Computer simulation of fracture dynamics in a fiber-reinforced composite

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Abstract. Calculation models are developed to examine dynamic stress concentration and fracture in a plane elastic fiber-reinforced composite sheet. The sheet consists of a regular system of alternating extensible fibers and pliable adhesive layers. In calculations, special algorithms are built preventing or minimizing parasite distortions caused by the mesh dispersion. Two engineering models of the adhesive are considered: (i) inertionless bonds of constant stiffness, and (ii) an inertial medium perceived shear stresses. Clarified solutions of the problem are obtained in the case that the sheet pre-stretched along fibers.

1. Introduction
Fiber-reinforced composites of a wide range of applications can be subjected to intense dynamic loadings [1-5]. A set of dynamic problem have been studied on the basis of the homogenization approach averaging local properties of microstructure (see e.g. [5-8]). Within such approach, features inherent in impact processes are, as a rule, explored by methods and models of solid dynamics well developed for analysis of a longwave spectrum, where microstructure peculiarities play a secondary role. Significant rise in influence of microstructure on the wave picture essentially restricts capabilities of analytical modeling. Wave front gaps and high-gradient components responsible for dynamic stress concentrations and brittle fracture propagation in a composite have not yet been adequately examined and remain a subject of contemporary research (see e.g. [9-10]). There exist a set of numerical approaches intended for comprehensive simulations of composite dynamics in practice [11-13]. The work [14] reviews new software upgrades advantages in the composites marketplace.

At the same time, explicit-time algorithms used in computer codes come across specific obstacles, which do not allow to calculate accurately wave fronts and high-gradient components. We mean the mesh dispersion responsible for the emergence of short-wave "parasite" oscillations at calculation of transient processes and, notably, of fracture dynamics in reinforced composites: breaking of fibers and cracking of adhesive result in appearing of multiple wave fronts and high-frequency oscillations.

In this work, computational algorithms are designed on the basis of the Mesh Dispersion Minimization (MDM) technique, which allows drawbacks caused by the Mesh Dispersion (MDM) to be eliminated or significantly decreased. The idea behind MDM is to properly adjust the so-called domains of influence determined by continuous and discrete hyperbolic equations. The MDM approach proposed in [15] for 1D initial-boundary problems has been used in diverse dynamic problems (see, e.g., diffraction of elastic waves on buried structures [16, 17], impact driving of piles...
[18], dynamics and high-speed penetration of layered shields [19, 20], impact indentation of a rigid body into an elastic slab [21]). To calculations of multidimensional initial-boundary problems the MDM was upgraded in [22]. Recently, the MDM algorithm was used in [23] in a transient problem of wave propagation in a fiber upon a non-linear foundation.

In this work we have presented a version of MDM intended for calculations of 2D wave processes and some results related to wave/fracture processes in pre-stretched reinforced composite sheet.

2. Mechanical and mathematical models

2.1 Physical assumptions. Description of possible fracture events.

We consider in-plane elastic waves and dynamic fracture in a periodic structure shown in figure 1 (a). The generating element formulation of infinite periodic system (m = 0±1,±2,...) of unbounded high strength fibers alternated with pliable adhesive layers. The mechanical model assumes that fibers function in tension-compression, while the adhesive is under shear stress. This assumption often used in statics of unidirectional composites has wide range of practical application (for example, in aircraft and ship engineering). Although the stress state of structure components is, in fact, more complex, such approach correctly expresses the concept of the efficient performance of a reinforced material: high strength fibers are oriented along the tensile stress lines, while the adhesive facilitates a more uniform distribution of these loads between fibers, preventing stress concentrations.

Parameters of fibers are: width, h, Young modulus, E, and density, ρf; the adhesive has width, H, shear modulus, G, and density, ρa. Axis x is directed across the fibers, and axis y – along them.

The following problem formulation is considered. At time t < 0 (figure 1,a) all fibers are stretched by a constant tensile stress $\sigma_f = \sigma_\infty$ applied in infinity (y = ±∞). Let one of fibers (say fiber $m = 0$) be instantly breaks in a cross section y = 0 at time $t = 0$ – figure 1,b. At the same time cross-section $y = 0$ of fiber $m = 0$ becomes free of stresses: $\sigma_f = 0$ (t = 0, $m = 0$, y = 0). It is suitable to use the following transformation: we subtract the initial static stress field in fibers ($\sigma = \sigma_\infty$) from the new one and come to the dynamic problem of the state shown in figure 1,b: in surfaces of the initial slit (y = ±0) in fiber $m = 0$, the stresses $\sigma_f = \sigma_\infty$ are applied (and remains at $t > 0$), while $\sigma_f = 0$ in other cross-sections of fiber $m = 0$ and in other fibers at $t = 0$. The gap grows with time, and progressive fracture pattern can be realized with time in fibers and the adhesive depending on their strengths – figure 1(c).

Figure 1. (a) t < 0: The sheet pre-stretched by tensile stresses $\sigma_\infty$; (b) t = 0: Breaking of fiber $m = 0$, and the transformed stress state; (c) Growing the initial slit with time and a schematic fracture pattern.
The problem is to describe the dynamic stress concentration, subsequent fracture process occurring with time and to reveal the “trauma” area after the (possible) fracture arrest.

We introduce local coordinate $X$ varied in interval $0 < X < H$, then $x = mH + X$. Displacements and stresses in fibers are denoted by $u_m(y,t)$ and $\sigma_m(y,t)$, in adhesive $-v_m(X,y,t)$ and $\tau_m(X,y,t)$. The initial stress state in the fibers $\sigma_m = \sigma^*_m (t < 0)$ is less than given critical value: $\sigma^*_m < \sigma^*$. If $\sigma^*_m (y,t) \geq \sigma^*$, the initial fracture can excite a consequence breaking of fibers.

Besides, we introduce the critical value of the shear stress in adhesive, $\tau^*$: if $\tau^*_m > \tau^*$, the shear crack appears in a some point of the adhesive and can propagate away from the area of the initial fracture.

The physical pattern of the process can be developed due to the following schematic scenario:

- $t = 0$, figure 1 (b). Two free boundaries appear in the fractured cross-section: $\sigma_0(0,t) = 0$ and $\sigma_0(-0,t) = 0$.
- $t > 0$, figure 1 (c). The unloading wave begins propagating in the fiber, and its free ends move along axis $y$ in opposite directions. Along with unloading, the broken fiber pulls adhesive adjacent to this area. Under the condition that adhesive does not reveal resistance to normal stresses, the line $y = 0$ in adhesive begins to open free: a transverse crack appears in adhesive between fibers $m = \pm 1$ and $m = 1$, and the splitting area increases with time resulting in the intensification of shear waves in adhesive, see Fig. 1(b). The shear waves reach fibers $m = \pm 1$ nearest to broken fiber $m = 0$, overload them (in addition to $\sigma_m$) and involve them in motion. In their turn, fibers $m = \pm 1$ excites shear waves in adhesive that propagate upward, to the next fibers, and backward, to the broken one. Shear waves reflected from the first intact fibers reach the broken one and decelerate it, while next intact fibers begin to overload. Together with this, free ends in fibers $m = \pm 1$ and shear cracks in the nearest adhesive can be appeared if normal and shear stresses are to be exceed critical values $\sigma$ and $\tau$, respectively. These boundaries surround a domain associated to the trauma despite the fact that in some cases such trauma is not continued but can be formed from a set of alternative fractured and intact domains.

The dynamic process is developed with time farther on according to the described scheme, more and more volume of fibers neighbouring to broken one are being fractured and involved into the motion, while intact those get additional load. At some particular moment of time, energy of the initial fracture is completely spent on tension-compression waves in fibers and shear waves in adhesive propagated away of the initial impact area. After that, strains and displacements of the composite reach a new static state, and dynamic process is finished. As a result, a trauma remains in this static state.

The used model of the fiber dynamics describes the one-dimensional wave process in a thin rod embedded into adhesive, while two models are considered for adhesive:

(i) Model I. The model is simplified one with the assumption that adhesive can be represented by inerterless bonds of constant stiffness $K = G/H$. Note, the inertia of bond can be taken into account due to distribution of the bond mass along the fiber in each the fiber-adhesive layer.

(ii) Model II. The model is more precise. The adhesive is described by inertial bonds perceived shear stresses, while tension-compression stresses in bonds are neglected. Such a theoretical treatment of the nature of the components performance is partially justified by the fact that the shear modulus of adhesive is hundreds times less than that of fiber (see e.g. [1-4]), while their stretches have roughly the same level due to the cohesion of the fibers and the adhesive.

In Model I, strains in the adhesive are completely described by fibers displacements, and displacements of adhesive is calculated from the linear approximation:
\[ v_m(X, y, t) = u_m(y, t) + \left( \frac{X}{H} \right) \left[ u_{m+1}(y, t) - u_m(y, t) \right]. \]

The following patterns can be realized in considered models depending on strength conditions:
- the composite remains intact: maximal stresses reached in fibers and adhesive do not exceed critical limits: \( \sigma_m < \sigma^* \), \( \tau_m < \tau^* \). These conditions are used to calculate the parameters of stress concentration in intact fibers and adhesive associated to them;
- fibers are fractured, while adhesive remains intact: \( \sigma_m \geq \sigma^*, \tau_m < \tau^* \);
- adhesive is fractured, while fibers (with \( m \neq 0 \)) remain intact: \( \sigma_m < \sigma^*, \tau_m \geq \tau^* \);
- both fibers and adhesive are fractured: \( \sigma_m \geq \sigma^*, \tau_m \geq \tau^* \).

### 2.2 Mathematical formulation of the problem.

In the mathematical sense, we are dealing with a non-linear hyperbolic problem possessing non-classical boundary conditions. Due to the symmetry, a quarter of plane \( x, y \) can be considered in the calculation algorithm (let it be \( x \geq 0, y \geq 0 \)) with the symmetry condition in mind. Displacements and strains of the composite at the intact static state \( t < 0 \) are

\[ u_m(y) = y \sigma_\infty / E, \quad \varepsilon_m(y) = \varepsilon_\infty / E; \quad v_m(X, y) = 0 \quad (m = 0, \pm 1, \pm 2, \ldots), \]

where \( \varepsilon_m(y) = \partial u_m / \partial y \) is the strain in \( m \)th fiber \( (m \neq 0) \), and the fracture event of fiber \( m = 0 \) at \( t = 0 \) changes (1) by adding condition \( \varepsilon_0(0, t) = 0 \):

\[ \varepsilon_0(0, t) = -\sigma_\infty / E, \quad \varepsilon_m(0, t) = 0 \quad (m \neq 0) \]

Let us reformulate the problem for the additional dynamic state. For this we subtract the static strains \( (1) \) from \( (2) \). Then boundary conditions for strains in fibers are the following:

\[ y = 0: \quad \varepsilon_0(0, t) = -\sigma_\infty / E, \quad \varepsilon_m(0, t) = 0 \quad (m \neq 0) \]

The motion of fibers is described by 1D wave equation

\[ \rho_h \partial^2 u_m / \partial t^2 - E_h \partial^2 u_m / \partial y^2 + \tau_m^+ (y) - \tau_m^- (y), \quad m = 0, \pm 1, \pm 2, \ldots \]

where \( \tau_m^+ \) and \( \tau_m^- \) correspond to reactive shear forces at the fiber-adhesive interface on the right and the left, respectively. Their expressions in \( M_1 \) are:

\[ \tau_0^+ = -\tau_0 = K(u_0 - u_m) \quad (m = 0), \quad \tau_m^+ = K(u_{m+1} - u_m), \quad \tau_m^- = K(u_m - u_{m-1}) \quad (m > 0), \]

while in \( M_2 \) (the adhesive is considered as an array of inertial bonds), reactive forces are the following:

\[ \tau_0^+ = -\tau_0 = G \left( \partial \varepsilon_0 / \partial X \right)_{x=0}, \quad \tau_m^+ = G \left( \partial \varepsilon_m / \partial X \right)_{x=0}, \quad \tau_m^- = G \left( \partial \varepsilon_{m-1} / \partial X \right)_{x=H} \quad (m > 0), \]

and displacements in adhesive described by wave equations

\[ \partial^2 v_m (X, y, t) / \partial t^2 = c^2 \partial^2 v_m (X, y, t) / \partial X^2 \quad (c_a = \sqrt{G / \rho_a}, \quad 0 \leq X \leq H), \quad m = 0, 1, 2, \ldots \]

with the following boundary conditions:

\[ v_m(0, y, t) = u_m(y, t), \quad v_m(H, y, t) = u_{m+1}(y, t) \]

Then we add fracture conditions to system \( (3) - (8) \) as follows. If in current cross section \( y \) of \( m^\text{th} \) fiber at the moment of time \( t = \hat{t}_m \) tension stress \( \sigma_m (y) \) reaches critical value \( \sigma^* \), this cross-section breaks, and new boundaries appear resulting in the following conditions for equations \( (4) \):

\[ \sigma_m (y, t) \geq \sigma^* \Rightarrow t_m^* = t; \quad \sigma_m (y+0, t) = \sigma_m (y-0, t) = 0 \]

In the case of the fiber-adhesive splitting, we, in addition to \( (7) \), have the additional expressions for reactive forces in equations \( (4) \) and boundary conditions for \( v_m(0, y, t) \) and/or \( v_m(H, y, t) \):

\[ \tau_m^+ (\xi^+, y, t) \geq \tau^* \Rightarrow \hat{t}_m^* = t; \quad \tau_m^- (\xi^-, y, t) = 0, \quad \partial v_m^+ (\xi^+, y, t) / \partial X = 0, \quad \partial v_m^- (\xi^-, y, t) / \partial X = 0, \]

where \( \xi^+ \) and \( \xi^- \) are concentration in intact adhesive associated to them; \( \tau_m^+ \) and \( \tau_m^- \) correspond to reactive shear forces at the fiber-adhesive interface on the right and the left, respectively.
where $\xi^+=0$, $\xi^-=H$, and indices $\pm$ at $t^+_m$ denote right and left interfaces, respectively.

It is evident that each possible scenario of wave-fracture pattern is saturated by reflected waves with discontinuities appeared due to snaps of fibers and rupture of the adhesive.

Our goal is to calculate such processes as precise as possible. Below we have presented the practical calculation devices based on the MDM technique allowing this goal to be reached.

3. A fiber embedded into an adhesive layer. MDM calculation algorithms.

Let values $E, \rho_f, h$ be measurement units (then $c_f = \sqrt{E/\rho_f}$ is the velocity unit), while adhesive constants $H, G, \rho_a$ and criterions $\sigma_-, \sigma_+$ remain as free parameters of the problem.

The explicit finite difference algorithms for calculating a discrete analog of system (3) – (10) are built on the basis of the MDM technique elaborated in [15] and upgraded in [22, 23].

Consider a single semi-infinite fiber embedded in an adhesive layer: the problem formulation above is reduced to the motion of $0^\text{th}$ fiber, while neighboring fibers remain at rest: $u_m(y,t) = 0, m \neq 0$. The motion of this system is described in the Model I by Eqsns. (4), (5) with $m = 0$. We denote: $g$ is the stiffness of ‘elastic foundation’, then the governing equation together with the boundary condition is:

$$\ddot{u} = c_f^2 u^* + gu, \quad u(y,t) = u_0(y,t), \quad g = 2K; \quad \sigma(0,t) = E \dot{u}'(0,t) = F(t),$$

where $F(t)$ is the loading form.

Substituting the form of the steady state Fourier solution,

$$u = U \cdot \exp(i \omega t - i q x),$$

in Eqn (11) we obtain the Dispersion Relation (DR)

$$c(q) = \omega / q = \sqrt{1 + g / q^2}. \quad (13)$$

(Introduced above notations are: $U$ is an additive constant, $\omega = q c$ is the frequency, $c$ is the phase velocity, $q = 2\pi/l$ is the wave number, $l$ is the wave length). Relation (13) reveals the wave dispersion in the considered system – the dependence of the phase velocity (or frequency) on the wave length.

The aim of the MDM procedure is to built a difference scheme possessing DR maximally closed to that in (13). The conventional explicit cross type finite-difference scheme applied to (12) results in its difference analogue as the following:

$$u_{j+1}^{k+1} = 2u_j^k - u_{j-1}^{k-1} + \lambda^2 (u_{j+1}^k - 2u_j^k + u_{j-1}^k) + g (\Delta t)^2 u_j^k, \quad \lambda = \Delta y / \Delta t,$$

where $k$ and $j$ are current numbers of temporal and spatial mesh nodes, $y = j \Delta x, \ t = k \Delta t; \ j, k = 0, 1, 2, ..., \ \text{while} \ \lambda$ is the Courant number.

We note, the main requirement to MDM algorithms is the satisfaction of equality $\lambda = 1$ (see [15]).

Let $\Delta y$ and $c_0$ be measurement units, then $\lambda = \Delta t$. The discrete analogue of DR (13) obtaining from (14),

$$c = (2 / q \Delta t) \arcsin \left[ \Delta t \sqrt{\sin^2 \left( q / 2 \right) + g / 4} \right],$$

determines the stability condition of (14) as $\Delta t \leq 1 / \sqrt{1 + g / 4}$ and manifests presence of the MD.

To prevent MD we, in contrast to the conventional finite difference approximation $u \sim u_j^k$, use the so-called three-point-approximation of $u$ initially proposed in [15]:

$$u \sim (1/4) (u_{j+1}^k + 2u_j^k + u_{j-1}^k). \quad (16)$$

Then the difference equation (14) turns out into the following form:

$$u_{j+1}^{k+1} = 2u_j^k - u_{j-1}^{k-1} + \lambda^2 \left[ (u_{j+1}^k - 2u_j^k + u_{j-1}^k) + (g / 4) (u_{j+1}^k + 2u_j^k + u_{j-1}^k) \right]. \quad (17)$$
As shown in [20], the order of difference approximation for equations (14) and (17) is the same:

$$(\Delta t)^2 + (\Delta y)^2$$. The DR for Eqn. (17) is the following:

$$c = (2/q\Delta t)\arcsin\left[\Delta t\sqrt{\sin^2(q/2) + (g/4)\cos^2(q/2)}\right].$$

(18)

In contrast to Eqn. (14), scheme (17) has the same stability condition $\lambda \leq 1$ as in dispersionless wave equation. The MDM requirement $\lambda = 1$ ($\Delta t = 1$) being substituting in (18) results in the dispersionless propagation of extremely short waves of length $i = 2$ ($q = \pi$): $c = 1$—like to continual DR (13) at $q \to \infty$. The comparison of DRs (13) and (18) conducted in [22] shows their practical coincidence in the entire spectrum. The inconsistency of continual and difference length-wave spectrums, correspondingly $0 \leq q < \infty$ and $0 \leq q \leq \pi$, is not manifested in calculations of front zones.

The snapshots of stress distributions are presented in figure 2, computed with conventional scheme (14) and MDM scheme (17) in the problem of pulse propagation along the fiber. One can reveal that conventional scheme (14) containing spurious oscillations is not suitable to calculate wave fronts and high gradients, while the MDM scheme gives us the precise (in the integer mesh nodes) solution. The results of computer simulations below are obtained with the use of described MDM-algorithms.

![Figure 2. Stress distributions in a fiber embedded into adhesive ($g = 0.001$) under action of pulse $\sigma(0,t) = H(50 - t)$; (a) – the conventional algorithm, $\lambda = 0.9$; (b) – the MDM algorithm, $\lambda = 1$.](image)

4. MDM algorithms to calculation of wave/fracture pattern in the fiber-reinforced composite.

4.1 Model I.

The approximation (16) together with MDM condition $\lambda = 1$ results in representation of shear strains in adhesive as follows:

$$u_{m+1} - u_m \sim \left(1/4\right)\left[(u_{i+1} + 2u_i + u_{i-1})_{m+1} - (u_{i+1} + 2u_i + u_{i-1})_m\right].$$

(19)

The system of governing equations, Eq. (4), has the following calculation algorithm:

$$u_{m,j}^{k+1} = u_{m,j}^k + u_{m,j-1}^k - u_{m,j+1}^k - K\left(\varphi_{m,j}^k - \varphi_{m,j}^*\right),$$

$$\varphi_0 = \varphi_1 = (1/4)\left[u_{m+1,j}^k + 2u_{m,j}^k + u_{m-1,j}^k - (u_{m,j+1}^k + 2u_{m,j}^k + u_{m,j-1}^k)\right]$$ (m = 0),

$$\varphi_m = (1/4)\left[u_{m+1,j}^k + 2u_{m,j}^k + u_{m-1,j}^k - (u_{m,j+1}^k + 2u_{m,j}^k + u_{m,j-1}^k)\right]$$ (m > 0).

4.2 Model II.

Let mesh step in adhesive be $\Delta x$. Then the MDM condition are $\Delta t = \Delta y = 1$, $\Delta x = c_n$.

The MDM-algorithm for calculation of waves in adhesive, as the analogue of Eqn. (7), is
\( v_{m,j,i}^{k+1} = v_{m,j,i+1}^k + v_{m,j,i-1}^k - v_{m,j,i}^{k-1} \quad (0 \leq i \leq s = H/\Delta x), \) (21)

while the analogue of Eqn. (4) is the following:

\[
u_{m,j}^{k+1} = u_{m,j+1}^k + u_{m,j-1}^k - u_{m,j}^{k-1} + \kappa F, \quad F = \left( v_{m,j,i}^k + v_{m-1,j,i}^k \right) (m > 0), \quad F = 2v_{i,j}^k (m > 0), \quad \kappa = GH/(Eh\mu), \quad \mu = h\rho_f \Delta y + H\rho_y \Delta x.
\] (22)

Interface conditions (8) and fracture conditions (9), (10) have the conventional approximations.

5. Results of computer simulations

In calculation examples below parameters of composite are: \( H = 5, \ G = 0.025, \ \rho_s = 0.4; \) stresses are normalized to \( \sigma_m, \) critical values \( \sigma \) and \( \tau \) are varied.

First, we present results obtained for. In figure 3, stresses vs. time are depicted for the case when \( \sigma \) and \( \tau \) less than maximal values \( \sigma_{\text{max}}, \ \tau_{\text{max}} \) detected in the composite within the calculation time.

![Figure 3. Model I. Stresses vs. time in the intact composite.](image)

The maximal overloading of the composite is detected in cross-section \( y = 0: \ \left( \sigma_1 \right)_{\text{max}} = 1.482 \) and \( \left( \tau_{01} \right)_{\text{max}} = 0.073. \) Here \( \sigma_m(0), \) the normal stress in \( m^{th} \) fiber at \( y = 0, \) and \( \tau_{01}(0), \) the shear stress in the adhesive between \( 0^{th} \) and \( 1^{th} \) fibers, become (at \( t > 25H/c_f \)) close enough to their static limits: \( \left( \sigma_1 \right)_{\text{st}} = 4/3 \) and \( \left( \tau_{01} \right)_{\text{st}} = 0.053 \) that were analytically obtained in [24].

The next example is related to fracture development with time – figure 4. One can observe processes of propagation and arrest of normal cracks – (a) and (b), and shear cracks – (c).
In figure 4(a) – the intact adhesive, fiber numbers are bold; stress development vs. time is shown for fibers with \( m = 24, \ldots, 39 \). One can detect time values when \( m \)th fiber becomes broken if equality \( \sigma_m^{(0,m)} = \sigma' \) realized. Here, the 39th fiber is the last broken one, while \( \sigma_{40}^{(0)} < \sigma' \). After the last breaking the static state occurs. At the beginning stage of fracture, the crack growth is practically linear – figure 4(b), then it decelerated with time, short stops are observed and finally crack stopped. In the case (c) – the intact fibers, such a pattern occurs with adhesive cracking.

The stress pattern shown in figure 5 (Model II) is calculated in the case of intact composite. The peaks in \( \sigma_m(0,t) \) correspond to gaps of shear stresses, which related to their reflections from the intact 1th fiber at \( X = H \). The shear gap remains equal to 0.2 in the whole process. After next reflections, the influence of gaps to normal stresses decreases. As comparison shows, the maximal and static stresses in fibers are practically the same as those obtained in the case of Model I. The significant difference in maximal shear stresses (~ 3 times) will result in different fracture processes.

Thus, Model I can be used at the pre-fracture stage, where the wave pattern is calculated, while fracture propagation processes should be computed with Model II.
The features of fracture propagation in the case $\sigma^* = 1.2$ and $\tau^* = 0.1$ are shown in figure 6. After the initial rupture of $0^{th}$ fiber ($t = 0$), the fracture in the adhesive occurs at the fiber-adhesive interfaces, $X = 0$ and $X = H$, and possesses a high-speed avalanche-like pattern; the speed of fracture propagation decreases with time and the adhesive fracture is stopped at $t = 30$ in interface $X = 0$. Adhesive layers with $m \neq 0$ remain intact. Rapture of fibers occurs at $y = 0$ and propagates with a practically constant speed up to $t = 63H/c_f$, then it arrested.

![Figure 6](image.png)

**Figure 6.** Model II. Fracture of composite vs. time ($\sigma^* = 1.2$, $\tau^* = 0.1$)

The Model II could be used within computer simulators allowing an optimization problem to be explored. Let limit $\sigma^*$ be constant, while limit $\tau^*$ is varied within the interval $(\tau_o, \tau_{\text{max}})$. Our aim is to find such a value of $\tau^*$, for which the volume of breaking fibers, $m^*$ is less than the given value $M$.

In the Table below, we present some related results obtained in the case $\sigma^* = 1.2$, $M = 10$ and varied $\tau^*$. In the two lower rows we set the amount $m^*$ of fractured fibers (or rather its half due to symmetry) and lengths of shear cracks in adhesive, $y^*/h$. One can see that the required result is $\tau^* = 0.1$.*Note that in the presented example, shear cracks are observed only in the layer $m = 0$.

| $\tau^*$  | 0.07  | 0.075 | 0.08  | 0.085 | 0.1   | 0.15  | 0.25  |
|-----------|-------|-------|-------|-------|-------|-------|-------|
| $m^*$     | 0     | 0     | 5     | 7     | 9     | 13    | 13    |
| $y^*/h$   | $\infty$ | 208   | 65    | 23    | 11    | 3     | 0     |

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