RESEARCH ARTICLE

Quantile causality and dependence between crude oil and precious metal prices

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Abstract
This paper examines long-run dependence and causality between oil and precious metal (gold, silver, platinum, palladium, steel, and titanium) prices across quantiles by exploiting their time series properties with the help of novel econometric techniques. The empirical results for the period 1990–2019 indicate that oil and metal prices are nonstationary across different quantiles and that cointegration patterns differ widely across quantiles. Causality running from oil to metal prices is quantile-dependent and differs according to the metal, whereas upward and downward movements in metal prices have no causal effect on oil prices. These results have implications for investors and policymakers in terms of portfolio and risk management decisions.

KEYWORDS
crude oil, metal commodities, quantile regression

1 | INTRODUCTION

Oil and precious metal markets are inextricably related through economic and financial channels. Surges in oil prices typically trigger inflationary pressures, raise growth concerns, and impact stock prices. This worries investors and they resort to precious metals—particularly gold—to protect the real value of investments by managing portfolio risk. Likewise, oil price oscillations modify the composition of the international reserve portfolios of oil-exporting countries, which typically use gold and other precious metals to manage their portfolio risk. However, oil and precious metals are connected through exchange rates in such a way that U.S. dollar (USD) depreciations tend to reduce the value of both oil and precious metals. Therefore, understanding oil price oscillations and causal effects between oil and precious metal prices is of interest to producers, to investors and to policymakers. The producers are interested because oil and precious metals co-movement has implications for the level of stock they hold for these commodities. The investors are interested because they are concerned about their portfolio composition and risk management of their portfolios and finally the policymakers are interested because they use precious metals as a store of wealth.

The relationship between oil prices and metal prices has been extensively investigated. Some studies have focused on the relationship between crude oil and gold
prices.\textsuperscript{1} Zhang and Wei (2010) demonstrated causality from oil to gold prices but no causality from gold to oil prices, whereas Beckmann and Czudaj (2013), using a cointegrated variance-at-risk (VaR) model, found a positive relationship between gold and oil. Reboredo (2013), using copula functions, reported that gold is not a useful hedge against oil price change but is a safe-haven during extreme movement of oil prices.\textsuperscript{2} Using generalized forecast error variance decompositions and the generalized impulse response functions for four metals, oil and the USD/EUR exchange rate, Sari, Hammoudeh, and Soytas (2010) found that oil prices positively impact precious metal prices in the short run but not over the long run. Similarly, Jain and Ghosh (2013) studied co-movement of precious metals, oil and the USD/Indian rupee exchange rate using the autoregressive distributed lag (ARDL) bounds tests for cointegration and Toda and Yamamoto (1995) version of Granger causality, finding that all price series were cointegrated and that causality mainly runs from exchange rates to all the other variables, with mixed evidence of causality between precious metal and oil prices. Likewise, Bildirici and Turkmen (2015) analysed cointegration and causality among crude oil and other precious metals, finding evidence of nonlinear causality. Bampinas and Panagiotidis (2015) found evidence of a bidirectional nonlinear relationship between oil and gold prices, whereas Kumar (2017) reported an analogous nonlinear and asymmetric relationship between crude oil and gold markets in India. Contrarily, Zhang and Tu (2016) found that crude oil price shocks have a symmetric impact on China’s metal markets. Relying on the exchange traded funds (ETFs), Lau, Vigne, Wang, and Yarovaya (2017) analyse return spillovers derived from an exponential generalized autoregressive conditional heteroskedasticity (EGARCH) model and highlight the role of gold ETFs as the most influential market in the sample. More recently, Mei-Se, Shu-Jung, and Chien-Chiang (2018) studied the dynamic relationship among oil prices and three metals using recursive cointegration, finding no long-run relation between gold and oil prices and that dependence largely changes over time, mainly at times of financial market crises.

Other studies have investigated oil and metal price volatilities. Plourde and Watkins (1998) showed that oil prices are more volatile than gold and silver prices. Using a Bayesian Markov-switching vector error correction model, Balcilar, Hammoudeh, and Asuba (2015) found that gold prices are more informative in a high-volatility scenario, while palladium and platinum prices are more informative in low-volatility scenario. Bouri, Jain, Biswal, and Roubaud (2017) examined cointegration and causality among the implied volatilities of oil, gold and the Indian stock exchange, reporting that the implied volatilities of oil and gold are cointegrated and that those volatilities impact on the implied volatility of the Indian stock exchange. Kuruppuarachchi and Premachandra (2016), using the conditionally heteroskedastic common factor approach, found a significant impact of energy markets on other markets during the global financial crisis (GFC) in 2008–2009 and the European sovereign debt crisis in 2010–2012, whereas Sensoy, Hacihasanoglu, and Nguyen (2015) found that the GFC had some impact on contagion in commodity futures markets. Using a dynamic conditional correlation-generalized autoregressive conditional heteroskedasticity (DCC-GARCH) model, Jain and Biswal (2016) found that oil and gold price oscillations are inter-related and are also related to the Indian rupee exchange rate and the stock market index. Rehman, Shahzad, Uddin, and Hedström (2018) use structural vector autoregression (SVAR) and show time varying effect of oil shocks on precious metals and this effect increases significantly during the financial crisis. Finally, Mokni (2018) use fractional integrated exponential generalized autoregressive conditional heteroskedasticity (FIEGARCH)-copula framework and find positive relationship between oil and precious metals returns, volatilities and market risk.

While above mentioned studies analyse the relationship between oil and precious metal prices from a range of different perspectives, little is known about long-run dependence and causality between oil and precious metal prices across different quantiles. This paper fills this gap in the extant literature and uses a quantile regression approach to determine how oil prices and precious metals commove under different market moves (e.g., bull or bear markets). We focus on monthly prices for oil and six major precious metals—gold, silver, platinum, palladium, steel and titanium—for the period of January 1990 to September 2019.

Specifically, our paper contributes to the literature in the following ways. First, we introduce an appropriate methodology, that is, quantile regression, to study comovement of oil prices and precious metal prices. The quantile regression approach is more thorough in their analysis of the data than the usual mean regression analysis as the former consider the distributional aspects of the data. As such, a quantile approach can capture nonlinearity in dependence between variables that may originate due to many factors, including structural breaks. Second, we innovate in using the following tests: (a) the quantile autoregressive unit root test as proposed by Galvao (2009), (b) the Kuriyama (2016) quantile cointegration test, and (c) the Granger-causality in quantiles test introduced by Troster (2018). The quantile
autoregressive unit root test of Galvao (2009) provides deeper insights into the nonstationarity properties of variables in their respective quantiles. The Kuriyama (2016) approach tests for the null hypothesis of quantile cointegration, which validates the ARDL bounds test estimates by considering changes in the distributional structure of long-run equilibrium while taking into account serial correlation and regressor endogeneity. Troster (2018) tests the direction of causality across all quantiles. The Troster (2018) method holds several advantages of not requiring the smoothing parameters to be selected as well as remaining consistent against different fixed alternatives and being robust against Pitman deviations from the null hypothesis. These tests able to show whether oil and metal prices are nonstationary and co-moving in the mean and across different quantiles. Finally, we use the ARDL bounds test along with diagnostic tests such as residual normality, autocorrelation, heteroskedasticity and Ramsey (RESET) tests, which indicate that the results are unbiased, efficient, and reliable. The ARDL bounds test overcomes the limitations of earlier approaches of Engle and Granger (1987), Johansen (1991), and Toda and Yamamoto (1995) by incorporating both I(0) and I(1) variables, by disparating lag orders for each regressor, by having a good small-sample performance, and by correcting for residual serial correlation.

Our empirical evidence indicates that both oil and metal prices are nonstationary in the mean and across different quantiles. With the use of the quantile autoregressive unit root test as proposed by Galvao (2009), the Kuriyama (2016) quantile cointegration test, and the Granger-causality in quantiles test introduced by Troster (2018), we find that oil price is cointegrated with the different metal prices and that these relationships vary widely in distributional structure across quantiles and shows no specific patterns for the different metal-crude oil prices pairs. In other words, long-run equilibrium (cointegration) relationships between the different metal- crude oil prices pairs is non-monotonic. Furthermore, our quantile causality analysis shows that there is bidirectional causality between oil and metal prices. Nevertheless, we find that causality from crude oil prices to metal prices is quantile-dependent, where the distribution is specific for each metal. In particular, our evidence points to causality from oil to metal prices in the lower intermediate quantiles and noncausality in the extreme lower quantiles. In terms of causality from each metal price to oil price, the distributional structure is uniform for all metals with causation running in the extreme lower and higher quantiles. Our empirical evidence has implications for investors in terms of portfolio design and risk management: some precious metals (e.g., gold and silver) are revealed to be useful for managing downside oil price risk, whereas other metals are good tools for hedging oil price risks. Our evidence has implications for policymakers who use precious metals to preserve the wealth value of oil resources. It also has important implications for risk management decisions regarding hedging and downside risk given the financial usefulness of precious metals may vary according to market circumstances. Finally, our analysis has implications for the predictability of metal prices across different quantiles on the basis of oil price information.

The paper is structured as follows: in Section 2, we describe our data sample. In Section 3, we outline our methodological approach to quantile analysis of long-run dependence and causality. In Section 4, we discuss the empirical evidence, and, finally, in Section 5, we summarize our results and discuss their implications for investors and policymakers.

2 | DATA

We used monthly price data for crude oil and six major metal commodities: gold, palladium, platinum, silver, steel, and titanium for the period of January 1990 to September 2019. Prices are expressed in USD per troy ounce (1 oz = 31.1034768 g) for gold, palladium, platinum, silver, and titanium, in USD per metric ton for steel, and in USD per barrel for oil. Oil prices were sourced from the U.S. Energy Information Administration (EIA), whereas metal prices were sourced from World Metal Statistics, World Bureau of Metal Statistics and World Bank staff estimates (World Bank, 2019). The monthly data series was seasonally adjusted using the X12 approach. A natural logarithmic transformation of the variables was implemented before conducting the empirical analysis. Figure 1 depicts the dynamics of all the series, showing that oil prices closely co-move with metal prices, mainly by gold, steel, palladium, crude oil, silver, and titanium. The standard deviation of prices from the mean, followed by gold, steel, palladium, crude oil, silver, and titanium.

Table 1 provides the descriptive statistics for the (log) variables used in the empirical study. It can be observed that the highest mean price is that of platinum, followed by gold, steel, palladium, crude oil, silver, and titanium. The maximum price is for platinum followed by gold, steel, palladium, crude oil, silver, and titanium. Similar to the mean, the minimum price is the highest for platinum, followed by gold, steel, palladium, crude oil, silver, and titanium. The standard deviation of prices from the highest to lowest are: palladium, silver, titanium, gold, crude oil, steel, and platinum. The Jarque and Bera (1987)
FIGURE 1 Prices for crude oil and six precious metals. Source: World Bank (2019) [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 1 Descriptive statistics

|               | Crude oil | Gold  | Palladium | Platinum | Silver | Steel  | Titanium |
|---------------|-----------|-------|-----------|----------|--------|--------|----------|
| Mean          | 3.669     | 6.395 | 5.767     | 6.606    | 2.201  | 6.206  | 1.075    |
| Maximum       | 4.829     | 7.488 | 6.921     | 7.573    | 3.734  | 7.483  | 2.757    |
| Minimum       | 2.464     | 5.557 | 4.383     | 5.842    | 1.305  | 5.262  | 0.193    |
| SD            | 0.634     | 0.636 | 0.724     | 0.548    | 0.686  | 0.624  | 0.652    |
| Jarque-Bera²  | 27.156*   | 42.417* | 25.448*   | 30.850*  | 33.042* | 28.783* | 43.918*  |
| p-Value       | .000      | .000  | .000      | .000     | .000   | .000   | .000     |

²Jarque and Bera (1987) test for variable normality.
*A p-value less than .05 rejects the null hypothesis of normality.
test statistics reject the null hypothesis of a normal distribution at the 5% significance level for all the series.

Table 2 provides the correlation coefficients (in conditional means) of the crude oil and metal prices. The precious metal prices are found to be highly correlated with the crude oil price. In particular, the highest correlation between crude oil-metal price pairs is for platinum, followed by that of silver, gold, steel, palladium, and titanium. The precious metals also appear to be highly correlated among themselves, with the highest correlation coefficient recorded for the gold-silver pair while the lowest is for the palladium-titanium pair. This demonstrates the existence of positive association between the crude oil and precious metal prices.

| TABLE 2 | Correlation coefficients |
|-----------------|--------------------------|
|                | Crude oil | Gold      | Palladium | Platinum | Silver | Steel | Titanium |
| Crude oil      | 1.0000    |           |           |          |        |       |          |
| Gold           | 0.8390    | 1.0000    |           |          |        |       |          |
| Palladium      | 0.6591    | 0.6665    | 1.0000    |          |        |       |          |
| Platinum       | 0.9618    | 0.8473    | 0.7693    | 1.0000   |        |       |          |
| Silver         | 0.8820    | 0.9631    | 0.7351    | 0.9088   | 1.0000 |       |          |
| Steel          | 0.6776    | 0.8934    | 0.5975    | 0.6503   | 0.7960 | 1.0000 |          |
| Titanium       | 0.6312    | 0.5657    | 0.5191    | 0.5834   | 0.5619 | 0.6621 | 1.0000   |

3 | METHODOLOGY

The empirical analysis involves testing the long-run equilibrium and spillover of information between the price of crude oil and that of each of the six precious metals. As such, the crude oil price is paired with each metal price and each pair of variables is tested for long-run equilibrium (cointegration), and Granger causality to gauge the direction of information spillover between the respective markets. This bivariate approach allows us to observe the long-run comovements and dependence between the crude oil and precious metals markets, without burdening the discussion with the interplay between the different metal prices. This is in line with the recent and relevant literature— including Bildirici and Turkmen (2015), Reboredo and Ugolini (2016), Kumar (2017), Shahbaz, Balcilar, and Ozdemir (2017), and Kuruppuarachchi and Premachandra (2016), inter alia—that aim to observe the long-run equilibrium and spillover of information between commodities/markets.

We tested for the presence of unit roots, cointegration and causal relationships across quantiles using quantile regression-based methods. In considering different supports of the distribution of the data, those methods have the advantage of analysing the data more thoroughly than the usual mean regression analysis and so capture nonlinearity in dependence between variables.

We first tested for the presence of quantile unit roots using the quantile unit root test as introduced by Galvao (2009). This test, which naturally extends the traditional unit root tests by Dickey and Fuller (1979) and Phillips and Perron (1988), provides deeper insights into the nonstationarity properties of variables in their respective quantiles. Galvao’s quantile unit root test, with appropriate finite sample properties, can be conducted by calculating the following quantile autoregressive (QAR) equation:

\[ Q_{\tau}(y_t | y_{t-1}) = \mu + \beta \tau^t + \alpha y_{t-1} + \sum_{j=1}^{p} \alpha_j y_{t-j} + \sum_{t=-q_2, t \neq -q_1, \ldots, t \neq q_1}^q \gamma x_{t-l} + F_{\tau}^{-1}(\tau), \]

where the \( \tau \)-th quantile conditional value of \( y_t \) is represented by \( Q_{\tau} \); \( y_{t-1} \) represents \( \tau \)-field that is generated using \( \{ u_t, s < t, x_{t-q_2}, \ldots, x_{t+q_1} \} \); and \( F_{\tau} \) represents the errors’ common distribution function. The linear QAR equation can be estimated by solving the following minimization problem:

\[ \min_{\beta \in \mathbb{R}^{p+q_2}} \sum_{t=1}^{n} \left| y_t - z_t \beta \right|, \]

where \( \beta \) and \( z_t \) are defined as \( \beta(\tau) = (\mu(\tau), \tau, \alpha_1, \ldots, \alpha_p, \gamma_{q_1}, \ldots, \gamma_{-q_1}) \) and \( z_t = (1, t, y_{t-1}, \Delta y_{t-1}, \ldots, \Delta y_{t-p}, x_{t-q_2}, \ldots, x_{t+q_1}) \), respectively. Following Koenker and Bassett Jr (1978), \( \rho(u) = u(\tau - I(u < 0)) \). The null hypothesis is defined as nonstationarity, which requires \( \alpha(\tau) = 1 \). As part of the unit root test procedure, the null hypothesis is tested for a range of quantiles: \( \tau \in T \). Galvao (2009), following Koenker and Xiao (2004), specifies the following Kolmogorov–Smirnov (KS) test on the quantile regression for \( \tau \in T = [\tau_0, 1-\tau_0] \) for which \( 0 < \tau_0 < \frac{1}{2} \).

\[ Q_{KS} = \sup_{\tau \in T} | I_n(\tau) |. \]

Second, if variables are found to be nonstationary, tests for cointegration between crude oil prices and metal
prices need to be applied. We tested for cointegration between oil prices and metal prices using the ARDL bounds test—introduced by Pesaran, Shin, and Smith (2001)—based on estimates of conditional means. This test requires the calculation of an unrestricted error correction equation (UECM), say \( y_t \) on \( x_t \), such as the following:

\[
\Delta y_t = \alpha + \beta_1 y_{t-1} + \beta_2 x_{t-1} + \sum_{i=1}^{q} \gamma_i \Delta y_{t-i} + \sum_{i=0}^{q} \delta_i \Delta x_{t-i} + u_t.
\]  

(4)

Under the null hypothesis of no cointegration the bounds test procedure involves testing \( \beta_1 \) and \( \beta_2 \) for joint significance using critical values from Pesaran et al. (2001): if the estimated \( F \)-statistic is higher than the upper critical bound, cointegration is confirmed, otherwise the variables are not cointegrated. However, when the \( F \)-statistic falls between the upper and lower critical bounds, the ARDL test is inconclusive. Of the cointegration tests for the conditional mean, the ARDL method is known for its various advantages over commonly used approaches such as Engle and Granger (1987), Johansen (1991), and Toda and Yamamoto (1995). The ARDL bounds test overcomes the limitations of these past approaches by being able to incorporate both \( I(0) \) and \( I(1) \) variables, have disparate lag orders for each regressor, good small-sample performance, and correct for residual serial correlation. In addition, it is possible to estimate the reduced form ARDL UECM for different precious-metal price pairs instead of estimating a system of equations in vector autoregression (VAR)-based methods such as Johansen (1991) and Toda and Yamamoto (1995). These improve the degrees of freedom while estimating and allow the ARDL bounds test to determine the presence of cointegration more efficiently and reliably in contrast to the conventional tests.

We explored the distributional aspects of cointegration across quantiles using Kuriyama’s (2016) test for the null hypothesis of quantile cointegration, which validates the ARDL bounds test estimates by considering changes in the distributional structure of long-run equilibrium taking into account serial correlation and regressor endogeneity. This test involves the following quantile regression:

\[
y_t = \alpha' \tau d_t + \beta' \tau x_t + u_{\tau}(\tau) = \theta' \tau z_t + u_t(\tau),
\]

(5)

where for \( \tau \)-th quantile, \( \theta(\tau) = (\alpha' (\tau), \beta' (\tau))^T \), the error term \( u \) is defined as \( u_{\tau}(\tau) = y_t - \alpha(\tau) d_t - \beta(\tau) x_t \), such that the conditional quantile \( Q_{\alpha(\tau)}(\tau|x_t) = 0 \), and \( z_t = (d_t, x_t') \). Here, until time \( t \), the information set is denoted by \( F_t \) so that \( x_t \in F_t \). It is also possible to model a nonlinear relationship between \( y_t \) and \( x_t \) using the quantile dependent regression coefficient vector \( \theta(\tau) \). This unknown coefficient vector is estimated by minimizing the sum of asymmetric weighted residuals:

\[
\hat{\theta}(\tau) = \arg\min_\theta \sum_{t=1}^{T} \rho_{\tau}(y_t - z_t' \theta(\tau)),
\]

(6)

where \( \rho_{\tau}(\cdot) = u(\tau - I(u < 0)) \) is the check function (Koenker & Bassett Jr, 1978). Then, the residuals from the fully modified quantile regression are computed as \( \hat{u}_t^+ (\tau) = y_t^+ - z_t' \hat{\theta}^+ (\tau) \), where \( y_t^+ = y_t - \bar{Q}_{\rho_{\tau}} \bar{Q}_{\rho_{\tau}} - 1 \Delta x_t \), and the cumulated sum (CS) test statistic is defined as:

\[
CS_T(\tau) = \max \left\{ \frac{1}{\hat{Q}_{\rho_{\tau}} \hat{Q}_{\rho_{\tau}}}(\sum_{t=1}^{n} \psi_{\tau}(\hat{u}_t(\tau))) \right\},
\]

(7)

where \( \psi_{\tau}(\hat{u}_t(\tau)) = \tau - I(\hat{u}_t(\tau) < 0) \) and \( \hat{Q}_{\rho_{\tau}}^2 \) is the long-run variance of \( \psi_{\tau}(\hat{u}_t(\tau)) \), which is estimated by nonparametric kernel smoothing.

Finally, we tested for Granger causality in the conditional quantiles of oil and metal prices using the procedure developed by Troster (2018), which tests the direction of causality across all quantiles. This approach remains functionally equivalent to testing for the direction of Granger-causality in distribution. The Troster (2018) method for Granger-causality in quantiles holds several advantages, such as the not requiring the smoothing parameters to be selected as well as remaining consistent against different fixed alternatives and being robust against Pitman deviations from the null hypothesis. Troster’s (2018) Granger-causality test involves testing the null hypothesis of noncausality between two variables, say from \( Z_t \) to \( Y_t \),

\[
H_0^{Q_{\rho_{\tau}}^Y}: F_Y(y | I_t^Y, I_t^Z) = F_Y(y | I_t^Y), \quad \text{for all } y \in \mathbb{R}
\]

(8)

where \( I_t \) is an explanatory vector satisfying the identity \( I_t \equiv (I_t^Y, I_t^Z) \in \mathbb{R}^d \), \( d = s + q \), in which \( I_t^Y = (Y_{t-1}, ..., Y_{t-q})' \in \mathbb{R}^s \) and \( I_t^Z = (Z_{t-1}, ..., Z_{t-q})' \in \mathbb{R}^q \). The test for Granger (non)-causation from \( Z_t \) to \( Y_t \) in distribution—that is, across \( \tau \)-quantiles—for Equation (8):

\[
H_0^{Q_{\rho_{\tau}}^{Q_{\rho_{\tau}}^Z}}: Q_{\tau}^{Y_Z}(Y_t | I_{t-1}^Y, I_{t-1}^Z) = Q_{\tau}^{Y}(Y_t | I_t^Y) \quad \text{for all } \tau \in \mathcal{T}
\]

(9)

where the \( \tau \)-quantiles of \( F_Y(y | I_t^Y, I_t^Z) \) and \( F_Y(y | I_t^Y) \) are represented by \( Q_{\tau}^{Y_Z}(y | I_t^Y, I_t^Z) \) and \( Q_{\tau}^{Y}(y | I_t^Y) \), respectively. In addition, \( \mathcal{T} \subset [0,1] \) is a compact set and the following restrictions have to be satisfied by \( Y_t \)’s conditional \( \tau \)-quantiles:
Here, for \( I_t \) \( P(Y_t \leq Q_t(Y_t|I_t)|I_t) = E\{1|Y_t \leq Q_t(Y_t|I_t)|I_t\} \) holds, and for the event \( a \) being less than or equal to \( b \), the indicator function is denoted as \( 1(a \leq b) \). The test statistic for direction of Granger-causality proposed by Troster (2018) is as follows:

\[
S_T = \frac{1}{Tn} \sum_{j=1}^{n} |\psi_j^t W \psi_j^t| \tag{11}
\]

where \( n \) denotes the equidistributed points over the grid \( T_n = \{\tau_j\}_{j=1}^{n} \), \( W \) is a \( T \times T \) matrix containing the element \( w_{t_1 t_2} = \exp(-0.5(L_t - L_0)^2) \), \( \Psi \) is a \( T \times n \) matrix containing the element \( \psi_{j t} = \Psi_t(Y_t - m(I_t^0, \theta_T(\tau_j))) \) in which \( \theta_T \) denotes \( \theta_0(\tau) \)'s \( \sqrt{T} \)-consistent estimator satisfying all \( \tau \in T \). The null hypothesis of Granger noncausality in distribution is rejected whenever the estimated values of \( S_T \) are large. Troster (2018) proposed a subsampling method to compute the \( p \)-values for \( S_T \). For a sample size of \( T \), subsamples numbered \( B = T - b + 1 \) and with size \( b = \lfloor kT^{2/5} \rfloor \) are taken (without replacement) to calculate \( S_T \) and the corresponding \( p \)-values for each subsample. The \( p \)-values are then averaged over \( B \) number of subsamples. To compute the \( S_T \) in Equation (11), the following QAR equation of order 1 \( (m^1) \) is calculated:

\[
QAR(1): m^1(\tau, \theta(\tau)) = \mu_1(\tau) + \mu_2(\tau)Y_{t-1} + \sigma_t \Phi^{-1}_u (\tau) \tag{12}
\]

where a maximum likelihood estimator is used to generate the parameters \( \theta(\tau) = (\mu_1(\tau), \mu_2(\tau), \mu_3(\tau), \mu_4(\tau), \sigma_t) \) over \( \tau \)-quantiles positioned at regular intervals, and where the inverse function of a standard normal distribution is denoted by \( \Phi^{-1}_u (\cdot) \).

### 4 | EMPIRICAL RESULTS

Table 3 reports the empirical evidence from the augmented Dickey–Fuller (ADF) and Phillips–Perron (PP) unit root tests, showing that the null hypothesis of a unit root process is accepted at the 5% significance level, both in level and trend. By considering price series in first differences, we observe that crude oil prices and metal prices are stationary, implying an integration of crude oil prices and metal prices of order 1. As the ADF and PP tests are stationarity tests for the conditional mean, we

| TABLE 3 | Unit root evidence from conventional unit root tests |
|----------|---------------------------------|
| **Form** | **ADF test** | **PP test** |
| **Test statistic** | **Test statistic** |
| **p-Value** | **p-Value** |
| Crude oil Level | -1.6515 | -1.3313 |
| | .4551 | .6159 |
| First diff. | -14.5212* | -14.3829* |
| | .0000 | .0000 |
| Gold Level | 0.3121 | 0.2267 |
| | .9787 | .9740 |
| First diff. | -17.0711* | -17.1779* |
| | .0000 | .0000 |
| Palladium Level | -1.5422 | -1.2425 |
| | .5111 | .6569 |
| First diff. | -6.0357* | -19.8339* |
| | .0000 | .0000 |
| Platinum Level | -1.0391 | -1.1138 |
| | .7401 | .7114 |
| First diff. | -6.8107* | -15.4354* |
| | .0000 | .0000 |
| Silver Level | -1.1783 | -8.0087 |
| | .6848 | .8173 |
| First diff. | -4.1811* | -16.4603* |
| | .0000 | .0000 |
| Steel Level | 0.7697 | 0.5600 |
| | .9934 | .9885 |
| First diff. | -6.5729* | -15.6060* |
| | .0000 | .0000 |
| Titanium Level | -0.9371 | -0.4489 |
| | .7757 | .8976 |
| First diff. | -6.1431* | -13.7018* |
| | .0000 | .0000 |

*Note: Unit root tests include a deterministic term (an intercept) for the null hypothesis of non-stationarity. For the augmented Dickey–Fuller (ADF) test, lag length is determined by minimizing the Akaike Information Criterion (AIC). For the Phillips–Perron (PP) test, the choice of bandwidth is derived by using the Newey–West Bartlett kernel. *Denotes a \( p \)-value below .05, rejecting, thus, the null hypothesis at the 5% significance level.
### Table 4: Results for Galvao's conditional quantile autoregressive unit root test

| $\tau$ | Crude oil $\hat{\alpha}$ | Gold $\hat{\alpha}$ | Palladium $\hat{\alpha}$ | Platinum $\hat{\alpha}$ | Silver $\hat{\alpha}$ | Steel $\hat{\alpha}$ | Titanium $\hat{\alpha}$ |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.05  | 0.9749          | 1.0001          | 0.9730          | 0.9962          | 1.0113          | 1.0394          | 1.0770          |
|       | $-1.2465$       | 0.0147          | $-1.1296$       | $-0.1869$       | 0.9848          | 1.4471          | 1.2171          |
| 0.10  | 1.0010          | 0.0679          | 0.9869          | 0.9968          | 1.0009          | 1.0318          | 1.0422          |
|       | $-0.3126$       | $-0.7472$       | $-0.2720$       |                  | 2.2817          |                  |                  |
| 0.15  | 1.0057          | 0.4172          | 0.9858          | 0.9945          | 0.9989          | 1.0234          | 1.0300          |
|       | $-0.6656$       | $-1.0292$       | $-0.6640$       |                  | $-0.1265$       | 4.5010          |                  |
| 0.20  | 1.0117          | 1.1245          | 0.9973          | 0.9974          | 1.0019          | 1.0999          | 1.0163          |
|       | $-0.4572$       | $-0.2191$       | $-1.2194$       |                  | 4.4790          |                  | 1.4929          |
| 0.25  | 1.0073          | 0.8495          | 0.9982          | 0.9975          | 1.0005          | 1.0016          |                  |
|       | $-0.5332$       | 1.0002          | $-1.3805$       |                  | 4.1698          |                  | 0.5932          |
| 0.30  | 1.0016          | 0.2073          | 0.9990          | 0.9931          | 1.0029          | 1.0144          | 1.0006          |
|       | $-0.3266$       | 1.0056          | $-1.3341$       |                  | 4.4560          |                  | 0.0734          |
| 0.35  | 1.0032          | 0.4095          | 0.9996          | 0.9918          | 1.0039          | 1.0110          | 1.0028          |
|       | $-0.1377$       | 1.0083          | $-1.8416$       |                  | 3.3465          |                  | 0.4102          |
| 0.40  | 0.9982          | $-0.2583$       | 1.0005          | 0.9974          | 1.0037          | 1.0088          | 1.0040          |
|       |                  | 0.1607          | 0.9218          | $-0.5957$       | 0.6923          | 3.0655          | 0.7099          |
| 0.45  | 0.9976          | $-0.3489$       | 1.0004          | 0.9984          | 1.0022          | 1.0066          | 1.0024          |
|       |                  | 0.1454          | 0.6322          | $-0.3458$       | 0.4576          | 2.4257          | 0.4471          |
| 0.50  | 0.9999          | $-0.0126$       | 1.0032          | 0.9978          | 1.0014          | 1.0020          |                  |
|       |                  | 1.0899          | 0.9208          | $-0.4362$       | 0.6923          |                  | 0.9989          |
| 0.55  | 1.0017          | 0.2247          | 1.0023          | 0.9952          | 0.9995          | 0.9995          |                  |
|       | 0.7265          | $-0.8599$       | 0.9969          | $-0.6498$       | $-0.0965$       | $-0.1656$       |                  |
| 0.60  | 0.9982          | $-0.2328$       | 1.0024          | 0.9928          | 0.9994          | 0.9967          |                  |
|       |                  | 0.6754          | $-1.3641$       | $-0.2328$       | $-0.1078$       | $-1.1085$       |                  |
| 0.65  | 0.9963          | $-0.4528$       | 1.0031          | 0.9987          | 0.9994          | 0.9946          |                  |
|       |                  | 0.8320          | $-2.2335$       | $-0.6651$       | $-0.0670$       | $-1.7858$       |                  |
| 0.70  | 0.9906          | $-1.1427$       | 1.0045          | 0.9880          | 0.9916          | 0.9908          |                  |
|       |                  | 1.1839          | $-1.9711$       | $-0.1255$       | $-1.3460$       | $-3.0210^*$.    |                  |
| 0.75  | 0.9827          | $-2.0057$       | 1.0061          | 0.9830          | 0.9909          | 0.9866          |                  |
|       |                  | 1.4518          | $-2.2555$       | $-1.4710$       | $-3.9378^*$.    | 0.9714          |                  |
| 0.80  | 0.9809          | $-2.1375$       | 1.0032          | 0.9807          | 0.9904          | 0.9838          |                  |
|       |                  | 0.6783          | $-2.1696$       | $-1.2244$       | $-3.4744^*$.    | 0.9514          |                  |
| 0.85  | 0.9682          | $-3.1887^*$.    | 1.0045          | 0.9781          | 0.9928          | 0.9791          |                  |
|       |                  | 1.0400          | $-2.1021$       | $-0.8044$       | $-3.0632^*$.    | 0.9385          |                  |
| 0.90  | 0.9657          | $-2.6569^*$.    | 1.0027          | 0.9634          | 0.9869          | 0.9741          |                  |
|       |                  | 0.3875          | $-2.5565^*$.    | $-1.3683$       | $-1.6827$       | 0.9217          |                  |
| 0.95  | 0.9339          | $-2.7558^*$.    | 0.9891          | $-0.8789$       | 0.9635          | $-1.0817$       |                  |
|       |                  |                  | 0.9635          | $-1.0895$       | 0.9805          | 0.9472          |                  |

*Note: $\tau$ refers to the respective quantile. $\hat{\alpha}$ refers to the persistence parameter. $t$-stat. refers to $t$-value of the respective test statistic. Model contains an intercept but not a trend. The null hypothesis is that the series is nonstationary in a specific quantile. $^*$Denotes a $p$-value below .05, rejecting, thus, the null hypothesis at the 5% significance level.
We first examined cointegration at the mean level using the Pesaran et al. (2001) ARDL bounds test for cointegration between crude oil prices and respective metal prices. Given that (log) oil and metal prices are non-stationary, we checked for the existence of a cointegration relationship between oil and metal prices. We then applied the Kuriyama (2016) quantile cointegration test, considering 19 quantiles and providing the estimated test statistics \( CS^2(\tau) \) at the 5 and 10% significance levels. The empirical results (reported in Table 6) indicate that the null hypothesis of a cointegrated system for gold prices is not rejected for the eight lower quantiles (except the 5th percentile) or for the last quantile (95th percentile), as the test statistics are either lower than the respective critical bounds for all estimated models. This implies structural stability of the estimated ARDL bounds test results. Since the tests do not reject any of the adequate specification or structural stability tests at the 5% critical bounds for all estimated models. This implies structural stability of the empirical results (reported in Table 6) indicate that the null hypothesis of a cointegrated system for gold prices is not rejected for the eight lower quantiles (except the 5th percentile) or for the last quantile (95th percentile), as the test statistics are either lower than the

Note: Panel A provides the estimated results from the Pesaran et al. (2001) ARDL bounds test for cointegration between crude oil prices and respective metal prices. \( H_0 \): non-cointegration in respective model. The lag order is chosen using the Akaike Information Criterion (AIC) and indicates the number of lags of crude oil and respective metal prices chosen for each model. Panel B provides the diagnostic test results for each of the ARDL models estimated in Panel A. The respective null hypotheses for these tests are as follows: Jarque and Bera (1987): \( H_0 \): residual normality; Breusch (1978) and Godfrey (1978a): \( H_0 \): correct functional specification. p-values are reported in round parentheses: a p-value below .05 rejects the null hypothesis at the 5% significance level.

* A test statistic greater than the upper bound critical value, in square brackets, indicates the number of lags of crude oil and respective metal prices chosen for each model. Panel B provides the diagnostic test results for each of the ARDL models estimated in Panel A. The respective null hypotheses for these tests are as follows: Jarque and Bera (1987): \( H_0 \): residual normality; Breusch (1978) and Godfrey (1978a): \( H_0 \): correct functional specification. p-values are reported in round parentheses: a p-value below .05 rejects the null hypothesis at the 5% significance level.
5 or 10\% critical values or both. Consequently, we find cointegration between gold prices and crude oil prices in eight, mostly lower, quantiles. Regarding palladium, the empirical results confirm the existence of cointegration in the quantiles between the 35th and 85th percentiles. Therefore, crude oil and palladium prices are
cointegrated in 11, mostly mid-range and upper, quantiles. In considering platinum prices, we find cointegration in 10 quantiles between the 25th and 75th percentiles, excluding the 35th percentile. We also find cointegration between platinum and oil prices in the 90th and 95th percentiles. Our estimated results for silver point to cointegration in 13 quantiles, namely, the first six and last seven quantiles. For steel, cointegration is evident only in five quantiles, including the 20th to 25th percentiles, the 70th percentile and the 90th to 95th percentiles. Finally, the empirical results for titanium indicate the existence of a cointegration relationship for three of the first four quantiles (except the 10th percentile), between the 55th and 65th percentiles, and the last (95th) quantile. This validates the presence of cointegration between the crude oil and titanium prices in a total of seven quantiles, which are in the lower and upper tails as well as the upper middle quantiles. Thus, the Kuriyama (2016) cointegration test indicates mixed evidence for cointegration, which varies according to quantiles and depending on the metal.

Table 7 shows the fully modified coefficient estimates $\hat{\beta}^+ (r)$ from Kuriyama’s (2016) procedure. Those coefficients are long-run elasticities given that the variables are all in natural logarithmic form. Evidence for gold price for the second quantile (10th percentile) to the eighth quantile (40th percentile) show that the elasticities are positive and range in magnitudes from 0.6366 to 0.8406. Here, for the second quantile, it can be interpreted as a response (increase) in the gold price by approximately 0.73% when the crude oil price increased by 1%. The intensity of this effect of oil prices on gold price was found to increase for the higher quantiles, with the 95th percentile (highest quantile) recording an elasticity above 1 (>unit elasticity). A decline in the estimated elasticities can be found for palladium price, where the highest value of 0.8716, estimated for the 45th percentile, declines to 0.5258 for the 85th percentile. For platinum price, we observed a relative stability in the elasticities, from a high of 0.8804 for the 25th percentile to a low of 0.8502 for the 45th percentile, with some recovery in magnitude for the 90th and 95th percentiles.

The silver price elasticities were found to increase from the first six quantiles and for three quantiles starting from the 65th to 75th percentiles. The elasticities of silver price then declined slightly for the next four quantiles (80th–95th percentiles), reaching a value of 0.9964 in the final quantile. The estimated elasticities of silver prices with respect to oil price cross the unit elasticity threshold seven times and are among the highest in magnitude across all six models (metals). The steel price elasticities decreased from 0.6649 to 0.5422 around the 20th to 25th percentiles, then rose to 0.6017 for the 70th percentile, and crossed the unit-elasticity threshold with
values of 1.3359–1.3499 for the 90th–95th percentile quantiles. Finally, the titanium price elasticities increased for three of the first four quantiles (excluding the 10th percentile), from 0.3501 for the 5th percentile to 0.5234 for the 20th percentile, rising between 0.5979 and 0.7982 in the 55th–65th percentiles, and to 1.3583 for the 95th percentile. Overall, we can conclude that the elasticities of metal prices with respect to crude oil price are positive and change over the respective distributions. In the cases of gold, steel, and titanium, the elasticity is above 1 for the 95th percentile. In contrast, the elasticity of silver price with respect to crude oil price exceed unit elasticity threshold in the lower-middle and upper tail quantiles.

Panel A in Table 8 reports the \( p \)-values for causality from crude oil price to metal prices estimated using Troster’s (2018) quantile causality test. Considering all 19 quantiles combined, the estimated \( p \)-values clearly reject the null hypothesis of noncausality for all metal prices, as can be seen from the first row (Panel A in Table 8). Oil price Granger-caused gold price in eight of the 19 quantiles: including the 20th–30th percentiles, and the 45th–65th percentiles. However, we found no evidence of causality in the upper and lower tail quantiles. The palladium price is Granger-caused by oil price in the 15th–40th and 70th–95th percentiles, for a total of 12 quantiles. For platinum, we found causality to be significant from the 10th to 30th percentiles, in the 75th percentile, and between the 85th and 90th percentile quantiles. As such, oil price Granger-caused platinum price in lower and upper tail quantiles. For silver, Granger causality is statistically significant for the quantiles in the 10th–15th percentiles, 25th–35th percentiles, 55th percentile, 65th–70th percentiles, and 80th percentile. This brings a total of 9 quantiles in which causality ran from crude oil to silver price, with no evidence of causality in the middle and upper quantiles. Finally, causality from oil price is significant in all quantiles for steel, and also for titanium with the exception being its 50th and 55th percentiles.

Panel B in Table 8 reports the \( p \)-values for quantile causality running from metal prices to oil price. For all quantiles combined, we find consistent evidence of causality for all metals at the 5% significant level. However,
in considering different quantiles, the evidence consistently points to causality for all metals in the 15th–25th, 35th, 65th–75th, and 85th–90th percentiles. No causality runs from metals prices to crude oil price in the remaining quantiles, in particular, for the upper and lower quantiles. This brings a total of nine quantiles in which the precious metal prices Granger-cause oil price. Overall, our evidence indicates that there is little distributional difference between the six metals, which is consistent with the fact that metal prices are often associated with strong economic activity, which, in turn, has a transmission effect on crude oil price through macroeconomic factors such as inflation and industrial production (Pindyck & Rotemberg, 1990; Wang & Chueh, 2013). This transmission effect can be expected to be approximately uniform, due to the macroeconomic transmission mechanism as well as to excessive metal price co-movements. Contrarily, the causality running from crude oil price to metals prices shows substantial diversity across the support of the distribution, not only for any particular metal but also across metals.

The general substance of the empirical results is that the dependence and causality between crude oil and precious metal prices is unique for each metal and dependent on the respective quantile. Overall, the quantile cointegration analysis revealed some interesting patterns. The long-run equilibrium relationships between oil price and that of gold, silver, steel, and titanium, respectively, are concentrated more in the upper and lower quantiles than in the middle quantiles. In particular, the cointegrating relationship between the price of crude oil and that of steel and titanium, respectively, are located in the extremities of their distributions—i.e., the upper and lower tail quantiles. In contrast, the cointegrating relationships between oil and respective palladium and platinum prices are positioned heavily in the middle quantiles. The estimated elasticities of each metal price with respect to oil price are positive and generally higher in magnitude in the upper quantiles (except for palladium). This shows the extreme price fluctuations of gold, silver, steel, and titanium have long-run equilibrium comovements with oil price while deviation of palladium and platinum prices from the mean (and/or median) results in greater comovements in the crude oil market.

The quantile Granger causality test findings are a contrast to the respective findings of the quantile

| τ   | Gold | Palladium | Platinum | Silver | Steel | Titanium |
|-----|------|-----------|----------|--------|-------|----------|
| 0.05| —    | —         | —        | 0.7931 | —     | 0.3501   |
| 0.10| 0.7348| —         | —        | 0.8304 | —     | —        |
| 0.15| 0.7103| —         | —        | 0.8671 | —     | 0.4787   |
| 0.20| 0.7208| —         | —        | 0.9147 | 0.6649| 0.5234   |
| 0.25| 0.6880| —         | 0.8804   | 0.9354 | 0.5422| —        |
| 0.30| 0.6377| —         | 0.8802   | 1.0167 | —     | —        |
| 0.35| 0.6366| 0.8256    | —        | —      | —     | —        |
| 0.40| 0.8406| 0.8595    | 0.8664   | —      | —     | —        |
| 0.45| —    | 0.8716    | 0.8502   | —      | —     | —        |
| 0.50| —    | 0.8658    | 0.8735   | —      | —     | —        |
| 0.55| —    | 0.8441    | 0.8721   | —      | —     | 0.5979   |
| 0.60| —    | 0.7359    | 0.8650   | —      | —     | 0.6099   |
| 0.65| —    | 0.6207    | 0.8652   | 1.0015 | —     | 0.7982   |
| 0.70| —    | 0.5269    | 0.8721   | 1.0498 | 0.6017| —        |
| 0.75| —    | 0.5271    | 0.8550   | 1.0497 | —     | —        |
| 0.80| —    | 0.5531    | —        | 1.0163 | —     | —        |
| 0.85| —    | 0.5258    | —        | 1.0203 | —     | —        |
| 0.90| —    | —         | 0.8640   | 1.0153 | 1.3359| —        |
| 0.95| 1.0940| —         | 0.8579   | 0.9964 | 1.3499| 1.3583   |

Note: This table reports the statistically significant coefficient estimates of $\beta^+(\tau)$ for those quantiles in which oil and metal price series are cointegrated (as reported in Table 6). The model contains an intercept ($\alpha$) but not a trend.
### TABLE 8  Evidence for Troster’s quantile causality test between crude oil price and respective metal prices

#### Panel A. Causality from crude oil price to metal prices

| τ     | Gold | Palladium | Platinum | Silver | Steel | Titanium |
|-------|------|-----------|----------|--------|-------|----------|
| [0.05, 0.95] | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 |
| 0.05  | 0.238 | 0.729 | 0.310 | 0.294 | 0.010 | 0.043 |
| 0.1   | 0.109 | 0.191 | 0.003 | 0.003 | 0.003 | 0.003 |
| 0.15  | 0.112 | 0.003 | 0.003 | 0.007 | 0.003 | 0.003 |
| 0.2   | 0.003 | 0.003 | 0.003 | 0.056 | 0.003 | 0.003 |
| 0.25  | 0.036 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 |
| 0.30  | 0.030 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 |
| 0.35  | 0.630 | 0.013 | 0.066 | 0.013 | 0.003 | 0.003 |
| 0.40  | 0.660 | 0.023 | 0.125 | 0.310 | 0.003 | 0.003 |
| 0.45  | 0.040 | 0.066 | 0.086 | 0.383 | 0.003 | 0.003 |
| 0.50  | 0.020 | 0.076 | 0.779 | 0.191 | 0.007 | 0.221 |
| 0.55  | 0.026 | 0.673 | 0.063 | 0.007 | 0.003 | 0.224 |
| 0.60  | 0.007 | 0.267 | 0.079 | 0.096 | 0.003 | 0.003 |
| 0.65  | 0.003 | 0.056 | 0.069 | 0.003 | 0.003 | 0.003 |
| 0.70  | 0.125 | 0.010 | 0.135 | 0.007 | 0.003 | 0.003 |
| 0.75  | 0.129 | 0.003 | 0.030 | 0.162 | 0.003 | 0.003 |
| 0.80  | 0.168 | 0.003 | 0.099 | 0.003 | 0.003 | 0.003 |
| 0.85  | 0.587 | 0.003 | 0.030 | 0.264 | 0.003 | 0.003 |
| 0.90  | 0.828 | 0.013 | 0.010 | 0.386 | 0.003 | 0.003 |
| 0.95  | 0.406 | 0.003 | 0.274 | 0.251 | 0.003 | 0.003 |

#### Panel B. Causality from metal prices to crude oil price

| τ     | Gold | Palladium | Platinum | Silver | Steel | Titanium |
|-------|------|-----------|----------|--------|-------|----------|
| [0.05, 0.95] | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 |
| 0.05  | 0.495 | 0.495 | 0.495 | 0.495 | 0.495 | 0.495 |
| 0.10  | 0.257 | 0.132 | 0.257 | 0.257 | 0.257 | 0.257 |
| 0.15  | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 |
| 0.20  | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 |
| 0.25  | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 |
| 0.30  | 0.102 | 0.102 | 0.102 | 0.102 | 0.102 | 0.102 |
| 0.35  | 0.007 | 0.007 | 0.007 | 0.007 | 0.007 | 0.007 |
| 0.40  | 0.320 | 0.320 | 0.320 | 0.320 | 0.320 | 0.320 |
| 0.45  | 0.634 | 0.634 | 0.634 | 0.634 | 0.634 | 0.634 |
| 0.50  | 0.290 | 0.290 | 0.290 | 0.290 | 0.290 | 0.290 |
| 0.55  | 0.102 | 0.162 | 0.162 | 0.149 | 0.102 | 0.152 |
| 0.60  | 0.063 | 0.063 | 0.063 | 0.063 | 0.063 | 0.063 |
| 0.65  | 0.026 | 0.026 | 0.026 | 0.026 | 0.026 | 0.026 |
| 0.70  | 0.036 | 0.036 | 0.036 | 0.036 | 0.036 | 0.036 |
| 0.75  | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 |
| 0.80  | 0.102 | 0.102 | 0.102 | 0.102 | 0.102 | 0.102 |
| 0.85  | 0.017 | 0.017 | 0.017 | 0.017 | 0.017 | 0.017 |
| 0.90  | 0.007 | 0.007 | 0.007 | 0.007 | 0.007 | 0.007 |
| 0.95  | 0.343 | 0.343 | 0.343 | 0.343 | 0.343 | 0.353 |

*Note:* The table reports *p*-values for Troster’s (2018) quantile causality test under the null hypothesis of no Granger causality. Subsample size is denoted by *b* and equals 52. Bold values indicate rejection of null hypothesis at 5% significance level.
cointegration tests. The price of crude oil is found to Granger-cause the price of gold and silver, respectively, mainly in the lower and middle quantiles. Oil price caused steel prices in all quantiles, while also causing titanium prices in all but two middle quantiles. For the case of palladium and platinum, the upper and lower quantiles are found to be Granger-caused by crude oil in the upper and lower quantiles for the most part, but hardly in the middle quantiles. Contrarily, crude oil is Granger-caused by all metal prices in mostly the lower and upper tail quantiles. This reveals an interesting observation: the long-run fluctuations have little or no causal linkage from the respective metal prices to the crude oil price. In other words, there is little spillover of information from the individual precious metals markets to the crude oil market, despite the appearance of long-run comovements between them.

5 | CONCLUSIONS AND IMPLICATIONS

We have analysed how long-range dependence and causality between oil and metal prices change across different quantiles of the price distribution function. Our methodological approach was based on the novel quantile unit root test by Galvao (2009), and the recent quantile cointegration test proposed by Kuriyama (2016) and the quantile Granger causality test introduced by Troster (2018).

For a sample set of oil prices and metal prices—including gold, silver, palladium. Platinum, steel and titanium—for the monthly period of January 1990 to September 2019, we find that these commodity prices display unit roots across different quantiles. Although oil and precious metals are cointegrated on the basis of mean cointegration tests, our evidence for quantiles indicates that cointegration largely differs across quantiles and metal prices; thus, gold and silver are not cointegrated with oil prices in the lower quantiles, whereas the opposite occurs in the upper quantiles. Likewise, we find bidirectional causality between oil and metal prices even though there are substantial differences across metals regarding causality from oil prices, whereas causality from metal prices is quite uniform across different metals.

Our empirical findings have interesting implications for investors operating in oil and metal markets in terms of portfolio design and risk management. Our quantile results reveal that oil and gold/silver markets are disconnected at times of downward market movements, so investors in those markets can use those commodities to protect themselves against risk arising from extreme downward price movements. Our quantile evidence also indicates that distributional differences in terms of cointegration and causality provide hedging opportunities for investors and presents useful information for investors with different investment horizons who base their investment strategies on fundamental and technical analyses. Risk managers can build efficient risk-hedging models aimed at applying appropriate risk management strategies that avoid the adverse effects of volatility. Thus, an investor with a portfolio containing many metals should closely monitor crude oil prices, given the virtually identical distributional structure of causality from crude oil prices to other metals. However, an investor with a portfolio containing crude oil as well as metals would need to closely monitor other metal prices, given the asymmetric effect on crude oil prices. Similarly, the fact that distributional cointegration and causality differences provide detailed knowledge on commodity co-movement means that more efficient portfolios can be built by traders and portfolio managers, as information on distribution differences will assist them in diversifying and rebalancing their portfolios during periods of turmoil. Finally, the evidence from causality across quantiles also allows investors to more accurately forecast prices in specific market circumstances, for example, during periods of abrupt movements in prices.

Specific knowledge of the cointegration and causality across quantiles of these commodities may also provide ammunition to policymakers in developing a strong policy framework to protect against contagion risks and foster market stability. Since we observed substantial variation in distributions for each metal, if causality runs from metals to crude oil prices, a uniform policy framework for metals will have asymmetric effect on crude oil prices. The distributional structures of causality towards crude oil prices are virtually identical across most metals, so a uniform policy framework regarding the impact of crude oil prices on the prices of other metals would prove beneficial. We observed very little distributional change in causality from a particular metal to crude oil prices, which supports the demand side argument regarding crude oil prices, given that low crude oil prices are associated with weak economic activity. To induce economic growth during recessions, policymakers could target crude oil prices as crucial for industry and could use information on distributional cointegration and causality structures to block speculative activities if the market started to heat up. Furthermore, manufacturers who use metals as inputs need to take into account the additional source of risk arising from crude oil price movements, given the asymmetric impact on metal prices. Finally, because of the asymmetric impact of crude oil prices on metal prices, measures to stabilize crude oil prices could prove beneficial to users of precious metals.
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DATA AVAILABILITY STATEMENT
Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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ENDNOTES
1 See O’Connor, Lucey, Batten, and Baur (2015) for a survey on gold.

2 Other studies that point to gold as a safe haven include Worthington and Pahlavani (2007), Tully and Lucey (2007), Blose (2010), Wang and Lee (2011) and Reboredo and Uddin (2016). Some studies that find gold as a safe haven relative to stock markets include Baur and McDermott (2010), Baur and Lucey (2010), Miyazaki, Toyoshima, and Hamori (2012). A recent study by Lucey and Li (2015) found that silver, platinum and palladium also act as safe havens when gold is not a safe haven.

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