Steady-State Entanglement for Distant Atoms by Dissipation in Coupled Cavities

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We propose a scheme for the generation of entangled states for two atoms trapped in separate cavities coupled to each other. The scheme is based on the competition between the unitary dynamics induced by the classical fields and the collective decays induced by the dissipation of two delocalized field modes. Under certain conditions, the symmetric or asymmetric entangled state is produced in the steady state. The analytical result shows that the distributed steady entanglement can be achieved with high fidelity independent of the initial state, and is robust against parameter fluctuations. We also find out that the linear scaling of entanglement fidelity has a quadratic improvement compared to distributed entangled state preparation protocols based on unitary dynamics.

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There have been various practical applications for quantum entangled states, ranging from quantum teleportation\textsuperscript{1, 2} to universal quantum computation\textsuperscript{3, 4}. The main obstacle in preserving entanglement is decoherence induced by the environment. Recently, dissipative state preparation has become a focus in quantum computation and entanglement engineering\textsuperscript{5–20}, which uses the competition between the transitions induced by the microwave fields and the two collective atomic decay channels to drive atoms to a symmetric or asymmetric entangled state. Analytical and numerical results show that the distributed steady entanglement can be obtained compared to any known entangled state preparation protocol for coupled-cavity systems\textsuperscript{27–31}, whose optimal value is $1 - F \propto C^{-1/2}$.

The experimental setup, as shown in Fig. 1, consists of two identical $A$-type atoms each having two ground states $|0\rangle$ and $|1\rangle$, and an excited state $|2\rangle$ trapped in one detuned cavity. An off-resonance optical laser with detuning $\Delta$ drives the transition $|0\rangle \leftrightarrow |2\rangle$ and a microwave field resonantly drives the transition $|0\rangle \leftrightarrow |1\rangle$. The cavity mode is coupled to the $|0\rangle \leftrightarrow |2\rangle$ transition with the detuning $\Delta - \delta$, where $\delta$ is the cavity detuning from two photon resonance.

The Hamiltonian of the whole system in the interaction picture reads

$$H_I = H_0 + H_g + V_+ + V_-,$$

and propose a scheme for producing distributed entanglement for two atoms trapped in coupled cavities. Due to the coherent photon hopping between the two cavities, the system is mathematically equivalent to that involving two atoms collectively interacting with two common nondegenerate field modes symmetrically and asymmetrically, respectively. Each delocalized field mode induces a collective atomic decay channel. The present scheme uses the competition between the transitions induced by the microwave fields and the two collective atomic decay channels to drive atoms to a symmetric or asymmetric entangled state.

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$$H_0 = \delta(a_1^\dagger a_1 + a_2^\dagger a_2) + \Delta(|2\rangle_1\langle 2|_2 + |2\rangle_2\langle 2|_1) + J(a_1^\dagger a_2 + a_2^\dagger a_1),$$

$$H_g = \frac{\Omega_M}{2}(e^{i\theta_M}|1\rangle_1\langle 0|_2 + |1\rangle_2\langle 0|_1) + H.c.,$$

$$V_+ = \frac{\Omega}{2}(|2\rangle_1\langle 0| + |2\rangle_2\langle 0|),$$

$$V_- = \frac{\Omega}{2}(|0\rangle_1\langle 1| + |2\rangle_2\langle 2|).$$

In this paper, we generalize the idea of Refs.\textsuperscript{5, 0} and propose a scheme for producing distributed entanglement for two atoms trapped in coupled cavities. Due to the coherent photon hopping between the two cavities, the system is mathematically equivalent to that involving two atoms collectively interacting with two common nondegenerate field modes symmetrically and asymmetrically, respectively. Each delocalized field mode induces a collective atomic decay channel. The present scheme uses the competition between the transitions induced by the microwave fields and the two collective atomic decay channels to drive atoms to a symmetric or asymmetric entangled state. Analytical and numerical results show that the distributed steady entanglement can be obtained compared to any known entangled state preparation protocol for coupled-cavity systems\textsuperscript{27–31}, whose optimal value is $1 - F \propto C^{-1/2}$.
decay and atomic spontaneous emission can be expressed

\[ \gamma \] denoted as

\[ \rho \]

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our coupled cavity system is governed by the effective perturbation theory [5, 6, 32]. Then the dynamics of the coupling constant, \( \Omega \) and \( \Omega' \) describes cavity and cavity coupling, \( g \) atoms trapped in two coupled cavities. The atom in each detuned cavity has two ground states \( |1\rangle \) and \( |0\rangle \), and one excited state \( |2\rangle \), which is driven by the same off-resonance optical laser. The microwave fields applied to the two atoms differ by a relative phase of \( \theta_M \).

\[ V = (V_i)^4, \quad a_i \] is the cavity field operator in cavity \( i = 1, 2 \), \( J \) is the photon-hopping strength which describes cavity and cavity coupling, \( g \) is the atom-cavity coupling constant, \( \Omega \) and \( \Omega_M \) represent the classical laser driving strength and the microwave driving strength, respectively. \( \theta_M = \pi \) (or 0) guarantees a high fidelity for asymmetric steady-state \( |S\rangle = (|01\rangle - |10\rangle)/\sqrt{2} \) (or symmetric steady-state \( |T\rangle = (|01\rangle + |10\rangle)/\sqrt{2} \). Let us introduce two delocalized bosonic modes \( c_1 \) and \( c_2 \), and define asymmetric mode \( c_1 = (a_1 - a_2)/\sqrt{2} \) and symmetric mode \( c_2 = (a_1 + a_2)/\sqrt{2} \), which are linearly related to the field modes of two cavities. In terms of the new operators, the Hamiltonian \( H_0 \) can be rewritten as

\[
H_0 = \frac{g}{\sqrt{2}} |2\rangle_1 \langle 1| (c_1 + c_2) + |2\rangle_2 \langle 1| (c_2 - c_1) + H.c. \\
+ \frac{\delta - J}{2} c_1^+ c_1 + \frac{\delta + J}{2} c_2^+ c_2 + \Delta \sum_{i=1,2} |2\rangle_i \langle 2|.
\]

The Hamiltonian \( H_0 \) describes the asymmetric coupling for the two atoms to the delocalized field mode \( c_1 \) and the symmetric coupling to \( c_2 \). Due to the photon hopping these two delocalized field modes are nondegenerate and each induces a collective atomic decay channel. The photon decay rate of cavity \( i = 1, 2 \) is denoted as \( \kappa_i \) and the spontaneous emission rate of the atoms is denoted as \( \gamma_j \) \( (j = 1, 2, 3, 4) \). Under the condition \( \kappa_1 = \kappa_2 = \kappa \), the Lindblad operators associated with the cavity decay and atomic spontaneous emission can be expressed as \( L^{\kappa_1} = \sqrt{\kappa_1} c_1, \quad L^{\kappa_2} = \sqrt{\kappa_2} c_2, \quad L^{\gamma_2} = \sqrt{\gamma_2} \frac{1}{2} |1\rangle_2 \langle 2|, \quad L^{\gamma_4} = \sqrt{\gamma_4} \frac{1}{2} |1\rangle_4 \langle 2| \). We assume \( \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma/2 \) for simplicity.

Under the condition of weak classical laser field, we can adiabatically eliminate the excited cavity field modes and excited states of the atoms when the excited states are not initially populated. To tailor the effective decay processes to achieve a desired steady-state, we introduce an effective operator formalism based on second-order perturbation theory [3, 4, 32]. Then the dynamics of our coupled cavity system is governed by the effective Hamiltonian \( H_{eff} \) and effective Lindblad operator \( L_{eff} \)

\[
H_{eff} = -\frac{1}{2} V[H^{-1}_{NH} + (H^{-1}_{NH})^\dagger]V + H_g, \\
L_{eff} = L^x H^{-1}_{NH} V + H_g,
\]

where \( H^{-1}_{NH} \) is the inverse of the non-Hermitian Hamiltonian \( H_{NH} = H_0 - \frac{i}{2} \sum_x (L_x^x)^\dagger L_x^x \). The resulting effective master equation in Lindblad form is

\[
\dot{\rho} = i[\rho, H_{eff}] + \sum_x (L_x^x)^\dagger L_x^x \rho - \frac{1}{2} [(L_x^x)^\dagger L_{eff}^x] \rho + \rho(L_{eff}^x)^\dagger L_x^x \rho,
\]

\[
H_{eff} = -Re\left[\frac{\Omega^2}{4} \frac{\gamma_4}{\gamma_2} |00\rangle \langle 00| \right] + H_g,
\]

where \( Re[ ] \) denotes the real part of the argument.

\[
geff = \frac{g\Omega}{\Delta}, \quad \left( \frac{\delta - J}{\Delta} \right) \left( \frac{\delta - J}{\Delta} \right), \quad R_1 = \frac{\delta J - \delta g + \Delta (\delta - J)}{g^2 - \Delta (\delta - J) (\delta - J)}, \\
R_2 = \frac{\delta J - \delta g + \Delta (\delta - J)}{g^2 - \Delta (\delta - J) (\delta - J)}, \quad R_3 = \frac{\delta J - \delta g + \Delta (\delta - J)}{g^2 - \Delta (\delta - J) (\delta - J)}.
\]
As shown in Fig. 2 (a) and (b), the loop-like elements $O_{00}(\Omega^2)$, $O_{01}(\Omega^2)$ and $O_{02}(\Omega^2)$ represent the effective-Hamiltonian evolution in three triplet states $|00\rangle$, $|T\rangle$ and $|S\rangle$ without microwave fields, respectively. For weak optical driving $\Omega$, $H_{\text{eff}} \simeq H_g$. There exist two effective decay channels characterized by $L_{c1}^{(\text{eff})}$ and $L_{c2}^{(\text{eff})}$ through the two delocalized bosonic modes $c_1$ and $c_2$ as compared with the case of Ref. [5] in which only one decay channel is mediated. It is the photon hopping that lifts the degeneracy of the two delocalized field modes and leads to the two independent decay channels. $L_{c1}^{(\text{eff})}$ indicates the effective decay from $|00\rangle$ to $|S\rangle$ at a rate $\kappa_{c1}$, and from $|S\rangle$ to $|11\rangle$ at a rate $\kappa_{c2}$ caused by asymmetric $c_1$ mode, and $L_{c2}^{(\text{eff})}$ denotes the effective decay from $|00\rangle$ to $|T\rangle$ at a rate $\kappa_{c2}$, and from $|T\rangle$ to $|11\rangle$ at a rate $\kappa_{c2}$ caused by symmetric $c_2$ mode simultaneously. The decay rates $\kappa_{c1}$ ($\kappa_{c2}$) and $\kappa_{c2}$ ($\kappa_{c2}$) equal to the square of the first (second) coefficient in the right side of Eq. (9) and Eq. (10), respectively. Set $A_{c1} = A_{c2} = 0$, decays from $|S\rangle$ to $|11\rangle$ and from $|T\rangle$ to $|11\rangle$ can be both largely suppressed. On the other hand, the microwave fields drive the transition between the three states $|00\rangle$, $|T\rangle$ ($|S\rangle$) and $|11\rangle$ for $\theta_M = 0(\pi)$. The dynamics of the full master equation in Fig. 3 (a) and (b) illustrates that we can obtain state $|S\rangle$ or $|T\rangle$ of high fidelity, and the time needed for reaching the entangled steady-state $|T\rangle$ is about two times as large as that of $|S\rangle$. This is because that the optimal ratio of $\kappa_{c1}/\kappa_{c2}$ is about two times as large as $\kappa_{c2}/\kappa_{c2}$. The errors imposed by all possible atomic spontaneous emissions should also be taken into account. We apply Eq. (6) again to derive four analytic expressions of effective spontaneous emissions with the other Lindblad operators $L_{\gamma_1}^{(T)}$, $L_{\gamma_2}^{(T)}$, $L_{\gamma_3}^{(T)}$ and $L_{\gamma_4}^{(T)}$

$$
L_{c1}^{(\text{eff})} = -\frac{\gamma}{2} - \frac{g^2}{4\Delta} - (\delta - J_1^2),
B_{c1} = B_{c2} = \kappa(\delta - \frac{g^2}{4\Delta}) + \frac{\gamma(\delta - J)^2}{2\Delta},
C_{c1} = \frac{g^2}{4\Delta} - (\delta - J),
C_{c2} = \frac{g^2}{4\Delta} - (\delta + J),
\kappa_{c1} = \frac{\kappa}{2} + \frac{\gamma(\delta - J)}{2\Delta},
\kappa_{c2} = \frac{\kappa}{2} + \frac{\gamma(\delta + J)}{2\Delta}.
$$

(11)

FIG. 3. (Color online) The populations of four states $|S\rangle$, $|T\rangle$, $|00\rangle$ and $|11\rangle$ versus the dimensionless parameter $gl$ for a random initial state. Both curves are plotted for $C = 200$, $\kappa = \gamma/2$, $\Omega_M = 2\pi/5$, $\Omega = g/20$ with $\Delta$, $\delta$ and $J$ being the optimal values for two entangled steady-states. (a) $\theta_M = 0$. (b) $\theta_M = \pi$. (c) The fidelity $F_{g[S]}$ for steady-state $|S\rangle$ versus $C$, and the coefficient of the linear scaling in $F_{g[S]}$ as a function of $C$ with different ratios $\kappa/\gamma$ is plotted in the inset.

and the operators of that for $|T\rangle$ state are

$$
L_{c1}^{(\text{eff})} = L_{c1}^{(\text{eff})} = \sqrt{\gamma} |J_i = 1, 2 \rangle |S\rangle |T\rangle,
L_{c2}^{(\text{eff})} = L_{c2}^{(\text{eff})} = \sqrt{\gamma} |J_i = 3, 4 \rangle |S\rangle |T\rangle,
\gamma_{c1} = \frac{\gamma}{2} [4(\Delta^2 - J^2) + (\delta - J)^2]^{1/2},
\gamma_{c2} = \frac{\gamma}{2} [4(\Delta^2 - J^2) + (\delta + J)^2]^{1/2}.
$$

(18)

and $\gamma_{S,i=1,2} = \gamma_{T,i=1,2} = \gamma_{eff}/16$, $\gamma_{S,i=3,4} = \gamma_{T,i=3,4} = \gamma_{eff}/8$. Then we use the rate equation to evaluate the fidelity for the state ($j = S$ or $T$)
The optimal fidelity of the entanglement can be obtained. $P$ is nearly 1 and the probability in each of the other three states $\propto \delta g$ of the state population decaying into the state $|S\rangle$.

The first term on the right side of Eq. (19) represents the population decaying into the state $j$ with the rate $\kappa_a$, while the other terms express the population leaking out of the state $j$ with the rate $\kappa_b + \sum_i \gamma_{j,i}$. Suppose $P_j \approx 1$ and the probability in each of the other three states is nearly $P_0$, then

$$\hat{P}_j = \kappa_a P_0 - (\kappa_b + \sum_{i=1}^{4} \gamma_{j,i}) P_j,$$

where $P_j$ is the probability to be in the state $j$. The effective two-qubit system in the inset of Fig. 3 (c) shows that the fidelity scaling of state $|S\rangle$ is independent of different ratios $\kappa/\gamma$, then we find out the actual constants for maximizing the fidelity as follows

$$1 - F_{|S\rangle} \approx 12.8 C^{-1}.$$  \hfill (21)

The influences of different parameter fluctuations on the fidelity $F_{|S\rangle}$ of entangled state are considered. As shown in Fig. 4 (a) and (b), $F_{|S\rangle}$ keeps above 90% even 5% fluctuations in these parameters. The preparation process of state $|T\rangle$ is similar to that of $|S\rangle$.

Photonic band gap cavities coupled to atoms or quantum dots are suitable candidates for realizing the proposal. Cooperativity of value $C \approx 100$ has been realized. \hfill (22) The cavity modes can be coupled via the overlap of their evanescent fields or via an optical fiber, and photon hopping between two cavities has been observed. \hfill (23)

In conclusion, we have proposed a scheme for dissipative preparation of entanglement between two atoms that are distributed in two coupled cavities. We find the linear scaling of the fidelity is a quadratic improvement compared with distributed entangled state preparation protocols based on unitary dynamics.

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