Electroweak baryogenesis with electroweak strings

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ABSTRACT: If stable electroweak strings are copiously produced during the electroweak phase transition, they may contribute significantly to the presently observed baryon to entropy ratio of the universe. This analysis establishes the feasibility of implementing an electroweak baryogenesis scenario without a first order phase transition.
1. INTRODUCTION

Recently there has been a revival of interest in studies of the electroweak phase transition. Two of the main reasons are the discovery that a nonvanishing baryon to entropy ratio can be generated at the electroweak scale,$^{1-4}$ and the rediscovery of solitonic solutions (electroweak strings$^5$) in the standard electroweak theory (which were initially discussed by Nambu$^6$).

Electroweak baryogenesis has immense relevance for cosmology. The goal is to explain the observed nonvanishing baryon to entropy ratio $n_B/s$, $n_B$ being the net baryon number density, $s$ the entropy density. As realized by Sakharov,$^7$ the three necessary ingredients to be able to generate a nonvanishing $n_B/s$ are the existence of baryon number violating interactions, $CP$ violation and the presence of out of equilibrium processes.

Until recently, the canonical implementation of baryogenesis has been in the context of grand unified models. The out of equilibrium decay of superheavy gauge and Higgs particles$^8$ was the main physical mechanism operating, making use of the explicit $n_B$ violation in the Lagrangian. A recently discovered variant$^9$ of GUT baryogenesis is based on the collapse of topological defects—in particular cosmic strings.

Of late, however, some problems have arisen for this scenario. Most importantly, it was realized that electroweak anomaly effects at temperatures above the critical temperature of the phase transition effectively erase$^{10}$ any primordial baryon symmetry if $B - L = 0$ ($B$ and $L$ being baryon and lepton numbers respectively). Hence it is particularly important to study the possibility of regenerating a nonvanishing $n_B/s$ ratio after the electroweak phase transition.

The mechanisms suggested so far for electroweak baryogenesis all rely on having a first order phase transition. The resulting bubble walls were required in order to obtain a region of unsuppressed baryon number violation occurring out of thermal equilibrium. Our
work is based on the observation that topological defects forming in a second order phase transition may play a similar role to the bubble walls. We propose a specific mechanism in which electroweak strings are responsible for baryogenesis.

Electroweak strings\textsuperscript{5}) are nontopological solitons which arise in the standard electroweak theory (and extensions thereof). They are essentially Nielsen-Olesen\textsuperscript{11}) strings of $U(1)_Z$ embedded in the $SU(2) \times U(1)$ theory ($U(1)_Z$ is the Abelian subgroup which is broken during the electroweak phase transition). For certain ranges of the parameters of the standard model, electroweak strings are energetically stable.\textsuperscript{12}) Like semilocal strings,\textsuperscript{13}) electroweak strings are not topologically stable.

If, however, we are in a region of parameter space in which electroweak strings are stable, a network of such strings will form during the electroweak phase transition—even if it is second order. Inside the strings, anomalous baryon number violating processes are unsuppressed. If the strings move, the out of thermal equilibrium condition will be satisfied. Finally, the standard model contains $CP$ violation. Hence all of Sakharov’s criteria are satisfied. As we shall demonstrate, it is in fact possible to generate a substantial $n_B/s$ using electroweak strings.

In the following we shall first briefly review the proposed electroweak baryogenesis scenarios (section 2) and electroweak strings (section 3). In section 3 we propose two specific mechanisms of baryogenesis based on electroweak strings. We conclude with a discussion of the results. Units in which $c = \hbar = k_B = 1$ are used throughout.

2. ELECTROWEAK BARYOGENESIS

Although at a classical level, the electroweak theory conserves baryon number, there are baryon number violating quantum effects due to the electroweak anomaly.\textsuperscript{14}) The
standard model also contains \( CP \) violating effects. Hence, provided some of the \( CP \) and baryon number violating processes occur out thermal equilibrium, it is possible to generate a nonvanishing \( n_B/s \) ratio after the electroweak phase transition.

Following some initial suggestions\(^1\) interesting specific mechanisms were proposed by Turok and Zadrozny\(^2\) and by Cohen, Kaplan, and Nelson.\(^3\)\(^,\)\(^4\) These mechanisms\(^2\)\(^−\)\(^4\) rely on having a first order electroweak phase transition producing bubbles of broken symmetry phase expanding into the unbroken phase. The bubble walls form the region where the out-of-equilibrium, \( CP \)-violating effects take place and where the net baryon number is generated.

The Turok and Zadrozny mechanism makes use of nontrivial winding number (“local texture”) configurations which are in equilibrium in the unbroken phase. Unwinding of such a configuration involves a change in Chern-Simons (and hence baryon) number. Inside the bubble wall the change in baryon number has a preferential direction (because of \( CP \) violation in the Higgs sector), and this leads to a net baryon asymmetry.

Cohen, Kaplan, and Nelson have proposed two mechanisms for electroweak baryogenesis. In the first,\(^3\) a detailed thermodynamic argument is presented showing that an effective chemical potential for baryon number is generated by the \( CP \) violating effects inside the bubble wall. The second mechanism\(^4\) is based on fermions incident from the unbroken phase scattering off the bubble walls. \( CP \) violation in the walls leads to an asymmetry in particle-antiparticle scattering, and hence to a lepton excess in the unbroken phase which is—via equilibration—converted into a baryon asymmetry. The baryon asymmetry does not change as the bubble wall passes by.

The three mechanisms discussed above are all inequivalent.\(^15\) The maximal baryon to entropy ratio \( n_B/s \) which can be obtained is of the order\(^4\)

\[
\frac{n_B}{s} \sim \epsilon \alpha^4_w
\]
where $\epsilon$ measures the strength of CP violation (in the two Higgs doublet model$^{,2-4}$) $\epsilon$ can be as large as 1) and $\alpha_w = g_w^2/4\pi$, $g_w$ being the weak coupling constant. Thus, it is not too hard to generate the observed baryon to entropy ratio at the electroweak scale.

However, it is unclear$^{16}$ whether the electroweak phase transition is sufficiently strongly first order for the above mechanisms to work. It is therefore interesting to explore the possibility of generating a nonvanishing $n_B/s$ below the electroweak symmetry breaking scale assuming that the transition proceeds without the formation of bubbles. We shall propose a concrete mechanism which makes use of electroweak strings.

3. ELECTROWEAK STRINGS

Electroweak strings$^{5,6}$ are essentially an embedding of the Nielsen-Olesen $U(1)$ string$^{11}$ in the standard electroweak theory. They are solutions of the field equations for all electroweak parameters, but are stable only for a narrow range of these parameters.$^{12}$

The Lagrangian for the bosonic part of the Weinberg-Salam model contains $SU(2)$ gauge fields $W^i$, $i = 1, \ldots, 3$, a $U(1)$ gauge field $B$ and a complex scalar doublet $\phi$. The vortex solution which extremizes the action is

\begin{equation}
\phi = f_{NO}(r)e^{im\theta}\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \vec{Z} = \vec{A}_{NO}
\end{equation}

and $\vec{A} = 0 = \vec{W}^a(a = 1, 2)$. Here, $r$ and $\theta$ are polar coordinates in the plane perpendicular to the string, $\vec{A}_{NO}$ is the Nielsen-Olesen gauge field,

\begin{equation}
\phi_{NO} = f_{NO}(r)e^{im\theta}
\end{equation}

is the Nielsen-Olesen Higgs field and

\begin{equation}
\vec{Z} = \cos \theta_w \vec{W}^3 - \sin \theta_w \vec{B}, \quad \vec{A} = \sin \theta_w \vec{W}^3 + \cos \theta_w \vec{B}.
\end{equation}
\( \theta_w \) is the weak mixing angle, and \( f_{NO}(r) \) is a function which approaches the symmetry breaking scale \( \eta \) at large \( r \) and vanishes for \( r = 0 \). It has been shown\(^{12}\) that these vortex solutions are stable for \( \sin^2 \theta_w \simeq 1 \), in which case the electroweak string is essentially the \( U(1)_B \) Nielsen-Olesen vortex.

In order to obtain a sufficiently large baryon to entropy ratio, the standard electroweak model must be extended by adding new terms in the Lagrangian which contain explicit \( CP \) violation. An often used prototype theory is the two Higgs model.\(^{2-4}\)

The above construction of nontopological vortex solutions in theories which do not satisfy the topological criterion for strings is not specific to the minimal standard model. Thus, we expect electroweak strings to exist also in extensions of the Weinberg-Salam model (This has recently been demonstrated in the two Higgs model \(^{17}\)). It is possible that these strings could be stable even for experimentally allowed values for the model parameters. In the following we shall assume that electroweak strings exist and are stable.

In models admitting stable electroweak strings, a network of such strings will form during the electroweak phase transition. If we consider a theory with Higgs potential

\[(5) \quad V(\phi) = \lambda(\phi^+ \phi - \eta^2/2)^2,\]

then the initial correlation length (mean separation of strings) will be\(^{18}\)

\[(6) \quad \xi(t_G) \simeq \lambda^{-1} \eta^{-1},\]

where \( t_G \) is the time corresponding to the Ginsburg temperature of the phase transition.

The initial network of electroweak strings will be quite different from that of cosmic strings, the reason being that electroweak strings can end on local monopole and anti-monopole configurations. From thermodynamic considerations,\(^{19}\) we expect most of the strings to be short, \( i.e., \) of length \( l \simeq \xi(t_G) \), since this maximizes the entropy of the network for fixed energy.
After the phase transition, the vortices will contract along their axes and decay after a time interval

\[ \Delta t_s \simeq \frac{1}{v} (\lambda \eta)^{-1} \]

where \( v \) is the velocity of contraction (expected to be \( \simeq 1 \)). In the following, we shall demonstrate that the string contractions will produce a net baryon symmetry.

4. THE BARYOGENESIS MECHANISM

We shall consider an extension of the standard electroweak theory in which there is additional \( CP \) violation in the Higgs sector. An example is the two Higgs model used in Refs. 2-4. We assume that electroweak strings can be embedded in this model \(^{17}\), and we choose the values of the parameters in the Lagrangian for which these strings are stable. Furthermore, the phase transition is taken to be second order.

A key issue is the formation probability of electroweak strings. In the following, we make the rather optimistic assumption that both the mean length and average separation of electroweak strings at \( t_G \) will equal the correlation length \( \xi(t_G) \). For topological defects, this result follows from the Kibble mechanism \(^{18}\). When applied to electroweak strings, the Kibble mechanism implies that the vortex fields \( \phi \) and \( Z \) have the correlation length \( \xi(t_G) \). However, to form an electroweak string, the other fields must be sufficiently small such that the configuration relaxes to the exact electroweak string configuration. Obviously, the restriction this imposes (and the consequent increase in the mean separation of electroweak strings) is parameter dependent - the more stable the strings, the smaller the increase in the mean separation. Pieces of string are bounded by monopole-antimonopole pairs. Energetic arguments tell us that the string will shrink. We now argue that the moving string ends will have the same effects on baryogenesis as the expanding bubble walls in Refs. 2&3.
We can phrase our argument either in terms of the language of Ref. 2 or of Ref. 3. The phase of the extra $CP$ violation is nonvanishing in the region in which the Higgs fields $\phi$ are changing in magnitude, i.e., at the edge of the string. Since $|\phi|$ increases in magnitude, $CP$ violation has a definite sign. Hence, in the language of Ref. 3, a chemical potential with definite sign for baryon number is induced at the tips of the string (where $|\phi|$ is increasing). This chemical potential induces a nonvanishing baryon number.

In the language of Ref. 2, the $CP$ violation with definite sign at the tips of the string leads to preferential decay of local texture configurations with a definite net change in Chern-Simons (i.e., baryon) number.

Let us now estimate the magnitude of this effect. The rate of baryon number violating events inside the string (in the unbroken phase) is

$$\Gamma_B \sim \alpha_w^4 T^4.$$  \hspace{1cm} (8)

The volume in which $CP$ violation is effective changes at a rate ($g$ is the gauge coupling constant)

$$\frac{dV}{dt} = g^2 w^2 V,$$  \hspace{1cm} (9)

where $w \simeq \lambda^{-1/2} \eta^{-1}$ is the width of the string and $v$ is its contraction velocity. The factor $g$ comes from the observation that baryon number violating processes are unsuppressed only if $|\phi| < g \eta$. \hspace{1cm} (20)

The rate of baryon number generation per string is

$$\frac{dN_B}{dt} \sim w^2 v \Gamma_B \epsilon \Delta t_c,$$  \hspace{1cm} (10)

where $\epsilon$ is a dimensionless constant measuring the strength of $CP$ violation and

$$\Delta t_c = \frac{g w}{v \gamma(v)}$$  \hspace{1cm} (11)

is the time a fixed point in space is in the transition region. Here, $\gamma(v)$ is the usual relativistic $\gamma$ factor. Since there is one string per correlation volume $\xi(t_G)^3$, the resulting
rate of increase in the baryon number density $n_B$ is

$$\frac{dn_B}{dt} \sim \lambda^{-3/2} \eta^{-3} g^3 \alpha^4 w T^4 \frac{1}{\gamma(v)} \xi(t_G)^{-3}.$$  

(12)

The net baryon number density is obtained by integrating (12) from $t_G$, the time corresponding to the Ginsburg temperature, and $t_G + \Delta t_S$ (see (7)). The result is

$$n_B \sim \frac{\lambda}{v \gamma(v)} g^3 \alpha^4 w T^4 G^3 \epsilon.$$  

(13)

Our result (13) must be compared to the entropy density at $t_G$:

$$s(t_G) = \frac{\pi^2}{45} g^* T^3 G^3,$$  

(14)

where $g^*$ is the number of relativistic spin degrees of freedom. From (13) and (14) we obtain

$$\frac{n_B}{s} \sim \frac{45}{\pi^2 g^* \gamma(v) v} \frac{\lambda}{\epsilon g^3 \alpha^4 w}.$$  

(15)

For $\lambda \sim v \sim 1$ and $\epsilon \sim 1$, the ratio obtained is only slightly smaller than the observational value.

In order for our mechanism to work, the core radius of the string ($|\phi| < g \eta$) must be large enough to contain the nonperturbative configurations which mediate baryon number violating processes. This leads to the condition $\lambda < g^4$, i.e. small Higgs mass. In addition, the sphaleron must be sufficiently heavy such that sphaleron transitions in the broken symmetry phase are suppressed for $T = T_G$. For small values of $\lambda$, this condition will automatically be satisfied. Finally, the model parameters must be such that the phase transition is of second order. In the standard electroweak theory, this condition is incompatible with $\lambda \ll g^4$. In any extended electroweak theory, the consistency of the above conditions must be satisfied in order for our baryogenesis mechanism to be effective.
5. DISCUSSION

We have presented a counterexample to the “folk theorem” stating that electroweak baryogenesis requires a first order electroweak phase transition. We propose a mechanism in which finite length electroweak strings during their contraction generate a nonvanishing net baryon number. The strings play a similar role to the expanding bubble walls in a first order phase transition: they provide out of equilibrium processes, and also a region where $CP$ violation occurs.

The mechanism presented here requires stable electroweak strings and an extra source of $CP$ violation (which is present in the two Higgs models used in Refs. 2-4). Based on the stability analysis of electroweak strings in the standard model,$^{12}$ it is unlikely that these strings will be stable for experimentally allowed values of the parameters in the Lagrangian.

Note that translational or rotational motion of the strings will not generate any net asymmetry since in this case the absolute value of the Higgs fields will increase at some points in space (those leaving the string) and decrease at others (those entering the string). Hence, the contributions to the net baryon number should cancel out. In our mechanism, it is essential that below $T_G$ there is a distinguished direction to the evolution of $|\phi|$, and hence a distinguished sign for the chemical potential for baryon number.

We hope that this work will point toward more realistic mechanisms of electroweak baryogenesis using second order phase transitions.

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