Modeling Repeatedly Flaring \(\delta\) Sunspots

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Active regions (AR) appearing on the surface of the Sun are classified into \(\alpha\), \(\beta\), \(\gamma\), and \(\delta\) by the rules of the Mount Wilson Observatory, California on the basis of their topological complexity. Amongst these, the \(\delta\)-sunspots are known to be super-active and produce the most X-ray flares. Here, we present results from a simulation of the Sun by mimicking the upper layers and the corona, but starting at a more primitive stage than any earlier treatment. We find that this initial state consisting of only a thin sub-photospheric magnetic sheet breaks into multiple flux-tubes which evolve into a colliding-merging system of spots of opposite polarity upon surface emergence, similar to those often seen on the Sun. The simulation goes on to produce many exotic \(\delta\)-sunspot associated phenomena: repeated flaring in the range of typical solar flare energy release and ejective helical flux ropes with embedded cool-dense plasma filaments resembling solar coronal mass ejections.

Delta-sunspots are formed when two sunspots of opposite polarity magnetic field appear very close to each other and reside in the same penumbra, the radial filamentary structure outside the umbra region of the strongest magnetic fields. Strong shear and horizontal magnetic fields often exist at the polarity-inversion line separating the two polarities \([1]\). The subsurface processes which form the \(\delta\)-sunspots are still debated. Early observational studies \([2, 3]\) propose that \(\delta\)-sunspots form from collision-merging of topologically separate dipoles, while numerical simulations by \([4, 5]\) show that kink-unstable magnetic flux-tube – helical field lines winding around a central axis – emerging from the subsurface can have a \(\delta\)-sunspot like structure. More recently, attempts to model the \(\delta\)-spot in the NOAA AR 11158 utilized a uniformly twisted sub-surface flux-tube initially buoyant in two adjacent regions along its length \([6, 7]\). Also, \([8]\) found a magnetic flux concentration resembling a \(\delta\) sunspot in their stratified helical dynamo simulation. These studies did not report any flaring activity. On the other hand \([9]\) initialized their simulation with two parallel flux-tubes each lying at a different depth from the surface and with a different value of the initial magnetic twist which later evolved into a \(\delta\)-sunspot like structure and powered multiple reconnection events. Delta-sunspots are highly flare-productive – 95% of the strongest (X-class) X-ray flares originate from these regions \([3]\). Using a realistic numerical simulation \([10]\) showed that interaction between adjacent expanding magnetic dipoles pressing against each other can lead to the formation of strong current layers in the atmosphere which in turn lead to repeated flaring in the region. Here, we report on a three-dimensional magneto-hydrodynamic (MHD) simulation of the formation of a \(\delta\)-sunspot like region as a result of the break-up of a cool magnetic layer embedded in the upper solar convection zone into several flux-tubes due to the growth of three-dimensional unstable modes excited in the layer. This instability is well known in the literature as the undular instability (UI) \([11, 12]\). The magnetic layer initially inserted is thinner than the local pressure scale height – a precondition for UI. A detailed study of UI was performed by \([13, 14]\) in a very similar setup but without convection. They also calculated the amount of magnetic twist that is generated by the UI inside a thin magnetic layer with zero initial twist in a stratified, and rotating plasma. The latter study also found that the tubes formed are more twisted with increasing rotation. Also, the sign of the mean twist in the domain changes with the sense of the rotation vector. As it is not yet possible to observationally discern the subsurface structure of the sunspots, most simulations so far employ only uniformly twisted kink-unstable cylindrical flux tubes as the initial condition. The amount of twist applied is a free parameter and so are the segments where the tubes must be initially buoyant. Our simple initial condition alleviates the need for such free parameters.

We solve the equations of compressible magnetohydrodynamics in a \(36 \text{ Mm} \times 36 \text{ Mm} \times 25 \text{ Mm}\) Cartesian box using the higher-order finite difference code, the Pencil Code\([15]\). The box rotates with a solar-like angular velocity \(\Omega = 2.59 \times 10^{-6} \text{ s}^{-1}\) making an angle of 30° with the vertical z-direction. The box is resolved using a uniformly spaced grid with \(dx = dy = 96 \text{ km}\) and \(dz = 48 \text{ km}\). The initial state is a convectively relaxed state with the vertical profiles of density, \(\rho\), and temperature, \(T\), given by Figure 1(a). The domain consists of a sub-photospheric super-adiabatic layer in the lower 8.5 Mm of the box. The layer above (\(0 < z < 2 \text{ Mm}\)) is cooled by a radiative cooling term \(\propto \rho^2 \Lambda(T)\) in the entropy equation to drive the surface convection and mimic the photosphere and the chromosphere. The temperature in this layer connects via a transition region to an isothermal corona (3.5 Mm \(z < 16.5 \text{ Mm}\)) maintained at \(8.0 \times 10^5 \text{ K}\) by a Newtonian cooling term. Into this steady state atmosphere we introduce a horizontal magnetic sheet at \(z_0 = -7.75 \text{ Mm}\) with the magnetic field...
vector, $\mathbf{B}$, strongly oriented in the x-direction as shown in Figure 4(b). The horizontal extent of the sheet is about $-3 \text{ Mm} < y < 3 \text{ Mm}$ and the maximum half-width, $R$, is $0.3 \text{ Mm}$ at $y = 0$. The sheet is neutrally buoyant with respect to the surroundings to prevent it from rising immediately. However, for maintaining magneto-static equilibrium we cool it by reducing the specific entropy inside the sheet. The initial plasma-\(\beta\), which is the ratio of gas pressure to magnetic pressure is $\sim 0.6$. Simulations of the generation of such a magnetic sheet due to dynamo action and its subsequent break-up into buoyant flux-tubes have been performed [16]. It is also likely that the apex of an $\Omega$-shaped ribbon rising from deep inside the convection zone as in Figure 3 of [17] can be described in terms of such a sheet. Furthermore, we introduce an ambient magnetic field in the form of a potential field arcade at $z > 0$ also shown in Figure 4(b) to facilitate the perturbations formed due to the insertion of the strong magnetic sheet to travel along the field lines and out of the domain. Initially, the magnetic field at the photospheric foot points of the parallel arcade is $10 \text{ G}$. The lower boundary at $z = -8.5 \text{ Mm}$ is closed whereas the upper boundary only allows mass outflow with a vertical magnetic field condition. The x-boundaries are periodic, while the y-boundaries are perfectly conducting walls. Further details about the MHD equations, initial conditions, and the dissipation coefficients used here are given in [18].

Our numerical simulation was run for 263 min of solar time. We show the breaking up of the magnetic sheet into tubes and its subsequent evolution in Figure 2-b using volume rendering of a scalar quantity $B\rho^{-1/4}$, where $B$ is the magnetic field strength. Here, the subsurface convection excites several spatial modes at the onset of the UI. As a result, the wave number along x is between one and two and the wave number in y is larger than twelve, which is the number of tubes counted from Panel (b). The dominant mode excited will likely depend on the aspect ratio of the initial magnetic sheet as well as on the boundary conditions in the x-direction. The magnetic field strength inside the newly formed flux tubes at $z = -7.75 \text{ Mm}$ is about $25 \text{ kG}$. Figure 2(b) depicts the progenitors of the $\delta$-sunspot as separate fronts of positive and negative $B_z$ emerging at $z = 0$ which successively move closer. The progenitors consist of different flux systems even though we traced some field lines directly connecting the spots below the photosphere. Figure 3 shows snapshots of the photospheric vertical magnetic field before and after the formation of the $\delta$-sunspot region upon collision of the opposite polarities of similar sizes and magnetic field strength. The unsigned magnetic flux emerging into the black square reaches a maximum of $7.25 \times 10^{20} \text{ Mx}$ at $t = 200 \text{ min}$ and decreases slightly thereafter. The thread-like patterns of mixed polarities seen on both sides of the $\delta$-sunspot in the photospheric magnetogram indicate that there are two emerging and expanding bipolar regions side-by-side, the lateral extremes of which may be imagined to go beyond the periodic x-boundaries. The collision is marked by several flares. A solar flare occurs when magnetic energy is suddenly released in the form of heat, radiation, and energetic particle emission inside very thin current sheets - that are regions of very strong magnetic field gradients and thus sites where the magnetic field topology undergoes a major change. These flares thus reconnect field lines, ultimately leading to the components of the $\delta$-sunspot becoming increasingly directly connected by field lines above the region. This prevents the component polarities of the $\delta$-sunspot from separating during the rest of the evolution. Out of several flares we have been able to isolate only two strong ones, which can be located in Figure 3(a,b) by the white coloured contours of temperature at $z = 3.25 \text{ Mm}$. The plasma is heated to a maximum temperature of $2.5 \text{ MK}$ at this height where the average temperature is $54,000 \text{ K}$. The onset of the two flares can also be identified as the locations of peaks (dashed lines) in the temporal evolution of the magnetic energy, $\mathcal{E}_B$ (dashed-dotted line in Figure 4(a)), inside a sub volume of the domain with $0 < z < 16.5 \text{ Mm}$ and the horizontal extent demarcated by a black square in Figure 3(a). The flares are powered by the magnetic energy transported from the convection zone to the solar atmosphere. The rate of the magnetic energy input, or the Poynting flux, integrated over the faces of the same sub volume is, $\dot{\mathcal{E}}_{PF} = \int \mathbf{E} \times \mathbf{B} \cdot d\mathbf{S}/4\pi$ where, $c$ is the speed of light, $\mathbf{E}$ is the electric field vector, and $d\mathbf{S}$ is the area element. The time evolution of $\dot{\mathcal{E}}_{PF}$ is shown by the dashed-dotted line in Figure 4(a). The maximum possible flare energy, $\mathcal{E}_{\text{flare}}$ is related to the difference in magnetic energy, $\Delta \mathcal{E}_B$, after and before the flare as,

$$\Delta \mathcal{E}_B = -\mathcal{E}_{\text{flare}}^{\text{max}} + \int_{\Delta \mathcal{E}_{\text{flare}}} \dot{\mathcal{E}}_{PF} dt,$$

with $\Delta \mathcal{E}_{\text{flare}}$ being the duration of the flare. From Figure 4 as well as the animation of Figure 5, the flare occurring at $t = 167.5 \text{ min}$ lasts for 5 min and the one at $t = 197.2 \text{ min}$ lasts for 25 min. We estimate the magnetic energy release, $\mathcal{E}_{\text{flare}}^{\text{max}}$ in the two cases to be $3.3 \times 10^{29} \text{ ergs}$ and $1.7 \times 10^{30} \text{ ergs}$, respectively. The rate of energy release amounts to $1.1 \times 10^{27} \text{ ergs s}^{-1}$ in both cases, which agrees very well with the estimate made by [19] for a C-class flare that occurred on November 16, 2000. The magnetic energy dip at $t = 240.2 \text{ min}$ is due to the eruption of a highly twisted flux rope releasing at least $2.3 \times 10^{30} \text{ ergs}$. We also observe good temporal correlation between the onset of energy release and bipolar reconnection jets appearing as pairs of maximum negative and positive values ($\sim \pm 270 \text{ km s}^{-1}$) of vertical velocity (panel (b)).

The magnetic field lines at the key moments before and during the onset of flaring are visualised in Figure 5. Panel (a) illustrates the formation of a flux rope (FR-1) due to several reconnections above the large opposite po-
larities during the first of these flares at $t = 168.9$ min. Twenty-five minutes later an inverse-$S$-shaped flux-rope forms corresponding to a left-handed magnetic twist. The flux-rope erupts later, denoted as EFR-1 in panel (c) possibly because it has been de-stabilised due to reconnections with the almost vertical magnetic field in the approaching positive polarity spot. The field lines, with red (blue) corresponding to upward (downward) velocity, clearly indicate the presence of a bipolar reconnection jet. These field lines pass through a current sheet, shown by a pink surface, with a thickness of $4\delta z$. The cusped reconnected field loops formed at this stage develop into a new sigmoidal flux rope (FR-2) at $t = 240.2$ min (panel(d)). In panels (b)-(d), we note a sigmoid-cusp-sigmoid morphology transition over the region, also modelled by [20], and often observed in the coronal soft X-rays above the source regions of homologous eruptions [21]. Here, the inverse-$S$-shaped flux rope is formed due to reconnections inside the current sheet followed by the shearing photospheric foot point motions, flux convergence and cancellation; supporting earlier observations by [22, 23] and simulations of [24, 25], rather than sub-surface flux tubes of Figure 2 (b) bodily emerging into the photosphere. Figure 5 (e) shows strong shear at the polarity-reversal line between the spots which continuously pump twist and magnetic energy into the atmosphere in the form of a sustained Poynting flux $\sim 6 \times 10^8$ ergs s$^{-1}$ cm$^{-2}$ or $2 \times 10^{27}$ ergs s$^{-1}$ over the area of the box shown in Figure 3. This value compares well with Figure 2 of [26] where they calculate the Poynting flux for C-class flares producing NOAA AR 11560 within an area $\sim 145 \times 10^5$ Mm$^2$. After this time, the vertical-component of the Poynting flux at $z = 0$ is clearly dominated by the shearing foot-point motions rather than sub-surface flux emergence. We also note the formation and eruption of a dense and cool filament-like structure above the $\delta$-sunspot region between $z = 2.3 - 4.5$ Mm as shown in Figure 6 (a). At the beginning of the filament formation, the dense region grows, supported by the tilted field loops underneath. At a later time during the filament evolution, the tilted field loops develop dips into which the plasma flows. This scenario is similar to 2.5-dimensional simulation of funnel prominence formation from an arcade like geometry [27] but is distinct from the cavity prominences where dipped or concave field lines supporting the denser plasma pre exist as part of an emerged flux rope in the corona [28]. The temperature inside the filament at $z = 2.75$ Mm is $1.1 \times 10^5$ K which is one-third of the mean temperature in that layer. Hot plasma patches at $6.3 \times 10^5$ K exist close to the cooler filament. This phase separation of plasma into neighbouring regions of hot and cold has long been attributed to the onset of thermal instability due to the radiative loss function [29, 30]. Further, the evolution of
FIG. 3. Vertical component of the magnetic field, $B_z$, at $z = 0$ (shaded contours) at times indicated. Overlaid line contours (white) in (a) represent temperature contour levels at $z = 3.26$ Mm for values 90,000K, 1MK, 1.2MK whereas the respective temperature contour levels in (b) are at 90,000K, 1MK, 2.2MK. This figure is available as an animation at http://www.mn.uio.no/astro/english/people/aca/piyali/fluxemerge/fig3prl.mp4.

FIG. 4. Evolution of (a) magnetic energy, $E_B$ (solid), and $\dot{E}_{PF}$ (dashed-dotted) and that of (b) maximum positive (solid) and negative values (dashed) of vertical velocity above the active region. The three vertical lines denote the times of the two flares and a flux rope eruption respectively.

these plasma condensations in long low-lying flux tubes is governed by the presence of both steady as well as impulsive heating just above the chromosphere [51]. We refer the readers to [52] for a detailed review on filament formation and structure. We define a quantity, $DE$, to describe the fractional density enhancement in the filament as $\rho/\bar{\rho}$, where $\bar{\rho} = \exp((\ln \rho))$ with angular brack-

FIG. 5. (a)–(d) Magnetic field lines coloured by the vertical velocity at times indicated to illustrate the formation and eruption of two flux ropes (FR). The field lines traced in (c) pass through the current-layer iso-surface (pink) with a value $J/B = 1/4\delta z$, where $cJ/4\pi$ is the current density, $c$ being the speed of light. (e) Horizontal velocity vectors in the $\delta$-sunspot region. The grey shaded surface represents the vertical magnetic field at $z = 0$. A fly-by animation of panel (c) also including the sub-surface structure of the $\delta$-sunspot is available at http://www.mn.uio.no/astro/english/people/aca/piyali/fluxemerge/fig5c.mp4. Also available is an animation of the time evolution of the magnetic field in this region at http://www.mn.uio.no/astro/english/people/aca/piyali/fluxemerge/fig5prl.mp4.
ets denoting horizontal averaging. The region of plasma condensation has an inverse-$S$ shape and the maximum density is 116 times the ambient or $DE_{\text{max}} = 116$. The total mass, at this stage, inside a volume bounded by the $DE = 10$ surface is $1.2 \times 10^{13}$ kg. Panel (b) of Figure 6 shows a part of the filament erupting at a speed of 50 km s$^{-1}$ along with the twisted flux rope depicted in panel (d) of Figure 5. The eruption speed is lower than observed, likely because of the large viscosity and thermal diffusion used in this numerical simulation makes the conversion of the magnetic energy released to kinetic energy inefficient. The other possibility is the presence of a very weak pre-existing magnetic field incapable of confining the flux rope [33, 34] long enough to build sufficient non-potential magnetic free energy in the system before its eruption. The magnetic free energy is a measure of the maximum energy available to drive eruptions. The larger the free energy, the faster may be the ejecta speeds. This may explain the absence of more energetic flares of class M and X in the simulated $\delta$-spot.

This numerical simulation started from a very primitive configuration, making no assumptions about the properties of sub-surface flux tubes, demonstrates the formation of a $\delta$-sunspot from the collision of two or more young flux emerging regions developing in close vicinity. It is very similar to what is often seen in the solar photospheric magnetograms e.g., the widely studied active region with NOAA number 11158. However the two neighbouring regions in the vicinity occur not by mere chance, but emerge almost simultaneously as they are part of the same initial subsurface structure. The collision leads to repeated flaring which according to us causes the pair to lock together throughout the evolution even though a major part of the component $\delta$-spots originate from topologically different flux tubes in the subsurface. This result validates the observational finding of [35] and [3] for $\delta$-sunspots from solar cycle 22. Another striking common feature of several observed $\delta$-sunspots e.g. NOAA AR 11158, 10488 [36] and 10808 [37] and our simulation is the Yin-Yang structure of the interpenetrating positive and negative $B_z$ in the late evolutionary phase (inset of Figure 6 (b)). Even though the treatment of the solar atmosphere is very simplified here, we believe it captures the essential physics of magnetic flux emergence and evolution into a flaring $\delta$-sunspot. There is scope for improvement, for instance by including self consistent Ohmic heating of the corona, ionization, and detailed radiative transfer; this will be our future work.

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