Reliability Analysis on Domestic Special CNC Machine Tools Based on Weibull Distribution

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Abstract. Mean time between failures (MTBF), regarded as an important target to evaluate the performance of machine tool, is widely applied in analysis of reliability, especially in reliability model building. Taking Huazhong CNC HNC-848C Complex Machine Tool as research object, this paper conducted censored test to collect MTBF data during a period of 226 days. Based on the data and presupposed Weibull reliability model. Parameters of the model were estimated by least square method. Graph test Method and Kolmogorov-Smirnov Method are implemented to perform the hypothesis test on obtained parameters, which provides evaluation on MTBF of complex machine tool with theoretical basis and data reference.

Keywords. Complex machine tool; Reliability model; Weibull Distribution; Censored test.

1. Introduction
Machine tools, especially high-end machine tools take an indispensable part in current manufacture [1, 2]. However, as a kind of complex mechanical-electronic-hydraulic system, machine tool dose not resemble electronic product or mechanical product with relatively perfect reliability theory and technology.

Meanwhile, comparing with aerospace products and weaponry, the industry of machine tool has no complete reliability technology system. Low reliability in China-made machine tool has become main bottleneck that restricts the development of Chinese machine tool industry. Unlike industrially advanced country, such as German or Japan, researches of machine tool reliability in China have the shortcomings of weak technology accumulation and few people involved because of late start. There is some data indicating that mean time between failures (MTBF) of machine tool made in China is twice as shorter as that of machine tool reached the international leading level [3]. Carrying out research on machine tool reliability is of great significance to break foreign monopoly and realize the localization of machine tool production.

In China, based on fuzzy comprehensive evaluation method, Xia and Shu calculated the correction coefficient of reliability evaluation of CNC machine tools and verified it through comparing theoretical MTBF value and actual MTBF value [4]. Yang proposed a time-varying reliability modeling method which realized the quantitative characterization of the reliability level of the machine tool at any time [5]. Zi researched machine tool spindle reliability based on the theoretical analysis of performance degradation data [6].
For abroad research, researcher of Former Soviet Union proposed a tool machine optimal design method targeting economy and reliability [7]. In America, Jones et al. carried out on-site tracking tests on 35 machine tools for up to 3 years, then recorded and sorted out all fault data, finding that the MTBFs of machine tools obey Weibull distribution of which shape parameter is between 0.8 and 1.07 [8]. Rezvani et al. performed fault analysis on machining units of flexible manufacturing cell by comprehensive use of reliability block diagram, fault tree, discrete Markov model and other methods to calculate the MTBF of machining units [9]. Michael Vineyard et al. conducted analysis on fault frequency of flexible production line, and summed up 6 types of equipment failures which were modeled relatively [10]. In South Korea, Kim et al. used related software to process fault data of machining center and summarized a variety of failure modes, in accordance with which, the system of machining center was evaluated [11].

Along the stream that machine tools are becoming complex and multifunctional, lots of factors result machine tool failures during its running time. The research, which is for China-made machine tool service life, is not mentioned yet. Besides, there are little case focusing on collection and analysis for comprehensive adaptability in small samples.

Taking Huazhong CNC HNC-848C complex machine tool as research object, this paper conducted the evaluation on machine tool reliability using the field censoring test data. The whole paper is divided into 7 sections. In section 2, the time between failures collected during test is outlined. Then the model of MTBF of machine tool is built and the parameters of model are identified in section 3 and section 4, relatively. The identified parameters are tested by hypothesis test in section 5. The observed value and point estimate of MTBF are calculated in section 6. Section 7 is the conclusion.

2. Time between Failures Statistics

This test recorded all 14 failures happened on Huazhong CNC HNC-848C complex machine tool between 8:00 am, 3 Jul, 2020 to 9:30 pm, 14 Feb, 2021.

![Complex machine tool](image1)

Figure 1. Complex machine tool.

After processing by fault handing module, 13 pieces of data of time between failure are outlined in table 1, sorted by size in ascending order.

| Time between failures $t_i$ (h) |
|---|
| 14 | 23.8 | 25.4 | 89.8 | 101.1 | 119.5 | 129.5 |
| 161.5 | 236.4 | 260.4 | 308 | 445.8 | 564 |
3. MTBF Distribution Model
In accordance with data listed in table 1, the data of time between failures in the range of 0 h to 600 h roughly is divided into 6 groups, which were compiled in table 2 of fault frequency.

Table 2. Fault frequency table.

| Number | Upper interval | Down interval | Frequency number | Frequency | Cumulative frequency |
|--------|----------------|---------------|------------------|-----------|---------------------|
| 1      | 0              | 100           | 4                | 0.3076    | 0.3076              |
| 2      | 100            | 200           | 4                | 0.3076    | 0.6152              |
| 3      | 200            | 300           | 2                | 0.1538    | 0.7690              |
| 4      | 300            | 400           | 1                | 0.0769    | 0.8459              |
| 5      | 400            | 500           | 1                | 0.0769    | 0.9228              |
| 6      | 500            | 600           | 1                | 0.0769    | 1.0000              |

Figure 2. Fitting curve of cumulative distribution function of working time between faults.

In the curve drawing in figure 2, the MTBF of complex machine tool obey non-normal distribution or While typical Weibull distribution is shown in figure 3.

It is not difficult to infer there is similarity between MTBF curve [12,13] shown in figure 2 and Weibull Distribution curve in figure 3, which underpins the hypothesis that MTBF of complex machine tool obeys Weibull Distribution.

4. Identification Parameters of MTBF Distribution Model

4.1. Weibull Distribution
The probability density function of Weibull Distribution [14, 15] is:

\[
f(t) = \begin{cases} \frac{\beta}{\alpha} \left(\frac{t-\gamma}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{t-\gamma}{\alpha}\right)^{\beta}\right], & t \geq \gamma \\ 0, & t < \gamma \end{cases}
\]

where \(\beta\) is shape parameter which is greater than 0; \(\alpha\) is scale parameter which is greater than 0; \(\gamma\) is position parameter which is greater than 0.

The distribution function is:
\[ F(t) = \begin{cases} \int_0^t f(t) dt = 1 - \exp \left[ -\left( \frac{t - \gamma}{\alpha} \right)^\beta \right], & t \geq \gamma \\ 0, & t < \gamma \end{cases} \]  

(2)

In the fault analyze of product, \( \beta \) is related with fault principle, which means \( \beta \) varies with fault principle. In engineering practice, \( \beta \) less than 1 means analyze tends to perform early machine fault early distribution. When \( \beta \) is equal to 1, it means accidental faults take a large part of analyze. If it is supposed to analyze loss fault error, \( \beta \) should be greater than 1.

Working condition is in related with \( \alpha \). The value of \( \alpha \) is smaller when the huge payload is applied on machine and vice versa.

Function curve position is influence d by \( \gamma \). There is no fault happens while \( t < \gamma \) and fault emerges only on the condition that usage time exceeds \( \gamma \).

The identification on Position parameter \( \gamma \) is complicated. Consequently, model built by this study includes 2 parameters of \( \alpha \) and \( \beta \), in which \( \gamma \) is eliminated by assuming fault happens at \( t = \gamma \).

Thus, equation (2) and (3) change into:

\[ f(t) = \begin{cases} \frac{\beta}{\alpha} \left( \frac{t - \gamma}{\alpha} \right)^{\beta-1} \exp \left[ -\left( \frac{t}{\alpha} \right)^\beta \right], & t \geq 0 \\ 0, & t < \gamma \end{cases} \]  

(3)

\[ F(t) = \begin{cases} 1 - \exp \left[ -\left( \frac{t}{\alpha} \right)^\beta \right], & t \geq \gamma \\ 0, & t < \gamma \end{cases} \]  

(4)

4.2. Identification on Parameters in Model

In general, there are two methods used to identify the parameters in machine tool reliability model, which are analysis method and linear regression method [16]. In the identification of parameters of distribution with integral terms, analysis method is widely applied since its remarkable effectiveness on identification precision [17]. However, as for some kinds of distribution without integral terms, such as Weibull Distribution and Extremum Distribution, linear regression based on least square method characterized by simplicity and intuitiveness in calculate process, has gained many implements in parameter identification. Hence, it is used to identify the parameter in Weibull model constructed before.

Transform (4) to linear form:

\[ \ln \left[ \ln \left( \frac{1}{1 - F(t)} \right) \right] = \beta \ln t - \beta \ln \alpha, \quad t \geq 0 \]  

(5)

Letting \( x' = \ln t \), \( y' = \ln \left[ \ln \left( \frac{1}{1 - F(t)} \right) \right] \), convert equation (5) to:

\[ y' = \beta x' - \beta \ln \alpha \]  

(6)

where \( \alpha \) and \( \beta \) is the slope and intercept of (6).

Least square method is utilized to identify \( \alpha \) and \( \beta \). The introduction of least square method is as follows: suppose several experiment results were obtained: \((x_1, y_1), (x_2, y_2), ..., (x_n, y_n)\) while a linear relationship is presented between \( x \) and \( y \). Assume \( y \) as a function of \( x \) is:

\[ \hat{y} = A + Bx \]  

(7)

where \( A \) and \( B \) are the slope and intercept of equation (8).

Let:

\[
\begin{align*}
\bar{x} &= \frac{1}{n} \sum_{i=1}^{n} x_i \\
\bar{y} &= \frac{1}{n} \sum_{i=1}^{n} y_i
\end{align*}
\]  

(8)
The estimates of A and B are:

\[
\begin{align*}
A &= \hat{y} - B\bar{x} \\
B &= \frac{\sum_{i=1}^{n} x_i n_x y_i}{\sum_{i=1}^{n} x_i^2 n_x^2}
\end{align*}
\]

Based on least square method, the value of \(\alpha\) and \(\beta\) in equation (6) is estimated: \(\alpha=198.3434\), \(\beta=0.9483\).

Under the data given in table 1, detailed data in calculation is shown in table 3:

**Table 3.** Collation table of failure test data of composite machining machine tools.

| i  | t(i) | F(t)    | x    | y    | xx   | yy   | xy   |
|----|------|---------|------|------|------|------|------|
| 1  | 14   | 0.04861 | 2.6391 | -2.9991 | 6.9646 | 8.9945 | -7.9147 |
| 2  | 23.8 | 0.11805 | 3.1697 | -2.0744 | 10.0469 | 4.3033 | -6.5753 |
| 3  | 25.4 | 0.18750 | 3.2347 | -1.5719 | 10.4636 | 2.4710 | -5.0848 |
| 4  | 89.8 | 0.25694 | 4.4976 | -1.2141 | 20.2283 | 1.4739 | -5.4604 |
| 5  | 101.1 | 0.32638 | 4.6161 | -0.9286 | 21.3085 | 1.4739 | -4.2865 |
| 6  | 119.5 | 0.39583 | 4.7833 | -0.6853 | 22.8801 | 0.4697 | -3.2783 |
| 7  | 129.5 | 0.46528 | 4.8637 | -0.4684 | 23.6554 | 0.2194 | -2.2781 |
| 8  | 161.5 | 0.53472 | 5.0845 | -0.2677 | 25.8522 | 0.0717 | -1.3612 |
| 9  | 236.4 | 0.60416 | 5.5655 | -0.0761 | 29.8719 | 0.0127 | 0.6287 |
| 10 | 260.4 | 0.67361 | 5.5622 | 0.1130 | 30.9382 | 0.0940 | 1.7573 |
| 11 | 308  | 0.74305 | 5.7301 | 0.3067 | 32.8340 | 0.2654 | 3.1426 |
| 12 | 445.8 | 0.81250 | 6.0999 | 0.5152 | 37.2084 | 0.5764 | 4.8097 |
| 13 | 564  | 0.88194 | 6.3351 | 0.7592 | 40.1329 | 0.5764 | 4.8097 |

MTBF of Complex machine tool obeys Weibull distribution is verified by test.

Then distribution function \(F(t)\), probability density function \(f(t)\), reliability function \(R(t)\) and failure rate function \(\lambda(t)\) are obtained:

\[
F(t) = 1 - \exp\left[-\left(\frac{t}{198.3434}\right)^{198.3434}\right]
\]

\[
f(t) = \frac{0.9483}{198.3434}\left(\frac{t}{198.3434}\right)^{-0.0517} \exp\left[-\left(\frac{t}{198.3434}\right)^{198.3434}\right]
\]

\[
R(t) = 1 - F(t) = \exp\left[-\left(\frac{t}{198.3434}\right)^{198.3434}\right]
\]

\[
\lambda(t) = \frac{f(t)}{R(t)} = \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1} = \frac{0.9483}{198.3434}\left(\frac{t}{198.3434}\right)^{-0.0517}
\]

**4.3. Complex Machine Tool Reliability Index**

According to the model built previous [18], curves of distribution function \(F(t)\), probability density function \(f(t)\), reliability function \(R(t)\) and failure rate function \(\lambda(t)\) are shown in figures 3-6.
Figure 3. Probability density function curve of failures interval time.

Figure 4. Curve of distribution function of time between failures.
5. Hypothesis Test of Weibull Distribution

5.1. Pattern Test
A figure taking the observed reliability value as abscissa and the calculated reliability value by fitting model as ordinate, is called PP Figure. If fitting model is appropriate, the curve of PP Figure would be a straight line.

Figure 7 shows the PP Figure based on data in this study. It is not difficult to indicate that curve of
PP Figure performs a straight line while failure model based on Weibull Distribution was verified by pattern test.

![Figure 7. PP data analysis and verification method of fault data.](image)

5.2. **K-S Test Method**

Assume the MBTF of complex tool machine obeys Weibull Distribution:

\[
F_0(t) = 1 - \exp \left[ - \left( \frac{t}{198.3434} \right)^{198.3434} \right]
\]  
(14)

Let empirical distribution function marked as \(F_n(t)\):

\[
F_n(t) = \begin{cases} 
0 & t < t_i \\
\frac{i}{n}, t_{i-1} < t < t_{i+1} \\
1 & t < t_n 
\end{cases}
\]  
(15)

By K-S test method, Define test statistics is the maximum of absolution of difference between \(F_i(t)\) and \(F_n(t)\), noted as \(D_n\):

\[
D_n = \sup_{-\infty < x < \infty} |F_n(t_i) - F_0(t_i)|
\]  
(16)

Compare \(D_n\) and critical value \(D_{n,a}\). If \(D_n < D_{n,a}\) is satisfied, hypothesis would be accepted while if not, hypothesis would be rejected.

Assign significance level \(a_0 = 0.10\). According to empirical formula, when \(n=14\):

\[
D_{n,a} = \frac{1.22}{\sqrt{n}} = 0.326
\]  
(17)

In this study \(D_n = 0.46627\), which is less than \(D_{n,a} = 0.326\). Hence, origin hypothesis which stands MBTF obeys Weibull distribution, is accepted.

6. **Calculation of MTBF**

6.1. **The Observed Value of MTBF**

The observed value is calculated by maximum likelihood estimation [19]:

\[
\text{MTBF}_0 = \frac{1}{N_0} \sum_{i=1}^{N_0} t_i
\]  
(18)

where \(N_0\) is accumulated valued of frequency during test cycle, \(t_i\) is the interval time between one single failure and latest one, of which unit is hour.

Based on the data outlined in table.1, complex machine tool had been running for 2479.2h and the
observed valued MTBF is 190.71h.

6.2. Point Estimate of MTBF
Point estimate of MTBF is:

\[
\text{MTBF}_{P} = \int_{0}^{\infty} tf(t)dt = a \Gamma\left(\frac{1}{\beta} + 1\right)
\]  \hspace{1cm} (19)

where \( \alpha = \exp\left(-\frac{A}{\hat{B}}\right) = 198.3484, \beta = \hat{B} = 0.9483. \) As a result, \( \text{MTBF}_{P} = 203.7551. \)

Because of point estimate varies with sample, there is error between estimated value and actual value. It is necessary to assess the error in point estimate. The precision of point estimate is expressed in confidence interval where actual value is included with a certain level of confidence \([20, 21]\). Set certain level of confidence as 90%, then two-sided confidence interval of \( \text{MTBF}_{P} \) is:

\[
\frac{2T}{x_{0.95}^{2}(2n+2)} < \theta < \frac{2T}{x_{0.05}^{2}(2n)}
\]  \hspace{1cm} (20)

where \( n \) is failure happening times; \( T \) is the total time for which complex machine tool running during test.

According \( \chi^2 \) distribution table, \( T=2479.2h \) and time interval of failures obeys Weibull distribution, when certain level of confidence is 90%, the two sided confidence interval of MTBF estimate is: [127.514h, 292.911h].

7. Conclusion
For distribution law of failures interval time of Huazhong CNC HNC-848C Complex Machine Tool, this paper analyzed complex machine tool censored failures data. Referring to Weibull distribution, a model of failures interval time was built, then identified the 2 parameters of scale parameter, which is 198.3484, and shape parameter, which is 0.9483. Evaluation is taken on MTBF of complex machine tool and judgment of point estimate and two-sided confidence interval was obtained.

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