Peculiar Fields in Maxwell’s Equations.

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Abstract—A theoretical analysis of the excitation of an infinitely long solenoid by oscillating current has revealed the existence of specific potentials in the space outside the solenoid, which can affect electron diffraction in an experiment similar to the Aharonov-Bohm effect. Thus, these time-dependent potentials are physical fields possessing a number of specific features, which set them off from the fields known heretofore.

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The peculiar phenomenon, predicted in 1939 and 1949 [1, 2] and rediscovered and studied theoretically in considerable detail in 1959 [3], was subsequently called the Aharonov-Bohm (AB) effect. It consists essentially in that in propagating through a region with no magnetic or electric field present, but where the vector or scalar potential is nonzero, the de Broglie wave corresponding to a quantum charged particle is acted upon by the latter. These conditions are best realized in a static regime, which was exactly the case studied before the 1990s. While a long discussion has certainly contributed to a proper understanding of the AB effect (see, e.g., reviews [4]), heated debates on this issue are still continuing in the literature. Most clear and sequentially theory of effect AB enunciated in paper of prof. D. H. Kobe [5].

Based on the totality of the experiments performed, one has to admit that the AB effect can exist only if there are potentials, which do not generate fields and cannot be removed by gauge transformation. We have termed them “zero-field potentials”. Note that zero-field potentials, which transform only
the phase of a wave function, are responsible for the AB effect in all the papers published heretofore and dealing with the static case. In a general case, such potentials satisfy the relations

\[-c^{-1}\partial A^0/\partial t - \text{grad} \varphi^0 = 0 \text{ and } \text{rot} A^0 = 0, \]  

(1)

where the upper indices of the potentials refer to the zero-field potentials. Because such potentials should obviously have the form

\[ A^0 = \text{grad} \chi \quad \varphi^0 = -\frac{1}{c} \frac{\partial \chi}{\partial t}, \]  

(1)

the \( \chi \) function was erroneously identified in practically all publications with the gradient potential transformation function, and this is what gives rise frequently to misunderstanding.

After the convincing experiments of Tonomura et al. [6], the possibilities of studying the static AB effect at the present level of technology were apparently exhausted, and the researchers turned their attention to the investigation of the time-dependent, or quasi-AB effect [7]. However, in this work, which has certainly produced fruitful results, the potentials responsible for the quasi-AB effect were introduced artificially, without discussing in any way their nature. Nevertheless, the origin of these potentials (fields) is a major issue in the separation of the AB effect from the general variation of the de Broglie wave-interference pattern due to Lorenz force.

We maintain that in the regions of space with no currents present the total potentials can be presented, generally speaking, in the form

\[ A = A^f + A^0 \quad \text{and} \quad \varphi = \varphi^f + \varphi^0, \]  

(3)

where index \( f \) refers to “field” potentials corresponding to nonzero electromagnetic fields:

\[ E = -c^{-1}\partial A^f/\partial t - \text{grad} \varphi^f, \quad B = \text{rot} A^f. \]  

(2)

Index 0 in Eq. (3) identifies zero-field or excess potentials defined by relations (1). Note that the “excess” potentials have been long in use in mathematical physics [8]; they are necessary when solving Maxwell’s equations with boundary conditions.

We are going to demonstrate the above in a specific example. Consider circular currents flowing in a region of space to form an infinitely long cylinder of radius \( R \) (a solenoid with circular currents). Choose a cylindrical reference frame \((\rho, \alpha, z)\) with the axis \( z \) coinciding with the solenoid axis. In the magnetostatic case, the solution within the infinite solenoid \((0 \leq \rho < R)\)
can be chosen in the form \( A_{1\alpha} = c_1 \rho \) (\( A_1 = A_{1\alpha} \), \( A_1^0 = 0 \)). In the outer region (\( \rho > R \)), the solution has the form \( A_{2\alpha} = c_2 / \rho + c_3 \rho \). The system being infinite, one cannot require the potential to vanish at large distances. As is clear from purely physical considerations, the magnetic field outside the solenoid is zero, i.e., \( A_2^I = 0 \). Therefore, the only potential that can exist in the outer region is \( A^0 \), which satisfies the additional condition \( \text{rot} A^0 = 0 \), and it is this condition that identifies the correct solution \( A_{2\alpha} = c_2 / \rho \).

The potential in the outer region is essentially the zero-field potential, so that \( A_2 = \nabla \chi \), but because this region is doubly connected, the \( \chi \) function is multivalued, and \( \oint_L A_2 \, dl \neq 0 \), despite the fact that in this region (\( R < \rho < \infty \)) \( \text{rot} A_2 \equiv 0 \).

The above separation of the potentials into the field and zero-field ones permits one to find the zero-field potentials for a time-dependent current as well. As before, we assume that circular currents flow in a region of space to form an infinitely long cylinder. The reference frame will be left unchanged.

The current can be described by the following relations

\[
\begin{align*}
  j_\alpha (\rho, \alpha, z) &= I_0 \delta (\rho - R) \exp i \omega t, \\
  j_\rho &= j_z = 0, \quad (5)
\end{align*}
\]

where \( R \) is the solenoid radius, \( \omega \) is the cyclic frequency of the current, and \( I_0 = J / 2\pi R \); here \( J \) is the current in the cylinder wall per unit length of the solenoid.

The nonzero vector-potential components \( A_\rho \) and \( A_\alpha \) can be written [9]

\[
\begin{align*}
  A_\rho &= \int_V j_\alpha (\rho') \sin (\alpha - \alpha') G (\rho, \rho') \, dV', \\
  A_\alpha &= \int_V j_\alpha (\rho') \cos (\alpha - \alpha') G (\rho, \rho') \, dV',
\end{align*}
\]

where \( G (\rho, \rho') = -\frac{i\pi}{c} H_0^{(2)} (k|\rho - \rho'|) \) is the Green function of the Helmholtz equation [9], \( H_0^{(2)} \) is the Hankel function, \( k = \omega / c \), and \( dV' = \rho' \, dp' \, d\alpha' \). Here and in what follows, the harmonic dependence on time is omitted. The integrals entering Eq. (6) can be easily calculated using the rules of the totals for the Hankel functions [9]

\[
H_0^{(2)} (k\sqrt{\rho^2 + R^2 - 2\rho R \cos (\alpha - \alpha')}) =
\]

\[
= \sum_{m=-\infty}^{\infty} e^{-im(\alpha - \alpha')} \left\{ \begin{array}{ll}
  H_m^{(2)} (kR) J_m (kp), & \rho < R \\
  J_m (kR) H_m^{(2)} (kp), & \rho > R
\end{array} \right.
\]

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As a result, we obtain
\[ A_\alpha = -\frac{2\pi^2 I_0}{c} \begin{cases} H_1^{(2)}(kR) J_1(k\rho), & \rho < R \\ J_1(kR) H_1^{(2)}(k\rho), & \rho > R \end{cases} \]
and \( A_\rho = 0 \). (7)

In the static case \((\omega \to 0)\), one obtains from these relations the well-known expressions
\[ A_\alpha = J_1(k\rho)/cR (\rho < R) \text{ and } A_\alpha = J_1(kR)/c\rho (\rho > R), \]
(8)

Consider in more detail the potential of Eq. (7) in the outer region, which is of major interest for us here
\[ A_\alpha = Q H_1^{(2)}(k\rho) \equiv Q[J_1(k\rho) - i Y_1(k\rho)] = \]
\[ = Q \left\{ \frac{2i}{\pi k\rho} + \left[ 1 - \frac{2IC}{\pi} - \frac{2i}{\pi} \ln \left( \frac{k\rho}{2} \right) \right] \sum_{m=0}^{\infty} \frac{(-1)^m}{m\Gamma(m+2)} \left( \frac{k\rho}{2} \right)^{2m+1} + \right. \]
\[ + \frac{i}{\pi} \sum_{m=0}^{\infty} \frac{(-1)^m}{m! (m+1)!} \left( \frac{k\rho}{2} \right)^{2m+1} \left[ \sum_{j=1}^{m} \frac{1}{j} \frac{m}{j} + \sum_{j=1}^{m+1} \frac{1}{j} \right] \}
(9)

where \( Q = -\frac{2\pi^2 I_0}{c} J_1(kR) \), \( C \) is Euler’s constant, and \( Y_1 \) is the Neumann function.

As seen from Eq. (9), the curl of the first term in braces is zero. One can readily verify that the curls of the other terms in the braces are nonzero. Thus, in this case the total potential can be separated into the field and the zero-field potential. As follows from Eq. (1)
\[ \varphi^0 = \begin{cases} 0, & \rho < R \\ -\frac{4\pi I_0 R}{c} J_1(kR) \alpha, & \rho > R \end{cases} \]
(5)

Separation of the real part of the components of the potentials in Eq. (9) yields [10]
\[ \text{Re} A_f^\alpha = W \{ \pi J_1(k\rho) \sin \omega t - \left[ \frac{2}{k\rho} + \pi Y_1(k\rho) \right] \cos \omega t \}, \]
(11a)
\[ \text{Re} A_0^\alpha = W \frac{2}{k\rho} \cos \omega t, \]
(11b)
where \( W = \frac{2\pi I_0 R J_1(kR)}{c} \).

Consider now the geometry of the Aharonov-Bohm experiment, in which electrons move around a solenoid along a circle of a given radius. We shall limit ourselves to the case where the electrons meet on their way nonzero zero-field potentials, while field potentials are not present. This situation can be realized by enclosing the solenoid in cylindrical screens, or, as follows from Eq. (11a), by choosing the trajectory radii of the electrons and by matching properly their transit with the current variation in the solenoid. Substituting now the zero-field potentials in the Schrödinger equation and
using the procedure of the solution proposed in (Appendices B and D in [7])
but, in contrast to [7], performing time averaging, we come to the following
expression for the intensity of the interference pattern [11]

$$
\mathcal{P} = 0.5 \, P_0 \, \{1 + J_0(S) \cdot \cos(\omega e \tau)\},
$$

(6)

where $S = 16 \pi^3 I_0 R \mu_0^{-1} \omega^{-1} J_1(kR)$, $\mu_0 = \frac{\hbar}{|e|}$ and $J_0$ and $J_1$ are the
Bessel functions. For $I_0 = 158 \, mA/cm$ ; $R = 5 \, \mu m$ ; $\omega/2\pi < 10^{10} Hz$,
we obtain $S = 2.45$. This means that the interference pattern should vanish
for these parameters. To verify experimentally this conclusion, one should
use preferably electrons in metallic mesoscopic rings or cylinders [4]

Thus, we believe that the Aharonov-Bohm experiment in both the static
and the time-dependent case is actually an experiment on detection of a
field of a new type in classical electrodynamics. This field has none of the
characteristics inherent in the classical electromagnetic fields, namely, the
energy, the momentum, and the angular momentum. Therefore, these fields
have a high penetration capacity and can be used for information transfer,
with its detection by the AB effect.

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