ESTIMATING THE RADIUS OF THE CONVECTIVE CORE OF MAIN-SEQUENCE STARS FROM OBSERVED OSCILLATION FREQUENCIES

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ABSTRACT

The determination of the size of the convective core of main-sequence stars is usually dependent on the construction of models of stars. Here we introduce a method to estimate the radius of the convective core of main-sequence stars with masses between about 1.1 and 1.5 $M_\odot$ from observed frequencies of low-degree $p$-modes. A formula is proposed to achieve the estimation. The values of the radius of the convective core of four known stars are successfully estimated by the formula. The radius of the convective core of KIC 9812850 estimated by the formula is $0.140 \pm 0.028 R_\odot$. In order to confirm this prediction, a grid of evolutionary models was computed. The value of the convective-core radius of the best-fit model of KIC 9812850 is $0.149 R_\odot$, which is in good agreement with that estimated by the formula from observed frequencies. The formula aids in understanding the interior structure of stars directly from observed frequencies. The understanding is not dependent on the construction of models.

Key words: convection – stars: evolution – stars: interiors – stars: oscillations (including pulsations)

1. INTRODUCTION

By matching the luminosity, atmospheric parameters, and oscillation frequencies of models with the observed ones, asteroseismology is used to determine fundamental parameters of stars. Asteroseismology is also used to probe physical processes in stars and diagnose internal structures of stars (Roxburgh & Vorontsov 1994, 1999, 2001, 2003, 2004, 2007; Cunha & Metcalfe 2007; Brandão et al. 2010, 2014; Christensen-Dalsgaard & Houdek 2010; Deheuvels et al. 2010; Yang & Meng 2010; Cunha & Brandão 2011; Silva Aguirre et al. 2011; Yang et al. 2012, 2015; Silva Aguirre et al. 2013; Chaplin et al. 2014; Ge et al. 2014; Guenther et al. 2014; Liu et al. 2014; Yang 2016). Asteroseismology is a powerful tool for studying the structure and evolution of stars.

Stars with a mass larger than 1.1 $M_\odot$ are considered to have a convective core during their MS stage. Due to the fact that the overshooting of the convective core can bring more hydrogen-rich material into the core, the evolution of a star could be significantly affected by the overshooting. Thus, determining the size of the convective core including the overshooting region is important for understanding the structure and evolution of stars. However, the size of the convective core has never been determined directly from observed data of stars. Generally, the understanding of the size of the convective core derives from the computation of evolutionary models of stars.

When seeking to probe the internal structures of stars with low-$l$ $p$-modes, the small separations, $d_{10}$, $d_{01}$, $d_{02}$, and $d_{13}$, and the ratios of the small separations to the large separations, $r_{10}$, $r_{01}$, $r_{02}$, and $r_{13}$ (Roxburgh & Vorontsov 2003; Yang & Bi 2007b, and references therein), are considered to be the very useful diagnostic tools. The small separations $d_{10}$ and $d_{01}$ are defined as (Roxburgh & Vorontsov 2003)

$$d_{10}(n) = -\frac{1}{2}(-\nu_{n,0} + 2\nu_{n,1} - \nu_{n+1,0})$$

and

$$d_{01}(n) = \frac{1}{2}(-\nu_{n,1} + 2\nu_{n,0} - \nu_{n-1,1}).$$ (2)

But in calculation, the smoother five-point separations are adopted.

Stars with masses between about 1.1 and 1.5 $M_\odot$ have a convective core during their MS. The discontinuity in density at the edge of the convective core increases with the evolution of the stars. The rapid variation of density with depth in a stellar core can distort acoustic wave propagation in stellar interiors, producing a reflected wave (Roxburgh & Vorontsov 2007). The reflectivity can come from the rapid density change at the edge of the convective core (Roxburgh & Vorontsov 2007). For the modes with frequencies larger than a critical frequency, they can penetrate into the convective core. Partial wave reflection at the core boundary could lead to acoustic resonances in the convective core (Roxburgh & Vorontsov 2004). As a consequence, at high frequencies, we would see a periodic variation in the small separations with frequency (Roxburgh & Vorontsov 2004). If this periodic component is determined from observations, it can be used for constraining the size of the convective core (Roxburgh & Vorontsov 1999, 2001, 2004).

Roxburgh & Vorontsov (1994, 2000a, 2000b, 2001) developed the theory of semiclassical analysis that can more accurately describe the low-degree $p$-modes and the small separations. However, Roxburgh & Vorontsov (2004) pointed out that their expressions for the perturbations in the phase shifts are not transparent enough to serve as a basis for simple estimates. The effects of the convective core on $d_{10}$, $d_{01}$, $r_{10}$, and $r_{01}$ are also studied by other authors (Cunha & Metcalfe 2007; Brandão et al. 2010, 2014; Deheuvels et al. 2010; Cunha & Brandão 2011; Silva Aguirre et al. 2011, 2013; Liu et al. 2014; Yang et al. 2015). The conclusion is that the ratios $r_{01}$ and $r_{10}$ can be affected by the presence of the convective core.

In order to isolate the frequency perturbation produced by the edge of the convective core, Cunha & Metcalfe (2007) and
Cunha & Brandão (2011) defined a tool \( r_{213} = r_{02} - r_{13} \). They have shown that the tool can potentially be used to infer information about the amplitude of the discontinuity in the sound speed at the edge of the convective core, but it is unable to fully isolate the frequency perturbation.

Yang et al. (2015) show that the ratios \( r_{01} \) and \( r_{10} \) of a star with a convective core can be described by equation

\[
B(\nu_{n,l}) = \frac{2A\nu_{n,l}}{2\pi^2(\nu_0^2 - \nu_{n,l}^2)} \sin \left( \frac{2\pi \nu_{n,l}}{\nu_0} \right) + B_0, \tag{3}
\]

where the quantities \( A \) and \( B_0 \) are two parameters, and \( \nu_0 \) is the frequency of the \( l = 1 \) mode whose inner turning point is located on the boundary between the radiative region and the overshooting region of the convective core.

In this work, we propose a method to estimate the radius of the convective core of MS stars with masses between about 1.1 and 1.5 \( M_\odot \) from observed frequencies of low-degree \( p \)-modes. The estimated radius is comparable with that obtained from the evolutionary model. Individual frequencies of \( p \)-modes of KIC 9812850 have been extracted by Appourchaux et al. (2012). The mass of KIC 9812850 estimated by Metcalfe et al. (2014) is 1.39 \( \pm \) 0.05 \( M_\odot \). Thus, KIC 9812850 could have a convective core. We determined the radius of the convective core of KIC 9812850 in two ways. One is estimated from the observed frequencies; the other is determined from the best model for KIC 9812850. In Section 2, a formula that can be used to determine the radius of the convective core from oscillation frequencies is proposed and is applied to different stars. In Section 3, based on finding the maximum likelihood of models of a grid of evolutionary tracks, the best-fit model of KIC 9812850 is found; then, we compare the radius of the convective core of the best model with that determined from oscillation frequencies. Finally, we give a discussion about the domain of the validity of the method and a summary in Section 4.

2. ESTIMATING THE RADIUS OF THE CONVECTIVE CORE FROM OSCILLATION FREQUENCIES

The inner turning point, \( r_t \), of the mode with a frequency \( \nu_{n,l} \) is determined by

\[
r_t = f_0 \frac{c_s(r_t) \sqrt{l(l + 1)}}{2\pi}, \tag{4}
\]

where \( c_s(r_t) \) is the adiabatic sound speed at radius \( r_t \), and the value of the parameter \( f_0 \) is 2.0 (Liu et al. 2014). For the modes with \( l = 1 \), the frequency \( \nu_0 \) of the mode whose inner turning point is just located on the boundary between the radiative region and the overshooting region of the convective core can be estimated by Equation (3) from observed frequencies and the ratios computed from the frequencies. Thus, the radius of the convective core including the overshooting region, \( r_c \), can be determined by

\[
r_c = \frac{c_s(r_c) \sqrt{2}}{\nu_0}, \tag{5}
\]

In the middle stage of MS stars with masses between about 1.1 and 1.5 \( M_\odot \), the magnitude of \( c_s(r_c) \) is of the order of about \( 5 \times 10^7 \) cm s\(^{-1}\). Thus, for the MS stars with a frequency \( \nu_0 \) that is determined from observed frequencies of low-degree \( p \)-modes by using Equation (3), the radius \( r_c \) can be estimated by

\[
r_c \approx \frac{5 \times 10^7 \sqrt{2}}{\nu_0 \times 10^{-6}} \frac{1}{\pi} \times 6.9598 \times 10^{15}
= \frac{323}{\nu_0(\mu\text{Hz})}(R_\odot) \tag{6}
\]

from the observed data.

By using the function of nonlinear curve fitting of the Origin software where chi-square fitting is used and a Hessian matrix is calculated,\(^1\) from the observed frequencies (Benomar et al. 2009) and ratio \( r_{10} \) of HD 49933, we obtained that the value of the parameter \( \nu_0 \) of Equation (3) is 1920 \( \pm \) 46 \( \mu \text{Hz} \) for HD 49933. From the observed frequencies of KIC 6225718 (Tian et al. 2014) and ratio \( r_{10} \), the value of \( \nu_0 \) is estimated to be about 5764 \( \pm \) 312 \( \mu \text{Hz} \). From the frequencies of KIC 2837475 and KIC 11081729 given by Appourchaux et al. (2012), the value of \( \nu_0 \) is estimated to be 913 \( \pm \) 36 \( \mu \text{Hz} \) for KIC 2837475 and 795 \( \pm \) 21 \( \mu \text{Hz} \) for KIC 11081729.

Using Equation (6), one can obtain that the radius of the convective core including the overshooting region is about 0.056 \( R_\odot \) for KIC 6225718, 0.168 \( R_\odot \) for HD 49933, 0.354 \( R_\odot \) for KIC 2837475, and 0.406 \( R_\odot \) for KIC 11081729. The models of the four stars were determined (Liu et al. 2014; Tian et al. 2014; Yang 2015; Yang et al. 2015). The radius of the convective core including the overshooting region is around 0.056 \( R_\odot \) for model 14 of KIC 6225718 (Tian et al. 2014), 0.17 \( R_\odot \) for model M52 of HD 49933 (Liu et al. 2014), 0.358 \( R_\odot \) for model M14 of KIC 2837475 (Yang et al. 2015), and 0.37 \( R_\odot \) for model M6 of KIC 11081729 (Yang 2015). The values of radius \( r_c \) of these stars, estimated from observed data, are in good agreement with those obtained from the models of the stars. This indicates that the radius of the convective core \( r_c \) of the MS stars could be determined directly from observed frequencies and ratios \( r_{01} \) and \( r_{10} \) by using Equations (3) and (6).

Appourchaux et al. (2012) extracted the frequencies of low-\( l \) \( p \)-modes of KIC 9812850. Using the ratio \( r_{10} \) computed from the frequencies and the observed frequencies \( \nu_{n,l} \), the values of \( A, \nu_0 \), and \( B_0 \) in Equation (3) are estimated to be 264 \( \pm \) 62 \( \pi \), 2309 \( \pm \) 141 \( \mu \text{Hz} \), and 0.043 \( \pm \) 0.005, respectively, for KIC 9812850. Thus, the radius \( r_c \) of KIC 9812850 is predicted to be about 0.14 \( R_\odot \) by Equation (6).

3. THE RADIUS OF THE CONVECTIVE CORE OF THE BEST MODEL OF KIC 9812850

3.1. Evolutionary Models

In order to compare the value of \( r_c \) estimated by Equations (3) and (6) for KIC 9812850 with that of the evolutionary model of KIC 9812850, we searched for the best model of KIC 9812850 that matches both nonseismic constraints and seismic characteristics in a grid of evolutionary models. We used the Yale Rotation Evolution Code (YREC; Pinsonneault et al. 1989; Yang & Bi 2007a; Yang et al. 2015) to construct the models. For the microphysics, the OPAL equation-of-state table E02S005 (Rogers & Nayfonov 2002) and OPAL opacity table G93 (Iglesias & Rogers 1996) were adopted, supplemented by the Alexander & Ferguson (1994) opacity tables at low temperature. The models with a mass less

\(^1\) http://www.originlab.com/doc/Origin-help/NLFIt-theory
than $1.30 M_\odot$ take into account the diffusion and settling of both helium and heavy elements by using the diffusion coefficients of Thoul et al. (1994). The standard mixing-length theory is adopted to treat convection. The mixing-length parameter $\alpha$ is a free parameter in this work. For the Sun, the value of the $\alpha$ for the YREC is 1.74. The distance of the overshooting of the convective core is defined as $\delta_{ov}$, where $\delta_{ov}$ is a free parameter and $H_p$ is the local pressure scale height. The full mixing of material is assumed in the overshooting region. The initial helium mass fraction is fixed at the standard big bang nucleosynthesis value 0.248 (Spergel et al. 2007) and 0.295. All models are evolved from zero-age MS to the end of MS. The values of the input parameters, mass, $\alpha$, $\delta_{ov}$, and $Z_i$, for the calculations are summarized in Table 1.

The adiabatic oscillation frequencies $\nu_{nl}$ of models were computed by using the pulsation code jig7 of Guenther (1994). The effects of the near-surface effects of a model on the frequencies were calculated by using the method of Kjeldsen et al. (2008).

### 3.2. Observational Constraints on Models

KIC 9812850 is an F8 star (Wright et al. 2003). The value of [Fe/H] given by Ammons et al. (2006) is $0.00^{+0.15}_{-0.16}$, but that given by Bruntt et al. (2012) is $-0.16 \pm 0.06$ for KIC 9812850. Combining the value of 0.023 (Grevesse & Sauval 1998) of $(Z/X)_c$ of the Sun, the value of $(Z/X)_c$ of KIC 9812850 is estimated to be between 0.016 and 0.033 for the [Fe/H] of Ammons et al. (2006), or in the range of 0.014–0.018 for the [Fe/H] of Bruntt et al. (2012). The effective temperature of KIC 9812850 is 6297 ± 70 K (Ammons et al. 2006) or 6330 ± 70 K (Bruntt et al. 2012). The estimated atmospheric parameters of stars hotter than 6000 K could be affected by the method of spectral analysis (Molenda-Zakowicz et al. 2013). Therefore, the atmospheric parameters determined by both Ammons et al. (2006) and Bruntt et al. (2012) were considered in this work.

The parallax of KIC 9812850 is in the range between about 5.9 and 17.5 mas (Kharchenko 2001; Ammons et al. 2006). The bolometric correction of KIC 9812850 is estimated from the tables of Flower (1996). The extinction of KIC 9812850 is given by Ammons et al. (2006). The visual magnitude of this star is $9.5 \pm 0.3$ mag (Ammons et al. 2006; Droge et al. 2006). Thus, the luminosity of KIC 9812850 is estimated to be about $2.6 \pm 2.3 L_\odot$.

In order to find the best model for KIC 9812850, we calculated the likelihood function of all models. The likelihood function is defined as (Basu et al. 2010)

$$
\mathcal{L} = \frac{1}{(2\pi)^{N/2} \prod_{i=1}^{N} \sigma(C_{i}^{ob})} \exp \left( -\frac{1}{2} \chi^2 \right),
$$

where

$$
\chi^2 = \sum_{i=1}^{N} \left[ \frac{C_{i}^{ob} - C_{i}^{th}}{\sigma(C_{i}^{ob})} \right]^2,
$$

the quantity $C_{i}^{ob}$ indicates the observed $T_{\text{eff}}, L/L_\odot, (Z/X)_c$, and $\nu_{nl}$, while the $C_{i}^{th}$ corresponds to the $T_{\text{eff}}, L/L_\odot, (Z/X)_c$, and $\nu_{nl}$ of models. The quantity $\sigma(C_{i}^{ob})$ represents the observational error of $C_{i}^{ob}$. The value of $N$ is 45.

Moreover, the values of classical $\chi^2$ and $\chi^2_{e}$ of models were also computed as a reference. The $\chi^2_e$ and $\chi^2_{e}$ are defined as

$$
\chi^2_e = \frac{1}{3} \sum_{i=1}^{N} \left[ \frac{C_{i}^{th} - C_{i}^{ob}}{\sigma(C_{i}^{ob})} \right]^2,
$$

and

$$
\chi^2_{e} = \frac{1}{42} \sum_{i=1}^{42} \left[ \frac{\nu_{i}^{th} - \nu_{i}^{ob}}{\sigma(\nu_{i}^{ob})} \right]^2,
$$

respectively, where $C_{i} = (T_{\text{eff}}, L/L_\odot, (Z/X)_c), \sigma(C_{i}^{ob})$ denotes the observational error, and $\nu_{i}$ corresponds to frequencies. The observational error of $\nu_{i}^{ob}$ is indicated by $\sigma(\nu_{i}^{ob})$.

When the model evolves to the vicinity of the error box of luminosity and effective temperature in the Hertzsprung–Russell diagram, the time step of the evolution for each track is set as small as 1 Myr, which ensures that the consecutive models have an approximately equal $\chi^2_e$.

### 3.3. The Best Models of KIC 9812850

For a given mass, the model that maximizes $\mathcal{L}$ is chosen as a candidate for the best-fit model. Table 2 lists four models that have a larger $\mathcal{L}$ in the calculations and shows that model M3 has the maximum $\mathcal{L}$.

Figure 1 compares the distributions of the observed $r_{o1}$ and $r_{o10}$ with those calculated from the models listed in Table 2. The distributions of $r_{o1}$ and $r_{o10}$ of KIC 9812850 are reproduced well by models M2 and M3. The right panels of Figure 1 show that there are periodic variations in the differences between the observed ratios and those of models. This may come from the effects of the helium ionization region and the base of the

| Model | M (M$_\odot$) | T$_{\text{eff}}$ (K) | L (L$_\odot$) | R (R$_\odot$) | Age (Gyr) | Z$_c$ | X$_c$ | $\alpha$ | $\delta_{ov}$ | $r_c$ (R$_\odot$) | $\nu_{o1}^{ob}$ (mHz) | $\chi^2_e$ | $\chi^2_{e}$ | $\mathcal{L}$ |
|-------|--------------|---------------------|---------------|--------------|------------|-------|-------|--------|-----------|-----------------|-------------------|---------|---------|-------|
| M1    | 1.44         | 6267                | 4.75          | 1.851        | 2.861      | 0.024 | 0.728 | 1.75   | 0.2       | 0.144           | 2364              | 1.5     | 0.7     | $1.8 \times 10^{-22}$ |
| M2    | 1.46         | 6188                | 4.58          | 1.864        | 2.950      | 0.028 | 0.724 | 1.75   | 0.2       | 0.147           | 2248              | 1.0     | 2.0     | $9.1 \times 10^{-19}$ |
| M3    | 1.48         | 6408                | 5.28          | 1.867        | 2.606      | 0.024 | 0.728 | 1.95   | 0.2       | 0.149           | 2246              | 0.7     | 1.6     | $1.0 \times 10^{-15}$ |
| M4    | 1.50         | 6320                | 5.07          | 1.880        | 2.620      | 0.030 | 0.722 | 1.95   | 0.2       | 0.152           | 2184              | 1.0     | 1.8     | $1.6 \times 10^{-18}$ |

Note. The symbol $r_c$ represents the radius of the convective core of models, while $\nu_{o1}^{ob}$ indicates the frequency at which ratios $r_{o1}$ and $r_{o10}$ of models reach the minimum.

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*Table 1: Range of the Input Parameters for the Evolutionary Tracks*

| Variable | Minimum | Maximum | Resolution |
|----------|---------|---------|------------|
| M/M$_\odot$ | 1.00 | 1.60 | $\leq 0.02$ |
| $\alpha$ | 1.65 | 2.05 | 0.1 |
| $\delta_{ov}$ | 0.0 | 1.8 | 0.2 |
| Z$_c$ | 0.010 | 0.040 | 0.0002 |
Figure 1. Left panels show the distributions of ratios $r_{01}$ and $r_{10}$ and the results of Equation (3) as a function of frequency. The values of $A$, $v_0$, and $B_0$ for $B(\nu)$ are $180\pi$, $2180$ $\mu$Hz, and 0.0465, respectively. Right panels represent differences between observed $r_{01}$ and $r_{10}$ and those of models, in the sense (observed value – model value)/observational error.
convective envelope on the observed frequencies (Roxburgh & Vorontsov 2004; Mazumdar et al. 2014). Model M3 not only maximizes the likelihood function in the calculations but also reproduces the distributions of observed $r_{01}$ and $r_{10}$ of KIC 9812850. Therefore, M3 is chosen as the best-fit model of KIC 9812850.

Moreover, Figure 1 shows that the distributions of $r_{01}$ and $r_{10}$ of M3 are reproduced well by Equation (3) with $A = 180\pi$, $v_0 = 2180$ $\mu$Hz, and $B_0 = 0.0465$. This indicates that ratios $r_{01}$ and $r_{10}$ can be described by Equation (3).

The value of $r_c$ of model M3 is 0.149 $R_\odot$, which is in good agreement with $0.140 \pm 0.028$ $R_\odot$ estimated by Equation (6). The radius of the convective core including the overshooting region of the best model of KIC 9812850 is successfully estimated by Equations (3) and (6) from observed oscillation frequencies.

Moreover, the value of $\nu_{\text{max}}$ of KIC 9812850 is 1186 $\mu$Hz (Appourchaux et al. 2012), which is less than the value of 2309 $\mu$Hz for $\nu_0$. The value of $\delta_{\nu}$ of M3 is 0.2, which is consistent with the deduction that if the value of $\nu_{\text{max}}$ of a star is less than the value of $\nu_0$, the star may have a small $\delta_{\nu}$ (Yang 2015).

4. DISCUSSION AND SUMMARY

4.1. Discussion

When angular frequencies of modes are larger than a critical frequency $\omega_0$, the modes can penetrate into the convective core of stars. Assuming that the effects of the convective core on oscillations are related to $-A \cos(\omega_0 t)$, where $A$ is a free parameter, Yang (2015) obtained Equation (3) as the result of the Fourier transform of $-A \cos(\omega_0 t)$. Thus, Equation (3) is
invalid for stars whose core is radiative. Figure 2 shows the distributions of H mass fraction, adiabatic sound speed, and $r_{10}$ of core-radiative models in different evolutionary stages. The ratio $r_{10}$ decreases with an increase in frequency. The distributions cannot be reproduced by Equation (3). The core of model S2 in Figure 3 is also radiative. The distribution of $r_{10}$ of the model cannot be reproduced by Equation (3) either.

Roxburgh & Vorontsov (2004, 2007) pointed out that the discontinuity in density at the boundary of a convective core can distort acoustic wave propagation in the stellar interior, producing a reflected wave. The effects of the convective core on oscillations are related to the fact that the discontinuity reflects acoustic waves. Therefore, Equation (3) is invalid for stars with a convective core but without the discontinuity in density or sound speed at the edge of the convective core. Model S3 in Figure 3 has a small convective core but has no an obvious discontinuity in density or sound speed at the edge of the convective core (see Figure 3). The distribution of $r_{10}$ of the model cannot be reproduced by Equation (3). Model S4 has an obvious sound-speed discontinuity at the edge of the convective core. The distribution of $r_{10}$ of model S4 is almost

![Figure 4](image1.png)

**Figure 4.** Similar to Figure 3, but for the models with $M = 1.16 M_\odot$ and $\delta_m = 0.2$. The distribution of $r_{10}$ of model S3 is almost completely reproduced by Equation (3) with $\nu_0 = 2590 \mu Hz$, which is approximately equal to 2600 $\mu Hz$ determined by Equation (5). The cores of these models are convective.

![Figure 5](image2.png)

**Figure 5.** Similar to Figure 3, but for the models with $M = 1.40 M_\odot$ and $\delta_m = 0$. The cores of these models are convective.

| Star     | $\nu_0/\mu Hz$ | $r_{c}/R_\odot$ | $r_{c,model}/R_\odot$ |
|----------|----------------|----------------|------------------------|
| KIC 6225718 | 5764 ± 312    | 0.056 ± 0.011   | 0.056                  |
| KIC 9812850 | 2309 ± 141    | 0.140 ± 0.028   | 0.149                  |
| HD 49933     | 1920 ± 46     | 0.168 ± 0.034   | 0.170                  |
| KIC 2837475  | 913 ± 36      | 0.354 ± 0.070   | 0.358                  |
| KIC 11081729 | 795 ± 21      | 0.406 ± 0.081   | 0.370                  |

**Table 3**

Radius of the Convective Core of Five Stars

Note.

* The uncertainty is estimated by assuming that there is an uncertainty of 10% in $c_s$ and $\nu_0$. The values of $r_c$ are estimated by using Equation (6), while the values of $r_{c,model}$ are obtained from the best models of the stars.
reproduced by Equation (3). The cores of model S1 in Figures 4 and 5 are also convective, but there is no an obvious discontinuity in density or sound speed of the models. The distributions of $r_{10}$ of the models cannot be reproduced by Equation (3) either. However, the distributions of $r_{10}$ of models with a convective core and an obvious discontinuity in sound speed at the edge of the convective core are almost reproduced by Equation (3) (see Figures 4 and 5).

The modes with $l = 0$ are considered to be able to reach the center of a star. According to Equation (4), it is more difficult to arrive at the convective core for the modes with $l \geq 2$ than for the modes with $l = 1$. Thus, the frequency $\nu_0 = \omega_0 / 2\pi$ could be the frequency of the $l = 1$ mode whose inner turning point is located on the boundary between the radiative region and the overshooting region of the convective core. Roxburgh & Vorontsov (1994, 1999, 2004, 2007) show that there are periodic variations in small separations with period determined approximately by the acoustic diameter of the convective core, i.e., the period

$$T_c \approx 2 \int_0^{r_c} \frac{dr}{c_s}(11)$$

The larger the value of $r_c$, the longer the $T_c$, i.e., the smaller the frequency $\nu_c = 1/T_c$. According to Equation (5), the larger the value of $r_c$, the smaller the frequency $\nu_0$. Thus, the frequency $\nu_c$ of Roxburgh could be related to the frequency $\nu_0$. The value of sound speed decreases from $5.60 \times 10^7$ cm s$^{-1}$ to $4.85 \times 10^7$ cm s$^{-1}$ in the convective core of model S3 of a star with $M = 1.16 M_\odot$ (see Figure 4). If $c_s$ in Equation (11) is replaced by $c_s(r_c)$, the value of $\nu_c$ can be estimated to be about $0.5 c_s(r_c) / r_c$. From Equation (5), one can obtain

$$\nu_0 = 0.45 c_s(r_c) / r_c,$$

which is very close to $\nu_c$.

In the stellar interior, sound speed decreases with an increase in radius. The value of $c_s(r_c)$ varies with the mass and the age of stars and is affected by overshooting. The value of $c_s(r_c)$ for most MS stars with masses between about 1.1 and 1.5 $M_\odot$ is mainly in the range of $(4-6) \times 10^7$ cm s$^{-1}$ (see Figures 4 and 5). For example, for a star with $M = 1.16 M_\odot$, $X_c = 0.7$, $Z_i = 0.02$, and $\delta_{ov} = 0.02$, when it evolves from central hydrogen abundance $X_c = 0.59$ to $X_c = 0.16$, the value of $c_s(r_c)$ decreases from about $5.2 \times 10^7$ cm s$^{-1}$ to $4.5 \times 10^7$ cm s$^{-1}$; for a star with $M = 1.4 M_\odot$, $X_c = 0.7$, $Z_i = 0.02$, and $\delta_{ov} = 0$, when it evolves from $X_c = 0.5$ to $X_c = 0.1$, the value of $c_s(r_c)$ decreases from about $5.5 \times 10^7$ cm s$^{-1}$ to $4.5 \times 10^7$ cm s$^{-1}$. Therefore, in most of the MS stages of stars with masses between about 1.1 and 1.5 $M_\odot$, a model in the middle stage of the MS as a reference, one can assume that there is a change of 10% in $c_s(r_c)$, i.e.,

$$c_s(r_c) \sim (5.0 \pm 0.5) \times 10^7 \text{ cm s}^{-1}.
$$

For our sample, the relative uncertainty of $\nu_0$ determined by chi-square fitting from observed data is between about 2.4% and 6.2%. But our sample is small. The relative uncertainty of $\nu_0$ of other stars might be larger than 6.2%. In order to estimate the uncertainty of the estimated $r_c$ of other stars when this method is applied to the stars, we assume that the relative uncertainty of $\nu_0$ for other stars is of the order of 10% and apply the uncertainty of 10% to all cases. As a consequence, the relative uncertainty of the estimated $r_c$ is about 20%. Table 3 shows that the values of the radius of the convective core determined by Equations (3) and (6) from observed frequencies of different stars are in good agreement with those obtained from the best models of the stars.

4.2. Summary

Combining Equation (3), we propose here for the first time using Equation (6) to estimate the radius of the convective core including the overshooting region of MS stars with masses between about 1.1 and 1.5 $M_\odot$ from observed frequencies and ratios. The estimated values of the radius of the convective core of four stars are consistent with those of the best models of the four stars. Using the observed frequencies and ratios of KIC 9812850, Equations (3) and (6) predict that the radius of the convective core of KIC 9812850 is $0.140 \pm 0.028 R_\odot$. In order to confirm this prediction, we constructed a grid of evolutionary tracks. Basing on finding the maximum likelihood of models, we obtained the best-fit model of KIC 9812850 with $M = 1.48 M_\odot$, $R = 1.867 R_\odot$, $T_{eff} = 6408$ K, $L = 5.28 L_\odot$, $I = 2.606$ Gyr, $r_c = 0.149 R_\odot$, and $\delta_{ov} = 0.2$. The best model can reproduce asteroseismic and nonasteroseismic characteristics of KIC 9812850. The value of the radius of the convective core of the best-fit model is in good agreement with that predicted by Equation (6). Equations (3) and (6) aid in understanding the structure of stars directly from the observed frequencies.

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