Comment On “On observation of neutron quantum states in the Earth’s gravitational field” [1]

V.K.Ignatovich

Criticism in [1] of experiments [2] is not valid. It is based on misunderstanding of difference between classical and quantal behavior of particles in classical gravitational potential. I decided to write this comment because my name is mentioned in Acknowledgement of [1], so the impression can appear that I approve the interpretation of gravity experiments given in [1]. Below I present my vision of ideas of the experiments made in [2], and make some critical remarks on the content of the paper [1]. The more detailed theory can be found in the papers [3, 4] that are referenced in [1].

A neutron motion along a horizontal infinitely thick mirror ($z < 0$) with the ideal plain interface at $z = 0$ and with account of the Earth gravity field is described by the stationary Schrödinger equation

$$\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - u\Theta(z < 0) - mgz - E \right) \Psi(r) = 0,$$  \hspace{1cm} (1)

where $E$ is the total energy of the neutron, $m$ is its mass, $g$ is the free fall gravity acceleration, $u = 4\pi N_0 b$ describes neutron matter interaction with the horizontal mirror, $b$ is the coherent amplitude of the neutron scattering from the matter atoms, $N_0$ is atomic density, and $\Theta(z < 0)$ is a step function equal to unity for $z < 0$ and to zero otherwise. After division of this equation by $\hbar^2/2m$ we reduce it to the form

$$\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - u\Theta(z < 0) - az - k^2 \right] \Psi(r) = 0,$$  \hspace{1cm} (2)

where $a = 2m^2 g/\hbar^2$ and $k^2 = 2mE/\hbar^2$.

The variables $x, y, z$ can be separated. Therefore solution can be represented as

$$\Psi(r) = \psi_n(z) \exp(i k_n r_\parallel),$$  \hspace{1cm} (3)

where $r_\parallel = (x, y, 0)$ is two-dimensional radius vector parallel to the mirror surface and $\psi_n(z)$ is an eigen solution of the one dimensional equation

$$\left[ \frac{d^2}{dz^2} - u\Theta(z < 0) - az\Theta(z > 0) - k^2_n \right] \psi_n(z) = 0$$  \hspace{1cm} (4)

with eigen value denoted here by $k^2_n$. In this equation we introduced the step function $\Theta(z > 0)$ to avoid consideration of neutron tunnelling through the floor. The tunnelling transforms the bound states to resonant ones.
The wave vector $k_\parallel$ in Eq. (3) is parallel to the mirror surface and according to Eq. (2) and (4) its length is $k_\parallel = \sqrt{k^2 - k_n^2}$. The neutron propagation is free along the mirror surface and is bound or, in other words, quantized along its normal, which is parallel to the vertical $z$-axis.

The separation of variables is natural and it does not depend on how large is $k_\parallel$ comparing to $k_n^2$. We do not denote $k_\parallel$ as $k_{n\parallel}$ for not to give an impression that the motion parallel to the mirror surface is also quantized. It is not quantized because there is no potential in the horizontal direction and $k^2$ can acquire an arbitrary value. However, of course, for a given $k^2$ different modes $n$ propagate with different $k_{n\parallel}$. In experiments [2] it was not essential, because there was $k^2 \gg k_n^2$.

We derived all that in details to show that the second of the two below sentences of the abstract in [1] is not correct:

*The Airy functions describe the quantum bouncer (QB), the concept of which is subject to theoretical study of toy 1D models of gravitationally bound particles in nonrelativistic quantum mechanics (QM). This is essentially different from the 3D nonstationary QM object, "the running QB," investigated in the experiment.*

If the horizontal mirror has some static imperfections then for a given energy $E$, i.e. given value of $k$, the general neutron wave function because of elastic scattering becomes a superposition of modes with different $k_n^2$ and different wave vectors $k_{n,\phi}$ of the length $|k_{n,\phi}| = \sqrt{k^2 - k_n^2}$:

$$\Psi(r, t) = \exp(-iEt/\hbar) \left[ \psi_{n_0}(z) \exp(ik_{n_0}r_\parallel) + \sum_{n=1}^{\infty} \int d\phi a(n, \phi) \psi_n(z) \exp(ik_{n,\phi}r_\parallel) \right].$$

(5)

Here we separated an initial state in the mode $n_0$ as an incident one. All the coefficients $a(n, \phi)$ ($\phi$ is azimuthal angle with respect to the initial wave vector $k_{n_0}$) can be calculated, say, by perturbation theory. If imperfections are not static, say mechanical vibration of the mirror position or elastic waves, then the Schrödinger equation becomes nonstationary and scattering can be inelastic.

The idea of the experiments [2] criticized in [1] is very simple. Because of quantization along $z$ axis propagation of neutrons in $n$-th mode along the horizontal mirror is analogous to propagation of waves along a waveguide of width $z_n$, where $z_n$ is the point, after which $\psi_n(z)$ described by Airy functions starts to decay exponentially. The larger is $n$, the larger is $z_n$.

If one puts at some height $z_a$ above the reflecting horizontal mirror an additional horizontal plate with rough lower surface and complex potential the Schrödinger equation changes. The scattering (their role is investigated in [5]) and absorption (also the tunnelling through the floor plate) both lead to losses, which can be described by an imaginary potential $iv$, and equation...
for a quantum state $n$ formed at the entrance point satisfies the equation

$$ \left[ d^2/dz^2 - u\Theta(z < 0) + iv\Theta(z > z_a) - az\Theta(z > 0) - K_n^2 \right] \psi_n(z) = 0. \quad (6) $$

With such a potential the eigen value $K_n^2$ becomes a complex number: $K_n^2 = k_n'^2 - ik_n''^2$. From energy conservation we immediately find that the propagation wave number $k_\parallel$ along the plates for every mode also becomes the complex number:

$$ k_\parallel = \sqrt{k^2 - K_n^2} = \sqrt{k^2 - k_n'^2 + ik_n''^2} = k'_n + ik''_n. \quad (7) $$

Therefore the wave of $n$-th mode propagating between the plates decays proportionally to $\exp(-k''_n L)$, where $L$ is the neutron path along the plates. For all $z_n \geq z_a$ absorption is high, and for $z_a \ll z_a$ the absorption is low, because the Airy functions exponentially decay at $z_n < z < z_a$. Therefore the gap between two horizontal plates is a filter, which transmits only modes with $z_n < z_a$. If $z_a < z_1$ no neutrons are transmitted through such a filter. The zero transmission at $0 < z_a < z_1$ is the indication of quantization in $z$ direction.

Quantization can be observed even without gravity. The neutron spectrum between two ideal mirrors with identical potential barrier $u$, separated by a distance $z_a$ contains quantized and non quantized parts. The quantized states propagate along plates. Non quantized states correspond to neutrons penetrating the plates. If the plates are sufficiently thick and long, and their potential contains an imaginary part due to absorption or scattering, the neutrons in non bound states are absorbed. So the neutrons can be transmitted along the gap between plates only, if they are in bound states. The bound state levels $E_n = k_n^2/2$ are determined from the equation $R^2 \exp(2ik_n z_a) = 1$, where $R$ is reflection amplitude from one of the plate. The minimal distance, at which the bound state exists, is equal to $l_1 = \pi/\sqrt{u}$, which is of the order of 50 nm. Therefore at $z_a < l_1$ transmission is zero, and at $z_a = l_1$ there must be a step in transmission curve.

The gravity level is observed at $z_a = z_1 \sim 10\mu$ which is 200 times larger than $l_1$. At such distances there are a lot of bound levels between the plates, and without gravity transmission would increase linearly in the range $l_1 < z_a < z_1$. Of course the roughnesses and imperfections degrade the quantum step on transmission curve because after $z_a > l_1$ without gravity, or $z_a > z_1$ with gravity there is exponential attenuation $\exp(-2k''_n L)$ of neutron flux along the plates, however, it is important to note that these imperfections do not increase transmission below $z_1$ if not to take into account that they smear the value of $z_1$ itself.

It is necessary to comment the blue curve in Fig.4 of [4]. In the legend of the insert of the figure it is said that this curve presents transmission calculated quantum mechanically, but
without gravity. This is a misleading. If we consider an ideal situation when only gravity is switched off, then in this situation the optical potentials of the mirrors must be considered unchanged. In Fig.4 of [4] the blue curve is calculated for a different case: the case when gravity is switched off, but potentials of the mirrors become infinitely large. So this blue curve does not correspond to the absence of gravity.

To explain what did they do, let’s look, how the losses due to roughnesses on the surface of the upper mirror are calculated in [4]. It was supposed that we can find channelling wave function $\psi_n(z)$ between two ideal mirrors in presence of gravity. This wave function is expressed via Airy functions. If distance between ideal mirrors is $l$, and height of roughnesses on the upper mirror is $2\sigma$ then probability of scattering on roughnesses is estimated as [4]

$$\Gamma = \alpha \int_{l-2\sigma}^{l} dz |\psi_n(z)|^2,$$

(9)

where $\alpha$ is some loss parameter, which depends on character of roughnesses. The density of neutrons $|\psi_n(z)|^2$ near roughnesses is determined by Airy functions. The red curve in [4] was calculated with such loss coefficient.

The blue curve in fig.4 of [4] is calculated with the wave function [4]

$$\psi_n(z) = \sqrt{\frac{2}{l}} \sin \left( \frac{\pi nz}{l} \right).$$

(21)

I.e. this calculation is not related to the case without gravity. It is related to the case with gravity like the red curve, but neutron density near roughnesses is approximated by functions (4.21) instead of Airy functions. So it is an approximation for the same case with gravity. The choice of the function (4.21) is based on assumption that the potential of the mirrors is infinite. Since it is not true, the choice is voluntary one and is not related to absence of gravity.

However the blue curve is really some defect of the paper [4]. If the losses in the range $l_1 < z_a < z_1$ calculated with the help of rigorous theory without gravity would give transmission below the background, i.e. unmeasurable in the experiment, then we cannot be sure that the start of count rate at 10 $\mu$ is really due to the gravity level. The higher losses at $z_a > z_1$ for blue curve than for red one are not so much spectacular and can be attributed to some normalization effect. Nevertheless the criticism of [1] is aimed not at this defect but at all the idea of the experiment, and such a criticism is wrong.

The green curve in fig.4 of [4], which shows transmission when the rough mirror is at bottom instead of ceiling shows nothing new. It corresponds only to higher losses, which now can be calculated with formula like

$$\Gamma = \alpha \int_{0}^{2\sigma} dz |\psi_n(z)|^2,$$
instead of (4.9). These losses are higher because $|\psi_n(0)|^2 > |\psi_n(l)|^2$. The only sign of quantization in gravity is the absence of transmission at $l < \sim 10 \mu$.

The factor $\exp(-2k''_n l)$, or $\exp(-\Gamma t)$ in (4.8), where $t = L/v$ and $\Gamma = 2k''_n v$, is the result of quantization and absorption of quantized states. It can be estimated by Eq. (3) of [1], but not replaced. The author of [1] replaced it by his classical expression (1.3) and lost any relation to quantum mechanics. His claim (p. 5 before section B)

*If one doubts our predictions, arguments from rigorous numerical studies of non-stationary dynamics inside the slit must be given.*

is incorrect, because the problem of bound states is purely stationary, and propagation along the channel is not a classical bouncing of a point particle between up and down mirrors.

Regrettably there are almost no formula given in [1] therefore we have to analyze only words. We will do it by quoting some parts of conclusion [1]:

1. According to [1] the authors of [2] failed to see quantization of the "running quantum bouncer"

   *because the experiment methodology was not originally formulated in QM rigor terms.*

   Above derivation shows that it is not true.

2. The next claim is not founded:

   *We criticize both the model and the measurement method as being inadequate to the claimed objective of the experiment.*

3. The next sentence is surprising:

   *The authors do not attempt to justify their methodology by conducting a detailed investigation of the problem (Monte Carlo simulation, for example) as a necessary part of the work.*

   It is not explained how the Monte Carlo method can be used for solution of the stationary Schrödinger equation.

4. The following sentence
We also note that the authors do not attempt to gain into a deeper insight of the QB concept, first of all, the problems of its definition, existence, and observation. The matter is that physics of layer of quantum bouncers on a surface of perfect mirror embraces many aspects well beyond the Airy equation and a neutron guide problem [23].

contains a reference to my work and I cannot understand what does this sentence mean.

5. The next claim

that the authors’ claim, that neutron quantum levels in the gravitational field of Earth are observed for the first time, is neither theoretically nor experimentally substantiated.

is absolutely not true.

We would like to finish our comment by citation of an experimental proposal in [1]:

the experiment setup in the quantum regime must be arranged to measure directly, by definition, a probability of finding a neutron in the space volume \(dV(z) = \Delta x \Delta y dz\) above the mirror behind the slit. It can be realized, for example, with the use of microscopic detectors sensitive to neutron wave properties that is, in a coordinate system comoving with the neutron bouncer in the \(x\) direction. The main point is that the observable QM object should be the QB rather than ”a running QB” to allow standing wave conditions to be satisfied. If so, the transverse (Airy) mode would be the main quantum mode with energy eigenvalues being physically meaningful.

and we leave it to readers to decide how feasible is it.

I. HISTORY OF SUBMISSION

I had a correspondence with the author of [1] prior his publication. I tried to explain him the essence of the experiment, but failed. After I saw [1] I proposed to the author to submit an errata. He refused then I said that I will submit my comment on his paper. The comment was submitted to Phys.Rev.D on 18.06 of 2010. One referees tried to defend paper [1], the second one agreed that the paper [1] is wrong, but tried to find defects in my paper. I corrected some points according to his criticism, however he recommended to reject my comment because I did not criticized enough the blue curve of Fig.4 in [4]. So finally the editors rejected my comment.
I appealed, but appellation system of APS is wrong. There was a member of the editorial board who wrote that he concurred with the second referee, and on 8 December of 2010 my appellation was declined. I have no opportunity to present here the complete referee reports and the name of the editorial board member who declined my appeal, because of restrictive ArXiv policy. In this submission to ArXiv I added a paragraph started with the words “However the blue curve is really some defect”. All the other text is not changed. The readers can see how hard the Phys.Rev.D defended their incompetence, which is demonstrated by publication of [1]. Later I will provide the internet address, where all the referee reports can be found.

[1] Anatoli Andrei Vankov. On observation of neutron quantum states in the Earth’s gravitational field. Phys.Rev.D 81 052008 (2010).

[2] V.V. Nesvizhevsky and K.V. Protasov, Quantum States of Neutrons in the Earth’s Gravitational Field: State of the Art, Applications, Perspectives, Trends in Quantum Gravity Research, edited by D. C. Moore (Nova Science Publishers, New York, 2006), pp. 65-107.

[3] A. E. Meyerovich, V. V. Nesvizhevsky, Gravitational quantum states of neutrons in a rough waveguide. Phys. Rev. A 73, 063616 (2006)

[4] A.Westphal1, H.Abele, S. Baeßler, V.V. Nesvizhevsky, K.V.Protasov, A.Y. Voronin, A quantum mechanical description of the experiment on the observation of gravitationally bound states. Eur. Phys. J. C 51 367-375 (2007).

[5] R. Adhikari, Y. Cheng, and A. E. Meyerovich, Quantum size effect and biased diffusion of gravitationally bound neutrons in a rough waveguide. Phys. Rev. A 75, 063613 (2006)

[6] Vladimir K. Ignatovich, Masahiko Uturo, Handbook on Neutron Optics, Wiley-VCN Verlag GmbH, & Co. KGaA, 2009.