Rigidity of automorphisms and spherical
CR structures

Jih-Hsin Cheng *
Institute of Mathematics, Academia Sinica, Taipei

Abstract
We establish Bochner-type formulas for operators related to CR automorphisms and spherical CR structures. From such formulas, we draw conclusions about rigidity by making assumptions on the Tanaka-Webster curvature and torsion.

1 Statement of results
It is now clear (e.g. [CL1], [CL2], [CT], [Rum]) that certain distinguished second-order partial differential operators and their fourth-order “Laplacians” play important roles in the study of three-dimensional CR geometry. In this paper we will establish Bochner-type formulas for operators related to CR automorphisms and spherical CR structures. From such formulas, we can draw conclusions about rigidity by making assumptions on the so-called Tanaka-Webster curvature and torsion.

To be precise, let $(M, J, \theta)$ be a smooth, closed (compact without boundary) 3-dimensional strictly pseudoconvex pseudohermitian manifold. (see, e.g. [Web],[Le1]) Here $J$ denotes a CR structure and $\theta$ is a contact form, i.e. a non-vanishing real 1-form defining the underlying contact structure. Associated to $(M, J, \theta)$, we have a canonical affine connection and notions of Tanaka-Webster scalar curvature and torsion, denoted $R$ and $A$ (a tensor with coefficient $A_{1\bar{1}}$ or $A_{\bar{1}}^1$) respectively. ([Le1],[Tan],[Web])

Let $T$ be the characteristic vector field of $\theta$ defined by $\theta(T) = 1, \mathcal{L}_T \theta = 0$. (some authors call $T$ the Reeb vector field) By choosing suitable complex vector fields $Z_1, Z_{\bar{1}}$ such that $JZ_1 = iZ_1, JZ_{\bar{1}} = -iZ_{\bar{1}}$, we form a (unitary) frame $\{Z_1, Z_{\bar{1}}, T\}$. ($h_{1\bar{1}} = 1$) The covariant derivatives are taken with respect to this frame and indicated by $_{1, \bar{1}, 0}$ and so on. Let $\text{Aut}_0(J)$ denote the identity component of the CR automorphism group with respect to $J$. 

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Now we can state our first result.

**Theorem A.** Let \((M,J,\theta)\) be a smooth, connected, closed 3-dimensional strictly pseudoconvex pseudohermitian manifold.

(a) Suppose \(R < 0, \sqrt{3}R_0 - 2\text{Im}(A_{11,11}) > 0\). Then \(\text{Aut}_0(J)\) consists of only the identity diffeomorphism.

(b) Suppose \(R < 0, \sqrt{3}R_0 - 2\text{Im}(A_{11,11}) = 0\). Then \(\text{dimAut}_0(J) \leq 1\).

We remark that the torsion \(A = 0\) implies the second condition in (b) holds due to the Bianchi identity: \(R_{0,0} = A_{11,11} + A_{11,11}\). And the result is compatible with Prop.4.8(a) in [CT]. On the other hand, we do not know any examples satisfying curvature conditions in (a).

The idea of proving Theorem A goes as follows. Consider a certain second-order linear partial differential operator \(D_J\) and its “Laplacian” \(D^*_J D_J\) acting on functions. Here \(D^*_J\) denotes the adjoint of \(D_J\). The kernel of \(D_J\) parametrizes infinitesimal \(CR\) automorphisms. (see [CL1] where \(D_J\) was denoted \(B'_J\)) Establish a suitable Bochner-type formula for \(<D^*_JD_Jf,f>\) by using commutation relations and integration by parts repeatedly. (see section 2 for details) Theorem A then follows easily from the final formula. Note that the kernel of \(D^*_J\) parametrizes the infinitesimal slice in the study of \(CR\) moduli spaces. ([CL2])

Next we consider deformation of spherical \(CR\) structures. A \(CR\) structure or \(CR\) manifold is called spherical if it is locally \(CR\)-equivalent to the unit sphere with the standard \(CR\) structure. (e.g. [BS],[CL1]) In dimension 3 it can be characterized quantitatively by the vanishing of a certain fourth-order partial differential operator, the so-called Cartan (curvature) tensor, denoted \(Q_J\). (while in higher dimensions, Chern’s curvature tensor plays the similar role ([CM]), which is of second order)

A spherical \(CR\) structure \(J\) is called rigid if there is no infinitesimal deformation up to diffeomorphisms, i.e. for any smooth family of spherical \(CR\) structures \(J(\xi)\) on the base manifold \(M\) with \(J(0) = J, d/dt|_{\xi=0}J(\xi) = L_X J\) for some vector field \(X\) of \(M\).

To study the rigidity of spherical \(CR\) structures, we consider the linearization of \(Q_J\) plus a symmetry-breaking term provided by \(D_J D^*_J\). In section 3 we work out a Bochner-type formula and analyze it to obtain pointwise conditions for \(J\) to be rigid.

**Theorem B.** Let \((M,J)\) be a smooth, closed, spherical \(CR\) 3-manifold. Suppose there is a contact form \(\theta\) such that \(R > 0, (3.11)\) and \(3.12\) hold. Then \(J\) is rigid.

Note that conditions (a),(b) in Theorem A and (3.11),(3.12) in Theorem B are independent of positive constant multiples of \(\theta\). When the torsion vanishes, we have the simplified expression as follows.

**Corollary C.** Let \((M,J)\) be a smooth, closed, spherical \(CR\) 3-manifold. Suppose there is a contact form \(\theta\) such that the torsion \(A = 0\) and
\[ R > 0, 4R(5R^2 + 3\Delta_b R) - 3|\nabla_b R|^2 > 0 \]

Then \( J \) is rigid.

The sublaplacian and subgradient operators \( \Delta_b, \nabla_b \) acting on (smooth) functions are defined by \( \Delta_b f = -f,^{11}_1 - f,^{11}_1 \) and \( \nabla_b f = f,^1_1 Z_1 + f,^1_1 Z_\bar{1} \) respectively. (cf. [Le1] or [Che]) Also we define \( |\nabla_b f|^2_\theta = 2f,^{11}_1 f,^{11}_1 \) for real \( f \).

Observe that \( A = 0 \) and \( R \) being a positive constant satisfy conditions in Corollary C. In this case the universal cover of \( (M,J) \) must be compact by Rumin’s pseudohermitian version of Myers’ theorem ([Rum]), and hence CR-equivalent to the standard \( S^3 \). It follows that the fundamental group \( \Gamma \) of \( M \) is finite. Hence the group cohomology \( H^1(\Gamma, G) \) in deformation theory (p.232 in [BS]) vanishes. So in this special case our result is compatible with the result obtained by “Lie theoretical” argument. Note that a small perturbation of \( A = 0 \) and \( R \) being a positive constant still satisfies the conditions in Theorem B.

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## 2 Proof of Theorem A

Let \( \{\theta^1, \theta^\bar{1}, \theta\} \) be the coframe dual to the “unitary” frame \( \{Z_1, Z_\bar{1}, T\} \). (with \( h^{1\bar{1}} = h_{1\bar{1}} = 1 \) in mind, hereafter, we’ll write tensors with only lower indices)

Recall ([CL2] or [CL1]) that \( D_J f = 2Re((f,^{11}_1 + iA^{11}_1 f)\theta^1 \otimes Z_1) \) and the adjoint operator \( D_J^* \) acts on a deformation tensor \( E = 2Re(E,^{11}_1\theta^1 \otimes Z_1) \) by

\[ D_J^* E = E,^{11\bar{1}}_1 + iA^{11}_1 E,^{1\bar{1}}_1 + \text{conjugate}. \]

Also the generalized Folland-Stein operator \( L_\alpha \) is defined by \( L_\alpha f = \Delta_b f + i\alpha f,^0 \) for a function \( f \). By Lemma 2.1 in [CL2], we have

\[ D_J^* D_J = (1/2)L_\alpha^* L_\alpha + \mathcal{O}_2 \]

with \( \alpha = i\sqrt{3} \), where \( \mathcal{O}_2 \) is an operator of weight \( \leq 2 \). In the rest of this section, we’ll look into the details of \( \mathcal{O}_2 \). (Note that \( L_\alpha \) in the leading term of (2.2) is subelliptic. This implies the existence of the “infinitesimal slice decompositions” in the study of \( CR \) moduli spaces without knowing details of the lower-weight term ([CL2]))

A direct computation shows that for a real-valued function \( f \), we have

\[ D_J^* D_J f = f,^{11\bar{1}}_1 + f,^{1\bar{1}1}_1 + 2Re[2iA_{11} f,^{11}_1 + 2iA_{11} f,^{1}_{1\bar{1}} + iA_{11} f] + 2iA_{11} f,^{1}_{1\bar{1}} + iA_{11} f,^{11}_1 + |A_{11}|^2 f]. \]
We’ll frequently use the following commutation relations.

**Lemma 2.1** (Ricci identities in pseudohermitian geometry) Let \( c_I \) be a coefficient of some tensor with multi-indices \( I \). Suppose \( I \) consists of only 1 and \( \bar{1} \), and \( \alpha = (# \text{ of } 1 \text{ in } I) - (# \text{ of } \bar{1} \text{ in } I) \). Then

\[
\begin{align*}
&c_{I,11} - c_{I,\bar{1}1} = ic_{I,0} + \alpha c_I R \\
&c_{I,01} - c_{I,10} = c_{I,\bar{1}1} A_{11} - \alpha c_I A_{11,\bar{1}} \\
&c_{I,01} - c_{I,10} = c_{I,1\bar{1}} A_{\bar{1}11} + \alpha c_I A_{\bar{1}11,1}
\end{align*}
\]

(2.4) (2.5) (2.6)

(this lemma generalizes Lemma 2.3 in [Le2] for the three-dimensional case)

By using (2.4), (2.5), (2.6) repeatedly, we obtain

\[
\frac{1}{2} L^*_{X} f = f_{11\bar{1}1} + f_{\bar{1}111} + 2Re(\sqrt{3} - i)(A_{\bar{1}11,f_{11}},1 - (Rf_{11},\bar{1})]
\]

(2.7) with the choice of \( \alpha = i\sqrt{3} \) eliminating terms having covariant derivative in the direction \( T \). Comparing (2.3) with (2.7) and taking \( L^2 \) inner product with \( f \) give

\[
\begin{align*}
\|D_{J} f\|^2 &= \frac{1}{2} \|L_{i\sqrt{3}} f\|^2 - \int_M R|\nabla f|^2_\theta dv_\theta \\
&\quad + 2\int_M [Re(\sqrt{3} + i)A_{11,\bar{1}1} + |A_{11}|^2] f^2 dv_\theta \\
&\quad + 2\int_M Re([-i - \sqrt{3})A_{\bar{1}11,f_{11}}] f dv_\theta
\end{align*}
\]

(2.8)

Here the volume form \( dv_\theta = \theta \wedge d\theta \). With respect to \( dv_\theta \), we have the divergence theorem and hence integration by parts in calculus of pseudohermitian geometry. ([Le2],[Che]) For instance,

\[
\int_M A_{\bar{1}11,f_{11}} f dv_\theta = \frac{1}{2} \int_M A_{\bar{1}11}(f^2)_{1} dv_\theta \text{ (if being real)}
\]

\[
= -\frac{1}{2} \int_M A_{11,11} f^2 dv_\theta \text{ (integration by parts)}
\]

was used in deducing (2.8). Now suppose \( \phi_t \in Aut_0(J) \) is a smooth family of CR automorphisms with \( \phi_0 = \text{identity} \). Then \( X = \frac{d}{dt}|_{t=0}\phi_t \in \text{Lie } Aut_0(J) (= \text{Lie algebra of } Aut_0(J)) \) is an infinitesimal CR automorphism, in particular, an infinitesimal contact automorphism. According to Lemma 3.4 and 3.5 in [CL1], \( X = X_f \) is determined by a function \( f = -\theta(X) \) and satisfies the relation: \( L_X J = 2D_{J} f \). Since \( L_X J = 0 \), we get
(2.9) \[ 0 = \mathcal{D}_J f = f_{,11} + i A_{11} f, \]

and hence

(2.10) \[ A_{1\bar{1},11} = -i |A_{11}|^2 f. \]

Substituting (2.9), (2.10) in (2.8), we finally obtain

(2.11) \[ 0 = \| L_{i\sqrt{3}f} \|^2 - 2 \int_M R |\nabla_b f|^2 d\theta \\
+ \int_M [\sqrt{3} R_{,0} + i (A_{11,\bar{1}\bar{1}} - A_{1\bar{1},11})] f^2 d\theta \]

Now it is easy to see from (2.11) that the condition in (a) of Theorem A implies \( f = 0 \). Therefore \( X = X_f = 0 \). For (b), the condition implies \( \nabla_b f = 0 \). So \( f_{,0} = 0 \) by (2.4). Thus \( f \) is constant since \( M \) is connected. It follows that \( \dim(\text{Aut}_0(J)) = \dim(\text{Lie Aut}_0(J)) \leq 1. \)

3 Proof of Theorem B

Recall ([CL1]) that the Cartan tensor \( Q_J = i Q_{11} \theta^1 \otimes Z_{\bar{1}} - i Q_{\bar{1}1} \theta^\bar{1} \otimes Z_1 \) where

\[ Q_{11} = \frac{1}{6} R_{,11} + \frac{i}{2} RA_{11} - A_{11,0} - \frac{2i}{3} A_{11,\bar{1}\bar{1}} \]

and \( J \) is spherical if and only if \( Q_J = 0 \). (note that we have lowered indices using \( h_{1\bar{1}} = 1 \); also \( Q_J \) changes “tensorially” when we make a different choice of contact form) Let \( \tilde{J}_{(t)} \) be a smooth family of spherical CR structures with \( \tilde{J}_{(0)} = J \). By a theorem of Gray ([Gra] or [Ham]), there exists a smooth family of diffeomorphisms \( \phi_t \) with \( \phi_0 = \text{identity} \) so that for all \( t \), \( J_{(t)} = \phi_t^* \tilde{J}_{(t)} \) has the same underlying contact structure as \( J \) does. Write infinitesimal deformation

\[ \frac{d}{dt}|_{t=0} J_{(t)} = 2E = 4 \Re (E_{11} \theta^1 \otimes Z_{\bar{1}}) \]

and compute \( DQ_J(2E) = \partial_t Q_J|_{t=0} \) as we did in [CL1]. There appears a “bad” term \( E_{\bar{1}1,11\bar{1}1} \) in the formula, so we add a “symmetry-breaking” term \( \mathcal{D}_J \mathcal{D}_J^* E \) to cancel it. The final formula including terms of lower weights reads

(3.1) \[ -DQ_J(2E) + \frac{1}{6} \mathcal{D}_J \mathcal{D}_J^* f = 2 \Re \left( \frac{1}{3} E_{11,\bar{1}\bar{1}11} \right) \\
- E_{11,00} - \frac{2i}{3} \Re (E_{11,0\bar{1}\bar{1}} + i (A_{11} E_{1\bar{1}1} + i E_{1\bar{1}1} E_{\bar{1}1})) \]

\[ - \frac{1}{6} E_{11} R_{,11} + \frac{1}{6} E_{1\bar{1}1} R_{,11} - \frac{1}{6} (E_{1\bar{1}1} R_{,11}) \]

5
By the local slice theorem ([CL2]), there exists a smooth family of contact kernels of $D$ is subelliptic, the above expression was used in [CL1] to show the short-time solution of a certain regularized evolution equation. Using Lemma 2.1 repeatedly, we can write the highest-weight term of (3.1) as follows:

$$\frac{1}{3} E_{\bar{i}i,1} - E_{11,00} - \frac{2i}{3} E_{11,011} = \frac{1}{3} E_{11,1i1} - E_{11,0i1} + \frac{i}{3} (E_{11,1} A_{ii}),,1 + \frac{2i}{3} (E_{11} A_{ii1},,1) - \frac{1}{3} (RE_{11}),,1$$

On the other hand, we compute

$$\int_{M} R |E_{11},i|^2 dv_{\theta} = -\int_{M} (RE_{11},i E_{i\bar{i}} + R_{1i} E_{11},i E_{i\bar{i}}) dv_{\theta}$$

(by integration by parts)

$$= -\int_{M} [R(E_{11,11} + iE_{11,0} + 2RE_{11})E_{11} + R_{1i} E_{11},i E_{i\bar{i}}] dv_{\theta} \text{ (by (2.4))}$$

To see how we treat (3.1) in general, we first deal with the torsion= 0 case. By the local slice theorem ([CL2]), there exists a smooth family of contact diffeomorphisms $\psi_{t}$ with $\psi_{0} = \text{identity}$ so that $J_{t}^{} = \psi_{t}^{-1} J_{t} \psi_{t}$ lies in the local slice passing through $J$. Since the infinitesimal slice at $J$ is parametrized by the kernel of $D_{J}^*$, we have $D_{J}^* E = 0$ for $2E = \frac{d}{dt} |_{t=0} J_{t}$. 

Now applying (3.1) to such a deformation tensor $E$: $D_{J}^* E = 0$, substituting (3.2) in (3.1), and taking $L^{2}$-inner product with $E$, we obtain

$$\int_{M} 2Re\left(\frac{1}{3} |E_{11,11}|^2 + |E_{11,0}|^2 - iE_{11,0} E_{11,11} + \frac{1}{6} R|E_{11,1}|^2ight)$$

$$+ \frac{2i}{3} RE_{11,0} E_{i\bar{i}} - \frac{1}{6} RE_{11,i1} E_{i\bar{i}} + \frac{1}{6} R_{1i} E_{11,1} E_{i\bar{i}}$$

$$+ \frac{1}{6} R_{i1} - 2R_{i1} + \frac{1}{3} |R_{i1}^2| E_{11,1}^2 dv_{\theta}$$

by putting $A_{ii} = 0$, using (3.3) and integration by parts repeatedly.
Note that we reduce (3.4) to the formula (6.3) in [CL1] for \( R \) being a constant \( \hat{R} \). Furthermore, if \( \hat{R} > 0 \), the right-hand side of (3.4) is a positive definite quadratic hermitian form in \( E_{11,11}, iE_{11,0}, \) and \( \hat{R}E_{11} \).

It follows that \( E = 0 \) and \( 0 = \left. \frac{d}{dt} \right|_{t=0} J(t) = \left. \frac{d}{dt} \right|_{t=0} (\phi_t \circ \psi_t)^* \tilde{J}(t) = \mathcal{L}_X J + \left. \frac{d}{dt} \right|_{t=0} \tilde{J}(t) \) where the vector field \( X = \left. \frac{d}{dt} \right|_{t=0} (\phi_t \circ \psi_t) \). So \( J \) is rigid. For general \( R \), we require that the integrand in (3.4) is a (pointwise) positive definite quadratic hermitian form in \( E_{11,11}, iE_{11,0}, E_{11,1}, E_{11} \). Now the conditions in Corollary C can be deduced from basic linear algebra.

When the torsion does not vanish, the formula for \( -DQ_J(2E), E > \) with \( D_J^* E = 0 \) reads

\[
(3.5) \quad 0 = -DQ_J(2E), E > = \left. \int_M \left( \frac{2}{3} |E_{11,11}|^2 + 2|E_{11,0}|^2 \right. \right. \\
\quad \quad + \frac{1}{3} R |E_{11,1}|^2 + \frac{1}{3} R^2 + \frac{1}{6} \mathcal{A}_B R + 6|A_{11}|^2 + \frac{8i}{3} (A_{11,11} - A_{11,11}) |E_{11}|^2 \right. \\
\quad \quad + 2 R^2 \left. \left. -iE_{11,0} E_{11,11} + 2i \right| \left. \left. R E_{11,0} E_{11} - \frac{1}{6} R E_{11,11} E_{11} \right. \right. \right. \\
\quad \quad \left. \left. \left. + \left( \frac{1}{6} R \right) \right. \left. - 2i A_{11,11} E_{11,11} \right. \right. \right. \\
\quad \quad \left. \left. \left. - \frac{2i}{3} A_{11,11} E_{11,1} E_{11} - \frac{5i}{3} A_{11} E_{11,1} E_{11,1} \right) \right) \right) \right) dv_0
\]

There are non-cross terms like \( A_{11,1} E_{11,1} E_{11,1} \) and \( A_{11} E_{11,1} E_{11,1} \) in (3.5). We need an inequality to deal with \( A_{11} E_{11,1} E_{11,1} \).

**Lemma 3.1.** Let \( \lambda, \rho \) be real numbers. Then

\[
(3.6) \quad 2 \lambda \int_M R |E_{11,1}|^2 dv_0 \leq \lambda^2 \int_M |E_{11,1}|^2 dv_0 + \rho^2 \int_M R^2 |E_{11}|^2 dv_0 \\
\quad \quad - \lambda \rho \int_M (R \left. (\frac{2i}{3} A_{11,1} E_{11,1} E_{11} + R_\tilde{E}_{11,1} E_{11}) \right) dv_0
\]

For the term \( A_{11,1} E_{11,1} E_{11,1} \), we use the following estimate:

\[
(3.7) \quad 2 \mathcal{R}(\frac{2i}{3} A_{11,1} E_{11,1} E_{11}) \geq - 2 \frac{2}{3} |A_{11,1}|^2 |E_{11,1}|^2 - 2 \frac{2}{3} |A_{11,1}|^2 |E_{11}|^2.
\]

(To deduce (3.7), replace \( a, b \) by \( 2R \) in the basic inequality \( 2R(ab) \geq -|a|^2 - |b|^2 \) with \( a = -iE_{11,1}, b = E_{11}, \) and \( a^2 = A_{11,1} \))

The reason for taking fractional exponents in (3.7) is to make our conditions invariant under the scale change of contact form by a positive constant multiple as we’ll see later. Take a small amount of \( \int_M |E_{11,11}|^2 dv_0 \) and \( \int_M R^2 |E_{11}|^2 dv_0 \) to gain the term \( \int_M R |E_{11,1}|^2 dv_0 \) by (3.6) in the right-hand side of (3.5) while keeping the quadratic hermitian form in \( E_{11,11}, iE_{11,0}, E_{11,1}, E_{11,1}, E_{11}, \) positive definite at least for \( R = constant > 0, A_{11} = 0 \). For instance, we can take \( \lambda = \rho = \frac{3}{2} \) in (3.6) and then use it and (3.7) in estimating the right-hand side of (3.5). The final result reads
Theorem B is larger than 0. Here change by multiplying $k^{-3}$ while $A_{11,1}A_{\bar{1}1,1}$, and $R_{1,1}$ change by multiplying $k^{-\frac{1}{2}}$. Similarly $A_{11,11}, A_{\bar{1}1,11}$, and $\Delta_b R$ change by multiplying $k^{-2}$. So the conditions (3.11),(3.12) are invariant under the change of contact form by a positive constant multiple. Now Theorem B follows from (3.8) under the conditions (3.11),(3.12).
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