\) such that \(L'_i = 0\), then the protocol is restarted from the very beginning with the new vectors sampled from \(\mathcal{O}'\).

**Output:** The players output \((L', R')\).

**Views:** The view \(\text{view}_L\) of player \(P_L\) is \((L, A, V, \tilde{A}, \tilde{V})\) and the view \(\text{view}_R\) of player \(P_R\) is \((R, B, V, \tilde{B}, \tilde{V})\).

**Fig. 1.** Protocol \(\text{Refresh}_n^o\). Oracle \(\mathcal{O}'\) samples random vectors \((A, \tilde{A}, B, \tilde{B}) \in (\mathbb{F} \setminus \{0\})^n \times \mathbb{F}^n \times \mathbb{F}^n \times (\mathbb{F} \setminus \{0\})^n\) such that \(\langle A, B \rangle = -\langle \tilde{A}, \tilde{B} \rangle\). Note that the inverses in Steps 2 and 5 always exist, because \(L, R \in (\mathbb{F} \setminus \{0\})^n\). Steps 4 and 7 guarantee that this condition is preserved under the execution of the protocol \(\text{Refresh}_n^o\). It can be easily proven that the protocol is restarted with a bounded probability regardless of \(n\) (but keeping \(|\mathbb{F}| \geq 4n\)), so it changes the efficiency of the algorithm only by a constant factor.
Protocol $\text{ReconstructRefresh}_n^P((L, R), (L', R'))$:

Input $((L, R), (L', R'))$: $L, L' \in (F \setminus \{0\})^n$ are given to $P_L$ and $R, R' \in (F \setminus \{0\})^n$ are given to $P_R$.

Offline sampling: Vectors $V$ and $\tilde{V}$ are independently and uniformly sampled from $(F \setminus \{0\})^n$ and given to both players.

Reconstructing the “Refreshing the share of $P_R$” phase:
1. The player $P_L$ computes a vector $A$ such that $A_i := V_i \cdot L_i$ for $1 \leq i \leq n$.
2. The player $P_R$ sets $X := R' - R$ and computes a vector $B$ such that $B_i := V_i^{-1} \cdot X_i$ for $1 \leq i \leq n$.

Reconstructing the “Refreshing the share of $P_L$” phase:
3. The player $P_R$ computes a vector $\tilde{B}$ such that $\tilde{B}_i := \tilde{V}_i \cdot R'_i$ for $1 \leq i \leq n$.
4. The player $P_L$ sets $\tilde{X} := L' - L$ and computes a vector $\tilde{A}$ such that $\tilde{A}_i := \tilde{V}_i^{-1} \cdot \tilde{X}_i$ for $1 \leq i \leq n$.

Views: The view $\text{view}_L$ of player $P_L$ is $(L, A, V, \tilde{A}, \tilde{V})$ and the view $\text{view}_R$ of player $P_R$ is $(R, B, V, \tilde{B}, \tilde{V})$.

Fig. 2. Protocol $\text{ReconstructRefresh}_n^P$

Experiment $\text{ExpRefresh}(L, R)$:
Run the protocol $\text{Refresh}_n^P((L, R))$.
Output $(L', R', \text{view}_L, \text{view}_R)$.

Experiment $\text{ExpReconstructRefresh}(L, R)$:
Sample $L', R' \leftarrow (F \setminus \{0\})^n$ such as $(L, R) = (L', R')$.
Run the protocol $\text{ReconstructRefresh}_n^P((L, R), (L', R'))$.
Output $(L', R', \text{view}_L, \text{view}_R)$.

Fig. 3. Experiments $\text{ExpRefresh}(L, R)$ and $\text{ExpReconstructRefresh}(L, R)$. 
Lemma 2. For every $L, R \in (\mathbb{F} \setminus \{0\})^n$ we have that
\[
\text{ExpRefresh}(L, R) \overset{d}{=} \text{ExpReconstructRefresh}(L, R).
\]

Proof. We only show that the equality of distributions holds for the variables involved in the “Refreshing of the share of $P_R$ phase” (the same fact for the other phase is proven analogously). These variables are
\[ L, R, A, B, V, X, R'. \]

We prove it showing that each of the above variables has an identical conditional distribution given the previous variables in the series:

1. **L, R**: Clearly in both experiments $(L, R)$ is constant and identical;
2. **A**: $A$ is uniformly distributed over $(\mathbb{F} \setminus \{0\})^n$ independently of $(L, R)$. In the first experiment it comes from the way it is sampled from $O'$. In the second scenario it is defined by the equation $A_i := V_i \cdot L_i$ for $1 \leq i \leq n$. Hence, each $A_i$ is a product of $V_i$ distributed uniformly over $(\mathbb{F} \setminus \{0\})$ and some fixed non-zero $L_i$. Therefore $A_i$ has a uniform distribution over $(\mathbb{F} \setminus \{0\})$.
3. **B**: $B$ is uniformly distributed over $\mathbb{F}^n$ independently of $(L, R, A)$. In the first experiment it comes from the way it is sampled from $O'$. In the second scenario it is defined by the equation $B_i := V_i^{-1} \cdot A_i$ for $1 \leq i \leq n$. Notice that $R'$ has a uniform distribution over $\mathbb{F}^n$ independent of $(L, R, A)$, so $X$ defined by $X := R' - R$ is also uniform over $\mathbb{F}$. Hence, each $B_i$ is a product of some non-zero $V_i^{-1}$ and $A_i$ distributed uniformly over $\mathbb{F}$ and independently of $V$. Therefore $B_i$ has a uniform distribution over $\mathbb{F}$.
4. **V**: $V$ is uniquely determined given $(L, R, A, B)$ by the equation $V_i := L_i^{-1} \cdot A_i$ for $1 \leq i \leq n$ (Step 2 in Fig. 1 and Step 4 in Fig. 2).
5. **X**: $X$ is uniquely determined given $(L, R, A, B, V)$ by the equation $X_i = V_i \cdot B_i$ for $1 \leq i \leq n$ (Step 3 in Fig. 1 and Step 5 in Fig. 2).
6. **R'**: $R'$ is in both experiments equal to $L + X$.

\[ \square \]

Acknowledgments

The author wishes to thank his supervisor Stefan Dziembowski for the guidance in carrying out the research and writing this note.

References

BKKV10. Zvika Brakerski, Yael Tauman Kalai, Jonathan Katz, and Vinod Vaikuntanathan. Overcoming the hole in the bucket: Public-key cryptography resilient to continual memory leakage. In *FOCS*, pages 501–510, 2010.

DDV10. Francesco Davì, Stefan Dziembowski, and Daniele Venturi. Leakage-resilient storage. In *Security and Cryptography for Networks, 7th International Conference, SCN 2010, Amalfi, Italy, September 13-15, 2010. Proceedings*, volume 6280 of *Lecture Notes in Computer Science*, pages 121–137. Springer, 2010.
DF11. Stefan Dziembowski and Sebastian Faust. Leakage-resilient cryptography from the inner-product extractor. In ASIACRYPT, pages 702–721, 2011. Full version appears on the Cryptology ePrint Archive http://eprint.iacr.org/.

DF12. Stefan Dziembowski and Sebastian Faust. Leakage-resilient circuits without computational assumptions. In Ronald Cramer, editor, Theory of Cryptography, volume 7194 of Lecture Notes in Computer Science, pages 230–247. Springer Berlin / Heidelberg, 2012.

DHLAW10. Yevgeniy Dodis, Kristiyan Haralambiev, Adriana López-Alt, and Daniel Wichs. Cryptography against continuous memory attacks. In FOCS, pages 511–520, 2010.

DP08. Stefan Dziembowski and Krzysztof Pietrzak. Leakage-resilient cryptography. In FOCS ’08: Proceedings of the 49th Annual IEEE Symposium on Foundations of Computer Science, Washington, DC, USA, 2008. IEEE Computer Society.

FKPR10. Sebastian Faust, Eike Kiltz, Krzysztof Pietrzak, and Guy N. Rothblum. Leakage-resilient signatures. In Daniele Micciancio, editor, Theory of Cryptography, 7th Theory of Cryptography Conference, TCC 2010, Zurich, Switzerland, February 9-11, 2010. Proceedings, volume 5978 of Lecture Notes in Computer Science, pages 343–360. Springer, 2010.

FRR+10. Sebastian Faust, Tal Rabin, Leonid Reyzin, Eran Tromer, and Vinod Vaikuntanathan. Protecting circuits from leakage: the computationally-bounded and noisy cases. In Henri Gilbert, editor, Advances in Cryptology - EUROCRYPT 2010, 29th Annual International Conference on the Theory and Applications of Cryptographic Techniques, French Riviera, May 30 - June 3, 2010. Proceedings, volume 6110 of Lecture Notes in Computer Science. Springer, 2010.

GR10. Shafi Goldwasser and Guy N. Rothblum. Securing computation against continuous leakage. In Tal Rabin, editor, Advances in Cryptology - CRYPTO 2010, 30th Annual Cryptology Conference, Santa Barbara, CA, USA, August 15-19, 2010. Proceedings, volume 6223 of Lecture Notes in Computer Science, pages 59–79. Springer, 2010.

GR12. Shafi Goldwasser and Guy N. Rothblum. How to compute in the presence of leakage. Electronic Colloquium on Computational Complexity (ECCC), 19:10, 2012. To be presented on FOCS 2012.

LLW11. Allison Lewko, Mark Lewko, and Brent Waters. How to leak on key updates. to appear at STOC 2011, 2011.

MR04. Silvio Micali and Leonid Reyzin. Physically observable cryptography (extended abstract). In Mon Naor, editor, TCC, volume 2951 of Lecture Notes in Computer Science, pages 278–296. Springer, 2004.