Collisionally Induced Transport in Periodic Potentials

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(Dated: January 19, 2022)

We study the transport of ultra cold atoms in a tight optical lattice. For identical fermions the system is insulating under an external force while for bosonic atoms it is conducting. This reflects the different collisional properties of the particles and reveals the role of inter-particle collisions in establishing a macroscopic transport in a perfectly periodic potential. Also in the case of fermions we can induce a transport by creating a collisional regime through the addition of bosons. We investigate the transport as a function of the collisional rate and we observe a transition from a regime in which the mobility increases with increasing collisional rate to one in which it decreases.

We compare our data with a theoretical model for electron transport in solids introduced by Esaki and Tsu.

PACS numbers: 05.60.-k, 03.75.Ss, 03.75.Lm

The motion of particles in periodic potentials is the underlying process for fundamental transport phenomena like electric current in metals. Quantum mechanically, particles in a periodic potential can be described with Bloch states. Without an external force, the particles can move freely through the potential and in the case of a non-interacting sample the system acts like a perfect conductor. Under a constant external force, the periodic potential is tilted and the new stationary states are localized Wannier-Stark states.[1] In the absence of interactions the particles cannot change their quantum state and the latter system behaves like an insulator for DC currents. Instead, in the presence of interactions collisions can change the quantum state of the particles and a macroscopic current is established. At the onset of interactions an increasing collisional rate is therefore expected to favor a current through the potential whereas at high collisional rate the current is hindered by collisions. The latter regime is well known from solids where scattering with phonons and impurities provide an extremely large collisional rate and the conductivity decreases linearly with increasing collisional rates.[2] However the limit of low collisional rate, where the role of collisions is reversed, is experimentally not accessible in solids. With the development of semi-conductor superlattices,[3] this regime could be entered and phenomena like negative electric conductivity could be observed.[4, 5, 6] a completely non-interacting system is not achievable even in superlattices.

In this work we use ultra cold atoms in an optical lattice to investigate the transport in periodic potentials induced by an external force starting from the limit of zero interaction. Such kind of systems have already been used with success to study solid state phenomena like the Wannier-Stark ladder.[7] or Josephson junctions.[8] Here we take advantage of the unique possibility of controlling both the scattering process and the parameters of the lattice to study the transition from an ideal insulator to a real conductor in a perfectly periodic potential. The use of indistinguishable fermionic atoms allows us to create a completely non-interacting system since inter-particle collisions are forbidden at ultra low temperatures. On the other hand we can add a scattering mechanism in a controllable way by using a mixture of fermions and bosons or pure bosonic samples.

For the experiments we employ a mixture of ultra cold fermionic $^{40}$K and bosonic $^{87}$Rb atoms in a magnetic trapping potential. The spin polarized fermions are sympathetically cooled by forced evaporation of the bosons.[3, 10] For potassium (rubidium) atoms, the magnetic trap has an axial and radial oscillation frequency of $\omega_\lambda = 2\pi \times 24(16)$ s$^{-1}$ and $\omega_r = 2\pi \times 280(190)$ s$^{-1}$. The experiments were carried out with samples at temperatures between 300 and 400 nK. The number fermions can be varied between $2 \times 10^4$ and $10^5$ which corresponds to a Fermi temperature of $300 - 400$ nK. By changing the final ramp of the radio frequency evaporation, we can adjust the number of bosons in the mixture and with a sweep below the trap bottom it is also possible to remove the bosons. The temperature of the mixture is always above the critical temperature for Bose-Einstein condensation. During the last 500 ms of the evaporation ramp, the optical lattice is switched on adiabatically allowing for thermalization within the lattice. The lattice is formed by two counter-propagating laser beams and is aligned in the horizontal plane along the weak axis of the magnetic trap.

The potential is given by $U(x) = U_0/2(1 - \cos(4\pi x/\lambda))$, where $\lambda = 830$ nm is the wavelength of the laser. The lattice depth is measured in units of the recoil energy $U_0 = sE_r$ with $E_r = \hbar^2k^2/2m$ and $k = 2\pi/\lambda$. In the axial direction, the combined periodic and harmonic potential has two different kinds of eigenstates that belong to the first energy band. On both sides of the potential localized stationary states develop for sufficiently high magnetic field gradients. These states are the analog to the Wannier-Stark states in the case of a linear potential and typically extend over a few lattice sites. In the trap center, the states extend symmetrically around
the potential minimum and resemble harmonic oscillator eigenstates which are modulated by the periodic potential \[ |1\rangle. \] The temperature of the samples is comparable to the recoil energy and is chosen in order to have a significant occupation of the localized states. The typical \( 1/e^2 \)-radius of the cloud in the direction of the lattice is 100 \( \mu \)m corresponding to roughly 250 lattice sites. In the two radial directions, the atoms occupy the radial harmonic oscillator states. To study the transport of the particles along the lattice, the magnetic trap is suddenly shifted \[ |12\rangle \] in the direction of the lattice by a fraction of the extension of the cloud (displacement \( x_d \)). The harmonic confinement acts like an external driving force and we monitor the center of mass (CM) position of the cloud (see Fig. 1b).

The evolution of the CM motion for an initial displacement of \( x_d = 35 \mu \)m is shown in Fig. 1b. The open circles show the motion of a pure fermionic sample. The filled squares show the evolution of the fermionic sample in the presence of bosons. After an initial damped oscillation, which is attributed to particles in the harmonic oscillator like states, the pure fermionic cloud remains displaced in the trapping potential. This is due to the asymmetric occupation of the localized states after the displacement which gives rise to an offset of the cloud with respect to the equilibrium position. Without collisions, the fermions cannot change the population of the single particle states. Because Landau-Zener tunnelling into higher bands is also negligible for our parameters, the offset cannot vanish and the system is insulating. With an admixture of bosons however, the fermions rapidly move towards the equilibrium position \[ |13\rangle. \] This macroscopic transport corresponds to a DC current. To quantify this current, we fit an exponential decay to the long time tail of the data. For the fermions in the mixture we find a decay time of \( \tau = 260 \pm 30 \) ms, whereas for the pure fermionic sample the decay time is longer than 5 s which is comparable to the lifetime of the atoms in the optical potential. This experiment proofs that in a perfect lattice interactions between the particles are needed to establish a macroscopic current under an external force.

To investigate the role of collisions in more detail, we have changed the collisional rate by changing the number of bosons in the mixture. Fig. 2 shows the decay time for the fermionic cloud in a range where the collisional rate was changed over more than one order of magnitude. Starting from the collisionless regime, we observe a decrease of the decay time with increasing collisional rate. This is what one expects if the collisions assist the hopping between different localized states. For high collisional rates, the experimental data show a slight increase...
of the decay time with increasing collisional rate. In this regime the number of bosons is much higher than the number of fermions and the bosons can be regarded as a thermal bath for the fermions. The fermions exhibit a drift motion and the collisions with the bosons impede the current through the potential like in a electric conductor. This change in the mobility of the current carriers is also known from negative differential conductivity in semiconductor superlattices (NDC), which is related to the phenomenon observed here. In the regime of NDC, the current through the potential decreases when the applied voltage is increased. This is due to the tighter localization of the electron wave function which reduces the transition probability of a hopping event between the localized states. In our experiment, we do not change the transition probability but the rate of transition inducing collisions. Despite the different mechanism for NDC only the product of the transition probability and the collisional rate determines the final hopping rate. This formal identity allows us to compare our experimental data with the theoretical model that was introduced by Esaki and Tsu to describe NDC. The authors calculate the drift velocity of electrons in a periodic potential under a constant external force. They introduce a phenomenological scattering rate \( \gamma \) and show that the drift velocity depends on the ratio of the Bloch oscillation frequency in the linear potential \( \omega_{BO} \) and the scattering rate \( \gamma \):

\[
v_d = v_0 / \left( 4 \frac{\omega_{BO}/\gamma}{1 + (\omega_{BO}/\gamma)^2} \right),
\]

with \( v_0 = \lambda \Delta E / h \) being the tunnelling speed through the potential and \( \Delta E \) being the width of the first band. A direct adaptation of the above equation to our dynamics is rather complicated because we have a spatially varying Bloch oscillation frequency and an inhomogeneous system. However, we can compare the initial velocity \( v_i \) of the center of mass that we observe in the experiment with the drift velocity calculated from Eq. (1) for a uniform system in a linear potential. To determine the Bloch oscillation frequency in this potential, we take the force that initially acts on the center of mass after the displacement and we identify the scattering rate \( \gamma \) with the average collisional rate between the fermions and the bosons. Because the initial velocity of the center of mass is connected to the decay time by \( \tau = x_d / v_i \) we can also compare the decay time \( \tau \) with the inverse of the drift velocity \( v_d \). The result is shown in Fig. 4 where we have computed the solid line leaving \( v_0 \) in (1) as a free fitting parameter. In spite of the simplification the model reproduces well both the drop of the decay time at low collisional rates and its slight increase for high collisional rates. This supports the phenomenological interpretation given above.

Because we have to deal with inter-particle collisions, our damping mechanism is different from that in superposed lattices and we have to ask if the assumptions made in Ref. 2 to derive expression (1) are fully valid in our system. The phenomenological scattering rate \( \gamma \) describes dissipative scattering processes, where the electrons can arbitrarily exchange energy and momentum with an external thermal bath. In our system, no energy exchange with the lattice is possible because the lattice is free of impurities or excitations and momentum can only be transferred to the lattice in multiples of the Bragg momentum via umklapp scattering processes. As mentioned above, in the case of a mixture of fermions and bosons, the latter might be regarded as a thermal bath for high atom numbers and a dissipative scattering channel exists. For small numbers of admixed bosons and for pure bosonic samples, where the assumption of having a thermal bath is questionable, we find the same phenomenology. This indicates that also in this case a dissipative mechanism is still present, possibly related to the coupling to the two radial degrees of freedom.

We have also studied how the transport of the atoms is affected by different lattice potentials which can be easily tuned by changing the laser intensity. In experiments carried out with thermal bosons, we have measured the dependence of the decay time on the lattice height, such as shown in Fig. 3. The data show a rapid increase of the decay time with increasing lattice height. This can be explained with a reduction of the tunnelling probability between neighboring lattice sites with increasing lattice height. In expression (1) the tunnelling speed appears as a scaling factor for the drift velocity. Even if one takes into account a spatially varying Bloch oscillation frequency \( \omega_{BO}(x) \) and an inhomogeneous scattering rate \( \gamma(x) \), the role of \( v_0 \) does not change. Thus, we can write for each single particle a differential equation of the form \( \dot{x}/v_0 = f(\omega_{BO}(x), \gamma(x)) \), whose solution scales
in the time domain with \( v_0 \). Consequently, also the behavior of the center of mass scales with \( v_0 \) and the decay time must be proportional to the inverse of the bandwidth. For a sinusoidal potential, the bandwidth can be expressed in terms of Mathieu functions and we find that for \( s < 10 \) the bandwidth is well described (the maximum error is smaller than 10 percent) with an exponential drop of the form \( \Delta E = E_v e^{-s/3.8} \). One therefore expects an exponential increase of the decay time with increasing lattice height. The exponential fit in Fig. 3 demonstrates well that this dependence is accomplished. We find a numerical value for the factor in the exponent of the fit of 1.6. For other experimental data sets with different temperatures and initial displacements we derive values ranging from 1.5 to 4.5.

We can now identify two crucial processes that are needed for the macroscopic transport through a periodic potential in the presence of an external force. The first one is the tunnelling from one lattice site to the next one. However, the coherent nature of the tunnelling process leads to a localization of the particle. Therefore an additional dissipative process is needed to destroy the localization of the particle wave function. If one of these two mechanisms is missing, the system is insulating, as we observe it for non-interacting fermions and in the limit of deep lattices.

In conclusion we have demonstrated that under an external force the macroscopic transport of particles in a perfect periodic potential requires an interaction between the particles. Indeed, in the non-interacting case, we find an insulating behavior of the system. In the interacting regime, we observe a transport, which is mediated by collisions. We have investigated the dependence of the transport velocity on the collisional rate and on the lattice height. A comparison with a semiclassical band model introduced for electrons in superlattices reveals a good qualitative agreement although the microscopic dissipative mechanism is different. In this first study of the transport of fermionic atoms in optical lattices the interaction was provided by an admixture of bosons. In the future it will be interesting to prepare the fermions in two different spin states and to extend the study to the regime of high quantum degeneracy. An even more flexible way of tuning the interaction can be achieved by using a Feshbach resonance. Such a system of interacting fermions would be interesting for possible studies of the influence of fermionic superfluidity on the transport along a periodic potential.

We want to thank G. La Rocca for illuminating discussions and T. Ban for contributions to the experiment. This work was supported by MIUR, by EU under Contract. No. HPRICT1999-00111, and by INFM, PRA "Photonmatter". H.O. was supported by EU with a Marie Curie Individual Fellowship under Contract No. HPMF-CT-2002-01958.

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[17] We calculate a collisional rate which is averaged over the extension of the cloud taking into account the spatial overlap of both clouds and the additional compression due to the periodic potential. For the K-Rb scattering length we take the value \(-395 \pm 15 a_0 \).