RESPONSE OF RANDOM FIELD ISING MODEL DRIVEN BY AN EXTERNAL FIELD

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We study the dynamics of spin flipping at first order transitions in zero temperature two-dimensional random-field Ising model driven by an external field. We find a critical value of the disorder strength at which a discontinuous sharp jump in magnetization first occurs. We discuss growth morphology of the flipped-spin domains at and away from criticality.

Keywords: disorder; threshold-dynamics; domain-growth.

1. Introduction

The Random Field Ising Model (RFIM) has been widely studied in recent years in the context of hysteresis [1-3], return-point memory [4,5] and Barkhausen noise [6-11]. Hysteresis is a common phenomenon, exhibited, for example, by most magnetic materials. In hysteresis the magnetization lags behind the applied field and the phenomenon has received lots of attention in the past. Moreover, if one observes carefully, the change in magnetization in systems like that in ferromagnetic alloys and amorphous ferromagnets, takes place in a series of irregular pulses as the external field is slowly varied. In experiments these pulses manifest themselves as acoustic emissions and is known as Barkhausen noise. Interestingly, Barkhausen signals show scale-free behavior similar to that of systems at criticality. Barkhausen noise has been studied much in recent years in the context of driven disordered systems for example of which include martensite transformation in shape-memory alloys [12], earthquakes [13], deformation of granular materials [14] and the breakdown of solids [15]. Various experiments [8] show that the distribution of size, duration and energy associated with Barkhausen noise show power-law behavior. Sethna et al, in order to account for the hysteresis and Barkhausen noise incorporated disorder in the form of random fields into the otherwise pure spin systems such as the Ising model [3].

In the RFIM, the Ising spins \{s_i\} with nearest neighbor ferromagnetic interac-
tion $J$ are coupled to the on-site quenched random magnetic fields $\{h_i\}$ and the external field $h_a$. The Hamiltonian of the system is

$$H = -J \sum_{<ij>} s_is_j - \sum_i h_is_i - h_a \sum_i s_i$$  \hspace{1cm} (1)$$

where $\{h_i\}$ are quenched independent random variable drawn from a distribution of zero mean and variance $\Delta$. In the study of hysteresis and Barkhausen jumps the applied field is ramped up or down adiabatically (so that the rate of spin flips is much larger than the rate of change of $h_a$). In the absence of disorder ($\{h_i\} = 0$), the model exhibits a sharp first order transition taking the magnetization $m(h_a)$ from -1 to +1 as $h_a$ passes through $zJ$, $z$ being the coordination number of the underlying lattice. In presence of random field, the sharpness of the transition gets smeared out and the change in magnetization with field takes place in a series of “spin-cluster” flips and the magnetization changes in sporadic jumps as is observed in $BN$.

Simulations of RFIM in three-dimensions show that for large values of $\Delta$ (greater than a critical value $\Delta_c$), the spins flip in small clusters resulting in avalanches of small sizes. The sizes of these avalanches are distributed in a power-law. At $\Delta = \Delta_c$ avalanches of all sizes occur. For $\Delta < \Delta_c$, the magnetization shows a first order jump (corresponding to an infinite avalanche) as is the case if there is no disorder. The behavior of avalanches and avalanche size distribution at and near $\Delta_c$ has been studied in the mean field limit [3] and the power law relating to the avalanche size distribution is known exactly in RFIM in one-dimension and on Bethe lattices [11]. For a linear chain and a Bethe lattice of coordination number $z = 3$ the avalanche size ($s$) distribution decreases exponentially for large $s$. No first order jump has been observed for any finite amount of disorder (unbounded) for $z \leq 3$. Thus the low $\Delta$ behavior depends on the coordination number of the underlying lattice. For $z \geq 4$ and for small disorder the magnetization shows a first order discontinuity for several continuous and unimodal distributions of the random fields. The avalanche distribution $P(s)$ varies as $s^{-\frac{5}{2}}$ for large $s$ near the discontinuity.

This growth of magnetization is essentially due to the motion of an interface of a cluster of positive spins through a disordered media. Renormalization group studies of the interface (starting from a continuum model and an interface without any backbends) motion in disordered medium shows that the interface at the critical value of the strength of the disorder becomes rough with a roughness exponent $\zeta = (5 - d)/3$ [16] for a d-dimensional system. According to this relation the upper critical dimension is five and the lower critical dimension is two. At $d = 2$, it is considered that the interface may have backbends and can encompass bubbles as it moves through the medium. Simulation results show the following generic features of the interface growth in random media [17-24]: (i) For large strengths of the disorder the interface growth is percolation-like. The fractal dimension of the magnetic-interface corresponds to that ordinary site percolation. (ii) At the critical value of the disorder the interface becomes self-affine with a roughness exponent...
which closely matches the RG result. (iii) For low strength of the disorder the interface growth is faceted.

We study the spin flipping dynamics in RFIM in two-dimensions at zero temperature as the external field is ramped up slowly from a high negative value. Our intention is to see if there is any finite value of the disorder strength $\Delta_c$ in two-dimension. At $\Delta_c$ and at a particular field $h_a = h_a^*$, the spin starts flipping from a site and does not end till the flipped spins span the entire system. The domain wall that separates the flipped spin regions from the rest is highly tortuous (fractal like). The spin flipped region has bubbles of unflipped spins, so that $m(h_a)$ does not jump from -1 to +1, instead attains a value $\sim 0.76$ irrespective of system size. In the regime when the flipped spin domain grows, the number $M$ of flipped spins and the interface length $I$ both grow with time $t$ as $M \sim I^2 \sim t^2$. For $\Delta < \Delta_c$, $M$ jumps from -1 to +1 at a particular field and the flipped spin domain remains compact as it grows following $M \sim I^2 \sim t^2$. For $\Delta > \Delta_c$, $m(h_a)$ grows continuously with the field in jumps of different sizes. These jumps correspond to flipping of small spin clusters at various places in the lattice.

2. The Model and Simulation

We have studied RFIM in two-dimensions on a square lattice of size $L$ with a classical Ising spin $s_i = \pm 1$ and a quenched random field $h_i$ on every site $i$ of the lattice. Each spin interacts with each of its nearest neighbor spin through a ferromagnetic interaction $J = 1$. The random field $h_i$ is drawn from a uniform distribution

$$p(h_i) = \frac{1}{2\Delta} \quad \text{if} \quad -\Delta \leq h_i \leq \Delta$$
$$= 0 \quad \text{otherwise}.$$ 

(2)

The spins are subjected to an applied field $h_a$ which is varied. We study the model at zero temperature. We start with a configuration of all down spins which corresponds to a high negative value of $h_a$. The field $h_a$ is then slowly raised and the resulting spin flips are recorded. The spin flipping process follows the rule of zero temperature Glauber dynamics [25], where a spin is flipped only if the flipping lowers the energy of the system as calculated according to the Hamiltonian given by “Eq (1)”.

We use periodic boundary conditions in our simulation. For a certain $L$, $\Delta$ and a configuration of the random field $h_i$ we first find out the spins $\{\alpha\}$ which can be most easily flipped and the corresponding value of the applied field $h_a^*$ required to flip the spins. We then set $h_a = h_a^*$ and flip all the spins in $\{\alpha\}$ simultaneously. This constitutes one time step in our simulation. Next we check if this primary set of spin flips introduce secondary spin flips and so on, always flipping all the flippable spins simultaneously. If there is no spin that can be flipped, we increase $h_a$ to trigger next generation of spin flipping. We continue this process till all the spins of the lattice are flipped. We have taken results for $L = 500$ to 7000 and for various values $\Delta$. 

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from 2.0 to 2.8. For a certain $\Delta$, our results are averaged over 50 initial random configurations of the on-site fields.

3. Results and Discussion

Our simulation results show that there is a critical value $\Delta_c$ of the strength of the disorder. The growth morphology of domains of flipped spins are distinctly different for $\Delta < \Delta_c$, $\Delta > \Delta_c$ and for $\Delta = \Delta_c$. We discuss the three cases below:

(i) For $\Delta < \Delta_c$ as the external field is ramped up there is a sharp first order transition (like that happens in the absence of disorder) at a particular field ($h_a = 4J - \Delta$) that takes the magnetization from -1 to +1 as is shown in Fig.1. This infinite avalanche nucleates from a single spin and invades the system in course of time. The flipped spin cluster remains compact while it grows: the mass $M$ (the number of upturned spins) of the cluster grows with time $t$ (in Monte Carlo steps) as $M \sim t^2$ (Fig.2). The number of spins $I$ along the interface grows as $I \sim t$ (Fig.2).

![Fig. 1. The magnetization curve for $\Delta(= 2.2) < \Delta_c$. The magnetization shows a discontinuous jump at the critical field $h_a = 4J - \Delta$.](image)

(ii) For $\Delta > \Delta_c$ an infinite avalanche never happens. The magnetization increases in irregular steps with the increase of external field as shown in Fig.3. In Fig.4 we show a snapshot of the upturned spins at a particular field when the flipping of the spins has stopped. In this regime we see nucleation of many domains as opposed to the nucleation of a single domain for $\Delta < \Delta_c$. Such a feature is also observed in dilute RFIM [10].
Fig. 2. The variation of the total number of spins, $M$ and the number of spins along the interface $I$ with time $t$ (in units of 300 Monte Carlo steps) for $\Delta < \Delta_c = 2.2$ and $h_a = h^*_a = 1.800$. $M$ varies as the square of the time and $I$ varies linearly with time. The dotted line has a slope of two and the dashed line has a slope of unity and both are a guide to the eye.

Fig. 3. The magnetization curve for $\Delta(=2.5) > \Delta_c$. The magnetization increases in small irregular jumps as the field is increased.

For any field, when the system has stopped evolving, a typical distribution of the random fields over the unturned spins along the interface is as shown in Fig.5. It
Fig. 4. The domain of up spins (black) is shown for $\Delta(=2.5) > \Delta_c$, $h_u = h_u^* = 1.5000$.

consists of two steps. The higher step is for those sites that have large negative random fields that would require three upturned neighbors for its flipping. The lower step corresponds to the random fields of those sites which can flip if two of its neighbors have already flipped. This shows that the interface spins are strongly pinned.

The non-existence of sites (at the interface) that can flip when one of its neighbor is up is responsible for the freezing of further spin flipping at the particular field.

(iii) At the critical value of $\Delta = \Delta_c$ we see the growth of a single domain and a discontinuous first order jump in the magnetization at a particular value of the applied field. However, unlike the situation for $\Delta < \Delta_c$ here the domain of flipped spins encompasses bubbles of unturned spins of various sizes at different stage of its growth and the interface has overhangs and is fractal like. As a result magnetization does not jump from -1 to +1 but to (around) 0.76 irrespective of the system size (we have checked it for $L = 500, 800, 2000, 4000, 5000$ and 7000). The change in magnetization with field is shown in Fig. 6.

A typical up-spin cluster at a certain time of its development is shown in Fig. 7. It shows the tortuous interface with bubbles of unturned spins (white region). In this case both the mass and interface develops as $M \sim I \sim t^2$ as is shown in Fig. 8.

At any time during the growth of the up-spin domain the probability distribution of the random fields of the unturned spins along the interface has three steps as is shown in Fig. 9. The lowest step corresponds to the random fields on those sites that can flip if one of its neighbor is up. This guarantees the growth of domains
Fig. 5. The probability distribution of “negative” spins at a field when the growth of domains is arrested. This is for $\Delta (= 2.5) > \Delta_c$.

Fig. 6. The magnetization curve for $\Delta = \Delta_c = 2.4$. The magnetization increases to about 0.76 in an infinite jump after which the field has to be increased for the magnetization to increase.

as along the interface one neighbor will always be up. However when the up-spins span the system the probability distribution of the random fields along the interface corresponds to those for the bubbles only and is similar to that for $\Delta > \Delta_c$. This shows that the spins along the interface of the bubbles are strongly
Fig. 7. The domain of up spins (black) is shown for $\Delta = \Delta_c (= 2.4)$, $h_a = h_a^* = 1.60000$.

Fig. 8. The variation of the total number of spins, $M$, and the number of spins along the interface $I$ with time $t$ (in units of 300 Monte Carlo steps) for $\Delta = \Delta_c = 2.4$ and $h_a = h_a^* = 1.6000$. Both $M$ and $I$ vary as the square of the time. The dotted line has a slope of unity and is a guide to the eye.

pinned. The external field has to be continuously increased for the magnetization to change from 0.76 to +1 which will eventually fill up the bubbles (Fig. 6).
Fig. 9. Distribution $p(h_i)$ of the random field $h_i$ over the spins along the interface between the up and the down spins for $\Delta = \Delta_c = 2.4$ and $h_a = 1.6000$. (The distribution remains invariant with time and is marked by three steps as is shown)

4. Conclusion

We find a finite, non-zero $\Delta_c$ for bounded distribution. This $\Delta_c$ demarcates a region ($\Delta > \Delta_c$) where infinite avalanches never happens to that from a region where magnetization shows a jump discontinuity from -1 to +1. Earlier studies indicate $\Delta_c = 0$ for unbounded distribution. We find at $\Delta_c$ the interface around the flipped domains have backbends and is fractal like. It encloses bubbles of unflipped spins in all length scales.

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