RECONSTRUCTING A CEPHEID LIGHT CURVE WITH FOURIER TECHNIQUES. I. THE FOURIER EXPANSION AND INTERRELATIONS

Chow-Choong Ngeow and Shashi M. Kanbur
Astronomy Department, University of Massachusetts, Amherst, MA 01003; ngeow@nova.astro.umass.edu

Sergei Nikolaev
Institute for Geophysics and Planetary Physics, Lawrence Livermore National Laboratory, Livermore, CA 94550

Nial R. Tanvir
Department of Physical Science, University of Hertfordshire, College Lane, Hatfield, AL10 9AB, UK

AND

Martin A. Hendry
Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, UK

Received 2002 October 17; accepted 2002 November 25

ABSTRACT

Fourier decomposition is a well-established technique used in the study of stellar pulsation. However, the quality of reconstructed light curves using this method is reduced when the observed data have uneven phase coverage. We use simulated annealing techniques together with Fourier decomposition to improve the quality of the Fourier reconstruction for many Optical Gravitational Lensing Experiment LMC fundamental-mode Cepheids. This method restricts the range that Fourier amplitudes can take. The ranges are specified by well-sampled Cepheids in the Galaxy and Magellanic Clouds. We also apply this method to reconstructing Cepheid light curves observed by the Hubble Space Telescope (HST). These typically consist of 12 V-band and four I-band points. We employ a direct Fourier fit to the 12 V-band points using the simulated annealing method mentioned above and explicitly derive and use Fourier interrelations to reconstruct the I-band light curve. We discuss advantages and drawbacks of this method when applied to HST Cepheid data over existing template methods. Application of this method to reconstruct the light curves of Cepheids observed in NGC 4258 shows that the derived Cepheid distance ($\mu_0 = 29.38 \pm 0.06$ mag, random error) is consistent with its geometrical distance ($\mu_0 = 29.28 \pm 0.09$ mag) derived from observations of its water maser.

Subject headings: Cepheids — distance scale — galaxies: distances and redshifts — methods: data analysis

1. INTRODUCTION

The study of Cepheid light curves has two major applications: (1) distance determinations to nearby galaxies via the well-calibrated period-luminosity ($P-L$) relations (see, e.g., Madore & Freedman 1985, 1991; Feast & Walker 1987; Saha, Labhardt, & Prosser 2000) and (2) the understanding of the pulsational behavior of Cepheids by comparing the observed light curves to their theoretical counterparts (Simon & Davis 1983; Buchler, Moskalik, & Kovác 1990). Therefore, reconstructing the Cepheid light curves from the observed photometric data is important regarding these two aspects.

Because Cepheid light curves are periodic, the data points from well-observed Cepheids can be described by Fourier expansion. This technique was first introduced by Schaltenbrand & Tammann in 1971 and developed by Simon & Lee (1981) to study the structural properties of Cepheid light curves. In general, the $n$th-order Fourier expansion has the following form:

$$m(t) = A_0 + \sum_{j=1}^{n} A_j \cos(j\omega t + \phi_j),$$

where $\omega = 2\pi/P$ is the frequency and $P$ is the period of the Cepheid in days. The $A_0$ term is the mean value of the light curve; $A_j$ and $\phi_j$ are the Fourier amplitudes and phases for the $j$th order, respectively. In this paper we develop the use of simulated annealing techniques to improve the quality of light-curve reconstruction in nearby galaxies such as the LMC. We also apply these techniques in the light-curve reconstruction of Cepheids observed by the Hubble Space Telescope (HST), which is necessary for distance determinations. In particular, we use Fourier expansion, as given by equation (1) but with modifications, to reconstruct the V-band light curves, and we use Fourier interrelations to reconstruct the I-band light curves. These Fourier techniques have been established in stellar pulsation studies. We also present some potential advantages and drawbacks of the application of our method to this problem.

Sections 2 and 4 describe the Fourier expansion and interrelations in detail, respectively. Section 3 briefly describes the application of these Fourier techniques to HST data. The error analysis of the light-curve reconstruction procedures is presented in § 5, followed by the discussion and conclusion in § 6. In the Appendix, we show another Fourier method, Fourier intrarelations, that can be applied to reconstruct the Cepheid light curves. However, this method is less preferable than Fourier interrelations.

2. FOURIER EXPANSION

The form of Fourier expansion is given by equation (1), which has been extensively used in studying the light curves of pulsating stars (see Antonello et al. 1987 and references therein). Some applications of equation (1) in the study of the Cepheids can be found, for example, in Simon & Lee...
(1981), Simon & Davis (1983), Simon & Moffett (1985),
Antonello & Poretti (1986), Andreasen & Petersen (1987),
Buchler & Moskalik (1994), Poretti (1994), and Hintz &
Joner (1997). Obviously, equation (1) has 2n + 1 unknown
parameters, which require at least the same number of data
points to solve for these parameters (in this paper we assume
all of the periods for the Cepheids are given in the literature
because they can be determined with other methods). Most
of the observations of Galactic (see, e.g., Schaltenbrand &
Tammann 1971; Simon & Lee 1981; Moffett & Barnes 1985;
Ferro et al. 1998) and Magellanic Cloud Cepheids (see, e.g.,
Andreasen & Petersen 1987; Moffett et al. 1998; Udalski et
al. 1999a, 1999b) include large numbers of epochs, which
permit the higher order of Fourier expansion, up to n = 8.
In contrast, there are typically about 12 V-band epochs in
the HST observations of extragalactic Cepheids (see § 3),
which permit a fourth-order Fourier expansion. The fourth-
order fit is a compromise between the goodness of fit and
available data: higher order fits may not result in a better fit,
and lower order fits may not capture the structural proper-
ties of the light curves. Since there are only four I-band data
points in the HST observations (see § 3), the fourth-order
Fourier expansion cannot be applied.

Since the period is known from the literature, we can
fold the time observation into phase as \( \Phi(t) = \frac{(t - t_0)}{P} \),
with \( t_0 \) being a common starting
epoch. The value of \( \Phi \) is from 0 to 1, corresponding to a full
cycle of pulsation. Hence, equation (1) can be written as
(Schaltenbrand & Tammann 1971)

\[
m(t) = A_0 + \sum_{j=1}^{4} A_j \cos[2\pi\Phi(t) + \phi_j].
\]

(2)

Fig. 1.—Example of the reconstructed light curve for a Cepheid in NGC
1326A. The solid curve shows the reconstructed light curve from the least-
squares solution. It is clear that the least-squares solution for this Cepheid
does not provide a satisfactory fit to the data. Rather than solving for the
parameters in eq. (2), we fit the parameters in eq. (2) with an appropriate
choice for the ranges (as given in § 2.2) for Fourier amplitudes, \( A_j \).
The resulting light curve is drawn with a dashed curve, for comparison. The
original data are indicated with error bars.

Fig. 2.—Two typical examples for the constructed light curves (in the V band)
by using the simulated annealing method for LMC Cepheids with similar
periods. The ranges for the Fourier phases that are used in fitting the data points
are from 0 to 2\( \pi \), and the ranges for the Fourier amplitudes are \( A_1 \): 0–0.7, \( A_2 \),
\( A_3 \), and \( A_4 \): 0–0.5. This choice of range is relatively large, as demonstrated in § 2.1. (a) Well-constructed light curve. The data points are distributed relatively
uniformly in phase. This LMC Cepheid has \( \log P = 0.4576 \). (b) Poorly constructed light curve due to the bad phase coverage; it exhibits a numerical bump at
phase ~0.5. This LMC Cepheid has \( \log P = 0.4580 \).
In general, one approach to solve the unknown parameters in equation (2) is by using the linear least-squares method together with the available data. However, we find that in some cases the solution does not provide a good fit to the data and produces some numerical bumps in the light curves. Figure 1 illustrates this problem. The solid curve is the resulting light curve from the least-squares solution to a Cepheid in NGC 1326A. The light curve does not fit the data well and exhibits numerical bumps around phases ~0.2 and ~0.5. These numerical bumps are not physically associated with the true light curves but result from the sparsely sampled data points. Instead of solving for the unknown parameters, we specify the range of each parameter in equation (2) and fit the data to obtain the best-fit values for each parameter. All the parameters are fitted simultaneously with the simulated annealing method (see, e.g., Press et al. 1992) to minimize the corresponding $\chi^2$ value: $\chi^2 = \sum (m_{\text{obs}} - m_{\text{fit}})^2 \sigma_{\text{obs}}^2$. The simulated annealing method searches the best-fit values for a group of parameters within a reasonable amount of computational time. The dashed curve in Figure 1 shows that fitting the parameters in equation (2) to the observed data with an appropriate choice for the ranges (given in § 2.2 for Fourier amplitude, $A_j$), greatly improves the reconstructed light curve. In the following subsections, we demonstrate how the choice of the ranges for Fourier parameters affect the reconstructed light curves.

2.1. The Ranges of the Fourier Parameters

In order for the simulated annealing method to fit the available data points, certain ranges of the fitted parameters have to be imposed. The range for the mean magnitude ($A_0$) term in equation (2) is trivial (say, ±5 mag from the mean magnitude of the data points), and the range for the phase ($\phi_j$) is between 0 and $2\pi$. For the Fourier amplitudes, the following ranges are initially used to fit the Fourier expansion:

$A_1$: 0.0–0.7; $A_2$: 0.0–0.5; $A_3$: 0.0–0.5; $A_4$: 0.0–0.5.

These initial ranges are selected based on the guess that some Cepheids may have large Fourier amplitudes, but Simon &
Lee (1981) show the Fourier amplitudes are generally smaller than 0.4. Therefore, we pick the above ranges as a starting point. By fitting the data with the ranges given above, most of the light curves can be reconstructed fairly well with good agreement to the data points. Figure 2a shows an example of the reconstructed light curve for a Cepheid in the LMC with this initial choice for the ranges of Fourier amplitudes. However, these initial ranges also produce some poorly reconstructed light curves due to the bad phased coverage of the data points, as illustrated in Figure 2b. Comparing the two cases in Figure 2 (which have similar periods), it is clear that some of the Fourier amplitudes for poorly reconstructed light curves are relatively large, indicating that the choice of the initial ranges is inappropriate.

The quality of the reconstructed light curve can be improved by further narrowing the ranges of the Fourier amplitudes when fitting equation (2) to the data points. In order to determine the proper ranges for the Fourier amplitudes, we use the Optical Gravitational Lensing Experiment (OGLE) LMC data (Udalski et al. 1999a), which are obtained from the OGLE Web site.¹ There are 771 fundamental-mode Cepheids in the data, based on the judgment made by the OGLE team (Udalski et al. 1999a). The numbers of epochs in the V and I bands are 12–50 and more than 80, respectively. The large numbers of epochs are sufficient to construct the light curves for individual Cepheids by using equation (2). However, the 10 Cepheids that do not have V-band data and the Cepheids that are not fundamental-mode Cepheids were removed from the final sample. Because the Fourier amplitudes are independent of distance, we also include a “calibrating set” of Cepheids (Hendry, Tanvir, & Kanbur 1999), which is comprised mostly of Galactic and some LMC/SMC Cepheids. In addition, most of the Cepheids observed by the OGLE team have periods shorter than 10 days, with peaks around 0.5 < log \( P \) < 0.7, and terminating at log \( P \) = 1.5. In contrast, the calibrating set Cepheids occupy a wider range of period, from log \( P \) ~ 0.2 to 2.0. Therefore, including the calibrating set Cepheids can be used to determine the ranges of Fourier amplitudes beyond log \( P \) = 1.5. There are about 118 Cepheids in this sample. Since J-band data for the OGLE LMC Cepheids have a large number of epochs, we mainly impose ranges for the Fourier amplitudes in the V band.

¹ See http://bulge.princeton.edu/~ogle.

---

**Fig. 4.**—Same as Fig. 3, but for Cepheids with well-constructed light curves (crosses). The Cepheids in the calibrating set are shown as filled circles, for comparison.
The \(V\)-band Fourier amplitudes.—After constructing the light curves of the OGLE LMC Cepheids with initial (and relatively large) Fourier amplitudes, we divided the \(V\)-band light curves into two groups by visual inspection: those with well-constructed light curves (as in Fig. 2a) and those with poorly constructed light curves (as in Fig. 2b). The distributions of the Fourier amplitudes as a function of \(\log P\) are plotted in Figure 3. This shows the positions of Fourier amplitudes for well-constructed light curves (crosses) and poorly constructed light curves (triangles) as a function of period. As can be seen from the figure, the distributions of the Fourier amplitudes for well-constructed light curves occupy a smaller range than those of the poorly constructed light curves, which have relatively larger amplitudes, especially for higher order harmonics. The \(V\)-band light curves are further trimmed to contain only the really well constructed light curves, exemplified by Figure 2a. We then replot the distributions in Figure 4, which shows evidence of well-defined ranges for the \(V\)-band Fourier amplitudes.

The \(I\)-band Fourier amplitudes.—Since the number of epochs for most of the OGLE LMC Cepheids in the \(I\) band is more than 80, the constructed light curve is expected to be well behaved, and the Fourier amplitudes are assumed to fall within the appropriate ranges. The plots of the Fourier amplitudes are shown in Figure 5, with some suspected outliers marked as triangles and labeled. We also plot out some of the Fourier light curves for these outliers in Figure 6. As seen from the figure, some of them have acceptable light curves, with slightly larger amplitudes when compared to the Fourier light curves from Cepheids with similar periods. The exception is C686, which exhibits a tip around phase 0.1 from the original data. The simple fourth-order Fourier expansion cannot reproduce this tip, and hence, the constructed light curves do not show good agreement with the data. Nevertheless, the difference in amplitudes between the outliers and the majority of the data points is very small because the \(I\)-band amplitudes for Cepheids are generally smaller (Freedman 1988).

2.2. The Adopted Ranges for Fourier Amplitudes

From Figures 3–5, it is clear that the Fourier amplitudes occupy certain ranges in the amplitude versus \(\log P\) plots. Therefore, we can determine the appropriate ranges of Fourier amplitudes for a given period from these figures.
The adopted ranges for the Fourier amplitudes in the $V$ and $I$ bands are presented in Table 1. Because of the fact that the OGLE LMC data only contain Cepheids with periods shorter than 32 days ($\log P < 1.5$), the ranges for Cepheid with $\log P > 1.5$ are determined from the calibrating set, while the ranges for shorter period Cepheids are determined from both OGLE LMC and calibrating set data. In addition, the ranges of the Fourier amplitudes are set to start from zero to account for possible low-amplitude Cepheids. Note that the upper limit of the ranges given in Table 1 is an approximation since there is no exact upper limit for a given period. Experience shows that sometimes a slightly larger range of the Fourier amplitudes than the one given in Table 1 can reconstruct the light curve better. An iterative process

**TABLE 1**

**A D O P T E D R A N G E S F O R T H E F O U R I E R A M P L I T U D E S**

| Period Ranges | $A_1(V)$ | $A_2(V)$ | $A_3(V)$ | $A_4(V)$ |
|---------------|----------|----------|----------|----------|
| $0.0 < \log P \leq 1.0$              | 0.0–0.40 | 0.0–0.18 | 0.0–0.11 | 0.0–0.08 |
| $1.0 < \log P \leq 1.1$              | 0.0–0.44 | 0.0–0.14 | 0.0–0.08 | 0.0–0.07 |
| $1.1 < \log P \leq 1.2$              | 0.0–0.48 | 0.0–0.16 | 0.0–0.10 | 0.0–0.08 |
| $1.2 < \log P \leq 1.3$              | 0.0–0.52 | 0.0–0.19 | 0.0–0.13 | 0.0–0.09 |
| $1.3 < \log P \leq 1.4$              | 0.0–0.52 | 0.0–0.23 | 0.0–0.16 | 0.0–0.10 |
| $1.4 < \log P \leq 1.5$              | 0.0–0.52 | 0.0–0.26 | 0.0–0.16 | 0.0–0.10 |
| $1.5 < \log P \leq 1.6$              | 0.0–0.49 | 0.0–0.25 | 0.0–0.14 | 0.0–0.10 |
| $1.6 < \log P \leq 1.7$              | 0.0–0.48 | 0.0–0.23 | 0.0–0.13 | 0.0–0.09 |
| $1.7 < \log P \leq 1.8$              | 0.0–0.43 | 0.0–0.20 | 0.0–0.12 | 0.0–0.08 |
| $1.8 < \log P \leq 1.9$              | 0.0–0.40 | 0.0–0.17 | 0.0–0.11 | 0.0–0.07 |
| $1.9 < \log P \leq 2.0$              | 0.0–0.35 | 0.0–0.13 | 0.0–0.10 | 0.0–0.06 |

| Period Ranges | $A_1(I)$ | $A_2(I)$ | $A_3(I)$ | $A_4(I)$ |
|---------------|----------|----------|----------|----------|
| $0.0 < \log P \leq 1.0$              | 0.0–0.25 | 0.0–0.12 | 0.0–0.08 | 0.0–0.045 |
| $1.0 < \log P \leq 1.1$              | 0.0–0.28 | 0.0–0.10 | 0.0–0.07 | 0.0–0.04 |
| $1.1 < \log P \leq 1.2$              | 0.0–0.31 | 0.0–0.11 | 0.0–0.08 | 0.0–0.05 |
| $1.2 < \log P \leq 1.3$              | 0.0–0.33 | 0.0–0.12 | 0.0–0.09 | 0.0–0.06 |
| $1.3 < \log P \leq 1.4$              | 0.0–0.34 | 0.0–0.13 | 0.0–0.10 | 0.0–0.065 |
| $1.4 < \log P \leq 1.5$              | 0.0–0.34 | 0.0–0.15 | 0.0–0.12 | 0.0–0.07 |
| $1.5 < \log P \leq 1.6$              | 0.0–0.34 | 0.0–0.16 | 0.0–0.11 | 0.0–0.07 |
| $1.6 < \log P \leq 1.7$              | 0.0–0.31 | 0.0–0.15 | 0.0–0.10 | 0.0–0.065 |
| $1.7 < \log P \leq 1.8$              | 0.0–0.27 | 0.0–0.13 | 0.0–0.09 | 0.0–0.06 |
| $1.8 < \log P \leq 1.9$              | 0.0–0.24 | 0.0–0.11 | 0.0–0.08 | 0.0–0.055 |
| $1.9 < \log P \leq 2.0$              | 0.0–0.21 | 0.0–0.10 | 0.0–0.07 | 0.0–0.05 |

**Note.**—Determined from Figs. 3–5 for both the $V$ and $I$ bands.
can be made, if necessary, to find the most suitable upper limits to reconstruct satisfactory light curves. The reduction in ranges can improve the quality of the Fourier fit since some of the numerical bumps are removed in the reconstructed light curve, provided that the problem of bad phase coverage is not too severe (see details in §2.3).

The distributions of the Fourier amplitudes for OGLE LMC Cepheids and the calibrating set (or mostly Galactic) Cepheids appear to coincide, as seen in Figures 3–5. This may imply that the ranges of the Fourier amplitudes depend weakly on metallicity. However, this conclusion is only based on the analysis of two galaxies and may not reflect the assumption that metallicity can affect the distribution of Fourier amplitudes. Van Genderen (1978) showed that the upper limits of $B$-band amplitudes are different for Cepheids in the Galaxy/M31, LMC, and SMC. In addition, Paczyński & Pindor (2000) showed that the OGLE LMC Cepheids, in the period range of $1.1 < \log P < 1.4$, have

Fig. 7.—Examples of the reconstructed light curves fitted with the ranges of Fourier amplitudes given in Table 1. These light curves are indicated as solid curves. The dotted curves show the light curves fitted with initial ranges of Fourier amplitudes (i.e. $A_0$: 0–0.7; $A_1$, $A_2$, and $A_3$: 0–0.5). The periods of the OGLE LMC Cepheids are also given in upper right-hand corner. Original data points are also indicated.

Fig. 8.—Examples of the reconstructed light curves that exhibit numerical bumps/dips, regardless of fitting the data with smaller (solid curves, as given in Table 1) or larger (dotted curves) ranges of the Fourier amplitudes. This is mainly due to the bad phase coverage in the data, with gaps in between the data points. Original data points are also indicated.
larger $I$-band amplitudes than OGLE SMC Cepheids with the same period range. Because the $V$-band amplitudes can be scaled from $B$- and $I$-band amplitudes (Freedman 1988), the different $V$-band upper limits for Cepheids in different galaxies could also exist. Nevertheless, the ranges of Fourier amplitudes given in Table 1 are assumed to cover the Fourier amplitudes for different metallicity environments because these ranges go from zero to the upper limit, which is slightly larger than the ranges defined by OGLE LMC and calibrating set Cepheids. The detailed analysis of the relationship between the Fourier amplitudes and the metallicity environments will be presented in future work.

2.3. Results and Problems of the Fits

Using the techniques of fourth-order Fourier expansion with the appropriate ranges for the Fourier amplitudes, some of the poorly reconstructed light curves can be improved, and the results show good agreement with observational data. Figure 7 shows some examples of the improved reconstructed light curves when fitting equation (2) to the OGLE LMC $V$-band data by narrowing the Fourier amplitudes as given in Table 1. As can be seen from the figure, some of the obvious bumps (or dips) in the original reconstructed light curves can be removed with this simple technique, resulting in smoother light curves and better fits to the data. The estimation of the mean magnitudes would be closer to the true mean without the influence of the numerical bumps/dips.

In certain extreme cases in which either the data points are heavily clustered in certain phases or there is a large gap between two adjacent data points, the Fourier expansion fails to construct a satisfactory light curve, regardless of the choice of the ranges for Fourier amplitudes. Figure 8 shows some

![Figure 7](image1)

![Figure 8](image2)

![Figure 9](image3)

Fig. 9.—Interrelations among the Fourier amplitudes in the calibrating set Cepheids. Crosses represent corresponding Fourier amplitudes and standard errors for each Cepheids. The dashed lines show the best-fit straight lines to the data, with the slopes and zero points given in Table 2. Filled and open squares show bump and nonbump Cepheids, respectively.
examples of this case. Using either the initial ranges (dotted curves) or the narrower ranges (solid curves) to fit the Fourier amplitudes with observed data points does not yield satisfactory reconstructed light curves because of the phase clustering of the original data. When using this technique to estimate means for the $P$-$L$ relations, these Cepheids should be treated with caution or rejected. In fact, the rejection of Cepheids with bad phase coverage has been practiced in the past (see, e.g., Simon & Lee 1981; Antonello & Poretti 1986).

3. APPLICATION TO HST DATA

Since Cepheids are primary distance indicators, the Cepheid $P$-$L$ relations can be used to calibrate secondary distance indicators, such as the Tully-Fisher relation, Type Ia supernovae, the fundamental plane relation, and surface brightness fluctuations. This builds up the distance ladder and indicates the distance to more remote galaxies. Together with the measurements of the recession velocities of these more distant galaxies, the Hubble constant can be determined: this was the main goal of the Hubble $H_0$ Key Project (Freedman et al. 2001, hereafter H0KP).

The conventional way of applying the Cepheid $P$-$L$ relations is by measuring the mean magnitude of the Cepheid over the pulsational cycle for a given period. In recent years, a great deal of effort has been made to discover extragalactic Cepheids and determine the distance by using the $HST$ (as in H0KP and Saha et al. 1999, 2001a, 2001b). In most of the $HST$ observations, in order to optimize the phase coverage of Cepheids within the $HST$ observational windows, the number of $V$-band observations is chosen to be 12 (Freedman et al. 1994; Kennicutt, Freedman, & Mould 1995). To correct for the extinction and reddening, $I$-band observations are included (see, e.g., H0KP and Kennicutt et al. 1995). However, only four (or five) $I$-band observations are

![Graphical representation of Cepheid light-curve reconstruction](image-url)

Fig. 10.—Same as Fig. 9, but for the interrelations among the Fourier phases in the calibrating set Cepheids.
needed because the $I$-band amplitudes are smaller than the $V$-band amplitudes (Freedman 1988; Freedman et al. 1994; Kennicutt et al. 1995). Existing techniques to estimate the mean from the sparsely sampled HST data include (1) taking the phase-weighted intensity mean (Saha & Hoessel 1990) in the $V$ band and using an empirical relation developed by Freedman (1988) in the $I$ band (see, e.g., Silbermann et al. 1996) and (2) adopting template-fitting procedures (Stetson 1996; Tanvir, Ferguson, & Shanks 1999).

In the application of the methods to this problem developed in the previous sections, we use Fourier expansion, as given by equation (1) but with modifications, to reconstruct the $V$-band light curves, and we use Fourier interrelations to reconstruct the $I$-band light curves. Some results of applying these methods to determine the Cepheid distance in nearby galaxies are presented in § 6.4.

### 4. FOURIER INTERRELATIONS

Because the $I$-band observations of the HST typically consist of four epochs, equation (2) cannot be applied to reconstruct the light curves. However, statistical relationships between the $j$th-order Fourier coefficients for the $V$ and $I$ bands have been introduced by Hendry et al. (1999) and Kanbur & Nikolaev (2001), known as the Fourier interrelations. We explicitly derive and present them here because of their applicability in stellar pulsation studies. These allow the reconstruction of $I$-band light curves up to fourth order. There also exist correlations of the first-order Fourier amplitude ($A_1$) to subsequent higher order Fourier amplitudes in the same bands (see, e.g., Antonello et al. 1987), known as the Fourier intrarelations (presented in the Appendix); this approach is less preferable than Fourier interrelations. The Fourier interrelations are the linear

---

**Fig. 11.**—Interrelations among the Fourier amplitudes in the OGLE LMC (fundamental-mode) Cepheids. The solid lines show the best-fit straight lines to the data, with the slopes and zero points given in Table 3. The dashed lines show the best-fit model for calibrating set Cepheids, for comparison.
relation of \( j \)th-order Fourier amplitude and phases from the \( V \) band to the \( I \) band and have the following form:

\[
A_j(I) = \alpha_j + \beta_j A_j(V), \quad j = 1, \ldots, 4, \quad (3)
\]

\[
\phi_j(I) = \gamma_j + \eta_j \phi_j(V), \quad j = 1, \ldots, 4. \quad (4)
\]

The coefficients of the interrelations are determined from the calibrating set, by minimizing the \( \chi^2 \) value of the fit. Fourier interrelations for fundamental-mode Cepheids are presented in Table 2. The table also shows the rms of the straight-line fit to the data. The fits were derived, taking full account of errors in both variables, with the standard model-fitting procedures (see, e.g., Press 1992, pp. 666–681). The Fourier interrelations for equations (3) and (4) are clearly shown in Figures 9 and 10, respectively. The standard errors of the Fourier coefficients, derived from inverting the Hessian matrix, are also shown in the figures. Despite the appearance of strong correlations in the figures, the relations have large \( \chi^2 \) per degree of freedom, i.e., very

| Relation | \( \alpha \) | \( \beta \) | \( \chi^2 \) |
|----------|-------------|-------------|-------|
| \( A_1 \) | \(-0.006 \pm 0.002\) | \(0.643 \pm 0.007\) | \(2.69 \times 10^2\) |
| \( A_2 \) | \(0.001 \pm 0.001\) | \(0.600 \pm 0.011\) | \(9.74 \times 10^2\) |
| \( A_3 \) | \(0.000 \pm 0.001\) | \(0.645 \pm 0.023\) | \(6.05 \times 10^3\) |
| \( A_4 \) | \(0.000 \pm 0.001\) | \(0.631 \pm 0.039\) | \(4.85 \times 10^3\) |

| Relation | \( \gamma \) | \( \eta \) | \( \chi^2 \) |
|----------|-------------|-------------|-------|
| \( \phi_1 \) | \(-0.178 \pm 0.033\) | \(0.996 \pm 0.001\) | \(2.21 \times 10^2\) |
| \( \phi_2 \) | \(-0.048 \pm 0.010\) | \(1.005 \pm 0.003\) | \(6.89 \times 10^2\) |
| \( \phi_3 \) | \(-0.015 \pm 0.021\) | \(1.003 \pm 0.004\) | \(1.02 \times 10^2\) |
| \( \phi_4 \) | \(-0.000 \pm 0.034\) | \(1.004 \pm 0.006\) | \(7.10 \times 10^1\) |

**Note.**—Coefficients correspond to those defined in eqs. (3) and (4). Parameter \( \chi^2 \) characterizes the goodness of fit:

\[
\chi^2 = \sum (y_i - \alpha - \beta x_i)^2 / (\sigma^2_{y_i} + \beta^2 \sigma_x^2).
\]
small statistical significance. Nevertheless, using interrelation to recover sparsely sampled light curves works quite well, as shown in 6. For the Galactic data in the calibrating set, we display bump and nonbump Cepheids with filled and open squares, respectively (because of the \( P_2/P_0 = 0.5 \) resonance around 10 days on Hertzsprung progression, the periods of bump Cepheids normally lie between 8 and 14 days). For the LMC/SMC data, we do not differentiate between bump and nonbump Cepheids.

Besides the Fourier interrelations among the calibrating set Cepheids, similar linear relations also exist for the OGLE LMC and SMC Cepheids. The data for the OGLE LMC Cepheids are as in 2, and the data for the OGLE SMC Cepheids (Udalski et al. 1999b) are downloaded from the OGLE Web site. The number of the fundamental-mode Cepheids, as judged by Udalski et al. 1999b, observed in the SMC is \( \sim 1300 \), with most of them in short period ranges. Table 3 presents the best-fit results of Fourier interrelations to the OGLE LMC data, and the plots of the Fourier interrelations are given in Figures 11 and 12 for Fourier amplitudes and phases, respectively. The preliminary results of the OGLE SMC Fourier interrelations are presented in Table 4, with the plots of Fourier amplitudes and phases in Figures 13 and 14, respectively. Again, a clear linear relation exists with increasing scatter as we go to higher order parameters. There is little difference in the slopes of the best-fit lines to those found for calibrating set Cepheids. Hence, the Fourier interrelations given in Table 2 are applied to reconstruct the \( I \)-band light curves.

The procedures for reconstructing the \( I \)-band light curves are similar to those for \( V \)-band light curves. Instead of fitting all the parameters (the mean magnitude and the Fourier parameters), we use the Fourier interrelations to obtain the \( I \)-band Fourier parameters from the \( V \)-band fit. This gives the shapes of the \( I \)-band light curves, and the mean \( I \)-band magnitudes are found by using the observed \( I \)-band data to minimize the corresponding \( \chi^2 \) values.
**TABLE 3**

Coefficients of the Fourier Interrelations for OGLE LMC (Fundamental-Mode) Cepheids

| Relation | α         | β         | χ^2    |
|----------|-----------|-----------|--------|
| A_1      | 0.004 ± 0.005 | 0.601 ± 0.002 | 2.64 \times 10^3 |
| A_2      | 0.001 ± 0.004 | 0.601 ± 0.004 | 1.68 \times 10^3 |
| A_3      | 0.000 ± 0.001 | 0.606 ± 0.006 | 1.36 \times 10^3 |
| A_4      | -0.001 ± 0.001 | 0.625 ± 0.015 | 1.16 \times 10^3 |

| Relation | α         | β         | χ^2    |
|----------|-----------|-----------|--------|
| φ_1      | -0.208 ± 0.002 | 1.001 ± 0.001 | 4.79 \times 10^3 |
| φ_2      | -0.067 ± 0.004 | 1.001 ± 0.001 | 1.49 \times 10^3 |
| φ_3      | -0.029 ± 0.009 | 1.001 ± 0.001 | 1.20 \times 10^3 |
| φ_4      | -0.063 ± 0.022 | 1.008 ± 0.003 | 9.15 \times 10^2 |

Note.—The columns are the same as in Table 2.

**TABLE 4**

Coefficients of the Fourier Interrelations for OGLE SMC (Fundamental-Mode) Cepheids

| Relation | α         | β         | χ^2    |
|----------|-----------|-----------|--------|
| A_1      | -0.003 ± 0.001 | 0.608 ± 0.002 | 3.87 \times 10^3 |
| A_2      | -0.000 ± 0.001 | 0.623 ± 0.003 | 2.91 \times 10^3 |
| A_3      | -0.001 ± 0.001 | 0.650 ± 0.005 | 2.65 \times 10^3 |
| A_4      | -0.002 ± 0.001 | 0.647 ± 0.010 | 2.28 \times 10^3 |

| Relation | α         | β         | χ^2    |
|----------|-----------|-----------|--------|
| φ_1      | -0.214 ± 0.003 | 1.000 ± 0.001 | 5.46 \times 10^3 |
| φ_2      | -0.075 ± 0.004 | 1.001 ± 0.001 | 3.06 \times 10^3 |
| φ_3      | -0.018 ± 0.008 | 1.000 ± 0.001 | 2.08 \times 10^3 |
| φ_4      | -0.089 ± 0.016 | 1.008 ± 0.002 | 1.54 \times 10^3 |

Note.—The columns are the same as in Table 2.

Fig. 14.—Same as Fig. 13, but for the interrelations among the Fourier phases in the OGLE SMC Cepheids.
There is little difference between the Fourier interrelations for Cepheids in the calibrating set data and in OGLE LMC or SMC data, as shown in Tables 2, 3, and 4, respectively. The good agreement between these indicates that the relative changes in Fourier parameters are almost unaffected by metallicity. This does not mean, however, that the light-curve shape is independent of metallicity. Rather, it means that change in metallicity affects the Fourier parameters in different bands in the same way. Therefore, given a well-sampled light curve in the V band, one can estimate Fourier parameters in the I band reasonably well regardless of the metallicity of the parent galaxy. This result is useful in reconstructing the light curves of extragalactic Cepheids in a broad range of metallicity environments.

The empirical relation that H0KP used to obtain the I-band mean magnitude is based on the observational results of Freedman (1988) since the amplitude ratio of the I and V bands is about 0.5. However, Tanvir (1997) recommended the use of an amplitude ratio of 0.6 because this ratio can improve the estimation of the I-band mean magnitude. We note that the slopes in the Fourier interrelations (β term in Tables 2 and 3) are approximately equal to 0.6, which is close to Tanvir’s value. This is the main reason why Fourier interrelations exist and work well in reconstructing the light curves.

5. ERROR ANALYSIS FOR LIGHT-CURVE RECONSTRUCTION

The Fourier techniques described in this paper are now applied to reconstruct the Cepheid light curves in nearby galaxies observed by HST, particularly for H0KP galaxies. Perhaps the most important question regarding the application of our work to this problem is the following: Does a fourth-order fit with nine free parameters to 12 V-band data points represent anything physically? In order to estimate the errors associated with the light-curve reconstruction procedures presented in §§ 2 and 4, we performed an error analysis based on Monte Carlo simulations.

Because of the large number of Cepheids, as well as the large number of observations in the V and I bands, the simulations are performed primarily with OGLE LMC Cepheids that have good light curves (as in Fig. 2a), represented by crosses in Figure 3. These OGLE LMC Cepheid light curves, constructed by using all the available data points in both bands, are referred to as the original light curves. We assume that when we fit a fourth-order Fourier expansion to these data, the resulting Fourier amplitudes are very close to their true values.

In order to mimic the published HST photometric data, we performed three simulations for the error analysis. The procedures for each simulation are as follows:

**Simulation 1.**—First a Cepheid is picked randomly (with a replacement) from the data set, then 12 points in the V band and four points in the I band were randomly selected without replacement. In addition to the OGLE LMC photometric errors, we add Gaussian noise of \( \mu_{\text{noise}} = 0.05 \) mag and \( \sigma_{\text{noise}} = 0.10 \) mag to these randomly selected data.

**Simulation 2.**—This is the same as in simulation 1, but with larger Gaussian noise of \( \mu_{\text{noise}} = 0.15 \) mag and \( \sigma_{\text{noise}} = 0.10 \) mag.

**Simulation 3.**—We pick one Cepheid with a large number of epochs \( \left( N_e = 34, N_f = 188 \right) \), then 12 points in the V band and four points in the I band were randomly selected without replacement from this Cepheid. The additional Gaussian noise is same as in simulation 1.

Then the data from each simulation are used to reconstruct the light curves with the procedures described in §§ 2 and 4, i.e., a fourth-order Fourier fit in the V band and interrelations in the I band. The 12 randomly selected V-band points could, of course, be uniformly distributed or concentrated around one particular phase point. Mean magnitudes and the Fourier amplitudes obtained from our reconstruction procedures for these simulated data are then compared to the mean magnitudes and Fourier amplitudes from the original light curves. This is repeated a large number of times to build up an error distribution.

We emphasize that we are simulating the photometric data published in the literature. It has been suggested by an anonymous referee that even this is not a real simulation of HST Cepheid data since we neglect the possibility of events such as “warm pixels” or cosmic rays. However, we contend that such data points will be rejected by the photometric reduction package used, and, if not, the point sources responsible for these should not be used anyway.

After running \( N = 1000 \) trials in each simulation, the error histograms for the mean magnitudes, as well as for the Fourier amplitudes, in both the V and I bands are constructed. Gaussian distributions with parameters of \( \mu \) and \( \sigma \) are then fitted to these error histograms, where \( \mu \) represents the mean offsets between the simulated and original data and \( \sigma \) represents the errors in either the means or the Fourier amplitudes. The results of the Gaussian fits from each simulation for the V and I bands are presented in Tables 5 and 6, respectively. The error histograms for the mean values resulting from simulation 1 are presented in Figure 15, with abscissa to be the difference between the simulated means and original means. Similarly, the error histograms for Fourier amplitudes in the V and I bands are presented in Figures 16 and 17, respectively. The parameters from Gaussian fits are listed in the upper left-hand corners of these figures. We did not include the error histograms for simulations 2 and 3 in this paper because they all look similar to the error histograms of simulation 1 (Figs. 15, 16, and 17).

To check the simulations, we also plot the V-band \( R_{21} (= A_2/A_1) \) parameters (Simon & Lee 1981) resulting from simulation 1 as a function of log \( P \) in Figure 18. The filled circles show the original data, and the crosses show the simulated data. Those crosses with triangles show the simulated data when either \( A_1 \) or \( A_2 \) is about 2.0 \( \sigma \) away from the original values. It can be seen from the figure that the simulated data do indeed trace the original data. In addition, the \( P_2/P_0 = 0.5 \) resonance around 10 days can be clearly seen from this figure.

From Tables 5 and 6, it can be seen that the errors of the means and each Fourier amplitude are consistent in all three simulations. As expected, simulation 2 gave the largest errors because the Gaussian noise generated in this simulation is bigger that in the other two simulations. Simulation 3 has the smallest errors because this simulation is performed for one star. The errors for simulation 1 are between these two cases, which is typical. In addition, Tables 5 and 6 show that the mean offsets of the simulated data and original data are very small. This can be seen in Figures 15–17 since all histograms are almost centered at zero. Therefore, no bias is introduced by the reconstruction procedure because all four
Fourier amplitudes are tightly clustered around the "true" values given by the original well-sampled Cepheid. The simulations suggest that with typical HST Cepheid data, it is meaningful to fit a fourth-order Fourier expansion and that our direct Fourier-fitting procedures for estimating the $V$- and $I$-band means are not biased.

6. DISCUSSION AND CONCLUSION

The light curve of a Cepheid can be reconstructed with the Fourier techniques described in this paper. For OGLE LMC Cepheid data, we have demonstrated that we can frequently improve the quality of the reconstructed light curve by restricting the range that Fourier amplitudes can take in the Fourier fit. These ranges are obtained from well-sampled Cepheid light curves in the Galaxy and Magellanic Clouds.

We have then applied these techniques to HST Cepheid data. Such data typically consist of 12 and four $V$- and $I$-band points, respectively. In particular, Cepheids with sufficient observations permit a fourth-order Fourier expansion. However, the Cepheids in HST $I$-band observations that have insufficient data points require the application of Fourier interrelations to reconstruct the $I$-band light curves from the $V$ band. In summary, the Fourier techniques for light-curve reconstruction procedures, as applied to the HST data, include two main parts:

1. Fourth-order Fourier expansion to 12 $V$-band data points. This is reconstructing the light curves by a direct fit to the data points, with appropriate ranges for each of the Fourier parameters.
2. Fourier interrelation to four $I$-band data points. This is using the linear relations of Fourier parameters between the $V$ and $I$ bands to reconstruct the light curves.

In general, these relatively simple techniques can reconstruct the Cepheid light curves quite well, given that the data points are relatively uniform and well sampled. These techniques serve as an alternative method to obtain the mean magnitude of Cepheids besides phase-weighted intensity mean and template-fitting procedures. However, the detailed comparisons between these methods are beyond

---

**Table 5**

| Simulation | $\mu_{\text{mean}}$ | $\sigma_{\mu}$ | $\sigma_{A1}$ | $\sigma_{A2}$ | $\sigma_{A3}$ | $\sigma_{A4}$ |
|------------|---------------------|----------------|--------------|--------------|--------------|--------------|
| 1          | 0.0014              | 0.0121         | -0.0063      | 0.0002       | 0.0063       |
| 2          | 0.0006              | 0.0217         | -0.0062      | 0.0013       | 0.0095       |
| 3          | -0.0018             | -0.0226        | -0.0154      | -0.0071      | 0.0059       |

**Table 6**

| Simulation | $\mu_{\text{mean}}$ | $\sigma_{\mu}$ | $\sigma_{A1}$ | $\sigma_{A2}$ | $\sigma_{A3}$ | $\sigma_{A4}$ |
|------------|---------------------|----------------|--------------|--------------|--------------|--------------|
| 1          | 0.0012              | 0.0071         | -0.0053      | 0.0018       | 0.0048       |
| 2          | 0.0010              | 0.0109         | -0.0052      | 0.0020       | 0.0064       |
| 3          | -0.0042             | -0.0132        | -0.0039      | 0.0018       | -0.0004      |

Note.—The Gaussian parameters $\mu$ and $\sigma$ are mean offsets and errors, respectively, for the means and the Fourier amplitudes.

---

**Fig. 15.**—Histograms of offsets for mean magnitudes in OGLE LMC Cepheids from light-curve reconstruction procedures. The x-axes indicate the offsets of the simulated means from the original values, and the y-axes indicate the normalized counts. The dashed curves show the fitted Gaussian distribution to the histograms, with the parameters given in upper left-hand corners. The left- and right-hand panels show the $V$ and $I$ bands, respectively.
the scope of this paper. Some examples of the reconstructed light curves for extragalactic Cepheids (observed by H0KP) with these Fourier techniques are presented in Figure 19, which shows good agreement with the observed data points.

We also performed Monte Carlo simulations to determine the errors associated with our light-curve reconstruction procedures. The errors of the mean magnitudes and Fourier amplitudes are determined from the fits of Gaussian distribution to the error histograms. These Gaussian fits show that our procedure is unbiased. The errors are as follows:

1. \( V \) band. \( \sigma_{\text{mean}} = 0.034, \sigma_{A1} = 0.039, \sigma_{A2} = 0.033, \sigma_{A3} = 0.028, \) and \( \sigma_{A4} = 0.023. \)
2. \( I \) band. \( \sigma_{\text{mean}} = 0.035, \sigma_{A1} = 0.025, \sigma_{A2} = 0.021, \sigma_{A3} = 0.017, \) and \( \sigma_{A4} = 0.014. \)

Could it be the case that the distribution of 12 points is so severe that a decent fit to the \( V \)-band data is not possible? Certainly, and it could be that in this case a template-type technique will produce a \( V \)- and \( I \)-band mean. However, we again contend that in this case, this point source will probably not be used in the final analysis.

6.1. The Correlations of Fourier Amplitudes

It has been suggested by the anonymous referee that the higher order Fourier amplitudes shown in Figures 3 and 5 are correlated with the first-order Fourier amplitude (\( A_1 \)) in the same bands (the Fourier intrarelations). These have been examined, for example, by Antonello et al. (1987) and in the Appendix. However, the Fourier intrarelations are less preferable than Fourier interrelations for reconstructing the Cepheid light curves. The reasons are discussed in following examples and in the Appendix.

Here, we reexamine the results of Antonello et al. (1987) with our data sets and compare them to the Fourier interrelations. We use all calibrating set Cepheids and pick only the “good” OGLE LMC Cepheids (crosses) from Figure 3. Then we replot the \( A_1 \) versus \( A_2 \) Fourier interrelations and the \( A_2(V) \) versus \( A_1(I) \) Fourier interrelation in Figure 20. The calibrating set Cepheids and OGLE LMC Cepheids are in the left- and right-hand panels, respectively. In the figure, we distinguished Cepheids with different period ranges: crosses show Cepheids with periods shorter than 8 days (short-period Cepheids), triangles show Cepheids with...
periods between 8 and 14 days (bump Cepheids), and filled circles show Cepheids with periods longer than 14 days (long-period Cepheids). Error bars for each data point are omitted in the figure for clarity. The top two panels in Figure 20 show Fourier intrarelations in the $V$ and $I$ bands, respectively, and the bottom panel shows the Fourier interrelations of $A_1(V)$ and $A_1(I)$.

It is clear from the figure that the scatter of the $A_1$ versus $A_2$ intrarelations is larger than the scatter of the $A_1(V)$ versus $A_1(I)$ interrelations. Furthermore, the short-period, bump, and long-period Cepheids populate different regions in the plots of intrarelations, as is also seen in Antonello et al. (1987). In contrast, the tightness of correlations in the Fourier interrelations is clear and less dependent on period distribution. These two properties make Fourier interrelations more applicable in reconstructing the light curves than Fourier intrarelations.

6.2. Reconstructing Light Curves for Short-Period Cepheids

The relatively large range of Fourier amplitudes for Cepheids with periods less than 10 days (or $\log P < 1.0$) is one reason for using the Fourier techniques to reconstruct the Cepheid light curves. From Figures 3–5, it can be seen that the distribution of the Fourier amplitudes at periods longer than 10 days (or $\log P > 1.0$) shows certain trends, which can be used to construct template light curves as a function of period. However, the Fourier amplitudes for Cepheids with periods less than 10 days do not show any obvious trends but scatter around certain ranges (as given in Table 1). For example, at a given short period, $A_1(V)$ may occupy the range from $\sim 0.1$ to $\sim 0.4$. Therefore, extra care has to be taken when applying template fitting to the Cepheids with periods less than 10 days. In fact, at periods shorter than $\log P < 0.85$, the template techniques do not apply (Stetson 1996), but the Fourier techniques still hold good.

Although the extragalactic Cepheids discovered in the past $HST$ observations generally have periods longer than 10 days, some short-period Cepheids have been discovered in Local Group galaxies IC 1613 (Dolphin et al. 2001) and Leo A (Dolphin et al. 2002) with $HST$ and ground-based observations, respectively. Hence, for observing Cepheids at short period, the Fourier techniques are still useful to reconstruct the light curves. With the new installation of the Advanced Camera for Survey on $HST$, the discovery of more short-period Cepheids in other galaxies is probable.
6.3. The Effect of Metallicity

One motivation for studying direct Fourier techniques to reconstruct Cepheid light curves is to deal with the possible metallicity dependence in Cepheids. Antonello, Fugazza, & Mantegazza (2000) have shown that the metallicity may affect the shapes of light curves close to 10 days, based on the comparison of Cepheid light curves in two galaxies. The appropriate range of the Fourier parameters (as given in Table 1) is assumed to cover the possible ranges due to the

![Fig. 18. Plot of $R_{31} = A_2/A_1$ in the $V$ band for simulated data (crosses). The crosses with triangles show the simulated data points with the values of either $A_1$ or $A_2$ being $\sim 2.0 \sigma$ away from the original values. The filled circles show the original OGLE LMC Cepheid data.](image)

![Fig. 19. Examples of the reconstructed light curves in nearby galaxies. The $V$-band light curves (left) are reconstructed via fourth-order Fourier expansion, and the $I$-band light curves (right) are reconstructed via the Fourier interrelations.](image)
different metallicity environments. In addition, the Fourier interrelations are shown to be only weakly dependent on the metallicity, although this does not mean that the actual light-curve shape is independent of metallicity.

The template light curves defined by Stetson (1996) are obtained from the Galaxy ($Z = 0.020$), LMC ($Z = 0.008$), and SMC ($Z = 0.004$), with a sample of $\sim$30–45 Cepheids in each galaxy. Stetson (1996) looked for and failed to find differences between the Galactic, LMC, and SMC Cepheids in terms of their amplitude and light-curve shape sufficient to change the estimation of mean magnitudes. However, much more data are available now. Paczynski & Pindor (2000) found statistically significant different amplitudes between the Galactic, LMC, and SMC Cepheids in the period range $1.1 < \log P < 1.4$, in the sense that higher metallicity Cepheids have higher amplitudes. Moreover, in the period range $0.0 < \log P < 0.95$, the amplitude ratios of $R_{31}$ and $R_{33}$ for Cepheids in the LMC (Andreasen & Petersen 1987) and SMC (Andreasen 1988) are found to be larger than those for the Galactic Cepheids. This may be due to the different metallicity environments, at least for the case of the SMC (Buchler & Moskalik 1994). Further, some established workers in the field have found varying period-color relations between the Galaxy, LMC, and SMC and, subsequently, different $P$-$L$ relations (Tammann et al. 2002; Tammann & Reindl 2003). Thus, while it may be that template methods yield a sufficiently accurate mean in varying metallicity environments, we contend that the issue of light-curve shape and metallicity needs to be revisited in light of the data now currently available.

6.4. Applications in Determining the Cepheid Distances

An immediate application of these Fourier techniques is to reconstruct the Cepheid light curves in NGC 4258 and derive its distance modulus. NGC 4258 is a spiral galaxy that has an accurate geometrical distance of $7.2 \pm 0.3$ Mpc (corresponding to a distance modulus of $29.28 \pm 0.09$ mag) from water maser measurement (Herrnstein et al. 1999). By using the published data of 15 Cepheids discovered from HST observations (Newman et al. 2001) and following the H0KP procedures (including the same $P$-$L$ relations and reddening correction), we derived a distance modulus of $29.38 \pm 0.06$ mag (random errors only) to NGC 4258, which is about 1.1 $\sigma$ away from the water maser distance. If we apply the $-0.07$ mag correction due to the WFPC2 calibration of ‘‘long versus short’’ exposures (this is caused by the charge transfer efficiency of WFPC2; see H0KP and reference therein for details), then the final distance modulus becomes $29.31 \pm 0.06$ mag, which is consistent with the water maser distance. In addition, Kanbur et al. (2002) have also derived the Cepheid distance to NGC 4258 with the same techniques presented here but slightly different $P$-$L$ relations; the result of $29.36 \pm 0.06$ mag (random errors only) is also consistent with the water maser distance and the distance derived in this paper.

When we apply these techniques to determine Cepheid distances to 16 galaxies using published photometry by the H0KP, our distance moduli, on average, are very similar to H0KP results, with a difference of $\sim0.01$ mag (Kanbur et al. 2002). On the other hand, application of these techniques to three Sandage-Tammann-Saha galaxies (NGC 3627: Saha et al. 1999; NGC 3982: Saha et al. 2001b; NGC 4527: Saha et al. 2001a) yields shorter distance moduli of $0.07$ mag, on average (Kanbur et al. 2002). The good agreements between the results in Kanbur et al. (2002) and in both H0KP and the Sandage-Tammann-Saha galaxies suggest that our method is not too far off base.

This work has been supported by HST grant AR 08752.01-A. Part of S. N.’s work was performed under the auspices of the US Department of Energy, the National Nuclear Security Administration at the University of California, and the Lawrence Livermore National Laboratory under contract W7405-Eng-48. We thank an anonymous referee for many helpful suggestions that made the paper more relevant.
APPENDIX

THE FOURIER INTRARELATIONS

The technique of Fourier interrelations, as described in § 4, uses the linear relations between the Fourier parameters in equation (1) in different wave bands. Similarly, there exists another group of linear relations between the Fourier parameters in the same wave band, known as the Fourier intrarelations. The reason for developing the Fourier intrarelations is the same as for Fourier interrelations, mainly to reconstruct the I-band light curves that only contain four epochs. Therefore, the Fourier intrarelations are the linear relation between the first-order Fourier parameter and the higher order Fourier parameters in one particular wave band, since four data points only permit the first-order Fourier expansion. The Fourier intrarelations have the following expression for either the V or I band:

\[
A_j = a_j + b_j A_1, \quad j = 2, \ldots, 4, \tag{A1}
\]

\[
\phi_j = c_j + d_j \phi_1, \quad j = 2, \ldots, 4. \tag{A2}
\]

As in the case of Fourier interrelations, the coefficients are determined from the calibration set Cepheids. The results of the fits to the data with equations (A1) and (A2) are presented in Table 7, which only listed the third- and fourth-order Fourier intrarelations. The second-order Fourier intrarelations were omitted because, for sparse, 12 epoch V-band data, we can fit the data with second-order Fourier expansion and expand to fourth order with Fourier intrarelations to reconstruct V-band light curves. Then we can use the Fourier interrelations to reconstruct the I-band light curves from the V-band light curves.

Fig. 21.—Intrarelations among the Fourier amplitudes in the calibrating set Cepheids. The dashed lines show the best-fit straight lines to the data.
TABLE 7
Coefficients of the Fourier Intrarelations, as Determined from the "Calibrating Set," for Fundamental-Mode Cepheids

| Relation | a            | b            | \(\chi^2\) |
|----------|--------------|--------------|-------------|
| \(A_3(V)\) | \(-0.043 \pm 0.002\) | \(0.281 \pm 0.006\) | \(3.97 \times 10^2\) |
| \(A_4(V)\) | \(-0.021 \pm 0.002\) | \(0.139 \pm 0.006\) | \(2.97 \times 10^2\) |
| \(A_3(I)\) | \(-0.025 \pm 0.002\) | \(0.275 \pm 0.010\) | \(2.77 \times 10^2\) |
| \(A_4(I)\) | \(-0.014 \pm 0.002\) | \(0.145 \pm 0.010\) | \(1.98 \times 10^2\) |

| Relation | c            | d            | \(\chi^2\) |
|----------|--------------|--------------|-------------|
| \(\phi_3(V)\) | \(2.544 \pm 0.020\) | \(3.017 \pm 0.008\) | \(2.42 \times 10^3\) |
| \(\phi_4(V)\) | \(0.202 \pm 0.034\) | \(4.061 \pm 0.013\) | \(1.40 \times 10^3\) |
| \(\phi_3(I)\) | \(3.122 \pm 0.027\) | \(3.043 \pm 0.011\) | \(1.72 \times 10^3\) |
| \(\phi_4(I)\) | \(0.887 \pm 0.044\) | \(4.122 \pm 0.020\) | \(9.02 \times 10^3\) |

Note.—Coefficients are corresponding to those defined in eqs. (A1) and (A2). The \(\chi^2\) parameter is same as in Table 2.

Fig. 22.—Same as Fig. 21, but for the intrarelations among the Fourier phases in the calibrating set Cepheids
The plots for the Fourier intrarelations in calibrating set Cepheids are presented in Figures 21 and 22 for Fourier amplitudes and phases, respectively. From these figures, although a relation clearly exists, it may not be linear, although we show the best-fit linear relation. In the plots of $A_3$ against $A_1$ for both the $V$ and $I$ bands, there are some stars that lie well below the best-fit linear relation. The $A_4$ versus $A_1$ plots also show some evidence of a nonlinearity. Since the error bars have been plotted on these diagrams, the trends described here are real. This nonlinearity may be due to differences in long- and short-period Cepheids.

As in the case of Fourier interrelations, the Fourier intrarelations in OGLE LMC Cepheids have also been found and presented in Table 8. The corresponding plots of the Fourier amplitudes and phases are given in Figures 23 and 24, respectively. The possible nonlinearity of the Fourier intrarelations that seem to be on the calibrating set Cepheids also shows up in these figures, as do some stars that lie below the best-fit linear regression ($A_3$ versus $A_1$) and the indications of nonlinearity in the $A_4$ versus $A_1$ plots. Furthermore, by comparing the Fourier intrarelations in the calibrating set and OGLE LMC Cepheids, we see clearly that the slope of the best-fit line increases from the calibrating set Cepheids to the LMC Cepheids. The difference in slope between calibrating set and LMC Cepheids found in the intrarelations plots is real and is probably attributable to the metallicity differences.

Because of the possible of nonlinearity and metallicity dependence on the parent galaxy, the Fourier intrarelations are less preferable than the Fourier interrelations (§ 4) for reconstructing the $I$-band light curves with only a few data points available.

![Fig. 23. Intrarelations among the Fourier amplitudes in the OGLE LMC (fundamental-mode) Cepheids. The solid lines show the best-fit straight lines to the data. The dashed lines show the best-fit model for calibrating set Cepheids, for comparison.](image)
TABLE 8
Coefficients of the Fourier Intrarelations for OGLE LMC
(Fundamental-Mode) Cepheids

| Relation | $a$       | $b$       | $\chi^2$  |
|----------|-----------|-----------|-----------|
| $A_3(V)$ | $-0.051 \pm 0.001$ | $0.364 \pm 0.005$ | $2.53 \times 10^3$ |
| $A_4(V)$ | $-0.034 \pm 0.002$ | $0.214 \pm 0.005$ | $1.71 \times 10^3$ |
| $A_3(I)$ | $-0.031 \pm 0.001$ | $0.355 \pm 0.002$ | $1.42 \times 10^4$ |
| $A_4(I)$ | $-0.019 \pm 0.001$ | $0.198 \pm 0.002$ | $8.24 \times 10^3$ |

| Relation | $c$       | $d$       | $\chi^2$  |
|----------|-----------|-----------|-----------|
| $\phi_3(V)$ | $2.076 \pm 0.029$ | $3.106 \pm 0.009$ | $2.59 \times 10^3$ |
| $\phi_4(V)$ | $0.072 \pm 0.055$ | $4.084 \pm 0.017$ | $1.74 \times 10^3$ |
| $\phi_3(I)$ | $2.901 \pm 0.011$ | $3.042 \pm 0.003$ | $1.57 \times 10^4$ |
| $\phi_4(I)$ | $1.193 \pm 0.019$ | $3.995 \pm 0.006$ | $7.81 \times 10^3$ |

Note.—The columns are same as in Table 4.

Fig. 24.—Same as Fig. 23, but for the intrarelations among the Fourier phases in the OGLE LMC Cepheids
REFERENCES

Andreasen, G. 1988, A&A, 191, 71
Andreasen, G., & Petersen, J. 1987, A&A, 180, 129
Antonello, E., Broglio, P., Conconi, P., & Mantegazza, L. 1987, A&A, 171, 131
Antonello, E., Fugazza, D., & Mantegazza, L. 2000, A&A, 356, L37
Antonello, E., & Poretti, E. 1986, A&A, 169, 149
Buchler, R., & Moskalik, P. 1994, A&A, 292, 450
Buchler, R., Moskalik, P., & Kovacs, G. 1990, ApJ, 351, 617
Dolphin, A., et al. 2001, ApJ, 550, 554
———. 2002, AJ, 123, 3154
Feast, M., & Walker, A. 1987, ARA&A, 25, 345
Ferro, A., Arellano, E. R., González-Bedolla, S., & Rosenzweig, P. 1998, ApJS, 117, 167
Freedman, W. 1988, ApJ, 326, 691
Freedman, W., et al. 1994, ApJ, 427, 628
———. 2001, ApJ, 553, 47 (H0KP)
Hendry, M., Tanvir, N., & Kanbur, S. 1999, in ASP Conf. Ser. 167, Harmonizing Cosmic Distance Scales in a Post-Hipparcos Era, ed. D. Egret & A. Heck (San Francisco: ASP), 192
Herrnstein, J., et al. 1999, Nature, 400, 539
Hinz, E., & Joner, M. 1997, PASP, 109, 639
Kanbur, S., Ngeow, C., Nikolaev, S., Tanvir, N., & Hendry, M. 2002, ApJ, submitted
Kanbur, S., & Nikolaev, S. 2001, BAAS, 197, 104.02
Kennicutt, R., Jr., Freedman, W., & Mould, J. 1995, AJ, 110, 1476
Madore, B., & Freedman, W. 1985, AJ, 90, 1104
———. 1991, PASP, 103, 933
Moffett, T., & Barnes, T. 1985, ApJS, 58, 843
Moffett, T., Gieren, W. P., Barnes, T. G., & Gomez, M. 1998, ApJS, 117, 135
Newman, J., Ferrarese, L., Stetson, P. B., Maoz, E., Zepf, S. E., Davis, M., Freedman, W. L., & Madore, B. F. 2001, ApJ, 553, 562
Paczyński, B., & Pindor, B. 2000, ApJ, 533, L103
Poretti, E. 1994, A&A, 285, 524
Press, W., Teukolsky, S., Vetterling, W., & Flannery, B. 1992, Numerical Recipes in C (2d ed.; Cambridge: Cambridge Univ. Press)
Saha, A., & Hoessel, J. 1990, AJ, 99, 97
Saha, A., Labhardt, L., & Prosser, C. 2000, PASP, 112, 163
Saha, A., Sandage, A., Tammann, G. A., Dolphin, A. E., Christensen, J., Panagia, N., & Macchetto, F. D. 2001a, ApJ, 562, 314
Saha, A., Sandage, A., Tammann, G. A., Labhardt, L., Macchetto, F. D., & Panagia, N. 1999, ApJ, 522, 802
Saha, A., Sandage, A., Thim, F., Labhardt, L., Tammann, G. A., Christensen, J., Panagia, N., & Macchetto, F. D. 2001b, ApJ, 551, 973
Schaltenbrand, R., & Tamman, G. 1971, A&AS, 4, 265
Silverman, N., et al. 1996, ApJ, 470, 1
Simon, N., & Davis, C. 1983, ApJ, 266, 787
Simon, N., & Lee, A. 1981, ApJ, 248, 291
Simon, N., & Moffett, T. 1985, PASP, 97, 1078
Stetson, P. 1996, PASP, 108, 851
Tammann, G., & Reindl, B. 2003, in XXXVIIth Moriond Astrophysics Meeting. The Cosmological Model, in press (astro-ph/0208176)
Tammann, G., Reindl, B., Thim, F., Saha, A., & Sandage, A. 2002, in ASP Conf. Ser. 283, A New Era in Cosmology, ed. N. Metcalfe & T. Shanks (San Francisco: ASP) (astro-ph/0112489)
Tanvir, N. 1997, in The Extragalactic Distance Scale, ed. M. Livio, M. Donahue, & N. Panagia (Cambridge: Cambridge Univ. Press), 91
Tanvir, N., Ferguson, H., & Shanks, T. 1999, MNRAS, 310, 175
Udalski, A., Soszyński, I., Szymański, M., Kubiak, M., Pietrzyński, G., Woźniak, P., & Zebrun, K. 1999a, Acta Astron., 49, 223
Udalski, A., Soszyński, I., Szymański, M., Kubiak, M., Woźniak, P., & Zebrun, K. 1999b, Acta Astron., 49, 437
van Genderen, A. 1978, A&A, 65, 147