Universal Inverse seesaw mechanism as a source of the SM fermion mass hierarchy

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We build a renormalizable theory where the inverse seesaw mechanism explains the pattern of SM fermion masses. To the best of our knowledge, our model corresponds to the first implementation of the inverse seesaw mechanism for the charged fermion sector. In our theory, the inverse seesaw mechanism is implemented at the tree and one-loop levels in order to generate the masses for the second and first families of the SM charged fermions, respectively. The third family of SM charged fermions obtain tree-level masses from the Higgs doublets $\phi_1$ (for the top quark) and $\phi_2$ (for the bottom quark and tau lepton). The masses of the active light neutrinos are generated from a two-loop level inverse seesaw mechanism. Our model successfully explains the observed SM fermion mass hierarchy, the tiny masses of the active light neutrinos, contains the necessary means for efficient leptogenesis and is in accordance with the constraints resulting from meson oscillations, as well as with the measured values of the observed dark matter relic density and of the muon and electron anomalous magnetic moments.

I. INTRODUCTION

Despite the remarkable success of the Standard Model (SM) in describing the strong and electroweak interactions with a high degree of accuracy, as confirmed by the experiments at the Large Hadron Collider (LHC), it does not address several questions. One of them is the unexplained hierarchy of the SM fermion masses, which extends over a range of 13 orders of magnitude, from the light active neutrino mass scale up to the top quark mass. Other unaddressable issues of the SM are the number of SM fermion families, the current amount of dark matter relic density, lepton asymmetry of the Universe, and the muon and electron anomalous magnetic moments.

Models with extended symmetries, enlarged particle content, and radiative seesaw mechanisms, are frequently used to tackle the limitations of the SM. [1–97]. Furthermore, several extensions of the SM have been constructed to explain the experimental value of the muon anomalous magnetic moment [73, 84, 88, 91–94, 96, 98–168], an anomaly not explained by the SM and recently confirmed by the Muon $g - 2$ experiment at FERMILAB [169].

Intending to address the drawbacks as mentioned earlier of the SM, we propose an extension of the Two Higgs Doublet Model (2HDM) with enlarged particle spectrum and symmetries, which allows for a successful implementation of the inverse seesaw mechanism to explain the SM fermion mass hierarchy. Unlike most of the works considered in the literature, in the proposed theory the inverse seesaw mechanism is implemented not only for the neutrino sector, but also for the charged fermion sector. Specifically, the charged fermions of the first and the second families receive masses via one-loop and tree-levels inverse seesaw mechanisms, respectively. For the third family, the charged fermions obtain a mass at tree-level, namely the t-quark mass depends on the VEV of the Higgs doublet, $\phi_1$, while the masses of both tau-lepton and b-quark depend on the VEV of $\phi_2$. The light active neutrinos gain masses at two loop level. The content of this paper goes as follows. In Sec. II we describe our proposed model. The implications of the model in the SM fermion mass hierarchy are discussed in section III, while we study the new contributions to the muon and
II. THE MODEL

We start this section by explaining the reasoning behind the inclusion of extra scalars, fermions, and symmetries required for the implementation of tree and one-loop level inverse seesaw mechanisms to generate the masses of the second and first families of SM charged fermions, respectively, and a two-loop level inverse seesaw mechanism to produce the tiny active neutrino masses. In our model, which is an extended 2HDM theory, the top quark mass will arise at tree level from a renormalizable Yukawa interaction involving a $SU(2)$ Higgs doublet, i.e., $\phi_1$. In contrast, the bottom quark and tau lepton will get tree-level masses from the second $SU(2)$ Higgs doublet $\phi_2$. Such required Yukawa interactions, which generate the tree level masses for the top and bottom quarks as well as for the tau lepton, are:

$$
\overline{q}_{3L} \phi_1 u_{iR}, \quad \overline{q}_{3L} \phi_2 d_{iR}, \quad \overline{l}_{iL} \phi_2 l_{3R}, \quad i = 1, 2, 3
$$

The $SU(2)$ Higgs doublets $\phi_1$ and $\phi_2$ need to be distinguished by a symmetry, which can be a $U(1)_X$ local symmetry, assumed to be non universal in the quark sector, as it will be shown below. Such symmetry will distinguish the left handed quark doublets $q_{nL}$ ($n = 1, 2$) from the $q_{3L}$. Furthermore, a discrete symmetry $Z_4$ is required to allow only the above given Yukawa interactions, thus allowing to forbid the operators:

$$
\overline{q}_{nL} \phi_2 u_{iR}, \quad \overline{q}_{nL} \phi_1 d_{iR}, \quad \overline{l}_{iL} \phi_2 l_{nR}, \quad i = 1, 2, 3, \quad n = 1, 2
$$

which would give rise to Dirac type masses for the first two generations of SM fermions. Furthermore, the $Z_4$ discrete symmetry, together with the $U(1)_X$ gauge symmetry, will also permit the implementation of tree and one-loop level inverse seesaw mechanisms to generate the masses for the second and first generation of SM charged fermions, respectively. The advantage of the local $U(1)_X$ gauge symmetry, with respect to a cyclic symmetry, is that it allows more freedom in the particle assignments. The presence of this symmetry means that heavy non SM fermions can be produced via a Drell-Yan portal mediated by a heavy $Z'$ gauge boson. The masses of the active light neutrinos, obtained from an inverse seesaw mechanism when the full neutrino mass matrix is expressed in the basis $(\nu_L, \nu_R^C, N_R^C)$, has the following structure:

$$
M_\nu = \begin{pmatrix}
0_{3\times 3} & m_{\nu_D} & 0_{3\times 3} \\
0_{3\times 3} & M & 0_{3\times 3} \\
0_{3\times 3} & M^T & \mu
\end{pmatrix},
$$

where $\nu_{iL}$ ($i = 1, 2, 3$) correspond to the active neutrinos, whereas $\nu_{iR}$ and $N_{iR}$ ($i = 1, 2, 3$) are the sterile neutrinos. Furthermore, the entries of the full neutrino mass matrix should fulfill the hierarchy $\mu_j \ll (m_{\nu_D})_{ij} \ll M_{ij}$ ($i, j = 1, 2, 3$). It is worth mentioning that in the case $\mu = 0$, the light active neutrinos remain massless and the sterile neutrinos $\nu_{iR}$ and $N_{iR}$ become degenerate, forming Dirac neutrinos whose corresponding mass matrix is $M$, and their contribution to the light active neutrino masses vanishes. Thus, by analogy with the neutrino sector, the implementation of the tree-level inverse seesaw mechanism for the SM charged fermions requires that its corresponding mass matrix in the basis $(\bar{f}_{1L}, \bar{f}_{2L}, \bar{f}_{3L}, \bar{F}_L, \bar{F}_R)$ should have the form:

$$
M_F = \begin{pmatrix}
0_{3\times 3} & F_F & 0_{3\times 1} \\
0_{1\times 3} & G_F^T & 0 \\
0_{1\times 3} & Y_F & m_F
\end{pmatrix},
$$

where $f_i$ ($i = 1, 2, 3$) correspond to the SM charged fermions, whereas $F$ and $\bar{F}$ are the exotic charged fermions. Let us note that in the limit $m_F \to 0$ ($F = T, D, E$), the model under consideration has an accidental $U_X(1)$ symmetry under which the charges of the $f_{1L}, f_{iR}, F_L, F_R, \bar{F}_L, \bar{F}_R$ are given by:

$$
Q_{UX(1)}(f_{iL}) = Q_{UX(1)}(\bar{F}_L) = Q_{UX(1)}(F_R) = a, \quad Q_{UX(1)}(f_{iR}) = Q_{UX(1)}(\bar{F}_R) = Q_{UX(1)}(F_L) = b, \quad a \neq b.
$$

It is worth mentioning that unlike the neutrino sector, in the charged fermion sector the tree level inverse seesaw mechanism is only implemented to generate the masses of the second family of SM charged fermions. This is due to
the fact that the third family of SM charged fermions obtain their masses from renormalizable interactions involving the \( SU(2) \) Higgs doublets \( \phi_1 \) and \( \phi_2 \), whereas the first family of SM charged fermions get their masses from radiative corrections, as it will be shown below. Because of this reason there is only one family of exotic charged fermions \( F \) and \( \bar{F} \) involved in the tree level inverse seesaw mechanism, whereas the sterile neutrino spectrum resulting from Eq. (3) is composed of three copies of the \( \nu_R^C \) and \( N_R^C \) fields. It is worth mentioning that the Dirac type masses of the first two generations of SM charged fermions that would result from the \( q_{nL}\phi_2 u_{rR} \) and \( q_{nL}\phi_1 d_{rR} \) operators will be forbidden by the \( Z_4 \) discrete symmetry.

To generate the charged fermion mass matrix of Eq. (4), one has to include the following operators:

\[
\begin{align*}
\bar{q}_{nL}\phi_2 u_{rR}, & \quad T_L\eta U_R, & \quad m_T T_L T_R, & \quad \bar{U}_{LX}^T T_R, & \quad \bar{U}_{L}\sigma^* u_{rR}, & i = 1, 2, 3, \\
\bar{q}_{nL}\phi_1 d_{rR}, & \quad D_L\eta^* D_{1R}, & \quad m_B B_L B_R, & \quad \bar{D}_{1L} \chi^* B_R, & \quad \bar{D}_{1L} \sigma d_{rR}, \\
\bar{l}_{iL}\phi_2 E_{1R}, & \quad \bar{E}_{2L}\rho^* E_{1R}, & \quad m_E E_{2L} E_{2R}, & \quad \bar{E}_{1L} \rho E_{2R}, & \quad \bar{E}_{1L} S l_{nR}, & n = 1, 2,
\end{align*}
\]

Here \( q_{iL}, l_{iL} \) \( (i = 1, 2, 3) \) are the \( SU(2) \) left handed SM quark and lepton doublets, respectively, whereas \( u_{rR}, d_{rR} \) and \( l_{rR} \) stand for the right SM up-type quarks, down type quarks, and right-handed leptons. Furthermore, the SM fermion sector has to be extended to include the exotic fermions: up type quarks \( U, T \), down type quarks \( D_1, D_2, B \), charged exotic leptons \( E_1, E_2 \), as well as the right-handed Majorana neutrinos \( \nu_R^C \) \( (i = 1, 2, 3) \), \( N_{iR}, \Psi_{mR} \) and \( \Omega_{mR}^C \), in singlet representations under \( SU(2)_L \) with appropriate \( U(1)_X \) charges (to be specified below), which will allow the implementation of inverse seesaw mechanisms to explain the SM charged fermion mass hierarchy and the tiny values of the light active neutrino masses, respectively, in a way consistent with the cancellation of chiral anomalies. Up to this point, the first generation of SM charged fermions remain massless up to loop corrections. In order to generate the masses for the first generation of SM charged fermions, the one-loop level corrections to the SM charged fermion mass matrices should have different vertices used for generating the resulting tree-level mass matrices arising from the inverse seesaw. The most economical solution to this problem requires considering one-loop level corrections involving electrically charged scalars running in the internal lines of the loop. So we have to introduce extra electrically charged scalar singlets in the scalar spectrum, generating the following interactions:

\[
\begin{align*}
\bar{U}_L\zeta_1^+ d_{iR}, & \quad \bar{D}_{1L} \zeta_1^- u_{iR}, & \quad \bar{N}_{iR}^C \zeta_2^+ l_{nR}, & i = 1, 2, 3, & n = 1, 2.
\end{align*}
\]

Then, the up quark mass can be generated at one-loop level from the first four operators of the second line of Eq. (6), as well as from the \( \bar{D}_{1L} \zeta_1^- u_{iR} \) interaction. Similarly, one-loop diagrams which generate a mass for down quark, arise from the first four operators of the first line of Eq. (6), as well as from the \( \bar{U}_L\zeta_1^+ d_{iR} \) interaction. In order to close the one-loop Feynman diagrams giving rise to the first generation SM charged fermions, the following scalar interactions are required:

\[
\varepsilon_{ab}\phi_1^a\phi_2^b\zeta_3 \sigma^*, \quad \zeta_1^+ \zeta_3 S^2, \quad \zeta_2^- \zeta_3 \sigma^*, \quad a, b = 1, 2.
\]

Moreover, the operators required for the implementation of the two loop level inverse seesaw mechanism that produces the tiny active neutrino masses are:

\[
\begin{align*}
\bar{l}_{iL}\phi_2 \nu_{rJ}, & \quad \nu_{iR} \sigma^C \bar{N}_{kR}^C, & \quad N_{rR} \Psi_{mR}^C \varphi_1, & \quad \bar{N}_{mR} \varphi_2 \Omega_{mR}^C, & \quad \bar{N}_{mR} \Omega_{mR}^C \eta, & i, j, k, r = 1, 2, 3, & n, m = 1, 2.
\end{align*}
\]

It is worth mentioning that the first two operators of Eq. (9), together with the \( \bar{N}_{iR} \zeta_2^+ l_{nR}, \varepsilon_{mn}\phi_1^m\phi_2^n\zeta_3 \sigma^* \) and \( \zeta_2^- \zeta_3 \sigma^* \) interactions, are crucial for generating a nonvanishing one-loop level electron mass.

Implementing the above-described inverse seesaw mechanism requires to add a preserved \( Z_2 \) symmetry, under which the right-handed Majorana neutrinos \( \Psi_{mR} \), the exotic down type quark fields \( D_{2L, 2R} \) and the gauge scalar singlets \( \varphi_1 \) and \( \varphi_2 \) are charged, whereas the remaining fields are neutral. Additionally, we have extended the scalar sector of our 2HDM theory by including the electrically neutral gauge singlet scalars \( \sigma, \chi, \eta, \rho, S, \varphi_1, \varphi_2 \) as well as the electrically charged gauge singlet scalars \( \zeta_1^+, \zeta_3^+ \) and \( \zeta_3 \). Notice that the electrically neutral gauge singlet scalars \( \sigma \) and \( S \) are needed to generate mixing between the right-handed SM charged fermions and left-handed heavy fermionic fields. On the other hand, the singlet scalars \( \chi, \eta, \rho \) generate mixings between the heavy charged exotic fermionic fields. Those mixings are crucial for implementing a tree-level universal seesaw mechanism that produces the masses for the second generation of SM charged fermions. As mentioned earlier, we will be able to implement seesaw mechanisms useful for explaining the SM fermion mass hierarchy by suitable charge assignments, specified below. Due to those mentioned above exotic charged fermion spectrum, the SM charged fermion mass matrices would feature a proportionality between rows and columns, thus implying that the first generation SM charged fermions would be massless at tree level. The one-loop level corrections to these matrices involving different vertices that the
SU(3)$_C$\times SU(2)$_L$\times U(1)$_Y$\times U(1)$_X$\times Z$_2$\times Z$_4$.

Table I: Scalar assignments under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \times Z_2 \times Z_4$.

|  | SU(3)$_C$ | SU(2)$_L$ | U(1)$_Y$ | U(1)$_X$ | Z$_2$ | Z$_4$ |
|---|----------|----------|----------|----------|-------|-------|
| $\phi_1$ | 1 | 2 | $\frac{1}{3}$ | $\frac{2}{3}$ | 0 | −1 |
| $\phi_2$ | 1 | 2 | $\frac{1}{3}$ | $\frac{2}{3}$ | 0 | 1 |
| $\sigma$ | 1 | 1 | 0 | $\frac{1}{3}$ | 0 | −1 |
| $\chi$ | 1 | 1 | 0 | $\frac{2}{3}$ | 0 | −2 |
| $\eta$ | 1 | 1 | 0 | −1 | 0 | 2 |
| $\rho$ | 1 | 1 | 0 | 2 | 0 | −1 |
| $S$ | 1 | 1 | 0 | 0 | 0 | 1 |
| $\zeta^+_1$ | 1 | 1 | 1 | $\frac{2}{3}$ | 0 | −1 |
| $\zeta^+_2$ | 1 | 1 | 1 | 1 | 0 | 0 |
| $\zeta^+_3$ | 1 | 1 | 1 | $\frac{2}{3}$ | 0 | 1 |
| $\phi^+_1$ | 1 | 1 | 0 | 1 | 1 | 0 |
| $\phi^+_2$ | 1 | 1 | 0 | 0 | 1 | 1 |

Table II: Quark assignments under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \times Z_2 \times Z_4$. Here $i = 1, 2, 3$.

|  | SU(3)$_C$ | SU(2)$_L$ | U(1)$_Y$ | U(1)$_X$ | Z$_2$ | Z$_4$ |
|---|----------|----------|----------|----------|-------|-------|
| $q_{iL}$ | 3 | 2 | $\frac{1}{3}$ | 0 | 0 | 1 |
| $q_{iR}$ | 3 | 2 | $\frac{1}{3}$ | $\frac{2}{3}$ | 0 | 0 |
| $u_{iR}$ | 3 | 1 | $\frac{2}{3}$ | $\frac{1}{3}$ | 0 | 1 |
| $d_{iR}$ | 3 | 1 | $\frac{2}{3}$ | $\frac{1}{3}$ | 0 | −1 |
| $U_L$ | 3 | 1 | $\frac{2}{3}$ | $\frac{1}{3}$ | 0 | 2 |
| $U_R$ | 3 | 1 | $\frac{2}{3}$ | $\frac{1}{3}$ | 0 | 2 |
| $T_L$ | 3 | 1 | $\frac{2}{3}$ | $\frac{1}{3}$ | 0 | 0 |
| $T_R$ | 3 | 1 | $\frac{2}{3}$ | $\frac{1}{3}$ | 0 | 0 |
| $D_{1L}$ | 3 | 1 | $\frac{2}{3}$ | 0 | 0 | 2 |
| $D_{1R}$ | 3 | 1 | $\frac{2}{3}$ | $\frac{1}{3}$ | 0 | 2 |
| $D_{2L}$ | 3 | 1 | $\frac{2}{3}$ | $\frac{1}{3}$ | 0 | −1 |
| $D_{2R}$ | 3 | 1 | $\frac{2}{3}$ | $\frac{1}{3}$ | 1 | 0 |
| $B_L$ | 3 | 1 | $\frac{2}{3}$ | $\frac{2}{3}$ | 0 | 0 |
| $B_R$ | 3 | 1 | $\frac{2}{3}$ | $\frac{2}{3}$ | 0 | 0 |

ones used to implement the tree level universal seesaw will break such proportionality, thus giving rise to one-loop level masses for the up and down quarks, as well as for the electron. In order for this to happen, the electrically charged scalar fields $\zeta_1^+$, $\zeta_2^+$ and $\zeta_3^+$ have to be introduced in the scalar spectrum. Furthermore, the inert electrically neutral scalar singlets $\phi_1$ and $\phi_2$ are needed to generate the Majorana $\mu$ term at the two-loop level. Thus, such scalar and exotic charged fermion spectrum is the minimal required so that no massless charged SM-fermions would appear in the model, provided that one loop level corrections are taken into account. Furthermore, as mentioned earlier, exotic neutral lepton content is the minimal one required to generate the masses for two active light neutrinos, as required from the neutrino oscillation experimental data.

Our proposed model corresponds to an extension of a 2HDM based on the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ gauge symmetry, supplemented by the $Z_2 \times Z_4$ discrete group. The $SU(2)_L \times U(1)_Y \times U(1)_X$ gauge symmetry, as well as the $Z_4$ discrete symmetry, are spontaneously broken, whereas the $Z_2$ symmetry is preserved, which allows the implementation of a two loop level inverse seesaw mechanism to produce the tiny active neutrino masses. The scalar sector of our extended 2HDM model is composed of two Higgs doublets (having different $U(1)_X$ and $Z_4$ charges) plus several electrically neutral gauge singlet scalars. In our model one Higgs doublet, i.e., $\phi_1$, provides a tree level mass to the top quark, whereas the other one, i.e., $\phi_2$, generates tree level masses for the bottom quark and tau lepton. The second and first families of SM charged fermions get their masses from tree and one loop level inverse seesaw mechanisms, respectively. The tiny active neutrino masses arise from a two loop level inverse seesaw mechanism. The scalar, quark and lepton content with their assignments under the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ group are shown in Tables I, II and III, respectively.
With the particle content previously specified, we have the following relevant Yukawa terms invariant under the symmetries of the model:

\[
-\mathcal{L}_Y^{(q)} = \sum_{i=1}^{3} y_i^{(u)} \overline{q}_{3L} \tilde{\phi}_1 u_i R + \sum_{i=1}^{3} y_i^{(d)} \overline{q}_{3L} \tilde{\phi}_2 d_i R + \sum_{n=1}^{2} x_n^{(U)} \overline{q}_{nL} \tilde{\phi}_2 U_R + \sum_{n=1}^{2} x_n^{(D)} \overline{q}_{nL} \tilde{\phi}_1 D_R
\]

\[
+ z_D \mathcal{D}_{2L} \sigma D_{2R} + \sum_{i=1}^{3} x_i^{(u)} \overline{U}_L \sigma^* u_i R + z_T \overline{U}_L \chi T_R + z_T \overline{T}_L \eta U_R + m_T \overline{T}_L T_R
\]

\[
+ \sum_{i=1}^{3} x_i^{(d)} \mathcal{D}_{1L} \sigma d_i R + z_B \mathcal{D}_{1L} \chi^* B_R + z_B \overline{B}_L \eta^* D_{1R} + m_B \overline{B}_L B_R
\]

\[
+ \sum_{i=1}^{3} w_i^{(u)} \overline{D}_{1L} \zeta_1^* u_i R + \sum_{i=1}^{3} w_i^{(d)} \overline{U}_L \zeta_1^* d_i R + H.c.
\]

(10)

\[
-\mathcal{L}_Y^{(l)} = \sum_{i=1}^{3} y_i^{(l)} \overline{l}_{iL} \phi_2 l_{3R} + \sum_{i=1}^{3} y_i^{(E)} \overline{l}_{iL} \phi_2 E_{1R} + \sum_{n=1}^{2} x_n^{(l)} \overline{E}_{1L} S l_{nR} + y_E \overline{E}_1 \rho E_{2R} + x_E \overline{E}_2 \rho^* E_{1R} + m_E \overline{E}_2 E_{2R}
\]

\[
+ \sum_{i=1}^{3} \sum_{j=1}^{2} y_i^{(E)} \overline{l}_{iL} \phi_2 \nu_{jR} + \sum_{i=1}^{3} \sum_{j=1}^{2} x_{n}^{(l)} \overline{E}_{1L} \chi_2 l_{nR} + \sum_{i=1}^{3} \sum_{j=1}^{2} y_{ij}^{(N)} \nu_{iR} \sigma^* \overline{N}_{jR} + \sum_{i=1}^{3} \sum_{n=1}^{2} (x_{N})_{in} \overline{N}_{iR} \Psi_{nR}^C \phi_1
\]

\[
+ \sum_{n=1}^{2} \sum_{m=1}^{2} (x_{\Psi})_{nm} \overline{N}_{nR} \phi_2 \Omega_{mR}^C + \sum_{n=1}^{2} \sum_{m=1}^{2} (y_{\Omega})_{nm} \overline{N}_{nR} \Omega_{mR}^C \eta + h.c.
\]

(11)
III. FERMION MASS MATRICES

From the charged fermion Yukawa terms we find that the up and down type quark mass matrices in the basis $(u_{1L}, u_{2L}, u_{3L}, U_L, T_L, T_R)$ and $(d_{1L}, d_{2L}, d_{3L}, D_L, B_L, B_R)$ are respectively given by:

$$M_U = \begin{pmatrix} C_U + \Delta_U & F_U & 0_{3 \times 1} \\ G_U^T & 0 & X_U \\ 0_{1 \times 3} & Y_U & m_T \end{pmatrix}, \quad (F_U)_{33} = 0, \quad (F_U)_n = x_n^{(u)} \frac{v_2}{\sqrt{2}}, \quad (G_U^T)_i = x_i^{(u)} \frac{v_\sigma}{\sqrt{2}},$$

$$C_U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_1^{(u)} & y_2^{(u)} & y_3^{(u)} \end{pmatrix} \frac{v_1}{\sqrt{2}}, \quad X_U = z_T \frac{v_\chi}{\sqrt{2}}, \quad Y_U = z_U \frac{v_\eta}{\sqrt{2}}, \quad i = 1, 2, 3, \quad n = 1, 2. \quad (12)$$

Figure 1: Feynman diagrams contributing to the entries of the SM charged fermion mass matrices. Here, $n = 1, 2$ and $i, j = 1, 2, 3$. 
the charged lepton mass matrix, written in the basis $\left( \Psi_n, \Omega_{k}, \Omega_{l}, \Psi_{r}, \Psi_{r}^{c}, N_{s}, N_{p} \right)$. Here we assume that the entries of the charged fermion mass matrices fulfill the hierarchy:

The hierarchy, as mentioned above, allows for the implementation of the inverse seesaw mechanisms at the tree and one-loop level to generate the masses of the second and first families of SM charged fermions, respectively. Thus, the

Figure 2: Two-loop Feynman diagram contributing to the Majorana neutrino mass submatrix $\mu$. Here $n, k, l, r = 1, 2, s, p = 1, 2, 3, a, b, c = R, I$, with $\varphi_{nR}$ and $\varphi_{nI}$ corresponding to the CP even and CP odd parts of the scalar field $\varphi_{n}$, respectively.

$$M_D = \begin{pmatrix} C_D + \Delta_D & F_D & 0_{3 \times 1} \\ G_D^T & 0 & X_D \\ 0_{1 \times 3} & Y_D & m_D \end{pmatrix}, \quad (F_D)_{3} = 0, \quad (F_D)_{n} = x_n^{(D)} \frac{v_1}{\sqrt{2}}, \quad (G_D^T)_{i} = x_i^{(d)} \frac{v_\rho}{\sqrt{2}},$$

$$C_D = \begin{pmatrix} 0 & 0 & 0 & 0 & y_1^{(d)} & y_2^{(d)} & y_3^{(d)} \\ 0 & 0 & 0 & 0 & y_2^{(d)} & y_2^{(d)} & y_3^{(d)} \\ \frac{v_2}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} \end{pmatrix}, \quad X_D = z_B \frac{v_{\chi}}{\sqrt{2}}, \quad Y_D = z_D \frac{v_{\eta}}{\sqrt{2}}, \quad i = 1, 2, 3, \quad n = 1, 2, \quad (13)$$

Furthermore, notice that due to the $Z_2$ symmetry, the exotic $D_2$ down type quark does not mix with the remaining down type quarks. This exotic $D_2$ quark gets a tree level mass equal to $z_D \frac{v_{\eta}}{\sqrt{2}}$, which is at the $v_\rho$ scale. Furthermore, the charged lepton mass matrix, written in the basis $(l_{1L}, l_{2L}, l_{3L}, E_{1L}, E_{2L})$ versus $(l_{1R}, l_{2R}, l_{3R}, E_{1R}, E_{2R})$, takes the form:

$$M_E = \begin{pmatrix} C_E + \Delta_E & F_E & 0_{3 \times 1} \\ G_E^T & 0 & X_E \\ 0_{1 \times 3} & Y_E & m_E \end{pmatrix}, \quad (F_E)_{i} = y_i^{(E)} \frac{v_2}{\sqrt{2}}, \quad (G_E^T)_{n} = x_n^{(E)} \frac{v_S}{\sqrt{2}}, \quad (G_E^T)_{3} = 0,$$

$$C_E = \begin{pmatrix} 0 & 0 & 0 & 0 & y_1^{(E)} & y_1^{(E)} & y_1^{(E)} \\ 0 & 0 & 0 & 0 & y_2^{(E)} & y_2^{(E)} & y_2^{(E)} \\ \frac{v_2}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} \end{pmatrix}, \quad X_E = y_E \frac{v_\rho}{\sqrt{2}}, \quad Y_E = x_E \frac{v_\rho}{\sqrt{2}}, \quad i = 1, 2, 3, \quad n = 1, 2, \quad (14)$$

Here we assume that the entries of the charged fermion mass matrices fulfill the hierarchy:

$$m_{TF}, m_{DF} \ll \ll (G_Q^T)_{i}, (G_Q^T)_{n} < X_Q \sim Y_Q \sim \mathcal{O}(\text{TeV}), \quad Q = U, D,$$

$$m_{E} \ll \ll (F_E)_{i}, (G_E^T)_{n} < X_E \sim Y_E \sim \mathcal{O}(\text{TeV}), \quad i = 1, 2, 3, \quad n = 1, 2. \quad (15)$$

where $\Delta_U$, $\Delta_D$ and $\Delta_E$ correspond to the one-loop level corrections to the SM charged fermion mass matrices. The one-loop level Feynman diagrams, generating the $\Delta_U$, $\Delta_D$ and $\Delta_E$ submatrices, are shown in Figure 1.

The hierarchy, as mentioned above, allows for the implementation of the inverse seesaw mechanisms at the tree and one-loop level to generate the masses of the second and first families of SM charged fermions, respectively. Thus, the
resulting SM charged fermion mass matrices are given by:

\[ \tilde{M}_U = C_U + \frac{m_F}{X_U Y_U} F_U G^T_U + \Delta_U, \]
\[ \tilde{M}_D = C_D + \frac{m_D}{X_D Y_D} F_D G^T_D + \Delta_D, \]
\[ \tilde{M}_E = C_E + \frac{m_E}{X_E Y_E} F_E G^T_E + \Delta_E. \]

where the second and third terms of Eqs. (16), (17) and (18) correspond to the tree and one-loop levels, which contribute to the SM charged fermion mass matrices arising from the inverse seesaw mechanism. The first term in Eqs. (16), (17) and (18) corresponds to the dominant contribution to these matrices, arising from the renormalizable Yukawa interactions involving the SU(2) scalar doublets \( H_1 \) (for the up type quark sector) and \( H_2 \) (for the down type quark and charged lepton sector), which generate the masses for the third family of SM charged fermions. Furthermore, the resulting physical charged exotic fermion mass spectrum is composed of two nearly degenerate heavy charged fermions with masses at the \( O(\text{TeV}) \) scale and a small mass splitting of the order of \( m_F \) (\( F = U, D, E \)). The subGeV mass scale of the second family of SM charged fermions, arising from a tree-level inverse seesaw mechanism, can naturally be explained by considering \( (F_Q)_n (G^T_Q)_n \sim (F_E)_n (G^T_E)_n \sim O(10^{-2}\text{TeV}^2) \), \( m_F \sim O(10^{-1}\text{GeV}) \). Therefore, the inverse seesaw mechanism presented here can naturally explain the SM fermion mass hierarchy.

The one loop level contributions to the SM charged fermion mass matrices are given by:

\[ \Delta_U = \frac{m_T}{16\pi^2} \sum_{i=1}^3 \begin{pmatrix} r_{11}^{(T)} & r_{12}^{(T)} & r_{13}^{(T)} \\ r_{11}^{(T)} & r_{22}^{(T)} & r_{23}^{(T)} \\ r_{12}^{(T)} & r_{23}^{(T)} & r_{33}^{(T)} \end{pmatrix} \frac{m^2_{H^+_i}}{m^2_{H^+_i} - m^2_T} \ln \left( \frac{m^2_{H^+_i}}{m^2_T} \right), \]
\[ \Delta_D = \frac{m_B}{16\pi^2} \sum_{i=1}^3 \begin{pmatrix} r_{11}^{(B)} & r_{12}^{(B)} & r_{13}^{(B)} \\ r_{11}^{(B)} & r_{22}^{(B)} & r_{23}^{(B)} \\ r_{12}^{(B)} & r_{23}^{(B)} & r_{33}^{(B)} \end{pmatrix} \frac{m^2_{H^-}_i}{m^2_{H^-}_i - m^2_B} \ln \left( \frac{m^2_{H^-}_i}{m^2_B} \right), \]
\[ \Delta_E = \frac{m_E}{16\pi^2} \sum_{i=1}^3 \begin{pmatrix} r_{11}^{(E)} & r_{12}^{(E)} & r_{13}^{(E)} \\ r_{11}^{(E)} & r_{22}^{(E)} & r_{23}^{(E)} \\ r_{12}^{(E)} & r_{23}^{(E)} & r_{33}^{(E)} \end{pmatrix} \frac{m^2_{H^+_i}}{m^2_{H^+_i} - m^2_E} \ln \left( \frac{m^2_{H^+_i}}{m^2_E} \right), \]

The experimental values of the SM quark masses [170, 171] and Cabibbo-Kobayashi-Maskawa (CKM) parameters are:

\[ m_u(\text{MeV}) = 1.24 \pm 0.22, \quad m_d(\text{MeV}) = 2.69 \pm 0.19, \quad m_s(\text{MeV}) = 53.5 \pm 4.6, \]
\[ m_c(\text{GeV}) = 0.63 \pm 0.02, \quad m_t(\text{GeV}) = 172.9 \pm 0.4, \quad m_b(\text{GeV}) = 2.86 \pm 0.03, \]
\[ \sin \theta_12 = 0.2245 \pm 0.00044, \quad \sin \theta_23 = 0.0421 \pm 0.00076, \quad \sin \theta_13 = 0.00365 \pm 0.00012, \]
\[ J = (3.18 \pm 0.15) \times 10^{-5}, \]

which can be well reproduced for the following benchmark point:

\[ m_{H^+_1} = m_{H^+_2} = 1.5\text{TeV}, \quad v_1 \simeq 244.2\text{GeV}, \quad v_2 \simeq 30\text{GeV}, \quad v_{\tau} \simeq 6\text{TeV}, \quad v_\eta = v_\chi \simeq 5\text{TeV}, \]
\[ m_T \simeq 1.7\text{TeV}, \quad m_B \simeq 1.3\text{TeV}, \quad m_T \simeq 9.7\text{GeV}, \quad m_D = 1.6\text{GeV}, \quad j = 1, 2, 3, \]
\[ y^{(u)}_1 \simeq 0.585, \quad y^{(u)}_2 \simeq 0.717, \quad y^{(u)}_3 \simeq -0.368, \quad y^{(d)}_1 \simeq 0.0713, \quad y^{(d)}_2 \simeq 0.060, \quad y^{(d)}_3 \simeq -0.067 \]
\[ x^{(U)}_1 \simeq -4.566 - 1.209i, \quad x^{(U)}_2 \simeq -1.177 + 4.256i, \quad x^{(D)}_1 \simeq -0.732 - 0.053i, \quad x^{(D)}_2 \simeq 0.292 + 0.883i, \]
\[ x^{(u)}_1 \simeq -0.238, \quad x^{(u)}_2 \simeq 0.141, \quad x^{(u)}_3 \simeq -0.114, \quad z_T = z_u \simeq -0.481, \quad z_D = z_B \simeq -0.441 \]
\[ x^{(d)}_1 \simeq -0.038 - 0.158i, \quad x^{(d)}_2 \simeq 0.037 - 0.035i, \quad x^{(d)}_3 \simeq 0.040 + 0.145i, \]
\[ r^{(T)}_{1j} \simeq 0.139, \quad r^{(T)}_{2j} \simeq 0.087, \quad r^{(B)}_{1j} \simeq 0.009 - 0.038i, \quad r^{(B)}_{2j} \simeq 0.083 + 0.081i, \quad w^{(B)}_{3i} \simeq 0.012 \]
\[ w^{(T)}_{1j} \simeq 0.011, \quad w^{(T)}_{2j} \simeq 0.014, \quad w^{(T)}_{3j} \simeq 0.009, \quad w^{(B)}_{1i} \simeq -0.009, \quad w^{(B)}_{2i} \simeq -0.013, \]

Thus, the proposed model can numerically reproduce the existing pattern of the observed quark spectrum. Furthermore, in the benchmark point shown in Eq. (23), the exotic up and down type quark masses are close to about 1.7 TeV and 1.6 TeV, respectively, which are values larger than the ATLAS exclusion limits of 1.6 TeV and 1.42 TeV [172]. Besides that, in the simplified benchmark scenario considered above, the electrically charged scalars are assumed to
be degenerate, and their masses are set to be equal to 1.5 TeV, a value that exceeds the upper limit of [80, 160] GeV arising from collider searches [173, 174].

On the other hand, from the neutrino Yukawa interactions and considering $\Omega_{nR}$ ($n = 1, 2$) as physical neutral leptonic fields, we find the following neutrino mass terms:

$$ -\mathcal{L}_{\text{mass}}^{(\nu)} = \frac{1}{2} \left( \tilde{\nu}_L^c \nu_R \nu_R \tilde{\nu}_R^c \right) M_{\nu} \left( \nu_L^c \nu_R^c \nu_R^c \right)^T + \sum_{n=1}^{2} (m_{\Omega})_n \tilde{\nu}_{nR} \Omega_{nR}^c + H.c. $$

(24)

where $(m_{\Omega})_n = (y_{\Omega})_n \frac{v_n}{\sqrt{2}}$ ($n = 1, 2$) and the neutrino mass matrix reads:

$$ M_{\nu} = \begin{pmatrix} 0_{3 \times 3} & m_{\nu D} & 0_{3 \times 3} \\ m_{\nu D}^T & 0_{3 \times 3} & M \\ 0_{3 \times 3} & M^T & \mu \end{pmatrix}, $$

(25)

and the submatrices are:

$$ (m_{\nu D})_{ij} = y_{ij}^{(N)} \frac{v_\sigma}{\sqrt{2}}, \quad M_{ij} = y_{ij}^{(N)} \frac{v_\sigma}{\sqrt{2}}, \quad i, j, s, p = 1, 2, 3, \quad n, k, r = 1, 2, $$

(26)

$$ \mu_{sp} = \sum_{k=1}^{2} \frac{(x_{\nu N})_{sn} (x_{\nu N})_{nk} (x_{\nu N})_{kr} (x_{\nu N})_{rp} m_{\Omega k}}{4(4\pi)^4} \int_0^{1/\alpha} d\alpha \int_0^{1/\alpha} d\beta (1/\alpha) \left[ I \left( m_{\Omega k}^2, m_{R R}^2, m_{R I}^2 \right) - I \left( m_{\Omega k}^2, m_{R R}^2, m_{I I}^2 \right) \right], $$

with the loop integral given by [175]:

$$ I(m_1^2, m_2^2, m_3^2) = \frac{m_1^2 m_2^2 \log \left( \frac{m_1^2}{m_2^2} \right)}{(m_1^2 - m_2^2)(m_1^2 - m_3^2)} + \frac{m_2^2 m_3^2 \log \left( \frac{m_2^2}{m_3^2} \right)}{(m_2^2 - m_1^2)(m_2^2 - m_3^2)}, $$

(27)

$$ m_{ab}^2 = \frac{\beta_m^2 (x_{1})_a + \alpha_m^2 (x_{2})_b}{\alpha (1/\alpha)} \quad (a, b = R : \text{or} : I), $$

The active light neutrino masses are generated from an inverse seesaw mechanism at the two-loop level, and the physical neutrino mass matrices are given by:

$$ \tilde{M}_{\nu} = m_{\nu D} (M^T)^{-1} \mu M^{-1} m_{\nu D}^T, $$

(28)

$$ M_{\nu}^{(-)} = -\frac{1}{2} (M + M^T) + \frac{1}{2} \mu, $$

(29)

$$ M_{\nu}^{(+)} = \frac{1}{2} (M + M^T) + \frac{1}{2} \mu. $$

(30)

where $\tilde{M}_{\nu}$ is the mass matrix for the active light neutrinos ($\nu_a$), whereas $M_{\nu}^{(-)}$ and $M_{\nu}^{(+)}$ are the mass matrices for sterile neutrinos. In the limit $\mu \rightarrow 0$, which corresponds to unbroken lepton number, the active light neutrinos become massless. The smallness of the $\mu$ parameter yields a small mass splitting for the two pairs of sterile neutrinos, thus implying that the sterile neutrinos form pseudo-Dirac pairs.

IV. MUON AND ELECTRON ANOMALOUS MAGNETIC MOMENTS

The experimental data shows that the muon and electron anomalous magnetic moments deviate significantly from their SM values

$$ \Delta a_\mu = a_\mu^{\exp} - a_\mu^{\text{SM}} = (2.51 \pm 0.59) \times 10^{-9} $$

[60, 169, 176–181]

$$ \Delta a_e = a_e^{\exp} - a_e^{\text{SM}} = (-0.88 \pm 0.36) \times 10^{-12} $$

[182], (4.8 \pm 3.0) \times 10^{-13} [183]

(31)

(32)

where the above given value of $a_\mu^{\exp}$ is a combined result of the BNL E821 experiment [184] and the recently announced FNAL Muon g-2 measurement [169], showing the 4.2$\sigma$ tension between the SM and experiment. The last positive value for $\Delta a_e$ corresponds to the recently published new measurement of the fine-structure constant, with an accuracy
of 81 parts per trillion [183]. In this section, we will analyze the implications of our model in the muon and electron anomalous magnetic moments.

Muon and electron anomalous magnetic moments receive contributions from one-loop diagrams involving the exchange of electrically neutral CP even and CP odd scalars and charged exotic leptons as well as electrically charged scalar and sterile neutrinos running in the internal lines of the loop. To simplify our analysis, we will consider a simplified benchmark scenario close to the alignment limit, where $\phi_{21}^0 (\phi_{22}^0)$ and $S_R (S_f)$ are mainly composed of two orthogonal combinations involving two heavy CP even (odd) $H_1 (A_1), H_2 (A_2)$ physical scalar fields. We consider the alignment limit in which the 125 GeV SM like Higgs boson is mainly composed of the CP even neutral part of the SU(2) scalar doublet $\phi_1$. This component does not appear in the neutral scalar contribution to the muon and electron anomalous magnetic moment shown in Eq. (33). Thus, in the chosen benchmark scenario, the couplings of the 125 GeV SM like Higgs boson are very close to the SM expectation, which is consistent with the experimental data [171]. Then, the leading contributions to the muon and electron anomalous magnetic moments take the form:

$$\Delta a_{e,\mu} \simeq \frac{\text{Re} \left(\alpha e^{\mu} \beta e^{\mu}\right) m_{e,\mu}^2}{8\pi^2} \left[I_{S}^{(e,\mu)} (m_{E}, m_{H_1}) - I_{S}^{(e,\mu)} (m_{E}, m_{H_2}) + I_{P}^{(e,\mu)} (m_{E}, m_{A_1}) - I_{P}^{(e,\mu)} (m_{E}, m_{A_2})\right] \sin \theta \cos \theta + \frac{1}{8\pi^2} \sum_{i=1}^{3} \sum_{j=1}^{3} \text{Re} \left[\frac{(ij)}{m_{i}^2} m_{e,\mu} m_{\phi_j} G_2 \left(\frac{m_{\phi_j}^2}{m_{i}^2}\right)\right],$$

where $H_1 \simeq \cos \theta S_R + \sin \theta \phi_{21}^0$, $H_2 \simeq -\sin \theta S_R + \cos \theta \phi_{22}^0$, $A_1 \simeq \cos \theta P S_f + \sin \theta \phi_{21}^0$, $A_2 \simeq -\sin \theta P S_f$ and for the sake of simplicity we have set $\theta_* = \theta_2$ and $\theta_1$ is the mass of the nearly degenerate physical charged leptons. Besides that, $m_{\phi_1}^\pm$ and $m_{\phi_2}^\pm (i, j = 1, 2, 3)$ are the masses of the physically charged scalars and $Z_2$ even sterile neutrinos, respectively. Furthermore, the $I_{S(P)} (m_{E}, m)$ and $G_2 (r)$ loop functions have the form: [185–189]:

$$I_{S(P)}^{(e,\mu)} (m_{E}, m) = \int_0^1 \frac{x^2 \left(1 - x \pm \frac{m_E}{m_{e,\mu}}\right)}{m_{e,\mu}^2 x^2 + \left(m_{E}^2 - m_{e,\mu}^2\right) x + m_{S,P}^2 (1 - x)} \, dx,$$

$$G_2 (r) = \frac{-1 + r^2 - 2r \ln r}{(r - 1)^3}$$

Requiring that the muon and electron magnetic anomalies acquire values in the ranges shown in Eqs. (31) and (32), respectively, we display in Figure 3 the correlation between the masses of the scalars $H_1$ and $H_2$ consistent with the experimental data on $(g - 2)_{e,\mu}$. These masses have been taken to range from 1 TeV up to 2 TeV. In contrast, the electrically charged scalar masses are varied from 0.5 TeV up to 1.5 TeV, and the CP odd scalar masses have been set to be equal to 1 TeV. Furthermore, the masses for the heavy vector-like leptons have been varied from 2 TeV up to 3 TeV, a range of values more extensive than the expected reach of 600 GeV for the High Luminosity Large Hadron Collider (HL-LHC). The above values for the scalar masses are consistent with constraints arising from collider searches [171], and accommodate a nearly degenerate spectrum of heavy scalar masses which is favored by electroweak precision tests [190]. Figure 3 shows that our model is consistent with the experimental values of the muon and electron anomalous magnetic moments.

V. MESON OSCILLATIONS

In this section, we analyze the consequences of our proposed theory in the $K^0 - \bar{K}^0$, $B^0_d - \bar{B}^0_d$ and $B^0_s - \bar{B}^0_s$ meson oscillations. These meson oscillations are caused by flavour violating down type quark interactions mediated by the tree level exchange of electrically neutral CP even and CP odd scalars as well as by the tree level $Z'$ exchange. The $K^0 - \bar{K}^0$, $B^0_d - \bar{B}^0_d$ and $B^0_s - \bar{B}^0_s$ meson oscillations are described by the following effective Hamiltonians:

$$\mathcal{H}_{eff}^{(K)} = \sum_{j=1}^{3} c_j^{(K)} (\mu) \mathcal{O}_j^{(K)} (\mu) + \frac{g_X^2}{9m_{Z'}^2} |(V_{DL})_{32} (V_{DL})_{31}|^2 \mathcal{O}_4^{(K)},$$

$$\mathcal{H}_{eff}^{(B_d)} = \sum_{j=1}^{3} c_j^{(B_d)} (\mu) \mathcal{O}_j^{(B_d)} (\mu) + \frac{g_X^2}{9m_{Z'}^2} |(V_{DL})_{33} (V_{DL})_{31}|^2 \mathcal{O}_4^{(B_d)},$$

$$\mathcal{H}_{eff}^{(B_s)} = \sum_{j=1}^{3} c_j^{(B_s)} (\mu) \mathcal{O}_j^{(B_s)} (\mu) + \frac{g_X^2}{9m_{Z'}^2} |(V_{DL})_{32} (V_{DL})_{33}|^2 \mathcal{O}_4^{(B_s)},$$
and the Wilson coefficients are:

\[
\begin{align*}
\mathcal{O}_1^{(K)} &= \left( \bar{\sigma}_R d_L \right) \left( \bar{\sigma}_R d_L \right), \\
\mathcal{O}_2^{(K)} &= \left( \bar{\sigma}_L d_R \right) \left( \bar{\sigma}_L d_R \right), \\
\mathcal{O}_3^{(K)} &= \left( \bar{\sigma}_R d_L \right) \left( \bar{\sigma}_L d_R \right), \\
\mathcal{O}_4^{(K)} &= \left( \bar{\sigma}_L \gamma_\mu d_L \right) \left( \bar{\sigma}_L \gamma_\mu d_L \right), \\
\mathcal{O}_1^{(B_d)} &= \left( \bar{d}_R b_L \right) \left( \bar{d}_R b_L \right), \\
\mathcal{O}_2^{(B_d)} &= \left( \bar{d}_L b_R \right) \left( \bar{d}_L b_R \right), \\
\mathcal{O}_3^{(B_d)} &= \left( \bar{d}_R b_L \right) \left( \bar{d}_L b_R \right), \\
\mathcal{O}_4^{(B_d)} &= \left( \bar{d}_L \gamma_\mu b_L \right) \left( \bar{d}_L \gamma_\mu b_L \right), \\
\mathcal{O}_5^{(B_d)} &= \left( \bar{s}_R b_L \right) \left( \bar{s}_L b_R \right), \\
\mathcal{O}_6^{(B_d)} &= \left( \bar{s}_L \gamma_\mu b_L \right) \left( \bar{s}_L \gamma_\mu b_L \right), \\
\mathcal{O}_7^{(B_d)} &= \left( \bar{s}_R b_L \right) \left( \bar{s}_L b_R \right), \\
\mathcal{O}_8^{(B_d)} &= \left( \bar{s}_L \gamma_\mu b_L \right) \left( \bar{s}_L \gamma_\mu b_L \right),
\end{align*}
\]  

where the operators relevant for the \( K^0 - \bar{K}^0 \), \( B^0_d - \bar{B}^0_d \) and \( B^0_s - \bar{B}^0_s \) meson mixings have the form:

\[
\begin{align*}
c_1^{(K)} &= \frac{\gamma_\mu}{m_h^2} + \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{H_j}^2} - \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{A_j}^2}, \\
c_2^{(K)} &= \frac{\gamma_\mu}{m_h^2} \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{H_j}^2} - \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{A_j}^2}, \\
c_3^{(K)} &= \frac{\gamma_\mu}{m_h^2} \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{H_j}^2} + \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{H_j}^2} - \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{A_j}^2} + \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{A_j}^2}, \\
c_1^{(B_d)} &= \frac{\gamma_\mu}{m_h^2} + \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{H_j}^2} - \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{A_j}^2}, \\
c_2^{(B_d)} &= \frac{\gamma_\mu}{m_h^2} \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{H_j}^2} - \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{A_j}^2}, \\
c_3^{(B_d)} &= \frac{\gamma_\mu}{m_h^2} \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{H_j}^2} + \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{H_j}^2} - \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{A_j}^2}, \\
c_4^{(B_d)} &= \frac{\gamma_\mu}{m_h^2} \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{H_j}^2} + \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{H_j}^2} - \sum_{j=1}^2 \frac{\gamma_\mu_j}{M_{A_j}^2}.
\end{align*}
\]
In our numerical analysis, we use the following numerical values of the meson parameters \[191–197\]:

\[
c_{1}^{(B_s)} = \frac{\frac{2}{2} \bar{\pi}_b h_{l}}{m_{h}^2} + \sum_{j=1}^{2} \frac{2}{2} \bar{\pi}_l h_{l} - \frac{2}{2} \frac{z_{A_{j} \bar{\pi}_b h_{l}}}{M_{A_{j}}^2},
\]

\[
c_{2}^{(B_s)} = \frac{z_{A_{j} \bar{\pi}_b h_{l}}}{m_{h}^2} + \sum_{j=1}^{2} \frac{2}{2} \bar{\pi}_l h_{l} - \frac{2}{2} \frac{z_{A_{j} \bar{\pi}_b h_{l}}}{M_{A_{j}}^2},
\]

\[
c_{3}^{(B_s)} = \frac{z_{A_{j} \bar{\pi}_b h_{l}}}{m_{h}^2} + \sum_{j=1}^{2} \frac{2}{2} \bar{\pi}_l h_{l} - \frac{2}{2} \frac{z_{A_{j} \bar{\pi}_b h_{l}}}{M_{A_{j}}^2},
\]

Furthermore, the following relations have been taken into account:

\[
\tilde{M}_{f} = (M_{f})_{\text{diag}} = V_{f}^\dagger M_{f} V_{f}, \quad f_{(L,R)} = V_{f(L,R)} \tilde{f}_{(L,R)},
\]

\[
\bar{f}_{iL} (M_{f})_{ij} f_{jR} = \tilde{f}_{kL} V_{f}^\dagger (M_{f})_{ij} V_{f} f_{jR} = \tilde{f}_{kL} \tilde{f}_{jR} = \tilde{f}_{kL} (M_{f})_{ij} f_{jR} = m_{f} \tilde{f}_{kL} \tilde{f}_{kR},
\]

\[k = 1, 2, 3.\]

Here, \(\tilde{f}_{k(L,R)}\) and \(f_{k(L,R)}\) \((k = 1, 2, 3)\) are the SM fermionic fields in the mass and interaction bases, respectively.

On the other hand, the \(K - \bar{K}, B_{d}^{0} - \bar{B}_{d}^{0}\) and \(B_{s}^{0} - \bar{B}_{s}^{0}\) meson mass differences are given by:

\[
\Delta m_{K} = \Delta m_{K}^{(SM)} + \Delta m_{K}^{(NP)}, \quad \Delta m_{B_{d}} = \Delta m_{B_{d}}^{(SM)} + \Delta m_{B_{d}}^{(NP)}, \quad \Delta m_{B_{s}} = \Delta m_{B_{s}}^{(SM)} + \Delta m_{B_{s}}^{(NP)},
\]

where \(\Delta m_{K}^{(SM)}, \Delta m_{B_{d}}^{(SM)}\) and \(\Delta m_{B_{s}}^{(SM)}\) stand for the SM contributions, while \(\Delta m_{K}^{(NP)}, \Delta m_{B_{d}}^{(NP)}\) and \(\Delta m_{B_{s}}^{(NP)}\) arise from new physics effects.

In the model under consideration, we find the following new physics contributions to the \(K - \bar{K}, B_{d}^{0} - \bar{B}_{d}^{0}\) and \(B_{s}^{0} - \bar{B}_{s}^{0}\) meson mass splittings:

\[
\Delta m_{K}^{(NP)} = \frac{g_{X}^{2}}{9 m_{Z}^2} \left| (V_{DL})_{32} (V_{DL})_{31} \right|^2 f_{K} B_{K} \eta_{K} m_{K} + \frac{8}{3} f_{K} B_{K} \eta_{K} m_{K} \left[ k_{2}^{(K)} c_{3}^{(K)} + k_{1}^{(K)} \left( c_{1}^{(K)} + c_{2}^{(K)} \right) \right]
\]

\[
\Delta m_{B_{d}}^{(NP)} = \frac{g_{X}^{2}}{9 m_{Z}^2} \left| (V_{DL})_{32} (V_{DL})_{31} \right|^2 f_{B_{d}} B_{d} \eta_{B_{d}} m_{B_{d}} + \frac{8}{3} f_{B_{d}} B_{d} \eta_{B_{d}} m_{B_{d}} \left[ k_{2}^{(B_{d})} c_{3}^{(B_{d})} + k_{1}^{(B_{d})} \left( c_{1}^{(B_{d})} + c_{2}^{(B_{d})} \right) \right]
\]

\[
\Delta m_{B_{s}}^{(NP)} = \frac{g_{X}^{2}}{9 m_{Z}^2} \left| (V_{DL})_{32} (V_{DL})_{31} \right|^2 f_{B_{s}} B_{s} \eta_{B_{s}} m_{B_{s}} + \frac{8}{3} f_{B_{s}} B_{s} \eta_{B_{s}} m_{B_{s}} \left[ k_{2}^{(B_{s})} c_{3}^{(B_{s})} + k_{1}^{(B_{s})} \left( c_{1}^{(B_{s})} + c_{2}^{(B_{s})} \right) \right]
\]

In our numerical analysis, we use the following numerical values of the meson parameters \[191–197\]:

\[
\Delta m_{K} = (3.484 \pm 0.006) \times 10^{-12} \text{MeV}, \quad \Delta m_{K}^{(SM)} = 3.483 \times 10^{-12} \text{MeV}
\]

\[
f_{K} = 160 \text{MeV}, \quad B_{K} = 0.85, \quad \eta_{K} = 0.57, \quad m_{K} = 497.614 \text{MeV},
\]

\[
(f_{K})_{\text{exp}} = (3.337 \pm 0.033) \times 10^{-10} \text{MeV}, \quad \Delta m_{K}^{(SM)} = 3.582 \times 10^{-10} \text{MeV},
\]

\[
k_{1}^{(K)} = -9.3, \quad k_{2}^{(K)} = 30.6, \quad m_{K} = 5279.5 \text{MeV},
\]

\[
(k_{1}^{(B_{d})})_{\text{exp}} = (104.19 \pm 0.8) \times 10^{-10} \text{MeV}, \quad \Delta m_{B_{d}}^{(SM)} = 121.103 \times 10^{-10} \text{MeV},
\]

\[
f_{B_{d}} = 225 \text{MeV}, \quad B_{B_{d}} = 1.26, \quad \eta_{B_{d}} = 0.55, \quad m_{B_{d}} = 5366.3 \text{MeV},
\]

\[
(k_{1}^{(B_{s})})_{\text{exp}} = (104.19 \pm 0.8) \times 10^{-10} \text{MeV}, \quad \Delta m_{B_{s}}^{(SM)} = 121.103 \times 10^{-10} \text{MeV},
\]

We plot in Figure 4 the allowed region in the \(m_{Z'} - g_{X}\) plane, consistent with the constraint arising from \(B_{d}^{0} - \bar{B}_{d}^{0}\) (left-plot) and \(B_{s}^{0} - \bar{B}_{s}^{0}\) (right-plot) mixings. In our numerical analysis we have considered a simplified benchmark.
The considered model contains two kinds of candidates: the fermion singlet $\Psi_{nR}$ both the $SU(2)_L \times U(1)_Y$ gauge symmetry and the $Z_4$ discrete symmetry are spontaneously broken, whereas the $Z_2$ symmetry is preserved. This $Z_2$ conservation implies that the particles carrying a non trivial $Z_2$ charge always couple in pairs, and therefore the lightest of the electrically neutral $Z_2$ odd particles is a dark matter candidate. The considered model contains two kinds of candidates: the fermion singlet $\Psi_{nR}$ and the scalar singlets that are either $\varphi_1$ or $\varphi_2$. The fields, $\varphi_1, \Psi_{nR}$, carry the X charge but the $\varphi_2$ does not. Except for the Yukawa interaction of $\varphi_2$ with new fermions $\Omega_{nR}, \Psi_{nR}$, the $\varphi_2$ has only quartic scalar interactions. Thus, the $\varphi_2$ mainly annihilates into $W^+W^-, ZZ, tt, bb, HH$ via a scalar portal interaction, and the relic density is governed by Higgs portal interactions and takes the form [198]

$$\Omega h^2 \simeq 0.1 \left( \frac{m_{\varphi_2}}{\lambda_{eff} \times 1.354 \text{TeV}} \right)^2,$$

where $\lambda_{eff}$ is an effective coupling, which depends on all trilinear Higgs couplings of $\varphi_2$ with the remaining Higgs. In order to consistently reproduce the experimental value of the dark matter relic density [199], $\Omega h^2 = 0.1198 \pm 0.0026$, the mass $m_{\varphi_2}$ has to fulfill the constraint, $m_{\varphi_2} < \lambda_{eff} \times 1.5 \text{TeV}$. If the effective coupling is in the range $0.5 < \lambda_{eff} < 1.5$, the dark matter mass satisfies $0.75 \text{TeV} < m_{\varphi_2} < 2.25 \text{TeV}$. Since $\varphi_2$ is a gauge singlet scalar, it is electrically neutral, and then it only scatters off in a quark antiquark pair via SM Higgs portal interaction, which has a rate proportional
to the quartic coupling of $\varphi^2 H^2$, denoted as $\lambda_D$. The tree-level SM Higgs exchange produces a spin-independent cross section given by [200]:

$$\sigma_{\varphi_2-p,n} \simeq 3.88 \times 10^{-45} \left(\frac{\lambda_D}{0.5}\right)^2 \left(\frac{2 \text{TeV}}{m_{\varphi_2}}\right)^2 \text{cm}^2.$$  (62)

This scattering cross-section reaches the direct detection limit from the XENON1T experiment [201] for dark matter mass around 2 TeV and effective coupling $\lambda_D \simeq 0.5$. Unlike $\varphi_2$, $\varphi_1$ carries a $X$–charge. Thus, if $\varphi_1$ is a dark matter candidate, it will scatter off a nucleon not only through the exchange of the Higgs, but also via the exchange of a new neutral gauge boson. This obeys direct detection limits from the XENON1T experiment [201], and yields the correct relic density if the dark matter mass is heavier than 3 TeV, (see in [202]).

Let us now assume that the dark matter is the neutral fermion, denoted $\Psi$. This is a $SU(2)_L$ singlet which does not carry hypercharge, but carries a $U(1)_X$ charge. Thus, the interaction of two dark matter candidates $\Psi$ with a new neutral gauge boson, called $Z_X$, determines the dark matter phenomenology. $\Psi_n$ annihilates into SM particles through the exchange of a new gauge boson. The relic density is given as [203]

$$\Omega_\Psi h^2 \simeq 0.1 \text{pb} \left(\frac{\alpha}{150 \text{GeV}}\right)^{-2} \left(\frac{m_\Psi}{2.86 \text{TeV}}\right)^2,$$  (63)

where $\left(\frac{\alpha}{150 \text{GeV}}\right)^2 \simeq 1 \text{pb}$. This relic density satisfies the experimental value [199] if and only if $m_\Psi > 3.13 \text{TeV}$. At the tree-level, the dark matter $\Psi_N$ scatters off nuclei via the exchange of the gauge boson $Z_X$. In the limit, $m_\Psi > 3.13 \text{TeV}$, the cross-section of this scattering is predicted to be consistent with the XENON1T experiment (see [95, 198, 202]).

VII. LEPTOGENESIS

Before counting flavored CP–asymmetry parameters from the decay of each heavy pseudo-Dirac neutrinos, we must rotate the sterile neutrinos $\nu_{aR}, N_{aR}$, into their mass basis. As previously mentioned, the sterile neutrinos form three pairs of quasi-degenerate pseudo-Dirac fermions due to the small $\mu$–parameter, which is generated at two loop level. The eigenstates ($N_{aR}^+, N_{aR}^-$), corresponding to the eigenvalues ($M_\nu^+, M_\nu^-$), are related to ($\nu_{Ra}, N_{Ra}$) through:

$$N_{aR}^+ = \frac{1}{\sqrt{2}} (\nu_{aR} + N_{aR}) , \quad N_{aR}^- = \frac{i}{\sqrt{2}} (\nu_{aR} - N_{aR}).$$  (64)

Henceforth, the Yukawa interactions of $\nu_{aR}, N_{aR}$ can be modified and written on the new basis as follows

$$- \mathcal{L}_Y \supset \sum_{i=1}^3 \sum_{a=1}^3 y_{\nu_i a} \bar{\nu}_i \phi_2 \left( \frac{N_{aR}^+ - i N_{aR}^-}{\sqrt{2}} \right) + \sum_{i=1}^2 \sum_{n=1}^2 (x_{\nu_i n})_{aL} \left( \frac{N_{aR}^+ + i N_{aR}^-}{\sqrt{2}} \right) \psi_{aR}^0 \varphi_1$$  (65)

The lepton asymmetry is generated from the decay of the lightest pair of pseudo-Dirac neutrinos, called ($N_{\pm}$), and it can be enhanced due to a resonance effect [204–206]. If ($N_{\pm}$) couples only with a SM lepton, the washout factor is determined from the inverse decay of the SM lepton and Higgs into the pair of pseudo-Dirac neutrinos. Since the washout factor has a quadratic suppression with the $\mu$–parameter [207, 208], the smallness of $\mu$ can naturally suppress the washout factor. However, in our model, the $\mu$–parameter can be small in a technically natural way since it is generated at the two-loop level. Therefore, in addition to the inverse decay $lH^\pm \rightarrow N_{\pm} \rightarrow lH^\pm$, the model creates new washout processes: $lH^+ \rightarrow N_{\pm} \rightarrow \Psi_R \varphi_1$ (see Eq.(65)). In the high-temperature region (temperature larger than the inverse see-saw scale), new washout processes can be avoided if the Yukawa couplings ($x_{\nu_i n}$) are very suppressed. This is unreasonable, because the $\mu$–parameter is generated at two loop level, as shown in Eq.(26). If the temperature of the Universe drops below the see-saw scale, the inverse decays producing $N_{aR}$ fall out of thermal equilibrium, and thermal leptogenesis can happen. Assuming that the fermions $\Psi_{aR}$ are heavier than the lightest pseudo-Dirac pair, ($N_{\pm}^\mp$), the lepton asymmetry generates via the decay of the lightest pair of pseudo-Dirac ($N_{\pm} \equiv N_{1\pm}^\mp$) to the SM Higgs and lepton doublets and has the following form

$$\epsilon_{\pm} \equiv \sum_{a=1}^3 \left\{ \frac{\Gamma (N_{\pm} \rightarrow l_\alpha H^+) - \Gamma (N_{\pm} \rightarrow l_\alpha H^-)}{\Gamma (N_{\pm} \rightarrow l_\alpha H^+) + \Gamma (N_{\pm} \rightarrow l_\alpha H^-)} \right\} \text{Im} \left( \frac{(y_{N_{\pm} l_\alpha})^\dagger}{y_{N_{\pm} l_\alpha}} \right) \frac{r}{\pi A_{\pm}} \sqrt{\frac{F_{\varphi_1}^2}{m_{N_{\pm}^\mp}}},$$  (66)
where \( y_{N \pm} = \frac{y'}{\sqrt{2}} \left( 1 \pm \frac{1}{4} M^{-1} \mu \right) \), \( r \equiv \frac{m_{N_+} - m_{N_-}}{m_{N_+} + m_{N_-}} \), \( A_{\pm} = (y') y' \), \( \Gamma_{\pm} = \frac{A_{\pm} m_{N_{\pm}}}{8 \pi} \). In the weak and strong washout region, the approximate baryon asymmetry is estimated as

\[
\eta_B = \frac{\epsilon N_{\pm}}{g_*} \text{ for } K_{N\pm}^{eff} \ll 1,
\]

\[
\eta_B = \frac{0.3 \epsilon N_{\pm}}{g_* K_{N\pm}^{eff} (\ln K_{N\pm}^{eff})^{0.6}} \text{ for } K_{N\pm}^{eff} \gg 1.
\]

(67)

Here, \( g_* \approx 118 \) is the number of relativistic degrees of freedom. In the leptogenesis epoch, the effective washout parameter is defined as

\[
K_{N\pm}^{eff} \simeq \left( \frac{\Gamma_+ + \Gamma_-}{H} \right) \left( \frac{m_{N_+} - m_{N_-}}{\Gamma_\pm} \right)^2
\]

(68)

where \( H = \sqrt{\frac{4 \pi g_*}{45 M_{Pl}^2}} \) is the Hubble constant. For a successful leptogenesis and inverse see-saw mechanism, we have to choose the parameter space including four Yukawa couplings \( y' \), \( y'N \), \( x_N \), \( x_\Psi \), two VEVs \( v_2 \), \( v_\sigma \), and as well as the masses of \( \Omega_\sigma \), \( \Psi_\Omega \), \( \phi \), \( \phi' \). To study this result more quantitatively it is convenient to use the Casas-Ibarra parametrization [209, 210] of the Yukawa coupling \( y' \), as follows

\[
y' = v_\sigma \left( \frac{v_2}{v_\sigma} \right) \left( \frac{U_{\text{PMNS}} \mathcal{M}_2^T R \mathcal{M}_2^{-1}}{y\nu} \right).
\]

(69)

In this parametrization, \( R \) is a complex orthogonal matrix that has the general form

\[
R = \begin{pmatrix}
c_y c_z & -s_x c_z s_y - c_x s_z & s_x s_z - c_x s_y c_z \\
c_y s_z & c_x c_z - s_x s_y s_z & -c_x s_z - c_x s_y s_z \\
s_y & s_x c_y & c_x c_y
\end{pmatrix}
\]

(70)

where \( c_x = \cos x, s_x = \sin x \) and so on, with \( x, y, z \in C \). Here \( U_{\text{PMNS}} \) is the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix for the lepton sector, while \( \mathcal{M}_2 = \text{Diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \) is a diagonal light active neutrino mass matrix. The current best fit values for the light neutrino masses, mixing angles and the CP violation phase of \( U_{\text{PMNS}} \) are given in [211]. In order to reduce the number of free parameters, we need to take some assumptions. Therefore, we fix the three complex mixing angles \( x = 0, y = z = \Re[\theta] + i \Im[\theta] \). Notice that the values of the Yukawas \( y_{ij}' \) are sensitive to \( \Im[\theta] \) but not sensitive to \( \Re[\theta] \). Additionally, we assume that \( Y(N) \) is a diagonal matrix and is fixed as \( Y(N) = \text{Diag}(0.5, 0.9, 1.8) \). Two VEVs are chosen as \( v_2 = 24.6 \text{ GeV} \), \( v_\sigma \approx 5 \times 10^{12} \text{ GeV} \). These choices guarantee that the gauge interactions of \( N_\pm \) decouple at the leptogenesis epoch [212, 213]. In the perturbative region, the Yukawa coupling given in (65) has to satisfy the condition: \( (y')^2 < 4 \pi \). Thus, the large-value domain of \( \Re[\theta] \) is inhibited, namely \( -2 < \Im[\theta] < 2 \). With the above choices, the mass of light neutrinos can approach the experimental limit [211] when \( \mu \) satisfies: \( \mu \approx 1 \text{ keV} \).

Figs. (5) show the strong sensitivity of the washout parameter with respect to \( \Im[\theta] \), especially in the region of small values of \( \Im[\theta] \). The strong washout regime, \( K_{eff} \gg 1 \), corresponds to small values of \( \Im[\theta] \) (left panel), while the weak washout regime, \( K_{eff} \ll 1 \) corresponds to large values of \( \Im[\theta] \) (right panel). For fixed values of \( \Im[\theta] \), the washout parameter oscillates over \( \Re[\theta] \) (see Figs. (6)), and the amplitude of oscillation increases as the value of \( \Im[\theta] \) decreases. The washout effect is not affected much by \( \Re[\theta] \). Figs. (7) show the generated baryon asymmetry from a numerical study of the approximate results of the Boltzmann equation, given in Eq.(67). In the weak washout regime, the predicted baryon asymmetry barely reaches the observed values \( \eta_B \approx 6.08 \times 10^{-10} \).

In the strong-washout regime, the baryon asymmetry generation can reach the observed value, for the small phase entries of the R matrix. For small values of \( \Im[\theta] \), we obtain the strong washout effect for every choice of \( \Re[\theta] \) (see the left panel of Fig.(5)). However, not every value of \( \Re[\theta] \) predicts a sufficient amount of baryon asymmetries for the Universe. The right-panel of Figs. (8) shows the amount of baryon asymmetry as a function of \( \Re[\theta] \), which seems to drastically change under variations of \( \Re[\theta] \). Fixing \( \Re[\theta] \), the behavior of the baryon asymmetry as a function of the imaginary part of the complex angle \( \theta \) appears in the left panel of Figs. (8). It drastically decreases as \( \Im[\theta] \) increases. We conclude that in the present model leptogenesis is viable with a strong-washout regime if we include the small phase of the R matrix. The amount of baryon asymmetry oscillates according to \( \theta \), and the oscillation amplitude can reach the observed value.
We have built a 2HDM theory in which both particle content and symmetry are enlarged. We added several gauge singlet scalars and electrically charged vector-like fermions, as well as right-handed Majorana neutrinos, with the SM gauge symmetry being supplemented by a $U(1)_X \times Z_2 \times Z_4$ family symmetry. We have built a renormalizable

\[ \text{Re}[q] = -5 \quad \text{Re}[q] = -3.5 \quad \text{Re}[q] = -2 \]

\[ \text{Im}[q] = -1.5 \quad \text{Im}[q] = -2 \]

\[ \text{Re}[\theta] = 0 \quad \text{Re}[\theta] = 0.05 \quad \text{Re}[\theta] = -2 \]

\[ \text{Im}[\theta] = 0.005 \quad \text{Im}[\theta] = 1.5 \]

\[ \text{Re}(\theta) = -3 - 2 - 1 0 1 2 3 \]

\[ \text{Im}(\theta) = -6 - 4 - 2 0 2 4 6 \]

\[ \text{Baryon Asymmetry} = \text{Log-Plot of the baryon asymmetry as a function of the } \Im[\theta]. \]

\[ \text{Baryon Asymmetry} = \text{Log-Plot the baryon asymmetry as a function of the } \Re[\theta]. \]

\[ \text{VIII. CONCLUSIONS} \]

We have built a 2HDM theory in which both particle content and symmetry are enlarged. We added several gauge singlet scalars and electrically charged vector-like fermions, as well as right-handed Majorana neutrinos, with the SM gauge symmetry being supplemented by a $U(1)_X \times Z_2 \times Z_4$ family symmetry. We have built a renormalizable
theory based on the given particle content, where to the best of our knowledge, for the first time an inverse seesaw mechanism produces the SM fermion mass hierarchy. The nonuniversal $U(1)_X$ gauge symmetry and the discrete $Z_4$ symmetry are spontaneously broken, whereas the $Z_2$ symmetry is preserved, thus allowing to have stable scalar and fermionic dark matter candidates. Our proposed theory is consistent with the observed SM fermion mass hierarchy, the tiny values for the light active neutrino masses, the lepton and baryon asymmetries of the Universe, the dark matter relic density, the meson oscillation experimental data as well as the muon and electron anomalous magnetic moments.

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