Low-lying $\Omega$ states with negative parity in an extended quark model with Nambu-Jona-Lasinio interaction

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Here we investigate mixing of the low-lying three- and five-quark $\Omega$ states with spin-parity quantum numbers $\frac{1}{2}^-$ and $\frac{3}{2}^-$, employing the quark-antiquark creation triggered by Nambu-Jona-Lasinio (NJL) interaction. Wave functions of the three- and five-quark configurations are constructed using the extended constituent quark model, within which the hyperfine interaction between quarks is also taken to be the NJL interaction induced one. Numerical results show that the NJL-induced pair creation results in vanishing mixing between three- and five-quark $\Omega$ configurations with spin-parity $1/2^-$, but mixing between three- and five-quark $3/2^-$ $\Omega$ states should be very strong. And the mixing decreases energy of the lowest $3/2^-$ $\Omega$ state to be $1785 \pm 25$ MeV, which is lower than energy of the lowest $1/2^-$ state in this model. This is consistent with our previous predictions within the instanton-induced quark-antiquark creation model.

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I. INTRODUCTION

Recently, spectrum of low-lying $\Omega$ resonances with negative parity was investigated employing an extended constituent quark model$^1, 2$, within which the $\Omega$ resonances were considered as admixtures of three- and five-quark components, and the hyperfine interaction between quarks was taken to be of three different kinds, namely, one gluon exchange (OGE)$^3, 6$, Goldstone boson exchange (GBE)$^7,$ and instanton-induced interaction (INS)$^8, 11$. In Ref. $2$, mixing of three- and five-quark $\Omega$ states was calculated by treating the $q\bar{q}$ creation mechanism as the one induced by instanton interaction. It is shown that the mixing between three- and five-quark components in $\Omega$ resonances with spin-parity $1/2^-$ is very small and negligible, but in the $3/2^-$ $\Omega$ resonances the mixing is very strong, and the mixing decreases the energy of the lowest $3/2^-$ state to be around $1750 \pm 50$ MeV. It is very interesting that this energy is lower than energy of the lowest spin-parity $1/2^-$ $\Omega$ resonance.

As shown in $2$, the instanton quark-antiquark pair creation precludes transitions between $s^3$ and $s^4\bar{s}$ configurations, while the instanton-induced hyperfine interaction between a quark and an antiquark could lead to mixing between five-quark $\Omega$ configurations with light quark-antiquark pair and $s\bar{s}$ pair$^1$. Therefore, Once we take the instanton-induced hyperfine interaction and quark-antiquark pair creation simultaneously, mixing between $s^3$ and $s^4\bar{s}$ configurations will not vanish. But if the hyperfine interaction between quarks is chosen as OGE or GBE, the instanton-induced quark-antiquark pair creation mechanism cannot result in mixing between $s^3$ and $s^4\bar{s}$ configurations. Generally, even if probability of $s^4\bar{s}$ component in $\Omega$ resonances may be small, it should not be exactly 0. This may indicate that once we take the instanton-induced quark-antiquark pair creation, we have to keep the hyperfine interaction between quarks being based on the same model.

In the present work, we try to calculate mixing between three- and five-quark components in low-lying $\Omega$ resonances with negative parity using the Nambu-Jona-Lasinio (NJL) approach$^{12, 13}$, which was originally constructed for nucleons that interact via an effective two-body contact interaction, and later developed to include the quark freedom$^1$. Analogous to the instanton interaction$^{15, 17}$, the NJL model can describe various aspects of QCD related to the dynamical and explicit breaking of chiral symmetry and the axial anomaly very well$^{18}$. As discussed above, here we take the hyperfine interactions between quarks to be also the NJL induced one for model consistency.

The present paper is organized as follows. In Section II we present our theoretical framework, which includes explicit forms of the NJL-induced quark-quark hyperfine interactions, and quark-antiquark pair creation mechanism. Numerical results for the spectrum of the states under study and the mixing of three- and five-quark configurations in our model are shown in Section III. Finally, Section IV contains a brief conclusion.

II. THEORETICAL FRAMEWORK

In the present model, the Hamiltonian is almost all the same as that using in$^1, 2$ only except for the parts

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describe the quark-quark hyperfine interaction and mechanism for transition between three- and five-quark components in Ω resonances. For completeness, here we also repeat the same parts. The Hamiltonian describing Ω resonances as admixtures of three- and five-quark components is of the following form:

\[ H = \begin{pmatrix} H_3 & T_{1Ω_3+Ω_5} \\ T_{1Ω_3+Ω_5} & H_5 \end{pmatrix}, \]

where \( H_3 \) and \( H_5 \) are the Hamiltonians for three-quark and five-quark systems, respectively, and \( T_{1Ω_3+Ω_5} \) denotes the transition between three- and five-quark systems. Here we discuss the diagonal and non-diagonal terms of the Hamiltonian \( \text{II} \) in Secs. \( \text{IIA} \) and \( \text{IIB} \) respectively.

### A. Diagonal terms of the Hamiltonian

The Hamiltonian for a \( N \)-particle system in the constituent quark model can be written as the following form

\[ H_N = H_o + H_{hyp} + \sum_{i=1}^{N} m_i, \]

where \( H_o \) and \( H_{hyp} \) represent the Hamiltonians for the quark orbital motion and for the hyperfine interactions between quarks, respectively, \( m_i \) denotes the constituent mass of the \( i \)th quark. The first term \( H_o \) can be written as a sum of the kinetic energy term and the quark confinement potential as

\[ H_o = \sum_{i=1}^{N} \frac{\vec{p}_i^2}{2m_i} + \sum_{i<j}^{N} V_{conf}(\vec{r}_{ij}). \]

In \( \text{II} \) the quark confinement potential was taken to be

\[ V_{conf}(\vec{r}_{ij}) = -\frac{3}{8} \lambda_i \cdot \lambda_j \left[ C^{(N)}(\vec{r}_{i} - \vec{r}_{j})^2 + V_0^{(N)} \right], \]

where \( C^{(N)} \) and \( V_0^{(N)} \) are constants. In principle these two constants can differ for three- and five-quark configurations. \( H_{hyp} \) denotes the hyperfine interaction between quarks, here we take \( H_{hyp} \) to be the NJL interaction induced one. The NJL interaction between quarks can be described by

\[ \mathcal{L}_{NJL} = \frac{1}{2} g_s \sum_{a=0}^{8} \left( \bar{q}_i \lambda^a q_j \right)^2 + \left( \bar{q}_i \gamma_5 \lambda^a q_j \right)^2 \]

where \( \lambda^a \) (\( a = 1 \ldots 8 \)) are the Gell-Mann matrices in the flavor \( SU(3) \) space, and \( \lambda^0 = \sqrt{\frac{1}{3}} \mathbb{I} \), with \( \mathbb{I} \) the unit matrix in the three dimensional flavor space. In the non-relativistic approximation, the NJL induced quark-quark interaction can be obtained as

\[ H_{NJL}^{qq} = \sum_{i<j}^N \sum_{a=0}^8 \hat{g}_{ij} \lambda_i \lambda_j [1 + \frac{1}{4m_i m_j} \hat{s}_i \cdot (\vec{p}_i - \vec{p}_j) \hat{s}_j \cdot (\vec{p}_j - \vec{p}_i)], \]

where \( \hat{g}_{ij} \) is an operator which distinguishes the coupling strength between two light quarks \( q_\rho \), one light and one strange quarks \( q_s \), and two strange quarks \( q_s \). In principle, once the \( SU(3) \) breaking effects are taken into account, the three coupling strength should be different. One may find that the present hyperfine interaction is very similar to the one mediated by Goldstone boson exchange, which includes the pseudoscalar and scalar mesons exchange \([7]\). Therefore, here we take the relationship of the three different coupling strength as in \([7]\)

\[ g_{qq}; g_{qs}; g_{ss} = 1: \frac{m}{m_\rho}; \frac{m^2}{m_\rho^2}, \]

where \( m \) and \( m_s \) represent the constituent masses of the light and strange quarks. The empirical value for the constituent mass of light quark is in the range 300 ± 50 MeV, and for strange quark is \( \sim 120 - 200 \) MeV higher than \( m \). In the traditional \( qqq \) constituent quark model, \( m \) is often taken to be 320 – 340 MeV \([\text{F}2]\). In the present case, since we take the baryons to be admixtures of three- and five-quark components, in general, the constituent quark mass should be lower than that in the three-quark model. Accordingly, here we take the value \( m = 310 \) MeV for the light quarks, and \( m_s = 460 \) MeV for the strange quark.

Generally, one can divide the interaction \( \text{II} \) by different spin dependences \([\text{F}10]\). If we neglect the tensor term in the quark-quark interaction, the NJL-induced hyperfine interaction between quarks \( H_{hyp}^{NJL} \) should be the following form

\[ H_{hyp}^{NJL} = \sum_{i<j}^N \sum_{a=0}^8 \hat{g}_{ij} \lambda_i \lambda_j \left(1 - \frac{1}{12} \hat{s}_i \cdot \hat{s}_j \right), \]

Accordingly, one can obtain the three coupling strength by reproducing the mass splitting between \( \Sigma \) and \( \Lambda \) baryons

\[ g_{qq} = 69 \text{ MeV}, \quad g_{qs} = 51 \text{ MeV}, \quad g_{ss} = 38 \text{ MeV}. \]

As discussed in \([\text{F}2]\), there are two low-lying \( \Omega \) resonances with negative parity in \( N \leq 2 \) band within the \( qqq \) three-quark model \([\text{F}4, \text{F}5, \text{F}20]\), one is with spin 1/2, and the other 3/2, correspond to the two first orbitally excited states of \( \Omega(1672) \). And these two states should be degenerate in a given hyperfine interaction model if the L-S coupling hyperfine interaction is not taken into account. The matrix elements of the sub-matrix \( H_3 \) in \( \text{II} \) obtained by using the OGE, GBE and INS hyperfine interactions in \([\text{F}2]\) are \( \langle H_3^{OGE} \rangle_{-} = \langle H_3^{OGE} \rangle_{\frac{1}{2}^-} = 2020 \text{ MeV}, \langle H_3^{OGE} \rangle_{\frac{3}{2}^-} = 1991 \text{ MeV}, \) and \( \langle H_3^{INS} \rangle_{\frac{1}{2}^-} = \langle H_3^{INS} \rangle_{\frac{3}{2}^-} = 1887 \text{ MeV}, \) respectively. In present work, with the above given NJL-induced hyperfine interaction strength, by reproducing the mass of the ground state \( \Omega(1672) \), one can obtain that \( V_0^{(3)} = -188 \text{ MeV} \), which is smaller than the value \(-140 \text{ MeV} \) in GBE model. With these parameters, we
obtain that the matrix elements of $H_3$ in present model are $(H_3^{N,J,L})^2 = (H_3^{N,J,L})^4 = 1942$ MeV.

On the other hand, to get the value of the parameter for the $V^{(5)}_0$, we fit the lowest five-quark $\Omega$ configuration to be $\sim 1810$ MeV, which value was proposed to be the energy of the lowest $K\Xi$ bound state with spin-parity $1/2^-$. This method yields $V^{(5)}_0 = -294$ MeV, which is also smaller than the value $-269$ MeV in GBE model.

### B. Non-diagonal terms of Hamiltonian

The non-diagonal term $T_{\Omega_3+\Omega_5}$ depends on the explicit quark-antiquark pair creation mechanism. In present work, we take the quark-antiquark pair creation mechanism to be the one based on a nonrelativistic reduction of the amplitudes found from the NJL interaction. One may find that the two terms in Eq. (5) just correspond to the quark-antiquark pairs with quantum numbers $0^+$ and $0^-$ creation, respectively. Accordingly, in present case, only the second term will contribute. And in the nonrelativistic limit, the second term in Eq. (5) reduces to

$$T_{qq} = -\frac{2}{m_s} g_{qs} \xi_i^\dagger \hat{\sigma} \cdot (\vec{p}_i - \vec{p}_j) \xi_i \xi_j \tilde{\eta}_{ij}$$

for light $qq$ creation and

$$T_{s\bar{s}} = \frac{1}{m_s} g_{ss} \xi_i^\dagger \hat{\sigma} \cdot (\vec{p}_i - \vec{p}_j) \xi_i \xi_j \tilde{\eta}_{ij}$$

for $s\bar{s}$ creation, where $\xi_{f(i)}$ and $\vec{p}_{f(i)}$ denotes the final (initial) spin and momentum operators of quark which emits a $qq$ or $s\bar{s}$ pair, $\xi_{q(s)}$ is the spin operator of the created light (strange) quark and $\eta_{ij}$ the spin operator of the created light (strange) antiquark.

If we treat the other two quarks as spectators, then the non-diagonal term $T_{\Omega_3+\Omega_5}$ in Hamiltonian (11) can be obtained as

$$T^{qq}_{\Omega_3+\Omega_5} = -\frac{2}{m_s} g_{qs} \sum_{i=1}^3 \sum_{j=1}^4 \mathcal{C}_F \mathcal{C}_S \mathcal{C}_C \mathcal{C}_O \xi_i^\dagger \hat{\sigma} \cdot (\vec{p}_i - \vec{p}_j) \xi_i \xi_j \tilde{\eta}_{ij}$$

$$T^{ss}_{\Omega_3+\Omega_5} = \frac{1}{m_s} g_{ss} \sum_{i=1}^3 \sum_{j \neq i}^4 \mathcal{C}_F \mathcal{C}_S \mathcal{C}_C \mathcal{C}_O \xi_i^\dagger \hat{\sigma} \cdot (\vec{p}_i - \vec{p}_j) \xi_i \xi_j \tilde{\eta}_{ij}$$

for transitions $sss \leftrightarrow sssq\bar{q}$ and $sss \leftrightarrow sss\bar{s}$, respectively. Where $\mathcal{C}_F$, $\mathcal{C}_S$, $\mathcal{C}_C$ and $\mathcal{C}_O$ are operators for the calculation of the corresponding flavor, spin, color and orbital overlap factors, respectively.

### III. NUMERICAL RESULTS

As what we have done in [1, 2], to show our numerical results clearly, here we denote the two three-quark configurations as $|3, \frac{1}{2}^-angle$ and $|3, \frac{3}{2}^-angle$, respectively, and five-quark configurations with spin-parity quantum number $1/2^-$ as

$$|\frac{5}{2}^-, \frac{-1}{2}\rangle_1 = |s^3 q([4]_x [211]_c [31]_F S [31]_F [22]_S) \otimes \bar{q}\rangle,$$

$$|\frac{5}{2}^-, \frac{-1}{2}\rangle_2 = |s^3 q([4]_x [211]_c [31]_F S [31]_F [31]_S) \otimes \bar{q}\rangle,$$

$$|\frac{5}{2}^-, \frac{-1}{2}\rangle_3 = |s^3 q([4]_x [211]_c [31]_F S [4]_F [31]_S) \otimes \bar{q}\rangle,$$

$$|\frac{5}{2}^-, \frac{-1}{2}\rangle_4 = |s^4 ([4]_x [211]_c [31]_F S [4]_F [31]_S) \otimes \bar{s}\rangle.$$ (14)

and those with spin-parity quantum number $3/2^-$ as

$$|\frac{5}{2}^-, \frac{-1}{2}\rangle_1 = |s^3 q([4]_x [211]_c [31]_F S [31]_F [31]_S) \otimes \bar{q}\rangle,$$

$$|\frac{5}{2}^-, \frac{-1}{2}\rangle_2 = |s^3 q([4]_x [211]_c [31]_F S [31]_F [4]_S) \otimes \bar{q}\rangle,$$

$$|\frac{5}{2}^-, \frac{-1}{2}\rangle_3 = |s^3 q([4]_x [211]_c [31]_F S [4]_F [31]_S) \otimes \bar{q}\rangle,$$

$$|\frac{5}{2}^-, \frac{-1}{2}\rangle_4 = |s^4 ([4]_x [211]_c [31]_F S [4]_F [31]_S) \otimes \bar{s}\rangle.$$ (15)

The main parameters involved in the transitions between three- and five-quark $\Omega$ states are the ratio of harmonic oscillator parameters $R_{35} = \omega_5 / \omega_3$, and the NJL interaction strength $g_{qs}$ and $g_{ss}$. We present the numerical results by taking the parameters to be empirical values in Sec. III-A and those by treating the the ratio $R_{35}$ and interaction strength $g_{qs}$ and $g_{ss}$ to be free parameters in Sec. III-B.

### A. Numerical results with fixed parameters

Firstly, we take a tentative value $R_{35} = \sqrt{3}/6$, and the values for NJL interaction strength given in Sec. III-A to show the mixing between three- and five-quark $\Omega$ states within the NJL-induced quark-antiquark pair creation model. With the notations in Eqs. (14) and (15), the matrix elements of the Hamiltonian (11) including $H_3$, $H_5$ and $T_{\Omega_3+\Omega_5}$ are

$$\langle H^{N,J,L}_{1/2} \rangle_1 = \begin{pmatrix}
1942.0 & 0 & 0 & 0 & 0 \\
0 & 1809.9 & 0 & 0 & 0 \\
0 & 0 & 1816.2 & -6.1 & 0 \\
0 & 0 & -6.1 & 2254.8 & 0 \\
0 & 0 & 0 & 0 & 2474.7
\end{pmatrix}$$

(16)

$$\langle H^{N,J,L}_{3/2} \rangle_1 = \begin{pmatrix}
1942.0 & 38.5 & -60.9 & -27.2 & 20.3 \\
38.5 & 1816.2 & 0 & -6.1 & 0 \\
-60.9 & 0 & 1821.5 & 0 & 0 \\
-27.2 & -6.1 & 0 & 2254.8 & 0 \\
20.3 & 0 & 0 & 0 & 2474.7
\end{pmatrix}$$

(17)

As we can see in Eq. (16), the NJL-induced quark-antiquark pair creation does not contribute to transitions between three- and five-quark $\Omega$ states with spin 1/2. This is because of that the quark-antiquark pairs in corresponding five-quark $\Omega$ states are with the quantum number $^3S_1$, while the created quark-antiquark pair in
NJL approach should be with the quantum number $^1S_0$, therefore, transitions between the studied spin 1/2 three- and five-quark configurations vanish in the NJL-induced pair creation model. One may find this is consistent with the results obtained in [2] using the instanton pair creation model. In Ref. [2], although the obtained mixing between three- and five-quark configurations with spin 1/2 is not 0, it is very small and negligible, in fact, the mixing is proportional to $1/m - 1/m_s$, with $m$ and $m_s$ the constituent masses of the light and strange quarks, so the mixing will also be 0 in the flavor $SU(3)$ limit. For the non-diagonal matrix elements from transitions between three- and five-quark spin 3/2 $\Omega$ states, Eqs. (17) shows that the transition matrix elements are smaller than those caused by instanton-induced quark-antiquark creation [2].

On the other hand, as shown in Eqs. (16) and (17), the NJL-induced hyperfine interaction between quarks leads to only two very small nonvanishing non-diagonal matrix elements for both spin 1/2 and 3/2 cases. This is the same as the results obtained within GBE hyperfine interaction model [1]. As we have discussed in Sec. 11, the present hyperfine interaction model is very similar to the GBE model.

Diagonalization of Eqs. (16) and (17) results in the numerical results of the energies and corresponding probability amplitudes of three- and five-quark configurations for the obtained $\Omega$ states shown in Table I. As shown in this table, mixing between three- and five-quark spin 3/2 $\Omega$ configurations are very strong, while since the transition matrix elements listed in Eq. (17) are not large, so the mixing does not decreases energy of the lowest state so much as that obtained in [2]. Nevertheless, the obtained energy of the lowest spin 3/2 state is lower than energy of the lowest spin 1/2 state, this is consistent with our previous results obtained within the instanton-induced quark-antiquark pair creation model [2]. And one can find that the mixing between three-quark $\Omega$ state and the $s^4\bar{s}$ is very small, the largest mixing appears in the third state with probability of $s^4\bar{s}P_{s^4\bar{s}} = 0.0364^2 \approx 0.1\%$. This is because of that on the one hand the transition matrix element between $s^4$ and $s^4\bar{s}$ configurations is smaller than the other ones, and on the other hand energy of the $s^4\bar{s}$ configuration is much larger than the three-quark $\Omega$ state.

### Table I: Energies and the corresponding probability amplitudes of three- and five-quark configurations for the obtained $\Omega$ states in the NJL-induced hyperfine interaction model. The upper and lower panels are for states with quantum numbers $\frac{1}{2}^+$ and $\frac{3}{2}^+$, respectively, and for each panel, the first row shows the energies in MeV, others show the probability amplitudes.

| $\frac{1}{2}^+$ | $\frac{3}{2}^+$ |
|-----------------|-----------------|
| $|\frac{1}{2}^+\rangle$ | 1810 | 1816 | 1942 | 2255 | 2475 |
| $|\frac{3}{2}^+\rangle_1$ | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |
| $|\frac{3}{2}^+\rangle_2$ | 0.0000 | 0.9999 | 0.0000 | -0.0140 | 0.0000 |
| $|\frac{3}{2}^+\rangle_3$ | 0.0000 | 0.0140 | 0.0000 | 0.9999 | 0.0000 |
| $|\frac{3}{2}^+\rangle_4$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |
| $\frac{3}{2}^+$ | 1786 | 1818 | 1972 | 2257 | 2475 |
| $|\frac{3}{2}^+\rangle_1$ | 0.4227 | -0.0354 | -0.9902 | 0.0905 | 0.0389 |
| $|\frac{3}{2}^+\rangle_2$ | -0.5385 | -0.8135 | -0.2185 | 0.0217 | 0.0023 |
| $|\frac{3}{2}^+\rangle_3$ | 0.7286 | -0.5803 | 0.3635 | -0.0127 | -0.0036 |
| $|\frac{3}{2}^+\rangle_4$ | 0.0175 | -0.0136 | -0.0916 | -0.9955 | -0.0049 |
| $|\frac{3}{2}^+\rangle_5$ | -0.0125 | 0.0011 | 0.0364 | -0.0085 | 0.9992 |

![FIG. 1: (Color online) Energies of Ω resonances with spin 3/2 as functions of $g_{qs}$.](image1.png)

![FIG. 2: (Color online) Energies of Ω resonances with spin 3/2 as functions of $R_{35}$.](image2.png)

B. Dependence of numerical results on parameters

To show the dependence of mixing between three- and five-quark $\Omega$ states with spin 3/2, we present the energies of the obtained states in Fig. 1 and Fig. 2, the former one shows $M_{\Omega}$ as functions of the NJL interaction strength $g_{qs}$ with $g_{qs}$ varying from 25 MeV to 75 MeV, and the...
latter shows dependence of $M_{\Omega}$ on the ratio $R_{35}$ with $R_{53}$ being in the range $0.5-2$. Note that here we just show the dependence of mixing effects on parameters, since there is no mixing between three- and five-quark configurations in the obtained $\Omega$ states with spin 1/2, so we only give the numerical results for $\Omega$ states with spin 3/2 in this section. In addition, we keep the diagonal terms being constants when varying the parameters.

As we can see in Fig. 4, the energy of the lowest state shows a little sensitivity on $g_{qs}$, $M_{\Omega}^{\text{lowest}}$ falls in the range $1785 \pm 25$ MeV when $g_{qs}$ varying from 25 to 75 MeV, this obtained energy is higher than the one $M_{\Omega}^{\text{lowest}} = 1750 \pm 50$ MeV in [2] by taking the quark-antiquark pair creation mechanism to be the instanton interaction induced one, since the transition matrix elements between three- and five-quark $\Omega$ configurations in present work are smaller than those obtained in [2], as we have discussed in Sec. III A. While in any case, the obtained value for $M_{\Omega}^{\text{lowest}}$ is decreased to be lower than energy of the lowest spin 1/2 state in present model, this conclusion is consistent with that in the instanton-induced interaction model. On the other hand, energies of the other states are not so sensitive to $g_{qs}$, especially the highest two states, whose energies are even constants, this is because of that the mixing effects are very small in these two states, as shown in Table I. In fact, energy of the next-to-lowest state is also not sensitive to $g_{qs}$, as we can see in Fig. 1. This is because of that the main components in this state is the first two five-quark configurations, whose masses are very close to each other, lying at $\sim 1820$, and mixing between three- and five-quark configurations in this state is not strong, as shown in Table I there is only $\sim 0.1\%$ three-quark component in this state.

Fig. 2 shows almost the same features as Fig. 1 only energies of two obtained states are sensitive to $R_{35}$, and the lowest energy falls in the range $1790 \pm 20$ MeV.

Comparing to the numerical results in Ref. [2], one can find that the present obtained energies for spin 3/2 states are very different to those obtained by taking both quark-quark hyperfine interaction and quark-antiquark pair creation to be the instanton induced ones, almost all the present obtained energies are lower than those in [2]. This is because of that the one hand the instanton-induced hyperfine interaction is very different to the present NJL interaction induced one, the obvious two differences are those firstly the instanton-induced hyperfine interaction can only exist between two quarks whose flavor wave function is antisymmetric, and secondly instanton-induced hyperfine interaction leads to strong mixing between five-quark configurations with light and strange quark-antiquark pairs. And on the other hand the transition matrix elements between three- and five-quark spin 3/2 states in instanton-induced quark-antiquark pair creation model are larger than present ones.

While in any case, the mixing between three- and five-quark spin 1/2 $\Omega$ states is very small and negligible in INS model, and is 0 in NJL model, while both INS and NJL induced quark-antiquark pair creation mechanism result in strong mixing between three- and five-quark spin 3/2 $\Omega$ states, and the mixing decreases energy of the lowest spin 3/2 to lower than that of the lowest spin 1/2 state.

IV. CONCLUSION

In this work, we investigate the mixing of the low-lying three- and five-quark $\Omega$ configurations with negative parity in a NJL interaction induced quark-antiquark pair creation model. Hyperfine interaction between quarks is also taken to be based on the NJL interaction for model consistency. Numerical results show that the three- and five-quark configurations with spin 1/2 do not mix to each other in present model, this is because of that the spin structure of the five-quark $\Omega$ states with spin 1/2 results in vanishing matrix elements for the transition $\text{sss} \leftrightarrow \text{sssq}$. This is consistent with the results obtained in the instanton induced quark-antiquark pair creation model, within which mixing between three- and five-quark $\Omega$ states with spin-parity 1/2$^-$ is very small and negligible [2].

The NJL interaction induced quark-antiquark pair creation results in strong mixing between three- and five-quark spin 3/2 $\Omega$ states, and the mixing decreases energy of the lowest state to $1785 \pm 25$ MeV, which is lower than energy of the lowest spin 1/2 state. On the other hand, mixing between $s^3$ and $s^3\bar{s}$ configurations is very small. This is also consistent with our previous predictions within the instanton-induced quark-antiquark pair creation model.

While at present time, the experimental data of the $\Omega$ resonances spectrum is very poor, so we cannot say which model is more appropriate. Recently, BESIII Collaboration at Beijing Electron Positron Collider (BEPC) reported an interesting result that $\psi(2S) \rightarrow \Omega \bar{\Omega}$ was observed with a branch fraction of $(5 \pm 2) \times 10^{-7}$ [22]. Now with the upgraded BEPC, i.e., BEPCII, BESIII Collaboration [22] is going to take billions of $\psi(2S)$ events, which is two orders of magnitude higher than what BESII experiment got. We hope the BESIII experiment [23] will provide us more information through $\psi(2S) \rightarrow \Omega \Omega^*$ reaction.

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