Development of Pump-Drive Turbine Module with Hydrostatic Bearing for Supercritical CO\textsubscript{2} Power Cycle Application

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Abstract: The turbomachinery used in the sCO\textsubscript{2} power cycle requires a high stable rotor-bearing system because they are usually designed to operate in extremely high-pressure and temperature conditions. In this paper, we present a pump-drive turbine module applying hydrostatic bearing using liquid CO\textsubscript{2} as the lubricant for a 250 kW supercritical CO\textsubscript{2} power cycle. This design is quite favorable because stable operation is possible due to the high stiffness and damping of the hydrostatic bearing, and the oil purity system is not necessary when using liquid CO\textsubscript{2} as the lubricant. The pump-drive turbine module was designed to operate at 21,000 rpm with the rated power of 143 kW. The high-pressure liquid CO\textsubscript{2} was supplied to the bearing, and the orifice restrictor was used for the flow control device. We selected the orifice diameter providing the maximum bearing stiffness and also conducted a rotodynamic performance prediction based on the designed pump-drive turbine module. The predicted Campbell diagram indicates that a wide range of operation is possible because there is no critical speed below the rated speed. In addition, an operation test was conducted for the manufactured pump-drive turbine module in the supercritical CO\textsubscript{2} cycle test loop. During the operation, the pressurized CO\textsubscript{2} of the 70 bar was supplied to the bearing for the lubrication and the shaft vibration was monitored. The successful operation was possible up to the rated speed and the test results showed that shaft vibration is controlled at the level of 2 \(\mu\text{m}\) for the entire speed range.

Keywords: supercritical CO\textsubscript{2} cycle; hydrostatic bearing

1. Introduction

Owing to stricter environmental regulations and energy depletion worldwide, the demand for power generation systems with higher efficiencies and reduced capital and operating cost has increased. With these aims, power cycle systems using supercritical CO\textsubscript{2} (sCO\textsubscript{2}) have been examined as potential alternatives to conventional steam Rankine power cycle systems. In the sCO\textsubscript{2} power generation cycle, friction losses are very low due to the small viscosity of the working fluid, and the size of the modular system can be significantly reduced because of the high density of sCO\textsubscript{2} and the high-pressure operation characteristics. Accordingly, owing to these advantages of sCO\textsubscript{2} power generation, numerous studies are underway, primarily in developed countries. Sandia National Lab (SNL) proposed and tested a 250 kW sCO\textsubscript{2} Brayton cycle [1,2]; Bechtel Marine Propulsion Corporation (BMPC) also proposed a 100-kW CO\textsubscript{2} Brayton cycle integrated system [3,4]. Echogen developed a 7.3-MW sCO\textsubscript{2} power cycle for waste heat recovery system [5]. They used hydrostatic CO\textsubscript{2}-supported bearings for the compressor and reported approximately 3.5 MW of generated power. The Korea Institute of Machinery and Materials (KIMM) investigated a sCO\textsubscript{2} cycle for a heat recovery system [6]. To evaluate the performance of the designed core modules, the integrated test facility is designed as a 250-kW class sCO\textsubscript{2} recuperated Rankine cycle. These studies demonstrated the technical viability of sCO\textsubscript{2} power generation systems.
The turbomachinery used in the sCO$_2$ power cycle is generally designed to operate in extreme conditions of high temperature, high pressure, and high speed; consequently, the bearings and the lubrication system require high reliability and stability. Therefore, the selection of bearing type and compatible design for the required operating conditions is important for the safe operation of the sCO$_2$ power cycle. In this manner, the development of a reliable bearing system for the sCO$_2$ turbomachinery has been identified as one of the major challenges.

In previous research, an appropriate bearing system can be found, from small-scale to commercial-scale systems [7]. For the sCO$_2$ turbomachinery developed to date, oil lubrication bearings such as tilting pad bearings, of which the reliability has already been demonstrated in steam turbine generation system, have been widely adopted. Additionally, in small-scale facilities or prototype demonstration cases, various types of bearings such as magnetic bearings [8], bump type foil bearings [1], and hydrostatic bearings have been adopted. Among the aforementioned bearings, hydrostatic bearings are operated by supplying a pressurized lubricant to the bearing. Due to these operating characteristics, hydrostatic bearings can provide high stiffness and damping, which result in the precise operation of the rotating shaft and superior stability of the rotor bearing system. Owing to these advantages, hydrostatic bearings have been widely applied to many rotating machines to date and thorough investigations have been performed to investigate the bearing characteristics.

Rowe et al. outlined a procedure to optimize the design parameters of multi-recess hydrostatic bearings based on minimum power consumption; they suggested an optimal ratio of the land part to recess of 0.25 [9]. Singh et al. calculated the dynamic coefficients of capillary-compensated bearings in a pure hydrostatic operation [10]. Ghosh calculated the stiffness and damping coefficients of pure hydrostatic operation according to changes in recess pressure ratio and eccentricity [11]. He showed that there exists an optimum value of the pressure ratio at which load capacity is maximum. Rowe compared the dynamic properties caused by the hydrostatic, hydrodynamic and squeeze effects for various types of restrictors [12]. Chen et al. calculated stability threshold speeds for hybrid operation and demonstrated superior stability at a low eccentricity ratio [13]. In addition, Rhode et al. reported that the dynamic characteristics of a hydrostatic bearing can change considerably with the compressibility of the lubricant inside the recess [14]. They also analytically showed that there is a break frequency beyond which bearing stiffness increases sharply. Subsequently, Ghosh et al. extended this study [15]. They showed that the fluid compressibility in the recess affects the dynamic behavior of the hydrostatic bearing, and cross coupled stiffness and damping increase with rotating speed. As application fields for hydrostatic bearings increased, bearings operating in turbulent regions were investigated. Through numerical analysis, Heller described the static and dynamic performance of a hydrostatic bearing, considering turbulent effects [16]. Kim et al. reported the effects of changes in the physical properties of the lubricant on a cryogenic hydrostatic bearing [17]. San Andres proposed an approximate solution for the static and dynamic properties of hydrostatic bearings considering the flow inertia effect [18]. He reported that approximate solutions show good agreement with the full numerical solution and that maximum direct stiffness occurs at pressure ratio of 0.6, and that maximum direct damping is present at different pressure ratios depending on the rotor speed. By comparing critical mass, Ghosh and Satish demonstrated that a lobe bearing with offset factor of more than one can achieve a better stability than that of circular bearings [19,20]. Owing to the increasing demand for environmentally friendly fluid machines, studies regarding hydrostatic bearings using water as the lubricant have been conducted. Ren et al. theoretically investigated the performance of water-lubricated hydrostatic bearings for compressors through operating tests [21]. Subsequently, Du and Liang analyzed the dynamic performance of water-lubricated hydrostatic bearings [22].

Owing to their numerous aforementioned advantages, hydrostatic bearings are widely utilized in many rotating machines. However, to the best of the authors’ knowledge, the application of hydrostatic bearings to sCO$_2$ turbomachinery has not been studied sufficiently. This study pertains to the development of a pump-drive turbine module for sCO$_2$ cycle application with hydrostatic bearings using liquid CO$_2$ as lubricant. The proposed design is unique and quite favorable because
stable operation is possible due to the high stiffness and damping of the hydrostatic bearing and there is no oil contamination on the working fluid by using liquid CO$_2$ as the lubricant. It is known that most oils dissolve well in sCO$_2$; thus, for reliable operation of the cycle, a CO$_2$ purity control system is necessary when using oil lubrication bearings. This paper presents a design approach for a rotor bearing system with hydrostatic bearings for sCO$_2$ turbomachinery application. The detailed design parameters for the rotor-bearing system were provided. We performed bearing performance prediction and rotordynamic analysis of the pump-drive turbine module. In addition, to show the viability of the designed pump-drive turbine in sCO$_2$ cycle applications, we conducted operating tests in a sCO$_2$ test facility.

2. Pump-Drive Turbine Module Design

Figure 1 provides a schematic diagram of the pump-drive turbine module for sCO$_2$ cycle application. As shown in the figure, the pump is driven by a drive turbine directly connected to a rotor; the rotor is supported by a set of hydrostatic bearings. During operation, high-temperature and high-pressure CO$_2$ is supplied to drive the turbine, leading to the generation of a driving force, and thereby increasing the pressure of the liquid CO$_2$ flowing into the pump. The pump-drive turbine module is installed in the vertical direction; the pump is assembled at the bottom, the turbine is located at the top, and two radial bearings and a thrust bearing are placed between them. The bearings supporting the rotor comprise two radial bearings and a pair of thrust bearings. Because it was installed and driven in the vertical direction, the weight of the rotating body was supported by the thrust bearings. The pump-drive turbine module was designed to operate in the KIMM 250 kW sCO$_2$ test facility; the operating conditions are determined by cycle analysis of the simple recuperated Rankine cycle [6]. The rated speed of the pump-drive turbine was 21,000 rpm, and it was designed to generate a pump power of 143 kW at the rated speed.

![Figure 1. Pump-drive turbine module.](image)

Figure 2 shows a schematic diagram of a hydrostatic radial bearing for a pump-drive turbine. From a CO$_2$ pump installed outside of the pump-drive turbine, the bearing was supplied with liquid CO$_2$ at a pressure of 70 bar and temperature of 20 °C. The bearing discharge pressure was maintained at 60 bar. An orifice restrictor was installed as a flow control device and a recess was machined around the supply hole to enhance the bearing’s load capacity. As shown in the figure, two rows of recesses were machined in the axial direction, and 12 supply holes were created in the circumferential direction for each row. The axial recess location was selected such that the land width ratio equals 0.25. Table 1 shows the other design parameters of the radial bearing.
3. Theoretical Model

3.1. Bearing Performance Analysis

When operating the hydrostatic bearing in a steady state, the pressure of the bearing land part is calculated using the Reynolds equation, which can be expressed as shown in Equation (1).

$$\frac{\partial}{\partial \theta} \left( H^3 \frac{\partial P}{\partial \theta} \right) + \frac{\partial}{\partial Z} \left( H^3 \frac{\partial P}{\partial \theta} \right) = \Lambda \frac{\partial H}{\partial \theta} \quad (1)$$

The dimensionless variables used in Equation (1) are defined in Equation (2)

$$\theta = \frac{x}{R}, \ Z = \frac{z}{R}, \ p = \frac{p}{p_a}, \ H = \frac{h}{C}, \ \Lambda = \frac{6 \mu \omega}{p_a (R C)^2} \quad (2)$$

where $C$ is the bearing clearance, $R$ is the bearing radius, $h$ is the film thickness, $\mu$ is the viscosity of the lubricant, and $\omega$ is the rotating speed of the shaft.

The boundary conditions of the Reynolds equation above are as follows

$$P(\theta, 0) = P(\theta, L) = P_e$$
$$P(\theta, z) = P(2\pi + \theta, z)$$
$$P = P_e \ at \ recess \quad (3)$$
where $P_e$ is the dimensionless pressure outside of the bearing and $P_r$ is the dimensionless recess pressure. The dimensionless recess pressure $P_r$ is calculated using the flow continuity relationship between the dimensionless supply pressure $P_s$ and the orifice restrictor, as follows

$$
\int_{L_i} \left( \Lambda H_i - H^3 \cdot \nabla P \right) \cdot dl = \Gamma_i \left[ 2(P_s - P_r) \right]^{1/2} \tag{4}
$$

The dimensionless flow coefficient is defined as follows

$$
\Gamma_i = \frac{12\mu}{C_A d A_o} \sqrt{\frac{p_s}{\rho}} \tag{5}
$$

Bearing performance analysis was conducted based on the governing equation above, and the bearing stiffness and damping were calculated using the governing equation derived, using the perturbation method from Equation (1) [21,22]. The finite element method was used to conduct a numerical analysis, and 120 × 60 grids were used in the circumferential and axial directions.

3.2. Rotordynamic Analysis

Figure 3 presents a rotor dynamics analysis model for predicting the vibration characteristics of a pump-drive turbine. The pump-drive turbine can be considered an anisotropic rotor system that consists of a symmetric rotor and anisotropic stator. The equation of the motion of the anisotropic rotor system can be written as

$$
[M]\ddot{q}(t) + [C]q(t) + [K]q(t) = f(t) \tag{6}
$$

where $[M]$, $[C]$ and $[K]$ denote generalized mass, damping, and stiffness matrix, and each component of the matrix can be derived by finite element method. In this study, the rotor was modeled using Euler–Bernoulli beam elements, whereas the pump, turbine impeller, and thrust collar were modeled using the equivalent inertia. In the analytical model, each node had two translational degrees of freedom and two rotational degrees of freedom. The stiffness and damping of bearing were calculated based on the aforementioned bearing performance analytical theory.

![Figure 3. Rotordynamic analysis model for pump-drive turbine. (C.G.: center of gravity)](image)

4. Results and Discussion

4.1. Validation of Theoretical Model

To validate the hydrostatic bearing analysis model developed in the present study, the computed results were compared with the data available in [22]. The hydrostatic bearing in [22] was a water-lubricated
radial bearing with four recesses; specifications of the bearing are listed in Table 2. For other information, the same conditions as those in [22] were applied.

Table 2. Design parameters of hydrostatic bearing [22].

| Properties               | Symbol | Unit | Value |
|--------------------------|--------|------|-------|
| Diameter                 | D      | mm   | 20    |
| Length                   | L      | mm   | 14    |
| Radial clearance         | C      | mm   | 0.039 |
| Rotating speed           | ω      | rpm  | 50,000|
| Lubricant                | -      | -    | Water |
| Density                  | ρ      | kg/m³| 998.6 |
| Viscosity                | µ      | µPas | 1005  |
| Discharge coefficient    | C_d    | -    | 0.875 |
| Supply pressure          | p_s   | bar  | 17    |
| Recess size              | -      | mm   | 10 × 8|
| Recess number            | n      | EA   | 4     |
| Orifice hole diameter    | d      | mm   | 0.64  |

Figure 4 shows the predicted stiffness and damping according to changes in the rotating speed. The results in [22] and those calculated using the analysis program of the current study are presented herein. The analytical results demonstrate that the results from [22], and the calculations from the analysis program used in this study were within 5% of each other.

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Figure 4. Stiffness and damping for hydrostatic radial bearing in [22]. (a) Stiffness; (b) Damping.

4.2. Determination of Orifice Diameter

To select a proper orifice diameter for a hydrostatic bearing, the bearing stiffness and the flow rate according to the recess pressure ratio were calculated. As shown in Figure 1, because the pump-drive turbine to be developed was installed and operated in the vertical direction, the shaft center operated at the concentric location of the bearing. Therefore, analysis was conducted for a case with an eccentricity of 0, and the rotating speed was the rated speed of 21,000 rpm.

Figure 5 shows the bearing stiffness and flow rate according to the recess pressure ratio. In this study, two translational degrees of freedom were considered for the rotor, and the bearing stiffness described in Figure 5a indicates the value of $k_{xx}$ as defined in the coordinate system shown in Figure 2. Figure 5a also shows the orifice diameter corresponding to the recess pressure ratio. The analytical
results show that there is a recess pressure ratio that results in the highest stiffness. In this study, the highest stiffness was present at the recess pressure ratio of 0.6, at which the orifice diameter was 1.8 mm. In addition, as shown in Figure 5b, the flow rate increased linearly with the pressure ratio. The orifice diameter was set at 1.8 mm according to the analytical results above and the bearing flow rate was predicted to be 0.9 kg/s.

![Figure 5](image_url)

**Figure 5.** Stiffness and flow rate for recess pressure ratio \((p_s = 70 \text{ bar}, p_e = 60 \text{ bar}, \omega = 21,000 \text{ rpm})\). (a) Stiffness; (b) Flow rate.

### 4.3. Bearing Performance Analysis

Figure 6 shows the pressure distribution of the designed radial bearing. The shaft speed was 21,000 rpm, and the eccentricity was 0. The bearing supply pressure is 70 bar, and discharge pressure is 60 bar. Under these operating conditions, the recess pressure was predicted to be 65.9 bar, and the pressure distribution in the circumferential direction was periodic around the recess. Furthermore, as shown in the figure, the highest pressure was found at the recess and the pressure decreased toward both ends in the axial direction. This indicates that, owing to the pressure difference, the CO₂ lubricant supplied to the recess flowed out through both ends in the axial direction. Additionally, a high-pressure region was created in the surrounding central area with the recess where the pressure was relatively uniform; this is usually called a virtual recess.

![Figure 6](image_url)

**Figure 6.** Pressure distribution of radial bearing \((p_s = 70 \text{ bar}, p_e = 60 \text{ bar}, \omega = 21,000 \text{ rpm})\).

Figure 7 shows the minimum film thickness according to the applied load to the radial bearing. As shown in the figure, the minimum film thickness decreases as the applied load increases. When a load of 2000 N was applied, the minimum film thickness was predicted to drop to 20 µm, corresponding to
the eccentricity ratio of 0.5. Accordingly, the designed bearing was predicted to be able to support radial loads up to 2000 N at the rated speed.

Figure 7. Minimum film thickness versus applied load.

Figure 8 shows the bearing stiffness and damping versus the rotor speed. The predicted results demonstrate that the stiffness and damping in the vertical direction (x direction in Figure 2) were identical to those in the horizontal direction (y direction in Figure 2) at all rotating speeds. Such a symmetric dynamic characteristic occurs because the rotor was operated at a concentric position. Since both the stiffness and damping in the vertical direction are identical to those in the horizontal direction, it is expected that the pump-drive turbine will have the characteristics of an isotropic rotor. In addition, when the rotating speed is low, the hydrostatic effects are dominant compared to the hydrodynamic effects, resulting in small cross-coupled stiffness. However, as the rotor speed increases, cross-coupled stiffness increases almost linearly. The damping coefficients were almost identical for all rotor speeds and the cross-coupled terms of damping were predicted to be trivial compared to the direct terms.

Figure 8. Stiffness and damping versus rotor speed. (a) Stiffness; (b) Damping.

4.4. Rotordynamic Analysis

Figure 9 shows a Campbell diagram and the damping ratio calculated from the rotordynamic analysis. In Figure 9a, the mode shapes corresponding to the damped natural frequencies are also shown. All modes shown in Figure 9a were rigid body mode. The conical mode was found at the lowest natural frequency and the translational mode was found at the higher frequency. However, these values were predicted to be located at higher frequencies than the rated speed, and thus it is
expected that there are no critical speeds below the rated speed of 21,000 rpm. Moreover, the analysis results showed that the first critical speed was at 27,000 rpm, and thus the separation margin between the rated speed and the first critical speed is more than 20%. These results indicate that a large vibration will not occur at the rated speed. In addition, as shown in Figure 9b, the damping ratios of all rigid body modes are larger than 0.1, and thus unstable vibrations will not occur. This phenomenon results from the designed hydrostatic bearing’s large stiffness of \(1 \times 10^8\) N/m and its large damping of \(2 \times 10^4\) Ns/m, as shown in Figure 8. Therefore, it is expected that the designed pump-drive turbine can be stably operated at any speed below the rated speed, indicating a large operation speed range. This rotordynamic characteristic is favorable for the pump-drive turbine operation because a large operation range is possible depending on the cycle condition.

Figure 9. Campbell diagram and damping ratio. (a) Damped natural frequency; (b) Damping ratio.

Figure 10 shows the predicted unbalance response of the pump-drive turbine rotor. In the figure, the shaft vibrations at the two radial bearings are presented and the vibration amplitudes in the vertical and horizontal direction (x and y directions in Figure 2) are shown. For the prediction, the unbalance of the ISO G2.5 was applied at the node of turbine and pump wheels, as shown in Figure 3. The vibration amplitude in the vertical direction is the same as that in the horizontal direction at both bearing locations because of the symmetric stiffness and damping of the bearing. In addition, the vibration amplitude increases to 25,000 rpm and no critical speed is present at up to 25,000 rpm, as predicted in the Campbell diagram. Vibration amplitude at the pump side bearing is larger than that at the turbine side at all speeds; the maximum amplitude at the rated speed is predicted to be 3 \(\mu\text{m}\) at the pump side bearing.

Figure 10. Predicted unbalance response at the two bearings (ISO G2.5).
4.5. Pump-Drive Turbine Operating Test

Figure 11 shows the bearings and rotor of the manufactured pump-drive turbine. The internal flow path was designed such that pressurized liquid CO$_2$ supplied from the outside can be supplied to the radial and thrust bearings and discharged to the outside. As shown in the figure, the thrust bearing consists of six recesses; five supply holes were made in each recess. The radial and thrust bearings were made of brass and the rotor was made of STS 420. The pump and turbine impeller were attached to both sides of the rotor through a tie rod, as shown in Figure 11b. In addition, to measure the shaft vibration, proximity probes were installed vertically and horizontally near the turbine-side radial bearing. This manufactured pump-drive turbine was installed on the 250 kW sCO$_2$ test cycle facility in KIMM, as shown in Figure 12, and an operation test was conducted. To supply pressurized liquid CO$_2$ to the bearing, a CO$_2$ pump was also installed in the test cycle and piping was constructed to the pump-drive turbine. The supply pressure of the liquid CO$_2$ for the bearing is 70 bar and bearing discharge pressure is maintained at 60 bar.

![Figure 11. Pump-drive turbine assembly. (a) Radial and thrust bearing; (b) Rotor and impeller.](image1)

![Figure 12. Test loop for pump-drive turbine. (a) Test loop for supercritical CO$_2$ cycle; (b) Pump-drive turbine module.](image2)

In the test, high-temperature, high-pressure CO$_2$ (84 bar, 136 °C) generated through a pump and a heater (boiler) was supplied to the drive turbine; the flow rate was adjusted through a valve to increase the rotating speed. The discharged high-temperature CO$_2$ from the drive turbine flowed into the cooler (condenser). After flowing through the cooler, CO$_2$ passed through the liquid separator and flowed into the pump.

Figure 13 shows the rotor vibration measured by the displacement sensor installed near the turbine side radial bearing with increase in rotating speed to the rated speed of 21,000 rpm. In the figure, waterfall diagrams of the vibration amplitude in the vertical and horizontal direction are presented.
No subsynchronous vibration is found in the entire speed range, indicating that the hydrostatic bearing is operating in a stable state, as predicted in Figure 8. It was also observed that 1X synchronous vibration was dominant for the entire test speed range and no critical speed was present under the rated speed. This is consistent with the predicted rotordynamic analysis results shown in Figure 9. In addition, the measured vibration amplitude in the x-direction was similar to that in the y-direction at entire speeds, indicating that the stiffness of the manufactured bearing is symmetrical, as predicted in Figure 8.

![Figure 13. Rotor vibration measurement results. (a) x-direction; (b) y-direction.](image)

Using unbalance response prediction, Figure 14 compares measured and predicted 1X vibration amplitude at the turbine side bearing. As can be seen in the figure, predicted rotor vibration increases with rotating speed and amplitude is around 1 μm at the rated speed. However, measured rotor vibration maintains a value of 2 μm at all rotating speeds. These test results are believed to have occurred because the rotor unbalance is relatively small, and the measured 1X vibration was mainly caused by shaft runout occurring due to a machining error in the rotor.

![Figure 14. Measured and predicted rotor vibration at the turbine side bearing.](image)

5. Conclusions

In this study, to design a pump-drive turbine for sCO₂ cycle application, the performance of hydrostatic bearings using liquid CO₂ as lubricant was analyzed and the dynamic characteristics of the rotor were predicted. To demonstrate the viability of the designed pump-drive turbine, an operating test was performed, and the rotor vibration was measured during the test. The following conclusions were obtained from this study.
1. The results predicted using the hydrostatic bearing analysis program of this study were similar to those of previous research, and the reliability of the developed analysis model was confirmed; 
2. Under the design conditions, a pressure ratio existed that maximized the stiffness of the radial bearing, in consideration of which the orifice diameter was determined; 
3. Based on the rotordynamic analysis, it was predicted that no critical speed would be presented below the rated speed and no instability would occur, indicating a wide range of operating speeds; 
4. Tests performed on the manufactured pump-drive turbine in the sCO$_2$ test facility confirmed that successful operation is possible for the designed sCO$_2$ cycle, and the measured rotor vibration was 2 $\mu$m at the rated speed; 
5. The test results showed that hydrostatic bearing with liquid CO$_2$ as the lubricant can be successfully applied to the sCO$_2$ turbomachinery.

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**Nomenclature**

- $A_o$: orifice area
- $C$: radial clearance
- $[C]$: damping matrix
- $C_d$: orifice discharge coefficient
- $D$: bearing diameter
- $f$: force vector
- $h$: film thickness
- $[K]$: stiffness matrix
- $L$: bearing length
- $[M]$: mass matrix
- $p$: film pressure
- $p_a$: ambient pressure
- $p_e$: discharge pressure
- $p_s$: supply pressure
- $p_r$: recess pressure
- $q$: displacement vector
- $R$: bearing radius
- $x$, $y$, $z$: Cartesian coordinates
- $\mu$: viscosity of lubricant
- $\rho$: density of lubricant
- $\omega$: rotating speed
- $\theta$: circumferential coordinate

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