Developing non-routine problems for assessing students' mathematical literacy

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Abstract. The purpose of this study is to develop non-routine problems for assessing the mathematics literacy skills of students, which is valid, practical, and effective. It is based on the previous research said that Indonesian students' mathematical literacy is still low. The results of this study can be used as a guide in developing the evaluation questions that can train students to improve the ability of solving non-routine problems in everyday life. This research type is formative evaluation that consists of preliminary, self evaluation, expert reviews, one-to-one, small group, and field test. The sample of this research is grade 8 students at one of Junior High School in Yogyakarta. This study results in mathematics literacy problems prototype consisting of level 1 to level 6 problems similar to PISA problems. This study also discusses the examples of students’ answer and their reasoning.

1. Introduction

The ability to apply mathematics in real world situation is considered as a main goal of mathematics education in all around the world [1-2]. This ability can be called mathematical literacy. Mathematical literacy is a skill that must be mastered by students in order to compete in 21st century [3]. The results of study in PISA (Program for International Students Assessment) for 5 times in a row indicate that Indonesian students have low mathematics literacy skills. In fact, the results in PISA 2012 show that 75.7% of Indonesian students are only able to work on PISA problems at level 2 and only 0.3% of students can work on contextual mathematics problems at the two highest levels of PISA [4].

In relation to the results of the PISA survey, many factors have contributed to the low literacy skills of Indonesian students. These factors include students not accustomed to solve similar problems of PISA in mathematics learning activities [5] because they are accustomed to use formal mathematical knowledge and apply the formulas taught in the classroom. This fact leads to the lack of problem solving skills of students to solve non-routine problems or high difficulty levels of problems because most of the questions given in the lessons only focus on problems with low difficulty levels [6].

The results of research from Sampoerna Foundation [7] shows that the questions of the National Exam only revolve around the level of memorizing the formula, using procedures, and showing a formal understanding. While the other two aspects of making conjecture/ generalize/ prove and solving non-routine problem are not touched at all in the matter of the mathematics National Exam. Though both aspects are the highest level in the cognitive aspect.

Based on the above description, it is necessary to conduct a research in non-routine problems development to assess students' mathematical literacy skills with a research question: How to develop valid, practical, and effective non-routine problems to assess students' mathematical literacy skills.
1.1. Non-routine problems in problem solving

Problem solving is the most important cognitive activity in everyday life [8]. One of the characteristics in problem solving is that students can work in a flexible way and can modify the strategies used in accordance with changing circumstances [9]. In mathematics education, students continuously face new situations and new problems which require them not only to know and apply various strategies, but also to be flexible [10]. Student flexibility determines whether he or she can adapt well to new problems or non-routine problems.

A non-routine problem occurs when students encounter a situation but do not know directly the way to solve problems in the situation. Contrary to routine problems that require applications of routine or regular calculations, non-routine problems have no direct way to deal with the question, but require creative thinking and application of some strategies to understand the problem and to find the best way to solve the problem [11]. Therefore, non-routine problems tend to be more complex and more difficult than routine problems.

Sweller [12] stated that problem solving is taught independently of basic tools and basic thinking. Over time, students build up a repertoire of problem solving techniques. Ultimately, the difference between someone who is good and someone who is bad at solving non-routine problems is not that the good problem solver has learned to solve novel, previously unseen problems. It is more the case that, as students increase their expertise; more non-routine problems appear to them as routine.

1.2. Mathematical literacy skill

Mathematical literacy skill is popularized by PISA. PISA is a project of the Organization for Economic Cooperation and Development (OECD). PISA aims to measure the extent to which students approaching the end of compulsory education have the essential knowledge and skills to participate actively in the learning community. PISA tests 15-year-old students in reading, mathematics, and science skills, and measures factors that affect such abilities such as student talent, interest, and motivation. The PISA test focuses on the skills of students in using their knowledge and skills to face life's challenges, not just to master the school curriculum. This approach is called "literacy" [13]. PISA uses a concept of mathematical literacy that is concerned with the capacity of students to analyze reason and communicate effectively as they pose, solve and interpret mathematical problems in a variety of situations including quantitative, spatial, probabilistic or other mathematical concepts [13].

The ability of mathematical literacy according to PISA is divided into 6 levels [13]. At level 1, students can answer question involving familiar contexts where all relevant information is present. They are able to carry out routine procedures according to direct instructions. At level 2, students can interpret and recognize situations in contexts that require no more than direct inference. They can employ basic algorithms, formulas, procedures, or conventions. At level 3, students can select and apply simple problem solving strategies. They can interpret and use representations based on different information sources. At level 4, students can work effectively with explicit models for concrete situations. They can select and integrate different representations and link them to real world’s situations. At level 5, students can develop and work with models, identifying constraints and specifying assumptions. They can compare and evaluate appropriate problem solving strategies for dealing with complex problems. At level 6, students can utilize information based on their investigations and modeling of complex problem situations. They are capable of advances mathematical thinking and reasoning.

2. Methods

This research type is formative evaluation [14] that consists of preliminary, self evaluation, expert reviews, one-to-one, small group, and field test [15]. The product of this study is non-routine problems that have valid, practical, and effective classification to assess students' mathematical literacy skills. Non-routine problems are said to be valid if they meet a good classification of the expert validators scoring sheets. Non-routine problems are said to be practical if they meet a good classification of the
students response questionnaires and interviews. Non-routine problems are said to be effective if they meet a good classification of the students work tests. The sample of this study is grade 8 students at one of Junior High School in Yogyakarta.

3. Result and Discussion

3.1. Non-routine problems development process
Non-routine problems development is carried out with the formative evaluation method consisting of the following stages.

3.1.1. Preliminary stage. At this stage, resources gathering is done to develop non-routine problems. The sources referenced are from curriculum manuals, textbooks for Junior High Schools, as well as a collection of PISA questions from year to year.

3.1.2. Self-evaluation stage. At this stage, non-routine problems are made and evaluated by the researcher herself. This stage produces Prototype 1.

3.1.3. Expert reviews stage. At this stage, non-routine problems are validated by 2 expert lecturers. This phase guaranteed the validity of non-routine problems that have been made based on expert validity.

3.1.4. One-to-one stage. At this stage, non-routine problems are tested on 3 students with different abilities that are high, medium, and less. At this stage, giving questionnaires and interviews are also conducted with students whether they could understand the problem and find a solution to ensure the practicality of non-routine problems that have been made. Expert Reviews and One-to-One stage generated Prototype 2. Revisions from Prototype 1 to Prototype 2 are changing some problems and making problems and place to answer the problems on the same sheet.

3.1.5. Small group stage. At this stage, non-routine problems are tested on 6 students and the result of student's answer are analyzed to find out if there is a misconception to non-routine problems. This phase guaranteed the effectiveness of non-routine problems. Revisions from Prototype 2 to Prototype 3 are changing some sentence formulations on the problems and enlarging the place to answer the problems.

3.1.6. Field test stage. At this stage, non-routine problems which have valid, practical, and effective qualification are tested on 31 students of Junior High School grade 8.

The non-routine problems generated are “PISA like” problems from level 1 to level 6 shown in Figure 1, Figure 2, Figure 3, Figure 4, Figure 5, and Figure 6.

![Figure 1. Non-routine problem level 1](image-url)
2. **MANGO JUICE**
   To make 2 glasses of mango juice is needed 1 mango and 2 spoons of sugar. Ani wants to make 5 glasses of mango juice for her friends. How many mango that Ani needs to make those juice?

**Figure 2.** Non-routine problem level 2

3. **STUDENTS COMPETITION**
   Students A, B, C, D and E are following Student Competition at Harapan Bangsa Junior High School. To be a winner, all students are asked to solve mathematics, science, bahasa, and English questions. The score obtained by each student can be seen in the following table:

| Student | Math | Science | Bahasa | English |
|---------|------|---------|--------|---------|
| A       | 7    | 8       | 9      | 8       |
| B       | 8    | 9       | 8      | 7       |
| C       | 7    | 8       | 8      | 8       |
| D       | 8    | 8       | 7      | 8       |
| E       | 9    | 8       | 8      | 7       |

To determine the winner, the final score of each student is obtained by the formula: (3 x Math) + (2 x Science) + (3 x bahasa) + (2 x English). Who is the winner? Write down your steps in getting the winner.

**Figure 3.** Non-routine problem level 3

4. **PARKING AREA**
   On Sunday, Rania's family went on vacation to the zoo. There are two parking areas. Parking lot A can accommodate 200 cars and there already contains 185 cars. While parking lot B can accommodate 300 cars, but there already contains 255 cars.
   Compare parking lot A and B. Which parking lot is more crowded? Explain your answer.

**Figure 4.** Non-routine problem level 4

5. **FISH POND**
   Mr. Hasan has a wire that is 32 meters long and wants to use the wire to create a barrier area that will be used as a fish pond. We can call the wire as a perimeter of a fish pond.
   Mr. Hasan created four fish pond designs like in the picture below.

![Fish pond designs](cited from PISA_Take_the_Test_Sample_ Questions_from_OECs_PISA_Assessments_2009)

Which fish pond design can be limited by 32 meters of the wire? Write down your steps in getting the answer.

**Figure 5.** Non-routine problem level 5
3.2. Non-routine problems test result

Based on the result of field test, all students got right answer for question level 1, 90.32% students got right answer for question level 2, and 83.87% students got right answer for question level 3. In the other hand, only 19.35% students can answer correctly for question level 4, only 16.13% students can answer correctly for question level 5, and 9.68% students can answer correctly for question level 6.

The examples of students’ answer and their reasoning for question level 4, level 5 and level 6 are discussed as follows.

3.2.1. Question level 4 related to the meaning of proportion. Student 1 already knew the meaning of proportion by not only recognizing the parts that have been used to park, but also recognizing the whole parking place. While, most of the other students’ only saw the used parking place (Figure 7 and Figure 8). They only compared 185 and 255 and ignored relevant information those are 200 and 300. So, they answered that the more crowded parking place is B. This finding showed that improving the task comprehension of students requires a focus not only on students’ language competence, but also on the ability to select and to use relevant information [16].

3.2.2. Question level 5 related to the perimeter of 2-dimensional figure. Student 2 already understood the concept of perimeter by calculating all sides of 2-dimensional figure given with adding the sides one by one. While, most of the other students only did the procedure to find the perimeter of parallelogram and rectangle. They only calculated the perimeter of picture B and picture D. So, they answered that design with 32 meters around is picture D. This finding point out that students’ procedural knowledge is not compatible with proper conceptual knowledge. Several researchers were in common argument that both procedural knowledge and conceptual knowledge were important aspects to observe and evaluate mistakes when solving problems. Focusing only on procedural knowledge might impede development of intuitive sense and conceptual knowledge itself [17-19].

3.2.3. Question level 6 related to the probability of an event. Student 3 already understood the meaning of probability of an event by calculating probability of each event, then multiplying the probability and the point to get the total score. While, most of the other students only compared the point of two dices multiplication result, whether odd or even. So, they answered that the game is not a fair game. They didn’t understand that the question is regarded to probability problem (Figure 9). This finding indicated that understanding problems process was the early step from solving mathematic literacy problems process. This step will influence the next steps. Because of that, the low understanding skill can be one of the factors of low mathematic literacy skill [20].
4. Conclusion
The process of developing non-routine problems with formative evaluation methods consisting of preliminary, self-evaluation, expert reviews, one-on-one, small group, and field test had resulted in mathematics literacy problems prototype consisting of level 1 to level 6 problems similar to PISA problems that is valid, practical, and effective. The results of non-routine problems tests showed that most students can only work correctly on problems level 1 to level 3. However, most students still have difficulty working on problems level 4, 5, and 6 [4].

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