In a gedankenexperiment about the generalized second law (GSL) of black hole thermodynamics, the buoyant force by black hole atmosphere (the acceleration radiation) plays an important role, and then it is significant to understand the nature of the buoyant force. Recently, Bekenstein criticizes that the fluid approximation of the acceleration radiation which is often used in the estimation of the buoyant force is invalid for the case that the size of the target is much less than a typical wavelength of the acceleration radiation, due to the diffractive effect of wave scattering. He calculated the buoyant force as a wave scattering process and found that the buoyant force as a wave scattering process is weaker than in the fluid approximation. And he asserts that while the buoyant force by black hole atmosphere is insufficient for the GSL to hold, the Bekenstein’s entropy bound is enough.

In this letter, we argue that even if it is correct that we should calculate the buoyant force as a wave scattering process, its implication in the GSL strongly depends on whether there exists any massless scalar field, that is, S-wave scattering. By reconsidering the diffractive effect by S-wave scattering, we show that if some massless scalar field exists, then the GSL can hold without invoking a new physics, such as an entropy bound for matter.

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I. INTRODUCTION

We believe that black hole is a thermodynamical object and has entropy in a sense. This belief is based on analogy with ordinary thermodynamics; mathematical relationship between black hole mechanics [1] and ordinary law of thermodynamics, and the existence of thermal radiation from a black hole.

In order to transmute the mathematical relationship into physical one, that is, black hole mechanics into black hole thermodynamics [1], understanding of statistical origin of black hole entropy has progressed from the various points of view [3]. It is any theory of quantum gravity which controls a measure of the density of the states that we need to understand genuinely statistical black hole entropy without divergent quantities. On the other hand, black hole entropy can be derived in metrical theories of gravity by a classical method such as Noether charge method [4] or by semiclassical methods such as Euclidean path integral method [4], though these methods by no means count quantum degrees of freedom that are responsible for black hole entropy. Since any successful quantum gravity theory comes to a corresponding metrical theory of gravity in suitable low energy limit, we expect that the value of black hole entropy obtained by classical or semiclassical methods should be also derived by counting quantum degrees of freedom. Thus, we may regard success of statistical derivation of black hole entropy as a benchmark test of a proposed quantum gravity theory.

On the other hand, it is also necessary to understand black hole entropy better, even at the level of thermodynamics. For instance, it is the second law that characterizes the peculiar property of entropy in ordinary thermodynamics because of its referring to a direction in time. Therefore it is very important for understanding of black hole entropy to establish the second law of black hole thermodynamics.

Since we cannot regard a black hole as an isolated system owing to the universal interaction with ordinary matter outside the black hole by gravity, any second law of black hole thermodynamics should refer to total entropy of self-gravitating system including black holes. Therefore we are led to the generalized second law (GSL) of black hole thermodynamics, which asserts that in any process, the generalized entropy

$$S_G := S_{BH} + S_M = \frac{1}{T_{BH}} \left( \kappa A_{BH} \right) + S_M$$

(1.1)
never decreases, where \( S_{BH} \) and \( S_M \) denote the entropy of the black holes and that of ordinary matter outside the black hole, and then \( \kappa, A_{BH} \) and \( T_{BH} \) are the surface gravity, area of the event horizon and temperature of the black hole, respectively. The validity of the GSL is essential for the consistency of black hole thermodynamics and for the interpretation of the horizon area as representing the physical entropy of a black hole, because it is nothing but the ordinary second law for self-gravitating systems containing black holes. Thus, the GSL is a cornerstone of black hole thermodynamics.

Although an explicit general proof of the GSL has not been given until now, the various attempts for special cases have been performed \(^1\) \(^2\) \(^3\). Considering a process which transfers an infinitesimal energy \( \delta E \) and entropy \( \delta S \) in the external region into the black hole adiabatically, we obtain the change in the total entropy \( \delta S_G = \delta E/T_{BH} - \delta S \).

In classical theory, we may argue as follows. Since a black hole can classically export nothing outside the horizon, it is natural to give zero temperature \( T_{BH} = 0 \) to the black hole. Therefore, if it were so, by dominance of the first term in Eq. (1.1), the GSL in classical theory should be no more than the second law for the black hole entropy alone and then it would amount to the area increasing law in black hole mechanics \( \delta A_{BH} > 0 \) which holds by energy condition \( \delta E > 0 \) \(^4\).

However, it is awkward to assign \( T_{BH} = 0 \) to the black hole, since the black hole entropy or the change in it becomes divergent and ill-defined. Thus, the physical analogy appears end in classical theory. In order to have non-zero black hole temperature and well-defined black hole entropy, it is indeed essential to incorporate quantum effects even semiclassically. Due to the breakdown of the energy condition of quantum fields, black holes can radiate and acquire non-zero temperature, and then the thermodynamic quantities of a black hole can be fixed as \( T_{BH} = \kappa/2\pi \) and \( \delta S_{BH} = A_{BH}/4 \) \(^5\). Therefore, it is important to investigate the validity of the GSL by consistent arguments with taking account of quantum effects.

An observer accelerating with acceleration \( a \) detects isotropic thermal radiation with temperature \( T_U = ha/2\pi \) by the Unruh radiation (acceleration radiation) \(^6\). An object suspended near a black hole is accelerated by virtue of its being prevented from following a geodesic. Unruh and Wald \(^7\) \(^8\) suggested that this object will likewise see Unruh radiation. Since its acceleration (i.e. temperature) varies with distance from the horizon, they surmised that the object will be subject to a buoyant force by the acceleration radiation fluid and the buoyancy affects the energetics of a process which exchange entropy and energy between the black hole and outer matter. They concluded that quantum buoyancy is sufficient by itself to protect the GSL.

Recently, Bekenstein reconsidered the nature of acceleration radiation and its implication on the GSL \(^9\). He pointed out that the wave nature (diffractive effect) of the acceleration radiation cannot be neglected in the case that the size of the object lowered toward the black hole is smaller than a typical wavelength of the acceleration radiation and that the fluid approximation of the acceleration radiation is invalid. For such a case, he estimated the buoyant force as a wave scattering process and found that the buoyant force as a wave scattering process is weaker than in the fluid approximation. Therefore, the diffractive effect alters energetics of exchange process of the entropy and energy compared with that in fluid picture, and then the quantum buoyancy is insufficient by itself to protect the GSL. A breakdown of the GSL in the existing physics leads us to a new physics, such as an entropy bound for matter, if we take granted that the GSL holds. Thus, the question of the validity of the GSL is still be opened even in a simple gedankenexperiment.

In this letter, we observe that if a massless scalar field exists, the quantum buoyancy is sufficient to protect the GSL, rather strengthen the validity of the GSL, even though we take account of the wave nature of the acceleration radiation.

### II. A GEDANKENEXPERIMENT

In this section, we specify a gedankenexperiment investigated in this letter and review two independent reasonings for the GSL to hold.

We consider a static black hole which area of the event horizon is \( A \) and a box of proper height \( b \) and geometrical cross-sectional area \( A \). Far from the black hole, the box is filled with matter, so that the total energy of the box and contents is \( E_0 \) and its total entropy \( S_0 \). Subsequently, the box is lowered adiabatically toward the black hole by a weightless rope to some height \( l \) that is the proper distance between the horizon and the center of mass of the box. And then, the box and contents are released and dropped into the black hole.

\(^1\) Someone may doubt additivity of entropy of self-gravitating system, due to long-range nature of gravity. Instead of no conclusive argument about the additivity, we assume the validity of the additivity. See \(^3\) for argument validating the additivity.
Because of the process to be adiabatic, the total entropy of the box and contents remains constant. Therefore, the change in the total entropy becomes:

$$\Delta S_G = \Delta S_{BH} - S_0 = \frac{\Delta M(l)}{T_{BH}} - S_0 , \quad (2.1)$$

where \( T_{BH} \) is the (non-zero) black hole temperature.

On the other hand, the energy \( \Delta M(l) \) delivered to the black hole decreases during the lowering process, because the gravitational energy of the box and contents is lost by the work against the tension of the rope. As denoting the redshift factor \( \xi(l) \), we obtain the equation,

$$\Delta M(l) = E_0 + (\text{work done by the rope}) = E_0 + W_{\infty}(l) = E_0 + \int_{\infty}^{l} (-F_{\infty}^G) \, dl$$

$$= E_0 + E_0 [ \xi(l) - 1 ] = E_0 \xi(l) , \quad (2.2)$$

where we use the relation \(-F_{\infty}^G \, dl = dE_{\infty} = d(E_0 \xi)\), that is derived from \( E_{\infty} = E_0 \xi \). Thus, the energy delivered to the black hole is “redshifted away”, due to the negative gravitational potential.

Therefore, the change in the total entropy is

$$\Delta S_G = \frac{E_0}{T_{BH}} \xi(l) - S_0 = \frac{E_0}{T(l)} - S_0 , \quad (2.4)$$

where \( T(l) := T_{BH}/\xi(l) \) means the locally measured temperature of the black hole atmosphere. Because, if the box can be close to the horizon without limit, \( \xi \) can be arbitrary small near the horizon, we can make the value of \( \Delta S_G \) negative at will.

If we take granted that the GSL holds, then we need any mechanism which prevents the box from the horizon. At present, there exist two reasonings: one is invoking to an entropy bound for matter and the other makes use of the buoyant force by the black hole atmosphere. It is essential to recognize that the box must have a finite size which is greater than its Compton wavelength.

The argument of the first reasoning invoking an entropy bound is as follows: The finiteness of the box size imposes a constraint, \( l \geq b/2 \), that is,

$$\xi(l) \sim \kappa l = 2\pi T_{BH} l \geq \pi T_{BH} b$$

$$\Rightarrow 2\pi T_{BH} b \geq \frac{4\pi r_g^2}{3} \leq 4\pi r_g^2 . \quad (2.5)$$

Thus, the entropy of any matter in this case is bounded above by its energy and size. Since, obviously, the size \( b/2 \) is greater than its gravitational radius \( r_g = 2E \), we obtain

$$S \leq 2\pi E \, r_g \leq \frac{4\pi r_g^2}{4} . \quad (2.6)$$

Thus the maximum entropy of any matter is bounded above by its gravitational radius and the saturated state is attained by the black hole state. This relation is called holographic bound, which the validity of Eq.(2.7) is open problem and has actively been discussed in the different viewpoint, holographic principle \[18\]. Even though it is finally true that there exists the entropy bound for matter or the holographic bound, it is important to investigate to what degree the GSL is protected by the known physics and whether the validity of the GSL implies the entropy bound or the holographic bound.

Another reasoning invoking the known physics makes use of quantum effect of matter field outside black holes, that is, the buoyant force by the black hole atmosphere, which has been neglected in the argument of the first reasoning. We may start with two main working hypothesis \[4,5,20\].

---

\[2\] Here we implicitly assume that processes after the box released preserves, the total energy and entropy of the box and contents.
**A1.** The black hole atmosphere is describable by radiation fluid of a unconstrained thermal matter which is defined to be the state of matter that maximizes entropy density at a fixed energy density and the radiation fluid has the locally measured temperature $T(l)$.

**A2.** The buoyant force on the box exerted by the black hole atmosphere is equal to the pressure gradient of the radiation fluid of unconstrained thermal matter.

The assumption A1 means that the Gibbs-Duhem relation holds,

$$
\begin{align*}
\rho_{\text{rad}} + P_{\text{rad}} - T(l) s_{\text{rad}} &= 0, \\
d\rho_{\text{rad}} &= d(T(l) s_{\text{rad}}),
\end{align*}
$$

(2.8)

and by Eqs.(2.8) and $T(l) = T_{\text{BH}}/\xi(l)$, we obtain balance equation between gravitational force and pressure gradient force of the radiation fluid

$$
\frac{d}{dl}(\xi P_{\text{rad}}) = -\rho_{\text{rad}}(l) \frac{d\xi}{dl}.
$$

(2.9)

Using Eq.(2.9) and the assumption A2, we obtain “Archimedean principle”,

$$
F_{\infty}^B = A[ (\xi P_{\text{rad}})_{\text{bottom}} - (\xi P_{\text{rad}})_{\text{top}} ] = -V \frac{d}{dl}(\xi P_{\text{rad}}) = V \rho_{\text{rad}}(l) \frac{d\xi}{dl}.
$$

(2.10)

Therefore, the work done by the total force $F_{\infty} = F_{\infty}^G + F_{\infty}^B$ becomes

$$
W_{\infty} = \int_{\infty}^{d} (-F_{\infty}) 
= E_0[ \xi(l) - 1 ] + V \xi(l) P_{\text{rad}}(l).
$$

(2.11)

And then, the energy delivered into the black hole is

$$
\Delta M = E_0 + W_{\infty} = E_0 + V P_{\text{rad}}(l) \xi(l) = V \rho_0 - \rho_{\text{rad}} + T(l) s_{\text{rad}} \xi(l),
$$

(2.12)

where $\rho_0 := E_0/V$ is the average energy density of the box and the contents. The change in the total entropy becomes

$$
\Delta S_G = V \left( \frac{\rho_0 - \rho_{\text{rad}}}{T(l)} + s_{\text{rad}} - s_0 \right),
$$

(2.13)

where $s_0 := S_0/V$ is the average entropy density of the box and the contents.

The critical situation for the positivity of $\Delta S_G$ is the case of minimizing $\Delta M$,

$$
0 = \frac{d}{dl} \int_{\infty}^{d} (-F_{\infty}) 
= F_{\infty}^G + F_{\infty}^B
= V \left( \rho_0 - \rho_{\text{rad}} \right) \frac{d\xi}{dl},
$$

(2.15)

so that, it is the most dangerous for the validity of the GSL when the box is dropped into the black hole at the floating point $\rho_0 = \rho_{\text{rad}}(l)$.

Nevertheless the positivity of $\Delta S_G$ holds by the definition of the radiation fluid, that is, we can show the validity of the GSL (2.16) without invoking a new physics,

$$
\Delta S_G \geq V (s_{\text{rad}} - s_0) \geq 0,
$$

(2.16)

where the last inequality follows the definition A1 of the radiation fluid, because of $\rho_0 = \rho_{\text{rad}}(l)$ at the floating point.

Now we should check the validity of our assumptions, especially, the validity of the assumption A2. It is natural to think that if a typical wavelength of the acceleration radiation $\lambda$ is much bigger than the box size $b$, the assumption A2 is invalid due to the breakdown of the fluid picture, such as diffractive effect. Therefore, it is doubtful to consider that the fluid picture is still valid far from the black hole, such as $b < \lambda \sim T^{-1}(l)$. 

4
III. THE BUOYANT FORCE BY LONG WAVELENGTH SCATTERING

Recently, Bekenstein pointed out the breakdown of the fluid picture far from the horizon [17].

Strictly speaking, the true pressure exerted on the surface of the box is given by integrating true stress tensor over the surface and the true stress tensor must be obtained by inclusion of the boundary condition of the surface. However, in the previous section, we estimated the pressure by the fictitious stress tensor, which means that the stress tensor is estimated by neglecting the surface, exclusive of the boundary condition. In order to estimate the true pressure, it is often useful to calculate the change in momentum flux on the surface and it is essential for calculating the change in the momentum flux to estimate the reflection coefficient, that is, to include the boundary condition on the surface. For example, a perfectly transparent glass is not exerted by photons, even though the momentum flux across the glass does not vanish.

Thus, we need to estimate the scattering cross section of the box for the acceleration radiation. If \( b \ll R_H \), where \( R_H \) is the curvature radius at the horizon, then we can acquire a large local (Lorentz) frame including the target (the box) in which the target is at rest. Therefore, we can approximate the scattering process in the black hole spacetime by the scattering process in flat spacetime and at first estimate quantities in interest, such as the momentum transfer, in the local frame. A remained task is to transform quantities obtained in the local frame into ones in the global frame, that is, quantities as measured at infinity [17].

We calculate physical quantities in the long wavelength limit \( b \ll \lambda =: 2\pi/k \), because we are especially interested in scattering phenomena in the situation that the fluid picture of the acceleration radiation is suspicious. In this limit, the differential cross section is indifferent to details such as the shape of the target. Hereafter we assume that the the shape of the target is spherically symmetric.

A. the buoyant force by dipole scattering

According to the above procedure, Bekenstein estimated the buoyant force exerted by dipole scattering. In order to estimate the buoyant force, we calculate the differential cross-section of the target object with the size \( b \) by the dipole scattering. For the dipole scattering which transfers the incident wave with the wave vector \( \vec{k} \) into the scattered wave with \( \vec{k}' \) and preserves the magnitude of the momentum, \( k := |\vec{k}| = |\vec{k}'| \), we have

\[
\frac{d\sigma}{d\Omega} = b^2 \left( \frac{k}{kb} \right)^4 F(\vec{n}, \vec{n}'),
\]

where \( \vec{n} \) and \( \vec{n}' \) are a pair of the unit vectors denoting the incident and scattering directions, \( \vec{n} := \frac{\vec{k}}{k} \) and \( \vec{n}' := \frac{\vec{k}'}{k} \), respectively. And \( F \) is some dimensionless function, which, for example, is given by \( F(\vec{n}, \vec{n}') = \pi^{-2} \left\{ \frac{3}{8} \left[ 1 + \cos^2 (\vec{n} \cdot \vec{n}') \right] - \cos(\vec{n} \cdot \vec{n}') \right\} \) for electromagnetic scattering from a conducting sphere. The fourth order dependence of the cross-section on the wave vector is attributed to the fact that the dipole part is dominant in the scattering of the electromagnetic wave.

Given a distribution function of the incident wave as \( f(k) = 1/\left[ \exp(hk/T) - 1 \right] \), the incident momentum flux carried in the acceleration radiation in the vicinity of the wave vector \( \vec{k} = k\vec{n} \) becomes

\[
\vec{n} I(k, \vec{n}) \frac{d^3k}{(2\pi)^3} = \frac{\hbar}{\pi} k^3 f(k) \frac{dk d\vec{n}}{(2\pi)^3},
\]

where \( d^3k = k^2 dk \, d\vec{n} \).

Because the fraction \( d\sigma/d\Omega' \) among the incident flux \( I \, dk d\vec{n} \) is scattered into the direction \( \vec{n}' \), we obtain the momentum transfer of the box in the local frame,

\[
\frac{d\vec{P}}{d\tau} = \int d\vec{n} I(k, \vec{n}) \frac{d\sigma}{d\Omega}(\vec{n} - \vec{n}'),
\]

where \( \tau \) is time measured in the local frame, that is, the proper time of the target.

Since the acceleration radiation has the temperature gradient, the radiation going to the direction \( \vec{n} \) hits on the box with the local temperature \( T(l, \vec{n}) = T_{BH}/\xi(l, \vec{n}) = \hbar/2\pi[l + (\vec{e}_l \cdot \vec{n})b] = T(l)/[1 + (\vec{e}_l \cdot \vec{n})b/l] \), where \( \vec{e}_l \) is the unit vector that is directed to the center of mass of the box from the black hole. Therefore, the buoyant force \( F_{\infty}^{Scatt} \) by the dipole scattering in the global frame becomes
\[ F_{\infty}^{\text{Scatt}} = \xi(l) \left| \frac{d\vec{P}}{d\tau} \right| \sim (2\pi l T_{BH}) \left[ \frac{T(l)}{b} \right]^8 b^6 \int d\vec{n} \left[ 1 + \frac{b}{l} (\vec{e}_l \cdot \vec{n}) \right]^{-8} \sim \frac{T_{BH}}{b} \left( \frac{b}{l} \right)^8, \] (3.4)

where we neglect numerical factor. Thus, the buoyant force by the dipole scattering is proportional to the seventh power of the size \( b \), not to the volume (non-Archimedean) and proportional to the eighth inverse power of the proper distance \( l \) from the event horizon.

On the other hand, the buoyant force in fluid picture is estimated by,

\[ F_{\infty}^{B} = V \rho_{\text{rad}}(l) \frac{d\xi}{dl} \sim b^3 \left[ T(l) \right]^4 T_{BH} \sim \frac{T_{BH}}{b} \left( \frac{b}{l} \right)^4. \] (3.5)

Thus, in the case of the dipole scattering, Archimedean character of buoyant force that the force is proportional to the volume of the box does not work for \( b \ll \lambda \) and the force rapidly decreases with the distance from the horizon than in the fluid picture.

Since it is possible to saturate the inequality Eq.(2.16) by lowering the radiation matter, even in the case for the fluid picture to be valid, the fact that the buoyant force by dipole scattering is weaker than in the fluid picture suggests that buoyant force alone is not enough for the GSL to be valid. Indeed, we can show that there exists cases satisfying both of the breakdown of the GSL and the validity of the approximation used [17].

**B. the buoyant force by S-wave scattering**

The non-Archimedean character of buoyant force shown in the previous subsection is attributed to the dipole dominant scattering. If we assume that a massless scalar field exists in nature, the argument based on dipole scattering does not work and implication on the GSL by wave nature of the acceleration radiation is drastically changed, due to S-wave scattering.

By mode decomposition of the equation of motion of the scalar field \( \square \phi = 0 \) with respect to the plane wave in the local frame, we obtain

\[ \left[ k^2 + \Delta \right] \Psi_{\vec{k}} = 0, \quad \phi = \frac{\exp(-ik\tau)}{\sqrt{2k}} \frac{\Psi_{\vec{k}}(\vec{x})}{(2\pi)^{3/2}}. \] (3.6)

Since we would like to consider the case that the total entropy of the box and contents remains constant, we regard the surface of the box as infinite potential barrier. Therefore, we solve a scattering problem by the infinite potential barrier at the radius \( b \) in quantum mechanics. We easily obtain the result [19],

\[ \Psi_{\vec{k}}(\vec{x}) \sim \exp(ik \cdot \vec{x}) + g(\Omega) \frac{\exp(ikr)}{r}; \quad \frac{d\sigma}{d\Omega} = |g(\Omega)|^2, \] (3.7)

\[ g(\Omega) = \sum_{l=0}^{\infty} \frac{2l+1}{2ik} \left( 1 + \frac{h_l^{(2)}(kb)}{h_l^{(1)}(kb)} \right) P_l(\cos \theta), \] (3.8)

where \( h_l^{(n)} \) and \( P_l \) are the spherical Hankel function of the \( n \)-th kind and the Legendre one of the first kind, respectively. In the long wavelength limit, the reflection coefficient \( g(\Omega) \) is approximated by

\[ g(\Omega) \sim -b \sum_{l=0}^{\infty} \frac{(kb)^{2l}}{[(2l-1)!!]^2} P_l(\cos \theta). \] (3.9)

If S-wave scattering occurs, we have the differential cross section independent of wavelength,

\[ \left( \frac{d\sigma}{d\Omega} \right)_{l=0} \sim b^2 \sim \frac{\sigma_T}{4\pi}, \quad \sigma_T \sim 4\pi b^2 = 4A, \] (3.10)

\(^{3}\)Since we concentrate on the case that the lowering process goes on far from the black hole in order to make the used approximations valid, massive fields less contribute to buoyant force. The reason is that massive quanta far from the black hole are much less “excited” for a stationary observer than massless ones.
where $\sigma_T$ is the total scattering cross section of the target.

In this connection, if the dipole dominated scattering occurs, such as the electromagnetic field, we have the differential cross section

$$\left( \frac{d\sigma}{d\Omega} \right)_{l=1} \sim b^2 (kb)^4 \left[ P_1(\cos \theta) \right]^2 ,$$

which depends on the fourth power of $k$ as Eq.(3.1).

The contribution of the S-wave scattering to the momentum transfer of the target in the local frame is

$$\frac{d\vec{P}}{d\tau} = \int dk d\vec{n} \int d\vec{n}' I(k, \vec{n}) \left( \frac{d\sigma}{d\Omega'} \right)_{l=0} (\vec{n} - \vec{n}') = \int dk [ \vec{n} \cdot I(k, \vec{n}) ] ,$$

where the last equality is due to the spherical symmetric scattering of S-wave. Since the quantity $\sigma_T \int dk \left[ \vec{n} \cdot I (k, \vec{n}) \right]$ is nothing but the momentum flux across the surface with the area $\sigma_T$ into the direction $\vec{n}$, Eq.(3.12) gives momentum transfer four times larger than the total momentum flux across the surface of the box.

Therefore, the buoyant force in the global frame exerted on the box by S-wave scattering is four times larger than that in the fluid picture, because of the diffractive effect,

$$F^\infty_{\text{Scatt}} = 4 F^\infty_{B \text{(fluid)}} = 4 V \rho_{\text{rad}} \frac{d\xi}{dt} .$$

(3.13)

Since the diffractive effect of S-wave scattering strengthens the buoyant force than in the fluid picture, the above fact suggests that the GSL is protected by the buoyant force alone.

Indeed, following the argument in Sec. II for this case, we obtain the inequality from Eq.(2.14)

$$\frac{\Delta S_G}{V} \geq 3 \frac{\rho_{\text{rad}}}{T} + s_{\text{rad}}(\rho_{\text{rad}}) - s_0$$

$$\geq 3 \frac{\rho_{\text{rad}}}{T} + s_{\text{rad}}(\rho_{\text{rad}}) - s_{\text{rad}}(4\rho_{\text{rad}}) ,$$

(3.14)

(3.15)

where we explicitly denote the dependency of $s_{\text{rad}}$ on $\rho_{\text{rad}}$. The first line is given by $\rho_0 = 4\rho_{\text{rad}}$ at the floating point and the second comes from the assumption A1, $s_0 \leq s_{\text{rad}}(\rho_0) = s_{\text{rad}}(4\rho_{\text{rad}})$. Using the equations,

$$s_{\text{rad}}(\rho_{\text{rad}}) = \frac{4}{3} \frac{\rho_{\text{rad}}}{T} ,$$

$$s_{\text{rad}}(\rho_{\text{rad}}) = \frac{\rho_{\text{rad}}}{3/4} ,$$

(3.16)

(3.17)

we can show the validity of the GSL

$$\frac{\Delta S_G}{V} \geq s_{\text{rad}}(\rho_{\text{rad}}) \left( \frac{13}{4} - 2 \frac{\rho_{\text{rad}}}{\rho_{\text{rad}}} \right) > 0 .$$

(3.18)

Thus, if some massless scalar field exists, then without invoking a new physics such as an entropy bound for matter, the GSL holds thanks to the buoyant force strengthened by the diffractive effect of S-wave scattering of black hole atmosphere.

For the completeness, we should check the validity of the fluid picture in short wavelength limit. In this limit, we obtain the differential cross section

$$\frac{d\sigma}{d\Omega} \sim \frac{A}{4\pi} ,$$

(3.19)

and finally obtain the momentum transfer in the local frame as

$$\frac{d\vec{P}}{d\tau} = \int dk d\vec{n} \int d\vec{n}' I(k, \vec{n}) \left( \frac{d\sigma}{d\Omega'} \right) (\vec{n} - \vec{n}') = \int \frac{d\vec{n}}{A} \int \frac{dk}{4\pi} [ \vec{n} \cdot I(k, \vec{n}) ] .$$

(3.20)

As expected, the buoyant force in the geometrical optics approximation is equal to that in the fluid picture.
IV. SUMMARY

In this letter, we briefly reviewed a gedankenexperiment to test the validity of the GSL, which is any process composed of adiabatically lowering the object toward the black hole and dropping into. In the analysis of this gedankenexperiment, the buoyant force by the black hole atmosphere plays an important role and the buoyant force is usually estimated by the pressure gradient of the radiation fluid. However, since the pressure exerted on the target is given by the change in the momentum flux, it is necessary to estimate the reflection coefficient on the surface of the target, in order to get the correct buoyant force. In the case that the size of the target is larger than a typical wavelength of the black hole atmosphere, the pressure exerted on the surface of the box is well estimated by the fluid picture for the black hole atmosphere. On the other hand, in the case that the lowering process goes on with satisfying $b < \lambda$, we cannot complete the reasoning which makes the GSL to hold by the buoyant force estimated in based on the fluid picture, because the fluid picture breaks down by diffractive effect of wave scattering.

For buoyant force far from the black hole, massless fields dominate over massive ones, due to less acceleration of the quasi-stationary target compared with their masses. Furthermore, in the long wavelength limit, the dependence of the scattering cross section on wavelength much varies according to the spin of the scattered wave. Therefore, it much depends on the spin of massless fields in nature whether we need to invoke a new physics such as an entropy bound for matter, in order to hold the GSL, or not. If some massless scalar field exists in nature, then the GSL can hold, due to the buoyant force alone by black hole atmosphere. If not so, the validity of the GSL might suggest the existence of some new physics such as an entropy bound.

The above conclusion is based on the viewpoint of an accelerated observer who rest on the box lowering adiabatically. Although the energy-momentum tensor normalized by the accelerated observer is different from the true one, we can expect that the calculation of buoyant force by the viewpoint of the accelerated observer gives correct estimation. It is because the essential quantity in the calculation is gradient of the energy-momentum tensor, not value itself, and the difference between the energy-momentum tensor normalized by the accelerated observer and the true one is divergence free.

In the Ref. [7], it was shown that in two dimensional spacetime, the estimation of $\Delta M(l)$ delivered to the black hole in an accelerating viewpoint with the fluid approximation is equivalent to that in an inertial point of view. Does this equivalence suggest that the estimation of buoyant force by wave scattering is different from that in an inertial point of view, that is, not physical? Since, in two dimensional spacetime, the reflection coefficient of wave scattering by infinite potential is unity, two estimations of buoyant force in an accelerating viewpoint with and without the fluid approximation are equal to one another. Therefore, three estimations, including in an inertial point of view, are consistent and this result is due to the peculiarity of two dimensional spacetime. For completeness, it is worthwhile to estimate energetics $\Delta M(l)$ from an inertial point of view for higher dimensional spacetimes.

Furthermore, although we regard the mere sum of black hole entropy and matter one as the total entropy, we have not yet obtained the foundation. Since gravity is long range force, it may be doubtful to assume the additivity of entropies of individual systems in self-gravitating system. It is future work to reconsider the GSL without the assumption of the additivity of entropies [10].

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