Investigation to Reduce Flux Variance Effects Using Partial Coherent Beam in Free Space Optical Communication

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Abstract. This paper focus on the employment of partial coherent beam (PCB) in free space optical communication to reduce the flux variance effects in the system. The implementation of PCB in the free space optical system is significantly able to reduce the flux variance effect. The reduction of the flux variance effect by using PCB in free space optical communication by using Gamma-Gamma distribution is presented in this paper. The analysis result shows that the FSO system with PCB have an improved in performance compares with FSO system without PCB implemented on it as it capable to decrease the flux variance and scintillation index. This can help FSO system to encounter with severe turbulence effect for optimum operation.

1. Introduction
The FSO communication is strongly influenced by the atmospheric channel effect which can cause the beam signal fading and wander. The study in the spatially Partial Coherent Beam (PCB) has an attractive number of researchers due to the capability for improving the performance of laser communication systems was indicated recently in a number of publications [1]–[4]. In this paper, the theoretical model for PCB focus on the scintillation index calculation in turbulence. This is because the scintillation index is the most important statistic for practical application FSO systems.

In this research study, the Kolmogorov spectrum model will be used in the case of weak atmospheric turbulence. By using the theory of Andrews [5], [6] the weak scintillation index of a PCB can be used in all atmospheric conditions. Therefore, the scintillation index depends on the strength of the diffuser and the strength of the atmospheric turbulence.

2. Partially Coherent Beam
The figure 1 shows the diffuser model for PCB with propagation channel. For the transmitted beam wave of the diffuser is characterized by TEM00 Gaussian beam wave parameters.

\[
\Lambda_0 = \frac{2L}{kW_0^2} \quad (1)
\]

\[
\Theta_0 = 1 - \frac{L}{F_0} \quad (2)
\]
where \( \Lambda_0(L) \) represent the initial Fresnel ratio and the \( \Theta_0(L) \) represent the initial phase curvature. The \( k = \frac{2\pi}{\lambda} \) is the laser wave number, \( \lambda \) (in meters) is wavelength, \( L \) (in meters) is propagation distance to the collecting (Gaussian) lens, \( F_0 \) (in meters) is the phase front radius of curvature, and \( W_0 \) (in meters) is the laser exit aperture radius. From the Andrews and Phillips [5] the parameters \( \Theta_1 \) and \( \Lambda_1 \) for the beam incident on the collecting lens can be written as:

\[
\Lambda_1 = \frac{2L}{kW_1^2} = \frac{\Lambda_0}{\Theta_0^2 + \Lambda_0^2} \tag{3}
\]

\[
\Theta_1 = 1 + \frac{L}{F_1} = \frac{\Theta_0}{\Theta_0^2 + \Lambda_0^2} \tag{4}
\]

![Figure 1. PCB's model and its configuration.](image)

The Gaussian lens at the receiver is assumed has radius \( W_G \) and phase front radius of curvature \( F_G \). After propagating through the lens to the detector located in the image plane at distance \( L_f \) behind the lens, the beam has radius \( W_2 \) and radius of curvature \( F_2 \), which are characterized by beam parameters:

\[
\Lambda_2 = \frac{2L_f}{kW_2^2} = \frac{L}{L_f} \left[ \frac{\lambda_1 + \Omega_G}{\left( L/L_f - L/F_G + \Theta_1 \right)^2 + (\Lambda_1 + \Omega_G)^2} \right] \tag{5}
\]

\[
\Theta_2 = 1 + \frac{L_f}{F_2} = \frac{L}{L_f} \left[ \frac{L/L_f - L/F_G + \Theta_1}{\left( L/L_f - L/F_G + \Theta_1 \right)^2 + (\Lambda_1 + \Omega_G)^2} \right] \tag{6}
\]

where \( \Theta_1 = 1 - \Theta_1 \) and the nondimensional parameter \( \Omega_G \) is defined by

\[
\Omega_G = \frac{2L}{kW_G^2} \tag{7}
\]

The model the diffuser in front of the laser transmitter by a thin random phase screen diffuser can be characterized by a single-scale Gaussian spectrum model [7], [8]:

\[
\Phi_s(k) = \frac{\langle n_1^2 \rangle l_c^2}{8\pi^2} \exp \left( -\frac{1}{4} l_c^2 k^2 \right) \tag{8}
\]

where \( k \) is wave number, and \( l_c \) is directly related to the variance \( \sigma_g^2 \) to describe the partial coherence properties of the source [1]:

\[
l_c^2 = 2\sigma_g^2 \tag{9}
\]
The parameter $\langle n^2 \rangle$ is the fluctuation in the index of refraction induced by the diffuser and the nondimensional parameter analogous that relating the strength of diffuser is defined as:

$$ q_c = \frac{L}{k l_c^2} $$  \hspace{1cm} (10)

Therefore the ‘new’ Gaussian beam from the diffuser effect can be characterized by an effective set of beam that are $\Theta_e, \Lambda_e$ and $N_s$ represent the number of speckles cells.

$$ N_s = 1 + \frac{2 W_0^2}{l_c^2} = 1 + \frac{4 q_c}{\Lambda_0} $$  \hspace{1cm} (11)

$$ \Lambda_e = \frac{\Theta_0^2}{\Theta_0^2 + \Lambda_0^2} $$  \hspace{1cm} (12)

$$ \Theta_e = \frac{\Theta_0}{\Theta_0^2 + \Lambda_0^2} $$  \hspace{1cm} (13)

For the effective spot beam $W_\zeta(L)$ for PCB is described by [9] to consider the global coherent parameter $\zeta$. It can be written as

$$ W_\zeta(L) = w_o \left( \Theta_0^2 + \zeta \left( \frac{2L}{k w_o^2} \right)^2 \right)^{1/2} $$  \hspace{1cm} (14)

$$ \zeta = \zeta_s + \frac{2 w_0^2}{\rho_o^2} $$  \hspace{1cm} (15)

where the $\zeta_s = 1 + \frac{w_0^2}{\sigma_{fl}^2}$ represent the source of the coherence parameter of the laser beam emitted by the transmitter, $\rho_0 = \left[0.545C_n^2 L^2 k^2 L\right]^{-3/5}$ is the coherence length through turbulence atmosphere and $\sigma_{fl}^2$ is the variance of the Gaussian describing the ensemble average of the random phases which relate to diffuser strength. If the $\zeta_s = 1$, the beam is categorized as a perfectly coherent beam and the beam is categorized as a partially coherent beam if the $\zeta_s > 1$ [10]. If without diffuser effect the beam spot radius at the receiver can be expressed as [11].

$$ W(L) = W_o (\Theta_o + \Lambda_o)^{1/2} $$  \hspace{1cm} (16)

where the $\Theta_o$ and $\Lambda_o$ are the initial phase curvature and initial Fresnel ratio respectively as shows in Equation (1) and Equation (2).

3. Flux Variance

The reduction of the scintillation index due to PCB alone would not be sufficient for the high-quality data transfer calculated in terms of the bit error rate (BER) in communication systems. However, the combination of partial coherence with large enough collecting lens (aperture averaging effect) can provide the required BER level. The flux variance of the intensity fluctuations at the detector plane that calculated for the collecting lens with normalized radius $\Omega_G$ is valid for all atmospheric conditions as given in [6]:

$$ \sigma_{flux}^2 (L + L_f, \Omega_G) = \exp \left[ \sigma_{flux}^2 (L + L_f, \Omega_G) + \sigma_{fl}^2 (L + L_f, \Omega_o) \right] - 1 $$  \hspace{1cm} (17)

where $\sigma_{fl}^2$ is the flux variance associated with large-scale fluctuations [12].
The effect of aperture averaging can be described as when the receiving aperture is larger than a spatial scale size that produces the intensity fluctuations, the receiver will average the fluctuations over the aperture and the scintillation will be less compared to scintillation measured with a point receiver [13], [14]. It has been shown that aperture averaging causes a shift of the relative spatial frequency content of the intensity spectrum toward lower frequencies, since the fastest fluctuations caused by small-scale sizes average out [15]. Hence, the scintillation measured by a receiving aperture is produced by scale sizes larger than the aperture. The most significant advantage of increasing the aperture size is in collecting more light. However, as the aperture size exceeds multiple times the coherence length of incident light on it, the scintillation index gets reduced, this phenomenon is known as aperture averaging [16], [17].

Under strong turbulence, there is two-scale behaviour in the intensity flux variance obtained by a finite size receiving aperture. A sharp decrease in scintillation is observed as the aperture sizes increase up to the coherence scale, $\rho_0$, after which there is a levelling effect followed by a second decrease in the intensity flux variance when the aperture exceeds the scattering disk $L/K_0$ [13], [15]. The levelling effect arises since medium-scale sizes are inefficient in producing scintillation in strong turbulence. Hence, if a receiving aperture is larger than $\rho_0$ in strong turbulence, the small-scale scintillation is mostly averaged out. This would affect the PDF of the intensity fluctuations since existing models developed for point receivers are obtained by modulating the small and large scale distributions. Therefore in this paper we use the Gamma-Gamma distribution and it is suitable for all fluctuation regime [18].

The log intensity due to large scale eddies is given as:

$$
\sigma_{lnx}^2(L + L_f, \Omega_c) = 0.49\sigma_1^2 \left(\frac{\Omega_c - \Lambda_e}{\Omega_c + \Lambda_e}\right)^2 \times R \left[\frac{\eta_x}{1 + 0.4\eta_x(1 + \Theta_e)/(\Lambda_e + \Omega_c)}\right]^{7/6}
$$

(18)

where, $R = 1/3 - (1/2)(1 - \Theta_e)(1/5)(1 - \Theta_e)^2$

The quantity $\eta_x$ in Equation (18) is the normalized large-scale cutoff frequency determined by the asymptotic behaviour of $\sigma_{lnx}^2$ in weak turbulence and saturation regime [12], [19]:

$$
\eta_x = \frac{R^{-6/7} \left(\sigma_B/\sigma_1\right)^{12/7}}{1 - 0.56\sigma_B^{12/5}}
$$

(19)

Meanwhile, the log intensity due to small scale eddies is given as:

$$
\sigma_{lny}^2(L + L_f, \Omega_c) = \frac{1.27\sigma_1^2 \eta_y^{-5/6}}{1 + 0.4\eta_y/(\Lambda_1 + \Omega_c)}
$$

(20)

where the corresponding cutoff frequency is

$$
\eta_y = 3(\sigma_1/\sigma_B)^{12/5} \left(1 + 0.69\sigma_B^{12/5}\right)
$$

(21)

### 4. Results and Discussion

In terms of the effect of diffuser strength relating aperture averaging effect, the figure 2 shows the effect normalize collecting lens, $\Omega_c$ versus the several partially coherent beams with $q_e = 0.1$, 1, and 10, and coherent beam for the weak turbulent condition. The propagation distance $L = 1000 \text{m}$, wavelength, $\lambda$
As we can see, the flux variance value is low when under this region turbulent with the highest flux is at 0.136 for coherent beam. Meanwhile the lowest flux is at 0.0054 for $q_c = 10$. This indicates the flux variance reduce when the $q_c$ increase which relating to the strength of the diffuser.

![Graph](image)

Figure 2. Comparison of flux variance $\sigma_{\text{flux}}^2 (L + L_f, \Omega_G)$ versus the collecting lens radius, $\Omega_G$ between weak and strong turbulence condition for partially coherent beam (diffuser effect).

Similar curves are generated in weak turbulence regime and strong turbulence regime in figure 3 and figure 4, respectively. In strong turbulence regime, the effect of the diffuser is declining as shown in figure 3. Consequently the flux variance value is high and can lead to increase of signal to noise ratio (SNR). To be compared with figure 4, the highest value of flux is at 0.09 for coherent beam and the lowest flux is produced for $q_c = 10$ with value 0.052. This indicates that the reduction of flux is reduced even the increasing of $\Omega_G$. The flux variance effect decrease with the increase of the collecting aperture size.
Figure 3. Effect of normalize collecting lens, $\Omega_G$ vs. flux variance for $q_c = 0.1, 1, 10$ and coherent beam in strong turbulence regime.

Figure 4. Effect of normalizing collecting lens, $\Omega_G$ vs. flux variance for $q_c = 0.1, 1, 10$, and coherent beam in weak turbulence regime.

The figure 5 shows the comparison of flux variance versus the various of collecting lens radius, $W_G$ between weak and strong turbulent condition. The propagation distance $L = 2000\text{m}$ and for weak and strong turbulent strength $C_n^2 = 10^{-15}\text{m}^{-2/3}$ and $C_n^2 = 10^{-13}\text{m}^{-2/3}$ respectively which leading to Rytov variance $\sigma_I^2 = 0.07$ and $\sigma_I^2 = 7.09$. The significant reduction of the flux variance occurs in weak turbulent the flux varies from 0 to 0.05 but for the strong turbulence the flux value varies from 0 to 0.4. This illustrated that the effect of flux variance is greater when the strength of turbulence increase and the reduction of flux become less. From the graph also shows that the effect of flux variance can be
reduced by increase the collecting lens radius. As the collecting lens radius increase the value of flux variance will decrease.

![Figure 5. Comparison scintillation index due to strong turbulence between with and without diffuser](image)

5. Conclusion

The effect of atmospheric turbulence from flux variance can be reduce by using partial coherent beam. Without combining it with diffuser, it can be seen at distance of $L = 2000m$ that the scintillation index is at high value. However, by using the diffuser, the scintillation index can be reduced and the reduction of scintillation index where estimated approximately 19 percent.

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