STATE FEEDBACK FOR SET STABILIZATION OF MARKOVIAN JUMP BOOLEAN CONTROL NETWORKS

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ABSTRACT. In this paper, the set stabilization problem of Markovian jump Boolean control networks (MJBCNs) is investigated via semi-tensor product of matrices. First, the conception of set stabilization is proposed for MJBCNs. Then based on the algebraic expression of MJBCN, a necessary and sufficient condition for set stabilization is provided by a linear programming problem, which is simple to solve. Moreover, by solving this linear programming problem, an algorithm for designing a state feedback controller is developed. Finally, two examples are presented to illustrate the feasibility of the obtained results.

1. Introduction. Boolean networks (BNs) and Boolean control networks (BCNs), originally proposed by Kauffman for modeling gene regulatory networks [11], have recently witnessed an increasing interest. This renewed interest is mainly owing to two aspects: on the one hand, BNs and BCNs have been proved to be widely used in many fields, ranging from biology community [12], to game theory [22] and consensus problems [7]. On the other hand, a dramatic concept, semi-tensor product (STP), is introduced by Cheng and his colleagues. By virtue of this powerful algebraic tool, a BN (BCN) can be converted into a discrete-time linear system [2], whose states, inputs and outputs are all canonical vectors. Indeed, the variables in BNs (BCNs) only display two levels (off and on) and the interaction between them is determined by logical functions. By using STP, many fundamental control problems on BCNs are studied and excellent results have been derived, including stability and stabilization [36], optimal control [6], controllability and observability [3, 43, 44], and other research topics [37, 42, 38]. It is worth mentioning that STP also successfully applied in other various fields, such as game theory, logical networks, fuzzy systems, graph theory, etc. [24, 25, 26].

To describe the stochastic and uncertain nature of gene regulatory networks, the concept of probabilistic Boolean networks (PBN) is proposed by Shmulevich [33]. A PBN can be regarded as a switched BN whose switching signals have probability distribution. In recent years, lots of excellent study results have been obtained for
PBNs [23, 45, 13]. Since in gene regulatory, the evolutionary processes are always influenced by internal stimulus and environmental changes, Markovian jumping process has been employed to depict these dynamics [31, 32]. The Markovian jump Boolean network (MJBN) presented in [27] is a model for dynamically tracking the gene activity profile with uncertainty in the relationship between different genes. In an MJBN, for a given initial state, the subsequent states evolve according to a priori determined probabilities. Very recently, the stabilization problem of BNs with Markovian switching time delay and the controllability problem of MJBCNs have been investigated in [28] and [29], respectively. In fact, Markovian jump systems are one class of stochastic systems, about which, there are many results. For instance, Literature [34] discussed leader-following mean square consensus of stochastic multi-agent systems with input delay via event triggered control. Literature [40] studied output tracking control of delayed switched systems. The stabilization problem of BCNs with stochastic impulses is studied in [10].

Stability is a basic concept of any dynamic system. And it has been widely investigated for many years and still draw much research attention[41, 14, 4, 15]. As the generalization of stabilization problem, the set stabilization problem was proposed by Guo [8], which focus on whether a system converges to a target state set, instead of a single point. It soon caught the attention of many scholars and a large of results are obtained [16, 21, 17]. Moreover, in [8, 46], it has presented that many problems, including output tracking, partial stabilization and synchronization can be converted into set stabilization problem. The feedback control strategy is widely used in industry and biological systems. For example, Literature [1] considered a multistage feedback control strategy for the production of 1,3-propanediol(1,3-PD) in microbial fermentation. Literature [9] studied the set stabilization of PBCNs via state feedback control. Xu at.el. studied the set stabilization with probability one of PBCNs via sampled-date state feedback control in [39]. However, the set stabilization problem of MJBCNs is a challenging problem due to the combination of stochastic properties and switching properties. As an extension of PBCNs, the research on MJBCNs can enlarge the applications in genetic networks. Hence the set stabilization problem of MJBCNs is of great significance.

Motivated by the analysis above, we study the set stabilization of MJBCNs via state feedback control. The main contributions of this paper are summarized:

- We first introduce the conceptions of set stabilization and control invariant subset for MJBCNs. Meanwhile, an algorithm is obtained to calculate the largest control invariant subset of MJBCNs.
- Based on the largest control invariant subset, a necessary and sufficient condition for set stabilization with probability one is presented. And an equivalent condition for the existence of the feedback controller is given by the convex programming problem, which is facilitated to design the control law.
- A new set stabilization criterion based on convex programming is established and the controller designing process is considerably easier than by defining k-step reachable set [35].

The paper is organized as follows. Some preliminaries and the problem formulation are presented in Section 2. The main results are obtained in Section 3, including a necessary and sufficient condition for set stabilization of MJBCNs and a set of stabilizer designing scheme. Illustrative examples are given in Section 4. A brief conclusion is provided in Section 5.
2. Preliminaries.

2.1. Notations for logical functions. We first introduce some notations which will be used in this paper.

- \( \mathbb{R}^n \): the set of \( n \)-dimensional column real vectors.
- \( \mathcal{M}_{m \times n} \): the set of \( m \times n \) real matrices.
- \( \mathbb{N}_+ \): the set of positive integers.
- Given \( k, n \in \mathbb{N}_+ \) with \( k < n \), \([k, n]\) denotes the set \( \{k, k+1, \ldots, n\} \).
- \( \mathcal{B} := \{0, 1\} \) and \( \mathcal{B}^n := \mathcal{B} \times \ldots \times \mathcal{B} \).

- \( \delta_i^n \): the \( i \)-th column of \( I_n \), where \( I_n \) is the \( n \times n \) identity matrix.
- \( \Delta_n := \text{Col}(I_n) \) denotes all columns of \( I_n \).
- \( A_{ij} \): the \((i,j)\)-th entry of a matrix \( A \).
- \( 1_n \) and \( 0_n \) represent \( n \)-dimensional vectors which all entries are equal 1 and 0, respectively.
- \( 0_{m\times n} \): the \( m \times n \)-dimensional matrix which all entries are 0.
- \( |S| \): the number of elements in the set \( S \).
- \( A < (\leq, >, \geq) 0 \): all elements of a matrix or a vector \( A \) is negative (nonpositive, positive, nonnegative).

Note that there is a bijective correspondence between Boolean variable \( X \in \mathcal{B} \) and vector \( x \in \Delta_2 \), defined by

\[
x = \begin{bmatrix}
X \\
\neg X
\end{bmatrix}.
\]

\( x \) is called the vector form of Boolean variable \( X \). We first introduce the STP of matrices.

**Definition 2.1.** [2] Let \( A \in \mathcal{M}_{m \times n} \) and \( B \in \mathcal{M}_{p \times q} \). The STP of \( A \) and \( B \) is defined as:

\[
A \triangledown B = (A \otimes I_{\alpha/n})(B \otimes I_{\alpha/p}) \in \mathcal{M}_{(m\alpha/n) \times (q\alpha/p)},
\]

where \( \alpha \) is the least common multiple of \( n \) and \( p \) and \( \otimes \) is the Kroncker product.

This product is traditional product if \( n = p \), hence the STP is a generation of traditional product. In the following, the “\( \kappa \)” is omitted without of ambiguity and a more comprehensive analysis of STP can be found in [2].

By resorting to the STP, we can extend the previous bijective correspondence between \( \mathcal{B}^n \) and \( \Delta_2^n \). Thus the algebraic form of BNs can be obtained.

**Lemma 2.2.** [2] Let \( f(X_1, X_2, \ldots, X_n) : \mathcal{B}^n \to \mathcal{B} \) be a logical function. Then there exists a unique logical matrix \( L \in \Delta_2 \times \Delta_2^n \), called the structure matrix of \( f \), such that

\[
f(x_1, x_2, \ldots, x_n) = L \kappa_{i=1}^n x_i, \quad x_i \in \Delta_2,
\]

where \( \kappa_{i=1}^n x_i = x_1 \kappa x_2 \kappa \cdots \kappa x_n \).
2.2. Problem formulation. An MJBCN with $n$ nodes, $m$ control inputs and $\tau$ stochastic switching signals can be described by:

\[
\begin{align*}
X_1(t+1) &= f_1^{\tau(t)}(X(t), U(t)), \\
X_2(t+1) &= f_2^{\tau(t)}(X(t), U(t)), \\
& \vdots \\
X_n(t+1) &= f_n^{\tau(t)}(X(t), U(t)),
\end{align*}
\]  

(2)

where $X(t) := (X_1(t), X_2(t), \ldots, X_n(t)) \in \mathcal{B}^n$ denotes the $n$-dimension state variable, $U(t) := (U_1(t), U_2(t), \ldots, U_m(t)) \in \mathcal{B}^m$ is the $m$-dimension control input, and $f_i^{\tau(t)} : \mathcal{B}^{n+m} \to \mathcal{B}$, $i = [1, n]$ are logical functions. In this paper, the stochastic switching signal $\tau(t)$ is modeled by a finite homogeneous Markov chain that takes values in $\Omega = \{1, \ldots, \tau\}$. And the corresponding transition probability matrix $\Pi \in \mathcal{M}_{\tau \times \tau}$ is given by

\[
\pi_{ij} = P(\tau(t+1) = j | \tau(t) = i)
\]  

(3)

where $\pi_{ij} \geq 0$ and $\sum_{i=1}^{\tau} \pi_{ij} = 1$. For convenient, we assume that Markov chain $\tau(t)$ in this paper is irreducible, ensuring that each state of Markov process is reachable with probability from any other state in $\Omega$.

Let $x(t) = \kappa_{i=1}^n x_i(t) \in \Delta_{2^n}$ and $u(t) = \kappa_{i=1}^m u_i(t) \in \Delta_{2^m}$, according to Lemma 2.2, MJBCN (2) can be equally represented as the following algebraic form

\[
x(t+1) = L_{\tau(t)} x(t) u(t),
\]  

(4)

where $L_i \in \mathcal{L}_{2^n \times 2^n+m}$, $i \in [1, \tau]$ is the structure matrix of the $i$-th subnetwork. For simplicity, we denote $N = 2^n, M = 2^m$. Given a control sequence $U(u(0), u(1), \ldots)$, starting from the initial state $x(0)$, the trajectory of system (4) is denoted by $x(t; x(0), U)$.

Consider the type of closed-loop control for MJBCN (4) and the controller is described by

\[
u(t) = K_{\tau(t)} x(t),
\]  

(5)

where $K_i \in \mathcal{L}_{M \times N}$, $i \in [1, \tau]$.

Combine (4) and (5), we have the closed-loop system

\[
x(t+1) = L_{\tau(t)} (I_N \otimes K_{\tau(t)}) \Phi_N x(t),
\]  

(6)

where $\Phi_N = \text{diag}(\delta_N^1, \delta_N^2, \ldots, \delta_N^N)$ is the $N$-dimension power-reducing matrix. Denote $L_i = L_i(I_N \otimes K_i) \Phi_N$.

The aim of this paper is to present a necessary and sufficient condition for the existence of control law (5), which can drive system (4) to converge on a target state set forever from any initial state. Based on this condition, the corresponding control gain matrices are designed.

3. Main results. First, the definitions of set stabilization and control invariant subset for MJBCNs are given.

**Definition 3.1.** Let $\mathcal{M} \subset \Delta_N$, MJBCN (4) is said to be $\mathcal{M}$-stabilization with probability one if for any initial state $x(0)$ and any distribution of $\tau(0)$, there exists a control sequence $U$ such that

\[
\lim_{t \to \infty} P\{x(t; x(0), U) \in \mathcal{M} | x(0) \in \Delta_N\} = 1.
\]  

(7)
Particularly, MJBCN (4) is called $\mathcal{M}$-stabilization with probability one by state feedback if the control sequence $U$ is determined by control law (5).

**Remark 1.** As the PBCN is a special case of MJBCN, Definition 3.1 is a natural extension of the existing set stabilization of PBCNs [47]. When $\mathcal{M}$ is a signal point, Definition 3.1 degenerates to the definition of stabilization with probability one of MJBCN (4).

**Definition 3.2** (Control invariant subset). Let $S \subset \Delta_N$, $S$ is called a control invariant subset of MJBCN (4) if for any initial state $x(0) \in S$ and any distribution of $\tau(0)$, there exists a control sequence $U$ determined by (5) such that $P\{x(1; x(0), U) \in S\} = 1$.

The union of two control invariant subsets is still a control invariant subset. For a given subset $\mathcal{M} \in \Delta_N$, the union of all control invariant subsets contained in $\mathcal{M}$ is called the largest control invariant subset of $\mathcal{M}$ for MJBCN (4) and is denoted by $I_c(\mathcal{M})$. Based on the logical matrices $L_i, i \in [1, \tau]$, Algorithm 1 is designed to determine $I_c(\mathcal{M})$. First split $L_i = [L_{i1}, L_{i2}, \ldots, L_{iN}], $ where $L_{ij} \in \mathcal{L}_{N \times M}$. The main idea of Algorithm 1 is excluding method. For given state set, we first exclude the point that cannot be reachable from initial state in given state set at one step for every subsystem, which is shown in Line 3. Then the remaining state set can be regarded as the given set. Repeat the above process, which is finite since set $\mathcal{M}$ is finite. The most advantages of this algorithm is avoiding the probability in computation. Lemma 3.3 has explained the set obtained in Algorithm 1 is the largest control invariant subset.

**Algorithm 1** Find $I_c(\mathcal{M})$

**Require:** : Subset $\mathcal{M}, \Omega$

**Ensure:** : $S_w$

1: Initialize $S_0 = \mathcal{M}$
2: while $k \leq |\mathcal{M}|$ do
3: \hspace{1em} $S_{s+1} = \{\delta^i_N \in S_s | \text{Col}(L_{ij}) \cap S_s \neq \emptyset, \forall i \in \Omega\}$
4: \hspace{1em} end while
5: if $S_{s+1} = S_s$ then
6: \hspace{1em} return $S_w = S_s$
7: \hspace{1em} end if

**Lemma 3.3.** The set $S_w$ obtained in Algorithm 1 is the largest control invariant subset of $\mathcal{M}$.

**Proof.** By the construction of $S$ in Line 3, for any $\delta^i_N \in S_w$ and any $i \in [1, \tau]$, there must be an $ij$ such that $\text{Col}_{ij}(L_{ij}) \in S_w$. Noting that $\text{Col}_{ij}(L_{ij}) = L_i \delta^i_N \delta^i_M$, hence there must be $\delta^i_M$ such that $P\{L_i \delta^i_N \delta^i_M \in S_w\} = 1$ for any $i \in [1, \tau]$. Consequently, $P\{x(1) = L_\tau(0) \delta^i_N \delta^i_M \in S_w\} = \sum_{i=1}^\tau w_i(0)P\{L_i \delta^i_N \delta^i_M \in S_w\} = 1$ for any distribution $w(0) = (w_1(0), \ldots, w_\tau(0))^T$ of $\tau(0)$. Therefore, $S_w$ is a control invariant subset.

On the other hand, assume that there exists another control invariant subset $S_\star \not\subset S_w$. According to the construction of $S_w$ in Line 3, we have sequence $S_w \subset \cdots \subset S_1 \subset S_0$. The sequence is finite since $|\mathcal{M}|$ is finite. Since $S_\star \subset S_0$, there must be an integer $v$ such that $S_v \subset S_{v+1}$ and $\tau(0) = i$,

\[ S_v \subset S_{v+1} \quad \tau(0) = i, \]
there exists $\delta^i_M$ such that $P\{x(1; \delta^i_N, \delta^i_M) = L_i \delta^i_N \delta^i_M \in S_\nu\} = 1$ by the definition of control invariant subset. Consequently $\text{Col}(L_{ij}) \cap S_\nu \neq \emptyset$ for all $i \in \Omega$. Then $\text{Col}(L_{ij}) \cap S_\nu \neq \emptyset$. Therefore we have $\delta^i_N \in S_{\nu+1}$ because of construction of $S_{\nu+1}$. That contradicts $\delta^i_N \in S_\nu \setminus S_{\nu+1}$.

**Remark 2.** The largest control invariant subset for system (4) is in fact the largest common control invariant subset of all subsystems. Furthermore, Algorithm 1 can be directly applied to calculate the largest invariant subset of a given set for the closed-loop system (6). The computational complexity of Algorithm 1 is $O(M|\mathcal{M}|^3)$.

According to Algorithm 1, the corresponding state feedback gain matrices can be designed. Let

$$P_i(j) = \{\delta^i_M | \exists i_j \text{ s.t. Col}_{ij}(L_{ij}) \in I_c(\mathcal{M})\}, j \in I_c(\mathcal{M}), i \in \Omega. \quad (8)$$

Then matrices $K_i, i \in \Omega$ is designed by $\text{Col}_{ij}(K_i) \in P_i(j), j \in I_c(\mathcal{M})$.

Based on the largest control invariant subset $I_c(\mathcal{M})$, the following result is given.

**Proposition 1.** Let $\mathcal{M} \subset \Delta_N$, $MJBCN$ (4) is $\mathcal{M}$-stabilization with probability one under control law (5) if and only if this system is $I_c(\mathcal{M})$-stabilization with probability one under the same controller.

**Proof.** Assume that for any initial state $x_0$ and any distribution of $\tau(0)$, there exists a sequence of control gain matrices $K_i, i \in \Omega$ such that $\lim_{t \to \infty} P\{x(t) \in M\} = 1$. Then there must be a subset $S \subset \mathcal{M}$ satisfying

$$\lim_{t \to \infty} P\{x(t) \in S\} = 1, \quad (9)$$

$$\lim_{t \to \infty} P\{x(t) = \delta^i_N\} \neq 0, \forall \delta^i_N \in S. \quad (10)$$

We claim that $S$ is a control invariant subset of system (4).

Consider the closed-loop system (6) and review the results of Markov chain [5], we denote $\xi_i(t) = E\{x(t)1_{\tau(t)=i}\} \in \mathbb{R}^N$, where $1_{\tau(t)=i}$ stands for the indicator function of the set $\{\tau(t) = i\}$. Obviously the expected value $E\{x(t)\} = \sum_{i=1}^{\tau} \xi_i(t)$.

We can obtain

$$\xi_j(t+1) = \sum_{i=1}^{\tau} E\{x(t+1)1_{\tau(t+1)=j}1_{\tau(t)=i}\}$$

$$= \sum_{i=1}^{\tau} E\{\pi_{ij}L_i x(t)1_{\tau(t)=i}\}$$

$$= \sum_{i=1}^{\tau} \pi_{ij}L_i \xi_i(t). \quad (11)$$

Split $L_i = [L_{i1}, L_{i2}, \ldots, L_{iN}]$, then we have

$$[\xi_j(t+1)]_k = \sum_{i=1}^{\tau} \pi_{ij}[L_i \xi_i(t)]_k$$

$$= \sum_{i=1}^{\tau} \sum_{v=1}^{N} \pi_{ij}[L_{iv}]_k[\xi_i(t)]_v.$$
Since $x(t) \in \Delta_N$, combing equations (9) and (10), we have
\[
1 = 1^T_N x(t)
= \sum_{\delta_N^k \in S} \lim_{t \to \infty} P\{x(t) = \delta_N^k\}
= \sum_{j=1}^{\tau} \sum_{\delta_N^k \in S} \lim_{t \to \infty} [\xi_j(t + 1)]_k
= \sum_{i=1}^{\tau} \sum_{\delta_N^k \in S} (\sum_{j=1}^{\tau} \sum_{\delta_N^k \in S} \pi_{ij}[T_{iv}]_k) \lim_{t \to \infty} [\xi_i(t)]_v.
\]
(12)

Recalling back to the assumption in this paper, Markov chain $\tau(t)$ is irreducible, which means that for any $\delta_N^k \in \Delta_N$, if $\lim_{t \to \infty} P\{x(t) = \delta_N^k\} \neq 0$, then $\lim_{t \to \infty} [\xi_i(t)]_j = 0$ for all $i \in \Omega$. Considering $\lim_{t \to \infty} [\xi_i(t)]_v = 1$, we obtain
\[
\sum_{i=1}^{\tau} \sum_{\delta_N^k \in S} \pi_{ij}[T_{iv}]_k = \sum_{j=1}^{\tau} \pi_{ij} P\{T_i \delta_N^v \in S\} = 1
\]
for all $i \in \Omega$. According to $\sum_{j=1}^{\tau} \pi_{ij} = 1$, $P\{T_i \delta_N^v \in S\} = 1$ for all $\delta_N^v \in S$. Consequently, $S$ is a control invariant subset for MJBCN (4). Subsequently, $I_c(M)$-stabilization with probability one under controller (5), the considered system must be $I_c(M)$-stabilization with probability under the same control law.
The necessity is obviously by $I_c(M) \subseteq M$. \hfill \Box

Without loss of generality, assume that $I_c(M) = \{\delta_N^1, \ldots, \delta_N^s\}$ and $|I_c(M)| = s$, otherwise, a coordinate transformation guarantees this assumption. Split the state $x(t) \in \Delta_N$ as $x(t) = (w^T(t), v^T(t))^T$ with $w(t) \in \mathbb{R}^s$ and $v(t) \in \mathbb{R}^{N-s}$, then the closed-loop system (6) can be equivalently expressed by
\[
\begin{cases}
w(t + 1) = T_{i1} \tau_{11} w(t) + T_{i2} \tau_{12} v(t) \\
v(t + 1) = T_{i2} \tau_{21} w(t) + T_{i2} \tau_{22} v(t)
\end{cases}
\]
(13)

where
\[
\begin{bmatrix}
T_{i1} & T_{i2} \\
L_{21} & L_{22}
\end{bmatrix} = T_i, \ T_{i1} \in \mathcal{M}_{s \times s}, \ T_{i2} \in \mathcal{M}_{(N-s) \times (N-s)}.
\]

Based on the largest control invariant subset $I_c(M)$ and the above system (13), the following result can be easily established.

**Lemma 3.4.** MJBCN (4) is $I_c(M)$-stabilization with probability one under controller (5) if and only if for any initial state $x(0)$ and any distribution of $\tau(0)$, there exists a control sequence of matrices $K_i, i \in [1, \tau]$ such that
\[
\lim_{t \to \infty} E\{v(t)\} = 0_{N-s}.
\]
(14)

**Proof.** In MJBCN (4), if there exists a sequence of matrices $K_i, i \in [1, \tau]$ such that for any initial state $x(0) \in \Delta_N$ and any distribution of $\tau(0)$, $\lim_{t \to \infty} P\{x(t) \in \Delta_N\}$ converges to one with probability one. Therefore, for any initial state $x(0) \in \Delta_N$ and any distribution of $\tau(0)$, the considered system must be $I_c(M)$-stabilization with probability under the same control law.

The necessity is obviously by $I_c(M) \subseteq M$. \hfill \Box
\( I_c(\mathcal{M}) \) = 1, that means \( \lim_{t \to \infty} E\{1_\mathcal{M}^T w(t)\} = 1 \). We have
\[
1 = 1_N^T x(t) = \lim_{t \to \infty} E\{1_\mathcal{M}^T w(t) + 1_{\mathcal{M}}^T v(t)\} = 1 + \lim_{t \to \infty} 1_N^T E\{v(t)\},
\]
hence \( \lim_{t \to \infty} E\{v(t)\} = 0_{N-s} \).

On the other hand, if \( \lim_{t \to \infty} E\{v(t)\} = 0_{N-s} \), then \( \lim_{t \to \infty} 1_N^T E\{w(t)\} = 1 \) according to (15). We have \( \lim_{t \to \infty} P\{x(t) \in I_c(\mathcal{M})\} = 1 \). Therefore MJBCN (4) is \( I_c(\mathcal{M}) \)-stabilization under the controller (5).

Next we present the main result of this section.

**Theorem 3.5.** Let \( \mathcal{M} \subseteq \Delta_N \), MJBCN (4) is \( \mathcal{M} \)-stabilization with probability one under control law (5) if and only if there exists a sequence of matrices \( K_i, i \in \Omega \) such that the following conditions are satisfied:

1. The largest control invariant subset \( I_c(\mathcal{M}) \) is not empty.
2. Assume that \( I_c(\mathcal{M}) = \{\delta^L_N, \delta^L_{N-1}, \ldots, \delta^L_1\} \), there exist vectors \( \lambda_i \in \mathbb{R}^{N-s}, i \in [1, \tau] \) satisfying
\[
\sum_{i=1}^{\tau} \pi_{ij} T_{22}^i \lambda_i - \lambda_j < 0, \tag{16}
\]
\[
\lambda_i > 0. \tag{17}
\]

**Proof.** Sufficiency. If MJBCN (4) is \( \mathcal{M} \)-stabilization with probability one under control law (5), Proposition 1 has proved the existence of \( I_c(\mathcal{M}) \). Then for any \( \delta^L_N \in \Delta_N \), there must be a sequence of matrices \( K_i, i \in [1, \tau] \) such that the closed-loop system (6) satisfying \( L_i \delta^L_N \subseteq I_c(\mathcal{M}) \) for all \( i \in \Omega \). Therefore, \( L_{21} = 0_{(N-s) \times s} \).

Denote \( \alpha_i(t) = E\{w(t) 1_{r(t)=i}\} \) and \( \beta_i(t) = E\{v(t) 1_{r(t)=i}\} \), we have \( E\{w(t)\} = \sum_{t=1}^{\tau} \alpha_i(t), E\{v(t)\} = \sum_{t=1}^{\tau} \beta_i(t) \) and
\[
\alpha_j(t+1) = E\{w(t+1) 1_{r(t+1)=j} 1_{r(t)=i}\} = \sum_{t=1}^{\tau} \pi_{ij} T_{11}^i \alpha_i(t) + \sum_{t=1}^{\tau} \pi_{ij} T_{12}^i \beta_i(t). \tag{18}
\]

In the same way, we have
\[
\beta_j(t+1) = \sum_{t=1}^{\tau} \pi_{ij} T_{21}^i \alpha_i(t) + \sum_{t=1}^{\tau} \pi_{ij} T_{22}^i \beta_i(t). \tag{19}
\]

Submitting \( T_{21}^i = 0_{(N-s) \times s} \) into (19), this equation can be expressed as
\[
\beta_j(t+1) = \sum_{t=1}^{\tau} \pi_{ij} T_{22}^i \beta_i(t). \tag{20}
\]

Denote \( \beta(t) = (\beta_1^T(t), \beta_2^T(t), \ldots, \beta_\tau^T(t))^T \), then we obtain
\[
\beta(t+1) = \Gamma \beta(t), \tag{21}
\]
where \( \Gamma = (\Pi^T \otimes I_{N-s}) \text{ diag}\{T_{22}^1, T_{22}^2, \ldots, T_{22}^\tau\} \).
According to Lemma 3.4, \( \lim_{t \to \infty} E\{v(t)\} = 0_{N-s} \). Therefore we have \( \lim_{t \to \infty} \beta(t) = 0_{\tau(N-s)} \). Noting that \( H \geq 0 \) and \( \beta(t) \geq 0 \), the system (21) is positive and stable. According to [18], there exists a positive vector \( \lambda = (\lambda_1^T, \lambda_2^T, \ldots, \lambda_T^T) > 0 \) with \( \lambda_i \in \mathbb{R}^{N-s} \), \( i \in \Omega \) such that

\[
(\Gamma - I_{\tau(N-s)})\lambda < 0.
\]  
(22)

Then we have inequalities (16) and (17).

Necessity. If \( I_c(\mathcal{M}) \neq \emptyset \), there exist \( K_i, i \in [1, \tau] \) such that the closed-loop system (6) satisfying \( \mathcal{T}_{21} = 0_{(N-s) \times s} \), then we have the positive system (21). Since there exists \( \lambda = (\lambda_1^T, \lambda_2^T, \ldots, \lambda_T^T) > 0 \in \mathbb{R}^{\tau(N-s)} \), \( \lambda_i > 0 \) such that (16) is satisfied. According to [18], the positive system (21) is stable. Therefore \( \lim_{t \to \infty} E\{v(t)\} = 0_{N-s} \). Then MJBCN (4) is \( I_c(\mathcal{M}) \)-stabilization with probability one under control law (5) by virtue of Lemma 3.4. That completes the proof.

Remark 3. Recalling back to the equation (11), let \( \xi(t) = (\xi_1^T(t), \xi_2^T(t), \ldots, \xi_T^T(t))^T \), we obtain

\[
\xi(t + 1) = \mathcal{F}\xi(t),
\]  
(23)

where \( \mathcal{F} = (\Pi^T \otimes I_N) \text{diag}\{L_1, L_2, \ldots, L_{\tau}\} \), and \( \mathcal{F} \) is a transition probability matrix. Then for any given matrices \( K_i, i \in [1, \tau] \), the closed-loop system (6) can be equivalently converted into a PBN with 1-step transition probability matrix \( \mathcal{F} \). While this method will enlarge the dimension of state spaces from \( N \) to \( \tau N \), making it difficult to construct \( k \)-reachable set.

In Theorem 3.5, the condition for existence of controller is provided by a linear programming problem with \( \tau(N-s) \) variables, and the computational complexity of this problem is \( O(\tau(N-s)) \) [30].

Algorithm 1 has provided the control law to guarantee that \( I_c(\mathcal{M}) \) is a control invariant subset for MJBCN (4). Then the control gain matrices can be designed as

\[
K_i = \delta_M[i_1, \ldots, i_s, i_{s+1}, \ldots, i_N],
\]

where \( \delta_M^j \in \mathcal{P}_i(j), j \in [1, s], i \in \Omega \), \( \mathcal{P}_i(j) \) is well defined in (8). However, it is difficult to design \( i_j, j \in [s+1, N] \), \( i \in [1, \tau] \) since every control gains \( K_i \) is a \( 0 \)-1 matrix and coupled by \( \lambda_i \) from Theorem 3.5. Therefore, we will give another equivalent result to design the control law. First, partition \( L_i \in \mathcal{L}_{N \times NM} \)

\[
L_i = \begin{bmatrix} L_{11}^i & L_{21}^i \\ L_{12}^i & L_{22}^i \end{bmatrix},
\]  
(24)

where \( L_{11}^i \in \mathcal{M}_{s \times s} \) and \( L_{22}^i \in \mathcal{M}_{(N-s) \times (N-s)} \) for all \( i \in \Omega \).

Observe \( \mathcal{L}_i = L_i(I_N \otimes K_i)\Phi_N \). That is

\[
\mathcal{L}_i = L_i \text{diag}\{K_i, \cdots, K_i\} \text{diag}\{\delta_N^1, \cdots, \delta_N^N\}
= L_i \text{diag}\{\text{Col}_1(K_i), \cdots, \text{Col}_N(K_i)\}.
\]  
(25)

Hence \( \mathcal{L}_{11}^{(i)} \) and \( \mathcal{L}_{22}^{(i)} \) can be written as

\[
\mathcal{L}_{11}^{(i)} = L_{11}^{(i)} \text{diag}\{\text{Col}_1(K_i), \cdots, \text{Col}_s(K_i)\}
\]  
(26)

\[
\mathcal{L}_{22}^{(i)} = L_{22}^{(i)} \text{diag}\{\text{Col}_{s+1}(K_i), \cdots, \text{Col}_N(K_i)\}.
\]  
(27)

Then the following result is derived.
Theorem 3.6. Let $M \subset \Delta_N$, MJBCN (4) is $\mathcal{M}$-stabilization with probability one under control law (5) if and only if there exists a sequence of matrices $K_i$, $i \in \Omega$ such that the following conditions are satisfied:

1. The largest control invariant subset $I_c(M)$ is not empty.
2. Assume that $I_c(M) = \{\delta_{i1}, \delta_{i2}, \ldots, \delta_{iN}\}$, there exist vectors $\lambda_i = (\lambda_{is+1}, \lambda_{is+2}, \ldots, \lambda_{iN})^T \in \mathbb{R}^{N-s}$, $i \in \Omega$ and $p_{i,j} \in \mathbb{R}^M$, $i \in \Omega$, $j \in [s+1, N]$ satisfying

$$\sum_{i=1}^{\tau} \pi_{ij} L_{22}^i p_{i} - \lambda_j < 0, \tag{28}$$

$$\lambda_{i,k} \leq \|p_{i,k}\|_\infty, \forall i \in \Omega, k \in [s+1, N], \tag{29}$$

$$\lambda_i > 0, \tag{30}$$

where $p_i = (p_{i,s+1}^T, p_{i,s+2}^T, \ldots, p_{i,N}^T)^T \in \mathbb{R}^{M(N-s)}$. Moreover, the control gain matrices $K_i = \delta_M[i_1, i_2, \ldots, i_N], i \in \Omega$ can be computed as $\text{Col}_j(K_i) \in \mathcal{P}_i(j)$ if $j \in [1, s]$ and

$$\text{Col}_j(K_i) = \delta_{ij}^i_M, \quad j \in [s+1, N], i \in \Omega, \tag{31}$$

where the $i_j$-th element is the maximal element of vector $p_{i,j}$.

Proof. If MJBCN (4) is $\mathcal{M}$-stabilization with probability one, then by Theorem 3.5, there must be vectors $\lambda_i \in \mathbb{R}^{N-s}$, $i \in \Omega$ satisfying (16) and (17). Submitting (27) into Inequality (16), we obtain

$$\sum_{i=1}^{\tau} \pi_{ij} L_{22}^i \lambda_i - \lambda_j < 0,$$

Let $p_{i,k} = \lambda_{i,k} \text{Col}_k(K_i) \geq 0$. Noting that $K_i$ is a logical matrix, only one element is nonzero in $\text{Col}_k(K_i)$. Hence Inequality (29) is satisfied.

On the other hand, if inequalities (28)–(30) have a feasible solution set, $p_{i}, i \in \Omega$ and $\lambda_j, j \in \Omega$. Owing to the computation of $K_i$ in (31), $\lambda_{ij} \text{Col}_j(K_i) \leq p_{ij}$. Consequently, (16) and (17) are satisfied. By Theorem 3.5, MJBCN (4) is $\mathcal{M}$-stabilization with probability one.

Remark 4. The condition 2 in Theorem 3.6 is the form of convex programming with $\tau(M+1)(N-s)$ variables. Meanwhile, the $i_j$-th row of control gains $K_i = [i_1, \ldots, i_s, i_{s+1}, \ldots, i_N], i \in [1, \tau], j \in [s+1, N]$ can also be computed by a feasible solution of the convex programming problem (28)-(30), which can be simple solved.

4. Illustrative examples. In this section, two examples are given to present the feasibility of our obtained results.
Example 1. Consider an MJBCN with $n = 3$, $m = 1$ and $\Omega = \{1, 2\}$. And two subsystem of this MJBCN are described as
\[
\begin{align*}
X_1(t) &= X_1(t) \lor X_3(t), \\
X_2(t) &= X_1(t) \lor X_3 \lor U(t), \\
X_3(t) &= (X_2(t) \land X_3(t)) \lor U(t), \\
X_1(t) &= X_1(t) \lor X_3(t), \\
X_2(t) &= X_2(t) \lor X_3, \\
X_3(t) &= X_2(t) \lor U(t).
\end{align*}
\]
The switching signal $\tau(t)$ is modeled by an irreducible Markov chain with the transition probability matrix being $\begin{pmatrix} 0.1 & 0.9 \\ 0.7 & 0.3 \end{pmatrix}$.

According to Lemma 2.2, the algebraic form of this system can be expressed as
\[
x(t+1) = L_{\tau(t)}x(t)u(t),
\]
\[
L_1 = \delta_8[1, 1, 1, 1, 5, 1, 6, 4, 1, 2, 2, 5, 1, 6, 4],
\]
\[
L_2 = \delta_8[1, 1, 1, 1, 5, 1, 5, 1, 3, 3, 4, 4, 5, 1, 6, 2],
\]
are the structure matrices of two subsystems. Now consider the problem of finding a control law of form $u(t) = K_{\tau(t)}x(t)$ such that the system is $\mathcal{M} = \{\delta_8, \delta_8^2, \delta_8^0\}$-stabilization with probability one.

First by Algorithm 1, we can find the largest control invariant subset $I_c(\mathcal{M}) = \{\delta_8^1, \delta_8^2\}$, and $P_1(1) = \{\delta_8^1, \delta_8^2\}, P_2(1) = \{\delta_8^1, \delta_8^2\}, P_1(2) = \{\delta_8^1, \delta_8^2\}, P_1(2) = \{\delta_8^1, \delta_8^2\}$. Then based on Theorem 3.6, we can find a feasible solution for (28)-(30),
\[
\begin{align*}
\lambda_1 &= [3.804, 3.705, 2.31, 1, 1, 1], \\
\lambda_2 &= [1.636, 7.945, 2.71, 2.11, 1, 1], \\
p_1 &= [1, 6.608, 1, 6.41, 1, 3.62, 1, 1, 1, 1, 1, 1], \\
p_2 &= [1, 2.272, 1, 14.89, 1, 4.42, 1, 3.22, 1, 1, 1, 1].
\end{align*}
\]
Then control gain matrices can be computed as:
\[
\begin{align*}
K_1 &= \delta_2[1, 1, 2, 2, 2, 1, 1, 1], \\
K_2 &= \delta_2[1, 1, 2, 2, 2, 2, 1, 1].
\end{align*}
\]
Taking initial conditions $x(0) = \delta_8^6$ and $\tau(0) = 2$, the state trajectory is shown in Figure 1, where $P\{h(t) = 4\} = 7 \times 10^{-1}, t \geq 2$. We can see that $\lim_{t \to \infty} P\{x(t) = \delta_8^1\} = 1$.

Example 2. Consider a reduced model of the lac operon in the Escherichia coli [19],
\[
\begin{align*}
X_1(t) &= -U_1(t) \land (X_2(t) \lor X_3(t)), \\
X_2(t) &= -U_1(t) \land U_2(t) \land X_1(t), \\
X_3(t) &= -U_1(t) \land (U_2(t) \lor (U_3(t) \land X_1(t))).
\end{align*}
\]
where state variables $X_1(t)$, $X_2(t)$, $X_3(t)$ represent lac mRNA, lactose, medium concentration of lactose, respectively. Input variables $U_1(t), U_2(t), U_3(t)$ denote the extracellular glucose, low concentration of extracellular lactose, and medium concentration of extracellular lactose, respectively.
When $U_1(t) \equiv 0$, the system is as follows
\[
\begin{cases}
X_1(t) = X_2(t) \lor X_3(t), \\
X_2(t) = U_1(t) \land X_1(t), \\
X_3(t) = U_1(t) \lor (U_2(t) \lor X_1(t)).
\end{cases}
\]

As considered in [20], we assume that $X_3(t)$ does not update its value, then this system is depicted as
\[
\begin{cases}
X_1(t) = X_2(t) \lor X_3(t), \\
X_2(t) = U_1(t) \land X_1(t), \\
X_3(t) = X_3(t).
\end{cases}
\]

We consider the switching signal is modeled by Markov chain and the transition probability matrix of $\tau(t)$ is given as
\[
\begin{pmatrix}
0.4 & 0.6 \\
0.8 & 0.2
\end{pmatrix}.
\]

Based on Lemma 2.2, the algebraic form of this MJBCN is
\[
x(t + 1) = L_{\tau(t)}x(t)u(t),
\]
where $L_i$ is the logical matrix of $i$-th subsystem and they are listed as
\[
L_1 = \delta_{h}[1, 1, 3, 4, 1, 1, 3, 4, 1, 1, 3, 4, 5, 7, 8, 3, 3, 4, 4, 3, 4, 3, 4, 7, 7, 8, 8],
\]
\[
L_2 = \delta_{h}[1, 1, 3, 3, 2, 2, 4, 4, 1, 1, 3, 3, 6, 6, 8, 8, 3, 3, 4, 4, 4, 4, 3, 3, 3, 3, 8, 8, 8, 8].
\]

Assume that $\mathcal{M} = \{\delta_1^h, \delta_2^h\}$, then according to Algorithm 1, $L(\mathcal{M}) = \{\delta_1^h, \delta_2^h\}$ and $\mathcal{P}_1(1) = \{\delta_1^h, \delta_2^h\}, \mathcal{P}_2(1) = \{\delta_1^h, \delta_2^h\}, \mathcal{P}_1(8) = \{\delta_1^h, \delta_2^h\}, \mathcal{P}_2(8) = \{\delta_1^h, \delta_2^h, \delta_1^h, \delta_2^h\}$. From Theorem 3.6, we have the feasible solution
\[
p_1 = [1, 3.344, 1.1, 1, 44.528, 1.1, 1, 1, 1, 32.944,
\]
\[
1, 1, 1, 1, 1, 1, 3.44, 1, 1, 1, 1],
\]
\[
p_2 = [1, 1, 1, 1, 1, 24.912, 1, 1, 1, 1, 1, 1, 26.896,
\]
\[
1, 1, 1, 1.84, 1, 1, 1, 1, 1, 1, 1, 1],
\]
\[
\lambda_1 = [1.61, 12.882, 8.986, 1, 1.61, 1],
\]
\[
\lambda_2 = [1, 6.978, 7.474, 1.21, 1, 1].
\]
Then the control gain matrices can be designed as
\[ K_1 = \delta_4[1, 2, 4, 1, 3], \]
\[ K_2 = \delta_4[1, 2, 4, 1, 3]. \]
Taking initial conditions \( x(0) = \delta_8^5 \) and \( \tau(0) = 2 \), the state trajectory is shown in Figure 2.

5. Conclusion. The set stabilization problem of MJBCNs via STP has been studied in this paper. The concepts of set stabilization and control invariant subset have been extended to MJBCNs from PBCNs. And an algorithm has been designed to calculate the largest control invariant subset. This algorithm was only involved logical matrices and logical variables, avoiding the probabilities in subsystems. Based on this subset, a necessary and sufficient condition for the set stabilization with probability one has been given by a convex programming problem. By solving this convex programming problem, the number of the different feedback controllers can be easily obtained. This method is also valid for the set stabilization problem of BCNs and PBCNs. Finally, two examples have been given to show the effectiveness of the main results in this paper.

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