Dynamical model of expansion free dissipative perfect fluids in general relativity

Rajesh Kumar and S. K. Srivastava
Department of Mathematics & Statistics,
Deen Dayal Upadhya Gorakhpur University, Gorakhpur, INDIA.

Abstract

This paper deals with the spherically symmetric self-gravitating star which is considered to be expansion free dissipative perfect fluids distribution. Some recent research reveals that expansion free dynamical star must be accelerating and dissipating. We adopted some conjectures to obtain the analytical solution for the dynamical model of such stars. Firstly, it has shown that density of dynamical star is homogeneous and Λ– dominated under quasi-static diffusion approximation. Secondly, the self-similar solution is also discussed to describe the dynamical model.

Mathematics Subject Classification 2010: 83C05, 83F05, 83C75.
PACS numbers: 04.40.-b, 04.20.-q, 04.40.Dg, 04.40.Nr
Keywords: Expansion scalar; Spherically-symmetric; Dissipative perfect fluids, Cavity evolution.

1 Introduction

The study of evolution of self-gravitating celestial objects (e.g. star, galaxies, etc.) are the most fascinating among relativists and astrophysicists. During the processes, self-gravitating objects may pass through phases of intense dynamical activities which can be observed by exploring the dynamical equations[1], [2]. During the evolution of compact stars after explosion under

---

* rkmath2009@gmail.com
†rajeshkumar.mathstat@ddugu.ac.in
‡sudhirpr66@rediffmail.com
the expansion-free condition, an interesting phenomenon of cavity formation has been observed. The evolution of expansion free self-gravitating object provides interesting cosmological significances and it consents to for the obtention of a wide range of solutions in general relativity and cosmology (3 - 8 and references their in).

In 1960, Skripkin [9] studied the very fascinating problem of the evolution of a spherically symmetric fluid distribution following a central explosion which result a Minkowskian cavity surrounding the centre of fluid distribution. Herrera et al. [4] discussed this problem by showing that under Skripkin conditions (constant energy density) the scalar expansion vanishes. It was further shown that the assumption of vanishing expansion scalar requires the existence of a vacuum cavity within the fluid distribution (of any kind). A systematic study on shearing expansion-free spherically symmetric distributions was presented in [10] and shown that in general for any non-dissipative fluid distribution, the expansion-free condition requires the energy density to be inhomogeneous. Recently, some authors (11, 16) have obtained interesting results for the spherically-symmetric self gravitating expansion-free fluids with a vacuum-cavity. Some works (7, 11 - 14) were presented the dynamical stability of the expansion-free spherical collapse. Sharif and Yousaf have studied the inhomogeneous non-dissipative dust model with expansion free conditions and the model is completely integrable [15]. Some analytical solutions for the self gravitating spherically-symmetric dissipation-less fluids with the expansion free motion have been obtained Di Prisco et al. [11]. Recently, Kumar and Srivastava [16, 17] have studied the gravitational collapse of self-gravitating star (dissipative/non-dissipative) which discussed the model of vacuum cavity, which showed the potential of expansion free motion. Recently, Sherif et al. [6] have studied the general features of expansion free dynamical star and showed that non-zero acceleration and dissipation are necessary for the evolution of such kinds of star. In present studies, authors are interested in the dynamics of the self-gravitating radiating (dissipative perfect fluid) star only in its evolution once cavity is already formed and The main contribution of this work to investigate the dynamical solution of such relativistic expansion free star.
2 Fluids distribution, Einstein’s field equations, Kinematical parameters and the junction conditions

Consider a spherically symmetric radiating compact star, which is closed by spherically hypersurface $\Sigma^r$. The matter distribution is taken to be perfect fluid concerning dissipation in form of heat flow (diffusion approximation) and outgoing null fluid (streaming out limit). For the bounded or closed system, a space time line-element concerning to the fluids inside hypersurface $\Sigma^r$,

$$ds^2 = -A(t, r)^2 dt^2 + B(t, r)^2 dr^2 + R(t, r)^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

(1)

where the metric coefficient $B$ is dimension-less and $R$ defines the areal-radius of spherically-symmetric surface and $\dot{R} < 0$ in case collapsing configuration. The coordinates is label as $x^\lambda = (t, r, \theta, \phi)$, $\lambda = 0, 1, 2, 3$.

Here the fluids - distribution are considered to be dissipative perfect fluid expressed by the energy-momentum tensor (inside $\Sigma^r$ if the system being closed/bounded)

$$T^\alpha_{\beta} = (p + \rho)v^\alpha v_\beta + p\delta^\alpha_\beta + q^\alpha v_\beta + v^\alpha q_\beta + \epsilon l^\alpha l_\beta$$

(2)

where $p \rightarrow$ pressure, $\rho \rightarrow$ energy-density and $\epsilon \rightarrow$ radiation energy. And $q_\alpha \rightarrow$ heat-flux, $v_\alpha \rightarrow$ four-velocity vector and $l_\alpha \rightarrow$ is a radial null four vector which satisfies

$$v^\alpha v_\alpha = -1, \quad v^\alpha q_\alpha = 0, \quad l^\alpha v_\alpha = -1, \quad l^\alpha l_\alpha = 0$$

(3)

and system of the comoving coordinate,

$$v^\alpha = A^{-1}\delta^\alpha_0, \quad q^\alpha = qB^{-1}\delta^\alpha_1, \quad l^\alpha = A^{-1}\delta^\alpha_0 + B^{-1}\delta^\alpha_1$$

(4)

where $q$ is a function of $t$ an $r$ and $q^\alpha = q\chi^\alpha$, $\chi^\alpha$ is a unit four vector along radial direction, satisfying

$$\chi^\alpha \chi_\alpha = 1, \quad \chi^\alpha v_\alpha = 0, \quad \chi^\alpha = \frac{1}{B}\delta^\alpha_1$$

(5)

I. Einstein field equations

Consider the evolution of cavity inside spherically symmetric self-gravitating radiating star. The general formalism of such dynamics is deployed details in [4], [16]-[17]. Following the work it can be seen that as the consequence of the vanishing expansion scalar, the line element (1) reduces to the form

$$ds^2 = -A^2 dt^2 + R^{-4} dr^2 + R^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

(6)
Thus, the Einstein’s field equations

$$R^i_j - \frac{1}{2} R \delta^i_j = 8 \pi T^i_j$$

yield following,

$$8 \pi \dot{\tilde{\rho}} A^2 = -3 \frac{\dot{R}^2}{R^2} - A^2 R^4 \left( -\frac{1}{R^6} + 5 \frac{R''}{R^2} + 2 \frac{R'''}{R} \right)$$ (7)

$$8 \pi \dot{\tilde{q}} A^2 = 2 \left( \frac{A'}{A} \dot{R} - \frac{2 R' \dot{R}}{R^2} + \frac{\dot{R}'}{R} \right)$$ (8)

$$8 \pi \ddot{\tilde{p}} A^2 = -\frac{A^2}{R^2} + A^2 R' R^3 \left( 2 \frac{A'}{A} + \frac{R'}{R} \right) - 2 \frac{\dot{R}}{R} + \frac{\dot{R}}{R} \left( 2 \frac{\dot{A}}{A} - \frac{\ddot{R}}{R} \right)$$ (9)

$$8 \pi p A^2 = A^2 R^4 \left[ 2 \frac{A'}{AR} + \frac{R'}{R} \left( \frac{A'}{A} + 2 \frac{R'}{R} \right) + \frac{A''}{A} + \frac{R''}{R} \right] - \frac{\dot{A} R}{AR} - 4 \frac{R^2}{R} + \frac{\dot{R}}{R}$$ (10)

where $\dot{\tilde{\rho}} = \rho + \epsilon$, $\dot{\tilde{p}} = p + \epsilon$ and $\dot{\tilde{q}} = q + \epsilon$. The Bianchi identities $T^\alpha_{\beta \dot{\tau}} = 0$ for the space time metric (6),

$$\dot{\tilde{\rho}} - 2 \epsilon \frac{\dot{R}}{R} + \dot{\tilde{q}} R^2 + 2 R^2 \dot{\tilde{q}} \frac{(AR)'}{AR} = 0$$ (11)

$$\dot{\tilde{q}} A - 2 \frac{\dot{\tilde{q}} R}{AR} + R^2 [\tilde{p}' + \frac{A'}{A} (p + \rho + 2 \epsilon) + 2 \frac{R'}{R}] = 0$$ (12)

II. Mass-function, Kinematic-parameters and Weyl curvature tensor

The mass-function $m(t, r)$ introduced in [18], is for space time metric (6) is

$$m(t, r) = \frac{1}{2} R \left( \frac{\dot{R}^2}{A^2} - R^4 R'^2 + 1 \right)$$ (13)

In view of the equations (7) - (11), one can obtain from (13)

$$\dot{m} = -4 \pi (\dot{\tilde{p}} \dot{R} + \dot{\tilde{q}} AR' R^2)$$ (14)

$$m' = 4 \pi (\dot{\tilde{p}} R' R^2 + \dot{\tilde{q}} A R')$$ (15)

On integration of Equ(15) gives

$$\frac{3 m}{R^3} = 4 \pi \tilde{\rho} - 4 \pi \int R^3 (\tilde{p}' - 3 \frac{\tilde{q} R'}{AR^3}) dr$$ (16)
The acceleration vector \((a^\alpha)\) and shear tensor \((\sigma_{\alpha\beta})\) are defined by
\[
a^\alpha = v^\alpha_{\beta} v^\beta \\
\sigma_{\alpha\beta} = \frac{1}{2} (v_{\alpha;\beta} + v_{\beta;\alpha})
\]
For the metric (6) above yields,
\[
a^\alpha = a\chi^\alpha = (0, aR^2, 0, 0), \quad a = \frac{A'}{A} R^2
\]
and,
\[
\sigma = -3 \frac{\dot{R}}{AR} 
\]
where \(\sigma^2 = \frac{3}{2} \sigma^{\alpha\beta} \sigma_{\alpha\beta}\).

Also, it can be found from Equ (8) and (20),
\[
4\pi \dot{q} + \frac{1}{3} \sigma' R^2 + \sigma RR' = 0
\]
The Weyl curvature tensor \((C^\rho_{\alpha\beta\gamma})\) is defined via the Riemann curvature tensor \(R^\rho_{\alpha\beta\gamma}\), the Ricci tensor \(R_{\alpha\beta}\) and scalar curvature \(\mathcal{R}\) in four-dim space-time
\[
C^\rho_{\alpha\beta\gamma} = R^\rho_{\alpha\beta\gamma} - \frac{1}{2} (R_\beta^\rho g_{\alpha\gamma} - R_\alpha^\rho g_{\beta\gamma} + R_\gamma^\rho g_{\alpha\beta} + R_\beta^\rho g_{\gamma\alpha}) + \frac{1}{6} \mathcal{R} (\delta_\beta^\rho g_{\alpha\gamma} - g_{\alpha\beta} \delta_\gamma^\rho) 
\]
The electric-Weyl tensor (magnetic-Weyl tensor absent due to spherically-symmetry of the metric) is given by
\[
E_{\alpha\beta} = C_{\alpha\beta\chi\nu} v^\gamma v^\nu
\]
It can be seen that the electric Weyl tensor \(E_{\alpha\beta}\) can also be expressed as [5]
\[
E_{\alpha\beta} = E (\chi_\alpha \chi_\beta - \frac{1}{3} h_{\alpha\beta})
\]
where \(h^\alpha_\beta = g^\alpha_\beta + v^\alpha v_\beta\)
\[
2E = -\frac{1}{R^2} + R^4 \left[ \frac{A''}{A} - \frac{R''}{R} + \frac{R'}{R} \left( \frac{A'}{A} - \frac{R'}{R} \right) \right] + \frac{1}{A^2} \left[ \frac{\dot{R}}{R} - 9 \frac{\dot{R}^2}{R^2} - 3 \frac{\ddot{R}}{R A} \right]
\]
Also, in view of Equ. (7), (13) and (25) it yield [5]
\[
3 \frac{m}{R^3} = 4\pi \rho - E
\]
Sherif et al. [6] prove that expansion free dynamical star must be conformally flat \(E = 0\), then we have
\[
8\pi \rho = \frac{6m}{R^3}
\]
III. The Junction conditions

For the bounded configuration (self-gravitating system), exterior of spherically-symmetric surface $\Sigma^e$ (on $r = r_{\Sigma^e}$), the Vaidya metric (it has been assumed that all the outgoing-radiation is massless) is given by

$$ds^2 = -(1 - \frac{2M}{\rho})d\tau^2 - 2d\rho d\tau + \rho^2(d\theta^2 + \sin^2\theta d\phi^2)\quad (28)$$

$M = M(\tau)$ denote the mass, and $\tau$ the retarded-time.

Since the expansion free models construct a central vacuum-cavity (Minkowski space time) of the fluids distribution. If $\Sigma^i$ (on $r = r_{\Sigma^i}$) denotes boundary-surface between the vacuum-cavity and the fluids, then such evolution problem requires Darmois-junction conditions connect two distinct space-time metric into one such that the both the hypersurfaces $\Sigma^e$ and $\Sigma^i$ demarcate the distribution of fluids. The former causes distribution of fluids from Vaidya metric and the later distinguish from space-time metric of vacuum cavity.

The junction of space-time (26) to the Vaidya-metric (28) and Minkowskian space-time over the surface $\Sigma^e$ and $\Sigma^i$ respectively has studied in [4]-[7],[16],[19] and [20], as

$$m(t, r_{\Sigma^e}) = M(\tau),\quad q(t, r_{\Sigma^e}) = p(t, r_{\Sigma^e})\quad (29)$$

$$m(t, r_{\Sigma^i}) = 0,\quad q(t, r_{\Sigma^i}) = p(t, r_{\Sigma^i})\quad (30)$$

It is also interesting to note that during the expansion free evolution, the matter of the voids stream out, which decreases the density of void from inside and becomes zero at $\Sigma^i$ (it follows from (27), $\rho(t, r_{\Sigma^i}) = 0$). If it allows for the presence of thin shell on $\Sigma^i$, then it has to relax the junction condition (30) and consent to the discontinuity of the mass-function ([11]).

3 Diffusion approximation

A renewed attention in general relativity is that self gravitating gravitational collapsing system is a dissipative-process. Dissipation-processes are usually considered with two feasible approximations: diffusion-approximation and streaming-out limit ([21] - [23]). The diffusion-approximation is rational, since it pertains whenever the mean-free path of fluids responsible for the energy-propagation is little as compare with the usual length of the system, which instituted very regular in the cosmological scenarios.

This section discussed the purely diffusion approximation case which reveal that $q \neq 0, \epsilon = 0$ considering with quasi-static regime [5]. The quasi static
means that the sphere changes slowly in time scale that is very long compared
to the typical time in which the sphere reacts to a straight perturbation of
hydrostatic equilibrium and so in this case $\dot{A} = \ddot{R} = \dot{A} = \dddot{R} = 0$.[24]. Then from equ. (9) - (10)

$$R^4 \left( \frac{A'R'}{AR} + \frac{R'^2}{R^2} + \frac{A''}{A} + \frac{R''}{R} \right) + \frac{1}{R^2} = 0$$

(31)

Taking use of (27), it gives

$$\frac{A''}{A} + \frac{A'R'}{AR} = 0$$

(32)

which gives after integration

$$A = \int \frac{d_1(t)}{R} dr + d_2(t)$$

(33)

where $d_1$ and $d_2$ are arbitrary function of $t$. Introducing (33) into (31) gives

$$R^5 R'' + R^4 R'^2 + 1 = 0$$

(34)

The first integral of (34) gives

$$R' = \frac{\sqrt{1 + d_3(t)}}{R^2}$$

(35)

where $d_3(t)$ is an arbitrary function. Further, integration of (35) yields

$$R \sqrt{1 + d_3 R^2} - \frac{\text{Sinh}^{-1}(R\sqrt{d_3})}{2d_3} = d_4 + r$$

(36)

which determine $R(t, r)$, where $d_4$ is an integrating constant and $d_3 > 0$. Thus, in view of equ. (34) - (36), equations (7) - (9) yield

$$8\pi \rho = -3 d_3(t) = \rho(t)$$

(37)

$$8\pi p = 2d_1 \sqrt{1 + d_3 R^2} + d_3$$

(38)

$$8\pi q = \frac{-2\dot{R}}{\int \frac{d_1}{R} dr + d_2} \left[ \frac{d_1}{R^2} dr + d_2 + \frac{(4 + 3d_3 R^2)}{R^2 \sqrt{1 + d_3 R^2}} \right]$$

(39)

It also can be seen that Equs. (11 - 12) are satisfied identically. we have from equ. (19), the acceleration

$$a = \frac{d_1 R}{\int \frac{d_1}{R} dr + d_2}$$

(40)

Thus the solutions presented here have non-zero acceleration and dissipation which reveals the dynamical cavity model of star.
4 Self-similar regime

The existence of a homothetic- killing vectors described the self similar space-time. The self similar solutions of the field equations in general relativity was studied in extensive details ( see [25], [26] and references their in ). Any spherically-symmetric space-time is self similar if it contain a radial-coordinate $r$ and an orthogonal time-coordinate $t$ so that for the metric-coefficient $g_{tt}$ and $g_{rr}$ satisfied

$$g_{tt}(kt, kr) = g_{tt}(t, r)$$

$$g_{rr}(kt, kr) = g_{rr}(t, r)$$

for all constant $k > 0$. In the self similar regime, the field equations, a set of partial-differential equation transform into ordinary-differential equation. Self similarities are the strong constraint of geometry, which have been effectively expressed in various cosmological/physical scenarios ([25] - [29]). In this section it has studied that the appearance of a homothetic-killing vector field for a spherically-symmetric space-time entails the detachability of the space time metric components in terms of the comoving coordinates and that the line element can be expressed in a simplified unique form. A vector field $\xi^i$ is said to be homothetic if it follows

$$\mathcal{L}_\xi g_{ij} = 2g_{ij}$$

Consider that a spherically-symmetric space-time consist a homothetic-killing vector of the form

$$\xi^i = (0, \alpha(t, r), 0, 0)$$

Usually, a homothetic-Killing vector is expressible in the form

$$\xi^i = (\ell, \tau, 0, 0)$$

However, any vector of the form (44) can be transformed into the form (45) via a coordinate transformation ([28] - [29])

$$\ell = l(t)e^{\int \alpha^{-1} dr}, \quad \tau = k(t)e^{\int \alpha^{-1} dr}$$

If the line element (6) admits a homothetic killing vector of the form (44), then equ. (43) yields

$$\frac{A'}{A} = 4\alpha^{-1}$$

$$\dot{\alpha}(t, r) = 0$$
\[ \alpha' = 2(2 + \alpha \frac{R'}{R}) \]  
\[ \frac{R'}{R} = 4\alpha^{-1} \]  

Solving the above systems of equations, one obtain

\[ \alpha = 12r \]

\[ A(t, r) = k(t)r^{\frac{1}{2}} \]

\[ R(t, r) = l(t)r^{\frac{1}{2}} \]

where \( k(t) \) and \( l(t) \) are arbitrary constants. In view of Equ. (52) and (53), the metric (6) reduces to the form

\[ ds^2 = -k^3 r^{\frac{3}{2}} dt^2 + l^{-4} r^{-\frac{3}{2}} dr^2 + l^2 r^{\frac{5}{2}} (d\theta^2 + \sin \theta d\phi^2) \]

Thus, by virtue of this it can be obtain from equation (7) - (10)

\[ 8\pi \rho = \frac{1}{3r \frac{k^3 l^2}{2}} \left[ (-t^6 + 6)k^3 - 9i\dot{k}i + 9k(\ddot{i} - 2\dot{i}^2) \right] \]

\[ 8\pi p = \frac{1}{9r \frac{k^3 l^2}{2}} \left[ k^3 t^6 - 9i\dot{k} + 9k(\ddot{i} - 4\dot{i}^2) \right] \]

\[ 8\pi q = \frac{1}{9r \frac{k^3 l^2}{2}} \left[ k^3 (9 - 2t^6) - 12k^2 \dot{t}^3 i - 27l\dot{k}k - 27k(\dot{i}^2 - \ddot{i}i) \right] \]

\[ 8\pi \sigma = \frac{1}{9r \frac{k^3 l^2}{2}} \left[ k^3 (2t^6 - 9) + 27l\dot{k}k + 27k(\dot{i}^2 - \ddot{i}i) \right] \]

Also, the equations (13) and (20) yield

\[ m(t, r) = \frac{1}{2} r^{\frac{3}{2}} l \left( 1 - \frac{t^6}{9} + \frac{\dot{t}^2}{k^2} \right) \]

\[ \sigma = -3\frac{i}{r \frac{k}{kl}} \]

It follows from equ. (19), the acceleration

\[ a = \frac{1}{3} \dot{t}^2 r^{-\frac{3}{2}} \]

Now, apply the junction condition (29) over hypersurface \( \Sigma_c \)

\[ k(t) = -\frac{(M - 4)\dot{M}}{3^{2/3} \sqrt{(4 - 3M)^2}} \frac{M - 2}{(M - 2)^{7/6} M^{2/3}} \]
and thus the arbitrary functions $l(t)$ and $k(t)$ are completely determined. Also, from equations (50), (56), (57) and (59) it can be observed that the model (54) does not satisfy the Darmois junction condition over cavity hypersurface $\Sigma'$. Therefore, it might be pertinent to relax junction condition and it exhibits the presence of thin shell which allows for the discontinuities of mass function across $\Sigma'$. 

5 Discussion and Concluding remarks

In last decade the evolution of expansion free dynamical stars have taken considerable interest among relativists and has been applied to illustrate the physical and geometrical properties of radiating star (1-11, 13-17 and references therein). It can be emphasized that very limited dynamical models are investigated for dissipative cavity evolution. The existing work is in itself very remarkable to describe the cavity model of self-gravitating spherically symmetric dissipative perfect fluids. For obtaining solution, some alternatives are presented namely, quasi-static diffusion approximation and selfsimilar regime to elucidate the modelling of cavity evolution.

In this way, the first solution is investigated with diffusion approximation with quasi-static regime and it has been shown that the energy density is homogeneous, $\rho = \rho(t)$ and $\Lambda-$ dominated (with negative energy density which may be violated baryonic equation of state,30). As an important special case, self-similarity (homothetic) is also introduced here to discuss the solution of field equations. It is interesting to note that the model is explicitly described in terms of $r$ and arbitrary function $k(t), l(t)$. Since it is seen here describing the localized objects without the unusual topology of a spherical fluid without centre, $r \neq 0$, the centre is surrounded by a compact spherical surface of another space time suitably matched to the rest of the fluid1, 7. Therefore no real singularity occurs in this model at all ($r \neq 0$). Possibly the solutions presented here could be applied for the localized system of supernova explosion. Forthcoming researches of such models with numerical solutions of pertinent equation would produce more appropriate applications in astrophysics.
References

[1] Kommemi, J.: The Global Structure of Spherically Symmetric Charged Scalar Field Spacetimes, Commun. Math. Phys. 323, 35 (2013)

[2] M. Sharif and Khadija Iqbal: Spherically symmetric gravitational collapse, Modern Physics Letters A 24(19), 1533 (2009)

[3] L. Herrera , No. Santos: Local anisotropy in self-gravitating systems, Phys. Rep. 286 53 (1997).

[4] L. Herrera, N.O. Santos and A. Wang: Shearing expansion-free spherical anisotropic fluid evolution, Phys.Rev. D 78, (2008) 084026.

[5] L. Herrera, A. Di Prisco, J. Ospino and J. Carot: Lemaitre-Tolman-Bondi dust spacetimes: Symmetry properties and some extensions to the dissipative case, Phys. Rev D 82(2), 024021 (2010).

[6] Sherif, A., Goswami, R. and Maharaj, S.: Properties of expansion-free dynamical stars. Phys. Rev. D, 100(4), 044039 (2019).

[7] L. Herrera, G Le Denmat, N O Santo: Cavity evolution in relativistic self-gravitating fluids, Class. Quantum Grav. 27, (2010) 13.

[8] M. Sharif and Z. Yousaf: Evolution of expansion-free self-gravitating fluids and plane symmetry, International Journal of Modern Physics D 21(14), 1250095 (2012).

[9] V. A. Skripkin, Soviet Physics-Doklady 135, (1960) 1183.

[10] L. Herrera, G. Le Denmat, and N.O. Santos: Expansion-free evolving spheres must have inhomogeneous energy density distributions, Phys. Rev. D 79 (2009) 087505.

[11] Di Prisco, A., Herrera, L., Ospino, J., Santos, N.O., Vi na-Cervantes, V.M.: Expansion-free cavity evolution: some exact analytical models, Int. J. Mod. Phys. D 20 (2011) 2351.

[12] L.Herrera, J. Ospino, A. Di Prisco, E. Fuenmayor and O. Troconis: Structure and evolution of self-gravitating objects and the orthogonal splitting of the Riemann tensor, Phys. Rev D 79, 064025 (2009).

[13] L. Herrera, G. Le Denmat, and N.O. Santos: Dynamical instability and the expansion-free condition, Gen. Relativ. Grav. 44(5), 1143 (2012).
[14] M. Sharif and M. Azam: Role of anisotropy in the expansion-free plane gravitational collapse, Gen. Relativ. Grav. 46, 1647 (2014).

[15] Sharif, M., Yousaf, Z.: Expansion-free cylindrically symmetric models, Can. J. Phys. 90, 865 (2012).

[16] Kumar, R and Srivastava, S.K.: Evolution of expansion-free spherically symmetric self-gravitating non-dissipative fluids and some analytical solutions, Int. J. Geom. Methods Mod. Phys. 15(4), 1850058 (2018).

[17] Kumar, R and Srivastava, S.K.: Expansion-free self-gravitating dust dissipative fluids, Gen. Relativ. Grav., 50(8), 95 (2018).

[18] C. Misner and D. Sharp: Relativistic equations for adiabatic, spherically symmetric gravitational collapse, Phys. Rev. 136 (1964) B571.

[19] R. Chan: Radiating gravitational collapse with shear viscosity, Mon. Not. R. Astron. Soc. 316, 588 (2000).

[20] N. O. Santos: Non-adiabatic radiating collapse, Mon. Not. R. Astron. Soc. 216, 403 (1985).

[21] W. Israel and J. Stewart: Thermodynamics of nonstationary and transient effects in a relativistic gas, Phys. Lett. A 58, 213 (1976).

[22] W. Israel and J. Stewart: Transient relativistic thermodynamics and kinetic theory, Ann. Phys. (NY) 118, 341 (1979).

[23] J. Lattimer: Supernova theory and the neutrinos from SN1987a, Nucl. Phys. A 478, 199 (1988).

[24] L. Herrera, W. Barreto, A. Di Prisco and N. O. Santos, Phys. Rev. D, 65, 104004 (2002).

[25] M. E. Cahill and A. H. Taub: Spherically symmetric similarity solutions of the Einstein field equations for a perfect fluid, Commun. Math. Phys. 21, 1 (1971).

[26] Joshi, P.S. and Dwivedi, I.H: The structure of naked singularity in self-similar gravitational collapse, Commun. Math. Phys. 146, 333 (1992).

[27] P. S. Joshi, Global Aspects in Gravitation and Cosmology. (Oxford: Clarendon Press, Oxford University Press., 1993).

[28] B. J. Carr and A. A. Coley: Self-similarity in general relativity, Class. Quant. Grav. 16, R31 (1999).
[29] S. M. Wagh and K. S. Govinder: Spherically symmetric, self-similar spacetimes, Gen. Relativ. Grav. 38, 1253 (2006).

[30] Yousaf, Z.: Spherical relativistic vacuum core models in a $\Lambda$-dominated era, Eur. Phys. Jou. Plus, 132, 71 (2017).