2D full-wave simulations of different scenarios of ECR plasma heating at the L-2M stellarator

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Abstract. Different scenarios of electron cyclotron resonance (ECR) plasma heating under the conditions typical of L-2M experiments ($n_e(0) = 1.75 \times 10^{13} \text{ cm}^{-3}$, $T_e(0) = 1 \text{ keV}$, $\lambda_0 = 0.4 \text{ cm}$) are simulated using a 2D full-wave model with allowance for the nonlocal (differential) thermal correction to the plasma dielectric tensor. It is shown that, under central ECR heating, due to the specific shape of the resonance surface $\omega_0 = 2\omega_{ce}$, a significant fraction of the input microwave power is deflected downward and escapes onto the chamber wall. During off-axis heating at the midradius of the plasma column, about one-half of the microwave power is reflected upward from the resonance surface. Optimal conditions for the deposition of the microwave power in plasma are achieved under ECR heating at the vacuum magnetic axis, when the microwave beam is incident normally onto the resonance surface. In this case, the microwave power is almost completely ($\approx 99.5\%$) absorbed by the plasma, while the coefficient of microwave reflection into the aperture of the incident microwave beam amounts to $\sim 0.1\%$, which agrees with results of 1D full-wave simulations.

1. Introduction

Electron cyclotron resonance (ECR) heating at the first ($\omega_0 = \omega_{ce}$) or second ($\omega_0 = 2\omega_{ce}$) harmonic of the electron gyrofrequency [1–5] is widely used for plasma heating in toroidal magnetic confinement systems. In particular, an electron temperature of up to $\sim 1 \text{ keV}$ at plasma densities of $(1–2) \times 10^{13} \text{ cm}^{-3}$ was achieved at the L-2M stellarator under second-harmonic ECR heating by an extraordinarily (X) polarized microwave beam ($\lambda_0 = 0.4 \text{ cm}$, $P \leq 1 \text{ MW}$) launched from the outer side of the torus [6]. As a rule, various versions of the ray tracing method are used to calculate the propagation and absorption of the heating microwave beam [2, 7–9]. Since the microwave power is absorbed in a relatively narrow region $(\Delta x \sim L_\theta T_e / m_e c^2 << a)$, where $L_\theta = B_\theta / |\nabla B_\theta|$ is the characteristic inhomogeneity length of the magnetic field) near the resonance surface, whereas thermal corrections to the plasma dielectric tensor in the rest of the plasma column are small, a simplified version of the ray tracing method is often used in which thermal corrections are taken into account only through the absorption coefficient, while the ray trajectories are calculated in the cold plasma approximation [2, 6]. However, theoretical analysis and computer simulations [10–12] show that such a simplified model can fail under conditions typical of L-2M experiments.

The L-2M device is a classical $l = 2$ stellarator with a major radius of $R = 1 \text{ m}$ and an average plasma column radius of $a = 11.5 \text{ cm}$ [6, 13]. Figure 1a shows the L-2M vacuum magnetic configuration in the standard poloidal cross section, in which ECR heating is performed, for the case of central ECR heating. Here, the vacuum magnetic axis is shifted by $2.7 \text{ cm}$ to the left (toward the major axis) from the coordinate origin ($x = y = 0$), which coincides with the minor axis of the vacuum chamber. Due to the specific shape of the resonance surface, the microwave beam launched from the right along the $x$ axis propagates nearly parallel to the resonance surface before it reaches the
central region of the plasma column. As was shown in [11], in this case, thermal corrections to the plasma dielectric tensor can cause substantial refraction of microwave radiation, due to which an appreciable fraction of the microwave power is deflected downward, not reaching the absorption region.

Figure 1. Magnetic surfaces and contour lines of $B_0(x, y)$ ($\Delta B_0 = 0.04 B_{\text{res}}$) in the standard cross section of L2-M for the case of central ECR heating ($B_0(0, 0) = B_{\text{res}} = 1.34$ T): (a) vacuum magnetic configuration; (b) configuration with shifted magnetic surfaces in the presence of plasma with the density and temperature profiles shown in figure 2. Here and in the subsequent figures, the red and blue heavy lines show the positions of the plasma boundary and resonance surface $\omega = 2\omega_{ce}(x, y)$, respectively. The major axis of the torus is on the left. The magnetic field on the axis of the vacuum chamber $(x = y = 0)$ increases to the left upward.

In this study, we consider some other possible scenarios of ECR plasma heating at the L-2M stellarator, differing in the position of the resonance region (the value of the toroidal magnetic field $B_0$): heating on the vacuum magnetic axis and off-axis heating at the midradius of the plasma column. It is shown by means of 2D full-wave numerical simulations that optimal conditions for microwave power deposition in plasma are achieved under ECR heating on the vacuum magnetic axis, when the microwave beam is incident normally onto the resonance surface.

Figure 2. Radial profiles (in flux variables) of the electron density and temperature used in numerical simulations ($n_e(0) = 1.75 \times 10^{13}$ cm$^{-3}$, $T_e(0) = 1$ keV).
2. Formulation of the problem

The 2D problem is formulated in the same way as in [11, 12]. The radial profiles of the plasma density and electron temperature in terms of the magnetic flux coordinate \( \rho = (\Psi/\Psi_{\text{max}})^{1/2} \) (figure 2) are taken close to those in standard L-2M experiments [6].

It is assumed that the stellarator magnetic field \( \mathbf{B}_0 \) is directed along the \( z \) axis (the radial and poloidal components of the magnetic field are neglected). The structure of shifted magnetic surfaces [14] in the L-2M standard cross section for plasma with such density and temperature profiles is shown in figure 1b. The 2D X-polarized microwave beam (\( \lambda_0 = 0.4 \) cm) with a width of 2 cm at a level of \( e^{-1} \) in amplitude is launched from the right (from the outer side of the torus) along the \( x \) axis. The complex amplitudes of the wave fields \( E_x \) and \( E_y \) are calculated by solving the 2D full-wave equation

\[
\nabla \times (\nabla \times \mathbf{E}) - k_0^2 (\hat{\epsilon}_0 + \delta\hat{\epsilon}_\perp) \mathbf{E} = 0, \tag{1}
\]

where \( k_0 = \omega_0/c \), \( \hat{\epsilon}_0 \) is the dielectric tensor of cold magnetized plasma [15] and \( \delta\hat{\epsilon}_\perp \) is the nonlocal (differential) thermal correction to the dielectric tensor near the resonance \( \omega_0 = 2\omega_{ce} \) [10]. The problem is solved in a \( 24.96 \times 24 \) cm simulation box divided into 616 \times 600 cells with dimensions \( \Delta x = \Delta y = 0.04 \) cm. Outgoing conditions are imposed on the transmitted and reflected waves at the left and right boundaries, respectively, and smooth absorbing layers are introduced near the lower and upper boundaries to suppress reflection from them. Equation (1) with these boundary conditions was solved by the matrix sweep method [16]. The output parameters are the 2D distributions of the wave fields \( E_x \), \( E_y \), and \( B_z \) and \( Q \) profiles of the transmitted and reflected microwave powers; and the distribution of the absorbed microwave power \( Q(x, y) \).

3. Simulation results

3.1. Central heating

Let us remember the results obtained in [11] for the case of central heating \((B_0(0, 0) = 1.34 \text{ T})\), when the resonance point \( \omega_0 = 2\omega_{ce} \) on the \( x \) axis coincides with the center of the vacuum chamber. Figures 3a and 3b show the distributions of the wave electric field squared, \(|\mathbf{E}|^2 = |E_x|^2 + |E_y|^2\), and the wave magnetic field \( B_z \), respectively, in the \((x, y)\) plane for the density and temperature profiles shown in figure 2. The profile of the incident microwave power is shown on the right of each panel.

It is seen that, in this case, a fraction of the incident microwave power (about 13%) is deflected downward and escapes onto the wall in the form of a narrow beam. The rest power (about 87%) is almost completely absorbed just behind the resonance surface (the plasma region absorbing 75% of the total absorbed microwave power is marked with purple color). The reason for the downward deflection of microwave radiation is refraction caused by the thermal correction to the plasma dielectric tensor. Analysis of the effective refractive index of hot plasma [11, 12] shows that, just below the resonance surface, where \( \omega_0 > 2\omega_{ce} \), the refractive index has a negative vertical gradient, due to which the upper part of the microwave beam, which propagates nearly along the resonance surface, is deflected downward and begins to interfere with its lower part. Since the resonance surface gradually turns down, the deflected radiation is “guided” along this surface and finally escapes downward. The guiding of microwave radiation along the resonance surface is clearly seen in figure 3b, which illustrates the wave structure of the beam. Simulations show that the fraction of the deflected microwave power grows rapidly with increasing plasma density, reaching more than 50% at \( n_e(0) = 3 \times 10^{13} \text{ cm}^{-3} \) [11, 12].
Figure 3. Distributions of the (a) wave electric field squared $|E|^2$ and (b) wave magnetic field $B_z$ in the L-2M standard cross section for the density and temperature profiles shown in figure 2 for the case of central ECR heating ($B_0(0, 0) = 1.34$ T). Red (blue) regions in panel (b) correspond to positive (negative) values of $B_z$ at a certain moment of time. The profile of the incident microwave power is shown on the right of each panel. Here and in the subsequent figures, the plasma region absorbing 75% of the total absorbed microwave power is marked with purple color.

3.2. Heating on the vacuum magnetic axis
A decrease in the toroidal magnetic field $B_0$ results in the leftward shift of the resonance point along the $x$ axis. At $B_0(0, 0) = 1.305$ T, the point where the resonance surface intersects the $x$ axis ($x = -2.7$ cm, $\rho = 0.22$), coincides with the vacuum magnetic axis. Figure 4 shows the calculated distributions of $|E|^2$ and $B_z$ for this case.

Figure 4. Distributions of (a) $|E|^2$ and (b) $B_z$ in the ($x$, $y$) plane for the case of ECR heating on the vacuum magnetic axis ($B_0(0, 0) = 1.305$ T). The thin red line on the right of panel (a) shows the intensity profile of reflected radiation on the 1000 : 1 scale relative to the incident power.
It is seen that, on the right of the absorption region, the resonance surface is shifted upward compared to the case of central heating, due to which the influence of refraction caused by the thermal correction to the plasma dielectric tensor is substantially reduced and the microwave beam propagates nearly as in cold plasma. Moreover, since the beam is incident on the resonance surface almost normally, no downward deflection occurs. More than 99% of the incident microwave power is absorbed just behind the resonance surface. A small fraction (~0.1%) of the microwave power is reflected back into the aperture of the incident beam, which agrees with results of 1D numerical simulations for similar plasma parameters [17, 18].

3.3. Off-axis heating at the midradius of the plasma column

At $B_0(0, 0) = 1.253$ T, the resonance on the $x$ axis shifts into the point $x = -6.4$ cm ($\rho = 0.5$). The corresponding distributions of $|E|^2$ and $B_z$ are shown in figure 5. In this case, in contrast to the case of central heating, the beam is incident obliquely on the convex (rather than on the concave) resonance surface. As a result, only about one-half of the input microwave power passes through the resonance surface and is then absorbed, while the rest power is reflected and escapes onto the wall. The incident and reflected waves produce a clearly pronounced interference pattern in the $(x, y)$ plane.

![Figure 5](image_url)

**Figure 5.** Distributions of (a) $|E|^2$ and (b) $B_z$ in the $(x, y)$ plane for the case of ECR heating at the midradius of the plasma column ($B_0(0, 0) = 1.253$ T).

3.4. Hypothetical (yet unfeasible) heating scenarios

In all the above scenarios, the microwave beam was launched from the outer (low-field) side of the torus. Let us now consider some hypothetical heating scenarios in which the microwave beam is launched from the side of the higher magnetic field. There are, in principle, two ways to introduce the microwave beam in the camber from the high-field side between the L-2M stellarator windings: (i) in the equatorial plane from the inner side of the torus and (ii) vertically from the top. Although both these scenarios are yet unfeasible for technical reasons (there is no enough room on the inner side and there are no suitable ports on the top of the L-2M vacuum chamber), it is of interest to compare them with the above results.

Figures 6 and 7 show the distributions of $|E|^2$ and $B_z$ for the cases where the beam is launched from the inner side of the torus and from the top, respectively, for $B_0(0, 0) = 1.34$ T (central heating) and for the same density and temperature profiles as in the previous scenarios (see figure 2). In both cases, the beam before absorption propagates nearly as in cold plasma and is almost completely (>99%) absorbed near the resonance surface. No microwave power is reflected.
Figure 6. Distributions of (a) $|E|^2$ and (b) $B_z$ in the $(x, y)$ plane for the case of microwave beam launching from the inner side of the torus ($B_0(0, 0) = 1.34$ T).

Figure 7. Distributions of (a) $|E|^2$ and (b) $B_z$ in the $(x, y)$ plane for the case of microwave beam launching from the top of the vacuum chamber ($B_0(0, 0) = 1.34$ T).

4. Discussion and conclusions

Thus, 2D full-wave simulations of the propagation and absorption of the heating microwave beam in the L-2M stellarator plasma under the conditions typical of experiments on ECR plasma heating at the second harmonic of the electron gyrofrequency show that optimal conditions for the deposition of the microwave power in plasma are achieved under ECR heating at the vacuum magnetic axis. In this case, the microwave power is almost completely ($\approx$99.5%) absorbed in the region located not far from the center of the plasma column ($\rho \approx 0.2$).

In the cases of central heating and off-axis heating, an appreciable fraction of the introduced microwave power is deflected from the resonance surface and escapes onto the wall. Of course, after multiple reflections from the chamber wall, this power will eventually be absorbed by the plasma.
However, in this case, the absorbed power will be smeared out over the resonance surface, rather than deposited in the prescribed plasma region.

Of certain interest are yet unfeasible scenarios in which the microwave beam is launched from the side of the higher magnetic field (from the inner side of the torus or vertically from the top). In this case, the beam propagates regularly (with no self-interference or reflections) as in cold plasma and is almost completely absorbed near the resonance surface.

To conclude, let us consider the applicability of different versions of the ray tracing method to calculate the propagation and absorption of the heating microwave beam under the conditions of L-2M experiments. Figures 8a and 8b show ray trajectories calculated without and with allowance for the thermal correction to the real part of the refractive index, respectively, for the case of central heating \(B_0(0, 0) = 1.34 \, \text{T}\) for the density and temperature profiles shown in figure 2. The rays are cut off when 50% of the ray power is absorbed. It is seen from figure 8a that, in the simplified version of the ray tracing method [2, 6], in which thermal corrections to the real part of the refractive index are neglected (i.e., the rays propagate as in cold plasma), all rays starting below the resonance surface cross this surface and are then absorbed. In contrast to 2D simulations with allowance for thermal corrections to the dielectric tensor (see section 3.1), no reflection occurs.

When thermal corrections to the real part of the refractive index are taken into account (figure 8b), the pattern of ray trajectories resembles the distribution of the wave field intensity calculated in the 2D full-wave model (see figure 3). In this case, the ray tracing method not only adequately describes the formation of the downward-deflected microwave beam, but the power fraction of the deflected beam (~10%) turns out to be close to that in the 2D full-wave model. Thus, although this version of the ray tracing method fails to describe details of the wave structure of the heating microwave beam, it can well be used to calculate the propagation and absorption of microwave radiation under the conditions typical of L-2M experiments.

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