Outpatient Scheduling based on Bottleneck Analysis from the Perspective of Complex Network

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Abstracts: At present, most hospitals are facing the situation of overcrowding, and how to speed up the patient's medical service speed is an urgent problem. In order to solve the problem of long waiting time and low utilization rate of medical resources in outpatient, a priority scheduling algorithm based on bottleneck analysis in complex network is proposed. According to the patients’ tasks, the algorithm sets up two graphs of medical service and medical resources, and projects it into a complex network model of medical resources, considering the network characteristics such as betweenness, network efficiency, medical service and medical resource characteristics, the bottleneck of medical resources is calculated. Then, bottleneck medical resources and medical services related to bottleneck resources are determined based on bottleneck analysis. Finally, the patient scheduling is carried out according to the bottleneck priority rules. The rationality and superiority of the model and algorithm presented in this paper are verified by the example of an outpatient clinic in a certain hospital.

1. Introduction

Hospital medical resources are relatively scarce. How to balance the relationship between patients and medical resources, shorten patient waiting time, make full use of resources and improve the efficiency of diagnostic resources, has become an important problem for most hospitals.

Now, aiming at medical scheduling, scholars have conducted a series of studies. Day studied cardiac diagnosis center real time resource allocation scheduling for outpatient and inpatient, a discrete-time Markov decision process model with finite planned horizon is established, for coping with the growing state space, a fast heuristic rule is introduced to realize the on-line dynamic decision of diagnosis resource allocation; Kortbeek studies the scheduling of reserved and non-scheduled patients, two methods of full enumeration and heuristic algorithm are proposed, and simulation verification and comparative analysis are carried out in the allocation process of CT scanner.

The analytic model represented by complex network theory has obvious advantages in system modeling, and provides a new opportunity for solving resource scheduling problem. The arrival of outpatients has the characteristics of dynamics, randomness and uncertainty, in line with the characteristics of dynamic modeling of complex network, therefore, in this paper, the complex network theory is adopted to construct the complex network model of medical resources by
taking outpatients as scheduling objects, medical resources as nodes and medical pathways as connecting edges, meanwhile, using network characteristics and outpatient medical characteristics to identify bottleneck of medical resources [9], proposing an outpatient priority scheduling algorithm based on bottleneck analysis of medical network, reasonable scheduling of hospital resources and patient services, which has shortened the waiting time of the patients and ensured the medical resources utilization. Finally, this paper takes the outpatients scheduling of a hospital as an example to verify the rationality and superiority of the proposed algorithm model.

2. Construction of Medical Resource Complex Network (MRCN) Model

2.1 MRCN Network Characterization Definitions

The complex network theory is good at finding model, rules, which has obvious advantages in system modeling. The arrival of outpatients has the characteristics of dynamic, random and uncertainty, each patient usually contains a number of medical services, completely in conformity with the characteristics of dynamic complex network modeling. So, this paper uses complex network theory to model and analyze the scheduling problem.

This paper extracts three representative parameters of betweenness, network efficiency and medical flow to identify bottleneck medical resources in medical resource network. Specific definitions are as follows:

**Definition 1: Medical Resource Node Betweenness**

The node betweenness \( B_r \) is used to describe the centrality of the node \( r \) in the overall network, as shown in equation (1)

\[
B_r = \sum_{s,t \in m} \frac{\delta_{st}(r)}{\delta_{st}}
\]

Type: \( B_r \in [0, 1] \), The higher the centrality of the node \( R \), the closer its value is to 1. \( \delta_{st} \) is the number of medical tasks from node \( s \) to node \( t \). \( \delta_{st}(r) \) is the number of all medical tasks from node \( s \) to node \( t \) through node \( r \). The higher the betweenness of nodes, represent the probability of medical tasks through the resource node is greater.

**Definition 2: Medical Resource Network Efficiency**

The efficiency of network \( e_{st} \ (s, t \in m) \) refers to the reciprocal of the distance between any two nodes \( (s, t) \) (the shortest number of consecutive nodes), \( e_{st} = 1/\delta_{st} \). In MRCN, the degree of coupling efficiency of the network can be characterized by two medical services, the greater the distance, the smaller the coupling. Network efficiency is the most direct interaction between nodes, the most efficient form. Therefore, the efficiency of MRCN matrix with \( m \) nodes can be expressed as:

\[
E = \begin{bmatrix}
e_{11} & e_{12} & \ldots & e_{1m} \\
e_{21} & e_{22} & \ldots & e_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
e_{m1} & e_{m2} & \ldots & e_{mm}
\end{bmatrix}
\]

**Defining 3: Medical flow**

Node \( r \)'s medical flow refers to the ratio of the total time spent by all patients occupying \( r \) to the rated working time of \( r \) node, as shown in equation (3)

\[
W_r = \frac{\sum_{i=1}^{h} A_{ir} T_{ir}}{T_r}
\]

Among them, \( W_r \) said the medical flow node of \( r \), \( h \) is the total number of patients using the \( r \) resource, \( A_{ir} \) resources coefficient, if the \( j \) medical service of \( i \) patients can be completed in \( r \) resources, then \( A_{ir} = 1 \), and vice versa, \( A_{ir} = 0 \). \( T_{ir} \) handles the standard time for item \( j \) of the \( i \) patient for the \( r \) resource. \( T_r \) rated working time for resource \( r \).
3. MRCN Network Bottleneck

In MRCN, a node bottleneck degree depends on two aspects: (1) the importance of medical resources in MRCN, evaluate the calculation using the network efficiency ($B_\tau$); (2) The contribution of other nodes in the network to the bottleneck of the network, using the node flow value contribution to characterize the bottleneck of the whole network of reference nodes.

The extent to which each node in the network depends on the remaining nodes can be calculated by the contribution matrix, as shown in equation (4):

\[
S = \begin{bmatrix}
    s_1 \\
    s_2 \\
    \vdots \\
    s_m
\end{bmatrix} = E \cdot W_f
\]

Therefore, the relative importance of the $r$ node in the overall MRCN is

\[
S_r = \sum_{i=1,i \neq r}^{m} s_{ij}
\]

By considering the global centrality of the nodes to be evaluated--betweenness and other nodes' contribution to the bottleneck of the evaluation nodes, the bottleneck of the $r$ node is obtained:

\[
P_{BN} = B_\tau \cdot S_r
\]

According to the TOC theory, all the services in the medical process are divided into three parts: Bottleneck medical services, upstream medical services and downstream medical services.

4. Outpatient Medical Resource Scheduling Algorithm

4.1 Objective Function of Outpatient Scheduling Problem

The outpatient scheduling problem can be described as: there were $n$ patients, respectively $O_1$, $O_2$, ..., $O_n$, that is, there are $n$ medical tasks; Each patient required $m$ medical service, $O_1^1$, $O_1^2$, ..., $O_1^m$, that is, $m$ medical services; A total of $P$ medical resources, $P_1$, $P_2$, ..., $P_n$; $T_{ijk}$ is the standard medical examination time for $i$ patients in the medical services $j$ at the medical resource $k$. The outpatient visits are random and the examinations are independent of each other.

The objective function of scheduling is to minimize the average waiting time of patients and maximize the utilization rate of bottleneck medical resources:

\[
\min f_1 = \min \left\{ \text{average} \left\{ C_i | i = 1,2,...,n \right\} \right\}
\]

Among them, $f_1$ represents the average waiting time of patients, and $C_i$ represents the total waiting time of a patient.

\[
f_2 = \max \left\{ D_i | i = 1,2,...,n \right\}
\]

Among them, $f_2$ represents maximized bottleneck medical resource rate, and $D_i$ represents bottleneck medical resource rate. When the two scheduling objectives conflict, take the average waiting time of patients is minimized as the main objective.

4.2 Algorithm Constraint Function

1) There is precedence constraint between patients' medical services

\[
S_{ij} + t_{ij} - S_{ij+1} \leq 0, \forall i \in \{1,2,...,n\}, \forall j \in \{1,2,...,n-1\}
\]

Among them, $S_{ij}$ is the start time of $O_i^j$, and $t_{ij}$ is the medical time of $O_i^j$.

2) Medical resources can only be carried out in one medical service at the same time

\[
S_{ij} - T_{r(k-1)} \geq 0
\]

\[
\forall i \in \{1,2,...,n\}, \forall j \in \{1,2,...,O_i\}, \forall k \in \{1,2,...,n\}
\]
Among them, K represents the K service of the resource r, and \( T_r (k-1) \) is the end time of the previous service on the resource R.

4.3 Algorithm Flow
Scheduling rules are as follows:

1. The operation of non-bottleneck medical service should obey the optimal scheduling of bottleneck medical service unconditionally;
2. Non bottleneck medical services can not realize their own interests by impairing the interests of the bottleneck medical service. The specific scheduling process is shown in figure 2.

![Algorithm Flow Diagram](image)

**Figure 1** Scheduling process of complex network priority for medical resources.

The algorithm steps are as follows:

Step1: Basic data processing, quantification and normalization.
Step2: The medical service of all patients is presorted according to the scheduling objective, and then the sorted medical service flow, its starting time and the completion time are obtained. The scheduling time division is carried out.
Step3: Calculate the bottleneck of medical resource nodes in different time periods.
Step4: Identify medical services related to bottleneck resources, bottleneck based priority algorithm scheduling is carried out, and the starting time and the completion time of bottleneck medical service are obtained (For multiple bottlenecks in the same time period, according to the bottleneck size of scheduling).
Step5: Restart the upstream medical service and the downstream medical service start time and the completion time.
Step6: Go through the next time period and repeat step 3-6, with all patient medical services scheduled.
5. Verification

5.1 MRCN Model Generation
The algorithm was validated by medical service in ophthalmology clinic, and table 1 was the item list of ophthalmology clinic in a hospital.

Table 1: The check list of an eye clinic in a hospital.

| Examination room | Examination project          | Examination room | Examination project          |
|------------------|-----------------------------|------------------|-----------------------------|
| 1                | A/B-ultrasound              | 9                | Children with amblyopia training/synoptophore |
| 2                | Fundus camera               | 10               | Corneal thickness/corneal topography/Three position corneal topography/auto-refraction |
| ...              | ...                         | ...              | ...                         |
| 8                | Subjective trial of lens/Retinoscopy | 16 | Perimetry examination of visual field/Dry eye three items |

Patients usually check project by doctors are given, from a normal working day in all patients with medical data, the data contains 16 items of 82 outpatients in the medical service, establish arrival time - use of medical resources - medical service time table:

Table 2: Arrival time - Use of medical resources - Medical service time table.

| Patient | Arrival time - Use of medical resources - Medical service time |
|---------|---------------------------------------------------------------|
| 1       | 1.1,5 1.5,10 1.3,8 ... 0.0,0 0.0,0 |
| 2       | 1.4,1 1.2,10 2.5,17 ... 0.0,0 0.0,0 |
| ...     | ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... |
| 82      | 7.2,1 7.6,20 8.5,15 ... 0.0,0 0.0,0 |

Among them: 1.1,5 represents the arrival at 1, the use of medical resources 1’ time is 5 minutes. 0.0,0 represents the patient without the medical service.

5.2 Bottleneck Calculation
The bottleneck of each node at different time is calculated according to the established algorithm, and the result is shown in table 3:

Table 3: The bottleneck degree of medical resources.

| Medical resources | The bottleneck degree of medical resources |
|-------------------|-------------------------------------------|
| Node.1            | 0.29 0.27 1.55 0.97 0.96 0.21 0.41 0.80 |
| Node.2            | 0.92 0.91 0.89 0.86 0.79 1.34 0.87 0.86 |
| ...               | ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... |
| Node.15           | 0.53 0.41 0.39 0.42 0.54 1.43 1.54 1.57 |
| Node.16           | 0.23 0.31 0.09 0.04 0.16 0.21 0.27 0.19 |

5.3 Scheduling Simulation
The scheduling results for a given period of time are shown in figure 4 (During this time period, medical resources 4 are bottleneck resources, and there are 9 medical resources at work, 12 patients were treated, the 12 patients were divided into 6 groups for scheduling, each group had the same medical task, see table 4).
Table 4 Use of medical resources - Medical service time.

| Group | Use of medical resources - Medical service time |
|-------|------------------------------------------------|
| 1     | 1,10 2,6 4,16 6,18 |
| 2     | 2,6 3,4 4,14 5,14 |
| 3     | 1,10 4,18 5,14 7,10 |
| 4     | 1,10 2,18 3,4 4,12 |
| 5     | 3,4 6,4 8,10 9,10 |
| 6     | 1,10 3,4 8,10 9,10 |

Among them: 1,10 represents the use of medical resources 1 time is 10 minutes.

At the same time, for the same set of data, using FCFS (first come, first served) and GA (genetic algorithm) scheduling, in order to achieve the algorithm contrast, the results shown in figures 5 and 6:

Figure 2 Scheduling results Gantt chart based on bottleneck priority

Figure 3 Scheduling results Gantt chart of FCFS algorithm

Figure 4 Scheduling results Gantt chart of GA algorithm.

In the figure, the numbers in the box indicate a patient's medical service on a resource, such as "21" representing the medical services of first patients on Second Medical resources.

5.4 Result Analysis

Figure 5 The average patient waited in comparison.

Figure 6 Bottleneck medical.
Comparative analysis: as shown in Figure 7, the bottleneck based scheduling algorithm has the shortest and optimal scheduling results compared with other rules in the average waiting time of patients. As illustrated in Figure 8, the bottleneck resource scheduling algorithm based on bottleneck priority scheduling algorithm is higher than the GA scheduling results in the first five time periods, at the latter three moments, below the GA scheduling results, at time 1, 2, 6, 7, 8, higher than the FCFS algorithm, the rest of the time is lower than the FCFS algorithm. According to the FCFS algorithm, patients usually line up the inspection documents according to the doctor's order of inspection, which is easy to form congestion on the bottleneck project, the formation time of 3 patients average waiting time to peak, congestion usually does not disappear immediately, resulting in the bottleneck in 4,5,6 utilization of resources is too high; For GA scheduling, the algorithm has the longest total service time and the patient priority, so the average waiting time is greatly shortened, which is lower than the FCFS algorithm; For bottleneck based priority algorithm, bottleneck service priority arrangement, the bottleneck service waiting time is the shortest, and non-bottleneck service usually does not cause great waiting, so the average waiting time is the shortest.

6. Conclusion
From the point of view of complex network, a bottleneck algorithm for medical resource nodes is proposed, a quantitative description of the bottleneck of medical resources is realized, and a priority scheduling scheme for outpatient medical service is proposed based on bottleneck analysis, the algorithm reduces the average waiting time of the patients while guaranteeing the utilization of the bottleneck resources. Although this study is aimed at outpatient patients, the models and algorithms can be easily extended to other types of medical procedures. Aiming at the bottleneck of medical resources dynamic recognition, network analysis method can take into account the global and local, which has certain universality, and it can also be extended to other engineering fields, provide new methods and ideas for solving discrete and continuous optimization problems.

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