On the Viability of Two-temperature Accretion Flows

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ABSTRACT
Binney (2003) has argued that two-temperature radiatively inefficient accretion flow models are unphysical because the electron-ion equipartition time is much shorter than the accretion time. I show that this conclusion is incorrect because it relies on a misidentification of the electron-ion equipartition time. I also clarify what requirements must, in fact, be satisfied to maintain a two-temperature accretion flow.

Key words: accretion, accretion disks – black hole physics – galaxies: active – X-ray: stars – binaries

1 INTRODUCTION
Since the mid 1970’s accretion theorists have invoked two-temperature collisionless plasmas in models of accreting compact objects (e.g., Shapiro, Lightman, & Eardley 1976; Ichimaru 1977; Rees et al. 1982; Narayan & Yi 1995). In such models it is usually assumed that Coulomb collisions are the dominant mechanism for exchanging energy between electrons and ions. The inefficiency of collisional energy transfer at high temperatures and low densities (accretion rates), together with the much shorter radiative cooling time of relativistic electrons with respect to nonrelativistic ions, leads to the possibility of a two-temperature plasma with $T_e \gg T_i$.1

An outstanding problem is whether there are any mechanisms that might enforce electron-ion energy equipartition on a timescale much shorter than that of Coulomb collisions (e.g., Phinney 1983; Begelman & Chiueh 1988). If so, then two-temperature accretion flow models would not be viable.

Binney (2003; hereafter B03) has argued that the electron-ion equipartition time is, in fact, much shorter than the accretion time, and thus that two-temperature accretion flow models are unphysical. His argument is actually more general, and implies that electron-ion equipartition is a characteristic feature of plasmas in the presence of a time varying electromagnetic field (in the accretion context, MHD turbulence driven by the magnetorotational instability (MRI; Balbus & Hawley 1991) is an important and widely studied example of such a time varying electromagnetic field). In the next section I briefly review Binney’s argument and explain why it is incorrect. I then summarize the conditions that must be satisfied to maintain a two-temperature accretion flow, and what calculations can be done (and have been done) to address this issue.

2 ELECTRON-ION EQUIPARTITION
B03 estimates the electron-ion equipartition time by first evaluating the work done on a particle by an electromagnetic field (described by potentials $\psi$ and $A$). The Hamiltonian for the motion of a particle of mass $m$ and charge $q$ is

$$H = \frac{(p - qA)^2}{2m} + q\psi + m\Phi.$$  

(1)

where $\Phi$ is the gravitational potential. The change in the particle’s energy is given by $\dot{H}$, i.e.,

$$\frac{dH}{dt} = \frac{\partial H}{\partial \dot{\mathbf{r}}} = -q\frac{\mathbf{p} - q\mathbf{A}}{m} \cdot \frac{\partial \mathbf{A}}{\partial t} + q\frac{\partial \psi}{\partial t}$$

$$= -q\mathbf{v} \cdot \frac{\partial \mathbf{A}}{\partial t} + q\frac{\partial \psi}{\partial t}.$$  

(2)

where, as in B03, I have assumed that $\Phi$ is time-independent. Equation (2) is the work done by the electric field.

Using equations (1) and (2) we can define the timescale on which the energy of a particle changes as

$$t_E \equiv \frac{H}{|dH/dt|}.$$  

(3)

B03 identifies $t_E$ in equation (3) as the timescale for electrons and ions in a two-temperature plasma to come into equipartition, mediated by the electromagnetic field. His argument for this identification is that “since the right side of this equation [eq. (2)] is proportional to the charge $q$, if at some location energy is lost by one species, it is gained by the oppositely charged species. Thus this equation describes the mechanism by which equipartition is established between ions and electrons; the net direction of the energy flow is mandated by the general principles of statistical physics, and the rate of flow may be estimated from equation (2).”

This interpretation of equation (2) is incorrect, as is the resulting identification of equation (3) as the equipartition time. The essential problem is that equation (2) represents the instantaneous work done on a particle by the electric...
field. It does not, however, represent the net energy transfer integrated over time, i.e., the true heating or change in entropy. These differ because of the adiabaticity of particle motion in a plasma and because energy transfer occurs at discrete resonances (Landau and cyclotron) that are not accounted for in equation [3] (see below).

To elaborate on these points, it is useful to focus on a concrete example: consider a magnetic field that varies on a timescale $t$ that is much longer than $\Omega^{-1}$, where $\Omega = eB/mc$ is the cyclotron frequency. The magnetic moment of a particle is then an adiabatic invariant, i.e., $\mu = m\nu_{\perp}^2/B = \text{constant}$, where $\nu_{\perp}$ is the velocity perpendicular to the magnetic field (e.g., Sturrock 1994). Thus, if the magnetic field varies in an arbitrary (slow) manner, and returns to its initial value, the perpendicular kinetic energy (temperature) of each particle, and thus each particle species, is ultimately unchanged. This is in spite of the fact that the energy of every particle instantaneously changes with the magnetic field on the timescale $\sim t$. By contrast, B03’s argument, which considers only this instantaneous change in energy, incorrectly implies that 1. equipartition is established, and 2. that it is established on a timescale given by equation (3). This example highlights that the timescale given by equation (3) is in general the timescale for adiabatic changes, not true heating (let alone equipartition). A similar conclusion could be drawn by considering the energy of a particle in the presence of an undamped Alfvén wave: during the oscillation of the wave magnetic energy gets converted into particle energy and vice versa, but there is no net transfer of energy to or from the particles, and no tendency towards equipartition, during a period of the wave.

3 DISCUSSION

Although the arguments in the previous section show the problems with Binney’s claim that electron-ion equipartition is rapid, they do not specify how to in fact determine whether a two-temperature accretion flow can be maintained. I believe that there are two key calculations that must be done to address this. First, given that MHD turbulence generated by the MRI drives accretion, it is important to understand how the MRI operates in a collisionless plasma and how the resulting turbulent energy is transferred to the particles. There has been a fair amount of analytical work addressing this (e.g., Bisnovatyi-Kogan et al. 1997; Quataert 1998; Gruzinov 1998; Blackman 1999; Quataert & Gruzinov 1999; Medvedev 2000; Sharma et al. 2003). The key physics is that turbulent energy is transferred to particles at discrete wave-particle resonances, when $\omega - k_{\parallel}\nu_{\parallel} \approx n\Omega$, where $n$ is an integer, $\nu_{\parallel}$ is the velocity of a particle along the local magnetic field, and $\omega$ and $k_{\parallel}$ are the frequency and parallel wavevector of a wave comprising the turbulence. The details of which particles are heated depend on which waves are present in the turbulence and which particles are resonant; the net particle heating will in general be different for the ions and the electrons and so there is no reason for the system to approach equipartition. Existing calculations of particle heating using models for MRI-generated turbulence are somewhat uncertain and depend sensitively on $\beta$ (the ratio of the gas pressure to magnetic pressure in the flow), with $\beta \sim 1$ favoring electron heating while higher $\beta > 10$ favors ion heating. This uncertainty stems from (1) uncertainty in the relative importance of the three MHD waves in MRI-generated turbulence, (2) uncertainty in the importance of magnetic reconnection (e.g., Bisnovatyi-Kogan & Lovelace 1997; Blackman 1999), and (3) uncertainty in the behavior of Alfvénic turbulence on small scales comparable to the proton Larmor radius (see Quataert & Gruzinov 1999). Numerical simulations of turbulence in collisionless plasmas are currently underway that will help sort out these issues.

Even if MRI-generated turbulence predominantly heats the ions, there is no guarantee that the resulting two-temperature plasma is stable. There could be kinetic instabilities that transfer ion thermal energy to the electrons on a timescale much shorter than the accretion time, thus enforcing a one-temperature plasma. It is important to stress that the MRI is not an example of such an instability because it feeds on the free energy of differential rotation, not the ion thermal energy. It is also worth noting that a uniform two-temperature collisionless plasma containing particles with Maxwellian distribution functions is stable to linear perturbations (e.g., Stix 1992). Thus a candidate instability must feed on gradients in the background medium or velocity space anisotropies, not the mere presence of a two-temperature plasma. The question of whether there are any such instabilities in the accretion flow context is an open and difficult problem (see Begelman & Chiueh 1988 for the most detailed investigation to date). In the absence of theoretical calculations or numerical simulations to settle this issue, it may be useful to take observations as our guide: both the solar wind and the post-shock environment in supernova remnants show two-temperature plasmas in which it appears that the dominant mechanism for electron-ion energy exchange is Coulomb collisions, not a kinetic instability (see, e.g., Esser et al. 1999 for the solar wind and Michael et al. 2002 for supernova remnants). This qualitatively supports the assumptions of two-temperature accretion flow models, with the necessary caveat that the plasma conditions in these examples are quite different from those around compact objects.

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2 Because there is no general proof for or against the claim that electron-ion thermal equilibration in a collisionless plasma is rapid, this example is intended only to show why B03’s conclusions are incorrect; the more general problem of how to assess whether a two-temperature accretion flow can in fact be maintained is discussed in the next section.
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