Modulation and trapping of a test beam in a traveling wave tube

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Abstract. For wave-particle interaction studies, the paradigmatic 1D beam-plasma system can be advantageously replaced by a Traveling Wave Tube (TWT). We consider the case when the cold beam intensity is low enough to ensure that instabilities are ruled out. This allows to carefully study the interaction of a beam of essentially test electrons with a single wave propagating in the TWT. By recording the test beam energy distribution at the output of the tube, we report the experimental observation of the velocity modulation and synchronization of a non-resonant beam and of the beam trapping by a resonant wave.

1. Introduction
In plasma physics the wave-particle interaction is an important phenomenon, fundamental for numerous instabilities, wave heating, and many diagnostics. This complex phenomenon is at the essence of the well-known linear Landau damping [1, 2]. It also involves non-linear behavior such as particle trapping in a single wave or chaotic diffusion in a broad spectrum of waves.

A paradigm for the complete self-consistent description of wave-particle interaction is the 1D beam plasma system. In the case when the beam is sufficiently weak to allow to discriminate between resonant and non-resonant particles, this system can advantageously be replaced by a Traveling Wave Tube (TWT). The slow wave structure of the TWT plays essentially the same role as the plasma and supports the waves which interact with the electron beam.

This paper focuses on the study of test electrons interacting with a single wave propagating in a specially designed TWT. The test beam situation is obtained when the beam intensity is so low that the wave growth rate is negligible upon the length of the experiment. We are then able to explore the complex Hamiltonian dynamics of an electron in one wave.

The paper is organized as follows. Section 2 briefly recalls the theoretical description of the motion of a particle in a single wave. Section 3 describes the principle of the experiment in the TWT. Section 4 considers the experimental interaction of test electrons with one wave. Section 5 gives our conclusion and perspectives.

2. Motion of a test charged particle in one wave
The Hamiltonian for a particle, of mass $m$ and charge $q$, moving in a single electrostatic wave with wave number $k$ and frequency $\omega$ is

$$H = \frac{p^2}{2m} + q\phi \cos (kz - \omega t + \chi)$$

(1)
where \( \chi \) is a constant phase. The resonant phase velocity is \( v_\phi = \omega / k \). Classical mechanics tells that the motion of the charged particle in the frame moving at the phase velocity of the wave is the same as for the integrable nonlinear pendulum. For an energy lower than \( |q\phi| \), the particle is trapped in a potential trough of the wave, and the extension of this trapping, or resonant, domain in velocity space is equal to \( 2\sqrt{\eta\phi} \) around the phase velocity of the wave, with \( \eta = q/m \). This trapping motion of the particle corresponds to libration (or oscillation around the stable equilibrium position) of the nonlinear pendulum.

Let us now consider a particle with a velocity lying outside the trapping domain. Perturbation theory around the free particle motion yields a simple estimate of the particle velocity modulation. Let \( x = kz - \omega t + \chi, u = kv - \omega \) and \( \epsilon = \eta k^2 \phi \). Then the motion of a test electron with initial position \( x_0 \) and velocity \( u_0 \) is described by the differential equation \( \ddot{x} = \epsilon \cos(x) \) where the dot denotes derivation with respect to time \( t \). This equation of the classical non linear pendulum is analytically integrable using Jacobi elliptic functions for each particle. Using perturbation theory to second order in \( \epsilon \) around the unperturbed motion \( x = x_0 + u_0 t \), we estimate the velocity deviation and obtain

\[
\begin{align*}
u &= u_0 - \epsilon [\cos(x) - \cos(x_0) + u_0 t \sin(x_0)]/u_0^2 \\
&- \epsilon^2 [\cos(2x) - \cos(2x_0) - 4 \cos(u_0 t - 1)]/(4u_0^3) \\
&- t \sin(x_0) \cos(x)/u_0^3 \\
&+ \ldots
\end{align*}
\]

(2)

For the mean beam velocity obtained by averaging over initial position \( x_0 \) uniformly distributed in one wavelength, we get

\[
\langle u \rangle = u_0 + \epsilon^2 [\cos(u_0 t - 1)/u_0^3 + t \sin(u_0 t)/(2u_0^2)] + \ldots
\]

(3)

As expected, a first order estimate does not produce any secular term and Eq.(3) shows that the average velocity of the particles having a given initial velocity but arbitrary phase oscillates with the pulsation \( u_0 \) which corresponds to the velocity mismatch between the wave and the beam. These oscillations are physically very important and the first nonlinear synchronization of the test particle with the wave which occurs before the first minimum of the mean velocity deviation, i.e. for \( u_0 t \sim 4 \), is in fact responsible for the well-known Landau damping in the self-consistent case [3, 4].

Particle trapping and velocity modulation are universal phenomena. In this paper, we report a direct experimental evidence of the occurrence of these fundamental phenomena underlying the self-consistent theory of wave-particle interaction in a plasma [3, 5, 6].

3. Principle of the experiment

If we consider the 1D weak beam plasma system where the beam lies in the tail of the velocity distribution function, unstable waves propagate with phase velocities much larger than the plasma particles. Therefore the plasma particles are far from resonant with the waves and can be treated linearly as a dielectric which supports the waves. On the contrary beam particles interact non linearly with the waves. This statement leads to the fact that the 1D beam plasma system can be advantageously replaced by a Traveling Wave Tube (TWT). The bulk plasma is replaced by a slow wave structure which consists in a 4 m long helix along which the electromagnetic waves are propagating. Since the pitch of the helix is much smaller than its radius, the wave propagates along the axis of the helix with a phase velocity which is much smaller than the velocity of light and can be resonant with an electron beam [7]. As compared with the plasma, the TWT has the advantage that the slow wave structure remains linear for the wave amplitudes reached in the experiments; furthermore, it does not introduce noise. Saturation of the small–cold-beam-plasma instability due to the trapping of the electron beam inside the most unstable growing wave could thus be thoroughly studied [8] and the observations are in excellent agreement with the results of theory [9] and of numerical calculations [10].

More recently the TWT has been modified so that an electron beam with a controllable velocity spread [11] can propagate along the axis of the helix to mimic the gentle bump or weak-warm-beam-plasma instability whose saturation is currently described by quasilinear theory [5] and involves chaotic...
diffusion of electrons in a broad spectrum of unstable waves. This quasilinear scenario was confirmed experimentally in a plasma [12], but without much accuracy in the saturation regime. Beam-mediated mode-coupling effects [13] neglected in standard quasilinear theory could be observed in a TWT [14] but do not induce modification of the quasilinear prediction [15] leading to the so-called quasilinear paradox [16, 17].

In this paper, we consider a very weak cold beam. The beam intensity is so weak that the unstable waves exhibit a negligible growth upon the length of the experiment. The beam electrons can thus be considered as test particles submitted to externally excited waves with almost constant amplitude. The complex self-consistent effects above mentioned can be neglected and the Hamiltonian dynamics of a charged test particle can thus be experimentally explored.

The experiment is performed in a long Traveling Wave Tube (TWT) previously described in detail [18]. It consists of three main elements: an electron gun, a slow wave structure (SWS) formed by a helix with axially movable antennas, and an electron velocity analyzer. The principle of the test beam measurement is sketched in Fig. 1.

The electron gun creates a beam which propagates along the axis of the SWS, made of a 4 m long helix, and is confined by a strong axial magnetic field. Waves are launched with an antenna at a variable position along the helix, thus allowing to vary the interaction length. The antenna consists in a small piece of copper tangent to the vacuum glass chamber and capacitively coupled to the helix. The helix is long enough to allow nonlinear processes to develop. Launched electromagnetic waves travel along the helix with the speed of light; their phase velocities, $v_{\phi j}$, along the axis of the helix are smaller by approximately the tangent of the pitch angle, giving $2.8 \times 10^6 \text{m/s} < v_{\phi} < 5.3 \times 10^6 \text{m/s}$.

Finally the cumulative changes of the electron beam distribution are measured with a trochoidal velocity analyzer [19], located at the end of the interaction region and working on the principle that electrons undergo an $E \times B$ drift when passing through a region in which an electric field $E$ is perpendicular to a magnetic field $B$. A small fraction (0.5%) of the electrons passes through a hole in the center of the front collector, and is slowed down by retarding electrodes. Then the electrons having the correct drift energy determined by the potential difference on two parallel deflector plates are collected after passing through an off-axis hole at the back of the analyzer. The time averaged collected current is measured by means of a pico-ammeter. Retarding potential and measured current are computer controlled, allowing an easy acquisition and treatment with an energy resolution lower than 0.5 eV.
Figure 2. Measured velocity distribution function in a single wave at \( v_\phi = 3.55 \times 10^6 \text{ m/s} \) with increasing amplitude: 2D plot with 1st order estimates of modulation (continuous lines) domain.

4. Test particle experiments
In this paper, we study the interaction of the test beam with a single launched wave. The control parameters of the experiment are:

- the test beam entrance velocity,
- the emitted wave frequency (or phase velocity as determined from the SWS dispersion relation),
- the emitted wave amplitude,
- and the interaction length determined by the position of the emitting antenna.

The helix wave amplitude estimate is obtained by determining the emitting and receiving probe coupling coefficients using 3 probes measurements [20].

For a single wave, characterized by its amplitude \( \phi \) and frequency \( f \) launched by a fixed antenna at a given distance from the output of the device, the time-averaged velocity distribution function \((vdf)\) of the test electron beam is measured at the device output as sketched in Fig. 1. This procedure averages out the linear sloshing of particles due to the wave. The plots of the subsequent figures are the result of superposing measurements obtained by varying one of the controlling parameter keeping all the others constant. For each value of the varying control parameter \((vcp)\), the output beam \(vdf\) is recorded after interaction of the test beam with the wave propagating along the helix: the \(vdf\) is obtained by scanning the retarding voltage with a step of \(61 \text{ mV}\). The zero level of each \(vdf\) is defined as the mean trochoidal collector current averaged over 50 velocities in the tail of the \(vdf\). Each beam \(vdf\) is then normalized to keep the beam current constant. The final plot is obtained after an appropriate Matlab treatment of the recorded output \(vdfs\), giving a 3D plot of the \(vdf\) in \((vcp, v)\) plane, or a 2D contour plot of the amplitude of the distribution function in \((vcp, v)\) plane.

4.1. Non resonant test beam modulation
A single wave frequency \( f = 40 \text{ MHz} \), i.e. phase velocity \( v_\phi = 3.55 \times 10^6 \text{ m/s} \), is launched toward the entrance of the device by a fixed probe at \( L = 300 \text{ cm} \) from the device output. As explained before, Fig. 2 gives a 2D plot of the \(vdf\) detected by the trochoidal analyzer when the varying control parameter is the wave amplitude. It is the result of superposing measurements obtained for different wave amplitudes of the single wave varying from \(0 \text{ mV}\) to \(33 \text{ mV}\) by steps of \(2.75 \text{ mV}\).

When the wave amplitude is null, Fig. 2 shows that the \(vdf\) exhibits a single peak centered at \( v_b = 2.35 \times 10^6 \text{ m/s} \) which is the entrance velocity of the test beam with current \(I_b = 220 \text{ nA}\). Thus, in the absence of the wave, the test beam propagates unperturbed along the helix.

When the wave amplitude \( \phi \) is gradually increased, this single peak gives birth to two peaks whose separation increases. This is explained easily by the first order term of Eq. 2. As \( v_b < v_\phi - 2\sqrt{\eta \phi} \), the beam electrons are outside the trapping velocity region of the wave and are non resonant with the wave. Neglecting the wave spatial damping, their motion mainly consists in a velocity modulation with
amplitude \( \eta \phi / |v_\phi - v_b| \) around their initial velocity \( v_b \). Averaging over the arbitrary initial phase of the electron in the wave yields two peaks at the maximum and minimum electron velocity for the \( vdf \), as usual for a sinusoidal motion. The two lines (symmetric around \( v_b \)) in Fig. 2 correspond to the first order estimate of the amplitude of the velocity modulation given by Eq. 2.

The 2D contour plot of the \( vdf \) in \((\phi, v)\) plane of Fig. 2 is not exactly symmetric around \( v_b \). This lack of symmetry is still more apparent on the 3D contour plot obtained by using the interaction length as the \( vcp \) and keeping the wave amplitude constant \( \phi = 30 \) mV. To be more accurate, Fig. 3 is obtained by scanning the probe position \( z \) from 0 cm (arbitrarily set near the entrance of the helix) to 330 cm by step of 5 cm. The entrance beam velocity of the beam is \( v_b = 3.835 \times 10^6 \) m/s and its intensity is \( I_b = 10 \) nA. One main feature of this 3D plot is the periodic bunching in velocity with a period much longer than the wavelength. This can be explained by the fact that when the electron transit time \( L/v_b \) over a length \( L \) differs from the wave propagation time \( L/v_\phi \) by one wave period \( 1/f \) the electrons have undergone a complete acceleration-deceleration cycle and thus recover their initial velocity. The above condition gives \( L = v_b v_\phi / f |v_b - v_\phi| = 1.1 \) m which is the measured periodic length of Fig. 3 and corresponds to a periodic velocity bunching of the beam.

As for the already mentioned dissymmetry around \( v_b \), it can be better shown by calculating the mean beam velocity for each measured \( vdf \). The circles of Fig. 4 give a plot of the measured mean beam velocity versus the antenna position \( z \). The continuous curve of Fig. 4 (also superimposed to the 3D plot of Fig. 3) corresponds to the theoretical prediction of Eq. 3 for the measured wave amplitude of 30 mV when the small measured cold wave damping \( k_1 = 0.1 \) m\(^{-1}\) is also included (Eq. 3) corresponds to
3.3

Figure 5. Measured velocity domain for a test beam \((I_b = 120 \text{nA}, v_b = 3.55 \times 10^6 \text{ m/s})\) trapped in a single wave at 40 MHz with trapping domain (continuous curve) for increasing amplitude.

\(k_i = 0\). We observe a good agreement between theory and experiment for increasing interaction length (decreasing \(z\)). This non-resonant non-linear effect is at the essence of the well-known Landau damping [4].

4.2. Resonant test beam trapping

Fig. 5 is similar to Fig. 2 but for the case when the beam is trapped inside the potential trough of the wave (libration of the classical non-linear pendulum). Indeed the test beam with intensity \(I_b = 120 \text{nA}\) has an entrance velocity equal to the phase velocity of the wave at 40 MHz launched by a fixed antenna at \(L = 230 \text{ cm}\) from the device output. In Fig. 5 the 2D plot of the amplitude of the \(vdf\) detected by the trochoidal analyzer is the result of superposing measurements obtained for different wave amplitudes of the single wave varying from 0 mV to 45 mV by steps of 3 mV.

We first notice that the shape of the velocity domain in which the test beam electrons are spread is very different from Fig 2. The width of this domain does not increase linearly with the wave amplitude \(\phi\) but rather like the square root of \(\phi\). This is explained by the fact that the beam is trapped in the potential troughs of the wave. From the measured antenna coupling coefficients [20], the helix wave amplitude \(\phi\) can be estimated. The resonant or trapping domain in velocity can thus be deduced and is indicated by the continuous lines in Fig. 5 which correspond to \(v_{\phi} \pm 2\sqrt{\eta\phi}\). We observe a very good agreement with measurement.

Another feature appearing in Fig. 5 is a further velocity bunching of the electrons around their initial velocity for an amplitude equal to 27 mV. This phenomenon is also related to the trapping of the electrons in the wave. If we refer to the rotating bar model [21] to describe the trapped electrons motion, we expect oscillations between small spread in velocity and large spread in position (corresponding to initial conditions for a cold test beam) to large spread in velocity and small spread in position after half a bounce or trapping period equal to \(T_b/2 = v_{\phi}/2(f\sqrt{\eta\phi})\). To the bounce period, we can associate a bounce length \(L_b = v_{\phi}T_b\). For \(\phi = 27 \text{ mV}\), we get \(L_b/2 = 230 \text{ cm}\) which is precisely the interaction length \(L\) of Fig. 5. We thus confirm that Fig. 5 is displaying the trapping of the test beam in the single wave.

The 2D amplitude plot of Fig. 6a is again obtained by using the interaction length as the \(vcp\) and keeping the wave amplitude constant \(\phi = 15 \text{ mV}\). To be more accurate, Fig. 6a is obtained by scanning the probe position \(z\) from 0 cm (arbitrarily set near the entrance of the helix) to 330 cm by step of 5 cm. The entrance beam velocity of the beam is \(v_b = 3.52 \times 10^6 \text{ m/s}\) and its intensity is \(I_b = 140 \text{nA}\). In this case, the half bounce length is equal to 3.1 m. We notice, on this plot, a periodic length equal to 25 cm.
Figure 6. Velocity spread of a test beam ($I_b = 140 \text{nA}, v_b = 3.52 \times 10^6 \text{m/s}$) sloshing in a single wave at 40 MHz vs position $z$ of the emitter with a) with $z$ step of 3 cm, b) with $z$ step of 1 cm.

We can not find any physical meaning to this periodic length and, in fact, it appears as an artefact of the plot. This is best shown in Fig. 6b which is obtained by scanning the probe position $z$ from 0 cm to 100 cm with a much smaller step of 1 cm. We observe that the previous period does not show up anymore and we measure a new periodic length equal to 3 cm. This can be interpreted as half the wavelength of the helix mode and corresponds to the existence of a stationary wave produced by partial reflection of the launched wave at both ends of the helix.

4.3. Transition from modulation to trapping

Fig. 7 is again obtained after the same treatment by Matlab as Fig. 5, scanning from 0 mV to 42 mV by steps of 3 mV the entrance amplitude of the wave with frequency $f = 40 \text{MHz}$ launched by a fixed antenna at $L = 230 \text{cm}$ from the device output. But the test beam with intensity $I_b = 100 \text{nA}$ has a mean velocity $v_b = 3.66 \times 10^6 \text{m/s}$ differing from the wave phase velocity by a lower amount than in Fig. 2. For low wave amplitude, we recover velocity modulation of the test electrons similar to Fig. 2 as indicated by the two straight lines starting from the beam entrance velocity. As the wave amplitude increases, the velocity modulation is large enough to allow the electrons to enter into the trapping velocity domain associated to electron bouncing in the wave troughs indicated by the two curves starting from the wave phase velocity. For a wave amplitude $\phi = 30 \text{mV}$ the electrons have experienced half a bounce period as already shown in Fig. 5. As a consequence, Fig. 7 clearly shows the transition from particle non-resonant velocity modulation to resonant particle trapping when the wave amplitude increases. Fig. 7 also shows that, when the wave amplitude $\phi$ increases, the lower border of the measured velocity domain does not increase linearly with $\phi$ as predicted by test beam velocity modulation. This can be explained by the presence of a beam mode associated to a perturbation at $f = 40 \text{MHz}$ propagating in the beam reference frame at the beam velocity $v_b$ (in fact there are two such beam modes, the slow and the fast beam mode.
which cannot be resolved due to the low intensity of the test beam). We expect chaotic individual particle motion to occur due to the strong overlap of the resonant domains of the helix mode and of this beam mode [18].

5. Conclusion and perspectives
In conclusion we have propagated in a TWT a sufficiently weak beam to control the growth of any instability. Thus the beam electrons can be considered as test particles. We have studied their interaction with a perturbation at a single frequency. The beam intensity is nevertheless sufficient to allow reliable detection with a trochoidal energy analyzer situated at the output of the device. The observed spatial evolution can be directly related to the temporal evolution of the well-known classical non-linear pendulum.

When the entrance beam velocity is such that the electrons are not resonant with the wave, we have observed the 1st order velocity modulation proportional to the wave amplitude, the velocity bunching of the beam propagating at a velocity different from the wave phase velocity, and the second order mean velocity modulation responsible for beam synchronization for small interaction length.

When the entrance beam velocity is such that the electrons can be resonant with the wave, we have observed the trapping of the test electrons in a single wave.

The excitation of a single frequency in the TWT can in fact result in the propagation of a helix mode and a beam mode. The resonant domains of these two modes can overlap [22] as a consequence of the destruction of invariant K.A.M. tori in the velocity domain of the two waves [23]. This transition to large scale Hamiltonian chaos can be observed in the more complicated situation when two frequencies are excited [18]. The perfect knowledge of the TWT also allows to perform experiments aimed at testing new methods of control of Hamiltonian chaos [24]. In the still more complicated situation when a broad discrete spectrum of waves is excited by an arbitrary waveform generator, experiments can be performed to check the existence of a supra-quasilinear diffusion regime [25] reported in numerical simulations for intermediate overlap parameter [26].

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