Production of $a_0$-mesons in $pp$ and $pn$ reactions

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Abstract. We investigate the cross section for the reaction $NN \to NNa_0$ near threshold and at medium energies. An effective Lagrangian approach with one-pion exchange is applied to analyze different contributions to the cross section for different isospin channels. The Reggeon exchange mechanism is also considered. The results are used to calculate the contribution of the $a_0$ meson to the cross sections and invariant $K\bar{K}$ mass distributions of the reactions $pp \to pnK^+K^0$ and $pp \to ppK^+K^-$. It is found that the experimental observation of $a_0^+$ mesons in the reaction $pp \to pnK^+K^0$ is much more promising than the observation of $a_0^0$ mesons in the reaction $pp \to ppK^+K^-$. 

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1. Introduction

The excitations of the QCD vacuum with different quantum numbers as well as their
life times and decay modes are of fundamental interest in the physics of the strong
interaction. The masses of the pseudo-scalar mesons have been found to be essentially
due to a spontaneous breaking of the chiral $SU(3)_R \times SU(3)_L$ symmetry or the $U(1)_A$
anomaly (in case of the $\eta'$). The vector mesons $\rho, \omega, \phi, K^*, J/\Psi$ etc., which are the
dipole modes of the vacuum, have found increasing attention during the last two decades.
Especially their decay to dileptons is presently investigated in elementary and complex
(nucleus-nucleus) collisions in different laboratories all over the world (cf. the reviews
[1, 2, 3] and Refs. therein). On the other hand, the scalar sector of vacuum excitations
is not well known experimentally and theoretically, so far.

The structure of the lightest scalar mesons $a_0(980)$ and $f_0(980)$ is still under
discussion (see e.g. [4, 5, 6, 7, 8, 9, 10] and references therein). Different authors
interpreted them as unitarized $q\bar{q}$ states or as four-quark cryptoexotic states or as $K\bar{K}$
molecules or even as vacuum scalars (Gribov’s minions). Although it has been possible
to describe them as ordinary $q\bar{q}$-states (see Refs. [11, 12, 13]), other options cannot
be ruled out up to now. Another problem is the possible strong mixing between the
uncharged $a_0(980)$ and the $f_0(980)$ due to a common coupling to $K\bar{K}$ intermediate states
[14, 15, 16, 17, 18, 19, 20]. This effect can influence the structure of the uncharged component of the $a_0(980)$ and implies that it is important to perform a comparative
study of $a_0^0$ and $a_0^+$ (or $a_0^-$). There is no doubt that new data on $a_0^0$ and $a_0^+/a_0^-$ production
in $\pi N$ and $NN$ reactions are quite important to shed new light on the $a_0$ structure and
the dynamics of its production.

In our recent paper [21] we have considered $a_0$ production in the reaction $\pi N \to a_0 N$
near the threshold and at GeV energies. An effective Lagrangian approach as well as
the Regge pole model were applied to investigate different contributions to the cross-
section of the reaction $\pi N \to a_0 N$. Here we employ the latter results for an analysis
of $a_0$ production in $NN$ collisions. Our study is particularly relevant to the current
experimental program at COSY-Jülich [22, 23, 24].

Our paper is organized as follows: In Sect. 2 we discuss an effective Lagrangian
approach with one-pion exchange while the Reggeon exchange model is considered in
Sect. 3. Sect. 4 is devoted to the calculations of the cross section for the reaction
$NN \to NN a_0$. In Sect. 5 we analyze the contribution of the $a_0$ resonance to the cross
sections and invariant $K\bar{K}$ mass distributions for the reactions $pp \to pp K^+ K^-$ and
$pp \to pn K^+ K^0$. Our conclusions are presented in Sect. 6.

2. An effective Lagrangian approach with one-pion exchange

We consider $a_0^0$, $a_0^+$, $a_0^-$ production in the reactions $j = pp \to ppa_0^0$, $pp \to pna_0^+$,
$pn \to ppa_0^-$ and $pn \to pna_0^0$ using the effective Lagrangian approach with one-pion
exchange (OPE). For the elementary $\pi N \to Na_0$ transition amplitude we take into
account different mechanisms $\alpha$ corresponding to $t$-channel diagrams with $\eta(550)$- and $f_1(1285)$-meson exchanges ($\alpha = \tau(\eta), \tau(f_1)$) as well as $s$- and $u$-channel graphs with an intermediate nucleon ($\alpha = s(N), u(N)$) (cf. Ref. [21]). The corresponding diagrams are shown in Fig. 1. The invariant amplitude of the $NN \rightarrow NN\alpha_0$ reaction then is the sum of the four basic terms (diagrams in Fig. 1) with permutations of nucleons in the initial and final states

$$M_{j(\alpha)}^\pi[ab; cd] = \xi_{j(\alpha)}^\pi[ab; cd] M_{\alpha}^\pi[ab; cd] + \xi_{j(\alpha)}^\pi[ab; dc] M_{\alpha}^\pi[ab; dc]$$

$$+ \xi_{j(\alpha)}[ba; dc] M_{\alpha}^\pi[ba; dc] + \xi_{j(\alpha)}[ba; cd] M_{\alpha}^\pi[ba; cd],$$

where the coefficients $\xi_{j(\alpha)}$ are given in Table 1. The amplitude for the $t$-channel exchange with $\eta(550)$- and $f_1(1285)$-mesons are given by

$$M_{t(\eta)}^\pi[ab; cd] = g_{\eta\eta\pi} F_{\eta\eta\pi} \left( (p_a - p_c)^2, (p_d - p_b)^2 \right) g_{\eta NN} F_{\eta} \left( (p_a - p_c)^2 \right)$$

$$\times \frac{1}{(p_a - p_c)^2 - m_\eta^2} \bar{u}(p_c)\gamma_5 u(p_a) \times \Pi(p_b; p_d),$$

with

$$\Pi(p_b; p_d) = \frac{f_{\pi NN}^{\eta}}{m_\pi} F_\pi \left( (p_b - p_d)^2 \right) \frac{1}{(p_b - p_d)^2 - m_\pi^2}.$$  

The amplitudes for the $s$- and $u$-channels (lower part of Fig. 1) are given as

$$M_{s(N)}^\pi[ab; cd] = \Pi(p_b; p_d) \frac{f_{s NN}^\pi}{m_\pi} F_\pi \left( (p_d - p_b)^2 \right) \frac{g_{s NN} F_N ((p_a + p_b - p_d)^2)}{(p_a + p_b - p_d)^2 - m_N^2} \times (p_d - p_b) \bar{u}(p_c) [(p_a + p_b - p_d)\gamma_5 \gamma_\mu u(p_a)].$$

$$M_{u(N)}^\pi[ab; cd] = \Pi(p_b; p_d) \frac{f_{u NN}^\pi}{m_\pi} F_\pi \left( (p_d - p_b)^2 \right) \frac{g_{u NN} F_N ((p_c + p_d - p_b)^2)}{(p_c + p_d - p_b)^2 - m_N^2} \times (p_d - p_b) \bar{u}(p_c) \gamma_5 \gamma_\mu [(p_c + p_d - p_b)\delta \gamma_\delta + m_N] u(p_a).$$

Here $p_a, p_b$ and $p_c, p_d$ are the four momenta of the initial and final nucleons, respectively.

The effective Lagrangians involving $a_0$ and $f_1$ mesons were taken in the following forms:

$$\mathcal{L}_{\eta_0\eta \pi} = g_{\eta_0\eta \pi} \eta(x) \pi(x) a_0(x),$$

$$\mathcal{L}_{\eta_0 f_1 \pi} = g_{\eta_0 f_1 \pi} \epsilon_{\lambda}^1(x) \partial^\lambda \pi(x) a_0(x),$$

$$\mathcal{L}_{\eta_0 NN} = g_{\eta_0 NN} \bar{\Psi}_N(x) a_0(x) \Psi_N(x),$$

$$\mathcal{L}_{f_1 NN} = g_{f_1 NN} \epsilon_{\lambda}^1(x) \bar{\Psi}_N(x) \gamma^\lambda \Psi_N(x).$$

We mostly employ coupling constants and form factors from the Bonn-J"ulich potentials (see e.g. Refs. [25, 26, 27]).
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The functions $F_i$ in Eqs. (2)-(8) represent form factors for virtual mesons at the different vertices $i$ ($i = \pi, \eta, f_1$) and for each vertex they are taken in the monopole form

$$F_i(t) = \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - t},$$

(8)

where $\Lambda_i$ is a cut-off parameter. For the ‘effective’ $\pi$ exchange we use the coupling constant $f_{\pi NN}^2/4\pi = 0.08$ and cut-off parameter $\Lambda_{\pi NN} = 1.05 \div 1.3$ GeV. In the case of $\eta$ exchange we take $g_{\eta NN}^2/4\pi = 3$, $\Lambda_{\eta NN}=1.5$ GeV and $g_{a_0\eta\pi} = 2.46$ GeV, which results from the width $\Gamma(a_0 \rightarrow \eta\pi) = 80$ MeV.

The contribution of the $f_1$ exchange is calculated with $g_{f_1 NN} = 11.2$, $\Lambda_{f_1 NN} = 1.5$ GeV from Ref. [27] and $g_{a_0 f_1\pi} = 2.5$. The latter value for $g_{a_0 f_1\pi}$ corresponds to $\Gamma_{tot}(f_1) = 24$ MeV and $Br(f_1 \rightarrow a_0\pi) = 34\%$. The same parameters have been used in our previous study of $a_0$ production in $\pi N \rightarrow a_0 N$ and $pp \rightarrow a_0^+\pi^-$ reactions [21].

For the form factors at the $a_0 f_1\pi$ (as well as $a_0 \eta\pi$) vertex factorized forms are applied following the assumption from Refs. [28, 29],

$$F_{a_0 f_1\pi}(t_1, t_2) = F_{f_1 NN}(t_1) F_{\pi NN}(t_2),$$

(9)

where $F_{f_1 NN}(t)$, $F_{\pi NN}(t)$ are taken as in (8).

According to different versions of the Bonn potential the coupling constant $g_{a_0 NN}^2/4\pi$ can vary from 1.1075 to 2.67 [25, 27]. On the other hand, the unitary model for meson-nucleon scattering [30] gives a different range for this constant from 0.0026 to 0.88. In the latter model the $a_0$ only gives a contribution to the $\pi\eta$ background because there are no known resonances which decay to $a_0 N$. Since the model is extended only up to energies $\sqrt{s} \leq 1.9$ GeV, which is below the $a_0$ threshold, the meson-nucleon dynamics is not very sensitive to the $a_0 N N$ coupling. We note that a small value of $g_{a_0 NN}^2/4\pi$ certainly contradicts the experimental values of $Br(pp \rightarrow a_0\pi) = 0.69 \pm 0.12$ [31] and $Br(pp \rightarrow a_0\omega) = 0.354 \pm 0.028$ [22], which are quite large (see e.g. Refs. [26, 27]).

Having in mind these considerations we take (as well as in Ref. [21]) the minimal value suggested by the Bonn potential $g_{a_0 NN} \simeq 3.7$. This value is not very different from the upper value of 3.33 given by the model of Ref. [30].

Another problem is the treatment of a virtual nucleon. In this case – instead of the product of two monopole form factors (at the $a_0 N N$ and $\pi N N$ vertices) – we use a dipole-like form factor,

$$F_N(s) = \frac{\Lambda_N^4}{\Lambda_N^4 + (s - m_N^2)^2},$$

(10)

which is normalized at $s = m^2$ and has the same asymptotics at large $s$ (positive or negative) as $F_i(s)F_j(s)$.

There are a couple of arguments in favour of using the form factor (10) for virtual nucleons instead of those which are applied for virtual mesons. In the $t$-channel graph in elastic $NN$ scattering the value of $t$ is negative and the monopole form factor $F_\pi$ as given by Eq. (8) does not have a singularity in the physical region and decreases with $t$. For the $s$-channel graph with a nucleon exchange in the $\pi N \rightarrow a_0 N$ amplitude the
value of \( s \) is positive in the physical region and the conventional form factor

\[
\frac{\Lambda_N^2 - m_N^2}{\Lambda_N^2 - s}
\]

may have even a pole in the physical region (this happens for \( \Lambda_N = 2 \text{ GeV} \), which is used in the Bonn potential for a virtual \( a_0 \)). This undesirable property is absent in the form factor \((10)\), where we consider the cut-off \( \Lambda_N \) as a free parameter. In our previous work \([21]\) we fixed \( \Lambda_N \) in the interval 1.2-1.3 GeV using experimental data on the differential cross section of the reaction \(pp \to da_0^+ \) at \( p_{lab} = 3.8 \div 6.3 \text{ GeV}/c \) \([33]\); in this study we take \( \Lambda_N = 1.24 \text{ GeV} \) as an average value (see also the discussion in Section 4).

We recall that the functional form of the nucleon form factor given by \((10)\) was used in many papers, where meson production in \( \pi N, \gamma N \) and \( NN \) collisions has been discussed (see e.g. \([23, 30, 34, 35, 36, 37]\) and references therein).

The total cross section for \( a_0 \) production in the isospin reaction \( j \) is given as the coherent sum of the amplitudes \((1)\) over all channels \((\alpha = s(N), u(N), t(f_1), t(\eta))\) integrated over phase space

\[
\sigma_{a_0}^j(s) = \int dE_c \, dq_0 \, d\theta_q \, d\varphi_q \, \frac{1}{2^j \pi^4 \rho_a \sqrt{s}} \left| \sum_{\alpha} M_{j(\alpha)}^{\pi} \left[ a_b; cd \right] \right|^2. \quad (11)
\]

Here \( s = (p_a + p_b)^2 \) is the total energy of the \( NN \) system squared, \( E_c \) and \( q_0 \) are the energy of the outgoing nucleon and \( a_0 \) meson, respectively. \( \theta_q \) is the polar angle of the 3-momentum of the \( a_0 \)-meson \( \mathbf{q} \) in the cms of the initial nucleons defined as \( \theta_q = \hat{\mathbf{q}} \cdot \hat{\mathbf{P}}_a \), while \( \varphi_q \) is the azimuthal angle of \( \mathbf{q} \) in the cms.

As shown in the analysis in Ref. \([21]\) the contribution of the \( \eta \)-exchange to the amplitude \( \pi N \to a_0 N \) is small. Note that in Ref. \([38]\) only this mechanism was taken into account for the reaction \(pn \to ppa_0^-\). Here we also include the \( \eta \)-exchange because it might be noticeable in those isospin channels where a strong destructive interference of \( u \)- and \( s \)-channel terms can occur (see below).

3. The Reggeon exchange model

Here as in Ref. \([21]\) we also use the Regge-pole model for the amplitude \( \pi N \to a_0 N \) as developed by Achasov and Shestakov \([15]\). The \( s \)-channel helicity amplitudes for the reaction \( \pi^-p \to a_0^0n \) in this approach can be written as

\[
M_{\chi'_2\lambda_2}(\pi^-p \to a_0^0n) = \bar{u}_{\chi'_2}(p'_2) \left[ -A(s, t) + (p_1 + p'_1)_{\alpha} \gamma_{\alpha} \frac{B(s, t)}{2} \right] \gamma_5 u_{\lambda_2}(p_2) \quad \text{[12]}
\]

where the invariant amplitudes \( A(s, t) \) and \( B(s, t) \) do not contain kinematical singularities. The relations between the invariant and \( s \)-wave helicity amplitudes are given by

\[
M_{++} = -M_{--} = \cos \frac{\theta}{2} \left[ A(s, t) \sqrt{-t_{\text{min}}} - B(s, t) \sqrt{-t_{\text{max}}}s \right], \quad (13)
\]

\[
M_{+-} = M_{-+} = \cos \frac{\theta}{2} \left[ A(s, t) \sqrt{-t_{\text{max}}} - B(s, t) \sqrt{-t_{\text{min}}}s \right], \quad (14)
\]
where \( \theta \) is the c.m. scattering angle, while \( t_{\text{min}} \) and \( t_{\text{max}} \) are the values of \( t \) at \( \theta = 0^\circ \) and \( 180^\circ \), respectively.

In the model of Ref. \[13\] the s-channel helicity amplitudes are expressed through the \( b_1 \) and the conspiring \( \rho_2 \) Regge trajectories exchange as follows

\[
M_{++} = \gamma_{\rho_2}(t) \exp \left[ -i \frac{\pi}{2} \alpha_{\rho_2}(t) \right] \left( \frac{s}{s_0} \right)^{\alpha_{\rho_2}(t)},
\]

\[
M_{+-} = \sqrt{(t_{\text{min}} - t)/s_0} \gamma_{b_1}(t) i \exp \left[ -i \frac{\pi}{2} \alpha_{b_1}(t) \right] \left( \frac{s}{s_0} \right)^{\alpha_{b_1}(t)}.
\]

As in Ref. \[21\] we take the meson Regge trajectories in linear form \( \alpha_j(t) = \alpha_j(0) + \alpha'_j(0)t \) with \( \alpha_{b_1}(0) \approx -0.37, \alpha_{\rho_2}(0) \approx -0.6 \) and \( \alpha'_{b_1}(0) = \alpha'_{\rho_2}(0) = 0.9 \text{ GeV}^{-2} \). The residues are parametrized in a convential way, \( \gamma_{\rho_2}(t) = \gamma_{\rho_2}(0) \exp(b_{\rho_2}t), \gamma_{b_1}(t) = \gamma_{b_1}(0) \exp(b_{b_1}t); \) all parameters were taken the same as in Ref. \[21\]. They correspond to two fits of the Brookhaven data on \( d\sigma/dt \) at 18 GeV/c \[39\] found by Achasov and Shestakov \[13\]: a) with pure \( \rho_2 \) contribution and b) with combined \( \rho_2 + b_1 \) contribution.

The invariant amplitude corresponding to the diagram of Fig. 3 can be written as

\[
\mathcal{M}_{\text{Regge}}^{ab;cd} = \bar{u}(p_c) \left[ -A(s,t) + (p_{a_0} + p_d - p_b) \alpha \gamma_0 \frac{B(s,t)}{2} \right] \gamma_5 u(p_a) \times \bar{u}(p_d) \gamma_5 u(p_b) \times \Pi(p_b; p_d).
\]

4. The reaction \( NN \rightarrow NNa_0 \)

In order to demonstrate the sensitivity of the effective OPE model to the cut-off parameter \( \Lambda_{\pi NN} \) used in the \( \pi NN \) vertices we show in Fig. 3 the total cross section for the reaction \( pp \rightarrow pna_0^+ \) for \( u(N) \) and \( t(f_1) \) channels as a function of the excess energy \( Q = \sqrt{s} - \sqrt{s_0}, \) where \( \sqrt{s_0} = m_{a_0} + 2m_N, \) calculated for different cut-off parameters. The dotted lines correspond to \( \Lambda_{\pi NN} = 0.8 \text{ GeV}, \) the solid lines show the result for \( \Lambda_{\pi NN} = 1.05 \text{ GeV} \) whereas the dashed lines indicate \( \Lambda_{\pi NN} = 1.3 \text{ GeV} \). The results for \( \Lambda_{\pi NN} = 1.3 \text{ GeV} \) and \( 0.8 \text{ GeV} \) differ by a factor of \( \sim 5. \) For our subsequent calculation we choose \( \Lambda_{\pi NN} = 1.05 \text{ GeV} \) while keeping the uncertainty on \( \Lambda_{\pi NN} \) in our 'effective' approach in mind.

Since we have two nucleons in the final state it is necessary to take into account their final-state-interaction (FSI), which has some influence on meson production near threshold. For this purpose we adopt the FSI model from Ref. \[40\] based on the (realistic) Paris potential. We use, however, the enhancement factor \( F_{NN}(q_{NN}) \) – as given by this model – only in the region of small relative momenta of the final nucleons \( q_{NN} \leq q_0, \) where it is larger than 1. Having in mind that this factor is rather uncertain at larger \( q_{NN}, \) where for example contributions of nonnucleon intermediate states to the loop integral might be important, we assume that \( F_{NN}(q_{NN}) = 1 \) for \( q_{NN} \geq q_0. \)

In Fig. 3 we show the FSI effect on the total cross section for the reactions \( pp \rightarrow ppa_0^0 \) (upper part) and \( pp \rightarrow pna_0^+ \) (lower part) for \( u(N), s(N) \) and \( t(f_1) \) channels. The solid lines show the calculation without FSI whereas the dashed lines indicate the results with
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FSI. As seen from Fig. 4, the FSI effect is stronger for $pn$ than for $pp$ in the final state due to the Coulomb repulsive interaction in the $pp$ system and the isospin dependence of the $NN$ interaction at small relative momenta.

The results of our calculations for the total cross sections with FSI for the different isospin reactions are presented in Figs. 5, 6 as a function of $Q = \sqrt{s} - \sqrt{s_0}$. In Fig. 5 we show the total cross section for the $pp$ reactions $pp \rightarrow pna_0^+$ (upper part) and $pp \rightarrow pna_0^-$ (lower part), whereas in Fig. 6 we display the results for the $pn$ reactions $pn \rightarrow pna_0^+$ (upper part) and $pn \rightarrow pna_0^-$ (lower part). The solid lines with full dots and with open squares (r.h.s.) represent the results within the $\rho_2$ and $(\rho_2, b_1)$ Regge exchange model. The short dotted lines (l.h.s.) corresponds to the $t(f_1)$ channel, the dotted lines to the $t(\eta)$ channel, the dashed lines to the $u(N)$ channel, the short dashed lines to the $s(N)$ channel. The dashed line in the right upper part of Fig. 5 is the incoherent sum of the contributions from $s(N)$ and $u(N)$ channels ($s + u$).

As seen from Figs. 5 and 6, the $u$- and $s$-channels give the dominant contribution; the $t(f_1)$ channel is small for all isospin reactions. For the reactions $pp \rightarrow pna_0^+$, $pn \rightarrow pna_0^-$ and $pn \rightarrow pna_0^-$ the Regge exchange contribution (extended to low energies) becomes important and for the $pn \rightarrow pna_0^-$ reaction this contribution is even dominant near threshold. For the $pp \rightarrow pna_0^+$ channel the Regge model predicts no contribution from $\rho_2$ and $\rho_2, b_1$ exchanges due to isospin arguments (i.e. the vertex with a coupling of three neutral components of isovectors vanishes); thus only $s$-, $u$- and $t(f_1)$- channels are plotted in the upper part of Fig. 5.

Here we have to point out the influence of the interference between the $s$- and $u$-channels. According to the isospin coefficients from the OPE model presented in Table 1, the phase (of interference $\alpha$) between the $s$- and $u$- channels $M_{s(N)}^\pi + \exp(-i\alpha)M_{u(N)}^\pi$ is equal to zero, i.e. the sign between $M_{s(N)}^\pi$ and $M_{u(N)}^\pi$ is 'plus'. The solid lines in Figs. 5, 6 indicate the coherent sum of $s(N)$ and $u(N)$ channels including the interference of the amplitudes $(s + u + \text{int.})$. One can see that for $pp \rightarrow pna_0^+$, $pn \rightarrow pna_0^-$ and $pn \rightarrow pna_0^-$ reactions the interference is positive and increases the cross section, whereas for the $pp \rightarrow pna_0^+$ channel the interference is strongly destructive since we have identical particles in the initial and final states and the contributions of $s$- and $u$-channels are very similar.

Here we would like to comment about an extension of the OPE (one-pion-exchange) model to an OBE (one-boson-exchange) approximation, i.e. accounting for the exchange of $\sigma, \rho, \omega, \ldots$ mesons as well as for multi-meson exchanges. Generally speaking, the total cross section of $a_0$ production should contain the sum of all the contributions:

$$\sigma(NN \rightarrow NNa_0) = \Sigma_j \sigma_j,$$

where $j = \pi, \sigma, \rho, \omega, \ldots$. Depending on their cut-off parameters the heavier meson exchanges might give a comparable contribution to the total cross section for $a_0$ production. An important point, however, is that near threshold (e.g. $Q \leq 0.3$ GeV) the energy behaviour of all those contributions is the same, i.e. it is proportional to the three-body phase space $\sigma_j \sim Q^2$ (when the FSI is switched off and the narrow
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The resonance width limit is taken. In this respect we can consider the one-pion exchange as an effective one and normalize it to the experimental cross section by choosing an appropriate value of $\Lambda_\pi$. The most appropriate choice for $\Lambda_\pi$ is about $1.5 \div 1.3$ GeV. Another question is related to the isospin of the effective exchange. As it is known from a serious of papers on the reactions $NN \rightarrow NNX$, $X = \eta, \eta', \omega, \phi$ near threshold the most important contributions to the corresponding cross sections comes from $\pi$ and $\rho$ exchanges (see e.g. the review [11] and references therein). In line with those results we assume here that the dominant contribution to the cross section of the reaction $NN \rightarrow NNa_0$ comes also from the isovector exchanges (like $\pi$ and $\rho$). In principle, it is also possible that some baryon resonances may contribute. However, as mentioned above, there is no information about resonances which couple to the $a_0N$ system. Our assumptions thus enable us to make exploratory estimates of the $a_0$ production cross section without introducing free parameters that would be out of control by existing data. The model can be extended accordingly when new data on the $a_0$ production will be available.

Another important question is related to the choice of the form factor for a virtual nucleon, that – in line with the Bonn-Jülich potentials – we choose as given by (10), which corresponds to monopole form factors at the vertices. In the literature, furthermore, dipole-like form factors (at the vertices) are also often used (cf. Refs. [29, 30, 34, 35, 36]). However, there are no strict rules for the ‘correct’ power of the nucleon form factor. In physics terms, the actual choice of the power should not be relevant; we may have the same predictions for any reasonable choice of the power if the cut-off parameter $\Lambda_N$ is fixed accordingly. Note, that $\Lambda_N$ may also depend on the type of mesons involved at the vertices. Therefore, we can not simply employ the parameters from Refs. [29, 34] or others in case of the $a_0$ problem.

In our previous work [21] we have fixed $\Lambda_N$ for the monopole related form factor (10) in the interval 1.2-1.3 GeV fitting the forward differential cross section of the reaction $pp \rightarrow da_0^+$ from [33]. On the other hand, the same data can be described rather well using a dipole form factor (at the vertices) with $\Lambda_N = 1.55-1.6$ GeV (cf. Fig. 7). If we employ this dipole form factor with $\Lambda_N = 1.55-1.6$ GeV in the present case we obtain practically identical predictions for the cross sections of the channels $pp \rightarrow pna_0^+$, $pn \rightarrow pna_0^+$, $pn \rightarrow ppa_0^-$, where the $u$-channel mechanism is dominant and $u - s$ interference is not too important. In the case of the channel $pp \rightarrow ppa_0^-$ we obtain cross sections by up to a factor of 2 larger for the dipole-like form factor in comparison to the monopole one. This is related to the strong destructive interference of the $s$ and $u$ exchange mechanisms, which slightly depends on the type of form factor used. However, our central result, that the cross section for the $pna_0^+$ final channel is about an order of magnitude higher than the $ppa_0^0$ channel in $pp$ collisions, is robust (within less than a factor of 2) with respect to different choices of the form factor.

As seen from Figs. 1, 2, we get the largest cross section for the $pp \rightarrow pna_0^+$ isospin channel. For this reaction the $u$-channel gives the dominant contribution, the $s$-channel cross section is small such that the interference is not so essential as for the $pp \rightarrow ppa_0^0$
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The result within the Regge model is shown in Fig. 8 for the reactions $pp \to pna_0^+$, $pn \to pna_0$ (upper part) and $pn \to ppa_0$ (lower part) in a wide energy regime for $Q = 1$ MeV÷10 GeV. The total cross section is calculated with the $\rho_2$ (dashed lines) and $(\rho_2, b_1)$ (solid lines) Regge trajectories (with FSI) for a cut-off parameter $\Lambda_{\pi NN} = 1.05$ GeV. In order to show the influence of the cut-off parameter $\Lambda_{\pi NN}$ in the Regge model we present in the lower part of Fig. 8 also the results for $\Lambda_{\pi NN} = 1.3$ GeV (dotted line for $\rho_2$ exchange and the dot-dashed line for the $(\rho_2, b_1)$ trajectory). Changing the cut-off $\Lambda_{\pi NN}$ from 1.05 to 1.3 GeV gives a factor $\sim 2$ in the total cross section similar to the results within the effective Lagrangian model (cf. Fig. 3).

As it was already discussed in our previous study [21] an effective Lagrangian model cannot be extrapolated to high energies because it predicts the elementary amplitude $\pi N \to a_0 N$ to rise fast. Therefore, such model can only be employed not far from the threshold; at larger energies it has to be unitarized. On the other hand, the Regge model is valid at large energies and we have to worry how close to the threshold we can extrapolate corresponding amplitudes. According to duality arguments one can expect that the Regge amplitude can be applied at low energy, too, if the reaction $\pi N \to a_0 N$ does not contain essential s-channel resonance contributions. In this case the Regge model might give a realistic estimate of the $\pi N \to a_0 N$ amplitude even near threshold.

Anyway, as we have shown in our previous paper [21] the Regge and u-channel model give quite similar results for the $\pi^- p \to a_0^0 n$ cross-section in the near threshold region; some differences in the cross sections of the reactions $NN \to NNa_0$ – as predicted by those two models – can be attributed to differences in the isospin factors and effects of $NN$ antisymmetrization which is important near threshold (the latter was ignored in the Regge model formulated for larger energies).

5. The reaction $NN \to NNa_0 \to NN\bar{K}\bar{K}$

5.1. The $\bar{K}\bar{K}$ and $\pi\eta$ decay channels of the $a_0(980)$

The $a_0(980)$ meson production has not yet been measured in $NN \to NNa_0$ reactions. There are only a few $pp \to ppK\bar{K}$ and $pp \to p\bar{np}K\bar{K}$ experimental data points. Therefore, it is important to analyse a possible resonance contribution to $K\bar{K}$ production in the reactions $NN \to NNX$, using the calculated $NN \to NNa_0$ amplitudes and the experimental fits obtained for the $a_0$ resonance mass distribution in the $K\bar{K}$ decay channel.

The amplitude for the $a_0(980)$ decays into $K\bar{K}$ and $\pi\eta$ modes can be parametrized by the well-known Flatté formula [12] which satisfies both requirements of analyticity and unitarity for the two-channels $\pi\eta$ and $K\bar{K}$.

In the case of the $a_0(980)$ resonance the mass distribution of the final $K\bar{K}$ system can be written as a product of the total cross section for $a_0$ production (with the ‘running’
mass $M$) in the $NN \to NNa_0$ reaction \((\ddagger)\) and the Flatté mass distribution function

$$
\frac{d\sigma_{KK}}{dM^2}(s, M) = \sigma_{a_0}(s, M) C_F \frac{M_R \Gamma_{a_0KK}(M)}{(M^2 - M_R^2)^2 + M_R^2 \Gamma_{tot}^2(M)}
$$

(18)

with the total width $\Gamma_{tot}(M) = \Gamma_{a_0KK}(M) + \Gamma_{a_0\pi\eta}(M)$. The partial widths

$$
\Gamma_{a_0KK}(M) = g_{a_0KK}^2 \frac{q_{KK}}{8\pi M^2}, \\
\Gamma_{a_0\pi\eta}(M) = g_{a_0\pi\eta}^2 \frac{q_{\pi\eta}}{8\pi M^2}
$$

(19)

are proportional to the decay momenta in the center-of-mass (in case of scalar mesons),

$$
q_{KK} = \frac{[(M^2 - (m_K + m_K)^2)(M^2 - (m_K - m_K)^2)]^{1/2}}{2M}, \\
q_{\pi\eta} = \frac{[(M^2 - (m_\pi + m_\eta)^2)(M^2 - (m_\pi - m_\eta)^2)]^{1/2}}{2M}
$$

for a meson of mass $M$ decaying to $KK$ and $\pi\eta$, correspondingly. The branching ratios $Br(a_0 \to KK)$ and $Br(a_0 \to \pi\eta)$ are given by the integrals of the Flatté distribution over the invariant mass squared $dM^2 = 2MdM$:

$$
Br(a_0 \to KK) = \int_{m_K + m_K}^{\infty} \frac{dM}{2M} C_F M_R \frac{\Gamma_{a_0KK}(M)}{(M^2 - M_R^2)^2 + M_R^2 \Gamma_{tot}^2(M)},
$$

(20)

$$
Br(a_0 \to \pi\eta) = \int_{m_\pi + m_\pi}^{\infty} \frac{dM}{2M} C_F M_R \frac{\Gamma_{a_0\pi\eta}(M)}{(M^2 - M_R^2)^2 + M_R^2 \Gamma_{tot}^2(M)}
$$

(21)

$$
+ \int_{m_\pi + m_\pi}^{m_K + m_K} \frac{dM}{2M} C_F M_R \frac{\Gamma_{a_0\pi\eta}(M)}{(M^2 - M_R^2)^2 + M_R^2 \Gamma_{a_0KK}(M)^2 + M_R^2 \Gamma_{a_0\pi\eta}(M)^2}.
$$

The parameters $C_F, g_{KK}, g_{\pi\eta}$ have to be fixed under the constraint of the unitarity condition

$$
Br(a_0 \to KK) + Br(a_0 \to \pi\eta) = 1.
$$

(22)

Choosing the parameter $\Gamma_0 = \Gamma_{a_0\pi\eta}(M_R)$ in the interval $50 \div 100$ MeV as given by the PDG \((\ddagger\ddagger)\), one can fix the coupling $g_{\pi\eta}$ according to \((\ddagger\ddagger)\). In Ref. \([15]\) a ratio of branching ratios has been reported,

$$
\frac{Br(a_0(980) \to KK)}{Br(a_0 \to \pi\eta)} = 0.23 \pm 0.05,
$$

(23)

for $m_{a_0} = 0.999$ GeV, which gives $Br(a_0 \to KK) = 0.187$. In another recent study \([14]\) the WA102 collaboration reported the branching ratio

$$
\frac{\Gamma(a_0 \to KK)}{\Gamma(a_0 \to \pi\eta)} = 0.166 \pm 0.01 \pm 0.02,
$$

(24)

which was determined from the measured branching ratio for the $f_1(1285)$-meson. In our present analysis we use the results from \([15]\), however, keeping in mind that this branching ratio $Br(a_0 \to KK)$ more likely gives an ‘upper limit’ for the $a_0 \to KK$ decay.
Thus, the two other parameters in the Flatté distribution \( C_F \) and \( g_{a_0K\bar{K}} \) can be found by solving the system of integral equations, for example, Eq. (24) for \( Br(a_0 \rightarrow K\bar{K}) = 0.187 \) and the unitarity condition (22). For our calculations we choose either \( \Gamma_{a_0\pi\eta}(M_R) = 70 \) MeV or 50 MeV, which gives two sets of independent parameters \( C_F, g_{a_0K\bar{K}}, g_{a_0\pi\eta} \) for a fixed branching ratio \( Br(a_0 \rightarrow K\bar{K}) = 0.187 \):

\[
\begin{align*}
\text{set 1} \quad & (\Gamma_{a_0\pi\eta} = 70 \text{ MeV}) : \\
& g_{a_0K\bar{K}} = 2.297, \quad g_{a_0\pi\eta} = 2.189, \quad C_F = 0.365 \\
\text{set 2} \quad & (\Gamma_{a_0\pi\eta} = 50 \text{ MeV}) : \\
& g_{a_0K\bar{K}} = 1.943, \quad g_{a_0\pi\eta} = 1.937, \quad C_F = 0.354.
\end{align*}
\]

Note, that for the \( K^+K^- \) or \( K^0\bar{K}^0 \) final state one has to take into account an isospin factor for the coupling constant, i.e. \( g_{a_0K^+K^-} = g_{a_0K^0\bar{K}^0} = g_{a_0K\bar{K}}/\sqrt{2} \), whereas \( g_{a_0K^+K^0} = g_{a_0K^-\bar{K}^0} = g_{a_0K\bar{K}} \).

5.2. Numerical results for the total cross section

In the upper part of Fig. 3 we display the calculated total cross section (within parameter set 1) for the reaction \( pp \rightarrow pna_0^+ \rightarrow pnK^+\bar{K}^0 \) in comparison to the experimental data for \( pp \rightarrow pnK^+\bar{K}^0 \) (solid dots) from Ref. [16] as a function of the excess energy \( Q = \sqrt{s} - \sqrt{s_0} \). The dot-dashed and solid lines in Fig. 3 correspond to the coherent sum of \( s(N) \) and \( u(N) \) channels with interference \( (s + u + \text{int}) \), calculated with a monopole form of the form factor (10) with \( \Lambda_N = 1.24 \) GeV and with a dipole form of (10) with \( \Lambda_N = 1.35 \) GeV, respectively. We mention that the latter (dipole) result is in better agreement with the constraints on the near-threshold production of \( a_0 \) in the reactions \( \pi^-p \rightarrow K^-\bar{K}^0p \) and \( \pi^+p \rightarrow K^+\bar{K}^0p \) [17]. In the middle part of Fig. 3 the solid lines with full dots and with open squares present the results within the \( \rho_2 \) and \( (\rho_2, b_1) \) Regge exchange model. The short dashed line shows the 4-body phase space (with constant interaction amplitude), while the dashed line is the parametrization from Sibirtsev et al. [18]. We note, that the cross sections for parameter set 2 are similar to set 1 and larger by a factor \( \sim 1.5 \).

In the lower part of Fig. 3 we show the calculated total cross section (within parameter set 1) for the reaction \( pp \rightarrow ppa_0^0 \rightarrow ppK^+K^- \) as a function of \( Q = \sqrt{s} - \sqrt{s_0} \) in comparison to the experimental data. The solid dots indicate the data for \( pp \rightarrow ppK^0\bar{K}^0 \) from Ref. [19], the open square for \( pp \rightarrow ppK^+K^- \) is from the DISTO collaboration [19] and the full down triangles show the data from COSY-11 [20].

For the \( pp \rightarrow ppa_0^0 \rightarrow ppK^+K^- \) reaction (as for \( pp \rightarrow ppa_0^0 \)) there is no contribution from meson Regge trajectories; \( s \)- and \( u \)-channels give similar contributions such that their interference according to the effective OPE model (line \( s + u + \text{int} \)) is strongly destructive (cf. upper part of Fig. 3). The \( t(f_1) \) contribution (short dotted line) is practically negligible, while the \( t(\eta) \)-channel (dotted line) becomes important closer to the threshold.

Thus our model gives quite small cross sections for \( a_0^0 \) production in the \( pp \rightarrow \).
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$ppK^+K^-$ reaction which complicates its experimental observation for this isospin channel. The situation looks more promising for the $pp \rightarrow pna_0^+ \rightarrow pnK^+K^0$ reaction since the $a_0^+$ production cross section is by an order of magnitude larger than the $a_0^0$ one. Moreover, as has been pointed out with respect to Fig. 5, the influence of the interference is not so strong as for the $pp \rightarrow ppa_0^0 \rightarrow ppK^+K^-$ reaction.

Here we stress again the limited applicability of the effective Lagrangian model (ELM) at high energies. As seen from the upper part of Fig. 9, the ELM calculations at high energies go through the experimental data, which is not realistic since also other channels contribute to $K^+K^0$ production in pp reactions (cf. dashed line from Ref. [48]). Moreover, the ELM calculations are higher than the Regge model predictions which indicates, that the ELM amplitudes at high energies have to be reggeized or unitarized.

5.3. Numerical results for the invariant mass distribution

As follows from the lower part of Fig. 9, the $a_0^0$ contribution to the $K^+K^-$ production in the $pp \rightarrow ppK^+K^-$ reaction near the threshold is hardly seen. With increasing energy the cross section grows up, however, even at $Q = 0.111$ GeV the full cross section with interference $(s + u + int.)$ gives only a few percent contribution to the $0.11 \pm 0.009 \pm 0.046 \mu b$ ‘nonresonant’ cross section (without $\phi \rightarrow K^+K^-$) from the DISTO collaboration [49].

To clarify the situation with the relative contribution of $a_0^0$ to the total $K^+K^-$ production in pp reactions we calculate the $K^+K^-$ invariant mass distribution for the $pp \rightarrow ppK^+K^-$ reaction at $p_{lab} = 3.67$ GeV/c, which corresponds to the kinematical conditions for the DISTO experiment [49]. The differential results are presented in Fig. 10. The upper part shows the calculation within parameter set 1, whereas the lower part corresponds to set 2. The dot-dashed lines (lowest curves) indicate the coherent sum of $s(N)$ and $u(N)$ channels with interference $(s + u + int.)$ for the $a_0$ contribution. However, one has to consider also the contribution from the $f_0$ scalar meson, i.e. the $pp \rightarrow ppf_0 \rightarrow ppK^+K^-$ reaction. The $f_0$ production in pp reactions has been studied in detail in Ref. [51]. Here we use the result from this previous work [51] and show in Fig. 10 the contribution from the $f_0$ meson calculated with parameter set A from Ref. [51] as the solid line with open circles ($f_0$).

We find that when adding the $f_0$ contribution to the phase-space of nonresonant $K^+K^-$ production (the short dotted lines in Fig. 10) and the contribution from $\phi$ decays (resonance peak around 1.02 GeV), the sum (solid) lines almost perfectly describe the DISTO data. This means that there is no visible signal for an $a_0^0$ contribution in the DISTO data according to our calculations while the $f_0$ meson gives some contribution to the $K^+K^-$ invariant mass distribution at low invariant masses $M$, that is $\sim 12\%$ of the total ‘nonresonant’ cross section from the DISTO collaboration [49]. Thus the reaction $pp \rightarrow pmK^+K^0$ is more promising for $a_0$ measurements as it has been pointed out in the previous subsection.

For an experimental determination of the $a_0^+$ we present the invariant mass
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distribution of \(K^+ K^0\) in the reaction \(pp \to pnK^+\bar{K}^0\) at different \(Q\) (solid lines) in Fig. [1]. The dashed lines show the invariant mass distributions for 'background' (i.e. according to phase space with constant interaction amplitude) under the assumption that the integrals below the solid and dashed lines are the same for each \(Q\). We see that the shape of the solid and dashed lines are practically the same for \(Q \leq 50\) MeV. Noticeable differences between the lines can be found for \(Q \geq 100\) MeV. This means that a separation of the resonance contribution from the background very close to threshold can be done only in the case when the background is small or very well known.

6. Conclusions

In this work we have estimated the cross sections of \(a_0\) production in the reactions \(pp \to pna_0^+\), \(pp \to pna_0^-\), \(pn \to pp a_0^-\) and \(pn \to pna_0^+\) near threshold and at medium energies. Using an effective Lagrangian approach with one-pion exchange we have analyzed different contributions to the cross section corresponding to \(t\)-channel diagrams with \(\eta(550)\) and \(f_1(1285)\)-meson exchanges as well as \(s\) and \(u\)-channel graphs with an intermediate nucleon. We use the same parameters as in our previous paper where we describe rather well the Berkeley data [33] on the reaction \(pp \to d a_0^+\).

We additionally have considered the \(t\)-channel Reggeon exchange mechanism with parameters normalized to the Brookhaven data for \(\pi^- p \to a_0^- p\) at 18 GeV/c [33]. These results have been used to calculate the contribution of \(a_0\) mesons to the cross sections of the reactions \(pp \to pnK^+\bar{K}^0\) and \(pp \to ppK^+K^-\). Due to unfavourable isospin Clebsh-Gordan coefficients as well as rather strong destructive interference of the \(s\)- and \(u\)-channel contributions our model gives quite small cross sections for \(a_0^-\) production in the \(pp \to ppK^+K^-\) reaction. However, the \(a_0^+\) production cross section in the \(pp \to pna_0^+ \to pnK^+\bar{K}^0\) reaction should be larger by about an order of magnitude. Therefore the experimental observation of \(a_0^+\) in the reaction \(pp \to ppK^+\bar{K}^0\) is much more promising than the observation of \(a_0^-\) in the reaction \(pp \to ppK^+K^-\). We note in passing that the \(\pi\eta\) decay channel is experimentally more challenging since, due to the larger nonresonant background [52], the identification of the \(\eta\)-meson (via its decay into photons) in a neutral-particle detector is required.

We have also analyzed invariant mass distributions of the \(K\bar{K}\) system in the reaction \(pp \to pNa_0 \to pNK\bar{K}\) at different excess energies \(Q\) not far from threshold. Our analysis of the DISTO data on the reaction \(pp \to ppK^+K^-\) at 3.67 GeV/c has shown that the \(a_0^-\)-meson is practically not seen in \(d\sigma/dM\) at low invariant masses, however, the \(f_0\)-meson gives some visible contribution. In this respect the possibility to measure the \(a_0^+\) meson in \(d\sigma/dM\) for the reaction \(pp \to pnK^+\bar{K}^0\) (or \(dK^+\bar{K}^0\)) looks much more promising not only due to a much larger contribution for the \(a_0^+\), but also due to the absence of the \(f_0\) meson in this channel.

Experimental data on \(a_0\) production in \(NN\) collisions are practically absent (except of the \(a_0\) observation in the reaction \(pp \to dX\) [33]). Such measurements might give new information on the \(a_0\) structure. According to Atkinson et al. [52] a relatively strong
production of the $a_0$ (the same as for the $b_1(1235)$) in non-diffractive reactions can be considered as evidence for a $q\bar{q}$ state rather than a $qq\bar{q}\bar{q}$ state. For example, the cross section of $a_0$ production in $\gamma p$ reactions at 25–50 GeV is about 1/6 of the cross sections for $\rho$ and $\omega$ production. Similar ratios are found in the two-body reaction $pp \to dX$ at 3.8–6.3 GeV/c where $\sigma(pp \to da_0^+) = (1/4 \div 1/6)\sigma(pp \to d\rho^+)$. In our case we can compare $a_0$ and $\omega$ production. Our model predicts $\sigma(pp \to pna_0^+) = 30 \div 70\mu b$ at $Q \simeq 1$ GeV (see Fig. 8) which can be compared with $\sigma(pp \to pp\omega) \simeq 100 \div 200\mu b$ at the same $Q$. If such a large cross section could be detected this would be a serious argument in favour of the $q\bar{q}$ model for the $a_0$.

To distinguish between the threshold cusp scenario and a resonance model one can exploit different analytical properties of the $a_0$ production amplitudes in those approaches. In case of a genuine resonance the amplitude of $\eta\pi$ and $K\bar{K}$ production through the $a_0$ has a pole and satisfies the factorization property. This implies that the shapes of the invariant mass distributions in the $\eta\pi$ and $K\bar{K}$ channels should not depend on the specific reaction in which the $a_0$ resonance is produced (for $Q \geq \Gamma_{tot}$). On the other hand, for the threshold cusp scenario the $a_0$ bump is produced through the $\pi\eta$ final state interaction. The corresponding amplitude has a square root singularity and in general can not be factorized (see e.g. Ref. [40] were the factorization property was disproven for pp FSI in the reaction $pp \to ppM$). This implies that for a threshold bump the invariant mass distributions in the $\eta\pi$ and $K\bar{K}$ channels are expected to be different for different reactions and will even depend on kinematical conditions (i.e. initial energy and momentum transfer) at the same excess energy, e.g. $Q \simeq 1$ GeV.

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| Reaction $j$ (mechanism $\alpha$) | $\xi^\pi_{j(\alpha)}[ab; cd]$ | $\xi^\pi_{j(\alpha)}[ab; dc]$ | $\xi^\pi_{j(\alpha)}[ba; dc]$ | $\xi^\pi_{j(\alpha)}[ba; cd]$ |
|-----------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $pp \rightarrow ppa_0^0$ ($t(\eta), t(f_1)$) | $+1/\sqrt{2}$ | $-1/\sqrt{2}$ | $+1/\sqrt{2}$ | $-1/\sqrt{2}$ |
| ($s(N)$) | $+1/\sqrt{2}$ | $-1/\sqrt{2}$ | $+1/\sqrt{2}$ | $-1/\sqrt{2}$ |
| ($u(N)$) | $+1/\sqrt{2}$ | $-1/\sqrt{2}$ | $+1/\sqrt{2}$ | $-1/\sqrt{2}$ |
| Regge | $0$ | $0$ | $0$ | $0$ |
| $pp \rightarrow pna_0^+$ ($t(\eta), t(f_1)$) | $-\sqrt{2}$ | $0$ | $0$ | $+\sqrt{2}$ |
| ($s(N)$) | $0$ | $+\sqrt{2}$ | $-\sqrt{2}$ | $0$ |
| ($u(N)$) | $+2\sqrt{2}$ | $-\sqrt{2}$ | $+\sqrt{2}$ | $-2\sqrt{2}$ |
| Regge | $-1$ | $+1$ | $-1$ | $+1$ |
| $pn \rightarrow ppa_0^-$ ($t(\eta), t(f_1)$) | $+1$ | $-1$ | $0$ | $0$ |
| ($s(N)$) | $-2$ | $+2$ | $-1$ | $+1$ |
| ($u(N)$) | $0$ | $0$ | $+1$ | $-1$ |
| Regge | $+1/\sqrt{2}$ | $-1/\sqrt{2}$ | $-1/\sqrt{2}$ | $+1/\sqrt{2}$ |
| $pn \rightarrow pna_0^0$ ($t(\eta), t(f_1)$) | $-1$ | $0$ | $+1$ | $0$ |
| ($s(N)$) | $-1$ | $-2$ | $+1$ | $+2$ |
| ($u(N)$) | $-1$ | $+2$ | $+1$ | $-2$ |
| Regge | $0$ | $+\sqrt{2}$ | $0$ | $-\sqrt{2}$ |

Table 1. Coefficients in Eq. (1) for different mechanisms of the $pp \rightarrow ppa_0^0$, $pp \rightarrow ppa_0^+$, $pn \rightarrow ppa_0^-$ and $pn \rightarrow pna_0^0$ reactions.
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**Figure 1.** Diagrams for $a_0$ production in the reaction $NN \rightarrow a_0 NN$ near threshold as considered in the present study.

**Figure 2.** The diagram for $a_0$ production in the reaction $NN \rightarrow NN a_0$ within the Regge exchange model.
Figure 3. The total cross section for the reaction \( pp \rightarrow pna_0^+ \) for \( u(N) \) and \( t(f_1) \) channels as a function of the excess energy \( Q = \sqrt{s} - \sqrt{s_0} \) for different cut-off parameters \( \Lambda_{\pi NN} = 0.8 \text{ GeV} \) (dotted lines), \( \Lambda_{\pi NN} = 1.05 \text{ GeV} \) (solid lines) and \( \Lambda_{\pi NN} = 1.3 \text{ GeV} \) (dashed lines).
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Figure 4. The total cross section for the reactions $pp \rightarrow ppa_0^0$ (upper part) and $pp \rightarrow pna_0^+$ (lower part) as a function of the excess energy $Q = \sqrt{s} - \sqrt{s_0}$ for $u(N)$, $s(N)$ and $t(f_1)$ channels calculated without FSI (solid lines) and with FSI (dashed lines).
Production of $a_0$-mesons in $pp$ and $pn$ reactions

Figure 5. The total cross sections for the reactions $pp \rightarrow ppa_0^0$ (upper part) and $pp \rightarrow pna_0^+$ (lower part) as a function of the excess energy $Q = \sqrt{s} - \sqrt{s_0}$ calculated with FSI. The short dotted lines (l.h.s.) corresponds to the $t(f_1)$ channel, the dotted lines to the $t(\eta)$ channel, the dashed lines to the $u(N)$ channel, the short dashed lines to the $s(N)$ channel. The dashed line (upper part, r.h.s.) is the incoherent sum of the contributions from $s(N)$ and $u(N)$ channels ($s + u$). The solid lines indicate the coherent sum of $s(N)$ and $u(N)$ channels with interference ($s + u + \text{int}$). The solid lines with full dots and with open squares (lower part, r.h.s.) present the results within the $\rho_2$ and $(\rho_2, b_1)$ Regge exchange model.
Figure 6. The total cross sections for the reactions $pn \rightarrow p p a_0^-$ (upper part) and $pn \rightarrow p n a_0^0$ (lower part) as a function of $Q = \sqrt{s} - \sqrt{s_0}$ calculated with FSI. The assignment of the individual lines is the same as in Fig. 5. The results from the effective OPE model are shown on the l.h.s. while those from the $\rho_2$ and $\rho_2 b_1$ Regge approach are displayed on the r.h.s.
Production of $a_0$-mesons in $pp$ and $pn$ reactions

Figure 7. Forward differential cross section of the reaction $pp \rightarrow da_0^+$ as a function of $(p_{lab} - 3.29)$ GeV/c. The bold and thin solid curves are calculated at $\Lambda_{\pi NN}=1.05$ and 1.3 GeV, respectively. The solid curves correspond to a monopole nucleon form factor with $\Lambda_N=1.2$ (thin) and 1.24 GeV (bold). The long-dashed and short-dashed curves are calculated using the dipole nucleon form factor for different values of $\Lambda_N$ as shown in the figure. The experimental data are taken from Ref. [33].
Production of $a_0$-mesons in $pp$ and $pn$ reactions

Figure 8. The total cross sections for the reactions $pp \to pna_0^+$, $pn \to pna_0^0$ (upper part) and $pn \to ppa_0^-$ (lower part) as a function of $Q = \sqrt{s} - \sqrt{s_0}$ calculated within the $\rho_2$ (dashed lines) and $(\rho_2, b_1)$ (solid lines) Regge exchange model (with FSI) for cut-off parameters $\Lambda_{\pi NN} = 1.05$ GeV. The dotted and dot-dashed lines in the lower part show the results for $\Lambda_{\pi NN} = 1.3$ GeV within the $\rho_2$ and $(\rho_2, b_1)$ exchanges, respectively.
Production of $a_0$-mesons in $pp$ and $pn$ reactions

Figure 9. Upper part: the calculated total cross section (within parameter set 1) for the reaction $pp \rightarrow pna_0^0 \rightarrow pnK^+\bar{K}_0$ in comparison to the experimental data for $pp \rightarrow pnK^+\bar{K}_0$ (solid dots) from Ref. [46] as a function of $Q = \sqrt{s} - \sqrt{s_0}$. The dot-dashed and solid lines correspond to the coherent sum of $s(N)$ and $u(N)$ channels with interference ($s + u + int.$) calculated with a monopole form of the form factor (10) with $\Lambda_N = 1.24$ GeV and with a dipole form of (10) with $\Lambda_N = 1.35$ GeV, respectively. Middle part: the solid lines with full dots and with open squares represent the results within the $\rho_2$ and $(\rho_2, b_1)$ Regge exchange model. The short dashed line shows the 4-body phase space (with constant interaction amplitude): the dashed line is the parametrization from Sibirtsev et al. [48]. Lower part: the calculated total cross section (within parameter set 1) for the reaction $pp \rightarrow ppa_0^0 \rightarrow ppK^+K^-$ as a function of $Q = \sqrt{s} - \sqrt{s_0}$ in comparison to the experimental data. The solid dots indicate the data for $pp \rightarrow ppK_0\bar{K}_0$ from Ref. [46], the open square for $pp \rightarrow ppK^+K^-$ from Ref. [10]; the full down triangles show the data from Ref. [10].
Production of $a_0$-mesons in pp and pn reactions

Figure 10. The $K^+K^-$ invariant mass distribution for the $pp \to ppK^+K^-$ reaction at $p_{lab} = 3.67$ GeV/c. The short dotted lines indicate the 4-body phase space with constant interaction amplitude, the dot-dashed lines show the coherent sum of $s(N)$ and $u(N)$ channels with interference ($s + u + int.$). The solid lines with open circles correspond to the $f_0$ contribution from Ref. [51]. The thick solid lines show the sum of all contributions including the decay $\phi \to K^+K^-$. The experimental data are taken from Ref. [49].
Figure 11. The $K^+\bar{K}^0$ invariant mass distribution for the $pp \rightarrow pnK^+\bar{K}^0$ reaction at different $Q = \sqrt{s} - \sqrt{s_0}$. The solid lines describe the $a_0^+$ resonance contributions. The dashed lines show the invariant mass distributions for 'background' under the assumption that the integrals below the solid and dashed lines are the same for each $Q$. 