Reconstructions of the Strong Gravitational Lenses
MS 2137 and MS 1455 Using a Two-Stage Inversion Algorithm

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ABSTRACT

We propose a new, two-stage algorithm for inverting strong gravitational lenses. The key to the algorithm is decoupling the effects of lens magnification and intrinsic structure in the background source in the appearance of lensed arcs. First, the distribution of mass on the deflector plane and the geometry of the source-deflector-observer optical system are established. This is done by numerically simulating the lensing of light past a parametric mass model, and adjusting the handful of model parameters to match the positions and shapes of the observed and simulated lensed arcs. At the same time, this determines the magnification of the background source induced by the lensing process. The predicted magnification is then removed from the data to reveal the intrinsic, though still distorted, background distribution of light. After tracing each lensed ray back to the source plane, the data are combined to produce a surface brightness distribution of the source. This two-stage inversion scheme produces a parametric model of the deflector and a pixelized rendering of the background source which together mimic the observed gravitationally lensed features. We test the viability of scheme itself on a well-studied collection of lensed objects in the galaxy-cluster MS 2137. Confident in the algorithm, we apply it second time to predict the distribution of mass in the galaxy-cluster MS 1455 responsible for an observed triplet of lensed arcs. Our predictions about the lens in MS 1455 make it particularly interesting, for a single background source is responsible for both tangential arcs and a radial arc.

Subject headings: galaxies: clusters: individual (MS 2137, MS 1455)—gravitational lensing—methods: analytic, numerical

1. Introduction

The study of gravitational lensing has evolved from a novel application of General Relativity to an astronomical tool, for the analysis of lensing provides an additional, independent measure of the mass distribution of galaxies and clusters of galaxies. Comparison of the lensing mass and luminous mass, for instance, can begin to answer questions about the nature of the dark matter in these objects.
One way to extract the information encoded in the strong lensing behaviour of a cluster of galaxies is to produce a model of the mass distribution which, together with one or more luminous, background sources, produces the observed collection of lensed arcs and arclets. Several lens inversion schemes have been developed in the last 10 years to study lenses characterized by sets of compact lensed arcs, each set being multiple images of a background source. The complexity of the schemes and the strength of their predictions have increased along with advances in observations.

Kochanek & Narayan (1992) developed the LensClean algorithm, based on Kochanek’s earlier Ring Cycle (1989), to study lensing systems containing extended images, particularly Einstein Rings formed from background radio sources. The result is a discrete map of the mass distribution on the lens plane. The LensMEM routine of Wallington et al. (1996) introduces the maximum entropy method (MEM) into the inversion routine. Parametric model parameters are adjusted so that the background source needed to reproduce the observed lensed features is the “most probable...consistent with the data” and therefore the most “natural.”

Another family of lens inversion schemes is based upon parametric lens models where the nature of the deflector is specified, and only the values of the parameters are altered. These methods are based on the fact that when multiple images of a common background source are traced back through the parametric lens model, the pre-images must coincide on the source plane.

Mellier, Fort, & Kneib (1993) (hereafter M93) produce a parametric model of the mass distribution of the core of the galaxy-cluster MS 2137. A large pseudo-isothermal, elliptical mass distribution is postulated, and three lensed images are traced back to the source plane. Parameter values are selected by minimizing a $\chi^2$ statistic measuring the distances between the three pre-images on the source plane. Lensing in the galaxy-cluster A2218 is examined, first with ground-based data (Kneib et al. 1995) and later with HST observations (Kneib et al. 1996). In the latter, parameters describing four large galaxies and 30 smaller galaxies are fit through seven multiply-imaged background sources. Nair (1998) generates a parametric model reproducing the 10 lensed images observed in B1993 + 503 by minimizing distances between pre-images on the source plane, while at the same time demanding the lensed images show the correct parity. Tyson, Koczanski, & Dell’Antonio (1998) constrain a 512 parameter model of CL 0024+1654 by matching close to 4000 lensed pixels in HST and numerically simulated images. The stunning detail is the result of a very expensive computation.

High-resolution HST images of MS 2137 allow Hammer et al. (1997) (hereafter H97) to produce a more complex parametric model of the mass distribution of the cluster core, as well as reconstructions of the background sources. Values for the model parameters are estimated by selecting bright knots of light observed within multiple lensed images, a triplet of images of one source and a pair of images of a second source, and adjusting the parameters to make the pre-images of these knots coincide on the source plane. Then the lensed images in their entirety are traced back to the source plane to reconstruct the two sources.

It is critical in the lens inversion schemes of Mellier, Fort, & Kneib, Kneib et al., Hammer et al., and Nair that common structures or knots of light be identified in two or more lensed arcs. The model parameters are chosen by forcing these structures back to a common origin on source plane. Because of the extreme magnification that occurs near the critical lines of the lens, faint regions of the background which lie near the corresponding caustics may be greatly magnified and appear as bright knots of light. These

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1 Narayan & Nityananda 1986, 128.
2 Ibid., 137.
knots can be misidentified as coming from bright structures in the background source. Adjusting the model parameters to make coincident these faint regions and regions of the source which do show structure may lead to inconsistent models of the lens. This problem stems from the fact that the appearance of a lensed arc is the product of both the structure of background source and the effects of magnification of the lens.

The two-stage inversion algorithm is described in detail in Section 2, using the lensing observed in MS 2137 as a test case. In this Section, we introduce polar moments, statistics used to quantify the position and shapes of lensed arcs. In Section 3, the algorithm is applied to the galaxy-cluster MS 1455, producing a model which suggests that a single background source is responsible for both a tangential and a radial arc. Our model does not fully reproduce the lensing behaviour of MS 1455, revealing both shortcomings of the model and also the difficulties associated with inverse problems. In Section 4, we discuss strengths and weaknesses of the new inversion scheme, particularly the use of polar moments. Finally, our conclusions are summarized in Section 5.

2. A Two-Stage Inversion Algorithm

We introduce a new inversion algorithm for producing a model of the geometry of the gravitational lens, a parametric description of the cluster mass distribution, and a reconstruction of the background source. The key to this new approach is decoupling the effects of lens magnification and background source structure in the appearance of multiple lensed arcs. The algorithm is stated briefly here, and then illustrated with the well-studied gravitational lens MS 2137.

The first stage of the inversion is to build a parametric model of the deflector mass distribution, establish the redshifts of the lens and source planes, and determine the position of the background source, based only on the positions and shapes of the lensed arcs in the observations. As discussed below, the positions and shapes are characterized by polar moments. The model establishes a family of “conduits” through which the solutions to the lens equation pass. At the same time, the models fixes the magnification throughout the lens, so that the magnification factor at each point on the deflector plane can be calculated and removed from the data.

The second stage of the inversion algorithm is the reconstruction of the background source responsible for the observed arcs. Each pixel in the observations identified as containing lensed light is traced back to the source plane along the solution to the lens equation, and the magnification factor is removed. This produces a collection of points on the source plane, each point carrying the flux of the source as it would appear in the absence of lensing. There are many choices for interpolating this data across the source plane to build a pixelized image of the source. We adopt a simple strategy by choosing a uniform pixel size, and then setting the pixel size so that the number of pixels in the reconstructed source is comparable to the number of data, namely the number of lensed pixels identified in the observations.

As a test of consistency of the model, the reconstructed source is passed back through the model to check for spurious structure within the arcs, or spurious arcs altogether.

2.1. Arcs in MS 2137

To test the validity of the new lens inversion algorithm, the algorithm is applied to the lensed objects observed in the galaxy-cluster MS 2137, for which models have already been produced in M93 and H97.
A HST image of the centre of MS 2137 is shown in Figure 1. The central cD galaxy and a smaller cluster galaxy, identified as G1 and G7, respectively, in M93, have been removed to reveal more structure of the arcs. In previous models of MS 2137 and in the model below, the giant tangential arc A0 and two counter-images A2 and A4 are traced to one source in the background. The radial arc AR and its counter-image A6 are traced to a second source. As noted in H97, a third source produces the BR-B1 pair of arcs. Our model also predicts a fourth background source responsible for a further C1-CR pair of arcs.

To select values for the parameters in a parametric model of the lens, the positions and shapes of numerically simulated lensed arcs are “matched” to the positions and shapes of the observed arcs. Quantifying positions and shapes of lensed objects is the basis of the inversion-via-distortion techniques applied to weak lensing inversions (Kaiser & Squires 1993). In weak lensing analyses, the (flux-weighted) centroid and quadrupole moments of a weakly distorted background galaxy are calculated, and the galaxy is modeled as an equivalent ellipse. This analysis cannot be applied directly in the strong lensing regime because the arcs are not generally elliptical in shape (for example, the giant arc A0.) Instead, we introduce “polar moments” of the lensed images: The polar coordinates $(\theta, r)$ of pixels containing lensed light are interpreted as if they are coordinates in a Cartesian coordinate system. By summing over pixels about a chosen threshold, $I_o$, the following statistics are tabulated:

\[
Q_o = N \text{ (number of lensed pixels)}
\]
\[
\bar{r} = Q_o^{-1} \sum_{I_i > I_o} r_i
\]
\[
\bar{\theta} = Q_o^{-1} \sum_{I_i > I_o} \theta_i
\]
\[
Q_{rr} = Q_o^{-1} \sum_{I_i > I_o} (r_i - \bar{r})^2
\]
\[
Q_{r\theta} = Q_o^{-1} \sum_{I_i > I_o} (r_i - \bar{r})(\theta_i - \bar{\theta})
\]
\[
Q_{\theta\theta} = Q_o^{-1} \sum_{I_i > I_o} (\theta_i - \bar{\theta})^2
\]

These moments are not flux-weighted, but depend only on the positions of the lensed features on the image plane. The 0th moment $Q_o$ is the area of the image, in units of $\Omega = \Delta^2$ arcsec$^2$, where $\Delta$ is the arcsecond pixel length of the pixels in the observations. The radial moment $\bar{r}$ specifies the average radius of the lensed image, for there is just as much weight in the image outside the circle of radius $\bar{r}$ as there is inside this circle. In the case of giant arcs, $\bar{r}$ should closely approximate the Einstein Radius of the lens. The moment $\bar{\theta}$ specifies the average position angle of the lensed image: there is just as much weight clockwise from the line at position angle $\bar{\theta}$ as there is counter-clockwise from this line. As $\theta$ simply measures position angle on the sky, the magnitudes of $\bar{r}$ and $\bar{\theta}$ are incomparable.

The second polar moments of an arc characterize its shape: $Q_{rr}$ measures the radial spread of the arc,
while $Q_{\theta \theta}$ measures the angular spread. Analogous to the equivalent ellipse of weak lensing analyses, we construct a representative region based on the values of these components. A uniform rod of length $2L$ lying along the $x$-axis between $-L$ and $L$ has a second moment $Q_{xx} = \frac{1}{3}L^2$. The length can be recovered from the moment, $L = \sqrt{3Q_{xx}}$. We apply this result to the polar moments of the lensed arcs. The region lying between $\bar{r} \pm \sqrt{3Q_{rr}}$ and $\bar{\theta} \pm \sqrt{3Q_{\theta \theta}}$ is a rectangle in the Cartesian system, and what we refer to as an “annular sector” in the polar coordinate system.

The polar moments of the A0-A2-A4 and AR-A6 arcs are listed in Table 1. The quantities $r_i$ and $\theta_i$ which enter the polar moments in Equations (2)-(6) are simply the coordinates of the centre of each pixel containing lensed light, and are not based on the surrounding light distribution. The radial coordinate of any ray of light which strikes a lensed pixel is therefore accurate only to $\delta r = \Delta / 2$, which amounts to 0\'050 for the 0\'100 resolution of the HST observations. An uncertainty of $\Delta / 2$ arcseconds at a radius of $\bar{r}$ corresponds to an uncertainty in position angle

$$\delta \theta = \frac{90\Delta}{\pi \bar{r}} \text{ degrees} .$$

The annular sectors built from the quadrupole moments are shown in Figure 1, where each annular sector sits at the intersection of a circle of radius $\bar{r}$ and a radial line at position angle $\bar{\theta}$.

In weak lensing analyses, the coordinate frame in which the matrix of quadrupole moments is diagonal defines the principal axes of the equivalent ellipse. In the polar moments scheme, the off-diagonal moment $Q_{r \theta}$ cannot be interpreted so easily. This is due in part to the incomparable dimensions of the quadrupole moments. We can extract some information, nevertheless, from the sign of the $Q_{r \theta}$ moment. Spherically symmetric arcs have equal weight inside and outside the circle of radius $\bar{r}$, and equal weight clockwise and counter-clockwise from the radial line at position angle $\bar{\theta}$. The off-diagonal moment $Q_{r \theta}$ in Equation (5) vanishes. Outside (inside) the centroid circle, $r - \bar{r}$ is positive (negative); counter-clockwise (clockwise) from the centroid radial line, $\theta - \bar{\theta}$ is positive (negative). Thus the sign of $Q_{r \theta}$ shows any asymmetry of the image inside the annular sector. A lensed image rotated clockwise about the $(\bar{\theta}, \bar{r})$ centroid, like arc A6 in Figure 1, more heavily populates the regions where $Q_{r \theta} > 0$. Similarly, $Q_{r \theta} < 0$ for images rotated counter-clockwise with respect to the polar centroid, like the giant arc A0.

Comparing the polar moments tabulated for the different types of lensed arcs is revealing. In the weak lensing regime, the ratio of the major and minor axes of the equivalent ellipse gives a measure of the ellipticity of the distorted background galaxy. In the strong lensing regime, we define a shape parameter $\chi$ by measuring the ratio of the dimensions of the annular sector built from the polar quadrupole moments:

$$\chi = \frac{\text{tangential dimension}}{\text{radial dimension}} = \frac{\pi \bar{r} \sqrt{Q_{\theta \theta}}}{180 \sqrt{Q_{rr}}}. $$

A lensed feature which is circular produces $\chi \sim 1$. In the case of MS 2137, the giant tangential arc A0 shows $\chi \gg 1$, while the radial arc AR shows $\chi < 1$. The other arcs in the collection are tangentially distorted, with $\chi > 1$ in each case. The quantity $\chi$ may serve to distinguish between radial and tangential arcs, based only on their polar moments.

The small collection of polar moments defined in Equations (1)-(6) characterizes the position of the lensed arcs quite well. Four constraints can be extracted from each image: $\bar{r}$, $\bar{\theta}$, $Q_{rr}$, and $Q_{\theta \theta}$, or equivalently, $\bar{r}$, $\bar{\theta}$, the shape parameter $\chi$, and one dimension, $\sqrt{3Q_{rr}}$. Including the off-diagonal moment $Q_{r \theta}$ as a constraint is dubious, although matching its sign between the observations and simulations provides an additional check of the model.
2.2. Models of MS 2137

The arcs in MS 2137 are numerically simulated by ray-tracing through a parametric model. We model the dark-matter halo of cluster core with a large pseudo-isothermal, elliptical mass distribution (PID). The radial profile of the mass density is given by

\[ \rho_{PID}(r) = \frac{\sigma^2}{2\pi G r_c^2} \frac{1}{1 + (r/r_c)^2} \]

where \( \sigma \) is related to the line-of-sight velocity dispersion of the distribution, and \( r_c \) is the core radius of the distribution. A second, smaller mass distribution models the central cD galaxy, following a profile proposed by Miralda-Escudé (1995):

\[ \rho_{cD}(r) = \frac{\sigma^2}{2\pi G r_c^2} \frac{1 + r/r_c}{(1 + r^2/r_h^2)^2} \]

The parameter \( \sigma \) sets the mass of the cD, while the two scale parameters \( r_c \) and \( r_h \) control the shape. Both density profiles are adapted to elliptical distributions following the prescription of Schramm (1990).

The orientation and eccentricity of the cD are set to match observed values, where eccentricity is defined as \( \sqrt{1 - (b/a)^2} \) where \( a \) and \( b \) are the semi-major and semi-minor axes of the ellipse, respectively. The scale parameters of the cD are fit to the observed light profile through an iterative parameter estimation scheme. The mass parameter \( \sigma \), essentially the mass-to-light ratio, is a free parameter. The centre, orientation, and eccentricity of the PID are allowed to vary slightly from the central cD. The core radius \( r_c \) of the PID is also a free parameter. The mass of the PID \( \sigma \) is set to reproduce the observed line-of-sight velocity dispersion, following the description of Binney & Tremaine (1987) based on the Jeans’ Equation. The redshift of the deflector plane is set to the observed value of \( z_d = 0.313 \); the redshift \( z_s \) of the source plane is predicted by the model.

The free parameters are chosen by simulating the A0-A2-A4 triplet of arcs originating from a common source, S1. As only the positions of the lensed arcs are important at this stage, the lensed appearance of a uniform, elliptical background source is simulated and the polar moments of the resulting arcs are tabulated. The interactive simulation program immediately updates the lensing behaviour as the parameters are adjusted: coordinates in multiples of \( \Delta \), mass \( \sigma \) in steps of 25 km/s, redshifts in steps of 0.025, and each source’s semi-major axis in steps of 0.05, orientation in steps of 5°, and eccentricity in steps of 0.02. The position of the source is determined primarily by simultaneously matching the centroids \((\tilde{\theta}, \tilde{r})\) of the three arcs. Values for the size, eccentricity, and orientation of the background ellipse are set primarily by matching the quadrupole polar moments of the three arcs.

The values chosen for the parameters are contained in Table 2. The values are comparable to those found in M93 and H97, also listed in the Table. The simulation based on these model parameters is shown in Figure 2. Annular sectors around the simulated arcs are built from the moments listed in Table 3. The polar moments of the simulated arcs closely reproduce the observed moments of the A0-A2-A4 arcs. In particular, the shape parameter \( \chi = 11.4 \) identifies arc A0 as a giant tangential arc, \( \chi = 0.3 \) identifies AR as a radial arc, and the sign of the moment \( Q_{r\theta} \) correctly characterizes the asymmetry of each arc.

To further justify the choice of parameter values, a second source S2 is added at the same redshift as S1 (following H97) without changing the deflector mass distribution, to test the ability of the model to reproduce the AR-A6 pair of arclets. The second source and its two lensed images are included in Figure 2. The close match between the polar moments of the observed and simulated AR-A6 arcs in Tables 2 and 3 supports our selection of model parameters.
The two-mass, one-source model of MS 2137 is described by 20 parameters listed in Table 2. The redshift of the deflector plane, and the centre, orientation, eccentricity, and scale lengths of the cD galaxy are deduced from observations, leaving 13 free parameters. The three arcs A0, A2, and A4 provide 12 constraints on the model, leaving a one parameter family of models. Because the constraints do not directly measure the model parameters, there is not a particular parameter that can identified as the free parameter. Instead, some mixture of parameters varies in the family of models, for instance the product of distance and mass which enters the lens equation. The addition of a second background source S2, assumed to lie at the same redshift as S1, requires only 5 more parameters, while producing 8 data from the two new lensed features. Therefore, more information can be extracted from the model that is needed to produce it, and the results become predictive. The geometry of the model provides a ready explanation for lensed objects in the simulation which are independent of those used to constrain the model in the first place.

2.3. Reconstruction of the Sources

Uniform elliptical disks are used to model the background sources in the first stage of the lens inversion. With the lens geometry and deflector mass distribution established, these idealized sources can be replaced with distributions reconstructed from the data itself. The model for the lens specifies the origin on the source plane of each lensed pixel in the observations. Furthermore, the model parameters determine the magnification of the background at any point of the lens. To reconstruct the appearance of the background source(s), each pixel containing lensed light in the observations is traced back to the source plane along the solution to the lens equation and the magnification factor is removed from the data. This leaves a point on the source plane carrying the surface brightness of the source as it would appear in the absence of lensing. A coherent picture of the background source is constructed by interpolating between these points.

Data are uniformly spaced in the observations and cover the image plane exactly once. Because of the lensing distortions, data on the source plane do not inherit this simple structure, but cluster about the caustics of the lens. We choose a uniform pixel size, $\Delta_s$, to reconstruct a pixelized rendering of the background source. This is the simplest choice, and can surely be improved by exploiting the concentrations of data.

Several choices are available to set the size of the source pixels. Source pixels representing the same physical size on the source plane as the pixels in the observations represent on the deflector plane cannot contiguously cover the background plane, for the background is physically larger than the foreground. By choosing source pixels with the same angular size as the pixels in the observations, the background plane may be covered, but these source pixels extrapolate the information in the data over a much larger region. We adopt a strategy which is a compromise between these two choices: The size of the source pixels is set so that the total number of pixels in the reconstructed source is comparable to the number of data. The total number of pixels is used, not just those containing a signal, because blank (dark) regions on the source plane may be just as important as regions containing light.

The source plane is divided into a uniform grid, and a flux is assigned to each source pixel following these steps:

1. Source pixels which are not pierced by any backwards-traced rays are assigned a value of NaN, and appear in the result as pixels of zero flux.

2. A source pixel pierced by a single backwards-traced ray carrying an observed signal $I$ from a point
where the magnification is $\mu$ is assigned the de-magnified flux, $S = \mu^{-1}I$.

3. If $k$ backwards-traced rays, each carrying signal $I_i$, error $\sigma_i$, and magnification $\mu_i$, pierce the same source pixel, the pixel is assigned a flux $S$ which minimizes the error

$$\phi = \sum_{i=1}^{k} \left( \frac{I_i - |\mu_i|S}{\sigma_i} \right)^2.$$ 

In the observations of MS 2137, we identify 2800 pixels in arcs A0-A2-A4 coming from source S1, and 550 pixels in arcs AR-A6 coming from source S2, for a total of 3350 pixels containing lensed light. The two sources reconstructed from the 3350 pixels, following the strategy outlined above, are shown in Figure 3. The reconstruction of source S1 closely coincides the position of the elliptical disk used to simulate the arcs in Figure 2. The source lies on the astroid-shaped tangential caustic, producing the giant tangential arc A0. The reconstruction of source S2 coincides with the second elliptical disk added without altering the deflector mass parameters to check the consistency of the model. The second source crosses the radial caustic, producing the radial arc AR. Figure 3 contains 3400 pixels with length $\Delta_s = 0.092\,''$, slightly smaller than the $0.100\,''$ resolution of the HST data. The majority of the pixels in the reconstruction contain no signal, indicating the absence of additional background light on this source plane which could form additional arcs in the observations. This reconstruction is comparable to that shown in H97.

To better explore their internal structure, the two sources are reconstructed separately in Figure 4. In the reconstruction of source S2, there are 450 pixels, with length $\Delta_s = 0.074\,''$, comparable to the 550 data drawn from arcs AR and A6. There are only 513 pixels with length $\Delta_s = 0.074\,''$ in the reconstruction of source S1, despite the 2800 data coming from the A0-A2-A4 triplet of arcs. The reason for this discrepancy is that arcs A2 and A4 barely support a reconstruction at this resolution as they are only weak distortions of the data sampled at $0.100\,''$. The data arriving from the giant arc A0 samples the source plane at a much higher density because of the great distortion that occurs.

The reconstruction of source S1 in Figure 4 (right) shows a curious faint stripe which follows the caustic of the lens. It is inconceivable that the source truly has a dim region so perfectly aligned with the caustic, so the stripe must be a result of the modeling process. The numerical simulation of the lens in Figure 2 shows a peak in the brightness of arc A0 where the image crosses the tangential critical line and the magnification diverges. The observations of arc A0 in Figure 1 show the arc is very nearly uniform in brightness all the way along its length, however. The lack of a bright peak in the data along the critical line results in a dim reconstruction along the caustic. The stripe is also a result of the finite resolution of the simulation. The magnification is infinite along the critical line, and must be approximated. We impose an upper limit of $100\times$ magnification in the reconstruction routine. The approximation affects any pixel in observations through which the critical line passes. By running the simulation at twice the resolution, the chain of effected pixels remains, but with only one half the width. The effect remains at all finite resolutions, with the stripe becoming narrower and narrower. The overall change in brightness between the parts of the source inside and outside the caustic is due to the subtraction of galaxy G7 from the data.

### 2.4. Reconstruction of MS 2137

As a final test of the consistency of the model, the reconstructed sources shown in Figure 3 are passed back through the parametric model. The result is shown in Figure 6, where gaussian noise matching that in the data has been added. It is impossible to compare this Figure with the observations at a pixel-by-pixel
level without a complete model of the sky and a thorough understanding of the noise. It is apparent, though, that the prediction is consistent with the observations. We note in particular (i) the reproduction of brighter knots in arcs A2, A4, and A6, (ii) the twist in the radial arc AR, (iii) the double-ring structure in the giant arc A0, and (iv) the absence of any extraneous lensed objects.

In an ideal model, the re-lensed source perfectly reproduces the observations. Imperfections in the model are doubly amplified, though, once in each direction through the lens. The success of the results provides compelling evidence that gravitational lensing, at the level prescribed by this PID model, is actually occurring in the galaxy-cluster MS 2137.

More importantly, the results show that the two-stage inversion algorithm used to reconstruct MS 2137 is consistent with other algorithms that exist today. Fitting the lens with polar moments to decouple the effects of magnification and source structure in the observed arcs appears to be viable, at least in the cases of relatively simple mass distributions with a well defined centre-of-lensing. With this confidence, we turn to the collection of features attributed to gravitational lensing visible in the galaxy-cluster MS 1455.

### 3. A Radial Arc in MS 1455

The galaxy-cluster MS 1455+22, observed as part of the Einstein Medium Survey of X-ray clusters, lies at redshift $z = 0.257$. A candidate gravitationally lensed tangential arc was identified by LeFèvre et al. (1994). This prompted subsequent observations in May, 1995 at the Canada-France-Hawaii Telescope (CFHT) as part of a weak lensing survey of the cluster over a wide field of view. The core of the cluster appears in each of 12 overlapping 20-minute exposures. These frames are aligned and added with IRAF routines, resulting in an equivalent 4-hour exposure of the cluster core. A hint of a structure is visible in the envelope of the luminous central cD galaxy. When the cD galaxy is digitally removed, a collection of objects surrounding the core is revealed, as shown in Figure 6. These include several small cluster galaxies and an irregular radial feature labeled A1 in Figure 6, which we propose is a radial arc. The previously identified tangential arc is labeled A2. During the initial modeling phase of this gravitational lens, a third arc appeared in the simulations which closely matched the position of a third diffuse object in the observations. This arc, labelled A3 in Figure 6, is incorporated into the modeling strategy.

The lensed features in MS 1455 are similar to those seen in MS 2137. Both clusters contain a radial arc and a large tangential arc. The arcs in MS 2137 appear in two sets, the A0-A2-A4 triplet due to source S1, and the AR-A6 pair due to source S2. Our analysis suggests that the three arcs in MS 1455 are images of the same background source. Radial and tangential arcs are produced across the two different types of critical lines, which implies the single background source lies under both the tangential and radial caustic. As the caustics cross at only a limited number of points on the source plane, this greatly constrains the geometry of the lens.

From the position of 716 pixels in the observations containing lensed light, polar moments are tabulated for the three arcs, listed in Table 4. Pixels in the CFHT data are $\Delta = 0.207$ in length, producing an uncertainty of about $0.1$ in the radial positions. The shape parameter $\chi$ again distinguishes the radial arc A1 ($\chi = 0.4 < 1$) from the tangential arclet A2 ($\chi = 3.1 > 1$). The proximity of the radial arc to the centre-of-lensing produces a wide angular width $Q_{\theta\theta}$. The annular sectors built from the moments are included in Figure 6.
3.1. Parametric Models of MS 1455

A simple model of MS 1455 consists of a large mass distribution to model the halo of the cluster core, together with a smaller cD distribution at the centre, and a single background source. To begin to answer more astrophysical questions, we build two models with two different halo profiles. The first model contains a PID mass, while the second model uses a singular mass density profile proposed by Navarro, Frenk, & White (1995):

$$\rho_{NFW}(r) = \frac{\sigma^2}{2\pi G r_s} \frac{1}{(r/r_s)(1 + r/r_s)^2}.$$  

The NFW density diverges as $r^{-1}$ at the origin and falls off as $r^{-3}$ for $r \gg r_s$. The profile has a well-founded basis in the results of large numerical N-body simulations of cold dark matter.

In both the PID+cD and NFW+cD models, we allow the dark matter halo to wander slightly from the cD galaxy in position, orientation, and eccentricity. The redshift of the lens plane is observed to be $z_d = 0.257$, while the redshift $z_s$ of the source is a free parameter. The mass parameter $\sigma$ of the dark matter halo is set to reproduce the observed line-of-sight velocity dispersion of the several dozen cluster galaxies (Carlberg et al. 1996). The scale parameters of the cD are set to match the profile of the surface brightness. The values of the model parameters we choose are listed in Table 5. The significant difference in source redshift $z_s$ between the two models, 0.825 for the PID+cD model but only 0.620 for the NFW+cD model, may serve to distinguish between the two if future observations are made. Of the 20 parameters in the model, 8 are determined from the observations, leaving 12 free parameters. Three lensed arcs producing 12 constraints should be sufficient to constrain the parametric models presented here.

Figure 7 shows the numerical simulations of the PID+cD and NFW+cD lenses. Note how the single background source S1, represented by a dashed ellipse, lies under both the astroid caustic (forming arc A3) and the ovoid radial caustic (forming arc A1). The original tangential arclet A2 is the even parity image that forms outside the network of critical lines.

The polar moments of the arcs are listed in Table 6. In both models of the lens, the position of the tangential arclet A2 and the position angle of the radial arc A1 are more carefully matched to the observations. The polar moments of the third image, arc A3, are treated more as a consistency check. Because of the extreme distortion that occurs near the critical lines, the appearance of arc A3 in the simulations is very sensitive to small changes in the parameters, particularly in the position and shape of the background source. The magnification across the critical lines produces problems in the reconstruction discussed below.

3.2. Source Reconstruction

With the geometry of the lens fixed by the positions of the lensed arcs, the magnification of the background source is determined. Some 716 pixels in the observations are flagged as containing lensed light. These data are traced back to the source plane, and magnification factor is removed.

The source reconstructed behind the PID+cD lens is shown in Figure 8 (left). There are 780 pixels in the Figure with length $\Delta_s = 0'.145$, smaller than the 0'.207 resolution of the CHFT data. Figure 8 (right) shows the source reconstructed behind the NFW+cD model from 792 pixels with length $\Delta_s = 0'.132$.

Both reconstructions show the majority of the signal comes from a generally elliptical object with a brighter central bulge, very likely a spiral galaxy. This galaxy closely coincides with the position of the
uniform elliptical disk used to simulate lensed arcs. The data contained in the third lensed arc A3 are wholly responsible for the faint limb of the source which follows the astroid caustic to the upper-right. The flux is quite small because of the great magnification the signal experiences as it passes through the lens. It is likely the source is surrounded by a low surface brightness extension, but only a small portion of this can be seen through the high-magnification parts of the lens.

The collections of isolated pixels in the lower half of the plots are due to shortcomings of the model. We remove these spurious pixels by comparing each datum to its immediate neighbourhood, and discarding data straying farther than 3 standard deviations from the local average. The implications of this “cleaning” process are addressed below. The “cleaned” sources are shown in Figure 9. The spurious pixels are gone, but the faint limb responsible for the tangential arc A3, a feature supported by lensing within the context of the model, remains.

3.3. Reconstruction of MS 1455

According to our model of the lens and the source recovered from data, the lensed features in the observations should be reproduced by passing the reconstructed source back through the lens model. The results of this consistency check are shown in Figure 10. The simulations have been convolved against a Moffat point-spread function with $\beta = 2.5$ and radius $R = 0\farcs414$ (two pixels in the observations) corresponding to a FWHM of $0\farcs828$ (four pixels), recreating the seeing at the CFHT at the time of the observations. Gaussian noise matched to empty regions in the data has been added. There are two inconsistencies between the observations and the relensed, reconstructed source: The radial arc does not extend far enough towards the centre of the lens, and the third arc is much more extended than in the observations.

The first flaw can be traced directly to the data “cleaning” step. As each spurious datum is discarded, its origin in one of the arcs in the observations is flagged. These flagged data are concentrated entirely in the inner end of the radial arc. By discarding data on the source plane which originates in only one of three arcs, the reconstructed source fails to reproduce a portion of the radial arc without effecting the appearance of the two other arcs.

This behaviour shows that the models we propose for MS 1455 do not adequately model the mass distribution near the centre of the lens. Inconsistencies near the centre of the lens are not unexpected, however. Gravitational lensing does not uniformly measure the mass distribution of the deflector, but only the cumulative projected mass distribution. The lensing behaviour far from centre-of-lensing, but still within the strong regime, is insensitive to perturbations in the mass distribution at the lens centre. As demonstrated by Miralda-Escudé (1995) in the case of MS 2137, these same perturbations can remove the radial arc from the lens altogether, because of its proximity to the centre of the lens. In MS 1455, the other small galaxies in the vicinity of the cluster core and the radial arc undoubtedly play a role in the appearance of the radial arc. Refinements to the position and shape of the radial arc can be made by including more masses near the centre of the cluster. However, the small number of statistics we extract from the three lensed arcs does not support the inclusion of further masses.

Upon relensing the reconstructed source, the third arc A3 appears greatly extended. This can be traced to the effects of seeing in the data. Ideally, the data in any one arc can be traced back to the source plane, combined to reconstruct the source, and then traced forward through all three arcs. The ability of the data from one arc to reproduce three arcs is a measure of the success of the model. However, when the
data contained in arc A3 alone is used to reconstruct the source, only the faint limb of the source following
the astroid tangential caustic, and none of the elliptical bulge, is reconstructed. When this reconstructed
source is passed back through the lens, it forms a large tangential arc following the critical line of the lens,
the light having originated from the corresponding tangential caustic.

This behaviour indicates that the position of lensed light forming arc A3 in the observations is not due
entirely to gravitational lensing. Instead, as the simulations is Figure 7 suggest, a bright, very compact
arc forms at the location of arc A3. The image is smeared due to the effects of seeing. This broadens the
image in the observations, so that lensing is not wholly responsible for the position of lensed light in the
observations. To reproduce the enlarged arc, the inversion algorithm must reconstruct a larger background
source, which is then amplified into a larger relensed arc. This suggests that the data should be deconvolved
with a suitable point spread function before applying the inversion algorithm. Space-based observations of
MS 1455 may resolve this problem.

4. Discussion

The two-stage inversion algorithm described here decouples the effects of lens magnification and
intrinsic background source structure on the appearance of the lensed images. This allows for an independent
reconstruction of the background sources. That is, the natural appearance of the background sources is
not used to determine the parametric model. This is particularly important in the case of MS 1455, where
the faint limb which follows the tangential caustic does not coincide with the centre of the bright source
assumed to be responsible for all three arcs. Forcing the pre-images of the three arcs in MS 1455 to coincide
on the source plan leads to inconsistent lens reconstructions.

The polar moments approach was envisioned to describe galaxy-clusters with a single centre-of-lensing.
In some clusters, such as A2390 (Pierre et al. 1996), there appear to be more than one centre-of-lensing.
Because of the rapid decrease in deflection with distance from the mass centre, however, there may be arcs
formed primarily about one or the other centres-of-lensing, which could serve as the origin of the polar
analyses. Furthermore, relensing the reconstructed sources will test the consistency of the bimodel mass
distribution. Some suggestion of this occurs at the centre of MS 1455, where the relensed, reconstructed
radial arc suggests that additional masses are needed. In the case of A2390, a parametric lens model built
around two centres-of-lensing should still predict the existence of the “long straight arc” formed by light
squeezed between the two centres, even if the arc is not used to fit the parameters. Frye & Broadhurst
(1997) suggest that the “long straight arc” is in fact a superposition of two arcs, so this scenario might be
moot.

Further analysis of polar moments is needed, but already they have several favourable characteristics.
Only four statistics $\bar{r}$, $\bar{\theta}$, $Q_{rr}$, and $Q_{\theta\theta}$ are required to quite adequately characterize the position and shape
of a wide range of arcs. The shape parameter $\chi$ is a quantitative measure which can distinguish between
tangential and radial arcs. Finally, polar moments may act as a bridge between the strong and weak lensing
regimes, for the dimensions $\bar{r} \pm \sqrt{3Q_{rr}}$ and $\bar{\theta} \pm \sqrt{3Q_{\theta\theta}}$ of the annular sector built from the quadrupole
moments smoothly extend into the axes of the equivalent ellipse which characterizes the shape of a weakly
distorted background galaxy.

We have been unable to define a useful global statistic which can be minimized to produce a “best
model.” We could conceive of constructing a $\chi^2$-like measure which tabulates weighted differences between
the observed and simulated polar statistics. How the different statistics $\bar{r}$, $\bar{\theta}$, $Q_{rr}$, and $Q_{\theta\theta}$ should be
weighted, if at all, is unclear. This omission is due, in part, to the lack of a quantitative definition of a “best model” and a meaningful target value for the $\chi^2$-like statistic. At this time, we refrain from forming such a measure, and rely on the appearance of the relensed, reconstructed sources to check the consistency of the model.

5. Conclusions

We have introduced a two-stage lens inversion algorithm which decouples the effects of lens magnification and intrinsic source structure in the appearance of lensed arcs and arclets. The key to decoupling the deflector plane from the source plane is characterizing the positions and shapes of the lensed objects using polar moments. While these statistics are artificial, they have interesting and useful qualities. In reconstructing a background source from demagnified data, we adopt a simple strategy of generating a pixelized image of the source, where the resolution of the image is set so that the number of data in the reconstructed source is comparable to the number of data in the observations. Refinements of this strategy, such as adaptive gridding to take advantage of the concentration of data around the caustics, will be explored in the future.

It is not the goal of this paper to answer astrophysical questions about the nature of the cluster-galaxies MS 2137 and MS 1455. It is interesting to note, however, that both the non-singular PID and singular NFW halos are able to model the lensing behaviour of MS 1455. To begin to answer questions like these requires a detailed model of the gravitational lens. The algorithm described here offers an efficient and intuitive approach to generating such a model.

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Fig. 1.— WFPC2 image of MS 2137, showing lensed arcs A0-A2-A4, AR-A6, BR-B1, and CR-C1. The central cD galaxy and a smaller galaxy obscuring the upper end of arc A0 have been removed. A circle of radius $\bar{r}$ and a line at position angle $\bar{\theta}$ are drawn through the polar centroid of each arc. The shapes of the arcs are characterized by the annular sectors drawn around each arc.
Fig. 2.— Simulation of MS 2137. Dashed lines mark the critical lines of the lens; dotted lines trace the corresponding caustics. The PID mass M1 and cD mass M2 coincide at the centre of the frame. Small ellipses represent the positions of the two uniform, elliptical background sources S1 and S2. For each of the five arcs, a circle of radius $\bar{r}$, a radial line at position angle $\bar{\theta}$, and an annular sector are drawn.
Fig. 3.— Reconstruction of the two sources behind MS 2137, S1 at the bottom right and S2 at the top left. Dashed ellipses mark the location of the uniform elliptical disks used in the simulations. Both sources lie on caustics of the lens, traced with dotted lines. There are 3400 pixels in the frame, comparable to the 3350 data in the observations.
Fig. 4.— (left) Reconstruction of source S2. The 550 data in arcs AR and A6 are interpolated onto 450 source pixels. (right) Reconstruction of source S1. The 2800 data in arcs A0, A2, and A4 are interpolated onto only 513 source pixels because of the low density of data coming from the weakly distorted A2, A4 arcs.
Fig. 5.— The reconstructed sources are passed back through the lens, and noise mimicking the HST observations is added. Five arcs are reproduced.
Fig. 6.— MS 1455 with the central cD galaxy removed, revealing a candidate radial arc A1. Subsequent modeling of the lens suggests the object A3 is also a gravitationally lensed image. Circles and annular sectors show the polar moments of the arcs.
Fig. 7.— Simulations of MS 1455 with a single source behind a PID+cD lens (left) and the NFW+cD lens (right).

Fig. 8.— Source reconstructed behind the PID+cD lens (left) and the NFW+cD lens (right).
Fig. 9.— “Cleaned” source reconstructed behind the PID+cD lens (left) and the NFW+cD lens (right).

Fig. 10.— The relensed reconstructed sources in the PID+cD (left) and NFW+cD (right) models of MS 1455. In both cases, the radial arc does not extend far enough towards the centre of the lens, and the third arc is greatly extended.
Table 1. Polar Moments of Observed MS 2137 Arcs

| Arc | $\bar{r}$ (arcsec) | $\bar{\theta}$ (degrees) | $Q_{rr}$ | $Q_{r\theta}$ | $Q_{\theta\theta}$ | $\chi$ |
|-----|------------------|-----------------|--------|-------------|----------------|------|
| A0  | 15.40 ± 0.05     | −46.7 ± 0.2     | 0.15   | −2.1        | 222.7          | 10.4 |
| A2  | 13.07 ± 0.05     | −152.1 ± 0.2    | 0.37   | 0.2         | 11.9           | 1.3  |
| A4  | 19.45 ± 0.05     | 82.6 ± 0.1      | 0.09   | −0.5        | 19.9           | 5.0  |
| AR  | 5.43 ± 0.05      | −53.6 ± 0.5     | 1.07   | 1.5         | 5.7            | 0.2  |
| A6  | 24.20 ± 0.05     | 112.5 ± 0.1     | 0.19   | 0.4         | 3.4            | 1.8  |

Table 2. Model Parameters for Simulations of MS 2137

|           | Model | M93     | H97     |
|-----------|-------|---------|---------|
| Deflector redshift $z_d$ | 0.313 | 0.313   | 0.313   |
| Source redshift $z_s$     | 1.05  | 0.5 − 3 | 0.99 − 1.01 |

| Mass distribution | PID | cD | PID | cD | $\beta$-profile |
|-------------------|-----|----|-----|----|-----------------|
| Centre (arcsec)   | (0.00, 0.00) | (0.00, 0.00) | (0.0) | (0.0) | (0,0) |
| Orientation (degrees)$^a$ | 6.0 | 10.0 | 10.5$^{+5}_{-9}$ | 10.5 ± 5$^b$ | 4 − 10$^c$ |
| Eccentricity       | 0.65 | 0.45 | 0.62$^{+0.08}_{-0.11}$ | 0.59$^{+0.11}_{-0.18}$ | 0.583 ± 0.025$^e$ |
| Mass parameter $\sigma$ | 1150 | 500 | ... | ... | ... |
| $\sigma_{los}$ (km s$^{-1}$) | 1135 | 676 | 1000 | 350$^f$ | 1216 |

| Scale | $r_c = 4''0$ | $r_c = 0''51$ | $r_c = 8''0^{+0.5}_{-2.0}$ | $r_c = 1''5$ | $\beta = 0.875 ± 0.045$ | $r_c = 2''25 ± 1''75$ |
|-------|--------------|---------------|-----------------|-----|-----------------|-----------------|
|       | $r_h = 1''50$ |               |                 |     |                 |                 |

| Source model | S1 | S2 | S1 | S2 | S1 | S2 |
|--------------|----|----|----|----|----|----|
| Centre (arcsec) | (0.77, 1.78) | (−2.71, 5.63) | (1.0, 1.6)$^b$ | (−2.0, 4.7)$^b$ | 3.7 ± 0.3 separation |
| Semi-major axis (arcsec) | 0.65 | 0.65 | ... | ... | 0.58 ± 0.025 | ... |
| Orientation (degrees)$^a$ | 25.0 | 9.0 | ... | ... | ... | ... |
| Eccentricity | 0.74 | 0.74 | ... | ... | 0.81$^e$ | ... |

$^a$Counter-clockwise from positive $x$-axis.

$^b$Estimated.

$^c$Estimated, based on HST orientation.

$^d$Deduced from ellipticity $(a^2 − b^2)/(a^2 + b^2)$.

$^e$Deduced from ellipticity $1 − b/a$.

$^f$Derived from Faber-Jackson relation.
Table 3. Polar Moments of Simulated MS 2137 Arcs

| Arc | $\bar{r}$ (arcsec) | $\bar{\theta}$ (degrees) | $Q_{rr}$ | $Q_{r\theta}$ | $Q_{\theta\theta}$ | $\chi$ |
|-----|-------------------|--------------------------|----------|---------------|------------------|-------|
| A0  | 15.5 ± 0.2        | −44.7 ± 0.7              | 0.1      | −0.3          | 177.0            | 11.4  |
| A2  | 13.3 ± 0.2        | −151.8 ± 0.9             | 0.2      | 0.1           | 10.6             | 1.7   |
| A4  | 19.4 ± 0.2        | 84.9 ± 0.6               | 0.1      | −0.3          | 11.9             | 3.7   |
| AR  | 6.2 ± 0.2         | −54.5 ± 1.8              | 1.5      | 0.6           | 15.1             | 0.3   |
| A6  | 24.3 ± 0.2        | 109.2 ± 0.5              | 0.1      | 0.1           | 4.1              | 2.7   |

Table 4. Polar Moments of Observed MS 1455 Arcs

| Arc | $\bar{r}$ (arcsec) | $\bar{\theta}$ (degrees) | $Q_{rr}$ | $Q_{r\theta}$ | $Q_{\theta\theta}$ | $\chi$ |
|-----|-------------------|--------------------------|----------|---------------|------------------|-------|
| A1  | 5.1 ± 0.1         | −70.1 ± 1.2              | 3.9      | −3.9          | 61.9             | 0.4   |
| A2  | 20.5 ± 0.1        | 154.8 ± 0.3              | 0.4      | 1.8           | 29.8             | 3.1   |
| A3  | 18.3 ± 0.1        | 71.7 ± 0.3               | 0.7      | 1.1           | 6.7              | 1.0   |
Table 5. Model Parameters for Simulations of MS 1455

| Mass distribution | PID+cD | NFW+cD |
|-------------------|--------|--------|
| Centre (arcsec)   | PID    | cD     | NFW   | cD |
| Orientation (degrees) | 0.75   | 0.80   | 0.70  | 0.80 |
| Mass parameter σ  | 1143   | 500    | 2734  | 470 |
| σlos (km s⁻¹)     | 1133   | 687    | 1133  | 729 |
| Scale             | r_c = 4''0 | r_c = 0''0 | r_s = 40''0 | r_c = 0''0 | r_h = 1''50 | r_h = 1''50 |
| Source model      | S1     | S1     |
| Centre (arcsec)   | (-2''44, 3''59) | (-1''83, 2''65) |
| Semi-major axis (arcsec) | 1.50   | 1.20   |
| Orientation (degrees) | 25.0   | 30.0   |
| Eccentricity      | 0.82   | 0.78   |

aCounter-clockwise from positive x-axis.

Table 6. Polar Moments of Simulated MS 1455 Arcs

| Arc | $\bar{r}$ (arcsec) | $\bar{\theta}$ (degrees) | $Q_{rr}$ | $Q_{r\theta}$ | $Q_{\theta\theta}$ | $\chi$ |
|-----|---------------------|---------------------------|----------|---------------|-------------------|--------|
| PID+cD | A1 | 5.7 ± 0.2 | -69.4 ± 2.1 | 3.9 | -2.5 | 49.4 | 0.4 |
|       | A2 | 20.2 ± 0.2 | 153.6 ± 0.4 | 0.4 | 2.0 | 38.5 | 3.5 |
|       | A3 | 17.8 ± 0.2 | 70.1 ± 0.7 | 0.1 | 0.1 | 5.1 | 2.2 |
| NFW+cD | A1 | 6.4 ± 0.2 | -70.8 ± 1.9 | 4.8 | -1.4 | 31.6 | 0.3 |
|       | A2 | 20.3 ± 0.2 | 154.5 ± 0.6 | 0.3 | 1.8 | 37.6 | 4.0 |
|       | A3 | 17.9 ± 0.2 | 67.4 ± 0.7 | 0.1 | 0.1 | 11.6 | 3.4 |