Origin of the Thermal Radiation in a Solid-State Analog of a Black-Hole

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Abstract

An effective black-hole-like horizon occurs, for electromagnetic waves in matter, at a surface of singular electric and magnetic permeabilities. In a physical dispersive medium this horizon disappears for wave numbers with $k > k_c$. Nevertheless, it is shown that Hawking radiation is still emitted if free field modes with $k > k_c$ are in their ground state.

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Hawking discovered that black-holes emit thermal radiation with temperature $T_H = M_{Planck}^2/8\pi M_{BH}$. For large black-holes, with mass $M_{BH} \gg M_{Planck}$, spacetime curvature is extremely small near the horizon, and the Hawking temperature is much smaller than the Planck temperature. It can be argued therefore, that as long as $M_{BH} \gg M_{Planck}$, the Hawking effect is essentially a low energy process, which is decoupled from quantum-gravity effects. Nevertheless, the standard derivation relies on free field theory on curved space-time, and describes the Hawking radiation as originating from vacuum fluctuations on scales exponentially smaller than $1/M_{Planck}$. On such scales, free field theory may not be a valid description, and quantum-gravity may become important. A question arises, therefore, to what extent is the Hawking radiation dependent on the details of the short distance structure of quantum gravity?

In the absence of a theory of quantum-gravity, guiding toy models can be helpful. Unruh has suggested using a sonic analog of a black-hole. In his model, an effective event horizon is formed for sound waves on a flowing fluid background, on the surface where the flow becomes supersonic. Unruh has shown, that when a natural cutoff is implemented, sonic black-holes still gives rise to Hawking radiation. In his approach the cutoff is implemented by modifying the dispersion relation for sound waves at high frequencies. This, in turn, alters the motion of modes with frequency close to the cutoff scale and gives rise to a new type of trajectories, which approach the horizon but eventually ”reflect” back to infinity. It is not clear that a similar process is indeed realized for real black-holes. Further investigations, based on Unruh’s model have been carried out in Refs. 7–9.

In order to gain further insight into the problem, in this letter, we provide and examine, a different toy model, which is based on phenomenological electrodynamics in matter. It is known that Maxwell’s equations in a medium are formally analogous to electromagnetism in curved space-time. In this case, an effective event horizon for electromagnetic waves in matter occurs on a surface of singular electric and magnetic permeabilities. As we shall show, formally, thermal radiation can still be emitted, if the initial state of the field is chosen appropriately. Nevertheless, this requires, non-physical, singular electric polarization on
arbitrary short scales, far beyond the scale where the electromagnetic field and the medium still couple. The short distance modifications that must be introduced in this toy model are clear: at high momentum or frequency the medium becomes dispersive and gradually the electromagnetic field and the medium decouple. This implies that for sufficiently short wave-lengths, the effective event horizon disappears. We shall show however that inclusion of such dispersive behavior does not eliminate Hawking radiation. The process which generates Hawking’s radiation in the present toy model, differs however from that found for the sonic model. In the following we adopt units such that $\hbar = \kappa_B = c = 1$.

To begin with, let us consider the macroscopic Maxwell’s equations in a medium:

\[
\begin{align*}
\nabla \cdot \mathbf{B} &= 0 \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \\
\nabla \cdot \mathbf{D} &= 0 \quad \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = 0
\end{align*}
\]

where $\mathbf{D}$ and $\mathbf{H}$ are determined by the local electric and magnetic polarizations: $\mathbf{D} = \mathbf{E} + \mathbf{P}$ and $\mathbf{H} = \mathbf{B} - \mathbf{M}$. We shall consider the special case of linear constitution relation of the particular form:

\[
\begin{align*}
\mathbf{H}(r,t) &= \alpha z \mathbf{B}(r,t), \\
\mathbf{D}(r,t) &= (\alpha z)^{-1} \mathbf{E}(r,t), \quad (3)
\end{align*}
\]

i.e., the electric and magnetic permeabilities are $\varepsilon(z) = \mu(z) = 1/(\alpha z)$.

With the gauge choice $\varphi = 0$ and $\nabla \cdot \varepsilon \mathbf{A} = 0$, we obtain for the vector potential, the equation of motion

\[
\alpha^2 z \nabla \times z \nabla \times \mathbf{A} + \frac{\partial^2 \mathbf{A}}{\partial t^2} = 0. \quad (4)
\]

Thus, although space-time is flat, with respect to the rest frame of the medium, waves propagate in an effective Rindler geometry:

\[
ds_{\text{effective}}^2 = \alpha^2 z^2 dt^2 - dz^2 - dx^2 - dy^2. \quad (5)
\]

The latter describes the space-time geometry near a black-hole horizon, of mass $M_{BH} = (4\alpha)^{-1}$, for $r/2M_{BH} \ll 1$ [11]. The event horizon corresponds here to the singular surface,
z = 0, where \( \varepsilon \) and \( \mu \to \infty \). Although this model clearly breaks-down near the horizon, let us first examine its effect on electromagnetic waves and defer the discussion of a more physical realization to the sequel.

The Hamiltonian of the system can be written as

\[
H = H_I - H_{II} = \int_{-\infty}^{\infty} \varepsilon^{-1}(z) \left( D^2 + (\nabla \times A)^2 \right) dz - \int_{-\infty}^{0} |\varepsilon(z)|^{-1} \left( D^2 + (\nabla \times A)^2 \right) dz, \tag{6}
\]

where \( A \) and \( D \) are conjugate canonical coordinates. The energy of a wave is thus negative in the region \( z < 0 \). This indeed suggests, that like a black-hole, the present model can emit thermal radiation. To see this let us quantize the field.

For simplicity, we shall consider in the following only waves in the \( \hat{z} \)-direction, which correspond to radial transverse waves in the black-hole case. (Since we consider only the case \( r/2M_{BH} << 1 \), the vector nature of electromagnetic waves is unimportant.) The solutions of the wave equation \( \Box \) can be written as:

\[
g_I(\kappa) = \begin{cases} 
\frac{1}{\sqrt{4\pi\omega_\kappa}} e^{-i\omega_\kappa t + i\frac{\omega_\kappa}{\kappa} \ln|z|}, & \text{for } z > 0 \\
0, & \text{for } z < 0
\end{cases} \tag{7}
\]

\[
g_{II}(\kappa) = \begin{cases} 
0, & \text{for } z > 0 \\
\frac{1}{\sqrt{4\pi\omega_\kappa}} e^{i\omega_\kappa t + i\frac{\omega_\kappa}{\kappa} \ln|z|}, & \text{for } z < 0
\end{cases} \tag{8}
\]

where \( \omega_\kappa = |\kappa| \). The modes \( g_I(\kappa) \) correspond to an outgoing (right moving) wave for \( \kappa > 0 \) and ingoing wave for \( \kappa < 0 \). Similarly, \( g_{II}(\kappa) \) corresponds to an outgoing wave for positive \( \kappa \) and an ingoing wave for negative \( \kappa \). The modes \( \Box \) are orthogonal under the scalar product:

\[
(g_1, g_2) = i \int_{-\infty}^{+\infty} \varepsilon(z) g_1^* \partial_t g_2 dz, \tag{9}
\]

and can be used to express the vector potential as

\[
A(t, z) = \int_{-\infty}^{+\infty} (g_I(\kappa)a_I(\kappa) + g_{II}(\kappa)a_{II}(\kappa) + \text{h.c.}) d\kappa. \tag{10}
\]

Here, \( a_I \) and \( a_{II} \) are the annihilation operators in the domains \( z > 0 \) and \( z < 0 \), respectively, and \([a_I, a_{II}] = 0\).
Next, in order to determine what will be seen by a stationary observer at \( z > 0 \), we must specify an initial condition for the electromagnetic field at the effective horizon. Let us define the analogue Kruskal coordinates by \( U = -ze^{-\alpha t} \) and \( V = ze^{\alpha t} \). In terms of the \( U, V \) coordinates the effective metric is conformally flat and the solutions to the wave equation are outgoing right moving waves, \( \exp(-i\tilde{\omega}U) \), and ingoing left moving waves, \( \exp(-i\tilde{\omega}V) \). The Unruh initial condition [12] amounts to the requirement that at the past horizon, \( V = 0 \), the outgoing modes, are in their ground state. Thus, time is defined with respect to light ray trajectories, \( z = -Ue^{\alpha t} \), with constant \( U \).

The analyticity of the outgoing modes, in the lower half of the \( U \) complex plane, implies that the combinations

\[
f_1(\kappa) = e^{\pi \omega/2\alpha} g_I(\kappa) + e^{-\pi \omega/2\alpha} g_{II}^*(-\kappa),
\]

(11)

and

\[
f_2(\kappa) = e^{\pi \omega/2\alpha} g_{II}(\kappa) + e^{-\pi \omega/2\alpha} g_I^*(-\kappa),
\]

(12)

have only positive frequencies with respect to \( U \)-modes [12]. It then readily follows that the outgoing state is thermal with (global) temperature

\[
T = \frac{\alpha}{2\pi}.
\]

(13)

As in the standard derivation of Hawking radiation, this model involves ultra-high momenta. To see this, consider an outgoing photon described by the wave packet

\[
\int e^{-(\omega-\omega_0)^2} e^{-i\omega(t-\frac{1}{\alpha}\ln z)} d\omega \propto e^{-(t-\frac{1}{\alpha}\ln z)^2/4}.
\]

(14)

As we evolve the wave backward in time, the wave packet approaches the horizon like \( z \approx \exp \alpha t \), and gives rise to ultra-high momentum \( k \approx \omega \exp(-\alpha t) \). Clearly this phenomena is also related to the occurrence of singular polarization on the horizon.

Nevertheless, since the present model involves known physics, we do have some handle on these problems. When the wave length becomes comparable to the “molecular” length
scale, we will have to account for the dispersive properties of the medium, which so far have been ignored. In particular, for sufficiently large wave numbers, the electromagnetic waves decouples from the medium degrees of freedom, and the effective geometry for the electromagnetic field becomes in this limit flat. Since in our model, the momentum $k$ increases near the horizon, while the frequency, $\omega$, remains constant, we will introduce *spatial-dispersion* into the constitution equations ($\mathbb{E}$). As we shall see, this indeed eliminates the above mentioned difficulties.

In order to describe both the inhomogeneity (in the $z$ direction) and spatial dispersion, we shall now assume the following constitution relation for the electric field:

$$D(z, t) = \int \varepsilon(z, z - z') E(z', t) dz'.$$  \hspace{2cm} (15)

and a similar relation, with $\mu(z, z - z') = \varepsilon(z, z - z')$, for the magnetic field. The first argument in $\varepsilon(z, z - z')$ allows for spatial non-homogeneity of the electric polarization, which in turn induces a low-momentum effective Rindler geometry. The second argument gives rise to a spatial non-locality around the point $z$, which is taken to be homogeneous, i.e. to depends only on $z - z'$. This allows us to simplify eq. (15) by going to the momentum representation. Assuming further that $\varepsilon$ is analytic in its first argument, we obtain:

$$D(z, t) = \int e^{ikz} \varepsilon(i \partial_k, k) E(k, t) dk,$$  \hspace{2cm} (16)

where

$$\varepsilon(i \partial_k, k) = \int e^{-ikz} \varepsilon(i \partial_k, z) dz,$$  \hspace{2cm} (17)

and $E(k, t)$ is the Fourier transform of $E(z, t)$.

Using a similar relation between the magnetic field and the magnetic induction, we are led to the Hamiltonian

$$H = \int \left( D(k, t) \varepsilon^{-1}(i \partial_k, k) D(k, t) + k A(k, t) \varepsilon^{-1}(i \partial_k, k) k A(k, t) \right) dk.$$  \hspace{2cm} (18)

Here, $D(k, t)$ and $A(k, t)$ are the canonical coordinates. The equation of motion for the vector potential becomes
\[ \varepsilon^{-1}(i\partial_k, k)k\varepsilon^{-1}(i\partial_k, k)kA(k) - \omega^2 A(k) = 0, \]  
\[ (19) \]

where \( A(k, t) = \exp(\pm i\omega t)A(k) \). Eq. (19) can be further simplified, by noting that the solutions satisfy

\[ \varepsilon^{-1}(i\partial_k, k)kA = \pm \omega A. \]  
\[ (20) \]

The latter has a natural interpretation of an “eigenvalue-like” equation for the differential dielectric operator \( \varepsilon^{-1} \).

We shall now make some assumptions on the behavior of \( \varepsilon^{-1} \). For \( k << k_c \), where \( 1/k_c \) is the “molecular” length scale, we must recover the non-homogeneous dependence \( \varepsilon = 1/\alpha z \), thus: \( \varepsilon^{-1}(i\partial_k, k << k_c) \rightarrow \pm i\alpha \partial_k \). On the other hand, for \( k >> k_c \), we expect the electromagnetic field to decouple from the medium, thus: \( \varepsilon^{-1}(i\partial_k, k >> k_c) \rightarrow (\epsilon_c)^{-1} \), where \( \epsilon_c \) is a constant. Assuming that \( \varepsilon^{-1} \) interpolates smoothly between these two limiting cases, we shall represent the dielectric operator by

\[ \varepsilon^{-1}(i\partial_k, k) = \tilde{\theta}(k_c - k)i\alpha \partial_k + \tilde{\theta}(k - k_c)\epsilon_c^{-1}, \]  
\[ (21) \]

where \( \tilde{\theta}(k) \equiv \frac{1}{2}[1 + \tanh(k/\Delta)] \) is the Heaviside step-function, and \( \Delta \) determines the width of the transition.

With this choice the solutions of eq. (19) are given by:

\[ G_\eta(k, t) = \frac{1}{k} \exp\left\{ \frac{i}{\alpha \epsilon_c} \int_{k_0}^{k} \left[ e^{2(k' - k_c)/\Delta} - \frac{\eta \epsilon_c}{k'} (1 + e^{2(k' - k_c)/\Delta}) \right] dk' \right\} e^{\pm i\omega_\eta t} \]

\[ = \frac{1}{k} \exp\left\{ \frac{i\Delta}{2\alpha \epsilon_c} \left[ e^{2(k-k_c)/\Delta} - e^{2(k_0-k_c)/\Delta} \right] - i\frac{\eta}{\alpha} \ln(k/k_0) - i\frac{\eta}{\alpha} e^{-2k_c/\Delta} \left[ \text{Ei}(\frac{2}{\Delta}k) - \text{Ei}(\frac{2}{\Delta}k_0) \right] \right\} e^{\pm i\omega_\eta t}, \]  
\[ (22) \]

where \( \text{Ei}(x) = \int_{-\infty}^{x} (e^{t}/t) dt \) is the integral-exponential function, and \( \omega_\eta = |\eta| \).

For \( k << k_c \) (22) reduces to \( \frac{1}{k} \exp(\pm i\omega_\eta t) \exp(-i\frac{\eta}{\alpha} \ln |k|) \). Substituting this into the Hamiltonian, we find that in the limit of \( k << k_c, \eta k > 0 \) and \( \eta k < 0 \) correspond to positive and negative energy modes, respectively. In the other limiting case, \( k >> k_c \), we have free waves with \( \omega = |k|/\epsilon_c \), and the energy is positive for \( \omega > 0 \).
In order to relate these solutions with the modes (11,12), it will be useful to define the following low momentum modes:

\[ \tilde{g}_I(\eta) = \begin{cases} 
\frac{1}{\sqrt{4\pi\omega}} k^{-1} e^{-i\omega t - i\eta \frac{1}{\alpha} \ln k}, & \text{for } k > 0 \\
0, & \text{for } k < 0 
\end{cases}, \quad (23) \]

\[ \tilde{g}_{II}(\eta) = \begin{cases} 
0, & \text{for } k > 0 \\
\frac{1}{\sqrt{4\pi\omega}} k^{-1} e^{i\omega t + i\eta \frac{1}{\alpha} \ln |k|}, & \text{for } k < 0 
\end{cases}, \quad (24) \]

for positive \( \eta \). For negative \( \eta \) we replace above \( \omega \rightarrow -\omega \). These modes form in the limit \( k_c \rightarrow \infty \), an orthogonal set under the scalar product:

\[ (\tilde{g}_I, \tilde{g}_{II}) = i \int_{-\infty}^{+\infty} dk \tilde{g}_I^* \varepsilon(i\partial_k, k) \tilde{g}_{II}. \quad (25) \]

For finite \( k_c \), we need to replace \( \tilde{g}_{I,II} \) with the exact solutions \( G_\eta(k,t) \). In the domain \( |k| < k_c \), however, we can now expand the field as:

\[ A(t,k) = \int_{-\infty}^{+\infty} (\tilde{g}_I(\eta)\tilde{a}_I(\eta) + \tilde{g}_{II}(\eta)\tilde{a}_{II}(\eta) + \text{h.c.}) d\eta, \quad (26) \]

where \( \tilde{a}_I \) and \( \tilde{a}_{II} \) are the annihilation operators for \( k > 0 \) and \( k < 0 \), states respectively.

Next let us notice that although the spectrum of \( g_I \) or \( g_{II} \) in eqs. (11,12) contains both positive and negative momentum \( k \), the Fourier transform of the positive \( U \) frequency combination, \( f_1 \) in eq. (11), is:

\[ \int_{-\infty}^{+\infty} f_1 e^{-ikz} dz = 2i \sinh\left(\pi\omega_\eta \frac{\alpha}{\kappa}\right) \Gamma\left(\frac{i\omega_\eta}{\alpha} + 1\right)\tilde{g}_I(\eta), \quad (27) \]

where \( \kappa = \eta > 0 \), and \( \Gamma \) is the Gamma function. The Fourier transform of \( f_2 \) yields \( \tilde{g}_{II} \). Consequently, \( \tilde{g}_I \) and \( \tilde{g}_{II} \) correspond to positive frequency \( U \)-modes. In the limit of \( k_c \rightarrow \infty \), we conclude that outgoing radiation modes, \( f_1 \) and \( f_2 \), correspond in momentum space, to the modes \( \tilde{g}_I \) and \( \tilde{g}_{II} \).

When dispersion in high momentum is present this correspondence is no longer exact. \( g_I \) and \( g_{II} \) will be modified near the horizon in the regime \( |z| < 1/k_c \). Nevertheless, let us consider an outgoing wave packet, and a time \( t_0 \) such that the wave packet is localized in a domain sufficiently far away from the horizon. At \( t = t_0 \) we can describe the outgoing
Hawking photon and its negative energy pair by the mode $f_1$, up to a small correction. The latter can be estimated by noting that in the momentum representation, large momentum amplitudes are suppressed like $1/k$. Thus, if we instead start at $t = t_0$ with the mode $G_\eta(k) \approx \tilde{g}_H$, with $\eta > 0$, it will correspond to $f_1$ up to corrections of at most $O(\lambda_c/M_{BH})$ where $\lambda_c = 2\pi/k_c$. Notice that unlike $f_1$, which is composed of both positive and negative parts, the corresponding mode $G_\eta(k)$ has a positive energy, i.e., an outgoing pair of thermal radiation are described by $G_\eta(k)$ modes in their ground state.

To proceed with, let us now represent an outgoing Hawking photon and its pair at $t = t_0$, by a wave packet of the exact modes $G_\eta(k)$ with $\eta, k > 0$:

$$A_g(k, t) = \frac{1}{\sqrt{2\pi}} \int_0^{\omega_c} e^{-(\omega - \omega_0)^2/\delta^2} e^{-i\omega t} G_\eta(k) d\omega. \quad (28)$$

$t_0$ is chosen such that $A_g(k, t_0)$ is mostly peaked at $k < k_c$, and $\omega_c$ is taken to be much larger then $\omega_0$. We can then follow $A_g(k, t)$ as it evolves backwards in time, and observe the effects of dispersion. As is shown in the following figures, the momentum gradually increases and the wave packet, from which the radiation must have originated, is sharply peaked with $k \sim k_c$.

![Figure 1. The imaginary part of $kA_g(k, t)$, at $t = 0$, with $k_c = 20$, $\omega_0 = \alpha = \delta = 1$, $\Delta = 0.5$, $\epsilon_c = k_c/\omega_0$](image-url)
As can be seen from Figure 1, at this stage the dispersion causes only a minor modification at $k \approx k_c = 20$, where the amplitude drops down to zero.

![Figure 1](image1.png)

**Figure 2.** The wave packet at $t = -5$.

In the last figure, the wave packet is completely localized within $\sim 3$ standard deviations, away from $k_c$. It corresponds therefore to a wave packet of free waves which interact very

![Figure 3](image2.png)

**Figure 3.** The wave packet at $t = -10$. 
weakly with the medium. As we evolve $A_g$ even further back in time, the wave continues very slowly to shift towards larger $k$, and its width, $\Delta k$, narrows. For example at $t = -40$, the wave is localized at $k \approx 22$ with $\Delta k \approx 0.2$. The corresponding spread in space is $\Delta z \sim 1/\Delta k$.

Therefore, the wave packet which gives rise to Hawking radiation, is at sufficiently early time much larger than the size of the black hole.

The value of $\epsilon_c$ in the simulations above has been chosen as $\epsilon_c = k_c/\omega_0$. Since the transition occurs at $k \approx k_c$, and since for $k > k_c$ the frequency, $\omega_0 = k/\epsilon_c$, is fixed by the Hawking temperature, this choice corresponds to a minimal mismatch between the maximal dielectric constant, $\epsilon_{\text{max}} \sim 1/\alpha z_{\text{min}} \sim k_c/\alpha$, for low momentum waves, and $\epsilon_c$ for $k > k_c$. This choice implies that the velocity of light (or $g_{00}$) drops down to a minimal value, $1/\epsilon_c$, and remains constant for higher momenta. For other values, say $\epsilon_c = 1$, free waves at $k \sim k_c$ have frequency $\omega \sim k_c$ which does not correspond to the given Hawking frequency $\omega_0$. In this case we find that the Hawking radiation originates from a wave packet in the same range of $k$ and the same amplitude as above, but which is strongly oscillating as a function of $k$. Although only weakly interacting free modes with $\omega \sim k_c$ contribute, in the domain $k > k_c$, this still effectively gives rise to the exact mode $G_\eta$ with low frequency $\omega = \omega_0$. (A related phenomena which acts in the reverse direction, yielding an effective high frequency mode from low frequency modes, was described in [13]. We shall further describe this case elsewhere).

Before concluding let us comment on some open issues. First we note that although the inclusion of spatial dispersion has eliminated the singular electric polarization at $z = 0$, we still can have $\varepsilon \sim k_c/\alpha \sim \lambda_c/M_{BH} >> 1$. In other words, the use of linear constitution relations is questionable. Another question concerns the role of temporal dispersion, which has been so far ignored. In the present model, the latter does not seem to be essential in order to generate the Hawking radiation, since $\omega << k_c$ at any time. Nevertheless, if we do assume a cutoff scale $\omega_c$, then the dielectric constant will typically be $\varepsilon(t - t', z) \sim (z\alpha)^{-1}\sin(\omega_c(t - t'))/\omega_c$. Therefore, temporal dispersion induces a non-locality, $t - t' \sim 1/\alpha z\omega_c$, which greatly increases near the horizon. This phenomena may be relevant for
understanding the final stages of black hole evaporation.

In conclusion, we have established that outgoing radiation modes, originate in this model from nearly free modes, $G_η(k)$, with $k \approx k_c$, which are weakly coupled to the medium. The latter are analogous to the Kruskal modes, $\exp(-i\tilde{\omega}U)$, in the standard picture, but do not contain wave numbers with $k \gg k_c$. As a consequence, the initial state which gives rise to Hawking radiation, most naturally turns out to be the ground state of nearly free modes with $k \sim k_c$, in an effectively flat background geometry.

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[11] More general space-times can be easily obtained. For example the Schwarzschild geometry corresponds to the case $\varepsilon_{zz} = \mu_{zz} = 1$, $\varepsilon_{xx} = \varepsilon_{yy} = \mu_{xx} = \mu_{yy} = r/r_0 - r_0$, where the event horizon is at $r = r_0$.

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