Semileptonic $B$ and $D$ decays—latest developments

Giulia Ricciardi
Dipartimento di Scienze Fisiche, Università di Napoli Federico II
Compl. Univ. di Monte Sant’Angelo, Via Cintia, I-80126 Napoli, ITALY

1 Introduction

Semileptonic $B$ and $D$ decays are on a well deserved podium to extract CKM matrix elements, to validate theoretical tools and, more recently, even to look for hints of new physics. They provide information about the CKM matrix elements $|V_{cb}|$, $|V_{ub}|$, $|V_{cd}|$ and $|V_{cs}|$ through exclusive and inclusive processes driven by $b \to c(u)$ and $c \to s(d)$ decays, respectively. All semileptonic $B$ and $D$ decays share the theoretical advantages of being tree level dominated and of a better control of non-perturbative parameters, due to the possibility to factorize leptonic and hadronic currents, with respect to hadronic decays. Inclusive and exclusive decays differ on both theoretical and experimental grounds, and their comparison is a useful probe into strong interaction dynamics.

Here we review the most recent updates. Part of them is already covered in Refs. [1, 2], to which we refer for more details.

2 $b \to c l \nu$ decays

The $b \to c$ decay rates, which are proportional to the CKM matrix element $|V_{cb}|$ squared, are a useful handle to extract its value. Recent data on $rB \to D l \nu$, with $l = e, \mu$, come from Babar [3, 4], on $B \to D^* l \nu$ from Babar [3] and Belle [5]. The differential rate for the latter is more easily measured, since the rate is more than twice and there is no background from mis-reconstructed $B \to D^* l \nu$. The extraction of $|V_{cb}|$ requires the knowledge of the form factors that parameterize the matrix elements. They can be calculated by nonperturbative methods, and this is the main source of theoretical uncertainty. In the case of lepton masses zero, we have only one form factor, which considerably simplify the theoretical approach.

In Table 1 we compare exclusive values of $|V_{cb}|$ extracted by analyzing $B \to D^{(*)} l \nu$ decays. On the left, we list the Collaborations providing the experimental...
Table 1: Comparison of exclusive, inclusive and indirect determinations of $|V_{cb}|$.

In the exclusive section experimental and theoretical errors are listed, respectively, except in the second line, where the errors are respectively statistical, systematic and due to the theoretical uncertainty in the form factor.

| Exclusive Decays | $|V_{cb}|$ (10$^{-3}$) |
|------------------|---------------------|
| Data/Th $(B \to D l \nu)$ | |
| HFAG [6]/Fermilab & MILC [7] | 39.70 ± 1.42$^{\text{exp}}$ ± 0.89$^{\text{th}}$ |
| Babar [4]/lattice SSM (quenched) [8] | 41.6 ± 1.8$^{\text{stat}}$ ± 1.4$^{\text{syst}}$ ± 0.7$^{\text{FF}}$ |
| PDG [9]/non lattice BPS [10] | 40.7 ± 1.5$^{\text{exp}}$ ± 0.8$^{\text{th}}$ |
| $(B \to D^* l \nu)$ | |
| HFAG [6]/Fermilab & MILC [11] | 39.54 ± 0.50$^{\text{exp}}$ ± 0.74$^{\text{th}}$ |
| HFAG [6]/Sum Rules [12] | 41.6 ± 0.6$^{\text{exp}}$ ± 1.9$^{\text{th}}$ |

| Inclusive Decays |
|------------------|
| HFAG [6] (1S scheme) | 41.96 ± 0.45 |
| HFAG [6] (kinetic scheme) | 41.88 ± 0.73 |

| Global CKM fits |
|------------------|
| CKMfitter [13] | 40.69 ± 0.99 |
| UTfit [14] | 42.3 ± 0.9 |

In inclusive $B \to X_q l \nu$ decays, we sum over all possible final states $X_q$, no matter if single-particle or multi-particle states. Since inclusive decays do not depend on the details of final state, quark-hadron duality is generally assumed. In most of the phase space, long and short distance dynamics are factorized by means of the heavy quark expansion. However, the phase space region includes a region of singularity, also called endpoint or threshold region, plagued by the presence of large double (Sudakov-like) perturbative logarithms at all orders in the strong coupling (see e.g. [16, 17, 18, 19]). For $b \to c$ semileptonic decays, the effect of the small region of singularity is not very important; in addition, corrections are not expected as singular as in the $b \to u$ case, being cut-off by the charm mass (see e.g. [20, 21]).

In order to determine $|V_{cb}|$, a global fit may be performed to the width and all available measurements of moments in $B \to X_c l \nu$. A global fit has been recently accomplished in both the kinetic and the 1S scheme [6]. Each scheme has its own
non-perturbative parameters that have been estimated together with the charm and bottom masses. In Table 1 we report the fit results in both schemes. The inclusive averages are higher than the values extracted from exclusive decays, but in substantial agreement within the errors. The most precise measurements are from inclusive, that are below 2%. Still, the determination of $|V_{cb}|$ from $\bar{B} \to D^* \tau \nu$ has reached the relative precision of about 2%. In Table 1 we also compare with the global fit of the CKM matrix elements within the Standard Model, as calculated by the CKMfit and UTfit groups. In comparing with global fits, though, one should remind that global fits are also constrained from loop processes, potentially affected by new physics, while exclusive and inclusive studies are based on standard model tree level decays. Such considerations hold of course also for the $|V_{ub}|$ extraction, discussed in Sect 3.

Until 2007, only decays where the final lepton was an electron or a muon had been observed. The first observation of the decay $\bar{B} \to D^{*} \tau^{-} \nu_{\tau}$, by the Belle Collaboration [22], was followed by improved measurements and evidence for $\bar{B} \to D \tau^{-} \nu_{\tau}$, by both Babar and Belle Collaborations [23, 24]. The measured values for

$$R(D^{(*)}) = \frac{B(\bar{B} \to D^{(*)} \tau^{-} \nu_{\tau})}{B(\bar{B} \to D^{(*)} l^{-} \nu_{l})}$$ (1)

have been consistently exceeding the standard model (SM) expectations. This year, Babar has updated its older measurement [23] by using the full Babar data sample and increasing the signal efficiency by more than a factor of 3 [25]. The resulting $R(D) = 0.440 \pm 0.058 \pm 0.042$ and $R(D^{*}) = 0.332 \pm 0.024 \pm 0.018$ have been compared with the SM predictions, updating the calculations in Refs. [26, 27], and finding $R(D)_{SM} = 0.297 \pm 0.017$ and $R(D^{*})_{SM} = 0.252 \pm 0.003$, averaged over electrons and muons. The results exceed the standard model expectations by 2.0 $\sigma$ for $R(D)$ and by 2.7 $\sigma$ for $R(D^{*})$; taken together, they disagree at the 3.4 $\sigma$ level [25]. In the Babar experimental analysis [25], it is excluded that the excess can be explained by a charged Higgs boson in the type II two-Higgs-doublet model, for any value of $\sin \beta$.

The experimental results have prompted several theoretical studies, where new physics has been advocated to take into account this disagreement. Enhancements over the standard model values have been found in the framework of R-parity violating MSSM [28], in the two Higgs doublet model of type III [29] and in a model with four-fermi operators having vector/axial vector and scalar/pseudoscalar couplings [30]. Other interpretations require the presence of non minimal flavour violation right-right vector or right-left scalar currents; also leptoquark models or models with composite quarks and leptons (with nontrivial flavor structure) have been studied [31]. Besides checking constraints on new physics, the theoretical inputs leading to the determination of $R(D)$ within the standard model have also been requestioned [32, 33].
Table 2: Comparison of exclusive, inclusive and indirect determinations of $|V_{ub}|$. For the exclusive LCSR determination, the three uncertainties are statistical, systematic and theoretical, respectively. Inclusive values are taken by HFAG [6].

3 $b \to u l \nu$ decays

The analysis of exclusive charmless semileptonic decays, in particular the $B \to \pi l \nu$ decay, is currently employed to determine the CKM parameter $|V_{ub}|$, which plays a crucial role in the study of the unitarity constraints. Most studied approaches to calculate the form factors are once again lattice QCD (LQCD) and light-cone QCD sum rules (LCSR), whose domains of applicability are somewhat complementary, lying at high and low $q^2$, respectively. Very recent $|V_{ub}|$ estimates, all in agreement among them, have been reported by the Babar Collaboration, see Table VII of Ref. [34]. In Table 2 we compare two of them, an average estimate determined from the simultaneous fit to experimental data and the LQCD theoretical predictions, and an estimate obtained using a LCSR determination for the form factor [35].

The extraction of $|V_{ub}|$ from inclusive $B \to X_u l \nu$ decays would follow in the footsteps of the $|V_{cb}|$ determination, if not for the copious background from the $B \to X_c l \nu$ decay. To overcome this background, inclusive $B \to X_u l \nu$ measurements utilize restricted regions of phase space, where the $B \to X_c l \nu$ process is highly suppressed by kinematics. These regions overlap with the threshold one, complicating the theoretical issues considerably.

It is a long standing problem the discrepancy between the values of $|V_{ub}|$ extracted from inclusive and exclusive decays. On the experimental side, a lot of effort has been devoted to enlarge the experimental range, so to reduce on the whole the weight of the endpoint region. Latest results by Belle [36] access about the 90% of the $B \to X_u l \nu$ phase space, claiming an overall uncertainty of 7% on $|V_{ub}|$. A similar
portion of the phase space is covered also by the more recent Babar analysis \cite{37}. On the theoretical side several approach have been devised to analyze data in the threshold region, with differences in treatment of perturbative corrections and the parameterization of nonperturbative effects.

The latest experimental determinations of $|V_{ub}|$ come from Babar \cite{37} and HFAG \cite{6} Collaborations. Both Collaborations extract $|V_{ub}|$ from the partial branching fractions relying on at least four different QCD calculations of the partial decay rate, that is BLNP by Bosch, Lange, Neubert, and Paz \cite{38,39,40}; DGE, the dressed gluon exponentiation, by Andersen and Gardi \cite{41}; ADFR by Aglietti, Di Lodovico, Ferrara, and Ricciardi \cite{42,43,44}; and GGOU by Gambino, Giordano, Ossola and Uraltsev \cite{45}. In Table 2 we reports the estimates by the HFAG \cite{6} Collaboration, where the same inputs have been used for all frameworks; the results are roughly consistent among them. Other approaches have been discussed in \cite{46,47,48,49}. Notwithstanding all the experimental and theoretical efforts, the values of $|V_{ub}|$ extracted from inclusive decays maintain about two \si{\sigma} above the values given by exclusive determinations. Also indirect fits prefer a lower value of $|V_{ub}|$. Recent results from CKMfitter and UTfit Collaborations are listed in Table 2 as well.

4 $b \to s(d) l^+ l^-$ decays

The increased luminosity of the actual experimental facilities has made possible to explore flavour-changing neutral current (FCNC) decays in quantitative detail. In the SM, FCNC decays are forbidden at tree level and driven by loop diagrams, therefore they are particularly sensitive to non-standard virtual contributions.

In the inclusive $B \to X_s l^+ l^-$ decays the major theoretical uncertainties arise from the non-perturbative nature of the intermediate $\overline{c}c$ states. By cutting on the invariant dilepton mass around the masses of the $J/\psi$ and $\psi'$ resonances, rather precise determinations seem to be possible, since below or above the $\overline{c}c$ resonances, the inclusive decay is dominated by perturbative contributions.

The calculations of the perturbative contribution has been completed up to next-to-leading (NLO) order in QCD \cite{50,51,52,53,54}. In the last 10 years, it has been extended to the next-to-next-to leading order (see Ref. \cite{55} and Refs. within) greatly reducing the theoretical uncertainty, in particular the large matching scale uncertainty of 16\% at the NLO level. The inclusive $B \to X_s l^+ l^-$ decays have been measured at Belle and at Babar \cite{56,57,58}, and their branching ratios found of order $10^{-6}$, consistent with SM expectations. Also a recent model-independent fit of some short-distance couplings shows consistency with the SM \cite{59}.

In the case of inclusive $B \to X_d l^+ l^-$, the short distance analysis is very similar, once one keeps the CKM suppressed terms in the operator expansion. The first $b \to d l^+ l^-$ transition has been recently observed in the channel $B^+ \to \pi^+ \mu^+ \mu^-$ by
the LHCb Collaboration [60]. The predicted SM branching ratio is of order $10^{-8}$.

The NLO and NNLO QCD corrections for inclusive decays can of course be also used for the corresponding exclusive decays, that are easier to measure. The kinematic available phase space in $B \rightarrow K^{(*)} \mu^+ \mu^-$ is fully covered experimentally, with the exception of the $J/\psi$ and $\psi'$ resonances, which are removed by cuts. A lot of efforts has been devoted to the determination of the form factors. In the exclusive channel $B \rightarrow K^{*} l^+ l^-$, it has been shown that a systematic theoretical description using QCD factorization in the heavy quark limit is relevant for small invariant dilepton masses and reduces the number of independent form factors from 7 to 2 [61]. Spectator effects, neglected in naive factorization, also become calculable. The region of low $q^2$, the dimuon invariant mass squared, where the energy of the emitted meson is large in the $B$ meson rest frame, is also the region of applicability of LCSR to calculate form factors (see e. g. [62]). Also Soft Collinear Effective Theory (SCET) has been applied at large recoil of the $K^{(*)}$ system, typically in the range between 1 and 6 GeV (for a recent Ref., see [63]).

In the high $q^2$ region, QCD factorization is less justified, becoming invalid close to the endpoint of the spectrum at $q^2 = (m_B - m_K)^2$. Alternative approaches have been developed, based on expansions whose scale is set by the large value of $q^2$ [64, 65, 66, 67]. The large $q^2$ region is the domain of election for lattice QCD; unquenched calculations of form factors have been recently performed [68, 69]. In the same large $q^2$ region, ratios of $B \rightarrow K^*$ form factors have been extracted from angular variables recently measured [70, 71, 72, 73], precisely the fraction of longitudinally polarized vector mesons and the transverse asymmetry in the $B \rightarrow K^* l^+ l^-$ decay, and found consistent with lattice results [74]. In general, the study of angular observables can be used advantageously in the $B \rightarrow K^* l^+ l^-$ decay, even to explore the possibility of new physics [75, 76, 77, 78]. The angular distribution $B \rightarrow K^* (\rightarrow K \pi) l^+ l^-$ may be polluted by events coming from the distribution $B \rightarrow K^*_0 (\rightarrow K \pi) l^+ l^-$, where $K^*_0$ is a scalar meson resonance, and this possibility was analyzed in [79, 80].

LHCb has recently reported the most precise measurement of the branching ratio for the $B^+ \rightarrow K^+ \mu^+ \mu^-$ channel to date, together with a study of its angular distribution and differential branching fraction [81]. In the SM, the differential decay rate can be written as

$$\frac{1}{\Gamma} \frac{d\Gamma(B^+ \rightarrow K^+ \mu^+ \mu^-)}{d \cos \theta} = \frac{3}{4} (1 - F_H) (1 - \cos^2 \theta) + \frac{1}{2} F_H + A_{FB} \cos \theta$$  \hspace{1cm} (2)

Here $\theta$ is the angle between the $\mu^-$ and the $K^+$ in the rest frame of the dimuon system. The two parameters, $F_H$ and the forward-backward asymmetry of the dimuon system, $A_{FB}$, depend on $q^2$. In the SM, $A_{FB}$ is zero and $F_H$ highly suppressed, and their measured values are consistent with the SM expectations. The differential branching fraction of the $B^+ \rightarrow K^+ \mu^+ \mu^-$ decay is, however, consistently below the SM prediction at low $q^2$ [81]. LHCb reports also the actual more precise determinations of $A_{FB}$ for the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ [82].
5 Charm decays

In the past decade, charm semileptonic decays have not received the same first-rate attention than beauty decays, but they are rapidly gaining ground. The extraction of $|V_{cd}|$ and $|V_{cs}|$ follows in the footsteps of the just described extraction of $|V_{ub}|$ and $|V_{cb}|$ from semileptonic $B$ decays. Once again, the main theoretical hardship comes from the nonperturbative evaluation of the form factors. Lately, high statistics studies on the lattice have become available. The HPQCD Collaboration has estimated the value $|V_{cd}| = 0.225 \pm 0.006_{\text{exp}} \pm 0.010_{\text{lat}}$, with the first error coming from experiments and the second from their lattice computation. The result is in agreement with the value of $|V_{cd}|$ the same collaboration has recently extracted from leptonic decays and from determinations of $|V_{cd}|$ coming from neutrino scattering. Instead, their best, preliminary value $|V_{cs}| = 0.965 \pm 0.014$ shows a discrepancy with the average value from leptonic decays $|V_{cs}| = 1.010 \pm 0.017$. Both $|V_{cs}|$ and $|V_{cd}|$ semileptonic estimates are in agreement with indirect fits. Other $D$ and $D_s$ decays, such as $D_s \to \phi \ell \nu_\ell$ or $D \to \pi \ell \nu_\ell$, have been analyzed as well.

Results from QCD light-cone sum rules on $|V_{cs}|$ and $|V_{cd}|$ give substantial agreement on the averages and higher theoretical errors with respect to the previous quoted lattice results. Recently, a revised version of QCD sum rules reports reduced errors and an higher average value $|V_{cd}| = 0.244 \pm 0.005\pm 0.003 \pm 0.008$, the first and second errors being of an experimental origin and the third due to the theoretical uncertainty.

According to lattice determinations in the $|V_{cd}|$ the form factors are insensitive to the spectator quark: $D_s \to \eta_s \ell \nu_\ell$ and $D \to K \ell \nu_\ell$ form factors are essentially the same, and the same holds for $D_s \to K \ell \nu_\ell$ and $D \to \pi \ell \nu_\ell$ within 5%. This result, which can be tested experimentally, is expected to hold also for $B$ meson decays so that $B_s \to D_s$ and $B \to D$ form factors would be equal.

The decays driven by $c \to u\ell^+\ell^-$ are forbidden at tree level in the standard model (SM) and proceed by one loop diagram at leading order in the electroweak interactions. Virtual quarks in the loops are of the down type, and no breaking due to the large top mass occurs. The GIM mechanism works more effectively in suppressing FCNC decays than their strangeness and beauty analogues, leading to tiny decay rates, dominated by long distances contributions. They set the scale, with branching fractions of order $10^{-6}$, shielding possible enhancements due to new physics. A way out is to choose appropriate observables containing mainly short distance contributions, whose order of magnitude lays way behind $10^{-6}$.

This year, effects of possible new physics have been been investigated in $D^+ \to \pi^+ \mu^+\mu^-$ and $D^+_s \to K^+ \mu^+\mu^-$ decays, around the $\phi$ resonant peak in spectrum of dilepton invariant mass, concluding that in favourable conditions their value can be as high as 10% [88]. Older studies report investigations of semileptonic decays in the framework of other new physics models, such as R-parity violating supersymmetric...
models, extra heavy up vector-like quark models \[89\], Little Higgs \[90\], and leptoquark models \[91\]. Theoretical analysis of semileptonic four body decays $D^0 \rightarrow h_1^+ h_2^- l^+ l^-$, with $l = e, \mu$ and $h_i = \pi, K$, have been reported this year as well \[92, 93\].

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