Dynamic current-current susceptibility in 3D Dirac and Weyl semimetals

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We study the linear response of doped three dimensional Dirac and Weyl semimetals to vector potentials, by calculating the wave-vector and frequency dependent current-current response function analytically. The longitudinal part of the dynamic current-current response function is then used to study the plasmon dispersion, and the optical conductivity. The transverse response in the static limit yields the orbital magnetic susceptibility. In a Weyl semimetal, along with the current-current response function, all these quantities are significantly impacted by the presence of parallel electric and magnetic fields (a finite $\mathbf{E} \cdot \mathbf{B}$ term), and can be used to experimentally explore the chiral anomaly.

I. INTRODUCTION

Dirac and Weyl semimetals are materials with linearly dispersing bands touching at discrete Dirac/Weyl points1–4. Graphene is one of the most prominent example of a Dirac material in two dimensions (2D). In 3D materials, Dirac points appear due to accidental band crossings, and are robust against gap opening, only if protected by some crystallographic symmetry1,2. The presence of time reversal and crystal inversion symmetry forces the Dirac point to be four fold degenerate, with two degenerate pairs of linearly dispersing bands. Breaking of the time reversal (or crystal inversion) symmetry splits the Dirac node into a pair of Weyl nodes of opposite chiralities displaced in momentum (or energy). 3D Weyl fermions have been realized in TaAs5–10, NbP11, Mo,W1−xTe212 and photonic crystals13. 3D Dirac semimetals have been realized in Na3Bi14–16, Cd3As217–21 and ZrTe522,23.

A peculiar phenomena related to Weyl semimetals, is the chiral anomaly in crystals: pumping of charges between the nodes of opposite chirality in presence of parallel electric and magnetic fields (finite $\mathbf{E} \cdot \mathbf{B}$ term)24. This non-conservation of the number of particles in a given Weyl node, is a direct consequence of the lowest Landau level carrying only right or left movers (depending on the chirality of the Weyl node), as demonstrated explicitly in Ref. [24]. Alternately, it can also be obtained in a semi-classical transport framework as shown in Ref. [25], or from a field theoretic framework of Ref. [26]. There have been several proposals to detect the chiral anomaly: in collective density excitations or plasmons27,28, transport experiments29–33, optical conductivity34, circular and linear dichroism35,36 etc.

In this paper we study the response of a single node of Dirac and Weyl semimetals to static and dynamic vector fields, by explicitly calculating the current-current response function37–39. For each node, we consider a rotational invariant system in which the current-current correlation function can be expressed as a combination of longitudinal (wave-vector $\parallel$ to the vector field) and transverse (wave-vector $\perp$ to the vector field) response. The longitudinal current-current response function determines the optical conductivity40 of the system. It is also related to the density-density response via the current continuity equation and hence determines the dielectric properties and the spectrum of collective density excitations (plasmons) as well27,41–43. The transverse current-current response function determines the diamagnetic/orbital susceptibility44. We present analytical results for the wave-vector and frequency dependent longitudinal as well as transverse current current response function for for a single Dirac node, and then use it to explore the impact of chiral anomaly in Weyl semimetals. In particular the impact of chiral anomaly (a finite $\mathbf{E} \cdot \mathbf{B}$ term) can be observed via its impact on the plasmon dispersion, optical conductivity, and the diamagnetic susceptibility.

The paper is organized as follows: In Sec. II, we set up the calculation of the current-current response function for a single Dirac node. The results of the longitudinal response function are discussed in Sec. III, followed by the results for the transverse case in Sec. IV. In Sec. V we study the response of Weyl semimetals in context of the chiral anomaly. Section VI explores the implications for anisotropic systems, and we summarize our results in Sec. VII.

II. CURRENT-CURRENT RESPONSE FUNCTION OF A SINGLE DIRAC NODE

The effective low energy continuum Hamiltonian to describe a single isotropic massless 3D Dirac (or Weyl) node is given by

\[ \mathcal{H} = \hbar v_F \left( k_x \sigma_x + k_y \sigma_y + k_z \sigma_z \right), \tag{1} \]

where, $\sigma_i$ are the Pauli matrices denoting real spins, and $v_F$ is the Fermi velocity. The response of this system to an electromagnetic vector potential with spatio-temporal variations, $A(q, \omega)$, is determined by the current-current response function $\Pi_{ij}^q(q, \omega)$. In general $\Pi_{AB}^q(q, \omega)$ describes the response of the observable $A$ coupled to a second observable $B$, and is defined by standard Kubo...
product\textsuperscript{45},
\[ \Pi_{AB}(\omega) = -\frac{i}{\hbar S} \lim_{\epsilon \to 0} \int_0^\infty dt (\dot{A}(t) \dot{B}(t)) e^{i\omega t} e^{-\epsilon t}, \]  
where $S$ denotes the volume of the system. For the Hamiltonian given in Eq. (1), density operator is given by $\hat{\rho}_q = \sum_k \psi_{k-q,\alpha}^\dagger \psi_{k,\alpha}$ and the corresponding current operator is given by $\hat{j}_q = \nu F \sum_k \sigma_{\alpha\beta} \psi_{k,\alpha}^\dagger \sigma_{\alpha\beta} \psi_{k,\beta}$. Since the current operator depends on the spin operator, the current-current response function can be expressed in terms of the spin-spin response function via the relation $\Pi_{j_q j_i}(q,\omega) = \nu F \Pi_{\sigma \sigma}(q,\omega)$. The non-interacting spin-spin response function is explicitly given by
\[ \Pi^{(0)}_{\sigma \sigma}(q,\omega) = \frac{1}{S} \lim_{\epsilon \to 0} \sum_k \sum_{\lambda,\lambda'} \frac{n_{k,\lambda} - n_{k+q,\lambda'}}{\hbar \omega + \lambda \epsilon_k - \lambda' \epsilon_{k+q} + i\epsilon} \times |\langle \chi_k(\lambda) |\sigma_i \chi_{k+q}(\lambda + q)\rangle|^2, \]  
where $\epsilon_k = \hbar \nu_F |k|$, $\lambda, \lambda' = 1 (-1)$ are the band indices for conduction (valence) band and $\chi_k(\lambda)$ is the corresponding normalized eigen-spinor. In general, the spin-spin response function depends on both the magnitude and direction of the wave vector $q$. However for systems with rotational symmetry it can be broken into longitudinal and transverse components and both of them depend only on $q = |q|$. We calculate $\Pi^{(0)}_{\sigma \sigma}(q,\omega)$ considering $q$ along $z$ axis ($l$ to the applied vector field) for the longitudinal part and $q$ on the $x - y$ plane ($\perp$ to the applied field) for the transverse part.

III. LONGITUDINAL SPIN-SPIN RESPONSE FUNCTION

We now discuss the case of non-interacting longitudinal spin-spin response function, i.e., $\Pi^{(0)}_{\sigma \sigma}(q,\omega)$, in both the undoped and the doped scenario. Using the current continuity relation, $i\hbar \hat{\rho}_q = \hat{j}_q$, the longitudinal spin-spin response function can be related to the dynamical density-density response function $\Pi_{pp}(q,\omega)$ via the relation:
\[ \Pi_{pp}(q,\omega) = \frac{q}{\hbar \omega^2} \langle [\hat{j}_{z,q}, \rho_{-q}] \rangle + \frac{q^2 v_F^2}{\omega^2} \Pi_{\sigma \sigma}(q,\omega). \]  
Here the first term in the r.h.s. of Eq. (4) is the anomalous commutator and it arises due to presence of infinite sea of negatively charged electrons in the continuum version of the Dirac Hamiltonian\textsuperscript{37}. It turns out to be purely real and is given by
\[ \frac{1}{\nu F q} \langle [\hat{j}_{z,q}, \rho_{-q}] \rangle = \frac{q_{\text{max}}^2}{6\pi^2 \hbar \nu_F} = \epsilon_{\text{max}} \frac{q_{\text{max}}^2}{6\pi^2 \hbar^3 v_F^3}, \]  
where $q_{\text{max}} (= \epsilon_{\text{max}}/\hbar \nu_F)$ is wave vector corresponding to the ultraviolet energy cutoff. For details of the calculation of Eq. (5), see appendix A.

Similar to the case of the density-density response function\textsuperscript{43}, the total non-interacting spin-spin response function can also be expressed as a sum of contributions coming from the undoped part, and an additional doping dependent contribution:
\[ \Pi^{(0)}_{\sigma \sigma}(q,\omega) = \Pi^{(0)u}_{\sigma \sigma}(q,\omega) + \Pi^{(0)d}_{\sigma \sigma}(q,\omega). \]  

A. Undoped case

For the undoped case, the Fermi energy lies at the Dirac point and all the contributions are solely from the inter-band transitions occurring from the full valence band to empty conduction band. Equation (4) leads to the following relation between the intrinsic (undoped) parts of the spin and density response functions:
\[ \text{Im} \Pi^{(0)u}_{\sigma \sigma}(q,\omega) = \frac{\omega^2}{v_F^2 q^2} \text{Im} \Pi^{(0)u}_{pp}(q,\omega), \]  
where
\[ \text{Im} \Pi^{(0)u}_{pp}(q,\omega) = -\frac{q^2}{24\pi^2 \hbar \nu_F} \Theta(\omega - \nu F q), \]  
is the known density-density response function for undoped single Dirac node\textsuperscript{27,41}. In Eq. (8), $\Theta(x)$ denotes the step function. Note that the $\omega = \nu F q$ line also marks the boundary of the intra-band single particle-hole excitations in massless Dirac systems (both doped and undoped).

The real part of the response function $\text{Re} \Pi^{(0)u}_{\sigma \sigma}(q,\omega)$ can be calculated directly using Eq. (3). Upon integrating Eq. (3) and after simplification we get
\[ \text{Re} \Pi^{(0)u}_{\sigma \sigma}(q,\omega) = -\frac{q_{\text{max}}^2}{6\pi^2 \hbar \nu_F} + \frac{\omega^2}{v_F^2 q^2} \text{Re} \Pi^{(0)u}_{pp}(q,\omega), \]  
where the real part of the density response is given by\textsuperscript{27,41},
\[ \text{Re} \Pi^{(0)u}_{pp}(q,\omega) = -\frac{\omega^2}{24\pi^2 \hbar \nu_F} \log \left[ \frac{4v_F^2 q_{\text{max}}^2}{|v_F^2 q^2 - \omega^2|} \right]. \]  
Note that Eq. (9) also follows directly from Eqs. (4)-(5).

Before proceeding further, we note that the current current response function for the undoped case corresponds to the polarization bubble diagram of quantum electrodynamics (QED) in 3+1 dimensions. In QED - with an infinite energy spectrum- the dimensional regularisation scheme\textsuperscript{40} is generally used and it only gives logarithmic divergence. However in a lattice system, this cannot be implemented and an energy cutoff scheme has to be used. More importantly, condensed matter systems always have a finite bandwidth corresponding to the Bloch bands. In such systems an energy cutoff regularization scheme is also physically relevant, with the interpretation of the cutoff energy scale as the energy bandwidth.
accounts for the renormalisation of the scale dependent
in the Lagrangian for cancelling out the divergence. This
physical
the cutoff regularization scheme, in known to lead to un-
the polarization bubble diagram in 3+1 dimensions, with
citations. Regions 1B, 3A, and 3B do not have any single
 excitations. Regions 1A and 2A only have intraband particle-hole ex-
Region 1A and 2A only have intraband particle-hole excita-
tions, while regions 2B has only interband particle-hole ex-
citations.

In quantum electrodynamics (QED) the calculation of
the polarization bubble diagram in 3+1 dimensions, with
the cutoff regularization scheme, in known to lead to un-
physical $q^2$ divergence. In QED such divergences are
generally taken care of by adding a suitable counter term
in the Lagrangian for cancelling out the divergence. This
accounts for the renormalisation of the scale dependent
and screened electric charge: $e \rightarrow e_r$. See Appendix
F for details. A similar interpretation can be made for
our work as well. The unphysical quadratically diverging
cutoff dependent terms in Eq. (9), can be taken care of
by redefining the effective renormalized scale dependent
dependent charge of the Dirac quasiparticles.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Different regions in the $\omega - q$ plane used to define the
imaginary part of the longitudinal as well as the transverse
response function in a 3D Dirac semimetal.}
\end{figure}

B. Doped case

We now proceed to calculate additional contribution
to the non-interacting longitudinal spin-spin (or
spin-pseudospin) response function $\Pi^{(ld)}_{\sigma_\tau,\sigma_\tau}(q\mathbf{z}, \omega)$
which arises only at finite doping (say electron doping).
In this case the valence band is completely occupied and
conduction band is filled up to the Fermi energy level
$\mu = h v_F k_F > 0$, where $k_F$ is the Fermi wave-vector. Simi-
lar to the case of the undoped part, the doping depend-
ent part of the response function can also be expressed in
in terms of the corresponding density response function
using Eq. (4). For the imaginary part we have

$$ \text{Im} \Pi^{(ld)}_{\sigma_\tau,\sigma_\tau}(q\mathbf{z}, \omega) = \frac{\omega^2}{v_F^2 q^2} \text{Im} \Pi^{(ld)}_{\rho\rho}(q, \omega). $$

(11)

The imaginary component of the extrinsic part of the
density-density response function has already been calcu-
lated in Ref. [27 and 41]. Thus the imaginary component
of spin-spin response function can be expressed as

$$ \text{Im} \Pi^{(ld)}_{\sigma_\tau,\sigma_\tau}(q\mathbf{z}, \omega) = -\frac{\omega^2}{8\pi \hbar^2 v_F^4 q^2} \left( \zeta(q, \omega) - \zeta(q, -\omega) \right), $$

(12)

in Eq. (12), we have defined the function,

$$ \zeta(q, \omega) = \frac{1}{12\hbar^3 v_F^3 q} \left( 2(2\mu + \hbar \omega)^3 - 3h^2 v_F^2 q^2 (2\mu + \hbar \omega) + 2h^3 v_F^3 q^3 \right). $$

(13)

The various regions in the $\omega - q$ plane, specified by
Eq. (12) are defined as

1A: $0 < \omega < v_F q$ and $2\mu - h v_F q - \hbar \omega > 0$, 
2A: $0 < \omega < v_F q$ and $2\mu - h v_F q + \hbar \omega > 0$, 
3A: $0 < \omega < v_F q$ and $2\mu - h v_F q + \hbar \omega < 0$, 
1B: $0 < v_F q < \omega$ and $2\mu - h v_F q + \hbar \omega < 0$, 
2B: $0 < v_F q < \omega$ and $2\mu - h v_F q - \hbar \omega > 0$, 
3B: $0 < v_F q < \omega$ and $2\mu - h v_F q - \hbar \omega < 0$. 

(14)

These regions are also marked in the $\omega - q$ plane in
Fig. 1. Note that the finite contribution in the 1B region
in Eq. (12), cancels out the corresponding contribution
from the undoped part in Eq. (7). Thus in all, regions 1B
and 3A are the only regions without single particle-hole
excitations.

The real part of the doped response function can also be obtained using Eq. (4), and it is given by

$$ \text{Re} \Pi^{(ld)}_{\sigma_\tau,\sigma_\tau}(q\mathbf{z}, \omega) = \frac{\omega^2}{v_F^2 q^2} \text{Re} \Pi^{(ld)}_{\rho\rho}(q, \omega). $$

(15)

The real component of extrinsic density-density response
function in Eq. (15) is given by [27, 41],

$$ \text{Re} \Pi^{(ld)}_{\rho\rho}(q, \omega) = -\frac{q^2}{8\pi \hbar^2 v_F^4} \left[ \mathcal{C}(q, \omega) + \mathcal{D}(q, \omega) \right]. $$

(16)

In Eq. (16), we have defined the following functions:

$$ \mathcal{C}(q, \omega) = \frac{8\mu^2}{3h^2 v_F^2 q^2} - \frac{\zeta(q, \omega) H(q, \omega) - \zeta(-q, \omega) H(-q, \omega)}{q^2}, $$

$$ \mathcal{D}(q, \omega) = \zeta(q, -\omega) - \frac{8\mu^2}{3h^2 v_F^2 q^2}, $$

(17)

which in turn use $\zeta(q, \omega)$ defined in Eq. (13) and

$$ H(q, \omega) = \log \left[ \frac{2\mu + \hbar \omega - h v_F q}{\hbar v_F q - \hbar \omega} \right]. $$

(18)
C. Plasmon dispersion

The dispersion of the collective density excitations, or plasmon, of an interacting electron gas can be calculated within the random phase approximation. It is given by the zeros of the RPA dielectric function,

\[ \epsilon(q, \omega) = 1 - v_0 \text{Re} \Pi_{\nu \nu}^{\rho}(q \hat{z}, \omega_{\text{pl}}) = 0 , \]  

where \( v_0 = 4 \pi e^2 / \kappa q^2 \) is the 3D Fourier transform of Coulomb potential with \( \kappa \) being the surrounding dependent dielectric constant. Equation (19) can be expressed in terms of spin-spin response function:

\[ 1 - v_0 \frac{q^2}{\omega^2} \left[ \Pi_{\sigma z, \sigma z}(q \hat{z}, \omega) + \frac{q_{\text{max}}^2}{6 \pi^2 \hbar^2 v_F^2} \right] = 0 . \]

Using the calculated spin-spin response function and expanding the expressions in square bracket in Eq. (19) in powers of \( q \) up to fourth order, we obtain the plasmon frequency (\( \omega_{\text{pl}} \)) to be

\[ \omega_{\text{pl}} = \omega_0 \left[ 1 - \frac{\hbar^2 v_F^2 q^2}{8 \mu^2} \left( 1 + \gamma (\hbar \omega_0 / 2 \mu) \right) \right] . \]  

In Eq. (21), we have defined

\[ \hbar \omega_0 \equiv \mu \sqrt{\frac{2 \alpha_{\text{ee}}}{3 \pi \kappa^* (\omega_0)}} , \quad \text{and} \quad \gamma (x) = \frac{x^2 - 3/5}{x^2 (1 - x^2)^2} , \]  

where \( \alpha_{\text{ee}} = e^2 / (\kappa \hbar v_F) \) is the effective fine structure constant. The effective background dielectric constant of a Dirac node now becomes frequency dependent and it is given by \( \kappa^* (\omega) = 1 + \frac{\alpha_{\text{ee}}}{6 \pi} \log \left| \frac{4 \hbar^2 v_F^2 q_{\text{max}}^2}{\hbar^2 \omega^2 - 4 \mu^2} \right| \). This is unlike the case of 2D massless Dirac systems (such as graphene\(^{37,38,39,40}\) or 2D surface states of 3D topological insulators\(^{40}\)), where the long wavelength plasmon dispersion does not depend on the ultraviolet cutoff. The long wavelength plasmon dispersion varies linearly with the chemical potential \( \mu \) or as \( n^{1/3} \) with the electronic density.

A few earlier works (including work from our group) report a slightly different version of the plasmon dispersion: \( \omega_{\text{pl}} = \omega_0 \) in the \( q \to 0 \) limit\(^ {24,43,51} \). This is a consequence of using the long wavelength approximation in the calculation of the (approximate) polarization function itself, which leads to vanishing overlap function in the inter-band contribution. This is technically incorrect since the overlap function contributes significantly in the inter-band part of the full polarization function, and leads to the logarithmic terms in Eq. (21). Interestingly, in the very weak interaction limit (\( \kappa \to \infty \)), with \( \alpha_{ee} \ll 1 \), \( \kappa^* (\omega) \to 1 \) and Eq. (21) reduces to \( \omega_{\text{pl}} = \omega_0 \) to zeroth order in \( q \), which is consistent with the expressions in earlier works\(^ {37,42,43,51} \).

Finally we note that for massless Dirac systems RPA is not exact even in the \( q \to 0 \) limit, since the massless Dirac Hamiltonian of Eq. (1) is not invariant to Galilean boosts, unlike the case of typical 2D/3D electron gas with parabolic dispersion\(^ {37,52,53} \). Thus, similar to the case of graphene, the RPA plasmon dispersion of Eq. (21) can be expected to have some interaction induced renormalization correction, even in the long wavelength limit\(^ {37} \).

D. Longitudinal optical conductivity

Another observable connected to the longitudinal current response function is the optical conductivity. The real part of long-wavelength longitudinal conductivity \( \sigma (\omega) \), in the linear response regime, is given by

\[ \text{Re} \sigma (\omega) = -\frac{e^2 \omega}{24 \pi \hbar v_F} \lim_{q \to 0} \text{Im} \Pi_{\sigma x, \sigma x}(q \hat{z}, \omega) . \]

Using Eq. (7) and Eq. (12) in Eq. (23), the longitudinal conductivity for a single node of 3D Dirac semimetal can be obtained to be

\[ \text{Re} \sigma (\omega) = \frac{e^2 \omega}{24 \pi \hbar v_F} \Theta (\hbar \omega - 2 \mu) , \]

which is consistent with the results of Refs. [33 and 34]. Beyond the threshold of the Pauli blocked region, \( \omega > 2 \mu \), the interband optical conductivity (involving only vertical transitions) is linearly proportional to the frequency in an ideal (very clean) 3D massless Dirac system. Such a linear dependence of the optical conductivity on the frequency has already been experimentally reported for ZrTe\(_5\)\(^ {23} \), in Cd\(_3\)As\(_2\) with 001 orientation\(^ {54} \), and in Eu\(_3\)Ir\(_2\)O\(_7\)\(^ {55} \).

IV. TRANSVERSE SPIN-SPIN RESPONSE FUNCTION

To calculate the transverse spin-spin response function we use the expression as given in Eq. (3) with \( q = q \hat{x} \) (in the \( x - y \) plane) which is perpendicular to the direction of the applied vector potential. Since the calculation from Eq. (3), proceeds in a manner similar to that of the density-density response function also in terms of the density response function. Similar to the case of longitudinal response, we find that transverse response function (with \( q \) along the \( \hat{x} \) direction), can also be expressed as a sum of intrinsic (\( \mu = 0 \)) as well as extrinsic (\( \mu \neq 0 \)) contributions:

\[ \Pi_{\sigma x, \sigma x}^{(0)} (q \hat{x}, \omega) = \Pi_{\sigma x, \sigma x}^{(0 \omega)} (q \hat{x}, \omega) + \Pi_{\sigma x, \sigma x}^{(0d)} (q \hat{x}, \omega) . \]  

Let us consider the intrinsic (undoped) case first.

A. Undoped Case

To evaluate the transverse spin-spin response function, we follow an approach similar to that used for calculating
the density-density response function in Ref. [27]. The details of the calculations are presented in appendix C. For the undoped case, the imaginary and real component of transverse response function can be expressed in terms of the intrinsic density-density response function as

$$\text{Im}\Pi^{(0\mu)}_{\sigma\sigma}(q,\omega) = \frac{\omega^2 - v_F^2 q^2}{v_F^2 q^2} \text{Im}\Pi^{(0\mu)}_{\rho\rho}(q,\omega), \quad (26)$$

$$\text{Re}\Pi^{(0\mu)}_{\sigma\sigma}(q,\omega) = \frac{\omega^2 - v_F^2 q^2}{v_F^2 q^2} \text{Re}\Pi^{(0\mu)}_{\rho\rho}(q,\omega) - \frac{\eta_{\text{max}}^2}{6\pi^2\hbar v_F}. \quad (26)$$

It turns out that the relation between the density-density response and spin-spin response function is a bit cumbersome. It can be cured by subtracting the cut-off term \(q_{\text{max}}^2/(6\pi^2\hbar v_F)\). This is a direct consequence of the fact that gauge invariance of Eq. (1) is explicitly broken by the ultraviolet energy cutoff. In order to restore the gauge invariance of the system, the static response function of the system should be corrected by subtracting the cut-off term \(q_{\text{max}}^2/(6\pi^2\hbar v_F)\) from Eqs. (9) and (26). This issue also arises in massive 2D Dirac systems, and can be cured by taking a lattice Hamiltonian (tight-binding) instead of the continuum Hamiltonian.

Similar divergences also arise while calculating the polarization bubble diagram in QED in 3+1 dimensions, with a cutoff regularization scheme. These are generally taken care of by adding a counter term in the Lagrangian to cancel such divergences. This leads to scale dependent and renormalized couplings constant: the Dirac quasiparticle charge in our case, and restores the gauge invariance of the low energy theory – see the discussion following Eq. (9) and Appendix F.

## B. Doped Case

We now proceed to calculate the contributions to the transverse spin-spin response function \(\Pi_{\sigma\sigma}(q\hat{x},\omega)\) at finite doping with \(\mu > 0\). See appendix C for details.

The imaginary component of doped transverse spin-

$$\text{Im}\Pi^{(0\mu)}_{\sigma\sigma}(q\hat{x},\omega) = \frac{\beta(q,\omega) - \beta(q,-\omega)}{2\hbar v_F q}, \quad (27)$$

$$\text{Re}\Pi^{(0\mu)}_{\sigma\sigma}(q\hat{x},\omega) = \frac{\beta(q,\omega)}{2\hbar v_F q}, \quad (27)$$

In Eq. (27), the different regions are specified in Eq. (14)/Fig. (1), and we have defined the function:

$$\beta(q,\omega) = 2q\zeta(q,\omega) + q^2(2v_F k_F - v_F q + \omega), \quad (28)$$

where \(\zeta(q,\omega)\) is defined in Eq. (13).

The real part of the doped component of the transverse spin-spin response function is a bit cumbersome. It can be expressed as

$$\text{Re}\Pi^{(0\mu)}_{\sigma\sigma}(q\hat{x},\omega) = \frac{\omega^2 - v_F^2 q^2}{2v_F^2 q^2} \text{Re}\Pi^{(0\mu)}_{\rho\rho}(q,\omega) + \frac{\omega^2 - v_F^2 q^2}{16\pi^2\hbar v_F q^2} \sum_{p=\pm 1} [I_p^{(1)}(q) + I_p^{(1)}(-\omega)] + \frac{1}{16\pi^2\hbar v_F q^2} \sum_{p=\pm 1} I_p^{(2)}(q). \quad (29)$$

Here \(I_p^{(1)}(q)\) and \(I_p^{(2)}(q)\) are integrals defined as

$$I_p^{(1)} = \int_0^{k_F} dk \int_{l_1}^{l_2} dk' \frac{v_F}{v_F k + \omega + v_F k'}, \quad (30)$$

$$I_p^{(2)} = \int_0^{k_F} dk \int_{l_1}^{l_2} dk' (k' - pk)(k' + pk)^2 + q^2]. \quad (31)$$

Here \(l_1 = |k - q|, l_2 = |k + q|\). The exact analytic expression for these integrals are specified in Appendix-D.

## C. Diamagnetic Susceptibility

In this subsection, we use the obtained transverse current-current response function to calculate the diamagnetic susceptibility in 3D Dirac semimetals due to a static magnetic field. The noninteracting diamagnetic susceptibility \(\Pi_{\text{orb}}\) is given by:

$$\Pi_{\text{orb}} = -\frac{v_F^2 e^2}{c^2} \lim_{q \to 0} \frac{\Pi^{(0)}_{\sigma\sigma}(q\hat{x}, 0)}{q^2}, \quad (32)$$

where \(\Pi^{(0)}_{\sigma\sigma}(q\hat{x}, 0)\) is the static transverse spin-spin response function. The static transverse susceptibility is purely real and it is given by

$$\Pi^{(0)}_{\sigma\sigma}(q\hat{x}, 0) = \frac{1}{48\pi^2 q v_F} \left[ (4k_F^2 + 3k_F q^2) \log \left( \frac{|q - 2k_F|}{q + 2k_F} \right) - 2q^3 \log \frac{4k_F^2 - q^2}{4k_{\text{max}}^2 + 4k_F q} \right]. \quad (33)$$
In the limit \( q \to 0 \), Eq. (33) reduces to

\[
\lim_{q \to 0} \Pi^{(0)}_{\sigma_{x}\sigma_{s}} (q\hat{x}, 0) = \frac{q^2}{12\pi^2 \hbar v_F} \log \left( \frac{\varepsilon_{\text{max}}}{\mu} \right). \tag{34}
\]

Substituting this in Eq. (32) we obtain

\[
\Pi_{\text{orb}} = -\frac{v_F^2 e^2}{12\pi^2 \hbar^2 v_F} \log \left( \frac{\varepsilon_{\text{max}}}{\mu} \right). \tag{35}
\]

Note that since we have ignored the explicit structure of the Landau levels etc, the diamagnetic susceptibility obtained above should ideally hold in the weak field limit only. However it turns out that the diamagnetic susceptibility in Eq. (35) is consistent with a more rigorous calculation involving Landau levels, as it is consistent with the result of Eq. (59) of Ref. [44] in the \( \Delta \to 0 \) limit, and Eq. (38) of Ref. [56].

Similar anomalous divergence in diamagnetic susceptibility has been studied in detail in the context of Bi, which has an anisotropic Dirac node\(^{27}\). A simple physical way of understanding the anomalous and diverging contribution is based on the energy argument. In a regular metal with parabolic dispersion, the Landau levels are equispaced in energy, thus the total energy of the system (for levels away from the chemical potential) remains unchanged on the application of the magnetic field. Only the states in vicinity of the chemical potential are affected and give a finite contribution to the diamagnetic susceptibility. However in systems with a Dirac node, the Landau levels are not equispaced, and as a consequence the total energy of the system gets finite contribution from all filled states (primarily the infinite levels in the valance band), and thus the orbital/diamagnetic susceptibility diverges. This argument can also be quantified based on a crude estimate by second order perturbation theory\(^{58}\). The external magnetic field \( B \) couples to the orbital degrees of freedom via the vector potential \( A \approx B \times r \approx B/k \). Thus the correction to the total energy of the system upto second order in \( A \) is (for a given \( k \) mode) is \( \approx (B/k)^2/(v_F k) \), where \( 2v_F k \) is the transition energy from valance to conduction band. Summing over the allowed \( k \) modes now yields, \( \delta E(B) \propto B^2 \log(\varepsilon_{\text{max}}/\mu) \), consistent with Eq. (35). This crude estimate is also consistent with a more thorough calculation for the change in the total energy of a Dirac node in presence of a magnetic field\(^{58}\).

\section*{V. CURRENT-CURRENT RESPONSE IN WEYL SEMIMETALS WITH CHIRAL ANOMALY}

Having calculated the current-current (and spin-spin) response functions for a single Dirac node, we now proceed to calculate the corresponding response for a Weyl semimetal. For simplicity, we consider a Weyl semimetal with one pair of Weyl nodes. For an experimental system with \( g \) pairs of Weyl nodes, all our results should be multiplied by \( g \). The low energy Hamiltonian, for each of the Weyl node of chirality \( \chi = \pm 1 \), in vicinity of the Weyl node, is

\[
H_\chi = \hbar v_F \sigma \cdot k - I\mu_\chi \tag{36}
\]

where \( \sigma \) is the vector of the three Pauli matrices and \( I \) is the \( 2 \times 2 \) identity matrix and \( \mu_\chi \) denotes the chemical potential of the Weyl node of chirality \( \chi \).

Application of parallel electric (\( E \)) and magnetic (\( B \)) fields in a Weyl semimetal leads to a chiral anomaly: charge transfer from the \( \chi = -1 \) node to the \( \chi = +1 \) node for \( E \cdot B > 0 \) and vice versa for \( E \cdot B < 0 \). This charge transfer is eventually stabilized by some inter-node scattering mechanism with timescale \( \tau \). The amount of electron transferred from one Weyl node to other is given by \( \Delta n = \frac{e^2}{2\pi^2 \hbar v_F} E \cdot B \tau \). This leads to a shift in the respective chemical potentials in the two Weyl nodes. If initially both the nodes were doped with a chemical potential \( \mu = \hbar v_F k_F \), with \( k_F^2 = 6\pi^2 n \), then the modified densities in the two nodes with \( \chi = \pm 1 \) are \( n_{\pm} = n \pm \Delta n/2 \). Accordingly their modified Fermi wave-vectors are given by \( (k_F^\pm)^3 = 6\pi^2 n_{\pm} \), and the corresponding chemical potential is given by\(^{27}\),

\[
\mu_{\pm} = \left( \mu^3 \pm \frac{3e^2 \hbar v_F^3}{2} E \cdot B \tau \right)^{1/3}. \tag{37}
\]

For the rest of the article, we will be working in the weak magnetic field limit, whereby the discrete Landau levels structure of the Weyl semimetal can be ignored. Additionally we assume that \( E \parallel B \) (or \( E \cdot B > 0 \)) and consequently we have \( \mu_+ > \mu_- \).

Equation (37) implies that for a physical manifestation of the chiral anomaly to be seen in experiments, the splitting of the chemical potential should be of the order of the chemical potential. This implies that \( |\mu_+ - \mu_-| \approx \mu \) or alternatively to lowest order in \( E \cdot B \) term, \( e^2 \hbar v_F^3 E \cdot B \tau \approx \mu^3 \). Typically the inter-valley scattering time is of the order of \( \approx 10^{-9}s \) with the corresponding length scale of the order of a few microns - see Ref. [59], for a discussion. Assuming \( v_F \) to be of the order of \( 10^3 \text{m/s} \), this implies that \( E \cdot B \approx \mu^3 \times 10^6 \text{Tesla V/m} \), where \( \mu \) is expressed in eV. Thus if \( \mu \approx 0.1 \text{eV} \), then \( E \cdot B \) should be of the order of \( 10^3 \text{Tesla V/m} \) for the chiral anomaly to be distinguishable in experiments.

The longitudinal and the transverse spin-spin response functions for a Weyl semimetal can now be obtained simply by summing the contributions from the two Weyl nodes with the modified chemical potential\(^{27,36}\).
Following a similar procedure, the real part of longitudinal spin-spin response function of a Weyl semimetal with 2 nodes can be expressed as

$$\operatorname{Re} \Pi_{\sigma_z \sigma_z}^{(0d)}(q \hat{z}, \omega) = -\frac{\omega^2}{8\pi^2 \hbar v_F^2 q^2} \sum_{\chi=\pm 1} [C_\chi(q, \omega) + D_\chi(q, \omega)] .$$

Here we have defined the functions,

$$C_\chi(q, \omega) = C(q, \omega) |_{\mu=\mu_\chi},$$
$$D_\chi(q, \omega) = D(q, \omega) |_{\mu=\mu_\chi},$$

which in turn use the definitions from Eq. (17).

Substituting the above expression in Eq. (21), the plasmon dispersion in long-wavelength limit up to order of $q^2$ modifies to

$$\omega_{pl} = \omega_0 \left[ 1 - \frac{\hbar^2 v_F^2 q^2}{8[|\mu^+|^2 + |\mu^-|^2]} \sum_{\chi=\pm 1} \left( 1 + \gamma(\hbar \omega/2\mu_\chi) \right) \right].$$

In Eq. (42) we have used,

$$\hbar \omega_0 = \sqrt{\frac{2\alpha_{ee} |\mu^+|^2 + |\mu^-|^2}{3\pi \kappa^*(\omega_0)}} .$$

and the effective frequency dependent background dielectric constant is given by

$$\kappa^*(\omega) = 1 + \frac{\alpha_{ee}}{6\pi} \sum_{\chi=\pm 1} \log \left| \frac{4\hbar^2 v_F^2 q_{\text{max}}^2}{\hbar^2 \omega^2 - 4|\mu_\chi|^2} \right| .$$

In the absence of chiral anomaly, $|\mu^+| = |\mu^-|$ and Eq. (42) reduces to Eq. (21). The results for the plasmon dispersion in Eq. (42) are consistent with those derived in Ref. [27]. Substituting the value of $\mu_\pm$ from Eq. (37) in (43), to leading order in $\mathbf{E} \cdot \mathbf{B}$, $\omega_0$ can be expressed in terms of the external parallel electric and magnetic field as

$$\hbar \omega_0 = \mu \sqrt{\frac{4\alpha_{ee}}{3\pi \kappa^*(\omega_0)}} \left( 1 - \frac{\hbar^2 e^4 v_F^2 \tau^2 (\mathbf{E} \cdot \mathbf{B})^2}{8\mu^5} \right) .$$

Thus the $q \to 0$ plasmon dispersion is expected to have a dependence of $(\mathbf{E} \cdot \mathbf{B})^2$ to leading order in $\mathbf{E}$ and $\mathbf{B}$ fields.

Another experimental observable which carries signature of the chiral anomaly is the optical conductivity. In presence of the chiral anomaly, the longitudinal conductivity defined in Eq. (23) gets modified and it is given by [34],

$$\operatorname{Re} \sigma(\omega) = \frac{e^2 \omega}{24\pi \hbar v_F} \left| \Theta(\hbar \omega - 2\mu^+) + \Theta(\hbar \omega - 2\mu^-) \right| .$$

The additional step function appearing in Eq. (46) as compared to Eq. (24) is a consequence of different Pauli blocking of the optically excited carriers in the two Weyl

A. Longitudinal response

The imaginary part for longitudinal spin-spin response function at finite doping modifies to

$$\operatorname{Im} \Pi_{\sigma_z \sigma_z}^{(0d)}(q \hat{z}, \omega) = -\frac{\omega^2}{8\pi^2 \hbar v_F^2 q^2} \sum_{\chi=\pm 1} \left[ \zeta_\chi(q, \omega) \right] .$$

where

$$\zeta_\chi(q, \omega) = \zeta(q, \omega) - \frac{2q^2}{3} - \zeta_-(-q, \omega).$$

The different regions defined above can be evaluated by replacing $\mu \to \mu_\chi$ in Eq. (14) and are also displayed in Fig. (2). Additionally we have defined the function,

$$\zeta_\pm = \frac{1}{12\hbar^2 v_F^2 q} \left( [2\mu'_\pm]^3 - 3\hbar^2 v_F^2 q^2[2\mu'_\pm] + 2\hbar^3 v_F^3 q^3 \right) ,$$

with $2\mu'_\pm = 2\mu_\pm + \hbar \omega$. 

FIG. 2. Different regions in the $\omega - q$ plane used to define the imaginary part of the longitudinal as well as the transverse spin-spin response function in a Weyl semimetal with chiral anomaly. Here the regions $2B$, $3B$ and $4B$ have interband single particle excitations while the regions $1A$, $2A$, $3A$, $4A$ and $5A$ have intraband single particle excitations for either one or both the Weyl nodes with Fermi energy located at $\mu_\pm$. More specifically the $3B$ region has interband particle-hole excitation of both nodes, and the $3A$ has intraband particle-hole excitations of both the Weyl nodes. The regions $1B$, $5B$, $6B$ and $6A$ are without any single particle excitation.
nodes due to different chemical potential in presence of a chiral anomaly. In absence of chiral anomaly, there is linear optical conductivity beyond the chemical potential as a function of \( \omega \), while in presence of a chiral anomaly an extra step function with linear dependence on \( \omega \) appears in the optical conductivity, with the width of the step function being proportional to \( \mathbf{E} \cdot \mathbf{B} \).34

## B. Transverse response

In this subsection we explore the impact of chiral anomaly on the transverse current-current response function, and the orbital susceptibility. The imaginary component of the transverse spin-spin response function is given by,

\[
\Im \Pi_{(0d)}(q\mathbf{x}, \omega) = \frac{\omega^2 - v_F^2 q^2}{32\pi \hbar^2 q^3} \sum_{\chi=\pm} \left[ \beta_{\chi}(q, \omega) - \beta_{\chi}(q, -\omega) \right].
\]

where the different regions in \( \omega - q \) plane are shown in Fig. (2) and we have defined the function,

\[
\beta_{\pm}(q, \omega) = 2q \zeta_{\pm}(q, \omega) + q^2 (2\mu_{\pm} + \hbar \omega - \hbar v_F q) .
\]

The real component of the doped transverse spin-spin response function is explicitly given by

\[
\Re \Pi_{(0d)}(q\mathbf{x}, \omega) = \frac{\omega^2 - v_F^2 q^2}{16\pi \hbar^2 v_F^2} \sum_{\chi=\pm} [C_\chi(q, \omega) + \mathcal{D}_\chi(q, \omega)]
+ \frac{\omega^2 - v_F^2 q^2}{16\pi \hbar^2 v_F^2} \sum_{p=\pm1} \sum_{\chi=\pm} \left[ I^{(1)}_p(k_{F\chi}, \omega) + I^{(1)}_p(k_{F\chi}, -\omega) \right]
+ \frac{1}{16\pi \hbar^2 v_F^2} \sum_{p=\pm1} \sum_{\chi=\pm} I^{(2)}_p(k_{F\chi}, q) .
\]

Here we have defined the chiral Fermi wavevector \( k_{F\chi} = \mu_{\chi}/(\hbar v_F) \) and the explicit form of the integrals \( I^{(1)}_p \) and \( I^{(2)}_p \) are specified in Appendix-D.

The static part of transverse spin-spin response function of a Weyl semimetal with chiral anomaly modifies as

\[
\Pi^{(0)}_{\sigma_\alpha \sigma_\alpha}(q\mathbf{x}, 0) = \frac{1}{48\pi^2 \hbar v_F q} \sum_{\chi=\pm} \left[ 2q^3 \log \left( \frac{4q_{\max}^2}{|k_{F\chi}^2 - q^2|} \right) + [4k_{F\chi}^3 + 3k_{F\chi}^2 q^2] \log \left( \frac{|q - 2k_{F\chi}|}{q + 2k_{F\chi}} \right) + 4k_{F\chi}^2 q \right] .
\]

Using Eq. (50) to evaluate the diamagnetic susceptibility, we obtain

\[
\Pi_{\text{orb}} = -\frac{v_F^2 e^2}{12\pi^2 \hbar^2 v_F} \log \left( \frac{q_{\max}}{\mu_{\pm}} \right) .
\]

To leading order in \( (\mathbf{E} \cdot \mathbf{B}) \), the diamagnetic susceptibility is

\[
\Pi_{\text{orb}} = -\frac{v_F^2 e^2}{12\pi^2 \hbar^2 v_F} \left[ 2 \log \frac{q_{\max}}{\mu} + \frac{3\hbar^2 e^4 v_F^2 T^2 (\mathbf{E} \cdot \mathbf{B})^2}{4\mu^6} \right] .
\]

## VI. Implications for Anisotropic Dirac and Weyl Systems

While we have focussed primarily on isotropic systems, several actual experimental realization of Dirac/Weyl semimetals such as Na3Bi14–16, Cd3As217–21, PtTe290 etc., host anisotropic Dirac/Weyl fermions. Thus in this section we qualitatively discuss the implications of our isotropic calculations for systems with anisotropic dispersion.

Let the anisotropic velocities of the anisotropic Dirac cone be \( \{ v_x, v_y, v_z \} \). The anisotropy in the response functions is likely to be captured by the following replacement: \( q \to q' \) where

\[
q' = q \left[ \sin^2 \theta_q \left( \cos^2 \phi_q + \frac{v_y^2}{v_x^2} \sin^2 \phi_q \right) + \frac{v_z^2}{v_x^2} \cos^2 \theta_q \right]^{1/2} ,
\]

and \( v_x \to v_x \), in all the response functions. A similar replacement was shown to arise in the calculation of the density response function for an anisotropic and tilted Dirac cone in two dimensions, in the context of Borophene.61 Accordingly a similar direction dependent anisotropy factor will also appear in the plasmon dispersion. However the qualitative features of the gapped plasmon dispersion with \( \omega_{pl} \propto \mu \) or \( \omega_{pl} \propto n^{1/3} \) should not change.

As far as the optical conductivity (longitudinal response) of a given Dirac node is concerned, it should be still given by a form similar to Eq. (24), with the Fermi velocity being substituted by \( v_F \to (v_x v_y v_z)^{1/3} \). For the diamagnetic magnetic susceptibility (transverse response) also we expect the same functional form as Eq. (35), with the substitution of \( v_F \to v_z \). The qualitative behaviour involving logarithmic divergence in the orbital susceptibility, i.e., \( \Pi_{\text{orb}} \propto \log (\varepsilon_{\text{max}}^2/\mu) \), should persist even in the anisotropic case.
VII. SUMMARY

To summarize, we have presented the analytical results for the longitudinal and transverse current-current response function in the $(\omega, q)$ plane for a single Dirac/Weyl node. As expected, the current-current response function is related to the density-density response function due to the charge continuity equation. Additionally since the current operator is proportional to the spin operator, the current-current response function is also related to the spin-spin response function.

We find that for the undoped 3D Dirac node, the relation between spin-spin response function and density-density response function are identical to that for a 2D Dirac node as in graphene. However for the case of finite doping the relationship between the response functions differ between 2D and 3D. For a 3D Dirac node, the long wavelength plasmon dispersion is directly proportional to the chemical potential, the optical conductivity beyond the Pauli blocked regime is linearly proportional to the frequency and the diamagnetic susceptibility diverges logarithmically for vanishing chemical potential.

The current current response function of a single Dirac node is then used to obtain the response function of a Weyl Semimetal with chiral anomaly. For a Weyl semimetal in presence of parallel $E$ and $B$ fields, we find that the long wavelength plasmon dispersion (or gap) is proportional to $(E \cdot B)^2$ to leading order in $E \cdot B$, the optical conductivity displays a two step behaviour due to partial Pauli blocking in one of the Weyl nodes, and the diamagnetic susceptibility is found to vary as $(E \cdot B)^2$ to leading order in $E \cdot B$.

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Appendix A: The commutator term

Here we evaluate the commutator term appearing in Eq. (4). A similar calculation has already been done for the 2D case of graphene in Ref. [62] and we follow a similar approach here. For massless electrons described by unbounded linear relations the operators are also defined in the unbounded energy/momentum space. For unbounded operators in general $\sum_k (\mathcal{O}(k)) \neq \sum_{k+q} (\mathcal{O}(k + q))$. To overcome this, one defines 'bounded' operators by subtracting out the ground state contribution. This is done by defining normal ordered operators:

$$
:G(k) := G(k) - \langle 0 | G(k) | 0 \rangle.
$$

Expressing the commutator defined in Eq. (4) in terms of normal ordered operators yields,

$$
[qJ_q, \rho_{-q}] = -v_F \sum_k \left[ \langle 0 | \psi_{k-q} (q, \sigma) \psi_{k-q} | 0 \rangle - \langle 0 | \psi_k (q, \sigma) \psi_k | 0 \rangle \right],
$$

where the ground state comprises of the completely filled valence band. However for doped Weyl semimetals with $\mu > 0$, the contributions from electron above the Dirac points also have to be accounted for. The difference of the two infinite sums in Eq. (A2), can be computed by regularizing it with the high energy ultraviolet cutoff. To proceed further, we switch to a diagonal basis by diagonalizing the matrices defined in Eq. (A2) by using the unitary transformation that diagonalizes the Hamiltonian:

$$
U_k = \left( \frac{\cos \theta_k e^{-i\phi_k}}{\sqrt{2} e^{-i\phi_k}} \frac{\sin \theta_k e^{i\phi_k}}{\sqrt{2} e^{i\phi_k}} \cos \frac{\theta_k e^{i\phi_k}}{\sqrt{2} e^{i\phi_k}} - \cos \frac{\theta_k e^{-i\phi_k}}{\sqrt{2} e^{-i\phi_k}} \right).
$$

For the longitudinal case, taking $q$ along the $z$ direction Eq. (A2) simplifies to

$$
[qJ_{z,q}, \rho_{-q\hat{z}}] = v_F \sum_k [\cos \theta_{k+q} - \cos \theta_k].
$$

For evaluating the sum, the summation over $k$ is converted to the integral in the $k$ space using the ultraviolet cutoff $q_{\text{max}}$ for the maximum allowed $k$:

$$
\sum_k [\cos \theta_{k+q} - \cos \theta_k] = \frac{1}{(2\pi)^3} \int_{k=0}^{k_{\text{max}}} k^2 \cos \theta \, d\Omega,
$$

where $k_{\text{max}} = \sqrt{q^2 + q_{\text{max}}^2} + 2 q_{\text{max}} q \cos \theta$ and $d\Omega = \sin \theta \, d\theta \, d\phi$ is the integration over the solid angle. Note that in the l.h.s. of Eq. (A5), only the first term contributes to the integral and we have chosen $q$ along the $\hat{z}$ direction. Taking the leading order contribution in $q_{\text{max}}$ on the r.h.s. of Eq. (A5) we get

$$
\frac{1}{v_F q} [qJ_{z,q}, \rho_{-q\hat{z}}] = \frac{q_{\text{max}}^2}{6\pi^2}.
$$

Appendix B: The longitudinal and transverse overlap function

Here we briefly discuss the calculation of the overlap function appearing in our calculations. The eigenfunction of the Hamiltonian in Eq. (1) are given by,

$$
\chi_{\lambda}(k) = \left( e^{i\phi_k} \cos \left( \theta_{\lambda,k} / 2 \right) \lambda \sin \left( \theta_{\lambda,k} / 2 \right) \right).
$$

Here $\lambda = \pm 1$ denotes the conduction and valence band respectively and $\theta_{\lambda,k} = \theta_k$ for $\lambda = 1$ and $\pi - \theta_k$ for $\lambda = -1$. Here $\theta_k$ and $\phi_k$ are simply the angles related to the point $k$ in spherical coordinates, with cot $\theta_k =$
\( k_z / \sqrt{k_x^2 + k_y^2} \) and \( \tan \phi_k = k_y / k_x \). Using Eq. (B1), the overlap function can be evaluated to be,

\[
f^{\lambda\chi}(k, k') = \left| \langle \chi\lambda(k) | \sigma_z | \chi\lambda(k') \rangle \right|^2
\]

\[
= \frac{1}{2} \left( 1 + \lambda \left( \cos \theta_k \cos \theta_{k'} - \sin \theta_k \sin \theta_{k'} \cos \phi_{kk'} \right) \right).
\]

Here \( k' = k + q \) and \( \phi_{kk'} \) is the angle between \( k' \) and \( k \).

Specifically for the longitudinal case we have \( q = q \hat{z} \).

and the longitudinal overlap function is given by,

\[
f^{\lambda\chi}_L(k, k') = \frac{1}{2} \left[ 1 + \lambda \left( \frac{k \cos 2\theta_k + q \cos \theta_k}{k'} \right) \right].
\]

For the transverse case where we have chosen \( q = q \hat{x} \), and accordingly we have

\[
f^{\lambda\chi}_T(k, k') = \frac{1}{2} \left[ 1 + \lambda \left( \frac{k \cos 2\theta_k - q \sin \theta_k \cos \phi_k}{k'} \right) \right].
\]

Appendix C: The transverse spin-spin response function

The transverse spin-spin response function can expressed as a sum of the intrinsic (undoped) and extrinsic (doped) part:

\[
\Pi^{(0)}_{\sigma_z\sigma_z}(q\hat{x}, \omega) = \Pi^{(0u)}_{\sigma_z\sigma_z}(q\hat{x}, \omega) + \Pi^{(0d)}_{\sigma_z\sigma_z}(q\hat{x}, \omega).
\]

The extrinsic contribution is further decomposed into inter-band and intra-band transitions as

\[
\Pi^{(0d)}_{\sigma_z\sigma_z}(q\hat{x}, \omega) = \chi^-_{kp}(q\hat{x}, \omega) + \chi^+_{kp}(q\hat{x}, \omega).
\]

The functions, \( \chi^-_{kp}(q\hat{x}, \omega) \) and \( \chi^+_{kp}(q\hat{x}, \omega) \) are defined as

\[
\chi^-_{kp}(q\hat{x}, \omega) = -\frac{1}{L^3} \sum_{k < k_p} f_T(k, k + q) \left( \frac{1}{\hbar \omega + \varepsilon_k - \varepsilon_{k+q} + i\epsilon} - \frac{1}{\hbar \omega + \varepsilon_k + \varepsilon_{k+q} + i\epsilon} \right),
\]

\[
\chi^+_{kp}(q\hat{x}, \omega) = \frac{1}{L^3} \sum_{k < k_p} f_T(k, k + q) \left( \frac{1}{\hbar \omega + \varepsilon_k - \varepsilon_{k+q} + i\epsilon} - \frac{1}{\hbar \omega - \varepsilon_k + \varepsilon_{k+q} + i\epsilon} \right),
\]

where \( \varepsilon_k = \hbar v_F |k| \) and \( f_T(k, k + q) \) is the band overlap function of normalized eigen spinors. In terms of these \( \Pi^{(0u)}_{\sigma_z\sigma_z}(q\hat{x}, \omega) = -\chi^-_{\max}(q\hat{x}, \omega) \). Defining \( \varepsilon_{k+q}/(\hbar v_F) = k' \), and \( \omega' = (\omega + i\epsilon)/v_F \), the overlap function is given by

\[
f^{\pm}_{T}(k, k + q) \propto \frac{1}{4k'kq^2} \left[ (k' + k)^2 - q^2 \right] \left[ q^2 + (k' \pm k)^2 \right] \cos^2 \phi.
\]

Converting the sum into integrals, Eqs. (C3)-(C4), reduce to the following:

\[
\chi^-_{kp}(q\hat{x}, \omega) = -\frac{1}{32\pi^2 \hbar v_F q^3} \int_0^{k_p} \frac{dk}{l_2} \int_0^{l_2} \frac{dk'}{(k' + k)^2 - q^2} [(k' + k)^2 - (k' - k)^2] \left( \frac{1}{\omega' - k - k'} \right) + \omega' \rightarrow -\omega',
\]

\[
\chi^+_{kp}(q\hat{x}, \omega) = \frac{1}{32\pi^2 \hbar v_F q^3} \int_0^{k_p} \frac{dk}{l_2} \int_0^{l_2} \frac{dk'}{(k' - k)^2 - q^2} [(k' + k)^2 - (k' - k)^2] \left( \frac{1}{\omega' + k - k'} \right) + \omega' \rightarrow -\omega',
\]

where \( l_1 = |k - q| \) and \( l_2 = |k + q| \). Focussing on only \( \omega' > 0 \) case, we first evaluate the intrinsic contribution \( \Pi^{(0u)}_{\sigma_z\sigma_z}(q\hat{x}, \omega) \). Using the Sokhotski Plemelj theorem: \( 1/(x \pm i\epsilon) = \mathbb{P}(1/x) \mp i\pi \delta(x) \), with \( \mathbb{P} \) denoting the principal value of the integral, we obtain

\[
\text{Im} \, \Pi^{(0u)}_{\sigma_z\sigma_z}(q\hat{x}, \omega) = -\frac{1}{32\pi^2 \hbar v_F q^3} \int_0^{\omega_{\max}} \frac{dk}{l_1} \int_0^{l_1} \frac{dk'}{(k' + k)^2 - q^2} [(k' + k)^2 - (k' - k)^2] \delta(\omega - k - k'),
\]

\[
\text{Re} \, \Pi^{(0u)}_{\sigma_z\sigma_z}(q\hat{x}, \omega) = \frac{1}{32\pi^2 \hbar v_F q^3} \mathbb{P} \int_0^{\omega_{\max}} \frac{dk}{l_1} \int_0^{l_1} \frac{dk'}{(k' + k)^2 - q^2} [(k' + k)^2 - (k' - k)^2] \left( \frac{1}{\omega - k - k'} - \frac{1}{\omega + k + k'} \right).
\]
Evaluating the integrals in Eq. (C8)-(C9) yields,

\[ \text{Im } \Pi^{(0n)}_{\sigma\sigma'}(q\mathbf{x}, \omega) = -\frac{\omega^2 - v_F^2 q^2}{24 \pi \hbar v_F} \Theta(\omega - q) \]  
\[ \text{Re } \Pi^{(0n)}_{\sigma\sigma'}(q\mathbf{x}, \omega) = -\frac{\omega^2 - v_F^2 q^2}{24 \pi \hbar v_F} \log \left( \frac{4 \pi^2 q^2_{\text{max}}}{|v_F^2 q^2 - \omega^2|} \right) - \frac{q^2_{\text{max}}}{6 \pi^2 \hbar v_F} \]  

(C10)  
(C11)

Following the similar procedure one can calculate the imaginary and real component of the doped transverse spin-spin response function, by integrating Eq. (C6) and (C7).

**Appendix D: Integrals \( I_p^{(1)}(\omega) \) and \( I_p^{(2)}(q) \)**

The expression in Eq. (29) for real component of doped transverse spin-spin response function involves the integrals defined as

\[ I_p^{(1)}(k_F, \tilde{\omega}) = \int_0^{k_F^2} dk \int_{l_2}^{l_1} \frac{dk^\prime}{pk + k^\prime + \tilde{\omega}}, \]  
\[ I_p^{(2)}(k_F, q) = \int_0^{k_F^2} dk \int_{l_1}^{l_2} dk^\prime (k^\prime - pk)(k^\prime + pk)^2 + q^2 \]  

(D1)  
(D2)

where \( l_1 = |k - q|, l_2 = |k + q| \) and we have defined \( \tilde{\omega} = \omega/v_F \). Doing the integral over \( k^\prime \) and \( k \) we obtain

\[ I_p^{(1)}(k_F, \tilde{\omega}) = \left[ \eta(k_F, q, p - 1, q + \tilde{\omega}) - \eta(k_F, q, p + 1, -q + \tilde{\omega}) \right] \Theta(k_F - q) \]  
\[ + \left[ \eta(k_F, 0, p + 1, q + \tilde{\omega}) - \eta(k_F, 0, p - 1, q + \tilde{\omega}) \right]. \]  

(D3)

In Eq. (D3), we have defined the function

\[ \eta(a, b, c, d) \equiv \int_b^a \log|ck + d| \, dk = \frac{1}{c} [(b - a)c + (ac + d) \log(|ac + d|) - (bc + d) \log(|bc + d|)]. \]  

(D4)

For the other integral, performing the integration over \( k^\prime \) and \( k \) in Eq. (D2), gives

\[ I_p^{(2)}(k_F, q) = \left( 2k_F^2 q^3 - \frac{4}{15} k_F^5 p \right) + \frac{2p}{15} \left[ 2k_F^5 - 5k_F^2 q^3 + 3q^5 \right] \Theta(k_F - q). \]  

(D5)

Note that in the final expression for the real part of the extrinsic transverse function in Eq. (29), all the terms involving the \( \Theta(x) \) terms will cancel each other.

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**Appendix E: Kramers-Kronig relations and the current-current response function**

To validate the correctness of our current-current response function, we check that they satisfy the Kramers-Kronig relations. A response function \( f(q, \omega) \), which is analytic in the upper half complex plane and which vanishes in the limit of complex \( \omega \to \infty \), satisfies the standard Kramers-Kronig (KK) relation given by,

\[ \text{Re } f(q, \omega) = \frac{1}{\pi} \text{P} \int_{-\infty}^{\infty} d\xi \frac{\text{Im } f(q, \xi)}{\xi - \omega}. \]  

(E1)

However in general \( f(\omega) \) is of the form \( \mathcal{A}(q, \omega)^{n-1} \), with \( n \) being a positive integer and \( \mathcal{A}(q, \omega \to \infty) \) is finite along with \( \mathcal{A}(q, \omega \to \infty) \). Since the response function \( f(q, \omega) \) diverges as \( \omega^{n-1} \) in the \( \omega \to \infty \) limit, the function \( f(q, \omega)/\omega^n \) is used in the Cauchy relation to construct the generalized Kramers-Kronig relation of order \( n \) (KK\( _n \)) with one subtraction\textsuperscript{63}. The generalized KK\( _n \) can be obtained as follows:

\[ \frac{f(\omega)}{\omega^n} = \lim_{\epsilon \to 0} \frac{f(\omega + i\epsilon)}{\omega^n} = \frac{1}{2\pi i} \int \frac{f(\xi)}{\xi^n(\xi - \omega)}. \]  

(E2)

Here the contour in the integral is the standard contour in the upper half plane – parallel to the real axis (just above it) and closed around \( \xi \to \infty \) in a semicircle. Since \( f(\xi)/\xi^n \) vanishes for \( \xi \to \infty \), the r.h.s. of Eq. (E2) has three contributions: 1) half of the residue of \( f(\xi)/\xi^n \) at the point \( \xi \to \omega \) and 2) the principle value of the integral along the real line and 3) the contribution from the \( n \)’th
order pole at \( \xi \to 0 \). Combining these three, we obtain the generalized Kramers Kronig relation to be,

\[
\frac{f(\omega)}{\omega^n} = \frac{A(q,0)}{\omega} + \frac{1}{i\pi} P \int_{-\infty}^{\infty} d\xi \frac{f(\xi)}{\xi^n(\xi-\omega)} .
\]  

(E3)

The real part of Eq. (E3) is given by

\[
\frac{\text{Re}\ f(\omega)}{\omega^n} = \frac{\text{Re} \ A(q,0)}{\omega} + \frac{1}{i\pi} P \int_{-\infty}^{\infty} d\xi \frac{\text{Im} \ f(\xi)}{\xi^n(\xi-\omega)} .
\]  

(E4)

where \( A(q,0) = \lim_{\xi \to 0} [f(q,\xi)/\xi^{n-1}] \) is the residue of the function \( f(z)/z^{n-1} \) at \( z = 0 \). In case the response function is a sum of parts, each part of which has a different power law dependence as \( \omega \to \infty \), the different terms have to be considered separately using different KK

Note that there is an alternate way to construct the generalized Cauchy relations. For \( f(\omega) \) does not diverge more than \( \omega^{n-1} \) as \( \omega \to \infty \), instead of using the theory of the divergence by \( \omega^n \) in \( f(\omega) \) to cure the divergence at infinity, we can also use the product \( f(\omega) \prod_{m=1}^{n} \frac{1}{\omega - \omega_m} \) to cure the divergence. In this way an alternate generalized Kramers Kronig relation (of order \( n \)) can be obtained and it is given by

\[
f(\omega) \prod_{m=1}^{n} \frac{1}{\omega - \omega_m} = \sum_{l=1}^{n} \frac{f(\omega_l)}{\omega - \omega_l} \prod_{m=1, m \neq l}^{n} \frac{1}{\omega_l - \omega_m} + \frac{1}{i\pi} P \int_{-\infty}^{\infty} d\xi \frac{f(\xi)}{g(\xi)} + C_{\infty} .
\]  

(E5)

Where \( g(\xi) = (\xi - \omega) \prod_{m=1}^{n} (\xi - \omega_m) \) and the contribution from the semicircle at infinity vanishes, i.e. \( C_{\infty} \to 0 \). Here the choice of \( \omega_m \) is arbitrary, and Eq. (E5) is independent of the choice of \( \omega_m \).

It turns out that for a particular choice of \( \omega_m \) Eq. (E5) reduces to Eq. (E4), which is in general simpler to use. For even \( n \), we choose \( \omega_m = \epsilon \times \{ -n/2, -(n-1)/2, \ldots, -(n-1)/2 \}, \) and for odd \( n \) we choose \( \omega_m = \epsilon \times \{ -(n-1)/2, \ldots, -1, 0, 1, (n-1)/2 \} \). Using this choice in Eq. (E5), and then taking the limiting case of \( \epsilon \to 0 \) it is easy to see that the l.h.s. and the second term on the r.h.s. of Eq. (E5) reduce to corresponding l.h.s. and the second term on the r.h.s. of Eq. (E4). The first term on the r.h.s. of Eq. (E5) reduces to,

\[
\lim_{\epsilon \to 0} \sum_{p=0}^{k-1} \frac{(-\epsilon)^p(\epsilon - k - p)^k}{p!} \left[ \frac{1}{\epsilon(\epsilon - k - p)} \right]^{n-1} \int_{-\infty}^{\infty} d\xi \frac{f(\epsilon(\epsilon - k - p))}{\xi^n(\xi - \omega)} + \frac{f(-\epsilon(\epsilon - k - p))}{(-\epsilon(\epsilon - k - p))^{n-1} \xi^n(\xi - \omega)} = \frac{A(q,0)}{\omega}.
\]  

(E6)

where \( k = n/2 \) and \( (n-1)/2 \) for even and odd \( n \) respectively. In the limiting case of \( \epsilon \to 0 \), we have for all \( m \),

\[
\lim_{\epsilon \to 0} \frac{f(m \epsilon)}{(m \epsilon)^{n-1} \xi^n(\xi - m \epsilon)} = \frac{A(0)}{\omega} .
\]  

(E7)

In addition the first term in Eq. (E7) is simply given by

\[
\sum_{p=0}^{k-1} \frac{(-\epsilon)^p(\epsilon - k - p)^k}{p!} \left[ \frac{1}{\epsilon(\epsilon - k - p)} \right]^{n-1} = \frac{1}{2} .
\]  

(E8)

Substituting Eqs. (E7)-(E8), in Eq. (E6) shows that the first term in the r.h.s. of Eq. (E5) is identical to the first term on the r.h.s. in Eq. (E3).

Let us now consider the intrinsic response first. The imaginary part of the density-density response function is constant at \( \omega \to \infty \). So it satisfies KK1 as shown explicitly in Ref. [27]. For the longitudinal current-current response function the imaginary part diverges as \( \omega^2 \). So it satisfies KK3, or more explicitly,

\[
\text{Re} \ \Pi_{\sigma z,\sigma z}(q, \omega) = \omega^2 \lim_{\xi \to 0} \frac{\text{Re} \ \Pi_{\sigma z,\sigma z}(q, \xi)}{\xi^2} + \frac{\omega^3}{\pi} \int_{-\infty}^{\infty} d\xi \frac{\text{Im} \ \Pi_{\sigma z,\sigma z}(q, \xi)}{\xi^3(\xi - \omega)} .
\]  

(E9)

The first term on the right hand side of Eq. (E9), can be evaluated by using Eq. (9) and (10), and it is given by,

\[
\omega^2 \lim_{\xi \to 0} \frac{\text{Re} \ \Pi_{\sigma z,\sigma z}(q, \xi)}{\xi^2} = -\frac{\omega^2}{24\pi^2 \hbar v_F} \log \frac{4m_{\text{max}}}{q^2} .
\]  

(E10)

To evaluate the second term in Eq. (E9) we use Eq. (7) and Eq. (8) to obtain,

\[
\frac{\omega^3}{\pi} \int_{-\infty}^{\infty} d\xi \frac{\text{Im} \ \Pi_{\sigma z,\sigma z}(q, \xi)}{\xi^3(\xi - \omega)} = -\frac{\omega^2}{24\pi^2 \hbar v_F} \log \frac{v_F q^2}{\omega^2} .
\]  

(E11)

Combining Eqs. (E10)-(E11) reproduces the second term in Eq. (9). Note that the first term in Eq. (9), which arises from the anomalous commutator, is real and independent of \( \omega \). Thus it trivially satisfies KK1 with a vanishing imaginary part.

The imaginary part of the intrinsic transverse current-current response function can be split into two parts. First part diverges as \( \omega^2 \) while the other one is finite for \( \omega \to \infty \). Thus the first part obeys KK3 and the second part follows KK1. Following the procedure outlined for the longitudinal case, we have explicitly checked analytically that the real part of the intrinsic transverse current current response function can be obtained from the corresponding imaginary parts.

The extrinsic part of both the transverse and the longitudinal response function has no divergence problem and both vanish in the \( \omega \to \infty \) limit. Thus the extrinsic part satisfies Eq. (E4), or alternatively the modified version of the standard Kramers-Kronig relation given by,

\[
\text{Re} \ \Pi_{\sigma z,\sigma z}(q, \omega) = \frac{2}{\pi} \int_{0}^{\infty} d\xi \text{Im} \ \Pi_{\sigma z,\sigma z}(q, \xi) .
\]  

(E12)

We have checked numerically that the real part of the response function obtained using Eq. (E12), are identical to the analytical results for the real part of the longitudinal and transverse response function in Eq. (15) and Eq. (29), respectively.
Appendix F: QED and charge renormalization

The QED Lagrangian with arbitrary scaling factors for the vector field \((A^\mu \rightarrow \sqrt{Z_3} A^\mu)\) and the spinor wavefunction \((\psi \rightarrow \sqrt{Z_2} \psi_r)\) is given by

\[
L = -\frac{1}{4} Z_3 F^\mu_{\nu} F^{\nu}_{\mu} + i Z_2 \bar{\psi}_r \gamma^\mu \partial_\mu \psi_r + \sqrt{Z_3} Z_2 e \bar{\psi}_r \gamma_\mu A^\mu \psi_r ,
\]

where \(e\) denotes the bare charge. Here \(Z_3, Z_2\) are determined by renormalization to cancel the anomalous diverging terms in the loop integrals. Here the last term can be expressed in terms of renormalized charge by defining \(e_r Z_1 = e \sqrt{Z_3} Z_2\). Now the Ward-Takahashi identity implies that \(Z_1 = Z_2\). This also allows the last two terms of Eq. (F1) to be expressed as a covariant derivative:

\[
i Z_2 \bar{\psi}_r \gamma^\mu \partial_\mu \psi_r + \sqrt{Z_3} Z_2 e \bar{\psi}_r \gamma_\mu A^\mu \psi_r = i Z_2 \bar{\psi}_r \gamma^\mu (\partial_\mu - e_r A^\mu) \psi_r .
\]

The single loop polarization diagram or the self energy correction to the photon propagator [intrinsic current-current correlator in Eq. (9)] has an anomalous diverging term \(-i \sqrt{\pi} e^2 v F^2_{\mu \nu} \delta_{\mu \nu} / (6\pi^2 \hbar)\). This implies that the appropriate counter terms to cancel it, which renormalizes the vector field is given by

\[
\sqrt{Z_3} = \left( 1 - \frac{e^2 v_F q_{\text{max}}^2}{6 \pi^2 \hbar} \right)^{1/2} .
\]

Consequently the bare and the renormalized charges are related by

\[
e = e_r \times \left( 1 - \frac{e^2 v_F q_{\text{max}}^2}{6 \pi^2 \hbar} \right)^{-1/2} .
\]

The renormalized charge \(e_r\) is the scale dependent effective screened charge which is observed in experiments.

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