Gauge Theory of Maxwell-Weyl Group

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Abstract

Starting from Maxwell-Weyl algebra we found the transformation rules for generalized space-time coordinates and the differential realization of corresponding generators. By treating local gauge invariance of Maxwell-Weyl group, we presented the Einstein-Cartan-Weyl gravity with the additional terms containing the gauge fields associated with the antisymmetric generators.

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I. INTRODUCTION

Symmetries are of great importance in the formulation of theories describing a given physical system. Invariance of any physical system under a given symmetry transformation group determines its properties to a great extent. In order to get more information on the physical systems one has to find new symmetries or enlarge the symmetries of the system. For instance, there is a well-known theorem, the Coleman-Mandula No-Go theorem \cite{1} says that there is no non-trivial way to combine space-time and internal symmetries. In this case symmetry generators are bosonic and their Lie algebra includes only commutators. But there is one non-trivial way to get around this by incorporating anticommutators for fermionic generators. This type of extension leads to supersymmetry. There is a different extension of Poincare algebra bypassing another well-known theorem \cite{2} that does not allow non-central extension of this algebra. In this case one introduces a new anti-symmetric tensor generator by imposing non-commutativity of momentum generators and satisfying $[P_a, P_b] = iZ_{ab}$. Motivation behind this kind of extension is that symmetries of empty Minkowski space-time is described by Poincare algebra if such a space-time filled with some background field must lead to modification of the Poincare algebra. The extension of the Poincare algebra by six additional abelian generators is called the Maxwell algebra \cite{3, 8}. Addition of new generators to the Poincare algebra leads naturally to extended space-time geometry.

A decade ago, another possible tensor extension of Poincare algebra was proposed and its supersymmetric generalization was studied in \cite{9}, and two years later semi-simple extension and also supersymmetric generalization of Poincare algebra were presented in \cite{10}. After these articles, Maxwell symmetries has appeared in literature once more. Different deformation of Maxwell algebras, their supersymmetric generalization and their dynamical realization for massless superparticle model were investigated in \cite{11–14}. These extended symmetries were also applied to the planar dynamics of the Landau problem and to the description of higher spin fields \cite{15, 16}. N-extended Maxwell algebra was constructed via a contraction procedure of (extended Lorentz $\oplus$ extended AdS) superalgebras to arrive various type of N-extended Maxwell superalgebras \cite{17–20}. Based on these kind of Maxwell superalgebras, several Maxwell supergravity models were studied by the following authors \cite{21–25}.

J.A. de Azcarraga in his paper \cite{26} achieved the generalized cosmological constant problem by gauging the Maxwell algebra, later D.V. Soroka \cite{25} presented another approach to the problem but this time gauging semi-simple extension of Poincare algebra. The formulation of different types of Maxwell gravities has already been done in \cite{26–28}. With this motivation, we have presented the gauge theory of Maxwell-Weyl group denoted by $\mathcal{MW}(1, 3)$.

The paper is organized as follows: In Sec. II we consider the Weyl algebra and its extension by an antisymmetric tensor generator. Applying the method of non-linear coset realization \cite{29, 32} to Maxwell-Weyl group, the transformation rules for generalized coordinates are found and explicit expression for the generators of the Maxwell-Weyl algebra are given. In Sec. III we discussed local gauge theory based on the Maxwell-Weyl algebra by introducing vierbein, spin connection, dilatation, six additional geometric abelian gauge fields corresponding to the antisymmetric generator and a compensating field in order to preserve Weyl scaling. From these we deduced the transformations of the fields under local gauge transformation and their covariant curvatures that leave the Lagrangian invariant. After that we constructed invariant Lagrangian and finally, we obtained the equations of motion for all dynamical variables.
II. WEYL ALGEBRA AND ITS TENSOR EXTENSION

Scale transformation together with Poincare transformations form the eleven dimensional Weyl group, $W(1,3)$. Its Lie algebra is generated by the operators $P_a$, $M_{ab}$ and $D$ associated with space-time translations, restricted Lorentz transformations and dilatations, respectively. The non-zero commutation relations can be written in the following form:

$$
[M_{ab}, M_{cd}] = i (\eta_{ad} M_{bc} + \eta_{bc} M_{ad} - \eta_{ac} M_{bd} - \eta_{bd} M_{ac})
$$

$$
[M_{ab}, P_c] = i (\eta_{bc} P_a - \eta_{ac} P_b)
$$

$$
[P_a, D] = i P_a
$$

and differential realization of the generators are given by

$$
M_{ab} = i (x_a \partial_b - x_b \partial_a)
$$

$$
P_a = i \partial_a
$$

$$
D = ix^a \partial_a
$$

where $\eta_{ab}$ is the Minkowski metric $\eta_{ab} = \text{diag}(+1, -1, -1, -1)$ with the tangent space indices $a, b$ run over $0, 1, 2, 3$. The Weyl algebra can be extended with antisymmetric tensor generator $Z_{ab}$ by the use of expansion method introduced in [17, 33, 34], see also [35–37]. One could also consider Maxwell-Weyl algebra as the extension of Maxwell algebra by dilatation generator [13]. Here we present a novel 17-dimensional Maxwell-Weyl algebra with the following commutation rules:

$$
[M_{ab}, M_{cd}] = i (\eta_{ad} M_{bc} + \eta_{bc} M_{ad} - \eta_{ac} M_{bd} - \eta_{bd} M_{ac})
$$

$$
[M_{ab}, P_c] = i (\eta_{bc} P_a - \eta_{ac} P_b)
$$

$$
[P_a, P_b] = i Z_{ab}
$$

$$
[P_a, D] = i P_a
$$

$$
[M_{ab}, Z_{cd}] = i (\eta_{ad} Z_{bc} + \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac})
$$

$$
[Z_{ab}, D] = 2i Z_{ab}
$$

and all the other commutators vanish identically.

We introduce the coordinates $X^M = (x^a, \theta^{ab} = -\theta^{ba})$ as the group space coordinates which are dual to $P_a, Z_{ab}$. One can regard generalized space-time as a coset space of the Maxwell group with respect to the Lorentz group [13, 38]. In the case of Maxwell-Weyl symmetry, the variables $x^a, \theta^{ab}, \sigma$ parametrize the coset element as

$$
K(x, \theta, \sigma) = e^{ixp} e^{i\theta Z} e^{i\sigma D}
$$

The infinitesimal transformation of the coset parameters, generated by the constant group elements $a, \varepsilon, \lambda, u$ corresponds to a motion in tensorially extended space-time induced by the left multiplications

$$
g(a, \varepsilon, \lambda, u)K(x, \theta, \sigma) = K(x', \theta', \sigma') h(\omega)
$$

where

$$
h(\omega) = e^{-\frac{i}{2} \omega^M}
$$
and can be easily evaluated through the use of the well known Baker-Hausdorff-Campell
formula:

\[ e^A e^B = e^{A + B + \frac{1}{2}[A,B]} \]  

which holds when \([A, B]\) commutes with both \(A\) and \(B\). Infinitesimally, the action of the \(\mathcal{MW}(1,3)\) on group space coordinates reads

\[ \delta x^a = u^a x^b + \lambda x^a + \alpha^a \]
\[ \delta \theta^{ab} = \varepsilon^{ab} - \frac{1}{4}x^{[a} x^{b]} + 2\lambda \theta^{ab} + u^{[a} \theta^{cb]} \]
\[ \delta \sigma = \lambda \]
\[ \omega^{ab} = u^{ab} \]  

where anti-symmetrization is defined by \(A^{[a} B^{b]} = A^a B^b - A^b B^a\).

As seen from transformation rules we simply have Weyl transformation for space-
time coordinates and there is no contribution to that resulting from tensorial extension.
On the other hand as seen from transformation rules two translation yields a shift of
\(\theta^{ab}\), induces some additional terms on tensorial space. The transformation law for a scalar
field \(\Phi'(x^a, \theta^{ab}) = \Phi(x^a - \delta x^a, \theta^{ab} - \delta \theta^{ab})\) under the infinitesimal action of \(\mathcal{MW}(1,3)\) implies

\[ \delta \Phi = -\delta x^a \partial_a \Phi - \delta \theta^{ab} \partial_{ab} \Phi = i \left( a^a P_a + \varepsilon^{ab} Z_{ab} + \lambda D - \frac{1}{2} u^{ab} M_{ab} \right) \Phi \]

with

\[ P_a = i(\partial_a - \frac{1}{2} x^b \partial_{ab}) \]
\[ Z_{ab} = i\partial_{ab} \]
\[ D = i(x^a \partial_a + 2 \theta^{ab} \partial_{ab}) \]
\[ M_{ab} = i\left\{ x_a \partial_b - x_b \partial_a + 2(\theta_a^c \partial_{bc} - \theta_b^c \partial_{ac}) \right\} \]

where \(\partial_a = \frac{\partial}{\partial x^a}\), \(\partial_{ab} = \frac{\partial}{\partial \theta^{ab}}\) and one can check that these generators fulfil the Maxwell-Weyl algebra, \(mw(1,3)\), and satisfy the all Jacobi identities.

III. GAUGING THE MAXWELL-WEYL ALGEBRA

Gauge theories of ordinary Weyl group were treated in \[39-44\]. In this section we consider
gauge theory of the \(mw(1,3)\) algebra. In order to gauge this algebra one introduces a connection \(A_\mu(x)\) with

\[ A_\mu = e^a_\mu P_a + B^{ab}_\mu Z_{ab} + \chi_\mu D - \frac{1}{2} \omega^{ab}_\mu M_{ab} \]

where \(e^a_\mu(x)\) is the vierbein, \(\omega_\mu(x)\) is the spin connection, \(B^{ab}_\mu(x)\) is the gauge field corre-
spending to the antisymmetric tensor generator and \(\chi_\mu(x)\) is the gauge field corresponding to

dilatation generator. The variation of these fields under infinitesimal gauge transformations is given by

\[ \delta A_\mu = -\partial_\mu \zeta - i [A_\mu, \zeta] \]
with the gauge generator
\[
\zeta (x) = y^a(x) P_a + \varphi^{ab}(x) Z_{ab} + \rho(x) D - \frac{1}{2} \tau^{ab}(x) M_{ab}
\]  
where $y^a(x)$ are space-time translations, $\varphi^{ab}(x)$ are translations in tensorial space, $\rho(x)$ dilatation parameter and $\tau^{ab}(x)$ are the Lorentz transformation parameters. From Maxwell-Weyl algebra and Eq. (12) it follows
\[
\delta e^a_\mu = - \partial_\mu y^a - \omega^a_\mu y^b - \chi_\mu e^a + \rho e^a_\mu + \tau^a_\mu e^b
\]
\[
\delta B^{ab}_\mu = - \partial_\mu \varphi^{ab} - \omega^{[a}_\mu \varphi^{cb]} - 2 \chi_\mu \varphi^{ab} + \frac{1}{2} e^a_\mu y^b + 2 \rho B^{ab}_\mu + \tau^{[a}_c B^{cb]}_\mu
\]
\[
\delta \chi_\mu = - \partial_\mu \rho
\]
\[
\delta \omega^{ab}_\mu = - \partial_\mu \tau^{ab} - \omega^{[a}_\mu \tau^{cb]}
\]  
(14)

The curvature two-form $\mathcal{F}$ is given by the structure equation
\[
\mathcal{F} = dA + iA \wedge A = dA + \frac{i}{2} [A, A] = \frac{1}{2} e^a \wedge e^b \mathcal{F}_{ab}
\]  
whence writing
\[
\mathcal{F} = F^a P_a + F^{ab} Z_{ab} + f D - \frac{1}{2} R^{ab} M_{ab}
\]  
we find
\[
F^a = de^a + \omega^a_\mu e^b + \chi \wedge e^a
\]
\[
R^{ab}_\mu = d\omega^{ab}_\mu + \omega^a_\mu \omega^b_\nu + \omega^b_\mu \omega^a_\nu
\]
\[
F^{ab} = dB^{ab} + \omega^{[a}_\mu B^{cb]}_\nu + 2 \chi_\mu B^{ab} - \frac{1}{2} e^a_\mu e^b + \rho F^{ab}_\mu + \tau^{[a}_c F^{cb]}_\mu
\]
\[
f = d\chi
\]  
(17)

The explicit expressions of curvatures defined by Eq. (16) are
\[
F^{\mu\nu}_a = \partial_{[\mu} e^a_{\nu]} + \omega^a_{[\mu} b_{\nu]} + \chi_{[\mu} e^a_{\nu]}
\]
\[
R^{\mu\nu}_{ab} = \partial_{[\mu} \omega^{ab}_{\nu]} + \omega^a_{[\mu} \omega^{cb]}_{\nu]}
\]
\[
F^{\mu\nu}_{ab} = \partial_{[\mu} B^{ab}_{\nu]} + \omega^{[a}_{[\mu} B^{cb]}_{\nu]} + 2 \chi_{[\mu} B^{ab}_{\nu]} - \frac{1}{2} e^a_{[\mu} e^b_{\nu]}
\]
\[
f^{\mu\nu}_a = \partial_{[\mu} \chi_{\nu]}
\]  
(18)

These are the torsion tensor, curvature tensor, a component corresponding to the tensor generator $Z_{ab}$ and a component corresponding to the dilatation generator $D$ respectively.

Under an infinitesimal gauge transformation with parameters $\zeta$, the curvature 2-form $\mathcal{F}$ transform as
\[
\delta \mathcal{F}_{\mu\nu} = i [\zeta, \mathcal{F}_{\mu\nu}]
\]  
(19)

and hence one gets
\[
\delta F^{\mu\nu}_a = - y^a f_{\mu\nu} - R^{\mu\nu}_{ab} y^b + \rho F^{\mu\nu}_a + \tau^a_\mu F^{\nu b}_\mu
\]
\[
\delta F^{\mu\nu}_{ab} = - 2 \varphi^{ab} f_{\mu\nu} + \varphi^{[a}_{[\mu} R^{cb]}_{\nu]} - \frac{1}{2} y^{[a} F^{cb]}_{\mu\nu} + 2 \rho F^{\mu\nu}_{ab} + \tau^{[a}_c F^{cb]}_{\mu\nu}
\]
\[
\delta f_{\mu\nu} = 0
\]
\[
\delta R^{\mu\nu}_{ab} = \tau^{[a}_c R^{cb]}_{\mu\nu}
\]  
(20)
The field strengths Eq. (16) may be used to construct $\mathcal{MW}(1, 3)$ invariant free Lagrangians for the corresponding gauge fields. To construct Lagrangians which are locally invariant under the whole group, special care must be given to the scale invariance and its relation to dilatation subgroup of $\mathcal{MW}(1, 3)$. Localization of the dilatation symmetry bring us back to Weyl-gauge theory. One observes that infinitesimal action of dilatation on vierbein induces a change on the metric tensor as

$$\delta g_{\mu\nu}(x) = 2\rho(x) g_{\mu\nu}(x) \quad (21)$$

which is exactly the Weyl-gauge transformation and it considerably restricts the form of any action built from curvature tensors. Eq. (21) means that the metric tensor $g_{\mu\nu}(x)$ has scale(Weyl) weight two i.e. $w(g_{\mu\nu}) = +2$, consequently the reciprocal $g^{\mu\nu}(x)$ has weight $-2$ and $\sqrt{-g}$ is of weight $+4$. From transformation rules Eq. (14) and Eq. (20), we immediately infer that gauge fields $e^a_\mu, B^{ab}_\mu, \omega^{ab}_\mu$, and field strengths $F^a_\mu, R^{ab}_\mu, F^{ab}_\mu, f_{\mu\nu}$ have the following Weyl weights 1, 2, 0, 0 and 1, 0, 2, 0 respectively.

After these preliminaries for Weyl weight we can obtain the Bianchi identities corresponding to the curvature forms as

$$\mathcal{D}R^{ab} = 0 \quad (22)$$

$$\mathcal{D}F^{ab} = R^{ca}_b \wedge B^b_{\cdot c} + 2f \wedge B^{ab} - \frac{1}{2}F^{a[c} \wedge e^{b]_d} \quad (23)$$

$$\mathcal{D}F^a = R^a_b \wedge e^b + f \wedge e^a \quad (24)$$

where $\mathcal{D}\Phi = [d + \omega + w(\Phi)\chi] \Phi$ is the Lorentz-Weyl covariant derivative with $\omega = -i\omega^{\alpha\beta}\Sigma_{\alpha\beta}$ and $w$ being Weyl weight of the corresponding field.

The free gravitational action has the form

$$S_f = \int d^4x e\mathcal{L}_f \quad (25)$$

requiring that the action has Weyl weight zero. $w(\sqrt{-g}) = w(e) = +4$, implies that the Lagrangian density must satisfy the condition $w(\mathcal{L}_f) = -4$. Since the scalar curvature has $w(R) = -2$, it is not allowed to appear linearly in the action. In order to have consistent theory of gravity Weyl used quadratic terms in the action yielding the fourth order field equations [45]. Clearly, $R$ by itself inappropriate, however if we multiply it by a compensating scalar field introduced by Brans-Dicke [46] and elaborated by Dirac [47–49] one can form Weyl invariant action linear in $R$.

In our approach we will follow Dirac’s idea. The scalar field $\phi$ with Weyl weight $-1$ let $\phi^2R$ be regular part of $\mathcal{L}_f$ and hence the following combination (shifted curvature)

$$J^{ab} = R^{ab} + 2\gamma\phi^2 F^{ab} \quad (26)$$

have Weyl weight zero. With this combination we have Einstein Lagrangian that involves the curvature scalar linearly. Therefore we consider the following Lagrangian density 4-form as our starting point for the free gravitational part:

$$L_f = \frac{1}{2\kappa\gamma} J^{ab} \wedge J^{cd} = \frac{1}{4\kappa\gamma} \varepsilon_{abcd}J^{ab} \wedge J^{cd}$$

$$= \frac{1}{4\kappa\gamma} \varepsilon_{abcd}R^{ab} \wedge R^{cd} + \phi^2 \frac{1}{\kappa}\varepsilon_{abcd}R^{ab} \wedge F^{cd} + \phi^4 \frac{\gamma}{\kappa}\varepsilon_{abcd}F^{ab} \wedge F^{cd} \quad (27)$$
where $\gamma$ and $\kappa$ are constants and the first term can be ignored because it is a closed form. Introduction of compensating field forces us to add its kinetic term to the Lagrangian by means of full covariant derivative which in turn includes the gauge field $\chi_{\mu}$ hence one has to add one more Maxwell like kinetic term to Lagrangian. For completeness we can add to Lagrangian the further Weyl invariant self-interacting term $\frac{\lambda}{4} \phi^4$ with $\lambda$ another constant. We then get the total action for vacuum as follows:

$$L_0 = \frac{1}{4} f \wedge^* f - \frac{1}{2} D\phi \wedge^* D\phi + \frac{\lambda}{4} \phi^4$$

(28)

Here $\wedge^*$ denotes the Hodge duality operation then our complete action is the sum of the free gravity action and the vacuum action:

$$S = \int \phi^2 \kappa \epsilon_{abcd} R^{ab} \wedge F^{cd} + \phi^4 \gamma \kappa \epsilon_{abcd} F^{ab} \wedge F^{cd}$$

$$+ \frac{1}{4} f \wedge^* f - \frac{1}{2} D\phi \wedge^* D\phi + \frac{\lambda}{4} \phi^4$$

(29)

Since the local translation together with tensorial translation being traded for diffeomorphism invariance are not symmetries of the action [27], omitting both local space-time and tensorial-space translations the transformation rules for the curvature can be rewritten as:

$$\delta F_{\mu\nu}^a = \rho F_{\mu\nu}^a + \tau_{ab} F_{\mu\nu}^b$$

$$\delta F_{\mu\nu}^{ab} = 2 \rho F_{\mu\nu}^{ab} + \tau^{[a}_b F_{\mu\nu}^{b]}$$

$$\delta f_{\mu\nu} = 0$$

$$\delta R_{\mu\nu}^{ab} = \tau_{[a}^{[c} R_{\mu\nu}^{\cdots b]}$$

(30)

Clearly the components of the shifted curvature transform in a homogeneous way as

$$\delta J_{\mu\nu}^{ab} = \tau_{[a}^{[c} J_{\mu\nu}^{\cdots b]}$$

(31)

since the transformation rule for the scalar field is $\delta \phi = -\rho \phi$. This ensures that the free gravity part of the action is gauge invariant besides that the vacuum part is already gauge invariant by construction.

Invariance under local Maxwell-Weyl transformations (diffeomorphism) can be directly checked by using the explicit form of the Lie derivative. Since

$$\delta S_{\text{diff}} = \int \xi L = \int (d\xi L + i_{\xi} dL)$$

(32)

the first term is a total divergence which can be ignored as a surface integral and the second term is zero since the $5-$form $dL$ vanishes identically on the $4-$dimensional space-time hence $\delta S = 0$.

In order to show the diffeomorphism invariance of the action explicitly, one substitutes the transformation rules Eq. (14) to the following action integral

$$\delta S = \int \delta e^a P_a + \delta B^{ab} V_{ab} + \delta \chi Q + \delta \phi U + \delta \omega^{ab} Y_{ab}$$

(33)

then one gets the following identities (conservation rules)
\[ DP_a - V_{ab}e^b = 0 \]
\[ DV_{ab} = 0 \]
\[ e^a P_a + 2B^{ab}V_{ab} + DQ = 0 \]
\[ U = 0 \]
\[ e_a P_b - B_{[ad}V^d_{b]} - DY_{ab} = 0 \] (34)

The field equations are
\[ P_a = 0, \quad V_{ab} = 0, \quad Q = 0, \quad U = 0 \] and \[ Y_{ab} = 0 \] or they can be found by
varying the action Eq.(29) directly over the independent variables \( \omega, e, B, \) and \( \kappa. \) Varying
over the connection components \( \omega \) we obtain the following equation for the generalized
torsion tensor
\[ D \left( \phi^2 F^{ab} \right) - \phi^2 \mathcal{J}^{[a} \wedge B^{cb]} = 0 \] (35)
The variation of the action Eq.(29) with respect to \( e \) leads to
\[ -\phi^2 \frac{1}{\kappa} \varepsilon_{abcd} \mathcal{J}^{ab} \wedge e^d + \frac{1}{2} \left[ D_c \phi^* D \phi + D \phi \wedge^* (e_a \wedge e_c) D^a \phi \right] \]
\[ -\frac{1}{4} (f_{cb} e^b \wedge^* f - \frac{1}{2} \varepsilon_{abcd} f^{ab} e^d \wedge f) + \frac{\lambda}{4} \phi^* e_c = 0 \] (36)
Furthermore, the \( B \) variation of Eq.(29) gives
\[ D(\phi^2 \mathcal{J}^{ab}) = 0 \] (37)
while the \( \chi \) variation of Eq.(29) yields the following equations of motion
\[ \frac{2\phi^2}{\kappa} \epsilon_{abcd} \mathcal{J}^{ab} \wedge B^{cd} + \frac{1}{2} D^* f + \phi^* D \phi = 0. \] (38)
and finally we obtain equation of motion for the scalar field \( \phi: \)
\[ \frac{2\phi}{\kappa} \epsilon_{abcd} \mathcal{J}^{ab} \wedge F^{cd} + D^* D \phi + \lambda \phi^3 = 0. \] (39)
The equations obtained are invariant with respect to the local Maxwell-Weyl transfor-
mation considered above. By a straightforward calculation, one can show that all these
equations of motion verify each other. In terms of shifted curvature, Eq.(36) becomes
\[ \phi^2 \left( \mathcal{J}^a_{b} - \frac{1}{2} \delta^a_b \mathcal{J} \right) = -\frac{\kappa}{2} \left[ D^a \phi D_b \phi - \frac{1}{2} \delta^a_b \left( D_c \phi D^c \phi - \frac{\lambda}{2} \phi^4 \right) \right] \]
\[ + \left( f^{ac} f_{cb} - \frac{1}{4} \delta^a_b f_{cd} f^{cd} \right) \] (40)
and passing from tangent space to world indices one gets field equation with cosmological
term depending on dilaton field
\[ R^\mu_{\alpha} - \frac{1}{2} \delta^\mu_{\alpha} R - 3 \gamma \phi^2 T (B) ^\mu_{\alpha} - \frac{\kappa}{2} \phi^{-2} [T (\phi) ^\mu_{\alpha} + T (f) ^\mu_{\alpha}] \] (42)
where
\[ T (B) ^\mu_{\alpha} = e^\mu_a e^\beta_b D [\alpha B^{ab}_{\beta}] - \frac{1}{2} \delta^\mu_{\alpha} \left( e^\alpha_a e^\beta_b D [\beta B^{ab}] \right) \] (43)
\begin{equation}
T(\phi)_{\mu \alpha} = D^\mu \phi D_{\alpha} \phi - \frac{1}{2} \delta^\mu_{\alpha} \left(D_{\gamma} \phi D^\gamma \phi - \frac{\lambda}{2} \phi^4\right) \tag{44}
\end{equation}

\begin{equation}
T(f)_{\mu \alpha} = f_{\mu \beta} f_{\beta \alpha} - \frac{1}{4} \delta_{\mu \alpha} f_{\gamma \delta} f^{\gamma \delta} \tag{45}
\end{equation}

are the energy-momentum tensors for the $B$-gauge field, dilaton field and $\chi$-field respectively.

It is an interesting exercise to rewrite the above system of equations in terms of torsion-free connection. In such a reformulation, the torsional effects due to the scalar field coupling to gravity may be interpreted as an additional contribution to the stress energy forms.

IV. CONCLUSION

In the present paper we considered the algebra of generators and constructed a non-linear realization of the Maxwell-Weyl group on its coset space with respect to the Lorentz group. We proposed here the gauge theory formulation of Maxwell-Weyl gravity in order that we introduced the corresponding gauge fields, presented field transformations and found the equations of motion. Defining scale invariant shifted curvature, we achieved extension of Einstein-Cartan-Weyl field equation with variable cosmological term and additional source term. From source term we derived the stress energy-momentum tensor and from there we concluded that introduction of dilatation to the theory does affect the cosmological constant and contributes to energy-momentum of the B-field.

Appearance of the cosmological constant term as a dynamical variable in the presence of constant background field forces us to interpret the a part of stress energy-momentum tensor as the dark energy. Recall that cosmological constant problem can be explained by extending Minkowski space-time to the de Sitter space. Due to the close connection between cosmological constant and dark energy, one can infer that dark energy can be related to the B-gauge field.

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