X-RAY AND OPTICAL FLUX RATIO ANOMALIES IN QUADRUPLY LENSED QUASARS. I. ZOOMING IN ON QUASAR EMISSION REGIONS

DAVID POOLEY, JEFFREY A. BLACKBURNE, SAUL RAPPAPORT, AND PAUL L. SCHECHTER

Received 2006 July 28; accepted 2006 December 21

ABSTRACT

X-ray and optical observations of quadruply lensed quasars can provide a microarcsecond probe of the lensed quasar, corresponding to scale sizes of $\sim 10^{-10}$ gravitational radii of the central black hole. This high angular resolution is achieved by taking advantage of microlensing by stars in the lensing galaxy. In this paper we use X-ray observations of 10 lensed quasars recorded with the Chandra X-Ray Observatory as well as corresponding optical data obtained with either the Hubble Space Telescope or ground-based optical telescopes. These are analyzed in a systematic and uniform way with emphasis on the flux ratio anomalies that are found relative to the predictions of smooth lens models. A comparison of the flux ratio anomalies between the X-ray and optical bands allows us to conclude that the optical emission regions of the lensed quasars are typically larger than expected from basic thin-disk models by factors of $\sim 3-30$.

Subject headings: gravitational lensing — quasars: general

1. INTRODUCTION

In this paper we carry out a systematic, uniform study of 10 quadruply gravitationally lensed quasars (hereafter “quads”) for which one or more Chandra X-Ray Observatory and optical images exist. We show how such observations can probe the lensed quasar on angular scales of a few microarcseconds.

Simple models for the gravitational potentials of lensing galaxies are usually sufficient to reproduce the positions of four-image gravitationally lensed quasars. But these same models—a monopole plus a quadrupole—fail to reproduce the optical fluxes of those images. Such “flux ratio anomalies” are thought to be the product of small-scale structure in the gravitational potentials of galaxies (Witt et al. 1995; Mao & Schneider 1998; Chiba 2002; Metcalf & Madu 2001; Dalal & Kochanek 2002; Schechter & Wambsganss 2002). Of the two leading explanations, the more intriguing is that we are seeing the effects of dark matter condensations of subgalactic mass. The more prosaic explanation (although exciting for very different reasons) is that the anomalies are largely the result of microlensing by stars in the intervening galaxy.

A mass condensation can produce an anomaly only if the radius of its Einstein ring (a “circle of influence”) is large compared to the emitting region of the lensed source. To set the scale for the discussion, the Einstein radius of a star in a typical lensing galaxy is $\sim 3 (m/M_\odot)(\text{Gpc}/D_L)^{1/2} \mu\text{as}$, where $D_L$ is the angular diameter distance of the lens and $m$ is the mass of the star. If the optical continuum emission from the typical quasar originated from something like a Shakura & Sunyaev (1973) thin accretion disk, we estimate that the Einstein ring of a 0.7 $M_\odot$ star would be much bigger than the optical emitting region. The stars in the lensing galaxy would then be expected to produce microlensing of the optical continuum (Mortonson et al. 2005). As X-ray emission is generally thought to arise from an even smaller region than the optical continuum, the X-ray fluxes of quasar images ought likewise to exhibit such anomalies when observed with Chandra.

This expectation for the X-rays is borne out by observations. Morgan et al. (2001) found that in the quadruple system RX J0911+0551 the C/D ratio was 0.19 in the X-ray compared with 1.28 in the optical. Blackburne et al. (2006) found that in the quadruple system RX J1131−1231 the A/B ratio was 0.18 in the X-ray compared with 1.10 in the optical and model predictions in the vicinity of 1.70. More recent X-ray observations show considerable changes in the flux ratios of the images in this lens (Kochanek et al. 2007). Pooley et al. (2006) find that in the quadruple system PG 1115+080 the $A_2/A_1$ ratio is roughly 0.16 in the X-ray compared with 0.68 in the optical and model predictions more nearly equal to unity.

In each of the above cases, the sense of the anomaly is that the X-ray flux ratios are yet more anomalous (in the sense of disagreeing with the models) than the optical flux ratios. In previous work (cited above) we showed that this could happen only if the optical continuum emitting region were substantially larger than predicted for Shakura-Sunyaev disks. In a number of individual studies—the three aforementioned cases, as well as MG J0414+0534 (Chartas et al. 2002), H1413+117 (Chartas et al. 2004), and Q2237+0305 (Dai et al. 2003)—microlensing has been invoked to explain some of the observed X-ray properties. In this paper we report the results of a systematic study of a larger sample of X-ray imaged quad lenses.

The above discussion illustrates how microlensing permits at least some resolution of a quasar on microarcsecond scales, 2 orders of magnitude better than VLBI. This corresponds to physical scales in the accretion disk of just a few thousand AU, or $\sim 1000$ gravitational radii for a $\sim 3 \times 10^8 M_\odot$ black hole at an angular diameter distance of 1 Gpc.

1 Based on observations obtained with the Magellan Consortium’s Clay Telescope.
2 Astronomy Department, University of California at Berkeley, 601 Campbell Hall, Berkeley, CA 94720; dave@astron.berkeley.edu.
3 Chandra Fellow.
4 Department of Physics and Kavli Institute for Astrophysics and Space Research, Massachusetts Institute of Technology, 70 Vassar Street, Cambridge, MA 02139; jeffb@space.mit.edu, sar@mit.edu, schech@achernar.mit.edu.

5 The systems are the quadruple lenses HE 0230−2130 (Wisotzki et al. 1999), MG J0414+0534 (Hewitt et al. 1992), RX J0911+0551 (Bade et al. 1997), SDSS J0924+0219 (Inada et al. 2003a), PG 1115+080 (Weymann et al. 1980), RX J1131−1231 (Sluse et al. 2003), H1413+117 (Magan et al. 1988), B1422+231 (Patnaik et al. 1992), WFI J2033−4723 (Morgan et al. 2004), and Q2237+0305 (Huchra et al. 1985).
In addition, as suggested above, lensed systems present unique opportunities to study not only the lensed object but also the lensing object. As we will discuss in a forthcoming paper, these same observations of X-ray flux ratio anomalies permit measurements of the dark matter content of the lensing galaxies (Schechter & Wambsganss 2004).

In §2 we discuss the analysis of the Chandra archival data for 10 quads. In §3 we describe the properties of the selected group of lenses in the optical band. We also present a uniform set of models for these lenses produced with the same software and for a common set of model assumptions. The flux ratio anomalies are compared between the X-ray and optical images in §4. In §5 we draw conclusions concerning the sizes of optically emitting quasar images since it would have removed substantially more source flux than background flux within the small extraction regions. Finally, in §6 we summarize our findings. Throughout this paper we adopt a cosmology with $\Omega_{\Lambda} = 0.7$, $\Omega_M = 0.3$, and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

2. X-RAY OBSERVATIONS

The data were downloaded from the Chandra archive, and reduction was performed using the CIAO version 3.3 software provided by the Chandra X-Ray Center.6 The data were reprocessed using the CALDB version 3.2.2 set of calibration files (gain maps, quantum efficiency, quantum efficiency uniformity, and effective area) including a new bad pixel list made with the acis_run_hotpix tool. The reprocessing was done without including the pixel randomization that is added during standard processing. This omission slightly improves the point-spread function. The data were filtered using the standard ASCA grades and excluding both bad pixels and software-flagged cosmic-ray events. Intervals of strong background flaring were searched for, and a few were found. In all cases, the flares were mild enough that removing the intervals would have decreased the signal-to-noise ratio of the quasar images since it would have removed substantially more source flux than background flux within the small extraction regions. Therefore, we did not remove any flaring intervals. The observation IDs, dates of observation, and exposure times are given in Table 1.

2.1. Determining X-Ray Flux Ratios

For each observation, a sky image was produced in the 0.5–8 keV band with a sampling of 0.0246 pixel$^{-1}$. Because of the significant overlap of the lensed images (especially the close image pairs) in many cases, the intensities were determined by fitting to each sky image a two-dimensional model consisting of four Gaussian components plus a constant background. The background component was fixed to a value determined from a source-free region near the lens. The relative positions of the Gaussian components were fixed to the separations given in the CASTLES (CFA-Arizona Space Telescope Lens Survey) online database,7 but the absolute position was allowed to vary. Each Gaussian was constrained to have the same full width at half-maximum (FWHM), but this value was allowed to float. The fits were performed using Cash (1979) statistics and the Powell minimization method in Sherpa (Freeman et al. 2001).

From the best-fit four-Gaussian model, the image flux ratios were calculated for the high-magnification pair (saddle point and minimum, HS and HM, respectively) as well as for each image relative to the less magnified minimum (LM) image. The uncertainties on these ratios were determined with Sherpa via the projection command, which varies each ratio in turn along a grid of values while all other parameters are allowed to float to the new best-fit values. The results are given in Table 1.

Because the Chandra point-spread function is only approximately described by a Gaussian, we sought to test this method by using a Chandra observation (ObsID 5794) of the large-separation quad SDSS J1004+4112, for which all four images are well separated.8 We extracted counts from the 90% encircled energy region of each image, as determined by ACIS Extract version 3.94 (Broos et al. 2002), and formed a number of flux ratios. We also followed the above method of fitting Gaussians. The agreement in flux ratios is excellent (see Table 2).

Finally, a spectrum of the LM image was extracted for each observation with ACIS Extract and fit in Sherpa via a simple absorbed power law. The absorption consisted of a fixed Galactic component (Dickey & Lockman 1990) plus a variable component. This simple model provided an acceptable fit in all cases, and the additional absorption component was usually consistent with zero. The 0.5–8 keV flux of the unabsorbed power law is given in Table 1.

2.2. X-Ray Variability

As the numbers in Table 1 indicate, many of the flux ratios vary to some degree for the quads that have been observed multiple times. This may be due to varying degrees of microlensing or to normal quasar variability combined with time delays among the images. In fact, variability plus a time delay could masquerade as a flux ratio anomaly. Figure 1 shows the X-ray light curves of the sum of the high-magnification pair of images for each system, which are seen to be fairly constant in all systems; only small-amplitude (factor of 2 or less) variability is observed, even in cases in which the length of the observation exceeds the predicted time delay between the brightest images.9

For the rest of the analysis, we used the observation with the highest signal-to-noise ratio for the quads observed multiple times by Chandra. We chose not to average over multiple epochs in order to avoid averaging out variations due to changes in microlensing. We use ObsID 3419 for MG J0414+0534, ObsID 419 for RX J0911+0551, ObsID 363 for PG 1115+080, ObsID 5645 for H1413+117, ObsID 4939 for B1422+231, and ObsID 431 for Q2237+0305.

3. OPTICAL IMAGES AND LENS MODELS

We turned to the existing literature for optical data with which to compare our X-ray flux ratios. For each lens, we used data near 8000 Å, either Sloan $i'$, Cousins $I$, or Hubble Space Telescope (HST) F814W. An effort was made to choose the observations closest in time to the deepest Chandra observation. The dates of the observations, along with the optical bandpasses and the image magnitudes, can be found in Table 3. The images are arranged according to their magnifications and parity (see §4).

Under ideal circumstances, the X-ray and optical observations would have been made on the same day, in order to minimize systematic errors resulting from quasar variability and microlensing variability. But for most of these lenses, such contemporaneous observations have not been made. Three lenses have X-ray and optical observations separated by about 6–10 yr, three
Comparison of Gaussian Fitting to Aperture
SDSS J0924+0219
MG J0414+0534
A H1413+117

Table 1
X-Ray Fluxes and Flux Ratios

| Lensed Quasar | ObsIDb | Date       | Exp. (s) | B/A   | B/C   | A/C   | A/B   | C/B   | D/C   | F_b   |
|---------------|--------|------------|----------|-------|-------|-------|-------|-------|-------|-------|
| HE 0230−2130  |        |            |          |       |       |       |       |       |       |       |
| 1642...........| 2000 Oct 14.4 | 14764     |          | 0.44±0.08 | 0.70±0.13 | 1.6 ± 0.2 | 0.45±0.08 | 0.76±0.12 | 5.9±2.8 |       |
| MG J0414+0534  |        |            |          |       |       |       |       |       |       |       |
| 417.............| 2000 Jan 13.7 | 6578      |          | 0.82±0.10 | 1.9 ± 0.4 | 2.3±0.5 | 0.25±0.07 | 0.9 ± 0.4 | 12±7   |       |
| 418.............| 2000 Apr 2.9 | 7437      |          | 0.50±0.10 | 1.3 ± 0.3 | 2.6±0.4 | 0.63±0.07 | 1.0 ± 0.3 | 13±4   |       |
| 421.............| 2000 Aug 16.9 | 7251      |          | 0.38±0.12 | 0.96±0.33 | 2.5±0.5 | 0.45±0.09 | 0.7 ± 0.3 | 15±5   |       |
| 422.............| 2000 Nov 16.6 | 7504      |          | 0.67±0.12 | 1.8 ± 0.4 | 2.6±0.4 | 0.65±0.10 | 0.7 ± 0.3 | 14±5   |       |
| 1628...........| 2001 Feb 5.1 | 9020      |          | 0.35±0.08 | 0.89±0.20 | 2.5±0.3 | 0.35±0.06 | 1.6 ± 0.5 | 16±5   |       |
| 3395...........| 2001 Nov 9.3 | 28413     |          | 0.90±0.06 | 1.8 ± 0.2 | 1.9±0.2 | 0.53±0.05 | 1.3 ± 0.3 |       |       |
| 3419...........| 2002 Jun 9.0 | 96664     |          | 0.61±0.04 | 1.3 ± 0.1 | 2.1±0.1 | 0.42±0.02 | 1.4±3   |       |       |
| RX J0911+0551   |        |            |          |       |       |       |       |       |       |       |
| 419.............| 1999 Nov 2.7 | 28795     |          | 2.7±2.27 | 3.4±0.6 | 1.3±0.4 | 0.35±0.13 | 0.11±0.13 |       |       |
| 1632...........| 2000 Oct 29.8 | 9826      |          | 0.14±0.07 | 0.45±0.25 | 3.2±0.6 | 0.42±0.19 | 0.14±0.14 | 1.2±0.3 |       |
| SDSS J0924+0219 |        |            |          |       |       |       |       |       |       |       |
| 363.............| 2000 Jun 2.8 | 26492     |          | 0.16±0.03 | 0.62±0.13 | 3.9±0.3 | 1.1±0.1 | 6.8±1.1 |       |       |
| 1630...........| 2000 Nov 3.3 | 9826      |          | 0.28±0.06 | 1.2 ± 0.3 | 4.4±0.5 | 1.0±0.2 | 8.3±2.9 |       |       |
| RX J1131−1231   |        |            |          |       |       |       |       |       |       |       |
| 4814...........| 2004 Apr 12.2 | 10047     |          | 0.10±0.01 | 0.22±0.02 | 2.2±0.1 | 0.30±0.02 | 50±5   |       |       |
| H1413+117       |        |            |          |       |       |       |       |       |       |       |
| 930.............| 2000 Apr 19.7 | 38185     |          | 1.8±0.4 | 4.0±1.4 | 2.2±0.9 | 1.2±0.3 | 2.7±0.7 |       |       |
| 5645...........| 2005 Mar 30.1 | 88863     |          | 1.7±0.4 | 1.5±0.9 | 0.9±0.2 | 0.7±0.2 | 3.3±0.5 |       |       |
| B1422+231       |        |            |          |       |       |       |       |       |       |       |
| 367.............| 2000 Jun 1.6 | 28429     |          | 0.68±0.06 | 1.1 ± 0.1 | 1.6±0.1 | 0.11±0.01 | 37±5   |       |       |
| 1631...........| 2001 May 21.5 | 10651     |          | 0.62±0.08 | 0.87±0.12 | 1.4±0.2 | 0.08±0.02 | 40±10  |       |       |
| 4939...........| 2004 Dec 1.6 | 47729     |          | 0.55±0.04 | 0.95±0.08 | 1.7±0.1 | 0.10±0.01 | 33±5   |       |       |
| WFI J2033−4723  |        |            |          |       |       |       |       |       |       |       |
| 5603...........| 2005 Mar 10.1 | 15420     |          | 1.1±0.3 | 1.0 ± 0.2 | 0.87±0.16 | 0.64±0.11 | 3.5±0.9 |       |       |
| Q2237+0305      |        |            |          |       |       |       |       |       |       |       |
| 431.............| 2000 Sep 6.7 | 30287     |          | 0.17±0.01 | 0.85±0.09 | 5.0±0.4 | 2.1±0.2 | 5.9±1.3 |       |       |
| 1632...........| 2001 Dec 8.8 | 9538      |          | 0.20±0.03 | 0.95±0.20 | 4.7±0.7 | 1.7±0.3 | 6.1±2.6 |       |       |

^a HS: Highly magnified saddle point; HM: highly magnified minimum; LS: less magnified saddle point; LM: less magnified minimum. See § 4.
^b The observation identifier of the Chandra data set.
^c The unabsorbed flux of the LM image is computed from the best-fit power-law model described in § 2.1.

Table 2
Comparison of Gaussian Fitting to Aperture
Extraction of SDSS J1004+4112

| X-Ray Image Flux Ratios | Method         | A/B       | A/C       | B/C       | D/C       |
|-------------------------|----------------|-----------|-----------|-----------|-----------|
| Gaussian fit...         | 0.77±0.03      | 0.96±0.04 | 1.24±0.04 | 0.57±0.03 |          |
| Extraction...           | 0.77±0.03      | 0.93±0.04 | 1.20±0.05 | 0.55±0.03 |          |

by 2—4 yr, and four by 15 months or less. One of these, RX J1131−1231, was observed in both bands on the same day (Sluse et al. 2006).

These delays between observations can add systematic uncertainty to the results. However, there are reasons to believe that their effect is not a strong one. The general lack of strong quasar variability seen in X-rays (see § 2.2), coupled with the limited success of campaigns to measure lens time delays (which rely on quasar variability), suggest that quasars do not often vary by the factors that would be required to explain the flux ratio anomalies.
The fact that RX J1131–1231 has an extremely strong discrepancy between X-ray and optical flux ratios despite simultaneous observations in both bands shows that time variability cannot fully explain the anomalous ratios.

We used Keeton’s (2001) lensmodel software, version 1.06, to model each of the 10 lenses as a singular isothermal sphere (SIS) with an external shear. This model has seven free parameters (lens strength, shear strength $\gamma$, and direction $\phi$, and the positions of source and lens), making it overconstrained by the 10 input measurements (the positions of four images and the lensing galaxy). The position measurements were obtained from the online CASTLES database. The observed fluxes of the lens images were not used as constraints.

The models fit the image positions fairly well in all cases except that of HE 0230–2130, where the position of the $D$ image is

![X-ray light curves](https://example.com/xray_light_curves.png)

**Fig. 1.** X-ray light curves of the high-magnification pair of images in each quad. All observations in Table 1 were used to make these 0.5–8 keV light curves, with multiple observations of the same quad separated by hash marks. The time delay between the pair (from the models described in § 3 and Table 4) is given and shown as a thick horizontal bar in the cases in which it fits on the plots.
TABLE 3

| Lensed Quasar | Optical Magnitudes |
|---------------|--------------------|
|               | HS     | HM     | LS     | LM     | Filter |
| HE 0230−2130  | B      | A      | D      | C      |        |
| 2002 Jul 29   | 19.22  | 19.02  | 21.21  | 19.59  | F814W  |
| MG J0414+0534 | A_2    | A_1    | C      | B      |        |
| 1994 Nov 08   | 21.36  | 20.43  | 22.10  | 21.24  | F814W  |
| RX J0911+0551 | A      | B      | C      | D      |        |
| 2000 Mar 02   | 18.38  | 18.64  | 19.36  | 19.66  | F814W  |
| 1994 Dec 22   | 17.77  | 17.84  | 18.15  | 18.06  | F814W  |
| 2003 Aug 01   | 19.14  | 18.68  | 19.41  | 19.32  |        |
| SDSS J0924+0219 | D    | A      | C      | B      |        |
| 2003 Nov 19   | 21.59  | 18.69  | 19.86  | 19.52  | F814W  |
| PG 1115+080   | A_2    | A_1    | C      | B      |        |
| 2004 Feb 22   | 15.86  | 16.08  | 17.68  | 17.26  |        |
| RX J1131−1231 | A      | B      | D      | C      |        |
| 2004 Apr 12   | 17.43  | 17.42  | 19.72  | 18.44  |        |
| H1413+117     |        |        |        |        |        |
| 1994 Dec 22   | 17.77  | 17.84  | 18.15  | 18.06  | F814W  |
| B1422+231     | B      | A      | D      | C      |        |
| 1999 Feb 06   | 15.85  | 15.88  | 19.68  | 16.41  | F814W  |
| WFI J2033−4723 | A_2   | A_1    | C      | B      |        |
| 2003 Aug 01   | 19.14  | 18.68  | 19.41  | 19.32  |        |
| Q2237+0305    | D      | A      | C      | B      |        |
| 1999 Oct 20   | 17.39  | 15.92  | 16.77  | 17.21  | F814W  |

* See http://www.cfa.harvard.edu/castles/.
* Keeton et al. (2006).
* Relative magnitudes from Pooley et al. (2006); zero point from this work.
* Sluse et al. (2006).
* Morgan et al. (2004).


4. COMPARISON OF ANOMALOUS FLUX RATIOS: X-RAY VERSUS OPTICAL

Figure 2 provides a visual guide to the optical-to-model and X-ray–to–model flux ratios of each quad. It shows representations of each system using two-dimensional Gaussians, the positions of which come from the CASTLES database. As a point of reference, the left panel for each quad shows Gaussians of unit amplitude. The middle panel represents the optical-to-model ratio of the images, normalized by each rms (described below). The amplitude $A_i$ of image $i$ is given by

$$A_i = \frac{F_{\text{opt},i}}{\mu_i}$$

where $i = 1, 2, 3, 4$; $F_{\text{opt},i}$ is the (linear) optical flux of image $i$; and $\mu_i$ is the image magnification from Table 4. The right panel gives a similar representation for the X-rays. The rms of the optical (and X-ray) observations is first computed as

$$\sqrt{\frac{1}{4} \sum_{i=1}^{4} \frac{(F_{\text{opt},i})^2}{\mu_i}}.$$
However, because the rms can be dominated by one highly anomalous image, we remove the largest deviator and then recompute the rms. The largest deviator is defined as the image $i$ with the maximum value of $|\log A_{i,\text{opt}} + \log A_{i,X}|$. This new rms is then used in equation (1) to compute the amplitudes, the values of which are given in Table 5.

In every case save one, the most anomalous image was the highly magnified saddle point image. This is not surprising, since Schechter & Wambsganss (2002) have shown that microlensing is likely to affect high-magnification saddle points most strongly. In order to give the lenses a uniform treatment, we have classified the four images in each lens according to their magnifications and the local morphology of the travel-time surface. Henceforth in this paper, “HS” designates the highly magnified saddle point and “HM” the highly magnified minimum. Likewise, “LS” designates the less magnified saddle point and “LM” the less magnified minimum.

In this work, we are most interested in the optical-to-model and X-ray-to-model ratios of the HS/HM flux ratio. The comparison between optical and X-ray ratios is shown for each quad in Figure 3. The left panel shows the observed HS/HM ratio relative to the model HS/HM ratio, and the middle and right panels show how each of HS and HM compare to the less magnified minimum image (LM). In almost all cases, the HS/HM ratio is more extreme in X-rays than in the optical; when the observed ratio is greater than the model ratio, the X-ray ratio is greater than the optical, and when the observed ratio is less than the model ratio, the X-ray ratio is less than the optical.

### Table 5: Flux-to-Model Ratios Normalized by RMS

| QUASAR       | BAND | HS   | HM   | LS   | LM   |
|--------------|------|------|------|------|------|
| HE 0230−2130 | Optical | 0.76 | 0.94 | 0.62 | 1.02 |
|               | X-ray | 0.46 | 1.06 | 1.50 | 1.24 |
| MG J0414+0534 | Optical | 0.43 | 1.10 | 2.68 | 1.86 |
|               | X-ray | 0.58 | 1.03 | 2.33 | 1.72 |
| RX J0911+0551 | Optical | 1.49 | 0.64 | 0.82 | 1.14 |
|               | X-ray | 1.76 | 0.36 | 0.24 | 1.29 |
| SDSS J0924+0219 | Optical | 0.09 | 1.10 | 0.80 | 1.25 |
|               | X-ray | 0.20 | 1.20 | 0.34 | 0.92 |
| PG 1115+080   | Optical | 1.02 | 1.15 | 1.04 | 1.19 |
|               | X-ray | 0.20 | 1.14 | 1.48 | 1.10 |
| RX J1131−1231 | Optical | 1.03 | 1.76 | 1.87 | 0.72 |
|               | X-ray | 0.10 | 1.72 | 2.08 | 0.82 |
| H1413+117     | Optical | 1.12 | 0.99 | 1.23 | 0.88 |
|               | X-ray | 1.23 | 0.69 | 0.96 | 0.87 |
| B1422+231     | Optical | 0.76 | 1.00 | 0.77 | 0.96 |
|               | X-ray | 0.49 | 1.22 | 1.78 | 1.10 |
| WFI J2033−4723| Optical | 0.71 | 0.93 | 1.65 | 1.25 |
|               | X-ray | 0.84 | 0.65 | 1.66 | 1.81 |
| Q2237+0305    | Optical | 0.33 | 1.36 | 1.08 | 0.48 |
|               | X-ray | 0.22 | 1.41 | 1.90 | 0.33 |

**Note:** The rms values were computed from the three least anomalous images in each quad. See § 4 for details.
the X-ray ratio is less than the optical. The middle and right panels show whether the discrepancy with the model comes from the HS or the HM image (or a combination of the two). In general, the LM image is much less susceptible to microlensing than either the HS or HM image (Kochanek & Dalal 2004).

The group statistics for the flux ratio anomalies presented in Figure 3 and Table 5 are summarized in Figure 4. The error bars represent the ±rms spread in the logarithm of the flux ratios (normalized by the smooth model values) between various image pairs for our quasar sample. The black outer bars result from including all 10 quasars; the thick blue bars result when we exclude the systems Q2237+0305 and SDSS J0924+0219. Q2237+0305 is excluded because the uniquely small redshift of its lensing galaxy causes the projected microlens Einstein radius to be bigger than any region of the source, while SDSS J0924+0219 might also be excluded because the source size is thought to be so small that even its broad-line region is partially microlensed (Keeton et al. 2006).

It can be seen from the blue bars in Figure 4 that the ratios of the HS to HM images deviate more (from their expected values) in the X-ray band than in the optical band by a factor of ~2.4. The discrepancy is somewhat smaller for the HS/LM ratios at a factor of ~1.7. The HM/LM and LS/LM ratios are not as anomalous in either band, but the X-ray ratios still have a wider range than do the optical ratios. It is on the larger anomalies in the X-ray band for the HS/HM and HS/LM ratios, as compared to those for the optical band, that we base our quantitative analysis of the size of the optically emitting regions of the accretion disks in the next section.

5. Sizes of Quasar Emission Regions

For the purpose of interpreting our results, we adopt the working hypothesis that the anomalous flux ratios presented in this paper are the result of microlensing. Microlensing by stars in the lensing galaxy can account for the observed flux ratio anomalies, but only if the source is small compared to the Einstein radii of the microlensing stars. Figure 3 shows dramatic evidence for microlensing in the X-ray band for at least 7 of the 10 lensing systems in our study. In general, the optical emission of these same systems, while still being microlensed, has less extreme flux ratio anomalies than in the X-ray band by a factor of ~2 (see Fig. 4 and the discussion above). Since the X-rays are expected to be emitted very near the black hole, the condition for microlensing is easy to meet—the source should indeed be quite small compared to the Einstein radius of the microlensing stars. By contrast, the markedly lower degree of microlensing in the optical band implies that the size of the optical emission region in many of these sources is roughly comparable to the size of the stellar microlens Einstein radius.

Many authors have studied the effect of source size on the microlensing of quasars by intervening galaxies. Typically the results are presented as plots of microlensing light curves (e.g., Wambsganss & Paczynski 1991) rather than rms fluctuations in the logarithm of the flux.10 There are no analytic techniques for estimating rms fluctuations, so one must simulate the microlensing process.

Ideally we would run point-source simulations for each of the 40 images in our sample, taking into account the theoretical magnification (which in turn depends on two independent parameters, a convergence and a shear) and the fraction of baryonic matter. Each simulation would produce a magnification map, which might then be convolved with sources of different sizes, producing magnification histograms.

Such an effort lies beyond the scope of the present paper, but we can draw on such simulations that have been carried out. In particular we use the work of Mortonson et al. (2005), who studied in detail the effect of source size on minima and saddle points with magnifications of ~6 and ~2, respectively, assuming that the convergence (a dimensionless surface density) is due entirely to equal-mass stars and taking the convergence to be equal to the shear, as would be the case for an unperturbed isothermal sphere. The magnifications for our highly magnified

---

10 While the rms microlensing fluctuations in flux are formally divergent, rms fluctuations in the logarithm of the flux are not (e.g., Witt et al. 1995).
minima and saddle points are larger than this, typically by a factor of 2, while our less magnified images are typically fainter than this by a factor of 2. Moreover, there is reason to think that the stellar component comprises only a fraction—somewhere between $\frac{1}{10}$ and $\frac{1}{5}$—of the mass surface density. In the absence of a complete set of simulations we take those of Mortonson et al. as representative.

They find that, independent of the detailed radial profile of the source, the rms logarithmic fluctuations depend only on the ratio of the half-light radius of the source to the Einstein radius, $r_{1/2}/r_{\text{Ein}}$. The rms logarithmic fluctuations decrease from their maximum value at $r_{1/2}/r_{\text{Ein}} = 0$ to one-half that value at $r_{1/2}/r_{\text{Ein}} \approx \frac{1}{2}$. Since our optical fluctuations are roughly one-half the amplitude of the X-ray fluctuations (which we take to arise from a region of negligible extent), we infer that the radius of the (projection of the) optical region is roughly $\frac{1}{2}$ the Einstein radius of the stars.

To estimate a rough size for the expected region of the optical emission from quasar accretion disks, we adopt a generic thin-disk model (see, e.g., Shakura & Sunyaev 1973). In such a model the gravitational energy release is redistributed via internal viscous stresses in such a way that, independent of the detailed nature of the origin of the viscosity, the rate of energy release per unit area of the disk at radius, $r$, is

$$f = \frac{3GM\dot{M}}{8\pi r^3} \left(1 - \frac{r_0}{r}\right),$$

(3)

where $M$, $\dot{M}$, and $r_0$ are the black hole mass, the accretion rate, and the inner radius of the accretion disk, respectively. Note that in this formulation neither special nor general relativistic effects are included, except implicitly via the location of $r_0$. In our context, such relativistic effects are unimportant in the case of a Schwarzschild black hole. Relativistic corrections, including those for accretion disks around Kerr black holes (e.g., Page & Thorne 1974), are only likely to exacerbate the difficulties with understanding the size of the optical emission regions discussed below.

In the context of the thin-disk model around a Schwarzschild black hole, the fractional luminosity that emerges within a radial distance $r$ is

$$f_L(<r) = 3r_0 \int_{r_0}^{r} \left(1 - \frac{r_0}{r}\right) r^{-2} dr = 1 - \frac{3}{r/r_0} + \frac{2}{(r/r_0)^{3/2}}.$$

(4)

The complement of this quantity, $1 - f_L(<r)$, i.e., the fraction of the luminosity released at radii $r$, is plotted in Figure 5. Here we have labeled the $x$-axis in physical units starting at $r_0 = 6GM/c^2 = 2.5 \times 10^{14}$ cm, i.e., the last stable orbit about a Schwarzschild black hole of $3 \times 10^8 M_\odot$, an illustrative quasar mass. We also show curves for other possible black hole masses. For black holes with appreciable angular momentum, the value of $r_0$ moves progressively inward, and radii at which equal fractions of the luminosity are emitted do likewise.

Also overplotted on Figure 5 are nine arrows, one for each of our sources with known redshifts, marking the physical size of the Einstein radius of a 0.7 $M_\odot$ star in the lensing galaxy as projected back onto the lensed quasar. What we see is that the arrows are virtually all located at radii where only a tiny fraction of the quasar luminosity can emerge from the disk—at least for our fiducial black hole mass of $3 \times 10^8 M_\odot$. These fractional luminosity values are typically $\leq 2\%$ for sizes comparable to the back-projected stellar Einstein radii. Only for black hole masses $\geq 3 \times 10^9 M_\odot$ does a significant fraction of the luminosity (i.e., $\sim 20\%$) originate from radial distances comparable in size to the Einstein radius. However, even then, as we showed in Pooley et al. (2006) and demonstrate below, much of this radiation should be emitted at wavelengths well beyond the optical or near-IR. Given that the optical radiation (e.g., $0.4 - 1.5 \mu$m) typically comprises a substantial fraction of quasar luminosities, e.g., $\sim 15\%$ (Elvis et al. 1994), it appears difficult for the optical emission to be released from a thin disk at radii that are sufficiently large to allow for the partial suppression of microlensing—as observed. We further quantify this conclusion below.

Figure 5 and equation (4) imply effective upper limits to the size of thin accretion disks in the optical by evaluating the bolometric luminosity emitted within a radial distance $r$ of the central black hole. We now proceed to compute more quantitatively how large the accretion disk is expected to appear for a fixed wave band, e.g., $V$, $R$, or $I$. Based on the relativistic invariant $I/\nu^2$, we find the following expression for the half-light radius, $r_{1/2}$, of a thin accretion disk in a wave band centered at $\nu$ (in the Earth’s frame):

$$r_{1/2} = \frac{1}{2} \left[ \frac{6 \pi M_\text{BH}}{8GM} \right]^{1/2} \left[ \frac{1}{\nu} \right]^{3/2},$$

(5)

where $r_0$ is the location of the inner edge of the accretion disk and $T(r)$ is the local temperature of the accretion disk, which in the Shakura & Sunyaev (1973) model is

$$T(r) = \left[ \frac{3GM_{BH} M}{8\pi \sigma T^4} \right]^{1/4} \left(1 - \frac{r_0}{r}\right)^{1/4}.$$

In this simple picture, calculation of the half-light radius requires knowledge of three parameters: $M_{BH}$, $M$, and $r_0$. We use primarily the optical-based method of Kaspi et al. (2000; discussed below) to estimate the bolometric luminosity of each of the 10 sources in our sample. We also use the X-ray luminosity, coupled with a bolometric correction factor (also discussed below) to provide a sanity check on the Kaspi et al. approach. We further assume that all of the quasars are operating at the same fraction, $f^\text{edd} \approx \frac{1}{3}$, of their respective Eddington limits (Kollmeier et al. 2006). We show below from a simple scaling argument that our final results for $r_{1/2}$ are relatively insensitive to this choice. Finally, we assume that the radiation efficiency (rest mass to radiant energy conversion efficiency, $\eta$) of all the quasars in our sample is $\eta = 0.15$ (see,
e.g., Yu & Tremaine 2002). For this choice of efficiency, the dimensionless black hole spin parameter would be $a = 0.88$ and the innermost stable orbit would be located at $r_{\text{in}} \approx 2.5R_{\text{p}} = 2.5G M_{\text{BH}} c^2$ (e.g., Bardeen 1970). However, in our simple non-relativistic disk model, we can only fix $r_0$ and accept whatever the nonrelativistic energy release is. For $r_0 = 2.5R_{\text{p}}$, this turns out to yield an equivalent $\eta = 0.2$, which is sufficiently close to the Kerr value to provide the desired accuracy in computing $r_{1/2}$.

We summarize the computed and inferred properties of our quasar sample in Table 6. Column (2) gives the bolometric luminosity as calculated from the Kaspi et al. (2000) prescription. In this approach, $L_{\text{bol}}$ is taken to be $9[\lambda F_{\lambda}]_{1500} 4\pi d_L^2$. To estimate the 5100 Å flux in the rest frame of the quasar, we used the flux measured in the closest available broadband filter, usually the HST NICMOS F160W band, and extrapolated using an assumed power-law spectrum $F_{\lambda} \sim \lambda^{-1.7}$ (Kollmeier et al. 2006). Column (3) in Table 6 gives an independent estimate of the bolometric luminosity for each quasar based on the measured X-ray luminosity and a bolometric correction factor of 20, as inferred from the composite active galactic nucleus (AGN) spectrum of Elvis et al. (1994). Column (4) provides the black hole mass inferred from the bolometric luminosity (in col. [2]) divided by the Eddington fraction, $f_{\text{Edd}} = \frac{1}{3}$, which then yields $L_{\text{Edd}}$, and thence $M_{\text{BH}}$. It should be noted that since Kollmeier et al. (2006) derive $f_{\text{Edd}} = \frac{1}{4}$ using the prescription of Kaspi et al. (2000) and since we follow suit, the masses we derive are independent of the dimensionless factor (of 9) in Kaspi’s prescription. The error bars on the mass represent the uncertainties inferred from the $\pm$1rms (logarithmic) spread in the bolometric luminosities obtained via three different estimates: (1) the Kaspi et al. (2000) method, (2) the X-ray luminosity, and (3) in seven of the nine cases the mass estimate directly provided by Peng et al. (2006; based on a “virial” method involving broad-line widths and sizes of broad-line regions). The values of the half-light radius, $r_{1/2}$, computed with equations (5) and (6) for the I band are given in column (5). In column (6) are Einstein ring radii of typical 0.7 $M_\odot$ microlenses, projected onto the plane of the source. Finally, in column (7) we give the logarithm of the ratio of the half-light radius to the microlens Einstein radius.

The results of our thin-disk estimates for the ratio $r_{1/2}/r_{\text{Edd}}$ (Table 6, col. [7]) are plotted in Figure 6. The central filled circle within each error bar is based on the black hole mass given in column (4) of Table 6. The error bars on the ratio $r_{1/2}/r_{\text{Edd}}$ are propagated from the black hole mass uncertainties given in Table 6. The blue vertical dashed line at $r_{1/2}/r_{\text{Edd}} = \frac{1}{3}$ is our estimate of the ratio of the required to suppress microlensing in the optical band by the (logarithmic) factor of $\sim 2$ discussed in § 4. The red vertical dashed line at $r_{1/2}/r_{\text{Edd}} = \frac{1}{10}$ represents a more conservative lower limit on $r_{1/2}/r_{\text{Edd}}$ that might plausibly still be consistent with the suppressed microlensing in the optical. An inspection of Figure 6 shows that seven of the nine systems (for which $r_{1/2}/r_{\text{Edd}}$ could be calculated) lie below the limit of $\frac{1}{10}$, and therefore the disk size in the optical that we calculate appears to be too small to explain the reduced microlensing. Only one of the systems, B1422+231, has a ratio of $r_{1/2}/r_{\text{Edd}}$ that slightly exceeds the $\frac{1}{10}$ value that we think is reasonable to account for the reduced microlensing in the optical for this particular source.

Arguably the largest contribution to the uncertainty in the calculation of $r_{1/2}$ arises from the errors in estimating the bolometric luminosities of the quasars in our sample. We believe we can make a fairly robust estimate of the uncertainty in $r_{1/2}$—due

| Quasar | $L_{\text{bol, opt}}$ | $L_{\text{bol, x}}$ | log $M_{\text{BH}}$ | $r_{1/2}$ | $r_{1/2}$ | Stellar $r_{\text{Edd}}$ | log $(r_{1/2}/r_{\text{Edd}})$ |
|--------|----------------|-----------------|-----------------|--------|----------|-----------------|-----------------------------|
| HE 0230–2130 | 2.9 | 6.3 | 7.95 ± 0.24 | 0.93 | 70 | 43 | −1.66 ± 0.16 |
| MG J0414+0534 | 36 | 28 | 9.04 ± 0.17 | 3.8 | 23 | 31 | −0.91 ± 0.11 |
| RX J0911+0551 | 13 | 13 | 8.60 ± 0.18 | 1.9 | 32 | 35 | −1.26 ± 0.12 |
| SDSS J0924+0219 | 0.6 | 0.3 | 7.27 ± 0.56 | 0.42 | 152 | 48 | −2.06 ± 0.37 |
| PG 1115+0800 | 11 | 13 | 8.53 ± 0.37 | 2.5 | 50 | 55 | −1.35 ± 0.25 |
| RX J1131–1231 | 0.80 | 1.3 | 7.39 ± 0.19 | 0.84 | 230 | 38 | −1.65 ± 0.13 |
| H1413+117 | 56 | 6.5 | 9.24 ± 0.51 | 5.4 | ... | ... | ... |
| B1422+231 | 250 | 135 | 9.89 ± 0.18 | 13 | 11 | 47 | −0.55 ± 0.12 |
| WFI J2033–4723 | 5.7 | 3.8 | 8.24 ± 0.12 | 1.6 | 62 | 36 | −1.35 ± 0.08 |
| Q2237+0305 | 32 | 2.7 | 8.99 ± 0.76 | 5.5 | 38 | 150 | −1.43 ± 0.51 |

* Bolometric luminosities computed using $L_{\text{bol}} = 9[\lambda F_{\lambda}]_{1500} 4\pi d_L^2$. Computed from HM, LS, and LM images and corrected for magnification (Kaspi et al. 2000).
* Approximate bolometric luminosities derived from the X-ray (0.5–8 keV) luminosities (computed from LM image) with a bolometric correction factor of 20 (see § 5).
* Calculated from the bolometric luminosities in col. (2). See § 5.
* $r_{1/2}$ is computed according to eq. (5) for the $I$ band.
to the uncertainties in $L_{bol}$—by inspection of Figure 6. As discussed above, the plotted error bars are derived from the rms (logarithmic) scatter among the three (two) different and independent methods we have employed to infer $L_{bol}$ (see above discussion) for seven (two) of the sources. As Figure 6 indicates, the uncertainties in $r_{1/2}$ range between factors of 1.2 and 3.2, with an average value of a factor of 1.7. We take this to be a fairly reliable estimate of the uncertainty in our values for $r_{1/2}$ due to errors in estimating $L_{bol}$.

In our calculations leading to the set of values for $r_{1/2}/r_{Ein}$ we assumed values for two key parameters of the quasars: (1) the radiative efficiency $\eta$ and (2) the fraction $f_{edd} \equiv L_{bol}/L_{edd}$. Based on a simple scaling argument, we can show how our results for $r_{1/2}$ depend on $\eta$ and $f_{edd}$. Equation (5) provides the exact definition of $r_{1/2}$ that we use. However, if we use the expression for $T(r)$ in equation (6) to find the radius where $T/(1 + z) = h \nu/k_B$, where $\nu$ is the center of the observation band, this is to a good approximation proportional to $r_{1/2}$. If we further neglect the factor $1 - (r_0/r)^{1/2}$ in equation (6), we find a handy scaling relation for $r_{1/2}$:

$$r_{1/2} \propto (M_{BH} \dot{M})^{1/3}.$$  

(7)

If we consider the bolometric luminosity to be a measured quantity for each system, then $M_{BH} \propto L_{bol}/f_{edd}$ and $\dot{M} \propto L_{bol}/\eta$. Combining these, we can see how $r_{1/2}$ depends on the assumed parameters $f_{edd}$ and $\eta$:

$$r_{1/2} \propto (f_{edd} \eta)^{-1/3},$$  

(8)

which is a fairly weak dependence and not likely to lead to uncertainties in $r_{1/2}$ of more than an additional factor of $\sim 2$.

6. SUMMARY AND CONCLUSIONS

We have presented a study of 10 quadruply gravitationally lensed quasars for which high spatial resolution X-ray and optical data are available, paying particular attention to the differences between the observed flux ratios of the high-magnification pair of images to the predicted flux ratio from smooth lensing models. The Chandra data were analyzed in a uniform and systematic manner, and the X-ray flux ratios were determined via two-dimensional Gaussian fits. The optical fluxes and image positions were found in the existing literature, with the bulk coming from the CASTLES project. We also modeled each lensing system as a singular isothermal sphere with external shear (except for HE 0230−2130, where a second mass component was necessary), and these simple models fit the image positions quite well.

As illustrated in Figures 2−4, almost all systems show evidence for an anomaly in the ratio of high-magnification saddle point and minimum images (HS/HM) as compared to the smooth model prediction. In the systems that show a pronounced anomaly, the X-rays are generally seen to be more anomalous than the optical.

For a number of reasons, we believe that the anomalous flux ratios, and the differences between these ratios in the X-ray and optical bands, are best explained by microlensing. In previous work (Blackburne et al. 2006; Pooley et al. 2006) we have shown that extinction in the visible band and absorption of soft X-rays cannot provide the explanation. Second, we show in this study (as well as in previous work) that temporal variability intrinsic to the source, in conjunction with lens time delays, also cannot, in most cases, explain the observed anomalies. Third, since images in both the X-ray and optical bands exhibit these flux ratio anomalies, but to differing degrees, no smooth lens model can reproduce these anomalies. Finally, we find that in the preponderance of systems, it is the highly magnified saddle point image (HS) whose flux is anomalous. This is in agreement with microlensing magnification distributions (Schechter & Wambsganss 2002). Since there is no reason for the HS location to systematically produce larger optical extinctions or X-ray absorptions, this is another argument against differential extinction/absorption being the cause of the flux ratio anomalies.

Under the hypothesis that the anomalies are produced via microlensing by stars (of typical mass 0.7 $M_\odot$) in the lensing galaxy, the implication is that the optical emitting region, which suffers rms (logarithmic) microlensing variations only half as big as those of the X-ray region, must have a typical size $\sim 3^{1/2}$ of the Einstein radius of the microlensing stars (see discussion in § 5). Likewise, the X-ray emitting region, being more severely microlensed, must be substantially smaller than this.

In the context of a thin accretion disk around a black hole, the X-ray requirement is easily satisfied, as this emission likely arises from the inner parts of the disk. However, the optical emission poses something of a problem. It is generally thought to arise from a region not much larger than the X-ray region, but this is in conflict with the observed microlensing results that require larger optical emitting regions by factors of $\sim 3$−30 (see Fig. 6) than are commonly accepted.

Therefore, we are left with a conundrum. Either there is a mechanism to transport the optical radiation to larger radii (and that does not affect the X-rays), or there is a missing piece of the puzzle. Regardless, we have demonstrated how the X-ray and optical observations can provide a microarcsecond probe of the lensed quasars and thereby yield potentially important results.

From the work in this paper and the above discussion we draw three summary conclusions:

1. Microlensing is the primary cause of the flux ratio anomalies.

2. The optical emitting regions in the quasars involved in this study have sizes of $\sim 3^{1/2}$ of a stellar Einstein radius, i.e., $\sim 1\mu$as, corresponding to $\sim 1000$ AU.

3. Millilensing (e.g., by dark matter halos) is ruled out as an explanation of the flux ratio anomalies by virtue of the above conclusion since this implies that both the X-ray and optical emission regions are small compared to the milliarcsecond scale and should therefore be lensed by the same amount.

Finally, as mentioned above in the paper, these same flux ratio anomalies can be used to provide valuable information on the ratio of stellar matter to dark matter in the lens galaxy in the vicinity of its Einstein radius. Schechter & Wambsganss (2004) used optical flux ratio anomalies for a sample of 11 quads to derive a projected stellar/dark mass ratio at the typical impact parameter of a quasar image. They first assumed that the optical emission region was small compared to the Einstein rings of the stars in the lensing galaxy. The result was very heavily influenced by the inclusion (or exclusion) of the system with the most extreme flux ratio anomaly, SDSS J0924+0219. They found less discordant results if they instead assumed that half the optical light came from a pointlike source and half came from a more extended source. But allowing for a fraction of the light to come from a more extended region adds a second parameter to the problem, making determination of the stellar/dark matter ratio more uncertain. If, as we have argued here, the X-ray emission comes from a region substantially smaller than the optical emission region, the use of X-ray flux ratio anomalies in the analysis of Schechter & Wambsganss would eliminate that second parameter and more
uniquely determine the ratio of stellar matter to dark matter. The same study would give a better idea of the emission region size required to attenuate microlensing variations. Such an analysis is the subject of a forthcoming paper.

We thank the anonymous referee for useful suggestions and Julian Krolik for extensive and very helpful discussions about this work. We are grateful to Josiah Schwab for helping with the numerical accretion-disk calculations. D. P. gratefully acknowledges the support provided by NASA through Chandra Postdoctoral Fellowship grant PF4-50035 awarded by the Chandra X-Ray Center, which is operated by the Smithsonian Astrophysical Observatory for NASA under contract NAS8-03060. S. R. received some support from Chandra grant TM5-6003X. J. A. B. and P. L. S. acknowledge support from NSF grant AST 02-06010.

REFERENCES

Bade, N., Siebert, J., Lopez, S., Voges, W., & Reimers, D. 1997, A&A, 317, L13
Bardeen, J. M. 1970, Nature, 226, 64
Blackburne, J. A., Pooley, D., & Rappaport, S. 2006, ApJ, 640, 569
Broos, P. S., Townsley, L. K., Getman, K., & Bauer, F. E. 2002, ACIS Extract, An ACIS Point Source Extraction Package (University Park: Pennsylvania State Univ.)
Cash, W. 1979, ApJ, 228, 939
Chartas, G., Agol, E., Eracleous, M., Garmire, G., Bautz, M. W., & Morgan, N. D. 2002, ApJ, 568, 509
Chartas, G., Eracleous, M., Agol, E., & Gallagher, S. C. 2004, ApJ, 606, 78
Chiba, M. 2002, ApJ, 565, 17
Dai, X., Chartas, G., Agol, E., Bautz, M. W., & Garmire, G. P. 2003, ApJ, 589, 100
Dalal, N., & Kochanek, C. S. 2002, ApJ, 572, 25
Dickey, J. M., & Lockman, F. J. 1990, ARA&A, 28, 215
Elvis, M., et al. 1994, ApJS, 95, 1
Freeman, P., Doe, S., & Siewiginowska, A. 2001, Proc. SPIE, 4477, 76
Hewitt, J. N., Turner, E. L., Lawrence, C. R., Schneider, D. P., & Brody, J. P. 1992, AJ, 104, 968
Holder, G. P., & Schechter, P. L. 2003, ApJ, 589, 688
Huchra, J., Gorenstein, M., Kent, S., Shapiro, I., Smith, G., Horine, E., & Perley, R. 1985, AJ, 90, 691
Inada, N., et al. 2003a, AJ, 126, 666
———. 2003b, Nature, 426, 810
Kaspi, S., Smith, P. S., Netzer, H., Maoz, D., Jannuzi, B. T., &Giveon, U. 2000, ApJ, 533, 631
Keeton, C. R. 2001, preprint (astro-ph/0102340)
Keeton, C. R., Burles, S., Schechter, P. L., & Wambsganss, J. 2006, ApJ, 639, 1
Kochanek, C. S., Dai, X., Morgan, C., Morgan, N., Poindexter, S., & Chartas, G. 2007, in Statistical Challenges in Modern Astronomy IV, ed. G. J. Babu & E. D. Feigelson (San Francisco: ASP), in press (astro-ph/0609112)
Kochanek, C. S., & Dalal, N. 2004, ApJ, 610, 69
Kollmeier, J. A., et al. 2006, ApJ, 648, 128
Magain, P., Surdej, J., Swings, J.-P., Borgeest, U., & Kayser, R. 1988, Nature, 334, 325
Mao, S., & Schneider, P. 1998, MNRAS, 295, 587
Metcalfe, R. B., & Madau, P. 2001, ApJ, 563, 9
Metcalfe, R. B., & Zhao, H. 2002, ApJ, 567, L5
Morgan, C. W., Kochanek, C. S., Morgan, N. D., & Falco, E. E. 2006, ApJ, 647, 874
Morgan, N. D., Caldwell, J. A. R., Schechter, P. L., Dressler, A., Egami, E., & Rix, H.-W. 2004, AJ, 127, 2617
Morgan, N. D., Chartas, G., Malm, M., Bautz, M. W., Burud, I., Hjorth, J., Jones, S. E., & Schechter, P. L. 2001, ApJ, 555, 1
Mortonson, M. J., Schechter, P. L., & Wambsganss, J. 2005, ApJ, 628, 594
Page, D. N., & Thorne, K. S. 1974, ApJ, 191, 499
Patnaik, A. R., Browne, I. W. A., Walsh, D., Chaffee, F. H., & Foltz, C. B. 1992, MNRAS, 259, 1P
Peng, C. Y., Impey, C. D., Rix, H.-W., Kochanek, C. S., Keeton, C. R., Falco, E. E., Lehar, J., & McLeod, B. A. 2006, ApJ, 649, 616
Pooley, D., Blackburne, J. A., Rappaport, S., Schechter, P. L., & Fong, W. 2006, ApJ, 648, 67
Schechter, P. L., & Wambsganss, J. 2002, ApJ, 580, 685
———. 2004, in IAU Symp. 220, Dark Matter in Galaxies, ed. S. D. Ryder et al. (San Francisco: ASP), 103
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Sluse, D., et al. 2006, A&A, 449, 539
———. 2003, A&A, 406, L43
Wambsganss, J., & Paczyński, B. 1991, AJ, 102, 864
Weymann, R. J., et al. 1980, Nature, 285, 641
Wisotzki, L., Christlieb, N., Liu, M. C., Maza, J., Morgan, N. D., & Schechter, P. L. 1999, A&A, 348, L41
Witt, H. J., Mao, S., & Schechter, P. L. 1995, ApJ, 443, 18
Yu, Q., & Tremaine, S. 2002, MNRAS, 335, 965