A NEW LORENTZ-VIOLATING MODEL OF NEUTRINO OSCILLATIONS

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A new model for neutrino oscillations is introduced, in which mass-like behavior is seen at high energies, but various behavior can be predicted at low energies. The model employs no neutrino masses, but instead relies on the Lorentz-violating parameters \(a\) and \(c\). Oscillations into sterile neutrinos and into antineutrinos are also considered.

1. Introduction

Neutrino oscillations have been experimentally observed in a variety of situations, and are among the first evidence of physics beyond the Standard Model. Typically, these oscillations are explained by attributing mass to neutrinos; however, not all experiments can be explained using the same masses - notably, LSND\(^1\) and MiniBooNE\(^2\) require a larger mass-squared difference than the other experiments, and cannot be explained using a three-flavor theory of mass. Furthermore, recent results at MINOS and MiniBooNE have hinted at an asymmetry between neutrinos and antineutrinos,\(^2,3\) which would be evidence for Lorentz violation.

It has already been shown that models incorporating Lorentz violations can reproduce many of the results of the mass model.\(^4\) Examples include the Bicycle Model\(^5\) and the Tandem Model.\(^6\) Here, a new model is introduced to attempt to explain these experiments.

2. Theory

We consider three generations of light, left-handed neutrinos, and three generations of light, sterile, right-handed neutrinos, and their antiparticles. We allow for small mass and small general Lorentz violations. To first order,
the general Hamiltonian is a $12 \times 12$ matrix, given in block form by

$$H = \frac{1}{E} \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$

(1)

where

$$H_{11} = \begin{bmatrix} -c^{\mu \nu} p_\mu p_\nu + a_1^{\mu \nu} p_\mu p_\nu + a_2^{\mu \nu} p_\mu \\ -c_3^{\mu \nu} p_\mu p_\nu + a_4^{\mu \nu} p_\mu \end{bmatrix}$$

(2)

$$H_{12} = \begin{bmatrix} -ig^{\mu \nu} \rho_\mu \rho_\nu + i H_1^{\mu \nu} \rho_\mu \rho_\nu + i H_2^{\mu \nu} \rho_\mu \rho_\nu \\ -ig_1^{\mu \nu} \rho_\mu \rho_\nu + i H_1^{\mu \nu} \rho_\mu \rho_\nu + i H_2^{\mu \nu} \rho_\mu \rho_\nu \\ -ig_2^{\mu \nu} \rho_\mu \rho_\nu + i H_1^{\mu \nu} \rho_\mu \rho_\nu + i H_2^{\mu \nu} \rho_\mu \rho_\nu \end{bmatrix}$$

(3)

$$H_{21} = \begin{bmatrix} -c_1^{\mu \nu} p_\mu p_\nu + a_1^{\mu \nu} p_\mu p_\nu + a_2^{\mu \nu} p_\mu \\ -c_3^{\mu \nu} p_\mu p_\nu + a_4^{\mu \nu} p_\mu \end{bmatrix}$$

(4)

$$H_{22} = \begin{bmatrix} -c^{\mu \nu} p_\mu p_\nu + a_1^{\mu \nu} p_\mu p_\nu + a_2^{\mu \nu} p_\mu \\ -c_3^{\mu \nu} p_\mu p_\nu + a_4^{\mu \nu} p_\mu \end{bmatrix}$$

(5)

in the basis \{\nu_L, \bar{\nu}_R, \nu_R, \bar{\nu}_L\}. Note that mass does not appear because it enters only at second order. Such a Hamiltonian allows many unusual features, including neutrino-antineutrino mixing, neutrino-sterile neutrino mixing, strange energy dependence and direction dependence.

3. Model

We propose a Tricycle Model, in which we assume that $g = H = 0$ so that the off-diagonal terms can be ignored. This allows us to restrict our attention to one quadrant of the matrix, the \{\nu_L, \bar{\nu}_R\} sector \((H_{11})\). Note that we have only considered the isotropic part of each of the Lorentz-violating coefficients.

In particular, the model to be investigated has the form

$$H_T = \begin{bmatrix} 0 & A \\ A^T & CE \end{bmatrix}$$

(6)

where $A$ is taken to be Hermitian and $A$ and $C$ commute, so that they can be simultaneously diagonalized. We assume that the diagonalizing matrix has the conventional form

$$U = \begin{bmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} \cos \theta_{23} & \cos \theta_{12} \cos \theta_{23} & \sin \theta_{23} \\ \sin \theta_{12} \sin \theta_{23} & -\cos \theta_{12} \sin \theta_{23} & \cos \theta_{23} \end{bmatrix}$$

(7)

(we are assuming that $\theta_{13} = 0$). The model is then fixed by 8 parameters: the two mixing angles and the three eigenvalues of each block, which we
call \( \{a_i\} \) and \( \{c_i\} \) respectively. The eigenvalues of the Hamiltonian are

\[
\lambda_{i \pm} = c_i E \pm \sqrt{(c_i E)^2 + 4a_i^2}/2
\]

(8)

The model employs a seesaw mechanism to produce very different behavior at high and low energies. At high energy the \( CE \) matrix dominates, which cuts off left-right oscillations and allows the left-handed neutrinos to oscillate among themselves as normal. At low energies, however, the \( A \) terms dominate, and oscillations into sterile neutrinos are predicted. Observe that

\[
\lim_{c_i E \to \infty} \lambda_{i -} = -\frac{a_i^2}{c_i E}
\]

(9)

so that three of the eigenvalues have the expected \( E^{-1} \) energy dependence at high energies.

Transition probabilities can be calculated exactly. For example,

\[
P_{e\mu} = 4 \sin^2 \theta_{12} \cos^2 \theta_{12} \cos^2 \theta_{23} \times \]

\[
\left[ -\sin^2 \alpha \cos^2 \beta \sin^2 \left( \frac{\Delta \lambda_{41} L}{2} \right) - \sin^2 \beta \cos^2 \beta \sin^2 \left( \frac{\Delta \lambda_{52} L}{2} \right) \right. \]
\[
+ \sin^2 \alpha \sin^2 \beta \sin^2 \left( \frac{\Delta \lambda_{21} L}{2} \right) + \sin^2 \alpha \cos^2 \beta \sin^2 \left( \frac{\Delta \lambda_{53} L}{2} \right) \]
\[
+ \cos^2 \alpha \sin^2 \beta \sin^2 \left( \frac{\Delta \lambda_{24} L}{2} \right) + \cos^2 \alpha \cos^2 \beta \sin^2 \left( \frac{\Delta \lambda_{54} L}{2} \right) \right] \]

(10)

where

\[
\sin^2 \alpha = \frac{1}{2} \left[ 1 - \frac{1}{\sqrt{1 + \left( \frac{2a_1}{c_1 E} \right)^2}} \right]
\]

(11)

and

\[
\sin^2 \beta = \frac{1}{2} \left[ 1 - \frac{1}{\sqrt{1 + \left( \frac{2a_2}{c_2 E} \right)^2}} \right]
\]

(12)

The mixing angles and two of the eigenvalues are determined by the high-energy, mass-like behavior widely detected. There remain four independent parameters in the model, which can be adjusted to control the low energy behavior without disrupting the high-energy limit. As energy decreases, the probabilities diverge smoothly from the standard mass predictions.
4. CPT violation

The model introduced above (6) will never produce observable CPT violations. This is because $A$ is a CPT-odd variable, but $C$ is CPT-even, so that under CPT transformations, $H$ goes to

$$H' = \begin{bmatrix} 0 & -A \\ -A^\dagger & CE \end{bmatrix}$$

However, the eigenvalues and mixing angles on which the probability depends do not observe the sign of $A$, as can be seen from their definitions (8, 11, 12). This causes the probabilities to be the same whether $A$ or $-A$ is used. In fact, even if $A$ does not commute with $C$, CPT symmetry will still be preserved; to introduce CPT violation, $A$ and $C$ terms must be mixed (for example, an ordinary rotation of (6) will introduce CPT violations).

5. Conclusion

This model is intended to show that behavior typical of mass models can be reproduced by a Lorentz-violating model without mass. A variety of low-energy behavior is consistent with the same behavior at high energy. However, it remains difficult to explain all experiments, even with four free parameters.

Acknowledgments

We would like to thank Swarthmore College for funding the research reported here. Thanks also to Matt Mewes for his advice and assistance in this project.

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