Anisotropic flows from colour strings: Monte-Carlo simulations

M.A.Braun\textsuperscript{a,b}, C.Pajares\textsuperscript{a}, V.V.Vechernin\textsuperscript{b}
\textsuperscript{a} University of Santiago de Compostela, Spain,
\textsuperscript{b} S.Petersburg State University, Russia

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Abstract By direct Monte-Carlo simulations it is shown that the anisotropic flows can be successfully described in the colour string picture with fusion and percolation provided anisotropy of particle emission from the fused string is taken into account. Quenching of produced particles in the strong colour field of the string is the basic mechanism for this anisotropy. The concrete realization of this mechanism is borrowed from the QED. Due to dependence of this mechanism on the external field strength the found flows grow with energy, with values for $v_2$ at LHC energies greater by 15\% than at RHIC energies.

1 Introduction

The study of anisotropic flows in the spectra of particles produced in high-energy heavy-ion collisions proves to be very informative as to the dynamics of the underlying emission mechanisms. Observed experimental values for the anisotropic flow coefficients $v_n$ impose strong restrictions on the models which try to describe the data. Of special importance are the coefficients with odd values of $n$, which disappear on the average and can only be observed on the event-by-event basis. They carry information about the details of the evolution of the hot nuclear matter in the overlap and its fluctuations during the collision. These fluctuations in principle may come from both initial conditions for the nuclear matter evolution and dynamics of the successive evolution itself. Simple phenomenological initial conditions based on fluctuations in the distribution of participant nucleons followed by the hydrodynamical evolution allow to describe the third harmonic coefficient $v_3$ quite successfully \cite{8, 9, 10, 11}. Remarkably less phenomenological initial conditions which follow from the preasymptotic evolution of emitted gluons in the Colour Glass Condensate approach seriously underestimate $v_3$ \cite{8, 11} unless this evolution is treated in a rigorous non-perturbative manner and the subsequent evolution has a non-vanishing viscosity \cite{12}.

In \cite{13} we drew attention to the fact that the elliptic flow could also be naturally explained in the colour string approach, with string fusion and percolation \cite{14, 15, 16, 17}, quite successful in the description of particle production and correlations in the soft part of spectra \cite{18, 19, 20}. In this approach partons emitted at some point have to pass a certain length before they appear outside and are observed. On their way they cross a strong colour field inside the strings and emit gluons, so that their energy diminishes. As a result, the observed particle energy turns out to be smaller than at the moment of its creation and this energy quenching depends on the path length passed inside the nuclear matter and so on the direction of its transverse momentum $p$. Very crude
estimates made in [13] confirmed that a sizable elliptic flow followed from this mechanism. Its centrality and transverse momentum dependence qualitatively agreed with the behaviour of the RHIC data [21, 22, 23, 24]. These results also supported the ones obtained in a similar framework using different simplified methods [25, 26, 27].

It was stressed in [13] that any trustworthy and comprehensive quantitative results could only be achieved in more elaborate studies based on detailed Monte-Carlo simulations. This is especially true for odd flow harmonics which, as mentioned, can only be studied on the event-by-event basis, with all sources of fluctuations taken into account. Calculations along these lines constitute the subject of this paper. On the general they confirm crude predictions made in [13] and also allow for a comprehensive study of azimuthal anisotropy in particle production and thus determination of higher anisotropic coefficients \( v_n \) with \( n > 2 \). Note that the colour string approach combines both the initial conditions created in the course of collisions and the subsequent evolution of the nuclear matter in the collision zone modeled by the fusion of produced strings and their final decay into observed hadrons. Fluctuations may be present at all stages of this process. Our calculations show that the decisive role for the successful description of both \( v_2 \) and \( v_3 \) is played by the fluctuations in the initial geometry of the collision plus those in the string fusion. Fluctuations in the final production of observed particles play a minor role. Our model has a single new parameter, the quenching coefficient, which characterizes energy loss of the produced parton in the string matter. We adjust this parameter to agree with coefficient \( v_2 \) observed in mid-central Au+Au collisions at 200 GeV and integrated over the transverse momenta. With thus adjusted parameter we are able to explain \( v_2 \) at all centralities and transverse momenta for collisions at 62.4 GeV, 200 GeV and 2.76 TeV. The third harmonic \( v_3 \) comes out a little smaller than the average observed values. However the event-by-event fluctuations in \( v_3 \) (as well as in \( v_2 \)) are calculated to be quite large so that the observed values are found to be within the calculated ones when these fluctuations are taken into account.

The structure of the paper is as follows. In the next section we briefly discuss our model and also present our quenching mechanism borrowed from QED. In Section 3 we describe our Monte-Carlo procedure to study the anisotropic flow on the event-by-event basis. Section 4. presents our numerical results. In Conclusion we draw some lessons from our study and point out some possible further refinements.

## 2 The model

One needs to combine two ingredients to have anisotropic flows. First, string fusion has to generate clusters which are azimuthally asymmetric and emit particles anisotropically. But by itself this will not produce any elliptic flow unless the distribution of these clusters in the transverse plane is also azimuthally asymmetric. This latter phenomenon can only occur if the clusters are large enough to feel the asymmetric form of the overlapping region in the collision. Such clusters arise in the process of percolation of fused strings.

In our model it is assumed that at the moment of the collision color strings are stretched between partons of the colliding nuclei. Since they are many and so overlap in the transverse space, they fuse and percolate to form macroscopic clusters at some critical string density \( \rho = \rho_c \) where

\[
\rho = \frac{N \Omega_0}{\Omega},
\]

(1)

\( \Omega_0 \) is the transverse area of simple strings, \( N \) is their number and \( \Omega \) is the nuclear overlap area. Starting from the moment of their formation strings decay into particles (quark-antiquark pairs),
which process we describe using the well-known formalism for pair creation in a strong electromagnetic field. According to this mechanism, in its simplest version, the particle distribution at the moment of its production by the string is

\[ P(p, \phi) = Ce^{-\frac{p_0^2}{2T}}. \]  

(2)

where \( p_0 \) is the particle initial transverse momentum, \( T \) is the string tension (up to an irrelevant numerical coefficient) and \( C \) is the normalization factor. However \( p_0 \) is different from the observed particle momentum \( p \) because the particle has to pass through the fused string area and emit gluons on its way out. So in fact in Eq. (2) one has to consider \( p_0 \) as a function of \( p \) and path length \( l \) inside the nuclear overlap: \( p_0 = f(p, l(\phi)) \) where \( \phi \) is the azimuthal angle. Note that Eq. (2) describes the spectra only at very soft \( p_0 \). To extend its validity to higher momenta one may use the idea that the string tension fluctuates, which transforms the Gaussian distribution into the thermal one [29, 30]:

\[ P(p, \phi) = Ce^{-\frac{p_0}{\sqrt{T}}} \].

(3)

Radiative energy loss has been extensively studied for a parton passing through the nucleus or quark-gluon plasma as a result of multiple collisions with the medium scattering centers [31]. In our case the situation is somewhat different: the created parton moves in the external gluon field inside the string. In the crude approximation this field can be taken as being constant and orthogonal to the direction of the parton propagation. In the same spirit as taken for the mechanism of pair creation, one may assume that the reaction force due to radiation is similar to the one in the QED when a charged particle is moving in the external electromagnetic field. This force causes a loss of energy which for an ultra-relativistic particle is proportional to \([\text{its momentum} \times \text{field}]^{2/3}\) [32]:

\[ \frac{dp(x)}{dx} = -0.12e^2 \left( eEp(x) \right)^{2/3}, \]  

(4)

where \( E \) is the external electric field. Eq. (4) leads to the quenching formula

\[ p_0(p, l) = p \left( 1 + \kappa p^{-1/3}T^{2/3}l \right)^3, \]  

(5)

where we identified \( eE/\pi = T \) as the string tension. As mentioned in the Introduction, the quenching coefficient \( \kappa \) is adjusted to give the experimental value for the coefficient \( v_2 \) in mid-central Au+Au collisions at 200 GeV, integrated over the transverse momenta.

Of course the possibility to use electrodynamic formulas for the chromodynamic case may raise certain doubts. However in [33] it was found that at least in the \( N = 4 \) SUSY Yang-Mills case the loss of energy of a coloured charge moving in the external chromodynamic field was given by essentially the same expression as in the QED.

Note that from the moment of particle creation to the moment of its passage through other strings a certain time elapses depending on the distance and particle velocity. During this time strings decay and the traveling particle will meet another string partially decayed, with a smaller colour \( Q \) than at the moment of its formation. So one has to consider a non-static string distribution with string colours evolving in time and gradually diminishing until strings disappear altogether. The time scale of this evolution is estimated to be considerably greater than time intervals characteristic for partons traveling inside the string matter. However the effect of string decay with time is noticeable and we take it into account in our calculations.
To study the time evolution of strings we again turn to the Schwinger mechanism. For it one has the probability of pair creation in unit time and unit volume as  

\[
\Gamma_{Vt} = \frac{1}{4\pi} T^2 e^{-\frac{p_0^2}{T}},
\]  

(6)

where again \(T\) stands for \(eE/\pi\) in QED. For a realistic string the volume \(V = SL_z\) where \(S\) is the string transverse area and \(L_z\) is the longitudinal dimension of the string. For the string of colour \(Q\) is \(T = QT_0\) where \(T_0\) is the string tension of the ordinary string with \(Q = 1\). The average transverse momentum squared of the emitted quark-antiquark pair \(\langle p_0^2 \rangle\) is just \(T\). To estimate \(L_z\) we assume that the string emits a pair when its energy is equal to 2 \(\langle p_0 \rangle = \sqrt{T}\), which gives 

\[L_z = \frac{1}{\sqrt{T}},\]

so that we get the average probability in unit time

\[
\Gamma_t = \frac{1}{4\pi} T^{3/2} S.
\]  

(7)

The string colour diminishes by unity with each pair production. So we find an equation which describes the time evolution of the string colour \(Q(t)\)

\[
\frac{dQ(t)}{dt} = -\alpha Q^{3/2}(t)
\]  

(8)

with the solution

\[
Q(t) = \frac{Q_0}{(1 + \frac{3}{2}\alpha t \sqrt{Q_0})^2}
\]  

(9)

where \(Q_0\) is the initial colour at the moment of the string creation. Coefficient \(\alpha = T_0^{3/2} S/(2\pi)\) depends on the string transverse area \(S\). As will be explained in next section, we use the picture in which the fused string is in fact modeled by a set of "ministrings" formed at intersections of simple strings with the same area as the simple string, but greater color. This gives \(\alpha = 0.03\) 1/fm. This value has been used in our calculations. The average color of ministrings is of the order 2 - 3. So it changes only by 30 -50 % even when the emitted parton travels 5 fm of distance. Still this effect is felt in the calculations. In fact it practically does not change the results but changes the value of the quenching coefficient \(\kappa\), which in any case is to be adjusted, as explained above. In this sense our results are practically independent of the concrete choice of \(\alpha\) in the reasonable interval of values.

3 Monte-Carlo simulations

3.1 Generalities

In principle Monte-Carlo simulations of the quenching in the fusing string scenario seem to be straightforward. One models strings in the nuclear overlap by discs of a given radius \(r_0\). In an event \(N\) discs are assumed to be distributed in the transverse plane in agreement with the distribution of the participant ("wounded") nucleons, given by product of the nuclear profile functions. Inside the transverse area of each pair of colliding nucleons strings may be taken to be distributed according to the nucleon density of the Gaussian form. The number of discs is to be chosen in agreement with the observed value of the percolation parameter \(\rho\) given by Eq. (11) Values of the percolation parameter \(\rho\) can be taken from [34] for Au-Au collisions at 62.4 and 200 GeV. They were extracted from the observed distributions in the transverse momentum at different centralities and so depend
on the impact parameter $b$. They are shown in Fig. 1. For the LHC energy $E = 2.76 TeV$ we used the conclusions in [35, 36] that the values of $\rho$ are roughly 4 times larger than at RHIC energies.

The number of participant nucleons for Au-Au collisions at 200 GeV and given centralities, or equivalently, $b$ can be borrowed from [11]. They are reproduced in Table 1.

Taking the radius of both colliding nuclei $R = A^{1/3} R_0$ with $R_0 = 1.2$ fm and $r_0 = 0.3$ fm, for Au – Au collisions at 200 GeV one gets from Eq. (1) and Fig. 1 the number of strings given at a given $b$. Comparison with Table 1. then shows that for Au-Au collisions at 200 GeV each pair of participant nucleons gives rise to approximately 7 colour strings. Assuming that the number of participant nucleons is purely geometrical and does not change significantly with energy we conclude that for Au+Au collisions at 62.4 GeV and 2.76 Tev the numbers of strings per each pair of participant nucleons are 4 and 28 respectively.

The strings modeled by discs may overlap in the transverse area and form clusters of different number $n$ of fused strings and form. Observed particles are emitted from each cluster with the average multiplicity

$$\mu_{nk} = \sqrt{\frac{n \Omega_{nk}}{\Omega_0}} \mu_0$$

and average transverse momentum squared

$$p_{nk}^2 = \sqrt{\frac{n \Omega_0}{\Omega_{nk}} p_0^2}$$

for the $k$-th cluster of $n$ fused strings. Here $\Omega_{nk}$ is the transverse area of the cluster and $\mu_0$ and $p_0^2$ are the multiplicity and transverse momentum squared for a simple string. In our picture each particle emitted from a given point in a cluster has to pass a certain path in the overlap area before being observed. A part of it has to pass through the same or different clusters and so looses...
its energy as described in the previous sections. During the time of its passage strings partially decay and lose their colour. This effect is taken into account according to Eq. (9). The average length $l_{nk}$ traveled by the particle emitted from the $k$-th cluster of $n$ strings depends both on the distribution of clusters and on the direction of the emission. Due to azimuthal asymmetry of the cluster distribution following from the asymmetry of the overlap area the average distribution of emitted particles will depend on the azimuthal angle and lead to non-vanishing anisotropic flows.

In the simulation of an event one has, first, to determine the distribution of clusters in the overlap area and, second, for each cluster to find the average length $l_{nk}$ of the path which the emitted particle has to travel inside the cluster matter. The final distribution of emitted particles in the transverse momentum and azimuthal angle will be given as a sum

$$P(p, \phi) = C \sum_{n,k} \mu_{nk} e^{-\frac{ra(p,l_{nk})}{\sqrt{\gamma_{nk}/2}}}$$  \hspace{1cm} (12)

and the distribution in the azimuthal angle only

$$P(\phi) = C \int dp^2 P(p, \phi).$$  \hspace{1cm} (13)

(We use the thermal distribution (3) to be able to move into the region of $p$ of the order of several GeV/c)

With the number of strings $N$ not very large (below 100) the described procedure is realizable on a computer for a reasonable processing time. However this time grows very fast with the number of strings (as approximately $\sim N^3$). This motivated a somewhat simplified approach to our simulations. Instead of taking clusters at different locations in the transverse space and of different geometric forms, we assumed them to form a square lattice in the transverse space with the side length $a = \sqrt{\pi r_0^2}$. Throwing of discs transforms these primitive structures into clusters of strings (”ministrings”) with a variable number of fused simple strings equal to the number of disc centers found in a given lattice cell and thus with a variable cell-dependent percolation parameter $\rho$. In this manner we avoid the study of all complicated geometric structures which arise when
a large number of simple strings overlap. Calculations of the momentum distribution of emitted particles in such a picture shows that it models the actual cluster formation quite successfully [37].

Note that in Eq. (12) it is assumed that multiplicity $\mu_{nk}$ of the cluster is fixed by its area and colour according to Eq. (10). This multiplicity is the average over events. On the event-by-event basis one may take the multiplicity distributed according to Poisson’s law around this average, as advocated in [38], where it was shown that such distribution leads to the negative binomial distribution of observed particles. We shall see that such stochastic emission does not change significantly the final results for the flow coefficients although naturally somewhat complicates numerical calculations.

Note that the fluctuations in the distribution of strings and multiplicities are not the only ones in the dynamics of particle production. Also the numbers of participant nucleons $N_{part}$ and strings inside each of them $N_{str}$ fluctuate as well as the values of the impact parameter $b$. However these additional fluctuations seem to have little to do with the azimuthal anisotropy, as supported by our results, which show that fluctuations in the multiplicity do not influence the final anisotropy. Therefore in our calculations we have not taken into account fluctuations of $N_{part}$, $N_{str}$ nor $b$, although they can be essential in other context [39].

In our model, with the coefficient $\alpha$ for the string color decay fixed at the value 0.03, as explained in the previous section, only a single dimensionless parameter $\kappa$ remains, which characterizes the loss of energy in passing through the string field and is to be extracted from the experimental data. It was adjusted to fit the experimental value of $v_2$ integrated over the transverse momenta for Au-Au collisions at 200 GeV in mid-central events.

On the event-by-event basis the final flow coefficients $v_n$ are found from the distribution of emitted particles $P(p, \phi)$ at a given $p$ or $P(\phi)$, integrated over all $p$ by the standard formulas. Let in a given event

$$a_n = \int d\phi \cos(n \phi) P(\phi), \quad b_n = \int d\phi \sin(n \phi) P(\phi)$$

then

$$v_n = \frac{\sqrt{a_n^2 + b_n^2}}{a_0}, n = 1, 2, ....$$

## 4 Numerical results

Our numerical calculations were performed for Au-Au collisions with the profile function taken according to the Saxon-Woods formula and cluster multiplicities distributed according to the Poisson law. The ordinary string tension parameter $T_0$ was taken to agree with the slope of the spectra in the soft region: $T_0 = 0.08 \text{ (GeV/c)}^2$ The adjusted value of the quenching coefficient turned out to be $\kappa = 0.058$. The number of simulation was taken 100. Our Monte-Carlo results obtained for $v_2$ and $v_3$ integrated over the transverse momenta and averaged over events are shown as a function of $b$ in Figs. 2-4 for Au-Au collisions at energies 62.4, 200 and 2760 GeV respectively. In these and the following figures error bars show fluctuations of the flow coefficients around their average values from event to event, not the insufficient precision of the calculations due to the finite number of simulations. One observes that these event-by-event fluctuations are quite large.

To show higher harmonics we present $v_n$ for $n = 1, ..8$ integrated over $p_T$ at centrality 40–50 % in Figs. 5-7 again with their event-by-event fluctuations from the average.

Finally in Figs. 8-11 we present our results for the $p_T$ dependence of $v_2$ and $v_3$ at two centralities 10–15 % and 40–50 % for Au+Au collisions at energies 200 GeV and 2.76 TeV.
Figure 2: $v_2$ and $v_3$ integrated over $p_T$ as a function of $b$ for Au-Au collisions at $E = 62.4$ GeV.

Figure 3: Same as for Fig. 2 at $E = 200$ GeV.
Figure 4: Same as for Fig. 2 at $E = 2.76$ TeV.

Figure 5: $v_n$, $n = 1,...,8$ integrated over $p_T$ at centrality 40–50 % for Au+Au collisions at 62.4 GeV
Figure 6: Same as for Fig. 5 at $E = 200$ GeV

Figure 7: Same as for Fig. 5 at $E = 2.76$ TeV
Figure 8: $v_2$ and $v_3$ as a function of $p_T$ for Au+Au collisions at 200 GeV and centrality 10–15 %

Figure 9: $v_2$ and $v_3$ as a function of $p_T$ for Au+Au collisions at 200 GeV and centrality 40–50 %

Figure 10: $v_2$ and $v_3$ as a function of $p_T$ for Au+Au collisions at 2.76 TeV and centrality 10–15 %

Figure 11: $v_2$ and $v_3$ as a function of $p_T$ for Au+Au collisions at 2.76 TeV and centrality 40–50 %
Inspecting these results we see that the $b$- and $p_T$ dependence of $v_2$ as well as their absolute values well agree with the experimental data (which of course is based on the proper adjustment of coefficient $\kappa_0$ at $E = 200$ GeV and medium centrality). The behaviour of the triangular coefficient $v_3$ is also found to agree with the experimental observations. Its average values are calculated to be somewhat smaller than the experimental values. However their event-by-event fluctuations from the average turn out to be very large, so that taking them into account may put the experimental values well within the calculated ones.

To analyze the important components of fluctuations leading to relatively large values of $v_3$ we repeated our calculations substituting Poisson distributed cluster multiplicities by their average. For Au+Au collisions at 200 GeV and centrality 40−50 % this gives values for $v_2$ and $v_3$ integrated over $p_T$ $0.594E-01 \pm 0.145E-01$ and $0.574E-02 \pm 0.403E-02$ respectively to be compared with $0.588E-01 \pm 0.186E-01$ and $0.668E-02 \pm 0.399E-02$ with Poisson distributed multiplicities. As one observes the difference is insignificant. On the other hand the importance of quenching is found to be overwhelming. Taking $\kappa_0 = 0$ but keeping the Poisson distributed multiplicities one obtains values for $v_2$ and $v_3$ $0.180E-02 \pm 0.839E-03$ and $0.112E-02 \pm 0.622E-03$, that is practically zero.

5 Conclusions

We have performed detailed Monte-Carlo simulations to study anisotropic flows in the percolating string scenario. We have confirmed that the colour string model with fusion and percolation can successfully describe the observed elliptic and triangular flows in high-energy heavy-ion collisions. An important ingredient in this description is anisotropy of the string emission spectra in the azimuthal direction which follows from quenching of the emitted partons in the strong colour field inside the string. As a mechanism for this quenching we used the radiation energy loss during propagation of a fast charged particle in a constant field, borrowed from the QED. Upon adjusting the parameter of quenching, this allowed to describe the data quite well both in their energy centrality and transverse momentum dependence. We have also studied higher anisotropic flow coefficients from $v_4$ to $v_8$ which are found to be small as compared to $v_2$.

To compare with our earlier calculations in the grossly oversimplified picture in [13] we have found more pronounced energy dependence. This follows from the explicit dependence of the quenching in the QED on the field strength, which is translated into dependence on the string tension in the colour string picture. So, in contrast to our phenomenological formula adopted in [13], this quenching is not purely geometrical but grows with the percolation parameter even when all strings are fused into a single cluster occupying the whole overlap area.

Our results are based on strings which have infinite dimensions in rapidity. So from the start they refer to the central region of nearly zero rapidity and thus do not allow to study the rapidity dependence of the flow coefficients. To do this we have to introduce strings of finite rapidity length and in this way take into account energy conservation. This complicates our picture substantially and will be the object of our further studies.

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References

[1] P.F.Kolb and U.Heinz, in "Quark-Gluon Plasma 3", Eds. R.C.Hwa and X.N.Wang, World Scientific, Singapore, 2004.

[2] N.Borghini and U.A.Wiedemann, J.Phys., G 35 (2008) 023001.

[3] D.A.Teaney, [nucl-th/0905.2433].

[4] J.Y.Ollitrault, Phys. Rev., D 46 (1992) 229.

[5] S.Afanasiev et al, PHENIX collab., Phys.Rev. C 80 (2009) 024909 [nucl-ex/0905.1070].

[6] K.Aamodt et al, ALICE collab., Phys.Rev.Lett. 107 (2011) 032301 [nucl-ex/1105.3865].

[7] A.Adare et al, PHENIX collab.,[nucl-ex/1105.3928].

[8] T.Hirano, U.Heinz, D.Kharzeev, B. Lacey and Y.Nara, Phys.Rev., C 83 Phys.Lett B 636 (2006) 299 [nucl-th/0511046].

[9] H.Holopainen, H.Niemi and K.J.Eskola, Phys.Rev. C 83 (2011) 034901 [hep-ph/1007.0368].

[10] H.Petersen,G-Y.Quin, S.A.Bass and B.Mueller, Phys.Rev, C 82 (2010) 041901,064903 [nucl-th/1008.0625;1009.1847].

[11] Zhi Qiu and U.Heinz, Phys.Rev.C 84 (2011) 024911 [nucl-th/1104.0650].

[12] B.Schenke, P.Tribody and R.Venugopalan,[nucl-th/1202.6646].

[13] M.A.Braun and C.Pajares, Eur. Phys. J., C 71 (2011) 1558.

[14] N.Armesto, M.A.Braun, E.G.Ferreiro and C.Pajares, Phys. Rev. Lett. 77 (1996) 3736; M.Nardi and H.Satz. Phys. Lett. B 442 (1998) 14.

[15] M.A.Braun and C.Pajares, Phys. Rev. Lett., 85 (2000) 4864.

[16] M.A.Braun and C.Pajares, Eur. Phys. J., C 16 (2000) 349.

[17] M.A.Braun, C.Pajares and J.Ranft, Int. J. Mod. Phys., A 14 (1999) 2689.

[18] M.A.Braun, F.del Moral and C.Pajares, Phys. Rev., C 65 (2002) 024907.

[19] J.Dias de Deus, E.G.Ferreiro, C.Pajares and R.Ugoccioni, Eur. Phys. J., C 40 (2005) 229.

[20] L.Cunqueiro, J.Dias de Deus, E.G.Ferreiro and C.Pajares, Eur. Phys. J., C 53 (2008) 585.

[21] S.S.Adler et al, PHENIX collab., Phys. Rev. Lett., 91 (2003) 182301; Phys. Rev. C 77 (2008) 014906.

[22] A.Adare et al, PHENIX collab., Phys. Rev. Lett., 98 (2007) 242302.
[23] B.Alver et al, PHOBOS collab., Phys. Rev. Lett., 98 (2007) 162301.
[24] S.A.Voloshin (for the STAR collab.), J.Phys., G 34 (2007) S883.
[25] I.Bautista, L.Cunqueiro, J.Dias de Deus and C.Pajares, J.Phys., G 37 (2010) 015103.
[26] I.Bautista, J.Dias de Deus and C.Pajares, Phys. Lett., B 693 (2010) 362.
[27] I.Bautista, J.Dias de Deus and C.Pajares, AIP Conf.Proc. 1343 (2011) 495 [hep-ph/1011.1870].
[28] A.I.Nikishov, Nucl. Phys. B 21 (1970) 346.
[29] A.Bialas, Phys. Lett. B 466 (1999) 301.
[30] J.Dias de Deus and C.Pajares, Phys. Lett., B 642 (2006) 455.
[31] R.Baier, Y.L.Dokshitzer, A.H.Mueller, S.Peigne and D.Schiff, Nucl. Phys. B 483 (1997) 291; B 484 (1997) 265.
[32] A.I.Nikishov, V.I.Ritus, Sov. Phys. Uspekhi, 13 (1970) 303.
[33] A.Mikhailov, [hep-th/0305196].
[34] T.J.Tarnowsky, B.Srivastava, R.Scharenberg (for the STAR collab), Nukleonika 51S3 (2006) S109-S112 [nucl-ex/0606019].
[35] K.Aamodt et al, ALICE collab, Phys. Rev. Lett., 107 (2011) 032301.
[36] J.Dias de Deus, A.S.Hirsch, C.Pajares, R.P.Scharenberg and B.K.Srivastava, [hep-ph/1106.4271]
[37] M.A.Braun, R.S.Kolevatov, C.Pajares and V.V.Vechernin, Eur. Phys. J. C 32 (2004) 535.
[38] M.A.Braun, C.Pajares and V.V.Vechernin, Phys. Lett., B 493 (2000) 54.
[39] V.V.Vechernin abd R.S.Kolevatov. Phys. Atom. Nucl. 70 (2007) 1797.
[40] K.Aamodt at al., Phys.Rev.Lett., 105 (2010) 252302.