MRS Response of Water-bearing Geological Body of Coal in Uniform Conductivity

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Abstract

The MRS forward calculation of 3D water-contained medium involves two difficulties. First of them is to calculate the expression of the exciting field, which is an integration including 2 Bessel functions’ product. In this paper, the integration was divided into 2 integral intervals. In the first interval, using the Hankel expression of Bessel functions and the latter asymptotic properties of large volume, the double bessel integration was converted into Fourier sine(cosine) transformations and fast calculation to the transformations was adopted to finish the integration in this interval. Traditional calculation method can achieve high accuracy in the second interval. The second difficulty is the space dicretization of the 3D water-contained medium. Dicretization methods in both cylindrical and rectangular coordinates system were introduced in this paper. The cubic grids in rectangular coordinates system can construct 3D water-contained bodies of any shape. At last, MRS forward calculation of 3D water-contained bodies in uniform media achieved. Based on what mentioned above, water-filled tunnel and water-filled gob models are build, and both models MRS forward curves are calculated.

Keywords: Bessel function; Magnetic Resonance Sounding; 3D Forward; Water-bearing Geological Body of Coal; water-filled tunnel; water-filled gob.
1. Introduction

Magnetic Resonance Sounding (MRS) method is, currently, the only geophysical method that detects the groundwater directly. Its theory is based on 1D detection and assumes that the groundwater distributes in layers. The groundwater information is acquired layer by layer and the result is given in the same way. The reason for the assumption is that the status of MRS theory nowadays is focusing on 1D detection and interpretation. The calculation of MRS signals in more than 1D is subjected to the difficulty of the complexity of ground conditions, which will affect the distribution of excitation field, and the excitation degree of hydrogen nuclei is not simply symmetric. Thus it makes the calculation simpler to assume the groundwater distributes in layers. So there is a certain margin of error between this result and the actual occurrence of groundwater. To eliminate such errors, and improve the detection accuracy of MRS, the work to carry out the numerical simulation of MRS response for complex underground water body is greatly required.

The difficulties involved in the forward calculation of MRS include fast calculation of the vertical excitation field and the forward integration of the water-contained spaces. The expression of the vertical excitation field is an infinite integration which contains double Bessel functions. The integration cannot be calculated directly because of the high vibration and low attenuation of Bessel functions. At present, fast Hankel transform and continuous fraction method by Chave[1] and Hanggi[2]. By comparison, method by Hua[3] was selected to calculate the underground vertical fields excited by a round loop in uniform medium. On the basis of this, the water-contained model was divided into cubic grids in rectangular coordinates system. By adding the MRS forward integration of each grid, the response of the water-contained body was achieved.

2. Forward calculation method

2.1. Theory of MRS

The MRS method is based on the NMR characteristic of protons, information of groundwater is acquired by forcing the protons into NMR status. This procedure can be described as following: the magnetic moments of protons in stable geomagnetic field are static in the same direction with the geomagnetic field. If a exciting field with Lamor frequency is given in direction perpendicular to the geomagnetic field, the protons will absorb the energy from the excitation field, and their magnetic moments will leave the direction of the geomagnetic field, entering a resonance status. After a while, the excitation field is taken out, the protons will release the energy they absorbed, and the magnetic moments return to the direction of geomagnetic field.

In MRS method, the excitation field is transmitted by a loop that carries current with Lamor frequency. After a certain period of time when the protons entering resonance status, the current is cut off, then, the loop starts to receive the field from protons. The field changes the flux in the loop, and inductive voltage is the SNMR signal. Obviously, the signal contains information of protons, information of ground water. The original magnitude of MRS signal \( P_0 \) can be stated as follows:

\[
E_0 = \int K_{sd} \cdot n(p) \, dv
\]

Where: \( K_{sd} = \omega_0 \cdot M_0 \cdot b_{\perp}(p) \cdot e^{-i2\delta(p)} \cdot \sin(0.5\gamma_p b_{\perp}(p) \cdot Q) \)

\[
M_0 = 3.287 \times 10^{-3} \times B_0(20^\circ C)
\]
\[ Q = I \cdot \tau \]

- \( K_{3d} \): called nuclear function;
- \( I \): the current in the loop;
- \( \tau \): the time that \( I \) lasts;
- \( \omega_L \): Larmor frequency;
- \( M_0 \): the magnetic moment of water of unit volume;
- \( n(p) \): content of water at point \( p \);
- \( Q \): pulse moment;
- \( b_\perp(p) \): the vertical component of the magnetic field excited by unit of current in the loop at point \( p \);
- \( \varphi(p) \): is the phase of the magnetic field at point \( p \);
- \( \gamma_p \): is the magnetogyric ratio.

In cylindrical coordinates system, the equation can be discrete as:

\[ E_0 = \sum_{r} \sum_{z} K_{3d} \cdot n \cdot r \cdot d\tau \cdot d\varphi \cdot d\theta \cdot dz \]

Where \( r, z \leq 4a, 0 \leq \theta \leq 2\pi \)

- \( a \): Stands for the radius of the loop.

2.2. The excitation field

The expression of MRS signal involves calculation of magnetic field, which is excited under ground by a loop carrying current with Larmor frequency. By method by Cao[4] we can obtain the expression of magnetic field excited by a surface round loop in isotropic medium:

\[ H_r(r, z) = \int_0^{\infty} \frac{I(a \lambda^2)}{k_0 + k_1} e^{-k_1^2} J_0(a \lambda) J_1(r \lambda) \lambda d\lambda \]

\[ H_\varphi(r, z) = \int_0^{\infty} \frac{I(a \lambda^2)}{k_0 + k_1} e^{-k_1^2} J_0(a \lambda) J_1(r \lambda) \lambda d\lambda \]

Where \( k_0 = \sqrt{\lambda^2 - \mu_0 \varepsilon_0 \omega^2} \)

\[ k_1 = \sqrt{\lambda^2 - \left( \mu_0 \varepsilon_0 \omega^2 + j \mu_\sigma \omega \right)} \]

Where \( \mu_0 \) is the magnetic permeability, \( \varepsilon_0 \) and \( \varepsilon_i \) are specific inductive capacities of the air and ground, \( \sigma_i \) is the conductivity of the earth.

2.3. Calculation method

The excitation field expression contains the infinite integration of two Bessel functions. It is known that Bessel function has strong vibration and slow attenuation, the product of 2 Bessel functions will have stronger vibration and slower attenuation, thus it is difficult to calculate the integration of double Bessel functions with usual methods. In this paper, the method proposed by Hua[3] to calculate the double Bessel function. In this method the integration interval \([0, \infty)\) will be divided into \([0, K_0]\) and \([K_0, \infty)\),
using the Hankel function expression of Bessel functions and the latter asymptotic properties of large
volume, the double Bessel function integral in the interval \([K_0, \infty)\) can be converted into Fourier
sine(cosine) transform; the calculation of the double Bessel function integral in the interval \([0, K_0]\) can be
applied directly in general numerical integration method and the accuracy is higher [5].

The MRS response of water contained body can be understood as the integration of the function over
the volume. For horizontal water containing layers, it can be divided in cylindrical coordinates system,
but for other shapes, rectangular coordinates system will be capable to construct the model[5].

The calculation of excitation field takes a long time. It is repetitive in MRS forward calculation if the
ground electric conditions are the same. To avoid the repetition and improve the efficiency, taking the
NUMIS software as reference, the vertical magnetic field value of every grid in the integration space were
calculated firstly to form the excitation field matrix. The value of a corresponding grid was quoted in the
following calculation. The time cost of this method decreases while the size of loop and grid increase, and
normally acceptable. After the excitation field matrix being finished, the MRS forward calculation of the
model became very fast, being finished in a flash.

2.4. Grid method

Cartesian grid method is used for groundwater models which are a cube shape (Fig.1). The accuracy of
the method associated with the grid size. For a coil of 100m diameter, \(100 \times 50 \times 100\) nodes need to
calculate and the data is about 31M if grid size is \(2m \times 2m \times 2m\).

![Fig.1. Sketch map of rectangular coordinate system](image)

3. Forward modeling and curve analysis

A gob is an empty zone left after mining in coal mines. Gobs are very common in China. If the gobs
exist for a long time, they will be filled with groundwater, which make them like underground reservoirs.
If a tunnel of a new coal mine meets the water-filled gob, there will be killing accidents. Therefore, it is
important to research the MRS response of water-filled gobs and detect them by this method.

To describe the space expanded range of the 3D water-contained model, a triangular coordinates
system was built, taking the canter of the loop as the origin, north-south as X axis, east-west as Y axis and
vertical up-down as Z axis, and set MRS loop radius is 50m, earth resistivity is 200\(\Omega\cdot m\), Larmor
frequency is 2100Hz, geomagnetic inclination is 50\(^\circ\).

Model I: Model of Water-filled tunnel

Water-filled tunnels are important targets in groundwater exploration. The tunnels are usually in form
of a cuboids lying horizontally. The model built here is from -2m to 2m in direction of east to west, and
from -100m to 100m in direction of south to north, depths are 10m to 14m, 20m to 24m and 46m to 50m, 
water content is 100% and the attenuation time is 1000ms.

Figure 2 shows a MRS response of water-filled tunnel model. Similarly, as the pulse moment increases, 
there will be a maximum in every initial amplitude curves, with the model depth increasing, the amplitude 
maxima become smaller, and need greater pulse moments. After further study on the curves, we can find 
that it is similar to curve of a horizontal water-contained layer. It means shape of a water-filled tunnel is 
difficult be distinguished from the form of the curve.

![Figure 2](image)

Fig.2. (a) Schematic diagram of water-filled tunnel models; (b) MRS response of water-filled tunnel models

**Model II: Model of water-filled gobs of different sizes**

The modern gobs in coal mines are usually in shape of long and thin cuboids, the scales are usually 
bigger than those in small coal mines. The model built here has a scale becoming bigger and bigger in 
order to fetch MRS responses from water-filled gobs in different scales. The height and depth of the gobs 
stay unchanged, from 20 to 22m. the scale from east to west and south to north are from -10m to 10m, 
-20m to 20m, -40m to 40m, -60m to 60m, -80m to 80m and infinite (a horizontal layer of water-contained 
body), the water content is 100% and \(T_2 = 1000\text{ms}\).

Figure 3 points out the MRS amplitude responses of water-filling gob models in different scale, what 
can be seen from the diagram is that every curve has maximum amplitude, and with the scale of the 
model increasing, the amplitude maximum values become higher, indicating that the water quantity can 
be known from the amplitude. After further study on the curves, we can find that it is similar to curve of a 
horizontal water-contained layer or a water-filled tunnel. So it means shape of a water-filled gob is 
difficult be distinguished from the form of the curve.

![Figure 3](image)

Fig. 3. (a) Schematic diagram of water-filled gobs models; (b) MRS response of water-filled gob models
4. Conclusion and discussion

In this paper, the infinite-integral integration containing the product of a double Bessel function is divided into two parts and each part is calculated by used method of Hua[3]. So, rapid calculation of the excitation field is achieved. In order to do MRS forward of 3D body, Water-filled model is broken down into many discrete cubes in Cartesian coordinates, MRS response of each cube is calculated and the sum of all the cubes is the response of the model.

Models of water-filled tunnel and gob that in different depth are established and the MRS attitude response of those models are calculated by above forward method. Forward results show that with the model depth increasing, the amplitude maxima become smaller, and need greater pulse moments. It means the depth of model can be determined from the pulse moments. In addition, shape of 3D model is difficult to be distinguished from the form of the forward curves because of almost the same similarity between curves of water-filled tunnel models or water-filled gobs modes and curve of horizontal water-contained layer.

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