Surface Phonon Polariton Mediated Thermal Conduction of a Micrometric Glass Waveguide

Laurent Tranchant¹*, Takuro Tokunaga², Beomjoon Kim², Nobuyuki Takama², Yann Chalopin¹ and Sebastian Volz¹
¹Laboratoire d’Energétique Moléculaire et Macroscopique, Combustion, UPR CNRS 288, Ecole Centrale Paris, Grande Voie des Vignes, 92295 Chatenay Malabry, France
²Institute of Industrial Science, The University of Tokyo, CIRMM (Center for International Research on MicroMechatronics) 4-6-1 Komaba, Meguro-ku, Tokyo 153-8505, Japan

E-mail: laurent.tranchant@ecp.fr

Abstract. Calculations of the dispersion relation and of the propagation length of surface phonon-polariton modes in a micrometric glass waveguide have been performed. The dispersion relation was solved in two ranges of frequency where SPP appear in glass. We succeeded in showing a maximal propagation length of 35 µm in a 10 µm radius waveguide with a 1 µm thick wall.

1. Introduction
Near-field radiative heat transfer appears when two opposite surfaces supporting surface phonon-polariton (SPP) waves interact with a distance of separation less than 1 µm. As a result the exchanged radiative heat flux increases by several orders of magnitude in comparison with the far-field one [1-6]. Moreover this flux was proven to be monochromatic with the frequency of the optical phonons [6]. However it is difficult to benefit from this near-field radiation flux to enhance heating or cooling at the nanoscale in relevant applications requiring heat management, because of the very short gap widths involved, i.e. a few nanometers to several tens of nanometers [2]. Another way to use SPPs consists in taking advantage of the in plane propagation to guide heat but the propagation length of the SPP waves usually does not exceed a few micrometers along the air/solid interface [7]. In order to increase this propagation length, we intend to design a cylindrical waveguide to increase the SPP propagation length and heat transfer efficiency.

* To whom any correspondence should be addressed.
2. System Geometry and Theoretical Model

The model consists in a cylindrical structure with internal and external radii $a$ and $b$ respectively defining three distinct regions: air if $r \leq a$; glass if $a \leq r \leq b$ and air if $r \geq b$. The SPP waves are propagating along the axis of the cylinder.

\[ E_z^{(1)}(a) = E_z^{(2)}(a) \ i.e. \ A_1 I_0(p_1a) = A_2 I_0(p_2a) + A_3 K_0(p_2a) \]

and

\[ H_\theta^{(1)}(a) = H_\theta^{(2)}(a) \ i.e. \ \frac{j\omega\epsilon_1}{p_1} A_1 I'_0(p_1a) = \frac{j\omega\epsilon_2}{p_2} [A_2 I'_0(p_2a) + A_3 K'_0(p_2a)] \]

\[ E_z^{(2)}(b) = E_z^{(3)}(b) \ i.e. \ A_2 I_0(p_2b) + A_3 K_0(p_2b) = A_4 K_0(p_3b) \]

and

\[ H_\theta^{(2)}(b) = H_\theta^{(3)}(b) \ i.e. \ \frac{j\omega\epsilon_2}{p_2} [A_2 I'_0(p_2b) + A_3 K'_0(p_2b)] = \frac{j\omega\epsilon_3}{p_3} A_4 K'_0(p_3b) \]

where $E_z^{(i)}$ and $H_\theta^{(i)}$ denotes the z-component of the electric field and the angular component of the magnetic field in region $i$ respectively. $\omega$ denotes the field angular frequency, $j$ refers to the complex number and $A_i$ to constants. The resolution of the four boundary equations leads to cancel the determinant of this system, which can be expressed as $AB-CD$ with:

\[
A = \frac{I_0(p_2a)}{I_0(p_1a)} - \frac{\epsilon_2 p_1}{\epsilon_1 p_2} \frac{I'_0(p_2a)}{I'_0(p_1a)}
\]

\[
B = \frac{\epsilon_3 p_3}{\epsilon_2 p_2} \frac{K'_0(p_2b)}{K'_0(p_3b)} - \frac{K_0(p_2b)}{K_0(p_3b)}
\]

\[
C = \frac{K_0(p_2a)}{I_0(p_1a)} - \frac{\epsilon_3 p_1}{\epsilon_2 p_2} \frac{K'_0(p_2a)}{I'_0(p_1a)}
\]
$$D = \frac{\varepsilon_2 p_3 \ I'_0(p_2b) \ - \ I_0(p_3b)}{\varepsilon_3 p_2 \ K'_0(p_3b)} \ K_0(p_3b)$$ [8]$$

with \( p_i^2 = \beta^2 - \omega^2 \mu_0 \varepsilon_i \), \( i = 1, 2, 3 \) (1). \( I_0 \) and \( K_0 \) are the modified Bessel functions of zeroth order, the prime corresponds to the derivative with respect to the referred function argument. \( \mu_0 \) represents the magnetic permittivity of vacuum and \( \varepsilon \) is the dielectric constant. \( p \) is the cross-plane wavevector while \( \beta \) corresponds to the in-plane wavevector. The structure is a glass cylinder surrounded by air, so that \( p_1 = p_3 \) and \( \varepsilon_1 = \varepsilon_3 \). No other assumption was made.

Glass is a lossy, linear and isotropic material, therefore its optical behavior has to be described by using a complex dielectric constant \( \varepsilon(\omega) \). SPPs appear in two frequency ranges, i.e. when \( \text{Re}(\varepsilon(\omega)) < 0 \) [9]. The first range is defined by \( 8.75 \times 10^{13} \text{rad/s} < \omega < 9.5 \times 10^{13} \text{rad/s} \) and the second by \( 2.02 \times 10^{14} \text{rad/s} < \omega < 2.34 \times 10^{14} \text{rad/s} \) [10]. Thus, the dispersion relations were calculated in these two ranges of frequency because the two surface phonon-polariton resonances for glass are included in those latter [11]. More precisely, the basic conditions for the existence of surface type modes is:

\[
\frac{\omega^2}{c^2} < \beta^2 < \frac{\varepsilon \omega^2}{c^2}
\]

where \( \varepsilon \frac{\omega^2}{c^2} \) is the wavevector of the bulk phonon polariton [12]. The geometry of the structure is also fixed with an internal radius \( a = 10 \mu m \) and an external radius \( b = 11 \mu m \).

The resolution of the determinant was performed over a set of frequencies in the chosen ranges by using Newton’s method. Because the dielectric constant is a complex number, complex cross-plane wavevectors \( p_1 \) and \( p_2 \) are computed whereas the frequency is assumed to be a real number [13].

Unlike the cross-plane wavevector \( p \), the in-plane wavevector \( \beta \) of the SPP presents the property of continuity at the surfaces. Using equation (1), the determinant can be expressed in terms of \( \beta \), which will be the only unknown of the equation. The real part of \( \beta \) corresponds to the wavelength of the SPP while the imaginary part is linked to its propagation length \( \Lambda = \frac{1}{2 \text{Im}(\beta)} \) [5,14]. The dispersion relation curve corresponds to the relation between the frequency \( \omega \) and the real part of \( \beta \) while the attenuation of the SPP is the propagation length versus frequency \( \omega \).

### 3. Results and Discussion

Both frequency intervals are complex because the determinant presents several zeros of interest with propagation lengths higher than one hundred nanometers. Two zeros present a dispersion relation curve with a back bending, we therefore assume that they correspond to SPP modes [13].

The SPP dispersion relation and the propagation length in the \( 8.8 \times 10^{13} - 9.5 \times 10^{13} \text{rad/s} \) range are reported in Figures 2(a,b). Two solutions have been plotted. The SPP dispersion relation obtained for a cylindrical structure is quite similar with these obtained on a plane surface assuming a real frequency with a back bending of the curve [13]. SPP modes have a wavelength included between 10 and 30 \( \mu m \) using the equation \( \text{Re}(\beta) = \frac{2\pi}{\lambda} \), \( \lambda \) being the wavelength. The propagation length of a few microns for the solution 1 remains small compared to the one predicted for a film configuration [14] and decreases when the frequency increases. The propagation length of the other solution is negligible (with order of magnitude of tens of nanometers).
Since transport properties are targeted, the group velocity was also presented in Figure 3 by calculating the numerical wavevector derivative of the dispersion relation curve.

![Figure 2](image1.png)

**Figure 2.** Two solutions of the determinant are examined in the $8.8 \times 10^{13} - 9.5 \times 10^{13}$ rad/s range for a glass waveguide with an internal radius of 10 µm and a 1 µm thin wall; one is plotted with a dash-dotted line and the other with a dotted line. (a) Surface Phonon-Polariton dispersion relation (b) SPP propagation length spectrum.
Figure 3. Group Velocity corresponding to the SPP dispersion relation of Figure 2a.

The SPP group velocity value ranges between $5 \times 10^6$ and $2.5 \times 10^7$ m/s, which is less than light velocity but more than the typical velocity of acoustic phonons (~$10^4$ m/s). Because of the singularity in the dispersion relation curve, the group velocity diverges for the frequency $9.25 \times 10^{13}$ rad/s. The range of frequency between $9.25$ and $9.5 \times 10^{13}$ rad/s is worth of interest: the group velocity is negative. This phenomenon of antiparallel group and phase velocities has been already observed for surface plasmon polariton producing negative refraction at visible frequencies [15]. The studied frequency interval does not yield a significant enhancement of the SPP propagation length, but the second interval provides different outcomes as shown next.
In this second range of frequency two groups of SPPs can propagate with similar wavelengths included between 5 and 10 µm (Figure 4.a) but again with very different propagation lengths, one does not exceed 10 µm while the second reaches 35 µm (Figure 4.b).

Because of the singularities in the dispersion relation, the group velocity diverges for several frequencies. Both solutions have globally a higher group velocity than these observed in the lowest range of frequency, moreover the solution with the highest propagation length has also the highest group velocity. Above the resonance frequency we observed again negative group velocities.

In the studied geometry, the propagation length of the SPP modes is not as enhanced as in the case of a silicon carbide thin layer [14]. The influence of the radius and the wall thickness still needs to be clarified in order to optimize the design of the waveguide.

4. Conclusion
The dispersion relation of a surface phonon-polariton has been investigated in a cylindrical glass structure. We have shown that SPPs propagate inside this structure with wavelengths in the near and mid infrared range e.g. between 3 µm and 30 µm (Figures 2.a and 4.a) with a propagation length, which can reach 35 µm (Figure 4.b). Several solutions of the dispersion equation have been identified, most of them have a negligible propagation length, but we presume that the wall thickness and the
internal radius have a strong influence on the properties of the SPP modes allowed inside the structure. Thus, it is possible to optimize the structure to enhance the propagation of SPP modes.

References

[1] Domingues D, Volz S, Joulain K and Greffet J-J 2005 Heat Transfer between Two Nanoparticles Through Near Field Interaction Phys Rev Letters 94 085901
[2] Rousseau E, Siria A, Jourdan G, Volz S, Comin F, Chevrier J and Greffet J-J 2009 Radiative heat transfer at the nanoscale Nature Photonics, 3, 514
[3] Chapuis P-O, Laroche M, Volz S and Greffet J-J 2008 Near-field induction heating of metallic nanoparticles due to infrared magnetic dipole contribution Phys Rev B vol. 77 (12) pp. 125402
[4] Domingues G, Rochais D and Volz S 2008 Thermal Contact Resistance between two Nanoparticles Journal of Computational and Theoretical Nanosciences 5 153
[5] Chapuis P-O, Laroche M, Volz S, et al. 2008 Radiative heat transfer between metallic nanoparticles Applied Physics Letters 92 201906
[6] Mulet J-P, Joulain K, Carminati R and Greffet J-J 2002 Enhanced radiative heat transfer at nanometric distances Nanoscale and Microscale Thermophysical Engineering 6:3 pp. 209-222
[7] Chen D-Z A, Narayanaswamy A and Chen G 2005 Surface phonon-polariton mediated thermal conductivity enhancement of amorphous thin films Phys Rev B vol. 72 pp. 155435
[8] Yeh C, Shimabukuro F 2008 The Essence of Dielectric Waveguides Springer pp. 373-382
[9] Agranovich V M, Mills D L 1982 Surface Polaritons: Electromagnetic Waves at Surfaces and Interfaces North-Holland Pub. Co. pp. vii-xii
[10] Palik E D 1985 Handbook of Optical Constants of Solids Academic Press Orlando
[11] Shen S, Narayanaswamy A, and Chen G 2009 Surface Phonon Polaritons mediated Energy Transfer between Nanoscale Gaps Nanoleters vol 9 n°8 pp 2909-2913
[12] Khosravi H, Tilley D R and Loudon R 1991 Surface Polaritons in Cylindrical Optical Fibers JOSA A vol 8 issue 1 pp 112-122
[13] Le Gall J, Olivier M and Greffet J-J 1997 Experimental and Theoretical Study of Reflection and Coherent Thermal Emission by a SiC Grating supporting a Surface Phonon Polariton Phys Rev B vol.55 (15) pp. 10 105-10 114
[14] Chen D-Z A, Chen G 2010 Heat Flow in Thin Films via Surface Phonon-Polaritons FHMT 1 023005
[15] Lezec H J, Dionne J A and Atwater H A 2007 Negative Refraction at Visible Frequencies Science vol 316 n°5823 pp. 430-432

Acknowledgments

This work was supported by KAKENHI (22360085), Grant-in-Aid for Scientific Research (B) of Japan Society for the Promotion of Science.