Single- and two-photon interference within the simulated photon wave function in coordinate representation

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Abstract. The main provisions of the construction of the single-particle wave function of a photon in the coordinate representation are expounded and its application to the description of single-photon interference phenomena is discussed. The connection between the concepts of electromagnetic field strengths used in quantum electrodynamics and the characteristics of wave packets in photon quantum mechanics is substantiated. For illustration, we consider the relationship between the structure of the photon wave function in coordinate representation and the polarization of photon in the formalism of quantum transition amplitudes. The meaning of the concept of linear polarization of a photon in the state of a wave packet is explained. The wave function is constructed in the coordinate representation of the state of two entangled photons at the exit from a nonlinear crystal by modeling the polarization vectors and Gaussian momentum distribution for each of the photons.

1. Introduction

Over the past about a decade and a half, new directions related to quantum physics have been rapidly developing in the scientific world. Special attention is paid to the development of quantum computers, the implementation of quantum cryptography and teleportation, the formulation and understanding of concepts such as classical and quantum realism, nonlocality of interactions, etc. The development of these trends leads to ideas that are outside the framework of orthodox quantum mechanics. Considerable importance is attached to the implementation and use of single-photon and "entangled" two-photon states. From a metaphysical point of view, “single-particle” interference phenomena, which are conventionally considered equivalent to Young's double-slit experiment, are still of great interest, for example, for photons - in the Mach-Zehnder interferometer, in which a photon or a particle with a mass interferes “with itself”.

In such phenomena, light can no longer be considered in the form of a classical electromagnetic wave, and the results of experiments with photons are explained in terms of quantum transition amplitudes, just like the results of experiments with particles having mass. At the same time, both for photons and for particles, the riddle of wave-particle duality remains unresolved. However, if for particles this "duality" can be formally accepted, remaining within the framework of the Copenhagen interpretation of quantum mechanics, then the concept of a photon-corpuscle is highly doubtful, "in the usual sense", due to the impossibility of localizing a photon in space. Meanwhile, on the other hand, the
use of the language of quantum amplitudes just contributes to the "formation" of the idea that a photon supposedly manifests itself as a corpuscle in certain states in experiments with single photons.

To eliminate this inconsistency and, in particular, to significantly smooth out the problem of wave-particle duality, in our opinion, the photon wave function in the coordinate representation should be used to describe single-photon interference. Substantially, this was done in a number of works [1–6]. The purpose of this article is to demonstrate the description using the wave function in the coordinate representation of the entangled state of two photons at the initial moment of time as a result of their formation when leaving a nonlinear crystal.

2. The photon wave function in the coordinate representation

Apparently, the history of the discussion of the construction of the photon wave function in the coordinate representation goes back to the pioneering work [7], in which, however, it was "forbidden" for several decades (see also [8, 9]), and now the point of view still dominates (see, for example, [10–12]) that the construction of this function is impossible. However, with the advent of sources [13–16] and detectors [17–21] of single photons, the relevance of constructing the wave function of a photon in the coordinate representation began to manifest itself again [22–30]. At the same time, the emphasis in its interpretation gradually shifted from the concept of photon localization to the spatial point of its detection at the corresponding moment in time. The question was also raised about the possibility of normalizing this function to the total unit probability of detecting a photon in the entire space (see, for example, [31]). Further development of the mathematical apparatus and physical ideas related to the wave function of a photon in the coordinate representation can be traced from the works [32–37] and the references therein.

The main provisions of the construction of the photon wave function in the coordinate representation, in accordance with [31, 36, 37], are summarized below.

In [31, 36, 37], the eigenvectors ("circularly polarized plane waves") of the operators of energy, momentum, and helicity of a photon were obtained:

$$\xi^{(\pm)} \xi_{\pm 1}(\mathbf{t}, \mathbf{r}) = \frac{(Oe)e^{i\mathbf{k} \cdot \mathbf{r}}}{\sqrt{2\pi^2}} e^{i(\mathbf{k} \cdot \mathbf{r} + \mathbf{t} \cdot \mathbf{c})}; \quad \eta^{(\pm)} \eta_{\pm 1}(\mathbf{t}, \mathbf{r}) = \frac{(Oe)e^{i\mathbf{k} \cdot \mathbf{r}}}{\sqrt{2\pi^2}} e^{i(\mathbf{k} \cdot \mathbf{r} + \mathbf{t} \cdot \mathbf{c})}. \quad (1)$$

These vectors are solutions of Maxwell's equations presented by Majorana [38] in quantum-mechanical form (two of the equations are similar to the Schrödinger equation). They are used to construct the photon wave function - the wave packet with some specified coefficients $b(k, \pm 1)$:

$$\Psi^{(\pm)}(\mathbf{r}, \mathbf{t}) = \int \frac{b(k, \pm 1)}{(2\pi)^{3/2}} \left( \begin{array}{c} 1 \\ 0 \end{array} \right) e^{i(k \cdot \mathbf{r} + \mathbf{t} \cdot \mathbf{c})} d^3k + \int \frac{b(-k, \mp 1)}{(2\pi)^{3/2}} \left( \begin{array}{c} 0 \\ 1 \end{array} \right) e^{i(-k \cdot \mathbf{r} + \mathbf{t} \cdot \mathbf{c})} d^3k, \quad (2)$$

where $(Oe)$ is the measure unit (Oersted) of $\zeta$, $\eta$; the signs "$\pm$" in the upper indices respond to photon with positive and hypothetical negative energy, and "$\mp$" or "$\mp$" in the lower indices and in coefficients $b(k, \pm 1)$ respond to the photon helicity $\lambda$, according to the upper signs. In (1) the $\mathbf{e}_j(k) = [\mathbf{e}_j(k) + i\lambda \mathbf{e}_\perp(k)]/\sqrt{2}$ are the polarization vectors, where $\mathbf{e}_j$, $\mathbf{e}_\perp$ are the real mutually orthogonal unit vectors constituting the right-handed triad with the vector $\mathbf{n} = \mathbf{k}/\mathbf{k}$:

$$|\mathbf{e}_j| = |\mathbf{e}_\perp| = 1; \quad (\mathbf{e}_j, \mathbf{n}) = (\mathbf{e}_\perp, \mathbf{n}) = (\mathbf{e}_j, \mathbf{e}_\perp) = 0; \quad \mathbf{e}_\perp = [\mathbf{n} \times \mathbf{e}_j]; \quad \mathbf{n} = i\lambda [\mathbf{e}_j \times \mathbf{e}_\perp]. \quad (3)$$

Moreover, the vector $\mathbf{e}_j$ does not change when the direction of the $\mathbf{n}$ is changed: $\mathbf{e}_j(\mathbf{n}) = \mathbf{e}_j(-\mathbf{n})$; from (3) the other relations also follow:

$$\left( \mathbf{e}_\perp, \mathbf{e}_\lambda \right) = \delta_{\perp\lambda}; \quad \mathbf{e}_\perp^* \mathbf{e}_\lambda = \delta_{\perp\lambda}; \quad \mathbf{e}_\lambda(\mathbf{n}) = \mathbf{e}_\perp(-\mathbf{n}); \quad \left[ \mathbf{e}_\lambda(\mathbf{k}) \right]^* = \mathbf{e}_\perp(\mathbf{k}) = \mathbf{e}_\lambda(-\mathbf{k}). \quad (4)$$
Function (2) is a solution of the Schrödinger-type equation established in [31, 36, 37] and it satisfies the unit probability normalization and the continuity equation [31, 36, 37] for the probability density 

\[ \rho_p^{(\pm)}(r,t) \] of detecting a photon near a point \( r \) at the moment \( t \). For a free or diffracting photon, the coefficients \( b(k,\pm) \) must be determined from the initial and boundary conditions. However, the behavior of the photon can be modeled by setting these coefficients for some physical reasons and then comparing the results with real experiments. In any case, these coefficients must satisfy the condition of normalization to the unit probability:

\[ \langle \Psi^{(\pm)} | \Psi^{(\pm)} \rangle = \int \rho_p^{(\pm)}(r,t) d^3r = \int d^3k \rho_p^{(\pm)}(k) = \int d^3k \left\{ |b(k,\pm)|^2 + |b(-k,\mp)|^2 \right\} = 1. \]  

(5)

3. Modeling the femtosecond laser radiation

In [1, 37, 39, 40], the modeling of wave packet with Gaussian distribution in photon momenta was carried out. For this, we have chosen \( b(k,\pm) \) in the form

\[ b(k,\pm) = [b(-k,\mp)]^* = \frac{\alpha_1\alpha_2\alpha_3}{2\pi^3} \exp \left[ -\frac{1}{2} \left( \alpha_1^2 k_x^2 + \alpha_2^2 k_y^2 + \alpha_3^2 (k_z \mp k_0)^2 - ikr_0 \right) \right], \]  

(6)

and the vectors of polarization were written as follows (in matrix form): 1) at \( 0 \leq \theta \leq \pi/2 \) – as

\[ e_I (k) = \begin{pmatrix} e_{Iz} \\
 e_{Iy} \\
 e_{Ix} \end{pmatrix} = \begin{pmatrix} 1-(1-\cos \theta)\cos \varphi \\
 -(1-\cos \theta)\sin \varphi \cos \varphi \\
 -\sin \theta \cos \varphi \end{pmatrix}, \quad e_{II} (k) = \begin{pmatrix} e_{IIz} \\
 e_{IIS} \\
 e_{IIY} \end{pmatrix} = \begin{pmatrix} -(1-\cos \theta)\sin \varphi \cos \varphi \\
 \cos \theta + (1-\cos \theta)\cos \varphi \cos \varphi \\
 -\sin \theta \sin \varphi \end{pmatrix}. \]  

(7)

(where \( \theta \) and \( \varphi \) define the vector \( k \) in spherical coordinate system), and 2) at \( \pi/2 < \theta \leq \pi \) – as

\[ e_I (k) = \begin{pmatrix} e_{Iz} \\
 e_{Iy} \\
 e_{Ix} \end{pmatrix} = \begin{pmatrix} 1-(1+\cos \theta)\cos \varphi \\
 -(1+\cos \theta)\sin \varphi \cos \varphi \\
 \sin \theta \cos \varphi \end{pmatrix}, \quad e_{II} (k) = \begin{pmatrix} e_{IIz} \\
 e_{IIS} \\
 e_{IIY} \end{pmatrix} = \begin{pmatrix} (1+\cos \theta)\sin \varphi \cos \varphi \\
 \cos \theta - (1+\cos \theta)\cos \varphi \cos \varphi \\
 -\sin \theta \sin \varphi \end{pmatrix}. \]  

(8)

It was established in [1, 37, 39, 40] that for \( t = 0 \) the case when \( \alpha_1 = \alpha_2 = \alpha_3 = \alpha \) gives the wave packet spatial shape being as practically spherically symmetric. On axis \( z \) of the wave packet symmetry, the points of the “cloud” of probability density of detecting a photon at the forefront of the packet move almost precisely at the light speed in vacuum. The further away from \( z \) axis, the less the movement velocity of the accordant probability density. Thus, the original “spherical” packet shape is reformed into a “cone-shaped” one, reminding Cherenkov radiation process. The rate of spreading of the packet (2) is higher the less its starting “radius”, in accordance with the common positions of quantum mechanics. These conclusions are also in accordance [40] with the experimentally established [41] “reduction” of the “group” velocity of photons in comparison with the light phase speed in vacuum, what indicates the adequacy of the model (6)–(8).

4. Photon wave function of atom electric dipole radiation and the Young’s experiment

If, however, it is necessary to construct a single-photon state so that it corresponds to a certain state of the electromagnetic field, initially described by the intensities \( E \) and \( H \), then first one should compose a vector \( \xi = E + iH \), and then find [36, 37] the coefficients \( b(k,\lambda) \) that determine the wave function of the photon (2):

\[ b(k,\lambda) = \frac{\langle 0E | B(k,\lambda) }{\sqrt{8\pi\hbar c}} = \frac{1}{\langle 0E | \sqrt{8\pi\hbar c} \int d^3r \left[ \bar{\xi}^{(\pm)} e^{i\bar{\xi}^{(\pm)}(k,\pm)} (r,t) \right] \xi(r,t). \]  

(9)
Consider, for example, an electric dipole oriented along the z-axis and performing harmonic oscillations with a frequency \( \omega_0 = k_0 c \). The intensities of the electromagnetic field emitted by it in the wave zone, as is known, are

\[
E_r = E_\varphi = 0, \quad E_\theta = A \sin \theta, \quad H_r = E_\varphi, \quad H_\varphi = A \sin \theta, \quad H_\theta = -A \cos \theta
\]

(10)

Moving in (10) from spherical to Cartesian components of the field, we have

\[
\xi = \left( \frac{\xi_x}{\xi_y}, \frac{\xi_z}{\xi_y} \right) = A \frac{\sin (\omega_0 t - k_0 r) \cos \varphi}{r} \left( \begin{array}{c} \cos \theta \cos \varphi - i \sin \varphi \\ \cos \theta \sin \varphi + i \cos \varphi \\ -\sin \theta \end{array} \right).
\]

(11)

Although in classical electrodynamics the coefficient \( A \) in (10) is expressed in terms of the dipole parameters, it is advisable to find it, for example, based on the time of radiation of the atom, equating all the radiated energy to the value \( h \nu_0 \). In this article we leave the coefficient \( A \) free.

Substituting \( \xi_{k, z}^{(r, t)}(r, t) \) from (1), and (7), (8), (11) into (9), may obtain \([6]\) the wave function (2) in the form:

\[
\Psi^{(+)}(r, t) = \frac{A e^{-ik_0ct}}{4a} \sqrt{\frac{k_0}{\pi \hbar c}} \left[ -i \left( \begin{array}{c} y I_1 - x I_2 \\
-ix I_1 - y I_2 \\
-ia I_3 \end{array} \right) + \left( \begin{array}{c} 0 \\
0 \\
1 \end{array} \right) \left( \begin{array}{c} i y I_1 + x I_2 \\
-ix I_1 + y I_2 \\
-ia I_3 \end{array} \right) \right],
\]

(12)

where

\[
I_1 = \left( \begin{array}{c} 2 \\
0 \\
-2 \end{array} \right) \sum_{n=0}^{\infty} \frac{(-2)^n (k_0 z)^{2n+1}}{(ak_0)^{n+1}(2n)!} J_n \left( \frac{z}{ak_0} \right),
\]

(13)

\[
I_2 = \left( \begin{array}{c} 2 \\
0 \\
-2 \end{array} \right) \sum_{n=0}^{\infty} \frac{(-2)^n (k_0 z)^{2n}}{(ak_0)^{n+1}(2n)!} J_n \left( \frac{z}{ak_0} \right),
\]

(14)

\[
I_3 = 4 \sum_{n=0}^{\infty} \frac{(-1)^n (k_0 z)^{2n+1}}{(2n+1)(2n+3)(2n)!} F_2 (2; 1, n+5/2; -(ak_0)^2/4),
\]

(15)

where \( F_2 \) is the generalized hypergeometric function. The case, when \( ak_0 >> 1 \) and \( |z| << a \), where \( a = \sqrt{x^2 + y^2} \), is applicable to explain Young’s experiment, for instance, since series (13)–(15) quickly converge. Taking into account the terms only with \( n = 0 \) and supposing \( z/a \approx z/r = \cos \theta \), we obtain \([6]\) from (12):

\[
\Psi^{(+)}(r, t) = \frac{A e^{-ik_0ct}}{4a \sqrt{\pi \hbar k_0 c}} \left[ \begin{array}{c} 1 \\
0 \\
-1 \end{array} \right] \left( \begin{array}{c} -i \cos (ak_0) \\
\sin (ak_0) \cos \theta \\
-\sin (ak_0) \cos \theta \end{array} \right).
\]

(16)

Using the same method as in \([2, 4]\), it can be shown that this function describes a single-photon interference picture of Young’s experiment.

From general physical considerations, taking into account (10) and the expression for the energy density of the electromagnetic field of classical electrodynamics, in formula (16) the value \( a \) should be replaced by \( r \), taking into account that formula (16) was obtained in the zero approximation. The resulting expression is similar in structure to the photon wave function obtained in \([2, 4]\), proceeding...
from other considerations and for a different type of simulation. Then the wave function of a photon passing "through both holes" in the first screen of Young’s experiment takes the form

$$\Psi(r, t) = \Psi_1(r, t) + \Psi_2(r, t) = \frac{B e^{-ik_0ct}}{r_1} \left[ \begin{pmatrix} -i \cos(r_1k_0) \\ \sin(r_1k_0) \cos \theta_1 \\ -i \sin(r_1k_0) \end{pmatrix} + \begin{pmatrix} -i \cos(r_2k_0) \\ -\sin(r_2k_0) \cos \theta_2 \\ -i \sin(r_2k_0) \end{pmatrix} \right] + \frac{B e^{-ik_0ct}}{r_2} \left[ \begin{pmatrix} 1 \\ 0 \\ -i \sin(r_2k_0) \end{pmatrix} \right] \begin{pmatrix} -i \cos(r_1k_0) \\ \sin(r_1k_0) \cos \theta_1 \\ -i \sin(r_1k_0) \end{pmatrix} + \begin{pmatrix} -i \cos(r_2k_0) \\ -\sin(r_2k_0) \cos \theta_2 \\ -i \sin(r_2k_0) \end{pmatrix} \right] = \Psi_1(r + \frac{d}{2} - t) + \Psi_2(r - \frac{d}{2} - t)$$

where $B$ includes all constants, vector $d$ connects holes; $r_1, r_2$ – the distances from the holes to the observation point $P$ located on the second screen (at a distance $\ell$ from the first). Writing down the probability density of photon detection [31, 36, 37] as $\Psi^* \Psi$, we obtain an interference term in it, which, after transformations and neglect of the term including the product $\cos \theta_1 \cos \theta_2$, reduces to the form

$$\rho_{\text{int}} = \frac{4B^2}{r_1r_2} \left[ \sin(k_0r_1) \sin(k_0r_2) + \cos(k_0r_1) \cos(k_0r_2) \right] = \frac{4B^2}{r_1r_2} \cos(k_0(r_2 - r_1)) = \frac{4B^2}{r_1r_2} \cos \delta, \quad (18)$$

where it is assumed that $r_1 + r_2 \approx 2\ell, \quad r_2 - r_1 = \Delta$, where $\Delta$ is the optical path difference of the rays emanating from both holes, $\delta = 2\pi \Delta/\lambda_0$ is their phase difference from the point of view of classical electrodynamics, $k_0 = 2\pi/\lambda_0$.

Thus, the wave function of a photon in the coordinate representation explains wave phenomena on an equal basis for all quantum particles and photons emitted in the experiment, obviously one by one.

5. Relation of the structure of the photon wave function in coordinate representation and the polarization of photon in the formalism of quantum transition amplitudes

Strictly speaking, the concepts of the intensities of the electromagnetic field are inapplicable to an individual photon due to the fact that they are concepts of “purely” classical electrodynamics. However, in quantum electrodynamics, they are nevertheless used as sources of the secondary quantization of the field, as well as in the formal description of the states of photons (of so-called modes) and of the amplitudes of photon transitions from one state to another. One of the characteristics of the modes is photon polarization. By itself, this concept can be thought of as a certain quantum number, which is a common attribute of quantum mechanics. However, when one-photon and two-photon interference phenomena are analyzed, then, in fact, the language of classical electrodynamics is used. For example, passing photons through a polarizer oriented in a certain way, they achieve a change in the “polarization” of a photon and its probability of passage, in accordance with Malus’s law. This law is strictly established only in classical electrodynamics, and in quantum it is postulated. By the way, on the other hand, it can be formally substantiated within the framework of quantum physics, based on the quantum mechanics of the photon [42]. Obviously, when we talk about linear polarization, for example, horizontal or vertical, in the state of entangled photons (and superposition in this state), we mean, first of all, the orientation of the plane in which, "being considered in classical electrodynamics, the electric field intensity vector must oscillate". And although the field intensity is not applicable to the photon, it is nevertheless in fact thus used in some metaphysical sense.

The question arises, is it possible to give this concept a “more physical meaning” than “just” metaphysical? The answer is yes. Indeed, one of the complete sets of physical quantities characterizing the state of a free photon includes its energy, momentum, and helicity [31, 36]. A state of a photon with a certain helicity in classical electrodynamics corresponds to a circularly polarized wave. And the linear polarization of a classical wave in quantum mechanics does not correspond to any quantity that would be included in any complete set and would have a definite value in a state with “linear polarization” of
a photon. But then the question arises, how to characterize, for example, wave packet (2) with coefficients (6)? The average value of the helicity of a photon in this state is equal to zero – this is all that can be asserted using only quantum characteristics. It can also be added that the polarization vectors were chosen in the form (7), (8), and they also, obviously, characterize the state of the photon (2). However, it is much easier to say that the “electric field intensity” extracted by a certain procedure described in [31, 36] from all formulas (2), (6), (7), (8) has only one (“almost”) nonzero component – $E_x$, the expression for which for any moment of time $t \geq 0$ has the form:

$$E_x = \text{Re} \left\{ \sqrt{\frac{\hbar c}{4\pi^3 t^4}} \exp(S) \left[ -\frac{q \pi \exp(Q)}{(-q^2)^{1/4}} (2\zeta F_1 + q F_2) - \right. \\
- \left. (1 - i)\sqrt{p}(2\zeta F_3 - pF_4) \exp(P) + 2(1 - i)(ct)^{3/2} F_5 \exp(R) \right] \right\},$$

where

$$b = \pm \alpha^2 k_0 + iz, \quad \zeta = -i\alpha^2 k_0, \quad p = ct + \zeta, \quad q = ct - \zeta, \quad P = -\frac{p^2}{4\alpha^2}, \quad Q = -\frac{q^2}{4\alpha^2}, \quad R = -\frac{\alpha^2 k^2}{2},$$

$$F_1 = (1 + 4Q) [I_{-1/4}(Q) + I_{1/4}(Q)] + 4Q [I_{-3/4}(Q) + I_{3/4}(Q)],$$

$$F_2 = I_{-1/4}(Q) + I_{1/4}(Q) + I_{-3/4}(Q) + I_{3/4}(Q),$$

$$F_3 = (1 + 4P) K_{1/4}(P) - 4P K_{3/4}(P), \quad F_4 = K_{3/4}(P) - K_{1/4}(P), \quad F_5 = K_{3/4}(R) - K_{1/4}(R);$$

$I_v$ and $K_v$ are the Bessel functions of the imaginary argument. For $t = 0$, formula (11) has a much simpler form given in [43].

Thus, the field strength, in particular $E_x$, for the case of vectors (7), (8), can serve as an additional characteristic of the state of the photon, expressing a certain property of “its polarization” corresponding to the linear polarization of a classical wave. Its capabilities, “adequacy” and “legitimacy”, to a certain extent, are illustrated in [43].

Is it possible to construct a wave packet in which the average helicity would again be zero, and the wave packet would be no different from the previous one, except that the photon in this state was characterized by only one (“almost”) nonzero component $E_y$? The answer is yes again. To do this, choose the polarization vectors in the following form:

- at $0 \leq \theta \leq \pi/2$

$$\vec{e}_x(k) = \begin{pmatrix} \hat{e}_{Ix} \\ \hat{e}_{Iy} \\ \hat{e}_{Iz} \end{pmatrix} = \begin{pmatrix} -1 - \cos \theta & \sin \varphi \cos \varphi \\ 1 + \cos \theta & -\sin \varphi \cos \varphi \\ 0 & \sin \varphi \sin \varphi \end{pmatrix}; \quad \vec{e}_y(k) = \begin{pmatrix} \hat{e}_{Hy} \\ \hat{e}_{Hy} \\ \hat{e}_{Hz} \end{pmatrix} = \begin{pmatrix} -\cos \theta - (1 - \cos \theta) \sin^2 \varphi \\ (1 - \cos \theta) \sin \varphi \cos \varphi \\ \sin \theta \cos \varphi \end{pmatrix}.$$  

- at $\pi/2 < \theta \leq \pi$

$$\vec{e}_x(k) = \begin{pmatrix} \hat{e}_{Ix} \\ \hat{e}_{Iy} \\ \hat{e}_{Iz} \end{pmatrix} = \begin{pmatrix} -1 + \cos \theta \sin \varphi \cos \varphi \\ 1 - \cos \theta \sin \varphi \cos \varphi \\ 0 & \sin \varphi \sin \varphi \end{pmatrix}; \quad \vec{e}_y(k) = \begin{pmatrix} \hat{e}_{Hy} \\ \hat{e}_{Hy} \\ \hat{e}_{Hz} \end{pmatrix} = \begin{pmatrix} -\cos \theta + (1 + \cos \theta) \sin^2 \varphi \\ (1 + \cos \theta) \sin \varphi \cos \varphi \\ \sin \theta \cos \varphi \end{pmatrix}.$$  

In this case, the intensity $E_z$ will again be determined by formula (11) for points for the $z$ axis in the state of a photon (2) with coefficients (5).

Thus, we will assume that formulas (13), (14) specify the horizontal ($H$) polarization, and (6), (7) – the vertical ($V$) in the photon state (2).
6. Wave function of two photons in an entangled state

The source of two entangled photons is usually a nonlinear crystal on which a laser beam falls on. As a result of spontaneous parametric scattering, two polarization cones $H$ and $V$ appear at the output, carrying pairs of photons in an entangled state (biphotons). In a particular case, the state most entangled in photon polarizations is described by the vector

$$|\Psi\rangle = \frac{1}{2} \left( |H\rangle_1 |m\rangle_1 |H\rangle_2 |n\rangle_2 + |H\rangle_1 |n\rangle_1 |H\rangle_2 |m\rangle_2 + |V\rangle_1 |m\rangle_1 |V\rangle_2 |n\rangle_2 + |V\rangle_1 |n\rangle_1 |V\rangle_2 |m\rangle_2 \right), \quad (22)$$

where numbers 1 and 2 refer to the first and second photons of one pair, $|m\rangle$ and $|n\rangle$ describe the spatial states in the first and second beam. Passing to the wave function of the system in the coordinate representation, for each of the eight single-particle modes contained in (22), one should compare the wave function in the form of a wave packet (2) with a Gaussian, for example, distribution (6) in momenta and the corresponding choice of polarization vectors. In this case, one should, first, take into account that in (22) single-particle modes mean the states of a photon with a certain momentum and polarization, and the wave packet (2) for each photon is a superposition of plane waves. State (2) can be approached arbitrarily close to a plane wave, directing the parameters $\alpha_1, \alpha_2, \alpha_3$ to infinity. Second, in (22), the states $|m\rangle$ and $|n\rangle$ form an orthonormal basis for each photon separately. To implement this property, when writing a single-particle wave function corresponding to each of the eight modes in (22), type of $|H\rangle_1 |m\rangle_1$, $|H\rangle_2 |n\rangle_2$, etc., the wave packet (2) can be supplemented with a factor $\left( \begin{array}{c} 1 \\ 0 \end{array} \right)$ — for the state $|m\rangle$ and $\left( \begin{array}{c} 0 \\ 1 \end{array} \right)$ — for $|n\rangle$. Since function (2) is 6-component, the resulting construction with such an additional factor will have 12 complex components.

Let us write the wave function of two entangled photons in the coordinate representation, assuming that at $t=0$ their mean values of coordinates $(x, y, z)$ are respectively equal to $(0, -y_0, 0)$ and $(0, y_0, 0)$, mean values of momentum projections $(p_x, p_y, p_z)$ are equal to $(0, -h k_{0y}, h k_{0z})$ and $(0, h k_{0y}, h k_{0z})$ and determine the directions of the corresponding axes of the beams $|m\rangle$ and $|n\rangle$: the axis of symmetry of the entire system coincides with the $z$ axis. Let the beam $|m\rangle$ forms $H$-polarization, and $|n\rangle$ — $V$-polarization. Then, for the beam $|m\rangle$, one should choose formulas (13) and (14), and for the beam - (6) and (7). As a result, in the case $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$, that corresponds to the same (Gaussian) spread of momentum projections for each photon, we obtain for the first and second factors in the first term in (22):

$$\Psi_{H=1m}(r_1, t) = C \frac{1}{(2\pi)^{1/2}} \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \int \exp \left[ -\frac{\alpha^2}{2} \left( k_x^2 + (k_y - \mp k_{0y})^2 + (k_z + \mp k_{0z})^2 \right) + i(kr_1 \mp kct - k_y y_0) \right] \frac{\tilde{\epsilon}_{\mp 1}(k)}{\tilde{\epsilon}_{\mp 1}(k)} \; d^3 k,$$

$$\Psi_{H=1n}(r_2, t) = C \frac{1}{(2\pi)^{1/2}} \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \int \exp \left[ -\frac{\alpha^2}{2} \left( k_x^2 + (k_y - \pm k_{0y})^2 + (k_z + \mp k_{0z})^2 \right) + i(kr_2 \mp kct + k_y y_0) \right] \frac{\tilde{\epsilon}_{\pm 1}(k)}{\tilde{\epsilon}_{\pm 1}(k)} \; d^3 k.$$

Two factors of the second term in (22) correspond to the functions

$$\Psi_{H=2m}(r_1, t) = C \frac{1}{(2\pi)^{1/2}} \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \int \exp \left[ -\frac{\alpha^2}{2} \left( k_x^2 + (k_y - \pm k_{0y})^2 + (k_z + \mp k_{0z})^2 \right) + i(kr_1 \pm kct + k_y y_0) \right] \frac{\tilde{\epsilon}_{\pm 1}(k)}{\tilde{\epsilon}_{\pm 1}(k)} \; d^3 k,$$

$$\Psi_{H=2n}(r_2, t) = C \frac{1}{(2\pi)^{1/2}} \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \int \exp \left[ -\frac{\alpha^2}{2} \left( k_x^2 + (k_y - \mp k_{0y})^2 + (k_z + \mp k_{0z})^2 \right) + i(kr_2 \pm kct - k_y y_0) \right] \frac{\tilde{\epsilon}_{\mp 1}(k)}{\tilde{\epsilon}_{\mp 1}(k)} \; d^3 k.$$
The next 4 factors will differ from the previous 4 only by replacing the polarization vectors \( \vec{e}_{\pm 1}(k) \) and \( \vec{e}_{\mp 1}(k) \), determined by formulas (20), (21), by the corresponding vectors \( e_{\pm 1}(k) \) and \( e_{\mp 1}(k) \), given by formulas (7), (8).

Summing up all four terms, in accordance with (22), we obtain the wave function in the coordinate representation, normalized to the unit probability, which describes the state of two entangled photons immediately after their escape from the crystal until the moment of their separation along different fiber channels.

7. Conclusion
The presented modeling of a photon wave packet makes it possible to illustrate the single-photon and two-photon approaches to the description of the phenomena of light interference, in particular, Young's interference experiment with two slits, which are usually described either in the language of classical electrodynamics, or in quantum mechanics with the participation of secondary quantization of the electromagnetic field. Obviously, the application of the stated “primary quantization” of the electromagnetic field significantly expands the capabilities of quantum mechanics and simplifies the solution of the problem of wave-particle duality at the present stage of our knowledge. It is suggested in [44] that the ideas of constructing the wave function of a free photon in the coordinate representation can find relevance in the field of quantum cryptography.

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