Chiral vortical wave and induced flavor charge transport in a rotating quark-gluon plasma

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(Dated: April 14, 2015)

We show the existence of a new gapless collective excitation in a rotating fluid system with chiral fermions, named as the Chiral Vortical Wave (CVW). The CVW has its microscopic origin at the quantum anomaly and macroscopically arises from interplay between vector and axial charge fluctuations induced by vortical effects. The wave equation is obtained both from hydrodynamic current equations and from chiral kinetic theory and its solutions show nontrivial CVW-induced charge transport from different initial conditions. Using the rotating quark-gluon plasma in heavy ion collisions as a concrete example, we show the formation of induced flavor quadrupole in QGP and estimate the elliptic flow splitting effect for Λ baryons that may be experimentally measured.

PACS numbers: 11.40.Ha,12.38.Mh,25.75.Ag

Introduction.— Anomalous transport effects in many body systems with exact, approximate, or effective chiral fermions have generated wide interests recently. Such phenomena span a wide range of physical systems 1, from semimetals to cold atomic systems, and from hot quark-gluon plasma (QGP) created in heavy ion collisions to cold dense matter in neutron stars. A salient feature of chiral fermions is the chiral anomaly and remarkably it has been found that such microscopic quantum effect can manifest itself in macroscopic dynamics. These systems also provide possible environments with nonzero macroscopic chirality. Such a chiral matter allows anomalous transport processes that would be normally forbidden by symmetries like parity invariance.

One way to induce the anomalous transport effects in such chiral system is to apply external electromagnetic fields. A famous example is the so-called Chiral Magnetic Effect (CME) in which an electric current (a parity-odd vector quantity) can be generated in parallel to an external magnetic field (a parity-even axial-vector quantity). The CME could lead to experimentally measurable effects both for the QGP in heavy ion collisions 2 and for certain Dirac and Weyl semimetals 3. Other interesting examples include e.g. the Chiral Separation Effect (CSE) 4,5 and the Chiral Electric Separation Effect (CESE) 6,7. Specifically for the QGP, the CME predicts a charge separation effect, and the resulting charge-dependent azimuthal correlations have been measured 8,9,10. A lot of studies are done for disentangling background effects and quantifying possible CME signals 11,12; see reviews in e.g. 13,14.

The anomalous transport effects can also occur when the system is undergoing a global rotation. Such fluid rotation can be quantified by a vorticity \( \omega = \frac{1}{\tau} \nabla \times \mathbf{v} \) where \( \mathbf{v} \) is the flow velocity field. Interesting analogy between velocity field in fluid dynamics and vector potential in electrodynamics as well as between vorticity and magnetic field, was emphasized in 15 and vortex-driven effects similar to CME and CSE were suggested there. Such vortical effects were quantified in a newly developed anomalous hydrodynamic framework in 16. For given vorticity \( \omega \), both a vector current and an axial current may be generated, depending on local fluid temperature \( T \) as well as vector and axial chemical potentials \( \mu, \mu_5 \). The Chiral Vortical Effect (CVE) quantifies the generated vector current \( J_V \) as

\[
J_V = \frac{1}{\pi^2} \mu \mu_5 \omega , \tag{1}
\]

and the generated axial current \( J_A \) as

\[
J_A = \left[ \frac{1}{6} T^2 + \frac{1}{2\pi^2} (\mu^2 + \mu_5^2) \right] \omega . \tag{2}
\]

It was suggested that such CVE may lead to baryon charge separation in heavy ion collisions 17,18.

The vorticity-driven anomalous transport effects in Eqs. (1), (2) couple together the vector and axial densities/currents. It is natural to wonder if certain collective modes may arise from mutually induced vector/axial density fluctuations in the presence of fluid vorticity. Such hydrodynamic modes, like well-known sound waves and charge density waves, are robust gapless excitations in a fluid system. A recent example in the context of anomalous transport is the Chiral Magnetic Wave (CMW), which stems from the interplay between CME and CSE in an external magnetic field 19,20. In this Letter, we show that there indeed exists a new wave mode for vector and axial density fluctuations induced by the vortical effects, which we therefore call a “Chiral Vortical Wave” (CVW). We will derive this new wave equation and determine the CVW propagation speed in both the hydrodynamic and kinetic theory frameworks.
Furthermore by solving the wave equation we show how a fermion charge quadrupole distribution arises via the CVW from initial vector density and quantify its dependence on wave speed. Finally we will make predictions for possible signatures of CVW in heavy ion collisions and suggest future measurements.

**The Chiral Vortical Wave.**— Let us start by rewriting the vortical effects \[ \delta n \] in terms of chiral currents \( J_{L/R} = \frac{1}{2}(J_{\pi} \mp J_{\lambda}) \):

\[
J_{L/R} = \mp \left( \frac{1}{12} T^2 + \frac{1}{4\pi^2} \mu^2_{L/R} \right) \omega .
\]  

(3)

Correspondingly we have introduced the chiral densities \( n_{L/R} = \frac{1}{2}(n_{\pi} \mp n_{\lambda}) \) and chemical potentials \( \mu_{L/R} = \mu_{\pm} \mu_{S} \). Intuitively the above vortical effects can be understood as follows. In the presence of global rotation, the underlying fermions experience an effective interaction of the form \( -\omega \cdot \mathbf{S} \) in their local rest frame, with \( \mathbf{S} \) the spin of fermions. This causes a spin polarization effect (as indeed found in other context \[27, 28\]), namely the fermions will have their spins preferably aligned with \( \omega \). Such spin polarization is *charge-blind*, which is different from the polarization caused by magnetic field. Assume \( \omega \) is along positive \( \hat{x} \), and denote the number of fermions as \( n_{p, s} \), with \( p = \pm \) and \( s = \pm \) indicating the particle’s momentum and spin directions in parallel/antiparallel to \( \hat{x} \) direction. The polarization effect implies that for right-handed fermions \( n_{p+, s+} > n_{p-, s-} \) thus leading to a current \( J_{R} \propto (n_{p+, s+} - n_{p-, s-}) > 0 \) (i.e. in parallel to \( \omega \)) while for left-handed fermions \( n_{p+, s-} > n_{p-, s+} \) thus leading to a current \( J_{L} \propto (n_{p+, s-} - n_{p-, s+}) < 0 \) (i.e. in anti-parallel to \( \omega \)).

Let us then consider the small fluctuations of left- and right-handed densities on top of a uniform equilibrium background. For simplicity we consider \( \omega \) to be constant and neglect fluctuations of temperature (which is controlled by linearized hydro equations for energy momentum tensor). By combining the continuity equations \( \partial_t n_{L/R} + \nabla \cdot J_{L/R} = 0 \) with the above anomalous transport effects \[ \delta n \] one obtains the following equations:

\[
\partial_t n_{L/R} = \pm \frac{1}{4\pi^2} \omega \partial_x (\mu^2_{L/R}) = \pm \frac{\omega \mu_{L/R}}{2\pi^2} \partial_x \mu_{L/R}.
\]  

(4)

where we have set vorticity along \( x \)-direction \( \omega = \omega \hat{x} \). Clearly there are two modes, one for right-handed density and the other for left-handed density, that propagate in opposite direction. For later convenience let us also introduce the susceptibilities for the corresponding densities: \( \chi_{\mu}^{L/R} \equiv \partial n_{L/R}/\partial \mu_{L/R} \).

At this point, two possibilities may occur. For simplicity we focus on the right-handed mode below.

1. The background fluid is charge neutral, i.e. \( \mu_0 = 0 \).

   In this case the density fluctuation \( \delta n = \chi_{0} \delta \mu \) and is governed by the nonlinear wave equation below:

\[
\partial_t (\delta n) + \frac{\omega}{4\pi^2 \chi_0} \partial_x (\delta n^2) = 0 .
\]  

(5)

This takes the form of inviscid Burgers’ equation, for which the (implicit) solution can be formally written as:

\[
\delta n(x, t) = F_i \left( x - \frac{\omega t}{2\pi^2 \chi_0} \right),
\]  

(6)

with \( F_i(x) = \delta n(x, t = 0) \) given by the initial distribution of density fluctuation.

2. The background has nonzero charge density \( \mu_0 \neq 0 \).

   In this case one can linearize the equation for the evolution of small density fluctuations on top of the background density and obtain a linear wave equation:

\[
\partial_t (\delta n) + \frac{\mu_0 \omega}{2\pi^2 \chi_{0}} \partial_x (\delta n) = 0.
\]  

(7)

This is just like the familiar sound wave equation, and a Fourier-mode expansion (with frequency \( \nu \) and wavevector \( k = k \hat{x} \)) of the fluctuation \( \delta n \) immediately reveals a gapless dispersion relation for this wave:

\[
\nu = V_\Omega |k| , \quad V_\Omega = \frac{\mu_0 \omega}{2\pi^2 \chi_{0}}.
\]  

(8)

This is the *Chiral Vortical Wave* (CVW) with the wave speed \( V_\Omega \) defined above. More precisely this is the right-handed wave mode that propagates along the \( \omega \) direction. The left-handed wave mode propagates in opposite direction to \( \omega \), with a speed given by a similar formula albeit replacing \( \mu_0 \) and \( \chi_{0} \) with the left-handed quantities.

In short, the Chiral Vortical Wave found above is essentially a hydrodynamic density wave arising from slowly varying vector and axial density fluctuations that are coupled together through vortical effects. The wave equation \[ \delta n \] comes from a leading-order gradient expansion in the hydrodynamic sense. Diffusion effects of charge densities may be included via higher-order gradient expansion terms, i.e. longitudinal and transverse (with respect to \( \omega \) direction) diffusion terms like \(- (D_L \partial_x^2 + D_T \partial_T^2) (\delta n) \) may be added to the LHS of \[ \delta n \] where \( D_L \) and \( D_T \) are the corresponding diffusion constants.

**CVW from Chiral Kinetic Theory.**— Kinetic theory provides a general framework for understanding transport processes, and recently the physics of chiral anomaly has been incorporated into this framework \[32, 37\]. In such chiral kinetic theory, anomalous transport effects such as the CME and CVE arise in a transparent way. Collective excitations can also be studied in this approach. For example, a recent study \[36\] has shown that the Chiral Magnetic Wave emerges naturally in the hydrodynamic regime and the so-obtained CMW velocity reproduces previous results. It is therefore desirable to understand how the newly found Chiral Vortical Wave may arise in the chiral kinetic theory and whether the CVW velocity is consistent with the above derivation.

Let us consider a rotating system of noninteracting right-handed (denoted by \( \mu^+ \)) Weyl fermions as well as their left-handed (denoted by \( \mu^- \)) anti-particles. (The
discussion for a system of left-handed fermions with their right-handed anti-fermions will be very similar.) Taking a similar approach as in [24][25], we start from the equation of motions for these fermions in their local rest frame

\[ \sqrt{G_\pm} \dot{\mathbf{x}} = \frac{\mathbf{p}}{p} \pm \frac{\mathbf{\omega}}{p}, \quad \sqrt{G_\pm} \dot{\mathbf{p}} = 2 \mathbf{p} \times \mathbf{\omega}, \]

(9)

where \( \sqrt{G_\pm} = 1 \pm \mathbf{p} \cdot \mathbf{\omega}/p^2 \) and \( \mathbf{\omega} = |\mathbf{\omega}| \) is the global rotational angular speed of the system. The corresponding kinetic equations can be written as:

\[ \partial_t f_\pm + \mathbf{x} \cdot \partial_x f_\pm + \mathbf{p} \cdot \partial_p f_\pm = C_\pm [f_+, f_-]. \]

(10)

The time evolution of their densities follows from integrating the above equations, for which we obtain:

\[ \partial_t \int_p \sqrt{G_\pm} f_\pm + \int_p \left( \frac{\mathbf{p}}{p} \pm \frac{\mathbf{\omega}}{p} \right) \cdot \partial_x f_\pm 
+ \int_p 2(\mathbf{p} \times \mathbf{\omega}) \cdot \partial_p f_\pm = \int_p \sqrt{G_\pm} C_\pm. \]

(11)

Note we have used the equations of motion (9) in the above. The last term of L.H.S is zero after integration by part. Also \( \int_p \equiv \int \frac{d^3p}{(2\pi)^3} \).

We are interested in examining small fluctuations in the net (vector) density on top of certain background equilibrium distribution \( f_0 \pm (p; T, \mu_0) \). Similar to the analysis in [26], the density fluctuations can be parameterized by the following form:

\[ f_\pm(t, \mathbf{x}, \mathbf{p}) = f_{0\pm}(p) + \delta f_\pm(t, \mathbf{x}, \mathbf{p}), \]

\[ \delta f_\pm = \pm [\partial_p f_{0\pm}] \int d\nu d^3k e^{i(\nu - k \cdot \mathbf{x})} h(\nu, \mathbf{k}, \mathbf{p}) \]

(13)

where the \( \delta f_\pm \) have been expanded in Fourier modes. Subjecting the above into Eq.(11), and taking a difference to yield the time evolution of net density, we obtain the following relation in linear order of fluctuations:

\[ \nu \int_p [\partial_p f_{0+}(p) + \partial_p f_{0-}(p)] h(\nu, \mathbf{k}, \mathbf{p}) \]

\[ = k \cdot \int_p \frac{\mathbf{\omega}}{p} [\partial_p f_{0+}(p) - \partial_p f_{0-}(p)] h(\nu, \mathbf{k}, \mathbf{p}). \]

(14)

In deriving the above, we have used the facts that (a) the equilibrium distribution \( f_{0\pm} \) is a space-time independent fixed point of collision kernel, (b) \( \int_p \mathbf{p} z(p) = 0 \) for any \( z(p) = |p| \), and (c) the collision terms from fluctuation vanish because of charge conservation constraint.

Let us then examine the low frequency, long wavelength limit in the above equation, \( \nu \rightarrow 0 \) and \( k \rightarrow 0 \). The hydrodynamic zero mode in this limit arises from \( \delta f_\pm \rightarrow \pm [\partial_p f_{0\pm}(p)] H(t, \mathbf{x}) \) which implies \( h \) becomes independent of \( \mathbf{p} \). This allows one to perform integrations over \( \mathbf{p} \) in (14), and obtain the following:

\[ [\chi_{\mu_0} \nu - C \mathbf{k} \cdot \mathbf{\omega}] h(\nu, \mathbf{k}) = 0, \]

(15)

where \( \chi_{\mu_0} \) is the thermodynamic susceptibility defined in equilibrium, \( \chi_{\mu_0}(T, \mu_0) = \partial n/\partial \mu|_{T, \mu_0} \) with net charge density \( n = \frac{1}{(2\pi)^3} \int_p \sqrt{G}(f_{0+} - f_{0-}). \) The constant above is defined by \( C = -\int_p (1/p) [\partial_p f_{0+}(p) - \partial_p f_{0-}(p)] = \frac{1}{8\pi^2} \int dp [f_{0+}(p) - f_{0-}(p)] \), and for the Fermi-Dirac distribution it gives \( C = \mu_0/(2\pi^2) \). This allows one to immediately identify a hydrodynamic collective excitation that propagates along the vorticity direction, \( \mathbf{k} \parallel \mathbf{\omega} \), with the following dispersion relation:

\[ \nu = V_\Omega |\mathbf{k}|, \quad V_\Omega = \frac{\mu_0 \mathbf{\omega}}{2\pi^2 \chi_{\mu_0}}. \]

(16)

Notably, the so-obtained Chiral Vortical Wave speed \( V_\Omega \) agrees exactly with that in Eq.(5).

**CVW-induced charge transport.** — We now discuss interesting charge transport phenomena induced by the CVW. This depends on the initial conditions. To be concrete we consider different initial density fluctuations on top of a background medium with vorticity \( \mathbf{\omega} = \mathbf{\omega_0} \) and CVW speed \( V_\Omega \) given by (16).

Let’s first consider the initial condition as a purely axial charge density fluctuation \( F_i^A(x) \) at \( t = 0 \) (noting that we have suppressed “trivial” coordinates \( y, z \)). This can be cast into initial conditions for right-handed and left-handed density initial conditions \( F_i^{R/L}(x) = \pm F_i^A(x) \). The subsequent evolution via the wave equations simply yields \( \delta n_i^{R/L}(x) = \pm F_i^A(x + V_\Omega t)/2 \). We are interested in the transport of vector charge density which is an observable quantity. This can be obtained as follows:

\[ (\delta n_i^V) \equiv \left[ F_i^A(x - V_\Omega t) - F_i^A(x + V_\Omega t) \right]/2 \]

\[ \approx \left[ -\partial_x F_i^A(x) \right] V_\Omega t \]

(17)

where the second line is true for a smallish \( V_\Omega t \). This implies a separation effect of vector charge along the vorticity direction: when initial axial charge fluctuation is positive/negative vector charge along the vorticity direction. Consider the transport process leads to a quadrupole moment of vector charge along the vorticity direction.
initial fluctuation to be positive and concentrated around \(x = 0\) (e.g. a Guassian form), then \[\varphi^2 F_i(V(x))/2\] is positive (negative) at large (small) \(|x|\), implying concentration of positive charge away from \(x = 0\) toward both directions along \(\omega\). The resulting vector charge quadrupole \(|q_f\)| is proportional to \((V_{\Omega})^2\).

**Experimental observable in heavy ion collisions.** — Our discussions on the CVW and its induced charge transport effects so far have been rather general. It is of great interest to find possible experimental manifestation in a concrete system. To that end, we consider the rotating quark-gluon plasma created in off-central heavy ion collisions. In such a QGP, CVW occurs for each light flavors, e.g. \(u, d\) quarks and possibly \(s\) quarks as well. The global rotation points in the out-of-plane direction. Given initial fluctuations in their respective densities, each flavor develops its CVW along \(\omega\). This transports flavor charges toward the two “tips” of the QGP fireball, thus leading to a quadrupole charge distribution on the transverse plane. Here we make a first estimate of such effect.

Let us first quantify the quadrupole moment \(q_f^l\) resulting from CVW for a single quark flavor. We use the participant density from Glauber model as an initial condition for the flavor charge fluctuation and study the dependence of \(q_f^l\) on the key parameter \(V_{\Omega}\) by solving the CVW equation. An illustration of the CVW-transported flavor charge density distribution at \(\tau = 8\)fm (with beam energy \(\sqrt{s} = 200\) GeV, impact parameter \(b = 7\)fm, initial time \(\tau_0 = 0.6\)fm) is shown in Fig. 1: a quadrupole pattern is evident. The quadrupole moment can be obtained by integrating the density distribution \(q_f^l = \int dxdy(\delta n_f \cos(2\phi_s))/\int dxdy(\delta n_f)\). We have computed this quantity with the results: \(q_f^l \simeq -0.03(V_{\Omega}\Delta \tau)^2\) with \(\Delta \tau\) (in fm/c) the propagation time in QGP. The numerical coefficient is checked to depend only very mildly on centrality and beam energy. The minus sign is merely due to convention of defining azimuthal angle \(\phi_s\) with respect to the in-plane direction.

Clearly we need a plausible estimate of \(V_{\Omega}\). AMPT simulations suggest an initial value of \(\omega\) at about \(0.5\) fm\(^{-1}\), and lattice susceptibility at initial temperature \(T_0 \sim 350\) MeV is about \(\chi_f \sim 3\) fm\(^{-2}\), with both decreasing as QGP expands. Using an estimate of background density \(\mu_0\) of 0.1 \(1\) fm\(^{-1}\), one gets an estimate \(\langle V_{\Omega}\rangle \sim 10^{-2} \sim 10^{-3}\). Given this, one may estimate the induced quadrupole \(|q_f^l|\) at \(\sim 10^{-4}\) level. Such estimate is very sensitive to \(\mu_0\). By going to lower beam energy or by selecting events with large baryon asymmetry in given centrality bin, the background \(\mu_0\) could be considerably increased thus magnifying the CVW-induced effects.

**FIG. 1:** CVW-induced flavor charge density profile.

**FIG. 2:** Normalized \(\bar{\Lambda}\) and \(\Lambda\) elliptic flow splitting, \([v_2^3 - v_2^\Lambda]/(|q_f^l|A_{\Lambda}^2)|\), for symmetric 2-flavor (2-F) and 3-flavor (3-F) cases. The mid-central and the peripheral correspond to for 15 – 30\% and 60 – 92\% centrality class (see [38]).

So how could such an effect be detected by experiments? This may be measurable by the elliptic flow splitting of identified particles/anti-particles with nonzero baryonic charges. Flavor quadrupole implies more quarks on the out-of-plane tips of fireball and less on the in-plane equator, and therefore more baryons will be formed on the tips than on the equator. The stronger in-plane radial flow will thus translate the quadrupole into baryon/anti-baryon \(v_2\) splitting. This mechanism is in analogy to the electric charge quadrupole induced by CMW [30]. To quantify such baryon flow splitting, one needs to combine quarks of varied flavors into hadrons. Suppose at the freeze-out, the flavor-wise chemical potential for quarks contains the CVW-induced quadrupole contribution 
\[\delta \mu_f \propto 2q_f^l \cos(2\phi_s)\] (with \(f = u, d, s\)). The corresponding chemical potential for a given type of hadron can be determined from its constituent quark content, e.g. for a \(\Lambda\) baryon 
\[\delta \mu_{\Lambda} \propto 2(q_u^l + q_d^l + q_s^l) \cos(2\phi_s)\]. We particularly propose to use \(\Lambda\) baryon which is electric charge neutral thus unaffected by possible CMW effect. We then use the STAR blast-wave model [38] to compute the resulting differential flow splitting. As it is unclear how much the \(s\) quark mass may reduce their chiral effects, we consider two extreme cases: a symmetric two-flavor (2-F) case \(q_u^l = q_d^l = q_s^l = q_0\) with \(q_0 = 0\), or a symmetric three-flavor (3-F) case \(q_u^l = q_d^l = q_s^l = q_0^l\). From Cooper-Frye scheme it is easy to see \(\Delta v_2 = v_2^{\Lambda} - v_2^{\bar{\Lambda}} \propto |q_0^l|A_{\Lambda}^2\) with...
$A_\Lambda^f = (N^\Lambda - N^{\bar\Lambda})/(N^\Lambda + N^{\bar\Lambda})$ the $\Lambda$-asymmetry that is directly related to background density $\mu_0$ (in analogy to a similar relation in CMW case [30]). The results for normalized flow splitting $\Delta v_2/\langle q_0^2 \rangle_{A_\perp}$ are shown in Fig. 2. Note that while the curves for the two centralities appear close, they have rather different normalization as the $\langle q_0^2 \rangle \sim V_f^2$ strongly depends on centrality. Needless to say, the present estimate is not yet a sophisticated hydrodynamic modeling of CVW which will be a future task. Given that, our results suggest that a CVW-induced signal could be experimentally measurable and may give indications on chiral effects of strange flavor.

Summary. — In summary we have found a new gapless collective excitation in a rotating fluid system with chiral fermions, named as the Chiral Vortical Wave. The CVW stems from the microscopic quantum anomaly and macroscopically arises from the interplay between vector and axial charge fluctuations induced by vortical effects. We derive the equation that governs the evolution of CVW from both hydrodynamic current equations and from chiral kinetic theory and determine the wave speed by using fluid thermodynamic quantities. We have demonstrated nontrivial CVW-induced flavor charge transport from different initial conditions. Using the rotating quark-gluon plasma in heavy ion collisions as a concrete example, we show the formation of induced charge transport from different initial conditions. Using

We have demonstrated nontrivial CVW-induced flavor effects. We derive the equation that governs the evolution of CVW from both hydrodynamic current equations and from chiral kinetic theory and determine the wave speed by using fluid thermodynamic quantities. We have demonstrated nontrivial CVW-induced flavor charge transport from different initial conditions. Using the rotating quark-gluon plasma in heavy ion collisions as a concrete example, we show the formation of induced flavor quadrupole in QGP and estimate the elliptic flow splitting effect for $\Lambda$ baryons. Such proposal could be experimentally measurable and may give indications on chiral effects of strange flavor.

Acknowledgments. — We thank Shu Lin and Yi Yin for discussions. The research of YJ and JL is supported by National Science Foundation (Grant No. PHY-1352368). The research of XGH is supported by Fudan University (Grant No. EZH1512519) and Shanghai Natural Science Foundation (Grant No. 14ZR1403000). JL also thanks the RIKEN BNL Research Center for partial support.

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