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Global design optimization for an axial-flow tandem pump based on surrogate method

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Abstract. Tandem pump, compared with multistage pump, goes without guide vanes between impellers. Better cavitation performance and significant reduction of the axial geometry scale is important for high-speed propulsion. This study presents a global design optimization method based on surrogated method for an axial-flow tandem pump to enhance trade-off performances: energy and cavitation performances. At the same time, interactions between impellers and impacts on the performances are analyzed. Fixed angle of blades in impellers and phase angle are performed as design variables. Efficiency and minimum average pressure coefficient (MAPC) on axial sectional surface in front impeller are the objective function, which can represent energy and cavitation performances well. Different surrogate models are constructed, and Global Sensitivity Analysis and Pareto Front method are used. The results show that, 1) Influence from phase angle on performances can be neglected compared with other two design variables, 2) Impact ratio of fixed angle of blades in two impellers on efficiency are the same as their designed loading distributions, which is 4:6, 3) The optimization results can enhance the trade-off performances well: efficiency is improved by 0.6%, and the MAPC is improved by 4.5%.

1. Introduction

Traditional optimizations are conducted as open-loop, feed-forward processes. Parameters are conducted one by one. One design iteration is conducted for one parameter, which usually takes up to several weeks, from geometry design to experiments or simulations. At the same time, impact of each parameter on the performance of the pump can’t be well understood. Global design optimization for an axial-flow tandem pump can significantly improve the productivity and shorten the design cycle.

Global design optimization based on surrogate method has been widely used for aerodynamics. Shyy W [1] gave an introduction about optimization processes, and dealt some aerodynamics and rocket propulsion components. Yongsheng Lian [2] took a scope of design optimization using evolutionary algorithms for aerodynamics problems, and examples of turbo pump, compressor, and micro-air vehicles are conducted. Now, with the advantages, global design optimization is becoming more and more popular in hydraulic systems. John. S [3] developed a fast numerical method for blade design in centrifugal pump impellers, the results verified that the optimization process can converge fast and to reasonable optimal values. Mustafa Gölcü [4] used an artificial neural network (ANN) to model the performance of deep well pumps with splitter blades based on experimental data, and...
different ANN methods were trained to determine the effect of the transfer function. J. Fan [5] used moving least squares method for the surrogate optimization of a jet pump, the approach resulted in pump efficiency increasing from 29% to 33% and enable the energy requirements of the pump to be reduced over 20%. Jinya Zhang [6] developed a multi-objective optimal method, which combined the ANN with non-dominated sorting genetic algorithm-II (NSGA-II), to improve the prototype’s performance of the helicon-axial multiphase pump. The result shows compared to the original design, the pressure rise increased by ~10% and the efficiency has increased by ~3%. J H Kim [7] presented an optimization procedure based on a radial basis neural network (RBNN) surrogate model for design of an diffuser in a mixed-flow pump.

Tandem cascade, compared with multistage cascade, goes without guide vanes between impellers. Better cavitation performance, well reduction of boundary layer separation, higher power density and smaller design space make tandem cascade widely used in aeronautics, astronautics [8] and water jet propulsion system. Tandem pump is a typical application of tandem cascade. Significant reduction of the axial geometry scale, resulting from lack of guide vanes, makes great sense to high-speed propulsion. Direct interactions between front and rear impellers may lead to special flows, which are different from those in a multistage pump. There are few studies of these differences.

This paper presents an optimization procedure based on surrogate method and make use to improve performance of an axial-flow tandem pump; impacts of parameters on the performance are also analyzed.

2. Optimization Procedure

The flowchart of the optimization procedure based on surrogate model is shown in figure 1. The procedure begins with problem setup, during which reasonable design variables and objective functions should be selected to construct the design space [9].

2.1. Design of Experiments

Design of experiment is the sampling plan design space, at which training data for surrogate model are obtained. The key question is how to assess the goodness of such designs. For this optimization design problem, the relationship between design variables and objective functions are unknown beforehand, so it may be simplest to use random sampling. Also, considering the computing cost, a more efficient approach is desired. In this study, Latin Hypercube Sampling (LHS) [10] and face centered composite
design (FCCD) [11] are taken, which provide a random sampling and ensure a stratified sample within the full range of each dimension of the sample space.

2.2. Surrogate models
There are both parametric and non-parametric alternatives for constructing surrogate models. The parametric approaches deal with all sampling points at the same time, while the non-parametric approaches use different samples to build different local models in different regions, with which to build up a overall surrogate model. Different methods may work well for different problems. In this study, not knowing which surrogate may perform best, two parametric methods (Polynomial Response Surface and Kriging) and a non-parametric method (Radial Based Neural Network) are used to construct the surrogate models, from which the best is adopted to get future analysis.

(a) Polynomial Response Surface (PRS)
In a PRS model, objective function $f_i(z)$ is approximated as a linear combination of polynomial basis functions $z_j$ [12],

$$f_i(z) = \sum_{j=1}^{N_{PRG}} \beta_j z_j + \varepsilon_i$$

Where the errors $\varepsilon_i$ are considered independents with expected value $E(\varepsilon_i)$ equals to zero. $\beta_j$ represent estimated parameters, which are unbiased and have minimum variance. $N_{PRG}$ is the maximum degree of $z_j$, determined by the order of the PRG model. There often exits an "optimum" order for any problem, but with the increase of the order, the response surface may suffer from excessive curvature, which will hind accuracy and show inconsistent trends in the objective functions from actual data. Considering both accuracy and computing cost, $N_{PRG}$ equals to 2 in this study.

(b) Kriging (KRG)
A Kriging model consists of two components: a linear model and a systematic departure representing low (large scale) and high frequency (small scale) variation components, respectively. The systematic departure component depends only on the distance between the locations under consideration [13].

$$f_p(x) = \mu + \varepsilon(x), \quad E(\varepsilon) = 0,$$

$$\text{cov}(\varepsilon(x_i), \varepsilon(x_j)) \neq 0, \forall i, j$$

Where, $\mu$ is the mean of response at sampled design points, $\varepsilon$ is the error with zero expected value and the correlation structure is a function of a generalized distance between sampling points. In this paper, the correlation structure is shown as follows,

$$\text{cov}(\varepsilon(x_i), \varepsilon(x_j)) = \sigma^2 \exp \left( -\sum_{k=1}^{N_{dv}} \phi_k (x_i^k - x_j^k)^2 \right)$$

Where, $N_{dv}$ means number of design variables, $\sigma$ represents the standard deviation of the response at design points, $\Phi_k$ is a parameter which measures the degree of correlation among the data along the $K$ direction.

(c) Radial Based Neural Network (RBNN)
A Radial Based Neural Network model approximates the objective function as a linear combination of radial basis functions, also known as neurons [14],

$$f_p(x) = \sum_{i=1}^{N_{RBNN}} w_i h_i(x)$$

Where, $W_i$ represent coefficients of the linear combinations, $h_i(x)$ represents the radial function, which using Gaussian function in this study.
\[ h_i(x) = \exp\left( -\left( \|s_i - x\| \right) \beta \right)^2 \]

2.3. Model Validation
To select appropriate surrogate models for analysis, unified method for evaluating and comparing the accuracy of the models are required. Cross validation method [15] is used in this study. The main idea of this method is to divide the data into \( K \) subsets (\( K \)-fold cross validation) of approximately equal size. A surrogate model is constructed \( K \) times, each time leaving out one of the subsets from training, and using the omitted subset to compute the error. The generalization error is described by mean square error:

\[
GMSE = \frac{1}{K} \sum_{i=1}^{K} \left( f_i - f_i^{(-i)} \right)^2
\]

Where, \( f_i^{(-i)} \) represents the predicion at \( (x^{(i)}, f_i) \) using the surrogate constructed using all samples expect the point \( (x^{(i)}, f_i) \). \( K \) equals to 1 in this study.

2.4. Global Sensitivity Analysis and Pareto Front
Sensitivity is a measure of the contributions of an independent variable to the total variance of the dependent data. There are alternative approaches for sensitivity analysis [16]. Sobol method [17] is employed in this study. A single continuous objective function obtained through surrogate-based modeling may be future optimized by simply searching the design space for the minimum or maximum value of the objective functions. However, if multiple competing objectives are present, there may be no single optimal design, but many designs in which one objective is improved at the cost of another. Pareto-optimal solutions (also known as Pareto-efficient solutions) comprise the set of designs that are not dominated by any other design [9].

3. Optimization design of an axial-flow tandem pump

3.1. Problem setup

| Table 1. Designed parameters of the tandem pump |
|-----------------------------------------------|
| \( D \) (m) | \( n \) (rpm) | \( Q \) (m\(^3\)/s) | \( H \) (m) | \( n_s \) (mkw) | \( \eta \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.3             | 1450            | 0.461           | 13.8            | 500             | 84.7\%          |

| Table 2. Essential parameters of impellers |
|--------------------------------------------|
| \( N_b \) | \( \bar{d}_s \) | \( H \) (m) | \( n_s \) (mkw) |
|-----------------|-----------------|-----------------|-----------------|
| Front impeller  | 3               | 0.33–0.40       | 5.5             | 1000            |
| Rear impeller   | 6               | 0.40–0.47       | 8.3             | 735             |

Tandem pump investigated in this work consists of two impellers and a stator with 11 blades, shown as picture. Table 1 shows some essential design parameters of the impellers and essential parameters of impellers are shown in table 2. Among which, \( D \) is the diameter of the pump, \( n \) is rotating speed, \( Q \)
is volume flow rate, \( H \) is the head rise, \( n_s \) is the specific speed, \( \eta \) is efficiency, \( N_b \) is number of blades, \( \bar{D}_h \) is the relative hub diameter. The blade tip of the impellers is \( 2.5 \times 10^{-4} \) m.

**Figure 2.** Schematic concept of fixed angle of blades in front impeller (-5°, 0°, +5°)

**Figure 3.** Schematic concept of phase angle (0°, +20°, -20°)

Compared with single stage axial pump and multi-stage axial pump, interactions between two impellers become more complex. To get better understand of the interacts between impellers, and the influence on the performance of the whole pump, the fixed angle of blades in front and rear impellers are chosen as design variables. At the same time, phase angle, which represents position in the circle direction, may influence the interactions between impellers. So phase angle is also chosen as design variable. Efficiency is usually the important parameter to evaluate the performance of pumps, so it’s chosen to be the first objective function. Considering that minimum average pressure coefficient (MAPC) on axial sectional surface in front impeller can represent cavitation performance well, it’s be chosen as the second objective function.

**Table 3.** Objective functions and design variables

| Objective functions | Description | Designed value |
|---------------------|-------------|----------------|
| Efficiency          |             | 83.7%          |
| MAPC                | \( C_{map} = \frac{(\rho_{max} - \rho_{min})}{(0.5\rho U^2)^{1/2}} \) | -1.43 |

| Design variables | Description          | Minim um | Origin | Maximum |
|------------------|----------------------|----------|--------|---------|
| MAPC             | Based on designed position | -10°     | 0°     | +10°    |
| Fixed angle of front impeller | Based on designed position | -10°     | 0°     | +10°    |
| Fixed angle of rear impeller | Based on designed position | -10°     | 0°     | +10°    |
| Phase angle      | Based on designed position | -30°     | 0°     | +30°    |

3.2. Design of Experiment
A design experiment consisting of 40 training data points is selected using a combined FCCD strategy (15 points), and LHS (35 points).
3.3. Numerical Simulations
This study utilizes commercial CFD code. Turbulence is modeled by using the FBM turbulence model[9]. Second order, upwind discretizations have been used for convection terms and central difference schemes for diffusion terms. For these calculations, stationary and rotating components are analyzed simultaneously. Quasi-steady rotor-stator computation with appropriate modeling is adopted.

![Computational mesh](image)

**Figure 4. Computational mesh**

The structure computational grid is used for the whole domain. The whole domain is divided into four different parts for mesh generating to get higher mesh quality, which are the inlet, impellers and the stator respectively. O-grid zone is generated around the impeller blades and stator blades. Different meshes are generated for different parts. These meshes are then put together. In order to check the influence of the grid on the results, meshes with different numbers of nodes are tested, and grid independence is achieved. Fig. 3 shows the grid sizes and the computed overall quantities, which are efficiency and head coefficient. It can be observed that the results on successive grids become closer to each other as the grid is refined and with further grid refinement the results become grid independent. The final computational grid number for one of the whole domain is $2.21 \times 10^6$.

![Effect of grid refinement](image)

**Figure 5. Effect of grid refinement**

A constant velocity, normal to the inflow boundary was prescribed at the inlet to specify the discharge. An average static pressure of 1 atm was put at the outlet. Shroud casing of the impeller is set as absolutely stationary, and the blade and hub of the impeller are relatively stationary. Steady calculation with a frozen rotor interface model between two impellers and a stage interface model between rear impeller and stator are used. Solution convergence is determined by monitoring the normalized rates of change for each dependent variable. In addition to the normalized rates of change, the mass balance for the model is checked, as well as location and value of the minimum and maximum for each dependent variable. When all these are stable from one iteration to the next, the solution is considered converged.
3.4. Model Construction

Efficiency and MAPC can be obtained from the simulation results of the designed points. Some results that the efficiency is lower than 20% should be neglected. At last 35 designed points are used to construct PRS, KRG and RBNN surrogate models. Cross validation is summarized in following chart.

|                        | GMSE (%) | PRS  | KRG  | RBNN |
|------------------------|----------|------|------|------|
| Efficiency             | 10.88    | 12.53| 58.30|      |
| MAPC                   | 43.83    | 43.25| 57.36|      |

The results in the chart show that errors of efficiency are small, while the errors of MAPC perdition are too large to get accurate enough optimization results. So it’s necessary to get a refinement of previous design space. Before that, some analysis about efficiency based on the PRS model can be taken, considering the errors are not so large.

3.5. Future Analysis

Global sensitivity analysis based on the PRS models is presented here. From the figure, it’s shown that, Sensitivity of phase angle about efficiency can be neglected compared with other two design variables. Ratio of sensitivity of front impeller fixed angle and that of rear impeller fixed angle is about 4:6, same as designed loading ratio.

Figure 6. Global sensitivity analysis of
In objective function space, we can easily get the optimal region with high efficiency and high MAPC shown as the figure. Mapping objective function space to design space, we can get optimal region in design variables space: (-1°, -8°) and (-1°, -6°) for fixed angle of blades in front and rear impellers.

3.6. Design space Refinement

With analysis above, Influence from phase angle on performances can be neglected, so the design variables for the refinement are fixed angle of blades in impellers, ranges of which are refined shown as following table. Objective functions are still the same as before.

| Table 5. Objective functions and design variables |
|-----------------------------------------------|
| Objective functions | Designed value |
| Efficiency         | 83.7%          |
| MAPC              | -1.43          |

| Design variables        | Range       |
|-------------------------|-------------|
| Minimum                 | Maximum     |
| Fixed angle of front impeller | -8°      | -1°      |
| Fixed angle of rear impeller  | -6°       | -1°      |

A design experiment consisting of 20 training data points is selected using LHS method. PRS, KRG and RBNN surrogate models are constructed. Errors are shown as follows.

| Table 6. GMSE of surrogate models |
|-----------------------------------|
| GMSE (%) | PRS | KRG | RBNN |
| Efficiency | 0.34 | 2.01 | 1.59 |
| MAPC   | 0.17 | 0.40 | 0.24 |

The error for the PRS model is the smallest, which means the PRS model can provide accurate enough prediction of the relationship between design variables and objective functions. Pareto Front method is using again to get the final optimization results.
Figure 8. Pareto Front in objective function space

Figure 8 shows clearly the tradeoff of efficiency and high MAPC, which represent energy and cavitation performances, respectively. Pareto Front can be found, consisting of points with high efficiency and high MAPC in objective functions space. Considering both efficiency and MAPC, the optimum case can be pointed out. Mapping the point in design variables space, optimization results can be obtained: fixed angle of front impeller are -3.7° and -2.2°, respectively. Take simulations of the optimum case, efficiency and MAPC are 84.3% and -1.36, which improved 0.6% and 4.5% compared with prototype, respectively.

Figure 9 shows average static pressure contour of meridian interface in front and rear impellers

Figure 9 shows average static pressure contour of meridian interface with designed flow rate in front and rear impellers of prototype and optimum pump. It’s shown clearly that the average pressure increasing near the inlet of front impeller, which means cavitation performance enhancement for the optimum pump.
4. Conclusions
1) Global design optimization based on surrogate method can work well with efficiency and MAPC being improved simultaneously.
2) Influence from phase angle on performances can be neglected compared with other two design variables. Impact ratio of fixed angle of blades in two impellers on efficiency is the same as their designed loading distributions, which is 4:6.
3) The optimization results can enhance the tradeoff performances well: efficiency and the MAPC are improved by 0.6% and 4.5%, respectively.

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