Consistency test of the fine-structure constant from the whole ionization history

Ke Wang\textsuperscript{a,b} and Lu Chen\textsuperscript{c,*}

\textsuperscript{a}Lanzhou Center for Theoretical Physics, Key Laboratory of Theoretical Physics of Gansu Province, School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China
\textsuperscript{b}Institute of Theoretical Physics & Research Center of Gravitation, Lanzhou University, Lanzhou 730000, China
\textsuperscript{c}School of Physics and Electronics, Shandong Normal University, Jinan 250014, China

E-mail: wangkey@lzu.edu.cn, chenlu@sdnu.edu.cn

Received July 15, 2022
Revised September 10, 2022
Accepted October 2, 2022
Published October 21, 2022

Abstract. In cosmology, the fine-structure constant can affect the whole ionization history. However, the previous works confine themselves to the recombination epoch and give various strong constraints on the fine-structure constant. In this paper, we also take the reionization epoch into consideration and do a consistency test of the fine-structure constant from the whole ionization history. From the data combination of Planck 2018, BAO data, SNIa samples, SFR density from UV and IR measurements, and the Q_{HI} constraints, we find the constraint on the fine-structure constant during the recombination epoch is $\alpha_{\text{rec}}/\alpha_{\text{EM}} = 1.00149^{+0.00204}_{-0.00206}$ and its counterpart during the reionization epoch is $\alpha_{\text{rei}}/\alpha_{\text{EM}} = 0.85403^{+0.03167}_{-0.02721}$ at 68\% C.L.. They are not consistent with each other by 4.64\%σ. A conservative explanation for such a discrepancy is that there are some issues in the data we used. We prefer a calibration of some important parameters involved in reconstructing the reionization history.

Keywords: cosmological parameters from CMBR, reionization

ArXiv ePrint: 2207.06719
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1 Introduction

The fine-structure constant $\alpha_{\text{EM}} = \frac{e^2}{4\pi\epsilon_0\hbar c}$ is a fundamental constant in physics, which indicates the strength of electromagnetic interaction. Its value has been measured with various experiments in laboratory, such as the measurements of the neutron de Broglie wavelength, the quantum Hall effect, the electron anomalous magnetic moment, local atomic clock measurements and so forth [1, 2]. The full 2018 CODATA Committee on Data for Science and Technology [3] gives the numerical value of $\alpha_{\text{EM}} = 1/137.035999084(21)$, its relative standard uncertainty is $1.5 \times 10^{-10}$. In astrophysical and cosmological systems, we also can get relatively precise measurements of the fine-structure constant with high-resolution astrophysical spectroscopy measurements [4–9], big bang nucleosynthesis [10] and cosmic microwave background (CMB) [11–14] respectively. For reviews of the other measurements, see [15]. Since these measurements may come from different epochs of our universe, it is possible for people to probe a varying fine-structure constant [4, 11, 16]. Theoretically, the fine-structure constant can be dynamical in a more general framework than the standard model of particle physics. If it is actually a dynamical quantity, the basic equations related to $\alpha_{\text{EM}}$ we are using are approximations of some more general equations. In practice, comparing all above measurements, one can find that these measurements are consistent with each other and there is no evidence to support a varying fine-structure constant now.

However, there is a caveat for measurements with CMB [11–14] that these previous constraints on the fine-structure constant are only obtained from recombination epoch although the fine-structure constant can affect the whole ionization history. For recombination epoch, the story is simple. The fine-structure constant influences the Thomson scattering cross-section and the energy levels of hydrogen and helium directly. As a result, its derived effects on the visibility function can leave footprints on the CMB power spectra, which can be researched with cosmological observations. As for the reionization epoch, the situation is complicated. Because there is a lack of knowledge about the reionization, such as its sources and its outset. Here we suppose that the star-forming galaxies are the major sources of the reionization. Therefore, there are two important parameters for reconstructing the reionization history: the ionizing photon emission rate of a star-forming galaxy $\xi_{\text{ion}}$ and the escape fraction of ionizing photons $f_{\text{esc}}$. To obtain the constraints on the fine-structure constant from the reionization epoch, there are two essential prerequisites. On the one hand, we should know the values of these two parameters with respect to $\alpha_{\text{EM}} = 1/137.035999084(21)$. On
the other hand, we should know the dependence of these two parameters on $\alpha/\alpha_{\text{EM}}$, where $\alpha$ is the fine-structure constant in question.

In this work, we make the fine-structure constant free during the whole ionization history and constrain its values with observational data in two models. In the $\Lambda$CDM+$\alpha/\alpha_{\text{EM}}$ model, the fine-structure constant remains a constant $\alpha$ at all redshifts. In the $\Lambda$CDM+$\alpha_{\text{rec}}/\alpha_{\text{EM}} + \alpha_{\text{rei}}/\alpha_{\text{EM}}$ model, there is a constant $\alpha_{\text{rec}}$ at the recombination epoch and another constant $\alpha_{\text{rei}}$ at the reionization epoch. In the latter model, we can do a consistency test of the fine-structure constant from the whole ionization history and have a glance at the redshift-varying fine-structure constant.

This paper is organized as follows. In section 2, the ionization history of our universe is reconsidered with the rescaled fine-structure constant. In subsection 2.1, we introduce the recombination calculation related to $\alpha_{\text{rec}}$. In subsection 2.2, modifications of the reionization history are presented. In section 3, we put constraints on free parameters in previous models with the same data combination including CMB measurement from Planck 2018 [17–19], SFR density from UV data and IR data [20], $Q_{\text{HII}}$ constraints from observations of quasars [21–23], baryon acoustic oscillation (BAO) data [24–29], as well as the measurements of Type Ia supernova [30]. Finally, a brief summary is included in section 4.

2 Effects of the fine-structure constant on ionization history

In this section, the effects of varying fine-structure constant on the ionization history are introduced and the evolution of ionized fraction is shown in figure 1. Note we denote the fine-structure constant as $\alpha$ uniformly for both recombination epoch and reionization epoch.

2.1 Effects of the fine-structure constant on recombination

The evolution of ionized fraction $x_e = x_p + x_{\text{HeII}}$ is described by a set of differential equations during the recombination epoch [12]:

$$\frac{dx_p}{dz} = \frac{C_H}{H_0(1+z)E(z)} \left[ x_e x_p \langle n_H \rangle A_H - B_H (1 - x_p) e^{-h\nu_{1H}T_M/kT_M} \right],$$

$$\frac{dx_{\text{HeII}}}{dz} = \frac{C_{\text{HeI}}}{H_0(1+z)E(z)} \left[ x_e x_{\text{HeII}} \langle n_H \rangle A_{\text{HeI}} - B_{\text{HeI}} (f_{\text{He}} - x_{\text{HeII}}) e^{-h\nu_{1\text{HeI}}T_M/kT_M} \right],$$

$$\frac{dT_M}{dz} = \frac{8\sigma_T a_R T_M^4}{3H_0 E(z)(1+z) m_e c} (T_M - T) + \frac{2T_M}{1+z}.$$  (2.3)

Here $A_i$ and $B_i \ (i = \text{H, HeI})$ are the effective recombination rates and photoionization rates, respectively. The coefficients $C_i$ are

$$C_H = \frac{1 + K_H A_H \langle n_H \rangle (1 - x_p)}{1 + K_H (A_H + B_H) \langle n_H \rangle (1 - x_p)},$$

$$C_{\text{HeI}} = \frac{1 + K_{\text{HeI}} A_{\text{HeI}} \langle n_H \rangle (f_{\text{He}} - x_{\text{HeII}}) e^{h\nu_{1\text{HeI}}T_M/kT_M}}{1 + K_{\text{HeI}} (A_{\text{He}} + B_{\text{HeI}}) \langle n_H \rangle (f_{\text{He}} - x_{\text{HeII}}) e^{h\nu_{1\text{He}}/kT_M}},$$

where the so-called “$K$-quantities” $K_i$ represent the effective transition rates for the main resonances of hydrogen and helium. $A_i$ describe the two-photon decay rates.

As mentioned above, the fine-structure constant $\alpha$ implies the strength of electromagnetic interaction. Therefore it affects the energy levels of hydrogen and helium atoms directly.
as $E_i \propto (\alpha/\alpha_{\text{EM}})^2$. This leads to two main sequent changes. Firstly, the transition frequencies $\nu_i$ are altered, such as the atomic transition rates of H $2s\rightarrow 1s$, He $2^1p \rightarrow 1^1s$, and $2^3p \rightarrow 1^1s$. Likewise, values of $\Lambda_i$, $A_i$, $B_i$, and $K_i$ are also rescaled. Secondly, this leads to the effective temperature $T_{\text{eff}} \propto (\alpha/\alpha_{\text{EM}})^{-2}$ when we calculate $A_i$ and $B_i$. In addition, the Thomson scattering cross-section $\sigma_T$ ought to be modified. Overall, above modifications can be summarized as following [13]:

$$
\Lambda_i \propto \left(\frac{\alpha}{\alpha_{\text{EM}}}\right)^8, \quad A_i \propto \left(\frac{\alpha}{\alpha_{\text{EM}}}\right)^2, \quad B_i \propto \left(\frac{\alpha}{\alpha_{\text{EM}}}\right)^5, \\
K_i \propto \left(\frac{\alpha}{\alpha_{\text{EM}}}\right)^{-6}, \quad T_{\text{eff}} \propto \left(\frac{\alpha}{\alpha_{\text{EM}}}\right)^{-2}, \quad \sigma_T \propto \left(\frac{\alpha}{\alpha_{\text{EM}}}\right)^2.
$$

(2.6)

As the right figure of figure 1 shows, recombination begins earlier with higher $\alpha$.

### 2.2 Effects of the fine-structure constant on reionization

Previous researches reveal stars and galaxies contributed the majority of ionizing photons during the epoch of reionization [20, 31–37]. In this work, we reconstruct the reionization history with star formation history, instead of the instantaneous reionization model.

The Thomson optical depth is given by

$$
\tau(z) = c\sigma_T \int_0^z (1 + f_{\text{He}})Q_{\text{HII}}(z')n_H(z')(1+z')^{-1}H^{-1}(z')dz'.
$$

(2.7)

Here $c$ is the speed of light and $H(z)$ is the redshift-dependent Hubble parameter. The ionization fraction $f_{\text{He}}$ is 0.08 (or 0.16) for the singly (or fully) ionized Helium. The ionized fraction $Q_{\text{HII}}$ (equivalent to $x_p$ physically) can be calculated by the following differential equation [31]:

$$
\dot{Q}_{\text{HII}} = -\frac{Q_{\text{HII}}}{t_{\text{rec}}} + \frac{n_{\text{ion}}}{\langle n_H \rangle}.
$$

(2.8)
Here $t_{\text{rec}}$ is the recombination time defined by [23, 31]

$$
t_{\text{rec}} = \left[ C_{\text{HI}} A_{\text{HI}} \left( 1 + \frac{Y_{\text{p}}}{4X_{\text{p}}} \right) \langle n_{\text{H}} \rangle (1 + z)^3 \right]^{-1}
$$

$$
= 0.88 \text{ Gyr} \left( \frac{1 + z}{7} \right)^{-3} \left( \frac{T_0}{2 \times 10^4 \text{K}} \right)^{-0.7} \left( \frac{C_{\text{HI}}}{3} \right)^{-1},
$$

(2.9)

where $T_0 \sim 2 \times 10^4 \text{K}$ is the temperature of the ionizing hydrogen gas and $C_{\text{HI}} \sim 3$ is the clumping factor of ionized hydrogen. The second term of eq. (2.8) represents the effect of SFR. The growth rate of number density for ionized photons is [31]

$$
\dot{n}_{\text{ion}} = f_{\text{esc}} \xi_{\text{ion}} \rho_{\text{SFR}},
$$

(2.10)

where $f_{\text{esc}}$ is the escaping fraction, which indicates the fraction of the number of escaping Lyman continuum photons to that of Lyman continuum photons produced in a galaxy. $\xi_{\text{ion}}$ transforms a UV luminosity density into the Lyman continuum photon emission rate of a star-forming galaxy. We adopt the widely used values $f_{\text{esc}} = 0.2$ [38] and $\log_{10} \xi_{\text{ion}} = 53.14$ [21–23].

Considering a varying fine-structure constant in star formation, the recombination time $t_{\text{rec}}$ should be rescaled as $t_{\text{rec}} \propto A_{\text{HI}}^{-1} \propto (\alpha/\alpha_{\text{EM}})^{-2}$. The escaping rate of ionizing photons $f_{\text{esc}}$ would be inversely proportional to the Thomson cross-section, which implies $f_{\text{esc}} \propto \sigma_T^{-1} \propto (\alpha/\alpha_{\text{EM}})^{-2}$. The ionizing photon production rate $\dot{\xi}_{\text{ion}}$ would be rescaled as $\dot{\xi}_{\text{ion}} \propto (\alpha/\alpha_{\text{EM}})^{-2}$ because the Lyman limits shift shortward by $(\alpha/\alpha_{\text{EM}})^2$. We show the ionized fraction during reionization epoch in the left picture of figure 1 for different $\alpha/\alpha_{\text{EM}}$.

### 3 Results

In this section, we put constraints on the fine-structure constant and other cosmological parameters by reconstructing the ionization history of our universe from observational data. We apply the following data combination: the latest measurements of CMB power spectra — Planck 2018 TT,TE,EE+lowE+lensing [17–19], the SNIa Pantheon sample [30], the BAO measurements at redshifts $z = 0.106, 0.15, 0.32, 0.57, 1.52, 2.34$ (named 6dFGS [24], MGS [25], DR12 [26], DR14 [27–29] respectively), the SFR density from UV and IR data [20], as well as the $Q_{\text{HI}}$ constraints between $5.0 \leq z \leq 8.0$ from observations of Gunn-Peterson optical depth and Ly-$\alpha$ emission in galaxies [21–23]. In this work, we use the SFR data below $z \sim 8$ to reconstruct the reionization history. Certainly, some researches, such as refs. [23, 42], have provided the SFR densities extending to $z \sim 10$. However, the data at high-$z$ vary dramatically with different truncation magnitudes $M_{\text{trunc}}$ and their errors also get larger with higher redshifts. So, our choice would not impact the results significantly. We modify CAMB [43] and CosmoRec [44] packages to calculate the optical depth and CMB power spectra. Then the Markov Chain Monte Carlo sampler — CosmoMC [45, 46] is applied to constrain the free parameters. In the first model, named the $\Lambda$CDM+$\alpha/\alpha_{\text{EM}}$ model thereafter,
there are ten free parameters: \( \{ \Omega_b h^2, \Omega_c h^2, 100\theta_{\text{MC}}, \ln (10^{10} A_s), n_s, a_p, b_p, c_p, d_p, \alpha/\alpha_{\text{EM}} \} \). Five of them are basic parameters in the \( \Lambda \)CDM model: \( \Omega_b h^2 \) and \( \Omega_c h^2 \) are the density of baryons and cold dark matter today respectively, \( 100\theta_{\text{MC}} \) is 100 times the ratio of angular diameter distance to the large scale structure sound horizon, \( A_s \) is the amplitude of the power spectrum of primordial curvature perturbations, and \( n_s \) is the scalar spectrum index. Four of them, \( a_p, b_p, c_p, d_p \) are parameters of SFR density as described in section 2.2. Lastly, \( \alpha/\alpha_{\text{EM}} \) indicates the ratio of fine-structure constant \( \alpha \) to its fiducial value \( \alpha_{\text{EM}} \). It remains unchanged during the whole ionization history.

In the second model, named the \( \Lambda \)CDM+ \( \alpha_{\text{rec}}/\alpha_{\text{EM}} + \alpha_{\text{reion}}/\alpha_{\text{EM}} \) model, there are eleven free parameters: \( \{ \Omega_b h^2, \Omega_c h^2, 100\theta_{\text{MC}}, \ln (10^{10} A_s), n_s, a_p, b_p, c_p, d_p, \alpha_{\text{rec}}/\alpha_{\text{EM}}, \alpha_{\text{reion}}/\alpha_{\text{EM}} \} \). We use \( \alpha_{\text{rec}} \) and \( \alpha_{\text{reion}} \) to denote the fine-structure constant during recombination and reionization epoch respectively. Our results are summarized in table 1. We have listed the 68\% limits of the free parameters and necessary derived parameters in these two models. The ratio of fine-structure constant reads \( \alpha/\alpha_{\text{EM}} = 1.000049^{+0.001983}_{-0.001972} \) at 68\% C.L. in the \( \Lambda \)CDM+ \( \alpha/\alpha_{\text{EM}} \) model, which is well consistent with 1. The uncertainties decrease slightly than previous works which modify the fine-structure constant at only the recombination epoch. For example, the Planck Collaboration released \( \Delta \alpha/\alpha_{\text{EM}} = (3.6 \pm 3.7) \times 10^{-3} \) at 68\% C.L. [12] and ref. [13] obtained \( \alpha/\alpha_{\text{EM}} = 0.9999 \pm 0.0025 \) at 68\% C.L.. Comparing other measurements from astrophysics, our constraints are tighter than \( \Delta \alpha/\alpha_{\text{EM}} < 10^{-2} \) \( -10^{-3} \) from the abundance of light elements during BBN [47], but weaker than constraints of \( \Delta \alpha/\alpha_{\text{EM}} \sim 10^{-7} - 10^{-8} \) from the 1.8 billion-year-old natural nuclear reactor at the Oklo Uranium Mine in Gabon [48], \( \Delta \alpha/\alpha_{\text{EM}} \sim 10^{-5} - 10^{-6} \) from the spectral lines of quasars [49–51] and so on [16]. However, in the \( \Lambda \)CDM+ \( \alpha_{\text{rec}}/\alpha_{\text{EM}} + \alpha_{\text{reion}}/\alpha_{\text{EM}} \) model, our results show \( \alpha_{\text{rec}}/\alpha_{\text{EM}} = 1.001494^{+0.002041}_{-0.002063} \) and \( \alpha_{\text{reion}}/\alpha_{\text{EM}} = 0.8540342^{+0.031678}_{-0.027209} \) at 68\% C.L.. It indicates \( \alpha_{\text{rec}} \) is inconsistent with \( \alpha_{\text{reion}} \) by 4.64\( \sigma \). The evolution of ionized fraction \( x_e(z) \) in these two models is shown in figure 2. They coincide with each other almostly during the recombination epoch, but differ slightly at the reionization epoch. We also provide a separate figure to show the small difference. The straightforward explanation for this inconsistency is that the fine-structure constant may vary with redshift. However, the value at low redshift is much smaller than \( \alpha_{\text{EM}} \). Therefore, we prefer another conservative explanation that the observational data about SFR we used are not good enough and a calibration of some parameters about SFR is necessary. Since \( f_{\text{esc}} \) and \( \xi_{\text{ion}} \) appear in product form, for example, above inconsistency can be solved by rescaling this product to \( f_{\text{esc}}\xi_{\text{ion}} \left( \frac{\alpha_{\text{rec}}}{\alpha_{\text{EM}}} \right)^{-4} \) if all the difference between \( \alpha_{\text{reion}} \) and \( \alpha_{\text{EM}} \) is due to the uncertainty in the product of \( f_{\text{esc}} \) and \( \xi_{\text{ion}} \). In fact, this calibration is acceptable. Because the reasonable constraints on \( f_{\text{esc}} \) and \( \xi_{\text{ion}} \) at the epoch of reionization have not been obtained so far [42].

Moreover, we find the optical depth \( \tau = 0.0573^{+0.0005}_{-0.0006} \) at 68\% C.L. in the \( \Lambda \)CDM+ \( \alpha/\alpha_{\text{EM}} \) model and \( \tau = 0.0559^{+0.0006}_{-0.0007} \) in the \( \Lambda \)CDM+ \( \alpha_{\text{rec}}/\alpha_{\text{EM}} + \alpha_{\text{reion}}/\alpha_{\text{EM}} \) model. These little larger values derived by integrating from \( z = 0 \) to \( z = 22 \) are still consistent with \( \tau = 0.054 \pm 0.007 \) in the instantaneous reionization model given by Planck 2018. Also, we list values of Hubble constant, \( H_0 = 67.80 \pm 0.67 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} \) in the \( \Lambda \)CDM+ \( \alpha/\alpha_{\text{EM}} \) model and \( H_0 = 68.17 \pm 0.70 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} \) in the \( \Lambda \)CDM+ \( \alpha_{\text{rec}}/\alpha_{\text{EM}} + \alpha_{\text{reion}}/\alpha_{\text{EM}} \) model. Therefore, the varying fine-structure constant can relieve the so-called Hubble tension slightly. Furthermore, we show the triangular plots of several parameters in previous two models in figure 3 and figure 4. We find that there is an expected strong degeneracy between \( 100\theta_{\text{MC}} \) and the fine-structure constant during the recombination as well as an expected degeneracy between \( \tau \) and the fine-structure constant during the reionization.
Figure 2. The left picture shows the evolution of ionized fraction $x_e(z)$ in the $\Lambda$CDM+$\alpha/\alpha_{EM}$ model and the $\Lambda$CDM+$\alpha_{rec}/\alpha_{EM}$ + $\alpha_{rei}/\alpha_{EM}$ model. The right one shows the reionization histories. The black solid curve corresponds to the $\Lambda$CDM+$\alpha/\alpha_{EM}$ model and the red dashed one illustrates the $\Lambda$CDM+$\alpha_{rec}/\alpha_{EM}$ + $\alpha_{rei}/\alpha_{EM}$ model. Values of all the parameters are set to their mean values as table 1 shows.

|                         | $\Lambda$CDM+$\alpha/\alpha_{EM}$ | $\Lambda$CDM+$\alpha_{rec}/\alpha_{EM}$ + $\alpha_{rei}/\alpha_{EM}$ |
|-------------------------|-----------------------------------|-------------------------------------------------|
| $\Omega_b h^2$          | 0.02244 ± 0.00014                 | 0.02241 ± 0.00014                               |
| $\Omega_c h^2$          | 0.1191 ± 0.0013                   | 0.1198 ± 0.0013                                 |
| $100\theta_{MC}$        | 1.04107 ± 0.00272                | 1.04305 ± 0.00280                              |
| $\ln(10^{10} A_s)$      | 3.0484 ± 0.0057                  | 3.0492 ± 0.0064                                 |
| $n_s$                   | 0.9670 ± 0.0063                  | 0.9630 ± 0.0064                                 |
| $a_p [M_{\odot} \cdot yr^{-1} \cdot Mpc^{-3}]$ | 0.01772 ± 0.00070               | 0.01633 ± 0.00070                              |
| $b_p$                   | 2.968 ± 0.121                    | 2.897 ± 0.124                                   |
| $c_p$                   | 2.554 ± 0.088                    | 2.736 ± 0.098                                   |
| $d_p$                   | 5.125 ± 0.097                    | 5.791 ± 0.140                                   |
| $\alpha/\alpha_{EM}$    | 1.000042 ± 0.001983              | -                                               |
| $\alpha_{rec}/\alpha_{EM}$ | -                                | 1.001494 ± 0.002041                           |
| $\alpha_{rei}/\alpha_{EM}$ | -                              | 0.854034 ± 0.031678                           |
| $\tau$                  | 0.0573 ± 0.0005                  | 0.0559 ± 0.0069                                 |
| $10^4 Q_{HI}(z = 22)$   | 2.133 ± 0.019                    | 2.125 ± 0.020                                   |
| $H_0[km \cdot s^{-1} \cdot Mpc^{-1}]$ | 67.80 ± 0.67 | 68.17 ± 0.70 |

Table 1. The 68% limits for the cosmological parameters in the $\Lambda$CDM+$\alpha/\alpha_{EM}$ and $\Lambda$CDM+$\alpha_{rec}/\alpha_{EM}$ + $\alpha_{rei}/\alpha_{EM}$ model.
Figure 3. The triangular plot of $100\theta_{\text{MC}}, \tau, H_0, a_p, b_p, c_p, d_p, \alpha/\alpha_{\text{EM}}$ in the $\Lambda$CDM+$\alpha/\alpha_{\text{EM}}$ model from observational data.

4 Summary and discussion

In this paper, we investigate how the fine-structure constant $\alpha$ affect the whole ionization history of our universe, including both the recombination and reionization epoch. $\alpha$ influences the Thomson scattering cross-section and atomic energy levels, then modify the ionization history, which are reflected in the CMB power spectra. Therefore we constrain the fine-structure constant and other free parameters in two $\alpha$-related models from data combination of Planck 2018, BAO data, PANTHON samples, SFR density from UV and IR measurements, and the $Q_{\text{HI}}$ constraints, using CosmoRec+CAMB+CosmoMC packages. Comparing previous studies on constraining $\alpha$ from CMB, we take the influence of $\alpha$ on the reionization epoch into consideration and reconstruct the reionization history with SFR density. In the $\Lambda$CDM+$\alpha/\alpha_{\text{EM}}$ model, the fine-structure constant behaves as a constant and we
Figure 4. The triangular plot of $100\theta_{MC}, \tau, H_0, a_p, b_p, c_p, d_p, \alpha_{rec}/\alpha_{EM}, \alpha_{rei}/\alpha_{EM}$ in the $\Lambda$CDM+$\alpha_{rec}/\alpha_{EM} + \alpha_{rei}/\alpha_{EM}$ model from observational data.

find its value is well consistent with the standard one. In the $\Lambda$CDM+$\alpha_{rec}/\alpha_{EM} + \alpha_{rei}/\alpha_{EM}$ model, the fine-structure constant reads $\alpha_{rec}/\alpha_{EM} = 1.001494^{+0.002041}_{-0.002063}$ at 68% C.L. during the recombination epoch and $\alpha_{rei}/\alpha_{EM} = 0.854034^{+0.031678}_{-0.027209}$ at the reionization epoch. The value of $\alpha_{rei}$ deviates from $\alpha_{rec}$ by 4.64σ.

If above such an inconsistency results from the uncertainty in the product of $f_{esc}$ and $\xi_{ion}$, more refined calibration is necessary. That is to say, instead of the typical values of these two parameters $f_{esc} = 0.2$ and $\log_{10} \xi_{ion} = 53.14 [\text{Lyc \cdot photons \cdot s}^{-1} \cdot \text{M}_\odot^{-1} \cdot \text{yr}]$, we should turn to their improved counterparts. Given enough star-forming galaxies at different redshifts, we can use the observed Hα and UV-continuum fluxes of them to improve the constraints on $\log_{10} \xi_{ion}$ [52] and use the direct detection of Lyman-continuum emission from them to improve constraints on $f_{esc}$ [53, 54]. Also a better fitting formula and measurements
with a higher precision of SFR density would help us reconstruct the reionization history and acquire a more precise value of $\alpha$. It would be interesting to evaluate how a new fitting formula including the steeper slopes of $\log \rho_{SFR}$ at $z \sim 8-10$ [42] affects the results. Moreover, the fine-structure constant may be a redshift-dependent variation which is beyond the simple models of $\alpha$ assumed in the present paper. We expect the problem of fine-structure constant to obtain a much more robust description with future researches using future high precision data.

Acknowledgments

We acknowledge the use of HPC Cluster of Tianhe II in National Supercomputing Center in Guangzhou. Ke Wang is supported by grants from NSFC (grant No. 12005084) and grants from the China Manned Space Project with NO. CMS-CSST-2021-B01. Lu Chen is supported by grants from NSFC (grant No. 12105164). This work has also received funding from project ZR2021QA021 supported by Shandong Provincial Natural Science Foundation.

References

[1] T. Kinoshita, The Fine structure constant, Rept. Prog. Phys. 59 (1996) 1459 [inSPIRE].
[2] R. Lange et al., Improved limits for violations of local position invariance from atomic clock comparisons, Phys. Rev. Lett. 126 (2021) 011102 [arXiv:2010.06620] [inSPIRE].
[3] https://physics.nist.gov/.
[4] V. da Fonseca et al., Fundamental Physics with ESPRESSO, Constraining a simple parametrisation for varying $\alpha$, arXiv:2204.02930 [inSPIRE].
[5] J.K. Webb, J.A. King, M.T. Murphy, V.V. Flambaum, R.F. Carswell and M.B. Bainbridge, Indications of a spatial variation of the fine structure constant, Phys. Rev. Lett. 107 (2011) 191101 [arXiv:1008.3907] [inSPIRE].
[6] M.T. Murphy and K.L. Cooksey, Subaru Telescope limits on cosmological variations in the fine-structure constant, Mon. Not. Roy. Astron. Soc. 471 (2017) 4930 [arXiv:1708.00014] [inSPIRE].
[7] M.R. Wilczynska et al., Four direct measurements of the fine-structure constant 13 billion years ago, Sci. Adv. 6 (2020) eaay9672 [arXiv:2003.07627] [inSPIRE].
[8] D. Milaković, C.-C. Lee, R.F. Carswell, J.K. Webb, P. Molaro and L. Pasquini, A new era of fine structure constant measurements at high redshift, Mon. Not. Roy. Astron. Soc. 500 (2020) 1 [arXiv:2008.10619] [inSPIRE].
[9] M.T. Murphy et al., Fundamental physics with ESPRESSO: Precise limit on variations in the fine-structure constant towards the bright quasar HE 0515−4414, Astron. Astrophys. 658 (2022) A123 [arXiv:2112.05819] [inSPIRE].
[10] M. Deal and C.J.A.P. Martins, Primordial nucleosynthesis with varying fundamental constants — Solutions to the lithium problem and the deuterium discrepancy, Astron. Astrophys. 653 (2021) A48 [arXiv:2106.13989] [inSPIRE].
[11] L. Hart and J. Chluba, Varying fundamental constants principal component analysis: additional hints about the Hubble tension, Mon. Not. Roy. Astron. Soc. 510 (2022) 2206 [arXiv:2107.12465] [inSPIRE].
[12] PLANCK collaboration, Planck intermediate results — XXIV. Constraints on variations in fundamental constants, Astron. Astrophys. 580 (2015) A22 [arXiv:1406.7482] [inSPIRE].
[13] L. Hart and J. Chluba, *New constraints on time-dependent variations of fundamental constants using Planck data*, *Mon. Not. Roy. Astron. Soc.* **474** (2018) 1850 [arXiv:1705.03925] [inSPIRE].

[14] L. Hart and J. Chluba, *Updated fundamental constant constraints from Planck 2018 data and possible relations to the Hubble tension*, *Mon. Not. Roy. Astron. Soc.* **493** (2020) 3255 [arXiv:1912.03986] [inSPIRE].

[15] J.-P. Uzan, *Varying Constants, Gravitation and Cosmology*, *Living Rev. Rel.* **14** (2011) 2 [arXiv:1009.5514] [inSPIRE].

[16] Z.-E. Liu, W.-F. Liu, T.-J. Zhang, Z.-X. Zhai and K. Bora, *Probing the Time Variation of a Fine Structure Constant Using Galaxy Clusters and the Quintessence Model*, *Astrophys. J.* **922** (2021) 19 [arXiv:2109.00134] [inSPIRE].

[17] Planck collaboration, *Planck 2018 results. VI. Cosmological parameters*, *Astron. Astrophys.* **641** (2020) A6 [Erratum ibid. **652** (2021) C4] [arXiv:1807.06209] [inSPIRE].

[18] Planck collaboration, *Planck 2018 results. VIII. Gravitational lensing*, *Astron. Astrophys.* **641** (2020) A8 [arXiv:1807.06210] [inSPIRE].

[19] Planck collaboration, *Planck 2018 results. V. CMB power spectra and likelihoods*, *Astron. Astrophys.* **641** (2020) A5 [arXiv:1907.12875] [inSPIRE].

[20] P. Madau and M. Dickinson, *Cosmic Star Formation History*, *Ann. Rev. Astron. Astrophys.* **52** (2014) 415 [arXiv:1403.0007] [inSPIRE].

[21] X.-H. Fan et al., *Constraining the evolution of the ionizing background and the epoch of reionization with z~6 quasars. 2. a sample of 19 quasars*, *Astron. J.* **132** (2006) 117 [astro-ph/0512082] [inSPIRE].

[22] M.A. Schenker, R.S. Ellis, N.P. Konidaris and D.P. Stark, *Line Emitting Galaxies Beyond a Redshift of 7: An Improved Method for Estimating the Evolving Neutrality of the Intergalactic Medium*, *Astrophys. J.* **795** (2014) 20 [arXiv:1404.4632] [inSPIRE].

[23] R.J. Bouwens et al., *Reionization after Planck: The Derived Growth of the Cosmic Ionizing Emissivity now matches the Growth of the Galaxy UV Luminosity Density*, *Astrophys. J.* **811** (2015) 140 [arXiv:1503.08228] [inSPIRE].

[24] F. Beutler et al., *The 6dF Galaxy Survey: Baryon Acoustic Oscillations and the Local Hubble Constant*, *Mon. Not. Roy. Astron. Soc.* **416** (2011) 3017 [arXiv:1106.3366] [inSPIRE].

[25] A.J. Ross, L. Samushia, C. Howlett, W.J. Percival, A. Burden and M. Manera, *The clustering of the SDSS DR7 main Galaxy sample — I. A 4 per cent distance measure at z = 0.15*, *Mon. Not. Roy. Astron. Soc.* **449** (2015) 2617 [arXiv:1607.03155] [inSPIRE].

[26] BOSS collaboration, *The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological analysis of the DR12 galaxy sample*, *Mon. Not. Roy. Astron. Soc.* **470** (2017) 2617 [arXiv:1607.03155] [inSPIRE].

[27] V. de Sainte Agathe et al., *Baryon acoustic oscillations at z = 2.34 from the correlations of Lyα absorption in eBOSS DR14*, *Astron. Astrophys.* **629** (2019) A85 [arXiv:1904.03400] [inSPIRE].

[28] M. Ata et al., *The clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample: first measurement of baryon acoustic oscillations between redshift 0.8 and 2.2*, *Mon. Not. Roy. Astron. Soc.* **473** (2018) 4773 [arXiv:1705.06373] [inSPIRE].

[29] J. Hou et al., *The clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample: anisotropic clustering analysis in configuration-space*, *Mon. Not. Roy. Astron. Soc.* **480** (2018) 2521 [arXiv:1801.02656] [inSPIRE].
[30] PAN-STARRS1 collaboration, The Complete Light-curve Sample of Spectroscopically Confirmed SNe Ia from Pan-STARRS1 and Cosmological Constraints from the Combined Pantheon Sample, *Astrophys. J.* **859** (2018) 101 [arXiv:1710.00848] [SPIRE].

[31] B.E. Robertson, R.S. Ellis, S.R. Furlanetto and J.S. Dunlop, Cosmic Reionization and Early Star-forming Galaxies: a Joint Analysis of new Constraints From Planck and the Hubble Space Telescope, *Astrophys. J. Lett.* **802** (2015) L19 [arXiv:1502.02024] [SPIRE].

[32] A. Gorce, M. Douspis, N. Aghanim and M. Langer, Observational constraints on key-parameters of cosmic reionisation history, *Astron. Astrophys.* **616** (2018) A113 [arXiv:1710.04152] [SPIRE].

[33] S. Mitra, T.R. Choudhury and A. Ferrara, Cosmic reionization after Planck II: contribution from quasars, *Mon. Not. Roy. Astron. Soc.* **473** (2018) 1416 [arXiv:1606.02719] [SPIRE].

[34] D.K. Hazra, D. Paoletti, F. Finelli and G.F. Smoot, Joining Bits and Pieces of Reionization History, *Phys. Rev. Lett.* **125** (2020) 071301 [arXiv:1904.01547] [SPIRE].

[35] D. Paoletti, D.K. Hazra, F. Finelli and G.F. Smoot, Dark twilight joined with the light of dawn to unveil the reionization history, *Phys. Rev. D* **104** (2021) 123549 [arXiv:2107.10693] [SPIRE].

[36] B.E. Robertson et al., New Constraints on Cosmic Reionization from the 2012 Hubble Ultra Deep Field Campaign, *Astrophys. J.* **768** (2013) 71 [arXiv:1301.1228] [SPIRE].

[37] J.S. Dunlop et al., The UV continua and inferred stellar populations of galaxies at $z \sim 7-9$ revealed by the Hubble Ultra Deep Field 2012 campaign, *Mon. Not. Roy. Astron. Soc.* **432** (2013) 3520 [arXiv:1212.0860] [SPIRE].

[38] L. Chen and K. Wang, Does the reionization model influence the constraints on dark matter decay or annihilation?, *JCAP* **12** (2021) 034 [arXiv:2106.05509] [SPIRE].

[39] J.D. Bowman, A.E.E. Rogers, R.A. Monsalve, T.J. Mozdzen and N. Mahesh, An absorption profile centred at 78 megahertz in the sky-averaged spectrum, *Nature* **555** (2018) 67 [arXiv:1810.05912] [SPIRE].

[40] J.R. Shaw and J. Chluba, Precise cosmological parameter estimation using CosmoRec, *Mon. Not. Roy. Astron. Soc.* **415** (2011) 1343 [arXiv:1102.3683] [SPIRE].

[41] A. Lewis, A. Challinor and A. Lasenby, Efficient computation of CMB anisotropies in closed FRW models, *Astrophys. J.* **538** (2000) 473 [astro-ph/9911177] [SPIRE].

[42] J.R. Shaw and J. Chluba, Precise cosmological parameter estimation using CosmoRec, *Mon. Not. Roy. Astron. Soc.* **415** (2011) 1343 [arXiv:1102.3683] [SPIRE].

[43] A. Lewis, A. Challinor and A. Lasenby, Efficient sampling of fast and slow cosmological parameters, *Phys. Rev. D* **87** (2013) 103529 [arXiv:1304.4473] [SPIRE].

[44] M.E. Mosquera and O. Civitarese, New cosmological constraints on the variation of fundamental constants: the fine structure constant and the Higgs vacuum expectation value, *Astron. Astrophys.* **551** (2013) A122 [SPIRE].
[48] T. Damour and F. Dyson, *The Oklo bound on the time variation of the fine structure constant revisited*, *Nucl. Phys. B* 480 (1996) 37 [hep-ph/9606486] [InSPIRE].

[49] M.T. Murphy, J.K. Webb, V.V. Flambaum, M.J. Drinkwater, F. Combes and T. Wiklind, *Improved constraints on possible variation of physical constants from HI 21cm and molecular QSO absorption lines*, *Mon. Not. Roy. Astron. Soc.* 327 (2001) 1244 [astro-ph/0101519] [InSPIRE].

[50] M.T. Murphy et al., *Possible evidence for a variable fine structure constant from QSO absorption lines: Motivations, analysis and results*, *Mon. Not. Roy. Astron. Soc.* 327 (2001) 1208 [astro-ph/0012419] [InSPIRE].

[51] M.T. Murphy, J.K. Webb, V.V. Flambaum, J.X. Prochaska and A.M. Wolfe, *Further constraints on variation of the fine structure constant from alkali doublet QSO absorption lines*, *Mon. Not. Roy. Astron. Soc.* 327 (2001) 1237 [astro-ph/0012421] [InSPIRE].

[52] R.J. Bouwens et al., *The Lyman-Continuum Photon Production Efficiency $\xi_{\text{ion}}$ of $z \sim 4$–5 Galaxies from IRAC-based $H_{\alpha}$ Measurements: Implications for the Escape Fraction and Cosmic Reionization*, *Astrophys. J.* 831 (2016) 176 [arXiv:1511.08504].

[53] C.C. Steidel, M. Pettini and K.L. Adelberger, *Lyman continuum emission from galaxies at $Z \sim 3.4$*, *Astrophys. J.* 546 (2001) 665 [astro-ph/0008283] [InSPIRE].

[54] A.E. Shapley, C.C. Steidel, M. Pettini, D.K. Erb and K.L. Adelberger, *The Direct Detection of Lyman-Continuum Emission from Star-forming Galaxies at $z \sim 3$*, *Astrophys. J.* 651 (2006) 688 [astro-ph/0606635] [InSPIRE].