Universal critical exponents of the magnetic domain wall depinning transition

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Magnetic domain walls
Magnetic domain wall motion

depinning transition

- **thermal rounding**: \( v(f_c) \sim T^\gamma \)
- **creep**: \( \ln v \sim f^{-\mu} \)
- **T > 0**: \( v \sim (f - f_c)^\beta \)
- **T = 0**: \( v \sim (f - f_c)^\beta \)

- equilibrium
- depinning
- fast flow \((f \to \infty)\)
Magnetic domain wall motion

depinning transition

Elastic string model

universality
Quenched Edwards-Wilkinson universality class (q-EW)

\[ \frac{\partial u(z, t)}{\partial t} = c \frac{\partial^2 u(z, t)}{\partial z^2} + F + \sqrt{T} \eta(z, t) + D \xi(u, z) \]

Quenched Kardar-Parisi-Zhang universality class (q-KPZ)

\[ \frac{\partial u(z, t)}{\partial t} = c \frac{\partial^2 u(z, t)}{\partial z^2} + \lambda \left( \frac{\partial u(z, t)}{\partial z} \right)^2 + F + \sqrt{T} \eta(z, t) + D \xi(u, z) \]
Universal exponents

Quenched Edwards-Wilkinson universality class (q-EW)

\[
\frac{\partial u(z, t)}{\partial t} = c \frac{\partial^2 u(z, t)}{\partial z^2} + F + \sqrt{T} \eta(z, t) + D \xi(u, z)
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\]

| exponent                  | qEW               | qKPZ              |
|---------------------------|-------------------|-------------------|
| order parameter \( \beta \) | \( v \sim (H - H_d)^\beta \) | 0.245 ± 0.006* 0.33 ± 0.02† | ~ 0.64‡ |
| correlation length \( \nu \) | \( \xi \sim (H - H_d)^\nu \) | 1.333 ± 0.007* | ~ 1.73‡ |
| thermal rounding \( \psi \) | \( v \sim T^{\psi} \) | 0.15 ± 0.01** | ? |

* Ferrero, Bustingorry, Kolton, PRE (2013)
† Duemmer, Krauth, PRE (2005)
** Bustingorry, Kolton, Giamarchi, EPL (2008)
‡ Tang, Kardar, Dhar, PRL (1995)
Universal exponents

Pt/Co/Pt

\[ \psi = 0.15 \]
Measuring DW motion

GdFeCo
Velocity measurements

Athermal depinning transition below 70 K!
The velocity exponent

\[ v(H, T = 0K) = v_H \left( \frac{H - H_d}{H_d} \right)^\beta \]
The velocity exponent

\[ \nu(H, T = 0 K) = \nu_H \left( \frac{H - H_d}{H_d} \right)^\beta \]

(c) 14.6 mT

(f) 15.0 mT

\Delta t = 1.5 \mu s

100 \mu m

Albornoz, Ferrero, Kolton, Jeudy, Bustingorry, Curiale, PRB L060404 2021
The velocity exponent

$$v(H, T = 0K) = v_H \left( \frac{H - H_d}{H_d} \right)^\beta$$

(a) $1.1 \chi_{\text{min}}$

(b) $\mu_0 H_d = (14.8 \pm 0.2) \text{ mT}$

(c) $\beta = 0.28 \pm 0.08$

$\ln \left( \frac{v_H}{(\text{m/s})} \right) + 3 \ln \left( \frac{(H - H_d)}{H_d} \right)$

$\mu_0 H_d = 14.8 \text{ mT}$

$\beta = 0.28$

$v_H = 270 \text{ m/s}$
The velocity exponent

(Alternatively: fitting $\beta$ as a single global parameter, independent of $T$, gives a consistent result $\beta = 0.33 \pm 0.04$)
The correlation length exponent

\[ C_v(x) = \frac{1}{\Delta t^2} \sum [\Delta(x + x) - \bar{\Delta}] [\Delta(x') - \bar{\Delta}] \]

\[ \Delta = \frac{1}{N} \sum \Delta(x) \]

\[ C_v(x = \xi) = \frac{1}{2} C_v(x = 0) \]
The correlation length exponent

\[ C_v(x) = \frac{1}{\Delta t^2} \sum \left[ \Delta(x' + x) - \Delta \right] \left[ \Delta(x') - \Delta \right] \]

\[ \Delta = \frac{1}{N} \sum \Delta(x) \]

\[ C_v(x = \xi) = \frac{1}{2} C_v(x = 0) \]
The correlation length exponent

$GdFeCo$

$C_v(x = \xi) = \frac{1}{2}C_v(x = 0)$

\[
\xi(H, T = 0K) = \xi_0 \left( \frac{H - H_d}{H_d} \right)^{-\nu}
\]

\[
v(H, T = 0K) = v_H \left( \frac{H - H_d}{H_d} \right)^\beta
\]

$\nu \sim \xi^{-\beta/\nu}$
The correlation length exponent

\[ \nu = \nu_H \left( \frac{\xi}{\xi_0} \right)^{-\beta/\nu} \]
### Universal exponents

| exponent | qEW         | qKPZ   | Measured   |
|----------|-------------|--------|------------|
| $\beta$  | $0.245 \pm 0.006^*$ | $\sim 0.64^§\dagger$ | $0.30 \pm 0.03$ |
|          | $0.33 \pm 0.02^†$   |        |            |
| $\nu$    | $1.333 \pm 0.007^*$ | $\sim 1.73^‡$ | $1.3 \pm 0.3$ |
| $\psi$   | $0.15 \pm 0.01^{**}$ | $?$      | $0.15 \pm 0.03^{***}$ |
| $\zeta$  | $1.250 \pm 0.005^*$ | $\sim 0.63^§$ | $1.2 \pm 0.2$ |
| $z$      | $1.433 \pm 0.007^*$ | $\sim 1^‡$   | $1.5 \pm 0.2$ |
| $\tau$   | $1.11 \pm 0.04^◊$   | $\sim 1.26^§\dagger$ | $1.11 \pm 0.07$ |

* Ferrero, Bustingorry, Kolton, PRE (2013)
† Duemmer, Krauth, PRE (2005)
** Bustingorry, Kolton, Giamarchi, EPL (2008)
*** Gorchon, Bustingorry, Ferré, Jeudy, Kolton, Giamarchi, PRL (2014)
§ Rosso, Hartmann, Krauth, PRE (2003)
‡ Tang, Kardar, Dhar, PRL (1995)
◊ Ferrero, Foini, Giamarchi, Kolton, Rosso, PRL (2017)
### Universal exponents

| exponent | qEW             | qKPZ          | Measured    |
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| $\beta$  | $0.245 \pm 0.006^*$  
$0.33 \pm 0.02^\dagger$ | $\sim 0.64^\S\ddagger$ | $0.30 \pm 0.03$ |
| $\nu$    | $1.333 \pm 0.007^*$ | $\sim 1.73^\ddagger$ | $1.3 \pm 0.3$ |
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| $\tau$   | $1.11 \pm 0.04^\Diamond$ | $\sim 1.26^\S\ddagger$ | $1.11 \pm 0.07$ |

$$B(r) = \langle [u(x + r) - u(x)]^2 \rangle = B_0 \left( \frac{r}{r_0} \right)^{2\zeta}$$

$$\ell(t) \sim t^{-1/\zeta}$$

$$P(S) \sim S^{-\tau}$$

$$\nu = \frac{1}{2 - \zeta}$$

$$\beta = \nu(z - \zeta)$$

$$\tau = 2 - \frac{2}{1 + \zeta}$$
### Universal exponents

| exponent | qEW                  | qKPZ                | Measured   |
|----------|----------------------|---------------------|------------|
| $\beta$  | $0.245 \pm 0.006^*$  | ~ $0.64^§\dagger$   | $0.30 \pm 0.03$ |
|          | $0.33 \pm 0.02^\dagger$ |                     |            |
| $\nu$    | $1.333 \pm 0.007^*$  | ~ $1.73^\dagger$    | $1.3 \pm 0.3$   |
| $\psi$   | $0.15 \pm 0.01^{**}$ | ?                   | $0.15 \pm 0.03^{***}$ |
| $\zeta$  | $1.250 \pm 0.005^*$  | ~ $0.63^§$          | $1.2 \pm 0.2$   |
| $\zeta$  | $1.433 \pm 0.007^*$  | ~ $1^\dagger$       | $1.5 \pm 0.2$   |
| $\tau$   | $1.11 \pm 0.04^\diamond$ | ~ $1.26^§\dagger$  | $1.11 \pm 0.07$ |
| What we know… |
|----------------|
| DW motion is compatible with the EW universality class (… mostly) |

| What we partially know… |
|-------------------------|
| Relationship with micromagetics, temperature dependent parameters. |
| Roughness exponents at different scales (multiple exponents, negative KPZ?). |

| What we don’t… |
|----------------|
| Plasticity effects |
| Scaling relations with the thermal rounding exponent |
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