GERT-network optimization model for technologies of hazardous industry management

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Abstract. The paper introduces GERT-network optimization model for the technologies of hazardous manufacturing process management supported by the software of distributed production control systems. The development and implementation of the formal means for optimizing processes for the formation of managing hazardous industries using GERT-networks technology is presented. It allows to include into the model random deviations and uncertainty arising directly during the execution of each separate task of hazardous industries management technology.

1. Introduction

Development and operation of modern control systems for hazardous industries, implemented on the basis of distributed real-time computing systems, is directly related to the development of software [1]. Currently, the development and operation of control systems use a combination of serial methods with individual production, which, on the one hand, is characterized by a high degree of unification of the basic software, engineering methods and techniques for its design, and on the other - by the use of unique software [2].

The process of operation and maintenance of hazardous production management systems is regulated by cyclograms [1], which determine the temporal and information relationships between the individual stages and tasks of information management and processing, which is expressed in the establishment of periodicity, the planned duration and order of solving information processing and production management tasks. In the process of operation, it is possible to change the characteristics of production processes and modes of operation due to technical malfunctions or changes in the programs of work, which is reflected both in the composition and algorithms of the control software. At the same time, in case of manufacturing output from abnormal conditions (in accordance with non-standard cycles), a number of characteristics of the control software, as well as the process of its development, in particular reliability, fault tolerance, speed, determine the probability of successful completion of this operation and the amount of damage [1].

Nowadays great importance is paid to such requirements of the software product, the methods of its development and support, as speed and cost effectiveness of development, external support for execution, testing and analysis, as well as accessibility of methods and tools to a wide range of specialists, responsible for production management and management cyclograms in general [3].
Thus, there is an important task of technology for managing hazardous production optimisation, supported by software distributed production control systems. For a successful solution, it is necessary to develop models and research methods that reflect the management processes and allow for the synthesis, analysis, correction of control schedules, accounting for these factors as part of models and dialog procedures that are targeted at domain specialists.

The paper suggests the development and implementation of formal means for optimizing the processes of technology formation for managing hazardous production.

2. Problem statement

Let us consider systems aspect of management technology formation [4]. The system level model of management cyclograms as a whole should represent the dynamics of interacting information-algorithmic processes in the management system. It is shown that the solution of the forming control technology task is primarily connected with the need to provide interaction with the object (production) in accordance with the time diagrams in real time. A simpler form of real-time mode involves limiting the response time to a request from an object. The reaction time restrictions are associated in this case with the implementation of periodic actions within the framework of control schedules. For example, in multiprogramming systems, the real time scale is set by the time limit for application stay in the system, which includes waiting time in queues and servicing by processors and other devices. The natural mathematical interpretation of distributed, asynchronous, multiprocessor and multiprogramming systems represent network models that allow reflecting the structure distribution, the network nature of the relationships between processes and resources, as well as between the hardware and software components of the control system, technological processes and automated production. In this regard, it is advisable to involve the apparatus of network analysis, including GERT-analysis to solve the problems of the formation of control technology [5].

The network representation of control technology provides ample opportunities to use analysis of the cyclograms of a mathematical apparatus in models of reliability that takes probabilistic aspects into account [6]. One of these possibilities is associated with the introduction of uncertainty into the duration of task implementation management (similar to the use of PERT-methodology in network analysis). However, such model is a typical deterministic network, for the full implementation of which it is necessary to perform all the arcs, that is, by this we mean the unconditional execution of the corresponding control operations.

This condition implies that such model cannot contain operations with feedback since they are represented by loops whose existence in their turn means that the end node of the operation must be performed before its initial node. In the field of deterministic networks there are two models which are studied mostly carefully. In the first one which is the model of the critical path, the execution time of each arch is fixed. The second method of PERT-analysis implies that there are several possible time periods for the execution of each arch.

The model interpretation under consideration allows to manage multiprocessor systems and organize monitoring and keeping records of the causes of failures and emergencies in network models. While ensuring efficient management it is necessary to ensure the following quality criteria: the execution of the program of technological operations, while observing the directive terms, regulated by the control schedules; rational use of equipment, computing resources of the control system; minimization of material and time costs, etc.

To provide automation of displaying the spatio-temporal picture of the production process or the technological process, the formation of control technology can be realized by dynamic interpretation of network control models. The network model can be the main electronic document of the system for the formation of control schedules, and the resulting model is developed at the algorithmic level of the reproduction of processes.

Dynamic interpretation of the management model allows to automate the construction of operational graphs, including cyclograms of technological operation sequence, to formalize a number of statements.
of cyclograms optimization problems and control technology as a whole in the form of optimization problems for parameters and the structure of the reference control model.

Let us consider the possibility of stochastic GERT-analysis to develop management technology. As it was mentioned above, this possibility is related to the use of networking models with stochastic structure since they prove to be the most flexible and useful in practice. In our case, while analyzing the realization of management cyclograms let us define the stochastic network as a network which can be developed only by realization of a subset of arches; additionally, the time of developing each arch (management task) is chosen in accordance with the probabilistic distribution. In such stochastic network in order to develop a node it is not necessary to develop all arches included into it. Therefore, cycles and loops exist in such a model [6].

3. Solution method
So, for the determined case it is possible to consider a simple acyclic deterministic graph that has a GERT-like nodal logics. Such a model will be called a network for the design (or solution) of the management cyclogram [7]. The term "design / solution" shows what we choose, i.e. we make a decision about which management tasks should be performed to minimize some objective function. In order to take into account, the probabilistic characteristics of the management cyclogram implementation, the concept of random shares is introduced and the possibility of a multiple-series sequential implementation of the management chart up to the point of successful completion is considered.

Let \( N \) be an acyclic network control model with the sources and sinks (actions corresponding to management tasks are represented by arcs), where the node set is denoted by \( V \), and the set of arcs is \( E \). The network has only one source, which is denoted by \( r \) and corresponds to the beginning of the control cycle. It is assumed that one of the \( N \) stocks represents a successful completion of the management process and is denoted by \( s \). The remaining stocks, if any, can represent different types of failed completion or management process interruption.

**Definition 1.**

An acyclic network management model with only one source and sinks will be called a network for the design/solution of the control sequence if each node \( i \) of \( N \) is defined by the input characteristic \( CH_i \in \{0,1,...,P(i)\} \) and the output characteristic \( CH'_i \in \{0,1,...,S(i)\} \).

These characteristics forming the node logics have the following values: (a) the node is activated immediately after the input activities \( CH_i \); (b) as soon as the node \( i \) is activated, not more than \( CH'_i \), of the output activities start. In case \( i \) node is not activated, no output activities are executed.

Let us suppose for \( r \) source \( CH_r = 0 \), i.e. it is always activated. Besides, \( CH'_r = 0 \) for \( i \in S \), where \( S \) is a subset of stocks.

Note, that if \( CH'_r = 1 \), the node \( i \) has IOR-input, and if \( CH'_r = |P(i)| \), then \( i \) has AND-input. And if “not more” is replaced to “exactly” in the condition (b), then \( CH'_i = 1 \) corresponds to the probabilistic output, and \( CH'_i = |S(i)| \) corresponds to deterministic output.

If the given network \( N \) for the development of the management technology has got a variety of \( R \) (\(|R|>1\)) sources \( h \) and \( R \subseteq R \), \( R \neq 0 \) variety is activated at the beginning of the management process, it is possible to formally transfer \( N \) to the corresponding one-source network.

Let us denote the arc variables through \( w_{ij} \), setting \( w_{ij} = 1 \), if \((i, j) \in E \), and \( w_{ij} = 0 \), otherwise. Node variables \( u_i = 1 \) (\( i \in V \)), if \( i \) is activated. Otherwise, \( u_i = 0 \), with \( u_r = 1 \), i.e. the source is always activated. Then the conditions of the node logics ((a) and (b)) are presented in the following form:

\[
\sum_{k \in P(i)} w_{ki} \geq CH_i; \quad u_i; \quad (i \in V\setminus \{r\}),
\]

\[
\sum_{k \in P(i)} w_{ki} < CH'_i + M_i; \quad (i \in V\setminus \{r\}),
\]

где \( M_i > |P(i) - CH'_i| \),
\[ \sum_{j \in S(i)} w_{ij} \geq CH_i^t u_i ; (i \in V_S). \]  

(3)

**Definition 2.** Since the decision logics is acyclic, each task of the corresponding management cycle is either performed only once, or never at all. Thus, each implementation of the control sequence (or implementation of the network model) can be correlated with the set of control tasks performed (performed network actions) or with the function \( w : E \rightarrow \{0, 1\}; ((i, j) \in E), \) which are given as \( w((i, j)) =: w_{ij} = 1, \) if \( (i, j) \) is satisfied, and \( 0, \) otherwise.

From the other hand, if some \( w \)-th management implementation is specified, then both node and arc variables for this case are specified and we can talk about the feasible implementation of the management cyclogram, if \( w \) satisfies the conditions of the node logics. Then \( e = \{ w : E \rightarrow \{0, 1\} \mid w_{ij} \) satisfies (2.1)-(2.3); \((i, j) \in E\}, \) and \( e \) is the set of all admissible control realizations.

If the arc weight \((i, j) \in E \) is defined as the duration \( d_{ij} \in R_+ \) of the corresponding control problem in the decision network, then the earliest possible time for the \( w \)-th realization of the control sequence, starts at the earliest possible time for a given \( w \)-th implementation of the management cycle under consideration. It is necessary to minimize \( d_w \) under the following conditions: \( w \) activates \( s; \) \((w \in e)\). We will denote variety of successful management implementations by \( e^* := \{ w \in e \mid w \) activates \( s \}. \) For \( e^* \neq 0 \)

\[ d^* = \min_{w \in e} d_w \]  

(4)

corresponds to the minimum value of the task target function.

Assuming that each implementation of the control loop begins with the source \( r \) being activated at time 0, we assume for \( w \in e \), that \( t^w_i \) is the activation time for the node \( j \in V \) for \( w \)-th implementation, with \( t^w_0 = 0 \) and \( t^w_i = \infty, \) if \( j \) is not activated during \( w \)-th cyclogram implementation. For \( j \in V \setminus \{r\} \) we have \( t^w_j = \min\{ t \geq 0 \mid w \) activates \( s; \) \((w \in e)\}, \) different from \( i \in P(j) \) so, that \( w_{ij} = 1 \) and \( t^w_j + d_{ij} \leq t\).

In addition, we have \( d_w \geq t^w_j \forall w \in e^* \). Then \( d_w > t^w_j, \) if stock \( s \) is activated, while some actions \((i, j) \) при \( w_{ij} = 1 \) are still being performed.

\[ t'_{rj} = \min_{w \in e^*} t^w_j \]  

(5)

Since the earliest possible time for the \( j \) node to be invoked during any possible implementation of the cyclogram, it is obvious

\[ t'_{rs} = \min_{w \in e^* \cap \{(i, j) \in E \}} \{ t^w_i + d_{ij} \}. \]

\( E_w = \{(i, j) \in E \mid w_{ij} = 1\} \) is the set of tasks performed during the \( w \)-th implementation of the control sequence. Taking into account the above, it can be stated that the minimum duration of a successful implementation of the control sequence is the earliest time of the run off stock \( s; \) \( d^* = t'_{rs} \) for \( e^* \neq 0. \) Thus, we can find the value of \( d^* \) by calculating the minimum possible activation times of \( t'_{rs} \) \((j \in V)\).

Considering the moments \( t_i \) as components of the time-warping vector (TWV) of the control sequence, which satisfy \((t_i, t_{ij}, d_{ij}) \) \( w_{ij} \geq 0, \) \((i, j) \in E; \) \( t_i \geq 0, \) \( i \in V \setminus \{r\}; \) \( t_r = 0, \) we can assert that for some \( w \)-th realization of the cyclogram \((w \in e^*), t^w_i \) correspond to these constraints, and the earliest \( t^w_i \) satisfy them for the corresponding minimal \( w \in e^* \). The corresponding optimization problem looks like:

To minimize \( \max_{(i, j) \in E} \{ t^w_i + d_{ij} \} \)

while implementing (1), (3) and restricting TWV \((u_i = 0). \)
Compared with minimizing the costs of management, we have not only additional limitations, but also a more complex objective function. Moreover, in addition to the nodal and arc variables, there are moments \( t_i \) (\( i \in V \)), which are also variables of this optimization problem.

Taking into account algorithmically defined constraints for TWV, procedures are developed for the analysis phase of reliability and correction of control schedules, which correspond to a common problem of distinguishability and include the study of the joint properties of control processes and the hardware and software of the control system with a given implementation vector. Thus, it is necessary to repeat over and over again successive stages of the target functional optimization (when choosing the optimal time management implementation, according to the hardware and software of the system), and the general analytical and optimization procedure, except for the actual analysis and correction stage, includes such steps as the choice for a given control schedule (or a set of cyclograms) of the composition of software modules using approximate methods; critical path method (CPM) for the suboptimal implementation of the control and the precise optimization of the TWV.

It should be noted, that within the proposed procedure of the control technology formation between all stages of the problem solution, it is necessary to organize a dialog interface that will provide the user (an expert in control systems or control technology) with sufficient information for making a decision. In addition, a stochastic control analysis is provided—finding of a mathematical expectation and standard deviation of the directive time for the implementation of control schedules in conditions of uncertainty.

For a complex process that is a technological cycle, it is reasonable to consider the directive time as a random variable with finite mathematical expectation and variance, described by an appropriate distribution function. To obtain the variance estimates, some assumptions are required concerning the stochastic characteristics of each element of the structure of the control cyclogram (information processing and control tasks) in standard and, if necessary, supernumerary situations. This description of control tasks in comparison with the case when only time characteristics are given is more complex and, obviously, more accurate.

The procedure we use to analyse the probabilistic characteristics of control charts is based on stochastic networks such as GERT and combines flow theory in graphs, torque generation functions and PERT-analysis to obtain a result. The following relationship exists between PERT-type networks, graphs with flows and stochastic networks.

1. PERT-type networks correspond to stochastic GERT-networks in which all nodes are determined by "And-Nodes" (And-Deterministic);
2. Graphs with flows correspond to stochastic networks with a simple multiplicative parameter (all additive parameters, such as time, are set equal to zero). The probability interpretation for the multiplicative parameter is excluded from consideration.

The stages of the problem solution by means of stochastic GERT-networks are the following:
1. Transfer a qualitative description of control technology into the GERT-network model.
2. Collect the necessary structural data to describe the relationships in the GERT-network (including the characteristics of information processing and object management tasks).
3. Apply a topological equation to determine the equivalent function (or functions) of the GERT-network.
4. Calculate through the equivalent function the following two characteristics of the network functioning:
   • probability of a specific node;
   • the torque generation function for the time associated with the node, if it is running.
5. Draw conclusions on management technology based on the information received in paragraph 4.

So, the stochastic representation of control technology in the form of a GERT-network allows to obtain a sufficient amount of useful information about the temporal characteristics of the control schedule implementation. Using Chebyshev's inequality, it is possible to show the limits in which the real time of the control implementation will change, and also to obtain stronger assertions. In addition, if the real indicator does not correspond to these estimates, it is possible to construct a number of criteria.
for testing hypotheses that allow determining the perspective characteristics of the management implementation time.

4. Conclusion
In conclusion, we will note that the considered GERT-model is a kind of alternative to the traditional methods of determining the directive times for the implementation of the control schedules. When using traditional methods, it is assumed that the execution time of each individual task is constant. After summing these times, a correction is made to the result obtained in order to take into account random oscillations or to eliminate the instability of real times. The system of GERT-models allows to include random deviations and uncertainty arising directly during execution of each separate control task.

Consequently, all random fluctuations are already included into the result, and there is no need to introduce additional amendments into it, except for those that correspond to abnormal situations or accidents at work.

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