EARLY-AGE EVOLUTION OF THE MILKY WAY RELATED BY EXTREMELY METAL-POOR STARS

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ABSTRACT

We exploit the recent observations of extremely metal-poor (EMP) stars in the Galactic halo and investigate the constraints on the initial mass function (IMF) of the stellar population that left these low-mass survivors of \(\text{[Fe/H]} \lesssim -2.5\) and the chemical evolution that they took part in. In previous study, the high-mass nature of the IMF with typical mass \(\approx 10 M_\odot\) for the stars of the EMP population and the overwhelming contribution of low-mass members of binaries to the EMP survivors are derived from the statistics of carbon-enriched EMP stars with and without the enhancement of \(\alpha\)-process elements. We first examine the analysis to confirm their results for various assumptions on the mass-ratio distribution function of binary members. As compared with the uniform distribution they used, the increase or decrease function of the mass ratio gives a higher- or lower-mass IMF. For the independent distribution a lower-mass IMF results in both the members in the same IMF, but the derived ranges of typical mass differ less than by a factor of 2 and overlap for the extreme cases. Furthermore, we prove that the same constraints are placed on the IMF from the surface density of EMP stars estimated from the surveys and the chemical evolution consistent with the metal yields of theoretical supernova (SN) models. We then apply the derived high-mass IMF with the binary contribution to show that the observed metallicity distribution function (MDF) of EMP stars can be reproduced not only for the shape but also for the number of EMP stars. In particular, the scarcity of stars below \(\text{[Fe/H]} \sim -4\) is naturally explained in terms of hierarchical structure formation, and there is no indication of significant changes in the IMF for the EMP population. The present study indicates that three hyper-metal-poor/ultra-metal-poor stars of \(\text{[Fe/H]} < -4\) are the primordial stars that were born as the low-mass members of binaries before the host clouds were polluted by their own SNe.

Key words: Galaxy: formation – Galaxy: halo – stars: abundances – stars: carbon – stars: luminosity function, mass function – stars: Population II

1. INTRODUCTION

To reveal the nature of extremely metal-poor (EMP) stars in the Galactic halo is the key to the understanding of the formation process of the Galaxy as well as of the mechanism of star formation in the primordial and very metal-poor gas clouds. Because of the very low abundances of iron and other metals, these stars are thought to be survivors from the early days, and hence, are expected to carry precious information about the early universe when they were born while they reside in our nearby space. Over the past decade, a lot of EMP stars have been discovered by the HK survey (Beers et al. 1992) and the Hamburg/ESO (HES) survey (Christlieb et al. 2001), which enables us to use halo EMP stars as a probe into the early universe. The number of known EMP stars exceeds several hundreds even if we limit the metallicity range below \(\text{[Fe/H]} < -2.5\) and disclose the metallicity distribution function (MDF) of these stars (Beers & Christlieb 2005b).

One of their observed characteristics is a very low frequency of stars below the metallicity \(\text{[Fe/H]} \sim -4\). Despite that more than \(\sim 160\) stars have been registered in the metallicity range of \(-4 \lesssim \text{[Fe/H]} \lesssim -3\) by high-dispersion spectroscopy (e.g., see the SAGA Database; Suda et al. 2008), only three stars were found well below this metallicity; two hyper-metal-poor (HMP) stars of \(\text{[Fe/H]} < -5\), HE 0107−5240 (\(\text{[Fe/H]} = -5.3\); Christlieb et al. 2002) and HE 1327−2326 (\(\text{[Fe/H]} = -5.4\); Frebel et al. 2005), and one ultra-metal-poor (UMP) star of \(-5 < \text{[Fe/H]} < -4\), HE 0557−4840 (\(\text{[Fe/H]} = -4.8\); Norris et al. 2007). This has attracted wide interest, in particular, before the discovery of HE 0557−4840 in between the metallicity of \(-5 < \text{[Fe/H]} < -4\). Karlsson (2005) points out that such a metallicity cut-off can be interpreted as a result of the metal spreading process in the stochastic and inhomogeneous chemical-enrichment model. Karlsson (2006) then introduce a period of low or delayed star formation due to negative feedback by the Population III stars, during which metals spread to explain the very low iron-abundance of HMP stars with the carbon yield from rotating stellar models by Meynet & Maeder (2002). Prantzos (2003) argues an early infall phase of primordial gas to alleviate the paucity of low-metallicity stars. Tumlinson (2006) adopts a semi-analytic approach for the hierarchical structure formation and presents the model of inhomogeneous Galactic chemical evolution in an attempt to reproduce the statistical features of EMP stars and the re-ionization of the universe. He addresses the constraints on the initial mass function (IMF) of population III stars, arguing a high-mass IMF of mean mass at \(\langle M \rangle \simeq 8-42 M_\odot\). Salvadori et al. (2007) also take a similar approach to investigating the chemical evolution of our Galaxy with the mass outflow from mini-halos. In these former works, the low-mass star formation under the metal-deficient condition is introduced in rather arbitrary ways, and a proper explanation is yet to be devised about the nature and origin of HMP/UMP stars.

One of the decisive ingredients in studying the structure formation and chemical evolution of the Galactic halo is the IMF of stars in the early days. Most of existing studies have assumed the IMF of EMP stars to be more or less similar to that of metal-rich populations except for HMP and UMP stars. From the observations, however, we know that the EMP stars have the distinctive feature that a fair proportion of them show surface carbon enhancement relative to iron, the proportion by far larger than the stars of younger populations (Rossi et al. 1999). In
addition, it is revealed that carbon-enhanced extremely metal-poor (CEMP) stars are divided into two sub-groups, CEMP-s and CEMP-nos, according to the presence and absence of the enhancement of s-process elements (Ryan et al. 2005; Aoki et al. 2007). This also forms a striking contrast with the fact that their correspondences among the younger populations, CH stars and Ba stars, are all observed to exhibit the enhancement of s-process elements. Since the EMP survivors are low-mass stars, the enrichment of these elements are expected only through the mass transfer and/or the wind accretion from the asymptotic giant branch (AGB) primaries in the binaries. Assuming this binary scenario and the same carbon enhancement as the stars of younger populations, Lucatello et al. (2005) argue an IMF with a typical mass of $M_{\text{md}} \sim 0.79 M_\odot$ for EMP stars from the surplus of CEMP-s stars. Previously, Abia et al. (2001) have also asserted an IMF peaking in the intermediate-mass range of 4–8 $M_\odot$ for population III stars from the consideration of Galactic chemical evolution with CN enrichment among the EMP stars. Furthermore, an IMF with $M_{\text{md}} \sim 1.7$–2.3 $M_\odot$ has been discussed for the old halo stars from the MACHO observation in relation to the prospect that the observed microlensing may be caused by an alleged population of white dwarfs (Adams & Laughlin 1996; Chabrier et al. 1996).

In order to use the carbon-enhancement to constrain the IMF, we should properly take into account the evolutionary peculiarity of EMP stars. It is known that for the stars of [Fe/H] $< -2.5$, there are two mechanisms of carbon enhancement, while only one mechanism for the stars of younger populations, Populations I and II, and also, that a different mode of s-process nucleosynthesis works (Fujimoto et al. 2000; Suda et al. 2004; Iwamoto et al. 2004; Nishimura et al. 2008). Applying these theoretical understandings to the binary scenario, Komiya et al. (2007, referred to as Paper I in the following) find that the IMF for EMP stars has to be high-mass with a typical mass of $M_{\text{md}} \sim 10 M_\odot$ to explain the observed statistic features of both CEMP-s and CEMP-nos stars. In particular, as a consequence, it follows that the majority of EMP stars, including CEMP stars, were born as low-mass members of binary systems with the primary stars which have shed their envelope by mass loss to be white dwarfs and have exploded as supernovae (SNe). Tumlinson (2007a) discusses the binary scenario for EMP stars in a similar way.

The purpose of this paper is twofold: first to demonstrate the robustness of the high-mass IMF derived in Paper I, and then to discuss the implications for the formation and early evolution of Galaxy. In the following, we make a distinction between the total assembly of EMP stars that were born in the early Galaxy, including massive stars which have already exploded as SNe, and the low-mass EMP stars that are still alive in the nuclear burning stages by calling the former “EMP population” and the latter “EMP survivors.” In deriving the constraints on the IMF of stars for the EMP population, one has to make assumptions on binary characteristics, among which the most crucial is the distribution function of the mass ratio between the primary and secondary stars in binaries. Paper I adopts a flat distribution for simplicity. It seems plausible from the observations of the stellar systems of younger populations (Duquennoy & Mayor 1991; Mayor et al. 1992), and yet it is true that the mass-ratio distribution is yet to be properly established both observationally and theoretically even for the binaries of younger populations. Several different mechanisms have been proposed for binary formation, such as the fragmentation during the collapse and capture of formed stars, and are thought to give different mass-ratio distributions (see also, e.g., Goodwin et al. 2007 and the references therein). The distribution may increase or decrease with the mass ratio, or the two stars may form in the same IMF as suggested for the capture origin. In this paper, we examine the dependence of the resultant IMF on the assumed mass-ratio distributions of various functional forms, including the independent coupling of both the stars in the same IMF to demonstrate that the high-mass nature of the IMF of the EMP population is essentially unaltered.

The recent large-scaled surveys of EMP stars provide additional information on the early history of the Galactic halo. A fairly large number of known metal-poor stars (144 and 234 stars of [Fe/H] $< -3$ by the HK and HES surveys, respectively) makes it feasible to discuss the MDF (Beers et al. 2005). Moreover, the significant coverage of the celestial sphere (6900 and 8225 deg$^2$ by the HK and HES surveys, respectively; Christlieb 2003; Beers & Christlieb 2005b) allows us to consider the total number of EMP survivors in the Galactic halo. We demonstrate that the latter also places an independent constraint on the IMF of the EMP population in combination with the metal yields produced by EMP SNe if the binary contribution is properly taken into account.

We then apply the IMF, thud derived, to discuss the chemical evolution in which the stars of the EMP population take part. It is shown that the resultant IMFs can reproduce the number and slope of observed MDFs for EMP stars, and also, to give an explanation for the scarcity and origin of HMP/UMP stars with the effects of the hierarchical structure formation process included. In this paper, we deal only with iron production by SNe since we are interested in the MDF, and discuss the basic characteristics of hierarchical structure formation by using simple analytic approximations. Tumlinson (2007b) studies low-mass star formation taking into account the contribution of binary stars, but his approach is different from ours in the uses of the hypothesized IMF with the effect of the cosmic microwave background. In addition, he considered only the CEMP-s stars, but not CEMP-nos stars or the MDF.

This paper is organized as follows. In Section 2, we discuss the constraints on the IMF of the EMP population from the statistics of CEMP stars and from the total number of EMP survivors in our Galaxies. In Section 3, we investigate the metallicity distribution of EMP stars in the Galactic halo with the formation process of the Galaxy taken into account. Then our conclusions follow with a discussion of the origin of the observed MDF and also of HMP stars. In the Appendix, we re-discuss the relationship between the number of EMP survivors, estimated from the surveys, and the metal production by the EMP SNe with the binary contribution taken into account, to demonstrate that they entail the same IMFs as drawn independently from the statistics of CEMP stars.

2. CONSTRAINTS ON THE IMF OF EMP STARS

In this section, we revisit the problem of constraining the IMF for the stars of the EMP population from the observations of EMP survivors studied in Paper I. The method is based on an analysis of statistics of CEMP stars in the framework of the binary scenario, and hence involves the assumptions of EMP binary systems. We start by reviewing the method and assumptions used in Paper I in deriving the constraints on the IMF of EMP population stars. We first investigate the dependence of the resultant IMF on these assumptions, in particular of the mass-ratio distribution of binary members. We then discuss iron production by EMP population stars in relation
to the total number of EMP survivors, estimated from the HK and HES surveys, to assess the constraints on the IMF through the chemical evolution of the Galactic halo.

2.1. Method and Basic Assumptions

We give an outline of our method in studying the statistics of CEMP stars and the chemical evolution of the Galactic halo with a discussion of the assumptions involved, and a brief summary of the observational facts that our study relies on.

2.1.1. Statistics of CEMP Stars

Our method is founded on the results of stellar evolution that the stars of \([\text{Fe}/\text{H}] \lesssim -2.5\) and of mass \(< 3.5 \, M_\odot\) undergo hydrogen mixing into the helium convection during the helium core or shell-flashes differently from the stars of younger populations, Populations I and II (Fujimoto et al. 1990, 2000). This triggers the helium-flash-driven deep mixing (He-FDDM) to carry out carbon to the surface (Hollowell et al. 1990). It is necessarily accompanied by \(s\)-process nucleosynthesis in the helium convection as mixed protons are transformed into neutrons (Suda et al. 2004; Iwamoto et al. 2004; Nishimura et al. 2008). For EMP stars, He-FDDM works as a mechanism to enrich both carbon and \(s\)-process elements in their surface in addition to the third dredge-up (TDU), the latter of which works in the stars of \(M \sim 1.5 \, M_\odot\) in common with the stars of younger populations.

Consequently, the origins of two subgroups of CEMP stars are identified with these two mechanisms. The CEMP-s and CEMP-no stars stem from the low-mass members of EMP binaries with the primaries in the mass ranges of \(0.8 \, M_\odot < M < 3.5 \, M_\odot\) and \(3.5 \, M_\odot \leq M \leq M_{\text{up}}\), respectively. Here \(M_{\text{up}}\) is the upper limit to the initial mass of stars for the formation of white dwarfs. We take \(M_{\text{up}} = 6.0 \, M_\odot\) (Cassisi & Castellani 1993, see also Siess 2007), which is also taken to be the lower mass limit to the stars that explode as SNe. This is the fundamental premise of our study. Among the CEMP-no stars, there are stars that show different characteristics such as CS22892–052 with a large enhancement of \(r\)-process elements. They may have different origins according to the scenarios such as proposed in connection to SNe yields (e.g., Tsujimoto & Shigeyama 2001; Umeda & Nomoto 2003; Wänajo et al. 2006). Accordingly, the CEMP-no stars may be the admixture of the stars of different origins. Since the ratio between the CEMP-s and CEMP-no stars depends strongly on the IMF, as shown from Paper I, our results will not be affected as long as the CEMP-no stars contain those with the AGB mass transfer origin that we propose.

For the formation of CEMP stars in binary systems, the initial separation, \(A\), has to be large enough to allow the primary stars to evolve through the AGB stage without suffering from the Roche lobe overflow, but small enough for the secondary stars to accrete a sufficient mass of the wind to pollute their surface with the envelope matter processed and ejected by the AGB companion. The lower bound, \(A_{\text{min}}(m_1, m_2)\), to the initial separation is estimated from the stellar radii of EMP stars taken from evolutionary calculation (Suda & Fujimoto 2007), where \(m_1\) and \(m_2\) are the masses of primary and secondary stars. The AGB star is assumed to eject the carbon enhanced matter of \([\text{C}/\text{H}] = 0\) with the wind velocity \(v_{\text{wind}} = 20 \, \text{km s}^{-1}\) until it becomes a white dwarf, and we define CEMP stars as \([\text{C}/\text{Fe}] \lesssim 0.5\). The upper bound, \(A_{\text{max}}(m_1, m_2)\), is estimated by the amount of accreted matter calculated by applying the Bondi–Hoyle accretion rate,

\[
\frac{dm_2(t)}{dt} = - \frac{G^2 m_2(t)^2}{A(t)^2 v_{\text{rel}}(t)^2} v_{\text{wind}} \times \frac{dm_1(t)}{dt},
\]

in the spherically symmetric wind from the companion, and \(v_{\text{rel}}\) is the relative velocity of the secondary star to the wind. Accreted matter is mixed in surface convection of depth \(0.35 \, M_\odot\) and 0.0035 \(M_\odot\) in mass for giants and dwarfs, respectively. For example, for the stellar metallicity \([\text{Fe}/\text{H}] = -3.5\), the mass of accreted matter has to be larger than \(3.5 \times 10^{-4} \, M_\odot\) and \(3.5 \times 10^{-6} \, M_\odot\), and hence, the upper bounds are \(\sim 100 \, \text{AU}–1000 \, \text{AU}\) for dwarfs, respectively. It is pointed out that the molecular diffusion (Weiss et al. 2000) and/or the thermohaline mixing (Stancliffe et al. 2007) work in the envelope of EMP dwarfs to lower the surface abundance of accreted matter by nearly an order of magnitude. If these effects are included, it demands a larger accreted mass, and hence, a decrease of the upper bound of binary separation to the carbon enrichment for the dwarf low-mass members with carbon by \(-0.5\) dex. This will not affect our results so much since the upper bound is itself sufficiently large to exceed the separation at the peak of period distribution (see below). In addition, it has little effects on the giants which we mainly deal with in the following because of far deeper surface convection.

If we specify the IMF, \(\xi_s(m)\), and the distributions of binary parameters, we can evaluate the frequency of CEMP-s and CEMP-no stars, and through comparison with the observations, we may impose the constraints on the IMF and on the binary parameters. The numbers of CEMP-s and CEMP-no stars currently observable in flux-limited samples are given by

\[
\psi_{\text{CEMP-s}} = f_b \int_{0.8 M_\odot}^{3.5 M_\odot} dm_2 N_s(L[m_2]) \int_{0.8 M_\odot}^{3.5 M_\odot} dm_1 \xi_b(m_1) \frac{n(q)}{m_1} \times \int_{A_{\text{up}}(m_1, m_2)}^{A_{\text{up}}(m_1, m_2)} f(P) \frac{dP}{da} \, da,
\]

\[
\psi_{\text{CEMP-no}} = f_b \int_{0.8 M_\odot}^{3.5 M_\odot} dm_2 N_s(L[m_2]) \int_{0.8 M_\odot}^{3.5 M_\odot} dm_1 \xi_b(m_1) \frac{n(q)}{m_1} \times \int_{A_{\text{up}}(m_1, m_2)}^{A_{\text{up}}(m_1, m_2)} f(P) \frac{dP}{da} \, da,
\]

where \(f_b\) is the binary fraction, \(n(q)\) is the distribution of the mass ratio, \(q \equiv m_2/m_1\), and \(f(P)\) is the distribution of the period of binaries, and \(N(L)\) is the probability of the stars in the Galactic halo with the luminosity \(L\) in the survey volume of the HES survey. Note that all of them stem from the low-mass members of the binary since it is only a very small fraction of stars that have experienced He-FDDM to develop CEMP-s characteristics in themselves and now stay on the AGB. Similarly the total number of EMP survivors is given by

\[
\psi_{\text{surv}} = \int_{0.8 M_\odot}^{3.5 M_\odot} dm_1 N_s(L[m_1]) (1 - f_b) \xi_b(m) + f_b \int_{0.8 M_\odot}^{3.5 M_\odot} dm_1 \xi_b(m) \frac{n(q)}{m_1},
\]

\[
\times \int_{0.8 M_\odot}^{3.5 M_\odot} dm_2 N_s(L[m_2]) \int_{m_1}^{\infty} \frac{n(q)}{m_1} dm_1 \xi_b(m_1) \frac{n(q)}{m_1},
\]

(4)
with the contribution of the stars born as single under the IMF $\xi$, (the first term). The second and third terms give the number of EMP survivors formed as binary, $\psi_{\text{binary}}$. The stellar luminosity and lifetime are taken from the evolution calculation of EMP stars by Suda & Fujimoto (2007). The AGB primaries dredge up to increase their surface helium abundances, and hence, may cause the surface enrichment of helium to the companion stars in the binaries at the same time with the carbon enrichment, though both suffering the diﬀusion in the envelope convection (Catelan et al. 1996; Suda et al. 2006). The surface enrichment increases the luminosity during the red giant branch (RGB) evolution to shorten their RGB lifetime of polluted EMP stars nearly in inverse proportion of the luminosity, but the survey volume increases with the luminosity. For the flux limited sample, the observed number of EMP giants may rather increase with the surface enrichment in proportion to a half power under a constant density distribution. On the other hand, the increase in the luminosity occurs only after the hydrogen burning shell comes to take place in the shell to which the pollutants are carried in by surface convection, and hence, in the later stages for metal-poorer stars. Accordingly, helium enhancement will little affect our results of constraints on the IMF since the HES survey is thought to reach far enough so that the spatial distribution of halo stars decreases.

2.1.2. Model Parameters

In this paper, we assume that binary primary stars and single stars are born under the same IMF, i.e., $\xi(m) = \xi_s(m) = \xi_2(m)$. For the form of the IMF, we may well assume a lognormal function with the medium mass, $M_{\text{nd}}$, and the dispersion, $\Delta_{M}$, as parameters

$$\xi(m) \propto \frac{1}{m} \exp \left[ -\frac{(\log m - \log M_{\text{nd}})^2}{2\Delta_{M}^2} \right]. \quad (5)$$

In addition, we assume the binary fraction $f_b = 0.5$ in this paper. Our results are little affected by the assumption about $f_b$ since not only the CEMP stars but also most of the EMP survivors come from the secondary companions of binaries unless $M_{\text{nd}} < 0.8 M_\odot$, as seen later. As for the binary period, we may adopt the distribution derived for the nearby stars by Duquennoy & Mayor (1991),

$$f(P) \propto \frac{1}{P} \exp \left[ -\frac{(\log P - 4.8)^2}{2 \times 2.3^2} \right], \quad (6)$$

where $P$ is the period in units of days. The binary fractions and period distributions of halo stars are observed to be not significantly different from those of nearby disk stars (Latham et al. 2002; Carney et al. 2003). Additionally, it is shown in Paper I that this period distribution is consistent with the observations of CEMP stars for the periods of $P \lesssim 10$ yr confirmed to date (see Figure 3 in Paper I).

The mass-ratio distribution is an essential factor in discussing the evolution of binary systems, and yet it is not well understood. The mass-ratio distribution of metal-poor halo stars is investigated observationally (e.g., see Goldberg et al. 2003; Abt 2008), and yet subject to large uncertainties. Especially for the binary with intermediate-mass or massive primary stars, it is hard to know the mass-ratio distribution from the observations. Theoretically, neither the fragmentation of gas cloud nor the accretion process onto protobinaries is yet well understood even for Population I stars (e.g., Bate & Bonnell 1997; Ochi et al. 2005; Machida 2008). In order to test the assumption on the mass-ratio distribution, we investigate the constraints on the IMF for different mass-ratio distributions. In Paper I, the simplest flat distribution is assumed in Paper I among the possible distributions. In this paper, we test some other assumptions and discuss the dependence of IMF parameters on the mass-ratio distributions, as stated in Section 2.2.

We may define the coupling mass distribution function, $\chi(m_1, m_2)$, as the fraction of the binaries with a primary and secondary star in the mass range of $[m_1, m_1 + dm_1]$ and $[m_2, m_2 + dm_2]$, and write it in the form

$$\chi(m_1, m_2)dm_1 dm_2 = \xi(m_1)n(q)dm_1 \int_{m_1}^{m_1 + dm_1} \xi(m_2)\frac{dm_2}{m_1 dm_1}. \quad (7)$$

Here the initial mass function, $\xi$, of the primary star is assumed to be the same as the IMF of single stars, $n(q)$ is the mass-ratio distribution, for which we assume both extremities of increase and decrease functional forms in addition to the constant one, adopted in Paper I:

$$n(q) = \begin{cases} 1/(1 - 0.08 M_\odot/m_1) & \text{(Case A)} \\ 2q/[1 \times (0.08 M_\odot/m_1)^2] & \text{(Case B)} \\ q^{-1}/\ln(m_1 / 0.08 M_\odot) & \text{(Case C)} \end{cases} \quad (8)$$

Furthermore, we take up a different type of mass-ratio distribution that the primary and secondary stars independently obey the same IMF such as assumed by Lucatello et al. (2005). In this case, the coupling mass distribution function is given as a product of the same IMF as

$$\chi(m_1, m_2)dm_1 dm_2 = 2\xi(m_1)\xi(m_2)dm_1 dm_2 \quad \text{(Case D).} \quad (9)$$

We shall refer to this distribution function as “independent” coupling. From a comparison with Equation (7), we may write the mass-ratio function in the form $n(m_2, m_1) = 2m_1 \xi(m_2)$; it is should be noted, however, that the frequency of binaries with a primary star of mass $m_1$ is not normalized and increases with $m_1$ from zero to 2, as given by the integral

$$\int_{0.08 M_\odot/m_1}^{1} n(m_1, m_2)dm_2 = 2 \int_{0.08 M_\odot/m_1}^{m_1} \xi(m_2)dm_2.$$ 

With these specifications and with the assumed mass-ratio distribution function, we may compute the fractions of EMP survivors, $\psi_{\text{EMP}}(M_{\text{nd}}, \Delta_{M})$, and of both EMP-s stars, $\psi_{\text{EMP}-s}(M_{\text{nd}}, \Delta_{M})$, and $\psi_{\text{EMP}-no}(M_{\text{nd}}, \Delta_{M})$, and $\psi_{\text{EMP}}$ of the IMF parameters, medium mass $M_{\text{nd}}$ and dispersion $\Delta_{M}$, that can reproduce the statistics of CEMP stars consistent with observations.

2.1.3. Total Iron Yield of EMP Supernovae

We can pose another constraint from the total iron yield, $M_{Fe}$, of the EMP population and the total number, $N_{\text{EMP}}$, of the giant EMP stars. For $N_{\text{EMP}}$, estimated from the results of existing surveys, the total stellar mass, $M_{\text{EMP}}$, of the EMP population for an assumed IMF is given by

$$M_{\text{EMP}}(M_{\text{nd}}, \Delta_{M}) = \bar{m} N_{\text{EMP}} f_G. \quad (10)$$

where $f_G$ is the fraction of giant EMP survivors in all the stellar systems, born as EMP population, and $\bar{m}$ is the averaged mass of EMP population stars:

$$f_G = \left[ \xi(0.8 M_\odot) + f_b \int_{0.8 M_\odot}^{m_1} \xi(m_1)n(0.8 M_\odot/m_1)dm_1 \right] \Delta M_{\text{G}}, \quad (11)$$

where $\Delta M_{\text{G}}$ is the averaged mass of EMP population stars:
\[ \bar{m} = \int dm_{1}\{m_{1}\xi(m_{1}) + \frac{f_{n}}{m_{1}} \int m_{2}n(q)dm_{2} \}. \] (12)

The first terms of both equations denote the contributions by the stars born as single stars and as primary stars in the binaries and the second terms denote the contributions by the stars born as secondary stars in the binaries. The mass and mass range of EMP stars now on the giant branch are taken to be \( M = 0.8 \, M_{\odot} \) and \( \Delta M_{G} = 0.01 \, M_{\odot} \), based on the stellar evolution calculation of stars with \([Fe/H] = -3\), as in Paper I.

The massive stars of the EMP population have exploded as SNe to enrich the interstellar gas with metals. The amount of iron, \( M_{Fe,EMP} \), ejected by all the SNe of the EMP population of the total mass, \( M_{EMP} \), is given by

\[ M_{Fe,EMP} = \frac{M_{EMP}}{m} f_{SN}(Y_{Fe}) = N_{EMP,G} \frac{f_{SN}}{f_{G}} (Y_{Fe}), \] (13)

where \( f_{SN} \) is the fraction of the SNe that have exploded as SNe and given by

\[ f_{SN} = \int_{M_{SN}} dm_{1}\xi(m_{1})[1 + \frac{f_{S}}{m_{1}} \int_{M_{SN}} n(q)dm_{2}]. \] (14)

and \( (Y_{Fe}) \) is the averaged iron yield per SN, taken to be \( (Y_{Fe}) = 0.07 \, M_{\odot} \) in the following calculations.

These evaluations and the observed number of EMP giants, we can give the total iron yield of stars of the EMP population as a function of IMF parameters. Comparison with the total amount of iron estimated from the chemical evolution of the Galactic halo may impose constraints on the IMF parameters.

2.1.4. Observational Constraints

The first constraint is the number fraction of CEMP-s stars. HK and HES observations show that the EMP stars with \([\zeta/Fe] > 1 \) account for 20\%—25\% of EMP stars (e.g., Beers 1999; Rossi et al. 1999; Christlieb 2003). Cohen et al. (2005) suggest a slightly lower fraction of 14.4\% ± 4\% with the errors in the abundance analysis taken into account, while Lucatello et al. (2006) obtain a larger frequency of 21\% ± 2\% for the HERES (HES r-process enhanced star) survey sample, both for \([Fe/H] < -2.0\). It is claimed that the frequency of CEMP is higher at a lower metallicity of \([Fe/H] < -2.5\), but we have to subtract the contribution from the CEMP-no stars. In this paper, we adopt the observational constraint on the fraction of the CEMP-s stars at 10\%—25\%:

\[ 0.1 < \frac{\psi_{CEMP-s}(M_{md}, \Delta M)}{\psi_{surv}(M_{md}, \Delta M)} < 0.25. \] (15)

The second constraint is the number ratio between CEMP-no and CEMP-s stars. The observed frequency of CEMP-no to CEMP-s stars is \( \sim 1/3 \) or more (e.g., Ryan et al. 2005; Aoki et al. 2007). Aoki et al. (2007) point out that it increases for lower metallicity, reporting the ratio to be as large as 9/14 for \([Fe/H] \leq -2.5\). In addition, EMP stars enriched with nitrogen are found in number comparable with, or more than, CEMP-no stars (“mixed” stars; Spite et al. 2005), whose origin can be interpreted in terms of the same mechanism but with more massive primary companions that experience the hot bottom burning (HBB) in the envelope of the AGB. Some other scenarios for CEMP stars have been proposed (Umeda & Nomoto 2005; Meynet et al. 2006), but we assume that all CEMP stars are formed in binaries with the AGB in this paper.

We adopt the observational constraint on the relative frequency of CEMP-no to CEMP-s stars at 1/3 — 1:

\[ 1/3 < \frac{\psi_{CEMP-no}(M_{md}, \Delta M)}{\psi_{CEMP-s}(M_{md}, \Delta M)} < 1. \] (16)

We note that the above two constraints are not dependent on the total mass or on the spatial distribution of the stellar halo because they are concerned with the relative number ratios.

The third constraint is the total iron yield from the EMP population. The HES survey obtained 234 stars of \([Fe/H] < -3\) (Beers et al. 2005) as a result of the medium-resolution, follow-up observations of 40\% of the candidates, selected by the objective-prism survey of the nominal area \( S = 8225 \, \text{deg}^{2} \) (Beers & Christlieb 2005b). Taking the relative frequency between the giants and dwarfs (1.093) and the ratio of the stars of \([Fe/H] < -3\) and \([Fe/H] < -2.5\) (6\%:20\%) from their Table 3, we may estimate the total number of EMP giants in the Galactic halo in the flux limited sample at

\[ \sigma_{EMP,G} \simeq 410 \, \text{sr}^{-1}. \] (17)

We assume that all giant stars in the survey areas are observed because of the fairly large limiting magnitude of the HES survey (\( B = 17.5 \), about 2 mag deeper than for the HK survey), and neglect the spatial distribution of EMP giants for simplicity since sufficient information is not yet available (see Section 7.3 in Paper I for details). Then we have a total number of EMP giants \( N_{EMP,G} = 5.2 \times 10^{5} \) in the Galaxy.

On the other hand, the amount of iron necessary to promote the chemical evolution of the whole gas in the Galaxy of mass, \( M_{h} = 10^{11} \, M_{\odot} \), up to \([Fe/H] = -2.5\) is as much as

\[ M_{Fe,halo} = M_{h} X_{Fe,\odot} 10^{-2.5} = 10^{5.5} \, M_{\odot}. \] (18)

and the SNe of the EMP population should have provided this amount of iron unless there were other population(s) of stars that made iron without producing low-mass stars. Using Equation (13), this is transferred into a constraint on the IMF as

\[ M_{Fe,EMP} = 0.07 \, M_{\odot} \times 5.2 \times 10^{5} \frac{f_{SN}}{f_{G}} (M_{md}, \Delta M) \simeq 10^{5.5} \, M_{\odot}. \] (19)

The estimated number of EMP survivors may be subject to significant uncertainties. If we take into account the EMP stars in the outer halo and in the Galactic bulge that the HES survey cannot reach, \( N_{EMP,G} \) can be larger, which demands a smaller amount of iron produced per EMP survivor, and hence, a smaller number of SNe, leading to a lower-mass IMF. A lower-mass IMF also results if the binary fraction is smaller and/or if there is another source(s) of iron that does not accompany the low-mass star formation. On the other hand, if part of the SNe ejecta is other source(s) of iron that does not accompany the low-mass star formation, it demands a larger amount of iron produced per EMP survivor, and hence, a higher-mass IMF. Despite such uncertainties of both observations and theoretical assumptions, the constraints on the IMF derived from Equation (19) are rather robust since the ratio, \( f_{SN}/f_{G} \), is a rapidly varying function of the IMF.

2.2. Dependence on Mass-Ratio Distributions

For the four mass-ratio distributions formulated in Section 2.1.2, we can figure out, as a function of \( M_{md} \) and \( \Delta M \), the
obtained in Paper I. In Figure 1, the CEMP- mass-ratio distribution of Case A, which reproduces the results of the Galactic spheroid component (Chabrier 2003). Figures 2 and 3 present contour maps on the $M_{\text{md}}$–$\Delta M$ diagram for the fractions of CEMP-s stars and the ratio between the CEMP-no and CEMP-s stars, respectively.

The left top panels of these figures show the results for the flat mass-ratio distribution of Case A, which reproduces the results obtained in Paper I. In Figure 1, the CEMP-s fraction peaks at $M_{\text{md}} = 4.8 M_\odot$, slightly above the upper mass limit of the primary stars for CEMP-s. Note that when the secondary mass is specified, the mass distribution of primary stars peaks at mass smaller than $M_{\text{md}}$ for this mass-ratio function $[\propto \xi(m_1)/m_1$, see Figure 12 in Paper I]. Two ranges of $M_{\text{md}}$, 0.6–2.8 $M_\odot$ and 7.6–15.3 $M_\odot$ (light shaded parts), give IMFs compatible with the observations, separated by the overproduction of CEMP-s stars. The relative frequency of CEMP-no to CEMP-s stars is a steep increase function of $M_{\text{md}}$, and excludes the lower range of $M_{\text{md}}$ compatible with the CEMP-s fraction. The IMFs with $M_{\text{md}} = 4.8$–11.6 $M_\odot$ (dark shaded part) gives a compatible ratio with the observations. This range of $M_{\text{md}}$ lies in the mass range of primary stars of CEMP-no stars or even larger. Accordingly, the intersection of the light and dark shaded parts designates the ranges $M_{\text{md}} = 7.6$–11.6 $M_\odot$ that can explain both the statistics of CEMP stars, and hence, high-mass IMFs result for a dispersion $\Delta M = 0.33$.

In the $M_{\text{md}}$–$\Delta M$ diagram of Figure 2, the parameter space compatible with the observed CEMP-s fraction separates into two ranges for dispersion smaller than $\Delta M \approx 0.43$, converging to the narrow ranges around $M_{\text{md}} \approx 1$ and 4 $M_\odot$, respectively, as $\Delta M$ decreases. For larger dispersion, on the other hand, it merges into one part to a cover wider range. As for the ratio between the CEMP-no and CEMP-s stars, Figure 3 shows that the medium mass compatible with the observed ratio increases with dispersion to cover a wider range, from $M_{\text{md}} = 3.2$–3.7 $M_\odot$ at $\Delta M = 0.1$ through $M_{\text{md}} = 12.3$–100 $M_\odot$ at $\Delta M = 0.54$. Accordingly, for the IMFs that satisfy both the statistical constraints on the IMF of the EMP population for the four different assumptions on the mass-ratio distributions: Case A of $n(q) = \text{const}$ (top left panel), Case B of $n(q) \propto q$ (top right panel), Case C of $n(q) \propto 1/q$ (bottom left panel), and Case D of independent coupling (bottom right panel). Thin and thick solid lines denote the fraction of CEMP-s in the EMP survivors as a function of the medium mass $M_{\text{md}}$ with a dispersion of $\Delta M = 0.33$ for the EMP stars born as binaries $[\psi_{\text{CEMP-s}}(M_{\text{md}}, 0.33)/\psi_{\text{binary}}(M_{\text{md}}, 0.33)]$ and for the total EMP stars including the single stars born in equal number to the binaries $[\psi_{\text{EMP}}(M_{\text{md}}, 0.33)/\psi_{\text{binary}}(M_{\text{md}}, 0.33)]$, respectively. The broken line denotes the ratio of CEMP-no to CEMP-s stars $[\psi_{\text{CEMP-no}}(M_{\text{md}}, 0.33)/\psi_{\text{CEMP-s}}(M_{\text{md}}, 0.33)]$. Light and dark shaded areas denote the parameter ranges of $M_{\text{md}}$ for the IMFs that can give rise to the observed fraction of CEMP-s stars in the EMP survivors (10%–25%) and the observed ratio of CEMP-no to CEMP-s stars (1/3 – 1), respectively.
As a result, the IMFs consistent with both the statistics of EMP population with the use of the four cases of mass-ratio distributions as indicated on the top left corner of each panel. Numbers attached to solid lines denote the observed fractions of CEMP-\(s\) stars in proportion to \(\psi_{\text{CEMP-s}}(M_{\text{md}}, \Delta M)/\psi_{\text{surv}}(M_{\text{md}}, \Delta M)\). The shaded area denotes the parameter space for the IMFs compatible with the observed fraction of CEMP-\(s\) stars.

In summary, the medium mass range of EMP stars is larger for a given \(M_{\text{md}}\) and the range of \(\Delta M\) is larger for a given \(M_{\text{md}}\) than for Case A (\(\Delta M = 0.2\)) and beyond \(M_{\text{md}} = 100 M_{\odot}\) for \(\Delta M > 0.62\).

For the mass-ratio distribution function decreasing with \(q\) of Case B (right top panel), the portion of binaries that have secondary stars surviving to date decreases with the mass of primary stars in proportion to \((m_2/m_1)^{-2}\), more steeply than in proportion to \((m_2/m_1)^{-1}\) for a flat mass-ratio distribution in Case A. Since the average mass of the primary stars is smaller for a given EMP star, the fraction of CEMP-\(s\) stars is larger for a given \(M_{\text{md}}\), and the peak shifts to larger \(M_{\text{md}}\), as compared with Case A. In Figure 1, the \(M_{\text{md}}\) of IMFs compatible with the observed fraction of CEMP-\(s\) stars separates into two ranges, as in Case A, but the in-between gap is larger; the higher mass range shifts upward in mass to a greater extent \((M_{\text{md}} = 14.5–28 M_{\odot})\) than the smaller mass range shifts downward \((M_{\text{md}} = 0.38–2.4 M_{\odot})\). This also causes a smaller ratio of CEMP-\(s\) to CEMP-\(s\) stars for a given \(M_{\text{md}}\), and hence, the IMFs consistent with the observed ratio shift to a larger mass of \(M_{\text{md}} = 8.9–21.6 M_{\odot}\), as compared with that for Case A. As a result, the IMFs consistent with both the statistics of CEMP stars turn out to be higher mass by a factor of \(\sim 2\) than for Case A \((M_{\text{md}} = 14.5–21.6 M_{\odot}\) for \(\Delta M = 0.33\)). In Figure 2, we see that the range of \(M_{\text{md}}\) compatible with the observed fractions of CEMP-\(s\) star (shaded area), separates into two and the higher range shifts to larger mass for a given \(\Delta M\). Similarly, in the Figure 3, the observed ratio of the CEMP-\(s\) stars also demands larger \(M_{\text{md}}\), and the range of \(M_{\text{md}}\) of IMFs compatible with the observation increases rapidly with \(\Delta M\) to exceed 100 \(M_{\odot}\) for \(\Delta M = 0.62\). In order to satisfy both conditions of CEMP star observations, the IMFs fall in the range of higher medium mass and in a rather narrow range of dispersion, lying in the parameter space of \(M_{\text{md}} > 7.1 M_{\odot}\), larger by a factor of \(\sim 1.3\) than for Case A, and \(\Delta M > 0.2\) and of \(\Delta M = 0.45–0.58\) for \(M_{\text{md}} = 100 M_{\odot}\).

For a mass ratio function decreasing with \(q\) of Case C, we see the opposite tendency of Case B (the bottom left panels of Figure 1–3). The portion of EMP binaries whose low-mass members survive to date depends only weakly on the primary mass \((\propto \log m_1)\) so that the fraction of CEMP-\(s\) stars reduces because of larger contributions from the binaries with more massive primaries. As seen in Figure 1, the fraction of CEMP-\(s\) stars in the total EMP survivors is well below the upper bound of the observations, and hence, the \(M_{\text{md}}\) compatible with the observations merges into one narrower range of \(M_{\text{md}} = 1.1–15.6 M_{\odot}\) for \(\Delta M = 0.33\). The observed ratio of CEMP-\(s\) to CEMP-\(s\) stars can be reproduced also by the IMFs with a smaller \(M_{\text{md}}\) by a factor of \(\sim 2\) than in Case A \((M_{\text{md}} = 3.2–7.5 M_{\odot})\). Accordingly, the \(M_{\text{md}}\) for the IMFs consistent with both CEMP star statistics are smaller.
by a factor of 1.5–2.4 than for Case A (the mass range \(M_{\text{md}} = 3.2–7.5 M_\odot\) for \(\Delta_M = 0.33\)). In Figure 2, the range of \(M_{\text{md}}\) for the IMFs, compatible with the observed CEMP-s fraction varies only little with \(\Delta_M\), and is restricted in the range between \(M_{\text{md}} = 1.1–23 M_\odot\), though it separates into two for small \(\Delta_M < 0.31\). As shown in Figure 3 the dependence of the ratio of CEMP-no to CEMP-s stars on \(\Delta_M\) is also weaker than for Case A. Consequently, the IMFs can reproduce both CEMP star statistics with the mass as small as \(M_{\text{md}} = 3.3 M_\odot\), smaller by a factor of \(\sim 0.6\) than for Case A, but differently from the above two cases, an upper bound is placed at \(M_{\text{md}} = 23 M_\odot\), regardless of the dispersion with a lower bound of \(\Delta_M > 0.21\).

The bottom right panels depict the results for the “independent” coupling of Case D. For this case, the number of EMP survivors produced per binary is independent of the mass, \(m_1\), of primary stars, while the binary frequency itself increases with \(m_1\). The former is similarly to Case C, and then, the production of EMP survivors from the binaries with massive primary poses a severe constraint on the high-mass side of IMFs. On the other hand, the latter favors the production of CEMP-s stars as compared with the low-mass binaries of \(m_1 \lesssim 0.8 M_\odot\). They both shift the IMFs, compatible with the observed fraction of CEMP-s stars, to smaller \(M_{\text{md}}\). In addition, the single stars, born in the same number of binaries, contribute to a significant fraction of EMP survivors, increasing from 1/3 up to 1/2 for smaller \(M_{\text{md}}\) for \(M_{\text{md}} < 0.8 M_\odot\) since the low-mass binaries are counted as one object. As a result, the maximum fraction of CEMP-s stars remains below the upper limit of the observed range, which makes the \(M_{\text{md}}\) for the IMFs that can reproduce the observation lie in a single range within a relatively small upper bound.

The observed ratio of CEMP-no to CEMP-s stars demands also lower-mass IMFs, as for Case C. Accordingly, the IMFs that can reproduce both CEMP star statistics fall in the narrowest range of \(M_{\text{md}} = 2.5–7.0 M_\odot\) with a rather small upper mass limit, almost irrespective of the dispersion, in the \(M_{\text{md}}–\Delta_M\) diagram in Figure 3. The CEMP-s star fraction remains smaller than \(\sim 20\%\) because of the contribution of the stars born as single.

In conclusion, the statistics of CEMP stars demand the IMFs for the EMP population, peaking at the intermediate-mass stars or the massive stars, by far higher mass than those of Population I and II stars, irrespective of the assumed mass-ratio distribution. The presence of CEMP-no stars in a significant number of the CEMP-s stars excludes the IMFs of small mass. The derived mass range varies by a factor of \(\sim 2\), from the highest \(M_{\text{md}} > 7.1 M_\odot\) for the mass-ratio distribution of increase function of the mass ratio (Case B) to the lowest \(2.5 < M_{\text{md}} / M_\odot < 7\) for the mass-ratio distribution of “independent” coupling (Case D). This tendency is explained in terms of the difference in the averaged mass of the primary companion of the EMP survivors; if the contributions to the EMP survivors decrease rapidly with the mass of primaries, relatively higher-mass IMFs result without an upper mass limit imposed, while if the

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**Figure 3.** Contour maps of the number ratio between CEMP-s stars and CEMP-no stars on the \(M_{\text{md}}–\Delta_M\) plane for the four cases of mass-ratio distributions, as indicated in the top left corner of each panel. Attached numbers denote the ratios \([\psi_{\text{CEMP-no}}/(M_{\text{md}}, \Delta_M)]/\psi_{\text{CEMP-s}}(M_{\text{md}}, \Delta_M)\]. The dark shaded area, superimposed on the light shaded area taken from Figure 2, designates the parameter ranges for the IMFs of the EMP population that fulfill both constraints on the fraction of CEMP-s and on the ratio between CEMP-no and CEMP-s.
2.3. Constraints from Galactic Chemical Evolution

In this section, we discuss that additional constraints can be derived from the relationship between the total number of EMP survivors and the iron yields from the EMP population.

Figure 4 shows contour maps of the total stellar mass, \( M_{\text{EMP}} \), and the IMF properties of EMP population, in Equation (10), necessary to leave the observed number of EMP survivors in the \( M_{\text{md}}-\Delta M \) diagram. The total stellar mass increases for higher mass IMFs to produce the given number of low-mass survivors. For the flat mass-ratio distribution (Case A; left panel), the total stellar mass of the EMP population is essentially determined by \( M_{\text{md}} \) in proportion to \( M_{\text{md}}^2 \) and only weakly dependent on \( \Delta M \) for large \( M_{\text{md}} \), since almost all EMP survivors formed as secondary (i.e., less massive) stars, since the dependence of the SN fraction differs across the border of \( M_{\text{md}} \approx M_{\text{up}} \); the amount of produced iron increases (or decreases) with \( \Delta M \) for \( M_{\text{md}} < M_{\text{up}} \) (or \( M_{\text{md}} > M_{\text{up}} \), little dependent on \( \Delta M \) for \( M_{\text{md}} \approx M_{\text{up}} \).

For the “independent” coupling (Case D; left panel), the fractions of giant EMP survivors and SNe among EMP population stars are given by

\[
f_G = (1 + f_b)\xi(0.8)\Delta M_G \tag{20}
\]

\[
f_{\text{SN}} = (1 + f_b) \int_{M_{\text{md}}}^{M_{\text{up}}} dM \xi(m_1). \tag{21}
\]

The total mass and the amount of iron production of the EMP population are sensitive both to \( M_{\text{md}} \) and \( \Delta M \), especially for small \( \Delta M \) and large \( M_{\text{md}} \) in contrast to the other cases. For small \( \Delta M \), therefore, the fraction of low-mass stars varies greatly with \( M_{\text{md}} \), and both the contours of \( M_{\text{EMP}} \) and \( M_{\text{Fe}}(M_{\text{EMP}}) \) converge to \( M_{\text{md}} \approx 2-3 M_\odot \). As \( \Delta M \) increases, the differences from Case A diminish since the IMFs tend to extend into the low-mass stars, and in particular, for \( \Delta M \geq 0.4 \) and \( M_{\text{md}} \approx 3 M_\odot \), the contours in both panels resemble each other to run through similar parameter spaces.

From comparison with the total amount of iron \( M_{\text{Fe,EMP}} \), necessary for the chemical evolution, in this diagram, the parameter space where \( M_{\text{Fe,EMP}}(M_{\text{md}}, \Delta M) \gg M_{\text{Fe,halo}} = 10^{-5} M_\odot \) is excluded by the overproduction of iron or by the underproduction of EMP survivors. For the parameter space where \( M_{\text{Fe,EMP}}(M_{\text{md}}, \Delta M) \ll M_{\text{Fe,halo}} \), on the other hand, the stars of the EMP population can leave the number of EMP survivors currently observed but are short of iron production, so that the chemical evolution demands other sources of iron production without producing low-mass stars that survive to date. For a flat mass-ratio distribution, the IMFs that can satisfy the condition of iron production coincide the IMFs, derived above, from the statistics of CEMP stars (shaded area) in the parameter range of \( M_{\text{md}} \approx 10-16 M_\odot \) and \( \Delta M \approx 0.3-0.6 \). For “independent” coupling, the parameter range of IMFs that satisfy the condition of iron production also overlaps the shaded area of the parameter range, derived above from the statistics of CEMP stars, but with mass \( M_{\text{md}} \approx 3.5-5.1 M_\odot \), slightly smaller than for Case A and only for a small dispersion of \( \Delta M \approx 0.2-0.35 \). For larger \( \Delta M \), even the highest-mass IMFs of \( M_{\text{md}} = 7.5 M_\odot \) are slightly short of, or at most marginally sufficient at, iron production.

For the other two mass-ratio distributions of \( n \approx q \) (Case B) and \( n \approx 1/q \) (Case C), the iron production, \( M_{\text{Fe,EMP}} \), with a given IMF resulting to be larger or smaller than for Case A because of the difference in the number of massive stars exploded as SN per low-mass survivor (e.g., by factors of 1.38 and 0.47, respectively, per a star of \( m = 0.8 M_\odot \) and the IMF of \( M_{\text{md}} = 10 M_\odot \) and \( \Delta M = 0.4 \)). The iron production then demands smaller-mass (higher-mass) IMFs for Case B (Case C) as compared with Case A, the shift of IMFs in an opposite direction, discussed from the statistics of CEMP stars. Accordingly, for these two extreme cases, the parameter ranges for the IMFs derived from the statistics of CEMP stars and the iron production are marginally overlapped (Case C) or are dislocated with a narrow gap (Case B), although a definite conclusion waits for future observations, in particular, to improve the estimate of the total number of EMP stars (see the Appendix).

The relative production rate of carbon to iron may also impose additional constraints since the intermediate-mass stars enrich intergalactic matter with carbon through the mass loss on the AGB, as discussed by Abia et al. (2001). In particular, when \( \Delta M \) is small and \( M_{\text{md}} \) is in the range of intermediate- and low-masses, the intermediate-mass stars much surpass the massive stars in number and eject more carbon than the latter eject iron. We compute the amount of carbon ejected by AGB stars by taking the carbon abundance in the wind ejecta of AGB stars, and the remnant mass at \( 1 M_\odot \). Contours of \( [\text{C}/\text{Fe}] = 2, 1, 0 \) are plotted in the figure (dashed lines), for which only the carbon from the AGB stars is taken into account. The overabundance of carbon excludes the IMFs with low dispersion and low medium mass; it excludes the parameter space in a range of \( \Delta M < 0.2 \), derived by the CEMP star statistics for Case D, but has nothing to do with the high-mass IMFs derived for Case A.

We demonstrate that the IMFs, derived from the observed properties of CEMP stars, have the parameter ranges that can explain the chemical evolution and the production of low-mass stars, consistent with the observations, both for the flat mass-ratio distribution and for the “independent” coupling. In the Appendix, we will discuss the converse to demonstrate that the argument based on the total number of EMP survivors and the total iron production can potentially provides more stringent constraint on IMFs, independent of the argument based on the CEMP star statistics.

Relative abundances of other elements may also be affected by the IMF. A theoretical study of SN nucleosynthesis suggests the peculiarities and variations of yields for metal-free and EMP stars (Woosley & Weaver 1995; Umeda & Nomoto 2002; Heger & Woosley 2002; Tominaga et al. 2007; Heger & Woosley 2008). The SN yields are, however, sensitive to the assumption of model parameters such as the explosion energy and the treatment of nonaxisymmetric effects, and are currently subject to large uncertainties. In this paper, therefore, we are concerned with the iron yields as an indicator of the chemical evolution, and defer detail study about the abundance pattern variations elements in future works. As for the iron yield, recently, Type Ia SNe with short delay time and their contribution of iron production are discussed by some authors (e.g., Scannapieco & Bildsten 2005), although the evolutionary scenario is not yet
clear. The iron reduction is suggested to be several times larger than that by Type II SNe, and yet will hardly affect our results since the ratio, \( f_{SN}/f_{g} \) depends strongly on the IMF.

**2.4. Distinctive Features of EMP Survivors**

The different assumptions on the mass-ratio distributions admit the parameter ranges of high-mass IMFs that can reproduce the statistics of CEMP stars and the chemical evolution, consistent with existent observations. The predicted mass ranges differ by a factor of 2 or more between \( M_{\text{md}} \approx 5-20 M_{\odot} \). Although hardly distinguishable from the observations discussed so far, they surely make the differences in the properties of EMP survivors. We discuss the imprints that the mass-ratio distributions have left on the current EMP survivors and investigate the possibility of discriminating the mass coupling of binary systems in the EMP population, especially for the two distinct distributions of the flat mass-ratio distribution and the “independent” coupling.

Firstly, an obvious difference is the mass distribution function of EMP survivors. For a given IMF, \( \xi(m) \), the mass distribution, \( \xi_{\text{EMP-surv}}(m) \), of EMP survivors is given by

\[
\xi_{\text{EMP-surv}}(m) = (1 - f_{b})\xi(m) + f_{b}\xi(m) \int_{m}^{m} n(m_{2}/m) / m_{2} dm_{2}
\]

\[+ f_{b} \int_{0.8 M_{\odot}}^{m} \xi(m_{1}) n(m/m_{1}) / m_{1} dm_{1}, \quad (22)\]

Here a low-mass binary, whose components are both less massive than \( 0.8 M_{\odot} \), is counted as one object with the primary star. Figure 6 shows the mass distributions of EMP survivors \( (m \leq 0.8 M_{\odot}) \) for different assumptions of mass-ratio distributions Cases A–C. For these mass-ratio functions, the mass distribution of EMP survivors is nearly proportional to the mass-ratio distribution \( n(q) \) because almost all of them come from the secondary stars; the contribution from the primary components is denoted by a thin solid line, and the same contribution comes from the stars born as single. For the “independent” coupling, in contrast, the \( \xi_{\text{EMP-surv}}(m) \), has the same form as the IMF and the number of EMP survivors decreases rapidly as the stellar mass decreases.

Secondly, the fraction of double-lined binary and the contribution of stars born as single among EMP survivors may differ according to the mass-ratio distribution. The EMP survivors born as binaries are divided into three categories according to the mass of the primary stars: (1) the low-mass binaries with the primary of mass \( m_{1} \leq 0.8 M_{\odot} \), (2) the white-dwarf binaries of primary stars of mass between \( 0.8 M_{\odot} < m_{1} \leq M_{\text{up}} \), and (3) the SN binaries of primary stars of mass \( m_{1} > M_{\text{up}} \). The fraction of low-mass binaries with the primary stars of mass \( m \leq 0.8 M_{\odot} \) in the EMP survivors of mass between \( m \) to \( m + dm \) is given by

\[
\varphi_{\text{surv, LMB}}(m) = f_{b}\frac{\xi(m)/m}{\xi_{\text{EMP-surv}}(m)} \int_{m}^{m} n(m_{2}/m) dm_{2} / \xi_{\text{EMP-surv}}(m). \quad (23)\]
They can be detected as double-lined binary. For the flat mass-ratio distribution, this gives a significant fraction of \( \phi_{\text{LMB}}(0.8 M_\odot) = 7.3\% \) for \( M_{\text{up}} = 10 M_\odot \) and \( \Delta M = 0.4 \), and increases with \( \Delta M \) to 16\% for \( \Delta M = 0.5 \) and with decreasing \( M_{\text{up}} \) to 18\% for \( M_{\text{up}} = 5 M_\odot \), respectively. We note that these values depend weakly on \( f_b \), since most of the EMP survivors are from the binaries. The number of low-mass binary decreases rapidly for smaller masses while the number of EMP survivors, formed as the low-mass members of white dwarf binaries or SN binaries, remains constant.

For the “independent” coupling, the fraction of low-mass binaries in the EMP survivors reduces to:

\[
\phi_{\text{LMB}}(m) = 2 f_b \int_{m}^{M_{\text{up}}} \xi(m_2) d m_2 \left[ (1 + f_b) - 2 f_b \int_{m}^{0.8 M_\odot} \xi(m_1) d m_1 \right].
\]

which gives a much smaller fraction of \( \phi_{\text{LMB}}(0.8 M_\odot) = 1.6\% \) for \( M_{\text{up}} = 5 M_\odot \) and \( \Delta M = 0.4 \) as compared with the flat mass-ratio distribution. The fraction may increase for smaller medium mass, to 5.5\% at \( M_{\text{up}} = 3 M_\odot \), and for larger dispersion, to 3.9\% and 9.5\% at \( \Delta M = 0.5 \) and 0.7, respectively, although these may cause underproduction of iron, in particular for smaller \( M_{\text{up}} \), as seen from Figure 5 (bottom panel). In this case, the proportion of the EMP survivors, born as single stars, is fairly large as given by

\[
\phi_{\text{sing}}(m) \simeq (1 - f_b) \int_m^{M_{\text{up}}} \left[ (1 + f_b) - 2 f_b \int_m^{0.8 M_\odot} \xi(m_1) d m_1 \right].
\]

Consequently, nearly one-third of EMP stars were born as single stars, for \( f_b = 0.5 \), which is much larger fraction than in the case of the flat mass-ratio distribution.

Thirdly, the fraction, \( \phi_{\text{SNB}} \), of SN binaries with the primary stars of mass \( m_1 > M_{\text{up}} \) also differs between the two mass-ratio distributions. For the flat mass-ratio distribution, almost all of the EMP survivors belong, or have been belonged, to the binary systems, and the fraction is given by

\[
\phi_{\text{SNB}}(m) = f_b \int_{M_{\text{up}}} m (n/m_1) \xi(m_1)/m_1 d m_1 / \xi_{\text{EMP-surv}}(m),
\]

and amounts to \( \sim 50\% \). For the “independent” coupling, on the other hand, one-third of EMP survivors are single stars from their birth, and the percentage of SN binaries is relatively small, as given by

\[
\phi_{\text{SNB}}(m) = 2 f_b \int_{M_{\text{up}}} \xi(m_1) d m_1 \left[ (1 + f_b) - 2 f_b \int_{m}^{0.8 M_\odot} \xi(m_1) d m_1 \right].
\]

and turns out to be \( \sim 20\% \). The EMP survivors from the SN binaries have experienced an SN explosion of the erstwhile primary stars at close distances and are thought to suffer from some abundance anomalies, affected by SN ejecta. Accordingly, these stars, in particular from the binaries of sufficiently small separations, may be discriminated by a large enhancement of elements, characteristic to the SN yields.

These differences in the properties of remnant EMP survivors may potentially serve as tools to inquire into the nature of EMP binaries and to distinguish the mass-ratio distributions. Among the EMP stars, several double-lined spectroscopic binaries are reported in the literature. If we restricted to the metallicity range of \([\text{Fe/H}] < -3\), for which the observations with high-resolution spectroscopy may be regarded as unbiased, there are two stars CS22876−032 ([\text{Fe/H}] \simeq -3.6, \( V = 12.84, P = 424.7 \) d, and \( m_2/m_1 \simeq 0.89 \); Thorburn & Beers 1993; Norris et al. 2000; González et al. 2005) and CS 22873−139 ([\text{Fe/H}] \simeq -3.0)}.  

Figure 6. Comparison of the mass function of EMP survivors for the different assumptions on the coupling mass distribution of binaries. Here the parameters of IMFs are taken to be \( M_{\text{up}} = 10 M_\odot \) and \( \Delta M = 0.4 \) (Cases A–C). Legends are given at top right corner, where a thin solid line denotes the contribution from the stars born as single.
the MDFs with low-mass IMFs for the Salpeter’s power-law mass function (dashed line) and for the log-normal IMF (solid line). Figure 7.

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observed MDF, obtained by the HES survey (shaded columns; Beers et al. 2005). The number distribution, $n_\nu$, is computed with the log-normal IMF of medium mass $M_{\text{med}} = 10 M_\odot$ and dispersion $\Delta M = 0.4$ with the 50% binary fraction under the same flux-limited condition as the HES survey. Dashed and dotted lines illustrate the MDFs with low-mass IMFs for the Salpeter’s power-law mass function (dashed line) and for the log-normal IMF ($M_{\text{med}} = 0.79 M_\odot$ and $\Delta M = 1.18$, dotted line), derived by Lucatello et al. (2005) from the CEMP-s star statistics alone; both of them bring about the overproduction of low-mass survivors, more numerous by a factor of $\sim 100$ than observed.

-3.4, $V = 13.8$, $P = 19.165$ d, $m_2/m_1 \simeq 0.92$; Preston 1994, 2000; Spite et al. 2000) with the detailed analyses and one star HE 1353−2735 ([Fe/H] $\simeq -3.2$, $V = 14.7$; Depagne et al. 2000) without the binary parameter. So far 39 dwarf stars of [Fe/H] $< -3$ are confirmed by the high-resolution spectroscopy (we define the dwarf as $\log g [\text{cm s}^{-2}] \geq 3.5$), and hence, the fraction of low-mass binaries, composed of two unevolved EMP stars, turns out to be $\geq 3/39 \simeq 7.7%$. It seems to be compatible with the estimated fraction $\phi_{\text{binary}} \text{LMC}(0.8 M_\odot) = 7.3%$ for the frat mass-ratio distribution but to be a little larger, or marginal, for the “independent” coupling. We note, however, that only with two samples, the above fraction may be subject to significant observational selection effects. These stars have to be concentrated near to the upper end of the main sequence since they are found among the candidates, selected from the flux limited surveys, and the mass ratio has to be sufficiently large for the lines of two components to be observed. The actual fraction has to be larger than observed if we take into account the detection probability due to the orbital phase and to the inclination angle, and the rather narrow range of mass-ratios for the observed double-lined binaries. On the contrary, the larger survey volume by a factor up to $2^{3/2}$ for the double-lined binaries due to the sum of luminosities may reduce the actual fraction. More observations for the main-sequence EMP binaries and the bias corrections are necessary to discriminate the mass-ratio distributions.

It may be more straightforward to compare our results with the mass distribution function of EMP survivors. From the existent observations, however, it is rather hard to determine since the observed dwarfs are mostly concentrated near to the upper end of main sequence. An exception is a carbon dwarf G77-61 of [Fe/H] = 4.03 (Plez & Cohen 2005) whose mass is inferred at 0.3−0.5 $M_\odot$, but it was found among the proper-motion-parallax stars (Dahn et al. 1977), not from the surveys. We have to wait for the larger-scaled surveys in near future to reveal the distribution of EMP survivors of low masses. As for the SNe binaries, they are expected to be related to the large star-to-star variations in the surface elemental abundances, in particular, with those of $r$-process elements, ranging by more than two orders of magnitude. It is necessary, however, to understand the nature of interactions between the SN ejecta colliding at very high velocity and the nearby low-mass stars before the meaningful conclusions can be drawn from the observations.

3. METALLICITY DISTRIBUTION FUNCTION OF EMP STARS

We have shown that the high-mass IMFs with the binary provide a reasonable explanation of the observed properties of EMP stars in the Galactic halo, revealed by the recent large-scaled HK and HES surveys. In this section we discuss the consequence of derived IMF on the metal enrichment history of the Galactic halo up to [Fe/H] = -2.5 to study their relevance to the MDF, observed for the EMP stars. As seen above, the derived IMFs using the different assumptions on the mass-ratio distribution function are hardly distinguishable by the current observations. In the following, therefore, we assume the IMF with the medium mass $M_{\text{med}} = 10 M_\odot$ and the dispersion $\Delta M = 0.4$ in the lognormal form, which is at least compatible with any of the assumed mass-ratio distributions.

3.1. Simple Model of Chemical Evolution

Under the assumption that matter ejected from SNe spreads homogeneously and is recycled instantaneously, the iron
abundance, $X_{Fe}$, of our Galaxy of (baryonic) mass $M_8$ can simply be related to the cumulative number, $N(X_{Fe})$, of the stars born before the metallicity reaches $X_{Fe}$ as:

$$M_8 X_{Fe} = \langle Y_{Fe} \rangle N(X_{Fe}) f_{SN},$$

where $\langle Y_{Fe} \rangle$ is the averaged iron yield per SN and $f_{SN}$ is the fraction of EMP stars that have exploded as SNe, defined in Equation (14). By differentiating it with respect to $[Fe/H] = \log(X_{Fe}/X_{Fe,0})$, the number distribution of EMP survivors is written as a function of metallicity in the form

$$n([Fe/H]) = \frac{dN(X_{Fe})}{d[Fe/H]} = \frac{M_8}{\langle Y_{Fe} \rangle f_{SN}} \ln(10) X_{Fe,0} 10^{[Fe/H]}.$$  

This shows that the number distribution of EMP survivors is simply proportional to the iron abundance apart from the variation of $\langle Y_{Fe} \rangle$ through the IMF and the latter is small enough to be neglected for $M_{md} \lesssim 20 M_\odot$ (see Figure 9 in the Appendix).

Figure 7 depicts the number distribution of EMP survivors and compares it with the observed MDF provided by the HES survey (Beers et al. 2005). We assume stars of mass $m > M_{up} = 8 M_\odot$ become Type II SN and eject $\langle Y_{Fe} \rangle = 0.07 M_\odot$ of iron. In this figure, the theoretical MDF, $v_{surv}$, is evaluated under the same flux-limited condition as the observed MDF is derived: $v_{surv}([Fe/H]) = n([Fe/H]) f_{\odot} \times (40\%) \times (8225 \text{ degree}^2/4\pi \text{ sr}) \times 1.93$. Here the fraction of follow-up observation and the sky coverage are taken into account: as for the contribution of TO stars, we take the same ratio to the giants as in the observed sample under the assumption that the giant survivors are all reached in the survey area. Solid line shows the MDF for the IMF with $M_{md} = 10 M_\odot$ and $\Delta M = 0.4$ with the 50% binary fraction, derived above for EMP population stars for the flat mass-ratio distribution, and it is similar to the other mass-ratio functions, as discussed in Section 2.3. This reasonably reproduces the observed MDF between the metallicity $-4 \lesssim [Fe/H] \lesssim -2.5$, as expected from the discussion in the previous section.

In this figure, we also plot the MDF using the low-mass IMFs, the Salpeter’s power-law mass-function as observed among the present-day stellar populations and that derived only from the statistics of CEMP-s stars by Lucatello et al. (2005, $M_{md} = 0.79 M_\odot$ and $\Delta M = 1.18/\ln 10$). They bring about the overproduction of EMP survivors by a factor of more than a few hundreds not only from the low-mass members of binaries but also from the primary stars and the single stars; both the IMFs give the similar MDF since our flux-limited samples are dominated by the giants and luminous dwarfs of mass $M \simeq 0.8 M_\odot$. This means that the EMP survivors is by far a small population as compared with the stellar systems of Populations I and II, and it is only with the high-mass IMFs that can make the EMP population produce sufficient amount of metals to enrich the early Galactic halo without leaving too many low-mass survivors now observable in the Galactic halo.

In addition, we see in this figure that the slope of observed MDF is consistent with the prediction from the simple one-zone approximation at least for $[Fe/H] > -4$. It implies that the IMFs have little changed while the Galactic halo has evolved through these metallicities. Beyond $[Fe/H] \simeq -2$, the observed MDF derived from the HK and HES surveys seems to be underestimated since those objects are out of the metallicity range sought after by the survey and subject to imperfect selection.

3.2. Effect of Hierarchical Galaxy Formation

The observed MDF of Galactic halo stars has a sudden drop at $[Fe/H] \lesssim -4$, and only three stars are found below it. We propose the mechanism responsible for this depression of low-metallicity stars from the consideration of the Galaxy formation process.

In the current cold dark matter (CMD) model, galaxies were formed hierarchically. They started from low mass structures and grew in mass through merging and accreting matter, finally to be large-scale structures like our Galaxy. In the hierarchical structure formation scenario with $\Lambda$CDM cosmology, the typical mass of first star-forming halos is $\sim 10^8 M_\odot$ for the dark matter and $\sim 2 \times 10^5 M_\odot$ for the gas (e.g., see Tegmark et al. 1997; Spergel et al. 2007).

In these first collapsed gas clouds, the first stars contain no pristine metals except for lithium. When the first star explodes as SN, it ejects $\sim 0.07 M_\odot$ of iron, which enriches the gas cloud of mass $\sim 2 \times 10^5 M_\odot$ where it was born up to the metallicity of $[Fe/H] \sim -3.5$ if the ejecta is well mixed in the gas cloud. We call this event the “first pollution.” Consequently, the second generation stars have the metallicity of $[Fe/H] \sim -3.5$.

In the course of time, the mini-halos that host the gas clouds merge with each other and accrete the intergalactic gas to form an early Galactic halo with a baryonic mass of $10^{11} M_\odot$.

---

3 There are two more stars with the iron abundances reported below $[Fe/H] < -4$: CD-38 245 with $[Fe/H] = -4.19 \pm 0.10$ (Carayol et al. 2004) and G77-61 with $[Fe/H] = -4.03 \pm 0.1$ (Pléz & Cohen 2005). We will omit these two stars in our discussion since larger abundances of $[Fe/H] = -4.07 \pm 0.15$ (François et al. 2003) and $-3.98 \pm 0.15$ (Norris et al. 2001) have been reported for the former, and hence, their abundances are closer to the EMP stars of $[Fe/H] \gtrsim -4$ than to the other three HMP/UMP stars.
We may take the metallicity of this early Galactic halo to be 
\[ [\text{Fe/H}] \simeq -4 \] because of the scarcity of stars of metallicity 
\[ [\text{Fe/H}] < -4 \]. The cumulative number of stars born before 
the early Galactic halo is enriched up to \([\text{Fe/H}] = -4\) is estimated 
at \(N(10^{-4}X_{\text{Fe,H}}) = 7 \times 10^5\) with taking into account the SN 
fraction \(f_{\text{SN}}\). If the mini-halos of larger masses stand between 
the first collapsed halos and the Galactic halo, the dilution of 
iron with unpolluted primordial gas can give birth to the stars of 
smaller metallicity of \([\text{Fe/H}] \simeq -4\), and then, the metallicity at 
the formation of the Galactic halo can be larger to increase the 
cumulative number of stars in accordance (see below).

We may estimate the fractions of both the first generation stars 
without metals and the second generation stars of the metallicity 
\([\text{Fe/H}] \sim -3.5\), respectively, assuming that stars are born with 
an equal probability whether in the gas clouds, polluted with 
metals, or in the primordial gas clouds. Here it is worth noting 
that some recent computations of star formation demonstrate 
that the low-mass stars can be formed as the binary members 
even out of metal-free gas (Clark et al. 2008; Machida et al. 
2008). The accumulated number, \(N_{\text{PopIII}}\), of Pop III stars, born 
of gas unpolluted by SN ejecta, when the average metallicity 
reaches \(X_{\text{Fe,H}}\), is given by

\[
N_{\text{PopIII}} = \frac{M_\odot}{M_{\text{SN}}} \left[ 1 - \exp \left( -\frac{M_\odot X_{\text{Fe,H}}}{(Y_{\text{Fe,H}})^{\frac{3}{2}}} \right) \right],
\]

where \(M_\odot(= 2 \times 10^5 M_\odot)\) is the mass of gas in the first 
star forming clouds. If we assume the same IMF and binary 
parameters as in the stars of the EMP population, then, we 
eventually expect that the number of Pop III stars is

\[
N_{\text{PopIII}}(10^{-4}X_{\text{Fe,H}}) = 3.1 \times 10^5,
\]

and the number of Pop III survivors is

\[
3.1 \times 10^5 \times \int_{0.8 M_\odot}^{0.8 M_\odot} dm \left[ \xi(m) \right] + f_{\text{SN}} \int_{0.8 M_\odot}^{0.8 M_\odot} n(m/m_1) \xi(m_1) \frac{dm_1}{m_1} = 1.3 \times 10^4,
\]

and similarly we have \(5.5 \times 10^4\) and \(2.3 \times 10^3\) of the second 
generation stars and their survivors, formed before the averaged 
metallicity of the Galaxy reaches \([\text{Fe/H}] \simeq -4\). The IMF of 
Population III stars may differ from EMP stars but the existence 
of the stars with \([\text{Fe/H}] < -5\) suggests that the low-mass stars 
can be formed before the first pollution.

Figure 8 illustrates an expected MDF with the hierarchical 
structure formation. After the formation of a large Galactic 
halo, the metal enrichment process is thought to follow the 
argument of the previous subsection. Thus, we can explain the 
cutoff around \([\text{Fe/H}] \sim -4\) naturally. Shaded columns indicate 
the initial distributions of Population III stars and of the second 
generation stars formed in the low-mass clouds. The second stars 
were mixed and observationally lost their identities among 
the stars formed in the merged halo. On the other hand, Population 
III stars should form a distinctive class. From the above estimate, 
we expect \(~ 23\) Population III survivors in the existing flux-
limited samples of HES surveys. It is true, however, that there 
is no star with zero metallicity among the stars detected by 
the existent surveys. We may propose one scenario to explain 
this absence that Population III survivors are no longer remain 
metal-free at present since their surface are polluted by the 
accretion of interstellar matter, enriched with metals ejected by 
the SNe of the first and subsequent generations. With the surface 
pollution of \([\text{Fe/H}] \sim -5\), they are observed as HMP stars. We 
discuss about evolution of Population III stars with pollution 
Section 4.1. Similarly, the number of second generation of 
stars is estimated at \(~ 4\) in the same flux-limited HES sample, 
indicative that most of EMP stars are formed of mixture of the 
ejecta from plural SNe. This has direct relevance to the study of 
the nucleosynthetic signatures on the EMP survivors and the 
impacts of SNe of the first and subsequent generations.

4. CONCLUSIONS AND DISCUSSION

We have studied the IMF and low-mass star formation with 
the chemical evolution of the Galactic halo population on the 
basis of the characteristics of EMP stars, revealed by the recent 
large-scaled HK and HES surveys; the observational facts that 
we make use of are: (1) the overabundance of CEMP stars, 
(2) the relative frequencies of CEMP stars with and without 
the enrichment of s-process elements, (3) the estimate of the surface 
density or total number of EMP stars in the Galactic halo, and (4) 
the MDF. We take into account the contribution of binary stars 
properly, as expected from the younger populations. In Paper I, a 
high-mass IMF peaking around \(~ 10 M_\odot\) is derived for the stars 
of the EMP population and it is shown that the binary population 
plays a major role in producing the low-mass stars that survive 
to date, but by using the flat mass-ratio distribution between the 
component stars. In this paper, we examine these properties of 
the stars of the EMP population and EMP survivors for different 
types of mass-ratio distributions, and investigate the constraints 
on the IMFs of the stars of the EMP population and discuss the 
observational tests of discriminating them. The derived IMFs 
are applied to understand the characteristics of the MDF and the 
nature of EMP stars including HMP/UMP stars, provided by 
the surveys.

Our main conclusions are summarized as follows:

1. The statistics of CEMP stars are explained by the high-mass 
IMFs with the binaries of significant fraction. The predicted 
typical mass is significantly larger than Population I or 
II stars, irrespective of the assumptions of the mass-ratio 
distribution. The mass-ratio distribution with a preference 
for nearby equal masses demands an IMF with higher 
typical mass \(M_{\text{md}} > 7 M_\odot\) and smaller dispersion \((\Delta M < 
0.6)\). While the mass-ratio distribution in favor of smaller 
mass secondary or the independent combination of two stars 
with the same IMF demands smaller typical masses around 
\(M_{\text{md}} \sim 5 M_\odot\) irrespective of \(\Delta M\).

2. High-mass IMFs with \(M_{\text{md}} \sim 5-20 M_\odot\) derived from the 
statistics of CEMP stars agree with those derived from 
the low-mass star formation and the chemical evolution of 
the Galactic halo based on the number of giant EMP sur-

3. The mass-ratio distribution of binaries in the EMP pop-
ulation can be discriminated by the imprints left on the 
EMP survivors such as the mass function, the binary 
function, and the fraction of stars influenced by the SN 
explosion of primary stars. In particular, the flat mass-
ratio distributions predict significant fraction \(~ 7.7\% for 
\(M_{\text{md}} = 10\)\) of double-lined spectroscopic binaries while 
the mass-ratio distribution of “independent” coupling pre-
dict much lower fraction. Among the 39 unevolved stars of
[Fe/H] < −3, studied spectroscopically to date, three
double-lined binaries are found, but there may be signif-
cient uncertainties and biases for the existent surveys
and future observations can discriminate the distributions.
4. The observed MDF of EMP survivors is consequent upon
the derived IMF with the contribution of the binaries. There
is no indication of significant change in the IMFs between
the metallicity of −4 ≤ [Fe/H] ≤ −2. The depression of
stars below [Fe/H] < −4 is naturally explicable within
the current favored framework of the hierarchical structure
formation model. Then, the Population III stars born of
primordial gas, and also, the stars in the primordial clouds
before they are contaminated by their own SNe, should
form the distinct class other than EMP stars, and may have
the relevance to HMP and UMP stars observed at lower
metallicity, as discussed below in this subsection.

Originality in the present work is to take into account the stars
born in binary systems properly in discussing the low-mass star
formation in the early universe. In addition, we make full use
of available information from the existent large-scaled surveys
and to draw the maximal constraint on the early evolution of our
Galactic halo. The combination of stars born in high-mass IMF
with those in the binary components enables us to reproduce not
only the shape of the MDF but also the total number of low-mass
survivors, observed by the surveys, consistently with the statist-
cics of chemically peculiar, CEMP-s and CEMP-nos stars. Now
known EMP stars ([Fe/H] ≲ −2.5) with the detailed stellar parameters amount to ∼ 400 in number (SAGA Database; Suda
et al. 2008), and allow us to discuss the averaged properties stud-
ied as in this paper. Yet the sample size is not large enough to
study the detailed properties; one of the limitations is the spatial
distributions of EMP stars. In this work, we have only treated
the spatially averaged properties since sufficient information is
not available as to the spatial variations of EMP survivors from
the surveys. The spatial distribution may affect the estimate of
the total number of EMP survivors in the Galactic halo, and
hence, may influence the discussion in Section 2.3 quantita-
tively through the iron production consistent with it. In order
to improve and sharpen our conclusions, we have to wait for
the future larger-scaled surveys such as SDSS/SEGUE (Beers
et al. 2004) and LAMOST (Zao et al. 2006). Also high-
dispersion spectroscopy is necessary to understand the char-
acteristics of EMP stars.

The constraints on the IMFs derived in this work may serve
as the basis of understanding the formation and early evolution
of the Galaxy. We find that the EMP population has a high-mass
IMF and there is no indication of changes in the IMF for the
MDF derived from the HK and HES surveys. Although both
surveys may suffer from imperfect selections for [Fe/H] ≳ −2,
Ryan & Norris (1991) have found that the metallicity distribu-
tion is continuous up to the peak at [Fe/H] ≃ −1.6 without a
break for the halo subdwarfs in the kinematically selected samples. Other evidence, suggesting that the IMF biased toward
high masses, is reported mostly from cosmological observa-
tions (e.g., see the review by Elmegreen 2008). On the other
hand, a low-mass IMF of a characteristic mass of 0.2–0.3 M⊙ is
derived for the Galactic spheroid population of the metallicity
[Fe/H] ≃ −1.7–1.4 (Chabrier 2003). Accordingly, there should be
the transition from the high-mass IMF to the low-mass one.
Note that this is different from the transition from the massive
primordial IMF such as discussed by Tumlinson (2006) and
Salvadori et al. (2007). Our result suggests that the transition is
postponed until high metallicity even beyond [Fe/H] ≃ −2 is
reached. It is likely that the transition may not be simply deter-
mined by the metallicity alone; however, since the globular clusters,
which extend in the metallicity down to [Fe/H] ≃ −2.5, have a low-mass IMF similar to that of the Galactic spheroid. For
proper understanding of the transition, more theoretical work is
necessary, taking into account the effects of hierarchical structure
formation properly, where such processes as the stochastic and
inhomogeneous chemical-enrichment (Karlsson 2005) and
the early infall phase (Prantzos 2003) are naturally realized by the
formation of first collapsed objects and their merging with accretion. In discussing the primordial stars or HMP/UMP stars
in the present work, we assume the metallicity at the formation of
the Galactic halo at [Fe/H] ≃ −4. The detailed chemical evolu-
tion with the merger history taken into account is discussed in
a subsequent paper (Komiya et al. 2009), in similar ways to those of Tumlinson (2006) and Salvadori et al. (2007), but
taking into account the high-mass IMF, derived above, and the
contribution of binaries.

4.1. Origin of HMP/UMP Stars

We end by discussing the consequences of the present study
on the understanding of the origin of stars found below the cut-off
of MDF.

In our model, the stars made after the first pollution have the metallicity [Fe/H] ≃ −3.5 and the stars with slightly
lower metallicity of [Fe/H] ≃ −3.5–−4 are made in the merged clouds where metals are diluted with the primordial
gas unpolluted by SN ejecta. After the halos merge, the second
generation stars mingle and observationally lose their identities
among the stars formed in the merged halo. This means that the
HMP and UMP stars are the stars formed before the first
metal pollution by the Type II SN in their host mini-halos. One
possible scenario for these stars is that they are the survivors of
Population III stars and the metal abundances at their surface are
influenced not only by the matter from their binary companions
but also by the interstellar matter, enriched with metals ejected
by the SNe after their birth. In fact, it is shown that their
peculiar abundance patterns of light elements from lithium,
carbon through aluminum, including s-process elements such
as Sc and Sr, observed for three known HMP/UMP stars can be
reproduced by neutron-capture nucleosynthesis during the AGB
phase of their primary stars even under the pristine metal-free
condition (Nishimura et al. 2008). As for iron group elements
(Ca–Zn), Suda et al. (2004) argue the effects of surface pollution
of Population III stars through the accretion of interstellar gas
to show that the main-sequence Population III stars can be polluted to be [Fe/H] ≃ −3 while the giants to be [Fe/H] ≃ −5 since the pollutant is diluted by the surface convection deepening ∼ 100 times in mass on the giant branch. Thus, the Population III survivors have evolved to giants to be observed as HMP/UMP stars. As for a subdwarf HMP star HE1327−2623, the dilution of the accreted iron group elements has occurred in the envelope of primary star on the AGB because of the low-mass nature of its primary star (M ≲ 1.5 M⊙, Nishimura et al. 2008).

The majority of the Population III survivors have also to be
the secondary members of binary systems similar to the EMP
survivors if their IMF and binary parameters are similar to EMP
stars. Then some of Population III stars become carbon-enriched
HMP/UMP stars with [Fe/H] ∼ −5 through binary mass
transfer. If the mass of primary star is 0.8 M⊙ < m1 < 3.5 M⊙
and 3.5 M⊙ ≲ m1 < M⊙, the primary star enhances the surface abundances of carbon and nitrogen though the He-FDDM and of
carbon and/or nitrogen through TD and hot bottom burning
in the envelope, respectively, which are transferred onto the secondary stars through the wind accretion. It is to be noted that the primary stars of $M_1 > 2 M_\odot$ have the accreted pollutants mixed inward into the whole hydrogen-rich envelope at the second dredge-up, and thereafter, evolve like the stars with the pristine metals. At the same time, the accreted matter is diluted in the envelope and the iron abundance is reduced to $[\text{Fe}/\text{H}] \sim -5$ in the primary stars. We estimate that $\sim 35\%$ of Population III stars become carbon-rich HMP under the same assumptions on the binary parameters as in Paper I. The surface abundances of main sequence stars can be smaller than stated in Paper I, however, since the accreted matter mixes down and are reduced by an order of magnitude if the diffusion and thermohaline mixing works (Weiss et al. 2000; Stancliffe et al. 2007).

In Figure 8, solid lines denote the expected MDF at the present days with the surface pollution taken into account. The basic form of observed MDF is reproduced, i.e., the cutoff around $[\text{Fe}/\text{H}] \sim -4$, the scarcity of stars for the metallicity below it and the existence of a few HMP/UMP stars.

From the above estimates, there should be $\sim 23$ Population III stars in the existent flux-limited samples of HES surveys; about a half of them may be discovered as giants with the surface metal pollution and one third as carbon stars. In actuality, only three HMP/UMP stars (two giants and one subdwarf) are found to date, all enriched with carbon, among 153 stars of $[\text{Fe}/\text{H}] < -3$, registered in SAGA database (Suda et al. 2008). Since such low metal abundances can be discriminated only with high-dispersion spectroscopy, $\sim 4.6$ HMP/UMP stars are expected among the whole HES samples of 234 stars of $[\text{Fe}/\text{H}] < -3$. The observed numbers are significantly smaller than prediction from our model. The above estimates are made, however, under the assumption that the Population III stars are formed in the same IMF as EMP stars and with the same binary parameters. This may not be warranted and rather we may take that this deficiency may suggest a still higher-mass IMF and/or less efficiency of binary formation for Population III stars than the EMP stars.

In the discussion, we assume the closed box chemistry in the collapsed object before merging. It is shown that the hypernovae, exploded with a large energy of $10^{52}$ erg, blow off the first collapsed objects of mass $M \simeq 10^6 M_\odot$ (Machida et al. 2005); if the first stars are sufficiently massive, the metal yields are spread into larger masses, and pollute the ambient gas before they collapse to form mini-haloes, as discussed by Salvadori et al. (2007). After that, the first stars in the collapsed clouds are no longer metal-free. Nevertheless, those stars which are formed before each collapsed cloud is polluted by its own SN form a distinct class from those which suffer from the first explosion. Further study is necessary to make clear the present appearance of the possible Pop III survivors and to settle the origin of HMP/UMP stars, in particular, for tiny amounts of iron-group metals and the overwhelming carbon-enhancement, shared by all these stars known to date.

We benefit greatly from discussion with Dr. W. Aoki. This paper is supported in part by a Grant-in-Aid for Scientific Research from the Japan Society for the Promotion of Science (grant 18104003 and 18072001). T.S. has been supported by a Marie Curie Incoming International Fellowship of the European Community Fp7 Program under contract number PIIF-GA-2008-221145.

Figure 9. Average iron yields per SN, $\langle Y_{\text{Fe}} \rangle_{\text{EMP}}$, demanded by the chemical evolution of the Galactic halo that leaves the EMP survivors consistent with the observed flux-limited samples, and the theoretical iron yields, $\langle Y_{\text{Fe}} \rangle_{\text{SN}}$, computed from the theoretical SN models with use of iron yields by Umeda & Nomoto (2002), and by Woosley & Weaver (1995) and Heger & Woosley (2002) as a function of $M_{\text{md}}$ with $\Delta M = 0.4$. The top panel compares $\langle Y_{\text{Fe}} \rangle_{\text{EMP}}$ computed for the different samples of EMP stars, i.e., giants, dwarfs, and total stars from the HES survey and total stars from HK survey, with the flat mass ratio function, while the bottom panel compares the results with the different mass-ratio distributions for the giant samples of the HES survey.

APPENDIX

LOW-MASS STAR FORMATION AND IRON PRODUCTION BY EMP POPULATION

One of the important findings of the recent large-scaled surveys is the scarcity of EMP stars in the Galactic halo. The HES survey gives the total number of EMP stars in our Galactic halo at $\sigma_{\text{EMP}} \simeq 796$ sr$^{-1}$ (giants of $\sigma_{\text{EMP.G}} \simeq 412$ sr$^{-1}$ in Equation (17) plus turn-off stars $\sigma_{\text{EMP.TO}} \simeq 384$ sr$^{-1}$) within the limiting magnitude $B \simeq 17.5$. Similarly, the HK survey gives $\sigma_{\text{EMP}} \simeq 528$ sr$^{-1}$ within the limiting magnitude of $B \simeq 15.5$; 114 stars of $[\text{Fe}/\text{H}] < -3$ are found by the medium-resolution, follow-up spectroscopy of 50% of the candidates, selected from the objective prism survey covering the 2800 deg$^2$ and 4100 deg$^2$ areas in the northern and southern hemispheres (Beers & Christlieb 2005b). Because of the significantly large areas covered by these surveys ($\sim 20\%$ of all sky with the follow-up observations), we may place reliance on these results, granted that they may not be complete. This also constrains on the IMF of stellar population that promoted the chemical evolution, or more specifically, the formation of metals and the low-mass survivors. In the paper, we have discussed the
chemical evolution starting with the statistics of CEMP stars. In this appendix, we show that the chemical evolution with the total number of EMP survivors provides more stringent constraints on the IMF of the EMP population with the aid of the amount of ejecta from SN models, independently of the statistics of CEMP stars.

Our basic premise is that the same stellar population is responsible both for the production of metals and of low-mass survivors. In discussing the low-mass survivors, it is indispensable to take into account the contribution from the binaries. This is one of the major conclusions in Paper I. We assume that the stars are born not only as single stars but also as the members of binaries in an equal number and with the primary assume that the stars are born not only as single stars but also as the members of binaries in an equal number and with the primary assumption.

The total number, \( N_{\text{EMP,surv}} \), of EMP survivors, currently observed in the Galactic halo, is related to the cumulative number, \( N_{\text{EMP}} \), of stars of EMP population as

\[
N_{\text{EMP,surv}} = N_{\text{EMP}} f_{\text{surv}} = N_{\text{EMP}} \int_{0.8 M_\odot}^{m_1} dm \left[ \xi(m) + f_b \int_{0.8 M_\odot}^{m_1} dm_1 \frac{m_1}{m} \right],
\]

and hence, to the cumulative number of EMP SNe as \( N_{\text{EMP,SN}} = N_{\text{EMP}} f_{\text{SN}} = N_{\text{EMP,surv}} (f_{\text{SN}} / f_{\text{surv}}) \). These SNe have to supply the amount of iron, \( M_{\text{Fe,EMP}} \), in Equation (18), in order to enrich the gas in the Galaxy of mass \( M_\odot \), with iron to promote the chemical evolution up to the metallicity \([Fe/H] = -2.5\). Then, we may derive the averaged iron yield, \( \langle Y_{\text{Fe,EMP}} \rangle \), per SN of EMP population, necessary to explain the chemical evolution of Galaxy, by the relation \( \langle Y_{\text{Fe,EMP}} \rangle = M_{\text{Fe,EMP}} / N_{\text{EMP,SN}} \) for an assumed IMF with the mass-ratio distribution function.

We show in Figure 9 the averaged yield, \( \langle Y_{\text{Fe}} \rangle_{\text{EMP}} \), as a function of \( M_{\text{nd}} \) for \( \Delta m = 0.4 \); upper panel for the observations of EMP stars of different evolutionary stages from the HES survey and of the total EMP stars from the HK survey with use of the IMFs with the flat mass-ratio function, and lower panel for the different mass-ratio functions with use of the observation of EMP giants from the HES survey. In order to compare the stars of different evolutionary stages, we include the effects of the limiting magnitude of the surveys by assuming the de Vaucouleurs density distribution, \( \rho \propto r^{-\alpha} \), with the radial distance, \( r \), from the Galactic center, the same as the stars in the Galactic halo and by assigning the luminosity of \( L = L_\odot (M / M_\odot)^{2.5} \) and 100 \( L_\odot \) to dwarfs and giants, respectively.

The amount of iron demanded by the chemical evolution turns out to be a steep decrease function of \( M_{\text{nd}} \) since in order to leave a fixed number of low-mass survivors, the total number of stars of EMP populations, and hence, the SN fraction increase rapidly with \( M_{\text{nd}} \), in particular near \( M_{\text{nd}} \approx M_{\text{up}} \). The necessary yields computed from the different samples in upper panel show a fairly good agreement with each other. The difference between the giants and dwarfs for the HES samples is indicative of a relatively deficiency of dwarf stars compared with giants by a factor of \(~2.2\) in number, which may be attributed to rather crude assignment of averaged giant luminosity, and/or to the different efficiency of identifying giants and turn-off stars in the survey plates, and/or to the uncertainties in the spatial density distribution. The results for the HK survey and the HES survey also agree within the difference by a factor of \(~2.3\) in number despite the difference in the limiting flux by 2 mag, and hence, to the difference in the searched volume by a factor of \(~20\).

The variations with the mass-ratio functions in the lower panel are caused by the difference in the number of SNe per EMP survivor. As compared to the flat mass-ratio function, the mass-ratio function increasing (decreasing) with \( q \) gives a larger (smaller) number of SNe to produce one EMP survivors; the difference of which increases for higher-mass IMFs.

These iron yields necessary to promote the chemical evolution may be compared with the theoretical iron yields predicted from the SN models. The IMF-weighted iron yields, \( \langle Y_{\text{Fe,SN}} \rangle \), per SN is given by using the iron mass, \( Y_{\text{Fe}}(m) \), ejected from a massive star of initial mass \( m \) as

\[
\langle Y_{\text{Fe,SN}} \rangle = \frac{\int_{M_{\text{up}}}^{M_\odot} dm_1 \xi(m_1) [Y_{\text{Fe}}(m_1) + f_b \int_{m_1}^{1} dm_2 \xi(m_2) n(q) dq]}{\int_{M_{\text{up}}}^{M_\odot} dm_1 \xi(m_1) [1 + f_b \int_{m_1}^{1} n(q) dq]}.\tag{A2}
\]

The IMF averaged yield \( \langle Y_{\text{Fe}} \rangle_{\text{SN}} \) is also shown in this figure, for which the theoretical yields are taken from the metal-deficient SN models computed by Umeda & Nomoto (2002), and by Woosely & Weaver (1995) and Heger & Woosley (2002). It is a slowly increase function of \( M_{\text{nd}} \) for \( M_{\text{nd}} \lesssim 20 M_\odot \), with the increase in the fraction of more massive stars that ended as SNe, while beyond it, the gradient grows steeper owing to the contribution of the electron pair-instability SNe of \( M > 100 M_\odot \).

The averaged yields, demanded by the chemical evolution, and the theoretical IMF-weighted iron yields both meet with each other near \( M_{\text{nd}} \approx 6-11 M_\odot \) and with the iron yield \( \langle Y_{\text{Fe}} \rangle \approx 0.04-0.06 M_\odot \) per SN. As typically seen for the flat mass-ratio distribution, the parameter range coincides with that we have derived for the IMFs from the CEMP statistics in Figures 1–3. There are discernible differences arising from the mass-ratio distributions, though not large enough to differentiate these mass-ratio distributions in view of the uncertainties of current observations. As compared with the flat distribution, the mass-ratio distribution increasing (decreasing) with \( q \) demands smaller (larger) \( M_{\text{nd}} \). The opposite tendency derived from the CEMP statistics. These distributions prefer smaller (larger) number of EMP survivors, larger (smaller) fraction of CEMP-stars and smaller (larger) ratio of CEMP-stars to CEMP-s stars. In principle, however, we can discriminate the mass-ratio functions in the EMP binaries, including those destroyed already by the evolution, with use of the survey and observations of EMP stars in sufficiently large number and with sufficient accuracy, which waits for future works.

In this paper, we discuss about the stellar population that can be expressed in terms of a single log-normal IMF with the binary fraction. For the IMFs of higher-mass than derived above, the stellar population cannot produce the sufficient number of low-mass survivors by themselves even if the low-mass member of binaries are taken into account, while for lower-mass IMFs, it results short of iron production. It is possible to assume the bi-modal IMF and to explain the production of iron and the formation of low-mass stars, separately, in terms of the bi-modal IMF and to explain the production of iron and the higher-mass IMF responsible for the iron production and the other with a lower-mass IMF for the low-mass survivors, respectively. In the case of bi-modal IMFs, the constraints, derived here, place an upper mass limit to the IMF of lower-mass population and an lower mass limit to the IMF of higher-mass population. It is to be noted that the IMF with the binary mass function of Cases A–C is regarded as a sort of bi-modal IMF with the primary plus single stars as the higher-mass population and the secondary stars as the lower-mass population (see Figure 12 in Paper I);
the separation of two IMFs differs with the mass-ratio function and the relative contributions of two populations vary with the binary fraction. In any case, as for the EMP stars in the Galactic halo, the statistics of CEMP stars, in particular, the ratio of CEMP-nos and CEMP-s stars, place the lower mass limit to the IMF of the lower-mass population, and hence, endorses the high-mass IMF, which narrows, if anything, the contribution of the higher-mass population.

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