Co-optimization of power line de-energization and restoration under high wildfire ignition risk

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Abstract—Electric power infrastructure has ignited several of the most destructive wildfires in recent history. Preemptive power shut-offs are an effective tool to mitigate the risk of ignitions from power lines, but at the same time can cause widespread power outages. Electric utilities are thus faced with the challenging trade-off of where and when to implement these shut-offs, as well as how to most efficiently restore power once the wildfire risk is reduced. This work proposes a mathematical optimization problem to help utilities make these decisions. Our model co-optimizes the power shutoff (considering both wildfire risk reduction and power outages) as well as the post-event restoration efforts related to inspection and energization of lines. It is implemented as a rolling horizon optimization problem that is resolved whenever new forecasts of load and wildfire risk become available. We demonstrate our method on the IEEE RTS-GMLC test case using real wildfire risk data and forecasts from US Geological Survey, and investigate the sensitivity of the results to the forecast quality, decision horizon and system restoration budget. The software implementation is available in the open source software package PowerModelsWildfire.jl.

Index Terms—Public Safety Power Shutoff, Power Grid Restoration, Mixed-Integer Optimization, Model Predictive Control

I. INTRODUCTION

Wildfire ignitions caused by electrical equipment are an increasing concern for power grid operators. A study of Australia’s bushfires found that electrical infrastructure accounts for 30% of ignitions during droughts and heat-waves [1]. In California, the state firefighting organization’s annual wildfire activity report Redbook reports that between 2015-2020, the 10% of wildfires that were ignited by electrical equipment were responsible for more than 70% of damages ($17.5 billion) [2].

Research on interactions between wildfires and the electric grid after a fire has been ignited include derating of power line capacity near a progressing wildfire [3], [4], operations when wildfire approach electric systems, [5]–[7], and fast simulations of wildfire spread to provide early warnings for transmission line shutdowns [8]. However, the focus has recently shifted towards how to reduce the risk of wildfire ignitions. Refs. [9]–[12] review approaches to reduce the risk of ignitions, and adaptations of this kind are already being planned by utilities. For example, Pacific Gas & Electric (PG&E) announced an initiative to underground 10,000 miles of distribution lines over the course of more than a decade at a cost of $20 billion [13], along with other measures such as more frequent vegetation clearing within the right of way of power lines. Unfortunately, the significant cost and need for qualified crews and equipment means that these risk reduction approaches take time to implement.

Meanwhile, power system operators rely on short-term measures to reduce risk, including changes to the protection system settings (i.e. disabling of automatic reclosing) [3] or Public Safety Power Shutoffs (PSPS), which de-energize grid equipment during high wildfire risk events. While highly effective in reducing wildfire risk [14], PSPS leads to intentional disconnection of customers, causing large scale power outages that may last for days [15]. Since the economic and health impacts of power outages can be significant, it is important to balance the benefits of wildfire risk reduction with the impacts of power outages. To this end, [16] proposed a framework to optimally balance wildfire risk reduction and power outage sizes when deciding which lines to shut off. Other approaches include data-driven methods to plan PSPS by training machine learning models with optimization problems results [17]–[19], or using a dynamic programming approach to optimize a PSPS [20]. These methods focus on accelerating the solution time of a planned PSPS for improved use on large networks in real time. Ref. [21] proposes an optimal investment model that considers installation of batteries and under-grounding power lines to mitigate and reduce the impact of PSPS and [22] studies how microgrids can improve resilience against unplanned outages due to events like wildfires. Further research on PSPS include [23], which expands on time-varying risk and operational factors such as energy storage for temporal load-shifting, and [24], which considers the fairness of repeated PSPS events for utility customers. Another related research includes improved risk forecasting for electric grid wildfire risk [19], [25].

Although the above methods may be effective in identifying locations where the implementation of PSPS is required, they do not explicitly consider how to effectively restore the system once the wildfire risk is reduced. A main reason for the prolonged outages following a PSPS event is that utility crews must manually inspect the de-energized power lines for damage that could cause ignitions before they can be placed back into operation. Thus, while de-energization of a line can happen instantaneously, the re-energization (or restoration) can take hours or days [26]. To make optimal decisions about which lines to de-energize, it is therefore necessary to assess how de-energization impacts the risk of wildfire ignitions and power outages both during the initial shut-down and throughout the restoration process.

In this paper, we aim to address this gap by developing a multi-period optimal power shut-off (MOPS) problem which...
co-optimizes power line shut-offs and restoration for the power grid in situations with high wildfire risk. The formulation combines and extends our prior work on balancing wildfire risk and power outages for a single time period [16] and post-disaster restoration planning [27]. The objective function maximizes the electric load served to customers, while minimizing total wildfire risk, and accounting for utility capacity to perform line inspections.

Since the optimal operational decisions depend on the current and forecasted risk and must be updated as new information becomes available (e.g. every day), we formulate the problem as a rolling horizon optimization problem. The problem is challenging to solve due to the presence of binary variables representing the on/off status of the transmission lines, as well as the large number of grid constraints across multiple time periods [27]. In our case study, we therefore investigate the impact of the decision horizon on the solution quality. We also assess how forecast errors for the wildfire risk and different restoration budgets impact the solution.

In summary, the contributions of this paper are 1) a rolling horizon, multi-period optimization model which co-optimizes public safety power shutoffs and the subsequent grid restoration, 2) analysis of the sensitivity of the problem to several parameters including the forecast errors, forecast horizon and the restoration budget.

The remainder of the paper is organized as follows. Section II introduces the problem formulation. Section III presents the case study and problem setup, and analysis of the results. Section IV concludes the work.

II. MODELING AND PROBLEM FORMULATION

In this section, we formulate the Multi-period Optimal Power Shutoff (MOPS) problem. MOPS is a rolling horizon optimization model that provides an optimized schedule for both power line shut-offs and restoration. It combines and extends the single-period optimal power shut-off model in [16] and the grid restoration planning model in [27].

A. Rolling Horizon Formulation

We envision a grid operator of a power grid with a set of buses $B$, lines $L$, generators $G$ and loads $D$. This grid has elevated wildfire risk, and the operator is considering a public safety power shutoff to mitigate the wildfire threat. The operator must make decisions on the power lines they may shutdown, customers they may disconnect, and how to bring the grid back online in the future. Their goal is to reduce shutdown, customers they may disconnect, and how to bring the grid back online in the future. Their goal is to reduce wildfire risk while minimizing customer power outages and maintaining grid reliability.

Wildfire risk depends on ambient conditions such as vegetation cover, humidity and wind speed, and is updated on a regular basis. For example, the Wildland Fire Potential Index (WFPI) [28] is released daily and provides both current and 7-day forecasts for the contiguous United States. Due to the evolving nature of these forecasts, it makes sense to (i) take the forecasts into account when deciding on an optimal schedule for public safety power shut-offs and restoration, and (ii) update the schedule daily as new information becomes available. We therefore formulate our problem as a rolling horizon optimization problem with forecast horizon $H$, as shown in Model [1] (and further explained below). For a given day $T$, a Multi-period Optimal Power Shutoff (MOPS) problem is solved including all timesteps $t \in T$ where $T = \{T,...,T + H\}$. The power shut-off and restoration actions for the first day $t = T$ are then implemented on the power grid. The next day, we repeat the optimization process with $T \leftarrow T + 1$, given the current status of the grid and updated forecast information. Below, we present the MOPS model for an individual day.

B. Objective function

The system operator is pursuing three different objectives, namely maximizing load, minimizing wildfire risk and maintaining grid reliability. We first describe how these three objectives are evaluated.

1) Load served: The load $D_{\text{Served}}$ is calculated as

$$D_{\text{Served}} = \sum_{t \in T} \sum_{d \in D} x_{dt} w_{dt} P_d^D.$$  \hspace{1cm} (1)

Here, $P_d^D$ is a parameter expressing the total demand for electricity from load $d$ and time period $t$ and $w_{dt}$ is a weight used to express increased priority for certain load (e.g. a hospital may have a higher weight). We assume that load can be continuously shed, with the continuous decision variable $x_{dt}$ representing the percentage of load that is served. The total demand served $D_{\text{Served}}$ is obtained by summing over all loads $d \in D$ and all time periods $t \in T$.

2) Wildfire risk: We express the total risk of wildfire ignitions based on the wildfire risk associated with each transmission line in the network, i.e.

$$R_{\text{Fire}} = \sum_{t \in T} \sum_{ij \in L} z_{ijt} R_{ijt}.$$  \hspace{1cm} (2)

Here, $R_{ijt}$ represents the risk of a wildfire ignition from line $ij$ at time $t$ if the line is energized (we discuss how to obtain these risk values in the case study). The binary decision variable $z_{ijt}$ represents the status of the line, with $z_{ijt} = 1$ indicating that it is on and $z_{ijt} = 0$ indicating that it is off. By choosing to de-energize the line, i.e. setting $z_{ijt} = 0$, we can thus reduce the wildfire risk of line $ij$ at time $t$ to zero.

3) Grid Vulnerability: Because the grid benefits from having many transmission lines and multiple paths for power to flow along, we want to avoid disabling too many lines. In particular, we would like to keep low risk lines in operation even if they do not contribute to serving more load, because removing too many lines from service reduces redundency in the transmission network. To characterize the effect of de-energizing lines on system security, we define the system vulnerability as

$$V_{\text{System}} = \sum_{t \in T} \sum_{ij \in L} (1 - z_{ijt}) V.$$  \hspace{1cm} (3)

Here, $V$ represents the vulnerability associated with turning an individual line off and $V_{\text{System}}$ represents the total vulnerability of the network.
Next, we combine $R_{\text{Fire}}$ and $V_{\text{System}}$ in a single term,
\[ R_{\text{Fire}} - V_{\text{System}} = |L|/T \forall V + \sum_{t \in T} \sum_{ij \in L} z_{ijt} (R_{ijt} - V) \]

This expression highlights that $V$ represents the minimum wildfire risk for which it is beneficial to disable a line, and we will hence refer to $V$ as the vulnerability threshold. Choosing a non-zero value for $V$ thus provides an incentive to keep low risk lines, or lines that may be low risk in the future, energized to increase redundancy.

We recognize that expressing grid vulnerability on a line-by-line basis is less comprehensive and meaningful than using other metrics such as N-1 security. However, it does allow us to express a preference for keeping low risk lines in operation. A more in-depth modeling and investigation of grid vulnerability and operational security in the context of high wildfire risk is left for future work.

Given the above modeling considerations, we formulate the objective function (5a). Note that the objective function uses the total load $D_{Tot}$ and the total wildfire risk $R_{Tot}$ before a power shut-off as normalization factors, with $D_{Tot}, R_{Tot}$ defined as
\[ D_{Tot} = \sum_{t \in T} \sum_{d \in D} P_{dt}^D, \quad R_{Tot} = \sum_{t \in T} \sum_{ij \in L} R_{ijt} \]

This normalization implies that the objective functions expresses the percentage of load or wildfire risk after implementation of the PSPS, with $0 \leq D_{Served} / D_{Tot} \leq 1$ and $0 \leq R_{Fire} / R_{Tot} \leq 1$.

The trade-off parameter $\alpha \in [0, 1]$ allows us to express a preference for either focusing on serving load or mitigating wildfire risk. The preference for mitigating wildfire risk vs limiting grid vulnerability is expressed by our choice of the vulnerability threshold $V$.

\section{C. Restoration Constraints}
An important and novel aspect of our formulation is the consideration of a limited restoration budget, i.e. a limited capacity to inspect and restore power lines after a public safety power shut-off. The limits on how many miles of lines can be restored in each time period is described by constraints (5j)-(5n). Constraints (5j) set the indicator variable for restoration $y_{ijt}$ to 1 if a line is offline in the previous period $z_{ijt-1}^L = 0$ and on in the current period $z_{ijt}^L = 1$. These logical constraints are implemented in the optimization problem by (5c), where the three inequalities form the logical negation of $z_{ijt-1}^L$ as well as the and operation,
\[ y_{ijt}^L \leq (1 - z_{ijt-1}^L) \quad (4a) \]
\[ y_{ijt}^L \leq z_{ijt}^L \quad (4b) \]
\[ y_{ijt}^L \geq (1 - z_{ijt-1}^L) + z_{ijt}^L - 1 \quad (4c) \]

Constraint (5j) implements the same constraint for the initial condition $z_{ij0}$ of the line in the first period. Constraint (5j) limits the amount of restoration that can occur in a single period according to the budget available $Y_t$. The cost of restoring a line scales with length $\ell_{ij}$, because the primary action in restoring a line is to inspect the line for damage

\section{D. Power Flow Constraints}
To model how de-energization and restoration of lines impact the amount of electricity served to customers, it is important to model the power flows in the system. Equations (5j)-(5n) uses a DC power flow formulation to model the power flow in each time period, accounting for shedding load and de-energized/restored lines.

Nodal power balance is enforced by (5j) where total generation $P_{gt}^G$, transmission line power $P_{ijt}^L$, and load served $x_{dt}^D$ must sum to zero at each bus. The sets $G, L, D$ represent the sets of generators, lines and load demands at bus $i$. Eq. (5j) ensures that the load shed proportion $x_{dt}^D$ is constrained to be within 0 and 1. The constraint in (5k) constrains the power $P_{gt}^G$ from each generator $g$ to between 0 and its upper power limit $\overline{P}_{gt}^G$. The upper limit may vary with the time period $t$ for renewable energy sources, based on the forecasted maximum output. While this problem formulation can support non-zero lower bounds on generation by including additional binary variables for generator on/off status, we remove the unit commitment aspect in this paper for simplicity.
Equation (5l) constrains the power flow $P_{ijt}^L$ on the line from $i$ to $j$ between the power limits $-P_{ijt}^{L\max}$ and $P_{ijt}^{L\max}$ when the line is energized and $z_{ijt}^L = 1$. When a line is de-energized, $z_{ijt}^L = 0$, the power flow across the line is 0. Equations (5m) and (5n) define the relationship between voltage angle differences and line power flow, while accounting for when a line becomes disabled. When a line is energized, $z_{ijt}^L = 1$, these constraints reduce to ordinary linearized DC power flow,

$$P_{ijt}^L = -b_{ijt} (\theta_{it} - \theta_{jt}),$$

where $b_{ijt}$ is the line susceptance, and $\theta_{it}$ and $\theta_{jt}$ are the bus voltage angles for bus $i$ and $j$ at time $t$. When the line is de-energized, $z_{ijt} = 0$, the power flow is decoupled from the voltage angle difference through the big-M values $\theta_{ijt}^\Delta$ and $\theta_{ijt}^\Delta$, which can be calculated as in [29]. This allows the power flow to be constrained to 0 in $\theta_{ijt}$ without constraining the voltage angle differences.

III. CASE STUDY

We next demonstrate the use of our proposed model and study the sensitivity to the forecast horizon, forecast accuracy, restoration budget and the vulnerability threshold of disabling lines.

A. Case study setup

We first describe our implementation and the data used to set up our case study.

1) Implementation: The optimization model is available in the open source package PowerModelsWildfire.jl [16], implemented in the Julia language [30], and solved using the Gurobi v9.1 optimization solver [31].

2) Test System: We base our case study on the RTS-GMLC [32] system. This synthetic test system has geographic coordinates located in southern California, a region which has been affected by Public Safety Power Shut-Offs. The system has one year of hourly load and renewable energy profiles, and we choose to use data from the month of October. For each day, we pick the hour with the highest load forecast. To obtain a case with an interesting level of congestion, we increased the active power demand using the API method in [33], which proportionally increases each load in the system until transmission line limits are constrained. This increased the load and generation in the network by a factor of 2.14, and lead to a higher utilization of the transmission network. We use the a uniform load priority weight $w_{dt} = 1$.

3) Wildfire risk data: Wildfire risk data was obtained from the United States Geological Survey’s (USGS) Wildland Fire Potential Index (WFPI) [28] which estimates the potential a one-acre fire will spread and burn more than 500-acres. This measure incorporates fuel models and forecast information for precipitation, dry bulb temperature and wind speeds to create a daily forecast of the WFPI for up to 7 days. For each line and each day, we calculate the wildfire risk as the highest WFPI value along the line, in accordance with [34]. Using the maximum risk implies that we focus on minimizing risk at particularly risky locations.

To provide an overview of the wildfire risk data, Figure 1a shows a histogram of the wildfire risk of individual transmission lines, based on all realized wildfire risk observed in the month of October. Figure 1b shows the realized (black) and forecasted (orange) wildfire risk for the overall system, calculated as the sum of all wildfire risk values across all lines. Each forecast is represented with a line that covers the next 7 days.

To provide an overview of the wildfire risk data, Figure 1a shows a histogram of the all the realized wildfire risk values (i.e. not including forecasted data) for all lines across all days in the month of October. We observe that many lines have either zero risk or risk in the range between 70-120. There is a small number of lines that have very high risk > 160. Figure 1b shows the total system risk before any power shut-offs (i.e., the sum of all wildfire risk values for all lines, assuming all lines are energized) based on the forecasted (in red) and realized (in black) wildfire risk values. We observe that there are substantial forecast errors, with the forecast sometimes underestimating and sometimes overestimating the risk.

4) Baseline Parameters: We set $\alpha = 0.7$ as we found this to provide a reasonable trade-off between load shed and risk, based on previous work [16]. We further use the following parameter selections, unless otherwise specified:

- Optimal Gap: 0.01%
- Solver Time Limit: 1 hour
- Restoration Budget: 75 miles/day
- Forecast Horizon: 4 days
- Vulnerability Threshold: 100 WFPI

B. Baseline Solution

We first present the output of running this method in on the RTS-GMLC system using the baseline parameter settings. The MOPS problem is solved for each day, using information about the current on/off status of the grid and the forecasted load and wildfire risk. The output is a set of optimal shutoff and
restoration actions, along with a DC power flow dispatch and load shed. Recall that the optimization problem considers a 4-day forecast horizon, but that we only implement the actions for the current day (before reoptimizing the next day).

1) Total system risk and load shed: We first analyze the impact on total system risk and total load served for this baseline case. Fig. 2(a) shows the total system risk value without (orange) and with (blue) power shut-offs, and Fig. 2(b) shows the total load without (orange) and with (blue) power shut-offs. In the first few days of the case study, the grid risk is moderate. A small number of lines are de-energized to reduce the risk, but no load shed occurs. On Oct 29th, a large spike in risk occurs, leading to a significant grid shutoff, and resulting in around 5% load shed for this day. On Oct 30th, the wildfire risk values return to moderate levels, and sufficiently many lines are restored to avoid significant load shed. However, we observe that the total system risk after accounting for power shut-offs is lower than the total system risk without accounting for power shut-offs because many lines remain de-energized due to limited restoration resources.

2) Geographical allocation of risk and load shed: Next, we analyze how the load shed and wildfire risk is allocated geographically in the system. Figure 3(a) shows the state of the grid (top) and the wildfire risk values for each line (bottom) on Oct 28, 29 and 30. On Oct 28, there are several lines with risk values $R_{ijt} > V$ that are de-energized, but these de-energizations do not result in any load shed. On Oct 28, the wildfire risk has increased dramatically in the eastern region of the grid in 3(a). This results in a large portion of the lines in this region to be turned off, as seen in 3(b), but high risk lines remain energized to continue serving power. In total, there is partial load shed at 2 buses and total load shed at 3 buses. By Oct 30, the risk is reduced and many lines have been restored. There is still partial load shed at 2 buses, but no buses with total load shed.

3) Solution times: The baseline experiment involved solving the rolling horizon MOPS problem with a 4 day forecast horizon for 21 days, and solved in 29 seconds. However, we note that the solve time is highly dependent on the problem parameters, in particular to the vulnerability threshold $V$ and $\alpha$. In some instances it could take more than 12 hours to solve the problem.

C. Sensitivity to Forecast Horizon

Next, we investigate the impact of the forecast horizon on the solution. The USGS provides a seven day WFPI risk forecast, and we solve the rolling horizon problem with forecast horizons ranging from 1 to 7 days. We expect that a longer horizon may improve the decision making, but also increase the size of the optimization problem and associated solve time. In our experiments, the solve time increases from 1.93s 1-day horizon to 28.67s for a 4-day horizon and to 213.63s for the 7-day horizon.

We first assess the impact on the objective function. We calculate the total load served with Eq. (1), total system risk with (2) and system vulnerability with (3), using the realized decisions and true wildfire risk for each time step and summing the values across the entire decision horizon. Table I(a) show the 3 elements of the objective function (after scaling by the factor $\alpha$) and the final objective function value for forecast horizons from 1 to 7 days. Figure 4 shows the percentage of load served $D_{Served}/D_{Tot}$, the percentage of wildfire risk $R_{Served}/R_{Tot}$ and the normalized system vulnerability $V_{System}/R_{Tot}$.

From Table I(a), we observe that the total objective function value improves by approximately 4.2% as the forecast horizon increases from 1 to 7 days, with the most significant improvements achieved within 3 days with a reduction of 3.2%. Figure 4 shows that the forecast horizon has a very small impact on the total load served, but that for shorter horizons, the total system risk is smaller, while the system vulnerability is higher.

We next consider the total line energization and de-energization events that occur during the 21 day period, shown in Table I(b). The forecast horizon has a major impact on the number of de-energized lines. We observe 93 de-energizations with a 1-day horizon, which is reduced to 74 de-energizations when using a 7-day horizon. The number of line restorations is more similar for the two problems, with 61 restorations when using a 1-day horizon compared with 54 restorations when using a 7-day horizon. The total miles of restored power lines do not follow the same pattern. The two scenarios that restore the fewest miles of lines are with a 1-day (1,316 miles) and 7-day horizon (1,307 miles). A 3-day horizon results in the most miles of restored power lines, restoring 1,436 miles. The total length of restored lines is likely impacted by the ability to use as much of the daily restoration budget as possible.

Table II(c) shows the average, maximum and minimum risk for the lines. Notably, both the average and minimum line risk for de-energized lines is smaller for shorter forecast horizons, indicating that more low-risk lines remain de-energized due to an inadequate restoration budget.
Overall, we conclude that fewer lines are de-energized when a longer horizon is used, resulting in higher overall wildfire risk but a smaller system vulnerability. The reason behind this trend is that a shorter time horizon does not allow the optimization problem to consider system vulnerability in future time periods and thus limits its ability to adequately plan restoration efforts.

### Table I: Results for different forecast horizons.

| Horizon (days) | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|---------------|-----|-----|-----|-----|-----|-----|-----|
| Load Obj      | 0.297| 0.298| 0.299| 0.299| 0.299| 0.299| 0.299|
| Risk Obj      | -0.380| -0.423| -0.455| -0.466| -0.496| -0.510| -0.510|
| Vulnerability Obj | -0.322| -0.271| -0.235| -0.224| -0.193| -0.178| -0.177|
| Total Objective | -0.404| -0.396| -0.391| -0.391| -0.389| -0.388| -0.387|

### D. Sensitivity to Forecast Accuracy

As observed in Fig. 1b, there are substantial forecast errors associated with the wildfire risk forecast. To test the impact of forecast errors on the results, we solve the same MOPS problem as above with forecast horizons varying from 1 to 7.
TABLE II: Objective values obtained with realized risk for different forecast horizons.

| Horizon (days) | 1       | 2       | 3       | 4       | 5       | 6       | 7       |
|---------------|---------|---------|---------|---------|---------|---------|---------|
| Load Obj      | 0.297   | 0.298   | 0.299   | 0.299   | 0.299   | 0.299   | 0.299   |
| Risk Obj      | -0.380  | -0.423  | -0.455  | -0.466  | -0.496  | -0.510  | -0.510  |
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| Total Objective | -0.404  | -0.396  | -0.391  | -0.391  | -0.389  | -0.388  | -0.387  |

Fig. 5: Wildfire risk of each line for a 7-day horizon Multi-period Optimal Power Shutoff problem. In orange is the realized risk and in blue is the risk forecast made on October 25 for the following week.

To explain this surprising result, we provide a deeper investigation of the optimization solutions for the scenarios using forecasted and realized risk. We consider the result of one multi-period optimization problem for October 25th, using the risk forecast and the realized daily risk values, respectively. We choose to investigate this day because the forecast was particularly poor, and under-estimated the wildfire risk for the following 7 days. Figure 5 shows the risk values for both cases, with forecasted risk in blue, realized risk in orange and the vulnerability threshold in red. The forecasted risk for each individual line was almost entirely below the vulnerability threshold, while the realized daily risk was above the vulnerability threshold for many days. On Oct 29, many lines exhibited a wildfire risk that was twice as high as the vulnerability threshold. Because of the difference between the forecasted risk and the realized daily risk, we might expect a very different shutoff plan to mitigate the risk for the problem based on the forecasted vs the realized wildfire risk. For the initial conditions, we use the state of the grid on Oct 25, while line 118 has been re-energized. Thus, the optimization problem for the next day (when the forecasted risk values are updated) will have the same starting point.

From these results investigation, we conclude that the use of the current, realized risk values in day 1 of the optimization problem makes the problem significantly less sensitive to forecast errors. However, there are situations where even the first day solutions based on forecasted and realized risk deviate. For example, if the forecasted risk is significantly overestimating the future wildfire risk, the problem based on the forecasted risk may choose to shut down more lines than the problem with perfect information and struggle with restoring them sufficiently quickly.

E. Sensitivity to Restoration Budget

The restoration budget \( Y_t \) determines how many miles of transmission lines can be restored each day, and impacts both the number and length of lines that are de-energized and restored. To assess the impact of the restoration budget on the solutions, we solve the MOPS problem for each day in our 21 day horizon with different restoration budgets. Table IV shows the result of varying the restoration budget on the objection value (top), the number of de-energized and restored

Table III shows the total number of line de-energizations and re-energizations using for each solution, along with the miles of restored lines. Over the course of the horizon, the solution based on the forecasted risk de-energizes only 4 lines, while the perfect information solution based on realized wildfire risk de-energizes a total of 55 lines. This indicates that the forecast errors have a significant impact on the solution for later time steps in the forecast horizon.

However, only the decisions for the first day (which are based on the true realized risk in both solutions) are actually implemented before the de-energization schedule is re-optimized. We observe that the de-energization decisions made here are identical: lines 29, 54, and 68 are still de-energized on Oct 25, while line 118 has been re-energized. Thus, the optimization problem for the next day (when the forecasted risk values are updated) will have the same starting point.

IV shows the result of varying the restoration budget on the number of restored lines. Over the course of the horizon, the solution based on the forecasted risk de-energizes only 4 lines, while the perfect information solution based on realized wildfire risk de-energizes a total of 55 lines. This indicates that the forecast errors have a significant impact on the solution for later time steps in the forecast horizon.

Fig. 6: Wildfire risk forecast for a 7-day horizon Multi-period Optimal Power Shutoff problem. In blue is the forecasted risk, and orange shows the realized risk for the 7 days. The dashed line is the risk without a shutoff, and the solid line shows the risk after components have been de-energized.

TABLE III: Comparing line energization and de-energization events when using forecasted and realized wildfire risk.

| Wildfire Risk Data Source | Forecast Risk | Realized Risk |
|---------------------------|---------------|---------------|
| Line Energized            | 4             | 21            |
| Line De-energized         | 4             | 55            |
| Miles of Restored Lines   | 151           | 385           |

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E. Sensitivity to Restoration Budget

The restoration budget \( Y_t \) determines how many miles of transmission lines can be restored each day, and impacts both the number and length of lines that are de-energized and restored. To assess the impact of the restoration budget on the solutions, we solve the MOPS problem for each day in our 21 day horizon with different restoration budgets. Table IV shows the result of varying the restoration budget on the objection value (top), the number of de-energized and restored
TABLE IV: Results for different restoration budgets.

(a) Objective Value Components (scaled by $\alpha$)

| Restoration Budget | 25    | 50    | 75    | 100   | 125   |
|--------------------|-------|-------|-------|-------|-------|
| Load Obj           | 0.299 | 0.299 | 0.299 | 0.299 | 0.299 |
| Risk Obj           | -0.436| -0.459| -0.466| -0.481| -0.493|
| Vulnerability Obj  | -0.264| -0.235| -0.224| -0.206| -0.191|
| Total Objective    | -0.402| -0.395| -0.391| -0.388| -0.385|

(b) Line Energization and De-energization Events

| Restoration Budget | 25    | 50    | 75    | 100   | 125   |
|--------------------|-------|-------|-------|-------|-------|
| Line Energized     | 28    | 45    | 60    | 75    | 83    |
| Line De-energized  | 66    | 77    | 85    | 95    | 99    |
| Miles of Restored Lines | 308     | 890    | 1381   | 1907   | 2325   |

(c) Risk of Energized and De-energized Lines

| Restoration Budget | 25    | 50    | 75    | 100   | 125   |
|--------------------|-------|-------|-------|-------|-------|
| Energized Line Risk | Avg | 54 | 54 | 55 | 55 | 56 |
|                    | Min  | 0   | 0   | 0   | 0   | 0   |
|                    | Max  | 208 | 208 | 206 | 206 | 206 |
| De-energized Line Risk | Avg | 100 | 103 | 104 | 107 | 108 |
|                     | Min  | 1   | 8   | 8   | 40  | 48  |
|                     | Max  | 215 | 215 | 215 | 215 | 215 |

The vulnerability threshold $V$ can be interpreted as a threshold beyond which the wildfire risk is considered sufficiently high to justify shutting down a line, even if it reduces redundancy of the electric grid. If the wildfire risk $R_{ijt} < V$, we pay a penalty in the objective function for de-energizing line $ij$ at time $t$. Thus, the result of increasing the vulnerability threshold is to keep higher risk lines energized. In the following, we first compare solutions with and without the vulnerability threshold. Then, we assess how changing the vulnerability threshold changes the solution.

1) Comparison of solutions with and without vulnerability threshold: To demonstrate the benefit of including a vulnerability threshold that penalizes the shutoff of low-risk lines, we make two different comparisons.

First, we set the trade-off parameter $\alpha = 0.7$ and solve the rolling horizon problem for 21 days using two different vulnerability thresholds. Figure 7(a) and (b) shows the network status on October 29th, the highest risk day, for $V = 0$ and $V = 100$. We observe that the solution with vulnerability threshold $V = 0$ is incentivized to turn off even comparatively low risk lines, leading to many more de-energized lines and significantly more load shed (in red). In comparison, the solution with $V = 100$ only disables a few lines are disabled in the eastern edge of the grid and many redundant lines are maintained, maintaining some resiliency in the network. The two solutions serve 1413 MW and 2038 MW of load and have a wildfire risk of 4,920 and 114,220, respectively. We thus conclude that a non-zero vulnerability threshold promotes more system redundancy and higher load served for a given value of $\alpha$, but also has significantly higher wildfire risk.

Next, we want to compare solutions with different vulnerability thresholds that serve a similar amount of load. To achieve this, we use $V = 0$ and decrease $\alpha$ until the solution serves 2038 MW of load (the same as we obtained for $V = 100$). This result is obtained when $\alpha = 0.2$, and the resulting system configuration for October 29 is shown in Fig. C. We observe that with $V = 0$, the network configuration is poorly connected and results in 10 islands and 6 single-bus microgrids. In addition, within the islands all network connections are radial with no redundancy, leaving the system vulnerable to any unplanned outages that may occur. However, the risk of wildfire in this network is 50,379, less than half of the solution with $V = 100$ that serves a comparable amount of power. This trade-off between system vulnerability and redundancy on the one hand and wildfire risk reduction on the other highlights the inherent challenges of operating power systems in areas with high wildfire risk.

2) Comparison of solutions with different vulnerability thresholds: After demonstrating the benefit of incorporating a non-zero vulnerability parameter $V$, we investigate how the choice of $V$ impacts the solution. We solve the MOPS problem for the 21 day period with three different values $V = \{75, 100, 125\}$ and compute similar comparison metrics as presented for other cases above. The impact of the vulnerability threshold on the objective function components is listed in Table IV(a) and shown graphically in Fig. 8. The total objective value is reduced by 25\% as the vulnerability threshold is increased. This happens primarily because the risk is three times higher for the case with a vulnerability threshold of 125 compared with a vulnerability threshold of 75. This wildfire risk increase outweighs the decreased in

F. Sensitivity to Vulnerability Threshold

The vulnerability threshold $V$ can be interpreted as a threshold beyond which the wildfire risk is considered sufficiently high to justify shutting down a line, even if it increases as the restoration budget increases. This is average and minimum transmission line risk for de-energized lines increases, while the maximum risk decreases. Both the number of de-energized and restored lines increase as the restoration budget increases, thus keeping few lines energized or restored, as well as the miles of restored lines. We observe that a larger restoration budget results in a larger total system risk, but a smaller system vulnerability.

To achieve this, we use $V = 0$ and $\alpha = 0.7$ until the solution serves 2038 MW of load (the same as we obtained for $V = 100$). This result is obtained when $\alpha = 0.2$, and the resulting system configuration for October 29 is shown in Fig. C. We observe that with $V = 0$, the network configuration is poorly connected and results in 10 islands and 6 single-bus microgrids. In addition, within the islands all network connections are radial with no redundancy, leaving the system vulnerable to any unplanned outages that may occur. However, the risk of wildfire in this network is 50,379, less than half of the solution with $V = 100$ that serves a comparable amount of power. This trade-off between system vulnerability and redundancy on the one hand and wildfire risk reduction on the other highlights the inherent challenges of operating power systems in areas with high wildfire risk.
Without a vulnerability threshold on disabling low-risk lines, the network becomes fragmented and all networks are radial.

A higher vulnerability threshold reduces the number of de-energized lines as seen in Table V(b). With $V = 125$, only 21 high-risk lines are de-energized, and all lines are restored by the end of the horizon. Further, the minimum risk of any de-energized line is 140, as seen in Table V(c). Thus, we can conclude that the group of de-energized lines corresponds to the few lines with a very high risk in Figure 1a.

One major impact of increasing the vulnerability threshold is a decrease in the solve time of the problem. A vulnerability threshold of 75 results in a total solve time of 2,676 seconds, compared to 7.5 seconds when the vulnerability threshold is 125. We believe a higher threshold limits the number of lines that are beneficial to de-energize, and allowing the mixed-integer solver to identify optimal solutions more quickly.

IV. CONCLUSION

Wildfire threats to power grid infrastructure are increasing, and public safety power shutoffs (PSPS) represent an effective short-term method to reduce the risk of wildfire ignitions by the electric grid. However, PSPS are challenging to plan due to the time-varying nature of wildfire risk, as well as the need to limit the size of power outages that may last for several days while utility crews work to restore power. To address this challenge, we propose a rolling horizon optimization problem, the Multi-period Optimal Power Shutoff (MOPS) problem, that co-optimizes decisions on where and when to de-energize power lines while also providing a schedule for how to restore them. The MOPS problem is implemented in a rolling horizon framework where the schedules are re-optimized on a daily basis as new information about wildfire risk become available.

We apply the proposed method to the RTS-GMLC system to analyze the performance as well as the sensitivity to the forecast horizon, forecast errors, restoration budget, and vulnerability threshold. We find that (i) a longer forecast horizon...
better accounts for the impact of de-energizing a line on load shed, (ii) the solution is relatively insensitive to forecast errors related to the wildfire risk, (iii) a larger restoration budget allows for disabling and restoring more lines, and (iv) a higher vulnerability threshold incentivizes more system redundancy, thus increasing operational security but also wildfire risk.

The proposed framework has some limitations and provides several avenues for future work. First, the vulnerability threshold is not a comprehensive metric for system security, and additional modeling efforts are needed to capture more standard metrics such as N-1 security. Second, the model uses a linear approximation of the power flow to accelerate the solution time, but this may lead to solutions that are not AC power flow feasible. In addition, despite the linear approximation, the problem can still be very slow to solve even on a small network. The development of heuristic solution techniques may be necessary to plan a PSPS on a realistic scale power network. In future work, we aim to develop algorithms that can solve MOPS in real-time for large systems during wildfire risk events and incorporate constraints that guarantee the resiliency and reliability of PSPS solutions to additional contingency events. We hope to eventually solve an AC-feasible MOPS problem with N-1 security constraints for a synthetic version of the California grid.

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