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Impact of XPM and FWM on the digital implementation of impairment compensation for WDM transmission using backward propagation

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Abstract: The impact of cross-phase modulation (XPM) and four-wave mixing (FWM) on electronic impairment compensation via backward propagation is analyzed. XPM and XPM+FWM compensation are compared by solving, respectively, the backward coupled Nonlinear Schrödinger Equation (NLSE) system and the total-field NLSE. The DSP implementations as well as the computational requirements are evaluated for each post-compensation system. A 12 × 100 Gb/s 16-QAM transmission system has been used to evaluate the efficiency of both approaches. The results show that XPM post-compensation removes most of the relevant source of nonlinear distortion. While DSP implementation of the total-field NLSE can ultimately lead to more precise compensation, DSP implementation using the coupled NLSE system can maintain high accuracy with better computation efficiency and low system latency.

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1. Introduction

In long-haul fiber transmission systems, fiber chromatic dispersion (CD), Kerr nonlinearity and amplifier noise are responsible for signal degradation, limiting the capacity of wavelength-division multiplexed (WDM) transmission systems [1]. Over the past few years, optical techniques such as dispersion management or mid-span phase conjugation [2], have been extensively studied and deployed to mitigate the degrading effects in fibers [3, 4]. Recently, electronic pre- and/or post-compensation of transmission impairments have attracted significant attention due to the fast development of coherent detection and digital signal processing (DSP), which constitute the enabling technologies for electronic impairment compensation [5, 6, 7].

In particular, post-compensation schemes offer a great flexibility since adaptive compensation can be incorporated, improving the robustness against modifications on the physical layer. In the context of WDM transmission, our group recently proposed a universal post-processing scheme [8] where, for the first time to our knowledge, dispersive and nonlinear intra- and inter-channel impairments are fully compensated using electronic backward propagation. In [8], the total optical DWM signal is backward propagated using a full time-domain split-step method to solve the $z$-reversed nonlinear Schrödinger equation (NLSE). Although this method has been proven effective, reducing the number of computations required and its impact on the system latency is desirable for the eventual implementation of the post-compensation method.

In this paper, the impact of nonlinear inter-channel effects, i.e. cross-phase modulation (XPM) and four-wave mixing (FWM), are studied in the framework of backward propagation post-compensation systems. For this purpose, a WDM transmission system with 12 channels, each of them modulated in a 100 Gbits/s 16-QAM format, is simulated. This modulation format is selected because it provides high spectral efficiency and nonlinear impairment compensation becomes necessary for long-distance transmission. The impact of XPM and FWM on post-compensation using backward propagation will be evaluated individually, by solving respectively, the coupled NLSE system (C-NLSE) and the total-field NLSE (T-NLSE) using the split-step Fourier method (SSFM). The impact of both effects on the detected $Q$-factor, optimum launching power and channel spacing will be analyzed in detail. Additionally, generalized conclusions about the DSP computational requirements for each compensation scheme will be
presented for WDM systems.
This paper is structured as follows. In section 2, the governing equations for XPM and XPM+FWM compensation will be introduced, focusing on the numerical aspects involved on the SSFM for each case. In section 3, the results of this work will be presented followed by the discussion and conclusion.

2. Theory of backward propagation compensation and digital implementation

2.1. Backward propagation equations and numerical procedure

In a coherent detection system, a full reconstruction of the optical field can be achieved by beating the received field with a co-polarized local oscillator. The reconstructed field will be used as the input for backward propagation in order to compensate the transmission impairments. Let \( \hat{E}_m \) be the envelope of the \( m \)-th-channel field where \( m \in I, I = \{1, 2, \cdots, N\} \) and \( N \) is the total number of WDM channels. By rewriting the field expression as, \( E_m = \hat{E}_m \exp(i\Delta \omega t) \), where \( \Delta f = \Delta \omega / 2\pi \) is the channel spacing, the expression of the full optical field can be expressed as, \( E = \sum_m E_m \). The total-field back propagation equation, i.e T-NLSE, is given by [9],

\[
\frac{\partial E}{\partial z} + \frac{\alpha}{2} E + \frac{i\beta_2}{2} \frac{\partial^2 E}{\partial t^2} - \frac{\beta_3}{6} \frac{\partial^3 E}{\partial t^3} - i\gamma |E|^2 E = 0, \tag{1}
\]

where \( \beta_j \) represent the \( j \)-th-order dispersion, \( \alpha \) is the absorption coefficient, \( \gamma \) is the nonlinear parameter and \( t \) is the retarded time [9]. Equation (1) governs the backward propagation of the total field including second and third order dispersion, SPM, XPM and FWM compensation.

Alternatively, the effect of FWM can be omitted in backward propagation by introducing the expression for \( E \) into Eq. (1), expanding the \( |E|^2 \) term and neglecting the so-called FWM terms, that is,

\[
- \frac{\partial E_m}{\partial z} + \frac{\alpha}{2} E_m + \frac{i\beta_2}{2} \frac{\partial^2 E_m}{\partial t^2} - \frac{\beta_3}{6} \frac{\partial^3 E_m}{\partial t^3} - i\gamma \left( \sum_{q \neq m} \frac{1}{2} |E_q|^2 - |E_m|^2 \right) E_m = 0. \tag{2}
\]

The system of coupled equations (2) describe the backward evolution of the WDM channels where dispersion, SPM and XPM are compensated.

The above equations are solved by means of the split-step Fourier method (SSFM) [10, 11]. Here, the dispersive and nonlinear contributions are considered to be independent in a short segment of propagation. The length of this segment, i.e. step size, is determined according to the characteristics of the system under study and it should be selected carefully in order to preserve the accuracy of the results. In this work, we will use the symmetric step size where the nonlinear part is estimated by averaging the optical power over the the step length in a iterative way (the details of this procedure can be found in [8, 9]). Although this method increases the number of computations per step, it allows to increase the step size in such a way that the total number of operations is reduced without accuracy penalty. In Fig. 1 it is depicted a block diagram of the SSFM for a single step and for Eq. (1), where two iterations (sub-steps) for the power averaging are performed.

The involved operators in the SSFM are given by, \( D(x) = \mathcal{F}^{-1} \left[ H \mathcal{F}(x) \right], P(x) = |x|^2 \) and \( E(x) = \exp(i\gamma ch) \), where the transfer function \( H \) for fiber dispersion and loss is given by,

\[
H(\omega) = \exp \left[ \left( \frac{\alpha}{2} + \frac{i\beta_2}{2} \frac{\omega}{|\omega|} + i\frac{\beta_3}{6} \frac{\omega^3}{|\omega|^3} \right) h \right],
\]

with \( \omega \) being the angular frequency and \( h \) the step size.

Likewise, Eqs. (2) are also solved by the SSFM, following the procedure depicted in Fig. 2, again, for a single step. In contrast to the total-field NLSE, a sum operation is included at each
Fig. 1. Block diagram of the DSP implementation of the total-field NLSE.

Fig. 2. Block diagram of the DSP implementation of coupled NLSEs.

sub-step to include the XPM contribution on each channel. Finally, for a given optical link with \( M \) spans, the iterative procedure for SSFM backward propagation is given by the block diagram shown in Fig. 3 where \( n_s \) is the number of steps per span (note that an attenuation element has been introduced to compensate the amplification stages).

Fig. 3. SSFM backward propagation diagram for a \( M \)-spans optical link.

2.2. SSFM Step size and digital implementation efficiency

As it was mentioned before, the number of operations required for SSFM backward propagation is of great importance in digital post-processing. The number of operations for a given span depends on the number of steps \( (n_s) \), and hence, on the step size \( (h = L/n_s) \) where \( L \) is the span length. The SSFM accuracy depends fundamentally on the mutual influence of dispersion and nonlinearity within the step length. Due to the nature of the dispersion and nonlinearity operators, the step size has to make sure that: (1) the nonlinear phase shift is small enough to preserve the accuracy of the dispersion operation and (2) the optical power fluctuations due to
dispersion effects are small enough to preserve the accuracy of the nonlinear operation.

One way to set the upper bound for the step size is to identify the characteristic physical lengths of the transmission system, which correlate the optical field fluctuations with the propagation distance. Three physical lengths are of interest here, namely, the nonlinear length $L_{\text{nl}}$, the walk-off length $L_{\text{wo}}$ and the four-wave mixing length $L_{\text{fwm}}$. The nonlinear and walk-off lengths can be defined, for a multi-channel system, as follows,

$$L_{\text{nl}} = \frac{1}{\gamma P_T \frac{2N-1}{N}}, \quad L_{\text{wo}} = \frac{1}{2\pi|\beta_2|(N-1)\Delta f B},$$

where, $P_T = \sum_m |E_m|^2$ is the total launched power and $B$ is the symbol rate (effectively the inverse of the pulse width). The nonlinear length has been defined as the length after which an individual channel experiences a 1 radian phase shift due to SPM and XPM. The walk-off length is defined as the distance after which the relative delay of pulses from the edge channels is equal to the pulse width. The above characteristic lengths are well known [12] and widely used to qualitatively describe the optical field behavior through fiber propagation. However, when FWM is considered, the nonlinear and walk-off lengths are not enough to qualitatively identify the range where the fastest field fluctuations take place. In order to identify the fastest field fluctuations due to FWM, the total optical field should be rewritten as $E = \sum_m E_m \exp(ik_m z)$ where $k_m$ is the linear propagation constant of the $m$th-channel. By following the same procedure as for Eq. (2), the nonlinear term, now including FWM, can be expressed as follows for the $m$th-channel,

$$-i\gamma \left( 2\sum_{q \in I} |E_q|^2 - |E_m|^2 \right) E_m - i\gamma \left[ \sum_{[rlm] \in I} E_r E_s E_l^* \exp(i\delta k_{rlm} z) \right],$$

whith the following conditions $l = r + s - m$, $[m,r,s] \in I$ and $r \neq s \neq m$. The first condition neglects fast time-oscillating terms (frequency matching). The second condition forces the new generated waves to lay within the WDM band. Finally, the third condition excludes SPM and XPM terms. $\delta k_{rlm}$ is the phase mismatch parameter, given by,

$$\delta k_{rlm} = k_r + k_s - k_l - k_m = \frac{1}{2} \beta_2 \Delta \omega^2 [r^2 + s^2 - (r + s - m)^2 - m^2].$$

In order to identify the fastest $z$-fluctuations for the $m$th-channel, let us set $r = 1$ and $s = N$ corresponding to the indexes of the edge channels. By maximizing Eq. (5), the expression for the maximum phase-mismatch is given by,

$$\delta k_{\text{max}} = \frac{1}{4} |\beta_2|(N-1)^2 \Delta \omega^2.$$ (6)

The above expression leads to the following definition for the FWM length,

$$L_{\text{fwm}} = \frac{1}{\pi^2 |\beta_2|(N-1)^2 \Delta f^2}.$$ (7)

The above expression represents the length after which the argument of the fastest FWM term is shifted by 1 radian; hence, it can be understood as the distance after which power fluctuations due to FWM start to take place. The definition of the FWM length assumes that the FWM-induced variations on a given channel are governed by the linear (dispersive) phase mismatch. However, nonlinearity also contribute to the overall phase mismatch through SPM.
and XPM. This contribution is only relevant in high power regimes and it is not expected to play a role in the analysis of fiber transmission. The SSFM step size will be correlated to the minimum characteristic length involved in each case; thus, for the compensation of SPM and XPM effects via coupled equations, the step size is limited by the walk-off length whereas the FWM length will be the parameter limiting the step size for the total-field NLSE. For WDM systems, the nonlinear length is longer than the FWM and walk-off lengths for the typical launch powers of interest in communications.

Although variable step-size schemes have been used to reduce the number of operations required to describe FWM in fibers [13], a constant step size will be considered is this paper. The variable step size technique corrects the spurious FWM tones generated by a wrong step size distribution. However, the computational efficiency of this technique rapidly decreases in multi-channel and/or large dispersion systems [11], where the generation of spurious tones is not the only source of error. In addition, it is important to note that a constant step size presents a great practical advantage in the eventual realization of the DSP post-processor.

The following relationships define the characteristic step sizes for XPM and FWM compensation via the iterative symmetric SSFM, i.e.,

\[
\begin{align*}
  h_{wo} &= \tau_r L_{wo}, \\
  h_{fwm} &= \phi_{fwm} L_{fwm},
\end{align*}
\]

where the dimensionless parameters \( \tau_r \) and \( \phi_{fwm} \) represent, respectively, the maximum inter-pulse relative delay (with respect to the pulse width) and the maximum phase-mismatch allowed within one step.

The values of \( \tau_r \) and \( \phi_{fwm} \) may depend on several factors including the impact of non-deterministic fluctuations on the system (ASE and laser phase noise), the desired degree of accuracy and most importantly, the numerical procedure (in our case the iterative symmetric SSFM). Thus, those parameters are usually determined \textit{a posteriori} within the simulation results. However, it is important to stress that, although \( \tau_r \) and \( \phi_{fwm} \) can be understood as phenomenological parameters, their values are related to the numerical procedure itself and they are expected to be independent on the general WDM transmission parameters, such as channel spacing or number of channels, whose effect over the step size is governed by the physical lengths. In addition, provided that the SSFM is used for both the T-NLSE and the C-NLSE, the ratio \( \kappa = \tau_r / \phi_{fwm} \) can be regarded as a constant for any variant of the split-step Fourier method.

In order to gain a perspective on the computational requirements for each method, it is interesting to evaluate the number of operations required for FWM and/or XPM compensation. For simplicity, the number of operations in the XPM modules shown in Fig. 2 is neglected. This is justified because, in general, dispersion modules require a large computational load. Since the total number of computations is inversely proportional to the step size,

\[
\frac{C_{fwm}}{C_{xpm}} = \frac{h_{xpm}}{Nh_{fwm}} = \frac{\pi \kappa (N - 1) \Delta f}{2 NB},
\]

where \( C \) represents total number of operations. Note that the the XPM characteristic step size dictates the number of operations \textit{per channel}; thus, the total number of operations for XPM compensation is multiplied in Eq. (9) by the total number of channels.

It is interesting to note that this ratio becomes asymptotically independent of the number of channels for \( N \gg 1 \). This is an expected result since the scaling of \( L_{wo} \) and \( L_{fwm} \) with \( N \) is compensated by the number of coupled equations that have to be solved for XPM compensation. On the other hand, the number of computations for FWM compensation grows with the ratio between the channel spacing and the symbol-rate. This means that the cost for FWM compensation decreases in systems with a high spectral efficiency, where the channels are highly phase-matched and larger step sizes can be used to describe the field propagation. It is also
interesting to note that the above ratio is independent of the fiber chromatic dispersion. This indicates that the above ratio can be generalized to dispersion-managed systems provided that the dominant characteristic lengths are related to dispersive effects, as expected in WDM systems.

Together with number of computations, the latency of the post-processor is of great relevance in communications, especially for real-time applications. A fundamental difference arises between the impairment compensation via coupled or total-field equations. As shown in Fig. 2, the coupled equations for XPM compensation naturally parallelize post-processing, which drastically reduces the latency of the system. For FWM compensation via T-NLSE, the system latency is proportional to the number of operations per step ($\tau_{\text{fwm}} \propto C_{\text{fwm}}$). However, for XPM compensation via C-NLSE, the system latency ($\tau_{\text{xpm}}$) is proportional to the number of operations per step and per channel; hence, the following relationship can be obtained for the processing latency,

$$\frac{\tau_{\text{fwm}}}{\tau_{\text{xpm}}} = \frac{h_{\text{xpm}}}{h_{\text{fwm}}} = \frac{\pi \kappa (N - 1) \Delta f}{2 B}. \quad (10)$$

Again, the XPM module is omitted in the analysis. This module is expected to add a small correction to the processing latency for XPM compensation. Note that the above expression is independent of any additional parallelization, which can be done to the C-NLSE and T-NLSE systems without distinction. Eq. (10) indicates that FWM compensation leads to a latency that grows with the total WDM optical bandwidth.

The results obtained so far, show the great importance of individually identify the impact of XPM and FWM on the WDM transmission in order to be able to select the most efficient post-processing scheme.

3. Simulation results

In this section, a transmission simulation is used to evaluate the impact of XPM and FWM on backward propagation impairment compensation. For that, a 12 channel WDM is considered in which each channel is modulated at 100 Gb/s in a 16-QAM format, corresponding to a symbol rate of $B = 25$ Gbaud. The total transmission distance is 1000 km, divided into $M = 10$ spans of 100 km. Two channel spacing values will be considered according to the ITU-T standards for WDM systems, i.e. $\Delta f = 50, 100$ GHz. The schematic of the transmission system is shown in Fig. 4, where post-compensation is performed in the digital domain. The 16-QAM/WDM signals are transmitted over multiple amplified fiber spans; after transmission, the received signals are mixed in a 90° optical hybrid with a set of co-polarized local oscillators (LOs). The in-phase and quadrature components of each WDM channel are obtained by balanced photo-detectors.
Analog-to-digital (A/D) conversion is followed by DSP field reconstruction, backward propagation, demultiplexing and data recovery. Using coherent detection, each channel is translated to baseband and sampled at 25 Gsa/s (i.e. 1 sample per symbol). Then, 32/64 samples are added to each symbol (respectively for $\Delta f = 50/100$ GHz) and the 12 channels are combined to reconstruct a final optical waveform of 800/1600 GHz bandwidth (corresponding to an upsampled transmitted bandwidth of 600/1200 GHz). The details of the DSP implementation of the up-sampling procedure can be found in [8].

The transmission channel is a non-zero dispersion shifted fiber, with: $\beta_2 = -5.63$ ps$^2$/km ($D = 16$ ps/km/nm), $\beta_3 = 0.083$ ps$^3$/km, $\alpha = 0.046$ km$^{-1}$ (0.2 dB/km) and $\gamma = 1.46$ W$^{-1}$km$^{-1}$. The signal is amplified after each span with an EDFA with a noise figure of 5 dB. For simplicity, the laser phase noise is neglected.

Clearly, backward propagation requires an accurate knowledge of the link parameters. From the practical point of view, training experiments can be done to set the link parameters which optimize the performance. In addition, small fluctuations on the optical link due to environmental effects are expected to have a small impact on the performance [15].

Forward transmission simulations have been made using VPItransmissionMaker where two different channel spacings and nine different input power values had been considered. Backward propagation algorithms are developed in Matlab where C-NLSE and T-NLSE are solved with different step sizes for each case. As a preliminary illustration, in Fig. 3 are shown the constellation and eye diagrams as well as the $Q$-factor values for, respectively, back-to-back, dispersion compensation only and XPM compensation via C-NLSE. The results are given for one of the central channels (which presents the highest inter-channel nonlinear distortion). The $Q$-factor is obtained from the constellation diagram as in [16].

![Fig. 5. Constellation and eye diagrams for one of the central channels ($P_T = 9$ dBm). (A) Back-to-back ($Q = 29.8$ dB); (B) Dispersion compensation ($Q = 8.3$ dB) and (C) XPM compensation with 30 steps per span ($Q = 13.3$ dB).](image-url)

The compensation of XPM clearly gives rise to a well defined constellation and better eye opening, as shown in Fig. 3-(C), in comparison with no nonlinearity compensation as shown in Fig. 3-(B). It should be noted, though, that the number of operations for only dispersion compensation is drastically reduced, corresponding to a single-step linear operation on the SSFM. Next, an exhaustive analysis of the two compensation schemes is made, starting with a channel spacing of 50 GHz.
A. Results for $\Delta f = 50$ GHz

Fig. 6 shows the $Q$-factor values as a function of the total input power for different step sizes.

The results show the well-known optimum power for optical transmission in nonlinear systems, where the post-compensation of deterministic effects provide the maximum $Q$-factor. Above the optimum power, the non-deterministic nonlinear distortion due to signal-ASE beat starts to offset the SNR growth with launching power.

The characteristic step sizes for the 50 GHz channel spacing take the values of $h_{ww} = 3.08$ km and $h_{fwm} = 178$ m, with $\tau_r = 3/2$ and $\phi_{fwm} = 3$ respectively according to Eqs. (8, 7 and 3). It is important to recall, here, that the values of $\tau_r$ and $\phi_{fwm}$ are strongly correlated with the numerical methods depicted in Figs. 1 and 2, where three dispersion operations are made per step, relaxing consequently, the step size requirements.

The results in Fig. (6) show the impact of the SSFM step size on the received $Q$-factor as well as on the optimum power. Lines in blue correspond to step sizes close to the respective characteristic step size. In this case, the results show that the optimum $Q$-factor (i.e. for the characteristic step size and for the optimum power) increases by approximately 1 dB when FWM is compensated, indicating in a very small impact of FWM in the optimum operation point. Likewise, the compensation of FWM allows to increase the total launching power by 1 dBm. The improved performance with the correction of FWM requires approximately 1.4 times more computations than XPM compensation and incurs a latency 17 times larger than for XPM compensation according to Eqs. (9 and 10).

A more detailed analysis can be extracted from the contour maps depicted in Fig. (6). Here, a linear interpolation of the $Q$-factor has been made from simulations with 11 different step sizes. The optimum operation points are indicated by white spots within the figures, corresponding to the optimum power and characteristic step size. Qualitatively, a similar pattern is obtained for XPM and FWM compensation indicating that FWM effects are weak and XPM is the dominant nonlinear impairment. Quantitatively with respect to the XPM pattern, the FWM compensation pattern is slightly shifted to higher power levels and remarkably shifted towards smaller step sizes. To estimate the numerical error induced by an improper step size in the FWM compensation, the $Q$-factor for XPM and FWM compensation can be compared for the XPM characteristic step size. For $P_T = 12$ dBm and $h = 3$ km the $Q$-factor is reduced by 7 dB in the FWM case, which confirms the great distortion that is induced due to a wrong estimation of FWM, even when XPM is properly compensated.

From the numerical point of view, the dark and homogeneous regions located around the respective optimum powers, show the expected asymptotic behavior of the $Q$-factor for step sizes below the characteristic step size. Such behavior (see also Fig. 9) sets the values of $\tau_r$
Fig. 7. $Q$-factor map, as a function of the launched power and the step size ($\Delta f = 50$ GHz). (A) XPM compensation, (B) FWM compensation. The white spot indicates the optimum power and characteristic step size location.

and $\phi_{\text{fwm}}$, which provide an optimum ratio between the $Q$-factor and the computational load. Two patterns depicted in Fig. 7 are worth mentioning. First, a flat transition with respect to the step size is observed in the high $Q$-factor region. This flatness indicates the independence of the step size with the power, which confirms that linear dispersion effects, such as walk-off and dispersive phase-mismatch, limit the step size values. On the other hand, in the upper right sides of the maps, a diagonal pattern of the iso-$Q$s can be observed. This pattern show a correlation between the step size and the optical power, suggesting that nonlinear effects start to be compensated. Those diagonal transitions do not appear in the left hand side of the map, where nonlinearity does not play a significant role and the $Q$-factor grows with the power regardless of the step size.

**B. Results for $\Delta f = 100$ GHz**

To assess the impact of WDM on the nonlinearity and, eventually, on the backward propagation impairment compensation, the above analysis is made now, for a channel spacing of 100 GHz. Likewise, this will confirm the validity and generality of the step size requirements and its relation with the characteristic physical lengths.

Fig. 8 shows the $Q$-factor values as a function of the total input power for different step sizes, including the results for the characteristic step size of the system (lines in blue).

From Eqs. (8, 7 and 3), the characteristic step sizes are: $h_{\text{wo}} = 1.54$ km and $h_{\text{fwm}} = 44$ m. The

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values $\tau_r = 3/2$ and $\phi_{fwm} = 3$ are preserved, confirming that those parameters are independent on the WDM system. Note that the XPM and FWM characteristic step sizes, are respectively halved and quartered with respect to the 50 GHz channel spacing, according to the scaling of the walk-off and phase mismatch with the channel spacing. To illustrate this, Fig. 9 shows the $Q$-factor with respect to the step size for both channel spacings. Here, the above mentioned asymptotic behavior of the $Q$-factor is observed as well as the characteristic step size locations. Both Figs. 8 and 9 show that FWM has a negligible influence for $\Delta f = 100$ GHz, having effectively the same $Q$-factor for XPM and FWM compensation. The larger channel spacing gives rise to a higher phase-mismatch, which rapidly averages to zero the contribution of the FWM products. Because of this small contribution, the optimum power is also effectively equal for both compensation schemes. Regarding DSP requirements, and according to Eqs. (9 and 10) the correction of FWM requires approximately 2.8 times more computations than XPM compensation. In addition, FWM compensation incurs a latency 34 times larger than XPM compensation. The figure 10 show the $Q$-factor map for the 100 GHz channel spacing.

The results depicted in Fig. 10 are qualitatively similar to those shown in Fig. 7. Quantitatively, it is observed that the optimum power is increased roughly 1 dBm with respect to $\Delta f = 50$ GHz, confirming the well known reduction of both XPM and FWM effects when the channel spacing grows [17]. This is also confirmed by considering the optimum $Q$-factor values, which
are increased 2.4 and 1.5 dB respectively for the XPM and FWM compensation case (as it is also depicted in Fig. 9).

C. Enhanced DSP implementation for XPM compensation.

So far, we have analyzed the impact of nonlinearity on the digital implementation of distributed back propagation in a WDM system. For that, the same numerical method has been implemented for the T-NLSE and the C-NLSE system allowing: (1) an equitable comparison of the results and (2) a general description of the impact of WDM parameters on the digital compensation schemes. However, the differences between the two implementations may not only arise from the physical restrictions imposed on the step size. In fact, a fundamental difference exists because XPM compensation is performed through a system of coupled equations, which allows selective operations to each channel. On the contrary, the T-NLSE describe the evolution of all the channels as a single wave.

As shown in Figs. (1 and 2), three dispersion operations are required per step and per channel. The second dispersion operator is used to calculate more accurately the nonlinear propagation by using the trapezoidal rule [9]. Moreover, the physical dispersion is compensated by the first and third dispersion modules. Since the step size is limited by the walk-off length, the second dispersion operator can be replaced by a delay operator to account solely for the walk-off. This operation will preserve high accuracy since the inter-channel delay is the most relevant intra-step dispersive effect. Within one sub-step, each \( m \)th channel undergoes a relative time delay given by \( T_m = d_m h_{\text{XPM}} / 2 \), where \( d_m = 2\pi\beta^2 m\Delta f \) is the walk-off parameter. Therefore, a DSP delay operator will shift each channel data array by \( K_m \) samples, where \( K_m = [S_r/T_m] \) being \( S_r \) the sampling rate and \( \lfloor x \rfloor \) the nearest integer of \( x \).

![Fig. 11.](image-url) Received Q-factor as a function of the step size for XPM post-compensation using intra-step walk-off modeling.

Fig. 11 shows the results for the received \( Q \)-factor for the DSP implementation of Fig.(2) with dispersion operators and the same implementation with delay operators. The results show that almost no penalty is incurred by using delay operators indicating that this approach can be applied to reduce the number of operations required for XPM compensation. Since the delay operators do not contribute to the total number of operations, the ratio \( C_{\text{FWM}}/C_{\text{XPM}} \) is now increased by a factor of 3/2 according to the elimination of one dispersion operator. Consequently, for 50(100) GHz channel spacing, XPM compensation via C-NLSE requires 2.1(4.2) times less operations than FWM compensation via T-NLSE.

So far, physical and numerical differences between XPM and FWM compensation have been analyzed. For simplicity, the same optical hardware scheme has been used for both compensation schemes. However, some aspects regarding the coherent receiver are worth to mention.
As shown in Fig. 4, a set of phased-locked LOs is required for full reconstruction of the optical field before back-propagation. Such phase-locking represents a practical shortcoming for backward propagation since rather complex techniques like frequency comb generation or electronic phase tracking should be implemented (see more details in [8]). However, since XPM is a phase-insensitive nonlinear effect, phase-locked LOs are not required for XPM compensation. This represents an important practical advantage with respect to FWM compensation.

4. Discussion

From the physical perspective, the results confirm that XPM is the dominant source of distortion in WDM systems transmitted through non-zero dispersion shifted fibers, as it was predicted in works such as [18]. It must be noted that the analysis of backward propagation compensation through both coupled and total-field NSLE gives a clear and reliable picture of the impact of different nonlinear effects on the optical transmission system. A discussion from the DSP computational perspective, can be outlined by considering the influence of the different WDM transmission parameters.

Modulation format

In this work a 16-QAM modulation format has been used. In order to extend the obtained results, a comment on the modulation format is worthwhile. In this sense, the impact of the modulation format on nonlinearity is usually associated with the constant or non-constant characteristic of the optical power. Constant-power formats ideally present less sensitivity to SPM and XPM since the absence of power fluctuations keeps the phase free of nonlinear noise [19]. Moreover, due to the multiplicative character of the FWM efficiency, constant power formats are expected to be less tolerant to FWM. In systems with non-negligible chromatic dispersion, variations on the phase are translated into amplitude fluctuations through dispersion [20]. In such cases, the nonlinear distortion becomes effectively independent of the modulation format for long haul transmission systems.

Channel spacing

We have evaluated the impact of the channel spacing both from the physical and the computational points of view. The results suggest than only in the limit of OWDM [14], an eventual increase of the FWM-induced distortion may justify the use of the total-field NLSE for backward propagation. In this case, we show that although the number of computations is almost the same as for the XPM compensation case, the latency is still remarkably higher (see Eqs. (9-10)). A different approach can be investigated to compensate FWM without noticeably increasing the latency. One strategy is to perturb the coupled NLSE with FWM terms corresponding to the interaction of neighboring channels. Since only highly phase-matched terms will be considered, the step size could be kept above the walk-off limit, avoiding a latency penalty. Despite the fact that the total number of computations is increased, this enhanced coupled-equations approach presents a promising technique to increase the $Q$-factor in a stronger FWM environment without paying a high price in term of DSP efficiency.

Number of WDM channels

It has been shown that the number of SSFM steps for XPM and FWM compensation grows, respectively, linearly and quadratically with the number of channels. On the other hand, if the number of channels is sufficiently high, widely separated channels will induce both a high walk-off and phase-mismatch, which consequently, reduces the nonlinear impact on transmission. This suggests that any target channel will only be distorted, via inter-channels effects, by the
channels located within a limited optical bandwidth. One approach following this idea is the so-called Mean Field Approach (MFA) [21], which neglects the time and $z$-variations of the channels outside the effective bandwidth. To test this approach, we monitored the received $Q$-factor for one of the central channels, in the XPM compensation case, by reducing the number of channels included in the C-NLSE (i.e. sequentially removing the most separated channels). For the 100 GHz channel spacing and with $P_T = 13$ dBm, we observed that the received $Q$-factor is consistently reduced as the effective bandwidth is reduced. The fact that no convergence is observed, means that all the channels contribute to XPM and hence, the MFA is inefficient in this system. This can be explained by considering the fact that, even though the walk-off effect averages the XPM interaction between well-separated channels, there is a cascaded effect between channels which propagates the XPM distortion from edge to central channels. Such effect, might require an increase in the effective bandwidth, even for largely spaced channels.

5. Conclusions

An investigation of the impact of XPM and FWM on electronic impairment compensation via backward propagation has been carried out. The relative impact of both effects has been evaluated by means of the coupled NLSE and the total-field NLSE, which have been solved by using the symmetric iterative SSFM. The DSP implementation of the post-processor has been presented stressing the parallel character of the coupled NLSE system. The results show that the impact of FWM is weak compared to XPM, which is the most important source of non-linear distortion. Analytical expressions for the characteristic SSFM step sizes, have been used to evaluate the computational requirements of each compensation scheme. A 12×100 Gb/s 16-QAM transmission system with coherent detection is simulated to evaluate the efficiency of the impairment post-compensation schemes. For a channel spacing of 50 GHz, XPM compensation has 1 dB of penalty on the received $Q$-factor with respect to FWM compensation. The different physical restrictions that are imposed for the FWM and XPM characteristic step sizes give rise to a digital XPM post-compensation that requires 1.4 times less number of operations and performs 17 times faster in terms of latency. For the $\Delta f=100$ GHz case, almost no improvement in the $Q$-factor is observed when FWM is compensated. In this case, the total-field NLSE solution requires almost 3 times more computations with a latency 34 times larger than the coupled NLSE scheme for XPM compensation. Finally, the possibility of modeling dispersive walk-off (as a pure delay) in the XPM compensation scheme, allows an additional reduction of the number of operations by a factor of $3/2$. 

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