Nonlocal Condensate Model for QCD Sum Rules

Ron-Chou Hsieh and Hsiang-nan Li

1 Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China,
2 Department of Physics, National Tsing-Hua University, Hsinchu, Taiwan 300, Republic of China
3 Department of Physics, National Cheng-Kung University, Tainan, Taiwan 701, Republic of China and
4 Institute of Applied Physics, National Cheng-Chi University, Taipei, Taiwan 116, Republic of China

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We include effects of nonlocal quark condensates into QCD sum rules (QSR) via the Källén-Lehmann representation for a dressed fermion propagator, in which a negative spectral density function manifests their nonperturbative nature. Applying our formalism to the pion form factor as an example, QSR results are in good agreement with data for momentum transfer squared up to \( Q^2 \approx 10 \text{ GeV}^2 \). It is observed that the nonlocal quark condensate contribution descends like \( 1/Q^2 \), different from the exponential decrease in \( Q^2 \) obtained in the literature, and contrary to the linear rise in the local-condensate approximation.

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In QCD sum rules (QSR) nonperturbative contributions are taken into account via vacuum expectation values of nonlocal operators, such as \( \langle \overline{q}(0)q(z) \rangle \) and \( \langle G(0)G(z) \rangle \), where \( q \) is a quark field and \( G \) is the gluon field strength. In the standard approach vacuum effects are assumed to be sufficiently soft to allow the Taylor expansion of, for instance, the quark condensate \( \langle \overline{q}(0)q(z) \rangle \), at \( z = 0 \) by means of local composite operators,

\[
\langle \overline{q}(0)q(z) \rangle = \langle \overline{q}q \rangle + z^\mu \langle \overline{q} \partial_\mu q \rangle + \frac{z^\mu z^\nu}{2} \langle \overline{q} \partial_\mu \partial_\nu q \rangle + \ldots \tag{1}
\]

A local condensate \( \langle \overline{q}q \rangle \), i.e., the first term of the above expansion, prohibits momentum flow. A loop diagram then turns into a tree diagram as shown in Fig. 1 when inserting the local quark condensate into the lower (nonperturbative) line. The external momentum \( q \) flows only through the upper (perturbative) line, and one has the loop integral approximated by the product of the propagator \( 1/q^2 \) and the condensate \( \langle \overline{q}q \rangle \). With this localization assumption, simple hadronic properties including masses, decay constants, moments of hadronic wave functions, and form factors have been calculated in QSR.

It has been known that nonperturbative contributions from local quark condensates grow with the momentum transfer squared \( Q^2 \) in form factor calculations, whereas perturbative contributions decrease [3, 4]. This is the reason why the standard QSR approach encounters difficulty, when applied to form factors in the region with high \( Q^2 > 3 \text{ GeV}^2 \). It has been observed that the \( Q^2 \) dependence of nonperturbative contributions is moderated by employing the nonlocal quark condensate \( \langle \overline{q}(0)q(z) \rangle \). Moreover, using local quark condensates in QSR analysis of more complicated processes, such as Compton scattering which involves four-point correlation functions [6], infrared divergences appear. Consider the box diagram in Fig. 2 where a light hadron is scattered by an on-shell photon of momentum \( q_1 \). The external momentum \( q_1 \) flows through the upper horizontal quark line, when the local quark condensate is inserted into the left vertical quark line. The upper line then gives a divergent propagator proportional to \( 1/q_1^2 \to \infty \), and the evaluation of the Wilson coefficient associated with the quark condensate makes no sense. A resolution of the above difficulties is to relax the localization assumption. Including the nonlocal condensates, a finite loop momentum \( k \) is allowed to flow through the box diagram, and the above infrared divergence is smeared into

\[ \text{FIG. 1: Loop diagram with the insertion of the local quark condensate, where } k \text{ denotes the loop momentum.} \]
This is our motivation to investigate effects of the nonlocal quark condensates in QSR. In this letter we shall set up the framework by studying simpler processes like the pion form factor, and compare the results with the local condensates and with the nonlocal condensates.

Nonlocal condensate models have been applied to QSR for the pion wave function, whose outcome was then treated as an input of the perturbative QCD factorization formula for the pion form factor. Recently, Bakulev, Pimikov and Stefanis calculated the space-like pion form factor based on QSR with nonlocal condensates. They parameterized the nonlocal quark condensate as

$$\langle q(0) q(z) \rangle = \langle q q \rangle \exp(-|z|^2/\lambda_q^2)$$

where \(\lambda_q^2\) is related to the average virtuality of the condensed quarks. Our formalism is different, which starts from the Källén-Lehmann (KL) representation for a dressed propagator of the quark

$$\langle \Omega | T(q(z)q(0)) | \Omega \rangle = i \int \frac{d^4k}{(2\pi)^4} e^{-ik:z} \int_0^\infty d\mu^2 \frac{k\rho_1^q(\mu^2) + \rho_2^q(\mu^2)}{\mu^2 - \mu^2 + i\epsilon}.$$  

where \(\Omega\) represents the exact QCD vacuum, \(T\) denotes the time ordering, the spectral density functions \(\rho_1^q, \rho_2^q(\mu^2)\) describe the glutinous medium effect, and \(\mu\) is the effective mass. The KL representation can be deemed as a superposition of free quark propagators for all mass eigenstates with the weights \(\rho_1^q(\mu^2)\).

Equation (2) is recast into

$$\langle \Omega | T(q(z)q(0)) | \Omega \rangle = \frac{1}{16\pi^2} \int_0^\infty ds \exp\left(\frac{s^2}{4}\right) \int_0^\infty d\mu^2 \exp\left(-\frac{\mu^2^2}{s}\right) \left[ i\frac{s}{2} \rho_1^q(\mu^2) + \rho_2^q(\mu^2) \right] .$$

We decompose the above matrix element into the perturbative and nonperturbative pieces

$$\langle \Omega | T(q(z)q(0)) | \Omega \rangle \equiv iZS(z, m_q) + \langle \Omega \rangle : q(z)q(0) : | \Omega \rangle ,$$

respectively, with \(Z\) being a renormalization constant, \(S(z, m_q)\) being the quark propagator in perturbation theory, \(m_q\) being the quark mass. The nonperturbative piece collects the contribution from large \(\mu^2\),

$$\langle \Omega \rangle : q(z)q(0) : | \Omega \rangle = \frac{1}{16\pi^2} \int_0^\infty ds \exp\left(\frac{s^2}{4}\right) \int_{\mu_q^2}^\infty d\mu^2 \exp\left(-\frac{\mu^2^2}{s}\right) \left[ i\frac{s}{2} \rho_1^q(\mu^2) + \rho_2^q(\mu^2) \right] .$$

The lower bound for the integration variable \(\mu^2\) is usually set to the multi-particle threshold \(m^2_{\gamma}\) in the KL representation. In the above expression we have modified it into \(\mu^2 = \max(cs, m^2_{\gamma})\), where the free parameter \(c\) of order unity will be fixed later. This choice respects the multi-particle threshold, and at the same time guarantees a finite integral in Eq. (5). Note that the integration over \(\mu^2\) in Eq. (4) develops a divergence as the variable \(s\) approaches infinity without the above modification. A negative spectral density function implies confinement, and we indeed have the property \(\rho_1^q(\mu^2) < 0\) as shown in our formalism.
We define the distribution functions

\[ f_v(s) = \frac{3}{2\pi^2(q\bar{q})} \int_{\mu_*^2}^\infty d\mu^2 \exp\left(-\frac{\mu^2}{s}\right) s \rho_v^0(\mu^2), \]

\[ f_s(s) = \frac{-3}{4\pi^2(q\bar{q})} \int_{\mu_*^2}^\infty d\mu^2 \exp\left(-\frac{\mu^2}{s}\right) \rho_s^0(\mu^2), \]

and assume the spectral density functions to be

\[ \rho_v^0(\mu^2) = N_1 \exp(-a\mu^2)/\mu, \quad \rho_s^0(\mu^2) = N_2 \exp(-a\mu^2), \]

with the free parameters \( a, N_1 \) and \( N_2 \). It is then easy to see that

\[ f_s(s) \propto \frac{s}{1 + as} \exp(-\mu_c^2/s - a\mu_c^2), \]

exhibits the limiting behaviors \( \exp(-m_\gamma^2/s) \) at small \( s \) and \( \exp(-acs) \) at large \( s \), consistent with \( \exp(-m_\gamma^2/s) \) and the exponential ansatz \( \exp(-\sigma_q s) \) proposed in the literature \[11, 16\], respectively. Hence, we set the threshold mass to a constituent quark mass \( m_\gamma \sim 0.36 \text{ GeV} \[17\]. Comparing the Taylor expansion of the nonlocal quark condensates \[2, 16, 18\]

\[ \langle \bar{q}(0)q(z) \rangle = -\text{Tr} \left[ \langle \Omega : q(z)\bar{q}(0) : \langle \Omega \rangle \right] 
= \langle \bar{q}q \rangle \left[ 1 + \frac{z^2}{4} \left( \frac{\lambda_q^2}{2} - \frac{m_q^2}{2} \right) + \cdots \right], \]

\[ \langle \bar{q}(0)\gamma_\mu q(z) \rangle = -\text{Tr} \left[ \gamma_\mu \langle \Omega : q(z)\bar{q}(0) : \langle \Omega \rangle \right] 
= -i\frac{z_\mu}{4} \langle \bar{q}q \rangle (m_q + \cdots), \]

with Eq. (5), we obtain the constraints

\[ \int_0^\infty f_v(s)ds = m_q, \]
\[ \int_0^\infty f_s(s)ds = 1, \quad \int_0^\infty sf_s(s)ds = \frac{1}{2}(\lambda_q^2 - m_q^2), \]

which determine \( a, N_1 \) and \( N_2 \) in Eq. (5), given values of \( \lambda_q \) and \( m_q \).

The dressed propagator includes both the perturbative and nonperturbative contributions,

\[ S^q(p) = \frac{\not{p} + m_q}{p^2 - m_q^2} - \frac{1}{2} \left[ (\gamma^\alpha G^\alpha_{\mu\beta} - m_q\gamma_\alpha G^{\alpha\beta}) \right] \frac{G^{\alpha\beta}m_q \not{p}}{p^2 - m_q^2} \]
\[ - \frac{\pi \alpha_s G^{\alpha\beta}m_q \not{p}}{(p^2 - m_q^2)^4} + \left[ \hat{I}^1 + \hat{I}^2 \right] \frac{\exp[-c(p^2 - \mu^2)/\mu^2]}{p^2 - \mu^2}. \]

with the definitions

\[ \hat{I}^0_{1,2}f(\mu) = \int_{m_q^2}^\infty d\mu^2 \rho^{0}_{1,2}(\mu^2)f(\mu). \]

The second and third terms on the right hand side of Eq. (12) arise from the background gluon field \[19, 20\], and the forth term comes from the nonlocal quark condensates with the integrations over \( \mu^2 \) and \( s \) being exchanged in Eq. (5).

As stated before, local quark condensates lead to contributions linear in \( Q^2 \), which are more serious than the constant contributions from local gluon condensates at large \( Q^2 \[3, 14\]. Gluon condensate contributions to the pion form factor are actually negligible, and quark-gluon-antiquark condensates vanish in the localization limit. Besides, the potential infrared divergences in four-point correlation functions are generated from local quark condensates. Therefore, only the nonlocal quark condensates are taken into account here.

Inserting Eq. (12) into the triangle diagrams for the three-point correlation function, we derive the perturbative and nonperturbative contributions to the pion form factor \( F_\pi(Q^2) \),

\[ - f_\pi^2 F_\pi(Q^2) \exp \left( -\frac{2m_q^2}{M^2} \right) = \frac{1}{\pi^2} \left\{ \int_0^{s_0} ds_1 ds_2 \rho^\text{pert}(s_1, s_2, Q^2) \exp \left( -\frac{s_1 + s_2}{M^2} \right) + \Delta^\text{quark} + \Delta^\text{gluon} \right\}. \]
FIG. 3: (a) Perturbative contribution, (b) two-quark condensate contribution, (c) four-quark condensate contribution, and (d) gluon condensate contribution to the pion form factor.

In the above expression $f_\pi$ is the pion decay constant, $m_\pi$ is the pion mass, $M$ is the Borel mass, and $s_0$ is the duality interval. The calculation of the spectral function $\rho^{pert}$ associated with the perturbative contribution, and of the quark (gluon) condensate contribution $\Delta^{\text{quark}}$ ($\Delta^{\text{gluon}}$) involves four types of diagrams displayed in Fig. 3. The perturbative spectral function and the gluon condensate contribution are given by \[3, 4\]

\[
\rho^{pert} = \frac{N_c(e_u - e_d)}{2\lambda^{7/2}} Q^4 \left\{ s_1 (Q^2 + s_1)^3 + s_2 (Q^2 + s_2)^3 - s_1 s_2 [2Q^4 + Q^2 (s_1 + s_2) - 2(s_1^2 + s_2^2) + 6s_1 s_2] \right\},
\]

\[
\Delta^{\text{gluon}} = -\frac{\alpha_s}{12\pi M^2} (G_\alpha^2 S_{\alpha\beta}),
\]

respectively, with $N_c$ being the number of colors, $e_u$ ($e_d$) being the charge of the $u$ ($d$) quark, and the variable

\[
\lambda = (s_1 + s_2 + Q^2)^2 - 4s_1 s_2.
\]

We compute the quark condensate contribution, obtaining

\[
\Delta^{\text{quark}} = \langle \bar{q} q \rangle \int_0^{s_0} ds_1 ds_2 \left[ (\bar{u}_d \hat{T}_1^d - \bar{u}_d \hat{T}_1^d) \rho_{2qc}^a + (\bar{u}_d \hat{T}_1^d - \bar{u}_d \hat{T}_1^d) \rho_{2qc}^b \right] \exp \left( -\frac{s_1 + s_2}{M^2} \right) + \alpha_s \langle \bar{q} q \rangle^2 \Delta^{\text{quark}}_{4qc}.
\]

The two-quark condensate spectral functions $\rho_{2qc}^{a,b}$ and the four-quark condensate function $\Delta^{\text{quark}}_{4qc}$ are written as

\[
\rho_{2qc}^a = \frac{N_c}{\lambda^{7/2}} \left[ Q^6 \mu^2 (Q^2 + \mu^2)^2 - (Q^4 - Q^2 \mu^2 + \mu^4) (s_1^4 + s_2^4) - Q^2 (3Q^4 + 2Q^2 \mu^2 - 5\mu^4) (s_1^2 + s_2^2) 
-3Q^2 (Q^6 + 2Q^4 \mu^2 - 2\mu^6) (s_1^2 + s_2^2) - Q^4 (Q^6 + 2Q^4 \mu^2 + 4Q^2 \mu^4 + 3\mu^6) (s_1 + s_2) 
+6Q^3 \mu^2 \mu^4 - (s_1 + s_2)^2 - 2Q^2 (Q^6 + 5Q^4 \mu^2 - 2Q^2 \mu^4 - 6\mu^6) (s_1 s_2)
\right],
\]

\[
\rho_{2qc}^b = \frac{N_c}{\lambda^{7/2}} \left[ 2Q^2 \mu^2 (3Q^4 - 12Q^2 \mu^2 + 10\mu^4) (s_1^4 + s_2^4) + 6Q^4 \mu^2 \mu^4 - (s_1 s_2)^2 - 2Q^2 (Q^6 + 6Q^4 \mu^2 - 18\mu^4) (s_1 + s_2) 
+3Q^4 - 2Q^2 \mu^2 + 2\mu^4) (s_1^2 + s_2^2) + Q^2 (Q^4 + 6Q^2 \mu^2 - 18\mu^4) (s_1 + s_2) 
+6(s_1^2 s_2 + s_2^2 s_1) - (Q^6 - 6\mu^2) (s_1 s_2 + s_2 s_1) - 6s_1^2 s_2^2 - 2(Q^4 - 12Q^2 \mu^2 + 6\mu^4) s_1 s_2
\right],
\]

\[
\Delta^{\text{quark}}_{4qc} = (e_u \hat{T}_2^d - e_d \hat{T}_2^d) \lim_{m^2 \to 0} \frac{\partial}{\partial m^2} \left( \int_{m^2}^{s_0} ds_1 \int_{m^2}^{s_0} ds_2 + \int_{0}^{\mu^2} ds_1 \int_{0}^{\mu^2} ds_2 \right) \left( e^{-s_1/M^2} - 1 \right) g_1 e^{-s_2/M^2}
+ (e_u \hat{T}_2^d - e_d \hat{T}_2^d) \lim_{m^2 \to 0} \frac{\partial}{\partial m^2} \left( \int_{\mu^2}^{s_0} ds_1 \int_{\mu^2}^{s_0} ds_2 + \int_{0}^{\mu^2} ds_1 \int_{0}^{\mu^2} ds_2 \right) \left( e^{-s_2/M^2} - 1 \right) g_2 e^{-s_1/M^2},
\]
with the functions
\[
g_1 = \frac{8\pi N_c}{3\lambda^{5/2}s_1} \{ Q^4[-6m^4 - \mu^2(Q^2 + \mu^2) + m^2(4Q^2 + 6\mu^2)] + (Q^2 + \mu^2)s_1 + (Q^2 - \mu^2)s_2^2 \\
- (2m^2Q^2 - 2Q^4 - 3Q^2\mu^2 + \mu^4)s_2^2 (2m^2Q^2 - 2Q^4 - Q^2\mu^2 + \mu^4)s_1^2 \\
+ Q^2[Q^4 - Q^2\mu^2 - 2\mu^4 + 2m^2(Q^2 + 3\mu^2)]s_1 + Q^2[Q^4 + 3Q^2\mu^2 + 4\mu^4 + 2m^2(Q^2 - 3\mu^2)]s_2 \\
- (Q^2 + \mu^4)s_1^2s_2 - (Q^2 - 3\mu^2)s_2s_1 + 2s_1s_2(2m^2Q^2 - Q^4 - 2Q^2\mu^2 + \mu^4) \},
\]
\[
g_2 = \frac{8\pi N_c}{3\lambda^{5/2}s_2} \{ Q^4[Q^4 + m^4 - 6\mu^2(Q^2 - \mu^2) + 2m^2(Q^2 - 3\mu^2)] + (m^4 - 4m^2Q^2 + Q^4)s_1^2 + (Q^2 + m^2)^2s_2^2 \\
+ 2Q^2s_2(Q^2 + m^2)(Q^2 + m^2 - 3\mu^2) - 2Q^2s_1[2m^4 - (Q^2 - m^2)(Q^2 - 3\mu^2)] \\
- 2s_1s_2(m^4 - m^2Q^2 - 2Q^4) \},
\]
(21)
and the variables
\[
\alpha = (m^2Q^2 + \mu^2s_2) \left( \frac{1}{Q^2 + \mu^2} + \frac{1}{s_2 - m^2} \right),
\]
\[
\beta = (\mu^2Q^2 + m^2s_1) \left( \frac{1}{Q^2 + m^2} + \frac{1}{s_1 - \mu^2} \right).
\]
(22)
Note that the singularity from \(s_1 \to 0\) \((s_2 \to 0)\) in the function \(g_1\) \((g_2)\) is removed by the factor \((e^{-s_1/M^2} - 1)\) \([e^{-s_2/M^2} - 1]\) in Eq. (20). It is observed that the contributions from the nonlocal quark condensates must be power-like in \(Q^2\) in the asymptotic limit, no matter how to parameterize \(\rho_{1,2}(\mu^2)\). The dominant contribution \(\Delta_{q_{4c}}^{q_{4c}}\) descends like \(1/Q^2\) as \(Q^2 \to \infty\), which is different from the exponential decrease in \(Q^2\) obtained in [3], and contrary to the linear rise in the local condensate approximation [3, 4].

The local condensates appearing in Eqs. (16) and (18) are taken to be [21]
\[
\frac{\alpha_s}{\pi} \langle G_{2\rho}^a \rangle = 0.005 \pm 0.004 \text{ GeV}^2,
\]
\[
\langle \bar{q}q \rangle = -(1.65 \pm 0.15) \times 10^{-2} \text{ GeV}^3
\]
\[
\alpha_s \langle \bar{q}q \rangle^2 = (1.5 \pm 0.2) \times 10^{-4} \text{ GeV}^6.
\]
(23)
The duality interval \(s_0(Q^2)\) at a given \(Q^2\) is determined by the requirement that the form factor is least sensitive to the Borel mass \(M\). The average virtuality \(\lambda_q\) and lower bound \(c\), being not known with certainty, are fixed by fits to the data of the pion form factor \(F_\pi(Q^2)\) which is \(0.179 \pm 0.021\) at \(Q^2 = 1.99 \text{ GeV}^2\) [22, 23]. In figure 4a we display the allowed values of \(c\) and \(\lambda_q\) as a curve in the \(c-\lambda_q\) plane. The range of \(\lambda_q\) is consistent with \(\lambda_q = 0.63 \text{ GeV}\) from QSR [20] and \(\lambda_q = 0.83 \text{ GeV}\) from the instanton analysis [27]. Below we adopt \(\lambda_q = 0.745 \text{ GeV}\) and \(c = 0.25\) to produce the central values of our predictions for the pion form factor. Choosing the light quark masses \(m_u = 4.2 \text{ MeV}\) and \(m_d = 7.5 \text{ MeV}\), we solve for the free parameters \(a\), \(N_1\) and \(N_2\) from the constraints in Eq. (11), whose results are listed in Table I. The product \(ac \approx 7 \text{ GeV}^{-4}\) is in agreement with the value of \(ac \approx 10 \text{ GeV}^{-4}\) postulated in [28]. The opposite signs of \(N_1\) and \(N_2\) imply the violation of positivity, which can be interpreted as a manifestation of confinement [15].

| \(
\begin{array}{cccc}
\lambda_q \text{ (GeV)} & m_q \text{ (MeV)} & a \text{ (GeV}^{-4}) & N_1/\langle q\bar{q} \rangle \text{ (GeV}^{-4}) \\
\hline
\text{u quark} & 0.745 & 4.2 & 28 & 40.32 & -18,790.09 \\
\text{d quark} & 0.745 & 7.5 & 28 & 72.00 & -18,790.09 \\
\end{array}
\)

TABLE I: Parameters associated with the quarks \(u\) and \(d\) in our formalism.

Figure 4b indicates the best choice of \(s_0 = 0.715 \text{ GeV}^2\) with \(\lambda_q = 0.745 \text{ GeV}\) and \(c = 0.25\), at which the pion form factor \(F_\pi(Q^2 = 1.99 \text{ GeV}^2)\) becomes independent of \(M\) for \(M > 1.5 \text{ GeV}\). In the calculation below, we simply set the Borel mass to \(M = 1.5 \text{ GeV}\). In Fig. 4c we present the \(Q^2\) dependence of the best choice \(s_0(Q^2)\) with the same inputs, whose curve is close to a straight line:
\[
s_0(Q^2) = 0.6 + 0.06Q^2 - 0.002Q^4,
\]
(24)
for \(Q^2 > 1 \text{ GeV}^2\). It is seen that \(s_0\) drops rapidly in the region of low \(Q^2 < 1 \text{ GeV}^2\), where QSR are supposed to be inapplicable. \(s_0\) in Fig. 4c, increasing from 0.65 GeV2 to 1.05 GeV2 for 1 GeV2 < Q2 < 10 GeV2, shows a bit
stronger $Q^2$ dependence compared to that in [5]. Nevertheless, its range obeys the postulation [5] that it should not be lower than the middle point 0.6 GeV$^2$ of the interval between the meson masses $m_\pi^2 = 0$ and $m_{A_1}^2 = 1.6$ GeV$^2$. Besides, we have confirmed that the pion decay constant squared takes the value $f_\pi^2 \simeq 0.0171$ GeV$^2$ for $s_0 \approx 0.7$ GeV$^2$ in our formalism with the nonlocal quark condensates.

Our results of the pion form factor $F_\pi(Q^2)$ are displayed in Fig. 5(a) for three values of $\lambda_q = 0.765$, 0.745, and 0.725 GeV, corresponding to the curves from top to bottom, respectively. Their difference indicates the theoretical uncertainty of our analysis. It is obvious that all three curves are well consistent with the experimental data for $Q^2 > 1$ GeV$^2$, the region where QSR are applicable. We investigate the perturbative and condensate contributions to the pion form factor $F_\pi(Q^2 = 1.99$ GeV$^2$) at different Borel mass $M$, as shown in Fig. 5(b). It is observed that the former increases with $M$, and the latter decreases with $M$ for $M > 1$ GeV. The gluon condensate contribution becomes negligible for $M > 1$ GeV, justifying the sole modification from the nonlocal quark condensates. Although the magnitudes of different contributions vary with $M$, their sum is almost constant for $M > 1.5$ GeV. The quark condensates contribute 23% of the pion form factor $F_\pi(Q^2 = 1.99$ GeV$^2$) at $M = 1.5$ GeV, which is slightly higher than the percentage 17% in the localization approximation [5].

In this letter we have included the nonlocal quark condensates into QSR via the KL parametrization for a dressed fermion propagator, which is decomposed into the perturbative and nonperturbative pieces. The negative spectral density function implies that the contribution from higher effective quark masses is nonperturbative. The parametrization of the spectral density functions leads to the known exponential ansatz for the nonlocal condensate model in our formalism. We have analyzed the pion form factor as an example, and the results are in good agreement with the data for $Q^2$ between 1-10 GeV$^2$. The fitted virtuality $\lambda_q = 0.745$ GeV and the derived duality interval $s_0(Q^2)$ are also consistent with those reported previously. The nonlocal quark condensates remedy the improper dependence of the nonperturbative contribution in the localization approximation at large $Q^2$: the quark condensate effects decrease like $1/Q^2$, which is different from the exponential decrease obtained in the literature. Viewing the success of this approach to the pion form factor, we shall extend it to more complicated processes, including Compton scattering and two-photon hadron production [29].

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FIG. 4: (a) Curve for the allowed values of $c$ and $\lambda_q$ that produce the data of $F_\pi(Q^2 = 1.99$ GeV$^2$). (b) $M$ dependence of $F_\pi(Q^2 = 1.99$ GeV$^2$) for different $s_0$ with $\lambda_q = 0.745$ GeV and $c = 0.25$. (c) $Q^2$ dependence of $s_0$ for the pion form factor. The function of the fitting curve is presented in Eq. (24).

FIG. 5: (a) $F_\pi(Q^2)$ for the pion form factor. The function of the fitting curve is presented in Eq. (24).
FIG. 5: (a) $Q^2$ dependence of $F_\pi$ for, from top to bottom, $\lambda_q = 0.765, 0.745, \text{ and } 0.725 \text{ GeV}$. The data points are referred to \[22-25\]. (b) $M$ dependence of the perturbative and condensate contributions to $F_\pi(Q^2 = 1.99 \text{ GeV}^2)$.

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