Higher neutrino mass allowed if Cold Dark Matter and Dark Energy are coupled

G. La Vacca\textsuperscript{a,b}, S. A. Bonometto\textsuperscript{a,c}, L. P. L. Colombo\textsuperscript{d}

\textsuperscript{a}I.N.F.N., Sezione di Milano–Bicocca, Piazza della Scienza 3, 20126 Milano, Italy
\textsuperscript{b}Nuclear \& Theoretical Physics Department, Pavia University, Via Ugo Bassi 3, 22100 Pavia, Italy
\textsuperscript{c}Physics Department G. Occhialini, Milano–Bicocca University, Piazza della Scienza 3, 20126 Milano, Italy
\textsuperscript{d}Department of Physics \& Astronomy, University of Southern California, Los Angeles, CA 90089-0484

Abstract

Cosmological limits on neutrino masses are softened, by more than a factor 2, if Cold Dark Matter (CDM) and Dark Energy (DE) are coupled. In turn, a neutrino mass yielding $\Omega_\nu$ up to $\sim 0.20$ allows coupling levels $\beta \simeq 0.15$ or more, already easing the coincidence problem. The coupling, in fact, displaces both $P(k)$ and $C_l$ spectra in a fashion opposite to neutrino mass. Estimates are obtained through a Fisher–matrix technique.

Key words: cosmology: theory, neutrinos
PACS: 98.80.-k, 98.65.-r

1 Introduction

There seem to be little doubt left: at least one neutrino mass eigenstate or, possibly, two of them exceed $\simeq 0.055$ eV (direct or inverse hierarchy). This follows solar \cite{1} and reactor \cite{2} neutrino experiments, yielding $\Delta m_{1,2}^2 \simeq 8 \times 10^{-5}$eV$^2$ and, namely, atmospheric \cite{3} and accelerator beam \cite{4} experiments yielding $\Delta m_{2,3}^2 \simeq 3 \times 10^{-3}$eV$^2$.

Email address: lavacca@mib.infn.it (G. La Vacca).
Cosmology is also sensitive to neutrino mass. Since 1984, Valdarnini & Bonometto [5] made a detailed analysis of transfer functions in cosmologies where a part of Dark Matter (DM) is due to massive neutrinos, so proposing mixed DM models, where neutrinos play an essential role in adjusting CMB (Cosmic Microwave Background) anisotropies and matter fluctuation spectra to fit observations. A large deal of work on this subject took place in the Nineties; mixed models were widely tested, using both the linear and the non-linear theory.

Hubble diagram of SNIa [6] showed then an accelerated cosmic expansion, while advanced data on CMB [7] and large scale structure [8] required a spatially flat cosmology with a matter density parameter $\Omega_{o,m} \approx 0.27$, so that the gap up to unity was to be filled by a smooth non-particle component dubbed Dark Energy (DE).

All that relegated neutrinos to a secondary role in shaping cosmic data while, by using such advanced astrophysical data, increasingly stringent limits on neutrino masses could be computed (see e.g. [9]), also combining cosmological and laboratory data [10]. Moreover, data coming from future weak lensing surveys seems to be powerful probes of neutrino masses [16].

Standard limits on neutrino masses were recently summarized by Komatsu et al (2008) [11], within the WMAP5 release, and are quoted in Table 1. More stringent but more speculative limits are suggested in [12], who make a more extensive use of 2dF [13] or SDSS [14] data, and in [15], by using Ly$\alpha$ forest data.

These limits, clearly, rely on implicit assumptions concerning the dark cosmic sector, whose knowledge still fully relies on astrophysical data, requiring two components characterized by state parameters $w \simeq 0$ and $\simeq -1$. But the assumption that no energy exchange between them occurs, tested vs. data, leads just to coupling limits.

In this paper we show that spectral distortions due to CDM–DE coupling and to neutrino mass tend to compensate. We tentatively estimate how far we can go, simultaneously increasing coupling and mass, by using a Fisher Matrix (FM) technique. On that basis we perform a preliminary exploration of the parameter space.

Table 1
Summary of the 2–σ (95% C.L.) constraints on the sum of $\nu$ masses, from WMAP 5-year and other cosmological data sets.

|                  | $w = -1$ | $w \neq -1$ |
|------------------|----------|-------------|
| WMAP5            | < 1.3 eV | < 1.5 eV    |
| WMAP5+BAO+SN    | < 0.67 eV| < 0.80 eV   |
A large deal of work dealt with the coupling option (see, e.g. [17,18,19,20]). One of its motivations is the attempt to overcome the coincidence paradox (see [21]), i.e. the fact that DE becomes relevant just at the eve of structure formation. All that makes our epoch unique and, unless one indulges to anthropic views, apparently requires an explanation. However, also independently from this conceptual issue, our very ignorance of the physics of the dark sector requires that all reasonable options consistent with basic physics and data are explored.

It is also important to outline that neutrino mass limits can be softened if DE with a state parameter $w < -1$ is considered [22,23]. Unfortunately, this kind of state equations, yielding the so–called phantom–DE, can be justified only making recourse to unconventional physics.

Here, starting from dynamical DE, we shall preliminarily discuss how a CDM–DE coupling can lead to a context similar to phantom DE. In this framework, however, no unconventional physics is involved; on the contrary, thanks to coupling, coincidence could be eased.

For the sake of definiteness, in this paper we use the potential

$$V(\phi) = (\Lambda^{\alpha+4}/\phi^\alpha) \exp(4\pi \phi^2/m_p^2)$$

admitting tracker solutions. This potential has been shown to fit WMAP data at least as well as $\Lambda$CDM [24] and takes origin within the context of SUGRA theories [25].

The plan of the paper is as follows. In Section 2 we shall review coupled DE (cDE) models, comparing some aspects of its physics to phantom–DE. In Section 3 we show some spectra of a number of cosmological models, showing how CDM–DE coupling and non–vanishing neutrino mass can be selected so to (approximately) compensate their effects. In Section 4 we debate technical aspects of a FM approach. In Section 5 we give the result of such approach. Section 6 is then devoted to a guided exploration of the parameter space, while in Section 7 we present our conclusions.

## 2 Coupled–DE models

The essential feature of the scalar field $\phi$, in order that it yields DE, is its self–interaction through a potential $V(\phi)$. The simplest form of possible coupling is a linear one. It can be formally obtained by performing a conformal transformation of Brans–Dicke theory (see e.g. [27]), where gravity is modified by adding a $\phi R$ term ($R$ is the Ricci scalar) to the Lagrangian.
Interactions with baryons are constrained by observational limits on violations of the equivalence principle (see, e.g. [26]). No similar constraints hold for CDM–DE interactions. In this case, constraints will follow from cosmological observations.

In the coupled DE (cDE) scenario, as for dynamical DE, a self–interacting scalar field $\phi$ yields a cosmic component which does not cluster and has negative pressure. As a matter of fact, its energy density and pressure read

$$\rho = \rho_k + V(\phi) \, , \quad p = \rho_k - V(\phi) \, ,$$

(2)

where $V(\phi)$ is the self–interaction potential and

$$\rho_k = \dot{\phi}^2 / 2a^2 \, .$$

(3)

Here dots indicate differentiation in respect to $\tau$, while the background metrics reads

$$ds^2 = a^2(\tau) \left[ d\tau^2 - d\lambda^2 \right] \quad \text{with} \quad d\lambda^2 = dr^2 + r^2(d\theta^2 + \cos^2\theta \, d\phi^2) \, .$$

(4)

If $\rho_k \gg V$, the DE state parameter approaches +1 \textit{(stiff matter)} so that DE energy density rapidly dilutes during expansion ($\rho \propto a^{-6}$). In the opposite case $V \gg \rho_k$, the state parameter approaches −1 and DE allows the observed cosmic acceleration. In cDE models, an energy transfer occurs from CDM to DE, so allowing DE to have a non–negligible density since the matter–radiation decoupling era. However, $\rho_k$ is then dominant and the transferred energy is soon diluted. A so–called $\phi$–matter dominated period then occurs, when CDM density however declines more rapidly than $a^{-3}$. The increase of $\dot{\phi}$ then brings $\phi$ to approach $m_p$ (the Planck mass) and $V(\phi)$ to exceed $\rho_k$. DE dilution then stops and DE eventually exceeds CDM density.

Within this picture, CDM and DE stress–energy tensors ($T^{(c,de)}_{\mu\nu}$, let their traces read $T^{(c,de)}$) no longer obey separate equations; although still being

$$T^{(c)}_{\mu\nu} + T^{(de)}_{\mu\nu} = 0 \, ,$$

(5)

it ought then to be

$$T^{(de)}_{\mu\nu} = + CT^{(c)}_{\phi,\nu} \, ,$$

(6)

$$T^{(c)}_{\mu\nu} = - CT^{(c)}_{\phi,\nu} \, .$$

(7)

When the metric is $[4]$, these equations yield

$$\ddot{\phi} + 2\frac{\dot{a}}{a} \dot{\phi} + a^2 V'_{\phi} = + C a^2 \rho_c \, ,$$

(8)

$$\dot{\rho}_c + 3\frac{\dot{a}}{a} \rho_c = - C \rho_c \dot{\phi} \, ,$$

(9)
\(\rho_c\) being CDM energy density. General covariance requires \(C\) to be a constant or to evolve as a function of \(\phi\) itself. Here, instead of \(C\), we shall use the dimensionless parameter
\[
\beta = \left(\frac{3}{16\pi}\right)^{1/2} m_p C .
\] (10)

Let us then define the coupling function \(f(\phi)\), through the relation
\[
C(\phi) = \frac{d\log f}{d\phi}
\] (11)
so that CDM energy density scales according to
\[
\rho_c(a) = \rho_{o,c}(a_0/a)^3 f(\phi) .
\] (12)

Then, if we set \(\bar{V} = V + \rho_c\), the \(\phi\) eq. of motion takes the (standard) form,
\[
\ddot{\phi} + 2\dot{a}\frac{\dot{\phi}}{a} + a^2 \bar{V}' = 0 ,
\] (13)
as though CDM and DE were decoupled, once the effective potential \(\bar{V}\) is used.

The CDM evolution (12), implying a density decline faster than in the absence of coupling, together with Eq. (5), implying that \(\rho_c + \rho_{de}\) has the same evolution as in the absence of coupling, means that \(\rho_{de}\) scale dependence is different from what would follow from the state parameter \(w = p_{de}/\rho_{de}\) deducible from the expressions (2). The effective behavior, obtainable by using the potential \(\bar{V}\), mimics a phantom–like state equation, yielding a DE density increase with \(a\), as we would find for \(w < -1\).

This makes reasonable to expect that neutrino mass limits can be relaxed in a cDE context, as they are in the presence of phantom DE. This option, however, does not lead to requiring unconventional physics. On the contrary, if we are allowed to consider fairly high \(\beta\) values, the coincidence problem is also eased.

3 Some angular and linear spectra

The point of this paper can be appreciated through the spectra in Figures 1 and 2. We compare a model with zero coupling and zero neutrino mass (00–model, hereafter) with: (i) a model with 2 massive neutrinos with mass \(m_\nu = 0.119\) eV, yielding \(\Omega_\nu = 0.005\) (plus 1 massless neutrino); (ii) a model with a CDM–DE coupling \(\beta = 0.049\); (iii) a model with both neutrino mass and coupling (CM–model, hereafter). All models are spatially flat, have adimensional Hubble parameter \(h = 0.71\), density parameters \(\Omega_b = 0.04, \Omega_{de} =\)
Fig. 1. Transfer functions in cosmologies with/without coupling and with/without 2 massive neutrinos. Coupling and mass are selected so to yield an approximate balance. The functions are multiplied by $k^{1.5}$, to help the reader to distinguish different cases.

0.73, spectral index $n_s = 0.96$ and a cosmic opacity $\tau_{opt} = 0.089$. DE is due to a SUGRA potential with $\Lambda = 1.1$ GeV, fitting WMAP and other data at least as well as $\Lambda$CDM. The slope $\alpha$ of the SUGRA potential is then fixed by requiring that the field density today matches $\Omega_{de}$.

Angular and spatial spectra are computed with an extension of the program CAMB [28], allowing to treat coupled DE models also in the presence of massive neutrinos.

DE treatment requires that 4 first order differential equations are added to the basic budget. In the CAMB implementation used for this paper we account for the dynamical evolution of DE by using the variables $\phi(\tau)$ and $\dot{\phi}(\tau)$ for the background DE field, as well as the variables $\varphi(\tau,k)$ and $\dot{\varphi}(\tau,k)$ for DE fluctuations.

Also CDM dynamics shall be modified when coupling is considered. The equa-
Fig. 2. Angular anisotropy spectra for the same models of the previous figure. Due to intrinsic $C_l$ oscillations, this Figure is slightly harder to read. In the lower frame we also give the spectral differences between 00– and CM–models. Large $l$ oscillations could be further damped by a shift by 1 or 2 units along $l$. The dotted lines represent the cosmic variance interval.

...tions, reported in Appendix A, are easily obtainable, e.g., from [29].

Before running CAMB we need a precursor program, to determine the value of $\alpha$ consistent with the assigned $\Omega_{de}$ and $\Lambda$.

Initial conditions for background variables are easily set according to known tracker solutions. In the presence of coupling, the tracking regime for density fluctuations is complex and includes different alternatives. We however found that, if we set $\varphi = 0$ and $\dot{\varphi} = 0$ at an initial time $\tau_{in}$, or we choose their expressions according to an arbitrary alternative, fluctuations accommodate rapidly on the right tracking and are completely independent from the initial choice at a time $\tau(k)$ when the $k$ scale enter the horizon, provided that $\tau_{in} \ll \tau(k)$. 
In the linear theory, however, DE fluctuations matter just about the horizon scale and are rapidly damped afterwards.

Both \( l \) and \( k \) ranges are selected for being those physically most significant. At lower \( l \)'s model discrepancies essentially vanish. In the \( l \) region shown, we have the sequel of maxima and minima due to primeval compression waves. The \( k \) range covers the scale explored by deep samples, as 2dF or SDSS, up to \( k \) values where non-linear effects become important.

In the plots, spectra are multiplied by suitable powers of the abscissa \( l \) or \( k \), so to reduce the ordinate range. In spite of that, in the \( C_l \) plot different spectra are not easy to distinguish. We then plot also the ratio \( \Delta C_l / C_l \) at constant \( l \); shifts would however appear even smaller if slight shifts along the \( l \) axis (by 1 or 2 units) were performed.

The Figures are principally meant to show that the effects of neutrino mass and coupling are opposite. The coupling intensity, in fact, is selected so to (approximately) balance neutrino masses.

We took, however, \( \sum m_\nu \ll 0.67 \) and \( \beta \ll 0.075 \) (see [17,18,19]); each of these values, by itself, is within current observational limits. Accordingly, even the difference between thin and thick solid–line spectra cannot be appreciated through current data.

In particular, let us outline how the BAO (baryonic acoustic oscillation) structure is faithfully reproduced when passing from the 00–mode ls (thick solid line) to the CM–models (thick dashed line).

4 Fisher matrix (data and technique)

We then aim to test how far we can go, simultaneously increasing \( \beta \) and \( \Omega_\nu \), without conflicting with data. This can be estimated by using a FM analysis.

This approach allows a rapid, semi–analytic estimate of the confidence limits for a specific experiment. It assumes a reference model as the most probable one, i.e. as the maximum of the likelihood distribution \( \mathcal{L}(\bar{x} | \bar{\theta}) \) of the data system \( \bar{x} \) given the model, described by parameters \( \bar{\theta} \equiv (\theta_i) \). Exploiting this hypothesis, one can approximate \( \mathcal{L} \) by a multivariate Gaussian distribution, built using its second derivatives in respect to the parameters \( (\bar{\theta}_i) \) at the reference model. Nevertheless, as is known, this technique is limited by the actual non–Gaussian behavior of data.
FM is nothing but the Hessian of the log-likelihood function:

$$F_{ij} = -\left( \frac{\partial}{\partial \theta_i} \frac{\partial}{\partial \theta_j} \log L \right)_{\hat{\theta}} = \sum_{i \neq j} \frac{\partial x_i}{\partial \theta_i} \frac{\partial x_j}{\partial \theta_j} \text{Cov}^{-1}_i(\hat{\theta}) \frac{\partial x_i}{\partial \theta_i}.$$  \hspace{1cm} (14)

In the literature, cosmological models are constrained by using a large number of observables. To our present aims we shall directly consider the spectrum of matter fluctuations $P(k)$ and the CMB angular spectra $C_{l}^{XY}$ ($XY = TT, TE, EE$). In their recent analysis, Komatsu et al (2008) made a more restricted use of $P(k)$, using only BAO’s, while they used SNIa Hubble diagrams, so significant also for being the first signal of DE.

Here we chose observables directly coming from the model, in the attempt to leave apart observational biases, focusing just on the level of sensitivity of possible experiments. We consider then two different experimental contexts. The first one assumes that CMB spectra are measured at WMAP sensitivity and $P(k)$ is measured with the sensitivity of the 2dF experiment (case W). The second assumes PLANCK sensitivity for CMB spectra and SDSS sensitivity for $P(k)$ (case P). The observational features for each mission considered are listed in Table 2 for the case of CMB experiments and the galaxy surveys.

Let us now consider first the use of CMB data only and let $C_{l}^{XY}$ be the angular spectra of the input model, to which we must add a white noise signal, to obtain

$$\bar{C}_{l}^{XY} = C_{l}^{XY} + N_{l}^{XY} \quad \text{with} \quad N_{l}^{XY} = \delta_{XY} \sigma_{X}^{2} \exp \left[ l(l+1) \frac{\theta_{FWHM}^{2}}{8 \ln 2} \right].$$  \hspace{1cm} (15)

The expressions of the Fisher matrix $F_{ij}^{C}$ components are then obtainable according to the relation

$$F_{ij}^{C} = \sum_{i} \sum_{XY,XY'} \frac{\partial}{\partial \theta_i} C_{l}^{XY} [\text{Cov}^{-1}_i]^{XYXY'} \frac{\partial}{\partial \theta_j} C_{l}^{XY'}.$$  \hspace{1cm} (16)

Table 2
CMB (upper table) and galaxy surveys (lower table) specifications used in the paper. In the lower table, scales and volumes are in Mpc/h and (Mpc/h$^3$, respectively.

| Mission | $l_{max}$ | $f_{sky}$ | $\theta_{FWHM}$ | $\sigma_T$ | $\sigma_P$ |
|---------|-----------|-----------|-----------------|-------------|-------------|
| WMAP    | 1000      | 0.8       | 13'             | 260         | 500         |
| PLANCK  | 2500      | 0.8       | 7.1'            | 42          | 80          |
| 2dF     | 0.02      | 0.1       | 10$^8$          |             |             |
| SDSS    | 0.02      | 0.15      | 0.72×10$^9$     |             |             |
On the contrary, when dealing with matter power spectra, we used the following definition for the FM [34]

\[
F_{ij}^P = \sum_{\alpha, \beta} \frac{\partial}{\partial \theta_i} P(k_\alpha) [\text{Cov}^{-1}_P]_{\alpha \beta} \frac{\partial}{\partial \theta_j} P(k_\beta) \tag{18}
\]

with

\[
[\text{Cov}_P]_{\alpha \beta} \simeq \delta_{\alpha \beta} \frac{V_f}{V_s(k_\alpha)} 2P^2(k_\alpha), \tag{19}
\]

where \(V_f = (2\pi)^3/V\) is the volume of the fundamental cell in \(k\) space, \(V\) is the volume of the survey and \(V_s(k_\alpha) = 4\pi k_\alpha^2 \delta k\) is the volume of the shell of width \(\delta k\) centered on \(k_\alpha\) [35,36]. In Eq. (19) we left aside the contribution of the trispectrum, because in our analysis we considered only the linear scales, where the trispectrum is expected to be negligible.

The cosmological model we consider is characterized by 9 parameters:

- \(\omega_b = \Omega_b h^2\): Physical baryon density
- \(\omega_c = \Omega_c h^2\): Physical CDM density
- \(H_0\): Hubble constant
- \(A_s\): Scalar fluctuation amplitude
- \(n_s\): Scalar spectral index
- \(\tau_{\text{opt}}\): Reionization optical depth
- \(\log(\Lambda/\text{GeV})\): Decimal logarithm of the energy scale in SUGRA potential
- \(\beta\): CDM – DE coupling strength
- \(\sum m_\nu/eV\): sum of neutrino masses

We estimate the neutrino mass density parameter, \(\Omega_\nu h^2\), converting it from the total neutrino mass via

\[
\Omega_\nu h^2 = \frac{\sum m_\nu}{93.5 \text{ eV}}. \tag{20}
\]

We compute the CMB anisotropies (temperature and polarisation) power spectra and the transfer functions, used to calculate linear matter power spectrum, using a modified version of CAMB. Double sided numerical derivatives were evaluated considering a 5% stepsize, except for \(\Lambda\), where we adopted a 5% stepsize on \(\lambda \equiv \log(\Lambda/\text{GeV})\).
Fig. 3. 1– and 2–σ confidence levels for a 2–massive–neutrino model, assuming that the true cosmology is a SUGRA model with \( \log(\Lambda/\text{GeV}) = 1.1 \), while \( \beta = 0 \) and \( \Omega_\nu \simeq 0 \). Thick (thin) curves show the constraints deriving from CMB and deep sample data (from CMB data only). Solid curves refer to the (i) case (WMAP+2dF). Dashed curves refer to the (ii) case (PLANCK+SDSS). In the sequel we shall examine in detail models corresponding to the points labeled a, b, c, d, e and others. The location of the CM–model of Figs. 1 and 2 is indicated by an open box. The two locations indicated by an open circle and a cross will also be considered in detail below. This Figure is somehow analogous to Fig. 2 in Hannestad, 2005.

5 Fisher matrix (results)

In Figure 3 we then report the expected 1– and 2–σ likelihood curves on the \( \sum m_\nu/\beta \) plane, for both cases W and P. In either case we analyse the constraints coming just from CMB data and those arising from the joint exploitation of CMB and deep sample data. We performed the analysis either assuming 3 equal mass neutrinos, or 1 massless and 2 massive neutrinos. The plots shown in the Figure are obtained for the latter case, but discrepancies are just a minor effect.
Fig. 4. Correlation between $\beta$ and the other model parameters for the W case (WMAP+2dF); dashed lines refer to CMB data only.

Fig. 5. Correlation between $\beta$ and the other model parameters for the P case (PLANCK+SDSS); dashed lines refer to CMB data only.

The Fisher–matrix results, for the W case, substantially confirms known 1– and 2–$\sigma$ limits on $\beta$ \cite{18,19,37}, yielding $\beta < 0.05$ and $\beta < 0.075$, respectively, along the $\beta$ axis (i.e. with $\sum m_\nu = 0$).

On the other axis, with $\beta = 0$, $\sum m_\nu$ seems to be more constrained than what we know from current limits ( $\sum m_\nu < 0.35$ eV vs. $\sum m_\nu < 0.8$ eV with $w \neq -1$). These discrepancies can be read as an indication of the level or reliability that Fisher–matrix estimate can have. In particular, they may be partially due to the impact of using the whole $P(k)$ information, as well as to the fact that the reference cosmology is SUGRA instead of $\Lambda$CDM.
Fig. 6. 1– and 2–σ limits, for the P–case, assuming that the true cosmology corresponds to the points marked by cross or open circle. The dashed lines report the same limits around the 00–model, as shown in Fig. similarly, the dotted line is the 1–σ limit around the 00–model in the W–case, as shown in Fig. This Figure shows that cosmologies, comprising CDM–DE coupling and neutrino masses, presently compatible with the 00 option, will be easily discriminated, at the P sensitivity level.

However, the CMB 2–σ constraint we find, $\sum m_\nu < 1.65$ eV, is close to the 95% confidence limit $\sum m_\nu < 1.5$ eV obtained through a full MonteCarlo analysis of WMAP data only, with $w \neq -1$.

The likelihood plots have the expected shape. Taken at face value they yield upper limits $\beta \lesssim 0.22$ and $\sum m_\nu \lesssim 1.05$ eV, in the case W. With the value of $H_0$ used here this would correspond to $\Omega_\nu \simeq 0.022$, more than 50% of baryon density.

On the contrary, in the case P, constraints are more severe, as only CM–models with $\beta < 0.07$ and $\sum m_\nu < 0.4$ eV appear consistent with the 00–model, at the 2–σ level. These limits are close to the maximum coupling and neutrino mass separately admitted in the present observational constraints.
In Figure 4 we also show the correlations between $\beta$ and the whole set of parameters considered, in the W case. Correlations can be considered negligible for the parameters $A_s$, $n_s$, $\Lambda$, $\tau_{\text{opt}}$. The correlations with the parameters $\omega_c$, $\omega_b$, $H_0$, as expected, are stronger. Figure 5 yields analogous results for the P case.

It may also be useful to consider Figure 6 showing that models, including CDM–DE coupling and neutrino masses, compatible with the 00 option at the W sensitivity level, at the P sensitivity level will be well discriminated from it and/or also between them.

6 Exploring the parameter space

Of course, a FM analysis gives just a basic idea of the precision with which current/future data can constrain our parameter set and relax current bound on $\Omega_{\nu}$; on the contrary, by no means a FM analysis can tell us whether data favor $\beta = 0$ or $\neq 0$: here, when computing the FM from WMAP+2dF–like data, we choose arbitrarily a fiducial model with $\beta = 0$, $\Omega_{\nu} = 0$, assuming it to have a high likelihood on the $\beta$–$\Omega_{\nu}$ plane.

*While a FM approach allows no likelihood estimate*, in this Section we wish to provide a few examples, exploring the parameter space that FM results apparently allow and, in Figures 7 and 8 we exhibit the spectra for a set of models.

As stated in the frame of Fig. 7, the models yielding maximum CDM–DE coupling (and vanishing $\nu$ mass) or maximum neutrino mass (and vanishing coupling) have the thick line spectra. Model discrepancy is enhanced by taking the same amplitude $A_s$, instead of normalizing them to the same $\sigma_8$.

The setting of models a, b, c, d, e on the $\sum m_{\nu}$–$\beta$ plane is indicated in Fig. 3. They are typically within 1–$\sigma$ boundaries. The best performance, perhaps, can be ascribed to models d and e. Both of them yield a present hot dark matter density exceeding 1% of the critical density and 5% of the whole DM.

One of the scopes of this Section was testing that mildly mixed DM models, in the presence of a significant CDM–DE coupling, are reasonably consistent with data.
Fig. 7. Spectra for a number of cosmologies. All of them are obtained setting $n_s = 0.96$ and $\log A_s = -8.64$, so to enhance model differences. Thick lines correspond to models presently considered in agreement with data, and yielding maximum values either for $\sum m_\nu$ or $\beta$. The other lines yield models corresponding to the points a, b, c, d, e in Fig. 3 consistent with the 00–model at the 1–σ level.

7 Conclusions

This paper performs a first inspection on the possibility that high $\nu$ masses and CDM–DE coupling yield compensating distortions of matter fluctuations and CMB spectra. This compensation is highly effective for small masses and couplings, as shown in Figs. 1 and 2. We then address the most significant question concerning the limits on coupling and $\nu$ masses, when simultaneously considered.

This question should be carefully addressed by using MonteCarlo techniques and considering all available observational constraints. Unfortunately, to do so, we should widen the usual parameter space, by adding 3 extra degrees of freedom: the coupling parameter $\beta$, neutrino mass, and the energy scale $\Lambda$ in the SUGRA model (or another equivalent parameter, in the same or in
Fig. 8. Spectra of CMB anisotropies for the same cosmologies of Fig. 7 compared with WMAP error amplitudes. Let us remind that all of them are obtained keeping the same values $n_s = 0.96$ and $\log A_s = -8.64$, so to enhance model differences. The relative difference of the thick line models from the 00-model appears not so wide as for some of the other models. Among them, however, the solid and dashed line models seem to perform quite well. Their performance can be improved by adjusting the $H_o$ value, slightly modifying Fisher matrix outputs.

This is among the reasons that led previous authors to perform a preliminary test by using a Fisher matrix technique. As is known, a FM analysis assumes that a given cosmology has a top likelihood and explores its neighbours. We can imagine, e.g. that the data favor $\beta \neq 0$ in association with a significant $\nu$ mass. A FM analysis would hardly tell us that. This calls for a MCMC analysis that we reserve for future work.

Taking FM outputs at face value leads to state that models with $\Omega_\nu \lesssim 0.022$ and $\beta \lesssim 0.22$ are observationally consistent with a $\Omega_\nu = 0$ and $\beta = 0$ model.
Fig. 9. Density parameters for CDM, hot DM ($\nu$'s), DE and radiation in models with $\Omega_\nu$ and $\beta$ taking the values 0.005–0.049 (short dashed), 0–0.07 (long dashed), 0.011–0.1 (dotted), 0.02–0.21 (solid), respectively. In the last case, the DE plateau, extending up to the equality redshift, occurring slightly above $z \sim 10^3$, shows a DE density keeping $\sim 1/20$ of CDM.

Models with cDE were initially considered to overcome the coincidence problem, in the presence of DE. To achieve this aim, one has to accept that DE is characterized by two different scales. For a coupled SUGRA cosmology like the one explored here, they are about the EW and the Planck scales. No ad–hoc scale is then apparently introduced, but the whole framework appears somehow artificial and demands for a more basic scheme, to reduce its complexity. An example is the double–axion model [38], which however leads to features different from the ones considered here.

Still working at a phenomenological level, in Figure 9 we however show the scale dependence of the density parameters, for various models with different $\sum m_\nu$ and $\beta$.

In the usual case, with negligible $m_\nu$, a CDM–DE coupling compatible with data hardly eases the coincidence problem. Such easing is represented by the plateau in the $\Omega_{\text{de}}$ curve, whose proportions are then almost insignificant. This does not mean that $\beta \neq 0$ is not to be considered among the possible degrees of
freedom; e.g., in [39] it is shown that a cosmology with $\beta$ as small as $\sim 0.05$, if inspected assuming $\beta \equiv 0$, can yield wrong values for some cosmic parameters, including $\omega_{oc}$. Figure 9 however indicates that, when $\beta \sim 0.2$ is recovered, in the presence of suitably massive $\nu$’s, a significant plateau is present and DE density keeps at the level $\sim 1–2\%$ of the critical density up to $z \sim 10^3$.

Acknowledgements

Thanks are due to R, Mainini for discussions. LPLC was supported by NASA grant NNX07AH59G and Planck subcontract 1290790 for this work, and would like to thank the Physics Department G. Occhialini, for hospitality.

References

[1] Q. R.Ahmad et al., Phys. Rev. Lett.89, 011301 (2002); S. N.Ahmed et al., Phys. Rev. Lett.92, 181301 (2004).
[2] K. Eguchi et al., Phys. Rev. Lett.90, 021802h (2003); T. Araki et al., Phys. Rev. Lett.94, 081801 (2005).
[3] W. W. Allison et al., Phys. Lett.B449, 137 (1999); M. Ambrosio et al., Phys. Lett.B517, 59 (2001).
[4] M. H. Ahn et al., Phys. Rev. Lett.90, 041801h (2003); D. G. Michael et al., Phys. Rev. Lett.97, 191801 (2006).
[5] R. Valdarnini & S. Bonometto, A&A 146, 2 235 (1985); R. Valdarnini & S. Bonometto, Astrophys. J.299, L71 (1985).
[6] A. G. Riess, R. P. Kirshner, B. P. Schmidt, S. Jha, et al., Astrophys. J.116, 1009 (1998); S. Perlmutter, G. Aldering, G. Goldhaber, et al., Astrophys. J.517, 565 (1999); A. G. Riess, L.G. Strolger, J. Tonry, et al., Astrophys. J.607, 665 (2004).
[7] P. de Bernardis, P. Ade, J. Bock, et al., Nat. 404, 955 (2000); S. Padin, J. Cartwright, B. Mason, et al., Astrophys. J.549, L1 (2001); J. Kovac, E. Leitch, c. Pryke, et al., Nat. 420, 772 (2002); P. Scott, P. Carreira, K. Cleary, et al., Mon. Not. R. Astron. Soc.341, 1076 (2003); D. Spergel, R. Bean, Dorè et al., Astrophys. J.Suppl. 170, 377 (2007).
[8] M. Colless, G. Dalton, S. Maddox, et al., Mon. Not. R. Astron. Soc.329, 1039 (2001); M. Colless, B. Peterson, C. Jackson, et al., Preprint astro–ph/0306581; J. Loveday (the SDSS collaboration), Contemp. Phys.43, 437 (2002); M. Tegmark, M. Blanton, M. Strauss, et al., Astrophys. J.606, 702 (2004); J. Adelman–McCarthy, M. Agueros, S. Allam, et al., Astrophys. J.Suppl. 162, 38 (2004).
[9] A. Dolgov, Phys. Rev. 370, 333 (2002); O. Elgaroy & O. Lahav, New J. Phys. 7, 61t (2005); J. Lesgourgues & S. Pastor, Phys. Rev. 429, 307 (2006).

[10] G. L. Fogli et al., Phys. Rev. D 78, 033010 (2008); G. L. Fogli et al., Phys. Rev. D 75, 053001 (2007).

[11] E. Komatsu et al. [WMAP Collaboration], Preprint [arXiv:0803.0547] [astro-ph].

[12] A. Goobar, S. Hannestad, E. Mortsell & H. Tu, JCAP 0606, 019 (2006); A. G. Sanchez & S. Cole, Mon. Not. R. Astron. Soc., 385, 830 (2008).

[13] W. Percival et al., Monthly Notices of the Royal Astronomical Society 327, 1297 (2001).

[14] D. G. York et al., Astronomical Journal 120, 1579 (2000); C. Stoughton et al., Astronomical Journal 126, 485 (2002); K. Abazajian et al., Astronomical Journal 123, 2081 (2003).

[15] U. Seljak et al. [SDSS Collaboration], Phys. Rev. D 71, 103515 (2005); U. Seljak, A. Slosar and P. McDonald, JCAP 0610, 014 (2006).

[16] K. N. Abazajian & S. Dodelson, Phys. Rev. Lett. 91, 041301 (2003); S. Hannestad, H. Tu & Y. Y. Y. Wong, JCAP 0606, 025 (2006); T. D. Kitching, A. F. Heavens, L. Verde, P. Serra & A. Melchiorri, Phys. Rev. D 77, 103008 (2008).

[17] Wetterich C., A&A 301, 321 (1995); Amendola L., Phys. Rev. D 62, 043511 (2000).

[18] L. Amendola and C. Quercellini, Phys. Rev. D 68, 023514 (2003).

[19] Maccio’ A. V., Quercellini C., Mainini R., Amendola L., Bonometto S. A., Phys. Rev. D 69, 123516 (2004).

[20] J. Ellis, S. Kalara, K. A. Olive & C. Wetterich, Phys. Lett. B 228, 264 (1989); M. Gasperini, F. Piazza & G. Veneziano, Phys. Rev. D 65, 023508 (2002); D. Comelli, M. Pietroni and A. Riotto, Phys. Lett. B 571, 115 (2003); L.P. Chimento, A.S. Jakubi, D. Pavon & W. Zimdahl, Phys. Rev. D 67, 083513 (2003); Z. K. Guo, N. Ohta and S. Tsujikawa, Phys. Rev. D 76, 023508 (2007); E. Abdalla, L. R. W. Abramo, L. J. Sodre and B. Wang, Preprint [arXiv:0710.1198] [astro-ph]; M. Manera and D. F. Mota, Mon. Not. R. Astron. Soc. 371, 1373 (2006).

[21] J. Khoury and A. Weltman , Phys. Rev. Lett. 93 (2004) 17110; S.S. Gubser and J. Khoury, Phys. Rev. D 70 (2004) 104001; P. Brax et al., Phys. Rev. D 70 (2004) 123518; D. F. Mota and D. J. Shaw, Phys. Rev. Lett. 97, 151102 (2006); P. Brax, C. van de Bruck, A. C. Davis, D. F. Mota and D. J. Shaw, Phys. Rev. D 76, 124034 (2007).

[22] S. Hannestad, Phys. Rev. Lett. 95, 221301 (2005).

[23] A. De La Macorra, A. Melchiorri, P. Serra & R. Bean, Astropart. Phys. 27, 406 (2007).
A Fluctuation evolution in models with dynamical and coupled DE

Uncoupled DE equations – Let

$$\Phi(\tau, k) = \phi(\tau) + \varphi(\tau, k)$$

(A1)
be the dynamical DE field. In eq. (A1) the background component $\phi$, independent from space coordinates, and its fluctuations $\varphi$ are outlined. The dependence on spatial coordinates is also Fourier–transformed, so to have a $\varphi$ field dependent on the time $\tau$ and the wavenumber $k$.

In the absence of CDM–DE coupling, the background equation

$$\ddot{\phi} + 2(\dot{a}/a)\dot{\phi} + a^2 V'(\phi) = 0$$

(A2)

is known to hold, together with the equation

$$\ddot{\varphi} + 2(\dot{a}/a)\dot{\varphi} + \dot{\varphi}h/2 + k^2 \varphi + a^2 V''(\phi)\varphi = 0$$

(A3)

for its fluctuations. Here $\dot{h}$ is the usual variable describing the gravitational field due to density fluctuations in the synchronous gauge.

Aside of them, the equations

$$\dot{\rho}_c + 3(\dot{a}/a)\rho_c = 0$$

(A4)

$$\dot{\delta}_c + k\nu_c + \dot{h}/2 = 0$$

(A5a)

$$\dot{v}_c + (\dot{a}/a)v_c = 0.$$  

(A5b)

will hold, in a generic synchronous gauge, for the CDM density $\rho_c$ and its fluctuations $\delta_c$. Here $v_c$ is the (gauge dependent) velocity field in CDM.

The presence of dynamical DE clearly implies that the equation fulfilled by the scale factor is also modified into

$$\dot{a}^2 = a^4 H_o^2 \left[ \Omega_{\gamma} a^4 + (\Omega_{ob} + \Omega_{oc})/a^3 + (\rho_k + V)/\rho_{oc,cr} + \rho_{\nu}/\rho_{oc,cr} \right].$$  

(A6)

The symbols $H_o$, $\Omega_{\gamma}$, $\Omega_{oc}$, $\Omega_{ob}$ have their obvious meaning; $\rho_{oc,cr}$ is the present critical energy density, while

$$\rho_k = \dot{\phi}^2/2a^2$$

(A7)

is the kinetic energy density of the DE field. Finally

$$\rho_{\nu} = \frac{\mathcal{N}_\nu T^4}{2\pi^2} \int_0^\infty dx \, x^3 \frac{\epsilon(m_\nu/x T_\nu)}{\exp \epsilon(m_\nu/x T_\nu) + 1} \quad \text{with} \quad \epsilon(\mu) = \sqrt{1 + \mu^2}$$

is the energy density of $\nu$’s with mass $m_\nu$ and $\mathcal{N}_\nu$ spin states.

This equation shall not be modified by the presence of coupling, as well as the equations for gravitational field fluctuations, to whose source $\varphi$ contributes. We shall omit these last equations.

**Coupled DE equations** – In the presence of a constant CDM–DE coupling, eqs. (A2)–(A5) are modified so that their r.h.s.’s no longer vanish. More in
detail:

\[
\begin{align*}
(A2)-(A4) \rightarrow & \quad \ddot{\phi} + \ldots = Ca^2 \rho_c, \quad \dot{\rho}_c + \ldots = -C \rho_c \dot{\phi} \\
(A3) \rightarrow & \quad \ddot{\varphi} + \ldots = Ca^2 \rho_c \delta_c \\
(A5) \rightarrow & \quad \ddot{\delta}_c + \ldots = -C \phi, \quad \dot{v}_c + \ldots = -kC \varphi
\end{align*}
\]

All other dynamical equations keep unmodified.