Three-nucleon interactions: dynamics

M. R. Robilotta

Instituto de Física, Universidade de São Paulo, São Paulo, SP, Brazil

Abstract. A discussion is presented of the dynamics underlying three-body nuclear forces, with emphasis on changes which occurred over several decades.

Keywords: nuclear forces, pion, chiral symmetry.

PACS: 13.75.Cs, 13.75.Gx, 21.30.-x, 21.45.Bc, 21.45.Ff

A WARNING AS INTRODUCTION

In this work, I sketch the development of nuclear three-body forces over a rather large span of time and concentrate narrowly on dynamics. As a consequence, many important subjects are completely omitted, especially those concerning experimental facts and facilities, phenomena and calculation techniques. Even within the restricted domain of dynamics, many relevant contributions could not be mentioned or discussed, owing to limitations of space. A more comprehensive list of references can be found in a previous related presentation[1].

This work is definitely not a historical reconstruction, even if many developments are presented in a rough chronological order. The various sections are organized around themes possessing a kind of internal coherence, which yield something like a research mood. I have tried to convey these moods and to suggest that they change from one period to another. With that purpose in mind, most technicalities were avoided and many original quotations were made, indicated by both quotation marks and italic fonts.

CLASSICAL THREE-BODY FORCES

Classical Electrodynamics, as represented by Maxwell’s equations, is the paradigm for describing particle interactions mediated by fields. In Electrostatics, the substructure dealing with systems of charges at rest, the Coulomb law holds and the potential energy $U$ is a meaningful concept. In the case of just a pair of particles 1 and 2, the potential energy has the form $U_{12} = KQ_1Q_2/r_{12}$. When several particles are present, one usually invokes the linear superposition principle in order to generalize this result. Accordingly, the potential energy of a system with three particles would be given by

$$U = U_{12} + U_{23} + U_{31}.$$  \hspace{1cm} (1)

The manifestation of the superposition principle in this result is the fact that the potential energy of the system is not written as

$$U' = U'_{12} + U'_{23} + U'_{31},$$  \hspace{1cm} (2)
with $U'_{ij} \neq U_{ij}$. The principle implements the idea that the interaction between particles 1 and 2 is completely independent of the presence of the third particle. This idea is based on a tacit assumption, namely that the charge distributions of the particles involved are rigid and do not change in the interaction process. In Nature, however, this kind of condition is satisfied only in a few special cases.

Quite generally, if a particle has parts, it must also contain internal forces, responsible for the binding of these parts. As a necessary consequence, when the particle is placed in the presence of intense enough external forces, deformations can be produced. Only elementary particles, which by definition do not have parts, cannot be deformed by external interactions. They also cannot have size, for this would imply the existence of parts. Elementary particles must therefore be point-like, with the word point used in its mathematical sense.

Going back to Electrostatics, one sees that the linear superposition principle can hold only for truly elementary particles. In the case of systems, such as molecules, atoms or nucleons, one indeed has $U'_{ij} \neq U_{ij}$ and eq. (1) corresponds to an approximation. The convenience of this approximation depends on the problem considered and, in many instances, precision demands the inclusion of corrections. These are usually taken into account by means a three-body potential $W$, such that $W \equiv U' - U$. This definition ensures that the full interaction is suitably recovered by the joint use of $U$ and $W$. Physically, this class of three-body forces is associated with deformations of the interacting objects.

Three-body interactions can also be generated by another kind of mechanism, which depends on the complete Maxwell’s picture. When two charged particles 1 and 2 are free to move, both electric and magnetic interactions occur. The force acting on one of the particles depends then on the relative distance $r_{12}$ and on the individual velocities $v_1$ and $v_2$. Moreover, as the particles are accelerated, interaction through radiation is also possible. Even when the velocities involved are not very high, the complete force $F_1$ acting on particle 1 is a complicated function of the form

$$ F_1 = F_1(r_{12}, v_1, v_2, a_2). $$

(3)

If a third charge is brought into the system, it will give rise to new forces over particles 1 and 2. In particular, the new force over charge 2 will induce a change $a_2 \rightarrow a'_2$ in its acceleration. The inclusion of this effect into eq. (3) promotes an indirect influence of the new particle over charge 1, which also corresponds to a three-body force.

**EARLY THREE-BODY FORCES**

The first work to deal with quantum three-body forces was entitled "Many-Body Interactions in Atomic and Nuclear Systems" and produced in 1939, by Primakoff and Holstein[2]. They begin by considering the second classical mechanism mentioned in the previous section, associated with eq. (3), and then move on to quantum electromagnetic interactions. The conclusion is reached that "classical many-body potentials" correspond to the case in which an electron "simultaneously emits two virtual quanta". On the other hand, they note that the process in which "one electron emits two virtual light
quanta in succession”, gives rise to "specifically quantum-mechanical many-body potentials". The same kind of interactions are considered in the section "Mesotron Field Theory of Nuclear Interactions”, with the light quanta replaced by vector Yukawa particles. By estimating the typical sizes and velocities of atomic and nuclear systems, they infer that "the description of the electromagnetic interaction of electrons in atomic systems, by means of action-at-a-distance two-body potentials is an extraordinarily good approximation [...]", whereas "The usual description of nuclei in terms of two-body potentials cannot [...] be considered satisfactory, except in the case of the deuteron".

An extension of this discussion, together with an application to the trinucleon system, were the object of another paper, published in the same year[3].

**EARLY STRONG THEORY**

The discoveries[4] of the charged pions, in 1947, and of their neutral counterpart, in 1948, were followed by a consistent effort aimed at producing a theory of nuclear forces. This research program relied on the idea of a range expansion, outlined by Taketani, Nakamura and Sasaki[5], in 1951. Soon afterwards, field theory was being employed in nuclear calculations, with great vigour.

In 1952, Lévy published a very influential paper[6], dealing with the description of nuclear forces by means of pion exchanges. The basic $\pi N$ interaction was assumed to be pseudoscalar, "since it is only in this case that the intrinsic field theoretical infinities can be separated and re-interpreted consistently", and several Feynman diagrams describing the two-body system were calculated. In a subsequent work, which appeared in 1953, Klein[7] corrected some of Lévy’s results and considered the role of three-body forces. He also adopted pseudoscalar $\pi N$ coupling and a pattern that would become important later began to appear, namely that dominant contributions were associated with diagrams containing $N\bar{N}$ intermediate states. Another remark that would have future impact was hidden in a note added in proof: "Numerical calculations show that the potential obtained in the paper does not agree with experiment. Further work, to be published, indicates moreover, that the perturbation theory doesn’t even converge."

Almost simultaneously, Drell and Huang[8] were investigating the problem of nuclear saturation, which requires a mechanism for short range repulsion. In their abstract, they state: "[... ]The leading term in the n-body potential depends only on the interparticle distance and is repulsive (attractive) for n odd (even)." This finding is materialized in their interesting fig 6 and the special role of three-body forces is stressed in the concluding section: "[...]we find that many-body forces, and in particular the three-body repulsion, provide a satisfactory qualitative understanding of nuclear saturation.”

The qualitative findings by both Klein and Drell-Huang were confirmed and summarized by Wentzel[9]: "In pseudoscalar meson theory (PS coupling), the saturation character of nuclear forces is accounted for by the cooperative action of many-body forces. In particular, the repulsive 3-, 5-, 7-, ... body forces keep the nucleus from collapsing into a very small volume. These nuclear interactions result mainly from matrix elements describing virtual nucleon-pair creation and annihilation processes[...]."

These conclusions were disputed, already in 1953, by Brueckner and Watson[10]. In their own words: "We have seen that it is possible to derive a nucleon-nucleon poten-
tial, working entirely with a nonrelativistic approximation to the pseudoscalar meson theory. [...] We have depended rather strongly on the suppression of nucleon pair formation by radiative effects; the contributions to the potential from pair formation in high order calculated without taking into account such effects otherwise tend to be so large as to invalidate the power series expansions usually used.” This perception underlies their ad hoc calculation procedure, which became known as pair suppression and was widely adopted. In another publication along this line, by Brueckner, Levinson and Mahmoud[11], one finds: “The application of these techniques to a specific problem of considerable interest, namely the two-body potentials of pseudoscalar meson theory (adjusted to fit the low-energy scattering parameters) shows that these potentials also have the necessary characteristics to give an approximately correct description of nuclear saturation.” In the same paper these authors also argue that many-body forces produce negligible effects.

This intense debate about nuclear forces raised many problems regarding the applicability of ordinary perturbation theory to strong interactions. As a consequence, in the mid-fifties, lagrangians had fallen into disgrace and theoretical interest shifted to the analytic structure of the S-matrix, investigated by means of dispersion relation techniques. The rehabilitation of strong lagrangians had to wait for the development of chiral symmetry.

**THE FIRST MODERN THREE-BODY FORCE**

In the second half of the fifties, the idea of deriving potentials from first principles was put aside and replaced by the more pragmatic attitude of blending theory and empirical information. At that time, the relationship between the off-shell πN amplitude and important components of the potential, as indicated in fig. 1 had already been suggested. Also, many measurements of elastic πN scattering, performed in the same period, had led to a considerable improvement of the empirical information about the on-shell amplitude. If suitably treated by means of dispersion relations, a rather valuable information about the off-shell πN amplitude could be obtained from empirical data.

![FIGURE 1.](a) Free πN amplitude; (b) two-pion exchange two-body potential; (c) two-pion exchange three-body potential.)

At the end of 1956, a paper incorporating the new attitude was published by Miyazawa[12]. Its title was "Interaction of P- and S-Wave Pions with Fixed Nucleons” and, in the abstract, one finds the clear statement: "A method is given of replacing pion scattering parts in a Feynman diagram by experimentally observed quantities. [...] Two examples are given: (1) The anomalous magnetic moment of the proton is rigorously
expressed in terms of pion-nucleon scattering amplitudes [...]. (2) The internucleon potential is also expressed by means of scattering quantities. In this case the number of virtual pions exchanged between the two nucleons is limited to two, although the number of pions emitted and absorbed by the same nucleon is not limited.

In the cases of $P$- and $S$-waves, the scattering matrix elements for the process $\pi^i(k)N \rightarrow \pi^j(q)N$ were written respectively as

$$\langle j, q|S|i, k \rangle = 2\pi i \delta(k_0 - q_0) \left[ A(k_0) \tau_i \tau_j \sigma \cdot k \sigma \cdot q + B(k_0) \left( \tau_i \tau_j \sigma \cdot q \sigma \cdot k + \tau_j \tau_i \sigma \cdot k \sigma \cdot q \right) + C(k_0) \tau_j \tau_i \sigma \cdot q \sigma \cdot k \right] e^{i(k-q)\cdot x},$$

and dispersion relations were used to derive the functions $A$, $B$ and $C$ from empirical total cross sections, whereas $D$ and $E$ were obtained from scattering lengths. The importance of these results lies in the fact that they hold for both real and virtual pions. The derivation of the two-pion exchange component of the $NN$ potential, fig. 1b, was performed with great care, so as to avoid various possibilities of double counting.

In 1957, Fujita and Miyazawa published the work "Pion Theory of Three-Body Forces"[13], in which the off-shell $\pi N$ amplitude also plays an essential role, as in fig. 1c. Their construction of the two-pion exchange three-nucleon potential ($TPE-3NP$) is described: "The process being considered is this. Particle (1) emits a pion which is scattered by particle (2) and then absorbed by particle (3). [...] For the scattering by particle (2), experimental values can be used. Actually, this is a scattering of a virtual pion having zero energy. However, the use of dispersion relations makes it possible to correlate this scattering with real scatterings. In this way all virtual transitions on particle (2) are correctly taken into account."

This clear understanding of the role played by the off-shell $\pi N$ amplitude in the $TPE-3NP$ establishes the work of Fujita and Miyazawa as the first one in the modern tradition. With variations, the essence of their approach is present in the leading terms of potentials constructed ever since.

On the more technical side, an explicit analytic form for the potential was presented in terms of Yukawa functions and numerical values for the constants $A(0)$, $B(0)$ and $D(0)$ were derived from $\pi N$ scattering data. A little of manipulation allows their original expressions to be recast in the form of $TPE-3NP$ adopted nowadays, given by

$$V_L(123) = -\frac{\mu}{(4\pi)^2} \left\{ \delta_{ab} \left[ a \mu - b \mu^3 \nabla_{12} \cdot \nabla_{23} \right] + d \mu^3 i \varepsilon_{bac} \tau^c \cdot \nabla_{12} \times \nabla_{23} \right\},$$

where $\mu$ is the pion mass and $a$, $b$ and $d$ are strength constants. The first one is associated with $S$-waves and the other two, with $P$-waves. The interpretation of this result is rather simple. The term within curly brackets describes the off-shell $\pi N$ scattering on nucleon 2, those within parentheses represent the emission and absorption of a single pion on nucleons 1 and 3, whereas the Yukawa functions $Y$ describe pion propagation. To my knowledge, this structure for the potential was first obtained by Fujita and Miyazawa, indicating the power and generality of their approach.
A final comment about their work is in order. It is well known that \( \Delta \) intermediate states play a dominant role in some low-energy \( \pi N \) cross sections and, by extension, the same happens in the three-body force. On the other hand, the structure of the actual Fujita-Miyazawa potential also incorporates other effects and is rather general. Therefore the widespread identification of the F-M force with \( \Delta \) intermediate states corresponds, at least, to an undue over-simplification of their results.

The off-shell \( \pi N \) amplitude derived in ref.\[12\] was also used in the study of more involved components of the three-nucleon force, by Fujita, Kawai and Tanifuji\[14\], in 1962. In that work, particular attention was paid to both the so called ring diagrams and to processes containing exchanges of one pion with one of the nucleons and two pions with the other, which continue to be object of attention nowadays.

**THE AGE OF CHIRAL SYMMETRY**

The discovery, in 1956, of parity non-conservation in weak interactions, motivated a great interest about the nature of weak currents. The well known electromagnetic current is represented by a vector \((J^\mu)\) and conserves parity. In the case of weak interactions, on the other hand, one finds both vector \((V^\mu)\) and axial-vector \((A^\mu)\) currents, which respectively conserve and do not conserve parity. The electromagnetic current is conserved \((\partial \cdot J = 0)\), but the same does not happen with the weak ones. Nevertheless, in the late fifties, the notion was developed that, in some special limits, weak currents could be considered as being approximately conserved. This was translated into approximate symmetries of the interaction lagrangian, associated with transformations that can be schematically represented as

\[
[V, V] \rightarrow V, \quad [V, A] \rightarrow A, \quad [A, A] \rightarrow V. \tag{7}
\]

Loosely speaking, changes in parity occur in images produced by mirrors and one may say that the action of an axial operator is analogous to transforming a right hand into a left one. A vector operator does not change parity and corresponds to transforming a right hand into itself. This kind of analogy led the transformations indicated in eqs.(7) to be called **chiral**, a word derived from **hand** in Greek.

In 1960, **chiral symmetry**, implemented by means of the linear-sigma model\[15\], has been applied with great success to strong interactions. This model assumes the existence of a scalar-isoscalar particle called \( \sigma \) and proved to have a very rich structure. One of its beautiful features is the picture of a strong vacuum, which is both covariant and not empty.

Another important result was the natural explanation produced for the observed smallness of the \( \pi N \) scattering lengths, which puzzled research in the early fifties. This achievement is crucial for Nuclear Physics. The outer layers of nucleon interactions are due to pion exchanges and many important components of the force depend on the intermediate \( \pi N \) amplitudes shown in fig. 1. In the \( \sigma \)-model, the \( \pi N \) amplitude is given by the three diagrams shown in fig.[2] with pseudoscalar \( \pi N \) coupling. The first two diagrams correspond to the model adopted in the early fifties[6] and explicit calculation of their contributions yields \((a_{PS}^+, a_{PS}^-) = (-1.84, 0.136)\mu^{-1}\), where the superscripts \( \pm \) refer to isospin channels. The comparison of these results with the experimental values[16]...
\[(a_+^{exp}, a_-^{exp}) = (-0.008, 0.092)\mu^{-1}\] shows that the prediction for \(a_+\) is too large by a factor of about 200. The third diagram, containing the exchange of the \(\sigma\), is the signature of chiral symmetry in this process and produces \((a_+^{\sigma}, a_-^{\sigma}) = (1.83, 0)\mu^{-1}\). A large cancellation occurs in \((a_+^{PS} + a_+^{\sigma}, a_-^{PS} + a_-^{\sigma}) = (-0.01, 0.136)\mu^{-1}\), turning both sums compatible with empirical orders of magnitude. Related cancellations also happen in the case of nuclear forces and a pedagogical discussion can be found in ref.[17].

\[FIGURE 2.\] (Color online) Structure of the \(\pi N\) amplitude in the linear-sigma model.

Two remarks about the chiral \(\pi N\) amplitude are important at this point. The first one is that the smallness of the full \(\sigma\)-model result is a direct consequence of chiral symmetry. The linear \(\sigma\)-model corresponds to just one among many possibilities for implementing the symmetry in hadronic systems and the order of magnitude of the chiral \(\pi N\) amplitude is independent of the method employed. The second remark is that the symmetry, powerful as it is, cannot predict the full empirical content of the \(\pi N\) interaction. At low-energies, empirical information about this process is usually encoded into a polynomial in the variables \(v\) and \(t\), proposed by Höhler and collaborators[16]. The coefficients of this polynomial are obtained by extrapolating experimental information to the region below threshold, by means of dispersion relations. For this reason, they are called \textbf{subthreshold coefficients}. So, in order to construct a \(\pi N\) amplitude suitable to be employed in nuclear interactions, one uses chiral symmetry, supplemented by subthreshold information, as indicated in fig. 3.

\[FIGURE 3.\] (Color online) Chiral structure of the \(\pi N\) amplitude; the dashed (green) bubble represents a polynomial contribution.

The incorporation of chiral symmetry into hadronic amplitudes is performed rigorously and many results have the status of \textbf{theorems}. In the simplest versions, these theorems are exact for low-energy interactions of massless pions. The masses of actual pions, although small, are non-vanishing and it is in this sense that chiral symmetry is approximate. However, the transposition of chiral theorems to the case of massive pions is also performed rigorously and, as a consequence, predictions from the approximate symmetry are both under control and unambiguous.

In general, chiral theorems or amplitudes have the form of power series\(^1\) in a typical scale \(q\), set by either pion four-momenta or nucleon three-momenta, such that \(q \ll 1\).

\(^{1}\) This series also includes chiral logarithms and other non-analytic terms.
GeV. As power series\footnote{In a Taylor series such as $\sin x \simeq x - x^3/3! + \cdots$, the r.h.s. is obtained by means of a well defined set of operations over the l.h.s.: $\sin x$. On the other hand, in the case of a chiral amplitude $A_X$, one has schematically $A_X \simeq A_L q^k + A_{NL} q^{k+1} + \cdots$ and the coefficients $A_i$ must be determined by means of field theory, since one does not know the l.h.s.}, these theorems involve both leading order terms and corrections. The former have been evaluated for a variety of problems since the sixties, whereas the latter is the object of the **chiral perturbation theory**, which gained momentum in the last two decades.

**THE LEADING ORDER THREE-BODY FORCE**

The leading chiral three-body force begins at $\mathcal{O}(q^3)$ and is due to the process shown in fig.1c. It took a long time between the first derivations were performed and a kind of consensus about the uniqueness of the result could be reached by practitioners of the field.

The first application of chiral symmetry to three-body forces appeared in 1968, in a paper by Brown, Green and Gerace\cite{18}, in the case of nuclear matter. They state in the abstract: "[...] it is shown that long-range three-body forces in nuclear matter are essentially zero. This means that they are also small in nuclei." This conclusion was soon disputed in a letter by McKellar and Rajaraman\cite{19}, where one finds: "[...] we present below two models [...], both of which indicate that three-body forces in nuclei are about as strong as the two-body forces." A compromise between these extremes was reached in a third paper, by Brown and Green\cite{20}, whose abstract begins with: "An earlier claim about the smallness of three-body effects in nuclear matter is examined in the light of a criticism. Our conclusion is that, whereas the lowest-order three-body effect is small, second order effects are larger[...]."

The next step was given in 1974, by Yang\cite{21}, who shifted attention to the triton and was the first to use the so called **chiral dynamics**. As he explains in the abstract: "We have derived a two-pion exchange three-body potential using a chiral invariant Lagrangian for the pion-nucleon interaction. Only the three dominant lowest order processes which give rise to a three-body interaction are included: (a) The second pion is emitted before the first pion is absorbed and the nucleon which emits the second pion goes into a positive-energy state; (b) one nucleon exchanges a $\rho$ meson with a pion; (c) one nucleon is scattered into an $N^*$ (1236)." In the main text, one finds an instructive description of how effective lagrangians were striking back. After emphasizing the role of the off-shell intermediate $\pi N$ amplitude, he says: "The PCAC (partially conserved axial-vector current) and the current algebra developed in the late 1960's have been quite successful in describing the soft-pion process, e.g., the Adler's consistency conditions\footnote{This is a famous paper by Weinberg\cite{22}. Another passage on strong interactions and low-energy parameters\footnote{This is a famous paper by Weinberg\cite{22}.} of the $\pi N$ scattering are found to be in good agreement with the experiments. It was noted that for soft-pion processes the predictions obtained with current algebra can also be derived by a different method: Just use the lowest-order graphs generated by a chiral-invariant Lagrangian. We shall apply this method[...]." His reference is a famous paper by Weinberg\cite{22}. Another passage
shows how chiral symmetry was displacing the old pair-suppression mechanism of the fifties: "The three-body force arising from a virtual nucleon-antinucleon pair [...] is suppressed, since the soft-pion theory tells us explicitly that $T^{\text{Born}}$ is computed according to the gradient coupling scheme." Approximations done in the treatment of process (a), however, does not allow his results to be cast in the form of eq.(6).

Just a little later, in 1975, another paper focused on nuclear matter appeared, by Coon, Scadron and Barrett[23]. Their strategy for dealing with dynamics is presented in the abstract: "We re-examine the off-shell $\pi N$ amplitude occurring in the two-pion exchange three-body force, subject to all the constraints of current algebra. This amplitude is not dominated by the $\Delta(1231)$ isobar; instead, if the $\sigma-$term is known, it can be determined from on-shell scattering." In the third section, "Off-shell pion-nucleon scattering", one finds a detailed description of their approach, as well as a comparison with results from other works. With hindsight, this work can be considered as being a precursor of the Tucson-Melbourne (TM) potential[24], published in 1979. In this later paper, previous results were generalized and its abstract reads: "We derive the complete three-nucleon potential of the two-pion exchange type, suitable for nuclear structure calculations, by extending away from the forward direction the subthreshold off-pion-mass-shell $\pi N$ amplitude of Coon, Scadron and Barrett. The off-mass-shell extrapolation, subject to current algebra and PCAC constraints, yields approximately model independent amplitudes (in that they depend primarily on $\pi N$ data) in the complete potential. [...]". This version of the $3NP$ was restricted to a particular reference frame, but this condition was lifted a little later[25]. In those papers, the notation used in eq.(6) was introduced.

The TM force also included another structure, represented by a parameter $c$ within the curly bracket, which was written as $\{\delta_{ab}[a \mu - b \mu^3 \nabla_{12} \cdot \nabla_{23} - c \mu^3 (\nabla^2_{12} + \nabla^2_{23})] + d \mu^3 i\varepsilon_{bac} \tau(2) \cdot \nabla_{12} \times \nabla_{23}\}$. This term proved to be problematic in numerical calculations and has been identified with short range effects[26, 27]. It was eventually removed[3] in a paper by Coon and Han[28], published in 2001, where parameters for a new version, known as TM', were presented.

In 1983, the $TPE-3NP$ known as Brazil potential was produced[4] by Coelho, Das and Robilotta[29]. It was based on effective lagrangians and relied on a chiral model for the $\pi N$ amplitude derived by Olsson and Osypowski[30] which included, besides the same $\rho$ and $\Delta$ intermediate processes considered by Yang[21], a parametrized form for the $S$-wave interaction. In that model, all the parameters were tuned to empirical subthreshold coefficients and the resulting structure conveyed model independent information. This gave rise to a potential in agreement with eq.(6). In 1986, the possibility that the long range physics in both the TM and Brazil forces could be contaminated by the inclusion of form factors, required by numerical calculations, was discussed[27].

It is worth recalling that chiral expansions are performed by means of well defined theoretical rules and hence their results must be unique. This holds, in particular, for leading terms. In the case of three-body forces, the leading term has the generic structure given by eq.(6), whereas free parameters are determined by subthreshold coefficients. A

---

3 For further details, the reader is directed to Prof. Coon’s contribution to this volume.

4 The last term of eq.(67) in that paper contains a misprinted sign, which was corrected in ref.[27].
sample of values adopted for potential parameters along five decades is given in table 1. At present, variations in those values reflect just the empirical information used as input and are totally unrelated with chiral symmetry.

| ref. | year | potential | $a \mu$ | $b \mu^3$ | $c \mu^3$ | $d \mu^3$ |
|------|------|-----------|--------|----------|----------|----------|
| [13] | 1957 | F-M       | -0.27  | -1.24    | 0        | -0.31    |
| [24] | 1979 | TM        | 1.13   | -2.58    | -1.05    | -0.75    |
| [29] | 1983 | BR        | -1.05  | -2.29    | 0        | -0.77    |
| [28] | 2001 | TM'-93    | -0.74  | -2.53    | 0        | -0.72    |
| [28] | 2001 | TM'-99    | -1.12  | -2.80    | 0        | -0.75    |

CHIRAL PERTURBATION THEORY

Chiral perturbation theory (ChPT) is the art of deriving systematic corrections to leading order terms in chiral expansions. In the case of Nuclear Physics, the research program was outlined by Weinberg[31] and one of his papers deals specifically with three-body forces[32]. In ChPT, emphasis is put on a lagrangian which possesses approximate chiral symmetry and is written as a string of terms with different orders in the power-counting parameter $q$. The coefficients of higher order terms are known as low-energy constants (LECs) and their presence is felt in final results for chiral amplitudes. The values of the LECs cannot be fixed by the symmetry and must be determined from experimental information. In the case of nuclear potentials, the basic source of information are the usual $\pi N$ subthreshold coefficients. For this reason, the result for the leading term, which was already unique and given by eq.(6) before ChPT, continues to be so afterwards, in spite possible apparent changes in form due to rephrasing. On the other hand, as stated in the first paper to deal with applications of ChPT, by Ordóñez and van Kolck[33], in 1992, the new method has the clear advantage of stressing the model independence of the results. ChPT is also important for preventing whimsical diagramatics in complex problems.

From an operational point of view, the power of ChPT in nuclear forces is that it allows the generation of new structures within a unified theoretical framework. One possible direction for new developments concerns short-range physics. In the case of three-body forces, short-range chiral interactions were considered in 1994 by van Kolck[34] and by Friar, Huber, and van Kolck[35], in 1999. In the later paper[35], the abstract points to the solution of an old problem: "It is demonstrated that the short-range c term of the Tucson-Melbourne force is unnatural in terms of power counting an should be dropped."

The second possible direction is related with the production of new mathematical structures, which go beyond eq.(6). At present, next-to-leading order corrections to the three-body force, which involve a rather large number of diagrams, are being calculated.

---

5 One notes that the values quoted for the F-M and Brazil forces in table II of ref.[35] are in qualitative disagreement with those given in table of the present work.
by Epelbaum, Meissner and collaborators\[36\]. The reader is directed to the contribution by Prof. Meissner to this volume for a comprehensive account of their results.

A taste of what is about to come can be felt in a recent work by Ishikawa and Robilotta\[37\], in which a selected subset of corrections to the leading term, associated with the long-range part of the $TPE-3NP$, was considered. The final result has the structure $V(123) = V_L(123) + [V_{\delta L}(123) + \delta V(123)]$, where $V_L$ is the old leading term given by eq.\(6\), whereas the factor within square brackets includes corrections due to ChPT. The term $V_{\delta L}$ can be obtained directly from $V_L$ by replacing $(a, b, c)$ with $(\delta a, \delta b, \delta c)$. In other words, this part of the correction corresponds to shifts in the parameters of the leading term, which are numerically smaller than 10%. The factor $\delta V(123)$, on the other hand, represents effects described by new mathematical functions, whose actual forms are too cumbersome to be displayed here. For the present purposes, it suffices to say that they involve both non-local operators and the replacement of the Yukawa functions in eq.\(6\) by more complicated propagators involving loop integrals. The strengths of these new functions are described by a set of new parameters $e_i$, which are also typically smaller than 10% of the leading ones. So, the impact of ChPT in table\[1\] is to produce both small modification in already existing coefficients and the appearance of many new columns. The latter are by far the most interesting ones, since they may contain the explanation for effects such as the $A_y$ puzzle\[6\].

**CONCLUDING REMARKS**

Progress in our understanding of the world always involves both conceptual ruptures and continuities. In this work, I have tried to recall that this feature was also present in the development of three-body forces, a highly collective task. If one compares the discussions of strong interactions held in the fifties with those occurring nowadays, one may be tempted to stress discontinuity. On the other hand, reflecting about the work by Fujita and Miyazawa, one notices that their formulation of the problem in terms of dispersion relations corresponds more to a bending of its path than to a complete break with the past. It is therefore best described as a turning point. Chiral symmetry came next and has promoted a solid understanding of both meanings and values of the strength parameters of the force. And it is remarkable that chiral perturbation theory, which guides the present wave of research, is not replacing, but rather extending and complementing the results derived fifty years ago by Fujita and Miyazawa.

**ACKNOWLEDGMENTS**

It was a great pleasure participating in the FM50 Symposium and I would like to thank the organizers for this nice opportunity.

---

\[6\] Discussed by Prof. Tornow in this volume.
REFERENCES

1. M.R. Robilotta, Few-Body Systems, Suppl. 2, 35 (1987).
2. H. Primakoff and T. Holstein, Phys. Rev. C 55, 1218 (1993).
3. L. Jánossy, Proc. Cambridge Phil. Soc. 35, 616 (1939).
4. C.M.G. Lattes, G.P.S. Occhialini and C.F. Powell, Nature 160, 453, 486 (1947); E. Gardner and C.M.G. Lattes, Science 107, 270 (1948).
5. M. Taketani, S. Nakamura, and T. Sasaki, Prog. Theor. Phys. 6, 581 (1951).
6. M.M. Lévy, Phys. Rev. 88, 725 (1952).
7. A. Klein, Phys. Rev. 90, 1101 (1953).
8. S.D. Drell and K. Huang, Phys. Rev. 91, 1527 (1953).
9. W. Wentzel, Phys. Rev. 91, 1573 (1953); Helv. Phys. Acta 15, 111 (1942); 25, 569 (1952).
10. K.A. Brueckner and K.M. Watson, Phys. Rev. 92, 1023 (1953).
11. K.A. Brueckner, C.A. Levinson and H.M. Mahmoud, Phys. Rev. 95, 217 (1954).
12. H. Miyazawa, Phys. Rev. 104, 1741 (1956).
13. J. Fujita and H. Miyazawa, Progr. Theor. Phys. 17, 360 (1957).
14. J. Fujita, M. Kawai and M. Tanifuji, Nucl. Phys. 29, 252 (1962).
15. M. Gell-Mann and M. Lévy, N. Cim. 16, 705 (1960).
16. G. Höhler, group I, vol.9, subvol.b, part 2 of Landölt-Bornstein Numerical data and Functional Relationships in Science and Technology, ed. H.Schopper, 1983; G. Höhler, H. P. Jacob, and R. Strauss, Nucl. Phys. B39, 273 (1972).
17. J-L. Ballot, M. R. Robilotta and C.A. da Rocha, Int. J. Mod. Phys. E 6, 83 (1997).
18. G.E. Brown, A.M. Green and W.J. Gerace, Nucl. Phys. A 115, 435 (1968).
19. B.H.J. McKellar and R. Rajaraman, Phys. Rev. Lett. 21, 450, 1030 (1968).
20. G.E. Brown and A.M. Green, Nucl. Phys. A 137, 1 (1969).
21. S-N. Yang, Phys. rev. C 10, 2067 (1974).
22. S. Weinberg, Phys. Rev. Lett. 18, 507 (1967).
23. S.A. Coon, M.S. Scadron and B.R. Barrett, Nucl. Phys. A 242, 467 (1975).
24. S.A. Coon, M.D. Scadron, P.C. McNamee, B.R. Barrett, D.W.E. Blatt and B.H.J. McKellar, Nucl. Phys. A 317, 242 (1979).
25. S. A. Coon and W. Glöckle, Phys. Rev. C 23, 1790 (1981).
26. M.R. Robilotta, M.P. Isidro Filho, H.T. Coelho and T.K. Das, Phys. Rev. C 31, 646, (1985); M.R. Robilotta and M.P. Isidro Filho, Nucl. Phys. A 451, 581 (1986).
27. M.R. Robilotta and H.T. Coelho, Nucl. Phys. A 460, 645 (1986).
28. S.A. Coon and H.K. Han, Few Body Syst. 30, 131 (2001).
29. H. T. Coelho, T. K. Das, and M. R. Robilotta, Phys. Rev. C 28, 1812 (1983).
30. M. G. Olsson and E. T. Osypowski, Nucl. Phys. B 101,136 (1975); E. T. Osypowski, Nucl. Phys. B 21, 615 (1970).
31. S. Weinberg, Phys. Lett. B 251, 288 (1990); Nucl. Phys. B 363, 3 (1991).
32. S. Weinberg, Phys. Lett. B 295, 114 (1992).
33. C. Ordonez and U. van Kolck, Phys. Lett. B 291, 459 (1992).
34. U. van Kolck, Phys. Rev. C 49, 2932 (1994).
35. J. L. Friar, D. Huber, and U. van Kolck, Phys. Rev. C 59, 53 (1999).
36. V. Bernard, E. Epelbaum, H. Krebs and Ulf-G. Meissner, preprint nucl-th/0712.1967.
37. S. Ishikawa and M.R. Robilotta, Phys. Rev. C 76, 014006 (2007).