Inferred 4.4 eV Upper Limits on the Muon- and Tau-Neutrino Masses

V. Barger\textsuperscript{1}, T.J. Weiler\textsuperscript{2}, and K. Whisnant\textsuperscript{3}

\textsuperscript{1}Department of Physics, University of Wisconsin, Madison, WI 53706, USA
\textsuperscript{2}Department of Physics and Astronomy, Vanderbilt University, Nashville, TN 37235, USA
\textsuperscript{3}Department of Physics and Astronomy, Iowa State University, Ames, IA 50011, USA

Abstract

By combining experimental constraints from atmospheric and solar neutrino oscillations and the tritium beta decay endpoint, we infer upper limits of 4.4 eV on the $\nu_\mu$ and $\nu_\tau$ masses, if the universe consists of three neutrinos. For hierarchical mass spectra $m_3 \gg m_1, m_2$ or $m_3 \approx m_2 \gg m_1$ we infer that $m_{\nu_\alpha} \lesssim 0.08$ eV for $\alpha = e, \mu, \tau$. In a four neutrino universe, and assuming neutrino oscillations also account for the LSND experimental results, $m_{\nu_\alpha} \lesssim 5.4$ eV. We also obtain lower limits on the masses.
The quest to learn whether neutrinos are massive has been long and arduous. The present laboratory endpoint measurements only give upper limits that are not very restrictive in the $\nu_\mu$ and $\nu_\tau$ cases. However, atmospheric and solar oscillation experiments have recently obtained positive results, but oscillation experiments measure only mass-squared differences $\delta m_{ij}^2 \equiv m_i^2 - m_j^2$, leaving the overall mass scale unknown. Phenomenologically, we have the two inequalities

$$\delta m_{\text{sun}}^2 \ll \delta m_{\text{atm}}^2 \ll m_\beta^2 ,$$

where $m_\beta = 4.4$ eV is the upper limit on the effective $\nu_e$ mass from tritium beta decay. We show in this Letter that these constraints may be combined to place an upper bound of $m_\beta$ on the $\nu_\mu$ and $\nu_\tau$ masses in a three-neutrino universe. To also accommodate the indications for $\nu_\mu \rightarrow \nu_e$ oscillations in the LSND experiment with one sterile and three active neutrinos, we obtain an upper bound of 5.4 eV on all four neutrino masses.

The flavor eigenstates $\nu_e, \nu_\mu, \nu_\tau$ in a three-neutrino universe are related to the mass eigenstates $\nu_1, \nu_2, \nu_3$ by a unitarity transformation

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},
$$

where the elements $U_{\alpha i}$ can be expressed in terms of 3 angles and one (three) phase(s) for Dirac (Majorana) neutrinos. It is convenient to introduce the notation

$$P_{\alpha j} \equiv |U_{\alpha j}|^2 .$$

The unitarity constraint on the $P_{\alpha j}$ is

$$\sum_j P_{\alpha j} = 1 .$$

Without loss of generality we take the mass eigenvalues to be real and positive in the following discussion, and order them as

$$m_3 > m_2 > m_1 > 0 .$$

The atmospheric neutrino oscillations have a substantially larger $\delta m^2$ than the solar oscillations, so we define

$$1 \text{The explanation of the atmospheric data requires $\nu_\mu \rightarrow \nu_\tau$ oscillations (or $\nu_\mu \rightarrow \nu_s$ oscillations to a sterile neutrino $\nu_s$) with mass–squared difference $0.5 \times 10^{-3} \lesssim \delta m_{\text{atm}}^2 \lesssim 6 \times 10^{-3}$ eV$^2$ and mixing angle $\sin^2 2\theta_{\text{atm}} \gtrsim 0.8$. The solar neutrino anomaly requires a mass–squared difference of either $\delta m_{\text{sun}}^2 \approx 3.5 \times 10^{-6} - 1 \times 10^{-5}$ eV$^2$ with small-angle vacuum mixing $\sin^2 2\theta_{\text{sun}} = 1.5 \times 10^{-3} - 1.0 \times 10^{-2}$ or a mass–squared difference of $\delta m_{\text{sun}}^2 \approx 1 \times 10^{-10}$ eV$^2$ with large-angle vacuum mixing $\sin^2 2\theta_{\text{sun}} \gtrsim 0.6$ \cite{3}.}$$
\[
\delta m_{\text{atm}}^2 = m_3^2 - m_1^2, \\
\delta m_{\text{sun}}^2 = m_2^2 - m_1^2 \quad \text{or} \quad m_3^2 - m_2^2,
\]
where the two possibilities for \(\delta m_{\text{sun}}^2\) correspond to the two possible mass-squared scenarios shown in Fig. 1. By convention we take \(\delta m_{\text{atm}}^2\) and \(\delta m_{\text{sun}}^2\) to be positive since negative signs can be absorbed into the mixing matrix \(U\). The relation
\[
m_3 = \sqrt{m_1^2 + \delta m_{\text{atm}}^2} \geq \sqrt{\delta m_{\text{atm}}^2},
\]
gives a lower limit on the largest mass via
\[
m_3 - m_1 = \frac{\delta m_{\text{atm}}^2}{m_3 + m_1} \leq \frac{\delta m_{\text{atm}}^2}{m_3} \leq \sqrt{\delta m_{\text{atm}}^2}.
\]
These equations apply equally to the \(\delta m_{\text{sun}}^2 = m_2^2 - m_1^2\) or \(\delta m_{\text{sun}}^2 = m_3^2 - m_2^2\) cases. Thus the three masses are all degenerate to within (see footnote 1)
\[
|m_i - m_j| \leq \sqrt{\delta m_{\text{atm}}^2} \simeq 0.02 - 0.08 \text{ eV},
\]
and are therefore indistinguishable in the tritium endpoint measurement. The tritium endpoint constraint is
\[
\sum_j P_{e_j} m_j^2 < m_\beta^2.
\]
Using Eq. (3) and \(m_j^2 = m_3^2 - \delta m_{3j}^2\), Eq. (12) can be expressed as
\[
m_3 - P_{e_1} \delta m_{31} - P_{e_2} \delta m_{32} < m_\beta^2.
\]
It immediately follows from Eqs. (3) and (13) and the unitarity condition \(P_{e_1} + P_{e_2} \leq 1\) that
\[
m_3 < \sqrt{m_\beta^2 + \delta m_{\text{atm}}^2},
\]
and since \(\delta m_{\text{atm}}^2\) is small compared to \(m_\beta^2\), Eqs. (3) and (14) constrain \(m_3\) to the range
\[
\sqrt{\delta m_{\text{atm}}^2} \leq m_3 < m_\beta,
\]
which numerically bounds \(m_3\) to the range
\[
0.02 \text{ eV} \lesssim m_3 \lesssim 4.4 \text{ eV}.
\]
Also, we have
\[
\sqrt{\delta m_{\text{sun}}^2} \leq m_2 \leq m_3.
\]
In hierarchical mass spectra in which \(m_1 \simeq m_2 \ll m_3\) or \(m_1 \ll m_2 \simeq m_3\), one obtains the equality
\[
m_3 \simeq \sqrt{\delta m_{\text{atm}}^2} \simeq 0.02 - 0.08 \text{ eV}.
\]
The effective mass-squared of a flavor state is
\[ m_{\nu\alpha}^2 = \sum_j P_{\alpha j} m_j^2. \] (19)

When combined with Eqs. (5) and (6), we obtain
\[ m_{\nu\alpha} < \sim m_3, \] (20)
for \( \alpha = e, \mu, \) or \( \tau. \)

This derivation of mass bounds is independent of the values of the mixing-matrix; unitarity of the mixing matrix is sufficient. Values or bounds for the \( P_{ej} \) inferred from solar and atmospheric neutrino data may be input, but as seen in (13), they affect the bound only to order \( (\delta m_{atm}^2/m_3^2). \) The tritium endpoint constraint in Eq. (12) is also quite general, given the sizes of the oscillation mass scales \( \delta m_{atm}^2 \) and \( \delta m_{sun}^2; \) within the precision of the tritium experiment, all three mass eigenstates would appear at the same place in the tritium decay spectrum. If the common mass is too large to have sufficient phase space to contribute to tritium decay, then tritium decay could not occur at all; since tritium decay is observed, the limit applies in all possible cases.

The above results are valid for Majorana or Dirac neutrinos, because neutrino oscillations and beta decay are determined by the mass–squared eigenvalues of the Hermitian matrix \( MM^\dagger. \) However, neutrinoless double–beta decay measurements offer the possibility of further limiting elements of the mass matrix itself for the case of Majorana neutrinos. For Majorana neutrinos, non-observation of neutrinoless double-beta decay \( \text{[8]} \) gives a bound of
\[ |M_{ee}| = |\sum_j m_j P_{ej} e^{i\phi_j}| < 0.46 \text{ eV}, \] (21)
where the \( \phi_j \) are possible phases from the \( U_{ej} \) or the entries in the diagonal mass matrix. Since \( m_3 \) is the largest mass eigenvalue, unitarity leads to the condition
\[ |\sum_j m_j P_{ej} e^{i\phi_j}| \leq m_3. \] (22)

If \( m_1 < 0.38 \text{ eV}, \) then from Eq. (11), \( m_3 < 0.46 \text{ eV}; \) it then follows from Eq. (22) that Eq. (21) is satisfied for all values of the mixing matrix elements \( U_{ej} \) and phases \( \phi_j. \) On the other hand, if \( m_1 \geq 0.38 \text{ eV,} \) Eq. (21) may give an additional constraint, depending on the values of the \( m_j, U_{ej}, \) and \( \phi_j. \)

These considerations can be extended to the four–neutrino case, assuming the fourth neutrino is sterile and that neutrino oscillations also explain the recent LSND results \( \text{[9]} \) (see, e.g., Ref. \( \text{[7]} \)). Equation (12) may then be written as
\[ m_4^2 - \sum_{j<4} P_{ej} \delta m_{kj}^2 < m_3^2, \] (23)
\[ ^2 \text{The LSND data} \text{[9]} \text{indicate a mass–squared difference in the range} 0.3 \text{ eV}^2 < \delta m_{LSND}^2 < 10 \text{ eV}^2, \text{although the KARMEN experiment [9]} \text{rules out part of the LSND allowed region.} \]
where we assume
\[ m_4 > m_3 > m_2 > m_1 > 0. \] (24)

In this case \( m_4^2 - m_1^2 = \delta m_{LSND}^2 \) defines the mass-squared difference that describes the LSND results. Then from Eqs. (23) and (24), and the unitarity condition \( P_{e1} + P_{e2} + P_{e3} \leq 1 \), it follows that
\[ \sqrt{\delta m_{LSND}^2} \leq m_4 < \sqrt{m_3^2 + \delta m_{LSND}^2} \lesssim 5.4 \text{ eV}, \] (25)

which numerically bounds \( m_4 \) to the range
\[ 0.5 \text{ eV} \lesssim m_4 \lesssim 5.4 \text{ eV}. \] (26)

This four-neutrino result is likely to be stable even if more neutrinos exist. The generalization of Eqs. (12), (24), and (25) to a more-neutrinos universe is
\[ m_4 < \left( \sum_j^* P_{ej} \right)^{-1/2} \left[ m_3^2 + \delta m_{41}^2 \sum_{j<4} P_{ej} - \sum_{j>4} P_{ej} \delta m_{j4}^2 \right]^{1/2}, \] (27)

where the * on the sum reminds us to include only neutrino masses small enough to appear in the phase space near the end point spectrum of tritium decay. We know that \( \sum_j P_{ej} \), which is the overlap of \( \nu_e \) with the mass states kinematically accessible to beta decay, is close to unity from the universality of the coupling strength in beta decay and in other weak interaction channels. Setting \( \sum_j P_{ej} \approx 1 \) in Eq. (27) reveals that if \( \delta m_{41}^2 \) is \( \lesssim \delta m_{LSND}^2 \), then \( m_4 \) still satisfies the bound in Eq. (25).

In the same way, it can be shown that Eqs. (14)–(18) continue to hold if more than three neutrinos exist, if \( \delta m_{31}^2 \) is \( \lesssim \delta m_{atm}^2 \). Then, Eq. (20) follows from universality of the weak coupling in processes which produce the \( \nu_e, \nu_\mu, \) and \( \nu_\tau \), e.g., in beta, muon, and tau decay, respectively.

The bounds in Eqs. (14), (18), and (25) have very different implications for cosmology. The fractional contribution from neutrinos to the closure density of the universe is \( \Omega_\nu = 0.02(\sum \nu/\text{eV})h_{70}^{-2} \), where \( h_{70} \) is the Hubble constant in units of 70 km/s/Mpc. With the three–neutrino mass hierarchy \( m_1 \ll m_3 \), neutrino mass makes an insignificant contribution to hot dark matter. On the other hand, with the three–neutrino degenerate case or with four neutrinos, the contribution to hot dark matter may be relevant to large-scale structure formation [10].

In summary, if oscillations of three neutrino flavors explain the atmospheric and solar neutrino data, then all the differences in masses must satisfy \( |m_i - m_j| \lesssim 0.08 \text{ eV}. \) The tritium beta decay endpoint constraint then leads to \( m_{\nu_\alpha} \lesssim 4.4 \text{ eV} \) for \( \alpha = e, \mu, \tau \), and in the case of a hierarchical spectrum with one or two neutrino mass eigenstates much lighter than \( m_3 \), the upper limit becomes \( m_{\nu_\alpha} \lesssim 0.08 \text{ eV}. \) If oscillations of one sterile and three active neutrinos explain the LSND, atmospheric, and solar data, the tritium beta decay endpoint constraint then leads to \( m_{\nu_\alpha} \lesssim 5.4 \text{ eV} \) for \( \alpha = e, \mu, \tau, s \).
Acknowledgements. We thank Sandip Pakvasa for collaboration on previous related work. This work was supported in part by the U.S. Department of Energy, Division of High Energy Physics, under Grants No. DE-FG02-94ER40817, No. DE-FG05-85ER40226, and No. DE-FG02-95ER40896, and in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation and the Vanderbilt University Research Council.

REFERENCES

[1] Review of Particle Properties, Eur. Phys. Jour. C3, 1 (1998).
[2] Kamiokande collaboration, K.S. Hirata et al., Phys. Lett. B328, 146 (1992); Y. Fukuda et al., Phys. Lett. B335, 237 (1994); IMB collaboration, R. Becker-Szendy et al., Nucl. Phys. Proc. Suppl. 38B, 331 (1995); Soudan-2 collaboration, W.W.M. Allison et al., Phys. Lett. B391, 491 (1997); Super-Kamiokande Collaboration, Y. Fukuda et al., hep-ex/9803006, hep-ex/9805006, hep-ex/9807003; T. Kajita, talk at Neutrino-98, Takayama, Japan, June 1998.
[3] B.T. Cleveland et al., Nucl. Phys. B (Proc. Suppl.) 38, 47 (1995); Kamiokande collaboration, Y. Fukuda et al., Phys. Rev. Lett, 77, 1683 (1996); GALLEX Collaboration, W. Hampel et al., Phys. Lett. B388, 384 (1996); SAGE collaboration, J.N. Abdurashitov et al., Phys. Rev. Lett. 77, 4708 (1996); Super-Kamiokande Collaboration, talk by Y. Suzuki at Neutrino–98, Takayama, Japan, June 1998.
[4] See, e.g., N. Hata and P. Langacker, Phys. Rev. D56, 6107 (1997); J. Bahcall, P. Krastev, and Y. Smirnov, hep-ph/9807216.
[5] A.I. Belesev et al., Phys. Lett. B350, 263 (1995); V.M. Lobashev et al., in Proc. of Neutrino-96, ed. by K. Enqvist, K. Huitu, and J. Maalampi (World Scientific, Singapore, 1997).
[6] Liquid Scintillator Neutrino Detector (LSND) collaboration, C. Athanassopoulos et al., Phys. Rev. Lett. 75, 2650 (1995); ibid. 77, 3082 (1996); nucl-ex/9706000.
[7] See, e.g., D.O. Caldwell and R.N. Mohapatra, Phys. Rev. D48, 3259 (1993); J.T. Peltoniemi and J.W.F. Valle, Nucl. Phys. B406, 409 (1993); S.M. Bilenky, C. Giunti and W. Grimus, Eur. Phys. J. C1, 247 (1998); V. Barger, T.J. Weiler, and K. Whisnant, Phys. Lett. B427, 97 (1998); V. Barger, S. Pakvasa, T.J. Weiler, and K. Whisnant, hep-ph/9806328, and references therein.
[8] H.V. Klapdor-Kleingrothaus, hep-ex/9802007; J. Phys. G24, 483 (1998).
[9] KARMEN collaboration, talk by B. Zeitnitz at Neutrino–98, Takayama, Japan, June, 1998.
[10] See, e.g., J. Primack, astro-ph/9707288.
FIG. 1. The two possible mass scenarios if the atmospheric and solar neutrino data are described by the oscillations of three neutrinos.