Computational analysis of water based $Cu - Al_2O_3/H_2O$ flow over a vertical wedge

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Abstract
Theoretical and numerical investigation of the fluctuating mixed convection of hybrid nanofluid flow over a vertical Riga wedge is considered in this analysis. Two kinds of solid nanoparticles with base fluid at vertical Riga wedge is studied. Thermal and velocity slip impacts on vertical Riga wedge are investigated in the current study. We discussed both the unsteady and steady cases. The water has low thermal conductivity. We added the nanoparticle $Cu$ and $Al_2O_3$ which increases the thermal conductivity of the base fluid. This phenomena increase the heat transfer rate at the surface Riga plate. Partial differential equations are reduced into an ordinary differential equation by means of dimensionless similarity variables. The resulting ordinary differential equations are further solved through numerical and perturbation methods. Thickness of momentum boundary layer is reduced because of the solid nanoparticle rises in all cases of $\xi = 0, (2i\xi)^0, (2i\xi)^1$ and $(2i\xi)^n$. Our results are more agreeing with the decay results of Bachok et al. and Yacob et al. when rest of the physical parameters dimensions.

Keywords
Vertical Riga wedge, hybrid nanofluid, shooting method, thermal slip

Introduction
Heat transfer and boundary layer flow at vertical wedge have been considered much attention in recent days because of various physical applications of metallurgical and engineering, namely: plastic and metal extrusion, hot rolling, manufacturing of the fiberglass, crystal growing, wire drawing, manufacturing of paper and continuous casting. In the early time, Crane$^1$ pioneered boundary layer flow caused by stretching the surface. Carragher and Crane$^2$ highlighted that the temperature difference between the ambient fluid and surface was proportional to the power of the distance from the fixed point. The conduct of Newtonian nanofluids could be valuable in assessing the chance of heat transfer improvement in different procedures of these industries. Wu$^3$ studied the boundary layer flow of water entry of twin wedges numerically. Micropolar fluid flow at convective stretching sheet discussed by Haq et al.$^4$. Ramesh$^5$ explored the results of slip effects with convective condition of peristaltic flow in a porous medium. Ramesh$^6$ investigated the effects of inclined MHD in the inclined asymmetric channel. Gireesha et al.$^7$ investigated the hall impacts with MHD nanofluid and also worked on irregular heat consumption/generation.

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Gireesha et al. discussed the impacts of hall and thermal radiation impacts at porous medium in the presence of suspension of fluid particles. Mahanthesh explored the influence of dusty liquid with hall effects at the stretching sheet. Gireesha et al. analyzed the hall effects on nanomaterial fluid at linear stretching sheet under the hall effects. Gireesha et al. examined the dusty fluid flow with hall impacts under the nonlinear radiation at the stretching plate. Shalini and Mahanthesh explained the nanomaterial dusty fluid with Rayleigh-Benard convection without/with Coriolis force. Recently, few investigators have discussed the behavior of fluid at various stretching surfaces under various assumptions at sheet see Ramesh et al., Mahanthesh et al. and Wakif et al.

Within the past few a protracted time, researchers have confronted a substantial issue of thermal efficiency in numerous engineering and industrial applications. Advance technology such as microfluidics, optical, chemical synthesis, high speed microelectronics, transportation, microsystems including electrical components and mechanical bears high thermal loads. In such days, cooling specific systems is a key problem. The rate of heat transfer is typically enhanced by adding the region accessible for heat transfer. A further approach is to use a thermally effective fluid, which is typically distributed inside traditional heat transfer fluids, such as water, propylene glycol, ethylene glycol, etc. Heat transfer is most important in these days due to an energy crisis in the whole world. Choi and Eastman was the first person who worked on nanofluid. He used the nanoparticle with base fluid like water, oil, and ethylene glycol. Before this idea, several authors analyzed the heat transfer rate enhancement. But Choi and Eastman achieved heat transfer rate more than two times as compared to conventional flow because low thermal conductivity. Lee et al. used the oxide nanoparticle. The properties of the nanofluid discussed by Das et al. They additionally brought a physically and chemically method for synthesizing nanofluids and also defined the methods for making a stable suspension of nanoparticles with the base fluids. They also analyzed the stagnation point flow at the convective stretching sheet. Mixed convection of nanomaterial fluid flow at porous wedge explored by Ellahi et al. Most interesting research has done by few authors on nanofluid in Ellahi et al., Usman et al., Sheikholeslami et al., Ramesh and colleague, Zeeshan et al., Liu et al.

Hybrid nanofluid more important role in the heat transfer enhancement because of their lots of applications namely, biomedical, aselectronic cooling, defense, generator cooling, automobile radiators, nuclear system cooling, lubrication, coolant in machining, welding, thermal storage, solar heating, cooling and heating in buildings, drug reduction and biomedical etc. At the mechanical level, there are positive highlights like chemical soundness and tall thermal efficiency, which empowers them to perform efficiently as compared to nanofluids. Furthermore, hybrid words were introduced by Suresh et al. He claimed that two different kinds of particles are used with fluid. He proved that the heat transfer rate enhancement was two times more than nanofluid. The experimental results of hybrid nanofluid explored by Suresh et al. and Labib et al. The turbulent flow of hybrid nanofluid analyzed at a low volume concentration by Sundar et al. Stagnation point flow of hybrid nanomaterial fluid flow explored by Nadeem et al. Slip effects of hybrid nanomaterial fluid flow at cylinder analyzed by Nadeem and Abbas. Several scientists are being researched the heat transfer of hybrid nanofluid for different physical assumptions reported in Nadeem and colleagues, Mahanthesh and colleagues, Nehad et al. Mixed convection of hybrid nanofluid flow dusty fluid over a vertical Riga wedge plate is considered. Mathematical model has been constructed on the assumptions of fluid flow. The partial differential equations have been converted into systems of ordinary differential equations using the suitable similarity transformations. Nonlinear ordinary differential equations are solved through perturbation scheme and numerically through BVP4C method. The solutions are deliberated for numerous physical parameters.

Mathematical formulations

Consider the mixed convection fluctuating flow of hybrid nanofluid over a vertical Riga wedge in the presence of solid nanoparticle. The coordinate system and flow configuration reveals in Figure 1. The thermal and momentum boundary layer thickness is represented by δ_M and δ_T. It is considered that both the free stream and surface temperature present small amplitude oscillations in time about steady, non-zero free stream velocity and mean temperature. The electro-magneto-hydrodynamic applied on the vertical wedge see Wakif et al. Under the assumption and usual boundary layer approximation, the following mathematical system has been considered as below

$$\frac{\partial V}{\partial X} + \frac{\partial W}{\partial Y} = 0,$$  \(1\)
\[
\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + W \frac{\partial V}{\partial y} = - \frac{\partial P}{\partial x} + \left( \rho_{l/hf} \right) \frac{\partial^2 V}{\partial y^2} + \left( \rho \right) \beta g \cos \left( \frac{\pi}{4} \right) \frac{T}{T_{\infty}} + \frac{\pi J_m M_0}{\delta \rho_{l/hf}} \exp \left( - \frac{\pi}{a} Y \right),
\]

\[
= \alpha_{l/hf} \frac{\partial^3 T}{\partial y^2},
\]

(2)

Where, \( T = T - T_{\infty} \), the boundary conditions are as following

\[
V = \lambda \frac{\partial V}{\partial x}, \quad W = 0, \quad \bar{T} = T_w + \lambda_1 \kappa_{l/hf} \frac{\partial T}{\partial y}, \quad \text{at} \ Y \to 0
\]

\[
V \to V_0(X, t), \quad \bar{T} = T_{\infty} = 0, \quad \text{at} \ Y \to \infty,
\]

(4)

The coefficient of the volumetric expression for temperature expresses as \( \beta, V_0(X, t) \) be the fre stream velocity, \( V \) and \( W \) be the local velocity components of the fluid, \( \rho_{l/hf}, \beta_{l/hf}, \kappa_{l/hf} \) and \( \rho C_{p_{l/hf}} \) is the density, viscosity, thermal conductivity and heat capacity of hybrid nano fluid. Under these conditions, we combine the energy equations (3) and (6), momentum equations (2) and (5) for the two phases which we expressed as

\[
\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + W \frac{\partial V}{\partial y} = - \frac{\partial P}{\partial x} + \left( \rho_{l/hf} \right) \frac{\partial^2 V}{\partial y^2} + \left( \rho \right) \beta g \cos \left( \frac{\pi}{4} \right) \frac{T}{T_{\infty}} + \frac{\pi J_m M_0}{\delta \rho_{l/hf}} \exp \left( - \frac{\pi}{a} Y \right),
\]

\[
= \alpha_{l/hf} \frac{\partial^3 T}{\partial y^2},
\]

(6)

Corresponding boundary conditions are

\[
V = \lambda \frac{\partial V}{\partial x}, \quad W = 0, \quad \bar{T} = T_w + \lambda_1 \kappa_{l/hf} \frac{\partial T}{\partial y}, \quad \text{at} \ Y \to 0
\]

\[
V \to V_0(X, t), \quad \bar{T} = T_{\infty} = 0, \quad \text{at} \ Y \to \infty,
\]

(8)

Where

\[
V_0(X, t) = V_0(X)[1 + \epsilon \cos(\omega t)] \quad \text{and} \quad T_0(X, t) = T_0(X)[1 + \epsilon \cos(\omega t)]
\]

(9)

The mean surface temperature and mean velocity represent as \( T_0(X) \) and \( V_0(X) \), \( \epsilon \) is the amplitude of oscillation which is \( \epsilon \ll 1 \) while the frequency of oscillation is \( \omega \).

**Solution procedure**

On the behalf of the boundary conditions given in equation (9), we deliberate solutions of the equations (6–8) in the following form

\[
V(X, Y, t) = V_1(X, Y) + \epsilon V_2(X, Y) \exp(\imath \omega t),
\]

(10)

\[
W(X, Y, t) = W_1(X, Y) + \epsilon W_2(X, Y) \exp(\imath \omega t),
\]

(11)

\[
\bar{T}(X, Y, t) = T_1(X, Y) + \epsilon T_2(X, Y) \exp(\imath \omega t),
\]

(12)

Where \( V_1, V_2 \) are the fluctuating parts and \( W_1, W_2 \) are the velocity components. \( V, W \) and \( T_1, T_2 \) are the real and fluctuating part of the temperature function \( T \). The predictable solution functions are the real parts of the functions known in equation (10). The thermophysical properties are presented in Tables 1 and 2. After applying the equation (10) in equations (6–9) and getting the terms and conditions to 0(\( \epsilon \)), we obtained succeeding equations

\[
\frac{\partial V}{\partial x} + \frac{\partial W}{\partial y} = 0,
\]

(13)

\[
V_1 \frac{\partial V}{\partial x} + W_1 \frac{\partial V}{\partial y} = \frac{\partial^2 V}{\partial y^2} + \frac{\pi J_m M_0}{\delta \rho_{l/hf}} \exp \left( - \frac{\pi}{a} Y \right),
\]

\[
= \alpha_{l/hf} \frac{\partial^3 T}{\partial y^2},
\]

(14)

With the boundary conditions are
Table 1. Thermo-physical properties.

| Properties          | Nanofluid   | Hybrid nanofluid   |
|---------------------|-------------|-------------------|
| Density             | \( \rho_d = (1 - \phi) \rho_t + \phi \rho_f \) | \( \rho_{hf} = \left\{ \left[ (1 - \phi^2) (1 - \phi) \rho_1 \right. \right. | \rho_f \rho_s \right\} + \phi \rho_f \rho_s \) |
| Heat capacity       | \( (\rho C_p)_d = (1 - \phi) (\rho C_p)_t + \phi (\rho C_p)_s \) | \( (\rho C_p)_{hf} = \left\{ \left[ (1 - \phi^2) (1 - \phi) (\rho C_p)_1 \right. \right. | \rho_f \rho_s \right\} + \phi \rho_f (\rho C_p)_s \) |
| Viscosity           | \( \mu_d = \frac{\mu_t}{(1 - \phi)^{2T}} \) | \( \mu_{hf} = \left\{ \left[ (1 - \phi^2) (1 - \phi) \mu_t \right. \right. | \phi \mu_f \right\} + (1 - \phi) \mu_t \) |
| Thermal conductivity| \( \kappa_d = \frac{k_1 + (n - 1)k_t - (n - 1) \phi (k_1 - k_t)}{k_1 + (n - 1)k_t + \phi (k_1 - k_t)} \) | \( \kappa_{hf} = \frac{k_1 + (n - 1)k_t + \phi (k_1 - k_t)}{k_1 + (n - 1)k_t + \phi (k_1 - k_t)} \) |

Now we consider functions \( V_0 = V_0(X, t) \) like as \( V_0(X) = V_c X^{1/2} \) and \( T_0(X) = T_c \) while \( V_c \) and \( T_c \) are the constants. We introduced the similarity following transformations are

\[
\Psi_1 = \nu Y \sqrt{Re} \left( 1 + Pr \right)^{3/4} F(\eta), \quad T_1 = T_n \Theta(\eta),
\]

\[
\Psi_2 = \nu Y \sqrt{Re} \left( 1 + Pr \right)^{3/4}, \quad T_2 = T_n \left( \eta, \xi \right), \quad \eta = Y \sqrt{Re} \left( 1 + Pr \right)^{1/4}, \quad \xi = \frac{\omega}{\sqrt{X}}
\]

Where \( \Psi_1 \) and \( \Psi_2 \) are the stream function which satisfy the equations (11) and (15) correspondingly. The similarity and local frequency variables are \( \eta \) and \( \xi \). By applying the similarity transformation equation (23) on equations (20–22), we get the dimensionless form

\[
(1 + Pr) \nu_{hf} F'''' = \frac{3}{4} \sqrt{F''} F'''' - \frac{1}{2} \left( F' F'' - 1 - Pr \right)
\]

\[
+ \frac{\rho_t \text{Ri}}{(1 + Pr) \rho_{hf}} \Theta + \left( 1 + Pr \right) \frac{\rho_t M}{\rho_{hf}} \exp \left( -\sigma(1 + Pr)^{1/4} \right), \quad F(\eta) = 0, \quad F'(\eta) = \gamma \rho_{hf} \mu_f \left( 1 + Pr \right)^{3/4} F'(\eta),
\]

\[
\Theta(\eta) = 1 + \delta \frac{k_{hf}}{k_t} \left( 1 + Pr \right)^{1/4} \Theta(\eta), \quad \eta \to 0,
\]

\[
F'(\eta) \to \left( 1 + Pr \right)^{1/2}, \quad \Theta(\eta) \to 0, \quad \eta \to \infty.
\]

The steady part of the flow presented as follows

\[
(1 + Pr) \rho_{hf} G'''' + \frac{3}{4} (FG'' + F''G) - F' G'' + i \xi
\]

\[
\left\{ 1 + Pr - (1 + Pr)^{3/4} G' \right\} + 1 + Pr (1 + Pr) \frac{\rho_t \text{Ri}}{\rho_{hf}} H
\]

\[
+ (1 + Pr) \frac{\rho_t M}{\rho_{hf}} \exp \left( -\sigma(1 + Pr)^{-1/4} \right) = \frac{1}{2} \xi \left( F' \frac{\partial G'}{\partial \xi} - F'' \frac{\partial G}{\partial \xi} \right),
\]
Putting these values in equations (25–26), than equating of terms are only valid for the small frequency range’.

Respected boundary conditions

\[ G(\eta) = 0, G'(\eta) = \gamma \frac{\mu_{\text{nf}}}{\mu_f} (1 + Pr)^{\frac{3}{2}} G''(\eta), \]

\[ H(\eta) = 1 + \delta \frac{K_{\text{nf}}}{K_f} (1 + Pr)^{1/4} H'(\eta), \eta \to 0, \]

\[ G'(\eta) \to (1 + Pr)^{1/2}, H(\eta) \to 0, as \eta \to \infty, \] (27)

**Perturbation solutions for small \( \xi \)**

To gain the impacts of mixed convection near the leading edge, the results based on the series ‘finite numbers of terms are only valid for the small frequency range’. So

\[ G(\eta, \xi) = \sum_{n=0}^{\infty} (2\xi)^n G_n(\eta), H(\eta, \xi) = \sum_{n=0}^{\infty} (2\xi)^n H_n(\eta) \] (28)

Putting these values in equations (25–26), than equating the coefficients of \((2\xi)^n\)

\[ (1 + Pr)v_{\text{nf}} G_0'' + \frac{3}{4}(FG_0'' + F''G_0) \]

\[ - F'G_0' + 1 + Pr + (1 + Pr) \frac{\rho Ri}{\rho_{\text{nf}}} H_0 = 0, \]

\[ + (1 + Pr) \frac{\rho M}{\rho_{\text{nf}}} \exp\left(-\sigma(1 + Pr)^{-1/4}\right) = 0, \] (29)

\[ 1 + \frac{Pr}{Pr} \alpha_{\text{nf}} H_0'' + \frac{3}{4}(FH_0'' + \Theta'G_0) = 0, \] (30)

Respected boundary conditions are

\[ G_0(\eta) = 0, G_0'(\eta) = \gamma \frac{\mu_{\text{nf}}}{\mu_f} (1 + Pr)^{\frac{3}{2}} G_0''(\eta), \]

\[ H_0(\eta) = 1 + \delta \frac{K_{\text{nf}}}{K_f} (1 + Pr)^{1/4} H_0'(\eta), \eta \to 0, \]

\[ G_0'(\eta) \to (1 + Pr)^{1/2}, H_0(\eta) \to 0, as \eta \to \infty, \] (31)

Equating the coefficients of \((2\xi)^n\)

\[ (1 + Pr)v_{\text{nf}} G_1'' + \frac{3}{4}FG_1'' + \frac{5}{4}F''G_1 - \frac{3}{2}F'G_1' \]

\[ + \frac{\rho Ri}{\rho_{\text{nf}}} (1 + Pr)H_1 + \frac{1}{2}(1 + Pr) - \frac{1}{2}(1 + Pr)^{1/2}G_1'(\eta) = 0, \] (32)

\[ \frac{1 + Pr}{Pr} \alpha_{\text{nf}} H_1'' + \frac{3}{4}FH_1'' + \frac{5}{4}FG_1'' - \frac{1}{2}F'H_1 \]

\[ - \frac{1}{2}(1 + Pr)^{1/2}H_0 = 0 \] (33)

Relevant boundary conditions are

\[ G_1(\eta) = 0, G_1'(\eta) = \gamma \frac{\mu_{\text{nf}}}{\mu_f} (1 + Pr)^{\frac{3}{2}} G_1''(\eta), \]

\[ H_1(\eta) = \delta \frac{K_{\text{nf}}}{K_f} (1 + Pr)^{1/4} H_1'(\eta), \eta \to 0, \]

\[ G_1'(\eta) \to 0, H_1(\eta) \to 0, as \eta \to \infty, \] (34)

Equating the coefficients of \((2\xi)^n\)

\[ (1 + Pr)v_{\text{nf}} G_n'' + \frac{3}{4}FG_n'' + \frac{3}{4}F''G_n - \frac{1}{2}F'G_n' \]

\[ + \frac{\rho Ri}{\rho_{\text{nf}}} (1 + Pr)H_n + \frac{1}{2}(1 + Pr) - \frac{1}{2}(1 + Pr)^{1/2}G_n'(\eta) = 0, \]

\[ \frac{1 + Pr}{Pr} \alpha_{\text{nf}} H_n'' + \frac{3}{4}FH_n'' + \frac{5}{4}FG_n'' - \frac{1}{2}F'H_n \]

\[ - \frac{1}{2}(1 + Pr)^{1/2}H_{n-1} = 0 \] (35)

Relevant boundary conditions are

\[ G_n(\eta) = 0, G_n'(\eta) = \gamma \frac{\mu_{\text{nf}}}{\mu_f} (1 + Pr)^{\frac{3}{2}} G_n''(\eta), \]

\[ H_n(\eta) = \delta \frac{K_{\text{nf}}}{K_f} (1 + Pr)^{1/4} H_n'(\eta), \eta \to 0, \]

\[ G_n'(\eta) \to 0, H_n(\eta) \to 0, as \eta \to \infty, \] (37)

**Result and discussions**

The influence of the physical parameters on the heat transfer and skin fraction is presented in Table 3. The skin fraction improved because of improving the solid nanoparticle. Heat transfer rate declines as well as solid nanoparticle enhances which reveals in Table 3. The heat transfer rate \((2\xi)^n\) (Unsteady flow) achieve much higher than that of the \( \xi = 0 \) (Steady flow) in case of the \( Re^{1/2} \) \( C_f \) and \( Re^{1/2} N_{uf} \). Skin friction reduces when the slip parameter rises. The thickness of momentum is reduced by cause of the enhancing velocity slip parameter. While the heat transfer improved with improving the velocity slip parameter. Because thickness of thermal boundary rises by cause of velocity slip enhances. Incase of \( \xi = 0 \), variation of the modified Hartman number on the skin friction and heat transfer revealed in Table 3. Skin friction reduces when the modified Hartman number rises. The thickness of momentum is reduced by cause of the enhancing modified Hartman number but opposite results are achieved for the \((2\xi)^n\). While the heat transfer reduced with improving the modified Hartman number because
thickness of thermal boundary declines by cause of velocity slip enhances but opposite results are achieved for the \((2i\xi)^n\). In case of \(\xi = 0\), variation of the Richardson number on the skin friction and heat transfer revealed in Table 3. Skin friction improves when the Richardson number rises. The thickness of momentum improves by cause of the enhancing Richardson number, but opposite results are achieved for the \((2i\xi)^n\). In case of \(\xi = 0\), the heat transfer improved with improving the dimensionless number because thickness of thermal boundary improves by cause of velocity slip enhances but opposite results are achieved for the \((2i\xi)^n\). Impacts of the physical parameters on the modified Hartman number, thermal slip, velocity slip and solid nanoparticle on temperature gradient highlighted in Figures 2–5 for the various cases \(\xi = 0\), \((2i\xi)^0\), \((2i\xi)^1\) and \((2i\xi)^n\). Figure 2 reveals the variation of thermal thickness and thermal slip. Results are achieved that thermal thickness reduced by cause of improving the thermal slip parameter for all cases of \(\xi = 0\), \((2i\xi)^0\), \((2i\xi)^1\) and \((2i\xi)^n\). But in case of the thermal thickness reduced \((2i\xi)^0\), \((2i\xi)^1\) and \((2i\xi)^n\) respectively, as the values of \(n\) enhances at the surface of Riga vertical wedge.  

Table 3. Numerical results of heat transfer and skin friction.

| Physical parameters | Steady case | Unsteady case |
|---------------------|-------------|---------------|
| \(\Phi_2\) | \(\gamma\) | \(\delta\) | \(M\) | \(R_i\) | \(\sigma\) | \(Re_{\chi}^{1/2}Cf\) | \(Re_{\chi}^{1/2}Nu_x\) | \(Re_{\chi}^{1/2}Cf\) | \(Re_{\chi}^{1/2}Nu_x\) |
| 0.005 | 0.3 | 0.4 | 0.5 | 0.4 | 0.4 | 1.3863 | 0.6330 | 17.8755 | 1.3177 |
| 0.02 | | | | | | 1.4122 | 0.5991 | 18.1203 | 1.2363 |
| 0.04 | | | | | | 1.4489 | 0.5572 | 18.4219 | 1.1373 |
| 0.06 | | | | | | 1.4880 | 0.5185 | 18.6809 | 1.0877 |
| 0.04 | 0.0 | 0.3 | 3.0803 | 0.4774 | 53.3095 | 1.0877 |
| 0.6 | 0.8782 | 0.5690 | 10.5880 | 1.1343 |
| 0.3 | 0.0 | 1.5126 | 1.2494 | 18.2691 | 1.7630 |
| 0.2 | 1.4689 | 0.7694 | 18.1219 | 2.3232 |
| 0.4 | 1.4489 | 0.5572 | 17.8755 | 1.1373 |
| 0.6 | 1.4373 | 0.4370 | 17.4383 | 0.6821 |
| 0.4 | 0.0 | 1.7618 | 0.4498 | 1.4169 | 0.1913 |
| 0.5 | 1.4373 | 0.4370 | 18.2691 | 0.6821 |
| 1.0 | 1.0372 | 0.4172 | 40.1399 | 1.5455 |
| 0.5 | 0.0 | 1.3998 | 0.4354 | 18.7622 | 0.7086 |
| 0.4 | 1.4373 | 0.4370 | 18.2691 | 0.6821 |
| 0.8 | 1.4789 | 0.4385 | 17.8242 | 0.6584 |
| 0.4 | 0.0 | 1.2104 | 0.4244 | 25.2105 | 1.0087 |
| 0.2 | 1.3646 | 0.4333 | 20.7774 | 0.7801 |
| 0.4 | 1.4373 | 0.4370 | 18.2691 | 0.6821 |
| 0.6 | 1.4817 | 0.4392 | 16.5176 | 0.6145 |

Table 4. Numerical results of Bachok et al.\(^35\) and Yacob et al.\(^36\) compared with present results with \(\Phi_1 = 0\).

| Cu/H\(_2\)O | Present results | Bachok et al.\(^35\) | Yacob et al.\(^36\) |
|-------------|----------------|---------------------|---------------------|
| \(\Phi_2\) | \(\lambda\) | \(\sqrt{Re_{\chi}Cf}\) | \(\sqrt{Nu_x\over Re_{\chi}}\) | \(\sqrt{Re_{\chi}Cf}\) | \(\sqrt{Nu_x\over Re_{\chi}}\) | \(\sqrt{Re_{\chi}Cf}\) | \(\sqrt{Nu_x\over Re_{\chi}}\) |
| 0.1 | 0.0 | 1.7968 | 1.4043 | 1.8843 | 1.4043 | 1.8843 | 1.4043 |
| 0.2 | 0.5 | 2.4589 | 1.6421 | 2.6226 | 1.6692 | 2.6226 | 1.6692 |
| 0.1 | 0.0 | 1.0795 | 1.7895 | 1.0904 | 1.8724 | | |
| 0.2 | 0.5 | 1.5004 | 2.0987 | 1.5177 | 2.1577 | | |
increases due to enhancing the thermal conductivity of the fluid increases. When the solid nanoparticles added in the base fluid which enhances the thermal conductivity. But in case of the thermal thickness improved \(2i\xi^0\), \((2i\xi)^1\) and \((2i\xi)^n\) respectively, as the values of \(n\) enhances at the surface of Riga vertical wedge. Figure 4 reveals the variation of thermal thickness and modified Hartman number. Results are achieved that thermal thickness raised by cause of improving the modified Hartman number for all cases of \(\xi = 0\), \((2i\xi)^0\), \((2i\xi)^1\) and \((2i\xi)^n\). But in case of the thermal thickness improved \((2i\xi)^0\), \((2i\xi)^1\) and \((2i\xi)^n\) respectively, as the values of \(n\) enhances at the surface of Riga vertical wedge. This phenomena exist due to Lorentz forces \(F_m = (-F_m, 0, 0)\) which explained physically. The electromagnetic forces \(F_m = \left(\frac{1}{\rho_m}\right) \frac{\pi h M_v}{\beta} \exp \left(-\frac{\pi}{\eta} \gamma\right)\) worked as opposite along to the motion of fluid as drag-like forces which successful extents are rotting exponentially along the \(y\) axis. Figure 5 reveals the variation of thermal thickness and velocity slip. Results are achieved that thermal thickness reduced by cause of improving the velocity slip parameter for all cases of \(\xi = 0\), \((2i\xi)^0\), \((2i\xi)^1\) and \((2i\xi)^n\). But in case of the thermal thickness reduced \((2i\xi)^0\), \((2i\xi)^1\) and \((2i\xi)^n\) respectively, as the values of \(n\) enhances at the surface of Riga vertical wedge. Impacts of the physical parameters on
the modified Hartman number, thermal slip, velocity slip and solid nanoparticle on the velocity distribution highlighted in Figures 6–9 for the various cases $\xi = 0$, $(2i\xi)^0$, $(2i\xi)^1$ and $(2i\xi)^n$. Variation of the thermal slip and velocity profile presented in Figure 6. Thickness of momentum reduced by cause of the thermal slip rises in all cases of $\xi = 0$, $(2i\xi)^0$, $(2i\xi)^1$ and $(2i\xi)^n$. Momentum thickness decreases by cause of enhancing the values of $n$. Variation of the velocity slip and velocity profile presented in Figure 7. Thickness of momentum improved by cause of the velocity slip rises in all cases of $\xi = 0$, $(2i\xi)^0$, $(2i\xi)^1$ and $(2i\xi)^n$. Momentum thickness decreases by cause of enhancing the values of $n$. Variation of the modified Hartman number and velocity profile presented in Figure 8. Thickness of momentum reduced by cause of the solid nanoparticle rises in all cases of $\xi = 0$, $(2i\xi)^0$, $(2i\xi)^1$ and $(2i\xi)^n$. because thermal thickness increases due to enhancing the thermal conductivity of the fluid increases. Momentum thickness decreases by cause of enhancing the values of $n$. Variation of the modified Hartman number and velocity profile presented in Figure 9. Thickness of momentum improved by cause of the modified Hartman number rises in all cases of $\xi = 0$, $(2i\xi)^0$, $(2i\xi)^1$ and $(2i\xi)^n$. Momentum thickness decreases by cause of enhancing the values of $n$.

**Final remarks**

We considered the mixed convection fluctuating flow of hybrid nanofluid over a vertical Riga wedge. Major
effects of the physical parameters on the temperature, velocity, heat transfer and skin friction on the Riga vertical wedge are highlighted as below:

- The thickness of momentum is reduced by cause of the enhancing velocity slip parameter while the heat transfer improved with improving the velocity slip parameter.
- In case of $\xi = 0$, the heat transfer improved with improving the Richardson number because thickness of thermal boundary improves by cause of velocity slip enhances but opposite results are achieved for the $(2i\xi)^n$.
- Momentum thickness decreases by cause of enhancing the values of $n ((2i\xi)^0$, $(2i\xi)^1$ and $(2i\xi)^n$).
- Thickness of momentum reduced by cause of the solid nanoparticle rises in all cases of $\xi = 0$, $(2i\xi)^0$, $(2i\xi)^1$ and $(2i\xi)^n$.
- Our results are more agreeing with the decay results of Bachok et al.35 and Yacob et al.36 when and rest of the physical parameters dimensions.

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Appendix

Notation

\[ V, W \] Velocity components, \( m/s \)

\[ \mu_{\text{hnf}} \] Dynamic viscosity of hybrid nanofluid, \( kg/ms \)

\[ \nu_{\text{hnf}} \] Kinematic viscosity of hybrid nanofluid, \( m^2/s \)

\[ \rho_{\text{hnf}} \] Density of hybrid nanofluid, \( kg/m^3 \)

\[ \alpha_{\text{hnf}} \] Thermal diffusivity of hybrid nanofluid

\[ k_{\text{hnf}} \] Thermal conductivity of hybrid nanofluid pressure, \( kg/ms^2 \)

\[ \rho c_p_{\text{hnf}} \] Heat capacitance of hybrid nanofluid

\[ T \] Temperature of the fluid

\[ t \] Dimensional time, \( s \)

\[ Nu \] Nusselt number

\[ \Phi_1, \Phi_2 \] Nanoparticles concentration

\[ Ri \] Richardson number

\[ \mu_{\text{nf}} \] dynamic viscosity of nanofluid, \( kg/ms \)

\[ \nu_{\text{nf}} \] kinematic viscosity of nanoluid, \( m^2/s \)

\[ \rho_{\text{nf}} \] Density of nanofluid, \( kg/m^3 \)

\[ \alpha_{\text{nf}} \] thermal diffusivity of nanofluid

\[ k_{\text{nf}} \] Thermal conductivity of nanofluid

\[ \sigma \] Dimensionless parameter

\[ \eta \] Dimensionless similarity variable

\[ \rho c_p_{\text{nf}} \] Heat capacitance of nanofluid

\[ M \] Modified Hartman number

\[ Re \] Reynolds number

\[ Pr \] Prandtl number