STUDY OF SINGLE TOP PRODUCTION
AT HIGH ENERGY ELECTRON
POSITRON COLLIDERS

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Top production will play a important role in future high energy electron–positron colliders. Detailed and precise calculations are readily available for the process $e^+e^- \rightarrow t\bar{t}$, but single top quark production is often not included. We evaluate the relevance of these events and advocate the exploration of the related process $e^+e^- \rightarrow W^+bW^-\bar{b}$.

1 Introduction

A high-luminosity, high-energy, linear $e^+e^-$ collider yields excellent opportunities for precision tests of the Standard Model of particle physics. The combination of precisely calculable electroweak production and strict control of the initial state with the relatively benign experimental environment and state-of-the-art detector systems allow for a characterization of Standard Model and new physics processes with a precision that goes well beyond what can be achieved at hadron colliders.

Two projects of linear electron-positron colliders are being considered: the International Linear Collider (ILC [1, 2]) and the Compact Linear Collider (CLIC [3]). The physics case for a linear $e^+e^-$ machine has been made in great detail in References [4–11]. The specific case of a multi-TeV $e^+e^-$ collider is discussed in References [12–14]. In both cases, the center-of-mass energy will exceed $\sqrt{s} = 350$ GeV, the threshold for top quark pair production. Unlike other quarks, the top quark has never been produced in $e^+e^-$ machines, and therefore a precise measurement of electroweak top quark pair production is missing. The study of top quark properties is therefore one of the most exciting prospects for a future linear collider [15]. Detailed full-simulation studies have been made of the prospects for a precise top quark mass measurement [16] and characterization of the $t\bar{t}Z$ and $t\bar{t}\gamma$ vertices [17].

Single top production, through the $e^+e^- \rightarrow W^-\bar{b}, W^+\bar{t}$ process depicted in the central panel of Figure 1, is abundant at $e^+e^-$ colliders that operate at $\sqrt{s} > 300$ GeV. Note that for the $t \rightarrow bW$ decay, this process gives rise to the same $W^+bW^-\bar{b}$ final state as top pair production. The same is true for a third group of processes: $WWZ$, $WWh$ and $WW\gamma$.
Figure 1. Feynman diagrams for top quark pair production at a linear collider ($e^+e^- \to Z/\gamma^* \to t\bar{t}$, left panel), single top production ($e^+e^- \to W^-t\bar{b}, W^+\bar{t}b$, central panel), and triple gauge boson production ($e^+e^- \to W^+W^-Z$, right panel).

production, with $Z/\gamma/h \to b\bar{b}$. Ultimately, all three processes yield the same set of six-fermion final states. In many studies of the linear collider prospects for top physics, single top quark production has been neglected. Notable exceptions are found in References [18, 19].

A fully consistent analysis of the inclusive $e^+e^- \to t\bar{t}$ is currently impossible. Even if event generation for the ILC TDR routinely included the full $2 \to 6$ matrix element at leading order, higher-order corrections for this process are not available for the extraction of top quark properties and couplings.

In this note we investigate the impact of single top events in the study of top quark pair production.

2 Distinguishing single top from top quark pair production

As top quark pair production and single top quark production give rise to the same six-fermion final state, the question arises as to how one can distinguish both sources. At a fundamental level the single top and top quark pair production processes are entangled by interference between the different diagrams. No algorithm can ever separate them fully. However, one could hope to use some of the marked features of the $e^+e^- \to t\bar{t}$ process to make it stand out among the other processes that give rise to the $W^+bW^-\bar{b}$ final state. One could then hope to isolate samples that are enriched in top quark pairs or single top quark events.

In Figure 2 we present the invariant mass of the $W^+b$ and $W^-\bar{b}$ combinations at truth level using $e^+e^- \to t\bar{t} \to W^+bW^-\bar{b}$ events generated with MADGRAPH [20] at $\sqrt{s} = 500$ GeV, without initial state radiation. The top quark candidates are mostly on mass shell, We observe however a significant fraction of the events off-shell. This figure suggests an (ad-hoc) truth-level categorisation of events according to the number of on-shell top quark candidates. In the following we consider a top quark pair production whenever $|m_{Wb} - m_{tMC}^c| < 15$ GeV is satisfied for both $Wb$ combinations. Events that meet only once this criterium are labeled as single top quark events, and the remaining events are considered as non-top events.
Figure 2. Reconstructed invariant mass at truth level of the hadronic versus the leptonic side of $t\bar{t} \rightarrow W^+bW^-\bar{b}$ events. The events cluster around the input top mass value, but one can observe a significant amount of off-shell events.

In Figure 3 we present the reconstructed beam energy of the $Wb$ decaying leptonically versus the $Wb$ decaying hadronically, at truth level and using $e^+e^- \rightarrow W^+bW^-\bar{b}$ events generated with WHIZARD [21, 22] at $\sqrt{s} = 500$ GeV, including initial state radiation. The beam energies cluster at about half the center-of-mass energy, as expected for double-top events. We observe however a significant fraction of events along the diagonal $E_{lep} + E_{had} = \sqrt{s}$. Those are mostly single single top events. This figure suggests a potential criterium for partial separation of single and double-top events using the reconstructed beam energy.

3 Experimental study at $\sqrt{s} = 500$ GeV

In the continuum, double-top cross-sections are available at NNLO. Cross-sections for the process $W^+bW^-\bar{b}$ are for the moment only available at LO, but are required at least at NLO, in order to have a precise evaluation of the single-top contamination in double-top events (note
that NLO calculations are already available for the LHC). On the other hand, at $\sqrt{s} = 500$ GeV, the WHIZARD generator is able to provide fully reconstructed events, so in this section we concentrate on the analysis of reconstructed $W^+bW^-\bar{b}$ events. Our aim is to compare selection efficiencies for single and double-top events.

A study of cross-sections and asymmetries at $\sqrt{s} = 500$ GeV has been presented in [17] using Monte Carlo events with polarized beams and integrated luminosity of 500 fb$^{-1}$, as expected for ILC. Final states of the type $l\nu q_1q_2b_1b_2$ are generated using the WHIZARD Monte Carlo program. These events are mostly $W^+bW^-\bar{b}$ events, where one $W$ decays hadronically and the other leptonically. Some 100,000 MC events were generated. In these samples the beams are 100% polarized.

We have checked at truth level the composition of this sample of input events. A top quark is defined as the combination of 3 quark 4-vectors, provided their invariant mass equals

\textbf{Figure 3.} Reconstructed beam energy at truth level of the hadronic versus the leptonic side of $W^+bW^-\bar{b}$ events. The center-of-mass energy is 500 GeV, so double-top events cluster at beam energies of 250 GeV. In single-top events, the beam energy follows the diagonal $E_{lep} + E_{had} = 500$ GeV.
the top predefined mass in the Monte Carlo within a window of 15 GeV. We find in this way that the fraction of $t\bar{t}$ events is equal to 89%, single tops are 10% and non-top events 1%. Using MADGRAPH, we observe that $W^+bW^-\bar{b}$ events are produced by some 60 Feynman diagrams, where 2 of them are double-tops, but only 10 of them contain a single top quark, the rest being mostly $WWZ$, $ZZh$ and $WW\gamma$ events. The fraction of single-tops is far from negligible and may have a significant impact on the measurement of top quark properties and the search for signs of new physics in $t\bar{t}$ production.

The selection cuts include a $\chi^2$ formed by the reconstructed top mass, beam energy and $b$–quark energy in the top rest frame. Only the hadronically decaying top is used in the analysis. The details of this analysis can be found in [17].

Table 1 summarizes the selection efficiencies at various stages of the analysis:

|       | WbWb | single t | $t\bar{t}$ |
|-------|------|----------|------------|
| $\epsilon_1$ | 51.6% | 50.3% | 51.8% |
| $\epsilon_2$ | 27.2% | 13.5% | 29.0% |
| $\epsilon_3$ | 26.8% | 14.6% | 28.5% |

where $\epsilon_1$ includes kinematical and identification cuts, $\epsilon_2$ the hadronic top $\chi^2$, and $\epsilon_3$ both hadronic and leptonic top $\chi^2$. The $\chi^2$ cut used to obtain $\epsilon_2$ and $\epsilon_3$ is adjusted to obtain the same efficiency on the total sample. We observe that single top events have less efficiency than double top events, as expected, since a beam energy compatible with half the center-of-mass energy is included in the $\chi^2$ definition. The inclusion of the leptonic top in the $\chi^2$ does not improve the rejection. This can also be explained: once an event passes the beam energy constraint, momentum conservation (forced by the neutrino reconstruction) imposes a similar invariant mass on both the hadronic and leptonic sides of the event, whether the event is a single or a double top. In other words, the leptonic side does not provide any additional background rejection. We conclude that the selected sample contains 5.6% of single top events with the corresponding increase in the measured cross-section. The forward-backward asymmetry is even more sensitive, since the obtained value is 0.25 instead of the expected 0.34.

Finally we discuss briefly the single top content as a function of beam polarization. The fraction of $W^+bW^-\bar{b}$ events that corresponds to single top quark production is quite sensitive to the polarization of the electron and positron beams. For unpolarized beams at 500 GeV the sample contains nearly 90% of events with two on-shell top quarks, 9% of events with a single top quark and 1% processes without on-shell top quarks. For a fully left-handed electron beam and fully right-handed positron beam ($e^-_L e^+_R$) the total cross-section is nearly three times larger. The top quark pair fraction drops to 88%. The single top and non-top fractions grow to 10% and 2%, respectively. For the opposite ($e^-_R e^+_L$) configuration, the total
cross-section is similar to the unpolarized result. The top quark pairs make up 94% of the sample, with only 5% of single top quarks and about 1% for non-top production.

4 Analysis of top mass at threshold

As pointed out a long time ago, the cross-section of the $e^+e^- \rightarrow t\bar{t}$ at threshold could be used to measure in a precise and well defined way the top mass [23]. Detailed calculations are present in Reference [24] and a complete study of the extraction of top quark properties was performed in Reference [25].

A recent reanalysis includes realistic beam energy spectra for the ILC and CLIC [16]. In the following we use the latter analysis as a reference. The selection of events includes a kinematic fit, but as we showed in the previous section, the leptonic side of the event brings no additional rejection of single top events, since the neutrino can always be adjusted to fake a second top. This analysis is very detailed, but single top events have not been included. This paper includes for the case of ILC the program TOPPIK, a NNLO calculation of the cross-section including the 1S resonance, initial state radiation and the ILC package ILC350LS without beam polarization. For a mass input value of 174 GeV and luminosity of 10 fb$^{-1}$ per data point, a fit is performed in the range 344–354 GeV of center of mass energies. The result is an impressive 27 MeV statistical error for the top mass, and systematic errors below 100 MeV.

NLO calculations for the $W^+bW^-\bar{b}$ process have been recently implemented in WHIZARD around the double-top production threshold\(^1\). We start however our analysis with a LO calculation using MadGraph, in order to understand the stability of the results versus radiative corrections, and also in order to study the effect of beam polarization. Since beam effects are not included, and we work at truth level, our results cannot be directly compared with [16] but they give a first estimate of the effect of single-top events in the measurement of the top mass. Table 2 summarizes our cross-sections for polarized and unpolarized beams. We only consider the $e^+_Le^+_R$ full polarization, since it provides the largest possible cross-section. More precisely, this fully polarized cross-section is 3 times larger than the the unpolarized one. The table provides the content of single top and non-top events in the $W^+bW^-\bar{b}$ sample for the center-of-mass energy range that is used in [16] to fit the cross-section. The MC top mass is set to 174 GeV, as in this paper.

We note therefore that the presence of single top events modifies significantly the cross-section in the threshold region. This does not mean, however, that the top mass measurement is affected. According to the method proposed in [16], a fit is performed to the cross-section measurement, using a predefined function obtained from the double-top calculation at NNLO. As long as the shape of the curve is not modified, the result is not affected. We observe that according to the NLO calculation of the $W^+bW^-\bar{b}$ cross-section, the $t\bar{t}$ cross-section may be obtained quite exactly by shifting down the $W^+bW^-\bar{b}$ cross-section by some 0.020 pb. There is only a residual shift at 348 GeV of 0.004 pb that would shift the top mass by at most

\(^1\)J.Reuter and F.Bach, private communication.
Table 2. Single top and non-top content of the $W^+bW^-\bar{b}$ sample, for unpolarized and polarized beams, as a function of the center-of-mass energy.

| $\sqrt{s}$ (GeV) | $e^-e^+$ (LO) | $e^-e^+$ (LO) | $e^-e^+$ (NLO) |
|------------------|----------------|----------------|----------------|
| 344              | 23%            | 36%            | 32%            |
| 345              | 19%            | 30%            | 22%            |
| 346              | 13%            | 26%            | 15%            |
| 347              | 9%             | 19%            | 9%             |
| 348              | 7%             | 14%            | 6%             |
| 349              | 5%             | 10%            | 5%             |

Figure 4. Cross-sections at NLO, according to WHIZARD, for $W^+bW^-\bar{b}$ and $t\bar{t}$ production in $e^+e^-$ annihilations with unpolarized beams, as a function of center-of-mass energy, around the double-top threshold region. The 1S-top mass is set to 174 GeV.

20 MeV. The $W^+bW^-\bar{b}$ and $t\bar{t}$ curves are displayed Figure 4. This result is also supported by a recent calculation at NNLO [26], where unfortunately initial state radiation is missing. One can see however that the $t\bar{t}$ cross-section is, as before, shifted down from the inclusive
$W^+bW^-\bar{b}$ cross-section by 0.050 pb, with a residual shift at 348 GeV of 0.010 pb. Taking into account that this later cross-section is about twice the first one, and assuming a perfect scaling of the cross-sections, both results seem compatible with a top mass shift of the order of 20 MeV.

The single top content of the samples depends strongly on the energy, as seen in table 2. In analyses where the top quark is reconstructed and cuts are applied on the mass and/or energy of the candidates, the selection efficiency can be quite different for top quark pair and single top events (up to 50% smaller for single top in the example above). This means that, after correcting for efficiency, the shape of the curve will be deformed if single top events are ignored. Note that this does not apply to Reference [16], since in this analysis no selection cuts based on reconstructed top quark mass and energy are applied.

Finally, a polarized threshold scan should be favoured due to the increased cross-section. None of the studies of future collider prospects so far has taken into account the full $W^+bW^-\bar{b}$ process, where statics is larger and, furthermore, there are no systematic effects due to single top contamination. We therefore suggest to repeat the study of the top quark mass extraction of future lepton colliders using $W^+bW^-\bar{b}$ events and polarized beams, as soon as theoretical
calculations at NNLO are implemented in a MC generator.

5 Energy dependence

![Graph showing cross-section as a function of center-of-mass energy]

**Figure 6.** Cross-section for $W^+bW^−\bar{b}$ and $t\bar{t}$ events produced in unpolarized electron-positron annihilations as a function of the center-of-mass energy. The curves are obtained using WHIZARD at LO.

The composition of the $W^+bW^−\bar{b}$ sample is energy dependent as shown in Fig. 6, that presents the unpolarized LO cross-sections for $e^+e^− \rightarrow W^+bW^−\bar{b}$. The solid line corresponds to the full $2 \rightarrow 4$ process, including single top production and non-top production. The dashed line presents the results for the $e^+e^− \rightarrow t\bar{t} \rightarrow W^+bW^−\bar{b}$ process. All results have been obtained using WHIZARD. The difference between the $t\bar{t}$ cross-section and the full $2 \rightarrow 4$ result is 10% at 500 GeV and rises to 20% at 1 TeV and nearly 50% at 3 TeV.

The relatively small contribution of the $e^+e^− \rightarrow t\bar{t}$ process at large center-of-mass energy is confirmed by an analysis of the number of on-shell top quarks ($|m_{Wb} - m_{t}^{MC}| < 15$ GeV) in $W^+bW^−\bar{b}$ events at 3 TeV. The fraction of events with two on-shell top quarks is 48%, events with a single on-shell top quark make up 35% of the sample and the remaining 17% of events has no on-shell top quarks. We conclude that while the rate for the $e^+e^− \rightarrow t\bar{t}$ process...
drops at very large center-of-mass energy, single-top and non-top production increase rapidly. The three processes become comparable in size for $e^+e^-$ colliders operating in the multi-TeV regime.

6 Summary and conclusions

The prospects for precision top quark physics at future lepton colliders relies on a comparison of measured cross-sections to very precise predictions of top quark pair production, including NLO and sometimes NNLO calculations. However, as we have shown in this note, single top events, that are ignored in most prospect studies, may lead to significant effects. A precise experimental separation of single top and top quark pair production processes seems difficult. For this reason we advocate the analysis of $W^+bW^−b$ production, that includes single top quark production, and leads to increased statistics and possibly smaller systematic errors. To achieve the ultimate possible precision at the ILC, observables for the $W^+bW^−b$ final state must be calculated with a precision well below 1%, at NNLO, both at production threshold and in the continuum.

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References

[1] T. Behnke, J. E. Brau, B. Foster, J. Fuster, M. Harrison, et al., The International Linear Collider Technical Design Report - Volume 1: Executive Summary, arXiv:1306.6327.
[2] The ILC community Collaboration, E. Brau, James et al., ILC Reference Design Report: ILC Global Design Effort and World Wide Study, arXiv:0712.1950.
[3] M. Aicheler et al., A Multi-TeV Linear Collider based on CLIC Technology: CLIC Conceptual Design Report, .
[4] H. Baer, T. Barklow, K. Fujii, Y. Gao, A. Hoang, et al., The International Linear Collider Technical Design Report - Volume 2: Physics, arXiv:1306.6352.
[5] J. E. Brau, R. M. Godbole, F. R. L. Diberder, M. Thomson, H. Weerts, et al., The Physics Case for an e+e- Linear Collider, arXiv:1210.0202.
[6] The ILC community Collaboration, G. Aarons et al., ILC Reference Design Report Volume 2: physics at the ILC, arXiv:0709.1893.
[7] American Linear Collider Working Group Collaboration, T. Abe et al., Linear collider physics resource book for Snowmass 2001. 1: Introduction, hep-ex/0106055, hep-ex/0106056, hep-ex/0106057, hep-ex/0106058.
[8] American Linear Collider Working Group Collaboration, T. Abe et al., Linear collider physics resource book for Snowmass 2001. 2: Higgs and supersymmetry studies, hep-ex/0106056.
[9] American Linear Collider Working Group Collaboration, T. Abe et al., Linear collider physics resource book for Snowmass 2001. 3: Studies of exotic and standard model physics, hep-ex/0106057.

[10] American Linear Collider Working Group Collaboration, T. Abe et al., Linear collider physics resource book for Snowmass 2001. 4: Theoretical, accelerator, and experimental options, hep-ex/0106058.

[11] ECFA/DESY LC Physics Working Group Collaboration, J. A. Aguilar-Saavedra et al., TESLA Technical Design Report Part III: Physics at an e+e- Linear Collider, hep-ph/0106315.

[12] L. Linssen, A. Miyamoto, M. Stanitzki, and H. Weerts, Physics and Detectors at CLIC: CLIC Conceptual Design Report, arXiv:1202.5940.

[13] P. Lebrun, L. Linssen, A. Lucaci-Timoce, D. Schulte, F. Simon, et al., The CLIC Programme: Towards a Staged e+e− Linear Collider Exploring the Terascale : CLIC Conceptual Design Report, arXiv:1209.2543.

[14] CLIC Physics Working Group Collaboration, E. Accomando et al., Physics at the CLIC multi-TeV linear collider, hep-ph/0412251.

[15] D. Asner, A. Hoang, Y. Kiyo, R. Pschl, Y. Sumino, et al., Top quark precision physics at the International Linear Collider, arXiv:1307.8265.

[16] K. Seidel, F. Simon, M. Tesar, and S. Poss, Top quark mass measurements at and above threshold at CLIC, Eur.Phys.J. C73 (2013) 2530, [arXiv:1303.3758].

[17] M. Amjad, M. Boronat, T. Frisson, I. Garcia, R. Poschl, et al., A precise determination of top quark electro-weak couplings at the ILC operating at √s = 500 GeV, arXiv:1307.8102.

[18] E. Boos, M. Dubinin, A. Pukhov, M. Sachwitz, and H. Schreiber, Single top production in e+ e-, e- e-, gamma e and gamma gamma collisions, Eur.Phys.J. C21 (2001) 81–91, [hep-ph/0104279].

[19] P. Batra and T. M. Tait, Measuring the W-t-b Interaction at the ILC, Phys.Rev. D74 (2006) 054021, [hep-ph/0606068].

[20] F. Maltoni and T. Stelzer, MadEvent: Automatic event generation with MadGraph, JHEP 02 (2003) 027, [hep-ph/0208156].

[21] W. Kilian, F. Bach, T. Ohl, and J. Reuter, WHIZARD 2.2 for Linear Colliders, arXiv:1403.7433.

[22] W. Kilian, T. Ohl, and J. Reuter, WHIZARD: Simulating Multi-Particle Processes at LHC and ILC, Eur.Phys.J. C71 (2011) 1742, [arXiv:0708.4233].

[23] J. H. Kuhn, Weak Interactions of Quarkonia, Acta Phys.Polon. B12 (1981) 347.

[24] K. Fujii, T. Matsui, and Y. Sumino, Physics at t anti-t threshold in e+ e- collisions, Phys.Rev. D50 (1994) 4341–4362.

[25] M. Martinez and R. Miquel, Multiparameter fits to the t anti-t threshold observables at a future e+ e- linear collider, Eur.Phys.J. C27 (2003) 49–55, [hep-ph/0207315].

[26] A. H. Hoang, C. J. Reisser, and P. Ruiz-Femenia, Phase Space Matching and Finite Lifetime Effects for Top-Pair Production Close to Threshold, Phys.Rev. D82 (2010) 014005, [arXiv:1002.3223].