The flavor of product-group GUTs

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The doublet-triplet splitting problem can be simply solved in product-group GUT models, using a global symmetry that distinguishes the doublets from the triplets. Apart from giving the required mass hierarchy, this “triplet symmetry” can also forbid some of the triplet couplings to matter. We point out that, since this symmetry is typically generation-dependent, it gives rise to non-trivial flavor structure. Furthermore, because flavor symmetries cannot be exact, the triplet-matter couplings are not forbidden then but only suppressed. We construct models in which the triplet symmetry gives acceptable proton decay rate and fermion masses. In some of the models, the prediction $m_b \sim m_\tau$ is retained, while the similar relation for the first generation is corrected. Finally, all this can be accomplished with triplets somewhat below the GUT scale, supplying the right correction for the standard model gauge couplings to unify precisely.

I. INTRODUCTION

The doublet-triplet splitting problem can be elegantly solved in Grand Unified Theories (GUTs) based on semi-simple GUT groups \cite{1,2,3,4}. If the standard-model (SM) Higgses originate from GUT fields that transform under different factors of the GUT group, these theories can accommodate a global symmetry, which we will refer to in the following as a “triplet symmetry”, that allows a triplet mass at the GUT scale while forbidding a doublet mass. Furthermore, the triplet symmetry may also forbid some of the triplet couplings to standard-model (SM) matter fields \cite{3,4}. This eliminates dangerous contributions to the proton decay rate, so that GUT-scale triplets are consistent with current bounds on proton decay, unlike in minimal SU(5). In fact, even triplets below the GUT scale are allowed, and one can construct models in which the triplets are around $10^{15}$ GeV, so that they provide precisely the right threshold correction for successful coupling unification \cite{5,6}.

As mentioned above, the standard-model Higgses transform under different group factors in these models. Likewise, the SM matter fields can transform under different GUT group factors. This has two immediate consequences for the fermion mass matrices. First, some fermion mass terms involve fields transforming under different group factors. Because of the GUT gauge symmetry, such a mass term must come from a higher-dimension term that includes GUT breaking fields, and is therefore suppressed by a power of $\epsilon = M_{\text{GUT}}/M_{\text{Pl}}$. Thus, the GUT gauge symmetry automatically generates some non-trivial fermion mass textures \cite{4}. Second, in such models the triplet symmetry is necessarily horizontal—or generation-dependent \cite{5}. It therefore dictates a non-trivial structure of fermion mass matrices on top of the texture generated by the GUT gauge symmetry.

In this paper, we will study the flavor structure of SU(5)×SU(5) models. As we will see, the triplet symmetry can generate viable mass matrices. The picture that emerges is very attractive. The same global symmetry generates a doublet-triplet mass hierarchy, suppresses the triplet contribution to proton decay, and gives viable fermion masses. In addition, some of the models partially break the usual GUT “ Yukawa unification”, so that the successful relation $m_b \sim m_\tau$ is maintained while a similar relation for the first generation is avoided.

Since the triplet symmetry is generation dependent, it must be broken—otherwise some fermion mass splittings and/or mixings vanish \cite{7,8}. Therefore, triplet-matter couplings that would have been forbidden had the symmetry been exact, are no longer zero. In fact, the proton decay rate may even be larger in some SU(5)×SU(5) models compared to minimal SU(5), since the triplet couplings to matter may be enhanced relative to the doublet couplings. Therefore, apart from checking the flavor parameters of the models, we must check the proton decay rate.

The paper is organized as follows. We start (section I) with a brief review of the basics of SU(5)×SU(5) models. In section II we list the possible Higgs sectors of the models, and discuss the effects of the triplet symmetry on flavor and on proton decay. We find that (a) the up sector mass matrix hierarchies, (b) the mass ratio of a lepton and corresponding down-sector quark, (c) the ratio of the triplet Yukawa couplings and corresponding doublet coupling, are all governed by a single parameter. Model building is then reduced to finding a parameter that satisfies flavor and proton decay constraints. In section III we apply this to specific models. Finally, motivated by gauge coupling
unification, we complete our analysis in section [V] by looking into the possibility of having the triplets at $10^{14} - 10^{15}$ GeV. We summarize our results in section [VI]. In the Appendix we discuss a model that is ruled out only by the combined constraints from proton decay and flavor.

II. BASIC STRUCTURE

We consider supersymmetric models with gauge group $G = \text{SU}(5)_1 \times \text{SU}(5)_2$, that have an additional $Z_n$ global symmetry. Refs. [1, 2, 3, 4] show how this setup may be used to account for the doublet-triplet splitting. The basic scenario of [3] is briefly described below. We consider $G$ breaking down to $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \times Z'_n$ by the following VEVs of bifundamental fields $\Phi_t$ and $\Phi_d$ \cite{[5]}, \cite{[5], \cite{[5]}}

$$
\langle \Phi_t \rangle = \langle \bar{\Phi}_t \rangle = \text{diag}(v_t, v_t, v_t, 0, 0), \quad \langle \Phi_d \rangle = \langle \bar{\Phi}_d \rangle = \text{diag}(0, 0, v_d, v_d).
$$

As shown in [4], these VEVs may correspond to exact or approximate flat directions with $v_t \sim v_d \sim M_{\text{GUT}}$, so we will take $v_t = v_d = v$ for simplicity. Since the SM gauge group is contained in the diagonal subgroup of $G$, the low energy couplings $\alpha_1, \alpha_2, \alpha_3$ unify even when the couplings of the two SU(5) factors are different, and charge quantization is maintained.

The MSSM Higgs fields, now embedded in a GUT multiplet, are taken to transform under different group factors. For example, let us consider $h$ and $h'$, transforming as $(5,1)$ and $(1,\bar{5})$. The superpotential terms

$$h\bar{\Phi}_t h', \quad h\bar{\Phi}_d h',
$$

give a triplet mass term and a doublet mass term respectively. Clearly, if

$$Z_n(\Phi_t) - Z_n(\Phi_d) = Z_n(\bar{\Phi}_t) - Z_n(\bar{\Phi}_d) \neq 0,$$

then the doublet and triplet masses cannot be allowed simultaneously.

The bifundamental VEVs \cite{[1]} leave a combination of $Z_n$ and the hypercharge of SU(5)$_1$ unbroken. This combination,

$$Z'_n = Y_1^k \times Z_n,$$

with some integer $k$ and with

$$Y_1 = \text{diag}(e^{-2i\frac{\pi}{5}}, e^{-2i\frac{\pi}{5}}, e^{-2i\frac{\pi}{5}}, e^{+3i\frac{\pi}{5}}, e^{+3i\frac{\pi}{5}}),$$

is the triplet symmetry. The Higgs triplets and doublets transform differently under $Z'_n$,

$$
\begin{align*}
(h_t, \ h_d) & \rightarrow (e^{i \frac{2\pi}{5} (Z_n(h) - 2k)} h_t, \ e^{i \frac{2\pi}{5} (Z_n(h) + 3k)} h_d), \\
(\bar{h}_t', \ \bar{h}_d') & \rightarrow (e^{i \frac{2\pi}{5} Z_n(h')} \bar{h}_t', \ e^{i \frac{2\pi}{5} Z_n(h')} \bar{h}_d').
\end{align*}
$$

The matter fields, too, may be split between the two group factors. This has immediate consequences for the quark and lepton masses \cite{[3], \cite{[3]}}. Denoting,

$$\begin{align*}
T & \sim (10,1), \quad T' \sim (1,10), \\
\bar{F} & \sim (5,1), \quad \bar{F'} \sim (1,\bar{5}),
\end{align*}
$$

with the Higgs field $h \sim (5,1)$, the following mass terms may arise (depending on the matter content of the specific model)

$$TTTh \sim (10,1)(10,1)(5,1),$$

$$\frac{1}{M_{\text{Pl}}} T'Th\Phi \Phi \sim \frac{1}{M_{\text{Pl}}} (1,10)(10,1)(5,1)(5,\bar{5})(5,\bar{5}),$$

where $M_{\text{Pl}}$ is the Planck scale. Some mass terms are then suppressed by powers of

$$\epsilon = \frac{v}{M_{\text{Pl}}} \sim \frac{M_{\text{GUT}}}{M_{\text{Pl}}} \sim 10^{-2},$$
and a fermion mass hierarchy is generated. We refer to this structure of the mass matrices as \( \epsilon \)-dependence.

Models that have matter generations transforming under different group factors have some Yukawa couplings mediated by \( \Phi_t \) (\( \Phi_T \)) and some mediated by \( \Phi_d \) (\( \Phi_D \)). Since the two bifundamentals carry different \( Z_n \) charges, the discrete symmetry distinguishes Yukawa couplings that would have been equal in minimal SU(5). This may partially break the lepton-down sector Yukawa unification \(^4\) and suppress the couplings of the Higgs triplet to matter, thus suppressing proton decay mediated by Higgsino exchange \(^3\, ^5\).

A further consequence of splitting matter between the two group factors is that in such models \( Z_n' \) is necessarily horizontal. Suppose a model has 10s coming from \( T \sim (10, 1) \) and \( T' \sim (1, 10) \). For \( Z_n' = Z_n \times Y_1^k \) to be generation blind, we have to require

\[
Z_n'(Q) = Z_n'(Q') ,
\]

where \( Q \) and \( Q' \) are the SU(2) doublet superfields coming from \( T \) and \( T' \) respectively. From the definition of \( Z_n' \) it follows that fields coming from \( T' \) have the same \( Z_n' \) charge, so that

\[
Z_n'(u^c) = Z_n'(Q') ,
\]

with \( u^c \) the up sector SU(2) singlet contained in \( T' \). However, fields coming from \( T \sim (10, 1) \) are then distinguished by \( Z_n' \),

\[
Z_n'(u^c) = Z_n'(Q) - kY_1(Q) + kY_1(u^c) \neq Z_n'(u^c) ,
\]

so the triplet symmetry is generation dependent. Therefore, it must be broken, otherwise degenerate quarks or zero mixing angles result \(^6\, ^8\). Thus, the mass matrices acquire an additional hierarchy, and Yukawa couplings that would have been forbidden if the symmetry were exact are now allowed.

We assume then that \( Z_n' \) is broken by the VEV of a gauge singlet field \( S \), with charge \( Z_n(S) = s \), and that the standard-model Yukawa couplings originate from higher-dimension terms involving some power of \( S \), which is dictated by the triplet symmetry. Below the scale \( \langle S \rangle \), the resulting Yukawa couplings acquire a flavor hierarchy parameterized by powers of \( \delta = \langle S \rangle / M_{\text{Pl}} \), on top of their \( \epsilon \)-dependence. Since the bifundamentals carry zero \( Z_n' \) charge, the \( c \)'s are \( Z_n' \) neutral, and the \( \epsilon \)- and \( \delta \)-dependences can be studied separately. We also assume that \( \delta \) is not much smaller than \( \sin \theta_c \), and will take \( \delta \sim 0.1 \) for concreteness. We stress that in our analysis we ignore order one coefficients and stick to counting powers of \( c \)'s and \( \delta \)'s.

For later convenience we define the parameter \( w \in \{0, 1, \ldots, n - 1\} \), such that

\[
wS = Z_n(\Phi_t) - Z_n(\Phi_d) .
\]

The doublet triplet splitting condition becomes \( wS \neq 0 \). Consider the ratio of two Yukawa couplings that would have been equal in minimal SU(5). If one involves \( \Phi_t \), and the other \( \Phi_d \), their ratio is \( \delta^n \). This parameter will enter in the suppression of the triplet couplings relative to the doublet couplings, as well as in the ratio of lepton- and down-quark masses, as we show in section \(^7\).

### III. THE EFFECT OF THE TRIPOLET SYMMETRY

#### A. Higgses and model classification

As discussed above, the MSSM matter and Higgs fields can transform under either one of the SU(5)’s. Following Ref. \(^2\), we list below all possible assignments such that (a) the model is anomaly free and (b) the top and up type Higgs transform under the same SU(5), so that the top Yukawa is renormalizable. In some cases we add additional \((5, 1) + (1, 5)\) to cancel anomalies.

A. In this class the MSSM Higgses come from \( h \sim (5, 1) \) and \( h' \sim (1, 5) \), with no additional \((5, 1)\) and \((1, 5)\) pairs. The triplet mass term \( h_d \Phi_t h_d^* \) is assumed to be \( Z_n' \) neutral so the triplets acquire \( M_{\text{GUT}} \) mass. (We consider lower triplet mass consistent with gauge coupling unification in section \(^7\)). Consequently the doublet mass term \( h_d \Phi_d h_d^* \) has a \( Z_n \) charge of \( +wS \), so the doublet mass is suppressed by \( \delta^n \). We can then arrange for the doublets to be at the electroweak scale by taking \( n \) sufficiently large. The MSSM matter fields come from

A.1: \( 3 \times (\bar{5}, 1) + 2 \times (10, 1) + (1, 10) \),
A.2: \( 2 \times [(\bar{5}, 1) + (1, 10)] + (10, 1) + (1, 5) \).
B. The model contains $h(5,1), \tilde{h}(5,1), h'(1,5)$ and $\tilde{h}'(1,5)$. Of these, the MSSM Higgses come from $h$ and $\tilde{h}'$; $h'$ and $\tilde{h}$ do not couple to the MSSM fields (this can be ensured by imposing an additional symmetry). $h$ and $\tilde{h}'$ gain mass through a mutual mass term, and the Higgs-sector spectrum is as in A. The MSSM matter generations come from

B.1: $3 \times [(5,1) + (10,1)]$,
B.2: $2 \times [(5,1) + (10,1)] + (1,5) + (1,10)$,
B.3: $(5,1) + (10,1) + 2 \times (1,5) + (1,10)$.

C. This model contains $h, \tilde{h}, h'$ and $\tilde{h}'$, with all triplets heavy. The matter fields are as in models B. The Higgs mass terms are (neglecting $O(1)$ coefficients):

$$\Delta W = \frac{s^A}{M_{Pl}} h \tilde{h} + \frac{s^A}{M_{Pl}'} h' \tilde{h}' + \langle \Phi_1 \rangle h \tilde{h}' + \langle \Phi_2 \rangle h' \tilde{h} + \frac{\kappa}{M_{Pl}} \langle \Phi_3 \rangle h \tilde{h} + \frac{\kappa}{M_{Pl}'} \langle \Phi_4 \rangle h' \tilde{h}$$

where $A \in Z_n$ is a free parameter, and we have to require $A \neq 0$, or all Higgses, triplets and doublets alike, get $M_{Pl}$ mass.

Inspecting the resulting triplet- and doublet-mass matrices it is easy to see that the only acceptable choices are $A = 1, 2$. In this case, one triplet pair and one doublet pair are at or above the GUT scale, with mass $\delta^A M_{Pl}$, and the second triplet pair is at or just below the GUT scale, with mass $\epsilon/\delta^A M_{GUT}$. Finally the MSSM Higgs doublets, which are made predominantly of $h_d$ and $h_d$, are light, with mass $\delta^{n-A} M_{Pl}$.

We are interested in the flavor structure of models in which the triplet symmetry is horizontal. Models B.1 and C.1 are not of this type. Hence, the flavor structure of these models must come from another mechanism. As for the proton decay rate, in model C.1, all Yukawa couplings come from renormalizable terms, so the triplet and doublet Yukawa couplings are equal as in minimal SU(5). Therefore the proton decay rate in this model is the same as in minimal SU(5) and it is ruled out $[6, 9, 10, 11]$. Unlike model C.1, in model B.1 all dangerous dimension-5 operators are zero and the model is viable. We will not consider this model further.

Of the remaining models we show that models A.1, A.2, B.2, and B.3 exhibit viable flavor parameters and suppressed proton decay rate. Models C.2 and C.3 are shown to be more severely constrained by proton decay. Model C.2 is shown to be ruled out by the combination of flavor and proton decay constraints.

B. Flavor

We now turn to a systematic analysis of the models starting with the general flavor structure, and continuing with down lepton splitting in section III C and proton decay in III D.

Consider first the relative hierarchy and mixing between fields that come from $10s$ charged under the same SU(5), say $T_1, T_2$ (the indices denote generations), as in models A.2, B.3, and C.3. It is easy to show that

$$\frac{m_{u_1}}{m_{u_2}} \sim \left( \frac{m_{u_2}}{m_{u_2}} \right)^2.$$  (14)

These models therefore imply that

$$\frac{m_u}{m_c} \sim (V_{us})^2,$$  (15)

which is off by 10–40. Thus the up-quark mass is generically too high in these models, and must be further suppressed. The suppression, however, cannot be done with an additional $Z_n$ symmetry because $[11]$ will still hold.

The corresponding equality in models A.1, B.2, and C.2, that have the $10s$ $T_2$ and $T_3$, is more successful

$$\frac{m_c}{m_t} \sim (V_{cb})^2,$$  (16)

which is consistent within our order of magnitude analysis.

Now let us consider the flavor parameters of two generations that have $10s$ transforming under different SU(5)’s—$T$ and $T'$. Specifically, let’s consider the up sector mass ratio: $m_{Qw'/Qw}$ where, $Q', w' \in T'$, and $Q, w \in T$, and
the mixing of the two generations $m_{Q'^x}/m_{Qx}$, where $x = u', w', d', d''$. We define the parameters, $z$ and $r$, by
\[
\frac{m_{Q'x}}{m_{Qu}} \sim \epsilon \delta^z, \quad \frac{m_{Qx}}{m_{Qx}} \sim \epsilon^\# \delta^r,
\] (17)
where $\epsilon^\# = \epsilon^0, \epsilon^{\pm 1}, \epsilon^{\pm 2}$ depending on $x$ and the Higgses of the specific model. A priori, the ambiguity in the power of $\epsilon$ makes the choice of $r$ ambiguous. However, in all our models the leading contribution to the CKM element $V_{u'd}$, will come from Yukawa terms that obey $V_{u'd} \sim m_{Q'^x}/m_{Qx} \sim \epsilon^0 \delta^r$, so that $r$ may related to the experimental data by $V_{u'd} \sim \delta^r$. Note also that, since the bifundamental VEVs are $Z'_n$ neutral, the $\delta$-dependence of any Yukawa coupling is only determined by the $Z'_n$ charges of the low energy matter fields and Higgses and does not depend on which bifundamental mediates the coupling. Specifically, $r$ is determined only by the $Z'_n$ charge difference of $Q$ and $Q'$. We can rewrite eqn. (17) as,
\[
Z'_n(Q') + Z'_n(u'^c) = Z'_n(Q) + Z'_n(u^c) - zs
\]
\[
Z'_n(Q') = Z'_n(Q) - rs \mod n,
\] (18)
using (4) this neatly comes down to
\[
(2r - z)s = 5k \mod n. \tag{19}
\]
The parameters $r$ and $z$ may be related to $w$ by noting that since the non-zero expectation values of the bifundamentals transform trivially under $Z'_n$ then
\[
\{
0 = k(-2) + Z_n(\Phi_t) \\
0 = k(+3) + Z_n(\Phi_d) \mod n
\}
\]
so that
\[
w = (Z_n(\Phi_t) - Z_n(\Phi_d)) = +5k \mod n. \tag{20}
\]
Thus, assuming for simplicity $\gcd[s, n] = 1$,
\[
w = (2r - z) \mod n. \tag{21}
\]
Since $r$ and $z$ are input parameters that are determined by the observed masses and mixings [see eqn. (17)], $w$ is now determined. We show next that $w$ is also related to the down quark-lepton mass splitting, and to the suppression of proton decay.

C. Electron-down splitting

In any model that has different numbers of $10$s and $\bar{5}$s coming from the first SU(5), there is a diagonal down-sector mass term that involves a $10$ and a $\bar{5}$ of different SU(5)'s and is mediated by $\Phi$ (or $\Phi'$). Furthermore, the mass term of the down quark in these models, involves $\Phi_t$ (or $\Phi'')$, while the lepton mass term involves $\Phi_d$ (or $\Phi_d$). It follows that
\[
Z_n(m^T_l) = Z_n(m_d) \pm (Z_n(\Phi_d) - Z_n(\Phi_t)),
\] (22)
where $Z_n(m^T_l)$ is the total $Z_n$ charge of the lepton mass term and the minus sign is used whenever the mass term is mediated by $\Phi'$ rather than $\Phi$. This means that (keeping in mind the modulo-$n$ math)
\[
m^T_l \sim m_d \delta^w.
\] (23)
In models A, one then gets
\[
\frac{m_e}{m_d} \sim \delta^w, \tag{24}
\]
by taking the first generation matter representations to be $T'_1, \bar{F}_1$. Yukawa unification is maintained for the two heavier generations.
One may wish to use this mechanism in models that have equal numbers of $\bar{F}$s and $T$s by taking the following generation representations\(^1\): $T_1$, $T_2$, $\bar{F}_1$, $\bar{F}_2$. Although this may a-priori enable splitting the masses of the two light generations, we could not build such models.

Splitting the down and lepton masses introduces a constraint on $w$. If we want to arrange for $m_c/m_d \sim 0.1$ with $\delta = O(0.1)$, we have to take $w = 1$. Other charge assignments will result in an electron mass that is too small. This limits the flavor parameters in these models may give. For example, in model A.1 (with the MSSM generations coming from $T_1$, $T_2$, $T_3$, $\bar{F}_1$, $\bar{F}_2$, $\bar{F}_3$) taking $V_{us} \sim \delta$ forces $m_u/m_c \sim \epsilon\delta$, in good agreement with observation. However, in model A.2 (with the MSSM generations coming from $T_1$, $T_2$, $T_3$, $\bar{F}_1$, $\bar{F}_2$, $\bar{F}_3$), taking $m_c/m_t \sim \epsilon\delta$ forces $V_{cb} \sim \delta$, while $V_{ub} \sim \delta^2$ would have been more appropriate. Larger mixing angles in model A.2 will result in values of $m_c$ or $m_t$ that are too low. We show below that the dangerous dimension-5 operators inducing proton decay in models A and B are suppressed by $\delta^w$. Hence, in models A.1, A.2 and B.2 the leading LLLL and RRRR operators are suppressed by $m_c/m_d$ relative to the corresponding operators in minimal SU(5) and thus fit, but marginally so, the constraints by Ref. \[312\].

### D. Proton decay

As we have seen above, triplet-matter couplings and doublet-matter couplings originating from higher dimension terms that involve some bifundamentals, are distinguished by the triplet symmetry. Consequently, the triplet-matter Yukawa couplings can be suppressed compared to minimal SU(5). As we will see, in models C, the triplet couplings can also be enhanced compared to their SU(5) values.

It is convenient to study the triplet couplings by starting with 2-generation examples. Consider then two generations with fields $T_1$, $T_2$, $\bar{F}_1$, $\bar{F}_2$. The triplet-matter couplings in models A and B are then related to the down and up sector doublet couplings, $y^u$ and $y^d$ by

$$y_{QQ} \sim \begin{pmatrix} y^n_1 \delta^w & y^n_2 \delta^w \\ y^n_2 \delta^w & y^n_{22} \delta^w \end{pmatrix}, \quad y_{e^e} \sim \begin{pmatrix} y^n_1 \delta^w & y^n_{12} \delta^w \\ y^n_{12} \delta^w & y^n_{122} \delta^w \end{pmatrix},$$

$$y_{u^d} \sim \begin{pmatrix} y^n_1 \delta^w & y^n_2 \delta^w \\ y^n_2 \delta^w & y^n_{22} \delta^w \end{pmatrix}, \quad y_{QL} \sim \begin{pmatrix} y^n_1 \delta^w & y^n_{12} \delta^w \\ y^n_{12} \delta^w & y^n_{122} \delta^w \end{pmatrix}.$$

(25)

Since model C has four massive triplets that couple to matter, we have to consider not only $y_{QQ}$, $y_{e^e}$, $y_{QL}$, $y_{u^d}$, the couplings of $h_t$ and $\tilde{h}_t$ to matter, but also $y'_{QQ}$, $y'_{e^e}$, $y'_{QL}$, $y'_{u^d}$, the couplings of $h'_t$ and $\tilde{h}'_t$.

$$y_{e^e} \sim \begin{pmatrix} y^n_1 \delta^w & y^n_{12} \delta^w \\ y^n_{12} \delta^w & y^n_{122} \delta^w \end{pmatrix}, \quad y'_{e^e} \sim \delta^A \begin{pmatrix} y^n_1 \delta^w & y^n_{12} \delta^w \\ y^n_{12} \delta^w & y^n_{122} \delta^w \end{pmatrix},$$

$$y_{QQ} \sim \begin{pmatrix} y^n_1 \delta^w & y^n_{12} \delta^w \\ y^n_{12} \delta^w & y^n_{122} \delta^w \end{pmatrix}, \quad y'_{QQ} \sim \delta^A \begin{pmatrix} y^n_1 \delta^w & y^n_{12} \delta^w \\ y^n_{12} \delta^w & y^n_{122} \delta^w \end{pmatrix},$$

$$y_{QL} \sim \begin{pmatrix} y^n_1 \delta^w & y^n_{12} \delta^w \\ y^n_{12} \delta^w & y^n_{122} \delta^w \end{pmatrix}, \quad y'_{QL} \sim \delta^A \begin{pmatrix} y^{d_1} \delta^w & y^{d_2} \delta^w \\ y^{d_2} \delta^w & y^{d_{22}} \delta^w \end{pmatrix},$$

$$y_{u^d} \sim \begin{pmatrix} y^n_1 \delta^w & y^n_{12} \delta^w \\ y^n_{12} \delta^w & y^n_{122} \delta^w \end{pmatrix}, \quad y'_{u^d} \sim \delta^A \begin{pmatrix} y^{d_1} \delta^w & y^{d_2} \delta^w \\ y^{d_2} \delta^w & y^{d_{22}} \delta^w \end{pmatrix}.$$

(26)

Note that while in models A and B the dangerous $QQQL$ and $e^e u^d$ dim-5 operators are suppressed by $\delta^w$, some of the operators in model C seem to be a-priori enhanced. The problematic couplings, say $(y'_{u^d})_{11}$, can only be suppressed if $w$ is large enough so that $(y'_{u^d})_{11} \sim \delta^{n-k}$ with $k \ll n$. This will serve as a strict requirement on models C.2 and C.3 and will rule out model C.2.

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\(^1\) Since we require the top mass term to be renormalizable, all our models have the MSSM $H^U$ coming from $h \sim (5, 1)$, and since we wish to maintain $m_t \sim m_b$, the third generation representations have to be $T_3$, $\bar{F}_3$. Thus, the above option for c-d splitting is relevant to models B.2 and C.2. However, the latter model is not viable (see Appendix [A]).
IV. RESULTS

Applying our results to specific models we find that the models of the different classes A, B, and C show qualitatively different behavior. Models A.1 and A.2 are the only models that necessarily break the unification of lepton and down quark masses for a single generation. The mass splitting, however, sets $\delta^w \sim 0.1$. The resulting flavor parameters are viable in model A.1 but in model A.2 this leads to $V_{cb} \sim 0.1$. The leading dimension-5 operators, suppressed by $m_e/m_d \sim 0.1$, are consistent with triplets at $M_{\text{GUT}}$, but not lighter.

Models of class B (B.1, B.2, and B.3) are valid for a wider range of $w$, allowing for a wider range of flavor parameter and stronger suppression of dimension-5 operators. For an appropriate choice of $w$, these models also allow triplets below than $M_{\text{GUT}}$. In model B.2, one can choose to partially break Yukawa unification. With this choice, this model is essentially the same as model A.1 in all other respects.

In models C (C.2 and C.3) it is more difficult to find a value of $w$ that satisfies both proton decay and flavor constraints. In model C.2 this is impossible and the model is ruled out (see Appendix). In model C.3, however, such a choice is possible and the model is viable. In this model, too, the triplets may be made lighter than $M_{\text{GUT}}$.

In order to illustrate the above, we give one example for each class.

A. Model A.1

This model has matter representations:

$$T'_1, T_2, T_3, \bar{F}_1, \bar{F}_2, \bar{F}_3,$$

and the Higgses are $h, h'$. As discussed in section III.C, to have $m_e/m_d \sim 0.1$, we must take $w = 1$. The requirement $V_{us} \sim \sin \theta_c$, gives $r = 1$, and thus also $z = 1$. We then get

$$m_u \sim \langle h \rangle \left( \begin{array}{ccc} c \delta^3 & c^2 \delta^2 & c^2 \delta^2 \\ c^2 \delta^2 & \delta & 1 \\ c^2 \delta & c^2 & 1 \end{array} \right), \quad m_d \sim \langle h' \rangle \epsilon \left( \begin{array}{ccc} \delta^4 & \delta^3 & \delta^3 \\ \delta & \delta & \delta \\ \delta & \delta & \delta \end{array} \right),$$

and the charged lepton sector mass matrix

$$m_l^T \sim \langle h' \rangle \epsilon \left( \begin{array}{ccc} \delta & \delta & \delta \\ \delta & \delta & \delta \\ \delta & \delta & \delta \end{array} \right).$$

The resulting CKM matrix is

$$V \sim \left( \begin{array}{ccc} 1 & \delta & \delta^2 \\ \delta & 1 & \delta \\ \delta^2 & \delta & 1 \end{array} \right).$$

The dominant dimension-5 operators $Q_1Q_1Q_2L_2$, $Q_2Q_1Q_1L_2$, $u_1^c \bar{d}_1 u_2^c$, and $u_2^c \bar{d}_1 u_1^c$ are all suppressed by $m_e/m_d \sim \delta$. Thus, the triplet mass can be lower than the minimal SU(5) bound [12] of $m_{h_i} \geq 7.2 \times 10^{16}$ GeV. However, we can not lower the triplet mass to the range given in Ref. [6] for coupling unification.

B. Model B.3

The matter field and Higgs representations in this model are

$$T'_1, T'_2, T_3, \bar{F}_1, \bar{F}_2, \bar{F}_3, h, h',$$

Let us set $V_{cb} \sim \delta^2$ and $m_e/m_d \sim \epsilon \delta$ so that $r = 2, z = 1$ and hence $w = 3$. We also take $V_{us} \sim \delta$; this determines the up sector mass matrix. The down sector mass matrix is determined by taking $m_d \sim \delta^6, m_s \sim \delta^4, m_b \sim \epsilon$. The
resulting mass matrices are

\[ m_u \sim \langle h_d \rangle \begin{pmatrix} \epsilon & \epsilon & \epsilon^2 \delta^2 \\ \epsilon & \epsilon & \epsilon^2 \delta^2 \\ \epsilon^2 \delta^n - 2 & \epsilon^2 \delta^n - 2 & 1 \end{pmatrix}, \]

\[ m_d, m_l^T \sim \langle h_d \rangle \begin{pmatrix} \delta^6 & \delta^5 & \epsilon^2 \delta^n \\ \delta^5 & \delta^4 & \epsilon^2 \delta^n \\ \epsilon^2 \delta^n - 2 & \epsilon^2 \delta^n - 2 & 1 \end{pmatrix}. \]

The CKM matrix is

\[ V \sim \begin{pmatrix} 1 & \delta & \delta^3 \\ \delta & 1 & \delta^2 \\ \delta^3 & \delta^2 & 1 \end{pmatrix}. \]

The dominant dimension-5 operators, \( Q_1Q_2L_2, Q_2Q_1L_2, u_1d_1d_2e_2 \), and \( u_2d_1d_1e_2 \) are all suppressed by \( \delta^3 \sim 10^{-3} \). The strong suppression of the dimension-5 operators in this model allows for triplets as light as \( \mathcal{O}(\delta^2 M_{GUT}) \), compatible with gauge coupling unification.

C. Model C.3

In this model the matter and Higgs fields are \( T_1', T_2', T_3', \bar{T}_1', \bar{T}_2', \bar{T}_3', h, \bar{h} \). In section III D we have seen that to avoid enhanced dimension 5 operators we have to take \( w \) as large as possible. We therefore take \( z = 0 \) (meaning that \( m_e/m_t \sim \epsilon \delta^0 \)) and \( r = 2 \) (corresponding to \( V_{cb} \sim \delta^2 \)). The dangerous triplet couplings are all suppressed if we take \( A = 1 \) [see equation (14)], and

\[ m_u \sim \langle h_d \rangle \begin{pmatrix} \epsilon \eta^2 & \epsilon \eta & \epsilon^2 \delta^2 \eta \\ \epsilon \eta & \epsilon \eta & \epsilon^2 \delta^2 \eta \\ \epsilon^2 \delta^n - 2 \eta & \epsilon^2 \delta^n - 2 \eta & 1 \end{pmatrix}, \]

\[ m_d, m_l^T \sim \langle h_d \rangle \begin{pmatrix} \epsilon \delta^2 \eta & \epsilon \delta^2 \eta & \epsilon^2 \delta^4 \eta \\ \epsilon \delta^2 \eta & \epsilon \delta^2 \eta & \epsilon^2 \delta^4 \eta \\ \epsilon^2 \delta^n - 2 & \epsilon^2 \delta^n - 2 & 1 \end{pmatrix}. \]

For example, \( (y_{QL})_{11} \sim \delta^{-2} \) and \( (y_{QL}')_{11} \sim \delta^{-1} \). The first and second generation terms may be split by introducing an additional non triplet, horizontal symmetry broken by a small parameter \( \eta \),

\[ \tilde{m}_u \sim \langle h_d \rangle \begin{pmatrix} \epsilon \eta^2 & \epsilon \eta & \epsilon^2 \delta^2 \eta \\ \epsilon \eta & \epsilon \eta & \epsilon^2 \delta^2 \eta \\ \epsilon^2 \delta^n - 2 \eta & \epsilon^2 \delta^n - 2 \eta & 1 \end{pmatrix}, \]

\[ \tilde{m}_d, \tilde{m}_l^T \sim \langle h_d \rangle \begin{pmatrix} \epsilon \delta^2 \eta & \epsilon \delta^2 \eta & \epsilon^2 \delta^4 \eta \\ \epsilon \delta^2 \eta & \epsilon \delta^2 \eta & \epsilon^2 \delta^4 \eta \\ \epsilon^2 \delta^n - 2 & \epsilon^2 \delta^n - 2 & 1 \end{pmatrix}. \]

Note that the quark masses and mixing angles are all greater than or of the order of the experimental parameters, so this model has viable flavor parameters. However, the u-quark mass, which is significantly larger than the experimental value, \( m_u \sim \epsilon \eta^2 \) (note that since \( V_{us} \sim \eta \), then \( \eta \gg 0.1 \)), cannot have its mass suppressed further by a horizontal symmetry, triplet or non-triplet, without suppressing \( V_{us} \) below its measured value.

V. NATURALLY LIGHT TRIPLETS

If the triplet couplings to matter are small, we can try to construct models with triplets below \( M_{GUT} \), and still have an acceptable proton-decay rate. In particular if the triplets are around \( 10^{14} - 10^{15} \) GeV the standard model couplings unify precisely \( \frac{4}{3} \) with no need for any new thresholds or other corrections. To construct such models, we
can introduce a new global symmetry $\tilde{Z}$, that suppresses both the doublet and triplet masses and leaves the fermion masses unchanged. For example, this may be done for models A and B, by taking $\tilde{Z}(d^c) = -\tilde{Z}(h') \neq 0$, with all other fields neutral under $\tilde{Z}$. In order to keep the doublets at the electroweak scale, the doublet-triplet mass ratio must be modified. This can be easily accomplished by reducing the size $n$ of the triplet symmetry.

However, we can also lower the triplet mass using the triplet symmetry at hand. As we have already seen, models C can have triplets at $\delta \cdot M_{\text{GUT}}$. To get lighter triplets in models A and B we must make sure that the doublets remain light.

Consider for example the realization of model B.3 of section IV.B. The Higgs masses in the model came from $h\Phi_1 h' + h\Phi_d h' \delta^n - w$, with $w = 3$. Using the triplet symmetry to suppress the triplet mass by $\delta^n$ the superpotential Higgs mass terms become $h\Phi_1 h' \delta^n + h\Phi_d h' \delta^n - w + \alpha$. Clearly, if $\alpha \geq w$, the doublets become heavy.

It is interesting to note that in models A and B the triplet symmetry can be used to lower the triplet mass (while keeping the doublets light) only in the models where the dimension-5 operators are appropriately suppressed. In these models, where the leading dimension-5 operators are suppressed by $\delta^w$, the triplet symmetry can suppress the triplet mass by up to $\delta^{w-1}$; otherwise the doublets are made heavy. This is the same condition that we would have imposed thinking about proton decay— if the leading dim-5 operators are to be suppressed by at least an order of magnitude $\delta^w/\delta^\alpha \lesssim \delta$, then $\alpha \leq w - 1$.

Thus models with $w = 1$, such as models A, can not be viable with triplets below the GUT scale. Models B.2 and B.3, however, can have triplets at $O(\delta^{w-1}M_{\text{GUT}})$, with dimension-5 operators sufficiently suppressed and with light Higgs doublets.

VI. CONCLUSIONS

We studied the fermion masses and the Higgsino mediated proton decay rate in SU(5) $\times$ SU(5) models. As we explained, the two are tied together by the triplet symmetry. We constructed a few examples in which viable quark masses arise as a result of the combination of the SU(5) $\times$ SU(5) gauge symmetry and the triplet symmetry.

We also exhibited models in which, by virtue of the large suppression of the proton decay rate, the triplets can be below the GUT scale, and supply the required threshold correction for gauge coupling unification. As in all our models, the triplet mass in this case is naturally obtained using the triplet symmetry.

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APPENDIX A: MODEL C.2

This model is ruled out by a combination of constraints from flavor and proton decay. We show below how this models parameters have to be chosen so that flavor is viable, and further show that this choice leaves some triplet couplings enhanced thus enhancing proton decay.

In this model the generation representations are $T'_1, T_2, T_3, \bar{F}_1', \bar{F}_2, \bar{F}_3$. The $\epsilon$-dependence of the mass matrices is

$$m_u \sim \langle h_d \rangle \begin{pmatrix} \epsilon & \epsilon^2 & \epsilon^3 \\ \epsilon^2 & 1 & 1 \\ \epsilon^2 & 1 & 1 \end{pmatrix}, \quad m_d \sim \langle \bar{h}_d \rangle \begin{pmatrix} \epsilon & \epsilon^2 & \epsilon^3 \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}. \quad (A1)$$

Consider the $T'_1, T_1$ mixing between the first and second generations. The maximal value of $r$ we can take is $r = 1$, corresponding to $V_{us} \sim \delta$. Note that both $V_{us} \sim \delta$, and $V_{ub} \sim \delta^3$, can only come from the mixing terms of the first column in $m_d$, the rest of the mixings are too small— at least $O(\epsilon^2)$. Thus, none of the terms of first column in $m_d$ should be of $O(\delta^{n-k})$ (for any $k << n$), and furthermore, the second and third column terms should be at least of the same order as the first column terms. Substituting $r = 1$, and taking the minimal order for the second and third
column terms, we are left with three free parameters in the mass matrices (which we denote by x,y,z),

\[
m_u \sim \langle h_d \rangle \begin{pmatrix}
\epsilon^2 y^{2y+z} & \epsilon^2 y^{y+1} & \epsilon^2 y^{y+1} \\
\epsilon^2 y^{y+z-1} & \delta^2 y & \delta y \\
\epsilon^2 y^{y+z-1} & \delta y & 1
\end{pmatrix},
\]

(A2)

\[
m_d \sim \langle h_d \rangle \begin{pmatrix}
\epsilon^2 \delta^{x-1} & \epsilon^2 \delta^{x+2} & \epsilon^2 \delta^{x+2} \\
\epsilon^2 \delta^{x-1} & \delta^{x+1} & \delta^{x+1} \\
\epsilon^2 \delta^{x-1-\delta y} & \delta^{x+1-\delta y} & \delta^{x+1-\delta y}
\end{pmatrix}.
\]

The Yukawa triplet couplings may be directly related to the doublet Higgs couplings using eq. (26). Since \( x, A \geq 1 \), we have to take \( w = (2r - z) \geq 3 \), in order to suppress the \( y_{QL} \) and \( y'_{u,d} \) first generation couplings. This may only be done by taking \( z < 0 \). We take the minimal value of \( z \) requiring non-vanishing mass for the u-quark, \( z = -2y \), so \( w \) is maximal. Requiring that \( m_{d31} \sim \epsilon^0 \), we get the minimum value of \( x, x = y + 1 \). Finally, in order to avoid enhancement of \( y_{QL11} \) and \( y'_{u,d'-11} \) we set \( y=1 \). The resulting mass matrices are

\[
m_u \sim \langle h_d \rangle \begin{pmatrix}
\epsilon^2 y^0 & \epsilon^2 y^3 & \epsilon^2 y^2 \\
\epsilon^2 y^{n-1} & \delta^2 & \delta^1 \\
\epsilon^2 y^{n-2} & \delta^1 & 1
\end{pmatrix},
\]

(A3)

\[
m_d \sim \langle h_d \rangle \begin{pmatrix}
\epsilon^2 \delta^2 & \epsilon^2 \delta^4 & \epsilon^2 \delta^4 \\
\epsilon^2 \delta^1 & \delta^3 & \delta^3 \\
\epsilon^2 \delta^0 & \delta^2 & \delta^2
\end{pmatrix}.
\]

Of the eight triplet Yukawa matrices, two still have enhanced couplings,

\[
y'_{u,d'} \sim \begin{pmatrix}
y_{11}' \delta^{n-3} & y_{12}' \delta & y_{13}' \delta \\
y_{21}' \delta & y_{22}' \delta & y_{23}' \delta \\
y_{31}' \delta & y_{32}' \delta & y_{33}' \delta
\end{pmatrix},
\]

(A4)

\[
y_{QL} \sim \begin{pmatrix}
y_{11} \delta^{n-3} & y_{12} \delta & y_{13} \delta \\
y_{21} \delta & y_{22} \delta & y_{23} \delta \\
y_{31} \delta & y_{32} \delta & y_{33} \delta
\end{pmatrix}.
\]

The decays through \( y_{QL12} \) and \( y'_{u,d'-12} \) are at least as dangerous as the corresponding decays in minimal SU(5). Thus, model C.2 can either have viable flavor parameters or have suppressed dimension-5 operators— it cannot have both! We conclude that model C.2 cannot be made viable.

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