Dispersion relation of square lattice waves in a two-dimensional binary complex plasma

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Binary complex plasmas consist of microparticles of two different species and can form two-dimensional square lattices under certain conditions. The dispersion relations of the square lattice waves are derived for the longitudinal and transverse in-plane modes, assuming that the out-of-plane mode is suppressed by the strong vertical confinement. The results are compared with the spectra obtained in Langevin dynamics simulations. Furthermore, we investigate the dependence of the dispersion relation on the charge ratio and mass ratio of the two particle species.

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Complex plasmas consist of a mixture of weakly ionized gases and microparticles. The latter acquire a charge due to the flow of the surrounding ions and electrons which are negative, owing to the higher thermal velocity of electrons.1 Considering the plasma screening effect, the interaction between the microparticles can be described via the Yukawa potential.2 In the laboratory, the charged particles are usually suspended in the sheath above the lower electrode of a radio-frequency (rf) discharge, where the gravity force is balanced by the electric force. In strongly coupled complex plasmas, monodisperse microparticles can be vertically confined to a single layer and form a hexagonal lattice, known as plasma crystals.3 Due to the stretched time scales and slow damping, two-dimensional (2D) complex plasma crystals provide an unique opportunity to study generic processes in solids and liquids at the kinetic level.5 With external manipulations by electric fields or laser beams, various phenomena such as melting6 and recrystallization7,8,9,10 microstructure under shear Mach cone excitations11,12 and entropy production13 have been investigated both experimentally and theoretically.

One of the most defining properties of plasma crystals is the dispersion relation of the microparticles’ collective oscillations, in the form of lattice waves. This has been derived analytically and measured directly using video microscopy in the case of monodisperse complex plasma16,17,18,19. Remarkably, due to the strong ion flow in the sheath, the interactions between microparticles are altered by the so-called wake effect, resulting in the coupling of the horizontal and vertical modes.16,18 This eventually triggers a mode-coupling instability (MCI) and causes the crystal to melt.20,21

A binary complex plasma consists of microparticles of two different species. With an appropriate selection of their mass and size, these particles can form, in the laboratory, a bilayer22 or a quasi-two-dimensional (q2D) crystalline suspension23,24,25. The phonon spectra for these structures have been measured experimentally and studied by a quasi-localized charge approximation approach and molecular dynamics simulations26,27,28. A mode coupling between the horizontal modes in the two layers, mediated by the plasma wakes, has been proposed.29

Meanwhile, taking advantage of the plasma etching effect, the two particle species can be suspended at the same height for a certain amount of time.30 Under these conditions, binary complex plasmas have been found to form square lattice structures with a quadruple symmetry28,31 as the one presented in Fig. 1. A strong vertical confinement can efficiently suppress the vertical motions and thus the expected MCI.21,22 A natural question that then arises is how the horizontal wave modes are modified in such 2D square lattices compared to the well-studied case of hexagonal lattices in monodisperse 2D complex plasma crystals.

In this article, we derive the dynamical matrix of a 2D binary complex plasma crystal in order to study the dispersion

![FIG. 1. (a) The sketch of a square lattice with particle spacing \( a \) in a 2D binary complex plasma consisting of the particle species \( A \) and \( B \). (b) The reciprocal lattice in the \( k \)-space, where the basis vectors are \( \mathbf{b}_{1,2} = 2\pi a^{-1}(1/2, \pm 1/2) \). Due to the square symmetry, we consider the wave vectors \( \mathbf{k} \) at \( 0^\circ \) and \( 45^\circ \) in the first Brillouin zone (the shaded area surrounded by a dashed line).]
In this expression we have denoted the mass ratio as \( \Lambda_m \) with a screening length \( \lambda_D \) assumed to be of a Yukawa form. Thus, each particle approach we assume a very strong confinement, resulting in dependence on the involved particle charges and masses. In our approach we assume a very strong confinement, resulting in dependence on the involved particle charges and masses. In our

\[
\phi_{p,i}(r) = \frac{Q_p}{|r-r_{p,i}|} \exp(-|r-r_{p,i}|/\lambda_D)
\]  

(1)

with a screening length \( \lambda_D \) at its 2D position \( r_{p,i} = x_{p,i}\hat{x} + y_{p,i}\hat{y} \). Consequently the equations of motion for the \( i \)-th particle of species \( A \) and the \( i \)-th particle of species \( B \), in the 2D plane of confinement, read

\[
\frac{d^2 r_{A,i}}{dt^2} + v \frac{dr_{A,i}}{dt} = - \frac{Q_A}{M_A} \nabla_{A,i} \left( \sum_{j \neq i} \phi_{A,j}(r_{A,i}) + \sum \phi_{B,j}(r_{A,i}) \right),
\]

\[
\frac{d^2 r_{B,i}}{dt^2} + v \frac{dr_{B,i}}{dt} = - \frac{Q_B}{M_B} \nabla_{B,i} \left( \sum_{j \neq i} \phi_{A,j}(r_{B,i}) + \sum \phi_{B,j}(r_{B,i}) \right),
\]

(2)

where \( v \) is the frictional drag coefficient and \( M_{A,B} \) and \( Q_{A,B} \) are the mass and the charge of the particle species \( A \) and \( B \), respectively. Note also that the symbols \( \nabla_{A,i} \) and \( \nabla_{B,i} \), denote the gradients with respect to the vectors \( r_{A,i} \) and \( r_{B,i} \), accordingly.

Our scope here is to investigate the vibrational properties of the 2D binary complex plasma crystal around its square lattice equilibrium, shown in Fig. 1. In this crystalline configuration, the microparticles of the species \( p = A \) or \( B \) are located at the positions \( R_p = x_p\hat{x} + y_p\hat{y} \) with \( X_A = (m+n)\alpha \), \( Y_A = (m-n)\alpha \), \( X_B = 1 - X_A \) and \( Y_B = Y_A \) where \( m,n \) are arbitrary integers and \( \alpha \) denotes the square lattice constant (Fig. 1). Linearizing around this equilibrium and introducing the plane wave ansatz

\[
d_p = d_p^{(0)} \exp[-i\omega t + i(k_xX_p + k_yY_p)]
\]

(3)

for the displacement \( d_p \) of the particles of species \( p \) from their equilibrium positions \( R_p \), we arrive, in view of the square lattice symmetry, at the \( 4 \times 4 \) dynamical matrix

\[
D = \frac{Q_A^2}{M_A} \begin{pmatrix}
F_A^x + \frac{1}{\Lambda_Q} F_B^x & G_A^x + \frac{1}{\Lambda_Q} G_B^x & -\frac{1}{\Lambda_Q} F_A^x & -\frac{1}{\Lambda_Q} G_B^x \\
G_A^y + \frac{1}{\Lambda_Q} G_B^y & F_A^y + \frac{1}{\Lambda_Q} F_B^y & -\frac{1}{\Lambda_Q} G_A^y & -\frac{1}{\Lambda_Q} F_B^y \\
-\Lambda_m F_A^x & -\Lambda_m G_B^x & \Lambda_m F_A^y & \Lambda_m G_B^y \\
-\Lambda_m G_A^x & -\Lambda_m F_B^x & \Lambda_m G_A^y & \Lambda_m F_B^y
\end{pmatrix}.
\]

(4)

In this expression we have denoted the mass ratio as \( \Lambda_m = M_A/M_B \) and the charge ratio as \( \Lambda_Q = Q_A/Q_B \). The elements \( F_p^x, F_p^y, \) and \( F_p^i \) with \( p = A \) or \( B \) are given by the following sums of the effective spring constant \( F(X,Y) \) over the lattice positions \( (X_p,Y_p) \), excluding the central position:

\[
F_p^x = \sum_{x_p,y_p} F(X_p,Y_p),
\]

\[
F_p^y = \sum_{x_p,y_p} F(X_p,Y_p) \cos(k_xX_p + k_yY_p),
\]

\[
F_p^i = F_p^o - F_p^l.
\]

(5)

Such summations also apply to the rest of the matrix elements \( \hat{T}_p^{s,i,l} \) and \( \hat{G}_p^{o,l,s} \). The corresponding effective spring constants

\[
F(X,Y) = R^{-5}e^{-R/\lambda_D}[X^2(3 + 3R/\lambda_D + R^2/\lambda_D^2)]
\]

\[
- R^{-2}(1 + R/\lambda_D)],
\]

\[
\mathcal{F}(X,Y) = F(Y,X),
\]

\[
G(X,Y) = (XY/R^2)e^{-R/\lambda_D}(3 + 3R/\lambda_D + R^2/\lambda_D^2)
\]

with \( R = \sqrt{X^2 + Y^2} \).

The eigenvalues \( \Omega_j^2 \) of the dynamical matrix \( D \) are connected with the eigenfrequencies \( \omega_j \) of the crystal through the relation \( \Omega_j^2 = \omega_j(\omega_j + iv) \). For simplicity, we approximate in the following theoretical results the \( \omega_j \) with the respective values of \( \Omega_j \), under the assumption that the damping in our system is very small \( (v \ll \omega_j) \). Since \( D \) is a \( 4 \times 4 \) matrix its eigenvalues yield 4 branches \( \Omega_j(k_x,k_y) \), two of which are transverse and two longitudinal.

An example of these spectra for \( \Lambda_m = 1 \) and \( \Lambda_Q = 8 \), at the two characteristic wave vector angles for the square lattice, \( \theta = 0^\circ \) and \( \theta = 45^\circ \), is shown in Fig. 2(a),(b) and
The wave spectra $V_{k,\omega}$ of the simulated square lattices in the binary complex plasma are computed using the 2D Fourier transform

$$V_{k,\omega} = 2/ST \int_0^S \int_0^T v(r,t) \exp[-i(k \cdot r + \omega t)] dt ds,$$  \hspace{1cm} (7)
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