Simulation of the dependence of integrated fire risks in the residential area on social-economic factors

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Abstract. It is proposed to calculate the integral fire risks for the residential area basing on the mathematical model of the fire safety rational offender. Rational behaviour of offender means that violation of the fire safety requirements by the members of homestead occurs only in case when the expected additional usability of offending exceeds the risk of damages occurring as a result of fire in a homestead. Analysis of the mathematical model allows to obtain a trend in the changes of value for the integral fire risks in the residential area under the changes of statistically registered regional social-economic factors. Results of statistical analysis of the panel data related to the fire and social-economic statistics over the regions of Russia obtained basing on the model with random effect are in a good agreement with the results of applying fire safety rational offender model to the calculation of the fire risks.

1. Introduction

Results of the integrated study concerned with fire risks in Russia presented in [1,2], basing on the statistical data in Russian Federation showed that the main part of fire cases and major part of the fire victims were referred to the buildings arranged in the residential area. Note that the main part of the fire cases happened due to the "human factor". It means that their reasons were stipulated by social factors [1,2]. Similar conclusions on the decisive effect of “social factor” on the situation with the fires were also obtained according to the results of foreign studies [3-6], performed by the correlation and regression analysis and with the use of the general statistics methods. The fires stipulated by social factors are considered as those ones related with the violations of the fire safety by the people occupying their homesteads, i.e. accomplishment of the actions or outages of the individuals which could facilitate the possibility of fire occurrence in the homesteads. Hence, it seems reasonable to represent a theoretical model for the estimation of the fire risks estimations in the residential area determined by a behaviour of the homestead members relative to the fire safety requirements in a dependence of the statistically registered social-economic factors.
2. Theoretical model

The number of fire cases $K$ occurring in a residential area per time unit (year) at the certain territory (in a region) is determined by a sum of fire cases number stipulated by social factors $K_s$, as well as by a number of the fire cases that are independent as on the social as on the social-economic factors

$$K = K_s + K_n.$$  \hspace{1cm} (1)

Similarity of social reasons in the fire and criminal risks is confirmed by the linear dependence between the number of fires per 1 thousand of men and the number of registered offences calculated per 100 men of the population obtained on the basis of statistical analysis in [7]. Since the probability of the fire occurrence $p_s$ is connected with the violations of the fire safety by the members of homesteads then these violations can be considered as a variety of offences or crimes in the realm of public safety. At the same time it seems actual to apply a theory of the rational offender by Becker [8] in order to analyze the probability of offences and crimes. In the considered situation the rationality of an offender means that offence occurs only in the case if the expected additional usability $b$ of its realizing exceeds the possible losses $u$ in case of the fire occurrence:

$$(1 - p) \cdot b > p \cdot u.$$ \hspace{1cm} (2)

Application of the model of rational fire safety offender to the analysis of the integral fire risks at the object of economics is presented in [9,10]. Let us consider the model of the rational fire safety offender for the investigation of the integral fire risks in the residential area.

According to the fire service statistics absolute majority of the registered fires in the residential area occurs and can be localized at the isolated homesteads. Therefore, one can assume a linear dependence of $K_s$ on the total number of the homesteads:

$$K_s = k \cdot N_z = k \cdot C \cdot N_z,$$ \hspace{1cm} (3)

where $C$ - is the economic factor characterizing a part of homesteads with the violations of fire safety rules. In this case the expected additional utility of the homestead members’ behavior connected with non-compliance of the fire safety rules exceeds the risk of fire in the homestead taking into account the probability of the fire occurrence and economic equivalent of the losses as a result of the fire; $N_z$ - is a total number of homesteads in the region; $N_z$ - is a number of homesteads in the region with the violations of fire safety by the members of homesteads; $k$ - proportionality coefficient between the number of fire cases in the homesteads due to the social factors and the number of homesteads with the violations of fire safety requirements and it depends on the regional factors.

Probability $p$ of the fires occurring in the homesteads of a region during a certain unitary interval of time (a year) with the account of (1), (2) and statistical determination of the fires rate can be written as

$$p = \frac{K}{N_z} = \frac{K_n}{N_z} + \frac{K_s}{N_z} = p_n + p_s = p_n + k \cdot C,$$ \hspace{1cm} (4)

where $p_n$ is the probability of fire occurrence in a residential area of the region due to the reasons that are not stipulated by social and social-economic factors, respectively; $p_s$ is the probability of fire occurrence in a residential area of the region due to the reasons that are just stipulated by social and social-economic factors.

Let us designate the integral fire risk connected with the probability of encountering for a man with a fire in a residential area per a year (number of fires per $10^3$ people) as $R_{iz}$ [1]. The relationship between risk $R_{iz}$ and the probability of fire occurrence per year in the homesteads of the residential area within a region can be determined by the expression:
\[ R_{xz} = p \cdot 10^3 \frac{N_x}{N}, \]  

where \( N_x \) is the number of homesteads in a region; \( N \) is the population size.

While estimating the probability of offences in the realm of the fire safety in the residential area basing on a hypothesis of the rational offender it should be taken into account that the later one can consider as the expected additional usability b the following: economy on the expenses for providing of the homestead fire safety, economical equivalent of self-satisfying when smoking, drinking strong spirits or due to other habits provoking fire danger, while losses as a result of the fire occurrence in the homestead u can be written as:

\[ u = u_m + \frac{E \cdot R_{xz}}{10^3}, \]  

where \( u_m \) are direct material damages from the fire in a homestead; \( R_{xz} \) is an integral fire risk connected with the probability of the loss of a man’s life during fire in the residential area of a region (number of the dead per \( 10^3 \) of fires) [1]; \( E \) is the economical equivalent of the human life [11].

\[ E = \frac{12 \cdot D_r}{K_{mort}} = \frac{12 \cdot r \cdot D}{K_{mort}}, \]  

where: \( D_r \) is the disposable income of a person; \( D \) is a monthly income of a person; \( r \) - is a mean coefficient of the rest (disposable) income after paying of compulsory payments and mandatory contributions; \( K_{mort} \) - is the total mortality coefficient (per a year).

It is also possible to represent the averaged model of relation of the direct material damages due to fire occurrence in a homestead \( u_m \) with the monthly incomes of a person \( D \):

\[ u_m = g \cdot d \cdot n \cdot \tau \cdot r \cdot D, \]  

where: \( g \) - is a mean share of a tangible property lost in a fire; \( d \) is a mean share of a disposable income provided for the accumulation of the tangible property; \( n \) is a mean number of people in a homestead with an income; \( \tau \) is a mean time of accumulation (in months) of a tangible property before fire occurrence; \( r \) is a mean coefficient of the rest (disposable) income after paying of compulsory payments and mandatory contributions.

Then it may be assumed that the damages due to fire occurrence \( u \) are proportional to the incomes \( D \):

\[ u = (g \cdot d \cdot n \cdot \tau + \frac{R_{xz} \cdot 12}{10^3 \cdot K_{mort}}) \cdot r \cdot D. \]

From analysis of the data on the economical statistics it follows that the population's incomes, including \( D \) (incomes of the homestead members) have lognormal distribution density [12]. Moreover, the studies performed in [13] according to the materials of the Russian and foreign statistics demonstrated that both the number of aggrieved persons and the value of material damages due to fire occurrence or explosions can be correctly described by a lognormal model of the distribution density:

\[ p_{\mu, \sigma_{\ln u}}(u) = \rho_{\mathcal{N}(\ln \mu, \sigma_{\ln u})}(\ln(u)) = \frac{1}{\sqrt{2\pi} \sigma_{\ln u} \mu} \exp\left(-\frac{(\ln(u) - \ln(\mu))^2}{2\sigma_{\ln u}^2}\right), \]

where \( p_{\mu, \sigma_{\ln u}}(u) \) is a density function of lognormal distributed random value of the damages \( u \) for the homesteads as a result of fires, \( \mu \) is a median value for the corresponding distribution of the
damages for homesteads in a region due to the fires, \( \sigma^2_{\ln(u)} \) is a dispersion of the normal logarithm distribution for the value of damages from the fires in the homesteads of a region, \( \ln(u) \).

Applying this representation to the owners of homesteads one can assume that the lognormal distribution of the value of additional usability \( b \) stipulated by failure to comply the requirements of the fire safety in the residential area of a region with a median value \( \eta \) and dispersion \( \sigma^2_{\ln(b)} \) of the normal distribution \( \ln(b) \) is true and it takes the form of:

\[
\rho_{\eta,\sigma_{\ln(u)}}(b) = \rho_{N(\ln(\eta),\sigma^2_{\ln(u)})}(\ln(b)) = \frac{1}{\sqrt{2\pi} \sigma_{\ln(b)}} \exp \left( -\frac{[\ln(b) - \ln(\eta)]^2}{2\sigma^2_{\ln(b)}} \right),
\]

(11)

With the account of (2), (10), (11) under stationary values of the social-economic factors multiplier \( C \) can be determined as

\[
C = \int_0^\infty \rho_{\eta,\sigma_{\ln(u)}}(b) \int_0^\infty \rho_{\eta,\sigma_{\ln(u)}}(u) du \, db =

\int_0^\infty \rho_{N(\ln(\eta),\sigma^2_{\ln(u)})}(\ln(b)) \int_{-\infty}^\infty \rho_{N(\ln(\mu),\sigma^2_{\ln(u)})}(\ln(u)) du \, d\ln(b),
\]

(12)

where \( \rho_{N(\ln(\eta),\sigma^2_{\ln(u)})}(\ln(b)) \) and \( \rho_{N(\ln(\mu),\sigma^2_{\ln(u)})}(\ln(b)) \) are the densities of normal distribution functions for the values of \( \ln(u) \) and \( \ln(b) \) with the dispersions of \( \sigma^2_{\ln(u)} \) and \( \sigma^2_{\ln(b)} \).

In order to elucidate the character of the changes for \( C \) factor under the variation of the values \( \ln(\mu) \) and \( \sigma^2_{\ln(u)} \) one should note the derivatives: \( \frac{dC}{d\ln(\mu)} \) and \( \frac{dC}{d\sigma_{\ln(u)}} \). While considering \( \frac{dC}{d\ln(\mu)} \) one should start from the calculations of a derivative:

\[
\frac{d\rho_{N(\ln(\mu),\sigma^2_{\ln(u)})}(\ln(u))}{d\ln(\mu)} = -\frac{2(\ln(u) - \ln(\mu))}{2\sigma^2_{\ln(u)}} \cdot \frac{1}{\sqrt{2\pi} \sigma_{\ln(u)}} \exp\left( -\frac{(\ln(u) - \ln(\mu))^2}{2\sigma^2_{\ln(u)}} \right) = -\frac{d\rho_{N(\ln(\mu),\sigma^2_{\ln(u)})}(\ln(u))}{d\ln(u)},
\]

(13)

then, with the account of (12) and (13), we obtain:

\[
\frac{dC}{d\ln(\mu)} = -\int_{-\infty}^\infty \rho_{N(\ln(\eta),\sigma^2_{\ln(u)})}(\ln(b)) \frac{d}{d\ln(u)} \int_{-\infty}^\infty \rho_{N(\ln(\mu),\sigma^2_{\ln(u)})}(\ln(u)) du \, d\ln(b) =

\int_{-\infty}^\infty \rho_{N(\ln(\eta),\sigma^2_{\ln(u)})}(\ln(b)) \rho_{N(\ln(\mu),\sigma^2_{\ln(u)})}(\ln(\frac{1-p}{p})) db =

\int_{-\infty}^\infty \frac{1}{\sqrt{2\pi} \sigma_{\ln(b)}} \exp\left[ -\frac{(\ln(b) - \ln(\mu))^2}{2\sigma^2_{\ln(b)}} \right] \frac{1}{\sqrt{2\pi} \sigma_{\ln(b)}} \exp\left[ -\frac{(\ln(b) - \ln(\eta))^2}{2\sigma^2_{\ln(b)}} \right] db =
\]
\[
= - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi \sigma_{\ln(u)}}} \exp \left[ \frac{\ln(b) - \ln \left( \frac{\mu p}{1 - p} \right)}{2\sigma_{\ln(u)}^2} \right]^2 \frac{1}{\sqrt{2\pi \sigma_{\ln(b)}}} \exp \left[ \frac{\ln(b) - \ln(\eta)}{2\sigma_{\ln(b)}^2} \right] d\ln(b) =
\]

Applying lemma from [14] to (14):

\[
\rho_{N(n_{m_2},\sigma^2_{u})}(\ln(b)) \rho_{N(n_{m_1},\sigma^2_{b})}(y) = \rho_{N(n_{m_2} - s \cdot m_{1},\sigma^2_{u} + s^2 \cdot \sigma^2_{b})}(0) \cdot \rho_{N(M,\sigma^2)}(y),
\]

where: \( M = \frac{m_2 \cdot \sigma^2_{u} - s \cdot (1 \cdot x - m_{1}) \cdot \sigma^2_{b}}{\sigma^2_{u} + s^2 \cdot \sigma^2_{b}} \); \( \sigma^2 = \frac{\sigma^2_{u} \cdot \sigma^2_{b}}{\sigma^2_{u} + s^2 \cdot \sigma^2_{b}} \).

In (14) the product: \( \rho_{N(n_{m_2},\sigma^2_{u})}(\ln(b)) \rho_{N(n_{\eta},\sigma^2_{b})}(\ln(b)) \) is similar to (15) under the substitution:

\[
y = \ln(b) \; ; \; s = 1 \; ; \; l = 0 \; ; \; m_1 = \ln(\eta) \; ; \; m_2 = \ln \left( \frac{\mu \cdot p}{1 - p} \right) ; \; \sigma_1 = \sigma_{\ln(b)} ; \; \sigma_2 = \sigma_{\ln(u)}. \]

Then, basing on the relations of (14 - 16) we obtain:

\[
\frac{dC}{d\ln(\mu)} = -\rho_{N(n_{m_2},\sigma^2_{u})}(0) \cdot \int_{-\infty}^{\infty} \rho_{N(n_{m_2} - s \cdot m_{1},\sigma^2_{u} + s^2 \cdot \sigma^2_{b})}(0) d\ln(b) =
\]

\[
= -\rho_{N(n_{m_2},\sigma^2_{u})}(0) < 0.
\]

After that on the basis of the expression (4) taking into account the independence of \( p \) on the social-economic factors:

\[
\frac{dp}{d\ln(\mu)} = -k \cdot \rho_{N(n_{m_2},\sigma^2_{u})}(0) < 0,
\]

and it means a decrease of the probability \( p \) for the fire occurrence in the residential area of a region under the growth of the median value \( \mu \) for the lognormal distribution of the damages value in the homesteads of a region occurring as a result of fire.

The relation between the mean \( u_{mn} \) and median \( \mu \) values for the fire damages can be determined as [15]:

\[
u_{mn} = \exp \left( \ln(\mu) + \frac{\sigma^2_{\ln(u)}}{2} \right), \text{ and hence: } \ln(\mu) = \ln(u_{mn}) - \frac{\sigma^2_{\ln(u)}}{2} ;
\]

then:

\[
\frac{dR_{1z}}{d\ln(u_{mn})} = \frac{dR_{1z}}{d\ln(\mu)} \cdot \frac{d\ln(u_{mn})}{d\ln(\mu)} = \frac{dR_{1z}}{d\ln(\mu)} ;
\]

\[
\frac{dR_{1z}}{d\ln(u_{mn})} = \frac{dR_{1z}}{d\ln(\mu)} \cdot \frac{d\ln(u_{mn})}{d\ln(\mu)} = \frac{1}{u_{mn}} \cdot \frac{dR_{1z}}{d\ln(\mu)} \cdot \frac{1}{u_{mn}} \cdot \frac{10^{l \cdot N_{m} \cdot dp}}{N} < 0.
\]
From the formulas of (6) and (9) concerned with the mean damages of the homesteads in a region as a result of fire occurrence it follows that:

$$u_{m} = u_{m} + \frac{E_{m} \cdot R_{2z}}{10^2} = \left( g \cdot d \cdot n \cdot \tau + \frac{R_{2z} \cdot 12}{10^2} \cdot K_{mort} \right) \cdot r \cdot D_{m} ,$$  \hspace{1cm} (20)

where: $u_{m}$ are the mean direct damages due to the fire in the homesteads of a region; $E_{m}$ - is a mean economic equivalent of the human life; $D_{m}$ are the mean monthly population’s incomes in a region.

Then with the account of (19) and (20) it follows:

$$\frac{dR_{1z}}{dR_{2z}} = \frac{du_{m}}{du_{m}} = \frac{dR_{1z}}{dR_{2z}} = \frac{E_{m}}{10^2} < 0; \hspace{1cm} (21)$$

$$\frac{dR_{1z}}{du_{m}} = \frac{dR_{1z}}{du_{m}} = \frac{dR_{1z}}{du_{m}} < 0; \hspace{1cm} (22)$$

When considering $\frac{dC}{d\sigma_{ln(u)}}$ we start from the calculation for the expression:

$$\frac{d}{d\sigma_{ln(u)}} \ln \left\{ \frac{\left( 1-p \right)_b^\mu}{\sigma^2 \ln(u)} \right\} \left\{ \ln(u) \right\} \frac{d\ln(u)}{du} =$$

$$= - \frac{1}{\sigma_{ln(u)}} \left\{ \int_{-\infty}^{\ln \left( \frac{1-p}{\mu} \right)} \left[ I - \frac{(\ln(u) - \ln(\mu))^2}{\sigma^2 \ln(u)} \right] \frac{\rho_{N(\ln(\mu),\sigma^2_{ln(u)})}}{(\ln(u))} \frac{d\ln(u)}{du} \right\} +$$

$$+ \frac{1}{\sigma_{ln(u)}} \left\{ \int_{-\infty}^{\ln \left( \frac{1-p}{\mu} \right)} \left[ \frac{(\ln(u) - \ln(\mu))^2}{\sigma^2 \ln(u)} \right] \frac{\rho_{N(\ln(\mu),\sigma^2_{ln(u)})}}{(\ln(u))} \frac{d\ln(u)}{du} \right\}.$$

(23)

Let us calculate an integral in a separate way

$$\int_{-\infty}^{\ln \left( \frac{1-p}{\mu} \right)} \left[ \frac{(\ln(u) - \ln(\mu))^2}{\sigma^2 \ln(u)} \right] \frac{\rho_{N(\ln(\mu),\sigma^2_{ln(u)})}}{(\ln(u))} \frac{d\ln(u)}{du} =$$

$$= - \int_{-\infty}^{\ln \left( \frac{1-p}{\mu} \right)} (\ln(u) - \ln(\mu)) \frac{d\rho_{N(\ln(\mu),\sigma^2_{ln(u)})}}{(\ln(u))} (\ln(u)) =$$
\[
= \left( \ln \left( \frac{1-p}{p} \right) b - \ln(\mu) \right) \rho_{N(\ln(\mu), \sigma^2_{\ln(u)})} \left( \ln \left( \frac{1-p}{p} \right) b \right) + \\
\ln \left( \frac{1-p}{p} b \right)
\]
\[
+ \int_{-\infty}^{\infty} \rho_{N(\ln(\mu), \sigma^2_{\ln(u)})} \left( \ln( u ) \right) d\ln( u ).
\] (24)

Substituting (24) to (23) we obtain
\[
\frac{d}{d\ln(\ln(u))} \left( \ln \left( \frac{1-p}{p} \right) b - \ln(\mu) \right) \rho_{N(\ln(\mu), \sigma^2_{\ln(u)})} \left( \ln \left( \frac{1-p}{p} \right) b \right)
\]
\[
= - \frac{1}{\sigma_{\ln(u)}} \left( \ln \left( \frac{1-p}{p} \right) b - \ln(\mu) \right) \rho_{N(\ln(\mu), \sigma^2_{\ln(u)})} \left( \ln \left( \frac{1-p}{p} \right) b \right).
\] (25)

Then:
\[
\frac{dC}{d\sigma_{\ln(u)}} = - \frac{1}{\sigma_{\ln(u)}} \left( \ln \left( \frac{1-p}{p} \right) b - \ln(\mu) \right) \times
\]
\[
\times \rho_{N(\ln(\mu), \sigma^2_{\ln(u)})} \left( \ln \left( \frac{1-p}{p} \right) b \right) \rho_{N(\ln(\eta), \sigma^2_{\ln(b)})} \left( \ln( b ) \right) d\ln( b )
\]
\[
= - \frac{1}{\sigma_{\ln(u)}} \left( \ln \left( \frac{1-p}{p} \right) b - \ln(\mu) \right) \rho_{N(\ln(\mu), \sigma^2_{\ln(u)})} \left( \ln \left( \frac{1-p}{p} \right) b \right) \rho_{N(\ln(\eta), \sigma^2_{\ln(b)})} \left( \ln( b ) \right) d\ln( b ).
\] (26)

Applying equations (14 - 16) to (26) we obtain
\[
\frac{dC}{d\ln(\ln(u))} = - \frac{\rho_{N(\ln(\mu), \sigma^2_{\ln(u)}), \sigma^2_{\ln(\ln(u))}}(0)}{\sigma_{\ln(u)}} \times
\]
\[
\times \int_{-\infty}^{\infty} \left( \ln \left( \frac{1-p}{p} \right) b - \ln(\mu) \right) \rho_{N(\ln(\eta), \sigma^2_{\ln(u)}), \sigma^2_{\ln(b)}} \left( \ln \left( \frac{1-p}{p} \right) b \right) \rho_{N(\ln(\eta), \sigma^2_{\ln(b)}} \left( \ln( b ) \right) d\ln( b )
\]
\[
= - \frac{\rho_{N(\ln(\mu), \sigma^2_{\ln(u)}), \sigma^2_{\ln(\ln(u))}}(0)}{\sigma_{\ln(u)}} \times
\]
\[
\times \left[ \ln \left( \frac{1-p}{p} \right) + \frac{\ln(\mu) - \ln \left( \frac{1-p}{p} \right)}{\sigma^2_{\ln(u)} + \sigma^2_{\ln(b)}} - \ln(\mu) \right]
\]
\[
= \rho_{N(\ln(\mu), \sigma^2_{\ln(u)}), \sigma^2_{\ln(\ln(u))}}(0) \cdot \frac{\sigma_{\ln(u)}}{\sigma^2_{\ln(u)} + \sigma^2_{\ln(b)}} \cdot \ln \left( \frac{1-p}{p} \eta \right).
\] (27)

Since Gini coefficient [16]:
\[ J_u = \operatorname{erf}\left( \frac{\sigma_{\ln(u)}}{2} \right) = \frac{2}{\sqrt{\pi}} \int_{0}^{\sigma_{\ln(u)}} \exp(-t^2) \, dt , \]  
(28)

Then, using the relationship:

\[ \frac{dR_{iz}}{dJ_u} = \frac{dR_{iz}}{d\sigma_{\ln(u)}} \cdot \frac{dJ_u}{d\sigma_{\ln(u)}} , \]  
(29)

we obtain:

\[ \frac{dR_{iz}}{dJ_u} = \frac{dR_{iz}}{d\sigma_{\ln(u)}} \cdot \sqrt{\pi} \cdot \exp\left( \frac{\sigma_{\ln(u)}^2}{4} \right) . \]  
(30)

It means that, \( \frac{dR_{iz}}{dJ_u} \) is of the same sign as \( \frac{dR_{iz}}{d\sigma_{\ln(u)}} \).

From (30), (27) and (4), with the account of independency for \( p_n \) on the social-economic factors it follows that:

\[ \frac{dC}{d\sigma_{\ln(u)}} < 0 , \quad \frac{dp}{d\sigma_{\ln(u)}} < 0 , \quad \frac{dR_{iz}}{dJ_u} < 0 , \quad \text{if } (1-p) \cdot \eta > p \cdot \mu ; \]  
(31)

or, moving to the mean values if \( (1-p) \cdot b_{mn} \cdot \exp\left( -\frac{\sigma_{\ln(b)}^2}{2} \right) > p \cdot u_{mn} \cdot \exp\left( -\frac{\sigma_{\ln(u)}^2}{2} \right) ; \)

\[ \frac{dC}{d\sigma_{\ln(u)}} > 0 , \quad \frac{dp}{d\sigma_{\ln(u)}} > 0 , \quad \frac{dR_{iz}}{dJ_u} > 0 , \quad \text{if } (1-p) \cdot \eta < p \cdot \mu ; \]  
(32)

or, moving to the mean values if \( (1-p) \cdot b_{mn} \cdot \exp\left( -\frac{\sigma_{\ln(b)}^2}{2} \right) < p \cdot u_{mn} \cdot \exp\left( -\frac{\sigma_{\ln(u)}^2}{2} \right) ; \)

\[ \frac{dC}{d\sigma_{\ln(u)}} = 0 , \quad \frac{dp}{d\sigma_{\ln(u)}} = 0 , \quad \frac{dR_{iz}}{dJ_u} = 0 , \quad \text{if } (1-p) \cdot \eta = p \cdot \mu ; \]  
(33)

or, moving to the mean values if \( (1-p) \cdot b_{mn} \cdot \exp\left( -\frac{\sigma_{\ln(b)}^2}{2} \right) = p \cdot u_{mn} \cdot \exp\left( -\frac{\sigma_{\ln(u)}^2}{2} \right) . \)

Distribution of the logarithm of the value for the legal incomes \( \ln(D) \) for the homestead members in a region can be represented by the normal distribution function \( \rho_n(\ln(D); \sigma^2_{\ln(D)}) \) [12].

Hence, with the account of (20):

\[ \frac{dR_{iz}}{dD_{mn}} = \frac{dR_{iz}}{d\sigma_{\ln(u)}} \cdot \frac{du_{mn}}{dD_{mn}} = \frac{dR_{iz}}{d\sigma_{\ln(u)}} \cdot \left( g \cdot d \cdot n \cdot \tau + \frac{R_{2z} \cdot 12}{10^2 \cdot K_{\text{mort}}} \right) \cdot r < 0 . \]  
(34)

Similarly (20), for the most widespread (modal) damages \( u_{\text{mod}} \) of the homesteads in a region occurred due to the fire and modal monthly population’s incomes \( D_{\text{mod}} \) in a region it is possible to write down the relationship:

\[ u_{\text{mod}} = u_{\text{mnmod}} + \frac{E_{\text{mod}} \cdot R_{2z}}{10^2} = \left( g \cdot d \cdot n \cdot \tau + \frac{R_{2z} \cdot 12}{10^2 \cdot K_{\text{mort}}} \right) \cdot r \cdot D_{\text{mod}} . \]  
(35)

Then from (20) and (35), taking into account of [15], it follows:
$$\sigma_{\ln(u)} = \frac{2}{\sqrt{3}} \left( \ln(u_{\text{rm}}) - \ln(u_{\text{mod}}) \right) = \frac{2}{\sqrt{3}} \left( \ln(D_{\text{rm}}) - \ln(D_{\text{mod}}) \right) = \sigma_{\ln(D)};$$

(36)

Since Gini coefficient [16]:

$$J_D = \text{erf}(\frac{\sigma_{\ln(D)}}{2}) = \frac{\sigma_{\ln(D)}}{2} \cdot \frac{1}{\sqrt{\pi}} \cdot \int_0^\infty \exp(-t^2) \, dt;$$

(37)

then, using the relationship:

$$\frac{dR_{1z}}{d\sigma_{\ln(D)}} = \frac{dR_{1z}}{dJ_D} \cdot \frac{dJ_D}{d\sigma_{\ln(D)}};$$

(38)

We obtain obtain:

$$\frac{dR_{1z}}{dJ_D} = \frac{dR_{1z}}{d\sigma_{\ln(D)}} \cdot \sqrt{\pi} \cdot \exp\left( \frac{\sigma^2_{\ln(D)}}{4} \right);$$

(39)

Respectively, since from (36): $\sigma_{\ln(u)} = \sigma_{\ln(D)}$, and, hence:

$$\frac{dR_{1z}}{dJ_D} = \frac{dR_{1z}}{d\sigma_{\ln(u)}} \cdot \sqrt{\pi} \cdot \exp\left( \frac{\sigma^2_{\ln(u)}}{4} \right) = \frac{dR_{1z}}{dJ_u};$$

(40)

and so, $\frac{dR_{1z}}{dJ_D}$ is of the same sign as $\frac{dR_{1z}}{dJ_u}$.

From (31 - 33), (36) and (40) it follows that:

$$\frac{dR_{1z}}{dJ_D} < 0 \text{, if: } (1 - p) \cdot \eta > p \cdot \mu; \text{ or: } (1 - p) \cdot b_{\text{rm}} \cdot \exp(-\frac{\sigma^2_{\ln(b)}}{2}) > p \cdot u_{\text{rm}} \cdot \exp(-\frac{\sigma^2_{\ln(u)}}{2});$$

(41)

$$\frac{dR_{1z}}{dJ_D} > 0 \text{, if: } (1 - p) \cdot \eta < p \cdot \mu; \text{ or: } (1 - p) \cdot b_{\text{rm}} \cdot \exp(-\frac{\sigma^2_{\ln(b)}}{2}) < p \cdot u_{\text{rm}} \cdot \exp(-\frac{\sigma^2_{\ln(u)}}{2});$$

(42)

$$\frac{dR_{1z}}{dJ_D} = 0 \text{, if: } (1 - p) \cdot \eta = p \cdot \mu; \text{ or: } (1 - p) \cdot b_{\text{rm}} \cdot \exp(-\frac{\sigma^2_{\ln(b)}}{2}) = p \cdot u_{\text{rm}} \cdot \exp(-\frac{\sigma^2_{\ln(u)}}{2}).$$

(43)

3. Analysis of panel data

In order to determine the effect of the factors registered by statistics on the level of fire risks in a residential area over the regions of Russia let us perform the analysis of panel data basing on the model with a random effect [17]. Models with a random effect are described by the equation $y_{it} = bx_{it} + u_i + c + \varepsilon_{it}$, where $y_{it}$ is an explained variable; $x_{it}$ is an explanatory variable.; $u_i$ are the random invariant in time values for each economic unit, $\varepsilon_{it}$ is the error of the model, and $c$ is a constant. In the models with a random effect it is assumed that the individual differences are of a random character. In order to calculate models with random effects the generalized least square (GLS) method has to be applied; it is realized in plm suite of R program.

Information base for the analysis was formed of the panel data over 82 regions of Russian Federation (RF) (one of the regions was excluded from the analysis since it did not provide a complete set of data) for the period of 2006 – 2012. Information on the fire statistics and indicators of the time showing operative response to the fire calls and alerts was obtained from the data registered by the State fire fighting service of the Ministry of emergency of Russia, while social and economic
indicators for the regions and the values of inflation rates were taken from the publications of Russian statistical agency.

Assuming that the risk $R_{1i}$ of happening a case that a person occurs into a fire in the residential area per a year is a linear function depending on the various factors. Thus, aggregating data over the population of a region one can make a linear model 1 with a random effect for the description of the fire risks in a dependence of a set of the independent variables:

$$
R_{1it} = a_j u_{mit} + a_2 R_{2it} + a_3 D_{it} + a_4 J_{Dit} + a_5 A_{it} + a_6 Z_{it} + a_7 G_{it} + a_8 S_{it} + a_9 T_{it} + a_{10} f_{it} + C, \tag{44}
$$

where the lower indices $i$ and $t$ designate region and a year, respectively; dependent variable in the equation (44) $R_{1it}$ - risk for a person to occur into fire in the residential area per a year; $u_{mit}$ - mean material damage due to a fire in the residential area, in thousands of rubles with the account of inflation relative to 2006, and taking a mean material damage of one fire in the residential area in 2006 as a reference one; $R_{2it}$ is a risk of death of a person in a fire in the residential area; $D_{it}$ are monetary population’s incomes in thousands of rubles, with the account of inflation relative to the 2006 rates, taking the mean monetary incomes in thousands of rubles in 2006 as the reference ones; $J_{Dit}$ is Gini coefficient for the regions (inequality in the incomes measure); $A_{it}$ is a number of the patients diagnosed for the first time as having a psychotic disorder as a result of drinking and alcoholic-dependent syndrome pertained to the prophylaxy by psychoneurologic and narcological dispensaries per 10^5 of the regional population (according to the reasons provided in [14], this index may characterize the alcohol abuse level among the population of the corresponding region); $Z_{it}$ is a part of a slum and hazardous dwelling in the region; $G_{it}$ is a part of the urban population in a region; $S_{it}$ is a part of students in the institutions of the higher professional education relative to the population in the region (this index characterizes the level of education in the region); $T_{it}$ is a mean temperature of January (in Celsius degrees) in the region; $t_{it}$ is the time averaged over the region (in minutes) showing the time of arriving for the first fire teams to start fighting; $C$ is a constant including unaccounted factors.

Results of identification for the parameters of the linear model 1 with a random effect obtained when utilizing plm suite of R program are presented in table 1. Coefficients of the linear dependence on a set of the independent variables were obtained for the fire risk $R_{1it}$. Besides the values of the coefficients the table shows the values of the standard errors (in brackets). High values of Waald’s statistics mean an overall adequacy of the model: 499.33.

**Table 1.** Results of the regression analysis of panel data on dependence of level data on dependence of level of fire risk $R_{1i}$ in a residential area over the regions of Russia from socio-economic factors and response time of fire divisions.

| Factors | Model 1 equation (44) for $R_{1it}$ [fire / (10^5 man year)] |
|---------|---------------------------------------------------------------|
| $R_{2it}$ [victim / (10^5 fires)] | $a_f = -0.0179946 *** (0.0040385)$ |
| $u_{mit}$ [thousand rubles / fire] | $a_2 = -0.0001957 *** (0.0000999)$ |
| $D_{it}$ [thousand rubles] | $a_3 = -0.0503379 *** (0.00458)$ |
| $J_{Dit}$ |
| $A_{it}$ [patients / 10^5 man] | $a_4 = 1.749431 *** (0.2553084)$ |
| $Z_{it}$ [%] | $a_5 = 0.0023804 *** (0.0002213)$ |
| $G_{it}$ [%] | $a_6 = 0.0041696 (0.0067839)$ |
| $S_{it}$ [%] | $a_7 = 0.0163211 *** (0.0029772)$ |
| $T_{it}$ [degrees. C°] | $a_8 = 0.0158513 (0.011199)$ |
| $t_{it}$ [minute] | $a_9 = 0.0005644 (0.0013093)$ |
| $C$ | $a_{10} = 0.008715 *** (0.0022977)$ |
| Waald test, $\chi$ | $C = -0.5286148 (0.2332259)$ |

In the table 1 asterisks have designated levels of credibility: *** – 1%, ** – 5%, * – 10%.
Comparison of the results of calculation on the basis of the economic model for the rational offender by the effect of the changes of economic factors on the variations of an integral fire risk \( R_{it} \) with the results of the calculations for the model with a random effect for \( R_{it} \), presented in table 1, showed an agreement between the calculations according to the model of a rational offender and calculations based on the statistical analysis of the panel data. In fact:

\[
dR_{it}/dR_{it} = a_1 = -0.018 < 0, \quad \text{conforms with (21)};
\]

\[
dR_{it}/du_{it} = a_2 = -0.0002 < 0, \quad \text{conforms with (22)};
\]

\[
dR_{it}/D_a = a_3 = -0.05 < 0, \quad \text{conforms with (34)};
\]

\[
dR_{it}/D_{it} = a_4 = 1.75 > 0, \quad \text{conforms with (42)},
\]

that is realized provided that:

\[
(1 - p) \cdot b_{mn} \cdot \exp(-\frac{\sigma_{ln(b)}^2}{2}) < p \cdot u_{mn} \cdot \exp(-\frac{\sigma_{ln(u)}^2}{2}).
\]

Fulfillment of (42) for a statistics of Russia is confirmed by the analysis of the results presented in [18]. In fact, as it was shown in [18], for the average state expenditures \( g_{mn} \) utilized for the prevention of the fire risks and the mean damage \( u_{in} \) incurred as a result of fires in Russian Federation if those risks are not prevented the following relationship is satisfied:

\[
(1 - p) \cdot g_{mn} < p \cdot u_{mn}.
\]

But from the idea of the rational offender violating fire safety rules it follows that the owners of homesteads would be quite rational in their expenditures for prevention of the fire risks and would not surrender to the state in this point of interests. Hence, for Russia the following expression can be written as:

\[
(1 - p) \cdot b_{mn} \leq (1 - p) \cdot g_{mn} < p \cdot u_{mn},
\]

that is in agreement with the fulfillment of (42).

The value of economic equivalent of the human life \( E \) for the homestead in Russia can be estimated representing (44) in the form of:

\[
R_{it} = a_1(u_{mn} + a_2 R_{2it} / a_1) + a_3 D_{it} + a_6 J_{Dis} + a_5 A_{it} + a_7 G_{it} + a_9 S_{it} + a_8 T_{it} + a_{10} f_{it} + C_i.
\]

With the account of the relation (6), equation (47) can be also written in the form of:

\[
R_{it} = a_1(u_{mn} + E \cdot R_{2it} / 10^2) + a_3 D_{it} + a_6 J_{Dis} + a_5 A_{it} + a_7 G_{it} + a_9 S_{it} + a_8 T_{it} + a_{10} f_{it} + C_i.
\]

Comparing (47) and (48) one can conclude that the value of economic equivalent of a human life \( E \) for the residential area in Russia can be determined by the formula:

\[
E = 10^2 \cdot a_2 / a_1.
\]

With the account of the values and dimension of the coefficients \( a_1 \) and \( a_2 \) from the table 1:

\[
E = 9195 \quad \text{[thousands rubles/victim]}, \quad \text{subject to the inflation relative to its level at 2006}.
\]

The obtained estimation of the economic equivalent of the human life in the residential area of Russia is close to that one calculated for Russia according to the technique for estimation of the economic equivalent of the human life. It is based on the idea that the economic equivalent of the human life for a typical man is equal to the ratio of the average per capita disposable annual income relative to the mean probability of a death per year [11,19]. According to the calculations of [19], this value for Russia was of 12472 thousands rubles. This value corresponded to 9038 thousands rubles accounting for the official inflation relative to the level of 2006. Such an agreement of the \( E \) value obtained from the model 1, with the value of the economic equivalent for the human life obtained with the use of technique from [19], confirms validity of the model 1 when it was applied to the estimation of the risk for a man to occur in the fire case when this happens in the residential area of the regions in Russian Federation per year. And it also complied with the model of a rational offender.
4. Conclusion
Mathematical model of the rational offender violating fire safety in the residential area is presented based on the economic approach. Analysis of this model allows obtaining the direction of variation of the value for integrated fire risk in a region. It is connected with the probability of a person to happen in the situation of fire occurrence in the residential area per a year. This probability is calculated under the following statistically registered regional social-economic factors: an integral fire risk related with the probability of a person death in the fire occurred in the residential area; mean value of the amount of direct material damages caused by the fire in the residential area; mean value of the monthly population’s incomes; Gini coefficient for the distribution of the monthly population’s income.

Results of statistical analysis of the panel data on the fire and social-economic statistics collected over the regions of Russia and obtained basing on the linear model with a random effect are in a reasonable agreement with the results of analysis for the mathematical model of the rational offender violating fire safety requirements.

References
[1] Brushlinsky N N, Sokolov S V, Klepko E A, Belov V A, Ivanova O V and Popkov S Yu 2012 Basics of the fire risks theory and its applications (Moscow: Academy of the State Fire Service of Russia) 192 p
[2] Brushlinsky N, Sokolov S and Wagner P 2010 Humanity and Fires (Leipzig: Protection Association) 353 p
[3] Jennings C R 1999 Socioeconomic characteristics and their relationship to fire incidence: a review of the literature Fire Technology 35 1 pp 7–34
[4] Hasofe A and Thomas I 2006 Analysis of Fatalities and Injuries in Building Fire Statistics Fire Safety Journal 41 1 pp 2-14
[5] Runyan C W, Bangdiwala S T, Linzer M A, Sacks J J and Butts J 1993 Risk Factors for Fatal Residential Fires Fire Technology 29 2 pp 183–193
[6] Lizhong Y, Xiaodong Z, Zhihua D, Weicheng F, and Qing’an W 2002 Fire situation and fire characteristic analysis based on fire statistics of china Fire Safety Journal 37 8 pp 785–802
[7] Klimkin V I, Matyushin A V, Poroshin A A, Lupanov S A, Bobrinev E V, Kondashov A A, Ivanova G G 2012 Analysis of influence of consequences of fires on stability of socio-economic development of regions of Russian Federation Fire safety 1 pp 74–84
[8] Becker G 1968 An Economic Approach Journal of Political Economy 76 pp 169–217
[9] Trostiansky S N, Zenin Yu N, Skryl S V and Kalatch A V 2013 Fires and emergency situations: prevention, elimination Fire and explosion safety 4 28
[10] Trostiansky S N, Zenin Yu N and Bakayeva G A 2014 Mathematical model and algorithms of calculation of level of regional fire risks on objects of supervision Control systems and information technologies 1 204
[11] Kharisov G Kh and Teterin I M 2008 Problems of forecasting Economical equivalent of the human life (Moscow: Academy of GPS Emergency Ministry of Russia) 57 p
[12] Suvorov A V 2001 Problems of analysis in the differentiation of the population’s incomes and building of the differential balance of the money incomes and expenses of population Problems of forecasting 1 pp 58–74
[13] Akimov V A, Bykov A A, Shchetinin E Yu 2009 Introduction to the statistics of the extreme values and its applications (Moscow: FGU VNII GOChS (FC)) 524 p
[14] Andrienko Yu V 2003 Economics of a crime: theoretical and empirical study of the determining factors of criminality (crime-metric approach): PhD dissertation (Economic science) (Moscow: Center for Economic and Financial Research) pp 108–109
[15] Kolmakov I B 2006 Prognostics of the factors for the differentiation of the money population’s incomes Prognostics problems 1 pp 136–162
[16] Zolotukhina L A, Tikhonenko D M and Fridman G M 2015 The study of dependences between the factors of population’s incomes differentiation Finances and business 3 pp 55–64

[17] Magnus Ya R, Katyishev P K and Peresetskiy A A 2004 Econometrics. Beginner’s course: student book (Moskow: Delo) 576 p

[18] Firsov A V 2013 Models and algorithms for substantiation of the value for the individual fire risk in order to manage security of the people in the buildings and facilities: Extended abstract of PhD (Technical science) (Moscow: Academy of GPS Emergency Ministry of Russia) 26 p

[19] Vostokov V Yu, Minaeva Ya V and Tchasnavichus Yu K 2011 On the problem of determination of the economic equivalent for the cost of living of a typical man Herald of St. Petersburg University of the State Firefighting Service at the Ministry of emergency Situations (GPS MChS) of Russia 1 pp 38–42