Phase-Only Synthesis for Large Planar Arrays via Zernike Polynomials and Invasive Weed Optimization

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Abstract—A synthesis method for the design of large planar array antennas with phase-only control is here presented. The synthesis is based on the use of Zernike polynomials, as global basis function for the phase, to reduce the number of optimization variables with respect to the number of elements of the array. Invasive weed optimization (IWO) is applied to polynomial coefficients’ optimization to circumvent non-linearity and local trapping issues typical of phase-only problems. The periodicity of the array factor is exploited to reduce the optimization to a Voroni cell of the grating-lobes’ lattice and non-uniform meshing is introduced to best adapt the control stations to the beam shape requirements. The technique is applied to the design of shaped beams for continental coverage from geostationary satellites.

Index Terms—Beam shaping, evolutionary optimization algorithm, large phased arrays, phase-only synthesis

I. INTRODUCTION

SPACEBORNE antennas have always been on the edge of technology, needing extreme performances in terms of beam shaping, efficiency, and reliability. In the continuous evolution of satellite communication systems active array antennas have become the preferred configuration being capable to adapt the antenna performance to the mission needs which evolve during the satellite lifetime [1].

Active arrays have distributed amplification (i.e. typically one amplifier per radiator or per subarray) and offer several advantages, among them full power allocation flexibility in transmission and re-configurable generation of multiple beams.

The beams to be generated often need to cover a specific geographical coverage with gain requirements within the region of interest and isolation requirements in areas reuseing the same satellite resources (i.e. frequency and polarization). Several amplitude and phase optimization methods are known in the literature and could be used for this purpose. Nevertheless, in transmitting arrays, phase-only optimization can provide maximum amplifier efficiency and is attracting growing interest.

A non-exhaustive list of phase-only optimization approaches addressing all the individual phases of the array include gradient methods [2], [3], [19], integer programming [5], alternate projections [6], more recently semi-definite programming relaxations [7], the steepest descent method [8], [9], Taguchi method [10] and the non uniform FFT [11]. Furthermore, for these tough problems, evolutionary algorithm as particle swarm [12], [13], bat algorithm [14] or cross entropy [15] are usually preferred since they perform a global optimization, and do not easily get trapped in local minima, as deterministic techniques do, especially in complex non-convex optimization problems as the one faced here. These techniques consent optimization of spot or contoured beams (which are necessary for broadcasting applications) with control of gain in the beam and sidelobes in co-frequentional areas.

Main limitation of phase-only stands in the reduced space of the degrees of freedom (all the excitations have the same amplitude) and in the high non-linearity of the optimization problem which may trap the algorithm in largely sub-optimal local minima. Additionally, the large increase of the number of elements in future active antennas requires paying attention to the computational aspects and efficiency of the optimization methods. As an example, phased arrays antennas in the Ka frequency band can have dimensions larger than 2 meters and can be composed by more than 1000 elements.

It is important to note that other antenna architectures have been also adopted to generate shaped beams. In particular, shaped reflector antennas have been successfully used in several missions [16]. An excellent overview of synthesis techniques for this problem is offered in [17]. While simplified synthesis techniques exploiting the first-order approximation of the surface variation as a phase variation were initially explored in [18], more accurate synthesis tools had to be developed mainly due to the coupling of the amplitude and phase in the projected aperture field which forbids the problem to be considered as a strictly phase-only optimization problem [19].

Part of the complexity and cost associated to the manufacturing of shaped reflectors can be overcome by reflectarray antennas. They are typically based on flat panels and to modify the required coverage only the dimensions of printed elements are varied [20], [21] with advantages in terms of lead time.

Active phased arrays represent a further evolution because they permit reconfiguring the coverage when the antenna is operational; on the other hand, the beamforming network

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tend to be complex and the power efficiency and thermal management is critical. For these reasons, especially for a phased array to be embarked on a satellite with limited power resources, phase-only excitation represent a suitable solution. In this respect equi-amplitude excitation have been proposed together with aperiodic placement of the array elements for generating shaped beams [22-23]. The objective of the paper is to present a design method for very large phased array antennas based on Zernike polynomials and the Invasive Weed optimization (IWO) evolutionary algorithm.

Zernike polynomials will be used to represent an auxiliary continuous wave-front which can be sampled to obtain the array phase excitations. The truncation order of the Zernike representation substantially limits the number of variables to be optimized with respect to the number of elements and allows obtaining a relatively smooth phase wave-front while a drastic variability is usually observed when implementing a direct optimization of the phase excitation variables. The use of Zernike polynomials is well established in optical aberration analysis [24] and has been used in shaped-reflector synthesis to represent the reflector metallic surface deviation with respect to a nominal conic profile [17].

Early examples of the use of global basis functions for phase optimization problems can be found in [18], in relation to shaped reflector synthesis, and in [25], in relation to linear phased arrays, respectively. Notwithstanding the reduction in the number of optimization variables, the synthesis problem remains highly non linear, non convex, and subject to local traps. Previous studies in electromagnetics [26], applications to reflector antennas [27], [28] or to arrays given fixed amplitude tapering and optimizing for phase [29] proved IWO validity in electromagnetics. A more recent paper [30] on phase-only synthesis of linear arrays, showed that with phase distribution expressed in terms of a reduced set of full-domain basis functions in order to decrease the variables to be optimized, IWO outperformed other state-of-the-art evolutionary algorithms like particle swarm or grey wolf optimizer.

The method proposed in this paper can be implemented for the design of phased array antennas generating spot beams and shaped beams. The paper is organized as follows: Section II describes the problem geometry with regards to planar array in the Voronoi space, in Section III the expansion of the phase distribution as a combination of basis function is presented, while in Section IV the IWO optimization algorithm together with implemented cost function and optimization parameters are sketched. In Section V the main results are presented for several cases from the easiest pencil beam to the most complex shaped and irregular coverages.

II. PROBLEM GEOMETRY

An array of isotropic sources will be considered, to keep tractation simpler. The element factor, which, anyway, has a reduced impact on large arrays, can be added trivially to all subsequent reasonings. If the array elements are arranged periodically, the periodicity of their array factor can be exploited to limit the control points of the radiation pattern (i.e. the points where the radiation pattern need to be evaluated and controlled) to a single periodic cell in the Fourier domain which corresponds to the first Brillouin zone (or equivalently to the Voronoi cell of the main beam in the grating-lobes’ lattice) [31]. The mathematical details are elaborated in the following.

The array lattice geometry is fully described by two linearly independent lattice vectors \( \mathbf{d}_1, \mathbf{d}_2 \), usually arranged in a single matrix \( \mathbf{D} = [\mathbf{d}_1, \mathbf{d}_2] \).

The set of all linear combinations of \( \mathbf{d}_1, \mathbf{d}_2 \) with integer coefficients defines a bi-dimensional lattice \( \mathbf{L}(\mathbf{D}) \) (Fig. 1).

By a bi-dimensional Fourier transform of the periodic lattice in the direct space \( \mathbf{D} \), a dual lattice is obtained, having a base \( \mathbf{G} \) given by:

\[
\mathbf{G} = (\mathbf{D}^{-1})^T
\]

The direct space periodicity matrix, \( \mathbf{D} \), and the Fourier domain periodicity matrix, \( \mathbf{G} \), indeed satisfy an orthogonality condition, \( \mathbf{D} \cdot \mathbf{G}^T = \mathbf{I} \), where \( \mathbf{I} \) represents the identity matrix. For a cophasal array the lattice \( \mathbf{L}(\mathbf{G}) \) corresponds to the periodic lattice of the main-lobe and of all grating lobes in the \((u, v)\) space (Fig. 2).

Indeed, if a Cartesian reference is used for \( \mathbf{d}_1, \mathbf{d}_2 \) and \( \hat{\mathbf{r}} = \sin(\theta) \cos(\phi) \hat{x} + \sin(\theta) \sin(\phi) \hat{y} + \cos(\theta) \hat{z} \) is the unit vector of the observation direction, being \((\theta, \phi)\) the angles of a spherical reference, then the array factor can be expressed as:

\[
P(\hat{\mathbf{r}}) = a \sum_{p=1}^{N_1} \sum_{q=1}^{N_2^+(p)} e^{j \frac{2\pi}{\lambda} [pd_1 \cdot \hat{\mathbf{r}} + qd_2 \cdot \hat{\mathbf{r}}]} e^{j \Phi_{pq}}
\]

having \( N_1 \) elements along \( \mathbf{d}_1 \) and \( N_2^+(p) - N_2^-(p) \) along \( \mathbf{d}_2 \). If \( N_2^- \) and \( N_2^+ \) are not a function of \( p \), then the array is a parallelogram, otherwise it can be of any shape, for example, circular as in our case. Amplitude \( a \) is the same for all elements, and \( \Phi_{pq} \) are the phases of every single element. The pattern, since \( \mathbf{d}_1 \) and \( \mathbf{d}_2 \) are on the \( xy \) plane, is more commonly expressed in the \((u, v)\) plane, defined by:

\[
\begin{align*}
  u &= \sin(\theta) \cos(\phi) \\
  v &= \sin(\theta) \sin(\phi)
\end{align*}
\]

as:

\[
P(u, v) = a \sum_{p=1}^{N_1} \sum_{q=N_2^-}^{N_2^+(p)} e^{j \frac{2\pi}{\lambda} [pd_1 + qd_2]} |u \hat{x} + v \hat{y}| e^{j \Phi_{pq}}
\]

The array factor \( P(u, v) \) is periodic in the \((u, v)\) plane with periodicity matrix \( \mathbf{G} \). The main beam and all the grating lobes can be considered the center of periodic cells determined by the Voronoi tessellation of \( \mathbf{L}(\mathbf{G}) \) (refer to Fig. 2). Due to the periodicity of the array factor, we can limit the evaluation of the pattern and its cost function to the Voronoi cell of the main beam which corresponds to the first Brillouin zone (i.e. the locus of points in reciprocal space that are closer to the origin of the reciprocal lattice). The phases \( \Phi_{pq} \) in (4) are the only degrees of freedom. They can be independent, and each is an optimization variable, in a so-called “brute force” approach; or they can be expressed as the sampling of a continuous function defined as a linear combination of a reduced number of basis functions \( S \), as detailed below.
Fig. 1: Planar array lattice in direct space.

Fig. 2: Planar array lattice in dual space with pertinent Voronoi tessellation. In green the Voronoi cell relative to the main beam.

Fig. 3: Triangular lattice with \( d_1 = [1, 0.5], d_2 = [0, \sqrt{3}/2] \) for an hexagonal array and relevant far-field Voronoi Cell

From the relation among lattices it is apparent how the closer the elements in the direct space, the farther away the grating lobes from the main lobe in the dual space.

In satellite applications, the main point is not to suppress grating lobes, but to guarantee that no grating lobe falls in the field of view (FoV) of the Earth as seen from the satellite.

Considering the limited FoV of the Earth, that for a geostationary satellite is of few degrees, a large distance between elements is possible. Indeed, if the Voronoi tessellation of the dual space is performed, and assuming the Earth FoV is within the Voronoi cell of the broadside beam, then it is also guaranteed that, by electronically scanning within the Earth FoV, and hence within the Voronoi cell, no grating lobe will ever enter the cell to cause undesired Earth illumination (Fig. 2).

In Fig. 3, direct space (top) and main beam Voronoi cell in the dual space (bottom) are shown for a triangular equilateral distribution of the radiating elements. Within the main beam Voronoi cell (black filled hexagon in the figure) a target beam contour can be defined (e.g. the red polygon representing an European coverage as seen by a Geostationary satellite). The figure also shows how the dual space can be sampled (far-field sampling grid) and how the cost function can be limited to the sampling points falling within the Voronoi cell (black dots). The mesh of points where the array factor is sampled for the evaluation of the cost function can be either structured (e.g. bottom of Fig. 3) or unstructured (e.g. in FEM-like discretization of Fig. 9).

Depending on the array lattice base, inter-element distance can be varied, as in Table I, in order to guarantee that the Voronoi cell is larger than the Earth FoV.

| TABLE I: Field of view for triangular and squared lattices |
|----------------------------------------------------------|
| **Triangular**   | **d = 3\lambda** | **d = 2\lambda** | **d = 1\lambda** | **d = 0.5\lambda** |
| \([u_{\min}, u_{\max}]\) | ±0.22 | ±0.33 | ±0.67 | ±1.33 |
| \([v_{\min}, v_{\max}]\) | ±0.19 | ±0.29 | ±0.58 | ±1.15 |
| **Square**       | **d = 3\lambda** | **d = 2\lambda** | **d = 1\lambda** | **d = 0.5\lambda** |
| \([u_{\min}, u_{\max}]\) | ±0.17 | ±0.26 | ±0.5 | ±1 |
| \([v_{\min}, v_{\max}]\) | ±0.17 | ±0.26 | ±0.5 | ±1 |

III. Phase Expansion into Zernike Polynomials

As stated earlier, phases \( \Phi_{pq} \) shall not be optimized independently but rather seen as a sampling of a continuous phase distribution described in terms of a limited number of full-domain polynomial bases.

In this paper Zernike polynomials will be used. Even if “polynomials” is the usual way of addressing Zernike...
functions, these are not polynomials strictly speaking since
trigonometric functions are present in their definition. These
form a class of orthogonal functions over the continuous unit
circle (see [32], [33]) and are particularly suitable for this
application, since the unit circle maps naturally with just a
radial degree over a circular array.

Indeed the radial-azimuth dependence of Zernike poly-
nomials, as opposed to 2D polynomials resulting by the product
of \(x\)—dependant and \(y\)—dependant polynomials in a Cartesian
reference, makes them the optimal choice when dealing with a
circular array since the domain is exactly covered. Other bases,
like Bessel functions, exhibit the same symmetry with no
additional advantage but at a higher computational complexity.

In polar coordinates, Zernike function \(Z_n^m(\rho, \theta)\) is expressed as:

\[
Z_n^m(\rho, \theta) = \begin{cases} 
N_n^m R_n^m(\rho) \cos(m\theta) & \text{if } m \geq 0 \\
-N_n^m R_n^m(\rho) \sin(m\theta) & \text{if } m < 0 
\end{cases}
\]  

being \(N_n^m\) a normalization factor and \(R_n^m(\rho)\) a polynomial in \(\rho\). In detail:

\[
N_n^m = \sqrt{\frac{2(n + 1)}{1 + \delta_n^m}} 
\]  

being \(\delta_n^m\) the Kronecker delta function (i.e. \(\delta_n^0 = 1\) for \(m = 0\),
and \(\delta_n^m = 0\) for \(m \neq 0\), and):

\[
R_n^m(\rho) = \sum_{k=0}^{\frac{n-m}{2}} \frac{(-1)^k (n-k)!}{k!(\frac{n-m}{2}-k)!} \frac{1}{(\frac{n+m}{2}-k)!} \rho^{n-2k} 
\]  

for \(n - m\) even, and zero otherwise

In Zernike functions definition, \(n\) is a positive integer named
the radial degree or order, and \(m\), named the azimuthal or
angular frequency, is a negative or positive integer such that
\(|m| \leq n\).

A continuous phase distribution is then defined as:

\[
\Phi(\rho, \theta) = \sum_{n,m} \alpha_n^m Z_n^m \left(\frac{\rho}{r}, \theta\right) 
\]  

being \(r\) the radius of the circle circumscribed to the array. The
discrete phases \(\Phi_{pq}\) in (1) are hence samples of \(\Phi\):

\[
\Phi_{pq} = \Phi \left(\|p\mathbf{d}_1 + q\mathbf{d}_2\|, \tan^{-1}\left(\frac{p\mathbf{d}_1 + q\mathbf{d}_2}{\|p\mathbf{d}_1 + q\mathbf{d}_2\|} \cdot \hat{\alpha}\right)\right) 
\]  

For the sake of simplicity the two indexes \(n, m\) are replaced
by a single-index \(k\):

\[
\Phi_{pq}(\rho, \theta) = \sum_{k=0}^{K} \alpha_k Z_k \left(\frac{\rho}{r}, \theta\right) 
\]  

To obtain the single index \(k\), it is convenient to arrange the
polynomials in a pyramid with row number \(n\) and column
number \(m\). It will be \(k = 1\) for \(n = 0, m = 0\) and the index
increases from top to bottom and from left to right (Fig. 4):

\[
k = \frac{n(n+2) + m}{2} \Leftrightarrow \begin{cases} 
n = \left\lfloor (\sqrt{8k+9} - 3) \right\rfloor \\
m = 2k - n(n+2) 
\end{cases}
\]  

Obviously, if

\[
N = \sum_{p=1}^{N_1} (N_2^+(p) - N_2^-(p)) 
\]  

is the total number of elements in the array, if \(K \ll N\) the
optimization will have a much lower number of variables to
work on, with respect to a brute force approach.

Furthermore, additional reductions can be obtained if sym-
metries in the desired patterns are present, by excluding
Zernike functions non complying to said symmetry. For ex-
ample if a broadside beam independent of \(\phi\) is desired then
only \(m = 0\) Zernike functions are necessary.

IV. OPTIMIZATION IMPLEMENTATION

There are two main key points in a numeric optimization
strategy: the definition of the cost function to be minimized
and the choice of the optimization algorithm. In our case a
third key point, related to the selection of an appropriate sub-
set of the complete Zernike basis set is analyzed.

A. Cost function

Cost function for the optimization procedure is defined in
terms of two or more masks to which the pattern must comply.
The simplest presented here is for a wide-angle uniform
circular beam, while the most complex is aimed at a South
America coverage with different pattern levels on different
countries.

Hence, in general, several masks \(C^{(i)}\) are defined with a
desired gain level for each zone within the coverage, plus an
outer mask \(O\) for controlling the side lobe maximum level.
Between the SLL mask and the coverage mask(s) a transition
zone in which the gain is unchecked allows faster convergence
(Fig. 5).
Pattern is sampled uniformly or non-uniformly within the main Voronoi cell to check for out-of-mask levels. Sampling points in \((u, v)\) plane are indexed with a single index \(s = 1, \ldots, S\), sampled pattern values are \(P_s = P(u_s, v_s)\) and the same notation is used for mask samples \(C_s^{(t)}(u_s, v_s)\) and \(O_s = O(u_s, v_s)\).

With this notation the cost function is:

\[
\begin{align*}
    c &= \sum_{t=1}^{T} \left[ \sum_{s=1}^{S} \eta_1^{(t)} |P_s - C_s^{(t)}| \delta_{P_s} + R^{(t)} \right] \\
    &+ \sum_{s=1}^{S} \eta_2 |P_s - O_s| \delta_{O_s} 
\end{align*}
\]

(13)

where:
- \(\delta_{P_s}^{a,b,c} = 1\) if \(a < b - c\) or \(a > b + c\) or if point \(q\) is not within the current mask definition area;
- \(\delta_{P_s}^{a,b,c} = 1\) if \(a < b\) and zero otherwise or if point \(q\) is not within the current mask definition area;
- \(\eta_1^{(t)}\) and \(\eta_2\) are appropriate weights of the coverage and side lobe masks, taken inversely proportional to the number of evaluation points within each sub-area.

The cost function is evaluated on the normalized pattern and evaluated in logarithmic scale. The coverage mask allows for a dynamic \(R^{(t)}\) within each coverage mask.

### B. IWO algorithm

Optimization is then performed by a single-objective implementation of the IWO algorithm [26], [34], [36]. Such an algorithm relies on the paradigm of an invasive weed specie colonizing a field, producing seeds proportionally to the richness of the soil (how good the cost function value is at the point where the plant grows) and spreading these seeds around its location with a Gaussian distribution.

The algorithm has a few control parameters:
- \(P_i\) : initial number of plants, randomly scattered on the domain;
- \(P_m\) : maximum number of plants. To avoid exponential grow of plants, worse specimens are discarded, so as to maintain this maximum number of plants at next generation.
- \(M_i\) : maximum number of iterations;
- \(M_s\) : stall; if the optimal solution is not updated within this number of steps the algorithm ends (with a warning) even if \(M_i\) is not reached;
- \(S_{min}\) : minimum number of seeds a plant produces at each iteration;
- \(S_{max}\) : maximum number of seeds a plant produces at each iteration;
- \(\sigma_i\) : initial variance for seed dispersion;
- \(\sigma_f\) : final variance (at iteration \(M_{iter}\)) for seed dispersion; variance is decreased linearly with iteration number;

Optimization variables are internally rescaled in the IWO algorithm to \([0, 1]\) so that \(\sigma_i\) and \(\sigma_f\) are not problem-dependent.

Details on the algorithm can be found also in [35], [37], [38]. It is worth mentioning that in [30] an accurate comparison between different optimization techniques was carried out on the problem of the phase-only synthesis of linear arrays and IWO over-performed the competitors. Hence it is used here.

### C. Criteria of polynomials selection

While in the linear case [30] a symmetrical mask would call for symmetrical phases and hence the number of basis functions at a given order could be reduced by selecting only even basis, in the planar case, we must distinguish patterns with a rotational symmetry, which would call for the same symmetry in the phases and hence only \(Z_{2n}^m\) functions and non-rotationally symmetrical masks which will need, in principle, all \(Z_{2n}^m\). In practice, if the beam is rotationally symmetric but not pointing broadside, just the two \(Z_{2n}^m\) functions, can be added since they allow for a linear phase and hence for the beam scanning to a generic \((u_0, v_0)\).

For what concerns the maximum radial degree \(n\) to be chosen, preliminary studies in [30] showed that the key issue is allowing enough variation for the phase, as a rule of thumb one full variation (from maximum to minimum or vice versa) in a \(4\lambda\) radial direction, as the following example will show. Variation of \(\alpha_k\) coefficients is constrained within an allowable range \([UB_k, LB_k]\), which, also according to the previous experience [30], are decreasing with the square of the radial order of the considered Zernike polynomials:

\[
UB_k, LB_k = \pm \frac{\Phi_{max}}{\alpha_k} \approx \frac{1}{n^2} \quad (14)
\]

with \(k\) the single index given by (11) and \(\Phi_{max}\) selected taking into account the overall array dimension and maximum order, as analyzed in the next section.

### V. Numerical Results

A list of results is presented for planar arrays using the combination of the Zernike functions for the IWO synthesis. Three cases are here presented, from the easiest case of a broadside wide-angle pencil beam followed by two more challenging masks: an Europe uniform coverage and a South America non-uniform coverage.

Table II shows IWO parameters for the three cases. \(P_i\), \(P_m\), \(S_{min}\), and \(S_{max}\) are chosen, according to the authors’
A. Pencil beam coverages

Firstly, a planar array with triangular lattice of diameter $D = 30\lambda$ and element spacing $d = 3\lambda$ with an internal circular mask centred in zero with radius $r_C = 0.05$ (which corresponds to an angular aperture $\theta_C = 2.87^\circ$) and an external concentric mask with radius $r_O = 0.06$ (an angular aperture $\theta_O = 3.43^\circ$) is considered. Only $Z_0^k$ polynomials are used, due to symmetry, and in particular those for $k = 5, 13, 25, 41$. Note that, since $Z_0^k$ would give a uniform phase distribution, that is a simple bias, the base for $k = 1$ is never considered in the optimization. Here $\eta_1 = 0.7, \eta_2 = 0.3$ and $\Phi_{\text{max}} = 8\pi$ are considered.

Patterns are presented in Fig. 6 in $(u, v)$ plane by selecting an appropriate parameter $w$ as linear coordinate along the cuts $(w = u$ and $w = v$ in the principal planes for the broadside pattern). The optimal $\alpha_k$ are reported in Table III, row (a). Due to the negligible amplitude of the $k = 25$ and $k = 38$ coefficients, a separate optimization with only two variables is also carried out, leading to the values in the row (b) of Table III. Anyway four coefficients produce a better pattern as compared to only two, as Fig. 6 clearly shows, while further coefficients do not significantly improve the compliance to the mask, while lengthening computational time and making convergence slower.

B. Irregular coverages

Focusing on irregular coverages, both even and odd polynomials are investigated in the synthesis of the pattern with the optimized coefficients within the phase distribution.

1) Typical European Coverage: the typical European coverage has a polygon defined in $(u, v)$ space for the internal mask, $w_{\text{poly}}$. The external mask is constructed by enlarging the internal mask in all direction by a quantity $\Delta = 0.01$ to allow for a transition zone (Fig. 7, where the internal mask is black and the external mask red). The weights in (13) are: $\eta_1 = 0.7, \eta_2 = 0.3$ and $\Phi_{\text{max}} = 8\pi$.

The analyzed planar array with triangular lattice has a diameter of $D = 60\lambda$ and intra-element spacing $d = 3\lambda$, $N_{el} = 363$ elements.

The amplitude of computed coefficients versus the Zernike order functions and their combination in the phase distribution is hence displayed in Table IV. There is still an important advantage in the reduction of the computational complexity as optimization variables are:

- 10 optimized weights with Zernike,
- 363 optimized weights with Brute Force technique.

| TABLE II: Invasive weed optimization parameters. |
|--------------------------------------------------|
| **Pencil beam** | **Europe** | **South America** |
| $P_i$ | 20 | 20 | 30 |
| $P_m$ | 50 | 50 | 60 |
| $M_1$ | 100 | 300 | 500 |
| $S_{\min}$ | 1 | 1 | 1 |
| $S_{\max}$ | 5 | 5 | 5 |
| $\sigma_j$ | 0.1 | 0.1 | 0.1 |
| $\sigma_j$ | 0.001 | 0.001 | 0.001 |

| TABLE III: Optimized amplitude coefficients for the pencil beam. |
|---------------------------------------------------------------|
| $\alpha_k$ | $k$ | $(a)$ | $(b)$ |
| 2 | 3 | 5 | 13 | 25 | 38 |
| 2.876 | 1.452 | -0.401 | -0.382 |
| 2.600 | 1.496 | - | - |

Fig. 6: Synthesis of the broadside pencil beam. Top: normalized gain on the $(u, v)$ plane, internal mask is defined by the black circle, external mask by the red circle; bottom, normalized gain on the principal planes (blue and red lines on top map).

| TABLE IV: Optimized amplitude coefficients for the European coverage. |
|------------------------------------------------------------------|
| $k$ | 2 | 3 | 4 | 5 | 6 |
| $\alpha_k$ | -19.847 | 4.719 | 1.116 | 2.591 | 2.098 |
| $k$ | 11 | 12 | 13 | 14 | 15 |
| $\alpha_k$ | -0.204 | -0.038 | -0.783 | -0.006 | -0.012 |
2) Multi-polygonal of South-America Test Case: The final challenging problem is to guarantee a multi-gain coverage of the regions of South-America exploiting the phase-only tapering, as in Fig. 8. Gain requirements, summarized in Table V are relative to an existing shaped reflector realized by AIRBUS currently on orbit. The same problem has been addressed by synthesizing a reflectarray comprising 6944 three by three elements. Within each coverage area a dynamic gain on neighbouring zones, as well as a maximum value of just 199 or 177 active elements. The results obtained shows a good matching with earlier results presented in literature, but using much less elements for the present array. Fig. 10 shows synthesized pattern and phase distribution by an array of D = 45λ diameter and distance between elements d = 3λ considering a triangular lattice (N_{elf} = 199) for the conventional structured discretization in the (u, v) plane. In this case the element factor is either with a $\cos^2(\theta)$ [41] or a more realistic Bessel-like pattern of an external polygon.

Indeed attaining simultaneously several desired levels of gain on neighbouring zones, as well as a maximum value of −20dB, with respect to the maximum, for the SLL out-of-coverage is extremely challenging. Although a transition zone is defined between the main beam and the out of coverage area, no transition is foreseen in the requirements between different areas of the coverage, this too makes the problem really tough. To ease convergence the cost function implemented is an extension of the (13) attained as a sum over $T = 6 + 1$ areas of single costs. Within each coverage area a dynamic with respect to the nominal value, $R^{(s)} = −1$dB is tolerated and the level of side lobes imposed is SLL = −20 dB.

Indeed the upper cap to gain could be avoided, leading to an easier optimization, but this would reflect into a lower efficiency of the system since, to guarantee the minimum power level in the lowest-illuminated zones, an excess of power would be delivered to zones where gain is unneededly high, hence requiring higher than necessary output power from the transmitter.

Each contribution of the cost function in this case is weighted proportionally to the number of evaluation points belonging to the area, hence achieving an error which is consistent between areas. As for the previous analysis on planar arrays, the discretization of the evaluation points for the pattern could be done using a regular grid in the (u, v) plane. Another approach is possible, and used here, that to define sampling points on an unstructured grid which is finer in critical areas and coarser in less-critical ones, for example out of coverage.

Optimization is computed by evaluating the cost function either on a structured (regular, square) and a unstructured mesh of points, this latter based on a FEM-like mesh of the domain [39], which allows for a coarser sampling in the SLL region and hence faster optimization (Fig. 9). Distance between elements is either 2λ or 3λ, in both cases grating lobes fall outside Earth and, of course, the latter requires less radiating elements for a given aperture area.

To ease convergence the cost function implemented is an extension of the (13) attained as a sum over $T = 6 + 1$ areas of single costs. Within each coverage area a dynamic gain on neighbouring zones, as well as a maximum value of just 199 or 177 active elements.
uniform aperture of radius $r_B$. The optimization has been addressed exploiting only 27 optimized coefficients related to the Zernike functions from $k = 2$ up to $k = 28$ and attaining the values in Table VI. In Table VII the compliances within each areas, defined as the percentage of sampling points where gain is equal or greater than the required, are given.

Using the unstructured discretization in the $(u, v)$ plane, and for an array of $D = 45\lambda$ diameter and distance between elements $d = 3\lambda$ but with a square lattice of only ($N_{\text{el}} = 177$), the performances in Fig. 11 are attained.

The number of coefficients used for the expansion is 20 (so from $k = 2$ to $k = 21$). The amplitude of the optimized coefficients are displayed in Table VIII.

Compliances for the unstructured discretization are presented in Table IX. Finally Table X gives computing time for an optimization run. The relative speed up linked to the limited number of samples, concentrated in critical area, is evident.

It is interesting to note how the unstructured lattice gives both better results and lower CPU times. This because pattern sampling points are better distributed: more concentrated in the coverage zone, and less concentrated in the SLL areas.

![FEM-like discretization of the $(u, v)$ plane for the multi-gain problem cost function evaluation.](image1)

![Synthesized denormalized gain and phase distribution using the structured discretization with element factor $\cos^2(\theta)$.](image2)

### VI. Conclusion

A novel synthesis of planar arrays exploiting the phase-only technique with an efficient number of basis functions within the phase distribution has been investigated. Considering that only phase tapering has been exploited, the results for shaped coverages and multi-gain problems are very satisfying. A reduced number of basis functions has been combined to synthesize the phase distribution in order to reduce the computational time in the optimization process. The proposed design procedure is particularly effective in the case of complex coverages and large arrays as the ones considered for satellite applications.

#### TABLE VI: Optimized amplitude coefficients for the South America coverage and structured discretization.

| $k$ | 2   | 3   | 4   | 5   | 6   | 7   |
|-----|-----|-----|-----|-----|-----|-----|
| $\alpha_k$ | 3.869 | 2.290 | 1.506 | -3.344 | 1.311 | -0.097 |

| $k$ | 8   | 9   | 10  | 11  | 12  | 13  |
|-----|-----|-----|-----|-----|-----|-----|
| $\alpha_k$ | 0.330 | -0.497 | 0.451 | 0.377 | -0.412 | 0.662 |

#### TABLE VII: Compliances of the multi-gain problem with structured discretization.

| Coefficients $\cos^2(\theta)$ (%) | $d = 3\lambda$ | $d = 2\lambda$ |
|-----------------------------------|----------------|----------------|
| $r_B = 1.3\lambda$ | 100.0 | 99.9 | 97.6 | 100.0 | 100.0 | 97.46 |
| $r_B = 0.8\lambda$ | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |

| Compliances Bessel (%) | $r_B = 1.3\lambda$ | $r_B = 0.8\lambda$ |
|------------------------|---------------------|---------------------|
| $d = 3\lambda$ | 100.0 | 99.9 | 97.6 | 100.0 | 100.0 | 97.46 |
| $d = 2\lambda$ | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |

| $r_B = 1.3\lambda$ | 100.0 | 99.9 | 97.6 | 100.0 | 100.0 | 97.46 |
| $r_B = 0.8\lambda$ | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 99.84 |
TABLE VIII: Optimized amplitude coefficients for the South America coverage and unstructured discretization.

| $k$ | $\alpha_k$ |
|-----|------------|
| 1   | 5.138      |
| 2   | -0.453     |
| 3   | 8          |
| 4   | 14         |
| 5   | 20         |
| 6   | 0.569      |
| 7   | 0.073      |

TABLE IX: Compliances of the multi-goal problem with unstructured discretization.

| Compliance $\cos^2(\theta)$ (%) | SA1 | SA2 | SB | SC1 | SC2 | SD |
|---------------------------------|-----|-----|----|-----|-----|----|
| $d = 3\lambda$                 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 98.82 |
| $d = 2\lambda$                 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |

| Compliance $Bessel$ (%)        | SA1 | SA2 | SB | SC1 | SC2 | SD |
|--------------------------------|-----|-----|----|-----|-----|----|
| $\rho_B = 1.3\lambda$ $d = 3\lambda$ | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 98.66 |
| $\rho_B = 0.8$ $d = 2\lambda$     | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 99.95 |

TABLE X: Computing times.

| Type          | Structured | Unstructured |
|---------------|------------|--------------|
| Samples       | 10000      | 2356         |
| Iterations    | 500        | 500          |
| Time          | 117m55s    | 29m51s       |

Fig. 11: Synthesized denormalized gain and phase distribution using the unstructured discretization (with element factor $\cos^2(\theta)$).

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