Formation of the guillotine cutting card of a sheet

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Abstract. The tasks of rational cutting-packaging have long attracted the attention of researchers and practitioners in various industries. They are classified as NP-complete problems. In this paper, two-dimensional orthogonal guillotine packaging in sheets of the same size is considered as the main one. An essential feature of the problem under consideration is technological limitations. You must place all the rectangles of different sizes and quantities, however, the layout template should be the same for all sheets. Thus, it is necessary to minimize the number of sheets in an acceptable time. To compare the effectiveness of various methods on the same class of tasks, the cutting coefficient is determined. However, an important issue in evaluating a particular method is the decision time. The paper presents a heuristic algorithm. The performed computational experiments confirm the effectiveness of the proposed method.

1. Introduction
One of the problems of many branches of modern production is the solution of cutting-packaging problems [1]. This class of tasks combines the need to establish a certain correspondence between two groups, as a rule, large and small objects. Rectangular blanks — rectangles of various sizes — act as small objects, and large ones — material coming in the form of strips, rolls, rectangular sheets or containers of various capacities.

There are problems of n-dimensional packaging ($n = 1, 2, 3$). One can also single out a problem of a larger dimension, where the solution time is used as the fourth dimension.

The most common problems are orthogonal packaging and cutting. It is characteristic of them that the sides of the rectangles are parallel to the edges of the strip. Necessary conditions are also the non-overlap of the rectangles with each other and the rectangles do not go beyond the edges of the strip.

The class of guillotine cutting tasks will also be distinguished. A guillotine cut is called a rectilinear cut, which is made from edge to edge of the sheet [2]. These tasks are a problem of both theoretical and practical plan. The reason for the growing interest in cutting-packaging tasks is their diversity and NP-complexity. Algorithms of polynomial complexity are not known for this class of problems [3]. There are exact methods of solution, which are based on a complete enumeration scheme, therefore they are rarely found in practice.

The essence of constructing optimal cutting is the development of feasible maps, in which the necessary set of blanks is obtained and the objective function is minimized under given constraints imposed by technological processes.
To compare the effectiveness of various methods on the same class of tasks, the cutting coefficient is determined. For the task of two-dimensional packaging, this is the percentage of the total area of all placed items to the area of the spent material [4].

However, an important issue in evaluating a method is the decision time. This is due to the fact that, in view of the NP-completeness and a large number of small and large objects, the temporal characteristics are quite significant.

2. Statement and methods for solving cutting-stock problems
A lot of methods have been developed for solving this class of problems with their own advantages and disadvantages. This is due to the size and quantity of small and large objects and imposed technological limitations [5–7].

In [8], modeling and solving the problem of minimizing waste for the woodworking industry is presented. Raw materials are presented in the form of beams of various sizes. The decision variables are the number of beams that need to be cut in accordance with a set of patterns.

Currently, there are various models for representing n-dimensional problems \(n=1, 2, 3\) of cutting-stock [9, 10].

In this paper, we consider the two-dimensional orthogonal guillotine packing into sheets of the same size as the main task. An essential feature of the problem under consideration is technological limitations. It is necessary to place all the rectangles of different size and quantity, however, the layout template is the same for all sheets. Thus, it is necessary to minimize the number of sheets in an acceptable time.

The mathematical model can be represented as follows. Let the sheet sizes \(L, W\) be given, - height and width, respectively. The sizes of \(n\) rectangles are denoted by \(l_i, w_i\). It is necessary to find a map \(\Omega (\langle \Pi, X, Y, E, R \rangle)\), where \(\Pi\) – converted list of rectangles; \(X = (x_1, x_2, ..., x_n)\), \(Y = (y_1, y_2, ..., y_n)\) – rectangle minimum coordinate vectors; \(E = (e_1, e_2, ..., e_n)\) – signs of turning: \(e = 1, \) if objects can be turned \(ε = 0\) otherwise; \(R\) – the number of sheets that converts input to output, and the following conditions are met:

- complete the entire order;
- orthogonal placement of rectangles on a sheet;
- non-overlapping borders of the sheet;
- non-overlapping rectangles with each other;
- minimizing the number of sheets;
- minimization of decision time.

3. Method algorithm
Basic steps of the algorithm:

- Sort rectangles by width and not ascending.
- Calculation of the optimal quantity of each type of element on one sheet: \(A_i = \frac{q_i}{S_a/S_i}\), where \(S_a\) is the area of all elements, \(S_i\) is the area of the sheet, \(q_i\) is the number of the elements of the \(i\) – type, \(S_{a} = \sum_{i=0}^{n-1} q_i w_i h_i\), where \(h_i\) is the height, \(w_i\) is the width, \(q_i\) is the number of rectangles on the sheet.
- Array formation \(W_i = F*(A_i)\).
- The starting point \(E\) is set with the coordinates \((0; 0)\).
- Selection of a column of maximum height from the \(W\) array using a modified number system.
- Changing \(Ex = Ex + \) column width.
- Operations 4 – 6 are repeated until an element can be placed.
- Calculation of the number of sheets: \(\text{Max } (F(h_0 / W_0... F (h_{n-1}/ W_{n-1}))\).
The more different rectangular elements, the more difficult it is to solve the problem, since at least one copy of an element of each type should be on the sheet. Thus, the effectiveness of the algorithm depends on the variety of elements.

For a more objective testing, we introduce the coefficient \( E \) and obtain a system of inequalities:

\[
\begin{cases}
S > 0 \\
S \leq S_\text{L} \\
S > S_\text{L} \ast E
\end{cases}
\]

where \( S \) is the sum of the areas of all types of elements without taking into account their quantity; \( S_\text{L} \) is the area of the sheet (\( S > 0 \) – at least 1 element is needed; \( S \leq S_\text{L} \) is a necessary condition for the solvability of the problem). Given these conditions, we will generate elements for testing the algorithm.

Example. For \( v_1 = 0.1 \), \( v_2 = 0.2 \); \( w_1 = 0.1 \), \( w_2 = 0.2 \); \( E = 0.25 \) we get such a set of different elements (figure 1).

![Figure 1](image1.png)

**Figure 1.** Types of rectangles with a value of \( E = 0.25 \).

By increasing the coefficient \( E \) to 0.75, we obtain a more diverse set (figure 2).

![Figure 2](image2.png)

**Figure 2.** Types of rectangles with a value of \( E = 0.75 \).

4. **Computational experiment**

To evaluate the effectiveness of the proposed series of calculations based on the methodology of G. Wescher. The basis of the division into different classes was taken.

The basis for splitting into various classes of objects is taken: the lower limit on the length of objects \( v_1 \), the upper limit on the length of objects \( v_2 \) \( (v_1 L \leq l_i \leq v_2 L, \ i =1,\ldots, n) \); lower item width limit \( w_1 \), top item width limit \( w_2 \) \( (w_1 W \leq w_i \leq w_2 W, \ i =1,\ldots, n) \). Values were randomly generated. The number of rectangles varied in the range \( n = 50, 100, 200, 250, 500, 1000 \).
Figures 3, 4 show diagrams of the dependence of the time of solution and the efficiency of solving the problem for averaged randomly generated data, respectively ($E = 0.75$).

![Figure 3](image1.png)  
**Figure 3.** The dependence of the time of solving the problem on the number of rectangles.

![Figure 4](image2.png)  
**Figure 4.** The effectiveness of solving the problem depending on the number of rectangles.

**Conclusion**

The lack of proven methods for solving NP-complete problems hinders the effective decision-making for a diverse class of cutting-packaging tasks, which determines the need to develop new methods and approaches.

In scientific works and practical recommendations, questions of the uniform placement of rectangles on sheets of the same size were not reflected. This determines the relevance of this work.

This approach can be used in various technical areas of production: woodworking, label production and others.

A computational experiment showed that the proposed heuristic shows the best results in terms of solving problems for objects of medium size, in efficiency - for generated objects of small sizes.

Further research should be directed to improve the performance of the solution, as well as for tasks not only of the guillotine cutting.

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