A general effective field theory description of $b \rightarrow s\ell^+\ell^-$ lepton universality ratios

Gino Isidori, Davide Lancierini, Abhijit Mathad, Patrick Owen, Nicola Serra, and Rafael Silva Coutinho

1 Physik-Institut, Universität Zürich, CH-8057 Zürich, Switzerland

We construct an expression for a general lepton flavour universality (LFU) ratio, $R_X$, in $b \rightarrow s\ell^+\ell^-$ decays in terms of a series of hadronic quantities which can be treated as nuisance parameters. This expression allows to include any LFU ratio in global fits of $b \rightarrow s\ell^+\ell^-$ short-distance parameters, even in the absence of a precise knowledge of the corresponding hadronic structure. The absence of sizeable LFU violation and the approximate left-handed structure of the Standard Model amplitude imply that only a very limited set of hadronic parameters hamper the sensitivity of $R_X$ to a possible LFU violation of short-distance origin. A global $b \rightarrow s\ell^+\ell^-$ combination is performed including the measurement of $R_{pK}$ for the first time, resulting in a significance of new physics of $4.2\sigma$. In light of this, we evaluate the impact on the global significance of new physics using a set of experimentally promising non-exclusive $R_X$ measurements that LHCb can perform, and find that they can significantly increase the discovery potential of the experiment.

I. INTRODUCTION

In recent years, a pattern of deviations with respect to Standard Model (SM) predictions has manifested in measurements of $b \rightarrow s\ell^+\ell^-$ processes. These include deviations in the angular distribution of the decay $B^0 \rightarrow K^{*0}\mu^+\mu^-$, a deficit in the decay rates $\sigma_{\mu\mu}$ and deviations in lepton flavour universality (LFU) ratios. Within the framework of effective field theories, these deviations are numerically consistent with each other, pointing to a well-defined hypothesis of new physics of short-distance origin. Even under highly conservative theoretical assumptions, the global significance of the new physics hypothesis is as large as $4.3\sigma$.

Among these deviations, the LFU ratios are particularly interesting as their SM uncertainty is very precise. They are defined within a region of squared dilepton invariant mass ($q^2$) as

$$R_X \equiv \frac{\int_{q^2_{\min}}^{q^2_{\max}} d\Gamma(H_b \rightarrow X_s \mu^+\mu^-) dq^2}{\int_{q^2_{\min}}^{q^2_{\max}} d\Gamma(H_b \rightarrow X_e e^+e^-) dq^2},$$

where $H_b$ represents a $b$-hadron (meson or baryon) and $X_s$ represents a well-defined hadronic system with strangeness, such that the transition satisfies $\Delta B = \Delta S$.

While the SM prediction $R^{\text{SM}}_X = 1$ is very robust, the precise cancellation of hadronic uncertainties can be broken in presence of new physics (NP). Namely, the interpretation of a new physics structure affecting these LFU ratios relies on the knowledge of the hadronic structure of the decays involved. This is why the LFU ratio $R_{pK}$ has not been included yet in $b \rightarrow s\ell^+\ell^-$ global fits, despite its clean SM prediction. The same problem holds for any LFU ratio which contains a mixture of overlapping/interfering hadronic resonances where the underlying structure is unknown, referred to in the following as non-exclusive $R_X$ ratios. Examples of this type are the LFU ratios $R_{K\pi\pi}$ and $R_{K\pi}$, where for the latter the $K\pi$ system has an invariant mass larger than the $K^*(892)^0$ resonance. The experimental prospects for these ratios are promising but their interpretation in terms of $b \rightarrow s\ell^+\ell^-$ short-distance dynamics is not obvious.

Here, we propose a new method that allows to interpret any LFU ratio within the framework of effective Lagrangians for the first time, even if the detailed structure of the hadronic matrix elements is unknown. The key observation that allows us to reduce the number of unknown hadronic quantities is the fact that the SM amplitude is both lepton flavour universal and approximately left-handed. These two properties imply that only a very limited set of NP amplitudes can yield sizeable non-standard contributions to $R_X$. Their contribution can be described in terms of very few combinations of hadronic parameters, which can in turn be treated as nuisance parameters.

The theoretical decomposition of $R_X$ following this logic is presented in Sect. Using this decomposition we perform a global $b \rightarrow s\ell^+\ell^-$ combination including the measurement of $R_{pK}$ for the first time, improving upon the global estimate of the significance presented in Ref. Using this method we also explore the potential impact of the expected measurements of $R_{pK}$, $R_{K\pi\pi}$, and $R_{K\pi}$ with the full dataset collected so far by LHCb (Sec. IV). The conclusions of our analysis are summarised in Sect. V.

1 We assume the $q^2$ range extends well above the dilepton mass threshold.
II. GENERAL EXPRESSION OF $R_X$ IN TERMS OF WILSON COEFFICIENTS

In the limit of heavy new physics, we can describe both SM and NP effects in $b \to s\ell^+\ell^-$ decays by means of an effective Lagrangian containing only light SM fields. We normalise it as

$$\Delta \mathcal{L}_{\text{eff}}^{b \to s\ell^+\ell^-} = \frac{4G_F \alpha}{\sqrt{2}} \frac{V_{ts}^* V_{tb}}{4\pi} \sum_i C_i \partial_i + \text{h.c.,} \quad (2)$$

where $G_F$ and $\alpha$ denote the Fermi constant and the electromagnetic coupling, respectively, and $V_{ij}$ denotes the elements of the Cabibbo-Kobayashi-Maskawa matrix. The only difference between the SM and NP cases lies in the number of effective operators, which is larger in a generic NP framework. In full generality the dimension-six operators with a non-vanishing tree-level matrix element in $b \to s\ell^+\ell^-$ decays can be composed into three sets: i) dipole operators,

$$O_7 = \frac{m_b}{e} (\bar{s}_L \gamma_\mu b_R)(\bar{\ell}_L \gamma^\mu \ell), \quad \mathcal{O}_7 = \frac{m_b}{e} (\bar{s}_R \gamma_\mu b_L)(\bar{\ell}_L \gamma^\mu \ell), \quad (3)$$

vector operators,

$$O_9^f = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}_R \gamma^\mu \ell), \quad O_9^{10} = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}_R \gamma^\mu \ell), \quad (4)$$

and ii) scalar operators,

$$O_S^f = (\bar{s}_R b_R)(\bar{\ell}_L \ell_L), \quad \mathcal{O}_S^f = (\bar{s}_R b_L)(\bar{\ell}_L \ell_R). \quad (5)$$

In the NP case the $\ell = e$ and $\ell = \mu$ terms should be treated separately. The scalar operators lead to $b \to s\ell^+\ell^-$ amplitudes which are helicity suppressed and can be safely neglected in most of the observables we are interested in. The only exception being the (single) effective combination which contributes to the $B_s \to \mu^+\mu^-$ helicity-suppressed rate. The dipole operator $\mathcal{O}_7$ is negligible in the SM and is severely constrained by $\Gamma(B \to K^{*}\gamma)$ and $\Gamma(B \to K^{*}\ell^{+}\ell^{-})$ at low $q^2$ [20]. To describe SM and NP effects in the $R_X$ ratios, we can thus limit our attention to the SM dipole operator ($O_7$) and the four vector operators in Eq. (4).

As pointed in [27], to elucidate general properties of the LFU ratios beyond the SM, irrespective of the detailed structure of the hadronic matrix elements, it is convenient to write the decay amplitudes in a basis of chirally projected operators. To do so, we introduce the combinations

$$C_L^f = C_9^f - C_{10}^f, \quad C_R^f = C_9^f + C_{10}^f, \quad C_L^{\prime f} = C_9^{\prime f} - C_{10}^{\prime f}, \quad C_R^{\prime f} = C_9^{\prime f} + C_{10}^{\prime f}. \quad (6)$$

With this notation, the generic $H_b \to X_s\ell^+\ell^-$ transition amplitude can be decomposed as

$$\mathcal{A}(H_b \to X_s\ell^+\ell^-) \propto \langle \mathcal{M}_{X,L} \rangle^J (J_L)_\alpha + \langle \mathcal{M}_{X,R} \rangle^J (J_R)_\alpha \quad (7)$$

where

$$(J_L)_\alpha = \bar{\ell}_L \gamma^\alpha \ell_L, \quad (J_R)_\alpha = \bar{\ell}_R \gamma^\alpha \ell_R, \quad (8)$$

and

$$\langle \mathcal{M}_{X,L} \rangle^J = C_L^f J_X^f + C_L^{\prime f} J_X^{\prime f} + C_7 J_X^c, \quad \langle \mathcal{M}_{X,R} \rangle^J = C_R^f J_X^f + C_R^{\prime f} J_X^{\prime f} + C_7 J_X^c \quad (9)$$

with

$$J_X^f = \langle X_s | \bar{s}_L \gamma^\alpha b_L | H_b \rangle, \quad J_X^{\prime f} = \langle X_s | \bar{s}_R \gamma^\alpha b_R | H_b \rangle, \quad J_X^c \propto \frac{1}{q^2} \langle X_s | \bar{s}_L \sigma_{\mu\nu} b_R | H_b \rangle. \quad (10)$$

In the limit where we neglect small lepton mass effects, the terms in Eq. (7) proportional to the left-handed and right-handed leptonic currents do not interfere. Moreover, the following relation holds

$$|\langle \mathcal{M}_{X,L} \rangle^J|^2 = |\langle \mathcal{M}_{X,L} \rangle^J (C_L^f \rightarrow C_R^f, C_L^{\prime f} \rightarrow C_R^{\prime f})|^2 \quad (11)$$

Integrating over all kinematic variables but for $q^2$, we can thus decompose the decay rate as

$$\frac{d\Gamma_{X,L}}{dq^2} = \frac{d\Gamma_{X,L}}{dq^2} + \frac{d\Gamma_{X,R}}{dq^2}, \quad (12)$$

with

$$\frac{d\Gamma_{X,R}}{dq^2} = \frac{d\Gamma_{X,L}}{dq^2} \bigg|_{(C_L^f \rightarrow C_R^f, C_L^{\prime f} \rightarrow C_R^{\prime f})}. \quad (13)$$

The explicit expression of $d\Gamma_{X,L}/dq^2$ in terms of Wilson coefficients is

$$\frac{d\Gamma_{X,L}}{dq^2} = f_X^L(q^2) \left\{ |C_L^f|^2 + |C_L^{\prime f}|^2 + \text{Re} \left[ \eta_X^L(q^2) C_L^f C_L^{\prime f} \right] + \eta_X^{77}(q^2) |C_7|^2 + \text{Re} \left[ \eta_X^{79}(q^2) C_7 C_L^f + \eta_X^{97}(q^2) C_7 C_L^{\prime f} \right] \right\}, \quad (14)$$

where $f_X^L(q^2)$ and the four $\eta_X^L(q^2)$ are channel-dependent hadronic parameters. The hadronic matrix elements $J_X^f$ and $J_X^{\prime f}$ are transformed into each other under the action of parity, which is a unitary operator. As a result, integrating over the phase space of $|X_s|$ for any $q^2$ value, and summing (averaging) over the spin configurations of both $|X_s|$ and $|H_b|$, leads to the same coefficients in Eq. (14) for $|C_L^f|^2$ and $|C_L^{\prime f}|^2$. Moreover, the positivity of the squared matrix element implies

$$|\eta_X^L(q^2)| \leq 2, \quad \eta_X^{77}(q^2) > 0. \quad (15)$$

Given the definition of $R_X$ in Eq. [1], it is convenient to define the following $q^2$-integrated hadronic parameters:

$$F_X^L = \int_{q^2_{\text{min}}}^{q^2_{\text{max}}} f_X^L(q^2) dq^2,$$
As can be seen, only two types of NP effects can lead to a sizeable deviation of \( R_X \) from unity: a lepton non-universal shift in either \( C_L^\mu \) or \( C_L^{\mu'} \). Note also that the only hadronic parameter with direct impact on the extraction of NP constraints from \( R_X \) is \( \eta_X^L \), which is bounded by Eq. (15). The \( \eta_X^L \) and \( \eta_X^R \) parameters have a minor role: they control the dilution of the LFV violation in the rate due to the lepton-universal contribution by \( \mathcal{O}_7 \). Finally, the effect of \( \eta_X^{\mu_\tau} \) is always subleading.

The approximate expression in Eq. (21) is shown for illustrative purposes only, in the following numerical analysis we use the complete expression in Eq. (15), treating all the \( \langle \eta_X^L \rangle \) as nuisance parameters. In order to define a range for the \( \langle \eta_X^L \rangle \), we use a channel where we are able to compute the values of the \( \langle \eta_X^L \rangle \) parameters explicitly and where the impact of the dipole operator is maximal, namely the \( B^0 \to K^*(892)^0 \ell^+ \ell^- \) decay. In this mode, characterised by a spin-one final state, the dipole operator is maximally enhanced by the \( q^2 \to 0 \) pole. In multi-body channels, such as \( B^0 \to K^{\pi^-} \ell^+ \ell^- \) and \( B^+ \to K^+ \pi^- \ell^+ \ell^- \), with a sizeable \( S \)-wave component of the hadronic final state, we expect a significantly smaller contribution of \( \mathcal{O}_7 \) to the total decay rate. The values for the \( \langle \eta_X^L \rangle \) for this channel as a function of \( q^2_{\text{min}} \), setting \( q^2_{\text{max}} = 6 \text{ GeV}^2 \), are shown Fig. 1. The corresponding ranges for the hadronic parameters used in the numerical analysis are shown in Table 3.

We conclude this section with a few observations related to the theoretical expression of \( R_X \):

- In Eq. (14) we ignored the contribution to the rate of four-quark operators. In the \( q^2 \) region far from the narrow charmonia, dominated by perturbative contributions, their effect is small and cannot induce a violation of LFU. Similarly to \( \mathcal{O}_7 \), four-quark operators can only induce a dilution of the LFV contribution. Their effect can indeed be described as a \( q^2 \) dependence modification of coeffi-

\[
\langle \eta_X^{i,\ell} \rangle = \frac{1}{F_X^\ell} \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} f_X^\ell(q^2)\eta_X^L(q^2) dq^2. \tag{16}
\]

The normalization factor \( f_X^\ell(q^2) \) depends on the lepton mass via kinematic effects, which are sizeable only close to the endpoint (i.e. for \( q^2 \to 4m_q^2 \)). If the \( q^2 \) range of the measurement extends well above the di-lepton mass threshold, the lepton mass dependence is safely neglected

\[
R_X = \left\{ \left| C_L^\mu \right|^2 + \left| C_L^{\mu'} \right|^2 + \text{Re} \left[ \langle \eta_X^L \rangle C_L^\mu C_L^{\mu'} + C_L^7 \left( \langle \eta_X^L \rangle C_L^7 + \langle \eta_X^{\ell \tau} \rangle C_L^{\ell \tau'} + \langle \eta_X^{\ell \mu} \rangle C_L^{\ell \mu'} + \langle \eta_X^{\mu \tau} \rangle C_L^{\mu \tau'} \right) \right] \right\} + \left\{ L \to R \right\}. \tag{18}
\]

This implies that in the SM, and in all models where the Wilson coefficients are lepton universal, \( R_X \approx 1 \) up to corrections due to QED and/or residual kinematic effects which are at most of \( 1\% \) \cite{24, 25}.

The key observation of the present work is that \( R_X \) retains a significant discriminating power with respect to NP models even in the absence of a precise knowledge of the hadronic parameters, i.e. even when treating the \( \langle \eta_X^L \rangle \) as nuisance parameters. This statement emerges quite clearly by the following two observations:

- Sizeable deviations of \( R_X \) from unity can only be attributed to non-universal Wilson coefficients, i.e. \( |R_X - 1| \neq 0 \) only if \( |\Delta C_i| \neq 0 \) for some \( i \), where

\[
\Delta C_i = C_i^\mu - C_i^L, \quad i = L, L', R, R'. \tag{19}
\]

- Other observables constrain NP effects to be a small perturbation over the SM: this implies that large NP effects in \( R_X \) can arise only by non-vanishing \( \Delta C_i \) interfering with the SM amplitude. The latter has a peculiar structure,

\[
\begin{align*}
|C_L^{SM}| &= O(10) \gg |C_L^\mu|, |C_L^L|, \\
|C_{L,R}^{SM}| &= 0,
\end{align*}
\]

hence only a very limited set of NP amplitudes can lead to \( |R_X - 1| \gg 0 \).

These two observations become evident when linearising the theoretical expression of \( R_X \) with respect to the \( \Delta C_i \) and neglecting the interference of \( \Delta C_i \) with suppressed SM amplitudes. In this limit we obtain

\[
R_X - 1 \approx \frac{\text{Re} \left( \frac{2 \Delta C_i^\mu}{C_i^\mu} + \langle \eta_X^L \rangle \frac{\Delta C_i^L}{C_i^L} \right)}{1 + \langle \eta_X^L \rangle \left| \frac{C_{i,\ell}^{SM}}{C_i^L} \right|^2 + \text{Re} \left[ \langle \eta_X^L \rangle \frac{C_{i,\ell}^{SM}}{C_i^L} \right]}.
\tag{21}
\]

As can be seen, only two types of NP effects can lead to a sizeable deviation of \( R_X \) from one: a lepton non-universal shift in either \( C_L^\mu \) or \( C_L^{\mu'} \). Note also that the only

and we can set

\[
F_X^e = F_X^e \equiv F_X, \quad \langle \eta_X^{i,\ell} \rangle \equiv \langle \eta_X^L \rangle. \tag{17}
\]

In this limit the overall normalization factor drops out in \( R_X \) and the same hadronic parameters appear in both numerator and denominator:
III. GLOBAL COMBINATION OF CURRENT MEASUREMENTS

In this section we present a combination of $b \to s \ell^+ \ell^-$ measurements following the procedure described in Ref. 22. We include the following three sets of observables: i) the LFU ratios $R_K$ [15], $R_{K^*}$ [13] and $R_{pK}$ [29], ii) the branching ratio for the rare dilepton mode $B^0 \to \mu^+ \mu^-$ [5, 6, 30, 31] and, iii) the normalised angular distribution in $B^0 \to K^{(*)0} \mu^+ \mu^-$ decays [3, 4]. In the case of $R_K$ and $R_{K^*}$, where the structure of the hadronic matrix elements is well understood, we use the standard theoretical expressions in terms of Wilson coefficients and form factors using the Flavio package [32].

The ratio $R_{pK}$ is described by means of Eq. (18).

As discussed in Ref. 22, we employ a highly generic NP hypothesis and a highly conservative approach towards hadronic uncertainties. We generate pseudo-experiments according to the SM, fluctuating the measurements according to their experimental uncertainties, and calculate the likelihood ratio between the NP and SM hypotheses. The distribution of the likelihood ratio is then used to calculate the p-value of a fit to data. Long-distance charm contributions are treated by allowing for a lepton universal shift of $\mathcal{O}_R^0$ in the SM definition.

The lepton universality ratio $R_{pK}$ has been measured by the LHCb collaboration to be consistent with unity in the $q^2$ region $0.1 < q^2 < 6.0$ GeV$^2$/c$^4$ [29]. We include it in the combination by means of Eq. (18), using the limits reported in Table I for the hadronic parameters. In fact, preliminary results on the differential branching fraction intervals of the dimuon invariant mass further confirms the smaller contribution of $\mathcal{O}_7$ to the total rate [33], if compared to the benchmark $B^0 \to K^{(*)0} \mu^+ \mu^-$ decay [34]. As four nuisance parameters are included with only one measurement, degeneracies in the likelihood can occur due to multiple solutions. To counteract this, loose Gaussian constraints, whose width is the size of the physical ranges, are placed on each parameter to ensure the likelihood has a well-defined minimum. The exact value of these ranges has a very small effect on the numerical results.

The distribution of the likelihood ratio for the SM pseudoeperiments is shown in Fig 2 along with the value obtained from data. The inclusion of the measurement of $R_{pK}$ increases the effective degrees of freedom by 0.6 units. This increase represents the uncertainty on the $\langle \eta_K \rangle$ which allows for potentially different NP sensitivity compared to the existing $R_K$ and $R_{K^*}$ ratios. Compared with the results from Ref. 22, we observe a small reduction in significance, from 4.3$\sigma$ to 4.2$\sigma$ when including the observable $R_{pK}$. This is due to the fact that the value of $R_{pK}$ is not perfectly consistent with the other LFU ratios and the hadronic uncertainties allow to accommodate deviations from the SM amplitude in other directions, within a general NP hypothesis.

Using the same approach we test the specific hypothesis of a violation of lepton universality, considering all
FIG. 2. Distribution of the likelihood ratio for pseudo-experiments under the SM hypothesis along with the value obtained from data. Results are shown under the same conditions as in Ref. [22] and also when the measurements of $R_{pK}$ is included.

$R_X$ ratios measured so far, i.e. including $R_K$, $R_{K^*}$ and $R_{pK}$, and ignoring all other observables. This results in a local significance of 4.1σ for the hypothesis of a LFU violation, which is very close to the global significance of NP in $b \to s\ell^+\ell^-$ decays. This small variation in the significance can be understood as follows: the analysis of LFU observables has a smaller trial factor compared to the generic NP analysis; however, with present data, this effect is compensated by the lack of inclusion in the fit of $\mathcal{B}(B_s \to \mu^+\mu^-)$ [5, 6, 10, 11, 30], which enhances the significance in the generic NP case.

IV. IMPACT OF FUTURE MEASUREMENTS

In addition to assessing the significance with the current measurements, we calculate the expected gain in discovery potential by using this approach with other non-exclusive $R_X$ measurements that can be performed at LHCb in the near future. To this end, we estimate the experimental sensitivity of these ratios and include the hypothetical measurements in a fit with the current measurements.

We estimate the experimental sensitivity of three modes with the full run I and run II dataset of 9fb$^{-1}$ for the following ratios:

$$R_{pK} = \frac{B(A^0 \to pK^-\mu^+\mu^-)}{B(A^0 \to pK^-e^+e^-)},$$

$$R_{K\pi\pi} = \frac{B(B^+ \to K^+\pi^-\pi^+\mu^+\mu^-)}{B(B^+ \to K^+\pi^-\pi^+e^+e^-)},$$

$$R_{K\pi} = \frac{B(B^0 \to K^+\pi^-\mu^+\mu^-)}{B(B^0 \to K^+\pi^-e^+e^-)},$$

where for the $R_{K\pi}$ case, the $K^+\pi^-$ invariant mass is required to be above 1 GeV to separate it from the comparatively well understood $K^*(892)^0$ resonance.

The sensitivity for non-exclusive $R_X$ measurements depends primarily on the precision of the electron mode. Given the ratio $R_{pK}$ has already been measured, the precision can easily be predicted assuming it scales with luminosity, resulting in a precision of 12.2%. As the decays $B^+ \to K^+\pi^-\pi^+e^+e^-$ and $B^0 \to K^+\pi^-e^+e^-$ have yet to be observed, their yields are extrapolated from the corresponding muonic decay modes from Refs. [33, 34], by scaling with luminosity and the centre-of-mass energy. These muon yields are compared to the corresponding yield in the $R_K$ measurement [37] to scale the resulting precision of the LFU ratio. A statistical uncertainty on $R_{K\pi}$ and $R_{K\pi\pi}$ of 7.7% and 13.5% is expected for the full run I-II datasets in the range of $1.1 < q^2 < 6.0$ GeV$^2$/c$^4$. The estimated uncertainty on $R_{K\pi}$ turns out to be comparable with that of $R_{K^*}$, as can be expected given there are many significant contributions above the $K^*(892)^0$ resonance [35, 36].

Information on the differential branching fraction in intervals of the dimuon invariant mass can provide insights on the underlying dynamics of the non-exclusive hadronic system, which allows us to check the limits of the $\langle \eta_X^i \rangle$ parameters. For instance, for the $B^0 \to K^{*0}\mu^+\mu^-$ decay a relative increase of the differential branching fraction between the $0.1 < q^2 < 0.98$ and $1.1 < q^2 < 6.0$ GeV$^2$/c$^4$ regions by a factor of three is reported in Ref. [33]. Similar inspection can be performed for the non-exclusive channels and are found to be at the order of 2.0 and 2.6 for the $K\pi$ and $K\pi\pi$ hadronic systems, respectively [35, 36]. As a result, this confirms the conclusion of Sec [1] that the limits obtained for the $\langle \eta_X^i \rangle$ parameters involving the $K^*(892)^0$ resonance can be used as a proxy for these channels.

The impact of these future measurements is examined by repeating the procedure from the previous section introducing two benchmark points common to all non-exclusive LFU ratios: $R_X = 1.0$ (SM) and $R_X = 0.8$ (NP). The latter is chosen being broadly consistent with current global fits. Figure 3 (top) shows the distribution of the likelihood ratio when including these new $R_X$ observables under the NP hypothesis. A large increase in the significance from 4.3σ to 5.4σ when including the $R_X$ observables is seen. If the new measurements are set to the SM prediction of $R_X = 1.0$, a reduction to 3.8σ can be expected. These measurements can therefore have a large impact on the clarification of lepton universality violation in $b \to s\ell^+\ell^-$ decays.

In order to investigate the dependence of the significance with respect to the freedom given to the hadronic parameters, we have repeated the fit fixing the $\langle \eta_X^i \rangle$ to their central values. The result is also shown in Fig. 3 (bottom). As expected, in this case the additional measurements do not increase the effective degrees of freedom in the system. The exact knowledge of all hadronic nui-
sance parameters would lead to a significance of 5.9\(\sigma\), i.e. an increase in significance of 0.5\(\sigma\) compared to when they are treated as nuisance parameters. This relatively small increase provides an a posteriori confirmation that they play a minor role in the fit. Finally, we also decrease the limits allowed for \(\langle \eta_{\ell\ell}^{2} \rangle\) to 60, which would be appropriate if the \(R_{pK}\) ratio were measured setting \(q_{\text{min}}^{2}\) above 1 GeV\(^{2}\). A negligible difference in discovery potential is seen, which indicates that the exact kinematic range is not crucial for the subsequent interpretation.

V. CONCLUSIONS

In summary, we have introduced a method to include any LFU ratio in global fits by treating the hadronic uncertainties as nuisance parameters. This method is not designed to replace the existing theoretical description of \(R_{K}\) or \(R_{K^{*}}\), where we can take advantage of a detailed knowledge of all the components of the transition amplitudes. It is conceived for interpreting LFU ratios where we lack precise information about the underlying hadronic dynamics.

To demonstrate the method, we have updated the global fit of Ref. [22] to include the LHCb measurement of \(R_{pK}\). With current data, we find that \(R_{pK}\) has a marginal effect on the global significance of new physics in \(b \to s\ell^{+}\ell^{-}\) decays. However, when extrapolating to the full LHCb dataset, and including also hypothetical measurements of \(R_{K\pi}\) and \(R_{K^{*}\pi}\), we find that the increase in the significance can be large.

In this paper we concentrated on the three non-exclusive LFU ratios which are more promising from the experimental point of view. However, the method proposed here can be extended to include many other channels, such as \(B^{+} \to K^{+}K^{-}K^{+}\ell^{+}\ell^{-}\). An interesting experimental feature of some of the non-exclusive channels is that, due to the large invariant mass of the hadronic systems, they suffer much less from partially reconstructed backgrounds compared to the golden modes \(B^{0} \to K^{*0}\ell^{+}\ell^{-}\) and \(B^{+} \to K^{+}\ell^{+}\ell^{-}\). This additional experimental advantage reduces the risk of hypothetical mis-modelling of backgrounds, which right now are among the leading systematic uncertainties in the LFU measurements. The inclusion of the non-exclusive \(R_{X}\) using the method proposed here will therefore not only increase the new-physics sensitivity from a pure statistical point of view, but also enhance the redundancy of the experimental results.

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