A calculation for vector dark matter direct detection

Natsumi Nagata\textsuperscript{1,2}
\textsuperscript{1} Department of Physics, Nagoya University, Nagoya 464-8602, Japan
\textsuperscript{2} Department of Physics, University of Tokyo, Tokyo 113-0033, Japan
E-mail: natsumi@eken.phys.nagoya-u.ac.jp

Abstract.
We evaluate the elastic scattering cross section of vector dark matter with nucleon based on the method of effective field theory. The dark matter is assumed to behave as a vector particle under the Lorentz transformation and to interact with colored particles including quarks in the Standard Model. After formulating general formulae for the scattering cross sections, we apply them to the case of the first Kaluza-Klein photon dark matter in the minimal universal extra dimension model. The resultant cross sections are found to be larger than those calculated in previous literature.

1. Introduction
The existence of dark matter (DM) has been established by cosmological observations\textsuperscript{2}. One of the most attractive candidates is what we call Weakly Interacting Massive Particles (WIMPs), which are stable particles with masses of the electroweak scale and weakly interact with ordinary matters. This interactions enable us to search for WIMP DM by using the scattering signal of DM with nuclei on the earth. Such kind of experiments are called the direct detection experiments of WIMP DM.

For the past years, a lot of efforts have been dedicated to the direct detection of WIMP DM, and their sensitivities have been extremely improving. The XENON100 Collaboration, for example, gives a severe constraint on the spin-independent (SI) elastic scattering cross section of WIMP DM with nucleon $\sigma_{N}^{SI}$ ($\sigma_{N}^{SI} < 2.0 \times 10^{-45}$ cm$^2$ for WIMPs with a mass of 55 GeV/c$^2$)\textsuperscript{3}. Moreover, ton-scale detectors for the direct detection experiments are now planned and expected to have significantly improved sensitivities.

In order to study the nature of DM based on these experiments, we need to evaluate the WIMP-nucleon elastic scattering cross section precisely. In this work, we assume the WIMP DM to be a vector particle, and evaluate its cross section scattering off a nucleon. Several candidates for vector DM have been proposed in various models, and there have been a lot of previous work computing the scattering cross sections\textsuperscript{4,5,6}. However, we found that in the calculations some of the leading contributions to the scattering cross section are not evaluated correctly, or in some cases completely neglected. Taking such situation into account, we study the way of evaluating the cross section systematically by using the method of effective field theory.

\textsuperscript{1} This talk is based on the work with Junji Hisano, Koji Ishiwata, and Masato Yamanaka\textsuperscript{1}. 
2. Direct detection of vector dark matter

In this section we discuss the way of evaluating the elastic scattering cross section of vector DM with nucleon. First, we write down the effective interactions of vector DM ($B_\mu$) with light quarks and gluon [1]:

\[ \mathcal{L}^\text{eff} = \sum_{q=u,d,s} \mathcal{L}^\text{eff}_q + \mathcal{L}^\text{eff}_G, \]

with

\[ \mathcal{L}^\text{eff}_q = f_q m_q B^\mu B_\mu \bar{q} q + \frac{d_q}{M} \epsilon^{\mu\nu\rho\sigma} B_\mu i \partial^\nu B^\rho \bar{q} \gamma^\sigma q + \frac{g_q}{M^2} B_\mu i \partial^\nu \partial^\rho B_\rho \mathcal{O}^q_{\mu\nu} \]
\[ \mathcal{L}^\text{eff}_G = f_G B^\rho B_\rho G^{a\mu\nu} G^a_{\mu\nu}, \]

where $m_q$ are the masses of light quarks, $M$ is the DM mass, and $\epsilon^{\mu\nu\rho\sigma}$ is the totally antisymmetric tensor defined as $\epsilon^{0123} = +1$. The covariant derivative is defined as $D_\mu \equiv \partial_\mu + ig_s A^a_\mu T_A$, with $g_s$, $T_A$ and $A^a_\mu$ being the strong coupling constant, the SU(3)$_C$ generators, and the gluon fields, respectively. The gluon field strength tensor is denoted by $G^{a\mu\nu}$, and $\mathcal{O}^q_{\mu\nu} \equiv \frac{i}{2} \bar{q} \left( D_\mu \gamma_\nu + D_\nu \gamma_\mu - \frac{i}{2} g_{\mu\nu} \partial_5 \right) q$ are the twist-2 operators of light quarks. When we write down the effective Lagrangian, we consider the fact that the scattering process is non-relativistic. The coefficients of the operators are to be determined by integrating out the heavy particles in high energy theory. The second term in Eq. (2) gives rise to the spin-dependent (SD) interaction, while the other terms yield the spin-independent (SI) interactions. We focus on the SI interactions hereafter, because the experimental constraint is much severe for the SI interactions, rather than for the SD interactions.

In order to obtain the effective coupling of the vector DM with nucleon induced by the effective Lagrangian, we need to evaluate the nucleon matrix elements of the quark and gluon operators in Eqs. (2) and (3). First, the nucleon matrix elements of the scalar-type quark operators are parametrized as

\[ f_{Tq} \equiv \langle N | m_q \bar{q} q | N \rangle / m_N , \]

with $|N\rangle$ and $m_N$ the one-particle state and the mass of nucleon, respectively. The parameters are called the mass fractions and their values are obtained from the lattice simulations [7, 8]. Second, for the quark twist-2 operators, we can use the parton distribution functions (PDFs):

\[ \langle N(p) | \mathcal{O}^q_{\mu\nu} | N(p) \rangle = \frac{1}{m_N} (p_\mu p_\nu - \frac{1}{4} m_N^2 g_{\mu\nu}) (q(2) + \bar{q}(2)) , \]

where $q(2)$ and $\bar{q}(2)$ are the second moments of PDFs of quark $q(x)$ and anti-quark $\bar{q}(x)$, respectively, which are defined as $q(2) + \bar{q}(2) = \int_0^1 dx \ x [q(x) + \bar{q}(x)]$. These values are obtained from Ref. [9]. Finally, the matrix element of gluon field strength tensor can be evaluated by using the trace anomaly of the energy-momentum tensor in QCD [10]. The resultant expression is given as

\[ \langle N | G^{a\mu\nu} G^a_{\mu\nu} | N \rangle = -\frac{8\pi}{9\alpha_s} m_N f_{TG} \]

with $f_{TG} \equiv 1 - \sum_{q=u,d,s} f_{Tq}$. Note that the right hand side of the expression is divided by the strong coupling constant, $\alpha_s$. For this reason, although the gluon contribution is induced by higher loop diagrams, it can be comparable to the quark contributions [11]. Briefly speaking, the enhancement comes from the large gluon contribution to the mass of nucleon. As a result, the SI effective coupling of vector DM with nucleon, $f_N$, is given as

\[ f_N/m_N = \sum_{q=u,d,s} f_q f_{Tq} + \sum_{q=u,d,s,c,b} \frac{3}{4} (q(2) + \bar{q}(2)) g_q - \frac{8\pi}{9\alpha_s} f_{TG} f_G . \]
Using the effective coupling, we eventually obtain the SI scattering cross section of DM with nucleon:

$$\sigma_N^{(SI)} = \frac{1}{\pi} \left( \frac{m_N}{M + m_N} \right)^2 |f_N|^2.$$  

(8)

Now, all we have to do reduces to evaluate the coefficients of the effective operators by integrating out the heavy fields in the high-energy theories. For example, we take the case where the interaction Lagrangian of the vector DM has a generic form as

$$\mathcal{L} = \bar{\psi}_2 \left( a_{\psi_2\psi_1} \gamma^\mu + b_{\psi_2\psi_1} \gamma^\mu \gamma^5 \right) \psi_1 B_\mu + \text{h.c.},$$

(9)

where $\psi_1$ and $\psi_2$ are colored fermions with masses $m_1$ and $m_2$ ($m_1 < m_2$), respectively. In this case, the vector DM is scattered by light quarks at tree-level. The relevant interaction Lagrangian is given by taking $\psi_1 = q$ in Eq. (9), and the corresponding diagrams are shown in Fig. [1]. After integrating out the heavy particle $\psi_2$, we obtain

$$f_q = \frac{a_{\psi_2q}^2 - b_{\psi_2q}^2}{m_q} \frac{m_2}{m_2^2 - M^2} - \left( a_{\psi_2q}^2 + b_{\psi_2q}^2 \right) \frac{m_2^2}{2(m_2^2 - M^2)^2},$$

(10)

$$g_q = \frac{2M^2(a_{\psi_2q}^2 + b_{\psi_2q}^2)}{(m_2^2 - M^2)^2}. $$

(11)

One can easily find that the effective couplings obtained here are enhanced when the vector DM and the heavy colored fermion are degenerate in mass [12].

The effective coupling of the vector DM with gluon is induced by 1-loop diagrams illustrated in Fig. [2]. In those processes, all the particles $\psi_1$ and $\psi_2$ which couple with $B_\mu$ run in the loop.
Figure 3. Tree-level diagrams for the elastic scattering of the KK photon DM $B^{(1)}$ with light quarks: (a) Higgs boson exchange contribution, and (b) KK quark exchange contributions.

Figure 4. One-loop diagrams for the effective interaction of $B^{(1)}$ with gluon.

The resultant expressions are somewhat complicated, and thus we just quote Ref. [1] for their complete formulae as well as their derivation.

3. Application and Results

Next, we deal with a particular model for vector DM as an application. We carry out the calculation for the first Kaluza-Klein (KK) photon DM in the minimal universal extra dimension (MUED) model. In this model, an extra dimension is compactified on an $S^1/Z_2$ orbifold with the compactification radius $R$, and all of the Standard Model (SM) particles propagate in the dimension. The lightest KK-odd particle (LKP) is prevented from decaying to the SM particles, so it becomes DM. This model has just three undetermined parameters: the radius of the extra dimension $R$, the mass of Higgs boson $m_h$, and the cutoff scale $\Lambda$.

In general, extra dimensional models give rise to the degenerate mass spectrum at tree-level, which is broken by radiative corrections. By evaluating the radiative corrections, one finds that the first KK photon $B^{(1)}$ is the lightest KK-odd particle, thus, becomes the DM in the Universe. Moreover, since the mass difference is induced by radiative corrections, the mass degeneracy is tight for a small cut-off scale. In such a case, although it is difficult to probe the MUED model at the LHC because of the soft QCD jets, the direct detection rate of dark matter is expected to be enhanced, as we shall see soon later.

Now we evaluate the SI scattering cross section of the KK photon DM with nucleon. The effective interaction of the KK photon DM $B^{(1)}$ with light quarks is induced by the tree-level diagrams shown in Fig. 3. Here, $h^0$ and $q^{(1)}$ are the Higgs boson and the first KK quark, respectively. Also, there are one-loop diagrams we should evaluate for the gluon contribution. They are illustrated in Fig. 4. In these diagrams, all of the KK quarks run in the loop. Taking

\footnote{Indirect DM searches also might be a powerful alternative in such a case [16] [17].}
Figure 5. Each contribution in effective coupling $f_N/m_N$ given in Eq. (7). Here we set $m_h = 125$ GeV and $(m_{1st} - M)/M = 0.1$.

Figure 6. Spin-independent cross section with proton for $m_h = 125$ GeV. Each line corresponds to $A = 5/R$, $20/R$, and $50/R$, respectively.

all the contributions into account, we obtain the effective interactions. Their expressions are given in Ref. [1].

In Fig. 5 we plot each contribution to the SI effective coupling of DM with a proton. The solid line shows the contribution of the Higgs boson exchanging diagrams, the dashed line indicates the twist-2 type contribution, the dash-dotted line corresponds to the scalar-type contribution (except for the Higgs-exchanging contribution), and the dotted line represents the gluon contribution (except for the Higgs-exchanging contribution). Here we set the Higgs boson
mass equal to 125 GeV, and the mass difference between the DM and the first KK quark to be 10% by hand. We find that all of the contributions have the same sign thus they are constructive. The twist-2 contribution is dominant when $M \lesssim 800$ GeV, while that of Higgs boson exchanging process becomes dominant above it. Moreover, although the tree-level quark contributions are dominant, it is found that the gluon contribution is not negligible at all.

By using the effective couplings obtained above, we evaluate the SI scattering cross sections. In Fig. 6, we plot the SI cross section of KK photon DM with a proton as a function of the DM mass. Here again, the Higgs boson mass is set to be 125 GeV. We take the cut-off scale $\Lambda = 5/R, 20/R, 50/R$ from top to bottom. We find that the scattering cross sections reduce as the cut-off scale is taken to be large, as expected. As a result, we obtain the SI scattering cross section which is larger than those obtained by the previous calculations. Note that the DM mass with which the thermal relic abundance is preferred by the WMAP observation is around 1300 GeV [20]. It corresponds to $\sigma_N^{SI} = (2.5-5) \times 10^{-47}$ cm$^2$, so the direct detection experiments with ton-scale detectors might be able to probe the DM in the future.

4. Conclusion
We calculate the spin independent elastic scattering cross sections of vector dark matter with nucleon based on the effective field theory. It is found that the interaction of dark matter with gluon as well as quarks yields sizable contribution to the scattering cross section, though the gluon contribution is induced at loop level. The scattering cross section of the first Kaluza-Klein photon dark matter in the MUED model turns out to be larger than those obtained by the previous calculations.

Acknowledgments
The author would like to thank Junji Hisano, Koji Ishiwata, and Masato Yamanaka for collaboration. This work is supported by Research Fellowships of the Japan Society for the Promotion of Science for Young Scientists.

References
[1] J. Hisano, K. Ishiwata, N. Nagata and M. Yamanaka, Prog. Theor. Phys. 126, 435 (2011).
[2] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. 192, 18 (2011).
[3] E. Aprile et al. [XENON100 Collaboration], arXiv:1207.5083 [astro-ph.CO].
[4] H. C. P. Cheng, J. L. Feng and K. T. Matchev, Phys. Rev. Lett. 89, 211301 (2002).
[5] G. Servant and T. M. P. Tait, New J. Phys. 4, 99 (2002).
[6] A. Birkedal, A. Noble, M. Perelstein and A. Spray, Phys. Rev. D 74, 035002 (2006).
[7] R. D. Young and A. W. Thomas, Phys. Rev. D 81, 014503 (2010).
[8] H. Ohki, et al. [JLQCD Collaboration], arXiv:1208.4185 [hep-lat].
[9] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. M. Nadolsky and W. K. Tung, JHEP 0207, 012 (2002).
[10] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Phys. Lett. B 78, 443 (1978).
[11] J. Hisano, K. Ishiwata and N. Nagata, Phys. Rev. D 82, 115007 (2010).
[12] J. Hisano, K. Ishiwata and N. Nagata, Phys. Lett. B 706, 208 (2011).
[13] T. Appelquist, H. -C. Cheng and B. A. Dobrescu, Phys. Rev. D 64, 035002 (2001).
[14] H. -C. Cheng, K. T. Matchev and M. Schmaltz, Phys. Rev. D 66, 036005 (2002).
[15] S. Arrenberg, L. Baudis, K. Kong, K. T. Matchev and J. Yoo, Phys. Rev. D 78, 056002 (2008).
[16] M. Asano, T. Bringmann and C. Weniger, Phys. Lett. B 709, 128 (2012).
[17] M. Garny, A. Ibarra, M. Pato and S. Vogl, arXiv:1207.1431 [hep-ph].
[18] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716, 1 (2012).
[19] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716, 30 (2012).
[20] G. Belanger, M. Kakizaki and A. Pukhov, JCAP 1102, 009 (2011).

3 Recent searches for the Standard Model Higgs boson at the LHC indicate its mass to be around 125 GeV [18, 19].