A Note on Rich Incomplete Argumentation Frameworks

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Abstract

Recently, qualitative uncertainty in abstract argumentation has received much attention. The first works on this topic introduced uncertainty about the presence of attacks, then about the presence of arguments, and finally combined both kinds of uncertainty. This results in the Incomplete Argumentation Framework (IAFs). But another kind of uncertainty was introduced in the context of Control Argumentation Frameworks (CAFs): it consists in a conflict relation with uncertain orientation, i.e. we are sure that there is an attack between two arguments, but the actual direction of the attack is unknown. Here, we formally define Rich IAFs, that combine the three different kinds of uncertainty that were previously introduced in IAFs and CAFs. We show that this new model, although strictly more expressive than IAFs, does not suffer from a blow up of computational complexity. Also, the existing computational approach based on SAT can be easily adapted to the new framework.

1 Introduction

Abstract argumentation [16] is an important topic in the Knowledge Representation and Reasoning community. Intuitively, an abstract argumentation framework (AF) is a directed graph where nodes are arguments and edges are relations (usually attacks) between these arguments. The outcome of such an AF is an evaluation of the arguments’ acceptance (through extensions [16, 3], labellings [7] or rankings [1]). In such an AF, the assumption of complete information is made: an argument that appears in the graph is sure to actually exist, and similarly, an edge (or the absence of an edge) in the graph means that the attack between arguments certainly exists (or certainly does not).

The question of how to incorporate uncertainty in AFs has then arisen. Two kinds of approaches have been proposed. If a quantitative evaluation of the uncertainty is available, it seems natural to use it in the definition of reasoning mechanisms. This corresponds (e.g.) to Probabilistic Argumentation Frameworks [20]. But such a quantitative information about uncertainty may not be available. The other approach is then the Incomplete Argumentation Frameworks (IAFs) [9, 5, 4], where the uncertainty is only qualitative. In an IAF, some arguments are identified as uncertain, i.e. there is a doubt whether the argument actually appears in the framework. Similarly, attacks may be uncertain. However, another form of uncertainty in AFs has been defined in the
literature. Control Argumentation Frameworks [13] integrate uncertainty and argumentation dynamics [15] in a single framework. Besides the two aforementioned forms of uncertainty, a third one has been proposed: a symmetric conflict relation is defined, such that there is an uncertainty about the actual direction of the attack: either it appears in one direction, or in the other one, or in both directions at the same time. We investigate how this third kind of uncertainty can be added to IAFs.

The report is organized as follows. Section 2 describes the background notions on abstract argumentation and Incomplete Argumentation Frameworks (IAFs). In Section 3, we introduce Rich Incomplete Argumentation Frameworks (RIAFs), that generalize IAFs by adding a new kind of uncertainty over the attacks. Section 4 concludes the report by mentioning several interesting research tracks about (R)IAFs.

2 Background

2.1 Abstract Argumentation

Abstract argumentation was introduced in [16], where arguments are abstract entities whose origin or internal structure are ignored. The acceptance of arguments is purely defined from the relations between them.

Definition 1 (Abstract AF). An abstract argumentation framework (AF) is a directed graph $F = \langle A, R \rangle$, where $A$ is a set of arguments, and $R \subseteq A \times A$ is an attack relation.

We say that $a$ attacks $b$ when $(a, b) \in R$. If $(b, c) \in R$ also holds, then $a$ defends $c$ against $b$. Attack and defense can be adapted to sets of arguments: $S \subseteq A$ attacks (respectively defends) an argument $b \in A$ if $\exists a \in S$ that attacks (respectively defends) $b$.

Example 1. Let $F = \langle A, R \rangle$ be the AF depicted at Figure 1, with $A = \{a, b, c, d, e\}$ and $R = \{(b, a), (c, a), (c, d), (d, b), (d, c), (e, a)\}$. Each arrow represents an attack. $d$ defends $a$ against both $b$ and $c$, since these are attackers of $a$ that are, in turn, both attacked by $d$.

[16] introduces different semantics to evaluate the acceptability of arguments. They are based on two basic concepts: conflict-freeness and defence.

Definition 2 (Conflict-freeness and Admissibility). Given $F = \langle A, R \rangle$, a set $S \subseteq A$ is:

- conflict-free iff $\forall a, b \in S$, $(a, b) \notin R$;

![Figure 1: The AF $F$](image)
• admissible iff it is conflict-free, and defends each \( a \in S \) against its attackers.

We use \( \text{cf}(F) \) and \( \text{ad}(F) \) for denoting the sets of conflict-free and admissible sets of an argumentation framework \( F \).

The intuition behind these principles is that a set of arguments may be accepted only if it is internally consistent (conflict-freeness) and able to defend itself against potential threats (admissibility). The semantics proposed by Dung are then defined as follows.

**Definition 3** (Extension Semantics). Given \( F = \langle A, R \rangle \), an admissible set \( S \subseteq A \) is:

- a complete extension iff it contains every argument that it defends;
- a preferred extension iff it is a \( \subseteq \)-maximal complete extension;
- the unique grounded extension iff it is the \( \subseteq \)-minimal complete extension;
- a stable extension iff it attacks every argument in \( A \setminus S \).

The sets of extensions of an AF \( F \), for these four semantics, are denoted (respectively) \( \text{co}(F) \), \( \text{pr}(F) \), \( \text{gr}(F) \) and \( \text{st}(F) \).

Based on these semantics, we can define the status of any (set of) argument(s), namely skeptically accepted (belonging to each \( \sigma \)-extension), credulously accepted (belonging to some \( \sigma \)-extension) and rejected (belonging to no \( \sigma \)-extension). Given an AF \( F \) and a semantics \( \sigma \), we use (respectively) \( \text{sk}_\sigma(F) \), \( \text{cr}_\sigma(F) \) and \( \text{rej}_\sigma(F) \) to denote these sets of arguments.

**Example 2.** We consider again \( F \) given at Figure 1. Its extensions for the different semantics, as well as the sets of accepted arguments, are given at Table 1.

| \( \sigma \) | \( \sigma(F) \) | \( \text{cr}(F) \) | \( \text{sk}(F) \) |
|------------|----------------|----------------|----------------|
| co         | \{e\}, \{d, e\}, \{b, c, e\} | \{b, c, d, e\} | \{e\}           |
| pr         | \{d, e\}, \{b, c, e\} | \{b, c, d, e\} | \{e\}           |
| gr         | \{e\}             | \{e\}         | \{e\}           |
| st         | \{d, e\}, \{b, c, e\} | \{b, c, d, e\} | \{e\}           |

Table 1: Extensions and Accepted Arguments of \( F \) for \( \sigma \in \{ \text{co, pr, gr, st} \} \)

For more details about argumentation semantics, we refer the interested reader to [16, 3].

Now, we introduce Incomplete Argumentation Frameworks [9, 5, 4], i.e. AFs with qualitative uncertainty about the presence of some arguments or attacks.

**Definition 4** (Incomplete AF). An Incomplete Argumentation Framework (IAF) is a tuple \( I = \langle A, A', R, R' \rangle \), where \( A \) and \( A' \) are disjoint sets of arguments, and \( R, R' \subseteq (A \cup A') \times (A \cup A') \) are disjoint sets of attacks.

Elements from \( A \) and \( R \) are certain arguments and attacks, i.e. the agent is sure that they appear in the framework. On the opposite, \( A' \) and \( R' \) represent uncertain arguments and attacks. For each of them, there is a doubt about their actual existence.
Example 3. Let us consider \( \mathcal{I} = \langle A, A', R, R' \rangle \) given at Figure 2. We use plain nodes and arrows to represent certain arguments and attacks, i.e. \( A = \{a, b, c, d, e\} \) and \( R = \{(b, a), (c, a), (d, b), (d, c)\} \). Uncertain arguments are represented as dashed square nodes (i.e. \( A' = \{f\} \)) and uncertain attacks are represented as dotted arrows (i.e. \( R' = \{(e, a), (f, d)\} \)).

\[\begin{array}{c}
\Rightarrow
\end{array}\]

Figure 2: The IAF \( \mathcal{I} \)

The notion of completion in abstract argumentation was first defined in [9] for Partial AFs (i.e. IAFs with \( A' = \emptyset \)), and then adapted to IAFs. Intuitively, a completion is a classical AF which describes a situation of the world coherent with the uncertain information encoded in the IAF.

Definition 5 (Completion of an IAF). Given \( \mathcal{I} = \langle A, A', R, R' \rangle \), a completion of \( \mathcal{I} \) is \( \mathcal{F} = \langle A', R' \rangle \), such that

- \( A \subseteq A' \subseteq A \cup A' \);
- \( R_{|A'} \subseteq R' \subseteq R_{|A'} \cup R'_{|A'} \);

where \( R_{|A'} = R \cap (A' \times A') \) (and similarly for \( R'_{|A'} \)).

The set of completions of an IAF \( \mathcal{I} \) is denoted \( \text{comp}(\mathcal{I}) \).

Example 4. We consider again the IAF from Figure 2. Its set of completions is described at Figure 3.

The number of completions of an IAF \( \mathcal{I} = \langle A, A', R, R' \rangle \) is bounded by \( 2^n \), with \( n = |A'| + |R'| \). However, this upper bound may not be reached, as it is the case in the previous example. Indeed, the uncertain attack \( (f, d) \) cannot appear in completions where the uncertain argument \( f \) does not appear.
2.2 Incomplete Argumentation Frameworks

To conclude this section, let us introduce the different reasoning problems for IAFs that have been studied in the literature, as well as their complexity.\(^1\)

They are the adaptation to IAFs of three classical reasoning problems for AFs:

- Verification: given an AF, a set of arguments, and a semantics, is the set an extension of the AF under the chosen semantics?
- Credulous acceptance: given an AF, an argument, and a semantics, is the argument a member of some extension under the chosen semantics?
- Skeptical acceptance: given an AF, an argument, and a semantics, is the argument a member of each extension under the chosen semantics?

Adapting these problems to IAFs requires to take into account the set of completions. Indeed, an argument being accepted in one completion is much less demanding than being accepted in all the completions. This is why there are two variants of these problems for IAFs: the possible and the necessary variant. The definition of the possible variant quantifies existentially over the set of completions, while the necessary variant quantifies universally.

Verification for IAFs was first studied in [6]:

\[
\sigma\text{-IncPV} \quad \text{Given } I = \langle A, A^?, R, R^? \rangle \text{ an IAF and } S \subseteq A \cup A^?, \text{ is there a completion } F = \langle A', R' \rangle \text{ such that } S \cap A' \text{ is a } \sigma\text{-extension of } F? 
\]

\[
\sigma\text{-IncNV} \quad \text{Given } I = \langle A, A^?, R, R^? \rangle \text{ an IAF and } S \subseteq A \cup A^?, \text{ for each completion } F = \langle A', R' \rangle, \text{ is } S \cap A' \text{ a } \sigma\text{-extension of } F? 
\]

In [19], a set of arguments for which the answer to \(\sigma\text{-IncPV}\) (respectively \(\sigma\text{-IncNV}\)) is called a possible (respectively necessary) \(i\)-extension. The authors identify some issues with this definition (for instance, a set of arguments could be identified as an \(i\)-extension even if it is not conflict-free). To remedy this issue, they define so-called \(i^*\)-extensions, and the corresponding verification problems:

\[
\sigma\text{-IncPV}^* \quad \text{Given } I = \langle A, A^?, R, R^? \rangle \text{ an IAF and } S \subseteq A \cup A^?, \text{ is there a completion } F = \langle A', R' \rangle \text{ such that } S \text{ is a } \sigma\text{-extension of } F? 
\]

\[
\sigma\text{-IncNV}^* \quad \text{Given } I = \langle A, A^?, R, R^? \rangle \text{ an IAF and } S \subseteq A \cup A^?, \text{ for each completion } F = \langle A', R' \rangle, \text{ is } S \text{ a } \sigma\text{-extension of } F? 
\]

We refer the interested reader to [19] for a detailed discussion of the difference between \(i\)-extensions and \(i^*\)-extensions.

Finally, the (possible and necessary) variants of credulous and skeptical acceptance are studied in [4]:

\[
\sigma\text{-PCA} \quad \text{Given } I = \langle A, A^?, R, R^? \rangle \text{ an IAF and } a \in A, \text{ is there a completion } F = \langle A', R' \rangle \text{ such that } a \text{ is credulously accepted in } F \text{ under } \sigma? 
\]

\[
\sigma\text{-NCA} \quad \text{Given } I = \langle A, A^?, R, R^? \rangle \text{ an IAF and } S \subseteq A \cup A^?, \text{ for each completion } F = \langle A', R' \rangle, \text{ is } a \text{ a credulously accepted in } F \text{ under } \sigma? 
\]

\(^1\)We suppose that the reader is familiar with basic concepts of computational complexity, like (non-)deterministic polynomial algorithms, and the classes of the polynomial hierarchy: \(P, NP, coNP, \Sigma_k^P, \Pi_k^P\), where \(k \in \mathbb{N}\). Otherwise, we refer the interested reader to, e.g., [2].
Given $I = \langle A, A^c, R, R^c \rangle$ an IAF and $a \in A$, is there a completion $F = \langle A', R' \rangle$ such that $a$ is skeptically accepted in $F$ under $\sigma$?

Given $I = \langle A, A^c, R, R^c \rangle$ an IAF and $S \subseteq A \cup A^c$, for each completion $F = \langle A', R' \rangle$, is $a$ a skeptically accepted in $F$ under $\sigma$?

Now, let us give the complexity of these problems under the classical semantics.

| $\sigma$ | IncPV | IncNV | IncPV* | IncNV* | PCA | NCA | PSA | NSA |
|---|---|---|---|---|---|---|---|---|
| ad | NP-c | P | P | P | NP-c | $\Pi_2^c$ | trivial | trivial |
| st | NP-c | P | P | P | NP-c | $\Pi_2^c$ | $\Sigma_2^c$ | coNP-c |
| co | NP-c | P | P | P | NP-c | $\Pi_2^c$ | NP-c | coNP-c |
| gr | NP-c | P | P | P | NP-c | coNP-c | NP-c | coNP-c |
| pr | $\Sigma_2^c$ | coNP-c | $\Sigma_2^c$ | coNP-c | NP-c | $\Pi_2^c$ | $\Sigma_3^c$ | $\Pi_2^c$ |

Table 2: Complexity of Reasoning with IAFs for Various Problems under $\sigma \in \{\text{ad, st, co, gr, pr}\}$

### 3 Rich Incomplete Argumentation Frameworks

#### 3.1 Definition and Expressivity of RIAFs

Now, we enrich the definition of IAFs.

**Definition 6 (Rich IAF).** A **Rich Incomplete Argumentation Framework (RIAF)** is a tuple $\mathcal{R} = \langle A, A^c, R, R^c, \leftrightarrow^? \rangle$, where $A$ and $A^c$ are disjoint sets of arguments, and $R, R^c, \leftrightarrow^? \subseteq (A \cup A^c) \times (A \cup A^c)$ are disjoint sets of attacks, such that $\leftrightarrow^?$ is symmetric.

The new relation $\leftrightarrow^?$ is borrowed from Control Argumentation Frameworks [13]. It is a symmetric (uncertain) conflict relation: if $(a, b) \in \leftrightarrow^?$, then we are sure that there is a conflict between $a$ and $b$, but not of the direction of the attack. This new relation impacts the definition of completions.

**Definition 7 (Completion of a RIAF).** Given $\mathcal{R} = \langle A, A^c, R, R^c, \leftrightarrow^? \rangle$, a completion of $\mathcal{R}$ is $F = \langle A', R' \rangle$, such that

- $A \subseteq A' \subseteq A \cup A^c$;
- $R_{|A'} \subseteq R' \subseteq R \cup R^c \cup \leftrightarrow^?$;
- if $(a, b) \in \leftrightarrow^?$, then $(a, b) \in R'$ or $(b, a) \in R'$ (or both);

where $R_{|A'} = R \cap (A' \times A')$.

Again, we use $\text{comp}(\mathcal{R})$ to denote the set of completions of a RIAF $\mathcal{R}$.

**Example 5.** We present a slight modification of the IAF from Example 3, where the (certain) attack $(b, a)$ is replaced by a symmetric uncertain conflict between $a$ and $b$. The resulting RIAF is given at Figure 4.
Figure 4: The RIAF $\mathcal{R}$

Figure 5: Three Completions of $\mathcal{R}$

We do not give here the full set of completions of $\mathcal{R}$. Let us focus on one option for each of $(e, a)$, $(f, d)$ and $f$, and we only illustrate the three options for $(a, b)$. These three completions are given at Figure 5.

Similarly, for each other configuration of $(e, a)$, $(f, d)$ and $f$ (i.e. each completion at Figure 3), there are three options for the conflict between $a$ and $b$, leading to three different completions.

Observation 1. Let us notice that $\leftrightarrow^?$ is not defined as a symmetric relation in [13]. However, its meaning imposes the relation to be symmetric. Indeed, $(a, b) \in \leftrightarrow^?$ means that there is a conflict between $a$ and $b$ whose direction is uncertain. Formally, it means that (ceteris paribus) there are three completions with (respectively) $(a, b) \in \mathcal{R}'$, or $(b, a) \in \mathcal{R}'$, or both. This is obviously equivalent to “there is a conflict between $b$ and $a$ whose direction is uncertain”, i.e. $(b, a) \in \leftrightarrow^?$. A non-symmetric relation can be used as a more compact representation of its symmetric counterpart.

Now, we prove that RIAFs are strictly more expressive than IAFs. Said otherwise, it means that the new relation $\leftrightarrow^?$ cannot be equivalently represented with a combination of fixed and uncertain attacks.

Proposition 2 (Relative Expressivity of IAFs and RIAFs).

- For any IAF $\mathcal{I}$, there exists a RIAF $\mathcal{R}$ such that $\text{comp}(\mathcal{I}) = \text{comp}(\mathcal{R})$.

- There exists a RIAF $\mathcal{R}$ such that there is no IAF $\mathcal{I}$ with $\text{comp}(\mathcal{I}) = \text{comp}(\mathcal{R})$.

Proof. The first item is straightforward: any IAF is a RIAF with $\leftrightarrow^? = \emptyset$. For the second item, consider $\mathcal{R} = \langle\{a, b\}, \emptyset, \emptyset, \{(a, b), (b, a)\}\rangle$. This RIAF and its three completions are given at Figure 6.

Figure 6: A RIAF and its Completions
Now, let us prove that there is no IAF with the same set of completions. Reasoning towards a contradiction, suppose that such a IAF $I = \langle A, A', R, R' \rangle$ exists. Since all the completions have the same set of arguments $\{a, b\}$, there cannot be uncertain argument, i.e. $A' = \emptyset$.

Let us now consider the different options for $R$ and $R'$. If $(a, b) \in R$ (respectively $(b, a) \in R$), then there is an attack from $a$ to $b$ (respectively from $b$ to $a$) in every completion. This is not the case. Similarly, there cannot be any self attack in $R$ (since there is no such attack in any completion). Thus $R = \emptyset$.

In the case where only $(a, b)$ (respectively $(b, a)$) belongs to $R$, then the completions with $(b, a)$ (respectively $(a, b)$) do not belong to $\text{comp}(I)$. On the contrary, if both $(a, b)$ and $(b, a)$ belong to $R'$, then a fourth completion where there is no attack between $a$ and $b$ belongs to $\text{comp}(I)$. Of course, self-attacks in $R'$ are not possible, since they would yield addition completions (with the same self-attack appearing in them).

So we can conclude that $I$ does not exist.

3.2 Computational Issues

3.2.1 Complexity

While we have shown that RIAFs are strictly more expressive than IAFs, now we prove that this expressivity is not at the price of a complexity blow up. Let us recall that the complexity results for IAFs [6, 4, 19] are summarized at Table 2. The fact that any IAF is a RIAF with $\leftrightarrow' = \emptyset$ is enough to prove that reasoning with RIAFs is at least as hard as reasoning with IAFs. But also, we notice that the upper bounds of the complexity coincides with the upper bound for IAFs. Roughly speaking, this can be explained by the fact that non-deterministically guessing a completion is not different for RIAFs than for IAFs.

Possible Verification Let us start with $\text{IncPV}$ and $\text{IncPV}^*$. The problem for RIAFs can be solved by the non-deterministic guess of a completion, and then checking whether the queried set of arguments $S$ is a $\sigma$-extension of the given completion (for $\text{IncPV}^*$), or $S \cap A'$ where $A'$ is the set of arguments that appear in the completion (for $\text{IncPV}$). Since verification of an extension in AFs is polynomial for $\sigma \in \{\text{ad, st, co, gr}\}$ and coNP-complete for $\sigma = \text{pr}$, the results for $\text{IncPV}$ from Table 2 are valid for RIAFs, as well as the result for $\text{IncPV}^*$ under the preferred semantics. For $\text{IncPV}^*$ under the other semantics, the reasoning from [19] applies. First, if there are $(a, b), (b, a) \in \leftrightarrow'$ such that $a, b \in S$, then $S$ is not conflict-free in $R$, thus it is not an extension (for $\sigma = \text{ad, st, gr, co}$). Otherwise, for $(a, b), (b, a) \in \leftrightarrow'$ with $a \in S$ and $b \notin S$, include only $(a, b)$ in the completion built by the algorithm.

Necessary Verification Then, let us focus on $\text{IncNV}^*$, and $\sigma = \text{pr}$. A negative instance for this problem can be identified by non-deterministically guessing a completion and a superset $S'$ of the queried set of arguments $S$. Then, polynomially checking whether $S'$ is admissible allows to conclude that $S$ is not necessary a preferred $i^*$-extension. Thus $\text{IncNV}^* \in \text{coNP}$. For $\text{IncNV}$, the reasoning is the same, except that $S'$ must be a superset of $S \cap A'$ instead of a superset of $S$. 

8
For the other semantics, the reasoning from [6, 19] holds for RIAFs: IncNV and IncNV* can be solved polynomially by reducing the problem to reasoning with Argument-Incomplete AFs, or by constructing the adequate completion. If there are \((a, b), (b, a) \in \leftrightarrow\) such that \(a, b \in S\), then \(S\) is not conflict-free in \(R\), thus it is not an extension (for \(\sigma = \text{ad, st, gr, co}\)). Otherwise, for \((a, b), (b, a) \in \leftrightarrow\) with \(a \in S\) and \(b \not\in S\), include only \((a, b)\) in the Argument-Incomplete AF or completion built in the proof.

**Credulous Acceptance**  Now, we look at acceptance problems. For PCA, the problem is solved by non-deterministically guessing a completion \(C\) and a set of arguments \(S\) that contains the queried argument \(a\). Then, it can be checked polynomially whether \(S\) is a \(\sigma\)-extension, for \(\sigma \in \{\text{ad, st, co, gr}\}\). Moreover, if \(S\) is an admissible extension, then \(a\) also belongs to some preferred extension (since each admissible set is included in some preferred extension).

For NCA, let us non-deterministically guess a completion and check whether the queried argument is credulously accepted in it. This check is doable in polynomial time for \(\sigma = \text{gr}\), and with an NP oracle for the other semantics under consideration, hence the result.

**Skeptical Acceptance**  For PSA and NSA, the admissible semantics remains a trivial case: since the empty set is admissible for any AF, there is no skeptically accepted argument under ad. For PSA, we can non-deterministically guess a completion, and check whether \(a\) is skeptically accepted in it. This check is polynomial for \(\sigma \in \{\text{gr, co}\}\), in coNP for \(\sigma = \text{st}\), and in \(\Pi^p_2\) for \(\sigma = \text{pr}\), so we obtain the result. For NSA, the reasoning described in [4] still holds: checking the necessary skeptical acceptance of an argument can be represented by a universal quantification over the set of completions and the set of sets of arguments \((S)\) that do not contain the queried argument \(a\). The universal quantifiers are followed by a (deterministic) polynomial check for \(\sigma \in \{\text{st, co, gr}\}\). For \(\sigma = \text{pr}\), the universal quantifiers are followed by an existential quantifier over the supersets of \(S\), and finally a (deterministic) polynomial. Hence the results.

So we can conclude that the complexity of reasoning with RIAFs, for the various problems introduced in Section 2.2 and for \(\sigma \in \{\text{ad, st, co, gr, pr}\}\), is the same as in the case of IAFs.

**Proposition 3.** The complexity results for IAFs given at Table 2 also hold for RIAFs.

### 3.2.2 Algorithm

Now we show how to the SAT-based algorithms for reasoning with IAFs [21] to RIAFs. Roughly speaking, the encoding is made of one part that represents the structure of the IAF, i.e. the existence of (uncertain or not) arguments and attacks; and one part that maps this structure with the arguments acceptance evaluation (with respect to a chosen semantics). A slight modification to take into account \(\leftrightarrow\) is enough to reason with RIAFs instead of IAFs. Following the definition of encodings in [21], we define, for a RIAF \(R = (A, A', R, R', \leftrightarrow')\) the Boolean variables:

- \(y_a\) is true if and only if \(a \in A'\);
• \( r_{a,b} \) is true if and only if \((a, b) \in R' \);

• \( x_a \) is true if and only if \( a \in S \), for some \( S \in \sigma(\mathcal{F}) \);

where \( \mathcal{F} = \{A', R'\} \) is a completion of \( \mathcal{R} \) corresponding to the \( y_a \) and \( r_{a,b} \) variables. While \( y_a \) and \( r_{a,b} \) are necessarily true for \( a \in A \) and \((a, b) \in R \), they may be true or false for \( a \in A' \) and \((a, b) \in R' \). We also need to specify that either \( r_{a,b} \) or \( r_{b,a} \) is true if \((a, b), (b, a) \in \leftrightarrow \). Finally, an argument \( a \) must be in the completion (i.e. \( y_a \) is true) in order to appear in an extension (i.e. \( x_a \) is true) or in some attacks (i.e. \( r_{a,b} \) or \( r_{b,a} \) is true for some \( b \)). This corresponds to the following formula, for \( \mathcal{R} = \{A, A', R, R', \leftrightarrow'\} \):

\[
\phi_{cf}(\mathcal{R}) = (\bigwedge_{a \in A} y_a) \land (\bigwedge_{(a, b) \in R} r_{a,b}) \land (\bigwedge_{(a, b) \in \leftrightarrow} r_{a,b} \lor r_{b,a}) \\
\land (\bigwedge_{a \in A'} (\neg y_a \rightarrow (\neg x_a \land \bigwedge_{(a, b) \in R'} \neg r_{a,b} \land \bigwedge_{(b, a) \in R'} \neg r_{b,a})))
\]

Then, a formula encoding the semantics is given, for instance:

\[
\phi_{st}(\mathcal{R}) = \bigwedge_{(a, b) \in R \cup R' \cup \leftrightarrow} (y_a \land y_b \land r_{a,b}) \rightarrow (\neg x_a \lor \neg x_b)
\]

encodes conflict-freeness: if both arguments appear in the completion, as well as the attack between them, then they cannot be both accepted. Similarly, \( \phi_{ad} \) and \( \phi_{sta} \) are provided in [21], we show their adaptation to RIAFs:

• \( \phi_{ad}(\mathcal{R}) = \phi_{cf}(\mathcal{R}) \land \bigwedge_{a \in A \cup A'} \bigwedge_{(b, a) \in R \cup R' \cup \leftrightarrow}((x_a \land y_a \land y_b \land r_{b,a}) \rightarrow z_b), \)

• \( \phi_{sta}(\mathcal{R}) = \phi_{cf}(\mathcal{R}) \land \bigwedge_{a \in A \cup A'} (\neg x_a \land y_a \rightarrow z_a), \)

where \( z_a \) is a newly introduced variable for each argument \( a \in A \cup A' \), meaning that \( a \) is defeated. This is formally encoded by \( z_a \rightarrow \bigvee_{(b, a) \in R \cup R' \cup \leftrightarrow} (x_b \land y_b \land r_{b,a}) \).

\( \phi_{ad} \) and \( \phi_{sta} \) can be used directly for solving problems at the first level of the polynomial hierarchy, or as NP abstraction in CounterExample Guided Abstract Refinement algorithms. We refer the interested reader to [21] for more details.

4 Conclusion

This report introduces Rich Incomplete Argumentation Frameworks, that generalize IAFs by adding a new kind of uncertainty. We have shown that this model is strictly more expressive than IAFs, but not at the price of an increase of complexity. Moreover, a slight modification of existing logical encodings allows to use the algorithms described in the literature.

While complexity of reasoning with (R)IAFs has been well studied, there are still many open questions regarding this formalism. For instance, as far as we know, the only algorithms proposed (and implemented) for IAFs concern the acceptance problems, as mentioned in the Section 3.2.2. Other problems mentioned in Section 2.2 have not been tackled yet. Also, this study only considers the initial semantics defined by Dung, but other semantics have not received interest in the context of (R)IAFs, e.g. semi-stable [8], stage [22] or ideal [17] semantics.

Several works about the revision [10, 11], the update [14] or the merging [9, 12]
of AFs have faced the difficulty to represent the uncertainty of the result as a single AF, and chose to return a set of AFs as output. Let us recall that Partial AFs (that are a special case of RIAFs with \( A' = \emptyset \) and \( \leftrightarrow？ \)) were defined as a part of the process for computing the merging of AFs [9], but did not appear in the result of the operation. RIAFs might provide an interesting solution in order to have a more compact output for these operations. Related to these questions, the issue of realizability of extension sets [18] in the context of (R)IAFs is also interesting.

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