A Graph Model for the quantum mechanics of a moving cyclic disturbance interacting at a spatial position

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Abstract

A statistical approach based on a directed cyclic graph, is used to calculate the alternative positions in space and state of a moving disturbance for a given observed time. The probability for a freely moving entity interacting in a particular spatial position is calculated and a formulation derived for the minimum locus of uncertainty in position and momentum. This accords with calculations for quantum mechanics. The model has proven amenable to computer modelling.

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1 Introduction

An earlier paper\textsuperscript{2} detailed interrelated connections between Space, State, alpha-time and beta-Time using a directed cyclic graph. This lead to a potential locus in Space for a defined Time Magnitude (comprising both alpha-time(rst') and beta-time(t*)) where:

\[ |T| = \sqrt{(nt*)^2 + (npst' + rst')^2} \]  \hspace{1cm} (1)

The occurrence - at the State→Space trigger (ph) - of the bifurcation of identity to both a change in beta-time(t*) at an adjacent Space position, and the change in alpha-time associated with its change in State was shown to result in a fundamental ambiguity for a given magnitude of time: where an entity is located in Space(nd) and what its State(rh) is. Since n and r are variables, a range of alternative combinations of State and Space positions can combine to form the same total time magnitude |T| from variable components of alpha-time and beta-time. This can be represented for a fixed |T| of magnitude |rst'| - assuming a null Space trigger point (ie. a photon) - as a “temporal arc” (see diagram 1 below):

DIAGRAM 1 - temporal arc for a photon at a time magnitude |rst'|

All points on the temporal arc have same time magnitude |T|.

It was noted that we can represent time: (\(\alpha\)Time,\(\beta\)Time) as a complex vector. We use a notation of beta-time(t*) as real and alpha-Time(urst') as imaginary:

\[ T = nt* + i(np + r)st' \]  \hspace{1cm} (2)

or where \( z = (p + r/n)s \):

\[ T = n(t* + izt') \]  \hspace{1cm} (3)

\textsuperscript{2}see Brown (2003)
2 The probability for a freely moving entity interacting in a particular spatial position

For a defined time magnitude $|T|$ a range of possible combinations of State and Space positions exist along its arc. When the time magnitude measured is very small then specific state and spatial positions cannot accurately be divined. A very small $|T|$ is quite likely to be formed entirely from alpha-time changes or entirely from beta-time changes.

Since alternative possible compositions of beta-time$(nt^*)$ and alpha-time$(rst')$ exist for a given time magnitude $|T|$ then only a probabilistic method can reference the position in space and state of the IFE disturbance.

Calculation of $P(x)$ the probability of the IFE disturbance being located (through an interaction) at a specific spatial position is somewhat more intricate than might at first be expected.

There are two core issues to consider in the calculation:

1. the probability of an interaction occurring at a specific spatial position and specific alpha-time.

2. The probability of non interaction up to the point at which interaction occurs.

We can therefore represent the total probability for an IFE being found to be located in a specific position for a specific time as: $P_T = P_L P_N$ where $P_L$ is the probability of the interaction occurring in a particular alpha-time and beta-time and $P_N$ is the probability that there has not been an interaction up to that point.

To cover the first issue, assume that the probability of an interaction in a specific given spatial position and alpha-time is given by $Adx$ (where $A$ may be dependent on $x$ and $t$).

A toy model where $t^* = t' = dx = 1$ illustrates the probability of the IFE being found to interact with another IFE at a given spatial position (e.g. $x=1$) with the time magnitude undefined.

At time $|t| = 0$, $P_L(x = 1) = 0$

At time $t=1$ $P_T(x = 1) = \frac{1}{2}AP_N$ where $P_N(x, t)$ is the probability that an interaction has NOT occurred up to time $|t| = \sqrt{(nt^*)^2 + (rst')^2}$ (for $x=1$, $t=1$, $P_N = 1$). This assumes that there is an equal likelihood of the IFE taking a path in alpha-time or beta-time.
At time \(|t| = 2\) \(P_T(x = 1) = \frac{1}{4}AP_N\)

Different probabilities exist for the IFE being located at spatial position \(x=1\) for all possible time magnitude values. If the time magnitude is undefined, then all these probabilities have to be summed to establish the total probability of \(x = 1\).

To calculate \(P_L(x, t)\), we consider the end result of the movement to have been a random walk across a 2 dimensional time plane. The probability for a spatial position \(x\) with undefined time magnitude is the sum of a set of probabilities for all possible alpha times in that spatial position for all possible time magnitudes. Thus the probability that the IFE is located in spatial position \(1\) for a time magnitude of \(1\) is equivalent to the probability that the alpha time is \(0\) for a time magnitude of \(1\). The probability that the IFE is located in spatial position \(1\) for a time magnitude of \(2\) is equivalent to the probability that the alpha time is \(1\) for a time magnitude of \(2\) etc...

The alpha-time can be considered to reach a value \(rst'\) through a sequence of increments each \(r_i\). Thus \(rst' = \sum_{i=1}^{R} r_i st'\)

The probability of arriving at an alpha-time \(rst'\) in a particular sequence is \((P_1 dr_1)(P_2 dr_2)(P_3 dr_3)\ldots(P_R dr_R)\) where \(P_i\) is the probability density for a particular increment \(r_i\) in alpha time.

The probability of arriving at \(rst'\) in any sequence requires that all possible sequences are considered.

\[ P_L(rst') = \int \ldots \int (P_1 dr_1)(P_2 dr_2)(P_3 dr_3)\ldots(P_R dr_R) \] with the restriction that \(r < \sum_{i=1}^{R} r_i < (r + dr)\)

This restriction can neatly be encapsulated using the Dirac delta:

\[ P(rst') = \int \ldots \int (P_1 dr_1)(P_2 dr_2)(P_3 dr_3)\ldots(P_R dr_R) \delta(\sum_{i=1}^{R} r_i - r) \]

i.e. \[ P(rst') = \int \ldots \int (P_1 dr_1)(P_2 dr_2)(P_3 dr_3)\ldots(P_R dr_R) \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{iqr(\sum_{i=1}^{R} r_i - r)} dq \]

\[ = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iqr} dq \int_{-\infty}^{\infty} (P_1 e^{iqr_1} dr_1) \int_{-\infty}^{\infty} (P_2 e^{iqr_2} dr_2) \int_{-\infty}^{\infty} (P_3 e^{iqr_3} dr_3) \ldots \int_{-\infty}^{\infty} (P_R e^{iqr_R} dr_R) \]

i.e. \[ P_L(rst') dr = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iqr} dq \prod_{i=1}^{R} \int_{-\infty}^{\infty} (P_i dr_i) e^{-iqr_i} dr_i \]

\[ = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iqr} dq \int_{-\infty}^{\infty} (P_c dr_c) e^{-iqr_c} dr_c \] if all increments assumed the same.

Expanding \(e^{iqr_c}\) takes Taylor’s series gives:

\[ \int_{-\infty}^{\infty} (P_c dr_c) e^{-iq r c} dr_c = \int_{-\infty}^{\infty} P_c (1 + iqr_c - \frac{1}{2} q^2 r_c^2 + \ldots) dr_c = 1 + i\langle r_c \rangle q - \frac{1}{2} \langle r_c^2 \rangle q^2 \ldots \]

where \(\langle r_c^R \rangle = \int_{-\infty}^{\infty} dr_c P_c r_c^R\) is a constant for the Rth moment of \(r_c\).

Then \(ln[\int_{-\infty}^{\infty} (P_c dr_c) e^{-iq r c} dr_c]^R = R ln[1 + i\langle r_c \rangle q - \frac{1}{2} \langle r_c^2 \rangle q^2 \ldots] \)

Using Taylor’s series for \(y \\ll 1\):

\[ \ln(1 + y) = y - \frac{1}{2} y^2 \ldots \]

\[ \ln[\int_{-\infty}^{\infty} (P_c dr_c) e^{-iq r c} dr_c]^R = R[i\langle r_c \rangle q - \frac{1}{2} \langle r_c^2 \rangle q^2 - \frac{1}{2} (i\langle r_c \rangle q)^2 \ldots] \]
= R[t⟨rc⟩ − 1/2⟨rc⟩2 − ⟨rc⟩2]q...
= R[t⟨rc⟩ − 1/2(Δrc)2q2...] where (Δrc)2 = ⟨rc⟩2 − ⟨rc⟩2
Thus ln[∫−∞∞(Pcdr)c] = e−q⟨rc⟩q−1/2R(Δrc)2q2
And PL(rst′) = 1/2π∫−∞∞e−r0rσkr σk 2 e−ikx dx (4)
From Integral tables PL(r) = 1/2πσk e−(r−r0)2/2σ2
where r0 = R⟨rc⟩ and σr = R(Δrc)2 and ∫T = 1/(rst′) = k dx
Then e−(r−r0)2/2σ2 = e−([T]k dx−R(r0))2/2R(⟨rc⟩2−⟨rc⟩2) = e−(k−k0)2/2σ2

This allows us to define a function in k which we can label the ‘probability function’ that associates a probability with a complex vector:

We then sum all values of PL(rst′) for all values of t, associated with values of the vector position in time which we can define as a ‘probability function’...

\[ \psi(x) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi} \sigma_k} e^{-\frac{(k-k_0)^2}{2\sigma_k^2}} e^{-ikx} dk \] (4)

We now consider the probability of NOT having an interaction at any of the previous times (r − 1)st′, (r − 2)st′, ...

For an IFE disturbance starting from an initial time magnitude |T| = 0, to calculate the probability of an interaction at spatial position x each of the temporally precedent spatial positions where an interaction did NOT occur: NOT(n − 1)dx, NOT(n − 2)dx... must be considered, where there could have been but was no interaction.

For a probability of interaction in space that was identically and uniformly distributed in a one dimensional line this would be straightforward: the n possible positions could be examined - each separated by a very small distance dx prior to the interaction at x.

The probability of interaction in a very short space dx can be defined as (Bdx) where B is the probability density (of an interaction with another IFE disturbance).

So the probability of non-occurrence in a very short space is (1 − Bdx)
If a distance x = ndx is travelled before an interaction then where PN(x) is the probability for no interaction up to x: PN(x) = (1 − Bdx)n
i.e. PN(x) = (1 − Bdx)n = (1 − Bσ/n)n
For a large x then n = x/dx → ∞. i.e. it might at first be expected:
\[ P_N(x) = e^{-Bx} \]  

However, B, the probability density of an interaction in each short spatial position varies according to the number of alternative State positions at each possible Space position x. The range of possible State positions itself will vary at different spatial positions.

Assume that for each occasion that the IFE disturbance moves from one State position to another or from one spatial position to another there is a uniform (arbitrary) probability A of interaction with another (group of) IFE disturbance (that depends on the state of the other IFE disturbance).

To calculate the probability for an interaction at a specific State position (rh) at a spatial position x all of the probabilities for each possible State position at x can be summed (see Diagram 2 below).

**Diagram 2:** possible State positions for interaction at spatial position x

\[ r_x h \]

\[
\begin{array}{c}
\ast \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
x \end{array}
\]

For an interaction to occur at State position \((r_x h)\) at a given spatial position x, there must have been no interactions at each of the previous possible and temporally precedent State points \((r_x - 1)h, (r_x - 2)h\ldots\) etc.

To calculate the probability of an interaction at a particular State position a similar method can be used to that initially assumed for spatial position.

Define probability of an interaction at a State position= A

\[ P_N(rst') = (1 - Adst')^r = (1 - A_{\text{dist}})^r = e^{-A_{\text{dist}}} \]

The probability of NOT having an interaction up to time \(|rst|\) is:

\[ P_N(rst'/x = 0) = e^{-A_{\text{dist}}} \]  

(6)

We can calibrate between A and B: \( P_N(rst') = e^{-Brst'dx} \)
It is straightforward to calculate mean and variance using this\(^3\).
However, further alternative possible spatial positions such as at \(x = (n-1)d, (n-2)d...\) etc must be covered.

The total time magnitude \(|T|\) can be composed in more than one way (through variations in State and Space positions). Therefore for a particular State position (rh) not only non-occurrences at (r-1)h, (r-2)h..., but also for each of these State positions, the non-occurrences at all the coterminous spatial positions which provide the same time magnitude \(|T| = |rst'|\) must be considered.

To establish \(P(|T|)\), the probability of an interaction in a time \(|T|\), all the ways in which \(|T|\) can be formed from the combination of the first spatial position, the second spatial position etc...must be calculated.

For more than one possible Space positions, all possible State positions also must be accounted for at the second Space position which in combination with the beta-time (caused by the movement in spatial position) can comprise the same time magnitude equal to \(|rst'|\) in the first Space position.

These possible combinations of certain State position (rh) and specific Space position \(x (=nd)\) potentially exist only for those combinations which have the same time magnitude \(|T| = |rst'|\) such that \(|T| = \sqrt{(nt*)^2 + (npst' + rst')^2}\) where \(r\) is the State position that can occur at any Space position.

For each possible interaction at a Space position \(x\) and State position (rh), all possible non-interactions must be covered for State and Space positions on an associated temporal arc. To calculate possible positions are on this arc, a fundamental point lattice calculation originated by Gauss (see Appendix) can be used. This shows that \(C(|T|)\), the number of exactly permissable (integer) points on a temporal arc that can compose a time magnitude \(|T|\),

\(^3\)For simplicity let \(l = \frac{rst'}{rt}\). For a function \(B(x)\) mean \(<x> = \int_{-\infty}^{\infty} xB(x)dx\)

\[\text{i.e. mean } = l_o = <l> = \int_{-\infty}^{\infty} le^{-lt}dl = \frac{A}{A^2} = \frac{1}{A}\]

\[\text{variance } = \sigma^2 = <l^2> - (<l>)^2\]

\[<l^2> = \int_{-\infty}^{\infty} l^2e^{-lt}dl = \frac{2A}{A^3} = \frac{2}{A^2}\]

\[\text{i.e. } \sigma^2 = \frac{1}{A^2}\]
is:
\[ C(|T|) = 2\pi|T| \]  

For a particular State position \((rh)\), not only all of the potential interactions that did \textit{not} occur at alpha-times \((r−1)ist', (r−2)ist'\)...must be accounted for. Additionally all of the feasible interactions that could have, but did not occur at alternative beta-times must be included - such as \(|rst' − t*|, (|rst' − t* − 1|)... at a second spatial position - and further \(|rst' − nt*|, (|rst' − nt* − 1|)... at the nth spatial position.

\textbf{Calculation of the probability of NON-interactions requires summation of the area of the arc of every possible State position at every possible spatial position}

The mechanics for this calculation are facilitated by working backwards and investigating historically the non-occurrences of interactions for Space and State positions.

A convolution method enables aggregation of all the possible probabilities. To illustrate this, we can first calculate notionally for two spatial positions only.

We sum for every State position against all the non-events at all possible State positions.

From equation (6) and using (7) to locate the exactly permissible (integer) values only:

\[ P_N(rst') = \int_{-\infty}^{\infty} Ae^{-Ar'} . Ae^{A(r-r')} dr' \]

with the constraint that \(r'\) and \((r-r')\) are not negative - i.e. \(Ae^{-Ar'}\) and \(Ae^{A(r-r')}\) can be represented as \(H(r')\) and \(H(r-r')\) (Heaviside step functions):

\[ \int_{-\infty}^{\infty} Ae^{-Ar'} . Ae^{A(r-r')} . H(r') . H(r-r') dr' = A^2 re^{-Ar} \]

Similarly using this last result for 3 positions:

\[ \int_{-\infty}^{\infty} A^2 r e^{-Ar'} . H(r') . Ae^{A(r-r')} . H(r-r') dr' = \frac{A^3}{2} r^2 e^{-Ar} \]

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And for all possible n positions across the temporal arc, through inference:

\[ P(r) = \frac{A^n r^{n-1}}{(n-1)!} e^{-Ar} \]  

Note that (as with the earlier calculations for Mean and Variance):

\[ \text{Mean} = r_o = < r > = \int_{-\infty}^{\infty} \frac{(rA)^n}{(n-1)!} e^{-rA} dr = \frac{n}{A} \]  

similarly \( \text{Variance} = \sigma_r^2 = \frac{n}{A^2} \) (10)

let \( a = \frac{Ar}{n} \), then from (8)

\[ P(r) = \frac{A^n r^{n-1}}{(n-1)!} e^{-na} = \frac{A^n}{n!} a^{n-1} e^{-na} \]

If we replace with \( z = a - 1 \) then

\[ P(r) = \frac{A^n}{n!} (1 + z)^{n-1} e^{-n(1+z)} \]

Assuming that \( n \) is large, \( P(r) \) can be expressed more conveniently using Stirling’s factorial expansion:⁴

\[ n! = \sqrt{(2\pi)n}n^n e^{-n} \]

Then

\[ P(r) = \frac{A}{\sqrt{2\pi n}} (1 + z)^{n-1} e^{-nz} \]

But \( e^{-nz} = 1 - \frac{nz}{1!} + \frac{(nz)^2}{2!} \ldots \)

And from binomial expansion: \( (1 + z)^{n-1} = 1 + (n-1)z + \frac{(n-1)(n-2)z^2}{2} \ldots \)

Then collecting polynomials:

\[ P(r) = \frac{A}{\sqrt{2\pi n}} (1 - z - \frac{1}{2} (n-2)z^2 + \ldots) \]

Ignoring \( \frac{1}{n} \) for large \( n \) denominators and using the above series for \( e^{-nz} \):

\[ P(r) = \frac{A}{\sqrt{2\pi n}} e^{\frac{1}{2} n(z - \frac{1}{n})^2} = \frac{A}{\sqrt{2\pi n}} e^{-\frac{1}{2} n(a - \frac{1}{n})^2} \]

⁴See Jeffreys (1)
Substituting back for $a = \frac{A}{n}$

$$P(r) = \frac{A}{\sqrt{2\pi n}} e^{-\frac{1}{2} n \left(\frac{A}{n} - 1\right)^2} = \frac{A}{\sqrt{2\pi n}} e^{-\frac{1}{2} \left(\frac{Ar-(n+1)}{n}\right)^2}$$

For large $n$, $(n + 1) \sim n$ and:

$$P(r) = \frac{A}{\sqrt{2\pi n}} e^{-\frac{1}{2} \frac{(Ar-n)^2}{n}} = \frac{A}{\sqrt{2\pi n}} e^{-\frac{1}{2} \left(\frac{r-r_0}{\sigma_r}\right)^2}$$

But from (9) and (10) $r_0 = \frac{n}{A}$ and $\sigma_r^2 = \frac{n}{A^2}$

$$P_N(r) = \frac{1}{\sqrt{2\pi \sigma_r}} e^{-\frac{1}{2} \frac{(r-r_0)^2}{(\sigma_r)^2}} \quad (11)$$

This expresses the probability of a specific interaction at a specific state position but does not account for the spatial location.

As before $e^{-\frac{(r-r_0)^2}{2\sigma_r^2}} = e^{-\frac{(k-k_0)^2}{2\sigma_k^2}}$

Which allows us to define a further probability function in $k$:

$$\psi(x) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi \sigma_k}} e^{-\frac{1}{2} \frac{(k-k_0)^2}{(\sigma_k)^2}} e^{-ikx} dk \quad (12)$$

Thus from earlier $P(x) = P_L(x)P_N(x)$ and

$$P(x) = \psi(x)\psi^*(x) \quad (13)$$

The interplay between $P(k)$ and $\psi(x) = FT(P(k))$ produces a property of the differential of $P(x)$ indicated by $P'(x)$:

$$\int_{-\infty}^{\infty} P'(x)e^{-ikx}dx = e^{-ikx}P(x)|_{-\infty}^{\infty} + ik \int_{-\infty}^{\infty} P(x)e^{-ikx}dx$$

And because $P(x) = e^{-x^2}$ and $e^{-ikx}$ is an oscillating function:

$$\int_{-\infty}^{\infty} P'(x)e^{-ikx}dx = ikFT(P(k)) \quad (14)$$

Through a combination of such probability functions - say $P(k)$ and another similar probability function in $k$ $Q(k)$ - we can establish an interesting relationship between the square of their product and the product of their squares (see Rae 2002) since the integral of a magnitude must always be positive:
\[ \int_{-\infty}^{\infty} \| P(k) \{ \int_{-\infty}^{\infty} |Q(k)|^2 dk \} - Q(k) \{ \int_{-\infty}^{\infty} P(k)Q^*(k)dk \} \|^2 dk \geq 0 \]

Expanding this squared magnitude as the product of a function and its conjugate:
\[ \int_{-\infty}^{\infty} [P(k) \{ \int_{-\infty}^{\infty} |Q(k)|^2 dk \} - Q(k) \{ \int_{-\infty}^{\infty} P(k)Q^*(k)dk \}] \nonumber \]
\[ \{ [P^*(k) \{ \int_{-\infty}^{\infty} |Q(k)|^2 dk \} - Q^*(k) \{ \int_{-\infty}^{\infty} P^*(k)Q(k)dk \}] \} \nonumber \]
\[ dk \geq 0 \]

Multiplying out the square brackets obtains:
\[ \{ \int_{-\infty}^{\infty} P(k)Q(k)dk \}^2 \leq \int_{-\infty}^{\infty} |P(k)|^2 dk \int_{-\infty}^{\infty} |Q(k)|^2 dk \quad (15) \]

For the variance of \( x \) and \( k \) \( \sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 \) and \( \sigma_k^2 = \langle k^2 \rangle - \langle k \rangle^2 \) and assuming that \( \langle x \rangle = \langle k \rangle = 0 \) we form the product:
\[ \sigma_x^2 \sigma_k^2 = \int_{-\infty}^{\infty} x^2 |P(x)|^2 dx \int_{-\infty}^{\infty} k^2 |(P(k))|^2 dk \]

However, we can show that:
\[ \int_{-\infty}^{\infty} |(P(k))|^2 dk = \int_{-\infty}^{\infty} |FT(P(k))|^2 dk \]

Hence \( \sigma_x^2 \sigma_k^2 = \int_{-\infty}^{\infty} |xP(x)|^2 dx \int_{-\infty}^{\infty} |kFT(P(k))|^2 dk \)

From (15):

\[ ^{5}\text{For the case where } \langle x \rangle \neq 0 \text{ then we can perform a displacement function such that } \langle x' \rangle = 0 \text{ and it can be shown (e.g. Jeffreys 1939) that the product } \sigma_x^2 \sigma_k^2 \text{ then remains the same as for } \langle x \rangle = \langle k \rangle = 0. \]

\[ ^{6}\text{This is the “Parseval” identity:} \]
\[ \int_{-\infty}^{\infty} P(k)P^*(k)dk = \int_{-\infty}^{\infty} P(k)\{ \int_{-\infty}^{\infty} FT(P^*(k))e^{-i\omega} \}dk \]
\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(k)e^{i\omega} P^*(k)dk \]
\[ = \int_{-\infty}^{\infty} FT(P(k))FT^*(P(k)) \]
\[ = \int_{-\infty}^{\infty} |xP(x)|^2 \, dx \int_{-\infty}^{\infty} |P'(x)e^{-ikx}|^2 \, dx = \int_{-\infty}^{\infty} |xP(x)|^2 \, dx \int_{-\infty}^{\infty} |P'(x)|^2 \, dx \]

From (16) we have \( \{ \int_{-\infty}^{\infty} P(k)Q(k)\, dk \}^2 \leq \int_{-\infty}^{\infty} |P(k)|^2 \, dk \int_{-\infty}^{\infty} |Q(k)|^2 \, dk \)

\[ \sigma_x^2 \sigma_k^2 \geq \int_{-\infty}^{\infty} |x(P(x)P'(x))| \, dx \]

\[ \sigma_x^2 \sigma_k^2 \geq \int_{-\infty}^{\infty} \frac{x}{2} \frac{d}{dx} |(P(x))|^2 \, dx \]

\[ \sigma_x^2 \sigma_k^2 \geq \frac{1}{4} \int_{-\infty}^{\infty} |P(x)|^2 \, dx \]

And since \( \int_{-\infty}^{\infty} |P(x)|^2 \, dx \) is the probability of finding the IFE disturbance anywhere = 1. Then:

\[ \sigma_x \sigma_k \geq \frac{1}{2} \]  \hspace{1cm} (16)

### 3 Mass and Momentum

The velocity \( v = \frac{d}{\sqrt{(t^*)^2 + (ps t')^2}} \) of an IFE disturbance moving away from a notional fixed reference point can be combined with that of another IFE disturbance moving away in the opposite direction from the fixed reference point at a velocity \( u = \frac{d}{\sqrt{(t^*)^2 + (qs t')^2}} \). This produces a calculation for the resultant velocity from two independent velocities.

In time \( \sqrt{(t^*)^2 + (ps t')^2} \) the distance \( D \) travelled by both disturbances is:

\[ D = d + \frac{d}{\sqrt{(t^*)^2 + (qs t')^2}} \sqrt{(t^*)^2 + (ps t')^2} \]

However, during the period of time \( \sqrt{(t^*)^2 + (ps t')^2} \) which accounts for a movement in space \( dx \) for the first IFE disturbance, the number of beta-time increments must be accounted for by the second IFE disturbance (determined by its State→Space trigger-point \( qst' \)) which may overlap with those of the first.

To establish how many “extra” incidents of beta-time(\( t^* \)) occur in this time, in a theoretical amount of time stretching across \( \sqrt{(t^*)^2 + (ps t')^2} \sqrt{(t^*)^2 + (qs t')^2} \) there will be an extra number \( N \) of incidents of \( t^* \) where:
$$N = \sqrt{(t^*)^2 + (pst')^2 + \sqrt{(t^*)^2 + (qst')^2}^2} - \left\{ (\sqrt{(t^*)^2 + (pst')^2})^2 + (qst')^2 \right\}$$

This gives a rate of discrepancy of extra $t^*$ per unit of time such that:

$$rate = \frac{\sqrt{(t^*)^2 + (pst')^2 + \sqrt{(t^*)^2 + (qst')^2}^2} - \left\{ (\sqrt{(t^*)^2 + (pst')^2})^2 + (qst')^2 \right\}}{\sqrt{(t^*)^2 + (qst')^2}^2}$$

In an amount of time $\sqrt{(t^*)^2 + (pst')^2}$ there will be $\frac{\sqrt{(t^*)^2 + (pst')^2}}{\sqrt{(t^*)^2 + (qst')^2}}$ opportunities for an extra “skip” of beta-time.

The total number of extra incidents of $t^*$ will be:

$$\frac{\sqrt{(t^*)^2 + (pst')^2 + \sqrt{(t^*)^2 + (qst')^2}^2} - \left\{ (\sqrt{(t^*)^2 + (pst')^2})^2 + (qst')^2 \right\}}{\sqrt{(t^*)^2 + (qst')^2}^2}$$

Then the amount of time $t$ we have to consider when calculating the combined velocity of the two IFE disturbances is:

$$t = \sqrt{(t^*)^2 + (pst')^2}^2 + \left\{ (t^*)^2 + \frac{\sqrt{(t^*)^2 + (pst')^2}^2 + \sqrt{(t^*)^2 + (qst')^2}^2 - \left\{ (\sqrt{(t^*)^2 + (pst')^2})^2 + (qst')^2 \right\}}{\sqrt{(t^*)^2 + (qst')^2}^2} \right\}$$

$$= \sqrt{(t^*)^2 + (pst')^2}^2 + (t^*)^2 \frac{2(\sqrt{(t^*)^2 + (pst')^2})^2 + (t^*)^2}{\sqrt{(t^*)^2 + (qst')^2}^2}$$

$$= \sqrt{(t^*)^2 + (pst')^2}^2 + \frac{(t^*)^2}{\sqrt{(t^*)^2 + (qst')^2}^2} + 2(t^*)^2 \frac{\sqrt{(t^*)^2 + (pst')^2}}{\sqrt{(t^*)^2 + (qst')^2}^2}$$

$$= \frac{\sqrt{(t^*)^2 + (pst')^2}^2 + (t^*)^2}{\sqrt{(t^*)^2 + (qst')^2}^2} + \frac{(t^*)^2}{\sqrt{(t^*)^2 + (qst')^2}^2}$$

Then the combined velocity $V$ of the two IFE disturbances is:

$$V = \frac{d + \frac{d}{\sqrt{(t^*)^2 + (qst')^2}^2} \sqrt{(t^*)^2 + (pst')^2}}{\sqrt{(t^*)^2 + (pst')^2} + \frac{(t^*)^2}{\sqrt{(t^*)^2 + (qst')^2}^2}}$$

Consider two IFE disturbances of equal rest mass $m_0$ and equal velocity $u$ colliding in a non-elastic way from opposite directions (say a mass moving from the left and a mass moving from the right), resulting in a stationary object of mass $M_0$. Suppose that mass is not necessarily constant so that the moving mass $m_u$ may be different from the stationary rest mass $m_0$.

From the perspective of the second IFE disturbance mass moving from the right then the disturbance moving from the left has an effective velocity $V$ (of the combined velocities) and a mass $m_V$. If it then hits the second disturbance of mass $m_0$ this results in an IFE disturbance of mass $M_u$ moving with a velocity $u$. 

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From equation 18 the effective velocity of two combined equal velocities each moving towards one another with velocity $u = \frac{d}{\sqrt{(t)^2+(pst')^2}}$ is:

$$ V = \frac{2d\sqrt{(t)^2+(pst')^2}}{2(t)^2+(pst')^2} $$ \hfill (18)

Employing two fundamental laws (through empirical experience):
(1) Conservation of Momentum i.e. $m_V V = M u$
(2) Conservation of Mass i.e. $m_V + m_0 = M_u$
Combining these two conservation laws and eliminating $M_u$:

$$ \frac{m_V}{m_0} = \frac{u}{V-u} $$ \hfill (19)

Making use of $u = \frac{d}{\sqrt{(t)^2+(pst')^2}}$ and equation 18:

$$ \frac{m_V}{m_0} = \frac{d}{2d\sqrt{(t)^2+(pst')^2}} = \frac{d}{\sqrt{(t)^2+(pst')^2}} \frac{2d\sqrt{(t)^2+(pst')^2}}{2(t)^2+(pst')^2} - d $$

Then

$$ \frac{m_V}{m_0} = \frac{2(t)^2+(pst')^2}{(pst')^2} $$ \hfill (20)

If we multiply by $\frac{d^2}{(t)^2}$:

$$ m_V \frac{d^2}{(t)^2} = m_0 \frac{d^2}{(t)^2} + 2m_0 \frac{d^2}{(pst')^2} $$ \hfill (21)

The second expression on the right indicates a multiple of the rest mass with some form of the square of the velocity.

The traditional Newtonian formulation of kinetic energy is $\frac{1}{2}m_0 V^2$ and from eq (19) $V = \frac{2d\sqrt{(t)^2+(pst')^2}}{2(t)^2+(pst')^2}$. For speeds much less than the speed of light $\frac{1}{2}m_0 V^2 \sim \frac{2m_0 d^2}{(pst')^2}$. This is the last expression on the right of equation (22) which suggests that equation 22 refers to the energy of the IFE disturbance, where the term $2m_0 \frac{d^2}{(pst')^2}$ indicates its kinetic energy. Consequently for the rest energy $E_0$ of the disturbance:

$$ E_0 = m_0 \frac{d^2}{(t)^2} $$ \hfill (22)

And for the total energy $E_T$ of the IFE disturbance:
\[ E_T = m_V \frac{d^2}{(t^*)^2} \]  

These are, of course, instances of Einstein’s familiar expression \( E = mc^2 \). Since \( E_T = \frac{h}{st'} \) then \(^7\) from (22):

\[ m_0 = \frac{h(t^*)^2}{d^2(st') \{(pst')^2 + 2(t^*)^2 \}} \]  

A term can be defined in line with kinetic energy and designated “pure kinetic mass” \( m_k \):

\[ m_k = m_V - m_0 = \frac{h(t^*)^2}{d^2(st') \{(pst')^2 + 2(t^*)^2 \}} \]  

A neat balance is therefore exhibited. When \((pst')\) is very large compared to \(t^*\) then rest mass will dominate the total mass (i.e. \(m_0 \rightarrow \frac{h(t^*)^2}{d(st')}\)) and the pure kinetic mass will be negligible. As \((pst')\) decreases, however, then the proportion of pure kinetic mass to rest mass will increase. Ultimately, for a photon - which has no trigger point - then \((pst')\) is 0 and its mass comprises pure kinetic mass only: it has no rest mass.

An interesting formulation for the total energy can be obtained noting that:

\[ (m_VV)^2 c^2 = \frac{4h^2(t^*)^2(\frac{(t^*)^2}{(pst')^2})^2}{(st')^2} \text{ and } (m_0)^2 c^4 = \frac{h^2(t^*)^4(pst')^4}{(st')^2(2(t^*)^2 + (pst')^2)}. \] Thus

\[ (m_VV)^2 c^2 + (m_0)^2 c^4 = \frac{h^2}{(st')^2} = (e_T)^2 \]

This implies that the total energy squared is equal to sum of the momentum squared multiplied by the speed of light squared and the rest energy squared. This occurs once again because we have to consider what a moving “particle” is: for which there will only be a series of state changes which are included in the calculation of the time. Whereas from the perspective of another IFE disturbance this disturbance is moving and the time taken for movement in Space must also be considered.

Viewing energy as rate of change of State, the perception of the magnitude of this quantity will vary from different vantages moving at different

\(^7\) see earlier paper Brown (2003)
velocities. From the perspective of a mass moving at speed \( V \) its pure kinetic mass can be isolated from a vantage point moving at the same speed as the pure kinetic mass (i.e. we can neglect the inertial rest mass). The pure kinetic mass itself comprises a moving IFE disturbance moving from a notional fixed point with a certain speed. This moving IFE disturbance could assume a range of speeds with respect to the possible speeds of the fixed point, whilst nevertheless maintaining the effective speed \( V \). Average quantities enable calculation of the speed of the fixed point and the speed of the internal IFE disturbance which springs from it as both having the same speed \( u \).

If we now calculate the momentum, from (19) and (24):

\[
m_V V = \frac{2h}{\lambda (s't') \left\{2(t^*)^2 + (pst')^2\right\}} \sqrt{(t^*)^2 + (psts')^2}
\]

Yet from the above discussion this represents the product of the pure kinetic energy (which from the perspective of an entity moving at speed \( u = \frac{d}{\sqrt{(t^*)^2 + (psts')^2}} \) is its total energy) of the moving mass and the inverse \( \frac{1}{u} \) of the internal IFE disturbance velocity. Hence, expressing total energy using \( s' \) from the point of view of the moving entity \( E_T = \frac{h}{s't'} \), and using \( u = \frac{\lambda}{s't'} \):

\[
\text{Momentum } P = m_V V = \frac{h}{\lambda}
\]

Since \( P = \frac{h}{\lambda} = \frac{\hbar}{2\pi} \) then from (17)

\[
\sigma_x \sigma_p \geq \frac{h}{4\pi}
\]

This is the familiar expression of Heisenberg’s Uncertainty Principle.

### 4 Conclusions

A mechanism for formalising the statistical underpinnings of quantum calculations provides both a means for calculation and a rationale for the quantum uncertainty of position and momentum. A later paper is intended on the application of this method to the theory of gravity. Detailed computer models and discussion are available from the author on request.
Appendix: The number of potential positions precisely lying on the temporal arc

The goal is to locate the number of potential positions on the temporal arc formed through the time magnitude $|T| = \sqrt{(nt^*)^2 + (npst' + rst')^2}$. Since $t^*$ and $t'$ are finite numbers, and since $n$, $p$, $s$ and $r$ are integers then only a small subset of positions on the temporal arc can exist to form $|T|$. Since this can effectively be represented as the root of a sum of two squares, then we effectively want to estimate the number of lattice points $C(|T|)$ on the circumference of a circle of radius $|T|$.

A theory of point lattices can determine the number of possible lattice points in and on a circle $C(|T|)$ of radius $|T|$. If we consider the circle at the origin of a fundamental point lattice with each lattice point as the centre of a unit square with sides parallel to the axes $t^*$ and $t'$, then the area of all the squares whose centres are inside or on $C(|T|)$ can be analysed. This area $L(|T|)$ comprises a number of complete squares entirely within the circle, and also a number of squares that are divided by the circle of radius $|T|$. Some parts of squares with centres inside the circle of radius $|T|$ will remain outside of the circumference. Equally some squares with centres outside the circle have boundaries fitting partly within the circle’s perimeter. If we theoretically shade in all the complete squares whose centres are in or on the circle, then we can bound the shaded area $L(|T|)$ from below and above - we find the largest disk whose interior is completely shaded, and the smallest disk whose exterior is completely unshaded. Since the diagonal of a unit square is $\sqrt{2}$ then all shaded squares must be contained in a circle of radius $|T| + (\sqrt{2}/2)$. Similarly the circle whose radius $= |T| - (\sqrt{2}/2)$ is contained entirely within the shaded squares. Consequently

$$\pi(|T|^2 - \sqrt{2}|T| - \frac{1}{2}) \leq \pi(|T|^2 - \sqrt{2}|T| + \frac{1}{2}) \leq L(|T|) \leq \pi(|T|^2 + \sqrt{2}|T| + \frac{1}{2})$$

Which implies that

$$\frac{L(|T|)}{|T|^2} - \pi \leq \pi(\sqrt{\frac{2}{|T|^2}} + \frac{1}{2|T|^2})$$

Since $(\sqrt{\frac{2}{|T|^2}} + \frac{1}{2|T|^2})$ tends to 0 as $|T| \to \infty$ then $L(|T|)/|T|^2 \to \pi$

i.e. $L(|T|) = \pi|T|^2$.

This defines the number of lattice points both in and on a circle of radius $|T|$. The number of points solely on the circle of radius $|T|$ is simply $C(|T|) = 2\pi|T|$.

Whilst different arcs will have volatile numbers of potential composi-
tions through nt* and rst’ (and some arcs will be effectively prime composed through only a single instance of n and rst’) an average value for the number of possible positions on a variable temporal arc, will be effective if summed over a large/infinite series - probabilities will be summed. We therefore sum the first n values of $L(|T|)$ (the number of possible lattice positions on a circle of radius $|T|$) and divide by n to obtain an associated average for the total number of of ways for combining the two axes of time to form the single time magnitude:

$$\frac{C(|T|)}{|T|} = \frac{C(0) + C(1) + C(2) + \ldots + C(|T|)}{|T|}$$

Therefore $C(|T|) = 2\pi|T|$ can be utilised.

6 Acknowledgements

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7 References

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