DYNAMICS OF THE MAPPING CLASS GROUP ON THE MODULI OF A PUNCTURED SPHERE WITH RATIONAL HOLONY

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Abstract. Let $M$ be a four-holed sphere and $\Gamma$ the mapping class group of $M$ fixing the boundary $\partial M$. The group $\Gamma$ acts on $\mathcal{M}_B(\text{SL}(2, \mathbb{C})) = \text{Hom}^+_B(\pi_1(M), \text{SL}(2, \mathbb{C}))/\text{SL}(2, \mathbb{C})$ which is the space of completely reducible $\text{SL}(2, \mathbb{C})$-gauge equivalence classes of flat $\text{SL}(2, \mathbb{C})$-connections on $M$ with fixed holonomy $B$ on $\partial M$. Let $B \in (-2, 2)^4$ and $\mathcal{M}_B$ be the compact component of the real points of $\mathcal{M}_B(\text{SL}(2, \mathbb{C}))$. These points correspond to $\text{SU}(2)$-representations or $\text{SL}(2, \mathbb{R})$-representations. The $\Gamma$-action preserves $\mathcal{M}_B$ and we study the topological dynamics of the $\Gamma$-action on $\mathcal{M}_B$ and show that for a dense set of holonomy $B \in (-2, 2)^4$, the $\Gamma$-orbits are dense in $\mathcal{M}_B$. We also produce a class of representations $\rho \in \text{Hom}^+_B(\pi_1(M), \text{SL}(2, \mathbb{R}))$ such that the $\Gamma$-orbit of $[\rho]$ is finite in the compact component of $\mathcal{M}_B(\text{SL}(2, \mathbb{R}))$, but $\rho(\pi_1(M))$ is dense in $\text{SL}(2, \mathbb{R})$.

1. Introduction

Let $M$ be a 4-holed sphere and $\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\} \subset \pi_1(M)$ be the elements in the fundamental group corresponding to the four boundary components. Denote by $\text{Hom}^+_B(\pi_1(M), \text{SL}(2, \mathbb{C}))$ the space of completely reducible $\text{SL}(2, \mathbb{C})$-representations. Let $G$ be a semi-simple subgroup of $\text{SL}(2, \mathbb{C})$. Assign each $\gamma_i$ a conjugacy class in $G$: For each $B = (a, b, c, d) \in \mathbb{C}^4$, let

$$\mathcal{H}_B(G) = \{ \rho \in \text{Hom}^+_B(\pi_1(M), G) \subset \text{Hom}^+_B(\pi_1(M), \text{SL}(2, \mathbb{R})) : (\text{tr}(\rho(\gamma_1)), \text{tr}(\rho(\gamma_2)), \text{tr}(\rho(\gamma_3)), \text{tr}(\rho(\gamma_4))) = B \},$$

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where $\text{Hom}_B^+(\pi_1(M), G) \subset \text{Hom}_B^+(\pi_1(M), \text{SL}(2, \mathbb{C}))$ is the subspace of completely reducible $G$-representations. We restrict $B$ to be in $(-2, 2)^4$.

The group $G$ acts on $\mathcal{H}_B$ by conjugation.

**Definition 1.1.** The moduli space with fixed holonomy $B$ is

$$\mathcal{M}_B(G) = \mathcal{H}_B(G)/G.$$ 

Let $\mathcal{M}_B$ be either $\mathcal{M}_B(\text{SU}(2))$ or $\mathcal{M}_B(\text{SL}(2, \mathbb{R}))^c$, the compact component of $\mathcal{M}_B(\text{SL}(2, \mathbb{R}))$, depending on $B$ (see [1]).

Let $\text{Diff}(M, \partial M)$ be the group of diffeomorphisms of $M$ fixing $\partial M$.

The group $\Gamma$ acts on $\pi_1(M)$ naturally and the action induces an action:

$$\Gamma \times \mathcal{H}_B(G) \rightarrow \mathcal{H}_B(G)$$

with $\gamma(\rho)(X) = \rho(\gamma^{-1}(X))$. This, in turn, gives an action $\Gamma \times \mathcal{M}_B \rightarrow \mathcal{M}_B$.

The space $\mathcal{M}_B$ possesses a natural symplectic structure which gives rise to a finite measure $\mu$ on $\mathcal{M}_B$. Goldman first showed that $\Gamma$ acts ergodically on $\mathcal{M}_B(\text{SU}(2))$ (see [3, 4]). The same technique of [4] immediately gives ergodicity of the $\Gamma$-action on $\mathcal{M}_B$.

The space $\mathcal{M}_B$ has a natural topology and one can study topological dynamics of the $\Gamma$-action [5, 6, 7]. In this paper, we show that for a dense subset of boundary holonomies, the $\Gamma$-action is minimal:

**Theorem 1.2.** Let $M$ be a four-holed sphere. Suppose $B = (a, b, c, d) \in (-2, 2)^4$ such that two of $(ab + cd), (ac + bd), (ad + bc)$ are in the set $\mathbb{Q} \setminus \{i \in \mathbb{Z} : -8 \leq i \leq 8\}$. Then every $\Gamma$-orbit is dense in $\mathcal{M}_B$.

**Corollary 1.3.** There exists a dense subset $D \subset [-2, 2]^4$ such that $B \in D$ implies that the $\Gamma$-action is minimal in $\mathcal{M}_B$.

Suppose $G \subset \text{SU}(2)$ (resp. $G \subset \text{SL}(2, \mathbb{R})$) is a closed proper subgroup and $\rho \in \mathcal{H}_B(G)$. Then the $\Gamma$-orbit of $[\rho]$ is in $\mathcal{H}_B(G)$. Hence the $\Gamma$-action is not, in general, minimal. However, if $M$ is a surface of positive genus and if the image of $\rho \in \mathcal{H}_B(\text{SU}(2))$ is dense in $\text{SU}(2)$ then the $\Gamma$-orbit of $[\rho]$ is dense in $\mathcal{M}_B(\text{SU}(2))$ [6]. This is no longer true without the $g > 0$ hypothesis as examples in [7] illustrate. In this paper, we also produce similar examples of $\rho \in \mathcal{H}_B(\text{SL}(2, \mathbb{R}))$ having dense images in $\text{SL}(2, \mathbb{R})$ but with discrete $\Gamma$-orbits in $\mathcal{M}_B(\text{SL}(2, \mathbb{R}))^c$.

We adopt the following notational conventions: For a fixed $\rho \in \mathcal{H}_B$ and $X \in \pi_1(M)$, we write $X$ for $\rho(X)$ when there is no ambiguity. A small letter denotes the trace of the matrix represented by the corresponding capital letter.

2. The $\Gamma$-Action on Moduli Spaces

We first review some results that appear in [1] and [3]. Suppose $M$ is a four-holed sphere. Then the fundamental group $\pi_1(M)$ admits a
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presentation

\( \langle A, B, C, D : ABCD = I \rangle \).

Let \( X = AB, Y = BC \) and \( Z = CA \). For a fixed holonomy (trace) \( B \in (-2, 2)^4 \) on the four punctures and with the above coordinates, the moduli space satisfies

\[
x^2 + y^2 + z^2 + xyz = (ab + cd)x + (ad + bc)y + (ac + bd)z - (a^2 + b^2 + c^2 + d^2 + abcd - 4).
\]

The \( x \)-level sets \( X(x) \subset \mathcal{M}_B \) satisfy:

\[
\frac{2 + x}{4} \left[ (y + z) - \frac{(a + b)(d + c)}{2 + x} \right]^2 + \frac{2 - x}{4} \left[ (y - z) - \frac{(a - b)(d - c)}{2 - x} \right]^2 = \frac{(x^2 - abx + a^2 + b^2 - 4)(x^2 - cdx + c^2 + d^2 - 4)}{4 - x^2}.
\]

There are similar descriptions for the \( y \)- and \( z \)-level sets \( Y(y) \) and \( Z(z) \) respectively (see [3]).

Let

\[
I^-_{a,b} = \frac{ab - \sqrt{(a^2 - 4)(b^2 - 4)}}{2},
\]

\[
I^+_{a,b} = \frac{ab + \sqrt{(a^2 - 4)(b^2 - 4)}}{2},
\]

(similarly for \( I^-_{a,c}, I^+_{b,c} \), etc.).

The moduli space \( \mathcal{M}_B \) is \( \mathcal{M}_\mathcal{B}(SU(2)) \) if for some \((x, y, z) \in \mathcal{M}_B\)

\[
x^2 - abx + a^2 + b^2 - 4 < 0,
\]

\[
x^2 - cdx + c^2 + d^2 - 4 < 0.
\]

These two inequalities imply that \( I^-_{a,b} < I^+_{c,d} \) (or \( I^-_{c,d} < I^+_{a,b} \)). Moreover, for \( x \in S = (I^-_{a,b}, I^+_{c,d}) \) (resp. in \((I^+_{c,d}, I^-_{a,b})\)), the level set \( X(x) \) is an ellipse.

The moduli space \( \mathcal{M}_B \) is \( \mathcal{M}_\mathcal{B}(SL(2, \mathbb{R}))^c \) if for some \((x, y, z) \in \mathcal{M}_B\),

\[
x^2 - abx + a^2 + b^2 - 4 > 0,
\]

\[
x^2 - cdx + c^2 + d^2 - 4 > 0.
\]

In this case, \( I^+_{a,b} < I^-_{c,d} \) (or \( I^+_{c,d} < I^-_{a,b} \)). Moreover, for \( x \in S = (I^+_{a,b}, I^-_{c,d}) \) (resp. in \((I^-_{c,d}, I^+_{a,b})\)), the level set \( X(x) \) is an ellipse.

For fixed \( B = (a, b, c, d) \), the \( x \)-coordinates in \( \mathcal{M}_B \) take on all the values inside \( S \) (see [1]). In particular,

\[
\mathcal{M}_B = \bigcup_{x \in \text{closure}(S)} X(x).
\]

By symmetry, there exist similar constructions for the \( y \)- and \( z \)-coordinates.
2.1. The mapping class group action. The mapping class group \( \Gamma \) is generated by the maps \( \tau_X \) and \( \tau_Y \) induced by the Dehn twists in \( X, Y \in \pi_1(M) \). In local coordinates, the actions of \( \tau_X, \tau_Y, \tau_Z \) are
\[
\begin{bmatrix} y \\ z \end{bmatrix} \tau_X \rightarrow \begin{bmatrix} ad + bc - x(ac + bd - xy - z) - y \\ ac + bd - xy - z \end{bmatrix},
\begin{bmatrix} z \\ x \end{bmatrix} \tau_Y \rightarrow \begin{bmatrix} bd + ca - y(ba + cd - yz - x) - z \\ ba + cd - yz - x \end{bmatrix},
\begin{bmatrix} x \\ y \end{bmatrix} \tau_Z \rightarrow \begin{bmatrix} cd + ab - z(cb + ad - zx - y) - x \\ cb + ad - zx - y \end{bmatrix}.
\]
These three actions preserve the ellipses \( X(x) \subset M_B, Y(y) \subset M_B \), and \( Z(z) \subset M_B \), respectively. After coordinate transformations, these are rotations by angles \( 2 \cos^{-1}(x/2) \), \( 2 \cos^{-1}(y/2) \), and \( 2 \cos^{-1}(z/2) \), respectively [3].

3. The Irrational Rotations and Infinite Orbits

The Dehn twist \( \tau_Y \) acts on the (transformed) subsets \( Y(y) \) via a rotation of angle \( 2 \cos^{-1}(y/2) \). Thus there is a filtration of the \( y \)-coordinates that yields finite orbits under \( \tau_Y \) [6]. Let \( Y_n \subset (-2, 2) \) such that \( y \in Y_n \) if and only if the \( \tau_Y \)-action on non-fixed points \( (x, y, z) \in M_B \) is periodic with period less than or equal to \( n \). This gives a filtration
\[
\{0\} = Y_2 \subset Y_3 = \{0, \pm 1\} \subset Y_4 = \{0, \pm 1, \pm \sqrt{2}\}
\subset Y_5 = \{0, \pm 1, \pm \sqrt{2}, \pm \frac{1 + \sqrt{5}}{2}\}
\subset Y_6 = \{0, \pm 1, \pm \sqrt{2}, \pm \frac{1 + \sqrt{5}}{2}, \pm \sqrt{3}\} \subset ... \subset Y_n \subset ...
\]
By symmetry, there are similar filtrations \( X_n \) and \( Z_n \), with \( X_n = Y_n = Z_n \) as sets.

The global coordinates provide an embedding of \( M_B \) in \( \mathbb{R}^3 \). We consider the box metric
\[
D((x_1, y_1, z_1), (x_2, y_2, z_2)) = \max(|x_1 - x_2|, |y_1 - y_2|, |z_1 - z_2|),
\]
which generates the usual topology on \( M_B \).

**Definition 3.1.** For \( \epsilon > 0 \), a set \( U \) is \( \epsilon \)-dense in \( V \) if for each \( p \in V \subset M_B \), there exists a point \( q \in U \) such that \( 0 < D(p, q) < \epsilon \).

**Lemma 3.2.** For \( \epsilon > 0 \) there exists \( N(\epsilon) > 0 \) so that if \( y \notin Y_{N(\epsilon)} \), then the \( \tau_Y \)-orbit of \( (x, y, z) \) is \( \epsilon \)-dense in \( Y(y) \) for any \( (x, y, z) \) in any \( M_B \).
Proof. Since the non-degenerate ellipses $Y(y)$ have uniformly bounded circumferences, there exists $N(\epsilon) > 1$ such that for any $y \notin Y_{N(\epsilon)}$, the $\tau_Y$-orbit is $\epsilon$-dense in $Y(y)$. \hfill $\square$

Effectively, for any fixed $\epsilon$, there is a finite number of $y$-coordinates (resp. $x$) whose $\tau_Y$ (resp. $\tau_X$) actions are not $\epsilon$-dense in $Y(y)$ (resp. $X(x)$).

**Theorem 3.3.** If the $\Gamma$-orbit of $u_0 = (x_0, y_0, z_0)$ is infinite, then it is dense in $M_B$.

Proof. Let $u' = (x', y', z') \in M_B$ and $\epsilon > 0$. By compactness, the $\Gamma$-orbit of $u_0$ has an accumulation point $u_* = (x_*, y_*, z_*)$. Let $B_\epsilon(u_*)$ be the ball of radius $\epsilon > 0$ centered at $u_*$, with respect to the box metric. Then either the set of $x$-coordinates or the set of $y$-coordinates of the points in $\Gamma(u_0) \cap B_\epsilon(u_*)$ are infinite. Since $\tau_X$ and $\tau_Y$ are rotations by angles $2 \cos^{-1}(x/2)$ and $2 \cos^{-1}(y/2)$, by Lemma $3.2$, $\Gamma(u_0) \cap B_\epsilon(u_*)$ contains an infinite subset whose points have their $x$-coordinates and $y$-coordinates distinguished from each other.

Recall that $X(x)$ (resp., $Y(y)$) is the $x$–cross section (resp. $y$–cross section) of $M_B$, which is topologically a circle or a point. Let

$$X_\epsilon(x) = \bigcup_{|r| < \epsilon} X(x + r).$$

By compactness, there is a finite chain of sets $S_0 = X_\epsilon(x_*)$, $S_1 = Y_\epsilon(y_1)$, $S_2 = X_\epsilon(x_2)$, ..., $S_n = Y_\epsilon(y')$ with $S_i \cap S_{i+1} \neq \emptyset$.

As $\Gamma(u_0)$ contains an infinite number of points in $S_0$ with distinct $x$–coordinates, there is (by using $\tau_X$) an infinite number of points in $\Gamma(u_0) \cap S_0 \cap S_1$ having distinct $y$–coordinates. Continuing in this fashion, one generates an infinite number of points inside $\Gamma(u_0) \cap S_i \cap S_{i+1}$ which leads to an infinite number of points inside $\Gamma(u_0) \cap B_\epsilon(u')$. \hfill $\square$

### 4. Minimality

By Theorem $3.3$, the problem amounts to ensuring that some Dehn twist corresponds to an irrational rotation along a circle in $M_B$. For a given representation $\rho$, we use subscript notation to denote the actions on the coordinates $x, y, z$ of $[\rho]$. For instance, $x_y$ is short for the $x$-coordinate of $\tau_Y([\rho])$. Since $x_y = ab + cd - yz -x$, for the orbit to be finite, $2 \cos^{-1}(\frac{x}{2})$ and $2 \cos^{-1}(\frac{y}{2})$ must also be rational multiples of $\pi$. Hence

$$\cos(\theta_{x_y}) + 2 \cos(\theta_x) \cos(\theta_y) + \cos(\theta_z) = \frac{ab + cd}{2},$$
or equivalently,

\[
\cos(\theta_{x'y'}) + \cos(\theta_z + \theta_y) + \cos(\theta_z - \theta_y) + \cos(\theta_x) = \frac{ab + cd}{2},
\]

where all angles are rational multiples of \(\pi\), \(0 \leq \theta_z + \theta_y \leq 2\pi\), and 
\(-\pi \leq \theta_z - \theta_y \leq \pi\). We also obtain two more equations from the \(\tau_X\) and 
\(\tau_Z\) actions.

Equation (1) is a trigonometric Diophantine equation, the solutions 
to which are few as shown by Conway and Jones:

**Theorem 4.1** (Conway, Jones). Suppose that we have at most 
four distinct rational multiples of \(\pi\) lying strictly between 0 and \(\pi/2\) 
for which some linear combination of their cosines is rational, but no 
proper subset has this property. That is,

\[
A \cos(a) + B \cos(b) + C \cos(c) + D \cos(d) = E,
\]

for \(A, B, C, D, E\) rational and \(a, b, c, d \in (0, \pi/2)\) rational multiples of 
\(\pi\). Then the appropriate linear combination is proportional to one from 
the following list:

\[
\begin{align*}
\cos(\pi/3) & = 1/2 \\
\cos(t + \pi/3) + \cos(\pi/3 - t) - \cos(t) & = 0 \ (0 < t < \pi/6) \\
\cos(\pi/5) - \cos(2\pi/5) & = 1/2 \\
\cos(\pi/7) - \cos(2\pi/7) + \cos(3\pi/7) & = 1/2 \\
\cos(\pi/5) - \cos(\pi/15) + \cos(4\pi/15) & = 1/2 \\
- \cos(2\pi/5) + \cos(2\pi/15) - \cos(7\pi/15) & = 1/2 \\
\cos(\pi/7) + \cos(3\pi/7) - \cos(\pi/21) + \cos(8\pi/21) & = 1/2 \\
\cos(\pi/7) - \cos(2\pi/7) + \cos(2\pi/21) - \cos(5\pi/21) & = 1/2 \\
- \cos(2\pi/7) + \cos(3\pi/7) + \cos(4\pi/21) + \cos(10\pi/21) & = 1/2 \\
- \cos(\pi/15) + \cos(2\pi/15) + \cos(4\pi/15) - \cos(7\pi/15) & = 1/2.
\end{align*}
\]

The angles in equation (1) are not necessarily in \((0, \pi/2)\). By applying 
the identities \(\cos(\pi/2 - t) = -\cos(\pi/2 + t)\) and \(\cos(\pi - t) = 
\cos(\pi + t)\), we derive from equation (1) a new four-term cosine equation 
whose arguments are in \([0, \pi/2]\). That is, by a possible change of sign, 
each term in equation (1) may be rewritten with angles in \([0, \pi/2]\). Notice 
that if the resulting equation has non-distinct angles or an angle 
being 0 or \(\pi/2\), then we will obtain a rational trigonometric Diophantine 
equation of shorter length.
Proposition 4.2. Suppose two of \( ab + cd, ac + bd, ad + bc \in \mathbb{Q} \setminus \{ i \in \mathbb{Z} : -8 \leq i \leq 8 \} \). Then Equation (1) has no rational (in the sense of Conway-Jones) solution.

Proof. Let \((x, y, z) \in \mathcal{M}_B\). Since \( B \in (-2, 2)^4 \), \( \mathcal{M}_B \) is a smooth 2-sphere. Hence two of the level sets \( X(x), Y(y), Z(z) \) are circles. We assume that \( Y(y) \) is a circle and \( ac + bd \in \mathbb{Q} \setminus \{ i \in \mathbb{Z} : -8 \leq i \leq 8 \} \). Then a finite \( \tau_Y \)-orbit on \( Y(y) \) must yield a rational solution (in the sense of Conway-Jones) to Equation (1). However, with the given constraint on \( \frac{ab+cd}{2} \), Theorem 4.1 implies that Equation (1) has no solution. The other two cases are similar. \( \square \)

Theorem 1.2 follows immediately from Proposition 4.2.

5. Exceptional discrete orbits

In this section, we construct representations \( \rho \in \mathcal{H}_B(\text{SL}(2, \mathbb{R})) \) such that \( \rho(\pi_1(M)) \) is dense in \( \text{SL}(2, \mathbb{R}) \), but the \( \Gamma \)-orbit of \([\rho] \) is discrete in \( \mathcal{M}_B(\text{SL}(2, \mathbb{R}))^c \). The construction closely parallels the construction in [7].

Let \( F \) be the set of \( B = (a, a, c, -c) \in (-2, 2)^4 \) satisfying the following conditions:

1. \( a^2 + c^2 > 4 \),
2. \( \cos^{-1}(a) \pi \) or \( \cos^{-1}(c) \pi \) is irrational.

Consider the space \( \mathcal{M}_B \) with \( B \in F \). The orbit

\( \mathcal{O} = \{(a^2 - 2, 0, 0), (2 - c^2, 0, 0)\} \subset \mathcal{M}_B \)

is \( \Gamma \)-invariant. Let \( \rho \in \mathcal{H}_B(\text{SL}(2, \mathbb{R})) \) with \([\rho]\) having the coordinate \((a^2 - 2, 0, 0)\) or \((2 - c^2, 0, 0)\). Then condition (1) guarantees that \( \rho(\pi_1(M)) \) is not abelian since it has a non-trivial \( \Gamma \)-action. Condition (2) implies that \( \rho(\pi_1(M)) \) is dense in \( S^1 \subset \text{SL}(2, \mathbb{R}) \). Since \(-2 < a, c < 2\), \( \rho(\pi_1(M)) \) is not contained in the group of affine transformations of the real line. Hence \( \rho(\pi_1(M)) \) is dense in \( \text{SL}(2, \mathbb{R}) \).

For a concrete example of one such case, let \( B = (1, 1, \frac{2}{5}, -\frac{2}{5}) \). The special orbit \( \mathcal{O} \) consists of the two points that are intersections of the \( x \)-axis with \( \mathcal{M}_B \), i.e. \( \mathcal{O} = \{(-1, 0, 0), (-\frac{17}{13}, 0, 0)\} \). Below is a representation in the conjugacy class \((-1, 0, 0) \in \mathcal{O} \subset \mathcal{M}_B\):

\[
A = B = \begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{7}{5} & \frac{1}{5} \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & -\frac{1}{4} \\ 1 & \frac{3}{4} \end{bmatrix}.
\]
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