SELF-TUNING OF THE COSMOLOGICAL CONSTANT AND EINSTEIN GRAVITY ON THE BRANE *

JIHN E. KIM
School of Physics, Seoul National University
Seoul, 151-747 Korea

BUMSEOK KYAE, QAISAR SHAFI
Bartol Research Institute, University of Delaware,
Newark, DE 19716 USA

We show that a self-tuning mechanism of the cosmological constant could work in 5D non-compact space-time. The standard model matter fields live only on the 4D brane. The change of vacuum energy on the brane just gives rise to dynamical shifts of the profiles of the background metric and a bulk scalar field in the extra dimension, keeping 4D space-time flat. To obtain the Newtonian potential on the brane at long distances, we introduce an additional brane-localized 4D Einstein-Hilbert term.

1. Introduction and Summary

The problem of the cosmological constant or the absolute scale of the potential energy has been considered as the most serious mass hierarchy problem in modern particle physics. In recent years, however, there has been a hope to understand the vanishing cosmological constant in extra dimensional field theories 1,2,3. In the Randall-Sundrum-II (RS-II) type five dimensional (5D) theories with a 3-brane embedded in a non-compact 5D space, the 5D gravity sector contains a 5D cosmological constant \( \Lambda_b \) and a brane vacuum energy \( \Lambda_1 \). For a flat four dimensional (4D) subspace, a fine-tuning relation between \( \Lambda_b \) and \( \Lambda_1 \) is inevitable in the RS-II model. However, by introducing a massless scalar field in the bulk, it was shown that a self-tuning of the cosmological constant is possible in the RS-II type setup 2,1.

*Talk presented by B. KYAE at SUSY 2003: Supersymmetry in the Desert, held at the University of Arizona, Tucson, AZ, June 5-10, 2003. To appear in the Proceedings.
To guarantee a massless scalar field, the scalar can be regarded as the dual field of a 5D 3 form tensor field $A_{MNP}$.

To obtain $-\frac{1}{r}$ gravity potential, one might make the extra space compact by employing two branes. However, the introduction of two branes accompanies a fine-tuning relation between two brane cosmological constants $\Lambda_i$. In this paper, we employ only one brane, and introduce an additional brane-localized 4D Einstein-Hilbert term. As shown in Refs. 5, 6, if one restricts matter fields only on the brane, the $-\frac{1}{r}$ potential could be obtained even with an infinite volume extra dimension by introducing a brane-localized gravity term. In this paper, we will discuss only the $\Lambda_b = 0$ case.

2. Self-tuning of the cosmological constant

We consider the following action in 5D space-time $(x^\mu, y)$,

$$S = \int d^4x dy \left[ \sqrt{|g_5|} \left( \frac{M_5^3}{2} R_5 - \frac{1}{2} \partial_M \phi \partial^M \phi + L_m \right) \right. $$

$$\left. + \delta(y) \sqrt{|\bar{g}_4|} \left( \frac{M_4^2}{2} \bar{R}_4 + L_1^1 - \Lambda_1 \right) \right],$$

where $\bar{R}_4$ is the 4D Einstein-Hilbert term $\bar{g}_{\mu\nu} \bar{R}^c_{\mu\rho\sigma}$ constructed with a metric different from the bulk metric $g_{MN}$: $\bar{g}_{\mu\nu}(x, y) = (1 + \frac{M_4^2}{M_5^2} (\partial_y^2) g_{\mu\nu}(x, y) \times \Omega^{-2}(x, y)$ with a dimensionless coupling $\bar{a}$. At the linearized level, the $z$ derivative terms coming from $\delta(y) M_4^2 \sqrt{|\bar{g}_4|} \bar{R}_4$ maintain the 4D gauge symmetry required for the spin-2 field $^6$. A 4D general coordinate transformation still can be defined as $\partial_2^2 \bar{g}^\nu_{\mu'} = \frac{\partial z^\mu}{\partial x^\rho} \frac{\partial z^{\nu}}{\partial x^{\sigma}} \partial_2^2 \bar{g}_{\rho\sigma}$ with $\partial_2^2 (\frac{\partial z^{\nu}}{\partial x^{\sigma}})|_{z=0} = 0$. The ordinary standard model (SM) matter fields are assumed to live only on the brane. The presence of brane terms with $\delta(y)$ in Eq. (1) explicitly breaks the 5D general covariance into the 4D one at $y = 0$. The introduction of $\delta(y) M_4^2 \sqrt{|\bar{g}_4|} \bar{R}_4$ does not spoil any given symmetry $^5, 6$. We suppose that $M_4$ is comparable to $M_5$. We introduce a $Z_2$ symmetry in the extra space, and assign even (odd) parities to $g_{\mu\nu}$, $g_{55}$ ($g_{\mu5}$, $\phi$) under $y \leftrightarrow -y$.

The 4D flat background solutions turn out to be $^1$

$$ds^2 = \beta^2(y) \eta_{\mu\nu} dx^\mu dx^\nu + dy^2,$$

$$\beta(y) = \left[ 4a |y| + c \right]^{1/4}, \quad \Omega(y) = \beta(y),$$

$$\phi(y) = \sqrt{\frac{12 M_5^2}{4}} \ln \left[ \frac{4a}{c} |y| + 1 \right] \times \text{sgn}(y),$$

where $a (>0)$ is a dimensionful arbitrary constant. $\text{sgn}(y)$ is defined as $+1$
(-1) for $y > 0$ ($y < 0$). The integration constant $c$ should be determined by the boundary conditions:

$$c = -\left(\frac{6M_3^3a}{\Lambda_1}\right) > 0.$$  

(5)

To avoid naked singularities at some points in the bulk, we should take only positive values of $c$. Hence, the brane vacuum energy is required to be negative. In the boundary condition Eq. (5), we note that $\Lambda_1$ is never completely fixed by the Lagrangian parameters. Any arbitrary negative value of $\Lambda_1$ allows a 4D flat space-time solution, by adjusting $c$ and $a$ such that the boundary conditions at the brane are fulfilled. Even if the electroweak and QCD phase transitions change the brane vacuum energy $\Lambda_1 \rightarrow \Lambda_1'$, a flat 4D space-time could be maintained by proper shifts $c \rightarrow c'$ and/or $a \rightarrow a'$. It means that the profiles of the metric and the scalar field in the extra dimension are changed. Since they are dynamical fields, it is always possible. We note that our self-tuning mechanism is not associated with any parameter sensitive to low energy physics. As will be shown, the 4D Newtonian constant is given by $M_4$ rather than $M_5$ in our model.

According to Ref. 7, the flat universe is quantum mechanically most probable in 4D space-time, even though universe with a non-zero 4D curvature is also classically allowed. Hence, the flat universe can always be dynamically chosen through the self-tuning mechanism.

3. Metric perturbation and Conclusion

The brane-localized gravity term does not affect the self-tuning mechanism, but it is essential in gravity interaction on the brane. Let us consider metric perturbation near the background solutions. For convenience of the further analysis, we change the $(x, y)$ coordinate into the conformal coordinate $(x, z)$. With the gauge choices $\partial^\mu(h_{\mu
u} - \frac{1}{2}\eta_{\mu\nu}h) = h_{\mu5} = 0$, the $(\mu\nu)$ component of the linearized Einstein equation turns out to be

$$\frac{1}{2}\left[\left(\nabla_5^2 - 3\frac{K'}{K}\partial_z\right) + \frac{M_4^2}{M_5^2}\delta(z)K\nabla_4^2\left(1 + \frac{\alpha}{M_4^2}\partial_z^2\right)\right]\left[h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right]$$

$$- \frac{1}{6}\left[\partial_\mu\partial_\nu - \eta_{\mu\nu}\nabla_4^2\right]\left[f - h^0\right] + \frac{1}{M_5^2}T_{\mu\nu}^b + \frac{1}{M_5^2}\delta(z)KT^1_{\mu\nu},$$

(6)

where $K(z) = \frac{1}{\beta(\phi(z))} = (3a|z| + c^{3/4})^{-1/3}$ with $|z| = \int_0^z dy/\beta(y) = \frac{1}{a_n}|(4a|y| + c)^{3/4} - c^{3/4}|$. $'$ denotes the derivative with respect to $z$, and $\nabla_5^2$ and $\nabla_4^2$ indicate $\eta^{\mu\nu}\partial_\mu\partial_\nu + \partial_z^2$ and $\eta^{\mu\nu}\partial_\mu\partial_\nu$, respectively. $h$ is defined as $\eta^{\mu\nu}h_{\mu\nu}$, and $h^0(x) \equiv h(x, z = 0)$. The SM matter fields only contribute
to $T_{\mu\nu}^1$. For simplicity, we assume that another massless bulk scalar field independent of $z$ makes a dominant contribution to $T_{bMN}^b$. $f(x)$ satisfies $\nabla_4^2 f(x) = \frac{2}{M_4^2} T^b(x)$. Indeed, gravity interaction on the brane is insensitive to $T_{bMN}^b$ in our model. In Eq. (6) we note that at $z = 0$, the equation of motion for the trace mode is $\nabla_4^2 h^0(x) = \frac{2}{M_4^2} T^1(x)$, whereas at $z \neq 0$, it is $[\nabla_4^2 - 3\frac{K'K}{N} \partial_z](h - h^0) = 0$. The bulk solution of Eq. (6) in 4D momentum space $(p, z)$ is given by Bessel functions of order zero.

The terms with the coupling $\alpha$ in Eq. (6) require to satisfy a non-trivial boundary condition at $z = 0$: $\partial_z \tilde{h}_{\mu\nu} |_{z=0+} = \partial_z \tilde{h}_{\mu\nu} |_{z=0-} = 0$. Otherwise highly singular terms proportional to $\delta^2(z)$ arise, which can not be matched to the right handed side of Eq. (6). Then, since $\partial_z^2 \tilde{h}_{\mu\nu}$, $\partial_z^2 h$ do not induce any delta function, we have only to compare the coefficients of the delta functions appearing in Eq. (6) to fulfill the boundary condition at $z = 0$.

We found the graviton solution at $z = 0$ is given by

$$\tilde{h}_{\mu\nu}(p, z = 0) = \frac{2}{M_4^2 p^2} \left[ \tilde{h}_{\mu\nu}^1(p) - \frac{1}{2} \eta_{\mu\nu} \tilde{T}^1(p) \right] + O\left( \frac{\alpha T_{\mu\nu}^1}{M_4^4}, \frac{\alpha \tilde{S}_{\mu\nu}}{M_4^5 M_4^2} \right),$$

where $\tilde{T}^1 \equiv \eta_{\mu\nu} \tilde{h}_{\mu\nu}^1$, and $\tilde{S}_{\mu\nu} \equiv -\frac{M_5^2}{6} (p_{\mu} p_{\nu} - \eta_{\mu\nu} p^2) (\tilde{f} - \tilde{h}^0) + \tilde{T}_{\mu\nu}^b$. The unspecified part of order $\alpha$ are negligible at low energies. Hence, at low energies $\tilde{h}_{\mu\nu}$ in Eq. (7) reproduces the same result that the 4D Einstein gravity theory predicts with $G_N \equiv 1/(8\pi M_4^2)$.

In conclusion, as we have shown, 4D flat solution is always possible independent of the magnitude of 4D brane cosmological constant in 5D non-compact space-time, through dynamics by the 5D gravity and a massless scalar. Since the SM fields are localized on the brane, 4D vacuum energy by them and its variation do not destroy the 4D flatness. An additional brane-localized gravity term successfully induces $-\frac{1}{2}$ potential on the brane.

References

1. J. E. Kim, B. Kyae and Q. Shafi, arXiv:hep-th/0305239.
2. J. E. Kim, B. Kyae and H. M. Lee, Phys. Rev. Lett. 86, 4223 (2001).
3. N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, Phys. Lett. B 480, 193 (2000); S. Kachru, M. B. Schulz and E. Silverstein, Phys. Rev. D 62, 045021 (2000).
4. S. Forste, Z. Lalak, S. Lavignac and H. P. Nilles, Phys. Lett. B 481, 360 (2000); JHEP 0009, 034 (2000).
5. G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B 485, 208 (2000).
6. B. Kyae, arXiv:hep-th/0312161.
7. E. Baum, Phys. Lett. B 133, 185 (1983); S. W. Hawking, Phys. Lett. B 134, 403 (1984).