The study of hollow cylinder on inclined plane to determine the cylinder moment of inertia

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Abstract. The dynamic motion of cylinder that roll on an inclined plane is an exciting topic, rolling motion is found in landslide phenomena and other rolling phenomena. The cylinder has a variety of diameters including hollow cylinder holes. In this research, the effect of diameter hollow cylinder to the moment of inertia of the cylinder was investigated by rolling the cylinder on the inclined plane. Hole in the center of the cylinder. The ratio between hole diameter and cylinder diameter (η) is an important parameter in this research. Data collection has been done by video recording with a variation of travel distance and angle of the inclined plane. The video was imported into the logger pro software to obtain the traveling time for each case. The experimental results of the moment of inertia were compared to the analytical value. The lowest relative error is 0.13% for the distance variation experiment that was obtained by using η of 0.1011. The moment of inertia is 1912.8 g.cm². The lowest relative error is 0.80% for the angle variation experiment that was obtained by using η of 0.3419. The moment of inertia is 1708.9 g.cm². Hence, the size of the hole diameter is affected by the experimental results for the distance and angle variation.

1. Introduction

The rotation of a rigid body is a topic of study that remains interesting to date. Rotation can be encountered in everyday life, such as rotating wheels of vehicles either on horizontal roads, uphill roads and hillsides or mountains, and landslides on hillsides or mountains. In rotational motion, the moment of inertia of a rigid body has an important role in the dynamics of the object. One example of the dynamics of a rigid body is the rigid ball movement down the inclined plane. The ball undergoes rotational and translational motion [1]. The cylinder can also experience rotational and translational motion as it slopes down the incline. In this motion, the kinetic and static friction coefficients can be measured [1-11]. In addition to the kinetic and static friction coefficients [6], can also be measured the coefficient moment of inertia of solid ball using Tracker software [1] and research determining the coefficient moment of inertia of hollow cylinder (PVC Pipe) and solid cylinders (coins) using Logger Pro software [12].

With the background described, research has been conducted to determine the value of hollow cylindrical inertia moment with the variation of the cavity with the material of iron. The software of Logger Pro and Microsoft Excel 2007 were employed to process the data.
1.1 Hollow cylinder motion on an inclined plane
Suppose a hollow cylinder is released downhill from a stationary state on an inclined plane. It hollow rolls without slippage under the influence of two forces. They are a gravitational force and frictional force \( f \) between the spherical edge and the inclined planes [13]. The description of the forces on the hollow cylinder that moves on the inclined plane is shown in Figure 1.

![Figure 1](image)

Figure 1. The forces on the hollow cylinder move in the incline.

The description of hollow cylinder motion can be done mathematically by using two basic equations. The first equation is the translational motion equation, assuming that all external forces are working in the center of hollow cylinder mass. The Newton’s law II can be written as [5,6,8]:

\[
\sum F = ma
\]  
(1)

The second equation is the equation of rotational motion against the center of the mass of the hollow cylinder torque whose direction is perpendicular to the axis is rotation [5,7,12]:

\[
\tau = FR
\]  
(2)

with \( \tau \) is torque, force \( F \) and \( R \) are arm styles respectively.

In Fig. 1 it appears that the force which is directed perpendicular to the rotation axis is the frictional force \( f \), and the hollow cylinder force arm rotating with the rotary axis of one of its diameters is the radius of the hollow cylinder \( r \), so Equation (2) becomes [5,6,11]:

\[
\tau = fr
\]  
(3)

or the relationship between torque \( \tau \), the moment of inertia \( I \) and the angular accent \( \alpha \) of the rotating object are [5,6,7,11,14, 18]:

\[
\tau = I\alpha
\]  
(4)

and [8,9,11]

\[
a = r\alpha
\]  
(5)

where in the general equation of the moment of inertia \( I \) am theoretically \((I_T)\) a hollow cylinder of mass \( m \) and the radius \( r \) of the center axis is clarified [15,16]:

\[
I_T = \frac{1}{2} m (r_1^2 + r_2^2)
\]  
(6)

We can use 0.5 as coefficient \( k \) moment of inertia of hollow cylinder and \((r_1^2 + r_2^2)\) as \( r^2 \). Hence, Equation (6) becomes:

\[
I = kmr^2
\]  
(7)

By using Equation (7), the Equation (4) becomes:

\[
\tau = kmra
\]  
(8)

Equation (3) gives us the relationship between torque and force. Hence, the force can be written as

\[
f = km\alpha
\]  
(9)

The gravitational force and force equation of Equation (9) can be substituted to the Equation (1) [4,6,11]:

\[
mgsin\theta - f = ma
\]  
(10)
with $m$ is a mass of the hollow cylinder, $g$ is the acceleration of the gravitational force of Earth, $a$ is the acceleration of the movement of the hollow cylinder, and $\theta$ is slope angle and $f$ is a frictioanal force. Substitution Equation (9) to Equation (10) so:

$$gsin\theta = a(1 + k) \quad (11)$$

Then the acceleration of the translation in the center of the mass of the hollow cylinder:

$$a = \frac{gsin\theta}{(\frac{1}{m(r_2^2 + r_3^2)})} \quad (12)$$

$g$ is the acceleration of the gravitational force of earth. The length of the trajectory of $s$ associated with the shortest $t$ [17]:

$$s = \frac{1}{2}at^2 \quad (13)$$

Substitute Equation (12) to Equation (13), then:

$$s = \frac{1}{2} \left( \frac{gsin\theta}{\frac{1}{m(r_2^2 + r_3^2)}} \right) t^2 \quad (14)$$

for the equation of hollow cylinder inertia in the experiment:

### 1.1.1 The fixed angle $\theta$ and the distance varied.

The Equation (14) becomes:

$$t^2 = \left( \frac{2t + 2m(r_2^2 + r_3^2)}{mgasin\theta(r_1^2 - r_2^2)} \right) s \quad (15)$$

From Equation (15) the slope equation is obtained:

$$\alpha_2 = \frac{2t + 2m(r_2^2 + r_3^2)}{mgasin\theta(r_1^2 - r_2^2)} \quad (16)$$

and the equations of moment of inertia experimentally ($I_E$) is

$$I_E = \left( \frac{\alpha_2 mgasin\theta(r_1^2 - r_2^2)}{2} \right) - m(r_1^2 + r_2^2) \quad (17)$$

$\alpha_1$ = slope graph relation of $t^2$ to $s$

### 1.1.2 Fixed distance by varied angle $\theta$.

The Equation (14) becomes:

$$t^2 = \left( \frac{2t + 2m(r_2^2 + r_3^2)}{mg(r_2^2 - r_3^2)} \right) \frac{1}{sin\theta} \quad (18)$$

From Equation (18) the slope equation is obtained:

$$\alpha_2 = \frac{2t + 2m(r_2^2 + r_3^2)}{mg(r_2^2 - r_3^2)} \quad (19)$$

so the equations of moment of inertia experimentally ($I_E$) is

$$I_E = \left( \frac{\alpha_2 mg(r_2^2 - r_3^2)}{2s} \right) - m(r_1^2 + r_2^2) \quad (20)$$

$\alpha_2$ = slope graph relation of $t^2$ to $1 / sin \theta$

### 1.1.3 Correction of relative error.

$$\% \text{ Correction of relative error } = \left| \frac{I_E - I_f}{I_f} \right| \times 100\% \quad (21)$$
1.1.4 Acceleration of earth's gravity.
To calculate the acceleration value of gravity \( g \), the equation used is:
\[
g = -\frac{4\pi^2}{a}
\]  
(22)
and the correction is calculated by error propagation:
\[
s_g = \sqrt{\left(\frac{\partial g}{\partial a}\right)^2 s_a^2}
\]  
(23)

1.2 Video-based laboratory (vbl) and logger pro software 
Video-Based Laboratory (VBL) is a laboratory-based on video with real symptoms physics which is documented through video recording and computer help, the symptoms can be analyzed to determine the relationship between the physical variables. VBL can present the real physical physics and various forms of its representation (quantitative data, graphics, and equations) simultaneously, which can be done interactively. VBL is a tool which able to combine theoretical and experimental aspects in Physics learning [6,16,19,20]. Recently, some software for VBL is available, such as Tracker and Logger Pro. Logger Pro has an analytics video facility for creating and analyzing the visible graph of motion representation in the video so that it can be used in physics learning.

2. Methods
Logger pro, laptops, camera, tripod, the trajectory in the form of an inclined plane, the balance of o'haus, hollow cylinder material in the form of iron.

![Diagram of experimental setup](image)

**Figure 2.** The composition of the data collection tool determining the moment of inertia hollow cylinder.

Arrange the experimental device and prepare the material as shown above, the hollow cylinder can roll on a path in the form of an inclined plane. Recording using the camera when the hollow cylinder starts to be released in the inclined plane with variations in slope angle and variation in track length. Save the recorded video in a folder that will be analyzed using Logger Pro software. Repeats the recording of the motion of a hollow cylinder in a tilted path with five different slope angles and five different distances for a hollow cylinder.
2.1 Data analysis method

Determine the gravitational acceleration values with Equations (22) and (23) carried out using a mathematical pendulum. Using the fitting data approach used is a linear function between the quadratic swing period \( T^2 \) to the length of the \( L \) path. After that the recording of hollow cylinder motion for each hollow cylinder is five kinds of videos with variations in slope angle and variation in track length, then tracking the hollow cylinder in the video using Logger Pro software, so that \( t \) data is obtained. Then the data obtained is then processed in Microsoft Excel 2007 to obtain a graph of the relation \( \rho^2 \) to \( 1 / \sin \theta \) and \( \rho^2 \) to \( s \). Analyze experimental data by the fitting graph of the relation \( \rho^2 \) to \( 1 / \sin \theta \) and \( \rho^2 \) to \( s \). The event of a hollow cylinder rolling in an inclined plane is an irregularly moving motion event, so that the fitting data approach used is a linear function. The purpose of data fitting is to obtain the slope value from the graph of the relation \( \rho^2 \) to \( 1 / \sin \theta \) and \( \rho^2 \) to \( s \). After the gradient value of each relationship graph is obtained, then determine the value of the moment of inertia with Equations (17) and (20). Then the measurement of relative error of the moment of inertia of the experiment with the theory is done by the Equation (21).

3. Results and discussion

3.1 Acceleration of gravity

Determination of gravitational acceleration is done using a mathematical pendulum. Using the fitting data approach used is a linear function between the quadratic swing period \( T^2 \) to the length of the \( L \) path. Gravity acceleration values obtained are \( g = (974 \pm 76.8) \text{ cm} / \text{s}^2 \) with Equations (22) and (23).

3.2 Angle \( \theta \) remained with distance divided

In this study, each object was performed 5 times experiment for the distance of the varied path, that is 60 cm, 65 cm, 70 cm, 75 cm, 80 cm, 85 cm, and 90 cm.

![Figure 3](image-url)  
**Figure 3.** Graph of \( \rho^2 \) relation to \( s \) with distance varied by 60 cm, 65 cm, 70 cm, 75 cm, 80 cm, 85 cm, and 90 cm: (a) hollow cylinder with a radius ratio \( \eta \) of 0.1011 (b) hollow cylinder with a radius ratio \( \eta \) of 0.2212 (c) hollow cylinder with a radius ratio \( \eta \) of 0.3419 (d) hollow cylinder with a radius ratio \( \eta \) of 0.4592 (e) hollow cylinder with a radius ratio \( \eta \) of 0.5812 (f) hollow cylinder with a radius ratio \( \eta \) of 0.7001 (g) hollow cylinder with a radius ratio \( \eta \) of 0.8187.

From Figure 3, the slope is obtained for each hollow cylinder ratio, from the slope can be determined the moment of inertia of hollow cylinder experimentally with Equation (17) for each
hollow cylinder ratio. Table 1 shows the calculation of the moment of inertia corresponding to each ratio, and the best distance for data retrieval is 65 cm, 70 cm, 75 cm, 80 cm, and 85 cm.

Table 1. The calculation results with the inclination angle of the plane (\( \theta \)) fixed

| No | \( s \) (cm) | \( \eta \) | \( I_E \) (g.cm²) | \( I_T \) (g.cm²) | Relative Error (%) |
|----|-------------|---------|-----------------|-----------------|-------------------|
| 1  | 60          | 0.1011  | 1912.8          | 1915.3          | 0.13              |
| 2  | 65          | 0.2212  | 1822.5          | 1836.6          | 0.76              |
| 3  | 70          | 0.3419  | 1676.4          | 1695.4          | 1.12              |
| 4  | 75          | 0.4592  | 1534.4          | 1500.9          | 2.3               |
| 5  | 80          | 0.5812  | 1281.0          | 1252.7          | 2.3               |
| 6  | 85          | 0.7001  | 935.05          | 929.18          | 0.63              |
| 7  | 90          | 0.8187  | 562.31          | 563.50          | 0.21              |

Table 1 shows that the value of the moment of inertia experimentally matches the theoretical value with equation (6).

3.3 Fixed distance with angle \( \theta \) divided

In this study, each object was performed 5 times experiment for the angle of the inclined plane, i.e., 10°, 15°, 20°, 25°, 30°, 35°, and 40°.

From Figure 4, the slope is obtained for each hollow cylinder ratio, from the slope can be determined the moment value of the inertia of the hollow cylinder experimentally with Equation (20) for each hollow cylinder ratio. Table 2 shows the calculation of the moment of inertia corresponding to each ratio. Seen from the figure the best angles for data retrieval is 15°, 20°, 25°, 30°, and 35°.
Table 2 shows that the value of the moment of inertia experimentally matches the theoretical value with Equation (6).

| No | \(\theta\) (°) | \(\eta\) | \(I_E\) (g.cm²) | \(I_T\) (g.cm²) | Relative Error (%) |
|----|----------------|--------|-----------------|-----------------|-------------------|
| 1  | 10             | 0.1011 | 1935.2          | 1915.3          | 1.0               |
| 2  | 15             | 0.2212 | 1854.3          | 1836.6          | 0.96              |
| 3  | 20             | 0.3419 | 1708.9          | 1695.4          | 0.80              |
| 4  | 25             | 0.4592 | 1516.6          | 1500.9          | 1.0               |
| 5  | 30             | 0.5812 | 1282.8          | 1252.7          | 2.4               |
| 6  | 35             | 0.7001 | 954.25          | 929.18          | 2.7               |

4. Conclusion

Based on the result of data analysis, the experimental value of the moment of inertia of hollow cylinder following the theoretical on the variation of distance and angle variation. The best distance for data retrieval is 65 cm, 70 cm, 75 cm, 80 cm and 85 cm for distance variation. The best angle for data retrieval is 15°, 20°, 25°, 30°, and 35° for angle variation. Relative distance measurement error is the ratio at radius 0.1011 with 0.13%, and the relative error is the radius ratio of 0.3419 with 0.64%. The biggest relative error in variation in distance is the radius ratio 0.5812 with 2.3% and the biggest relative error in angle variation is the radius ratio 0.7001 with 2.7%. The smaller the relative error, the better the moment value of inertia of the hollow cylinder.

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