Theoretical study of an unsteady ciliary hemodynamic fluid flow subject to the Newton’s boundary conditions

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Abstract
This article addresses the hemodynamic flow of biological fluid through a symmetric channel. Methachronal waves induced by the ciliary motion of motile structures are the main source of Couple stress nanofluid flow. Darcy’s law is incorporated in Navier-Stokes equations to highlight the influence of the porous medium. Thermal transport by the microscopic collision of particles is governed by Fourier’s law while a separate expression is obtained for net diffusion of nanoparticles by using Fick’s law. A closed-form solution is achieved of nonlinear differential equations subject to Newton’s boundary conditions. Moreover, the current findings are compared with previous outcomes for the limiting case and found a complete coherence. Parametric study reveals that nanoflow is resisted by employing Newton’s boundary conditions. Thermal profile enhancement is contributed by the viscous dissipation parameter. Finally, one infers that hemodynamic flow of non-Newtonian fluid is an effective mode of heat and mass transfer especially, in medical sciences for the rapid transport of medicines in drug therapy.

Keywords
Ciliary motion, methachronal wave, Newton’s boundary conditions, porous medium, drug therapy

Introduction
The hemodynamic flow of non-Newtonian fluids is of great importance. Generally speaking, it is referred to the flow of blood to different of the human body. Hemodynamic flows are usually governed by the process of peristaltic or ciliated walls of the different organs, arteries, and tissues. Unlike pre-existing reports on nanofluids, the main emphasis of Ellahi et al.\textsuperscript{1} is to further enhance the heat transfer rate of nano species. In this connection, the simultaneous contribution of activation energy and chemical reaction are taken into account which improves the thermal properties of gold particles. They devised the medical use of gold particles, owing to the best metallic thermal conductivity. In Akbar et al.\textsuperscript{2} magnetohydrodynamic (MHD) flow of viscoelastic fluid with heat is presented. Since, Casson

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fluid model is the robust non-Newtonian fluid model which is the most appropriate one, to predict viscoplastic behavior of fluid that is transported due to cilia beating on the inner walls of the channel. Their findings include that large values of the Casson fluid parameter is consequently, elevates the quantity and size of the trapped boluses. Whereas, the short length of the cilia structure has an opposite effect on the pressure rise, in the pumping region. On the contrary, Mann et al.\textsuperscript{3} does not merely, consider the metachronal wave which generates the motion of the fluid in any geometry. But, they have elaborated the impacts of two separate types of propagation of waves namely; symplectic wave and antiepileptic wave on the motion of Burgers fluid. Since it was difficult to analyze the nonlinear flow phenomenon, therefore, the fractional Adomian decomposition method is employed to evaluate the pressure gradient. Nazeer et al.\textsuperscript{4,5} are relevant to the bio-magnetic fluid bounded by ciliated walls. Two-dimensional cilia-driven non-Newtonian fluid has been discussed under the influence of transverse magnetic fields. Closed-form solutions are obtained to elaborate the altering behaviors of field variables. Some other notable findings of Couple stress fluid can be seen.\textsuperscript{6–8}

Flows through a porous medium and bounded by slippery walls of the channel are reported, to highlight the significant role in mechanical and industries.\textsuperscript{9–15} In El-Dabe et al.\textsuperscript{16,17} explore the magnetohydrodynamics of respectively Power-law fluid and Williamson fluid, respectively. For an effective role of porous medium separately, non-Darcy law and simply Darcy law are employed. Zaman et al.\textsuperscript{18} prefer a numerical solution rather than an analytic solution for Numerical simulation of the pulsatile flow of blood in a porous-saturated over-lapping stenosed artery. However, in Machireddy and Kattamreddy\textsuperscript{19} an analytic solution for an MHD peristaltic flow is achieved by applying partial boundary conditions on a porous channel. Ellahi et al.\textsuperscript{20} used “Separation of variables” for a two-dimensional Jeffrey fluid. Gradual decline in the momentum of non-Newtonian fluid is reported against the porous media, which is quite unlike the previous cases. In Ellahi et al.\textsuperscript{21} authors had to opt numerical scheme since, it was difficult to predict the characteristics of the flow, due to the pertinent variables. The two-phase flow of Couple stress fluid is simulated with the help of the Range-Kutta method with the shooting technique, which highlights the resistive contribution of lubricated walls of the channel.

Heat and mass transfer through fluids is not a casual routine in mechanical and chemical engineering.\textsuperscript{22–26} Awais et al.\textsuperscript{27} simulated the effects of heat and mass transfer on the flow of Casson fluid. A permeable duct of uniform shape is chosen for dual expressions for the profiles of velocity, temperature, and mass fraction have been computed numerically by exploitation of explicit Runge-Kutta procedure. Analytic solution by Krishna and Chamkha\textsuperscript{28} interprets the simultaneous impacts of electric fields and magnetic fields. Viscoelastic fluid revolves due to the rotation of opposite plates bounding the fluid. Two different hemodynamics is given in Srinivas et al.\textsuperscript{29} and Hayat et al.\textsuperscript{30} deal with the transport of heat under influence of lubrication governed by the flow mechanism of peristalsis. An exact solution of a complex non-Newtonian fluid model is presented by Chu et al.\textsuperscript{31} The study deals with the properties of thermal radiation, heat generation, and the effect of convective boundary conditions through a duct with the Rabinowitsch fluid. In another attempt, Chu et al.\textsuperscript{32} have focused on minimizing the entropy production on Rabinowitsch fluid through a tilted channel. To achieve this goal, two different cases have been assumed. In the first case, the viscosity and thermal conductivity of the fluid are treated as a constant. While in lateral case both viscosity and thermal conductivity vary.

In view of the above, a brief and compact study of existing literature indicates that heat and mass transfer utilizing the hemodynamic flow of a biological fluid is not reported, yet. Moreover, a comparative analysis of two separate locomotion of nanofluids subject to Newton’s boundary conditions is a new idea. Finally, the main objective of the current study is to decide whether the Newtonian fluid highlights hemodynamic flow or not.

**Problem formulation**

Consider a two-dimensional and unsteady flow of a biological fluid with heat and mass transfer through an asymmetric channel as shown in Figure 1. Biological flow is caused by the ciliary motion of tiny motile structures, which propagate metachronal waves on the opposite walls having a constant speed $c$.

The back and forth motion of tiny hair cilia\textsuperscript{4,5} may be pretended from the following equation
\[ Y_2 = F_2(Y_1, t_1) = \pm (a + a\cos(2\pi \lambda (Y_1 - c t_1))) = \pm H. \quad (1) \]

The elliptical movement of cilia verified by Sleigh in his experimental study.\textsuperscript{33} Therefore, the vertical position of the cilia tips can be expressed as

\[ Y_1 = F_1(Y_1, t_1) = Y_0 + a\alpha \sin\left(\frac{2\pi \lambda}{\lambda} (Y_1 - c t_1) \right) \quad (2) \]

Fluid velocities are assembled by cilia tips without slip conditions are given as Nazeer et al.\textsuperscript{4,5}

\[ W_1 = \frac{\partial Y_1}{\partial t_1} |_{t_0} = \frac{\partial F_1}{\partial t_1} + \frac{\partial F_1}{\partial Y_1} \frac{\partial Y_1}{\partial t_1} = \frac{\partial F_1}{\partial t_1} + \frac{\partial F_1}{\partial Y_1} W_1, \quad (3) \]

\[ W_2 = \frac{\partial Y_2}{\partial t_1} |_{t_0} = \frac{\partial F_2}{\partial t_1} + \frac{\partial F_2}{\partial Y_1} \frac{\partial Y_1}{\partial t_1} = \frac{\partial F_2}{\partial t_1} + \frac{\partial F_2}{\partial Y_1} W_2. \quad (4) \]

Longitudinal and transverse velocities are easily illustrated in view of equations (1) to (4) respectively

\[ W_1 = \frac{-\left(\frac{\pi}{\lambda}\right) a \alpha c \cos(\frac{2\pi \lambda}{\lambda} (Y_1 - c t_1))}{1 - \left(\frac{\pi}{\lambda}\right) a \alpha c \sin(\frac{2\pi \lambda}{\lambda} (Y_1 - c t_1))}, \quad (5) \]

\[ W_2 = \frac{-\left(\frac{\pi}{\lambda}\right) a \alpha c \sin(\frac{2\pi \lambda}{\lambda} (Y_1 - c t_1))}{1 - \left(\frac{\pi}{\lambda}\right) a \alpha c \cos(\frac{2\pi \lambda}{\lambda} (Y_1 - c t_1))}. \quad (6) \]

To mathematically model the ciliary motion of biological fluid, stress tensor of Couple stress fluid is brought under consideration. Therefore, the governing equations relevant to the heat and mass transfer of Couple stress fluid are listed as:

\[ \frac{\partial W_1}{\partial Y_1} + \frac{\partial W_2}{\partial Y_2} = 0, \quad (7) \]

\[ \rho \left( \frac{\partial W_1}{\partial t_1} + W_1 \frac{\partial W_1}{\partial Y_1} + W_2 \frac{\partial W_1}{\partial Y_2} \right) = - \frac{\partial P}{\partial Y_1} + \mu_0 \left( \frac{\partial^2 W_1}{\partial Y_1^2} + \frac{\partial^2 W_1}{\partial Y_2^2} \right) - \frac{\mu_0}{k_1} W_1 \quad \left( \frac{\partial^2 W_1}{\partial Y_1^2} + \frac{\partial^2 W_1}{\partial Y_2^2} \right), \quad (8) \]

Transport of heat and mass through a non-Newtonian fluid is studied by employing Newton’s boundary conditions,\textsuperscript{18,34,35}

\[ \frac{\partial W_1}{\partial Y_1} \bigg|_{t_0} = \frac{\partial W_1}{\partial Y_1} = \frac{\partial W_2}{\partial Y_1} = \frac{\partial W_1}{\partial Y_1} = 0, \quad \text{at } Y_2 = 0. \quad (12) \]

\[ \frac{\partial W_1}{\partial Y_1} = 0, \quad W_1 + m_4 \frac{\partial W_1}{\partial Y_1} = -w_0, \quad (10) \]

\[ \frac{\partial W_2}{\partial Y_2} = 0, \quad \frac{\partial W_1}{\partial Y_2} = \frac{\partial W_2}{\partial Y_2} = D_m \left( \frac{\partial W_1}{\partial Y_1} + \frac{\partial W_2}{\partial Y_2} \right), \quad (11) \]

The most appropriate approach to investigating cilia-driven flow is to first introduce the following transformations in the above set of governing equations (7) to (11),

\[ y_1 = Y_1 - c t_1, \quad y_2 = Y_2, \quad \left\{ \begin{array}{l} p(y_1, y_2) = P(Y_1, Y_2), \\ w_1(y_1, y_2) = W_1(Y_1, Y_1, t_1) - c, \\ w_2(y_1, y_2) = W_2(Y_1, Y_1, t_1). \end{array} \right. \quad (14) \]

And, subsequently using the dimensionless quantities defined in (15) in the transformed form of the differential equations

\[ \frac{\partial W_1}{\partial Y_1} = \frac{\partial W_2}{\partial Y_1} = 0, \quad (12) \]

\[ \frac{\partial W_1}{\partial Y_1} = \frac{\partial W_2}{\partial Y_2} = \frac{\partial W_1}{\partial Y_1} = \frac{\partial W_1}{\partial Y_2} = 0, \quad \text{at } Y_2 = 0. \quad (12) \]

\[ \frac{\partial W_1}{\partial Y_1} = 0, \quad W_1 + m_4 \frac{\partial W_1}{\partial Y_1} = -w_0, \quad (10) \]

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The most appropriate approach to investigating cilia-driven flow is to first introduce the following transformations in the above set of governing equations (7) to (11),

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And, subsequently using the dimensionless quantities defined in (15) in the transformed form of the differential equations

\[ \frac{\partial W_1}{\partial Y_1} = \frac{\partial W_2}{\partial Y_1} = 0, \quad (12) \]
Finally, one can easily yield the dimensionless form of differential equations are expressed as:

\[
\text{Re} \left( \frac{\partial \hat{w}_1}{\partial t_1} + (\hat{w}_1 + 1) \frac{\partial \hat{w}_1}{\partial y_1} + \hat{w}_2 \frac{\partial \hat{w}_1}{\partial y_2} \right) = -\frac{\partial \hat{p}}{\partial y_1} + \nu \left( \eta^2 \frac{\partial^2 \hat{w}_1}{\partial y_1^2} + \frac{\partial^2 \hat{w}_1}{\partial y_2^2} \right) - \frac{1}{m_1^2} \left( \eta^4 \frac{\partial^4 \hat{w}_1}{\partial y_1^4} + 2\eta^2 \frac{\partial^2 \hat{w}_1}{\partial y_1^2} \frac{\partial^2 \hat{w}_1}{\partial y_2^2} + \frac{\partial^4 \hat{w}_1}{\partial y_2^4} \right) - \frac{1}{m_2} (\hat{w}_1 + 1),
\]

\( (\eta \ll 1), \) the reduced form of momentum, thermal, and mass diffusion equations are:

\[
\frac{\partial ^2 \hat{w}_1}{\partial y_2^2} + \frac{1}{m_1^2} \left( \frac{\partial^4 \hat{w}_1}{\partial y_2^4} \right) - \frac{1}{m_1^2} \left( \frac{\partial^2 \hat{w}_1}{\partial y_2^2} \right) + \frac{1}{m_2^2} (\partial^2 \hat{w}_1) \frac{\partial \hat{w}_1}{\partial y_2^2} + \frac{1}{m_2} (\hat{w}_1 + 1) + m_3 = 0,
\]

\[
\frac{\partial ^2 \hat{\theta}}{\partial y_2^2} + \frac{1}{m_7^2} \left( \frac{\partial^4 \hat{\theta}}{\partial y_2^4} \right) - \frac{1}{m_7^2} \left( \frac{\partial^2 \hat{\theta}}{\partial y_2^2} \right) + \frac{1}{m_8} (\partial^2 \hat{\theta}) \frac{\partial \hat{\theta}}{\partial y_2^2} = 0.
\]

Similarly, the set of dimensionless momentum, thermal, and mass flux boundary conditions are:

\[
\frac{\partial \hat{w}_1}{\partial y_2} = \frac{\partial ^2 \hat{w}_1}{\partial y_2^2} = \frac{\partial \hat{\theta}}{\partial y_2} = \frac{\partial \hat{\varphi}}{\partial y_2} = 0, \text{ at } y_2 = 0,
\]

\[
\frac{\partial ^2 \hat{w}_1}{\partial y_2^2} = 0, \quad \hat{w}_1 + m_4 \frac{\partial \hat{\theta}}{\partial y_2} = -1 - w_0, \quad m_6 \frac{\partial \hat{\varphi}}{\partial y_2} + \hat{\varphi} = 0, \text{ at } y_2 = h,
\]

**Solution of the problem**

Since, the thermal differential equation is of nonlinear form and coupled with momentum equation which is solved exactly, along with concentration, subject to the boundary condition (24) and (25). The closed-form solutions for the ciliary motion of Couple stress fluid is achieved as

\[
\hat{w}_1 = 1 - m_2 m_3 + \frac{1}{\text{cosh}[\hat{h}_1 a_{10}] \text{cosh}[\hat{v}_2 a_{20}] (a_{10}^2 - a_{20}^2) + a_{10}a_{20}} \left( \text{cosh}[\hat{h}_1 a_{10}] \text{cosh}[\hat{v}_2 a_{20}] (1 + m_2 m_3 - w_0) \right) \text{cosh}[\hat{h}_1 a_{10}] \text{cosh}[\hat{v}_2 a_{20}] (a_{10}^2 - a_{20}^2) + a_{10}a_{20} \right). \]

A mathematical equation that describes the thermal transport of the biological fluid is given as

\[
\hat{\theta} = b_{10} \text{cosh}[2\hat{v}_1 a_{10}] + b_{20} \text{cosh}[2\hat{v}_2 a_{20}]
+ b_{30} \text{cosh}[\hat{v}_1 a_{10}] \text{cosh}[\hat{v}_2 a_{20}]
+ b_{40} \text{sinh}[\hat{v}_1 a_{10}] \text{sinh}[\hat{v}_2 a_{20}] + b_{50} v^2 + b_{60}.
\]

Similarly, the mass diffusion through Couple stress fluid can be achieved as

\[
\hat{\varphi} = c_{10} - m_7 m_8 \left( \text{cosh}[2\hat{v}_1 a_{10}] b_{10} + \text{cosh}[2\hat{v}_2 a_{20}] b_{20}
+ \text{cosh}[\hat{v}_1 a_{10}] \text{cosh}[\hat{v}_2 a_{20}] b_{30}
+ \text{sinh}[\hat{v}_1 a_{10}] \text{sinh}[\hat{v}_2 a_{20}] b_{40} + v^2 b_{50} \right).
\]
The value of \( w_0, a_{10}, a_{20}, b_{10}, b_{20}, b_{30}, b_{40}, b_{50}, b_{60} \) and \( c_{10} \) are given in Appendix.

**Results and discussion**

This segment of the article is furnished to carry out a comprehensive parametric study of heat and mass transfer of Couple stress fluid. The pertinent parameters which are taken into account are the porosity parameter \( m_2 \), wave number \( \beta \), velocity slip parameter \( m_4 \), eccentricity parameter \( \alpha \), viscous heating parameter \( m_5 \), Biot number \( m_6 \), Schmidt number \( m_7 \), Soret number \( m_8 \), and Couple stress parameter \( m_1 \). Furthermore, the flow dynamics of heated Couples stress nanofluid are compared with Newtonian nanofluid for the limiting case \((m_1 \to 0)\). Comparison between Newtonian and non-Newtonian nanofluid flows are highlighted by dashed sketches \((m_1 \neq 0)\) for Couples stress nanofluid, while solid sketches \((m_1 \to 0)\) for Newtonian nanofluid flow.

Variations in the momentum of the Couple stress nanofluid against the pertinent parameters are shown in Figures 2 to 5. The effective role of the porous medium is displayed in Figure 2. It is seen that by increasing the permeability of the medium, the flow of Couples stress fluid gradually increases its velocity. This phenomenon is apprehended as the resistive force is dampened by increasing the value of \( m_2 \). By increasing the number of waves, the velocity of the biological fluid gets more momentum as shown in Figure 3. Similarly, the momentum of the fluid caused by the back and forth motion of tiny hair-like structure enhances in Figure 4, as well, subject to the variation in eccentricity parameter. On the contrary, a gradual decline in the velocity of the biological nanofluid is witnessed in Figure 5. It is noted that lubrication effects at
the cilia walls, introduce the resistive force on the fluid which results to impede the flow. Nevertheless, the significant feature of metachronal motion of Couple stress nanofluid through the lubricated walls, is that non-Newtonian nanofluid is much supported by the motile cilia, which is evident from the comparison Newtonian nanofluid. Transportation of heat is analyzed in Figures 6 to 8 against effectively contributing parameters. Effects of viscous heating parameter on the heat transfer of Couple stress nanofluid are shown in Figure 6. Variation in the dimensionless quantity contributes to generating more thermal energy, due to the fact that the temperature is directly related to the dimensionless quantity $m_s$. Increasing the value of the viscous heating parameter results in dominant viscous dissipation. This in return yields a weaker heat transfer. Hence, more thermal energy is added to the system of nanofluid flow. An opposite trend in the temperature profile is observed in Figures 7 and 8, respectively. It is seen that the thermal energy of nanofluid flow expunges...
by increasing the velocity slip effects in Figure 7. On the other hand, weak thermal conductivity is the consequence of a higher Biot number, which also results in a thermal decline in Figure 8. The trapping phenomenon is one of the most important outcomes of cilia-driven flow. The emergence of circulating boluses is the known as “Trapping phenomenon.” This always indicates the presence of resistance at the edge or boundary of the channel. Variation of pertinent parameters describes the change in shape and size of the bolus in Figures 9 to 12. Figures 9 and 10 shows the variation of porosity parameter on Newtonian nanofluid and non-Newtonian nanofluid. Since, Figure 2, clearly indicates that Couple stress fluid flow is more compressed, by the cilia walls of the channel. Therefore, the number of boluses increase, as compared to the case of Newtonian flow, when $m_2$ is varied from 0.3 to 0.8, respectively. Application of slip effects at the opposite walls hampers the motion of nanofluid flow as to be seen in Figure 5. Since velocity slip parameter $m_4$ resists the velocity of both types of nano flows, therefore, varying the dimensionless parameter produces more number of circulating boluses for Couple stress nanofluid flow in Figures 11 and 12.

**Numerical findings**

Since the present study is a comparative investigation of two different types of biological flows. Couple stress fluid is considered as the biological which is the main source of heat transfer and mass transfer utilizing metachronal waves. In this regard, some significantly important dimensionless quantities, due to their implementations in science and engineering have emerged during the process. Two of such significance are defined as:

\[
Nu = \frac{\partial h}{\partial y_1} \left( \frac{\partial \theta}{\partial y_2} \right)_{y_2 = h}. \tag{29}
\]

\[
Sh = \frac{\partial h}{\partial y_1} \left( \frac{\partial \phi}{\partial y_2} \right)_{y_2 = h}. \tag{30}
\]

Numerical data showing the variation in Nusselt number and Sherwood number against different parameters have been computed in Tables 1 and 2, respectively. Table 1 is constructed to numerically analyze the contribution of porous medium parameters $m_2$, velocity slip parameter $m_4$, and viscous heating $m_5$, on the behavior of Nusselt number. Computed data suggests that convective heat transfer is dominant for porosity.
parameters and viscous heating for both kinds of flows. However, the heat transfer rate is inversely affected by the excessive lubrication effects. Moreover, the heat transfer rate in Couple stress fluid is higher than Newtonian fluid. Table 2 displays the mass transfer rate against porous medium parameters $m_2$, Schmidt number $m_7$, and Soret number $m_8$. This is vivid from the computed data that all concerned parameters effectively contribute to improving mass transfer rate. However, it is inferred that non-Newtonian fluids are more suitable for heat mass transfer than Newtonian fluid, especially in medical sciences to deal with drug therapy.

### Comparative analysis

The obtained solution is further compared with the existing literature. Ramesh\textsuperscript{36} pertains to the effects of slip and convective conditions on the peristaltic flow of Couple stress fluid through a porous medium. Since the previous study merely discusses the transport of non-Newtonian fluid due to the flexibility of the opposite walls devoid of cilia structure. Therefore, the comparison is carried out for the limiting case cilia length parameter $e \to 0$. One can easily conclude from Table 3 that the momentum of Couple stress fluid subject to the applied constraints are in good agreement.

### Conclusions

Heat and mass transfer of a biological fluid is investigated. Methachronal waves induced by the ciliary motion of motile structure are the main source of

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**Table 1.** Variation in Nusselt number coefficient against different parameters.

| $m_2$ | $m_4$ | $m_5$ | Couple stress fluid | Newtonian fluid |
|-------|-------|-------|---------------------|-----------------|
| 0.1   | 0.2   | 0.2   | 2.29927             | 0.30962         |
| 0.3   |       |       | 4.66861             | 0.34623         |
| 0.5   |       |       | 7.31077             | 0.36304         |
| 0.2   | 0.1   | 0.15  | 6.80048             | 0.38680         |
| 0.3   |       |       | 3.99104             | 0.17376         |
| 0.5   |       |       | 2.62125             | 0.09826         |
| 0.2   | 0.1   |       | 3.41220             | 0.16609         |
| 0.3   |       |       | 10.23660            | 0.49829         |
| 0.5   |       |       | 17.06101            | 0.83049         |

**Table 2.** Variation of Sherwood number coefficient against different parameters.

| $m_2$ | $m_7$ | $m_8$ | Couple stress fluid | Newtonian fluid |
|-------|-------|-------|---------------------|-----------------|
| 0.1   | 0.1   | 0.2   | 8.86161             | 0.001657        |
| 0.3   |       |       | 26.18454            | 0.002148        |
| 0.5   |       |       | 43.52814            | 0.002360        |
| 0.15  | 0.15  | 0.1   | 9.89226             | 0.001376        |
| 0.30  | 0.30  |       | 19.78452            | 0.002752        |
| 0.45  | 0.45  |       | 29.67679            | 0.004138        |
| 0.2   | 0.2   |       | 26.37937            | 0.003670        |
| 0.4   | 0.4   |       | 52.75874            | 0.007340        |
| 0.6   | 0.6   |       | 79.13811            | 0.011010        |

**Table 3.** Current findings vs previous findings.

| $Y_2$  | Ramesh\textsuperscript{36} | Our results |
|--------|-----------------------------|-------------|
| -1.50  | -0.99999                    | -1.00000    |
| -1.20  | -0.46005                    | -0.46006    |
| -0.9   | -0.04000                    | -0.04008    |
| -0.6   | 0.26003                     | 0.26003     |
| -0.3   | 0.44005                     | 0.44005     |
| 0.0    | 0.50005                     | 0.50006     |
| 0.3    | 0.44005                     | 0.44005     |
| 0.6    | 0.26003                     | 0.26003     |
| 0.9    | -0.04000                    | -0.04000    |
| 1.20   | -0.46005                    | -0.46006    |
| 1.50   | -0.99999                    | -1.00000    |
Couple stress nanofluid flow. To dampen the skin friction of opposite walls of the symmetric channel lubrication effects are incorporated. An exact solution is achieved for the set of nonlinear differential equations. Some significant findings of the investigation are listed below:

- The velocity of nanofluid is resisted by employing Newton’s boundary conditions.
- Porous medium contributes to rising the momentum of the flow.
- Thermal energy rises due to the viscous dissipation parameter.
- Couple stress fluid is more suitable for hemodynamic flow than Newtonian fluid.
- Heat transfer rate is inversely affected by the excessive lubrication effects.
- Wall resistance is more prominent on the nanoflow of Couple stress fluid.
- The computed results of the current investigation are in full agreement with the previous findings subject to the applied constraints.

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Appendix

\[
\begin{align*}
\omega_1 &= 1 + \frac{2\pi e \alpha \beta \cos[2\pi \rho_1]}{1 - 2\pi e \alpha \beta \cos[2\pi \rho_1]}, \\
\alpha_{10} &= \sqrt{\frac{m_1^2 - m_1^2\sqrt{-4 + m_1^2m_2}}{m_2}}, \\
\alpha_{20} &= \sqrt{\frac{m_1^2 + m_1^2\sqrt{-4 + m_1^2m_2}}{m_2}}, \\
\cosh[\alpha_{10}]\sinh[\alpha_{20}]a_{10}&\left(1 - (f + h)a_{20}^2m_4 + w_0\right) \\
&\quad + \cosh[\alpha_{20}]a_{20}\left(-\sinh[\alpha_{10}]a_{10}^2(-1 + (f + h)a_{10}^2m_4 + w_0)\right) \\
m_1 &= \frac{m_2\left(\cosh[\alpha_{10}]\sinh[\alpha_{20}]a_{10}^2(1 - a_{20}^2m_4) + \cosh[\alpha_{20}]a_{20}\left(h\cosh[\alpha_{10}]a_{10}(-a_{10}^2 + a_{20}^2) + \sinh[\alpha_{10}]a_{10}a_{20}(-1 + (f + h)a_{10}^2m_4)\right)\right)}{\sqrt{\frac{m_1^2 - m_1^2\sqrt{-4 + m_1^2m_2}}{m_2}}}, \\
b_{10} &= \left(-\frac{1}{8}\pi_0^2m_5 - \frac{a_{10}^2a_{20}^2m_5}{8m_1^2}\right), \\
b_{20} &= \left(-\frac{1}{8}\pi_0^2m_5 - \frac{a_{10}^2a_{20}^2m_5}{8m_1^2}\right), \\
b_{30} &= \left(\frac{2a_{10}^2a_{20}^2m_5}{(a_{10}^2 - a_{20}^2)^2}, \\
b_{40} &= \left(-\frac{2a_{10}^2a_{20}^2a_{40}a_{50}m_5}{(a_{10}^2 - a_{20}^2)^2}, \\
b_{50} &= \left(\frac{1}{4}a_{20}^2a_{40}^2m_5 + \frac{1}{4}a_{10}^2a_{50}^2m_5\right)\right).
\end{align*}
\]
\[ b_{60} = -\frac{1}{m_6} \left( 2 \sinh[2ha_{10}]a_{10}b_{10} + 2 \sinh[2ha_{20}]a_{20}b_{20} + \cosh[ha_{20}] \sinh[ha_{10}]a_{10}b_{30} + \cosh[ha_{10}] \sinh[ha_{20}]a_{10}b_{40} + \cosh[ha_{20}] \sinh[ha_{10}]a_{10}b_{50} + \cosh[2ha_{10}]b_{10}m_6 + \cosh[2ha_{20}]b_{20}m_6 + \cosh[ha_{10}] \cosh[ha_{20}]b_{30}m_6 + \sinh[ha_{10}] \sinh[ha_{20}]b_{40}m_6 + h^2b_{50}m_6 \right) \]

\[ c_{10} = m_7m_8 \left( bh_{50}(h + 2a_9) + b_{10}(\cosh[2ha_{10}] + 2 \sinh[2ha_{20}]a_{20}m_9) + \sinh[ha_{20}](\sinh[ha_{10}]b_{40}) + \cosh[ha_{10}](\cosh[ha_{10}]b_{30}) + \sinh[ha_{10}](a_{10}b_{30} + a_{20}b_{40})m_9 \right) \]

**Notation**

- \( \varepsilon \): parameter of cilia length (m)
- \( m_1 \): couple stress parameter
- \( m_2 \): porous medium parameter
- \( m_3 \): pressure gradient
- \( m_4 \): velocity slip parameter
- \( m_5 \): viscous heating parameter
- \( m_6 \): heat transfer Biot number
- \( m_7 \): Schmidt number
- \( m_8 \): Soret number
- \( m_9 \): mass transfer Biot number
- \( \mu \): the viscosity of the fluid (kg m\(^{-1}\) s\(^{-1}\))
- \( (W_1, W_2) \): components of velocity along \((Y_1, Y_2)\) directions (m\(^{-1}\) s\(^{-1}\))
- \( \beta \): wavenumber
- \( \alpha \): account the eccentricity
- \( Re \): Reynolds number
- \( t_1 \): time (s)
- \( c \): speed of wave (m\(^{-1}\) s\(^{-1}\))
- \( \lambda \): wave-length (m)
- \( \rho \): the density of fluid (kg m\(^{-3}\))
- \( k_1 \): permeability of the porous medium
- \( k_2 \): thermal conductivity
- \( a_1 \): mean width of the channel (m)
- \( (w_1, w_2) \): dimensionless velocity components along \((y_1, y_2)\) direction
- \( \vartheta \): dimensionless form of temperature
- \( \phi \): stream Function
- \( D_m \): coefficient of mass diffusivity
- \( E_C \): Eckert number
- \( k_m \): thermal conductivity coefficient
- \( k_h \): mass diffusivity coefficient
- \( T_m \): mean of temperature fluid
- \( Pr \): Prandtl number
- \( h_h \): convective heat coefficient
- \( h_m \): mass transfer coefficient
- \( \varphi \): concentration