A microlensing event may exhibit a second brightening when the source and/or the lens is a binary star. Previous study revealed 19 such repeating event candidates among 4120 investigated microlensing light curves of the Optical Gravitational Lensing Experiment (OGLE). The same study gave the probability \( \approx 0.0027 \) for a repeating event caused by a binary lens. We present the simulations of binary source lensing events and calculate the probability of observing a second brightening in the light curve. Applying to simulated light curves the same algorithm as was used in the analysis of real OGLE data, we find the probability \( \approx 0.0018 \) of observing a second brightening in a binary source lensing curve. The expected and measured numbers of repeating events are in agreement only if one postulates that all lenses and all sources are binary. Since the fraction of binaries is believed to be \( \leq 50\% \), there seems to be a discrepancy.

**Key words:** Gravitational lensing - Galaxy structure - binaries: general

### 1. Introduction

Investigations of the Galactic disk show that the fraction of stars in binary systems is high, reaching 57% for solar-type stars (Duquennoy and Major, 1991). For later spectral types the multiplicity fractions seem to be lower (Fisher and Marcy 1992; Reid and Gizis 1997). Lada (2006) summarizes the existing observations, concluding that two thirds of the main sequence stars in the disk have no companions.

The shape of the microlensing light curve may be changed if the observed source is a binary star. In particular the event may look as subsequent two events corresponding to microlensing of two binary components. The probability of observing a binary source microlensing has been investigated theoretically by several authors (Griest and Hu, 1992; Dominik, 1998; Han and Jeong, 1998; Han 2005, to cite few). The typical conclusion of such studies is that in few percent of cases where the source is a binary, this fact should have observable consequences. Assuming that the stars in the Galactic bulge form binary systems with probability
similar to the stars in the disk (which has not been proved), one expects that several tens of events among several thousands discovered to date have binary source characteristics.

In a recent paper Skowron et al. (2009) investigate the problem of repeating microlensing events using the data of the Optical Gravitational Lens Experiment (e.g., Udalski 2003). Majority of the events have been discovered by the Early Warning System (EWS - see Udalski et al. 1994; Udalski 2003 for details). The definition of an repeating microlensing event adopted by Skowron et al. requires that the two brightenings in the light curve caused by microlensing are well separated, i.e. the observed luminosity comes to the baseline between the peaks. There are 19 such cases among 4120 events investigated, and 12 of them are interpreted as a result of lensing by wide binary systems, while 6 allow concurrent binary lens/binary source interpretations. Even if all ambiguous cases were binary source events, the probability of observing a binary source repeating event would be only 0.15%.

Not all the binary source events comply with the above definition of repeating events. If the binary source separation is of the order of the lens Einstein radius, the peaks in the light curve partially overlap. This may produce smooth light curves of various shapes, some indistinguishable from those produced by single source approaching cusps of the binary lens caustics. Even the binary lens caustic crossing event may be classified as due to binary source if the observations are sparse. Examples of several binary source / binary lens events are given by Jaroszyński et al. (2004, 2006) and Skowron et al. (2007).

In this paper we simulate the microlensing light curves with binary sources to find probability of producing a repeating event. Our aim is to obtain realistic light curves, and we include several details typical for the OGLE team observations. (Sampling rates, observational errors and their dependence on observed luminosity, span of observations etc). Also the classification of simulated light curves and fitting procedures closely follow the approach of Skowron et al. (2009). We expect, that the probability of finding a repeating event among simulated light curves would be directly comparable with the number obtained as a result of analyzing all EWS events by Skowron et al. (2009), and some limits on the binary stars population in the Galactic bulge will be possible to place.

2. The simulations

2.1. Parametrization of the light curves

A binary source event light curve can be modeled using eight parameters: the times of closest approaches of the lens to the source components $t_{01}$ and $t_{02}$, the characteristic Einstein time $t_E$, the two dimensionless impact parameters $b_1$ and $b_2$, the two source components energy fluxes $F_1$ and $F_2$, and the blended flux of
stars in the seeing disk $F_b$. The observed flux is a given function of time:

$$F(t) = A(u_1(t))F_1 + A(u_2(t))F_2 + F_b \tag{1}$$

where $A(u)$ describes the so called Paczyński curve (e.g. Paczyński, 1991):

$$A(u) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}} \quad u_1(t) = \sqrt{\left(\frac{t-t_0}{t_E}\right)^2 + b^2_1} \tag{2}$$

The model parameters listed above are related to even larger number of parameters characterizing the lens and source distances $d_L$, $d_S$, lens mass $m$, observer, lens and source velocities $v_O$, $v_L$, $v_S$, and source components intrinsic luminosities $L_1$, $L_2$. The Einstein time is is given as

$$t_E = \frac{r_E}{v_\perp} \quad r_E = \sqrt{\frac{4Gm}{c^2} \frac{d_L(d_S-d_L)}{d_S}} \tag{3}$$

where $r_E$ is the Einstein radius and $v_\perp$ is the velocity of the lens relative to the line joining the observer and the source, measured in the plane perpendicular to this line:

$$v_\perp = (v_L - (1-x)v_O - xv_S)_\perp \quad x \equiv \frac{d_L}{d_S} \tag{4}$$

The Einstein radius is a unit of length in the lens plane; in the source plane the projected Einstein radius $\tilde{r}_E \equiv r_E/x$ plays the same role.

The shape of the light curve depends on the binary source separation or, more precisely, on the projected into the sky distance between the two components in the time of interest, expressed in Einstein radius units, $d \equiv a_\perp/\tilde{r}_E$. Since we investigate a population of binary sources with large scatter of separations and the microlensing events may happen in any phase of binary motion, we replace the separation by the binary semimajor axis.

The angle between the source trajectory and the line joining its components, both projected into the sky, has a random value $0 \leq \beta \leq 2\pi$. Once it is chosen, the separation perpendicular to the trajectory $d_\perp \equiv |d \sin \beta|$ can be calculated.

Since we are interested in observed fraction of binary source microlensing events among events which may be interpreted as due to a single source, we assume that the lens passes within projected Einstein radius from at least one of the binary components. Depending on the value of $d_\perp$, the region of interest may consist of one ($d_\perp < 2$) or two parts ($d_\perp > 2$). We assume that the possible trajectories cross any point of the region of interest with the same probability. The choices of $d$ and $\beta$ imply the following relations between the parameters:

$$b_2 = b_1 + d \sin \beta \quad t_{02} = t_{01} + d t_E \cos \beta \tag{5}$$

and (by construction) at least one of the inequalities $|b_1| \leq 1$ or $|b_2| \leq 1$ holds. We choose $t_{01}$ at random from the time interval during which the observations were performed, so $t_{\text{start}} \leq t_{01} \leq t_{\text{end}}$, where $t_{\text{start}}$, $t_{\text{end}}$, denote the beginning and end of observations.
2.2. Distributions of physical parameters

Among the physical parameters, only the Einstein time \( t_E \), the dimensionless binary separation \( d \) and the fluxes \( F_1 \), \( F_2 \), and \( F_b \) have a direct influence on the shape of the observed (simulated) light curve and may be correlated with each other. The other light curve parameters depend on random choices of values and are independent of the parameters listed above.

We find the distribution of the Einstein time-scales using the simulations of the Galaxy structure (see below). The influence of the extinction and its dependence on position makes it difficult to obtain the distribution of apparent luminosities for the OGLE sources from a simple Galaxy model, and we use the distribution of baseline fluxes from the OGLE observations. The distribution of binary sources separations must be found independently, based on the model of binary systems population. Since the dimensionless separation \( d \) depends also on the value of the Einstein radius, the simulations of the Galaxy structure are needed again. We also find the distribution of distance moduli to sources, to check their correlations with other parameters.

The population of Galaxy disk binary stars has been studied by many authors (Abt 1983, Duquennoy and Mayor 1991, Fisher and Marcy 1992 Lada 2006 and references therein). We adopt a log-normal distribution of binary component separation \( a \) with \( \langle \log a \rangle \approx 1.5 \) and \( \sigma_{\log a} \approx 1.5 \) after Han and Jeong (1998), who use the study of Duquennoy and Mayor (1991). Little is known about the binaries in the bulge, and we assume that their population is similar to the disk one.

We use the Han and Gould (2003) model of the density distribution in the Galaxy disk. We also adopt their mass function (the same for the disk and the bulge), which includes brown dwarfs and main sequence stars with continuous distributions of masses, as well as discrete contributions from black holes (5\( M_\odot \)), neutron stars (1.35\( M_\odot \)) and white dwarfs (0.6\( M_\odot \)). For the bulge density distribution we use the triaxial exponential model E2 of Dwek et al. (1995) with parameters given by Stanek et al. (1997).

We basically follow Wood and Mao (2005) description of the disk and bulge kinematics. The adopted parameters are: disk rotation speed \( v_\phi = 220 \, \text{km/s} \), with the dispersion \( \sigma_\phi = 30 \, \text{km/s} \), \( \sigma_z = 20 \, \text{km/s} \) in the azimuthal and perpendicular to the disk plane directions respectively. Inside the central 0.5kpc the flat rotation curve is replaced by the solid body rotation curve. For the bulge we assume solid body rotation up to 0.5kpc, followed by flat rotation curve with velocity \( v_\phi = 50 \, \text{km/s} \) at greater radii. The velocity dispersion for the bulge is: \( \sigma_{x,y,z} = (110, 82.5, 66.3) \, \text{km/s} \). Both the disk and the bulge kinematics descriptions are simplified (compare e.g. Sofue, Honma and Omodaka 2008 for the disk, and Clarkson et al. 2008 for the bulge), but, as our numerical experiments show, the details have little influence on the obtained distributions of the Einstein times and radii.

We use Monte Carlo method to get the relations between the intrinsic physical
parameters influencing the microlensing phenomena. Since the observed distribution of parameters depends on their probability of occurrence, we always weight any set of intrinsic parameters by their contribution to the rate of events (Griest 1991, Kiraga and Paczyński 1994, Wood and Mao 2005):

$$\Delta \Gamma \propto n_L(m) \Delta m \Delta d_L n_S d_S^2 \Delta d_S v_\perp r_E$$  \hspace{1cm} (6)

where $n_L(m)$ is the number density of lenses with masses in the range $m - m + \Delta m$, at the distance $d_L$ from the observer, $n_S$ is the number density of "observable" sources at the distance $d_S$, $v_\perp$ is the lens velocity, and $r_E$ - the Einstein radius. The number density of observable sources is proportional to mass density times the fraction of all stars, which are brighter than $M_I = I_{\text{lim}} - 5 \lg(d_S/10\text{pc})$, where we use $I_{\text{lim}} = 20$ as a faint limit of OGLE photometry. We use Holtzman et al. (1998) luminosity function for the bulge and Gould, Bahcall, and Flynn (1997) for the disk.

We investigate the vicinity of the Galaxy center choosing at random locations with Galactic longitudes $-8^\circ \leq l \leq 10^\circ$ and latitudes $-7.5^\circ \leq b \leq -1.5^\circ$, which includes (but is not identical to) the majority OGLE fields. The source distance is also chosen at random from the range $1 \leq d_S \leq 13$ kpc, and the lens has a random location between the observer and the source. We assume that the probability that an object at given position belongs to the bulge or to the disk population is proportional to the respective mass densities, and each choice is done with random number generator. Thus four different combinations (source in the bulge and lens in the bulge i.e. "bulge - bulge", and similarly "bulge - disk", "disk - bulge", and "disk - disk") are possible. The velocities are assigned depending on the population to which the source/lens belong. The Gaussian part of each velocity component is chosen using Gaussian number generator. The lens mass is assigned using reversed cumulative mass distribution. The double source separation has log-normal distribution, so it also can be obtained with Gaussian number generator. We use random number and Gaussian generators as advocated by Press et al. (2007).

The choices of intrinsic parameters described above define the values of parameters closely related to observations, such as the Einstein time $t_E$, the dimensionless source separation $d$, or the source distance modulus $m - M = 5 \lg(d_S/10\text{pc})$. We construct three-dimensional histograms for the expected occurrence of these parameters, storing independently information for the four combinations of the source/lens belonging to the bulge/disk populations.

The results are shown in Fig.1. We treat the Einstein time, which can be directly measured in the model fits to light curves, as a primary variable. The histograms in the bottom panel show its distribution for the four cases mentioned. The relative importance of the four cases is given as the ratio: b-b:b-d:d-b:d-d = 57:27:11:5, where "b" stays for the bulge and "d" - for the disk. The resulting distribution of the Einstein times logarithm is well approximated by a Gaussian with the average $< \lg(t_E) > = 1.16$ and the dispersion $\sigma(\lg(t_E)) = 0.30$. This is the simulated (in a sense intrinsic) distribution of Einstein times.
In the upper panel the averaged dimensionless source separation is shown as a function of the Einstein time. The dependence is strong and slightly different for the "cases" considered. On the other hand the distribution of separations in each small $\Delta t_E$ interval remains log-normal, and the dispersion is practically independent of the time scale or case considered.

The middle panel shows that the distance modulus to the source is only weakly correlated with the Einstein timescale, except the least important case of the source and the lens both belonging to the disk. This can be understood as a direct consequence of the limited spatial extent of the bulge.

The relations found above suggest, that it is safe to simulate binary source light curves ignoring the correlations of observed luminosity with the timescale, but it is necessary to take into account the dependence of the source components separation on this parameter.

Assuming the similarity of the bulge binaries to better known binaries from Solar neighborhood, we postulate the mass luminosity relation for component stars in the form $L \sim m^\alpha$, which implies that the luminosity ratio, $w \equiv L_2/L_1$ is the function of the mass ratio $q \equiv m_2/m_1$: $w = q^\alpha$. Using stellar $I$ - band magnitudes one has:

$$\Delta I \equiv I_2 - I_1 = 2.5\alpha \lg q$$ (7)
Assuming that the logarithm of the mass ratio is uniformly distributed in the range corresponding to $0.1 \leq q \leq 1$, (compare Skowron et al., 2009) and postulating that $\alpha \approx 3.2$, we see that magnitude difference is uniformly distributed in the range $0 \leq \Delta I \leq 8$. We use this distribution of luminosity ratios in our simulations.

The definition of the blended flux in the case of a binary source is not obvious. For the binary sources undergoing microlensing and interpreted as such the definition of blending parameter is natural:

$$f \equiv \frac{F_1 + F_2}{F_1 + F_2 + F_b} \quad \text{(8)}$$

where the values of fluxes are obtained from the fit. If the same source undergoes microlensing interpreted as single source event involving its first component, the fitted blending parameter would be:

$$f \equiv \frac{F_1}{F_1 + F_2 + F_b} \quad \text{(9)}$$

and $F_2 + F_b$ would serve as blended flux (compare Dominik, 1998). Thus it is not clear which blending parameter should be used and what distribution for its values should be adopted. For the sake of simplicity we use the first of the definitions, thus avoiding possible dependencies between components flux ratio and blending parameter. We also assume the blending parameter to be uniformly distributed, as suggested by fits to a limited sample of binary lens events (Jaroszyński et al. 2004).

We get the baseline flux $F_{\text{base}}$ from the condition $P(> F_{\text{base}}) = R$, where $R$ symbolizes the random number generator and a result of its call, and $P()$ denotes the cumulative probability distribution valid for all OGLE EWS events. Next the blending parameter $f = R$ is obtained with the subsequent random number.

In principle the components stellar magnitude difference could also be chosen at random, since we have assumed its uniform distribution, but it is less time consuming to consider a finite number of fixed flux ratios, and derive the probability of discovering a repeating event for each of them. Binaries of similar luminosity components produce binary source events with higher probability (Han and Jeong, 1998), as compared to binaries with large flux ratios, so they require smaller number of simulations for the same accuracy of the result.

The fluxes relevant for the light curve simulation are given as:

$$F_1 = \frac{1}{1 + w} f F_{\text{base}} \quad F_2 = \frac{w}{1 + w} f F_{\text{base}} \quad F_b = (1 - f) F_{\text{base}} \quad \text{(10)}$$

where $w$ is one of the considered values of the flux ratio.

### 2.3. Light curves

Once all relevant parameters are chosen the model light curve of a binary source lensing event can be obtained using Eqs. 1 and 2. To simulate the observations
we use a reference light curve from the OGLE EWS database. (We have employed several reference light curves for several independent series of simulations.) The reference light curve gives the observed luminosities $I_{\text{ref}}$ and estimated errors $\Delta I_{\text{ref}}$ at a series of observation times $t_i$. Using our equations we can get the model luminosities $I_{\text{mod}}$ for each $t_i$. The simulated observational errors are obtained by rescaling the reference errors according to the formula of Wyrzykowski (2005):

$$\Delta I_{\text{mod}} = \Delta I_{\text{ref}} 10^{0.33875(I_{\text{mod}}-I_{\text{ref}})}$$

which describes the dependence of the estimated observational errors on measured luminosities.

The comparison between the fluctuations in luminosity of a set of constant flux stars and estimated measurement errors given by the same OGLE photometry pipeline (Wyrzykowski et al. 2008) suggests a further rescaling of the errors. The OGLE errors are slightly underestimated and we adopt:

$$\Delta I = \sqrt{(1.38\Delta I_{\text{ref}})^2 + 0.0052^2}$$

as a measure of observational errors, following Skowron et al. (2009).

Finally we add a Gaussian noise to the model light curve:

$$I = I_{\text{mod}} + \Delta I \cdot \mathcal{G}$$

where $\mathcal{G}$ is a Gaussian number generator with zero mean and unit dispersion. The set of $I(t_i)$ and $\Delta I(t_i)$ represents a simulated light curve. We use its flux equivalent for further analysis.

2.4. Looking for repeating microlensing events

The strategy used here closely follows the method of analysis applied to the real OGLE EWS light curves by Skowron et al. (2009) in search for repeating microlensing events. The search has been done with the help of an automated algorithm which is also employed here. Since the non-microlensing light curves are absent among our simulated set, the visual inspection of the events picked up by the algorithm does not seem to be necessary.

First we fit a constant flux model to the simulated light curve and calculate its $\chi^2(\text{constant}) = \chi^2_0$, which serves as reference for other models. Next we find the main peak in the light curve and try a single source fit, obtaining $\chi^2(\text{single}) = \chi^2_1$. Our previous experience shows that for $\sim 10^3$ observations, the attempt of fitting a single source model to the constant flux + noise light curve typically improves the fit by $\Delta \chi^2 \approx 15$, and $\Delta \chi^2 > 55$ happens in only 1% of cases. Thus if $\chi^2_0 - \chi^2_1 < 55$ we conclude that the synthetic light curve does not represent a microlensing event, which may be caused by excessive blending, passing close to the very faint binary component, or due to the fact that the event takes place out of observational season.
If the single source model improves the fit sufficiently, we try to remove the main peak to look for another one. To remove the main peak we use the single source model and find the time span where model light curve is more than $3\sigma$ above the baseline flux level. All simulated fluxes from this time interval are replaced by the baseline flux plus Gaussian noise, with observational errors rescaled to the base level. A constant flux model is again fitted to the modified light curve for comparison with the single source fit applied next. Since we are going to compare simulation results with the Skowron et al. (2009) analysis of real microlensing events, we follow their approach in identifying and evaluating the second peak. We require the following inequality to hold:

$$
\chi^2_0 - \chi^2_1 \geq 0.2\chi^2_1
$$

where $\chi^2_0$ and $\chi^2_1$ are related to the constant flux and single source models of the modified light curve respectively. This (rather strong) requirement produces samples of repeating microlensing events similar to obtainable by visual light curve inspection, as shown by Skowron et al. (2009).

If the improvement of the fit does not meet our requirement, we conclude that the second peak is absent, and that we probably deal with a single source event. If the original single source fit for the main peak is satisfactory ($\chi^2/N_{DOF}$ not too high) we write down its parameters as well as the original simulation parameters for future analysis. Otherwise the case is rejected.

On the other hand, if the existence of the second peak is established according to our criterion, we fit a double source model to the original light curve. We use the positions of the peaks, their impact parameters, and the fluxes obtained in the single source fits as starting parameter values for the final model. The starting value for the Einstein time is taken from the single source fit to the main peak, and the blended flux is estimated from the flux budget ($F_1 + F_2 + F_b \equiv F_{\text{base}}$ for a correct model). The goodness of fit for the binary source is checked. Finally we check whether the double source model light curve comes to the baseline, finding its minimum flux $F_{\text{min}}$ between the peaks and checking whether $F_{\text{min}} \leq F_{\text{base}} + 3\sigma_{\text{base}}$. If the case passes both tests we treat it as a repeating double source event. If goodness of fit test is passed but baseline test is not, the case is treated as non-repeating double source event. In any case the parameters of the fits and original simulation parameters are written down for further analysis. It must be admitted that our method is primarily looking for repeating events and many double source events are lost by our algorithm. In particular, if the peaks are partially overlapping, they would probably be both fitted by a single source model and the second peak would be lost. In consequence the case may be classified as a single source event or rejected due to the low fit quality.
3. Results

Using the described methods we have simulated few millions of binary source lensing light curves. About one third to one half of the simulated cases (depending on the flux ratio) have been classified by our algorithm as microlensing event candidates, while the other have been rejected because of too weak or missing microlensing signatures. This may have few reasons. We simulate binary source events and in half of the cases the lens passes close to the fainter component, thus leading to possibly insignificant magnification of the total flux. The added blended flux may have similar effect. The simulated magnification may happen off season, also preventing identification.

The distribution of the Einstein times used in simulations has been obtained from the Galaxy models. We always use distributions of time-scale logarithms in our calculations, and the mean values and standard deviations reported below are always mean logarithms and their deviations. (See also Discussion.)

Our Galaxy model gives the intrinsic distribution of Einstein times for all would-be lenses with mean logarithm corresponding to \( \approx 14^d \). The subsample of microlensing candidates has the average corresponding to \( 18^d \), where the simulation parameter values are used. Finally, the average for parameters fitted to the same subsample corresponds to \( 22^d \). The three distributions are shown in Fig.2. The standard deviations for two model distributions are the same (0.3 in logarithm corresponding to factor 2 in Einstein times) and for the distribution of fitted parameters slightly higher (0.5 or factor 3). This systematic shift to longer time-scales may be understood as the effect of preferential rejection of short lasting events, since they are easier to miss, and as a result of Woźniak and Paczyński (1997) degeneracy, which allows longer lasting, more blended and more strongly amplified models of the same fit quality. The small subsample of binary source lensing repeating events has still longer time-scales, with average corresponding to \( 26^d \), without significant difference between simulations and fits, and with deviation \( \sigma_{\log} = 0.3 \).

The effectiveness of finding a repeating double source event as a function of the binary components stellar magnitude difference is shown in Fig.3. It is a sharply decreasing function of luminosity difference and systems with \( \Delta I > 4^m \) have negligible chance of producing a repeating event. This tendency is in agreement with the results of Han and Jeong (1998).

The main result of our study is the evaluated probability of finding a repeating microlensing event among large number of events caused by sources belonging to the population of binaries of assumed properties. The effectiveness averaged over the interval of eight stellar magnitudes is 0.18% and this number should be compared with the result obtained for the sample of OGLE EWS events by Skowron et al. (2009).
Fig. 2. The histograms of the Einstein time. The dashed line represents the distribution for all simulated models. The dotted line also shows the distribution of model parameter values, but is limited to cases accepted as microlensing events. The solid line shows the distribution of fitted parameter values, again limited to accepted models.

Fig. 3. The effectiveness of discovering a repeating microlensing event as a function of the binary source components flux ratio. The flux ratio is expressed in stellar magnitudes. ($\Delta l = 2.5\log(F_1/F_2)$). The error bars are estimated based on the Poisson statistics and the number of repeating events found for given flux ratio.
4. Discussion

We have calculated the probability of observing a repeating microlensing event among the sample of OGLE EWS events toward the Bulge, caused by the binary nature of the source star. If all the sources included in the sample were binary, there should be $4120 \times 0.0018 \approx 7$ repeating microlensing events of this kind, where the estimate is done for the 4120 events investigated by Skowron et al. (2009). (The events of season 2008 have been excluded from their study.)

Analogous calculation by Skowron et al. (2009) shows that if all the lenses causing OGLE sample events were binary, they should produce another $4120 \times 0.0027 \approx 11$ repeating events. The analysis of light curves from the sample by the same authors gives 19 repeating events, 12 probably due to binary lenses, 6 possibly due to binary sources, and one leading to unsatisfactory models.

Since not all stars belong to multiple systems, there seems to be a problem with the interpretation of the repeating events. If half of the systems were binary, both in the disk and in the bulge (which is probably an overestimation – compare Introduction), the expected numbers of repeating events would be roughly 6 plus 3 due to binary lenses and binary sources respectively. According to Poisson statistics probability of observing 18 cases instead of expected 9 is only 0.0029.

Our result is valid as long as our assumptions are true. The strong dependence of the probability of observing a repeating event on the binary source flux ratio, as well as a similar dependence of the probability on the binary lens mass ratio obtained by Skowron et al. (2009) suggests, that one may avoid the discrepancy narrowing the ranges of the flux / mass ratios. For example assuming that population of binary sources is limited to $0 \leq \Delta I \leq 4$ with uniform distribution increases the averaged effectiveness twofold, since extreme flux ratio binaries with $4 \leq \Delta I \leq 8$ can produce only negligible number of repeating events. (Similarly narrowing the range of binary lenses mass ratios increases their probability of producing repeating events). The simple narrowing of the parameter ranges is only an example, but it shows that a different distributions of flux / mass ratios may improve the agreement between the theory and observations.

As a byproduct of our simulations we have the following comment related to the distribution of time scales. Many authors (cf. Wood and Mao 2005, Sumi et al. 2006, Popowski et al. 2005) discuss distribution of time-scales of microlensing event, providing and comparing the values of mean time-scale in days. It is worth to notice that both, observational histograms (e.g. Sumi et al. 2006) as well as simulated distributions (e.g. Paczyński 1991, Wood and Mao 2005) are usually presented on logarithmic abscissa. In this way the plots are better confined and more symmetric than their equivalent linear representation, and thus easier to compare with a Gaussian distribution. This is the reason we postulate using a mean logarithm of time-scales instead of mean time-scales in any application. Mean values derived from distributions far from Gaussian can be mis-guiding, and in the case of microlensing, can cause greater apparent discrepancies between models and ob-
servations, as small deviations at long time-scales tail give big difference in mean value.

It is also worth mentioning that the distribution of time-scales given by the Galaxy model, and the distribution of the observed (or, more precisely, fitted) values are not the same (see e.g., Wyrzykowski 2005), since longer lasting events have a greater chance to be included in the sample of microlensing event candidates, both in real observations and in simulations using realistic algorithm of event selection. In case of simulations it is also possible to investigate the distribution of intrinsic time scales in the subsample selected as microlensing event candidates. Our calculations give the mean logarithmic Einstein time-scale of $\approx 14^d$ for all simulated events, $\approx 18^d$ for simulated events, that were selected as microlensing candidates, and $\approx 22^d$ for time scales fitted to the selected subsample. It is interesting that the logarithmic means for OGLE II and OGLE III EWS events (Udalski 2003) are $\approx 22^d$ and $\approx 18^d$ respectively, in good agreement with our results.

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