Holographic Quantum Quench

Sumit R. Das
Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506, USA
E-mail: das@pa.uky.edu

Abstract. We discuss recent results in the study the evolution of strongly coupled field theories in the presence of time dependent couplings using the holographic correspondence. The aim is to understand (i) thermalization and (ii) universal behavior when the coupling crosses a critical point. Our emphasis is on situations where a subset of bulk fields can be treated in a probe approximation. We consider two different setups. In the first, defect conformal field theories are described by probe branes in AdS space-times, and an initial vacuum state evolves due to a time dependent coupling in the probe sector. While a black hole formation is invisible in this approximation, we show that thermalization can nevertheless happen - this is signalled by formation of apparent horizons on the brane worldvolume. In the second setup, we consider a probe bulk scalar field in the background of a AdS black brane. In equilibrium, this system undergoes a critical phase transition at some temperature when the source for the dual operator vanishes. For a time dependent source which goes across this critical point, we show that a zero mode of the bulk field dominates the dynamics and leads to scaling behavior of the order parameter as a function of the rate of change.

1. Introduction
The problem of quantum quench, i.e. the response of a system to a time dependent coupling, has recently attracted a lot of attention in several areas of many-body physics, particularly because of progress in cold atom experiments [1],[2],[3]. This problem is interesting for at least two reasons. The first relates to the question of thermalization. Suppose we start with the ground state and then turn on a time dependent coupling which again approaches a constant at late times. Does the system evolve into some kind of steady state ? If so, is the state ”thermal” in any sense ? The second question deals with the situation where the quench takes place across a value of the parameter where there is an equilibrium critical point. In this case there is some evidence that the time evolution carries some universal features of the critical point. There are very few theoretical tools available to study such systems when they are strongly coupled. It is natural to explore if the AdS/CFT correspondence [4] - [7] is useful in this problem. In this contribution we will summarize recent results in this direction [8],[9].

In the AdS/CFT correspondence, couplings of the boundary field theory are boundary values of a bulk field. In the regime where supergravity is valid, the problem of quantum quench then
reduces to a classical problem with given initial and boundary conditions. This problem has
been studied when the time dependent coupling is the boundary metric or the gauge theory
coupling (i.e. the boundary value of the dilaton). Suppose this coupling is a constant in the
far past and future, and has a smooth time dependent profile at intermediate times. In the
bulk description this corresponds to a disturbance created on the boundary which propagates in
the bulk. Under suitable conditions, this leads to black hole formation in the bulk [10, 11, 12].
The correlators at future time would then be thermal with a temperature characterized by the
Hawking temperature. The time scale after which this happens depends on the nature of the
correlators, but turns out to be always smaller than what one would expect from a conformally
invariant system evolving to a thermal state. Thus, in this case thermalization of the field theory
is signalled by black hole formation. For homogeneous planar collapse (i.e. a space independent
coupling in Poincare patch), a black hole is always formed. In the case of homogeneous collapse
in global AdS, a black hole is formed when the rate of change is fast enough compared to the
scale set by the radius of the sphere on which the boundary theory lives.

In other situations, e.g. a slow variation of the coupling for global AdS, a black hole is not
formed so long as the supergravity approximation is valid. Rather, if the coupling becomes
weak at some time, the bulk string frame curvature grows large, leading to a breakdown of the
supergravity approximation - thus mimicking a space-like singularity [13]. For the case of a slow
variation of the coupling it turns out that the gauge theory remains well defined and may be
used to show that a smooth passage through this region of small coupling is possible without
formation of a large black hole. Related scenarios appear in [14] and [15].

One of our main aims is to study quantum quench across critical points. Many such critical
points can be studied in setups where a subset of the bulk fields can be treated in a probe
approximation. In this approximation, these probe fields provide the essential physics and
their backreaction to the background gravity can be ignored, typically suppressed by 1/N.
This motivates us to study the problem of quantum quench in situations where such a probe
approximation is valid.

We will first consider defect field theories which arise as dual descriptions of a set of probe
branes in the $AdS \times S^b$ bulk [16]. This approach has been used extensively to study flavor
physics, as well as models with possible applications to condensed matter systems. The nice
feature of this approach is that the boundary field theory is known, though they typically
have supersymmetry. In this case a quantum quench of couplings in this subsector becomes a
classical motion of these probe branes, with specified time dependent boundary conditions at
the $AdS$ boundary [8]. We investigate the question of thermalization in this context. Since the
background geometry is unchanged in this approximation any black hole which is formed due to
the quench is not visible. We will find that thermalization is nevertheless visible - this manifests
itself as the formation of an apparent horizon on the brane worldvolume.

In the second setup we consider a ”bottom-up” bulk theory of gravity with a neutral scalar
field [17], where one writes down a bulk theory and assume that there is some dual field theory
on the boundary. In this specific instance, he background is a $AdS_4$ charged black brane and
the coupling of the scalar is large, so that its backreaction to the geometry is small. When the
mass of the scalar lies in the range $-\frac{3}{4} < m^2 < -\frac{3}{2}$ there is always a critical temperature below
which the trivial solution is unstable. Equivalently, for a given temperature, whenever the mass is below a certain value, the trivial solution is unstable. In this regime there is a new nontrivial static, stable solution whose “non-normalizable” part vanishes. This means that in the dual theory there is a new phase where the expectation value of the operator dual to this scalar is non-zero, even in the absence of any source. For any nonzero temperature there is a continuous phase transition with mean field exponents at the critical mass. At zero temperature (i.e. when the background is an extremal brane) the transition persists, but is of the Berezinskii-Kosterlitz-Thouless type. This setup is similar to that of holographic superconductors [18, 19, 20] and has been proposed as models for antiferromagnetic transitions. We consider quench across this critical point by working at the critical mass, but turning on a time dependent source which crosses zero (i.e. the critical point) at some time [9]. We show that the dynamics of the bulk scalar is dominated by a zero mode of the radial operator in the critical region when the rate of change of the source is small, This leads to a Landau-Ginsburg type dynamics with dynamical critical exponent $z = 2$, and a resulting scaling behavior of the order parameter.

2. Probe Branes and Thermalization

Probe branes in the bulk of $AdS$ have been used to introduce flavor in the standard AdS/CFT correspondence. Consider for concreteness $AdS_5 \times S^5$ whose dual is $\mathcal{N} = 4$ super-Yang-Mills in $3 + 1$ dimensions with gauge group $SU(N_c)$. Let us introduce $N_f$ Dp branes which wrap a $AdS_m \times S^{p+1-m}$. Possible supersymmetric wrappings are summarized in Table (1).

| Brane | Wrapping | Dual Theory |
|-------|----------|-------------|
| D1    | $AdS_2$  | 0 + 1 dim   |
| D3    | $AdS_3 \times S^1$ | 1 + 1 dim |
| D5    | $AdS_4 \times S^2$ | 2 + 1 dim |
| D7    | $AdS_5 \times S^3$ | 3 + 1 dim |

The Dp branes give rise to new hypermultiplet fields. These live on the intersection of the Dp branes with the D3 branes which gave rise to the $AdS_5 \times S^5$ geometry. From the point of view of the $\mathcal{N} = 4$ theory the hypermultiplets live on a lower dimensional defect. In the strong coupling regime, the bulk theory is the original supergravity together with the action of branes coupled to it.

In the limit of $N_f \ll N_c$ the backreaction of the probe branes on the background $AdS_5 \times S^5$ geometry can be ignored and the entire bulk theory is given by the action of these branes moving in the fixed background geometry. We will take the brane action to be of the DBI type. In the dual theory this means we can consider the defect field theory by itself, and ignore the effect of hypermultiplet loops.
As is standard in the AdS/CFT correspondence, the boundary values of the DBI fields are identified with sources for the dual operators in the dual field theory. Consider for example the case of a D5 brane. Let us write the $AdS_5 \times S^5$ metric in the form
\[ ds^2 = (y^2 + r^2)[-dt^2 + dx_1^2 + dx_2^2 + dx_3^2] + \frac{1}{y^2 + r^2}[dr^2 + r^2d\Omega_2^2 + dy^2 + y^2d(\Omega_2')^2] \] (2.1)

The D5 brane is wrapped along $\xi^a = (t, r, \Omega_2, x_1, x_2)$. The fields in the DBI action are $y(\xi), \Omega_3(\xi), x_3(\xi)$, which are the transverse coordinates to the brane. The value of $y(r = \infty)$ is then the mass of the hypermultiplet fields coming from $(3, 5)$ open strings joining the D5 brane with the stack of $N_c$ three branes which produce the background geometry. Thus a time dependent boundary value of $y$ is a time dependent mass for the hypermultiplets.

Therefore a quantum quench in this dual theory may be implemented simply by providing a time dependent boundary condition for the DBI field. However the DBI fields are the transverse coordinates of the branes - so this corresponds to a motion of the edge of the brane. This disturbance sets up a wave along the brane and therefore corresponds to an excited state of the defect field theory. Our aim is to figure out the nature of this state at late times.

In the full theory, such a disturbance would lead to a deformation of the background geometry and possibly lead to black hole formation, which would appear as thermalization in the boundary field theory. We want to explore if any signature of thermalization remains in the probe approximation.

2.1. Rotating D1 branes

The essential physics is in fact apparent in the simplest example - D1 branes in $AdS_5 \times S^5$. For this purpose it is convenient to write the $AdS_5 \times S^5$ metric as
\[ ds^2 = 2drdv - f(r)dv^2 + r^2d\tilde{s}^2 + (d\theta^2 + \sin^2 \theta d\varphi^2 + \cos^2 \theta d\Omega_3^2) \] (2.2)

If we are using the Poincare patch, $f(r) = r^2$ and $d\tilde{s}^2$ is the flat metric on $R^3$, while in the global patch $f(r) = 1 + r^2$ and $d\tilde{s}^2$ is the round metric on $S^3$. We have used Eddington-Finkelstein coordinates in the $AdS_5$ part. The D1 brane is along $(r, v)$ and its action is given by the standard DBI action obtained from the induced metric. The dynamical fields on the brane are $\theta(r, v), \varphi(r, v), \Omega_3(r, v)$ and the coordinates contained in $d\tilde{s}^2$. It is clear from the symmetries that one can have a class of solutions of the form
\[ \varphi(r, v), \quad \theta = \frac{\pi}{2} \] (2.3)

with all the other coordinates held constant. The equations of motion which follow from the DBI action is best written by using the advanced EF coordinate $u = v - 2\int \frac{dr}{f(r)}$ as well as $v$
\[ \partial_u \partial_v \varphi + \frac{2}{L} \partial_v \varphi \partial_u \left( \frac{\partial_u \varphi \partial_v \varphi}{f(r)} \right) + \frac{2}{L} \partial_u \varphi \partial_v \left( \frac{\partial_u \varphi \partial_v \varphi}{f(r)} \right) = 0 , \] (2.4)

where
\[ L = 1 - \frac{4}{f(r)} \partial_u \varphi \partial_v \varphi . \] (2.5)
It is easy to see that any $\varphi$ which satisfies either $\partial_u \varphi = 0$ or $\partial_v \varphi = 0$ is a solution of (2.4). In particular, solutions which are functions of $v$ alone is the retarded effect of a boundary value of $\varphi$.

The induced metric produced by such a retarded solution is given by

$$ds^2_{\text{ind}} = -f(r) du dv + (\partial_v \varphi)^2 dv^2 = 2dardv - [f(r) - (\partial_v \varphi)^2] dv^2 .$$ (2.6)

This is a two-dimensional AdS Vaidya metric which has an apparent horizon at $f(r) = (\partial_v \varphi)^2$, provided this equation has a solution for real $r$. In the Poincare metric there is always a solution for real $r$, while in global coordinates, this is not guaranteed.

Depending on the profile of $\varphi(v)$, the apparent horizon may or may not develop into an event horizon. An example where it does is given by the profile

$$\varphi(v) = \varphi_0 (v + \frac{1}{k} \log \cosh(kv))$$ (2.7)

which leads to the following equation for the location of the apparent horizon for the Poincare patch

$$r = \varphi_0 (1 + \tanh(kv))$$ (2.8)

This asymptotes to an event horizon at $r = 2\varphi_0$. This profile represents a D1 brane which starts from rest and spins with an increasing spin, asymptoting to a constant rotation rate. The function (2.7) and the location of the apparent horizon is shown in Figure (1) and (2).

![Figure 1.](image1.png)  
**Figure 1.** The profile (2.7) as a function of $v$

![Figure 2.](image2.png)  
**Figure 2.** Location of the apparent horizon. $v$ as a function of $r$ from equation (2.8). At late time this becomes an event horizon.

For strings which eventually stop spinning, the apparent horizon does not develop into an event horizon, but recedes back to $r = 0$. For example if

$$\varphi(v) = \varphi_0 (1 + \tanh(kv))$$ (2.9)

the apparent horizon is located at

$$r = \frac{k\varphi_0}{\cosh^2(kv)}$$ (2.10)

The function (2.9) and the location of the apparent horizon is shown in Figure (3) and (4).
Figure 3. The profile (2.9) as a function of $v$

Figure 4. Location of the apparent horizon. $v$ as a function of $r$ from equation (2.10). The apparent horizon now recedes back to $r = 0$

In global $AdS$ the equation which determines the apparent horizon is given by $1+r^2 = (\partial_v \varphi)^2$ which does not always have a solution for real $r$. For example, a brane whose end point is uniformly rotating has $\varphi(v) = \omega v$ - this would lead to an event horizon only when $\omega > 1$.

The Poincare patch solutions represent injection of energy from the boundary, which flows into the Poincare horizon. In the global solutions, the energy flows from a point of the boundary to the antipodal point of the $S^3$.

Fluctuations of the brane around this classical solution will feel the effect of an apparent horizon on the worldsheet. Let us choose a static gauge where the worldsheet coordinates are identified with two of the space-time coordinates $\xi^a = x^a, \ a = 0, 1$. The transverse coordinates are $x^I, I = 2 \ldots 9$. The $AdS \times S^3$ metric can be then written as

$$ds^2 = g_{ab}(x^a, x^I)dx^a dx^b + G_{IJ}(x^a, x^K)dx^I dx^J.$$ \hspace{1cm} (2.11)

Expanding around a classical solution $x^I_0(x^a)$,

$$x^I(x^a) = x^I_0(x^a) + y^I(x^a),$$ \hspace{1cm} (2.12)

This leads to the following action for quadratic fluctuations

$$S_2 = \frac{T D_{D1}}{2} \int d^2 \xi \sqrt{-\gamma_0} \gamma_0^{ab} G_{IJ}(\xi^a, x^I_0) \partial_a y^I \partial_b y^J.$$ \hspace{1cm} (2.13)

where $\gamma_0^{ab}$ denotes the induced metric due to the background solution $x^I_0$. In particular, the fluctuations of $\varphi, \theta$ i.e. all fluctuations in $S^3$ directions are minimally coupled massless scalars on the worldsheet, while the fluctuations of the boundary gauge theory spatial directions $x^i, i = 1 \ldots 3$ have an additional factor of $r^2$ coming from the fact that $G_{ij} = r^2 \delta_{ij}$ along these directions.

It is well known that fields which live on a space-time with an apparent horizon behave approximately thermally if the apparent horizon lasts long enough [21]. While the standard derivation of Hawking radiation assumes the presence of an event horizon, the essential physics is the large redshift, which is present near an apparent horizon as well. In our case, profiles
like (2.7) lead to exact thermality at late times since the apparent horizon evolves into an event horizon. On the other hand profiles like (2.9) lead to an effective “time dependent temperature” in the dual theory, which of course makes sense when the time variation is slow enough.

The thermal nature of the state produced by time dependence becomes clear from a calculation of the fluctuation of the end-point of the string. In [22] it has been shown that the fluctuations of a string suspended from the horizon of a AdS black brane ended at a flavor D-brane near the boundary of AdS are dual to Brownian motion of the corresponding quark in the hot \(\mathcal{N} = 4\) gauge theory. In this case the bulk black brane metric induces a worldsheet metric which has a horizon. The fluctuations then reflect Hawking radiation from the worldsheet horizon.

In the D-brane solutions considered above, the bulk metric has no horizon. However due to the motion of the D-brane, the induced metric on the worldvolume can develop a horizon. Since the fluctuations of [22] comes purely from properties of the induced metric it is natural to expect that a similar phenomenon appears in our case.

The result for this calculation for fluctuations in the \(\varphi\) direction is

\[
\langle (\Delta y^\varphi(t - t'))^2 \rangle \sim \frac{\pi(t - t')^2}{12\beta^2}, \quad \pi(t - t') \ll \beta, \\
\langle (\Delta y^\varphi(t - t'))^2 \rangle \sim \frac{(t - t')^2}{2\beta} - \frac{1}{2\pi} \log[2\pi(t - t')/\beta], \quad \pi(t - t') \gg \beta,
\]

while for fluctuations in the \(\vec{x}\) direction are

\[
\langle (\Delta y^i(t - t'))^2 \rangle \sim \frac{(t - t')^2}{m\beta}, \quad t \ll m\beta^2, \\
\sim \beta|t - t'|, \quad t \gg m\beta^2.
\]

The rotating \(D1\) brane solution corresponds to a time dependent coupling in the \(N = 4\) theory coupled to hypermultiplets living on the zero dimensional defect. The \(D1-D3\) system is 1/4-BPS and the \(D1-D3\) open strings lead to the two complex scalars \(Q, \tilde{Q}\) of hypermultiplets which belong to the fundamental and anti-fundamental representations of the color SU\((N)\) gauge group. Let us express the three complex adjoint scalar fields in the \(N = 4\) super Yang-Mills by \((\Phi_1, \Phi_2, \Phi_3)\). These correspond to cartesian coordinates in the transverse \(C^3\) composed of \(r, \Omega_5\) where \(\Omega_5\) represents the 5-sphere. We choose \(\Phi_3\) such that its phase rotation describes the one in the \(\varphi\) direction and that \(\theta = \pi/2\) is equivalent to \(\Phi_1 = \Phi_2 = 0\). The time dependent coupling term which corresponds to a uniformly rotation \(D1\)-brane is given by

\[
\int dt \left[ Q \left[ \text{Im}(\Phi_3 e^{-i\omega t}) \right]^2 + \tilde{Q} \left[ \text{Im}(\Phi_3 e^{-i\omega t}) \right]^2 \right].
\]

The justification for this is given in [8]. For non-uniform rotation with a profile \(\varphi(v)\) the exponential factors are simply replaced by \(e^{\pm \varphi(t)}\).

Thus, from the boundary theory point of view we have a time dependent coupling - this leads to thermalization. Note that this is thermalization of only the hypermultiplet sector - the vector multiplet sector is unchanged in the lowest order of this approximation.
2.2. Higher dimensional probes
Thermalization in higher dimensional field theories can be also investigated by considering higher
dimensional branes in the bulk of $AdS \times S$. It is possible to construct uniformly rotating D7
and D5 branes using a combination of analytic and numerical methods. While these D5 and D7
solutions have been obtained earlier in [23], the implications to thermalization was not realized.
Quench-like solutions (i.e. solutions where the rotation vanishes in the asymptotic past and the
asymptotic future) can be obtained numerically as well [8].

Rotating D5 branes are dual to a time dependent mass of the hypermultiplets in the dual
2+1 dimensional defect field theory. Rotating D7 branes are dual to a time dependent phase of
the mass of fermions in the hypermultiplet, as well as a time dependent bosonic potential. The
essential physics is similar to the D1 brane described in the previous subsection, viz. an apparent
horizon is formed and the fluctuations respond in a thermal fashion.

An additional signature of dissipation appears when we consider probe D3 branes obtained
by performing T-duality on the D1 brane solution described above along $x_1$ and $x_2$ directions.
In this case the worldvolume is a 3 + 1 dimensional theory and the dual defect field theory
is 2 + 1 dimensions. It turns out that in this case one can turn on a background electric
field on the worldvolume of a uniformly rotating D3 brane. This means we are turning on a
chemical potential and a charge density for the corresponding global charge in the dual field
theory, in addition to a time dependent coupling. The fluctuations of the gauge field around
this background now react to the apparent horizon of the induced metric. This leads to an
electrical conductivity $\sigma(\nu)$, whose behavior as a function of the frequency $\nu$ is quite similar to
Drude theory at low frequencies. However the real part of $\sigma(\nu)$ approaches a constant at large
frequency, as is typical in a 2+1 dimensional critical theories. This is shown in Figures (5) and
(6)

![Figure 5](image1.png) ![Figure 6](image2.png)

**Figure 5.** Real part of $\sigma(\nu)$.

**Figure 6.** Imaginary part of $\sigma(\nu)$.

2.3. Time Dependent Chemical Potential
An interesting example of the process of thermalization due to formation of an apparent horizon
on the worldvolume concerns the thermalization of the meson sector of $N = 4$ Yang-Mills theory
due to a time dependent chemical potential [24]. As usual, quarks are introduced by placing a
set of D7 branes and a chemical potential corresponds to a worldvolume electric field. This is
made time dependent by coupling to a time dependent external current, i.e. by injecting quarks
from outside. As in the previous examples this results in the formation of an apparent horizon with a characteristic temperature. For other effects of this phenomenon see [25].

Finally the formation of apparent horizons in these examples is similar to acceleration horizons on worldvolumes discussed in [27] and related phenomena have been studied in [26].

3. Quench across a Holographic Critical Point
As we discussed above, quantum quench is particularly interesting when the time dependent coupling crosses a critical point. The probe approximation has turned out to be quite useful for studying holographic critical points. Such phase transitions are known to occur for various probe branes [28, 29, 30].

Another class of probe fields appear in discussions of holographic superconductors [18, 19, 20]. This setup consists of a charged scalar field in the presence of a charged black brane. When the gauge coupling is large, the backreaction of the scalar and the gauge field to the background geometry can be ignored. In this case, for a given mass of the scalar field there is a critical temperature below which the scalar condenses - this is interpreted as superfluidity in the boundary theory. The phase transition between the ordered and disordered phases is a critical point.

An even simpler setting consists of a neutral scalar field with a quartic coupling which is large enough to ensure that a probe approximation is reliable. As shown in [17] for suitable values of the parameters, this model displays a critical point of the type encountered in antiferromagnetic phase transitions. In this section we will describe a recent study of quantum quench in this model [9].

3.1. The equilibrium phase transition
The model of [17] has a neutral scalar field $\phi(t, r, \vec{x})$ in the background of a charged $AdS_4$ black brane. The lagrangian is given by

$$\mathcal{L} = \frac{1}{2\kappa^2\Lambda} \sqrt{-g} \left( -\frac{1}{2} (\partial\phi)^2 - \frac{1}{4} (\phi^2 + m^2)^2 - \frac{m^4}{4} \right)$$

(3.17)

The background metric is given by (in $R_{AdS} = 1$ units)

$$ds^2 = [-r^2 f(r) dt^2 + r^2 d\vec{x}^2] + \frac{dr^2}{r^2 f(r)}$$

(3.18)

where

$$f(r) = \left[ 1 + \frac{3\eta r_0^4}{r^4} - \frac{(1 + 3\eta) r_0^3}{r^3} \right] \quad 0 \leq \eta \leq 1$$

(3.19)

The associated Hawking temperature is then given by

$$T = \frac{3}{4\pi r_0} (1 - \eta)$$

(3.20)

In the following we will replace $r \to r r_0$. In the limit of large $\Lambda$ the field $\phi$ can be regarded as a probe field.
In [17] it was shown that when the mass lies in the range
\[ -\frac{9}{4} < m^2 < -\frac{3}{2} \] (3.21)
there is a critical phase transition at some value of \( T = T_c(m) \) when the source to the dual operator vanishes. Conversely, for a given \( T \) there is a value of \( m^2 = m^2_c \) where the theory is critical.

The upper limit in (3.21) is the BF bound for the near-horizon \( AdS_2 \) geometry which appears in the extremal (\( \eta = 0 \)) metric. (Note that the AdS scale for this infrared \( AdS_2 \) is given by \( 1/\sqrt{6} \) in our units). The lower bound is the BF bound for the asymptotic \( AdS_4 \). Field configurations which are translationally invariant in the \( \vec{x} \) directions satisfy the equations of motion
\[ \frac{1}{r^2} \left[ -\frac{1}{f(r)} \partial^2_r + \partial_r (r^4 f(r) \partial_r) \right] \phi - m^2 \phi - \phi^3 = 0 \] (3.22)

Near the \( AdS_4 \) boundary the asymptotic behavior of the solution to the linearized equation is of the form
\[ \phi(r) = J(t) r^{-\Delta} [1 + O(1/r^2)] + < \mathcal{O} > (t) r^{-\Delta} [1 + O(1/r^2)] \] (3.23)
where \( \Delta \) is given by
\[ \Delta_\pm = \frac{3}{2} \pm \sqrt{m^2 + \frac{9}{4}} \] (3.24)
In the range of masses of interest, both the solutions are normalizable, so that there is a choice of quantization. The standard quantization considers the coefficient \( J(t) \) as the source in the dual field theory and \( B(t) \) then gives the expectation value of the dual operator. In the alternative quantization the expectation and source change the role.

Consider first the linearized problem, ignoring the cubic term. By a standard change of coordinates to tortoise coordinates \( \rho \) and a field redefinition to \( \chi \),
\[ d\rho = \frac{dr}{r^2 f(r)} \quad \phi(r, t) = \frac{\chi(\rho, t)}{r} \] (3.25)
The horizon is then at \( \rho = \infty \) and the boundary is at \( \rho = 0 \). At the linearized level, the equation (3.22) becomes
\[ -\partial^2_\rho \chi = -\partial^2_\rho \chi + V_0(\rho) \chi \equiv \mathcal{P}_\rho \chi \] (3.26)
with
\[ V_0(\rho) = r^2 f(r) [(m^2 + 2) - \frac{6\eta}{r^4} + \frac{1 + 3\eta}{r^4}] \] (3.27)
where in \( V_0(\rho) \) we need to express \( r \) in terms of \( \rho \) using (3.25).

For solutions of the type \( \chi \sim e^{-i\omega t} \), equation (3.27) is a Schrödinger problem in a potential \( V(\rho) \). The potential goes to zero at the horizon \( \rho = \infty \) and behaves as \( \frac{(m^2+2)}{\rho^2} \) near the boundary \( \rho = 0 \). Note that for a brane background at any finite temperature, \( f(r) \sim (r-1) \) near the boundary. This can be done as long as \( \Delta > 0 \) or \( m^2 < 0 \).
horizon, while $\rho \sim -\log(r-1)$ so that $V_0 \sim e^{-\rho}$ as we approach the horizon. In contrast, for the extremal background $f(r) \sim (r-1)^2$ while $\rho \sim 1/(r-1)$ so that $V_0 \sim 1/\rho^2$. This makes the analysis for the extremal background rather subtle. We will work with the non-extremal case.

In [17] it was shown that when $m^2$ is below a critical value, $m_c^2$ there are bound states of this Schrodinger problem, showing that the trivial $\phi = 0$ is unstable. At $m^2 = m_c^2$ a zero energy bound state appears, which vanishes in an appropriate fashion at the boundary and is in addition purely ingoing at the horizon. In the complex frequency plane some quasinormal mode(s) hit the origin at $m^2 = m_c^2$. This critical mass is $m_c^2 = -\frac{3}{2}$ when $\eta = 1$ and decreases with decreasing $\eta$ or increasing temperature.

For $m^2 < m_c^2$ there is a stable nontrivial static solution $\phi_0(r)$ of the full nonlinear equation of motion with the condition that $J = 0$. This means that in the conventional quantization, the expectation value of the dual operator is nonzero even in the absence of a source, i.e. the field condenses. The critical point is at $m^2 = m_c^2$ and $J = 0$. Similarly there is a nontrivial solution with $B = 0$ which means that there is a condensate in the alternative quantization as well.

The critical point has mean field exponents at any finite $T$. If the operator dual to the field $\phi$ is $\mathcal{O}$ and the source is $J$ then an analysis identical to that presented in [17] leads to (for $m^2 - m_c^2 \to 0^+$)

$$< \mathcal{O} >_{J=0} \sim (m^2 - m_c^2)^{1/2} \quad \frac{d < \mathcal{O} >}{dJ} \bigg|_{J=0} \sim (m^2 - m_c^2)^{-1} \quad < \mathcal{O} >_{m=m_c} \sim J^{1/3}$$

Exactly at zero temperature the phase transition is of BKT type and the order parameter depends exponentially

$$< \mathcal{O} >_{J=0} \sim \exp \left[ -\frac{\pi \sqrt{6}}{2 \sqrt{m_c^2 - m^2}} \right]$$

### 3.2. Quenching across the Critical Point

In the following, we will study quench across this critical point by considering a time dependent source $J(t)$ which asymptotes to constant values at early and late times and crosses zero at some time, e.g.

$$J(t) = J_0 \tanh(\nu t)$$

In the conventional quantization this means that in the dual boundary field theory, we have a source coupling to the operator dual to $\phi$. For any static $J \neq 0$ we of course have a nontrivial $\phi(r)$ and hence a nonzero $< \mathcal{O} >$. Our first aim is to get some insight into the time dependence of $< \mathcal{O}(t) >$ when we have a nontrivial $J(t)$.

#### 3.2.1. Breakdown of Adiabaticity

If the time dependence is slow enough one would expect that far away from the critical point the dynamics is adiabatic, while near the critical point adiabaticity should break down. It is instructive to examine the way this happens.

It is well known that to study low frequency modes in the background of a black brane it is convenient to use ingoing Eddington-Finkelstein coordinates,

$$u = t - \rho, \quad \rho$$
where \( \rho \) is defined in (3.25). In terms of these coordinates the equation of motion 3.22) becomes

\[
-2\partial_u \partial_\rho \chi = -\partial_\rho^2 \chi + V(\rho, \chi).
\]  

(3.32)

where

\[
V(\rho, \chi) = V_0(\rho)\chi + f(r)\chi^3.
\]  

(3.33)

This equation has to be solved with the boundary condition that the field is regular at the horizon, which at the linearized level is equivalent to requiring that the waves are purely ingoing at the horizon [31, 32]. We need to solve (3.32) with the condition

\[
\chi(u, \rho) \to \rho^{-1+\Delta_+} J(u) \quad \text{as} \quad \rho \to 0
\]  

(3.34)

where \( \Delta_\pm \) are defined in (3.24). To perform the adiabatic expansion, let us decompose the field \( \chi(\rho, u) \) as

\[
\chi(\rho, u) = \chi_I(\rho, u) + \chi_s(\rho, u)
\]  

(3.35)

Where \( \chi_I(\rho, u) = J(u)\rho^{-1+\Delta_-} \) and \( \chi_s(\rho, u) \sim \rho^{-1+\Delta_+} \) as \( \rho \to 0 \). If \( \chi_I(\rho, u) = \chi_I(\rho) \) is time independent there is a static solution \( \chi_s(\rho, u) = \chi_0(\rho) \). In the presence of a slowly varying source the adiabatic expansion of the solution for \( \chi_s \) is of the form

\[
\chi_s(\rho, u) = \chi_0(\rho, J(u)) + \epsilon \chi_1(\rho, u) + \cdots.
\]  

(3.36)

Here \( \epsilon \) is an adiabaticity parameter which keeps track of the adiabatic expansion. If we scale \( u \to u/\epsilon \), each \( u \) derivative is of order \( O(\epsilon) \). The idea then is to insert (3.36) into the equations of motion and obtain equations for \( \chi_1, \chi_2, \cdots \) order by order in \( \epsilon \). To the lowest order one gets

\[
\mathcal{D}_\rho^{(1)} \chi_1 = \{[-\partial_\rho^2 + V_0(\rho)] + f(r)(3\chi_0^2 + 6\chi_I\chi_0 + 3\chi_0^2)\} \chi_1 = -2\partial_u \partial_\rho \chi_I - 2\partial_u \partial_\rho \chi_0
\]  

(3.37)

The solution to this equation is

\[
\chi_1 = \int dp' G(\rho, \rho') \partial_u \partial_\rho' (\chi_0 + \chi_I)(\rho').
\]  

(3.38)

where \( G(\rho, \rho') \) is the Green’s function of the operator \( \mathcal{D}_\rho^{(1)} \) with the boundary conditions \( G(0, \rho) = 0 \) and \( G(\infty, \rho) \) is regular :

\[
G^{(1)}(\rho, \rho') = \begin{cases} 
\frac{1}{W(\xi_1, \xi_2)} \tilde{\xi}_1(\rho')\tilde{\xi}_2(\rho), & \rho < \rho' \\
\frac{1}{W(\xi_1, \xi_2)} \tilde{\xi}_2(\rho')\tilde{\xi}_1(\rho), & \rho > \rho',
\end{cases}
\]  

(3.39)

where \( \tilde{\xi}_1 \) and \( \tilde{\xi}_2 \) are solutions of homogeneous part of eqn (3.38) satisfying appropriate boundary condition at the horizon \( \rho = \infty \) and the boundary \( \rho = 0 \) respectively, and \( W(\xi_1, \xi_2) \) is the Wronskian which is independent of \( \rho \) in this case. We have normalized \( \tilde{\xi}_1(\rho) \) and \( \tilde{\xi}_2(\rho) \) in such a fashion that \( \tilde{\xi}_1 = 1 \) at the horizon and \( \tilde{\xi}_2 \to \rho^{-1+\Delta_-} \) near the boundary. Regularity of the functions \( \chi_I \) and \( \chi_0 \) mean that \( \partial_\rho \chi_I, \partial_\rho \chi_0 \) are finite at the horizon. Since \( (r - 1) \sim e^{-\rho} \), this implies that \( \partial_\rho(\chi_I + \chi_0) \sim \exp(-\rho) \). This ensures that the integral in (3.38) is finite even though
the Green’s function approaches a constant in the region near the horizon \( \rho' \to \infty \). Furthermore, near the horizon \( \xi_2 \) can be expressed as a linear combination of a regular and irregular solution, i.e. \( \xi_2(\rho \to \infty) = a\rho + b \). This implies that \( W(\xi_1, \xi_2) = a \). Thus \( \chi_1(u, \rho) \) is finite so long as \( a \) is finite.

At the critical point, \( J \) becomes small. Then \( \chi_0 \) and \( \chi_1 \) in the left hand side of (3.37) vanish, and the operator is identical to the operator acting on the linearized small fluctuations at \( m^2 = m_c^2 \) around the trivial solution \( \chi_0 = 0 \), i.e. the operator \( \mathcal{P}_\rho \) which appears on the right hand side of (3.26). We know that this operator has a zero mode which is regular at the horizon and vanishes as \( \rho^{-1+\Delta_-} \) at the boundary \( \rho = 0 \). This means that at this point \( a = 0 \). Therefore, the first adiabatic correction diverges. For small \( J(u) \), the leading departure from the critical operator comes from the term which is proportional to \( \chi_0^2 \sim J^{2/3} \). Thus we can use perturbation theory in \( J \) to estimate \( a \propto J^{2/3} \). As argued before, \( \chi_0 \sim (-J)^{1/3} \), while \( \chi_1 \sim J \). Hence the leading divergence in \( \chi_1 \) can be estimated as \( \chi_1(\rho, u) \sim J^{-4/3} \). Adiabaticity breaks down when

\[
\chi_1(\rho, u) \sim \chi_0 \Rightarrow \dot{J} \sim J^{5/3}
\]

In particular for profiles of \( J(u) \) where \( J(u) \sim vu \) near the critical point at \( J = 0 \), adiabaticity breakdown occurs at

\[
u \sim u^{-2/5}
\]

### 3.3. Quench in a LG Model

The scaling behavior found above is identical to that in a Landau-Ginsburg dynamics with dynamical critical exponent \( z = 2 \). The dynamics of an spatially homogeneous order parameter \( \varphi \) is given by

\[
\frac{d\varphi}{dt} + m^2 \varphi + \varphi^3 + J(t) = 0
\]

The equilibrium critical point is at \( m = J = 0 \). For \( m^2 = 0 \) the equilibrium value of the order parameter is

\[
\varphi_0(J) = (-J)^{1/3}
\]

As usual an adiabatic expansion is of the form

\[
\varphi(t) = \varphi_0(J(t)) + \epsilon \varphi_1(t) + \cdots
\]

and to lowest order

\[
\varphi_1 = \frac{1}{2\varphi_0^2} J \frac{\partial \varphi_0}{\partial J}
\]

and adiabaticity breaks down when \( \varphi_1 \sim \varphi_0 \) which becomes the condition

\[
J \frac{\partial \varphi_0}{\partial J} \sim J^{1/3} \Rightarrow \dot{J} \sim J^{5/3}
\]

exactly as in our system.

When adiabaticity breaks down our system enters a scaling region. Suppose the function \( J(t) \) behaves linearly with time in the critical region. Then it is straightforward to see from (3.42) that in this region the solution is of the form

\[
\varphi(t, v) = v^{1/5} \varphi(tv^{2/5}, 1)
\]
This means, in particular, that the time at which the order parameter hits zero scales as $v^{-2/5}$ while the value of the order parameter at $t = 0$ scales as $v^{1/5}$. A numerical solution of the equation (3.42) with adiabatic initial conditions is consistent with this scaling, as shown in Figure (7).

![Figure 7](image-url)

**Figure 7.** The scaled order parameter as a function of scaled time for a $J(t) = \tanh(vt)$ at $m^2 = 0$ with $v = 10^{-0.5}, 10^{-1}, 10^{-1.5}, 10^{-2}$ (from the bottom on the left). The adiabatic solution (dashed) is also shown as a comparison.

Note that the order parameter hits zero *later* than the location of the equilibrium critical point. This is a manifestation of the phenomenon of raising the critical temperature when the temperature is time dependent [33] which has been holographically realized in [34].

We will now argue that the behavior of our holographic system in the critical region is fairly well described by such a LG dynamics.

### 3.4. Scaling in the Holographic Model

We now study the dynamics of the bulk field in the critical region in the presence of a linear quench $J(u) = vu$ for small $v$. To do this, first substitute (3.35) in the equation (3.32) and rescale

$$
\chi_s \rightarrow v^{1/5} \tilde{\chi}_s, \ u \rightarrow v^{-2/5} \tilde{u}
$$

The equation (3.32) then becomes

$$
[-\partial_\rho^2 + V_0(\rho)]\tilde{\chi}_s + v^{2/5} [f(r)(\tilde{\chi}_s)^3 + \tilde{u}[-\partial_\rho^2 + V_0(\rho)]\tilde{\chi}_l + 2\partial_\rho \partial_\rho \tilde{\chi}_s] + \cdots = 0
$$

(3.49)

The ellipsis denote terms which contains higher powers of $v$.

Let us expand the sub-leading part of the scalar field in terms of eigenfunctions of the operator $\mathcal{P}_\rho$ (defined in equation (3.26) at the critical point,

$$
\tilde{\chi}_s(\rho, u) = \int \tilde{a}_k(u)\chi_k(\rho)dk
$$

(3.50)
The $\chi_k$ satisfy

$$P_\rho^v\chi_k = [-\partial_\rho^2 + V_0^v(\rho)]\chi_k = k^2\chi_k$$  (3.51)

where $V_0^v$ denotes the potential in (3.27) at $m^2 = m_c^2$. The eigenfunctions $\chi_k(\rho)$ are delta function normalized and obey the condition

$$\lim_{\rho \to 0}[\rho^{1-\Delta} \chi_k(\rho)] = 0$$  (3.52)

In terms of the eigen-coefficients $a_k(u)$ the equation (3.49) becomes,

$$k^2\tilde{a}_k + v^{2}\left(\tilde{u}\mathcal{J}_k - \int b_{kk'}\partial_\rho\tilde{a}_{k'}dk' - \int \tilde{a}_{k'}\tilde{a}_{k''}C_{k,k',k'',k'''\rho}dk'dk''dk'''\right) = 0$$  (3.53)

where

$$\mathcal{J}_k = \int \chi_k(\rho)[-\partial_\rho^2 + V_0(\rho)]\chi_k d\rho$$

$$b_{kk'} = \int d\rho \chi_k\partial_\rho\chi_k$$

$$C_{k,k',k'',k'''} = \int d\rho \chi_k\chi_{k'}\chi_{k''}\chi_{k'''}f(r).$$  (3.54)

The equation (3.53) suggests that there is a solution in a perturbation expansion of powers of $v^2$,

$$\tilde{a}_k(\tilde{u}) = \delta(k)\tilde{\zeta}_0(\tilde{u}) + v^{2}\tilde{\eta}_k(\tilde{u}) + \cdots,$$  (3.55)

The equations then become

$$\tilde{u}\mathcal{J}_0 - b_{k0}\frac{d}{d\tilde{u}}\tilde{\zeta}_0(\tilde{u}) - C_{0000}\tilde{\zeta}_0(\tilde{u})^3 = 0$$

$$\tilde{\eta}_k(\tilde{u}) = -\frac{1}{k^2}\left(\tilde{u}\mathcal{J}_k - b_{k0}\frac{d}{d\tilde{u}}\tilde{\zeta}_0(\tilde{u}) - C_{k000}\tilde{\zeta}_0(\tilde{u})^3\right)$$  (3.56)

It is useful to rewrite the second equation by subtracting the first from it,

$$\tilde{\eta}_k(\tilde{u}) = -\frac{1}{k^2}\left(\tilde{u}(\mathcal{J}_k - \mathcal{J}_0) - (b_{k0} - b_{k0})\frac{d}{d\tilde{u}}\tilde{\zeta}_0(\tilde{u}) - (C_{k000} - C_{0000})\tilde{\zeta}_0(\tilde{u})^3\right)$$  (3.57)

The expansion in powers of $v^{2/5}$ would be valid if $\eta_k(\tilde{u})$ remains finite. Since $k$ is a continuous parameter starting from zero, this appears unlikely.

Indeed, for generic potential $V_0$ the numerator on the right hand side of (3.57) behaves as $k$ for small $k$, so that $\eta_k$ indeed diverges at $k = 0$. However exactly at the critical point, the small $k$ behaviour changes to $k^2$ so that $\eta_k$ remains finite and the expansion in $v^{2/5}$ remains valid.

Consider the eigenvalue problem

$$[-\partial_\rho^2 + V_0(\rho)]\chi_k = k^2\chi_k$$  (3.58)

As discussed above the potential $V_0(\rho) \to -e^{-\rho}$ as $\rho \to \infty$. Writing $V_0(\rho) = -V_1e^{-\rho}$ in this region, the real solution is

$$\chi_k^{(1)}(\rho) = B_kJ_{2k}(2e^{-\rho/2}\sqrt{V_1}) + c.c.$$  (3.59)
For \( \rho \to \infty \) this becomes

\[
\chi_k^{(1)}(\rho) = \frac{B_k}{\Gamma(2ik + 1)}(V_1 e^{-\rho})^{ik} + \text{c.c.}
\]

which should become a sinusoidal solution with a \( k \)-independent coefficient. For this to happen we must have

\[
B_k = C \Gamma(2ik + 1) e^{i\alpha(k)}
\]

where \( C \) is real and \( \alpha(k) \) is a real function of \( k \).

In the region \( V_1 e^{-\rho} \gg k^2 \) one can treat the term proportional to \( k^2 \) perturbatively, leading to a solution of the form

\[
\chi_k^{(2)}(\rho) = A(k)[\chi_0(\rho) + O(k^2)]
\]

where \( \chi_0(\rho) \) is the solution of the equation for \( k = 0 \) and \( A(k) \) is some constant to be determined shortly.

For small enough \( k^2 \) there is a matching region which overlaps the above two regions, \( 1 \ll \rho \ll \log(V_1/k^2) \). In this region, the function \( \chi_0(\rho) \) in (3.62) has to be of the form

\[
\chi_0(\rho) = a \rho + b
\]

with some constants \( a \) and \( b \). On the other hand we can also use the form (3.62) for \( \chi_k^{(1)} \). Matching these two solutions leads to the following conditions to lowest order in \( k \)

\[
B_k + B_k^* = A(k)b
\]

\[
2ik(B_k - B_k^*) = A(k)a
\]

Generically \( a \neq 0 \), so that \( A(k) \sim k \). However exactly at the critical point the \( k = 0 \) mode is regular at the horizon, i.e. \( a = 0 \). In this case \( A(k) \sim O(1) + O(k^2) \).

Consider now integrals of a function \( K(\rho) \) with compact support and define

\[
K(k) = \int d\rho \chi_k(\rho) K(\rho)
\]

The integrals which appear in the definitions of \( J_k, b_{kk'}, C_{kk',k''k'''} \) in (3.54) are of this form. The integral over \( \rho \) is dominated by the small \( \rho \) region, where we can replace \( \chi_k \to \chi_k^{(1)} \) in (3.62). This immediately leads to the following behavior

\[
K(k) \sim O(k) \quad \text{(generic)}
\]

\[
K(k) \sim O(1) + O(k^2) \quad \text{(critical)}
\]

Thus the quantities like \( K(k) - K(0) \) behave as \( O(k) \) generically, while at the critical point \( K(k) - K(0) \sim O(k^2) \). The equation (3.57) then shows that \( \tilde{\eta}_k(\tilde{u}) \) are finite for the critical theory, and an expansion in \( v^{2/5} \) exists.

In this expansion the solution for \( \chi_s \) is

\[
\chi_s(\rho, u) \approx v^{1/2} \xi_0(v^2 u)\chi_0(\rho) + v^{2/5} \int \tilde{\eta}_k(v^2 u)\chi_k(\rho)dk + \cdots
\]
Thus the dynamics for small $v$ is dominated by the zero mode in the critical region. The equation for $\tilde{\xi}_0$ is, however, exactly the same as the equation for the order parameter $\phi$ in the LG model in the previous subsection. Thus in this region the system behaves as one with dynamical critical exponent $z = 2$. Therefore we conclude that to leading order in small $v$, we have

$$\langle \mathcal{O} \rangle (u) \sim \lim_{\rho \to 0} [\rho^{1-\Delta_s} \chi_s(\rho, u)] \sim v^{1/5}$$  \hspace{1cm} (3.68)

and the time scale behaves as $v^{-2/5}$.

The fact that the dynamics in the critical region is governed by an equation with a first order time derivative is made manifest in our treatment using Eddington-Finkelstein coordinates. The whole analysis can be of course performed in principle in the $(t, r)$ coordinates. However, we suspect that in this case one has to exercise extreme care, just as one had to extract out a leading horizon behavior in the linearized problem of fields in a black brane background [35].

Note that beyond the critical region, the expansion in powers of $v^{2/5}$ is no longer valid. Now all the modes are important, and integrating out higher modes can give rise to higher time derivatives in the effective equation for $\langle \mathcal{O} \rangle$.

3.5. Mass quench

An analysis similar to the above can be carried out when the quenching is performed by making the mass parameter of the bulk theory a function of the retarded time $u$, keeping $J(u) = 0$. This does not have a direct interpretation in the boundary field theory. However, as explained in [17], the field $\phi$ can acquire a mass because of a coupling to some other field $\phi'$. A time dependent boundary value of the field $\phi'$ can then lead to a time dependent mass. When $m^2(u) \sim m_c^2 + vu$ near the critical point, we now get a behavior $\langle \mathcal{O} \rangle \sim v^{1/4}$.

3.6. Numerical Results

We have performed some preliminary numerical work for the case of a mass quench. Our results clearly display the propagation of disturbances towards the horizon along a light cone which is shown in Figure(8), and a decay of the order parameter in a fashion similar to the $z = 2$ LG dynamics, which is shown in Figure (9). However our results are not accurate enough to verify the scaling behavior found above. The late time decay should be governed by the quasinormal modes [32, 37].

4. Outlook

We have shown that holographic methods are useful in providing insight into various questions related to quantum quench in strongly coupled field theories which have a gravity dual. Interestingly, this is possible in the probe approximation, which is typically much earlier to study.

So far we have been able to study in some detail quench dynamics near holographic critical points which are described by mean field exponents. Not surprisingly, we found scaling behavior characteristic of Landau-Ginsburg models with $z = 2$. This value of the dynamical critical exponent is consistent with the results of [36].
Figure 8. A typical plot of $\phi(x,t)$ with $x = 1/r$.

Figure 9. The order parameter $<\mathcal{O}>_c(t)$ in boundary theory.

It is important to perform our quench analysis for zero temperature, where the equilibrium transition is of the BKT type [17], similar to brane models of zero temperature chiral symmetry breaking transition in [30]. This case is rather subtle, but can be studied using similar methods. We expect that in this case we will have a $z = 1$ theory dominating the critical region. This would provide results for quench dynamics which are not easily obtainable by other methods. Finally, a more extensive numerical investigation should throw light on the question of thermalization at late times, after the system has crossed the critical region.

5. Acknowledgements

I would like to thank my collaborators Pallab Basu, Tatsuma Nishioka and Tadashi Takayanagi for very enjoyable collaborations and many insightful discussions. I would also like to thank Karl Landsteiner, Satya Majumdar, Gautam Mandal, Shiraz Minwalla, Takeshi Morita, Ganpathy Murthy, Omid Saremi, Alfred Shapere, Sandip Trivedi and especially Kristan Jensen and Krishnendu Sengupta for discussions. S.R.D. would like to thank Institut de Fisica Teorica at Madrid, Tata Institute of Fundamental Research at Mumbai and Indian Association for the Cultivation of Science at Kolkata for hospitality during the final stages of this work. This work is partially supported by National Science Foundation grants PHY-0970069 and PHY-0855614.

References

[1] For reviews and references see S. Mondal, D. Sen and K. Sengupta, arXiv:0908.2922; J. Dziarmaga, arXiv:0912.4034; A. Polkovnikov, K. Sengupta, A. Silva and M. Vengalattore, arXiv:1007.5331.

[2] P. Calabrese and J. L. Cardy, J. Stat. Mech. 0504 (2005) P010 [arXiv:cond-mat/0503393]; P. Calabrese and J. L. Cardy, Phys. Rev. Lett. 96 (2006) 130601 [arXiv:cond-mat/0601225]

[3] P. Calabrese and J. Cardy, [arXiv:0704.1880 [cond-mat.stat-mech]]; S. Sotiriadis and J. Cardy, J. Stat. Mech. (2008) P11003, [arXiv:0808.0116 [cond-mat.stat-mech]]; S. Sotiriadis, P. Calabrese and J. Cardy, EPL 87 (2009) 20002, [arXiv:0903.0895 [cond-mat.stat-mech]]; S. Sotiriadis and J. Cardy, arXiv:1002.0157 [quant-ph].

[4] J. M. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231 [Int. J. Theor. Phys. 38 (1999) 1113] [arXiv:hep-th/9711200];

[5] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B 428, 105 (1998) [arXiv:hep-th/9802109].

[6] E. Witten, Adv. Theor. Math. Phys. 2, 253 (1998) [arXiv:hep-th/9802150].

[7] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, Phys. Rept. 323, 183 (2000) [arXiv:hep-th/9905111].

[8] S. R. Das, T. Nishioka, T. Takayanagi, JHEP 1007, 071 (2010). [arXiv:1005.3348 [hep-th]].

[9] P. Basu, S. R. Das, [arXiv:1109.3909 [hep-th]].

[10] R. A. Janik and R. B. Peschanski, Phys. Rev. D 74, 046007 (2006) [arXiv:hep-th/0606149]; R. A. Janik, Phys. Rev. Lett. 98, 022302 (2007) [arXiv:hep-th/0610144]; P. M. Chesler and L. G. Yaffe, Phys. Rev. Lett.
For a review and references to the original literature see e.g. J. Erdmenger, N. Evans, I. Kirsch, E. Threlfall, Eur. Phys. J. A35, 131-133 (2008). [arXiv:0711.4467 [hep-th]].

For a review and references to the original literature see e.g. J. Erdmenger, N. Evans, I. Kirsch, E. Threlfall, Eur. Phys. J. A35, 131-133 (2008). [arXiv:0711.4467 [hep-th]].

T. Hertog and G. T. Horowitz, JHEP 0407, 073 (2004) [arXiv:hep-th/0406134]; T. Hertog and G. T. Horowitz, JHEP 0504, 005 (2005) [arXiv:hep-th/0503071]; N. Turok, B. Craps and T. Hertog, arXiv:0711.1824 [hep-th]; B. Craps, T. Hertog and N. Turok, arXiv:0712.4180 [hep-th].

J. McGreevy and E. Silverstein, JHEP 0508, 090 (2005) [arXiv:hep-th/0506130]; E. Silverstein, Phys. Rev. D 73, 086004 (2006) [arXiv:hep-th/0510044]; G. Horowitz, A. Lawrence and E. Silverstein, JHEP 0907, 037 (2009) [arXiv:0904.3922 [hep-th]].

S. A. Hartnoll, Class. Quant. Grav. 26 (2009) 224002 [arXiv:0903.3246 [hep-th]]; C. P. Herzog, J. Phys. A 42, 343001 (2009) [arXiv:0904.1975 [hep-th]].

See e.g. M. Visser, “Essential and inessential features of Hawking radiation,” Int. J. Mod. Phys. D 12, 649 (2003) [arXiv:hep-th/0106111].

J. de Boer, V. E. Hubeny, M. Rangamani and M. Shigemori, “Brownian motion in AdS/CFT,” JHEP 0907, 094 (2009) [arXiv:0812.5112 [hep-th]]; A. N. Atmaja, J. de Boer and M. Shigemori, “Holographic Brownian Motion and Time Scales in Strongly Coupled Plasmas,” arXiv:1002.2429 [hep-th]; D. T. Son and D. Teaney, “Thermal Noise and Stochastic Strings in AdS/CFT,” arXiv:0901.2388 [hep-th].

A. O'Bannon, JHEP 0901, 074 (2009). [arXiv:0811.0198 [hep-th]]; N. Evans, E. Threlfall, [arXiv:0807.3679 [hep-th]]; N. Evans, E. Threlfall, Phys. Rev. D79, 066008 (2009). [arXiv:0812.3273 [hep-th]].

K. Hashimoto, N. Iizuka, T. Oka, arXiv:1012.4463 [hep-th].

K.-Y. Kim, J. P. Shock, J. Torrio, JHEP 1106, 017 (2011). [arXiv:1103.4581 [hep-th]]; S. Prem Kumar, Phys. Rev. D84, 026003 (2011). [arXiv:1104.1405 [hep-th]]; S. Janiszewski, A. Karch, arXiv:1106.4010 [hep-th]; C. Hoyos, T. Nishioka, A. O'Bannon, arXiv:1106.4030 [hep-th].

U. Gursoy, E. Kiritsis, L. Mazzanti, F. Nitti, JHEP 1012, 088 (2010). [arXiv:1006.3261 [hep-th]].

J. G. Russo, P. K. Townsend, Class. Quant. Grav. 25, 175017 (2008). [arXiv:0805.3488 [hep-th]]; M. Chernicoff and A. Guijosa, JHEP 0806 (2008) 005 [arXiv:0803.3070 [hep-th]]; A. Paredes, K. Peeters and M. Zaanen, JHEP 0904, 015 (2009) [arXiv:0812.0981 [hep-th]]; C. Athanasiou, P. M. Chesler, H. Liu, D. Nickel and K. Rajagopal, arXiv:1001.3880 [hep-th]; E. Caceres, M. Chernicoff, A. Guijosa and J. F. Pedraza, arXiv:1003.5332 [hep-th]; T. Hirata, S. Mukohyama and T. Takayanagi, JHEP 0805 (2008) 089 [arXiv:0804.1176 [hep-th]]; T. Hirayama, P. W. Kao, S. Kawamoto and F. L. Lin, arXiv:1001.1289 [hep-th].

D. Mateos, R. C. Myers and R. M. Thompson, “Holographic phase transitions with fundamental matter,” Phys. Rev. Lett. 97, 091601 (2006) [arXiv:hep-th/0605046]; S. Kobayashi, D. Mateos, S. Matsuura, R. C. Myers and R. M. Thompson, “Holographic phase transitions at finite baryon density,” JHEP 0702, 016 (2007) [arXiv:hep-th/0611099]; D. Mateos, S. Matsuura, R. C. Myers and R. M. Thompson, “Holographic phase transitions at finite chemical potential,” JHEP 0711, 085 (2007) [arXiv:0709.1225 [hep-th]].

K. Jensen, A. Karch, E. G. Thompson, JHEP 1005, 015 (2010). [arXiv:1002.2447 [hep-th]].

K. Jensen, A. Karch, D. T. Son, E. G. Thompson, Phys. Rev. Lett. 105, 041601 (2010) [arXiv:1002.3159 [hep-th]]; K. Jensen, Phys. Rev. D82, 046005 (2010). [arXiv:1006.3066 [hep-th]]; N. Evans, K. Jensen, K.-Y. Kim, Phys. Rev. D82, 105012 (2010). [arXiv:1008.1889 [hep-th]].

S. Bhattacharyya, V. E. Hubeny, S. Minwalla, M. Rangamani, JHEP 0802, 045 (2008). [arXiv:0712.2456 [hep-th]].
S. Bhattacharyya, R. Loganayagam, S. Minwalla, S. Nampuri, S. P. Trivedi, S. R. Wadia, JHEP 0902, 018 (2009). [arXiv:0806.0006 [hep-th]].

32 G. T. Horowitz, V. E. Hubeny, Phys. Rev. D62, 024027 (2000). [hep-th/9909056].

33 G.M. Eliashberg, J. E. T. P. Letters 11 (1970) 114.

34 N. Bao, X. Dong, E. Silverstein, G. Torroba, [arXiv:1104.4098 [hep-th]].

35 G. Policastro, D. T. Son, A. O. Starinets, JHEP 0209, 043 (2002). [arXiv:hep-th/0205052 [hep-th]]; G. Policastro, D. T. Son, A. O. Starinets, JHEP 0212, 054 (2002). [hep-th/0210220].

36 K. Maeda, M. Natsuume, T. Okamura, Phys. Rev. D79, 126004 (2009). [arXiv:0904.1914 [hep-th]].

37 R. A. Konoplya, Phys. Rev. D66, 084007 (2002). [gr-qc/0207028]. I. Amado, M. Kaminski, K. Landsteiner, JHEP 0905, 021 (2009). [arXiv:0903.2209 [hep-th]].