On Light Mapping and Certain Concepts by Using $m_XN$-Open Sets

Haider Jebur Ali $^{1*}$  
Raad F. Hassan $^{2}$

Received 4/3/2019, Accepted 15/1/2020, Published 18/3/2020

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Abstract:

The aim of this paper is to present a weak form of $m$-light functions by using $m_XN$-open set which is $mN$-light function, and to offer new concepts of disconnected spaces and totally disconnected spaces. The relation between them have been studied. Also, a new form of $m$-totally disconnected and inversely $m$-totally disconnected function have been defined, some examples and facts was submitted.

Key words: $m_XN$-disconnected space, $m_XN$-Hausdorff space, $mN$-light function, $m_XN$-open set, $mN$-totally disconnected function

Introduction:

In (2016) Abass and Ali (1) introduced the definition of $m$-light function, Humadi and Ali (2) presented the $m^*$-light function. Al Ghour and Samarah (3) defined $N$-open set. In this research we defined the set $m_XN$-open set, we submitted a new type of functions by using $m_XN$-open sets, it is weaker than $m$-light function and we named it $mN$-light function. In (4) Carlos Carpintero, Jackeline Pacheco, Nimitha Rajesh, Ennis Rafael Rosas and S. Saranyasri defined $N$-connected space, by the same manner $m$-disconnected, $m_XN$-disconnection, $mN$-disconnected, $m_N$-connected and $m_XN$-totally disconnected spaces have been defined, additionally, many types of functions in $m$-structure spaces such as $mN$-totally disconnected, $mN^*$-totally disconnected, $mN^{**}$-totally disconnected, inversely $mN$-totally disconnected function have been introduced. In (5) Enas Ridha Ali, Raad Aziz Hussain introduced the definition of $N$-Hausdorff, and in the same way, $mN$-Hausdorff has been defined. Also $mNT_1$-spaces and zero dimension $m$-spaces have been provided. The relation between these concepts has been discussed. Moreover the relation between $m$-homeomorphism functions (6) and the $mN$-light functions has been illustrated. Examples, theorems and some facts supported our study.

Main Results:

In this section, $mN$-totally disconnected, $mN$-light functions and some spaces by using $m_XN$-open sets have been presented.

Definition 1 (7), (8)

A subcollection $m_X$ of the power set $P(X)$ of a non-empty set $X$ is called a minimal structure on $X$ if $\emptyset, X \in m_X$, the pair $(X, m_X)$ is called $m$-structure space (in short $m$-space). Each element in $m_X$ is said to be $m_X$-open set and its complement is said to be $m_X$-closed set.

Remark 1 (9)

Every topological space $(X, \mathcal{T})$ is $m$-space, but not conversely, because $\emptyset, X$ belong to $\mathcal{T}$.

Example 1

If $X = \{n, m, f\}$ and $m_X = \{\emptyset, X, \{n\}, \{m\}\}$, we observe that $m_X$ is not a topology, since $\{n\} \cup \{m\} = \{n, m\} \notin m_X$.

Definition 2 (10), (11)

The $m_X$-closure to a subset $B$ of $m$-space $(X, m_X)$ is the intersection of all closed sets $\mathcal{F}$ in $X$ which containing $B$ and we denote it by $m_X-closure(B)$, by symbols $m_X-closure(B)$=$\cap \{\mathcal{F}; B \subseteq \mathcal{F}\},$ where $\mathcal{F}$ is $m_X$-closed subset of $X$. While the $m_X$-interior to a subset $B$ of $m$-space $(X, m_X)$ is the union of all open sets $K$ in $X$ which contained in $B$ and we denote it by $m_X$-Int(B), by symbols $m_X-Int(B)=\cup \{K; K \subseteq B\}$ where $K$ is $m_X$-open set in $X$.

Definition 3

A subset $B$ of $m$-space $X$ is called $m_X$-$N$-open set if for each element $a \in B$ there exists an $m_X$-$N$-open set $K$ in $X$ containing $a$ such that $K-B$ is finite, the complement of $m_X$-$N$-open set is called $m_X$-$N$-closed. The family of all $m_X$-$N$-open sets in $X$ is symbolized as $m_N$.
Example 2
Any subset of a finite m-structure space \((X, m_X)\) is \(m_X\) N-open and \(m_X\) N-closed set.

Lemmal
If \(\{K_i \mid i \in I\}\) is a collection of \(m_X\) N-open subsets of \(m_X\)-space, then \(\bigcup_{i \in i} K_i\) is \(m_X\) N-open too.

Proof
Consider \(x \in \bigcup_{i \in i} K_i\), so there is an \(m_X\) N-open set \(K_j\) containing \(x\) for some \(j \in I\), so \(W_j-K_j\) is finite, where \(W_j\) is \(m_X\)-open subset of \(X\) containing \(x\), then \(W_j-\bigcup_{i \in i} K_i\) is also finite since \(W_j-\bigcup_{i \in i} K_i \subseteq W_j-K_j\), (a subset of finite set is finite), therefore \(\bigcup_{i \in i} K_i\) is \(m_X\) N-open set.

Definition 4 (1)
An \(m\)-space \(X\) is said to be \(m\)-disconnected, if there are non-empty \(m_X\)-open sets \(H\) and \(L\) in \(X\) such that \(H\bigcup L=X\) and \(H\cap L=0\), if \(X\) is \(m\)-disconnected space then it is called \(m\)-connected space.

Example 3
The discrete \(m\)-space \((Z, m_D)\), is \(m\)-disconnected space.

Definition 5
Let \((X, m_X)\) be an \(m\)-space and \(H, L\) are two non-empty \(m_X\)-open subsets of \(X\), we call \(H\bigcup L\) to be \(m_X\)-disconnection to \(X\), if \(H\bigcup L=X\) and \(H\cap L=0\). In example 3 \(Z\{-x\}\) and \(\{x\}\) where \(x \in Z\), are \(m_X\)-disconnection to \(Z\).

Definition 6
An \(m\)-space \(X\) is \(mN\)-disconnected if we can find an \(m_X\)-disconnection to it, if there is no such \(mN\)-disconnected so \(X\) is \(mN\)-connected space.

Example 4
The finite indiscrete \(m\)-space \((X, m_{ind})\) is \(mN\)-disconnected, but not \(m\)-connected.

Proposition 1
An \(m\)-space \(X\) is \(mN\)-disconnected if and only if there is a non-empty \(m_X\)-N-clopen subset \(G\) in \(X\) such that \(G\neq X\).

Proof
Suppose \(G\) is a non-empty \(m_X\)-N-clopen subset of \(X\) such that \(G\neq X\). Let \(U=G^c\), so \(U\) is a subset of \(X\) and \(U\neq 0\) (because \(G\neq X\), and \(G\bigcup U=X\), \(G\bigcap U=0\)). Also \(U\) is \(m_X\)-N-clopen because \(G\) is \(m_X\)-N-clopen, therefore \(X\) is \(mN\)-disconnected space. Conversely, if \(X\) is \(mN\)-disconnected space, so there is an \(m_X\)-disconnection \(GUU\) to \(X\), hence \(G=U^c\) which implies \(G\) is \(m_X\)-N-closed subset of \(X\), therefore \(G\) is \(m_X\)-N-clopen subset of \(X\) and \(G\neq X\) since \(U\) is non-empty subset of \(X\), and then \(G\) is a non-empty \(m_X\)-N-clopen subset of \(X\) such that \(G\neq X\).

Proposition 2
An \(m\)-space \(X\) is \(mN\)-connected space if and only if \(0\) and \(X\) are the only \(m_X\)-N-clopen set in \(X\).

Proof
If \(X\) is an \(mN\)-connected space, and \(U\) is a non-empty proper \(m_X\)-N-clopen subset of \(X\), then \(U^c\) is also \(m_X\)-N-clopen subset of \(X\), and since \(UU\bigcup U^c=X\), where \(U^c\neq 0\), therefore \(X\) is \(mN\)-disconnected space and that is a contradiction, so \(0\) and \(X\) are the only \(m_X\)-N-clopen set in \(X\). Conversely, suppose \(X\) is \(mN\)-disconnected space, so there is \(m_X\)-disconnection \(LUU\) to \(X\), but \(L\) is \(m_X\)-N-closed (since \(L=H^c\)) which is a contradiction, therefore \(X\) is \(mN\)-connected.

Definition 7
The \(m\)-space \((X, m_X)\) is called an \(mN\)-totally disconnected space. If for every pair of distinct points \(a\) and \(b\) in \(X\), there are two \(m_X\)-open sets \(N, M\) such that \(N\neq \emptyset, M\neq \emptyset, a \in N, b \in M, N \bigcup M=X\) and \(N \bigcap M=0\).

Example 5
For any distinct points \(x, y\) in the discrete \(m\)-space \((Z, m_D)\), the sets \(\{x\}\) and \(\{Z\{-x\}\}\) are \(m_X\)-open sets containing \(x\), \(y\) respectively such that \(\{x\}\bigcup (Z\{-x\})=\emptyset\) and \(\{x\}\bigcup (\{Z\{-x\}\})=Z\), so \((Z, m_D)\) is \(mN\)-totally disconnected space.

Remark 2
Let \(X\) be an \(m\)-space, then:-
1- Every \(m_X\)-open subset of \(X\) is \(m_X\)-N-open, but the converse is not true, since if \(K\) is \(m_X\)-open subset of \(X\), then for each \(x \in K\) there is an \(m_X\)-open subset \(M\) of \(X\), pick \(M=K\) then \(M\) containing \(x\) and \(M\bigcap K=0\) (finite), so \(K\) is \(m_X\)-N-open set.
2- Every \(m_X\)-closed subset of \(X\) is \(m_X\)-N-closed.

Example 6
Let \(K=\mathcal{R}\{-0\}\) be a subset of the indiscrete \(m\)-space \((R, m_{ind})\), then \(K\) is \(m_R\)-N-open set, but not \(m_X\)-open set.

Remark 3
I- Every \(mN\)-connected space is \(m\)-connected but the converse is not true, since if \((X, m_X)\) is an \(mN\)-connected space, and suppose it is \(m\)-disconnected space then there is \(m_X\)-disconnection \(NUM\) to \(X\), and then it is \(mN\)-disconnected (by Remark 2) which is a contradiction, hence \(X\) is \(m\)-connected.
II- Every \(m\)-disconnected space is \(mN\)-disconnected, but the converse is not true, since if \((X, m_X)\) is \(m\)-disconnected space, then there is \(m_X\)-disconnection \(MUN\) for \(X\), and by Remark 2 it is \(m_X\)-disconnected, therefore \(X\) is \(mN\)-disconnected space.

Example 7
The finite indiscrete \(m\)-space \((X, m_{ind})\) is \(m\)-connected and \(mN\)-disconnected space, but neither \(m\)-N-connected nor \(m\)-disconnected space.

Proposition 3
A subset \(G\) of \(m\)-space \(X\) is \(m_X\)-N-disconnected if and only if there is \(m_X\)-N-open subsets \(N\) and \(M\) of
Proposition 6

There is an $m$-function $f$ from $m$-space $X$ into $m$-space $Y$ called an $m$-continuous function if and only if $f^{-1}(M)$ is $m$-open set in $X$, for every $m_1$-open set $M$ in $Y$.

Definition 7

The $m_1$-continuous image of $m_1$-connected set in $X$ is $m_1$-connected set in $Y$.

Proof

Let $f: X \to Y$ be an $m$-continuous function and $T$ is $m_1$-connected set in $X$, and suppose that $f(T)$ is not $m_1$-connected, so there is an $m_1$-disconnection $N \cup M$ of $f(T)$, since $f$ is $m$-continuous function, then $f^{-1}(N) \cup f^{-1}(M)$ are $m$-open sets in $X$, with $f^{-1}(N) \cup f^{-1}(M) = T$, so $T = f^{-1}(N) \cup f^{-1}(M)$, and $N \cap M \cap f^{-1}(T) = \emptyset$, then $f^{-1}(N)$ and $f^{-1}(M)$ are disjoint and separation of $T$, that is a contradict the hypothesis that $T$ is $m$-connected set in $X$, so $f(T)$ is $m_1$-connected set.

Proof

If $K$ is a subset of an $m$-space $X$, then $K$ is $m_1$-open set if and only if any point in $K$ is an $m_1$-interior point of it.

Proof

Consider $K$ is an $m_1$-open set and $x \in K$, since $K$ is a subset of itself, so $x$ is an $m_1$-interior point.

Conversly, since $K$ is a union of all its points which are $m_1$-interior point, for each $x$ in $K$ there is an $m_1$-open set $W$ in $X$ with $x \in W \subseteq K$, then $K = \bigcup_{x \in K} W_x$, for each $x \in X$, and by lemma 1 we get $K$ is $m_1$-open set.

Proposition 6

Let $K$ be a subset of an $m$-space $X$, then $K$ is $m_1$-closed if and only if $m_1$-$N$-$d(K)\subseteq K$.
in $X$, such that $N \cap M = \emptyset$, and $a \in N$, $b \in M$, since $N \cap M = \emptyset$, so $b \notin N$ and $a \notin M$, hence $X$ is $mNT_2$-space.

**Example 11**

Let $(Z, \ m_{\text{ind}})$ be the indiscrete $m$-space, let $x, y \in Z$ with $x \neq y$, then we can find two $m_N$-open sets $U$ and $V$ in $Z$ such that $U=Z\{-x\}$ and $V=Z\{-y\}$ which containing $x$ but not $x$, and $V=Z\{-y\}$ which containing $x$ but not $x$, so $(Z, \ m_{\text{ind}})$ is $mNT_1$-space, but not $mNT_2$-space since $(Z\{-x\}) \cap (Z\{-y\}) = \emptyset$. Also $(\mathcal{R}, \ m_{\text{cof}})$ is $mNT_1$-space but not $mNT_2$-space.

**Remark 6**

Every $mN$-totally disconnected space is $mN$-Hausdorff space, but the converse is not true, since if $X$ is $mN$-totally disconnected space then for each distinct points $a, b$ in $X$, we can find two $m_N$-open sets $N, M$ contained $a, b$ respectively with $N \cap M = \emptyset$ and $N \cup M = X$, so $X$ is $mN$-Hausdorff space.

**Example 12**

Let $(\mathcal{R}, \ m_u)$ be the usual $m$-space, it is $mN$-Hausdorff space, but not $mN$-totally disconnected.

**Remark 7**

Every $m$-Hausdorff space is $mN$-Hausdorff, but the converse is not true, since if $X$ is $m$-Hausdorff space, so there are $m_N$-open sets $N$ and $M$ in $X$, such that $N \neq \emptyset, M \neq \emptyset, a \in N, b \in M$, by Remark 2 $X$ is $mN$-Hausdorff.

**Example 13**

Let $X= \{1, 2, 3\}$ and $m_X= \{\emptyset, X, \{1, 2\}, \{2\}, \{3\}\}$, 1 and 2 are distinct points in $X$, and there exist $m_N$-open sets $U= \{1\}$ and $V= \{2\}$ in $X$ containing 1, 2 respectively, and $U \cap V = \emptyset$, also 1 and 3 are distinct points in $X$, there exist $m_N$-open sets $U= \{1\}$ and $V= \{3\}$ in $X$ containing 1, 3 respectively, and $U \cap V = \emptyset$, by the way we 2 and 3 are distinct points in $X$, and 1 and 3 are distinct points in $X$, there exist $m_N$-open sets $U= \{2\}$ and $V= \{3\}$ in $X$ containing 2, 3 respectively, and $U \cap V = \emptyset$, so $(X, m_N)$ is $mNT_2$-space which is not $mNT_2$-space since there is no two disjoint $m_N$-open sets containing 1, 2 respectively.

**Remark 8**

Every $m$-totally disconnected space is $mN$-disconnected but the converse is not true, since if $X$ is $m$-totally disconnected space, then for any two points $a, b \in X$ where $a \neq b$ we can find $m_N$-open sets $N$ and $M$ in $X$, with $N \neq \emptyset, M \neq \emptyset, N \cap M = \emptyset$, and they containing $a, b$ respectively such that $N \cup M = X$, so $X$ is $m$-disconnected and then $mN$-disconnected (by remark 4)).

**Example 14**

Let $X= \{a, b, c\}$ and $m_X= \{\emptyset, X, \{a\}, \{b, c\}\}$, then $X$ is $mN$-disconnected space and not $m$-totally disconnected.

**Remark 9**

Let $f: (X, m_X) \rightarrow (Y, m_Y)$ be an $m$-continuous function and $K$ be $m_N$-totally disconnected subset of $X$, then $f(K)$ is not $m_N$-totally disconnected subset of $Y$.

**Example 15**

Let $I_2: (Z, m_D) \rightarrow (Z, m_{\text{cof}})$ where $I_2$ is the identity function, $(Z, m_D)$ is $mN$-totally disconnected space, while $(Z, m_{\text{cof}})$ is not $mN$-totally disconnected.

**Definition 11**

The $m$-function $f: (X, m_X) \rightarrow (Y, m_Y)$ is called $mN$-totally disconnected if the image of each $m_X$-totally disconnected set in $X$ is $m_Y$-totally disconnected in $Y$.

**Definition 12**

The $m$-function $f: (X, m_X) \rightarrow (Y, m_Y)$ is called $mN'$-totally disconnected if the image of each $m_N$-totally disconnected set in $X$ is $m_Y$-totally disconnected in $Y$.

**Definition 13**

The $m$-function $f: (X, m_X) \rightarrow (Y, m_Y)$ is called $mN''$-totally disconnected if the image of each $m_N$-totally disconnected set in $X$ is $m_Y$-totally disconnected in $Y$.

The following Example satisfying Definitions 11, 12 and 13.

**Example 16**

The identity $m$-function $I_X: (X, m_X) \rightarrow (X, m_D)$ is $mN$-totally disconnected function.

**Definition 14**

The surjective $m$-function $f: (X, m_X) \rightarrow (Y, m_Y)$ is called $mN$-light function if the inverse image of any $b \in Y$ is $m_X$-totally disconnected set in $X$.

**Example 17**

The identity $m$-function $I_{\mathcal{R}}: (\mathcal{R}, m_D) \rightarrow (\mathcal{R}, m_{\text{cof}})$ is $mN$-light function.

**Remark 10**

Every $m$-light function is $mN$-light function, but the converse is not true, since if $f: (X, m_X) \rightarrow (Y, m_Y)$ is $m$-light function, then $f^{-1}(b)$ is $m_N$-totally disconnected for any $b$ in $Y$, then it is $m_N$-totally disconnected set in $X$ (by Remark 2), so $f$ is $mN$-light function.

**Example 18**

The $m$-function $f: (X, m_{\text{ind}}) \rightarrow (X, m_{\text{cof}})$, which defined by $f(x) = c$, for each $x \in X$, where $X = \{1, 2, 3\}$, is $mN$-light function but not $m$-light function.

**Remark 11**

Every $m$-homeomorphism function is $mN$-light function, but the converse is not true, since if $f: (X, m_X) \rightarrow (Y, m_Y)$ is $m$-homeomorphism function, then for any $b$ in $Y$ there is a unique $a$ in $X$ where $f(a) = b$ (since $f$ is bijective), so $f^{-1}(b) = \{a\}$ which is $m_X$-totally disconnected, so $\{a\}$ is $m_X$-totally disconnected set in $X$. 


totally disconnected (by Remark 2), and then $f$ is $mN$-light.

**Example 19**

The function $f:(X, m_\delta) \to (Y, m_\gamma)$, where $X = \{a, b, c, d, e, f\}$ and $Y = \{g, h, i\}$ such that $f(a) = f(b) = g$, $f(c) = f(d) = h$, and $f(e) = f(f) = i$, is $mN$-light function but not $m$-homeomorphism.

**Theorem 1**

If $f:(X, m_\chi) \to (Y, m_\gamma)$ is $mN$-light function and $G \subseteq X$, so $f|_G: G \to f(G)$ is $mN$-light function too.

**Proof**

If $g \in f(G)$, so $g \in Y$ (because $f(G) \subseteq Y$), and since $f$ is $mN$-light function so $f^{-1}(g)$ is $m\chi_N$-totally disconnected set in $X$. To prove that $f^{-1}(g) \cap G$ is $m\gamma_N$-totally disconnected set in $G$ for any $g \in f(G)$. Let $a, b \in f^{-1}(g) \cap G$, then $a, b \in f^{-1}(g)$, since $f^{-1}(g)$ is $m\chi_N$-totally disconnected set in $X$, and there is an $m\gamma_N$-disconnection $N \cup M$ to $f^{-1}(g)$ with $(N \cap f^{-1}(g)) \cup (M \cap f^{-1}(g)) = f^{-1}(g)$ and $(N \cap f^{-1}(g)) \cap (M \cap f^{-1}(g)) = \emptyset$, such that $N$ and $M$ are $m\gamma_N$-open sets in $X$, and $a \in N$, $b \in M$. To show that $f^{-1}(g) \cap G$ is $m\gamma_N$-totally disconnected set in $G$.

Since $((G \cap f^{-1}(g)) \cap N) \cup ((G \cap f^{-1}(g)) \cap M) = (G \cap f^{-1}(g)) \cap N \cup (G \cap f^{-1}(g)) \cap M) = G \cap (f^{-1}(g) \cap N) \cup (f^{-1}(g) \cap M) = G \cap (f^{-1}(g) \cap N) \cap (f^{-1}(g) \cap M) = G \cap \emptyset = \emptyset$, such that $a \in (G \cap f^{-1}(g)) \cap N$ and $b \in (G \cap f^{-1}(g)) \cap M$, hence $(G \cap f^{-1}(g)) \cap N$ and $(G \cap f^{-1}(g)) \cap M$ are disjoint $m\gamma_N$-open sets and the union of them is equal to $f^{-1}(g) \cap G$, so $f^{-1}(g) \cap G$ is $m\gamma_N$-totally disconnected set in $G$, therefore $f|_G$ is $mN$-light function.

**Definition 15**

A surjective $m$-function $f:(X, m_\chi) \to (Y, m_\gamma)$ is called inversely $mN$-totally disconnected function if the inverse image of any $m\gamma_N$-totally disconnected set in $Y$ is $m\gamma_N$-totally disconnected set in $X$.

**Example 20**

The identity $m$-function $i_x:(X, m_{ind}) \to (X, m_\chi)$, where $X$ is a finite set is inversely $mN$-totally disconnected function.

**Proposition 8**

Every inversely $mN$-totally disconnected function is $mN$-light function.

**Proof**

Let $f:(X, m_\chi) \to (Y, m_\gamma)$ be inversely $mN$-totally disconnected function and $b \in Y$, since $f$ is surjective $m$-function (since it is inversely $mN$-totally disconnected) and $f^{-1}\{b\}$ is $m\chi_N$-totally disconnected set in $X$, where $\{b\}$ is $m\gamma_N$-totally disconnected set in $Y$ which implies $f$ is $mN$-light function.

**Proposition 9**

The $m$-function $h:(X, m_\chi) \to (Y, m_\gamma)$, where $h \circ f$ is $mN$-light function if $f:(X, m_\chi) \to (Z, m_\delta)$ is inversely $mN$-totally disconnected function and $g:(Z, m_\delta) \to (Y, m_\gamma)$ is $mN$-light function.

**Proof**

Let $b \in Y$, so $h^{-1}(b) = (g \circ f)^{-1}(b) = f^{-1}(g^{-1}(b))$, but $g^{-1}(b)$ is $mN$-totally disconnected set (because $g$ is $mN$-light function), and then $f^{-1}(g^{-1}(b))$ is $m\chi_N$-totally disconnected set in $X$ (since $f$ is inversely $mN$-totally disconnected function), so that $h^{-1}(b)$ is $m\chi_N$-totally disconnected set in $X$, which means $h$ is $mN$-light function.

**Proposition 10**

If $g:(Z, m_\delta) \to (Y, m_\gamma)$ is bijective $m$-function and $f:(X, m_\chi) \to (Z, m_\delta)$ is $mN$-light function, then the surjective $m$-function $h:(X, m_\chi) \to (Y, m_\gamma)$ where $h = g \circ f$ is $mN$-light function.

**Proof**

Let $b \in Y$, then there is an element $z \in Z$ such that $g(z) = b$ (since $g$ is bijective $m$-function), now $h^{-1}(b) = (g \circ f)^{-1}(b) = f^{-1}(g^{-1}(b)) = f^{-1}(g^{-1}(g(z)) = f^{-1}(z)$, but $f^{-1}(z)$ is $m\chi_N$-totally disconnected set in $X$ (because $f$ is $mN$-light function), which implies $h^{-1}(b)$ is $m\chi_N$-totally disconnected set in $X$, so that $h$ is $mN$-light function.

**Proposition 11**

If $g:(Z, m_\delta) \to (Y, m_\gamma)$ is one-to-one $m$-function, $f:(X, m_\chi) \to (Z, m_\delta)$ is $m$-function and $h:(X, m_\chi) \to (Y, m_\gamma)$ is $mN$-light function such that $h = g \circ f$, then $f$ is $mN$-light function.

**Proof**

Since $g(z) \in Y$, for each $z \in Z$ and $h^{-1}(g(z))$ is $m\chi_N$-totally disconnected set in $X$ (because $h$ is $mN$-light function), and since $h^{-1}(g(z)) = (g \circ f)^{-1}(g(z)) = f^{-1}(g^{-1}(g(z))) = f^{-1}(z)$, so $f^{-1}(z)$ is $m\chi_N$-totally disconnected set in $X$, hence $f$ is $mN$-light function.

**Proposition 12**

If $f:(X, m_\chi) \to (Z, m_\delta)$ is $mN$-totally disconnected function and $h:(X, m_\chi) \to (Y, m_\gamma)$ is a surjective $mN$-light function such that $h = g \circ f$, then $g:(Z, m_\delta) \to (Y, m_\gamma)$ is $mN$-light function.

**Proof**

Since $h^{-1}(y)$ is $m\chi_N$-totally disconnected set in $X$ for each $y \in Y$ (because $h$ is $mN$-light function), and $f(h^{-1}(y))$ is $m\gamma_N$-totally disconnected set in $Z$ (since $f$ is $mN$-totally disconnected function), but $f(h^{-1}(y)) = f((g \circ f)^{-1}(y)) = f(f^{-1}(g^{-1}(y)) = f(h^{-1}(y))$. 

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$g^{-1}(y)$, hence $g^{-1}(y)$ is $m_{X}\omega$-totally disconnected set in $Z$, so that $g$ is $mN$-light function.

**Definition 16**

The $m$-space $(X, m_X)$ is called a zero dimension $m$-space if it has a base of $m_X\omega$-clopen sets.

**Lemma 2**

Every zero dimension metric space is $mN$-totally disconnected space.

**Proof**

Let $X$ be a zero dimension metric $m$-space and $a, b$ are points in $X$ with $a \neq b$, then $X$ is $m$-Hausdorff space and since it is metric $m$-space, then $a$ has a neighbourhood $K$ with $b \notin K$, then there exists a basic $m_X$-open set $W$ which is also $m_X$-closed set in $X$ (since $X$ is zero dimensional $m$-space) and then $W$ is $m_XN$-clopen set (by Remark 2 and since the complement of $m_XN$-open set is $m_XN$-closed set), where $a \in W \subseteq K$, and $W^C$ is $m_XN$-clopen set in $X$ such that $b \in W^C, X=\emptyset \cup W^C$ and $W \cap W^C=\emptyset$, so $X$ is $mN$-totally disconnected space.

**Proposition 13**

Let $X, Y$ be metric $m$-spaces and $f : (X, m_X) \rightarrow (Y, m_Y)$ be a surjective $m$-function where $X$ is $mN$-compact space, then $f$ is $mN$-light function if the inverse image for each $b \in Y$ is a zero dimension a subspace of $X$.

**Proof**

Let $b \in Y$, so $f^{-1}(b)$ is zero dimension metric $m$-subspace of $X$ (since metric is hereditary property), so it is $m_XN$-totally disconnected subspace of $X$ (by lemma (3-63)) and so that $f$ is $mN$-light function.

**New subjects and future work.**

**Definition 17 (12)**

A subset $F$ of $m$-space $X$ is said to be $m_Xg$-closed if for each $m_X$-open set $U$ with $F \subseteq U$, then $m_Xcl(F) \subseteq U$.

**Definition 18**

A subset $G$ of $m$-space $X$ is said to be $m_Xg$-open if $F \subseteq m_X$-Int $(G)$ for each $m_X$-closed set $F$ with $F \subseteq G$.

**Definition 19**

A subset $A$ of $m$-space $X$ is said to be $m_X$-$Ng$-open set if for each $x \in A$, there exists $m_Xg$-open set $U$ containing $x$ such that $U-A$ is finite. The complement of $m_X$-$Ng$-open set is $m_X$-$Ng$-closed set.

There is a relation between Definition 19 and $m_X$-$N$-open set as follows.

**Remark 12**

Every $m_X$-$N$-open set is $m_X$-$Ng$-open, but the converse is not true in general.

**Example 21**

The subset $\{x\}_{x \in R}$ in $(R, m_{ind})$ is $m_X$-$Ng$-open but it is neither $m_X$-open nor $m_X$-$N$-open set.

**Question 1**

Is there a relation between Definition 19 and $m_X$-open set? if there is a relation, is there an example to the converse?

**Question 2**

If we use $m_X$-$Ng$-open set instead of $m_X$-$N$-open in this research, will we get approach results? Now we will use the previously presented set to define another type of $m$-disconnected space, which is:

**Definition 20**

An $m$-space $X$ is said to be $m$-$Ng$-disconnected if it is union of two disjoint $m_X$-$Ng$-open sets.

**Question 3**

What is the relation between $m$-$N$-disconnected and $m$-$Ng$-disconnected space?

In a same way and by using $m_X$-$Ng$-open set, new type of $m$-light function have been defined, which is:

**Definition 21**

A function $f$ from $m$-space $X$ into $m$-space $Y$ is said to be $m$-$Ng$-light if for every $y \in Y$, $f^{-1}(y)$ is $m$-$Ng$-totally disconnected.

**Question 4**

What is the relation between $mN$-light and $m$-$Ng$-light function?

**Remark 13**

There is a definition in the topological space to Nadia Kadum Humadi (13), we can exploit it by using the definition of $m_X$-$\omega$-$g$-open set.

**Conclusions:**

In this research, new spaces namely $mN$-disconnected, $mN$-totally disconnected, $mN$-Hausdorff, $mNT_1$-spaces, have been defined and $mN$-light and inversely $mN$-totally disconnected functions have been introduced.

**Acknowledgements:**

The authors would like to thank Mustansiriyah University (www.uomustansiriyah.edu.iq) Baghdad – Iraq for its support in the present work. We also express our gratitude to the referees for carefully reading the paper and for their valuable comments.

**Conflicts of Interest: None.**

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حول التطبيقات الواهنة وأنماط من الفضائيات باستخدام المجموعات المفتوحة

حيدر جبر علي
1 قسم الرياضيات، كلية العلوم، الجامعة المستنصرية، بغداد، العراق .
2 مديرية تربية الرصافة الثالثة، وزارة التربية، بغداد، العراق.

المقدمة:
قدمنا صيغة ضعيفة من الدوال m-واهنة باستخدام المجموعة mXN -المفتوحة والتي هي الدالة mXN - الواهنة، وقدمنا مفاهيم جديدة للفضاءات غير المتصلة و الفضاءات غير متصلة كليا، العلاقة بينهما قد درست كذلك عرفنا صيغة جديدة من الدوال m-grey المترابطة ودالة m-grey المترابطة كلياً. تتضمن هذه الصيغة بعض الأمثلة والحقائق.

المصطلحات المفاهيمية: الفضاء غير المتصل – mXN -، الفضاء هاوسدورف – mXN -، المجموعة المفتوحة – mXN -، الدالة الواهنة – mXN -، الفضاء غير المتصل كليا – mN -، الدالة غير المتصل كليا – mN -.