Fermion Mass Hierarchy and Electric Dipole Moments

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Abstract

We show that in supersymmetric models with a gauged flavor symmetry explaining the hierarchy of fermion masses and mixings, the electron, muon, neutron and the deuteron acquire sizable electric dipole moments (EDM) through loop diagrams involving the flavor gaugino/gauge boson near the Planck scale. These EDMs are proportional to the phases in the fermion Yukawa couplings and are typically much larger than the neutrino seesaw induced EDM for the leptons. In a popular class of models based on anomalous $U(1)$ flavor symmetry of string origin, we find $d_e \sim (10^{-26} - 10^{-27}) \text{ e cm}$, $d_\mu \sim (10^{-24} - 10^{-26}) \text{ e cm}$, $d_n \sim 10^{-27} \text{ e cm}$, and $d_D \sim (10^{-26} - 10^{-27}) \text{ e cm}$, which are within reach of next generation experiments.

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1 Introduction

The observed hierarchy in the masses and mixings of quarks and leptons is one of the most puzzling features of Nature. A plausible explanation is provided by flavor-dependent gauge symmetries. In a popular class of such models one extends the Standard Model (SM) to include a family-dependent \(U(1)\) factor. Upon spontaneous breaking of the \(U(1)\) symmetry effective Yukawa couplings of the form \(y_{ij} \epsilon^{n_{ij}} f_i f_j^c H\) are induced, where \(\epsilon \sim 0.2\) is a small parameter, and \(n_{ij}\) are positive integers related to the family dependent \(U(1)\) charges. Even when the fundamental Yukawa couplings \(y_{ij}\) are all of order one, a hierarchical spectrum is realized due to the suppression in powers of \(\epsilon\) \(\|\). Such flavor \(U(1)\) symmetries can be naturally identified with the anomalous \(U(1)_A\) symmetry of string theory \([2]\). Models using anomalous \(U(1)_A\) symmetry for fermion mass and mixing hierarchy abound in the literature \([3]–[6]\). Most models of this type also assume low energy supersymmetry (SUSY) to stabilize the Higgs boson mass. Novel phenomena which are amenable to experimental tests can arise in such contexts. The purpose of this paper is to analyze one such effect, viz., the electric dipole moments of elementary fermions \([7]\).

Low energy supersymmetry can potentially induce excessive flavor violation in processes such as \(K^0 - \bar{K}^0\) mixing and \(\mu \to e\gamma\) decay if the soft supersymmetry breaking Lagrangian takes its most general form. This potential problem is usually avoided by assuming a universal form for the soft SUSY breaking terms. Even with universality, the \(CP\)-violating phases present in the soft SUSY breaking Lagrangian can induce electric dipole moments (EDM) for the neutron and the electron at a level exceeding the current experimental limits. These effects have been extensively studied in the literature \([8]–[14]\). We will assume in the present work a universal SUSY breaking spectrum that is also \(CP\)-invariant so that excessive EDMs are not induced from the fundamental soft SUSY breaking parameters.

The EDMs that we find in the context of models of fermion mass hierarchy are induced purely by complex Yukawa couplings. The phases in the Yukawa couplings are believed to be the source for the observed \(CP\)-violation in the \(K\) and \(B\) meson systems (CKM \(CP\)-violation). It is thus reasonable to assume all Yukawa couplings, including the leptonic Yukawa couplings, to be complex. As we will see, it is natural that the flavor \(U(1)\) gauge symmetry responsible for explaining the fermion mass hierarchy breaks spontaneously at a scale \(M_F\) slightly below the fundamental Plank (or string) scale, \(M_F \sim M_{st}/50\). In the momentum regime \(M_F < \mu < M_{st}\) the flavor gauge sector will be active and will contribute
to flavor violation in the squark and slepton sectors. These effects would survive down to the SUSY breaking scale and can lead to observable phenomena. In a previous paper we have studied leptonic rare decays $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ induced by the flavor gauge sector \cite{6}. With complex Yukawa couplings, this flavor violation will also lead to EDMs for the electron ($d_e$), muon ($d_\mu$), the neutron ($d_n$), and the deuteron ($d_D$) even with universal and $CP$–conserving soft SUSY breaking terms at the string scale. In a popular class of anomalous $U(1)$ models which explains the fermion mass hierarchy, including bi–large neutrino mixing, we find $d_e \sim (10^{-26} - 10^{-27})\ e\ cm$, $d_\mu \sim (10^{-24} - 10^{-26})\ e\ cm$, $d_n \sim 10^{-27}\ e\ cm$, and $d_D \sim (10^{-26} - 10^{-27})\ e\ cm$, which are within reach of next generation experiments. There are proposals to improve the current limit on electron EDM, $|d_e| \leq 1.6 \times 10^{-27}\ e\ cm$ \cite{15}, by about two to four orders of magnitude \cite{16, 17}. It is expected that the current limit on the muon EDM, $|d_\mu| \leq 1.9 \times 10^{-18}\ e\ cm$, will be improved by six orders of magnitude or even more in the not too distant future \cite{18}. There are also proposals which would improve the current neutron EDM limit from $|d_n| \leq 6.3 \times 10^{-26}\ e\ cm$ \cite{19} by a factor of 5 \cite{20}. The deuteron EDM is expected to be probed to the level of $10^{-27}\ e\ cm$ in the near future \cite{21}. Supersymmetry may reveal itself in these experiments before direct discovery at LHC, if the current ideas of solving the fermion mass hierarchy problem are correct.

Lepton EDMs may arise even without flavor gauge symmetry from complex neutrino Yukawa couplings responsible for the seesaw mechanism in the context of low energy SUSY. This effect has received much attention recently \cite{14, 22, 23}. We have computed such effects for $d_e$ and $d_\mu$, but found them to be much less significant compared to the flavor $U(1)$ induced effects. For example, we find $d_e \sim 10^{-29}\ e\ cm$ for large $\tan\beta$ from the neutrino Yukawa coupling effects, to be compared with $d_e \sim 10^{-26}\ e\ cm$ from the flavor $U(1)$ sector. Similar effects from GUT threshold has been studied in Ref. \cite{24}.

The paper is organized as follows. In Section 2 we review the class of models based on anomalous $U(1)_A$ symmetry for fermion mass hierarchy \cite{6}. In Section 3 the EDMs induced by the flavor $U(1)_A$ gauge sector is analyzed. In 3.1 a qualitative discussion of the radiative corrections to the soft masses and $A$–terms is given. In 3.2 we present our full numerical results for the EDMs. Section 4 has our conclusions. In Appendix A.1 we give the relevant expressions for the $\beta$–functions for the soft SUSY breaking parameters including corrections from the $U(1)_A$ gauge sector. In Appendix A.2 the fermion mass fit for the model used in the numerical analysis is presented. Appendix A.3 lists the formulas needed for the calculation of EDMs.
2 Fermion Masses and Anomalous $U(1)$ Symmetry

In this section we review briefly the idea of explaining fermion mass hierarchy with a flavor dependent $U(1)$ symmetry. We focus on a specific class of anomalous $U(1)_A$ models discussed in Ref. [6] to address the fermion EDM. Most models of Ref. [3]–[5] will also fall into this category and will lead to similar results. In these models families are distinguished by their anomalous $U(1)$ charges. The $U(1)_A$ symmetry is broken spontaneously by an MSSM singlet flavon field $S$ which acquires a vacuum expectation value (VEV) slightly below the string scale $M_{st}$. This provides a small expansion parameter $\epsilon = \langle S \rangle / M_{st}$ needed for explaining the fermion mass hierarchy. $U(1)$ invariance forbids renormalizable Yukawa couplings for the light families, but would allow them through effective nonrenormalizable couplings suppressed by a factor $(S/M_{st})^{n_{ij}}$ (for the fermion mass operator connecting flavors $i$ and $j$) with $n_{ij}$ being positive integers. Even with all couplings being of order one, hierarchical masses for different flavors are naturally realized [1]. Although this mechanism will work with any flavor $U(1)$, anomalous $U(1)$ models are attractive since they would also provide a natural understanding for the smallness of $\epsilon \sim 0.2$ [3], which arises from the one–loop induced Fayet–Illiopoulos $D$–term [2a].

Consider the following fermion mass matrices studied in Ref [6]:

$$M_u \sim \langle H_u \rangle \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \quad M_d \sim \langle H_d \rangle \epsilon^p \begin{pmatrix} \epsilon^5 & \epsilon^4 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix},$$

$$M_e \sim \langle H_d \rangle \epsilon^p \begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon \\ \epsilon^4 & \epsilon^2 & 1 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \quad M_{\nu D} \sim \langle H_u \rangle \epsilon^s \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix},$$

$$M_{\nu C} \sim M_R \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \Rightarrow M_{\nu}^{light} \sim \frac{\langle H_u \rangle^2}{M_R} \epsilon^{2s} \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}. \quad (1)$$

Here $M_u$, $M_d$ and $M_e$ are the up–quark, the down–quark and the charged lepton mass matrices in the basis $f M_f f^c$ ($f = u, d, e, \nu$). Complex order one coefficients multiplying each entry of the matrices are not shown. $M_{\nu D}$ and $M_{\nu C}$ are the neutrino Dirac and Majorana mass matrices. The light neutrino mass matrix $M_{\nu}^{light}$ is derived from the seesaw mechanism. $p$ and $s$ are integers and are chosen differently for different values of $\tan \beta = \langle H_u \rangle / \langle H_d \rangle$. The choice of $p = 0, 1, 2$ corresponds to large ($\sim 50$), medium...
Table 1: The flavor $U(1)_A$ charge assignment for the MSSM fields and the flavon field $S$ in the normalization where $q_s = -1$. In the third column we list the notation for the charges used in the paper.

\[
\begin{array}{|c|c|c|}
\hline
\text{Field} & U(1)_A \text{ Charge} & \text{Charge notation} \\
\hline
Q_1, Q_2, Q_3 & 4, 2, 0 & q^Q_i \\
L_1, L_2, L_3 & 1 + s, s, s & q^L_i \\
u^c_1, u^c_2, u^c_3 & 4, 2, 0 & q^u_i \\
d^c_1, d^c_2, d^c_3 & 1 + p, p, p & q^d_i \\
e^c_1, e^c_2, e^c_3 & 4 + p - s, 2 + p - s, p - s & q^e_i \\
\nu^c_1, \nu^c_2, \nu^c_3 & 1, 0, 0 & q^\nu_i \\
H_u, H_d, S & 0, 0, -1 & (h, \bar{h}, q_s) \\
\hline
\end{array}
\]

\[\sim (20)\text{ and small } \sim (5)\text{ values of } \tan \beta \text{ respectively. The quark and lepton masses and mixings arising from Eq. (1) are fully consistent with experimental observations if } \epsilon \sim 0.2.\]

Note that the CKM mixing angles are small, while the leptonic mixing angles relevant for solar and atmospheric neutrino oscillations are of order unity. The lopsided nature of the matrices $M_d$ and $M_e$ of Eq. (1) enables such a disparity to be realized [26].

The superpotential which would lead to the Yukawa coupling structure in Eq. (1) has the following general form near the fundamental scale $M_{st}$:

\[W = \sum_f y^f_{ij} \frac{f_i H f_j^c}{n_{ij}!} \left( \frac{S}{M_{st}} \right)^{n_{ij}} + \frac{M_{Rij}}{2 n_{ij}^{\nu^c_i \nu^c_j}} \left( \frac{S}{M_{st}} \right)^{n_{ij}^{\nu^c}} + \mu H_u H_d + W_A (S, X_k). \] (2)

Here $i, j = 1, 2, 3$ are the generation indices and $n_{ij}^{f}$ ($f = u, d, e, \nu$) are positive integers fixed by the choice of the $U(1)_A$ charge assignment. We choose all charges to be integers with the charge of $S$ being negative. $y^f_{ij}$ are the Yukawa couplings which we take to be complex and of order one for all $i, j$. $H$ stands for the MSSM Higgs doublets $H_u$ and $H_d$. $S$ is the MSSM singlet flavon field whose VEV determines the expansion parameter $\epsilon$. $W_A$ in Eq. (2) contains MSSM singlet fields $X_k$ which would be needed for anomaly cancelation. The $U(1)_A$ charge assignment shown in Table 1 will lead to the texture of Eq. (1).

The $U(1)_A$ anomalies are cancelled by the Green–Schwarz mechanism [2] which requires

\[\frac{A_1}{k_1} = \frac{A_2}{k_2} = \frac{A_3}{k_3} = \frac{A_F}{3k_F} = \frac{A_{\text{gravity}}}{24}. \] (3)

Here $A_1, A_2, A_3, A_F$ and $A_{\text{gravity}}$ are $U(1)_Y \times U(1)_A, SU(2)_L \times U(1)_A, \ SU(3)_C \times U(1)_A,$
$U(1)^3_\text{A}$ and $(Gravity)^2 \times U(1)_A$ anomaly coefficients. All other anomalies (such as $U(1)^2_\text{A} \times U(1)_Y$) must vanish. $k_i (i = 1, 2, 3)$, $k_F$ are the Kac-Moody levels, with the Non–Abelian levels $k_2$ and $k_3$ being integers. The factor $1/3$ in front of the cubic anomaly $A_F$ has a combinatorial origin owing to the three identical $U(1)_A$ gauge boson legs.

We require string unification of all the gauge couplings including that of the $U(1)_A$, $g_F$, at the fundamental scale $M_{st}$ [27]:

$$k_i g_i^2 = k_F g_F^2 = 2 g_{st}^2.$$  \hspace{1cm} (4)

For a clear discussion of the coefficients in Eqs. (3)–(4) see Ref. [28]. Here $g_i$ are the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ gauge couplings for $i = 1, 2, 3$. With the choice $k_2 = k_3 = 1$, consistency with the observed unification of gauge couplings within MSSM would require $k_1 = 5/3$, corresponding to the $SU(5)$ normalization of hypercharge. This will be an automatic consequence of the Green–Schwarz anomaly cancelation conditions with our choice of texture. The small discrepancy (in the absence of a covering GUT group) between the unification scale derived from low energy data and the string scale may be understood in the context of $M$–theory by making use of the radius of the eleventh dimension [29].

With $k_2 = k_3 = 1$ we find from Table I $A_2 = (19 + 3s)/2$ and $A_3 = (19 + 3p)/2$. Eq. (3) then requires $p = s$, i.e., a common exponent for the charged lepton and the neutrino Dirac Yukawa coupling matrices. With $p = s$, the condition $A_1/k_1 = A_2/k_2$ fixes $k_1$ to be $5/3$, consistent with $SU(5)$ unification. Note also that the charges given in Table I become compatible with $SU(5)$ unification. Since $\text{Tr}(Y) = 0$ for the fermion multiplets of $SU(5)$, and since the Higgs doublets carry zero $U(1)_A$ charge, the anomaly coefficient $[U(1)_A]^2 \times U(1)_Y$ vanishes, as required. The last equality in Eq. (3) requires

$$A_{\text{gravity}} = \text{Tr} (q) = 12(19 + 3p).$$  \hspace{1cm} (5)

This cannot be satisfied with the MSSM fields alone, since $\text{Tr}(q)_{\text{MSSM}} = 5(13 + 3p)$, which does not match Eq. (5). We cancel this anomaly by introducing MSSM singlet fields $X_k$ obeying $\text{Tr} (q)_X = A_{\text{gravity}} - \text{Tr} (q)_{\text{MSSM}} = 163 + 21p$. If all the $X_k$ fields have the same charge equal to $+1$, they will acquire masses of order $M_{st} \epsilon^2$ through the coupling $X_k X_k S^2/M_{st}$ and will decouple from low energy theory. For other choices of the charge of $X_k$ these masses can be different. For example, if the charge is equal to $+1/2$, their masses will be of order $M_{st} \epsilon$; if the charge is $+2$ the masses will be of order $M_{st} \epsilon^4$. We will consider only the case where the $X_k$ fields have charge $+1$. 

5
With the charges of all fields fixed, we are now in a position to determine the \( U(1)_A \) charge normalization so that \( g_F^2 = g_2^2 = g_3^2 \) at the string scale, (We take \( k_2 = k_3 = 1 \).) This normalization factor, which we denote as \( |q_s| \), is given by \( |q_s| = 1/\sqrt{k_F} \). All the charges given in Table 1 are to be multiplied by \( |q_s| \).

From the Green–Schwarz anomaly cancelation condition \( A_F/(3k_F) = A_2/k_2 \), we have

\[
\text{Tr} \left( \frac{q^3}{3k_F} \right) = \frac{19 + 3p}{2k_2},
\]

from which we find the normalization of the \( U(1)_A \) charge \( |q_s| \) to be

\[
|q_s| = (0.179, 0.186, 0.181) \text{ for } p = (0, 1, 2).
\]

The Fayet–Iliopoulos term for the anomalous \( U(1)_A \), generated through the gravitational anomaly, is given by

\[
\xi = \frac{g_{st}^2 M_{st}^2}{192\pi^2} |q_s| A_{\text{gravity}},
\]

where \( g_{st} \) is the unified gauge coupling at the string scale (see Eq. (4)). By minimizing the potential from the \( U(1)_A \) \( D \)-term

\[
V = \frac{|q_s|^2 g_F^2}{8} \left( \frac{\xi}{|q_s|} - |S|^2 + \sum_a q_a^f \tilde{f}_a^f + \sum_k q_k X_k \right)^2,
\]

in such a way that supersymmetry remains unbroken, one finds for the VEV of \( S \)

\[
\langle S \rangle / M_{st} = \sqrt{g_{st}^2 A_{\text{gravity}}/192\pi^2}.
\]

For the fermion mass texture in Eq. (1), corresponding to the \( U(1)_A \) charges given in Table 1 we find

\[
\epsilon = (0.177, 0.191, 0.204) \text{ for } p = (0, 1, 2).
\]

The masses of the \( U(1)_A \) gauge boson and the corresponding gaugino are obtained from \( M_F = |q_s| g_F \langle S \rangle / \sqrt{2} \) and found to be

\[
M_F = \left( \frac{M_{st}}{54.5}, \frac{M_{st}}{52.5}, \frac{M_{st}}{53.9} \right) \text{ for } p = (0, 1, 2).
\]

In the momentum range below \( M_{st} \) and above \( M_F \), these gauge particles will be active and will induce flavor dependent corrections to the sfermion soft masses and the \( A \)-terms. It is these effects which induce EDMs for the electron, muon and the neutron at low energies.
3 Electric Dipole Moments from Anomalous $U(1)$

In the Standard Model the electric dipole moments of the electron, muon and the neutron are predicted to be extremely small and beyond reach of planned experiments. In the presence of low energy supersymmetry these EDMs can exceed the current experimental limits if soft SUSY breaking parameters are complex [8]–[14]. To focus on the anomalous $U(1)$ induced effects we shall adopt the minimal supergravity scenario with universal and $CP$–conserving soft SUSY breaking parameters. Specifically, at the string scale we assume a universal scalar mass $m_0$, a common gaugino mass $M_{1/2}$, and trilinear $A$–terms proportional to their respective Yukawa couplings. We assume $m_0$, $M_{1/2}$ and $A_0$, and the Higgs mass parameters $\mu$ and $B\mu$ to be real. Thus the only source of $CP$–violation is in the complex Yukawa couplings. This is needed for the CKM $CP$–violation in the quark sector and it is natural to assume that the leptonic Yukawa couplings are complex as well. In Sec. 3.1 we give a qualitative estimate of the EDMs induced by the radiative corrections involving the $U(1)_A$ gauge sector in such a SUSY context. Our numerical results are presented in Sec. 3.2.

3.1 $U(1)_A$ Correction to the Soft Parameters and EDM

We now give approximate expressions for the $U(1)_A$ gauge sector RGE corrections to the soft parameters between the string scale and the $U(1)_A$ breaking scale $M_F$. The full RGE expressions for the soft parameters in the presence of higher dimensional operators as in Eq. (2) have been derived in Ref. [6]. In Appendix A.1 we summarize the relevant expressions. The $U(1)_A$ corrections to the soft masses for the left–handed slepton are obtained from Eq. (21) to be

$$
\delta \left( m^2_{\tilde{L}} \right)_{ij}^A \simeq 4(q_i^L M_{\lambda^F})^2 - q_i^L m_0^2 \text{Tr}(q) \langle |q_s| g_F \rangle^2 \delta_{ij} \frac{\log (M_{st}/M_F)}{8\pi^2},
$$

and a similar expression for the right–handed slepton masses with the interchange $(\tilde{e}, q^e) \rightarrow (\bar{e}, q^e)$. There are analogous corrections in the squark sector. The corrections to the $A$–terms are obtained from Eq. (22) as

$$
\delta A^e_{ij} \simeq - M_{\lambda^F} g_F^2 Y^e_{ij} Z^e_{ij} \frac{\log (M_{st}/M_F)}{4\pi^2},
$$

where $Z_{ij}$ are bilinear combinations of the flavor charges given by [6]

$$
Z^e_{ij} = q_i^L q_j^L + q_i^L \bar{h} + q_j^L \bar{h} + n_{ij} q_s(q_i^L + q_j^L + \bar{h}) + \frac{1}{2} n_{ij} (n_{ij}^e - 1) q_s^2.
$$
Numerical values of $Z_{ij}^e$ for our model are given in Eq. (26) of Appendix A.1. Note that these corrections, Eqs. (13) and (14), are flavor dependent. Due to the flavor dependent nature of these corrections, the fermion and the corresponding sfermion mass matrices cannot be diagonalized simultaneously. This was the source of the flavor violation studied in Ref. [6]. For the same reason, with complex Yukawa couplings $Y_{ij}^f$, nonzero EDMs for the fermions will be induced.

Let us now estimate the EDM of the electron arising from the corrections in Eqs. (13) and (14). There are three flavor dependent matrices in the leptonic sector, not including the neutrino Yukawa matrix $Y_\nu$. They are the leptonic Yukawa matrix $Y_e$ and the matrices of $U(1)_A$ charges $\hat{Q}_L = \text{diag}(1 + p, p, p)$ and $\hat{Q}_e = \text{diag}(4, 2, 0)$ for the lepton doublets and singlets (see Table I). In the mass eigenbasis for the charged leptons $\hat{Q}_L$ and $\hat{Q}_e$ will develop complex off diagonal entries, with the phases arising from $Y_e$ through the unitary matrices that diagonalize $Y_e$. This is the basic source for the EDM.

The corrections given in Eq. (13) will generate EDM of the electron through the product of slepton mixings in $(1i)_{LL}$, $(ii)_{LR}$ and $(i1)_{RR}$ (for $i = 2, 3$). The induced EDM will be $d_e \propto \text{Im} \left[ \left( U^\dagger \hat{Q}_L Y_e \hat{Q}_e V \right)_{11}\right]$, where $U$ and $V$ are unitary matrices which diagonalize $Y_e$, $Y_e = U Y_{e\text{diag}} V^\dagger$. There are additional corrections which are quadratic in $\hat{Q}_L$ and $\hat{Q}_e$.

The corrections to the $A$–terms in Eq. (14) will also induce EDM directly through $(LR)$ mixings. Combining these effects with the formula for the EDM given in Eq. (42) of Appendix A.3 we arrive at the following approximate expression for $d_e$:

$$d_e/e \approx \frac{\alpha v d M_B}{8 \pi \cos^2 \theta_W} \frac{1}{m_i^2} \mathcal{A} \left( \frac{M_B^2}{m_i^2} \right) \frac{\text{Im} \left[ \left( Y_e \hat{Q}_e \right)_{11} \right]}{8 \pi^2} \sum_{i=2,3} \left[ C_{i}^m + C_{i}^A \right],$$

where $C_{i}^m$ and $C_{i}^A$ denote the contributions from the soft masses and the $A$–terms respectively. They are given by

$$C_{i}^m = \frac{(|q_s| g_F)^2 \log (M_{st}/M_F) m_0 (A_0 - |\mu| \tan \beta)}{8 \pi^2} \frac{m_0^6}{m_i^6} H_i^L H_i^R,$$

$$H_i^L = 4 \left( M_{1/2}/m_0 \right)^2 \left((q_i^L)^2 - (q_i^L)^2\right) - (q_i^L - q_i^L)^2 \text{Tr} q,$$

$$C_{i}^A = 2 \frac{M_{1/2}}{m_i^2} (Z_{i1}^e - Z_{i1}^e).$$

Here $H_i^R$ is obtained from $H_i^L$ by the replacement $q_i^L \rightarrow q_i^e$. $m_i$ is the average slepton mass and $M_B$ is the Bino mass. The function $\mathcal{A}(X)$ is given in Eq. (13) in Appendix A.3. We see explicitly that the complex Yukawa couplings along with nonuniversal $U(1)_A$ charges lead to nonzero EDM.
To estimate the size of this effect we choose the approximations $m_0 = M_{1/2} \simeq M_{\text{SUSY}}$. Following the mass matrices given in Eq. (1) we take $|Y_{ij}^e| \simeq e^{\nu_{ij} + p}$. We consider here only the contribution from the (13) mixing, since the $U(1)_A$ charge difference is the largest between the first and the third generations in $\hat{Q}^e$. Then we find

$$d_e/e \sim \left(10^{-27}\text{cm}\right) \times \left(\frac{500\text{GeV}}{m_{\tilde{t}}^6}\right)^2 \times M_B \left(O(10)\frac{M^4_{\text{SUSY}}(|\mu| \tan \beta)}{m_{\tilde{t}}^6} + O(1)\frac{M_{\text{SUSY}}}{m_{\tilde{t}}^2}\right) \text{Arg}[Y_{13}^e Y_{31}^e].$$

From this estimate we see that the electron EDM induced by the $U(1)_A$ gauge corrections is in the experimentally interesting range and already puts constraint on the soft SUSY breaking parameters. The actual numerical result is quite sensitive to the choice of $m_0$ and $M_{1/2}$. In our numerical calculations we have chosen $m_0 = M_{1/2}/4.4$ for low $\tan \beta$ for cosmological reason. In this case the $O(10)$ coefficient in Eq. (18) will be reduced to an $O(1)$ number. For large $\tan \beta$ this coefficient will remain as $O(10)$.

Let us now compare the anomalous $U(1)$ induced EDM with the right–handed neutrino induced effects pointed out in Ref. [14] and studied further in Ref. [22, 23]. The latter effects induce EDM which are given by

$$d_i \propto [(Y^{\nu}_i)\dagger Y^{\nu}, (Y^{\nu})\dagger Y^{\nu}],$$

where $\Lambda_{ij} = \log(M_{\text{GUT}}/(M_{\nu i})) \delta_{ij}$. Here $(M_{\nu i})_i$ is the mass of the right–handed neutrino of flavor $i$. With our texture for the neutrino mass matrices dictated by $U(1)_A$ symmetry we find the right–handed neutrino induced EDM to be $d_e \sim 10^{-29} e\text{ cm}$, which is two to three orders magnitude smaller than the anomalous $U(1)_A$ induced effects. In our numerical analysis we present separately our results for the electron EDM arising from the right–handed neutrino effects.

### 3.2 Numerical Results

In this sub-section we present our numerical results for the electron, muon, neutron and the deuteron electric dipole moments. We adopt the minimal supergravity scenario for supersymmetry breaking. At the string scale, taken to be $M_{st} = 10^{17}$ GeV, we assume a universal scalar mass $m_0$ and a common gaugino soft mass $M_{1/2}$. All SUSY breaking parameters $(m_0, M_{1/2}, A_0, B_0)$ and the $\mu$ term are taken to be real at the string scale. We choose $\mu > 0$ for all cases except in Fig. where we also show results for $\mu < 0$. 

The anomalous $U(1)$ gauge coupling $g_F$ is chosen to be $g_F^2/4\pi = 1/24$, consistent with string unification. The soft SUSY breaking parameters are evolved from $M_{st}$ to the $U(1)_A$ breaking scale $M_F \simeq M_{st}/50$ (see Eq. (12)) including the $U(1)_A$ gaugino/gauge boson corrections.

We present our results for the EDM for three values of the parameter $\tan \beta$, small (5), medium (20) and large (50). We take $m_0 = M_{1/2}/4.4$ for low and medium values of $\tan \beta$. This is motivated by the requirement that the right abundance of cosmological cold dark matter be generated. With $m_0 = M_{1/2}/4.4$, the right–handed charged sleptons will have masses slightly above that of the neutralino LSP. The relic abundance of neutralinos is in the right range with such a spectrum, as a result of coannihilation [30]. For large $\tan \beta$ we also allow the choice $m_0 = M_{1/2}$, since alternative mechanisms for reproducing the right relic abundance of LSP become available in this case [31].

We vary $M_{1/2}$ in the range 250 GeV to 1 TeV. The results are presented for two different values of $A_0$, 0 and 300 GeV. The lepton EDMs induced by the flavor $U(1)$ gaugino/gauge boson contribution are plotted against the universal gaugino mass $M_{1/2}$ in Figures 12. In Figure 3 the electron EDM induced purely by the right–handed neutrino effects is plotted. In Figure 45 we plot the EDM of the neutron and the deuteron arising from the flavor $U(1)$ gauge boson/gaugino effects.

As input at $M_{st}$ we choose the Yukawa coupling matrices given in Eqs. (37)–(40) of Appendix A.2 (for $\tan \beta = 5$). These are obtained by extrapolating the low energy Yukawa couplings to $M_{st}$ and applying bi-unitary transformations at $M_{st}$ to generate the texture given in Eq. (1). The low energy Yukawa couplings and their extrapolation are discussed in Appendix A.2. As for the neutrino Dirac Yukawa couplings, we choose $Y^\nu$ to be such that in the flavor basis (after the bi–unitary rotations) it exhibits approximately the structure given in Eq. (1) with $(Y^\nu)_{33} \sim \epsilon^p$. For a given choice of hierarchical light–neutrino spectrum this would uniquely fix the right–handed neutrino mass matrix through the seesaw mechanism. $M_{\nu}$ will then have the form given in Eq. (1). We set $(M_{\nu})_{33} = M^0_{R}e^{2p}$ with $M^0_{R} \simeq 4 \times 10^{14}$ GeV. The eigenvalues of the right–handed neutrino mass matrix are important for the lepton EDMs induced by the right–handed neutrino threshold effects. It should be noted that the unitary rotations applied on the diagonal Yukawa matrices at $M_{st}$ are not unique, except that they should conform to the fermion mass matrix structure shown in Eq. (1). So our fits should be taken only as indicative, and not definitive. We expect differences of order one in our numerical results on EDM arising from the arbitrariness in these unitary matrices.
In Figure 1 the electron EDM induced by the $U(1)_A$ gaugino/gauge boson contributions to the soft masses and $A$–terms are plotted as a function of $M_{1/2}$ for three values of $\tan \beta$. We see that some parts of the parameter space are already excluded by the current experimental upper bound $d_e \leq 1.6 \times 10^{-27} \text{e cm}$ and that the other parts are in the range which will be tested by next generation electron EDM experiments [16].

In Figure 2 we plot the muon EDM as a function of SUSY breaking parameters. We find $d_\mu$ to be in the range $(10^{-25} - 10^{-28}) \text{e cm}$ for most of the parameter space. This value is somewhat smaller than than $d_e(m_\mu/m_e)$, which would be the naive expectation based on the scaling of lepton masses. This happens for the following reason. The second and third family left–handed charged sleptons have the same $U(1)_A$ charge, so the flavor gauge bosons/gaugino will not generate any mass splitting between these sleptons. The mixing in the right–handed charged slepton sector is suppressed by a factor $\epsilon^2$ for all $\tan \beta$, compared to the suppression factor $\epsilon$ between the first and the second generations. On the other hand, we find quite an enhancement of the muon EDM for the choice $m_0 = M_{1/2}$ and $\tan \beta = 50$. For this choice, the electron EDM is well above the experimental bound. Since the two EDMs are induced by independent phases, it is possible to choose the parameters such that the electron EDM is below the experimental limit and at the same time the muon EDM is at the level of $\sim (10^{-25} - 10^{-24}) \text{e cm}$, although we do not attempt such an explicit solution here. It should also be pointed out that parts of the parameter space where $d_\mu$ is large is already ruled out by the experimental upper limit for the radiative decay $\mu \to e\gamma$ for the numerical fits shown [6]. The remaining regions will be put to experimental scrutiny by future experiments [18].

In Figure 3 we present for comparison, the electron EDM arising solely from the right–handed neutrino threshold effects [14]. With the proper decoupling of the right–handed neutrinos [23] we find our results to be in rough agreement with those in Ref. [14, 22, 23]. Nevertheless these effects, which yield at most $d_e \sim 10^{-29} \text{e cm}$, are much smaller compared to the $U(1)_A$ effects.

In Figure 4 we plot the neutron EDM versus $M_{1/2}$. In Figure 5 we plot the deuteron EDM. Details of the calculations are given in Appendix A.3 In both cases our numerical results are in the interesting range which should be accessible to proposed experiments in the near future. We find the contributions from the CKM phase to be of the same order as the contributions from the $U(1)_A$ gaugino/gauge boson sector. Figures 4, 5 include both these effects. The flavor sector contribution to the neutron EDM is somewhat smaller compared to the leptonic EDM due to the gluino focusing effect. (The squarks receive
flavor universal contributions for their masses below $M_F$ from the gluino, which tends to suppress flavor violation and thus $d_n$.)

We have also studied the constraint on the chromoelectric dipole moment for the strange quark $d_s^C$ arising from $^{199}$Hg EDM \cite{32,33}. This bound reads as $|d_s^C| \leq 5.8 \times 10^{-25}$ e cm. This constraint is easily satisfied in our model. The down–type squark mixing in the $(23)$ sector is suppressed by a factor $\epsilon^2$ for the right–handed squarks, and is vanishing to leading order for the left–handed squarks, similar to the case of $\mu – \tau$ mixing. Consequently, we find the chromoelectric EDM of the strange quark to be about two to three orders of magnitude below the experimental limit.

The soft SUSY breaking bilinear $B$–term and the gaugino masses will develop complex phases via the one–loop and two–loop RGE corrections respectively arising from the $A$–term contributions. In our model we find these corrections to be negligible compared to the $U(1)_A$ flavor gaugino/gauge boson effects.

![Figure 1: Electric Dipole Moment of the electron induced by the flavor gaugino/gauge boson. The (red) horizontal line shows the current experimental limit on $d_e$. We have chosen here $m_0 = M_{1/2}/4.4$. For $\tan \beta = 50$ we show an additional case with $m_0 = M_{1/2}$ (the uppermost curve). For $\tan \beta = 20$ and $A_0 = 300$ GeV we find a cancellation between the $A$–term contributions given in Eq. 24 and the soft left/right mass contributions in Eq. 21 for our particular fit of the Yukawa couplings. This cancellation disappears for the choice of negative $\mu$–term (the curve labeled by $\mu < 0$).](image)
Figure 2: Electric Dipole Moment of the muon induced by the flavor gauge corrections. Here \( m_0 = M_{1/2}/4.4 \). For \( \tan \beta = 50 \), we also present results for the case \( m_0 = M_{1/2} \).

Figure 3: Electric Dipole Moment of the electron induced purely by the right–handed neutrino threshold corrections. The notation is the same as in Fig. 2.
Figure 4: Electric Dipole Moment of the neutron induced by the flavor gaugino/gauge boson corrections. Here $m_0 = M_{1/2}/4.4$, with an additional case $m_0 = M_{1/2}$ shown for $\tan \beta = 50$. The horizontal line is the current experimental limit.

Figure 5: Electric Dipole Moment of the deuteron induced by the flavor gaugino/gauge boson corrections. Here $m_0 = M_{1/2}/4.4$, with an additional case $m_0 = M_{1/2}$ shown for $\tan \beta = 50$. 
4 Conclusions

In this paper we have studied the electric dipole moments of the electron, muon and the neutron induced by a flavor dependent $U(1)$ symmetry which explains the hierarchy of fermion masses and mixings in a natural way via the Froggatt-Nielsen mechanism. This $U(1)$ symmetry may be identified as the anomalous $U(1)$ of string theory. This symmetry is broken spontaneously at a scale $M_F$ slightly below the string scale, $M_F \sim M_{st}/50$. In the momentum regime $M_F \leq \mu \leq M_{st}$, the flavor $U(1)_A$ gauge boson sector will be active and will contribute to the soft SUSY breaking parameters in a flavor dependent fashion. We adopt the minimal supergravity scenario for SUSY breaking, and assume that the soft SUSY breaking parameters are universal and real. The complex Yukawa couplings will still induce phases in the soft SUSY masses and the $A$-parameters, leading to the generation of EDM. This is the main source of the EDM that we have studied here.

We have presented our numerical results for the electron, muon, neutron and the deuteron EDMs in Figures 1-5 as functions of supersymmetry breaking parameters. $d_e$ and $d_n$ are very close to the current experimental limits, $d_e \sim (10^{-26} - 10^{-27})$ e cm and $d_n \sim 10^{-27}$ e cm. For the deuteron, our prediction, $d_D \sim (10^{-26} - 10^{-27})$ e cm, would make it within reach of proposed experiments. For the case of the muon, although $d_\mu$ is rather small for low $\tan \beta$, in the case of large $\tan \beta \sim 50$, for certain choices of phases in the Yukawa couplings, we have found the induced EDM to be as large as $d_\mu \sim (10^{-25} - 10^{-24})$ e cm, which might be accessible to future experiments. In the leptonic sector, these EDMs are much larger than the ones induced by the neutrino seesaw sector, which yields, for example, $d_e \sim 3 \times 10^{-29}$ e cm with our texture of fermion mass matrices dictated by flavor symmetries. In Figure 4 we present our results for the induced $d_e$ arising from the neutrino seesaw sector. Discovery of electric dipole moments for the electron, muon and the neutron can shed light on one of the fundamental puzzles of Nature, viz., the origin of mass for elementary particles.

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A Appendix

A.1 RGE for Soft Parameters including $U(1)_A$ Corrections

Here we list all the $\beta$–functions for the soft masses and $A$–terms including contributions from the anomalous $U(1)$ gaugino/gauge boson sector (see Ref. 6 for details).

The one–loop $\beta$–functions for the soft sfermion masses are given by

$$\beta \left( m_f^2 \right)_{ij} = \beta \left( m_f^2 \right)_{ij}^{MSSM} + \beta \left( m_f^2 \right)_{ij}^{A} + \beta \left( m_f^2 \right)_{ij}^{N},$$

where the superscripts $MSSM$, $A$ and $N$ denote the contributions from the MSSM sector, the anomalous $U(1)$ gauge sector, and the right–handed neutrino sector respectively. $\beta \left( m_f^2 \right)_{ij}^{N}$ are present only for the left–handed slepton and the sneutrino soft masses and their explicit form is given elsewhere. The anomalous $U(1)$ part $\beta \left( m_f^2 \right)_{ij}^{A}$ is

$$\beta \left( m_f^2 \right)_{ij}^{A} = \frac{1}{16\pi^2} 2q_f^i g_F^2 \delta_{ij} \left( \sigma - 4q_f^i (M_{\lambda_F})^2 \right).$$

Here $\sigma$ is defined as

$$\sigma = 3 \text{Tr} \left( 2q^Q \tilde{m}_Q^2 + q^u \tilde{m}_u^2 + q^d \tilde{m}_d^2 \right) + \text{Tr} \left( 2q^L \tilde{m}_L^2 + q^e \tilde{m}_e^2 + q^\nu \tilde{m}_\nu^2 \right) + q_s \tilde{m}_s^2 + \sum_k q_k^X \tilde{m}_{X_k}^2,$$

where $\tilde{m}_{X_k}$ are the soft masses of the extra particles $X_k$ introduced for anomaly cancellation via the Green–Schwarz mechanism. The trace is taken over family space. Here $\beta \left( \tilde{m}_L^2 \right)_{ij}^{MSSM}$ etc., stand for the MSSM $\beta$–functions without the neutrino and the flavor $U(1)_A$ contributions.

Introducing a notation

$$A^f_{ij} \equiv a^f_{ij} q^f_{n_{ij}},$$

the $U(1)$ gaugino part of $A$–term $\beta$–functions is given by

$$\beta(A^f)_{ij}^A = -\frac{1}{8\pi^2} g_F^2 A^f_{ij} \left( (q^f_i)^2 + (q^f_j)^2 + h^2 \right) + \frac{1}{4\pi^2} g_F^2 Z^f_{ij} Y^f_{ij} M_{\lambda_F},$$

where $g_F$ and $M_{\lambda_F}$ are the $U(1)_A$ gauge coupling and the gaugino mass respectively. $h$ is the $U(1)_A$ charge of the up–type (down–type) higgs doublet if $f^c$ is up–type (down–type). Here we defined the combination of the $U(1)_A$ charges $Z^f_{ij}$ as

$$Z^f_{ij} = q^f_i q^f_{jc} + q^f_j h + q^f_{jc} h + n^f_{ij} q_s (q^f_i + q^f_j + h) + \frac{1}{2} n^f_{ij} (n^f_{ij} - 1) q_s^2.$$
From the $U(1)_A$ charge assignments for the MSSM fields as given in Table 1 one has

$$Z^e = -\begin{pmatrix}
(11, 13, 16) & (4, 6, 9) & (1, 3, 6) \\
(10, 11, 13) & (3, 4, 6) & (0, 1, 3) \\
(10, 11, 13) & (3, 4, 6) & (0, 1, 3)
\end{pmatrix}, \quad (26)$$

$$Z^d = -\begin{pmatrix}
(11, 13, 16) & (9, 10, 12) & (9, 10, 12) \\
(4, 6, 9) & (3, 4, 6) & (3, 4, 6) \\
(1, 3, 6) & (0, 1, 3) & (0, 1, 3)
\end{pmatrix}, \quad (27)$$

$$Z^\nu = -\begin{pmatrix}
(2, 4, 7) & (1, 3, 6) & (1, 3, 6) \\
(1, 2, 4) & (0, 1, 3) & (0, 1, 3) \\
(1, 3, 4) & (0, 1, 3) & (0, 1, 3)
\end{pmatrix}, \quad (28)$$

for the three different values of $p = (0, 1, 2)$ and

$$Z^u = -\begin{pmatrix}
20 & 13 & 10 \\
13 & 6 & 3 \\
10 & 3 & 0
\end{pmatrix}, \quad (29)$$

independent of $p$.

The terms we are interested in are the ones proportional to the $U(1)_A$ gaugino mass $M_\lambda$ and the term proportional to $\sigma$ in the $\beta$–functions. Beside these, the $A$–term $\beta$–functions contain a flavor dependent piece which arises from the wave–function renormalization. Since the corresponding Yukawa $\beta$–functions contain the same terms, these are simultaneously diagonalized, and do not lead to flavor violation.

### A.2 Fermion Mass Fit

Here we present the numerical fits to the fermion masses and mixings adopted for the calculation of the EDMs. As input at low energy we choose the following values for the running quark masses [34]:

$$m_u(1\text{ GeV}) = 5.1\text{ MeV}, \quad m_c(m_c) = 1.27\text{ GeV}, \quad m_t(m_t) = 167\text{ GeV},$$

$$m_d(1\text{ GeV}) = 8.9\text{ MeV}, \quad m_s(m_s) = 130\text{ MeV}, \quad m_b(m_b) = 4.25\text{ GeV}. \quad (30)$$

The CKM mixing matrix is chosen in the standard parametrization with $\theta_{12} = 0.221$, $\theta_{13} = 0.005$, $\theta_{23} = 0.043$ and the complex phase $\delta = 0.86$. We use two–loop QED and QCD renormalization group equations to evolve these masses from the low energy scale to the
scale of SUSY breaking. We obtain the following running factor \( r_f \equiv m_f(M_{SUSY})/m_f(m_f) \)
for the fermion masses at the SUSY breaking scale \( M_{SUSY} \), initially chosen to be 500 GeV
with \( \alpha_s(M_Z) = 0.118 \):

\[
(r_t, r_b, r_\tau, r_u, r_c, r_{d,s}, r_{e,\mu}) = (0.943, 0.605, 0.991, 0.395, 0.442, 0.395, 0.398).
\] (31)

Then these masses at \( M_{SUSY} \) are used to calculate the Yukawa couplings in \( \overline{DR} \) scheme.
Using one–loop SUSY RGE evolution above \( M_{SUSY} \) we obtain the Yukawa couplings at the \( U(1)_A \) breaking scale (\( M_F \sim 10^{15} \) GeV) to be

\[
(Y_u, Y_c, Y_t) = (5.2699 \times 10^{-6}, 1.4634 \times 10^{-3}, 0.55498),
\]
\[
(Y_d, Y_s, Y_b) = (3.4415 \times 10^{-5}, 5.0222 \times 10^{-4}, 2.8247 \times 10^{-2}),
\]
\[
(Y_e, Y_\mu, Y_\tau) = (1.0388 \times 10^{-5}, 2.1481 \times 10^{-3}, 3.6239 \times 10^{-2}),
\]
\[
(Y_{\nu_1}, Y_{\nu_2}, Y_{\nu_3}) = (7.4112 \times 10^{-4}, 3.5135 \times 10^{-3}, 4.7212 \times 10^{-2}),
\] (32)

for \( \tan \beta = 5 \),

\[
(Y_u, Y_c, Y_t) = (5.0919 \times 10^{-6}, 1.4140 \times 10^{-3}, 0.53090),
\]
\[
(Y_d, Y_s, Y_b) = (1.3834 \times 10^{-4}, 2.0189 \times 10^{-3}, 0.11508),
\]
\[
(Y_e, Y_\mu, Y_\tau) = (4.1778 \times 10^{-5}, 8.6456 \times 10^{-3}, 0.14818),
\]
\[
(Y_{\nu_1}, Y_{\nu_2}, Y_{\nu_3}) = (3.5720 \times 10^{-3}, 1.6934 \times 10^{-2}, 0.2275),
\] (33)

for \( \tan \beta = 20 \) and

\[
(Y_u, Y_c, Y_t) = (5.3161 \times 10^{-6}, 1.4764 \times 10^{-3}, 0.59610),
\]
\[
(Y_d, Y_s, Y_b) = (4.2226 \times 10^{-4}, 6.1621 \times 10^{-3}, 0.41186),
\]
\[
(Y_e, Y_\mu, Y_\tau) = (1.2720 \times 10^{-4}, 2.6645 \times 10^{-2}, 0.51807),
\]
\[
(Y_{\nu_1}, Y_{\nu_2}, Y_{\nu_3}) = (1.7822 \times 10^{-2}, 8.4492 \times 10^{-2}, 1.1354),
\] (34)

for \( \tan \beta = 50 \).

We have chosen the Dirac neutrino Yukawa couplings such that in the flavor basis
(after the bi–unitary rotations) it exhibits the assumed structure in Eq. (II). The light–
neutrino masses and the lepton mixing matrix are chosen to be \( m_{\nu_1} = 2.7 \times 10^{-3} \) eV,
\( m_{\nu_2} = 6.4 \times 10^{-3} \) eV and \( m_{\nu_3} = 8.6 \times 10^{-2} \) eV and

\[
V_{\text{MNS}} = \begin{pmatrix}
0.8494 & -0.5262 & -0.04 \\
0.3915 & 0.5775 & 0.7164 \\
-0.3539 & -0.6242 & 0.6965 \\
\end{pmatrix},
\] (35)

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which are compatible with the observed light–neutrino oscillation parameters. From this
the right–handed neutrino mass matrix is fixed uniquely with \((M_\nu)_{33} \simeq M_R^0 e^{2\beta}\) where
\(M_R^0 = 4 \times 10^{14}\) GeV. The eigenvalues of \(M_\nu\) are used to calculate the lepton EDM
induced by the right–handed neutrino threshold effects.

In the following, we present our fits to the texture of Eq. (11) which have been used
in our numerical calculations for \(\tan \beta = 5\) (We have similar fits upto an overall factor
for \(\tan \beta = 20, 50\). This fit is not unique, and so can lead to order one uncertainty in
our EDM results. The following fit is found by applying bi-unitary transformations with
complex phases on the diagonal Yukawa coupling matrices at the \(U(1)_A\) breaking scale
\(M_F\). We introduce a notation for the Yukawa couplings:

\[
Y^f_{ij} \equiv y^f_{ij} e^{\kappa^f_{ij}},
\]  

(36)

At \(M_F\) we find the following fit (for \(\tan \beta = 5\)):

\[
Y^u = \begin{pmatrix}
(1.45 + 1.60 i) e^8 & (-0.563 - 1.24 i) e^6 & (1.50 - 0.397 i) e^4 \\
(-0.769 - 0.584 i) e^6 & (0.765 - 0.109 i) e^4 & (-0.255 - 0.261 \times 10^{-2} i) e^2 \\
(-0.282 - 0.204 i) e^4 & (0.274 - 4.40 \times 10^{-2} i) e^2 & 0.554 - 2.80 \times 10^{-5} i
\end{pmatrix},
\]  

(37)

\[
Y^d = e^2 \begin{pmatrix}
(1.87 - 1.69 i) e^5 & (1.93 + 0.849 i) e^4 & (1.29 + 0.957 i) e^4 \\
(-0.404 - 0.248 i) e^3 & (0.552 + 1.54 \times 10^{-2} i) e^2 & (0.702 - 0.546 i) e^2 \\
(-1.52 - 0.435 i) & 0.312 + 0.314 i & 0.543 - 4.74 \times 10^{-4} i
\end{pmatrix},
\]  

(38)

\[
Y^e = e^2 \begin{pmatrix}
(3.52 \times 10^{-2} + 0.480 i) e^5 & (-1.85 - 1.74 i) e^3 & (-0.539 - 0.579 i) e \\
(-0.170 - 0.612 i) e^4 & (1.15 - 4.65 \times 10^{-2} i) e^2 & 0.319 - 0.321 i \\
(0.538 - 0.421 i) e^4 & (-0.419 - 0.536 i) e^2 & 0.784 - 9.73 \times 10^{-4} i
\end{pmatrix},
\]  

(39)

\[
Y^{\nu} = e^2 \begin{pmatrix}
(0.232 - 0.190 i) e^2 & (0.217 - 6.09 \times 10^{-2} i) e & (-0.206 - 0.637 i) e \\
(0.638 - 0.652 i) e & -7.82 \times 10^{-2} + 0.537 i & 0.804 + 0.296 i \\
(0.305 - 0.392 i) e & -4.41 \times 10^{-3} + 0.277 i & 0.404 - 3.89 \times 10^{-2} i
\end{pmatrix}.
\]  

(40)

Note that all coefficients multiplying \(e^{\kappa^f_{ij}}\) in Eqs. (37)–(40) are of order unity.

A.3 Formulas for Electric Dipole Moments

We list here the formulas for the electric dipole moments of leptons and quarks in the
MSSM from Ref. [10], which we have used in our numerical analysis.
The EDMs of elementary fermions are sum of neutralino, chargino and for quarks gluino contributions which we denote as $d^N_f$, $d^C_f$ and $d^G_q$. In addition to these, the quarks receive contributions from chromoelectric and purely gluonic dimension–six operators \cite{[36]}. We have not considered here the latter one, since these effects turn out to be small. The effective EDM operator $d_f$ for a spin–\(\frac{1}{2}\) particle is given by

$$\mathcal{L} = -\frac{i}{2} d_f \bar{\psi} \sigma_{\mu \nu} \gamma_5 \psi F^{\mu \nu} \quad (41)$$

The EDM $d_f$ in general has the following components in a supersymmetric theory:

$$d^N_f/e = \frac{\alpha}{8 \pi \sin^2 \theta_W} \sum_{i=1}^{4} \sum_{a=1}^{6} \text{Im} \left( N^f_{xa} \right) \frac{M_{\tilde{s}a}}{m^2_{f_i}} Q_{f_i} \left( \frac{M^2_{\tilde{s}a}}{m^2_{f_i}} \right),$$

$$d^C_u/e = \frac{-\alpha}{8 \pi \sin^2 \theta_W} \sum_{i=1}^{2} \sum_{b=1}^{2} \text{Im} \left( C^u_{xb} \right) \frac{M_{\tilde{\chi}^+_b}}{m^2_{d_i}} \left( B \left( \frac{M^2_{\tilde{\chi}^+_b}}{m^2_{d_i}} \right) - \frac{1}{3} A \left( \frac{M^2_{\tilde{\chi}^+_b}}{m^2_{d_i}} \right) \right),$$

$$d^C_d/e = \frac{-\alpha}{8 \pi \sin^2 \theta_W} \sum_{i=1}^{2} \sum_{b=1}^{2} \text{Im} \left( C^d_{xb} \right) \frac{M_{\tilde{\chi}^+_b}}{m^2_{u_i}} \left( \frac{2}{3} A \left( \frac{M^2_{\tilde{\chi}^+_b}}{m^2_{u_i}} \right) - B \left( \frac{M^2_{\tilde{\chi}^+_b}}{m^2_{u_i}} \right) \right),$$

$$d^G_q/e = \frac{\alpha_s}{3 \pi} \sum_{x=1}^{6} \text{Im} \left( G^q_x \right) \frac{M_{\tilde{g}}}{m^2_{q_i}} Q_{\tilde{g}} \left( \frac{M^2_{\tilde{g}}}{m^2_{q_i}} \right), \quad (42)$$

where

$$A(X) = \frac{1 - X^2 + 2X \log X}{(1 - X)^3},$$

$$B(X) = \frac{3 - 4X + X^2 + 2 \log X}{(1 - X)^3}. \quad (43)$$

Here $M_{\tilde{s}a}$, $M_{\tilde{\chi}^+_b}$ and $M_{\tilde{g}}$ are the neutralino, chargino and the gluino masses respectively. $m^2_{f_i}$ ($i = 1, ..., 6$) are the eigenvalues of the sfermion mass matrices. The coefficients $N^f_{xa}$, $C^f_{xb}$ and $G^q_x$ are given by

$$N^f_{xa} = \left[ \sqrt{2} \tan \theta_W Q_{f_i} \left( O^N \right)_{a1} U^f_{i3,x} - K_f \left( O^N \right)_{a'a} U^f_{i,x} \right]$$

$$\times \left[ -\sqrt{2} \left\{ \tan \theta_W \left( Q_{f_i} - T_{f_i} \right) \left( O^N \right)_{a1} + T_{3f_i} \left( O^N \right)_{a2} \right\} U^f_{i,x} \right]$$

$$- K_f \left( O^N \right)_{a'a} U^f_{i+3,x},$$

$$C^u_{xb} = K_u \left( O^C_u \right)_{b2} U^d_{1,x} \left[ \left( O^C_L \right)_{b1} U^d_{4,x} - K_d \left( O^C_R \right)_{b2} U^d_{4,x} \right]^*,$$

$$C^d_{xb} = K_d \left( O^C_d \right)_{b2} U^u_{i1} \left[ \left( O^C_L \right)_{b1} U^u_{1,x} - K_u \left( O^C_R \right)_{b2} U^u_{1,x} \right]^*,$$

$$G^q_x = U^q_{i,x} U'^q_{i+3,x}, \quad (44)$$

where $K_u = m_u/(\sqrt{2}M_W \sin \beta)$ and $K_{l,d} = m_{l,d}/(\sqrt{2}M_W \cos \beta)$. $O^N$, $O^C_L$, $O^C_R$ matrices diagonalize the neutralino and chargino mass matrices respectively. The index $a'$
of $O^N$ in the neutralino contribution formula takes value of 3(4) for $T_{3f} = -\frac{1}{2}(\frac{1}{2})$. The chromoelectric dipole moments $\tilde{d}_q$ for quarks are defined as

$$\mathcal{L}_{CEDM} = -\frac{i}{2}g_s \tilde{d}_q \bar{q} T^a \sigma_{\mu\nu} \gamma_5 q G^{\mu\nu}. \quad (45)$$

The contributions to $\tilde{d}_q$ from neutralino, chargino and gluino are given by

$$\tilde{d}^N_{qi} = \frac{g^2}{32\pi^2} \sum_{x=1}^{6} \sum_{a=1}^{4} \text{Im} (N^q_{xa}) \frac{M_{\chi_0^a}}{m_{\tilde{q}_x}^2} A \left( \frac{M_{\chi_0^a}^2}{m_{\tilde{q}_x}^2} \right),$$

$$\tilde{d}^C_q = -\frac{g^2}{32\pi^2} \sum_{x=1}^{6} \sum_{b=1}^{2} \text{Im} (C^q_{xb}) \frac{M_{\chi^+_{b}}}{m_{\tilde{q}_x}^2} A \left( \frac{M_{\chi^+_{b}}^2}{m_{\tilde{q}_x}^2} \right),$$

$$\tilde{d}^G_{qi} = \frac{\alpha_s}{4\pi} \sum_{x=1}^{6} \text{Im} (G^q_{xi}) \frac{M_{\tilde{g}}}{m_{\tilde{q}_x}^2} C \left( \frac{M_{\tilde{g}}^2}{m_{\tilde{q}_x}^2} \right), \quad (46)$$

where

$$C(X) = \frac{1}{6 (1 - X)^2} \left( 10X - 26 + \frac{2X\log X}{1 - X} - \frac{18\log X}{1 - X} \right). \quad (47)$$

We use the QCD sum rule based estimate of Ref. [37] to evaluate the neutron and the deuteron EDMs:

$$d_n = 0.7 (d_d - 0.25d_u) + 0.55 \left( \tilde{d}_d + 0.5\tilde{d}_u \right),$$

$$d_D = 0.5 (d_d + d_u) - 0.6 \left( \tilde{d}_d - \tilde{d}_u + 0.3 \left( \tilde{d}_d + \tilde{d}_u \right) \right). \quad (48)$$

Here the running factors are $\tilde{d}_q (1\text{ GeV}) \simeq 0.91\tilde{d}_q (M_Z)$ and $d_q (1\text{ GeV}) \simeq 1.2 d_q (M_Z)$. 

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References

[1] C.D. Froggatt and H.B. Nielsen, Nucl. Phys. B147, 277 (1979).

[2] M.B. Green and J.H. Schwarz, Phys. Lett. B149, 117 (1984); Nucl. Phys. B255, 93 (1985); M.B. Green, J.H. Schwarz and P. West, Nucl. Phys. B254, 327 (1985).

[3] L. E. Ibanez, G. G. Ross, Phys. Lett. B332, 100 (1994); P. Binetruy and P. Ramond, Phys. Lett. B350, 49 (1995); P. Binetruy, S. Lavignac and P. Ramond, Nucl. Phys. B477, 353 (1996).

[4] K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. 74, 2418 (1995); V. Jain and R. Shrock, Phys. Lett. B 352, 83 (1995); Y. Nir, Phys. Lett. B354, 107 (1995); E. Dudas, S. Pokorski and C.A. Savoy, Phys. Lett. B356, 45 (1995); E. J. Chun, A. Lukas, Phys. Lett. B387, 99 (1996); Y. Nir and R. Rattazzi, Phys. Lett. B382, 363 (1996); A. H. Chamseddine and H. K. Dreiner, Phys. Lett. B389, 533 (1996); K. Choi, E. J. Chun and H. D. Kim, Phys. Lett. B394, 89 (1997); Z. Berezhiani and Z. Tavartkiladze, Phys. Lett. B396, 150 (1997), Phys. Lett. B409, 220 (1997); P. Binetruy, S. Lavignac, S.T. Petcov and P. Ramond, Nucl. Phys. B496, 3 (1997); P. Binetruy, E. Dudas, S. Lavignac and C.A. Savoy, Phys. Lett. B422, 171 (1998); J. K. Elwood, N. Irges and P. Ramond, Phys. Rev. Lett. 81, 5064 (1998); Y Grossman, Y. Nir and Y. Shadmi, JHEP 9810, 007 (1998); G. Altarelli and F. Feruglio, JHEP 9811, 021 (1998), Phys. Rept. 320, 295 (1999); Q. Shafi and Z. Tavartkiladze, Phys. Lett. B451, 129 (1999), Phys. Lett. B482, 145 (2000); G. Eyal, Phys. Lett. B461, 71 (1999); M. E. Gomez, G. K. Leontaris, S. Lola and J. D. Vergados, Phys. Rev. D59, 116009 (1999); K. Choi, K. Hwang and E. J. Chun, Phys. Rev. D60, 031301 (1999); J. Feng and Y. Nir, Phys. Rev. D61, 113005 (2000); A.S. Joshipura, R. Vaidya and S.K. Vempati, Phys. Rev. D62, 093020 (2000); M.S. Berger, K. Siyeon, Phys. Rev. D62, 033004 (2000), Phys. Rev. D64, 053006 (2001); N. Maekawa, Prog. Theor. Phys. 106, 401 (2001); J. Sato and K. Tobe, Phys. Rev. D63, 116010 (2001); M. Tanimoto, Phys. Lett. B501, 231 (2001); I. Gogoladze and A. Perez-Lorenzana, Phys. Rev. D65, 095011 (2002); I. Jack, D.R.T. Jones and R. Wild, Phys. Lett. B535, 193 (2002); T. Ohlsson and G. Seidl, Nucl. Phys. B643, 247 (2002); S.M. Barr and I. Dorsner, Phys. Rev. D65, 095004 (2002).
[5] T. Kobayashi, H. Nakano, H. Terao and K. Yoshioka, Prog. Theor. Phys. 110, 247 (2003); K.S. Babu, I. Gogoladze and K. Wang, Nucl. Phys. B660, 322 (2003); H.K. Dreiner, M. Thormeier, arXiv:hep-ph/0305270; H.K. Dreiner, H. Murayama and M. Thormeier, arXiv:hep-ph/0312012.

[6] K.S. Babu, Ts. Enkhbat and I. Gogoladze, Nucl. Phys. B678, 233 (2004).

[7] For a recent review see N. Fortson, P. Sandars and S. Barr, Phys. Today 56N6, 33 (2003).

[8] J.R. Ellis and D.V. Nanopoulos, Phys. Lett. B110, 44 (1982); J.R. Ellis, S. Ferrara and D.V. Nanopoulos, Phys. Lett. B114, 231 (1982); W. Buchmuller and D. Wyler, Phys. Lett. B121, 321 (1983); J. Polchinski, Phys. Lett. B125, 393 (1983).

[9] F. del Aguila, M.B. Gavela, J.A. Grifols and A. Mendez, Phys. Lett. B126, 71 (1983), Erratum-ibid. B129 473 (1983); E. Franco and M. L. Mangano, Phys. Lett. B135, 445 (1984); P. Nath, Phys. Rev. Lett. 66, 2565 (1991); Y. Kizukuri and N. Oshimo, Phys. Rev. D45, 1806 (1992); R. Garisto and G. L. Kane, arXiv:hep-ph/9302282; C. Hamzaoui and M.E. Pospelov, Phys. Lett. B357, 616 (1995); T. Inui, Y. Mimura, N. Sakai and T. Sasaki, Nucl. Phys. B449, 49 (1995); T. Falk and K.A. Olive, Phys. Lett. 375, 196 (1996); T. Falk, K.A. Olive and M. Pospelov, Nucl. Phys. B560, 3 (1999); D. Chang, W. Y. Keung and A. Pilaftsis, Phys. Rev. Lett. 82, 900 (1999) [Erratum-ibid. 83, 3972 (1999)]; D. A. Demir, O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Nucl. Phys. B 680, 339 (2004).

[10] T. Ibrahim and P. Nath, Phys. Rev. D57, 478 (1997); Erratum-ibid. D58, 019901 (1998); Erratum-ibid. D60 079903 (1999); Erratum-ibid. D60 119901 (1999); Phys. Rev. D58, 111301 (1998); Erratum-ibid. D60, 099902 (1999); Phys. Lett. 418, 98 (1998); Phys. Rev. D61, 093004 (2000).

[11] R. Arnowitt, J.L. Lopez and D.V. Nanopoulos, Phys. Rev. D42, 2423 (1990).

[12] V.D. Barger, T. Falk, Tao Han, J. Jiang, T. Li and T. Plehn, Phys. Rev. D64, 056007 (2001); A. Pilaftsis, Nucl. Phys. B644, 263 (2002); A. Bartl, W. Majerotto, W. Porod and D. Wyler, Phys. Rev. D68, 053005 (2003).

[13] K.S. Babu, B. Dutta and R.N. Mohapatra, Phys. Rev. Lett. 85 5064 (2000); E. Accomando, R. Arnowitt and B. Dutta, Phys. Rev. D61, 115003 (2000); K.S. Babu,
S.M. Barr and I. Dorsner, Phys. Rev. D64, 053009 (2001); S. Abel, S. Khalil and O. Lebedev, Nucl. Phys. B606, 151 (2001); Y. Farzan, Phys. Rev. D 69, 073009 (2004); T. Appelquist, M. Piai and R. Shrock, arXiv:hep-ph/0401114

[14] J. Ellis, J. Hisano, M. Raidal and Y. Shimizu, Phys. Lett. B528, 86 (2002).

[15] B.C. Regan, E.D. Commins, C.J. Schmidt and D. DeMille, Phys. Rev.Lett. 88, 071805 (2002).

[16] D. Kawall, F. Bay, S. Bickman, Y. Jiang and D. DeMille, Phys. Rev. Lett. 92, 133007 (2004).

[17] S.K. Lamoreaux, Talk at “Lepton–Moments”, Cape Cod, 9–12 June 2003.

[18] Y.K. Semertzidis et al, Int. J. Mod. Phys. A16S1B, 690 (2001).

[19] P.G. Harris et al., Phys. Rev. Lett. 82, 904 (1999).

[20] See for example, T. Soldner, arXiv:hep-ex/0405062

[21] Y. K. Semertzidis et al. [EDM Collaboration], AIP Conf. Proc. 698, 200 (2004) arXiv:hep-ex/0308063.

[22] I. Masina, Nucl. Phys. B 671, 432 (2003).

[23] Y. Farzan and M.E. Peskin, arXiv:hep-ph/0405214

[24] F. Borzumati and A. Masiero, Phys. Rev. Lett. 57, 961 (1986); L. J. Hall, V. A. Kostelecky and S. Raby, Nucl. Phys. B 267, 415 (1986); S. Dimopoulos and L. J. Hall, Phys. Lett. B 344, 185 (1995).

[25] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B 289, 589 (1987); J. Atick, L. Dixon and A. Sen, Nucl. Phys. B 292, 109 (1987).

[26] K. S. Babu and S. M. Barr, Phys. Lett. B 381, 202 (1996); C. H. Albright, K. S. Babu and S. M. Barr, Phys. Rev. Lett. 81, 1167 (1998); J. Sato and T. Yanagida, Phys. Lett. B 430, 127 (1998); N. Irges, S. Lavignac and P. Ramond, Phys. Rev. D 58, 035003 (1998);

[27] P. Ginsparg, Phys. Lett. B 197, 139 (1987); V. S. Kaplunovsky, Nucl. Phys. B 307, 145 (1988), Erratum-ibid. B 382, 436 (1992);
[28] H. K. Dreiner, H. Murayama and M. Thormeier, M. Cvetic, L. L. Everett and J. Wang, Phys. Rev. D59, 107901 (1999).

[29] E. Witten, Nucl. Phys. B471, 135 (1996).

[30] See for example, J. Ellis, T. Falk, K. Olive and M. Srednicki, Astropart. Phys. 13, 181 (2000); R. Arnowitt, B. Dutta and Y. Santoso, Nucl. Phys. B606, 59 (2001).

[31] See for example, H. Baer, C. Balazs, A. Belyaev and J. O’Farrill, JCAP 0309, 007 (2003).

[32] M.V. Romalis, W.C. Griffith and E.N. Fortson, Phys. Rev. Lett. 86, 2505 (2001).

[33] J. Hisano and Y. Shimizu, Phys. Lett. B581, 224 (2004); M. Endo, M. Kakizaki and M. Yamaguchi, arXiv:hep-ph/0311072.

[34] See for example J. Gasser and H. Leutwyler, Phys. Rept. 87, 77 (1982).

[35] S.P. Martin, Phys. Rev. D61, 035004 (2000).

[36] S. Weinberg, Phys. Rev. Lett. 63, 2333 (1989); E. Braaten, C.S. Li and T.C. Yuan, ibid. 64, 1709 (1990); J. Dai, H. Dykstra, R.G. Leigh, S. Paban and D.A. Dicus, Phys. Lett. B237, 216 (1991).

[37] O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, arXiv:hep-ph/0402023.