Hyperbolic and trigonometric hypergeometric solutions to the star-star equation

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Abstract We construct the hyperbolic and trigonometric solutions to the star-star relation via the gauge/YBE correspondence by using the three-dimensional lens partition function and superconformal index for a certain $\mathcal{N} = 2$ supersymmetric gauge dual theories. This correspondence relates supersymmetric gauge theories to exactly solvable models of statistical mechanics. The equality of partition functions for the three-dimensional supersymmetric dual theories can be written as an integral identity for hyperbolic and basic hypergeometric functions.

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1 Introduction

The gauge/YBE correspondence \cite{1,2} connecting supersymmetric gauge theories and integrable lattice models of statistical mechanics provides a powerful tool for studying spin models. It turns out that most of integrable edge-interacting (Ising-like) models in statistical mechanics \cite{3–5} (and some IRF models \cite{6–8}) can be obtained by this correspondence.

We will start with a very short account of this topic, the interested reader can find an exhaustive review on the subject in \cite{9,10}, where necessary information about the correspondence presented. Similar identities appear in integrable models of statistical mechanics. In this work, we present some new hypergeometric integral identities of hyperbolic and trigonometric types.

One of the striking features of recent developments in non-perturbative supersymmetric gauge theories is their deep relationship with interesting mathematical structures, see, e.g. \cite{11–13}. At present, they provide the main source of many new identities for hypergeometric functions \cite{14–18}.

In a recent work \cite{19}, the authors constructed a new solution to the star-triangle equation. This was achieved by using a certain three-dimensional supersymmetric dual theories on the lens space $S^3_b/Z_r$. The sufficient condition for the integrability of the lattice spin models is the star-star relation \cite{20}. In this paper we present the corresponding star-star relation for the model studied in \cite{19} and also for models discussed in \cite{21–23} (the corresponding gauge theories live on the squashed sphere $S^3_b$ and $S^2 \times S^1$). In the context of the gauge/YBE correspondence, this relation can be obtained from the equality of three-dimensional $\mathcal{N} = 2$ supersymmetric partition functions for a certain dual SQED theories. Our first two solutions to the star-star relation are given in terms of hyperbolic hypergeometric integrals (they are written in terms of hyperbolic gamma functions) and the last solution is a trigonometric type written in terms of basic hypergeometric integrals.

The paper is organized as follows. In Sect. 2 we briefly recall the star-star relation for the IRF models. Then we present solutions to the star-star equation.
2 Star-star relation

We deal here with the integrable interaction-round-a-face (IRF) lattice spin models [24,25]. In the IRF models, spin variables are located on the sites of the square lattice and interact via face. The sufficient condition for the integrability is the Yang–Baxter equation. In [26] it was shown that the Yang–Baxter equation for the face models can be reduced to the integrability definition (2.3) to be convinced with the pictorial representation of the star-star relation in Fig. 1

\[
R \left( \frac{\sigma_4 \sigma_3}{\sigma_1 \sigma_2} \right) = \frac{W(\sigma_1, \sigma_2) W(\sigma_3, \sigma_4)}{W(\sigma_1, \sigma_3) W(\sigma_2, \sigma_4)} R \left( \frac{\sigma_4 \sigma_3}{\sigma_1 \sigma_2} \right).
\]

(2.5)

where \( R \left( \frac{\sigma_4 \sigma_3}{\sigma_1 \sigma_2} \right) \) functions differ with the spectral parameters [20] omitted in this study.

By using the star-star relation one obtains the following IRF Yang–Baxter equation (it is depicted in Fig. 2)

\[
\sum_{m_1 \in \mathbb{Z}} \int dx_1 \ R \left( \frac{\sigma_5}{\sigma_2} \right) R \left( \frac{\sigma_6}{\sigma_1} \right) R \left( \frac{\sigma_4}{\sigma_3} \right) = \sum_{m_2 \in \mathbb{Z}} \int dx_2 \ R \left( \frac{\sigma_4}{\sigma_3} \right) R \left( \frac{\sigma_5}{\sigma_2} \right) R \left( \frac{\sigma_6}{\sigma_1} \right) R \left( \frac{\sigma_4}{\sigma_3} \right) .
\]

(2.6)

where the summation and integration stand for the discrete and continuous spin variables, respectively. Note that there are several solutions to the IRF Yang–Baxter equation obtained via gauge/YBE correspondence [2,3,5,7,8].

One can also see (2.4) as the following way with the definition (2.3) to be convinced with the pictorial representation of the star-star relation in Fig. 1

\[
R \left( \frac{\sigma_4 \sigma_3}{\sigma_1 \sigma_2} \right) = \frac{W(\sigma_1, \sigma_2) W(\sigma_3, \sigma_4)}{W(\sigma_1, \sigma_3) W(\sigma_2, \sigma_4)} R \left( \frac{\sigma_4 \sigma_3}{\sigma_1 \sigma_2} \right).
\]

(2.5)

3 Solutions to the star-star equation

By using the gauge/YBE correspondence one can systematically derive solution of the Yang–Baxter equation from calculations of supersymmetric gauge theory. In the context of this correspondence the Yang–Baxter equation expresses the identity of partition functions for supersymmetric dual pairs. Therefore the main step is to choose appropriate supersymmetric duality. Here we consider the following three-dimensional \( N = 2 \) dual theories [19,23,41]

- theory A has \( U(1) \) gauge group, six chiral multiplets with \( SU(3) \times SU(3) \times U(1) \) global symmetry group
- theory B consists of nine free “mesons” with the same global symmetry group as theory A.

The supersymmetric localization technique\(^2\) [29] enables us to calculate the partition function on different manifolds. The results of Coulomb branch localization on \( S^4_h \), \( S^4_h/\mathbb{Z}_n \) and the \( S^2 \times S^1 \) are known (see, e.g. [30–32]) and we will use these results in order to construct the star-triangle relation and corresponding star-star relation in the next sections.

\(^2\) The short review of the three-dimensional supersymmetric localization can be found in [27,28].
Let us introduce some definitions and notations of special
functions which we use in the paper. The $q$-Pochhammer
symbol is defined as follows
\[(z; q)\infty = \prod_{i=0}^{\infty} (1 - zq^i) . \tag{3.1}\]
We use the shorthand notation
\[(z, x; q)\infty = (z; q)\infty (x; q)\infty . \tag{3.2}\]
We also use hyperbolic gamma function which can be
defined as
\[\gamma(2)(z; \omega_1, \omega_2) = e^{\frac{\alpha_1}{2} B_{2,2}(z; \omega_1, \omega_2)} \left( e^{-2\pi i \frac{z}{\omega_1}} q; \tilde{q} \right)_{\infty} \left( e^{-2\pi i \frac{z}{\omega_2}} q; \tilde{q} \right)_{\infty} , \tag{3.3}\]
with the parameters $\tilde{q} = e^{2\pi i \omega_1/\omega_2}$ and $q = e^{-2\pi i \omega_2/\omega_1}$ and
the $B_{2,2}(z; \omega_1, \omega_2)$ stands for the Bernoulli polynomial
\[B_{2,2}(z; \omega_1, \omega_2) = \frac{z^2 - z(\omega_1 + \omega_2)}{\omega_1 \omega_2} + \frac{\omega_1^2 + 3\omega_1 \omega_2 + \omega_2^2}{6\omega_1 \omega_2} . \tag{3.4}\]
The hyperbolic gamma function also has an integral rep-
tresentation\footnote{One can find different integral representations in \[33,34\].}
\[\gamma(2)(z; \omega_1, \omega_2) = \exp \left( -\int_0^{\infty} \frac{dx}{x} \left[ \frac{\sinh(x(2z - \omega_1 - \omega_2))}{\sinh(x\omega_1) \sinh(x\omega_2)} - \frac{2z - \omega_1 - \omega_2}{2x\omega_1 \omega_2} \right] \right) , \tag{3.5}\]
where $Re(\omega_1), Re(\omega_2) > 0$ and $Re(\omega_1 + \omega_2) > Re(z) > 0$.
Additionally, we will use the following reflection property
for hyperbolic gamma function
\[\gamma(2)(\omega_1 + \omega_2 - z; \omega_1, \omega_2) \gamma(2)(z; \omega_1, \omega_2) = 1 . \tag{3.6}\]

3.2 Solution $S_b^3$ supersymmetric partition function

The equivalence of the partition functions for dual theories
on $S_b^3$ gives the following hyperbolic hypergeometric
integral identity [1,35,36]
\[\int_{-\infty}^{\infty} \prod_{i=1}^{3} \gamma(2)(a_i - x; \omega_1, \omega_2) \gamma(2)(b_i + x; \omega_1, \omega_2) \frac{dx}{\sqrt{\omega_1 \omega_2}} = \prod_{i,j=1}^{3} \gamma(2)(a_i + b_j; \omega_1, \omega_2) , \tag{3.7}\]
with the balancing condition $\sum_{i=1}^{3} (a_i + b_i) = \omega_1 + \omega_2$. In
[21,37] it was shown that this integral gives the star-triangle
relation for the Faddeev–Volkov model which has the following
Boltzmann weight
\[W_{\alpha}(x_i, x_j) = \gamma(2)(-\alpha + x_i - x_j; \omega_1, \omega_2) \gamma(2)(-\alpha - x_i + x_j; \omega_1, \omega_2) , \tag{3.8}\]
where we introduced new variables $a_i = -\alpha_i + x_i$ and $b_i = -\alpha_i - x_i$. Here $\alpha$ is a spectral parameter, $x_i$ is a spin variable
and $\omega_1, \omega_2$ are temperature-like parameters.
The corresponding star-star relation for this model (for details, see Appendix A) has the following form

$$\int_{-\infty}^{\infty} \prod_{i=1}^{4} \gamma^{(2)}(a_i - x; \omega_1, \omega_2) \gamma^{(2)}(b_i + x; \omega_1, \omega_2) \frac{dx}{\sqrt{\omega_1 \omega_2}} = \prod_{i,j=1}^{4} \gamma^{(2)}(\hat{a}_i + \hat{b}_j; \omega_1, \omega_2)$$

with the balancing condition $\sum_{i=1}^{4} (a_i + b_i) = 2(\omega_1 + \omega_2)$ and we used the following notations

$$\hat{a}_i = a_i + s, \quad \hat{b}_i = b_i + s, \quad \text{if} \quad i = 1, 2,$$

$$\hat{a}_i = a_i - s, \quad \hat{b}_i = b_i - s, \quad \text{if} \quad i = 3, 4,$$

with

$$s = \frac{1}{2}(\omega_1 + \omega_2 - a_1 - a_2 - b_1 - b_2)$$

$$= \frac{1}{2}(-\omega_1 - \omega_2 + a_3 + a_4 + b_3 + b_4).$$

This integral identity was obtained in [38]. The physical interpretation of this identity discussed in [39].

3.3 Solution via $S^3_b/\mathbb{Z}_r$ supersymmetric partition function

We again start with the equivalence of the partition functions for dual theories. This time we consider the dual theories on $S^3_b/\mathbb{Z}_r$ and obtain the following hyperbolic hypergeometric integral identity

$$\sum_{y=0}^{[r/2]} \epsilon(y) e^{-\pi i y} \int_{-\infty}^{\infty} \prod_{i=1}^{3} \gamma^{(2)}(-i(a_i - x) - i\omega_1 (u_i - y); -i\omega_1 r, -i\omega) \gamma^{(2)}(-i(a_i - x) - i\omega_2 (r - (u_i - y)); -i\omega_2 r, -i\omega) \gamma^{(2)}(-i(b_i + x) - i\omega_1 (v_i + y); -i\omega_1 r, -i\omega) \gamma^{(2)}(-i(b_i + x) - i\omega_2 (r - (v_i + y)); -i\omega_2 r, -i\omega)$$

$$\times -i\omega_2 r, -i\omega) \frac{dx}{\sqrt{-\omega_1 \omega_2}} = e^{\frac{\pi i}{2} \sum_{i=1}^{3} (a_i - v_i)} \prod_{i,j=1}^{3} \gamma^{(2)}(-i(a_i + b_j) - i\omega_1 (u_i + v_j)); -i\omega_1 r, -i\omega) \gamma^{(2)}(-i(a_i + b_j) - i\omega_2 (r - (u_i + v_j)); -i\omega_2 r, -i\omega)

with the balancing conditions $\sum_{i=1}^{3} a_i + b_i = \omega_1 + \omega_2$ and $\sum_{i=1}^{3} u_i + v_i = 0$. The $\epsilon(y)$ function is defined as $\epsilon(0) = 1$ and $\epsilon(y) = 2$ otherwise. The main difference from the expression (3.7) is that here the summation is over the holonomies $y = \frac{r}{2} \int A_\mu dx^\mu$, where the integration is over a non-trivial cycle on $S^3_b/\mathbb{Z}_r$ and $A_\mu$ is the gauge field, see, e.g. [31].

By introducing new variables $a_i = -a_i + x_i$ and $b_i = -a_i - x_i$ with the condition $u_i = -v_i$ for $i = 1, 3$, one can rewrite the integral identity (3.12) as the star-triangle equation with the following Boltzmann weight

$$W_a(x_i, x_j, u_i, u_j) = e^{-\pi i (u_i + u_j)} \gamma^{(2)}(-i(-\alpha + x_i - x_j) - i\omega_1 (u_i - u_j); -i\omega_1 r, -i\omega) \times \gamma^{(2)}(-i(-\alpha + x_i - x_j) - i\omega_2 (r - (u_i - u_j)); -i\omega_2 r, -i\omega) \times \gamma^{(2)}(-i(-\alpha + x_i - x_j) - i\omega_1 (u_j - u_i); -i\omega_1 r, -i\omega) \times \gamma^{(2)}(-i(-\alpha + x_i - x_j) - i\omega_2 (r - (u_j - u_i)); -i\omega_2 r, -i\omega).$$

The model with the Boltzmann weight (3.13) is an exactly solvable lattice spin model with discrete and continuous spin variables living on sites, where $x_i$ represents continuous spin and $u_i$ represents discrete spin variable. The $r = 1$ case corresponds to the Faddeev–Volkov model from the previous section.

Using the similar technique presented in Appendix A one can construct the star-star relation for this model

$$\sum_{y=0}^{[r/2]} \epsilon(y) \int_{-\infty}^{\infty} \prod_{i=1}^{4} \gamma^{(2)}(-i(a_i - x) - i\omega_1 (u_i - y); -i\omega_1 r, -i\omega) \times \gamma^{(2)}(-i(a_i - x) - i\omega_2 (r - (u_i - y)); -i\omega_2 r, -i\omega) \times \gamma^{(2)}(-i(b_i + x) - i\omega_1 (v_i + y); -i\omega_1 r, -i\omega) \times \gamma^{(2)}(-i(b_i + x) - i\omega_2 (r - (v_i + y)); -i\omega_2 r, -i\omega) \frac{dx}{\sqrt{-\omega_1 \omega_2}} = e^{\frac{\pi i}{2} \sum_{i=1}^{3} (a_i - v_i)} \prod_{i,j=1}^{4} \gamma^{(2)}(-i(a_i + b_j) - i\omega_1 (u_i + v_j)); -i\omega_1 r, -i\omega) \gamma^{(2)}(-i(a_i + b_j) - i\omega_2 (r - (u_i + v_j)); -i\omega_2 r, -i\omega) \gamma^{(2)}(-i(a_i + b_j) - i\omega_1 (u_i + v_j)); -i\omega_1 r, -i\omega) \gamma^{(2)}(-i(a_i + b_j) - i\omega_2 (r - (u_i + v_j)); -i\omega_2 r, -i\omega) \gamma^{(2)}(-i(a_i + b_j) - i\omega_1 (u_i + v_j)); -i\omega_1 r, -i\omega) \gamma^{(2)}(-i(a_i + b_j) - i\omega_2 (r - (u_i + v_j)); -i\omega_2 r, -i\omega).$$

Note that this integral identity was obtained via the reduction procedure and it needs to be proven rigorously.
In this section we present a new trigonometric solution to the star-triangle relation by introducing the new fugacities $a_i = \alpha_i^{-1} x_i$ and $b_i = \alpha_i^{-1} x_i^{-1}$ and using the condition $u_i = -v_i$. The resulting Boltzmann weight then has the following form

$$W_\alpha(x_i, x_j, u_i, u_j) = \frac{(q^{1+u_i-u_j})/2(\alpha^{-1} x_i x_j^{-1})^{-1}; q)_{\infty}}{(q^{u_i-u_j})/2\alpha^{-1} x_i x_j^{-1}; q)_{\infty} \times (q^{1+(u_j-u_i)/2(\alpha^{-1} x_i x_j^{-1})^{-1}}; q)_{\infty} \times (q^{(u_i-u_j)/2\alpha^{-1} x_i x_j^{-1}}; q)_{\infty},$$

where $\alpha$ stands for the spectral parameter. The corresponding statistical mechanics model is a square lattice model with edge interaction and discrete and continuous spin variables. This model is a special case (with broken gauge symmetry) of the integrable lattice spin model considered in [3] and it is a trigonometric analogue of the Faddeev–Volkov model. One we can construct the star-star relation for this model

$$3.4 \text{ Solution via } S^2 \times S^1 \text{ supersymmetric partition function}$$

In this section we present a new trigonometric solution to the star-triangle equation and to the star-star equation. In

$$\sum_{y=-\infty}^{\infty} \int_{-\infty}^{\infty} \prod_{i=1}^{3} \frac{(q^{1+(y_i+u_i)/2}(a_i x_i)^{-1}, q^{1+(u_i-y_i)/2} b_i x_i^{-1}; q)_{\infty}}{(q^{u_i-y_i})/2\alpha^{-1} x_i x_j^{-1}; q)_{\infty} \times (q^{1+(u_j-u_i)/2(\alpha^{-1} x_i x_j^{-1})^{-1}}; q)_{\infty} \times (q^{(u_i-u_j)/2\alpha^{-1} x_i x_j^{-1}}; q)_{\infty},$$

with the new balancing conditions $\prod_{i=1}^{2} a_i b_i = q^2$ and $\sum_{i=1}^{4} u_i + v_i = 0$. In the latter expression we used the following notations

$$\tilde{a}_i = a_i s, \tilde{b}_i = b_i s, \tilde{u}_i = u_i + p,$$

$$\tilde{v}_i = v_i + p, \text{ if } i = 1, 2,$$

$$\tilde{a}_i = a_i s^{-1}, \tilde{b}_i = b_i s^{-1}, \tilde{u}_i = u_i - p,$$

$$\tilde{v}_i = v_i - p, \text{ if } i = 3, 4.$$

---

6 This identity can be written as a pentagon identity which is related to the triangulation of 3-manifolds.
where
\[ s = \sqrt{\frac{q}{a_1a_2b_1b_2}} = \sqrt{\frac{a_3a_4b_3b_4}{q}}, \]
\[ p = \frac{1}{2}(u_1 + u_2 + v_1 + v_2) = \frac{1}{2}(u_3 + u_4 + v_3 + v_4). \]

(3.21)

4 Conclusions

In this work, we constructed hyperbolic and trigonometric solutions to the star-star equation. We obtained new solutions from the equality of three-dimensional \( \mathcal{N} = 2 \) supersymmetric partition functions for certain dual SQED theories via the gauge/YBE correspondence.

There are several ways of constructing solutions to the star-star equation. One can use the Bailey pair construction starting from the star-triangle relation for the models discussed here. It is possible to obtain the solution by breaking the gauge symmetry from \( SU(2) \) group to the \( U(1) \) for the supersymmetric dual theories with \( SU(2) \) gauge group and \( SU(6) \) flavor group considered in [42].

There are many interesting limits of the solutions considered here, for instance it would be interesting to construct solution to the star-star equation in terms of Euler gamma functions [43].

The gauge/YBE correspondence has revealed various interesting connections among integrable models and supersymmetric gauge theories. There are underlying mathematical structures such as quantum algebras related to the solutions of the Yang–Baxter equations obtained via gauge/YBE correspondence. It would be interesting to pursue this direction for these star-star solutions.

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Appendix A: Derivation of the star-star relation (3.9)

Here we follow the approach presented in [38]. Let us consider the following double integral

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \gamma^{(2)}(a_i - x)\gamma^{(2)}(b_i + x) dx dz
\]

(3.22)

\[
\times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \gamma^{(2)}(a_i - s - z)\gamma^{(2)}(b_i - s + z) dz dx
\]

(3.23)

\[
\times \gamma^{(2)}(s + z - x)\gamma^{(2)}(s - z + x) \frac{dx}{i\sqrt{\omega_{1}\omega_{2}}} \frac{dz}{i\sqrt{\omega_{1}\omega_{2}}}. \]

(A.1)

Here we used the shorthand notation \( \gamma^{(2)}(x) = \gamma^{(2)}(x; \omega_1, \omega_2) \). First we integrate the integral (A.1) over the \( x \) variable using the identity (3.7). We end up with the following result

\[
\gamma^{(2)}(2s) \prod_{i,j=1}^{2} \gamma^{(2)}(a_i + b_j)
\]

(3.24)

\[
\times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \gamma^{(2)}(a_i - s - z)\gamma^{(2)}(b_i - s + z) dz dx
\]

(3.25)

\[
\times \prod_{i=1}^{2} \gamma^{(2)}(s + z + b_i) \prod_{i=1}^{2} \gamma^{(2)}(a_i + s - z) \frac{dz}{i\sqrt{\omega_{1}\omega_{2}}}. \]

(A.2)

Then integrating (A.1) over the \( z \) variable one finds that

\[
\gamma^{(2)}(2s) \prod_{i,j=3}^{4} \gamma^{(2)}(a_i + b_j - 2s)
\]

(3.26)

\[
\times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \gamma^{(2)}(a_i - x)\gamma^{(2)}(b_i + x) dx dz
\]

(3.27)

\[
\times \prod_{i=3}^{4} \gamma^{(2)}(b_i + x) \prod_{i=3}^{4} \gamma^{(2)}(a_i - x) \frac{dx}{i\sqrt{\omega_{1}\omega_{2}}}. \]

(A.3)

The latter two expressions are results of the same integral expression, therefore we find that

\[
\gamma^{(2)}(2s) \prod_{i,j=1}^{2} \gamma^{(2)}(a_i + b_j)
\]
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