Mass ratio of composite Higgs bosons from bottom-up holographic Wilson loop modeling of beyond the Standard Model strong sector

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Abstract

It is often assumed that the electroweak symmetry breaking is driven by some confining “strong sector” beyond the Standard Model and that the Higgs boson arises from this sector as a composite particle. The underlying strongly coupled gauge theory may be similar to QCD and could be modeled holographically. We construct a bottom-up holographic model for description of the spectrum of composite Higgs particles. The model is based on the holographic Soft Wall model and the Wilson confinement criterion. The constructed model predicts the mass of the second Higgs boson near $0.5 \text{ TeV}$ that coincides with the maximal branching ratio for decay of SM like Higgs boson into the $t\bar{t}$ pair.

1 Introduction

The Standard Model (SM) of particle physics postulates the scalar Higgs boson as a fundamental particle. This theory, however, contains many input parameters and for this reason is known to be rather phenomenological. It does not explain, for instance, the dynamical origin of the electroweak symmetry breaking and the observed value of Higgs mass. There is an old but still attractive idea that the Higgs boson might represent a bound state of a new, beyond the SM (BSM) strongly-interacting dynamics not much above the weak scale (see, e.g., the review [1]). This could solve the SM hierarchy problem, as quantum corrections to its mass would be saturated at the compositeness scale. Considerable theoretical progress in constructing such dynamical models was made from the possible holographic connection between gravity in five-dimensional curved space-times and four-dimensional strongly-coupled gauge theories [2]. The holographic approach was originally inspired by the AdS/CFT correspondence in string theory [3–5] but turned out to be unexpectedly successful in various areas of physics where the problem of strong coupling arises.

In the present work, we propose a new bottom-up holographic model that predicts the mass of the second Higgs particle. We will apply to the BSM Higgs sector the recent ideas put forward in Ref. [6] for holographic description of the scalar mesons.

By assumption, the BSM strongly-interacting dynamics above the weak scale 246 GeV is described by some strongly coupled field theory with the gauge group $SU(N)$ or related to $SU(N)$. The theory is analogous to QCD — it is confining and, what is especially important for us, does not change much if the large-$N$ limit is taken. The large-$N$ limit is well known in QCD [7,8]. In this limit, the only singularities of the two-point correlation function of a hadron...
current operator $O$ are one-hadron states \cite{8}. In the case of mesons, the two-point correlator has
the following structure in the momentum space,
\begin{equation}
\langle O(q)O(-q) \rangle = \sum_{n=1}^{\infty} \frac{F_n^2}{q^2 - M_n^2 + i\varepsilon},
\end{equation}
where the large-$N$ scaling of appearing quantities is: $M_n = \mathcal{O}(1)$ for masses, $F_n^2 = \langle 0|O|n \rangle^2 = \mathcal{O}(N)$ for residues, $\Gamma = \mathcal{O}(1/N)$ for the full decay width \cite{8}. The last scale tells us that the
large-$N$ limit describes the zero-width approximation. According to the principles of AdS/CFT
correspondence \cite{3-5}, the large-$N$ limit is necessary for building a holographic dual theory. In
the practical holographic models, the infinite sum over radially excited states in (1) is identified
with the infinite tower of 4D Kaluza-Klein excitations of a 5D field. This identification, however,
has certain problems \cite{9,10}. A physically motivated way for description of the radially excited
hadrons is to introduce operators of higher dimensions which correspond to different 5D fields
in dual theory \cite{10}. This path (additional higher dimensional operators) is used in QCD lattice
calculations of radial hadron spectra. The particle states corresponding to $n > 1$ in (1) should be
then discarded as redundant solutions, more strictly, they should be replaced by “perturbative
continuum”. Exactly this ansatz “one infinitely narrow resonance + perturbative continuum”
has been successfully used in QCD sum rules since the late 1970s \cite{11}. The physical reason is
that the resonance width grows with mass and in practice the excited states overlap so strongly
that often become almost indistinguishable from the perturbative background. As the BSM field
theory is expected to be more strongly coupled than QCD, the resonances should be much wider
in this theory, thus the approximation “one resonance + smooth background” should be even
more justified.

A naive bottom-up holographic five-dimensional description of Higgs sector inevitably pre-
dicts an infinite tower of “radially excited” Higgs particles\footnote{A possible exception could consist in consideration of specific holographic models describing a finite number of normalizable modes, such an example was constructed in Ref. \cite{12}.}. Following the discussion above, we
will consider only the ground state as a physical one, other states corresponding to $n > 1$ will
be discarded as an artifact of the large-$N$ approximation.

If the BSM strongly coupled theory is similar to QCD, it should lead to linearly growing with
distance energy between two static sources when the distance is large enough,
\begin{equation}
E \simeq \sigma r + \text{const},
\end{equation}
where $\sigma$ is the tension of “flux tube” which is usually approximated by a string. In a pure Yang-
Mills theory, the strings are closed. If the fermions are introduced, a string between fermion and
antifermion is open and its tension is half of tension of the closed string. This property can be
used for prediction of spectrum of composite states since the mass spectrum is expressed (in a
model-dependent way) via $\sigma$. We will further follow the method of Ref. \cite{6}.

Suppose that the fermion effects can be parametrized by a parameter $b$ within some dynamical
model. This will entail the dependence of tension $\sigma$ on this parameter, $\sigma = \sigma(b)$. The physical
value of $b$ is such that the tension is halved in comparison with the case $b = 0$ when the fermion
effects are absent. This gives the equation for $b$,
\begin{equation}
\sigma(b) = \frac{1}{2} \sigma(0).
\end{equation}

For a specific implementation of this idea we need a specific model. Perhaps the most suc-
cessful bottom-up holographic model describing the confining properties of QCD is the Soft Wall
If the BSM strongly coupled theory is analogous to QCD, the phenomenological holographic models for this BSM strongly coupled theory should look similar to the phenomenological holographic models for QCD. Below we give a brief overview of the scalar SW holographic model and its generalization, which is used further, and then apply this model for a holographic description of composite Higgs sector.

2 Generalized SW holographic model

The SW holographic model proposed by Son et al. [13] is defined by the action

$$S = \int d^4x dz \sqrt{G} e^{cz^2} \mathcal{L},$$

where $G = |\det G_{MN}|$, $G_{MN}$ is the metric of Poincare patch ($z > 0$) of five-dimensional anti-de Sitter (AdS$_5$) space that has the line element

$$ds^2 = G_{MN} dx^M dx^N = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2).$$

Here $R$ is the radius of the AdS$_5$ space and $z$ is the fifth (called holographic) coordinate. The function $e^{cz^2}$ represents the dilaton background, $c$ is a constant introducing the mass scale, and $\mathcal{L}$ is the Lagrangian density of some fields in AdS$_5$ space. By assumption, these fields are dual on the conformal boundary of AdS$_5$ space (situated at $z = 0$) to some QCD operators. The model is defined in the probe approximation, i.e., the metric is not backreacted by matter fields and dilaton (in some sense, this backreaction is effectively parametrized by the form of the dilaton background [14]).

Since the background space is curved, the model is nontrivial even in the case of free fields. The Lagrangian density $\mathcal{L}$ is then quadratic in fields and following the prescriptions of AdS/CFT correspondence [4,5] one can obtain nontrivial two-point correlation functions while higher-order correlation functions will not appear. Remembering that the holographic approach is formulated only in the large-$N$ limit of conformal field theories with $SU(N)$ gauge group, the absence of higher-order correlation functions reflects the fact that they are suppressed by positive powers of $1/N$ in confining $SU(N)$ gauge theories at $N \gg 1$ [8].

Consider the case of free scalar fields,

$$\mathcal{L} = \frac{1}{2} (G^{MN} \partial_M \Phi \partial_M \Phi - m_5^2 \Phi^2).$$

According to the prescriptions of AdS/CFT correspondence [4,5] the five-dimensional mass $m_5$ of scalar fields is related with the canonical dimension $\Delta$ of corresponding scalar operators of dual four-dimensional gauge theory living on AdS$_5$ boundary as

$$m_5^2 R^2 = \Delta(\Delta - 4).$$

The spectrum of normalizable modes of this model is (see, e.g., [10])

$$M_n^2 = 4|c|(n + \Delta/2),$$

where $n = 0, 1, 2, \ldots$. There are alternative formulations of SW model [4] which lead to same classical equations of motion and correlation functions. They are summarized and discussed in detail in the recent
work [14]. For our purpose, we will use a formulation without dilaton background. In general, if a SW model is given by the action

$$S = \int d^4 x d z \sqrt{g} B(z) \mathcal{L},$$  

(9)

where $B(z)$ is some $z$-dependent dilaton background, one can redefine the fields so that the action (9) takes the form

$$S = \int d^4 x d z \sqrt{\tilde{g}} \tilde{\mathcal{L}},$$

(10)

with some modified metric. In the scalar case, the modified metric becomes [14]

$$\tilde{g}_{MN} = B^{2/3} g_{MN}.$$  

(11)

The linear scalar spectrum (8) can be generalized to

$$M_i^2 = 4|c|(n + \Delta/2 + b),$$  

(12)

where $b$ is a free intercept parameter. The inclusion of this parameter into the scalar SW model requires generalization of the dilaton background $e^{cz^2}$ to $B = e^{cz^2} U^2(b, -1, cz^2)$, where $U(b, -1, cz^2)$ is the Tricomi function [14]. The transformation (11) leads then to the following modification of the metric (5),

$$ds^2 = f(z) \frac{R^2}{z^2} \left(\eta_{\mu\nu} dx_\mu dx_\nu - dz^2\right),$$

(13)

$$f(z) = e^{2cz^2/3} U^{4/3}(b, -1, cz^2).$$

(14)

The considered extension of holographic SW model by introducing the intercept parameter $b$ was first proposed in Ref. [15]. The quantity $b$ parametrizes the fermion (namely quark) effects like the impact of chiral symmetry breaking on the mass spectrum [14].

3 Confining behavior and breaking of effective string

The holographic calculation of potential between two sources from the expectation value of the Wilson loop was suggested by Maldacena [16]. Consider a rectangular Wilson loop located in the 4D boundary of the Euclidean AdS$_5$ space. In the limit of large Euclidean time, $T \to \infty$, the expectation value of such Wilson loop is proportional to

$$\langle W(C) \rangle \sim e^{-TE(r)},$$

(15)

where $E(r)$ corresponds to the energy of the fermion-antifermion pair. On the other hand, it can be computed as [16]

$$\langle W(C) \rangle \sim e^{-S},$$

(16)

where $S$ represents the area of a string world-sheet which produces the loop $C$. The static energy follows from comparing (15) and (16), $E = S/T$.

Within the bottom-up holographic QCD, the given idea was developed by Andreev and Zakharov for the case of vector SW holographic model [17]. The analysis is based on the Nambu-Goto string action,

$$S = \frac{1}{2\pi \alpha'} \int d^2 \xi \sqrt{\det G_{MN} \partial_\alpha X^M \partial_\beta X^N},$$

(17)
where $g_{MN}$ is the modified AdS$_5$ metric, $\alpha'$ is the tension of fundamental string. This action is known to describe the area of string world-sheet. The expectation value of Wilson loop can be calculated for this area and after that the dependence $E(r)$ can be extracted. For the case of generalized scalar SW model, the corresponding calculation was done in Ref. [6], the final answer is given in a parametric form by the expressions

$$r = 2\sqrt{\frac{\lambda}{c}} \int_0^1 \frac{dv}{v^2} \frac{U^{4/3}(b, -1, \lambda)}{U^{4/3}(b, -1, \lambda v^2)} \left( 1 - v^4 \frac{e^{2\lambda(1-v^2)/3}}{U^{8/3}(b,-1,\lambda v^2)} \right)^{1/2},$$  \tag{18}

$$E = \frac{R^2}{\pi \alpha'} \sqrt{\frac{\lambda}{c}} \int_0^1 \frac{dv}{v^2} \left( \frac{e^{2\lambda v^2/3}U^{4/3}(b, -1, \lambda v^2)}{1 - v^4 \frac{e^{2\lambda(1-v^2)/3}}{U^{8/3}(b,-1,\lambda v^2)}} - D \right) - D,$$  \tag{19}

where $D \equiv U^{4/3}(b, -1, 0)$ is the regularization constant and

$$z_0 \equiv \left. z \right|_{y=0}, \quad v \equiv \frac{z}{z_0}, \quad \lambda \equiv cz_0^2.$$  \tag{20}

The rectangular Wilson loop in this calculation was parametrized by the coordinate choice $\xi_1 = t$ and $\xi_2 = y$, where $0 \leq t \leq T$.

Combining (18) and (19), the large distance asymptotics for the energy can be extracted [6],

$$E \sim \frac{R^2}{2\pi \alpha'} \frac{e^{2\lambda x/3}U^{4/3}(b, -1, x)}{x} \text{cr.}$$  \tag{21}

The parameter $x$ is equal to one of the roots of derivative of the expression under the square root in the integrals above. The effective string tension between static sources as a function of intercept parameter $b$ follows from (21),

$$\sigma(b) \equiv \frac{R^2}{2\pi \alpha'} \frac{e^{2\lambda x/3}U^{4/3}(b, -1, x)}{x} \text{cr.}$$  \tag{22}

The condition [3] for $\sigma(b)$ taken from (22) results in the equation,

$$\frac{3e^{2\lambda x/3-1}U^{4/3}(b, -1, x)}{2x} = \frac{1}{2}.$$  \tag{23}

For consistency of the calculation, the expressions under the square roots in (18) and (19) must be non-negative in the whole interval of integration variable $v$. This reality condition leads to the second equation,

$$1 - \frac{2}{3} x - \frac{4}{3} x^2 U'(b, -1, x) = 0.$$  \tag{24}

We obtained thus two nonlinear equation for two unknown variables $x$ and $b$. The graphical solution is displayed in Fig. 1. The system has two solutions (the apparent intersection at $(x, b) = (0, -2)$ in Fig. 1 is not a solution to the Eq. (23)), the corresponding numerical solutions for the intercept parameter $b$ are

$$b_1 \approx -1.859, \quad b_2 \approx 0.394.$$  \tag{25}
4 Higgs masses

According to our previous reasoning we associate the two solutions (25) with two possible scalar spectra in which only the ground states correspond to physically distinguishable particles. The operator describing the coupling of Higgs field $h$ to a fermion-antifermion pair $\bar{\psi}\psi$ has the canonical mass dimension $\Delta = 4$. Within the framework of the proposed model, the relation (12) yields then the following mass of the SM Higgs boson,

$$M^2_h = 4|c| (2 + b_1). \quad (26)$$

The mass of the second Higgs boson $h'$ is given in terms of (25) and (26) by

$$M^2_{h'} = \frac{2 + b_2}{2 + b_1} M^2_h. \quad (27)$$

Substituting the corresponding numerical values from (25) and $M_h = 125$ GeV [18] we get

$$M_{h'} \approx 515 \text{ GeV}. \quad (28)$$

A scalar particle with the given mass value was not observed. This is not surprising because a SM like Higgs boson with a mass near 500 GeV should have the total decay width near 70 GeV [19]. Such a broad resonance can hardly be distinguished from the experimental background, especially taking into account that $h'$ should be even much broader due to additional decays into the light Higgs bosons. On the other hand, our prediction should be considered as a prediction made in the large-$N$ limit of underlying $SU(N)$ gauge theory, i.e. deviations of the order of 20% are acceptable. Several searches for additional Higgs bosons at the LHC performed by the CMS and ATLAS Collaborations show an excess of events of about $3 \sigma$ standard deviations above the background expectation around a mass scale $M_{h'} \approx 400$ GeV [20]. If this observation is interpreted in the future as the second Higgs boson, it is conceivable that we got a rough estimate for its mass.

It is interesting to take a closer look at the predicted value (28). We show in Fig. 2 the position of Higgs masses under consideration on the plot of Higgs branchings. It is seen from the right part of Fig. 2 that the predicted mass value of (28) corresponds to the maximal $t\bar{t}$ branching ratio. In other words, it corresponds to the point in the region $M > M_h$ where the direct decay to fermion-antifermion pair is most favorable. Even if the genuine second Higgs particle does not exist, we find the given holographic prediction highly nontrivial.

It is curious to observe that the standard Higgs boson mass corresponds to the maximal branching ratio for decay into massless gauge bosons, see the left part of Fig. 2. Simultaneously, the branching ratios for decays into fermions begin to rapidly decrease in the given region. We do not know a physical explanation but it is clear that the Higgs mass appears to be located at the point where the production of Higgs boson via the gluon fusion (as in $pp$-collisions at CERN) is most favorable. Within the presented model, one could speculate that the first solution describes the splitting into two massless gauge bosons. The canonical mass dimension $\Delta = 4$ refers then to the gauge-invariant field operator $O^2_{\mu\nu}$ and the energy $E(r)$ should be interpreted as a static energy between two “colored” sources.

This observation might shed light on the physical reason for the existence of two composite Higgs bosons: One can construct two gauge and renormalization group invariant scalar operators in the standard gauge theories, $O_1 = \beta G^2_{\mu\nu}$, where $\beta$ is the Gell-Mann–Low beta-function, and $O_2 = m_\psi \bar{\psi}\psi$. Both operators have the canonical dimension $\Delta = 4$. The two-point correlation
functions $\Pi_1 = \langle O_1O_1 \rangle$ and $\Pi_2 = \langle O_2O_2 \rangle$ calculated in the background of BSM strong sector may have poles. Then $M_h$ and $M_{h'}$ could arise from poles of different correlation functions, $\Pi_1$ and $\Pi_2$.

Finally it is curious to note that the numerically obtained relation $M_{h'} \approx 4M_h$ is close to the approximate relation between masses of first scalar excitations in QCD, $M_\sigma \approx 4M_\pi$, where the broad scalar $\sigma$-meson is identified with the resonance $f_0(500)$ \[18\].

5 Conclusion

Assuming the existence of confining “strong sector” beyond the Standard Model and that this sector can be described holographically, we applied the Wilson loop analysis of confining behavior of generalized holographic Soft Wall model \[14\] in the scalar sector to description of the composite Higgs particles. The generalization consists in introducing the Regge “intercept” parameter which, in a close analogy with the holographic description of standard strong interactions, parametrizes the fermion effects on the masses of composite Higgs bosons. The constructed model predicts the mass of the second SM like Higgs boson near $515$ GeV. This prediction has an unexpected and nontrivial feature — the given value coincides with the maximal branching ratio for decay into the $tt$ pair.

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![Figure 2](image-url): The branching ratios of the dominant decay modes of SM Higgs boson as a function of its mass (in GeV). All known QCD and leading electroweak radiative corrections are included. The plot is taken from Ref. \[19\]. The values of the SM Higgs mass and the predicted one \[28\] are shown in blue (the dashed and dotted lines, correspondingly).
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References

[1] B. Bellazzini, C. Csáki and J. Serra, Composite Higgses, Eur. Phys. J. C 74 (2014) no.5, 2766, [1401.2457]

[2] R. Contino, The Higgs as a Composite Nambu-Goldstone Boson, [1005.4269]

[3] J. M. Maldacena, The Large N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. 2 (1998), 231-252, Int. J. Theor. Phys. 38, 1113 (1999), [hep-th/9711200]

[4] E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. 2 (1998), 253-291, [hep-th/9802150]

[5] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Gauge theory correlators from noncritical string theory, Phys. Lett. B 428 (1998), 105-114, [hep-th/9802109]

[6] S. Afonin and T. Solomko, Gluon string breaking and meson spectrum in the holographic Soft Wall model, Phys. Lett. B 831 (2022), 137185, [2112.00021]

[7] G. ’t Hooft, A Planar Diagram Theory for Strong Interactions, Nucl. Phys. B 72 (1974), 461

[8] E. Witten, Baryons in the 1/n Expansion, Nucl. Phys. B 160 (1979), 57-115

[9] C. Csaki, M. Reece and J. Terning, The AdS/QCD Correspondence: Still Undelivered, JHEP 05 (2009), 067, [0811.3001]

[10] S. S. Afonin, Towards reconciling the holographic and lattice descriptions of radially excited hadrons, Eur. Phys. J. C 80 (2020), 723, [2008.05610]

[11] L. J. Reinders, H. Rubinstein and S. Yazaki, Hadron Properties from QCD Sum Rules, Phys. Rept. 127 (1985), 1-97

[12] S. S. Afonin, AdS/QCD models describing a finite number of excited mesons with Regge spectrum, Phys. Lett. B 675 (2009), 54-58, [0903.0322]

[13] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Linear confinement and AdS/QCD, Phys. Rev. D 74 (2006), 015005, [hep-ph/0602229]

[14] S. S. Afonin and T. D. Solomko, Towards a theory of bottom-up holographic models for linear Regge trajectories of light mesons, Eur. Phys. J. C 82 (2022), 195, [2106.01846]

[15] S. S. Afonin, Generalized Soft Wall Model, Phys. Lett. B 719 (2013), 399-403, [1210.5210]

[16] J. M. Maldacena, Wilson loops in large N field theories, Phys. Rev. Lett. 80 (1998), 4859-4862, [hep-th/9803002]

[17] O. Andreev and V. I. Zakharov, Heavy-quark potentials and AdS/QCD, Phys. Rev. D 74 (2006), 025023, [hep-ph/0604204]
[18] R. L. Workman et al. [Particle Data Group], *Review of Particle Physics*, Prog. Theor. Exp. Phys. **2022** (2022), 083C01

[19] M. Spira, A. Djouadi, D. Graudenz and P. M. Zerwas, *Higgs boson production at the LHC*, Nucl. Phys. B **453** (1995), 17-82, [hep-ph/9504378]

[20] T. Biektöter, A. Grohsjean, S. Heinemeyer, C. Schwanenberger and G. Weiglein, *Possible indications for new Higgs bosons in the reach of the LHC: N2HDM and NMSSM interpretations*, Eur. Phys. J. C **82** (2022) no.2, 178, [2109.01128]