A constitutive law based on the self-consistent homogenization theory for improved springback simulation of a dual-phase steel

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Abstract. It has been widely observed that below the flow stress of a plastically deformed material the stress–strain response of the material does not obey the linear relation assumed in classical elasto-plastic models. As a matter of fact, a closer observation indicates that the stress–strain response of the material is nonlinear upon unloading. This results in a larger strain recovery than predicted by the linear elastic law which consequently results in an error in springback prediction. Furthermore, when the material undergoes compression after tension, it exhibits Bauschinger effect, transient behavior and permanent softening. The accuracy of the springback prediction is dependent on the capability of the model in capturing the above mentioned phenomena. In this work a constitutive law based on the self-consistent homogenization method is developed. In this model the stress inhomogeneity in the material is realized through considering a distribution in yielding of individual material fractions. The model was calibrated using stress–strain curves obtained from tension–compression experiments. The model has shown to be capable of predicting the nonlinear unloading behavior and the Bauschinger effect while maintaining computational efficiency for FEM simulations.

1. Introduction
There has been an increasing interest by the automotive industry towards employing Advanced High Strength Steels (AHSS) in the past years. Thanks to the high strength offered by the AHSS the the weight of the produced parts can be reduced by using thinner steel sheets while meeting the crashworthiness requirements. Both reduced sheet thickness and the higher strength contribute to a larger springback in parts produced from AHSS than the conventional mild steels. Therefore, an accurate prediction and compensation of the springback are of a high importance for utilization of AHSS. Finite element method (FEM) is often used to optimize the forming process [1]. The accuracy of such simulations are dependent on the accuracy of the stress–strain prediction in the material during the forming process using the constitutive models employed in the simulations [2, 3]. During a typical deep-drawing process, the sheet material continuously undergoes loading, unloading and reverse loading. A material that is subjected to a loading-unloading-reverse loading typically exhibits nonlinear unloading behavior, the Bauschinger effect, transient behavior and permanent softening (see Figure 1). Therefore, for an accurate prediction of stress and strain in the material it is vital to incorporate the constitutive relations that can capture the above mentioned material behavior in the simulation. Over the
Figure 1. Schematic representation of stress-strain response of the material loading-unloading-reverse loading.

past years a wide range of phenomenological models have been developed to describe the material behavior upon load reversal [4–6] and the nonlinear unloading behavior [7, 8] independent from each other. Yet, the inhomogeneous deformation at the microstructure level is expected to be the governing mechanism for the observed material behavior.

The observed macroscopic behavior is directly linked to the material mechanics at microscale that is governed by complex interaction between the grains. Therefore, to study and describe a macroscopically observed behavior, it is important to understand the material behavior at microscale. Numerical simulations [9–11] and experimental observation [9, 12, 13] have shown that the material deformation at microscale is inhomogeneous. This is a consequence of inhomogeneous nature of the polycrystalline metals composed of grains with different orientations, sizes, phases, etc.

In this work a constitutive law based on the elasto-plastic self-consistent homogenization scheme is developed. In this model the stress inhomogeneity in the material is realized through considering a distribution in yielding of individual material fractions. The model was calibrated using a tension–compression stress–strain curve. The model has shown to be capable of predicting the nonlinear unloading behavior as well as the tension–compression response of the material.

2. Material characterization

The material used in this study was DP600 from the family of AHSS. The specimens were cut from a 1.2 mm thin sheet parallel to the rolling direction. The uniaxial tension–compression experiments were conducted using an in-house anti-buckling fixture to prevent the specimen from failing due to buckling under compressive load. For the tension–compression experiments, the material was stretched to various levels of pre-strain and then compressed back to zero strain. The second type of experiment was dedicated to characterize the nonlinear unloading/reloading behavior of the material. To that aim the specimen was uniaxially stretched to different levels of pre-strain and then unloaded. Subsequently the material was repeatedly loaded, unloaded and
reloaded to the next level of pre-strain. The loading/unloading/reloading (LUR) cycles were performed at pre-strains of 2, 4 and 6 % engineering strain.

3. Model description
As was pointed out earlier, the plastic deformation in the material is partitioned among the grains. The onset of the plastic deformation in each grain is dependent on its orientation with respect to the applied load and its strength. In order to consider the relative contribution of yielding at the grain scale to the macroscopic stress response, a yield probability distribution function is introduced. In this manner the net effect of the microstructural inhomogeneities is condensed into a yield stress distribution. Accordingly, the material can be considered as a collection of fractions that yield, each at a certain stress.

In this work the contribution of microscopic yielding on the macroscopic response of the material is resembled by assigning a normal distribution to yield stress of every fraction in the elasto-plastic self-consistent (EPSC) model. In the self-consistent scheme every fraction is idealized as a spherical inclusion embedded in a homogeneous infinite matrix. The matrix has the effective properties of the polycrystalline material that is felt by each inclusion. The interaction between the spherical inclusions and the homogeneous matrix was given by Eshelby [14]. The schematic representation of the self-consistent representative volume element (RVE) approximation of the polycrystalline material is shown in Figure 2.

The local mechanical behavior of each inclusion is related to the macroscopic behavior via the strain concentration tensor according to:

\[
\dot{\varepsilon}_i = A_i : \dot{\varepsilon}
\]  \hspace{1cm} (1)

with

\[
A_i = \left[ E : (C_i^{-1} : C - I) + I \right]^{-1}.
\]  \hspace{1cm} (2)

Here E is the Eshelby tensor and C and C_i are the modulus of the matrix and the ith inclusion respectively. The modulus of the matrix is calculated from the weighted sum of the inclusions moduli as:

\[
C = \sum_k f_i C_i : A_i
\]  \hspace{1cm} (3)

here \(f_i\) is the volume fraction of the inclusion. Equation (2) and (3) are solved iteratively using an implicit solution algorithm. The details on the solution algorithm are given in [15].

In this study 20 element fractions were used. Every fraction was modeled as an elasto-plastic inclusion using the von Mises yield function in combination with the Ludwik hardening law according to:

\[
\sigma_f = \sigma_{yi} + C (\dot{\varepsilon}_i^p)^n
\]  \hspace{1cm} (4)

- Figure 2. The schematic representation of the self-consistent RVE.
where $\sigma_{yi}$ and $\overline{\varepsilon}_i^p$ are the initial yield stress and the equivalent plastic strain of the $i$th inclusion.

Finally, the linearized relation between the homogenized macroscopic stress and strain increment is obtained as:

$$\dot{\sigma} = C : \dot{\varepsilon}$$

(5)

4. Prediction of mechanical response
An optimization method was used to determine the material parameters. In such procedure an objective function is defined as the root mean square error (RMSE) between the simulated and the measured stress-strain data. For this purpose the FMINCON function from optimization toolbox in MATLAB was used. The procedure typically starts with an initial guess of the values and sequentially optimizing the parameters to minimize the objective function. Ludwik hardening parameters, the mean and the standard deviation of the distribution were identified using the tension-compression curve pre-deformed at 8%. In order to minimize the number of the fitting parameters, the hardening behavior of every element is assumed to be the same. The parameters obtained from the fitting procedure for DP600 is summarized in Table 1.

The simulated stress–strain curves using the EPSC material model and the experimental results are shown in Figure 3. Overall, a fairly good agreement between the experimental stress–strain curves with the ones predicted numerically exists. Noticeably, the Bauschinger effect, transient behavior and permanent softening are captured by the model for various levels of pre-straining.

| Table 1. EPSC model parameters for DP600. |
|------------------------------------------|
| $C$ (MPa) | $n$ | $\sigma_{mean}$ (MPa) | $std^*$ (MPa) |
|----------|----|-------------------|--------------|
| 910      | 0.87 | 450               | 500          |

$^*$ Standard deviation of the distribution.
In Figure 4 the result of the LUR experiment is compared with the model prediction that was calibrated using the tension–compression experiment. The model can predict the experimentally observed nonlinear unloading/reloading behavior. The magnitude of the unloading strain is in agreement with the experimental results at the lower pre-strain levels. At higher pre-strain levels the model deviates from the experimental data, under-predicting the nonlinear unloading strain. This can be attributed to the fact that as the material deforms further, due to the local interactions between the grains, the stress distribution in the microstructure becomes more inhomogeneous. This leads to a larger nonlinearity upon unloading and a stronger Bauschinger effect. However, the evolution of the stress distribution due to the local grain interactions or texture evolution is not directly accounted for in this model.

5. Conclusions
A computationally efficient model based on the mean-field elasto-plastic self-consistent (EPSC) approach was developed to predict the cyclic mechanical response of a DP600 dual phase steel from the family of Advanced High Strength Steels. To that aim, a yield distribution was incorporated in the EPSC model to mimic the inhomogeneous deformation at the microscale.

The model successfully captured the Bauschinger effect, transient behavior, permanent softening and nonlinear unloading behavior simultaneously. The key achievement of this approach is that the model is shown to be capable in predicting various features of the cyclic stress–strain responses only by adding two extra fitting parameters for the mean and the standard deviation of the yield distribution in addition to the isotropic Ludwik hardening parameters. These parameters where obtained by fitting the model to only one set of tension–compression experiment. The model was found to be efficient enough to be used for modeling an industrial forming process.

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