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Reflectionless design of optical elements using impedance-tunable transformation optics

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We report a strategy to remove the reflections resulted from the finite embedded transformation-optical design by proposing a theory of impedance-tunable transformation optics, on which the functions of impedance coefficients can be set in the original space without changing the refractive index. Based on the approach, two-dimensional reflectionless beam compressors/expanders, bends, shifters, and splitters are designed using the modified anisotropic medium. It is found that the reflections can be removed in magnetic response materials for TE polarization or dielectric response materials for TM polarization. The numerical simulations confirm that various reflectionless optical elements can be realized in the pure transformation optics. The proposed method can be generalized to three-dimensional cases and can be applied to other transformation-optical designs.

Transformation optics derived from invariant coordinate transformations of Maxwell’s equation, which has been reported by Pendry1 and Leonhardt,2 provides an unconventional technique to design optical devices with excellent functionalities such as the invisible cloak,3 bends,4–6 field rotators,7,8 concentrators,9,10 and lens.11,12

The finite embedded coordinate transformation introduced by Rahm13 accelerates the transformation-optical design step and provides more flexibilities to the controlling and guiding of EM waves. However, such coordinate transformation is not a continuous one where the impedance of the transformation medium mismatches to those of the surroundings in many cases as reported in literatures.14–17 The most typical case is a two-dimensional (2D) compressor/expander, suffering from the reflections inevitably as pointed out in Ref. 14, which was explained in terms of an inherent topological metric mismatch due to the transformation map itself. Thus, impedance matching is very necessary for the design of metric-mismatched elements without reflections. Combining conventional method of inserting an antireflective coating, the reflection can be suppressed for a 2D compressor/expander,18 but the coating is difficult to be realized in cases with complicated boundaries. A three-dimensional (3D) impedance-matched compressor/expander has been presented by Emiroglu and Kwon,19 which can only be applied to some special cases where the expanding/compressing rates are the same in two orthogonal directions. Using a nonlinear coordinate transformation with optimal parameters to the design of nonmagnetic cloak20 and lens,17 the reflection can be reduced to a lower level compared to the linear one; however, it has not been removed totally. Therefore, to remove the reflections resulted from the coordinate transformation is still an open question up to now.

In this Letter, we propose a theory of impedance-tunable transformation optics, where a tunable impedance is set in the original space without changing the usual linear coordinate transformation, based on which 2D reflectionless beam compressors/expanders, bends, shifters, and splitters can be designed using anisotropic materials.

In the traditional transformation-optical design,13–19 the original space was set to be an invariant isotropic medium. We argue that the ratio of permittivity and permeability in the original space can be adjusted before transformation in order to compensate the ratio change induced by the asymmetric coordinate transformation and make the transformation medium match the surroundings. We reset the permittivity $\varepsilon^{\bar{i}} = \varepsilon^{0}/k$ and the permeability $\mu^{\bar{i}} = k\mu^{0}$ in the original space, where the coefficient $k$ is spatial dependent and is a continuous function to ensure the continuity of both original space and transformed space. For a given coordinate transformation $x^{\prime} = x^{\prime}(x)$, the permittivity $\varepsilon^{\bar{j}}$ and permeability $\mu^{\bar{j}}$ of the transformation medium calculated by the prescription of Ref. 21 can be expressed as

$$\varepsilon^{\bar{j}} = \left| \det(A^{t}) \right|^{-1} A^{t}_{i} A^{t}_{j} \varepsilon^{0}/k,$$

$$\mu^{\bar{j}} = \left| \det(A^{t}) \right|^{-1} A^{t}_{i} A^{t}_{j} k \mu^{0},$$

(1)

where $A^{t}_{i} = \frac{\partial x^{\prime}(x,y,z)}{\partial(x,y,z)}$ denote the Jacobian tensor between the transformed space $(x^{\prime}, y^{\prime}, z^{\prime})$ and the original space $(x,y,z)$. Since in Eq. (1), the coefficient $k$ does not change the refractive index of the original space, as known in the geometrical optics,22 thus, the light rays does not been changed both in original space and transformed space in comparison to the nontunable case ($k = 1$) by the same coordinate transformation. However, the ratio of the electric field and the magnetic field has been changed by $k$, that is, the impedance coefficient is $k$ times as large as the original one. As we will see that pure transformation optics can manipulate the

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impedance match without combining conventional method of inserting an antireflective coating, nor the combining of field transformation approach,\textsuperscript{23} which is a need for bianisotropic medium. To obtain the reflectionless transformation medium, it is our task to figure out an appropriate function $k$.

A 2D beam compressor of thickness $d$ has been designed as a first example, whose structure is shown in Fig. 1(a). To compress the wave propagating in the $z$ direction without reflections, a linear coordinate transformation has been done, which compresses the rectangular region in original space to trapezoidal region (the middle area “I”) in transformed space. The transformation can be easily defined as $x' = x[1 - \frac{z}{d}(1 - \gamma)]$, $y' = y$, $z' = z$, where $\gamma$ is the compression coefficient, $\gamma < 1$ is for compressor and $\gamma > 1$ is for expander, respectively. For simplicity, without the loss of generality, the surrounding medium of the embedded transformation medium is supposed to be air, and the permittivity and permeability of the original medium are set to be $\varepsilon_0/k$ and $\mu_0\mu$ in all of our discussions. Considering an incident plane wave upon the entrance boundary $z = 0$ of the compressor at an angle $\alpha$, it transmits from the exit boundary $z = d$. One can easily obtain the reflectionless condition of $k = 1$ both for TE polarization ($E$-field along $y$ axis, associated with the parameters of $\varepsilon_{yy}, \mu_{xx}, \mu_{yz} = \mu_{zy}$, and $\mu_{zz}$) and TM polarization ($H$-field along $y$ axis, associated with the parameters of $\varepsilon_{yy}, \mu_{xx}, \varepsilon_{zz} = \varepsilon_{zx}$, and $\varepsilon_{zy}$) plane waves at the entrance boundary $z = 0$, where the transformation is continuous. In order to get the reflectionless condition, we need to calculate the reflection coefficient $R$ of the waves at the exit boundary to solve appropriate coefficient $k$. Applying the continuity of the total tangential electric and magnetic fields, at the exit boundary $z = d$

$$R_{TE} = \frac{1 - \gamma k}{1 + \gamma k} \sqrt{1 + \tan^2 z \left(1 - \frac{1}{\gamma^2}\right)}$$

$$R_{TM} = \frac{1 - \gamma k}{1 + \gamma k} \sqrt{1 + \tan^2 z \left(1 - \frac{1}{\gamma^2}\right)}$$

From Eq. (2), one can obtain $k = \frac{1}{\gamma \sqrt{1 + \tan^2 z (1 - \frac{1}{\gamma^2})}}$ for $R_{TE} = 0$ and $k = \gamma \sqrt{1 + \tan^2 z (1 - \frac{1}{\gamma^2})}$ for $R_{TM} = 0$ at the exit boundary. To satisfy the reflectionless condition at both boundaries at the same time and the continuity of the transformation medium, impedance coefficient $k$ should be spatial continuous function, and can be set as

$$k_{TE} = \frac{d - z'}{d - z' \left(1 - \gamma \sqrt{1 + \tan^2 z \left(1 - \frac{1}{\gamma^2}\right)}\right)}$$

The function of $k$ is not unique, but should be easy for the realization of the transformation medium.

Note that for a vertical illumination ($z = 0$), which is the general case for many applications, our selection of function $k$ can make the parameter of relative permittivity be $\varepsilon_{yy} = 1$ for TE polarization, which is only a magnetic response. Therefore, one only needs to tune the relative permeability while keeping the permittivity unchanged, which can be easily realized in metamaterials using split-ring resonators (SRRs).\textsuperscript{3} Similarly, a dielectric response material ($\mu_{yy} = 1$) can also be obtained for TM polarization by the above function $k$, which can be realized by metal wires.\textsuperscript{24} The designed impedance-tunable compressor for vertical illumination can be transplanted to mode coupling in waveguide. Although some works\textsuperscript{25,26} have been done in the metallic waveguide using the traditionally coordinate transformation, our approach can improve the performance with a supper-high efficiency. The method can be further applied to remove reflection resulted from mode mismatch in dielectric waveguide, which is a vital problem highly desirable to be solved in fiber-to-chip coupling.

Beside a compressor, a bend is also one of the most important elements in the optical design. Using the strategy of impedance-tunable transformation optics, we propose a compact 2D beam bend design, as shown in Fig. 1(b). The bend transforms a rectangular region with a height of $h$ and a length of $d$ in the original space to trapezoidal region (the middle area “I”) with the changing height $h$ and the different lengths of $d_1$ and $d_2$ in the transformed space. The transformation is linear along $z$ axis and can be expressed by $x' = x$, $y' = y$, $z' = \frac{d_1 - d_2}{2} \frac{1}{k} (x + d_1)$. When a plane wave propagates through the transformed area along $+z$ axes, the equiphase surface will smoothly deflect an angle of $\beta = \tan^{-1} \left( \frac{d_2 - d_1}{h} \right)$, and the ray will be refracted to the air with the bending angle of $\beta$. For simplicity, a normal incident wave illuminating upon the bend is discussed here. Similar to the derivation of the compressor, the spatial continuous function $k$ for the reflectionless bend can be set as

$$k_{TE} = 1 + \frac{z'}{d_1 + x' \tan \beta} \left( \cos \beta - 1 \right)$$

$$k_{TM} = 1 + \frac{z'}{d_1 + x' \tan \beta} \left( \frac{1}{\cos \beta} - 1 \right).$$

Note that if only a single impedance-tunable bend is applied, the bending angle $\beta$ is limited from $0^\circ$ to $90^\circ$, and the size of the exit boundary of the bend is $1 \cos \beta$ times as large as that of the entrance boundary. The waves not only be bent but also be expanded with the decreasing magnitude due to the conservation of power; thus, the magnitude of the bending waves is dependent on the bending angles. Two impedance-tunable bends in series will expand the range of the bending angles from $0^\circ$ to $180^\circ$ and controlling the magnitude of the bending waves independently.
To validate our theoretical results, 2D numerical simulations using COMSOL Multiphysics are carried out to investigate the performance of the TE polarization plane waves (similar calculation for TM waves can also be done) illuminating upon the compressors/expanders (Fig. 2) and bends (Fig. 3), respectively. The calculation domain is bounded by perfectly matched layers. Figs. 2(a) and 2(b) plot the magnitude distributions of the electric field of an impedance-nontunable compressor and impedance-tunable compressor, respectively, with the compression coefficient $c = 0.25$ to a normal incidence. When the original space is set to be invariable vacuum, as shown in Fig. 2(a), reflections occur at the exit boundary of the compressor, resulting in an amplitude modulation of the incoming wave, and the simulated transmission efficiency is only 64%. Using impedance-tunable transformation optics by selecting the appropriate impedance coefficient $k$, as shown in Fig. 2(b), reflections have been suppressed with the simulated transmission efficiency near 100%. The magnitude distributions of the electric field doubles at the exit boundary of the compressor compared to that at the entrance boundary. An oblique illumination upon the impedance-nontunable compressor and the impedance-tunable compressor with $\alpha = 15^\circ$ is illustrated in Figs. 2(c) and 2(d). Note that a large compression rate ($\gamma \ll 1$) will lead to the total reflection even for moderate incidence angle of $\alpha$, so we let $\gamma = 0.5$. Fig. 2(c) shows the distorted wave profile resulting from the reflections at the boundary of the impedance-nontunable compressor. While the wave profile is undisturbed in the impedance-tunable compressor as shown in Fig. 2(d), with all the energy transmitted. A different impedance coefficient $k$ must be chosen to remove the reflections for a different incident angle $\alpha$, while for the expander ($\gamma > 1$), $k$ changes slowly with big range of $\alpha$ even for a moderate expanding $\gamma > 1$, which can be seen from Eq. (3). Fig. 2(e) shows zero reflections of the expanding waves for $\gamma = 2$ under a normal illumination ($\alpha = 0$), where the impedance coefficient $k$ equals to $\frac{d}{\alpha^2(1-\gamma)}$. Such coefficient of $k$ can also be applied to a large angle up to $30^\circ$ as shown in Fig. 2(f), where the expander shows a small amount of reflections, which has little influence on the performance.

Similar simulations of the impedance-tunable beam bends are performed for a normal TE polarization plane wave, as shown in Fig. 3. Selecting the appropriate impedance coefficient $k$, no reflections occur and all the transmission efficiencies are nearly 100%. Fig. 3(a) is a single bend with a bending angle of $60^\circ$; Figs. 3(b)–3(d) are the cases for the bending angle of $90^\circ$ of two bends connected in series. The bending waves are expanded in Fig. 3(b) and compressed in Fig. 3(c). Combining two symmetrical bends with the same bending angle in Fig. 3(d) where the transformation is continuous at the boundary, which is a special case designed in Ref. 4, the impedance coefficient $k = 1$ (the
original space is a vacuum) can satisfy the reflectionless condition. Introducing a tunable impedance, we can design reflectionless bends where direction and magnitude of the waves are tuned independently, while it cannot be realized in traditional transformation optics. Therefore, impedance-tunable transformation optics provides much more convenience in general applications of optical bends design.

As a further step, we can realize a reflectionless beam shifter by connecting the two impedance-tunable bends with the opposite bending direction and keeping the entrance boundary and the exit boundary in parallel. As shown in Fig. 4(a), despite the discontinuity in both the entrance boundary and the exit boundary, the transformation-optical design for the shifter is easily performed as well. A reflectionless beam splitter has also been realized by connecting the two impedance-tunable bends in parallel, as shown in Fig. 4(b). Combining multi impedance-tunable bends, a multi road reflectionless beam splitter can be designed with the arbitrary beam angles and energy ratios or magnitudes.

The impedance-tunable transformation optics can be applied to other transformation-optical designs including 3D cases, where the coordinate stretching rates in electric and magnetic direction are not consistent at the boundary. This method can also be easily applied to the flexible design of metric-matched elements, instead of the coefficient of $k$ keeping a constant of 1 in the traditional design, various impedance functions of $k$ with unchanged $k = 1$ at the boundary can be selected, which provides more opportunities to find realizable materials in the future.

In summary, to remove the reflections at the boundary of the transformation medium, we generalize the transformation optics by proposing a theory of impedance-tunable transformation optics, in which appropriate impedance coefficients are chosen in the original space. The method is applied to the design of 2D compressors/expanders, bends, shifters, and splitters. The numerical simulations confirmed our design with good performance.

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1. J. B. Pendry, D. Schurig, and D. R. Smith, Science 312, 1780 (2006).
2. U. Leonhardt, Science 312, 1777 (2006).
3. D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, Science 314, 977 (2006).
1 J. Huangfu, S. Xi, F. Kong, J. Zhang, H. Chen, D. Wang, B.-I. Wu, L. Ran, and J. A. Kong, J. Appl. Phys. 104, 014502 (2008).
2 D. A. Roberts, M. Rahm, J. B. Pendry, and D. R. Smith, Appl. Phys. Lett. 93, 251111 (2008).
3 W. X. Jiang, T. J. Cui, X. Y. Zhou, X. M. Yang, and Q. Cheng, Phys. Rev. E 78, 066607 (2008).
4 H. Chen and C. T. Chan, Appl. Phys. Lett. 90, 241105 (2007).
5 H. Chen, B. Hou, S. Chen, X. Ao, W. Wen, and C. T. Chan, Phys. Rev. Lett. 102, 183903 (2009).
6 W. X. Jiang, T. J. Cui, Q. Cheng, J. Y. Chin, X. M. Yang, R. Liu, and D. R. Smith, Appl. Phys. Lett. 92, 264101 (2008).
7 J. Yang, M. Huang, C. Yang, Z. Xiao, and J. Peng, Opt. Express 17, 19656 (2009).
8 D. Schurig, J. B. Pendry, and D. R. Smith, Opt. Express 15, 14772 (2007).
9 N. Kundtz and D. R. Smith, Nature Mater. 9, 129 (2010).
10 M. Rahm, S. A. Cummer, D. Schurig, J. B. Pendry, and D. R. Smith, Phys. Rev. Lett. 100, 063903 (2008).
11 M. Rahm, D. A. Roberts, J. B. Pendry, and D. R. Smith, Opt. Express 16, 11555 (2008).
12 W. X. Jiang, T. J. Cui, H. F. Ma, X. Y. Zhou, and Q. Cheng, Appl. Phys. Lett. 92, 261903 (2008).
13 D.-H. Kwon and D. H. Werner, New J. Phys. 10, 115023 (2008).
14 D.-H. Kwon and D. H. Werner, Opt. Express 17, 7807 (2009).
15 C. García-Meca, M. M. Tung, J. V. Galan, R. Ortuno, F. J. Rodríguez-Fortuno, J. Martí, and A. Martínez, Opt. Express 19, 3562 (2011).
16 C. D. Emiroglu and D.-H. Kwon, J. Appl. Phys. 107, 084502 (2010).
17 W. Cai, U. K. Chettiar, A. V. Kildishev, V. M. Shalaev, and G. W. Milton, Appl. Phys. Lett. 91, 111105 (2007).
18 D. Schurig, J. B. Pendry, and D. R. Smith, Opt. Express 14, 9794 (2006).
19 M. Born and E. Wolf, Principles of Optics (Cambridge University Press, Cambridge, 1999).
20 F. Liu, Z. Liang, and J. Li, Phys. Rev. Lett. 111, 033901 (2013).
21 W. Cai, U. K. Chettiar, A. V. Kildishev, and V. M. Shalaev, Nat. Photonics 1, 224 (2007).
22 P.-H. Tichit, S. N. Burokur, and A. Lustrac, Opt. Express 18, 767 (2010).
23 H. Y. Xu, B. Zhang, G. Barbastathis, and H. D. Sun, J. Opt. Soc. Am. B 28, 2633 (2011).