Effect of Tensor Interaction on Density Dependence of Nuclear Symmetry Energy within the BHF Framework

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The effect of tensor force on the density dependence of nuclear symmetry energy has been investigated within the framework of the Brueckner-Hartree-Fock approach. It is shown that the tensor force manifests its effect via the tensor $^3$SD$_1$ channel. The symmetry energy $E_{\text{sym}}$ turns out to be determined essentially by the tensor force from the $\pi$ meson and $\rho$ meson exchanges via the $^3$SD$_1$ coupled channel. Increasing the strength of the tensor component due to the $\rho$-meson exchange tends to enhance the repulsion of the equation of state of symmetric nuclear matter and leads to reduction of symmetry energy. The present results confirm the dominant role played by the tensor force in determining nuclear symmetry energy and its density dependence.

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I. INTRODUCTION

Equation of state (EOS) of asymmetric nuclear matter plays a central role in understanding many physical problems and phenomena in nuclear physics and nuclear astrophysics, ranging from the structure of rare isotopes and heavy nuclei \cite{Mayer, Ministre} to the astrophysical phenomena such as supernova explosions, the structure and cooling properties of neutron stars \cite{Nagle, Ng}. Nuclear symmetry energy describes the isovector part of the EOS of asymmetric nuclear matter. To determine the symmetry energy and its density-dependence in a wide range of density, especially at supra-saturation densities, is a new challenge in nuclear physics and heavy ion physics \cite{Boguta, Boguta2}.

Up to now, the density dependence of symmetry energy at low densities below $\sim 1.2\rho_0$ has been constrained to a certain extent by the experimental observables of heavy ion collisions and some structure information of finite nuclei, such as isospin-scaling of multifragmentation, isospin transport, neutron-proton differential collective flow in HIC and neutron skins, pygmy dipole resonances of finite nuclei, etc \cite{Boguta, Boguta2}. However, the density dependence of symmetry energy at high densities remains poorly known. Different groups \cite{Liang, Elster} have obtained completely different high-density behaviour of symmetry energy by comparing the experimental data measured by the FOPI Collaboration at GSI \cite{Liang} with their calculated results of transport models.

Theoretically, the EOS of asymmetric nuclear matter and symmetry energy have been investigated extensively by adopting various many-body methods, such as the Brueckner-Hartree-Fock (BHF) \cite{Pieper, Blaizot} and Dirac-BHF \cite{Boguta, Boguta2} approaches, the in-medium $T$-matrix and Green function methods \cite{Pieper, Blaizot}, and the variational approach \cite{Pieper, Blaizot}. Although almost all theoretical approaches are able to reproduce the empirical value of symmetry energy at the saturation density, the discrepancy among the predicted density-dependence of symmetry energy at high densities by adopting different many-body approaches and/or by using different nucleon-nucleon ($NN$) interactions has been shown to be quite large \cite{Pieper, Blaizot}. In order to clarify the above-mentioned discrepancy among different theoretical predictions, it is desirable to investigate the microscopic mechanism which controls the high-density behavior of symmetry energy.

The tensor interaction and its effect on finite nuclei and infinite nuclear matter have been discussed in connection with the evolution of nuclear spectra \cite{Pieper}, the evolution of nuclear shell structure \cite{Boguta, Boguta2}, the nuclear saturation mechanism \cite{Pieper, Blaizot}, and the contribution of isovector mesons to symmetry energy \cite{Kaplan, Kaplans}. Due to the tensor coupling is not well determined from experimental data, especially in short-range \cite{Pieper}, there still are a lot of open questions. One of the most important questions is: what is the role played by the short-range tensor force in determining the EOS of asymmetric nuclear matter? In Ref. \cite{Pieper}, the contributions of various components of $NN$ interaction to symmetry energy and its slope parameter $L$ at saturation density have been studied within the BHF framework and it has been shown that the tensor component is decisive for determining symmetry energy and $L$ around saturation density. In Ref. \cite{Kaplan}, the effect of the short-range tensor force due to the $\rho$ meson exchange has been investigated. In that paper, the tensor force have been added to the Gogny central force by hand and its effect has been discussed by adjusting the in-medium $\rho$ meson mass according to the Brown-Rho Scaling.

In the present paper, we shall investigate the effect of tensor force on the density dependence of symmetry energy, within the framework of the BHF approach.

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II. THEORETICAL APPROACHES AND FORMALISM

The present investigation is based on the BHF approach for asymmetric nuclear matter [13, 20]. Here we simply give a brief review for completeness. The starting point of the BHF approach is the reaction $G$-matrix, which satisfies the following isospin dependent Bethe-Goldstone (BG) equation,

$$G(\rho, \beta; \omega) = V_{NN} + V_{NN} \sum_{k_1 k_2} \frac{\langle k_1 k_2 \rangle \langle k_1 k_2 \rangle \langle k_1 k_2 \rangle}{\omega - \epsilon(k_1) - \epsilon(k_2)} \langle k_1 k_2 \rangle$$

where $k_1 \equiv (k_1, \sigma_1, \tau_1)$ denotes the momentum, the $z$-component of spin and isospin of nucleon, respectively. $V_{NN}$ is the realistic $NN$ interaction and $\omega$ is starting energy. For the realistic $NN$ interaction $V_{NN}$, we adopt the Bonn-$B$ interaction [15]. The Pauli operator $Q(k_1, k_2)$ prevents two nucleons in intermediate states from being scattered into their respective Fermi seas. The asymmetry parameter $\beta$ is defined as $\beta = (n - p)/n$, where $n$, $p$, and $p_n$ denote the total nucleon, neutron and proton number densities, respectively. The single-particle (s.p.) energy $\epsilon(k)$ is given by: $\epsilon(k) = h^2 k^2/2m + U(k)$. The auxiliary s.p. potential $U(k)$ controls the convergent rate of the hole-line expansion [16]. In the present calculation, we adopt the continuous choice for the auxiliary potential since it provides a much faster convergence of the hole-line expansion up to high densities than the gap choice [47].

In the BHF approximation, the EOS of nuclear matter (i.e., the energy per nucleon of nuclear matter) is given by [16]:

$$E_A(\rho, \beta) = \frac{E(\rho, \beta)}{A} = \frac{3}{5} \frac{k_F^2}{2m} + \frac{1}{2\rho} \sum_{k', \leq k_F} \langle kk' | G(\rho, \beta; \epsilon(k) + \epsilon(k')) | kk' \rangle A.$$  

One of the main purposes of this paper is to study the density dependence of symmetry energy which describes the isospin dependent part of the EOS of asymmetric nuclear matter and is defined generally as:

$$E_{sym} = \frac{1}{2} \left[ \frac{\partial^2 E_A(\rho, \beta)}{\partial \beta^2} \right]_{\beta = 0}.$$  

It has been shown by microscopic investigation [19-21, 29] that the energy per nucleon $E_A(\rho, \beta)$ of asymmetric nuclear matter fulfills satisfactorily a linear dependence on $\beta^2$ in the whole asymmetry range of $0 \leq \beta \leq 1$, indicating that the EOS of asymmetric nuclear matter can be expressed as:

$$E_A(\rho, \beta) = E_A(\rho, \beta = 0) + E_{sym}(\rho) \beta^2$$

The above result provides an microscopic support for the empirical $\beta^2$-law extracted from the nuclear mass table and extended its validity up to the highest asymmetry. Accordingly the symmetry energy can be readily obtained from the difference between the EOS of pure neutron matter and that of symmetric nuclear matter, i.e.,

$$E_{sym} = E_A(\rho, \beta = 1) - E_A(\rho, \beta = 0)$$

III. RESULTS AND DISCUSSIONS

The Bonn-$B$ two-body interaction adopted in the present calculation is an explicit one-boson-exchange potential (OBEP) and it describes the experimental $NN$ phase shifts with a high precision [43]. The Bonn potentials content the pseudoscalar $\pi$ and $\eta$ mesons, the scalar $\sigma$ and $\delta$ mesons, the vector $\rho$ and $\omega$ mesons. In the OBEP, the tensor components are determined by the competition between the $\pi$ meson and $\rho$ meson exchanges in the isospin singlet ($S = 1, T = 0$) neutron-proton channel, and they can be written explicitly in configuration space as follows [43]:

$$V_T^\pi(r) = \frac{1}{12} \frac{f_\pi^2}{4\pi} m_\pi Z(m_\pi r) \hat{S}_{12},$$

$$V_T^\rho(r) = -\frac{1}{12} \frac{f_\rho^2}{4\pi} m_\rho Z(m_\rho r) (1 + \frac{f_\rho}{g_\rho})^2 \hat{S}_{12},$$

where $\hat{S}_{12} = \sigma_{12} - 3 \left(\sigma_{11} \tau_1 \tau_2 + \sigma_{12} \tau_1 \tau_2\right)$ is the tensor operator; $f_\pi$ and $f_\rho$ denote the tensor coupling constants for the $\pi$ and $\rho$ meson exchanges, respectively; $g_\rho$ is the vector coupling constant; $Z(x) = (m_\alpha/m)^2(1+3/x^3+3/x^2)(e^{-x}/x)$. We notice that a large value of $f_\rho/g_\rho = 6.1$ has been confirmed and consistently adopted in the OBEP [43, 48]. Therefore, the tensor coupling of the $\rho$ meson exchange is much stronger than its vector coupling. And consequently it may suppress the tensor contribution from the $\pi$ meson exchange at short-range due to the fact that the mass of $\rho$ meson is much larger than the $\pi$ mass. The $\pi$ exchange provides a strongly attractive long- and mediate-range tensor component. A cancelation of the opposite contributions from the $\pi$ meson and $\rho$ meson exchanges is supposed to generate an mediate-range attractive and short-range repulsive tensor force. One of the most distinctive properties of tensor force is that it couples two-particle states with different angular momenta of $L = J \pm 1$.

In order to show the effect of the tensor force on the isospin-dependence of the EOS of asymmetric nuclear matter, in Fig. 1 we plot the contributions to the potential energy per nucleon of asymmetric matter from the isospin singlet $T = 0$ channel, the isospin triplet $T = 1$ channel and the $T = 0 \ 3SD_1$ tensor channel, respectively. In the figure, the results are obtained by adopting the Bonn-$B$ interaction, and the $T = 0 \ 3SD_1$ channel contribution due to the $\pi$ and $\rho$ exchange components in the Bonn-$B$ interaction is also given. It is seen from Fig. 1 that the contribution of the $T = 0$ channel
depends strongly on the asymmetry $\beta$ and it increases rapidly as a function of $\beta$ in the whole asymmetry range $0 < \beta < 1$. It increases rapidly from $-22.5$ MeV to 0 as the asymmetry $\beta$ goes from 0 to 1. Whereas the asymmetry-dependence of the $T = 1$ channel contribution turns out to be quite weak and it decreases slightly by only about 2.5 MeV from $-22.5$ MeV to $-25$ MeV as the asymmetry $\beta$ increases from 0 to 1. The above result is in good agreement with the previous investigation of Refs. [18,20] within the BHF framework by adopting the Paris and AV14 interactions, respectively. It indicates that the predominant contribution to the isovector part of the potential energy of asymmetric nuclear matter comes from the isospin singlet $T = 0$ channel. Similar conclusion has also obtained in Ref. [34]. According to Eq. (1), the isovector part of the EOS of asymmetric nuclear matter is completely described by the symmetry energy $E_{\text{sym}}$ and therefore the $T = 0$ channel contribution plays a decisive role in determining the symmetry energy and its density dependence. In Fig. 1 it is worth noticing that the corresponding filled squares, empty squares and empty triangles are almost coincident with one another, which not only indicates that the $T = 0$ channel contribution to the symmetry energy $E_{\text{sym}}$ is almost fully comes from the $^{3}SD_{1}$ tensor channel, but also implies that the $T = 0$ channel contribution is provided almost completely by the $\pi$- and $\rho$-exchange interactions in the tensor $^{3}SD_{1}$ channel. Therefore, we may readily conclude that the potential part of symmetry energy $E_{\text{sym}}$ is essentially governed by the tensor force in $NN$ interaction via the $^{3}SD_{1}$ channel.

The tensor interaction, especially its short-range part, has not been well determined consistently from the deuteron properties and/or the nucleon-nucleon scattering data. The tensor coupling ($D$-state probability $P_{D}%$) is one of the most uncertain low-energy parameters, and has estimated to be between 4% and 8% in various $NN$ potentials [22]. The tensor force, especially its short-range part, is expected to affect strongly the high density behavior of symmetry energy [41]. In the following, we shall investigate the short-range tensor force effect on the density-dependence of symmetry energy by varying the strength of the tensor force due to the $\rho$ meson exchange as follows:

$$V_{T}^{\rho} = \alpha V_{T}^{\rho}$$

where $V_{T}^{\rho}$ is specified as the original tensor component due to the $\rho$-meson exchange in the Bonn-B interaction. By varying the parameter $\alpha$, we may change the strength of the short-range tensor force from the $\rho$-meson exchange and study its effect. The calculated results for symmetric nuclear matter are displayed in Fig. 2. In the left panel of Fig. 2 the EOSs of symmetric nuclear matter (i.e., the energy per nucleon of symmetric nuclear matter vs. density) obtained by adopting various $\alpha$ values are plotted. As expected, increasing the strength of
The density dependence of symmetry energy by varying the energy, respectively from the isospin right panel, we show the contributions to the potential energy of nuclear matter. In the force from the densities. This is readily understood since the tensor component in the Bonn interaction. One may notice that the calculated symmetry energy is rather sensitive to the \(\rho\)-meson tensor force, especially at high densities and for large strength parameter \(\alpha > 1\). The variation of the symmetry energy with varying the tensor component due to the \(\rho\)-meson exchange turns out to be opposite to that of EOS of symmetric nuclear matter, i.e., increasing the tensor force of the \(\rho\)-meson exchange leads to a reduction of symmetry energy and a softening of the density dependence of symmetry energy. This can be explained easily in terms of Eq. (5) and the results in Fig. 2. According to Eq. (5), symmetry energy is determined by the difference between the energy per nucleon of pure neutron matter and that of symmetric nuclear matter. From Fig. 2 it is seen that the short-range \(\rho\) exchange tensor force plays its role almost fully via the \(T = 0\) channel which is absent in pure neutron matter. Therefore, varying the short-range tensor force may lead to opposite variations in symmetry energy and the EOS of symmetric nuclear matter.

### IV. SUMMARY

In summary, we have investigated the effect of tensor force on the isospin dependence of the EOS of asymmetric nuclear matter and nuclear symmetry energy within the framework of the BHF approach by adopting the Bonn-B interaction. The \(T = 0\) channel contribution is shown to depend sensitively on the isospin asymmetry \(\beta\), and it plays a predominate role over the \(T = 1\) channel contribution in determining the isospin dependence of the EOS of asymmetric nuclear matter (i.e., the isovector part of the EOS). The \(T = 0\) channel contribution stems almost fully from the \(3^3SD_1\) tensor channel. The tensor force manifests its effect via the \(3^3SD_1\) coupled channel, and the \(T = 0\) channel contribution turns out to be almost completely provided by the tensor component in \(NN\) interaction via the \(3^3SD_1\) coupled channel. The contributions to the EOS of symmetric nuclear matter and symmetry energy from the short-range tensor component due to the \(\rho\)-meson exchange in \(NN\) interaction have been calculated. The \(T = 1\) channel contribution to the EOS is almost independent of the strength of the \(\rho\)-meson exchange tensor component. Whereas, the \(T = 0\) channel contribution is shown to be affected strongly by the \(\rho\)-meson exchange tensor component. Increasing the tensor component due to the \(\rho\)-meson exchange in \(NN\) interaction tends to enhance the repulsion of the EOS of symmetric nuclear matter, and may leads to a reduction and softening of symmetry energy. The present results confirm the crucial role played by the tensor force in determining the isospin dependence of the EOS of asymmetric nuclear matter and the density dependence of nuclear symmetry energy.

![Graph showing symmetry energy vs. density](image)

**Fig. 3:** (Color online) Symmetry energy vs. density, obtained by adopting different strengths of the \(\rho\)-meson exchange tensor component in the Bonn-B interaction.

In Fig. 3, we report the short-range tensor effect on the density dependence of symmetry energy by varying the strength of the \(\rho\)-meson tensor component in the Bonn-B interaction. One may notice that the calculated symmetry energy is rather sensitive to the \(\rho\)-meson tensor force, especially at high densities and for large strength parameter \(\alpha > 1\). The variation of the symmetry energy with varying the tensor component due to the \(\rho\)-meson exchange turns out to be opposite to that of EOS of symmetric nuclear matter, i.e., increasing the tensor force of the \(\rho\)-meson exchange leads to a reduction of symmetry energy and a softening of the density dependence of symmetry energy. This can be explained easily in terms of Eq. (5) and the results in Fig. 2. According to Eq. (5), symmetry energy is determined by the difference between the energy per nucleon of pure neutron matter and that of symmetric nuclear matter. From Fig. 2 it is seen that the short-range \(\rho\) exchange tensor force plays its role almost fully via the \(T = 0\) channel which is absent in pure neutron matter. Therefore, varying the short-range tensor force may lead to opposite variations in symmetry energy and the EOS of symmetric nuclear matter.

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Acknowledgments

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