Finite Element Analysis of Overlapped Z-Purlins Restrained by Sheeting

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Abstract. Cold-formed steel Z-purlins with overlapping at the intermediate supports ensures the continuity of the bending moment. A continuous beam with two equal spans of 6 m is investigated under gravity loading using the finite element method. The numerical model includes the beneficial interaction between the Z-purlins and the trapezoidal sheeting. Finite element results show that the screw spacing and sheeting thickness has a limited influence on the resistance of the member. The effect of the overlapping length and detailing of the connection is studied through a parametric study. Six overlaps varying from 100mm to 1200mm (1.6% to 20% of one span) capture the change of the failure mode from the overlap edge to the support region of the purlin. The connection detailing is studied considering multiple bolt/screw patterns. The detailing and length of the overlap connection has a major influence on the moment distribution and consequently on the failure mode. The detailed finite element analysis shows that assuming the beam connection as continuous in 1D beam models lead to an unrealistic bending moment distribution and failure mode. Connecting the bottom flange using screws improves the overall load carrying capacity of the beam especially for short overlap lengths. The increase in resistance is attributed to the restraining effect on the free flange which fails due to distortional buckling.

1. Introduction

Cold-formed steel construction is a popular solution for secondary load bearing members due to their efficient material use, cross-sectional flexibility and ease of manufacturing and construction. Z-shaped cold formed profiles are traditionally used as purlins. The purlins commonly span continuously over rafters and splice through a sleeved or lapped connection. The thin-walled nature of cold formed members coupled with the complexity of the cross-sections usually require advanced numerical analysis such as [1] or extensive testing to yield accurate results. Moreover, the inclusion of the connection and roof sheeting makes large-scale testing of continuous purlins unfeasible from an economic and experimental point of view.

Extensive experimental and numerical analysis [2–7] performed on overlapped purlins conclude that the lap edge commonly governs the design. According to [8], assessing the resistance of the overlapped members must account for both the bending and shear capacities. [5] proposed an interaction criterion between bending moment and web crippling (BWC) to check the critical section at the edge of the overlap. In addition to the BWC check the static scheme of the continuous beam...
should account for the influence of the semi-rigid connection. The experimental program carried by [6] concludes that the resistance of lapped connections is mainly influenced by the bending moment. In addition, [6] recommends a simplified model to quantify the effect of the lap in the deflection and stress analysis, based on the moment of inertia. [4] confirms the limited influence of the shear force and puts forward a simplified design method based on the local and distortional critical buckling moment. Given the continuous nature of the purlins [9,10] proposed an approach to account for the bending moment redistribution based on advanced finite element models.

The stiffness of overlapped purlins can be predicted as a superposition of bending, shear and local bolt bearing deformation [11]. A detailed procedure for two bolt patterns is presented based on a non-linear relation of the bolt bearing behaviour. Experimental research of [12,13] shows that the nesting effect of the Z-purlins has a positive influence on the member stiffness. An increased linear bolt stiffness accounting for this effect is recommended to be used for elastic calculations. The case of vertical slotted holes is studied by [3] which introduced an effective flexural rigidity ratio based on the lap length, section depth and purlin thickness.

The aim of this paper is to identify the key parameters influencing the behaviour of overlapped Z-purlins based on validated numerical models. The analysis will be performed on a two-span continuous beam under gravity loading. To capture as much as possible the real behaviour of the purlin the sheeting is included in the model. The connection detailing, sheeting thickness and screw spacing influence is studied on set of six overlap lengths. The wide range of lap lengths aims to identify the key failure modes and the shift of the critical cross-section.

2. Finite element validation
The finite element models were developed using the commercially available software ABAQUS/CAE [22]. To accurately capture the behaviour of the thin-walled structure and connection the GMNIA method (geometrically and materially non-linear analysis with imperfections included) was used. The numerical model was validated against experimental data performed by [7,14]. (see Figure 1).

Figure 1. Test setup overview [7]
2.1. Model description
The Z-profiles were modelled using the S4R element which is defined as a 4-node shell element with reduced integration. A global mesh seed of 10mm was defined for the Z-profiles which was refined locally to ensure a minimum of 3 elements along the radius and the lip stiffener. The finite element mesh yields a good balance between the required computation time and the accuracy of the analysis.

The load was applied in a displacement-controlled manner at the mid-span bolts. In line with the test setup the specimen was constrained laterally using a series of stiff elastic springs. The springs are positioned at both the top and bottom flange as presented in Figure 2. The interaction was modelled using the General contact feature. The friction coefficient was defined equal to 0.3 while element penetration was prevented using the hard contact behaviour.

The experimental research reported for the purlins a tested yield strength of 541MPa and an elastic modulus of 199 GPa. The material model was described (see equation 1 and 2) by the basic Ramberg-Osgood stress-strain relation up to 0.2% proof stress $\sigma_{0.2}$ followed by a slope of $E/50$ to capture the strain hardening, where $E$ is the elastic modulus of the steel. It was found by [15] that the shape of the diagram up to the 0.2% proof stress has a significant influence on the accuracy of the results. The shape factor $n$ was recommended by [16] for 350 and 450 steel equal to 28. The determined engineering diagram was converted into a true stress-strain one using the relationships from the Annex C of [17]. The residual stresses and the increased yield strength in the corner region were neglected.

\[
\varepsilon = \frac{\sigma}{E} + 0.002 \left( \frac{\sigma}{\sigma_{0.2}} \right)^n \quad \text{for} \quad \sigma \leq \sigma_{0.2} \\
\varepsilon = \varepsilon_{0.2} + \frac{50(\sigma-\sigma_{0.2})}{E} \quad \text{for} \quad \sigma > \sigma_{0.2}
\]

2.2. Geometric imperfections
Geometric imperfections were included in the finite element model as an unstressed disturbed mesh. The imperfections were generated manually since the traditional method of scaling the elastic buckling shape requires many eigenmodes to capture separately the required local and distortional mode. It was observed the results are sensitive to the distribution of imperfections which emphasizes the need for a manual inclusion. The shape of the local and distortional buckling was obtained using the constrained finite strip method software CUFSM [18]. The maximum amplitude of the imperfection was defined equal to the values proposed by [19] for 50% CDF. For local buckling the amplitude of the imperfections was set to 0.34t and 0.94t for distortional buckling. The imperfections were distributed along the length of the beam in a sinusoidal fashion with a halfwave corresponding to each mode of buckling. The maximum amplitude of the local and distortional buckling was defined at the edge of the overlap in the compressed region of the beam. In Figure 2 the local imperfections are presented and magnified by a factor of 10 for illustration purposes.
### 2.3. Overlap connection

The bolted connection was modelled using non-linear springs to describe the bolt bearing relationship. The spring connects only the translational degrees of freedom of the nodes. The spring acting perpendicular to the web was defined an elastic stiffness of 100 kN/mm. The bolt bearing relation was defined by [11]. In Figure 3 the spring properties are plotted for the values used in the validation model, namely \( t = 1.61 \text{ mm}, f_u = 595 \text{ MPa}, d = 12\text{ mm}. \)

![Figure 3. Non-linear spring relationship for bolt bearing](image)

### 2.4. Validation

The model results in a safe prediction in terms of the final capacity of the overlap connection as shown in Table 1. The average deviation is on the conservative side making the finite element model safe for the parametric study.

|        | \( 2 \times L_p \) | \( M_{\text{exp}} \) | \( M_{\text{FEM}} \) | \( M_{\text{FEM}} / M_{\text{exp}} \) |
|--------|--------------------|---------------------|---------------------|----------------------------------------|
| ZA018R | 180                | 8.91                | 8.65                | 97%                                    |
| ZA024R | 240                | 9.59                | 9.68                | 101%                                   |
| ZA030R | 300                | 10.8                | 10.88               | 101%                                   |
| ZA060R | 600                | 14.41               | 13.8                | 96%                                    |
| ZA090R | 900                | 16.25               | 15.71               | 97%                                    |
| Average|                    |                     |                     | 98%                                    |

As shown in Figure 4 the model captures accurately the load deflection behaviour especially for overlap lengths higher than 150mm. The stiffer behaviour for the short overlap lengths of 90 mm and 180mm obtained from the numerical analysis could be explained by the nesting effect illustrated in Figure 5. The geometry of the setup was modelled such that the Z-purlins were in perfect contact from the start of the loading which facilitates the nesting of the sections. This effect was experimentally observed by [12] and showed to increase the bending stiffness by 20%. Regardless of this increase in stiffness, the model was able to capture accurately the bending moment resistance and the failure mode (see Figure 6) of the experimental setups.
Figure 4. Bending moment at mid-span vs. displacement curves (experimental [7] vs. finite element results)
3. Parametric study

3.1. Finite element model description

The validated model was developed to include a new cross-section, the profiled sheeting and the purlin support brackets as shown in Figure 7. The dimensions of the Z-section are presented in Figure 8a). The assumed steel grade is S350. The distance between the supports is equal to 6 m. The Z-purlins were connected together and to the support bracket using non-linear springs with the relationship presented in the previous section. A coarser mesh size was defined for the sagging bending moment region while the fine mesh was localized in the hogging moment region where the failure occurs.

The profiled sheeting is modelled using a linear elastic steel with an Youngs modulus of 210GPa. The mesh consists of a coarse distribution in the spanning region and is refined in the contact area to ensure the stability of the analysis. The gravity load is applied on the entire surface of the trapezoidal sheeting following a vertical vector. The sides of the sheeting were constrained using a symmetry condition perpendicular to the axis of the purlin. The sheeting to purlin connection consists of 5.5 mm screws which are modelled using a series of point-based fasteners. The load-displacement behaviour of the screws was determined experimentally by [20]. The connector behaviour is a function of the screw tilting resistance determined according to [21], connected sheeting thickness, screw diameter, screw thread pitch and tensile strength of the steel sheets. In Figure 7 the relationship is plotted for the connection between the 0.5 mm sheeting and the 1.2 mm purlin.
3.2. Results
The parametric study aimed to identify the influence of screw spacing, sheeting thickness and connection detailing for the studied overlapping lengths. The six lap lengths start from $L_p = 100\,\text{mm}$ (see Figure 1) followed by 300 mm, 450 mm, 600 mm, 900 mm and 1200 mm. The chosen lengths ensure that all the possible failure modes are included. The results will be detailed separately for each parameter in the following subsections.

3.2.1 Sheeting thickness and screw spacing
The trapezoidal sheeting has a crest height of 43 mm with a trough spacing of 180 mm. The studied thickness was 0.5 mm and 0.7 mm. The influence of the steel thickness was also accounted in the screw behaviour which is roughly 20% stiffer for the 0.7 mm sheet. Two screw spacing arrangements were placed at each trough and at each second trough. Finite element results show a limited influence on beam load carrying capacity by up to 5%. The variation was most significant for the 100 mm overlap.
3.2.2 Connection detailing

The length of the lap connection plays an influential role in both the behaviour of the purlin and the cost of the structure. Reducing the overlap length and the complexity of the connection can yield significant cost and time savings. The bolt/screw patterns presented in Figure 9 focus on minimizing the overlap length while at the same time reducing the complexity of the connection.

Figure 9. Connection detailing patterns

Numerical results emphasize the major influence of the overlapping length. Comparing the extreme laps, 100 mm vs 1200 mm, the difference in load carrying capacity is increased by roughly 61%. In addition, the bending stiffness of the beam increases by up to 41%. This significant improvement is attributed to the change in failure mode.

Table 2. Comparison between bending moment – experimental vs. FEM.

| $L_p$ [mm] | $M_{bd}$ [kNm] | $q_{bd}$ [kNm] | Stiffness ratio$^a$ [-] | $M_{bd}$ [kNm] | $q_{bd}$ [kNm] | Stiffness ratio$^a$ [-] | Failure$^b$ |
|-----------|---------------|---------------|-----------------|---------------|---------------|-----------------|---------|
| 100       | 3.17          | 0.90          | 87%             | 4.08          | 1.08          | 92%             | Edge    |
| 300       | 4.84          | 1.15          | 105%            | 4.94          | 1.15          | 107%            | Edge    |
| 450       | 5.64          | 1.28          | 111%            | 5.70          | 1.29          | 112%            | Edge    |
| 600       | 6.27          | 1.41          | 116%            | 6.38          | 1.41          | 117%            | Edge    |
| 900       | 6.99          | 1.48          | 123%            | 7.08          | 1.49          | 124%            | Support |
| 1200      | 7.07          | 1.45          | 128%            | 7.11          | 1.45          | 129%            | Support |

$^a$Reported to a 1D beam modelled with a single cross-section, $^b$Observed failure invariable of the pattern.

The difference between a simplified design considering a continuous beam modelled as a single profile is emphasized in Table 2 by the stiffness ratio. This assumption makes the simplified beam model unrealistic. Finite element results show the influence of the connection detailing is significant only for short overlap lengths ($L_p = 100$ mm). The difference between Pattern a and b is less than 2%. Connecting the bottom flanges by a screw prevents the early distortion of the free flange and increases the load carrying capacity by 20% and the bending stiffness by 5%. In Figure 10 the failure mode of the two situations is illustrated.
4. Conclusions
This paper investigates the influence of the overlapping length, connection detailing, sheeting connection and thickness on a two-span continuous beam under gravity loading. The developed numerical model was validated against experimental data yields accurate results in terms of bending moment resistance and failure mode. Based on the finite element results the following conclusion can be drawn:

a) the inclusion of geometric imperfections based on the scaled eigenmodes is not feasible for overlapped purlins and can affect the accuracy of the results,
b) the lapping length has the highest influence on both the stiffness and resistance of the member,
c) designing the connection to prevent the failure at the edge of the lap results in a major increase in stiffness and resistance,
d) the sheeting connection and thickness has a limited influence on the resistance of the member,
e) connecting the bottom flanges using screws has a significant effect only for short overlaps $L_p=100$mm where the free flange shows an early distortion.

The work presented herein represent the starting point of an extensive experimental program aiming to understand and develop design procedures for overlapped continuous beams.

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