SELF-SIMILAR BEHAVIOR IN GALAXY DYNAMICS AND DISTRIBUTIONS OF DARK MATTER

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ABSTRACT
Radial accelerations in galaxy dynamics are now observed over an extended range in redshift that includes model calculations in ΛCDM. In a compilation of data of the Spitzer Photometry and Accurate Rotation Curves (SPARC) catalogue, the recent sample of Genzel et al. (2017) and the McMaster Unbiased Galaxy Simulations 2, we report on self-similarity expressed in the variable ζ = a_N/a_dS, given by the Newtonian acceleration a_N based on baryonic matter content over the de Sitter scale of acceleration a_dS = cH, where c is the velocity of light and H is the Hubble parameter. At ζ = 1, smooth galaxy dynamics in ΛCDM models is found to be discrepant from the observed C0 galaxy dynamics, marking the transition to anomalous dynamics in weak gravity (ζ < 1) from Newtonian gravity (ζ ≥ 1) by baryonic matter content. At a level of confidence of 6σ, this poses a novel challenge to cold dark matter distributions on galactic scales models, here identified with a causality constraint on inertia imposed by the cosmological horizon.

Keywords: Dark energy – dark matter - gravitation
1. INTRODUCTION

Advances in high resolution spectroscopy of galaxy rotation curves across a range of redshifts give a detailed view on radial accelerations over an extended range in radius $r$ and redshift $0 \leq z \leq 2$ (Famae & McGaugh 2012; Lelli et al. 2016; McGaugh et al. 2016; Genzel et al. 2017). In ΛCDM, these observations suggest a diminishing of cold dark matter content with $z$, as observed accelerations $\alpha$ increasingly match the Newtonian acceleration

$$a_N = \frac{GM_b}{r^2}$$

by baryonic matter content $M_b$ within $r$, where $G$ is Newton’s constant. These results provide important benchmarks for galaxy models in ΛCDM from high resolution $N$-body simulations (Volker 2005). A recent comparison of the McMaster Unbiased Galaxy Simulations 2 (MUGS2) sample of models galaxy, for instance, suggests excellent agreement with the “missing mass” in galaxy rotation curves from the Spitzer Photometry and Accurate Rotation Curves (SPARC) (Keller & Wadsley 2017).

In a model-independent approach, redshift dependence in galaxy dynamics can be viewed as co-evolution with background cosmology. Described by the Hubble parameter $H = H(z)$, cosmological expansion carries a de Sitter scale of acceleration

$$a_{dS} = cH,$$

where $c$ is the velocity of light and $H$ is the Hubble parameter. For a galaxy such as the Milky Way, $a_N = a_{dS}$ corresponds to a distance

$$r_1 = R_H c H = 5.7 \text{kpc} M_{11}^{1/2},$$

where $R_H = c/H$ is the Hubble radius in a three-flat Friedmann-Robertson-Walker universe and $R_g = GM/c^2$ is the gravitational radius of a galaxy of mass $M = M_{11} M_\odot$. In quantum cosmology, $a_{dS}$ represents the surface gravity of the cosmological horizon at Hubble radius $R_H = c/H$ in de Sitter space (Gibbons & Hawking 1977). Based on dimensional analysis, this suggests evolution in galaxy dynamics in

$$\zeta = \frac{a_N}{a_{dS}},$$

where $\zeta = 1$ corresponds to a collison of Rindler and cosmological horizon (van Putten 2017b).

We here consider data on galaxy dynamics as a function of $\zeta$, of galaxy rotation curves of observed galaxies and numerical galaxy models in ΛCDM side-by-side (§2). This compilation highlights self-similar behavior in galaxy dynamics in $\zeta$ and a transition across $\zeta = 1$ to weak gravitation ($\zeta < 1$) from normal, Newtonian gravitation ($\zeta > 1$), where observed and modeled galaxy dynamics differ. These observations are interpreted in §3. In §4, we give our conclusions and outlook on future observations.

2. SELF-SIMILAR GALAXY DYNAMICS

![Figure 1. Compilation of radial accelerations in spiral galaxies from SPARC (red), Genzel et al.(2017) (black) and MUGS2 (blue, adapted from Lelli et al. (2016); Keller & Wadsley (2017)), plotted as $a_N/\alpha$ versus $\zeta = a_N/a_{dS}$ covering weak gravity ($a_N/a_{dS} < 1$) and Newtonian gravity $a_N/a_{dS} \geq 1$ of baryonic matter content (black dashed). Error bars are $3\sigma$ for data of SPARC and MUGS2 and from measurement and scatter in averaging over bins about common values of $\zeta$, and $1\sigma$ for Genzel et al. (2017). Data cover redshift zero (SPARC), $z_k = 0$, $0.4$, $1$ (MUGS2) and $z \leq 2$ (Genzel et al. (2017)). For MUGS2, small error bars in refer to $3\sigma$ in averaging over $k = 1, 2, 3$, while large error bars refer to scatter. The results for data and simulations show remarkable self-similarity in $\zeta$, in galaxy evolution tracing background cosmology with Hubble parameter increasing by about three. The onset to weak gravity appears to be $C^0$ (continuous with discontinuous derivative) at $\zeta = 1$ in SPARC data (van Putten 2017a,b), whereas MUGS2 provides a smooth interpolation with a $6\sigma$ discrepancy at $\zeta = 1$.](http://example.com/figure1.png)

Fig. 1 shows a compilation of SPARC (McGaugh et al. 2016; Lelli et al. 2016), Genzel et al. (2017) and MUGS2 (Keller & Wadsley 2017) data plotted as a function of $\zeta$, covering redshifts zero to about two. Here, MUGS2 data cover $z_k = 0, 0.4, 1$. Over this range of redshift, the Hubble parameter varies by a factor of about three, implying variations of order unity in dimensionful quantities such as $r_1$. 

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$\zeta$ represents the surface gravity of the cosmological horizon in de Sitter space, $c$ is the velocity of light, $H$ the Hubble parameter, $M$ the mass of a galaxy, $G$ Newton’s constant, $a_N$ the Newtonian acceleration, $a_{dS}$ the de Sitter scale of acceleration, $R_H$ the Hubble radius, $R_g$ the gravitational radius, $M_{11}$ the mass of the galaxy in units of $M_\odot$, $\zeta$ the parameter that characterizes the transition from normal to weak gravitation.
Plotting radial accelerations of $a_N/\alpha$ as a function of $\zeta$, results from observations and simulations each coalesce. For MUGS2, averaging over $(\zeta, a_N/\alpha)_{2k}$ over $k = 1, 2, 3$ leaves in particular a dispersion (small error bars) much smaller than scatter in the data (large error bars). The aforementioned agreement with the “missing mass” in galaxy rotation curves is hereby preserved over an extended range of redshift. Consequently, $\zeta$ effectively absorbs redshift dependence, thus arriving at a reduction in independent variables by one. $\zeta$ hereby is a self-similarity variable for galaxy dynamics in an evolving cosmology.

In the outskirts of galaxies, rotation curves satisfy Milgrom (1983)’s law, $\alpha = \sqrt{a_0 a_N}$ with \textbf{van Putten 2017b}

$$a_0 = \frac{\omega_0}{2\pi},$$

$$\omega_0 = \sqrt{1 - q a_{dS}},$$

where $q = q(z)$, $q(z) = -1 + (1 + z) H^{-1}(z) H'(z)$ is the deceleration parameter. In the asymptotic regime $\zeta << 1$, therefore, we have

$$\frac{a_N}{\alpha} = (1 - q)^{1/2} \zeta^{-1/2}.$$  \hspace{1cm} (6)

Deviations from self-similarity by ±20\% in $(1 - q)^{1/2}$ as $q$ varies over $-1 < q < 0.5$ in late-time cosmology are too small to be resolved in the present data.

3. A 6σ DEVIATION AT $\zeta = 1$

In transition from Newtonian gravity (1) ($\zeta >> 1$) to weak gravity ($\zeta << 1$), Fig. 1 shows an onset to the latter which is smooth in MUGS2 in contrast to what appears to be $C^0$ - continuous with discontinuous derivatives - in SPARC (\textbf{van Putten 2017b}). Smoothness in MUGS2 is expected and inherent to $N$-body simulations by diffusion due to small angle gravitational scattering. Consequently, the noticeable gap between MUGS2 and SPARC at $\zeta = 1$ is characteristic for galaxy models in $\Lambda$CDM, not limited to MUGS2.

It is perhaps paradoxical, that $\zeta$ is a similarity variable familiar from the theory of linear diffusion, yet the observed $C^0$ onset to weak gravity in SPARC runs counter to the same. We recently described inertia non-locally by an inertial mass-energy

$$U = m c^2$$

with the distance

$$\xi = \frac{c^2}{\alpha}$$  \hspace{1cm} (8)

to the Rindler horizon $h$ as a length scale accompanying the celebrated equivalence principle at accelerations $\alpha$. Here, $U$ is the gravitational binding energy in the gravitational field of $\alpha$, by integrating the inertial force $F = m\alpha$ over a distance $\xi$.

As a non-local quantity, $U$ may be identified with entanglement entropy $I_1 = 2 \pi \alpha \Delta \varphi_C$ in $h$, where $\Delta \varphi_C$ is the distance $\xi$ expressed in Compton phase (\textbf{van Putten 2015}), against the Unruh temperature $T_U = \alpha h / 2\pi c$ (Unruh 1976) of $h$ (rather than the local vacuum),

$$U = \int_0^\xi T_U dI_1,$$

where $h$ denotes the reduced Planck constant. The Rindler horizon herein appears as an apparent horizon surface. Apparent horizon surfaces are familiar concept in numerical relativity signaling black hole formation (Penrose 1965; Brewin 1988; Cook 2000; York 1989; Wald & Iyer 1991; Cook & Abrahams 1992; Thornburg 2007). In a three-flat Friedmann-Robertson-Walker universe, the cosmological horizon $H$ provides an apparent horizon in the background, whose Hubble radius $R_H$ puts a bound on $h$ in (7). As $h$ formally drops beyond $H$ ($\zeta > R_H$), $U$ in (7) drops below its Newtonian value $m = m_0$. At a given $a_N$, the observed acceleration

$$\alpha = \left( \frac{m_0}{m} \right) a_N$$

experiences a $C^0$ transition across $\zeta = 1$ (\textbf{van Putten 2017a,b}). $C^0$ galaxy dynamics in the SPARC data is hereby attributed to causality imposed on inertial mass-energy by $H$.

4. CONCLUSIONS

Empirically, we observe self-similar behavior in galaxy dynamics over an extended range in radial accelerations and redshifts, both in observations and simulations. The similarity variable $\zeta$ absorbs redshift dependence associated with the Hubble parameter $H(z)$, that varies appreciably over the redshift range $0 \leq z \lesssim 2$ considered here. When plotted as a function of $\zeta$, SPARC data point to distinct $C^0$ galaxy dynamics across $\zeta = 1$. The low apparent content of dark matter in the galaxies of Genzel et al. (2017) is identified with clustering of this sample close to $C^0$, obviating the need for exotic baryonic physics.

At $\zeta = 1$, the observed self-similar behavior in observed and model galaxies show a remarkable gap at the onset to weak gravity ($\zeta < 1$), away from Newtonian gravity based on baryonic matter ($\zeta > 1$). This discrepancy represents the distinct $C^0$ galaxy dynamics in SPARC data and smooth dynamics in $N$-body simulations. The latter is inherent to any $N$-body simulation due to diffusion from small angle scattering. At 6σ, this
appears to be fundamental to the nature of dark matter, here the absence thereof in galaxies in departure from ΛCDM.

Crucially, this self-similar and discrepant behavior in observed and modeled galaxies is found as a model-independent result, subject only to canonical observational and, respectively, computational uncertainties. We attribute the first with causality imposed on inertial mass-energy by $H$.

The $6\sigma$ gap reported here stands out in being more significant than the $H_0$ tension problem, currently at $\sim 3\sigma$, that appears in confrontation of ΛCDM on the largest scale with late-time cosmology (Riess et al. 2016; Ade et al. 2016; Anderson & Riess 2017; Freedman 2017; Riess et al. 2018). This may may represent an early manifestation of instability of $H$, arising from dispersive behavior at super-horizon scale fluctuations (van Putten 2017b), beyond the geometric optics limit of classical general relativity.

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