Assessment of Carrying Capacity of Timber Element Using SBRA Method

Michal Kraus 1

1 Institute of Technology and Business in České Budějovice, Department of Civil Engineering, České Budějovice, Czech Republic

info@krausmichal.cz

Abstract. Wood as a building material has a significant perspective in the context of non-renewable energy sources and production of greenhouse gas emissions. The subject of this paper is to verify the carrying capacity of the timber element using the probabilistic method Simulation Based Reliability Assessment (SBRA). The simulation is performed for one million cycles. Key factors decreasing the strength of wooden material at the time include the duration of the loads, and combinations thereof. Inconsiderable factor affecting the strength of wood is also the humidity. Continuous beam with three fields (length 15 m, glued laminated timber, and strength class GL 36 according to the DIN EN 1194) is placed in an environment with a thermal-humidity regime of the 2nd class according to the EC 5. Average life of carrying timber structure is estimated to be 50 years. The simulation results show that there is no risk of failure of wood during the first year. The probability of failure is common in the 10 years of its life. Then, wooden element already meets only a reduced level of reliability.

1. Introduction
Timber construction has a strong perspective in the context of scarcity of raw materials. In the context of sustainable development, wood as a building material is constantly more and more popular. It can be used as substituent of classic industrial materials [1]. Mechanical properties express ability to resist to external forces. Flexibility, strength, plasticity, toughness are essential mechanical properties of timber structures. The mechanical properties of wood are highly anisotropic. The arrangement of the wood structure is a major determinant of the mechanical properties. The properties of timber element can be modification by impregnation, chemical modification or thermal modification [2]. Life of timber structures is estimated at 40-80 years.

Assessments of the carrying load using SBRA method is based on analysis of reliability function for selected periods [3, 4]. Equation 1 expresses a function of reliability $SF$, where $R$ is the resistance of the structure expressed by histogram, and $S$ is the effect of loading expressed by histogram. Structural resistance $R$ and combined effects of loading $S$ are determined at any time $T_i$ using simulation technique and Monte Carlo method. If the reliability function $SF$ is negative (resistance of the structure is lower than the effect of the loading), it can be assumed fault of structure. The fault is the temporary or permanent loss of ability to perform its function [5]. Histogram of reliability function is the result of calculating all simulation steps. The probability of failure over time is determined by dividing the number of negative values of reliability and the total number of simulations [4, 6].
Assessments of the carrying load of timber element is based on the simplified assumption of time-dependent decrease of strength of wood in the first limit state of SBRA method. The duration of load and its influence on the strength of the timber element can be expressed by Madison curve [4, 7]. Madison curve can be expressed by the equation 2, where $f_t$ is the strength of the wood in time $t$ [MPa]; $f_0$ is characteristic strength of wood; $a$ and $b$ are constants determined short-term strength of the wood. Equation 3 expresses strength of the wood element for the basic unit of time $t = 1$ hour.

$$S_F = R - S$$ \hfill (1)

$$f_t = f_0 \cdot a \cdot t^b$$ \hfill (2)

$$f_t = f_0 \cdot 0,9313 \cdot t^{-0,0361}$$ \hfill (3)

**Figure 1.** Scheme of the accumulation of damage (Madison curve) according to Lokaj [4]

Failure probability $P_f$ is obtained by analysing the reliability function $S_F$. Probabilistic reliability assessment is carried out by comparing the calculated values with standard values in Table 1.

**Table 1.** Design failure probability $P_d$ according to CSN 73 1401 (1998), Appendix A.

| Level of reliability | Ultimate limit state | Serviceability limit state |
|----------------------|----------------------|----------------------------|
| Increased            | 0.000 008            | 0.023                      |
| Common               | 0.000 070            | 0.070                      |
| Decreased            | 0.000 500            | 0.160                      |
2. Specification and input data

Continuous beam with three fields is defined as a model example for simulation of assessment of carrying load using method SBRA. The total length of the beam is 15 meters. The beam is made of a glued laminated timber of strength class GL 36 according to CSN EN 1194. The beam is placed in an environment with temperature and humidity appropriate to service class 2 according to Eurocode 5. The beam is loaded by its own weight $q$ and a group of single loads $P$ (load of timber beams with spacing of 1.00 m). Figure 2 shows a static scheme the assessed continuous beam [8].

![Figure 2. Scheme of assessed timber continuous beam](image)

The final single loads $P$ includes a combination of design loads:

\[
P_{DL} = DL \cdot G_{DL, var} \tag{4}
\]

\[
P_{LL} = LL \cdot G_{LL, var} \tag{5}
\]

\[
q_{DL} = q \cdot Q_{DL, var} \tag{6}
\]

where $P_{DL}$ is dead load, $P_{LL}$ is long-lasting load. The values of $DL$ and $LL$ represents extreme design values of dead load ($DL = 7.04$ kN, including a floor surface layer, a cement screeds, hardened mineral wool, particle boards and beams 160x220 mm) and long-lasting load ($LL = 11.49$ kN, in category C - schools). The coefficients $G_{DL, var}$ and $G_{LL, var}$ reflect the variability of loads and there are represented by histograms DEAD1 and LONG1 (Figure 4). A continuous load $q$ expresses own weight of the girder ($q = 0.57$ kN/m).

Lateral and torsional rigidity of the beam is secured by bracing. Section of the beam is rectangular layout with dimensions $b = 300$ mm $h = 460$ mm. The normal stress can be determined from the following relationship, where $M$ is an extreme moment, $W$ is a flexural modulus.

\[
\sigma_x = \frac{M}{W_y} \tag{7}
\]

\[
W_y = \frac{1}{6} \cdot b \cdot h^2 \tag{8}
\]

Resistance of the die (R) is represented by the function of Madison curve, which can be expressed in the form of a power function (Equation 3). The partial factor for material glued laminated timber is $\gamma_m=1.25$. The final maximum torque of the die above the support expressed by the three-moment equations by using symmetry of the die.
3. Simulation calculation of carrying capacity

Assessment of carrying capacity of the beam is performed using a software AntHill for 1 million simulation cycles. The probability of failure at selected times during the 50 years, can be determined based on the rendered histogram from software AntHill. The probability of failure $P_f$ is determined as a ratio of steps, which corresponds to the negative value of $R-S$ and the total number of simulation steps. If the calculated failure probability is less than the design probability quoted in the standards for the selected level of confidence, then the proposal of element can be regarded as suitable.
4. Results and discussions

Figure 7 shows the function of reliability over time. The horizontal axis reflects time in 50 years (438,000 hours). The vertical axis shows the reliability function \( SF = R - S \) (MPa). The horizontal line in the bottom graph shows the threshold, when resistance of the structure (R) equals to effect of loads (S). The reliability function is negative below this limit. Negative reliability function constitutes a violation of the element. The vertical lines show the selected times for which the wooden element is assessed. The element is assessed at time 1 year, 10, 20, 30, 40 and 50 years.

Figure 7. The dependence of reliability function (SF) on the time during the 50 years.
Figure 8. Histogram of the reliability function at time $T=1$ year (8 760 hours); $P_f = 0.000 \ 000 \ 000$

Figure 9. Histogram of the reliability function at time $T=10$ year (87 600 hours); $P_f = 0.000 \ 051 \ 54$

Figure 10. Histogram of the reliability function at time $T=20$ year (175 200 hours); $P_f = 0.000 \ 112 \ 81$

Figure 11. Histogram of the reliability function at time $T=30$ year (262 800 hours); $P_f = 0.000 \ 246 \ 99$
5. Conclusions
The main goal of the paper is to verify the carrying capacity of the timber element using the probabilistic method Simulation Based Reliability Assessment (SBRA). The simulation results are compared with standard values. Continuous beam of glued laminated timber meets the increased requirements in the first year of operation. There is no failure during the first year of operation ($P_f = 0.000 00 < P_d = 0.000 008$). Girder meets the common requirements in the 10 years of its operation ($P_f = 0.000 05 < P_d = 0.000 07$). The beam meets the reduced requirements in the following years of the durability ($P_f = 0.000 11-0.000 45 < P_d = 0.000 50$).
References

[1] Y. Obata, T. Takeuchi, Y. Furuta, K. Kanayama, “Research on better use of wood for sustainable development: Quantitative evaluation of good tactile warmth of wood, “ Energy, vol 30, issue 8, pp. 1317-1328, 2005.

[2] M. H. Ramage, H. Burridge, M Busse-Wicher, G. Fereday, T. Reynolds, D. U. Stah, G. Wu, L. Yu, P. Fleming, D. Densley-Tingley, J. Allwood, P. Dupree, P. F. Linden, O. Scherman, “The wood from the trees: The use of timber in construction, “ Renewable and Sustainable Energy Reviews, vol. 687, part 1, pp. 333-359, 2017.

[3] Leonardo da Vinci Pilot Project CZ, Implementation of Eurocodes, Handbook 2, Reliability backgrounds, 2005.

[4] A. Lokaj, “Timber Beam Reliability Assessment,” Euro-SiBRAM’2002 Prague, 2002.

[5] M. Kraus, T. Vondráčková, V. Nývlt, “Defects, faults and accidents of contemporary constructions,” MATEC Web Conferences: 8th International Scientific Conference Building Defects (Building Defects 2016), vol. 93, 03004, 2017.

[6] C.R. Sundararajan, Probabilistic Structural Mechanics Handbook: Theory and Industrial Applications, 2012.

[7] L.W. Wood, Relation of strenght of wood to duration of stress, 1951.

[8] M. Kraus, Reconstruction and Outbuilding of the Academy - Havírov, 2009.