Mathematically gifted student’s ways of thinking on fractions

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Abstract. This research aims to analyse mathematically gifted student’s ways of thinking on fractions learning. The respondent was one mathematically gifted student of the 7th graders in the junior high school. The research approach was qualitative. The data were collected through paper and pencil measure, observation, and interview. The data were analyzed by grounded theory with coding and constant comparison. The results showed five mathematical thinking types; those are quantification, schematization, symbolization, concretization and analogical thinking. The findings are then elaborated using some related theories to justify the results.

1. Introduction

One group of learners who have different learning needs are gifted students. Learning for gifted students should be tailored to their gifted characteristics. Gifted students generally have the following characteristics: remarkable memories, detailed observers, deep curiosity, creativity, and the ability to learn teaching materials quickly and precisely with just a little practice and repetition [1].

Many experts have proposed the definition of giftedness. Renzulli states that giftedness is an interaction between the following three factors: above average ability, high creativity, and commitment to completing tasks [2]. Gagne stated that giftedness is a remarkable natural ability in the intellectual domain (residing on 10% highest position in the population) [3].

In mathematics education, some scholars and institutions have proposed the definition of giftedness in mathematics. Krutetskii states that mathematical giftedness is “a unique aggregate of mathematical abilities that opens up the possibility of successful performance in mathematical activity” [4]. According to Kapnik (in Singer, et al) mathematical giftedness is characterized by the following characteristics: remembering mathematical facts, structuring mathematical facts, mathematical sensitivity and mathematical fantasy, transferring mathematical structures, intermodal transfer, reversing lines of thoughts [5]. Based on Mann research, mathematically gifted students can be identified with the performance of students in the classroom, test scores, and recommendations [6]. NCTM stated mathematically gifted student with another language, i.e. mathematically promising student. NCTM introduces the term mathematical promising student characterized by its maximal abilities, motivations, beliefs, experiences and opportunities in mathematics [7].

The appropriate learning for the mathematically gifted student is very important to explore their potency. However, studies that examine the learning of mathematically gifted student are limited [8].

The problem of this research is how mathematically gifted student’s ways of thinking on fractions learning. In this study, we aim to analyse mathematically gifted student’s ways of thinking when she learns fractions. So, the focus of this research is to analyze mathematical thinking of mathematically gifted student on fractions learning.
Mathematical thinking can be divided into two categories, namely mathematical thinking in the form of mathematical methods and mathematical thinking in the form of mathematical content (idea). Both categories of mathematical thinking were supported by a mathematical attitude [9]. This paper will focus on mathematical thinking related to mathematical methods.

According to Isoda and Katagiri, mathematical thinking related to mathematical method consists of: inductive thinking, analogical thinking, deductive thinking, generalizing, specializing, symbolizing, quantification and schematization (thinking that represents with numbers, quantities, and figures) [9].

2. Methods
This research used a qualitative approach. This study used a case study with the single-case (holistic) designs [10]. The case study method is used to describe field findings related to the research problem formulation, i.e., how mathematically gifted student’s ways of thinking on fractions learning. The data collection techniques used in this study are paper and pencil measure (test), observation and interview. The data were analyzed by grounded theory, with coding and constant comparison [11].

2.1. Participant
This research is conducted in a Junior High School in West Java, Indonesia, with one mathematically gifted student in grade 7 as a respondent. The student is selected based on test results, observation and teacher recommendation [6]. The student gets the highest score during the test. She is a teenage girl who is very motivated to complete all tasks. The teacher also recommended her as a mathematically gifted student. The student is given research instruments, then her work was analyzed using Isoda and Katagiri theory.

2.2. Instruments
The instrument in table 1 has been used in research on student understanding of fractions in the inclusive school [12]. External experts have conducted content and construct validation of this instrument. This test instrument is given at the end of the learning session using the Indonesian national curriculum. The test indicator is the same as the fraction topic that has been studied by students. So, this test instrument evaluates the understanding of fraction material learned by the student.

Observations are carried out with the teacher when the student is working on the test. The purpose of this observation is to assess the student's commitment when working on test questions. While the interview was conducted on the teacher to get a recommendation that the student is mathematically gifted.

| No | Problems | Indicator |
|----|----------|-----------|
| 1  | Draw the following fractions: $\frac{1}{2}$ and $\frac{2}{3}$ with two different ways. | Students can understand the representation of fractions in various forms |
| 2  | Check which fraction images are larger. | Students can understand the fraction comparisons in various forms of representation |
| 3  | a. Explain which one is larger: $\frac{1}{2}$ or $\frac{2}{3}$<br>b. Explain which one is larger: $\frac{2}{3}$ or $\frac{3}{4}$ | Students can understand fraction comparisons |
4. Write down your way to find two fractions which are equal \( \frac{1}{2} \)?

5. Complete the following problems with the steps.
   a. \( \frac{1}{3} + \frac{1}{3} = \)
   b. \( \frac{1}{3} + \frac{1}{2} = \)

6. Complete the following problems with the steps.
   a. \( \frac{4}{5} \times \frac{1}{3} = \)
   b. \( \frac{3}{5} \div \frac{3}{5} = \)

7. Susi ran \( \frac{2}{5} \) km on Monday. On Tuesday, Susi ran \( \frac{3}{7} \) km. Explain how many kilometres (km) Susi ran on both days?

2.3. Data Processing Methods

In this study, we arrange the data as follows: the themes are mathematical thinking types, i.e. quantification, schematization, symbolization, etc. We created a code for each student’s answer; then the code is matched with the theme. For example, consider the student’s answer below:

![Figure 1](image1.png)

**Figure 1.** The example of student answer.

From figure 1 above, we record figures as a code. According to Isoda and Katagiri, thinking that is represented by figures is schematization. Therefore, we create a diagram that represents the interrelation between the code and the theme as follows:

![Diagram](image2.png)

**Figure 2.** The interrelation between the code and the theme.

In figure 2, the code (figure) corresponds to the theme (schematization), as Isoda and Katagiri statement: thinking that is represented by figures is schematization.
3. Results and Discussion
From the analysis of data, the student did mathematical thinking as follows:

3.1. Quantification
Quantification is thinking that represents with quantities and numbers. Thinking that represent with quantities means thinking that chooses an appropriate quantity based on the situation or objective [9]. While thinking that represents with numbers means thinking that uses numbers to express the amount of quantities [9]. From the test result, the student performs quantification as follows:

![Figure 3](image)

Figure 3. The example of student answer on quantification.

Figure 3 shows the student’s answer that uses quantification. The student is asked about which fraction representation image is larger. There are three pairs of images. It appears that students understand the problem and answer it by using quantification. The student's solution chooses the appropriate quantity based on the situation, i.e. the picture. We can also identify the student uses numbers to express the amount of quantities, i.e. $\frac{1}{4}$.

3.2. Schematization
Schematization is thinking that is represented by figures [9]. From the test result, the student performs schematization as follows:

![Figure 4](image)

Figure 4. The example of student answer on schematization.

In figure 4, the student is asked to describe fractions $\frac{1}{2}$ and $\frac{2}{5}$ in two different ways. Students' solutions are obtained by using thinking that is represented with figures, i.e. schematization, that produces a cycle and a rectangular picture with a half shaded as an interpretation of $\frac{1}{2}$. While the $\frac{2}{5}$ was interpreted as a cycle and a rectangular pictures whose two parts are shaded, while the other three parts are not shaded.

3.3. Symbolization
Symbolization is thinking that symbolize; this type of thinking try to express a problem with symbol and to refer to symbolize objects. Symbolization also includes the use of mathematical terms to express problems clearly. From the study of the student solution, we found the student’s answer which performed symbolization, as follows:
In figure 5, the student is asked to solved fractions addition word problem. In the student’s answer, it appears that the student performs thinking that symbolize by using fractions addition term to express the word problem briefly and clearly.

3.4. Abstraction (concretization)
According to Isoda and Katagiri, there are four types of abstraction, i.e. thinking that abstracts, thinking that concretizes, thinking that idealizes, and thinking that clarifies conditions [9]. Here is an example of student answers that use one type of abstraction, i.e. thinking that concretizes (concretization).

In figure 6, the student is asked to explain which fractions are larger between $\frac{1}{2}$ and $\frac{2}{3}$. The student’s explanation used the picture to concretizes her thinking that $\frac{2}{3}$ is larger than $\frac{1}{2}$. The student’s thinking is concretization.

Another example of the student’s answer that uses concretization as follows:

In figure 7, the student is asked to explain which fractions are larger between $\frac{2}{3}$ and $\frac{3}{4}$. The student’s explanation used the picture to concretizes her thinking that $\frac{3}{4}$ is larger than $\frac{2}{3}$. The student’s thinking is concretization.

3.5. Analogical Thinking
The fifth mathematical thinking that found from the analysis of student answers is analogical thinking. Analogical thinking is thinking to establish a perspective and to discover solutions [9]. From the analysis of the student’s answer, we find the example as follows:

In figure 8, the first example of student answer on analogical thinking.
In figure 8, the student is asked to write two fractions that equivalent to $\frac{1}{2}$. To answer this problem, the student does the analogical thinking by multiplying the first fraction with 2 and the second fraction with 3 for each numerator and denominator. Fractions $\frac{1}{2}$ is changed to $\frac{2}{4}$ and $\frac{1}{3}$ is changed to $\frac{3}{6}$.

We summarize the findings of students’ mathematical thinking in Table 1 below:

| Thinking Types       | Meaning                                                      |
|----------------------|--------------------------------------------------------------|
| Quantification       | thinking that represents with quantities and numbers         |
| Schematization       | thinking that is represented by figures                      |
| Symbolization        | thinking that symbolize                                      |
| Concretization       | thinking that concretizes                                   |
| analogical thinking  | thinking to establish a perspective and to discover solutions|

In Table 2, the findings of students’ mathematical thinking are based on general observable features, although in reality, they can intersect each other.

In the result, we find five mathematical thinking types minimally; there are quantification, schematization, symbolization, concretization and analogical thinking. The gifted student uses more methods to solve problems if compared with her peers. These findings are in line with Hong and Aqui result; they state that gifted children use more strategies to organize and transform information and use it more effectively [13]. The research result also shows that a gifted student uses schematization or thinking that is represented by figures; they can visualize the mathematics problem. This result is supported by Presmeg’s research; he stated that the gifted student could visualize problems and relations [14].

Furthermore, we also found that the gifted student uses analogical thinking, i.e. thinking to establish a perspective and to discover solutions. Another research finding from Polya and Kiesswetter (in Sriraman) stated the similar result; that is the gifted students have the ability to think analogically and heuristically and to pose related problems [15]. These findings are significant to be considered by the teacher when teaching the mathematically gifted student. Teachers should anticipate how students think when they teach the gifted student. So that teachers and students can achieve optimal learning outcomes.

4. Conclusion

In this research, we find five mathematical thinking types in the case of the mathematically gifted student; those are quantification, schematization, symbolization, concretization and analogical thinking. The results of this study can be made as one of didactic anticipation when teachers teach the concept of fractions to the mathematically gifted student.

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