I. INTRODUCTION

As the LHC keeps running, the searches of supersymmetry (SUSY) signals such as stop/gluino, sbottom and Higgs mass discovered at 126 GeV \cite{1} continue to push their mass bounds towards to multi-TeV range \cite{2,3}. On the other hand, the argument of naturalness requires the masses of third generation scalars, the Higgsinos and gluinos should be $\sim 1$ TeV. This is the present status of SUSY.

To reconcile the experimental limits and expectation of naturalness, either of them needs subtle reconsiderations. In this paper, we consider relaxing the upper bounds from argument of naturalness. The upper bounds on above soft breaking parameters arise from the significant cancellation among the RGE corrections arising from soft breaking parameters to $m_{H_u}^2$, although their input values are far beyond 1 TeV. This is known as focusing phenomenon \cite{4,5}.

Naively, low fine tuning implies the value of $\mu$ and $|m_{H_u}|$ at EW scale should be both near EW scale. But there exists an exception. In some cases, there is significantly cancellation among the RGE corrections arising from soft breaking parameters to $m_{H_u}^2$, although their input values are far beyond 1 TeV. This is known as focusing phenomenon \cite{6}.

The early attempts such as \cite{7,8} were mainly restricted to SUSY models of grand unification scale (GUT), such as supergravity. One recent work related to focus point SUSY deals with gaugino mediation \cite{9}. In this text, we consider gauge mediated (GM) SUSY models with intermediate or low messenger scale $M$ (for a review see, e.g., \cite{10}). Since the focusing phenomenon can be analytically estimated only if the gaugino masses dominate over all other soft breaking masses, or they are small in compared with the third-generation scalar masses \cite{7,8}, following this observation, we study a type of model of direct GM, in which the gaugino masses are naturally small \cite{10}.

As we will see, there are three free input parameters in our model. Two of them are fixed so as to induce focusing phenomenon, leaving an overall mass parameter $m_0$. The fit to 126 GeV Higgs boson discovered at the LHC then determines the magnitude of this parameter, with $m_0 \sim 4$–6 TeV. Thus, our model is highly predictive in mass spectrum.

In section IIA, we introduce the model in detail. In section IIB, we discuss the focusing phenomenon, the boundary conditions for such structure and the mass spectrum at EW scale. Finally we add some discussions in section III.

II. THE MODEL

A. Setup

The content of the model includes two chiral quark messengers $q + q'$ charged under SM gauge groups as $(3,1,1/3)$ and their bi-fundamental fields $\bar{q'}$; two chiral lepton messengers $l + l'$ charged under SM gauge groups as $(1,2,1/2)$ and their bi-fundamental fields $\bar{l} + \bar{l'}$. The model also includes two chiral singlets of SM gauge group $S + S'$ and their bi-fundamental fields, with which we have a messenger multiplet under $5 + \bar{5}$ representation of SM gauge groups. The superpotential in the model reads \cite{11}:

$$W = fX + Xq\bar{q} + m(q'\bar{q}' + qq') + X\bar{l} + m(l'\bar{l} + l\bar{l'}).$$

We also add a deformation to Eq. (2):

$$W = \lambda H_u s\bar{l}.$$  

\footnote{In \cite{12}, the author also discussed focusing phenomenon in SUSY models of non-minimal GM. In contrast to \cite{11}, we study SUSY models that don’t spoil the grand unification of SM gauge couplings.}

\footnote{It belongs to general Wess-Zumino model, which can be completed as effective theory of strong dynamics at low energy \cite{13}. The direct gauge mediation arises after gauging the global symmetries in the weak theory and identifying them as SM gauge groups.}
Here we have assumed unified mass parameter $m$ and ignored the Yukawa coefficients in Eq.(2) for simplicity. In what follows, we will consider $N$ copies of such messengers multiplets. Note that $N < 6$ so as to maintain the grand unification of SM gauge couplings. The deviation from mass spectrum of minimal GM due to Eq.(3) is controlled by the magnitude of Yukawa $\lambda$. This Yukawa term can be argued to be natural by imposing hidden parity. With this term added, we can uplift the soft mass $m_{H_u}^2$ as required for focusing in gauge mediation with small messenger scale (in compared with GUT scale). Without this Yukawa interaction, there is no possibility to satisfy the condition of focusing. Similar conclusion holds for $M \sim M_{GUT}$.

The SUSY breaking is restored in the spurion field $X = M + \theta^2 F$. One can examine that the gaugino masses at one loop of order $O(F)$ vanish due to the fact $\det M = \text{const}$ ($M$ being the mass matrix of messengers). So we expect that the RGE for $m_{H_u}^2$ is dominated by stop masses, $m_{H_u}^2$ and $A_t$ term induced by Eq.(4).

B. Focusing And Mass Spectrum

Following the observation [4, 5, 7] that the REGs for $A_t$ and scalar masses such as $m_{H_u}^2$ are affected by both themselves and gluino masses, while the RGE for gluino mass is only affected by itself, we can solve the RGEs for scalar masses,

$$
\begin{pmatrix}
    m_{H_u}^2(Q) \\
    m_{A_t}^2(Q) \\
    \lambda_2^2(Q) \\
    \lambda_2^2(Q)
\end{pmatrix}
= \kappa_1 \zeta^2(Q) \begin{pmatrix}
    3 \\
    2 \\
    1 \\
    1
\end{pmatrix}
+ \kappa_0 \zeta(Q) \begin{pmatrix}
    3 \\
    2 \\
    1 \\
    0
\end{pmatrix}
+ \kappa_0' \begin{pmatrix}
    1 \\
    0 \\
    -1 \\
    0
\end{pmatrix}
+ \kappa_0' \begin{pmatrix}
    0 \\
    1 \\
    -1 \\
    0
\end{pmatrix},
$$

(4)

for small gluino masses in compared with these scalar soft masses. Here,

$$
\zeta(Q) = \exp \left( \int_{\ln M}^{\ln Q} \frac{d \ln y^2(Q')}{8\pi^2} d \ln Q' \right)
$$

(5)

In the context of MSSM, $\zeta(1 \text{ TeV}) \simeq 0.527$ for $M = 10^8$ GeV. The condition for focusing phenomenon can be derived from Eq.(4) by imposing $m_{H_u}^2(1 \text{ TeV}) \simeq 0$. For $F/M^2 << 1$, we assume $m_{H_u}^2(M) = +m_0^2$, the mass spectrum which induces focusing at EW scale can be parameterized as,

$$
\begin{pmatrix}
    m_0^2 \\
    1.41 + x - 1.58y \\
    1.82 - x - 3.16y \\
    9y
\end{pmatrix}
\rightarrow
\begin{pmatrix}
    1 \times m_0^2 \\
    0.74 + x - 1.58y \\
    1.48 - x - 3.16y \\
    1.66y
\end{pmatrix}
$$

(6)

where $x$ and $y$ are two real numbers. The right side in Eq.(6) are soft masses at RG scale $\mu = 1$ TeV. Note that one can alternatively rescale the number $x$ as in [7] such that $m_{\tilde{Q}_3}^2$ only depends on $x$. Similarly, for $m_{H_u}(M) = -m_0^2$, Eq.(6) is instead of,

$$
\begin{pmatrix}
    -1 \\
    -1.41 + x - 1.58y \\
    -1.82 + x - 3.16y \\
    9y
\end{pmatrix}
\rightarrow
\begin{pmatrix}
    0 \\
    -0.74 + x - 1.58y \\
    -1.48 - x - 3.16y \\
    1.66y
\end{pmatrix}
$$

(7)

This parameterization appears when $F/M^2 \rightarrow 1$. In this limit, $m_{H_u}^2$ is dominated by the one-loop negative contribution proportional to Yukawa coupling $\lambda$. From Eq.(7), there is no consistent solution to $x$ and $y$ under this limit, we will focus on the case of small SUSY breaking.

From Eq.(2) and Eq.(4) we determine the input soft masses in Eq.(1), which are functions of parameters $\lambda$, $N$ and ratio $F/M^2$ in $X$. So we build the connection between $x$, $y$ and $\lambda$ and $N$ by matching the soft masses. For the three input parameters $m_0$, $x$ and $y$ for focusing in the model, two of them can be fixed by the choices of Yukawa $\lambda$ and messenger number $N$. We choose $x$ and $y$ for analysis. Fig.1 shows that plots of $x$ (dotted) and $y$ (solid) as function of $\alpha_3$ and $N$. Our parameters which give rise to focusing phenomenon are read from the crossing points between vertical line for each $N$ and solid curve for $y$ and dottted curve for $x$, respectively. Therefore, there is only one free parameter in the model we study, which is very predictive in the mass spectrum and signal analysis.

Since we performe our analysis in perturbative theory, in order to avoid Landau pole up to GUT scale, the Yukawa coupling $\alpha_3$ is upper bounded, $\sim 0.1$ for our choice of messenger scale. The dotted and solid horizontal lines in fig.1 refer to allowed ranges for $x$ and $y$, respectively. These ranges are derived from the requirement that the stop soft masses aren’t tachyon-like and the $A_t$ squared is positive. Following this fact we obtain,

$$
0 < y < 0.40, \quad -0.74 < x < 1.48, \quad 1.58y - 0.74 < x < 1.48 - 3.16y, \quad 1.58y - 1.41 < x < 1.82 - 3.16y,
$$

(8)

It is easy to verify that the crossing points for each case of $N$ satisfy the constraints above.
With focusing phenomenon we have single free parameter, namely $m_0$ at hand. It can be uniquely determined in terms of the mass of Higgs boson observed at the LHC.

$$m^2_h = m^2_Z \cos^2 2\beta + \frac{3m^4_t}{4\pi^2 v^2} \left\{ \log \left( \frac{M_S^2}{m^2_t} \right) + \frac{1}{2} \tilde{A}_t + \frac{1}{2} \log \left( \frac{M_S^2}{m^2_t} \right) \right\}$$

Here $v = 174$ GeV and $\tilde{A}_t = \frac{2X_t^2}{M^2_S} \left( 1 - \frac{X_t^2}{12M^2_S} \right)$, with $X_t = A_t - \mu \cot \beta$. We choose $\tan \beta = 20$ for simplicity. For larger values of $\tan \beta$, the fit to Higgs boson mass doesn’t change much. From fig.2 one observes that $m_0 \sim 4.0 - 7.0$ for interpretation of 126 GeV Higgs boson.

Substituting the values of $m_0$ from fig.2 and $x$, $y$ from fig.1 inot Eq. (9) we find the mass spectrum in our model, which is shown in table one. As for gaugino masses, the contribution of order $O(F)$ at one-loop level vanishes, so as well the two-loop contribution of order $O(F^2)$. So the leading term at one loop is of order $O(F^3/M^3)$ [10], and the next-to-leading order term is of order $O(F^5/M^5)$.

$$\text{Fig. 2 shows how } m_h \text{ changes as parameter } m_0 \text{ for different messenger numbers. We have used two-loop level Higgs boson mass in the MSSM [9],}$$

$$m^2_h = m^2_Z \cos^2 2\beta + \frac{3m^4_t}{4\pi^2 v^2} \left\{ \log \left( \frac{M_S^2}{m^2_t} \right) + \frac{1}{2} \tilde{A}_t + \frac{1}{2} \log \left( \frac{M_S^2}{m^2_t} \right) \right\}$$

The magnitude of gaugino mass relative to $m_{Q_3}$ under the limit $F/M^2 << 1$ is given by,

$$m_{\tilde{g}}_{,3} \mid_{M} \simeq 0.1 \times \frac{\sqrt{N_{\alpha_i}}}{\sqrt{2 \times (4\alpha_3^3(M) + 4\alpha_3^3(M) + \alpha_3^2(M))}}$$

Using one-loop RGEs for gluino masses, we find their values at the renormalization scale $\mu = 1$ TeV. One observes from table 1 that the gluino mass is below the 2013 LHC bound $\leq 1.2$ TeV. In this sense, the model is rather sensitive to the gluino mass bound. Roughly speaking, without extra significant modifications to the mass spectrum, gluino mass bound which exceeds 1 TeV will exclude this simple model, despite it provides a natural explanation of Higgs boson mass (which means low fine tuning) and it is consistent with present experimental limits.

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3 We refer the reader to [13] for the deviation to spectrum of minimal GM due to Eq. (9).
FIG. 2. \( m_h \) vs \( m_0 \) for different messenger numbers. \( N = 1, 2, 3, 4 \) from bottom to top, respectively. Multi-TeV \( m_0 \) is required for interpretation of 126 GeV Higgs boson.

| \( m_0 \)  | \( N = 1 \) | \( N = 2 \) | \( N = 3 \) | \( N = 4 \) |
|----------|----------|----------|----------|----------|
|          | 7.0      | 5.9      | 4.0      | 3.5      |
| \( m_{i_1} \) | 3.12     | 3.62     | 4.54     | 4.83     |
| \( m_{i_2} \) | 7.65     | 4.98     | 4.80     | 6.0      |
| \( A_f \) | 1.64     | 1.48     | 1.50     | 1.50     |
| \( m_2 \) | 0.6      | 0.80     | 0.67     | 0.67     |
| \( m_2 \) | 0.25     | 0.30     | 0.20     | 0.20     |
| \( m_1 \) | 0.13     | 0.12     | 0.10     | 0.10     |
| \( \mu \) | 0.50     | 0.42     | 0.28     | 0.24     |

TABLE I. Given a focus point, input mass parameter \( m_0 \) (in unit of TeV) required for \( m_h = 126 \) GeV and corresponding soft mass spectrum (in unit of TeV) at renormalization scale \( \mu = 1 \) TeV in the context of MSSM, for different vaules of messenger number \( N \). As for electroweak superparticles, since \( \mu \) is a few times of \( M_1,2 \) for each case, \( m_{S_1} \sim M_1, m_{S_2} \sim m_{\tilde{C}_1} \sim M_2 \) and \( m_{\tilde{C}_2} \sim \mu \).

Finally, we estimate the deflection to focusing phenomenon in the model. Gaugino masses in table one show that they are a few times smaller than stop soft masses. This means the deflection due to gaugino masses is small during the RG running. Another main source for deflection is due to \( S \) term, which vanishes at the input scale in the context of minimal GM. So its effect on focusing is also under control.

### III. DISCUSSION

From mass spectrum of table one, the main source for fine tuning arises from \( \mu \) term. The fine tuning parameter \( c \), which is defined as \( c = max\{c_i\} \), with \( c_i = | \partial \ln m_2^2 / \partial \ln a_i | \) and \( a_i \) being the soft breaking mass parameters, has been reduced from \( \sim 2000 \) to \( \sim 20 \) due to the focusing phenomenon.

As for other indirect experimental limits such as flavor changing neutral violation, the model feels confortable. Because the masses of the three-generation sleptons and first two-generation squarks are all of order \( \sim 1 \) to multi-TeV, with highly degeneracy in each sector.

What about the sensitivity of our results to the messenger scale? At first, assuming that there exists a completion of strong dynamics at high energy indicates that \( M \) should be smaller than the GUT scale. Typically, we have \( M < 10^{10} \) GeV in the context of direct gauge mediation. For the case of low-scale mediation, i.e, \( M < 10^8 \) GeV, the gluino mass is already close to the 2013 LHC mass bound. In other words, \( M = 10^8 \) GeV as we studied in detail is the cirtial value in this model. For intermediate scale \( M \sim 10^8 - 10^{10} \), there is no significant deviation from the results we have shown.

In summary, the promising signals for this simple and natural model include searching gluino, neutralinos and charginos at the LHC. The NLSP is mainly bino-like with mass \( \sim 150 \) GeV, and the light chargino is of order \( \sim 300 \) GeV. LHC bound on gluino mass above 1 TeV, if solid enough, will exclude such simple and natural model, although it is consistent with all other experimental limits.

**Acknowledgement** The work is supported in part by Natural Science Foundation of China under grant No.11247031.

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