Calculating method of MTBF for integrated circuit

Ying Zhang¹, Jianye Wang, Yichao Zeng and Jialiang Zhang
Air and Missile Defense College Force Engineering University, Xi’an, China
¹Email: 18362965670@163.com

Abstract. The integrated circuit have a great use in the electronic equipment, and the mean time between failures (MTBF) is an important index to reflect the stability of the system, this paper provides a general method to calculate the MTBF of the integrated circuit contracted to the previous methods. The general rule of integrated circuit failure is first introduced, which satisfies the law of bathtub curve; secondly, verify that the mean time of failure of the integrated circuit satisfy the Poisson distribution. Based on the two conditions, an algorithm of calculating the MTBF of the integrated circuit is provided. According to the mean time of failure of the 74HC00, its mean time of failure is calculated. Finally, verified the reliability of the provided method.

1. Introduction
In today’s information age, with the continuous development of electronic technology, integrated circuits are widely used in electronic systems. Usually, the mean time between failures (MTBF) is used to evaluate the reliability of the integrated circuits, so the MTBF of the integrated circuit directly affects the stability of the system. If an integrated circuit failed in the system, it will cause immeasurable loss. Therefore, the MTBF of the integrated circuits has been widely concerned by many engineers.

Generally, there are three ways to calculate the MTBF of electronic products: The first is reliability calculation. The designer mainly designs the products according to the client's requirements. The design staff's mastery and understanding of the efficiency model and correction factor is the key to accurate prediction. The second method is laboratory tests and calculations. It is mainly the production unit to evaluate the reliability of the product through laboratory test. Laboratory reliability tests are destructive and cannot be tested on all products, so there may be errors between the real data and the test. The third method is the field failure count calculation. It is mainly about the products that are put into the market, and the general level of MTBF is calculated through the statistics of the maintenance rate. This method has certain value for verify the laboratory test data and the continuous production of the same kind of products, but the time lag is insurmountable.

According to the index of MTBF, this paper explores a more general way to calculate the MTBF of the integrated circuit [1].

2. Bathtub curve
The bathtub curve refers to the whole life cycle of the products from input to scrap, and its reliability changes as the use time increase. Practices have proved that most of the integrated circuit failure rate is a function of time, the typical failure curve is called a bathtub curve (Bathtub curve, the failure rate curve), the shape of the curve shows two head high, intermediate low, has obvious stage, its shape is shown in figure 1. Time is used as the abscissa of the curve, failure rate as the ordinate of a curve, the
failure rate of the product is divided into three stages: early stage failure, accidental failure and loss failure period. Because the curve’s two head is high, and the middle is low, some like a bathtub, so called the "bathtub curve".

![Bathtub Curve Diagram](image)

**Figure 1.** Schematic diagram of bathtub curve.

As we can see from the figure 1, the first stage is the early failure stage, the reliability of this stage is very poor, but the time is very short. The main reason for the poor efficiency is the defects in the design, raw materials and manufacturing processes. In order to shorten the time of this period, the products should be tested run before putting into operation, find the problem early and eliminate the unqualified products. The reliability of the products after this period is guaranteed and can be used normally.

The second stage is the random failure period, which is more reliable and stable, and can be approximated as a constant. As the unreliability factors have been eliminated in the last period. This period is described as the product reliability index. The products of this period are in normal service period. If there are breaking failures, most of them are caused by improper use and environmental impact.

The third stage is the loss of failure period. Beyond the normal service period of the product, the failure rate of this phase with the extension of time and increased sharply, mainly by the wear, fatigue, aging and causes such as loss. The wastage of the general system requirements more than 10 years period. To use normally, that need to check for safety hazards and repair it, or extend the life of the products.

3. **Verify the life of the integrated circuit obeys the poisson process**

The algorithm provided in this paper is based on the following assumptions:
(1) The failure rate function is a bathtub curve; (2) The life of the integrated circuit obeys the Poisson process. Assuming that (1) has been described in the above section, it is necessary to verify the existence of hypothesis (2).

3.1. **An index distribution**

Collect the required validation data, the required validation of the product data if can approximate as consistent with the distribution function, by the nature of the distribution function that we can predict the development trend and the distribution regularity of life. The common cumulative distribution functions are exponential distribution, Weibull distribution, normal distribution, logarithmic normal distribution, etc. Therefore, according to the collection data, we should analyze the occurrence rules of the study, and find out whether the fault changes with time as a random variable and see whether it conforms to a certain cumulative distribution function. An index distribution is a common form of cumulative distribution, it has several notable features: system failures are additive, if component failure in the system obeys exponential distribution, the failure time of the system also obeys exponential distribution, the probability theory has proved that exponential distribution is the only
continuous distribution with "no memory". Most of the electronic devices and electronic components conform to the laws of exponential distribution. The next step is to verify that the theory is correct [2].

A set of failure time data of 15 IC 74HC00 to verify the cumulative distribution function of the integrated can be considered as exponential distribution. In this article, the service life of the integrated circuit is the time before it comes to failure, table 1 shows the failure time of the integrated circuit, and follows the order from small to large, mathematical expectation formula of empirical distribution function values.

\[
F_n = \frac{i}{n+1}
\]  

(1)

In the formula, \(n\) is the number of experimental samples, and the number of failed samples in order of failure order.

| Invalid sample number | \(t_n(i)\) | \(F_n(i)\) |
|-----------------------|------------|------------|
| 1                     | 7275.8     | 0.0625     |
| 2                     | 7311.2     | 0.1250     |
| 3                     | 7316.1     | 0.1875     |
| 4                     | 7344.4     | 0.2500     |
| 5                     | 7343.7     | 0.3125     |
| 6                     | 7362.4     | 0.3750     |
| 7                     | 7372.3     | 0.4375     |
| 8                     | 7379.6     | 0.5000     |
| 9                     | 7374.4     | 0.5625     |
| 10                    | 7437.8     | 0.6250     |
| 11                    | 7439.8     | 0.6875     |
| 12                    | 7441.2     | 0.7500     |
| 13                    | 7511.3     | 0.8125     |
| 14                    | 7598.2     | 0.8750     |
| 15                    | 7614.4     | 0.9375     |

Exponential distribution function

\[
F(t) = 1 - e^{-\eta t} \quad (t \geq \gamma)
\]

(2)

In the formula, \(\gamma\) is the correlation coefficient, and \(\eta\) is a constant.

I'm going to sort it out and take the logarithm

\[
\ln \frac{1}{1-F(t)} = \frac{1}{\eta} t - \gamma
\]

(3)

So

\[
\begin{align*}
\gamma_i &= \ln \frac{1}{1-F(t)} \\
x_i &= t_i \\
a &= \frac{1}{\eta} \\
b &= -\frac{\gamma}{\eta}
\end{align*}
\]

(4)

After transformation, we get the standard linear equation
\[ y = ax + b \]  

(5)

Take the data in table 1 and substitute it into the equation, as shown in table 2.

Table 2. The transformation of \( x \) and \( y \).

| Invalid sample number \( i \) | \( x_i \)     | \( y_i \)     |
|-------------------------------|-------------|-------------|
| 1                             | 7275.8      | 0.0645      |
| 2                             | 7311.2      | 0.1335      |
| 3                             | 7316.1      | 0.2076      |
| 4                             | 7344.4      | 0.2877      |
| 5                             | 7343.7      | 0.3747      |
| 6                             | 7362.4      | 0.4700      |
| 7                             | 7372.3      | 0.5754      |
| 8                             | 7379.6      | 0.6931      |
| 9                             | 7374.4      | 0.8267      |
| 10                            | 7437.8      | 0.9808      |
| 11                            | 7439.8      | 1.1632      |
| 12                            | 7441.2      | 1.3863      |
| 13                            | 7511.3      | 1.6740      |
| 14                            | 7598.2      | 2.0794      |
| 15                            | 7614.4      | 2.7726      |

By formula

\[
\begin{align*}
\bar{x} &= \frac{1}{n} \sum_{i=1}^{n} x_i \\
\bar{y} &= \frac{1}{n} \sum_{i=1}^{n} y_i 
\end{align*}
\]

(6)

We can get that: \( \bar{x} = 7408.2 \), \( \bar{y} = 0.9126 \).

The resulting data is substituted into the formula

\[
\gamma = \frac{L_{xy}}{\sqrt{L_{xx} \times L_{yy}}} \\
L_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})(x_i - \bar{x}) = \sum_{i=1}^{n} x_i^2 - \frac{1}{n} (\sum_{i=1}^{n} x_i)^2 \\
L_{yy} = \sum_{i=1}^{n} (y_i - \bar{y})(y_i - \bar{y}) = \sum_{i=1}^{n} y_i^2 - \frac{1}{n} (\sum_{i=1}^{n} y_i)^2 \\
L_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n} x_i y_i - \frac{1}{n} (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i) 
\]

(7)

Coefficient of the correlation is: \( \gamma = 0.9806 \).

Because the correlation coefficient is less than or equal to 1, the dimensionless statistic, the value \( |\gamma| \) gets closer to 1, the more linear the relationship is. In this case, \( \gamma = 0.9806 \), it's very close to 1. Therefore, it can be considered that the cumulative distribution function of the life of the integrated circuit can be considered as the exponential distribution function [3].
3.2. Poisson process

The Poisson process is one of the basic mathematical models to describe the occurrence of random events, and most random events in real life or nature can be described by the Poisson process.

The input process is a sufficient and sufficient condition for the Poisson process of a parameter: successive arrival intervals are independent of each other, and they are all subject to the exponential distribution of parameters.

\[ P(T_i \leq t) = \begin{cases} 1 - e^{-\lambda t}, & t \geq 0 \\ 0, & t < 0 \end{cases} \]  
(8)

In 3.1, the cumulative distribution function of the integrated circuit's failure rate is verified to satisfy the exponential distribution function, and the failure of the same type of integrated circuit can be considered as independent. Therefore, the stochastic process of the service life of the integrated circuit can be considered as the Poisson process.

4. Calculation of MTBF of IC

The algorithm used in this paper is only applicable to the product with the function of the failure rate function as the bathtub curve and the life of the Poisson process. The previous statement has shown that the life of the integrated circuit is satisfied with the two conditions, so the following algorithm can be used to calculate its MTBF [4].

4.1. The failure rate function of the bathtub curve

Based on existing research results, the failure density function based on the bathtub curve has the following forms:

\[ f(t) = \gamma \beta (t/\alpha)^{\beta-1} \exp\{(t/\alpha)^\beta + \gamma \alpha (1-e^{(t/\alpha)^\beta})\} \]  
(9)

Corresponding failure rate function is:

\[ \lambda(t) = \gamma \beta (t/\alpha)^{\beta-1} \exp\{(t/\alpha)^\beta\} \]  
(10)

Reliability function is:

\[ R(t) = \exp\{\gamma \alpha (1-e^{(t/\alpha)^\beta})\} \]  
(11)

![Figure 2. Failure rate curve.](image)

As can be seen from figure 2, the failure rate of the product is obviously a bathtub curve characteristic, which can be used to describe the failure rate of the product with the characteristics of the bathtub curve.
4.2. MTBF solution

As described in chapter 1, based on the definition of MTBF we can get the following formula:

$$MTBF = \int_{0}^{\infty} R(t) dt \int_{0}^{\infty} f(\tau) d\tau dt$$  \hspace{1cm} (12)

An integrated circuit of 74HC00 was used to calculate the MTBF of the integrated circuit. Because its failure rate obeys the bathtub curve characteristic, the maximum likelihood function can be constructed by using the fault density function of the bathtub curve, estimating the distribution parameters and constructing the likelihood function.

$$L(\alpha, \beta, \gamma) = \gamma^n \beta^n \prod_{i=1}^{n} \left( \frac{t_i}{\alpha} \right)^{\beta-1} \times \exp \{ \sum_{i=1}^{n} \left( \frac{t_i}{\alpha} \right)^{\beta} + \sum_{i=1}^{n} \gamma \alpha (1 - \exp \left( \frac{t_i}{\alpha} \right)^{\beta}) \}$$ \hspace{1cm} (13)

Use the data in table 1 to pass the equations:

$$\begin{align*}
\frac{\partial \ln L}{\partial \gamma} &= \frac{n}{\gamma} + na - \sum_{i=1}^{n} \frac{e^{(t_i/\alpha)^{\beta}}}{\gamma} = 0 \\
\frac{\partial \ln L}{\partial \beta} &= \frac{n}{\beta} + \sum_{i=1}^{n} \frac{t_i}{\alpha} + \sum_{i=1}^{n} \left[ \frac{(t_i/\alpha)^{\beta} \ln (t_i/\alpha)}{\alpha} \right] - \\
\gamma \alpha \sum_{i=1}^{n} e^{(t_i/\alpha)^{\beta}} (t_i/\alpha)^{\beta} \ln (t_i/\alpha) &= 0 \\
\frac{\partial \ln L}{\partial \alpha} &= -\frac{n(\beta-1)}{\alpha} + n\gamma - \frac{1}{\alpha} \sum_{i=1}^{n} (t_i/\alpha)^{\beta} - \\
\gamma \sum_{i=1}^{n} \left[ e^{(t_i/\alpha)^{\beta}} (1 - (t_i/\alpha)^{\beta}) \right] &= 0
\end{align*}$$ \hspace{1cm} (14)

We can get that: $\alpha = 0.8$, $\beta = 0.03$, $\gamma = 1.5$, 

So that:

$$\lambda(t) = 0.045(t/0.8)^{0.97} \exp[(t/0.8)^{0.03}]$$ \hspace{1cm} (15)

$$R(t) = \exp[1.2(1 - e^{(t/0.8)^{0.03}})]$$ \hspace{1cm} (16)

We can get that the $MTBF = 7426$ hours in the definition of MTBF.

An integrated circuit that works for a long period of time (usually a decade or more) will enter a depletion period, which will cause damage. According to the given data, the failure time of the forecast data and practical usage of the integrated chip, this paper provides the algorithm is practical and has reached the expected purpose [5].

5. Conclusions

This paper provides a general method for calculating the MTBF of the integrated circuit and provides the basis for the reliability analysis of the circuit system [6]. Compared with the existing method of calculating the MTBF of the integrated circuit, this paper provides the method is versatile, just meet the calculations required conditions, and provide the corresponding integrated circuit, a set of fault time, calculated its MTBF, can be analyzed and then the forecast for the high reliability requirement of system has a very heavy reference, can be in advance to test the components, to prevent the failure. The next step is to study other indices for the MTBF of the integrated circuit, and calculate the value of the same integrated circuit in different indexes of MTBF, comprehensive analysis of data under different indicators, a more accurate prediction will be obtained.
References

[1] Son J B, Zhou S Y, Chaitanya S, Du X Y and Zhang Y L 2016 Remaining useful life prediction based on noisy condition monitoring signals using constrained Kalman filter Reliability Engineering and System Safety 152 38-50

[2] Liao L X and Felix K 2016 A hybrid framework combining data-driven and model-based methods for system remaining useful life prediction Applied Soft Computing 44 191-199

[3] Han L and Nadarajah N 2011 An Accelerated Test Method for Predicting the Useful Life of an LED Driver IEEE TRANSACTIONS ON POWER ELECTRONICS 26 2249-2257

[4] Pan D H, Liu J B and Cao J 2016 Remaining useful life estimation using an inverse Gaussian degradation model IEEE TRANSACTIONS 18 64-72

[5] Long B, Wang H J, Miao Q and Michael P 2012 Prognostics and Health Management Strategy for Complex Electronic Systems Prognostics & System Health Management Conference (PHM-2012 Beijing)

[6] Long B and Michael P 2013 Diagnostics and Prognostics Method for Analog Electronic Circuits IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS 60 1-14