Coherent, time-correlated tunneling of density wave electrons

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Abstract. A growing body of evidence reveals that charge density wave (CDW) transport is a high-temperature cooperative quantum phenomenon. According to the time-correlated soliton tunneling (ST) model, quantum solitons, or electron-phonon correlates within the CDW condensate, act much like electrons tunneling through a Coulomb-blockade tunnel junction. Pair creation of charged fluidic soliton droplets is prevented by their electrostatic energy below a Coulomb-blockade threshold electric field. Above threshold, the quantum fluid flows in a periodic fashion, via a hybrid between Zener-like and coherent Josephson-like tunneling. We summarize the time-correlated ST model and compare model simulations with experiment. The ST model shows excellent agreement with coherent voltage oscillations, and with CDW current-voltage characteristics. Finally, we discuss implications for physics and potential applications.

1. Introduction
Charge density waves (CDWs) exhibit correlated flow of electrons at the highest known temperatures of any macroscopic electron condensate [1-4]. CDW dynamical behavior has been observed above room temperature [5], in some cases even above the boiling point of water [6-8]. In linear chain compounds, the CDW condensate modulates the charge along each of \( N \) parallel chains, \( \rho_l(x,t) = \rho_l^0(x,t) + \rho_l^1 \cos[2k_Fx - \phi_l(x,t)] \), where kinks in \( \phi_l \) carry charge and can transport electric current. Spin density waves are similar and can be viewed as two out-of-phase CDWs for the spin-up and -down sub-bands.

Considerable evidence shows that CDWs do not classically “slide,” but instead transport electric current as a fundamentally quantum process. In the classical picture, the CDW is expected to slide when the applied electric field tilts the washboard pinning potential enough for classical depinning. Just below this classical depinning field, the CDW should displace by at least a quarter wavelength, equivalent to a phase displacement of \( \pi/2 \), or 90°. The restoring force near threshold, moreover, should become vanishingly small, leading to a divergent dielectric response [3], as shown in figure 1. These classical predictions are refuted by: 1) NMR experiments [9] showing only 2° CDW phase displacement just below threshold; and flat 2) dielectric (figure 1 and ref. [3]), and 3) harmonic mixing [10] responses vs. bias field below threshold. These experiments reveal that the CDW remains near the bottom of the pinning potential well, and that the threshold field for nonlinear transport is often much smaller than the classical depinning field.

The Coulomb blockade effect in single electron tunneling shows how a threshold voltage or field can arise from a purely quantum process [11]. A similar Coulomb blockade mechanism was found to yield a threshold field for charged soliton pair creation [12]. This model, based on the (1+1)-D massive Schwinger model [13], was extended to a description of time-correlated soliton tunneling (ST) in a 1-D...
CDW model [14]. Oscillations of period $h/2e$, in CDW conductance vs. magnetic flux of TaS$_3$ rings, provided clear evidence for the quantum nature of CDW transport [15,16]. This Aharonov-Bohm-like behavior, first reported in 2009 [15], motivated further development of the time-correlated ST model. Later results, reported in 2012 [16], included telegraphic switching effects suggesting transitions between macroscopically distinct states. Additional evidence for quantum behavior includes linear ac admittance and mixing experiments showing agreement with photon-assisted tunneling theory [3,10].

Figure 1. Dielectric response vs. bias field [3], showing classical predictions vs. experiment. Classical models include classical sine-Gordon (s-G); random pinning (RP); renormalization group (NM); and incommensurate harmonic chain (CF) models. See [3] for details.

2. Time-correlated soliton tunneling model
We treat CDW transport as the periodic flow of a quantum fluid of electron–phonon correlates, or quantum solitons, within the condensate [2-4]. Pair creation of charged soliton droplets by an applied field $E$ is prevented by their electrostatic energy below a Coulomb blockade threshold field, $E_T = E^*/2$, where $E^*$ is the internal field produced by a soliton-antisoliton pair. This threshold corresponds to a “vacuum angle” $\theta = 2\pi E/E^* = \pi$, and is often much smaller than the classical depinning field. In a pinned CDW, the potential energy of the $i$th chain is $u[\phi_i] = 2u_0[1 - \cos[\phi_i(x)]] + u_E[\theta - \phi_i(x)]^2$. The quadratic term is the electrostatic energy resulting from the applied field and internal fields created by kinks at the boundaries. Above threshold, $\theta(t)$ is related to the evolving displacement charge $Q(t)$ by $\theta(t) = 2\pi Q(t)/Q_0$, where $Q_0 = 2eN$ is the charge of a fluidic soliton domain wall, an aggregate of many single-chain soliton dislocations.

Figure 2 (a) plots $u$ vs $\theta$ when the energy is minimized for $\phi \sim 2\pi n$ when $u_E \ll u_0$. The phases $\phi_i$ tunnel coherently into the next well via a matrix element $T$ [figure 2(b)] as each charging energy branch,
~ (θ − 2πn)2, crosses the next at the instability points θ = 2π(n + 1/2). The time-correlated soliton tunneling model [2-4] includes a shunt resistance R, representing normal, uncondensed electrons, in parallel with a capacitive tunnel junction depicting soliton tunneling [figure 2(c)]. Advancing the phases by 2πn creates multiple pairs of fluidic domain walls that are driven outward by the applied field. As in time-correlated single electron tunneling, the voltage is proportional to net displacement charge, \( V = (Q_0/2\pi C)[\theta − 2\pi n] \). If the expectation value ⟨ϕ⟩ among \( N \) parallel chains advances by a fraction or non-integer multiple of 2πn, the voltage and normal current are \( V = (Q_0/2\pi C)[\theta − ⟨ϕ⟩] \) and \( I_n = V/R \), respectively. Using \( I_{CDW} = I - I_n = (Q_0/2\pi)(d\theta/dt) \), and defining \( \omega = 2\pi I/Q_0 \) and \( \tau = RC \), yield the time evolution equation, \( d\theta/dt = \omega − [θ − ⟨ϕ⟩]/\tau \). We compute ⟨ϕ⟩ by solving the Schrödinger equation:

\[
\text{i}\hbar \frac{d\Psi_{n+1}}{dt} = U_0 \Psi_{n+1} + T\Psi_{n+1,n}.
\]

This describes Josephson-like coupling between successive branches via the tunneling matrix element \( T \), which has a Zener-like force dependence. Here \( \Psi_n \) depicts the macrostate amplitude for the system to be on branch \( n \) [figure 2(a)]. We interpret \( \Psi_n \) to be classically robust order parameters, whose magnitudes grow and diminish when the system evolves between successive branches [2-4].

\[ i\hbar \frac{d\Psi_{n+1}}{dt} = U_0 \Psi_{n+1} + T\Psi_{n+1,n}. \] (1)

\textbf{Figure 3.} (a) Theoretical (solid lines) vs experimental (dashed lines [17]) voltage oscillations of an NbSe}_3 crystal at 52 K for current pulse amplitudes (bottom to top, offset by 0, 0.25, 0.5, and 0.75 mV for clarity): 9:90, 10:89, 11:49, and 11:88 µA. (b) Simulated density wave (DW) current vs field for several values of \( q_0 \sim E_0/E_T \). Dotted lines: Bardeen’s modified Zener function [18], \( I_{DW} \propto [E - E_T] \exp[E_0/E] \). (c) Simulated differential resistance, \( R = dV/dI \), where \( I^* = E^* \ell/R_n \) and \( R_n \) is the normal resistance at zero bias. (d) Theoretical (solid lines) vs experimental (dotted lines) normalized \( dV/dI \) vs. \( I \) for NbSe}_3 [2,3].

The quantum fluid thus flows in drip-like fashion as microscopic entities tunnel coherently from one charging energy macrostate to the next. Using this model, we have performed simulations of coherent...
voltage oscillations, narrow-band noise, and current–voltage characteristics [2-4]. Some examples are shown in figure 3. Despite the model’s simplicity, we find unsurpassed agreement between theory and experiment. Work is in progress to use the ST model to study possible collective quantum CDW transport up to 474 K [8].

3. Concluding remarks
CDW transport is emerging as an extraordinary collective quantum phenomenon, at the highest known temperatures for any Earth-bound system. Revealing the underlying quantum mechanisms will likely constitute one of the most transformative breakthroughs in physics, with implications extending well beyond condensed matter. Practical implications include possible forms of quantum information processing, which do not require cooling with a dilution refrigerator. These include quantum reservoir computing, suggested by rapid CDW learning phenomena, in which the CDW would act as a physical reservoir between input and output neural network layers [4]. Related quantum processes include time-correlated tunneling of flux vortices in layered superconductors, important for applications of high-$T_c$ superconductivity [19]. Such advances will be enabled by understanding of the quantum transport mechanisms involving topological deformations in CDWs and related systems.

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5. References
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