Implosion-explosion in supernovae

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Supernovae explosions of massive stars are nowadays believed to result from a two-step process, with an initial gravitational core collapse followed by an expansion of matter after a bouncing on the core. This scenario meets several difficulties. We show that it is not the only possible one: a simple model based on fluid mechanics and stability properties of the equilibrium state shows that one can have also a simultaneous inward/outward motion in the early stage of the instability of the supernova. This shows up in the slow sweeping across a saddle-center bifurcation found when considering equilibrium states associated to the constraint of energy conservation. We first discuss the weakly nonlinear regime in terms of a Painlevé I equation. We then show that the strongly nonlinear regime displays a self-similar behavior of the core collapse. Finally, the expansion of the remnants is revisited as an isentropic process leading to shocks formation.

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I. INTRODUCTION

Although supernovae make one of the most spectacular phenomena displayed to us in the Universe, their understanding remains a challenge. As stated recently in a review on the subject [1] it is still “in an unsatisfactory state of affairs”. A major unsolved problem concerns the death of young massive stars, supposed to occur in two steps, first a collapse then emission of matter and radiation by an explosion. The problem with this picture, relying in part on extensive hydrodynamical simulations, is the difficulty to explain the reversal of the motion, from inward (collapse or implosion) to outward (the observed explosion of supernovae) which requires very large outward forces to turn the tide. According to most works on core collapse of supernovae, this reversal is due to a stiffening of the equation of state at the center, which requires very large outward forces to turn the tide. According to most works on core collapse of supernovae, this reversal is due to a stiffening of the equation of state at the center, which stops the collapse and leads to a bounce. Without bounce, namely if the shock remains at a more or less fixed location, matter keeps flowing inward and there is no explosion at all. In numerical studies, an outward propagating shock can be created, but typically this shock stalls at some radius, and it is hard to find a mechanism making it move again outward (see [1] [2] and references herein). Neutrino heating has been invoked but numerical simulations have shown that this is not generally sufficient to produce an explosion. More recently, 3D hydrodynamic instabilities have been discussed, that are still controversial. In summary, the revival of the stalled accretion shock remains an unexplained process since the early 1980’s.

In our theory, we focus on the pure fluid mechanical part of the physics of supernovae without considering the immensely complex network of possible nuclear reactions in the core. In particular, we do not add any mechanism of heating of compressed matter by increasing the rate of nuclear reactions in the evolving system, which should take into account the sensitivity of nuclear reactions to temperature and density, a direct consequence of the fact they are linked to quantum tunneling. Using the tools of bifurcation theory we show that an outgoing velocity field in the early stage of the loss of equilibrium of the star is compatible with a simple hydrodynamic model. In particular, it illustrates that a change from a canonical to microcanonical description does change the outcome of the evolution: the canonical model collapses without any outgoing flow as treated in our previous paper [3], although the microcanonical model studied here shows this outward flow! In both cases, supernova is described as a dynamical catastrophe due to a slow crossing of an instability threshold by the control parameter, the temperature in [3] and the total energy of the star here. These instabilities are manifested by a loss of balance between the inward pull of self-gravity and the outward pull of the pressure. The loss of balance is a global phenomenon that depends on the eigenvalues of a stability equation depending itself on the distribution of matter and energy in the star. Therefore, if the gravity is not dominant everywhere with respect to the pressure, one expects different orientations of the radial velocity as a
function of the radius in the early post-bifurcation stage, as found here.

Using inviscid compressible fluid equations with gravitation and a particular equation of state, we showed in [3] that the large difference of time scales between the slow evolution of a star and its quick explosion is explained by the slow crossing of a saddle-center bifurcation of its steady state. The dynamics close to the bifurcation is described by a “universal” Painlevé equation. Later, higher order terms come into play and require a full numerical study. In reference [3], the uniform temperature \( T \) of the star was the slowly varying control parameter, a situation which amounts to considering a star in contact with a thermostat whose temperature decreases very slowly with time. We refer to this model as the canonical Euler-Poisson model (CEP). In this case, we did observe a collapse of the whole mass of the star towards the center, as in published numerical studies of this phenomenon. In the present paper, using the same approach as in [3], we investigate the same hydrodynamical model, also with spherical symmetry, but in the microcanonical description (MEP model), namely by adding a condition of conservation of total energy. In other words, the control parameter is now the total energy \( E \) of the star, supposed to slowly decrease with time.

This change from given \( T \) to given \( E \) was motivated by previous studies concerning phase transitions in self-gravitating N-body systems (see the review in [4]) which may have applications in astrophysics where galaxies, globular clusters, dust gas, fermions gas (like electrons in white dwarfs or neutrons in neutron stars) are examples of self-gravitating systems. Using thermodynamics and statistical mechanics tools, it was found that very different dynamics characterize canonical ensembles (with fixed temperature) and microcanonical ensembles (with fixed energy) especially in the vicinity of phase transitions. In certain situations a collapsed core is formed in the canonical case, whereas a core surrounded by a halo is formed in the microcanonical case. Therefore, a question naturally sets up: what should be obtained with the fluid model proposed in [3] when passing from the canonical description which leads to a total collapse of the star with a growing singularity at its core, to the microcanonical one? We show below that we obtain simultaneously a collapse of the central part of the star, and emission of matter in the outer part. This simultaneous inward/outward motion occurs from the early stage of the bifurcation until the core collapse in the MEP model, but not in the CEP one. Indeed, if a constant temperature is assumed, there is an overall inward motion inside the whole star, from the very beginning of the loss of equilibrium. If one imposes with the same equation of state the constraint of energy conservation instead of the one of given temperature, the all inward-going velocity field is turned into an inward velocity field near the center of the star and outward in the rest of the star. This simple model opens the way to a new understanding of the explosion of stars, based on fluid mechanics, catastrophe theory, and bifurcation properties of their equilibrium state. It also provides a nice illustration of the property of inequivalence between canonical and microcanonical ensembles for systems with long-range interactions.

This Letter is organized as follows. We start by describing the first stage of the dynamics, where nonlinearities with respect to the amplitude of the instabilities are weak. We derive the normal form for the evolution of the unstable amplitude, which takes the generic form of a Painlevé I equation, and which agrees with the numerical solution of the full MEP system. Then, we show that beyond the Painlevé regime the numerical solution displays a self-similar behavior of the core collapse, with scaling laws illustrating, as expected, the dominance of the gravity forces over the pressure in the core. In this strongly nonlinear phase of the dynamics the inward and outward velocities increase with time, the acceleration of the inward motion in the core being larger than the one of the explosion of the outer shell. Finally, after a short self-similar post-collapse solution, we propose to interpret the expansion of the remnants as an isentropic process described by a Burgers-type equation leading to shocks formation.

II. SADDLE-CENTER IN THE MICROcanonical DESCRIPTION OF A SELF-GRAVITATING FLUID

The microcanonical model differs from the canonical one presented in [3] by an equation expressing the condition of fixed energy which has to be added. This simple change has noticeable consequences, it modifies the properties of the equilibrium states, especially the radial profile of the neutral mode. Using dimensionless variables such that \( G = \rho_\star = k_B = M = 1 \) (\( G \) and \( k_B \) being Newton’s and Boltzmann’s constants and \( M \) the total mass of the star), the Euler-Poisson system is

\[
\frac{\partial p}{\partial t} + \nabla \cdot (\rho u) = 0, 
\]

\[
\frac{\partial u}{\partial t} + (u \cdot \nabla) u = -\frac{1}{\rho} \nabla p - \nabla \Phi, 
\]

\[
\Delta \Phi = 4\pi \rho, 
\]

where \( u(r,t) \) is the radial fluid velocity and \( \rho(r,t) \) the mass density. We consider a barotropic equation of state of the form \( \rho(r,t) = T(t) g(\rho(r,t)) \), namely with uniform temperature as in [3]. However, differently from [3], we assume that the temperature \( T(t) \) evolves so as to conserve the total energy, with a simple energetic constraint of the form

\[
E = \frac{1}{2} \int \rho u^2 \, dr + \frac{3}{2} T(t) + \frac{1}{2} \int \rho \Phi \, dr, 
\]
which determines the uniform temperature $T(t)$ for a
given $E$.

We take the same equation of state as in [3],

$$ p(\rho, T) = T \left( \sqrt{1 + \rho} - 1 \right)^2, $$

which reduces to the isothermal equation $p = T \rho$ in the
high density region near the core, and to the polytropic
equation of state $p = T \rho^\gamma / 4$ with index $2$ at low den-
sity (edge of the star). This equation allows to confine
the system in a finite region, while having an isothermal
core. Solution with an isothermal core was chosen be-
cause previous studies on isothermal spheres enclosed in
a box have led to a saddle-center bifurcation [5, 6] as the
energy or the temperature varies.

Our first step is to investigate whether such a transition
exists within our model. To do that we compute the
equilibrium states, which are conveniently obtained from
the enthalpy $h$ defined by the relation $dh = dp/\rho$, or
$h(\rho) = \int_0^\rho \frac{\rho'(\rho')}{\rho'} d\rho$. Adding the relation $h(\rho = 0) = 0$
(which defines the radius $r_0$ of the star), one obtains
$h(\rho, T) = 2h_0 (1 + \sqrt{T + \rho} - 2 \ln(2))$.

Using the set of dimensionless variables and fields as
defined in [3] in order to have a single free parameter,
$h_0 = h(r = 0)/T$, varying the value of $h_0$ and returning
to the above dimensionless variables, we can draw the
spiralling curves depicting the steady states, such as the
radius of the star $r_0(E)$ or the inverse temperature $\beta(E)$
shown in Fig. 1. In curve (a), following the spiral from
large values of $E$ until the lowest value $E_\text{c} \approx -0.984$, one can
show that the solution is linearly stable before the
turning point $A'$ and becomes unstable after. The point
$A'$ is a saddle-center for the microcanonical description
because here a stable center merges with an unstable sad-
dle. Curve (b) displays the caloric curve $\beta(E)$ with its
two critical points $A$ and $C$ characterizing the first bi-
furcation for the CEP and MEP models respectively (see
the Figure caption). In between $A$ and $C$ (i.e., in the
region where the specific heat $C = dE/dT$ is negative)
the system is unstable for the CEP model, whereas it
remains stable for the MEP model.

FIG. 1: Equilibrium state radius versus energy $r_0(E)$ in (a);
caloric curve $\beta(E)$ in (b) with $\beta = 1/T$. The spiral in (b)
displays the two critical points $A$ and $C$ which characterize
the first bifurcation occurring respectively at $E_\text{c} \approx -0.984$ for
the MEP model, and at $\beta_E^{\text{min}} \approx 0.647$ for the CEP model.

III. PAINLEVÉ REGIME

Close to $A$ the weakly nonlinear analysis detailed in [7]
is performed by expanding the MEP solution in powers of
a small parameter $\epsilon$ associated to the slow decrease of
the energy of the form $E(t) = E_\text{c} - \epsilon^2 t$, written as

$$ E = E_\text{c} - \epsilon^2 E^{(2)}, $$

that amounts to defining $\epsilon^2 E^{(2)} = \gamma^2 t$ and taking $\epsilon$ small.

At first order, setting $\delta M(r, t) = \epsilon A^{(1)}(t) \delta M^{(c)}(r)$ and
similar expressions for other small variations, one finds
an ordinary integro-differential equation for the neutral
mode profiles. The radial profiles $S(r)$ of the velocity
field and mass deviation $\delta M^{(c)}(r) = \int_0^r \delta \rho^{(c)}(r') 4\pi r'^2 dr'$
shown in Fig. 2 (a) illustrate well the joint inward/outward motion of matter in this small amplitude
regime. The velocity is negative (inward motion) in the
core and positive (outward motion) in the halo. The
important point is that the double direction occurs si-
multaneously during the early stage of the bifurcation,
as observed numerically. For comparison, we present in
Fig. 2 (b) the neutral mode velocity profile of the CEP
model, which clearly shows an inward motion everywhere
in the star, as confirmed by the numerics [3]. This is the
key point of the present study, illustrating the very dif-
f erent behavior of the solution from the very beginning
of the approach to the phase transition, whether we con-
sider a model including the energetic constraint [4] or
not.

At order two, the solvability condition amounts also
to solving an integro-differential equation, which leads

FIG. 2: (a) MEP model: first order radial profiles for the
neutral mode of the velocity (or displacement) $S(r)$ and of
the mass deviation $\delta M^{(c)}(r)$ close to the saddle-center. (b) CEP model: neutral mode profile of the velocity.
ultimately to a normal form of Painlevé I type,

\[ \dot{A}(t) = \gamma t + KA^2, \quad (7) \]

for the time evolution of the deviations close to the critical point. We find \( \gamma = 46.63 \gamma' \) and \( K = 1055.98 \), that gives the temperature evolution drawn in Fig. 3 solid line, in good agreement with the numerical results (red dots) of the full MEP equations for this early stage of the implosion-explosion process. For comparison, we note that for the CEP model, the normal form (also of Painlevé I type) was found with coefficients \( \gamma \simeq 120.2 \) and \( K \simeq 12.3 \), leading to a slower growth of the amplitude. This relies on the fact that the critical density at \( r = 0 \) is much lower (by a factor 100) for the CEP model.

IV. POST-PAINLEVÉ DYNAMICS
(PRÉ-COLLAPSE)

After the weakly nonlinear Painlevé regime, the full MEP system of equations displays a solution which ultimately diverges, as illustrated in the insert of Fig. 3. This divergence of the temperature is associated to core-collapse, as illustrated in Fig. 4 where the density and velocity are drawn versus \( r \) for successive time values (the maxima of the density and absolute velocity profiles increase with time). In (a) the logarithmic scale shows that the solution in the core becomes self-similar. The singularity is of second kind in the sense of Zel’dovich, as already found in [3] for the CEP model leading to the collapse of the whole star. For both cases (MEP and CEP models) the core collapse is characterized by the fact that gravity dominates over pressure forces, but the exponents found here are different from the previous ones. Recall that for the gravity-dominating case, using the notations of [3] the self-similar density is of the form

\[ \rho(r, t) = (-t)^{-2}R(r(-t)^{-2/\alpha}), \quad (8) \]

and the velocity

\[ u(r, t) = (-t)^{-1+ \frac{2}{\alpha}}U(r(-t)^{-2/\alpha}), \quad (9) \]

where \( \xi = r(-t)^{-2/\alpha} \). That gives \( R(\xi) \sim \xi^{-\alpha} \), and \( U(\xi) \sim \xi^{-(\alpha/2-1)} \) for \( \xi \to +\infty \) in order to have a steady profile at large distances. The exponent \( \alpha \) is related to the behavior of the self-similar solution as \( \xi \) tends to zero [3]. More precisely, expanding \( R \) as \( R = R_0 + R_k \xi^k + ... \) (and \( U \) as \( U = U_1 \xi + U_k \xi^{k+1} + ... \)), one has \( \alpha(k) = \frac{2k}{2k+3} \). In the present case we find numerically that the asymptotic behavior of the density displays an exponent \( \alpha \) larger than two, that is equivalent to the gravity dominance (over pressure) property. More precisely, we obtain a best fit (solid line) with the value

\[ \alpha = 48/19 \quad (10) \]

which corresponds to \( k = 8 \), namely to the on-axis behavior \( R = R_0 + R_k \xi^8 + ... \). For comparison, we plot in dashed black line the slope corresponding to the value \( \alpha = 24/11 \), or \( k = 4 \), found for the CEP model [3].

In this strongly nonlinear regime standing from the Painlevé (weakly nonlinear) stage ending at \( t_s \) up to the divergence of the solution at \( t_s \), the absolute value of the inward and outward velocity increase everywhere in the star, as illustrated in Fig. 4 (b). The velocity
curves clearly show that the inward motion accelerates more rapidly (in the core) than the outward motion taking place in the outer shell. This remark allows us to deduce the temperature behavior in the post-Painlevé regime. For \( t_s < t < t_a \), using Eq. [3] and neglecting the contribution of the halo in the energy, we get \( T(t) \propto (t_s - t)^{15 - 2k}/3^k \), hence for \( k = 8 \),

\[
T(t) \propto (t_s - t)^{-1/24},
\]

which diverges at the collapse time. During the self-similar growth of density and inward velocity in the core, what happens in the halo? From Fig. 4(a) we observe by zooming on the halo region that the density decreases with time. Moreover, Fig. 4(b) clearly shows that the velocity which diverges in the core in the pre-collapse regime, barely increases and remains finite in the halo which expands radially by about 20%, a small evolution compared to the strong shrinking of the core. Investigating possible self-similar solutions of the form [8] and [9] for the halo, it is shown in [7] that no self-similar solution, if it exists, agrees with the numerical curves, either assuming that gravity is negligible with respect to the pressure, or the inverse, or else in the mixed case. We conclude that no self-similar solution is able to describe the dynamics of the halo before the collapse, the expansion of the halo being still in a preliminary stage.

V. POST-Collapse

Just after the collapse time when the mass at the center of the star is zero, self-similar solutions exist in the core and in the halo, as detailed in [7], and summarized below. They are obtained under the hypothesis that gravity overcomes pressure forces in the core, and the opposite in the halo. In the core, the post-collapse situation is then qualitatively the same as in the CEP model, and looks (mathematically) like the one of the dynamics of the Bose-Einstein condensation where the mass of the condensate begins to grow from zero after the time of the singularity [3, 9]. For the MEP model, the self-similar solution is the one derived in [3] but with the value of the exponent \( a \) found here, Eq. (10). We recall that the main change with respect to the pre-collapse study amounts to adding to the equations of density and momentum conservation, an equation for the mass at the center \( M_c(t) \) (with \( M_c(0) = 0 \)),

\[
\frac{dM_c}{dt} = \left[ -4\pi r^2 \rho(r) u(r) \right]_{r \to 0}.
\]

We get \( M_c(t) \propto t^b \), with \( b = 6/\alpha - 2 \) a positive exponent [3]. Because the scaling laws are the same before and after the singularity, Eq. (10) yields \( b = 3/8 \). If we replace the singular core by a relativistic compact object such as a neutron star, we get an estimate of the temperature evolution by the relation \( k_BT \sim M_c c^2 \) leading to the scaling

\[
T(t) \propto t^{3/8}.
\]

In the description of the halo expansion just after the explosion, we assume that the energy released during the collapse of the core heats the halo and provides its expansion. Indeed, as the gravitational energy \( W \) of the core decreases and becomes very negative, the temperature \( T \) of the halo given in [13] and its macroscopic kinetic energy \( E_{\text{kin}} \) increase and become very large \((T \sim E_{\text{kin}} \sim -W)\) as a result of energy conservation. Therefore, the pressure inside the halo can be high enough to accelerate its expansion. We assume that the pressure forces in the halo are stronger than the gravity forces, a condition checked in fine, and also that the velocity increases linearly with the radius, \( u(r,t) = H(t)r \). Within this frame, and for a polytropic equation of state \( P = K(t)\rho^\gamma \), where \( K(t) \) increases with time, we show in Appendix C of [7] that the radius \( R(t) \) of the halo is solution of

\[
\ddot{R}R^{3\gamma - 2} = K(t).
\]

The expansion rate \( \dot{R}(t) \) is time dependent, a result which differs from the common description of the remnant motion just after the explosion (supposed to expand with a constant velocity due to the conservation of kinetic energy). For \( \gamma = 1 \) and \( T(t) \propto t^{3/8} \), the radius increases as \( R(t) \propto t^{19/16} \), a solution that is expected to merge ultimately with the non self-similar Burgers solution suggested just below, which displays shocks.

VI. FREE EXPANSION STAGE (SHOCKS)

The last stage of the evolution of the supernova explosion is of interest, although it is often considered as rather uneventful. In this so-called “free expansion stage” the ejecta are generally believed to cool adiabatically because of their free expansion. We consider remnants as making a dilute gas, much denser than the interstellar medium. This free expansion stage lasts until the density of the remnants becomes of the order of magnitude of the one of the interstellar gas, so the expanding gas is dense enough to be interacting with itself (the attraction by the core of the exploded star is also considered), but not with the interstellar medium, and it makes a continuous fluid, not a Knudsen gas. Our argument relies on the fact that the mean-free path in interstellar matter may be as large as the size of a galaxy, that makes such event unrealistic. In that frame, the remnant is an entity which does not exchange energy and mass with the interstellar medium. Therefore, we assume that the expansion of a gas bubble in vacuo is an isentropic process with two constraints, the conservation of mass and energy. We start from the equations for an inviscid compressible ideal fluid with spherical symmetry, see section 6 of [10], which include the isentropic condition

\[
\frac{\partial}{\partial t}(r^2s\rho) + \frac{\partial}{\partial r}(ur^2s\rho) = 0,
\]

where \( s \) is the entropy per unit mass. The pressure \( P \) is now a function of \((\rho, s)\), \( P = K(s)\rho^\gamma \), with \( \gamma \) larger
than unity. Assuming that entropy is initially uniform, we try to find a possible self-similar solution with spherical symmetry of the form \( F(r,t) = r^a f(r/t^b) \), where the exponent \( a \) depends on the field \( F \) under consideration (that is either \( \rho \), \( u \) or \( s \)) although \( b \) is the same for all fields.

First, neglecting the gravity with respect to the pressure forces, we find that the two conservation laws impose that \( a = -3 \), \( b = -1 \), and \( \gamma = 1 \). The latter condition is incompatible with the definition of \( \gamma = c_p/c_v \). We conclude that a self-similar expanding solution for the halo is not physically meaningful, contrary to the free fall of dense molecular gas (where the opposite was assumed).

Secondly, we assume that gravity and pressure forces can be neglected, that reduces the momentum equation to

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} = 0, \tag{16}
\]

an equation well-known since Poisson to have the implicit solution

\[
u(r,t) = u_0(r - ut), \tag{17}\]

where \( u_0(r) \) is the initial radial velocity. This solution conserves the order of magnitude of \( u \) in the course of time, which is consistent with the conservation of energy. This Burgers-type equation is a prototype for creating shocks. The solution for the density is

\[
r^2 \rho(r,t) = \frac{\partial r_0}{\partial r} r^2 \rho_0(r_0(r,t)), \tag{18}\]

where the index 0 refers to the initial condition, as above. We check [7] that the term \( u \partial u/\partial r \) in Eq. (16), the dynamical pressure, is dominant over the term of thermodynamical pressure, \( P_r/\rho \), and over the gravitational term \( -4\pi G/r^2 \int_0^r \int_0^r \rho(r') \), during the expansion of a dilute gas before it enters into the Knudsen regime. Shocks forms if the radial velocity is larger for a given radius than for a larger one, because the larger velocities overcome the slower ones. Therefore, shocks are formed naturally inside the remnant, depending on the initial distribution of the fluid velocity inside the remnant, and they propagate inside this matter, the role of the interstellar medium being ignored.

This study shows that in order to create structures in an expanding gas volume, as observed in the remnants, there is no need to have interaction with an outside interstellar gas. This early stage of the expansion is, by far, the one that is the best known experimentally because it is a stage where the remnants are still far more luminous than the rest of the Galaxy. Based on the existence of such internal shock waves, we suggest an explanation for the very sharp luminous rings observed in the remnants of SN1987A.

Finally, we have to note that we have neglected a set of perhaps crucial phenomena, namely plasma effects due to the finite electric conductivity of the expanding gas (this yields Laplace forces which could supersede inertia and gravity in the expanding gas). We have also assumed spherical symmetry, not displayed by the observed remnants, except for their large-scale structure, but asphericity is not so crucial from the point of view of the present analysis.

VII. DISCUSSION

Presently, theories of supernova explosion focus on a physical phenomenon, the emission of neutrinos, or on 3D effects which we do not consider at all in our work. In our theory, we focus on an entirely different part of the complex physics of supernovae; namely the fluid mechanical part. We show that implosion and explosion taking place at the death of a massive star may occur simultaneously. This yields an alternative explanation to the yet unsolved problem of supernovae description where the two steps process is an unsatisfactory explanation. Using a simple model, we point out that the huge difference of time scales between the long life of a star and its abrupt death can be understood in the light of catastrophe theory, by a slow sweeping of a saddle-center bifurcation. Starting from the stable equilibrium state and approaching the saddle-center, we show that the weakly nonlinear analysis leads to a universal Painlevé equation for the amplitude of a bi-modal motion (with two opposite directions). Our study illustrates once more (see [4]) that a change from canonical to microcanonical description, not looking very important, does deeply change the outcome of the transition from stable to unstable state. Here the MEP model shows an explosive outer shell together with a core collapse although the CEP model studied in [3] shows a collapse of the star without any outgoing flow. This is a manifestation of ensembles inequivalence for systems with long-range interactions. Note that the CEP model could describe most of hypernovae since supermassive stars often collapse with emission of very intense gamma ray bursts, but without any explosion of the outer shell.

It is important to point out that the Painlevé analysis yields a definite sign for the velocity field at critical, contrary to what happens in “classical” transition from linearly stable to linearly unstable situation (where the unstable mode may have either positive or negative amplitude). This fair property of the definite sign of the growing Painlevé solution comes from the fact that in the case of a saddle-center bifurcation, the two stable and unstable equilibrium states merge at the critical point, beyond which no equilibrium state exists (neither stable nor unstable) that makes the difference with the “classical” case of a generic instability. Then, we describe the self-similar core-collapse regime where the acceleration is larger in the collapsing core than in the exploding envelope. Finally, after a short self-similar post-collapse solution, we propose to interpret the expansion of the remnants as an isentropic process which conserves the
energy and the mass of the halo. Within this description, there is no self-similar solution, contrary to the free fall of dense molecules, but another type of solution appears, of Burgers-type, which is a prototype for creating shocks. In our rough description, shocks are formed naturally inside the remnant, and they propagate inside this matter, the role of the interstellar medium being ignored.

[1] A. Burrows, Rev. Mod. Phys. 85 (2013) 245.
[2] P. Clavin, G. Searby, in Combustion Waves and Fronts in Flows (Cambridge University press, 2016), pp.32-40.
[3] Y. Pomeau, M. Le Berre, P.-H. Chavanis, B. Denet, Eur. Phys. J. E 37 (2014) 26.
[4] P.-H. Chavanis, Int. J. Mod. Phys. B 20 (2006) 3113.
[5] R. Emden, Gaskugeln Anwendungen der Mechanischen Wärmetheorie auf Kosmologische und Meteorologie Probleme (B.G. Teubner, Leipzig, 1907).
[6] P.-H. Chavanis, Astron. Astrophys. 381 (2002) 340.
[7] P.-H. Chavanis, B. Denet, M. Le Berre, Y. Pomeau, Supernova implosion-explosion in the light of catastrophe theory, in preparation.
[8] C. Josserand, Y. Pomeau, S. Rica, J. Low Temp. Phys. 145 (2006) 231.
[9] J. Sopik, C. Sire, P.-H. Chavanis, Phys. Rev. E 74 (2006) 011112.
[10] L.D. Landau, E.M. Lifshitz, Fluid Mechanics, Course of Theoretical Physics (Pergamon Press, Oxford, 1987).