Probing Supersymmetric Leptogenesis
with $\mu \to e\gamma$

Alejandro Ibarra and Cristoforo Simonetto*

Physik-Department T30d, Technische Universität München,
James-Franck-Strasse, 85748 Garching, Germany.

Abstract

Extending the Minimal Supersymmetric Standard Model with three right-handed neutrino superfields is one of the best motivated scenarios for physics beyond the Standard Model. However, very little is known from observations about the high energy parameters of this model. In this paper we show, under the plausible assumptions that the neutrino Yukawa eigenvalues are hierarchical and the absence of cancellations, that there exists an upper bound on the smallest Yukawa eigenvalue stemming from the non-observation of the rare lepton decay $\mu \to e\gamma$. Furthermore, we show that this bound implies an upper bound on the lightest right-handed neutrino mass of approximately $5 \times 10^{12}$ GeV for typical supersymmetric parameters. We also discuss the implications of this upper bound for the minimal leptogenesis scenario based on the decay of the lightest right-handed neutrino and we argue that an improvement of sensitivity of six orders of magnitude to the process $\mu \to e\gamma$ could rule out this mechanism as the origin of the observed baryon asymmetry, unless the neutrino parameters take very specific values.

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*E-mail addresses: alejandro.ibarra@ph.tum.de, cristoforo.simonetto@ph.tum.de
1 Introduction

One of the simplest and best motivated extensions of the Standard Model consists on adding to the particle content three right-handed neutrinos. Being singlets under the Standard Model gauge group, the most general Lagrangian has to include not only a Yukawa coupling with the lepton and the Higgs doublets, but also a Majorana mass term for the right-handed neutrinos. If, after the electroweak symmetry breaking, the right-handed Majorana masses are much larger than the Dirac neutrino masses, the active neutrinos will acquire effective masses which are much smaller than the electroweak symmetry breaking scale. This is the renowned see-saw mechanism [1].

Although very appealing theoretically, the see-saw mechanism faces the disadvantage of lacking predictability. The see-saw mechanism makes the definite prediction of the existence of neutrino masses and strongly suggests the existence of CP violation in the lepton sector. However, it cannot predict the concrete values of the neutrino masses, the mixing angles or the CP violating phases. Besides predicting non-vanishing neutrino masses, the see-saw mechanism makes a second definite prediction. If the interactions of the right-handed neutrinos with the lepton and Higgs doublets do not preserve CP, the out of equilibrium decays of the right-handed neutrinos in the primeval plasma will generate a baryon asymmetry through the leptogenesis mechanism [2], provided the mass of the lightest right-handed neutrino is larger than $\sim 100$ GeV, which is the temperature below which the sphaleron interactions can no longer convert the generated lepton asymmetry into a baryon asymmetry. Remarkably, the out of equilibrium decays of the right-handed neutrinos could account for all the observed baryon asymmetry if the mass of the lightest right-handed neutrino is larger than $10^9$ GeV [3,4]. However, the large right-handed neutrino masses required by the leptogenesis mechanism preclude any hope to test directly the see-saw mechanism.

On the other hand, the existence of such heavy particles interacting with the Higgs doublet strongly suggests the existence of supersymmetry (SUSY), in order to protect the Higgs mass against quadratically divergent quantum corrections. In the supersymmetric version of the see-saw mechanism, the flavour and CP violating effects of the neutrino Yukawa coupling typically propagate to the soft SUSY breaking terms through quantum corrections [5], thus reopening the possibility of probing this interesting scenario through lepton flavour violating processes, such as $\mu \rightarrow e\gamma$, or through
leptonic electric dipole moments.

Unfortunately, this new opportunity to probe the see-saw mechanism is hindered by the large number of parameters in the high energy Lagrangian. The complete leptonic Lagrangian depends on fifteen real parameters and six phases, of which only nine real parameters and three phases are in principle accessible at low energies (three charged lepton masses, three neutrino masses, three mixing angles and three CP phases). Furthermore, it can be shown that the see-saw mechanism can accommodate any observed rates for the rare lepton decays or the electric dipole moments, while being consistent with the observed neutrino parameters. Namely, there exists a one to one correspondence between the high energy see-saw parameters and the combinations of Yukawa couplings and right-handed masses which are relevant to low energy experiments [6]. In consequence, it is impossible to make any completely model independent prediction about the see-saw mechanism. Nevertheless, any assumption about the high-energy see-saw parameters will break this one-to-one correspondence and will lead to constraints among the low energy parameters, or well defined relations between the high energy see-saw parameters and observable quantities. Several works have appeared in the literature aiming to derive from laboratory experiments constraints on the see-saw parameters, either working in specific high-energy frameworks or pursuing a more phenomenological approach, where the constraints somehow rely on additional assumptions on the high-energy Lagrangian [7,8]. Other works have aimed to find connections between leptogenesis and observable quantities, such as the rates for the rare lepton decays or the leptonic electric dipole moments, [9–11], again imposing conditions on the high-energy theory.

The most minimal assumption that one could make on the high-energy see-saw parameters is the absence of artificial cancellations among terms when computing the low energy predictions. It is remarkable that this minimal assumption already leads to a correlation among low energy observables of the form $\text{BR}(\mu \rightarrow e\gamma) \gtrsim C \times \text{BR}(\tau \rightarrow \mu\gamma)\text{BR}(\tau \rightarrow e\gamma)$, where $C$ is a constant that depends on supersymmetric parameters [12,13]. It was shown in [12] that if present $B$-factories discover both $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$, the see-saw mechanism would be ruled out in large regions of the SUSY parameter space (assuming universal boundary conditions at the Grand Unification scale). This result is a proof of principle that popular supersymmetric scenarios incorporating the see-saw mechanism could be ruled out using low energy experiments,
with the only assumption of the lack of cancellations among parameters.

In this paper we would like to explore the implications of another well motivated assumption about the high energy see-saw parameters, namely that the neutrino Yukawa eigenvalues are hierarchical. Our motivation to consider this scenario is the observation of large hierarchies in the eigenvalues of all known Yukawa matrices. Therefore, although the particular mechanism which generates the Yukawa couplings is completely unknown, observations suggest that this putative mechanism prefers to generate Yukawa matrices with hierarchical eigenvalues.

In Section 2 we will review a parametrization of the neutrino Yukawa couplings which will prove to be useful in studying the implications of the see-saw mechanism for the rare decays, under the assumption of hierarchical neutrino Yukawa eigenvalues. In Section 3 we will derive lower bounds on the rates of the leptonic rare decays as a function of the eigenvalues of the Yukawa couplings and neutrino parameters, and we will use the experimental bounds on the rare decays to derive constraints on the neutrino Yukawa eigenvalues. In Section 4 we will discuss the implications of these bounds on the leptogenesis mechanism to generate the baryon asymmetry of the Universe. Lastly, in Section 5 we will present our conclusions.

2 The see-saw mechanism with hierarchical Yukawa eigenvalues

In the supersymmetric see-saw mechanism, the particle content of the Minimal Supersymmetric Standard Model (MSSM) is extended with three right-handed neutrino superfields, \( \nu_{Ri} \), \( i = 1, 2, 3 \), singlets under the Standard Model gauge group. Imposing \( R \)-parity conservation, the leptonic superpotential reads:

\[
W_{\text{lep}} = \epsilon_{Ri}^c Y_{eij} L_j H_d + \nu_{Ri}^c Y_{\nu ij} L_j H_u - \frac{1}{2} \nu_{Ri}^c M_{ij} \nu_{Rj}^c ,
\]

where \( H_u \) and \( H_d \) are the hypercharge +1/2 and −1/2 Higgs doublets, respectively, \( Y_e \) and \( Y_\nu \) are the matrices of charged lepton and neutrino Yukawa couplings, respectively, and \( M \) is a 3 × 3 Majorana mass matrix. It is natural to assume that the right-handed neutrino masses are much larger than the electroweak scale or any soft mass. If this is the case, the theory is well described at low energies by the following effective
superpotential:

\[ W_{\text{eff}} = e_R^c R_{ei} L_j H_d + \frac{1}{2} (Y_\nu^T M^{-1} Y_\nu)_{ij} (L_i H_u)(L_j H_u), \]

which generates neutrino masses after the electroweak symmetry breaking. In the phenomenological studies it is convenient to work in the leptonic basis where the charged lepton Yukawa coupling and the right-handed Majorana mass matrix are real and diagonal, namely \( Y_e = \text{diag}(y_e, y_\mu, y_\tau) \) and \( M = \text{diag}(M_1, M_2, M_3) \equiv D_M \), with \( M_1 \leq M_2 \leq M_3 \). Then, in this basis, the neutrino mass matrix is given by

\[ M = Y_\nu^T D_M^{-1} Y_\nu \langle H_u^0 \rangle^2, \]

where \( \langle H_u^0 \rangle = v \sin \beta \) and \( v = 174 \text{ GeV} \). The neutrino mass matrix can be diagonalized by a unitary matrix \( U \) yielding \( U^T M U = \text{diag}(m_1, m_2, m_3) \), being the eigenvalues, \( m_i \), naturally very small due to the suppression by the large right-handed neutrino mass scale.

We will work throughout this paper under the assumption that the neutrino Yukawa coupling has hierarchical eigenvalues. Therefore, it is convenient to parametrize the neutrino Yukawa coupling using the familiar singular value decomposition

\[ Y_\nu = V_R D_Y V_L^*, \]

where \( V_R \) and \( V_L \) are 3 \times 3 unitary matrices and \( D_Y \equiv \text{diag}(y_1, y_2, y_3) \) is the diagonal matrix of eigenvalues of the Yukawa coupling (with the convention \( y_1 \leq y_2 \leq y_3 \)).

Substituting this parametrization in Eq. (3) we find

\[ M = V_L^* D_Y V_R^T D_M^{-1} V_R D_Y V_L^* \langle H_u^0 \rangle^2, \]

from where the right-handed neutrino parameters \( D_M \) and \( V_R \) can be calculated in terms of the measurable neutrino mass matrix and the parameters \( D_Y \) and \( V_L \). To this end, we first rewrite the previous equation as

\[ V_R^T D_M^{-1} V_R = \frac{1}{\langle H_u^0 \rangle^2} D_Y^{-1} V_L^T M V_L D_Y^{-1} = \frac{1}{\langle H_u^0 \rangle^2} D_Y^{-1} \tilde{M} D_Y^{-1}, \]

where we have defined for convenience \( \tilde{M} \equiv V_L^T M V_L \). Then, barring cancellations and assuming a large hierarchy among the neutrino Yukawa eigenvalues, it follows that

\[ V_R^T D_M^{-1} V_R \approx \frac{1}{\langle H_u^0 \rangle^2} D_Y^{-1} \tilde{M} D_Y^{-1} = \frac{1}{\langle H_u^0 \rangle^2} y_1^2 \left( \begin{array}{ccc}
\frac{\tilde{M}_{11}}{y_1} & \frac{\tilde{M}_{12}}{y_2} & \frac{\tilde{M}_{13}}{y_3} \\
\frac{\tilde{M}_{21}}{y_1} & \frac{\tilde{M}_{22}}{y_2} & \frac{\tilde{M}_{23}}{y_3} \\
\frac{\tilde{M}_{31}}{y_1} & \frac{\tilde{M}_{32}}{y_2} & \frac{\tilde{M}_{33}}{y_3}
\end{array} \right), \]

(6)
from where it is straightforward to extract the smallest right-handed neutrino mass, $M_1$. On the other hand, taking the inverse of Eq. (5), the same set of assumptions leads to:

$$V_R^\dagger D_M^2 V_R = (H_u^0)^{4i} D_Y \tilde{M}^{-1} D_Y^\dagger (\tilde{M}^{-1})^\dagger D_Y$$

$$\simeq (H_u^0)^{4i} y_3^2 \left( \begin{array}{ccc} y_1^2 |\tilde{M}_{12}|^2 & y_1 y_2 |\tilde{M}_{12}^1|^2 & y_1 y_3 |\tilde{M}_{13}^1|^2 \\ y_1 y_2 |\tilde{M}_{12}^1|^2 & y_2^2 |\tilde{M}_{13}^1|^2 & y_2 y_3 |\tilde{M}_{13}^1|^2 \\ y_1 y_3 |\tilde{M}_{12}^1|^2 & y_2 y_3 |\tilde{M}_{13}^1|^2 & y_3^2 |\tilde{M}_{13}^1|^2 \end{array} \right), \quad (7)$$

from where the largest right-handed neutrino mass, $M_3$, can be extracted. Lastly, from taking the determinant of Eq. (5) the intermediate eigenvalue, $M_2$, can be derived. The approximate expressions for the three right-handed neutrino masses are [9,10,14,15]:

$$M_1 \simeq y_1^2 (H_u^0)^2 \frac{1}{|\tilde{M}_{11}|} ,$$

$$M_2 \simeq y_2^2 (H_u^0)^2 \frac{\tilde{M}_{11}}{|\tilde{M}_{12} - \tilde{M}_{11}\tilde{M}_{22}|} ,$$

$$M_3 \simeq y_3^2 (H_u^0)^2 |\tilde{M}_{33}^{-1}| = y_3^2 (H_u^0)^2 \left| \frac{\tilde{M}_{12}^2 - \tilde{M}_{11}\tilde{M}_{22}}{\det \tilde{M}} \right| . \quad (8)$$

Besides, the right-handed mixing matrix reads:

$$V_R = \text{diag}(e^{i\alpha_1/2}, e^{i(\alpha_2-\alpha_1)/2}, e^{i(\alpha_3-\alpha_2)/2}) \times W_R , \quad (9)$$

where $\alpha_1 = \text{arg}(\tilde{M}_{11})$, $\alpha_2 = \text{arg}(\tilde{M}_{11}\tilde{M}_{22} - \tilde{M}_{12}^2)$, $\alpha_3 = \text{arg}(\det \tilde{M})$ and

$$(W_R)_{12} \simeq \frac{y_1}{y_2} \frac{\tilde{M}_{12}}{\tilde{M}_{11}} , \quad (W_R)_{21} \simeq -(W_R)_{12}^* ,$$

$$(W_R)_{13} \simeq \frac{y_1}{y_3} \frac{\tilde{M}_{13}}{\tilde{M}_{11}} , \quad (W_R)_{31} \simeq \frac{y_1}{y_3} \frac{\tilde{M}_{12}\tilde{M}_{13} - \tilde{M}_{12}^2 - \tilde{M}_{11}\tilde{M}_{22}}{\tilde{M}_{12}^2 - \tilde{M}_{11}\tilde{M}_{22}} ,$$

$$(W_R)_{23} \simeq \frac{y_2}{y_3} \frac{\tilde{M}_{12}\tilde{M}_{13} - \tilde{M}_{12}^2}{\tilde{M}_{12}^2 - \tilde{M}_{11}\tilde{M}_{22}} , \quad (W_R)_{32} \simeq -(W_R)_{23}^* . \quad (10)$$

Thus, the high energy see-saw Lagrangian is parametrized in terms of the effective neutrino mass matrix, $\tilde{M}$, which is in principle accessible to low energy experiments, the neutrino Yukawa eigenvalues, $D_Y$, on which we can make the educated guess that they have a hierarchical structure, and $V_L$, whose structure is unknown.
3 Minimal rates for the rare lepton decays

The supersymmetric see-saw mechanism contains sources of lepton flavour violation in the superpotential, encoded in the neutrino Yukawa matrix $Y_\nu$, as well as in the soft SUSY breaking Lagrangian:

$$-\mathcal{L}_{\text{soft}} = (m_L^2)_{ij} \tilde{\nu}_R^i \tilde{\nu}_R^j + (m_e^2)_{ij} \tilde{\ell}_R^i \tilde{\ell}_R^j + (m_\nu^2)_{ij} \tilde{\nu}_R^i \tilde{\nu}_R^j + 
\left( A_{eij} \tilde{\ell}_R^i H_d \tilde{L}_j + A_{\nu ij} \tilde{\nu}_R^i H_u \tilde{L}_j + \text{h.c.} \right) + \text{etc}. \quad (11)$$

where $\tilde{L}_i$, $\tilde{e}_R^i$ and $\tilde{\nu}_R^i$ are the supersymmetric partners of the left-handed lepton doublets, right-handed charged leptons and right-handed neutrinos, respectively, $m_L^2$, $m_e^2$ and $m_\nu^2$ are their corresponding soft mass matrices squared, and $A_e$ and $A_\nu$ are the charged lepton and neutrino soft trilinear terms.

The flavour violation in the slepton sector contributes through one loop diagrams to different flavour violating processes such as rare muon and tau decays, $K_L^0 \rightarrow e^\pm \mu^\mp$ or $\mu - e$ conversion in nuclei. Clearly, the minimal rate for all those rare processes will arise in a scenario where the soft terms are strictly flavour universal at some high energy scale, $\Lambda$:

$$(m_L^2)_{ij} = m_L^2 \delta_{ij}, \quad (m_e^2)_{ij} = m_e^2 \delta_{ij}, \quad (m_\nu^2)_{ij} = m_\nu^2 \delta_{ij}, \quad (A_e)_{ij} = A_e Y_{eij}, \quad (A_\nu)_{ij} = A_\nu Y_{\nu ij}. \quad (12)$$

If this high energy scale is larger than the right-handed neutrino masses, the flavour violation in the neutrino Yukawa couplings will propagate through radiative effects to the soft terms [5]. Hence, even under the most conservative assumption for the soft terms from the point of view of lepton flavour violation, in many supersymmetric see-saw models some amount of flavour violation in the soft SUSY breaking terms is normally expected at low energies.

The off-diagonal elements of the soft SUSY breaking terms read at low energies, in the leading-log approximation, $^1$

$$\left( m_L^2 \right)_{ij} \simeq -\frac{1}{8\pi^2} \left( m_L^2 + m_e^2 + m_{H_u} + |A_\nu|^2 \right) P_{ij},$$

$^1$Note that the result for $(A_e)_{ij}$ differs from the one usually quoted in the literature, which is proportional $(2A_\nu + A_e)/(16\pi^2)$. The reason is that quantum corrections due to right-handed neutrinos also induce off-diagonal terms in the charged lepton Yukawa couplings. Hence, at low energies it is necessary to redefine the charged lepton basis in order to bring the charged lepton Yukawa coupling to its diagonal form. This introduces new sources of flavour violation in the soft terms, which are negligible in $m_L^2$ but not in $A_e$. Indeed, these new sources of flavour violation have the effect of removing the dependence in $A_e$ in the off-diagonal trilinear terms $(A_e)_{ij}$ [12].
\[ (m^2_{e})_{ij} \simeq 0, \]
\[ (A_e)_{ij} \simeq -\frac{1}{8\pi^2} A \nu Y_{ej} P_{ij}, \]  
where \( i \neq j \) and
\[ P_{ij} = \sum_k Y^*_{\nu ki} \log \left( \frac{\Lambda}{M_k} \right) Y_{\nu kj}. \]  

The size of the off-diagonal soft terms depends crucially on the flavour structure of the neutrino Yukawa couplings and on the scale of the cut-off, \( \Lambda \), which can be identified with the mass of the messenger particles which transmit supersymmetry breaking from the hidden sector to the observable sector. We will show in this paper that if thermal leptogenesis is the correct mechanism to generate the baryon asymmetry, a non-vanishing rate for the rare decays will be necessarily generated, unless artificial cancellations among different terms are taking place.

In the simplest version of the leptogenesis mechanism, the lightest right-handed neutrino is produced by thermal scatterings in the primeval plasma. Subsequently, the out of equilibrium decays of the right-handed neutrinos generate a lepton asymmetry, which is eventually converted by sphaleron processes into a baryon asymmetry. In order to produce the observed baryon asymmetry by this mechanism the reheating temperature of the Universe has to be larger than \( \sim 10^9 \) GeV [3,4]. At these very high temperatures gravitino thermal production is very efficient, therefore, in order to avoid overclosure of the Universe the gravitino mass has to be larger than \( m_{3/2} \gtrsim 5 \) GeV [16,17], which implies a rather large scale for the cut-off.

To show this, we recall that the gravitino mass is defined as
\[ m_{3/2} = \frac{|F|}{\sqrt{3} M_P}, \]  
where \( M_P = 2.4 \times 10^{18} \) GeV is the reduced Planck mass and \( \sqrt{|F|} \) is the scale of spontaneous supersymmetry breaking. On the other hand, SUSY breaking is transmitted to the observable sector by messenger particles with mass \( M_{\text{mes}} \), inducing soft masses which approximately read:
\[ m^2_{\text{soft}} \sim c \frac{|F|^2}{M^2_{\text{mes}}}, \]  
where \( c \sim 10^{-4} - 1 \) is a constant which depends on the details of the mediation mechanism. From Eqs. (15,16) it follows that
\[ M_{\text{mes}} \sim \sqrt{3c} \frac{m_{3/2}}{m^2_{\text{soft}} M_P}. \]
Therefore, the constraint on the gravitino mass from the requirement of successful
leptogenesis, $m_{3/2} \gtrsim 5$ GeV, and the assumption that the soft masses are $O(1$ TeV) 
imply that the messenger scale has to be larger than $10^{14} - 10^{16}$ GeV. This large scale 
for the cut-off suggests that at least one right-handed neutrino is coupled below the 
mediation scale and thus will contribute to the generation of off-diagonal soft terms 
via quantum effects \cite{18}.

Indeed, the experimental fact that the ratio of the atmospheric mass splitting to
the solar mass splitting is relatively mild, $\sqrt{\Delta m^2_{\text{atm}}/\Delta m^2_{\text{sol}}} \sim 6$, supports this conclusion. As discussed in \cite{20}, when the neutrino Yukawa eigenvalues are hierarchical, a
degenerate spectrum of right-handed neutrinos cannot reproduce the observed mild
neutrino mass hierarchy without a certain fine tuning. This is not the case, though, for 
a hierarchical spectrum of right-handed neutrinos, which could naturally explain the 
eutrino mass hierarchy for certain choices of the matrix $V_R$ without tunings. There-
fore, even assuming that the heaviest right-handed neutrino mass is around the Planck
scale, in view of the large hierarchy necessary to accommodate the ratio of the solar 
and atmospheric mass splittings without fine-tuning, it is plausible that at least the
lightest right-handed neutrino will have a mass smaller than $10^{14} - 10^{16}$ and hence will 
contribute to the radiative generation of off-diagonal terms in the leptonic soft terms.

The second necessary requirement to generate radiatively flavour violation in the 
soft SUSY breaking terms is a non trivial structure in the neutrino Yukawa couplings, 
encoded in the matrix $P$, Eq. (14). This equation can be conveniently rewritten as

$$
P_{ij} = (Y^\nu Y^\nu)^{ij} \log \left( \frac{\Lambda}{M_3} \right) + Y^\nu_{2i} Y^\nu_{2j} \log \left( \frac{M_3}{M_2} \right) + Y^\nu_{1i} Y^\nu_{1j} \log \left( \frac{M_3}{M_1} \right). \quad (18)$$

For generic neutrino Yukawa couplings, this expression is dominated by the first
term, which corresponds to the widely used approximation of decoupling all the right-
handed neutrinos altogether at the scale $M_3$. However, it is conceivable that the matrix
$Y^\nu Y^\nu$ could be exactly diagonal. If this is the case, the leading contributions to the
off diagonal elements of $P$ are determined by the subdominant terms proportional to
$Y^\nu_{2i}$ and $Y^\nu_{1i}$. In this scenario, that as we will see is consistent with present neutrino

\footnote{This bound on the messenger scale could be circumvented if the gravitino is ultralight so is in
thermal equilibrium in the early Universe, namely $m_{3/2} \lesssim 16$ eV, which corresponds to $M_{\text{mess}} \lesssim 260$ 
TeV \cite{19}. This scenario requires, though, an extension of the Minimal Supersymmetric Standard
Model in order to account for the cold dark matter of the Universe, since neither the gravitino nor
the lightest neutralino are any longer good dark matter candidates.}
experiments, the mixing in the left-handed sector is trivial, namely $V_L = 1$. However, in order to generate mixing in the effective neutrino mass matrix, there must exist mixing in the right-handed sector, $V_R \neq 1$. Therefore, even in this extreme scenario, a non-vanishing rate for the rare decays will always be generated through the subdominant terms $Y_{\nu 2i} = (V_R)_{2i} y_i$ and $Y_{\nu 1i} = (V_R)_{1i} y_i$, unless different terms cancel each other.

More concretely, using Eqs. (9,10) it follows that in the scenario with $V_L = 1$, when $\Lambda > M_3$,

$$P_{12} \simeq y_1^2 \frac{M_{12}}{M_{11}} \log \frac{M_2}{M_1},$$

$$P_{13} \simeq y_1^2 \frac{M_{12} M_{23} - M_{13} M_{22}}{M_{12}^2 - M_{11} M_{22}} \log \frac{M_3}{M_2} + \frac{M_{13}}{M_{11}} \log \frac{M_2}{M_1},$$

$$P_{23} \simeq y_2^2 \frac{M_{12} M_{13} - M_{11} M_{23}}{M_{12}^2 - M_{11} M_{22}} \log \frac{M_3}{M_2},$$

(19)

on the other hand, when $M_3 > \Lambda > M_2$, the expressions are identical with the substitution $M_3 \rightarrow \Lambda$. Lastly, when $M_2 > \Lambda > M_1$,

$$P_{12} \simeq y_1^2 \frac{M_{12}}{M_{11}} \log \frac{\Lambda}{M_1},$$

$$P_{13} \simeq y_1^2 \frac{M_{13}}{M_{11}} \log \frac{\Lambda}{M_1},$$

$$P_{23} \simeq y_2^2 \frac{M_{12} M_{13}}{|M_{11}|^2} \log \frac{\Lambda}{M_1},$$

(20)

where the right-handed neutrino masses are given in Eq. (8).

The off diagonal elements of the matrix $P$ induce through quantum corrections flavour violation in the soft mass matrices, Eq. (13), which in turn induce a non-vanishing rate for the rare lepton decays, which approximately reads:

$$\text{BR}(l_i \rightarrow l_j \gamma) \simeq \frac{\alpha^3}{G_F^2} \frac{|(m_L^2)_{ij}|^2}{m_S^2} \tan^2 \beta \text{BR}(l_i \rightarrow l_j \nu_j \bar{\nu}_j),$$

(21)

where $\text{BR}(\mu \rightarrow e \nu_\mu \bar{\nu}_e) \simeq 1$, $\text{BR}(\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu) \simeq 0.17$, $\text{BR}(\tau \rightarrow e \nu_\tau \bar{\nu}_e) \simeq 0.18$, and $m_S$ is a mass scale of the order of typical SUSY masses.

Among the scenarios compatible with the present neutrino experiments and thermal leptogenesis, the one presented here, with flavour universal soft terms at some cut-off scale and no flavour mixing in the left-handed sector, corresponds to the worst case for the detection of the rare decays or, conversely, to the scenario yielding the minimal rate for the rare decays. In any other scenario there will be additional sources of flavour
violation, either in the soft terms at the cut-off scale or in the left-handed mixing matrix $V_L$, thus yielding a larger rate for the rare decays, unless different terms cancel each other.

In what follows, let us illustrate our results calculating the minimal rates for the rare decays in the Constrained MSSM, which is defined at the Grand Unification scale by just five parameters: the universal scalar mass ($m_0$), gaugino mass ($M_{1/2}$) and trilinear term ($A_0$), $\tan \beta$ and the sign of $\mu$. As neutrino parameters, we will assume a hierarchical mass spectrum (which is the most plausible possibility under the assumption of hierarchical Yukawa eigenvalues [20]) and a neutrino mixing matrix approximately tri-bimaximal [21]:

$$U \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\frac{1}{\sqrt{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix} \times \text{diag}(e^{i\phi/2}, e^{i\phi'/2}, 1) \quad (22)$$

(assuming a non-vanishing $|U_{13}|$ will not change our conclusions). Then, the minimal rates for the rare decays can be straightforwardly computed using Eqs. (13,19,21), yielding

$$\text{BR}(\mu \to e \gamma) \gtrsim \frac{\alpha^3}{G_F} \left( \frac{3m_0^2 + |A_0|^2}{8\pi^2 m_S^4} \right)^2 y_1^4 \log^2 \frac{M_2}{M_1} \tan^2 \beta ,$$

$$\text{BR}(\tau \to e \gamma) \gtrsim \frac{\alpha^3}{G_F} \left( \frac{3m_0^2 + |A_0|^2}{8\pi^2 m_S^4} \right)^2 y_1^4 \left( 2 \log \frac{M_3}{M_2} + \log \frac{M_2}{M_1} \right)^2 \tan^2 \beta \text{BR}(\tau \to e \nu_{\tau} \bar{\nu}_{\mu}) ,$$

$$\text{BR}(\tau \to \mu \gamma) \gtrsim \frac{\alpha^3}{G_F} \left( \frac{3m_0^2 + |A_0|^2}{8\pi^2 m_S^4} \right)^2 y_2^4 \log^2 \frac{M_3}{M_2} \tan^2 \beta \text{BR}(\tau \to \mu \nu_{\tau} \bar{\nu}_e) , \quad (23)$$

which strongly depend on the size of the Yukawa eigenvalues and only logarithmically on the hierarchy of right-handed masses, or alternatively, on the hierarchy of the Yukawa eigenvalues, through

$$\frac{M_3}{M_2} \sim \frac{y_3^2}{y_3^2} \frac{m_3}{12m_1} , \quad \frac{M_2}{M_1} \sim \frac{y_2^2}{y_2^2} \frac{2m_2}{3m_3} . \quad (24)$$

For given neutrino Yukawa eigenvalues one can estimate using Eq. (23) a lower bound on the rates of the rare lepton decays. Conversely, one can derive constraints on the parameters of the high-energy Lagrangian $y_1$ and $y_2$ from the present bounds on
the rare lepton decays, $\text{BR}(\mu \to e\gamma) \leq 1.2 \times 10^{-11}$ [22], $\text{BR}(\tau \to \mu\gamma) \leq 4.5 \times 10^{-8}$ [23], $\text{BR}(\tau \to e\gamma) \leq 1.1 \times 10^{-7}$ [24]. The most stringent constraints on $y_1$ and $y_2$ stem from the non-observation of the processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$, respectively, and read:

$$y_1 \lesssim 4 \times 10^{-2} \left( \frac{\text{BR}(\mu \to e\gamma)}{1.2 \times 10^{-11}} \right)^{1/4} \left( \frac{m_S}{200 \text{ GeV}} \right) \left( \frac{\tan \beta}{10} \right)^{-1/2},$$

$$y_2 \lesssim 0.5 \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right)^{1/4} \left( \frac{m_S}{200 \text{ GeV}} \right) \left( \frac{\tan \beta}{10} \right)^{-1/2},$$

(25)

where we have conservatively assumed $M_3 : M_2 : M_1 = 100 : 10 : 1$ and $m_0 \sim A_0 \sim m_S$. Note that the bound on $y_2$ only applies when $\Lambda > M_2$.

This numerical estimate is confirmed by our numerical analysis of two typical points in the CMSSM parameter space. We have analyzed the SPS1a and SPS1b benchmark points [25], which correspond to typical CMSSM points with intermediate and relatively high values of $\tan \beta$, respectively. For these two benchmark points we find approximately the same result:

$$y_1 \lesssim 6 \times 10^{-2} \left( \frac{\text{BR}(\mu \to e\gamma)}{1.2 \times 10^{-11}} \right)^{1/4},$$

$$y_2 \lesssim 0.8 \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right)^{1/4},$$

(26)

which agrees with our general expectation, Eq. (25).

Alternatively, these bounds could be expressed in terms of the SUSY contribution to the muon $g - 2$, which depends on the same combination of SUSY masses and $\tan \beta$ [26],

$$\delta a_{\mu}^{\text{SUSY}} \simeq \frac{5g_2^2}{192\pi^2} \frac{m_\mu^2}{m_S^2} \tan \beta,$$

(27)

yielding

$$y_1 \lesssim 6 \times 10^{-2} \left( \frac{\text{BR}(\mu \to e\gamma)}{1.2 \times 10^{-11}} \right)^{1/4} \left( \frac{\delta a_{\mu}^{\text{SUSY}}}{10^{-9}} \right)^{-1/2},$$

$$y_2 \lesssim 0.8 \left( \frac{\text{BR}(\tau \to \mu\gamma)}{4.5 \times 10^{-8}} \right)^{1/4} \left( \frac{\delta a_{\mu}^{\text{SUSY}}}{10^{-9}} \right)^{-1/2},$$

(28)

These bounds demonstrate that it is possible to obtain information on the high energy see-saw parameters from low energy observations (namely the bounds on the rates of the rare decays, neutrino masses and mixing angles and supersymmetric parameters), under very general and well motivated assumptions about the high energy
theory, such as the absence of cancellations, hierarchical neutrino Yukawa eigenvalues and a large mediation scale (as suggested by thermal leptogenesis).

The resulting bound on $y_2$ is rather weak and lacks any practical interest. On the other hand, the bound on $y_1$ is fairly stringent (it corresponds to a Dirac neutrino mass of 7 GeV) and will be improved in the near future by a factor of three if the MEG experiment at PSI reaches the projected sensitivity $\text{BR}(\mu \rightarrow e\gamma) \sim 10^{-13}$ [27] without observing a positive signal. Furthermore, the bound on $y_1$ has important implications for leptogenesis, which will be discussed in the next section.

A similar rationale could be applied to calculate the minimal value of the leptonic electric dipole moments (EDMs). Following the analysis in [28], we estimate that in the worst case scenario for the detection of EDMs, again when the soft terms are flavour universal at the cut-off scale and when $V_{L} = 1$, the electron EDM reads:

$$d_{e} \sim e\frac{\alpha m_{e}}{\pi m_{S}^{2}} \left(\frac{1}{16\pi^{2}}\right)^{2} y_{1}^{4} \text{Im} \left[ \frac{M_{12}M_{13}M_{12}M_{13} - M_{11}M_{12}}{M_{11}^{2} - M_{11}M_{12}} \right] \log \frac{M_{2}}{M_{1}} \log \frac{M_{3}}{M_{2}},$$

(29)

where we have assumed $A_{0} \sim \mu \sim M_{1} \sim m_{S}$. Then, when the neutrino mass matrix has an approximate tri-bimaximal form and allowing a non-vanishing value for the 13 element of the leptonic mixing matrix, $U_{13} = \sin \theta_{13} e^{-i\delta}$, we find the following lower bound on the electron EDM:

$$|d_{e}| \gtrsim e\frac{\alpha m_{e}}{\pi m_{S}^{2}} \left(\frac{1}{16\pi^{2}}\right)^{2} y_{1}^{4} \left| 2\sqrt{2} \sin \theta_{13} \sin \delta + \frac{m_{1}}{m_{2}} \sin(\phi' - \phi) \right| \log \frac{M_{2}}{M_{1}} \log \frac{M_{3}}{M_{2}},$$

(30)

which can even be exactly zero if there is no CP violation at low energies or when both $m_{1}$ and $\sin \theta_{13}$ simultaneously vanish. Assuming generic CP violating phases and $\sin \theta_{13} = 0.2$, which corresponds to the present upper bound at the $2\sigma$ level [29], the following lower bound holds:

$$|d_{e}| \gtrsim 7 \times 10^{-29} \text{e cm} \left(\frac{m_{S}}{200 \text{ GeV}}\right)^{-2}.$$  

(31)

Finally, from the present experimental bound on the electron EDM, $|d_{e}| < 10^{-27} \text{ e cm}$ [30], we obtain the constraint on the smallest Yukawa eigenvalue $y_{1} \lesssim 2$ for $m_{S} = 200$ GeV, which is much weaker than the bound we derived in Eq. (25) from the non-observation of the process $\mu \rightarrow e\gamma$. More importantly, the constraint on $y_{1}$ from the electron EDM relies on assumptions about the size of the CP violating phases and $\sin \theta_{13}$, which are currently unknown. We find that even if future experiments determine that the CP
violating phases and sin $\theta_{13}$ are sizable, the best current proposal to improve the experimental sensitivity to the electron EDM will not provide bounds on $y_1$ competitive to the bounds stemming from the non-observation of $\mu \to e\gamma$. Namely, an improvement of sensitivity down to the level $d_e \sim 10^{-35}$ e cm [31], would translate into $y_1 \lesssim 0.02$, again for $m_S = 200$ GeV, which is comparable to the bound attainable by the MEG experiment at PSI, provided no positive signal is found.

4 Implications for leptogenesis

The baryon asymmetry generated through the leptogenesis mechanism depends, under the assumption of hierarchical neutrinos, essentially on two parameters: the lightest right-handed neutrino mass, $M_1$, and an effective neutrino mass $\tilde{m}_1$ [32], defined as

$$\tilde{m}_1 = \frac{(Y_\nu Y_\nu^T)_{11}}{M_1} \langle H_u^0 \rangle^2 ,$$

which measures the strength of the coupling of the lightest right-handed neutrino to the thermal bath.

In scenarios where the neutrino Yukawa couplings are hierarchical the lightest right-handed neutrino mass reads, for generic values of the matrix $V_L$,

$$M_1 \approx \frac{y_1^2 \langle H_u^0 \rangle^2}{|\tilde{M}_{11}|} ,$$

where $|\tilde{M}_{11}| = |(V_L^T M V_L)_{11}| = |\sum_k (U^\dagger V_L)^2_{1k} m_k|$. Strictly speaking, $|\tilde{M}_{11}|$ can range between 0 and $m_3$. Nevertheless, for generic values of the matrix $V_L$ the most natural range for $|\tilde{M}_{11}|$ is $\sqrt{\Delta m^2_{\text{sol}}} \lesssim |\tilde{M}_{11}| \lesssim \sqrt{\Delta m^2_{\text{atm}}}$. The only exception corresponds to the case when $(U^\dagger V_L)_{k1} \simeq \delta_{k1}$, which can arise in specific models and which leads to $|\tilde{M}_{11}| \ll \sqrt{\Delta m^2_{\text{sol}}}$ without cancellations. This special case will be discussed at the end of this section. In our numerical analysis we will take for the solar and atmospheric mass splittings the central values of the global fit to neutrino data [29], $\Delta m^2_{\text{sol}} = 7.65 \times 10^{-5}$ eV$^2$, $\Delta m^2_{\text{atm}} = 2.40 \times 10^{-3}$ eV$^2$.

In the generic case $\sqrt{\Delta m^2_{\text{sol}}} \lesssim |\tilde{M}_{11}| \lesssim \sqrt{\Delta m^2_{\text{atm}}}$. Then, from Eq. (33) it follows a natural range for $M_1$ as a function of $y_1$. More importantly, the lightest neutrino Yukawa eigenvalue, $y_1$, is bounded from above by the non-observation of the process $\mu \to e\gamma$, through $|P_{12}| \gtrsim y_1^2 \log M_2/M_1$, cf. Eq. (19). Therefore, in a supersymmetric
scenario with hierarchical neutrino Yukawa couplings, the following upper bound on the lightest right-handed neutrino mass holds for generic values of the matrix $V_L$:

$$M_1 \lesssim |P_{12}| \frac{|H_0^0|^2}{\sqrt{\Delta m_{\text{sol}}^2}} \log^{-1} \frac{M_2}{M_1},$$  \hspace{1cm} (34)

which numerically reads

$$M_1 \lesssim 5 \times 10^{12}\text{GeV} \left( \frac{\text{BR}(\mu \to e\gamma)}{1.2 \times 10^{-11}} \right)^{1/2} \left( \frac{m_S}{200 \text{GeV}} \right)^2 \left( \frac{\tan \beta}{10} \right)^{-1}. \hspace{1cm} (35)$$

This upper bound should be compared with the lower bound on the right-handed neutrino mass $M_1 \gtrsim 10^9$ GeV, leaving an allowed window of four orders of magnitude for the lightest right-handed neutrino mass. Alternatively, Eq. (35) could be rewritten as a lower bound on the rate for $\mu \to e\gamma$ as a function of the lightest right-handed neutrino mass,

$$\text{BR}(\mu \to e\gamma) \gtrsim 5 \times 10^{-19} \left( \frac{M_1}{10^9 \text{GeV}} \right)^2 \left( \frac{m_S}{200 \text{GeV}} \right)^{-4} \left( \frac{\tan \beta}{10} \right)^2. \hspace{1cm} (36)$$

Thus, exploring the allowed window of the thermal leptogenesis scenario requires an improvement in sensitivity to the process BR($\mu \to e\gamma$) of approximately eight orders of magnitude, which unfortunately does not seem feasible in the short or mid term. It is remarkable, though, that if supersymmetry is discovered at the LHC, the scenario of thermal leptogenesis with hierarchical neutrino Yukawa couplings and hierarchical right-handed masses could be tested using just low energy experiments.

Interestingly, following our premise of the absence of cancellations, the allowed mass window for leptogenesis can be further narrowed down. In the scenario with hierarchical neutrino Yukawa eigenvalues, the effective neutrino mass $\tilde{m}_1$ reads, following Eqs. (9,10):

$$\tilde{m}_1 \simeq \frac{|\tilde{M}_{11}|^2 + |\tilde{M}_{12}|^2 + |\tilde{M}_{13}|^2}{|\tilde{M}_{11}|},$$  \hspace{1cm} (37)

which ranges between $m_1 \leq \tilde{m}_1 < \infty$ for the Yukawa couplings consistent with the low energy neutrino experiments [3]. Using $|\tilde{M}_{11}| = |\sum_k (U^\dagger V_L)_{k1} (U^\dagger V_L)_{k1} m_k|$ it follows that the lower limit, $\tilde{m}_1 = m_1$, could be reached when $(V_L)_{k1} = U_{k1}$, which corresponds to the special case for the matrix $V_L$ which will be discussed at the end of this section. On the other hand, the upper limit, $\tilde{m}_1 \to \infty$ is reached when $\sum_k (U^\dagger V_L)_{k1}^2 m_k = 0$, which requires a cancellation among terms and is thus implausible. Therefore, in the generic case $\sqrt{\Delta m_{\text{sol}}^2} \lesssim |\tilde{M}_{11}| \lesssim \sqrt{\Delta m_{\text{atm}}^2}$, one expects a natural window for the
effective neutrino mass $\sqrt{\Delta m_{\text{sol}}^2} \lesssim \tilde{m}_1 \lesssim \sqrt{\Delta m_{\text{atm}}^2}$, which corresponds to the strong washout regime. On the other hand, from the analysis in [33,34] it follows that an effective neutrino mass $\tilde{m}_1 > \sqrt{\Delta m_{\text{sol}}^2}$ implies a lower bound on the right-handed neutrino mass $M_1 \gtrsim 3 \times 10^9$ GeV, which in turn implies, following Eq. (36), the lower bound on the rare muon decay $\text{BR}(\mu \rightarrow e\gamma) \gtrsim 5 \times 10^{-18}$ for typical SUSY parameters. Therefore, closing the natural window for leptogenesis requires, for generic neutrino parameters, an improvement in sensitivity to the process $\mu \rightarrow e\gamma$ of six orders of magnitude.

The required sensitivity is unfortunately below, although not far from, the sensitivity of the projected neutrino factory, where the high beam intensity may allow the observation of one $\mu \rightarrow e\gamma$ event if the branching ratio is $10^{-16}$. One should note, however, that the observation of this single event over the accidental background would require detector resolutions which are not currently available, and new technologies or new experimental ideas should be developed [35]. On the other hand, the PRISM/PRIME experiment at J-PARC aims to achieve a single event sensitivity to the process $\mu \rightarrow e\gamma$ at the level of $10^{-18}$ [36]. This is equivalent to a sensitivity to the process $\mu \rightarrow e\gamma$ at the level of $\sim 2 \times 10^{-16}$. Thus, if the LHC determines that $\tan \beta$ is large, the non-observation of muon flavour violation at PRISM/PRIME could rule out, for generic neutrino parameters and barring cancellations, the thermal leptogenesis scenario based on the decay of the lightest right-handed neutrino. If, on the contrary, $\tan \beta$ takes moderate values, it would be necessary a further improvement in sensitivity by more than one order of magnitude to close the leptogenesis window for $M_1$.

For large values of $\tilde{m}_1$, the upper bound on the lightest right-handed neutrino mass can be improved. From Eqs. (33,37) it follows that:

$$\tilde{m}_1 \approx \frac{y_{11}^2 (H_u^0)^2}{M_1} \left[ 1 + \left| \frac{\tilde{M}_{12}}{\tilde{M}_{11}} \right|^2 + \left| \frac{\tilde{M}_{13}}{\tilde{M}_{11}} \right|^2 \right].$$

(38)

Barring cancellations one expects in general $|\tilde{M}_{11}| \sim |\tilde{M}_{12}| \sim |\tilde{M}_{13}|$. Then, using the upper bound on the lightest Yukawa coupling, Eq. (25), we obtain:

$$M_1 \lesssim 10^{13}\text{GeV} \left( \frac{\tilde{m}_1}{9 \times 10^{-3} \text{eV}} \right)^{-1} \left( \frac{\text{BR}(\mu \rightarrow e\gamma)}{1.2 \times 10^{-11}} \right)^{1/2} \left( \frac{m_S}{200 \text{GeV}} \right)^2 \left( \frac{\tan \beta}{10} \right)^{-1}. \quad (39)$$

\(^3\)When the photon penguin diagram dominates the $\mu - e$ conversion in Ti, the conversion rate is approximately a factor $5 \times 10^{-3}$ smaller than the branching ratio of $\mu \rightarrow e\gamma$. 

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From Eq. (35) it is apparent that already large portions of the parameter space for $M_1$ are excluded. In particular this bound suggests that flavour effects should be always taken into account in leptogenesis. More concretely, the lepton asymmetry is mostly generated at the temperature $T_B$, defined through $\[4\]

$$
\frac{M_1}{T_B} \simeq 1 + \frac{1}{2} \log \left( 1 + \frac{\pi K^2}{1024} \left[ \log \left( \frac{3125 \pi K^2}{1024} \right) \right]^5 \right),
$$

(40)

where $K = \tilde{m}_1/m_*$ and $m_* \simeq 10^{-3}$ eV. For $T > T_B$ the asymmetry produced is essentially erased, while for $T < T_B$, washout is negligible. In the strong washout regime $K \gg 1$, which in turn implies $T_B \lesssim M_1 \lesssim 5 \times 10^{12}$ GeV. In this range of temperatures the tau Yukawa coupling is in equilibrium and thus flavour effects can be relevant. For a hierarchical left-handed neutrino spectrum there are two possibly relevant flavour effects, lowering the bound on $M_1$ in the strong washout regime [37].

If the lightest right-handed neutrino decays into different flavours, the washout in each flavour is not determined by $\tilde{m}_1$, but instead by

$$
\tilde{m}_{1\alpha} = \frac{|Y_{\nu}|^2_{1\alpha} (H_u^0)^2}{M_1}.
$$

(41)

As $\tilde{m}_1 = \sum_\alpha \tilde{m}_{1\alpha}$, the washout in each flavour is smaller than in the unflavoured case. In the approximation of flavours being CP eigenstates, the flavoured CP asymmetries are proportional to $\tilde{m}_{1\alpha}$ and the effect is maximized for equal $\tilde{m}_{1\alpha}$ in all flavours $\alpha$. This can lead to a relaxation of the lower bound on $M_1$ by a factor 2–3, depending on the number of families which have charged lepton Yukawa interactions in equilibrium with the thermal plasma. Secondly, for some specific neutrino textures it may occur that the CP asymmetry is sizable in one flavour, but the asymmetry is only weakly washed out, $\tilde{m}_{1\alpha} \sim m_*$. Using Eqs. (9,10) it follows that

$$
\tilde{m}_{1\alpha} \simeq \frac{|V_L^{T}\mathcal{M}|^2_{1\alpha}}{|\mathcal{M}_{11}|},
$$

(42)

from where it is apparent that this possibility requires $V_L$ with sizable off-diagonal entries (unless the low energy phases and $|U_{13}|$ take very special values), thus leading to an enhancement of $\text{BR} (\mu \rightarrow e\gamma)$. Since we are interested in scenarios yielding the minimal rate for $\mu \rightarrow e\gamma$ we will not consider this possibility.

In weak washout the lower bound on $M_1$ is not relaxed by flavour effects and especially the absolute lower bound $M_1 \gtrsim 10^9$ GeV is not affected [33,34].

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Figure 1: Allowed parameter space of thermal leptogenesis (in yellow, adopted from [33]), including the constraints on the relevant parameters which stem from the non-observation of the process $\mu \to e\gamma$, under the assumption of hierarchical neutrino Yukawa eigenvalues and barring cancellations. The orange region corresponds to the range of $\tilde{m}_1$ for generic neutrino parameters. In this plot it is assumed $m_S \simeq 200$ GeV and $\tan \beta \simeq 10$.

We show in Fig. 1 the impact of the bounds on the lightest right-handed neutrino mass stemming from the non-observation of the process $\mu \to e\gamma$, Eqs. (35,39), on the parameter space of thermal leptogenesis, spanned by $\tilde{m}_1$ and $M_1$. The yellow region corresponds to the allowed region found by Blanchet and di Bari, and shown in Fig. 1 of [33]. The thick solid lines encompass the allowed region assuming zero initial abundance of right-handed neutrinos, while the thin solid lines, the allowed region assuming thermal initial abundance. For each case we show the lower bound on $M_1$ for two scenarios. The left plot corresponds to the “alignment” scenario, where the final asymmetry is dominated by one flavor, and which amounts to neglecting flavour effects in leptogenesis. The right plot corresponds to the “democratic” scenario, where $\tilde{m}_{1\alpha} = \tilde{m}_1/3$, and which illustrates how flavour effects can relax the lower bound on $M_1$. On the other hand, the orange region corresponds to the range of $\tilde{m}_1$ for generic neutrino parameters, $\sqrt{\Delta m^2_{\text{sol}}} \lesssim \tilde{m}_1 \lesssim \sqrt{\Delta m^2_{\text{atm}}}$. We show as thick dashed lines the most stringent upper bound on $M_1$ for the projected sensitivity by the MEG experiment at PSI, $\text{BR}(\mu \to e\gamma) \sim 10^{-13}$, corresponding to $M_1 \lesssim 5 \times 10^{11}$ GeV, and the projected sensitivity by the PRISM/PRIME experiment at J-PARC, $R(\mu \text{Ti} \to e \text{Ti}) \sim 10^{-18}$. 

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which corresponds to $M_1 \lesssim 2 \times 10^{10}$ GeV. It should be stressed at this point that the main uncertainty in our calculation does not stem from the calculation of the baryon asymmetry, but from our present ignorance of the SUSY parameters, which can change considerably our numerical estimate of $\text{BR}(\mu \rightarrow e\gamma)$.

Lastly, we would like to discuss the case with $|\hat{M}_{11}| \ll \sqrt{\Delta m^2_{\text{sol}}}$, where the general discussion presented above does not apply. In the absence of cancellations, this situation corresponds to $(V_L)_{k1} \simeq (U_{k1})$, which could arise in certain models with $V_R$ very close to the identity. In this case, an upper bound on the lightest right-handed neutrino mass can be derived from taking the trace of

$$V^\dagger_R D_M^{-2} V_R = \frac{1}{\langle H^0_u \rangle^4} D^{-1}_Y \tilde{M}^\dagger D^{-2}_Y \tilde{M} D^{-1}_Y,$$

which gives

$$\frac{1}{M_1^2} + \frac{1}{M_2^2} + \frac{1}{M_3^2} = \frac{1}{\langle H^0_u \rangle^4} \sum_{ij} \frac{|\tilde{M}_{ij}|^2}{y^2_i y^2_j}.$$  

Therefore, a very conservative bound on $M_1$ is:

$$M_1 \leq \frac{y^2_2 \langle H^0_u \rangle^2}{|\tilde{M}_{22}|},$$

where $|\tilde{M}_{22}| \gtrsim \sqrt{\Delta m^2_{\text{sol}}}$. On the other hand, an upper bound on $y_2$ can be obtained from Eq. (18). Keeping the leading term, which in the absence of cancellations constitutes by itself a lower bound on $P_{12}$, we obtain

$$|P_{12}| \gtrsim y_3^2 (V_L)_{13}^* (V_L)_{23}^* + y_2^2 (V_L)_{12}^* (V_L)_{22}^* + y_1^2 (V_L)_{11}^* (V_L)_{21}^* \log \left( \frac{\Lambda}{M_3} \right).$$

where we have used the unitarity of $V_L$ and the fact that $(V_L)_{k1} \simeq (U_{k1})$. The lowest value is reached when $(V_L)_{13} (V_L)_{23}^* \simeq 0$, thus yielding

$$|P_{12}| \gtrsim \frac{y_2^2}{3} \log \left( \frac{\Lambda}{M_3} \right).$$

Substituting in Eq. (45) finally gives:

$$M_1 \lesssim 3|P_{12}| \frac{\langle H^0_u \rangle^2}{\sqrt{\Delta m^2_{\text{sol}}}} \log^{-1} \frac{\Lambda}{M_3},$$

which is comparable in magnitude to the result obtained for generic values of $V_L$, Eq. (34). Therefore, the bound for the lightest right-handed neutrino mass in terms
of \( \text{BR}(\mu \rightarrow e\gamma) \) derived in Eq. (35) also applies to the special case where \( |\tilde{\mathcal{M}}_{11}| \ll \sqrt{\Delta m_{\text{sol}}^2} \).

If the neutrino parameters satisfy the relation \((V_L)_{k1} \simeq U_{k1}\), the effective neutrino mass \(\tilde{m}_1\) can be much smaller than \(\sqrt{\Delta m_{\text{sol}}^2}\) without cancellations. Then, being the lepton asymmetry only weakly washed-out, the observed baryon asymmetry can be generated even when the absolute lower bound on the lightest right-handed neutrino mass, \(M_1 \gtrsim 10^9\) GeV, is saturated. As a consequence, following Eq. (36), it would be necessary an improvement in sensitivity to the process \(\mu \rightarrow e\gamma\) of eight orders of magnitude in order to close the leptogenesis window, which is below the sensitivity of any planned experiment.

One should note, however, that the lower bound Eq. (36) is very conservative and can be largely enhanced by the term proportional to \(y_3^2(V_L)_{13} (V_L)_{23}^*\) in Eq (46).\(^4\) Thus, even though this scenario is the worst case scenario for probing leptogenesis, which requires the non-observation of the process \(\mu \rightarrow e\gamma\), it is very favourable for observing a signal in future experiments searching for rare decays [8].

5 Conclusions

The see-saw mechanism is perhaps the most elegant explanation for the small neutrino masses, which in addition provides a potential solution to the longstanding puzzle of the origin of the matter-antimatter asymmetry of the Universe, through the mechanism of leptogenesis. However, although it is very appealing theoretically, it suffers the serious disadvantage of lacking predictability. Furthermore, being the scale of the new physics presumably very large, it also suffers the disadvantage of lacking testability. On the other hand, in the supersymmetric version of the see-saw mechanism, which is probably the most natural arena to implement it, the high-energy see-saw parameters leave an imprint on the slepton soft mass matrices through quantum effects, thus opening a unique opportunity to test the see-saw mechanism or the leptogenesis mechanism with low energy observations.

Working under very general and well motivated assumptions, namely the absence

\(^4\)It is interesting to note that the requirement of successful leptogenesis leads to a lower bound on \(y_3\), stemming from the condition \(m_3 \leq y_3^2 \langle H_u^0 \rangle^2 / M_1\). Therefore, \(y_3 \gtrsim \sqrt{m_3 M_1 / \langle H_u^0 \rangle}\), being \(m_3 \sim \sqrt{\Delta m_{\text{atm}}^2}\) and \(M_1 \gtrsim 10^9\) GeV, which gives \(y_3 \gtrsim 10^{-3}\). For generic values of \((V_L)_{13}\) and \((V_L)_{23}\), the contribution from the largest Yukawa coupling to \(P_{32}\) can be much larger than the minimal contributions from \(y_2\) considered here, thus yielding much larger rates for \(\text{BR}(\mu \rightarrow e\gamma)\).
of cancellations and a hierarchical pattern in the neutrino Yukawa eigenvalues, we have identified the scenario yielding the minimal rate for the rare decay $\mu \rightarrow e\gamma$. In this scenario, the rate depends essentially on the lightest neutrino Yukawa eigenvalue and on supersymmetric parameters. Using the experimental constraint on $\text{BR}(\mu \rightarrow e\gamma)$ we have derived an upper bound on the smallest neutrino Yukawa eigenvalue $y_1 \lesssim 4 \times 10^{-2}$ for typical soft SUSY breaking terms of 200 GeV and $\tan \beta = 10$.

We have shown that this upper bound on the smallest neutrino Yukawa eigenvalue in turn translates into an upper bound on the lightest right-handed neutrino mass, $M_1 \lesssim 5 \times 10^{12}$ GeV, which should be compared with the lower bound required by the thermal leptogenesis scenario, $M_1 \gtrsim 10^9$ GeV. The upper bound derived in this paper scales as $\text{BR}(\mu \rightarrow e\gamma)^{1/2}$, therefore, future improvements in sensitivity to the process $\mu \rightarrow e\gamma$ (and to $\mu - e$ conversion in nuclei) will have important implications for the thermal leptogenesis scenario if no positive signal is found. Namely, under the assumption of hierarchical eigenvalues and barring cancellations, if supersymmetry is discovered at the LHC, an improvement in sensitivity of six orders of magnitude to $\text{BR}(\mu \rightarrow e\gamma)$ (or seven orders of magnitude to the rate of $\mu - e$ conversion in nuclei) will suffice to rule out large classes of thermal leptogenesis models based on the decay of the lightest right-handed neutrino. Possible ways out are to accept that neutrino parameters take very special values or to invoke non-minimal scenarios of leptogenesis, such as leptogenesis induced by the decay of the next-to-lightest right-handed neutrino [38].

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**References**

[1] P. Minkowski, Phys. Lett. B 67 (1977) 421. M. Gell-Mann, P. Ramond and R. Slansky, Proceedings of the Supergravity Stony Brook Workshop, New York 1979, eds. P.
Van Nieuwenhuizen and D. Freedman; T. Yanagida, *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe*, Tsukuba, Japan 1979, eds. A. Sawada and A. Sugamoto; R. N. Mohapatra, G. Senjanovic, *Phys.Rev.Lett.* 44 (1980)912, *ibid. Phys.Rev.* D23 (1981) 165; S. L. Glashow, *The Future Of Elementary Particle Physics, In *Cargese 1979, Proceedings, Quarks and Leptons*, 687-713 and Harvard Univ.Cambridge - HUP-T-79-A059 (79,REC,DEC.) 40p. J. Schechter and J. W. F. Valle, *Phys. Rev. D* 22 (1980) 2227.

[2] M. Fukugita and T. Yanagida, *Phys. Lett. B* 174 (1986) 45.

[3] S. Davidson and A. Ibarra, *Phys. Lett. B* 535 (2002) 25;

[4] W. Buchmüller, P. Di Bari and M. Plümacher, *Annals Phys.* 315 (2005) 305.

[5] F. Borzumati and A. Masiero, *Phys. Rev. Lett.* 57, 961 (1986).

[6] S. Davidson and A. Ibarra, *JHEP* 0109 (2001) 013.

[7] J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, *Phys. Rev. D* 53 (1996) 2442.

J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, *Phys. Lett. B* 357 (1995) 579; J. Hisano and D. Nomura, *Phys. Rev. D* 59 (1999) 116005; M. E. Gomez, G. K. Leontaris, S. Lola and J. D. Vergados, *Phys. Rev. D* 59, 116009 (1999); J. A. Casas and A. Ibarra, *Nucl. Phys. B* 618 (2001) 171; S. Lavignac, I. Masina and C. A. Savoy, *Phys. Lett. B* 520, 269 (2001); T. Blazek and S. F. King, *Nucl. Phys. B* 662 (2003) 359; S. T. Petcov, S. Profumo, Y. Takanishi and C. E. Yaguna, *Nucl. Phys. B* 676 (2004) 453. S. T. Petcov, T. Shindou and Y. Takanishi, *Nucl. Phys. B* 738 (2006) 219; E. Arganda and M. J. Herrero, *Phys. Rev. D* 73 (2006) 055003; A. Ibarra, *JHEP* 0601 (2006) 064; L. Calibbi, A. Faccia, A. Masiero and S. K. Vempati, *Phys. Rev. D* 74 (2006) 116002; C. H. Albright and M. C. Chen, *Phys. Rev. D* 77 (2008) 113010.

[8] A. Masiero, S. K. Vempati and O. Vives, *Nucl. Phys. B* 649 (2003) 189.

[9] S. Davidson and A. Ibarra, *Nucl. Phys. B* 648 (2003) 345.

[10] S. Davidson, *JHEP* 0303, 037 (2003).
[11] M. Raidal and A. Strumia, Phys. Lett. B 553 (2003) 72; A. Ibarra and G. G. Ross, Phys. Lett. B 591 (2004) 285, Phys. Lett. B 575 (2003) 279; S. Pascoli, S. T. Petcov and C. E. Yaguna, Phys. Lett. B 564 (2003) 241; S. T. Petcov, W. Rodejohann, T. Shindou and Y. Takanishi, Nucl. Phys. B 739 (2006) 208; F. Depisch, H. Pas, A. Redelbach and R. Ruckl, Phys. Rev. D 73 (2006) 033004; G. C. Branco, A. J. Buras, S. Jager, S. Uhlig and A. Weiler, JHEP 0709 (2007) 004; F. R. Joaquim, I. Masina and A. Riotto, Int. J. Mod. Phys. A 22 (2007) 6253; S. Davidson, J. Garayoa, F. Palorini and N. Rius, Phys. Rev. Lett. 99 (2007) 161801, JHEP 0809 (2008) 053; M. Endo and T. Shindou, arXiv:0805.0996 [hep-ph].

[12] A. Ibarra and C. Simonetto, JHEP 0804 (2008) 102.

[13] A. Ibarra, T. Shindou and C. Simonetto, JHEP 0810 (2008) 021.

[14] G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim and M. N. Rebelo, Nucl. Phys. B 640 (2002) 202.

[15] E. K. Akhmedov, M. Frigerio and A. Y. Smirnov, JHEP 0309 (2003) 021.

[16] M. Bolz, A. Brandenburg and W. Buchmüller, Nucl. Phys. B 606 (2001) 518.

[17] J. Pradler and F. D. Steffen, Phys. Rev. D 75 (2007) 023509.

[18] K. Tobe, J. D. Wells and T. Yanagida, Phys. Rev. D 69 (2004) 035010.

[19] M. Viel, J. Lesgourgues, M. G. Haehnelt, S. Matarrese and A. Riotto, Phys. Rev. D 71 (2005) 063534.

[20] J. A. Casas, A. Ibarra and F. Jimenez-Alburquerque, JHEP 0704, 064 (2007).

[21] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B 530 (2002) 167.

[22] M. L. Brooks et al. [MEGA Collaboration], Phys. Rev. Lett. 83 (1999) 1521; M. Ahmed et al. [MEGA Collaboration], Phys. Rev. D 65 (2002) 112002.

[23] K. Hayasaka et al. [Belle Collaboration], Phys. Lett. B 666 (2008) 16.

[24] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 96 (2006) 041801.
[25] B. C. Allanach et al., in Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001) ed. N. Graf, Eur. Phys. J. C 25 (2002) 113.

[26] J. L. Lopez, D. V. Nanopoulos and X. Wang, Phys. Rev. D 49 (1994) 366; U. Chattopadhyay and P. Nath, Phys. Rev. D 53 (1996) 1648; T. Moroi, Phys. Rev. D 53 (1996) 6565 [Erratum-ibid. D 56 (1997) 4424].

[27] T. Mori et al. "Search for $\mu \to e\gamma$ Down to $10^{-14}$ Branching Ratio". Research Proposal to Paul Scherrer Institut. See also http://meg.web.psi.ch/

[28] I. Masina, Nucl. Phys. B 671 (2003) 432.

[29] T. Schwetz, M. Tortola and J. W. F. Valle, New J. Phys. 10 (2008) 113011.

[30] B. C. Regan, E. D. Commins, C. J. Schmidt and D. DeMille, Phys. Rev. Lett. 88 (2002) 071805.

[31] S. K. Lamoreaux, arXiv:nucl-ex/0109014; C. Y. Liu and S. K. Lamoreaux, Mod. Phys. Lett. A 19 (2004) 1235.

[32] For reviews on leptogenesis, see W. Buchmuller, R. D. Peccei and T. Yanagida, Ann. Rev. Nucl. Part. Sci. 55, 311 (2005); S. Davidson, E. Nardi and Y. Nir, Phys. Rept. 466 (2008) 105.

[33] S. Blanchet and P. Di Bari, JCAP 0703 (2007) 018.

[34] F. X. Josse-Michaux and A. Abada, JCAP 0710, 009 (2007)

[35] A. Bandyopadhyay et al. [ISS Physics Working Group], arXiv:0710.4947 [hep-ph].

[36] Letter of Intent to J-PARC, L25, An Experimental Search for the $\mu-e$ Conversion Process at an Ultimate Sensitivity of the Order of $10^{-18}$ with PRISM.

[37] A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada and A. Riotto, JCAP 0604, 004 (2006); E. Nardi, Y. Nir, E. Roulet and J. Racker, JHEP 0601, 164 (2006).

[38] O. Vives, Phys. Rev. D 73 (2006) 073006; G. Engelhard, Y. Grossman, E. Nardi and Y. Nir, Phys. Rev. Lett. 99, 081802 (2007).