WEAK COUPLING PHASE FROM DECAYS
OF CHARGED B MESONS TO $\pi K$ AND $\pi\pi$

Michael Gronau
Department of Physics
Technion – Israel Institute of Technology, Haifa 32000, Israel
and
Jonathan L. Rosner
Enrico Fermi Institute and Department of Physics
University of Chicago, Chicago, IL 60637
and
David London
Laboratoire de Physique Nucléaire
Université de Montréal, Montréal, PQ, Canada H3C 3J7

ABSTRACT

The theory of $CP$ violation based on phases in weak couplings in the
Cabibbo-Kobayashi-Maskawa (CKM) matrix requires the phase $\gamma \equiv \text{Arg} V_{ub}^*$ (in a standard convention) to be nonzero. A measurement of
$\gamma$ is proposed based on charged $B$ meson decay rates to $\pi^+K^0$, $\pi^0K^+$, $\pi^+\pi^0$, and the charge-conjugate states. The corresponding branching
ratios are expected to be of the order of $10^{-5}$.

\footnote{Submitted to Physical Review Letters}
At present direct evidence for CP violation comes exclusively from the decays of neutral $K$ mesons. One theory of this phenomenon is based on phases in the Cabibbo-Kobayashi-Maskawa (CKM) [1] matrix $V_{ai}$, which describes the weak charge-changing couplings of left-handed quarks $i = (d, s, b)$ of charge $-1/3$ with left-handed quarks $a = (u, c, t)$ of charge $2/3$. By choosing five relative quark phases, one can take the elements of $V$ along and just above the diagonal to be real (see, e.g., [2]). In this convention, taking account of the observed magnitudes of elements, only $V_{ub}$ and $V_{td}$ can have significant nonzero phases. The observed decays $K_L \rightarrow \pi^+\pi^-$ and $\pi^0\pi^0$ of the long-lived neutral kaon and the charge asymmetry in semileptonic $K_L$ decays can be ascribed to a CP-violating mixing of $K^0$ and $\bar{K}^0$ arising from these phases. The CKM model of CP violation also predicts small differences in the ratios $\eta_{+-} \equiv A(K_L \rightarrow \pi^+\pi^-)/A(K_S \rightarrow \pi^+\pi^-)$ and $\eta_{00} \equiv A(K_L \rightarrow \pi^0\pi^0)/A(K_S \rightarrow \pi^0\pi^0)$. Two recent experiments [3,4] reach different conclusions about whether $\eta_{+-} = \eta_{00}$, and a satisfactory alternative remains a “superweak” theory of direct $K^0 - \bar{K}^0$ mixing [5].

A fertile ground for testing the CKM model of CP violation involves the decays of mesons containing the fifth ($b$) quark [6]. Unequal rates for decays of the mesons $B^0 \equiv bd$ and $\bar{B}^0 \equiv b\bar{d}$ to CP eigenstates like $J/\psi K_S$ can be interpreted crisply in terms of the weak phase $\text{Arg} \, V_{td}$, without complications from strong final-state interactions. However, the presence of $B^0 - \bar{B}^0$ mixing, needed for the rate asymmetry, complicates the identification of neutral $B$ mesons.

The decays of charged $B$ mesons can manifest CP violation in the form of unequal rates for such processes as $B^+ \rightarrow \pi^0 K^+$ and $B^- \rightarrow \pi^0 K^-$. While the charge of a $B$ meson is easily determined, strong final-state interactions are required for such rate differences. Differences in strong final-state phases among different eigenchannels are expected to be small and uncertain. Thus, except in a few particular cases [7], it has usually been assumed that information on CKM phases cannot be extracted from the study of charged $B$ decays alone. Such decays can play useful auxiliary roles in the separation of final-state interaction effects from weak phases when decays of neutral $B$ mesons to CP eigenstates are also measured [8-10].

In this Letter we describe a way to obtain the weak phase $\gamma \equiv \text{Arg} \, V_{ub}^*$ from the rates for the decays of charged $B$ mesons to $\pi^+ K^0$, $\pi^0 K^+$, $\pi^+ \pi^0$, and the charge-conjugate states. We expect equal rates for $B^+ \rightarrow \pi^+\pi^0$ and $B^- \rightarrow \pi^-\pi^0$ on rather general grounds, and equal rates for $B^+ \rightarrow \pi^+ K^0$ and $B^- \rightarrow \pi^- \bar{K}^0$ as a result of a specific assumption to be noted below. The rates for $B^+ \rightarrow \pi^0 K^+$ and $B^- \rightarrow \pi^0 K^-$ can differ if CP is violated, but it is not necessary to measure a CP-violating observable in order to obtain $\gamma$. The corresponding branching ratios are expected to be of the order of $10^{-5}$, which is the level at which decays of $B$ mesons to two light pseudoscalars have already been seen [11].

The method relies upon an SU(3) relation between the amplitude for $B^+ \rightarrow \pi^+\pi^0$, which has isospin $I = 2$, and the isospin-3/2 amplitude in $B \rightarrow \pi K$. Both amplitudes belong to the same 27-dimensional representation of SU(3), and
are related by a Clebsch-Gordan coefficient. We perform the calculation using a convenient graphical method [12] which has been shown equivalent to a general decomposition into SU(3) representations. SU(3) breaking is also introduced, assuming that the two-body hadronic decay amplitudes are factorizable. Other applications of SU(3) to decays of B mesons to pairs of light pseudoscalars have been considered in Refs. [13-15]. A more general recent discussion is contained in Ref. [16], where several new tests of the SU(3) assumption are suggested.

The weak phase of the isospin-3/2 πK amplitude is expected to be ±γ for B± decays, while the strong phase does not change sign under charge conjugation. The weak phases of the amplitude for B+ → π0K± and B− → π−K0 are both expected to be π under the assumption that weak annihilation graphs do not contribute to the decay. (We shall suggest a test of this assumption.) Two triangle relations satisfied by amplitudes, which include information from the rates for B± → π0K±, then allow one to separate out the desired weak phase γ modulo a discrete ambiguity.

We consider charmless decays of B mesons to two light pseudoscalar mesons within SU(3) [12,13]. Adopting the same conventions as Ref. [12], we take the u, d, and s quark to transform as a triplet of flavor SU(3), and the −ū, ā, and ̅s to transform as an antitriplet. The mesons are defined in such a way as to form isospin multiplets without extra signs. Thus, the pions will belong to an isotriplet if we take

$$\pi^+ \equiv u\bar{d}, \quad \pi^0 \equiv (d\bar{d} - u\bar{u})/\sqrt{2}, \quad \pi^- \equiv -d\bar{u}, \quad (1)$$

while the kaons and antikaons will belong to isodoublets if we take

$$K^+ \equiv u\bar{s}, \quad K^0 \equiv d\bar{s}, \quad \bar{K}^0 \equiv s\bar{d}, \quad K^- \equiv -s\bar{u}, \quad (2)$$

The B mesons are taken to be B+ ≡ ̅bū, B0 ≡ ̅bd, and Bs ≡ ̅bs. Their charge-conjugates are defined as B− ≡ −bū, ̅B0 ≡ bd, and ̅Bs ≡ bs.

The operators associated with the four-quark transition ̅b → ̅qūū and the direct (“penguin”) transition ̅b → ̅q (q = d or s) transform as a 3*, 6, or 15* of SU(3). When combined with the triplet of B meson states, these operators lead to one singlet, three octets and one 27-plet, which appear in the symmetric product of two octets (the pseudoscalar mesons, which are in an S-wave final state). This leads to a decomposition of all strangeness-preserving and strangeness-changing decay processes in terms of five SU(3) reduced amplitudes.

As shown in Ref. [12], this algebraic decomposition is equivalent to a simpler graphical expansion. The six graphs which contribute are illustrated in Fig. 1 [14]. They consist of a “tree” amplitude T (T’), a “color-suppressed” amplitude C (C’), a “penguin” amplitude P (P’), an “exchange” amplitude E (E’), an “annihilation” amplitude A (A’) and a “penguin annihilation” amplitude PA (PA’). The unprimed amplitudes stand for strangeness-preserving decays, while
the primed ones represent strangeness-changing processes. These amplitudes are related by simple CKM factors. In particular:

\[ \frac{T'}{T} = \frac{C'}{C} = \frac{E'}{E} = \frac{A'}{A} = r_u, \]

(4)

where \( r_u \equiv \frac{V_{us}}{V_{ud}} \approx 0.23 \). The set of six graphs is overcomplete. They appear in all processes of the type \( B \to PP \) in the form of five linear combinations, corresponding to the five SU(3) reduced matrix elements.

To apply SU(3) to the three decay processes, \( B^+ \to \pi^+\pi^0, \pi^+K^0, \pi^0K^+ \), we write the corresponding amplitudes in terms of their graphical contributions:

\[ A(B^+ \to \pi^+\pi^0) = -\frac{1}{\sqrt{2}}(T + C), \]

(5)

\[ A(B^+ \to \pi^+K^0) = P' + A', \]

(6)

\[ A(B^+ \to \pi^0K^+) = -\frac{1}{\sqrt{2}}(T' + C' + P' + A'). \]

(7)

Here, for instance, the combinations \( C' + T' \) and \( P' + A' \) form two of the five linearly independent combinations of graphical contributions. We immediately find:

\[ \sqrt{2}A(B^+ \to \pi^0K^+) + A(B^+ \to \pi^+K^0) = r_u\sqrt{2}A(B^+ \to \pi^+\pi^0). \]

(8)

This relation is described by a triangle in the complex plane, as shown in Fig. 2. The corresponding triangle for the charge-conjugate process is also shown. Notice that the two triangles share one side. An SU(3) assumption has been made in order to obtain this simple result.

The diagrams denoted by \( E, A, PA \) involve contributions to amplitudes which should behave as \( f_B/m_B \) in comparison with those from the diagrams \( T, C, \) and \( P \) (and similarly for their primed counterparts). This suppression is due to the smallness of the \( B \) meson wave function at the origin, and it should remain valid unless rescattering effects are important. Such rescatterings indeed could be responsible for certain decays of charmed particles (such as \( D^0 \to \bar{K}^0\phi \)), but should be less important for the higher-energy \( B \) decays. In addition the diagrams \( E \) and \( A \) are also helicity suppressed by a factor \( m_{u,d,s}/m_B \) since the \( B \) mesons are pseudoscalars.

If rescattering effects are small and the amplitudes \( A, E, \) and \( PA \) can be neglected, the rate for \( B^0 \to K^+K^- \) will be suppressed relative to \( B^0 \to \pi^+\pi^- \), since the amplitudes for these processes are given by

\[ A(B^0 \to \pi^+\pi^-) = -(T + P + E + PA), \]

(9)

\[ A(B^0 \to K^+K^-) = -(E + PA). \]

(10)

Assuming that the amplitude \( A' \) can be neglected in (9) and (10), the phases in the decay amplitudes and those for the charge-conjugate processes have simple
relations to one another. The phase of the $P'$ amplitude, which is expected to be dominated by the top quark loop [17], should be approximately $\text{Arg} V_{cb}^* V_{ts} = \pi$. Then we may denote
\[
A(B^+ \to \pi^+ K^0) = A(B^- \to \pi^- \bar{K}^0) = P' = -a_P e^{i\delta_P},
\]
where $a_P$ is real. Note that the rates for the process and its charge conjugate are equal. Similarly, taking account of the factor which relates $T + C$ to $T' + C'$ and using $\text{Arg} V_{ub}^* V_{us} = \gamma$, we find
\[
r_u \sqrt{2} A(B^+ \to \pi^+ \pi^0) = -(T' + C') = a_T e^{i\delta_T} e^{i\gamma},
\]
(12)
while
\[
r_u \sqrt{2} A(B^- \to \pi^- \pi^0) = a_T e^{i\delta_T} e^{-i\gamma},
\]
(13)
with $a_T$ real. The rates for these two processes are equal because they involve a single weak phase and a single strong phase. The difference in phase between these two amplitudes is just $2\gamma$.

The third side of each amplitude triangle is provided by the rate for the decay $B^+ \to \pi^0 K^+$ or $B^- \to \pi^0 K^-$, as shown in Fig. 2. Here $A^{0+} \equiv A(B^+ \to \pi^0 K^+)$, $A^{+0} \equiv A(B^+ \to \pi^+ K^0)$, $A^{0-} \equiv A(B^- \to \pi^0 K^-)$, $A^{-0} \equiv A(B^- \to \pi^- \bar{K}^0)$, $A^{\pi\pi}_+ \equiv A(B^+ \to \pi^+ \pi^0)$, $A^{\pi\pi}_- \equiv A(B^- \to \pi^- \pi^0)$. Modulo a two-fold ambiguity which corresponds to flipping one triangle about the horizontal axis, the rates determine the shapes of the triangles and hence the difference $2\gamma$. The flipping of one triangle corresponds to interchanging $\gamma$ and $\delta_P - \delta_T$. In general, $CP$ violation is expected to show up as a difference in rates between $B^+ \to \pi^0 K^+$ and its charge conjugate, since two CKM amplitudes with different phases interfere in this process. The crucial point in determining $\gamma$ is that the magnitudes of these two amplitudes are separately measured in $B^+ \to \pi^+ K^0$ and $B^+ \to \pi^+ \pi^0$. If $\delta_P - \delta_T = 0$, we will not observe such a difference in rates. In that case, however, we would have to choose the lower of Figs. 2, since only this configuration would correspond to a nonzero value of $\gamma$.

One can take account of SU(3) breaking in factorizable amplitudes by noting that the decay $B^+ \to \pi^+ \pi^0$ involves a factor of the pion decay constant $f_\pi$, whereas the $I = 3/2$ amplitude in $B \to \pi K$ should involve a factor $f_K$. Thus, one should probably multiply $r_u$ in all the relations presented here by the factor $f_K/f_\pi \approx 1.2$. This prescription was adopted in Ref. [15].

Fig. 2 will permit the measurement of $\gamma$ if each of the decay rates can be measured with sufficient accuracy. Explicitly, defining $a \equiv |A^{+0}| = |A^{-0}|$, $b \equiv (f_K/f_\pi)r_u \sqrt{2}|A^{\pi\pi}_+|^2 = (f_K/f_\pi)r_u \sqrt{2}|A^{\pi\pi}_-|^2$, $c \equiv \sqrt{2}|A^{0+}|$, $c' \equiv \sqrt{2}|A^{0-}|$, one has
\[
4ab \sin \gamma = \pm \{[(a + b)^2 - c^2][c'^2 - (a - b)^2]\}^{1/2} \pm \{c \leftrightarrow c'\} .
\]
(14)

The present data on $B^0$ decays to pairs of pseudoscalars [11] do not allow one to distinguish between $\pi^- K^+$ and $\pi^+ \pi^-$ final states. The combined branching ratio is about $2 \times 10^{-5}$, with equal rates for $\pi^- K^+$ and $\pi^+ \pi^-$ being most likely. If
this is true, the amplitudes $T$ and $P'$ have about the same magnitude, so that the short sides of the triangles in Fig. 2 are probably about 1/4 to 1/3 ($\approx (f_K/f_\pi) r_\gamma$) the lengths of the other two sides. Then the “long” sides of the triangle must be measured with fractional accuracies of about $(f_K/f_\pi) r_\gamma \delta \gamma$ in order to achieve an accuracy of $\delta \gamma$ in the angle $\gamma$. For example, to measure $\gamma$ to a statistical accuracy of about $10^\circ$, one probably needs fractional errors of about $1/20$ in amplitudes, or $10\%$ in rates. This would require at least 100 decays in each channel of interest.

We end with some comments about other ways of measuring weak phases.

(1) Another measurement of $\gamma$ from charged $B$ decays uses the processes $B^\pm \to K^\pm D^0, K^\pm \bar{D}^0, K^\pm D_{CP}$, where $D_{CP}$ denotes a $CP$ eigenstate [7,18]. The three $B^+$ amplitudes and their charge-conjugates obey two triangle relations similar to the above. Here too the angle $\gamma$ can be measured without an observation of $CP$ violation in $B^\pm \to K^\pm D_{CP}$, even when the final-state phase differences are too small to detect. While $B^+ \to K^+ D^0$ may be strongly color-suppressed, all the measured rates are expected to be of comparable magnitudes in the method presented here.

(2) The present method uses $B$ decay modes with rates similar to $B^0 \to \pi^+ \pi^-$ decays. The use of $\pi^+ \pi^-$ decays requires tagging the neutral $B$ meson flavor at time of production, and suffers from uncertainties associated with penguin amplitudes [8]. These uncertainties can be eliminated by a complete isospin analysis of all charge states in $B \to \pi \pi$ decays [9], or at least estimated by relating via SU(3) the rates of $B^0 \to \pi^+ \pi^-$ and $B^0 \to \pi^- K^+$ [15]. Information from additional $\pi \pi$, $\pi K$, and $K \bar{K}$ branching ratios of charged and neutral $B$’s can be combined with the rates mentioned here to further eliminate ambiguities and constrain other weak phases [16].

To summarize, we have shown that measurements of the rates for charged $B$ decays to $\pi K$ and $\pi \pi$, together with a simple SU(3) relation, suffice to specify the geometry of amplitude triangles from which one can extract the weak phase $\gamma = \text{Arg} V_{ub}^*$, where $V_{ub}$ describes an element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. No final-state-interaction phases need be specified. A non-zero value of $\gamma$ in accord with other analyses of parameters in the CKM matrix would provide valuable confirmation of a popular model of $CP$ violation.

ACKNOWLEDGMENTS

We thank B. Blok for fruitful discussions. M. Gronau and J. Rosner respectively wish to acknowledge the hospitality of the Université de Montréal and the Technion during parts of this investigation. This work was supported in part by the United States-Israel Binational Science Foundation under Research Grant Agreement 90-00483/2, by the German-Israeli Foundation for Scientific Research and Development, by the Fund for Promotion of Research at the Technion, by the United States Department of Energy under Contract No. DE FG02 90ER40560, and by the N. S. E. R. C. of Canada and les Fonds F. C. A. R. du Québec.
REFERENCES

[1] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[2] J. D. Bjorken and I. Dunietz, Phys. Rev. D 36, 2109 (1987).

[3] Fermilab E731 Collaboration, L. K. Gibbons et al., Phys. Rev. Lett 70, 1203 (1993).

[4] CERN NA31 Collaboration, G. D. Barr et al., Phys. Lett. B 317, 233 (1993).

[5] L. Wolfenstein, Phys. Rev. Lett. 13, 562 (1964).

[6] See, e.g., Y. Nir and H. Quinn, Ann. Rev. Nucl. Part. Sci. 42, 211 (1992).

[7] M. Gronau and D. Wyler, Phys. Lett. B 265, 172 (1991). See also M. Gronau and D. London, Phys. Lett. B 253, 483 (1991); I. Dunietz, Phys. Lett. B 270, 75 (1991).

[8] D. London and R. Peccei, Phys. Lett. B 223, 257 (1989); M. Gronau, Phys. Rev. Lett. 63, 1451 (1989); B. Grinstein, Phys. Lett. B 229, 280 (1989); M. Gronau, Phys. Lett. B 300, 163 (1993).

[9] M. Gronau and D. London, Phys. Rev. Lett. 65, 3381 (1990).

[10] Yosef Nir and Helen R. Quinn, Phys. Rev. Lett. 67, 541 (1991); Michael Gronau, Phys. Lett. B 265, 389 (1991); H. J. Lipkin, Y. Nir, H. R. Quinn and A. E. Snyder, Phys. Rev. D 44, 1454 (1991).

[11] M. Battle et al. (CLEO Collaboration), Phys. Rev. Lett. 71, 3922 (1993).

[12] D. Zeppenfeld, Z. Phys. C 8, 77 (1981).

[13] M. Savage and M. Wise, Phys. Rev. D 39, 3346 (1989); ibid. 40, 3127(E) (1989).

[14] L. L. Chau et al., Phys. Rev. D 43, 2176 (1991).

[15] J. Silva and L. Wolfenstein, Phys. Rev. D 49, R1151 (1994).

[16] M. Gronau, O. Hernandez, D. London, and J. L. Rosner, Technion preprint TECHNION-PH-94-8, March, 1994, to be submitted to Phys. Rev. D.

[17] T. Inami and C. S. Lim, Prog. Theor. Phys. 65, 297, 1772(E) (1981); G. Eilam and N. G. Deshpande, Phys. Rev. D 26, 2463 (1982).
FIGURE CAPTIONS

FIG. 1. Diagrams describing decays of $B$ mesons to pairs of light pseudoscalar mesons. Here $\bar{q} = \bar{d}$ for unprimed amplitudes and $\bar{s}$ for primed amplitudes. (a) “Tree” (color-favored) amplitude $T$ or $T'$; (b) “Color-suppressed” amplitude $C$ or $C'$; (c) “Penguin” amplitude $P$ or $P'$ (we do not show intermediate quarks and gluons); (d) “Exchange” amplitude $E$ or $E'$; (e) “Annihilation” amplitude $A$ or $A'$; (f) “Penguin annihilation” amplitude $PA$ or $PA'$.

FIG. 2. $SU(3)$ triangles involving decays of charged $B$’s which may be used to measure the angle $\gamma$. Here $A^{0+} \equiv A(B^+ \rightarrow \pi^0 K^+), A^{+0} \equiv A(B^+ \rightarrow \pi^+ K^0), A^{0-} \equiv A(B^- \rightarrow \pi^0 K^-), A^{-0} \equiv A(B^- \rightarrow \pi^- K^0), A^{+\pi\pi} \equiv A(B^+ \rightarrow \pi^+ \pi^0), A^{-\pi\pi} \equiv A(B^- \rightarrow \pi^- \pi^0)$. The lower figure shows one of the triangles flipped about the horizontal axis. This solution must be chosen when $|A^{0+}| = |A^{0-}|$ if $\gamma \neq 0$. 

[18] S. L. Stone, in *Beauty 93*, Proceedings of the First International Workshop on B Physics at Hadron Machines, Liblice Castle, Melnik, Czech Republic, January 1993, ed. P. E. Schlein, Nucl. Instrum. Meth. 33, 15 (1993).
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9404282v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9404282v1