Computer models of pipeline systems based on electro hydraulic analogy

S V Kolesnikov, V A Kudinov, K V Trubitsyn, V K Tkachev, E V Stefanyuk

Samara State Technical University, 244, Molodogvardeyskaya St., Samara, 443100, Russia

E - mail: Totig@yandex.ru

Abstract. This paper describes the results of the development of mathematical and computer models of complex multi-loop branched pipeline networks for various purposes (water-oil-gas pipelines, heating networks, etc.) based on the electro hydraulic analogy of current spread in conductors and fluids in pipelines described by the same equations. Kirchhoff’s laws used in the calculation of electrical networks are applied in the calculations for pipeline systems. To maximize the approximation of the computer model to the real network concerning its resistance to the process of transferring the medium, the method of automatic identification of the model is applied.

1. Introduction

Mathematical and computer modeling of the processes of heat, mass and momentum transfer is widely used in the industry. The known models of these processes are complex and in many applications cannot be investigated using analytical methods. Therefore, the application of the analog method when the study of processes is performed on objects of a completely different nature but described by the same equations is promising. For example, Kirchhoff’s laws used in the calculations for electrical circuits can be used to calculate the pressures of moving fluids, considering the analog of electrical and hydraulic phenomena.

Mathematical models describing fluid flow in pipes contain, as a rule, nonlinear differential equations that are difficult to solve with the help of both analytical and numerical methods. Therefore, a theory based on the electro hydraulic analogy is an effective substitute for the process of direct mathematical modeling. There are some publications devoted to the development of this theory (Sukharev M.G., Merenkov A.G., Khasilev V.Y., Kovalenko A.G., Sokolov E.Y. et al), [1 - 6]. The basis of this theory is the method of calculating the flow rate on the sections of the network. In particular, the sequence of the use of Kirchhoff’s laws for these purposes is presented in the work [3]. It also shows that the application of this method for the calculation of multi-loop pipeline networks is associated with a large amount of calculations. Therefore, algorithms, based on iterative methods for calculating so-called balancing flow rates [1, 2, 4], are applied in this paper.

2. Theoretical justification of the method

Let us consider the main idea of the method by the example of finding the flow rate in the network including one loop with two branches (Figure 1). The flow rates for branches \( Q_1 \), \( Q_2 \) are specified, and
their sum is equal to the flow rate $Q$ at the inlet of the loop. It is required to find flow rates $Q_a$, $Q_b$, $Q_d$ on sections $a$, $b$, $d$ of the loop.

![Figure 1. Computational scheme of model.](image)

The first Kirchhoff’s law states that inflow and outflow of the medium at any node are equal:

$$\sum_{i=1}^{n} Q_i = 0,$$

According to the second Kirchhoff’s law, the summation of heads is equal to zero for any closed-loop system:

$$\sum_{i=1}^{n} H_i = \sum_{i=1}^{n} S_i Q_i^2 = 0,$$

where, $S_i$, $Q_i$, $(i=1, n)$ - hydraulic resistance and flow rate for $i$- sector; $n$ - the number of pipelines at a node (for the first Kirchhoff’s law) and the number of pipelines in a closed-loop system (for the second Kirchhoff’s law).

Based on above relations and using an iterative method for calculating given flow rates at the inlet of the loop and at the branches, there are the flow rates on the sections. At the first stage of iteration, the undefined flow rates are given on the loop sections. Consequently, according to the first Kirchhoff’s law:

$$Q_a = Q - Q_d; \quad Q_a = Q_i + Q_b. \quad (1)$$

According to the second Kirchhoff’s law:

$$\delta H = \sum_{i=1}^{3} S_i Q_i^2 = S_a Q_a^2 + S_b Q_b^2 - S_d Q_d^2. \quad (2)$$

From the last relation, there is the balancing flow rate if $\delta H = 0$. By neglecting the terms $(\delta Q)^2$ as relatively small, let us substitute (1) in (2) and with regard to arbitrary values of the flow rates on the sections at the first stage, there is:

$$\delta Q = \delta H \left( \sum_{i=1}^{3} S_i Q_i \right), \quad (3)$$

where $\sum_{i=1}^{3} S_i Q_i = S_a Q_a + S_b Q_b + S_d Q_d$.

Having calculated $\delta Q$ by formula (3) at the first stage of iteration, the flow rates on the loop sections are to be refined. Iterations continue until the flow rates of the last two iterations differ by a specified value.

The application of this method for a multi-loop hydraulic network is possible only with the use of computer technology. The graph theory [7] is applied resulting in the "tree" of the heat network (Fig. 2). In Fig. 2 the vertices are shown with the numbers 1, 2, 3, ..., 9, and arcs - with the letters $a$, $b$, $d$, .... The vertices are the points of junction of the pipelines, and the arcs are their sections. The "tree"
is designed in such a way that any other vertex can be reached from vertex 1. Every vertex is marked with the number, height of the location, the value of inflow or outflow of the medium. Every arc has the number, the length and the diameter of pipes, friction factor, etc.

![Figure 2. The scheme of the graph.](image)

In view of the iterative method of calculation for complex multi-loop networks the problem of iteration convergence occurs. In practice, a method of the full-system pressure drop balancing is used [1, 2, 6] characterized by rapid iteration convergence.

When constructing the pipeline network model, it is necessary to find hydraulic characteristics (the dependence of head losses on the flow rate) for all sections. For example, head losses in the pipeline include linear and local losses:

\[ \Delta h = \lambda \frac{l}{d} \frac{v^2}{2g} + \sum \xi \frac{v^2}{2g}, \]  

where \( \Delta h \) - head losses, m; \( \lambda \) - friction factor; \( l \) - length of the pipeline, m; \( d \) - inner diameter, m; \( v \) - average speed, m/s; \( \sum \xi \) - sum of local loss factors on the sector; \( g \) - acceleration of gravity, m/s².

If an equivalent length is introduced, then losses in local resistances are reduced to linear ones with the calculation of the equivalent length by ratio \( \lambda \sum \xi / \lambda \). Then formula (4) will be:

\[ \Delta h = \frac{v^2}{2g} \left( \lambda \frac{l}{d} + \lambda \frac{l}{d} \right) = \frac{\lambda v^2}{2dg} (l + l), \]

If \( v = 4Q / (\pi d^2) \), there is \( \Delta h = 8Q^2 \lambda (l + l) / (\pi^2 g d^5) \).

Then for the pipe section, the hydraulic characteristics is \( \Delta h = sQ^2 \), where \( s = 8\lambda (l + l) / (\pi^2 g d^5) \) is the hydraulic resistance of the section, s²/m⁵.

Pump sections in the model are presented by dependencies between the pump head and feed. The characteristic of the pump is determined by the formula \( H = H_o - Q_n S_o \), where \( H_o \) - shut-off head at outlet \( Q_n = 0 \); \( Q_n \) - pump feed, m³/s; \( S_o \) - hydraulic resistance of the pump, kg/m³. The value of \( m \) is assumed to be 2 or 1.85 in accordance with the characteristics of the pump.

The algorithm provided enables one to build a model with passport characteristics. However, the real network characteristics may differ from the passport ones. To bring the computer model as close to the real network as possible, its identification is performed [1,2]. For this purpose, the experimental pressure values at certain points of the network are used. The hydraulic resistance areas are changed so that the resulting pressures on the model do not differ much from the experimental ones. The identification process is an iterative algorithm, which accuracy is determined by the number of points with given experimental data.
3. Application of the method

Let us consider the application of the above-mentioned algorithm by the example of designing the combined computer model of the heating networks at Bezymyanskaia CHP and Samara CHP belonging to the district heating system of Samara city (Figure 3). Currently, the heating systems of these heat sources are divided by shut-off valves. Opening of any valve will result in an unpredictable imbalance in the work of combined heat networks where a specific pressure distribution can only be found experimentally. However, having models of these systems we can design the combined computer model for calculating various variants to identify the most appropriate ones. In particular, the calculations performed on the computer model of the heating network at Bezymyanskaia CHP showed a low pressure drop between the direct and return pipelines at the heat removal pipe 1-B (about 5 m H2O) at minimally acceptable level (normative) of 20 m H2O. The reasons may be as follows: undersized pipe diameters; the high consumption of heat agent (see Fig. 3, 4). At the same time, the calculations of the pressure distribution at the heat removal pipe 2-S at Samara CHP showed a significantly higher pressure drop between the direct and return pipelines (30 m H2O). Therefore, it would be more efficient to transfer part of the load from Bezymyanskaia CHP to Samara CHP. So, the load transfer of 800 t/h provides the opportunity to raise the pressure drop to 30 m H2O at the heat removal pipe 1 – B at Bezymyanskaia CHP (Figure 5). Thereby, the pressure drop at 2 – S at Samara CHP will decrease to 25 m H2O.

When performing the model identification of Samara CHP, two variants were studied with different number of points with known experimental pressures. The total number of nodes of the heat network at Samara CHP is 220. The identification accuracy is about 6% in case the known experimental data at 55 points (located regularly within the network) are used. If the number of experimental points is 70, the identification accuracy increases to 3%. However, the amount of computation for the process of iterative search of pipe resistance increases substantially. The search includes pipe resistances where the heads obtained from the calculations on the model differ from the experimental data by a given value. When the required accuracy is achieved, the computer stops working.

To enlarge pipe diameters at the heat removal pipe 1 – B at Bezymyanskaia CHP is the alternative to the load transfer from Bezymyanskaia CHP to Samara CHP. The calculations showed that the diameter increase from 800 mm to 1000 mm (on section L = 1.7 km - L = 2.8 km) makes it possible to raise the pressure drop to 30 m H2O.

![Diagram of heat networks](image)

Figure 3. Pressure distribution of centralized heat supply in Samara city.
BCHP - Bezymyanskaya CHP, SCHP - Samara CHP. \( \Phi \) - booster pump; \( \overline{\times} \) - valves; \( \_\_\_ \) - direct pipeline; \( \_\_\_\_\_ \) - return pipeline; \( P \) - pressure m H2O; \( G \) - consumption of heat agent, t/h; \( \vec{\gamma} \) - height mark, m; \( 1-S \), \( 2-S \) - heat removal pipes at Samara CHP; \( 1-B \), \( 2-B \) - heat removal pipes at Bezymyanskaya CHP.

4. Conclusion
The mathematical and computer model of the combined district heating network of Samara city, powered by Bezymyanskaya and Samara CHPs, has been developed by using an electrohydraulic analog. It was designed to calculate pressures, speeds, flow rates, head losses, electric power consumption for pump drives and so on.

To ensure more accurate approximation of the computer model to the real network, an automatic identification method, connected with the change in the resistances in some parts of the heating network, is applied. The purpose is to avoid the differences between the experimental data and the pressures obtained. The accuracy of identification depending on the used number of experimental points is 3-6%.

It is shown that the load transfer from removal pipe \( 1-B \) at Bezymyanskaya CHP in the amount of 800 t/h to removal pipe \( 2-S \) at Samara CHP provides raising the pressure drop at removal pipe \( 1-B \) at Bezymyanskaya CHP to 30 m H2O.

5. Acknowledgments
The work was carried out with the financial support of the Ministry of Education and Science of the Russian Federation within the framework of the basic part of the government order at FSBEI HPO "SSTU" (Project No. 1.5551.2017 / BP).

Figure 4. Pressure distribution in 1 – B removal pipe at Bezymyanskaya CHP before load transfer to 2 - S removal pipe at Samara CHP. \( P \) - pressure; \( L \) - length of pipelines; \( \_\_\_\_\_\_\_ \) - elevation mark on the terrain; \( \bullet \) - thermal chambers with given flow rate of the heat agent to consumers; \( \_\_\_\_\_\_\_\_ \) - direct pipeline; \( \_\_\_\_\_\_\_ \) - return pipeline.
Figure 5. Pressure distribution at 1 – B removal pipe at Bezymyanskaya CHP after load transfer to 2 - S removal pipe at Samara CHP.

References

[1] Zroychikov N A, Kudinov V A, Kovalenko A G, Kolesnikov S V, Moskvin A G and Lisitsa V I 2007 The development of a computer model and calculation of optimal operation for the circulating system at Mosenergo CHP-23 *Teploloenergetika* 12 7-15

[2] Kudinov I V, Kolesnikov S V, Eremin A V and Branfileva A N 2013 Computer models of complex multiloop branched pipeline systems *Teploloenergetika* 11 64-69

[3] Sokolov E Y 1982 *Heating and heating networks* (Moscow: Energoizdat) p 360

[4] Merenkov A P and Khaselev V Y 1985 *The theory of hydraulic circuits* (Moscow: Nauka) p 278

[5] Merenkov A P 1973 Differentiation of methods for calculating hydraulic networks *Journal of Computational Mathematics and Mathematical Physics* 13(5) 1237-48

[6] Sumarokov S V 1983 *Mathematical modeling of water supply systems* (Novosibirsk: Nauka) p 167

[7] Zykov A A 1969 *The theory of finite graphs* (Moscow: Nauka) p 543