Kink State in a Stack of Intrinsic Josephson Junctions in Layered High-\(T_c\) Superconductors and Terahertz Radiation

Shizeng Lin\textsuperscript{1,2} and Xiao Hu\textsuperscript{1,2,3}

\textsuperscript{1}WPI Center for Materials Nanoarchitectonics, National Institute for Materials Science, Tsukuba 305-0044, Japan
\textsuperscript{2}Graduate School of Pure and Applied Sciences, University of Tsukuba, Tsukuba 305-8571, Japan
\textsuperscript{3}Japan Science and Technology Agency, 4-1-8 Honcho, Kawaguchiko, Saitama 332-0012, Japan

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A new family of dynamic states are found in a stack of inductively coupled intrinsic Josephson junctions in the absence of an external magnetic field. In this state, \((2m_l + 1)\pi\) phase kinks with integers \(m_l\)'s stack along the \(c\) axis and lock neighboring junction together. Large dc power is pumped into plasma oscillation via kinks at the cavity resonance. The plasma oscillation is uniform along the \(c\) axis with the frequency satisfying the ac Josephson relation. Thus this state supports strong terahertz radiation and seems to be compatible with the recent experimental observations.

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The discovery of the intrinsic Josephson effect \cite{1} in highly anisotropic layered high-\(T_c\) superconductors has opened up a new direction for generation of terahertz electromagnetic (EM) waves, which have many promising applications in materials science, biology, security checking and so on. Much effort has been taken to investigate the feasibility of such a technique. One idea is to use motion of Josephson vortices lattice to excite Josephson plasma, and it attains certain degree of success \cite{4, 5, 6, 7, 8}. In 2007, it was demonstrated surprisingly that a coherent terahertz EM waves were emitted from a mesa of Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_{8+x}\) (BSCCO) single crystal even in the absence of magnetic field \cite{9}. The experiments \cite{9, 10} raise several questions on the mechanism of radiation. First, although the working principle is certainly the ac Josephson effect, it is not transparent how dc input power is pumped into ac plasma oscillation, rather than being dissipated as Joule heating. Secondly, how can in-phase plasma oscillation can be attained in all junctions?

The experiments attracted many theoretical attentions\cite{11, 12, 13, 14}. A new dynamic state is proposed to explain the experiments\cite{11, 12}. In this state, there are \((2m_l + 1)\pi\) kinks localized at the center of the mesa and stacked periodically along the \(c\) axis. The kinks help to pump large energy into plasma oscillation, and a part of which is emitted outside. The kink state has already captured the key observations of experiments, and is believed to be relevant to the emission in experiments.

The inductively coupled sine-Gordon equation appropriately describes the dynamics of gauge invariant phase difference \(P_l\) in a stack of intrinsic Josephson junctions (IJJs)

\[\partial_x^2 P_l = (1 - \zeta \Delta_l^{(2)})(\sin P_l + \beta \partial_t P_l + \partial_x^2 P_l - J_{\text{ext}}), \tag{1}\]

where \(P_l\) is the gauge-invariant phase difference at the \(l\)th junction, \(\Delta_l^{(2)} = Q_{l+1} + Q_{l-1} - 2Q_l\), \(\zeta = \lambda_{ab}^2/(s + D)^2\) the inductive coupling, \(\beta\) the normalized conductance, \(J_{\text{ext}}\) the bias current, \(\lambda_{ab}\) is the penetration depth along the \(c\) axis and \(s\) \((D)\) is the thickness of superconducting (insulating) layer. Length is normalized by penetration depth in the \(ab\) plane \(\lambda_c\) and time is normalized by Josephson plasma frequency \cite{11}. The typical value of \(\beta = 0.02\) and \(\zeta = 7.1 \times 10^4\) for BSCCO. Here we have neglected the variation of the amplitude of order parameter. The treatment is reasonable because the Josephson coupling is extremely small in comparison to the condensation energy. The change in \(P_l\), which is associated with that in Josephson energy, should not cause appreciable change in the amplitude, which is associated with the condensation energy.

In experiments, the thickness of a stack of IJJs is much smaller than the wavelength of electromagnetic wave, which makes the EM wave transmission from IJJs to outside very difficult. Same situation happens in a thin capacitor. Such a phenomenon is known as impedance mismatch. The mismatch in impedance was first formulated explicitly by L. N. Bulaevskii and A. E. Koshelev\cite{15}, and then confirmed by experiments\cite{16} with a cylindrical mesa\cite{17, 18}. As a good approximation, we may neglect the effect of radiation on the dynamics of phase. In the present paper, we will use the non-radiating boundary condition \(\partial_x P_l = 0\). A self-consistent treatment will be reported elsewhere.

Equation (1) allows for a variety of dynamic states, such as state without kink, state with kink and state with solitons, each of which has unique electrodynamics properties\cite{19}. The state with kink\cite{11} is described by

\[P_l(x,t) = \omega t + P_l^0(x) + \text{Re}\{-i g(x) \exp(i\omega t)\}, \tag{2}\]

where the first term at the r.h.s. is the rotating phase with frequency \(\omega\), the second term the static kink and the last term the plasma oscillation whose frequency obeys the ac Josephson relation. The plasma oscillation is uniform in all junctions which can be realized in a thick stack of IJJs. For the first cavity mode, \(g(x) = A_1 \cos k_1 x\) with
$k_j \equiv j\pi/L_x$. It will be shown later that $g(x)$ contains other modes, such as $k_0$, $k_2$, $k_3$... as well, but it is sufficient to take only $k_1$ mode when $A_1 < 1$. The solution with kink in Eq. (2) is not written artificially. Actually, it is observed repeatedly by computer simulations [19], which strongly suggests that this solution is very stable.

For ease of theoretically treatment, we consider the region with $A_1 < 1$. Substituting Eq. (2) into Eq. (1) and linearizing the plasma part [19], we have the equation for the static kink $P_i^s$

$$\partial^2 P_i^s = \frac{iA_1\zeta}{2} \cos(k_1x)\Delta(2) \exp(-iP_i^s). \quad (3)$$

Equation (3) has $(2m_1+1)\pi$ kink solutions. The operator $\Delta(2)$ allows for a variety of arrangements of $P_i^s$ along the $c$ axis.

Here we consider several typical periodic arrangements, i.e. $P_i^s = P_{i+s}$, with $\tau$ the period. The results for $\tau = 2, 4, 6$ are shown in Fig. 1. $(2m_1+1)\pi$ kinks are stacked periodically along the $c$ axis, which takes the advantage of huge inductive coupling to lock junction together. The kink runs sharply from 0 to $(2m_1+1)\pi$ in the region of width $\lambda_P = 1/\sqrt{\zeta|A_1|}$. In the BSCCO system, $\zeta \approx 10^5$ which renders the kink almost a step function. The huge inductive coupling also makes the kink very rigid. As shown in Fig. 1 even for the same period, there are many different ways to pile up the kinks. Thus the kink state occupies finite volume in the phase space, which makes the state easy to access. The phase kink for higher cavity modes can be straightforwardly constructed from the fundamental one. The kink is always at the nodes of oscillating electric field.

With the relation between magnetic field and phase

$$(1 - \zeta\Delta(2))B_1 = \partial_x P_1,$$

it is easy to show that flux associated with the static phase kink $P_i^s$ is

$$(1 - \zeta\Delta(2))F_i^s = (1 - \zeta\Delta(2))D \int_0^{L_x} B_i^s dx = D(2m_1 + 1)\pi.$$  

Thus the static vortices is quantized, depending on $P_i^s$ and the arrangement of $P_i^s$.

Since no external magnetic field is applied, the total dc flux is zero in the IJJs, i.e. $\sum_i \Phi_i^s = 0$. Thus the value of kink summed over all junctions vanishes, i.e. $\sum_i(2m_1 + 1) = 0$. For even period such as $\tau = 2, 4, 6$ shown in Fig. 1, the $(2m_1 + 1)\pi$ kinks commensurate with the period very well. But for odd period, there is no way to put $(2m_1 + 1)\pi$ kinks in junctions while keeping total dc flux zero, which makes the kinks incommensurate with IJJs. As depicted in Fig. 2 the kinks adjust themselves in order to eliminate the static flux. The resulting kinks deviate slightly from $(2m_1 + 1)\pi$.

From the equation for the frequency component $\omega t$, we obtain the expression for $A_1$

$$A_1 = \frac{F_1}{i\kappa_1^2 - i\omega^2 - \beta\omega} \quad (4)$$

with

$$F_1 = -\frac{2i}{L_x} \int_0^{L_x} \exp(iP_i^s) \cos(k_1x) dx \quad (5)$$

$F_1$ represents the coupling between the cavity mode and plasma. Here only the mode $k_1$ of $\exp(iP_i^s)$ is taken. In
principle, there are many different modes \( k_j \), and we have to write it down in \( g(x) \) at the very beginning. However only the mode \( k_1 \) is dominant when the plasma oscillation is not strong. The static kink \( P_1^n \) plays a role of pumping large dc energy into plasma oscillation. This becomes more transparent if we look at the total phase. The total phase is rotating with frequency \( \omega \) but it has kink over the \( x \) direction. This type of spatial structure is compatible with the oscillating electric field, that is the center of the kink coincides with the nodes of electric field. As the phase is rotating for the time being, the electric field is excited.

Adopting the step-function approximation for \( P_1^n \) \cite{11}, \( A_1 \) can be readily evaluated at different \( \omega \). The amplitude at other cavity modes \( A_j \) can be easily generalized from Eq. \( \text{11} \). The results are also depicted in Fig. 3. There is a big enhancement in the amplitude at each cavity resonance, which makes the higher frequency harmonics visible in the frequency spectrum as calculated by computer simulation\cite{11}. The higher frequency harmonics correspond one by one to the high modes in \( \exp(iP_1^n) \).

The radiation power can be calculated by using an effective impedance \( Z \) (for simplicity, \( Z \) is taken as real). When \( Z \) is large, the radiation can be treated as perturbation, and the radiation power is give by \( S = \omega^2|A_1|^2/2Z \). The dependence of \( S \) on \( Z \) is non-trivial. For a large \( Z \), much more power can be pumped into plasma oscillation, and \( |A_1| \) is larger than that of smaller \( Z \), which makes the radiation power large. In Ref. \( \text{11} \), by solving Eq. \( \text{11} \) with radiating boundary condition numerically, the maximum power at the first cavity mode is about 10mW with a mesa of size similar to the experiments\cite{6} even for \( Z = 1000 \). The power estimated by theoretical calculations is still much larger than that in experiments. There is an optimal value \( Z \) which presumes the maximum radiation power \cite{20}.

For theoretical tractability, we have neglected the effect of radiation on the phase dynamics. It does not mean that the radiation will kill the kink state. Actually, it was found that the kink state survives even for \( Z = 10 \), but the height of current steps is reduced because it is difficult to excite cavity resonance with small cavity quality factor\cite{11}.

The \( IV \) characteristics is given by the current conservation law

\[
J_{\text{ext}} = \beta \omega + \langle \sin P_1 \rangle_{xt} = \beta \omega + \frac{\beta \omega |F_1|^2/4}{(k_1^2 - \omega^2)^2 + \beta^2 \omega^2}, \quad (6)
\]

where \( \langle \cdots \rangle_{xt} \) represents the average over space and time. As considerable amount of dc power is converted into plasma oscillation at cavity resonances, the \( IV \) deviates significantly from linear ohmic behavior and self-induced current steps appear, as in Fig. 3. In the absence of irradiation, the input power \( J_{\text{ext}} \omega \) is consumed by dissipation by normal current \( \beta \omega^2 \) and by dissipation in plasma oscillation \( \beta \omega|A_1|^2/4 \).

In conclusion, \( (2m_\perp + 1)\pi \) kink state is formulated in the present paper. In this state, the plasma oscillation is uniform along the \( c \) axis, while the static phase kinks of \( (2m_\perp + 1)\pi \) sit periodically along the \( c \) axis. The kink induces a new type of current steps in the \( IV \) characteristics. This state supports intensive terahertz radiations. The recent experiments on THz radiations from BSCCO single crystals can be interpreted in terms of this state in a consistent way.

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