Improved H-Infinity Hybrid Model Predictive Fault-Tolerant Control for Time-Delayed Batch Processes Against Disturbances

HUI YI AND ZIYI CHEN

1 College of Electrical Engineering and Control Science, Nanjing Tech University, Nanjing 211816, China
2 School of Mathematics and Statistics, Hainan Normal University, Haikou 571158, China

Corresponding author: Hui Yi (jsyihui@126.com)

This work was supported in part by the National Project Funding for Key Research and Development programs under Grant 2018YFC0808500, and in part by the National Natural Science Foundation of China under Grant 61773190.

ABSTRACT An H-infinity model predictive fault-tolerant control strategy is proposed for multi-phase batch processes with interval delay and actuator failures. First, state variables, state errors and output tracking errors are introduced to establish an extended-state-space switched system model. Then, based on this model, a predictive fault-tolerant control law is designed for tracking the set point by the output that satisfies the requirements of the optimal performance index under input and output constraints. The feasibility conditions for the solvability of the control law are presented in the form of linear matrix inequalities. In addition, the designed switching law is constructed, and the gain of the control law is obtained via the optimization algorithm. This design has several advantages: the output tracking is faster, the tracking performance is superior, and the trace is smoother at the switching time. Finally, through a comparison with traditional methods, the effectiveness and feasibility of this method are demonstrated via injection molding simulation.

INDEX TERMS Batch production systems, added delay, fault tolerant control, linear matrix inequalities, Lyapunov methods, predictive models, switching systems, predictive control.

I. INTRODUCTION

As industrial production models exhibit a variety of characteristics, such as small scale, diversity, high added value and technological intensiveness, batch production technology has attracted increasing attention and has begun to play an important role in many fields. Although many studies have considered batch processes, the high-precision control of modern industrial processes remains a challenge. One reason for such challenges is the occurrence of time delays and perturbations, which may deteriorate the tracking performance and result in reduced production efficiency [1]–[3]. Furthermore, both the requirement for a high automation level and the complex process conditions increase the likelihood of system faults, including actuator faults, internal faults, and sensor faults. Actuator faults, which are the most frequently occurring faults, substantially impact the system. If these faults are not detected and corrected in a timely manner, then the production performance will deteriorate, and equipment and personnel safety issues may occur. Once a fault has been detected, the corresponding fault-tolerant control strategy must be quickly implemented to mitigate the impact of the fault on the control performance of the system. Fault-tolerant control refers to the tolerance of the system to faults such that, after a fault occurs, the control performance is not substantially affected, and the system can still operate stably for a period of time in the current state to ensure the quality of the products produced during this period.

In recent years, there have been many achievements in the research on fault-tolerant control for batch processes [4]–[8]. These results mainly occurred in two areas. In one area, the batch process is regarded as a one-dimensional (1D) system that is related only to time, as in [4] and the references therein. In the other area, the batch process is regarded as a two-dimensional (2D) system that is related to time and the batch direction. In [7], the author transformed a batch production process with unknown perturbations and actuator faults into a two-dimensional Fornasini-Marchesini (2D-FM)
model and designed a controller that ensures the closed-loop convergence of the system along the directions of time and batch. Regarding uncertainty, state delays and actuator failures, Wang et al. [8] proposed H-infinity learning fault-tolerant guaranteed cost control based on an equivalent 2D system description of these processes.

A series of research results have been presented for the fault-tolerant control of batch processes. The most popular methods for controlling batch processes are 2D iterative learning control (ILC) methods based on the repetitiveness of production processes [9]–[14], including single-phase batch processes [12], [13] and multi-phase batch processes [14]. However, as such methods adopt the same control law, if the deviation of the system output from the set value exceeds a threshold, then the deviation will continuously increase with time. Moreover, for safety, system constraints and restrictions must be considered during the design of the control system. It is imperative to develop new control methods. Model predictive control (MPC) with feedback correction and rolling optimization at every moment is widely used. In references [15]–[23], a set of control strategies that combine ILC with MPC are proposed. Recently, MPC was combined with fault-tolerant control (FTC) and applied [24]. Most research results focus on 2D systems theory.

As batch processes are processes that vary slowly with time, no perfect repetitiveness occurs; i.e., if system information cannot be repeated between batches, then the ILC method is no longer applicable. At this time, it is more suitable for the production process to be regarded as a 1D system [25]–[30]. Among these results, various design methods are combined with model predictive control, such as the neural network method and the extended-state control model method. The design strategy of the extended-state control model method is to introduce the state error and the output error along the time direction, expand the original model into a new error model, and design a control law under this model for the realization of system tracking control. This method has been widely applied since it has produced satisfactory control results. Another advantage of this design is that a time-delayed system can be transformed into a non-time-delayed system via dimension expansion. Another design method is available for time-delayed systems, in which the controller design depends on the upper and lower bounds of the time delay. Although it is difficult to design the Lyapunov function for the latter solution, its dimensionality and computing load are smaller than those of the former solution. If the control process involves uncertainties and external perturbations, then the stability of MPC will be negatively affected, while the H-infinity model predictive control strategy [31], [32] can analyze the stability of uncertain closed-loop systems, reduce the impact of perturbations on the system’s control performance, and enable the controlled objects to remain robust to external perturbations. Therefore, such control methods are widely used.

The above research on batch processes focused mainly on single-phase processes. However, most batch processes are multi-phase processes, and the phases influence one another. Multi-phase batch process control has become a hot topic in research [29], [33]–[36]. In [34], a switching system model was used to study a multi-phase batch production process, and a related predictive control strategy was proposed. In [35], the average dwell time method was used to study a multi-phase system, and an optimal control strategy is proposed. According to the research results, these models were all based on two-dimensional systems. As we have explained, the information of batch processes is not always repeated in the batch direction. In addition, few studies consider time delays, external disturbances and other issues. Therefore, the available research results cannot satisfy the market demand, and further research on multi-phase batch processes is urgently needed, especially for cases in which the batch information is not repeated and time delays occur.

This paper proposes an improved H-infinity hybrid model predictive fault-tolerant control method for time-delayed batch processes against disturbances. In contrast to control approaches for single-phase batch processes from the literature discussed above, for multi-phase batch processes, we have introduced not only the above variables but also new state variables that are related to output errors to form an extended switching system model. The main contributions of this work are as follows: (1) A new type of model is constructed, and the suitable switching conditions and running times in various phases are specified based on this model. (2) The solvability condition of model predictive control is specified in terms of linear matrix inequalities (LMIs), and the upper bound for the system’s optimal performance index is identified to improve the efficiency of the production process. (3) The designed controller is robust against disturbances and time delays, and it realizes superior tracking performance. Finally, to evaluate the performance of the proposed method, the proposed method is compared with the traditional method. The results demonstrate that the proposed method realizes a short running time and satisfactory tracking performance.

The remainder of this paper is organized as follows. Section II describes the system. In Section III, the design of model predictive fault-tolerant control is considered. In this part, the equivalent model is established, the sufficient conditions for the feasibility of model predictive control are established, and the optimization algorithm is constructed. An injection molding simulation example is presented to illustrate the performance of the proposed methods in Section IV. Finally, the conclusions of this study are presented in Section V.

\[ x(k + 1) = (A_\sigma(k) + \Delta_\sigma(k))x(k) + A_d(k)x(k - d(k)) + B_\sigma(k)u^F(k) + w_\sigma(k) \]

\[ y(k) = C_\sigma(k)x(k) \]
where $k$ is the finite discrete-time; $x(k)$, $y(k)$ and $u^f(k)$ represent the system state, system output and system control output under a fault condition, respectively; $v(k)$ is the unknown external perturbation; $\alpha(k) = [0, \infty) \rightarrow \tilde{p} = \{1, 2, \ldots, N\}$ is the time- or state-dependent piecewise constant switching signal; $\beta(k) = i$ denotes the activation of the $i$th system; $A^i$, $B^i$, $A^d_i$, $C^i$ are the coefficient matrices for the $i$th subsystem with suitable dimensions; $\Delta^i_d$ is the perturbation matrix of unknown parameters, which satisfies the condition $\Delta^i_d = D^iF^i(k)E^i$; $D^i, E^i$ are known real matrices with suitable dimensions; and $F^i(k)$ is an unknown matrix that satisfies the condition $F^i(k)F^iT(k) = I$.

The interval time delay $d(k)$ satisfies the condition

$$d \leq d(k) \leq \bar{d}$$

where $d$ and $\bar{d}$ denote the upper and lower bounds, respectively, of the interval time delay.

**Remark 1:** Some real systems can be expressed in the form of system (1). For example, the process of a continuous stirred tank reactor (CSTR) in [37] can be expressed in this form; in [38], when the update law with an input delay is designed, it can be converted into model (1).

In practical production operation, due to the long-term overload operation of the equipment, actuator faults in the system are unavoidable; hence, it is difficult for the input overload operation of the equipment, actuator faults in the system to operate smoothly even after the occurrence of a fault. In addition, the proposed strategy is a passive FTC that handles the fault occurrence as parameter uncertainties. Based on the conditions of activation at various phases, phase $i$ control input $u^f_i(k)$ represents the input signal under an actuator fault, which can be expressed as:

$$u^f_i(k) = \alpha^i u^i(k) \quad \text{with} \quad \alpha^i < \alpha^i < \bar{\alpha}$$

Therefore, the batch process with an interval time delay, a perturbation and an actuator fault can be expressed as:

$$x^{i+1}(k) = (A^i + \Delta^i_d(k))x^i(k) + A^i_d x^i(k-d(k)) + B^i \alpha^i u^i(k) + w^i(k)$$

$$y^i(k) = C^i x^i(k)$$

The following notations are introduced:

$$\beta = \text{diag} (\beta_1, \beta_2, \ldots, \beta_m), \quad \beta_0 = \text{diag} (\beta_{10}, \beta_{20}, \ldots, \beta_{mn}),$$

$$\beta^i = \frac{\bar{\alpha} + \alpha^i}{2}, \quad \beta_0 = \frac{\bar{\alpha} + \alpha^i}{2},$$

Therefore, $\alpha^i_0$ exists; hence,

$$\alpha^i = (I + \alpha^i_0) \beta^i, \quad |\alpha^i_0| \leq \beta_0 \leq I^i,$$

where

$$\alpha^i_0 = \text{diag} (|\alpha^i_0|, |\alpha^i_0|, \ldots, |\alpha^i_0|),$$

$$|\alpha^i_0| = \text{diag} (|\alpha^i_0|, |\alpha^i_0|, \ldots, |\alpha^i_0|).$$

For system (4), the controller will be designed using the novel model predictive fault-tolerant control method to ensure satisfactory control performance. The main steps are as follows: the error model and new state variables are introduced and used to transform the model into an equivalent model, and the novel controller and the switching law are designed on this basis. The main strategy is as follows: The difference operator $\Delta$ is introduced, and $\Delta x(k+1) = x(k+1) - x(k)$ is defined. Then, the following equation can be obtained from model (4):

$$\Delta x^i(k+1) = A^i(k) \Delta x^i(k) + A^i_d \Delta x^i(k-d(k)) + B^i \alpha^i \Delta u^i(k) + \tilde{w}^i(k)$$

where $\alpha^i_0$ is the control input at the current time.
where
\[
\hat{w}(k) = [\Delta J(x) - \Delta J(x-1)]x(k-1) + \Delta u(k)
\]
\[A'(k) = (A^T + \Delta A_1),\text{ and } e(k)\text{ is the error between the system output and the expected output at the } i^{th} \text{ phase:}
\]
e(k) = y(k) - y_r(k) \tag{6}

The following equation can be obtained from (5) and (6):
e(k + 1) = C'\Delta x(k) + C'\Delta u(k) + \tilde{G}'\tilde{w}(k) + e(k) \tag{7}

A new state variable is introduced:
\[
\hat{x}(k+1) = \tilde{x}(k) + e(k) \tag{8}
\]
where \(\tilde{x}(k)\) is selected based on the state of the extended information that is determined by \(e(k)\). Let
\[
\hat{x}(k) = [\Delta x(k) \quad \tilde{x}(k) \quad e^T(k)]^T,
\]
then the state model of the dimension-extended system with extended information at the \(i^{th}\) phase is obtained:
\[
\hat{x}(k + 1) = \tilde{A}'\hat{x}(k) + \tilde{A}_d\tilde{x}(k - d(k)) + \tilde{B}'u' + \tilde{G}'\tilde{w}(k) \tag{9}
\]
where
\[
\tilde{A}' = \begin{bmatrix} B' \\ 0 \\ C'\tilde{B}' \end{bmatrix}, \quad \tilde{C}' = \begin{bmatrix} I \\ 0 \\ C' \end{bmatrix}, \quad \tilde{A}_d = \begin{bmatrix} A_d' \\ 0 \\ 0 \end{bmatrix}, \quad \tilde{G}' = \begin{bmatrix} A_d' \\ 0 \\ 0 \end{bmatrix}.
\]

When the \(i^{th}\) phase is switched over to the \(i + 1^{th}\) phase, the phase transition can be expressed as:
\[
\tilde{x}(k + 1) = \tilde{A}'\tilde{x}(k) + \tilde{A}_d\tilde{x}(k - d(k) - 1) + \tilde{B}'u' + \tilde{G}'\tilde{w}(k) \tag{10}
\]
\[
\text{where } \tilde{K}' \text{ denotes the gain of the proposed controller. At this time, model (9) is transformed into:}
\]
\[
\tilde{x}(k + j + 1) = \tilde{A}'(k + j)\tilde{x}(k + j) + \tilde{A}_d\tilde{x}(k - d(k) + j) + \tilde{B}'\tilde{u}(k + j) + \tilde{G}'\tilde{w}(k + j) \tag{11}
\]

The optimal performance indices for system (11) are designed as follows:
\[
\min \Delta u(k + j | k) \Delta u^T(k + j | k), \sum_{j=0}^{\infty} \left[ \tilde{u}(k + j | k) + \Delta u^T(k + j | k)\tilde{r}_j \right] \tag{12}
\]

where the following condition must be satisfied:
\[
\left\| \Delta u^T(k + j | k) \right\| \leq \Delta u_m, \left\| \tilde{r}^2(k + j | k) \right\| \leq \tilde{y} \tag{13}
\]

The minimum upper-bound value of the objective function (performance index \(J_\infty(k)\)) is obtained under maximum perturbation and minimum control input. In the formulas above, \(Q_j'\) and \(R_j'\) are the process state weight matrix and the input weight matrix, respectively, and \(\Delta u_m^T\) and \(\Delta u_m\) are the upper-bound values of \(\Delta u^T\) and \(\tilde{r}^2\), respectively. The following notation is introduced to simplify the expression:
\[
V_i(\tilde{x}(k + j | k)) = V_i^T(\tilde{x}(k + j | k)) = V_{ij}, \quad \tilde{x}_j = \tilde{x}(k + j | k), \quad \tilde{x}_{ij} = \tilde{x}(r + j | k), \quad \tilde{x}_{d+1} = \tilde{x}(k + j - d(k), \quad \tilde{x}_{d+1} = \tilde{x}(k + j | k), \quad \tilde{x}_{d+j} = \tilde{x}(k + j + 1 | k) - \tilde{x}(k + j | k), \quad \tilde{x}_{d+j} = \tilde{x}(r + j + 1 | k) - \tilde{x}(r + j | k).
\]

Using the Lyapunov stability theorem, the controller design problem is transformed into the equivalent model stability problem. To demonstrate the system stability, the Lyapunov-Krasuski function (LKF) is defined as:
\[
V_i = \sum_{i=1}^{\infty} V_i(\tilde{x}(k + j | k)) = \tilde{x}_j^T P_i \tilde{x}_j = \tilde{x}_j^T \tilde{T}_1 \tilde{x}_j, \quad V_2 = \sum_{k=1}^{\infty} \tilde{x}_j^T Q_k \tilde{x}_j = \sum_{k=1}^{\infty} \tilde{x}_j^T M_k \tilde{x}_j, \quad V_3 = \sum_{k=1}^{\infty} \tilde{x}_j^T \tilde{M}_k \tilde{x}_j.
\]
\[ V_{dj} = \sum_{k=d}^{k-1} \sum_{r=k+s} \bar{x}_{rj}^T \bar{a}_j^k \theta^i \bar{T}_2^{-1} \bar{x}_{rj} \]
\[ V_{sj} = \bar{d} \sum_{s=d}^{s-1} \sum_{r=k+s} \bar{x}_{rj}^T \bar{a}_j^k \theta^i \bar{G}_2^{-1} \bar{x}_{rj} \] (14)

where \( \bar{P}_1^i, \bar{T}_1^i, \bar{M}_1^i, \bar{G}_1^i \) are positive-definite symmetric matrices, \( 0 < \bar{a}_i < 1 \), and \( \theta^i \) is a positive number. To ensure system stability, it is necessary to satisfy the following Lyapunov constraints:

\[ J^i_j = \bar{x}_{j}^T \bar{Q}_1 \bar{x}_{j} + \Delta u_j^T \bar{R}_1 \Delta u_j - \eta^i \bar{w}_j^T \bar{w}_j \]
\[ V_{j+1}^i - V_j^i \leq \bar{a}_j V_j^i \leq -J^i_j \] (15)

Moreover, if \( V^i(\bar{x}(\infty)) = 0 \), \( \bar{x}(\infty) = 0 \), and the upper bound of \( J^i_\infty(k) \) exists and satisfies \( \theta^i > 0 \), then:

\[ J^i_\infty(k) \leq V^i(k) \leq \theta^i \] (17)

**Definition 1:** For any \( t > t_0 \) and any switching signal \( \sigma(k) \), where \( t_0 \leq k < t \), \( N_1^i(t_0, t) \) denotes the number of switches of the \( i \)th subsystem within the time interval \((t_0, t)\). \( \bar{T}_i(t_0, t) \) is called the total operating time of the \( i \)th subsystem. For any specified \( t_i > 0 \), the following formula is established: \( N_1^i(t_i, t) \leq \frac{\bar{T}_i(t_i, t)}{t_i} \), where \( t_i > 0 \) is the average dwell time.

**Definition 2:** System (11) is said to be robustly exponentially stable under the switching signal if there exist positive constants \( a, b \), and \( 0 < v_i < 1 \) such that the following formula holds:

\[ \| \bar{x}_{T} \|_i = \sqrt{\frac{b}{a}} v_{-t_0} \| \bar{x}_{T} \|_i \]

**Lemma 1 (Schur Complement Lemma):** Let \( W, L \) and \( V \) be matrices of suitable dimensions in which \( W, V \) are real matrices. Then,

\[ L^T V L - W < 0 \]

if and only if

\[ \begin{bmatrix} -W & L^T \\ L & -V^{-1} \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -V^{-1} & L \\ L^T & -W \end{bmatrix} < 0. \]

**Lemma 2:** If \( D, F, E \) and \( M \) are positive-definite matrices with suitable dimensions, where \( M = M^T \) and \( F^T F < I \), then the following inequality is established, and if there exists \( \varepsilon > 0 \), then the sufficient and necessary condition for the establishment of the inequality is

\[ M + \varepsilon^{-1} D D^T + \varepsilon E^T E < 0. \]

**Lemma 3:** For any vector \( \delta(t) \in \mathbb{R}^n \), positive integers \( k_1 \) and \( k_2 \), and matrix \( R \in \mathbb{R}^{n \times n} \), the following matrix inequality is established:

\[-(k_1 - k_2 + 1) \sum_{i=k_1}^{i=k_2} \delta^T(t) R \delta(t) \leq -\sum_{i=k_1}^{i=k_2} \sum_{i'=k_1}^{i'=k_2} \delta^T(t) R \delta(t) \leq \sum_{i=k_1}^{i=k_2} \sum_{i'=k_1}^{i'=k_2} \delta^T(t) R \delta(t) \]

\[ V^i < \mu_i V^{i-1} \]

Theorem 1: If \( \bar{w}_i(k) \neq 0 \), for specified constants \( \bar{d} \leq \bar{d}(k) \leq \bar{d}, \theta^i > 0 \), and \( 0 < \bar{a}_i < 1 \), if there exist positive definite symmetric matrices \( \bar{P}_2^i, \bar{M}_2^i, \bar{T}_2^i, \bar{G}_2^i, \bar{T}_3^i, \bar{M}_3^i, \bar{G}_3^i, \) and \( \bar{Y}_1^i \) and positive real numbers \( \eta^i, \varepsilon_a, \) and \( \varepsilon_b \) such that the following matrix inequalities are feasible for the switching signal with an average dwell time that satisfies the following inequality (23)

\[ \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} & \bar{Y}_{13} & \bar{Y}_{14} & \bar{Y}_{15} \\
\bar{Y}_{22} \bar{Y}_{23} \bar{Y}_{24} \bar{Y}_{25} \bar{Y}_{26} \bar{Y}_{27} \bar{Y}_{28} \bar{Y}_{29} \bar{Y}_{30} \bar{Y}_{31} \bar{Y}_{32} \bar{Y}_{33} \bar{Y}_{34} \bar{Y}_{35} \bar{Y}_{36} \bar{Y}_{37} \bar{Y}_{38} \bar{Y}_{39} \bar{Y}_{40} \bar{Y}_{41} \bar{Y}_{42} \bar{Y}_{43} \bar{Y}_{44} \bar{Y}_{45} \bar{Y}_{46} \bar{Y}_{47} \bar{Y}_{48} \bar{Y}_{49} \bar{Y}_{50} \bar{Y}_{51} \bar{Y}_{52} \bar{Y}_{53} \bar{Y}_{54} \bar{Y}_{55} \end{bmatrix} < 0 \]

(18)

\[ \bar{P}_2^i \bar{G}_2^{-1} \bar{P}_2^i + \bar{G}_2^i \bar{P}_2^{-1} \bar{G}_2^i = \bar{T}_2^i \bar{P}_2^i \bar{M}_2^{-1} \bar{P}_2^i = \bar{M}_2^i, \bar{G}_2^i \bar{M}_2^{-1} \bar{G}_2^i = \bar{M}_2^i, \bar{G}_2^i \bar{P}_2^{-1} \bar{G}_2^i = \bar{P}_2^i, \bar{P}_2^i = \bar{P}_2^i \bar{T}_3^i \bar{M}_3^{-1} \bar{T}_3^i + (\bar{d} + \bar{d} + 1) \bar{T}_3^i - \bar{a}_i \bar{G}_3^i, \]

where \( * \) represents the transposed element in the symmetric position; then, the robust model predictive fault-tolerant control problem is solvable, and the fault-tolerant controller gain in the system is \( K^i = \bar{Y}_1^i \bar{P}_2^{-i} \). The LMI (18) is the sufficient condition. If the LMI (18) holds, then the system is stable.
Proof: Multiplying the left and right sides of inequality (18) by \( \text{diag} \left[ P^{-1}_2 T^{-1}_2 \bar{G}_2^{-1} \right] \), by using Schur complement Lemma 1 and Lemma 2, due to the occurrence of a perturbation and a fault in the control system, and letting
\[
\bar{A}' = A'(k) + B'\alpha_i\bar{K}' = A' + \Delta A' + B' \left( \beta' + \alpha_i\beta' \right) \bar{K}'
\]
we obtain:
\[
\left[ \begin{array}{c}
\varphi' \\
0 \\
\bar{G}_2^{-1} \\
0 \\
-\bar{G}_2^{-1}
\end{array} \right] \leq \left[ \begin{array}{c}
\bar{A}' \\
0 \\
0 \\
0 \\
0
\end{array} \right]
\]
\[
\leq \left[ \begin{array}{c}
\bar{A}' \\
0 \\
0 \\
0 \\
0
\end{array} \right]
\]

By summing the inequalities (26) and multiplying both sides by \( \theta^{-i} \), we obtain:
\[
\Delta V_j^i = \sum_{r=1}^{s} \Delta V_j^i(k+j|k) \\
\theta^{-i} \Delta V_j^i \leq \left[ \begin{array}{c}
\bar{e}'_j \\
0 \\
0 \\
0 \\
0
\end{array} \right] \Phi_2 \left[ \begin{array}{c}
\bar{e}'_j \\
0 \\
0 \\
0 \\
0
\end{array} \right] < -\theta^{-i} J_j^i(k)
\]

Hence,
\[
\Delta V_j^i < -J_j^i(k)
\]

The system is proved to comply with the Lyapunov inequality constraints. In addition, according to the performance function (15), \( J_j^i(k) > 0 \); therefore, \( \Delta V_j^i < 0 \), namely, \( V_j^i(k+1) < V_j^i(k) \). By setting \( V_j^i = V^i(k) \), the following holds for \( t_0 < k < t \):
\[
V^{\sigma_0}(t) \leq \alpha_i V^{\sigma_0}(t-1) \leq \alpha^{i-T_i^i} V^{i-1}(T_s^{i-1})
\]
where \( T_s^{i-1} \) denotes the switching time at the \( i \)th phase. From (19), we obtain
\[
V^{\sigma_0}(t) \leq \alpha_i^{i-T_i^i} \mu_i V^{i-1}(T_s^{i-1}) \leq \prod_{i=1}^{p} (\alpha(i)^{T_i(t_0-i)} \prod_{i=1}^{p} (u_i)^{T_i(t_0-i)} V^{\sigma_i}(t_0))
\]
and considering the definition of the Lyapunov function, we obtain:
\[
\alpha \|x_j^i(t+j|t)\| ^2 \leq V^{\sigma_0}(t) \leq \nu V^{\sigma_0}(t_0)
\]

where
\[
\|x_j^i(t+j|t)\| \leq \sqrt{\frac{b}{a}} \nu^{t-t_0} \|x_j^i(t_0+j|t_0)\|
\]

According to Definition 2, if the switching signal satisfies the condition \( \tau^i \geq -\frac{1}{\ln \alpha_i} \), then the time-delayed switched system is robustly stable.

In the following paragraphs, it will be proven that the upper bound \( \theta' > 0 \) of performance function \( J_{\infty}^i(k) \) exists, so that (17) can be established. Since \( 0 < \alpha_i < 1 \), in combination with formula (28), the following inequality is obtained:
\[
V_j^i(k+1) - V_j^i(k) < -J_j^i(k)
\]

The sum of the inequality above is calculated from \( j = 0 \) to \( j = \infty \):
\[
V^i(k+1) - V^i(k) < \sum_{j=0}^{\infty} J_j^i(k)
\]

\[
V^i(k+1) - V^i(k) < \sum_{j=0}^{\infty} J_j^i(k)
\]
namely,
\[ V_i^i(k + \infty |k) - V_i^i(k |k) < -J_i^i(k) \] (34)

If \( V_i^i(\bar{x}_i^i(\infty)) = 0 \) and \( \bar{x}_i^i(\infty) = 0 \), then:
\[ J_i^\infty(k) < V_i^i(k |k) \] (35)

Taking \( \bar{x}_i^i(k) = \max (\bar{x}_i^i(r), \bar{\delta}_i^i(r)) \) and \( r \in (k - \bar{d}, k) \), we obtain
\[ V_i^i(\bar{x}_i^i(1)) \leq \bar{x}_i^i(k)\bar{\psi}_i\bar{x}_i^iT(k) \] (36)

As \( 0 < \alpha_i < 1 \), from the definition of the Lyapunov function, it follows that:
\[ \bar{\psi}_i^j = \bar{P}_i^j + \bar{d}_i \left( \bar{T}_i^j + \bar{M}_i \right) + \frac{d + \bar{d}}{2} \left( \bar{d} - d + 1 \right) \bar{T}_i^j \]
\[ + \frac{\bar{d}^2}{2} \bar{G}_i^j \] (37)

The following inequality is obtained from the matrix inequality (20) in combination with the Schur complement lemma:
\[ \bar{x}_i^i(k)\bar{\psi}_i^j\bar{x}_i^iT(k) < 1 \leq \prod_{i=1}^{p} \left( \alpha_i \mu_i \right) \tilde{T}_{(t_0, t)} \bar{V}_i^\sigma(t_0) \] (38)

Letting \( v = \max \left( \frac{1}{\alpha_i \mu_i} \right) \), \( \bar{x}_i^i(k) = \max (\bar{x}_i^i(r), \bar{\delta}_i^i(r)), r \in (k - \bar{d}, k) \), \( \bar{\psi}_i^j \theta = \bar{\Phi}_i \). Then, the following inequality is obtained:
\[ \bar{x}_i^i(k)\left( \bar{\psi}_i^j \theta \right)^{-1}\bar{x}_i^iT(k) < 1 \] (39)

namely,
\[ \bar{x}_i^i(k)\bar{\psi}_i^j\bar{x}_i^iT(k) \leq \theta^i \] (40)

Combining (35) and (36), there exists an upper bound \( \theta^i \) of \( J_i^\infty(k) \) such that:
\[ J_i^\infty(k) \leq V_i^i(\bar{x}_i(1)) \leq \theta^i \] (41)

In the following section, the system constraints will be discussed.

For constraint (13), we obtain:
\[ \left\| \Delta u_i^j \right\|^2 = \left\| \bar{K}_i^j \bar{x}_i^j \right\|^2 = \left\| Y_i^j \theta^{-1} \bar{P}_i^j \bar{x}_i^j \right\|^2 \leq \bar{Y}_i^j \theta^{-1} \bar{\psi}_i^j \bar{x}_i^j \right\|^2 \]
\[ = \bar{Y}_i^j \bar{x}_i^j \bar{\Phi}_i^{-1} \bar{x}_i^j \bar{\Phi}_i^{-T} \bar{Y}_i^T \leq \bar{Y}_i^j \theta^{-1} \bar{\psi}_i^j \bar{x}_i^j \right\|^2 \] (42)

Formula (21) is obtained from Lemma 1.

For output constraint (13), we obtain:
\[ \left\| \Delta y_i^j \right\|^2 = \left\| \bar{C}^j \bar{x}_i^j \right\|^2 \leq \bar{x}_i^j \bar{C}^j \bar{C}^iT \bar{x}_i^j \leq \Delta y_{mi}^2 \] (43)

and we further obtain:
\[ \bar{C}^j \bar{C}^iT \leq \bar{x}_i^j \bar{C}^j \Delta y_{m} \bar{x}_i^j \leq \Delta y_{m} \bar{\Phi}_i^{-1} \] (44)

Output constraint (22) is obtained from Lemma 1.

Theorem 1 provides a sufficient condition for the solvability of the model predictive fault-tolerant control problem for batch processes with interval delays. If the lower bound of the interval delays is zero, then this control problem becomes the corresponding control problem with constant delays. As a special case of interval time-varying delay systems, the following corollary can be easily obtained from Theorem 1.

Set
\[ V_i^j = \sum_{l=1}^{3} V_i^j(k + j |k) \]
\[ V_i^j = \bar{x}_i^j \bar{P}_i^j \bar{x}_i^j = \bar{x}_i^j \theta_i^j \bar{P}_i^j \bar{x}_i^j \]
\[ V_i^j = \sum_{l=1}^{3} \bar{x}_i^j \theta_i^j \bar{P}_i^j \bar{x}_i^j = \sum_{l=1}^{3} \bar{x}_i^j \theta_i^j \bar{P}_i^j \bar{x}_i^j \]
\[ V_i^j = \bar{d}_i \sum_{l=1}^{3} \bar{G}_i^j \theta_i^j \bar{G}_i^j \]
\[ = \bar{d}_i \sum_{l=1}^{3} \bar{G}_i^j \theta_i^j \bar{G}_i^j \]

(45)

Similar to the proof of Theorem 1, Corollary 1 can be obtained. The content is represented as follows:

**Corollary 1:** If \( \bar{c}(k) \neq 0 \), then for specified constants
\( 0 \leq d \leq \bar{d}, \theta^i > 0 \), and \( 0 < \alpha_i < 1 \), if there exist positive-definite symmetric matrices \( \bar{P}_i^2, \bar{P}_i^3, \bar{M}_i, \bar{G}_i^j \), and \( \bar{P}_i^1 \) and positive real numbers \( \eta^j, \varepsilon_a, \) and \( \varepsilon_b \) such that the following matrix inequalities are feasible for the switching signal with an average dwell time that satisfies inequality (23),

\[
\begin{bmatrix}
\tilde{Y}_{11} & \tilde{Y}_{12} & \tilde{Y}_{13} & \tilde{Y}_{14} & \tilde{Y}_{15} \\
\ast & \tilde{Y}_{22} & \tilde{Y}_{23} & \tilde{Y}_{24} & 0 \\
\ast & \ast & \tilde{Y}_{33} & 0 & 0 \\
\ast & \ast & \ast & \tilde{Y}_{44} & 0 \\
\ast & \ast & \ast & \ast & \tilde{Y}_{55}
\end{bmatrix}
< 0
\] (46)

\[
\tilde{Y}_{11} = \begin{bmatrix}
\bar{Q}_i^1 \bar{G}_i^3 & -\bar{a}_i^3 \bar{G}_i^3 \\
0 & -\theta^i \eta^j
\end{bmatrix}
\tilde{Y}_{12} = \begin{bmatrix}
\bar{P}_i^2 \bar{A}_i^T + \tilde{Y}_i^j \beta_i^j \tilde{B}_i^T & \bar{P}_i^2 \bar{A}_i^T + \tilde{Y}_i^j \beta_i^j \tilde{B}_i^T - \bar{P}_i^2 \\
\bar{G}_i^j & \theta_i^j \tilde{B}_i^T
\end{bmatrix}
\tilde{Y}_{13} = \begin{bmatrix}
\bar{P}_i^2 \bar{E}_i^T & 0 \\
0 & 0
\end{bmatrix}
\tilde{Y}_{14} = \begin{bmatrix}
\tilde{Y}_i^j \beta_i^j & 0 \\
0 & 0
\end{bmatrix}
\tilde{Y}_{15} = \begin{bmatrix}
\tilde{Y}_i^j \bar{B}_i^T & 0 \\
0 & 0
\end{bmatrix}
\tilde{Y}_{22} = \begin{bmatrix}
-\bar{P}_i^2 & 0 \\
0 & -\bar{d}^{-2} \bar{G}_i^3
\end{bmatrix}
\tilde{Y}_{23} = \begin{bmatrix}
0 & \tilde{e}_i^j \bar{D}_i^T \\
\tilde{e}_i^j \bar{D}_i^T & 0
\end{bmatrix}
\tilde{Y}_{24} = \begin{bmatrix}
0 & \tilde{e}_i^j \bar{B}_i^T \\
\tilde{e}_i^j \bar{B}_i^T & 0
\end{bmatrix}
\]
\[ \gamma_{33} = \begin{bmatrix} -\varepsilon^2 & 0 \\ 0 & -\varepsilon^2 \end{bmatrix}, \quad \gamma_{44} = \begin{bmatrix} -\varepsilon^2 & 0 \\ 0 & -\varepsilon^2 \end{bmatrix}, \]

\[ V^i < \mu^i V^{i-1} \]

\[ \begin{bmatrix} -1 & \bar{x}^i_l \\ \bar{x}^i_r & -\Phi^i \end{bmatrix} < 0 \]

\[ \begin{bmatrix} -\Delta u^2_{\rho} & \bar{Y}^i_1 \\ \bar{Y}^i_1 & -\Phi^i \end{bmatrix} < 0 \]

\[ \begin{bmatrix} -\Delta y^2_{\rho} \bar{P}^i_1 \bar{C}^i \tilde{C}^i & \bar{P}^i_1 - I \end{bmatrix} < 0 \]

where \( \bar{c}^i = -\bar{a}^i \bar{P}^2 + \bar{T}^i - \bar{a}^i \tilde{G}^i, \bar{Y}^i = \bar{c}^i \) and \( \bar{Y}^i = \bar{c}^i + \bar{a}^i \tilde{G}^i, \) then the robust model predictive fault-tolerant control problem with constant delays is solvable, and the stable fault-tolerant controller gain in the system is \( \bar{K}^i = \bar{Y}^i \bar{P}^{-1}. \)

**C. OPTIMIZATION ALGORITHM**

In this part, we seek the controller design with the minimum upper bound under the maximum disturbance. The optimization problem at time \( k \) can be solved using the following formula:

\[ \min_{\Delta u(t+j|k)} \theta^i \]

subject to \( V(k|k) \leq \theta^i \), namely, subject to \( -1 < \bar{x}^i_l \bar{P}^i \bar{c}^i \lbar{\Phi}^i \)

In (18), \( \eta^i \) can be optimized. Since (18) is a bilinear inequality, let \( \zeta^i = \theta^i \eta^i \); thus, \( \zeta^i \) is optimized instead.

\[ \min_{\bar{P}^i, \bar{M}^i, \bar{Y}^i_1, \bar{Y}^i_2, \bar{M}^i_1, \bar{G}^i_1, \bar{G}^i_2, \bar{G}^i_3, \bar{M}^i_1, \bar{M}^i_2, \bar{M}^i_3} \zeta^i \]

subject to \( 18 - 22 \)

**IV. SIMULATION**

In this paper, fault-tolerant control for an injection molding process, which is a representative multi-phase batch process, is simulated. The injection molding process consists of five phases: mold closure, injection, packing, cooling, and mold opening. First, during the injection phase, the molten material is injected into the mold cavity until the cavity is filled. Then, the system is switched to the packing phase, and the polymer is filled into the contractions that are caused by cooling and curing to realize the objective of packing. After the packing phase, the cooling and mold opening phases begin, in which the polymer in the mold cavity cools until it is fully cured, and then the final product is ejected. The injection rate and the packing pressure are the two main variables to control because they have the largest impacts on the control efficiency in the corresponding phases, and errors tend to occur during these two phases. The injection rate and the packing pressure are controlled by the degrees of opening of the corresponding valves. When the mold cavity pressure reaches a threshold level at the injection phase, the system will switch to the next phase, namely, the packing phase.

In this paper, the model is transformed into a switched system, and the injection phase and packing phase of the injection molding process are controlled based on the principle of predictive fault-tolerant control under the conditions of time delay, perturbations and actuator faults.

The injection phase is defined as the first phase, and the packing phase is defined as the second phase. For the injection phase \( IV \) at the injection phase, the packing pressure \( NP \) at the packing phase and the valve opening degree \( VO \), the model is expressed as:

\[ (1 - 0.9291z^{-1} - 0.03191z^{-2})IV = (8.687z^{-1} - 5.617z^{-2})VO \\
(1 - 1.317z^{-1} + 0.3259z^{-2})NP = (171.8z^{-1} - 156.8z^{-2})VO \]

The model for the relationship between the mold cavity pressure \( NP \) and the injection rate at the injection phase is expressed as:

\[ NP(1 - z^{-1}) = 0.1054z^{-1}IV \]

For the injection rate \( IV \), packing pressure \( NP \) and valve opening degree \( VO \) in systems with real actuator faults, the model is expressed as:

\[ (1 - 0.9291z^{-1} - 0.03191z^{-2})IV = (6.950z^{-1} - 4.494z^{-2})VO \\
(1 - 1.317z^{-1} + 0.3259z^{-2})NP = (137.4z^{-1} - 125.4z^{-2})VO \]

Denote \( x^1 = 0.03191IV(k-1) - 4.4936VO(k-1), u^1(k) = VO(k), y^1(k) = NP(k), x^2 = 0.3259NP(k-1) - 125.44(k-1), u^2 = VO(k), y^2 = NP(k). \)

At the injection phase, the injection rate \( IV \) is set to 40 mm/s; at the packing phase, the packing pressure \( NP \) is set to 300 bar. Under the conditions of an actuator fault and the constraint conditions, the time-delayed extended state-space fault model for the injection phase is:

\[ x^1(k+1) = \begin{bmatrix} 0.9291 & 0 & 0 \\
0.03191 & 0 & 0 \\
0.1054 & 0 & 0 \end{bmatrix} x^1(k) + \begin{bmatrix} 0.02 \delta(k) & 0 & 0 \\
0.013 & 0 & 0 \\
0.0021 & 0 & 0 \end{bmatrix} u^1(k) + \begin{bmatrix} 8.687 \\
5.617 \\
0 \end{bmatrix} \]

\[ y^1(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x^1(k) \]

and the input and output constraints in this phase are selected as:

\[ \begin{bmatrix} y^1(t,k) \leq 45 \\
u^1(t,k) \leq 6 \end{bmatrix} \]
The packing phase model is:

\[
\begin{align*}
\dot{x}_2(k+1) &= \begin{bmatrix} 1.317 & 1 \\ -0.3259 & 0 \end{bmatrix} x_2(k) + \begin{bmatrix} 0.025 \delta(t, k) & 0 \\ 0.01 \delta(t, k) & 0 \end{bmatrix} u_2(k) \\
&+ \begin{bmatrix} 0.002 & 0 \\ 0.001 & 0 \end{bmatrix} x_2(k-d(k)) + \begin{bmatrix} 171.8 \\ -156.8 \end{bmatrix} \alpha u_2(k) + \omega_2(k) \\
y_2(k) &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_2(k)
\end{align*}
\]

and the input and output constraints in this phase are:

\[
\begin{align*}
\|y_2(t, k)\| &\leq 302 \\
\|u_2(t, k)\| &\leq 0.6
\end{align*}
\]

where \(\delta(k)\) is a random variable that is within the range \([0, 1]\), \(\alpha = 0.8\), and the switching condition is \(G_1(x(k)) = 350 - \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x_1(k) < 0\); hence, once the mold cavity pressure exceeds 350 Pa, the system will be switched from the injection phase to the packing phase. To examine the design performance in this paper, we determine the initial control law parameters. Then, MATLAB software is used for simulation. The initial control law is obtained via Theorem 1 and its optimization algorithm, and the controller gain is

\[
\begin{align*}
\bar{K}_1 &= [-0.1060, -0.1141, 0, 0.0433, 0.1323] \\
\bar{K}_2 &= [-0.0015, -0.0012, 0.0004, 0.0013]
\end{align*}
\]

To evaluate the performance of the proposed method, we compare it with that of the traditional control method [7]. In the traditional method, the controller design is of the following form: \(u(k+j|k)) = K_i \begin{bmatrix} x(k+j|k) \\ \hat{x}^i(k+j|k) \end{bmatrix}\) (where \(\hat{x}^i(k+j|k)\) is obtained from (7)). The sampling time of each step is 5 ms. Two kinds of faults are selected here to analyze the influence of faults on the control performance of the system. One is a constant fault (case 1); the other is a time-varying fault (case 2). The comparison results are as follows.

FIGURE 1. An output comparison between the proposed control and the traditional control strategy under constant faults.

Under case 1, \(\alpha = 0.8\). The results are presented in Figs. 1 and 2. As shown in Figs. 1, under the proposed control method, the system fluctuates less, output tends to stabilize more quickly at the second phase, and the phase running time is shorter. The operation time of the first phase is 88, while for the traditional control strategy, the operation time of the first phase is 93. In addition, According to Fig. 2, under random perturbations, both the predictive fault-tolerant control strategy and the traditional control strategy can enable the system to stabilize at the injection phase. However, from the 94th step, according the input chart, the system that adopts the predictive fault-tolerant control strategy is switched more stably at an earlier time, and the range of the curve’s fluctuation is smaller. Figs. 1 and 2 compare the output and input for a fault of 0.8. We present the comparison results under other fault conditions in Figs. 3 and 4. Here, the time-varying fault...
is chosen, and $\alpha = 0.6 + 0.2 \sin(k)$ (case 2). According to the
two figures, although the control performance of the system
has decreased, the control target can still be attained. In
addition, to analyze the impact of a time delay on the system
control performance, the parameters of systems with a time
delay and those without a time delay are compared under the
predictive fault-tolerant control strategy that is proposed in
this paper, as shown in Figs. 5 and 6. The two figures present
the comparison results of output $y$ and input $u$ for systems
with and without a time delay. The system with a time delay
is characterized by lower control performance and larger
fluctuations in the output and input at the time of system
switching; however, the system ultimately stabilizes under
the predictive fault-tolerant control strategy. The comparison
results demonstrate that the time delay affects the control
performance of the system.

In the common control method for a batch process, the pro-
cess is regarded as a two-dimensional (2D) system, and
its tracking control via the iterative learning control (ILC)
method is studied. Here, we compare our proposed method
with this method (2D-ILC). The tracking error is selected as
$DT(k) = \sqrt{\sum_{k=0}^{\infty} e^T(k)e(k)}$. The comparison results demon-
strate that the output has a short running time in each phase
when the method proposed in this paper is used, but the
fluctuations are larger at the initial time and the switching
time. According to the comparison of the tracking perfor-
mance, the tracking error of our proposed method is small,
as shown in Figs. 7 and 8. The predictive fault-tolerant con-

V. CONCLUSION

In this paper, an $H_\infty$-model predictive fault-tolerant con-

**FIGURE 5.** An output comparison with and without time delay under the
proposed method.

**FIGURE 6.** An input comparison with and without time delay under the
proposed method.

**FIGURE 7.** An output comparison between the proposed control
and 2D-ILC.

**FIGURE 8.** Tracking performance comparison between the proposed
control and 2D-ILC.
REFERENCES

[1] T. Liu, F. Gao, and Y. Wang, “IMC-based iterative learning control for batch processes with uncertain time delay,” J. Process Control, vol. 20, no. 2, pp. 173–180, Feb. 2010.

[2] L. Jia, T. Yang, and M. Chiu, “An integrated iterative learning control strategy with model identification and dynamic R-parameter for batch processes,” J. Process Control, vol. 23, no. 9, pp. 1332–1341, Oct. 2013.

[3] R. Zhang and F. Gao, “Improved infinite horizon LQ tracking control for injection molding process against partial actuator failures,” Comput. Chem. Eng., vol. 80, pp. 130–139, Sep. 2015.

[4] R. Zhang, R. Lu, A. Xue, and F. Gao, “New minmax linear quadratic fault-tolerant tracking control for batch processes,” IEEE Trans. Autom. Control, vol. 61, no. 10, pp. 3045–3051, Oct. 2016.

[5] R. Zhang, F. Gao, and P. D. Christofides, “An improved approach for $H_{\infty}$ design of linear quadratic tracking control for chemical processes with partial actuator failure,” J. Process Control, vol. 58, pp. 63–72, Oct. 2017.

[6] H. Tao, W. Paszke, E. Rogers, H. Yang, and K. Galkowski, “Iterative learning fault-tolerant control for differential time-delay batch processes in finite frequency domains,” J. Process Control, vol. 56, pp. 112–128, Aug. 2017.

[7] Y. Wang, J. Shi, D. Zhou, and F. Gao, “Iterative learning fault-tolerant control for batch processes,” Ind. Eng. Chem. Res., vol. 45, no. 26, pp. 9050–9060, Dec. 2006.

[8] L. Wang, S. Mo, D. Zhou, F. Gao, and X. Chen, “Delay-Range-Dependent method for iterative learning fault-tolerant guaranteed cost control for batch processes,” Ind. Eng. Chem. Res., vol. 52, no. 7, pp. 2661–2671, Feb. 2013.

[9] L. Wang, B. Liu, J. Yu, P. Li, R. Zhang, and F. Gao, “Delay-Range-Dependent-Based hybrid iterative learning fault-tolerant guaranteed control cost for multiphase batch processes,” Ind. Eng. Chem. Res., vol. 57, no. 8, pp. 2932–2944, Feb. 2018.

[10] L. Wang, R. Zhang, and F. Gao, Iterative Learning Stabilization and Fault-Tolerant Control for Batch Processes. Singapore: Springer Nature Singapore, 2020.

[11] L. Wang, B. Li, J. Yu, R. Zhang, and F. Gao, “Design of fuzzy iterative learning fault-tolerant control for batch processes with time-varying delays,” Optim. Control Appl. Methods, vol. 39, no. 6, pp. 1887–1903, Nov. 2018.

[12] L. Wang, F. Liu, J. Yu, P. Li, R. Zhang, and F. Gao, “Iterative learning fault-tolerant control for injection molding processes against actuator failures,” J. Process Control, vol. 59, pp. 59–72, Nov. 2017.

[13] L. Wang, S. Mo, D. Zhou, F. Gao, and X. Chen, “Robust delay dependent iterative learning fault-tolerant control for batch processes with state delay and actuator failures,” J. Process Control, vol. 22, no. 7, pp. 1273–1286, Aug. 2012.

[14] L. Wang, L. Sun, J. Yu, R. Zhang, and F. Gao, “Robust iterative learning fault-tolerant control for multiphase batch processes with uncertainties,” Ind. Eng. Chem. Res., vol. 56, no. 36, pp. 10099–10109, Sep. 2017.

[15] K. S. Lee, I.-S. Chin, H. J. Lee, and J. H. Lee, “Model predictive control technique combined with iterative learning for batch processes,” AIChE J., vol. 45, no. 10, pp. 2175–2187, Oct. 1999.

[16] S.-K. Oh and J. M. Lee, “Iterative learning model predictive control for constrained multivariable control of batch processes,” Comput. Chem. Eng., vol. 93, pp. 284–292, Oct. 2016.

[17] J. Shi, F. Gao, and T.-J. Wu, “Single-cycle and multi-cycle generalized 2D model predictive iterative learning control (2D-GPLC) schemes for batch processes,” J. Process Control, vol. 17, no. 9, pp. 715–727, Oct. 2007.

[18] Y. Wang, D. Zhou, and F. Gao, “Iterative learning model predictive control for multiphase batch processes,” J. Process Control, vol. 18, no. 6, pp. 543–557, Jul. 2008.

[19] L. Zhou, L. Jia, and Y.-L. Wang, “A robust integrated model predictive iterative learning control strategy for batch processes,” Sci. China Inf. Sci., vol. 62, no. 11, Nov. 2019, Art. no. 219202.

[20] C. Chen, Z. Xiong, and Y. Zhong, “Design and analysis of integrated predictive iterative learning control for batch process based on two-dimensional system theory,” Chin. J. Chem. Eng., vol. 22, no. 7, pp. 762–768, Jul. 2014.

[21] J. Lu, Z. Cao, Z. Wang, and F. Gao, “A two-stage design of two-dimensional model predictive iterative learning control for nonrepetitive disturbance attenuation,” Ind. Eng. Chem. Res., vol. 54, no. 21, pp. 5683–5689, Jun. 2015.

[22] J. Lu, “Nonlinear monotonically convergent iterative learning control for batch processes,” IEEE Trans. Ind. Electron., vol. 65, no. 7, pp. 5826–5836, Jul. 2018.