A comparative study of planetary mechanisms usable for power sources coupling

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Abstract. In the first part of the paper, a comparative study of the planetary mechanisms solutions which can be used in the coupling of power sources is made, this coming in addition to the previous works of the authors. It is analyzed the functioning mode of two planetary mechanisms used in different configurations and optimal variants are set.

1. Introduction
In the last decades, road traffic has increased considerably. Because motor vehicles are the main fossil fuel users and the source of greenhouse gases, it has been necessary to develop alternative methods of generating the energy required for propulsion. These methods and new technical solutions for the propulsion of motor vehicles are used to reduce the operating time of the thermal engine that could never reach a zero pollution level [5]. For this purpose, automotive engineers have conducted studies in the field of propulsion hybrid systems.

The hybrid vehicle is that vehicle that has at least two different energy sources that act as propulsion. The main elements of the propulsion system of the hybrid vehicle are the reversible energy storage system (for example an electric motor powered by a set of battery cells), the irreversible energy transformer (e.g. the internal combustion engine, whether it is a diesel engine or a spark-ignition engine) and a coupling system that connects the energy systems and conveys the movement to the wheels of the vehicle [1][6].

The coupling system can be classified into a hydraulic, electrical or mechanical coupling system.

This paper is dedicated to mechanical coupling systems used in the configuration of hybrid vehicle transmission, namely planetary mechanisms. These are mainly used by hybrid vehicle manufacturers due to their high efficiency, due to the fact that they are compact, they have a high-performance transmission ratio, a small size and a simple construction [2]. Planetary mechanisms are composed of central gear, crowns, planet carrier and one or more satellite pinions.

2. Mechanical systems for coupling power sources
The most commonly used planetary mechanism in power sources coupling is the power split mechanism with two degrees of freedom patented by Toyota (figure 1). The solution adopted by the Japanese manufacturer has changed to Hybrid Synergy Drive since 2004, when it started equipping vehicles outside Toyota brand, Lexus and Nissan vehicles [1].

The planetary mechanism components are the crown 2, the central gear 3, the satellite 4 and the planet carrier 1 [1][2][4]. For these, we have marked the numbers of teeth with \( z_2, z_3, z_4 \) and the absolute angular velocities with \( \omega_1, \omega_2, \omega_3, \omega_4 \).
Another solution that may be used for coupling power sources is by using planetary mechanisms with two degrees of freedom, double satellite and two eternal meshing [1][7] (figure 2a and b).

For these coupling systems we consider $z_2, z_3, z'_4, z'_4$ the number of teeth of the gears and $\omega_1, \omega_2, \omega_3, \omega_4$ the absolute angular speeds of the elements 1, 2, 3, 4 [7].

We will apply the Willis method [3] in order to obtain kinematic relations [1] [8].

\begin{equation}
\frac{\omega_3 - \omega_1}{\omega_4 - \omega_2} = \frac{z_4}{z_3} \quad ; \quad \frac{\omega_4 - \omega_1}{\omega_2 - \omega_1} = \frac{z_2}{z_4} ; \quad (1)
\end{equation}

from which it is obtained :

\begin{equation}
\frac{\omega_3 - \omega_1}{\omega_2 - \omega_1} = \frac{z_2}{z_3} . \quad (2)
\end{equation}

If we denote the ratio:

\begin{equation}
i = \frac{z_2}{z_3} ; \quad (3)
\end{equation}

by replacing into relations (2) it results:

\begin{equation}
\omega_3 = \omega_1 (1 - i) + i \omega_2 . \quad (4)
\end{equation}

If we consider the ratio $i_1$ :

\begin{equation}
i_1 = \frac{z_2}{z_4} ; \quad (5)
\end{equation}

from relation (1) and (5) we conclude:

\begin{equation}
\omega_4 = \omega_1 (1 - i_1) + i_1 \omega_2 . \quad (6)
\end{equation}

For the coupling systems in figure 2a and b, the ratios are [7]:

\begin{equation}
i_1 = \frac{z_2}{z_4} ; \quad (7)
\end{equation}
\[ i = \frac{z_2 z_4}{z_3 z_4} \]

By using the Willis method for the mechanisms in figure 2a and b one obtains:

\[
\frac{\omega_4 - \omega_1}{\omega_2 - \omega_1} = -\frac{z_2}{z_4}, \quad \frac{\omega_3 - \omega_1}{\omega_4 - \omega_1} = -\frac{z_4}{z_3}; \tag{8}
\]

by replacing equation (7) into equation (8) we obtain the following equations [7]:

\[
\begin{align*}
\omega_3 &= \omega_1 (1 - i) + i \omega_2; \\
\omega_4 &= \omega_1 (1 + i_1) + i_1 \omega_2. \\
\end{align*} \tag{9}
\]

3. A comparative study of Toyota planetary mechanism solution versus the planetary mechanism with double satellite solution

The powertrain’s operating mode (electric or hybrid propulsion) or the functioning regime of the two electric machines (engine or generator) can be explained by exemplifying the correlations of the rotations of the planetary mechanism elements (central gear, planet carrier and crown).

Next, we will compare mechanical systems solutions that use planetary mechanism with a double satellite and two eternal meshing gears with the solution used in the Prius configuration.

For the planetary mechanisms in the figure 1 and 2a, we consider:

- MG1 – Alternator: \( \omega_3, \omega_3 = \omega_1 (1 - i) + i \omega_2; \)
- MG2 – Electric motor: \( \omega_2; \)
- MT - Thermal engine: \( \omega_1. \)

For the configuration that has the planetary mechanism with a double satellite and the electric motor connected to the planet carrier, figure 2b, we note:

- MG1 – Alternator: \( \omega_3; \)
- MG2 – Electric motor: \( \omega_1; \)
- MT - Thermal engine: \( \omega_2. \)

We will consider solution I, the configuration described in figure 2a, for \( i = 2.5 \), solution II the same configuration in figure 2a but for \( i = 0.5 \) and solution III represented in figure 1c and \( i = 2.5 \).

All these solutions will be compared to the Prius solution, represented in figure 1 for which it was considered \( i = -2.5 \). From relation (10) one obtains:

Planetary mechanism used in the Prius configuration \( (i = -2.5) \) – figure 1:

\[ \omega_3 = 3.5 \omega_1 - 2.5 \omega_2; \tag{12} \]

Solution I - Planetary mechanism with double satellite \( (i = 2.5) \) - figure 2a:

\[ \omega_3 = -1.5 \omega_1 + 2.5 \omega_2; \tag{13} \]

Solution II - Planetary mechanism with double satellite \( (i = 0.5) \) – figure 2a:

\[ \omega_3 = 0.5 \omega_1 + 0.5 \omega_2; \tag{14} \]

Solution III - Planetary mechanism with double satellite \( (i = 2.5) \) - figure 2b:

\[ \omega_3 = -1.5 \omega_1 + 2.5 \omega_2. \tag{15} \]

The rotations of the two motors (electric and thermal) and of the electric generator are determined with the relations:

Planetary mechanism used in the Prius configuration \( (i = -2.5) \) – figure 1:

\[
\begin{align*}
\omega_1 &= (n_3 + 2.5n_2)/3.5; & \text{thermal engine;} \\
n_2 &= (3.5 \cdot n_1 - n_3)/2.5; & \text{electric motor;}
\end{align*}
\]

Solution I - Planetary mechanism with double satellite \( (i = 2.5) \) - figure 2a:

\[
\begin{align*}
n_1 &= (-n_3 + 2.5n_2)/1.5; & \text{thermal engine;}
n_2 &= (1.5 \cdot n_1 + n_3)/2.5; & \text{electric motor;}
n_3 &= 0.5n_1 + 0.5n_2; & \text{alternator;}
\end{align*}
\]

Solution II - Planetary mechanism with double satellite \( (i = 0.5) \) – figure 2a:

\[
\begin{align*}
n_1 &= (n_3 - 0.5n_2)/0.5; & \text{thermal engine;}
n_2 &= (-0.5 \cdot n_1 + n_3)/0.5; & \text{electric motor;}
n_3 &= -1.5n_1 + 2.5n_2; & \text{alternator;}
n_1 &= (-n_3 + 2.5n_2)/1.5; & \text{electric motor;}
\end{align*}
\]

Solution III - Planetary mechanism with double satellite \( (i = 2.5) \) - figure 2b:
\[ n_2 = (1.5 \cdot n_1 + n_3)/2.5 \] - thermal engine.

Next, we describe the functioning modes for these kinematic solutions in different driving stages.

### 3.1. Starting out the vehicle (electric mode)

After starting, if the driver presses the accelerator pedal, the vehicle starts to move, being propelled by MG2 that will be operating in the electric motor mode. This will determine MG1 to rotate in the opposite direction of MG2, and because the thermal engine is shut down, this will immobilize the planet carrier of the planetary mechanism. By using the equations (10) and (12), (13), (14), (15), one obtains:

**Planetary mechanism used in the Prius configuration \( i = -2.5 \) - figure 1:**
\[ \omega_1 = 0, \omega_3 = -2.5\omega_2; \] (20)

**Solution I - Planetary mechanism with double satellite \( i = 2.5 \) - figure 2a:**
\[ \omega_1 = 0, \omega_3 = 2.5\omega_2; \] (21)

**Solution II - Planetary mechanism with double satellite \( i = 0.5 \) - figure 2a:**
\[ \omega_1 = 0, \omega_3 = 0.5\omega_2; \] (22)

**Solution III - Planetary mechanism with double satellite \( i = 2.5 \) - figure 2b:**
\[ \omega_2 = 0, \omega_3 = -1.5\omega_1. \] (23)

### 3.2. Engine starting

In electric power mode, if the vehicle’s speed exceeds a certain limit (about 50 km / h), the thermal engine is started with the help of MG1. The inverter will power MG1 that will function in motor mode. The direction of rotation of MG1 and MG2 will be the same which will drive the thermal engine.

The equations (16) - (19) lead to:

**Planetary mechanism used in the Prius configuration \( i = -2.5 \) – figure 1:**
\[ n_1 = (1000 + 2.5 \cdot 2000)/3.5 = 1714 [\text{rot/min}]; \]
\[ n_2 = 2000 [\text{rot/min}]; \] (24)

**Solution I - Planetary mechanism with double satellite \( i = 2.5 \) - figure 2a:**
\[ n_1 = (-1000 + 2.5 \cdot 2000)/1.5 \]
\[ = 2666 [\text{rot/min}]; \]
\[ n_2 = 2000 [\text{rot/min}]; \]
\[ n_3 = 500 [\text{rot/min}]; \]
\[ n_3 = (500 - 0.5 \cdot 2000)/0.5 \]
\[ = -1000 [\text{rot/min}]; \] (25)

**Solution II - Planetary mechanism with double satellite \( i = 0.5 \) – figure 2a:**
\[ n_1 = (1000 - 0.5 \cdot 2000)/0.5 = 0 [\text{rot/min}]; \]
\[ n_3 = 1000 [\text{rot/min}]; \]
\[ n_3 = (1000 - 0.5 \cdot 2000)/0.5 = 0 [\text{rot/min}]; \] (26)

**the engine will initially spin in one direction and then change its direction of rotation. This shortcoming can be remedied with a directional coupling.**

**Solution III - Planetary mechanism with double satellite \( i = 2.5 \) - figure 2b:**
\[ n_1 = 2000 [\text{rot/min}]; \]
\[ n_2 = (1.5 \cdot 2000 + 1000)/2.5 = 1600 [\text{rot/min}]. \] (27)

### 3.3. Light acceleration

After starting the thermal engine if the driver desires a light acceleration of the vehicle, the thermal engine will increase its speed above the value of MG2. In this case MG1 will function in the electric generator mode. The electric energy produced by MG1 is used for MG2.

**Planetary mechanism used in the Prius \( n_2 = 2000 [\text{rot/min}], n_1 = 2500 [\text{rot/min}] \), configuration \( i = -2.5 \) – figure 1:**
\[ n_3 = 3.5 \cdot 2500 - 2.5 \cdot 2000 = 3780 [\text{rot/min}]; \] (28)
\[ n_2 = 2000[\text{rot/min}], n_1 = 2500[\text{rot/min}] \]

Solution I - Planetary mechanism with double satellite \(i = 2.5\) - figure 2a:
\[ n_3 = -1.5 \cdot 2500 + 2.5 \cdot 2000 = 1250[\text{rot/min}] \]
- the alternator does not charge;

Solution II - Planetary mechanism with double satellite \(i = 0.5\) – figure 2a:
\[ n_2 = 2000[\text{rot/min}], n_1 = 2500[\text{rot/min}], n_3 = 0.5 \cdot 2000 + 0.5 \cdot 2500 = 2250[\text{rot/min}] \]

Solution III - Planetary mechanism with double satellite \(i = 2.5\) - figure 2a:
\[ n_3 = -1.5 \cdot 2000 + 2.5 \cdot 2500 = 3250[\text{rot/min}] \]
- the alternator is charging.

According to the result obtained with relation (29) it is observed that the alternator does not charge in the case of a light acceleration when it is used the planetary mechanism with double satellite presented in the configuration in figure 2a, for \(i = 2.5\). The solution III is solving this drawback in this driving stage.

3.4. Light load cruise – light constant speed

In this driving stage, the vehicle is propelled in hybrid mode, by the internal combustion engine and the electric motor MG2. The MG1 generator, produces electric energy that it is supplied to MG2.

Planetary mechanism used in the Prius \(n_2 = 3000[\text{rot/min}], n_1 = 3000[\text{rot/min}]\), configuration \(i = 2.5\) – figure 1:
\[ n_3 = 3.5 \cdot 3000 - 2.5 \cdot 3000 = 3000[\text{rot/min}] \]

Solution I - Planetary mechanism with double satellite \(i = 2.5\) - figure 2a:
\[ n_3 = -1.5 \cdot 3000 + 2.5 \cdot 3000 = 3000[\text{rot/min}] \]

Solution II - Planetary mechanism with double satellite \(i = 0.5\) – figure 2a:
\[ n_2 = 3000[\text{rot/min}], n_1 = 3000[\text{rot/min}], n_3 = 0.5 \cdot 3000 + 0.5 \cdot 3000 = 3000[\text{rot/min}] \]

Solution III - Planetary mechanism with double satellite \(i = 2.5\) - figure 2a:
\[ n_3 = -1.5 \cdot 3000 + 2.5 \cdot 3000 = 3000[\text{rot/min}] \]

3.5. Full acceleration

If a full acceleration of the vehicle is desired, the thermal engine speed increases. The vehicle is powered in hybrid mode (electric and thermal). MG1 will produce electric power to power MG2. In addition, in order to get a maximum torque from the MG2, the battery will additionally supply the required power [1].

Planetary mechanism used in the Prius configuration \(i = 2.5\) – figure 1:
\[ n_2 = 3000[\text{rot/min}], n_1 = 5000[\text{rot/min}], n_3 = 3.5 \cdot 5000 - 2.5 \cdot 3000 = 10000[\text{rot/min}] \]

Solution I - Planetary mechanism with double satellite \(i = 2.5\) - figure 2a:
\[ n_3 = -1.5 \cdot 5000 + 2.5 \cdot 3000 = 0[\text{rot/min}] \]
- the alternator does not charge;

Solution II - Planetary mechanism with double satellite \(i = 0.5\) – figure 2a:
\[ n_2 = 3000[\text{rot/min}], n_1 = 5000[\text{rot/min}], n_3 = 0.5 \cdot 5000 + 0.5 \cdot 3000 = 4000[\text{rot/min}] \]

Solution III - Planetary mechanism with double satellite \(i = 2.5\) - figure 2b:
\[ n_3 = -1.5 \cdot 5000 + 2.5 \cdot 5000 = 8000[\text{rot/min}] \]
- the alternator charges.
It can be observed that even in the case of a full acceleration, according to the relation (37), the alternator does not charge in the configuration shown in solution II. Solution III is however a favourable option this time as well.

3.6. Full load cruise – constant speed
For a given speed of the thermal engine, in order to increase the speed of MG2, MG1 will be locked (zero speed). In this driving stage, the vehicle is propelled in hybrid mode, MG2 being powered by the battery pack.

\[
\begin{align*}
\text{Planetary mechanism used in the Prius} & \quad n_2 = 6000 [\text{rot/min}], n_3 = 0 [\text{rot/min}], \\
(\text{configuration } (i = -2.5)) & \quad n_1 = 2.5 \cdot 6000 / 3.5 = 4286 [\text{rot/min}]; \\
\text{Solution I - Planetary mechanism with double} & \quad n_2 = 6000 [\text{rot/min}], n_3 = 0 [\text{rot/min}], \\
\text{satellite } (i = 2.5) - \text{figure 1:} & \quad n_1 = 2.5 \cdot 6000 / 1.5 = 10000 [\text{rot/min}] \\
\text{solution II - Planetary mechanism with double} & \quad n_1 = -n_2 = 6000 [\text{rot/min}] \quad \text{- the thermal} \\
\text{satellite } (i = 0.5) - \text{figure 2a:} & \quad \text{motor turns backwards;}
\end{align*}
\]

We can choose the solution: \( n_3 = 6000, \) \( n_1 = (n_3 - 0.5n_2) / 0.5, \) \( n_1 = (6000 - 0.5 \cdot 6000) / 0.5 = 6000 [\text{rot/min}] \) – a directional coupling is needed, since it starts at negative speeds; \( n_2 = 6000 [\text{rot/min}], n_3 = 0 [\text{rot/min}], \) \( n_1 = -n_2 = 6000 [\text{rot/min}] \) – the thermal motor turns backwards;

3.7. Maximum speed
For the same speed of the thermal engine, MG2 will have a higher speed if MG1 is in engine mode and the rotational direction is reversed. The vehicle is propelled hybrid mode (electrical and thermal), the battery pack powering both MG2 and MG1 electric motors.

\[
\begin{align*}
\text{Planetary mechanism used in the} & \quad n_1 = 6000 [\text{rot/min}], n_3 = -3000 [\text{rot/min}], \\
\text{Prius configuration } (i = -2.5) - \text{figure 1:} & \quad n_2 = (3.5 \cdot 6000 + 3000) / 3.5 = 6857 [\text{rot/min}]; \\
\text{Solution I - Planetary mechanism} & \quad n_1 = 6000 [\text{rot/min}], n_3 = 3000 [\text{rot/min}], \\
\text{with double satellite } (i = 2.5) - \text{figure 2a:} & \quad n_2 = (1.5 \cdot 6000 + 3000) / 1.5 = 8000 [\text{rot/min}] \\
\text{Solution II - Planetary mechanism} & \quad n_1 = 6000 [\text{rot/min}], n_3 = 9000 [\text{rot/min}], \\
\text{with double satellite } (i = 0.5) - \text{figure 2a:} & \quad n_2 = (-0.5 \cdot 6000 + 9000) / 0.5 = 6000 [\text{rot/min}] \quad \text{- alternator with high speed;} \\
\text{Solution III - Planetary mechanism} & \quad n_2 = 6000 [\text{rot/min}], n_3 = 3000 [\text{rot/min}], \\
\text{with double satellite } (i = 2.5) - \text{figure 2b:} & \quad n_1 = (3000 + 2.5 \cdot 6000) / 1.5 = 8000 [\text{rot/min}] \\
\text{- it is not necessary to reverse the alternator's} & \quad \text{direction.}
\end{align*}
\]

In order to be able to meet the power requirements, it is necessary that the hybrid vehicle equipped with a planetary mechanism with double satellite from the solution I, to be equipped with a high power alternator. In the case of solution II it is necessary to use a high speed alternator.

3.8. Deceleration - regenerative braking
When the driver presses the brake pedal to reduce the speed of the vehicle, the thermal engine is switched off. In this mode, the MG2 will operate in electric generator mode, being driven by the vehicle's wheels (motor brake) [1]. The energy produced by MG2 is used to charge the battery pack. MG1 will not be powered, and will rotate in reverse direction to MG2 due to the zero speed of the thermal engine.

Planetary mechanism used in the Prius configuration \((i = -2.5)\) – figure 1:
\[ n_1 = 0 \, [\text{rot/min}], \, n_3 = -2.5n_2; \quad (48) \]

Solution I - Planetary mechanism with double satellite \((i = 2.5)\) - figure 2a:
\[ n_1 = 0 \, [\text{rot/min}], \, n_3 = 2.5n_2; \quad (49) \]

Solution II - Planetary mechanism with double satellite \((i = 0.5)\) – figure 2a:
\[ n_1 = 0 \, [\text{rot/min}], \, n_3 = 0.5n_2 \quad \text{- The energy recovered after braking the vehicle is very low; } \quad (50) \]

Solution III - Planetary mechanism with double satellite \((i = 2.5)\) - figure 2b:
\[ n_2 = 0 \, [\text{rot/min}], \, n_3 = -1.5n_1 . \quad (51) \]

3.9. Reverse

In this driving stage the vehicle is powered exclusively electric, MG2 being in electric motor mode but its rotational direction being opposite from the "Drive" mode. The thermal engine is turned off and MG1 will rotate freely. The electric energy for MG2 is supplied by the battery pack.

4. Conclusions

This paper reviews the general principles and features of hybrid propulsion systems, focusing on the mechanical systems used for coupling power sources.

The kinematic analysis made for the Prius planetary mechanism as well as for the planetary mechanism with a double satellite has helped determine the functioning modes of the hybrid solutions. Following the analysis of the functioning modes of the studied solutions, it can be concluded that only two of them can equip hybrid vehicle. One of these is, of course, the Prius mechanism, but also the planetary mechanism with a double satellite for which the electric motor is connected to planet carrier.

This solution, referred to in the paper as solution III, solves the drawbacks encountered in the use of solution I. For solution I, the planet carrier is connected to the thermal engine this time and is not a favourable solution to be used in the hybrid vehicle configurations.

Analyzing the functioning mode for the solutions with planetary mechanism, were determined the optimal constructions which can be used to become powertrain configuration for hybrid vehicles.

It is necessary to know the functioning characteristics in order to determine the dynamic performance of hybrid vehicle in road traffic but also to construct the driving cycles.

By using the explanations presented in point 3, of this current paper, we plan to determine the driving cycles for the two chosen architectures in a forthcoming paper.

By determining these driving cycles we will be able to compare the performances produced in each of these driving stages that will occur in real life driving traffic conditions.

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