Semi-Active Suspension for an Automobile

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Abstract—Handling characteristics and ride comfort of a vehicle are dictated by the suspension setup. There exists a design conflict in optimizing the two parameters. Hard suspension provides proper handling, but at the cost of reduced passenger comfort. On the other hand, soft suspension produces least disturbance to the passenger, but it becomes difficult to handle the vehicle. In recent times considerable interest is generated in literature developing different mechanisms for automatically controlling the speed of vehicle while travelling on rough roads. In this paper, a semi-active suspension system is proposed using inclined springs, whose effective stiffness can be varied. This variation is done, taking into account the pre-set comfortable acceleration level for the passenger, to be able to pass over a road hump at a reasonably high speed. The mechanism for the physical simulation of variable stiffness suspension is also addressed.

Keywords—Suspension; Damping; stiffness; Frequency ratio; Acceleration; Ride comfort; Sinusoidal road profile

I. INTRODUCTION

The main task of an automobile suspension system is to provide comfortable ride and proper handling characteristics for the passenger and driver. In order to achieve this, an optimal contact between the wheels and the road surface is needed under various driving conditions, including uneven road surfaces such as road humps. Comfort and road-handling performance of a passenger car are mainly determined by damping characteristic of the shock absorbers. Passive shock absorbers have a fixed damping characteristics determined by their design. Depending on the road excitation, however, it is desirable to adjust this characteristic to increase the performance. The suspension system can be an active or semi-active one. In case of active suspension, separate actuators are used which exert independent forces on the suspension to improve ride characteristics, when required. The active shock absorber hardware is designed such that its open-loop (uncontrolled) dynamic characteristic is comparable to that of a passive shock absorber tuned for the same type of car. The performance is then further improved by means of a controller that regulates the damping characteristic of the active shock absorber according to the road [1]. In case of semi-active system, either stiffness or damping of the suspension is altered. As presented in paper [2]: the conventional spring element is retained, but the damper is replaced with a controllable damper. Magnetorheological (MR) damper is a kind of semi-active device. A wide range of Magneto-rheological (MR) fluid based dampers are currently being explored for their potential implementation in various systems, such as vibration control devices and suspension system.

In this paper, a semi-active system is proposed, in which the suspension stiffness can be varied and comfort level [3] of the passengers can be maintained at acceptable levels even at reasonably high speeds of the vehicle. The whole vehicle is modeled as a single degree of freedom system (Fig.1), to simulate the vertical vibration characteristics and the response of passenger, while travelling on a sinusoidal road surface. The equations governing acceleration of the car (represented by body+passenger+Sprung mass) can be solved for any road input. However, in this paper the effort is only to develop and illustrate the concept of variable stiffness suspension, which can be easily extended to other road surfaces and parameters.

II. SIMULATION OF VEHICLE WITH VARIABLE SUSPENSION STIFFNESS

Figs.1 (a) and (b) represents a single degree of freedom system of an automotive vehicle moving at a constant speed ‘v’ on a typical sinusoidal road surface. The mass of the entire vehicle is represented by ‘m’. The suspension system consists of a primary spring of stiffness ‘k’ and two secondary springs, each of stiffness ‘k1’, located at an angle θ symmetrically with respect to the vertical. The spring stiffness k and k1 are constant. The resultant vertical stiffness ‘K_nr’ of the suspension can be varied by changing the angle θ from the initial position of 0° to a value φ°, which is currently fixed at 60°. Thus the effective stiffness can be brought down from initial maximum value to some pre-determined lowest value, thereby facilitating changing of natural frequency as per needs. The wave length of the road is λ, which determines the excitation frequency for a given speed of the vehicle; and the half-amplitude is Y, which decides the maximum road vibration input level The vertical displacement of the entire vehicle mass is X. The equivalent viscous damping ratio is ζ and the maximum passenger acceleration is A_max. Specific numerical values are assigned to the different parameters as suggested in [4]. This is only to demonstrate numerical calculations and can be changed depending on the actual problem in hand.

- Mass (Sprung+passenger+body) of the car model (m) = 1050 Kg
- Primary spring stiffness (k) = 77500 N/m
- Secondary spring stiffness (k1) = ak N/m
  a is varied from 0.1 to 0.6 in steps of 0.1 to arrive at the best possible equivalent stiffness.
- Angle of inclination of the secondary spring (θ) with respect to vertical is varied from 0° to 60° in steps of 10°
- Damping ratio ζ = 0, 0.05 and 0.1
- Road profile y = Y Sin (ωt), Y = 0.35m
- Maximum passenger acceleration (A_{max}) = 2g

![Figure 1(a). A single degree of freedom system model of a car](image)

![Figure 1(b). An equivalent single degree of freedom system model of a car with varying stiffness.](image)

### III. MATHEMATICAL FORMULATION

The effective spring stiffness of the suspension in the vertical direction is given by

\[ K_{eff} = \frac{1}{k} + \frac{1}{2k_{1}\cos^{2}\theta} \]  \hspace{1cm} (1)

\[ K_{eff} = \frac{2k_{a}\cos^{2}\theta}{1+2\alpha\cos^{2}\theta} \]  \hspace{1cm} (2)

The natural frequency of the system is given by

\[ \omega_{n} = \sqrt{\frac{K_{eff}}{m}} \]  \hspace{1cm} (3)

The ratio of amplitude of response X to that of the base motion Y for a single degree of freedom system comprising of spring mass damper units is called displacement transmissibility and its given by,

\[ X \over Y = \frac{\sqrt{1+(2\zeta r)^{2}}}{\sqrt{1-r^{2}}^{2}+(2\zeta r)^{2}} \]  \hspace{1cm} (4a)

\[ \ddot{x} = \omega^{2}x \quad \text{and} \quad \ddot{y} = \omega^{2}y \]  \hspace{1cm} (4b)

\[ r = \frac{\omega}{\omega_{n}} \]  \hspace{1cm} (4c)

The acceleration of the passenger is given by:

\[ \frac{X}{Y} = \frac{\dot{x}}{\dot{y}} = \frac{\sqrt{1+(2\zeta r)^{2}}}{(1-r^{2})^{2}+(2\zeta r)^{2}} \]  \hspace{1cm} (5)

Let \( r^{2} = a \)

Squaring both the sides of (5) and taking an inverse. We get,

\[ \frac{\dot{x}^{2}y^{2}\omega_{n}^{2}}{\dot{y}^{2}} = \frac{1+4\zeta^{2}a}{1+4\zeta^{2}a} \]

\[ \dot{x}^{2} - 2a(\dot{x}^{2}) + 4\zeta^{2}a(\dot{x}^{2}) \]

\[ = a^{2}y^{2}\omega_{n}^{2} + 4\zeta^{2}y^{2}\omega_{n}^{2}a^{3} \]

\[ (4\zeta^{2}y^{2}\omega_{n}^{2}a^{2} + (Y^{2}a^{2} - \dot{x}^{2})a^{2} + \]

\[ 2\dot{x}^{2} - 4\zeta^{2}\dot{x}^{2})a - \dot{x}^{2} = 0 \]  \hspace{1cm} (9)

This cubic equation can be solved for ‘a’ when all other parameters are given specific values.

Vehicle velocity \[ v = \frac{\omega_{n}^{2}}{2\pi} = \frac{\omega_{n}^{2}}{2\pi} \]  \hspace{1cm} (10)

### IV. RESULTS AND DISCUSSION

The solution of Eq (8) is studied from two separate aspects: (a) passenger response with respect to vehicle speed and (b) optimum suspension stiffness

#### A. Relationship between Vehicle Velocity and Passenger Acceleration:

Selecting a specific set of system parameters of natural frequency \( \omega_{n} \), the damping ratio \( \zeta \) and the road amplitude \( Y \), the cubic Eq (9) is solved to obtain a plot of passenger acceleration \( \dot{x} \) as a function of ‘a’, hence ‘r’ and subsequently \( v \), the vehicle velocity using Eq (10). These plots are shown in Fig.2 for different natural frequencies and damping ratios. They essentially represent how the passenger is subjected to acceleration, as the speed increases for a vehicle with fixed
suspension stiffness. The natural frequency $\omega_n$ is constant and the excitation frequency $\omega$ varies as the speed changes.

By drawing a horizontal line at specific passenger acceleration level, say $\ddot{x} = 2g$, as represented in Fig.2, three critical vehicle speeds $v_1$, $v_2$, $v_3$ are obtained, at which the passenger acceleration becomes equal to $2g$. These are obtained from the three roots of $'a'$ of Eq (9). The first root of $'a'$ corresponds to frequency ratio before resonance ($r < 1$), and hence critical speed $v_1$, is not of much practical significance. The other two roots corresponding to frequency ratios after resonance ($r > 1$) and the corresponding critical speeds $v_2$ and $v_3$, are important because of vibration isolation. When damping is zero, $v_2$ is finite, but $v_3$ tends to infinity. For different damping ratios, we get different $v_2$ and $v_3$. Between the critical speeds $v_2$ and $v_3$ the passenger acceleration is less than the specified value $2g$. Thus, $v_2$ is the minimum speed at which the vehicle should pass over the hump and $v_3$ is the maximum speed, for a given passenger comfort level of acceleration. Other conclusions that can be drawn from Fig.2, for a given passenger acceleration are that:

1. If the natural frequency $\omega_n$ can be decreased, the maximum speed $v_3$ can be increased.
2. If the damping ratio $\zeta$ can be decreased, the maximum speed $v_3$ can be increased. The lowest speed, however, does not depend on damping but only on the natural frequency of the system.

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**B. Optimum Suspension Stiffness**

Given a specific road profile, the speed of the vehicle determines the excitation frequency $\omega$. Given the primary and secondary spring constants $k$ and $k_1 = (\alpha k)$ and the included angle between secondary springs $\theta$, the effective vertical stiffness of suspension $K_{eff}$ is decided and therefore the natural frequency $\omega_n$. Choosing a specific value for the primary spring constant $k$, the effective stiffness can be varied through the parameter $\alpha$. Specific values are selected for $k$, $\alpha$, $Y$ and passenger acceleration $\ddot{x} = 2g$ and the damping ratio $\zeta$ and variation of critical vehicle speeds $v_1$, $v_2$ and $v_3$ for different angle of inclination of secondary spring ($\theta$) are obtained, by solving the cubic Eq (9) in $'a'$ for its 3 roots. The variation of $v_1$ and $v_2$ with $\theta$ is plotted in Fig. 3(a) and the variation of $v_3$, the maximum speed, with $\theta$ is plotted in Fig. 4, for different values of $\alpha$ and $\zeta$. Fig 3(a) & 3(b) shows the speeds $v_1$ and $v_2$ are too low and further decrease as $\theta$ is increased. They are not of much relevance.

The results of Fig. 4, show that the maximum critical speed $v_3$, which is the limiting speed for a specified passenger comfort level of acceleration can be increased as the angle $\theta$ is increased, which actually brings down the natural frequency. Further, maximum $v_3$ can also be increased for any given $\theta$ by decreasing damping. Zero damping, which is a limiting case, is not practical and therefore need not be considered. These are, however, the same observations from the results presented in Fig. 2.

Another important information that can be drawn from Fig. 4 is fixing the parameter value of $\alpha$ for a given ride comfort. It can be seen that the range of maximum speed $v_3$ (difference in speeds between $\theta = 60^0$ and $\theta = 0^0$) increases as $\alpha$ is decreased. And increasing the maximum angle $\varphi$ beyond $60^0$ also increases the range of $v_3$, but does not appear practical. Thus $\alpha$ can be chosen depending on practical considerations, keeping in mind the lower the value the better. Thus $\alpha = 0.2$ can be a good option in the current problem ($v_3 = 30$ to $60$ Km/h.)
Fig. 3(a): Variations of speed $V_1$ & $V_2$ with $\theta$ at 0% damping

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Fig 3(b) Variations of speed $V_1$ & $V_2$ with $\theta$ at 5% damping

Figure 4(a). Variation of Critical Speed $V_3$ (the maximum Speed) with $\theta$. (a) 5% damping

Fig 3(b) Variations of speed $V_1$ & $V_2$ with $\theta$ at 5% damping

Figure 4(b). Variation of Critical Speed $V_3$ (the maximum Speed) with $\theta$. (b) 10% damping
V. PHYSICAL SIMULATION OF VARIABLE STIFFNESS MECHANISM

A physical mechanism is proposed here conceptually to implement the variable suspension stiffness ideas presented earlier, in an automotive vehicle. Basically it requires simulation of inclined secondary springs and a mechanism to vary the angle of inclination with vertical as per requirements. The equivalent spring system for the whole vehicle shown in Fig. 1 can be implemented at each of the 4 wheel locations, with the k and k₁ spring constants reduced to 1/4th of the whole vehicle. A design configuration for implementing this is shown in Fig 5. The power required to rotate the inclined springs to change the angle is derived from a motor and transmitted to the secondary springs with the help of a transmission system.

The suspension system can be made semi-active as follows. The road profile is first sensed by a controller, which is programmed to take into account variation of passenger acceleration vs vehicle velocity (Fig 2), the limitation on critical speeds (Fig.3) and velocity vs angle of secondary spring (Fig.4). On the basis of road profile, controller (PID) decides the velocity of vehicle, the required natural frequency corresponding to velocity and finally the angle of springs, which is changed by running the motor. Fig.5 shows isometric view of proposed motor, gear and spring assembly. Controller sends the signal to motor which drives the belt transmitting power to the central pulley P mounted on the shaft S1. The gear on the left side G1 meshes with reverse gear G2 supported on shaft S2. A link L2 is attached to reverse gear which is similar to the link L1 attached to right side of bottom shaft. The link L1 attached to the right side of the shaft S1 is connected to top most point of the secondary spring P1 and further extends into the trail. Another link L2 driven by reverse gear G2 is connected to secondary spring P2, which undergoes movement in the direction opposite to P1.

Figure 5. Isometric View of Motor, Gear and Spring Assembly

VI. CONCLUSIONS

In this paper a design methodology and a conceptual implementation scheme for a semi-active variable stiffness suspension system is proposed to control the acceleration of the passenger of an automotive vehicle below a predetermined level while travelling over a sinusoidal road profile at different speeds. Though specific numerical values are used for different parameters, they can be changed as per requirements. This work can be carried forward to determine the power required from the engine to drive a motor and thus actuate the semi-active suspension.

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