Validity of Improved MTL for Effective Length of Counterpoise Wires under Low and High-Valued Lightning Currents

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Abstract

In this paper, an efficient modeling approach called improved MTL is used to predict effective length of counterpoise wires considering both ionization and dispersion of soils. This paper consists of two parts. At first part, validity of the model for computing effective length of counterpoise wires considering only soil ionization is investigated. The simulation results show that the improved MTL-based effective length of counterpoise wire are in good agreement with the existing formulae. Application of this modeling approach to include ionization and dispersion effects simultaneously (both-affected soil) is carried out in the second part. The simulation results show that in both-affected soils, the effective length with respect to only-ionized soils, is decreased especially in highly resistive soils under slow-fronted currents. This makes inclusion of both effects financially important in designing counterpoise wires.

Keywords: Ionization, dispersion, multi-conductor transmission line, counterpoise wires, lightning strike, and effective length.

1. Introduction

Counterpoise wires as typical grounding systems are widely used in discharging lightning currents to the soil. Fig. 1 shows different counterpoise wires consisting of horizontal electrode with different arms and injection points. Proper design of such devices are strictly dependent upon including complex phenomena such as ionization of soil [1], frequency variations of electrical parameters of soil (dispersion) [2] and so on. The former is occurred when the electric field of soil surrounding counterpoise wires is exceeded from its critical value inside soil, whereas the latter is taken place when the electrical parameters of soil is frequency-dependent.

There are a number of approaches for analysis and design of grounding systems including the frequency-domain approaches [3-5] for only-dispersive soils, and the time-domain approaches [6-8] for only-ionized soils. In the soils where ionization and dispersion of soils are simultaneously occurred, the mixed frequency-time domain approaches [9-15] should be used. All mentioned methods above are complex and time consuming.

One of important parameters in designing counterpoise wires is effective length which is defined as the length beyond which the impulse impedance (the ratio of peak values of induced voltage to lightning current) is no longer varied. In this regard a number of predicting formulae have been proposed for effective length of grounding electrodes in only-ionized soils [16, 17], only-dispersive soils [18], and neither-affected soils [19, 20].

In contrast with the mentioned accurate methods, J. L. Guardado et al [21] proposed multi-conductor transmission line model (MTL) and it was validated in soils having constant electrical parameters. In this model, each set of parallel conductors in the grounding systems is assumed as a multi-conductor transmission lines (MTL). A two-port network for each set of parallel conductors in the grounding system is then defined. Finally, the two-port networks are interconnected depending upon the pattern of connections in the grounding system and its representative equations are reduced. Through this approach, voltages and currents at any junction in the grounding system is easily extracted. Since the MTL is in the frequency domain, it was successfully applied in dispersive soil [22]. More recently it has been improved to include nonlinear phenomenon of ionization as well (improved MTL) [23] and was validated by comparing with full-wave methods and experiment. After then, predicting formulae for effective length of multiple vertical rods considering both ionization and dispersion were proposed [25]. Application of the proposed method for counterpoise wires was not addressed.

In this study, validity of the improved MTL for predicting effective length of counterpoise wires with and without considering ionization of soil is first investigated. The simulation results proves good agreement and very short runtime in comparison with [16, 19]. In addition, since the proposed approach is in the frequency domain, the soil dispersion effect can be easily included. Impact of both effects with respect to situation where only ionization effect is occurred, results in decreasing effective length that should be financially considered in designing such grounding systems especially in highly resistive soils.

This paper is organized as follows. In section 2, principles of the modified MTL is briefly explained with emphasis on the counterpoise wires. Section 3 is focused on validity of the modified MTL for predicting effective length of counterpoise wires buried in non-ionized and only-ionized soils. In section 4, significance of both dispersion and ionization on the effective length of counterpoise wires with respect to
situation where only ionization is considered is investigated. Finally, conclusion is given in section 5.

Figure 1: Different arrangements of horizontal electrodes under lightning current. (a): corner-injected one-arm electrode, (b): center-injected two-arm electrode, (c): center-injected four-arm electrode.

2. Improved MTL

Consider a horizontal electrode under high-valued impulse current so that ionization phenomenon is taken place as shown in Fig. 2(a). This phenomenon is conventionally modelled as gradually increasing radius as shown in Fig. 2(b) in which the electrode is divided into N segment of length \( l_k \) and equivalent radius \( a_1 \) [8]. Then, the sending and receiving voltage and current for each segment is defined as shown in Fig. 3(a). After then, the relation between voltage and current at the sending and receiving points is represented as the two-port network as shown in Fig. 3(b) and expressed in (4).

Figure 2: (a): Ionization phenomenon around the conductor, and (b): ionization model as gradually increasing radius. Adapted from [8].

Figure 3: (a): Definition of sending and receiving voltage and current for a conductor of length \( l_k \) and (b): Two-port network representation of a conductor of length \( l_k \).

\[
\begin{bmatrix}
I_{sk} \\
I_{rk}
\end{bmatrix} =
\begin{bmatrix}
A_k & B_k \\
C_k & D_k
\end{bmatrix}
\begin{bmatrix}
V_{sk} \\
V_{rk}
\end{bmatrix}
\]  
(4)

where \( V_{sk} \) and \( I_{sk} \) represent, respectively, the voltage and current at the sending point of the \( k \)th segment. Also, \( V_{rk} \) and \( I_{rk} \) are, respectively, the voltage and current at the receiving point of the \( k \)th segment. Now, if the above representation is applied to each segment in Fig. 2(b), and the lightning current is also denoted by current source \( \bar{I}_s \), the cascaded two-port networks as shown in Fig. 4 is consequently achieved.

\[
\bar{I}_s = \text{MTL}^{-1} \bar{I}_s
\]  
(5)

where \( \bar{V}_s \) is a vector including sending and receiving voltages of all segments, while \( \bar{I}_s \) is a vector containing the phasor of the lightning current at the injection point. Also, matrix of MTL includes two-port networks of all segments. From (5), once the lightning current \( \bar{I}_s \) is known, the sending and receiving voltages at any segment can be easily computed. Accordingly, the sending and receiving currents of each segment are computed from the individual two-port network as expressed in (4). Note that all computations are carried out at each frequency inside spectral content of lightning current.

The perpendicular component of electric field on the surface of \( k \)th segment is then computed as bellow [24]

\[
E_k = \frac{I_{lk}/l_k}{2\pi(1/p + j2\pi f)e_k}, \quad k = 1, 2, .., N
\]  
(6)

where \( \varepsilon \) and \( \rho \) are dielectric constant and resistivity of the lossy soil. \( I_{lk} \) is also leakage current of \( k \)th segment computed via subtracting currents at the sending and receiving points of each segment. If the magnitude of \( E_k \) is greater than its critical value \( (E_c = 300 \text{kV} / \text{m}) \), radius of each segment is increased. For the new value of radius, Eq. (5) is again solved up to \( E_k < E_c \) for each frequency inside spectral content of the lightning current. Finally, using inverse fast Fourier transform (IFFT), the sending voltage of each conductor in time domain, is easily computed.

As explained in [22], the mutual coupling effect among parallel conductors is only considered, and the other ones, i.e. echelon and collinear couplings, are low enough so that they can be neglected. Therefore, the analysis of center-fed electrodes with two and four arms are the same as corner-fed one-arm electrode except that the lightning current is equally
divided into arms. Flowchart of the MTL approach is illustrated as step by step in Fig. 5. Further information about the improved MTL can be found in [23].

3. Validity

In this section, the validity of the proposed method for analyzing grounding electrodes buried in only-ionized soils is investigated. Hence, a center-fed four-arm electrode (also called cross-arm electrode) with length of L = 5m which is injected by an impulse current 20 / 40µs with peak value of 7 kA, is considered. The soil where the electrode is buried consists of two layers with the upper and lower layer resistivity respectively 15.8Ωm and 2.6Ωm. The upper layer thickness is 6.2 m, whereas the lower layer thickness is infinite. The simulation results for transient voltage based on the improved MTL is shown in Fig. 5. In this figure, the measured results [11] are also included (see Fig. 5 in [11]). From this figure, one can easily observe that although the mutual coupling among collinear arms is ignored, good agreement is achieved. To better comparison, the peak value of transient voltage, i.e. grounding potential rise (GPR), and rise time of the transient voltages are compared in table 1 which is good agreement is observed.

![Figure 5: Transient voltage of center-fed four-arm electrode based on the improved MTL and measurement.](image)

**Table 1: GPR and T, of center-fed four-arm electrode based on improved MTL and measurement.**

| Cross-arm electrode | GPR(kV) | T, (μs) |
|--------------------|--------|---------|
| Improved MTL       | 9.2    | 19.5    |
| Measurement [11]   | 8.8    | 20      |

4. MTL-Based Analysis of Counterpoise Wires

In this section, the improved MTL is applied to counterpoise wires in Fig. 1, and its validity on the effective length in only-ionized soils is investigated. To this end, a horizontal electrode of length 12m and radius 12.5mm buried in a lossy soil with different resistivity and relative dielectric constants [25] is selected. The lightning current is also the same as [25] which is shown in Fig. 6. This current is a double-exponential waveform with rise time 8µs and peak value of 50kA, i.e. 8 / 20µs - 50kA. The transient voltage of counterpoise wires with and without ionization for different soils are shown in Fig. 7. As seen, the simulation results of corner-fed one-arm electrode in Fig. 7(a) is in excellent agreement with [25]. Also, from Figs. 7(b) and (c), when the number of arms are increased the ionization effect is decreased especially for poorly resistive soils. This is owing to decreasing the current injecting into the arms which results in decreasing the ionization effect. The impulse impedances (the ratio of peak values of transient voltage to injected current) are computed and shown in Fig. 8. The effective length, i.e. a starting length at which the impulse impedance is no longer varied [26], is easily extracted in Fig. 8 and compared with the individual ones in [16] in Fig. 9. From Fig. 9, the effective lengths are in good agreement. Tables 2 and 3 show the relative error percentage between the proposed method and the ones by L. Grecév [19] and J. He et al [16] respectively for non-ionized and only-ionized soils. Comparison shows small relative percentage in only-ionized soil. This error is more pronounced for non-ionized soil. This is owing to different expressions for the lightning current in this study and the one in [19]. Nevertheless, they are small enough from electromagnetic engineering point of view.

![Figure 6: Time domain representation of the lightning current used in this study.](image)

![Figure 7: Transient voltage of counterpoise wires with and without ionization for different soils.](image)
Figure 7: Transient voltage of (a): corner-fed electrode, (b): center-fed electrode with two arms, and (c): center-fed electrode with four arms for the three lossy soils with and without considering ionization of soil.

Figure 8: Impulse impedance of (a): corner-fed electrode, (b): center-fed two-arm electrode, and (c): center-fed four-arm electrode for the three lossy soils with and without ionization.
5. Sensitivity Analysis

In the previous section, the validity of the improved MTL for predicting transient voltage, and effective length of counterpoise wires buried in non-ionized and only-ionized soils was investigated. Evidently, due to its frequency-domain nature, the dispersion effect of soil can be easily incorporated. The dispersion model of soil here used is based on Alipio-Visacro measurement [4], that is

\[ p(f) = \rho_0 \left( 1 + \left( 2.2 \times 10^{-4} \times \rho_0^{0.75} \right) \right) (f - 1000)^{0.65} \]  

(7)

\[ \varepsilon'(f) = \begin{cases} 192.2 & f \leq 10kHz \\ 1.3 + 7.6 \times 10^{-5} \times f^{-0.4} & f \geq 10kHz \end{cases} \]  

(8)

where \( \rho_0 \) is low-frequency resistivity of lossy soil. Frequency variations of resistivity and relative permittivity are shown in Fig. 10. This figure shows that the soil dispersion is more pronounced for highly resistive soils (\( \rho_0 > 1000 \Omega \cdot m \)) and vice versa.

![Frequency variation of resistivity and relative permittivity](image)

Figure 10: Frequency variation of resistivity (left axis) and relative permittivity (right axis) based on Alipio-Visacro measurement [4].

Ability of the improved MTL for computing transient voltage in both-affected soils was validated [23] (see Fig. 8 in [23]). However, there is no research on the effective length of counterpoise wires buried in both-affected soils in comparison with only-ionized soil. Hence, to show the difference between the two situations, i.e. both-affected and only-ionized soils, a sensitivity analysis is carried out using defining the following decrement factor:

\[ DF = \frac{L_{eff} \text{ of both-affected soil}}{L_{eff} \text{ of only-ionized soil}} \]  

(9)

Now the effects of three parameters, i.e. low-frequency resistivity of soil (\( \rho_0 \)), rise time (\( T_M \)) and peak value (\( I_M \)) of the lightning current, on the DF are investigated. The simulation results for sensitivity analysis are shown in Figs. 11, 12 and 13. From these figures, the following key findings are inferred:...
1-From Fig. 11, when the low-frequency-resistivity of soil is increased, the effective length of counterpoise wires buried in both-affected soil become less than the individual one in only-ionized soil especially in highly resistive soils, whereas in poorly resistive soils ($\rho_0 \leq 10\Omega \cdot m$), since the dispersion effect is decreased, two situations results in approximately the same effective length. In addition, when the number of arms is increased, since the ionization effect is decreased, the decrement factor is less affected.

2-With reference to Fig. 12, when the peak value of lightning current is increased, two situations results in approximately the same effective length. This is physically because of decreasing/increasing the soil resistivity/conductivity surrounding the electrodes [1] which results in decreasing dispersion effect. This fact is more pronounced when the number of arms is increased.

3-As can be seen from Fig. 13, for low-valued rise times (fast-fronted currents), the two situations results in the same effective length, whereas for high-valued rise times (slow-fronted currents) the effective length in both-affected soil is less than the individual one in only-ionized soils. This fact was also observed in only-dispersive soils [18] where the effective length of electrodes under subsequent stroke current is less affected than that of the first stroke current. Physical reason can be found in [2]. The above extracted findings are financially important in power engineering point of view.

More recently Shariatinasab et al [15] proposed a hybrid method [15] which is based on combining MoM with harmonic balance method (MoM-HBM) considers both effects, it is, however, suffers from time-consuming computations of MoM and Newton’s Raphson algorithm in iteration process. These drawbacks are repeated when the weather conditions are changed. The computation times by the proposed method and MoM-HBM for only-ionized and both-effect soils are listed respectively in tables 4 and 5.

### Table 4: Comparing run-times of the different approximate methods for computing transient voltage.

| Counterpoise wires (only-ionized) | Run-time (sec) |
|------------------------------------|----------------|
| This paper                         | 1.4            |
| MoM-HBM [15]                       | 1.4            |
| MoM-HBM [15]                       | 1.4            |

| Counterpoise wires (both effects) | Run-time (sec) |
|-----------------------------------|----------------|
| This paper                        | 1.5            |
| MoM-HBM [15]                      | 1.5            |
| MoM-HBM [15]                      | 1.5            |

In table 4, ‘20’, ‘163’ and ‘+’ respectively mean the run-times of MoM, HBM and algebraic summation. From tables 4 and 5, one can see that the high computational efficiency of the improved MTL in comparison with MoM-HB. All computations were carried out on an Intel (R) Core (TM) i7-4702MQ CPU with 4GB of Ram.

### 6. Conclusion

In this study using a frequency-domain approach called improved MTL, significance of two aspects of lossy soils namely ionization and dispersion on the effective length of counterpoise wires was investigated. The simulation results show that when both effects are taken place, the effective length is generally decreased especially for highly resistive soils and slow-fronted currents which is is financially of importance. For poorly resistive soils and fast-fronted currents, however, the ionization effect is dominant so that the dispersion effect can be disregarded. The next step is to
extract closed-form solutions for effective length of counterpoise wires buried in both-affected soil using combining the proposed method with optimization algorithms [27-31], or fuzzy inference techniques [32-39] that is in progress.

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