How to Decipher the Seegers-Tisserand-Gerber-Einstein Formula and the Soldner-Einstein Formula?

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Abstract — The famous Seegers-Tisserand-Gerber-Einstein Formula describing correctly the Mercury perihelion advance passed through hands of many scholars who tried to decipher the physical meaning of the perturbation factor $\Omega$ introduced by Carl Seegers in 1864. Based on the Gauss’s law for gravity we have newly interpreted this perturbation factor $\Omega$ as the active solid angle of the Solar gravitational field $\Omega = 3$ steradians. We have inserted this model of the active solid angle of the Solar gravitational field the famous Soldner-Einstein Formula describing the light deflection in the vicinity of the Sun with $1 \leq \Omega \leq 8$. The enormous scatter of experimental data of the light deflections measured during the Solar eclipses was interpreted as the quantum jumps of the deflection angle with the quantum jump $0.44^{\circ}$. All known existing data on the light deflection was interpreted as the quantum jumps of the deflection angle during the individual runs of the Solar eclipse experiment. We propose to reanalyze all historical data taken for individual stars and to search for a hidden structure in these data. Moreover, we want to initiate new experimental activities for the coming Solar eclipses in order to collect more precise data that might guide us towards the model of quantum gravity.

Keywords — Active solid angle, light deflection, Mercury perihelion advance, quantum gravity, quantum jumps.

I. INTRODUCTION

The solution of difficult physical problems develops in three stages: Stage I: Both the mathematical formula and the physical model are wrong. Stage II: The mathematical formula is correct but the physical interpretation is wrong. Stage III: Both the mathematical formula and the physical model are correct.

The light deflection in the vicinity of the Sun (e.g., [1]-[4]) and the Mercury perihelion advance (e.g., [5]-[6]) played a very important role in the development of physics. Are we now with these topics in the Stage II or the Stage III? Can we add anything reasonable to these deeply studied Formulae?

II. EVOLUTION OF THE SEE ERS-TISSERAND-GERBER-EINSTEIN FORMULA (STGE FORMULA)

The first important step in the evolution of the STGE Formula made Carl Seegers in his PhD Thesis under the guidance of Wilhelm Weber in 1864 [7]. Carl Seegers stated: “Using Weber’s law, the orbit of the body deviates from the elliptical one and the perihelion of the attracted body moves in a circle around the Sun. After one orbit the perihelion shifts by this value” (written in the modern notation):

$$\omega = 2\pi \frac{GM_☉}{c^2 a (1 - e^2)}$$

where $\omega$ is the perihelion advance after one orbit, $\Omega$ is an un-known perturbation factor, $G$ is the Newtonian gravitational constant, $M_☉$ is the Solar mass, $c$ is the speed of gravity (Weber used the value $\sqrt{2} c$ in his model), $a$ is the semi-major axis, and $e$ is the eccentricity of that ellipse.

This was the starting formula for the interpretation of the famous experimental value $43^{\circ}$ found by Le Verrier. The main problem was to interpret that un-known perturbation factor $\Omega$, and many solutions were proposed [5]. There were developed several concepts how to find a physical interpretation for $\Omega$ — Table I.
TABLE I: THE EVOLUTION OF THE SEEIGERS-TISSERAND-GERBER-EINSTEIN FORMULA

| Year  | Scholar     | Perturbation coefficient Ω | Physical interpretation                                      | Predicted advance | Reference |
|-------|-------------|-----------------------------|-------------------------------------------------------------|-------------------|-----------|
| 1864  | Seegers     | 1                           | Weber’s law with $\sqrt{2}$ c                               | 7°.17             | [7]       |
| 1872  | Tisserand   | 1                           | Weber’s law with c                                          | 14°.3             | [8]       |
| 1890  | Tisserand   | 1                           | Gauss’s law with c                                          | 14°.3             | [9]       |
| 1898  | Gerber      | 3                           | Weber’s law with retarded potentials and with c             | 43"               | [10]      |
| 1915  | Einstein    | 3                           | Gauss’s law for gravity with the active solid angle $\Omega = 3$ sr | 43"               | [11]      |
| 2021  | This paper  | 3                           |                                                             |                   |           |

In our model the integral form of Gauss’s law for gravity states:

$$\frac{\Omega}{4\pi} \int \int \mathbf{g} \cdot d\mathbf{A} = -\Omega GM$$

(2)

where $\Omega$ in steradians represents the active solid angle of the gravitational source for that gravitational situation. Other symbols are taken from the well known Gauss’s law for gravity: a surface integral over a closed surface, $\mathbf{g}$ is the gravitational field, $d\mathbf{A}$ is a vector, whose magnitude is the area of an infinitesimal piece of surface, $G$ is the Newtonian gravitational constant, $M$ is the total mass enclosed within the surface.

Fig. 1 shows the gravitational situation for the $\Omega = 1$ steradian.

Fig. 1. The gravitational field with the active solid angle $\Omega = 1$ steradian (from Wikimedia with the permission licence: https://upload.wikimedia.org/wikipedia/commons/9/9b/Steradian_V2.svg).

There are different gravitational situations in which the source of the gravitational field acts on the attracted body. For the case of the Mercury perihelion advance we propose to insert into the Seeigers-Tisserand-Gerber-Einstein Formula the active solid angle $\Omega = 3$ sr. This situation is depicted in Fig. 2.

Fig. 2. The Solar gravitational field acting via $\Omega = 3$ steradians on the Mercury perihelion advance. L is the semi-latus rectum of that ellipse, $a$ is the semi-major axis, $e$ is the eccentricity.
III. LIGHT DEFLECTION IN THE VICINITY OF THE SUN

There are three different gravitational experiments for collecting experimental data of the photon deflection under the influence of the Solar gravitational field. We can sort them in the dependence of the intensity of the Solar gravitational field and the mass of deflected photons. Table II summarizes these three gravitational observations.

| TABLE II: THREE GRAVITATIONAL EXPERIMENTS IN THE SOLAR SYSTEM ON THE LIGHT BENDING |
|----------------------------------|-------------------|-----------------|-----------------|-----------------|
| Intensity of the Solar gravitational field | Photon mass | Method | Active solid angle Ω [sr] | Reference |
| HIGH (λ ≈ 5*10⁻² m) | LOW | VLBI | 4 | [12] |
| HIGH (λ ≈ 5*10⁻⁷ m) | HIGH | HIPPARCOS | 4 | [12] |
| HIGH (λ ≈ 5*10⁻⁷ m) | HIGH | Solar eclipse | 1 ≤ Ω ≤ 8 |

A deflection of light rays must take place if the gravitation influences the speed of light. This model was introduced by Albert Einstein in his Prague paper in 1911 [13] and later in the Schwarzschild metrics as:

\[
c_{R} = c_0 \left(1 - \frac{2GM}{c^2 R}\right)\]

where the light speed \(c_R\) depends on the distance \(R\) from the centre of the gravitational source and \(c_0\) is the far away distance.

Emil Wiechert in 1920 proposed to make this formula more general [14]:

\[
c_{R} = c_{\infty} \left(1 - \frac{\Omega G M}{c^2 R}\right)\]  
(4)

From this (4) we will get a more general expression for the light bending in the vicinity of the Sun:

\[
\theta = 1 - \frac{c_R}{c_\infty} = \Omega \frac{G M}{c^2 R_{\infty}} \quad 1 \leq \Omega \leq 8\]  
(5)

We predict the existence of quantum jumps with the deflection angle jump \(0^\circ.44\). There was already published one attempt by G.G. Nyambuya [15] to quantize the deflection angle jumps with the quantum jump \(0^\circ.87\). G.G. Nyambuya’s interpretation was based the influence of the photon spin on these quantum jumps.

This gravitational effect can be observed during the Solar eclipse events where both the Solar gravitational field and the photon mass are high enough to stimulate these quantum jumps.

![Fig. 3. Deflection of the light in the vicinity of the Sun with Ω = 4 steradians (Einstein shift).](image-url)
IV. ALL DATA FROM THE SOLAR ECLIPSE EXPEDITIONS

Many teams of professional astronomers took valuable data in the time span 1919-1973. However, the scatter of data was so high that no reasonable structure in these data was discovered and these professional activities were stopped in 1973. Table III surveys all these data for the light bending observed during the Solar eclipses.

| Number | Date            | Location | Average deflection [arcsecond] | Reference |
|--------|-----------------|----------|-------------------------------|-----------|
| 1      | May 29, 1919    | Sobral   | 1.98 ± 0.16                   | [16]      |
| 2      | May 29, 1919    | Principe | 1.61 ± 0.40                   | [16]      |
| 3      | September 21, 1922 | Wallal | 1.72 ± 0.11                   | [17]      |
| 4      | September 21, 1922 | Wallal | 1.82 ± 0.15                   | [17]      |
| 5      | September 21, 1922 | Wallal | 1.74 ± 0.3                    | [18]      |
| 6      | September 21, 1922 | Condillo | 1.77 ± 0.3                    | [19]      |
| 7      | May 9, 1929     | Takengon | 2.24 ± 0.10                   | [20]      |
| 8      | June 19, 1936   | Kubishev | 2.73 ± 0.31                   | [21]      |
| 9      | June 19, 1936   | Kosimizu | 1.71 ± 0.40                   | [22]      |
| 10     | May 20, 1947    | Brazil   | 2.01 ± 0.27                   | [23]      |
| 11     | February 25, 1952 | Sudan | 1.70 ± 0.10                   | [24]      |
| 12     | October 02, 1959 | Sahara | 2.17 ± 0.34                   | [25]      |
| 13     | February 15, 1961 | Italy | 1.98 ± 0.46                   | [26]      |
| 14     | June 30, 1973   | Mauritania | 1.66 ± 0.18                  | [27]      |
| 15     | August 21, 2017 | USA      | 1.751 ± 0.060                  | [28]      |

In 2020 Goldoni and Stefanini published the database for the light deflections of 170 individual stars taken during many of these expeditions with \( \theta_{\text{average}} = 1.98 \) [29]-[30].

This model \( \theta_{\text{average}} = 0.44 \times \frac{8 - 1}{2} > 0.44 = 1.98 \).

The dataset, published by Emanuele Goldoni [30] for 170 individual stars, was very inspirative for our research, and we tried to discover a hidden structure in that chaotic scatter of data. We have depicted individual stars for a given Solar eclipse and observed their quantum jumps of deflection angles in Fig. 4–14.

Fig. 4. Solar eclipse on May 29, 1919. Data from reference [30].
Fig. 5. Solar eclipse on September 21, 1922. Data from reference [30].

Fig. 6. Solar eclipse on May 9, 1929. Data from reference [30].

Fig. 7. Solar eclipse on June 19, 1936. Data from reference [30].
Fig. 8. Solar eclipse on June 19, 1936. Data from reference [21].

Fig. 9. Solar eclipse on May 20, 1947. Data from reference [30].

Fig. 10. Solar eclipse on February 25, 1952. Data from reference [30].
Fig. 11. Solar eclipse on October 02, 1959. Data from reference [25].

Fig. 12. Solar eclipse on February 15, 1961. Data from reference [26].

Fig. 13. Solar eclipse on June 30, 1973. Data from reference [30].
Fig. 14. Solar eclipse on August 21, 2017. Data from reference [30].

V. QUANTUM JUMPS DURING THE RUNNING TIME OF THE SOLAR ECLIPSE

The real situation with the interpretation of the light bending is more complicated. During the detailed analysis of several plates taken during the single Solar eclipse we can see in data “quantum jumps” of the deflection angles. Freundlich, von Klüber and von Brunn [20] made the very successful measurement on May 9, 1929 and took four plates during the single Solar eclipse. Their data are given in Table IV.

| Time since START | Plate  | Deflection angle | |Ω [sr]|
|------------------|--------|------------------|---|
| 10 seconds waiting | FT10   | 2.25 ± 0.19 | 5 |
| 40 seconds exposure | FT40   | 2.17 ± 0.20 | 5 |
| 15 seconds change | FT15   | 2.61 ± 0.26 | 6 |
| 60 seconds exposure | FT60   | 1.81 ± 0.19 | 4 |
| 10 seconds waiting | FT10a  |                |   |
| END | |

Interpretation: quantum jumps: 5 → 5 → 6 → 4

The detailed inspection of all data revealed a possible hidden structure in those light bending angles. For a better understanding of this structure we have to get new more precise data with the existing technology in our epoch.

VI. PREDICTIONS FOR THE LIGHT BENDING IN THE VICINITY OF THE SUN

Since 1801 there were done several predictions for the value of the light bending close to the surface of the Sun. These predictions are given in Table V.

| Year | Scholar | Interpretation | Predicted value of deflection angle | Active solid angle Ω [sr] | Reference |
|------|---------|----------------|-------------------------------------|--------------------------|-----------|
| 1801 | Soldner | Newtonian gravity | 0°.87 | 2 | [1] |
| 1899 | Ritz    | Ballistic theory | 1°.31 | 3 | [32] |
| 1915 | Einstein | Einsteinian gravity | 1°.75 | 4 | [33] |
| 1898 | Gerber  | Weber’s law with retarded potentials | 2°.62 | 6 | [34] |
| 2021 | This paper | Gauss’s law for gravity with active solid angle | 0°.44 ≤ θ ≤ 3°.52 | 1 ≤ Ω ≤ 8 | |

VII. CLASSROOM ACTIVITY OF GOLDONI AND STEFANINI

Goldoni and Stefanini [29] originally created their database as a possible activity for high-school students. It might happen that this activity could be very important for scholars searching for a model of
quantum gravity.

The target of this lesson is as follows [29]: “Two famous scientists A and B have proposed two alternative theories for an important astronomical phenomenon. According to A, the actual value should be 1°.75; on the contrary, B’s theory leads to 0°.87. The scientific community is split: different teams around the world have been asked to carry out experiments and gather useful data. Today we have received all the values obtained on the field. Now we have been asked to analyse them and choose which theory is the right one (or if both are wrong). The world is waiting for us!”

VIII. Conclusion

We might open a new road leading towards the model of quantum gravity. This contributin could stimulate some new activities of the international astronomical community to reanalyze all existing data on the bending of light in the vicinity of the Sun and to start preparation for the coming Solar eclipses to get more precise data with our existing technology. There is space enough for all participants on this Project.

However, K.J. Treschmann quoted [31]: “Anyone, no matter how well prepared can experience cloud at the crucial time of an eclipse.”

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Conflict of Interest

Authors declare that they do not have any conflict of interest.

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