A light plasmon mode in the color-flavor-locking phase

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We calculate the spectral densities of electric and magnetic gluons at zero temperature in color-superconducting quark matter in the color-flavor-locking (CFL) phase. We find a collective excitation, a plasmon, at energies smaller than two times the gap parameter and momenta smaller than about eight times the gap. The dispersion relation of this mode exhibits a minimum at some nonzero value of momentum, indicating a van Hove singularity.

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Single-gluon exchange between two quarks is attractive in the color-antitriplet channel. Therefore, sufficiently cold and dense quark matter is a color superconductor [1]. When the quark-chemical potential $\mu \gg \Lambda_{\text{QCD}}$, asymptotic freedom [2] implies that the strong coupling constant $g$ at the scale $\mu$ is much smaller than unity, $g(\mu) \ll 1$. This allows a controlled calculation of the color-superconducting gap parameter $\phi$ in the weak-coupling limit.

It is of interest to study the existence and the properties of collective gluonic excitations in a color superconductor. To this end, one has to compute the gluon self-energy, which allows to determine the gluon spectral density. At small temperatures $T \sim \phi \ll \mu$, the dominant contribution to the one-loop gluon self-energy comes from a quark loop; it is $\sim g^2 \mu^2$, while gluon (and ghost) loops contribute a term $\sim g^2 T^2$ and are thus suppressed [3]. In this note, we focus on the gluon self-energy in the color-flavor-locking (CFL) phase which is characterized by an order parameter of the form

$$\Phi_{fg}^{ij} = \epsilon^{ijk} \epsilon_{fh} \Phi_h^k$$

(1)

where $i, j, k$ are (fundamental) color indices and $f, g, h$ are flavor indices.

To calculate the gluon self-energy in the CFL phase, we start from Eq. (31a) (the self-energy of electric gluons) and Eq. (31b) (the self-energy of magnetic gluons) of Ref. [4]. We then take the zero-temperature limit and, using the Dirac identity

$$\frac{1}{x + i\eta} = P \frac{1}{x} - i \pi \delta(x)$$

(2)

where $P$ denotes the principal value prescription, we find the imaginary parts of the electric and magnetic self-energies. The contribution from antiquarks in the one-loop self-energies can be neglected, as the interesting range of gluon energies is $p_0 \ll \mu$. The imaginary part of the self-energies for electric and magnetic gluons is depicted in Fig. 1, together with the results for vanishing gap parameter, which corresponds to the hard-dense loop (HDL) limit. As in a two-flavor color superconductor, the imaginary parts vanish for values of the gluon energy smaller than twice the gap parameter $\phi$. This means that at energies smaller than $2\phi$ it is impossible to excite quasiparticle-quasihole pairs which would lead to non-vanishing imaginary parts. Above the light cone, for gluon energies $p_0 > 4\phi$, the imaginary part of the HDL self-energy vanishes. In the color-superconducting case, this no longer holds true, but at least the imaginary parts decrease rapidly.

In order to compute the real parts of the gluon self-energy, one can follow two approaches. Either, one computes it directly from Eq. (2) as a principal value integral, or one employs the dispersion integral

$$\text{Re} \Pi(p_0, p) = \frac{1}{\pi} P \int_{-\infty}^{\infty} d\omega \text{Im} \Pi(\omega, p) \left( \frac{1}{\omega + p_0} + \frac{1}{\omega - p_0} \right) + C$$

(3)

where $C$ is a subtraction constant. These constants are extracted from the behavior of the self-energies at large energies and thus have the same values as in the normal-conducting case, $C^{00} = 0$ and $C^\ell = m_\ell^2$, for details see Ref. [4]. The real parts are shown in Fig. 2.

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Let us first consider the results for the real part of the electric self-energy. In Fig. 2(a) there is a logarithmic singularity at $p_0 = 2\phi$ caused by the discontinuity of the respective imaginary part at the same point. This singularity moves toward $p_0 = 0$ as $\phi \to 0$. The change of the gradient of the imaginary part at $p_0 = 3\phi$ produces a corresponding change in the real part. The peak of the imaginary part at $p_0 = 4.5\phi$, followed by a rapid decrease, produces a cusp in the real part at the same energy. The cusp at $p_0 = 5\phi$ is due to the variation in the gradient of the imaginary part.

The discussion of the real part of the magnetic self-energy proceeds along similar lines. Because of the two discontinuities at $p_0 = 2\phi$ and $p_0 = 3\phi$ in Fig. 1(b) there are two logarithmic singularities at the same energies in the real parts. Because of the change in the gradient at $p_0 = 4\phi$ there is also a change of gradient in the real part. For $p_0 \gg \phi$ the real parts of the self-energies approach the corresponding HDL limit. As one would expect, deviations from the HDL limit show up only for $p_0 \sim \phi$.

From Eq. (46) of Ref. [5] we infer the spectral densities for the case $\text{Im}\Pi^{00,t}(p_0, p) \neq 0$,

$$
\rho^{00}(p_0, p) = \frac{1}{\pi} \frac{\text{Im}\Pi^{00}(p_0, p)}{|p^2 - \text{Re}\Pi^{00}(p_0, p)|^2 + |\text{Im}\Pi^{00}(p_0, p)|^2},
$$

$$
\rho^t(p_0, p) = \frac{1}{\pi} \frac{\text{Im}\Pi^t(p_0, p)}{p_0^2 - p^2 - \text{Re}\Pi^t(p_0, p)|^2 + |\text{Im}\Pi^t(p_0, p)|^2}.
$$

FIG. 1: The imaginary part of the gluon self-energy is plotted as a function of energy $p_0$ for a momentum $p = 4\phi$. Figure (a) shows the imaginary part for electric gluons and (b) the corresponding one for magnetic gluons. The solid lines are for the CFL phase and the dotted lines for normal-conducting matter in the HDL limit.

FIG. 2: The real parts of the gluon self-energy as a function of energy $p_0$ for a momentum $p = 4\phi$. Figure (a) shows the real part for electric gluons and (b) the corresponding one for magnetic gluons. The solid lines are for the CFL phase and the dotted lines for normal-conducting matter in the HDL limit.
If $\text{Im}\Pi_{00}^{00}(p_0, p) = 0$, the spectral densities have simple poles. For a given gluon momentum $p$, for electric gluons the pole is determined by

$$[p^2 - \text{Re}\Pi_{00}^{00}(p_0, p)]_{p_0=\omega_{00}^0(p)} = 0,$$

while for the magnetic gluons it is given by

$$[p_0^2 - p^2 - \text{Re}\Pi^t(p_0, p)]_{p_0=\omega^t(p)} = 0.$$

The results are shown in Fig. 3.

FIG. 3: The spectral densities for (a) electric and (b) magnetic gluons.

We observe a peak in the magnetic spectral density at energies below $2\phi$, which corresponds to a collective excitation with a rather small mass, i.e., to a very light plasmon. The dispersion relation of this mode is shown in Fig. 4. It was already predicted in Ref. [6] and exists only for energies smaller than two times the gap parameter and momenta smaller than about eight times the gap. It exhibits a minimum at some nonzero value of momentum, indicating a van Hove singularity.

FIG. 4: The plasmon dispersion relation for the regular plasmon above the light cone and the very light plasmon mode.

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