Wormholes in $f(R)$ gravity with a noncommutative-geometry background

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Abstract

This paper discusses the possible existence of traversable wormholes in $f(R)$ modified gravity while assuming a noncommutative-geometry background, as well as zero tidal forces. The first part of the paper aims for an overview via several shape functions by determining the corresponding wormhole solutions and their properties. The solutions are made complete by deriving the modified-gravity functions $F(r)$ and $f(R)$, where $F = df/dR$. It is subsequently shown that the violation of the null energy condition can be attributed to the combined effects of $f(R)$ gravity and noncommutative geometry. The second part of the paper reverses the strategy by starting with a special form of $f(R)$ and determining the wormhole solution and the concomitant $F(r)$. The approach in this paper differs in significant ways from that of Jamil et al.

Keywords: Wormholes; $f(R)$ modified gravity; Noncommutative geometry

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1 Introduction

Traversable wormholes, first proposed by Morris and Thorne [1], are perfectly valid solutions of the Einstein field equations. They have remained somewhat controversial, however, because their existence depends on a violation of the null energy condition, requiring the use of “exotic matter.” This requirement reduces the probability of the existence of naturally occurring wormholes if we adhere to classical general relativity. The most obvious alternative would therefore be a modification or extension of Einstein’s theory.

The best-known candidate for a modification is $f(R)$ gravity, strongly motivated by its success in the analysis of large-scale structures such as galaxies and clusters of galaxies to account for flat rotation curves without the need for dark matter and on a cosmological
scale to explain the accelerated expansion. Other key motivations are the possibility of realistic descriptions of strong gravitational fields near curvature singularities [2] and the possibility of threading wormholes with ordinary matter [3, 4], as opposed to the usual exotic matter. Another way to help eliminate the need for exotic matter is to assume a noncommutative-geometry background. As discussed below, noncommutative geometry, an offshoot of string theory, has a direct physical origin, thereby providing a simpler motivation.

An interesting counterpart is provided by $f(R, T)$ gravity [5, 6]. Here $R$ is the Ricci scalar (Sec. 2) and $T$ is the trace of the energy-momentum tensor. In Ref. [5] the extra curvature quantities can be interpreted as a gravitational entity that supports the wormhole without the need for exotic matter. Ref. [6] considers wormhole geometries filled with two physically different fluid configurations, one isotropic and another anisotropic, also leading to wormhole solutions without exotic matter. By contrast, in the present study, the avoidance of exotic matter can be attributed to two geometric rather than physical factors, the combined effects of $f(R)$ gravity and noncommutative geometry. Moreover, the combined effects may differ from the individual effects.

In the context of $f(R)$ gravity, another important variation, discussed in Ref. [7], is the construction of charged thin-shell wormholes by surgically grafting two cylindrically symmetric spacetimes via the cut-and-paste technique. Both logarithmic and exponential forms of $f(R)$ models are used in the stability analysis. It was concluded that stable solutions are possible due to the extra curvature invariants.

It should be stressed that the gravitational theories in the present paper, $f(R)$ modified gravity and noncommutative geometry are essentially independent and their effects on wormhole physics have been studied separately. On the other hand, the theories are not mutually exclusive and can therefore be valid simultaneously. So it is just as important to study the combined effects as the effects considered separately. A brief overview of these theories and their incorporation in the present study is discussed next.

2 Noncommutative geometry and $f(R)$ modified gravity

As noted in the Introduction, this paper discusses the possible existence of traversable wormholes by assuming two simultaneous modifications of gravity, noncommutative geometry and $f(R)$ modified gravity. The former refers to an important outcome of string theory, namely the realization that coordinates may become noncommutative operators on a $D$-brane [8, 9]. As discussed in Refs. [10, 11, 12], we are now dealing with a fundamental discretization of spacetime due to the commutator $[x^\mu, x^\nu] = i\theta^{\mu\nu}$, where $\theta^{\mu\nu}$ is an antisymmetric matrix. Thus noncommutativity replaces point-like objects by smeared objects with the aim of eliminating the divergences that normally appear in general relativity.

To describe the smearing, a model that naturally presents itself is a Gaussian distribution of minimal length $\sqrt{\alpha}$ instead of the Dirac delta function [10, 12, 13, 14]. The more convenient approach used in this paper is to assume that the energy density of the
static and spherically symmetric and particle-like gravitational source has the form
\[ \rho(r) = \frac{M \sqrt{\alpha}}{\pi^2 (r^2 + \theta)^2}. \] (1)

(See Refs. [15] and [16] for further details.) Here the mass \( M \) is diffused throughout the region of linear dimension \( \sqrt{\alpha} \) due to the uncertainty. The noncommutative geometry is an intrinsic property of spacetime and does not depend on particular features such as curvature.

The other modification, \( f(R) \) modified gravity, replaces the Ricci scalar \( R \) in the Einstein-Hilbert action
\[ S_{\text{EH}} = \int \sqrt{-g} R \, d^4x \]
by a nonlinear function \( f(R) \):
\[ S_{f(R)} = \int \sqrt{-g} f(R) \, d^4x. \]
(For a review, see Refs. [2, 17, 18].) Wormhole geometries in \( f(R) \) modified gravitational theories are discussed in Ref. [3].

To describe a spherically symmetric wormhole spacetime, we take the metric to be [1]
\[ ds^2 = -e^{\Phi(r)} dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2). \] (2)
(We are using units in which \( c = G = 1 \).) Here we recall that \( b = b(r) \) is called the shape function and \( \Phi = \Phi(r) \) the redshift function. For the shape function we must have \( b(r_0) = r_0 \), where \( r = r_0 \) is the radius of the throat of the wormhole. To form a wormhole, \( b(r) \) must be an increasing function that also satisfies the flare-out condition \( b'(r_0) < 1 \) [1], as well as \( b(r) < r \) near the throat. These restrictions automatically result in the violation of the null energy condition (and hence the need for “exotic matter”) in classical general relativity, but not in \( f(R) \) modified gravity, as we will see.

Regarding the redshift function, we normally require that \( \Phi(r) \) remain finite to prevent an event horizon. In the present study involving \( f(R) \) gravity, we need to assume that \( \Phi(r) \equiv \text{constant} \), so that \( \Phi' \equiv 0 \). Otherwise, according to Lobo [3], the analysis becomes intractable. On the positive side, the condition \( \Phi' \equiv 0 \), called the zero-tidal-force solution in Ref. [1], is a highly desirable feature for a traversable wormhole.

Next, let us state the gravitational in the form used by Lobo and Oliveira [3]:
\[ \rho(r) = F(r) \frac{b'(r)}{r^2}, \] (3)
\[ p_r(r) = -F(r) \frac{b(r)}{r^3} + F'(r) \frac{rb'(r) - b(r)}{2r^2} - F''(r) \left[ 1 - \frac{b(r)}{r} \right]. \] (4)
and
\[ p_t(r) = -F'(r) \left[ 1 - \frac{b(r)}{r} \right] + \frac{F(r)}{2r^3} [b(r) - rb'(r)]. \] (5)
where $F = \frac{df}{dR}$. The curvature scalar $R$ is given by

$$R(r) = \frac{2b'(r)}{r^2}. \quad (6)$$

Some aspects of wormholes in $f(R)$ modified gravity with a noncommutative-geometry background have already been discussed in Ref. [19] based on the Gaussian distribution. The emphasis is on wormhole solutions obtained from the power-law form $f(R) = aR^n$, particularly for higher powers. This paper uses a different approach: we assume various forms of the shape function, together with a noncommutative-geometry background via Eq. (1) to determine not only the wormhole solutions and their surprisingly similar properties, but also the functions $F(r)$ and $f(R)$ needed for a complete solution. It is shown in the process that the energy violations can be attributed to the combined effects of noncommutative geometry and $f(R)$ gravity, without requiring exotic matter. These ideas are discussed in Sec. 3. Sec. 4 reverses the strategy by starting with a form of $f(R)$ and determining both the wormhole solution and the function $F(r)$.

### 3 Solutions based on specific shape functions

Our starting point is the fairly standard shape function

$$b(r) = r_0 \left( \frac{r}{r_0} \right)^\beta, \quad 0 < \beta < 1, \quad (7)$$

which includes the important special case $\beta = \frac{1}{2}$:

$$b(r) = \sqrt{r_0 r}. \quad (8)$$

In both cases, $\lim_{r \to \infty} b(r)/r = 0$, so that the spacetimes are asymptotically flat. Also, in both cases, $b(r_0) = r_0$. Since

$$b'(r) = \beta \left( \frac{r}{r_0} \right)^{\beta - 1}, \quad (9)$$

we have $b'(r_0) = \beta < 1$, so that the flare-out condition is satisfied. It also follows from Eqs. (1) and (3) that

$$\frac{M \sqrt{\alpha}}{\pi^2 (r^2 + \alpha)^2} = F(r) \frac{\beta (r/r_0)^{\beta - 1}}{r^2}$$

and

$$F(r) = \frac{M \sqrt{\alpha}}{\pi^2 (r^2 + \alpha)^2} \frac{r^{3-\beta}}{r_0^{1-\beta}}. \quad (10)$$

The expression for $F(r)$ allows us to compute both the radial and transverse pressures from Eqs. (4) and (5), respectively. We also observe that in making use of Eq. (1), the right-hand sides of Eqs. (3)-(5) remain intact. Only the stress-energy tensor is modified; so the length scales are macroscopic [10].
As noted earlier, for any wormhole, the flare-out condition \( b'(r_0) < 1 \) must be met. For a Morris-Thorne wormhole this condition is sufficient to yield a violation of the null energy condition (NEC), \( \rho + p_r < 0 \), at the throat: if \( F(r) \equiv 1 \) in Eqs. (3) and (4), then

\[
\rho(r_0) + p_r(r_0) = r_0b'(r_0) - b(r_0) < 0
\]  

(11)
since \( b(r_0) = r_0 \). For \( f(R) \) gravity, this conclusion no longer holds because of the dependence on \( F(r) \). The violation must therefore be checked separately.

Since the general expression for \( \rho + p_r \) is rather lengthy, we will check the violation only at the throat and, by continuity, in its immediate vicinity:

\[
\rho(r_0) + p_r(r_0) = \frac{M\sqrt{\alpha}}{\pi^2(r_0^2 + \alpha)^2} \left( \frac{r_0^2 - \beta b(r_0)}{r_0^3} + F'(r_0) \frac{r_0b'(r_0) - b(r_0)}{2r_0^2} - F''(r_0) \left( 1 - \frac{b(r_0)}{r_0} \right) \right)
\]

(12)
since \( b(r_0) = r_0 \) and \( b'(r_0) = \beta \). Substituting

\[
F'(r_0) = -\frac{r_0^{2-\beta}[(\beta + 1)r_0^2 + \alpha(\beta - 3)]}{(r_0^2 + \alpha)^2},
\]

Eq. (12) can be reduced further to become

\[
\rho(r_0) + p_r(r_0) = \frac{M\sqrt{\alpha}}{\pi^2(r_0^2 + \alpha)^2} \left( 1 - \frac{1}{\beta} + \frac{1}{2\beta} r_0^2 - \beta^2 r_0^2 + 4\alpha \beta - 3\alpha + \alpha \beta^2 \right).
\]

(13)
The final form is

\[
\rho(r_0) + p_r(r_0) = \frac{M\sqrt{\alpha}}{\pi^2(r_0^2 + \alpha)^2} \frac{(\beta - 1)[(1 - \beta)r_0^2 + \alpha(5 - \beta)]}{2\beta(r_0^2 + \alpha)} < 0.
\]

(14)
since \( \beta < 1 \). So the NEC has been violated. (The dependence on Eqs. (11) and (3) shows that this conclusion can be attributed to the combined effects of noncommutative geometry and \( f(R) \) gravity, without requiring exotic matter.)

In the special case \( \beta = \frac{1}{2} \), which will be needed later, we get

\[
\rho(r_0) + p_r(r_0) = \frac{M\sqrt{\alpha}}{\pi^2(r_0^2 + \alpha)^2} \left( -1 + \frac{3r_0^2 - 5\alpha}{4r_0^2 + 4\alpha} \right) < 0.
\]

(15)
So all the conditions for a traversable wormhole have been met.

To finish the discussion, we need to return to Eq. (11),

\[
F(r) = \frac{M\sqrt{\alpha}}{\pi^2(r^2 + \alpha)^2} \frac{1}{\beta r^{1-\beta} - 3r_0^{1-\beta}}
\]

(16)
and
\[ R(r) = \frac{2b'}{r^2} = 2\beta r^{\beta - 3}r_0^{1 - \beta}. \quad (17) \]

So making use of Eq. (16), \( F(r) \) can now be written
\[ F(r) = \frac{2}{2\beta r^{\beta - 3}r_0^{1 - \beta}} \rho = \frac{2}{R} \rho. \quad (18) \]

To derive \( F(R) \), we need to invert \( R(r) \) to obtain \( r(R) \). From Eq. (17),
\[ r(R) = \left( \frac{2\beta r_0^{1 - \beta}}{R} \right)^{1/(3 - \beta)}. \quad (19) \]

Also needed is \( r_0(R) \):
\[ R_0 = R(r_0) = \frac{2\beta}{r_0}, \quad (20) \]
so that
\[ r_0 = \left( \frac{2\beta}{R_0} \right)^{1/2}. \quad (21) \]

Thus
\[ r = \left( \frac{2\beta(2\beta/R_0)^{1/2}(1 - \beta)}{R} \right)^{1/(3 - \beta)}. \quad (22) \]

Eq. (18) now yields
\[ F(R) = \frac{M \sqrt{\alpha}}{\pi^2} \frac{2}{R} \left( \left[ \frac{R}{2\beta} \left( \frac{2\beta}{R_0} \right)^{1/2(\beta - 1)} \right]^{2/(\beta - 3)} + \alpha \right)^{-2}. \quad (23) \]

For the special case \( \beta = \frac{1}{2} \), we get
\[ F(R) = \frac{2M \sqrt{\alpha}}{\pi^2} \frac{1}{R} \left( RR_0^{1/4} \right)^{-4/5} \left( -4/5 + \alpha \right)^{-2}. \quad (24) \]

Recalling that \( \frac{df}{dR} = F \), integration yields the following closed form:
\[ f(R) = \frac{10M \sqrt{\alpha}}{\pi^2} \frac{(\alpha R_0^{1/5} R^{4/5} + 1) \ln(\alpha R_0^{1/5} R^{4/5} + 1) - \alpha R_0^{1/5} R^{4/5}}{2\alpha^2(\alpha R_0^{1/5} R^{4/5} + 1)} + C. \quad (25) \]

The graph of \( f(R) \) is shown in Fig. 1 for \( C = 0 \).

For the purpose of comparison and gaining an overview, we now consider another shape function,
\[ b(r) = r_0 + ar_0 \left( 1 - \frac{r_0}{r} \right), \quad 0 < a < 1. \quad (26) \]

This time we get
\[ b'(r) = a \left( \frac{r_0}{r} \right)^2, \quad (27) \]
Figure 1: The graph of $f(R)$ corresponding to the shape function $b(r) = \sqrt{r_0}r$.

so that $b'(r_0) = a < 1$, and the flare-out condition has been met. Next, we find that

$$F(r) = \frac{M\sqrt{\alpha}}{a\pi^2 r_0^2 (r^2 + \alpha)^2},$$

(28)

while

$$\rho(r) + p_r(r)|_{r=r_0} = \frac{M\sqrt{\alpha}}{\pi^2 (r_0^2 + \alpha)^2} \left[ \frac{a - 1}{a} \left( 1 + \frac{2\alpha}{r_0^2 + \alpha} \right) \right] < 0$$

(29)

since $a < 1$. So the NEC is violated at the throat.

To obtain $f(R)$, we proceed as before, starting with $F(r)$:

$$F(r) = \frac{r^2 \rho}{b'(r)} = \frac{r^4}{ar_0^2} \rho,$$

(30)

$$R(r) = \frac{2b'}{r^2} = \frac{2ar_0^2}{r^4},$$

(31)

and

$$R_0 = R(r_0) = \frac{2a}{r_0^2},$$

(32)

resulting in $r_0^2 = 2a/R_0$. To obtain $r(R)$, we note that $r^4R = 2ar_0^2 = 2a(2a/R_0)$ and

$$r^2 = \frac{2a}{\sqrt{R_0}R}.$$  

(33)
Returning to $F(r)$, 
\[ F(r) = \frac{r^4}{a(2a/R_0)} \rho = \frac{R_0}{2a^2} (r^2)^2 \rho \]  
and 
\[ F(r) = \frac{R_0}{2a^2 R_0 R} \rho = \frac{2}{R} \rho, \]  
in agreement with Eq. (18). So 
\[ F(R) = \frac{2}{R} \frac{M \sqrt{\alpha}}{\pi^2} \left( \frac{2a}{\sqrt{R_0 R}} + \alpha \right)^{-2}. \]  
This yields the following closed form for $f(R)$: 
\[ f(R) = \frac{4M \sqrt{\alpha}}{\pi^2} \frac{\alpha \sqrt{R_0 R} + 2a}{\alpha^2 (\alpha \sqrt{R_0 R} + 2a)} + C. \]  
Continuing the comparison, suppose we consider the shape function 
\[ b(r) = r_0 + a(r - r_0), \quad a < 1, \]  
leading to $b'(r) = a < 1$, so that the flare-out condition is satisfied. This time let us simply list the basic features: 
\[ F(r) = \frac{M \sqrt{\alpha}}{\pi^2 a} \frac{r^2}{(r^2 + \alpha)^2}, \]  
\[ F'(r) = \frac{M \sqrt{\alpha}}{\pi^2 a} \frac{-2r(r^2 - \alpha)}{(r^2 + \alpha)^3}, \]  
and 
\[ \rho(r_0) + p_r(r_0) = \frac{M \sqrt{\alpha}}{\pi^2} \frac{a - 1}{\alpha^2 (r_0^2 + \alpha)^2} \left( 1 - \frac{r_0^2 - \alpha}{r_0^2 + \alpha} \right). \]  
So $\rho(r_0) + p_r(r_0) < 0$, since $a < 1$. Finally, $F(r) = (2/R) \rho$, as before, leading to 
\[ F(R) = \frac{2M \sqrt{\alpha}}{\pi^2} \frac{1}{R(2a/R + \alpha)^2} \]  
and 
\[ f(R) = \frac{2M \sqrt{\alpha}}{\pi^2} \frac{(\alpha R + 2a) \ln(\alpha R + 2a) - \alpha R}{\alpha^2 (\alpha R + 2a)} + C. \]  
These comparisons have shown that all the shape functions have yielded surprisingly similar results. The reason is that the violation of the NEC at the throat will occur under fairly general conditions since $b(r_0) = r_0$ and $b'(r_0) < 1$: 
\[ \rho(r) + p_r(r) \big|_{r=r_0} = \frac{M \sqrt{\alpha}}{\pi^2} \frac{1}{(r^2 + \alpha)^2} - F(r) \frac{b}{r^3} \]  
\[ + \frac{F'(r)}{2r^2} (b' r - b) - F''(r) \left( 1 - \frac{b}{r} \right) \big|_{r=r_0}. \]
will be less than zero if \( F(r_0) > 0 \) and \( F'(r_0) < 0 \), primarily because \( \alpha \ll 1 \).

An interesting contrast is provided by the shape function

\[
b(r) = \frac{r^2}{r_0}.
\]

(45)

Here we obtain

\[
F(r) = -\frac{M\sqrt{\alpha}}{\pi^2r_0^2} \frac{r^4}{(r^2 + \alpha)^2}
\]

and

\[
F'(r) = -\frac{M\sqrt{\alpha}}{\pi^2r_0^2} \frac{4\alpha r^3}{(r^2 + \alpha)^3}.
\]

(46)

(47)

The result is

\[
\rho(r_0) + p_r(r_0) = \frac{2M\sqrt{\alpha}}{\pi^2(r_0^2 + \alpha)^2} \left( 1 + \frac{2\alpha}{r_0^2 + \alpha} \right) > 0.
\]

(48)

Since the NEC is met, we do not get a wormhole.

On the other hand, Einstein’s theory with a noncommutative-geometry background yields

\[
\rho(r_0) + p_r(r_0) = \left. \frac{M\sqrt{\alpha}}{\pi^2(r^2 + \alpha)^2} \left( 1 + \frac{2\alpha}{r_0^2 + \alpha} \right) \right|_{r=r_0} < 0
\]

since \( \alpha \ll 1 \). Here the NEC is indeed violated.

As noted by Lobo and Oliveira [3], if we assume that the matter threading the wormhole satisfies the NEC, it is the higher-order curvature terms that sustain the wormhole. It is also clear from Eqs. (14), (15), (29), and (41) that noncommutative geometry is also a contributing factor due to the small value of \( \alpha \). We can see from Eqs. (48) and (49), however, that the combined effects of \( f(R) \) gravity and noncommutative geometry may differ from the individual effects.

4 Solutions based on \( f(R) \)

Ref. [19] considered the form \( f(R) = aR^n \) and obtained a number of wormhole solutions corresponding to different values of \( n \). Observe that \( \frac{df}{dR} = F(R) = bR^{n-1} \) for some constant \( b \). The case \( n = 1 \) corresponds to Einstein gravity (again assuming a noncommutative-geometry background). In this section we will consider \( R^2 \) (quadratic) gravity, in part because this is believed to be a physically viable model [20].

So we start with

\[
\frac{df}{dR} = aR = \frac{2ab'}{r^2} = F.
\]

(50)

From Eq. (6),

\[
b' = \frac{r^2\rho}{F} = \frac{r^2\rho}{2ab'/r^2}.
\]

Thus

\[
b'(r) = \frac{r^2\sqrt{\rho(r)}}{\sqrt{2a}}
\]

(51)
Continuing with the wormhole solution,

\[ b(r) = \left( \frac{M\sqrt{\alpha}}{2a\pi^2} \right)^{1/2} \int \frac{r^2dr}{r^2 + \alpha} = \frac{\sqrt{M\alpha^{1/4}}}{\sqrt{2a\pi}} \left( r - \sqrt{\alpha} \tan^{-1} \frac{r}{\sqrt{\alpha}} + C \right). \] (52)

To satisfy the condition \( b(r_0) = r_0 \), we must have

\[ C = r_0 \frac{\sqrt{2a\pi}}{\sqrt{M\alpha^{1/4}}} - r_0 + \sqrt{\alpha} \tan^{-1} \frac{r_0}{\sqrt{\alpha}}, \]

so that

\[ b(r) = \frac{\sqrt{M\alpha^{1/4}}}{\sqrt{2a\pi}} \left( r - \sqrt{\alpha} \tan^{-1} \frac{r}{\sqrt{\alpha}} + r_0 \frac{\sqrt{2a\pi}}{\sqrt{M\alpha^{1/4}}} - r_0 + \sqrt{\alpha} \tan^{-1} \frac{r_0}{\sqrt{\alpha}} \right). \] (53)

Next,

\[ b'(r) = \frac{\sqrt{M\alpha^{1/4}}}{\sqrt{2a\pi}} \left( 1 - \frac{1}{1 + r^2/\alpha} \right). \] (54)

So \( b'(r) > 0 \) and \( b'(r) < 1 \) since \( \alpha \ll 1 \). The flare-out condition is thereby satisfied.

It remains to determine \( F(r) \). Since \( b' = r^2\rho(r)/F \), we have from Eq. (51),

\[ \frac{r^2\sqrt{\rho(r)}}{\sqrt{2a}} = \frac{r^2\rho(r)}{F}, \]

hence

\[ F(r) = \sqrt{2a}\sqrt{\rho(r)} = \sqrt{2a\pi} \sqrt{M\alpha^{1/4}} \frac{1}{r^2 + \alpha}. \] (55)

Alternatively,

\[ \frac{df}{dR} = aR = \frac{2ab'}{r^2} = \frac{2a}{r^2} \frac{r^2\rho(r)}{\sqrt{2a}} = \sqrt{2a}\rho(r), \] (56)

as before. So \( \frac{df}{dR} \) reduces to \( F(r) \).

Finally, the NEC is violated at the throat since \( F(r) > 0 \) and \( F'(r) < 0 \).

5 Results and Discussion

This paper discusses the possible existence of traversable wormholes in the context of \( f(R) \) modified gravity, given a noncommutative-geometry background based on the Lorentzian distribution in Eq. (1).

The first part of this paper provides an overview by using several specific shape functions to show that the corresponding wormhole solutions share some fairly general common properties. An original objective was to obtain complete wormhole solutions by determining the modified-gravity functions \( F(r) \) and \( f(R) \). The second part of the paper reverses the strategy by starting with \( R^2 \) gravity, i.e., the power-law form \( f(R) = aR^2 \), which is considered to be a physically viable model [20]. This is followed by a determination of the wormhole solution, including the function \( F(r) \). Zero tidal forces are assumed throughout.
Implicit in the discussion is that these gravitational theories are essentially independent, so that their effects can be studied separately. On the other hand, they are not mutually exclusive, so it becomes equally important to study the combined effects. In particular, it is shown that the energy violation can be attributed to the combined effects of $f(R)$ gravity and noncommutative geometry, thereby avoiding the need for exotic matter. Moreover, the combined effects may differ from the individual effects.

6 Conclusions

Some aspects of wormholes in $f(R)$ modified gravity with a noncommutative-geometry background have already been considered in Ref. [19]. The discussion is based on a Gaussian distribution and concentrates mainly on higher powers of the power-law form $f(R) = aR^n$. By contrast, the purpose of the present study is to seek more complete solutions by determining the modified-gravity functions $F(r)$ and $f(R)$.

The two gravitational theories are independent without being mutually exclusive. As noted above, the combined effects may differ from the individual effects, as exemplified by the discussion of the violation of the null energy condition.

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