Supplementary Information for
Kirigami-based Metastructures with Programmable Multistability

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1. Kinematic Analysis of the Kirigami-inspired Foldable Cuboid

The kirigami shown in Fig.S1A consists of five hinges. As the mechanism is a parallel mechanism, the truss method (1) is used to determine the number of mobility. For a truss without external forces consisting of $b$ bars and $j$ joints, the equilibrium equation is $At = 0$, where $A$ is the $3j \times b$ equilibrium matrix, $t$ is a $b \times 1$ vector of bar axial forces. If $r$ is the rank of the matrix $A$, the number of mobility is

$$m = 3j - 6 - r, \quad \text{(S1)}$$

The truss form of the mechanism in Fig.S1A can be derived by using bars and nodes to replace edges of facets (the black solid lines in Fig.S1A). For triangular facets, three bars will make a facet rigid and for quadrilateral facets, arbitrary points $V_1$ and $V_2$ out of the facets ABCD and EFGH are introduced. By counting, the truss form in Fig.S1A contains $j = 18$ nodes and $b = 46$ bars. When taking the configuration in system 0 (Fig.S1B), the coordinate of nodes can be obtained (Dataset S3, truss method data) and the equilibrium matrix of the mechanism can be established according to (1). Then, we obtain the rank of the matrix as $r = 45$. The number of mobility is $m = 3$.

Fig. S1. The mechanism corresponding to the foldable cuboid for kinematic analysis. (A) The deployable configuration and the truss form. (B) The corresponding mechanism diagram with a limb.

As the mechanism needs three inputs to determine its configuration, it is not easy to analyze and control the motion of the mechanism directly. Here, we consider the motion with rotational symmetry in the mechanism. A deployable configuration shown in Fig.S1A is constructed where all the limbs are symmetric. Hence, their rotated angles are correspondingly equal. Here, one of the limbs with the top and bottom facets is picked up and its corresponding mechanism diagram (Fig.S1B) is also raised. Coordinate systems are
set up on this mechanism (2), as shown in Fig.S1B. Here, we chose limb 1 in Fig.S1A. The notation $x'_i, y'_i$ and $z'_i$ are the axes of the system $j$ on the limb $i$, where $j = 1, 2, 3, 4, 5$ and $i = 1, 2, 3, 4$. The axis $z'_i$ is along the axis of $j$th revolute joint on the limb $i$; the axis $x'_{j+1}$ is along the common normal line from $z'_j$ to $z'_{j+1}$; the axis $y'_j$ can be determined by the right-hand rule; $\theta'_j$ is the angle between $x'_j$ and $x'_{j+1}$ measured from $x'_j$ to $x'_{j+1}$ along the positive direction of $z'_j$; $\alpha_{i(j+1)}$ is omitted here for simplification and they can be measured from $z'_j$ and $z'_{j+1}$ along the positive direction of $x'_{j+1}$, where $\alpha_{12} = \frac{\pi}{2}, \alpha_{23} = \frac{3\pi}{2}$, $\alpha_{34} = \frac{\pi}{2}, \alpha_{45} = \frac{3\pi}{2}$ and $\alpha_{56} = 0$ in a limb. The mechanism with four limbs has three independent loops according to the Euler’s equation

$$l = g - n + 1.$$  

As any two limbs can form a loop, we choose the three loops, $L_1L_2, L_2L_3$ and $L_3L_4$ to carry on the further study.

For a single closed loop linkage consisting of $n$ links, the product of the transformation matrices is equal to the 4×4 identity matrix $I_4$, as

$$T_{21}T_{32}\cdots T_{n(n-1)}T_{1n} = I_4.$$  

Hence, we can obtain the following relationships from the three loops,

$$T_{10}T_{21}T_{32}T_{43}T_{54}T_{65}T_{76} = T_{10}T_{11}T_{22}T_{33}T_{44}T_{55}T_{66}T_{77},$$  

where limb $i = 1, 2, 3$ and the transformation matrix $T_{(j+1)j}$ about the $j$th coordinate system and the $j$th coordinate system are

$$T'_{21} = T_{21} \begin{bmatrix} 0, 0, & \alpha_{12} \\ \alpha_{12}, & 0, 0 \end{bmatrix} R_z \left( \theta'_1 \right) R_x \left( \alpha_{12} \right),$$

$$T'_{32} = T_{32} \begin{bmatrix} 0, 0, & a \\ a, & 0 \end{bmatrix} R_z \left( \theta'_2 \right) R_x \left( \alpha_{23} \right),$$

$$T'_{43} = T_{43} \begin{bmatrix} 0, 0, & a \\ a, & 0 \end{bmatrix} R_z \left( \theta'_3 \right) R_x \left( \alpha_{34} \right),$$

$$T'_{54} = T_{54} \begin{bmatrix} 0, 0, & a \\ a, & 0 \end{bmatrix} R_z \left( \theta'_4 \right) R_x \left( \alpha_{45} \right),$$

$$T'_{65} = T_{65} \begin{bmatrix} 0, 0, & a \\ a, & 0 \end{bmatrix} R_z \left( \theta'_5 \right) R_x \left( \alpha_{56} \right),$$

and
\[ T'_{10} = T_x \begin{pmatrix} \frac{a}{2} & 0 & 0 \end{pmatrix} R_z \left( \frac{\pi}{2} \right) R_y \left( \frac{\pi}{2} \right), \quad T'_{1} = T_y \begin{pmatrix} 0, \frac{a}{2}, 0 \end{pmatrix} R_z \left( \frac{3\pi}{2} \right), \quad T'_{67} = T_z \begin{pmatrix} 0, 0, \frac{a}{2} \end{pmatrix} R_y \left( \frac{3\pi}{2} \right) \]

Here, rotation matrices rotate vectors by an angle \( \theta \) about the x-, y-, or z-axis of a coordinate system in homogeneous coordinates are

\[
R_x(\theta) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta & -\sin \theta \\
0 & \sin \theta & \cos \theta
\end{bmatrix}, \quad R_y(\theta) = \begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}, \quad R_z(\theta) = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

and the translation matrix along vector \( v \) is

\[
T_r(v_x, v_y, v_z) = \begin{bmatrix} 1 & 0 & 0 & v_x \\
0 & 1 & 0 & v_y \\
0 & 0 & 1 & v_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

For the Eq. S4, we can rewrite it as

\[
E_{12} = T_{10}^1 T_{0}^1 T_{6}^1 T_{10}^1 T_{61}^2 T_{76}^2 = 0, \\
E_{23} = T_{10}^2 T_{10}^2 T_{61}^2 T_{76}^2 T_{10}^3 T_{61}^3 T_{76} = 0, \\
E_{34} = T_{10}^3 T_{10}^3 T_{61}^3 T_{76}^3 T_{10}^4 T_{61}^4 T_{76}^4 = 0
\]

where \( T_i^j = T_{21}^i T_{32}^i T_{43}^i T_{54}^i T_{65}^i \) with \( i = 1, 2, 3, 4 \).

The symmetric conditions of the motion indicate the corresponding rotated angles of the four limbs are equal, that is \( \theta_1^j = \theta_2^j = \theta_3^j = \theta_4^j \). Therefore, we can only consider the loop \( L_1 L_2 \). By introducing the symmetric condition \( \theta_1^i = \theta_3^i \) and \( \theta_2^i = \theta_4^i \) of the top and bottom facets and the equation \( Q_i = E_{12} + E_{23} \), we obtain the

\[
\cos^2 \theta_2 + (1 + \tan^2 \theta_1) \cos \theta_2 + \sin \theta_2 \tan \theta_2 \left( \sin \theta_1 \tan \theta_1 + 2 \mu + \cos \theta_1 \right) + \tan^2 \theta_1 = 0,
\]

\[
\tan \frac{\theta_3}{2} \tan \theta_1 - \cos \theta_2 = 0,
\]
from the following elements of the matrix \( Q \),

\[
Q(2, 1) = -2 \sin \theta_2 (\cos \theta_2 \cos \theta_3 - \sin \theta_1 \sin \theta_2) + \cos \theta_1 \cos \theta_2 \cos \theta_3,
\]

\[
Q(2, 4) = -a \begin{pmatrix} 2 \cos \theta_2 - \cos \theta_3 + \cos^2 \theta_2 + \cos^2 \theta_2 \cos \theta_3 + \cos \theta_2 \sin \theta_1 \sin \theta_2 \\
+ \cos \theta_1 \sin \theta_2 \sin \theta_3 + 2 \mu \sin \theta_2 \sin \theta_3 + \cos \theta_1 \cos \theta_2 \cos \theta_3 \sin \theta_2 + 1 \end{pmatrix}.
\]  

(S10)

According to coordinate systems and the geometry of the model, relationships between
dihedral angles and rotated angles are

\[
\theta_1 = \pi - \varphi_1, \quad \theta_2 = \varphi_2 - \pi, \quad \theta_3 = \pi - \varphi_3.
\]  

(S11)

Correspondingly, Eq. S10 can be rewritten as

\[
\cos^2 \varphi_2 - (1 + \tan^2 \varphi_1) \cos \varphi_2 - \sin \varphi_2 \tan \varphi_1 \sin \varphi_1 - 2 \mu \cos \varphi_1 + \tan^2 \varphi_1 = 0,
\]

\[
\tan \frac{\varphi_2}{2} \cos \varphi_2 - \tan \varphi_1 = 0.
\]  

(S12)

The motion paths of the mechanism are shown in Fig. 1C and F.

According to the coordinate systems of the mechanism in Fig. S1 A and B, we can obtain
the original coordinates of point D' and J' in system 0 at state \( \varnothing \), as

\[
D'_0 = \begin{pmatrix} -\frac{a}{2} \\
-\frac{a}{2} \\
2 \mu a \end{pmatrix}, \quad J'_0 = \begin{pmatrix} 0 \\
0 \\
2 \mu a \end{pmatrix}.
\]  

(S13)

The homogeneous coordinate of the vector \( J'_0 D'_0 \) is

\[
\overline{J'_0 D'_0} = \begin{pmatrix} -\frac{a}{2} \\
-\frac{a}{2} \\
0 \\
1 \end{pmatrix}.
\]  

(S14)

Besides, we can obtain the homogeneous coordinates of points D in system 5,
\( D_5 = \begin{pmatrix} 0 \\
0 \\
0 \\
1 \end{pmatrix} \). The homogeneous coordinates of points D and J in the coordinate
system 0 can be obtained with

\[
D_0 = T_{10}^1 T_{21}^1 T_{32}^1 T_{43}^1 T_{54}^1 D_5,
\]

\[
J_0 = T_{10}^1 T_{21}^1 T_{32}^1 T_{43}^1 T_{54}^1 T_{65}^1 T_{76}^1 J_7,
\]  

(S15)

where \( J_7 = \begin{pmatrix} 0 \\
0 \\
0 \\
1 \end{pmatrix} \) is the homogeneous coordinates of point J in the coordinate
system 7. According to the coordinates, the twist angle can be derived from

\[
\cos \theta_\omega = \frac{\overline{J'_0 D'_0} \cdot \overline{J'_0 D'_0}}{|J'_0 D'_0| \cdot |J'_0 D'_0|}.
\]  

(S16)

Then, we can obtain

\[
\cos \theta_\omega = -\frac{1}{2} \cos^2 \theta_2 + \frac{1}{2} \sin^2 \theta_1 \cos \theta_3 + \frac{1}{2} \cos \theta_1 \sin^2 \theta_2 + \frac{1}{2} \cos \theta_2 \sin^2 \theta_1 - \cos \theta_1 \cos \theta_2 \sin \theta_1 \sin \theta_3.
\]  

(S17)

and its corresponding equation with dihedral angles is
\[
\cos \theta_\omega = -\frac{1}{2} \cos^2 \phi_2 + \frac{1}{2} \sin^2 \phi_1 - \frac{1}{2} \cos^2 \phi_1 \cos \phi_3 - \frac{1}{2} \cos \phi_3 \sin^2 \phi_2 \\
- \frac{1}{2} \cos^2 \phi_2 \sin^2 \phi_1 + \frac{1}{2} \cos^2 \phi_2 \cos \phi_1 \sin^2 \phi_1 - \cos \phi_3 \cos \phi_2 \sin \phi_1 \sin \phi_3 .
\]  
(S18)

According to Eqs. S12 and S18, relationships of \(\theta_\omega\) vs. \(\phi_3\) in Fig.1D and G are derived. The z-axial coordinate of point J in the coordinate system 0 is the height of \(h\), which is

\[
h = a \mu \sin \theta_1 - a \cos \theta_1 \sin \theta_2 + a \mu \left( \cos \theta_1 \sin \theta_1 + \cos \theta_1 \cos \theta_2 \sin \theta_1 \right) \quad (S19)
\]

and its corresponding equation with dihedral angles is

\[
h = a \mu \sin \phi_1 - a \cos \phi_1 \sin \phi_2 - a \mu \left( \cos \phi_3 \sin \phi_1 - \cos \phi_1 \cos \phi_2 \sin \phi_3 \right) \quad (S20)
\]
for Fig.1E and H.

2. The Stored Energy of the System

A. The stored energy of foldable cuboid with torsional springs

For a spring hinge with a torsional spring, the potential energy can be derived from

\[
U = \frac{1}{2} K \left( \phi - \phi_0 \right)^2 .
\]  
(S21)

where \(\phi_0\) and \(\phi\) are the rest angle and real-time angle of the spring hinge, respectively, \(K\) is the stiffness of the torsional spring which is determined by modulus of elasticity of the material \(E\), wire diameter \(d_w\), mean spring diameter \(D_m\), number of active coils \(N\) and can be calculated from

\[
K = \frac{\pi E d_w^4}{11520 N D_m^4} .
\]  
(S22)

Here, the torsional spring is manufactured by 65Mn Spring Steel with \(E = 206000\) MPa, \(d_w = 1\) mm, \(D_m = 3.5\) mm, and \(N = 5\). Its spring constant is supposed to be \(K = 3.21\) N.mm/rad. For the torsional springs of stiffness \(K_{2,4} = K\) for creases 2 and 4 of four limbs in the kirigami cuboid with the rest angle of the springs \(\phi_{20} = \phi_{40} = 90^\circ\), the stored energy of each spring hinge can be derived from

\[
U = \frac{1}{2} K_{2,4} \left( \phi_2 - \phi_{20} \right)^2 .
\]  
(S23)

The total energy of the system with eight hinges can be derived from \(U_{2,4} = 8U\) whose curve is plotted in Fig.2A, where constant \(K_{s0} = 3.21\) N.mm/rad. Similarly, for the structure only with torsional springs (stiffness \(K_{1,5} = K\) on creases 1 and 5 of four limbs with rest angle, \(\phi_{10} = \phi_{50} = 180^\circ\), the total energy can be derived from \(U_{1,5} = 8 \times \frac{1}{2} K_{1,5} \left( \phi_1 - \phi_{10} \right)^2\) with curve shown in Fig.2B. When these two sets of creases 1, 5 and 2, 4 are replaced by the above-mentioned torsional springs, respectively and simultaneously, we will only obtain monostable configuration \(\mathbb{2}\) because the total energy of the system \(U_{1,5} + U_{2,4}\), as shown in Fig. 2C.
B. The stored energy of thick-panel foldable cuboid with elastic joints

In Fig. 2E, each elastic joint 1 (and 5) can be stretched by $\Delta x = 2 \sin \left(\frac{\Delta \phi_1}{2}\right) t_1$ and the potential energy of each elastic joint 1(and 5) can be derived from

$$U_{e1,e5} = \frac{1}{2} K_{e1,e5} (\Delta x)^2 = 2K_{e1,e5} \sin^2 \left(\frac{\Delta \phi_1}{2}\right) t_1^2 = 2K_{e1,e5} \sin^2 \left(\frac{\phi_1 - \pi}{2}\right) t_1^2.$$  \hspace{1cm} (S24)

An elastic joint 2 (and 4) in Fig. 2E can be stretched by $\Delta y = 2\sqrt{2} \sin \left(\frac{\Delta \phi_2}{2}\right) t_2$ and the potential energy of an elastic joint 2 (and 4) can be derived from

$$U_{e2,e4} = \frac{1}{2} K_{e2,e4} (\Delta y)^2 = 4K_{e2,e4} \sin^2 \left(\frac{\Delta \phi_2}{2}\right) t_2^2 = 4K_{e2,e4} \sin^2 \left(\frac{\phi_2 - \pi}{4}\right) t_2^2.$$  \hspace{1cm} (S25)

As there are eight elastic joints on 1(and 5) and eight elastic joints 2 (and 4), the whole stored energy of the system can be derived from

$$U_e = 8U_{e1,e5} + 8U_{e2,e4} = 16K_{e1,e5} \sin^2 \left(\frac{\Delta \phi_1}{2}\right) t_1^2 + 32K_{e2,e4} \sin^2 \left(\frac{\Delta \phi_2}{2}\right) t_2^2.$$  \hspace{1cm} (S26)

In the tristable structure with elastic joints in Fig. 2F, the energy equations (Eqs. 2 and 3) and twist angle equations (Eq. S34 and Eq. S41) can be derived from the results of dihedral angles from the two functional codes ‘IFDoTTHi1.m’ in Dataset S3 and ‘IFDoTTHi2.m’ in Dataset S3 by setting $\mu$, $a$ and thickness of panels. The corresponding relationships between the system energy $U_e$ and angle $\phi_3$ can be obtained from ‘F2G_TriStableStr_mu1_t3_50.m’ in Dataset S3, as shown in Fig. 2G.

In a tristable structure with elastic joints, the energy equations (Eqs. 2 and 3) and twist angle equations (Eq. S34 and Eq. S41) can be derived from the results of dihedral angles from the two functional codes ‘IFDoTTHi1.m’ in Dataset S3 and ‘IFDoTTHi2.m’ in Dataset S3 by setting $\mu$, $a$ and thickness of panels. The corresponding relationships between the system energy $U_e$ and twist angle $\theta_\omega$ can be derived from ‘F3AD_TriStableStr_mu1_t5_80.m’ in Dataset S3 to ‘F3AD_TriStableStr_mu02_t5_80.m’ in Dataset S3 under different $\mu = 1, 0.706, 0.388$ and 0.2, as shown in Fig. 3A.

Tension along the height direction of a foldable cuboid with elastic joints can activate the transformation of stable states. Its energy equations (Eqs. 2 and 3) and height equations (Eq. S36 and Eq. S42) can obtain the results with dihedral angles from the two functional codes ‘IFDoTTHi1.m’ in Dataset S3 and ‘IFDoTTHi2.m’ in Dataset S3 by setting $\mu$, $a$ and thickness of panels. The corresponding relationships between the system energy $U_e$ and height $h$ in Fig. 3D can be derived from codes ‘F3AD_TriStableStr_mu1_t5_80.m’ in Dataset S3 to ‘F3AD_TriStableStr_mu02_t5_80.m’ in Dataset S3 corresponding to $\mu = 1, 0.706, 0.388$ and 0.2, respectively.

3. Tension Experiment of the Elastic Joint for Stiffness

The elastic joints with elastic sheets of the foldable cuboid are constructed in Fig. 2E. Since the sheet is very thin, its bending stiffness can be negligible. Here, we carried out an experiment of an elastic joint with an elastic sheet under tension. The specimen consists of
panels, wedges and elastic sheets with dimensions shown in Fig. S2A.

**Fig. S2.** Tension experiment of the elastic joint. (A) Geometric parameters of the elastic joint with latex film (thickness 0.3 mm). (B) The flowchart for the fabrication process of the elastic joint specimen. (C) The experimental setup. (D) The diagram, results and deformation configurations of the elastic joint.
The manufacturing process includes three steps, as shown in Fig. S2B. First, two panels and two wedges are fabricated by a Stratasys Dimension Elite 3D printer with ABS (acrylonitrile-butadiene-styrene), and the elastic sheet (natural rubber latex film, thickness 0.3mm) was cut by a Trotec Speedy 300 laser cutter (power: 70%, speed: 2.5%, Hz: 2000). Second, two panels P1 and P2 are symmetrically placed and glued together with the elastic sheet (LOCTITE 401 glue). Finally, the wedges are bolted to the panels.

The setup of the tension experiment is shown in Fig. S2C. An Instron Universal Testing Machine 5982 with a load cell of 100N is adopted for the experiment. The specimen is connected to the two fixtures on the machine through PE (polyethylene) wires. The loading speed is 1mm/min and the tension displacement is 8mm. The displacement and forces are recorded by the data collection system. The deformation process is recorded by a digital camera (Canon 70D).

Under the experiment, we can obtain the stiffness of the joint with the elastic sheet from

\[ K_e = \frac{F_e}{\Delta y} = \frac{F_z}{z_e \Delta y} \] (S27)

whose relationships are shown in Fig. S2D (black solid polyline). This black polyline can be fitted by the red line with the equation \( F_e = 3.2187 \Delta y \) with the coefficient of determination \( R^2 = 0.9942 \). Hence, the stiffness of elastic joints (length 31.04 mm) with latex film (thickness \( t_e = 0.3 \) mm) is 3.2187 N/mm, and the per length (mm) stiffness is \( K_{e,0.3} = 0.1037N/mm^2 \). As the stiffness can also be derived from \( K_e = \frac{E_e t_e b_e}{l_e} \), where \( E_e \) is the modulus of elasticity of the material, \( t_e \) is the thickness of the film, \( b_e \) is the valid length of film on the crease, \( l_e \) is the rest width of the film in the joint, the per length stiffness of latex film with thickness \( t_e = 0.09 \) mm can be derived from \( K_{e,0.09} = K_{e,0.3} \frac{t_e = 0.09}{t_e = 0.3} = 0.3 K_{e,0.3} \). The per length stiffness of latex film with thickness \( t_e = 0.4 \) mm can be derived from \( K_{e,0.4} = K_{e,0.3} \frac{t_e = 0.4}{t_e = 0.3} = 4 K_{e,0.3}/3 \).

4. Kinematic Analysis of Foldable Cuboid with Thick Panels

Figure. 2E has shown the changes in hinges’ positions during the motion. During the motion between ① and ④, hinges 1 and 3 are kept while hinge 2 becomes the hinge 2’.

When the motion is in the range between ④ and ⑥, hinge 1’ is activated and hinges 2 and 3 are kept. Similar to Fig. S1, the thick-panel structures of the two moving parts are constructed in Fig. S3 with their corresponding mechanisms of a limb for the kinematic analysis.
Fig. S3. The mechanism corresponding to the thick-panel structure with elastic joints corresponding to Fig. 2E. (A-B) Deployable configuration of the structure and its corresponding mechanism diagram with a limb, which is the thick-panel theoretical model for the range between ① and ④; (C-D) Deployable configuration of the structure and its corresponding mechanism diagram with a limb, which is the theoretical model for the range between ④ and ⑥.

For the motion between ① and ④, its thick-panel structure and corresponding mechanism are shown in Fig. S3A and B. Similar to Eqs.S5a and S5b, the transformation matrix $T_{(j+1)j}$ about the $j+1$th coordinate system and the $j$th coordinate system are
\[ T_{21}^i = T_{21} \left( 0, 0, \frac{a}{2} + t_2 \right) R_z \left( \theta_1^i \right) R_y \left( \alpha_{12} \right) T_{21} \left( t_2, 0, 0 \right), \]
\[ T_{32}^i = T_{21} \left( 0, 0, b \right) R_z \left( \theta_2^i \right) R_y \left( \alpha_{23} \right) T_{21} \left( t_2, 0, 0 \right), \]
\[ T_{43}^i = T_{21} \left( 0, 0, a + 2t_2 \right) R_z \left( \theta_3^i \right) R_y \left( \alpha_{34} \right) T_{21} \left( t_2, 0, 0 \right), \]
\[ T_{54}^i = T_{21} \left( 0, 0, b \right) R_z \left( \theta_4^i \right) R_y \left( \alpha_{45} \right) T_{21} \left( t_2, 0, 0 \right), \]
\[ T_{65}^i = T_{21} \left( 0, 0, \frac{a}{2} + t_2 \right) R_z \left( \theta_5^i \right) R_y \left( \alpha_{56} \right) T_{21} \left( t_2, 0, 0 \right), \]

and
\[ T_{10}^i = T_{21} \left( \frac{a}{2}, 0, 0 \right) R_y \left( \frac{\pi}{2} \right) R_z \left( \frac{\pi}{2} \right), \]
\[ T_{10}^i = T_{21} \left( \frac{a}{2}, 0, 0 \right) R_y \left( \frac{\pi}{2} \right) R_z \left( \pi \right), \]
\[ T_{10}^i = T_{21} \left( \frac{a}{2}, 0, 0 \right) R_y \left( \frac{3\pi}{2} \right) R_z \left( \frac{3\pi}{2} \right), \]

Considering the symmetric condition \( \theta_1^i + \pi = \theta_5^i \) and \( \theta_2^i = \theta_4^i \) according to the systems and substituting Eqs.S28a and S28b into Eq.S4, according to \( Q_1 = E_{12} + E_{23} \), we obtain
\[
\cos \theta_2 \left( a \cos \theta_2 - a + 2t_2 \sin \theta_2 - 2t_2 + 2t_2 \cos \theta_2 \right) + a \left( \cos \theta_2 - 2\mu \right) \sin \theta_2 \tan \theta_i + (a + 2t_2)(a + 2t_2) \cos \theta_2 + (a \sin \theta_1 - 2t_2) \sin \theta_2 \right) \tan^2 \theta_i = 0, \] (S29)

from the following elements of the matrix \( Q_1 \)
\[
Q_1 \left( 2, 1 \right) = -2 \sin \theta_2 \left( - \cos \theta_2 \cos \theta_3 + \sin \theta_3 \sin \theta_3 + \cos \theta_2 \cos \theta_3 \right), \]
\[
Q_1 \left( 2, 4 \right) = -a \left( 2 \cos \theta_2 + \cos \theta_3 + \cos^2 \theta_2 - \cos^2 \theta_3 \cos \theta_2 \sin \theta_3 \sin \theta_2 \right) + \left( \cos \theta_2 \sin \theta_3 \sin \theta_3 + 2\mu \sin \theta_3 \sin \theta_3 + \cos \theta_2 \cos \theta_3 \sin \theta_2 \sin \theta_2 + 1 \right) \] (S30)

According to the relationships between dihedral angles and rotated angles
\[ \theta_1 = -\phi_1, \quad \theta_2 = \phi_2, \quad \theta_3 = -\phi_3, \] (S31)

Eq. S29 can be rewritten as
\[
\cos \phi_2 \left( a \cos \phi_2 - a + 2t_z \sin \phi_2 - 2t_z + 2t_z \cos \phi_2 \right) + a \left( -\cos \phi_1 + 2\mu \right) \sin \phi_2 \tan \phi_1 \\
+ \left( a + 2t_z \right) \cos \phi_2 - (a \sin \phi_1 + 2t_z) \sin \phi_2 \right) \tan^2 \phi_1 = 0,
\]
(S32)

\[
\tan \frac{\phi_2}{2} \cos \phi_2 = \tan \phi_1.
\]

For the twist angle of the structure, we can derive it from Eqs. S15 and S16 based on the homogeneous coordinates of points D in system 5, \( D_s = (0 \ 0 \ t \ 1)^T \), as

\[
\cos \theta_{\phi} = -\frac{1}{2} \cos^2 \theta_2 + \frac{1}{2} \sin^2 \theta_1 - \frac{1}{2} \cos^2 \theta_4 \cos \theta_3 - \frac{1}{2} \cos \theta_4 \sin^2 \theta_4 \\
- \frac{1}{2} \cos^2 \theta_4 \sin^2 \theta_1 + \frac{1}{2} \cos^2 \theta_2 \cos \theta_4 \sin^2 \theta_1 - \cos \theta_4 \cos \theta_2 \sin \theta_1 \sin \theta_3
\]
(S33)

and its corresponding equation with dihedral angles is

\[
\cos \theta_{\phi} = -\frac{1}{2} \cos^2 \phi_2 + \frac{1}{2} \sin^2 \phi_1 - \frac{1}{2} \cos^2 \phi_4 \cos \phi_3 - \frac{1}{2} \cos \phi_4 \sin^2 \phi_4 \\
- \frac{1}{2} \cos^2 \phi_4 \sin^2 \phi_1 + \frac{1}{2} \cos^2 \phi_2 \cos \phi_4 \sin^2 \phi_1 - \cos \phi_4 \cos \phi_2 \sin \phi_1 \sin \phi_3
\]
(S34)

For the height \( h \), it should be calculated by the z-axial coordinate of point J in the coordinate system 0 and plus the thickness of the top and bottom panels, as

\[
\begin{align*}
h &= t_z \cos \theta_1 - a \mu \sin \theta_1 + t_z \cos \theta_2 \left( \sin \theta_1 \sin \theta_3 + \cos \theta_4 \cos \theta_3 \cos \theta_1 \right) \\
&- t_z \sin \theta_2 \left( \sin \theta_1 \sin \theta_3 - \cos \theta_4 \cos \theta_3 \cos \theta_1 \right) + t_z \cos \theta_1 \cos \theta_2 \\
&+ t_z \sin \theta_1 \sin \theta_3 + au \left( \cos \theta_3 \sin \theta_1 - \cos \theta_4 \cos \theta_2 \sin \theta_3 \right) \\
&- (a + 2t_z) \cos \theta_1 \sin \theta_3 + t_z \cos \theta_1 \sin^2 \theta_2 + t_z \cos \theta_1 \cos \theta_3 + 2t_z.
\end{align*}
\]
(S35)

Its corresponding equation with dihedral angles is

\[
\begin{align*}
h &= t_z \cos \phi_1 + a \mu \sin \phi_1 + t_z \cos \phi_2 \left( \sin \phi_1 \sin \phi_3 + \cos \phi_4 \cos \phi_3 \cos \phi_1 \right) \\
&- t_z \sin \phi_1 \left( \sin \phi_1 \sin \phi_3 - \cos \phi_4 \cos \phi_3 \cos \phi_1 \right) + t_z \cos \phi_1 \cos \phi_2 \\
&+ t_z \sin \phi_1 \sin \phi_3 - a \mu \left( \cos \phi_3 \sin \phi_1 - \cos \phi_4 \cos \phi_2 \sin \phi_1 \right) \\
&- (a + 2t_z) \cos \phi_1 \sin \phi_3 + t_z \cos \phi_1 \sin^2 \phi_2 + t_z \cos \phi_1 \cos \phi_3 + 2t_z.
\end{align*}
\]
(S36)

For the motion between \( \odot \) and \( \odot \), its thick-panel structure and corresponding mechanism are shown in Fig. S3C and D. The transformation matrices are
\[ T^i_{21} = T_r \left( 0, 0, \frac{a}{2} \right) R_z (\theta^i_{12}) R_x (\alpha_{12}) T_r \left( t, 0, 0 \right), \]
\[ T^i_{32} = T_r \left( 0, 0, b \right) R_z (\theta^i_{13}) R_x (\alpha_{13}), \]
\[ T^i_{33} = T_r \left( 0, 0, a \right) R_z (\theta^i_{13}) R_x (\alpha_{14}), \]
\[ T^i_{34} = T_r \left( 0, 0, b \right) R_z (\theta^i_{14}) R_x (\alpha_{15}) T_r \left( t, 0, 0 \right), \]
\[ T^i_{65} = T_r \left( 0, 0, \frac{a}{2} \right) R_z (\theta^i_{16}) R_x (\alpha_{16}), \]

and
\[ T^i_{10} = T_r \left( \frac{a}{2}, 0, 0 \right) R_z \left( \frac{\pi}{2} \right) R_x \left( \frac{\pi}{2} \right), \]
\[ T^i_{67} = T_r \left( 0, \frac{a}{2}, 0 \right), \]
\[ T^i_{10} = T_r \left( 0, \frac{a}{2}, 0 \right) R_z \left( \frac{\pi}{2} \right) R_x (\pi), \]
\[ T^i_{67} = T_r \left( 0, 0, -\frac{a}{2} \right) R_z \left( \frac{3\pi}{2} \right), \]
\[ T^i_{10} = T_r \left( -\frac{a}{2}, 0, 0 \right) R_z \left( \frac{3\pi}{2} \right) R_x \left( \frac{3\pi}{2} \right), \]
\[ T^i_{67} = T_r \left( 0, 0, -\frac{a}{2} \right) R_x (\pi), \]
\[ T^i_{10} = T_r \left( 0, -\frac{a}{2}, 0 \right) R_z \left( \frac{3\pi}{2} \right), \]
\[ T^i_{67} = T_r \left( 0, 0, \frac{a}{2} \right) R_x \left( \frac{\pi}{2} \right), \]

where \( \alpha_{12} = \frac{\pi}{2}, \quad \alpha_{13} = \frac{3\pi}{2}, \quad \alpha_{14} = \frac{3\pi}{2}, \quad \alpha_{15} = \frac{\pi}{2}, \quad \alpha_{16} = 0 \) and \( \theta^i_{1} = \theta^i_{1} \), \( \theta^i_{2} = \pi + \theta^i_{2} \). According to the following elements of the matrix \( Q_{1} \)
\[ Q_{1}(2, 1) = -2 \sin \theta_{1} \left( \cos \theta_{1} \cos \theta_{2} + \sin \theta_{1} \sin \theta_{2} \cos \theta_{3} \cos \theta_{4} \right), \]
\[ Q_{1}(2, 4) = -a \left( 2 \cos \theta_{1} + \cos \theta_{3} + \cos^{2} \theta_{2} + \cos^{2} \theta_{3} \cos \theta_{1} - \cos \theta_{2} \sin \theta_{1} \sin \theta_{2} + \cos \theta_{1} \sin \theta_{2} \sin \theta_{3} - 2 \mu \sin \theta_{1} \sin \theta_{3} + \cos \theta_{2} \cos \theta_{3} \sin \theta_{1} \sin \theta_{2} + 1 \right) \]
+ \( t_{1} \sin 2 \theta_{2} - 2 \mu \cos \theta_{2} \cos \theta_{3} \sin \theta_{2}, \)

the relationships between kinematic variables can be derived as
\[ (-2t_{1} \sin \theta_{2} + a + a \cos \theta_{2} + a \sin \theta_{1} \sin \theta_{2}) \tan^{2} \theta_{1} + a \cos \theta_{2} + a \cos^{2} \theta_{2} \]
+ \( a \sin \theta_{2} \tan \theta_{1} \left( \cos \theta_{1} + 2 \mu \right), \quad \tan \frac{\theta_{1}}{2} \cos \theta_{2} = -\tan \theta_{1}. \)

Its corresponding equations with dihedral angles are
\[ (2t_{1} \sin \phi_{2} + a - a \cos \phi_{2} + a \sin \phi_{1} \sin \phi_{2}) \tan^{2} \phi_{1} - a \cos \phi_{2} + a \cos^{2} \phi_{2} \]
\[ -a \sin \phi_{2} \tan \phi_{1} \left( -\cos \phi_{1} + 2 \mu \right), \quad \tan \frac{\phi_{1}}{2} \cos \phi_{2} = -\tan \phi_{1}, \]

according to \( \theta_{1} = \pi - \phi_{1}, \quad \theta_{2} = \phi_{2} - \pi, \quad \theta_{3} = -\phi_{3}. \)
Similar to the Eqs. S34 and S36, the following equations are derived. For the twist angle based on $D_5=(0 0 0 1)^T$, we obtain
\[
\cos \theta_w = \frac{1}{2} \sin^2 \varphi_1 - \frac{1}{2} \cos^2 \varphi_2 - \frac{1}{2} \cos^2 \varphi_3 - \frac{1}{2} \cos \varphi_3 \sin^2 \varphi_2 \\
- \frac{1}{2} \cos^2 \varphi_1 \sin^2 \varphi_3 + \frac{1}{2} \cos^2 \varphi_2 \cos \varphi_3 \sin^2 \varphi_1 - \cos \varphi_1 \cos \varphi_2 \sin \varphi_3.
\] (S41)

For height $h$ based on point J, we get
\[
h = a \mu \sin \varphi_1 - t_1 \cos \varphi_2 \left( \sin \varphi_1 \sin \varphi_3 + \cos \varphi_1 \cos \varphi_2 \cos \varphi_3 \right) - t_1 \cos \varphi_1 - a \cos \varphi_1 \sin \varphi_2 - a \mu \left( \cos \varphi_1 \sin \varphi_1 - \cos \varphi_1 \cos \varphi_2 \sin \varphi_3 \right) - t_1 \cos \varphi_1 \sin^2 \varphi_2 + 2t_1.
\] (S42)

According to the Eqs. S29, S31, S33, function codes about the relationships of geometric and kinematic variables are constructed, named ‘IFDoTThi2.m’ in Dataset S3 for motion between state ① to state ④ considering the motion range of the angles. The relationships of geometric and kinematic variables can be derived, when the geometric parameters, length ratio $\mu$, length $a$ and thickness $t_2$. Similarly, codes named ‘IFDoTThi1.m’ in Dataset S3 for motion between state ④ to state ⑥ are constructed from the Eqs. S39. Combining the motion results from the two function codes, curves of Fig. 2H and I can be derived from ‘F2HI_TristableAnalysis_vspH3_t0t6.m’ in Dataset S3.

5. Analysis of Energy Barriers of Tristable Structures

According to Eqs. S24 to S26, the energy barriers of the system between states ① and ④, $U_{S3}$ at state 3, and between states ④ and ⑥, $U_{S5}$ at state 5, can be derived from
\[
U_{S3} = 32K_{e_2,e_4} \sin^2 \left( \frac{\Delta \varphi_{2,3}}{2} \right) \left( \frac{\varphi_{2,3}}{2} \right) t_2^2 = 16K_{e_2,e_4} \left( 1 - \cos \left( \frac{\varphi_{2,3}}{2} - \frac{\pi}{2} \right) \right) t_2^2 = 16K_{e_2,e_4} \left( 1 - \sin \varphi_{2,3} \right) t_2^2
\] (S43)

and
\[
U_{S5} = 16K_{e_1,e_5} \sin^2 \left( \frac{\Delta \varphi_{1,5}}{2} \right) t_1^2 = 16K_{e_1,e_5} \sin^2 \left( \frac{\varphi_{1,5}}{2} - \frac{\pi}{2} \right) t_1^2 = 8K_{e_1,e_5} \left( 1 + \cos \varphi_{1,5} \right) t_1^2.
\] (S44)

Here, $\Delta \varphi_{2,3}$ and $\varphi_{2,3}$ are angles $\Delta \varphi_2$ and $\varphi_2$ (Fig. 2E) in state ③, $\Delta \varphi_{1,5}$ and $\varphi_{1,5}$ are angles $\Delta \varphi_1$ and $\varphi_1$ (Fig. 2E) in state ⑤. From Eq. S32, we can obtain the angle $\varphi_{2,3}$ is only determined by $t_2$ and $\mu$ in state ③, when the section length $a$ is defined, i.e., $U_{S3}$ can be programmed by length ratio $\mu$ and thickness of panels $t_2$ without considering the stiffness of elastic joints. As the relationships of angles are implicit functions (Eqs. S32 and S40), $\varphi_{2,3}$ is the maximum $\varphi_1$ which can be derived from the kinematic analysis of the foldable cuboid under MATLAB function ‘max’, the energy barrier $U_{S3}$ can be derived from the code ‘F2J_EnergyUs3VSmu_thickness_80.m’ in Dataset S3 under varying length ratio $\mu$.
and thickness of panels $t_2$, as shown in Fig. 2J. Similarly, from Eq. S40, we can obtain the angle $\varphi_{1,5}$ is only determined by $t_1$ and $\mu$ in stable state $\circ$, when the section length $a$ is defined, i.e., $U_{SS}$ can be programmed by length ratio $\mu$ and thickness of panels $t_1$ without considering the stiffness of elastic joints. Similarly, the energy barrier $U_{SS}$ can be derived from the code ‘F2K_EnergyUs5VSmu_thickness_80.m’ in Dataset S3 under varying length ratio $\mu$ and thickness of panels $t_1$, as shown in Fig. 2K. Here, all the elastic joints are supposed to be with latex film (thickness 0.3 mm), the length $a = 80$ mm, and the real length of the elastic sheets in a joint is supposed to be $a_0 = 70$ mm, and the stiffness is supposed to be $K_{e2,e4} = \mu K_{e1,e5} = \mu K_{e0,a0}$.

6. The Relationships among Relative Position of States, Length Ratio and Thickness

Figures 1C and F show $\varphi_3$ gradually increases from $0^\circ$ to $180^\circ$ during the deployed sequence from state $\circ$ to state $\odot$, which indicates an arbitrary state of the folded cuboid corresponds to a unique value of angle $\varphi_3$. Therefore, the angle $\varphi_3$ can be used to represent the relative positions of the critical states $\odot$ to $\circ$. As the state $\odot$, the configuration with the maximum $h$, and state $\circ$, the one with maximum dihedral angles $\varphi_2 (= \varphi_4)$ which corresponds to the maximum energy state, are in the range between state $\odot$ and $\circ$, and do not always appear in order, angles $\varphi_3$’s in the state $\odot$ and state $\circ$ are chosen to analyze their relative positions under the variation of $\mu$ and thickness of panels $t_2$. The relationships among $\varphi_{32}$, $\varphi_{33}$, $\mu$ and $t_2$ are derived from the codes ‘F2L_muVSpositionOfstate2and3_3D.m’ in Dataset S3 with ‘IFDoTThi2.m’ in Dataset S3, as shown in Fig. 2L and Fig. S4A-B. From the relationships among $\varphi_3$ in state $\odot$ (the blue surface), $\varphi_3$ in state $\circ$ (the black surface), and $t_2$ in Fig. S4 A-B, we find the intersection of the two surfaces approaches a straight line which shows the influence of thickness on the relative position of two stable states $\odot$ and $\circ$ can be negligible. Then, a foldable cuboid with the panel thickness 5 mm is chosen to show the relative position of state $\odot$ and state $\circ$, the state $\odot$ is between $\odot$ and $\circ$ with $\mu > 0.388$, between $\circ$ and $\odot$ with $\mu < 0.388$, and coincident with $\circ$ with $\mu = 0.388$, as shown in Fig. S4C (‘FS4C_muVSpositionOfstate2and3.m’ in Dataset S3).
7. Torques and Forces of Thick-Panel Tristable Structures with Elastic Joints

A. Torques vs. twist $\theta \omega$ of the thick-panel tristable structures

In a tristable structure with elastic joints, the torques of a unit can be obtained by calculating the derivatives of $U_e$ to $\theta \omega$ based on Eq. S34 and Eq. S41, as

$$T = \begin{cases} 
T_{e2,e4} = \frac{dU_{e2,e4}}{d\theta \omega} = \frac{dU_{e2,e4}}{d\theta \omega} / d\varphi_2, & \text{between states 1 and 4} \\
T_{e1,e5} = \frac{dU_{e1,e5}}{d\theta \omega} = \frac{dU_{e1,e5}}{d\varphi_1} / d\varphi_1, & \text{between states 4 and 6} 
\end{cases} \quad \text{(S45)}$$

For the assembly between states 1 and 3, the derivative of energy $U_{e2,e4}$ to $\varphi_2$ can be derived from Eq. S26, as

$$\frac{dU_{e2,e4}}{d\varphi_2} = -16K e^2 f^2 \cos \varphi_2. \quad \text{(S46)}$$

From Eqs. S32 and S34, we find the $\theta \omega$ is the function of dihedral angles $\varphi_1$, $\varphi_2$, and $\varphi_3$, while $\varphi_1$ is the function of dihedral angles $\varphi_2$ and $\varphi_3$ is the function of dihedral angles $\varphi_1$, $\varphi_2$. With the derivative of $\varphi_1$ to $\varphi_2$ in Eq. S32, we obtained
According to the derivative of \( \varphi_3 \) to \( \varphi_2 \) in Eq. S32 with Eq. S47, we obtained
\[
\frac{d \varphi_3}{d \varphi_2} = \frac{2 \cos^2 \left( \frac{\varphi_3}{2} \right) \left( 2 t_2 \cos (2 \varphi_2) + (a + 2 t_2) \sec \varphi_2 \sin \varphi_2 - (a + 2 t_2) \sin (2 \varphi_2) \right) + 2 a \mu \cos \varphi_2 \tan \varphi_2 - a \cos \varphi_2 \sec \varphi_3 \tan \varphi_2 - 2 \cos \varphi_2 \cos \varphi_3 \sin \varphi_2}{\cos \varphi_2 \left( -4 (a + 2 t_2) \sin^2 \left( \frac{\varphi_3}{2} \right) \tan \varphi_2 + \sin \varphi_2 (-2 a \mu + a \sec \varphi_3 + 4 t_2 \tan \varphi_1 + a \sin \varphi_1 \tan \varphi_1) \right)}
\] (S48)

Then, from the derivative of \( \theta_\omega \) to \( \varphi_2 \) in Eq. S34, we obtained
\[
\frac{d \theta_\omega}{d \varphi_2} = -\frac{1}{2 \sin \theta_\omega} \left( \begin{array}{l}
2 \frac{d \varphi_1}{d \varphi_2} \cos \varphi_1 \sin \varphi_1 + 2 \frac{d \varphi_1}{d \varphi_2} \cos \varphi_1 \cos \varphi_3 \sin \varphi_1 \\
-2 \frac{d \varphi_2}{d \varphi_2} \cos \varphi_1 \cos \varphi_2 \cos \varphi_3 \sin \varphi_1 - 2 \cos \varphi_2 \cos \varphi_3 \sin \varphi_2 \\
-2 \cos \varphi_2 \left( -1 + (-1 + \cos \varphi_3) \sin^2 \varphi_1 \right) \sin \varphi_2 + \frac{d \varphi_3}{d \varphi_2} \cos^2 \varphi_3 \sin \varphi_3 \\
-2 \frac{d \varphi_1}{d \varphi_2} \cos^2 \varphi_1 \cos \varphi_2 \sin \varphi_3 + 2 \frac{d \varphi_1}{d \varphi_2} \cos \varphi_2 \sin^2 \varphi_1 \sin \varphi_3 \\
+2 \cos \varphi_1 \sin \varphi_1 \sin \varphi_2 \sin \varphi_3 + \frac{d \varphi_3}{d \varphi_2} \sin^2 \varphi_2 \sin \varphi_3 \\
+ \cos^2 \varphi_2 \left( 2 \frac{d \varphi_1}{d \varphi_2} \cos \varphi_1 \left( -1 + \cos \varphi_3 \right) \sin \varphi_1 - \frac{d \varphi_3}{d \varphi_2} \sin^2 \varphi_3 \sin \varphi_3 \right) \end{array} \right)
\] (S49)

Substituting Eqs. S47 and S48 to Eq. S49, and then combining Eqs. S46 and S49 according to Eq. 45, we can obtain the torque equation between states \( \textcircled{1} \) and \( \textcircled{3} \) in the codes ‘F3BE_DerivationsForForceTorqueEq_StableStates14.nb’ in Dataset S3. Similarly, the torque equation between states \( \textcircled{4} \) and \( \textcircled{6} \) can be obtained from codes ‘F3BE_DerivationsForForceTorqueEq_StableStates46.nb’ in Dataset S3. According to the torque equations, torques \( T \) vs. \( \theta_\omega \) with different \( \mu \) can be derived from Eq. S45, as shown in Fig. 3B (codes ‘F3_TorqueK_mu1_t5_80.m’ in Dataset S3 to ‘F3_TorqueK_mu02_t5_80.m’ in Dataset S3 corresponding to \( \mu = 1, 0.706, 0.388 \) and 0.2, respectively). In addition, the curves in Fig. 4A and D are derived from ‘F4A_TorqueK_mu0706_0706_LHU1_LHU2_t5_80.m’ and ‘F4D_TorqueK_mu0706_0706_LHU1_LHU2_t5_80.m’.
Here, critical torques for the transformations between stable states are noted by $T_{1-3}$, $T_{3-4}$, $T_{4-5}$. Normalized torques are derived from $T/K_{10}$, where $K_{10} = K_{ep0.3a^3} = 0.1037 \times 80^3 = 53094 \text{ N.mm}$.

**B. Forces vs. height $h$ of the thick-panel tristable structures**

Tension along the height direction of a foldable cuboid with elastic joints can activate the transformation of stable states. The tension forces can be derived from the derivation of the $U_e$ to $h$ based on Eq. S36 and Eq. S42, as

$$F = \begin{cases} 
    \frac{dU_{e2,e4}}{dh} = \frac{dU_{e2,e4}}{dh} / \frac{d\phi_2}{d\phi_2}, & \text{between states 1 and 4} \\
    \frac{dU_{e1,e5}}{dh} = \frac{dU_{e1,e5}}{dh} / \frac{d\phi_1}{d\phi_1}, & \text{between states 4 and 6}
\end{cases}$$

(S50)

The derivative of $h$ to $\phi_2$ in Eq. S36 is

$$\frac{dh}{d\phi_2} = \begin{cases} 
    \sin \phi_1 \left( \frac{d\phi_1}{d\phi_2} t_1 \cos \phi_1 + \frac{d\phi_2}{d\phi_2} \cos \phi_2 \right) + a \mu \frac{d\phi_2}{d\phi_2} \sin \phi_3 + t_1 (\cos \phi_2 - \sin \phi_1 \sin \phi_3) \\
    + \frac{d\phi_2}{d\phi_2} \cos \phi_1 \left( a \mu - a \mu \cos \phi_1 + t_1 (1 + \cos \phi_2 - \sin \phi_1 \sin \phi_3) \sin \phi_1 \right) \\
    - \frac{d\phi_2}{d\phi_2} \sin \phi_1 \left( t_2 + t_2 \cos^2 \phi_2 \cos \phi_3 - (a + 2t_1) \sin \phi_2 + t_2 \sin^2 \phi_2 \right) \\
    + \cos \phi_2 \left( t_2 + t_2 \sin \phi_2 + \cos \phi_3 (t_2 - t_2 \sin \phi_2) + a \mu \sin \phi_3 \right)
\end{cases}$$

(S51)

Substituting Eqs. S47 and S48 to Eq. S51, and then combining Eqs. S46 and S51, the force equation between states 1 and 4 can be obtained according to Eq. S50 with the codes ‘F3BE_DerivationsForForceTorqueEq_StableStates14.nb’. Similarly, the force equation between states 4 and 6 can be obtained from the codes ‘F3BE_DerivationsForForceTorqueEq_StableStates46.nb’. According to the force equations, forces vs. $h$ with different $\mu$ can be derived based on Eq. S50, as shown in Fig. 3E from the codes ‘F3_ForceK_mu1_t5_80.m’ in Dataset S3 to
‘F3_ForceK_mu02_t5_80.m’ in Dataset S3 corresponding to \( \mu = 1, 0.706, 0.388 \) and 0.2, respectively. In addition, the curves in Fig. 4G are derived from ‘F4G_ForceK_mu0706_0706_LHU1_RHU3_t5_80.m’. Here, critical forces for the transformations between stable states are noted by \( F_{5-6}, F_{4-5}, F_{3-4} \). Normalized forces are derived from \( F/K_0 \), where constant \( K_0 = K_{e_{0.3}a^2} = 0.1037 \times 80^2 = 663.68 \text{ N} \).

8. Experiment for Thick-Panel Tristable Structure and Their Assemblies

A. Fabrication of the foldable cuboids and tristable structures

Based on the geometric parameters in Fig. S5A, the panels of the specimen are fabricated by a Stratasys Dimension Elite 3D printer with ABS, whereas the elastic sheets are cut by a Trotec Speedy 300 laser cutter. Then the specimen is assembled following the process in Fig. S5B. First, one \( P_{top} \) panel is connected to four \( P_1 \) panels through four elastic sheets and PE wires for hinges 1 or 5 with LOCTITE 401 glue. Second, four \( P_2 \) panels are respectively connected to the four \( P_1 \) panels through four elastic sheets for hinges 2 and 4. Third, four \( P_2 \) and four \( P_3 \) panels are respectively joined by Tyvek paper. Fourth, the subassembly of \( P_1, P_2, P_3, P_{top} \) panels, and that of \( P_{bot} \) and \( P_4 \) panels are assembled with elastic sheets for hinges 2 and 4. Finally, sixteen wedges are installed at hinges 2 and 4.

The tristable wood structure in Fig. 2F is assembled similar to Fig. S5B with natural rubber latex film adhesive to plywood sheets. The assemblies of two units in Fig. 4 are assembled the same as that in Fig. S5B. For the assemblies of \( U_{10} & U_{12} \) (Fig. 4C) and \( U_{10} & U_{13} \) (Fig. 4F and I), the elastic joints are made of latex films (thickness 0.3mm), where the latex films for \( U_{12} \) and \( U_{13} \) are cut with holes (Fig. S5A) to obtain \( K_{e_{1e5}}^{U_{11}} / K_{e_{2e4}}^{U_{12}} = K_{e_{1e5}}^{U_{11}} / K_{e_{2e4}}^{U_{13}} = 2 \) and \( K_{e_{1e5}}^{U_{11}} / K_{e_{1e5}}^{U_{13}} = K_{e_{2e4}}^{U_{12}} / K_{e_{2e4}}^{U_{13}} = 2 \), respectively.

As the parameters \( \mu = 0, 0.2, 0.388, 0.417, 0.5, 0.706, 1, 1.5, 2 \) are specially selected, there is no fundamental difference in the curves of Fig. 1F-H. For the ideal case with \( \mu = 0 \) (\( b = 0, a \neq 0 \)), axes of hinges 1, 2 and 3 on each limb will intersect at a point, and axes of hinges 3, 4 and 5 on each limb will intersect at another point and there is physical interference of the corner joints at states ① and ⑧. Hence, the SolidWorks® model is constructed, as shown in Fig. S6A. When \( \mu = 2 \) a prototype with \( a = 80 \text{ mm} \) was fabricated according to the process in Fig. S5B, its stable states are shown in Fig. S6B. Here, a prototype of a smaller scale tristable unit with \( a = 8 \text{ mm} \) and \( \mu = 1 \) was fabricated according to the process in Fig. S5B, in which the panels of the tristable unit are made from carbon fiber board (thickness 0.2mm) and the elastic joints are made from Tyvek paper (thickness 0.142mm). Three stable states of this unit are shown in Fig. S6C.

B. Uniaxial tension experiment for a tristable structure

For the tristable structure with \( a = 80 \text{ mm} \) and \( \mu = 0.706 \), a specimen is fabricated (Fig. S5B) and a uniaxial tension experiment is carried out. A horizontal testing machine shown in Fig. S7A is utilized to avoid the influence of gravity. The load cell is 50N with an accuracy of 0.5%. Two fixtures are used to connect the specimen to the machine, where
the left one is fixed to the load cell and the right one has a rotational degree of freedom about the z-axis (Fig. 1A) of the specimen. A two-step loading process is adopted, one is tensioning the specimen by 136.72 mm from stable state ⑥ to the max height state ② and the other is compressing the specimen by 24.4 mm from state ② to states ①. The loading rate is selected to be 0.2mm/s. The deformation process is recorded by a digital camera (Canon 70D).

Fig. S5. The design and fabrication of a tristable structure with $\mu = 0.706$ and $a = 80$ mm. (A) The dimensions of the panels, elastic sheets, and wedges. (B) The fabrication process.
C. Experiment for the assemblies in Fig. 4

Units of the assembly with $a = 80\text{mm}$ and $\mu = 0.706$ are fabricated according to Fig. S5B. The setup of the experiment on a horizontal testing machine is shown in Fig. S7B. The fixed supporter is used to fix the left end of $U_t$ and the right supporter is used to support the specimen through a metal shaft to avoid the influence of gravity. For the tension experiment, the load cell was connected to the right end of $U_t$ through a rotational fixture by a PE wire (diameter 0.181 mm). The load cell is 50N with an accuracy of 0.5%. The loading rate is 0.2mm/s. For the tension process, displacement of 220 mm was set up and the assembly realise the transformation from $\theta$-$\xi$ to $\eta$-$\zeta$ via $\eta$-$\xi$ and $\xi$-$\zeta$, and overcome the state $\zeta$ of $U_t$. Then, for the release process, displacement of -40 mm was setup, the assembly went to state $\eta$-$\xi$. By tensioning 80 mm, $U_t$ overcame the state $\eta$, and the assembly got to state $\xi$-$\zeta$ by releasing displacement of -40 mm. The displacement and forces were recorded by the data collection system. The deformation process was recorded by a digital camera (Canon 70D). For the twisting process, the setup is similar to that in Fig. S7B, where the rotation is applied to the right end of the assembly through a cylinder with aramid wires and metal shafts connected to the assembly (Movie S5).

Fig. S6. Three typical foldable structures. (A) A SolidWorks® model constructed with $\mu = 0$. (B) A prototype with $a = 80\text{mm}$, $\mu = 2$. (C) A prototype with $a = 8\text{mm}$, $\mu = 1$. 

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9. Simulation for the Tristable Structures and Their Assemblies

SolidWorks® (Dassault Systèmes S.A, France) (3) is used to simulate the single unit and the assemblies. For a single unit, the panels are modeled as rigid bodies. Joints 1, 2, 4 and 5 are modeled as wire springs with zero original length and identical stiffness to that of the elastic sheet, whereas joint 3 is an ideal revolute joint. For the two-layer assembly, a cuboid with a thickness of 15 mm is used to connect the two units to avoid physical interference in the motion process. Solid contact is applied with setting Dry and penetration 0.001 mm. During deformation, the $P_{bot}$ is fixed and a linear or rotary motor is applied to the $P_{top}$ to realize tension or twist. In addition, a loading rate of 0.2 mm/s or 0.0115 rad/s is used to ensure a quasi-static process.

The single unit with $\mu = 0.706$ and $a = 80$ mm is first simulated, and the numerical results are presented together with the theoretical and experimental ones in Fig. 3E. A very good match is obtained, thus validating the theory and numerical model. Subsequently, the assemblies in Fig. 4 with different geometries and loading conditions are analyzed numerically using the same approach. Comparison of the theoretical, numerical, and experimental results are respectively presented in Fig. S8A-F. For each assembly, the numerically obtained transformation of stable states is identical to those obtained theoretically and experimentally. And the numerically predicted peak forces or torques at the transformation points match with the theoretical values very well. Therefore it can be concluded that the theoretical model can accurately predict the transformation process and required peak loads.
Fig. S8. The transformation of table states of assemblies with two tristable units corresponding to Fig. 4. (A) The assembly of left-handed units $U_{t1}$ and $U_{t2}$ with $\mu = 0.706$ and $K_{45}/K_{49} = 2$, and the corresponding $T$ vs. $\theta_\omega$ curve. (B) The relationships between the stable
states and the regions of normalized torques during the transformation from stable state □-□ in the assembly. (C-D) the same set of figures as (A-B) for the assembly of U₁ (left-handed, \( \mu = 0.706 \)) and U₃ (right-handed, \( \mu = 0.706 \)) with \( K_{e1,e5}^{-1}K_{e2,e5}^{-1} = K_{e1,e5}^{+}K_{e2,e5}^{+} = 2 \) under twist. (E) The numerical and experimental results of the assembly of U₁ and U₃ under tension. (F) The relationships between the stable states and the regions of normalized forces during the transformation from stable state ◊-◊ in the assembly. Here, Ten. 0 and Tw. 0 represent the positions of releasing the load.

10. The Tessellation of Thick-Panel Tristable Structures

A. The tessellation of two units with different length ratio

To discuss the variation of length ratio \( \mu \), we choose an assembly of two tristable units U₇ (right-handed, \( \mu = 1 \)) and U₁ (left-handed, \( \mu = 0.706 \)), where the thickness of latex film on the hinges are identical to 0.3mm. The schematic diagram of the assembly and curves of forces vs. \( h/a \) are shown in Fig. S9 which indicates \( F_{7,5}^{+} < F_{7,5}^{−} < 0 < F_{3,6}^{+} < F_{3,6}^{-} < F_{3,4}^{+} < F_{3,4}^{-} \). Here, the transformation from stable states ◊-◊ is analyzed with tension (Fig. S9B). Because the length ratio is different in the U₇ and U₁, the energy barriers of the two units are different which leads to the transformation path in the order of states ◊-◊-◊, ◊-◊-◊, ◊-◊-◊, ◊-◊-◊ and ◊-◊-◊ (Fig. S9C, Movie S6). Here, by changing the length ratio \( \mu \), the stable state ◊-◊ is obtained, which is different from the transformation path in Fig. 4I.

![Fig. S9](image)

Fig. S9. The tessellation of tristable units to program the multiple stable states controlled by input tensions. (A) The assembly of tristable structures U₇ (right-handed, \( \mu = 1 \)) and U₁ (left-handed, \( \mu = 0.706 \)), and the corresponding curves of forces \( F_{7} \) or \( F_{1} \) vs. \( h/a \) of the assembly. (B) The relationships between the stable states and the regions of normalized forces during the transformation from an arbitrary stable state in the assembly. (C) The transformation of stable states for the assembly of U₇ and U₁.
B. A programmed tessellation of three units with multiple stable states.

A multistable tessellation of three tristable units (left-handed Ut4 and right-handed Ut5 with \( \mu = 0.706 \), right-handed Ut6 with \( \mu = 1 \)) is carefully programmed through parameters in length ratio \( \mu \), elastic joint stiffness, cuboid chirality, loading modes, as shown in Fig. 5. For hinges 1 and 5, latex films with thickness 0.4 mm, 0.3 mm (cutting holes, see Fig. S5A), and 0.3 mm are used to Ut4, Ut5, Ut6, respectively. For hinges 2 and 4, films with thickness 0.3 mm, 0.4 mm, and 0.3 mm (cutting holes, see Fig. S5A) are used to Ut4, Ut5, Ut6, respectively. Then the stiffness in Fig. 5A is obtained, which determine the magnitude of critical forces/torques with length ratio \( \mu \) according to SI appendix, section 7, and relationships between the stable states and the regions of normalized forces/torques during the transformation from an arbitrary stable state in the assembly are obtained, as shown in Fig. 5B. The transformation path of 12 stable states (Fig. 5C) is obtained by controlling input rotations or tensions (Movie S7). The setup of the rotation and tension process is the same as that shown in Fig. S7B. The fixed supporter is used to fix the left end of Ut4 and the right supporter is used to support the specimen through the metal shaft to avoid the influence of gravity.

11. Frequency Reconfigurable Antenna

A. Design and fabrication

The antenna is designed using ANSYS High Frequency Structure Simulator (HFSS) according to the designed dimensions shown in Fig. S10A. The reconfigurable antenna includes two parts, antenna substrates with feeding network and radiator and a tristable metastructure, which is fed by an SMA connector for a coaxial cable. The FR4 with relative permittivity \( (\varepsilon_r) \) of 4.4, the thickness of 2 mm and loss tangent of 0.02 is selected as the antenna substrate. The antenna was fabricated by LPKF printed circuit broad (PCB) prototyping machine. The tristable metastructure was fabricated by Stratasys Dimension Elite 3D printer with ABS based on the dimensions in Fig. S10B, and the elastic sheet (length 24.2 mm and 50 mm, width 9 mm, thickness 0.3 mm) was cut by a Trotec Speedy 300 laser cutter. The assembly of the tristable metastructure is similar to that in Fig. S5B. The antenna and tristable metastructure are glued by 3M VHB double sticky tape.

B. Simulation

The antenna is simulated in HFSS. During the simulation, an airbox is employed and all six faces of the airbox are set as radiation boundaries. The distance between the antenna and radiation boundaries is around half wavelength at each resonant frequency, as shown in Fig. S10C.

C. Experiment

The setup of the electromagnetic experiment is shown in Fig. S10D. The antenna was measured in a near-field antenna pattern measurement chamber and connected to Rohde & Schwarz ZNA24 by the coaxial cable (Fig. S10E). By changing the state of the specimen and connecting the adjacent radiators with copper tape (3M 1181), frequencies can be measured and results are shown in Fig. 6D. Experiment and simulation radiation patterns at \( \text{zx} \)- and \( \text{yz} \)-planes of the three stable states resonant frequencies are exhibited in Fig. S10F. Measurement agrees with the simulations well.
Fig. S10. Frequency reconfigurable antenna. (A) The dimensions of the antenna. (B) The dimensions of parts of the tristable metastructure. (C) The setup of simulation. (D) The setup of the experiment. (E) The three stable states and the connection between the antenna and the
measurement system. (F) Experiment and simulation radiation patterns at xz- and yz-planes of the three stable states resonant frequencies.

12. Stiffness of The Thick-Panel Tristable Structures

The stiffness of a single unit can be obtained by calculating the derivatives of Eq. S50 about force, as

\[
K_p = \begin{cases} 
\frac{dF_{e2,e4}}{dh} = \frac{dF_{e2,e4}}{dh} / d\varphi_2, & \text{between states } 1 \text{ and } 4 \\
\frac{dF_{e1,e5}}{dh} = \frac{dF_{e1,e5}}{dh} / d\varphi_4, & \text{between states } 4 \text{ and } 6
\end{cases}
\] (S52)

Notice that only tension stiffness is considered at states 1 and 5 as the unit is folded at those two configurations. For stable states 4, the tension stiffness is different from the compressive one due to the different deformation paths.

Based on the Eq. S52 (codes ‘F3BE_DerivationsForForceTorqueEq_StableStates14.nb’ and ‘F3BE_DerivationsForForceTorqueEq_StableStates46.nb’ in Dataset S3), the stiffness of the unit is dependent on the unit geometry ratio \( \mu \) and joint stiffness. As shown in Fig. S11A-D, the tension stiffness at stable states 1 and 4 increases with \( \mu \), whereas the tension stiffness at 6 and the compressive stiffness at 4 reduce with \( \mu \). In contrast, an increase in joint stiffness leads to a higher stiffness at all the stable states.

When a number of units are connected in series, the stiffness of the assembly at each stable state can be obtained as

\[
\frac{1}{K_{ass}} = \frac{1}{K_{U11}} + \frac{1}{K_{U12}} + \cdots + \frac{1}{K_{Uir}} + \cdots
\] (S53)

in which \( K_{Uit} \) is the stiffness of the unit \( U_i \) \((i = 1, 2, \ldots)\).

Therefore, by adjusting the geometric and material parameters of each unit as well as the number of units, the stiffness of the assembly at each stable state can be programmed. To demonstrate this, the stiffnesses of all the stable states for the assembly in Fig. 4I are calculated and presented in Fig. S11E. The two units in the assembly have identical geometry but different joint stiffness, leading to different stiffnesses at different stable states.
Fig. S11. The programmable stiffnesses of the tristable structure and its assembly. (A-D) The stiffness vs. $h/a$ curve of the units with $\mu = 1, 0.706, 0.388$ and 0.2, and the corresponding stiffnesses at stable states. (E) The stiffnesses for all the stable states of the assembly in Fig. 4I.
**Movie S1.** Folding sequence of the right-handed kirigami cuboid.

**Movie S2.** Kirigami cuboid with torsional springs in Fig. 2A to Fig. 2C.

**Movie S3.** The wood tristable structures with $\mu = 1$ and 0.388.

**Movie S4.** The tension experiment of a tristable structure with $\mu = 0.706$.

**Movie S5.** Transformation path of stable states about assemblies of two units under twist in Fig. 4C and Fig. 4F.

**Movie S6.** Transformation path of stable states about assemblies of two units under tension in Fig. 4I and Fig. S9C.

**Movie S7.** Transformation path of stable states about the assembly of three units.
Dataset S1. The data about the experiment of an elastic joint in Fig. S2D.

Dataset S2. Multistable_structure_Datas_0.706

The data about the experiment and simulation of multistable structures with $\mu = 0.706$ in Fig. 3E, Fig. S8.

Dataset S3. Functional codes for the derivations of equations and curves in figures

Dataset S4. Antenna_Datas

The data about the experiment and simulation of the antenna in Fig. 6.
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