Feature Selection Based on Unique Relevant Information for Health Data

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Abstract
Feature selection, which searches for the most representative features in observed data, is critical for health data analysis. Unlike feature extraction, such as PCA and autoencoder based methods, feature selection preserves interpretability, meaning that the selected features provide direct information about certain health conditions (i.e., the label). Thus, feature selection allows domain experts, such as clinicians, to understand the predictions made by machine learning based systems, as well as improve their own diagnostic skills. Mutual information is often used as a basis for feature selection since it measures dependencies between features and labels. In this paper, we introduce a novel mutual information based feature selection (MIBFS) method called SURI, which boosts features with high unique relevant information. We compare SURI to existing MIBFS methods using 3 different classifiers on 6 publicly available healthcare data sets. The results indicate that, in addition to preserving interpretability, SURI selects more relevant feature subsets which lead to higher classification performance. More importantly, we explore the dynamics of mutual information on a public low-dimensional health data set via exhaustive search. The results suggest the important role of unique relevant information in feature selection and verify the principles behind SURI.

1 Introduction
Feature selection, which searches for the most representative features of the observed data, is critical for machine learning algorithms as different features can entangle and hide more or less the different explanatory factors of variation behind the data [1–5]. More importantly, the feature selection process preserves the interpretability of raw data. Hence, the selected feature subset provides useful information about which features are indicative of certain health conditions [6]. Understanding these relationships is important for domain experts, such as clinicians, to understand machine learning based predictive diagnosis, as well as improve their own diagnostic skills.

Mutual information (MI) measures the dependency between random variables [7] and is often used as a basis for feature selection. In this paper, we propose a novel mutual information based feature selection (MIBFS) method called Selection via Unique Relevant Information (SURI). The key differences between SURI and existing MIBFS methods [8–18] are as follows: (i) SURI is the first to use the unique relevant information (URI) of each individual feature in feature selection. It searches for the optimal feature subset by boosting features with high URI; (ii) Different from existing MIBFS methods which approximate the high-dimensional joint MI with low-dimensional MI terms, SURI directly estimates the high-dimensional joint MI via nearest-neighbor based MI estimators [19].

The main contributions of this paper are as follows:
• We propose SURI to boost features with high Unique Relevant Information during the feature selection process in order to achieve better classification performance.
• The proposed SURI algorithm outperforms other state-of-art MIBFS methods on 6 publicly available healthcare data sets for 3 classifiers. More importantly, the selected feature subset preserves the interpretability of raw data. Hence, it can provide useful information about which features are possibly indicative of certain health conditions.

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• Using a low-dimensional data set, we explore the dynamics of MI in feature selection via exhaustive search and establish the significance of URI. Based on experiments, we find that features with relatively large URI tend to be frequently present in the feature subsets with the highest classification accuracies (found via exhaustive search).

2 Proposed Algorithm

2.1 Notation & Data Set Description

Let $\Omega$ denote the set of all features and let $n$ be the total number of features. Let $S \subseteq \Omega$ be the set of selected features. Let $X \in \Omega$ be a feature and $Y$ be the classification target (label). Let $H(X_i)$ denote the entropy of feature $X_i$ and $I(X_i; Y)$ denote the mutual information between feature $X_i$ and label $Y$ \cite{7}.

Six publicly available data sets, obtained from the UCI Machine Learning Repository, are used in our study. The Z-Alizadeh Sani data set \cite{20}, Breast Cancer data set \cite{21}, SPECTF data set \cite{22}, Arrhythmia data set \cite{23} and EEG data set \cite{24} are used to examine the effectiveness of SURI and existing MIBFS methods. The Heart Disease data set \cite{25} is used to study, via exhaustive search, the dynamics of MI and explore the role of URI. The EEG data set is preprocessed by calculating mean, maximum, minimum, standard deviation, maximum adjacent change and minimum adjacent change for each channel. The Heart Disease data set is aggregated into 2 different classes. Class 0 has 164 patients without heart disease, Class 1 combines 139 patients who have 4 different level of heart disease into one class. Information about each data set is presented in Table 1.

2.2 Information Content: URI, ORI, II

The information content of a feature (i.e., entropy of a feature) can be divided into two parts: relevant information (RI) and irrelevant information (II). Relevant information can be further divided into unique relevant information (URI) and overlapped relevant information (ORI).

Irrelevant information (II) can be understood as the noise in the signal which confuses the classifier and leads to lower accuracy \cite{26, 27}.

Relevant information can be understood as the non-zero MI between feature $X_i$ and the label $Y$ (i.e., $I(X_i; Y)>0$). URI is the unique relevant information content of a feature with the label which is not shared by any other features in $\Omega$. Mathematically, the URI of a feature is $I(\Omega; Y) - I(\Omega \setminus \{X_i\}; Y)$. This notion of URI is equivalent to strong relevance defined in \cite{28, 16}. URI helps classifiers to differentiate labels and contributes to higher classification accuracy. In contrast to URI, ORI is the relevant information content of a feature with the label which is shared (or overlapped) with other features in $\Omega$ (i.e., ORI=RI-URI). This is equivalent to weak relevance defined in \cite{28, 16}.

We illustrate the idea of URI and ORI in Figure 1. Consider Feature A and Feature B shown in Figure 1. Assume that Feature B does not contain significant URI ($I(\Omega; Y) - I(\Omega \setminus \{X_B\}; Y) \to 0$). Thus, there exists a feature subset $P \subset \Omega$ that overlaps with Feature B’s total amount of relevant information. This means that the contribution of Feature B is a strict subset of $P$’s contribution. Therefore, when $P$ is selected, the ORI of Feature B will become redundant. Hence, selecting Feature B will not improve the classification performance (based on maximum relevance minimum redundancy criterion) \cite{17}, but include its undesired redundancy and irrelevance. On the other hand, Feature A contains non-negligible URI. So, even if Feature B (or any other feature) has been selected first, adding Feature A into the selected feature set can still improve the classification accuracy.

For each health data set in Table 1 we computed the percentage of features that carry non-negligible ($>10^{-8}$) URI with respect to the label. It can be seen that, for many of the data sets, the percentage of features that carry non-negligible URI is quite small (<20%). Therefore, it is likely that a feature...
containing no URI but high ORI will be selected first. To avoid this, features with high URI must be boosted during the feature selection process. This is the motivation behind SURI.

2.3 Proposed Algorithm: SURI

Our proposed feature selection algorithm, SURI, balances increasing relevancy and boosting URI. SURI assigns a score to each feature according to the scoring function given by

\[
J_{SURI}(X) = (1 - \beta)I(S; X; Y) + \beta(I(\Omega; Y) - I(\Omega \setminus X; Y)),
\]

where \(\beta\) is a weighting parameter. The first term corresponds to increasing relevancy and the second term rewards features with URI. The URI term allows SURI to have an overview of all features and helps to calibrate feature selection. The URI of each feature needs to be calculated only once and can be used in subsequent computations. Hence, the time complexity of SURI is \(O(n^2)\), which is comparable to many state-of-the-art MIBFS methods (e.g., JMI [12], GSA [16]). Different from existing MIBFS methods, which approximate the high-dimensional joint MI via low-dimensional MI, SURI directly estimates the high-dimensional joint MI via nearest neighbor based approach [19].

3 Performance Comparison with Existing MIBFS Methods

In this paper, we evaluate six representative MIBFS methods: Mutual Information Maximization (MIM) [11], Joint Mutual Information (JMI) [12], Mutual Information Maximization via Spectral Relaxation (SPEC\(_{CMI}\)) [14], Joint Mutual Information Maximization (JMIM) [13], High Order Mutual Information Feature Selection (HOMIFS) [15] and Greedy Search Algorithm (GSA) [16]. We clarify that GSA selects the candidate feature with the largest joint MI with the label at each step. The results of different MIBFS methods on the EEG data set via the random forest (RF) classifier are shown in Table 2. We shortlist GSA as it has the highest accuracy among existing MIBFS methods and compare SURI to GSA in Table 2, for various values of \(\beta\). The results for other data sets in Table 1 via RF, k-nearest neighbors (KNN) and support vector machine (SVM) are shown in Appendix A.

We observe that SURI has the highest performance over all existing methods studied.

4 Exploring the Dynamics of MI and URI on Heart Disease Data Set

In addition to the performance benefits of SURI, we also want to understand the dynamics of MI and the specific role of URI in performance. In this section, we explore a low-dimensional data set (Heart Disease data set [25]) via exhaustive search. We use the SVM classifier for this part of the study.

The different feature subsets obtained from exhaustive search are ranked based on their corresponding accuracies in descending order and the top 20 feature subsets are shown in Table 3. The optimal solution is the set containing Features \{0, 1, 2, 7, 11, 12\} with accuracy of 0.8567. The columns named HR(0) and HR(1) represent the hit rate of label 0 and label 1, respectively.

Table 4 shows some statistics derived from Table 3 for each of the 13 features in the data set. These include: (i) the URI of each feature measured by \(I(\Omega; Y) - I(\Omega \setminus X; Y)\), (ii) the frequency of each feature in the top 20 ranked feature sets, and (iii) the MI of the feature with the label, \(I(X; Y)\).

Using Table 4 we make the following observations which illustrate the dynamics of MI/URI and the working principles of SURI.
- Features with high MI tend to have non-zero URI as those features contain more relevant information. Those features (Features 2, 7, 8, 11, and 12) are more likely to have high frequencies in the top 20 ranking. This shows the correlation between MI and frequency and demonstrates that MI is a good criterion for feature selection.

- Features with similar MI, a feature with higher URI is more likely to have high frequency (Feature 7 & Feature 10, Feature 8 & Feature 9, Feature 1 & Feature 6). The frequency of Feature 7 is much higher than that of Feature 10 even though they have almost the same MI with the label. We observe that Feature 7 contains URI while Feature 10 does not. This phenomenon also happens with the pair Feature 8 and 9 and the pair Feature 1 and 6. For Features 1, 4 and 10, even though Features 4 and 10 have a higher MI than Feature 1, the frequency of Feature 1 is the highest among those 3 features. We observe that Feature 1 contains URI while the URI of Features 4 and 10 is negligible (\(< 10^{-8}\)).

- Features with low MI and low URI tend to have a low frequency (e.g., Feature 1). The frequency of Feature 1 is relatively low even though Feature 1 contains a small amount of URI. This is because low MI stands for high irrelevant information content, which is harmful to classification accuracy.

- Features with negligible URI and high MI can also have high frequency (e.g., Feature 9). Since \(X_2, X_8, X_{11}\), and \(X_{12}\) appear in most subset of top 20 ranks, we calculate \(I(X_2, X_8, X_{11}, X_{12}, X_9; Y)\) and find that \(I(X_2, X_8, X_{11}, X_{12}, X_9; Y) > I(X_2, X_8, X_{11}, X_{12}; Y)\), meaning that Feature 9 contributes to higher joint MI with respect to Features 2, 8, 11, & 12, explaining the relatively high frequency of Feature 9.

5 Reflections

In the experimental results above, we see the working principle of SURI in action. The first term in \(\bar{I}(S, X; Y)\), \(I(S, X; Y)\), guarantees the relevance of the selected subset and the second term in \(\bar{I}(S, X; Y)\), \(I(S; Y) - I(S; X; Y)\), is to reward features with relatively higher URI. Therefore, features with high joint MI and URI will be selected earlier. For features with similar joint MI, the second term will prioritize features with higher URI. For features with high URI and low joint MI, the first term will penalize them due to their joint URI.

Table 17 in Appendix B gives the physical description of each feature in the UCI Heart Disease data set. Referring to Tables 3 and 17, our findings indicate that the following features have high frequency and should have an effect upon heart disease and its diagnosis: Feature 2 (chest pain type), Feature 6 (resting ECG result), Feature 7 (maximum heart rate), Feature 8 (exercise induced angina), Feature 9 (ST depression induced by exercise relative to rest), Feature 11 (number of major vessels colored by fluoroscopy) and Feature 12 (Thallium stress test result). The majority of these features have non-negligible URI and are more likely to be chosen by SURI. Additionally, the Thallium stress test is administered in patients with heart disease, which is consistent with the fact that Feature 12 has the highest MI and significant URI (see Table 4).

To see how far SURI is from optimal, we compare it to the optimal solution and find that SURI is ranked 25th, which is highest amongst the MIBFS methods (see Figure 2 in Appendix B). So while MI and URI lead to good suboptimal feature selection solutions, our results suggest that there are other hidden variables which could further improve the performance. We also have not specified how to select the \(\beta\) parameter to optimally balance relevance and URI. We expect that these topics are worthy of investigation and will open up many interesting avenues for further exploration.
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A Supplementary Results for Section 3

Tables 5-8 show the performance metrics (peak accuracy, AUC-ROC and F1-score) for existing MIBFS algorithms and SURI (for different $\beta$ values) on five data sets using the KNN classifier.

Tables 9-12 show the performance metrics (peak accuracy, AUC-ROC and F1-score) for existing MIBFS algorithms and SURI (for different $\beta$ values) on five data sets using the SVM classifier.

Tables 13-16 show the performance metrics (peak accuracy, AUC-ROC and F1-score) for existing MIBFS algorithms and SURI (for different $\beta$ values) on five data sets using the Random Forest classifier.

| Data Set Name  | GSA   | MIM   | JMI   | SPEC_{CM1} | HOMIFS | JMIM |
|---------------|-------|-------|-------|------------|--------|------|
| Alizadeh     | 81.6% (25) | 77.7% (30) | 80.3% (28) | 80.6% (29) | 79.0% (31) | 80.3% (27) |
| Breast Cancer | 92.7% (6)  | 91.9% (8)  | 90.9% (9)  | 92.3% (7)  | 91.0% (10) | 90.7% (7)  |
| SPECTF Heart  | 60.8% (14) | 57.9% (16) | 59.1% (13) | 60.5% (15) | 58.8% (14) | 59.3% (15) |
| Arrhythmia    | 72.5% (33) | 69.4% (37) | 72.3% (34) | 68.1% (35) | 68.7% (38) | 67.4% (31) |
| EEG           | 90.3% (27) | 86.1% (32) | 87.3% (27) | 88.1% (31) | 89.1% (27) | 88.3% (31) |

Table 5: Peak Accuracy of Existing MIBFS Algorithms using KNN.

| Data Set Name  | GSA   | MIM   | JMI   | SPEC_{CM1} | HOMIFS | JMIM |
|---------------|-------|-------|-------|------------|--------|------|
| Alizadeh     | 0.821 | 0.799 | 0.805 | 0.813       | 0.802  | 0.809 |
| Breast Cancer | 0.931 | 0.922 | 0.907 | 0.926       | 0.915  | 0.911 |
| SPECTF Heart  | 0.613 | 0.581 | 0.588 | 0.603       | 0.591  | 0.595 |
| Arrhythmia    | 0.727 | 0.707 | 0.729 | 0.677       | 0.681  | 0.679 |
| EEG           | 0.899 | 0.857 | 0.861 | 0.878       | 0.893  | 0.871 |

Table 6: AUC-ROC & F1-Score Corresponding to the Peak Accuracy of Existing MIBFS Algorithms using KNN.
### Table 7: Peak Accuracy of SURI and GSA using KNN.

| Data Set Name | Alizadeh | Breast Cancer | SPECTF Heart | Arrhythmia | EEG |
|---------------|----------|---------------|--------------|------------|-----|
| GSA SURI(\(\beta=0.1\)) | 81.6% (25) | 92.7% (6) | 60.8% (14) | 72.5% (33) | 90.3% (27) |
| SURI(\(\beta=0.2\)) | 82.7% (26) | 93.1% (7) | 61.5% (15) | 74.7% (34) | 90.5% (37) |
| SURI(\(\beta=0.3\)) | 83.1% (26) | 93.8% (7) | 61.8% (15) | 73.9% (33) | 89.7% (33) |

Table 8: AUC-ROC & F1-Score Corresponding to the Peak Accuracy of SURI and GSA using KNN.

| Data Set Name | Alizadeh | Breast Cancer | SPECTF Heart | Arrhythmia | EEG |
|---------------|----------|---------------|--------------|------------|-----|
| GSA MIM JMI SPEC CMI HOMIFS JMIM | 0.821 | 0.931 | 0.613 | 0.727 | 0.899 |
| AUC-ROC | 0.827 | 0.933 | 0.617 | 0.743 | 0.898 |
| F1-Score | 0.803 | 0.922 | 0.606 | 0.709 | 0.887 |
| SURI(\(\beta=0.3\)) | 0.814 | 0.923 | 0.609 | 0.721 | 0.901 |
| SURI(\(\beta=0.2\)) | 0.817 | 0.922 | 0.614 | 0.741 | 0.898 |
| SURI(\(\beta=0.1\)) | 0.818 | 0.934 | 0.615 | 0.733 | 0.895 |

### Table 9: Peak Accuracy of Existing MIBFS Algorithms using SVM.

| Data Set Name | Alizadeh | Breast Cancer | SPECTF Heart | Arrhythmia | EEG |
|---------------|----------|---------------|--------------|------------|-----|
| GSA MIM JMI SPEC CMI HOMIFS JMIM | 83.2% (6) | 85.8% (4) | 61.9% (7) | 62.0% (25) | 83.3% (49) |
| Peak Accuracy (No. of Features Used) | 81.3% (7) | 83.1% (2) | 60.3% (9) | 58.2% (27) | 78.5% (22) |
| JMI SPEC CMI HOMIFS JMIM | 81.9% (9) | 82.4% (3) | 58.8% (9) | 60.0% (29) | 82.1% (37) |
| GSA MIM JMI SPEC CMI HOMIFS JMIM | 82.8% (10) | 85.5% (5) | 60.2% (8) | 58.6% (28) | 80.3% (41) |
| JMI SPEC CMI HOMIFS JMIM | 82.9% (8) | 83.4% (2) | 60.3% (10) | 60.6% (26) | 81.9% (46) |
| GSA MIM JMI SPEC CMI HOMIFS JMIM | 80.5% (9) | 85.5% (3) | 59.3% (11) | 58.9% (27) | 82.6% (52) |

Table 10: AUC-ROC & F1-Score Corresponding to the Peak Accuracy of Existing MIBFS Algorithms using SVM.

| Data Set Name | Alizadeh | Breast Cancer | SPECTF Heart | Arrhythmia | EEG |
|---------------|----------|---------------|--------------|------------|-----|
| GSA MIM JMI SPEC CMI HOMIFS JMIM | 0.836 | 0.861 | 0.621 | 0.625 | 0.835 |
| AUC-ROC | 0.817 | 0.843 | 0.607 | 0.591 | 0.791 |
| F1-Score | 0.829 | 0.857 | 0.606 | 0.619 | 0.825 |
| SPEC CMI HOMIFS JMIM | 0.808 | 0.836 | 0.595 | 0.587 | 0.825 |
| GSA MIM JMI SPEC CMI HOMIFS JMIM | 0.833 | 0.857 | 0.615 | 0.622 | 0.831 |
| JMI SPEC CMI HOMIFS JMIM | 0.823 | 0.833 | 0.609 | 0.595 | 0.784 |
| GSA MIM JMI SPEC CMI HOMIFS JMIM | 0.827 | 0.834 | 0.605 | 0.611 | 0.817 |
| JMI SPEC CMI HOMIFS JMIM | 0.801 | 0.843 | 0.607 | 0.598 | 0.827 |

### Table 11: Peak Accuracy of SURI and GSA using SVM.

| Data Set Name | Alizadeh | Breast Cancer | SPECTF Heart | Arrhythmia | EEG |
|---------------|----------|---------------|--------------|------------|-----|
| GSA SURI(\(\beta=0.1\)) | 83.2% (6) | 85.8% (4) | 61.9% (7) | 62.0% (25) | 83.3% (49) |
| SURI(\(\beta=0.2\)) | 82.7% (7) | 84.4% (4) | 60.3% (9) | 58.2% (27) | 78.5% (22) |
| SURI(\(\beta=0.3\)) | 83.9% (7) | 86.2% (4) | 58.8% (9) | 60.0% (29) | 82.1% (37) |
| Peak Accuracy (No. of Features Used) | 81.9% (9) | 82.4% (3) | 60.2% (8) | 58.6% (28) | 80.3% (41) |
| SURI(\(\beta=0.3\)) | 81.9% (8) | 83.4% (2) | 60.3% (10) | 60.6% (26) | 81.9% (46) |
| SURI(\(\beta=0.2\)) | 82.9% (8) | 85.5% (3) | 59.3% (11) | 58.9% (27) | 82.6% (52) |

Table 12: AUC-ROC & F1-Score Corresponding to the Peak Accuracy of SURI and GSA using SVM.
| Data Set Name | AUC-ROC | F1-Score |
|--------------|---------|---------|
|              | GSA     | SURI(β=0.1) | SURI(β=0.2) | SURI(β=0.3) | GSA     | SURI(β=0.1) | SURI(β=0.2) | SURI(β=0.3) |
| Alizadeh     | 0.836   | 0.833   | **0.845** | 0.827 | 0.833   | 0.825   | **0.836** | 0.807   |
| Breast Cancer| 0.861   | 0.843   | **0.863** | 0.851 | 0.857   | 0.843   | **0.859** | 0.853   |
| SPECTF Heart | 0.621   | 0.603   | 0.622   | **0.631** | 0.615   | 0.605   | 0.619   | **0.621** |
| Arrhythmia   | 0.625   | 0.617   | 0.629   | **0.630** | 0.622   | 0.619   | 0.623   | **0.624** |
| EEG          | 0.835   | 0.847   | 0.838   | **0.851** | 0.831   | 0.835   | 0.827   | **0.841** |

Table 12: AUC-ROC & F1-Score Corresponding to the Peak Accuracy of SURI and GSA using SVM.

| Data Set Name | Peak Accuracy (No. of Features Used) |
|--------------|--------------------------------------|
|              | GSA | MIM | JMI | SPEC | HOMIFS | JMIM |
| Alizadeh     | 85.4% (28) | 84.9% (31) | 85.1% (29) | 84.4% (31) | 85.1% (33) | 84.8% (30) |
| Breast Cancer| 95.8% (6)  | 94.1% (9)  | 95.3% (8)  | 94.4% (14) | 95.1% (13) | 94.8% (12) |
| SPECTF Heart | 64.3% (14) | 61.7% (17) | 63.3% (15) | 62.7% (16) | 64.6% (18) | 63.3% (16) |
| Arrhythmia   | 81.3% (39) | 79.2% (51) | 80.8% (46) | 80.4% (45) | 80.2% (55) | 78.6% (43) |
| EEG          | 90.1% (37) | 87.1% (47) | 87.4% (43) | 87.5% (42) | 88.1% (51) | 87.6% (49) |

Table 13: Peak Accuracy of Existing MIBFS Algorithms using Random Forest.

| Data Set Name | AUC-ROC | F1-Score |
|--------------|---------|---------|
|              | GSA     | SURI(β=0.1) | SURI(β=0.2) | SURI(β=0.3) | GSA     | SURI(β=0.1) | SURI(β=0.2) | SURI(β=0.3) |
| Alizadeh     | **0.806** | 0.801   | 0.802   | 0.797 | **0.901** | 0.893   | 0.890   | 0.894   | 0.898   | 0.886 |
| Breast Cancer| **0.945** | 0.934   | 0.943   | 0.938 | **0.938** | 0.922   | 0.923   | 0.925   | 0.920   | 0.913 |
| SPECTF Heart | 0.612   | 0.607   | 0.595   | 0.625 | 0.723   | 0.710   | 0.727   | 0.714   | 0.707   | **0.729** |
| Arrhythmia   | **0.765** | 0.703   | 0.752   | 0.715 | **0.742** | 0.711   | 0.763   | 0.740   | 0.732   | 0.715 |
| EEG          | **0.897** | 0.867   | 0.871   | 0.889 | **0.902** | 0.873   | 0.871   | 0.874   | 0.899   | 0.891 |

Table 14: AUC-ROC & F1-Score Corresponding to the Peak Accuracy of Existing MIBFS Algorithms using Random Forest.

| Data Set Name | Peak Accuracy (No. of Features Used) |
|--------------|--------------------------------------|
|              | GSA | SURI(β=0.1) | SURI(β=0.2) | SURI(β=0.3) |
| Alizadeh     | 85.4% (28) | 85.9% (32) | **86.3%** (33) | 85.3% (33) |
| Breast Cancer| 95.8% (6)  | 96.1% (8)  | **96.8%** (9)  | 95.0% (7)  |
| SPECTF Heart | 64.3% (14) | 63.9% (16) | **64.6%** (16) | 62.8% (17) |
| Arrhythmia   | 81.3% (39) | 81.8% (44) | **81.9%** (46) | 80.3% (45) |
| EEG          | 90.1% (37) | 90.7% (27) | **91.2%** (33) | 87.7% (32) |

Table 15: Peak Accuracy of SURI and GSA using Random Forest.

| Data Set Name | AUC-ROC | F1-Score |
|--------------|---------|---------|
|              | GSA     | SURI(β=0.1) | SURI(β=0.2) | SURI(β=0.3) | GSA     | SURI(β=0.1) | SURI(β=0.2) | SURI(β=0.3) |
| Alizadeh     | 0.806   | 0.808   | **0.813** | 0.810 | 0.901   | 0.904   | **0.909** | 0.901   |
| Breast Cancer| 0.945   | 0.946   | 0.944   | 0.937 | 0.938   | 0.935   | **0.942** | 0.936   |
| SPECTF Heart | **0.612** | 0.608   | 0.611   | 0.603 | 0.723   | 0.724   | 0.718   | 0.703   |
| Arrhythmia   | 0.765   | 0.763   | 0.767   | **0.771** | 0.742   | 0.756   | **0.763** | 0.749   |
| EEG          | 0.897   | 0.901   | **0.905** | 0.887 | 0.902   | 0.905   | **0.909** | 0.893   |

Table 16: AUC-ROC & F1-Score Corresponding to the Peak Accuracy of SURI and GSA using Random Forest.
B Additional Data

| Feature Number | Feature Description                                                                 |
|----------------|--------------------------------------------------------------------------------------|
| Feature 0      | Age in years (age)                                                                   |
| Feature 1      | Sex (sex)                                                                            |
| Feature 2      | Chest pain type (cp)                                                                 |
| Feature 3      | Resting blood pressure on admission to the hospital (trestbps)                      |
| Feature 4      | Serum cholesterol (chol)                                                             |
| Feature 5      | Fasting blood sugar (fbs)                                                            |
| Feature 6      | Resting ECG results (restecg)                                                        |
| Feature 7      | Maximum heart rate during Thalium stress test (thalach)                              |
| Feature 8      | Exercise induced angina (exang)                                                      |
| Feature 9      | ST depression induced by exercise relative to rest (oldpeak)                         |
| Feature 10     | Slope of the peak exercise ST segment (slope)                                        |
| Feature 11     | Number of major vessels colored by flouroscopy (ca)                                  |
| Feature 12     | Thalium stress test result (thal)                                                    |

Table 17: Feature Description for UCI Heart Disease Data Set

Figure 2: SVM average peak accuracies of various MIBFS methods and corresponding ranks based on average peak accuracy.