Strong magnetic field $B$ applied to a two-dimensional electron gas (2DEG) rearranges its single-particle density of states to a series of discrete Landau levels (LL$_m$). When the cyclotron gap $h\omega_c \propto B$ exceeds Coulomb energy $\epsilon^2/\lambda \propto \sqrt{B}$ ($\lambda = \sqrt{\hbar c/eB}$ being the magnetic length), the low-energy dynamics depends on interactions in one, partially filled LL. Despite reminiscence to atomic physics, macroscopic degeneracy and a distinct scattering matrix lead to very different, fascinating behavior[1].

Fractional quantum Hall effect $\nu$ reveals plethora of highly correlated electron phases at various LL filling factors $\nu_e = 2\pi \rho \lambda^2$ ($\rho$ being the 2D concentration). Among them are Laughlin[3] and Jain[4] incompressible liquids (IQLs) with fractionally charged quasiparticles (QPs) at $\nu_e = \frac{1}{3}$ or $\frac{2}{5}$. Wigner crystals[5] at $\nu_e < 1$, and stripes[6] in high LLs. Besides transport[2], they are probed by shot-noise (allowing detection of fractional charge of the QPs[7]) and optics (with discontinuities in photoluminescence energy related to the QP interactions[8]).

A key concept in understanding IQLs is Jain’s composite fermion (CF) picture[4]. The CFs are fictitious particles, electrons that captured part of the external magnetic field $B$ in form of infinitesimal tubes carrying an even number $2p$ of flux quanta $\phi_0 = h c/e$. The most prominent IQLs at $\nu_e = n(2p\pm1)^{-1}$ are represented by the completely filled $s$ lowest LLs of the CFs (CF-LL$_n$ with $n < s$) in a residual magnetic field $B^* = B - 2p\phi_0\rho$.

Not all IQLs are so easily explained by the CF model, e.g., Haldane–Rezayi[9] and Moore–Read[10] paired liquids proposed for $\nu_e = \frac{2}{5}$. Because of nonabelian statistics of its quasiholes (QHs), especially the latter state has recently stirred interest as a candidate for quantum computation in a solid-state environment[11].

Another family of IQLs discovered by Pan et al. [12] at $\nu_e = \frac{4}{11}$, $\frac{3}{7}$, and $\frac{5}{11}$ corresponds to fractional CF fillings $\nu_{CF} = \nu_e(1-2p\nu_e)^{-1} = \frac{4}{11}, \frac{3}{7}, \frac{5}{11}$ (with $p = 1$). Assuming spin polarization, all these states have a partially filled CF-LL$_4$. Their incompressibility results from residual CF–CF interactions. Familiar values of $\nu_{CF}$ suggested similarity between partially filled electron and CF LLs[13]. For $\nu_e = \frac{4}{11}$ and $\frac{5}{11}$ it revived the “QP hierarchy”[14], whose CF formulation consists of the CF → electron mapping followed by reapplication of the CF picture in CF-LL$_4$, leading to a “second generation” of CFs[13]. However, this idea ignored the requirement of a strong short-range repulsion[16,17]. Indeed, it was later excluded in exact diagonalization studies[18], in which a different series of finite-size $\nu_e = \frac{4}{11}$ liquids with larger gaps was identified. On the other hand, Moore–Read liquid of paired CFs was tested[19] for $\nu_e = \frac{3}{7}$, but it was eventually ruled out in favor of the stripe order[20,21].

In this Letter, we study two- and three-body correlations in several IQLs whose origin of incompressibility remains controversial. We find evidence for CF pairing in the $\nu_e = \frac{4}{11}$ liquid, hence interpreted as a condensate of (nonabelian) QHs of the “second generation” Moore–Read state of the CFs. The pair–pair or QH–QH correlations are not defined, but a Laughlin form is excluded. In Fig. 1(a, b), charge-density distributions of electrons are compared with three different CF quasiparticles at $\nu_e = \frac{4}{11}$. Laughlin liquid is a filled spin-polarized CF-LL$_4$, and its quasilectron (QE), quasihole (QH), and reversed-spin quasilectron (QE$_R$) correspond to a particle in CF-LL$_1$, a vacancy in CF-LL$_0$, and a spin-flip particle in CF-
LL₀, respectively. Particles/holes in CF-LL₀ resemble those in LL₀. However, the ring structure in CF-LL₁ makes the QEs different from the electrons and causes strong reduction of the QE–QE repulsion at short range [cf. Fig. 1(c)]. Such interaction cannot produce a Laughlin IQL of the QEs at the ν = 1 filling of CF-LL₁. Instead, other QE–QE correlations must be considered.

Spontaneous QE cluster formation would be somewhat analogous to the self-assembled growth of strained quantum dots [22]. A full CF-LL₀ representing the uniform-density Laughlin liquid plays the role of a “wetting layer.” Over this background, in analogy to atoms grouping into dots to minimize the elastic energy, QEs moving within CF-LL₁ arrange themselves into pairs or larger QE clusters easily pinned down by disorder. While in electronic “artificial atoms” the self-organization of real atoms serves a purpose of external confinement for the electrons, in their CF analogs both these roles are played by the QEs. Another distinction is the fractional charge electrons and composite fermions in different Landau levels.

Laughlin, Jain, or Moore–Read states (of QEs). The average QE–QE interaction energies (per particle) in these states overestimates the actual QE eigenenergies by at least 0.003 e²/λ (6–7%). Clearly, the microscopic origin of the observed QE incompressibility must be different.

What are these known correlations, excluded for QEs? Laughlin correlations result from strong short-range repulsion (such as between electrons in LL₁). They consist of the maximum avoidance of pairs states with the smallest R. E.g., Laughlin ν = 5/2 state is the zero-energy ground state of a model pseudopotential V = δ₁/₂₁. For more realistic interactions, the exact criterion is that V must rise faster than linearly when R decreases [17]. A linear decrease of V between R = 1 and 5 (such as in LL₁) leads to different correlations. E.g., Moore–Read ν = 1/2 liquid involves pairing and Laughlin correlations among pairs. It is the zero-energy ground state of a model three-body pseudopotential V = δ₁/₂₃ (T = 3l − L ≥ 3 is the relative triplet angular momentum, proportional to the area spanned by three particles) [10].

Weak QE–QE repulsion at R = 1 compared to R = 3 could force QEs into even larger clusters. As a simple classical analogy, consider a string of point particles, one per unit length, with a repulsive potential vₛ(r) = a + (1 − a)r for r < 1 and 1/r² otherwise. Equal spacing is favored for a > 1.64, and transitions to pairs, triplets, and larger clusters occur for decreasing a. A similar rearrangement might occur when going from LL₀ to LL₁ and CF-LL₁, with V(1) playing the role of vₛ(0) ≡ a.

In Fig. 2 we plot two leading “Haldane amplitudes” [24], G(1) and G(3). The discrete pair-correlation function G(R) is proportional to the number of pairs with a given R and normalized to 1. It connects many-body interaction energy with a pseudopotential, E = (ℏ²/2) ∑₉ G(R) V(R). Here, G is calculated in the ground states of N = 12 particles at 2l = 21 and 29 (corresponding to ν = 1/2 and 1/3 for the QEs) [18] with

FIG. 2: (color online). Haldane pair amplitudes G (~ number of pairs) at relative pair angular momenta R = 1 and 3, of N = 12 fermions in angular momentum shells with 2l = 21 (a) and 2l = 29 (b), as a function of parameter α of the interaction pseudopotential shown in (c). (d) Amplitudes G(R) of electrons and composite fermions in different Landau levels.
model interaction shown in the inset: \( V_\alpha(1) = \alpha \) and \( V_\alpha(R > 1) = 1/R^2 \). At \( \alpha > 0.3 \), \( G(1) \) takes on the minimum possible value, which means Laughlin correlations (no clusters). At \( \alpha < -0.25 \), \( G(1) \) reaches maximum, and the particles form one big quantum Hall droplet (QHD). The transition between the two limits occurs quasi-discontinuously through a series of well-defined states seen as plateaus in \( G(\alpha) \).

The cluster size \( K \) cannot be assigned to each state because the number of plateaus depends on the choice of \( V_\alpha \). The comparison of \( G(1) \) with the values predicted for \( N/K \) independent QHDs of size \( K = 2, 3, \) and \( 4 \) is not convincing because in a few-cluster system each QHD is relaxed by the cluster–cluster interaction, lowering \( G(1) \). Another problem is the contribution to \( G(1) \) from pairs of particles belonging to different clusters. Nevertheless, it is clear that the “degree of clustering” changes as a function of \( \alpha \) in a quantized fashion, supporting the picture of \( N \) particles grouping into various clustered configurations. Furthermore, the values of \( \alpha \) for which \( V_\alpha \) reproduces the exact ground states of QEs or electrons belong to different continuity regions, confirming different correlations in LL\( N_0 \), LL\( 1 \), and CF-LL\( 1 \) (except for a possible similarity of the \( \nu = 1/3 \) states in LL\( 1 \) and CF-LL\( 1 \)).

In Fig. 3(a) we compare \( N_2 = \binom{N}{2} G(1) \), the number of pairs with \( R = 1 \), calculated in the ground states of \( N = 12 \) CFs and electrons as a function 2\( l \). The downhill cusps in \( N_2(2l) \) at a series of Laughlin/Jain states in LL\( 0 \) are well understood. We also marked 2\( \nu = 2N - 3 \) and 3\( N - 7 \) corresponding to incompressible \( \nu = 1/2 \) and \( 7/3 \) ground states in LL\( 1 \) and CF-LL\( 1 \) [18], and their particle-hole conjugates \((N \rightarrow g - N)\) at 2\( \nu = 2N + 1 \) and \( 3/2 N + 2 \).

The comparison of \( N_2 \) tells about short-range pair correlations in different LLs. There are significantly more pairs in CF-LL\( 1 \) and in excited electron LLs than in LL\( 0 \). In LL\( 1 \), the Moore–Read state is known to be paired, and indeed \( N_2 \approx 1/2 N \) at \( \nu = 1/2 \). A similar value is obtained for the (not well understood) \( \nu = 1/3 \) state at \( 2l = 29 \). The CF-LL\( 1 \) is different (in terms of \( N_2 \)) from LL\( 0 \) or LL\( 1 \) in the whole range of \( 18 \leq 2l \leq 33 \). However, it appears similar to LL\( 2 \) at both \( 2l \leq 23 \) and \( 2l \geq 29 \). Also, LL\( 2 \) and LL\( 3 \) look alike for \( 23 \leq 2l < 29 \). While convincing assignment of \( \nu \) to a finite state \((N, 2l)\) requires studying size dependence (we looked at different \( N \leq 12 \)), notice that \( N/g = 1/7 \) at \( 2l = 23 \), and \( 2l = 29 \) is the \( \nu = 1/3 \) state in LL\( 1 \) and CF-LL\( 1 \). Note also that similar short-range correlations do not guarantee high wavefunction overlaps. Here, only (LL\( 2 \)[LL\( 3 \)]\(^2 \)) reaches 0.67 while all other overlaps, including (QE[LL\( n \)]\(^2 \), essentially vanish.

In Fig. 3(b) we plot \( N_3 \), the number of “compact” triplets with \( T = 3 \). It is proportional to the first triplet Haldane amplitude and tells about short-range three-body correlations. In both LL\( 0 \) and LL\( 1 \), \( N_3 \) decreases roughly linearly as a function of \( 2l \) and drops to essentially zero at \( 2l = 21 \), the smallest value at which the \( T = 3 \) triplets can be completely avoided. Exactly \( N_3 = 0 \) would indicate the Moore–Read state, but its accuracy for the actual \( \nu = 1/3 \) ground state in LL\( 1 \) depends sensitively on the quasi-2D layer width and on the surface curvature. Nevertheless, clusters larger than pairs clearly do not form in either LL\( 0 \) nor LL\( 1 \) at \( \nu \leq 1/3 \).

The number of QE triplets in CF-LL\( 1 \) is also a nearly linear function of \( 2l \), but it drops to zero at \( 2l = 3N - 7 = 29 \), earlier identified with \( \nu = 1/3 \) in this shell (i.e. with \( \nu_e = 1/7 \) [18]). In connection with having \( N_3 \approx 1/3 N \) pairs, vanishing of \( N_3 \) is the evidence for QE pairing at \( \nu_e = 1/7 \).

The elementary excitations that appear in the paired \( \nu = 1/3 \) Moore–Read state when \( 2l > 2N - 3 \) are the \( 1/7 \)-charged QHs (of the Laughlin liquid of pairs) and pair-breaking neutral-fermion excitations [10]. Being paired, the QE state at \( 2l = 3N - 7 \) can only contain the QHs but no pair-breakers. The interaction of Moore–Read QHs in CF-LL\( 1 \) is not known, but evidently it causes their condensation into an incompressible liquid at \( \nu = 1/3 \).

The “second generation” (to distinguish from \( \nu_e = 1/7 \) Moore–Read state of QEs) would occur at \( \nu = 1/3 \) in CF-LL\( 1 \) (i.e., at \( \nu_{CF} = 5/2 \) or \( \nu_e = 5/7 \)). Its instability [21] does not preclude reentrance with additional QHs at a lower \( \nu \) and, in particular, their condensation at \( \nu = 1/4 \) (i.e., at \( \nu_{CF} = 2/3 \) or \( \nu_e = 7/11 \)). A similar situation occurs with Jain \( \nu = 3/7 \) state, obtained (in Haldane hierarchy) from Laughlin \( \nu = 1/3 \) state by addition of “second generation” Laughlin QHs. There, stability of the \( \nu = 3/7 \) daughter does not require stability of the \( \nu = 1/3 \) parent.

The value of \( 2l = 3N - 7 \) precludes a Laughlin state of pairs (or, equivalently, of the QHs). To show it, let us use the following pictorial argument, equivalent to a more rigorous derivation. Laughlin \( \nu = 1/3 \) state (of individual particles) can be viewed as \( \bullet\bullet\bullet\bullet\bullet\equiv (\bullet\bullet\bullet)\equiv \bullet\bullet\bullet \), with “\( \bullet \)” and “\( o \)” denoting particles and vacancies. Counting the total LL degeneracy \( g \) leads to the correct value of \( 2l = 3N - 3 \). The Moore–Read state, i.e., the Laughlin state of pairs at \( \nu = 1/3 \), is represented by
pairs at \( \nu = \frac{1}{3} \) would correspond to \((\bigcirc\bigcirc\bigcirc\bigcirc\bigcirc)\), predicting (incorrectly) \( 2l = 3N - 5 \). Assumining pairing, \( 2l = 3N - 7 \) can only be obtained using a two-pair unit cell, \((\bigcirc\bigcirc\bigcirc\bigcirc\bigcirc\bigcirc\bigcirc\bigcirc\bigcirc\bigcirc\bigcirc\bigcirc\bigcirc)\), corresponding to more complicated pair–pair correlations.

At higher fillings of CF-LL, \( N_3 \approx \frac{2}{5}N \) at \( 2l \approx 20 \) suggests division of \( N \) QEs into \( \frac{1}{5}N \) triplets at \( \nu = \frac{2}{5} \), and \( N_3 \approx \frac{1}{3}N \) at \( 2l \approx 25 \) implies a more complicated cluster configuration (with mixed sizes) at \( \nu = \frac{1}{3} \). LL_2 and LL_3 look alike (and different from LL_0 or CF-LL_1) at \( 23 \leq 2l < 29 \), both having \( N_3 \approx \frac{2}{5}N \). At \( 2l = 29 \), \( N_3 \) for LL_2 drops rapidly to almost zero. This further supports similarity of the \( \nu = \frac{1}{3} \) states in LL_2 and CF-LL_1.

In Fig. 4(a) we replot \( N_2 \) as a function of \( N/g \sim \nu \). The quasi-linear dependences for LL_0, LL_1, and CF-LL_1 all aim correctly at \( N_2 = 2N - 3 \) for \( \nu = 1 \), but start from different values, \( N_2 \approx 0, \frac{2}{5}N, \) and \( \frac{1}{3}N \), at \( \nu = \frac{1}{3} \). Regular dependence allows subtraction of \( N_2 \) from the contribution from pairs belonging to different clusters. As reference we used ground states of \( V = \delta_{R,1} \). This short-range repulsion guarantees maximum avoidance of \( R = 1 \); its \( N_2^* \) contains only the inter-cluster contribution. To compare \( N_2 \) of QEs or electrons with \( N_2^* \), we: (i) calculated \( N_2 \) for a single \( K \)-size cluster, and multiplied it by \( N/K \) to obtain relation between \( N_2 \) and \( K \) in an idealized clustered state of \( N \) particles, (ii) using this relation (cf. Fig. 4c]), converted \( N_2 \) and \( N_2^* \) into the (average) cluster sizes \( K \) and \( K^* \); (iii) defined \( K = K - (K^* - 1) \) as the cluster size estimate free of the inter-cluster contribution.

The result in Fig. 4(b) indicates pairing in LL_1 at \( \frac{2}{5} \leq \nu \leq \frac{3}{5} \), and in both CF-LL_1 and LL_2 at \( \nu \leq \frac{1}{3} \). Triplets seem to form in CF-LL_1 at \( \nu = \frac{2}{5} \), in LL_2 at \( \frac{1}{3} \leq \nu \leq \frac{2}{3} \), and in LL_3 at \( \nu \leq \frac{1}{3} \). The \( \nu = \frac{1}{3} \) state of QEs falls between \( K = 2 \) and 3, suggesting mixed-size clusters.

In conclusion, we studied two- and three-body correlations of several quantum liquids. We found evidence for pairing of CFS at \( \nu_c = \frac{1}{11} \) and interpret this state as a condensate of “second generation” Moore–Read QHS.

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