SurvLIME-Inf: A simplified modification of SurvLIME for explanation of machine learning survival models

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Abstract

A new modification of the explanation method SurvLIME called SurvLIME-Inf for explaining machine learning survival models is proposed. The basic idea behind SurvLIME as well as SurvLIME-Inf is to apply the Cox proportional hazards model to approximate the black-box survival model at the local area around a test example. The Cox model is used due to the linear relationship of covariates. In contrast to SurvLIME, the proposed modification uses $L_\infty$-norm for defining distances between approximating and approximated cumulative hazard functions. This leads to a simple linear programming problem for determining important features and for explaining the black-box model prediction. Moreover, SurvLIME-Inf outperforms SurvLIME when the training set is very small. Numerical experiments with synthetic and real datasets demonstrate the SurvLIME-Inf efficiency.

Keywords: interpretable model, explainable AI, survival analysis, censored data, linear programming, the Cox model, Chebyshev distance.

1 Introduction

Deep machine learning models can be regarded as powerful tools for solving many applied problems, including medical diagnostics, finances, manufacturing etc. In spite of satisfactory performance of the deep learning models, they may be of limited use because their predictions are typically hard to be interpreted or explained by human. This is caused by the fact that many machine learning models work as black-box models. At the same time, there is a high demand for understanding predictions produced by deep learning models. For example, a doctor has to get an explanation of a diagnosis predicted by a black-box model in order to choose a corresponding treatment. She or he has to understand how the particular decisions are made by the deep learning model. Therefore, explainability of deep learning models is an topical direction of research nowadays, and, as a result, a lot of methods have been developed to address the interpretation problems and to get accurate explanations for obtained predictions [5, 11, 12, 21, 24, 37, 38, 39, 52].

There are two main groups of methods for explaining the black-box models. The first group includes local methods which aim to interpret a single prediction. The methods are based on using a local area around a test instance. The second group consists of global methods. In contrast to the local methods, they explain a black-box model on the whole input space or its part to take into account the overall behavior of the model. We study only the local models because our aim is to find
features which lead to the individual prediction. Moreover, we consider post-hoc explanation methods which are used to explain predictions of such black-box models after they are trained.

One of the well-known local explanation methods is the Local Interpretable Model-agnostic Explanations (LIME) [44], which uses simple and easily understandable linear models to locally approximate the predictions of black-box models. LIME provides an explanation for a single instance by perturbing it around its neighborhood and then by constructing a local surrogate model trained also on original training data. Garreau and Luxburg [20] proposed a thorough theoretical analysis of the LIME and derived closed-form expressions for coefficients of the explaining model for the case of the linear explanation function. Garreau and Luxburg [20] point out that LIME is flexible to provide explanations for different data types, including text and image data, while being model-agnostic, i.e., any details of the black-box model are unknown.

One of the peculiarities of LIME is that it explains point-valued predictions produced by the black-box model. However, there are models which produce functions as predictions instead of points. The well-known class of the model with these predictions is machine learning survival models [31, 59] which solve survival analysis tasks [25, 55].

One of the most widely-used regression models for the analysis of survival data is the well-known Cox proportional hazards model, which is a semi-parametric model that calculates effects of observed covariates on the risk of an event occurring, for example, death or failure [13]. The model assumes that a patient’s log-risk of failure is a linear combination of the instance covariates. This is a very important and at the same time very strong assumption.

There are many survival analysis models, for example, random survival forests, deep neural networks, etc., which relax this assumption and allow for more general relationships between covariates and the output parameters [55]. However, these models are the black-box ones and, therefore, they require to be explained. Taking into account that predictions of the models are functions, for example, the survival functions (SF), cumulative hazard functions (CHF), the original LIME cannot be used. Kovalev et al. [30] proposed an explanation method called SurvLIME, which deals with censored data. The basic idea behind SurvLIME is to apply the Cox model to approximate the black-box survival model at a local area around a test instance. The Cox model is chosen due to its assumption of the linear combination of covariates. Moreover, it is important that the covariates as well as their combination do not depend on time. Therefore, coefficients of the covariates can be regarded as quantitative impacts on the prediction.

SurvLIME includes a procedure which randomly generates synthetic instances around the tested instance, and the CHF is calculated for every synthetic instance by means of the black-box survival model. For every instance, the approximating Cox model CHF is written as a function of coefficients of interest. By writing the distance between CHFs provided by the black-box survival model and by the approximating Cox model, respectively, an unconstrained convex optimization problem for computing the coefficients of covariates is constructed. The $L_2$-norm is used in order to consider the distance between two CHF. As a result, the explanation by using SurvLIME is based on solving the convex optimization problem. In order to simplify the approach, we propose and investigate another explanation method which is based on using $L_\infty$-norm for the distance between CHFs. This modification is called SurvLIME-Inf.

The choice of this norm is caused by the fact that obtained optimization problems become rather simple from the computational point of view [35]. Indeed, we get the linear optimization problem for computing coefficients of the Cox model. The $L_\infty$-norm (Chebychev distance) is a measure of the approximation quality, which is defined as the maximum of absolute values of the difference between the function being approximated and the approximating function. Sim and Hartley [49] pointed out that $L_\infty$ minimization is not robust to outliers, i.e., $L_\infty$ minimization may fit the outliers and not the
good data. Nevertheless, our experiments have illustrated a perfect approximation of CHFs provided by the black-box survival model and the approximating Cox model by rather small datasets.

Numerical results using synthetic and real data illustrate SurvLIME-Inf.

The paper is organized as follows. A short survey of publications devoted to local explanation methods and machine learning models in survival analysis is given in Section 2. Basic concepts of survival analysis are considered in Section 3. A brief introduction to LIME can be found in Section 4. Basic ideas behind SurvLIME-Inf are proposed in Section 5. Section 6 contains a formal derivation of the linear programming problem implementing SurvLIME-Inf. Numerical experiments with synthetic and real data are given in Section 7. Concluding remarks are provided in Section 8.

2 Related work

Local explanation methods. LIME is one of the efficient and simple explanation methods. As a result, many modifications of LIME have been developed recently, including, DLIME [57], Anchor LIME [45], LIME-SUP [26], ALIME [17], NormLIME [5], LIME-Aleph [42], GraphLIME [27], MPS-LIME [48], Tree-LIME [32], SurvLIME [30]. Another popular method is the SHAP [50] which takes a game-theoretic approach for optimizing a regression loss function based on Shapley values [34]. It is pointed out by Aas et al. [1] that Shapley values explain the difference between the prediction and the global average prediction, while LIME explains the difference between the prediction and a local average prediction.

Another group of explanation methods is based on counterfactual explanations [54], which try to explain what to do in order to achieve a desired outcome by means of finding changes to some features of an explainable input instance such that the resulting data point called counterfactual has a different specified prediction than the original input. It is important to note that LIME was also modified to implement counterfactual explanations [43, 56].

Many explanation methods, including LIME, are based on perturbation techniques [14, 18, 19, 41, 53]. These methods assume that contribution of a feature can be determined by measuring how prediction score changes when the feature is altered.

Descriptions of many explanation methods and various approaches, their critical reviews can be found in survey papers [2, 4, 12, 21, 46].

Most methods explain point-valued predictions produced by black-box models, i.e., predictions in the form of some number (class, regression value, decision about anomaly, etc.). This fact restrict their use in survival models, where predictions are usually represented in the form of CHFs or SFs. Only SurvLIME [30] deals with these functions, but it may be computationally hard due to the optimization problem which has to be solved.

Machine learning models in survival analysis. A clear taxonomy of survival analysis methods and their comprehensive review can be found in [55]. Following the Cox model [13], a lot of its modifications have been proposed. Some modifications are based on the Lasso method [51], on the group Lasso penalty method [29], on the adaptive Lasso [58]. To relax assumptions of the Cox model, in particular, the linear relationship between covariates and the time of event, many models using neural networks, random forests, support vector machines, etc. have been developed starting from the pioneering work [16]. The corresponding review of the methods is proposed by Nezhad et al. [40]. One of the important class of survival models, illustrating their efficiency and accuracy especially by limited survival data, is the random survival forests (RSFs) which can be viewed as an extension of the original random forests [9]. A detailed review of RSFs is presented by Bou-Hamad et al. [8].

Most survival models except for those based on the Cox model can be regarded as black-box models.
Therefore, they require to be explained in many applications. At the same time, only the Cox model can be regarded as explainable one due to its linear relationship between covariates. Therefore, it will be used to approximate more powerful models, including survival deep neural networks and RSFs, in order to explain predictions of these models.

3 Basic concepts of survival analysis

In survival analysis, an instance (patient) $i$ is represented by a triplet $(x_i, \delta_i, T_i)$, where $x_i^T = (x_{i1}, \ldots, x_{id})$ is the vector of the patient parameters (characteristics) or the vector of the instance features; $T_i$ is time to event of the instance. If the event of interest is observed, then $T_i$ corresponds to the time between baseline time and the time of event happening, in this case $\delta_i = 1$, and we have an uncensored observation. If the instance event is not observed and its time to event is greater than the observation time, then $T_i$ corresponds to the time between baseline time and end of the observation, and the event indicator is $\delta_i = 0$, and we have a censored observation. Suppose a training set $D$ consists of $n$ triplets $(x_i, \delta_i, T_i)$, $i = 1, \ldots, n$. The goal of survival analysis is to estimate the time of event interest $T$ for a new instance (patient) with a feature vector denoted by $x$ by using the training set $D$.

The survival and hazard functions are key concepts in survival analysis for describing the distribution of event times. The survival function denoted by $S(t|x)$ as a function of time $t$ is the probability of surviving up to that time, i.e., $S(t|x) = \Pr(T > t|x)$. The hazard function $h(t|x)$ is defined as $h(t|x) = f(t|x)/S(t|x)$, where $f(t|x)$ is the density function of the event of interest.

Another important concept in survival analysis is the CHF $H(t|x)$, which is defined as the integral of the hazard function $h(t|x)$ and can be interpreted as the probability of an event at time $t$ given survival until time $t$. The survival function is determined through the hazard function and through the CHF as $S(t|x) = \exp(-H(t|x))$.

To compare survival models, the C-index proposed by Harrell et al. [23] is used. It estimates how good a survival model is at ranking survival times. In other words, this is the probability that the event times of a pair of instances are correctly ranking. C-index does not depend on choosing a fixed time for evaluation of the model and takes into account censoring of patients [36].

According to the Cox proportional hazards model [13], the hazard function at time $t$ given predictor values $x$ is defined as

$$h(t|x, b) = h_0(t) \exp(b^T x) = h_0(t) \exp\left(\sum_{k=1}^{d} b_k x_k\right).$$

(1)

Here $h_0(t)$ is a baseline hazard function which does not depend on the vector $x$ and the vector $b$; $b^T = (b_1, \ldots, b_d)$ is an unknown vector of regression coefficients or parameters.

In the framework of the Cox model, the survival function $S(t|x, b)$ is computed as

$$S(t|x, b) = \exp(-H_0(t) \exp(b^T x) = (S_0(t))^{\exp(b^T x)}.$$

(2)

Here $H_0(t)$ is the cumulative baseline hazard function; $S_0(t)$ is the baseline survival function. It is important to note that functions $H_0(t)$ and $S_0(t)$ do not depend on $x$ and $b$.

One of the ways for estimating parameters $b$ of the Cox model is the Cox partial likelihood function [13]. There are other methods, including the Breslow approximation [10] and the Efron approximation [15].
4 LIME

Before studying the LIME modification for survival data, this method is briefly considered below. LIME proposes to approximate a black-box model denoted as \( f \) with a simple function \( g \) in the vicinity of the point of interest \( x \), whose prediction by means of \( f \) has to be explained, under condition that the approximation function \( g \) belongs to a set of explanation models \( G \), for example, linear models. In order to construct the function \( g \) in accordance with LIME, a new dataset consisting of perturbed samples is generated, and predictions corresponding to the perturbed samples are obtained by means of the explained model. New samples are assigned by weights \( w_x \) in accordance with their proximity to the point of interest \( x \) by using a distance metric, for example, the Euclidean distance or a kernel. The weights are used to enforce locality for the linear model \( g \).

An explanation (local surrogate) model is trained on new generated samples by solving the following optimization problem:

\[
\arg\min_{g \in G} L(f, g, w_x) + \Phi(g). \tag{3}
\]

Here \( L \) is a loss function, for example, mean squared error, which measures how the explanation is close to the prediction of the black-box model; \( \Phi(g) \) is the model complexity. As a result, the prediction is explained by analyzing coefficients of the local linear model. The output of LIME, therefore, is a set of important features corresponding to coefficients of the linear model.

5 A general algorithm of SurvLIME and SurvLIME-Inf

Suppose that there are a training set \( D \) and a black-box model which produces an output in the form of the CHF \( H(t|x) \) for every new instance \( x \). An idea behind SurvLIME is to approximate the output of the black-box model with the CHF produced by the Cox model for the same input instance \( x \). With this approximation, we get the parameters \( b \) of the approximating Cox model, whose values can be regarded as quantitative impacts on the prediction \( H(t|x) \). The largest coefficients indicate the corresponding important features.

Denote the Cox CHF as \( H_{\text{Cox}}(t|x, b) \). Then we have to find such parameters \( b \) that the distance between \( H(t|x) \) and \( H_{\text{Cox}}(t|x, b) \) for the considered instance \( x \) would be as small as possible. In order to avoid incorrect results, a lot of nearest points \( x_k \) in a local area around \( x \) is generated. For every \( x_k \), the CHF \( H(t|x_k) \) of the black-box model is obtained as a prediction of the black-box model. Now optimal values of \( b \) can be computed by minimizing the weighted average distance between every pair of CHFs \( H(t|x_k) \) and \( H_{\text{Cox}}(t|x, b) \) over all points \( x_k \). Weight \( w_k \) assigned to the \( k \)-th distance depends on the distance between \( x_k \) and \( x \). Smaller distances between \( x_k \) and \( x \) produce larger weights of distances between CHFs.

It is important to point out that the optimization problem for computing parameters \( b \) depends on the used distance metric between CHFs \( H(t|x_k) \) and \( H_{\text{Cox}}(t|x, b) \). SurvLIME uses the \( L_2 \)-norm which leads to a convex optimization problem. SurvLIME-Inf uses the \( L_\infty \)-norm. We will show that this distance metric leads to the linear programming problem whose solution is very simple.

6 Optimization problem for computing parameters

Let \( t_0 < t_1 < \ldots < t_m \) be the distinct times to event of interest, for example, times to deaths from the set \( \{T_1,\ldots,T_n\} \), where \( t_0 = \min_{k=1,\ldots,n} T_k \) and \( t_m = \max_{k=1,\ldots,n} T_k \). The black-box model
maps the feature vectors $x \in \mathbb{R}^d$ into piecewise constant CHFs $H(t|x)$ such that $H(t|x) \geq 0$ for all $t$, $\max_t H(t|x) < \infty$. Let us introduce the time $T \geq t_m$ in order to restrict $H(t|x)$ and denote $\Omega = [0, T]$.

Interval $\Omega$ can be divided into $m + 1$ non-intersecting subsets $\Omega_0, ..., \Omega_m$ such that $\Omega_j = [t_j, t_{j+1}]$, $\forall j \in \{0, ..., m - 1\}$, $\Omega_m = [t_m, T]$. After introducing the indicator functions $I_j(t)$, which takes the value 1 when $t \in \Omega_j$, and 0 otherwise, we rewrite $H(t|x)$ as follows:

$$H(t|x) = \sum_{j=0}^{m} H_j(x) \cdot I_j(t). \quad (4)$$

Here $H_j(x)$ is a part of $H(t|x)$ in interval $\Omega_j$ under additional condition $H_j(x) > 0$. CHF $H_j(x)$ does not depend on $t$ in interval $\Omega_j$ because it is constant in this interval.

The same can be written for the Cox CHF:

$$H_{Cox}(t|x, b) = H_0(t) \exp(b^T x) = \sum_{j=0}^{m} [H_{0j} \exp(b^T x)] \cdot I_j(t). \quad (5)$$

It should be noted that the use CHFs for computing the distance between them leads to a complex optimization problem which may be non-convex. Therefore, we proposed to take logarithms of $H(t|x)$ and $H_{Cox}(t|x, b)$ denoted as $\phi(t|x)$ and $\phi_{Cox}(t|x, b)$, respectively. Since the logarithm is the monotone function, then there hold

$$\phi(t|x) = \sum_{j=0}^{m} (\ln H_j(x)) I_j(t), \quad (6)$$

$$\phi_{Cox}(t|x, b) = \sum_{j=0}^{m} (\ln [H_{0j} \exp(b^T x)]) I_j(t)$$

$$= \sum_{j=0}^{m} \{ \ln H_j(x) - \ln H_{0j} - b^T x \} I_j(t). \quad (7)$$

Let us consider the distance between $\phi(t|x_k)$ and $\phi_{Cox}(t|x_k, b)$ based on the $L_{\infty}$-norm for every generated point $x_k$:

$$D_{\infty,k} (\phi, \phi_{Cox}) = \| \phi(t|x_k) - \phi_{Cox}(t|x_k, b) \|_{\infty}$$

$$= \max_{t \in \Omega} | \phi(t|x_k) - \phi_{Cox}(t|x_k, b) |. \quad (8)$$

Hence, the weighted average distance between $\phi(t|x_k)$ and $\phi_{Cox}(t|x_k, b)$ for $N$ generated points $x_k$ has to be minimized over $b$. This can be written as the following optimization problem:

$$\min_b \left( \sum_{k=1}^{N} w_k \cdot \max_{t \in \Omega} | \phi(t|x_k) - \phi_{Cox}(t|x_k, b) | \right). \quad (9)$$

Let us introduce the optimization variables

$$z_k = \max_{t \in \Omega} | \phi(t|x_k) - \phi_{Cox}(t|x_k, b) |. \quad (10)$$
They are restricted as follows:

\[ z_k \geq |\phi(t|x_k) - \phi_{Cox}(t|x_k, b)|, \quad \forall t \in \Omega. \]  

(11)

The above constraints for every \( t \) can be represented as two constraints

\[ z_k \geq \phi(t|x_k) - \phi_{Cox}(t|x_k, b), \quad \forall t \in \Omega, \]  

(12)

\[ z_k \geq \phi_{Cox}(t|x_k, b) - \phi(t|x_k), \quad \forall t \in \Omega. \]  

(13)

Substituting (6)–(7) into (9) and taking into account (12)–(13), we get

\[ \min_{b} \sum_{k=1}^{N} w_k z_k, \]  

(14)

subject to \( \forall t \in \Omega \) and \( k = 1, \ldots, N \),

\[ z_k \geq \sum_{j=0}^{m} (\ln H_j(x_k) - \ln H_{0j} - b^T x_k) I_j(t), \]  

(15)

\[ z_k \geq \sum_{j=0}^{m} (\ln H_{0j} + b^T x_k - \ln H_j(x_k)) I_j(t). \]  

(16)

The last constraints can be rewritten as

\[ z_k \geq \ln H_j(x_k) - \ln H_{0j} - b^T x_k, \quad j = 0, \ldots, m, \]  

(17)

\[ z_k \geq b^T x_k + \ln H_{0j} - \ln H_j(x_k), \quad j = 0, \ldots, m. \]  

(18)

Note that term \( b^T x_k \) does not depend on \( j \). This implies that the constraints can be reduced to the following simple constraints:

\[ z_k \geq Q_k - b^T x, \quad k = 1, \ldots, N, \]  

(19)

\[ z_k \geq b^T x - R_k, \quad k = 1, \ldots, N, \]  

(20)

where

\[ Q_k = \max_{j=0,\ldots,m} \left( \ln H_j(x_k) - \ln H_{0j} \right), \]  

(21)

\[ R_k = \min_{j=0,\ldots,m} \left( \ln H_j(x_k) - \ln H_{0j} \right). \]  

(22)

Finally, we get the linear optimization problem with \( d + N \) optimization variables \( (z_1, \ldots, z_N \) and \( b ) \) and \( 2N \) constraints. It is of the form:

\[ \min_{b} \sum_{k=1}^{N} w_k z_k, \]  

(23)

subject to

\[ z_k \geq Q_k - x_k b^T, \quad k = 1, \ldots, N, \]  

(24)

\[ z_k \geq x_k b^T - R_k, \quad k = 1, \ldots, N. \]  

(25)

Finally, we write the following scheme of Algorithm [1]
Algorithm 1 The algorithm for computing vector $b$ for point $x$ in SurvLIME-Inf

**Require:** Training set $D$; point of interest $x$; the number of generated points $N$; the black-box survival model for explaining $f(x)$

**Ensure:** Vector $b$ of important features

1: Compute the baseline CHF $H_0(t)$ of the approximating Cox model on dataset $D$ by using the Nelson–Aalen estimator
2: Generate $N - 1$ random nearest points $x_k$ in a local area around $x$, point $x$ is the $N$-th point
3: Get the prediction of $H(t|x_k)$ by using the black-box survival model (the function $f$)
4: Compute weights $w_k = K(x, x_k)$ of perturbed points, $k = 1, ..., N$
5: Find vector $b$ by solving the convex optimization problem (23)-(25)

7 Numerical experiments

7.1 Synthetic data

In order to investigate the proposed method, random survival times to events are generated by using the Cox model estimates. For experiments, we randomly generate $N = 1000$ covariate vectors $x \in \mathbb{R}^d$ from the uniform distribution in the $d$-sphere with predefined radius $R = 8$. Here $d = 5$. The center of the sphere is $p = (0, 0, 0, 0, 0)$. There are several methods for the uniform sampling of points $x$ in the $d$-sphere with the unit radius $R = 1$, for example, [6, 22]. Then every generated point is multiplied by $R$.

We use the Cox model estimates to generate random survival times, applying results obtained by Bender et al. [7] for survival time data for the Cox model with Weibull distributed survival times. The Weibull distribution with the scale $\lambda = 10^{-5}$ and shape $v = 2$ parameters is used to generate appropriate survival times for simulation studies because this distribution shares the assumption of proportional hazards with the Cox regression model [7]. If we take the vector $b^T = (-0.25, 10^{-6}, -0.1, 0.35, 10^{-6})$, then the following expression can be used for generating survival times [7]:

$$T = \left(\frac{-\ln(U)}{\lambda \exp(b^T x)}\right)^{1/v},$$

(26)

where $U$ is the random variable uniformly distributed in interval $[0, 1]$.

It can be seen that vector $b$ has two almost zero-valued elements and three “large” elements which will correspond to important features. Generated values $T_i$ are restricted by the condition: if $T_i > 2000$, then $T_i$ is replaced with value 2000. The event indicator $\delta_i$ is generated from the binomial distribution with probabilities $\Pr\{\delta_i = 1\} = 0.9$, $\Pr\{\delta_i = 0\} = 0.1$.

Perturbations can be viewed as a step of the algorithm. According to it, $N$ nearest points $x_k$ are generated in a local area around $x$. These points are uniformly generated in the $d$-sphere with some predefined radius $r = 0.5$ and with the center at point $x$. Weights to every point are assigned as follows:

$$w_k = 1 - (r^{-1} \cdot \|x - x_k\|_2)^{1/2}.$$  

(27)

To compare vectors $b$, we introduce the following notation: $b_{\text{model}}$ are coefficients of the Cox model which is used as the black-box model; $b_{\text{true}}$ are coefficients used for training data generation (see (26)); $b_{\text{expl}}$ are explaining coefficients obtained by using the proposed explanation algorithm.

One of the aims of numerical experiments is to consider the method behavior by assuming that the black-box model is the Cox model. With these experiments, we have an opportunity to compare the
vector $b^{true}$ with vectors $b^{model}$ and $b^{expl}$ because the black-box Cox model as well as the explanation Cox model are expected to have close vectors $b$. We cannot perform the same comparison by using the RSF as a black-box model. Therefore, the results with the RSF will be compared by considering the proximity of SFs obtained from the RSF and the explanation Cox model.

To evaluate the algorithm, 900 instances are randomly selected from every cluster for training and 100 instances are for testing. In the test phase, the optimal explanation vector $b^{expl}$ is computed for every point from the testing set. In accordance with the obtained vectors $b^{expl}$, we depict the best, mean and worst approximations on the basis of the Euclidean distance between vectors $b^{expl}$ and $b^{model}$ (for the Cox model) and with Euclidean distance between $H(t_j|\mathbf{x}_i)$ and $H_{Cox}(t_j|\mathbf{x}_i, b^{expl}_i)$ (for the RSF). In order to get these approximations, points with the best, mean and worst approximations are selected among all testing points.

The three cases (best (pictures in the first row), mean (pictures in the second row) and worst (pictures in the third row)) of approximations for the black-box Cox model are depicted in Fig. 1. Left pictures show values of important features $b^{expl}$, $b^{model}$ and $b^{true}$. It can be seen from these pictures that all experiments show very clear coincidence of important features for all models. Right pictures in Fig. 1 show SFs computed by using the black-box Cox model and the Cox approximation. It follows from the pictures that the approximation is perfect even for the worst case.

Similar results for the black-box RSF model are shown in Fig. 2 where three pictures correspond to the best, mean and worst approximations. The important features are not shown in Fig. 2 because
RSF does not provide the important features like the Cox model. However, it follows from the SFs in Fig. 2 that the proposed method provides the perfect approximation of the RSF output by the Cox model.

It is interesting to point out that SurvLIME-Inf provides better results in comparison with SurvLIME for small amounts of training data. In order to study the method under this condition, the black-box Cox model and the RSF are trained on 10, 20, 30, 40 examples. The models are tested on 10 examples. Numerical results for the Cox model are shown in Fig. 3 where rows correspond to 10, 20, 30, 40 training examples, respectively, left pictures in every row show relationships between SFs obtained from the black-box Cox model and from the Cox approximation by using SurvLIME, the same relationships by using SurvLIME-Inf are depicted in right pictures. One can see from Fig. 3 that SurvLIME-Inf provides better approximations of SFs in comparison with SurvLIME for cases of 10, 20, 30 training examples. However, it can be seen from the last row (40 training examples) that SurvLIME becomes better with increase of the training set.

Measures $RMSE_{\text{model}}$ and $RMSE_{\text{true}}$ as functions of the sample size $n$ for SurvLIME and SurvLIME-Inf are provided in Table 1 for comparison purposes. They are defined for the Cox model from $n_{\text{test}}$ testing results as follows:

$$RMSE_{\text{type}} = \left( \frac{1}{n_{\text{test}}} \sum_{i=1}^{n_{\text{test}}} \left\| \mathbf{b}_{i,\text{type}} - \mathbf{b}_{i,\text{expl}} \right\|_2^2 \right)^{1/2},$$  \hspace{1cm} (28)

where is “model” and “true” is substituted into the above expression in place of “type”.

$RMSE_{\text{model}}$ characterizes how the obtained important features coincide with the corresponding features obtained by using the Cox model as the black-box model. $RMSE_{\text{true}}$ considers how the obtained important features coincide with the features used for generating the random times to events.

It can be seen from Table 1 that SurvLIME-Inf outperforms SurvLIME for small $n$, namely, for $n = 10$ and 20. At the same time, this outperformance disappears with increasing $n$, i.e., when $n = 30$ and 40. This is a very interesting observation which tells us that SurvLIME-Inf should be used when the training set is very small.

The same experiments are carried out for the RSF. They are shown in Fig. 4.

### 7.2 Real data

Let us apply the following well-known real datasets to study the method.
Figure 3: Comparison of the SFs relationship for SurvLIME and SurvLIME-Inf with using the black-box Cox model by 10, 20, 30, 40 training examples

Table 1: Approximation measures for four cases of using the black-box Cox model by the small amount of data for SurvLIME and SurvLIME-Inf

| n  | $RMSE_{model}$ | $RMSE_{true}$ | $RMSE_{model}$ | $RMSE_{true}$ |
|----|----------------|---------------|----------------|---------------|
| 10 | 0.719          | 0.809         | 0.290          | 0.575         |
| 20 | 0.659          | 0.664         | 0.358          | 0.460         |
| 30 | 0.347          | 0.428         | 0.398          | 0.432         |
| 40 | 0.324          | 0.344         | 0.388          | 0.451         |
Figure 4: Comparison of the SFs relationship for SurvLIME and SurvLIME-Inf with using the RSF by 10, 20, 30, 40 training examples
The Veterans’ Administration Lung Cancer Study (Veteran) Dataset [28] contains data on 137 males with advanced inoperable lung cancer. The subjects were randomly assigned to either a standard chemotherapy treatment or a test chemotherapy treatment. Several additional variables were also measured on the subjects. The number of features is 6, but it is extended till 9 due to categorical features.

The NCCTG Lung Cancer (LUNG) Dataset [33] records the survival of patients with advanced lung cancer, together with assessments of the patients performance status measured either by the physician and by the patients themselves. The data set contains 228 patients, including 63 patients that are right censored (patients that left the study before their death). The number of features is 8, but it is extended till 11 due to categorical features.

The Primary Biliary Cirrhosis (PBC) Dataset contains observations of 418 patients with primary biliary cirrhosis of the liver from the Mayo Clinic trial [17], 257 of whom have censored data. Every example is characterized by 17 features including age, sex, ascites, hepatom, spiders, edema, bili and chol, etc. The number of features is extended till 22 due to categorical features.

The above datasets can be downloaded via the “survival” R package.

Fig. 5 illustrates numerical results for the Veteran dataset. We provide only the case of the mean approximation in order to reduce the number of similar pictures. Fig. 5 contains three pictures: the first one illustrates the explanation important features and important features computed by using the Cox model; the second picture shows two SFs for the Cox model; the third picture shows two SFs for the RSF. It follows from Fig. 5 that the method provides appropriate results for the real dataset.

Similar numerical results for the LUNG and PBC datasets are shown in Fig. 6 and Fig. 7, respectively.

8 Conclusion

A new modification of SurvLIME using the $L_\infty$-norm for computing the distance between CHFs instead of the $L_2$-norm has been presented in the paper. The basic idea behind both the methods is to approximate a survival machine learning model at a point by the Cox proportional hazards model which assumes a linear combination of the instance covariates. However, this idea is differently implemented in SurvLIME and SurvLIME-Inf. SurvLIME-Inf extends the set of explanation methods dealing with censored data in the framework of survival analysis. Numerical experiments with
Figure 6: The mean approximation for the Cox model (the first and the second picture) and the RSF (the third picture) trained on the LUNG dataset.

Figure 7: The mean approximation for the Cox model (the first and the second picture) and the RSF (the third picture) trained on the PBC dataset.
synthetic and real datasets have clearly illustrated accuracy and correctness of SurvLIME-Inf.  

The main advantage of SurvLIME-Inf is that it uses the linear programming for computing coefficients of the approximating Cox model. This peculiarity allows us to develop new methods taking into account inaccuracy of training data, possible imprecision of data. This is an interesting and important direction for further research. Another problem, which can be solved by using SurvLIME-Inf, is to explain machine learning survival models by using the Cox model with time-dependent covariates. This is also an important direction for further research.

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