Current-induced transverse spin wave instability in a thin nanomagnet

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We show that an unpolarized electric current incident perpendicular to the plane of a thin ferromagnet can excite a spin-wave instability transverse to the current direction if source and drain contacts are not symmetric. The instability, which is driven by the current-induced “spin-transfer torque”, exists for one current direction only.

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Ferromagnets serve as spin filters for an electrical current passing through the magnet: the spin of the electrons that are transmitted through a ferromagnet becomes partially polarized parallel or antiparallel to the direction of the magnetization whereas spin current perpendicular to the magnetization direction is absorbed. Spin filtering is the root cause for the “spin-transfer torque”, the phenomenon that a polarized current impinging on a ferromagnet affects its magnetization direction. The source of the spin polarized current can either be a different ferromagnet, or, for a thick magnet, a region of the same ferromagnet upstream or downstream in the current flow. The “spin-transfer torque” gives rise to magnetization reversal in ferromagnet–normal-metal–ferromagnet trilayers, which has been observed experimentally by several groups. Dynamic manifestations of the spin-transfer torque include domain wall motion in bulk ferromagnets and the excitation of spin waves by polarized currents in ferromagnetic multilayers or wires. In all these manifestations, the current-induced spin torque can be distinguished from effects arising from the current induced magnetic field, the main difference being that spin-transfer torque effects depend on the current direction, whereas magnetic field induced effects do not.

In this letter, we show that an unpolarized current can also exert a spin-transfer torque on a ferromagnet, even if the magnet is so thin that its magnetization direction does not change along the current flow: Although an unpolarized current cannot exert a spin-transfer torque that changes the over-all magnetization direction, it can create a transverse spin wave instability for sufficiently high current densities if the source and drain contacts to the ferromagnet are not symmetric. This spin wave instability can be identified unambiguously as a spin-torque effect because of its dependence on current direction: the spin-wave instability is present for one current direction and absent for the other. The spin-wave instability should lead to a non-hysteretic feature in the current-voltage characteristic of the ferromagnetic film that exists for one current direction only. In thick ferromagnets, such features have been observed in recent experiments. The necessary criterion for the spin-wave instability, asymmetric contacts to source and drain, is generically fulfilled in experiments on nanoscale magnets. The issue of current-induced spin-wave excitation has significant practical relevance for devices based on the spin-torque effect in ferromagnetic multilayers. Whereas, experimentally, the presence of dynamical phenomena in these systems is well established, the precise nature of the excitations is not well known. A theoretical understanding of current-induced spin wave excitation in multilayer structures will shed light on the possibly useful application of the current-induced dynamical excitations, e.g., as GHz resonators, as well as on ways to avoid spin-wave excitation in devices that are designed to exhibit magnetization reversal only. For ferromagnet–normal-metal–ferromagnet trilayers, a comparison between theory and experiment is obstructed by the interplay of spin wave excitations and static magnetization reversal. Our finding that dynamical phenomena exist already in a single ferromagnetic layer allows for a study of spin-wave excitations in a much simpler geometry in which dynamical and static phenomena are well separated.

The geometry of the system under consideration is shown in Fig. 1: a nanomagnet of thickness \( d \), small
enough to be considered as single domain, is connected to source and drain reservoirs via diffusive normal-metal leads of lengths $L_-$ and $L_+$, respectively. The leads and the ferromagnet have width $W \gg d$. In the absence of an electrical current through the system, the ferromagnet has a uniform magnetization, with a direction determined by anisotropies and an external magnetic field. A current $j$ flows perpendicular to the ferromagnet. (Note that the electrical current $j$ points opposite to the electron flow.)

For a qualitative explanation of the mechanism of the spin-wave instability, we note that the passage of an electric current through a single ferromagnetic layer creates spin accumulations of opposite signs on both sides of the ferromagnet, see Fig. [1] where the sign of the spin accumulation depends on the current direction and on the spin-filtering properties of the ferromagnet. To first order in the spin-wave amplitude, the spin accumulation in the normal metal leads is not affected by the possible presence of spin waves in the ferromagnet, as long as their wavelengths are much smaller than the spin diffusion length in the normal metal. Each spin accumulation exerts a spin-transfer torque on the magnetization; the direction of the torque is to align the magnetization in the ferromagnet with the direction of the accumulated spins in the leads. Depending on the sign of the spin accumulation, such a torque either damps or enhances non-uniform spin wave excitations in the ferromagnet. The magnitudes of the spin accumulations on each side of the ferromagnet are, generically, different, since they depend on the spin diffusion length, scattering properties of normal-metal–ferromagnet interface, distance to reservoirs, etc. Therefore, an unpolarized current results, generically, in a net torque that either suppresses or enhances non-uniform spin waves in the ferromagnet, depending on current direction. The instability occurs when the current-induced enhancement of a spin wave amplitude overcomes the intrinsic spin-wave damping.

For a quantitative description of the spin-wave instability, we choose the $x$ axis along the leads, with the origin such that the normal-metal–ferromagnet interfaces are at $x = \pm d/2$, see Fig. [1]. We assume that the ferromagnet is so thin that its magnetization direction varies with respect to the transverse coordinates $y$ and $z$ only, but not with respect to $x$. In the diffusive normal-metal leads, the electron density is described by potentials $\mu_\alpha(x, t)$ for the electron density and $\mu_e(x, t)$ for the electron spin [18, 19, 20]. The current is separated into the charge current density $j_\alpha$, $\alpha = x, y, z$, and the spin current density $j_{s\alpha}$. Since electrons adjust to the changing magnetization on a time scale much faster than that of the magnetization dynamics, current and potential are related by means of the time-independent diffusion equation in the normal metal leads,

$$\nabla^2 \mu_\alpha = 0, \quad j_\alpha = (\sigma/e)\nabla_{\alpha} \mu_\alpha, \quad \alpha = x, y, z,$$

$$\nabla^2 \mu_s = \mu_s, \quad j_{s\alpha} = -(\hbar \sigma/2e^2)\nabla_{\alpha} \mu_s, \quad (1)$$

supplemented by boundary conditions for the source and drain reservoirs $\mu_e(-L_\pm) = -eV$, $\mu_e(L_\pm) = 0$, $\mu_s(\pm L_\pm) = 0$. Here $\sigma$ is the conductivity of the normal metal leads and $L_{sf}$ the spin-diffusion length. At the normal-metal–ferromagnet interfaces, the charge and spin current $j_x$ and $j_{sx}$ in the normal metal perpendicular to the interface are related to the potential drop $\Delta \mu$ over the interface as [14, 21]

$$j_x = \frac{(h/2e^2)}{\mu} \Re g_{\uparrow \downarrow} (2\Delta \mu_s \times m \pm \hbar \partial t m) \times m + (h/2e^2) \Im g_{\uparrow \downarrow} (2\Delta \mu_s \times m \pm \hbar \partial t m),$$

$$j_{sx} = -\frac{(g_{\uparrow \uparrow} + g_{\downarrow \downarrow})(h/2e^2)m \cdot \Delta \mu_s}{2} - \frac{(g_{\downarrow \uparrow} - g_{\uparrow \downarrow})(h/2e^2)\Delta \mu_s}{2},$$

$$j_{sx} = \frac{(g_{\uparrow \uparrow} + g_{\downarrow \downarrow})(\Delta \mu_s/e) + (g_{\downarrow \uparrow} - g_{\uparrow \downarrow})m \cdot (\Delta \mu_s/e).$$

Here $m(r, t)$ is the unit vector pointing in the direction of the magnetization of the ferromagnet, $(j_{sx})_\perp$ and $(j_{sx})_\parallel$ are the $x$-components of the spin current perpendicular and parallel to $m$, respectively, $g_{\uparrow \uparrow}$ and $g_{\downarrow \downarrow}$ are interface conductivities for spins aligned parallel and antiparallel to $m$, whereas $g_{\uparrow \downarrow}$ is the “mixing conductivity” [19, 21]. $\Delta \mu = \mu(\pm d/2 + 0) - \mu(\pm d/2 - 0)$ is the potential drop over the interface. (We assume identical conductivities and spin-flip lengths in the leads, and identical scattering properties of the ferromagnet–normal-metal interfaces, although our results are readily generalized to unequal values of $\sigma$, $L_{sf}$ and $g_{\alpha \beta}$.) At the normal-metal–ferromagnet interface, $(j_{sx})_\parallel$ and $j_x$ are continuous, whereas $(j_{sx})_\perp = 0$ in the ferromagnet. We assume that the ferromagnet is so thin that all potential drops occur at the interfaces, so that we can neglect the $x$-dependence of the potentials in the ferromagnet. Then Eqs. [1] and [2] fully determine the potentials and currents in the normal metal and the ferromagnet, as a function of $m$.

For a thin ferromagnet, variations of the magnetization direction $m$ along the $x$ axis will have a large energy cost and, hence, a large threshold for excitation. Therefore, we consider transverse modulation of $m$ only and take $m$ independent of $x$. The dynamics of $m(y, z, t)$ is determined by the Landau-Lifshitz-Gilbert equation (without an applied magnetic field [22])

$$\partial_t m = \alpha m \times \partial_t m + J_\gamma M \nabla^2 m \times m - (\gamma/M)(K_1 m_1 \hat{e}_1 + K_2 m_2 \hat{e}_2) \times m$$

$$+ (\gamma/Md)[j_{sx}(-d/2 - 0) - j_{sx}(d/2 + 0)],$$

where $\alpha$ and $J$ are the bulk Gilbert damping coefficient and spin stiffness (exchange constant) respectively, $\gamma = \mu_B g/h$ is the gyromagnetic ratio, $M$ is the magnetization per unit volume, and $K_1$ and $K_2$ are anisotropy constants along principal directions $\hat{e}_1$ and $\hat{e}_2$, respectively, obtained by expanding the magnet’s free energy around a preferred axis $\hat{e}_3$. The magnetization satisfies the boundary condition $(\hat{n} \cdot \nabla)m = 0$, where $\hat{n}$ is the normal to the ferromagnet’s surface. We neglect the effect of
the current-induced magnetic field on the magnetization dynamics, which is allowed if the width \( W \) is sufficiently small, \( \ll 1 \mu \text{m} \) for typical experimental parameters. The parameters in the Landau-Lifschitz-Gilbert equation define a length scale \( 1/q_1 \) and current scale \( j_1 \),

\[
q_1^2 = \frac{K_1 + K_2}{2JM^2}, \quad j_1^2 = \left( \frac{2e}{\hbar} \right)^2 J M^2 \frac{K_1 + K_2}{2}.
\]

The quantities \( 1/q_1 \) and \( h j_1/e \) are proportional to the width and energy of a domain wall, respectively. Order-of-magnitude estimates of the various parameters involved here are \( d \sim 10\,\text{nm}, \, W \sim 10^2\,\text{nm}, \, l_{sd}(\text{Cu}) \sim 10^2\,\text{nm} \) [24, 25]; \( \text{Im}\, g_{\uparrow \uparrow} \ll \text{Re}\, g_{\uparrow \downarrow} \simeq g_{\downarrow \downarrow} \sim 10^{14}\,\Omega^{-1}\,\text{m}^{-2} \) [24, 25] for Co/Cu and Fe/Cr interfaces; \( \sigma/l_{sd}(\text{Cu}) \sim 10^{15}\,\Omega^{-1}\,\text{m}^{-2} \), and typically \( q_1 \sim 10^{-1}\,\text{nm}^{-1}, \, j \sim 10^8\,\text{A/cm}^2 \) for Co, Fe or Ni [26].

We first solve these equations for a uniform magnetization, \( \textbf{m} \) independent of the transverse coordinates \( y \) and \( z \). The spin accumulation \( \mu_s \) in the normal metals leads close to the normal-metal–ferromagnet interface is

\[
\mu_s(\pm d/2) = \mp (e j_x/g_m) \tanh(L_{+}/l_{sd}) \textbf{m},
\]

where \( j_x \) is the charge current density and

\[
g_m = \frac{\sigma l_{sd}}{(e j_x/g_m)} (g_{\uparrow \uparrow} + g_{\downarrow \downarrow}) + 2g_{\uparrow \downarrow} g_{\downarrow \uparrow} \sum q \tanh(L_{+}/l_{sd}).
\]

The spin accumulation is shown schematically in Fig. 1.

In the case of uniform magnetization, the spin accumulation \( \mu_s \) is always parallel to \( \textbf{m} \), and no current-induced torque is applied to the magnetization. The situation changes if \( \textbf{m} \) varies in the transverse direction. In this case, transverse diffusion of spin in the normal-metal leads gives rise to an angle between \( \mu_s \) and \( \textbf{m} \), and, hence, to a current-induced torque. In order to study this scenario in detail, we analyze Eqs. (4)–(7) for a small deviation of \( \textbf{m} \) from the equilibrium direction \( \hat{e}_3 \). We assume a rectangular cross section of dimensions \( W_y \) and \( W_z \) in the \( y \) and \( z \) directions and perform a Fourier transform with respect to the transverse coordinates \( y \) and \( z \). The allowed wavevectors are \( q_y = \pi n_y / W_y, \, q_z = \pi n_z / W_z \), where \( n_y \) and \( n_z \) are non-negative integers. Representing \( \delta \textbf{m} \) through \( m_{\pm}(\textbf{q}) = 6 m_{1}(\textbf{q}) \pm i 6 m_{-}(\textbf{q}) \), one finds that, to first order in \( m_{n} \), the equations of motion of different Fourier modes separate,

\[
\left( \frac{\alpha}{\gamma} \pm \frac{i}{\gamma} \right) M \partial_t m_{\pm} = \frac{K_2 - K_1}{2} m_{\mp} - \frac{h j_1}{2e q_1} (q^2 + q_0^2) m_{\pm} - \frac{h j_x (S_2 + i S_1)}{2 e d} m_{\pm}.
\]

Here \( \alpha \) and \( \gamma \) are renormalized Gilbert damping parameter and gyromagnetic ratio,

\[
\frac{1}{\gamma} = \frac{1}{\gamma} + \frac{h^2}{2 M d c^2} \text{Im} \sum q \frac{g_{\uparrow \uparrow} G_{\pm}(q) + g_{\downarrow \downarrow}}{G_{\pm}(q) + g_{\uparrow \downarrow}},
\]

\[
\dot{\alpha} = \alpha + \gamma h^2 \frac{2}{2 M d c^2} \text{Re} \sum q \frac{g_{\uparrow \uparrow} G_{\pm}(q) + g_{\downarrow \downarrow}}{G_{\pm}(q) + g_{\uparrow \downarrow}},
\]

whereas the dimensionless quantities \( S_1 \) and \( S_2 \) set the magnitude of the current-induced torque,

\[
S_1 = \frac{\sigma}{g_m l_{sd}} \text{Re} \sum q \frac{g_{\uparrow \uparrow} G_{\pm}(q) - g_{\downarrow \uparrow}}{G_{\pm}(q) + g_{\uparrow \downarrow}} G_{\pm}(q),
\]

\[
S_2 = \frac{\sigma}{g_m l_{sd}} \text{Im} \sum q \frac{g_{\uparrow \uparrow} G_{\pm}(q) - g_{\downarrow \uparrow}}{G_{\pm}(q) + g_{\uparrow \downarrow}} G_{\pm}(q),
\]

where \( g_m \) was defined below Eq. (6) and

\[
G_{\pm}(q) = \frac{\sigma}{2} \sqrt{l_{sf}^2 + q^2} \text{coth}(L_{\pm} \sqrt{l_{sf}^2 + q^2}).
\]

In the limit \( q \to 0 \), Eqs. (6) coincide with the enhanced Gilbert damping and gyromagnetic ratio reported by Tserkovnyak et al. [21].

In the absence of a current, any spatial modulation of the magnetization is damped. It is the existence of the source terms \( S_1 \) and \( S_2 \) in Eq. (6) that leads to a spin-wave instability at sufficiently large current density \( j \). Note that the source terms exist only if the normal leads are asymmetric (different lengths, or different conductivities, spin-flip lengths or interface conductivities), and if the ferromagnet is a spin filter, \( g_{\uparrow \uparrow} \neq g_{\downarrow \downarrow} \). The source term \( S_2 \) gives rise to a small change of the ferromagnetic resonance frequency, whereas \( S_1 \) increases or decreases the amplitude of the spin wave, depending on the current direction. An instability occurs if the current-induced enhancement of the spin-wave amplitude overcomes the damping, i.e., if

\[
\left( \frac{\gamma S_1}{\alpha} - S_2 \right)(-j) > \frac{(q^2 + q_0^2) d}{q_1} j_1.
\]

Using the order-of-magnitude estimates listed below Eq. (6), the instability criterion considerably if we set \( \text{Im}\, g_{\uparrow \downarrow} = 0 \), take the limits \( d \to 0 \) and \( \sigma/l_{sd} \ll \text{Re}\, g_{\uparrow \downarrow} \), and consider the case \( L_{\pm} \gg l_{sd} \gg L_{+} \) of maximally asymmetric contacts,

\[
\frac{-j}{j_1} > \frac{h^2 g_m}{M q_1 e^2} \frac{q^2 + q_0^2}{1 - (1 + q_0^2)^{1/2}}.
\]

The r.h.s. of Eq. (10) is shown schematically in Fig. 2. For large \( q \gg 1/l_{sd} \), the spin-wave instability is dominated by the stiffness of the spin wave, which leads to a critical current density \( \propto q^2 \). For small \( q \ll 1/l_{sd} \), the magnetization accumulation in the normal metal leads tends to remain locally parallel to the magnetization, so
that the current-induced torque is strongly reduced and the critical current density for excitations at small $q$ is increased $\propto q^{-2}$. For a thick ferromagnet, the effect of bulk Gilbert damping becomes dominant, causing the critical current density to increase linearly with the thickness $d$.

In the experimentally relevant parameter regime $1/qt \ll l_{sd} \ll \sigma/Re g t_1$, the critical current density is

$$j_c = \frac{\hbar^2 \gamma g n q t}{M e^2} j_1, \quad q_c = \left(\frac{q_f^2}{2l_{sd}}\right)^{1/3}. \quad (11)$$

For a Cu/Co/Cu geometry, we estimate $\hat{d} \sim (0.1 \text{nm})/d$ and $\hbar^2 \gamma g n q t / M e^2 \sim 0.2$, as long as $dq \lesssim 1$. We then find $j_c \sim 10^7 \text{ A/cm}^2$, which is of the same order of magnitude as the critical current for magnetization reversal in a ferromagnet–normal-metal–ferromagnet trilayer. The estimate is valid as long as the width $W \gg 1/q_e$. For small $W$, the instability occurs at the lowest possible wavenumber, $q_e = \pi / W$.

In conclusion, we have shown that an unpolarized electric current passing through a thin ferromagnetic layer can create a spin wave instability with wavevector transverse to the current. The instability occurs at one current direction only and depends on the asymmetry of normal metal contacts. The mechanism for the instability is the same as the one believed to cause magnetization reversal in ferromagnetic multilayers, i.e., a spin-transfer torque arising from spin accumulation in the normal metal contacts perpendicular to the magnetization direction $\mathbf{m}$ and spin-filtering at the normal-metal–ferromagnet interfaces.

Current-induced excitations that occur for one current direction only have been observed in single ferromagnetic layers. For ferromagnets much thicker than the inverse wavevector $q_c^{-1} \sim 3 \times 10^4 \text{ nm}$ for transverse spin waves, cf. Eq. (11), spin wave excitations with wavevector along the current flow may have a lower threshold current than transverse excitations. Such thick ferromagnetic layers are common in the point-contact geometry. For the thin layers used in the nanopillar geometry, that we believe the transverse instability considered here is most significant.

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FIG. 2: Schematic picture of the current density $j_e$ required to excite a spin-wave at wavevector $q$. The spin-wave instability occurs at the wavevector $q_c$ for which $j$ is minimal.