Multiple flux jumps and irreversible behavior of thin Al superconducting rings

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An experimental and theoretical investigation was made of flux jumps and irreversible magnetization curves of mesoscopic Al superconducting rings. In the small magnetic field region the change of vorticity with magnetic field can be larger than unity. This behavior is connected with the existence of several metastable states of different vorticity. The intentional introduction of a defect in the ring has a large effect on the size of the flux jumps. Calculations based on the time-dependent Ginzburg-Landau model allows us to explain the experimental results semi-quantitatively.

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I. INTRODUCTION

Recently, Pedersen et al. observed jumps in the magnetization of superconducting rings which corresponds to changes of the vorticity larger than unity. This is in contrast to the behavior of superconducting disks where only changes in the value of the vorticity of unit size were observed. In some respect the observed behavior in rings is similar to vortex avalanches which were observed in superconductors with strong bulk pinning or to jumps in the magnetization when several vortices (in the form of a chain) enter in a superconducting film of width comparable to the coherence length. The occurrence of such jumps in a defect free superconducting ring originates from the fact that several metastable states with different vorticity \( L \) are possible for a given magnetic field. However the existence of such multiple stable states is not a sufficient condition to explain changes in the vorticity larger than unity (e.g. they also exist in the case of superconducting disks). An additional important requirement is to find the stability condition and finally the state to which the system relaxes to. This requires the study of the transition process from one state to another, i.e. it requires the analysing the time-dependent process.

The stability condition was studied numerically in Ref. for the case of a hollow cylinder, and in a number of works (see for example Ref. and references therein) for superconducting disks and rings by using the static Ginzburg-Landau (GL) equations. Unfortunately no analytical results were presented due to the rather general character of the studied systems in the above works.

Recently, we studied the transition process using the time-dependent Ginzburg-Landau equations. It was shown that transitions between different metastable states in a mesoscopic superconducting ring are governed by the ratio between the time relaxation of the phase of the order parameter \( \tau_\phi \) (which is inversely proportional to the Josephson frequency) and the time relaxation of the absolute value of the order parameter \( \tau_{|\psi|} \). We found that if the ratio \( \tau_{|\psi|}/\tau_\phi \) is sufficiently large the system will always transit from a metastable state to the ground state. This leads to an avalanche-type variation of \( L \) when the vorticity of the metastable state differs appreciably from the vorticity of the ground state. In contrast to the case of a superconducting film, in a ring the ‘vortex’ entry occurs through a single point and the vorticity increases one by one during the transition. In low-temperature superconductors like In, Al, Sn the ratio \( \tau_{|\psi|}/\tau_\phi \) is very large for temperatures far below the critical temperature \( T_c \) and hence, if such systems are driven far out of equilibrium they will always relax to the ground state.

In this work we investigate the conditions under which a state with a given vorticity becomes unstable in a finite width ring and we find how the superconducting order parameter in the ring changes with increasing applied magnetic field. We are able to find an analytical expression for the dependence of the order parameter on applied magnetic field, and hence for the upper critical field at which superconductivity vanishes in such a sample. We provide a direct comparison between the theoretical and experimental results on aluminium rings. Our theoretical calculations are based on a numerical solution of the time-dependent Ginzburg-Landau equations.

The paper is organized as following. In section II the theoretical formalism is presented and the two-dimensional time-dependent GL equations are solved. In Section III the experimental results are presented and compared with our theory. In Section IV we present our conclusions and our main results.

II. THEORY

We consider sufficient narrow rings such that we can neglect the screening effects. This is allowed when the
width of the ring \( w \) is less than \( \text{max} \lambda, \lambda^2/d \), where \( \lambda \) is the London penetration length and \( d \) is the thickness of the ring. In order to study the response of such a ring on the applied magnetic field we use the time-dependent Ginzburg-Landau equations

\[
\begin{align*}
  u \left( \frac{\partial \psi}{\partial t} + i \varphi \psi \right) &= (\nabla - i A)^2 \psi + (1 - |\psi|^2) \psi, \quad (1a) \\
  \Delta \varphi &= \text{div} (\text{Im}(\psi^*(\nabla - i A)\psi)), \quad (1b)
\end{align*}
\]

where all the physical quantities (order parameter \( \psi = |\psi|e^{i\varphi} \), vector potential \( A \) and electrostatic potential \( \varphi \)) are measured in dimensionless units: the vector potential \( A \) is scaled in units \( \Phi_0/(2\pi \xi \lambda^2) \) (where \( \Phi_0 \) is the quantum of magnetic flux), and the coordinates are in units of the coherence length \( \xi(T) \). In these units the magnetic field is scaled by \( H_{GL} \) and the current density, \( j \), by \( j_0 = c\Phi_0/8\pi \lambda^2 \xi \). Time is scaled in units of the Ginzburg-Landau relaxation time \( \tau_{GL} = 4\pi \sigma_0 \lambda^2 / \xi^2 \), the electrostatic potential, \( \varphi \), is in units of \( c\Phi_0/8\pi \xi \lambda \sigma_n \) (\( \sigma_n \) is the normal-state conductivity). Here the time derivative is explicitly included which allows us to determine the moment at which the state with given vorticity \( L \) becomes unstable. It is essential to include the electrostatic potential (which is responsible for the appearance of the Josephson time or frequency) in order to take into account the multi-vortex jumps. In some previous studies (see for example Refs. [13,14]) \( \varphi = 0 \) was assumed and as a consequence only transitions with unit vorticity jumps, i.e. \( \Delta L = 1 \), are possible in the ring. The coefficient \( u = 48 \) was chosen such that after the transition the system is in the thermodynamically equilibrium state. We assume that the width \( (w) \) of the ring is less than two coherence length \( \xi \), because: i) all experimental results presented here were performed for such type of samples; and ii) only in this case it is possible to obtain simple analytical expressions. For instance, the dependence of the order parameter on the applied magnetic field and the upper critical field \( H_{max} \).

For \( w \leq 2\xi \) the order parameter is practically independent of the radial coordinate. This is demonstrated in Fig. 1 where the dependence of the order parameter in the middle of the ring is compared with its value at the inner and outer boundary of the ring, i.e. \( r = R \pm w/2 \) (\( R \) is the mean radii of the ring), for two different rings. Notice that these two numerical examples correspond already to relative thick mesoscopic rings, i.e. \( R/w \sim 1 - 2 \). For the field \( H_{max} \) we are able to fit our numerical results to the expression

\[
H_{max} = 3.67 \frac{\Phi_0}{2\pi \xi w}. \quad (2)
\]

For rings with \( w \leq 2\xi \) and \( w/R < 1 \) this analytical expression is within 2\% of the numerical results. It is interesting to note that \( H_{max} \) does not depend on the radii of the ring. But the value of the vorticity of the system depends on \( R \). For example, for \( R = 5.5\xi(16.5\xi) \) \( L = 55(501) \) for \( w = \xi \) at \( H = H_{max} \).

Note that Eq. (2) has the same dependence on the superconducting parameters as in the case of a thin plate with thickness \( d < \sqrt{3} \lambda \) placed in a parallel magnetic field [14]. Even the numerical coefficient is quite close, i.e. for a thin plate it is equal to \( 2\sqrt{3} \approx 3.46 \). Furthermore, we found that the transition to the normal state of our rings at the critical field \( H_{max} \) is of second order as is also the case for a thin plate. A possible reason for this close similarity is that for a thin plate with thickness \( d < \sqrt{3} \lambda \) the screening effects are also very small. In the calculations of Refs. [13,14] an averaged value for the order parameter was used independent of the coordinate. Note that this is similar to our \( |\psi| \) which is practically independent on the radial coordinate (see Fig. 1).

The absolute value of the order parameter (in the middle of the ring) is, too a high accuracy, given by the expression

\[
|\psi|^2 = 1 - (H/H_{max})^2 - p(L,H)^2, \quad (3)
\]
with \( p(L, H) = L/R - HR/2 \), where the vorticity \( L \) depends on the history of the system. This result is similar to the one obtained in Refs. 13-15, with the exception of the last term in Eq. (3) which appears due to the closed geometry of the ring and hence leads to a nonzero \( L \).

FIG. 2: Magnetic field dependence of the magnetization (a), the order parameter (b) and the gauge invariant momentum (c) in the middle of the ring. Dotted curve in Fig. 2(b), is the expression \( \sqrt{1 - (H/H_{\text{max}})^2} \). Dotted curve in Fig. 2(c) is the expression \( \sqrt{1 - ((H - H_0)/H_{\text{max}})^2}/\sqrt{3} \), where \( H_0 \approx 13G \) - is the displacement of the maximum of \( M(H) \) from the \( H = 0 \) line.

All the above results were obtained for a ring which is in the ground state at any value of the magnetic field. However such a system can exhibit several metastable states at a given magnetic field, and consequently this may lead to hysteretic behavior when one sweeps the magnetic field up and down. Furthermore, with changing field the vorticity may jump with \( \Delta L > 0 \). An important question which arises is the condition of stability of the state with given vorticity. This question was studied earlier for one-dimensional rings, i.e. rings with zero width. It turns out that the system transits to a state with another vorticity when the value of the gauge-invariant momentum \( p = \nabla \phi - A \) reaches the critical value

\[
p_c = \frac{1}{\sqrt{3}} \sqrt{1 + \frac{1}{2R^2}}.
\]

At this condition it is easy to find the value of the field for the first ‘vortex’ entry

\[
H_{en}/H_{c2} = 2p_c/R = \frac{2}{\sqrt{3}R} \sqrt{1 + \frac{1}{2R^2}}.
\]

We will now generalize the results of Refs. 12,15 to the case of finite width rings with \( w \lesssim 2\xi \). First we will neglect the dependence of \( \psi \) on the radial coordinate in which case the GL equations reduce to one-dimensional expressions. But in order to include the suppression of the order parameter by an external field for a finite width ring we add the term \(- (H/H_{\text{max}})^2 \psi\) to the RHS of Eq. (1a), where \( H_{\text{max}} \) is given by Eq. (2). Using the stability analysis of the linearized Ginzburg-Landau equations near a specific metastable state as presented in Ref. 15 we obtain the modified critical momentum

\[
p_c = \frac{1}{\sqrt{3}} \sqrt{1 - \left(\frac{H}{H_{\text{max}}}\right)^2} + \frac{1}{2R^2}.
\]

Note that now \( p_c \) decreases with increasing magnetic field. This automatically leads to a decreasing value of the jump in the vorticity \( \Delta L \) at high magnetic field, because in Ref. 15 it was shown that

\[
\Delta L_{\text{max}} = \text{Nint}(p_cR),
\]

where \( \text{Nint}(x) \) returns the nearest integer to the argument.

In order to check the validity of Eq. (6) we performed a numerical simulation of the two-dimensional Ginzburg-Landau equations, Eqs. (1a,b), for a ring with \( R = 5.5\xi \) and \( w = 1.5\xi \) (for these parameters the theoretical findings fit the experimental results - see section below). In Fig. 2 the magnetization, the order parameter and the gauge-invariant momentum \( p \) are shown as function of the applied magnetic field. The magnetic field was cycled up and down from \( H < -H_{\text{max}} \) to \( H > H_{\text{max}} \). The condition (6) leads to an hysteresis of \( M(H) \) and to a changing value of the jump in the vorticity in accordance with the change in \( p_c \). The main difference between our theoretical prediction (6) and the results of our numerical calculations appears at fields close to \( H_{\text{max}} \). Apparently it is connected with the fact that for the considered ring the distribution of the order parameter along the width of the ring is appreciably nonuniform at \( H \approx H_{\text{max}} \) and as a consequence the one-dimensional model breaks down (see Fig. 1).

Finally, we also considered the same ring with a defect. The effect of the defect was modelled by introducing in the RHS of Eq. (1a) the term \(-\rho(s)\psi\) (\( s \) is the arc-coordinate) where \( \rho(s) = -1 \) inside the defect region...
radii. The rings - having Hall probes microfabricated from a high-mobility two-dimensional electron gas (2DEG). The rings - having approximately the same width - were placed directly on top of the microfabricated Hall cross sections, which had approximately the same width \( b \) of about 2 \( \mu m \) (see micrograph in Fig. 4 for a ring with an artificial defect). These experimental structures were prepared by multi-stage electron-beam lithography with the accuracy of alignment between the stages better than 100 \( nm \). The rings studied in this work were thermally evaporated and exhibited a superconducting transition at about 1.25K. The superconducting coherence length was \( \xi(T=0) \approx 0.18 \mu m \). The latter was calculated from the electron mean free path \( l \approx 25 \mu m \) of macroscopic Al films evaporated simultaneously with the Al rings. The Hall response, \( R_{xy} \), of a ballistic cross is given by the amount of magnetic flux \( \phi \) through the central square area \( b \times b \) of the cross. For simplicity, one can view the ballistic magnetometer as an analogue of a micro-SQUID, which would have a square pick-up loop of size \( b \) and superconducting rings placed in its centre. We present our experimental data in terms of the area magnetization \( M = \langle B \rangle - H \) which is the difference between the applied field \( H \) and the measured field \( < B > \approx R_{xy} \). Previously, we have studied individual superconducting and ferromagnetic disks and found excellent agreement with the above formulae. For further details about the technique, we refer to our earlier work.

![Micrograph of a superconducting ring](image)

**FIG. 4:** A micrograph of the superconducting ring placed on top of a Hall bar. An artificial defect (narrowing of the ring cross section) is intentionally made by electron beam lithography.

Rings with and without an artificial defect were studied. Let us consider first the ring without artificial defect. In Fig. 5(a) the full magnetization loop of such a ring with parameters \( R = 1.0 \pm 0.1 \mu m \) and \( w = 0.25 \pm 0.05 \mu m \) is shown. In Fig. 6 (solid curve) the low field part of the virgin curve is presented. From the virgin trace \( M(H) \) we can find the magnetic field for the first vortex entry, \( H_{en} \), and hence we estimate \( \xi \approx 0.19 \mu m \) at the given temperature \( T \approx 0.4K \) using Eq. (5) (this value of \( \xi \) is in agreement with the above experimental value \( \xi(0) \approx 0.18 \mu m \) obtained from the mean free path). Furthermore, we know from Fig. 5(a) that the vorticity changes with \( \Delta L = 3 \) for \( H \approx 0 \). This agrees with the fact that the radii of the ring is larger than 4 \( \xi \) (see Eq. (7)). Another important information which may be extracted from the virgin curve is that at the first vortex entry the magnetization drops considerably but it does not fall to zero.

### III. COMPARISON WITH EXPERIMENT

The measurements were performed on individual Al superconducting rings by using ballistic Hall micromagnetometry. The techniques employs small Hall probes microfabricated from a high-mobility two-dimensional electron gas (2DEG). The rings - having radii \( R \approx 1 \mu m \) and width \( w \) ranging from 0.1 to 0.3 \( \mu m \) - were placed directly on top of the microfabricated Hall cross sections, which had approximately the same width \( b \) of about 2 \( \mu m \) (see micrograph in Fig. 4 for a ring with an artificial defect). These experimental structures were...
not change sign. If we recall that at every vortex entry \( p \) decreases on \( 1/R \) (and hence the current density \( j \) and \( M \sim \int [j \times r] dV \) also changes proportionally) we can conclude that the radii of our ring should be in the range \( 5.5 \xi \lesssim R \lesssim 6.5 \xi \). This agrees with the experimental value \( R/\xi \approx 5.3 \pm 0.5 \).

Figs. 2(a) and 5(a) are qualitatively very similar. For example our theory describes: i) the hysteresis; ii) the non-unity of the vorticity jumps, i.e. \( \Delta L = 3 \) in the low magnetic field region, \( \Delta L = 2 \) in the intermediate \( H \)-region, and \( \Delta H = 1 \) near \( H_{max} \). Theoretically (experimentally) we found \( 6(5), 13(21), 22(18) \) jumps with respectively \( \Delta L = 3, 2, 1 \); and iii) the non symmetric magnetization near \( \pm H_{max} \) for magnetic field sweep up and down.

In the ring with approximately the same mean radii and width but containing an intentionally introduced artificial defect, jumps with \( \Delta L = 1 \) are mostly observed (see Fig. 5(b)). The reason is that an artificial defect considerably decreases the critical value \( p_c \) (and hence the field \( H_{cr} \) - see dotted curve in Fig. 6). From Figs. 2(c), 3(c) it is clear that the maximum value \( p_c^{d} \approx 0.54 \) for a ring without defect and \( p_c^{d} \approx 0.35 \) for a ring with a defect. The ratio \( p_c^{d}/p_c^{d} \approx 0.65 \) is close to the ratio of the field of first vortex entry \( H_{cr}^{d}/H_{cr}^{d} \approx 0.67 \) obtained from experiment (see Fig. 6). From Fig. 2(c) it is easy to see that for a ring without a defect at \( p \approx 0.35 \) there are only jumps with \( \Delta L = 1 \). But if we slightly increase \( p \) then jumps with \( \Delta L = 2 \) can appear in the system. So we can conclude that \( p = 0.35 \) is close to the border value which separates regimes with jumps in vorticity of \( \Delta L = 1 \) and \( \Delta L = 2 \). From our experimental data it follows that the maximum value of \( p_c \) is very close to this border. Thermal fluctuations may influence the value of \( \Delta L \), in particular for a \( p_c \) value close to this border value. This is probably the reason why in the experiment (Fig. 5(b)) occasional jumps with \( \Delta L = 2 \) are observed which are absent in our simulation (Fig. 3(a)).

![FIG. 5: Magnetic field dependence of the magnetization of the ring without (a) and with (b) an artificial defect at \( T \approx 0.4K \). Parameters of the rings (width and radii) are the same within experimental accuracy.](image-url)

If we take the above value for \( \xi \) and \( w \approx 1.5 \xi \) we obtain the maximum field of \( H_{max} \approx 223 \) G. This value is slightly smaller than the value obtained from Figs. 2 and 3 \( H_{max} \approx 240 \) G which we attribute to the large coordinate step which we used in our numerical calculations of Eq. 1(a,b). The value is also larger than the experimental value \( H_{max} \approx 185 \) G. This disagreement between theory and experiment is most likely connected to the semi-quantitative applicability of the Ginzburg-Landau equations in the considered temperature range. The range of applicability of the Ginzburg-Landau equations (even stationary ones) for this specific superconductor is very narrow. Nevertheless based on previous comparison between experiment and theory for mesoscopic superconducting disks, it was found that the GL equations provided a rather good description of the superconducting state even deep inside the \((H,T)\) phase diagram.

![FIG. 6: Magnetic field dependence of the virgin magnetization of a ring without (solid curve) and with (dotted curve) an artificial defect. The dotted curve is shifted for clarity by 0.6](image-url)
IV. CONCLUSION

We studied multiple flux jumps and irreversible behavior of the magnetization of thin mesoscopic Al superconducting rings. We have shown experimentally and theoretically that at low magnetic fields and for rings with sufficiently large radii the vorticity may change by values larger than unity. With increasing magnetic field the order parameter gradually decreases and thus leads to a decrease of the size of the jumps in the vorticity. For rings with width less than $2\xi$ analytical expressions were obtained for the dependence of the order parameter on the applied magnetic field. We have found that a state with a given vorticity becomes unstable when the value of the gauge-invariant momentum reaches a critical value $p_c$ which decreases with increasing magnetic field. This is responsible for the fact that $\Delta L$ decreases with increasing $H$. The introduction of an artificial defect in the ring leads to a decrease of $p_c$, in comparison to the case of a ring without a defect and also results in a decrease of $\Delta L$.

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