Propagation characteristics of low-frequency acoustic wave in full waveguide of shallow water based on time-domain finite element method

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Abstract. In order to clarify the propagation mechanism and characteristics of low-frequency sound signals in shallow sea with elastic bottom, using the Time-domain Finite Element Method (TDFEM), a time-domain numerical calculation model of the low-frequency sound field in shallow sea full-waveguide under the action of a point sound source is established. With this model, the influence of seabed acoustic parameters' and seabed topography' changes on the sound propagation characteristics in full-waveguide is simulated and analyzed. The results show that when there is a shear wave effect on the seabed, low-frequency sound waves will excite the Scholte wave propagating on the bottom boundary of the seabed. The greater the seabed acoustic impedance, the faster the excited Scholte wave velocity and the larger the amplitude. At the same time, there will be acoustic energy leakage phenomenon in the inclined seabed.

1. Introduction

The numerical calculation of acoustic propagation characteristics in shallow water has always been an important research work in the field of Marine acoustics. After nearly 50 to 60 years of development, a variety of sound field calculation methods have been established in the field of underwater acoustic propagation research, such as normal wave method, ray method, parabolic equation method, fast field method, and a series of derived calculation methods[1-2]. However, the calculation process of the previous method is mainly carried out in the frequency domain, and the seabed is usually assumed as the liquid medium with horizontal stratification when the model is assumed. When dealing with the seabed of non-horizontal stratified and elastic medium, these methods are limited to some extent.

With the development of underwater target detection technology and the requirement of monitoring various physical ocean phenomena, the research requirements for the propagation mechanism and characteristics of low-frequency sound waves in shallow sea are constantly proposed. Existing results have proved that when studying low-frequency acoustic propagation in shallow sea, the seabed is an elastic medium and the influence of seabed topographic relief cannot be ignored[3]. Finite Element Method (FEM) is more suitable for solving the acoustic field problems with various fluctuations in the model because it divides the model into discrete finite elements for calculation.
Based on the above reasons, this article is based on the Time domain finite element method (the Time-domain Finite Element Method, TDFEM), low frequency point source is established under the action of shallow water waveguide numerical calculating model of sound field in the Time domain, by using the model the simulation analysis of the underwater acoustic parameters and the change of seabed terrain on the shallow sea all the influence law of low frequency sound propagation in the waveguide characteristics.

2. Finite element form of sound field in time domain
The shallow-sea wave guide model is shown in Fig 1. $V_w$ is defined as seawater layer and $V_s$ as seabed layer. $B$ is the interaction boundary between the seawater layer and the shallow seabed layer. $c_w$ and $\rho_w$ are the wave velocity and density of compression wave in seawater layer, respectively. $c_p$, $c_s$ and $\rho_b$ are the compression wave velocity, shear wave velocity and density of the seabed respectively. $s(t)$ is the time-dependent sound source; PML (perfect matching layer) is a perfect matching layer, which is used to simulate the propagation of acoustic signals in the infinite Marine environment.

\[
\begin{align*}
K_\rho \phi + C_\rho \phi + K_\phi \phi + f &= 0 \\
M_\rho \dddot{\phi} + C_\rho \dddot{\phi} + K_\phi \dddot{\phi} &= 0 \\
M \dddot{\phi} + C \dddot{\phi} + K \dddot{\phi} + f &= 0
\end{align*}
\]

The value of the sound pressure field at $t + \Delta t$:

\[
K_c \phi + C_c \phi + K_\phi \phi = M \dddot{\phi} + C \dddot{\phi} + K \dddot{\phi} + f
\]

The fluid-elastic coupling equation is:

\[
\begin{bmatrix}
K_e & K_C \\
0 & K_w
\end{bmatrix} + j \omega \begin{bmatrix}
C_e & 0 \\
0 & C_w
\end{bmatrix} - \omega^2 \begin{bmatrix}
0 & M_e \\
M_e & 0
\end{bmatrix} \times \begin{bmatrix}
\phi \\
\phi
\end{bmatrix} =
\begin{bmatrix}
F_e \\
F_w
\end{bmatrix}
\]

By coupling the physical field equation, the fluid-shell coupling equation and the PML equation established by the above finite element method, the calculation of the sound pressure field $P$ in the water layer and the seabed layer in the shallow sea environment can be realized. The corresponding relationship between the sound pressure field $P$ and other physical fields in each layer can further obtain the values of other physical fields in the model, such as the vibration velocity field $V$.

Fig 1  Shallow water waveguide model

The finite element general form of the wave equation applicable to any spatial dimension problem:

\[
\begin{align*}
K_\rho \phi + C_\rho \phi + K_\phi \phi + f &= 0 \\
M_\rho \dddot{\phi} + C_\rho \dddot{\phi} + K_\phi \dddot{\phi} &= 0 \\
M \dddot{\phi} + C \dddot{\phi} + K \dddot{\phi} + f &= 0
\end{align*}
\]
3. The simulation verification

This section first compares the matching degree of the results obtained by using TDFEM method and the existing frequency domain sound field calculation method. The horizontal submarine environment model was selected in the verification, and the simulation conditions were shown in Table 1.

Table 1 Parameters in horizontal waveguide

|               | Longitudinal wave m/s | Shear wave m/s | Density kg/m³ | Depth m |
|---------------|-----------------------|----------------|---------------|---------|
| Seawater      | 1500                  | /              | 1000          | 400     |
| Seabed        | 2400                  | 1200           | 2000          | /       |

The sound source emits the Riker pulse wave with the main frequency of 30 Hz. Let the sound source depth be 395 m underwater.

Fig 2 shows the sound field at 0.5 s. It can be seen that the sound wave can be divided into compression wave and shear wave after reaching the sea bottom, and the interface wave is generated at the interface between the sea and the sea bottom. The compression wave propagates at the fastest speed. Meanwhile, it can be seen that the PML layer around the model well absorbs the incoming acoustic signals, thus realizing the simulation of the infinite boundary in a finite size.

![Acoustic signal propagation diagram of 30 Hz Riker pulse at 0.5 s](image)

Fig 3 Comparison of TDFEM and FFP propagation loss curves

![Comparison of TDFEM and FFP propagation loss curves](image)
Fig 3 shows the comparison between the propagation loss obtained by Fourier transform and the calculation results of the Fastfield method (FFM). The comparison of the sound propagation loss obtained by TDFEM and FFM shows that the calculation results of the two methods are basically consistent, which verifies the correctness of the calculation results of the TDFEM model.

4. The simulation research
In this section, on the basis of verifying the correctness of TDFEM calculation results, the effects of acoustic parameters and topographic changes on acoustic propagation characteristics under full waveguide in shallow sea are discussed. The parameters of three typical subsea media are shown in Table 2.

| Type of seabed | Moraine | Limestone | Basalt |
|----------------|---------|-----------|--------|
| Longitudinal wave m/s | 2000 | 2400 | 5000 |
| Shear wave m/s | 800 | 1200 | 2500 |
| Density kg/m$^3$ | 2000 | 2000 | 2500 |

4.1. Study on horizontal seabed model
Seawater parameters and sound source depth of the horizontal seabed model are consistent with those in Table 1. The parameters of the three submarine media are shown in Table 2. Riker pulse wave with the main frequency of 15 Hz is considered in the acoustic signal emitted by the sound source.

| Type of seabed | Moraine | Limestone | Basalt |
|----------------|---------|-----------|--------|
| Computing time/s | 4 | 4 | 4 |
| Time step/s | 2×10$^{-4}$ | 2×10$^{-4}$ | 2×10$^{-4}$ |
| Horizontal seabed/s | 1506 | 1510 | 1512 |
| Inclined seabed/s | 1520 | 1524 | 1528 |

As can be seen from Table 3, the wavelength of submarine acoustic wave increases along with the increase of the velocity of seabed compressional wave and shear wave, and a smaller grid is needed to resolve a wavelength to ensure the accuracy of the results.
It can be seen from Fig 4 that the velocity and amplitude of the interface wave increase with the increase of the shear wave impedance and compressional wave impedance of the seabed. Seabed interfacial wave is composed of non-uniform compression wave and non-uniform shear wave. With the increase of the two, the wave velocity of interface wave will naturally be faster and the impedance of submarine medium will increase. The increase of impedance will lead to the accumulation of sound energy at the interface, so the amplitude of interface wave will be larger.

4.2. Study on inclined seabed model
Consider the inclined seafloor as follows: the seafloor decreases linearly from 400 m depth at 2 km to 100 m depth at 4 km. The parameters are consistent with those in Tables 1 and 2.

As can be seen from Table 3, the more complex the seabed topography is, the greater the calculation amount of TDFEM will be, so the calculation time will be longer. This is because when the seabed topography fluctuates, more dense grids are needed to ensure the accuracy of the results, thus increasing the number of grids divided and the amount of calculation. As can be seen from Fig 5, with the increase of propagation time, the interface waves propagate upward along the inclined sea surface, and the
interface waves, compression waves and shear waves in seawater can be well distinguished. Meanwhile, there are continuous received acoustic signals at the inclined sea bottom interface. This is because influenced by the topography of the sea bottom, with the increase of the propagation distance, the reflection and transmission frequency of sound energy from the sea bottom and the sea surface increase, so there is a continuous received acoustic signal at the interface.

As can be seen from Fig 6, acoustic signal propagation is basically the same in the two terrains before 2 km. After 2 km, the propagation loss of the horizontal seabed is still very high and tends to increase slowly. However, when the inclined seabed is between 2 km and 3 km, the propagation loss decreases rapidly and is less than the horizontal seabed propagation loss, and begins to rise slowly and is greater than the horizontal seabed propagation loss until 3 km. The reason for the above phenomenon is that, influenced by the upslope seabed, the original low-order normal wave propagating at a small grazing Angle constantly transforms to the higher-order wave corresponding to a large grazing Angle, and the acoustic energy "leaks". However, sound energy accumulates for a short time at a distance, and then slowly begins to leak out. Therefore, the propagation loss under the inclined seabed will decrease first and then increase.

5. Conclusion
Based on the time-domain finite element method, this paper simulated and analyzed the influence of submarine acoustic parameters and submarine terrain changes on the low-frequency acoustic signal propagation characteristics in the shallow sea full-waveguide, and the following conclusions were drawn:

The acoustic signal propagation loss results obtained by the time-domain finite element method are consistent with the FFM calculation results, which proves the validity of the method. Meanwhile, the time-domain finite element method can realize the high-precision calculation of sound field in complex Marine environment.

Interfacial waves induced by low frequency and near seabed acoustic sources under the hard seabed can propagate long distances along the horizontal and inclined seabed. The greater the acoustic impedance, the greater the wave velocity and amplitude of the interfacial waves.

The tilt of the seabed will affect the propagation of sound energy, generally, it will propagate a distance and then slowly "leak" to the sea bottom.

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