Controlling magnon characteristics in thin ferromagnetic nanogratings by changing the direction of in-plane magnetic fields

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Micromagnetic simulations of magnons in ferromagnetic nanogratings for full range of directions of in-plane external magnetic fields are performed. The magnon dispersion and the modes’ spatial profiles depend strongly on the field orientation. As a result, by tuning this orientation it is possible to control the excitation and interaction efficiencies by selectively adjusting the spatial matching of the magnon modes with the other periodic excitations. Therefore, tuning the magnetic field direction is a powerful tool for controlling magnon characteristics in nanostructures.

Magnonics is a rapidly developing direction of modern magnetism that uses spin waves (or magnons) for data transfer and processing [1–3]. Structures with spatially periodic magnetic properties, referred to as magnonic crystals [4–14], and periodically patterned ferromagnetic films, referred to as surface modulated magnonic crystals [15–26], hold a central place in this field. The simplest pattern is a periodic set of grooves that form a (nano-) grating (NG) structure which, despite its simple design, already shows prospective for multiple applications, e.g. magnonic transistors [27], switches [28–30], filters [15, 31], grating couplers [32], magnetic field generators [33] and detectors [34].

An important feature of NG structures is the rich magnon spectrum, which can be tuned by an external magnetic field, \( H_{\text{ext}} \). It consists of discrete magnon modes with specific dispersions and spatial distributions (see e.g. [13, 25]). The dispersions give the wavevector dependencies of the magnon modes’ frequencies and their group velocities as well as the spectral positions and widths of the magnon band gaps. The spatial profiles of the magnon modes define the selectivity for their excitation and their interaction with other periodic excitations, such as electromagnetic [5, 7–9, 20, 21, 24, 25] or elastic [35–37] waves. The excitation and interaction efficiencies depend on the

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spatial matching of the magnon modes and other wave excitations, and can be fully suppressed in the case of poor matching.

An important fundamental phenomenon in magnonics is nonreciprocity, i.e. the difference in the localizations [38], and/or amplitudes [39, 40], and/or frequencies [13, 14, 22, 26, 41] of the spin waves with opposite directions of wave vectors. In NG structures, magnons can possess asymmetric dispersions with indirect band gaps in the Brillouin zone [13, 22]. In this case, spin waves with Bloch wavevectors corresponding to the center of the Brillouin zone are propagating waves. Thus, nonreciprocity provides an ability to excite propagating spin waves by means of spatially homogeneous excitations.

The main experimental tools for studying magnons in NG structures are Brillouin light scattering techniques [2, 5, 7–9] and ferromagnetic resonance (FMR) spectroscopy [19–21, 24, 25, 42]. Traditionally, only two directions of $H_{\text{ext}}$ are studied: either perpendicular (the Damon-Eshbach (DE) geometry) or parallel (the backward-volume (BV) geometry) to the magnon wavevector. However, to the best of our knowledge there is no information about the main magnon characteristics in NG structures for intermediate directions of $H_{\text{ext}}$. Moreover, time-resolved experiments on the excitation of a metallic ferromagnet by femtosecond laser pulses and detection of the coherent magnon response by means of the transient magneto-optical Kerr effect (MOKE) [33, 43–47], are typically performed at the intermediate directions of $H_{\text{ext}}$, where the excitation efficiency is maximal [46, 47]. Thus, an analysis of magnons at the intermediate directions of $H_{\text{ext}}$ is required.

In this Rapid Communication, we present a detailed theoretical analysis of magnons in a ferromagnetic NG using arbitrary directions of $H_{\text{ext}}$. We focus on the main magnon characteristics including the dispersions, spatial profiles and dependencies of magnon frequencies on the direction of $H_{\text{ext}}$. We find that in NG structures three optically active magnon branches are dominant in the spectra. We identify each branch and show the dependences of the mode profiles on the directions of in-plane magnetic fields. Our analysis shows that magnon spatial profiles are extremely tunable by magnetic field direction which can be used to control the excitation and interaction efficiencies by selectively adjusting the spatial matching. For the calculations, we use the COMSOL Multiphysics® software [48]. Our choice is based on the ability to solve and visualize the problem in both time and frequency domains with further integration of a specific external impact, such as the femtosecond laser excitation, monochromatic elastic wave or picosecond strain pulses, which are widely used in time-resolved magnonic experiments. The validity of our approach was successfully proved by solving standard micromagnetic problems [49, 50].

The system under study is a NG structure which consists of infinitely long parallel grooves with
Figure 1. Sketch of the nanograting, the used coordinate system, and the in-plane external magnetic field, $H_{\text{ext}}$.

depth $h$, introduced in a ferromagnetic film with thickness, $l$, (see Fig. 1) located on a nonmagnetic substrate, which is not shown for simplicity. The NG period is $d$, and the width of the grooves is $d - w$. The external magnetic field, $H_{\text{ext}}$, is applied in the plane of the ferromagnet at an angle $\varphi_H$ between the grooves and $H_{\text{ext}}$.

The magnetization $M$ in the ferromagnetic NG structure is described by the Landau-Lifshitz-Gilbert equation (see e.g. [51–53]). It is convenient to introduce a normalized magnetization $m = M/M_s$, where $M_s$ is the saturation magnetization and $m^2 = 1$. Moreover, it is reasonable to use units where the magnetic field, $M_s$ and the magnetic induction are given in Tesla. Then, the Landau-Lifshitz-Gilbert equation has the form:

$$\frac{\partial m}{\partial t} = -\gamma m \times H_{\text{eff}} + \alpha m \times \frac{\partial m}{\partial t},$$

(1)

where $\gamma$, $H_{\text{eff}}$ and $\alpha$ are the gyromagnetic ratio, effective magnetic field, and Gilbert damping parameter, respectively. For isotropic ferromagnetic materials the effective magnetic field is given by (see e.g. [51])

$$H_{\text{eff}} = H_{\text{ext}} + H_d + D \nabla^2 m,$$

(2)

where $H_{\text{ext}}$ and $H_d$ are the external and demagnetizing magnetic fields, respectively. The last term in (2) describes the exchange interaction with exchange stiffness constant $D$, and $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$.

We stress that both $m$ and $H_d$ are spatially inhomogeneous and time-dependent quantities. The connection between them is given by the magnetostatic Maxwell equations [51]:

$$\nabla \times H_d = 0, \quad \nabla \cdot (H_d + M_s m) = 0.$$  

(3)

The described macroscopic quasi-static approach is valid if the magnon wavelength is larger than the atomic spacing, and the magnon phase velocities are smaller than the velocity of light [51].

Equations (1)-(3) are the main set of nonlinear differential equations for the NG structure. In
addition, we use the standard electrodynamic boundary conditions and free boundary conditions for the magnetization, i.e. $\frac{\partial \mathbf{m}}{\partial n} = 0$, because this case describes experiments performed on similar structures [25].

The search for the solutions of Eqs. (1)-(3) can be divided into two stages: the steady-state (see [50] for details) and linear dynamics. In the steady-state, the magnetization, $\mathbf{m}^0$, is parallel to $\mathbf{H}_{\text{eff}}^0$ to minimize the free energy of the ferromagnet [51]. In the NG structure, the spatial distribution of $\mathbf{H}_{\text{eff}}^0$ is inhomogeneous as well as the spatial distribution of the magnetization (see e.g. the Eqs. (3)).

The dynamics of a ferromagnet consists of the spatially inhomogeneous precession of the magnetization around $\mathbf{H}_{\text{eff}}^0$. This time- and space-dependent precessional motion can be considered as a superposition of the magnon eigenmodes of the NG. In order to describe it, we have linearized the main set of equations by introducing a dynamic magnetization, $\delta \mathbf{m}$, with $\delta m \ll m^0$, and a dynamic demagnetizing field, $\delta \mathbf{H}_d$, with $\delta H_d \ll M_s$. In order to calculate the dispersion curves and spatial profiles of the magnon modes, we performed an eigenfrequency analysis with Floquet-Bloch periodic boundary conditions, i.e. $\mathbf{u}_d = \mathbf{u}_s \exp(-i k_x d)$, where $k_x$ is a Bloch wavenumber, and $\mathbf{u}_d$ and $\mathbf{u}_s$ are dependent variables at the destination and source boundaries, respectively.

The solutions of the eigenfrequency problem give the spatially inhomogeneous complex-valued Fourier components of the dynamic magnetization and demagnetizing field. For the characterization of the magnon modes’ spatial profiles let us choose the $z$-component of the dynamic magnetization, because it is nonzero for any in-plane $\mathbf{H}_{\text{ext}}$. Below we focus on the spatial profiles, which correspond to the center of the Brillouin zone. The real-valued solution for $\delta m_z$ in the time domain can be expressed in the form $\delta m_z = |\delta m_{z,\omega}| \cos(\omega t + \phi_z)$, where $\omega$ is the magnon angular frequency, $\delta m_{z,\omega}$ is the Fourier component of $\delta m_z$, and $\phi_z = \text{atan2}(\text{Im}(\delta m_{z,\omega}), \text{Re}(\delta m_{z,\omega}))$ is the phase. The latter is spatially inhomogeneous, thus the magnon modes’ spatial profiles change during propagation [25]. Moreover, the difference between supremum and infimum values of $\delta m_z$, $D_m(t) = \sup(\delta m_z) - \inf(\delta m_z)$, varies with time. However, we found that at the time, which corresponds to the maximum of $D_m(t)$, each mode profile becomes purely symmetric or antisymmetric [54]. Thus, we use the symmetry at this time for characterizing a magnon mode. This symmetry determines the possibility to excite the mode by a laser pulse or a homogeneous magnetic field in an FMR experiment.

For the calculations we chose polycrystalline Permalloy (Ni$_{80}$Fe$_{20}$) as the NG material. This isotropic ferromagnet possesses very weak magnetostriction, which allows us to exclude from consideration the lattice dynamics and consider only magnons. The magnetic parameters [25] for
the calculations are the saturation magnetization, $M_s = 0.9236$ T and the exchange stiffness, $D = 23.6$ Tnm$^2$. The optical parameters, required for simulating MOKE measurements, can be found in [50, 55, 56]. The parameter $\alpha$ is fixed to zero for the eigenfrequency analysis and to 0.01 for the MOKE simulations. The geometrical parameters are fixed to: $d = 300$ nm, $w = 140$ nm, $h = 13.2$ nm, and $l = 36.8$ nm. We find that the magnon dispersion and spatial profiles remain (topologically) similar for any surface modulated magnonic crystal with parameters that satisfy the criterium: $l_e d/l^2 > 1$, where $l_e = \sqrt{D/M_s}$ is the exchange length. The chosen parameters are the same as in [25] in order to compare the results for traditional DE and BV geometries [50]. We fix the magnetic field strength to $H_{\text{ext}} = 200$ mT in order to satisfy the condition that the average direction of the static magnetization, which is given by $\varphi_M = \frac{1}{V} \int \arctan \left( \frac{m_0^x}{m_0^y} \right) dV$, coincides with the direction of $H_{\text{ext}}$, i.e. $\varphi_M \approx \varphi_H$. Here $dV$ is the unit volume element.

Figures 2(a), (b) show the results of micromagnetic simulations for the DE geometry ($\varphi_H = 0^\circ$). At this field direction $\varphi_M = \varphi_H$, because the static demagnetizing field is zero [13, 50]. From Fig. 2(a) one can see that all magnon branches are nonreciprocal, i.e. $\omega(k_x) \neq \omega(-k_x)$, except the ground one. Moreover, indirect band gaps arise.

Figure 2(b) shows magnon modes’ spatial profiles which correspond to the center of the Brillouin zone at $k_x = 0$. The ground magnon mode is quasiuniform. The next two modes are symmetric and antisymmetric magnon modes with $k_x = 2\pi/d$. Then come the higher order modes. Magnon modes with wave vectors corresponding to the center of the Brillouin zone are propagating waves. Their group velocities $v_g = \frac{d\omega}{dk_x}$ are defined by the slopes of the dispersion curves. As an example, see Supplemental Video I [50], where the third mode which corresponds to $k_x = 0$, possessing negative group velocity, is visualized and its movement in the negative $x$-direction is clearly seen.

Figures 2(c), (d) shows magnon modes’ dispersion curves and their spatial profiles in the BV geometry ($\varphi_H = 90^\circ$). One can see, that this case drastically differs from the case of the DE geometry. In Fig. 2(c), all modes are reciprocal. The first eleven dispersion branches are nearly flat [24, 25]. Moreover, the existence of huge band gaps with respect to their frequencies is worth to mention. The largest three band gaps are between the second and third, fourth and fifth, and fifth and sixth magnon branches possessing values of 2.35 GHz, 1.67 GHz, and 1.7 GHz, respectively. The dispersions at intermediate directions of $H_{\text{ext}}$ can be found in [50].

Figure 2(d) shows magnon modes’ spatial profiles which correspond to the center of the Brillouin zone at $k_x = 0$. In the BV geometry all modes are standing due to the reciprocal dispersion. The first two modes are antisymmetric and symmetric edge modes. The next three are so-called wire modes. The 6-th and 7-th mode are symmetric and antisymmetric groove modes [25]. An example
Figure 2. Magnon dispersion and mode profiles at $H_{\text{ext}} = 200$ mT in (a), (b) the DE geometry and (c), (d) the BV geometry. (a), (c) Magnon dispersion curves. The vertical dotted line in (a), (c) at $k_x = 0$ indicates the center of the Brillouin zone. (b), (d) Spatial profiles of the magnon modes at $k_x = 0$ in the unit cell of the NG. The colored numbers correspond to the magnon dispersion branches.

of wire mode oscillations can be found in Supplemental Video II [50].

The nonreciprocity in the NG arises due to the symmetry of the structure and dynamical demagnetizing fields. In the DE geometry, both $\delta m_x$ and $\delta m_z$ contribute equally to $\delta H_d$. As a result, at a random time, $\delta H_d$ is neither symmetric nor antisymmetric, which causes the propagating behavior of the modes. In the BV geometry, $\delta H_d$ is defined by $\delta m_z$. As a result, the parity of the modes is conserved at any time and the dispersion is reciprocal.

Now we turn to the main part of our analysis, i.e. the intermediate directions of $H_{\text{ext}}$. The result of an eigenfrequency analysis is presented in Fig. 3(a), which shows the angle dependence of the magnon frequencies. Here, one can observe a complicated non-monotonic behavior with magnon-magnon interaction [25] and corresponding avoided crossings of the interacting modes, when their frequencies coincide. Despite that, the general tendency is typical for ferromagnets (see, for instance [46, 47]): the magnon frequency decreases with changing of the direction of $H_{\text{ext}}$ from the easy ($\varphi_H = 0^\circ$) to the hard ($\varphi_H = 90^\circ$) axis. Interestingly, the frequency of the ground magnon mode does not depend on the direction of $H_{\text{ext}}$ for $\varphi_H \lesssim 50^\circ$ and can be estimated with good accuracy, using the Kittel formula [13].

The magnetization precession can be excited and detected in the time domain by femtosecond laser pulses within a conventional magneto-optical pump probe experiment [44]. The pump pulse excites the precessional response of the magnetization by means of ultrafast modulation of the
Figure 3. (a) Magnetic field direction dependence of magnon frequencies. The lines’ colors correspond to the colored numbers and lines in Fig. 2. (b) Color map of the simulated MOKE spectral amplitude dependence on the direction of $H_{\text{ext}}$. (c)-(e) Evolutions of the dominant optically active magnon modes under rotation of the magnetic field. The values of $\varphi_H$ are in degrees. All quantities here correspond to the center of the Brillouin zone ($k_x = 0$) and $H_{\text{ext}} = 200$ mT.

magnetic anisotropy [44], and the linearly polarized probe pulse serves for monitoring $\delta m_z$ by means of the transient MOKE [43]. The Fourier analysis of the transient MOKE signal allows us to obtain the frequencies and spectral amplitudes of the optically excited magnon modes. Further, we focus on simulation of the simplest case of such a pump-probe experiment, in which the diameters of both pump and probe laser spots are much larger than the NG period [50]. In this case, the optically active magnon modes at the center of Brillouin zone can be excited [57]. The corresponding simulated MOKE spectra are shown in Fig. 3(b). Here one can see that three magnon mode branches are dominant in the spectra. Due to their symmetry they have strong overlap with the laser light. The other branches are weakly excited or not excited due to poor overlap with the optical excitation. Importantly, Fig. 3(b) clearly demonstrates that by simple rotation of the magnetic field it is possible to select which dominant optically active magnon modes to excite. The dependence presented in the Fig. 3(b) can be verified by conventional time-resolved optical excitation experiments (see e.g. [37, 58, 59]).

Figures 3(c)-(e) show the evolutions of the spatial profiles of the dominant optically active magnon modes under rotation of the magnetic field. One can see that the quasiuniform mode #1 in the DE geometry evolves to the edge mode #2 in the BV geometry (see Fig. 2(b), (d) and Fig. 3(c)-(e)). The magnon mode #3 with $k_x = 2\pi/d$ evolves to the wire mode #3. The mode #8 quantized in the z-direction in the DE geometry evolves to the high order magnon mode #11 in
the BV geometry. Therefore, the mode with \( k_x = 2\pi/d \) is dominant at \( \varphi_H = 25^\circ \), the z-quantized mode at \( \varphi_H = 50^\circ \), and the edge mode at \( \varphi_H = 75^\circ \). Figures 3(c)-(e) clearly show that the spatial profiles of magnon modes can be effectively modified by changing the direction of \( H_{\text{ext}} \). Moreover, all magnon modes’ spatial profiles evolve under rotation of \( H_{\text{ext}} \) [60]. The excitation selectivity at intermediate directions of \( H_{\text{ext}} \) is a direct evidence of this (see Fig. 3(b)). Furthermore, the intermediate directions of \( H_{\text{ext}} \) can be used for adjusting the spatial overlap of the selected magnon mode with another periodic excitation, such as an elastic wave, to maximize the magneto-elastic coupling strength [35–37]. In the latter case, the magneto-elastic interaction can occur between symmetric magnon and symmetric phonon modes, as well as between symmetric magnon modes and antisymmetric phonon modes [37] or in other combinations [35, 36].

To conclude, we used COMSOL Multiphysics\textsuperscript{®} to calculate magnon dispersion curves, mode profiles, and magnetic field direction dependencies for the system of a thin ferromagnetic nanograting. We show that the spatial profiles of magnon modes strongly depend on the direction of magnetic field. As a result, the spatial overlap of magnon modes with other excitations, such as electromagnetic or elastic waves, depend on the field orientation. The latter can be used for excitations of the selected optically active magnon modes or to adjust the spatial overlap of the selected magnon mode with another wave excitation. Moreover, one could imagine a magnonic device, which is inserted into a field of a permanent magnet and the direction of \( H_{\text{ext}} \) is controlled by a miniature translation stage. For instance, the band gap positions and widths could be control by the direction of \( H_{\text{ext}} \), which could be used for filtering spin waves. Our results illustrate that by simple rotation of a sample/device in an external magnetic field it is possible to achieve a huge tunability of the magnon characteristics in nanostructures.

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[1] B. Lenk, H. Ulrichs, F. Garbs, and M. Münzenberg, Phys. Rep. 507, 107 (2011).
[2] M. Krawczyk and D. D Grundler, J. Phys.: Condens. Matter 26, 123202 (2014).
[3] A. V. Chumak, V. Vasyuchka, A. A. Serga, and B. Hillebrands, Nat. Phys. 11, 453 (2015).
[4] S. A. Nikitov, P. Tailhades, and C. S. Tsai, J. Magn. Magn. Mater. 236, 320 (2001).
[5] M. P. Kostylev, A. A. Stashkevich, and N. A. Sergeeva, Phys. Rev. B 69, 064408 (2004).
[6] K. Y. Guslienko and A. N. Slavin, Phys. Rev. B 72, 014463 (2005).
[7] G. Gubbiotti, S. Tacchi, G. Carlotti, P. Vavassori, N. Singh, S. Goolaup, A. O. Adeyeye, A. Stashkevich, and M. Kostylev, Phys. Rev. B 72, 224413 (2005).
[8] G. Gubbiotti, S. Tacchi, G. Carlotti, N. Singh, S. Goolaup, A. O. Adeyeye, and M. Kostylev, Appl. Phys. Lett. 90, 092503 (2007).
[9] M. Kostylev, P. Schrader, R. L. Stamps, G. Gubbiotti, G. Carlotti, A. O. Adeyeye, S. Goolaup, and N. Singh, Appl. Phys. Lett. 92, 132504 (2008).
[10] N. I. Polushkin, Phys. Rev. B 77, 180401 (2008).
[11] M. Mruczkiewicz, M. Krawczyk, V. K. Sakharov, Y. V. Khivintsev, Y. A. Filimonov, and S. A. Nikitov, J. Appl. Phys. 113, 093908 (2013).
[12] M. Mruczkiewicz, M. Krawczyk, G. Gubbiotti, S. Tacchi, Y. A. Filimonov, D. V. Kalyabin, I. V. Lisenkov, and S. A. Nikitov, New J. Phys. 15, 113023 (2013).
[13] I. Lisenkov, D. Kalyabin, S. Osokin, J. W. Klos, M. Krawczyk, and S. Nikitov, J. Magn. Magn. Mater. 378, 313 (2015).
[14] J. Rychly, P. Gruszecki, M. Mruczkiewicz, J. W. Klos, S. Mamica, and M. Krawczyk, Low Temperature Physics 41, 745 (2015).
[15] A. V. Chumak, A. A. Serga, B. Hillebrands, and M. P. Kostylev, Appl. Phys. Lett. 93, 022508 (2008).
[16] A. V. Chumak, A. A. Serga, S. Wolff, B. Hillebrands, and M. P. Kostylev, J. Appl. Phys. 105, 083906 (2009).
[17] A. V. Chumak, A. A. Serga, S. Wolff, B. Hillebrands, and M. P. Kostylev, Appl. Phys. Lett. 94, 172511 (2009).
[18] A. A. Serga, A. V. Chumak, and B. Hillebrands, J. Phys. D: Appl. Phys. 43, 264002 (2010).
[19] P. Landeros and D. L. Mills, Phys. Rev. B 85, 054424 (2012).
[20] G. N. Kakazei, X. M. Liu, J. Ding, and A. O. Adeyeye, Appl. Phys. Lett. 104, 042403 (2014).
[21] G. R. Aranda, G. N. Kakazei, J. González, and K. Y. Guslienko, J. Appl. Phys. 116, 093908 (2014).
[22] V. D. Bessonov, M. Mruczkiewicz, R. Gieniusz, U. Guzowska, A. Maziewski, A. I. Stognij, and M. Krawczyk, Phys. Rev. B 91, 104421 (2015).
[23] M. Langer, F. Röder, R. A. Gallardo, T. Schneider, S. Stienen, C. Gatel, R. Hübner, L. Bischoff, K. Lenz, J. Lindner, P. Landeros, and J. Fassbender, Phys. Rev. B 95, 184405 (2017).
[24] R. A. Gallardo, T. Schneider, A. Roldán-Molina, M. Langer, J. Fassbender, K. Lenz, J. Lindner, and P. Landeros, Phys. Rev. B 97, 144405 (2018).
[25] M. Langer, R. A. Gallardo, T. Schneider, S. Stienen, A. Roldán-Molina, Y. Yuan, K. Lenz, J. Lindner, P. Landeros, and J. Fassbender, Phys. Rev. B 99, 024426 (2019).
[26] R. A. Gallardo, D. Cortés-Ortuño, T. Schneider, A. Roldán-Molina, F. Ma, R. E. Troncoso, K. Lenz, H. Fangohr, J. Lindner, and P. Landeros, Phys. Rev. Lett. 122, 067204 (2019).
[27] A. V. Chumak, A. A. Serga, and B. Hillebrands, Nat. Comm. 5, 4700 (2014).
[28] A. Khitun, M. Bao, and K. L. Wang, J. Phys. D: Appl. Phys. 43, 264005 (2010).
[29] K. Vogt, F. Y. Fradin, J. E. Pearson, T. Sebastian, S. D. Bader, B. Hillebrands, and H. Hoffmann, A. Schultheiss, Nat. Comm. 5, 3727 (2014).
[30] M. Balinskiy, H. Chiang, and K. A., AIP Adv. 8, 056628 (2018).
[31] S.-K. Kim, K.-S. Lee, and D.-S. Han, Appl. Phys. Lett 95, 082507 (2009).
[32] H. Yu, G. Duerr, R. Huber, M. Bahr, T. Schwarze, F. Brandl, and D. Grundler, Nat. Comm. 4, 2702 (2013).
[33] A. S. Salasyuk, A. V. Rudkovskaya, A. P. Danilov, B. A. Glavin, S. M. KukhtaruK, M. Wang, A. W. Rushforth, P. A. Nekludova, S. V. Sokolov, A. A. Elistratov, D. R. Yakovlev, M. Bayer, A. V. Akimov, and A. V. Scherbakov, Phys. Rev. B 97, 060404(R) (2018).
[34] M. Inoue, A. Baryshev, H. Takagi, P. B. Lim, K. Hatafuku, J. Noda, and K. Togo, Appl. Phys. Lett. 98, 132511 (2011).
[35] M. Bombeck, A. S. Salasyuk, B. A. Glavin, A. V. Scherbakov, C. Brüggemann, D. R. Yakovlev, V. F. Sapega, X. Liu, J. K. Furdyna, A. V. Akimov, and M. Bayer, Phys. Rev. B 85, 195324 (2012).
[36] R. Verba, I. Lisenkov, I. Krivorotov, V. Tiberkevich, and S. A., Phys. Rev. Appl. 9, 064014 (2018).
[37] F. Godejohann, A. V. Scherbakov, S. M. KukhtaruK, A. N. Poddubny, D. D. Yaremkevich, M. Wang, A. Nadzeyka, D. R. Yakovlev, A. W. Rushforth, A. V. Akimov, and M. Bayer, arXiv:1909.01886 (2019).
[38] R. W. Damon and J. R. Eshbach, J. Phys. Chem. Solids 19, 308 (1961).
[39] T. Schneider, A. A. Serga, T. Neumann, B. Hillebrands, and M. P. Kostylev, Phys. Rev. B 77, 214411 (2008).
[40] M. Kostylev, J. Appl. Phys. 113, 053907 (2013).
[41] M. Mruczkiewicz, P. Graczyk, P. Lupo, A. Adeyeye, G. Gubbiotti, and M. Krawczyk, Phys. Rev. B 96, 104411 (2017).
[42] M. Farle, Reports on Progress in Physics 61, 755 (1998).
[43] W. K. Hiebert, A. Stankiewicz, and M. R. Freeman, Phys. Rev. Lett. 79, 1134 (1997).
[44] M. van Kampen, C. Jozsza, J. T. Kohlhepp, P. LeClair, L. Lagae, W. J. M. de Jonge, and B. Koopmans, Phys. Rev. Lett. 88, 227201 (2002).
[45] V. N. Kats, T. L. Linnik, A. S. Salasyuk, A. W. Rushforth, M. Wang, P. Wadley, A. V. Akimov, S. A. Cavill, V. Holy, A. M. Kalashnikova, and A. V. Scherbakov, Phys. Rev. B 93, 214422 (2016).
[46] A. V. Scherbakov, A. P. Danilov, F. Godejohann, T. L. Linnik, B. A. Glavin, L. A. Shelukhin, D. P. Pattnaik, M. Wang, A. W. Rushforth, D. R. Yakovlev, A. V. Akimov, and M. Bayer, Phys. Rev. Applied 11, 031003 (2019).
[47] N. Khokhlov, P. Gerevenkov, L. Shelukhin, A. Azovtsev, N. Pertsev, M. Wang, A. Rushforth, A. Scherbakov, and A. Kalashnikova, Phys. Rev. Applied 12, 044044 (2019).
[48] COMSOL Multiphysics® v. 5.4. www.comsol.com. COMSOL AB, Stockholm, Sweden.
[49] µMAG, Micromagnetic Modeling Activity Group, www.ctems.nist.gov/∼rdm/mumag.org.html.
See Supplemental Material below for details related to the validation of micromagnetic calculations using Comsol, magnetic field dependencies of magnon frequencies in the BV and DE geometries, steady-states of the nanograting structure, dependencies of magnon dispersions and group velocities on the direction of magnetic field, and MOKE simulations. The magnon modes #3 in the DE and BV geometries are shown in the Supplemental Videos I and II, respectively. See Suplemental Videos at https://drive.google.com/file/d/1RSKe0hjDzzixrOAvYi11jVLMId0ASHgC/view?usp=sharing.

E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics, Part 2: Theory of the Condensed State*, Vol. 9 (Butterworth-Heinemann, 1980).

V. G. Bar’akhtar, Zh. Eksp. Teor. Fiz. 87, 1501 (1984).

O. Chubykalo-Fesenko, U. Nowak, R. W. Chantrell, and D. Garanin, Phys. Rev. B 74, 094436 (2006).

Hereafter, symmetric (even) and antisymmetric (odd) is always meant with respect to the center of the NG wire region.

G. Neuber, R. Rauer, J. Kunze, T. Korn, C. Pels, G. Meier, U. Merkt, J. Bäckström, and M. Rübhausen, Appl. Phys. Lett. 83, 4509 (2003).

K. K. Tikuišis, L. Beran, P. Cejpek, K. Uhlířová, J. Hamrlé, M. Vaňatka, M. Urbánek, and M. Veis, Materials & Design 114, 31 (2017).

T. Satoh, Y. Terui, R. Moriya, B. A. Ivanov, K. Ando, E. Saitoh, T. Shimura, and K. Kuroda, Nat. Photonics 6, 662 (2012).

J. Janušonis, C. L. Chang, T. Jansma, A. Gatilova, V. S. Vlasov, A. M. Lomonosov, V. V. Temnov, and R. I. Tobey, Phys. Rev. B 94, 024415 (2016).

C. L. Chang, R. R. Tamming, T. J. Broomhall, J. Janusonis, P. W. Fry, R. I. Tobey, and T. J. Hayward, Phys. Rev. Applied 10, 034068 (2018).

In particular, we found that the magnon mode #4 with $k_x = 4\pi/d$ evolves to the groove mode #6.
I. VERIFICATION OF MICROMAGNETIC SIMULATIONS USING COMSOL MULTIPHYSICS

In reference [S1] five µMAG standard problems of micromagnetics can be found. Among all of them, problem #4 deals with the case of a spatially nonhomogeneous and time-dependent magnetization precession in a thin Permalloy plate. Due to the link to the treated system in this work, in the following, the problem is formulated and solved using COMSOL Multiphysics® [S2] (Comsol). In order to verify the accuracy of the solved problem using Comsol, a comparison with different solutions given in [S1] is discussed, as well.

We want to emphasize, that we are not claiming the novelty of utilizing Comsol for micromagnetic simulations (see e.g. [S3, S4]). However, as far as micromagnetics is not a default module of Comsol we decided to verify our implementation of micromagnetics to Comsol by solving the standard micromagnetic problems.

The object of our study is given by a Permalloy plate, which is illustrated in Fig. S1. The used parameters are \( L = 500 \) nm, \( d = 125 \) nm and \( t = 3 \) nm. We take the same Permalloy material parameters as used in [S1] only in this section of Supplemental Material. It is important to emphasize that these parameters are different from the values in reference [S5], which have been used in the main text. The exchange stiffness constant is \( D = 3.25 \cdot 10^{-17} \) Tm\(^2\). It is worth to mention, that different authors defined the exchange stiffness constant in different ways. For example, in our case and in [S5], \( D = 2A \), where \( A \) is the exchange stiffness constant from [S1]. The saturation magnetization is \( M_s = 1.0053 \) T, and the Gilbert damping constant is \( \alpha = 0.02 \).

![Figure S1. Sketch of the geometry of the micromagnetic problem (not to scale!).](image)

The first part of the solution of problem #4 is given by the calculation of the so-called s-state, which is the specific steady-state spatial distribution of the magnetization (see Fig. S2(a)). In
order to do so, a saturating magnetic field is applied along the [111]-crystallographic direction and, afterwards, slowly decreased to zero. As one can see in Fig. S2, the calculated s-state using Comsol looks very similar to the results taken from [S1] (see Fig. S2(b)). The resulting s-state determines the initial state for the solution of the second part of problem #4, which deals with the time evolution of the magnetization.

For the second part of problem #4, another magnetic field is applied along the opposite direction of the equilibrium magnetization of the s-state in order to change the magnetization orientation. The main aim is to track the time evolution of the magnetization towards the new magnetic field direction [S1]. The applied magnetic field is characterised by $H_x = -24.6$ mT, $H_y = -4.3$ mT, and $H_z = 0.0$ mT. In other words, the magnetic field strength of $\approx 25$ mT is directed 170 degrees counterclockwise from the positive $x$-axis in the $x$-$y$ plane.

In this paragraph, we present the details of the calculation in Comsol which could be useful for Comsol users. For the magnetic field calculation we used the "AC/DC" module ("Magnetic Fields,
The Landau-Lifshits-Gilbert equation was implemented using the basic module of Comsol i.e. "Mathematics" ("Coefficient form PDE"). We introduce an air sphere of 2 µm radius around the Permalloy plate with a 200-nm thick external layer of "infinite element domain". The magnetic scalar potential on the surface of the sphere is set to zero. The maximum element size in the plate is set to 5 nm. For the sphere, we choose an "extremely coarse" mesh. We used the quartic Lagrange discretization in the Coefficient form PDE and quadratic in the "Magnetic Fields, No Currents". The maximum time step is defined as 0.5 ps.

Figure S3 summarizes calculated time-dependent spatially averaged magnetization components by our and other groups. Fig. S3(a) shows a comparison of results obtained using Comsol (lines) and using a finite difference software developed by E. Martinez, L. Torres and L. Lopez-Diaz (dots). One can see a very good agreement between the calculated lines and dots. Another comparison of results using Comsol and using a finite difference software developed by J. L. Martins and T. Rocha is presented in Fig. S3(b). In this case, a perfect agreement between both calculations is observed. The discrepancy between different calculations is due to the fact that the exact analytical solution of the considered problem is unknown. Thus, different approaches, different mesh sizes, etc., give slightly different results.

We want to emphasize, that the calculated amplitude of precession is in the order of the saturation magnetization (see Fig. S3), which is much larger than in usual experiments investigating magnons (see e.g. [S6–S9]). Hence, the three dimensional problem #4 is strongly nonlinear and reflects a case, which is much more complicated than the simulation of magnons in effectively two dimensional gratings with precession amplitudes much smaller than $M_s$.

The last validation of the solution of problem #4 is given by the comparison of the spatial distribution of the magnetization at the time, when the $x$-component of the spatially averaged magnetization first crosses zero ($\approx 0.1375$ ns). In Fig. S4(a) one can see the spatial distribution which was calculated by M. J. Donahue and D. G. Porter, using OOMMF eXtensible solver [S1], where Fig. S4(b) shows the spatial distribution of the magnetization using Comsol. Besides the different colors, there is a good agreement between both calculated distributions.

![Figure S4](image-url)
II. MAGNETIC FIELD DEPENDENCIES OF MAGNON FREQUENCIES IN A NANOGRATING

The magnetic field dependencies of magnon frequencies in nanograting for both the backward-volume (BV) geometry and Damon-Eshbach (DE) geometry have been already reported in [S5]. In this work, we complement these dependencies by additional antisymmetric magnon modes [S10] which cannot be accessed using a symmetric excitation and detection scheme [S5]. For calculations, all parameters are chosen as they were introduced in the main text.

The magnetic field dependence of the magnon frequencies in the BV geometry (\(\varphi_H = 90^\circ\)) is shown in Fig. S5(a). The corresponding inset shows the magnetic field dependence of the average steady-state in-plane magnetization angle \(\varphi_M\), indicating the saturation magnetic field at \(\approx 73\) mT. For \(H_{\text{ext}}\), smaller than the saturation magnetic field, one can observe complicated non-monotonic behavior with magnon-magnon interaction and corresponding avoided crossings. For \(H_{\text{ext}} > 73\) mT the behavior is much more simple and in most cases the magnon frequencies monotonically grow with increasing \(H_{\text{ext}}\).

Figure S5(b) shows the magnetic field dependence of the magnon frequencies in the DE geometry, i.e. \(\varphi_H = 0^\circ\). A similar inset as in Fig. S5(a) is not shown, because the magnetization always lies along the easy axis for any magnetic field strength (\(\varphi_M = 0^\circ\)). The presented dependence is similar as for magnon modes in thin ferromagnetic films for magnetization directions close to the easy axis, because the static demagnetizing field is zero [S5, S11] (see also the section III of Supplemental Material). The dependence of the ground magnon mode is often referred to as Kittel-like, because of the well-known Kittel formula for the precession frequency, \(\omega\), of an isotropic plane film: 
\[
\omega = \gamma \sqrt{H_{\text{ext}}(H_{\text{ext}} + M_s)}
\]
(see e.g. [S6, S11]).

The presented results in Fig. S5 are perfectly consistent with previously reported experimental (FMR) and numerical results [S5]. For the case of the BV geometry, Fig. S5(a) has to be compared to Fig. 5(d), (h) in [S5]. For the DE geometry, Fig. S5(b) has to be compared to Fig. 7 in [S5]. Note, that in this work the number of lines is larger than in [S5], because Fig. S5 shows both symmetric and antisymmetric magnon modes.

Figure S5. Magnetic field dependencies of magnon frequencies for the case of (a) the BV geometry (\(\varphi_H = 90^\circ\)) and (b) the DE geometry (\(\varphi_H = 0^\circ\)). The inset in (a) shows the magnetic field dependence of the average steady-state in-plane magnetization angle \(\varphi_M\). Note, that the vertical scales in (a) and (b) are different. The magnon mode branches correspond to the center of the Brillouin zone. The color schemes of the lines here and in the main text are the same.
III. THE NANOGRATING STRUCTURE IN THE STEADY-STATE

The steady-state stage of micromagnetic simulations is important, because, for instance, the steady-state demagnetizing field $H_d^0$ and magnetization $m^0$ are present in the linearized equations for dynamic magnetization. Thus, $H_d^0$ and $m^0$ are required to describe magnons.

Before we proceed with the steady-state case, we derive the general formula for a scalar magnetic potential of demagnetizing field, which we use below to analyze the steady-state case. Eqs. (3) from the main text are mathematically equivalent to the well-known Maxwell electrostatic equations. Thus, it is convenient to introduce the magnetic scalar potential $\psi$ using the following relation:

$$H_d = -\nabla \psi.$$  \hspace{1cm} (S1)

The general solutions for the magnetic scalar potential can be found using the well-known Green function for the Laplacian (see the similar formula e.g. in [S12]):

$$\psi(r, t) = -\frac{M_s}{4\pi} \int_{V'} \frac{\nabla' \cdot m(r', t)}{|r - r'|} dV' + \frac{M_s}{4\pi} \int_{S'} \frac{n \cdot m(r', t)}{|r - r'|} dS',$$  \hspace{1cm} (S2)

where $r$, $r'$ and $t$ are the radius vectors and time, respectively; $n$ is an outward-pointing unit vector normal to the surface, $S$, of a ferromagnet. In the first integral of Eq. (S2), $V$ is the volume of the considered ferromagnet. Using the Gauss-Ostrogradsky theorem Eq. (S2) can be rewritten in the simpler form:

$$\psi(r, t) = \frac{M_s}{4\pi} \int_{V'} \frac{m_x(x - x') + m_y(y - y') + m_z(z - z')}{|r - r'|^3} dV',$$  \hspace{1cm} (S3)

where $m_x(r', t), m_y(r', t),$ and $m_z(r', t)$ are the magnetization components. So far, Eqs. (S1), (S2), and (S3) are valid for any ferromagnet with an arbitrary shape.

Now, let us assume that the ferromagnet has the shape which is given by infinitely long parallel grooves introduced in the ferromagnetic film (see the Fig. 1 in the main text). As far as the nanograting is infinitely long along the $y$ direction it can be considered as homogeneous along this direction. Thus, after integration over $y'$, Eq. (S3) can be simplified to

$$\psi(x, z, t) = \frac{M_s}{2\pi} \int_{-\infty}^{\infty} dx' \int_{0}^{l} \frac{m_x(x - x') + m_z(z - z')}{(x - x')^2 + (z - z')^2} dz',$$  \hspace{1cm} (S4)

where $l$ is the thickness of the unpatterned film (see the Fig. 1 in the main text). For the nanograting structure, Eq. (S4) can replace the Eqs. (3) from the main text. As one can see from Eq. (S4), the $m_y$ component does not contribute to the magnetic scalar potential despite that, in general, $m_y \neq 0$. Moreover, the $y$-coordinate is absent, as well. This leads us to the general conclusion that $H_{d,y}$ is zero for any orientation of magnetization (or any direction of magnetic field) in the grating structure. Moreover, the demagnetizing field is zero if the magnetization is parallel to the grooves.

Let us discuss the symmetry properties of Eq. (S4) for the $x \rightarrow -x$ operation in the steady-state. We use upper zero indices to describe this case. The demagnetizing field imposes opposite symmetry for in-plane and out-of-plane magnetization components. Moreover, the symmetry of the $i$-th component of the magnetization reflects the symmetry of the $i$-th non-zero component of demagnetizing field ($i = x, y, z$). For example, in the case of an in-plane $H_{ext}, m_z^0(x, z)$ and
$m^0_x(x, z)$ are even and odd functions, respectively. Furthermore, the numerator of the integrand in Eq. (S4) consists of odd functions $m^0_x(x - x')$ and $m^0_z(z - z')$ (one needs to change also $x' \rightarrow -x'$). Thus, $\psi^0$ is an odd, $H_{d,x}^0$ is an even, and $H_{d,z}^0$ is an odd function, respectively. Similarly, one can see opposite symmetry properties if the magnetization is parallel to the $z$-direction. In the case of an arbitrary out-of-plane orientation of the magnetization, both $m^0$ and $H_d^0$ are neither symmetric nor antisymmetric functions.

Now we turn to the NG structure with the parameters discussed in the main text. The nonzero components of $H_d^0$ are shown in Fig. S6 for several important directions of $H_{\text{ext}}$. As one can see, both components of $H_d^0$ increase with increasing of $\varphi_H$. For the case of $\varphi_H = 90^\circ$, $H_{d,x}^0$ is similar as in [S5] (see Fig. 5(h) in [S5]). For this direction of $H_{\text{ext}}$ the arrows of the demagnetizing vector

![Figure S6. Spatial distributions of (a) $x$-components and (b) $z$-components of the static demagnetizing field for different directions of $H_{\text{ext}}$ at $H_{\text{ext}} = 200$ mT. The values of $\varphi_H$ are shown in between (a) and (b) in degrees. In the case of $\varphi_H = 90^\circ$ arrows represent the demagnetizing vector field. The length of arrows is proportional to $|H_d^0|$ in the center of each arrow.](image-url)
field are introduced. It is worth to emphasize that the demagnetizing field is nonzero outside the nanograting. The total magnetic field is the vector sum of $H_d^0$ and $H_{ext}$.

In the steady-state, the magnetization is parallel to $H_{0,eff}^d$. In the case of $H_{ext} = 0$, the magnetization tends to be parallel to the grooves of the NG. Indeed, if the magnetization would deviate from the grooves direction it will create a demagnetizing field, which acts in the opposite direction to the magnetization. Thus, the demagnetizing term in the free energy, $-m_0 \cdot H_d^0$, would be positive which is energetically inefficient. That is why the magnetization tends to be parallel to the grooves at $H_{ext} = 0$. In the case of $H_{ext} \neq 0$ the magnetization in the wire region tends to be parallel to the grooves while the magnetization in the film regions tends to be parallel to $H_{ext}$. Thus, the total spatial distribution of the steady-state magnetization is non-homogeneous for all directions of $H_{ext}$ except for $\varphi_H = 0^\circ$.

Let us introduce the quantities $\varphi_M(x, z) = \arctan(m_y^0/m_x^0)$ and $\theta_M(x, z) = \arccos m_z^0$ which are the local azimuthal and polar angles of the magnetization. The average value of $\varphi_M(x, z)$ is denoted as $\overline{\varphi_M}$ in the main text. The deviations of the magnetization angles from the direction of $H_{ext}$ are given by $\varphi_H - \varphi_M$ and $90^\circ - \theta_M$. The latter are shown in Fig. S7 with 15° step of $\varphi_H$. At $\varphi_H = 0$ the magnetization is homogeneous, hence $\varphi_H - \varphi_M = 0^\circ$ and $90^\circ - \theta_M = 0^\circ$. At $\varphi_H \neq 0$ the azimuthal deviation $\varphi_H - \varphi_M$ reaches $19.2^\circ$ with a maximum at $\varphi_H \approx 64^\circ$. The largest azimuthal deviations are localized at the outer corners of the NG wire. At $\varphi_H = 90^\circ$, the $y$-component of magnetization is zero, thus the azimuthal deviation is zero. Interestingly, these azimuthal deviations can have both signs. The positive sign means that the magnetization tends to be parallel to the grooves. A small negative azimuthal deviation (up to $-3.3^\circ$) arises in the region under the grooves, where $H_{d,x}^0$ is positive (see Fig. S6(a)). The latter, referred to as magnetizing field [S5], slightly rotates the magnetization towards the $x$-direction.

The polar deviation of the magnetization, $90^\circ - \theta_M$, is shown in Fig. S7(b). In this case, the distribution of the polar deviation is an odd function. This is because the symmetry of polar and azimuthal deviations (or magnetization components) corresponds to the symmetry of the $H_d^0$ components as it was discussed above. Note, that the positive polar deviations correspond to

![Figure S7](image-url)
positive $m_z^0$ and vice versa. It can be observed that such polar deviations increase with increasing of $\varphi_H$. Maximum deviation is reached at $\varphi_H = 90^\circ$. Interestingly, the polar deviations are localized in the inner corner of the NG, while the azimuthal deviations are localized in the outer corners. The reason for this is that the absolute value of the $z$-component of the demagnetizing field has its maxima at the sharp parts of the NG corners (see Fig. S6(b)). For the outer corners, $H_{d,z}^0$ is localized outside the nanograting structure and for inner corners, it is localized inside the structure and, thereby, it rotates the magnetization in the $z$-direction.

As one can see from the analysis above, the magnetization is not saturated (neither homogeneous nor parallel to $H_{\text{ext}}$) even at $H_{\text{ext}} = 200$ mT. However, it is saturated in average, i.e. $\vec{\varphi}_M \approx \varphi_H$ at $H_{\text{ext}} = 200$ mT. Due to the presence of the demagnetizing field, the magnetization in the NG saturates asymptotically with increasing of the external magnetic field. For example, at $H_{\text{ext}} = 10$ T and $\varphi_H = 90^\circ$, the polar deviation, $90^\circ - \theta_M$, changes in the range from $-1.5^\circ$ to $1.5^\circ$.

IV. DEPENDENCIES OF MAGNON DISPERSIONS AND GROUP VELOCITIES ON THE DIRECTION OF MAGNETIC FIELD

Magnon dispersions for the DE and BV geometries are shown in Fig. 2(a), (c) in the main text. Figure S8 (a)-(e) shows the magnon dispersions at the intermediate directions of $H_{\text{ext}}$ from $\varphi_H = 15^\circ$ to $\varphi_H = 75^\circ$ at $H_{\text{ext}} = 200$ mT. One can see that with increasing $\varphi_H$ the magnon dispersion branches shift to lower frequencies and several low-lying branches become flat. One may note that the slopes of the dispersion curves depend on $\varphi_H$ nonmonotonically. To show this explicitly, we calculated the group velocities $v_g = \frac{dk}{d\omega}$ at the center of the Brillouin zone.

The dependencies of $v_g(\varphi_H)$ for the seven lowest magnon modes and from the 8-th to 13-th magnon modes are shown in Fig. S8(f) and (g), respectively. Figure S8(f) shows that the $v_g$ of the ground magnon mode #1 is close to zero. The next pairs, namely #2 and #3, #4 and #5, #6 and #7, possess quite similar dependencies with opposite signs. For $\varphi_H = 90^\circ$ the group velocities for all shown 13 modes are zero. With decreasing $\varphi_H$ the group velocities of higher modes increase faster than for lower modes (compare Fig. S8(f) and (g) near $\varphi_H = 90^\circ$). Interestingly, the highest magnon group velocity is not observed for the DE geometry. The maximum of $v_g \approx 0.83$ km/s is realized for the 13-th magnon mode at $\varphi_H \approx 26.4^\circ$. The fastest optically active magnon mode is the mode #3 with $v_g \approx -0.5$ km/s around $\varphi_H = 30^\circ$.

In time-resolved optical excitation experiments, where the pump laser spot diameter, $d_p$, is comparable to the NG period, magnon modes with $k_x \neq 0$ become accessible. For example, in the case $d_p = 1 \mu$m, which is defined by the 1/ $\exp(2)$-level of the laser intensity with Gaussian distribution, a Fourier transform of the spatial intensity distribution [S13] gives the corresponding maximal wavevector $|k_x|d/\pi = 8d/(d_p\pi) \approx 0.76$. It is worth to mention, that the dependencies of $v_g(\varphi_H)$ at $k_x \neq 0$, as well as in the center of Brillouin zone are nonmonotonic.

The values of $v_g$ are quite low for the considered set of geometrical/material parameters. Indeed, at $\alpha = 0.01$, the magnon lifetimes are $\sim 1$ ns. In this case, the propagation length for the fastest optically active mode is $\sim 500$ nm, which is comparable with the NG period. Therefore, such magnon modes can not be used for data transfer. A search for other sets of geometrical/material parameters which give group velocities much larger than the considered set is beyond the scope of this work. We note that, the nonreciprocity and accompanying nonzero slope of the dispersion curves at $k_x = 0$ can potentially be used for data transfer and processing.
Figure S8. (a)-(e) Magnon dispersion evolution under rotation of $H_{\text{ext}}$ such that (a) $\varphi_H = 15^\circ$, (b) $\varphi_H = 30^\circ$, (c) $\varphi_H = 45^\circ$, (d) $\varphi_H = 60^\circ$, and (e) $\varphi_H = 75^\circ$. Note that the vertical scales are different in (a)-(e). (f) and (g) group velocities of magnon modes at $k_x = 0$ for the seven lowest magnon modes (f) and from the 8-th to 13-th magnon modes (g). The line colors correspond to the line colors in Fig. 3(a) in the main text.

V. OPTICAL EXCITATION AND DETECTION OF MAGNONs IN NANOGATING STRUCTURES

Most results of this work are obtained using the eigenfrequency analysis. Such analysis does not require any simulation of excitation or detection. The latter is required for the simulation of the transient magneto-optical Kerr effect (MOKE) (see Fig. 3(b) in the main text), where the introduced change of the polarization plane of light is used to estimate the Kerr rotation angle (see e.g. [S6, S14]).

In all-optical time-resolved experiments, the excitation of the metallic ferromagnet is realized by means of an absorbed femtosecond laser pulse. The ultrafast excitation is mainly given by two mechanisms: ultrafast demagnetization and ultrafast changing of the magnetic anisotropy constants [S6]. As Permalloy is an isotropic material, magnon excitation via the second mechanism is impossible. Thus, for the modeling of the magnon excitation we assume an excitation via ultrafast demagnetization. In general, however, this mechanism is not consistent with the Landau-Lifshitz-Gilbert (LLG) equation (Eq. (1) in the main text), because the ultrafast demagnetization decreases
the length of the magnetization vector, which is required to be constant. In this case, one has to use the Landau-Lifshits-Bar'yakhtar equation [S15] or the Landau-Lifshitz-Bloch equation [S16]. Nevertheless, the LLG equation still can be used as a good approximation.

Let us derive the criterion of validity of using the LLG equation in the case of ultrafast demagnetization. In the LLG equation, the length of the magnetization vector must be equal to $M_s$:

$$|M_s m| = |(M_s^0 - \delta M_s)(m^0 + \delta m)| \simeq \sqrt{(M_s^0)^2 + 2(M_s^0)^2 m^0 \cdot \delta m - 2M_s^0 \delta M_s},$$  \hspace{1cm} (S5)

where $M_s^0$ and $\delta M_s$ are the static and dynamic parts of saturation magnetization, respectively. In Eq. (S5) we assumed that $|m^0| = 1$ and neglected by quadratically small terms. The second term in the square root of Eq. (S5) is zero as the vectors $m_0$ and $\delta m$ are orthogonal. Thus, the LLG equation can be used if $2\delta M_s \ll M_s^0$.

The detection of the coherent magnon response is traced via the transient MOKE. For simulation of transient MOKE signals we assumed a linear dependence of $\delta M_s$ on the lattice temperature. The latter was found by implementing in Comsol the two-temperature model (see e.g. [S7]) for the electron and lattice temperatures of the nanograting. We assumed for simplicity that the NG is thermally isolated from the substrate. We considered excitation from the side of substrate and detection from the side of nanograting. The size of the pump and probe laser spots are assumed to be much larger than the grating period. The initial distribution of the electron temperature of the NG was found by solving Maxwell’s equations in Comsol. For calculation of the Kerr rotation angle we assumed that a perturbed dielectric permittivity tensor is given in linear approximation by

$$\delta \varepsilon = i\beta \begin{pmatrix} 0 & \delta m_z & -\delta m_y \\ -\delta m_z & 0 & \delta m_x \\ \delta m_y & -\delta m_x & 0 \end{pmatrix},$$  \hspace{1cm} (S6)

where the magneto-optical coefficient $\beta \approx -0.14 - i \cdot 0.2$ for the optical wavelength $\lambda_0 = 800$ nm [S17, S18]. For the case of normal incidence, $\delta m_z$ is the most important component of magnetization for detection. The angular dependence of the MOKE spectral amplitude (see Fig. 3(b) in the main text) was estimated by the following simplified expression:

$$\int \beta \delta m_{z,\omega} \cos \left(2k_0 n'(z - l)\right) \exp \left(2k_0 n''(z - l)\right) dV,$$  \hspace{1cm} (S7)

where $k_0 = 2\pi/\lambda_0$ is the wavevector of light in vacuum, $n = n' + in'' \approx 2.9 + i \cdot 4.5$ is the complex refractive index of permalloy [S17, S18], and $dV$ is the unit volume element. The Fourier component of $\delta m_{z,\omega}$ in Eq. (S7) is obtained from the solution of the problem in the frequency domain. We checked that solving the problem in the time domain and then performing the fast Fourier transform of the time-dependent signals give the same result for the spectral amplitude of the MOKE signal. However, we chose to perform the calculations in the frequency domain, because they are much faster than in the time domain.

[S1] µMAG, Micromagnetic Modeling Activity Group, www.ctcms.nist.gov/~rdm/mumag.org.html.
[S2] COMSOL Multiphysics® v. 5.4. www.comsol.com. COMSOL AB, Stockholm, Sweden.

[S3] M. Mruczkiewicz, M. Krawczyk, V. K. Sakharov, Y. V. Khivintsev, Y. A. Filimonov, and S. A. Nikitov, J. Appl. Phys. 113, 093908 (2013).

[S4] J. Rychly, P. Gruszecki, M. Mruczkiewicz, J. W. Klos, S. Manica, and M. Krawczyk, Low Temperature Physics 41, 745 (2015).

[S5] M. Langer, R. A. Gallardo, T. Schneider, S. Stienen, A. Roldán-Molina, Y. Yuan, K. Lenz, J. Lindner, P. Landeros, and J. Fassbender, Phys. Rev. B 99, 024426 (2019).

[S6] M. van Kampen, C. Joza, J. T. Kohlhepp, P. LeClair, L. Lagae, W. J. M. de Jonge, and B. Koopmans, Phys. Rev. Lett. 88, 227201 (2002).

[S7] V. N. Kats, T. L. Linnik, A. S. Salasyuk, A. W. Rushforth, M. Wang, P. Wadley, A. V. Akimov, S. A. Cavill, V. Holy, A. M. Kalashnikova, and A. V. Scherbakov, Phys. Rev. B 93, 214422 (2016).

[S8] A. S. Salasyuk, A. V. Rudkovskaya, A. P. Danilov, B. A. Glavin, S. M. Kukhtataruk, M. Wang, A. W. Rushforth, P. A. Nekludova, S. V. Sokolov, A. A. Elistratov, D. R. Yakovlev, M. Bayer, A. V. Akimov, and A. V. Scherbakov, Phys. Rev. B 97, 060404(R) (2018).

[S9] A. V. Scherbakov, A. P. Danilov, F. Godejohann, T. L. Linnik, B. A. Glavin, L. A. Shelukhin, D. P. Pattnaik, M. Wang, A. W. Rushforth, D. R. Yakovlev, A. V. Akimov, and M. Bayer, Phys. Rev. Applied 11, 031003 (2019).

[S10] Hereafter, symmetric (even) and antisymmetric (odd) is always meant with respect to the center of the NG wire region.

[S11] I. Lisenkov, D. Kalyabin, S. Osokin, J. W. Klos, M. Krawczyk, and S. Nikitov, J. Magn. Magn. Mater. 378, 313 (2015).

[S12] K. Y. Guslienko and A. N. Slavin, Phys. Rev. B 72, 014463 (2005).

[S13] T. Satoh, Y. Terni, R. Moriya, B. A. Ivanov, K. Ando, E. Saitoh, T. Shimura, and K. Kuroda, Nat. Photonics 6, 662 (2012).

[S14] W. K. Hiebert, A. Stankiewicz, and M. R. Freeman, Phys. Rev. Lett. 79, 1134 (1997).

[S15] V. G. Bar'yakhtar, Zh. Eksp. Teor. Fiz. 87, 1501 (1984).

[S16] O. Chubykalo-Fesenko, U. Nowak, R. W. Chantrell, and D. Garanin, Phys. Rev. B 74, 094436 (2006).

[S17] G. Neuber, R. Rauer, J. Kunze, T. Korn, C. Pels, G. Meier, U. Merkt, J. Bäckström, and M. Rübhausen, Appl. Phys. Lett. 83, 4509 (2003).

[S18] K. K. Tikuišis, L. Beran, P. Cejpek, K. Uhlířová, J. Hamrlé, M. Vaňatka, M. Urbánek, and M. Veis, Materials & Design 114, 31 (2017).