Local spherically symmetric perturbations of spatially flat Friedmann models

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Abstract. The spherically symmetric perturbations in the spatially flat Friedmann models are considered. It is assumed that the Friedmannian density and pressure are related through a linear equation of state. The perturbation is joined smoothly with an unperturbed Friedmann’s background at the sound horizon of perturbation. Such junction is in accordance with the “birth” of a local perturbation as a result of the redistribution of matter. The solution of the Einstein’s equations is obtained in linear approximation on a Friedmann’s background near the the sound horizon of perturbation.

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1. Introduction

The description of metric perturbations in relativistic cosmology is significant in the theory of creation of the universe large-scale structure. Spherically symmetric perturbations are an important and interesting class of that perturbations.

The study of small perturbations on the cosmological background has a long history. The first relativistic treatment was given by Lifshitz [1] and developed by himself and Khalatnikov [2]. The gauge-invariant formalism of perturbation was given in the fundamental paper of Bardeen [3] and was developed in the works [4].

Spherically symmetric perturbations on the cosmological background have been studied by a lot of authors [5, 6, 7]. As it was noted in [7] the spherically symmetric perturbation in fluid with nonzero pressure may be represent as outgoing and ingoing waves (travelling from and into the center of the configuration with the velocity of sound). The ingoing wave amplitude increases as soon as the wave approaches to the center of the configuration. Therefore the magnitude of spherically symmetric perturbation near the center of configuration must increase more quickly than the amplitude of Lifshitz’s plane-symmetric wave perturbations [1]. Moreover the local perturbation must be smoothly jointed with an unperturbed Friedmann’s background at the sound horizon of perturbation.

Our main difference from the paper by Ignat’ev and Popov [7] is that we consider a cosmological fluid with a linear equation of state \( \varepsilon = \gamma p \), where \( \varepsilon \) is density, \( p \) is pressure of fluid but leave arbitrary a constant \( \gamma (\gamma > 0) \).

In Sec. II we write out the rigorous equations describing the perfect fluid in spherically symmetric space-times and give the characterization a background space-time. In Sec. III we fix the gauge and set out the basic equations governing the spherically symmetric perturbations. The particular solutions of these equations are given in Sec. IV. In Sec. V we impose the boundary conditions on the perturbation and obtain the solution for perturbation near the boundary. Finally, the main conclusions raised in this paper are summarized in Sec. VI.

2. Perfect fluid in spherically symmetric space-time. Rigorous equations and background space-time

With the gravitational field being spherically symmetric, coordinates may be chosen so that the metric takes the form

\[
\begin{aligned}
ds^2 &= -e^\nu d\eta^2 + e^\lambda dr^2 + e^\mu r^2 \left( d\theta^2 + \sin^2 \theta \ d\varphi^2 \right),
\end{aligned}
\]

where \( \nu, \mu \) and \( \lambda \) are functions of the time coordinate \( \eta \) and a radial space coordinate \( r \). As we are interested in perfect-fluid distribution, the energy-momentum tensor is

\[
T^\mu_\nu = (\varepsilon + p)u^\mu u_\nu + p\delta^\mu_\nu,
\]
where $\varepsilon$ is density, $p$ is pressure and $u^\mu(u^\nu, u^r, 0, 0)$ is four-velocity of fluid. Take into account the equation of state

$$\varepsilon = \gamma p \tag{3}$$

one can obtain the following nontrivial Einstein equations ($c = G = 1$)

$$e^{-\lambda} \left( \ddot{\mu} + \frac{3\mu'^2}{4} + \frac{3\mu'}{r} - \frac{\mu'\lambda'}{r} + \frac{\lambda'}{r^2} \right) - \frac{e^{-\mu}}{r^2} - e^{-\nu} \left( \frac{\mu^2}{4} + \frac{\mu\dot{\lambda}}{2} \right) = -8\pi\varepsilon \left[ \gamma + (1 + \gamma)\nu^2 \right], \tag{4}$$

$$e^{-\lambda} \left( \frac{\mu^2}{4} + \frac{\mu'\nu'}{2} + \frac{\mu' + \nu'}{r} + \frac{1}{r^2} \right) - \frac{e^{-\mu}}{r^2} + e^{-\nu} \left( -\ddot{\mu} - \frac{3\mu^2}{4} + \frac{\mu\nu}{2} \right) = 8\pi p \left[ 1 + (1 + \gamma)\nu^2 \right], \tag{5}$$

$$e^{-\lambda} \left[ \frac{\mu'' + \nu''}{2} + \frac{\mu'^2 + \nu'^2 + \mu'\nu' - \mu'\lambda' - \nu'\lambda'}{4} + 2\frac{\mu' + \nu' - \lambda'}{2r} \right] + e^{-\nu} \left[ -\frac{(\ddot{\mu} + \ddot{\nu})}{2} + \frac{\dot{\nu}\dot{\lambda} + \nu\dot{\mu} - \ddot{\mu} - \ddot{\lambda}}{4} \right] = 8\pi p, \tag{6}$$

$$e^{-\nu} \left[ \frac{\mu'}{2} - \frac{(\mu'\dot{\lambda} + \dot{\mu}\nu)}{2} + \frac{\ddot{\mu} - \ddot{\lambda}}{r} \right] = 8\pi e^{(\lambda - \nu)/2}(1 + \gamma)p\nu\sqrt{1 + \nu^2}, \tag{7}$$

where $\nu = u^r e^{\lambda/2}$ is the frame projection of radial velocity, a dot and a prime denote partial derivatives with respect to $\eta$ and $r$, respectively.

In isotropic spherical coordinates the metric of the background space-time can be written as

$$ds^2 = a^2 \left\{ -d\eta^2 + dr^2 + r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2) \right\}, \tag{8}$$

where the function $a(\eta)$ satisfy the equation

$$2\gamma \frac{\ddot{a}}{a} = (\gamma - 3) \frac{\dot{a}^2}{a^2}. \tag{9}$$

The energy density $\varepsilon_0$, the pressure $p_0$ and the frame projection of radial velocity of the background fluid $v_0$ are

$$\varepsilon_0 = 3 \frac{\dot{a}^2}{8\pi a^4}, \quad \varepsilon_0 = \gamma p_0, \quad v_0 = 0. \tag{10}$$

3. Gauge transformation and perturbed equations

We shall only consider spherically symmetric perturbations of spatially flat Friedmann models, i.e.

$$\nu = \ln (a^2) + \delta \nu, \quad \lambda = \ln (a^2) + \delta \lambda, \quad \mu = \ln (a^2) + \delta \mu, \tag{11}$$

$$\varepsilon = \varepsilon_0 + \delta \varepsilon, \quad p = p_0 + \delta p. \tag{12}$$
Local spherically symmetric perturbations of spatially flat Friedmann models

where $\delta \nu, \delta \lambda, \delta \mu, \delta \varepsilon, \delta p$ are the functions of the coordinates $\eta, r$ such that

$$|\delta \nu|, |\delta \lambda|, |\delta \mu| \ll 1, \quad \delta \varepsilon \ll \varepsilon_0, \; \delta p \ll p_0.$$  \hspace{1cm} (13)

We shall assume that the frame projection of radial velocity of fluid $v$ is the small quantity too.

Let us note that the linearized gauge transformation

$$\eta = \tilde{\eta} + \delta \eta(\tilde{\eta}, \tilde{r}), \; r = \tilde{r} + \delta r(\tilde{\eta}, \tilde{r})$$  \hspace{1cm} (14)

gives

$$ds^2 = \frac{1}{a^2(\eta)} \left( (1 + \delta \nu) d\eta^2 + a^2(\eta) (1 + \delta \lambda) dr^2 + a^2(\eta) r^2 (1 + \delta \mu) d\Omega^2 \right)$$

$$= \frac{1}{a^2(\eta)} \left( 1 + \delta \nu + 2 \ddot{\eta} + 2 \frac{\dot{a}}{a} \delta \eta \right) d\eta^2 + 2 a^2(\eta) (\dot{\delta \lambda} + \dot{\delta \eta}) d\eta d\tilde{r}^2 + a^2(\eta) \tilde{r}^2 \left( 1 + \delta \mu + 2 \frac{\dot{\delta \lambda}}{r} + 2 \frac{\dot{a}}{a} \delta \eta \right) d\Omega^2$$  \hspace{1cm} (15)

Here the dot and the prime denote $\partial/\partial \tilde{\eta}$ and $\partial/\partial \tilde{r}$ respectively, $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$.

The functions $\delta \eta$ and $\delta r$ can be chosen such that

$$\frac{d(\delta r)}{d\tilde{\eta}} + \frac{d(\delta \eta)}{d\tilde{r}} = 0, \quad \frac{d(\delta r)}{d\tilde{r}} - \frac{\delta r}{\tilde{r}} = \frac{\delta \mu - \delta \lambda}{2}$$  \hspace{1cm} (16)

and consequently $\delta \tilde{\lambda} = \delta \tilde{\mu}$. Omitting the tilde over $\eta, r, \delta \lambda$ and $\delta \mu$ below we shall suppose

$$\delta \mu = \delta \lambda.$$  \hspace{1cm} (17)

Now one can to rewrite the metric in linear (with respect to the perturbations) approximation as

$$ds^2 = -a^2(\eta) \left\{ (1 + \delta \nu) d\eta^2 + (1 + \delta \lambda) \left[ dr^2 + r^2 (1 + \delta \mu) d\Omega^2 \right] \right\}$$  \hspace{1cm} (18)

Thus Einstein’s equations \[4\] are reduced in linear approximation to the following four equations

$$\delta \lambda'' + 2 \frac{\delta \lambda'}{r} - 3 \frac{\dot{a}}{a} \delta \lambda + 3 \frac{\dot{a}^2}{a^2} \delta \nu = -8\pi a^2 \delta \varepsilon,$$  \hspace{1cm} (19)

$$\frac{\delta \nu'}{r} - \frac{\delta \lambda'}{r} - \frac{3}{2} \frac{\dot{a}}{a} \delta \lambda + \frac{\dot{a}}{a} \delta \nu + \left( 2 \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \right) \delta \nu = 8\pi a^2 \delta \varepsilon \gamma,$$  \hspace{1cm} (20)

$$\frac{\delta \nu''}{2} + \frac{\delta \nu'}{2 r} - \frac{1}{2} \frac{\ddot{a}}{a} \delta \lambda + \frac{1}{2} \frac{\dot{a}}{a} \delta \nu + \left( 2 \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \right) \delta \nu = 8\pi a^2 \delta \varepsilon \gamma,$$  \hspace{1cm} (21)

$$\frac{\delta \lambda'}{a} = \frac{\ddot{a}}{a} \delta \nu' = 8\pi \left( 1 + \frac{1}{\gamma} \right) a^2 \varepsilon_0 \nu$$  \hspace{1cm} (22)

for the four unknown functions $\delta \nu, \delta \lambda, \delta \varepsilon, \nu$ (we take into account $\delta p = \delta \varepsilon / \gamma$).
Local spherically symmetric perturbations of spatially flat Friedmann models

Let us note that as the consequence of the symmetry of problem the perturbations described in this paper are scalar ones. The gauge-invariant amplitude of density perturbation with the notations of the paper [3] is

\[ \epsilon_g = \delta - 3(1 + w) \frac{1}{k} \frac{\dot{S}}{S} \left( B(0) - \frac{1}{k} \dot{H}(0) \right) \]  

(23)

The choice of the isotropic coordinates [18] in our paper corresponds to a longitudinal gauge. With the notations of the paper [3] this give

\[ B(0) = H_T(0) = 0. \]  

(24)

By comparing the corresponding expressions in two papers we find

\[ \epsilon_g Q(0) = \delta Q(0) = \delta \varepsilon / \varepsilon_0. \]  

(25)

The relations between the other perturbed quantities can be determined analogously

\[ v_s(0) Q(0) = \left[ v(0) - \frac{1}{k} \dot{H}_T(0) \right] Q(0) = v(0) Q(0) = v, \]  

(26)

\[ \Phi_A Q(0) = \left[ A + \frac{1}{k} \dot{B}(0) + \frac{1}{k} \frac{\dot{S}}{S} B(0) - \frac{1}{k^2} \left( \dot{H}_T(0) + \frac{\dot{S}}{S} \dot{H}_T(0) \right) \right] Q(0), \]  

(27)

\[ \Phi_H Q(0) = \left[ H_L + \frac{1}{3} H(0) + \frac{1}{k} \frac{\dot{S}}{S} B(0) - \frac{1}{k^2} \frac{\dot{S}}{S} \dot{H}_T(0) \right] Q(0) = H_L Q(0) = \frac{\delta \lambda}{2}. \]  

(28)

From this relations we find that the perturbation quantities introduced in this section are actualy physical as well as the gauge-invariant quantities \( \epsilon_g Q(0), v_s Q(0), \Phi_A Q(0), \Phi_H Q(0) \).

The difference of equations (20) (21) gives

\[ (\delta \lambda'' + \delta \nu'') = \frac{1}{r} (\delta \lambda' + \delta \nu'). \]  

(29)

The solution of this equation is

\[ \delta \lambda + \delta \nu = C_1 + C_2 r, \]  

(30)

where \( C_1 \) and \( C_2 \) are arbitrary functions of the time coordinate \( \eta \). If we require that

\[ \delta \lambda = \delta \nu = 0, \quad \text{for } r \to \infty, \]  

(31)

i.e. the space-time is asymptotically homogeneous and isotropic, then

\[ C_1 = C_2 = 0 \]  

(32)

and

\[ \delta \nu = -\delta \lambda. \]  

(33)
Local spherically symmetric perturbations of spatially flat Friedmann models

One can consider the expressions (19) and (22) as the definitions of $\delta \varepsilon$ and $\nu$ respectively

\[ 8\pi \delta \varepsilon = 8\pi \gamma \delta p = \frac{3}{a^3} \left( \delta \dot{\lambda} + \frac{\dot{a}}{a} \delta \lambda \right) - \frac{1}{a^2} \left[ \delta \lambda'' + \frac{2}{r} \delta \lambda' \right], \]

(34)

\[ 8\pi \nu = \frac{\gamma}{a^2 (1 + \gamma) \varepsilon_0} \left( \delta \frac{\dot{\lambda}}{a} + \frac{\dot{a}}{a} \delta \lambda' \right), \]

(35)

Thus the spherically symmetric perturbations can be described by the metric

\[ ds^2 = -a^2 (\eta) \left\{ (1 - \delta \lambda) d\eta^2 + (1 + \delta \lambda) \left[ dr^2 + r^2 (1 + \delta \mu) d\Omega^2 \right] \right\}, \]

(36)

where the single unknown function $\delta \lambda(\eta, r)$ is determined by the following combination of the expressions (19-21)

\[ \delta \lambda'' + \frac{2}{r} \delta \lambda' = \gamma \ddot{\lambda} + 3 \left( 1 + \gamma \right) \frac{\dot{a}}{a} \delta \lambda. \]

(37)

4. Particular solutions

First of all it is necessary to note that the equation (37) has a solution, which correspond to the Newtonian potential [8]

\[ \delta \lambda = -\delta \nu = \frac{2m}{ar}, \]

(38)

cauised by a particle of variable mass

\[ m(\eta) = C_3 a + C_4 a^{-3(1+\gamma)/(2\gamma)}, \]

(39)

where $C_3$ and $C_4$ are constants.

If the perfect fluid is a dust, i.e.

\[ p = 0 \quad (1/\gamma = 0), \]

(40)

then the metric perturbations are described by the equation

\[ \frac{\partial^2 (\delta \lambda)}{\partial a^2} + \frac{7}{2a} \frac{\partial \delta \lambda}{\partial a} = 0. \]

(41)

The solution of this equation is

\[ \delta \lambda = \frac{F_1}{a^{5/2}} + F_2, \]

(42)

where $F_1$ and $F_2$ are the arbitrary functions of $r$.

The case of the ultra relativistic fluid ($\varepsilon = 3p$) has been considered in [7]. The corresponding solution has a form

\[ \delta \lambda = \frac{1}{ar} \frac{\partial}{\partial \eta} \left\{ \frac{1}{a} \left[ \Phi_+ \left( \frac{\eta}{\sqrt{3}} + r \right) + \Phi_- \left( \frac{\eta}{\sqrt{3}} - r \right) \right] \right\}, \]

(43)

where $\Phi_+(x)$ $\Phi_-(x)$ are the arbitrary functions.
5. Boundary conditions and solution near the sound horizon

The hyperbolic equation (37) describes the outgoing and ingoing waves travelling with speed of sound from and into the center of the configuration. Therefore if we like to describe a local perturbation of radius \( r_0 \) (at the moment \( \eta_0 \)) we can impose the boundary conditions at the sound horizon of this perturbation

\[
\delta \lambda |_{r - r_0 - \tau + \tau_0 = 0} = \frac{\partial (\delta \lambda)}{\partial r} |_{r - r_0 - \tau + \tau_0 = 0} = \frac{\partial (\delta \lambda)}{\partial \eta} |_{r - r_0 - \tau + \tau_0 = 0} = 0, \tag{44}
\]

where

\[
\tau = \eta / \sqrt{\gamma}. \tag{45}
\]

Such boundary conditions are in accordance with the "birth" of a perturbation as a result of the redistribution of matter. If we make the substitution

\[
\delta \lambda = \frac{V(\tau, r)}{ra^\beta}, \tag{46}
\]

where

\[
\beta = \frac{3}{2} \left( 1 + \frac{1}{\gamma} \right), \tag{47}
\]

and introduce new coordinates

\[
x = \tau + r, \\
y = \tau - r - \tau_0 + r_0, \tag{48}
\]

then Eq. (37) can be rewritten in the form

\[
4 \frac{\partial^2 V(x, y)}{\partial x \partial y} - \beta \left( \frac{da}{ad\tau} |_{\tau = (x + y + \tau_0 - r_0)/2} \right)^2 V(x, y) = 0. \tag{49}
\]

We shall find the solution of this equation near the boundary \( y = 0 \) representing the functions \((da/(ad\tau))|_{\tau = (x + y + \tau_0 - r_0)/2}\) and \(V(x, y)\) as a power series

\[
\frac{da}{ad\tau} |_{\tau = (x + y + \tau_0 - r_0)/2} = \sum_{k=0}^{\infty} \frac{y^k}{k!} \left[ \left( \frac{\partial}{2 \partial \tau} \right)^k \frac{da}{ad\tau} \right] |_{\tau = (x + \tau_0 - r_0)/2}, \tag{50}
\]

\[
V(x, y) = \Theta(y) \sum_{n=2}^{\infty} V_n(x) y^n. \tag{51}
\]

Here we take into account that the first terms of last series vanish as a consequence of the boundary conditions (44). \( \Theta \)-function

\[
\Theta(y) = \begin{cases} 
0, & y < 0, \\
1, & y \geq 0
\end{cases} \tag{52}
\]
is introduced into expression (51) since the perturbation must be vanish out of the sound horizon. Then the equation (49) is rewritten as

$$\sum_{n=2}^{\infty} \left\{ 4n \frac{dV_n}{dx} y^{n-1} - \beta V_n \sum_{k,s=0}^{\infty} \frac{y^{k+s+n}}{k!s!} \left[ \left( \frac{\partial}{\partial \tau} \right)^{k} \frac{da}{d\tau} \right] \right\} = 0. \quad (53)$$

The first two terms of this series give

$$\frac{dV_2}{dx} = 0,$$

$$\frac{dV_3}{dx} - \frac{\beta}{12} V_2 \left( \frac{da}{d\tau} \right)^2 \big|_{\tau = (x + \tau_0 - r_0)/2} = 0. \quad (54)$$

The solutions of these equations are

$$V_2 = \Psi_2,$$

$$V_3 = \Psi_3 + \frac{\Psi_2 \beta}{6(1 - \beta)} \left( \frac{da}{d\tau} \right)^{2} \big|_{\tau = (x + \tau_0 - r_0)/2} \quad (55)$$

Here we take into account that

$$\int dx \left( \frac{da}{d\tau} \right)^2 \big|_{\tau = (x + \tau_0 - r_0)/2} = 2 \left[ \int d\tau \left( \frac{da}{d\tau} \right)^2 \right] \big|_{\tau = (x + \tau_0 - r_0)/2} \quad (56)$$

and

$$\int d\tau \left( \frac{da}{d\tau} \right)^2 = \beta \int d\tau \left( \frac{da}{d\tau} \right)^2 + \frac{da}{d\tau} \quad (57)$$

Thus

$$V(x, y) = \left\{ \Psi_2 + \left[ \Psi_3 + \frac{\Psi_2 \beta}{6(1 - \beta)} \left( \frac{da}{d\tau} \right)^2 \big|_{\tau = (x + \tau_0 - r_0)/2} \right] y \right. \right.$$

$$+ O \left( \frac{y^2}{(x + \tau_0 - r_0)^2} \right) \left\} \Theta(y). \quad (58)$$

The coefficient $1/(ra^\beta)$ in (46) must be expand in series of $y$ also

$$\frac{1}{ra^\beta(\tau)} = \frac{2}{(x - \tau_0 + r_0)a^\beta_{\tau = (x + \tau_0 - r_0)/2}} \left( 1 + \frac{1}{(x - \tau_0 + r_0)} \right)$$

$$- \frac{\beta}{2} \left( \frac{da}{d\tau} \right)^2 \big|_{\tau = (x + \tau_0 - r_0)/2} \bigg) y + O \left( \frac{y^2}{(x + \tau_0 - r_0)^2} \right) \bigg. \crl{59}$$

Thus

$$\delta \lambda(x, y) = \frac{2y^2}{(x - \tau_0 + r_0)a^\beta_{\tau = (x + \tau_0 - r_0)/2}} \left\{ \Psi_2 + \left[ \Psi_3 + \frac{\Psi_2}{(x - \tau_0 + r_0)} \right] \right. \right.$$

$$+ \Psi_2 \frac{\beta(3\beta - 2)}{6(1 - \beta)} \left( \frac{da}{d\tau} \right)^2 \big|_{\tau = (x + \tau_0 - r_0)/2} \bigg) y \right.$$

$$+ O \left( \frac{y^2}{(x + \tau_0 - r_0)^2} \right) \left\} \Theta(y), \quad (60)$$

The solutions of these equations are

$$V_2 = \Psi_2,$$

$$V_3 = \Psi_3 + \frac{\Psi_2 \beta}{6(1 - \beta)} \left( \frac{da}{d\tau} \right)^2 \big|_{\tau = (x + \tau_0 - r_0)/2} \quad (55)$$
Local spherically symmetric perturbations of spatially flat Friedmann models

or, if we take into account (47)

\[ \delta \lambda (\tau, r) = \frac{(\tau - r - \tau_0 + r_0)^2}{(\tau + r - \tau_0 + r_0) a^3(z)} \left\{ \Psi_2 + \left[ \frac{\Psi_3 + \Psi_2}{(\tau + r - \tau_0 + r_0)^2} \right] \right\} (\tau - r - \tau_0 + r_0) \]

\[ -\Psi_2 \frac{(9 + 14\gamma + 5\gamma^2) da(z)}{4\gamma(\gamma + 3)a(z)} \frac{dz}{dz} (\tau - r - \tau_0 + r_0) \]

\[ + O \left( \frac{(\tau - r - \tau_0 + r_0)^2}{(\tau + r + \tau_0 - r_0)^2} \right) \bigg|_{z=(\tau + r + \tau_0 - r_0)/2} \]

\[ \Theta (\tau - r - \tau_0 + r_0). \]

(61)

For the case \( \Psi_3 = \tau_0 = r_0 = 0, \gamma = 3 \), this expression coincides with the expansion of \( \delta \lambda \) from [6] near the sound horizon.

6. Conclusion

We have considered the spherically symmetric perturbations of a perfect fluid on the background of spatially flat Friedmann models. It was assumed that the fluid is described by the linear equation of state (3). It was shown that in longitudinal gauge the local perturbations of the energy density, the pressure, the frame projection of the radial velocity of fluid (34, 35) and the metric coefficients (17, 33) are determined by the single function. This function satisfies equation (37) of hyperbolic type and describes the outgoing and ingoing waves travelling with speed of sound from and into the center of the configuration. The boundary conditions (44) on the solution of this equation are imposed at the sound horizon of perturbation. The solution is represented as a power series (46, 51) near this horizon. First two terms of series are obtained obviously (61). The corresponding to these terms solution coincides with the expansion near sound horizon early known one in the case of ultrarelativistic equation of state (\( \varepsilon = 3p \)).

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