Distribution probability of galaxies in Local Group

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Abstract. The evolution of galaxies is a very important topic for understanding of the evolution of Universe, and this evolution results in that galaxies are not evenly distributed in space. The probability of distribution of galaxies in space can be computed according to probabilistic models. Although the evolution of galaxies has been studied from various angles, the probability of distribution of galaxies in space has yet to be studied with respect to their positions over time. Of galaxies, the positions of galaxies in Local Group have been studied more thoroughly. In this study, we computed the distribution probabilities of galaxies in Local Group, i.e. the distribution probabilities of galaxies at present and previous positions, and the probability for an even distribution of galaxies in space. The results show that galaxies in Local Group have an extremely small chance to distribute evenly, and the distribution probability of galaxies in Local Group increases from the past to the present. The implication is that the accelerating expansion of the universe is increasing the distribution probability of galaxies, i.e. the accelerating expansion of the universe is a probabilistically probable event.

1. Introduction

Studies on the distribution of galaxies are important to understand the evolution of the Universe, and the mass distribution of the Universe, because galaxies are not distributed evenly in space, which is often said not randomly distributed in space [1]. This is particularly evident when a large-scale distribution of galaxies in slice was studied [2, 3], where the distribution of galaxies appears to be something like the distribution of soap molecules in a pile of soap bubbles [4].

In past, this issue has been intensively studied using ΛCDM model [5], for example, the clustering of 264283 massive galaxies. In fact, the ΛCDM model is the most successful cosmological model. However, the ΛCDM model seems not to function well in small scale [6] for prediction on the small-scale structures. Therefore, it would be interesting to study the small-scale structures from different angles, for example, probabilistic models, because some of these models can be regarded as phenomenological models without consideration on dark matter, cold dark matter.

The cosmological principle is that the spatial distribution of matter in the universe is homogeneous and isotropic, and we are not sure whether or not the uneven distribution of galaxies could contradict this cosmological principle. Thus, it necessarily studies how a group of galaxies distributes in this aspect.

Two intriguing questions are (i) how large a chance is for those galaxies distributed in their current positions, and (ii) what a probability is for an even distribution of galaxies. Thereafter, an interesting question is how the distribution probability changes from the past to the present because the answer to this question can reveal how galaxies move in terms of distribution probability after their formation.
However, it is not easy to find the historical, even simulated positions of all galaxies in large-scale distribution of galaxies. Therefore, our attention is directed to the galaxies in Local Group, because these galaxies have been well studied [7] and we have relatively more knowledge on their positions. Thus they can serve as an important starting point for such studies. Indeed, the positions of galaxies in Local Group do change over time [8].

In this study, at first we model the distribution of galaxies from Local Group in the simplest scenarios with the probabilistic model, then we analyze the distribution probabilities in these scenarios, and finally we discuss the implication of the results.

2. Materials and methods

2.1. Data

The positions of galaxies in Local Group can be obtained from various resources, for example, the Local Group in Merrier Catalog lists 47 galaxies with different pieces of information [9].

2.2. Methods

The distribution of galaxies in Local Group in space can be described using probabilistic technique of subpopulations and partitions, and their application to occupancy problems [10]. This method for computation of distribution probability classifies the distribution probability of elementary particles in energy states according to three different assumptions with respect to whether or not to distinguish each particle and energy state, i.e., Maxwell–Boltzmann, Fermi–Dirac, and Bose–Einstein assumptions. In our case, this is an application of the Maxwell–Boltzmann distribution [11].

Accordingly, we can partition the space where galaxies in Local Group distribute into equal-sized sectors, count how many galaxies in each sector and compute the distribution probability using

\[ p = \frac{n!}{q_0! \times q_1! \times \ldots \times q_k! \times r_0! \times r_1! \times \ldots \times r_l!} \times \frac{1}{n^r} \]

where \( n \) is the number of equal-sized sectors, \( q_k \) is the number of sectors containing \( k \) galaxies, \( r \) is the number of galaxies, \( r_i \) is the number of galaxies in the \( i \)-th sector.

Needless to say, there are many different ways to partition the space where galaxies distribute in Local Group. Here we used two types of partition for simplicity, Type 1 and Type 2, to divide the space and compute the distribution probability.

In Type 1, the space is divided into equal-sized and fan-shaped sectors with the same angles, which is the simplest way to compute the distribution probability. To analyze the distribution of 47 galaxies in Local Group, for example, we divide the space into 47 equal-sized and fan-shared sectors with the angle of 7.66 degrees for each sector (panel A of Figure 1). In panel A of Figure 1, 26 sectors do not contain galaxies, 13 sectors contain one galaxy each, 3 sectors contain two galaxies each, 1 sector contains three galaxies, 2 sectors contain four galaxies, 1 sector contains six galaxies, and 1 sector contains eleven galaxies (0 degree column, Table 1). Consequently, we have \( q_0 = 26, q_1 = 13, q_2 = 3, q_3 = 1, q_4 = 2, q_6 = 1, \) and \( q_{11} = 1 \) for the first term of equation. We also have \( r_1 = 3, r_2 = 11, r_3 = 5, r_4 = 2, r_7 = 0, r_6 = 1, r_7 = 0, r_9 = 1, r_{10} = 0, r_{11} = 1, r_{12} = 0, r_{13} = 1, r_{14} = 0, r_{15} = 1, r_{16} = 0, r_{17} = 0, r_{18} = 0, r_{19} = 0, r_{20} = 6, r_{21} = 1, r_{22} = 1, r_{23} = 0, r_{24} = 0, r_{25} = 0, r_{26} = 1, r_{27} = 0, r_{28} = 0, r_{29} = 0, r_{30} = 1, r_{31} = 0, r_{32} = 0, r_{33} = 0, r_{34} = 1, r_{35} = 1, r_{36} = 0, r_{37} = 0, r_{38} = 1, r_{39} = 2, r_{40} = 0, r_{41} = 1, r_{42} = 0, r_{43} = 0, r_{44} = 1, r_{45} = 1, r_{46} = 3, \) and \( r_{47} = 1 \) for the second term of equation. As \( 0! = 1 \) and \( 1! = 1 \), and put these values into the equation, and we have:

\[ p = \frac{n!}{q_0! \times \ldots \times q_4! \times q_6! \times q_{11}! \times r_1! \times \ldots \times r_{47}!} \times \frac{1}{n^{r'}} = \frac{47!}{26! \times 13! \times 3! \times 2! \times 3! \times 11! \times 5! \times 2! \times 6! \times 2! \times 3!} \times \frac{1}{47^{r'}} = 4.2805 \times 10^{-13} \]
Table 1. Distributions of 47 galaxies of Local Group in 47 equal-sized and fan-shaped sectors for different scenarios.

| Galaxies in Sector | Rotation of Coordinates | Distribution with Maximal Probability | Even Distribution | Distribution in One Sector |
|--------------------|-------------------------|---------------------------------------|-------------------|---------------------------|
|                    | 0 degree | 0.5 degrees | 24 degrees | 47 degrees | |
| 0                  | 26       | 26         | 27        | 26        | 16 |
| 1                  | 13       | 11         | 11        | 11        | 18 47 |
| 2                  | 3        | 6          | 4         | 6         | 10 |
| 3                  | 1        | 1          | 1         | 1         | 3 |
| 4                  | 2        |            |           |           | 1 |
| 5                  |          | 1          | 1         | 1         | 1 |
| 6                  |          | 1          | 1         | 1         | 1 |
| 10                 |          |            | 1         | 1         | 1 |
| 11                 | 1        |            |           |           | 1 |
| 47                 |          |            |           |           | |
| Probability        | $4.281 \times 10^{-13}$ | $1.236 \times 10^{-11}$ | $1.236 \times 10^{-11}$ | $2.674 \times 10^{-2}$ | $6.669 \times 10^{-20}$ | $1.212 \times 10^{-77}$ |

In Type 2, the space is divided into equal-sized sectors with considering the distance from the observer. However, such partitioning is more complicated. First, from the center of the Local Group concentric circles are drawn using proportional increasing radii with the same distance. Second, the area of central circle is set as a unity, by which the space is partitioned into different sectors. Third, according to the relationship between the radium and the area, the number of sectors increases proportionally with the increase of the radium. So they are 3, 5, 7, 9, 11 and 13 when the radium increases from 2 to 7 times of the initial radium. In this partition, it is difficult to divide the space into 47 equal-sized sectors for 47 galaxies because the total equal-sized sectors are 49 and 36 for dividing 7 and 6 concentric circles, respectively. Thus there are two subtypes in Type 2: one is the number of galaxies is smaller than the number of equal-sized sectors, which is defined as Type 2a and showed in panel B of Figure 1; the other is the number of galaxies is larger than the number of equal-sized sectors, which is defined as Type 2b and showed in panel C of Figure 1. Thereafter, the distribution of the galaxies in Type 2a and 2b can be determined, and their distribution probabilities can be computed. For the example of Type 2a in panel B of Figure 1, we have the following:

$$p = \frac{49!}{36!} \times \frac{47!}{3! \times 5! \times 13! \times 17!} \times \frac{1}{49^{47}} = 2.6581 \times 10^{-32}$$

For the example of Type 2b in panel C of Figure 1, its distribution probability is $1.3663 \times 10^{-25}$.

In these ways, we can compare distribution probabilities of galaxies in Local Group with respect to different time points, and determined how the motions of galaxies in Local Group changed their distribution probability.

3. Results and discussion

The distribution probability of the 47 galaxies [11] in the 47 equal-sized and fan-shaped sectors in panel A of Figure 1 is $4.2805 \times 10^{-13}$, which is pretty small. However, the distribution probability that each sector has a galaxy is $6.6691 \times 10^{-20}$. This suggests that the 47 galaxies would have a far smaller chance to distribute evenly in space. Basically these probabilities are negligibly small. In plain words, this is because there are so many choices to distribute 47 items in 47 places. Therefore, importance is to compare probabilities between different choices.
Figure 1. Distribution of 47 galaxies of Local Group in 47 equal-sized sectors (A), in 49 equal-sized sectors (B), in 36 equal-sized sectors (C), in 47 equal-sized sectors (D), in 49 equal-sized sectors (E) and in 36 equal-sized sectors. The distribution probability is $4.2805 \times 10^{-13}$, $2.6581 \times 10^{-32}$, $1.3663 \times 10^{-25}$, $1.2360 \times 10^{-11}$, $7.6902 \times 10^{-30}$ and $1.2788 \times 10^{-22}$ of A to F, respectively. A, B and C without rotation; D, E and F with rotation.

On the other hand, we could expect that the actual distribution probability of the 47 galaxies may be different from that in panel A of Figure 1, because the coordinates in panel A of Figure 1 are designed according to astronomic convention, but nature should distribute galaxies without consideration of such coordinates. Perhaps, we could guess that the distribution probability of the 47 galaxies would be smaller after rotation of coordinates because the rotation of coordinates would result in that each sector would have different number of galaxies.

We, therefore, rotated the standard coordinates by 0.5 degrees each time and determine whether the distribution probability of the 47 galaxies changes. In this way, the biggest probability of $1.2360 \times 10^{-11}$ was found in eleven cases, whose coordinates were rotated 0.5, 24, 47, 123.5, 146.5, 169.5, 192.5, 215.5, 238.5, 315, and 338 degrees. Panel D of Figure 1 shows the distribution with 47-degree rotation of coordinates, and Table 1 lists the galaxies distributions without rotation and with rotations of 0.5, 24 and 47 degrees. For the rotations at 123.5, 146.5, 169.5, 192.5, 215.5, 238.5, 315, and 338 degrees, their distribution patterns are as the same as the rotation of 47 degrees. So the rotation of coordinates leads the distribution probability to increase two orders, $1.2360 \times 10^{-11}$ versus $4.2805 \times 10^{-13}$. How can we choose different distribution probabilities with different angles of rotation of coordinates? We definitely choose the maximal distribution probability because it easily occurs.
Table 2. Distributions of 47 galaxies of Local Group in 49 equal-sized sectors.

| Galaxies in sector | No Rotation of Sectors | Rotation of Sectors |
|--------------------|------------------------|---------------------|
| 0                  | 36                     | 35                  |
| 1                  | 9                      | 8                   |
| 2                  |                        | 2                   |
| 3                  | 1                      | 1                   |
| 5                  | 1                      | 1                   |
| 11                 |                        | 2                   |
| 13                 | 1                      | 1                   |
| 17                 | 1                      |                     |

The distribution probability of the 47 galaxies in the 49 sectors in panel B of Figure 1 is $2.6581 \times 10^{-32}$, however, the probability for no repetition of galaxies in a single sector is $1.1063 \times 10^{-17}$. Similar to Type 1, we would expect that the actual distribution probability of the 47 galaxies in the 49 equal-sized sectors would be different from that in panel B of Figure 1 because nature should not consider the coordinates. Panel E of Figure 1 shows the distribution of the 47 galaxies in the 49 sectors after rotating sectors (Table 2), which result in the biggest distribution probability of $7.6902 \times 10^{-30}$ that is two-order larger than the probability of $2.6581 \times 10^{-32}$ in panel B of Figure 1. Following the same reasoning, we also rotated sectors in panel C of Figure 1 for the 47 galaxies distributing in the 36 equal-sized sectors, and found a probability of $1.2788 \times 10^{-22}$ in panel F of Figure 1 that is three-order larger than the probability of $1.3663 \times 10^{-25}$ in panel C of Figure 1.

Table 3. Distribution probabilities with respect to the motions of the 28 galaxies in Local Group.

| Distribution type | Coordinates | Distribution probability at $1 + z_i = 10$ | Distribution probability at present |
|-------------------|-------------|---------------------------------------------|-----------------------------------|
| Type 1            | x-y         | $7.4800 \times 10^{-3}$                     | $3.8905 \times 10^{-2}$           |
|                   | x-z         | $6.4193 \times 10^{-2}$                     | $6.0790 \times 10^{-3}$           |
|                   | z-y         | $5.8357 \times 10^{-2}$                     | $1.6800 \times 10^{-3}$           |
| Type 2a           | x-y         | $5.0581 \times 10^{-25}$                    | $1.4935 \times 10^{-3}$           |
|                   | x-z         | $4.6234 \times 10^{-27}$                    | $3.7337 \times 10^{-5}$           |
|                   | z-y         | $7.2331 \times 10^{-24}$                    | $1.6152 \times 10^{-5}$           |
| Type 2b           | x-y         | $9.2242 \times 10^{-23}$                    | $5.8500 \times 10^{-3}$           |
|                   | x-z         | $7.1668 \times 10^{-31}$                    | $3.5455 \times 10^{-5}$           |
|                   | z-y         | $2.7198 \times 10^{-26}$                    | $4.9636 \times 10^{-5}$           |

The probabilities of $1.2360 \times 10^{-11}$, $7.6902 \times 10^{-30}$ and $1.2788 \times 10^{-22}$ are the largest probabilities for distribution of the 47 galaxies in their current positions for these Types of partition of space. Indeed, the comparison of panels A, B and C of Figure 1 with panels D, E and F of Figure 1 indicates that there is no change in any physical distance between any two galaxies, and no change in the physical distribution of those galaxies. Clearly, the only way that can change physical distances between any two galaxies and physical distribution of those galaxies is the motions of those galaxies, so let us consider several scenarios of galaxies in Local Group to get distribution probabilities with respect to positions of the galaxies at different time points.

1) If all the 47 galaxies had been initially distributed in a single sector in Type 1, which could be the sometime after formation of those 47 galaxies after Big Band, then the probability of such distribution is $1.2120 \times 10^{-77}$. Comparing with the current distribution probability of $1.2360 \times 10^{-11}$, the motion of 47 galaxies in Local Group approaches to a larger probability of distribution. When applying this case to Type 2a and Type 2b, the distribution probabilities are $1.7823 \times 10^{-78}$ for all the 47 galaxies in one of 49 sectors ($49 \times 46$) and $2.5709 \times 10^{-72}$ for the 47 galaxies in one of 36 sectors ($36 \times 46$), which are far smaller than the current distribution probabilities of $7.6902 \times 10^{-30}$ and $1.2788 \times 10^{-22}$. And we can reach the same
conclusion that the motion of galaxies in Local Group approaches to distributions with large probability of occurrence.

2) A dynamical study, which fits the motions of 28 galaxies in Local Group that have the distances less than 1.5 Mpc and are not apparently tight satellites [12], provides us with the data that contain the positions of those 28 galaxies at $1 + z_i = 10$ and at present in comoving supergalactic coordinates. Based on the data, the distribution probabilities were computed. Table 3 indicates how the distribution probability changes with respect to the motions of the 28 galaxies in Local Group. As can be seen, the distribution probability obtained from Type 1 distribution, which is the distribution of 28 galaxies in 28 equal-sized sectors, indicates that only one distribution probability increased from the past to the present, while the other two distribution probabilities decreased in one order from the past to the present. The distribution probability obtained from Type 2a distribution, which is the distribution of 28 galaxies in 36 equal-sized sectors, strongly indicates that all distribution probabilities increased in many orders from the past to the present for three coordinates. The distribution probability obtained from Type 2b, which is the distribution of 28 galaxies in 25 equal-sized sectors, also strongly indicates that all the distribution probabilities increased in many orders from the past to the present (Table 3).

3) Dynamical studies suggested that Andromeda Galaxy would join Milky Way Galaxy in the future [13]. Although this result is only related to Andromeda Galaxy and Milky Way Galaxy without consideration of positions of other galaxies in the future, we might still consider it as a weak case to compute the change in distribution probability with some compromise. For this case, the merger of two galaxies would lead the number of Local Group under our consideration to 46 galaxies in the future from 47 galaxies at the present. The probability for Type 1 distribution, which changes from 47 galaxies in 47 equal-sized sectors to 46 galaxies in 46 equal-sized sectors, is $1.2028 \times 10^{-10}$, which is larger than the current distribution probability of $1.2360 \times 10^{-11}$. The probability for Type 2a distribution, which is the distribution of 46 galaxies in 49 equal-sized sectors, is $8.4593 \times 10^{-28}$. And the probability for Type 2b distribution, which is the distribution of 46 galaxies in 36 equal-sized sectors, is $1.4067 \times 10^{-21}$. Both two probabilities are larger than the current distribution probabilities of $7.6902 \times 10^{-30}$ and $1.3663 \times 10^{-25}$. Again, we should emphasise that this computation is a weak case because it does not include the motions of other 45 galaxies [13]. However, this computation still suggests that the join of Andromeda Galaxy to Milky Way Galaxy in the future is a probabilistically possible scenario.

4) It is suggested that Leo I, Magellanic Clouds were once located near Andromeda, not only Leo I moved away but also Magellanic Clouds were captured by the Milky Way Galaxy around 6 billion years ago [13, 14]. Similarly, this is a weak case because it does not have the positions of other galaxies. However, it might still provide us with probabilistic insight. For Type 1 distribution, the distribution probability of 47 galaxies at the time when Leo I and Magellanic Clouds were located around Andromeda is $1.9074 \times 10^{-13}$, which is smaller than the current probability of $1.2360 \times 10^{-11}$. For Type 2 distribution, the distribution probabilities are somewhat similar to the current ones ($6.0423 \times 10^{-30}$ versus $7.6902 \times 10^{-30}$ for Type 2a distribution, and $1.5630 \times 10^{-22}$ versus $1.2788 \times 10^{-22}$ for Type 2b distribution).

The large-scale structure has been a major objective of many studies, which lay the foundation of how galaxies formed. After formation of galaxies either in groups or in clusters, their motions bring about different distributions in space at different time points. Indeed, those distributions were the objective of galaxy surveys. Therefore, it is interesting to study the probabilities for those distributions.

In the past, several studies addressed the fact that the samples of galaxy distributions are not directly traced back to the underlying matter distribution [15-17], and the distribution of Local Group galaxies has been studied with a different approach [18]. In our study, only four scenarios of motions of Local Group galaxies were studied because of difficulty in finding the positions of Local Group galaxies in the past.

The results in this study showed that motions of Local Group galaxies lead to an increased distribution probability in most cases. It is true that the galaxies in Local Group distribute in 3-dimension, but the distribution probability looks like to be in 2-dimension. Actually, it accounts 3-dimensional situation because Maxwell–Boltzmann, Fermi–Dirac, and Bose–Einstein assumptions are 3-dimensional cases.
It often says that the distribution of galaxies in space is not random, however how large difference between a random distribution and the present distribution has yet to quantify. At present, unfortunately it is still very difficult for both partitioning of the space in Field Galaxy Redshift Survey [19] and Sloan Digital Sky Survey [20] into equal-sized sectors, and computing of their distribution probability using abovementioned equation. Therefore, we have to direct our attention to the galaxies in Local Group because their motions and distributions are mainly subject to initial conditions and gravity. In this case, our study suggests that it is a very small chance that galaxies in Local Group would distribute evenly, if an even distribution can be regarded as pure random distribution. This implies that galaxies in Field Galaxy Redshift Survey and Sloan Digital Sky Survey are unlikely to distribute evenly.

A recent study demonstrates that an evolution of the large-scale clustering is consistent with dynamical passive evolution [21]. If the motion of galaxies in Local Group could represent a certain aspect of evolution of galaxies after their formation, then the results in this study suggest that the motion of galaxies in Local Group leads to an increase in distribution probability. If this would be case for distribution in Field Galaxy Redshift Survey and Sloan Digital Sky Survey, then the distribution of tens of thousands of galaxies goes along a probabilistic increasing pathway.

Although our study uses a small dataset in a slice of survey rather than a celestial sphere, the essential implication is that the accelerating expansion of the universe is increasing the distribution probability of galaxies. In other words, the accelerating expansion of the universe is a probabilistically probable event, whereas we have yet to know how this probabilistically probable event could be linked to the dark energy.

4. Conclusion
The evolution of galaxies is very important for the understanding of the evolution of Universe and the mass distribution of the Universe. Of various models to study galaxies, the model of distribution probability of galaxies has not yet been used. In this study, we apply the model of distribution probability to the positions of galaxies in Local Group in the past and the present. The results show that galaxies in Local Group have an extremely small chance to distribute evenly, and the distribution probability of galaxies in Local Group increases from the past to the present. The results suggest that the accelerating expansion of the universe is increasing the distribution probability of galaxies, i.e. the accelerating expansion of the universe is a probabilistically probable event.

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References
[1] Weinberg, D.H. (2005) Astronomy. Mapping the large-scale structure of the universe. Science, 309: 564–565.
[2] Springel, V., Frenk, C.S., White, S.D.M. (2006) The large-scale structure of the Universe., Nature, 440: 1137–1144.
[3] Geller, M.J., Huchra, J.P. (1989) Mapping the universe. Science, 246: 897–903.
[4] Soper, D.E. (2021) How are galaxies distributed in space? University of Oregon, https://pages.uoregon.edu/soper/Galaxies/distribution.html
[5] Anderson, L., et al. (2012) The clustering of galaxies in the SDSS-III baryon oscillation spectroscopic survey: Baryon acoustic oscillations in the data release 9 spectroscopic galaxy sample. Mon. Not. R. Astron. Soc., 427, 3435–3468.
[6] Nakama, T., Chluba, J., Kamionkowski, M. (2017) Shedding light on the small-scale crisis with CMB spectral distortions. Phys. Rev. D., 95, 121302(R).
[7] van den Bergh, S. (1999) The local group of galaxies. Astro. Astrophys. Rev., 9: 273–318.
[8] Grebel, E.K. (2000) The Local Group. In: Menzies, J.W., Sackett, P.D. (Eds.) Microlensing 2000: A New Era of Microlensing Astrophysics. ASP Conference Series, Vol. 000, 2000, Cape Town, South Africa, 21-25 February 2000, 280–298.

[9] Frommert, H., Kronberg, C. (2021) The Local Group of Galaxies. http://messier.seds.org/more/local.html

[10] Feller, W. (1968) An Introduction to Probability Theory and its Applications. 3rd Edition, John Wiley, New York, Vol I 34–40.

[11] Frommert, H., Kronberg, C. (2021) Local Group Dynamics. http://messier.seds.org/more/lg-dyn.html

[12] Peebles, P.J.E., Tully, R.B., Shaya, E.J. (2011) A Dynamical Model of the Local Group, arXiv: 1105.5596v1.

[13] Cox, T.J., Loeb, A. (2008) The collision between the Milky Way and Andromeda. Mon. Not. R. Astron. Soc., 386: 461–474.

[14] Byrd, G., Valtonen, M., McCall, M., Innanen, K.A. (1994) Orbits of the Magellanic Clouds and Leo I in Local Group history, Astron. J., 107: 2055–2059.

[15] Kaiser, N. (1984) On the spatial correlations of Abell clusters. Astrophys. J. Lett., 284: L9–L12.

[16] Davis, M., Efstathiou, G., Frenk, C.S., White, S.D.M. (1985) The evolution of large-scale structure in a universe dominated by cold dark matter. Astrophys. J., 292: 371–394.

[17] Bardeen, J.M., Bond, J.R., Kaiser, N., Szalay, A.S. (1986) The statistics of peaks of Gaussian random fields. Astrophys. J., 304: 15–61.

[18] Pasetto, S., Chiosi, C. (2009) Tidal effects on the spatial structure of the Local Group, Astron. Astrophys., 499: 385–394.

[19] Colless, M. et al. (2001) The 2dF Galaxy Redshift Survey: spectra and redshifts. Mon. Not. R. Astron. Soc., 328: 1039–1063.

[20] York, D.G. et al. (2000) The Sloan Digital Sky Survey: Technical summary. Astron. J., 120: 1579–1587.

[21] Tojeiro, R., et al. (2012) The progenitors of present-day massive red galaxies up to z ~ 0.7 - finding passive galaxies using SDSS-I/II and SDSS-III. Mon. Not. R. Astron. Soc., 424: 136–156.