Matrix approach for modeling of emission from multilayer spin-polarized light-emitting diodes and lasers

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Abstract
Spin-polarized light sources such as the spin-polarized light-emitting diodes (spin-LEDs) and spin-polarized lasers (spin-lasers) are prospective devices in which the radiative recombination of spin-polarized carriers results in emission of circularly polarized photons. The main goal of this article is to model emitted radiation and its polarization properties from spin-LED and spin-controlled vertical-cavity surface-emitting laser (spin-VCSEL) solid-state structures. A novel approach based on 4 × 4 transfer matrix formalism is derived for modeling of the interaction of light with matter in active media of resonant multilayer anisotropic structure and enables magneto-optical effects. Quantum transitions, which result in photon emission, are described using general Jones source vectors.

Keywords: spin-polarized lasers, spin-polarized light-emitting diodes, VCSEL, 4 × 4 matrix formalism, quantum selection rules, dipole radiation, stimulated emission

1. Introduction
Spin-polarized light sources such as the spin-polarized light-emitting diodes (spin-LEDs) and spin-polarized lasers (spin-lasers) are prospective devices in which the radiative recombination of spin-polarized carriers results in emission of circularly polarized photons. The control of optical polarization by spin polarization of the charge carriers can be applied in many practical areas. In experimental spintronics, spin-LEDs are already used for detection and characterization of spin-polarized carriers in a new generation of spintronic structures [1–8]. However spin-lasers promise lower threshold [9, 10], faster modulation dynamics [11], improved polarization stability, polarization determination [12, 13] as well as polarization switching [14] and reduced parasitic frequency modulation (chirp), which limits the high bit-rate in communication [15]. In this mind, the ability to control modulated circular light polarization together with improved properties opens new horizons in laser technology, cryptography and modern optical components.

For the aforementioned applications, a precise modeling of light emission from multilayer structure with active layer is strongly required. It could be based on two steps: (i) representation of active layer with dipole sources and (ii) modeling of light propagation in resonant multilayer structures by using an appropriate matrix approach fulfilling Maxwell equations in each layer. To this end, it is well established that spontaneous emission in a semiconductor can be represented as an electric dipole emission in the so-called weak-coupling regime. When atoms in the active area spontaneously emit electromagnetic radiation, each atom acts as a small randomly oscillating electric dipole resonant with the transition frequency. If the emission is stimulated, each atom acts as a resonant electric dipole, which is set oscillating by the
incident wave with the result that oscillations are now no longer random but coherent with the incident wave acting as an optical resonator [16]. In the case of the resonant-cavity LED (RCLED), the active layer is embedded in a multi-layered propagative media [17] also including possible ferromagnetic materials for electrical spin-pumping operation [14]. Optical interactions between the dipole field and the isotropic multilayer structure was efficiently described by Benisty [18] in the end of the nineties. Such approach uses transfer matrix formalism for description of light propagation in isotropic structure, while the vectorial problem of the dipole emission was decomposed in three scalar problems: (i) s- and p-field generated by a dipole parallel to the interface and (ii) a single p-field generated by a dipole perpendicular to the interfaces. This approach enables the calculation of both field amplitudes radiated from the structure as well as field in any point of the multilayer.

However, in spin-LEDs and spin-lasers, the optical polarization of the emitted photons is directly related to the spin polarization of the carriers through optical quantum selection rules [19]. Yet, the magnetic media used for electrical current polarization cause anisotropic magneto-optical effects and s-p mode conversion. Moreover, multiple quantum well structure used in lasers generally introduces an optical anisotropy caused by strain field as well as group point symmetry breaking, which is more realistically described using anisotropic permittivity tensor. Consequently, separate treatment of s- and p-polarized waves can not be applied for typical spin-LED and spin-laser structures.

The main goal of the present work is to model the polarization properties of the electromagnetic radiation emitted from spin-LED and spin-controlled vertical-cavity surface-emitting laser (spin-VCSEL) multilayer structures. We propose an approach, which (i) describes general polarization of emitted photons related to the quantum optical selection rules and consider spin polarization of injected current or generally polarized optical pumping field, (ii) describes the propagation of emitted field in general anisotropic multilayer system consisting of strained or magneto-optically active films, (iii) correctly defines phases of incoherent spontaneous emission and coherent stimulated emission, and (iv) describes the complete polarization state of emitted field from laser structure and conditions for laser resonance.

The article is organized as follows. In section 2, we first modify the 4 × 4 Yeh’s matrix formalism [20, 21] by including the active source layer described by general source amplitude vector and considering phases of stimulated and spontaneous emission. In section 3, we discuss the particular shape of the source amplitude distribution of the active layer. We introduce a method of general source vector emitting a certain degree of circularly polarized light with adaptable parameters possibly obtained by fitting experimental data or from quantum transition probability theory. Section 4 shows application of general approach to typical spin-VCSEL structures.

2. New approach for modeling of spin-LED and spin-VCSEL structures

The 4 × 4 Yeh’s matrix formalism provides an effective approach for investigation of the propagation of electromagnetic radiation in anisotropic layered media, where each layer may display anisotropic physical properties and is characterized by the complex relative permittivity tensor hereafter called \( \varepsilon^{(n)} \). In the case of anisotropic layered media, the electromagnetic field in an individual layer can be expressed as a linear superposition of monochromatic plane waves with four eigenmode polarizations obtained by non-trivial solutions of wave equation in each anisotropic layer. The boundary conditions requiring continuity of tangential components of the field vectors are expressed by using the 4 × 4 dynamical matrix \( \mathbf{D}^{(n)} \) as a relation between amplitudes of the four eigenmodes and the four tangential field components. The propagation of these partial waves in each layer is described by the diagonal matrix \( \mathbf{P}^{(n)} \) consisting of phase excursions [20, 21]. For more details, see appendix A.

Let us consider the multilayer system including the isotropic layer \( n \) with the emitting active region (see figure 1). Emitted field amplitudes in the respective halfspaces \((0)\) and \((N + 1)\) are related to the field amplitudes on both sides of the active region by well-known relationships

\[
\begin{pmatrix}
A_1^{(n)} \\
A_2^{(n)} \\
A_3^{(n)} \\
A_4^{(n)}
\end{pmatrix} = \begin{pmatrix}
M_{11}^{(d)} & M_{12}^{(d)} & M_{13}^{(d)} & M_{14}^{(d)} \\
M_{21}^{(d)} & M_{22}^{(d)} & M_{23}^{(d)} & M_{24}^{(d)} \\
M_{31}^{(d)} & M_{32}^{(d)} & M_{33}^{(d)} & M_{34}^{(d)} \\
M_{41}^{(d)} & M_{42}^{(d)} & M_{43}^{(d)} & M_{44}^{(d)}
\end{pmatrix}
\begin{pmatrix}
A_1^{(0)} \\
A_2^{(0)} \\
A_3^{(0)} \\
A_4^{(0)}
\end{pmatrix},
\]

and

\[
\begin{pmatrix}
A_1^{(n)} \\
A_2^{(n)} \\
A_3^{(n)} \\
A_4^{(n)}
\end{pmatrix} = \begin{pmatrix}
M_{11}^{(s)} & M_{12}^{(s)} & M_{13}^{(s)} & M_{14}^{(s)} \\
M_{21}^{(s)} & M_{22}^{(s)} & M_{23}^{(s)} & M_{24}^{(s)} \\
M_{31}^{(s)} & M_{32}^{(s)} & M_{33}^{(s)} & M_{34}^{(s)} \\
M_{41}^{(s)} & M_{42}^{(s)} & M_{43}^{(s)} & M_{44}^{(s)}
\end{pmatrix}
\begin{pmatrix}
A_1^{(N+1)} \\
A_2^{(N+1)} \\
A_3^{(N+1)} \\
A_4^{(N+1)}
\end{pmatrix},
\]

where the total matrices are calculated in the following way

\[
\mathbf{M}^{(n)} = \mathbf{P}^{(n)}^{-1} \mathbf{D}^{(n)} \mathbf{P}^{(n-1)} \mathbf{D}^{(n-2)} \ldots \mathbf{P}^{(1)}^{-1} \mathbf{D}^{(0)}
\]

and

\[
\mathbf{M}^{(s)} = \mathbf{P}^{(s)} \mathbf{D}^{(s)} \mathbf{P}^{(s-1)} \mathbf{D}^{(s-2)} \ldots \mathbf{P}^{(0)} \mathbf{D}^{(0)}
\]

The matrices \( \mathbf{P}^{(n)} \) and \( \mathbf{P}^{(s)} \) are respectively the propagation matrices of the \( n \)-th layer region above and below of the active plane. Let us write the complex amplitudes above and below active region in the form of the Jones vectors according to
where the amplitudes $A_{n1}$, $A_{n3}$, and $A_{n2}$, $A_{n4}$ correspond to the orthogonal eigen-polarizations propagating in the down (in positive $z$-axis direction) and in the up direction, respectively (see figure 2). The Jones vectors (3) are usually expressed in the basis of $s$- and $p$-polarized waves, however, any other basis, for instance, right and left circular polarizations, are possible. Easily defined basis eigenvectors is the reason for the requirement of isotropic layer close to the active region. In the case of anisotropic active layer we recommend treating fictional isotropic film in close proximity to the active dipole region.

In an active medium, field amplitudes traveling through the source plane are amplified by interactions with the active dipole region. First, we describe amplification of the waves propagating downwards using the Jones vector of the source

$$J_0 = \begin{pmatrix} A_1^{(n)} \\ A_2^{(n)} \end{pmatrix}$$

and similarly by using the source vector $J_0^T = [A_2^{(n)} A_1^{(n)}]^T$ for incident wave polarization $J_0'$ is different from the source polarization $J_0'$ only resulting in a partial amplification of the wave $J_0'$ by stimulated emission. Therefore, we had to express the incident Jones vector $J_0'$ as a weighted superposition of two orthogonal basis Jones vectors according to

$$J_0' = \alpha J_0 + \beta J_0', \quad \text{(4)}$$

where $J_0' = [A_1^{d'} A_3^{d'}]^T$ is the Jones vector of the source and $\beta J_0$ is its orthogonal counterpart. From orthogonality reasons, the Hermitian product vanishes

$$\begin{pmatrix} J_0' \end{pmatrix}^{*} = \begin{pmatrix} J_0 \end{pmatrix} = A_1^{d'} A_1^{s'} + A_3^{d'} A_3^{s'} = 0, \quad \text{(5)}$$

it follows that that the part of the incident wave, which is amplified by stimulated emission is weighted by

$$\alpha = \frac{\begin{pmatrix} J_0' \end{pmatrix}}{\begin{pmatrix} J_0 \end{pmatrix}}. \quad \text{(6)}$$

Considering the normalized source vectors $\begin{pmatrix} J_0' \end{pmatrix} = 1$, complex amplitudes traveling through the source plane are amplified in the following way:

$$J_0'' = J_0' + \gamma J_0', \quad \text{(7)}$$

where the second and third term correspond to stimulated and spontaneous emission, respectively. The factors $\gamma$ and $\delta$ represent here the weighted coefficients of spontaneous and stimulated emission, respectively. Thus, the field amplitudes on each side of the active region are linked by the relationship

$$J_0'' = T J_0' + \gamma J_0', \quad \text{(8)}$$

and similarly by using the source vector $J_0^T = [A_2^{d'} A_1^{d'}]^T$ for
upward propagating amplitudes

\[ \mathbf{J}'_t = \mathbf{T}' \mathbf{J}'_t + \gamma \mathbf{J}'_t \]

\[ \left( 1 + \delta A_{d}^d A_{d}^{d^t} \right) \frac{\partial A_{d}^d A_{d}^{d^t}}{\partial A_{d}^d A_{d}^{d^t}} \mathbf{J}'_t + \gamma \mathbf{J}'_t. \quad (9) \]

From equations (1), (2), (8) and (9), we then obtain a system of four algebraic equations corresponding to the four emitted field amplitudes expressed in the following compact matrix form

\[
\begin{pmatrix}
T_{11} M_{12} & T_{12} M_{24} & -M_{13}^d & -M_{33}^d \\
+T_{13} M_{24} & +T_{14} M_{34} & T_{11}^d M_{21} & T_{12}^d M_{23} \\
-M_{12}^d & -M_{23}^d & T_{11}^d M_{21} & T_{12}^d M_{23} \\
+T_{12} M_{24} & +T_{13} M_{34} & M_{13}^d & M_{33}^d \\
-M_{12}^d & -M_{23}^d & M_{13}^d & M_{33}^d
\end{pmatrix}
\times
\begin{pmatrix}
A_1^{(0)} \\
A_2^{(0)} \\
A_1^{(N+1)} \\
A_3^{(N+1)}
\end{pmatrix}
=
-\gamma
\begin{pmatrix}
A_1^d \\
A_2^d \\
A_1^{d^t} \\
A_3^{d^t}
\end{pmatrix}
\]

(10)

which can be solved by using standard matrix algebra.

In the case of an isotropic structure and for purely spontaneous emission (\( \delta = 0 \)), this approach corresponds to the approach described in [18]. However, note that the spontaneous emission is not a coherent process and thus the terms \( \gamma \mathbf{J}'_t \) and \( \gamma \mathbf{J}'^t_t \) have to be added incoherently to the incident wave. In the present method, we suppress possible interference effects originating from coherence spurious superposition. To that goal, we consider breaking phase factors \( e^{i\varphi} \), \( e^{i\varphi} \), \( e^{i\varphi} \), and \( e^{i\varphi} \) of the source vector on the right side of equation (10)

\[
\begin{pmatrix}
A_1^d \\
A_2^d \\
A_1^{d^t} \\
A_3^{d^t}
\end{pmatrix}
= e^{i\varphi} \begin{pmatrix}
A_1^d \\
A_2^d \\
A_1^{d^t} \\
A_3^{d^t}
\end{pmatrix}
= e^{i\varphi} \begin{pmatrix}
A_1^d \\
A_2^d \\
A_1^{d^t} \\
A_3^{d^t}
\end{pmatrix}
\equiv
\begin{pmatrix}
A_1^d \\
A_2^d \\
A_1^{d^t} \\
A_3^{d^t}
\end{pmatrix}
\]

(11)

Stokes vector components in the following way

\[
\begin{pmatrix}
S_0 \\
S_1 \\
S_2 \\
S_3
\end{pmatrix}
=
\begin{pmatrix}
I_1 + I_4 \\
I_1 - I_4 \\
I_{450} - I_{-450} \\
I_1 - I_4
\end{pmatrix}
= \begin{pmatrix}
A_1^{(0)} A_1^{(0)*} + A_2^{(0)} A_2^{(0)*} \\
A_1^{(0)} A_2^{(0)*} - A_2^{(0)} A_1^{(0)*} \\
A_1^{(0)} A_2^{(0)*} + A_2^{(0)} A_1^{(0)*} \\
i (A_1^{(0)} A_1^{(0)*} - A_2^{(0)} A_2^{(0)*})
\end{pmatrix}
\]

where \( I_1, I_4, I_{450}, \) and \( I_{-450} \) are the intensities of the emitted waves polarized along \( x, y, +45^\circ, \) and \( -45^\circ, \) respectively, and \( I_1 \) and \( I_4 \) denote intensities of right and left circularly polarized light [22–24]. Then we can define the degree of emitted polarization in the form

\[
\zeta = \frac{\sqrt{S_0^2 + S_1^2 + S_2^2}}{S_0}.
\]

(13)

By averaging outside Stokes vector components over \( \Delta \phi \),

\[
S_{\text{out}}^{\pm} = \left\langle S_{\text{out}}^{\pm} \left( \Delta \phi_{\pm} \right) \right\rangle_{\Delta \phi_{\pm}},
\]

(14)

we succeed to obtain outside Stokes vector of the structure with polarized source including the inherent incoherence property of spontaneously emitted waves.

Note, that resonance condition for laser structure is ensured if the determinant of the 4 x 4 matrix on the left side of equation (10) is equal to zero.

3. Distribution of the source amplitude

The probability of the photon emission is related to the exact mechanism of the radiative recombination process resulting, in most cases, to a relevant dependence on the angle \( \theta \). It is well established that emission in a semiconductor can be described by an electric dipole emission in the so-called weak coupling regime. The electric dipole is a vectorial point source, which in bulk material may possess any orientation. However, in quantum well structures, due to quantization effects, the emission by horizontal dipoles is enhanced [25]. For more detail, see appendix B. In the following section, we will discuss the optical quantum selection rules both for bulk and quantum well (QW) semiconductor structures. From angular momentum conservation, circularlypolarized photons are emitted due to spin-polarized carriers recombination. To model these transitions, we introduce system of two crossed linear sources (dipoles) with a characteristic phase shift \( \pi/2 \). This approach enables us to find source terms \( \mathbf{J}'_t = [A_1^d A_2^d]^T \) within the active region and to amplify incident amplitudes as described by equation (7).

3.1. Optical selection rules: normal emission

In this section, we give a simple description of the optical selection rules in the dipolar amiltonian approximation, well documented in the literature [19], in order to illustrate in the end the power of our technical method to model the properties
of light emission from quantum heterostructures. The optical selection rules are found by evaluating the dipole moment of the transition between the conduction band state \(|\psi_c\rangle\) and the valence band state \(|\psi_v\rangle\) at the center (\(\Gamma\)-point) of the Brillouin zone (BZ) using the transition matrix element

\[
D_{sv} = \langle \psi_v | \hat{D} | \psi_c \rangle.
\]  

(15)

where \(\hat{D}\) stands for the dipole moment operator at first order of perturbation. The electronic Bloch states may be expressed as \(|J, m\rangle\), where \(J\) denotes the total angular momentum and its projection onto the \(z\) axis described by the magnetic quantum number \(m\). The conduction band is represented by two different electron quantum states \(|1/2, \pm 1/2\rangle\), while the valence band is represented by two heavy hole states \(|1/2, \pm 3/2\rangle\) and two light hole states \(|1/2, \pm 1/2\rangle\) in the center of the zone at the \(\Gamma^+\) valley. The quantization axis for the angular momentum is fixed along the photon wave vector \(k\) and the crystal axis of the cubic \(A^{\text{III}}B^{\text{V}}\) crystal (001) in the \(z\) direction. An electron state of the conduction band can be described using the Bloch wave function

\[
\psi_{sv} = u_s e^{ikr}.
\]

(16)

where the Bloch amplitudes have the following form

\[
u_{1/2}^S = |S\uparrow\rangle, \quad \nu_{-1/2}^S = |S\downarrow\rangle.
\]

(17)

The \(|S\rangle\) denotes the \(s\)-type wavefunction and arrows denote spin functions. The Bloch amplitudes of the valence band can be described using the \(p\)-type wave functions \(|X\rangle\), \(|Y\rangle\) and \(|Z\rangle\) with symmetry in \(x, y, \) and \(z\) axes, respectively

\[
u_{1/2}^{HH} = -\frac{1}{\sqrt{2}}(|X\uparrow\rangle + i|Y\uparrow\rangle) \quad \nu_{-1/2}^{HH} = \frac{1}{\sqrt{2}}(|X\downarrow\rangle - i|Y\downarrow\rangle) \quad \nu_{1/2}^{LH} = \frac{1}{\sqrt{3}}\left[-\frac{1}{\sqrt{2}}(|X\downarrow\rangle + i|Y\downarrow\rangle) + \sqrt{2}|Z\uparrow\rangle\right] \quad \nu_{-1/2}^{LH} = \frac{1}{\sqrt{3}}\left[\frac{1}{\sqrt{2}}(|X\uparrow\rangle - i|Y\uparrow\rangle) + \sqrt{2}|Z\downarrow\rangle\right].
\]

(18-21)

The characteristic matrix elements given by the dipolar Hamiltonian coupling both the conduction and valence bands are in table 1 [19].

We remind the main conclusions concerning dipole transitions. As required from the conservation of the angular momentum, radiative recombinations lead to an emission of right- \((\sigma^+\rangle\) and left-circularly polarized photons \((\sigma^-\rangle\) with a projection of angular momentum along the direction of their \(k\) wave-vector equal to \(\pm 1\), respectively. Moreover, from intensity of dipoles follows, that the heavy hole (HH) transitions are three times larger than corresponding light hole (LH) transitions. Note, that the transition probability is proportional to \(|D_{sv}|^2\). Let us define normalized transition probabilities according to

\[
\xi_{HH} = 3/4 \quad \xi_{LH} = 1/4 \quad \xi_{HH} + \xi_{LH} = 1.
\]

(22)

| \(\) | \(\{1/2, + 1/2\}\) | \(\{1/2, + 1/2\}\) |
|---|---|---|
| \(cb\) | \(-\sqrt{3}/2 (\hat{x} + i\hat{y})\) | 0 |
| \(hh\) | \(-\sqrt{3}/2 (\hat{x} + i\hat{y})\) | \(\sqrt{1/2} (\hat{x} - i\hat{y})\) |
| \(lh\) | \(\sqrt{2/3} \hat{z}\) | \(-\sqrt{3}/2 (\hat{x} + i\hat{y})\) |

| \(\) | \(\{1/2, - 1/2\}\) | \(\{1/2, - 1/2\}\) |

(15)

In bulk semiconductor, the HH and LH bands are degenerate at the \(\Gamma\)-point and the present rules are valid for all direction of the emission. In the case of QW structures, this bulk-degeneracy between the HH and LH valence bands is lifted due to quantum confinement or by biaxial epitaxial strain. Moreover, these selection rules are valid only in the vertical (Faraday) geometry where the carrier spin orientation and the photon emission are oriented perpendicularly to the QW plane (see figure 3) [19, 26]. In the following, we will generalize the optical selection rules for any emission direction.

3.2. Method of two-crossed linear sources

We will first consider that the emission probability of the circularly polarized photons does not depend on the emission angle. Hence, for right- and left-circularly polarized waves propagating in the +z (\(\uparrow\)) and -z (\(\downarrow\)) direction, respectively, we obtain source Jones vectors of the form

\[
J_{L,R}^d = J_{L,R}^i = \left(\begin{array}{c} 1 \\ i \end{array}\right)
\]

(23)

\[
J_{L,L}^d = J_{L,L}^i = \left(\begin{array}{c} -1 \\ i \end{array}\right).
\]

(24)

In other words, circularly polarized part of incident waves, which travel through the active plane, are amplified (see equation (7)). Imaginary unit \(i\) is representative of a \(\pi/2\) phase shifting between two orthogonal (s- and \(p\)-polarized) linear waves.

3.3. More general approach: oblique direction of emission

In real cases, both photon emission probability and photon polarization depend on the incident angle \(\theta^\omega\) and related to the mechanism of radiative recombination and to the real optical parameters. Thus, a more general parametrized function describing the dependence on the emission angle \(\theta^\omega\) is generally required. In the case of QW structure, where most energy is emitted along the direction normal to the growth plane, we introduce the parametrization of the source Jones
by reciprocal argument. If one considers the densities of polarized photons with negative (σ−) and positive (σ+) helicity, respectively. Moreover, transitions involving heavy holes (HH) are three times more probable than those involving light holes (LH). When we consider 2D quantum system (b), the energetic splitting between HH and LH states appears as a consequence of the quantum confinement and epitaxial strain. In this case, the depicted selection rules are valid only for vertical geometry [19, 26].

Figure 3. (a) Selection rules in direct bulk semiconductor. Transitions for which Δm_j = ±1 result in the emission of circularly polarized photons with negative (σ−) and positive (σ+) helicity, respectively. Moreover, transitions involving heavy holes (HH) are three times more probable than those involving light holes (LH). When we consider 2D quantum system (b), the energetic splitting between HH and LH states appears as a consequence of the quantum confinement and epitaxial strain. In this case, the depicted selection rules are valid only for vertical geometry [19, 26].

vectors (23) and (24) according to

\[ J^{\pm}_{LL} = \begin{pmatrix} 1 \pm i & 0 \\ 0 & 1 \end{pmatrix} \]

\[ J^{\pm}_{LR} = \begin{pmatrix} 0 & 1 \pm i \\ 1 \pm i & 0 \end{pmatrix} \]

and

\[ J^{\pm}_{RR} = \begin{pmatrix} 1 \pm i & 0 \\ 0 & 1 \end{pmatrix} \]

\[ J^{\pm}_{RL} = \begin{pmatrix} 0 & 1 \pm i \\ 1 \pm i & 0 \end{pmatrix} \]

where \( A, B \) and \( l \) are non-negative parameters possibly determined by theory or obtained from fit of experimental data. Note, that from symmetry reason is sufficient to let \( \theta^{(n)} \in [-\pi/2, \pi/2] \).

We will now combine this approach together with the quantum selection rules discussed beforehand and depicted in figure 3. The source vectors \( J^{\pm}_{LR} \) and \( J^{\pm}_{RL} \) (26) correspond to transition \( \Delta m_j = 1 \) resulting in an emission of circularly-polarized photons of negative helicity (σ−) when propagating in the +z direction (↑) and positive-helicity (σ+) in the −z direction (↓) by reciprocal argument. If one considers the symmetry of the source

\[ A^1 + B^1 \cos \theta^{(n)} = A^1 + B^1 \cos \theta^{(n)} = A + B \cos \theta^{(n)} \]

and

\[ A^1 + B^1 \cos \theta^{(n)} = A^1 + B^1 \cos \theta^{(n)} = A^1 + B^1 \cos \theta^{(n)} \]

\[ A^1 + B^1 \cos \theta^{(n)} = A^1 + B^1 \cos \theta^{(n)} = A + B \cos \theta^{(n)} \]
combination of the dipole sources

\[ J_{1i}^0 = \frac{n_e}{n_e + n_l} \left[ \sqrt{\xi_{HH}} \left( -i \cos \theta^{(0)} \right) - \frac{1}{\sqrt{\xi_{HH}}} \right] + e^{i\phi} \sqrt{\xi_{LL}} \left( i \cos \theta^{(0)} \right) \]

\[ + \frac{n_e}{n_e + n_l} \left[ \sqrt{\xi_{HH}} \left( -i \cos \theta^{(0)} \right) \right] + e^{i\phi} \sqrt{\xi_{LL}} \left( i \cos \theta^{(0)} \right) \]

\[ \equiv \frac{1}{n_e + n_l} \left( - i \cos \theta^{(0)} \right) \frac{n_e + e^{i\phi} n_l}{n_e + n_l} \]

where \( \xi_{HH} \) and \( \xi_{HH} \) are normalized transition probabilities defined by equation (22). The effective polarization degree of the emitted photons from the dipole layer \( (n_e - n_l)/(n_e + n_l) \) is given by a product of the degree of transition probabilities \( (\xi_{HH} - \xi_{HH})/(\xi_{HH} + \xi_{HH}) \) and the spin-polarization of carriers

\[ P = \frac{n_e - n_l}{n_e + n_l} \]

We consider the outside angle \( \theta^{(0)} \in < 0, \pi/2 > \) for \( J_{1i}^0 \). By averaging final Stokes intensities over the random phase \( \theta^{(0)} \), we describe the independence of stochastic HH and LH transitions. If we consider anisotropic non-stochastic emission by specific broken symmetry, particular \( \phi \) can be chosen for describing preferential emitted polarization e.g. in the case of strain-induced broken symmetry of interfaces with GaAs quantum wells [27].

Recall from section 2 that the resonance condition for laser structure is ensured if the determinant of the 4 × 4 matrix on the left side of equation (10) is equal to zero. This resonance point (laser threshold) depends on the amplitudes of the general dipolar Jones vectors (25) and (26), which are functions of the densities of injected electrons in the 1/2 electron states \( n_e \). Therefore, without considering anisotropic effects, RCP and LCP lasing mode have different resonance point (threshold currents \( J_{RCP} \) and \( J_{LCP} \)) for given injected polarization \( P \). Thus, in the point between \( J_{RCP} \) and \( J_{LCP} \), we obtain 100% lasing circular polarization even for partially polarized current. This effect is analogous to spin-filtering in magnetic materials and is rigorously described in [28] without considering optical anisotropic effects. Specific dynamical effects, such as decreasing of spin relaxation time in (110) GaAs quantum well [29] and saturation phenomena, could be included into our model due to general definition of source vectors (25) and (26).

4. Model of realistic half spin-VCSEL

Spin-VCSELS consist of the active medium and the spin injector embedded by upper and lower distributed Bragg mirrors (DBM) and our modeling approach can be used for calculation of emitted polarization from these structures. From a technological point of view, the room-temperature electrically injected spin-VCSELS are beyond state of the art. First electrically injected spin-VCSEL with MQWs as the gain media was demonstrated by Holub at 50 K [30]. In the following we will demonstrate our approach on half spin-VCSEL structure, which consisted of only one DBM. Such structures can benefit from the external cavity degree of freedom ensured by an external concave mirror, which is placed outside to stimulate Fabry–Perot resonance. Similar structure was studied by Frougier [14].

For the following model, the particular form of the Jones source vector (34) will be used. Figure 4 schematically displays the model of the multilayered structure including, from top to bottom, the metallic capping layer (Au), the magnetic and magneto-optically active thin spin-injector (Co) described in a polar geometry \( e_{y}^{(0)} = e_{x}^{(0)} = -1.14 + 0.19i \) [31], the oxide tunnel barrier (MgO) dedicated to electrical spin-injection issue [32–34], the active layer GaAs with the active medium QW (plane) in the center and a Bragg reflector at the bottom. The energy of the emitted light was fixed to \( E = 1.6eV \) (typical 8 nm GaAs QW), corresponding to a wavelength of \( \lambda = 775nm \). Optical constants are

\[ n_{Au} = 0.17 - 4.86i \] [35], \( n_{Co} = 2.44 - 4.71i \) [36], \( n_{MgO} = 1.73 \) [37], \( n_{GaAs} = 3.69 - 0.01i \), \( n_{AlAs} = 3.02 \), and \( n_{BeGaAs} = 3.76 - 0.28i \) [37]. Note, that all plots describe emission to the halfspace (0) in direction ↑ with the parameters \( \gamma = 1 \) and \( \delta = 1 \).

Figure 5 demonstrates the changes of the Stokes vector components during varying of the injected carrier spin-polarization defined by equation (35). We observe two effects on outgoing emitted light polarization: (i) effect of the injected spin polarization described by the spin-polarization parameter \( P \) and (ii) polarization effects originating by transmission and reflection from the magneto-optical Co layer. The effect of the injected spin polarization is related to the transition probabilities for heavy and light holes (see figure 3). For the spin polarization \( P = 1 \) (subplot a), both heavy hole transition (resulting in RCP in -z direction) and light hole transition (resulting in LCP) polarization are involved, while HH transitions are three times more probable than LH transitions and thus polarization degree of emitted light \( \mathcal{P} = \langle S_{x}/S_{y} \rangle \) should be equal to 1/2. As can be seen on figures 5(a) and 6, the polarization degree is \( \mathcal{P} > 1/2 \).

Moreover the total intensity \( S_{y} \) changes slightly. These changes are caused by the selective polarization transmission through the structure related to Faraday magneto-optical effect in the ferromagnetic Co layer. Circularly polarized eigenmodes in Co layer with perpendicularly oriented magnetic field are absorbed differently and have different velocity.
due to the magnetic circular dichroism and birefringence, respectively. After crossing the Co layer, the input wave transforms in such a way that its azimuth is rotated and its polarization becomes generally elliptical. Thus even if unpolarized carriers \( P = 0 \) are injected (subplot c and d) and the source is linearly polarized, we can observe non-zero circular \( S_j \) component, which has different sign for opposite magnetic field orientation.

Reduced degree of polarization shows a depolarization effect, which is caused by incoherent summation of spontaneously emitted waves propagating in the direction up and down (see figure 6). The component \( S_j \) switches sign due to different transition probabilities for different current spin-polarization \( P \) (subplot e and f). Experimental measurement of the Stokes vector can thus bring valuable information about injected-spin-polarized current.

Figure 7 displays the evolution of the Stokes vector components during varying of the injected spin polarization in the case of the heavy hole transitions \((n=1, \xi=0)\). As expected for the injected current \( P = 1 \), 100% circularly-polarized light is emitted. Figure 7(d) demonstrates changes of the polarization degree during varying of the injected spin polarization. As can be seen on figure 8, the thickness of the GaAs film has an impact on outside field pattern due to the interference effects. One should note that if we set \( \gamma = 0 \), we obtain zero emitted intensity. In other words, the spontaneous emission is required to initialize the stimulated emission.

### 5. Conclusions

The matrix approach presented in this article offers a powerful method for modeling of spin-polarized light-emitting devices, which consists of source layer and generally anisotropic multilayer structure. We used Jones source vectors to describe coherence of stimulated and incoherence of spontaneous transitions with circularly polarized photons and included the impact of transitions probability and spin polarization of injected current. We demonstrated our approach on the model with magneto-optical layer used for spin injection. For modeling spin-devices with multiple quantum well structures (multiple source layers), we will introduce so-called scattering \( S \)-matrix formalism in the following developments.

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### Appendix A. The \( 4 \times 4 \) Yeh’s matrix formalism

Let us consider the multilayer system consisting of \( N \)-anisotropic layers embedded in isotropic halfspaces \((0)\) and \((N + 1)\) with the parallel interfaces normal to the \( z \) axis (see figure 1). Each layer \( n \) is characterized by the permittivity tensor \( \varepsilon^{(n)} \) and the thickness \( d^{(n)} \). Then, the wave equation for each layer takes the form [20, 21]

\[
\begin{align*}
\hat{\mathbf{k}}^{(n)} \varepsilon^{(n)} \mathbf{E}_{0}^{(n)} - k_{0}^{(n)} \mathbf{E}_{0}^{(n)} + k_{0}^{(n)} \left[ \hat{\mathbf{k}}^{(n)} \mathbf{E}_{0}^{(n)} \right] &= 0, \quad (A.1)
\end{align*}
\]

where \( \mathbf{E}_{0}^{(n)} \) is the amplitude of the electric field in each layer \( \mathbf{E}_{0}^{(n)} = E_{0 x}^{(n)} e^{i(w-k_{0}^{(n)}x)} \) and the wave vector \( \mathbf{k}^{(n)} = k_{0}^{(n)} \left( N_{x} \hat{\mathbf{x}} + N_{y} \hat{\mathbf{y}} + N_{z} \hat{\mathbf{z}} \right) \) obey the Snell’s law— invariance of the components \( N_{x} \) and \( N_{z} \) in each layer. Let us have \( N_{x} = 0 \) and \( N_{z} = N_{z}^{(n)} \sin \theta^{(n)} = \text{const.} \), which could be fulfilled by choosing appropriate coordinate system. Now, the wave equation (A.1) can be rewritten in the matrix form

\[
\begin{pmatrix}
\varepsilon_{xx}^{(n)} & \varepsilon_{xy}^{(n)} & \varepsilon_{xz}^{(n)} \\
\varepsilon_{yx}^{(n)} & \varepsilon_{yy}^{(n)} & \varepsilon_{yz}^{(n)} \\
\varepsilon_{zx}^{(n)} & \varepsilon_{zy}^{(n)} & \varepsilon_{zz}^{(n)}
\end{pmatrix}
\begin{pmatrix}
E_{0 x}^{(n)} \\
E_{0 y}^{(n)} \\
E_{0 z}^{(n)}
\end{pmatrix}
= 0,
\]

(A.2)

where the condition for existence of a nontrivial solution is ensured by

\[
\det(\bullet) = 0,
\]

which is generally a quartic equation. From (A.3) we obtain four solutions \( j = 1,3 \) and \( j = 2,4 \) corresponding to the forward and backward propagating modes, respectively, and the corresponding proper polarizations \( \mathbf{E}_{0}^{(n)} \), which can be rewritten in the form

\[
\mathbf{E}_{0}^{(n)} = A_{j}^{(n)} E_{0 j}^{(n)},
\]

(A.4)

where \( A_{j}^{(n)} \) is the amplitude of particular wave and \( E_{0 j}^{(n)} \) is the normalized eigen-polarization. Inside \( nth \) layer at the interface \( n(n + 1) \), we express the field vector \( \mathbf{E}^{(n)} \) as a linear
The propagation in the $n$th layer change the field vector according to a factor $\exp \left[ i k_N j \delta \right]$.

The boundary conditions require continuity of tangential components of the field vectors $E$ and $H$ at the interfaces.

**Figure 5.** The effect of the electron spin polarization. Subplots show emission pattern for varying injected spin polarization $P = 1$ (a), $P = 0.5$ (b), and $P = 0$ (c) with the magnetization of Co layer oriented in the $+z$ direction. Subplots d), e), and f) show emission pattern for varying injected spin polarization $P = 0$, $P = -0.5$ and $P = -1$, respectively, with the magnetization of Co layer oriented in the $-z$ direction.

Combination of these eigen-polarizations

\[
E^{(n)}_0 = \sum_{j=1}^{4} A_{ij}^{(n)} e_{ij}^{(n)}. \tag{A.5}
\]

The propagation in the $n$th layer change the field vector

\[
E^{(n+1)}_0 = \sum_{j=1}^{4} A_{ij}^{(n)} e_{ij}^{(n)} \exp \left[ i k_N j \delta \right]. \tag{A.6}
\]
Therefore

\[
\begin{align*}
(\alpha-1)E^{(\alpha)}_{00} &= (\alpha-1)E^{(\alpha)}_{01}, \\
(\alpha-1)H^{(\alpha)}_{10} &= (\alpha-1)H^{(\alpha)}_{01}, \\
(\alpha-1)E^{(\alpha)}_{30} &= (\alpha-1)E^{(\alpha)}_{31}, \\
(\alpha-1)H^{(\alpha)}_{30} &= (\alpha-1)H^{(\alpha)}_{31}.
\end{align*}
\]

These requirements can be rewritten in the compact matrix form

\[
D^{(\alpha)} A^{(\alpha)} = D^{(\alpha)} P^{(\alpha)} A^{(\alpha)},
\]

where \(D^{(\alpha)}\) (Dynamic matrix) and \(P^{(\alpha)}\) (Propagation matrix).

Figure 6. Degree of polarization of the emitted light for both heavy and light hole transitions.

Figure 7. The effect of the electron spin polarization in the case of the heavy hole transitions. Subplots show emission pattern for varying injected spin polarization \(P = 1, P = 0.5, \) and \(P = 0\) with the magnetization of Co layer oriented in the +z direction. Subplot d shows degree of polarization of the emitted light.
have the form
\[
D^{(a)} = \begin{bmatrix}
    e^{(a)}_{x1} & e^{(a)}_{x2} & e^{(a)}_{x3} & e^{(a)}_{x4} \\
    h^{(a)}_{x1} & h^{(a)}_{x2} & h^{(a)}_{x3} & h^{(a)}_{x4} \\
    e^{(a)}_{y1} & e^{(a)}_{y2} & e^{(a)}_{y3} & e^{(a)}_{y4} \\
    h^{(a)}_{y1} & h^{(a)}_{y2} & h^{(a)}_{y3} & h^{(a)}_{y4}
\end{bmatrix},
\]
\[
A^{(a)} = \begin{bmatrix}
    A^{(a)}_{1} \\
    A^{(a)}_{2} \\
    A^{(a)}_{3} \\
    A^{(a)}_{4}
\end{bmatrix}
\]
\[
P^{(a)} = \begin{bmatrix}
    ik_d N^{(a)}_{1} d^{(a)} & 0 & 0 & 0 \\
    0 & ik_d N^{(a)}_{2} d^{(a)} & 0 & 0 \\
    0 & 0 & ik_d N^{(a)}_{3} d^{(a)} & 0 \\
    0 & 0 & 0 & ik_d N^{(a)}_{4} d^{(a)}
\end{bmatrix}
\]

\[
M = \begin{bmatrix}
    D^{(0)} & D^{(1)} P^{(1)} & \ldots & D^{(N)} P^{(N)} & A^{(N+1)}
\end{bmatrix}^{-1}
\]

where \( M \) is so-called the total matrix of the system.

Note, that in the case of isotropic layered media, the electromagnetic field can be divided into two uncoupled modes: \( s \)-modes and \( p \)-modes with electric field vector perpendicular and parallel to the plane of incidence, respectively. Since they are uncoupled, characteristic equation for \( N^{(a)}_c \)
biquadratic with the solution
\[
N^{(a)}_{c,1,2,3} = n^{(a)} \cos \theta^{(a)},
\]
and the dynamic matrix takes the block diagonal form
\[
D^{(a)} = \begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & -n^{(a)} \cos \theta^{(a)} & 0 & 0 \\
    0 & 0 & \cos \theta^{(a)} & \cos \theta^{(a)} \\
    0 & 0 & -n^{(a)} & n^{(a)}
\end{bmatrix}
\]

**Appendix B. Dipole radiation**

It is well-known that spontaneous emission in a semiconductor active layer can be represented as the electric dipole emission. This representation is valid in the so-called weak-coupling regime. If the probability of re-absorption of a photon by the dipole is larger than its probability of escaping the resonant structure, the weak-coupling regime breaks and the so-called strong coupling takes place [38], which is not the case considered here.

Electric dipole is a vectorial point source, which in bulk material can have any orientation. However, in quantum well structures there is a preference for emission through horizontal dipoles [25]. Note, that the vector of the electric field emitted by a dipole lies in the plane defined by vector \( k \) and vector of the dipole moment \( \vec{\mu} \) [39]. The maximal field amplitude is emitted in the plane normal to the dipole moment \( \vec{\mu} \) and drops sinusoidally to 0 for the direction of the dipole moment. Thus, in the case of the horizontal dipoles (plane \( xy \) parallel to the interfaces), we could decompose the total emitted field into two scalar cases: \( s \)-polarized and \( p \)-polarized fields generated by horizontal dipoles [18]

\[
A^h = A \sin \varphi
\]
$A^h = A \cos \theta \cos \varphi = A \frac{k^{(s)}}{nk_0} \cos \varphi,$  \hspace{1cm} \text{(B.2)}$

where $\varphi$ and $\theta$ are the azimuth and the elevation, respectively. If we consider randomly oriented dipoles, we could use an average of the amplitude over $\varphi$ \[40\]

$$\langle P^h \rangle = A^2 \left[ \frac{k^{(s)}}{nk_0} \right]^2 < \cos^2 \varphi > = A^2 \left[ \frac{k^{(s)}}{nk_0} \right]^2 \left[ \frac{1}{2} \sqrt{\frac{2}{\pi}} \right] = A^2 \left[ \frac{k^{(s)}}{nk_0} \right]^2 \left[ \frac{1}{2} \sqrt{\frac{2}{\pi}} \right]$.$

and

$$\langle P^p \rangle = A^2 \left[ \frac{k^{(s)}}{nk_0} \right]^2 \left[ \frac{1}{2} \sqrt{\frac{2}{\pi}} \right] \int_0^{2\pi} \sin \varphi \, d\varphi = A^2 \left[ \frac{k^{(s)}}{nk_0} \right]^2 \left[ \frac{1}{2} \sqrt{\frac{2}{\pi}} \right].$$

Thus, we obtain

$$A^h = \frac{A}{\sqrt{2}},$$  \hspace{1cm} \text{(B.3)}$

$$A^p = \frac{A}{\sqrt{2}} \cos \varphi^{(s)} = \frac{A}{\sqrt{2}} \frac{k^{(s)}}{nk_0}.$$  \hspace{1cm} \text{(B.4)}$

We can express total radiated power through $4\pi \, sr$

$$P_{s,\text{total}} = \int_a \frac{A^2}{2} \cos \varphi^{(s)} \, d\varphi = \frac{A^2}{2} \sqrt{\frac{2}{\pi}} = \frac{2}{3} \sqrt{\frac{2}{\pi}},$$

$$P_{p,\text{total}} = \int_a \frac{A^2}{2} \cos^2 \varphi \, d\varphi = \frac{2}{3} \pi A^2.$$

To obtain normalized field amplitudes, let us consider the total radiated power through $4\pi \, sr$ equal to 1

$$P_{s,\text{total}} + P_{p,\text{total}} = 2\pi A^2 + \frac{2}{3} \pi A^2 = \frac{3}{3} \pi A^2 \Rightarrow A = \pm \sqrt{\frac{3}{8 \pi}}.$$

Thus, final normalized field amplitudes are in the form \[18\]

$$A^h = \pm \frac{3}{\sqrt{16\pi}},$$  \hspace{1cm} \text{(B.5)}$

$$A^p = \pm \frac{3}{\sqrt{16\pi}} \cos \theta = \pm \frac{3}{\sqrt{16\pi}} \frac{k^{(s)}}{nk_0}.$$  \hspace{1cm} \text{(B.6)}$

This approach enables calculation of the outside field amplitude of the classical RCLED anisotropic structures, where the active layer emits linearly polarized photons.

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