Nonlinear phase compensated OFDR based on Match Fourier Transform algorithm

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Abstract. The nonlinear phase of beat signal in Optical Frequency domain Reflectometry(OFDR) has become a bottleneck to its utilization. In this paper, a method to compensate nonlinear phase based on Match Fourier Transform algorithm(MFT) is presented. The nonlinear phase of beat signal is represented as a polynomial phase function, then the coefficients of the polynomial phase function are obtained from an auxiliary interferometer through the polynomial regression algorithm. Finally, the nonlinear phase in main OFDR interferometer can be compensated by Match Fourier Transform algorithm. The experimental results show that the method can effectively suppress the side lobe of the reflection peak and achieve a spatial resolution of 0.2m at 4.3km. The experiment results verify the effectiveness of the proposed algorithm.

1. Introduction
Optical Frequency domain Reflectometry, with the advantages of high spatial resolution and sensitivity, has attracted more and more attention in these years. It is widely used in distributed optical fiber sensing and optical communication networks monitoring[1-3]. For this technique, the beat signals from the interferometer are collected as a function of optical frequency of a tunable laser source. And then it is converted into the spatial domain signal from time domain by fast Fourier transform(FFT), where it requires that the beat signals are sampled at an equal time interval[4]. However, due to the presence of nonlinearities in the beat signals, the sampling interval of the optical frequency are different, which leads to the widening of the reflection peak in the spatial domain, deteriorating range resolution and reducing the peak amplitude.

To eliminate the nonlinearities of beat signals in OFDR system, two class methods have been adopted, hardware compensation and software compensation. The hardware compensation method is a frequency sampling method in which the output of an auxiliary reference interferometer is applied as a clock signal to trigger the data acquisition[5]. It can realize equal interval sampling and eliminate the influence of nonlinear phase noise. However, one of the drawbacks of this method is that the maximum sensing distance is limited to a quarter of the delay fiber length used in the auxiliary reference interferometer in order to satisfy the Nyquist sampling theorem[6]. So in remote distance application, this method may increase additional cost. To break through the limitation, nonlinear compensation methods based on software algorithm have been widely used, such as interpolation algorithm, non-uniform FFT and deskew filter algorithm[7-9]. A major advantage in the software method is that the maximum sensing distance is no longer limited by the delay fiber of the auxiliary interferometer. Song et al realized the spatial resolution of 0.3mm over a single-mode fiber sensing length of 300m by interpolation algorithm, the measurement range is about hundreds of meters[10].
Ding et al proposed the method using deskew filter algorithm to correct the nonlinearity and achieved a measurement range of 80 km and a spatial resolution of 1.6m. But the calculation efficiency of this method is low and the spatial resolution degrades severely for the long measurement range[9].

In this paper, a new nonlinear phase compensation algorithm based on Match Fourier Transform is proposed. The Match Fourier Transform is originally applied in a Frequency Modulated Continuous Wave Radar to compensate the Doppler phase of transmitted signals[11]. In the proposed technique, an auxiliary interferometer is implemented to obtain the nonlinear phase and the nonlinear phase is modeled as polynomial phase function. The coefficients of polynomial function can be estimated by the least square method. The beat signals from the main OFDR interferometer are also represented by polynomial phase function. Then a Match Fourier Transform is conducted to correct the nonlinearity. In theory the frequency tuning nonlinearity could be eliminated greatly when the sensing distance is not very long. Using this method, experiment results expressed that the spatial resolution of the fiber end reflection peak and the signal to noise of OFDR system can be improved.

2. Algorithm principle
In an OFDR system, the beat signals are produced by the optical interference between two arms of the interferometer in which the light signals are from the same linearly chirped light source. The reflection or backscattering light from the fiber under test(FUT) is called $E_s(t)$, while the light signal from reference arm is $E_r(t)$. For a linear sweep of slope $\gamma$, the optical field $E_r(t)$ can be written as

$$E_r(t) = E_0 \exp \left\{ j \left[ 2\pi f_0 t + \pi \gamma t^2 + e(t) \right] \right\},$$

where $E_0$ is the optical field amplitude, $f_0$ is the initial optical frequency, $e(t)$ is the nonlinear phase noise of the laser. Assumed that the reflection reflectivity is $R(\tau)$ at a time delay $\tau$, the reflected probe lightwave $E_s(t)$ from the FUT can be expressed as

$$E_s(t) = \sqrt{R(\tau)} E_0 \exp \left\{ j \left[ 2\pi f_0 (t-\tau) + \pi \gamma (t-\tau)^2 + e(t-\tau) \right] \right\},$$

Therefore, the coupled beat signal $I(t)$ generated by the interference of $E_s(t)$ and $E_r(t)$ can be written as

$$I(t) = 2\sqrt{R(\tau)} E_0^2 \cos \left\{ 2\pi \left[ f_0 \tau + \pi \gamma \tau^2 - \frac{1}{2} \gamma \tau^2 \right] + e(t) - e(t-\tau) \right\},$$

Because different $\tau$ correspond to the different fiber distances in a spatial domain, the last term $e(t) - e(t-\tau)$ is the nonlinear phase noise of the beat signal. For a short sensing distance, the third term of Eq.(3) can be neglected due to the small time delay. And the nonlinear phase noise can be approximated using Taylor series as

$$I(t) \approx 2\sqrt{R(\tau)} E_0^2 \cos \left\{ 2\pi \left[ f_0 \tau + \gamma \tau t + \tau \dot{e}(t) \right] \right\},$$

where $\dot{e}(t)$ is the derivative of the nonlinear phase noise. According to the Stone-Weierstrass theorem[12], the unknown nonlinear phase term $e(t)$ can be approximated by a K-order polynomial function as

$$e(t) \approx 2\pi \sum_{k=2}^{K} \frac{a_k}{k+1} t^{k+1},$$

Substituting the Eq.(5) into Eq.(4), the beat signal can be written as

$$I(t) \approx 2\sqrt{R(\tau)} E_0^2 \cos \left\{ 2\pi \tau \left[ f_0 + \gamma t + \sum_{k=2}^{K} a_k t^k \right] \right\},$$

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For convenience, equation (6) can also be expressed as

$$I(t) \approx 2\sqrt{R(\tau)}E_0^2 \cos\left\{2\pi\tau\sum_{k=0}^{K} b_k t^k \right\},$$

(7)

where $b_0 = f_0$, $b_1 = \gamma$. Because the nonlinearity is dependent on distance, $b_k$ can’t be estimated at the same time. But for the coupled signal from an auxiliary interferometer, the time delay $\tau$ is constant. The polynomial coefficients $b_k$ can be estimated with the polynomial regression algorithm. The normalized beat signal $I_{\text{ref}}(t)$ from auxiliary interferometer corresponding to a reference time delay $\tau_{\text{ref}}$ is

$$I_{\text{ref}}(t) = \cos\left\{2\pi\tau_{\text{ref}}\sum_{k=0}^{K} b_k t^k \right\},$$

(8)

The unwrapped phase of $I_{\text{ref}}(t)$ is

$$\phi(t) = \sum_{k=0}^{K} 2\pi\tau_{\text{ref}} b_k t^k = \sum_{k=0}^{K} B_k t^k,$$

(9)

In this polynomial regression model, the coefficients $B_k$ can be estimated through the least square method.

$$\hat{B}_k = \left( A^T A \right)^{-1} A^T \Phi_N,$$

(10)

where

$$A = \begin{bmatrix} t_1 & t_1^2 & \cdots & t_1^K \\ t_2 & t_2^2 & \cdots & t_2^K \\ \vdots & \vdots & \vdots & \vdots \\ t_N & t_N^2 & \cdots & t_N^K \end{bmatrix} \quad A^T = \begin{bmatrix} a^T (1) \\ a^T (2) \\ a^T (3) \\ \vdots \\ a^T (N) \end{bmatrix}, \quad \Phi_N = \begin{bmatrix} \phi(t_1) & \phi(t_2) & \cdots & \phi(t_N) \end{bmatrix}^T$$

is the measured phase.

Jointly with Eq.(9) and (10), the estimated coefficients $b_k = \frac{B_k}{2\pi\tau_{\text{ref}}}$ can be obtained.

With the polynomial coefficients $b_k$, a complex express is transformed from Eq.(7) by Hilbert transform that can be written as

$$I(t) \approx 2\sqrt{R(\tau)}E_0^2 \exp\left\{j2\pi\tau\sum_{k=0}^{K} b_k t^k \right\}$$

$$= 2\sqrt{R(\tau)}E_0^2 \exp\left\{j2\pi\tau b_0 + j2\pi\tau b_1 \sum_{k=1}^{K} b_k t^k \right\}$$

$$= 2\sqrt{R(\tau)}E_0^2 \exp\left\{j2\pi\tau b_0 + j2\pi\tau b_1 \varepsilon(t) \right\},$$

(11)

where $\varepsilon(t) = t + \sum_{k=2}^{K} \frac{b_k}{b_1} t^k$. Then a Match Fourier Transform can be conducted to correct the nonlinearity.

The Match Fourier Transform of the beat signal can be expressed as

$$F(f) = \mathcal{F} I(t) = \int_0^{T} I(t) e^{-j2\pi f(t)} d\varepsilon(t),$$

(12)
where $T$ is the time window, $e(t)$ is the integral path. Substituting the Eq.(11) into (12), the spectrum is

$$F_f = 2\sqrt{R(\tau)E_\phi^2e^2(T)}\exp\left(j2\pi \tau b_0\right)\text{sinc}\left(e(T)(f - \tau b_0)\right),$$  \hspace{1cm} (13)

From the Eq.(13), the energy has been focused in the Match Fourier Spectrum through match filtering and the nonlinearity is corrected.

3. Experiment results and analysis
The system structure of OFDR system is shown in Fig.1. An external modulation method is adopted to generate the frequency-swept lightwave. The main interferometer is used for measurement and the auxiliary interferometer is used for obtaining the nonlinear phase noise.

![Experiment setup diagram](image)

Figure 1. The system structure of OFDR system. EOM: Electro-optic modulator; OC: optical coupler; PD: photodetector.

The laser with linewidth of 2kHz is employed as the optical source and connected with an Electro-optic modulator. The instantaneous optical frequency is modulated by the RF synthesizer. The frequency-swept range is 1.3GHz corresponding to a theoretical spatial resolution of 0.08m. The chirped lightwave is then fed to a 90/10 optical coupler, with 10% lightwave launched into the auxiliary interferometer. Considering the linewidth of the light source in the experiment, the length of the fiber under test is set to 4.3km. The beat signals from the main interferometer and auxiliary interferometer are received by the photodetector.

In the experiment, the frequency-swept time is 900us, the corresponding frequency scanning rate is 1444GHz/s. In order to test the performance of OFDR system with the proposed method, the algorithm is applied to process the acquired data from OFDR system with measuring a 4.3km-long fiber. The results of the fiber end reflection with and without nonlinear phase compensation are shown in figure 2 and figure 3 respectively. The end zoom-ins for figure 2 and figure 3 is shown in figure 4.
Figure 2. The measured result of fiber end reflection without nonlinearity compensation

Figure 3. The measured result of fiber end reflection with nonlinearity compensation

As shown in Fig.2 and Fig.3, the spatial resolution of the fiber end reflection can be greatly enhanced after the nonlinearity compensation. The theoretical spatial resolution $\Delta z$ in the experiment is 0.08m based on the relationship $\Delta z = c / 2n\Delta F$, where $c$ is the light speed in vacuum, $n$ is the refractive index of fiber, $\Delta F$ is the frequency-swept range. The spatial resolution $\Delta z$ of fiber end reflection in Fig.2 is about 0.5m and the $\Delta z$ in Fig.3 is about 0.2m with the nonlinearity compensation. In addition, the interference spectrum peak has a relatively obvious broadening due to the effect of the nonlinear phase noise in Fig.2. With the nonlinearity compensation, the disturbing reflection peak can be eliminated to obtain a clear peak value as depicted in Fig.3. The experiment result verify the effectiveness of the proposed algorithm. But it should be noted that the nonlinearity can’t be eliminated completely due to the drawback of the algorithm.
Figure 4. The local zoom-ins of fiber end reflection for figure.2 and figure.3

4. Conclusion
In this paper, the nonlinear phase noise compensated algorithm based on Match Fourier Transform has been studied. The experiment results verify the effectiveness of the proposed method. By using this method, the spatial resolution of the fiber end reflection can be enhanced obviously. The clear peak value of the fiber end reflection can be obtained with the nonlinearity compensation.

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