Simulation research of the tire Basic Relaxation Model in conditions of the wheel cornering angle oscillations

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Abstract. A description of the tire Basic Relaxation Model (BRM) is presented in this paper. Simulation research of the tire BRM model in conditions of oscillatory changes of the wheel cornering angle were performed. During the simulation tests the courses of changes in the value of lateral reaction force, transmitted by the wheel, as a response to the sinusoidal changes in the value of the wheel cornering angle have been presented. There have been compared the simulation results obtained for the model of tire-road interaction in two modes: including and not including the BRM. The simulation results allowed to verify prepared BRM and also to determine the influence of the tire relaxation process on the tire behavior in conditions of dynamic changes of the wheel cornering angle.

1. Introduction
There are several reasons for changing the lateral reaction force transmitted by the tire, during the oscillatory dynamic changes in a value of the wheel cornering angle. The first reason is the shape of a tire cornering characteristics (figure 1a). The cornering angle oscillations can change the effective value of the lateral reaction force $F_y$ especially in a scope of the wheel cornering angle values, where the tire cornering characteristics is strongly nonlinear (figure 1a).

![Figure 1](image_url)

**Figure 1.** Example of the tire cornering characteristics - possible change in effective value of the lateral reaction force $F_y$ (a) and relaxation length $L_n$ (b) as a result of the cornering angle oscillation around its certain constant value.
The effective value of the lateral reaction force $F_y$ decreases when the cornering angle oscillates in the scope of values where the tire cornering characteristics is nonlinear and convex. But it is also possible that the effective value of the lateral reaction force $F_y$ does not change or even increases, when the tire cornering characteristics is linear or nonlinear but concave (e.g. in the scope of small values of the wheel cornering angle). In simulation research, when calculating the value of the lateral reaction force, that tire performance can be achieved using anyone of the available tire-road interaction model for steady-state wheel running conditions [7,1].

The second reason for changing in the effective value of the lateral reaction force $F_y$ transmitted by the tire during oscillatory changes of the wheel cornering angle can be the phenomenon of the tire relaxation. This is because the relaxation length $L_n$ which depends non linearly on the wheel cornering angle (figure 1b) [3,4]. Therefore, the growth rate of the lateral reaction force is higher than its decrease rate even than the cornering angle oscillates with constant amplitude. In addition, it is worth noting that due to the tire relaxation, the amplitude of the lateral reaction force transmitted by the tire can strongly decrease with increasing the cornering angle oscillation frequency. In simulation research, when calculating the value of lateral reaction force, such tire performance can be achieved using appropriate tire relaxation model [3,4,5,10].

Presented on the Figure 1 tire properties may be in opposition. It means that nonlinear tire cornering characteristics may decrease and the tire relaxation process may increase the effective value of the lateral reaction force transmitted by the tire during wheel cornering with oscillations of the wheel cornering angle. So when building the tire-road interaction model, as a module of the vehicle dynamics model, it is worth to take care of mentioned above tire properties. Especially the tire properties in transient cornering conditions may be very important when performing the vehicle simulation research in aspect of its safety or lateral dynamics. The achievement of tire properties in transient cornering conditions requires use of appropriate tire relaxation model and its properly matched coefficients.

The main purpose of this work is to assess the behavior of prepared tire Basis Relaxation Model in conditions of dynamic changes in the wheel cornering angle.

2. The mathematical description of the Basic Tire Relaxation Model

During work the tire relaxation model has been prepared. Its name is the Basic Relaxation Model (BRM) because it is based on the elementary tire properties, which are related to the lateral dynamics of the tire envelope. The essence of the model are both mathematical description and its coefficients which have been determined based on the experimental research results of real tires. Mathematical description of the BRM model is presented in the following expressions:

\[
F_{yu} = s_y \cdot k_\delta \quad (1)
\]

\[
m_{tr} \cdot \ddot{u}_y + \left( \frac{k_\delta}{u_x} + k_y \right) \dot{u}_y + k_{ys} \cdot u_y = F_{yu} \quad (2)
\]

\[
F_{ys} = k_{ys} \cdot \dot{u}_y \quad (3)
\]

\[
F_{yd} = k_{yd} \cdot \dot{u}_y \quad (4)
\]

\[
F_y = F_{ys} + F_{yd} \quad (5)
\]

Where particular markings mean:

$F_{yu}$ – the lateral reaction force transmitted in the contact area between the tire tread and the road surface in steady state cornering conditions – the value obtained from the tire-road interaction model without the tire relaxation (it is input to the BRM model),
\( \delta \) – the wheel cornering angle,
\( s_y \) – the wheel lateral slip \( s_y = \tan(\delta) \),
\( v_x \) – the longitudinal component of the wheel speed,
\( u_y \) – the tire lateral deflection,
\( \dot{u}_y \) – the speed of the tire lateral deflection,
\( m_0 \) – the mass of the tire envelope part deflected in lateral direction, reduced to its tread staying in contact with the road surface,
\( k_{yd} \) – the tire envelope lateral damping coefficient,
\( k_{ys} \) – the tire lateral stiffness,
\( F_{yd} \) – the lateral viscous damping force in the tire envelope,
\( F_{ys} \) – the lateral spring force in the tire envelope,
\( F_y \) – the lateral reaction force transmitted into the wheel rim, and further into the vehicle chassis (output from the BRM model).

Prepared model does not include the resistance of the tire envelope to deflection under the influence of its gyroscopic moment during wheel roll. The main differential equation of the model (2) is linear, but the model coefficients \((k_\delta, k_{yd})\), based on the tire experimental research results are under nonlinear influence of the wheel running conditions. Values of some model coefficients \((m_0, k_{yd})\) are established basing on the own experimental research results of the truck tire and also confirmed basing on information from the literature references [3,5,4,8].

The advantage of the prepared BRM is that there is no need to modify the main part of the model used for calculating the lateral reaction force \( F_{yu} \) in steady-state wheel cornering conditions. In this case, as the main part of the model, the DFS (Dugoff, Fancher, Segel) model have been used [1]. Prepared the BRM has been successfully verified by comparing its behavior with the real truck tire behavior, observed in different transient cornering conditions during the experimental laboratory tests. Therefore, it is possible to test the BRM in extended or extreme transient cornering conditions of the wheel, which are difficult to achieve in the laboratory.

3. Simulation based tests of the tire Basic Relaxation Model in extreme transient cornering conditions

In this case, the BRM model was tested by sinusoidal changing the wheel cornering angle about its constant non-zero value. During the tests the cornering angle oscillation frequency were increasing monotonously from 0 to 10 Hz. Whereas the oscillation amplitude were constant (Figure 2).

![Figure 2. Example of the wheel cornering angle waveform, entered into the BRM model as an input signal (the constant component of the cornering angle \( \delta = 4^\circ \), the amplitude of cornering angle oscillation \( A(\delta) = 2^\circ \), the oscillation frequency range \( f = 0 \div 10 \text{ Hz} \)).](image-url)
Particular test conditions were as follows:
- steady state value of the wheel cornering angle $\delta = 4^{\circ}$,
- values of the cornering angle oscillations amplitudes $A(\delta) = 1^{\circ}$ or $A(\delta) = 2^{\circ}$,
- the wheel rolling speeds $v=30$, $60$ or $90$ km/h.

Applied test method was based on a comparison between waveforms and values of the lateral reaction force $F_y$ obtained for the tire-road interaction model in two modes: with BRM or without BRM. So the tests results were described as obtained from the model with or without the tire relaxation.

4. Simulation tests results of the tire BRM
The example of the simulation research result is presented on the figure 3.

![Figure 3](image)

Figure 3. A single set of the test results obtained for the tire-road interaction model with and without the BRM (with and without the tire relaxation) ($v=30$ km/h, $\delta=4^{\circ}$, $A(\delta)=2^{\circ}$).

The figure 3 shows clear differences between waveforms of the lateral reaction force $F_y$ obtained for the model without and with the tire relaxation. For both tire model modes a decrease of the effective value of the lateral reaction force, after starting the cornering angle oscillation, is visible. However comparing the tire model with and without relaxation, there have been achieved:
- lower value of the lateral reaction force amplitude, decreasing with the oscillation frequency increase,
- time lag of the lateral reaction force oscillation,
- higher effective value of the lateral reaction force, increasing with the frequency increase.

Indicated differences between two tire model modes change in other wheel rolling conditions (figure 4). It means that the tire BRM reacts to changes in the wheel running conditions. Basing on presented simulation tests results, there have been calculated oscillation amplitudes $A(F_y)$ and effective values $F_{ye}$ of the lateral reaction force $F_y$. That values obtained for the different wheel running conditions have been presented due to the cornering angle oscillation frequency $f$ (Figure 5).

For the model without tire relaxation there have been obtained a permanent reduction of the lateral reaction force value $F_y$ as a result of the wheel cornering angle oscillation. That reduction does not depend on the speed and the oscillation frequency but is directly greater for the greater amplitude of the cornering angle oscillation. Whereas for model with the tire relaxation, the maximum reduction of the lateral reaction force $F_y$ effective value have been obtained from the beginning of the wheel cornering angle oscillation (at minimum frequency). However, that reaction force effective value clearly increases with the cornering angle oscillation frequency. Therefore, the tire model with the BRM behaves like getting stiffer with increasing frequency of the cornering angle oscillation.
Figure 4. Complete simulation research results of the tire model in two modes: without and with BRM model.
Observed reduction of the lateral reaction force effective value is greater for higher wheel rolling speeds and amplitude of cornering angle oscillation (figure 5c, d).

For the tire model with the BRM, there have been obtained decreasing amplitude of lateral reaction force $F_y$ oscillation with increasing frequency of the cornering angle oscillation (figure 5a, b.). That visible decrease of the lateral reaction force amplitude gets less with increasing wheel speed. Observed tire model behavior with BRM is quite similar to the real tires behavior in conditions of the cornering angle oscillation [2,6,9].

| a) Oscillation amplitudes of lateral reaction force $(A(\delta)=1^0)$ | b) Oscillation amplitudes of lateral reaction force $(A(\delta)=2^0)$ |
|---|---|
| ![Graph](image1.png) | ![Graph](image2.png) |

| c) Effective values of lateral reaction force $(A(\delta)=1^0)$ | d) Effective values of lateral reaction force $(A(\delta)=2^0)$ |
|---|---|
| ![Graph](image3.png) | ![Graph](image4.png) |

**Figure 5.** Influence of the wheel running conditions on the oscillation amplitude $A(F_y)$ and effective values $F_{ye}$ of the lateral reaction force $F_y$ calculated using the tire model in two modes: without and with the BRM model.

4. Conclusions

The simulation tests results show that including the tire relaxation model into a model of the tire-road interaction may significantly change the model properties in conditions of oscillatory changes in the wheel cornering angle values. However, it is necessary to achieve the required properties of the relaxation model by the appropriate mathematical description and data for modeling.

Prepared tire Basic Relaxation Model despite expanded mathematical description has many advantages. Important feature of the tire BRM is that it can be used as an addition module connected in series with the main part of the model established earlier for the tire-road interaction in steady-state
wheel running conditions, without need of its any modification. The tire BRM is specifically related by its coefficients with elementary properties of a tire. The tire BRM model features allow to achieve its very close behavior to behavior of real tire in transient states of the wheel side cornering. This gives the possibility of simulating the behavior of the tires in extreme wheel running conditions, which are difficult to achieve in laboratory but which may occur in the traffic conditions.

The tire BRM ensured the achievement of its expected behaviors, which are typical for the real tires. In this case of simulation research, by using the tire model with the BRM, special tire behavior in conditions of the wheel cornering angle oscillations have been achieved e.g.:

- expected reduction in the effective value of the lateral reaction force,
- clear relationship of the reduction in effective value of the lateral reaction force with the wheel speed, the amplitude and frequency of cornering angle oscillations,
- expected decrease of the amplitude of lateral reaction force due to frequency of the cornering angle oscillations and also due to the wheel running speed.

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