Critical Velocity for Vortex Shedding in a Bose-Einstein Condensate

Woo Jin Kwon, Geol Moon, Sang Won Seo, and Y. Shin

Department of Physics and Astronomy, and Institute of Applied Physics, Seoul National University, Seoul 151-747, Korea

We present measurements of the critical velocity for vortex shedding in a highly oblate Bose-Einstein condensate with a moving repulsive Gaussian laser beam. As a function of the barrier height \( V_0 \), the critical velocity \( v_c \) shows a dip structure having a minimum at \( V_0 \approx \mu \), where \( \mu \) is the chemical potential of the condensate. At fixed \( V_0 \approx 7\mu \), we observe that the ratio of \( v_c \) to the speed of sound \( c_s \) monotonically increases for decreasing \( \sigma/\xi \), where \( \sigma \) is the beam width and \( \xi \) is the condensate healing length. The measured upper bound for \( v_c/c_s \) is about 0.4, which is in good agreement with theoretical predictions for a two-dimensional superflow past a circular cylinder. We explain our results with the density reduction effect of the soft boundary of the Gaussian obstacle, based on the local Landau criterion for superfluidity.

In previous ultracold atom experiments, a similar situation was investigated by stirring superfluid samples with a repulsive laser beam [10–15]. The existence of finite critical velocities [10, 11, 14] and generation of vortex dipoles [12, 13] were successfully demonstrated. However, the measured values of \( v_c/c_s \) widely ranged from 0.1 to 0.45, which did not allow a quantitative study of the homogeneous 2D problem. Theoretical investigations showed that the inhomogeneous density distribution of a trapped sample [10, 15], three-dimensional (3D) vortex dynamics [19, 21], or the manner of stirring [13, 16, 21–24] should be critical in the measurements.

A superfluid flows without friction but it becomes dissipative above a certain critical velocity \( v_c \) via generating its elementary excitations such as phonons and vortices. The Landau criterion provides a conventional energetic consideration to determine the critical velocity, stating \( v_c = \min \{ \epsilon(p)/p \} \) [1], where \( \epsilon(p) \) is the energy of an elementary excitation of momentum \( p \). For a homogeneous case, this gives the speed of sound \( c_s \). However, dynamic response of a superfluid flow is significantly sensitive to the boundary condition of the system, and hence, quantitative understanding of the critical velocity has been a challenging task in the study of superfluidity.

One of the paradigmatic situations considered in fluid mechanics is a two-dimensional (2D) flow past a circular cylinder. For an incompressible flow, the local velocity is increased by a factor of 2 at the lateral sides of the cylinder [2] and the local Landau supersonic criterion suggests a critical velocity of \( v_c = 0.5c_s \) that is independent of the radius \( R \) of the cylinder. Theoretical studies showed that the onset of dissipation involves generating a counter-rotating vortex pair [3, 4]. More rigorous calculations, taking into account the compressibility of superfluid and quantum pressure near the boundary of the cylinder, predicted that the critical velocity converges to \( v_c = 0.37c_s \) in the large cylinder limit \( R \gg \xi \) [5–9], where \( \xi \) is the superfluid healing length. Experimental verification of the predictions on \( v_c/c_s \) is highly desirable.

In this Letter, we systematically study the critical velocity for vortex shedding in a Bose-Einstein condensate with a repulsive Gaussian potential. We measure the critical velocity as a function of the barrier height \( V_0 \) of the potential over a wide range of the beam width, \( 10 < \sigma/\xi < 55 \). In particular, in order to address the 2D homogeneous regime, we employ spatially large and highly oblate condensates, ensuring vortex dynamics in 2D.

The key difference of a Gaussian potential from a hard cylinder is its soft boundary. A Gaussian potential, \( V(r) = V_0 \exp(-2r^2/\sigma^2) \), produces a density-depleted hole in the condensate when \( V_0 > \mu \), where \( \mu \) is the chemical potential of the condensate. The radius of the hole and the potential slope at the hole boundary are given as

\[
R = \sigma \sqrt{\ln(V_0/\mu)/2},
\]

\[
S = -\frac{dV}{dr} \bigg|_{r=R} = 4\mu R/\sigma^2,
\]

respectively. In comparison to a hard cylinder, the soft boundary reduces the density in its proximity and consequently lowers the local speed of sound. Then, the local Landau criterion proposes that the critical velocity of the hard cylinder would define an upper bound for that of the obstacle formed by the Gaussian potential. When the hole radius \( R \) becomes larger with higher \( V_0 \) [Fig. 1(b)] and/or the beam width \( \sigma \) decreases for fixed \( R \) [Fig. 1(c)], the obstacle would converge to the hard cylinder with stiffening its boundary.

The main result of our measurements is that in the deep non-penetrable regime (i.e., \( V_0 \gg \mu \)), the critical velocity \( v_c \) increases for smaller \( \sigma/\xi \) and approaches about 0.4\( c_s \). This observation is consistent with the expectation based on the local Landau criterion and furthermore, the measured upper bound for \( v_c/c_s \) is in good agreement with theoretical predictions for a 2D superfluid flow past a hard cylinder.

Our experiment starts with a Bose-Einstein condensate
of 23Na atoms in a harmonic trap formed by combining optical and magnetic potentials [14]. The condensate fraction of the sample is over 90%. In a typical sample condition, where the trapping frequencies are \( \omega_{r,z} = 2\pi \times (9.0, 400) \) Hz and the atom number of the condensate is \( N_0 = 3.2(2) \times 10^6 \), the condensate healing length is \( \xi = h/\sqrt{2m\mu} \approx 0.46 \) \( \mu \)m and the speed of sound is \( c_s = \sqrt{\mu/m} \approx 4.3 \) mm/s at the trap center, where \( h \) is the Planck constant divided by \( 2\pi \) and \( m \) is the atomic mass. By adjusting the trapping frequencies or the atom number of the condensate, \( \xi \) is varied up to 0.9 \( \mu \)m. The Thomas-Fermi radius and thickness of the condensate are \( R_{TF}/\xi \geq 240 \) and \( Z_{TF}/\xi \leq 6 \), respectively. In this highly oblate condensate, vortex line excitations are strongly suppressed [21, 22] and the vortex dynamics is expected to be 2D.

We adiabatically ramp up the power of a repulsive Gaussian laser beam in 1 s and hold it for 0.2 s to ensure that the condensate is stationary. Then, we translate the laser beam horizontally by 24 \( \mu \)m by using a piezo-driven mirror [Fig. 2(a)]. The velocity \( v \) of the laser beam is kept constant during the translation and controlled by adjusting the moving time. The sweeping region by the laser beam is centered in the condensate [Fig 2(b)] and the density variation over the region is less than 2%. After completing the sweeping, we slowly ramp down the laser beam power for 0.5 s and take an absorption image of the condensate after time-of-flight to detect vortices [Fig. 2(c) inset].

The beam width \( \sigma \) of the laser beam is calibrated from the in-situ images of very dilute samples penetrated through by the laser beam, taking into account the imaging resolution of our system (≈ 5 \( \mu \)m). The beam waist of the laser beam is 9.1(12) \( \mu \)m at the focal plane and the Gaussian width \( \sigma \) of the optical obstacle is controlled by defocusing the laser beam at the sample plane. The sample thickness (< 3 \( \mu \)m) is much shorter than the Rayleigh length of the laser beam and we ignore the beam divergence.

The critical velocity \( v_c \) for vortex shedding is determined from the probability distribution \( P(v) \) for having vortex dipoles after sweeping with the laser beam. \( P(v) \) is obtained from 15 realizations of the same experiment with a given sweeping velocity \( v \). Even at low \( v \), we occasionally observe a single vortex with probability < 4%. Because the vortex life time is over 10 s [13] and vortices are generated as dipoles with the stirring laser beam [Fig. 2(c) inset], we attribute the single vortex to uncontrollable perturbations in sample preparation and we do not count it as a vortex dipole for \( P(v) \). The critical velocity \( v_c \) is determined by fitting a sigmoidal function to the probability distribution as \( \tilde{P}(v) = 1/(1 + e^{-(v-v_c)/\tau}) \) [Fig. 2(c)]. We use the value of 1.5\( \tau \) as the measurement uncertainty of \( v_c \), corresponding to the range of 0.2 \( \leq P \leq 0.8 \).

Figure 3(a) displays the results of the critical velocity as a function of the barrier height \( \tilde{V}_0 = V_0/\mu \) for various beam widths (10 < \( \tilde{\sigma} = \sigma/\xi \) < 55). The critical velocity shows a dip structure having a minimum at \( \tilde{V}_0 \approx 1 \), clearly distinguishing the two regimes: a pene-
the potential slope becomes steeper with higher $\tilde{\sigma}$. Further investigation on the functional form of $v_c(\tilde{V}_0; \tilde{\sigma})$ in the penetrable regime is warranted. However, in this work, we focus on the non-penetrable regime to address the 2D hard cylinder situation.

In Fig. 3(b), we recast the data for $\tilde{V}_0 > 1$ as a function of the hole radius $\tilde{R} = R/\xi$, together with additional data obtained for $\tilde{V}_0/\mu > 10$ with $\sigma/\xi = 20$ and 35. Note the slow dependence of $\tilde{R}$ on the barrier height as $\tilde{R} \propto \sqrt{\ln \tilde{V}_0}$. From the discussion based on the boundary stiffness $S \propto R/\sigma^2$ [Fig. 1(b)], one may expect that $v_c/c_s$ would be saturated to a certain value for large $\tilde{R}$ and the saturation behavior would be faster with smaller $\tilde{\sigma}$. In a small $\tilde{R}$ region, the experiment data roughly follows the expectation [Fig. 3(a)]. However, for further increasing $\tilde{R}$, the growth rate of the critical velocity slows down and becomes even reversed. At $\tilde{R} \sim 35$, the critical velocity with small $\tilde{\sigma} = 20$ is observed to be lower than that with large $\tilde{\sigma} = 54$.

One possible explanation for such climbing-over behavior is the imperfection of the laser beam profile. For example, if the beam profile is not perfectly Gaussian and in particular, the outer part of the laser beam has a slower intensity decay than exponential, then the potential slope $S$ at the obstacle boundary would become softer with higher $\tilde{R}$. We see that the climbing-over of $v_c$ occurs at $\tilde{R} \sim \tilde{\sigma}$ in both of the data sets with $\tilde{\sigma} = 20$ and 35, which seems to support the beam profile effect because in our experiment $\sigma$ is varied by defocusing the same laser beam. The $M^2$ factor of the laser beam is measured to be 1.2.

To further investigate the soft boundary effect, we take a different scanning trajectory in the parameter space of $S$: decreasing $\tilde{\sigma}$ for fixed $\tilde{V}_0$ [Fig. 1(a)]. In this setting, the hole radius $\tilde{R}$ is also varied with $\tilde{\sigma}$. In terms of excluding the effect due to the variation of $\tilde{R}$, a scanning with fixed $\tilde{R}$, as depicted in Fig. 1(c), might be more ideal. However, this would require exponentially high $\tilde{V}_0$ for small $\tilde{\sigma}$ as $\tilde{V}_0 \propto e^{2(\tilde{R}/\sigma)^2}$, necessarily recalling the outer part of the laser beam. We set $\tilde{V}_0 \approx 7$ with $\tilde{R} \approx \tilde{\sigma}$. Around this condition, $v_c(\tilde{V}_0)$ shows a maximum in the previous measurements. In Fig. 4, we see that the critical velocity monotonically increases as $\tilde{\sigma}$ decreases, which is consistent with our expectation from the boundary stiffness. It is worth noting that the data are obtained from samples with various healing lengths but well patched in the plane of the dimensionless parameters $\tilde{\sigma}$ and $v_c/c_s$. This demonstrates the 2D character of our system.

For our stiffest obstacle, $v_c/c_s$ is measured to be about 0.40, which is slightly higher than the predicted value of 0.37 for a 2D hard cylinder in the large $\tilde{R}$ limit. In our measurement, $\tilde{R} \approx \tilde{\sigma} \geq 10$ marginally satisfies the large obstacle condition, and we attribute the deviation of the measured value to the finite $\tilde{R}$ effect. The depen-

![Graph](image-url)
provide a new setting to study the role of thermal atoms in vortex nucleation [34–36]. A Gaussian potential with \( \tilde{\sigma} \) decreases (i.e., \( \tilde{\sigma} \) is for black solid squares and \( \xi \)) increases from the value of 0.37 as \( \tilde{R} \rightarrow 0 \). For comparison, in Fig. 4 we display the theoretical calculation result from Ref. [8] and the result of numerical simulations performed with a Gaussian potential with \( \tilde{V}_0 = 100 \) in Ref. [23]. We clearly observe that as \( \tilde{\sigma} \) decreases (i.e., the boundary stiffness of the obstacle becomes higher), our measurement results converges to the theoretical predictions. Finally, the \( 1/\tilde{R} \) dependence of \( v_c \) was predicted from analytic analyses on the stability of a superfluid flow past a hard cylinder [18, 32], which is not supported by our results.

FIG. 4: (color online). \( v_c/c_s \) versus \( \tilde{\sigma} / \xi \) for fixed \( \tilde{V}_0/\mu \approx 7 \). \( \tilde{\sigma} = 9.1(12) \mu \text{m} \) is for black solid squares and \( \xi = 0.46 \mu \text{m} \) is for black solid circles. The blue open circles show the theoretical calculation results for a 2D hard cylinder of radius \( \tilde{R} = \sigma \sqrt{\ln(\tilde{V}_0/\mu)} / 2 \) from Ref. [8]. The red open diamonds indicate the numerical results for a Gaussian potential with \( \tilde{V}_0/\mu = 100 \) from Ref. [23], where the potential slope \( \tilde{S} \) would be about two times higher than ours. The dashed line denotes the theoretically predicted value, \( v_c/c_s = 0.37 \) for a 2D homogeneous case with a hard cylinder in the large obstacle limit [4, 9].

In summary, we have presented the measurements of the critical velocity for vortex shedding in highly oblate Bose-Einstein condensates and investigated the soft boundary effect of the moving obstacle formed by a Gaussian potential. Our results are consistent with a picture based on the local Landau criterion and in good agreement with the theoretical predictions for a homogeneous 2D superflow past a cylindrical object. An intriguing extension of this work is to investigate the temperature dependence of the critical velocity, which might provide a new setting to study the role of thermal atoms in vortex nucleation [34, 36].

We thank Seji Kang for experimental assistance. This work was supported by the NRF of Korea (Grant No. 2011-0017527).
Phys. Rev. A 82, 033603 (2010).

[31] K. C. Wright, R. B Blakestad, C. J. Lobb, W. D. Phillips, and G. K. Campbell, Phys. Rev. A 88, 063633 (2013).

[32] J. S. Stießberger and W. Zwerger, Phys. Rev. A 62, 061601(R) (2000).

[33] G. W. Stagg, N. G. Parker and C. F. Barenghi, J. Phys. B: At. Mol. Opt. Phys. 47, 095304 (2014).

[34] J. E. Williams, E. Zaremba, B. Jackson, T. Nikuni, and A. Griffin, Phys. Rev. Lett. 88, 070401 (2002).

[35] A. A. Penckwitt, R. J. Ballagh, and C. W. Gardiner, Phys. Rev. Lett. 89, 260402 (2002).

[36] K. Kasamatsu, M. Tsubota, and M. Ueda, Phys. Rev. A 67, 033610 (2003).