Quantum enhanced optical phase estimation with a squeezed thermal state

Juan Yu¹, Yue Qin¹, Jiliang Qin¹,², Hong Wang¹, Zhihui Yan¹,², Xiaojun Jia¹,² and Kunchi Peng¹,²
¹State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan, 030006, P. R. China
²Collaborative Innovation Center of Extreme Optics, Shanxi University, Taiyuan 030006, P. R. China

Quantum phase estimation protocols can provide a measuring method of phase shift with precision superior to standard quantum limit (SQL) due to the application of a nonclassical state of light. A squeezed vacuum state, whose variance in one quadrature is lower than the corresponding SQL, has been pointed out a sensitive resource for quantum phase estimation and the estimation accuracy is directly influenced by the properties of the squeezed state. Here we detailly analyze the influence of the purity and squeezing level of the squeezed state on the accuracy of quantum phase estimation. The maximum precision that can be achieved for a squeezed thermal state is evaluated, and the experimental results are in agreement with the theoretical analyses. It is also found that the width of the phase estimation interval \(\Delta \theta\) beyond SQL is correlated with the purity of the squeezed state.

PACS numbers: 42.50.Dv; 03.65.Wj

I. INTRODUCTION

The question for measuring the phase of light has been a subject of great debate since the early work of Dirac [1]. Due to the existence of the phase Hermitian operator associated with phase, such as field- or intensity-based quantities interferometric devices [2][3], and then deduce the phase indirectly according to the measurement results. This indirect measurement process for the value of phase shift is called phase estimation. The accuracy of usual phase estimation is limited by standard quantum limit (SQL) because of the vacuum fluctuation of quantized electromagnetic field [8]. Phase estimation is a powerful measurement strategy to perform accurate measurements of various physical quantities including length, velocity, intensity-based quantities interferometric systems [13][14], gravitational wave detection [15][16], interferometry based on interacting systems [17][18], quantum imaging [19][20], atomic clock [21][22], and gravimetry [23][24].

Since Caves proposed that a quantum state can break the limit of shot noise in 1981 [23], many optical systems [24][35] have proved that a real quantum state, for instance a squeezed state and an entangled state, can greatly improve the accuracy of phase estimation with a given average photon number [36][37]. The accuracy of phase estimation is influenced by the properties of the quantum state. In the basic principle of quantum optics, the fluctuation added in one quadrature should be equal to that reduced in its orthogonal quadrature for an ideal squeezed state. However, a realistic squeezed state is difficult to be exactly pure especially for high-level squeezed state due to the existence of extra noise in its generation system [38]. Accordingly, it is quite necessary to explore the effect of the properties of the squeezed state on the phase estimation results.

The theory of quantum phase estimation provides the ultimate bound on precision of phase estimation in the form of quantum Cramér-Rao bound (QCRB), which is independent with detection strategies [29]. The QCRB is essentially determined by Heisenberg uncertainty and is given by the inverse of quantum Fisher information (QFI) associated with the resource. The theoretical analyses show that homodyne measurement is optimal for squeezed pure state but not optimal for squeezed thermal state, and the maximum precision that can be achieved for squeezed thermal state via homodyne measurement is called optimal Cramér-Rao bound (OCRB) [40]. In Ref. [41], a squeezed-enhanced phase estimation is realized with the help of feedback control. Here, we detailly analyze the influence of properties of squeezed state on the phase estimation results. By using a squeezed thermal state as the probe beam, the effects of the squeezing level and the purity of a squeezed state on the phase estimation results are given. Then, we experimentally implement that the absolute phase estimation can be enhanced with much higher squeezing level and squeezed pure state behaves the optimal resource to reach QCRB. Our research is of general interest in the sense of phase estimation based on the squeezing mechanism and the results provide a reference for multi phase estimation based on multipartite entanglement [42][43].

II. PHASE ESTIMATION WITH A SQUEEZED THERMAL STATE

An optical field can be represented by the annihilation operator \(\hat{a}\) in quantum mechanics. The orthogonal amplitude and phase operators can be represented in terms of the creation and annihilation operators as \(\hat{x} = (\hat{a} + \hat{a}^\dagger) / \sqrt{2}\) and \(\hat{p} = i (\hat{a}^\dagger - \hat{a}) / \sqrt{2}\). \(\hat{x}\) and \(\hat{p}\) satisfy the commutation relations \([\hat{x}, \hat{p}] = i\). Usually, a coherent state or a vacuum state is a minimum uncertainty state and the variances of the two quadrature components are equal: \(\langle \Delta^2 \hat{x} \rangle = \langle \Delta^2 \hat{p} \rangle = 1/2\). A squeezed state is defined as its variance of one quadrature \(\langle \Delta^2 \hat{x} \rangle\) is less than 1/2.
is reduced relative to the corresponding SQL while the variance of its orthogonal quadrature is amplified. The squeezing parameter \( r \) is used to indicate the squeezing level of the squeezed state, i.e., the variance in a squeezed quadrature, \( e^{-2r}/2 \), is always below the corresponding SQL \[44\]. The mean photon number of pure squeezed vacuum state is \( n = \sinh^2 r \). In the past decades, squeezed states of light have been obtained by several groups and squeezing level has been improved continually \[45,48\]. In the actual experimental generation processing, there is some inevitably extra noise in its antisqueezing quadrature component \[49\].

A squeezed state, i.e., the variance in a squeezed quadrature component \( \hat{x}_s \) and the covariance matrix of this squeezed state is expressed as \( \sigma_0 = 1/2 \) \[40\].

A general scheme of a quantum phase estimation with a squeezed state is shown in Fig. 1. A squeezed state \( \hat{\rho}(0) \) undergoes an unknown phase shift \( \theta \). A function of the data samples \( \hat{x}_a \) associated with the phase shift are measured by a detection strategy.

A general scheme of quantum phase estimation with a squeezed state is shown in Fig. 1. A squeezed state \( \hat{\rho}(0) \) undergoes a phase shift described by a unitary operator \( \hat{U}(\theta) = \exp(-i\theta \hat{n}) \),

\[
\hat{\rho}(\theta) = \hat{U}(\theta) \hat{\rho}(0) \hat{U}^\dagger(\theta),
\]

where \( \hat{n} = \hat{a}^\dagger \hat{a} \) is a number operator and \( \theta \) is the phase shift to be estimated. The output state is detected by a detection strategy and the obtained data samples associated with the phase shift are processed by Bayesian inference \[50\]. The essence of phase shift operation is a rotation operation on an initial state, and the quadrature operator is expressed as \( \hat{x}_s = (\hat{a}e^{-i\theta} + \hat{a}^\dagger e^{i\theta})/\sqrt{2} \). The variance of the squeezed state \( \rho(\theta) \) is associated with the phase shift acted on the probe beam, and the phase shift can be indirectly obtained by measuring the variance of quadrature \( \hat{x}_s \) via homodyne detection. Thus, the phase estimation protocol we provided here is only appropriate for a squeezed state. In general, the variance of phase shift \( \text{Var}[\theta] \) for any unbiased estimator is bounded at the times of measurements \( N \) by the Cramér-Rao theorem \[51\]:

\[
\text{Var}[\theta] \geq \frac{1}{NF(\theta)},
\]

where \( F(\theta) \) is the Fisher information (FI) \[52\], which is the observed information about the unknown parameter. The quantum Fisher information (QFI) \( H \) is the maximized FI over all possible detection schemes, i.e. \( F(\theta) \leq H \). According to Ref. \[53\], \( H \) can be fully expressed in terms of the covariance matrix of the Gaussian state and the QCRB of the quantum phase estimation for a single-mode squeezed thermal state is:

\[
\text{Var}_{\text{sq}}[\theta] = \frac{1}{8Nn_r(n_r+1)} \left( \frac{1}{2} + \frac{1}{2} e^{-2r} \right),
\]

Comparing with a coherent state to be used as probe beam in phase estimation \( (\sim 1/(4Nn)) \), where \( n \) is the mean photon number of probe beam \[40\], the estimation accuracy can be greatly improved with the help of a squeezed state. For a squeezed pure state, there is not any extra noise in the antisqueezing quadrature component, i.e. \( r' = 0 \), and the QCRB becomes \( \text{Var}[\theta] = 1/(8Nn_r(n_r+1)) \) \[54,55,55\].

### III. REACHABLE BOUND WITH HOMODYNE DETECTION AND BAYESIAN INFERENCE

Homodyne detection is a common detection strategy for state reconstruction in continuous-variable (CV) regime \[56\]. It is a kind of simple and accurate detection strategy because it can provide a phase reference for estimating the value of the phase shift. The data samples \( \{ \hat{x}_a \} \) associated with the phase shift \( \theta \) are obtained through a local projective Von Neumann measurement and then the true value of phase shift can be indirectly deduced according to the measurement results \[57\].

In order to evaluate the maximum precision that is achieved for a squeezed thermal state with homodyne measurement, we use the Wigner function to describe our system \[58,60\]. In the Wigner function description, the quadratures of the probe beam correspond to two phase-space coordinates \( x \) and \( p \), which can be grouped into a two-dimensional vector \( \mathbf{X} \), \( \mathbf{X}^T = (x, p) \). The Wigner function associated with the shifted squeezed thermal state \( \hat{\rho}_\theta, (\theta) \) is:

\[
W_\theta(\mathbf{X}) = \frac{\exp[-\frac{i}{2} \mathbf{X}^T \sigma_\theta^{-1} \mathbf{X} ]}{2\pi \sqrt{\text{Det}[\sigma_\theta]}},
\]

where \( \sigma_\theta \) is the covariance matrix after the phase shift, conditioned on single homodyne measurement outcome of a
shifted squeezed thermal state is calculated from the Wigner function [61]:

\[ p(x|\theta) = \int_R W_{\theta}(x) \, dy = \frac{1}{e^{\frac{x^2}{\sqrt{2\Sigma_0^2}}} \exp[-\frac{x^2}{2\Sigma_0^2}]}, \quad (6) \]

where \( \Sigma_0^2 = [e^{-2r-2r'} \cos^2 \theta + e^{2r} \sin^2 \theta] \) is the variance of the probe beam, \( \{x_0\} \) is the noise distribution of the squeezed state associated with \( \theta \) obtained from the homodyne measurement. The FI can be easily evaluated from its definition [62]:

\[
F(\theta) = \int_R p(x|\theta) \left( \frac{\partial \log p(x|\theta)}{\partial \theta} \right)^2 \, dx = \frac{\sin^2(2\theta) \left(e^{2r} - e^{-2r-2r'}\right)^2}{2 \left(\Sigma_0^2\right)^2}. \quad (7)
\]

It is obvious that the expression of the FI is dependent on the phase shift \( \theta \) and the squeezing parameters \( r \) as well as the extra antisqueezing parameter \( r' \). The maximum of the FI can be achieved at an optimal phase \( \theta_{\text{opt}} = 1/2 \arccos(\tanh(2r + r')) \) and \( F_{\text{max}} \) with homodyne measurement is,

\[
F_{\text{max}} = 2 \sinh^2(2r + r'). \quad (8)
\]

Then upon using the Cramér-Rao theorem, the variance of optimal phase estimation with homodyne measurement goes as:

\[
\text{Var}^{\text{hom}}[\theta] = \frac{1}{8N_{n_{r}}(n_{r} + 1)}. \quad (9)
\]

It means that the homodyne measurement is not optimal for a squeezed thermal state by comparing Eq. (3) and (9) and the estimation accuracy can attain the optimal Cramér-Rao bound (OCRB) [41].

Bayesian inference, which is known as “probability theory”, is the theory of how to combine uncertain information from multiple sources to make optimal decision under uncertainty. If \( x \) is the variable associated with the phase shift, then the Bayes’ rule states:

\[
p(\theta|x) = \frac{p(x|\theta) \, p(\theta)}{p(x)}, \quad (10)
\]

where \( p(\cdot|\cdot) \) are the conditional probabilities about parameters \( x \) and \( \theta \), \( p(x|\theta) \) is the marginal probability distribution of the shifted squeezed thermal state and \( p(\theta|x) \) is the posteriori probability distribution (PPD) of the phase shift. \( p(x) \) are the total probabilities to observe \( x \) and \( p(\theta) = 2/\pi \) is the prior information which is a flat distribution. The result of each measurement is used as a prior information for the next measurement. The PPD \( p(\theta|x) \) based on \( N \) sampled homodyne measurements is given by:

\[
p(\theta|x) = \frac{1}{N} \prod_{k=1}^{N} p(x_k|\theta), \quad (11)
\]

where \( N = \int_0^\pi p(\theta|x) \, d\theta \) is a normalization constant, \( p(x_k|\theta) \) is the individual marginal probability distribution conditioned on each homodyne measurement which are given by Eq. (6).

IV. EXPERIMENTAL SETUP AND RESULT

A schematic of experimental setup is illustrated in Fig. 2, which includes a source of squeezed state, a balanced homodyne detection system, a phase control system and a data acquisition system. A squeezed state is produced by a nondegenerate optical parametric amplifier (NOPA), which is pumped by a continuous wave intra-cavity frequency-doubled tunable single-frequency Nd:YAP/LBO solid-state laser provided by YuGuang company CDPSSFG-VIB (not shown in Fig. 2). The output fundamental wave at wavelength of 1080 nm is used for the injected seed beams of NOPA and the local oscillator beam of homodyne detection system. The second harmonic wave at 540 nm serves as the pump field of the NOPA. The NOPA consists of an \( \alpha \)-cut KTP crystal and a concave mirror, which can realize type-II non-critical phase matching without walk-off effect. The front face of the crystal is highreflection (HR) coated for 1080 nm and \( T_1 = 18\% \) coated for 540 nm, which serves as the input coupler. The end face of the KTP is cut to 1° along y-z plane of the crystal and antireflection coated for both 1080 nm and 540 nm. The concave mirror with a radius of curvature of 50 mm coated with \( T_2 = 12.5\% \) for 1080 nm and HR for 540 nm serves as the output coupler, which is mounted on a piezoelectric transducer to actively lock the cavity length of NOPA on res-
onance with the injected signal at 1080 nm. Through an intracavity frequency down conversion process in the NOPA, an Einstein-Podolsky-Rosen (EPR) entangled state of light or two single-mode squeezed states of light at 1080 nm with orthogonal polarizations can be generated separately [63]. The squeezed state with different squeezing parameter $r$ and different purity can be generated by controlling the experimental conditions.

The generated squeezed state acquires an unknown phase shift $\theta$ within a range of $[0, \pi/2]$ and then is combined with a strong local oscillator beam (5 mW) at a 50/50 beam splitter (BS) for homodyne measurement. The relative phase control between the local oscillator beam and probe beam is achieved by an improved Pound-Drever-Hall (PDH) technique [63]. The local oscillator beam is phase-modulated by an electro-optic modulator (EOM) with a sine signal at 7.3 MHz. The first-step error signal is obtained by mixing alternating-current (AC) signal detected by the homodyne detector and the sine signal modulated on the EOM. The final error signal to realize the phase locking of probe beam and the local oscillator beam to a specific degree is obtained by coupling the first-step error signal with the direct-current (DC) output from the homodyne measurement. The relative phase control between the local oscillator beam and probe beam is achieved by a certain percentage. Finally, the error signal is feedback to the piezo-electric transducer (PZT) attached on a high reflection mirror. The experimental data $\{ x_0 \}$ of quantum phase estimation is recorded by an oscilloscope via quantum tomography technique. A PPD of $\theta$ conditioned on the $N$ sampled homodyne measurements can be calculated according to Eq. (11).

![FIG. 3. Posteriori probability distributions for different values of the involved parameters at a fixed phase shift $\theta = 0.4$. (a) and (b) show PPDs versus phase shift for different numbers of homodyne samples $N$ when the squeezing parameter $r$ of 0.37 and 0.69, respectively.](image)

To investigate the performance for different times of measurements $N$ and squeezing parameter $r$, we fix the phase shift at $\theta = 0.4$ firstly in the experiment. The PPDs of the phase shift conditioned on the sampled homodyne data are obtained for different values of the involved parameters as a function of $\theta$ as shown in Fig. 3. The solid black, dash dot green, dot blue and dash red curves correspond to $N = 1000, 500, 300, 100$, respectively. A suitable estimator for the actual value of a fixed phase shift is given by the maximum of the distribution because of the symmetric form of the PPD. The definition of the variance is $\text{Var}[\theta] = \langle \theta^2 \rangle - \langle \theta \rangle^2$, and the $\text{Var}[\theta]$ is given by $1/N F(\theta)$, which is calculated from the homodyne measurements. Phase estimation can be enhanced with much higher squeezing parameter $r$ and more times of measurements $N$.

![FIG. 4. Estimation variance versus phase shift for three different purity squeezed states as the probe beam. (a) The noise suppression of one quadrature is measured as -3.21 dB relative to the SQL while the noise of the orthogonal quadrature is amplified by +3.41 dB and the purity of the probe beam is 0.977. (b) The purity is 0.891 and the noise suppression of one quadrature is the same as (a) while the noise of the orthogonal quadrature is increased to +4.23 dB. (c) The purity is 0.566 and the measured noise levels are -6.02 dB and +10.96 dB. The estimation variances $\text{Var}[\theta]$ for twelve phase shifts in the $[0, \pi/2]$ range are marked as the red circles with the standard deviations over 20 repetitions. The dot blue, dash origin and dash dot green curves correspond to the SQL, OCRB and QCRB, respectively.](image)

Then we analyze the effect of the purity of probe beam on the accuracy of phase estimation. The estimation variances $\text{Var}[\theta]$ for squeezed states with different purity are shown in Fig. 4. The estimation variances measured at twelve different phase shifts in $[0, \pi/2]$ range are marked as circles in the figure. The dot blue, dash origin and dash dot green curves correspond to the SQL, the OCRB and the QCRB which is calculated with the corresponding equations with same mean photon number, respectively. The noise power spectra of the probe beam at 3 MHz for different purity squeezed states measured by spectrum analyzers (SA) are shown in the inset of Fig. 4. It is obvious that the estimation accuracy attains OCRB only for one specific phase shift $\theta_{\text{opt}}$ and estimation accuracy beyond the SQL can be realized in a phase interval $\Delta \theta$ near the optimal phase shift $\theta_{\text{opt}}$ [59]. The larger the
squeezing parameter $r$, the higher the estimation precision. Because the mean photon number of probe beam is the main effect of the precision of quantum phase estimation, the estimation precision can also be enhanced with the increasing of factor of extra antisqueezing parameter $r'$ at the same $r$. For a squeezed state of same squeezing level $r$, the mean photon number of squeezed thermal state is more than that of pure squeezed state because the existence of extra noise $r'$ can increase the mean photon number of the squeezed state. Although the extra antisqueezing parameter has a helpful influence on the absolute precision of phase estimation, the estimation accuracy is further away from the QCRB with the increase of $r'$ due to the extra loss and phase fluctuation in antisqueezing quadrature. The accuracy of the quantum phase estimation can be enhanced with much higher squeezing level at a given fixed mean photon number and squeezed pure state behaves the optimal resource to reach QCRB, i.e. Heisenberg limit asymptotically.

Finally, we analyze the influence of purity of probe beam on the phase interval $\Delta \theta$ of phase estimation beyond the SQL. The range of quantum phase estimation beyond the SQL for four different purity probe beams with different squeezing level are shown in Fig. 5. The $\Delta \theta$ increases with the purity of the probe beam. The range of $\Delta \theta$ is 0.653 at purity of squeezed state with 0.977, which is more than twice of that at purity of 0.566 without any feedback control.

V. CONCLUSION

Through detailedly analyzing the influence of properties of squeezed thermal state on the precision of quantum phase estimation, it is found that the QCRB only can be reached with the help of a squeezed pure state and the absolute precision of quantum phase estimation can be enhanced with squeezed state of higher squeezing level. Through controlling the conditions of NOPA, squeezed states of light with different purity and squeezing level are used as a probe beam in the experiment of phase estimation. The experimental results are in good agreement with the theoretical analyses. This provides us a new direction of simple and convenient phase estimation scheme and it is also a good reference for the multi-parameter estimation with a multipartite entanglement.

ACKNOWLEDGMENTS

The authors would like to thank Kaikin Zheng and Shan Ma for helpful discussions. Our work was supported by the Key Project of the National Key R&D program of China (Grant No. 2016YFA0301402), the National Natural Science Foundation of China (Grants No. 61925503, No. 61775127, No. 11654002, No. 11804246, and No. 11834010), the Program for Sanjin Scholars of Shanxi Province, and the fund for Shanxi “1331 Project” Key Subjects Construction.

[1] P. A. M. Dirac, The quantum theory of the emission and absorption of radiation, Proc. R. Soc. A: Math. Phys. Eng. Sci. 114, 243 (1927).
[2] H. M. Wiseman and G. J. Milburn, Quantum measurement and control, Cambridge University, England (2010).
[3] G. Vittorio, L. Seth and M. Lorenzo, Quantum-enhanced measurements: beating the standard quantum limit, Science, 306, 1330 (2004).
[4] D. W. Berry and H. M. Wiseman, Adaptive quantum measurements of a continuously varying phase, Phys. Rev. A 65, 043803 (2002).
[5] Z. Y. Ou, Fundamental quantum limit in precision phase measurement, Phys. Rev. A 55, 2598 (1997).
[6] J. D. Zhang, Z. J. Zhang, L. Z. Cen, J. Y. Hu, and Y. Zhao, Nonlinear phase estimation: Parity measurement approaches the quantum Cramèr-Rao bound for coherent states, Phys. Rev. A 99, 022106 (2019).
[7] A. Lumino, E. Polino, A. S. Rab, G. Milani, N. Spagnolo, N. Wiebe, and F. Sciarrino, Experimental phase estimation enhanced by machine learning, Phys. Rev. Appl. 10, 044033 (2018).
[8] S. Boixo, M. J. Davis, and A. Shaji, Quantum metrology: dynamics versus entanglement, Phys. Rev. Lett. 101, 040403 (2008).
[9] V. Giovannetti, S. Lloyd, and L. Maccone, Advances in quantum metrology, Nat. Photonics 5, 222 (2011).
[10] S. L. Braunstein and C. M. Caves, Statistical distance and the geometry of quantum states, Phys. Rev. Lett. 72, 3439 (1994).
[11] T. Udem, R. Holzwarth, and T. W. Hänsch, Optical frequency metrology, Nature 416, 233 (2002).
[12] N. Hinkley, J. A. Sherman, N. B. Phillips, M. Schioppo, N. D. Lemke, K. Beloy, M. Pizzocaro, C. W. Oates, and A. D. Ludlow, An atomic clock with $10^{-18}$ instability, Science 341, 1215 (2013).
[13] LIGO Scientific Collaboration, Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light, Nat. Photonics 7, 613 (2013).
[14] U. L. Andersen, Quantum optics: squeezing more out of LIGO, Nat. Photonics 7, 589 (2013).
B. L. Higgins, D. W. Berry, S. D. Bartlett, H. M. Wiseman, F. Liu, Y. Y. Zhou, J. Yu, J. L. Guo, Y. Wu, S. X. Xiao, D. Wei, Z. X. Huang, K. R. Motes, P. M. Anisimov, J. P. Dowling, P. M. Anisimov, G. M. Raterman, A. Chiruvelli, W. N. Plick, S. M. Kacprowicz, R. Demkowicz-Dobrzański, W. Wasilewski, K. Banaszek and I. A. Walmsley, Quantum-enhanced estimation of a lossy phase shift, Nat. Photonics 4, 357 (2010).

H. Yonezawa, D. Nakane, T. A. Wheatley, K. Iwasawa, S. Takeda, H. Arai, K. Ohki, K. Tsumura, D. W. Berry, T. C. Ralph, H. M. Wiseman, E. H. Huntington, and A. Furusawa, Quantum-enhanced optical phase tracking, Science 337, 1514 (2012).

A. Monras, Optimal phase measurements with pure Gaussian states, Phys. Rev. A 73, 033821 (2006).

H. T. Dinani and D. W. Berry, Adaptive estimation of a time-varying phase with a power-law spectrum via continuous squeezed states, Phys. Rev. A 95, 063821 (2017).

W. H. Yang, X. L. Jin, X. D. Yu, Y. H. Zheng, and K. C. Peng, Dependence of measured audio-band squeezing level on local oscillator intensity noise, Opt. Express 25, 24262 (2017).

D. Li, B. T. Gard, Y. Gao, C. H. Yuan, W. P. Zhang, H. Lee, and J. P. Dowling, Phase sensitivity at the Heisenberg limit in an SU(1,1) interferometer via parity detection, Phys. Rev. A 94, 063840 (2016).

M. Aspachs, J. Calsamiglia, R. Muñiz-Tapia, and E. Bagan, Phase estimation for thermal Gaussian states, Phys. Rev. A 79, 033834 (2009).

A. A. Berni, T. Gehring, B. M. Nielsen, V. Händchen, M. G. A. Paris, and U. L. Andersen, Ab initio quantum-enhanced optical phase estimation using real-time feedback control, Nat. Photonics 9, 577 (2015).

M. Proietti, M. Ringbauer, F. Graffitti, P. Barrow, A. Pickston, D. Kundys, D. Cavalcanti, L. Aolita, R. Chaves, and A. Fedrizzi, Enhanced multiqubit phase estimation in noisy environments by local encoding, Phys. Rev. Lett. 123, 180503 (2019).

C. Zhang, T. R. Bromley, Y. Huang, H. Cao, W. Lv, B. Liu, C. Li, G. Guo, M. Cianciaruso, and G. Adesso, Demonstrating quantum coherence and metrology that is resilient to transversal noise, Phys. Rev. Lett. 123, 180504 (2019).

M. R. Huo, J. L. Qin, Y. R. Sun, J. L. Cheng, Z. H. Yan, and X. J. Jia, Generation of intensity difference squeezed state of light at optical fiber communication wavelength, Acta Sin. Quantum Opt. 24, 134 (2018).

M. Mehmet, S. Ast, T. Eberle, S. Steinlechner, H. Vahlbruch, and R. Schnabel, Squeezed light at 1550 nm with a quantum noise reduction of 12.3 dB, Opt. Express 19, 25763 (2011).

S. P. Shi, Y. J. Wang, W. H. Yang, Y. H. Zheng, and K. C. Peng, Detection and perfect fitting of 13.2 dB squeezed vacuum states by considering green-light-induced infrared absorption, Opt. Lett. 43, 5411 (2018).

H. Vahlbruch, M. Mehmet, K. Danzmann, and R. Schnabel, Detection of 15 dB squeezed states of light and their application for the absolute calibration of photoelectric quantum efficiency, Phys. Rev. Lett. 117, 110801 (2016).

J. Sun, X. C. Zhang, W. Z. Qu, E. E. Mikhailov, I. Novikova, H. Shen, and Y. H. Xiao, Spatial multiplexing of squeezed light by coherence diffusion, Phys. Rev. Lett. 123, 203604 (2019).

Y. Y. Zhou, J. Yu, Z. H. Yan, X. J. Jia, J. Zhang, C. D. Xie, and K. C. Peng, Quantum secret sharing among four players using multipartite bound entanglement of an optical field, Phys. Rev. Lett. 121, 150502 (2018).

N. Wiebe and C. Granade, Efficient Bayesian phase estimation, Phys. Rev. Lett. 117, 010503 (2016).

H. Cramér, Mathematical methods of statistics, Princeton University Press (1946).

M. Hassani, C. Macchiavello, and L. Maccone, Digital quantum estimation, Phys. Rev. Lett. 119, 200502 (2017).

Y. Gao and H. Lee, Bounds on quantum multiple-parameter estimation with Gaussian state, Eur. Phys. J. D 68, 347 (2014).
[54] R. Gaiba and M. G. A. Paris, Squeezed vacuum as a universal quantum probe, Phys. Lett. A 373, 934 (2009).
[55] M. G. Genoni, S. Olivares, and M. G. A. Paris, Optical phase estimation in the presence of phase diffusion, Phys. Rev. Lett. 106, 153603 (2011).
[56] S. Grandi, A. Zavatta, M. Bellini, and M. G A Paris, Experimental quantum tomography of a homodyne detector, New J. Phys. 19, 053015 (2017).
[57] M. R. Huo, J. L. Qin, J. L. Cheng, Z. H. Yan, Z. Z. Qin, X. L. Su, X. J. Jia, C. D. Xie, and K. C. Peng, Deterministic quantum teleportation through fiber channels, Sci. Adv. 4, eaas9401 (2018).
[58] O. Pinel, P. Jian, N. Treps, C. Fabre, and D. Braun, Quantum parameter estimation using general single-mode Gaussian states, Phys. Rev. A 88, 040102 (2013).
[59] S. Olivares and M. G. A. Paris, Bayesian estimation in homodyne interferometry, J. Phys. B: At. Mol. Opt. Phys. 42, 055506 (2009).
[60] B. Chen, J. P. Geng, F. F. Zhou, L. L. Song, H. Shen, and N. Y. Xu, Quantum state tomography of a single electron spin in diamond with Wigner function reconstruction, Appl. Phys. Lett. 114, 041102 (2019).
[61] U. Leonhardt, Measuring the quantum state of light, Cambridge University Press, Cambridge, (1997).
[62] M. G. A. Paris, Quantum estimation for quantum technology, Int. J. Quantum Inf. 07(supp01), 125 (2009).
[63] Y. Y. Zhou, X. J. Jia, F. Li, C. D. Xie, and K. C. Peng, Experimental generation of 8.4 dB entangled state with an optical cavity involving a wedged type-II nonlinear crystal, Opt. Express 23, 4952 (2015).
[64] X. W. Deng, S. H. Hao, H. Guo, C. D. Xie and X. L. Su, Continuous variable quantum optical simulation for time evolution of quantum harmonic oscillators, Sci. Rep. 6, 22914 (2016).