Pragmatic identification of the witness sets

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Abstract

Among the readings available for NL sentences, those where two or more sets of entities are independent of one another are particularly challenging from both a theoretical and an empirical point of view. Those readings are termed here as ‘Independent Set (IS) readings’. Standard examples of such readings are the well-known Collective and Cumulative Readings. (Robaldo, 2011) proposes a logical framework that can properly represent the meaning of IS readings in terms of a set-Skolemization of the witness sets. One of the main assumptions of Robaldo’s logical framework, drawn from (Schwarzschild, 1996), is that pragmatics plays a crucial role in the identification of such witness sets. Those are firstly identified on pragmatic grounds, then logical clauses are asserted on them in order to trigger the appropriate inferences. In this paper, we present the results of an experimental analysis that appears to confirm Robaldo’s hypotheses concerning the pragmatic identification of the witness sets.

Keywords: quantifiers, pragmatics, witness sets

1. Introduction

This paper is about the truth values of the Independent Set (IS) readings of NL sentences in the simple form ‘Subject-Verb-Object’. IS readings are interpretations where two or more sets of entities are independent of one another. Four kinds have been identified in the literature, since (Scha, 1981):

1. Branching Quantifier Readings, e.g. Exactly two students of mine have seen exactly three drug-dealers in front of the school.
2. Collective Readings, e.g. Exactly three boys made exactly one chair.
3. Cumulative Readings, e.g. Exactly three boys invited exactly four girls.
4. Cover Readings, e.g. Exactly three children ate exactly five pizzas.

The preferred reading of (1.a) is the one where there are exactly two students and exactly three drug-dealers and each of the students saw each of the drug-dealers. (1.b) may be true in case three boys cooperated in the construction of a single chair. In the preferred reading of (1.c), there are three boys and four girls such that each of the boys invited at least one girl, and each of the girls was invited by at least one boy. Finally, (1.d) allows for any sharing of five pizzas between three children. In Cumulative Readings, the single actions are carried out by atomic individuals only, while in (1.d) it is likely that the pizzas are shared among subgroups of children. For instance, the sentence is satisfied by the following extension of $\text{ate}'$ ($\oplus^1$ is the standard sum operator, from (Link, 1983)):

$$||\text{ate}'||^M = \{ \langle c_1\oplus c_2, p_1\oplus p_2 \rangle, \langle c_2\oplus c_3, p_3\oplus p_4 \rangle, \langle c_3, p_5 \rangle \}$$

In (2), children $c_1$ and $c_2$ (cut into slices and) shared pizzas $p_1$ and $p_2$, $c_2$ and $c_3$ (cut into slices and) shared $p_3$ and $p_4$, and $c_3$ also ate pizza $p_5$ on his own.

Branching Quantifier Readings have been the more controversial (cf. (Beghelli et al., 1997) and (Gierasimeczuk and Szymanik, 2009)), as many authors claim that those readings are always sub-cases of Cumulative Readings. Collective and Cumulative Readings have been largely studied; see (Link, 1983), (Beck and Sauerland, 2000), (Ben-Avi and Winter, 2003), and (Kontinen and Szymanik, 2008) to begin with. However, the focus here is on Cover readings. This paper assumes, following (van der Does and Verkuyl, 1996), (Schwarzschild, 1996), (Kratzler, 2007), that they are the IS readings, of which the three kinds exemplified in (1.a-c) are merely special cases. The name “Cover readings” comes from the fact that they are traditionally represented in terms of Covers, a particular mathematical structure. With respect to two sets $S_1$ and $S_2$, a Cover is formally defined as:

$$\text{Cov} \subseteq \text{Cov}_1 \times \text{Cov}_2,$$

where $\text{Cov}_1 \subseteq \wp(S_1)$ and $\text{Cov}_2 \subseteq \wp(S_2)$, s.t.

a. $\forall s_1 \in S_1, \exists \text{cov}_1 \in \text{Cov}_1 \text{ s.t. } s_1 \in \text{cov}_1$, and $\forall s_2 \in S_2, \exists \text{cov}_2 \in \text{Cov}_2 \text{ s.t. } s_2 \in \text{cov}_2$.

b. $\forall \text{cov}_1 \in \text{Cov}_1, \exists \text{cov}_2 \in \text{Cov}_2$ s.t. $\langle \text{cov}_1, \text{cov}_2 \rangle \in \text{Cov}$.

c. $\forall \text{cov}_2 \in \text{Cov}_2, \exists \text{cov}_1 \in \text{Cov}_1$ s.t. $\langle \text{cov}_1, \text{cov}_2 \rangle \in \text{Cov}$.

Covers may be denoted by 2-order variables called “Cover variables”. We may then define a meta-predicate Cover that, taken a Cover variable $C$ and two unary predicates $P_1$ and $P_2$, asserts that the extension of the former is a Cover of the extensions of the latter:

\[ \text{Cover}(C, P_1, P_2) \]
Cover variables, are pragmatically interpreted. Maximalization, provided that also Witness sets, besides
maximization. The present paper argues in favour of Local

\[ P \]

reading appears to be Cumulative.

two or more girls. Analogously, in (1.a) the fact that each

implicature that may be rewritten as in (5.b), where the sep-

were separated from the pigs. However, that is actually an

may be the case that two or more boys

thought as actions among pairs of individual persons. But it

precise separation is described by a contextually-dependent

variables whose extensions include the atomic individuals

variables whose extensions include the atomic individuals

Covers appears to be pragmatically determined, rather than

cusses numerous NL sentences where the identification of

Among the Cover approaches mentioned above, an inter-

Monotonicity of Generalized Quantifiers.

The previous section showed that, for representing IS read-

ings, it is necessary to reify the witness sets into relational

variables as \( P_1 \) and \( P_2 \). Separately, the elements of these

sets are combined as described by the Cover variables, in

order to assert the predicates on the correct pairs of (possi-

bly plural) individuals. As argued by (Sher, 1990), (Sher,

1997), (Steedman, 2007) and (Robaldo, 2010) the relational

variables must, however, be \textit{Maximized} in order to achieve

the proper truth values with any quantifier, regardless to its

monotonicity\(^2\) (cf. also (Dalrymple et al., 1998) and (Win-

ter, 2001)). To see why, let us consider (6.a-c), taken from

(Robaldo, 2010), that involve a single quantifier.

(6) a. At least two men walk.

b. At most two men walk.

c. Exactly two men walk.

In terms of reified relational variables, it seems that the

meaning of (6.a-c) may represented via (7.a-c), where \( \geq 2, \leq 2, \) and \( ^* 2 \) are, respectively, an \( M^* \), an \( M^+ \), and a non-M

Generalized Quantifier.

(7) a. \( \exists P[ \geq 2_4 (\text{man}'(x), P(x)) \land \forall x [P(x) \rightarrow \text{walk}'(x)] ] \)

b. \( \exists P[ \leq 2_4 (\text{man}'(x), P(x)) \land \forall x [P(x) \rightarrow \text{walk}'(x)] ] \)

c. \( \exists P[ = 2_4 (\text{man}'(x), P(x)) \land \forall x [P(x) \rightarrow \text{walk}'(x)] ] \)

Only (7.a) correctly yields the truth values of the corre-

sponding sentence. To see why, consider a model in which

three men walk. In such a model, (7.a) is true, while (7.b-c)

are false. Conversely, all formulae in (7) evaluate to true,
as all of them allow to choose \( P \) such that \( \| P \|^{M} \) is a set of

two walking men. Therefore, we cannot allow a free choice of \( P \). Instead, \( P \) must denote the Maximal set of indi-

viduals satisfying the predicates, i.e. the Maximal set of

walking men, in (7). This is achieved by changing (7.b-c)
to (8.a-b) respectively.

(8) a. \( \exists P[ \geq 2_4 (\text{man}'(x), P(x)) \land \forall x [P(x) \rightarrow \text{walk}'(x)] \land \forall P'[\forall x[P(x) \rightarrow P'(x)] \land \forall x[P'(x) \rightarrow \text{walk}'(x)] \rightarrow \forall x[P'(x) \rightarrow P(x)] ] ] \)

b. \( \exists P[ = 2_4 (\text{man}'(x), P(x)) \land \forall x [P(x) \rightarrow \text{walk}'(x)] \land \forall P'[\forall x[P(x) \rightarrow P'(x)] \land \forall x[P'(x) \rightarrow \text{walk}'(x)] \rightarrow \forall x[P'(x) \rightarrow P(x)] ] ] \)

The clauses \( \forall P[ \ldots ] \) in the second rows are Maximali-

ty Conditions asserting the non-existence of a superset \( P' \)
of \( P \) that also satisfies the predicate. There is a single choice

for \( P \) in (8.a-b): it must denote the set of \textit{all} walking men.

Note that, for the sake of uniformity, the Maximality con-

dition may be added in (7.a) as well: in case of \( M^+ \) quanti-

fiers, it does not affect the truth values.

\(^2\)See (Barwise and Cooper, 1981) for a survey on possible

monotonicities of Generalized Quantifiers.
2.1. Local Maximalization

Let us term the kind of Maximalization done in (8) as Local Maximalization. The Maximality conditions in (8) require the non-existence of a set \( |P^M_1| \) of walkers that includes \( |P^M_2| \). (Robaldo, 2010) proposed a logical framework for representing Branching Quantifier based on Local Maximalization. For instance, in (Robaldo, 2010), the two witness sets of students and drug-dealers in (1.a) are respectively reified into two variables \( P_1 \) and \( P_2 \), and the Maximality condition requires the non-existence of a Cartesian Product \( |P^M_1| \times |P^M_2| \), that also satisfies the main predicate and that includes \( |P^M_1| \times |P^M_2| \).

(9)

\[
\exists P_1 P_2 [\exists_2 (\text{stud}'(x), P_1(x)) \land \exists_3 (\text{drugD}'(y), P_2(y)) \land \\
\forall x y [(P_1(x) \land P_2(y)) \rightarrow \text{saw}'(x, y)] \\
\forall P_1' P_2' [\forall x y [(P_1'(x) \land P_2'(y)) \rightarrow (P_1'(x) \land P_2'(y))]] \\
\forall x y [(P_1'(x) \land P_2'(y)) \rightarrow \text{saw}'(x, y)] \\
\forall x y [(P_1'(x) \land P_2'(y)) \rightarrow (P_1(x) \land P_2(y))]] 
\]

As extensively argued in (Robaldo, 2011), in order to extend (Robaldo, 2010) to Cover readings we cannot simply require the inclusion of \( |P^M_1| \times |P^M_2| \) into the main predicate’s extension. Rather, we require the inclusion therein of a pragmatically-determined Cover \( |C|^{M,g} \) of \( |P^M_1| \) and \( |P^M_2| \). Furthermore, the (local) Maximality condition must require the non-existence of a superset of either \( |P^M_2| \) or \( |P^M_1| \) whose corresponding Cover is a superset of \( |C|^{M,g} \) that is also included in the main predicate’s extension. Thus, (1.d) is represented as 3:

(10)

\[
\exists P_1 [\forall x [P_1(x)] \rightarrow P_1'(x)] \\
\exists C [\forall x y [C(x,y) \rightarrow \text{ate}'(x,y)] \\
\forall P_1' [\forall x y [P_1'(x) \rightarrow P_1(x)]] \\
\forall P_2' [\forall y [P_2'(y)] \rightarrow P_2(y)] \\
\exists C [\forall x y [C(x,y) \rightarrow \text{ate}'(x,y)] \\
\forall P_1 [\forall x y [C(x,y) \rightarrow C'(x,y)] \\
\forall y [P_2'(y) \rightarrow P_2(y)]] \\
\forall x y [C'(x,y) \rightarrow \text{ate}'(x,y)]] \\
\forall P_2 [\forall y [P_2'(y) \rightarrow P_2(y)]] \\
\exists C [\forall x y [C(x,y) \rightarrow C'(x,y)] \\
\forall x y [C'(x,y) \rightarrow \text{ate}'(x,y)]] \\
\forall y [P_2'(y) \rightarrow P_2(y)] 
\]

Note that there are two Maximality conditions, i.e., \( \forall P_1 [\ldots] \) and \( \forall P_2 [\ldots] \), rather than a single one. Contrary to what is done with Cartesian Products, in Cover readings \( P_1 \) and \( P_2 \) must be Maximized independently, as it is no longer required that every member of the former is related with every member of the latter. Note also that \( P_1 \) and \( P_2 \) are pragmatically determined, as it is done with Cover variables in Schwarzschild’s, rather than being existentially quantified as in formula (9). In other words, their value is provided by an assignment function \( g \). This is the main point addressed (below) in this paper.

3. An experiment on IS readings

To summarize, in (Robaldo, 2011) witness sets are firstly identified on pragmatic grounds, then they are locally maximized. It is important to understand that, in Robaldo’s, Maximality conditions are not thought as “constraints that must be satisfied in order to judge the formula as true in the context”. Rather, they must be thought as “asserted knowledge needed to draw the appropriate inferences from the sentences’ meaning”. Conversely, the evaluation of the formula, i.e. the task of deciding whether a sentence is true or false in a certain model, is totally devolved upon the interpretation function \( g \).

What could be “the pragmatic grounds” that may affect the identification of the witness sets? As mentioned above, the use of certain determiners seems to affect the interpretation of the main predicate (Cumulative rather than Collective or each-all), i.e. the value of the Cover variables. Analogously, (Geurts and van der Silk, 2005) provide evidence that \( M^g_1 \) quantifiers are simpler to reason with, and this seems to explain why the identification of their witness sets is usually oriented towards the whole set of individuals in the model, rather than to specific sub-groups.

2.2. Global Maximalization

The other kind of Maximalization of the witness sets, termed here as ‘Global Maximalization’ has been advocated by (Schein, 1993), and formalized in most formal theories of Cumulativity, e.g. (Landman, 2000), (Hackl, 2000), and (Ben-Avi and Winter, 2003). With respect to IS readings involving two witness sets \( |P^M_1| \) and \( |P^M_2| \), Global Maximalization requires the non-existence of other two witness sets that also satisfy the predicate but that do not necessarily include \( |P^M_1| \) and \( |P^M_2| \). For instance, the event-based logic defined by (Landman, 2000) represents the Cumulative Reading of (1.c) as:

(11)

\[
\exists e \in \text{INVITE} : \exists x \in \text{BOY} : |x|=3 \land \text{Ag(e)=x} \\
\exists y \in \text{GIRL} : |y|=4 \land \text{Th(e)=y} \\
|\text{Ag(})| \in \text{INVITE} : |\text{Ag(e)=BOY} \land \text{Th(e)=GIRL})| = 3 \\
|\text{Th(}(|\text{e\in \text{INVITE} : |\text{Ag(e)=BOY \land \text{Th(e)=GIRL})})| = 4 
\]

Formula in (11) asserts the existence of a plural event \( e \) whose Agent is a plural individual made up of three boys and whose Theme is a plural individual made up of four girls. The two final conjuncts, in boldface, are Maximality conditions. Taken \( e_x \) as the plural sum of all inviting events having a boy as agent and a girl as theme, i.e.

\[
e_x \in \text{INVITE} : |e\in \text{BOY} \land \text{Th(e)=GIRL}} |= 4
\]

the cardinality of its agent \( \text{Ag}(e_x) \) is exactly three while the one of its theme \( \text{Th}(e_x) \) is exactly four.

Therefore, Landman’s Maximality conditions in (11) do not refer to the same events and actors quantified in the first row. Rather, they require that the number of the boys who invited a girl in the whole model is exactly three and the number of girls who were invited by a boy in the whole model is exactly four.
On the other hand, several cognitive experimental results showed that many other factors besides monotonicity, e.g., expressivity/computability, fuzzyness, the fact that quantifiers are cardinal rather than proportional, etc., may affect the interpretation of IS readings (cf. (Sanford and Paterson, 1994), (Szymanik, 2009), (Bott and Rad, 2009), (Musolino, 2009), (Szymanik and Zajenkowski, 2010), and (Szymanik, 2010)). As it is clear to understand, however, extra-linguistic factors seem the ones that mainly affect the interpretation of the variables. For instance, knowing that certain individuals are friends or are member of a team could induce the identification of sub-groups of individuals. In order to attest these hypotheses on empirical data, we carried out an online questionnaire. The experiment and its results are presented below.

3.1. Instructions
In the questionnaire, we show a set of sentences, each together with a figure. The subjects are asked to tell whether the sentence is true or false in the context depicted by the figure. There are eight target sentences, i.e. sentences for which we collect the results, plus twelve fillers, i.e. sentences whose answers are rather obvious and so they are not registered in the database. Fillers were used to prevent subjects from using some simplified strategy that could only work with specific experimental target items. The eight target sentences are:

(12)

a. Exactly three boys ate exactly three pizzas.

b. Exactly one boy ate exactly one pizza.

c. Fewer than three boys ate exactly one pizza.

d. More than three boys ate most pizzas.

e. Fewer than half of the boys ate exactly three pizzas.

f. Exactly two boys ate exactly three pizzas.

g. More than five boys ate more than four pizzas.

h. Fewer than three boys ate exactly one pizza.

The figures describe boys eating pizzas. Boys and pizzas are represented with stylized drawings, while the eating actions with lines connecting boys to pizzas. When a boy is connected by a line to a pizza, we mean that he ate the pizza. When two or more boys are connected to the same pizza, we mean that they ate it together, by cutting it into slices and sharing the slices. Boys are grouped into teams. Boys belonging to different teams are shown in the figures by means of different colors.

Each target sentence is associated with four figures. One of the figures is randomly chosen and shown to the subject together with the sentence. The four figures associated with a sentence include the same boys, the same pizzas and the same connections. They differ to each other for the presence/absence of two “pragmatic factors”: some distance may be added between sub-groups of boys, and/or the boys may belong to different teams rather than to a single one. Examples of the figures/scenarios used are shown below.

3.2. The questionnaire
We exploited the social network Facebook for inviting people to the questionnaire. We registered more than 23,000 participants.

Let us start by analyzing single experiment trials. The role of pragmatics in quantifiers’ interpretation is strongly visible in the analysis of sentence (12.f). The sentence was tested with respect to the four scenarios shown in fig.1. As pointed out above, the scenarios differ for the occurrence of two “pragmatic factors”: the subgroups of boys could have different colors and more distance may be added between the two pairs of witness sets. Each of the four scenarios corresponds to one of the available combinations: (A) does not include any pragmatic factor, (D) includes both, while (B) and (C) include only one of them. Obviously, our predictions were that the presence of pragmatic factors would induce the identification of the sub-structures, i.e., they would favor the local interpretation rather than the global one.

As said above, our predictions are met with respect to sentence (12.f) in the scenarios of fig.1 (see Table 1). Interestingly, also in scenario (A) a slight majority of subjects chose the local interpretation.

Table 1: Evaluation of (12.f) in scenarios fig.1 A-D

| Scenario | Yes  | No  | Don’t know | Yes%  |
|----------|------|-----|------------|-------|
| A        | 2830 | 2143| 292        | 56.91%|
| B        | 3316 | 1653| 264        | 66.73%|
| C        | 3352 | 1569| 260        | 68.12%|
| D        | 3525 | 1406| 291        | 71.49%|

In fig.2, we show the four scenarios associated with sentence (12.d). Those scenarios have been used to evaluate the sentence ‘More than three boys ate most pizzas’, that includes two $M^1\uparrow$ quantifiers. The results for the scenarios in fig.2 are shown in Table 2.

Also Table 2 appears to confirm our predictions. The sentence is logically true in all scenarios shown in fig.2, in line
with Schein’s theory. Nevertheless, most subjects answered it is false. Note also the high number of ‘Don’t know’ answers; in our view, many subjects simply found this example ‘confusing’, due to the high number of boys and pizzas occurring in the figures and the two pragmatic factors we inserted therein.

Note that sentence (12.g) is very similar to (12.d) as it also includes two $M\uparrow$ monotone quantifiers. The results of the (12.g)’s evaluation are very similar to the ones of (12.d).

In fig.3 we show the four scenarios where the sentence (12.b) is evaluated. Note that we inserted a different pragmatic factor in place of the greater distance between substructures. The sub-structure including one boy and one pizza only is crossed with respect to the other (bigger and more complex) one. The goal of the crossing is to avoid the identification of the witness sets making true the sentence. Nevertheless, we observe that in most cases subjects do manage to identify these witness sets. The results of fig.3 are shown in Table 3. In our view, these results may be explained by observing that the quantifier “Exactly one” has a very strong pragmatic preference towards the identification of sub-structures. Whenever a subject reads “Exactly one”, s/he most likely look for a single individual isolated from the others. In other two tests including the quantifier “Exactly one”, i.e. (12.c) and (12.h), the result are very similar.

In fig.4, we show the scenarios where sentence (12.e) has been evaluated. The crucial feature of this sentence is that it involves both a non-$M$ quantifier (‘Exactly three’) and a $M\downarrow$ one (‘Fewer than half of the boys’). In other words, it represents a mixed case. The results seem to confirm our hypotheses on the sentence’s preferred meaning. Most subjects do appear to identify the sub-structure of boys and pizzas making true the two quantifiers.

Finally, we show in fig.5 the single tuple of scenarios for which our hypotheses were not met. They are the four scenarios associated with sentence (12.a). The results are shown in Table 5. The percentages of ‘yes’ are very low in

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*Surprisingly, the percentage of ‘yes’ in (A) is superior to the one in (D). The visual effect given by the vertical line connecting the last boy on the right with the pizza below him appears to be a pragmatic factor even stronger than colors.*
each of the four scenarios.

![Figure 5: Four scenarios for sentence (12.a).](image)

In our view, these results are due to the fact that sentence (12.a) was the first sentence shown to the subjects. Note that (12.a) (and its associated scenarios) is very similar to sentence (12.f) (and its associated scenarios). But the results are quite different. Therefore, perhaps subjects had an initial inclination towards Global interpretation, but after evaluating some sentences for which the Local one is preferred, they tended to interpret also (12.f) locally.

The order along which sentences was evaluated can be observed as a further pragmatic factors affecting the interpretation.

### 4.1. Statistical analysis of the results

Below we only present a preliminary statistical analysis indicating that overall the interpretation depends on the pragmatic factors. We focus on two independent variables: Color’ with values ‘Non-colored’ and ‘Colored’, and ‘Distance’ with values ‘No-distance’ and ‘Distance’ (cf. figures). Let us describe the influence of those manipulations on the selection of local or global reading by our subjects.

- **Non-colored/Colored** Under No-colored condition 37% of all responses were global, 54% local, and 9% undecided. Under Colored condition: 34% global, 57% local, 9% undecided. The value of the color condition and the reading preferred by the subject are dependent ($\chi^2=231; df=2; p<0.001$). Therefore, in line with our predictions, sentences associated with pictures marking possible subgroups with different colors were more often interpreted locally.

- **Non-crossed/Crossed** Under No-crossed condition 29% of all responses were local, 67% global, and 4% undecided. Under Crossed condition: 35% global, 60% local, 5% undecided. The value of the crossed condition and the reading preferred by the subject are dependent ($\chi^2=227; df=2; p<0.001$). Therefore, in line with our predictions, sentences associated with pictures suggesting the whole group as a witness set were more often interpreted globally.

- **No-distance/Distance** Under No-distance condition 34% of all responses were global, 56% local, and 9% undecided. Under Distance condition: 36% global, 53% local, 10% undecided. The value of the distance condition and the reading preferred by the subject are moderately dependent ($\chi^2=7; df=2; p<0.05$). Therefore it seems that there is a statistical tendency towards the interpretation that added distance could trigger a preference for the local interpretation.

The main conclusion one can draw from our results is that the considered sentence do not have the absolute truth values. Their interpretation appears to be dependent on the possible pragmatic factors.

### 5. Conclusions

In this paper we presented an empirical study on Independent Set readings. The aim of the study was the one of comparing the two kinds of Maximalization proposed in the literature for handling the proper truth values of IS readings, termed here as ‘Local’ and ‘Global’ Maximalization respectively. The former requires the non-existence of any tuple of supersets of the witness sets that also satisfies the predication. The latter requires the witness sets to be the only tuple of sets that satisfies the predication.

The results of our experiment show that none of them suffices to properly handle the truth values of IS readings. The reason is that the identification of the witness sets appears to be highly subjective. Sometimes, subjects are able to focus on sub-structure of witness sets. Sometimes they are not, i.e. they consider all occurring individuals as a whole. Moreover, certain pragmatic factors, e.g. the knowledge that boys are divided into teams, a greater distance between offices to properly handle the truth values of IS readings. The latter requires the witness sets to be the only tuple of sets that satisfies the predication.

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an assignment function $g$ identifies the witness sets the sentence refers to, then (local) Maximality Conditions are asserted on them.

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