RADIATIVE CORRECTIONS TO NEUTRINO REACTIONS
OFF PROTON AND DEUTERON

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Radiative corrections are calculated for antineutrino proton quasielastic scattering, neutrino deuteron scattering, and the asymmetry of polarised neutron beta decay from which $G_A/G_V$ is determined. A particular emphasis is given to the constant parts that are usually absorbed into the coupling constants, and thereby those that appear in the processes that concern us are unambiguously tied among each other.

1. Introduction

Neutrino experiments have now entered the era of precision measurements with the accuracy reaching the level that radiative corrections cannot be ignored. In this talk we report our recent work\textsuperscript{1–3} on the radiative corrections to neutrino scattering, $\bar{\nu}_e + p \rightarrow e^+ + n$ as measured in KamLAND and $\nu_e + d \rightarrow e^- + p + p$, $\nu_x + d \rightarrow \nu_x + p + n$ that are measured at SNO, and those to the asymmetry in polarised neutron beta decay.

2. Radiative Corrections to $G_A/G_V$

The study of radiative corrections to weak processes has a long history, and the wisdom acquired for neutron beta decay rate\textsuperscript{4–7} will be transcribed in the calculation of neutrino scattering processes. With the local four-Fermi interactions, the $G_A/G_V$ ratio ($= g_A$) enters the tree-level cross sections as

$$\sigma(\bar{\nu}_e + p \rightarrow e^+ + n) \propto (f_V^2 + 3g_A^2) \quad (1)$$

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for neutrino-proton quasielastic scattering and
\[ \sigma(\nu_e + d \rightarrow e^- + p + p) \propto g_A^2, \]  
(\text{CC}) \quad (2)
\[ \sigma(\nu_x + d \rightarrow \nu_x + p + n) \propto g_A^2 \quad [x = e, \mu, \tau] \]  
(\text{NC}) \quad (3)

for scattering off deuteron. Here \( f_V = 1 \) is retained to trace the contribution of the weak vector current.

We separate the radiative corrections to charged current processes into the inner and outer parts as was done for neutron beta decay. The outer corrections depend on the \( e^\pm \) velocity in the final state and are free from UV and IR divergences. They are independent of the details of strong interactions and the calculation is straightforward. The inner part is velocity-independent but is plagued by the UV-divergence that is not cancelled within four-Fermi theory and depends on the details of the structure of nucleons. The inner corrections amount to replacing \( f_V^2 \) and \( g_A^2 \) by
\[ \bar{f}_V^2 = f_V^2 (1 + \delta_F^{\text{in}}), \quad \bar{g}_A^2 = g_A^2 (1 + \delta_{\text{GT}}^{\text{in}}). \]  
(4)
The evaluation of \( \delta_F^{\text{in}} \) and \( \delta_{\text{GT}}^{\text{in}} \) requires not only renormalisable Weinberg-Salam theory but receives complications from hadron structure. For the Fermi transitions \( \delta_F^{\text{in}} \) is already obtained\(^8-9\), but the evaluation of \( \delta_{\text{GT}}^{\text{in}} \) for the Gamow-Teller transitions has eluded the literature for long time.

The corrections to (3) differ from those to (1) and (2). There is no outer correction. The corrections are incorporated by the replacement of \( g_A^2 \) by
\[ g_A^2 \rightarrow \bar{g}_A^2 (1 + \Delta_{\text{GT}}^{\text{in}}). \]  
(5)

The prime purpose of our work is to give \( \delta_{\text{GT}}^{\text{in}} \) and \( \Delta_{\text{GT}}^{\text{in}} \), so that the \( g_A \) factor that appears in NC processes is related with that in the CC processes. Subsidiarily we show that the radiative correction to the polarised neutron beta decay asymmetry (from which we determine \( g_A \)) is described by the same factors as those that appear in (4) and in the neutron beta decay rate. This is expected, but we do not find any proofs. So we carried out explicit calculations. The \( G_A/G_V \) ratio measured in this process is
\[ \frac{g_A (1 + \delta_{\text{GT}}^{\text{in}})^{1/2}}{f_V (1 + \delta_F^{\text{in}})^{1/2}} \approx g_A \left( 1 + \frac{\delta_{\text{GT}}^{\text{in}} - \delta_F^{\text{in}}}{2} \right) \neq g_A. \]  
(6)

3. Calculational Strategy
Let us begin with the charged current processes (1) and (2). Following the procedure for the neutron beta decay rate\(^7\), we divide the integration region
of the exchanged gauge boson into

\[ (i) \ 0 < |k|^2 < M^2, \quad \text{and} \quad (ii) \ M^2 < |k|^2 < \infty, \]  

(7)

The mass scale \( M \) is supposed to be greater than the proton mass \( (m_p) \), but to be much smaller than the \( W \) and \( Z \) boson masses \( (m_W, m_Z) \), i.e., \( m_p \ll M \ll m_W, m_Z \). We use the four-Fermi interactions for nucleons in region (i), thereby dealing with the nucleons as point-like and only photons are exchanged between nucleons and charged leptons. In region (ii) we employ Weinberg-Salam theory for quarks and photons and \( Z \) bosons are exchanged. The mass scale \( M \) is the UV-cutoff for the four-Fermi theory, but is also regarded as the scale for the onset of the asymptotic behaviour to which Weinberg-Salam theory applies.

With four-Fermi theory, we end up with UV-divergences, i.e., \( \log M^2 \) terms in the calculation for (i). These divergences are classified into two types, the one eliminated by renormalisable gauge theories and the other that is rendered finite only by considering the structure of nucleons. It is known\(^5\)\(^6\) that such classification is possible for the Fermi transitions by using CVC and current algebra techniques. We showed in Ref. 1 that a parallel classification is possible for the Gamow-Teller transitions on the basis of CVC, PCAC and current algebra (see also Ref. 10).

In the first type \( \log M^2 \) terms are universal, their coefficients being independent of the details of strong interactions. These \( \log M^2 \) terms are cancelled when the integrals in (i) and (ii) are added. The \( \log M^2 \) terms that appear axial-vector vector interference terms cannot be cancelled. So, \( \log M^2 \) terms in (i) are tamed by introducing form factors at the electromagnetic and weak vertices in Feynman integrals. Weak magnetism cannot be ignored\(^1\) at the weak vertices, because the mass scale of the form factor is on the order of \( m_p \) and the loop integral over the weak magnetism form factor gives the same order of magnitude as does the \( V - A \) contribution.

4. Antineutrino Quasielastic Scattering off Proton

We write the differential cross section as

\[
\frac{d\sigma(\bar{\nu}_e + p \rightarrow e^+ + n)}{d(cos\theta)} = \frac{G^2_V}{2\pi} E^2 \beta \{A(\beta) + B(\beta)\beta cos\theta\}. 
\]  

(8)

Here \( E \) and \( \beta \) are the energy and velocity of the final positron, \( \theta \) is the angle between incident antineutrino and the positron and \( G_V = G_F\cos\theta_C \) is the vector coupling constant to nucleons.
After calculations we find that $A(\beta)$ and $B(\beta)$ are written
\begin{align}
A(\beta) &= \{1 + \delta_{\text{out}}(E)\} \left(f_V^2 + 3\bar{g}_A^2\right), \quad (9) \\
B(\beta) &= \{1 + \tilde{\delta}_{\text{out}}(E)\} \left(f_V^2 - \bar{g}_A^2\right), \quad (10)
\end{align}
where the inner corrections included in $f_V^2$ and $\bar{g}_A^2$ are given in (4) with
\begin{align}
\delta_{\text{in}}^F &= \frac{e^2}{8\pi^2} \left\{4\log \left(\frac{m_Z}{m_p}\right) + \log \left(\frac{m_p}{M}\right) + C^F\right\}, \quad (11) \\
\delta_{\text{in}}^{GT} &= \frac{e^2}{8\pi^2} \left\{4\log \left(\frac{m_Z}{m_p}\right) + \log \left(\frac{m_p}{M}\right) + 1 + C^{GT}\right\}. \quad (12)
\end{align}
The effect of the nucleons structure appears only in $C^F$ and $C^{GT}$. We computed these two numbers by introducing form factors. The contributions from the weak magnetism are nonnegligible at the weak vertex; it even dominates $C^{GT}$. Our results are $C^F = 2.160$ and $C^{GT} = 3.281$, and hence $\delta_{\text{in}}^F = 0.0237$ and $\delta_{\text{in}}^{GT} = 0.0262$ for $M \approx 1$ GeV.

One sees in (10) that the energy-dependent outer corrections are factored out. One of them, $\delta_{\text{out}}(E)$, has been known\textsuperscript{11}; the other $\tilde{\delta}_{\text{out}}(E)$ is new\textsuperscript{1}. The outer corrections for (2) are given in Ref. 12.

5. Asymmetry in Polarised Neutron Beta Decay
The $G_A/G_V$ ratio is determined from the asymmetry parameter $A$. The corrections are again separated into the inner and outer parts, as\textsuperscript{2}
\begin{equation}
A = 2 \{1 + C(E)\} \frac{f_V \bar{g}_A - \bar{g}_A^2}{f_V^2 + 3\bar{g}_A^2}. \quad (13)
\end{equation}
The energy-dependent factor $C(E)$ is given in Ref. 2. The important point is that exactly the same inner corrections appear in $f_V^2$ and $\bar{g}_A^2$, as given by (4) with (11) and (12), so that $\bar{g}_A$ from asymmetry can be used to predict the neutron decay rate without further corrections.

6. The Deuteron Neutral Current Process
Since the outer correction is absent, all we need is the evaluation of the inner part using Weinberg-Salam theory on the quark level. This type of calculation was done by Marciano and Sirlin\textsuperscript{13}. The effective interaction of quarks and neutrinos at low energy with an iso-singlet target is given by
\begin{align}
M_{\text{eff}} &= -i\frac{G_F}{\sqrt{2}}\gamma_{\text{NC}}^{(\nu,h)} \bar{\psi}_\nu \gamma^\mu (1 - \gamma^5)\psi_\nu \\
&\times \left\{\bar{\psi} I_3 \gamma_\mu (1 - \gamma^5)\psi - 2\kappa^{(\nu,h)} \sin^2\theta_W \bar{\psi} \gamma_\mu Q\psi\right\}, \quad (14)
\end{align}
where $\psi_\nu$ and $\psi$ are the neutrino and the quark doublet, and $\rho_{NC}^{(\nu;h)} - 1$ and $\kappa_{NC}^{(\nu;h)} - 1$ are the radiative corrections, which are found in Ref. 13.

If we sandwich (14) between the deuteron and two-nucleon states to evaluate the cross section (3), only the axial current in (14) contributes to the $^3S_1 \rightarrow ^1S_0$ transition. The effective interaction for the nucleon doublet $\psi_N$ reads

$$M_{\text{eff}} = i \frac{G_F}{\sqrt{2}} g_A \rho_{NC}^{(\nu;h)} [\bar{\psi}_\nu \gamma^\mu (1 - \gamma^5) \psi_\nu] [\bar{\psi}_N I_3 \gamma^\mu \gamma^5 \psi_N].$$

(15)

The term of $\kappa_{NC}^{(\nu;h)}$ does not contribute to (3). Eq. (15) indicates that $g_A$ is renormalised multiplicatively by $\rho_{NC}^{(\nu;h)}$. Writing $\rho_{NC}^{(\nu;h)} = (1 + \Delta_{\text{GT}}^{\nu})^{1/2}$, the radiative correction is the replacement (5). We compute $\rho_{NC}^{(\nu;h)} = 1.00955$ for Higgs boson mass $m_H = 1.5m_Z$ and 1.00862 for $m_H = 5m_Z$.

7. Summary

We computed radiative corrections to antineutrino proton quasielastic scattering, neutrino deuteron scattering, and the asymmetry of polarised neutron beta decay, with an emphasis given to the constant parts that are usually absorbed into the coupling constants. Hereby couplings that appear in the processes that concern us are unambiguously tied. For instance, the NC to CC ratio for neutrino-deuteron reactions receives the overall correction $(1 + \Delta_{\text{GT}}^{\nu})/(1 + \Delta_{\text{GT}}^{\nu}) = 0.992 \pm 0.001$ up to the outer correction for CC which is accounted for separately.

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