Transferring Fourier Multipliers from $\mathbb{R}^n$ to Compact Lie Groups

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Abstract

This paper uses the wrapping map of Dooley and Wildberger to prove $L^p$ boundedness of multipliers on compact Lie groups by transferring the estimate from $\mathbb{R}^n$. This improves the bounds in several cases, and simplifies the proofs of others.

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1 Introduction

Let $G$ be a compact connected Lie group and $\mathfrak{g}$ its associate Lie algebra.

There are many papers dealing with the $L^p$ boundedness of multipliers on compact Lie groups, including [10], [11], [12]. These papers have made use of one or both of the following techniques: the first is to try and transfer the analysis from the curved manifold to $\mathbb{R}^n$. The main drawback here is that this can only occur within a neighbourhood of where the exponential map is injective.

The second technique is to transfer the analysis directly to a maximal torus $T \subseteq G$ to appeal to existing (abelian) Fourier analysis. This has involved studying the integrability of negative powers of the Weyl function to transfer estimates on $G$ to estimates on $T$. Since this function vanishes at the origin, this creates many technical problems.

The wrapping map $\Phi$ introduced in [3] is a homomorphism of central measures or distributions under (abelian) convolution on $\mathfrak{g}$ (viewed as $\mathbb{R}^n$) to their

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corresponding counterparts on $G$, utilising the whole of the lift of the exponential map, not just its restriction to a fundamental domain. That is,

$$\Phi(\mu \ast g \nu) = \Phi(\mu) \ast_G \Phi(\nu)$$

As any Fourier multiplier can be written as a convolution with the associated kernel operator, the wrapping map provides a natural mechanism to transfer results concerning Fourier multipliers from $g \cong \mathbb{R}^n$ to $G$.

2 Notation and Preliminaries

Let $G$ be a compact connected Lie group, $\mathfrak{g}$ its Lie algebra, $T$ a maximal torus and $\mathfrak{t}$ the Lie algebra of $T$. Let $n$ be the dimension of $G$, and $l$ the dimension of $T$ (also known as the rank of $G$). Let $B(\cdot, \cdot)$ be the Killing form on $\mathfrak{g}$, with $\mathfrak{g}^*$ and $\mathfrak{t}^*$ the respective duals of $\mathfrak{g}$ and $\mathfrak{t}$ with respect to the Killing form.

We denote by $\Sigma^+$ the set of positive roots $\{\alpha_1, \ldots, \alpha_k\}$, with $k = |\Sigma^+|$, and thus we have $n = l + 2k$. Let $W$ denote the Weyl group, $\mathfrak{t}_+^*$ the positive Weyl chamber, and let $\rho = \frac{1}{2} \sum_{\alpha \in \Sigma^+} \alpha$. Let $\Lambda \subseteq \mathfrak{t}^*$ denote the set of integral weights, and $\Lambda^+ = \Lambda \cap \mathfrak{t}_+^*$ the set of positive integral weights.

Every irreducible representation $\pi \in \hat{G}$ is associated to a unique highest weight $\lambda \in \Lambda^+$. If $\chi_{\pi} = \chi_{\lambda}$ is the character of this representation, the Kirillov’s character formula is given by

$$j(X)\chi_{\lambda}(\exp X) = \int_{O_{\lambda+\rho}} e^{i\beta(X)}d\mu_{\lambda+\rho}(\beta), \text{ for all } X \in \mathfrak{g}$$

where $O_{\lambda+\rho}$ is the co-adjoint orbit through $\lambda + \rho \in \mathfrak{t}_+^*$, and $\mu_{\lambda+\rho}$ is the Liouville measure on $O_{\lambda+\rho}$ with total mass $d_\lambda = d_\pi = \dim \pi$, and where $j(X)$ is is the analytic square root of the Jacobian of exp with $j(0) = 1$, given by

$$j(X) = \prod_{\alpha \in \Sigma^+} \frac{\sin \alpha(X)/2}{\alpha(X)/2}$$

That is, for $f \in C_c^\infty(G)$:

$$\int_{\mathfrak{g}} f(\exp X)|j(X)|^2dX = \int_G f(g)dg :$$

where we have normalised the Haar measure $dg$ on $G$ to have total mass 1, with subsequent normalisation of the Lebesgue measure $dX$ on $\mathfrak{g}$. Similarly, we normalise the Haar measure and $dt$ on $T$ and the Lebesgue measure $dH$ on $\mathfrak{t}$ so that we have for $f \in C_c^\infty(T)$:

$$\int_T f(\exp H)dH = \int_T f(t)dt$$

Integrals over $G$ of central functions may be computed by integrals on $T$ by the Weyl integral formula:

$$\int_G f(g)dg = \frac{1}{|W|} \int_T f(t)|\triangle(t)|^2dt, \text{ where } \triangle(\exp H) = \prod_{\alpha \in \Sigma^+} 2\sin \alpha(H)/2$$

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Correspondingly, we also have a similar integral formula for an Ad-invariant function of compact support $\varphi$ on $g$:

$$\int_{g} \varphi(X) dX = \int_{t^+} \prod_{\alpha \in \Sigma_+} \alpha(H)^2 \varphi(H) dH.$$ 

We also denote by $L^p(g)$, $L^p(t)$, $L^p(G)$ and $L^p(T)$ the set of $p$-integrable functions on $g$, $t$, $G$ and $T$, respectively, and $\| \cdot \|_p$ the $p$-norm when the space concerned is unambiguous. We also denote by $L^p_I(g)$ and $L^p_I(G)$ the respective sets of $G$-invariant $L^p$ functions.

Let $\mu^\wedge$ denote the Fourier transform of $\mu$, with the convention

$$\mu^\wedge(\xi) = \int_{\mathbb{R}^n} \mu(x) e^{ix \cdot \xi} dx.$$ 

when $\mu \in L^1(\mathbb{R}^n)$. Correspondingly, let $\pi \in \hat{G}$ have highest weight $\lambda \in \Lambda^+$.

The Fourier transform of a central function, $\nu$, is a multiple $c_{\pi} I_{\pi}$ of the identity, where

$$c_{\pi} = \frac{1}{d_{\pi}} \int_{G} \nu(g) \chi_{\lambda}(g) dg$$

when $\mu \in L^1(G)$, such that we may write the Fourier series

$$\mu(g) = \sum_{\lambda \in \Lambda^+} c_{\lambda} d_{\lambda} \chi_{\lambda}(g).$$

We also employ the notation $X \lesssim Y$ to refer to the situation $X \leq CY$, where $C$ is an unspecified constant.

**The Wrapping Map**

Assume that $\nu$ is a distribution of compact support on $g$, or $j \nu \in L^1(g)$. We define the wrapping map, $\Phi$ by

$$\langle \Phi(\nu), f \rangle = \langle \nu, j\tilde{f} \rangle$$

where $f \in C^\infty(G)$, $\tilde{f} = f \circ \exp$. We call $\Phi(\nu)$ the *wrap* of $\nu$. The principal result is the wrapping formula ([4], Thm. 2) given by

$$\Phi(\mu *_g \nu) = \Phi(\mu) *_{G} \Phi(\nu)$$  (2.2)

What (2.2) shows us is that problems of convolution of central measures or distributions on a (non-abelian) compact Lie group can be “transferred” to Euclidean convolution of Ad-invariant distributions on $g$. Thus, since a Fourier multiplier operator may be regarded as a convolution operator, a natural question arises as to how we may “wrap” a Fourier multiplier operator. To explicitly show this connection, we must consider the formulation for $\Phi$.

**Proposition 1.** We have the following formulations for $\Phi$ for $G$-invariant distributions of compact support, firstly given as summations over the integer lattice:
Φ(μ)(exp H) = \sum_{\gamma \in \Gamma} \mu_j (H + \gamma) \quad \forall H \in t. \quad (2.3)

Φ(j\mu)(exp H) = \sum_{\gamma \in \Gamma} \mu(H + \gamma) \quad \forall H \in t. \quad (2.4)

and as the Fourier series:

\Phi(\mu)(exp H) = \sum_{\lambda \in \Lambda^+} d_\lambda \mu^\wedge (\lambda + \rho) \chi_\lambda (exp H) \quad (2.5)

\Phi(j\mu)(exp H) = \sum_{\lambda \in \Lambda^+} d_\lambda \int_G \mu^\wedge (\lambda + \rho - \text{ad}(g)(\rho)) \, dg \chi_\lambda (exp H) \quad (2.6)

where the values are given on H \in t

(2.4) is calculated in [4], and (2.3) follows naturally. In [4] it is shown that the Fourier transform of \Phi(\mu) at \pi is given by

\[ c_\pi = (\Phi(\mu))^\wedge (\lambda + \rho) = \mu^\wedge (\lambda + \rho) \]

by applying Kirillov’s character formula, that is:

\[ c_\pi = \frac{1}{d_\pi} \langle \Phi(\mu), \chi_\lambda \rangle = \frac{1}{d_\pi} \langle \mu, j\tilde{\chi}_\lambda \rangle = \langle \mu, \int_G e^{i\theta(\lambda + \rho)(\cdot)} \, dg \rangle \]

By the G-invariance of \mu this is

\[ c_\pi = \langle \mu, e^{i(\lambda + \rho)(\cdot)} \rangle = \mu^\wedge (\lambda + \rho) \quad (2.7) \]

which yields (2.5). Note that (2.2) also follows from (2.7), since

\[ (\Phi(\mu \ast \nu))^\wedge (\pi) = (\mu \ast \nu)^\wedge (\lambda + \rho) I_\pi \]
\[ = \mu^\wedge (\lambda + \rho) \nu^\wedge (\lambda + \rho) I_\pi \]
\[ = (\Phi(\mu))^\wedge (\lambda + \rho) (\Phi(\nu))^\wedge (\lambda + \rho) I_\pi \]
\[ = (\Phi(\mu) \ast \Phi(\nu))^\wedge (\pi) \]

We now have an analogue of (2.7) for Ad-invariant distributions with compact support of the form j\mu:

**Lemma 1.** Let \mu be an Ad-invariant distribution of compact support on g. Then the Fourier transform of \Phi(\mu) at \pi is a multiple \(c_\pi I_\pi\) of the identity, where

\[ c_\pi = (\Phi(\mu))^\wedge (\lambda + \rho) = \int_G \mu^\wedge (\lambda + \rho - \text{ad}(g)(\rho)) \, dg \]

**Proof:** From (2.7) we have that as a multiple of the identity:

\[ \Phi(\mu)^\wedge (\pi_\lambda) = \mu^\wedge (\lambda + \rho) \]
Thus,
\[
\Phi(j^\wedge(\mu;\pi)) = (j^\wedge(\mu;\pi)) + \rho
\]
\[
= (j^\wedge(\mu;\pi)) + \rho
\]
\[
= \int_G \mu^\wedge(\lambda + \rho - \text{ad}(g)(\rho)) \, dg
\]

Reconciling (2.3) with (2.5), or (2.4) with (2.6), gives the Poisson summation formula for compact Lie groups.

In the next section we will consider these two function spaces in the treatment of multipliers.

3 The Wrapping Map and Multipliers

Consider the Fourier multiplier operator, \( T \), on \( L^p \) by
\[
\hat{T}f(\xi) = m(\xi)\hat{f}(\xi)
\]
where \( m(\xi) \) is the symbol of the operator \( T \). By the standard kernel theorems, every Fourier multiplier \( T \) maybe expressed as a convolution operator with kernel \( K \), that is: \( Tf = K * f \).

On \( g \cong \mathbb{R}^n \), a multiplier \( T_\psi \) takes the form:
\[
(T_\psi f)(x) = \frac{1}{(2\pi)^n} \int_g \psi(\xi)f(\xi)e^{-iB(\xi,x)}d\xi
\]
with kernel
\[
K_\psi(x) = \int_g \psi(\xi)e^{iB(\xi,x)}d\xi
\]
that is
\[
\hat{K_\psi}(\xi) = \psi(\xi)
\]

On \( G \), a multiplier \( T_\Psi \) takes the form:
\[
(T_\Psi f)(g) = \sum_{\lambda \in \Lambda^+} \Psi(\lambda)f(\lambda)\chi_\lambda(g)
\]
with kernel
\[
K_\Psi(g) = \sum_{\lambda \in \Lambda^+} \Psi(\lambda)\chi_\lambda(g)
\]
that is
\[
\hat{K_\Psi}(\lambda) = \Psi(\lambda)
\]

Wrapping the multiplier \( T_\psi \) from \( g \cong \mathbb{R}^n \) to \( G \) we have
\[
\Phi(T_\psi f) = \Phi(K_\psi * f) = \Phi(K_\psi) * \Phi(f)
\]

To compute the kernel \( \Phi(K_\psi) \), we see that its Fourier coefficients are
\[
\Phi^\wedge(K_\psi)(\pi_\lambda) = \Psi(\lambda) = K_\psi^\wedge(\lambda)
\]
Thus, we need to compute $\Psi(\lambda) = K_\psi^*(\lambda)$. In light of (2.7), we see that

$$\Psi(\lambda) = \psi(\lambda + \rho)$$

(3.1)

Alternatively, if we wish to consider $\Phi(j \cdot K_\psi)$, we see by Lemma 1 that

$$\Psi(\lambda) = \int_G \psi(\lambda + \rho - \text{ad}(g)(\rho)) \, dg$$

(3.2)

These two expressions 3.1 and 3.2 for the multiplier $\Psi$ on $G$ constitute to two forms considered in [11], where they are referred to as “(*)” and “(**)”, respectively.

Thus, to obtain bounds for multipliers on $L_p^q(G)$, we now need to consider bound of the form

$$\|\Phi(\nu)\|_q \lesssim \|\nu\|_p \text{ and } \|\Phi(j \cdot \nu)\|_q \lesssim \|\nu\|_p$$

4 Main Results: $L_p^q - L^q$ bounds

In this section we consider the following problem: Suppose $\nu \in L^p(g)$. For what $q$ is the wrap of $\nu$ in $L^q(G)$? That is, for what $p$ and $q$ do we have

$$\|\Phi(\nu)\|_q \lesssim \|\nu\|_p$$

Furthermore, for what $p$ and $q$ do we have

$$\|\Phi(j \nu)\|_q \lesssim \|\nu\|_p$$

These will then be applied to the $L_p^q - L^q$ bounds of multipliers.

We firstly have the following key result:

**Lemma 2.** $j \in L^p(g)$ for $p > 2n/(n - l)$.

**Proof:** Firstly, note that $j$ is bounded at $0 \in g$. Denote a neighbourhood $B$ of $0 \in g$, with $\int_B |j(X)|^p \, dX = C$ being a finite quantity.

$$\|j(X)\|_p = \int_g |j(X)|^p \, dX$$

$$= \int_{g \setminus B} |j(X)|^p \, dX + C$$

$$\leq \int_{g \setminus B} \left| \prod_{\alpha \in \Sigma^+} \alpha(\alpha/2) \right|^{-p} \, dX + C$$

$$= \int_{l^+ \setminus B} \left| \prod_{\alpha \in \Sigma^+} \alpha(H/2) \right|^{2-p} \, dH + C$$

The function $\prod_{\alpha \in \Sigma^+} \alpha(H/2)$ is a homogeneous polynomial of degree $k$ on a space of dimension $l$, having positive coefficients, and thus will be bounded provided $k(2 - p) < l$. That is, $p > 2n/(n - l)$.

As a consequence of this bound on $j$, we have the following bounds on $p$ and $q$ for which $\|jf\|_q \lesssim \|f\|_p$:
Lemma 3. We have

a) \[ \|jf\|_p \leq \|f\|_p \] for \( 1 < p < \infty \).

b) \[ \|jf\|_p \leq \|f\|_{\infty} \] for \( p > 2n/(n-l) \).

c) \[ \|jf\|_1 \leq \|f\|_q \] for \( q < 2n/(n+l) \)

Proof: These follow from the Hausdorff-Young, Young, and Hölder inequalities, and are thus best possible.

Proof of a): Let \( p \) and \( q \) be conjugate exponents. We have
\[
\|jf\|_p \leq \|\hat{jf}\|_q \quad \text{(by Hausdorff-Young)}
\]
\[
= \|\hat{j} \ast \hat{f}\|_q
\]
\[
\leq \|\hat{j}\|_1 \|\hat{f}\|_q \quad \text{(by Young)}
\]
\[
= \|\mu\|_1 \|\hat{f}\|_p
\]
\[
= \|f\|_p
\]

since the mass of the Liouville measure \( \mu \) is equal to \( \text{dim} \pi_0 = 1 \).

Proof of b): Let \( p \) and \( q \) be conjugate exponents. We have
\[
\|jf\|_p \leq \|\hat{jf}\|_q \quad \text{(by Hausdorff-Young)}
\]
\[
= \|\hat{j} \ast \hat{f}\|_q
\]
\[
\leq \|\hat{j}\|_q \|\hat{f}\|_1 \quad \text{(by Young)}
\]
\[
= \|j\|_p \|f\|_{\infty}
\]

which is bounded so long as \( p > 2n/(n-l) \), since we have \( j \in L^p(g) \) by Lemma 2.

Proof of c): Let \( p \) and \( q \) be conjugate exponents. By Hölder we have:
\[
\|jf\|_1 \leq \|j\|_p \|f\|_q
\]

By Lemma 2, \( j \in L^p(g) \) for \( p > 2n/(n-l) \). Hence, the result follows if \( f \in L^q(g) \) for \( q < 2n/(n+l) \). \( \square \)

Regarding \( L^p \) bounds for \( \Phi \), we have the following from \[4\], which is almost obvious from the definition: If \( \nu \in L^1(g) \), then we have:
\[
\|\Phi(\nu)\|_1 \leq \|\nu\|_1 \quad \text{(4.1)}
\]
\[
\|\Phi(j\nu)\|_1 \leq \|\nu\|_1 \quad \text{(4.2)}
\]

We now prove a more general \( L^p \) bound:

Theorem 1. We have the bounds
\[
\|\Phi(j\mu)\|_p \lesssim \|\mu\|_{p^\prime}, \quad 1 \leq p \leq \infty \quad \text{(4.3)}
\]
\[
\|\Phi(\mu)\|_p \lesssim \|\mu\|_p, \quad p \geq 2 \quad \text{(4.4)}
\]
**Proof:** Let \( \Gamma \subseteq t \) be a fundamental domain for \( \Gamma \) in \( t \). We have:

\[
\| \Phi(j \mu) \|^p_p = \int_G |\Phi(j \mu)(g)|^p \, dg \\
= \frac{1}{|W|} \int_{t_r} |\Phi(j \mu)(t)|^p |\Delta(t)|^2 \, dt \\
= \frac{1}{|W|} \int_{t_r} \sum_{\gamma \in \Gamma} \mu(H + \gamma)|\Delta(\exp H)|^2 \, dH \\
\leq \frac{1}{|W|} \int_{t} |\mu(H)|^p |\Delta(\exp H)|^2 \, dH \\
= \int_{t} |\mu(H)|^p |j(H)|^2 \prod_{\alpha \in \Sigma'} \alpha(H/2)^2 \, dH \\
= \int_{\theta} |\mu(X)|^p |j(X)|^2 \, dX \\
\leq \int_{\theta} |\mu(X)|^p \, dX = \|\mu(X)\|^p_p
\]

Similarly for \( \Phi(\mu) \), we arrive at

\[
\| \Phi(\mu) \|^p_p = \int_{\theta} |\nu(X)|^p |j(X)|^2 \, dX \\
= \int_{\theta} |\mu(X)|^p |j(X)|^{2-p} \, dX \\
\leq \|\mu\|^p_p \|j(X)|^{2-p}\|_\infty \\
\leq \|\mu\|^p_p
\]

as long as \( p \geq 2 \). □

**Remark:** Compare this to [11] Lemma 3, which asserts these bounds as long as

\[
\sum_{\gamma \in \Gamma} |j(H + 2\pi \gamma)|^{np' - 2}
\]

is bounded, which is only explicitly addressed for the case of \( SU(2) \). Here, \( n = 1 \) is the case for \( \Phi(\mu) \), and \( n = 2 \) is the case for \( \Phi(j \mu) \). However, there appears to be an error in this calculation.

We can use this to prove further \( L^p - L^q \) for \( \Phi(j \mu) \) only.

**Theorem 2.** We have the \( L^q - L^1 \) bound

\[
\| \Phi(j \mu) \|_1 \lesssim \|\nu\|_q, \quad q < 2n/(n + l)
\]

and the \( L^\infty - L^q \) bound

\[
\| \Phi(j \mu) \|_q \lesssim \|\mu\|_\infty, \quad q > 2n/(n - l)
\]

**Proof:** Let \( \nu = j \mu \). From [11], we have that

\[
\|\Phi(\nu)\|_1 \lesssim \|\nu\|_1
\]
and so by setting $\nu = j\mu$, we have

$$\|\Phi(j\mu)\|_1 \lesssim \|j\mu\|_1$$

By Lemma 3 c), it is only possible to have

$$\|\Phi(j\mu)\|_1 \lesssim \|j\mu\|_1 \leq \|\mu\|_q$$

for $q < 2n/(n+l)$. Similarly, By lemma 3 b), it is only possible to have

$$\|\Phi(j\mu)\|_q \lesssim \|j\mu\|_q \leq \|\mu\|_\infty$$

for $q > 2n/(n-l)$. □

We concluded this section with an additional proof of the $L^2$ bound for $\Phi(j\nu)$ only. This uses the fact that the Fourier transform of $j$ leads to a region around each point $\lambda \in \Lambda^+$ to tessellate over $t^*$. We then apply Parseval.

**Theorem 3.**

$$\|\Phi(j\nu)\|_2 \leq \|\nu\|_2$$

**Proof:** From Lemma II we have that

$$d\nu \Phi(j\nu)^\wedge(\pi\lambda) = \int_G \nu^\wedge(\lambda + \rho - \text{ad}(\rho)) \, dg$$

Let $Q$ be the convex hull of $\{w\rho \mid w \in W\}$ and $E$ a bounded function on $Q$. We have,

$$\|\Phi(j\nu)\|_2^2 = \sum_{\lambda \in \Lambda^+} \left| \int_G \nu^\wedge(\lambda + \rho - \text{ad}(\rho)) \, dg \right|^2 \leq \sum_{\lambda \in \Lambda^+} \left| \int_Q \left( \prod_{\alpha \in \Sigma^+} \frac{\partial}{\partial \alpha} \right) \nu^\wedge(\lambda + \rho - \xi) E(\xi) \, d\xi \right|^2 \lesssim \int_{t^*} \left| \left( \prod_{\alpha \in \Sigma^+} \frac{\partial}{\partial \alpha} \right) \nu(\xi) \right|^2 \, d\xi = \int_{t^*} \left| \prod_{\alpha \in \Sigma^+} \alpha(H) \nu(H) \right|^2 \, dH \lesssim \int_{\mathbb{R}} |\nu(X)|^2 \, dX = \|\nu\|_2^2$$

□

5 An Application: Convergence of Fourier series

We firstly require some notation, which we adopt from [10]. Let $R$ be a Weyl-invariant, closed, convex polyhedral subset of $t$ which contains the origin. Let $tR = \{tX \mid X \in R\}$. As $t$ ranges over $[1, \infty)$, $(tR) \cap t^*_+$ generates only a countable
number of distinct sets, which is denoted by \( \{R_N\} \). Define the Weyl-invariant polygonal operator on \( g \cong \mathbb{R}^n \) by:
\[
K_t \mu(x) = \int_{tR} \exp(-isx)(\hat{\mu}(s))ds
\]
We define the \( N \)th partial sum of the Fourier series of a function \( f \in L^p(G), 1 \leq p \leq \infty \), as
\[
S_N f(g) = \sum_{\lambda \in R_N} d_{\lambda \chi} \ast f(g)
\]
Then we have

**Theorem 4.** Suppose \( R \) is a regular polyhedron, then if \( f \in L^p_1(G), \ p > 2n/(n+1) \), then \( S_N f \) converges to \( f \) almost everywhere.

**Proof:** Since \( \Phi(K_t) = S_N \), and from [6] the Fourier transform converges for any polygonal region containing the origin for \( 1 < p < \infty \), and therefore from section 4 \( \Phi(K_t) \) is bounded for \( 1 \leq p \leq \infty \). \( \square \)

6 Further Directions

The wrapping map devised in [3] has already been employed as a transference method in analysis. These could be further extended using the estimates (or the idea of transferring from the tangent space to a curved space) in this paper. For example:

- In [4] the concept of a ‘modulator’ was introduced. In particular, this allows one to consider the class of modulators that ‘wraps’ to a (generalised) character. Thus, one approach to considering the norms of characters on \( G \) would be to consider the norm of the modulator on \( g \).
- In [7] the wrapping map was used to ‘wrap’ not only heat kernels but also Brownian motion from \( g \cong \mathbb{R}^n \) to \( G \). The estimates in this paper could be used to compute \( L^p \) norms for solutions for these and other partial differential equations on compact Lie groups, as well as Brownian motions or other stochastic processes.
- In [8] the wrapping map was extended to complex Lie groups and compact symmetric spaces. \( L^p \) bounds on these spaces could again be computed by considering the \( L^p \) bounds on their respective Lie algebra and tangent space.

We will consider these in future work.

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