Extraction of the proton charge radius from experiments

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Abstract

Static properties of hadrons such as their radii and other moments of the electric and magnetic distributions can only be extracted using theoretical methods and not directly measured from experiments. As a result, discrepancies between the extracted values from different precision measurements can exist. The proton charge radius, $r_p$, which is either extracted from electron proton elastic scattering data or from hydrogen atom spectroscopy seems to be no exception. The value $r_p = 0.84087(39)$ fm extracted from muonic hydrogen spectroscopy is about 4\% smaller than that obtained from electron proton scattering or standard hydrogen spectroscopy. The resolution of this so called proton radius puzzle has been attempted in many different ways over the past six years. The present article reviews these attempts with a focus on the methods of extracting the radius.
I. INTRODUCTION

The structure of the proton plays an important role in atomic physics where experiments have reached a very high precision. The inclusion of the proton structure plays an important role in the accurate comparison of experimentally measured transition energies and very precise Quantum Electrodynamics (QED) calculations. Conversely, the unprecedented precision of atomic physics experiments allows one to probe some static properties of the proton such as its radius. Properties such as its charge and magnetization density are usually obtained as Fourier transforms of the Sachs form factors \[^1\] which are extracted from electron proton (e-p) scattering cross section measurements. One can deduce the radius and other moments from these densities and infer on the size of the proton. The radius thus extracted from e-p scattering and hydrogen spectroscopy seemed to be commensurate within error bars until a recent precision measurement of transition energies in muonic hydrogen changed the scenario. A comparison of the theoretical calculation of the Lamb shift in muonic hydrogen including all QED and finite size corrections (FSC) with the very precisely measured value of the shift \(\Delta E = E_{f=2}^P - E_{f=1}^P = 206.2949(32)\) meV, in muonic hydrogen, surprisingly led to a radius which was 4% smaller than the average CODATA value of 0.8768(69) fm \[^2\]. The extracted value of \(r_p = 0.84184(67)\) fm was much more accurate than the previous ones. This so-called “proton puzzle” was later reinforced \[^3\] with the precise value of \(r_p = 0.84087(39)\) fm from muonic hydrogen spectroscopy.

The puzzle gave rise to extensive literature which attempted solutions involving different approaches for the evaluation of finite size corrections \[^4\], off-shell correction to the photon-proton vertex \[^5\], the charge density being poorly constrained by data \[^6\], the existence of non-identical protons \[^7\] as well as problems in choosing the reference frame in the extraction of the radius \[^8\], \[^9\]. On the experimental side, accurate spectroscopic measurements of muonic deuterium and helium transition energies as well as additional scattering experiments are expected to shed light on the problem. For details of these plans and recent works we refer the reader to \[^10\], \[^11\]. The present article will focus on the theoretical aspects as well as the possible discrepancies arising from the methods used for the extraction of the proton radius.
II. PROTON CHARGE RADIUS AND OTHER MOMENTS

The size (or extension) of the proton is characterized by the moments of its charge density, \( \rho_p \) as

\[
<r^m> = \int r^m \rho_p(r) \, d^3r.
\]  

(1)

The charge density is conventionally defined as the Fourier transform of the electric form factor, \( G_E^p(q^2) \), namely, \( G_E^p(q^2) = \int e^{-i\vec{q} \cdot \vec{r}} \rho_p(r) d^3r/(2\pi)^3 \). Starting with this Fourier transform,

\[
G_E^p(q^2) = \frac{1}{2\pi^2} \int_0^\infty r^2 \rho_p(r) \frac{\sin(|q|r)}{|q|r} \, dr
\]

\[
= \frac{1}{2\pi^2} \frac{1}{|q|} \int_0^\infty r \rho_p(r) \left[ |q|r - \frac{|q|^3 r^3}{6} + ... \right]
\]

\[
= \frac{1}{2\pi^2} \int_0^\infty r^2 \rho_p(r) dr - \frac{1}{2\pi^2} \frac{q^2}{6} \int_0^\infty r^4 \rho_p(r) dr + ...
\]

(2)

it is easy to see that the radius defined above as \( \langle r_p^2 \rangle = \int r^2 \rho_p(r) \, d^3r \) can also be expressed in terms of the form factor \( G_E^p(q^2) \) as

\[
- \frac{6}{G_E^p(0)} \left. \frac{dG_E^p(q^2)}{dq^2} \right|_{q^2=0} = \langle r_p^2 \rangle.
\]

(3)

There exists another approach in order to extract the proton radius from experiment, one involving atomic spectroscopy. In this approach, one attempts to calculate the theoretical difference between atomic energy levels with the inclusion of all possible corrections from quantum electrodynamics (QED) as well as the proton finite size corrections (FSC). This difference is then compared with the experimentally measured transition energies in order to fit the radius which appears in the theoretical expression due to the inclusion of FSC. Such an approach was used in [3] and apart from the second moment of the charge density, the FSC in [3] also included the third Zemach moment [12] defined by

\[
\langle r^3 \rangle_{(2)} = \int d^3r \, r^3 \rho_{(2)}(r),
\]

(4)

where, \( \rho_{(2)}(r) = \int d^3z \, \rho_p(|z - r|) \rho_p(z) \). This inclusion introduced a small model dependence in the extraction and in fact has been discussed at length by several authors [13]. Some uncertainty depending on the approach to include the FSC was also found in [4].
A. Breit frame, Lorentz boost and relativistic corrections

In order to compare the radius extracted from the two methods mentioned in the previous subsection, we must ensure that the extractions are done in the same frame of reference. As mentioned in [14], the size and shape of an object are not relativistically invariant quantities: observers in different frames will infer different magnitudes for these quantities. The static relation \( \langle r_p^2 \rangle = \int r^2 \rho_p(r) \, d^3r \), defines the radius in the rest frame of the proton. The extraction of the radius from electron proton (ep) scattering is however not done in the proton rest frame. The ep scattering data is used to extract the invariant form factor \( G^p_E(q^2) \), where \( q^2 = \omega^2 - q^2 \) is the four-momentum transfer squared in ep elastic scattering. The radius is then evaluated using the following relation [15]:

\[
\langle r_p^2 \rangle = -\frac{6}{G^p_E(0)} \left. \frac{dG^p_E(q^2)}{dq^2} \right|_{q^2=0}.
\]  

(5)

This definition looks slightly different from that derived in Eq.(3) with the three momentum being replaced by the four momentum squared in (5). At first sight, Eq.(5) has the appearance of a Lorentz invariant quantity (and has even misled some authors to believe so [16]). However, if we examine the condition \( q^2 = 0 \), with \( q^2 = \omega^2 - q^2 \), it either means that \( \omega^2 = q^2 \neq 0 \) (in which case we have a real photon) or \( \omega = |q| = 0 \). It is impossible to exchange a real photon in the t-channel exchange diagram in elastic electron-proton scattering and hence we have to drop the first possibility. The second choice involving \( \omega = 0 \) is, however, equivalent to choosing the Breit or the so-called brick-wall frame in which the sum of the initial and final proton momentum is zero. This interpretation is consistent with what we find in the Breit equation where the same reference frame has to be chosen. The radius extracted in this frame should then be boosted to the proton rest frame before comparing it with the one extracted from atomic spectroscopy [8]. This and other relativistic corrections become important [8] with the improved precision of experimental data. Finally, we would like to comment that the extraction of the proton radius from atomic spectroscopy relies on formulas which start with the definition of the radius as given in (1).

The form factor \( G^p_E(q^2) \) is a Fourier transform of the density \( \rho_p(r) \) in the rest frame and hence \( G^p_E(q^2) = G^p_E(q^2) \) in the non-relativistic case but \( G^p_E(q^2) \neq G^p_E(q^2) \) in the relativistic case. There have been several attempts in literature in order to incorporate the above relations with relativistic corrections [17]. The fact that the structure of a bound system is independent of its motion in the non-relativistic case whereas it changes in the relativistic
case depending on how fast it moves, was taken into account in [18] for the calculation of the deuteron radius too. The authors in [8] found that incorporating the relativistic corrections (along with the Lorentz boost) could indeed remove the 4% discrepancy between the $ep$ scattering and $\mu p$ Lamb shift determinations of the radius.

B. Finite size effects

The corrections to the energy levels at order $\alpha^4$ due to the structure of the proton are usually included using first order perturbation theory with the point-like Coulomb potential modified by the inclusion of form factors [4]. The determination of the proton radius from accurate Lamb shift measurements in [3] relies for the FSC on a seminal calculation of Friar [19] based on a third order perturbation expansion of the energy which leads to an expression which depends on the proton radius rather than the form factors explicitly. Such an expression is a result of approximating the atomic wave function everywhere by its value at its centre and is useful in extracting the radius from spectroscopic measurements. In [19], the author finds,

$$\Delta E \simeq \langle 0 | \Delta V | 0 \rangle + \langle 0 | \Delta V | \Delta \phi \rangle + (\langle \Delta \phi | \Delta V | \Delta \phi \rangle - \langle 0 | \Delta V | 0 \rangle \langle \Delta \phi | \Delta \phi \rangle)$$

(6)

where $\Delta V$ is the perturbation and the wave function $|\Psi\rangle = |0\rangle + |\Delta \phi\rangle$ with $|0\rangle$ and $|\Delta \phi\rangle$ the unperturbed part and the first order perturbation respectively. Further, approximating the wave function $\Phi_n(r) = \langle r | 0 \rangle$ by its value at $r = 0$,

$$\Delta E_{FSC} = \frac{2\pi\alpha Z}{3}\Phi_n(0)^2 \left[ \langle r^2 \rangle - \frac{\alpha Z m_r}{2} \langle r^3 \rangle_{(2)} + \ldots \right].$$

(7)

The second term involves the third Zemach moment given by Eq.(4) which can be rewritten in terms of $\langle r_p^2 \rangle$ as:

$$\langle r^3 \rangle_{(2)} = \frac{48}{\pi} \int_0^\infty dq \frac{dq}{q^4} \left( G_E^2(q^2) - 1 + q^2 \frac{\langle r_p^2 \rangle}{3} \right).$$

(8)

The extraction of the radius from the muonic Lamb shift [3] was done using the above relation with a dipole form for $G_E(q^2)$ in order to rewrite $\langle r^3 \rangle_{(2)}$ in (7) in terms of $\langle r_p^2 \rangle$. Replacing all coefficients in (7) and including all QED corrections, the final expressions used in the two references in [3] in order to compare with the experimental values of $\Delta E(= E_{2S_{1/2}}^{f=1} - E_{2S_{1/2}}^{f=2}) = 206.2949(32)\,\text{meV}$ and $\Delta E_L(= E_{2P_{1/2}} - E_{2S_{1/2}}) = 202.3706(23)\,\text{meV}$ were
where the last term corresponds to the full two-photon exchange (TPE) contribution [20]. Note that the \( <r^3>_{(2)} \) term in Friar’s expression (7) is an order \( \alpha^5 \) correction and corresponds in principle to a two photon exchange diagram as shown in [21].

In order to confirm that the above formula (9) which relies on perturbative methods and is used to fit the proton radius does not change significantly due to the use of nonperturbative methods, the authors in [22] calculated the transition energies by numerically solving the Dirac equation including the finite-size Coulomb interaction and finite-size vacuum polarization. The point-like Coulomb potential was replaced by one including the proton charge distribution, \( \rho(r) \), given by

\[
V_C(r) = -\frac{Z\alpha}{r} \rightarrow -Z\alpha \int \frac{\rho(r')}{|\vec{r} - \vec{r}'|} d^3r',
\]

\[
\rho(r) = \frac{\eta}{8\pi} e^{-\eta r}; \quad \eta = \sqrt{\frac{12}{\langle r^2_p \rangle}},
\]

The energy shift was calculated by taking the difference between the eigenvalues calculated using the Dirac equation with the above potential for several values of \( \langle r^2_p \rangle \). These energy shifts were then interpolated and fitted to the function \( f = A\langle r^2_p \rangle + B\langle r^2_p \rangle^{3/2} \), in order to determine the coefficients \( A \) and \( B \). Their final result, namely,

\[
\Delta E(= E_{2S_{1/2}}^{f=1} - E_{2P_{3/2}}^{f=2}) = 209.9505 - 5.2345r^2_p + 0.0361r^3_p \text{ meV},
\]

as compared to (9) led to a radius which differed from the central value of 0.84184(67) fm but was well within the errors. Thus, no significant discrepancy between perturbative and nonperturbative methods was found. The authors in [23], on solving the Schrödinger equation numerically however found that the difference between perturbative methods and nonperturbative numerical calculations of the 2S hyperfine splitting in muonic hydrogen are larger than the experimental precision.

A different relativistic approach for the FSC based on the Breit equation with form factors was investigated in [4]. The method relies on the fact that all \( \vec{r} \) dependent potentials in Quantum Field Theory (QFT) are obtained by Fourier transforming an elastic scattering
amplitude suitably expanded in $1/c^2$. The Breit equation \[24, 26\] follows the very same principle for elastic $e^-\mu^+$, $e^+e^-$ (positronium), $e^-p$ (hydrogen) and $\mu^-p$ (muonic hydrogen) amplitudes. The one-photon exchange amplitude between the proton and the muon then leads to the Coulomb potential plus the fine and hyperfine structure (hfs), the Darwin term and the retarded potentials \[24, 25\]. The authors modified the standard Breit potential \[4, 27\] for the $\mu^-p$ system with the inclusion of the electromagnetic form factors of the proton. The FSC to the Coulomb, Darwin, fine and hyperfine energy levels for any $n, l$ were provided and performing an expansion of the atomic wave functions an alternative expression for $\Delta E = E_{2P_{3/2}}^{f=2} - E_{2S_{1/2}}^{f=1}$ was obtained. The main difference in their expression as compared to that of \[3\] arose due to the inclusion of the Darwin term with form factors. Since the use of a Dirac equation for energy levels would imply the inclusion of the Darwin term, the authors subtracted the point-like Darwin term from their calculations leaving only the effect of this relativistic correction with form factors. They obtained

$$\Delta E = E_{2P_{3/2}}^{f=2} - E_{2S_{1/2}}^{f=1} = 209.16073 + 0.1174r_p - 4.2585r_p^2 + 0.0203r_p^3 \text{ meV},$$

leading to a proton radius of $r_p = 0.83594(46)$ fm which was close to that obtained in \[3\] but hinted toward an uncertainty introduced due to the use of a different FSC approach.

A brief discussion of the FSC in the hyperfine splitting is in order here. The FSC to the hyperfine splitting in \[27\] was evaluated using

$$\Delta E_{hfs} = \int |\Phi_C(r)|^2 \hat{V}_{hfs}(r) \, dr$$

where $\Phi_C(r)$ is the unperturbed hydrogen atom wave function. The spin operators are included in the definition of $\hat{V}_{hfs}$ (see \[27\]). This correction seemed to be different from that used in \[3\] where it was calculated using the standard Zemach formula given by,

$$\Delta E_{hfs} = -\frac{2}{3} \mu_1 \mu_2 < \sigma_1 \cdot \sigma_2 > \int |\Phi(r)|^2 f_m(r) \, dr,$$

where $f_m(r)$ is the Fourier transform of $G_M(q^2)$. It was however shown in \[28\] that Eqs \[13\] and \[14\] would give the same result provided we replace $\Phi_C$ by $\Phi$ in \[13\]. Whereas $\Phi_C(r)$ in \[13\] is a solution of the point-like $1/r$ Coulomb potential, $\Phi(r)$ is the solution of the potential which includes the Coulomb potential with form factors and is given in \[12\] as, $\Phi(r) = \Phi_C(r) + m_1 \alpha \Phi_C(0) \int f_s(u)|u - r|du$. The difference thus lies in the usage of the unperturbed wave function in the energy correction. In other words, in \[27, 28\], the
total Hamiltonian is taken as \( H = H_0 + H_{CF}^F + H_{hfs}^{FF} \) with \( H_0 \) containing the \( 1/r \) Coulomb potential, \( H_{CF}^F \), the finite size correction to the Coulomb potential and \( H_{hfs}^{FF} \) the hyperfine interaction with form factors leading to the energy correction in first order perturbation theory given by \( \Delta E = \langle \Phi_C | H_{CF}^F | \Phi_C \rangle + \langle \Phi_C | H_{hfs}^{FF} | \Phi_C \rangle \). In [12] however, one finds \( H = \tilde{H}_0 + H_{hfs}^{FF} \), with \( \tilde{H}_0 \) which includes FSC to the Coulomb potential taken as the unperturbed Hamiltonian. We notice from the above discussion that the Breit equation and the Zemach method would lead to the same hyperfine correction if the time independent perturbation theory is handled in the same way. In a calculation which involves finite size corrections to the point-like Coulomb potential as well as hyperfine structure taken separately (as in [3, 24]) it seems reasonable to use the prescription with \( \Delta E = \langle \Phi_C | H_{CF}^F | \Phi_C \rangle + \langle \Phi_C | H_{hfs}^{FF} | \Phi_C \rangle \) in order to avoid double counting of the finite size corrections to the Coulomb term. The \( r_p^2 \) and \( r_p^3 \) terms in Eqs (12, 9) for example appear after the explicit inclusion of the FSC to the \( (1/r) \) Coulomb potential.

The proton radius extracted from the muonic hydrogen Lamb shift is much more accurate than that determined from standard (electronic) hydrogen. The procedure of extracting the radius from electronic hydrogen is slightly different and involves a simultaneous determination of the Rydberg constant and Lamb shift. Traditionally, the Lamb shift was actually a splitting (and not a shift) between the energy levels \( E(2S_{1/2}) \) and \( E(2P_{1/2}) \) which are degenerate according to the naive Dirac equation in the Coulomb field. The convention now is however to define the Lamb shift as any deviation from the prediction of the naive Dirac equation that arises from radiative, recoil, nuclear structure, relativistic and binding effects (excluding hyperfine contributions) [29] so that, \( E_{njl} = E_{njl}^{Dirac} + L_{njl} \). The measurement of the Lamb shift can be disentangled from the Rydberg constant by using two different intervals of hydrogen structure. For example, we can use the accurate measurements of \( f_{1S-2S} = 2466061413187.34(84) \) kHz and \( f_{2S_{1/2}-8D_{5/2}} = 770649561581.1(5.9) \) kHz along with the energy expressions

\[
\begin{align*}
E_{1S-2S} &= [E_{2S_{1/2}}^{Dirac} - E_{1S_{1/2}}^{Dirac}] + L_{2S_{1/2}} - L_{1S_{1/2}} \\
E_{2S-8D} &= [E_{8D_{5/2}}^{Dirac} - E_{2S_{1/2}}^{Dirac}] + L_{8D_{5/2}} - L_{2S_{1/2}} ,
\end{align*}
\]

(15)

to determine the radius. The first differences on the right hand side are dependent on the Rydberg constant \( R_\infty \) (through \( E_{nj}^{Dirac} = R_\infty E_{nj} \)), which can be eliminated using the two equations. The left hand side is replaced by accurate measurements and the Lamb shift
is determined independent of the Rydberg constant. Knowing the accurate value of the Lamb shift, it can be inserted back into the above equations to determine the Rydberg constant accurately. The value of the Rydberg constant is thus obtained to be \[ R_\infty = 10973731.568539(55) \text{ m}^{-1} \]. Knowing \( R_\infty \) accurately, one can now proceed to determine the radius as follows:

\[
\text{Measured energy splitting} = R_\infty E_{nj} + E(\text{Lamb shift})
\]

where \( E(\text{Lamb shift}) \) includes all QED as well as proton structure corrections. With a good knowledge of all QED related corrections (see for example [30]), the radius in the proton structure corrections appearing in \( E(\text{Lamb shift}) \) can be fitted to the measured energy splitting.

III. REANALYSES OF SCATTERING DATA

Apart from the various theoretical papers which attempted to explain the discrepancy between the proton radius from spectroscopy and scattering, there have also been some attempts at reanalysing the electron proton scattering data. Here, we shall mention some of the recent works and the criticisms too. In [31], the cross sections at the lowest \( q^2 \) were fitted using two single parameter models for form factors with one being the standard dipole given by \( G_E^2(q^2) = (1 + q^2/b_E)^{-4} \), \( G_M^2(q^2)/\mu_p^2 = (1 + q^2/b_M)^{-4} \) and the other involving a Taylor expansion given as, \( G_E^2 = 1 - c_E z \), \( G_M^2/\mu_p^2 = 1 - c_M z \), where \( z \) is the conformal mapping variable as defined in [31]. Following the philosophy that the charge radius of the proton is a small-\( q^2 \) concept, the authors analysed the low \( q^2 \) data using the simple fits and reached the conclusion that the proton radius could vary between 0.84 and 0.89 fm, thus making the spectroscopy and scattering results consistent.

In another attempt of a similar kind, instead of focussing on a reanalysis of recent data, the authors decided to review the older Mainz and Saskatoon data in [32]. They found that a dipole function with the muonic hydrogen radius of 0.84 fm, i.e., \( G_E(q^2) = (1 + q^2/0.66[\text{GeV}^2])^{-2} \), not only describes low \( q^2 \) \( G_E(q^2) \) results, but also reasonably describes \( G_E(q^2) \) to the highest measured \( q^2 \). The authors in [32, 33] performed a sharp truncation of the form factor expansion in momentum space which was strongly criticized for not being in accord with the basic facts of form factors and the extraction of radii from them in [34].
A completely novel point of view was chosen in [7] where the authors noted that the proton radius may not be unique but a quantity which is randomly distributed over a certain range. The standard definition of a “radius” of the proton is obviously based on the notion of the proton being spherical. Arguing that the definition of the radius could get blurred for a deformed proton and providing other literature in support of the idea of a fluctuating size of the proton, the authors performed a fit for a form factor of the so-called “non-identical” protons. Taking the standard dipole form factor as the basis, the authors introduced the fluctuation of the proton size by performing an average with the following form:

\[
\langle G_E^2(q^2, \Lambda_1) \rangle = \frac{1}{2\Delta\Lambda} \int_{\Lambda_1-\Delta\Lambda}^{\Lambda_1+\Delta\Lambda} G_E^2(q^2, \Lambda) d\Lambda,
\]

with the \( G_E \) in the integrand having the standard dipole form. Using the latest Mainz data to perform the fits, the authors determined an average \( \Lambda = 0.8203 \text{ GeV} \) with a variation \( \Delta\Lambda \) of 21.5%. They further studied the effects of such a radius variation in neutron star and symmetric nuclear matter. The electric form factor as defined in (16) can be evaluated analytically and using Eq. (5) leads to a radius given by

\[
r_p^2 = \frac{12}{\Lambda_1^2 - \Delta\Lambda^2},
\]

which with the substitution of the values from [7] gives a proton radius, \( r_p = 0.864 \text{ fm} \). On applying the relativistic correction (involving the Lorentz boost with \( \lambda_E = 1 \)) in [8], the radius reduces to a value of 0.844 fm which is quite close to that determined from muonic hydrogen spectroscopy [3]. In Fig. 1 we display the proton electric form factor at low momenta within the the three different parametrizations discussed above. In [7], the authors investigated the density dependence of the proton radius in nuclear matter. The right panel in the figure shows the behaviour of the proton radius using the parametrization in [7], with and without relativistic corrections (as found in [8]).

IV. BRIEF OVERVIEW OF PLANNED EXPERIMENTS

The discussion of the proton radius puzzle has so far revolved around the extractions from \( ep \) scattering measurements, standard hydrogen (electronic) as well as muonic hydrogen spectroscopy. The missing component in these analyses is then the data on muon-proton elastic scattering. The MUon proton Scattering Experiment (MUSE) at the Paul Scherrer
FIG. 1: Comparison of the parametrizations from Refs [7, 31, 32] for form factors at low $q^2$ (shown in the left panel). The right side panel displays the density dependent proton radius as calculated in [7] but with and without relativistic corrections included.

Institute is a simultaneous measurement of the $\mu^+ p$ and $e^+ p$ elastic scattering. The experiment is expected to decide if the $\mu p$ scattering and $\mu p$ Lamb shift experiment lead to the same proton radius. Another scattering experiment is the PRad which will measure the $ep$ scattering cross sections with higher precision and at low $q^2$. Besides these plans, the CREMA collaboration has been studying the spectroscopy of other exotic atoms such as muonic deuterium and muonic helium too. A detailed account of the future experiments can be found in [10, 11].

V. SUMMARY

The finite size of the proton is characterized fully by all the moments of its charge distribution. The second moment is however generally used to define the “radius” of the proton. The radius thus defined can either be extracted from spectroscopic measurements or lepton proton scattering data using theoretical methods. Until some time ago, there seemed to be an agreement between the radii extracted from spectroscopy (with standard electronic hydrogen) and scattering. However, the high precision muonic hydrogen spectroscopy revealed a 4% deviation from the average value obtained from all previous experiments. Since the
radius is an extracted and not directly measured quantity, a higher experimental precision should also be complemented by a higher confidence in the theoretical component. With this viewpoint, in this review we have examined the theoretical methods used for the extraction of the radius as well as the related literature which appeared in the form of possible solutions of the “proton radius puzzle”. These included checks on the validity of the perturbative methods used, approximations therein and the relevance of relativistic corrections. The latter is particularly of importance due to the fact that the relation between the charge density and the electric form factor is necessarily of a non-relativistic nature. This fact also makes it important that the comparison of radii extracted from different experiments is done in the same frame of reference. While the resolution of the puzzle is being attempted by reanalyses of old data and planning of new experiments, it is necessary to pay attention to the theoretical inputs involved in the extraction of the radius too.

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