Measuring Modular Weights in Mirage Unification Models at the LHC and ILC

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Abstract

String compactification with fluxes yields MSSM soft SUSY breaking terms that receive comparable contributions from modulus and anomaly mediation whose relative strength is governed by a phenomenological parameter $\alpha$. Gaugino and first/second generation (and sometimes also Higgs and third generation) scalar mass parameters unify at a mirage unification scale $Q \neq M_{\text{GUT}}$, determined by the value of $\alpha$. The ratio of scalar to gaugino masses at this mirage unification scale depends directly on the scalar field modular weights, which are fixed in turn by the brane or brane intersections on which the MSSM fields are localized. We outline a program of measurements which can in principle be made at the CERN LHC and the International Linear $e^+e^-$ collider (ILC) which can lead to a determination of the modular weights.

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Superstring theory provides a consistent quantum theory of gravity, together with all the necessary ingredients for a theory that potentially unifies all four forces of nature. However, in order to make any contact with phenomenology, it is essential to understand how the degeneracy associated with the many flat directions in the space of scalar fields (the moduli) is lifted to yield the true ground state, since many quantities relevant for physics at accessible energies are determined by the ground state values of these moduli. The discovery of a new class of compactifications, where the extra spatial dimensions are curled up to small sizes with fluxes of additional fields trapped along these extra dimensions has been exploited by Kachru et al. (KKLT)\(^1\) to construct a concrete model with a stable, calculable ground state with a positive cosmological constant and broken supersymmetry. This toy model is based on type-IIb superstrings including compactification with fluxes to a Calabi-Yau orientifold. While the background fluxes serve to stabilize the dilaton and the moduli that determine the shape of the compact manifold, it is necessary to invoke a non-perturbative mechanism such as gaugino condensation on a \(D7\) brane to stabilize the size of the compact manifold. Finally, a non-supersymmetric anti-brane \((D3)\) is included in order to break supersymmetry and obtain a de Sitter universe as required by observations. The resulting low energy theory thus has no unwanted light moduli, has a broken supersymmetry, and a positive cosmological constant, but of course does not yield the Standard Model (SM). The existence of these flux compactifications with stable calculable minima having many desired properties may be viewed as a starting point for the program of discovering a string ground state that may lead to the (supersymmetric) Standard Model at low energies, and which is consistent with various constraints from cosmology.

These considerations have recently motivated several authors to analyze the structure of the soft SUSY breaking (SSB) terms in models based on a generalization of the KKLT set-up \(^2\). The key observation is that because of the mass hierarchy,

\[
m_{\text{moduli}} \gg m_3/2 \gg m_{\text{SUSY}},
\]

that develops in these models, these terms receive comparable contributions via both modulus (gravity) and anomaly mediation of SUSY breaking\(^3\), with their relative size parametrized by one new parameter \(\alpha\). Moreover, the hierarchy \(^4\) that leads to this mixed modulus-anomaly mediated SUSY breaking (MM-AMSB) automatically alleviates phenomenological problems from late decaying moduli and gravitinos that could disrupt, for instance, the predictions of light element abundances from Big Bang nucleosynthesis. Upon integrating out the heavy dilaton field and the shape moduli, we are left with an effective broken supergravity theory of the observable sector fields denoted by \(\hat{Q}\) and the size modulus field \(\hat{T}\). The Kähler potential depends on the location of matter and Higgs superfields in the extra dimensions via their modular weights \(n_i = 0 (1)\) for matter fields located on \(D7\) (\(D3\)) branes, or \(n_i = 1/2\) for chiral multiplets on brane intersections, while the gauge kinetic function \(f_a = \hat{T}^{l_a}\), where \(a\) labels the gauge group, is determined by the corresponding location of the gauge supermultiplets, since the power \(l_a = 1 (0)\) for gauge fields on \(D7\) (\(D3\)) branes \(^4\).

Within the MM-AMSB model, the SSB gaugino mass parameters, trilinear SSB parameters and sfermion mass parameters, all renormalized just below the unification scale (taken to be \(Q = M_{\text{GUT}}\)), are given by,

\[
M_a = M_s \left( l_a \alpha + b_a y_a^2 \right),
\]
\[ A_{ijk} = M_s (-a_{ijk} \alpha + \gamma_i + \gamma_j + \gamma_k), \quad (3) \]
\[ m_i^2 = M_s^2 \left( c_i \alpha^2 + 4 \alpha \xi_i - \dot{\gamma}_i \right), \quad (4) \]

where \( M_s \equiv \frac{m_3}{2\sqrt{16\pi^2}} \), \( b_a \) are the gauge \( \beta \) function coefficients for gauge group \( a \) and \( g_a \) are the corresponding gauge couplings. The coefficients that appear in (2) – (4) are given by:

\[ c_i = 1 - n_i, \]
\[ a_{ijk} = 3 - n_i - n_j - n_k \]
\[ \xi_i = \sum_{j,k} a_{ijk} \frac{y_{ijk}}{4} - \sum_a l_a g_a^2 C_a^2 (f_i). \]
\[ \dot{\gamma}_i = 8\pi \frac{\partial \gamma_i}{\partial \log \mu}. \]

Expressions for the last two quantities involving the anomalous dimensions can be found in the Appendix of Ref. [5].

The MM-AMSB model is completely specified by the parameter set,

\[ m_{3/2}, \alpha, \tan \beta, \text{sign}(\mu), n_i, l_a. \quad (5) \]

The mass scale for the SSB parameters is dictated by the gravitino mass \( m_{3/2} \). The phenomenological parameter \( \alpha \), which could be of either sign, determines the relative contributions of anomaly mediation and gravity mediation to the soft terms, and as mentioned above \( |\alpha| \sim \mathcal{O}(1) \) is the hallmark of this scenario. Non-observation of large flavor changing neutral currents implies common modular weights of particles with the same gauge quantum numbers. Grand Unification implies matter particles within the same GUT multiplet have common modular weights, and that the \( l_a \) are universal. We will assume that all \( l_a = l \) and, for simplicity, a common modular weight for all matter particles, but allow a different (common) one for the two Higgs doublets of the MSSM. The main purpose of this analysis is to see to what extent it will be possible to confirm our assumptions and deduce the value of \( l \) and the modular weights, assuming that SUSY is discovered at the LHC and is further studied at a TeV \( e^+e^- \) linear collider. Other aspects of MM-AMSB phenomenology have been examined in the literature [4, 5, 6, 7, 8].

The universality of the \( l_a \) leads to the phenomenon of \textit{mirage unification} [4, 5] of gaugino masses. In other words, gaugino mass parameters \( M_i \) (assuming that these can be extracted from the data) when extrapolated using one loop renormalization group equations (RGEs) would unify at a scale \( Q = \mu_{\text{mir}} \neq M_{\text{GUT}} \), the scale of unification of gauge couplings. Indeed, the observation of gaugino unification at the mirage unification scale,

\[ \mu_{\text{mir}} = M_{\text{GUT}} e^{-8\pi^2/(l\alpha)}, \quad (6) \]

would strikingly point to such a scenario. If \( \alpha < 0 \), \( \mu_{\text{mir}} > M_{\text{GUT}} \), though one would have to continue extrapolation using MSSM RGEs to discover this! We assume here that \( l \neq 0 \), since this would be distinguished by a gaugino mass pattern as in the AMSB framework. While \( \mu_{\text{mir}} \) determines \( l\alpha \), the (unified) value of the the gaugino masses extrapolated to \( Q = \mu_{\text{mir}} \) is \( M_{\alpha}(\mu_{\text{mir}}) = M_s \times (l\alpha) \), and so gives the value of \( M_s \) (and so \( m_{3/2} \)).

We show the mirage unification scale versus \( l\alpha \) in Fig. 1 for \( l = 1 \). The existence of a mirage unification scale is taken to be a “smoking gun” signature for MM-AMSB models. If supersymmetry is discovered and the various soft parameters are precisely measured at the weak scale, then extrapolation of the soft parameters via the RGEs to a point of unification [9]
Figure 1: A plot of the mirage unification scale versus modulus-AMSB mixing parameter $\alpha$, assuming $l = 1$.

at a scale $\mu_{\text{mir}} \neq M_{\text{GUT}}$ would indicate that nature is in fact described by a MM-AMSB model with mirage unification! In the process, the scale $\mu_{\text{mir}}$, or equivalently $l\alpha$, would be measured.

In the MM-AMSB framework with universal matter modular weights (for the first two generations whose Yukawa couplings are negligible), the SSB matter mass parameters also unify at $Q = \mu_{\text{mir}}$, with $m^2_{\text{mi}}(\mu_{\text{mir}}) = M^2_{\text{mi}}c_i\alpha^2$. If the extrapolated values of selectron or first generation squark mass parameters indeed converge at the same unification scale as gaugino parameters, it would provide striking confirmation of this framework. Taking the ratio of first/second generation scalar to gaugino mass parameters yields,

$$\frac{m_i}{M_0}|_{\mu_{\text{mir}}} = \frac{\sqrt{c_i}}{l}.$$  \hspace{1cm} (7)

The obvious question is whether it is possible to disentangle the values of $c_i$ and $l$. A look at the boundary conditions for the gaugino and first/second generation SSB parameters shows that these depend only on the combinations $M_s$, $l\alpha$ and $c_i/l^2$: this is obvious for the gaugino masses, while for the scalar masses, this is clearly also the case since $\xi_i \propto l$ as long as the Yukawa couplings are negligible. Thus it is impossible even in principle to disentangle $c_i$ and $l$ from these measurements alone. To do so, even in principle, it is essential to determine either the SSB third generation mass parameters or the $A$-parameters. While it is clear that the boundary condition (3) depends on $c_i/l$ (together with $l\alpha$ and $M_s$), it is not difficult to check that the Yukawa coupling terms in $\xi_i$ also depend on $c_i/l$. A precise determination of third generation SSB or of $A$ parameters would, in principle, allow us to separately obtain $l$ and thus check whether or not this is unity. This may well be possible via a study of the stau sector at an electron-positron collider\cite{10}, and perhaps, via the stop sector if $e^+e^- \rightarrow \tilde{t}_1\tilde{t}_1$ pairs is accessible. We will not examine this any further but assume that $l = 1$ for the remainder of this paper.

In this case it is clear that the matter modular weights can be determined from (7) once SSB scalar and gaugino mass parameters are determined. What would it take to measure these?
It would likely take a combination of measurements from the CERN LHC and a linear \( e^+e^- \) collider such as the proposed International Linear Collider (ILC), which would operate at CM energies of around \( \sqrt{s} \sim 0.5 - 1 \) TeV, and/or by the CERN CLIC linear collider, which is proposed to operate in the multi-TeV regime [11].

The weak scale gaugino mass \( M_3 \) at tree level is the same as the gluino mass \( m_{\tilde{g}} \), although the relation between these quantities gets corrected by known loop effects that give corrections up to \( \sim 30\% \) [12] (though in the present framework, we do not expect very large corrections because the ratio \( m_{\tilde{q}}/m_{\tilde{g}} \) is not especially large). The gluino mass has been shown to be measurable at the LHC in several benchmark cases via \( \tilde{g}\tilde{g} \) production followed by gluino cascade decays [13].

Another measurement LHC can make is the \( m_{\tilde{Z}_2} - m_{\tilde{Z}_1} \) mass difference if \( \tilde{Z}_2 \to \tilde{Z}_1 \ell \bar{\ell} \) decays occur at a sufficient rate [14]. In this case, the dilepton invariant mass distribution will offer one strong constraint on the neutralino mass matrix, which depends on the gaugino masses \( M_1 \) and \( M_2 \), as well as on the superpotential Higgs mass term \( \mu \) and the ratio of Higgs vevs \( \tan \beta \). Moreover, from the shape of the end-point of the \( m_{\ell\ell} \) spectrum it may be possible to determine whether or not the higgsino component of the neutralinos is large or small, at least in the case that \( M_1/M_2 > 0 \) at the weak scale [17]: for very small higgsino components, \( m_{\tilde{Z}_2} - m_{\tilde{Z}_1} = M_2 - M_1 \).

The gaugino masses \( M_1 \) and \( M_2 \), and possibly the parameter \( \mu \), may be extracted at a LC by a combination of measurements of \( \tilde{W}_1^+\tilde{W}_1^- \) production, \( \tilde{Z}_1\tilde{Z}_2 \) production and \( \tilde{W}_1^\pm\tilde{W}_2^\mp \) production [15, 16, 17, 18]. While really a measurement of only two of the three SSB gaugino masses is necessary to establish the value of \( \mu_{\text{mir}} \) and \( M_a(\mu_{\text{mir}}) \), the measurement and extrapolation of the third gaugino mass would offer striking support for a mirage unification hypothesis.

Turning to matter scalar masses, the CERN LHC has some ability to measure squark masses, at least in some benchmark studies [13], although it will be difficult to tell the flavour or type of squark being produced. It may also be possible for LHC to extract some information on slepton masses, not so much from direct slepton production [19] as much as from their production in cascade decays in fortuitous cases, or via their influence on the shape of the dilepton invariant mass spectrum from \( \tilde{Z}_2 \to \tilde{Z}_1 \ell \bar{\ell} \) or \( \tilde{Z}_2 \to \ell \bar{\ell} \) decays [20].

For a LC, the first and second generation \( \tilde{\ell}_R, \tilde{\ell}_L \) and \( \tilde{\nu}_\ell \) masses should be readily measured if pair production of these scalars is allowed either through the lepton energy spectrum endpoints [15, 16] or via threshold measurements [18]. In addition, if squark pair production is accessible, then squark masses should be measurable to some degree, along with squark type, using the beam polarization tool [21]. Again, only two scalar masses (such as \( m_{\tilde{t}_L} \) and \( m_{\tilde{t}_R} \)) need be measurable to establish mirage unification at \( \mu_{\text{mir}} \) (which should coincide with the unification scale obtained via gauginos) and the associated soft term masses at \( \mu_{\text{mir}} \) that can yield information about the corresponding modular weights, and also serve to test our hypothesis that the modular weights are the same for all matter particles.

It would be interesting to be able to check that \( l = 1 \). As discussed above, this entails a determination of either the \( A \)-parameters or third generation SSB masses whose evolution receives sizeable contributions from Yukawa couplings. This appears to be very difficult at the LHC, though in some fortuitous cases where \( \tilde{b}_1 \) is light enough to be produced in gluino cascade decays some information may be possible [13]. Stau production at the ILC may offer the best access to the third generation parameters since, at least in favourable cases, the mass
as well as the stau mixing angle may be determined [10]. Unfortunately, unless tan β is also large, the effects of the Yukawa couplings that are essential for separating out the value of l will be small. Information about l can presumably be obtained via a study of t-squark system at an electron-positron collider with sufficiently high energy, but only if $n_H$ can be obtained via measurements in the Higgs sector.

Higgs scalar SSB mass parameters appear to be especially interesting because these can potentially be used to both determine the modular weights in the Higgs sector, and to obtain information on l (since their boundary condition depends also on the Yukawa couplings). These may be extracted at a linear collider if the heavy neutral and/or charged Higgs bosons are accessible. We note that one of the tree level MSSM scalar potential minimization conditions reads $\mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{(\tan^2 \beta - 1)} - \frac{M_Z^2}{2}$ while the pseudoscalar Higgs mass $m_A$ is given by $m_A^2 = m_{H_u}^2 + m_{H_d}^2 + 2\mu^2$, so that in principle a determination of $\mu$, $m_A$ and tan β would determine these quantities. Of course, these tree level relations suffer important loop corrections that depend on other sparticle masses, which would have to be taken into account. A variety of cases have been investigated at both the LHC and the ILC for measuring the heavier Higgs boson masses and the parameter tan β [22, 11], and as noted earlier, $\mu$ should be extractable at an ILC especially if $W_1^+ \bar{W}_2^-$ production is accessible. The extraction of Higgs modular weights is, however, more complicated than for first/second generation matter since, because of Yukawa coupling effects, the weak scale values of $m_{H_u}^2$ and $m_{H_d}^2$ are not expected to extrapolate (via one loop evolution) to a common value at $Q = \mu_{m_{\text{mir}}}$ except for the special cases 3 and 8 in Table 1 below; for these special cases, 11 applies, and the value of $m_H^2(\mu_{m_{\text{mir}}})$ yields $\sqrt{c_H/l}$. In principle, the GUT scale value of the Higgs SSB parameters depend on $c_l/l$ so it is possible that if these can be determined to a sufficiently good precision, these can be used to extract the value of l, and check that this is consistent with that obtained via a study of staus or top squarks. For the other cases in Table 1 the extraction of $n_H$ seems more difficult.1

We illustrate in Fig. 2(b) the gaugino mass unification in an MM-AMSB model with $\alpha = 6$, $m_{3/2} = 12$ TeV, tan β = 10 and $\mu > 0$ for $m_t = 175$ GeV, $n_{\text{matter}} = \frac{1}{2}$ and $n_H = 1$. It is apparent that $\mu_{m_{\text{mir}}} \sim 10^{11}$ GeV, while $M_a(\mu_{m_{\text{mir}}}) \sim 450$ GeV. In Fig. 2(b), we show the evolution of various matter and Higgs scalar soft masses from $M_{\text{weak}}$ to $M_{\text{GUT}}$. The soft parameters again unify at $\sim 10^{11}$ GeV, while matter scalars have a mass $\sim 320$ GeV and Higgs scalars have a mass $\sim 0$ GeV. We have checked that in fact the Higgs masses evolve to zero at $Q = \mu_{m_{\text{mir}}}$ if one-loop RGEs are used, so that the off-set of $m_{H_{u,d}}$ at $Q = \mu_{m_{\text{mir}}}$ is a consequence of the two-loop RGEs that are inherent in Isajet, which we use for our calculation of sparticle masses [23]. We stress that first/second generation masses always unify at $\mu_{m_{\text{mir}}}$, while the unification of third generation and Higgs SSB mass parameters is special to the choice of modular weights.

In Table 1 we show nine cases of matter and Higgs field modular weights. It is clear that a determination of matter modular weights from masses of gauginos and first generation sfermions at future colliders will localize the models in one of three groups where $m_i/M_a$ at the unification scale is 0 (cases 3, 6, 9), $1/\sqrt{2}$ (cases 2, 5, 8) or 1 (cases 1, 4, 7). Information

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1If we assume $l = 1$, and assume a universal value of $n_{\text{matter}}$, it should be possible to extract $n_H$ by extrapolating the Higgs SSB mass parameters to the GUT scale with sufficient precision. Since this requires a knowledge of many masses and their mixings, we do not make any representation that this can be done in practice.
about third generation, Higgs or trilinear SSB parameters will be essential to further separate the degeneracies. If, for instance, third generation mass parameters also unify at $\mu_{\text{mir}}$, we will know that we are in cases 3 or 8. In other cases, more careful scrutiny will be necessary since, for example, the distinction between cases 6 and 9 is only possible via the value of the Higgs or trilinear SSB parameters. As mentioned above, this may be possible if we assume $l = 1$ to determine $n_H$. If we can extrapolate the weak scale $A$-parameters to $M_{\text{GUT}}$, we can then test the consistency of this assumption: like the extrapolation of SSB Higgs mass parameters, this requires us to know masses and mixings of many sparticles, and detailed studies are needed to decide whether the extrapolation \cite{9} to $M_{\text{GUT}}$ can be done with the required precision. In the special cases 3 and 8, a complete determination of the modular weights along with the value of $l$ appears to be possible by combining the data from the LHC with that from an electron positron collider.

| case | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|---|---|---|---|---|---|---|---|---|
| $n_H$ | 0 | 0 | 0 | 1/2 | 1/2 | 1/2 | 1 | 1 | 1 |
| $n_{\text{matter}}$ | 0 | 1/2 | 1 | 0 | 1/2 | 1 | 0 | 1/2 | 1 |

Table 1: Nine cases of Higgs and matter modular weights which are explored in the text.

To summarize, in supersymmetric models with a KKLT type vacuum, SSB terms receive comparable contributions from modulus and anomaly mediated SUSY breaking resulting in
the phenomenon of mirage unification. The mirage unification scale should be measurable by extrapolation of soft SUSY breaking masses from $Q = M_{\text{weak}}$ to $Q = \mu_{\text{mir}}$ via one loop RGEs. The ratio of first/second generation soft masses to gaugino masses at the mirage unification scale offers a direct measurement of the scalar field modular weights which, in turn, provides information about the dimensionality of the branes on which these scalar fields reside.

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