Fate of false vacuum in a singlet-doublet dark matter model with RG improved effective action

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We study the effective potential and the Renormalization Group (RG) improvement of the effective potential of Higgs boson in a model with singlet-doublet fermion extension of the Standard Model (SM), which is motivated by the research of dark matter candidate. We study the stability of the electroweak vacuum with the RG improved effective potential in this model beyond the SM. We show that in this model the RG improved effective potential at high energy scale can be quite different from the Higgs potential aided with running quartic self-coupling which is usually considered in literatures, contrary to the case of the SM in which the difference at high energy scale is accidentally small. Then we study the decay of the electroweak vacuum using the RG improved effective potential in this model beyond the SM. In this study we consider the quantum correction to the kinetic term in the effective action and consider the RG improvement of the kinetic term. Combining all these effects, we find that the decay rate of the false vacuum is slightly changed when calculated using the RG improved effective action.

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I. INTRODUCTION

Quantum contribution to the effective potential is important in understanding the properties of scalar field, e.g. the property of the ground-state and the behavior at large energy scale. For example, radiative correction can make a vacuum unstable and trigger spontaneous symmetry breaking [1]. The RG improved effective potential, which re-sums contributions of large logarithms, is important in understanding the behavior of effective potential at large energy scale. For example, the RG-improved effective potential is crucial in reducing the dependence on the renormalization scale when calculating quantities related with physical parameters [2–4].

It is well known that a false vacuum can decay via tunneling [5–7] and become unstable. In the SM, the Higgs quartic self-coupling can become negative at an energy scale around $10^{10}$ GeV. This makes the electroweak (EW) vacuum unstable. The decay rate of the EW vacuum can be calculated using an approximate bounce solution of the Higgs potential with a negative Higgs quartic coupling [8, 9]. Calculation of vacuum decay using RG improved effective potential in the SM does not give much difference, because the bounce solution is dominated by behavior at high energy scale and the RG-improved effective potential in the SM at high energy scale is accidentally close to the Higgs potential with running quartic coupling [10].

In extension of the SM, the situation can be quite different. The RG improved effective potential is possible to be very different from the potential using a running Higgs quartic coupling. As an example, we consider a singlet-doublet fermionic dark matter (SDFDM) extension of the SM. We show that the RG improved effective potential can be quite different from the tree-level potential aided with running Higgs quartic coupling. Then we study the vacuum stability in this extension of SM. We study the false vacuum decay using RG improved effective potential in this SDFDM model. We also study quantum contribution to the kinetic term in the effective action in this model beyond the SM and consider the RG improvement of the kinetic term. After taking all these effects into account we find that the false vacuum decay rate is just slightly changed using RG improved effective action, although the RG effective potential is significantly different from the tree-level form of the Higgs potential with running quartic self-coupling.

The article is organized as follows. In section II, we first briefly review the SDFDM model. We study the threshold effect caused by the extra fermions in this model beyond the SM and study the running of Higgs quartic coupling in this model. Then we study the effective potential and the RG improved effective potential in this model. In Section III we study the vacuum stability in this model. We calculate the renormalization of the kinetic term in the effective action in the SM and in SDFDM model. We study the RG improvement of the kinetic term and calculate the decay rate of the false vacuum. Details of calculation are summarized in Appendix A, B and C. We summarize in conclusion.
II. EFFECTIVE POTENTIAL AND RG IMPROVED EFFECTIVE POTENTIAL IN SDFDM MODEL

A. The SDFDM model

In addition to the SM fields, the SDFDM model has SU(2) doublet fermions \( \psi_{L,R} = (\psi_{L,R}^0, \psi_{L,R}^-)^T \) with \( Y = -1/2 \) and singlet fermions \( S_{L,R} \). Here, \( L \) (\( R \)) refers to left(right) chirality. As a singlet, \( S \) can be either Dirac type or Majorana type fermion \([11, 12]\). In this model, the neutral fermion can be a dark matter candidate. The vacuum properties of SDFDM model with a Majorana type dark matter particle has been discussed in \([13]\). In this article we mainly work on Dirac type dark matter \([14]\).

The relevant terms of \( \psi \) and \( S \) in the Lagrangian are:

\[
\mathcal{L}_{\text{SDFDM}} = \bar{\psi} i \not{D} \psi + \bar{S} i \not{D} S \\
- M_D \bar{\psi}_L \psi_R - M_S \bar{S}_L S_R - y_1 \bar{\psi}_L \bar{H} S_R - y_2 \bar{\psi}_R \bar{H} S_L + \text{H.c.}
\]

where \( M_{D,S} \) are mass parameters, \( y_{1,2} \) the new Yukawa couplings, \( H \) is the SM Higgs doublet with \( Y = 1/2 \), and \( \bar{H} = i \sigma_2 H^* \). We impose a \( \mathbb{Z}_2 \) symmetry in the Lagrangian with the new fermions \( \psi \) and \( S \) odd and SM fermions even under the \( \mathbb{Z}_2 \) operation. This guarantees the lightest of these new fermions to be stable, and makes it a dark matter candidate if it is neutral.

After EW symmetry breaking, the mass matrix of \( S_{L,R} \) and the neutral component of \( \psi_{L,R}(\psi_{L,R}^0) \) is given as

\[
M = \begin{pmatrix} M_S & \frac{y_2 v}{\sqrt{2}} \\ \frac{y_1 v}{\sqrt{2}} & M_D \end{pmatrix},
\]

where \( v = 246 \text{ GeV} \). Mixings between these two neutral fermions are generated by this mass matrix. The mixing angles \( \theta_{L,R} \) appear in the diagonalization of the mass matrix using two bi-unitary mass matrices, that is

\[
M^d = \begin{pmatrix} M_{\chi_1}^0 & 0 \\ 0 & M_{\chi_2}^0 \end{pmatrix} = U_L^T M U_R
\]

with

\[
U_{L,R} = \begin{pmatrix} \cos \theta_{L,R} & \sin \theta_{L,R} \\ -\sin \theta_{L,R} & \cos \theta_{L,R} \end{pmatrix}
\]

and

\[
M_{\chi_1}^2 = \frac{1}{2} \left( T_M - \sqrt{T_M^2 - 4 D_M^2} \right),
\]
\[
M_{\chi_2}^2 = \frac{1}{2} \left( T_M + \sqrt{T_M^2 - 4 D_M^2} \right),
\]
where \( T_M = M_S^2 + \frac{1}{3}y_1^2v^2 + M_D^2 + \frac{1}{3}y_2^2v^2 \), \( D_M = \frac{1}{3}y_1y_2v^2 - M_SM_D \). \( \chi_1^0 \) and \( \chi_2^0 \) are neutral fermions in the diagonalized base with masses \( M_{\chi_1} \) and \( M_{\chi_2} \) respectively.

The mixing angles \( \theta_{L,R} \) can be solved as

\[
\tan 2\theta_L = \frac{\sqrt{2}v (M_S y_1 + M_D y_2)}{M_B^2 - M_S^2 + \frac{v^2}{2} (y_1^2 - y_2^2)}
\]

\[
\tan 2\theta_R = \frac{\sqrt{2}v (M_S y_2 + M_D y_1)}{M_B^2 - M_S^2 + \frac{v^2}{2} (y_2^2 - y_1^2)}
\]

Writing \( H = (0,(v+h)/\sqrt{2})^T \), the interaction Lagrangian of dark matter fields \( \chi_{1,2}^0 \) and the CP-even neutral Higgs field \( h \) is obtained from Eq. (1) as

\[
\Delta \mathcal{L} = -y_A \overline{\chi}_1^0 \chi_1^0 h - y_B \overline{\chi}_2^0 \chi_2^0 h - [y_C \overline{\chi}_1^0 P_R \chi_2^0 h + y_D \overline{\chi}_1^0 P_L \chi_2^0 h + h.c.]
\]

\[
\theta_{L,R} = (1 \pm \gamma_5)/2, \quad y_A = (-y_2 \cos \theta_L \sin \theta_R - y_1 \sin \theta_L \cos \theta_R)/\sqrt{2}, \quad y_B = (y_2 \cos \theta_R \sin \theta_L + y_1 \sin \theta_R \cos \theta_L)/\sqrt{2}, \quad y_C = (y_2 \cos \theta_L \cos \theta_R - y_1 \sin \theta_R \sin \theta_L)/\sqrt{2} \quad \text{and} \quad y_D = (-y_2 \sin \theta_R \sin \theta_L + y_1 \cos \theta_R \cos \theta_L)/\sqrt{2}.
\]

B. Effective potential and RG improved Effective potential

In the following, we take \( \phi \) to denote a neutral external field. \( \phi/\sqrt{2} \) corresponds to the CP-even neutral component of the Higgs doublet in the SM. The tree-level potential of \( \phi \) is

\[
V_0(\phi) = -\frac{m_\phi^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4, \quad (10)
\]

where \( \lambda \) is the Higgs quartic self-coupling in the SM and \( m_\phi \) the mass term. Coleman-Weinberg type quantum correction to the potential can be calculated using vacuum diagram by considering the quantum fluctuations around the external field \( \phi \). The one-loop contribution of extra fermion in the SDFDM model to the effective potential is calculated as

\[
V_1^{\text{Ext}} = -\frac{1}{64\pi^2} M_{\chi_1}^4(\phi) \left[ \ln \frac{M_{\chi_1}^2(\phi)}{\mu^2} - 3/2 \right] - \frac{1}{64\pi^2} M_{\chi_2}^4(\phi) \left[ \ln \frac{M_{\chi_2}^2(\phi)}{\mu^2} - 3/2 \right] \quad (11)
\]

where \( M_{\chi_1,\chi_2}^2(\phi) \) are obtained from Eqs. (5) and (6) by replacing \( v \) with \( \phi \). \( \mu \) is the renormalization scale chosen in this calculation. In the limit \( \phi \gg v \), we have approximately \( M_{\chi_1,\chi_2}^2(\phi) \approx y_1^2\phi^2/2, y_2^2\phi^2/2 \).

We arrive at an one-loop effective potential as follows

\[
V_{\text{eff}}(\phi, \lambda_i, \mu) = V_0^{\text{SM}}(\phi, \lambda) + V_1^{\text{SM}}(\phi, \lambda_i, \mu) + V_1^{\text{Ext}}(\phi, \lambda_i, \mu) \quad (12)
\]

where \( \lambda_i \) denotes various parameters in the model. \( V_0^{\text{SM}}(\phi, \lambda) \) is given in Eq. (10). \( V_1^{\text{SM}}(\phi, \lambda_i, \mu) \) is the one-loop contribution to the effective potential in the SM. The effective potential in the SM
is known up to two-loop \cite{15, 16}. \( V_1^{\text{Ext}}(\phi, \lambda_i, \mu) \) is given in Eq. (11).

In the vacuum stability analysis, we must consider the behavior of the effective potential for large external field. That is to say, we must deal with potentially large logarithms of the type \( \log(\phi/\mu) \) for a neutral external field \( \phi \). The standard way to solve the problem is by means of RG equation(RGE). \( V_{\text{eff}} \) satisfies the RGE

\[
\left( \mu \frac{\partial}{\partial \mu} + \beta_i \frac{\partial}{\partial \lambda_i} - \gamma \frac{\partial}{\partial \phi} \right) V_{\text{eff}} = 0, \tag{13}
\]

where \( \beta_i \) is the \( \beta \) function of parameter \( \lambda_i \), and \( \gamma \) the anomalous dimension of scalar field. Straightforward application of this method leads to a solution \cite{2}

\[
V_{\text{eff}}(\mu, \lambda_i, \phi) = V_{\text{eff}}(\mu(t), \lambda_i(t), \phi(t)) \tag{14}
\]

where

\[
\begin{align*}
\mu(t) &= \mu e^t \\
\phi(t) &= e^{\Gamma(t)} \phi
\end{align*} \tag{15}
\]

with

\[
\Gamma(t) = -\int_0^t \gamma(\lambda(t')) dt'
\tag{16}
\]

and \( \lambda_i(t) \) the running coupling determined by the equation

\[
\frac{d\lambda_i(t)}{dt} = \beta_i(\lambda_i(t)), \quad (17)
\]

with the boundary condition \( \lambda_i(0) = \lambda_i \). So the RG improved effective potential can be written by simply substituting \( \mu, \lambda_i, \phi \) in the original effective potential with \( \mu(t), \lambda_i(t), \phi(t) \).

The RG improved effective potential in the SDFDM model is obtained by implementing the substitution mentioned above into Eq. (12). We have

\[
V_{\text{eff}}(\phi, t) = V_0^{\text{SM}}(\phi, t) + V_1^{\text{SM}}(\phi, t) + V_1^{\text{Ext}}(\phi, t), \tag{18}
\]

with

\[
\begin{align*}
V_0^{\text{SM}}(\phi, t) &= -\frac{m_\phi^2(t)}{2} \phi^2(t) + \frac{1}{4} \lambda(t) \phi^4(t), \\
V_1^{\text{SM}}(\phi, t) &= \sum_i \frac{(-1)^i}{64\pi^2} \frac{M_i^4(\phi, t)}{M_i^2(\phi, t)} \left[ \ln \frac{M_i^2(\phi, t)}{\mu^2(t)} - c_i \right], \tag{20}
\end{align*}
\]

\[
\begin{align*}
V_1^{\text{Ext}}(\phi, t) &= \sum_i \frac{(-1)^i}{64\pi^2} \frac{M_{\chi_i}^4(\phi, t)}{M_{\chi_i}^2(\phi, t)} \left[ \ln \frac{M_{\chi_i}^2(\phi, t)}{\mu^2(t)} - 3/2 \right], \tag{21}
\end{align*}
\]

In Eq. (20) the index \( i = H, G, f, W, Z \) runs over SM fields in the loop, and \( c_{H,G,f} = 3/2, c_{W,Z} = 3/2 \).
5/6. In Eq. (21) the index \(i = \chi_1, \chi_2\) runs over extra neutral fermions in the SDFDM model. \(n_i\) is the number of degrees of freedom of the fields. In Eqs. (20), and \(M_i(\phi)\) is given by

\[
M^2_i(\phi, t) = \kappa_i(t)\phi^2(t) - \kappa'_i(t)
\]  

(22)

The values of \(n_i, \kappa_i\) and \(\kappa'_i\) in the SM can be found in Eq. (4) in Ref. [4] in the Landau gauge and in Ref. [17] both in the Fermi gauge and in the \(R_\xi\) gauge. For new contributions in the SDFDM model, we have \(n_i = 1\), and \(M^2_{\chi_1, \chi_2}(\phi, t) \approx y^2_{\chi_1}(t)\phi^2(t)/2, y^2_{\chi_2}(t)\phi^2(t)/2\). In Eqs. (20) and (21), \((-1)^i\) equals to \(\pm 1\). For gauge and scalar bosons \((-1)^i\) take a positive sign, while for fermionic fields it takes a negative sign.

In the limit \(\phi \gg v\), Eq. (18) can be written approximately as follows

\[
V_{\text{eff}}(\phi, t) \approx \frac{\lambda_{\text{eff}}(\phi, t)}{4}\phi^4,
\]

(23)

where \(\lambda_{\text{eff}}\) is an effective coupling. In vacuum stability analysis, we generally take \(\mu(t) = \phi\), so \(\lambda_{\text{eff}}(\phi, t)\) can be written as [10]

\[
\lambda_{\text{eff}}(\phi, t) \approx e^{4\Gamma(t)} \left[ \lambda(t) + \frac{1}{(4\pi)^2} \sum_i N_i \kappa_i^2(t) \left( \log \kappa_i(t)e^{2\Gamma(t)} - c_i \right) \right]
\]  

(24)

The values of coefficients \(N_i, \kappa_i\), and \(c_i\) appearing in Eq. (24) are listed in Table I.

| \(p\) | \(t\) | \(W\) | \(Z\) | \(h\) | \(G^+\) | \(G^0\) | \(C^\pm\) | \(C_Z\) | \(\chi_1\) | \(\chi_2\) |
|---|---|---|---|---|---|---|---|---|---|---|
| \(N_i\) | -12 | 6 | 3 | 1 | 2 | 1 | -2 | -1 | -1 | -1 |
| \(c_i\) | \(\frac{3}{2}\) | \(\frac{5}{6}\) | \(\frac{5}{6}\) | \(\frac{3}{2}\) | \(\frac{3}{2}\) | \(\frac{3}{2}\) | \(\frac{3}{2}\) | \(\frac{3}{2}\) | \(\frac{3}{2}\) | \(\frac{3}{2}\) |
| \(\kappa_i\) | \(\frac{y^2_{\chi_1}}{2}\) | \(\frac{g^2}{4}\) | \(\frac{g^2 + g'^2}{4}\) | \(3\lambda + \frac{\xi_W g^2}{4}\) | \(\xi_Z g^2\) | \(\xi_Z g^2\) | \(\xi_Z g^2\) | \(\frac{g^2}{2}\) | \(\frac{g^2}{2}\) |

TABLE I. The coefficients in Eq. (24) for the background \(R_\xi\) gauge [17]. \(\xi_W\) and \(\xi_Z\) are the gauge-fixing parameters in the background \(R_\xi\) gauge, \(G^+\) and \(G^0\) the goldstone bosons, \(C^\pm\) and \(C_Z\) the ghost fields, \(\chi_1\) and \(\chi_2\) are the dark matter particles in SDFDM model. For \(\xi_W = \xi_Z = 0\), Eq. (24) reproduces the one-loop result in the Landau gauge, and for \(\xi_W = \xi_Z = 1\), we get the result in the \'t Hooft-Feynman gauge.

C. Running parameters in the \(\overline{\text{MS}}\) scheme

To study the vacuum stability of a model at high energy scale, we need to know the value of coupling constants at low energy scale and then run them to the Plank scale according to RGEs. To determine these parameters at low energy scale, the threshold corrections must be taken into account. In this article we work with the modified minimal subtraction(\(\overline{\text{MS}}\)) scheme and use the strategy in [15, 18] to evaluate one-loop threshold corrections and determine the initial values for the RGE. The details of the corrections are summarized in Appendix A. Using these results, we find coupling constants in the \(\overline{\text{MS}}\) scheme at \(\mu = M_t\) scale which is different for the SM and for the SDFDM model. We list some of the results in Table II. Both the change of the Yukawa couplings \(y_{1,2}\) and the change of mass term have an effect on the corrections. We can see in Table III and
Table. II that changing the mass scale of dark matter particles does not give rise to change of the initial parameters as significant as changing Yukawa couplings. Therefore, we will always choose mass parameters as given in Table. II and concentrate on the impact of different Yukawa couplings \( y_{1,2} \) in the remaining part of the article. With these initial values in Table. II, we then run the parameters all the way up to \( M_{Pl} \) scale. For RGE running, we use three-loop SM \( \beta \) functions [15]. We also include one-loop contributions of new particles in the SDFDM model to the \( \beta \) functions of these SM parameters. For new parameters in the SDFDM model, we use one-loop \( \beta \) functions which can be extracted using PyR@TE 2[19]. The results are shown in Appendix B.

We can see that the evolution of \( \lambda(t) \) both in the SM and in the SDFDM model in Fig. 1(a). We see that the \( \lambda_{min} \) in the SDFDM model, the minimum of \( \lambda(t) \) in the RGE running, is negative and is more negative than in the SM. This indicates that in the SDFDM model the EW vacuum is unstable and lifetime of the EW vacuum would be much shorter owing to new physics effects. The greater the Yukawa couplings \( y_1 \) and \( y_2 \), the greater the destabilization effects of the SDFDM model.

As shown in Eq. (24), \( \lambda_{eff} \) differs from \( \lambda \). In the SM, the difference \( \lambda_{eff} - \lambda \) is always positive and is negligible near the Planck scale as shown in Ref. [10]. The situation is different in the SDFDM model. As we can see in Fig. 1(b), \( \lambda_{eff} - \lambda \) is not negligible in the SDFDM model. In fact, \( \lambda_{eff} \) is suppressed by the \( e^{4\Gamma(t)} \) factor in Eq. (24) which comes from the contribution of the anomalous dimension. As we can see, the instability scale \( \Lambda_I \), the energy scale at which \( \lambda_{eff}(t) \) or \( \lambda(t) \) becomes zero, is larger when determined by \( \lambda_{eff}(t) \). This is the case both in the SM and in the SDFDM model.

### TABLE II. Initial values in \( \overline{\text{MS}} \) scheme for RGE running

| \( \mu = M_t \) | \( \lambda \) | \( y_1 \) | \( y_2 \) | \( gy \) |
|---------------|-----|-----|-----|-----|
| SM_{LO} | 0.12917 | 0.99561 | 0.65294 | 0.34972 |
| SM_{NNLO} | 0.12604 | 0.93690* | 0.64779 | 0.35830 |
| SDFDM_{BMP1}^NLO | 0.12549 | 0.93526* | 0.64573 | 0.35752 |
| SDFDM_{BMP2}^NLO | 0.12554 | 0.93368* | 0.64574 | 0.35630 |
| SDFDM_{BMP3}^NLO | 0.12586 | 0.93269* | 0.64573 | 0.35553 |
| SDFDM_{BMP4}^NLO | 0.13126 | 0.92744* | 0.64573 | 0.35144 |

The parameters are renormalized at the top pole mass \( M_t \) scale in the \( \overline{\text{MS}} \) scheme. BMP1: \( y_1 = y_2 = 0.25, M_S = 1000 \text{ GeV}, M_D = 1000 \text{ GeV} \); BMP2: \( y_1 = y_2 = 0.35, M_S = 1000 \text{ GeV}, M_D = 1000 \text{ GeV} \); BMP3: \( y_1 = y_2 = 0.4, M_S = 1000 \text{ GeV}, M_D = 1000 \text{ GeV} \); BMP4: \( y_1 = y_2 = 0.6, M_S = 1000 \text{ GeV}, M_D = 1000 \text{ GeV} \). The superscript * indicates that the NNNLO pure QCD effects are also included. BMPs means benchmark points.

**III. VACUUM STABILITY AND LIFETIME OF THE VACUUM**

As we have seen in the last section, RG improvement to the effective potential can be quite significant in SDFDM model. We need to consider the effects of RG improved effective potential in the calculation of vacuum decay rate. The decay rate of the false vacuum can be computed by
Effects of different masses on initial values

| $\mu = M_t$ | $\lambda$ | $y_1$ | $y_2$ | $g_Y$ |
|------------|------|-----|-----|-----|
| $M_S = M_D = 800$ GeV | 0.12564 | 0.93402* | 0.64599 | 0.35650 |
| $M_S = M_D = 1000$ GeV | 0.12554 | 0.93368* | 0.64574 | 0.35630 |
| $M_S = M_D = 1200$ GeV | 0.12546 | 0.93340* | 0.64552 | 0.35613 |

TABLE III. $y_1 = y_2 = 0.35$ for all three cases with different masses of the new particles. The superscript * indicates that the NNNLO pure QCD effects are also included.

![Diagram](image1)

(a) $\lambda(t)$ up to $M_{Pl}$ for the SM and for various Yukawa couplings in the SDFDM model; (b) Running $\lambda(t)$ and $\lambda_{eff}(t)$ up to $M_{Pl}$ scale for the SDFDM model.

Finding a bounce solution to the field equations in Euclidean space [5–7]. For a potential $U(\phi)$, the decay rate per unit time per unit volume, $\Gamma_{t}$, can be expressed as

$$\Gamma_{t} = A_t e^{-S_{cl}}.$$  \hspace{1cm} (25)

where $S_{cl}$ is the Euclidean action of bounce solution and $A_t$ is the quantum correction. For fluctuation of $\phi$ field, $A_t$ is given as

$$A_t = \frac{S^2_{cl}}{4\pi^2} \left| \det \left[ -\partial^2 + U''(\phi_B) \right] \right|^{-1/2},$$  \hspace{1cm} (26)

where $\partial^2$ is operated in Euclidean space and $\det'$ the determinant omitting the zero mode contribution. $\phi_0$ is the field value in the false vacuum which can be taken as zero as an approximation. $\phi_B$ refers to the spherical symmetric bounce solution to the Euclidean field equation. $\phi_B$ satisfies

$$-\partial^2 \phi_B + U'(\phi_B) = -\frac{d^2 \phi_B}{dr^2} - \frac{3}{r} \frac{d \phi_B}{dr} + U'(\phi_B) = 0,$$  \hspace{1cm} (27)

where $U'$ means derivative of $U$ with respect to the field. In the case under consideration, $\phi/\sqrt{2}$ is the CP-even neutral component of the Higgs doublet in the SM. If there are other particles...
coupled to bounce field, their contributions to the determinants should also be taken into account, as happens in the SM and in the SDFDM model considered in this article.

For a potential \( U(\phi) = \frac{\lambda}{4} \phi^4 \) with a negative \( \lambda \), the calculation leads to [8]

\[
S_{cl} = \frac{8\pi^2}{3|\lambda|},
\]  

(28)

In the SM, there is a mass for the Higgs field. The Higgs mass can safely be neglected in this calculation because the bounce solution is dominated by the behavior at large field values, so that potential can be written approximately as a \( \phi^4 \) form. In quantum theory, \( \lambda \) is a quantity running with energy scale. To simplify calculation, \( \lambda \) can be taken at a sufficiently large energy scale \( M \) so that \( \lambda(M) \) is negative and varies slowly with energy scale. So \( S_{cl} = \frac{8\pi^2}{|\lambda(M)|} \) in this case. It has been shown that this scale dependence in \( S_{cl} \) in the false vacuum decay rate is cancelled when taking into account one-loop correction from the determinant [8, 9].

To fully take into account quantum corrections, we need to consider the effective action. As long as the field varies slowly with space and time, we can compute the effective action using derivative expansion [1]. Neglecting terms with higher derivative, we can write the effective action in Euclidean space for external field \( \phi \) as

\[
S_{eff}[\phi] = \int d^4x \left[ \frac{1}{2} (\partial_\mu \phi)^2 Z_2(\phi) + V_{eff}(\phi) \right]
\]  

(29)

\( Z_2 \) can be obtained from the \( p^2 \) terms in the Feynman diagrams as shown in Appendix. C. It is renormalized to make \( Z_2(\phi = 0) = 1 \) which makes the kinetic term going back to the standard form when there is no external field. The results in the ’t Hooft-Feynman gauge are summarized in Table. VI for the SM, and in Table. VII for the new contributions in SDFDM model. In the large \( \phi \) limit, we can simplify the result. We obtain \( Z_2 \) for the SM in Eq. (C15), and \( Z_2 \) for the SDFDM model in Eq. (C17). As we can see, the explicit dependences on \( \phi \) are cancelled in these results.

RG improvement of the kinetic term can be studied similar to the effective potential. The kinetic term in the effective action is the one-particle irreducible self-energy \( \Gamma_2 \). It satisfies the RG equation

\[
\left( \mu \frac{\partial}{\partial \mu} + \beta_i \frac{\partial}{\partial \lambda_i} - 2\gamma \right) \Gamma_2(\phi) = 0.
\]  

(30)

The equation can be solved in a way similar to solving \( V_{eff}(\phi) \). Solving this equation gives rise to \( Z_2(\phi, t) \) with all parameters \( \lambda_i \) in \( Z_2(\phi) \) substituted by \( \lambda_i(t) \) and with an \( e^{2\Gamma(t)} \) factor in the kinetic term. So we arrive at an Euclidean action

\[
S = \int d^4x \left[ e^{2\Gamma(t)} Z_2(\phi, t) \frac{1}{2} (\partial_\mu \phi)^2 + e^{4\Gamma(t)} \frac{\dot{\lambda}(t)}{4} \phi^4 \right],
\]  

(31)
FIG. 2. (a) The behavior of $Z_2$ at large energy scale in the SM; (b) The behavior of $Z_2$ at large energy scale with $y_1 = y_2 = 0.35$ and $M_S = M_D = 1000$ GeV in the SDFDM model.

where $\tilde{\lambda}$ is only different from Eq. (24) by a factor $e^{4\Gamma(t)}$, that is

$$\tilde{\lambda} = \lambda(t) + \frac{1}{(4\pi)^2} \sum_i N_i \kappa_i^2(t) \left( \log \kappa_i(t) e^{2\Gamma(t)} - c_i \right)$$

(32)

The Euclidean equation of bounce solution becomes

$$-Z_2 \partial^2 \tilde{\phi}_B + \tilde{\lambda} \tilde{\phi}_B^3 e^{2\Gamma(t)} = 0.$$  (33)

From the bounce action in Eq. (28), one can immediately deduce that the bounce action becomes

$$S_{cl} = e^{2\Gamma} Z_2 \times \frac{8\pi^2}{3|\lambda| e^{2\Gamma}/Z_2} = (Z_2)^2 \frac{8\pi^2}{3|\lambda|}.$$  (34)

$S_{cl}$ depends on $Z_2$ but is independent of the $e^{\Gamma(t)}$ factor. Similar to obtaining result in Eq. (28), running parameters in Eqs. (33) and (34) are understood to be at an arbitrary large energy scale $M$. The leading dependence on $M$ in decay rate would be cancelled by including quantum correction from the determinant, similar to analysis in Ref. [8, 9]

Similarly, one can find that $(S_{cl})^2$ factor in Eq. (26), which comes from the zero mode contribution, becomes $[8\pi^2/(3|\lambda|e^{2\Gamma}/Z_2)]^2$. The ratio of determinant in Eq. (26) becomes $|\text{det}'[-e^{2\Gamma} Z_2 \partial^2 + 3\tilde{\lambda} e^{4\Gamma} \tilde{\phi}_B^3]/|\text{det}[-e^{2\Gamma} Z_2 \partial^2]|^{-1/2}$ which equals to $|\text{det}'[-\partial^2 + 3(\tilde{\lambda}/Z_2) e^{2\Gamma} \tilde{\phi}_B^3]/|\text{det}[-\partial^2]|^{-1/2} \times (e^{2\Gamma} Z_2)^2$ when including effects omitting four zero modes. It’s easy to see that if taking $\phi_B = e^\Gamma \hat{\phi}_B$ the non-zero eigenvalues of operator $-\partial^2 + 3(\tilde{\lambda}/Z_2) e^{2\Gamma} \hat{\phi}_B^3$ for $\hat{\phi}_B$ satisfying Eq. (33) would be the same of the operator $-\partial^2 + 3(\tilde{\lambda}/Z_2) \phi_B^3$ for $\phi_B$ satisfaction

$$-Z_2 \partial^2 \phi_B + \tilde{\lambda} \phi_B^3 = 0.$$  (35)

So eventually we find that the decay rate is again expressed by Eq. (25) but with $S_{cl}$ expressed by
\[ A_t = \frac{S_{cl}^2}{4\pi^2} \left| \frac{\det\left[-\partial^2 + 3\left(\bar{\lambda}/Z_2\right)\phi_B^2\right]}{\det[-\partial^2]} \right|^{-1/2}, \]

in which \( \phi_B \) satisfies Eq. (35). We see that the final result depends on \( Z_2 \) but does not depend on \( e^{\Gamma(t)} \). The factor \( e^{\Gamma(t)} \) comes from the wave function renormalization but can be associated with an arbitrariness in relating \( \phi \) with a renormalization scale. So it is not surprising to see that the physical result does not depend on it. One can actually re-define the external field \( \phi \) from the very beginning in the Euclidean action in Eq. (31) in path integral and arrive at this conclusion.

\( Z_2 \) is a running parameter. As we can see in Fig. 2, \( Z_2 \) has a small deviation from unity at high energy scale, both in the SM and in the SDFDM model. So false vacuum decay rate is mainly controlled by the behavior of \( \bar{\lambda}(t) \). In the SDFDM model, the scale dependence appearing in \( S_{cl} \) is also cancelled by one-loop contribution from the determinant. This energy scale can be taken conveniently at the scale of the bounce \( \Lambda_B \) so that \( S_{cl}(\Lambda_B) \) takes care of the major contribution in the exponential [8–10]. \( \Lambda_B \) is determined as the scale at which the vacuum decay rate is maximized. In practice, this roughly corresponds to the scale at which the negative \( \bar{\lambda}(\Lambda_B) \) is at the minimum. If \( \Lambda_B > M_{Pl} \), we can only obtain a lower bound on the tunneling probability by setting \( \bar{\lambda}(\Lambda_B) = \bar{\lambda}(M_{Pl}) \).

In this way, the vacuum decay probability \( P_0 \) in our universe up to the present time can be expressed as [10, 20]

\[ P_0 = 0.15\frac{\Lambda_B^4}{H_0^2} e^{-S(\Lambda_B)} \]

where \( H_0 = 67.4 \text{ km sec}^{-1} \text{ Mpc} \) is the Hubble constant at the present time. \( S(\Lambda_B) \) is the action of the bounce of size \( R = \Lambda_B^{-1} \).

In vacuum stability analysis, we say the vacuum is stable if the potential at large \( \phi \) keeps positive. This requires \( \bar{\lambda} > 0 \) for energy scale up to the Planck scale. If \( \bar{\lambda} < 0 \) at an energy scale but with \( P_0 < 1 \), it means that the lifetime of the false vacuum is greater than the age of the Universe. In this case we call the vacuum is metastable. Other scenarios can be similarly defined.
In summary, we list them as follows:

- Stable: $\bar{\lambda} > 0$ for $\mu < M_{\text{Pl}}$;
- Metastable: $\bar{\lambda}(\Lambda_B) < 0$ and $P_0 < 1$;
- Unstable: $\bar{\lambda}(\Lambda_B) < 0$ and $P_0 > 1$;
- Non-perturbative: $|\lambda| > 4\pi$ before the Planck scale

Note that we classify states of EW vacuum in a way different from Ref. [10, 13], since $\bar{\lambda}(t)$ differs from $\lambda(t)$ significantly in the SDFDM model. We further note that the effective action we have used has imaginary part. The present work actually works on real part of the effective action and discusses the effect of the distortion of the bounce solution in the presence quantum correction to the effective action. A discussion on the effect of the imaginary part of the effective action would be interesting, e.g. as in Ref. [21]. In the present article, we will not elaborate on this topic.

Now we come to discuss the tunneling probability. As shown in Eqs. (25), (34) and (36), the false vacuum decay rate depends on $Z_2$ and $\bar{\lambda}(t)$ when including one-loop correction to the effective action. As mentioned before, the decay rate is mainly controlled by the behavior of $\bar{\lambda}(t)$. We first compare $\bar{\lambda}(t)$ and $\lambda(t)$ in the SM. As shown in Fig. 3(a), $\bar{\lambda}(t)$ and $\lambda(t)$ are very close at high energy scale in the SM. They both approach the minimum before the Planck scale. Both the values of their minima and the energy scales of the minima are very close, as can be seen in Table IV. This means that the one loop corrections to effective potential has little effects on tunneling probability in the SM. In the SDFDM model, the situation can be different. As can be seen in Fig. 3(b), $\bar{\lambda}(t)$ and $\lambda(t)$ at high energy scale are not as close as in the SM. In this plot, $\bar{\lambda}(t)$ and $\lambda(t)$ all approach their minima before the Planck scale. But their values at the minima and the energy scales of the minima are not as close as in the SM, as can be seen in Table IV. In Fig 4, we give more plots with larger $y_1$ and $y_2$. In these cases, the difference between $\bar{\lambda}(t)$ and $\lambda(t)$ is more significant. The larger the Yukawa coupling $y_1$ and $y_2$, the larger the difference. In these cases, both $\bar{\lambda}(t)$ and $\lambda(t)$ have no minimum for energy scale below the Planck scale. The energy scale of bounce, $\Lambda_B$, is chosen as the Planck scale for these two cases. We note that the positive sign of $\bar{\lambda} - \lambda$ shown in Fig 4 means that the lifetime calculated using $\bar{\lambda}$ in these plots is longer than that computed using $\lambda$.

In Table. IV, we list more numerical results for the SM and for some benchmark points in the SDFDM model. As a comparison, we also list results just using $\lambda(t)$. We can see that using $\bar{\lambda}(t)$ and $Z_2$ in the effective action leads to some differences in the probability of false vacuum decay. For SM case, we can see that the lifetime of EW vacuum computed using effective action is shorter than that just using $\lambda(t)$ although $\bar{\lambda}(t)$ and $\lambda(t)$ are very close at high energy scale. This is caused mainly by the presence of $Z_2$ in the effective action. In the SM, $(Z_2)^2$ term in Eq. (34) is about 0.9 which makes $S_{cl}$ smaller and leads to a larger decay rate. For SDFDM model, we find that in the region that $y_1$ and $y_2$ are less than about 0.3, the situation is similar to the SM case. When $y_1$ and $y_2$ are larger, the $Z_2$ factor becomes greater than 1. In this case the difference between $\bar{\lambda}$
FIG. 3. (a) Comparison between $\lambda$ and $\tilde{\lambda}$ in the SM; (b) Comparison between $\lambda$ and $\tilde{\lambda}$ with $y_1 = y_2 = 0.25$ and $M_S = M_D = 1000$ GeV in the SDFDM model.

FIG. 4. Comparison between $\lambda(t)$ and $\tilde{\lambda}(t)$ in the SDFDM model. (a) $y_1 = y_2 = 0.35$ and $M_S = M_D = 1000$ GeV; (b) $y_1 = y_2 = 0.4$ and $M_S = M_D = 1000$ GeV.

and $\lambda$ is also significant. Therefore, both the $Z_2$ factor and the increasing value of $\tilde{\lambda} - \lambda$ makes the lifetime calculated using effective action longer than that computed just using $\lambda$.

In Fig. 5, we compare the two ways of obtaining the tunneling probability. The green (blue) region indicates that the EW vacuum is metastable (unstable), and the red region means that the EW vacuum is non-perturbative. We find that the one-loop effect on effective potential slightly enlarges the parameter space for the vacuum to be metastable.
IV. CONCLUSION

In summary, we have studied the one-loop Coleman-Weinberg type effective potential of the Higgs boson in a single-doublet fermion dark matter extension of the SM. We have calculated the threshold effect of these fermions in this model beyond the SM and have studied the RG running of parameters in the \( \overline{\text{MS}} \) scheme. We have studied the RG improvement to the effective potential. We have studied the vacuum stability using the RG improvement effective potential. We find that in this model beyond the SM the RG improved effective potential at high energy scale can be quite different from the potential just using the running Higgs quartic self-couplings which is the quantity usually considered in literature. In the SM, the RG improved effective potential at high energy scale is accidentally close to the potential just using the running Higgs quartic self-couplings.

Using the method of derivative expansion, we have studied the quantum correction to the effective action. We have calculated the renormalization on the kinetic term in the effective action in the case with external field. We have studied the RG improvement of the kinetic term. Using the RG improved kinetic term and the RG improved effective potential, we calculate the decay rate of the false vacuum. We find that the factor arising from the anomalous dimension which appears in the kinetic term and the effective potential cancels in the decay rate. Taking all these considerations into account, we find that the decay rate of false vacuum is slightly changed by the effective action.

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Appendix A: Threshold effect and parameters in the $\overline{\text{MS}}$ scheme

1. General strategy for one-loop matching

To study the vacuum stability of a model at high energy scale, we need to know the value of coupling constants at low energy scale and then run them to Plank scale according to RGEs. To determine these parameters at low energy scale, the threshold corrections must be taken into account. In this article we use the strategy in [15, 18] to evaluate one-loop corrections. All the loop calculation are performed in the $\overline{\text{MS}}$ scheme in which all the parameters are gauge invariant and have gauge-invariant renormalisation group equations [22].

A parameter in $\overline{\text{MS}}$ scheme, e.g. $\theta(\mu)$, can be obtained from renormalized parameter $\theta$ in physical scheme which is directly related to physical observables. The connection between $\theta$ and $\theta(\mu)$ to one-loop order, can be found by noting that the unrenormalized $\theta_0$ is related to the renormalized couplings by

$$\theta_0 = \theta - \delta \theta = \theta(\mu) - \delta \theta_{\overline{\text{MS}}}$$ \hspace{1cm} (A1)

where $\delta \theta$ and $\delta \theta_{\overline{\text{MS}}}$ are the corresponding counterterms. By definition $\delta \theta_{\overline{\text{MS}}}$ subtracts only the divergent part proportional to $1/\epsilon + \gamma - \ln(4\pi)$ in dimensional regularization with $d = 4 - 2\epsilon$ being the space-time dimension. Since the divergent parts in the $\delta \theta$ and $\delta \theta_{\overline{\text{MS}}}$ counterterms are of the same form, $\theta(\mu)$ can be rewritten as

$$\theta(\mu) = \theta - \delta \theta|_{\text{fin}}$$ \hspace{1cm} (A2)

where the subscript fin denotes the finite part of the quantity $\delta \theta$, obtained after subtracting the terms proportional to $1/\epsilon + \gamma - \ln(4\pi)$. Difference at two-loop order has been neglected in this expression.

The physical parameters which would be used in Eq. (A2), such as $\mu^2$ and $\lambda$, the quadratic and quartic couplings in the Higgs potential, the vacuum expectation value $v$, the top Yukawa coupling $y_t$, the gauge couplings $g_2$ and $g_Y$ of SU(2)$_L \times$ U(1)$_Y$ group, can be determined from physical observables, such as the pole mass of Higgs boson ($M_h$), the pole mass of the top quark ($M_t$), the pole mass of the Z boson ($M_Z$), the pole mass of the W boson ($M_W$), and the Fermi constant ($G_\mu$). These physical observables are listed in Table. V. If knowing the corresponding counterterms in the physical scheme, the $\overline{\text{MS}}$ couplings are then obtained using (A2). For example, if knowing $\delta \lambda$, we then obtain $\lambda(\mu)$ in the $\overline{\text{MS}}$ scheme. More details are explained as follows.

We follow Ref. [18] to fix the notation. We write the classical Higgs potential in bare quantities as

$$V = -\mu_0^2 \Phi^\dagger \Phi + \lambda_0 \left( \Phi^\dagger \Phi \right)^2$$ \hspace{1cm} (A3)
Input values of SM observables

| Observables | Values               |
|-------------|----------------------|
| $M_W$       | 80.384 ± 0.014 GeV   |
| $M_Z$       | 91.1876 ± 0.0021 GeV |
| $M_h$       | 125.15 ± 0.24 GeV    |
| $M_t$       | 173.34 ± 0.76 GeV    |
| $v = (\sqrt{2}G_\mu)^{-1/2}$ | 246.21971 ± 0.00006 GeV |
| $\alpha_3(M_Z)$ | 0.1184 ± 0.0007     |

Table V. Input values of physical observables used to fix the SM fundamental parameters $\lambda$, $m$, $y_t$, $g_2$, and $g_Y$. $M_W$, $M_Z$, $M_h$, and $M_t$ are the pole masses of the W boson, of the Z boson, of the Higgs boson, and of the top quark, respectively. $G_\mu$ is the Fermi constant for $\mu$ decay, and $\alpha_3$ is the $SU(3)_c$ gauge coupling at the scale $\mu = M_Z$ in the $\overline{MS}$ scheme.

with

$$\Phi = \left( \sqrt{\frac{1}{2}} \left( \phi_1 + i\phi_2 + v_0 \right) \right)$$  \hspace{1cm} (A4)

Setting $\lambda_0 = \lambda - \delta \lambda, v_0 = v - \delta v, \mu_0^2 = \mu^2 - \delta \mu^2$, where $\lambda, v$ and $\mu$ are regarded as renormalized quantities, we write

\[ V = V(r) - \delta V \]  \hspace{1cm} (A5)

with

\[ V(r) = \lambda \left[ \phi^+ \phi^- \left( \phi^+ \phi^- + \phi_1^2 + \phi_2^2 \right) + \frac{1}{4} \left( \phi_1^2 + \phi_2^2 \right)^2 \right] + \lambda v \phi_1 \left[ \phi_1^2 + \phi_2^2 + 2\phi^+ \phi^- \right] + 2\lambda v^2 \frac{1}{2} \phi_1^2 \]  \hspace{1cm} (A6)

and

\[ \delta V = \delta \lambda \left[ \left( \phi^+ \phi^- \right) \left( \phi^+ \phi^- + \phi_1^2 + \phi_2^2 \right) + \frac{1}{4} \left( \phi_1^2 + \phi_2^2 \right)^2 \right] + \left[ \lambda \delta v + v \delta \lambda \right] \phi_1 \left[ \phi_1^2 + \phi_2^2 + 2\phi^+ \phi^- \right] + \delta \tau \left[ \phi^+ \phi^- + \frac{1}{2} \phi_2^2 \right] + \delta M_h^2 \frac{1}{2} \phi_1^2 + v \delta \tau \phi_1 \]  \hspace{1cm} (A7)

where

\[ \delta M_h^2 = 3v^2 \delta \lambda + 6\lambda v \delta v - \delta \mu^2, \]  \hspace{1cm} (A8)

\[ \delta \tau = v^2 \delta \lambda + 2\lambda v \delta v - \delta \mu^2. \]  \hspace{1cm} (A9)

$v$ is determined at tree-level by $G_\mu$ as shown in Table. V.
In order to determine $\delta \lambda$, $\delta v$ and $\delta \mu^2$ we need three constraints. The strategy is to adjust $\delta \tau$ so that the $v \delta \tau \phi_1$ term in Eq. A7 cancels the tadpole diagrams. Calling $iT$ the sum of the tadpole diagrams with the external legs extracted, we have the condition

$$\delta \tau = -T/v.$$  
(A10)

A second constraint is conveniently obtained by demanding that the coefficient of the term proportional to $\frac{1}{2} \phi_1^2$ in $V(r)$ be the physical mass of the Higgs boson. So we have

$$M_h^2 = 2\lambda v^2$$  
(A11)

and $\delta M_h^2$ is fixed by condition of on-shell renormalization, i.e.

$$\delta M_h^2 = \text{Re} \Pi_{hh} (M_h^2),$$  
(A12)

where $\Pi_{hh} (M_h^2)$ is the Higgs boson self-energy evaluated on shell. A third constraint is provided by Eq. (9b) of Ref. [23]

$$\delta M_W^2 = \text{Re} \Pi_{ww} (M_W^2),$$  
(A13)

where $\Pi_{ww} (M_W^2)$ is the W boson self-energy evaluated on shell. Recalling that the W-mass counterterm is given by [23]

$$\delta M_W^2 = \frac{1}{2} \left( v^2 g_2 \delta g_2 + g_2^2 v \delta v \right),$$  
(A14)

$\delta v$ is obtained using this expression with $\delta g_2$ known from other condition which can be found in Eq. (28a) of [23]. Putting $\delta v$ and Eqs. (A12) and (A14) into (A8) and (A9), one can then obtain $\delta \lambda$ and $\delta \mu^2$. They are as follows:

$$\delta \mu^2 = \frac{1}{2} \left[ \text{Re} \Pi_{hh} (M_h^2) + 3T/v \right],$$  
(A15)

$$\delta \lambda/\lambda = \frac{\left[ \text{Re} \Pi_{hh} (M_h^2) + T/v \right]}{M_h^2} - \frac{\text{Re} \Pi_{ww} (M_W^2)}{M_W^2} + \frac{2\delta g_2}{g_2},$$  
(A16)

$$\delta v/v = \text{Re} \Pi_{ww} (M_W^2) / \left( 2M_W^2 \right) - \frac{\delta g_2}{g_2},$$  
(A17)

We can get the expressions of the counterterms of the other parameters in a similarly way.

Ignoring the contribution of higher order, we list the one-loop results of counterterms as follows

$$\delta^{(1)} \lambda = \frac{G_\mu}{\sqrt{2}} M_h^2 \left\{ \Delta r_0^{(1)} + \frac{1}{M_h^2} \left[ \frac{T^{(1)}}{v} + \text{Re} \Pi_{hh} (M_h^2) \right] \right\},$$  
(A18)

$$\delta^{(1)} y_t = 2 \left( \frac{G_\mu}{\sqrt{2}} M_t^2 \right)^{1/2} \left( \frac{\text{Re} \Pi_{tt} (M_t^2)}{M_t} + \frac{\Delta r_0^{(1)}}{2} \right),$$  
(A19)

$$\delta^{(1)} g_2 = \left( \sqrt{2} G_\mu \right)^{1/2} M_W \left( \frac{\text{Re} \Pi_{ww} (M_W^2)}{M_W^2} + \Delta r_0^{(1)} \right),$$  
(A20)
\[
\delta^{(1)} g_Y = \left( \sqrt{2} G_\mu \right)^{1/2} \sqrt{M_Z^2 - M_W^2} \left( \frac{\text{Re} \Pi_{zz}(M_Z^2) - \text{Re} \Pi_{ww}(M_W^2)}{M_Z^2 - M_W^2} + \Delta r_0^{(1)} \right), \tag{A21}
\]

where superscripts 1 in these equations indicate that they are results at one-loop order. \( \Delta r_0^{(1)} \) in the above equations can be written as a sum of several terms \[15\]

\[
\Delta r_0^{(1)} = V_W^{(1)} - \frac{A_{WW}^{(1)}}{M_W^2} + \sqrt{2} G_\mu B_W^{(1)} + \mathcal{E}^{(1)}, \tag{A22}
\]

where \( A_{WW} \) is the \( W \) boson self-energy at zero momentum, \( V_W \) the vertex contribution in the muon decay process, \( B_W \) the box contribution, \( \mathcal{E} \) a term due to the renormalization of external legs. They are all computed at zero external momentum. Thus we eventually get the \( \overline{\text{MS}} \) parameter to one-loop order as follows \[13, 15\]:

\[
\lambda(\mu) = \frac{G_\mu}{\sqrt{2}} M_H^2 - \delta^{(1)} \lambda|_{\text{fin}}, \tag{A23}
\]

\[
y_t(\mu) = 2 \left( \frac{G_\mu}{\sqrt{2}} M_t^2 \right)^{1/2} - \delta^{(1)} y_t|_{\text{fin}}, \tag{A24}
\]

\[
g_2(\mu) = 2 \left( \sqrt{2} G_\mu \right)^{1/2} M_W - \delta^{(1)} g_2|_{\text{fin}}, \tag{A25}
\]

\[
g_Y(\mu) = 2 \left( \sqrt{2} G_\mu \right)^{1/2} \sqrt{M_Z^2 - M_W^2} - \delta^{(1)} g_Y|_{\text{fin}}. \tag{A26}
\]

2. \( \overline{\text{MS}} \) parameters in the SDFDM model

To determine the initial values of running couplings, we use the equations given in the last section. Since the threshold corrections have been done to NNLO in the SM, we only need to calculate the contribution of extra fermions in the SDFDM model. All the relevant Feynman diagrams for computing \( \delta^{(1)} \lambda|_{\text{fin}} \) with extra fermions are listed in Fig. 6.

As dark matter particles in SDFDM model do not couple to SM leptons, Eq. \( (A22) \) can be further simplified as

\[
\Delta r_0 = - \frac{A_{WW}}{M_W^2}. \tag{A27}
\]

Summing over all the loop contributions and using the matching conditions, we get coupling constants in the \( \overline{\text{MS}} \) scheme at \( M_t = 173 \) GeV energy scale and for the SDFDM model respectively.

We summarize here the one-loop corrections to \( \lambda \) from new particles in SDFDM model by using Eq. \( (A18) \). We write \( \delta^{(1)} \lambda_{\text{SDFDM}} \) in terms of finite parts of the the Passarino-Veltman functions

\[
A_0(M) = M^2 \left( 1 - \ln \frac{M^2}{p^2} \right), \quad B_0(M_1, M_2, p) = - \int_0^1 \ln \frac{xM_1^2 + (1 - x)M_2^2 - x(1 - x)p^2}{p^2} dx. \tag{A28}
\]
FIG. 6. Contributions of extra fermions to the self-energy for (a) the Higgs boson, (b) Z boson, and (d)(e) W bosons, as well as to (c) the tadpole of the Higgs boson. $\chi_{(1,2)}$ are dark sector fermions, L(R) means the chirality in the vertex.

The one-loop result is

$$\delta^{(1)} \lambda|_{fin} = \frac{G_\mu}{\sqrt{2}(4\pi)^2} \left\{ y_A^2 [4A_0(M_{\chi^0_1}) - 2(M_h^2 - 4M_{\chi^0_1}^2)B_0(M_{\chi^0_1}, M_{\chi^0_2}, M_h)] 
+ y_B^2 [4A_0(M_{\chi^0_2}) - 2(M_h^2 - 4M_{\chi^0_2}^2)B_0(M_{\chi^0_2}, M_{\chi^0_1}, M_h)] 
+ 2y_C^2 [A_0(M_{\chi^0_1}) + A_0(M_{\chi^0_2}) - (M_h^2 - M_{\chi^0_1}^2 - M_{\chi^0_2}^2)B_0(M_{\chi^0_1}, M_{\chi^0_2}, M_h)] 
+ 8y_C y_D M_{\chi^0_1} M_{\chi^0_2} B_0(M_{\chi^0_1}, M_{\chi^0_2}, M_h) \right\} 
+ \frac{G_\mu}{\sqrt{2}(4\pi)^2 v} \left[ -4y_A M_{\chi^0_1} A_0(M_{\chi^0_1}) - 4y_B M_{\chi^0_2} A_0(M_{\chi^0_2}) \right] 
+ \frac{G_\mu}{\sqrt{2}} M_h^2 \Delta r^{(1)}_0 |_{fin}$$ (A29)
where \( y_{A,B,C,D} \) has been given in the text following Eq. (9) and

\[
\Delta r_0^{(1)} \bigg|_{\text{fin}} = \frac{1}{(4\pi v)^2} \left\{ \sin^2 \theta_L + \sin^2 \theta_R \left[ \frac{2M^2_{\chi_1^0}}{M_{\chi^-} - M_{\chi_1^0}^2} A_0(M_{\chi^-}) - \frac{2M^2_{\chi_2^0}}{M_{\chi^-} - M_{\chi_2^0}^2} A_0(M_{\chi^-}) + M^2_{\chi^-} + M_{\chi_1^0}^2 \right] \right. \\
+ 8 \sin\theta_L \sin\theta_R \left[ \frac{M_{\chi^0} M_{\chi^-}}{M_{\chi^-} - M_{\chi_1^0}^2} \left( A_0(M_{\chi^-}) - A_0(M_{\chi_1^0}^0) \right) \right] \\
+ (\cos^2 \theta_L + \cos^2 \theta_R) \left[ \frac{2M^2_{\chi_2^0}}{M_{\chi^-} - M_{\chi_2^0}^2} A_0(M_{\chi_2^0}) - \frac{2M^2_{\chi^-}}{M_{\chi^-} - M_{\chi_2^0}^2} A_0(M_{\chi^-}) + M^2_{\chi^-} + M_{\chi_2^0}^2 \right] \\
\left. + 8 \cos\theta_L \cos\theta_R \left[ \frac{M_{\chi^0} M_{\chi^-}}{M_{\chi^-} - M_{\chi_2^0}^2} \left( A_0(M_{\chi^-}) - A_0(M_{\chi_2^0}) \right) \right] \right\}
\]

(A30)

Plugging Eq. (A29) into Eq. (A23) we obtain \( \lambda \) at one-loop order in the SDFDM model. Contributions of extra fermions to \( \delta^{(1)} y_{|\text{fin}}, \delta^{(1)} g_{2|\text{fin}} \) and \( \delta^{(1)} y_{Y|\text{fin}} \) can be similarly obtained. Plugging them into Eqs. (A24), (A25) and (A26) we obtain relevant parameters at one-loop order in the SDFDM model. Using these parameters in the \( \overline{\text{MS}} \) scheme, we then carry out calculation of the effective action in the \( \overline{\text{MS}} \) scheme.

### Appendix B: One-loop \( \beta \) and \( \gamma \) function in the SDFDM model

The \( \beta \)-function and the anomalous dimension can be decomposed into two parts:

\[
\beta^{\text{total}} = \beta^{\text{SM}} + \beta^{\text{SDFDM}}, \quad \gamma^{\text{total}} = \gamma^{\text{SM}} + \gamma^{\text{SDFDM}}
\]

(B1)

where \( \beta^{\text{SM}} \) and \( \gamma^{\text{SM}} \) are the \( \beta \) function and the anomalous dimension in the SM, while \( \beta^{\text{SDFDM}} \) and \( \gamma^{\text{SDFDM}} \) are the contributions from new particles in the SDFDM model.

The \( \beta \) functions in the SM are known to three-loop [15]. In this article we focus on the SDFDM model with Dirac type dark matter. Here we show one-loop contributions of new particles in the SDFDM model to the \( \beta \) functions of the SM parameters and the one-loop \( \beta \) functions of new parameters in the SDFDM model. They can be extracted using the python tool PyR@TE 2[19]. They are as follows.

The \( \beta \) functions of the SM parameters receive one-loop contributions of new particles in the SDFDM model as follows

\[
\beta^{\text{SDFDM}}(g_1) = \frac{1}{(4\pi)^2} \left( \frac{3}{5} \right) g_1^3,
\]

(B2)

\[
\beta^{\text{SDFDM}}(g_2) = \frac{1}{(4\pi)^2} g_2^3,
\]

(B3)

\[
\beta^{\text{SDFDM}}(y_\tau) = \frac{1}{(4\pi)^2} \left( y_1^2 + y_2^2 \right) y_\tau,
\]

(B4)

\[
\beta^{\text{SDFDM}}(y_b) = \frac{1}{(4\pi)^2} \left( y_1^2 + y_2^2 \right) y_b,
\]

(B5)
\[
\beta_{\text{SDFDM}}(y_t) = \frac{1}{(4\pi)^2} \left(y_1^2 + y_2^2\right) y_t, \quad (B6)
\]

\[
\beta_{\text{SDFDM}}(\lambda) = \frac{1}{(4\pi)^2} \left[ -2y_1^4 - 2y_2^4 + 4\lambda \left(y_1^2 + y_2^2\right) \right]. \quad (B7)
\]

The one-loop \(\beta\) functions of new parameters in the SDFDM model are as follows

\[
\beta_{\text{SDFDM}}(y_1) = \frac{1}{(4\pi)^2} \left[ \frac{5}{2} y_1^2 + y_1 y_2^2 - \frac{9}{20} g_1^2 y_1 - \frac{9}{4} g_2^2 y_1 + 3 g_2 y_1 + 3 y_2^2 y_1 + y_2^2 y_1 \right], \quad (B8)
\]

\[
\beta_{\text{SDFDM}}(y_2) = \frac{1}{(4\pi)^2} \left[ \frac{5}{2} y_2^2 + 4 y_1 y_2 - \frac{9}{20} g_1^2 y_2 - \frac{9}{4} g_2^2 y_2 + 3 g_2 y_2 + 3 y_1 y_2 + y_2^2 y_2 \right]. \quad (B9)
\]

Note here that \(g_1(g_2^2 = \frac{5}{4} g_Y^2)\), \(g_2\), \(g_3\) are the gauge couplings, \(y_t\), \(y_0\), \(y_r\), \(y_1\), and \(y_2\) are the Yukawa couplings, and \(\lambda\) is the Higgs quartic coupling. The one-loop anomalous dimension of the Higgs field is

\[
\gamma_{\text{total}} = \gamma_{\text{SM}} + \gamma_{\text{SDFDM}} = \frac{1}{(4\pi)^2} \left[ \frac{9}{4} g_2^2 + \frac{9}{20} g_1^2 - 3 y_1^2 - 3 y_2^2 - y_r^2 \right] + \frac{1}{(4\pi)^2} (y_1^2 - y_2^2). \quad (B10)
\]

### Appendix C: Renormalization of kinetic term in effective action

We compute effective action of an external field using derivative expansion. As long as the field varies slowly with respect to space and time, this is a valid approximation. Keeping derivatives up to second order, the Euclidean effective action for a neutral scalar \(\phi\) is written as

\[
S_{\text{eff}}[\phi] = \int d^4x \left[ V_{\text{eff}}(\phi) + \frac{1}{2} (\partial_\mu \phi)^2 Z_2(\phi) \right], \quad (C1)
\]

where \(V_{\text{eff}}\) is the effective potential. The one-loop result of \(V_{\text{eff}}\) in the SM in the background \(R_\xi\) gauge is given in [17]. \(Z_2\) can be obtained from the \(p^2\) terms in self-energy Feynman diagrams. We renormalize \(Z_2\) to make \(Z_2(\phi = 0) = 1\) which means that the kinetic term goes back to the standard form when there is no external field.

#### 1. Feynman rules in background \(R_\xi\) gauge

The Feynman rules with external field \(\phi\) in the SM and in the SDFDM model are given in Fig. 7. Here, we only list the vertices we need in \(Z_2\) calculation. We have introduced

\[
\overline{m}_G^2 = -m_\phi^2 + \lambda \phi^2, \quad \overline{m}_H^2 = -m_\phi^2 + 3\lambda \phi^2 \quad (C2)
\]

where \(m_\phi^2\) is the mass term in the Higgs potential given in (10). The other \(\phi\)-dependent masses can be obtained by substituting the vacuum expectation value \(v\) with \(\phi\).
FIG. 7. Propagators for SM fields with external field $\phi$ in background $R_\xi$ gauge. $m^2_{G} = -m^2_{\phi} + \lambda \phi^2$, $m^2_{H} = -m^2_{\phi} + 3 \lambda \phi^2$, $m^2_{W} = m^2_{\phi} + \frac{1}{2} g \phi$, $m^2_{Z} = \frac{1}{2} \sqrt{g^2 + g'^2} \phi$. $m^2_{W}$ is the mass term in the Higgs potential. $y_f$ is the Yukawa coupling for alternative SM Fermion. Note here that $G^\pm$ and $G_0$ are the goldstone bosons, $C_Z$ and $C^\pm$ are the ghost fields.

We define the field-dependent masses of goldstone bosons and ghost particles as:

$$m_{G^\pm}^2 = \xi_W m_W^2,$$  \hspace{1cm} (C3)

$$m_{C_Z}^2 = \xi_Z m_Z^2,$$  \hspace{1cm} (C4)

$$m_{G^+}^2 = m_G^2 + \xi_W m_W^2,$$  \hspace{1cm} (C5)

$$m_{G^0}^2 = m_G^2 + \xi_Z m_Z^2.$$  \hspace{1cm} (C6)

(C7)
FIG. 8. Vertices with external field $\phi$ for the SM in background $R_\xi$ gauge. $\bar{m}_W$ and $\bar{m}_Z$ are the $\phi$-dependent masses as given in Fig. 7. $\xi_W$ and $\xi_Z$ are the gauge fixing parameters in background $R_\xi$ gauge.

2. $Z_2$ factor in the SM

For simplicity, we calculate $Z_2$ in the 't Hooft-Feynman gauge with $\bar{\xi}_W = \bar{\xi}_Z = 1$. $Z_2$ comes from the $p^2$ term in the Higgs self-energy diagram in Fig. 9. Notations in [24] are used for the
integrals calculated in the modified minimal subtraction scheme:

\[
\frac{i}{16\pi^2} B_0(m_1, m_2, p^2) = \mu^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_1^2)( (k + p)^2 - m_2^2)},
\]

\[
\frac{i}{16\pi^2} B_0(m_1, m_2, p^2) = \mu^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_1^2)( (k + p)^2 - m_2^2)},
\]

\[
B_0(m_1, m_2, p^2) = B_0^0(m_1, m_2) + B_0^1(m_1, m_2) \cdot p^2 + O(p^4) + \ldots
\]

where \(B_0^0(m_1, m_2)\) and \(B_0^1(m_1, m_2)\) can be express as

\[
B_0^0(m_1, m_2) = \frac{m_1^2 \ln \frac{m_1^2}{\mu^2} - m_2^2 \ln \frac{m_2^2}{\mu^2}}{m_1^2 - m_2^2},
\]

\[
B_0^1(m_1, m_2) = \frac{1}{2} \left( \frac{m_1^2 + m_2^2}{(m_1^2 - m_2^2)^2} + \frac{m_1^2 m_2^2 \ln \frac{m_1^2}{m_2^2}}{(m_1^2 - m_2^2)^3} \right).
\]

When \(m_1 = m_2 = m\), \(B_0^0(m_1, m_2)\) and \(B_0^1(m_1, m_2)\) can be written as \(B_0^0(m)\) and \(B_0^1(m)\). They are expressed as

\[
B_0^0(m) = -\ln \frac{m^2}{\mu^2},
\]

\[
B_0^1(m) = \frac{1}{6m^2}.
\]

| a) \(\frac{1}{16\pi^2} (18\lambda^2 \phi^2 B_0^0(\bar{m}_H))\) |
|---|---|
| b) \(\frac{1}{16\pi^2} (4\lambda^2 \phi^2 B_0^1(\bar{m}_{G\pm}))\) |
| c) \(\frac{1}{16\pi^2} (2\lambda^2 \phi^2 B_0^1(\bar{m}_{G_0}))\) |
| d) \(\frac{1}{16\pi^2} (4g^2 \bar{m}_W^2 B_0^1(\bar{m}_W))\) |
| e) \(\frac{1}{16\pi^2} (\frac{4g^2}{\cos^2 \theta_W} \bar{m}_Z^2 B_0^1(\bar{m}_Z))\) |
| f) \(\frac{1}{16\pi^2} (\frac{g^2}{-4\cos^2 \theta_W} \bar{m}_{CZ}^2 B_0^1(\bar{m}_{CZ}))\) |
| g) \(\frac{1}{16\pi^2} (\frac{g^2}{4\cos^2 \theta_W} \bar{m}_{CZ}^2 B_0^1(\bar{m}_{CZ}))\) |
| h) \(\frac{1}{16\pi^2} (-\frac{g^2}{4}) [(-2m^2_{G-} + m^2_{W+}) B_0^1(\bar{m}_{G-}, \bar{m}_{W+}) - 2B_0^0(\bar{m}_{G-}, \bar{m}_{W+})]\) |
| i) \(\frac{1}{16\pi^2} (-\frac{g^2}{4}) [(-2m^2_{G+} + m^2_{W-}) B_0^1(\bar{m}_{G+}, \bar{m}_{W-}) - 2B_0^0(\bar{m}_{G+}, \bar{m}_{W-})]\) |
| j) \(\frac{1}{16\pi^2} (\frac{g^2 + g'^2}{4}) [(-2m^2_{C_{G_0}} + m^2_{Z}) B_0^1(\bar{m}_{C_{G_0}}, \bar{m}_{Z}) - 2B_0^0(\bar{m}_{C_{G_0}}, \bar{m}_{Z})]\) |
| k) \(\frac{1}{16\pi^2} (-g_t^2) [B_0^0(\bar{m}_t) + 4m_t^2 B_0^1(\bar{m}_t)]\) |

TABLE VI. \(p^2\) terms from the self-energy diagram in the SM which contribute to \(Z_2\). Note that here in these results we only list the fermion loop contribution from the top quark.

We list the \(p^2\) terms of each self-diagram in Table VI. Summing over all the \(p^2\) term contributions, we obtain the \(Z_2\) factor in the SM. Since the RG equation for the kinetic term in the effective action can be solved in a way similar to solving \(V_{\text{eff}}(\phi)\), we can obtain the RG improved kinetic term by replacing \(\phi, \mu, \lambda_i\) with \(\phi(t), \mu(t)\) and \(\lambda(t)\). Their expressions or equation are shown in Eqs. (15) and (17). Taking \(\mu(t) = \phi\) as mentioned before, we get the RG improved \(Z_2\) factor in
the SM for large \( \phi \) field.

\[
Z_2^{SM} = 1 + \frac{1}{16\pi^2} \left[ \lambda + \frac{8\lambda^2}{12\lambda + 3g^2} + \frac{4\lambda^2}{12\lambda + 3(g^2 + g'^2)} + \frac{2}{3} (2g^2 + g'^2) - \frac{(2g^2 + g'^2)}{24} \right] \\
+ \frac{1}{8\pi^2} \left[ \left( \frac{4\lambda g^2 + g^4}{16\lambda^3} \right) ln \left( \frac{4\lambda + g^2}{g^2 + g'^2} \right) + \frac{2\lambda + g^2}{4\lambda^2} \right] 8\lambda g^2 + g^4 \\
- \frac{(4\lambda g^2 + g^4) ln(\lambda e^{2\Gamma} + \frac{1}{4} g^2 e^{2\Gamma})}{8\lambda} - g^4 ln(\frac{1}{4} g^2 e^{2\Gamma}) \\
- \frac{1}{16\pi^2} \left[ \frac{(4\lambda (g^2 + g'^2) + (g^2 + g'^2)^2) ln(\frac{4\lambda + g^2 + g'^2}{g^2 + g'^2})}{16\lambda^3} + \frac{2\lambda + g^2 + g'^2}{4\lambda^2} \right] 8\lambda (g^2 + g'^2) + (g^2 + g'^2)^2 \\
+ \frac{(4\lambda (g^2 + g'^2) + (g^2 + g'^2)^2) ln(\lambda e^{2\Gamma} + \frac{1}{4} (g^2 + g'^2) e^{2\Gamma}) - (g^2 + g'^2)^2 ln(\frac{1}{4} (g^2 + g'^2) e^{2\Gamma})}{8\lambda} \\
- \frac{1}{16\pi^2} [ln(\frac{y_1^2}{2} e^{2\Gamma}) y_t^2 + \frac{2}{3} y_t^2] \\
\tag{C15}
\]

with

\[
\Gamma(t) = - \int_0^t \gamma (\lambda (t')) dt' \\
\tag{C16}
\]

3. Z\(_2\) factor in the SDFDM model

![Diagram](image)

FIG. 10. Self-energy diagrams contributing to \( Z_2 \) factor by extra fermions in the SDFDM model. \( \chi_1^0 \) and \( \chi_2^0 \) are the new extra fermions in the SDFDM model.

The contributions of by extra fermions in the SDFDM model to the Higgs self-energy are shown in Fig. 10. \( p^2 \) term contributions to \( Z_2 \) in these diagrams are summarized in Table. VII. Summing over all the \( p^2 \) term contributions in Table. VI and Table. VII, we obtain the \( Z_2 \) factor in the SDFDM model. In the large \( \phi \) limit, \( Z_2 \) factor in the SDFDM model can be expressed as

\[
Z_2^{SDFDM} = Z_2^{SM} - \frac{y_2^2}{16\pi^2} [ln(\frac{y_2^2}{2} e^{2\Gamma}) + \frac{2}{3}] - \frac{y_1^2}{16\pi^2} [ln(\frac{y_1^2}{2} e^{2\Gamma}) + \frac{2}{3}], \\
\tag{C17}
\]
TABLE VII. $p^2$ terms from the self-energy diagram contributed by extra fermions in the SDFDM model. Here we define $A = (-y_2 \cos \theta_L \sin \theta_R - y_1 \sin \theta_L \cos \theta_R)$, $B = y_2 \cos \theta_R \sin \theta_L + y_1 \sin \theta_R \cos \theta_L$, $C = y_2 \cos \theta_L \cos \theta_R - y_1 \sin \theta_L \sin \theta_R$, $D = -y_2 \sin \theta_L \sin \theta_R + y_1 \cos \theta_L \cos \theta_R$. $\overline{m}_{\chi_1}^0$, $\overline{m}_{\chi_2}^0$ are the masses of new particles under the external field $\phi$. They are obtained by substituting $v$ in Eqs. (5) and (6) with $\phi$.

where $Z_2^{SM}$ is given in Eq. (C15).

\[a) \frac{1}{16\pi^2}(-A^2)(4\overline{m}_{\chi_1}^0 m_{\chi_1}^0 B_1(\overline{m}_{\chi_1}^0) - B_0^0(\overline{m}_{\chi_1}^0))\]

\[b) \frac{1}{16\pi^2}(-B^2)(4\overline{m}_{\chi_2}^0 m_{\chi_2}^0 B_1(\overline{m}_{\chi_2}^0) - B_0^0(\overline{m}_{\chi_2}^0))\]

\[c) \frac{1}{16\pi^2}(-\frac{D^2}{2})(2(\overline{m}_{\chi_1}^0 m_{\chi_2}^0 B_1(\overline{m}_{\chi_1}^0, m_{\chi_2}^0) - B_0^0(\overline{m}_{\chi_1}^0, m_{\chi_2}^0))\]

\[d) \frac{1}{16\pi^2}(-\frac{C^2}{2})(2(\overline{m}_{\chi_1}^0 m_{\chi_2}^0 B_1(\overline{m}_{\chi_1}^0, m_{\chi_2}^0) - B_0^0(\overline{m}_{\chi_1}^0, m_{\chi_2}^0))\]

\[e) \frac{1}{16\pi^2}(-\frac{CD}{2})(\overline{m}_{\chi_1}^2 + \overline{m}_{\chi_2}^2)B_1(\overline{m}_{\chi_1}^0 m_{\chi_2}^0, \overline{m}_{\chi_2}^0 - B_0^0(\overline{m}_{\chi_1}^0, m_{\chi_2}^0))\]

\[f) \frac{1}{16\pi^2}(-\frac{CD}{2})(\overline{m}_{\chi_1}^2 + \overline{m}_{\chi_2}^2)B_1(\overline{m}_{\chi_1}^0 m_{\chi_2}^0, \overline{m}_{\chi_2}^0 - B_0^0(\overline{m}_{\chi_1}^0, m_{\chi_2}^0))\]

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