QCD sum rule studies on the \( sss\bar{s} \) tetraquark states of \( J^{PC} = 0^{-+} \)

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We apply the method of QCD sum rules to study the \( sss\bar{s} \) tetraquark states of \( J^{PC} = 0^{-+} \). We construct all the relevant \( sss\bar{s} \) tetraquark currents, and find that there are only two independent ones. We use them to further construct two weakly-correlated mixed currents. One of them leads to reliable QCD sum rule results and the mass is extracted to be \( 2.45^{+0.14}_{-0.10} \) GeV, suggesting that the \( X(2370) \) or the \( X(2500) \) can be explained as the \( sss\bar{s} \) tetraquark state of \( J^{PC} = 0^{-+} \). To verify this interpretation, we propose to further study the \( \pi\pi/\bar{K}K \) invariant mass spectra of the \( J/\psi \rightarrow \gamma\pi\pi\eta'/\gamma\bar{K}K\eta' \) decays in BESIII to examine whether there exists the \( f_0(980) \) resonance.

Keywords: exotic hadron, tetraquark state, QCD sum rules

I. INTRODUCTION

In the past twenty years there have been a lot of exotic hadrons observed in particle experiments [1], which can not be well explained in the traditional quark model [2–10]. Most of them contain one or two heavy quarks, and there are only a few exotic hadrons in the light sector composed only by up/down/strange quarks. However, this situation is changing now. With a large amount of \( J/\psi \) sample, the BESIII Collaboration are carefully examining the physics happening in the energy region around 2.0 GeV [11–17]. Such experiments can also be performed by Belle-II [18] and GlueX [19], etc.

In Ref. [11], the BESIII Collaboration observed two resonances \( X(2120) \) and \( X(2370) \) in the \( \pi\pi\eta' \) invariant mass spectrum of the \( J/\psi \rightarrow \gamma\pi\pi\eta' \) decay, together with the \( X(1835) \) [12–14]. Recently in Ref. [15], they further studied the \( J/\psi \rightarrow \gamma\bar{K}K\eta' \) decay, and observed the \( X(2370) \) in the \( \bar{K}K\eta' \) invariant mass spectrum with a statistical significance of \( 8.3\sigma \), but they did not observe the \( X(2120) \) in this process. This indicates that the \( X(2370) \) probably contains many \textit{strangeness} components, more than the \( X(2120) \). Besides, in Ref. [16], they observed another resonance \( X(2500) \) in the \( \phi\phi \) invariant mass spectrum of the \( J/\psi \rightarrow \gamma\phi\phi \) decay, which also contains many \textit{strangeness} components. The experimental parameters of the \( X(2370) \) and \( X(2500) \) were measured in these experiments to be:

\[
X(2370) : \quad M = 2341.6 \pm 6.5 \pm 5.7 \text{ MeV}/c^2, \quad (1)
\]
\[
\Gamma = 117 \pm 10 \pm 8 \text{ MeV},
\]
\[
X(2500) : \quad M = 2470^{+15}_{-19}^{+101}_{-23} \text{ MeV}/c^2, \quad (2)
\]
\[
\Gamma = 230^{+64}_{-35}^{+56}_{-33} \text{ MeV}.
\]

All these experimental observations inspire us to carefully investigate those hadrons containing many \textit{strangeness} components. One of the best candidates is the \( sss\bar{s} \) tetraquark states, and the advantages to study them are: a) experimentally the widths of these resonances, if exist, are possibly not too broad, so they are capable of being observed; b) theoretically their internal structures are simpler than other multiquark states due to the Pauli principle restricting on identical \textit{strangeness} quarks, so their potential number is limited (this also makes them easier to be observed).

In this paper we shall study the \( sss\bar{s} \) tetraquark states of \( J^{PC} = 0^{-+} \) using the method of QCD sum rules. We have used the same approach in Refs. [20–22] to study the \( sss\bar{s} \) tetraquark states of \( J^{PC} = 1^{++} \), where we found that there are only two independent \( sss\bar{s} \) tetraquark currents of \( J^{PC} = 1^{++} \) as well as two of \( J^{PC} = 1^{-+} \).

Similarly, in the present study we shall find that there are only two independent \( sss\bar{s} \) interpolating currents of \( J^{PC} = 0^{-+} \). This makes it possible to perform a rather complete QCD sum rule analysis using both their diagonal and off-diagonal two-point correlation functions, from which we can further construct two weakly-correlated currents. We shall use them to perform QCD sum rule analyses, and the obtained results will be used to check whether the \( X(2370) \) or the \( X(2500) \) can be explained as the \( sss\bar{s} \) tetraquark state of \( J^{PC} = 0^{-+} \).

Before doing this, we note that the \( sss\bar{s} \) tetraquark state is just one possibility, and there have been some other interpretations proposed to explain the \( X(2370) \) and \( X(2500) \). The \( X(2370) \) is explained as:

- a mixture of \( \eta'(4S_0) \) and glueball in Ref. [23] within the framework of \( 3P_0 \) model (see also discussions in Ref. [24]);
- the fourth radial excitation of \( \eta(548)/\eta'(958) \) in Ref. [25] using the quark pair creation model;
- a compact hexaquark state of \( I^G J^{PC} = 0^+0^{-+} \) in Ref. [26] using the flux tube model;
- a pseudoscalar glueball in Ref. [27] based on a chirally invariant effective Lagrangian and in Ref. [28] using lattice QCD in quenched approximation.
The $X(2500)$ is explained as the $^5S_0$ $ss$ state using the $^3P_0$ model in Refs. [29, 30] and using the flux-tube model in Ref. [31]. More Lattice QCD studies can be found in Refs. [32–35], and their relevant dynamical analyses can be found in Refs. [36–41].

This paper is organized as follows. In Sec. II, we systematically construct the $ss\bar{s}\bar{s}$ tetraquark currents of $J^{PC} = 0^{-+}$ and find two independent currents $\eta_1$ and $\eta_2$. We use them to perform QCD sum rule analyses in Sec. III, and calculate both their diagonal and off-diagonal two-point correlation functions. Then we perform numerical analyses using the two single currents $\eta_1$ and $\eta_2$ in Sec. IV, and using the two weakly-correlated mixed currents $J_1$ and $J_2$ in Sec. V. Sec. VI is a summary.

II. INTERPOLATING CURRENTS

In this section we construct the $ss\bar{s}\bar{s}$ tetraquark currents with the spin-parity quantum number $J^{P\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!
Thirdly, we perform the Borel transformation at both hadron and quark-gluon levels:
\[
\Pi^{(oI)}(M_B^2) \equiv B_{M_B^2} \Pi(p^2) = \int_{16m^2}^{\infty} e^{-s/M_B^2} \rho(s) ds. \tag{18}
\]
After approximating the continuum using the spectral density above a threshold value \(s_0\), we obtain the sum rule equation
\[
\Pi(s_0, M_B^2) \equiv f_X^2 e^{-M_X^2/M_B^2} = \int_{16m^2}^{s_0} e^{-s/M_B^2} \rho(s) ds, \tag{19}
\]
which can be used to calculate \(M_X\) through
\[
M_X^2(s_0, M_B) = \frac{\frac{\partial}{\partial (1/M_B^2)} \Pi(s_0, M_B^2)}{\Pi(s_0, M_B^2)} \tag{20}
\]
\[
= \int_{16m^2}^{s_0} e^{-s/M_B^2} \rho(s) ds
\]
\[
= \int_{16m^2}^{s_0} e^{-s/M_B^2} \rho(s) ds.
\]

In the present study we calculate OPEs up to the tenth dimension, including the perturbative term, the strange quark mass, the quark condensate, the gluon condensate, the quark-gluon mixed condensate, as well as their combinations:
\[
\Pi_{11} = \int_{16m^2}^{s_0} \left[ \frac{s^4}{15360\pi^6} - \frac{m_s^2}{192\pi^6} s^3 + \left( \frac{\langle g_s^2 GG \rangle}{3072\pi^6} + \frac{5m_s^4}{64\pi^6} + \frac{m_s\langle \bar{s}s \rangle}{24\pi^4} \right) s^2 + \left( \frac{m_s^2\langle g_s^2 GG \rangle}{256\pi^6} - \frac{3m_s^6}{8\pi^6} - \frac{m_s^2\langle \bar{s}s \rangle}{4\pi^4} \right) s \right] e^{-s/M_B^2} ds,
\]
\[
\Pi_{22} = \int_{16m^2}^{s_0} \left[ \frac{s^4}{2560\pi^6} - \frac{m_s^2}{32\pi^6} s^3 + \left( \frac{\langle g_s^2 GG \rangle}{768\pi^6} + \frac{15m_s^4}{32\pi^6} + \frac{m_s\langle \bar{s}s \rangle}{4\pi^4} \right) s^2 + \left( -\frac{m_s^2\langle g_s^2 GG \rangle}{64\pi^6} - \frac{9m_s^6}{4\pi^6} - \frac{3m_s^2\langle \bar{s}s \rangle}{2\pi^4} \right) s \right] e^{-s/M_B^2} ds,
\]
\[
\Pi_{12} = \int_{16m^2}^{s_0} \left[ \frac{s^4}{512\pi^6} - \frac{\langle g_s^2 GG \rangle}{128\pi^6} s^2 + \frac{3m_s^2\langle g_s^2 GG \rangle}{32\pi^4} s - \frac{3m_s^4\langle g_s^2 GG \rangle}{32\pi^4} s \right] e^{-s/M_B^2} ds + \frac{m_s^2\langle g_s^2 GG \rangle}{32\pi^4}. \tag{23}
\]

Based on these expressions, we shall use the two single currents \(\eta_1\) and \(\eta_2\) to perform QCD sum rule analyses in Sec. IV, and use the two mixed currents \(J_1\) and \(J_2\) to perform QCD sum rule analyses in Sec. V. In the calculations we shall use the following values for various quark and gluon parameters [1, 44–50]:
\[
m_s(2 \text{ GeV}) = 96.5^{+8}_{-8} \text{ MeV},
\]
\[
\langle \bar{s}s \rangle = -(0.8 \pm 0.1) \times (0.240 \text{ GeV})^3,
\]
\[
\langle g_s^2 GG \rangle = (0.48 \pm 0.14) \text{ GeV}^4,
\]
\[
\langle g_s\bar{s}sG \rangle = -M_0^3 \times \langle \bar{s}s \rangle,
\]
\[
M_0^2 = 0.8 \text{ GeV}^2,
\]
\[
\alpha_s(1.7 \text{ GeV}) = 0.328 \pm 0.03 \pm 0.025.
\]

IV. SINGLE CURRENTS \(\eta_1\) AND \(\eta_2\)

In this section we use the two single currents \(\eta_1\) and \(\eta_2\) to perform QCD sum rule analyses. When applying QCD sum rules to study multiquark states, one usually meets a serious problem, i.e., how to differentiate the multiquark state and the relevant threshold, because the current may couple to both of them. In the present study the relevant threshold is the \(\eta’(980)\) around 1950 MeV. Besides, \(\eta_1\) and \(\eta_2\) may also couple to the lower states of \(J^{PC} = 0^{+}\), such as the \(\eta(1475)\), etc.

If this happens, the resulting correlation function should be positive. Fortunately, as shown in Fig. 1, we find that the two correlation functions \(\Pi_{11}(M_B^2)\) and \(\Pi_{22}(M_B^2)\) are both negative in the region \(s_0 < 4.0 \text{ GeV}^2\) when taking \(M_B^2 = 1.5 \text{ GeV}^2\). This indicates that both \(\eta_1\) and \(\eta_2\) do not strongly couple to the \(\eta’(980)\) thresh-
old as well as the lower states. Hence, the state they
couple to, as if they can couple to some state, should be
new and possibly exotic.

To extract the mass of this exotic state, $M_X$, through
Eq. (20), we need to find proper working regions
for the two free parameters, the threshold value $s_0$ and the Borel
mass $M_B$. Taking $\eta_1$ as an example, first we investigate
the convergence of the operator product expansion by
requiring the $D = 10$ terms to be less than 5%:

\[
\text{CVG} \equiv \left| \frac{\Pi^{D=10}(s_0, M_B^2)}{\Pi(s_0, M_B^2)} \right| \leq 5\%.
\] (25)

This is the cornerstone of a reliable QCD sum rule
analysis. As shown in Fig. 2 using the solid curve, this
condition is satisfied in the region $M_B^2 > 1.53 \text{ GeV}^2$
when setting $s_0 = 8.0 \text{ GeV}^2$.

Then we investigate the one-pole-dominance assumption
by requiring the pole contribution (PC) to be larger
than 30%:

\[
\text{PC} \equiv \left| \frac{\Pi(s_0, M_B^2)}{\Pi(\infty, M_B^2)} \right| \geq 30\%.
\] (26)

As shown in Fig. 2 using the dashed curve, this condition
is satisfied in the region $M_B^2 < 1.78 \text{ GeV}^2$ when
setting $s_0 = 8.0 \text{ GeV}^2$. Altogether we obtain a Borel
window 1.53 GeV$^2 < M_B^2 < 1.78 \text{ GeV}^2$ when setting
$s_0 = 8.0 \text{ GeV}^2$. We change $s_0$ to redo the same
procedures, and find that there exist non-vanishing Borel
windows as long as $s_0 \geq 7.6 \text{ GeV}^2$.

Finally, we require the mass $M_X$ extracted from
Eq. (20) to have a dual minimum dependence on both
the threshold value $s_0$ and the Borel mass $M_B$. Still taking $\eta_1$
as an example, we show the mass $M_X$ in Fig. 3 as a function
of the threshold value $s_0$ (left) and the Borel mass $M_B$ (right).
We find $M_X$ has a minimum around $s_0 \approx 8.0 \text{ GeV}^2$, and its dependence on $M_B$ is moderate in the
Borel window $1.53 \text{ GeV}^2 < M_B^2 < 1.78 \text{ GeV}^2$. Accordingly,
we choose the working regions to be $7.0 \text{ GeV}^2 < s_0 < 9.0 \text{ GeV}^2$ and $1.53 \text{ GeV}^2 < M_B^2 < 1.78 \text{ GeV}^2$, where
the mass $M_X$ is evaluated to be

\[
M_{\eta_1} = 2.78^{+0.19}_{-0.12} \text{ GeV}.
\] (27)

Here the central value corresponds to $s_0 = 8.0 \text{ GeV}^2$ and
$M_B^2 = 1.65 \text{ GeV}^2$, and the uncertainty is due to the Borel
mass $M_B$ and the threshold value $s_0$ as well as various
quark and gluon parameters listed in Eqs. (24).

Similarly, we use $\eta_2$ to perform QCD sum rule analyses,
and find that there exist non-vanishing Borel windows as long as $s_0 \geq 6.8 \text{ GeV}^2$. Using the working regions
$6.5 \text{ GeV}^2 < s_0 < 8.5 \text{ GeV}^2$ and $1.44 \text{ GeV}^2 < M_B^2 < 1.80 \text{ GeV}^2$ (Borel window for $s_0 = 7.5 \text{ GeV}^2$), we obtain

\[
M_{\eta_2} = 2.54^{+0.14}_{-0.10} \text{ GeV},
\] (28)

where the central value corresponds to $s_0 = 7.5 \text{ GeV}^2$ and
$M_B^2 = 1.62 \text{ GeV}^2$. For completeness, we show the mass
obtained using $\eta_2$ in Fig. 4 as a function of the threshold value $s_0$ (left) and the Borel mass $M_B$ (right). The mass
dependence on $M_B$ is weak and acceptable in the
Borel window $1.44 \text{ GeV}^2 < M_B^2 < 1.80 \text{ GeV}^2$, which is slightly better than the previous result obtained using
$\eta_1$.

**V. MIXED CURRENTS $J_1$ AND $J_2$**

In the previous section we have used the two single
currents $\eta_1$ and $\eta_2$ to perform QCD sum rule analyses.
In this section we further study their mixing, and use the
two mixed currents $J_1$ and $J_2$ to perform QCD sum rule
analyses. We follow the procedures used in Refs. [21, 22]
to do this, where the mixing of $s\bar{s}s\bar{s}$ tetraquark currents
with $J^{PC} = 1^{\pm -}$ is carefully investigated.

Firstly, we examine how large is the off-diagonal term
$\Pi_{12}(M_B^2)$ defined in Eq. (14). As shown in Fig. 5 using
the solid curve, the ratio $\Pi_{12}^2/(\Pi_{11} \Pi_{22})$ is quite large, so the mixing should be taken into account. Accordingly, we diagonalize the matrix

$$
\begin{pmatrix}
\Pi_{11}(s_0, M_B^2) & \Pi_{12}(s_0, M_B^2) \\
\Pi_{12}(s_0, M_B^2) & \Pi_{22}(s_0, M_B^2)
\end{pmatrix}
$$

(29)

at around $M_B^2 = 1.5 \text{ GeV}^2$ and $s_0 = 7.0 \text{ GeV}^2$ (we shall see that these two values are both inside the working regions for the mixed current $J_2$). We obtain two new currents with the mixing angle $\theta = 16.3^\circ$:

$$
J_1 = \cos \theta \eta_1 + \sin \theta \eta_2,
$$

$$
J_2 = -\sin \theta \eta_1 + \cos \theta \eta_2,
$$

(30)

As shown in Fig. 5 using the dashed curve, the new ratio $(\Pi_{11} J_1) / (\Pi_{11} J_1 \Pi_{22} J_2)$ is significantly suppressed in the region $1.44 \text{ GeV}^2 < M_B^2 < 1.69 \text{ GeV}^2$ (Borel window for $J_2$ when setting $s_0 = 7.0 \text{ GeV}^2$), so $J_1$ and $J_2$ only weakly correlate with each other inside this region.

We separately use $J_1$ and $J_2$ to perform QCD sum rule analyses. When using $J_1$, we find that there exist non-vanishing Borel windows as long as $s_0 \geq 8.6 \text{ GeV}^2$, and the mass extracted is around 3.06 GeV, even larger than 3.0 GeV, so we shall not use it to draw any conclusion.

When using $J_2$, we find that there exist non-vanishing Borel windows as long as $s_0 \geq 6.5 \text{ GeV}^2$. Using the working regions $6.0 \text{ GeV}^2 < s_0 < 8.0 \text{ GeV}^2$ and $1.44 \text{ GeV}^2 < M_B^2 < 1.69 \text{ GeV}^2$ (Borel window for $s_0 = 7.0 \text{ GeV}^2$), we obtain

$$
M_{J_2} = 2.45^{+0.14}_{-0.10} \text{ GeV},
$$

(31)

where the central value corresponds to $s_0 = 7.0 \text{ GeV}^2$ and $M_B^2 = 1.57 \text{ GeV}^2$. For completeness, we show the mass obtained using $J_2$ in Fig. 6 as a function of the threshold value $s_0$ (left) and the Borel mass $M_B$ (right).
VI. SUMMARY AND DISCUSSIONS

In this paper we use the method of QCD sum rules to study the $ssss$ tetraquark states of $J^{PC} = 0^{-+}$. We systematically construct all the relevant diquark-antidiquark ($ss$) and meson-meson ($s\bar{s}$) interpolating currents, and derive their relations through the Fierz transformation. We find two independent currents $\eta_1$ and $\eta_2$, and calculate both their diagonal and off-diagonal two-point correlation functions. The obtained results suggest that these two single currents strongly correlate with each other. Hence, we use them to further construct two mixed currents $J_1$ and $J_2$, which only weakly correlate with each other.

We use the two single currents $\eta_1$ and $\eta_2$ as well as the two mixed currents $J_1$ and $J_2$ to perform QCD sum rule analyses. Our results suggest that they both couple weakly to the $\eta'f_0(980)$ threshold, so the state they couple to, as if they can couple to some state, should be new and possibly exotic. The masses extracted from $\eta_1$ and $\eta_2$ are $2.78^{+0.19}_{-0.12}$ GeV and $2.54^{+0.14}_{-0.10}$ GeV respectively, and the masses extracted from $J_1$ and $J_2$ are around $3.06$ GeV and $2.45^{+0.14}_{-0.16}$ GeV respectively.

Especially, the mass extracted from the mixed current $J_2$ is the lowest:

$$M_{J_2} = 2.45^{+0.14}_{-0.10} \text{ GeV}. \quad (32)$$

Use the Fierz transformation given in Eqs. (10), we can transform $J_2$ to be

$$J_2 = -\sin 16.3^\circ \eta_1 + \cos 16.3^\circ \eta_2 = 6.04 (\bar{s}_a s_a) (\bar{s}_b \gamma_5 s_b) - 0.55 (\bar{s}_a \sigma_{\mu\nu} s_a) (\bar{s}_b \sigma^{\mu\nu} \gamma_5 s_b). \quad (33)$$

This suggests that the state $X$, coupled by this current, can decay into:

- It can decay into the $\eta'f_0(980)$ channel, due to the $(\bar{s}_a s_a) (\bar{s}_b \gamma_5 s_b)$ operator [51, 52]. Considering that the $f_0(980)$ resonance can further decay into the $\pi\pi$ and $KK$ final states, we use the BaBar measurement [53]:

$$\frac{B(f_0(980) \to K^+K^-)}{B(f_0(980) \to \pi^+\pi^-)} = 0.69 \pm 0.32, \quad (34)$$

to further estimate and obtain

$$\frac{B(X \to \eta'f_0(980) \to \eta'KK)}{B(X \to \eta'f_0(980) \to \eta'\pi\pi)} = 0.92 \pm 0.43. \quad (35)$$

- It can also decay into the $\phi\phi$ final state, due to the $(\bar{s}_a \sigma_{\mu\nu} s_a) (\bar{s}_b \sigma^{\mu\nu} \gamma_5 s_b)$ operator.

In the three BESIII experiments [11, 15, 16], the $X(2370)$ was observed in both the $\eta'\pi\pi$ and $\eta'KK$ final states, and the $X(2500)$ was observed in the $\phi\phi$ final state, indicating that both of them contain many strangeness components. Accordingly, our results suggest that either the $X(2370)$ or the $X(2500)$ can be explained as the $ssss$ tetraquark state of $J^{PC} = 0^{-+}$ (they might even be the same state, so that its mass spectrum and decay properties can both be well explained).

To verify the above interpretation, we propose the BESIII Collaboration to further study the $\pi\pi$ and $KK$ invariant mass spectra of the $J/\psi \to \gamma\pi\pi\eta'$ and $J/\psi \to \gamma KK\eta'$ decays to examine whether there exists the $f_0(980)$ resonance.

Acknowledgments

We thank Cheng-Ping Shen for useful discussions. This project is supported by the National Natural Science Foundation of China under Grant No. 11722540 and the Fundamental Research Funds for the Central Universities.

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FIG. 6: Mass calculated using the current $J_2$, as a function of the threshold value $s_0$ (left) and the Borel mass $M_B$ (right). In the left panel the short-dashed/solid/long-dashed curves are obtained by setting $M_B^2 = 1.44/1.57/1.69$ GeV$^2$, respectively. In the right panel the short-dashed/solid/long-dashed curves are obtained by setting $s_0 = 6.0/7.0/8.0$ GeV$^2$, respectively.

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