Research on Non-Similarity about Thermal Deformation Error of Mechanical Parts in High-accuracy Measurement

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Abstract. Expanding with heat and contracting with cold are common physical phenomenon in the nature. The conventional theories and calculations of thermal deformation are approximate and linear, can only be applied in normal or low precision field. The thermal deformation error of mechanical parts doesn’t follow the conventional linear formula, it relates to all physical dimension of the mechanical part, and the deformation can be indicated by a nonlinear formula of physical dimensions. A theory on non-similarity about thermal deformation error of mechanical parts is presented. Studies on some common mechanical parts in precision technology have went on and the mathematical models have been set up, hollow piece, gear and cube are included. The experimental results also make it clear that these models are more logical than traditional models.

1. Introduction
Expanding with heat and contracting with cold are common physical phenomenon in the nature. Variation in temperature leads to corresponding variation in physical dimension of mechanical parts [1]. The conventional theories and calculations of thermal deformation are approximate and linear, can only be applied in normal or low precision field [2]. According to the research results of many years, a theory is given that the thermal deformation error of mechanical parts doesn’t follow the conventional linear formula, it relates to all physical dimension of the mechanical part, and the deformation can be indicated by a nonlinear formula of physical dimensions [3–5]. When the temperature changes, the thermal deformation rule of mechanical part don’t follow the initial detail on microscopic view, for example, planes change into complicate curved face, cubes change into drum cube, and hollow pieces change into complicate body. These phenomenon of not changing according to initial body can be referred to as thermal variation, and this is non-similarity of the mechanical parts. The influence on non-similarity of the mechanical parts can be neglected in the low precision field, but has great influence on the nano-scale precision field and should be valued. Meanwhile, thermal deformation of some isotropic mechanical parts also exist similarity, such as spheres, disks and annuluses. These parts don’t change their microforms after temperature variation.

2. Fundamentals and Main Problems of Non-Similarity Error about Thermal Deformation
According to our research results, three factors must be considered when building ideal thermal deformation error models. Except thermal expansion coefficient, body form parameter and surface
residual stress also influence, the former is determined by structure design, the latter by processing condition. Conventional thermal deformation error calculating formula, is determined by[6]:

$$\Delta L_t = \alpha L_0 (t - t_0) + \beta L_0 (t - t_0)^2 + \gamma L_0 (t - t_0)^3 + \cdots$$

(1)

Where $\alpha$, $\beta$, $\gamma$ are linear, quadratic and cubic thermal expansion coefficient, $t_0$ is the standard temperature, and $t$ is the temperature of the mechanical part, $L_0$ is the initial dimension of the part, $\Delta L_t$ is the thermal deformation of the dimension after temperature variation. Take the first item of equation (1), the linear approximate equation as the common used formula [7].

$$\Delta L_t = \alpha L_0 (t - t_0)$$

(2)

Equation(2) includes nonlinear errors and thermal expansion coefficient error, it is just an approximate formula, can be used in low precision field.

If, $L_0=200\text{mm}$, $\Delta t=\pm 0.5^\circ\text{C}$, the error of this equation result after calculated is 0.5$\mu$m, this is not allowed in high precision measurement.

Thermal deformation of every physical dimension of the mechanical part relates to the other physical dimension of the mechanical part. Different mechanical part has different form, so the thermal deformation of its physical dimension is different. On microscopic view, when the temperature changes, the form of the mechanical part is different with the initial form, it is non-similarity. Fig 1a shows the thermal deformation error of the short cylinder. The end faces of the cylinder are flat face, then the temperature changes, the end faces of the cylinder change too, they turn into convex face. Fig 1b shows the thermal deformation of long cylinder. Figure 2 shows the thermal deformation of hollow piece with square exterior surface. The thermal deformation of the internal hole relates to the exterior surface. When the temperature changes, the hole is no longer a circular hole, so there is non-similarity error of thermal deformation. If the hollow piece has a circular exterior surface, the hole is still a circular hole after the temperature change, so there is no non-similarity error of thermal deformation. Figure 3 shows a precise detection plane plate with ribbed plates, the ribbed plate affects the thermal deformation of the plate, when the temperature changes. Because there are many forms of ribbed plates, such as “#” form and “|” form, so the thermal deformation of the plane plate with ribbed plates are very complex, and there is non-similarity error. And the precise guide surface are quite similar to the plane plate, its thermal deformation includes non-similarity error too.
3. Results and Discussions
Mathematic thermal deformation models of mechanical parts must analyse non-similarity errors. The thermal deformation of one physical dimension relates with all physical dimensions of the part. After years of researching, mathematic models of some mechanical parts have been set up, hollow piece and cylindrical piece as well as gear are included.

The mathematic thermal deformation model of hollow piece is shown [8]:

\[
\Delta t = \sqrt{(1 + \alpha \Delta T)^2 d_0^2 - \alpha \Delta T D_0^2 - (D_0^2 - d_0^2)kH_0\Delta T} - d_0
\]  

(3)

where \( k = \rho g a E / 2E_0 \), \( d_0 \) is inside diameter of annulus, \( D_0 \) is outside diameter of annulus, \( H_0 \) is thickness of annulus, \( \rho \) initial elastic modular ratio is \( E_0 \), temperature coefficient of elastic modular ratio is \( aE \), thermal expansion coefficient of the material is \( \alpha \). \( \Delta t \) is the change amount of temperature.

The mathematic thermal deformation model of cylindrical piece is shown [9]:

\[
\frac{\Delta D}{D_0} = \alpha \Delta T + kH_0 \frac{\Delta T}{2}
\]  

(4)

Where \( D \) is the diameter of the cylindrical piece.

Figure 4 shows the sketch drawing of the hollow pieces with different outer diameter. All the pieces are processed in the same way. Their inside diameters are 30mm, and the lengths are all 40mm, the material is all 45 steels. Figure 5 shows the sketch drawing of the cylindrical pieces with different length. All the pieces are processed in the same way. Their diameters are 30mm, and the lengths are different, the material is all 45 steels. The results show the mathematic models of hollow piece and cylindrical piece are correct.

Figure 3. Sketch map of the thermal deformation of plane plate.

Figure 4. Relation graph of thermal deformation of hollow piece with different outside diameter.

Figure 5. Relation graph of thermal deformation of cylindrical pieces with different length.
The mathematic thermal deformation model of gear is shown \[10,11\]:

\[
\Delta f_t = -\frac{\Delta t\alpha}{2(1+\Delta t\alpha)} \left[ \frac{5}{r} - 2(\text{inv}\alpha_k - \text{inv}\alpha) \right] \left( 1 + \Delta t\alpha \right) r_b
\]

\[
= \Delta t\alpha r_b \text{inv}\alpha_k \cdot \frac{\Delta t\alpha r_b}{2} \left( \frac{s}{r} + 2\text{inv}\alpha \right)
\]

(5)

Where \(\alpha_k\) press angle of addendum circle, \(r_b\) base radius, \(s\) thickness of tooth, \(r\) reference radius, \(\alpha\) press angle of reference circle.

Fig6 shows the Sketch map of the involute flank. And table 1 shows the Theoretical and experimental results, where the \(r=100\text{mm}\), mode number \(m=4\text{mm}\), number of teeth \(Z=50\), and \(\alpha=11.6\times10^{-6}\), \(r_b=93.969\text{mm}\), \(s=6.283\text{mm}\), \(\alpha=20\), \(\alpha_k=28.2\), original temperature \(t_0=20^\circ\text{C}\), stable operating temperature \(t_1=100^\circ\text{C}\).

![Figure 6. Sketch map of the involute flank.](image)

| Stress angle (°) | 0   | 15  | 20  | 25  | 28   |
|-----------------|-----|-----|-----|-----|------|
| Theoretical values (µm) | -3.39 | -2.84 | -2.09 | -0.78 | 0.36 |
| experimental values(µm)   | -2.90 | -3.25 | -2.54 | -1.20 | 0.85 |

The theoretical and experimental values of heave motion \(\Delta f\) are nearly same, it is clear that the temperature variation can affect the flank of tooth, and operate errors. So the transmission efficiency and transmission precision will be reduced. In addition, the heave motion \(\Delta f\) relates to mismachining tolerance, and the striking angle error during measuring has effect on the \(\Delta f\) also.

4. Conclusion

To reduce and control the effects of thermal deformation errors without non-similarity error, the method of error correction is used. But it is difficulty to reduce and control the effects of thermal deformation errors which includes non-similarity errors, because different position has a different mathematic model of thermal deformation.

The models which are given before adopt some approximation calculation, and simplified some boundary conditions, so the experimental results is some different with the Theoretical ones, but the tendency is same [12]. If more studies to be done in the future, the accuracy of the mathematic models will be increased.

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