Chapter 1

NUCLEAR PASTA IN SUPERNOVAE AND NEUTRON STARS

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Abstract

In supernova cores and neutron star crusts, nuclei with exotic shapes such as rod-like and slab-like nuclei are expected to exist. These nuclei are collectively called nuclear “pasta”. For the past decades, existence of the pasta phases in the equilibrium state has been studied using various methods. Recently, the formation process of the pasta phases, which has been a long-standing problem, has been unveiled using molecular dynamics simulations. In this review, we first provide the astrophysical background of supernovae and neutron stars and overview the history of the study of the pasta phases. We then focus on the recent study on the formation process of the pasta phases. Finally, we discuss future important issues related to the pasta phases: their astrophysical evidence and consequences.

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Figure 1. Schematic pictures of nuclear “pasta”. The black regions show the liquid phase of nuclear matter, where protons and neutrons coexist. The gray regions show the gas phase, which is almost free of protons. Nuclear shape changes in the sequence from (a) to (e) with increasing density. Figure courtesy of K. Oyamatsu [6].

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1. Introduction

In terrestrial matter, atomic nuclei are roughly spherical. However, this common wisdom does not necessarily hold for matter in supernovae and neutron stars. There, density of matter is very high and can reach the value inside the atomic nuclei themselves, i.e., the normal nuclear density \( \rho_0 \approx 0.165 \) nucleons fm\(^{-3}\) corresponding to \( \approx 3 \times 10^{14} \) g cm\(^{-3}\).

In such a high-density environment, nuclei will adopt various shapes such as spaghetti-like rods and lasagna-like slabs [1] [2] (see Fig. 1). These non-spherical nuclei are collectively called nuclear “pasta” and the phases with these nuclei as “pasta” phases [1].

According to Bethe [3], the term “nuclear pasta” was coined by Cooperstein and Baron [4]. However, Ravenhall may be the first person who has coined this term [5].
and nuclei are closely packed, the effect of the electrostatic energy becomes comparable to that of the surface energy and the total energy of the system can be reduced by combining nearest neighbor nuclei and decreasing the total surface area of nuclei. Consequently, the energetically favorable configuration is expected to have remarkable structures of the pasta phases: the nuclear matter region (i.e., the liquid phase region) is divided into periodically arranged parts of roughly spherical (a), rod-like (b) or slab-like (c) shape, embedded in the gas phase and in a roughly uniform electron gas. Besides, there can be phases in which nuclei are turned inside out, with cylindrical (d) or spherical (e) bubbles of the gas phase in the liquid phase.

In the present article, we provide an overview of the recent progress of the study on the nuclear pasta phases. The structure of this article is as follows. In the remaining part of the present section, we provide a brief explanation about astrophysical background relevant to the pasta phases. In Section 2, we overview the study of the pasta phases so far, and then in Section 3 we explain the molecular dynamics approach of nucleon many-body systems. In Section 4, we explain the recent study about the dynamical formation of the pasta phases. Finally, we discuss the future prospects of the study of the pasta phases, especially focusing on their astrophysical consequences.

1.1. Astrophysical background

1.1.1. Core collapse supernovae

Core collapse supernova explosion is the final stage of the evolution of massive stars whose main-sequence mass is $\sim 8 - 30 M_\odot$ (for reviews, see, e.g., Refs. [3, 10, 11, 12]). In such stars, as the nuclear burning proceeds, iron core is formed, grows in mass, and finally becomes unstable to gravitational collapse triggered by electron capture on iron nuclei, $^{56}\text{Fe} + e^- \rightarrow ^{56}\text{Mn} + \nu_e$. This reaction leads to a reduction of the degeneracy pressure of electrons, which is a major pressure source supporting the iron core. Besides, when the temperature exceeds about $5 \times 10^9$ K, thermal energy generated by the core contraction is consumed by an endothermic reaction of the photodissociation of iron nuclei, $\gamma + ^{56}\text{Fe} \rightarrow ^{13}\text{He} + 4n - 124.4\text{MeV}$. This process acts as a positive feedback and accelerates the collapse.

As the gravitational collapse proceeds, density in the core becomes so high that even weakly interacting neutrinos are temporarily trapped in the core for $\sim 10$ msec. Namely, when the average density of the core exceeds $\sim 10^{11}$ g cm$^{-3}$, diffusion time scale $\tau_{\text{diff}}$ of neutrinos becomes comparable to or larger than dynamical time scale $\tau_{\text{dyn}}$ of the collapse. Here, $\tau_{\text{diff}}$ is given by the random-walk relation: $\tau_{\text{diff}} \sim R^2/c_\nu$, and $\tau_{\text{dyn}}$ by the free-fall time scale: $\tau_{\text{dyn}} \sim 1/\sqrt{G\rho_{\text{core}}} \sim 4 \times 10^{-3}(\rho/10^{12}\text{g cm}^{-3})^{-1/2}$ sec, where $l_\nu$ is the mean free path of neutrinos, $M \sim 1M_\odot$ and $R$ are the mass and the radius of the core, $\rho_{\text{core}} \sim M/(4\pi R^3/3)$ is the average density of the core, $c$ is the speed of light, and $G$ is the gravitational constant.

While neutrinos are trapped in the core, lepton (i.e., electron plus neutrino) number per nucleon (lepton fraction) is fixed around 0.3 – 0.4. Trapped neutrinos become degenerate and form a Fermi sea, which suppresses the electron capture reaction. Therefore, matter in

\footnote{In the relevant density region, the screening effect of electrons is negligible [7, 8, 9].}
supernova cores [i.e., supernova matter] is not so neutron rich and the typical value of the proton fraction $x$ (ratio of the number of protons to the number of nucleons) is $\simeq 0.3$. In Fig. 2(a), we show the schematic picture of the particle distributions in supernova matter: almost all protons and neutrons are bound in nuclei and charge-neutralizing electrons and trapped neutrinos are distributed uniformly in space.

After $O(100)$ msec from the onset of the collapse, the central density of the core reaches the normal nuclear density $\rho_0$. Then, the short-range repulsion in nucleon-nucleon interaction starts to act and the equation of state (EOS) suddenly becomes stiff. Due to this hardening, pressure becomes sufficiently high to halt the collapse, causing the inner region of the core to bounce. The outer region of the core continues to fall towards the center at supersonic velocities. Consequently, the bouncing inner core drives a shock wave outward into the infalling outer core. After the core bounce, temperature in the inner core becomes so high ($\gtrsim 10 - 20$ MeV) that nuclei melt.

In the process of the collapse, nuclear pasta phases are expected to be formed in the region corresponding to subnuclear densities. They would exist for $O(10 - 100)$ msec until they melt after the bounce of the core. The total amount of the pasta phases could be more than 20% of the total mass of the supernova core just before the bounce [14].

The shock wave propagates outward in the outer core. However, the shock wave is weakened by the following processes: 1) Energy dissipation due to dissociation of nuclei into free nucleons by the shock itself and 2) the pressure reduction behind the shock front due to the emission of neutrinos created by the electron capture on protons coming from the dissociated nuclei. Consequently, the shock wave stalls in the outer core. On the other hand, thermal neutrinos are emitted from the hot inner core (proto-neutron star) and they heat the material in the outer core (referred to as neutrino heating). If the energy transfer from neutrinos to the matter behind the shock front is large enough, the stalled shock can be revived and reach the surface of the outer core by $\sim 1$ sec after the bounce. The shock wave can propagate outside the core without obstacles and finally blow off the outer layer of the star including a part of the outer core after between a few hours and a few days; this explosive phenomenon is a core collapse supernova explosion.
1.1.2. Neutron stars

While the shock wave propagates in the outer core, the proto-neutron star contracts from \( \sim 100 \) km to \( \sim 10 \) km. Then, in the time scale of \( \sim 10 \) sec, thermal neutrinos trapped in the proto-neutron star diffuse out. As results, temperature of the proto-neutron star decreases and matter in the proto-neutron star becomes neutron rich: the proto-neutron star becomes a neutron star. When the temperature decreases to \( \lesssim 10 \) MeV, nucleons in the gas phase start to cluster to form nuclei below \( \sim \rho_0 \).

Neutron stars are dense and compact objects supported by the degeneracy pressure of neutrons (see, e.g., Refs. [7, 15] for reviews). The typical mass and the radius of a neutron star are \( \simeq 1.4M_\odot \) and \( \simeq 10\text{km} \), respectively. In Fig. 3 we show a schematic picture of the structure of a neutron star. Neutron stars roughly consist of two main regions: crust, which is made of crystalline lattices of neutron-rich nuclei, and core, which is made of liquid of nuclear matter (Deep inside the core corresponding to several times normal nuclear density is very uncertain and various hadronic phases and quark matter might exist. These are beyond the scope of the present article.).

In the crust of neutron stars, nuclei in deeper region (i.e., at higher densities) are more neutron rich due to larger chemical potential of electrons, which promotes electron capture. At a density \( \rho_{\text{drip}} \simeq 4 \times 10^{11}\text{g cm}^{-3} \), nuclei become so neutron rich that all neutrons cannot be bound in nuclei and they start to drip out. This neutron drip density divides the crust into two regions: outer crust \((\rho < \rho_{\text{drip}})\) and inner crust \((\rho > \rho_{\text{drip}})\).

The inner crust extends up to the boundary with a core at a density \( \simeq \rho_0 \), where nuclei merge into uniform nuclear matter. Schematic picture of the particle distribution in neutron star matter in the inner crust is shown in Fig. 2(b). Unlike the supernova matter in the same

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1Recently, a heavy neutron star with mass \( 1.97M_\odot \) has been discovered [17].
density region shown in Fig. 2(a), dripped neutrons exist outside the neutron-rich nuclei. Since there are no trapped degenerate neutrinos, neutron star matter is much more neutron rich than supernova matter; typical value of the proton fraction \( x \) in the deep region of the inner crust and in the core is \( x \lesssim 0.1 \).

Nuclear pasta phases are expected to exist in the bottom layer of the crust (thickness is around 100 m) above the crust-core boundary, which corresponds to the density region of \( 0.2 \lesssim \rho/\rho_0 \lesssim 0.5 \). Note that if the pasta phases exist in neutron stars, they can occupy half of the mass of the crust [18].

1.2. Astrophysical implications

Since the total amount of the pasta phases can be substantial both in supernova cores and neutron star crusts, pasta phases can affect various astrophysical phenomena related to supernovae and neutron stars. Here, let us briefly see the effects of the pasta phases.

In the collapse phase of a supernova, the mean free path of neutrinos in the core is reduced mainly by the coherent scattering from nuclei via a weak neutral current [13]. Suppose a neutrino is scattered by an isolated nucleus of mass number \( A \), the neutrino is coherently scattered by all the nucleons in the nucleus and the cross section \( \sigma \) is enhanced to be proportional to \( A^2 \) (cf. in the case of incoherent scattering, \( \sigma \propto A \)) provided the neutrino wavelength is much longer than the radius of the nucleus [19]. In the case of the neutrino scattering in matter, the cross section for coherent scattering is approximated to be proportional to the static structure factor \( S_{nn}(q) \) of neutrons, where \( q \) is the wave number of momentum transfer [20, 21, 14]. Thus existence of the pasta nuclei instead of uniform nuclear matter or spherical nuclei should strongly modify the neutrino cross section [22]. This would affect the onset and the duration of neutrino trapping, and the value of the lepton fraction during this period.

Regarding the pasta phases in neutron stars, we note that the crust is located close to the surface of the star (see Fig. 3). Therefore, the property of the crust naturally influences on many observed phenomena. Especially, cooling of the neutron star by neutrino emission is considered to be affected by existence of the pasta phases.

It is well known that the most efficient cooling process is the direct URCA process: \( n \rightarrow p + e^- + \bar{\nu}_e \) and \( p + e^- \rightarrow n + \nu_e \), but in neutron star matter, which is degenerate and neutron rich, this process is strongly suppressed by the energy and momentum conservation laws. However, it has been pointed out [18] and shown [23] that the direct URCA process can be open in the pasta phases with cylindrical bubbles and spherical bubbles because protons and neutrons move in a periodic potential created by inhomogeneous nuclear structures so that they can get lattice momenta needed to satisfy the momentum conservation (umklapp process). Cooling simulations of Ref. [23] show that the URCA process in the pasta phases could significantly accelerate the cooling of low-mass \( (M \approx 1.35M_\odot) \) neutron stars.

The elastic property of the pasta phases is very different from that of the crystalline solid; it is rather similar to that of the liquid crystal [24]. For example, in the pasta phases with rod-like nuclei and slab-like nuclei — analogous to the columnar and smectic \( A \) phases of the liquid crystal, respectively — there are directions in which the system is translationally invariant and any restoring forces do not act. Therefore, if the pasta phases exist between a crystalline solid in the crust and a liquid core, properties of the crust-core boundary
should be strongly modified, and consequently phenomena related to the crust-core boundary would be affected. The $r$-mode instability in rotating neutron stars is one of the important mechanisms for the gravitational wave radiation of neutron stars \cite{25}. This mechanism accompanies the relative motion between the crust and the core and crucially depends on the boundary condition between these two regions. If we neglect the pasta phases and assume that a viscous fluid core is bounded by a solid crust, there is a strong shear in the boundary layer and the $r$-mode is damped by the viscosity in this region \cite{26}. An existence of the liquid crystal-like pasta phases in the crust-core boundary relaxes the shear and reduces the viscous damping of the $r$-mode instability \cite{27}.

Related to the elastic property of the pasta phases, effect of the pasta phases on the torsional shear mode of the crust has been also studied \cite{28,29}. Due to the small shear modulus of the pasta phases compared to that of the crystalline solid, the shear mode frequency can be significantly reduced.

Finally, the pasta phases would have an influence also on glitch phenomena of pulsars. A glitch is a sudden decrease of the pulse period, i.e., a sudden spinup of neutron star crusts. In the crust, (dripped) neutrons are considered to form a superfluid and their angular momentum is carried by quantized vortices. On the other hand, nuclei in the crust, which mainly consist of a normal fluid, act as pinning centers for the vortices in the neutron superfluid. By electromagnetic radiation, the normal component of the crust is decelerated and the relative velocity between the nuclei and the neutron superfluid increases. Consequently, a strong Magnus force acts to unpin the pinned vortices. In the vortex pinning model \cite{30}, a neutron superfluid whose rotation velocity is higher than that of nuclei works as a reservoir of the angular momentum. Glitches are explained as the angular momentum transfer from the former to the latter by a catastrophic unpinning of vortices and their migration outward, which leads to a reduction of the angular velocity of the superfluid. It is easy to imagine that the structure of nuclei have a large influence on the pinning energy of vortices in this scenario.

2. Nuclear “pasta”

2.1. First-order phase transitions and inhomogeneous structures

Nuclear matter with pasta structures can be regarded as a structured mixed phase during the first-order liquid-gas phase transition. Here we briefly explain the basic notion of the mixed phase.

The simplest case of the first order liquid-gas phase transition is that in a single-component system such as water. Let us consider its liquid-gas phase transition by changing density at a constant temperature. In this case, the equation of state (EOS) during the liquid-gas phase transition is obtained by the Maxwell construction, which gives the phase equilibrium between the liquid and the gas phases at a constant pressure.

On the other hand, systems with multiple components like mixture of water and ethanol undergo a more complex phase transition. There, pressure in the mixed phase is no longer constant during the phase transition and the Maxwell construction cannot be applied. To obtain the EOS, equilibrium of partial pressures and chemical potentials, i.e., the Gibbs conditions, should be solved.
Nuclear matter, i.e., a mixture of protons and neutrons, neutralized by electrons is a system with two independent components with electric charge. Due to the electric charge, two phases in the mixed phase interact with each other. Another important factor in the mixed phase of nuclear matter is the surface tension of the interface between the liquid phase and the gas phase. In the case of nuclear matter, both the effects of the Coulomb force and the surface tension are substantial and therefore, due to the competition of these two, rich geometric structures can emerge. On the other hand, in the previous two cases (i.e., water and mixture of water and ethanol), while the effect of the surface tension is substantial, interaction between coexisting phases is negligible. Thus we have a simple phase separation in these systems (see Fig. 4). If both the effects are weak enough, the structure of mixed phases can be arbitrary and becomes amorphous.\footnote{If the surface tension is negligible but the effect of the Coulomb force is large, the mixed phase takes a Wigner crystal-like structure in which the phase with charged particles breaks up into a microscopic scale and form a crystalline lattice.}

2.2. Historical overview of the study of nuclear “pasta”

The first idea of the phase with non-spherical nuclei was brought out in a seminal paper by Baym, Bethe, and Pethick in 1971\cite{31}. They predicted “nuclei to turn inside out” close to the normal nuclear density: Bubbles of neutron gas form a crystalline lattice structure in the liquid of nuclear matter.

In early 1980’s, Ravenhall et al. and Hashimoto et al. have independently pointed out the five typical types of the pasta phases shown in Fig. 1 can be the ground state of matter. Using a liquid-drop model, they calculated the total energy of the system for the five phases and found that the structure of nuclear matter in the energy minimum state changes in the sequence from (a) to (e) of Fig. 1 with increasing density.

Since these pioneering works, existence of the pasta phases has been examined by various methods such as the liquid-drop model\cite{32,18,33,22} and the Thomas-Fermi approximation\cite{34,35,6,36,37,9,38} based on various nuclear forces including the Skyrme interactions\cite{34,35,18} and the relativistic mean-field models\cite{37,9,38}, etc. These studies have confirmed that, when the matter is less neutron rich (i.e., the average proton fraction

![Figure 4. Effects of the Coulomb force and the surface tension on the structure in the mixed phase.](image-url)

| Surface tension | Electrostatic force |   |
|-----------------|---------------------|---|
| strong          | pasta               |   |
| weak            | Wigner crystal like | amorphous |

| Electrostatic force |   |
|---------------------|---|
| strong              | weak |
| phase separation    |   |

Figure 4. Effects of the Coulomb force and the surface tension on the structure in the mixed phase.
of matter is $0.3 \lesssim x \leq 0.5$) as in the case of supernova matter, the pasta phases can be the ground state for almost all nuclear models. As for neutron-rich case such as neutron star matter, existence of the pasta phases in the ground state has also been confirmed by many nuclear models, except for those (e.g., the relativistic mean-field models [37, 9, 38]) which give relatively large energy density of neutron matter at higher densities of $\gtrsim \rho_0/2$. Actually, in the case of neutron star matter, symmetry energy and its density dependence is crucial for existence of the pasta phases. Within the Thomas-Fermi approximation, this issue has been studied systematically in Ref. [39].

After the early 2000’s, in addition to the macroscopic liquid-drop model and the semi-classical Thomas-Fermi approximation, the pasta phases have been studied by more elaborated calculations based on the Hartree-Fock (+BCS) approximation with the Skyrme interaction [40, 41, 42]. There, shell effects, which are absent in the previous calculations by the liquid-drop model and the Thomas-Fermi approximation, are naturally incorporated. Also in these Hartree-Fock calculations, existence of the pasta phases has been confirmed in both the neutron-rich [41] and less neutron-rich [42] cases.

Possibility of other exotic nuclear shapes besides the typical pasta structures has also been discussed [35, 43]. The double-diamond and the gyroid structures, which actually exist in the mixture of polymers, are good candidates. Although these structures have not been found in the ground state of dense matter so far, the energy difference between the ground state and the phases with these structures can be very small. This suggests that these structures might appear at nonzero temperatures.

All of the above studies are based on the static framework and focus only on the equilibrium state, mainly the ground state. Thus, dynamical problems such as the formation process of the pasta phases have been completely open. Besides, the shape of nuclei is assumed in all studies of the pasta phases before the early 2000’s except for the work by Williams and Koonin [34]. Even in their calculation, the size of the simulation box is so small that it can contain only one period of the pasta structures. This implicitly imposes a constraint on the nuclear structure, which prevents the appearance of complex shapes. In this situation, we studied the pasta phases using a dynamical framework [44, 45, 46, 47, 48, 49] called the quantum molecular dynamics (QMD) [50, 51, 52]. We will discuss QMD in the next section. Using this framework, we can simulate dynamical processes in a large size of the simulation box without any assumptions on the nuclear shape. This is the main scope of the present article and the results of our studies will be shown in Section 4.

### 3. Quantum Molecular Dynamics

#### 3.1. Microscopic simulation for heavy-ion collisions

Quantum molecular dynamics (QMD) [50] is one of the microscopic simulation methods developed for studying intermediate-energy ($\sim 100$ MeV per nucleon in the laboratory frame corresponding to the Fermi energy in the center of mass frame) heavy-ion collision processes in the late 1980’s. It describes the time evolution of all the constituent nucleons and physical quantities are analyzed using the information of each nucleon in the $6N$-dimensional phase space.

There are several other frameworks of the microscopic simulations used in the low
and intermediate energy regions: the time-dependent Hartree-Fock (TDHF) and kinetic approaches based on the Boltzmann equation such as the test-particle method of the Vlasov equation and that of the Vlasov-Uehling-Uhlenbeck (VUU) equation [53]. The TDHF deals with the time evolution of the wave function of many-body system which is approximated by a Slater determinant. This contains important features of a fermion many-body system such as the antisymmetrization of the total wave function and the deformation of single-particle wave functions. The Vlasov equation is a semi-classical approximation of the TDHF equation describing the time evolution of the one-body distribution function. The VUU equation is similar to the Vlasov equation, but the two-body (nucleon-nucleon) collision process taking account of the Pauli blocking is included. These frameworks are basically mean-field theories and cannot describe processes with higher-order correlations such as the formation of fragments.

QMD model has been proposed by Aichelin and Stöcker [50]. The main quantum aspect included in QMD is the stochastic nucleon-nucleon collision process taking account of the Pauli-blocking according to the phase-space occupation number. Technically, QMD simulation is similar to the test-particle method of the VUU equation (VUU simulation). However, unlike VUU simulations, QMD can well describe fragment formation processes including the particle spectrum and the fragment-mass distribution. It is known that the VUU simulation reduces to the QMD simulation when the number of test particles per nucleon is reduced to one. With decreasing the number of test particles, both fluctuation in the density distribution of a single event and event-by-event fluctuation become larger. On the other viewpoint, VUU simulation can be regarded as an ensemble average of many QMD simulation events. This averaging washes out the fluctuation among collision events, and consequently, the fluctuation in the density distribution. In the 1990's, intermediate energy heavy-ion collisions have been intensively analyzed using QMD simulations.

### 3.2. Antisymmetrization of the wave function in molecular dynamics

Although early version of QMD was successful in the intermediate energy region, its applicability was not guaranteed in the low energy region. Since the total wave function in QMD is not antisymmetrized, early QMD cannot describe the ground state of nuclei properly. In the energy minimum state of this model, all nucleons in nuclei have zero momentum and are deeply bound. To remedy this problem practically, nuclei with higher internal energy in which nucleons have nonzero velocities are often used in early QMD simulations. However, these nuclei can spontaneously evaporate nucleons by decaying into the lower energy state and such spurious evaporation becomes a serious problem for simulations of low-energy phenomena with longer time scales.

The fermionic molecular dynamics (FMD) [54] and the antisymmetrized molecular dynamics (AMD) [55] were developed to resolve the above problem of QMD. They use a Slater determinant for the total wave function and can describe the ground state of nuclei taking account of the Fermi statistics in a proper manner. FMD and AMD successfully describe nuclear structures as well as dynamical processes in low-energy heavy-ion collisions. The problem is, however, a huge amount of computing cost to solve the equations of motion

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Note that the QMD discussed here is different from “quantum molecular dynamics” used in material science.
of FMD and AMD, which is proportional to the fourth power of the particle number \( N \) (cf. \( \propto N^2 \) for QMD). Thus the use of FMD and AMD has been limited to small systems with the total number of particles up to a few hundreds.

### 3.3. Pauli potential in QMD

In this situation, a phenomenological way to mimic the Pauli principle using a repulsive two-body potential \([56]\) was introduced in QMD \([57, 51]\). This repulsive potential, so-called the Pauli potential, is a function of the distance not only in the coordinate space but also in the momentum space, and acts between nucleons with the same spin and isospin so that it prevents those particles from coming close in the phase space. Due to the momentum dependence of the Pauli potential, constituent nucleons have non-zero values of the momentum in the ground state of a nucleus keeping their velocities at zero; thus the above mentioned spurious evaporation is avoided.

A typical form of the Pauli potential \( V_{\text{Pauli}} \) is the Gaussian form,

\[
V_{\text{Pauli}}(\mathbf{R}_i, \mathbf{R}_j; \mathbf{P}_i, \mathbf{P}_j) = C_P \left( \frac{\hbar}{q_0 p_0} \right)^3 \exp \left[ -\frac{(\mathbf{R}_i - \mathbf{R}_j)^2}{2q_0^2} - \frac{(\mathbf{P}_i - \mathbf{P}_j)^2}{2p_0^2} \right],
\]

where \( \mathbf{R}_i \) and \( \mathbf{P}_i \) are the position and momentum of \( i \)th nucleon, \( C_P \) is the strength, and \( q_0 \) and \( p_0 \) are the width in the coordinate and the momentum space, respectively. The widths \( q_0 \) and \( p_0 \) determine the range of the phase space distance in which the exchange repulsion acts. Therefore, their product should be comparable to the volume element of the phase space: \( q_0 \cdot p_0 \sim \hbar \). All these parameters in the Pauli potential are determined by fitting the kinetic energy of the non-interacting Fermi gas \([6]\).

Unlike the early version of QMD, QMD with the Pauli potential can describe nuclear matter and finite nuclei in the ground state by appropriately setting the nuclear potentials. For example, in our model of Ref. \([52]\), nuclear potentials are determined to reproduce the energy, \((-16 \text{ MeV per nucleon})\) and the density \( (\rho_0 = 0.165 \text{ fm}^{-3}) \) of nuclear matter in the ground state, binding energies of nuclei, and the observed incident-energy dependence of the proton-nucleus potential. The symmetry energy at the normal nuclear density \( \rho_0 \) is 34.6 MeV. Since there is still an ambiguity in the stiffness of nuclear matter, we provide three sets of parameters giving the incompressibility \( K = 210, 280, \) and \( 380 \text{ MeV} \).

### 4. Recent progress of the study of the pasta phases — formation of the pasta phases

Since most of the previous works about the pasta phases assumed the shape of nuclei within a static framework, a fundamental problem whether or not the pasta phases are actually formed in young neutron stars in the cooling process and supernova cores in the stage of the gravitational collapse was unclear. In this situation, we have approached the above questions using the dynamical framework of QMD and have obtained the following results: 1) Pasta phases can be formed from hot uniform nuclear matter by decreasing temperature.

\[\text{6 The extra potential energy due to the Pauli potential itself is renormalized into the nuclear potential energies.}\]
2) Pasta phase with rod-like nuclei can be formed from a bcc lattice by compression. In the present section, we explain our method and results.

4.1. Formation of nuclear pasta from hot nuclear matter

Now we explain our works where we studied the formation of the pasta phases by cooling hot nuclear matter \[44, 45, 46, 48\]. This process corresponds to the formation of the pasta phases in young neutron stars in the cooling process.

In our simulations, we considered a system with neutrons, protons, and electrons in a cubic box with periodic boundary conditions. The system is not magnetically polarized, i.e., it contains equal numbers of protons (and neutrons) with spin up and spin down. Relativistic degenerate electrons which ensure charge neutrality can be regarded as a uniform background because electron screening is negligibly small at relevant densities around \(\rho_0\) \[8, 9\]. Consequently, one must take account of the long-range nature of the Coulomb interaction. We calculate the Coulomb interaction by the Ewald summation method, which enables us to sum up the contributions of long-range interactions in a system with periodic boundary conditions efficiently. For nuclear interaction, we used the QMD Hamiltonian of Ref. \[52\] with the standard medium-EOS parameter set and that of Ref. \[58\]. The qualitative results are the same for both models.

First we prepare a uniform hot nucleon gas at a temperature \(T \sim 20\) MeV for various densities. Then using the frictional relaxation method, we cool it down slowly for \(O(10^3 - 10^4)\) fm/c to keep the quasi-thermal equilibrium throughout the cooling process (the density is kept constant during the process).

In Fig. 5, we show the resulting nucleon distributions at \(T \simeq 0\) for various densities of \(\lesssim \rho_0\). Here we set the proton fraction of matter \(x = 0.3\); the total number of particles in this simulation is 2048 (614 protons and 1434 neutrons). We see that all the typical pasta phases, such as those with spherical nuclei (a), rod-like nuclei (b), slab-like nuclei (c), rod-like bubbles (d), and spherical bubbles (e) have been obtained. Note that, in this simulation, we did not impose any assumption of the nuclear shape, and these exotic structures were formed spontaneously.

In Figs. 6 and 7, we show the nucleon distributions at non-zero temperatures for \(x = 0.3\) and \(\rho = 0.175\rho_0\) and \(0.34\rho_0\), respectively. At \(T = 0\), we have obtained the phase with rod-like (slab-like) nuclei at the former (latter) density. In these figures, we can see how the pasta phases are formed from a hot uniform nucleon gas by decreasing temperature. At relevant densities, the liquid-gas phase separation occurs at \(T \sim 5\) MeV and the density inhomogeneity becomes significant at \(T \sim 3\) MeV. At \(T \simeq 2\) MeV, although the surface diffuseness of nuclei and the fluctuation of nuclear shape are large and there are still many evaporated nucleons, clustering of nucleons develops and the nuclear shape becomes recognizable. By further decreasing temperature, surface diffuseness, global fluctuation of nuclear shape, and the number of evaporated nucleons get smaller and finally clear rod-like and slab-like nuclei can be observed at \(T \lesssim 1\) MeV.

The proton fraction of the above results is \(x = 0.3\), which is higher than the typical value in the neutron star crust, \(\lesssim 0.1\). In Ref. \[45\], we also studied the case of \(x = 0.1\) as a more realistic condition for neutron star matter. There, as well as the phase with spherical nuclei (at \(\rho \lesssim 0.2\rho_0\)), the pasta phase with rod-like nuclei were obtained at \(\simeq 0.2\rho_0\) by
cooling down hot uniform nucleon gas (in this case, we took $\sim 10^4$ fm/$c$ to cool from 10 to 0 MeV). These results strongly support that the pasta phases can be formed dynamically in the cooling process of young neutron stars.

4.2. Formation of nuclear pasta by compression

4.2.1. Generally accepted scenario based on fission instability

Formation process of the pasta phases by compression of matter in collapsing supernova cores is a very non-trivial problem. Since formation of the pasta phases from a bcc lattice of spherical nuclei must be accompanied by dynamical and drastic changes of the nuclear structure, the fundamental question whether or not the pasta phases are formed in supernova cores was a long standing question.

A generally accepted scenario is that the pasta formation is triggered by fission instability of nuclei [7]: Namely, above some critical density, at which the volume fraction $u$ occupied by nuclei exceeds 1/8, the effect of the Coulomb repulsion between protons, which tends to deform the nucleus, dominates over that of the surface tension of nucleus to make the nucleus spherical. Consequently, nuclei are expected to undergo a quadrupole deformation and they would “eventually join up to form string-like structures” [7]. Below, we explain the fission instability in detail and derive the critical value of $u = 1/8$.

We consider a nucleus with the mass number $A$, the proton number $Z$, and the radius $r_N$. We approximate that the density inside the nucleus is constant and the self-Coulomb energy of the nucleus reads $E_{\text{Coul}}^{(0)} \equiv (3/5)(Z^2e^2/r_N)$. A condition for vanishing of the fission barrier with respect to a quadrupole deformation is given by

$$E_{\text{Coul}}^{(0)} \geq 2E_{\text{surf}} \quad (\text{Bohr-Wheeler's condition}),$$

where $E_{\text{surf}}$ is the surface energy of the nucleus. Physical meaning of this equation is that, when the Coulomb energy is sufficiently larger than the surface energy, the energy gain of the Coulomb repulsion due to the fission exceeds the energy cost of the surface tension by increase of the surface area. Now we consider a lattice of nuclei, where the
Figure 6. Nucleon distributions at $T = 1$, 2, and 3 MeV for $x = 0.3$, $\rho = 0.175\rho_0$, where the phase with rod-like nuclei is obtained at zero temperature. The total number of nucleons in this simulation is 16384 (4915 protons and 11469 neutrons). The upper panels show top views along the axis of the rod-like nuclei at $T = 0$, and the lower ones show side views. The red particles show protons and green ones neutrons. This figure is taken from Ref. [46].

Figure 7. The same as Fig. 6 for $x = 0.3$, $\rho = 0.34\rho_0$, where the phase with slab-like nuclei is obtained at zero temperature. These figures are shown in the direction parallel to the plane of the slab-like nuclei at $T = 0$. This figure is taken from Ref. [46].
charge of protons are neutralized by the background electrons. Within the Wigner-Seitz approximation, where the actual unit cell is replaced by a spherical cell of the same volume, the Coulomb energy $E_{\text{Coul}}$ per nucleus of the lattice is given by

$$E_{\text{Coul}} \simeq E_{\text{Coul}}^{(0)} \left(1 - \frac{3}{2} u^{1/3}\right). \tag{3}$$

Since $r_N \sim A^{1/3}$, the Coulomb and the surface energies per nucleon scale as $E_{\text{Coul}}/A \sim A^{2/3}$ and $E_{\text{surf}}/A \sim A^{-1/3}$ for fixed $Z/A$ and $u$. Therefore, the energy minimization with respect to $A$, $\partial_A (E_{\text{Coul}}/A + E_{\text{surf}}/A) = 0$, leads to $E_{\text{surf}} = 2E_{\text{Coul}}$. Using this equation and Eq. (3), Bohr-Wheeler’s condition (2) reads $u \geq 1/8$.

Although the above discussion provides a clear physical insight, we should keep in mind that Bohr-Wheeler’s condition (2) is derived for an isolated nucleus (in vacuum). In the actual situation of supernova matter and neutron star matter, there are background electrons which reduce the local net charge density inside nuclei. Thus the condition for the fission instability should be modified from Bohr-Wheeler’s one. This point has been studied by Brandt [59] (and revisited by Ref. [60]) and he has shown that the fission instability is suppressed by the existence of background electrons which decrease the Coulomb energy of nuclei. This result poses a doubt about the generally accepted scenario of the formation of the pasta phases based on the fission instability.

### 4.2.2. Simulations and results

To solve the above problem, we simulated the compression of the bcc lattice of spherical nuclei in the collapse of supernova cores using QMD [49]. Here we used the QMD Hamiltonian of Ref. [52] with the standard medium-EOS parameter set as in the previous section. Our simulations were carried out for the proton fraction $x \simeq 0.39$ and the total number of nucleons $N = 3328$ (with 1312 protons and 2016 neutrons).

In Fig. 8 we show the snapshots of the formation process of the pasta phase in adiabatic compression. Starting from an initial condition at $\rho = 0.15\rho_0$ and $T = 0.25$ MeV [Fig. 8(a)], we increased the density by changing the box size $L$ slowly (the particle positions were rescaled at the same time). Here the average rate of the compression was $\gtrsim O(10^{-6}) \rho_0/(\text{fm}/c)$ yielding the time scale of $\gtrsim 10^5 \text{ fm}/c$ to reach the typical density region of the phase with rod-like nuclei. While this time scale is, of course, much smaller than the actual time scale of the collapse, it is much larger than that of the change of nuclear shape (e.g., $\sim 1000 \text{ fm}/c$ for the nuclear fission) and thus the dynamics observed in our simulation should be determined by the intrinsic physical properties of the system, not by the density change applied externally.

At $t \simeq 57080 \text{ fm}/c$ and $\rho \simeq 0.243\rho_0$ in the compression process [Fig. 8(c)], the first pair of two nearest-neighbor nuclei started to touch and fused (dotted circle), and then formed an elongated nucleus. After multiple pairs of nuclei became such elongated ones, we observed a zigzag structure [Fig. 8(d)]. Then these elongated nuclei stuck together [see Figs. 8(e) and (f)], and all the nuclei fused to form rod-like nuclei at $t \lesssim 72700 \text{ fm}/c$ and $\rho \lesssim 0.267\rho_0$. At $t = 76570$ and $\rho = 0.275\rho_0$ [Fig. 8(g)], we stopped the compression; the temperature was $\simeq 0.5$ MeV at this point. Finally, we obtained a triangular lattice of rod-like nuclei after relaxation [Figs. 8(h-1) and (h-2)]. From the start of the structural
Figure 8. Snapshots of the formation process of the pasta phase with rod-like nuclei from the bcc lattice of spherical nuclei by compression of matter. The red particles show protons and the green ones neutrons. In panels (a)-(g) and (h-1), nucleons in a limited region [surrounded by the dotted lines in panel (h-2)] are shown for visibility. The vertices of the dashed lines in panels (a) and (d) show the equilibrium positions of nuclei in the bcc lattice and their positions in the direction of the line of sight are indicated by the size of the circles: vertices with a large circle, with a small circle, and those without a circle are in the first, second, and third lattice plane, respectively. The dotted circle in panel (c) shows the first pair of nuclei start to touch. The solid lines in panel (d) represent the direction of the two elongated nuclei: they take zigzag configuration. The box sizes are rescaled to be equal in the figures. This figure is taken from Ref. [49].
Figure 9. Snapshots of the simulation of compression at $T < 0.01$ MeV using the simplified model. The red particles show the centers of mass of nuclei and the nuclei within the distance less than $0.89r_{nn}^{(0)}$ are connected by a blue line. Here, $r_{nn}^{(0)}$ is the distance between nearest neighbor site of a bcc lattice for instantaneous density. Nuclei and connections within only two lattice planes normal to the line of sight are shown. This figure is taken from Ref. [49].

transition process triggered by the fusion of the first pair of nuclei, the transition process completed within the time scale of $O(10^5)$ fm/c.

Note that before nuclei deformed to be elongated due to the fission instability, they stuck together keeping their spherical shape [see Fig. 8(c)]. Our simulation shows that the pasta phases are formed without undergoing fission instability. Besides, in the middle of the transition process, pair of spherical nuclei got closer to fuse in a way such that the resulting elongated nuclei took a zigzag configuration and then they further connected to form wavy rod-like nuclei. This process is very different from the generally accepted scenario based on fission instability.

Let us now discuss the mechanism of the formation of the zigzag structure using a simplified model. Since nuclei start to connect before they are deformed, it is reasonable to treat a nucleus as a sphere and incorporate only its center-of-mass degrees of freedom. When the nearest neighbor nuclei are so close that the tails of their density profile overlap with each other, net attractive interaction between these nuclei starts to act due to the interaction...
between nucleons in different nuclei in the overlapping surface region. We thus consider a minimal model in which each nucleus is treated as a point charged particle interacting through the Coulomb potential and a potential of the Woods-Saxon form which describes the finite size of nuclei and models the internuclear attraction.

With this model, we performed compression of a bcc lattice with 128 nuclei of $^{208}\text{Pb}$, which corresponds to 8 times larger system than that of the QMD simulation. We show the snapshots of this simulation in Fig. 9. In the situation of Fig. 9(b), the first pair of nuclei started to get closer and then we stopped the compression and relaxed the system. We observed pairings in a zigzag configuration around the first pair [Fig. 9(c)] and finally we obtained a zigzag structure [Fig. 9(d)] similar to the one observed in the QMD simulation (Fig. 8). This result shows that the internuclear attraction caused by overlapping of the surface region of the neighboring nuclei leads to the spontaneous breaking of the bcc lattice.

What happens if the resulting pasta phase with rod-like nuclei is further compressed? This actually occurs in collapsing supernova cores. To answer this question, we have performed QMD simulations of the adiabatic compression of the pasta phases with rod-like nuclei and slab-like nuclei [47]. These simulations show that the pasta phase with rod-like (slab-like) nuclei turns into the phase with slab-like nuclei (rod-like bubbles) by compression. (From the start of the transition process, it completes within $O(10^4)\text{ fm/c.}$) According to these results, we can conclude that, starting from a bcc lattice of spherical nuclei, pasta phases with rod-like nuclei, slab-like nuclei, and rod-like bubbles are formed sequentially by increasing density during the collapse.

5. Summary and outlook

Nuclei with exotic structures such as rod-like and slab-like nuclei — nuclear “pasta” — are expected to exist in supernova cores and neutron star crusts. In this article, we have overviewed the study on nuclear pasta phases; special focus has been given to the recent progress about the formation process of the pasta phases studied by a molecular dynamics method called QMD (Quantum Molecular Dynamics). A great advantage of this method is that we can simulate dynamical processes in inhomogeneous nuclear matter using a large number of nucleons without any assumptions on the structure of nuclei. It is in contrast to many previous studies employing a static framework within the Wigner-Seitz approximation.

Even though the pasta phases were predicted to exist in the ground state of nuclear matter at subnuclear densities, it was unclear whether they are actually formed in the cooling process of hot neutron star crusts (Section 4.1.) and by the compression of matter in the collapse of supernova cores (Section 4.2.). In such a situation, we have shown that the pasta phases can be formed in both of the two cases using QMD simulations [44, 45, 46, 47, 48, 49].

Since the dynamical formation of the pasta phases has been shown, the next step to be made is to study the detectability of the pasta phases in astrophysical phenomena. Here is our proposal of a to-do list.

- There have been many studies about various elementary processes in the pasta phases and properties of the pasta phases. Now it is very important to study their effects on
the neutron star as a whole. Especially, effects of the pasta phases on the cooling curve and the oscillation of neutron stars are important problems. Recently, several authors have started to study these topics \cite{23, 28, 29}, but further study in this direction is needed.

- To study the $r$-mode instability based on the realistic crust-core boundary taking account of the pasta phases. Since the damping of the $r$-mode instability strongly depends on the condition of the crust-core boundary layer \cite{26, 27}, the existence of the pasta phases should affect whether or not the $r$-mode instability is efficient for the gravitational wave radiation.

- To make an EOS table for core collapse simulations taking account of the existence of the pasta phases including their effects on the neutrino opacity. There is an ongoing project in this direction \cite{61}. Core collapse simulations incorporating the pasta phases, which figure out their influence on the supernova explosion, is highly awaited.

- It is natural to consider that supernova cores and neutron star crusts are polycrystalline. Ultimately, it is important to understand the macroscopic properties of matter in supernova cores and neutron star crusts taking account of the polycrystalline structure. For example, even though the elastic properties of the bcc Coulomb crystal \cite{62} and the pasta phases \cite{24} have been studied, actual elastic properties of the macroscopic neutron star crust should be affected by the grain boundaries in the polycrystal and differ from those of the single crystal.

- It is also important to study the properties of nuclear matter at subnuclear densities in experiments. One possibility is as follows \cite{63}: By colliding two nuclei at sub-barrier energies, a low density region would be created between the nuclei at around their closest approach. The density of this region can be controlled by changing the incident energy. If fragments are formed in this low density region, they will be emitted in the transverse direction. The properties of these fragments such as the mass number and the proton fraction can be measured, which would tell us the information of low density nuclear matter.

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