THE FRACTAL STRUCTURE OF THE UNIVERSE: A NEW FIELD THEORY APPROACH

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ABSTRACT

While the universe becomes more and more homogeneous at large scales, statistical analysis of galaxy catalogs have revealed a fractal structure at small scales (λ < 100 h⁻¹ Mpc), with a fractal dimension D = 1.5–2. We study the thermodynamics of a self-gravitating system using the theory of critical phenomena and finite-size scaling, and we show that gravity provides a dynamical mechanism for producing this fractal structure. We develop a field theoretical approach for computing the galaxy distribution, assuming them to be in quasi-isothermal equilibrium. Only a limited (although large) range of scales is involved, between a short-distance cutoff, below which other physics intervene, and a large-distance cutoff, beyond which the thermodynamic equilibrium is not satisfied. The galaxy ensemble can be considered at critical conditions, with large density fluctuations developing at any scale. From the theory of critical phenomena, we derive the two independent critical exponents ν and η and predict the fractal dimension D = 1/ν to be either 1.585 or 2, depending on whether the long-range behavior is governed by the Ising or the mean-field fixed points, respectively. Both set of values are compatible with present observations. In addition, we predict the scaling behavior of the gravitational potential to be r⁻(1+η/2), that is, r⁻⁰.5 for mean field or r⁻⁰.519 for the Ising fixed point. The theory allows us to compute the 3 and higher density correlators without any assumption or Ansatz. We find that the N-point density scales as r_i^(N−1)(D-3) when r_i >> r_o, 2 ≤ i ≤ N. There are no free parameters in this theory.

Subject headings: galaxies: clusters: general — large-scale structure of universe — methods: statistical

1. INTRODUCTION

One obvious feature of galaxy and cluster distributions in the sky is their hierarchical character; galaxies gather in groups that are embedded in clusters, then in superclusters, and so on (Shapley 1934; Abell 1958). Moreover, galaxies and clusters appear to obey scaling properties, such as the power law of the two-point correlation function,

$$\xi(r) \propto r^{-\gamma},$$

with a slope γ of ≈ 1.7, the same for galaxies and clusters (e.g., Peebles 1980, 1993). This scale invariance suggested early on the idea of using fractal models for the clustering hierarchy of galaxies (de Vaucouleurs 1960, 1970; Mandelbrot 1975). Since then, many authors have shown that a fractal distribution does indeed reproduce quite well this aspect of galaxy catalogs, for example by simulating a fractal and observing it, as with a telescope (Scott, Shane, & Swanson 1954; Soneira & Peebles 1978). Sometimes the analysis has been done in terms of a multifractal medium (Balian & Schaeffer 1989; Castagnoli & Provenzale 1991; Martínez, Paredes, & Saar 1993; Dubrulle & Lachieze-Rey 1994).

There is some ambiguity in the definition of the two-point correlation function ξ(r) above, since it depends on the assumed scale beyond which the universe is homogeneous; indeed, it includes a normalization by the average density of the universe, which, if the homogeneity scale is not reached, depends on the size of the galaxy sample. Once ξ(r) is defined, one can always determine a length r_o where ξ(r_o) = 1 (Davis & Peebles 1983; Hamilton 1993). For galaxies, the most frequently reported value is r_o ≈ 5 h⁻¹ Mpc (where

$$h = H_0 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

but it has been shown to increase with the distance limits of galaxy catalogs (Davis et al. 1988). The term r_o is the “correlation length” in the galaxy literature. (The notion of correlation length ξ_0 is usually different in physics, where ξ_0 characterizes the exponential decay of correlations, N ~ e⁻¹/ξ_0. For power decaying correlations, it is said that the correlation length is infinite.)

The same problem arises for the two-point correlation function of galaxy clusters; the corresponding ξ(r) has the same power law as galaxies, their length r_o has been reported to be about r_o ≈ 25 h⁻¹ Mpc, and their correlation amplitude is therefore about 15 times higher than that of galaxies (Postman, Geller, & Huchra 1986; Postman, Huchra, & Geller 1992). The latter condition is difficult to understand unless there is a considerable difference between galaxies belonging to clusters and field galaxies (or morphological segregation). The other obvious explanation is that the normalizing average density of the universe was then chosen to be lower.

This statistical analysis of the galaxy catalogs has been criticized by Pietronero (1987), Einasto (1989), and Coleman & Pietronero (1992), who stress the uncomfortable dependence of ξ(r) and the length r_o upon the finite size of the catalogs, and on the a priori assumed value of the large-scale homogeneity cutoff. One way to circumvent these problems is to deal instead with the average density as a function of size (cf. § 2). It has been shown that the galaxy distribution behaves as a pure self-similar fractal over scales up to ~100 h⁻¹ Mpc, the deepest scale to which the data are statistically robust (Sylòs Labini et al. 1996; Sylòs Labini & Pietronero 1996). This is more consistent with the observation of contrasted large-scale structures, such as superclusters, large voids, or great walls of galaxies of ~200 h⁻¹ Mpc (de Lapparent, Geller, & Huchra 1986; Geller & Huchra 1989). After a proper statistical analysis of all available catalogs (CfA, SSRS, IRAS, APM, LEDA, etc., for galaxies, and Abell and ACO for clusters), Pietronero et al.
(1997) state that the transition to homogeneity might not yet have been reached up to the deepest scales probed until now. At best, this point is quite controversial, and the large-scale homogeneity transition is not yet well known.

Isotropy and homogeneity are expected at very large scales from the cosmological principle (e.g., Peebles 1993). However, this does not imply local or midscale homogeneity (e.g., Mandelbrot 1982; Sylos Labini 1994): a fractal structure can be locally isotropic but inhomogeneous. The main observational evidence in favor of the cosmological principle is the remarkable isotropy of the cosmic background radiation (e.g., Smoot et al. 1992), which provides information about the universe at the matter/radiation decoupling. There must therefore exist a transition from small-scale fractality to large-scale homogeneity. This transition is certainly smooth, and might correspond to the transition from linear perturbations to the nonlinear gravitational collapse of structures. The present catalogs do not yet show the transition, since they do not look sufficiently far back in time. It may be noticed that some recent surveys begin to see a different power-law behavior at large scales ($\lambda \approx 200-400$ h$^{-1}$ Mpc; e.g., Lin et al. 1996).

There are several approaches to understanding nonlinear clustering, and therefore the distribution of galaxies, in an infinite gravitating system. Numerical simulations have been widely used, in the hope of tracing the initial mass spectrum of fluctuations back from the observations, and testing postulated cosmologies such as CDM and related variants (cf. Ostriker 1993). This approach has not yet yielded definite results, especially since the physics of the multiple-phase universe is not well known. Numerical limitations (restricted dynamical range due to the softening and limited volume) have also often masked the expected self-similar behavior (Colombi, Bouchet, & Hernquist 1996). A second approach, which should work essentially in the linear (or weakly nonlinear) regime, is to solve the BBGKY equations for the comoving fractal in and apply our field theory approach in §4.

2. CORRELATION FUNCTIONS AND MASS DENSITY IN A FRAC TAL

The use of the two-point correlation function $\xi(r)$, widely spread in galaxy distributions studies, is based on the assumption that the universe reaches homogeneity on a scale smaller than the sample size. It has been shown by Coleman, Pietronero, & Sanders (1988) and Coleman & Pietronero (1992) that such a hypothesis could significantly perturb the results. The correlation function is defined as

$$\xi(r) = \left\langle \frac{n(r)n(r + r)}{n^2} \right\rangle - 1,$$

where $n(r)$ is the number density of galaxies, and $\langle \ldots \rangle$ denotes the volume average (over $d^3r$). The length $r_0$ is defined by $\xi(r_0) = 1$. The function $\xi(r)$ has a power-law behavior of slope $-\gamma$ for $r < r_0$, then turns down to zero rather quickly at the statistical limit of the sample. This rapid fall leads to an overestimate of the small-scale $\gamma$. Pietronero (1987) introduces the conditional density

$$\Gamma(r) = \frac{\langle n(r)n(r + r) \rangle}{\langle n \rangle},$$

which is the average density around an occupied point. For a fractal medium, where the mass depends on the size as

$$M(r) \propto r^D,$$

$D$ being the fractal (Hausdorff) dimension, the conditional density behaves as

$$\Gamma(r) \propto r^{D-3}.$$

This is exactly the statistical analysis used for the interstellar clouds, since the ISM astronomers from the start...
have not adopted any large-scale homogeneity assumption (cf. Pfenniger & Combes 1994).

The fact that the correlation \( \xi(r) \) can be highly misleading for a fractal is readily seen, since

\[
\xi(r) = \frac{\Gamma(r)}{\langle n \rangle} - 1 ,
\]

and since for a fractal structure the average density of the sample \( \langle n \rangle \) is a decreasing function of the sample length scale. In the general use of \( \xi(r) \), \( \langle n \rangle \) is taken as a constant, and we can see that

\[ D = 3 - \gamma . \]

If, for very small scales, both \( \xi(r) \) and \( \Gamma(r) \) have the same power-law behavior, with the same slope of \( -\gamma \), then the slope appears to steepen for \( \xi(r) \) when it approaches the length \( r_0 \). This explains why with a correct statistical analysis (Di Nella et al. 1996; Sylos Labini & Amendola 1996; Sylos Labini et al. 1996), the actual \( \gamma \) \( \approx 1-1.5 \) is smaller than that obtained using \( \xi(r) \). This also explains why the amplitude of \( \xi(r) \) and \( r_0 \) increase with the sample size, and also for clusters.

In the following discussion, we adopt the framework of the fractal medium that we used for the ISM (dVSC), and will no longer consider \( \xi(r) \).

3. EQUATIONS IN THE COMOVING FRAME

Let us consider the universe in expansion, with the characteristic scale factor \( a(t) \). For the sake of simplicity, we model the galaxies by points of equal mass \( m \), although they do have a mass spectrum (which may be responsible for a multifractal structure; see Sylos Labini & Pietronero 1996).

The present analysis can be generalized for galaxies of different masses following the lines of § 4 of de Vega et al. (1996b). We expect to come to this point in future work. (1996b).

As a first approximation, we assume in the following discussion that the characteristic time of the particle motions under the gravitational self-interaction are shorter than the time variation of \( a(t) \). We can then consider that this system of self-gravitating particles is at any time in approximate thermal equilibrium. This hypothesis is true, of course, for structures that have already decoupled from the expansion and are truly self-gravitating and virialized. It could also be valid for the whole nonlinear regime of the gravitational collapse. As for the linear regime, we already know that the primordial fluctuations are not forgotten in the large-scale structures, and therefore the resulting correlations will depend on initial conditions and not be entirely determined by self-gravity.

The above assumption introduces a natural upper limit to the scales of the theory developed below. Similar to the case of the interstellar medium, the fractal structure considered is bounded by a short-distance cutoff and also by a large-scale limit (dVSC).

The short-distance cutoff corresponds to the appearance of other physics, essentially dissipative, at short scales, that we do not need to introduce here. In addition, the short-distance cutoff avoids the gravothermal catastrophe. For the ISM, the cutoff was naturally the size of the smaller fragments, of the order of the Jeans length. Here the cutoff also corresponds to the size of the "particles" considered, i.e., the galaxy size below which another physics steps in, related to stellar formation and radiation.

The present treatment can be generalized when thermal equilibrium only holds region by region (de Vega et al. 1998). In such cases, we are led to a quenched average over the temperature, and we argue that the scaling properties are the same as in exact thermal equilibrium, provided that the temperature variations are smooth over the structure at the considered scale.

The fact that in the catalogs we are observing large-scale structures in projection at different epochs, with different values of the scale factor \( a(t) \), could slightly modify the fractal dimension. Even though fractal structures are self-similar and scale independent, the largest scales are systematically observed at a younger epoch, where the contrast has not grown up as high as at present. This evolution effect, however, should be significant only at high redshift \( (z > 1) \), and the present catalogs are not yet statistically robust so far back in time (the average redshift of optical catalogs is about 0.1).

4. APPLICATION OF RENORMALIZATION GROUP THEORY

As in all scale-independent problems where the fluctuations cannot be represented by analytical functions, the renormalization group theory developed in the 1970s for the study of critical phenomena appears to be perfectly adapted here (e.g., Wilson & Kogut 1974). We can consider
the fractal structure of the universe as the critical state of a system, where fluctuations develop at any scale, with a very large correlation length (asymptotically infinite). The fluctuations that are distributed as a fractal of dimension D are the large-scale structures of the universe (cf. Totsuji & Kihara 1969).

We have recently begun to tackle, with the tools of statistical field theory, the study of an N-body system only interacting through self-gravity (dVSC). We have found an exact mathematical connection between this statistical system and a scalar field with exponential self-coupling. We then turned to the powerful methods of field theory (e.g., Itzykson & Drouffe 1989; Parisi 1988; Zinn-Justin 1989). Using the renormalization group, the critical behavior of this gravitational system has been described and its critical exponents identified. This theory both explains the origin of the fractal structure and predicts its fractal dimension D. This has been successfully applied to the interstellar medium (dVSC). Another approach has been proposed for galaxy correlations (not for ISM) (Hochberg & Pérez Mercader 1996), but it yields different critical exponents.

Let us apply the theory to the system of galaxy points, already defined in the previous section. Since they are considered in approximate thermal equilibrium, we will use the grand canonical ensemble, which also allows a variable number of particles. The grand partition function of the system can be written as

\[ Z = \sum_{N=0}^{\infty} \frac{z^N}{N!} \prod_{i=1}^{N} \left( \frac{d^3p_i d^3x_i}{(2\pi)^3} e^{-\beta\mu N_i} \right), \]  

where \( z \) is the fugacity, which for an ideal gas of number density \( \rho_0 = z = \rho_0(h^2/2\pi m k T)^{3/2} \), where \( h \) is the Planck constant.

In dVSC, we found a functional integral representation for the grand partition function,

\[ Z = \int D\phi e^{-S(\phi, t)} , \]  

i.e., \( Z \) can be written as the partition function for a single scalar field \( \phi(x) \) with local action,

\[ S[\phi(t)] = \frac{1}{T_{\text{eff}}} \int d^3x \left[ \frac{1}{2} (\nabla \phi)^2 - \mu^2 e^{\phi(x)} \right] , \]  

where

\[ \mu^2 = \frac{\pi^{5/2}}{h^3} zG(2m)^{7/2} \sqrt{kT} \quad T_{\text{eff}} = 4\pi \frac{Gm^2}{kT} . \]  

In the \( \phi \) field representation, the mass-density equation (3) is expressed as

\[ \rho(x) = -\frac{m}{T_{\text{eff}}} \nabla^2 \phi(r) = \frac{\mu^2}{T_{\text{eff}}} e^{\phi(r)} , \]  

and the mass contained in a region of size \( R \) is given by

\[ M(R) = \frac{\mu^2}{T_{\text{eff}}} \int_0^R e^{\phi(x)} d^3x . \]  

The mass parameter \( \mu \) coincides at the tree level with the inverse of the Jeans length \( d_j \) (dVSC),

\[ \mu = \frac{12}{\pi} \frac{1}{d_j} . \]  

The functional representation for the grand partition function can be easily generalized for an arbitrary scale factor \( a(t) \). After the changes specified above in equation (6), the local action becomes

\[ S[\phi(t)] = \frac{a(t)}{T_{\text{eff}}} \int d^3x \left[ \frac{1}{2} (\nabla \phi)^2 - \mu^2 a(t)^2 e^{\phi(x)} \right] . \]  

Note that all quantities depend on time through the scale factor \( a(t) \) only. There is no integration over \( t \).

The mass parameter \( \mu \) in the \( \phi \) theory is effectively multiplied by the scale factor \( a(t) \). Since the Jeans length \( d_j \approx \mu^{-1} \), according to equation (13), in comoving coordinates \( d_j \) effectively becomes

\[ d_j = \sqrt{\frac{12}{\pi \mu a(t)}} \]  

as one would expect.

On the other hand, the dimensionless coupling constant

\[ g^2 = \mu T_{\text{eff}} \]  

is unchanged by the replacements of equation (6).

Therefore, for any fixed time \( t \) we find the same scaling behavior, after making the replacement

\[ \mu \rightarrow \mu a(t) \]  

and keeping the coupling \( g \) unchanged.

4.1. Scaling Behavior

As is well known in the theory of critical phenomena (e.g., Wilson 1975, 1983; Domb & Green 1976), physical quantities for infinite-volume systems diverge at the critical point as \( \Lambda \) to a negative power, where \( \Lambda \) measures the distance to the critical point. The correlation length \( \xi_0 \) diverges as

\[ \xi_0(\Lambda) \sim \Lambda^{-\nu} , \]  

and the specific heat \( C \) behaves as

\[ C \sim \Lambda^{-\alpha} . \]  

The critical exponents \( \nu \) and \( \alpha \) are pure numbers that depend only on the universality class of the problem considered (e.g., Binney et al. 1992).

For a finite-volume system, all physical quantities are finite at the critical point. For a system whose size \( R \) is large, the physical quantities take large, but finite, values at the critical point. Thus, for large critical systems, one can use asymptotically the infinite-volume theory. In particular, the correlation length \( \xi_0 \) can be identified with the relevant physical scale \( R \) as \( \xi_0 \sim R \). This implies that

\[ \Lambda \sim R^{-1/\nu} . \]  

These concepts apply to the gravitational \( \phi \) theory, since it exhibits scaling in a finite-volume \( \sim R^3 \) (dVSC). Scaling behavior was found for a continuum set of values of \( \mu^2 \) and \( T_{\text{eff}} \).

We have previously shown (dVSC) that it is possible to identify

\[ \Lambda \equiv \frac{\mu^2}{T_{\text{eff}}} = \frac{z}{h^3} (2\pi m k T)^{3/2} . \]  

Note that the critical point \( \Lambda = 0 \) corresponds to zero fugacity. Then, the partition function in the scaling regime
can be written as

$$\mathcal{Z}(\Lambda) = \int \mathcal{D} \phi e^{-S + \Lambda \int dx \phi(x)} ,$$  \hspace{1cm} (18)

where $S^*$ stands for the action at the critical point.

We define the renormalized mass density as

$$m\rho(x)_{\text{ren}} \equiv m e^{\phi(x)} ,$$  \hspace{1cm} (19)

and we identify it with the energy density in the renormalization group (also called the thermal perturbation operator).

Since the $\phi$ theory exhibits scaling (dVSC), the non-analytical part of the free energy is

$$\log \mathcal{Z}(\Lambda) \propto \Lambda^{2 - z} ,$$

so that its second derivative is $\mathcal{G} \sim \Lambda^{-z}$. Calculating the logarithmic derivative of $\mathcal{Z}(\Lambda)$ with respect to $\Lambda$ from equation (18), using the standard relation between critical exponents in a three-dimensional space, $x = 2 - 3\nu$, and equations (19) and (12), we find that the mass fluctuations inside a volume of radius $R$,

$$[\Delta M(R)]^2 \equiv \langle M^2 \rangle - \langle M \rangle^2 ,$$

will scale as

$$\Delta M(R) \sim R^{1/\nu} .$$  \hspace{1cm} (20)

The scaling exponent $\nu$ can be identified with the inverse Haussdorf (fractal) dimension $D$ of the system,

$$D = \frac{1}{\nu} .$$

4.2. Critical Exponents

As usual in the theory of critical phenomena, there are only two independent critical exponents. All exponents can be expressed in terms of two: for instance, the fractal dimension $D = 1/\nu$ and the independent exponent $\eta$, which usually governs the spin-spin correlation functions. The exponent $\eta$ appears here in the $\phi$-field correlator (dVSC), describing the gravitational potential, which scales as

$$\langle \phi(r) \rangle \sim r^{-(1 + \eta)/2} .$$

The values of the critical exponents depend on the fixed point that governs the long-range behavior of the system. The renormalization group approach applied to a single-component scalar field in three space dimensions shows the presence of only two fixed points: the mean field point and the Ising fixed point. The scaling exponents associated with the Ising fixed point are $\nu_{\text{Ising}} = 0.631 \ldots$, $D_{\text{Ising}} = 1.585 \ldots$, and $\eta_{\text{Ising}} = 0.037\ldots$. The mean field value for the critical exponents are $\nu_{\text{mf}} = 1/2$, $D_{\text{mf}} = 2$, and $\eta_{\text{mf}} = 0$.

The value of the dimensionless coupling constant $\nu = \mu T_{\text{eff}}$ should decide whether the fixed point chosen by the system is mean field (weak coupling) or Ising (strong coupling). At the tree level, we estimate $\nu \approx N^{1/2}$, where $N$ is the number of points in a Jeans volume $d_j$. The coupling constant then appears to be on the order of 1, and we cannot settle this question without effective computations of the renormalization group equations. At this point, the predicted fractal dimension $D$ should be between 1.585 and 2.

4.3. Three-Point and Higher Correlations

Our approach allows us to compute higher order correlators without any extra assumption (de Vega, Sánchez, & Combes 1998).

The two- and three-point densities,

$$D(r_1, r_2) \equiv \langle n(r_1)n(r_2) \rangle ,$$

$$D(r_1, r_2, r_3) \equiv \langle n(r_1)n(r_2)n(r_3) \rangle ,$$  \hspace{1cm} (21)

can be expressed in terms of the correlation functions,

$$D(r_1, r_2) = n_1 n_2 + C_{12} ,$$  \hspace{1cm} (22)

$$D(r_1, r_2, r_3) = n_1 n_2 n_3 + n_1 C_{23} + n_2 C_{13} + n_2 C_{13} + C_{123} .$$

Here,

$$n_i \equiv \langle n(r_i) \rangle , \hspace{1cm} i = 1, 2, 3 ,$$

and $C_{ij}$ and $C_{ijk}$ are the two- and three-point correlation functions, respectively:

$$C_{ij} \equiv C(r_i, r_j) , \hspace{1cm} C_{ijk} \equiv C(r_i, r_j, r_k) .$$

The behavior of $n_i$, $C_{ij}$, and $C_{ijk}$ in the scaling regime follows from the renormalization group equations at criticality (de Vega et al. 1998). If we do not impose homogeneity at all scales, we find

$$\langle n(r) \rangle \sim A r^{D - 3} ,$$

$$C(r_1, r_2) \sim B r_1^{2(D - 3)} ,$$

$$C(r_1, r_2, r_3) \sim C r_1^{3(2 - D)} (n_2 n_3 + C_{23}) + B r_1^{2(D - 3)} (n_2 n_3) + C r_1^{2(D - 3)} .$$  \hspace{1cm} (23)

Notice that this expression is dominated by the first term, since $D - 3 < 0$.

Higher point distributions can be treated analogously in our approach. We find that the dominant behavior in the $N$-point density is

$$C(r_1, r_2, \ldots, r_n) \sim r_1^{N - (D - 3)} .$$  \hspace{1cm} (25)

Note that when homogeneity is assumed to hold over all scales, the critical behavior of the $N$-point correlation function involves a factor $r_1^{2N - (3D - 3)}$ (Itzykson & Drouffe 1989).

Equations (24)-(25) are qualitatively similar, although not identical, to the behavior inferred by assuming the factorized hierarchical Ansatz (fhA) (Balian & Schaeffer 1989). That is,

$$D(r_1, r_2) \sim n^2 (1 + br_1^{D-3}) r_1 \sim n^2 (1 + r_1^{D-3}) ,$$

$$D(r_1, r_2, r_3) \sim n^3 + n^3 b (r_1^{D-3} + r_1^{D-3} + r_1^{D-3}) + n^3 Q_3 (r_1^{D-3} + r_1^{D-3} + r_1^{D-3})$$

$$\approx n^3 \left[1 + 2br_1^{D-3} + 2br_1^{D-3}(b + Q_3 r_1^{D-3}) + Q_3 r_1^{D-3} \right] ,$$

$$r_1 \gg r_2, r_3 .$$
where $r_{12} \equiv |r_1 - r_2|$, and so on, and $b$ and $Q_1$ are constants. Note that in the factorized hierarchical Ansatz, the fractal dimension $D$ is not predicted but is a free parameter.

We see that the dominant behaviors in equations (24) and (26) are similar when the scaling exponents $D - 3$ are the same.

5. CONCLUSIONS

The statistical analysis of the most recent galaxy catalogs, without the assumption of homogeneity at a scale smaller than the catalog depth, has determined that the universe has a fractal structure at least up to $\approx 100$ $h^{-1}$ Mpc (Syllos Labini et al. 1996). An analysis in terms of conditional density has revealed that the fractal dimension is between $D = 1.5$ and 2 (Di Nella et al. 1996; Syllos Labini & Amendola 1996). We apply a theory that we have developed to explain the fractal structure of the interstellar medium (dVSC), which has the same dimension $D$. The physics is based on the self-gravitating interaction of an ensemble of particles, over scales limited at both short and large distances. The short-distance cutoff is brought about by other physical processes, including dissipation. The long-range limit is fixed by the expansion timescale. In between, the system is assumed to be in approximate thermal equilibrium. The dynamical range of scales involved in this thermodynamic quasi-equilibrium is at present limited to 3–4 orders of magnitude, but it will increase with time.

The critical exponents found in the theory do not depend on the conditions at the cutoff, which determine only the amplitudes. The theory is based on statistical study of the gravitational field; it is shown that the partition function of the $N$-body ensemble is equivalent to the partition function of a single scalar field with a local action. This allows to use field theory methods and the renormalization group to find the scaling behavior. We find scaling behavior for a full range of temperatures and couplings. The theory then predicts for the system a fractal dimension $D = 1.585$ for the Ising fixed point or $D = 2$ in the case of the mean-field fixed point. Both are compatible with the available observations.

The $N$-point density correlators are predicted to scale with exponent $(N - 1)(D - 3)$ when $r_i \gg r_2, \; 2 \leq i \leq N$; that is, $-(N - 1)$ for the mean field or $-1.415(N - 1)$ for the Ising point.

We predict in addition a critical exponent $-\frac{1}{2}(1 + \eta)$ for the gravitational potential; that is, $-0.500$ for the mean field or $-0.519$ for the Ising fixed point.

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