Dyson treatment of NFT medium polarization processes in superfluid nuclei

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Abstract. We discuss the solution of the Dyson equation (also known as the Nambu-Gor’kov equation) for the case of medium polarization Nuclear Field Theory (NFT) processes in nuclei, and apply it to the nucleus \(^{120}\)Sn. Starting from a mean field obtained with the Sly4 effective nucleon-nucleon force, \((m_\hbar = 0.7)\), we renormalize the single-particle states and the pairing interaction through the coupling to collective vibrations of the system, obtaining a detailed description of the fragmentation of the quasiparticle strength and of the state-dependent pairing gap. These quantities are calculated in term of one- and two-particle spectroscopic amplitudes, amplitudes which can be used to calculate both one- and two-nucleon absolute transfer reaction cross sections to be directly confronted with the experimental findings.

1. Dyson equation and particle-vibration coupling

We calculate the pairing gap and other properties of the superfluid nucleus \(^{120}\)Sn considering both the N-N interaction and medium polarization effects due to the coupling of quasiparticles to collective surface vibrations (phonons) of the system within the framework of Nuclear Field Theory (cf.[1],[2],[3] and refs. therein and also, within the present context, ref.[4]). A key ingredient of the formalism is the particle-vibration coupling vertex, that is, the matrix element connecting a single-particle state of quantum numbers \(n_a, l_a, j_a\) with a single-particle \(n_b, l_b, j_b\) plus a vibrational state characterized by the deformation parameter \(\beta_\lambda\) (cf. [3] and refs. therein)

\[
h(a, b\lambda_n) = -\frac{i^{l_a-l_b+\lambda}}{\sqrt{4\pi}} (j_a \frac{1}{2}\lambda 0 | j_b \frac{1}{2} \beta_\lambda_n (j_a R_0 \frac{\partial U}{\partial r} | j_b),
\]

\(U(r)\) being the mean field acting on the nucleon.

Acting twice on the single-particle, and eventually iterating the coupling to all orders, the particle-vibration coupling vertex gives rise to self-energy processes. Such processes are taken into account in both normal and abnormal self-energies, \(\Sigma_{11}\) and \(\Sigma_{12}\), appearing in the Dyson-Nambu-Gor’kov equation,

\[
\left[ \begin{array}{cc} \varepsilon_a - \lambda & 0 \\ 0 & -\varepsilon_a + \lambda \end{array} \right] + \left( \begin{array}{cc} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{array} \right) \left( \begin{array}{c} u_a \\ v_a \end{array} \right) = \left( \begin{array}{c} u_a \\ v_a \end{array} \right),
\]
where,
\[ \Sigma_{11}(a, E) = -\Sigma_{22}(a, -E) = \sum_{b, \lambda, n} \left[ \frac{|h(a, b\lambda_n)u_b|^2}{E - E_b - \hbar\omega\lambda_n} + \frac{|h(a, b\lambda_n)v_b|^2}{E + E_b + \hbar\omega\lambda_n} \right], \quad (3) \]

and,
\[ \Sigma_{12}(a, E) = \Sigma_{21}(a, E) = \Sigma_{12}(a, -E), \]
\[ = -\sum_b u_b v_b \left[ \sum_{\lambda, n} \left( \frac{|h(a, b\lambda_n)|^2}{E - (E_b + \hbar\omega\lambda_n)} - \frac{|h(a, b\lambda_n)|^2}{E + (E_b + \hbar\omega\lambda_n)} \right) - \frac{g(2j_b + 1)}{2} \right], \quad (4) \]

from which the quasiparticle amplitudes \( u_a, v_a \) and the quasiparticle energies \( E_a \) can be obtained selfconsistently. The single-particle energies \( \varepsilon_a \) are the result of a Hartree-Fock calculation using the Sly4 effective interaction [5], while \( g = 0.16 \text{ MeV} \) is the strength of the monopole pairing force, \( -gP^T P \), value which has been chosen so as to reproduce the average BCS pairing gap (equal to about 1 MeV) obtained when using the \( v_{\text{low}-k+3\text{N}} \) force as pairing force [6].

The energy-dependence of Dyson equation (2), expressed by the energy denominators appearing in Eqs. (3) and (4) implies, among other things, that the normalization on the quasiparticle amplitudes \( u_a, v_a \) is given by the relation
\[ 1 = u_a^2 + v_a^2 = \frac{\partial\Sigma_{11}}{\partial E} u_a^2 - \frac{\partial\Sigma_{22}}{\partial E} v_a^2 - 2 \frac{\partial\Sigma_{12}}{\partial E} u_a v_a. \quad (5) \]

Consequently, as a rule, the pure quasiparticle strength \( N_a = u_a^2 + v_a^2 < 1 \), for any given solution. In other words, the original HF single-particle strength is now fragmented over a number of quasiparticle states.

For every quasiparticle a pairing gap may be defined [7],[8] according to
\[ \Delta_a = Z_a \Sigma_{12}(a, E_a), \quad (6) \]
where
\[ Z_a = \left( 1 + \frac{\Sigma_{11}(a, E_a) + \Sigma_{22}(a, E_a)}{2E_a} \right)^{-1}. \quad (7) \]

1.1. Mean Field and QRPA
The collective modes have been calculated in QRPA using the gap extracted from the odd-even mass difference (\( \Delta \approx 1.4 \text{ MeV} \)), together with a separable interaction displaying a surface peaked radial dependence given by \( R_0 dU/dr \). The coupling constants have been determined so as to reproduce the experimental energy and transition strength of the low-lying \( 2^+, 3^-, 4^+ \) and \( 5^- \) collective surface vibrations. We have not considered the coupling to spin modes, in keeping with the fact that in finite nuclei, these modes are, as a rule, not particularly collective. In any case, and as have been shown in [9], their contribution to the induced pairing interaction is repulsive, leading to a reduction of the density fluctuations mediated pairing by about 30%.

2. Solution of Dyson Equation
As mentioned above, the coupling of nucleons to collective vibrations leads to a fragmentation of the quasiparticle states. In most cases and near the Fermi energy, the quasiparticle strength remains concentrated in a single peak carrying typically 70% of the single particle strength. In such cases, the one-pole (quasiparticle) approximation is reasonably good. This is, for instance, the case of the \( h_{11/2} \) orbital, shown in Fig. 1(b). It may however happen that some levels become strongly fragmented. This is the case of the quasiparticle strength associated with the
The thick blue bars show the calculated spectroscopic factors obtained integrating the single-particle strength associated with the $h_{11/2}$ (a) and $d_{5/2}$ orbitals in bins of 1 MeV width. The thin red bars show the experimental spectroscopic factors obtained from one-neutron transfer reactions [10, 11].

$d_{5/2}$ orbital (see Fig.1(a)). The available experimental data [10, 11], also shown in Fig. 1, display less fragmentation than in theory. However, one should also consider that the data have low resolution and cover a small interval in excitation energy.

The quasiparticle energy spectrum resulting from the fragments carrying the largest single-particle strength is considerably denser than that obtained from Hartree-Fock plus BCS theory, in overall agreement with the experimental findings (see Fig.2). The spectrum is obviously affected by the details of the adopted Skyrme force. While the effective mass associated with the HF mean field should lie in the range $m_k \approx 0.7-0.8$ for a consistent calculation of renormalization effects, other properties of the force should be obtained by a careful refitting of the parameters including the renormalization effects. This is not attempted in the present work. However, we notice that a better agreement between theory and experiment, including renormalization effects, could be obtained by decreasing the value of the spin-orbit strength $W_0$. Reducing $W_0$ by about 15% increases the main peak of the $7/2^+$ strength by about 500 keV, as is required by the experiment, leaving the other quasiparticle energies almost unchanged. In fact a better agreement within theory and experiment can be achieved using SGII interaction, which has a smaller $W_0$ parameter. The total pairing gap is larger than that calculated in ref. [4], due to the different mean field and pairing force used in the present work.

Of notice that $\Sigma_{12}$ receives contributions from both the bare and the (phonon-mediated) induced interaction. This implies that the pairing gap is made out of the two corresponding contributions (see Eqs. (4) and (6)). In the case under discussion the induced and the bare contributions account for about 2/3 and 1/3 of the total gap, respectively (see Fig. 3). Within this context, it is important to consider the full particle-vibration coupling of the collective phonons, in particular to the low-lying surface modes. Coupling to uncorrelated particle-hole excitations (one-bubble approximation), leads to a much reduced renormalization effect (cf. Fig. 4), also due to cancellation effects (cf. e.g. [12]).

Of notice that the $Z$-factor plays a crucial role in the determination of the paring gap, since its value is about 0.6, thus renormalizing $\Sigma_{12}$ in an important way (see Fig.5).
2.1. One quasiparticle approximation

In Fig.6 we compare the solution of the Dyson equation with the results obtained taking into account only the fragments carrying the largest strength. This (single quasiparticle) approximation leads to a spectrum which is in overall agreement with the full solution, exception made for the $d_{5/2}$ orbital. In this case the one pole approximation does a very poor job indeed (see also Fig.1).

3. Hindsight

The Dyson equation provides a powerful framework to propagate to all orders the basic NFT processes which renormalize the motion of nucleons and their interaction. This is particularly important in connection with self-energy diagrams, in which it may be necessary to propagate the
Figure 4. Comparison between the phonon mediated contribution in the pairing gap (black squares) and first order particle-hole (one-bubble) mediated excitation (blue empty circles).

Figure 5. Comparison between $\Sigma_{12}$ and $\Delta = Z\Sigma_{12}$

basic second order process to all orders so as to obtain the full distribution of the single-particle strength (quasiparticle lifetime). Within this context, the case of the $d_{5/2}$ quasiparticle state of $^{120}$Sn is paradigmatic. These results provide the single-particle spectroscopic amplitudes needed to calculate absolute one-particle transfer cross sections. Quasiparticle fragmentation together with the induced pairing interaction can affect in an important way the structure of Cooper pairs and thus the absolute value of two-particle transfer cross sections.
Figure 6. Quasiparticle energy correction referred to the unperturbed value calculated making use of the one pole approximation (black dots) and taking into account the levels fragmentation (red squares).

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