A DIFFERENTIAL EQUATION FOR THE TRANSITION PROBABILITY

$B(E2) \uparrow$ AND THE RESULTING RECURSION RELATIONS

CONNECTING EVEN-EVEN NUCLEI*

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We obtain here a new relation for the reduced electric quadrupole transition probability $B(E2) \uparrow$ of a given nucleus in terms of its derivatives with respect to neutron and proton numbers based on a similar local energy relation in the Infinite Nuclear Matter (INM) model of Atomic Nuclei, which is essentially built on the foundation of the Hugenholtz-Van Hove Theorem of many-body theory. Obviously such a relation in the form of a differential equation is expected to be more powerful than the usual algebraic difference equations. Although the relation for $B(E2) \uparrow$ has been perceived simply on the basis of a corresponding differential equation for the local energy in the INM model, its theoretical foundation otherwise has been clearly demonstrated. We further exploit the differential equation in using the very definitions of the derivatives to obtain two different recursion relations for $B(E2) \uparrow$, connecting in each case three neighboring even-even nuclei from lower to higher mass numbers and vice-versa. We demonstrate their numerical validity using available data throughout the nuclear chart and also explore their possible utility in predicting $B(E2) \uparrow$ values.

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I. INTRODUCTION

From the very advent of nuclear physics, ground-state properties of nuclei mainly mass and binding energies have occupied special fancy of the physicists after the formulation of the celebrated Bethe-Weizsacker Semi-Empirical Mass formula, which of course is based on the classical liquid drop picture of the nucleus. In due course, the interest was confined to explore possible existence of symmetry in nuclear dynamics, in developing mass formulas such as the Garvey-Kelson \cite{1} mass formula that connects masses of six neighboring nuclei. Therefore it was natural to expect such symmetry to be manifested in the properties other than the ground-states and specifically in the excited states. In this regard Ross and Bhaduri \cite{2} have succeeded to some extent to find difference equations involving the reduced transition probabilities \(B(E2)\uparrow\) of the neighboring even-even nuclei. Patnaik et al. \cite{3} on the other hand have also succeeded in establishing even more simpler difference equations connecting these values of four neighboring even-even nuclei.

Here in the present work we have made an attempt first to obtain a possible relation involving the reduced transition probability of a given nucleus with its derivatives with respect to neutron and proton numbers, instead of the usual difference equations. We could achieve this based on the local energy \(\eta\)-differential equation of the Infinite Nuclear Matter (INM) model \cite{4,5,6} of atomic nuclei. In passing one may note, that the \(\eta\)-relation connecting the partial derivatives of the local energies in the INM model is primarily responsible for the success \cite{7} of the INM model as a mass-formula. Such a differential equation in \(\eta\) was possible in the model due to its very foundation, i. e., the Hugenholtz-Van Hove (HVH) theorem \cite{9,10} of many-body theory. Apart from this, the INM model has succeeded in resolving \cite{11,12} the so-called \(r_0\)-Paradox and also in determining the value of nuclear incompressibility from known masses of nuclei. It is needless to mention here that any relation in the form of a differential equation of any physical quantity is intrinsically sound enough to posses a good predictive ability. This philosophy has been well demonstrated in the INM model, specifically for the prediction \cite{7} of masses of nuclei throughout the nuclear chart.

The local energy of a nucleus as per the INM model basically constitutes the shell and deformation energies, and has been shown \cite{8} to carry the shell structure of a given nucleus [for a comprehensive account of all these aspects readers may refer our recently published book \cite{13}]. Therefore it is likely that this physical quantity \(\eta\) has a good one-to-one correspondence with the properties of excited states of a given nucleus in general, and in particular the reduced transition
probability. In view of all these facts, the local energy relation has been taken as the basis for formulating a similar differential equation connecting the $B(E2) \uparrow$ value of a given nucleus with its derivatives with respect to neutron and proton numbers. In Sec. II, we show how such a relation in the form of a differential equation can be formulated followed by its possible theoretical justification. Sec. III deals with how the same differential equation can be used to derive two recursion relations in $B(E2) \uparrow$, connecting in each case three different neighboring even–even nuclei. Finally we present in Section IV, their numerical validity when subjected to known [14] experimental data throughout the nuclear chart, and their possible utility for purpose of predictions.

II. DERIVATION OF THE DIFFERENTIAL EQUATION INVOLVING $B(E2)\uparrow$

It may be mentioned here that one of the basic physical quantities of a given nucleus as per the INM model is the the local energy $\eta$ that satisfies the INM equation [7]

$$\eta(N, Z)/A = \frac{1}{2} [(1 + \beta)(\partial\eta/\partial Z)_N + (1 - \beta)(\partial\eta/\partial N)_Z],$$

(1)

where $Z, N, A$ refer to proton, neutron and mass numbers respectively, while $\beta$ is the usual asymmetry parameter given by $(N-Z)/A$. $\eta$ in conjunction with two other physical quantities $E(N,Z)$ and $f(N,Z)$ defines the ground-state energy $E^F$ of a given nucleus given by

$$E^F(N, Z) = E(N, Z) + f(N, Z) + \eta(N, Z).$$

(2)

$E$ being the energy of asymmetric nuclear matter and $f$ being the energy of the INM sphere are global in nature, while $\eta$ characterizing a finite nucleus is considered local in contrast. Consequently these three quantities are considered distinct in the sense that each of them refers to a specific characteristic of the nucleus and as such, are more or less independent of each other. Conceptually the local energy equation (1) could be obtained when Eq. (2) is subjected to the generalized HVH theorem [10] of many-body theory given by

$$E/A = [(1 + \beta)\varepsilon_n + (1 - \beta)\varepsilon_p]/2.$$

(3)

$\varepsilon_n = (\partial E/\partial N)_Z$ and $\varepsilon_p = (\partial E/\partial Z)_N$ are respectively the neutron and proton Fermi energies of nuclear matter.

Physically the local energy $\eta$ embodies all the characteristic properties of a given nucleus, mainly the shell and deformation, and has been explicitly shown [8] to carry the shell-structure.
Therefore it is likely to have some characteristic correspondence with the properties of excited
states of a given nucleus in general and in particular, the reduced transition probability $B(E2)^\uparrow$.
Accordingly the above $\eta$-equation (1) can be used as an ansatz to satisfy a similar relation involv-
ing the $B(E2)^\uparrow$ of a given nucleus. As a result we write on analogy, a similar equation for $B(E2)^\uparrow$
as

$$B(E2)[N,Z]/A = \frac{1}{2} \left[ (1 + \beta) \left( \frac{\partial B(E2)}{\partial N} \right)_Z + (1 - \beta) \left( \frac{\partial B(E2)}{\partial Z} \right)_N \right].$$

(4)

Thus we see that we have a relation (4) that connects the $B(E2)^\uparrow$ value of a given nucleus (N,Z)
with its partial derivatives with respect to neutron and proton numbers N and Z. It is true that
our proposition of this differential equation for $B(E2)^\uparrow$ is purely on the basis of intuition and on
analogy with a similar relation for the local energy of a nucleus in the INM model. However to
what extent it is true, that needs to be established. This we show in the following.

For a theoretical justification of the above equation, we use the approximation of expressing
$B(E2)^\uparrow$ as the sum of two different functions $B_1(N)$ and $B_2(Z)$ as

$$B(E2)[N,Z] = B_1(N) + B_2(Z).$$

(5)

The goodness of this simplistic approximation can only be judged from numerical analysis of the
resulting equations that follow using the experimental data. Secondly we use the empirical fact
[see Fig. 1] that $B(E2)^\uparrow$s are more or less slowly varying functions of N and Z locally. This
assumption however cannot be strictly true at magic numbers and in regions where deformations
drastically change. In fact known $B(E2)^\uparrow$ values plotted as isolines for isotopes and isotones in
Fig. 1, convincingly demonstrate this aspect in most of the cases. The usual typical bending and
kinks at magic numbers like 50, 82 and also across $Z \approx 66$ [Fig. 1(f)] can be seen as a result of
sharply changing deformations. Consequently $B_1$ and $B_2$ can be written directly proportional to N
and Z respectively as

$$B_1(N) = \lambda N, \quad \text{and} \quad B_2(Z) = \nu Z,$$

(6)

where $\lambda$ and $\nu$ are arbitrary constants and vary from branch to branch across the kinks. Then
one can easily see that just by substitution of the above two Eqs. (5,6), the differential Eq. (4)
gets directly satisfied. Thus the proposed differential equation for $B(E2)^\uparrow$ analogous to the local
energy relation in the INM model gets theoretically justified. However, the differential Eq. (4)
has its own limitations, and need not be expected to remain strictly valid across the magic-number
nuclei as well as in regions where deformations sharply change because of the very approximations
involved in proving it.
Figure 1: Known $B(E2)^\uparrow$ values in standard units $e^2b^2$ plotted as isolines for even-even nuclei. Isolines drawn in the graphs (a-c) connect these values of various isotopes for $Z=34$ onwards with varying neutron number $N$, while the isolines drawn in the graphs (d-f) show the same for isotones for $N=40$ onwards with varying proton number $Z$. Other possible isolines are not shown here to avoid clumsiness of the graphs.

III. DERIVATION OF THE RECURSION RELATIONS IN $B(E2)^\uparrow$

In order to utilize the differential Eq. (4) for all practical purposes, it is desirable to obtain possible recursion relations in $B(E2)^\uparrow$ for even-even nuclei in $(N,Z)$ space. The partial derivatives
occurring in this equation at mathematical level are defined for continuous functions. However for finite nuclei, these derivatives are to be evaluated taking the difference of $B(E2)↑$ values of neighboring nuclei. Since our interest is to obtain recursion relations for even-even nuclei, we use in the above equation the usual forward and backward definitions for the partial derivatives given by

\[
\left( \frac{\partial B(E2)}{\partial N} \right)_{Z} \simeq \frac{1}{2} \left[ B(E2)[N+2,Z] - B(E2)[N,Z] \right],
\]

\[
\left( \frac{\partial B(E2)}{\partial Z} \right)_{N} \simeq \frac{1}{2} \left[ B(E2)[N,Z+2] - B(E2)[N,Z] \right],
\]

and

\[
\left( \frac{\partial B(E2)}{\partial N} \right)_{Z} \simeq \frac{1}{2} \left[ B(E2)[N,Z] - B(E2)[N-2,Z] \right],
\]

\[
\left( \frac{\partial B(E2)}{\partial Z} \right)_{N} \simeq \frac{1}{2} \left[ B(E2)[N,Z] - B(E2)[N,Z-2] \right].
\]

Substituting the above two pairs of definitions for the derivatives in Eq. (4) separately, we arrive at the following two recursion relations for $B(E2)↑$ connecting neighboring even-even nuclei. These are

\[
B(E2)[N,Z] = \frac{N}{A-2} B(E2)[N-2,Z] + \frac{Z}{A-2} B(E2)[N,Z-2],
\]

\[
B(E2)[N,Z] = \frac{N}{A+2} B(E2)[N+2,Z] + \frac{Z}{A+2} B(E2)[N,Z+2].
\]

The first recursion relation (9) connects three neighboring nuclei (N,Z), (N-2,Z) and (N,Z-2) while the second one (10) connects (N,Z), (N,Z+2) and (N+2,Z). Thus both these relations connect only three neighboring even-even nuclei. The first one relates $B(E2)↑$s of lower to higher mass nuclei while the second one relates higher to lower mass and hence they can be termed as forward and backward recursion relations termed as, $B(E2)$-F and $B(E2)$-B respectively. Thus depending on the availability of $B(E2)↑$ data, one can use either or both of these two relations to obtain the corresponding unknown values of neighboring nuclei.

**IV. NUMERICAL TEST OF THE RECURSION RELATIONS IN B(E2)↑**

Having derived the recursion relations in $B(E2)↑$ from the differential equation (7), it is desirable to test their numerical validity to see to what extent they stand the known experimental data throughout the nuclear chart. This would also numerically support the differential Eq. (4) from which the recursion relations are derived. For this purpose we use the experimentally adopted
Figure 2: Numerical Test of the Recursion Relations connecting $B(E2)$ values of neighboring nuclei. The percentage errors of the computed $B(E2)$ values of all the anchor nuclei are plotted against Proton Number $Z$ of those nuclei. The graph (a) shown as B(E2)-F corresponds to the results of the relation (9) while graph (b) marked as B(E2)-B shows those of the relation (10). Graph (c) marked as B(E2)-P presents those of Patnaik et al. [3] [see Eq. (11)]. The vertical solid lines are drawn just to focus larger deviations if any at the magic and semi-magic numbers.

$B(E2)$ data set of Raman et al. [14] in the above relations and compute the same of all possible anchor nuclei, that are characterized by the neutron and proton numbers (N,Z) occurring in the left hand sides of the relations (9,10) in the mass range of $A=10$ to 240. For better visualization of our results, we calculate the deviation of the computed $B(E2)$ values from those of the experimental data in terms of the percentage errors following Raman et al. [15]. The percentage error of a particular calculated quantity is as usual defined as the deviation of that quantity from that of the experiment divided by the average of the concerned data inputs and then expressed as the percentage of the average. Obviously the larger the percentage error larger is the deviation of the concerned computed value. These percentages so computed are plotted in the figures 2 and 3.
against proton and neutron number respectively. This is done to ascertain to what extent large deviations occur at proton and neutron magic numbers. From the presented results we see, that in most of the cases both forward and backward recursion relations \(^{(9,10)}\) give reasonably good agreement with experiment. The deviations in 119 out of 187 cases for the forward relation \([B(E2)-F]\) and 124 out of 186 cases for the backward relation \([B(E2)-B]\) lie within \(\pm 25\%\) error [shown within broken lines in the figures]. It should be noted that even the experimental data themselves vary quite widely from one experiment to another for the same nucleus [ see for instance Table II of the data compilation by Raman et al. \(^{14}\)]. Just to cite two examples of such wide variation, the \(B(E2)\) \(\uparrow\) value for the nucleus \(^{142}Nd\) lies in the range 0.256 to 0.437 units, while the adopted value is 0.265 units and the same for \(^{154}Sm\) varies from 3.45 to 6.8 units, the adopted value being 4.36 units \(^{14}\). Such wide variation in the experimental values are almost prevalent in most of the cases. In view of this, the agreement of the model recursion relations with those of experiment
can be considered rather good. However one can see from the figures 2 and 3, that the percentage errors (deviations) are relatively higher for some nuclei in the neighborhood of the magic numbers 20, 82, 126 and semi-magic number 40. Such increase in the vicinity of the magic numbers is expected as the differential Eq. (4) from which the recursion relations are derived, need not be strictly valid at the magic numbers.

For sake of comparison of our results with those of algebraic relations of other authors, we present here calculated values using the difference equations of Patnaik et al. [3]. One should note that they used the sum rule approach of the Garvey-Kelson type given by

$$B(E2)[N,Z] = B(E2)[N+2,Z] + B(E2)[N,Z+2] - B(E2)[N+2,Z+2].$$

(11)

It may be easily noted that this difference equation connects four nuclei namely (N,Z), (N+2,Z), (N,Z+2) and (N+2,Z+2) in contrast to three in our recursion relations (9,10). These results are presented as B(E2)-P in the graph (c) of figures 2 and 3. As can be seen that these results are almost similar to ours. Numerically we find that the deviations up to 25% error from experiment exists in 117 out of 178 cases.

To bring out the contrasting features of our results in a better way, we also present our results in Fig. 4 in the form of histograms, which represent number of cases having different ranges of percentage errors. As can be seen, the sharply decreasing heights of the vertical pillars with the increasing range of errors are a clear testimony of the goodness of our recursion relations. Similar behavior can be seen also with those of Patnaik et al. [3].

For exact numerical comparison, we present in Table I, results obtained in our calculation along with those of experiment [14] for some of the nuclides randomly chosen all over the nuclear chart. One can easily see that the agreement of the predictions with the measured values is exceedingly good in most cases. In few cases such as $^{114}Pd$, $^{124}Sn$ there exists some discrepancy in between the predicted value and those of experiment. But we should also note, that the actual experimental data for these two nuclei lie in the range of 0.203-0.380 and 0.133-0.220 units respectively, which we quote here from Table II of Raman et al. [14]. Therefore in the light of such variations in the experimental data, predictions of our recursion relations appear to be reasonably good and hence can be relied upon.

Now coming to the possible agreement in between the predictions of the two recursion relations (9) and (10), we see that more or less they agree well. However there exists some disagreement in between them in few cases such as $^{84}Kr$, $^{148}Ce$ and $^{236}U$. This may be attributed to the widely vary-
Figure 4: Vertical pillars showing number of cases having different ranges of absolute percentage errors. Those marked as B(E2)-F and B(E2)-B correspond to results of our relations (9) and (10) respectively, whereas those of B(E2)-P correspond to the Eq. (11) of Patnaik et al. [3].

The experimental inaccuracy of the concerned data inputs in our recursion relations. For instance, in case of $^{236}U$, the predictions differ from each other by about 17% while the corresponding data inputs $^{234}U$, $^{234}Th$ and $^{238}U$, $^{238}Pu$ have experimental error bars 0.20, 0.70, 0.20 and 0.17 units respectively. Thus all except the second data input has a relatively higher error, thereby possibly causing the difference in predictions. Thus in general both the recursion relations more or less can be relied upon for predicting the $B(E2)$ values.

Once we establish the goodness of the two recursion relations, it is desirable to compare our predictions with the latest experimentally adopted data of Pritychenko et al. [16]. It must be made clear that none of the values of the new experimental data set has been used in our recursion relations. Rather we use only the available data set of Raman et al. [14] to generate all possible values of a given nucleus employing the two recursions relations (9) and (10). One should note
Table I: Comparison of the predicted referred to as B(E2)-B [Eq. (10)] and B(E2)-F [Eq. (9)] and experimentally adopted [14] $B(E2)$ values presented in terms of the standard units $e^2 b^2$. The bracketed numbers refer to uncertainties in the last digits of the experimentally quoted values.

| Nucleus | Expt B(E2)-B | Expt B(E2)-F | Predicted B(E2)-B | Predicted B(E2)-F |
|---------|--------------|--------------|-------------------|-------------------|
| $^{26}\text{Ne}$ | 0.0228 (41) | - | 0.028 | 0.026 |
| $^{36}\text{Si}$ | 0.019 (6) | - | 0.019 | - |
| $^{46}\text{Ar}$ | 0.0196 (39) | 0.035 | - | 0.0095 (32) |
| $^{44}\text{Ti}$ | 0.065 (16) | 0.068 | - | 0.300 (6) |
| $^{80}\text{Se}$ | 0.253 (6) | - | 0.196 | 0.125 (6) |
| $^{98}\text{Sr}$ | 1.282 (39) | - | 1.274 | 1.66 (34) |
| $^{106}\text{Mo}$ | 1.31 (7) | - | 1.341 | 1.05 (12) |
| $^{114}\text{Pd}$ | 0.38 (12) | 0.881 | 0.585 | 0.48 (6) |
| $^{124}\text{Sn}$ | 0.166 (4) | 0.354 | - | 0.383 (6) |
| $^{134}\text{Xe}$ | 0.34 (6) | - | 0.375 | 1.05 (6) |
| $^{148}\text{Ce}$ | 1.960 (18) | 1.241 | 3.047 | 4.20 (28) |
| $^{154}\text{Sm}$ | 4.36 (5) | 3.807 | - | 5.02 (5) |
| $^{162}\text{Dy}$ | 5.35 (11) | 5.244 | 5.471 | 5.79 (10) |
| $^{174}\text{Yb}$ | 5.940 (6) | - | 5.228 | 4.82 (6) |
| $^{186}\text{W}$ | 3.500 (12) | - | 2.970 | 2.35 (6) |
| $^{196}\text{Pt}$ | 1.375 (16) | - | 1.034 | 0.612 (10) |
| $^{220}\text{Rn}$ | 1.86 (7) | - | 3.189 | 5.15 (14) |
| $^{232}\text{Th}$ | 9.28 (10) | - | 8.955 | 11.61 (15) |
| $^{242}\text{Pu}$ | 13.40 (16) | - | 13.949 | 14.94 (19) |

Here that each of these relations can be rewritten in three different ways just by shifting the three terms occurring in them from left to right and vice-versa. Thus altogether, these two relations in principle can generate up to six alternate values for a given nucleus subject to availability of the corresponding data. Since each of the values is equally probable, the predicted value for a given nucleus is then obtained by the arithmetic mean of all those generated values so obtained. Our predictions here are confined only for those isotopes for which measured values were quoted by
Pritychenko et al. [16]. The predicted values so obtained termed as the Model values are presented in Table II for various isotopes of Z=24, 26, 28 and 30 along with those of the latest experimental [16] data.

| Nucleus | Experiment [16] | Model    | Nucleus | Experiment [16] | Model    |
|---------|----------------|----------|---------|----------------|----------|
| 46Cr    | 0.093 (20)     | 0.1033   | 48Cr    | 0.137 (15)     | 0.131    |
| 50Cr    | 0.1063 (32)    | 0.1107   | 52Cr    | 0.0627 (27)    | 0.0735   |
| 54Cr    | 0.0879 (55)    | 0.1030   | 56Cr    | 0.055 (19)     | 0.1109   |
| 58Cr    | 0.099 (28)     | 0.057    | 54Fe    | 0.0608 (31)    | 0.0750   |
| 56Fe    | 0.0975 (27)    | 0.0781   | 58Fe    | 0.123 (4)      | 0.1115   |
| 60Fe    | 0.0938 (88)    | 0.1133   | 62Fe    | 0.1028 (90)    | 0.081    |
| 64Fe    | 0.178 (17)     | 0.039    | 54Ni    | 0.061 (12)     | 0.054    |
| 56Ni    | 0.0453 (86)    | 0.0332   | 58Ni    | 0.0673 (17)    | 0.0751   |
| 60Ni    | 0.0914 (17)    | 0.1110   | 62Ni    | 0.0893 (21)    | 0.1041   |
| 64Ni    | 0.0629 (32)    | 0.0743   | 66Ni    | 0.0611 (67)    | 0.0672   |
| 68Ni    | 0.0260 (40)    | 0.1084   | 62Zn    | 0.1224 (59)    | 0.1591   |
| 64Zn    | 0.1484 (52)    | 0.1341   | 66Zn    | 0.1371 (29)    | 0.1544   |
| 68Zn    | 0.1203 (25)    | 0.1626   | 70Zn    | 0.1525 (75)    | 0.1399   |
| 72Zn    | 0.174 (21)     | 0.261    | 74Zn    | 0.200 (10)     | 0.207    |

Table II: Comparison of the predicted and the latest experimental [16] $B(E2)^\uparrow$ values presented in terms of the standard units[$e^2b^2$]. The bracketed numbers refer to uncertainties in the last digits of the experimentally quoted values.

The same are also plotted in Fig. 5 to get a better view of the results. Our predictions are confined only for those isotopes for which measured values were quoted by Pritychenko et al. [16]. One can easily see that in all the cases except for $^{64}Fe$ and $^{68}Ni$, the agreement between the predictions with those of the experiment are remarkably good. For these two nuclei, the discrepancies may be attributed to the possible sub-shell effect as the neutron numbers are 38 and 40 respectively. For sake of comparison we have also presented in Fig. 5 results obtained from two shell-model calculations [16, 17] marked here as SM1 and SM2 by employing two different effective interactions GXPF1A [17] and JUN45 [18]. One should note here, that the first one because of its own limitations did not succeed in getting reliable values for nuclei having neutron number
beyond N=36. Hence the second shell-model with JUN45 effective interaction was performed by

| N  | 0.05 | 0.10 | 0.15 | 0.20 |
|----|------|------|------|------|
| SM1 | Expt. | Model | SM1 | Expt. | Model |
| 28  | 0.05 | 0.10 | 0.15 | 0.20 |
| 32  | 0.05 | 0.10 | 0.15 | 0.20 |
| 36  | 0.05 | 0.10 | 0.15 | 0.20 |

| N  | 0.00 | 0.10 | 0.20 |
|----|------|------|------|
| SM1 | Expt. | Model | SM1 | Expt. | Model |
| 24  | 0.00 | 0.10 | 0.20 |
| 28  | 0.00 | 0.10 | 0.20 |
| 32  | 0.00 | 0.10 | 0.20 |
| 36  | 0.00 | 0.10 | 0.20 |
| 40  | 0.00 | 0.10 | 0.20 |

| Neutron Number N | 0.00 | 0.10 | 0.20 |
|------------------|------|------|------|
| SM1 | Expt. | Model | SM1 | Expt. | Model |
| 30  | 0.00 | 0.10 | 0.20 |
| 32  | 0.00 | 0.10 | 0.20 |
| 34  | 0.00 | 0.10 | 0.20 |
| 36  | 0.00 | 0.10 | 0.20 |
| 38  | 0.00 | 0.10 | 0.20 |
| 40  | 0.00 | 0.10 | 0.20 |
| 42  | 0.00 | 0.10 | 0.20 |
| 44  | 0.00 | 0.10 | 0.20 |

| Z=24, Cr | Expt. | Model | SM1 | SM2 |
|---------|-------|-------|-----|-----|
| Z=26, Fe | Expt. | Model | SM1 | SM2 |
| Z=28, Ni | Expt. | Model | SM1 | SM2 |
| Z=30, Zn | Expt. | Model | SM1 | SM2 |

Figure 5: Both calculated (Model) and the latest experimental [16] $B(E2)$ values in standard units $e^2b^2$ [see text for details] are presented for various isotopes of Z=24, 26, 28 and 30 versus Neutron Numbers N. Values for different isotopes are connected by solid or dashed lines. Experimental uncertainties are shown as vertical error bars as usual. Recent shell-model calculated values [SM1 and SM2] are also presented for sake of comparison.

Pritychenko et al. [16] for the nuclei $^{64}Fe$, $^{68}Ni$ and $^{72,74}Zn$. One can easily see that the shell-model values SM1 almost agree with those of ours for almost all the isotopes. Even for $^{64}Fe$ where there is a little bit of discrepancy in between ours and the experiment, SM1 value is closer to ours. SM2 values more or less agree with those of experiment.

We have just demonstrated as shown above the utility of the recursion relations for predicting
\[ B(E2) \uparrow \] values for some of the even-even isotopes of Cr, Fe, Ni and Zn in agreement with the latest experimental \[ [16] \] data. Therefore it is desirable to find out whether the model is good enough for such predictions in the higher mass regions of the nuclear chart. However in these regions there is no new data to compare with and hence we can only compare with the data set of Raman et al. \[ [14] \]. With this view, we repeated our calculations for higher isotope series following the same methodology outlined above. Since the main aim of our present investigation is just to establish the goodness of our model, we present here results of only few such series for which experimental data exist for a relatively large number of isotopes. Accordingly we have chosen four isotope series Z=40, 48, 66 and 78 covering nuclei both in mid-mass and heavy-mass regions. Our choice of the first two series namely Z=40 and 48 is again to see to what extent our model works across the semi-magic number 40 and magic number 50. Our predictions along with those of experimental data of Raman et al. \[ [14] \] are presented in Fig. 6. From the presented results, it is fair enough to say that the agreement of the model values with experiment in most of the cases is good. It is remarkable that even sharply changing deformations at neutron numbers N=46 and 56 for Zr [Fig. 6. (a)] are well reproduced. However we find few exceptions such as \(^{116}Cd, ^{122}Cd, ^{156}Dy,\) and \(^{184}Pt\), for which some discrepancies exist. Since we use the recursion relations for our predictions, such random cases may be attributed to the possible experimental inaccuracy of \[ B(E2) \uparrow \] values in their neighborhood. For instance, the experimental inaccuracy in case of \(^{122}Cd\) is almost 66\% of the quoted value [Fig. 6 (b)], which is rather much larger than the discrepancy of our model value. Use of such input data in the recursion relations are possibly affecting predicted values in few cases. we have also shown in Fig. 6 results of the two well-known theoretical models namely Finite Range Droplet Model (FRDM) \[ [19] \] and Single-Shell Asymptotic Nilsson Model (SSANM) \[ [20] \] just for sake of comparison. One can see that in most of the cases for these two models, the discrepancies from experiment are rather high.

Now taking stock of all the results discussed so far, we can fairly say that the recursion relations work reasonably well almost throughout the nuclear chart. Even across the magic numbers and sharply changing deformations, these relations have succeeded in reproducing the experimental data to a large extent with a little bit of deviation here and there. Elsewhere there exists few exceptions but the discrepancies are not that large as to make the relations untrue. In a nutshell, the recursion relations for the reduced transition probability \[ B(E2) \uparrow \] derived here can be termed sound enough as to have passed the numerical test both in reproducing and predicting the experimental values, and thereby vouches for the goodness of the differential equation \[ (4) \] from which they
Figure 6: Similar to Fig. 5 but for Z=40, 48, 66 and 78. The experimental data points marked as Expt correspond to those of Raman et al. [14]. Data points marked as FRDM and SSANM correspond to predictions of the other two theoretical models (see text).
V. CONCLUDING REMARKS

In conclusion we note, that we have succeeded in deriving for the first time, a novel relation for the reduced transition probability $B(E2) \uparrow$ of a given nucleus in terms of its derivatives with respect to neutron and proton numbers. We could obtain such a differential equation on the basis of one-to-one correspondence with the local energy, that satisfies a similar relation formulated in the INM model of atomic nuclei. We have also succeeded in establishing its theoretical foundation using the empirical fact, that $B(E2) \uparrow$s are more or less slowly varying functions of neutron and proton numbers except across the magic numbers. We further exploited the usual definitions of the two derivatives with respect to neutron and proton numbers occurring in the equation, to derive two recursion relations in $B(E2) \uparrow$. Both these relations are found to connect three neighboring even-even nuclei from lower to higher mass and vice-verse. Their numerical validity was then established using known experimental data set compiled by Raman et al. in the mass range of A=10 to 240. Apart from this their utility was further established by comparing our predictions with the latest experimental data set of Pritychenko et al. for the isotopes of Cr, Fe, Si and Ni. The results so obtained convincingly show the goodness of the recursion relations and thereby their parent differential equation.

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