TURBULENT AMPLIFICATION AND STRUCTURE OF THE INTRACLUSTER MAGNETIC FIELD

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ABSTRACT

We compare DNS calculations of homogeneous isotropic turbulence with the statistical properties of intracluster turbulence from the Matryoshka Run and find remarkable similarities between their inertial ranges. This allowed us to use the time-dependent statistical properties of intracluster turbulence to evaluate dynamo action in the intracluster medium, based on earlier results from a numerically resolved nonlinear magneto-hydrodynamic turbulent dynamo. We argue that this approach is necessary (a) to properly normalize dynamo action to the available intracluster turbulent energy and (b) to overcome the limitations of low Re affecting current numerical models of the intracluster medium. We find that while the properties of intracluster magnetic field are largely insensitive to the value and origin of the seed field, the resulting values for the Alfvén speed and the outer scale of the magnetic field are consistent with current observational estimates, basically confirming the idea that the magnetic field in today’s galaxy clusters is a record of its past turbulent activity.

Key words: cosmology: theory – dynamo – magnetohydrodynamics (MHD)

1. INTRODUCTION

The hot intracluster medium (ICM) of galaxy clusters (GC) is known to be magnetized from radio observations. These reveal both the occurrence of the Faraday rotation effect on polarized radiation from background quasars (Clarke et al. 2001; Clarke 2004) and of diffuse synchrotron emission (Ferrari et al. 2008) from the ICM. Estimates of the magnetic field based on these observations range between a fraction and several 
μG. Measurements on the structural and spectral features are sparse and more difficult, but indicate steep power laws below a few tens of kiloparsecs (Laing et al. 2008; Kuchar & Enßlin 2011). For massive clusters, turbulence in the ICM is mainly driven by structure formation (Norman & Bryan 1999; Ryu et al. 2008; Vazza et al. 2011; Miniati 2014, 2015). The most important magnetic field amplification mechanism in the ICM is the small-scale or fluctuation dynamo (SSD), operating on scales smaller than the turbulence outer scale. The kinematic regime of SSD, i.e., when the back reaction of the magnetic field on the flow is negligible, has previously been studied in great detail (Kazantsev 1968; Kraichnan & Nagarajan 1967; Kulsrud & Anderson 1992). In the kinematic regime the magnetic energy grows exponentially, until the approximation breaks down, roughly in a dynamical time multiplied by Re−1/2, where Re is an effective Reynolds number. The extremely hot and rarefied plasmas of the cluster have very large collisional mean free paths, around

λ ≈ 103 pc(n/3 × 10−3 cm−3)−1(T/10 keV)−1/2,

but at the same time, given the observable magnetic fields around 3 μG, the Larmor radius is smaller by many orders of magnitude:

rl ≈ 10−9 pc(T/10 keV)(B/3 μG)−1.

Collisional transport, described by Braginsky viscosity and magnetic diffusivity, does not terminate a turbulent cascade before plasma scales dj and rl and the cascade continues as plasma turbulence, which is evidenced in another case of tenuous and magnetized plasma—the solar wind (see, e.g., Leamon et al. 1998). Such a situation, known as collisionless plasma, is challenging from a theoretical viewpoint, since nonlinear plasma effects are dominating the transport, which has been known since early Lab plasma experiments, when the explosion became clear that collisional “classic transport” is grossly insufficient to explain cross-field diffusion (see, e.g., Galeev & Sagdeev 1979). As a rule of thumb, the actual effective parallel mean free path is smaller than the one obtained by the collisional formula, but larger than the Bohm estimate (λeff ∼ rl). The search for this “mesoscale” for cluster conditions resulted in estimates for the mean free path of the proton in the ICM around 10−3−10−6 pc (Beresnyak & Lazarian 2006; Schekochihin & Cowley 2006; Schekochihin et al. 2008; Brunetti & Lazarian 2011b). From these estimates we expect clusters to be turbulent with Reynolds numbers Re exceeding 1012. Combining this with the above estimate of the kinematic SSD growth rates, for a dynamical time ∼ eddy turnover time ∼1 Gyr (Miniati 2014), we estimate that the exponents timescale will be smaller than 1 Gyr (Re)−1/2 ∼ 1 kyr.

The remainder of this paper is organized as follows: in Section 2 we discuss the properties of the nonlinear regime of the small-scale dynamo, which is supposed to dominate most of the cluster lifetime; in Section 3 we point to the inadequacy of current MHD cosmological simulations, as far as dynamos are concerned, and suggest a different approach; in Section 4 we describe new homogeneous dynamo simulations with intermittent driving; in Section 5 we explain our cosmological hydrodynamic model of the cluster; in Section 6 we combine the knowledge obtained in previous sections and analyze cluster simulations to derive the properties of the cluster magnetic fields; in Section 7 we discuss implications and compare with previous work.

2. NONLINEAR SMALL-SCALE DYNAMO

As the kinematic approximation of SSD breaks down very quickly, the dynamo spends most of the time in the nonlinear
regime. In this regime, inclusive of the back reaction of the magnetic field on the flow, the magnetic energy continues to grow as it reaches equipartition with the turbulent kinetic energy cascade at progressively larger scales (Schlüter & Biermann 1950). At this stage the magnetic energy is characterized by a steep spectrum and an outer scale, \( L_B \), a small fraction of the kinetic energy outer scale (Haugen et al. 2004; Brandenburg & Subramanian 2005; Ryu et al. 2008; Cho et al. 2009). This picture has been argued to be true in any high-Re flow, with the argument relying on the locality of energy transfer functions (Beresnyak 2012). It also followed from this study that the growth rate of the magnetic energy corresponds to a certain fraction of the turbulent dissipation rate, with this fraction being a universal dimensionless number around 0.05, and that the magnetic outer scale \( L_B \) grows with time as \( L_B \approx t^{3/2} \) (Beresnyak 2012). The growth of magnetic energy reaches final saturation when \( L_B \) is a substantial fraction of the outer scale of the turbulence. However, this never happens in clusters, as we show below.

3. LIMITATIONS OF COSMOLOGICAL DYNAMO SIMULATIONS

An important implication of the above picture is that the memory of the initial seed field is quickly lost and the cluster magnetic field is expected to depend only on the cluster turbulent history. While this theoretical insight was certainly useful, its applications to cluster formation were not immediately realized. There are two main reasons for this. First, while there has been considerable progress in computational models of structure formation, and GCs in particular, the level of dynamic range of spatial scales achieved so far is considerably below the threshold necessary for the turbulent dynamo to operate efficiently. In fact, numerical MHD models of GCs typically report rather weak magnetic field amplification, roughly by factors \( \lesssim 30 \) (Miniati et al. 2001; Dolag et al. 2002; Dubois & Teyssier 2008; Vazza et al. 2014), including significant contribution from adiabatic compression. As alluded to above, the reason is ascribed to the low Re of the simulated flows. The kinematic growth rate is \( \gamma \approx Re^{1/2} / 30 \tau_L \) (Haugen et al. 2004; Schekochihin et al. 2004; Beresnyak 2012), where \( \tau_L \) is the turnover time of the largest eddy. So even with Re of \( \sim \) several \( \times 10^3 \), typical for cluster simulations, the dynamo will be stuck for several dynamical times in a kinematic regime, i.e., several Gyr, while in nature this stage will be many orders of magnitude quicker than the dynamical time (see also Section 1). Figure 1 demonstrates the difference between the magnetic energy growth between the case with very large Re (straight line) and Re that are available with current numerical capabilities (actual growth obtained in simulations with Re = 1000 and 3300). The growth observed in simulations is delayed due to the grossly prolonged kinematic stage. Second, in view of the current understanding of MHD dynamos (Section 2), lack of detailed knowledge about the ICM turbulence precludes accurate estimates of both the magnetic energy and, in particular, the outer scale of the magnetic spectrum.

Below we report on the progress with the approach that is different from direct approach of a cosmological MHD simulation, which, given the present state of our numerical capabilities, is completely inadequate, as we argued above. We have recently employed a novel technique to model the formation of a massive GC with sufficient resolution to resolve the turbulent cascade (Miniati 2014, 2015). We have extracted the time-dependent properties of the turbulence and used this information in combination with independent results on turbulent dynamo obtained from high-resolution periodic box simulations. The novelty and advantage of our approach is that the turbulence is self-consistently estimated through a numerical hydrodynamic model of structure formation, while the magnetic field evolution is estimated based on theory, which was confirmed in large-scale homogeneous dynamo simulations, robustly tested by studying low Re effects in a scaling study. Importantly enough, such dynamo simulations, unlike cosmological cluster models, are not limited in the number of dynamical times one can simulate.

4. DYNAMO SIMULATIONS

We have extended the study of statistically homogeneous isotropic small-scale dynamo simulations in Beresnyak (2012) with a series of simulations with intermittent energy injection into the velocity field, with the period 1, 2, 4, and 8 self-correlation timescales of velocity, \( \tau_v \). All simulations have magnetic Prandtl number \( Pr_m = 1 \) and driving in Fourier space was limited to lower harmonics (\( |k| < 2.5 \)). We started each MHD simulation by seeding a low-level white noise magnetic field into the data set obtained from a driven hydrodynamic simulation that reached a statistically stationary state. This data set was further evolved by full incompressible MHD equations. Figure 2 shows the evolution of magnetic energy in time. The previously measured normalized growth rate \( C_E = 0.05 \) is roughly consistent with most of the data. An important prediction of Beresnyak (2012) was also that the magnetic outer scale is proportional to \( \nu_A / \epsilon \) and grows in time as \( t^{3/2} \).

Since we are going to use this conjecture to estimate the outer scale of cluster magnetic fields, we plotted the \( \nu_A / \epsilon \) versus the magnetic outer scale, which we determined from the peak of the magnetic spectrum. The constant driving simulation (upper panel of Figure 3) showed good agreement with the proposed scaling and we have determined the dimensionless coefficient \( c_1 \) in the relation \( L_B = c_1 \nu_A / \epsilon \) to be around 0.2 (best fit 0.18). The intermittently driven simulation has shown large scatter, which is due to the fact that turbulence spectra do not depend
5. CLUSTER SIMULATIONS

We use the Matryoshka run to extract the time-dependent turbulence properties of the ICM of a massive GC, with the total mass at redshift $z = 0$ of $1.3 \times 10^{13} M_\odot$, forming in a concordance $\Lambda$CDM universe (Komatsu et al. 2009). The simulation was carried out with CHARM, an Adaptive-Mesh-Refinement cosmological code (Miniati & Colella 2007). We use a concordance $\Lambda$CDM universe with normalized (in units of the critical value) total matter density, $\Omega_m = 0.2792$, baryonic mass density, $\Omega_b = 0.0462$, vacuum energy density, $\Omega_k = 1 - \Omega_m = 0.7208$, normalized Hubble constant $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$ = 0.701, spectral index of primordial perturbation, $n_s = 0.96$, and rms linear density fluctuation within a sphere with a comoving radius of $8 h^{-1}$ Mpc, $\sigma_8 = 0.817$ (Komatsu et al. 2009). The simulated volume has a comoving size of $L_{\text{Box}} = 240 h^{-1}$ Mpc on a side. The initial conditions were generated on three refinement levels with grafic++ (made publicly available by D. Potter). For the coarsest level we use $512^3$ comoving cells, corresponding to a nominal spatial resolution of $468.75 h^{-1}$ comoving kpc and $512^3$ particles of mass $6.7 \times 10^8 h^{-1} M_\odot$ to represent the collisionless dark matter component. The additional levels allow for refined initial conditions in the volume where the galaxy cluster forms. The refinement ratio for both levels is $n_{fi} = \Delta x_\ell/\Delta x_{\ell+1} = 2$, $\ell = 0, 1$. Each refined level covers $1/8$ of the volume of the next coarser level, with a uniform grid of $512^3$ comoving cells while the dark matter is represented with $512^3$ particles. At the finest level the spatial resolution is $\Delta x = 117.2 h^{-1}$ comoving kpc and the particle mass is $10^8 h^{-1} M_\odot$. As the Lagrangian volume of the galaxy cluster shrinks under self-gravity, three additional uniform grids covering $1/8$ of the volume of the next coarser level were employed with $512^3$, $1024^3$, and $1024^3$ comoving cells, respectively, and $n_{fi} = 2, 4, 2$, for $\ell = 2, 3, 4$, respectively. All of them were in place by redshift 1.4, providing a spatial resolution of $7.3 h^{-1}$ comoving kpc in a region of $7.5 h^{-1}$ Mpc, accommodating the whole virial volume of the GC. The ensuing dynamic range of resolved spatial scales is sufficiently large for the emergence of turbulence. The results of the cluster simulation are described in full detail in Miniati (2014, 2015).

6. ANALYSIS OF CLUSTER SIMULATIONS

Using a Hodge–Helmholtz decomposition it was found that between 60% and 90% of the kinetic energy of the cluster turbulence is in the solenoidal component (Miniati 2015, see also Federrath et al. 2011). This is the relevant component for the discussed small-scale dynamo mechanism and the key question is whether it resembles homogeneous isotropic turbulence in the inertial range.

In the upper panel of Figure 4 we checked the statistical isotropy of the cluster turbulence. We compared the longitudinal velocity structure function (SF) with the analytical expression that presumes statistical isotropy. Statistical isotropy seems to be satisfied quite well on all scales of interest consistent with results in (Miniati 2015). A more critical test is provided by the relation between structure functions of different order. For example the dimensionless ratio $\langle (\delta v_i)^2 \rangle^{3/2} / (\langle |\delta v_i|^3 \rangle^2)$ is of interest to relate the energy cascade rate with the energy content of the cascade. In the lower panel of Figure 4 we studied the comparison between this ratio in the cluster simulation and in the homogeneous incompressible
driven turbulence. For the latter we used data from fully resolved direct numerical simulations of incompressible hydrodynamic driven turbulence in a periodic box (see, e.g., Beresnyak & Lazarian 2009). Both cluster and box simulations exhibited a clear well-pronounced dissipation interval, which we used to convert box simulation units into physical scale units of the cluster simulation. Note, however, that no fitting has been involved on the y-axis. Given the relatively short inertial range, the correspondence between homogeneous isotropic turbulence statistics and cluster statistics is quite remarkable. From the above comparison we conclude that the second order structure function of the cluster simulations in the range of scales 0.14–0.4 Mpc could be reliably used to estimate the turbulent dissipation rate, \( \epsilon_{\text{turb}} \), associated with the incompressible velocity component.

The turbulence dissipation rate is then estimated as follows:

\[
\epsilon_{\text{turb}} = (c_1/c_2)(5/4)(\langle \delta v \rangle^2)^{3/2}/l,
\]

where \( c_2 \approx 27 \) is the ratio of the structure functions reported in Figure 4 and \( c_1 \approx 1.17 \) is a factor to correct for dissipation effects, as in our finite Re simulations the Kolmogorov’s −4/5 normalization slightly underestimates the turbulent dissipation rate.

As expected, at a given time \( \epsilon_{\text{turb}} \) is a rather constant function of \( l \) within the inertial range. The observed deviation was used to estimate the error of the measurement of \( \epsilon_{\text{turb}} \). We plotted the dissipation rate determined in this manner in the top panel of Figure 5. We used the velocity structure function calculated within 1/3 of the virial radius of the simulated cluster for each data-cube. As we see from this figure, the dissipation rate varies non-monotonically over roughly an order of magnitude in scale over the lifetime of the cluster. The error bars defined above indicate the deviation from Kolmogorov’s self-similarity and were rather small, except for the time intervals where the rate was changing rapidly, i.e., the cluster was either relaxing or experiencing a fresh injection of kinetic energy.

We then estimated magnetic energy density as (Beresnyak 2012)

\[
E_B = \int_0^t C_E \rho \epsilon_{\text{turb}} dt,
\]

with \( C_E = 0.05 \). This equation originally described the average magnetic energy density in a statistically homogeneous case, as in simulations from Section 4. When we apply it to the cluster simulation we have to take into account the fact that both \( \rho \) and \( \epsilon \) depend on the distance to the cluster center (radius) and on time. As we found, \( \epsilon \) has only a modest dependence on radius; however, \( \rho \) is highly peaked around the center, so \( E_B \) will exhibit similar behavior. The cluster’s density profile is almost invariant in time, if expressed in virial units, i.e., \( \rho(r, t) = \rho(t)f(r/R_{\text{vir}}) \), and the radial dependence can be pulled in front of the time integral in Equation (4). If, at any given time, we are interested only in Alfvén speed \( v_A = (2E_B/\rho)^{1/2} \), this quantity will only have weak dependence on radius and may well be characterized by the average within the radius of \( R_{\text{vir}}/3 \). We plotted such an averaged Alfvén velocity in the middle panel of Figure 5. Furthermore, as was shown in Beresnyak (2012), for statistically stationary turbulence the magnetic energy containing scale could be estimated as

\[
L_B = c_1 v_A^2/\epsilon_{\text{turb}},
\]

where \( c_1 \approx 0.18 \) is a universal coefficient, which could be determined in DNS, see Figure 3. Our cluster turbulence was rather non-stationary; however, as discussed in Section 4, the estimate Equation (5) can also be applied to non-stationary driven turbulence as long as the dissipation rate is averaged over a timescale around one dynamical time; see below. This is

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**Figure 4.** We used structure functions to test statistical isotropy of the given cluster simulations (upper panel). Also we compare the relation between the second order total structure function and the third order parallel structure functions in the cluster simulation (solid line) and in a periodic box simulation of statistically stationary driven turbulence (dashed line), after rescaling the dissipation scale to the same number (lower panel). Note that no fitting has been involved in making both figures. We conclude that the second order structure function in the range 0.14–0.4 Mpc could be used to robustly estimate the turbulent dissipation rate associated with the solenoidal velocity component.

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**Figure 5.** Evolution of the turbulent dissipation rate (upper plot), Alfvén speed (middle), and the outer scale of the magnetic field (bottom) inferred from the cosmological cluster simulation by using self-similar laws for turbulence and dynamo, tested and normalized in well-resolved high-Re DNS. In obtaining \( v_A \), we divided the magnetic energy \( E_B \) from Equation (4) to the average density within 1/3 of the virial radius.
because hydrodynamic cascade has a memory over around one dynamical time and the changes in the driving rate do not instantaneously affect turbulent rate on small scales (Section 4). So, in using Equation (5) we used the $\epsilon_{\text{turb}}$ averaged over 2 Gyr, which approximately corresponds to two dynamical times.

The middle and bottom panels of Figure 5 show the time evolution of the average rms Alfvén speed, $v_A = (2E_B/\rho)^{1/2}$ and the magnetic outer scale $L_B$. Note that while $v_A$ grows monotonically, $L_B$ can decrease somewhat during a prolonged increase of the turbulent activity, such as during several major mergers.

Our estimates for $z \sim 0$ characteristic values of Alfvénic speed $v_A \sim 10^7$ cm s$^{-1}$ and the outer scales $L_B \sim 30 \sim 50$ kpc are consistent with the observed values reported in the literature (Eilek & Owen 2002; Govoni et al. 2006, 2010; McNamara & Nulsen 2007; Bonafede et al. 2010). This indicates that the type of nonlinear dynamo described in Beresnyak (2012) is consistent with what we observe in clusters, irrespective of the initial conditions. On the other hand, kinematic models, as well as ad hoc MHD simulations with limited Re, e.g., Beck et al. (2012), would require a fine-tuning of initial conditions to achieve this.

One interesting conclusion from our results in Figure 5 is that the outer scale of the magnetic field grows relatively quickly after the beginning of the simulation. This is different from direct MHD cluster simulations that have mostly kinematic growth with a magnetic spectrum peaked on a numerical dissipation scale, e.g., Xu et al. (2012). Note that the scale of the magnetic field plays crucial role in cosmic ray escape times, therefore correctly estimating the magnetic outer scale is essential for models of particle acceleration in clusters (see, Brunetti & Lazarian 2007, 2011a; Beresnyak et al. 2013; Miniati 2015).

7. DISCUSSION

A similar idea based on post-processing of hydrodynamic data was also employed in Ryu et al. (2008), but with substantial differences. The turbulence in these early calculations was not as resolved as in ours, and the growth of magnetic energy and the Alfvén scale were not estimated from the turbulent dissipation rate and the precise estimate of $C_E$, as we did here.

One of the differences between cluster turbulence and the kind of statistically stationary turbulence studied in Beresnyak (2012) is the strong variations of the cascade rate over timescales of 1–2 dynamical timescales of the cluster. Our estimates of the efficiency in the case of intermittent driving from this work are roughly compatible with $C_E = 0.05$ and further work with higher Re is expected to clarify whether the differences between constant and intermittent driving are significant. We concluded that the effects of intermittent driving could probably be ignored at the level of precision of the $\epsilon_{\text{turb}}$ measurement. Note that although the Section 4 simulations were insightful for understanding the basic physics behind a high-Re small-scale dynamo, we did not use them directly as a subgrid model for the Section 5 simulations; instead we used Equation (4), which is a closer approximation for the astrophysical cases with $Re > 10^{12}$, this difference being especially critical for the cluster case, which had only a few dynamical times to evolve (see Figure 1 and corresponding discussion).

The actual calculation for the evolution of $v_A$ and $L_B$ was started at a time of 4.5 Gyr. This artificially assumes that $\epsilon_{\text{turb}}$ was zero for all times earlier than 4.5 Gyr. However, we find that despite this fairly unrealistic assumption, the values of $v_A$ and $L_B$ quickly converge to the asymptotic values, and as we argued above this initial state is quickly forgotten. All basic properties of the cluster, such as its mass, size, and thermal energy continue to grow along with its magnetic energy and magnetic outer scale. A detailed comparison between thermal, turbulent, and magnetic energy components of the cluster has been performed in our companion paper (Miniati & Beresnyak 2015). There it is found that the fraction of the thermal energy arising from the turbulent dissipation rate changes relatively little over the cosmological time and the turbulent Mach number is also rather stable. Since the magnetic energy is also a fraction of the accumulated turbulent dissipation rate, the plasma $\beta$ in our cluster fluctuates around a constant value of $\sim 40$ for the past 10 Gyr (Miniati & Beresnyak 2015).

Our treatment of cluster turbulence with ILES, as well as modeling the evolution of the magnetic energy with the model from Beresnyak (2012), relies on an assumption that the Reynolds numbers in clusters are high. For example, our comparison of the cluster simulation and the DNS leads to an estimate of an effective Kolmogorov (dissipation) scale for the cluster simulation of $\eta \approx 2.7$ kpc, corresponding to an effective Re around 3000. We actually expect clusters to have higher Re, as briefly discussed in Section 1, due to the collective microscopic scattering in the high-$\beta$ ICM plasma (Lazarian & Beresnyak 2006; Schekochihin & Cowley 2006; Schekochihin et al. 2008; Brunetti & Lazarian 2011b). An important observational test to the problem of the ICM viscosity is the measurements of Faraday rotation in active galactic nucleus (AGN) sources located in clusters, which allowed us to probe sub-kiloparsec scales due to the relatively high resolution of radio maps (Laing et al. 2008; Govoni et al. 2010; Kuchar & Enßlin 2011). The inferred magnetic spectrum in these measurements is negative and steep, typically around Kolmogorov in the range of scales below 5 kpc and down to the resolution limit. Such a magnetic spectrum is expected from MHD turbulence with small dissipation scales. It would be grossly inconsistent with magnetic spectra obtained in either kinematic dynamo models, due to their positive spectral indexes, or with MHD models using Spitzer viscosity, which would typically give a rather shallow spectrum with an index around $-1$ (see, e.g., Cho et al. 2002). Complementary to this observational constraint are the theoretical estimates leading to the “mesoscale,” i.e., the scale above which one can apply ordinary MHD. We discussed these estimates in the introduction; however, some additional explanation may be in order. The above-mentioned estimates assume that plasma is already magnetized to some degree (not necessarily strongly, e.g., a $1\text{nG}$ field still gives $r_L \sim 10^{-6}$ pc $\ll \lambda$ and the instability mechanisms should still work). The most agnostic and practical approach to this bootstrap problem is saying that ICM could obtain this tiny initial field through a variety of mechanisms, such as plasma turbulence, primordial field compressed into ICM, or the AGN fields being mixed by turbulence. A more minimalistic approach is to rely on plasma turbulence alone in the various mechanisms of plasma dynamo. One estimate (Schekochihin & Cowley 2006) suggests that the plasma dynamo grows on timescales of $\sim 1$ year. Similar work on small-scale dynamos based on closures for pressure-anisotropic...
plasma was done in Kowal et al. (2011), Santos-Lima et al. (2014), and Falceta-Gonçalves & Kowal (2015), and also indicates fast growth. It should be understood, however, that once the magnetic field reaches values such that Alfvén speed is comparable to the turbulent velocity perturbation on the mesoscale, the more generic and simple mechanism of nonlinear local MHD dynamo described in Beresnyak (2012) will start taking place and the outer scale of the magnetic field will start growing from a mesoscale up to the approximate values we derive in this paper. For example, if the mesoscale is $10^{-4}$ pc, and the density is $10^{-3}$ cm$^{-3}$, such an initial field is around 3 nG. The particular mechanism of the bootstrap makes little difference as far as our calculations are concerned, because either the timescale of 1 year predicted by the above plasma dynamo theories or the estimate for the maximum kinematic growth timescale of 1 kyr we mentioned in the introduction all could be considered essentially zero, i.e., the bootstrap happens “instantly,” as far as cosmological time-scales are concerned. From either observations alone or theory alone, the effective viscosity seems to not be large enough to affect the magnetic spectrum above 1 kpc. Therefore, we expect the Re in clusters to be at least $10^4$ and likely much higher. Our calculations relied on this fact and the results, grossly consistent with the current observational properties of clusters, provide further support for the picture of a turbulent ICM, as opposed to the earlier view of a viscous and laminar ICM.

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