On Sterile neutrino explanation of LSND and MiniBooNE anomalies

Claudio Dib, Juan Carlos Helo, Sergey Kovalenko, Ivan Schmidt

Universidad Técnica Federico Santa María,
Centro-Científico-Tecnológico de Valparaíso,
Casilla 110-V, Valparaíso, Chile

(Dated: January 25, 2013)

We examine the compatibility between existing experimental data and a recently proposed explanation of the LSND and MiniBooNE anomalies, given in terms of a sterile neutrino $N$ whose decay is dominated by a radiative mode. We find that current experimental data on $\tau \to \mu\nu\gamma$ decays are compatible with the sterile neutrino parameters required for the explanation of the anomalies, but $K \to \mu\nu\gamma$ shows a marginal tension with those parameters. We also propose experimental cuts on radiative $K$ decays that could test the sterile neutrino hypothesis better. Finally, we study the contribution of this sterile neutrino to $K \to \mu e e$, and find that measurements of this process would provide powerful tests for the sterile neutrino explanation of the LSND and MiniBooNE anomalies, if the experimental cut on the invariant mass of the $e^+e^-$ pair could be reduced from its current value of 145 MeV to a value below 40 MeV.

PACS numbers: 13.35.Hb, 13.15.+g, 13.20.-v, 13.35.Dx

Keywords: sterile neutrino, MiniBooNE, LSND, Karmen, radiative decays
Neutrino oscillation experiments have proven that neutrinos are massive, although very light particles, and that they exhibit flavor mixing. In order to give mass to neutrinos, most models introduce sterile (or right-handed) neutrinos, which generate the masses of the ordinary neutrinos via a see-saw mechanism or its modifications \cite{1, 2}. This mechanism gives masses to the three light neutrinos, leaving open the possibility of having one or more additional heavy neutrinos $N$, which would be sterile with respect to electroweak gauge interactions. If this is the case, the sterile neutrinos $N$ in general will contain a certain admixture of the active flavors $\nu_e, \nu_\mu, \nu_\tau$, parametrized by the corresponding elements of a neutrino mixing matrix $U_{eN}, U_{\mu N}, U_{\tau N}$. Therefore, $N$ can participate in charged and neutral current interactions of the Standard Model (SM), contributing to various processes. If a sterile neutrino with mass $m_N \lesssim 100$ MeV is produced in an intermediate state, it would typically decay into three leptons, but a radiative decay is also possible if a nonzero transition magnetic moment ($\mu_{tr}$) between the $N$ and $\nu$ mass states is introduced \cite{3-6}. Usually the radiative decay of the sterile neutrino is assumed to be negligible compared to its decay into three leptons. However, it has been recently proposed that a sterile neutrino $N$ with a dominant radiative decay mode $N \rightarrow \nu\gamma$ and with mass $m_N$, mixing strength $U_{\nu N}$ and lifetime $\tau_N$ in the range \cite{3-4}

$$40 \text{ MeV} \lesssim m_N \lesssim 80 \text{ MeV}, \quad 10^{-3} \lesssim |U_{\nu N}|^2 \lesssim 10^{-2}, \quad \tau_N \lesssim 10^{-9} \text{ s},$$

(1)

may be the source of the LSND \cite{7} and MiniBooNE \cite{8} experimental anomalies. In order to search for this sterile neutrino in an independent way, a new muon decay experiment \cite{5}, direct searches through $K$ meson decays \cite{4} and searches at neutrino telescopes \cite{9} have already been proposed. It was also shown that the sterile neutrino parameters with the values in the range \cite{1} are in some tension with the radiative muon capture \cite{10}. Other constraints relevant for the range \cite{1} have been derived \cite{11} from the accelerator and Super-Kamiokande results.

Here we consider the restrictions on the sterile neutrino $N$ parameters that can be deduced from the existing experimental data on radiative $K$-meson and $\tau$-lepton decays. The purpose of this note is to check whether these restrictions are consistent or exclude some of the values in Eq. (1), necessary for the explanation of the MiniBooNe and LSND anomalies. Specifically, we analyze the contribution of the sterile neutrino $N$ to the following decays:

$$K^+ \rightarrow \mu^+\nu\gamma, \quad \tau^- \rightarrow \mu^-\nu\nu\gamma.$$  

(2)

Here $\nu$ denotes the standard light neutrino or antineutrino, dominated by any of the neutrino flavors $\nu_e, \nu_\mu, \nu_\tau$. These decays receive their known SM contributions, which alone give good agreement with the experimental data. However, they also proceed according to the diagrams shown in Figs. (1a) and (1b), with the sterile neutrino $N$ as an intermediate particle. When $N$ is off-shell, the contribution of these diagrams is negligibly small \cite{12, 13} and far from experimental reach. On the other hand, there exist specific domains of sterile neutrino masses $m_N$ where $N$ comes close to its mass-shell, leading to an enormous resonant enhancement \cite{12, 13} of the diagrams in Fig. 1. These domains, for the $K$ and $\tau$ decays in Eq. (2) are respectively:

$$m_N < m_K - m_\mu, \quad m_N < m_\tau - m_\mu,$$

(3)

where the light neutrino mass, $m_\nu$, has been neglected. The mass domains in Eq. (3) cover completely the sterile neutrino mass range of Eq. (1), proposed for the explanation of the LSND and the MiniBooNe anomalies. Therefore, if there is a neutrino $N$ with a mass which is appropriate to explain the anomalies, then the $K$ or $\tau$ radiative decays will necessarily have a contribution from this neutrino $N$ close its mass-shell. This means that an intermediate sterilne neutrino is produced at the corresponding vertex on the left of the diagrams in Fig. 1, propagates as a free unstable particle, and then decays at the corresponding vertex on the right. Accordingly, the decay rate formulas for the reactions $K, \tau \rightarrow XN\gamma$ can be represented in the narrow width approximation ($\tau_N^{-1} \ll m_N$) as the product of two factors: the $K$ or $\tau$ decay rate into the sterile neutrino, $\Gamma(K \rightarrow \mu N)$ or $\Gamma(\tau \rightarrow \mu N)$, times the branching ratio $Br(N \rightarrow \nu\gamma)$. This approximation is clearly valid for $N$ with masses in the range of Eq. (1). The resulting decay rate formulas are then:

$$\Gamma(K^+ \rightarrow \mu^+N) = |U_{\mu N}|^2 \frac{G_F^2}{8\pi} |F_K|^2 |V_{us}|^2 m_K^3 \lambda^2 (x_1^2, x_2^2, 1),$$

(4)

$$\Gamma(\tau^- \rightarrow \mu^-\nu\gamma) = \frac{\Gamma(\tau^- \rightarrow \mu^-\nu\nu\gamma)}{\Gamma(\tau^- \rightarrow \mu^-\bar{\nu}_N)} \approx \frac{\Gamma(\tau^- \rightarrow \mu^-\bar{\nu}_N)}{\Gamma(\tau^- \rightarrow \mu^-\nu\nu\gamma)}.$$  

(5)

where the $K$ and $\tau$ decay rates into $N$ are \cite{14}

$$\Gamma(K^+ \rightarrow \mu^+N) = |U_{\mu N}|^2 \frac{G_F^2}{8\pi} \frac{m_5^5}{192\pi^3} I_1(z_N, z_\nu, z_\mu) \equiv |U_{\mu N}|^2 \Gamma_{\mu N}(\mu N),$$

(6)

$$\Gamma(\tau^- \rightarrow \mu^-\nu\tau) = |U_{\tau N}|^2 \frac{G_F^2}{192\pi^3} m_5^5 I_1(z_N, z_\nu, z_\mu) \equiv |U_{\tau N}|^2 \Gamma_{\tau}(\mu N).$$

(7)

$$\Gamma(\tau^- \rightarrow \mu^-\bar{\nu}_N) = |U_{\tau N}|^2 \frac{G_F^2}{192\pi^3} m_5^5 I_1(z_N, z_\nu, z_\mu) \equiv |U_{\tau N}|^2 \Gamma_{\tau}(\mu N).$$

(8)
Here $f_K = 159$ MeV and $V_{us} = 0.97377$. We denote $z_i = m_i/m_\tau$, $x_i = m_i/m_K$ with $m_i = m_N, m_\nu, m_\mu$, and we use the well known phase space function $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ac$ and the kinematical function $I_1(x, y, z)$ is defined as

$$I_1(x, y, z) = 12 \int_{(x+y)^2}^{(1-z)^2} \frac{ds}{s} (s - x^2 - y^2)(1 + z^2 - s) \lambda^{1/2}(s, x^2, y^2) \lambda^{1/2}(1, s, z^2),$$

(9)

In the scenario under consideration the decay mode $N \rightarrow \nu\gamma$ is dominant, and therefore as a reasonable approximation,

$$Br(N \rightarrow \nu\gamma) \approx 1.$$  

(10)

A general issue to take into account in the radiative decays in question is that the intermediate neutrino propagates as a real particle and decays at a certain distance from the production point. If this distance is larger than the size of the detector, the neutrino escapes before decaying and the signature of $\tau \rightarrow \mu \nu\gamma$ or $K \rightarrow \mu \nu\gamma$ cannot be recognized. Therefore, in order to calculate the rate of radiative $\tau$ or meson $K$ decays within the detector, one should multiply the theoretical rates (4) and (5) by the probability $P_N$ that the neutrino $N$ decays inside the detector. Roughly for a detector of length $L_D$, the probability $P_N$ takes the form (11):

$$P_N \approx 1 - e^{-L_D/\tau_N}.$$  

(11)

However, for short enough lifetimes such as $\tau_N \lesssim 10^{-9}$ (s) in Eq. 1, and detectors of size $L_D \gtrsim 70$ cm, which is typical for this kind of experiments, we can use $P_N \approx 1$.

In Ref. [4] the author studied the consistency of a sterile neutrino with parameters in the range given in Eq. 1 with the data of several experiments, and found no constraints for this part of the parameter space. Here, with the same purpose, we examine the following experimental data [10]:

$$Br(K^+ \rightarrow \mu^+\nu\gamma) = (6.2 \pm 0.8) \times 10^{-3},$$  

(12)

$$Br(\tau^- \rightarrow \mu^-\nu\gamma) = (3.6 \pm 0.4) \times 10^{-3}.$$  

(13)

These measured branching ratios agree with the SM prediction within the quoted experimental uncertainty, namely $\Delta^{exp} = 0.8 \times 10^{-3}$ and $0.4 \times 10^{-3}$, respectively. Therefore, the additional contribution of a sterile neutrino to these processes should not exceed the respective experimental uncertainties. Using (4), (5), (10), (12) and (13) we find the limits

$$|U_{\mu N}|^2 < \frac{\Delta^{exp}(K^+ \rightarrow \mu^+\nu\gamma)}{\Gamma_K/\Gamma_{(\mu N)}}$$  

(14)

$$|U_{\mu N}|^2 < \frac{\Delta^{exp}(\tau^- \rightarrow \mu^-\nu\gamma)}{\Gamma_{\tau}/\Gamma_{(\mu\nu N)}}$$  

(15)

valid for a sterile neutrino in the range given in Eq. (1). Here $\Gamma_K^{(\mu N)}, \Gamma_{\tau}^{(\mu\nu N)}$ were defined in [6], [7], [8]. The limits on $|U_{\mu N}|$ given in Eqs. (14) and (15) are plotted in Fig. 2 curves (a) and (b), respectively. As shown, The most stringent exclusion curve is Fig. 2.a, derived from the $K$ decay data [12]. Clearly, this bound is close, but is still unable to
definitely rule out the whole range of sterile neutrino parameters in Eq. (11) shown in Fig. 2 as the gray zone. On the other hand, the experimental data on radiative \( \tau \) decays shown in Eq. (15) is consistent with the required parameters.

Nevertheless, the following comment is in order. As we just saw, the experimental measurements of radiative \( K \) decays are marginally constraining the sterile neutrino parameters of Eq. (1). However, if experimental cuts were included to restrict the domain of the muon and photon energies, \( E_\mu \) and \( E_\gamma \), characteristic for this mechanism, more stringent bounds can be found. This is so because in the \( K \) rest frame the muon is monoenergetic with a value of kinetic energy determined by the sterile neutrino mass

\[
E_\mu(K) = \frac{(m_K - m_\mu)^2 - m_N^2}{2m_K}.
\]

(16)

For \( m_N = (40 - 80) \) MeV as specified in Eq. (1), the muon energy \( E_\mu(K) \) varies in a very narrow range \( E_\mu(K) = (146 - 151) \) MeV. In turn, the photon energy in the \( K \) rest frame ranges within the interval

\[
\frac{1}{2} (E_N - \sqrt{E_N^2 - m_N^2}) \leq E_\gamma \leq \frac{1}{2} (E_N + \sqrt{E_N^2 - m_N^2}),
\]

(17)

where \( E_N \) is the sterile neutrino energy, also a fixed value: \( m_N^2 \leq m_N \leq m_K - m_\mu \). The first subprocess, \( K^+ \to \mu^+ \nu e^+ e^- \), can be easily estimated from \( K_{\nu 2} \), except for a kinematic correction due to the neutrino mass \( m_N \) and a factor \( |U_{\mu N}| \) due to the \( \nu_\mu \) admixture in \( N \) (see Eq. (6)). The second subprocess is mediated by a photon coupled to the neutrino transition current, which depends on two form factors, \( F_1 \) and the transition magnetic moment \( \mu_tr \): \( J_{\nu}(N) = \bar{\nu} \{ F_1(q^2 \gamma^\mu - \beta q^\mu) + i \mu_tr \sigma^{\mu\nu} q_\nu \} N \).

(20)

For a real photon, only \( \mu_tr \) contributes, as in:

\[
\Gamma(N \to \nu \gamma) = \frac{\mu_tr^2 m_N^3}{8\pi},
\]

(21)

while for a virtual photon both \( F_1 \) and \( \mu_tr \) contribute without interfering, as in \( \Gamma(N \to \nu e^+ e^-) \). Consequently, \( \Gamma(N \to \nu e^+ e^-) \) has as lower bound the expression where \( F_1 \) is neglected, which can be written as:

\[
\Gamma(N \to \nu e^+ e^-) > \frac{8\alpha em}{3\pi} \left( \log \left( \frac{m_N}{2m_e} \right) - 2/3 \right) \Gamma(N \to \nu \gamma) \sim 10^{-2} \Gamma(N \to \nu \gamma).
\]

(22)

Since the experimental measurement [17]: \( Br(K^+ \to \mu^+ \nu e^+ e^-) = (7.06 \pm 0.31) \times 10^{-8} \)

(23)

confirms its SM theoretical estimate, then the extra contribution due to the sterile neutrino (see Eq. (19)) should be at most of the size of the quoted error, thus imposing the bound:

\[
Br(K^+ \to \mu^+ \nu e^+ e^-) \leq 0.31 \times 10^{-8}.
\]

(24)
Eqs. (19), (22) and (24) would then impose the bound $Br(K^+ \to \mu^+ N) \times Br(N \to \nu \gamma) < 3 \times 10^{-7}$. Recalling Eq. (6), $Br(K^+ \to \mu^+ N) = |U_{\mu N}|^2 \frac{\Gamma_K^{(\mu N)}}{\Gamma_K} \gtrsim 0.6 \times |U_{\mu N}|^2$, we could draw the following bound:

$$|U_{\mu N}|^2 \times Br(N \to \nu \gamma) < 0.5 \times 10^{-6}. \quad (25)$$

This stringent bound would rule out the explanation of the LSND and MiniBooNE anomalies in terms of a sterile neutrino with large transition magnetic moment. However, it is not applicable for $m_N$ in the required range of Eq. (1), because the experimental result, Eq. (23), is obtained using a cut on the invariant mass of the $e^+e^-$ pair $m_{ee} > 145$ MeV [17]. On the other hand, the limit in Eq. (25) shows that a new measurement of $K^+ \to \mu^+ \nu e^+ e^-$ would provide a stringent bound on the sterile neutrino hypothesis if the cut on $m_{ee}$ could be reduced below 40 MeV. This 145 MeV cut in the work of Ref. [17] had to be applied in order to suppress the background from the sequence of decays $K^+ \to \mu^+ \nu \pi^0$, $\pi^0 \to \gamma e^+ e^-$. Then an improvement in the efficiency of the veto for the photons from $\pi^0$-decay and measurements of the kaon tracks for better control of the missing mass may be required to achieve this goal.

**In conclusion.** We have shown that the existence of a sterile neutrino with mass and mixing in the range given in Eq. (1) is in tension with the existing experimental data on the radiative $K$ meson decay rate, given in Eq. (12). Future measurements of this rate with better precision will probably be able to derive a more decisive conclusion on the studied question. In addition, the purely leptonic 4-body $K$ decay $K^+ \to \mu^+ \nu e^+ e^-$ will be able to probe the parameter region of Eq. (1) required for the explanation of the LSND and MiniBooNE anomalies, if future measurements reduce the cut in the invariant mass of the $e^+e^-$ pair in the final state of this decay below 40 MeV.

![FIG. 2: The sterile neutrino mass $m_N$ and mixing $U_{\mu N}$ with $\nu_\mu$. In the gray region the resolution [3, 4] of the LSND and MiniBooNE anomalies is possible. The exclusion curves (a), (b) are derived from the experimental data [12, 13] respectively. The regions above these curves are excluded.](image-url)

**Acknowledgments**

We are grateful to William Brooks, Serguei Kuleshov and Marcela Gonzalez for discussions. We also thank Sergey Gninenko and Andrei Poblaguev for useful comments. This work was supported by FONDECYT projects 1100582,
110287, 1070227 and Centro-Científico-Tecnológico de Valparaíso PBCT ACT-028. C.D. acknowledges partial support from Research Ring ACT119, Conicyt.

[1] P. Minkowski, Phys. Lett. **B67**, 421 (1977); T. Yanagida, in Proc. of the Workshop on Grand Unified Theory and Baryon Number of the Universe, KEK, Japan, 1979; M. Gell-Mann, P. Ramond and R. Slansky in Sanibel Symposium, February 1979, CALT-68-709 [retroprint `arXiv:hep-ph/9809459`], and in Supergravity, eds. D. Freedman et al. (North Holland, Amsterdam, 1979); S. L. Glashow in Quarks and Leptons, Cargese, eds. M. Levy et al. (Plenum, 1980, New York), p. 707; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[2] J. Schechter and J. W. F. Valle, Phys. Rev. **D22**, 2227 (1980); Phys. Rev. **D25**, 774 (1982).

[3] S. N. Gninenko, Phys. Rev. Lett. **103**, 241802 (2009).

[4] S. N. Gninenko, Phys. Rev. **D83**, 015015 (2011); S. N. Gninenko, D. S. Gorbunov, Phys. Rev. **D81**, 075013 (2010).

[5] S. N. Gninenko, Phys. Rev. **D83**, 093010 (2011). [arXiv:1101.4004 [hep-ex]].

[6] P. Vogel, Phys. Rev. **D30**, 1505 (1984).

[7] A. Aguilar et al., Phys. Rev. **D64**, 112007 (2001), and references therein.

[8] A.A. Aguilar-Arevalo et al., Phys. Rev. Lett. **105**, 181801 (2010).

[9] Manuel Masip, Pere Masjuan, [arXiv:1103.0689v1 [hep-ph]].

[10] D. McKeen, M. Pospelov, Phys. Rev. **D82**, 113018 (2010). [arXiv:1011.3046 [hep-ph]].

[11] A. Kusenko, S. Pascoli, D. Semikoz, JHEP **0511**, 028 (2005). [hep-ph/0405198].

[12] C. Dib, V. Gribanov, S. Kovalenko and I. Schmidt, Phys. Lett. **B493**, 82 (2000).

[13] V. Gribanov, S. Kovalenko and I. Schmidt, Nucl. Phys. **B607**, 355 (2001).

[14] R. E. Shrock, Phys. Lett. **B96**, 159 (1980); R. E. Shrock, Phys. Rev. **D24**, 1232 (1981); R. E. Shrock, Phys. Rev. **D24**, 1275 (1981); L. S. Littenberg and R. E. Shrock, Phys. Rev. Lett. **68**, 443 (1992); Phys. Lett. **B491**, 285 (2000).

[15] A. Atre, T. Han, S. Pascoli and B. Zhang, JHEP **0905**, 030 (2009) [arXiv:0901.3589 [hep-ph]].

[16] K. Nakamura et al. (Particle Data Group), J. Phys. **G37**, 075021 (2010) (URL: `http://pdg.lbl.gov`).

[17] A.A. Poblaguev et al., Phys. Rev. Lett. **89**, 061803 (2002).

[18] A.A. Poblaguev, private communication.