Theta angle versus CP violation in the leptonic sector

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Assuming that the axion mechanism of solving the strong CP problem does not exist and the vanishing of $\theta$ at tree level is achieved by some model-building means, we study the naturalness of having large CP-violating sources in the leptonic sector. We consider the radiative mechanisms which transfer a possibly large CP-violating phase in the leptonic sector to the $\theta$ parameter. It is found that large $\theta$ cannot be induced in the models with one Higgs doublet as at least three loops are required in this case. In the models with two or more Higgs doublets the dominant source of $\theta$ is the phases in the scalar potential, induced by CP violation in leptonic sector. Thus, in the MSSM framework the imaginary part of the trilinear soft-breaking parameter $A_t$ generates the corrections to the theta angle already at one loop. These corrections are large, excluding the possibility of large phases, unless the universality in the slepton sector is strongly violated.

I. INTRODUCTION AND MOTIVATION

The strong CP problem whose existence was realized over twenty years ago remains a complete mystery. The theta term of the QCD Lagrangian breaks $P$ and $CP$ invariance, and thus induces a variety of $P,T$-odd observable effects, among which the electric dipole moments (EDMs) of the neutron and heavy atoms play a prominent role. The conflict between strong limits on $\theta$ resulting from experimental searches of EDMs and natural expectations of $\theta \sim 1$ presents a severe fine-tuning problem, usually referred to as the strong CP problem. Using the experimental limits on the EDM of the neutron together with the result of a recent QCD sum rule calculation of $d_n(\theta)$ one can place a very stringent limit on the theta term,

$$\theta < 6 \cdot 10^{-10}. \quad (1)$$

A common and universal solution to the strong CP problem may come through a dynamical relaxation mechanism which requires the existence of a light pseudoscalar (axion) in the particle spectrum. Negative results from experimental searches of an axion together with very restrictive astrophysical and cosmological bounds on its coupling constant stimulate searches for alternative solutions.

Another possibility is a model-building construction where $\theta$ can be naturally chosen to be zero at some high-energy scale due to exact parity or CP symmetry. In this case however, $\theta$ is not protected against radiative corrections at lower scales where parity and/or CP symmetry are spontaneously broken. Thus the theta term is extremely sensitive to the presence of additional, other than Kobayashi Maskawa, CP-violating sources in the hadronic sector. This sensitivity is unique: $\theta$ can receive contributions from the CP-violating phases in the “heavy” sector of the theory without power-like suppression, in contrast with other CP-violating operators. Thus, in the SUSY variants of these models large soft-breaking phases in the squark and gluino sectors are excluded, as they penetrate into the low-energy effective expression for $\theta$ already at one loop level. Therefore, a necessary consequence of these constructions seems to be a strong restriction on CP violation, i.e. no CP violation other than KM phase. Is this also true for CP violation which resides solely in the leptonic sector? In other words, how susceptible is $\theta$ to the CP-violation in the leptonic sector?

If the axion mechanism does not exist, the theta term is expected to be a dominant source of CP violation at low energy as it is the CP-odd operator of lowest dimension. What would be a signal of the “$\theta$-dominance” among CP-violating observables? Both neutron and mercury EDMs produce similar bounds on $\theta$ and one should naturally expect that

$$d_n \simeq 10^{-26} e \cdot \text{cm} \cdot \frac{\theta}{10^{-10}} \quad [4]$$

$$d_{Hg} \simeq 10^{-28} e \cdot \text{cm} \cdot \frac{\theta}{10^{-10}} \quad [4]$$

$$d_{Th} \sim 2 \cdot 10^{-29} e \cdot \text{cm} \cdot \frac{\theta}{10^{-10}} \quad [2]$$

Comparing the predictions of $\theta$-dominated EDMs with current experimental limits, one can easily see that for $\theta = 10^{-10}$, $d_n$ and $d_{Hg}$ are within a factor of 2-3 from the current experimental figures, whereas $d_{Th}$ is smaller by five orders of magnitude than its present limit. In other words, $\theta = 10^{-10}$ will produce thallium EDM at the level equivalent to $d_{Th}$, induced by the electron EDM $d_e \sim 4 \cdot 10^{-32} e \cdot \text{cm}$. Thus, it appears that the signal of $\theta$-dominance could be easily distinguished from the case of MSSM with large CP SUSY phases and axion-type solution to the strong CP problem. In the latter case $d_{Th}$ is expected to be much more important than in [2] and competitive with $d_n$ and $d_{Hg}$.

However, if CP violation is initially concentrated in the leptonic sector, the “$\theta$-signal” could be different. In this case the EDM of the electron and $d_{Th}$ could be enhanced relative to [2] and, at the same time, the $\theta$ term, induced by a lepton CP-phase via radiative corrections would still dominate $d_n$ and $d_{Hg}$.
The purpose of this note is to study the mechanisms of transferring CP violation from the lepton sector to the theta term in the context of different models without an axion. Assuming no fine tuning which would compensate an induced value of $\theta$, we find a “maximal” amount of CP-violation in the lepton sector, which can be consistent with the bound (1). At the same time, we study possible enhancement of $d_F$ and $d_{FI}$ due to the same sources of CP violation and the departure from the $\theta$-dominance signal, eq. (2).

II. NON-SUSY MODELS

We begin some remarks about the way how the low energy value of $\theta$ should be calculated in a generic theory with CP-violation. Besides the initial value of $\theta_{QCD}$, the relevant low energy parameter $\tilde{\theta}$ receives tree level contributions from the phases of the quark masses and other $SU(3)_c$-charged fermions.

$$\tilde{\theta} = \theta_{QCD} + \arg \det(M_uM_d) + \ldots \tag{3}$$

It is often assumed in the literature, that the radiative corrections to $\theta$ are simply contained in the imaginary parts of the quark and gluino masses. This is certainly true at the tree level, but at the loop level the structure of radiative corrections is more complicated. To give a simplest example, one can consider an effective Lagrangian for gluinos and quark field $q$ which arises after integrating out some unknown CP-violating physics at the scale $\Lambda$:

$$\mathcal{L}_{\text{eff}} = \theta(\Lambda) \frac{g_3^2}{16\pi^2} G_{\mu\nu}^a G^{a\mu\nu} + \bar{q}(i\partial_\mu \gamma^\mu - m - i\eta \gamma_5)q - \frac{i m''}{2\Lambda^2} \bar{q}G_{\mu\nu}^a \sigma^{\mu\nu} \gamma_5 q + \ldots \tag{4}$$

Here $\theta(\Lambda)$ denotes the theta term, coming from the scale $\Lambda$. Let us take for simplicity $m \gg \Lambda_{QCD}$ and $m' \ll m$. Then the field $q$ can be also integrated out and the theta parameter below the scale $m$ reads as

$$\tilde{\theta} = \theta(\Lambda) + \frac{m'}{m} + \frac{mm''}{\Lambda^2} \log(\Lambda^2/m^2). \tag{5}$$

The second term in this expression is the “usual” correction due to the phase of the mass term, whereas the third term is generated by the “chromoelectric dipole” in (4). It is usually smaller than the second term due to $\Lambda \gg m$, although not necessarily negligible. For example, the scale of new physics $\Lambda$ could be comparable to the mass of heaviest fermions (top quark) so that the ratio $mm''/\Lambda^2$ is not small, or $m'$ can be simply zero from additional symmetry arguments and then the third term dominates the expression for $\tilde{\theta}$. The latter is exactly the case in the minimal SM, where $\tilde{\theta}$ receives corrections from “dipole” contributions as it was first shown by Khriplovich [13]. Technically, the corrections to $\tilde{\theta}$ can be easily calculated within the external field formalism which will automatically account for all contributions.

In what follows we determine possible mechanisms of transmitting CP violation from the lepton sector into the theta term in various possible models [14]. As representative examples we take the Standard Model extended by right handed neutrino fields, dilepton Zee model [15], multi-Higgs models and the Minimal Supersymmetric Standard Model (MSSM) in particular.

It turns out that the main criterion which governs the efficiency of transmitting the CP violation from the leptonic sector into the theta term is the number of weak doublets which give masses to the quark fields. The contribution of the quark masses into $\tilde{\theta}$ can be separated into the contributions of Yukawa couplings and Higgs vevs:

$$\arg \det(M_uM_d) = \arg \det(Y_u) + \arg \det Y_d + 3(\arg v_u + \arg v_d). \tag{6}$$

We take the vanishing of this expression due to some symmetry arguments (for example, hermiticity of $Y_i$, reality of $v_i$) as the starting point for our analysis.

In the SM and in other models where $v_u \equiv v_d^*$, the contribution from the second line in (6) is identically zero, irrespective of the presence of CP-violation. Therefore the only way to insert CP-violation into the theta term is to “complexify” quark Yukawa couplings and/or create quark chromoelectric dipoles.

Nontrivial corrections to quark Yukawa couplings sensitive to a CP phase in the leptonic sector must be included via Yukawa and $SU(2) \times U(1)$ gauge interactions. Furthermore, it is clear that in the presence of only one Higgs doublet Yukawa interactions alone are not sufficient to achieve this. In any possible graph, involving a quark line and leptons in the loop, it is convenient to separate the loop part where actual CP violation takes place. Let us suppose now that the particles circulating in the loop are heavy (Majorana neutrinos, for example) and the lines, connecting the leptonic loop to a quark line are “soft”. Then it is possible to classify the effects of CP-violation in the leptonic sector in terms of effective CP-odd operators with dimension 6 and larger: $H^\dagger H (B_{\mu\nu}B_{\mu\nu})$; $H^\dagger H (W_{\mu\nu\rho}W_{\mu\nu\rho})$; $W_{\mu\nu}W_{\rho\sigma}W_{\alpha\beta}W_{\mu\nu}^{\rho\sigma\alpha\beta}$, etc. One needs at least two loops to attach these operators to a quark line with no external $SU(2)$ or $U(1)$ fields allowed. Together with at least one (leptonic) loop needed to generate these operators, three loops is the minimal order in which CP violation from the leptonic sector penetrates into $\tilde{\theta}$!

In practice, the loop level is often higher. In the SM with heavy Majorana neutrinos, singlets of the SM gauge group, one should have a minimum of four flavour-changing vertices on the lepton line. In the weak basis, which is more convenient because the momenta flowing in the loop are large, of the order of the heavy Majorana masses, these can only come from interactions with the
Thus the operators which introduce phases into the scalar potential. Different v.e.v. different doublets which give masses to quarks via two or more measurable level \[16\].

$\alpha \left( \frac{M}{16\pi^2} \right)^2 \frac{m_f}{M^2} \phi_{CP}$,

where $M$ is the relevant high energy scale, at least as heavy as $M_W$. No matter how large the CP violating combination of mixing angles $J_{\ell\nu}$ in the leptonic sector is, the result for $\theta$ is well within the experimental bound. Therefore all CP violating phenomena discussed in the literature such as CP violation in neutrino oscillations, CP violation in the heavy Majorana neutrino decay, needed for leptogenesis and others are entirely possible without causing problems for $\theta$.

Precisely the same estimates (in this crude approach) can be applied to the Zee model to produce similar conclusions, i.e. $\theta$ generated from the CP violation in the leptonic sector is small. However, unlike the SM with Majorana neutrinos, where the possible electron EDM is generated from the CP violation in the heavy Majorana neutrino sector, charged lepton EDMs are massless. A more detailed calculation may reveal further suppression factors. For our purposes it is sufficient to acknowledge that the suppression factor is at least

$\bar{\theta} < \left( \frac{\alpha}{4\pi} \right)^2 \left( \frac{1}{16\pi^2} \right)^2 \frac{m_f}{M^2} \phi_{CP}$,

III. MSSM WITH COMPLEX SOFT-BREAKING PARAMETERS IN THE LEPTON SECTOR

We concentrate only on the leptonic sector of the MSSM superpotential, i.e.,

$\mathcal{W} \supset \epsilon_{ab}(Y_e)_{ij} L^2 E_i^\dagger H^d_j$, \hspace{1cm} (8)

and the soft breaking terms,

$\mathcal{L}_{soft} \supset - \epsilon_{ab} \left[ (A_e Y_e)_{ij} \tilde{L}_i \tilde{H}^u_j \tilde{H}^a_j + h.c. \right] - \left[ \mu B \epsilon_{ab} H^u_j H^d_j + h.c. \right]$, \hspace{1cm} (9)

where as usual $\tilde{e}, \tilde{l}$ are the corresponding scalar components of the chiral superfields $L, E$ appeared in eq. (8).

Let us assume universality of the soft trilinear couplings at the GUT scale, $A_e = A_\mu = A_\tau$, and one common phase $\phi_A$ associated with them. We consider the third generation of leptons, i.e., $A_\tau$ where the Yukawa couplings are large as compared to those of the first and the second generation. Then the renormalization group running of the imaginary part of the parameter $A_\tau$, denoted as $\bar{A}_\tau$, induces an imaginary part of the parameter $B$, denoted as $\bar{B}$, at a scale below the GUT scale and their RGEs are given by (19),

$\frac{d \bar{A}_\tau}{dt} = \frac{8|Y_{\tau}|^2}{16\pi^2} \bar{A}_\tau$, \hspace{1cm} (10)

$\frac{d \bar{B}}{dt} = \frac{2|Y_{\tau}|^2}{16\pi^2} \bar{A}_\tau$. \hspace{1cm} (11)

All the other parameters of the SUSY or the soft SUSY breaking sector remain real. The tau lepton Yukawa coupling has a weak running (especially for small values of $\tan \beta$). Thus the system of differential equations of (11) can be solved trivially and gives,

$\bar{A}_\tau(Q) = \bar{A}_\tau(M_G) \left( \frac{Q}{M_G} \right)^\frac{|Y_{\tau}|^2}{2\pi^2}$, \hspace{1cm} (12)
\[
\bar{B}(Q) = -\frac{\bar{A}_r(M_G)}{4} \left[ 1 - \left( \frac{Q}{M_G} \right)^\frac{\nu^2}{2\pi^2} \right] .
\]

(13)

So even if all the parameters at the GUT scale are real apart from the leptonic trilinear coupling i.e., \( \bar{A}_r \), then this parameter affects the running of the \( \bar{B} \) parameter and generates a non-zero \( \bar{B} \). We can easily see from (13) that the running of the phase of the parameter \( B \), i.e., \( \phi_B \) at a scale \( Q \) is given by,

\[
\sin \phi_B(Q) = -\frac{1}{4} \frac{|A_r(M_G)|}{|B(Q)|} \sin \phi_A(M_G) \left[ 1 - \left( \frac{Q}{M_G} \right)^\frac{\nu^2}{2\pi^2} \right] .
\]

(14)

Let us now see what happens at the EW scale i.e., \( Q = M_Z \). It is reasonable to take \( |A_r(M_G)| \approx |B(M_Z)| \) in the MSSM with Radiative Electroweak breaking. This assumption of course depends on the choice of the other MSSM parameters, \( M_0, M_{1/2}, A_0 \) and \( \beta \). We display the numerical solutions below. For \( |Y_\tau|^2/4\pi \approx 4 \times 10^{-5} \) and \( \tan \beta = 2 \) with \( M_{GUT} = 3 \times 10^{16} \) GeV we get, from eq.(13)

\[
\sin \phi_B(M_Z) \approx -2 \times 10^{-4} \sin \phi_A .
\]

(15)

Now from the minimization conditions of the scalar Higgs potential we have,

\[
v_1 v_2 = \frac{\mu^* B^* |v|^2}{m_1^2 + m_2^2} ,
\]

(16)

where \( m_{1,2}^2 = m_{h,1,2}^2 + \mu^2 \) and \(|v|^2 = |v_1|^2 + |v_2|^2\). Note also that the parameter \( \mu \) remains real (if originally is real) at every scale because its renormalization is multiplicative.

The \( \theta \) angle is generated if \( B \) is complex and given by eq. (8). Putting the experimental bound (8) of \( \theta \) parameter into eq.(13) we get

\[
\phi_A(M_G) < 10^{-6} ,
\]

(17)
an unnatural small number at the GUT scale. We conclude that the phase in the leptonic sector produces large additive renormalization of \( \theta \)-QCD parameter which constitutes a fine tuning problem unless this phase is tiny, of the order of \( 10^{-6} \) or smaller.

Three remarks are in order: i) Even if we assume an appropriate phase for the parameter \( \mu \) at a scale \( Q \) which cancels the contribution of the \( \phi_B \) i.e., \( \phi_B(Q) = -\phi_B(Q) \) then eventually this pattern will be destroyed by the running of \( \phi_B \) of eq.(13) since \( \phi_B \) does not run, ii) the constraint (13) on \( \phi_A(M_G) \) is relaxed if we consider non-universality of the soft SUSY breaking trilinear couplings at the GUT scale in the case of the electron and muon Yukawa couplings, iii) if we start with the (trivial) case \( A = 0 \) GeV at the GUT scale then there is no contribution to the \( \theta \)-term and no CP-violation in the leptonic sector.

We perform a numerical analysis of the RGEs by also taking into account low energy threshold effects [20]. We present our results in Fig. 2. We see that \( \phi_A \lesssim 10^{-6} \) unless \( A_0 \) is exactly zero. Small departures from zero (see the line with \( |A_0| = 1 \) GeV in Fig. 2 for instance) put a strong bound on the phase \( \phi_A \). As \( |A_0| \) increases the bound becomes stronger; as strong as \( \phi_A \lesssim 10^{-8} \) for \( |A_0| \gtrsim 300 \) GeV. This happens because \( |A_0| = |A_r(M_G)| \) gets much larger than \( |B(M_Z)| \) which further enhances the value of \( \theta \), as seen from eq.(13).

![Fig. 2. The extracted value of the \( \theta \)-term as a function of the common phase \( \phi_A \) at the GUT scale of the lepton trilinear soft SUSY breaking couplings. The other SUSY breaking parameters have been fixed \( M_0 = M_{1/2} = 200 \) GeV and \( \tan \beta = 10 \). The shaded region is excluded by the experiment, see [8]. Results on \( \theta \) from different values of the modulo \( |A_0| = 1, 10, 50, 300, 600 \) GeV are also indicated.

Therefore we face two possible choices if we still want to keep large CP-violation in the leptonic sector: i) relax the universality pattern of the phases at the GUT scale (however, even in that case the phases of the \( \tau \) trilinear soft breaking coupling must be unnaturally small as we prove above) ii) introduce PQ symmetry and the axion solution to the strong CP problem.

IV. CONCLUSIONS

We have studied the question of naturalness for CP violation in the leptonic sector to be large without inducing large corrections to \( \theta \). This is an important question in the context of non-axionic solutions to the strong CP problem. We find that the main criterion dividing mod-
els into two classes is the number of Higgs doublets giving masses to quarks. In the case of one doublet the contribution of CP phases from the leptonic sector to $\theta$ is always small, being suppressed by at least a three-loop factor so that a “maximal” CP violation in the leptonic sector is allowed. In some of these models (dilepton model, for example), $d_e$ can be quite large, enhancing $d_{T1}$ with respect to “$\theta$-dominance” signal, eq. (2), usually expected when the axion mechanism is absent.

In the models with several doublets there is an efficient way of transmitting CP violation from the leptonic sector into $\theta$ via complex parameters in the scalar potential. In MSSM without an axion, a large phase of the leptonic $A_e$-parameter is excluded on the ground of naturalness, unless the lepton universality is broken in a peculiar way that only $A_e$ (or $A_\mu$) has the phase.

We thank R. R. Roberts for valuable discussions. AD acknowledges financial support from the Marie Curie Research Training Grant ERB-FMBI-CT98-3438. This work was supported in part by the Department of Energy under Grant No DE-FG-02-94-ER-40823.

[1] C. Callan, R. Dashen and D. Gross, Phys. Lett. B63 (1976) 334, R. Jackiw and C. Rebbi, Phys. Rev. Lett. 37 (1976) 172.
[2] I.B. Khriplovich and S.K. Lamoreaux, “CP Violation Without Strangeness”, Springer, 1997.
[3] K.F. Smith et al., Phys. Lett. B234 191 (1990); I.S. Altarev et al., Phys. Lett. B276 242 (1992); P.G. Harris et al., Phys. Rev. Lett. 82 904 (1999).
[4] M. Pospelov and A. Ritz, Phys. Rev. Lett. 83 (1999) 2526; M. Pospelov and A. Ritz, hep-ph/9908508.
[5] R.D. Peccei and H. Quinn, Phys. Rev. Lett. 38 (1977) 1440; Phys. Rev. D16 (1977) 1791; J. E. Kim, Phys. Rev. Lett. 43 (1979) 103; M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B166, 493 (1980); A.R. Zhitnitsky, Sov. J. Nucl. Phys. 31 (1980) 260; M. Dine, W. Fischler and M. Srednicki, Phys. Lett. B104 (1981) 199.
[6] S. Weinberg, Phys. Rev. Lett. 40, (1978) 223; F. Wilczek, Phys. Rev. Lett. 40, (1978) 279.
[7] C. Hagmann et al, in Eur. J. C3, (1998) 1.
[8] For reviews, see M. S. Turner, Phys. Rept. 197, (1990) 67; G. G. Raffelt, Phys. Rept. 198, (1990) 1.
[9] J. Preskill, M.B. Wise and F. Wilczek, Phys. Lett. B120 (1983) 127; L.F. Abbott and P. Sikivie, Phys. Lett. B120 (1983) 133; M. Dine and W. Fischler, Phys. Lett. B120 (1983) 137. For reviews, see J. E. Kim, Phys. Rept. 150, (1987) 1; M. S. Turner, Phys. Rept. 197, (1990) 68.
[10] M.A.B. Beg and H.S. Tsao, Phys. Rev. Lett. 41 (1978) 278; R.N. Mohapatra and G. Senjanovic, Phys. Lett. B79 (1978) 278.
[11] A. Nelson, Phys. Lett. B136 (1984) 387; S. Barr, Phys. Rev. Lett. 53 (1984) 329; P.H. Frampton, Phys. Rev. Lett. 68 (1992) 2129; H. Georgi and S.L. Glashow, Phys. Lett. B451 (1999).
[12] We are not aware of any work which calculates an EDM of a paramagnetic atom, $d_{T1}$ or $d_{CS}$, induced by the theta term. One way to estimate the effect (M. Pospelov, 1992, unpublished) is to determine a size of the effective T-odd nucleon-electron interaction $NNe\gamma_5e$, induced by $\theta$ via a $\pi_0$-exchange. In this case CP violation resides in the $\pi N N$ vertex, R. Crewther et al., Phys. Lett. B88 (1979) 123, and $\bar{e}\gamma_5e\pi_0$ interaction can be extracted from the $\pi_0 \rightarrow e^+e^-$ branching ratio. The final estimate is given in the third line of eq. (2).
[13] J.P. Jacobs et al., Phys. Rev. Lett. 71 (1993) 3782.
[14] E.D. Commins et al., Phys. Rev. A50 (1994) 2960.
[15] I.B. Khriplovich, Yad. Fiz. 44 (1986) 1019 (Sov. J. Nucl. Phys. 44 (1986) 659). The value of $\theta$, induced by KM phase $\delta$ at the electroweak threshold is much bigger than the renormalization group mixing of $\theta$ and $\delta$ considered in J. Ellis and M. Gaillard, Nucl. Phys. B150 (1979) 141.
[16] W. Bernreuther and M. Suzuki, Rev. Mod. Phys. 63, 313 (1991).
[17] A. Zee, Phys. Rev. Lett. 55, 2382 (1985).
[18] D. Ng and J. Ng, Mod. Phys. Lett. A11, 211 1996. The two-loop diagrams considered in this paper have additional suppression factors driving $d_e$ below the level of $10^{-23} e\cdot cm$.
[19] S. P. Martin and M. T. Vaughn, Phys. Rev. D 50, 2282 (1994); Y. Yamada, Phys. Rev. D50 (1994) 3537 hep-ph/940241; I. Jack and D.R. Jones, Phys. Lett. B333 (1994) 372 hep-ph/940523.
[20] A. Dedes, A.B. Lahanas and K. Tamvakis, Phys. Rev. D53 (1996) 3793 hep-ph/9504239.