Electric Dipole Polarizability in $^{208}$Pb as a Probe of the Symmetry Energy and Neutron Matter around $\rho_0/3$

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It is currently a big challenge to accurately determine the symmetry energy $E_{\text{sym}}(\rho)$ and pure neutron matter equation of state (EOS) $E_{\text{PNM}}(\rho)$ as well as pure neutron matter equation of state (EOS) $E_{\text{PNM}}(\rho)$ play key roles in the investigation from microscopic neutron-rich nuclei to macroscopic neutron stars [1–4] and even in new physics beyond the standard model [5]. Although significant progress has been made in recent years in understanding the $E_{\text{sym}}(\rho)$ and $E_{\text{PNM}}(\rho)$ due to a lot of experimental, observational and theoretical efforts, accurate determination of $E_{\text{sym}}(\rho)$ and $E_{\text{PNM}}(\rho)$, even their values around saturation density $\rho_0 \approx 0.16 \text{ fm}^{-3}$, remains a big challenge (see, e.g., Refs. [6–11]). While heavy ion collisions and astrophysical observations provide two important approaches to constrain the symmetry energy from sub- to supersaturation densities, nuclear structure probes usually can most effectively constrain the symmetry energy at sub-saturation densities. It has been established that nuclear mass can put stringent constraint on the magnitude of $E_{\text{sym}}(\rho)$ around $2\rho_0/3$ [12–17] and the neutron skin thickness $\Delta r_{np}$ of heavy nuclei can fix the density slope $L(\rho)$ of the symmetry energy around $2\rho_0/3$ [18–19]. At very low densities of $0.03\rho_0 < \rho < 0.2\rho_0$ and temperature in the range of $3 \sim 11 \text{ MeV}$ where the clustering effects are essential, the symmetry energy have been obtained using data from heavy ion collisions [20].

In contrast to the $E_{\text{sym}}(\rho)$, there has little experimental information on the $E_{\text{PNM}}(\rho)$ [17]. Pure neutron matter provides a unique system in which the $E_{\text{PNM}}(\rho)$ in principle can be precisely determined from microscopic theoretical studies based on chiral effective field theory (ChEFT) [21] and Quantum Monte Carlo (QMC) calculations [22–24]. In these theoretical studies, the main uncertainty is due to the poorly known many-nucleon interactions. Therefore, experimental constraints on $E_{\text{PNM}}(\rho)$ are extremely important to understand the largely uncertain many-nucleon interactions in microscopic calculations of neutron matter.

The nuclear electric dipole polarizability $\alpha_D$ [25] has been proposed to be a good probe of the symmetry energy [26]. However, their exact relationship has not yet been completely understood and even some controversial conclusions have been obtained in different analyses [26–31], which hinders us to extract exact information on the symmetry energy from the measured $\alpha_D$. In this Letter, we find that the $\alpha_D$ in $^{208}$Pb can be determined uniquely by the magnitude of the $E_{\text{sym}}(\rho)$ or almost equivalently the $E_{\text{PNM}}(\rho)$ at much lower densities around $\rho_0/3$, shedding a light upon the genuine correlation between the $\alpha_D$ and the $E_{\text{sym}}(\rho)$. This finding together with the $\alpha_D$ in $^{208}$Pb measured at the Research Center for Nuclear Physics (RCNP) [32] allow us to obtain quite precise constraints on $E_{\text{sym}}(\rho)$ and $E_{\text{PNM}}(\rho)$ around $\rho_0/3$. The present experimental constraint on $E_{\text{PNM}}(\rho)$ may be very useful in understanding the poorly known many-nucleon interactions in the microscopic theoretical calculations of pure neutron matter.

2. The symmetry energy and $\alpha_D$.—The EOS of an asymmetric nuclear matter, defined by its nucleon specific binding energy, can be expanded as

$$E(\rho, \delta) = E_0(\rho) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4),$$

where $\rho = \rho_n + \rho_p$ is nucleon density and $\delta = (\rho_n - \rho_p)/(\rho_p + \rho_n)$ is the isospin asymmetry with $\rho_n$ ($\rho_p$) denoting the neutron (proton) density; $E_0(\rho)$ represents the

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1. Introduction.—The symmetry energy $E_{\text{sym}}(\rho)$ as well as pure neutron matter equation of state (EOS) $E_{\text{PNM}}(\rho)$ play key roles in the investigation from microscopic neutron-rich nuclei to macroscopic neutron stars [1–4] and even in new physics beyond the standard model [5]. Although significant progress has been made in recent years in understanding the $E_{\text{sym}}(\rho)$ and $E_{\text{PNM}}(\rho)$ due to a lot of experimental, observational and theoretical efforts, accurate determination of $E_{\text{sym}}(\rho)$ and $E_{\text{PNM}}(\rho)$, even their values around saturation density $\rho_0 \approx 0.16 \text{ fm}^{-3}$, remains a big challenge (see, e.g., Refs. [6–11]). While heavy ion collisions and astrophysical observations provide two important approaches to constrain the symmetry energy from sub- to supersaturation densities, nuclear structure probes usually can most effectively constrain the symmetry energy at sub-saturation densities. It has been established that nuclear mass can put stringent constraint on the magnitude of $E_{\text{sym}}(\rho)$ around $2\rho_0/3$ [12–17] and the neutron skin thickness $\Delta r_{np}$ of heavy nuclei can fix the density slope $L(\rho)$ of the symmetry energy around $2\rho_0/3$ [18–19]. At very low densities of $0.03\rho_0 < \rho < 0.2\rho_0$ and temperature in the range of $3 \sim 11 \text{ MeV}$ where the clustering effects are essential, the symmetry energy have been obtained using data from heavy ion collisions [20].

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EOS of symmetric nuclear matter; $E_{\text{sym}}(\rho)$ is the symmetry energy and it can be expressed as

$$E_{\text{sym}}(\rho) = \frac{1}{2!} \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2}|_{\delta = 0}. \tag{2}$$

The $E_0(\rho)$ can be expanded around $\rho_0$ as $E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2!} (\frac{\rho}{\rho_0})^2 + O((\frac{\rho}{\rho_0})^3)$ where the $K_0$ is the so-called incompressibility coefficient. The $E_{\text{sym}}(\rho)$ can also be expanded around a reference density $\rho_r$ as $E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_r) + E_{\text{sym}}(\rho) - E_{\text{sym}}(\rho_r)$ with $E_{\text{sym}}(\rho_r) = \frac{3\rho_r}{\rho_r} \frac{dE_{\text{sym}}(\rho)}{d\rho}|_{\rho = \rho_r}$ denoting the density slope of the symmetry energy at $\rho_r$. Neglecting the higher-order terms in Eq. (1) leads to the well-known empirical parabolic approximation $E_{\text{sym}}(\rho) \approx E_0(\rho) + E_{\text{sym}}(\rho)$.

The electric dipole polarizability $\alpha_D$ is proportional to the inverse energy-weighted sum of the electric dipole response [25] which is dominated by the isovector giant dipole resonance (IVGDR) — a nuclear collective oscillation of all the protons against all the neutrons with the symmetry energy $E_{\text{sym}}(\rho)$ acting as the restoring force [33]. Since the neutron and proton densities in the nuclear interior essentially do not change in the IVGDR, the $\alpha_D$ thus probes the symmetry energy not at $\rho_0$ but rather at much lower densities around the nuclear surface where matter with extreme isospin or even pure neutron (proton) matter may form in the oscillation.

A more quantitative preview about the $\alpha_D$ can be obtained from the macroscopic hydrodynamical model which predicts [34] [35]

$$\alpha_D = \frac{e^2}{24} \int \frac{r^2}{v_{\text{sym}}(\rho)} d^3r, \tag{3}$$

where $r$ represents the radial coordinate in the nuclei and $v_{\text{sym}} = E_{\text{sym}}(\rho)/\rho$. Using an empirical radial density distribution of nuclei and a simple parametrization of $E_{\text{sym}}(\rho)$, one can find the $\alpha_D$ in Eq. (3) is dominated by the symmetry energy values at low densities around $\rho_0/3$ (see the supplemental material). Moreover, using a leptodermous expansion in Eq. (4), Lipparini and Stringari [34] derived a simple expression of $\alpha_D$ as

$$\alpha_D = \frac{e^2}{12 b_v} \frac{A(r^2)}{b_v} \left(1 + \frac{5}{3} \frac{b_v}{b_s} A^{-1/3}\right), \tag{4}$$

where $\langle r^2 \rangle$ is the mean-square radius of the nucleus with mass number $A$, and $b_v$ and $b_s$ are the volume and surface symmetry coefficients which are related to the symmetry energy coefficient $a_{\text{sym}}(A)$ of finite nuclei as [34]

$$a_{\text{sym}}(A) = \frac{1}{2} \frac{b_v}{1 + (b_s/b_v)A^{-1/3}}. \tag{5}$$

Eq. (1) and Eq. (4) can also be derived within the droplet model [15] [36] [37] by invoking the simple relations $b_v = 2E_{\text{sym}}(\rho_0)$ and $b_s/b_v = \frac{5}{3}E_{\text{sym}}(\rho_0)/Q$ with $Q$ being the surface stiffness coefficient [36]. Substituting Eq. (1) into Eq. (4), one then obtains the following relation

$$\alpha_D(A) = \frac{e^2}{24 a_{\text{sym}}(\frac{r^2}{A})}, \tag{6}$$

which suggests that the $\alpha_D$ of a nucleus with mass number $A$ is inversely proportional to the symmetry energy coefficient of a nucleus with mass number $(\frac{A}{2})^3$ (e.g., for $^{208}$Pb, $\alpha_D(A=208) \propto 1/a_{\text{sym}}(A=45)$. Considering the strong correlation between the $a_{\text{sym}}(A)$ and the $E_{\text{sym}}(\rho_A)$ at a specific density $\rho_A$ [12] [14] [28] [29] [30], one then expects that the $\alpha_D$ in $^{208}$Pb should be strongly correlated with $E_{\text{sym}}(\rho)$ at $\rho = \rho_A = 0.16$ fm$^{-3}$ [39].

The above discussions indicate that a model-independent linear correlation may exist between $1/\alpha_D$ in $^{208}$Pb and the magnitude of the symmetry energy around $\rho_0/3$. As will be shown in the following, this genuine correlation can be exactly confirmed by the microscopic random-phase approximation (RPA) calculations based on non-relativistic and relativistic mean-field models. Then given $E_{\text{sym}}(\rho_0/3) \approx E_{\text{sym}}(\rho_0) - \frac{2}{3} L(\rho_0)$ or even better $E_{\text{sym}}(\rho_0/3) \approx E_{\text{sym}}(2\rho_0/3) - \frac{2}{3} L(2\rho_0/3)$, one can easily understand the dependence of $\alpha_D$ in $^{208}$Pb on both $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ [28] or on both $E_{\text{sym}}(2\rho_0/3)$ and $L(2\rho_0/3)$ [29]. This can also explain the relatively weak correlation of $\alpha_D$ in $^{208}$Pb with $\Delta_{\rho_{np}}$ [27] since the latter is essentially determined by $L(2\rho_0/3)$.

3. The symmetry energy from $\alpha_D$ in $^{208}$Pb.—In order to study the correlation between the $\alpha_D$ in $^{208}$Pb and the symmetry energy value at different densities, we analyze the results from 62 representative non-relativistic and relativistic interactions which all give a good description of the ground state properties of finite nuclei but predict very different density dependence of the symmetry energy, including 47 Skyrme interactions [10] [10] [41] (i.e., BSK1, BSK2, BSK6, Es, Gs, KDE01v, MSK7, MSL0, MSL1, NRAPR, Rs, SAMi, SGI, SGII, SK255, SK272, SkA, SkI3, SkM, SkMP, SkP*, SKP, SkS1, SkS2, SkS3, SkS4, SkSC15, SkT7, SkT8, SkT9, SKX, SKXe, SKXm, Skxs15, Skxs20, SLy4, SLy5, SLy10, v070, v075, v080, v090, v105, v110, Zs, Zs*) and 15 relativistic interactions involving FSU, NL3 and TF families [42] [43] together with DD-ME2 [44]. For the Skyrme interactions, we evaluate the $\alpha_D$ using the Skyrme-RPA program by Colò et al [45] while for the relativistic interactions, we directly invoke the results of RPA calculations reported in Refs. [28] [43].

The obtained data-to-data relations between $10^3/\alpha_D$ in $^{208}$Pb and the $E_{\text{sym}}(\rho_0)$ at $\rho_0 = 0.02, 0.05, 0.08, 0.11$ and 0.16 fm$^{-3}$ are displayed in Fig. 1. Also included in Fig. 1 are the linear fits together with the corresponding Pearson correlation coefficient $r$ as well as the experimental result of $\alpha_D = 20.1 \pm 0.6$ fm$^{-3}$ obtained from a high-resolution measurement at RCNP via polarized proton inelastic scattering at forward angles [42]. Note that
since Coulomb excitation dominates at forward angles, the measured $\alpha_D$ in $^{208}$Pb should be a relatively clean isovector probe with less uncertainties from strong interaction. It is seen that the $1/\alpha_D$ exhibits a very strong linear correlation ($r = 0.977$) with the $E_{\text{sym}}(\rho)$ around $\rho_0/3$ (i.e., $\rho_{\text{r}} = 0.05$ fm$^{-3}$), confirming the predictions of both hydrodynamical and droplet models. The correlation remains strong at densities below $\rho_0/3$ but it drops rapidly when the density is above about $\rho_0/2$. For $\rho_{\text{r}} = 0.05$ fm$^{-3}$, the linear fit gives

$$10^3/\alpha_D = (18.94 \pm 0.94) + (1.94 \pm 0.05)E_{\text{sym}}(\rho_{\text{r}}),$$

with $\alpha_D$ in fm$^3$ and $E_{\text{sym}}(\rho_{\text{r}})$ in MeV. Eq. (7) together with the experimental value of $\alpha_D = 20.1 \pm 0.6$ fm$^3$ then lead to

$$E_{\text{sym}}(\rho_{\text{r}}) = 15.91 \pm (0.77)_{\text{exp}} \pm (0.63)_{\text{the}},$$

where the uncertainties with “exp” and “the” are obtained from the propagation of the experimental uncertainty of $\alpha_D$ and parameter errors in the linear fit, respectively. Consequently, one can obtain a stringent constraint of $E_{\text{sym}}(\rho_{\text{r}} = 0.05$ fm$^{-3}) = 15.91 \pm 0.99$ MeV. The similar analysis allows us to extract constraints on the symmetry energy from $\rho_{\text{r}} = 0.02$ fm$^{-3}$ to 0.11 fm$^{-3}$ and the results are shown as red hatched bands in Fig. 2 (see the supplemental materials for the detailed values). Shown in the inset in Fig. 2 is the density dependence of the $r$ value. At very low densities (e.g., less than 0.02 fm$^{-3}$), although the $r$ value is still large, effective constraints cannot be obtained since the clustering effects become important and are not considered in the present mean-field calculations. At higher densities (e.g., above 0.11 fm$^{-3}$), effective constraints cannot be obtained either as the $r$ value becomes much smaller.

For comparison, we also shown in Fig. 2 the constraints from transport model analyses of mid-peripheral heavy ion collisions of Sn isotopes (HIC) [46] and the Skyrme-Hartree-Fock (SHF) analyses of isobaric analogue states (IAS) as well as combing additionally the neutron skin “data” (IAS+NSkin) in Ref. [12], and six constraints on the value of $E_{\text{sym}}(\rho)$ around $2/3\rho_0$ from binding energy difference between heavy isotope pairs (Zhang) [16], Fermi-energy difference in finite nuclei (Wang) [14], properties of doubly magic nuclei (Brown) [17], the giant dipole resonance in $^{208}$Pb (Trippa) [47], the giant quadrupole resonance in $^{208}$Pb (Roca-Maza) [48] and the soft dipole excitation in $^{132}$Sn (Cao) [49]. In addition, we also show the experimental results of the symmetry energies at densities below $0.2\rho_0$ and temperatures in the range $3 \sim 11$ MeV from the analysis of cluster formation in heavy ion collisions (Wada and Kowalski) [20]. It is remarkable to see that a single data of $\alpha_D$ in $^{208}$Pb can give quite stringent constraints on $E_{\text{sym}}(\rho)$ around $\rho_0/3$ and the constraints are in very good agreement with other analyses. It is also interesting to see that the con-
strained $E_{\text{sym}}(\rho)$ around $\rho_0/7$ where the clustering effects just start to work is nicely consistent with the results extracted from heavy ion collisions (Wada).

4. Neutron matter from $\alpha_\text{D}$ in $^{208}\text{Pb}$.—From the empirical parabolic approximation $E_{\text{PNM}}(\rho) \approx E_0(\rho) + E_{\text{sym}}(\rho)$, one expects $E_{\text{PNM}}(\rho)$ should play a similar role as $E_{\text{sym}}(\rho)$ since $E_0(\rho)$ is relatively well determined, especially around $\rho_0/3$ (see the supplemental materials). Using the similar analysis as in Fig. 1 we indeed find a strong correlation between $1/\alpha_\text{D}$ in $^{208}\text{Pb}$ and $E_{\text{PNM}}(\rho)$ around $\rho_0/3$ (see the supplemental materials for the details). Therefore, we can constrain $E_{\text{PNM}}(\rho)$ around $\rho_0/3$ in the similar way as constraining the $E_{\text{sym}}(\rho)$ and the results are shown as red hatched band in Fig. 3 in the density interval $0.005 \text{ fm}^{-3} < \rho < 0.11 \text{ fm}^{-3}$ with the corresponding density dependence of $r$ in the inset (Detailed values can be found in the supplemental materials). Here $E_{\text{PNM}}(\rho)$ has been constrained down to very low density of $\rho = 0.005 \text{ fm}^{-3}$ as the clustering effects are negligible in neutron matter and the $r$ value remains close unit. At higher densities (e.g., above 0.11 fm$^{-3}$), the $r$ decreases rapidly and one cannot effectively constrain $E_{\text{PNM}}(\rho)$.

Also shown in Fig. 3 are the predictions from ChEFT using N$^3$LO potential (ChEFT) [21], Auxiliary-Field QMC (AFQMC) calculations employing local N$^3$LO ChEFT interaction with different cutoffs (QMC) [22], AFQMC calculations using adjusted nuclear force models by Gandolfi-Carlson-Reddy (GCR) [23], AFQMC calculations with the N$^3$LO 2-body interactions plus the N$^2$LO 3-body interactions (Wlazlowski) [24], configuration interaction Monte Carlo calculations using nonlocal N$^3$LO chiral interaction (Roggero) [24], variational calculations by Akmal-Pandharipande-Ravenhall (APR) [51], and the Bethe-Bruckner-Goldstone calculations using many-body expansion up to the three hole-line level of approximation with quark model interactions with the auxiliary potential of gap choice (GC) or continuous choice (BBG-QM 3h-gap and BBG-QM 3h-con) [52]. At $\rho = 0.1 \text{ fm}^{-3}$, a constraint from analyzing properties of doubly magic nuclei within SHF is also shown (Brown) [14].

One can see from Fig. 3 that our present analyses of the data on $\alpha_\text{D}$ in $^{208}\text{Pb}$ from RCNP give quite stringent constraints on $E_{\text{PNM}}(\rho)$ around $\rho_0/3$. To our best knowledge, our present results provide for the first time the experimental constraints on $E_{\text{PNM}}(\rho)$ around $\rho \approx \rho_0/3$. It is seen that our constraints are in excellent agreement with the predictions of APR and GCR as well as the constraint from Brown, and also consistent with other predictions. Interestingly, although our constraints are consistent with the predictions of both ChEFT and QMC within the uncertainty bands, there still exist some density regions where our constraints do not completely overlap with the uncertainty bands of ChEFT and QMC which are mainly due to the uncertainty of the many-body interactions. Therefore, our present experimental constraints on $E_{\text{PNM}}(\rho)$ are potentially useful for constraining the many-body interactions in ChEFT and QMC calculations.

5. Conclusion.—In summary, we have found that the electric dipole polarizability $\alpha_\text{D}$ in $^{208}\text{Pb}$ can be determined uniquely by the magnitude of the symmetry energy $E_{\text{sym}}(\rho)$ or almost equivalently pure neutron matter EOS $E_{\text{PNM}}(\rho)$ at subsaturation densities around $\rho_0/3$, significantly deepening the understanding of the relation between the $\alpha_\text{D}$ and the $E_{\text{sym}}(\rho)$. This finding together with the $\alpha_\text{D}$ in $^{208}\text{Pb}$ measured at RCNP have allowed us to obtain very stringent constraints on $E_{\text{sym}}(\rho)$ and $E_{\text{PNM}}(\rho)$ around $\rho_0/3$. The present constraints should be less model dependent since they are based on a large set of both non-relativistic and relativistic models. Our results provide for the first time the experimental constraints on $E_{\text{PNM}}(\rho)$ around $\rho_0/3$ which are potentially useful in constraining the many-nucleon interactions in microscopic calculations of neutron matter.

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In this supplement of the Letter, we include some more detailed results, i.e., I. The symmetry energy and \( \alpha_D \) in hydrodynamical model; II. Numeric values of the constraints on the symmetry energy from \( \alpha_D \) in \(^{208}\text{Pb} \); III. EOS of symmetric nuclear matter in non-relativistic and relativistic models; IV. Correlations between pure neutron matter EOS and \( \alpha_D \) in \(^{208}\text{Pb} \); V. Numeric values of the constraints on pure neutron matter EOS from \( \alpha_D \) in \(^{208}\text{Pb} \).

I. The symmetry energy and \( \alpha_D \) in hydrodynamical model

In hydrodynamical model, the electric dipole polarizability \( \alpha_D \) can be expressed as

\[
\alpha_D = \frac{e^2}{24} \int \frac{r^2}{v_{\text{sym}}} d^3r = \frac{\pi e^2}{6} \int \frac{\rho r^4}{E_{\text{sym}}(\rho)} dr,
\]

with \( v_{\text{sym}} = E_{\text{sym}}(\rho)/\rho \). Using this formula to estimate \( \alpha_D \) can shed light on the correlation between \( \alpha_D \) and the symmetry energy.

![Figure 4](image_url)

**Fig. 4**: (Color online). (a) 2PF nucleon density distribution in \(^{208}\text{Pb} \). (b) \( \rho \cdot r^4/E_{\text{sym}}(\rho) \) as function of radius \( r \) for different density dependence of the symmetry energy.

Shown in Fig. 4(a) is the nucleon density distribution of \(^{208}\text{Pb} \) by assuming a widely used 2-parameter Fermi distribution form

\[
\rho = \frac{\rho_1}{1 + \exp[(r - c)/a]}.
\]

Here we have used the radius parameter \( c = 6.7 \text{ fm} \) and the diffuseness parameter \( a = 0.51 \text{ fm} \). Then \( \rho_1 \) can be determined by the normalization condition \( \int \rho d^3r = 208 \), and that is \( \rho_1 = 0.1562 \text{ fm}^{-3} \).

To estimate the \( \alpha_D \), we employ the following simple parametrization for the symmetry energy

\[
E_{\text{sym}}(\rho) = 12.5 \left( \frac{\rho}{\rho_0} \right)^{2/3} + 20 \left( \frac{\rho}{\rho_0} \right)^\gamma,
\]

where \( \rho_0 = 0.16 \text{ fm}^{-3} \) is the saturation density. Then \( \rho r^4/E_{\text{sym}}(\rho) \) as a function of radius \( r \) can be easily obtained and the results are displayed in Fig. 4(b) for \( \gamma = 0.25, 0.5, 0.75 \) and 1, respectively. The red dotted line in Fig. 4 indicates the radius where \( \rho = \rho_0/3 \approx 0.055 \text{ fm}^{-3} \). One can see that \( \rho r^4/E_{\text{sym}}(\rho) \) has a peak at subsaturation densities around \( \rho = \rho_0/3 \) (for a stiffer symmetry energy, the peak shifts to even lower densities). Therefore, our present simple estimate indicates that the symmetry energy at much lower densities around \( \rho = \rho_0/3 \) essentially dominate the electric dipole polarizibility in \(^{208}\text{Pb} \), and this is confirmed by the more exact microscopic RPA calculations based on a number of non-relativistic and relativistic interactions.
II. Numeric values of the constraints on the symmetry energy from $\alpha_D$ in $^{208}$Pb

Table I lists the numeric values of the constraints on the symmetry energy $E_{\text{sym}}(\rho)$ and the Pearson correlation coefficient $r$ at different densities.

| $\rho$ (fm$^{-3}$) | $E_{\text{sym}}(\rho)$ (MeV) | $\sigma_{\text{exp}}$ (MeV) | $\sigma_{\text{the}}$ (MeV) | $\sigma_{\text{tot}}$ (MeV) | $r$ |
|---------------------|-------------------------------|-------------------------------|-----------------------------|-----------------------------|-----|
| 0.020               | 8.11                          | 0.55                          | 0.50                        | 0.74                        | 0.947 |
| 0.025               | 9.61                          | 0.62                          | 0.53                        | 0.81                        | 0.956 |
| 0.030               | 11.02                         | 0.67                          | 0.55                        | 0.87                        | 0.963 |
| 0.035               | 12.34                         | 0.71                          | 0.57                        | 0.91                        | 0.969 |
| 0.040               | 13.59                         | 0.74                          | 0.58                        | 0.94                        | 0.973 |
| 0.045               | 14.78                         | 0.76                          | 0.60                        | 0.97                        | 0.976 |
| 0.050               | 15.91                         | 0.77                          | 0.63                        | 0.99                        | 0.977 |
| 0.055               | 16.99                         | 0.77                          | 0.68                        | 1.03                        | 0.976 |
| 0.060               | 18.01                         | 0.77                          | 0.77                        | 1.09                        | 0.973 |
| 0.065               | 19.00                         | 0.77                          | 0.91                        | 1.19                        | 0.967 |

III. EOS of symmetric nuclear matter in non-relativistic and relativistic models

Fig. 5 displays the symmetry energy $E_{\text{sym}}(\rho)$, pure neutron matter EOS $E_{\text{PNM}}(\rho)$ and symmetric nuclear matter EOS $E_0(\rho)$ as functions of density $\rho$ predicted by the chosen 62 non-relativistic and relativistic models (see the text of the Letter for the details). In particular, we show their spread values at $\rho = 0.05$ fm$^3$ in this figure, namely, 1.4 MeV for $E_0$, 9.8 MeV for $E_{\text{PNM}}$ and 9.7 MeV for $E_{\text{sym}}$, respectively. It is thus indicated that symmetric nuclear matter EOS $E_0(\rho)$ is relatively well-determined since these non-relativistic and relativistic interactions are usually supposed to fit the ground state properties of finite nuclei, like binding energy, charge root-mean-square radius and so on. Given this and the well-known parabolic approximation $E_{\text{PNM}}(\rho) \approx E_0(\rho) + E_{\text{sym}}(\rho)$, it is thus expected that a strong correlation may also exist between $1/\alpha_D$ in $^{208}$Pb and pure neutron matter EOS $E_{\text{PNM}}(\rho)$ around $\rho_0/3$.

IV. Correlations between pure neutron matter EOS and $\alpha_D$ in $^{208}$Pb

Displayed in Fig. 6 are the data-to-data relations between $10^3/\alpha_D$ in $^{208}$Pb and pure neutron matter EOS $E_{\text{PNM}}(\rho_r)$ at $\rho_r = 0.02$, 0.05, 0.08, 0.11 and 0.16 fm$^{-3}$, respectively, from the microscopic RPA calculations based on the 62 non-relativistic and relativistic interactions. Again, we show linear fits as red lines with the corresponding Pearson correlation coefficient $r$ in Fig. 6 and the corresponding $r$ as a function of density $\rho$ has been displayed in the inset in Fig. 3 in the Letter. As expected, one indeed can see a very strong correlation between the $\alpha_D$ in $^{208}$Pb and pure neutron matter EOS $E_{\text{PNM}}(\rho_r)$ around $\rho_0/3$.

V. Numeric values of the constraints on pure neutron matter EOS from $\alpha_D$ in $^{208}$Pb

Table II lists the numeric values of the constraints on pure neutron matter EOS $E_{\text{PNM}}(\rho)$ and the Pearson correlation coefficient $r$ at different densities.
FIG. 5: (Color online). The symmetry energy $E_{\text{sym}}(\rho)$ (a), pure neutron matter EOS $E_{\text{PNM}}(\rho)$ (b) and symmetric nuclear matter EOS $E_0(\rho)$ (c) as functions of density predicted by a large number (62) of non-relativistic (SHF) and relativistic (RMF) models. The spread of $E_{\text{sym}}(\rho)$, $E_{\text{PNM}}(\rho)$ and $E_0(\rho)$ at $\rho = 0.05\text{fm}^{-3}$ are 9.7, 9.8 and 1.4 MeV, respectively.

FIG. 6: (Color online). Same as Fig. 1 in the Letter but for pure neutron matter EOS $E_{\text{PNM}}(\rho)$. 
TABLE II: The numeric values of binding energy per particle in pure neutron matter $E_{PNM}(\rho)$, experimental error $\sigma_{exp}$ from experimental value of $\alpha_D$, theoretical error $\sigma_{the}$ from linear fit, total error $\sigma_{tot}$ and Pearson correlation coefficients at different densities obtained in this work.

| $\rho$ (fm$^{-3}$) | $E_{PNM}(\rho)$ (MeV) | $\sigma_{exp}$ (MeV) | $\sigma_{the}$ (MeV) | $\sigma_{tot}$ (MeV) | $r$ |
|-------------------|------------------------|----------------------|----------------------|----------------------|----|
| 0.005             | 2.30                   | 0.19                 | 0.13                 | 0.23                 | 0.954 |
| 0.010             | 3.28                   | 0.32                 | 0.18                 | 0.37                 | 0.958 |
| 0.015             | 4.02                   | 0.44                 | 0.21                 | 0.48                 | 0.961 |
| 0.020             | 4.63                   | 0.53                 | 0.23                 | 0.58                 | 0.963 |
| 0.025             | 5.17                   | 0.60                 | 0.25                 | 0.65                 | 0.966 |
| 0.030             | 5.66                   | 0.66                 | 0.26                 | 0.71                 | 0.969 |
| 0.035             | 6.12                   | 0.70                 | 0.27                 | 0.75                 | 0.971 |
| 0.040             | 6.56                   | 0.73                 | 0.27                 | 0.78                 | 0.974 |
| 0.045             | 6.98                   | 0.75                 | 0.28                 | 0.80                 | 0.977 |
| 0.050             | 7.39                   | 0.76                 | 0.27                 | 0.81                 | 0.979 |
| 0.055             | 7.79                   | 0.77                 | 0.28                 | 0.81                 | 0.981 |
| 0.060             | 8.19                   | 0.76                 | 0.29                 | 0.81                 | 0.982 |
| 0.065             | 8.58                   | 0.75                 | 0.31                 | 0.81                 | 0.980 |
| 0.070             | 8.98                   | 0.74                 | 0.36                 | 0.82                 | 0.976 |
| 0.075             | 9.38                   | 0.72                 | 0.45                 | 0.85                 | 0.967 |
| 0.080             | 9.78                   | 0.71                 | 0.57                 | 0.91                 | 0.951 |
| 0.085             | 10.20                  | 0.70                 | 0.75                 | 1.03                 | 0.925 |
| 0.090             | 10.62                  | 0.71                 | 1.00                 | 1.23                 | 0.886 |
| 0.095             | 11.07                  | 0.74                 | 1.35                 | 1.54                 | 0.828 |
| 0.100             | 11.54                  | 0.81                 | 1.84                 | 2.01                 | 0.749 |
| 0.105             | 12.06                  | 0.94                 | 2.55                 | 2.72                 | 0.647 |
| 0.110             | 12.65                  | 1.18                 | 3.66                 | 3.85                 | 0.525 |