Unzipping flux lines from extended defects in type-II superconductors.

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With magnetic force microscopy in mind, we study the unbinding transition of individual flux lines from extended defects like columnar pins and twin planes in type II superconductors. In the presence of point disorder, the transition is universal with an exponent which depends only on the dimensionality of the extended defect. We also consider the unbinding transition of a single vortex line from a twin plane occupied by other vortices. We show that the critical properties of this transition depend strongly on the Luttinger liquid parameter which describes the long distance physics of the two-dimensional flux line array.

The competition between thermal fluctuations, pinning and interactions of vortices in type-II high-temperature superconductors leads to many interesting physical phenomena\textsuperscript{1}. These include the melting of the Abrikosov flux-lattice into an entangled vortex-liquid\textsuperscript{2} and the proposed existence of low temperature Bose-glass\textsuperscript{3}, vortex glass\textsuperscript{4} and Bragg glass\textsuperscript{5} phases.

Experimental probes range from decoration, transport and magnetization measurements, neutron scattering, electron microscopy, electron holography to Hall probe microscopes. While these experiments yield a wealth of information, the possibility of manipulating a single vortex, for example using magnetic force microscopy (MFM)\textsuperscript{6}, has received less attention. Such experiments could give rise to a direct measurement of microscopic vortex properties which, up to now, have been under debate or assumed. The possibility of performing such experiments is akin to single molecule experiments on motor proteins, DNA, and RNA which have transformed biophysics during the past decade by opening a window on phenomena inaccessible via traditional bulk experiments\textsuperscript{7}.

Such an experiment was recently proposed by Olson-Reichhardt and Hastings\textsuperscript{8}. These authors suggested using MFM to wind two vortices around each other, thus probing directly the energetic barrier for two vortices to cut through each other. A high barrier for flux line crossing has important consequences for the dynamics of the entangled vortex phase.

In this Letter we analyze theoretically several possible experiments in which a single vortex is depinned from extended defects using, for example, MFM. We first consider the situation where MFM is used to pull an isolated vortex bound to common extended defects such as a columnar pin or a twin plane in the presence of point disorder. Using a scaling argument we derive the displacement of the vortex as a function of the force exerted by the MFM near the depinning transition in an arbitrary dimension $d$. We argue that the transition can be characterized by a universal critical exponent, which depends only on the dimensionality of the defect. Hence unzipping experiments from a twin plane directly measure the free-energy fluctuations of a vortex in the presence of point disorder in $d = 1 + 1$ dimensions. To the best of our knowledge, there is only one, indirect, measurement of this important quantity in $d$. Our conclusions are supported by numerical and analytical calculations. Related results apply when a tilted magnetic field\textsuperscript{9} is used to tear away vortex lines in the presence of point disorder.

Next, we study the effect of interactions between vortices on the unzipping of a single vortex. We consider a system in which vortices are preferentially bound to a thin two dimensional slab, such as a twin plane, of say YBCO\textsuperscript{10}, in a three dimensional sample. A similar situation can be achieved by artificially inserting (with, for example, molecular beam epitaxy) a thin plane with a reduced lower critical magnetic field $H_{c1}$ into a bulk superconductor; we require that the density of vortices in the twin plane is much higher than in the bulk of the sample. We imagine that MFM is used to pull a single vortex out of the slab, thus creating an effective magnetic monopole inside it. The displacement of the vor-
text from the slab as the transverse force exerted by the MFM is increased then depends on the physics of the two dimensional vortex liquid which resides in the slab. Specifically, the “Luttinger liquid parameter” \[11\] which controls long-range behavior of the vortex liquid also controls critical properties of the unbinding transition of the vortex from the slab. We study the “unzipping of Luttinger liquids” both with and without point disorder, and argue that this setup can be used as a sensitive probe of the two dimensional physics.

Consider first the unzipping of a single vortex from an extended defect. We describe the flux line by a function \(r(\tau)\), where \(\tau\) denotes a coordinate parallel to the defect (see Fig. \[11\], and \(r\) denotes the \((d - 1)\)-dimensional coordinates perpendicular to it. In the absence of external force the appropriate free energy for a given configuration \(r(\tau)\) of the vortex is given by \[1\]:

\[
\mathcal{F}_0 = \int_0^L \left[ \frac{1}{2} (\partial_{\tau} r(\tau))^2 + V(r(\tau)) + \mu(r(\tau), \tau) \right].
\]

Here \(\gamma\) is the line tension and \(L\) is the length of the sample along the \(\tau\) direction. The point disorder \(\mu(r, \tau)\) is assumed uncorrelated and Gaussian distributed with \(\overline{\mu} = 0\) and \(\mu(r, \tau)\mu(r', \tau') = \sigma \delta(r-r') \delta(\tau-\tau')\), where the overbar denotes an average over realizations of the disorder. \(V(r)\) is a potential describing the \(d'\)-dimensional extended defect.

To study unzipping, we consider a total free energy \(\mathcal{F} = \mathcal{F}_0 + \mathcal{F}_1\), where \(\mathcal{F}_1 = -\int_0^L f \cdot \partial_{\tau} r(\tau) \, d\tau\). Here \(f\) stands for the local force exerted by the MFM in the transverse direction and we assume that \(r(0) = 0\): below the unzipping transition the flux line is unaffected by the boundary conditions at the far end of the sample.

As the force, \(f\), increases the free energy density of the unzipped portion of the vortex decreases. In contrast, the free energy density of the bound part is, clearly, independent of the force. The vortex will be unzipped when \(f = f_c\), such that the free-energy densities of the bound and unzipped states intersect. We study the behavior of \(s = \langle m \rangle\), where \(m\) is the length along the \(\tau\) direction which is unbound as the force approaches the critical value \(f_c\). The angular brackets and overbar denote thermal and disorder averages, respectively. Although we focus on disorder averages, particular disorder configurations will be characterized by a pattern of jumps and pauses in the unzipping \[12\]. Disorder averages can be probed experimentally by unzipping many different vortices. It is straightforward to show that \(\langle m \rangle\) is related linearly to the transverse displacement of the vortex at the top of the sample, as measured by MFM. We take the force to always act in the \(x\) direction. Without disorder it is known that \(s \sim (f_c - f)^{-1}\) in any dimension and for any dimensionality of the defect \[10\].

The universal properties of the unzipping transition with point disorder can be analyzed using a simple scaling argument adapted from \[12\] for the unzipping of DNA. Consider the free energy \(\mathcal{F}(m)\), with contributions from three sources. The first is linear in \(m\) and is due to the average free energy difference between a vortex on the defect and in the bulk of the sample \((f_c - f) m\). In addition there is a contribution, \(\delta \mathcal{F}_d\), from optimized free-energy fluctuations of the segment of the vortex which is bound to the defect. This term arises from point disorder which is localized on or near the defect and is expected to behave as \(m^{1/2}\) at large \(m\) for a \(d' = 1\) dimensional defect. For \(d' > 1\) we expect \(\delta \mathcal{F}_d \propto m^{\omega(d')}\), where \(\omega(d')\) is the exponent associated with the free-energy fluctuations of a directed path in the presence of point disorder in \(d'\) dimensions \[13\]. Finally, there is the interaction of the unzipped vortex with bulk point disorder, \(\delta \mathcal{F}_d \propto m^{\omega(d)}\), where \(d > d'\) is the dimensionality of the sample. Collecting these terms gives:

\[
\mathcal{F}(m) = a(f_c - f)m - bm^{\omega(d')} - cm^{\omega(d)}.
\]

Here \(a\), \(b\) and \(c\) are positive constants and the negative signs have been chosen since the behavior is expected to be dominated by the minima of the two random potentials. The exponent \(\omega(d)\) has been studied extensively in the past and it is well known that \(\omega(d') < \omega(d)\) for any \(d' < d\) \[12\]. For example \(\omega(d' = 2) = 1/3\) (twin planes) and \(\omega(d = 3) \simeq 0.22\) (bulk sample). Therefore, disorder on or close to the defect controls the unbinding transition for any dimension and the problem is equivalent to unzipping from a sample with disorder localized on the defect.

In practice, disorder is likely to be concentrated within real twin planes and near columnar damage tracks created by heavy ion irradiation, strengthening even more the conclusions of this simple argument. By minimizing the free energy with respect to \(m\) we find the typical value of length of the unzipped part of the flux line, which determines \(s = \langle m \rangle\):

\[
s \sim \frac{1}{(f_c - f)^\nu}, \quad \nu = [1 - \omega(d')]^{-1}.
\]

Thus we get \(\nu = 2\) (in agreement with \[12\]) for a columnar pin and a new result, \(\nu = 3/2\) for a twin plane.

The heuristic argument leading to Eq. \[3\] can be tested in a number of ways. First consider unzipping from an attractive hard wall in \((1 + 1)\)-dimensions with point disorder in the bulk. Upon applying an imaginary-gauge-transformation \[10\] (possible below the unzipping transition) to the Bethe-ansatz solution for \(f = 0\), obtained by Kardar (see Eq. (3.12) in Ref. \[14\]), we obtain

\[
s = \int_0^\infty dy \frac{\exp[-(\lambda - f/\gamma - \kappa) y]}{[1 - \exp(-\kappa y)]}.
\]

Here \(\kappa = \sigma/2\) and \(\lambda\) is the inverse localization length characterizing the pinning to the wall in the absence of point disorder. For \(\kappa \ll \lambda - f/\gamma\), Eq. \[4\] reduces to the clean result: \(s \sim (\lambda - f/\gamma)^{-1}\), while in the opposite limit
one has \( s \sim (\lambda - \kappa - f/\gamma)^{-2} \) as expected from Eq. 3. This result confirms our expectation that, even though disorder is present everywhere, the universal properties of the unzipping transition are determined by an effective disorder potential generated in the vicinity of the wall with \( \nu = 2 \).

To check the case of a symmetric attractive potential, representing a columnar pin, we performed numerical simulations for a lattice model in \( d = 1 + 1 \). The restricted partition function of this model, \( Z(x, \tau) \), which sums over the weights of all path leading to \( x, \tau \), satisfies the recursion relation

\[
Z(x, \tau + 1) = \delta_{x,0}(e^V - 1)Z(0, \tau) \\
+ e^{\mu(x, \tau + 1)} [Je^fZ(x - 1, \tau) + Je^{-f}Z(x + 1, \tau)].
\]

Here \( \mu(x, \tau) \) was taken to be distributed uniformly in \([-U_0, U_0]\) with a variance \( \sigma = U_0^2/3 \). The variable \( J \) controls the line tension \( (J = \exp \gamma/2) \) and was set to be \( J = 0.1 \) while we used \( V = 0.1 \). We work in units such that \( k_B T = 1 \), where \( k_B \) is the Boltzmann constant. The partition function was evaluated for each variance of the disorder for several systems of finite width \( w = 2L_x \) averaging over the time-like direction (typically \( \tau \approx 10^6 \) “time” steps) with the initial condition \( Z(0, 0) = 1 \) and \( Z(x, 0) = 0 \) for \( x \neq 0 \). In the vicinity of the transition we expect the finite size scaling prediction \( s = L_x f[L(f_c - f)]^\nu \). For finite \( L_x \) we expect a smooth interpolation between \( \nu = 1 \) (the clean result) and \( \nu = 2 \) (the asymptotic result with disorder) with increasing \( L_x \) or increasing strength of disorder. Using standard methods of finite-size scaling we extracted the exponent for a given strength of the disorder as a function of \( L_x \). The exponent for each value of \( L_x \) is obtained from the best collapse of the data of two systems sizes \( L_x \) and \( L_x/2 \). As shown in Fig. 2, the data is consistent with \( \nu \) saturating at \( \nu = 2 \) for large systems. The crossover to \( \nu = 2 \) is much more rapid if the point disorder is enhanced near the columnar pin (see the inset in Fig. 2), as might be expected for damage tracks created by heavy ion radiation.

FIG. 2: Effective exponent \( 1/\nu \) versus \( L_x \) for a fixed strength of point disorder \( \sigma = 0.03 \). The results are consistent with the general argument that this exponent should saturate at \( \nu = 2 \) as \( L_x \to \infty \). The inset shows the same exponent vs \( \sigma_c \), the variance of additional point disorder placed directly on the columnar pin extracted from two system sizes \( L_x = 600 \) and \( L_x = 1200 \). It is clear that \( \nu \to 2 \) as \( \sigma_c \) increases.

FIG. 3: Data collapse for unzipping from a disordered twin-plane with \( \nu = 3/2 \). See the text for more details.
walk which is pulled by a force $f$: $F_u(m) = -f^2m/2\gamma$. This allows a very efficient calculation of $s$ including averages over point disorder. The results agree very well with the prediction of Eq. (3).

Consider now unzipping from a two dimensional plane with many vortices in a three dimensional sample which is essentially free of vortices (see Fig. 4). A MFM tip pulls the top end of one of the vortices out of the plane with a force $f = f\dot{x}$. As in the single vortex problem, we expect an unzipping transition for $f > f_c$ such that the vortex is pulled out of the two dimensional slab. We again write the free energy of the vortex as a sum of two contributions: $F_u(m)$, which arises from the vortex segment separated from the two dimensional slab, and $F_0(m)$, the free energy change of the vortices which remain inside. Here $m$ is again the length along the $\tau$ direction which is unbound from the plane.

The free energy $F_u(m)$ is evaluated analytically as discussed above. As for $F_0(m)$, clearly, there is a linear contribution associated with the length $m$ removed from the attractive potential of the slab. In addition, a non-linear contribution in $\tau$ arises due to the dislocation created in the vortex array (see Fig. 4). The energy of this dislocation is determined by the long distance properties of a two-dimensional vortex liquid. These are known to be controlled by a single parameter $g$, which is related to the elastic moduli of the vortex lattice and temperature \cite{11}. For example, vortex density oscillations without point disorder decay as a power-law in distance with an exponent $\eta = 2g$. Due to an analogy with one dimensional quantum bosons, $g$ is often referred to as the “Luttinger liquid parameter”. Assuming that vortex lines must exit normal to the interface, the energetic cost of the dislocation, $F_d$, can be calculated using the method of images: $F_d = k_B T \ln m$, for large $m$ \cite{12}.

Having obtained $F_0(m)$ and $F_u(m)$ it is straightforward to calculate $\langle m \rangle = s$, (related to the mean displacement of the MFM tip by $x_m = fs/\gamma$) as a function of $f$ using the free energy

$$F(m) = r(f_e - f)m + \frac{k_B T}{4g} \ln(m).$$

where $r$ is a positive constant. At the transition point, $f = f_c$ and $e^{-F(m)/T}$ is a power law in $m$, so the results are sensitive to the value of $g$. For $g > 1/4$ we find the same divergence as for a clean twin plane without interactions,

$$s \sim \frac{T}{f_c - f}.$$ (7)

In contrast, for $1/8 < g < 1/4$ one finds a continuously variable exponent governing the transition

$$s \sim \left(\frac{T}{f_c - f}\right)^{\nu_g - 1},$$ (8)

Finally, for $g < 1/8$ we find that $s$ remains finite as $f \to f_c$ and the unzipping is presumably discontinuous. Higher moments of $m$ will also be sensitive to the value of $g$.

Finally, we consider the case of unzipping from a plane with many vortices in a three dimensional sample in the presence of point disorder. Here we expect the results to be sensitive to the boundary conditions imposed on the plane. When the boundary conditions are such that the number of flux lines is conserved (e.g. in a cylindrical geometry) we expect the free energy fluctuations to behave as $\delta F(m) \propto m^{1/2}$ due to the highly constrained, almost one dimensional, behavior of the flux lines. Thus, the universal exponent is expected to be $\nu = 2$ for any value of $g$. In slab geometries however, where flux lines can freely enter from the sides of the plane, the single flux line contribution to the free-energy fluctuations can be screened and the effective fluctuations come only from the dislocation energy. Near the vortex glass transition at $g = 1$, \cite{4, 13, 15} the calculation can be carried out using the replica approach. Although a rigorous treatment requires the method of images with reflected disorder, we do not expect this to affect the results and performed the calculations with uncorrelated disorder. We find that while the quenched average of the free energy is independent of the disorder, its fluctuations grow as $(\ln(m))^{1/2}$ for $g > 1$ and $(1 - g) \ln(m)$ for $g < 1$. Thus, in contrast to the single vortex problem and to the case when the number of vortices in the sample is conserved, here fluctuations in the bulk of the sample are dominant, growing as $m^{\omega_g(3)}$. Using $\omega(3) \approx 0.22$, this lead one to the prediction $s \sim (f_c - f)^{-1.28}$, allowing a direct measurement of $\omega(3)$.

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