Force prediction in ultrasonic vibration-assisted milling

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ABSTRACT

The use of ultrasonic vibration on milling has several benefits including reduction of the machining forces. However, the mechanism behind this phenomenon is unclear, and reported analytical studies are quite limited. An analytical predictive modeling work is presented in the current study. To describe the intermittent contact between tool and workpiece due to vibration, conditions of tool-workpiece separation are described by three types of criteria. The first criterion checks the instantaneous moving direction of cutting edge. The second criterion examines the radial displacement of cutting edge under vibration. The third criterion considers the smaller chip thickness due to extra displacement from previous tool path. If the material is being removed, the force prediction is performed through transformation of milling configuration, calculation of shear flow stress by mechanics of machining, and calculation of feed, cutting and axial force after coordinate transformation. The predicted forces are compared with experimental measurements on Aluminum alloy 2A12 for validation. The average percentage difference is 13.6\% in feed direction and 13.8\% in cutting direction. This is the first approach to mathematically describe the intermittent contact between tool and workpiece and combine the kinematic analysis with mechanics of machining to predict cutting forces.

KEYWORDS

Force; milling; modeling; ultrasonic vibration

Introduction

Non-conventional milling has been developed for years in order to reach high precision, extend tool life while keep the material removal rate. Ultrasonic vibration-assisted milling is a newly developed non-conventional milling process to achieve these targets in a more ecofriendly manner comparing to laser-assisted or electrical discharge milling. Vibration is applied on the tool or workpiece, with micro-scale high frequency and small amplitude, to realize the tool-workpiece separation during the milling process. Several research studies have concluded that the tool-workpiece separation in ultrasonic vibration-assisted milling is the main reason for several
benefits including improved surface quality (Chen et al., 2018), lower machining forces and extended tool life (Xu and Zhang, 2015). This tool-workpiece interaction is microscopically non-monotonic which facilitates chip separation and therefore reduces machining forces. In addition, the frequent separation reduces the deformation zone of workpiece during milling which is the main reason for improved surface finish. Several experimental investigations and statistical analysis have been conducted on ultrasonic vibration-assisted milling. Hsu et al. (2009) analyzed the effects of milling parameters on cutting forces in ultrasonic vibration-assisted milling of Inconel 718 and found that depth of cut was the dominant factor. Rasidi et al. (2016) studied the cutting force reduction on Aluminum Al6061, and a 32% reduction was recorded on mean forces while peak forces were at same level for conventional and ultrasonic vibration-assisted milling. Similarly, Halim et al. (2017) recorded a 10% reduction on maximum machining force in feed direction for ultrasonic vibration-assisted milling of carbon fiber reinforced plastic. These studies reflect the benefits of ultrasonic vibration-assisted milling, but the quantitative conclusions are limited to specific material and process parameter combinations and are unable to reveal the physics nature analytically.

Fundamental types of cutting force models dedicated for tradition non-vibration assisted milling have been developed for years. Xiao et al. (2015) applied molecular dynamics modeling to visualize the crack formation and stress distribution in the cutting zone. Wojciechowski and Mrozek (2017) developed mechanistic model including run out and variable edge forces to predict cutting forces in micro ball end milling. Wan et al. (2019) considers the dead metal zone in cutting force prediction during micro milling. Wojciechowski et al. (2019) predicted cutting forces during micro end milling considering chip thickness accumulation. All these models reached reasonable accuracy and explained the milling process well. However, there are few analytical models proposed for force prediction in ultrasonic vibration-assisted milling up to now. Verma et al. (2018) predicted static machining force in axial ultrasonic vibration-assisted milling. The instantaneous chip thickness is calculated based on the frequency and maximum velocity of vibration, and the mean cutting force for the specific tool angular position is derived from contact ratio and shear flow stress based on Johnson–Cook model. This method is able to calculate mean oblique cutting forces accurately through mechanics of machining. However, the tool-workpiece separation criterion in axial direction is relatively easy to decide, and the dynamics of moving tool or workpiece is simplified as a contact ratio. Ding et al. (2010) predicted cutting forces in two-dimensional vibration-assisted micro milling. The kinematic analysis is similar to current work, but the previous and current tool tip paths were calculated explicitly.
as opposed to tool-workpiece separation criteria in current study. In addi-
tion, the cutting force model applied was based on Tlusty–Macneil model, 
while the proposed model is based on Oxley’s mechanics of machining. 
Later, Shen et al. (2010) calculated cutting force of micro end milling with 
ultrasonic vibration in normal direction. Abootorabi Zarchi et al. (2012) 
predicted the critical cutting speed and tool-workpiece separation zones.
Elhami et al. (2015) also proposed analytical model on cutting force in 
two-dimensional vibration-assisted milling including a heat transfer model to 
incorporate the thermal effect. Verma and Pandey (2019) predicted 
machining forces with ultrasonic vibration through analysis of variance and 
regression equations. All the models so far are either not fully analytical or 
limited in specific vibration direction.

The proposed model is able to characterize vibration in feed, cutting or 
axial direction and consider the intermittent effect through three tool-
workpiece separation criteria (Feng et al., 2019). This is the first approach 
to mathematically describe the intermittent contact between tool and work-
piece and combine the kinematic analysis with mechanics of machining to 
predict cutting forces. At each specific tool angular position, the milling is 
considered as equivalent orthogonal cutting, and tool geometry as well as 
cutting parameters is recalculated (Pan et al., 2017a, 2017b, 2017c; Feng 
et al., 2018; Mirkoohi et al., 2018, 2019b; Ning et al., 2019). The shear zone 
strain and strain rate are decided based on contact mechanics (Ning and 
Liang, 2019), and the flow stress is calculated dependent on constitutive 
equation (Mirkoohi et al., 2019a). Flow stress over contact area decides cut-
ting force, and the coordinate transformation is applied to disassemble 
forces into feed, cutting and axial directions. The predicted forces are com-
pared with experimental measurements on Aluminum alloy 2A12 (Shen 
et al., 2012; Shen and Xu, 2018) for validation. The effects of different cut-
ting and ultrasonic parameters, including axial depth of milling, feed per 
tooth, ultrasonic frequency and spindle speed, on average forces are exam-
ined through sensitivity analysis.

**Force predictive model in ultrasonic vibration-assisted milling**

**Tool-workpiece separation criteria**

Kinematic analysis is conducted on tool under ultrasonic vibration, tool 
rotation and feed movement. According to the coordinate system in Figure 
1, the initial tool center position is chosen as origin, the position of tool 
center is changing with time as

\[
x(t) = A_x \sin (\omega_x t + \theta_x) - ft, \\
y(t) = A_y \sin (\omega_y t + \theta_y),
\]  

(1)  

(2)
\[ z(t) = A_z \sin(\omega_z t + \theta_z) + d, \]  
\[ x'(t) = \omega_x A_x \cos(\omega_x t + \theta_x) - f, \]  
\[ y'(t) = \omega_y A_y \cos(\omega_y t + \theta_y), \]  
\[ z'(t) = \omega_z A_z \cos(\omega_z t + \theta_z). \]

where \( d \) is the axial depth of milling, \( f \) is the feed rate, \( \omega_x \), \( \omega_y \) and \( \omega_z \) are the angular ultrasonic vibration frequency, \( \theta_x \), \( \theta_y \) and \( \theta_z \) are the phase angle, \( A_x \), \( A_y \) and \( A_z \) are the vibration amplitude in each direction, which is zero when no vibration is applied in that direction. The velocity of tool center and cutting edge are also calculated. The tool center velocity is described by the derivative of Equations (1)–(3) as

\[ V_x(t) = \omega_x A_x \cos(\omega_x t + \theta_x) - f - V_r \cos(\phi_r), \]  
\[ V_y(t) = \omega_y A_y \cos(\omega_y t + \theta_y) + V_r \sin(\phi_r), \]

where \( V_r \) is the cutting speed and \( \phi_r \) is the rotation angle.

There are three types of tool-workpiece separation criteria that decide whether there is contact between tool and workpiece at the moment. Type I criterion is described by Equations (4)–(8), which is satisfied when the cutting edge is moving away from the uncut surface (Abdur-Rasheed, 2011; Chen et al., 2018). \( V_n = -V_x \cos(\phi_r) + V_y \sin(\phi_r) \) is the instantaneous cutting velocity based on Equations (7) and (8) as in Figure 2a, and if there is vibration in axial direction, \( V_{ul} = z'(t) \) is the axial ultrasonic vibration

\[ Figure 1. \text{Coordinate system for kinematic analysis of cutting tool.} \]
velocity. $\beta$ is the helix angle. The type I intermittent effect is described by the transverse velocity perpendicular to the uncut surface. The cutting edge is moving away from the uncut surface when this transverse velocity is negative as in Figure 2b, and the tool will lose contact with workpiece as described by

\[
\text{Type I criterion : } V_n \cos(\beta) - V_{ul} \sin(\beta) < 0
\]

Type II criterion is met if the instantaneous tool center displacement in radial direction is larger than the instantaneous uncut chip thickness. If the tool center is at a position where it is further away from the workpiece comparing to the initial position, tool and workpiece are separated even
though Type I criterion is not satisfied. This criterion is mathematically expressed as

Type II criterion: \( x(t) \sin(\phi_r) + y(t) \cos(\phi_r) > t_{UVA,radial} \), \( (10) \)

where \( t_{UVA,radial} \) is the instantaneous uncut chip thickness.

Type III criterion describes the smaller uncut chip thickness in ultrasonic vibration-assisted milling due to overlaps between the current and previous tool paths. For conventional milling shown in Figure 3a, the material removal process is continuous, so the chip has large thickness. For ultrasonic vibration-assisted milling shown in Figure 3b, smaller chips are produced because of the extra radial displacement from previous cutting path. Therefore, the instantaneous uncut chip thickness is recalculated as

Type III criterion: \( t_{UVA,radial} = \max\{ (t_c - \max[A_x \sin(\phi_r) \sin(\omega_x t + \theta_x) + A_y \cos(\phi_r) \sin(\omega_y t + \theta_y)], 0]), 0 \}, \) \( (11) \)

where \( t_c \) is the instantaneous uncut chip thickness without vibration.

Based on the kinematic analysis, the tool-workpiece separation criteria are considered. If there is no contact, the machining forces are zero. If there is contact, the forces are calculated based on instantaneous cutting parameters.

**Instantaneous equivalent cutting and geometry parameters with ultrasonic vibration**

At each specific tool angular position, the milling is considered as equivalent orthogonal cutting. The average uncut chip thickness in conventional milling is given as
\[
\bar{t}_c = \frac{2}{\pi} \frac{f}{RPM/60},
\]

where \(RPM\) is the spindle speed. The instantaneous uncut chip thickness is calculated as

\[
t_c = \frac{\pi}{2} \times \bar{t}_c \times \sin(\phi_r).
\]

As shown in Figure 4, based on Equations (11)-(13), the instantaneous equivalent uncut chip thickness is calculated as

\[
t_{UVA} = t_{UVA,\text{radial}} \times \cos C_s^*,
\]

where \(C_s^*\) is the equivalent side cutting edge angle defined as

\[
C_s^* = C_s + \eta_c,
\]

where \(\eta_c\) is the chip flow angle calculated based on cutting parameters and tool geometry (Pan et al., 2017b).

As shown in Figure 5, the equivalent chip flow angle \(\eta_c^*\) is the same as the equivalent inclination angle \(i^*\) by
\[ \eta_{c}^* = i^* = \arcsin(\cos \eta_0 \sin i - \sin \eta_0 \sin \alpha \cos i), \]  \(16\)

where \(\alpha\) is the rake angle and \(i\) is the inclination angle. \(\eta^*\) is

\[ \eta_0 = \arccos \left( \frac{\sec i - \tan i \tan \eta_c \tan \alpha}{\sqrt{(\tan i - \tan \eta_c \tan \alpha \sec i)^2 + \sec^2 \eta_c}} \right). \]  \(17\)

And the equivalent rake angle \(\alpha^*\) is

\[ \alpha^* = \arcsin \left( \frac{\sec \eta_0 \sin i - \sin i^*}{\tan \eta_0 \cos i^*} \right). \]  \(18\)

As shown in Figure 4, the equivalent cutting width under orthogonal cutting condition is dependent on instantaneous axial depth of milling as

\[ w^* = \frac{z(t)}{\cos (C_s^*)}. \]  \(19\)

The cutting force and thrust force are then calculated under orthogonal cutting configuration, based on the equivalent cutting and tool geometry parameters.
Orthogonal cutting forces with ultrasonic vibration

As shown in Figure 6, the shear length along shear plane AB is given by

\[ l_s = \frac{t_{UVA}}{\sin \phi}, \]  

(20)

where shear angle \( \phi \) is decided through exhaustive search. For chip speed or the shear velocity \( V_s \), it is derived from cutting speed as

\[ V_s = \frac{\cos \alpha^*}{\cos (\phi - \alpha^*)} \sqrt{V_x^2 + V_y^2}. \]  

(21)

The plastic strain rate and strain are then calculated

\[ \dot{\varepsilon}_{AB} = \frac{C_{Oxley} V_s}{\sqrt{3} l_s}, \]  

(22)

\[ \varepsilon_{AB} = \frac{2 \sqrt{3} \sin \phi \cos (\phi - \alpha^*)}{\cos \alpha^*}, \]  

(23)

where \( C_{Oxley} \) is a model coefficient. The average temperature of the shear plane is described as

\[ T_{AB} = T_0 + \eta \Delta T, \]  

(24)

where \( T_0 \) is the room temperature, \( \eta \) is the plastic energy to enthalpy conversion ratio. The temperature increment is due to the shear energy in the primary shear plane \( F_s V_s \), the chip flow rate is \( \sqrt{V_x^2 + V_y^2} t_{UVA} w^* \). The temperature rise in the primary shear zone is obtained as (Pan et al., 2017a)

\[ \Delta T = \frac{(1 - \beta_c) F_s V_s}{\rho C_P \sqrt{V_x^2 + V_y^2} t_{UVA} w^*}, \]  

(25)

where \( F_s \) is the shear force in shear zone, \( F_s = k_{AB} \times l_s \times w^* \), \( \beta_c \) is the energy dissipation coefficient, \( \rho \) is the material density and \( C_P \) is the heat capacity. The average material flow stress in the shear plane is given by Johnson–Cook model as

\[ k_{AB} = \frac{1}{\sqrt{3}} \left( A + BE_{AB}^n \right) \left( 1 + C \ln \left( \frac{\dot{\varepsilon}_{AB}}{\dot{\varepsilon}_0} \right) \right) \left( 1 - \left( \frac{T_{AB} - T_w}{T_m - T_w} \right)^m \right), \]  

(26)

where \( A, B, C, m \) and \( n \) are model parameters based on material properties, \( \dot{\varepsilon}_0 \) is the reference strain rate, \( T_m \) is the melting temperature of workpiece. The values of the parameters \( (A, B, C, m \) and \( n) \) for the Aluminum alloy are 243.0, 618.8, 0.01, 1.6 and 0.2, respectively (Zhang et al., 2015). The angle \( \theta \) between resultant force R and shear plane AB is defined as follows:

\[ \theta = \tan^{-1} \left( 1 + 2 \left( \frac{\pi}{4} - \phi \right) - C_n \right), \]  

(27)
where $C_n = C_{Oxley} \frac{B_{e_A} n}{A + B_{e_A}}$. The friction angle $\lambda$ is denoted as

$$\lambda = \theta + \alpha^* - \phi.$$  

(28)

The tool and chip contact length is

$$h = \frac{t_{UVA} \sin \theta}{\cos \lambda \sin \phi} \left(1 + \frac{C_n}{3 \tan \theta} \right).$$  

(29)

By assuming the uniform stress distribution along the chip tool interface, the shear stress is calculated as

$$\tau_{int} = \frac{F}{hw^*},$$  

(30)

where $F$ is the shear force along the chip tool interface, $F = \frac{F_s}{\cos \theta} \sin \lambda$. With a similar approach, the temperature at the tool and chip interface is

$$T_{int} = \frac{F \sin \alpha^*}{\rho C_p t_{UVA} w^* \cos (\phi - \alpha^*)} + T_{AB}.$$  

(31)

The strain and strain rate in the chip are

$$\varepsilon_{int} = 2\varepsilon_{AB} + \frac{h}{\sqrt{3} \delta t_d},$$  

(32)

$$\dot{\varepsilon}_{int} = \frac{V_C}{\sqrt{3} \delta t_d},$$  

(33)

where $t_d$ is the deformed chip thickness and $\delta$ is a coefficient related to the thickness of plastic deformation zone on tool chip interface. Then the average flow stress in chip is

$$k_{chip} = \frac{1}{\sqrt{3}} \left(A + B_{e_{int}} n \right) \left(1 + C \ln \frac{\varepsilon_{int}}{\varepsilon_0} \right) \left(1 - \left(\frac{T_{int} - T_w}{T_m - T_w}\right)^m\right).$$  

(34)

Through the exhaustive search of $\phi$, $C_{Oxley}$ and $\delta$, these parameters are decided when $\tau_{int} = k_{chip}$ and the largest shear angle $\phi$ is selected. The orthogonal cutting forces are then calculated as

$$F_c = \frac{F_s}{\cos \theta} \cos (\lambda - \alpha^*),$$  

(35)

$$F_t = \frac{F_s}{\cos \theta} \sin (\lambda - \alpha^*).$$  

(36)

**Milling forces with ultrasonic vibration**

After the cutting force $F_c$ and radial force $F_t$ being calculated, the axial force is calculated as
\[ F_r = \frac{F_c (\sin i^* - \cos i^* \sin \alpha^* \tan \eta_c^*) - F_t \cos \alpha^* \tan \eta_c^*}{\sin i^* \sin \alpha^* \tan \eta_c^* + \cos i^*}. \]  

These three forces are first transferred into cutting, feed and axial directions as

\[
P_1 = F_c, \\
P_2 = -F_t \cos (C_\xi^*) - F_r \sin (C_\xi^*), \\
P_3 = F_t \sin (C_\xi^*) - F_r \cos (C_\xi^*).  
\]

Then, the forces are transferred into Cartesian coordinates as

\[
F_x(\phi) = P_1 \cos (\phi_r) + P_2 \sin (\phi_r), \\
F_y(\phi) = -P_1 \sin (\phi_r) - P_2 \cos (\phi_r), \\
F_z(\phi) = -P_3.  
\]

**Validation by experimental data**

The predicted forces are compared with experimental measurements on Aluminum alloy 2A12 (Shen et al., 2012; Shen and Xu, 2018) for model validation. The experiments are performed on DECKEL MAHO five-axis high-speed CNC machining center. The ultrasonic generator and vibrator, composed of a transducer and a booster, convert high-frequency oscillation electric energy into workpiece vibration along the feed direction. The dynamic cutting force signals are gathered through Kistler 9257B 3-component piezoelectric dynamometer. The cutting tool is a two-flute end mill made of cemented carbide. The diameter is 2 mm, and the helix angle is 30°. The feed per tooth is 3 \( \mu \)m, the spindle speed is 1000 r/min, and the axial depth of milling is 0.2 mm. For ultrasonic vibration, the vibration frequency is 19.58 kHz, and four slot-milling experiments are conducted with vibration amplitude of 0 (conventional milling), 4, 6 and 8 \( \mu \)m, respectively, as shown in Figure 7.

The machining forces in feed and cutting directions are calculated in analytical model between a rotation angle of 0° and 180°. Figure 8 shows the predicted instantaneous forces as well as measured raw data within half cutting cycle during conventional milling. When the vibration is applied, if the tool-workpiece separation criterion is met, both forces are recorded as zero. Otherwise, the forces are calculated based on methodology described. The first 1000 data points are plotted in Figure 9. The complete predicted force profiles are shown in Figure 10 with 30 data points within one ultrasonic vibration period and a total of 17,623 data points. The number of force data points predicted is later reduced to accommodate the number of measurements limited by the sampling frequency of dynamometer. The values of the parameters (A, B, C, m and n) for the aluminum alloy are 243.0,
618.8, 0.01, 1.6 and 0.2, respectively (Zhang et al., 2015). The measured force profiles are shown in Figure 11. As the force variation due to ultrasonic vibration is on the same scale of noisy signal, qualitative comparison is made between two waveforms through amplitude spectrum. Figure 12 shows single-sided amplitude spectrums of measured as well as predicted
When vibration amplitude is $8 \mu m$ in half cutting cycle based on Figures 10 and 11. The sampling frequency is 1750 Hz for dynamometer. There are two dominant frequencies. The fast Fourier transform of both experimental and predicted data shows presence of vibration around 1750 Hz due to intermittent cutting effect under the sampling frequency. Another peak is observed at near 0 Hz occurring due to tool engagement. The predicted spectrums in Figure 12b and d have more low frequency components since forces are considered zero when there is no contact between tool and workpiece, while measured force signals still have non-zero values at valleys. In addition, measured spectrums in Figure 12a and c have more high frequency components as a result of noise such as tool
Figure 10. Predicted force profiles of (a) $F_x$ and (b) $F_y$ when vibration amplitude is 8 $\mu$m in half cutting cycle.

Figure 11. Measured force profiles of (a) $F_x$ and (b) $F_y$ when vibration amplitude is 8 $\mu$m (Shen et al., 2012).
chatter. Overall, the predicted force amplitude spectrums have good agreements with experimental measurements on both feed and cutting forces.

**Figure 12.** Comparison of single-sided amplitude spectrum of $F_x$ between (a) experiment (Shen et al., 2012) and (b) predictive model, $F_y$ between (c) experiment (Shen et al., 2012) and (d) predictive model, during ultrasonic vibration-assisted milling with 8 $\mu$m amplitude.

Figure 13 shows the average cutting forces from experiments and predictive model under four different vibration amplitudes. The average forces
are 1.93 and 1.7 N for conventional milling in feed and cutting directions. With a vibration amplitude of 4 μm, the average forces in two directions are 1.17 and 1.1 N, respectively. The average values decrease by 39% in feed direction and 35% in cutting direction when the ultrasonic vibration is applied. In addition, the measured forces keep dropping gradually when

Figure 12. Continued.
the amplitude increases, since the tool-workpiece separation time is longer under higher vibration amplitude. The drop rate is approximately a constant as the average forces decrease linearly. The feed force is changing from 1.17 to 1.09 followed by 1.05 N as ultrasonic vibration amplitude increases from 4 to 6 and 8 \( \mu \text{m} \). Similarly, the cutting force is changing from 1.1 to 1.05 followed by 0.98 N as ultrasonic vibration amplitude increases from 4 to 6 and 8 \( \mu \text{m} \). Comparison of maximum force reduction between current work and literature is listed in Table 1. Up to 50% reduction in force is reported by different studies.

The predicted average forces are plotted as dashed lines in Figure 13. For conventional milling, both predicted forces are higher than measurements. In feed direction, the predicted value is 2.01 N which is 4.15\% higher. In cutting direction, the prediction is 2.14 N and the error is 25.88\% which is acceptable comparing with previous established conventional milling force predictive model (Pan et al., 2017b). When the ultrasonic vibration is applied, the average values decrease by 50\% in feed direction and 35\% in cutting direction. The feed force is changing from 1.01 to 0.96 followed by 0.79 N as ultrasonic vibration amplitude increases from 4 to 6 and 8 \( \mu \text{m} \). Similarly, the cutting force is changing from 1.39 to 1.07 followed by 0.97 N as ultrasonic vibration amplitude increases from 4 to 6 and 8 \( \mu \text{m} \). The maximum error in all cases is less than 27\%. Overall, the proposed force prediction model is able to match the trend with average error of 13.6\% in \( F_x \) and 13.8\% in \( F_y \). Comparison of accuracy between proposed and existing predictive models is presented in Table 2. Under similar condition, the proposed model shows the same or better effectiveness comparing with existing models.
Sensitivity analysis

In order to appreciate the proposed predictive model, sensitivity analysis is conducted to estimate average forces under the effects of different cutting and ultrasonic parameters including axial depth of milling, feed per tooth, ultrasonic frequency and spindle speed. A wide range of variation is selected for each parameter as aluminum alloy is relatively soft which lowers the possibility of tool chatter and wear in real life scenarios. The ultrasonic vibration amplitude is fixed at 6 μm, and other parameters are the same as in experiment. As shown in Figure 14a, a higher axial depth of milling will significantly increase the milling forces in both directions. \( F_x \) and \( F_y \) are doubled from 0.96 and 1.07 N to 1.92 and 2.14 N when the axial depth of milling changes from 0.2 to 0.4 mm. The axial depth of milling decides the cutting width according to Equation (19). With the increase of cutting width, the contact area is also expanded resulting in higher forces under same flow stress. Similarly, a doubled feed per tooth from 3 to 6 μm also approximately doubles \( F_x \) and \( F_y \) from 0.96 and 1.07 N to 1.88 and 2.20 N because of larger chip thickness according to Equation (12), as shown in Figure 14b. On the other hand, when the ultrasonic vibration frequency increases, the cutting speed is higher, leading to decreased milling forces. \( F_x \) and \( F_y \) are 25.4% and 15.0% smaller when the ultrasonic vibration frequency increases by 28.6% as depicted in Figure 14c. The spindle

| Vibration type     | Material                  | Maximum force reduction                  |
|--------------------|---------------------------|------------------------------------------|
| Current study      | Feed vibration            | Aluminum alloy 2A12                      |
| Rasidi et al. (2016) | 2 Dimensional            | Aluminum alloy 6061                       |
| Elhami et al. (2015) | Feed vibration          | Hardened AISI 4140                        |
| Halim et al. (2017) | Axial vibration           | Carbon fiber                              |
| Shen and Xu (2018)  | Feed vibration            | Aluminum alloy 2A12                      |
| Abootorabi Zarchi et al. (2012) | Feed vibration | X20Cr13 stainless steel                  |

| Vibration type     | Material                  | Maximum difference                      |
|--------------------|---------------------------|------------------------------------------|
| Current study      | Feed vibration            | Aluminum alloy 2A12                      |
| Verma et al. (2018) | Axial vibration           | Aluminum alloy 2A12                      |
| Ding et al. (2010) | 2-dimensional             | Aluminum alloy 6063                      |
| Shen et al. (2010) | Axial vibration           | Low carbon steel                         |
| Elhami et al. (2015) | Feed vibration          | Hardened AISI 4140                       |
| Verma and Pandey (2019) | Axial vibration | Aluminum alloy 6063                      |

**Sensitivity analysis**

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speed also decides cutting speed. $F_x$ and $F_y$ are 28.7% and 13.2% smaller when the spindle speed increases by 250% as depicted in Figure 14d. The results of sensitivity analysis are in good agreement with conclusions from Verma et al. (2018). The findings can be used as a guideline to design
future experiments on difficult-to-cut materials, where the selection of cutting and ultrasonic parameters becomes more critical. In return, the results on difficult-to-cut materials can be used to improve the model as high machining temperature and force will require the model to consider more factors such as tool wear and microstructure change.

**Conclusion**

A force predictive model is proposed during ultrasonic vibration-assisted milling. Three tool-workpiece separation criteria are considered based on kinematic analysis. If the material is being removed, the calculation is performed at each rotation angle under orthogonal cutting configuration. The cutting and thrust forces are predicted through the calculation of shear flow stress based on constitutive equation. After coordinate transformation, the feed, cutting and axial forces are calculated. The predicted forces are compared with experimental measurements on Aluminum alloy 2A12 for model validation. The following conclusions are summarized:

- The intermittent contact between tool tip and uncut surface is explained by three criteria: relative velocity between cutting edge and uncut surface, comparison between instantaneous tool center displacement in radial direction and instantaneous uncut chip thickness and overlaps between the current and previous tool paths. These criteria are described mathematically through kinematic analysis.
- With a vibration amplitude of 4 µm, the average forces in ultrasonic vibration-assisted milling are significantly lowered by 39% in feed direction and 35% in cutting direction comparing to conventional milling.
- The average forces keep decreasing by additional 10% as the vibration amplitude increases to 8 µm, since the tool-workpiece separation time is longer, and the effective cutting time is shorter.
The proposed model has high accuracy, with 13.6% difference in feed direction and 13.8% in cutting direction between predictions and measurements.

Based on the sensitivity analysis, a doubled axial depth of milling from 0.2 to 0.4 mm or feed per tooth from 3 to 6 μm will approximately double the milling forces in both directions from 1 to 2 N.

Based on the sensitivity analysis, a higher ultrasonic vibration frequency increased by 28.6% will lower the forces by 25.4% and 15.0% in feed and cutting directions.

Based on the sensitivity analysis, a higher spindle speed increased by 250% will result in lower milling forces by 28.7% and 13.2% in feed and cutting directions.

The proposed method is up to now the first approach to predict forces in milling with ultrasonic vibration in feed direction. And it is capable of providing an accurate and reliable reference and is applicable for force prediction in all types of vibration-assisted milling. Future works will focus on the validation of model in other types of vibration-assisted milling. And inverse analysis will be conducted to find optimal vibration parameters that give the minimum cutting forces.

**Nomenclature**

- $d$: axial depth of milling
- $\omega_x$, $\omega_y$, $\omega_z$: angular ultrasonic vibration frequency
- $\theta_x$, $\theta_y$, $\theta_z$: phase angle
- $A_x$, $A_y$, $A_z$: vibration amplitude
- $f$: feed rate
- $V_r$: cutting speed
- $\phi_r$: rotation angle
- $V_{ul}$: axial ultrasonic vibration velocity
- $\beta$: Helix angle
- $t_{UVA, radial}$: instantaneous uncut chip thickness
- $t_c$: instantaneous uncut chip thickness without vibration
- RPM: spindle speed
- $C_s$: side cutting edge angle
- $\eta_c$: chip flow angle
- $i$: inclination angle
- $\alpha$: rake angle
- $w$: cutting width
- $l_s$: shear length
- $\phi$: shear angle
- $V_s$: shear velocity
- $\varepsilon_{AB}$: plastic strain
- $C_{Oxley}$: Oxley’s model coefficient
- $T_{AB}$: average temperature of the shear plane
$T_0$ room temperature
$\eta$ plastic energy to enthalpy conversion ratio
$F_s$ shear force in shear zone
$\beta_e$ energy dissipation coefficient
$\rho$ material density
$C_p$ heat capacity
$k_{AB}$ average material flow stress
$\dot{\varepsilon}_0$ reference strain rate
$T_m$ melting temperature
$\lambda$ friction angle
$h$ tool and chip contact length
$\tau_{int}$ shear stress
$F$ shear force along the chip tool interface
$t_d$ deformed chip thickness
$k_{chip}$ average flow stress in chip
$F_c$ cutting force
$F_t$ radial force
$F_r$ axial force

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