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Structural Stability Analysis for Truss Bridge

Minhui Tong\textsuperscript{a,b}, Fei Mao\textsuperscript{a}, Huiqing Qiu\textsuperscript{a}

\textsuperscript{a} Tongji University, School of Mechanical Engineering, Shanghai and 201804, China
\textsuperscript{b} Shanghai Maritime University, School of Logistics Engineering, Shanghai and 200135, China

Abstract

In order to analyze truss bridge’s stability, Linear buckling analysis and nonlinear stability analysis have been done by ANSYS. Impact factor and nonlinearity are taken over, such as geometric nonlinearity, material nonlinearity and initial geometric defects. Then, the influence of the Lead Rubber Bearing (LRB) on structural stability was studied. The buckling modals and critical buckling load of truss bridge have been obtained. It shows that overall instability appears earlier than local instability; the critical bucking load of different types is less than linear buckling analysis result after considering nonlinearity; though its stability could be reduced by LRB, truss bridge has a good stability all the same.

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* Corresponding author. Tel.: +1-376-158-4005; fax: +0-215-885-5200.
E-mail address: tmh_cn@sohu.com.
1. Introduction

There are two ways to study truss bridge’s stability. One is Linear Buckling Stability Theory (The first type of stability theory), the other is Nonlinear Stability Theory (The second type of stability theory). Timoshenko has put forward Overall Stability Theory [1], which is based on the first type. Many scholars use this theory to solve stability problems [2] because it makes problems easier to solve. Due to overestimation about structure performance of anti-instability, the calculation result is usually greater than the actual value. Knot has firstly brought forward Nonlinear Optimization, and then Sun Huanchun and Wang Yuefang revised this optimization and put forward Linear Euler Theory and Nonlinear Euler Theory. In the automatic container terminal, truss bridge bears the container car’s moveable gravity. The stability of truss bridge is an important factor for structure safety, handling efficiency and working life. Considering the effect of impact factor, linear buckling stability and nonlinear stability were studied on truss bridge.

2. Automatic container terminal and truss bridge

Auto navigation car and container truck are usually used for transportation between handling equipment and rear yard in the common ports. But there are two big problems, high cost and low handling efficiency. However, 3D handling system has been brought forward in automatic container terminal (Fig.1) and high speed electrical car has been used for carrying containers on truss bridge. Truss bridge is composed of truss beam, pillar and pillar linkage (Fig.2), and the material is Q345. The geometric dimension of truss beam, pillar and pillar linkage is respectively 32m×2.8m×1.2m, 5.05m×1.75m×1.25m, and 6.2m×2.1m×0.2m.
LRB or rigid bearing connect pillar linkage with pillar. When all the connections between pillar linkage and pillar are rigid bearings, the support style is called Type 1.

When all the connections are LRB, it is Type 2. When both sides use LRB and the middle uses rigid bearing, it is Type 3. When both sides use rigid bearings and the middle uses LRB, it is Type 4. The connection between pillar and ground can be equal to rigid connection. LRB can be used as an equivalent linear model [4] (Tab.1)

Table 1. Characteristic parameters of LRB

| Type    | Mass \(m\) (kg) | Vertical stiffness \(k_v\) (t) | Horizontal stiffness \(k_h\) (N/m) |
|---------|-----------------|-----------------------------|-----------------------------|
| GZY500  | 228             | \(1.972 \times 10^9\)       | \(1.91 \times 10^8\)       |

3. Linear buckling stability analysis

3.1. Linear buckling stability theory [5]

In the condition of elastic state, small incremental displacement is the linear function of external incremental load. Incremental equilibrium equation about structure is as below:

\[
\Delta P = (K_e + \lambda K_g) \Delta u
\]  

Where \(\Delta P\) is external incremental load, \(\Delta u\) is small incremental displacement, \(K_e\) is elastic stiffness matrix, \(K_g\) is geometric stiffness matrix, and \(\lambda\) is eigenvalue.

When the structure begins to enter the state of instability \((\Delta P \approx 0)\), incremental equilibrium equation is equal to (2).

\[
(K_e + \lambda K_g) \Delta u = 0
\]  

In the case of \(\Delta u \neq 0\), we should solve a classic eigenvalue problem, that is (3).

\[
Det (K_e + \lambda K_g) = 0
\]
After solving (3), we can get critical stability factor ($\lambda_{cr}$) and corresponding eigenvector ($\Delta u_{cr}$). If external load equals to $P_0$, then we can get critical buckling load ($\lambda_{cr} P_0$).

3.2. Linear buckling analysis result in different support types

The calculation condition is the most dangerous working condition (Fig.1). Vehicles lie in the middle of truss beam; Vertical load includes gravity of vehicle (17t) and two full container (30.5t×2); Lateral load includes static wind load acting on truss bridge and containers, and lateral load.

The static wind load can calculate as follows according to “Crane Design Code” (GB 3811-2008).

$$P_w=CK_hQA$$  \hspace{1cm} (4)

Where $C$ is wind factor and equals to 1.6, $K_h$ is wind pressure coefficient in different height and equals to 1.0, $A$ is the front area; $q$ is wind pressure, proportional to $v^2$.

$$q=0.625v^2$$  \hspace{1cm} (5)

Lateral load ($P_s$) can be calculated as the following way.

$\lambda$ is the lateral load coefficient and we can choose the value in “Crane Design Code” [6].

$$P_s=0.5 \sum P \lambda$$  \hspace{1cm} (6)

Impact factor has been acquired from coupled vibration analysis of vehicle and truss bridge. Corresponding to Type 1, Type 2, Type 3 and Type 4, the vertical impact factor is respectively 2.95, 2.65, 2.32 and 2.56; the lateral impact factor is respectively 1.18, 1.06, 1.35 and 1.16.

In different support types, the middle pillar linkage and the middle pillar are distorted firstly, and then make truss beam bended. Overall instability appears earlier than local instability. Part of critical stability factor and buckling modal are list in the Table 2 (The number of truss beam refers to Fig.2) and Figure 3.

Table 2. Part of critical stability factor and buckling model
| Step | Eigenvalue | Buckling modal |
|------|------------|----------------|
| 1    | 1.878      | The middle pillar linkage and pillar are bended; All the truss beams are bended |
| 5    | 2.524      | The middle linkage and pillar between No.1 and No.2 truss beams are bended and twisted; All the truss beams are bended |
| 10   | 3.366      | The middle pillar linkage are bended; |
| 20   | 4.911      | No.2 truss beam is partly bended |
| 31   | 5.931      | No.3 truss beam is partly bended |

Fig. 3. Part of buckling modal
The first step is usually the most important because it can reflect the structure’s property. The critical stability factor of Type 2, Type 3 and Type 4 is respectively 1.268, 1.382 and 1.338. Corresponding to different type, the first buckling modal is respectively as follows.

Type2: the middle pillar linkage and pillar are bended and all the truss beams are bended;
Type3: the middle pillar linkage and pillar are bended; No.2 truss beam is bended and twisted; No.1 and No.3 truss beams are bended.
Type4: the middle pillar linkage is bended and all the truss beams are bended;

Compared to Type 1, LRB can reduce the truss bridge’s stability, however, the critical buckling load is 1.2~1.3 times more than the external load in the most dangerous working condition. Strictly speaking, the structure is nonlinear and has inevitable defects after loaded, so the problem of structure stability usually belongs to the second type. Because many structures’ instability is close to linear buckling in actual projects, the calculation result approaches the upper limit of the second type and can reflect stability to some extent. It is easier for us to solve, so buckling stability analysis has practical value.

4. Nonlinear stability analysis introduction

4.1. Nonlinear stability analysis introduction

The structure nonlinearity makes the above critical buckling load greater than the actual critical load. In order to reduce the error, nonlinear stability analysis can be used to trace the load-displacement relationship all the time and obtain the actual critical load. It is hard to solve the second type’s equilibrium equation because the tangent stiffness matrix is close to a singular matrix. Paper\(^8\) finishes nonlinear stability analysis about 3D truss arch considering nonlinearity, including geometric nonlinearity, material nonlinearity and initial geometric defects. Paper\(^9\) puts forward two methods to think over geometric defects, “mode method for random defects” and “mode method for the same defect”. Paper\(^10\) gets a useful conclusion that structural instability region could be predicted by the lowest buckling modes. If defect distribution coincides with the lowest buckling modes, it will be the most dangerous condition. The results calculated in this condition are close to experimental value.

The material nonlinearity relates with material constitutive relation. Q345 stress-strain bilinear isotropic hardening elastoplastic model is as follows (Fig.4).

![Fig. 4. Q345 stress-strain bilinear isotropic hardening elastoplastic model](image-url)
4.2. Nonlinear stability analysis result

Geometric nonlinearity, material nonlinearity and initial geometric defects (L/200, L/1000, L/500, L is span and equals to 32) have been thought over in this article, and “mode method for the same defect” has been used to simulate geometric defects. But other nonlinearity has not been considered, such as residual stress. Initial load is half the gravity of both vehicle and two full containers to multiply impact factor. That is 1.13×10^6 N, 1.01×10^6 N, 8.87×10^5 N and 9.78×10^5 N respectively according to Type 1, Type 2, Type 3 and Type 4. To be conveniently compared to linear buckling analysis result, critical buckling load and the multiples of initial load have been listed in Table 3. The relationship between stress and strain of Type 1 has been drawn in Figure 5 (ordinate is the multiples of initial load and abscissa is the lateral displacement of No.2 truss beam’s middle node).

Fig. 5. The relationship between stress and strain of Type 1

Table 3. Critical buckling load (N) of different types

| Geometric defect | Type 1 | Type 2 | Type 3 | Type 4 |
|------------------|--------|--------|--------|--------|
|                  | Value  | Multiple | Value  | Multiple | Value  | Multiple | Value  | Multiple |
| No initial       | 1.85×10^6 | 1.636   | 1.22×10^6 | 1.205 | 1.17×10^6 | 1.314 | 1.23×10^6 | 1.256 |
| geometric defect | L/200  | 1.80×10^6 | 1.594   | 1.20×10^6 | 1.189 | 1.15×10^6 | 1.295 | 1.21×10^6 | 1.241 |
|                  | L/1000 | 1.70×10^6 | 1.506   | 1.19×10^6 | 1.177 | 1.13×10^6 | 1.278 | 1.20×10^6 | 1.229 |
|                  | L/500  | 1.67×10^6 | 1.481   | 1.18×10^6 | 1.170 | 1.11×10^6 | 1.250 | 1.19×10^6 | 1.216 |

Results of Tab.3 and Fig.5 show that the critical buckling load of different types is less than linear buckling analysis result after considering geometric nonlinearity, material nonlinearity and initial geometric defects, but the deviation is only about 10%; the critical buckling load is smaller when the initial geometric defect is greater; however, it is 1.17~1.64 times more than the external load all the same, so truss bridge has good stability all the same; but when the initial geometric defect reaches L/500, Type 2’s stability margin is not enough to satisfy “Fundamental code for design on railway bridge and culvert” (TB1002.1-1999), which asks for that the vertical deflection-span ratio ≤ L/900 and the
horizontal deflection-span ratio \( \leq L/1800 \), so truss bridge’s structure safety is mainly controlled by stiffness index.

5. Conclusion

Linear buckling analysis and nonlinear stability analysis have been done by ANSYS, and the following conclusion could be drawn from buckling modal and critical buckling load.

1. In different support types, Overall instability appears earlier than local instability. Overall instability shows that the middle pillar linkage and the middle pillar are distorted, and truss beams are unstable.
2. The critical buckling load of different types is less than linear buckling analysis result after considering geometric nonlinearity, material nonlinearity and initial geometric defects, but the deviation is only about 10%.
3. LRB can reduce the truss bridge’s stability, but truss bridge has good stability all the same; truss bridge’s structure safety is mainly controlled by stiffness index.

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