Evolution of the density parameter in the anisotropic DGP cosmology

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Abstract

Evolution of the density parameter in the anisotropic DGP braneworld model is studied. The role of shear and cross-over scale in the evolution of $\Omega_\rho$ is examined for both the branches of solution in the DGP model. The evolution is modified significantly compared to the FRW model and further it does not depend on the value of $\gamma$ alone. Behaviour of the cosmological density parameter $\Omega_\rho$ is unaltered in the late universe. The study of deceleration parameter shows that the entry of the universe into self accelerating phase is determined by the value of shear. We also obtain an estimate of the shear parameter $\Sigma H_0 \sim 1.68 \times 10^{-10}$, which is in agreement with the constraints obtained in the literature using data.

1 Introduction

Results from the recent observations of the type-I supernovae and WMAP suggest that we are living in more or less flat type universe [1, 2, 3, 4]. These implies that the cosmological density parameter $\Omega$ must be equal to unity or very much near to it. The inflationary scenario also supports the value of the density parameter which is very much close to unity. For the estimation of the cosmological density parameter to be unity it is assumed that apart from the dark energy and baryonic matter the universe also contains

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dark matter. Therefore the contents of the universe are of paramount relevance in understanding the geometry and evolution of the universe. Hence determination of the density parameter is very important because it can put constraints on cosmological model building and that are subjected to observations. In general the density parameter can be considered as dynamical quantity rather than a constant. For the evolutionary study of the density parameter one has to take into account the role of the effective equation of the state of the epoch of the universe under consideration. Most of the recent observations estimate the value of total density parameter $\Omega_0$ to be close to unity, assuming the homogeneity and isotropy of the spacetime. However, one can also consider the universe that deviates from such idealized situation, to accommodate an anisotropic spacetime. Historically, anisotropic models of the universe have been studied to avoid assumptions of specific initial conditions in the Friedmann-Robertson-Walker (FRW) models and also it is believed that dynamics of early universe is profoundly influenced in presence of anisotropy just below the Planck or string scale [5, 6]. Bianchi models are also relevant in the context of Belinskii-Khalatnikov-Lifshitz (BKL) conjecture [7] which states that in the Einstein equations ‘terms containing time derivatives will dominate over those containing spatial derivatives’ as one approaches a space-like singularity, such a model is described by homogeneous and anisotropic cosmologies. Evolution of the cosmological density parameter in presence of anisotropy is studied and showed that the initial conditions are robust with respect to shear and viscosity [8]. In the present work our aim is to examine dynamical behaviour of the density parameter in the anisotropic Dvali-Gabadadze-Porrati (DGP) braneworld cosmology, which has the potential to explain the current accelerating phase of the universe. The accelerated expansion of the universe is again suggested by the recent observations such as type-I supernovae and WMAP [1, 2, 3, 4]. The DGP braneworld cosmological model has also been studied from observational point of view and these have put constraints on model parameters [9, 10].

The DGP braneworld cosmological model is basically inspired by higher dimensional theories that have been studied in great detail in recent times. In these models our observed four dimensional (4D) universe is a three dimensional hypersurface known as brane embedded in a higher dimensional spacetime called bulk (for a review of brane cosmology see [11]). The application of such scenario to the homogeneous and isotropic FRW brane leads to modification of the Friedmann equations with a quadratic correction to energy density at higher energies. Many issues in cosmology like inflation, dark energy, cosmological constant were investigated in the braneworld cosmological scenario and interesting results were obtained. Later the studies
were extended to anisotropic brane, such as cosmological solutions to the Bianchi-I and V braneworld models were obtained and it is shown that for matter obeying barotropic equation of state, there could be an intermediate phase of high anisotropy which in turn leads to an isotropic phase in the later universe [12]. Another interesting result is obtained in [13], with the scalar field as source, a large initial anisotropy induces significant damping in the dynamics of the scalar field resulting in greater inflation. The shear dynamics on Bianchi I brane cosmological model with a perfect fluid is considered and shown that shear parameter has a maximum during a transition period from non-standard to standard cosmology [14].

Motivated by these developments, we consider evolution of the density parameter in an anisotropic DGP braneworld model and also examine the late time acceleration in this model. Anisotropic brane in the DGP model is also considered in [15] and solutions are obtained without affecting the basic features of the model. The DGP cosmological model possesses two classes of solutions; one which is close to the standard FRW cosmology and the other is either a fully five dimensional regime or a self-inflationary solution which produces accelerated expansion. The self-accelerated branch is known to contain ghost and tachyonic like excitations [17, 18, 19, 20, 21], but nevertheless its ability to explain the late time acceleration makes it an interesting case for study and has generated enormous amount of interest in the recent years. We mainly focus on the effects of shear and cross over scale on the dynamics of dimensionless density parameter and the deceleration parameter. The evolution equation for $\Omega_\rho$ is derived in the self-accelerated branch and for the fully five dimensional branch and the corresponding deceleration parameter are also obtained.

2 DGP dynamics on Bianchi-I brane

In the DGP model our universe is a 3-brane embedded in five-dimensional (5D) Minkowski bulk and there is an induced four-dimensional Ricci scalar on the brane, due to radiative correction to the graviton propagator on the brane [22]. In this model gravity is modified at large length scales and there is a crossover scale between the four-dimensional and five-dimensional gravity given by

$$r_c = \frac{k_5^2}{2\mu^2}$$

where $k_5^2$ and $\mu^2$ are the 4D and 5D gravitational constants respectively. Below the crossover scale $r_c$, the potential has the usual Newtonian form and above which the gravity becomes five-dimensional (A similar scenario with
a scalar curvature term in the brane action is considered in [23, 24] with a brane embedded in AdS bulk. Here, it is shown that it is possible to generate a four-dimensional theory of gravity depending upon AdS lengths. We start with a generalized DGP model in which both bulk cosmological constant $\Lambda$ and brane tension $\sigma$ are non-zero. The effective Einstein equation on the brane can be written [15] as

$$\left(1 + \frac{\sigma k^2_5}{6\mu^2}\right) G_{\mu\nu} = -\left(\frac{k^2_5 \Lambda}{2} + \frac{k^2_5 \sigma^2}{12}\right) q_{\mu\nu} + \mu^2 T_{\mu\nu} + \frac{\sigma k^4_5}{6} \tau_{\mu\nu} + \frac{k^4_5}{\mu^2} F_{\mu\nu} + k^4_5 \Pi_{\mu\nu} + \frac{k^4_5}{\mu^2} L_{\mu\nu} - E_{\mu\nu},$$

(1)

$$\Pi_{\mu\nu} = -\frac{1}{4} \tau_{\mu\rho} \tau^\rho_\nu + \frac{1}{12} \tau\tau_{\mu\nu} + \frac{1}{8} q_{\mu\nu} \tau_{\alpha\beta} \tau^{\alpha\beta} - \frac{1}{24} q_{\mu\nu} \tau^2,$$

(2)

$$F_{\mu\nu} = -\frac{1}{4} G_{\mu\rho} G^\rho_\nu + \frac{1}{12} G G_{\mu\nu} + \frac{1}{8} q_{\mu\nu} G_{\alpha\beta} G^{\alpha\beta} - \frac{1}{24} q_{\mu\nu} G^2,$$

(3)

$$L_{\mu\nu} = \frac{1}{4} (G_{\rho\sigma} \tau^\rho_\mu \tau^\sigma_\nu + \tau_{\rho\sigma} G^\rho_\mu \tau^\sigma_\nu) - \frac{1}{12} (\tau G_{\mu\nu} + G \tau_{\mu\nu}) - \frac{1}{4} q_{\mu\nu} (G_{\alpha\beta} \tau^{\alpha\beta} - \frac{1}{3} G \tau),$$

(4)

where $\tau_{\mu\nu}$ is the energy-momentum tensor of the matter fields on the brane, $\sigma$ is the brane tension and $T_{\mu\nu}$ is the bulk energy-momentum tensor. It can be seen that apart from the quadratic matter field corrections $\Pi_{\mu\nu}$ to energy-momentum, there are corrections coming from the induced curvature term through $F_{\mu\nu}$ and $L_{\mu\nu}$. $E_{\mu\nu}$ are the non-local corrections, coming from extra dimensions which correspond to the projection of the bulk Weyl tensor on the brane. If we define the four velocity comoving with matter as $u^\mu$, the non-local term takes the following form,

$$E_{\mu\nu} = -4r_c^2 [U(u_\mu u_\nu + \frac{1}{3} h_{\mu\nu}) + P_{\mu\nu} + Q_\mu u_\nu + Q_\nu u_\mu].$$

(5)

Where $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ and

$$U = -\frac{1}{4r_c^2} E_{\mu\nu} u^\mu u^\nu,$$

(6)

is the effective non-local energy density on the brane.

$$P_{\mu\nu} = -\frac{1}{4r_c^2} \left[ h_{\mu}^\alpha h_{\nu}^\beta - \frac{4}{3} h_{\mu}^{\alpha\beta} h_{\nu}\right] E_{\alpha\beta}$$

(7)
is the effective non-local anisotropic stress and,

\[ Q_\mu = \frac{1}{4r_c^2} h_\mu^\alpha E_{\alpha\beta} u^\beta, \]

represents the effective non-local energy flux on the brane. The brane energy momentum satisfies the conservation equations \( \nabla^\mu \tau_{\mu\nu} = 0 \), the detailed description of conservation and dynamical equations is given in [25].

We are interested in anisotropic cosmological models on the brane, hence we consider the Bianchi I brane model. We start with the 5D metric of the following form

\[ ds^2 = dy^2 + q_{\mu\nu} dx^\mu dx^\nu, \]

where the brane is located at \( y = 0 \). The Bianchi metric on the brane is given by,

\[ ds^2|_{y=0} = q_{\mu\nu}(y = 0) dx^\mu dx^\nu = -dt^2 + a_i^2(t) dx^{i2}. \]

The conservation equation takes the following form,

\[ \dot{\rho} + \Theta (\rho + p) = 0 \]

\[ D^\nu P_{\mu\nu} = 0 \]

\[ \dot{U} + \frac{4}{3} \Theta U + \sigma^{\mu\nu} P_{\mu\nu} = 0 \]

where dot denotes the \( u^\nu \nabla_\nu \) and \( \Theta \) represents the volume expansion rate and \( \sigma^{\mu\nu} \) is the shear rate. There is no evolution equation for \( P_{\mu\nu} \), since it is bulk degree of freedom and cannot be predicted from the brane [25]. The Hubble parameters for the Bianchi I metric is given by \( H_i = \frac{\dot{a}_i}{a_i} \) and one can define the mean expansion factor as \( S = (a_1 a_2 a_3)^{1/3} \), thus

\[ \Theta \equiv 3H = \frac{\dot{S}}{S} \equiv \sum_i H_i. \]

The modified Raychaudhuri equation for the Bianchi-I metric on the brane is obtained as [15],

\[ \dot{\Theta} + \frac{1}{3} \Theta^2 + \sigma^{\mu\nu} \sigma_{\mu\nu} = -\frac{k_5^2}{2} \left[ \frac{1}{12\mu^2} \left( 3 \left( H^2 + \frac{K}{S^2} \right) - \mu^2 \rho \right)^2 + \frac{1}{4} \rho (\rho + 2p) \right. \\
\left. - \frac{1}{2\mu^2} \left( 3 \left( H^2 + \frac{K}{S^2} \right) (\rho + p) + G_{ii} \rho \right) \right. \\
\left. + \frac{1}{4\mu^2} \left( 3 \left( H^2 + \frac{K}{S^2} \right) \left( 3 \left( H^2 + \frac{K}{S^2} \right) + 2G_{ii} \right) + \frac{8r_c^2}{k_5^2} \right) \right]. \]
and Gauss-Codacci eqns are,
\[ \dot{\sigma}_{\mu \nu} + \Theta \sigma_{\mu \nu} = 4r_c^2 P_{\mu \nu} \]  
(16)

\[ \frac{2}{3} \Theta^2 + \sigma^{\mu \nu} \sigma_{\mu \nu} = \frac{\mu^4}{6 \mu^4} \left( 3 \left( H^2 + \frac{K}{S^2} \right) - \mu^2 \rho \right)^2 + 4r_c^2 U. \]  
(17)

The system of equations are not closed on the brane due to the presence of anisotropic stress term \( P_{\mu \nu} \) and complete bulk solutions are needed to determine brane dynamics. This problem is avoided (as in earlier papers \[13, 15, 25\]) by choosing a special case \( U = 0 \), this vanishing of non-local energy density leads via (13) to condition \( \sigma_{\mu \nu} P_{\mu \nu} = 0 \). This consistency condition implies a condition on evolution of \( P_{\mu \nu} \) as \( \sigma_{\mu \nu} \dot{P}_{\mu \nu} = 4r_c^2 P_{\mu \nu} P_{\mu \nu} \), following from Eq. (16). As there is no evolution equation for \( P_{\mu \nu} \) on the brane, this is consistent on the brane (for complete discussion see \[15, 25\]). This allows us to find the dynamics of shear scalar, equation (16) can be contracted with shear and integrated to get,
\[ \sigma^{\mu \nu} \sigma_{\mu \nu} = 2 \sigma^2 = \frac{6 \Sigma^2}{S^6}, \quad \dot{\Sigma} = 0. \]  
(18)

The modified Friedmann type equation in the DGP cosmology for the Bianchi-I metric (10) is obtained using eqns (17) and (18) as
\[ (H^2 + \frac{K}{S^2}) - \frac{\Sigma^2}{S^6} - \frac{2 \mu^2}{k_2^2} \epsilon \sqrt{\left( H^2 + \frac{K}{S^2} \right) - \frac{\Sigma^2}{S^6}} = \frac{\mu^2}{3} \rho, \]  
(19)

where \( \epsilon = \pm 1 \) corresponds to two possible embedding of the brane in the bulk space-time and gives rise to the two branches of solutions in the late universe. We can rewrite (19) as
\[ H^2 + \frac{K}{S^2} = \left( \frac{1}{2r_c^2} + \sqrt{\frac{1}{4r_c^2} + \frac{\mu^2 \rho}{3}} \right)^2 + \frac{\Sigma^2}{S^6}, \]  
(20)

and define the following dimensionless variables
\[ \Omega_\rho = \frac{\mu^2 \rho}{3H^2} \]  
(21)
\[ \Omega_r_c = \frac{1}{4H^2 r_c^2} \]  
(22)
\[ \Omega_K = -\frac{K}{H^2 S^2} \]  
(23)
\[ \Omega_\Sigma = \frac{\Sigma^2}{H^2 S^6}. \]  
(24)
The Friedmann type equation (20) takes the form
\[
1 = \left( \sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_\rho} \right)^2 + \Omega_\Sigma + \Omega_K
\] (25)
which implies the variables must take values in the interval [0,1] and further it is seen that the normalisation condition is modified from the usual FRW case. For the $K = 0$ isotropic DGP model, people have estimated values of $\Omega_{r_c}$ and $\Omega_\rho$ and compared with observation and which are in good agreement with theory [26]. For the Bianchi I case we have to take into account $\Omega_\Sigma$ to get correct estimate of values of $\Omega_{r_c}$ and $\Omega_\rho$, as can be seen from (25), except in the limit $S \to \infty$ when effects of shear are negligible.

![Figure 1. Evolution of $\Omega_\rho$ for $\epsilon = +1$, for different values of $\Omega_{r0}$, with $\Sigma = 0.001$, $K = 0$, $\gamma = 4/3$ and $r_c = 0.1$.](image)

3 Evolution of cosmological density parameter

Next we study the evolution of density parameter $\Omega_\rho$ and see its dependence on shear $\sigma$ and cross over scale $r_c$. For this we consider the two branches of solutions ($\epsilon = \pm 1$) of the DGP scenario separately. The branch $\epsilon = +1$ corresponds to the self-accelerating solution in the late universe, without a cosmological constant. Expanding equation (20) under the condition $\mu^2 \rho r_c^2 \ll 1$ and considering $\epsilon = +1$ case up to first order [15, 27], we obtain the Friedmann equation of the form,
\[
H^2 + \frac{K}{S^2} = \frac{1}{r_c^2} + \frac{2}{3} \mu^2 \rho + \frac{\Sigma^2}{S^6}. \tag{26}
\]
Using the definition (21) the above equation can be written as,
\[
H^2(2\Omega_\rho - 1) = -\frac{1}{r_c^2} - \frac{\Sigma^2}{S^6} + \frac{K}{S^2}, \tag{27}
\]
it can be noted from the above equation that $K = 0$ no longer implies that $\Omega_\rho = 1$, as in the standard FRW case, we see the extra dependence on $r_c$ which is a typical feature of the DGP model. Also the effects of shear $\Sigma$ cannot be neglected and value of $\Omega_\rho$ is constrained by these parameters. We also derive the following quantity and will use it later

$$\dot{H} = -H^2 - \mu^2(\rho + p) + \frac{1}{r_c^2} + \frac{2}{3} H^2 \rho - 2 \frac{\Sigma^2}{S^6}. \tag{28}$$

Figure 2. Evolution of $\Omega_\rho$ for $\epsilon = +1$ in the isotropic case ($\Sigma = 0$) and for different values of $\Sigma$, with $\Omega_{\rho_0} = 0.04$, $K = 0$, $\gamma = 4/3$ and $r_c = 0.1$. In order to study the evolution of $\Omega_\rho$, we differentiate eqn (21) and use eqns (26) and (28) to get,

$$\frac{d\Omega_\rho}{d\tau} = H\Omega_\rho \left[ (3\gamma - 2)(2\Omega_\rho - 1) - \frac{1}{H^2} \frac{2}{r_c^2} + \frac{1}{H^2} \frac{4\Sigma^2}{S^6} \right]. \tag{29}$$

Where we have assumed that the total energy density and pressure are related by linear barotropic equation of state, $p = (\gamma - 1)\rho$. Next, to obtain equation for the $\Omega_\rho(S)$ phase plane analysis we divide eqn (29) by $\frac{dS}{d\tau} = HS$ and substituting $H$ from (27) we get,

$$\frac{d\Omega_\rho}{dS} = \frac{\Omega_\rho}{S} \left[ (3\gamma - 2)(2\Omega_\rho - 1) + (2\Omega_\rho - 1) - \frac{2}{r_c^2} + \frac{4\Sigma^2}{S^6} \right]. \tag{30}$$

The above equation can be integrated to get,

$$\Omega_\rho = \left( 1 + \frac{1 - 2\Omega_{\rho_0} S^6}{\Omega_{\rho_0} S_0^6} - 3K S^4 r_c^2 + \Sigma^2 r_c^2 \left( \frac{S}{S_0} \right)^{3\gamma - 6} \right)^{-1}, \tag{31}$$

here $\Omega_{\rho_0}$ and $S_0$ denote the values of density parameter and scale factor at the present time. It is seen from (31) that evolution equation depends on the shear and its behaviour is significantly effected in the early universe, depending on the value of $\Sigma$. And the value of $\Omega_\rho$ as $S \to 0$ is around 0.5 for
all values of $\gamma$ in the range $(0,2)$, whereas in the FRW it depends on the $\gamma$ and equation of state. We analyse this further by giving phase plane plots of $\Omega_\rho$, Figure 1 shows behaviour of $\Omega_\rho$ as a function of $S$ for different values of $\Omega_{\rho 0}$ in the self accelerated branch of the solution. The value of $\Omega_{\rho}$ for $S$ very close to zero is same for different values of $\Omega_{\rho 0}$ and starts deviating for large values of $S$. Figure 2 shows the comparison of $\Omega_\rho$ in the isotropic case $^1$ along with different values of $\Sigma$ in the anisotropic case. It is observed that for the isotropic case $\Omega_\rho$ starts with a high value, whereas for anisotropic case $\Sigma = 0.1, 1$ the plot is peaked at intermediate value of $S$ and starts decreasing for large values of $S$. The dependence of density parameter on $\gamma$ is shown in the figure 3, the asymptotic value of $\Omega_\rho$ as $S \to 0$ is around 0.5 irrespective of the value of $\gamma$. And the density parameter decays faster for small values of $\gamma$. Finally the effects of cross over scale on the evolution of cosmological density parameter is shown in Figure 4. We have shown that changing the value of crossover scale i.e. $r_c = 0.1, 1, 2$ does not vary $\Omega_\rho$ much, whereas if we simultaneously change the value shear parameter $\Sigma$ to a higher value, the behaviour is significantly modified for the same values of $r_c$.

![Figure 3](image-url)

Figure 3. Evolution of $\Omega_\rho$ for $\epsilon = +1$ with different values of $\gamma$ and $\Omega_{\rho 0} = 0.04$, $K = 0$, $\Sigma = 0.001$, $r_c = 0.1$.

We can define the following dimensionless deceleration parameter $q = -\frac{\ddot{S}S}{\dot{S}^2}$ or $q = -\frac{H + H^2}{H^2}$ to study the accelerated phase of the universe, which can be written using (28) as,

$$q = (3\gamma - 2)\Omega_\rho - (2\Omega_\rho - 1)\frac{S^6 - 2\Sigma^2 r_c^2}{-S^6 - \Sigma^2 r_c^2 + K S^4}. \quad (32)$$

Acceleration takes place when $q < 0$, which is determined in the standard FRW case by the value of $\gamma$ and $\gamma = 2/3$ forms the critical value. It is clear from equation (32) in this case entry into the accelerating phase is determined

$^1$For plotting the isotropic case we have used the expression $\Omega_\rho = \frac{1}{1 + \frac{1 - 2\Omega_{\rho 0}}{\Omega_{\rho 0}} \frac{S^6 - H^2 r_c^2}{S^6 + H^2 r_c^2}} \left(\frac{S}{\Omega_{\rho 0}}\right)^{3\gamma - 6}$. 

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by values of the shear and cross-over scale. Figures 5 shows the behaviour of deceleration parameter as a function of scale factor, Fig.5(a) shows the dependance of $q$ on the shear and it is seen that higher value of the shear imply that the universe enters an accelerated phase at later time. In figure 5(b) the deceleration parameter is plotted for different values of $r_c$, it is seen the irrespective of the value of cross over scale, the universe enters a self accelerated phase at the same time. And in both the cases (5(a) and 5(b)) it is noted that in the limit $S \to 0$, $q$ approaches a value 2.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Evolution of $\Omega_\rho$ for $\epsilon = +1$ for different values of $r_c$ and $\gamma = 4/3$, $\Omega_{\rho_0} = 0.04$, $K = 0$. In the left panel we have taken the value of $\Sigma = 0.001$ and in the right panel $\Sigma = 0.01$, to show that a higher value of shear will lead to large variation in $\Omega_\rho$ with $r_c$.}
\end{figure}

Next, we consider the $\epsilon = -1$ case (which is also known as the fully 5D case [28]), again expanding (20) with condition $\mu^2 r_c^2 \ll 1$ we get the Friedmann type equation as,

$$H^2 + \frac{K}{S^2} = \frac{\mu^4 r_c^2}{9 \rho^2} + \frac{\Sigma^2}{S^6}. \quad (33)$$

Since it represents the fully 5D case, the Friedmann equation has only quadratic contribution from the energy density. We define new dimensionless parameter as

$$\Omega_{\rho^2} = \frac{\mu^4 r_c^2}{9 H^2} \rho^2. \quad (34)$$

this allows us to rewrite the equation (33) in the form

$$H^2(\Omega_{\rho^2} - 1) = \frac{K}{S^2} - \frac{\Sigma^2}{S^6}, \quad (35)$$

and

$$\dot{H} = -H^2 - \frac{\mu^4 r_c^2}{9} \rho(2\rho + 3p) - 2\frac{\Sigma^2}{S^6}. \quad (36)$$
Next, we discuss evolution equation for the dimensionless density parameter, for this we follow the method used in $\epsilon = +1$ case. Differentiating (34) with respect to time and using equations (35) and (36) we obtain,

$$
\frac{d\Omega_{\rho^2}}{d\tau} = H(\Omega_{\rho^2} - 1)\Omega_{\rho^2}\left[2(3\gamma - 1) + \frac{4\Sigma^2}{S^4K - \Sigma^2}\right].
$$

(37)

Integrating equation (37) gives,

$$
\Omega_{\rho^2} = \left(1 + \frac{1 - \Omega_{\rho^2}^0}{\Omega_{\rho^2}^0}\left[\frac{KS^4 - \Sigma^2}{K S^4_0 - \Sigma^2}\right]\left(\frac{S}{S_0}\right)^6(\gamma - 1)\right)^{-1},
$$

where $\Omega_{\rho^2}$ is the value at the present time. Let us analyse the effects of shear on the behaviour of $\Omega_{\rho^2}$. Fig.6(a) shows evolution of $\Omega_{\rho^2}$ for the the $\epsilon = -1$ case, it is seen that asymptotic value of $\Omega_{\rho^2}$ for $S \to 0$ is around 1 for all values of $\Omega_{\rho^2}^0$. But, if we take different values of $\gamma$ the behaviour changes drastically, figure 6(b) shows $\Omega_{\rho^2}$ for different values of $\gamma$ for $K = 0$. It is observed that for $\gamma = 4/3, 2$ in the limit $S \to 0$ the value of $\Omega_{\rho^2}$ is around 1 and decays from there. On the other hand for $\gamma = 2/3$ in the limit $S \to 0$ the value of $\Omega_{\rho^2} \to 0$ and increases for large values of $S$.

Now the deceleration parameter can be obtained in this case using (35) and (36) as,

$$
q = (3\gamma - 1)\Omega_{\rho^2} + 2(\Omega_{\rho^2} - 1)\frac{\Sigma^2}{S^4K - \Sigma^2}.
$$

(39)

Here it is noticed that when $K = 0$, entry in to the accelerated branch is determined by the value of $\gamma$. And, $\gamma = 1/3$ forms the critical value that separates the accelerating phase from the decelerating, compared to FRW case where $\gamma = 2/3$ is the critical value.

Figure 5. Plot for deceleration parameter $q$ for $\epsilon = +1$ (a) shows different values of shear parameter $\Sigma$ and $r_c = 0.1$, (b) shows evolution with different values of $r_c$ and $\Sigma = 0.001$ with $\Omega_{\rho^0} = 0.04$, $\gamma = 4/3$, $K = 0$. 

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Figure 6. Figure (a) shows behaviour of $\Omega_\rho$ in $\epsilon = -1$ case for different values of $\Omega_{\rho_0}$ and $\gamma = 4/3$. (b) for different values of $\gamma$ and $\Omega_{\rho_0} = 0.04$. In both the plots we use $\Sigma = 0.001$, $K = 0$ and $r_c = 0.1$.

4 Conclusions

We studied the evolution of cosmological density parameter in an anisotropic DGP braneworld model. The two branches of solutions are considered separately but we emphasized on the self accelerated branch with shear parameter and crossover scale. It is found that the evolution of dynamical density parameter is not altered much during the late universe by the presence of shear. It is seen in figure 1 that asymptotic value of $\Omega_\rho$ as $S \to 0$ approaches 0.5 and decreases for large values of $S$. The behaviour of density parameter for different values of shear is shown in figure 2 and is compared with the isotropic case. In the early universe shear plays a significant role and in the late universe evolution coincides with that of a isotropic DGP case. It is also observed that in the presence of shear irrespective of the value $\gamma$, the behaviour of $\Omega_\rho$ is same near $S = 0$. The density parameter $\Omega_\rho$ does not seem to vary much with value of cross over scale, for a smaller value of shear, as shown in the figure 4(a). Increasing the value of shear to a higher value ($\Sigma = 0.01$) the density parameter varies significantly, depending on cross over scale, in the early universe. But in the late universe is insensitive to the value of $r_c$. Deceleration parameter is plotted in figure 5, it shows that in the presence of shear universe enters a self accelerating phase at later time, for high values of shear. It is observed that value of $r_c$ does not change the evolution of $q$ significantly and universe enters a self-accelerated phase as shown in figure 5(b). Now, in the $\epsilon = -1$ or fully five dimensional branch of the solution, the density parameter shows interesting behaviour depending on the value of $\gamma$, in the limit $S \to 0$ the evolution is drastically changed for $\gamma = 2/3$. Looking at the deceleration parameter in this case, we found that $\gamma = 1/3$ forms the critical value that divides the accelerating phase from the decelerating. In this work we were interested in the $K = 0$ case for
both the branches of DGP model, the behaviour of $\Omega_\rho$ and $q$ would change significantly for the universe with a non-zero spatial curvature.

As the anisotropic effects are relevant at high redshifts and decreases with the expansion of the universe, we would like to estimate the value of the shear at the last scattering surface. We obtain the value of shear from our equations to be of the order $\Sigma H_0 \sim 1.68 \times 10^{-10}$, this is in agreement with the limits obtained for the other Bianchi models. Hence, the estimated value of shear from our model is compatible with the late time evolution of DGP model and the crossover scale in this case is consistent with the one obtained by others[9, 10]. Finally, we would like to conclude that the dynamical evolution of density parameter does not affect late time behaviour of the DGP model in the anisotropic case and the results are in agreement with isotropic case in the late universe.

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