A Comparative Study on the Exploration Depth and Resolution of Winner and Schlenberg Methods Based on Potential, Current and Field Fields

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Abstract. High-density electrical method is widely used in hydrology, engineering, environment, and other fields. It is of great practical significance to study its exploration depth and resolution. The traditional theoretical research is to analyze and explore the exploration depth and resolution through the current density. Based on the theory of field theory, this paper proposes to study the depth and resolution of high-density electrical exploration based on a stable DC electric field. First, the derivation of the three field quantities of potential, current, and field lines in the normal field was completed, and related numerical simulation calculations were realized. Through theoretical model calculation and analysis, qualitative comparison conclusions of the two methods of Winner and Schlumberger are obtained. It is believed that the detection depth and resolution of the Winner method are lower than those of the Schlumberger, but the observation accuracy of the Winner method is higher than the Schlumberger method. Furthermore, the reliability of this conclusion is verified by physical model experiments. At the same time, the feasibility of the research on exploration depth and resolution based on the three electric field components of potential, current, and field lines is also proved.

1. Introduction
Direct current exploration is an important branch of electrical exploration. It is based on the different resistivity of different rock formations or ore bodies, Using various electrical instruments, adopting point electrodes to supply power on the ground, the potential difference and current between the M and N electrodes were measured, Use apparent resistivity formula: \( \rho_a = K \frac{U}{I} \), to obtain the apparent resistivity [1]. Winner and Schlumberger resistivity method is a kind of direct current method exploration. Since the electrical survey is a volume survey, it uses different resistivity of rock formations to layer. Therefore, the apparent resistivity value obtained by each electrical survey is the average apparent resistivity of the volume it surveys [1]. This volume effect has brought many problems in determining the exploration depth and improving the resolution [2].

In the previous DC method exploration, the relationship between current density and resistivity was mainly used: \( \rho_z = K \frac{MN}{I} \cdot j_{MN} \cdot \rho_{MN} = \frac{j_{MN}}{j_0} \cdot \rho_{MN} \). This formula indicates that the depth of detection mainly depends on the ratio of apparent current density \( j_{MN} \) between MN and true current density \( j_0 \) in semi-
infinite space. Obviously, this proportionality factor \( \frac{J_{MN}}{J_0} \) determines the deviation between the apparent resistivity and the true resistivity. Because the apparent current density is related to the distance between the power supply electrodes A and B, the larger the distance between the power supply electrodes A and B, the smaller the apparent current density and the deeper of the exploration depth. Conversely, the smaller the distance between A and B, the larger the apparent current density and the shallower of the exploration depth. In addition, the higher the resistivity of the underlying rock formation, due to the high-resistance shielding effect; apparent current density becomes smaller, the smaller the exploration depth, on the contrary, the lower the resistivity of the underlying rock formation, apparent current density gets smaller, the greater of the exploration depth. In actual work, the situation will become more complicated, geological formation of the rock formation is not necessarily our imaginary horizontal layered formation, and the influence of the surrounding rocks should be considered. Therefore, the maximum exploration depth is generally set between \( \frac{AB}{6} - \frac{AB}{2} \) in realistic [3], As can be seen from the above analysis, This formula gives a qualitative explanation of the depth of exploration in electrical exploration only from the perspective of current density. Since we cannot get the current density \( J_{MN} \) and \( j_0 \), So it’s just a theoretical explanation formula, Moreover, the formula cannot explain how to explain the problem of exploration depth in the case where the distance between the power supply electrodes A and B is constant, and the distance between the receiving electrodes M and N is different, that is, when \( MN \ll AB \) is not satisfied.

In this paper, two methods of Wenner and Schlumberger in the high-density electrical method are selected. From the new perspective of potential, current, and field line, Assuming that the distance between the A and B electrodes is equal, the distance between the measuring electrodes M and N is changed to discuss the issue of exploration depth and resolution. This is a new exploration that will lay the foundation for quantitative calculation of exploration depth and resolution and point the way.

2. Potential, field line, and current at any point in the half-space geoelectric model

2.1. Calculate the potential at any point on the ground plane

According to field theory knowledge [4], the calculation formula for the potential generated by point power source A (x0, y0, z0) at any point in space P (x, y, z) is:

\[
U = \frac{I\rho}{4\pi r} \quad (1)
\]

I is the power supply current intensity of the point power supply, \( \rho \) is the resistivity at point P, \( r \) is the absolute value of the vector diameter from the point power source to the point P,

\[
|r| = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} \quad (2)
\]

Under the boundary conditions of the normal field, if the power is supplied by two points A and B on the ground, the potential calculation formula at any point below the ground is:

\[
U_p = \frac{I\rho}{2\pi} \left( \frac{1}{r_{wp}} - \frac{1}{r_{wp}} \right) \quad (3)
\]

\( r_{wp} \) Where is the absolute radius of the vector from point power A to point P, \( r_{wp} \) is the absolute value of the radius of the point power from point B to point P.
2.2. Calculate the field lines on the ground plane
Let P (x, y, z) be any point on the vector line. The direction of the field is the direction of the negative gradient of the potential, Field lines are perpendicular to the contours, Then the vector diameter is r. Then differentiate it.

\[
\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}
\]

(4)

\[
d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}
\]

(5)

That is the vector tangent to the vector line at the point M. According to the definition of the vector line, It must be collinear with the vector of the field at point M.

\[
\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}
\]

(6)

\[
dx = \frac{dy}{E_y} = \frac{dz}{E_z}
\]

(7)

Figure 1. the field graph of geoelectric section

2.3. Current Integration Formula and Solving Current in Tube Field
As shown in Figure 2, at the center point of the A and B electrodes (AB / 2), it makes an infinite plane S parallel to the y/z plane. If any area element ds is selected on the plane, ds = dy*dz, is the radius f the power supply electrode A pointing to this ds area element.

Figure 2. Solve the current integral plot

\[
dl = j_a^i d\hat{s} + j_b^i d\hat{s}
\]

(8)

\[
I_a = -I_b \cdot \cos[\vec{r}_a \cdot \hat{n}] = -\cos[\vec{r}_b \cdot \hat{n}]
\]

(9)

\[
dl = j_a^i d\hat{s} + j_b^i d\hat{s} = 2j_a^i d\hat{s}
\]

(10)

\[
\vec{j} = \lambda E
\]

(11)

\[
dl = 2\lambda E^i d\hat{s}
\]

(12)
\[ \mathbf{E}'_A = -\nabla (u'_A) = -\nabla u_A \]

\[ dl = 2\lambda E'_A \cdot d\mathbf{s} = -2\lambda (\nabla u_A) \cdot d\mathbf{s} \]

\[ \nabla u_A = \frac{I_A \rho}{2\pi} \frac{1}{r_a} = -\frac{I_A \rho}{2\pi} \frac{r_a}{r_a^3} \]

\[ dl = 2\lambda \frac{I_A \rho}{2\pi} \frac{r_a}{r_a^3} \cdot d\mathbf{s} = \frac{I_A \rho}{\pi} \frac{r_a}{r_a^3} d\mathbf{s} \]

\[ r_a \cdot d\mathbf{s} = r_a ds \cos \alpha \left( \cos \alpha = \frac{x}{\sqrt{x^2 + y^2 + z^2}} \right), d\mathbf{s} = dydz \]

\[ dl = \frac{I_A}{\pi r_a^2} ds \cos \alpha = \frac{I_A}{\pi} \frac{1}{x^2 + y^2 + z^2} \frac{x}{\sqrt{x^2 + y^2 + z^2}} dydz \]

\[ dl = \frac{I_A}{\pi} \frac{x}{(x^2 + y^2 + z^2)^{3/2}} dydz \]

\[ I = \iint_{\mathcal{S}^c} \frac{1}{\pi} \int \frac{x}{(x^2 + y^2 + z^2)^{3/2}} dydz + C \]

\[ I = -\frac{2I_A}{\pi} \left[ \arctan \frac{z}{x} \right]_{x=0}^{x=\bar{z}} \]

\[ I(AB/6) = -\frac{2I_A}{\pi} \left[ \arctan 1/3 \right]_{x=0}^{x=\bar{z}} = 0.2048 I_A \]

\[ I(AB/3) = -\frac{2I_A}{\pi} \left[ \arctan 2/3 \right]_{x=0}^{x=\bar{z}} = 0.3743 I_A \]

\[ I(AB/2) = -\frac{2I_A}{\pi} \left[ \arctan 1 \right]_{x=0}^{x=\bar{z}} = 0.5 I_A \]

\[ I(AB) = -\frac{2I_A}{\pi} \left[ \arctan 2 \right]_{x=0}^{x=\bar{z}} = 0.7048 I_A \]

**Figure 3.** Comparison of Wentner Method and Schlenberg’s Method
3. Theoretical model trial calculation and analysis

3.1. Field Line

Assuming there is a uniform semi-infinite conductive medium below the ground. We use the potential formula at any point in the existing uniform semi-infinite conductive medium and the relationship between field lines and field strength. The equivalent points equal to the potentials $U_m$ and $U_n$ of the ground measurement electrodes M and N on the ground plane and coordinates of field line points on the geoelectrical section are calculated. We could find the plot contours and field lines in Figure 3.

It can be seen from the figure that at $AB = 65 \text{ cm}$, the depth that the Wenner isoline can reach is $27 \text{ cm}$, Schlumberger isoline can reach is $68.5 \text{ cm}$, and they differ in depth by $41.5 \text{ cm}$. The depth of the contour of Winner is much shallower than the depth of the contour of Schlumberger, the difference is about $2.5$ times. The field line at the depth of $AB/2$ passes through the contour line of Wenner at a depth of $21.4 \text{ cm}$, and the contour line of Schlumberger at a depth of $31.2 \text{ cm}$; the field line at the depth of $AB/3$ is $16.2 \text{ cm}$ Cross the contour line of Winner everywhere and cross the Schlumberger contour line $21 \text{ meters}$ deep. Based on the field flowing through the $AB/2$ depth point, the depth difference between Winner and Schlumberger is $9.8 \text{ cm}$, and the field flowing through the $AB/3$ depth point is $4.8 \text{ cm}$. As we all know, the area where the electric charge does not exist in the steady DC electric field is the harmonic field, that is the tubular field. According to the properties of the tubular field, the currents passing through any cross-sectional area in the current tube are the same current. Therefore, according to the above calculation results and the property of the tubular field, it is obtained that the same current flows through the Wenner equipotential line at a shallow depth and through the Schlumberger equipotential line at a deep depth. As the field line moves up, this kind of depth difference is decreasing, while the field line is moving down, the depth difference is increasing. Similarly, the field line at the depth of $AB/2$ is calculated. The distance in the $x$ direction between the two equipotential lines passing by Wenner is $40.6 \text{ cm}$. The Schlumberger is $13.6 \text{ cm}$. The field line at the depth of $AB/2$ is calculated. The distance in the $x$ direction between the two equipotential lines passing by Wenner is $24.2 \text{ cm}$, and Schlumberger is $6 \text{ cm}$. This data shows that the volume of the same current flowing through the Wenner isopotential is much larger than the volume flowing through Schlumberger, so the resolution and detection depth of Winner is lower than Schlumberger.

3.2. Field Line

The potential formula in the field theory of the steady DC electric field is used to calculate the potential formula, and the relationship between the field strength equals the negative gradient and the differential expression of the current density. An expression for calculating the current $dI$ at any point on the $yz$ semi-infinite plane passing through the center of $AB$ is derived. By integrating this integrand on a semi-infinite plane, the expression of the current formula is obtained. Then use it to calculate the current in the tubular field and compare the ground-electrical section widths of the two methods, Wenner and Schlumberger, under the same current conditions (Figure 4).

![Figure 4. Comparison of Resolution and Exploration Depth between Winner and Schlenberg](image)
The current reaches 20.5% of the total current $IA$ between 0-AB/6, and the current reach 70.1% at 0-AB. Further divide the depth of 0-AB into 6 equal parts, and the current difference from shallow to deep is $I(AB/6) - I(0) = 0.205$, $I(AB/3) - I(AB/6) = 0.17$, $I(AB/2) - I(AB/3) = 0.13$, $I(AB/2) - I(AB/3) = 0.09$, $I(AB/6) - I(AB/3) = 0.065$ and $I(AB) - I(AB/6) = 0.039$. This shows that the increase of the current intensity decreases in order with the increase of depth. When the current decays to a certain extent, it also reaches the limit detection depth. Since Wenner's current decay rate is faster than Schlumberger, winner the detection depth is lower than the Schlumberger detection depth. In the range of supply current with a depth of AB/6, the cross-sectional area of the volume passing through the Wenner isopotential line is about five times the cross-sectional area of the isoelectric line passing Schlumberger. Therefore, the resolution of Wenner is far lower resolution than Schlumberger.

4. Physical model experiments

To test the correctness of the analysis results and the validity of the actual data proposed in this paper, the method was applied to the experiment of the physical model. The model electrical experiment water tank has a length of 3m, a width of 1m, and a height of 1m. The conductive background medium is tap water with a water depth of 1m (Figure 5). The test body is a plastic high-resistance board. The length of the plastic board is 0.645m, the width is 0.095m, and the thickness is 0.01m. The experimental instrument is a DUK-2 high-density electrical measurement system manufactured by Chongqing Geological Instrument Factory. During the experiment, the minimum pole spacing of the Wenner method is 4cm, the total number of electrodes is 30, the number of layers measured is $Ln = 1-8$, and the supply voltage is 10V. Schremberg method parameters are the same as above.

![Sink test model](image)

**Figure 5.** Sink test model

![The resistivity profile of winner method(A) and the schremberg method(B)](image)

**Figure 6.** The resistivity profile of winner method(A) and the schremberg method(B)

It can be clearly seen from the above two figures that the volume effect of the Wenner method is larger than that of the Schlumberger method. Due to the influence of this volume effect, the range of the high-resistance body reflected by the Winner method becomes larger, and resistivity value becomes lower. If the distance between electrodes A and B continues to increase, the apparent resistivity value obtained by the Wenner method will be mixed with the background value (Figure 6A). Due to the limited volume
of the experimental water tank, the distance between the electrodes A and B cannot continue to increase, but such signs have been seen in the figure obtained by the Wenner method. The Schlumberger method in the figure below is significantly better than the Winner method. At 32cm in the figure, the Schlumberger's method can also clearly see the existence of the subject (Figure 6B), while the Winner method has become blurred. This experimental result confirms our theoretical analysis above.

5. Conclusions
Through the analysis of the calculated large amount of qualitative data and the experimental verification of the model, we obtained the following knowledge. The Wenner detection method in high-density electrical surveying has a much lower resolution and a lower depth of detection than the Schlumberger detection method. The feasibility of the research on exploration depth and resolution based on the three electric field components of potential, current and field lines was proved.

References
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