Digital quantum simulation of real time dynamics of Yukawa coupling

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A mapping from Fock space boson states to qubits is given and an underlying digital quantum simulation algorithm of bosons is derived. We realize the algorithm in MPS (matrix product state) which simulating real time dynamics of Yukawa coupling in varies initial states and coupling constants. This proposal may be achieved in superconductivity NISQ (noisy intermediate-scale quantum) computer not far future.

I. INTRODUCTION

Quantum simulation [1–3] is a method to enhance our understanding of unfamiliar things by controllable and familiar quantum systems, such as the trapped ion quantum computers, ultra-cold atom systems and superconductivity quantum computers, etc. There are analogy quantum simulation and digital quantum simulation. Analogy quantum simulation can simulate particular system efficient and useful, especially when the system is ultra-strong coupling or needs to know real time dynamics information [10–15]. For example, in the trapped ion quantum computer [4, 5], the Dirac, Majorana and Weyl equation quantum simulation is given [6–9]; the quantum simulation of fermion anti-fermion boson interaction in quantum field theory is achieved [10] [11]; an excellent achievement is the theoretical proposal and experimental realization of quantum simulation of Schwinger model’s real time dynamics [12] [13]. There are theoretical proposals of quantum simulation of gauge theory by ultra-cold atoms and optical lattice [16] [17]. Digital quantum simulation can simulate almost any quantum multi-body [18] and quantum field theory models [19] efficiently, no matter the quantum systems are strong correlated, ultra-strong coupling or non-linear.

There are underlying algorithm of digital quantum simulation to simulate fermions and bosons. Almost all things and corresponding models are composed by fermions and bosons. For example, there are quarks, leptons, gauge bosons, Higgs and corresponding Standard Model of particle physics in high energy physics; proton, neutron, nuclear and corresponding NJL (Nambu–Jona-Lasinio) model in nuclear physics; electrons, atoms, ions, phonons, molecules, biomacromolecules and corresponding quantum optics, quantum chemistry and quantum biochemistry; superconductivities, superfluid states, topological insulators and corresponding low temperature physics. For fermions, there are well know underlying Jordan-Wigner and Bravyi-Kitaev transformation which mapping creation and annihilation operators of fermions to operations of qubits with complexities $O(n)$ and $O(\log(n))$, respectively, where $n$ is number of qubits. An experimental digital quantum simulation of fermionic mode in superconductivity quantum computer is realized [23]. An experimental digital quantum simulation Ising model in trapped ion quantum computer is performed [24]. For bosons, there is a classical one-to-one mapping from creation and annihilation operators of boson to operations of qubits with complexity $O(n)$. In this paper, we show another mapping from Fock space of boson to Hilbert space spanned by qubits. We derives a novel underlying quantum simulation algorithm of boson with complexity $O(\log(n))$ from this mapping. We realize a real time dynamics quantum simulation algorithm of Yukawa coupling (fermion anti-fermion boson interaction) by using the novel underlying boson simulation algorithm in MPS (matrix product state). We show the simulation results with varies initial states and parameters settings. The MPS [25] are quantum inspired algorithm developed at recent years in classical computer and can simulate one-dimension quantum multi-body problems and quantum field theory models efficiently and economically when entanglements between qubits not too many. There are many MPS simulation results about real time dynamics or phase diagram of Schwinger model [28] [35], Gross-Neveu model [36] [39], $\phi^4$ interaction [40] [42] and lattice gauge theory [43] [47]. At last of this paper, we analyze the required fidelity of the proposal and point out that the proposal may be simulated by superconductivity NISQ computer not far future.
II. MAPPING FROM BOSONS FOCK SPACE TO QUBITS

We span the truncated $2^t - 1$ dimension Fock space of boson in position $x$ by $t$ qubits as follow

$$|0\rangle_x = |0,0_1,\cdots,0_{t-1},0_t\rangle_x,$$
$$|1\rangle_x = |0,0_1,\cdots,0_{t-1},1_t\rangle_x,$$
$$|2\rangle_x = |0,0_1,\cdots,1_{t-1},0_t\rangle_x,$$
$$|3\rangle_x = |0,0_1,\cdots,1_{t-1},1_t\rangle_x,$$
$$\cdots$$
$$|2^t - 1\rangle_x = |1_1,1_2,\cdots,1_{t-1},1_t\rangle_x,$$

where $|i\rangle_x (i = 0, 1, \cdots, 2^t - 1)$ is the basis of $i$ bosons occupation space in position $x$ and for bosons

$$|\ldots,0_k,\cdots,1_t\rangle_x = |\ldots,\uparrow_k,\cdots,\uparrow_t\rangle_x,$$

where $|\downarrow\rangle$ and $|\uparrow\rangle$ are spin down and up qubits. The maximal occupation number of boson in position $x$ should be infinite in reality and we truncate it to $2^t - 1$ in this paper for quantum simulations. The matrix formulation of $t$ qubits truncated creation operator of boson field is

$$\hat{a}^\dagger_{t,x} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & \sqrt{2} & 0 & 0 & \cdots & 0 \\
0 & 0 & \sqrt{3} & 0 & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & 0 & 0 & \sqrt{2^t - 1} & 0
\end{pmatrix},$$

where $t$ means we need $t$ qubits to simulate a boson quantum state (truncation number is $t$). If we write the creation operator of boson as $2^t - 1$ terms

$$\hat{a}^\dagger_{t,x} = \sum_{i=1}^{2^t - 1} \sqrt{i!} \hat{c}_{1,i}^\dagger \bigotimes_{i=1}^{2^t - 1} x,$$

where $1, t$ indices upon $c_{1,i}^\dagger$ means the $\hat{c}_{1,i}^\dagger$ operator acting on qubits 1 to $t$ at site $x$. Equation (3) tells us that

$$\hat{a}^\dagger_{1,x} = (\sigma_z^1).$$

We find a recurrence relation to derive the $t$ truncation creation operator $\hat{a}^\dagger_{t,x}$ to $t + 1$ truncation creation operator $\hat{a}^\dagger_{t+1,x}$

$$\hat{a}^\dagger_{t+1,x} = \left( \sum_{i=1}^{2^t - 1} \sqrt{i!} \hat{c}_{1,i}^\dagger \bigotimes_{i=1}^{t} x + \sqrt{2^t} \sigma_-^1 \otimes \sigma_-^2 \otimes \cdots \otimes \sigma_-^{t+1} + \sum_{i=2^t+1}^{2^{t+1} - 1} \sqrt{i!} \hat{c}_{1,i}^\dagger \bigotimes_{i=2^t+1}^{t+1} x \right),$$

where $(\hat{c}_{1,i}^{2,t+1})_x$ acting on qubits from 2 to $t + 1$ at site $x$. The definition of $\sigma^+, \sigma^-, I^+$ and $I^-$ are listed

$$\sigma^+ = \frac{1}{2}(\sigma_x + i\sigma_y), \quad \sigma^- = \frac{1}{2}(\sigma_x - i\sigma_y), \quad I^+ = \frac{1}{2}(I + \sigma_z), \quad I^- = \frac{1}{2}(I - \sigma_z).$$

The Pauli matrix formulation of any $t$ truncation boson creation operator can be derived from [48, 49] and we show several examples in Appendix A.

III. DIGITAL QUANTUM SIMULATION ALGORITHM OF YUKAWA COUPLING

The discrete Hamiltonian in interaction picture of Yukawa coupling is ($h = c = 1$)

$$H_I = g \sum_x \kappa \psi^\dagger(x) \psi(x) \phi(x),$$

where $x$ is one-dimension lattice with latticing space $\kappa$. The fermion and scalar fields are represented by creation and annihilation operators [48, 49]

$$\phi(x) = \frac{1}{\sqrt{2\omega_0}} \left( \hat{a}_{t,x} e^{-i\omega_0 t} + \hat{a}^\dagger_{t,x} e^{i\omega_0 t} \right),$$
$$\psi(x) = \frac{1}{\sqrt{2\omega}} \left( \hat{b}_x e^{-i\omega t} + \hat{d}_x^\dagger e^{i\omega t} \right),$$

where $\hat{a}_{t,x}, \hat{b}_x$ and $\hat{d}_x (\hat{a}^\dagger_{t,x}, \hat{b}^\dagger_x$ and $d^\dagger_x$) are annihilation (creation) operators of boson, fermion and anti-fermion. $\omega_0$ and $\omega$ are masses of scalar and fermions. The operator $\hat{a}_{t,x}$ means there are $t$ qubits to span the boson truncated Fock space in position $x$. The creation and annihilation operators in position space are Fourier transform version of creation and annihilation operators in momentum space

$$\hat{a}_{t,x} = \int \frac{dp}{2\pi} \hat{a}_{t,p} e^{ipx},$$
$$\hat{b}_x = \int \frac{dp}{2\pi} \hat{b}_p e^{ipx}, \quad \hat{d}_x^\dagger = \int \frac{dp}{2\pi} \hat{d}_p^\dagger e^{-ipx}.$$
follows
\[ |0\rangle_{x,s} = |\downarrow\rangle_{x,s}, \quad |1\rangle_{x,s} = |\uparrow\rangle_{x,s}, \]
where \( s = 1, 2 = N, P \) is index of anti-fermion and fermion. The Jordan-Wigner mapping gives us a Pauli matrices representation of the creation operators of fermion and anti-fermion
\[
\hat{b}_x^\dagger = -\sigma_x^N \sigma_+^x, \quad \hat{d}_x^\dagger = \sigma_+^x ,
\]
(15)
where \( \sigma^P \) and \( \sigma^N \) acting on qubits “fermion” and “anti-fermion”, respectively, on site \( x \).

The Pauli matrices formulation of Yukawa coupling Hamiltonian in interaction picture is
\[
H_I = \eta \sum_x \left( \xi_1 I + \xi_2 \sigma_x^{x,P} + \xi_3 \sigma_x^{x,N} + \xi_4 \sigma_x^{x,P} \sigma_x^{x,N} + \xi_5 \sigma_x^{y,P} \sigma_x^{y,N} + \xi_6 \sigma_x^{y,P} \sigma_x^{y,N} + \xi_7 \sigma_x^{x,P} \sigma_x^{x,N} \right) \\
(\xi_1 \sigma_x^2 + \xi_2 \sigma_x^1 \sigma_y^1 + \xi_3 \sigma_x^2 \sigma_y^2 + \xi_4 \sigma_x^1 \sigma_y^2 \\
+\xi_5 \sigma_y^2 + \xi_6 \sigma_x^1 \sigma_y^2 + \xi_7 \sigma_y^2 \sigma_x^1 + \xi_8 \sigma_y^1 \sigma_y^2)x, \\
\]
where
\[
\eta = \frac{g \kappa}{8 \omega \sqrt{2} \omega_0}, \quad \xi_1 = 2, \quad \xi_2 = 1, \quad \xi_3 = -1, \\
\xi_4 = -\cos 2\omega t, \quad \xi_5 = \cos 2\omega t, \\
\xi_6 = \sin 2\omega t, \quad \xi_7 = \sin 2\omega t, \\
\xi_8 = \frac{1 + \sqrt{3}}{2} \cos \omega_0 t, \quad \xi_9 = \frac{1 - \sqrt{3}}{2} \cos \omega_0 t, \\
\xi_{10} = \frac{1}{2} \sin \omega_0 t, \quad \xi_{11} = \frac{1}{2} \sin \omega_0 t, \\
\xi_{12} = \frac{1 + \sqrt{3}}{2} \sin \omega_0 t, \quad \xi_{13} = \frac{1 - \sqrt{3}}{2} \sin \omega_0 t, \\
\xi_{14} = \frac{1}{2} \cos \omega_0 t. \quad \xi_{15} = \frac{1}{2} \cos \omega_0 t. \quad (17)
\]

The evolution of quantum state of quantum field theory in interaction picture is driven by the Dyson series time-evolution operator \( U(t, t_0) \)
\[
|\Psi(t)\rangle = U(t, t_0) |\Psi(t_0)\rangle. \quad (18)
\]
The \( |\Psi(t_0)\rangle \) is initial state and can be set by hand. The time evolution operator \( U(t, t_0) \) satisfy
\[
U(t, t_0) = U(t, t - \Delta t) \cdots U(t_0 + \Delta t, t_0), \quad (19)
\]
where \( n_t \cdot \Delta t = t - t_0 \) (\( \Delta t \) is very small) and
\[
U(t + \Delta t, t) \approx e^{-iH_I \Delta t}. \quad (20)
\]
The quantum circuit to simulate \( U(t, t_0) \) is shown in Fig. 1 and the parameters in Fig. 1 are listed in (17). The \( H \) in Fig. 1 is Hadamard operation and \( R \) is
\[
R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}. \quad (21)
\]

We measure the particle number operator of boson and fermions to get real time dynamics of occupation proba-
FIG. 2. Simulation results with varies initial states and coupling constants calculated by MPS are shown. The initial state of (a), (b), (c) and (d) is a pair of fermion anti-fermion in the site $x$. The initial state of (e) and (f) is 3 bosons on site $x$. The coupling constants of (a), (b), (c), (d), (e) and (f) are 1, 6.95, 10, 34.75, 1 and 34.75, respectively. The horizontal axis is time slice number $t = t/\Delta t$ and the vertical axis is probability for fermion, anti-fermion and boson. The blue, green and yellow lines are real time dynamics of occupation density in (23).

The Fig. 2(a), (b), (c) and (d) are real time dynamics evolutions with fermion, antifermion pair initial state in coupling constant 1, 6.95, 10 and 34.75, respectively. With the increasing of coupling constant, the fermion anti-fermion pair annihilating to bosons quickly. At Fig. 2(d), the non-linear dynamics of fermion anti-fermion pair and bosons are clearly emerging in ultra-strong coupling region. The initial state of Fig. 2(e) and (f) is 3 bosons in site $x$ with coupling constant 1 and 34.75. The real time dynamics of Fig. 2(e) and (f) show that the boson creates fermion anti-fermion pair from vacuum through Yukawa coupling. In ultra-strong coupling Fig. 2(f), the real time dynamics of fermion anti-fermion pair and bosons are non-linear.

IV. MPS SIMULATION RESULTS AND NISQ COMPUTER EXPERIMENTS PROPOSAL

We realize the algorithm to simulate the real time dynamics of Yukawa coupling in MPS and the default parameters are taken to be

\[ \kappa = 0.5, \quad \Delta t = 0.1, \quad n_x = 1, \quad n_t = 300, \]
\[ t = l \cdot \Delta t, \quad (l = 0, \cdots n_t), \quad \omega = 6.95, \quad \omega_0 = 0, \quad 1 \]

where we set just one position site $x$ such that $n_x$ is equal to 1. The $n_t$ is time slices number, the $\Delta t$ is time step length, the $\kappa$ is space latticing. The non-perturbative MPS calculation results are shown in Fig. 2.

The quantum algorithm of real time dynamics simulation of Yukawa coupling needs 355 - 238 = 117 CNOT operations in each $\Delta t$ unitary time evolution operator $U(t + \Delta t, t)$ from Fig. 1. The quantum circuits Fig. 1 can be ran in NISQ computer also. For example, we set the $\Delta t = 1/4, 1/3, 1/2$ and $n_t = 10$; then for total fidelity $70\%$, the fidelity of each CNOT operation should higher than

\[ \sqrt{70\%} = 99.9695\%. \]

At present, the average fidelity of CNOT operation in IBMQ-santiago 5 qubits superconductivity NISQ computer is 99.4395\%. We hope to see the digital quantum simulation algorithm of Yukawa coupling be ran in superconductivity NISQ computers such as IBMQ not far future.

V. SUMMARY

We show an novel mapping which reduces the complex of bosons simulation algorithm to $O(\log(n))$. As an example, the mapping derives a digital quantum simulation algorithm of Yukawa coupling. We realize the algorithm in MPS and show the non-perturbative calculation results. It proves that the MPS can work in ultra-strong coupling and non-linear regions of quantum field theory.
We consider an experimental realization in superconductivity NISQ computer for the digital quantum simulation of Yukawa coupling.

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APPENDIX A: THE PAULI MATRICES FORMULATIONS OF BOSONS CREATION OPERATORS IN SEVERAL TRUNCATION NUMBERS

For the boson creation operator, when truncation number $t = 3$, we have
\[
\hat{a}_3^\dagger = I_3^+ \otimes I_3^+ \otimes \sigma_3^+ + \sqrt{2}I_3^+ \otimes I_3^+ \otimes \sigma_3^+ + \sqrt{3}I_3^+ \otimes I_3^+ \otimes \sigma_3^+ + \sqrt{4}I_3^+ \otimes I_3^+ \otimes \sigma_3^+ + \sqrt{5}I_3^+ \otimes I_3^+ \otimes \sigma_3^+ + \sqrt{6}I_3^+ \otimes I_3^+ \otimes \sigma_3^+ + \sqrt{7}I_3^+ \otimes I_3^+ \otimes \sigma_3^+.
\]

The bosons creation operator with truncation number $t = 3$ can represented by Pauli matrices as follow
\[
\hat{a}_3^\dagger = \frac{1}{4}\left[ (1 + \sqrt{3} + \sqrt{5} + \sqrt{7})\sigma_z^2 + 2\sqrt{2} + \sqrt{3}\sigma_z^2 \otimes \sigma_z^2 + 2\sqrt{2} + \sqrt{3}\sigma_z^2 \otimes \sigma_z^2 + 2\sigma_z^1 \otimes \sigma_z^2 \otimes \sigma_z^2 - 2\sigma_z^1 \otimes \sigma_z^2 \otimes \sigma_z^2 \\
+ 2\sigma_z^1 \otimes \sigma_z^2 \otimes \sigma_z^2 + 2\sigma_z^1 \otimes \sigma_z^2 \otimes \sigma_z^2 + (1 + \sqrt{3} - \sqrt{5} - \sqrt{7})\sigma_z^2 \otimes \sigma_z^2 + (\sqrt{2} - \sqrt{6})\sigma_z^1 \otimes \sigma_z^2 \otimes \sigma_z^2 + (\sqrt{2} - \sqrt{6})\sigma_z^1 \otimes \sigma_z^2 \otimes \sigma_z^2 \\
+ (1 - \sqrt{3} - \sqrt{5} + \sqrt{7})\sigma_z^2 \otimes \sigma_z^2 \otimes \sigma_z^2 + (1 - \sqrt{3} + \sqrt{5} - \sqrt{7})\sigma_z^2 \otimes \sigma_z^2 \otimes \sigma_z^2 \\
+ i\left( 2\sqrt{2} + \sqrt{3}\sigma_z^2 \otimes \sigma_z^2 - 2\sqrt{2} + \sqrt{3}\sigma_z^2 \otimes \sigma_z^2 + (-1 + \sqrt{3} - \sqrt{5} + \sqrt{7})\sigma_z^2 \otimes \sigma_z^2 + 2\sigma_z^1 \otimes \sigma_z^2 \otimes \sigma_z^2 + 2\sigma_z^1 \otimes \sigma_z^2 \otimes \sigma_z^2 \\
- 2\sigma_z^1 \otimes \sigma_z^2 \otimes \sigma_z^2 + 2\sigma_z^1 \otimes \sigma_z^2 \otimes \sigma_z^2 + (\sqrt{2} - \sqrt{6})\sigma_z^1 \otimes \sigma_z^2 \otimes \sigma_z^2 - (\sqrt{2} - \sqrt{6})\sigma_z^1 \otimes \sigma_z^2 \otimes \sigma_z^2 \\
- (1 + \sqrt{3} + \sqrt{5} - \sqrt{7})\sigma_z^2 \otimes \sigma_z^2 \otimes \sigma_z^2 - (1 + \sqrt{3} - \sqrt{5} + \sqrt{7})\sigma_z^2 \otimes \sigma_z^2 \otimes \sigma_z^2 \right),
\]

and the particle number operator
\[
\hat{n}_3 = \hat{a}_3^\dagger \hat{a}_3 = \frac{1}{2}(7I - 4\sigma_z^1 - 2\sigma_z^2 - \sigma_z^3).
\]

Something usually appearing blocks in quantum simulation of bosons are
\[
\hat{n}_3 \hat{n}_3 = \frac{1}{2}(35I - 28\sigma_z^1 - 14\sigma_z^2 - 7\sigma_z^3 + 8\sigma_z^1 \sigma_z^2 + 4\sigma_z^1 \sigma_z^3 + 2\sigma_z^2 \sigma_z^3),
\]

and
\[
\hat{a}_3^\dagger \hat{a}_3^\dagger \hat{a}_3 \hat{a}_3 = \frac{1}{4}[(\sqrt{2} + \sqrt{6} + \sqrt{30} + \sqrt{42})\sigma_z^2 + (\sqrt{2} - \sqrt{6} - \sqrt{30} + \sqrt{42})\sigma_z^1 \sigma_z^2 \sigma_z^2 + (\sqrt{12} + \sqrt{20})\sigma_z^1 \sigma_z^2 \sigma_z^2 \\
+ (\sqrt{12} + \sqrt{20})\sigma_z^1 \sigma_z^2 \sigma_z^2 + (\sqrt{12} - \sqrt{20})\sigma_z^1 \sigma_z^2 \sigma_z^2 + (\sqrt{12} - \sqrt{20})\sigma_z^1 \sigma_z^2 \sigma_z^2 + (\sqrt{2} - \sqrt{6} \\
+ \sqrt{30} - \sqrt{42})\sigma_z^1 \sigma_z^2 \sigma_z^2 + (\sqrt{2} + \sqrt{6} - \sqrt{30} - \sqrt{42})\sigma_z^1 \sigma_z^2 \sigma_z^2].
\]

For truncation number $t = 4$, the Pauli matrices formulation of boson creation operator can be derived from recurrence relation (6)
\[
\hat{a}_4^\dagger = I_4^+ \otimes I_4^+ \otimes I_4^+ \otimes \sigma_4^+ + \sqrt{2}I_4^+ \otimes I_4^+ \otimes \sigma_4^+ \otimes \sigma_4^+ + \sqrt{3}I_4^+ \otimes I_4^+ \otimes I_4^+ \otimes \sigma_4^+ + \sqrt{4}I_4^+ \otimes I_4^+ \otimes I_4^+ \otimes \sigma_4^+ \\
+ \sqrt{5}I_4^+ \otimes I_4^+ \otimes I_4^+ \otimes \sigma_4^+ + \sqrt{6}I_4^+ \otimes I_4^+ \otimes I_4^+ \otimes \sigma_4^+ + \sqrt{7}I_4^+ \otimes I_4^+ \otimes I_4^+ \otimes \sigma_4^+ + \sqrt{8}I_4^+ \otimes I_4^+ \otimes I_4^+ \otimes \sigma_4^+ \\
+ \sqrt{9}I_4^+ \otimes I_4^+ \otimes I_4^+ \otimes \sigma_4^+ + \sqrt{10}I_4^+ \otimes I_4^+ \otimes I_4^+ \otimes \sigma_4^+ + \sqrt{11}I_4^+ \otimes I_4^+ \otimes I_4^+ \otimes \sigma_4^+ + \sqrt{12}I_4^+ \otimes I_4^+ \otimes I_4^+ \otimes \sigma_4^+ \\
+ \sqrt{13}I_4^+ \otimes I_4^+ \otimes I_4^+ \otimes \sigma_4^+ + \sqrt{14}I_4^+ \otimes I_4^+ \otimes I_4^+ \otimes \sigma_4^+ + \sqrt{15}I_4^+ \otimes I_4^+ \otimes I_4^+ \otimes \sigma_4^+ + \sqrt{16}I_4^+ \otimes I_4^+ \otimes I_4^+ \otimes \sigma_4^+,\]
and for truncation number $t = 5$

$$\begin{align*}
\hat{a}^\dagger_1 &= I_+^1 \otimes I_+^2 \otimes I_+^1 \otimes I_+^1 \otimes \sigma^2_+ + \sqrt{2}I_+^1 \otimes I_+^1 \otimes I_+^1 \otimes I_+^1 \otimes \sigma^2_+ + \sqrt{3}I_+^1 \otimes I_+^1 \otimes I_+^1 \otimes I_+^1 \otimes \sigma^2_+ \\
&+ \sqrt{4}I_+^1 \otimes I_+^1 \otimes \sigma^2_+ \otimes \sigma^2_+ + \sqrt{5}I_+^1 \otimes I_+^1 \otimes I_+^1 \otimes I_+^1 \otimes \sigma^2_+ + \sqrt{6}I_+^1 \otimes I_+^1 \otimes I_+^1 \otimes I_+^1 \otimes \sigma^2_+ \\
&+ \sqrt{7}I_+^1 \otimes I_+^1 \otimes \sigma^2_+ \otimes \sigma^2_+ + \sqrt{8}I_+^1 \otimes \sigma^2_+ \otimes \sigma^2_+ \otimes \sigma^2_+ + \sqrt{9}I_+^1 \otimes I_+^1 \otimes I_+^1 \otimes \sigma^2_+
\end{align*}$$

**APPENDIX B: THE PAULI MATRICES FORMULATIONS OF FIELD OPERATOR**

The scalar field is represented by Pauli matrices as follow

$$\phi(x) = \frac{1}{2\sqrt{2\omega_0}} \left[ \left( (1 + \sqrt{3})\sigma_x^2 + (1 - \sqrt{3})\sigma_z^2 \sigma_x^2 + \sqrt{2}\sigma_x^4 \sigma_y^2 + \sqrt{2}\sigma_x^4 \sigma_y^2 \right) \cos \omega_0 t + \left( (1 + \sqrt{3})\sigma_y^2 \right. \right.$$  

$$\left. + (1 - \sqrt{3})\sigma_x^4 \sigma_y^2 + \sqrt{2}\sigma_x^4 \sigma_y^2 \right] \sin \omega_0 t_{x, y}.$$  

As we have

$$I_\pm^1 = I_\pm, \quad \sigma_\pm^1 = \sigma_\mp,$$

then for fermions fields we write

$$\psi(x) = \frac{1}{2\omega} \left( I + \frac{1}{2} \sigma^x \right)^P + \frac{1}{2} \sigma^x \sigma^N - \frac{1}{2} \left( \sigma_x^P \sigma_x^N - \sigma_y^P \sigma_y^N \right) \cos 2\omega t + \frac{1}{2} \left( \sigma_x^P \sigma_y^N + \sigma_y^P \sigma_x^N \right) \sin 2\omega t.$$  

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